Electromagnetic properties of doubly-charmed pentaquark states

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(Dated: January 5, 2022)

Motivated by the latest discovery of doubly-charmed tetraquark $T_{cc}^{++}$ by LHCb collaboration, we have been studied the magnetic moments of the possible doubly-charmed pentaquark states with quantum numbers $J^{P} = 1/2^{-}$ and $J^{P} = 3/2^{-}$ within the light-cone sum rules method. In the analysis, these possible pentaquark states are considered in diquark-diquark-antiquark structure. The magnetic moments of hadrons encode helpful details about the distributions of the charge and magnetization inside the hadrons, which help us to figure out their geometric configurations. It will be interesting and useful to examine the magnetic moments of these possible doubly-charmed pentaquark states with different theoretical approaches.

Keywords: Doubly-charmed pentaquarks, magnetic moment, diquark-diquark-antiquark picture, light-cone sum rule

I. INTRODUCTION

The first experimental discovery came in 2003 with the discovery of the X(3872) state \cite{1}, although they were theoretically predicted long ago that states other than conventional hadrons could exist. Since then, scientist have paid more and more attention to the study of exotic states that are very different from conventional hadrons. The investigation of exotic states and how the quarks are got together inside plays a important role for comprehen- sion the low energy QCD, and it is very crucial to search for the new states beyond the standard hadrons, both experimentally and theoretically.

Compared to the doubly-charmed baryons, the doubly-charmed pentaquark states should be expected to be heavier. But, the sophisticated interactions within multi-quark states may lower the mass, which likely makes it hard to separate experimentally a traditional baryon from a pentaquark state just from the mass consideration. Examining other properties of these states along with mass, such as electromagnetic properties, can help shed light on the inner nature of these states. The magnetic dipole and higher moments of hadrons can help us to obtain helpful details on the charge and magnetization distributions as well as their geometric shape. The values of these parameters define whether the charge distribution inside the hadron is spherical or not and give knowledge about the geometric shape of the hadron whether it is spherical, oblate, prolate, etc. In this work, we study the magnetic moments of spin-1/2 and spin-3/2 doubly-charmed pentaquark (For short $P_{cc}^{1/2}$ and $P_{cc}^{3/2}$, respectively) states using the light-cone sum rule formalism \cite{22–24} in the compact pentaquark picture. The attractive interaction induced by one-gluon exchange supports formation of the diquarks in color antitriplet and the supported configurations are the scalar and axialvector diquark states from the QCD sum rules \cite{25–28}. In the heavy diquark systems, only the tensor and axialvector diquarks remain because of the Fermi-Dirac statistics, the axialvector diquarks are more stable than the tensor diquarks. Therefore, in this study, we choose the axialvector type heavy diquark interpolating currents. There are some theoretical estimations on the internal structure of $P_{cc}^{1/2}$ and $P_{cc}^{3/2}$ states, their masses, production mechanisms and decay channels using different configurations and models \textsuperscript{30–35}. It should be noted here that, magnetic moments of hidden-charm...
pentaquark states have been obtained via different models and substructures [36–43].

This work has the following structure. In Sec. II, we briefly discuss the formalism and calculate the light-cone sum rule for the magnetic moments under investigation. In Sec. III, the numerical analysis and discussions for the magnetic moments are presented. A brief summary of the article is presented in Sec. IV.

II. FORMALISM

For magnetic moment analysis, calculations are started by writing the appropriate correlation function in the light-cone sum rules. This correlation function allows us to calculate the physical quantity to be calculated, in our case, the magnetic moment, in terms of both QCD and hadron parameters. Then, the correlation function calculated in two different ways is equalized to each other using the quark-hadron duality. As a final step, Borel transform and continuum subtraction are performed to reduce continuum and higher states effects.

A. Formalism of the $P_{cc}^{1/2}$ states

The correlation function required for the computations magnetic moment has the following form:

$$\Pi(p, q) = i \int d^4xe^{ip\cdot x} \langle 0 | T \left\{ J_{P_{cc}^{1/2}}(x) \bar{J}_{P_{cc}^{1/2}}(0) \right\} | 0 \rangle_\gamma,$$

where $\gamma$ is the external electromagnetic field and the $J(x)$ stands for interpolating currents of the considered spin-1/2 doubly-charmed pentaquark states and its given as follows [32]:

$$J_{P_{cc}^{1/2}}(x) = \varepsilon^{abc} v^a \varepsilon \cdot g \left[ \gamma_5 \gamma \right] \left[ u^T_i y(x) c \gamma_5 d_{g}(x) \right] \gamma_5 \gamma^\mu C q^T(x),$$

where $q$ is u or d-quark.

In order to get the hadronic degrees of freedom of the correlation function, we insert a complete set of intermediate $P_{cc}^{1/2}$ states with the same quantum numbers as the interpolating currents into the correlation function. As a result, we get

$$\Pi^{Had}(p, q) = \frac{\langle 0 | J(x) | P_{cc}^{1/2}(p, s) \rangle}{(p^2 - m^2_{P_{cc}^{1/2}})} \langle P_{cc}^{1/2}(p, s) | P_{cc}^{1/2}(p + q, s) \rangle_{\gamma} \frac{\langle P_{cc}^{1/2}(p + q, s) | \bar{J}(0) | 0 \rangle}{(p + q)^2 - m^2_{P_{cc}^{1/2}}} + \cdots$$

The matrix element $\langle P_{cc}^{1/2}(p, s) | P_{cc}^{1/2}(p + q, s) \rangle_{\gamma}$ inserting Eq. (3) can be described in connection with form factors as follows:

$$\langle P_{cc}^{1/2}(p, s) | P_{cc}^{1/2}(p + q, s) \rangle_{\gamma} = \varepsilon^\mu \bar{u}(p, s) \left[ [F_1(q^2) + F_2(q^2)] \gamma_\mu + F_2(q^2) \frac{(2p + q)_\mu}{2m_{P_{cc}^{1/2}}} \right] u(p + q, s).$$

We insert Eq. (4) in Eq. (3). Then, after some calculations are made, we get the result for the hadronic side as follows

$$\Pi^{Had}(p, q) = \lambda^2_{P_{cc}^{1/2}} \gamma_5 \frac{\gamma_5}{(p^2 - m^2_{P_{cc}^{1/2}})} \varepsilon^\mu \left[ F_1(q^2) + F_2(q^2) \right] \gamma_\mu + F_2(q^2) \frac{(2p + q)_\mu}{2m_{P_{cc}^{1/2}}} \gamma_5 \frac{\gamma_5}{(p + q)^2 - m^2_{P_{cc}^{1/2}}}.$$
The next step is to obtain the correlation function in terms of QCD degrees of freedom. When calculating the correlation function in terms of QCD degrees of freedom, the explicit forms of the interpolating currents are substituted into the correlation function. Then, the relevant light and heavy quark fields are contracted via the Wick’s theorem, and the desired results are obtained. Consequently, we get

$$
\Pi^{QCD}(p, q) = -i \varepsilon_{abc} \varepsilon_{ade} \varepsilon_{bfg} \varepsilon_{a'de'} \varepsilon_{a'y'c} \varepsilon_{b'f'g'} \int d^4 x e^{ip \cdot x} \langle 0 | \gamma_5 \gamma^\mu \tilde{S}^{c'e'}_q (-x) \gamma^{\nu} \gamma_5 \\
\left\{ \mathrm{Tr} \left[ \gamma_\mu S^{c'e'}_c (x) \gamma_\nu \tilde{S}^{d'd'}_c (x) \right] \mathrm{Tr} \left[ \gamma_5 S^g_d (x) \gamma_5 S^f_f (x) \right] \\
- \mathrm{Tr} \left[ \gamma_\mu S^{c'e'}_c (x) \gamma_\nu \tilde{S}^{d'd'}_c (x) \right] \mathrm{Tr} \left[ \gamma_5 S^g_d (x) \gamma_5 \tilde{S}^{f'f'}_c (x) \right] \right\} | 0 \rangle_\gamma,
$$

(8)

where $\tilde{S}^{ij}_c (x) = C S^{ij}_c (x) C$ and $S_q (x)$ and $S_c (x)$ are the light and charm quark propagators, respectively. Their explicit expressions in the x-space are presented as

$$
S_q (x) = i \frac{\tilde{f}}{2 \pi^2 x^4} - \frac{\langle q \bar{q} \rangle}{12} \left( 1 - i \frac{m_q \tilde{f}}{4} \right) - \frac{\langle q \bar{q} \rangle}{192} \frac{m_q^2 x^2}{32 \pi^2 x^4} \left( 1 - i \frac{m_q \tilde{f}}{6} \right) - \frac{i g_s}{32 \pi^2 x^4} G^{\mu\nu} (x) \left[ \tilde{f} \sigma_{\mu\nu} + \sigma_{\mu\nu} \tilde{f} \right],
$$

(9)

$$
S_c (x) = \frac{m_c^2}{4 \pi^2} \frac{K_1 \left( m_c \sqrt{-x^2} \right)}{\sqrt{-x^2}} + \frac{\tilde{f}}{16 \pi^2} \int_0^1 dv G^{\mu\nu} (v x) \left[ (\sigma_{\mu\nu} + \tilde{f} \sigma_{\mu\nu}) \right] - g_s m_c K_0 \left( m_c \sqrt{-x^2} \right) + 2 \sigma_{\mu\nu} K_0 \left( m_c \sqrt{-x^2} \right).
$$

(10)

The first term of the light and massive quark propagators corresponds to the free or perturbative part, and the remaining part related to the interacting parts. In the light-cone sum rule, when a photon is emitted at long distances, the non-perturbative effects appears. To consider these effects, it is required to expand the light-quark propagator near the $x^2 \sim 0$. When this process is done, the matrix elements of two and three-particle non-local operators such as $\langle (\gamma (q) | q (x) \Gamma_1 q (0) | 0 \rangle$ and $\langle (\gamma (q) | q (x) \Gamma_2 G_{\mu\nu} q (0) | 0 \rangle$, appear between vacuum and the photon states. These matrix elements are described in connection with the photon distribution amplitudes (DAs), which were determined in Ref. [44]. The explicit expressions of these functions are presented in the Appendix A. The QCD side of the correlation function can be acquired associated with the quark-gluon features with the help of the photon DAs and then carrying out an integration over x, the expression of the correlation function in the momentum representation can be computed directly.

To find the desired sum rules, we obtain the invariant amplitude $\Pi^{QCD}(p, q)$ corresponding to the structure $f \bar{q}$, and match it to $\Pi^{Had}(p, q)$. We perform the double Borel transformation to both representations of the acquired equality, which is needed to suppress contributions of the higher resonances and continuum states. The last operation to be applied is continuum subtraction, which is obtained by invoking assumption on quark-hadron duality. After these steps, we acquire the required sum rules for the magnetic moments:

$$
\mu_{P^{1/2}} \lambda_{P^{1/2}}^2 m_{P^{1/2}} = e^{-m_{P^{1/2}}^2} \Delta^{QCD}.
$$

(11)

Explicit forms of the analytical expressions obtained for the $\Delta^{QCD}$ function are given in the Appendix B.

B. Formalism of the $P^{3/2}$ states

In the present subsection we derive the light-cone sum rule for the magnetic moments of the $P^{3/2}_{cc}$ pentaquark states. To do this, we begin with subsequent correlation function,

$$
\Pi_{\mu\nu}(p, q) = i \int d^4 x e^{ip \cdot x} \langle 0 | \mathcal{T} \{ J^{P^{3/2}}_{\mu} (x) \bar{J}^{P^{3/2}}_{\nu} (0) \} | 0 \rangle_\gamma,
$$

(12)

where the interpolating current of doubly-charmed pentaquark states with $J^P = \frac{3}{2}^-$ is denoted $J^{P^{3/2}}_{\mu (\nu)}$. In the compact pentaquark picture, it is given as [32]
The correlation function obtained depending on the hadron parameters is written as,

\[ \Pi^{Had}_{\mu \nu}(p, q) = \frac{\langle 0 | J^{p3/2}_{\mu cc}(x) | P^{p3/2}_{cc}(p, s) \rangle (P^{p3/2}_{cc}(p, s) | P^{p3/2}_{cc}(p + q, s))_\gamma \frac{\langle P^{p3/2}_{cc}(p + q, s) | J^{p3/2}_{\nu cc}(0) | 0 \rangle \frac{1}{(p + q)^2 - m^2_{p^{3/2}_{cc}}} + ...}{p^2 - m^2_{p^{3/2}_{cc}}}} \]

The matrix element of the interpolating current between the vacuum and the \( P^{3/2}_{cc} \) pentaquark is defined as

\[ \langle 0 | J^{p3/2}_{\mu cc}(x) | P^{p3/2}_{cc}(p, s) \rangle = \lambda_{p^{3/2}_{cc} \mu}(p, s), \]
\[ \langle P^{p3/2}_{cc}(p + q, s) | J^{p3/2}_{\nu cc}(0) | 0 \rangle = \lambda_{p^{3/2}_{cc} \nu}(p + q, s), \]

where the \( u_{\mu}(p, s) \), \( u_{\nu}(p + q, s) \) and \( \lambda_{p^{3/2}_{cc}} \) are the spinors and residue doubly-charmed \( P^{3/2}_{cc} \) pentaquark states, respectively.

The transition matrix element \( (P^{p3/2}_{cc}(p) | P^{p3/2}_{cc}(p + q))_\gamma \) entering Eq. (14) can be written as follows [45–48]:

\[ \langle P^{p3/2}_{cc}(p, s) | P^{p3/2}_{cc}(p + q, s) \rangle \cdot \gamma = -e \bar{u}_{\mu}(p, s) \left[ F_1(q^2)g_{\mu \nu} \gamma_{\mu} - \frac{1}{m_{p^{3/2}_{cc}}} \left[ F_2(q^2)g_{\mu \nu} + F_4(q^2) - \frac{q_{\mu}q_{\nu}}{2m_{p^{3/2}_{cc}}} \right] \gamma_{\mu} \right] \quad F_3(q^2) \left( \frac{1}{2m_{p^{3/2}_{cc}}} \right)^2 q_{\mu}q_{\nu} \gamma_{\mu} \quad u_{\nu}(p + q, s). \]

where \( F_1 \)'s are the Lorentz invariant form factors.

In principle, we can derive the hadronic representation of the correlation function employing Eqs. (12)-(16), but in this case we run into two undesirable problems. The first of these problems is that the Lorentz structures in the correlation function are not independent, and the second is that the correlation function also contains spin-1/2 contributions. To eliminate unwanted effects of the spin-1/2 states and acquire only independent Lorentz structures in the correlation function, we carry out the ordering for Dirac matrices as \( \gamma_4 \gamma_3 \gamma_2 \gamma_1 \) and eliminate expressions with \( \gamma_\mu \) at the beginning, \( \gamma_\nu \) at the end and those proportional to \( p_\mu \) and \( p_\nu \) [49]. Consequently, employing Eqs. (12)-(16) the hadronic side take the form,

\[ \Pi^{Had}_{\mu \nu}(p, q) = \frac{\lambda^3_{p^{3/2}_{cc}}}{(p + q)^2 - m^2_{p^{3/2}_{cc}}}[p^2 - m^2_{p^{3/2}_{cc}}] g_{\mu \nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \gamma_{\nu} \gamma_{\mu} \\nu(1 + \tau) \]

The final form of the hadron description associated with the chosen structures as follows:

\[ \Pi^{Had}_{\mu \nu}(p, q) = \Pi_{1}^{Had} g_{\mu \nu} \gamma_{\mu} \gamma_{\nu} + \Pi_{2}^{Had} g_{\mu \nu} \gamma_{\mu} \gamma_{\nu} + \ldots, \]

where \( \Pi_{1}^{Had} \) and \( \Pi_{2}^{Had} \) are functions of the form factors \( F_1(q^2) \) and \( F_2(q^2) \), respectively; and other independent structures and form factors are denoted by dots.

The magnetic form factor, \( G_M(q^2) \), is characterized with respect to the form factors \( F_1(q^2) \) and \( F_2(q^2) \), respectively; and other independent structures and form factors are denoted by dots.

\[ G_M(q^2) = [F_1(q^2) + F_2(q^2)](1 + \frac{4}{5} \tau) - \frac{2}{5} \tau F_2(q^2) \]

where \( \tau = -\frac{q^2}{4m_{p^{3/2}_{cc}}} \). At \( q^2 = 0 \), the magnetic moment is obtained with respect to the functions \( F_1(0) \) and \( F_2(0) \) form factors as:

\[ G_M(0) = F_1(0) + F_2(0). \]

The magnetic moment, \( \mu_{p^{3/2}_{cc}} \), is described as follows:

\[ \mu_{p^{3/2}_{cc}} = \frac{e}{2m_{p^{3/2}_{cc}}} G_M(0). \]
When we perform the above processes, the calculations in terms of hadronic parameters, which are the first step of light-cone sum rule calculations, are completed.

The second step in light-cone sum rule calculations is to evaluate the correlation function in Eq. (12) in connection with the hadron duality ansatz. By matching the coefficients of the structures $g_i^F$ and $g_i^g$, we get light-cone sum rules for these two form factors. Consequently, we acquire,

$$\Pi_{\mu\nu}^{QCD}(p, q) = \Pi_1^{QCD} g_{\mu\nu}\bar{\Phi} + \Pi_2^{QCD} g_{\mu\nu}\bar{\Phi} + \ldots.$$  \hspace{1cm} (23)

Since the $\Pi_{\mu\nu}^{QCD}$ functions are very lengthy, their explicit forms are not given here.

We have obtained the correlation function in terms of both QCD and hadronic parameters. For the magnetic moment calculations, the QCD and hadronic descriptions of the correlation function are equalized using the quark-gluon parameters as well as photon DAs. Repeating the processes in the previous subsection gives the subsequent result:

$$\Pi^{QCD}(p, q) = \int d^4x \, e^{ip\cdot x} (0 | \bar{S}^{\prime c}_{\gamma}(-x)$$

$$\left\{ \text{Tr} \left[ \gamma_\mu S^{\prime c}(x) \gamma_\nu \bar{S}^{\prime d}(x) \right] \right\} | 0 \rangle.$$  \hspace{1cm} (22)

Consequently, the QCD representation of the correlation function in connection with the chosen structures is computed as

$$\Pi^{QCD}_{\mu\nu}(p, q) = \Pi_1^{QCD} g_{\mu\nu}\bar{\Phi} + \Pi_2^{QCD} g_{\mu\nu}\bar{\Phi} + \ldots.$$  \hspace{1cm} (24)

Analytical expressions have also been obtained for the $P^{3/2}_{cc}$ doubly-charmed pentaquarks. The next step will be to perform numerical calculations for both $P^{1/2}_{cc}$ and $P^{3/2}_{cc}$ doubly-charmed pentaquarks.

\section{III. NUMERICAL ANALYSIS}

The light-cone sum rule for magnetic moments of the $P^{1/2}_{cc}$ and $P^{3/2}_{cc}$ states contain many input parameters that we need their numerical values. We use $m_s = 96^{\pm 8}$ MeV, $m_u = m_d = 0$, $m_c = 1.275 \pm 0.02$ GeV [50], $f_{3\gamma} = -0.0039$ GeV$^2$ [44], $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = (-0.24 \pm 0.01)^3$ GeV$^3$ [51], $m_0^2 = 0.8 \pm 0.1$ GeV$^2$ [51], $g_3^2G_3^2 = 0.88$ GeV$^4$ [47]. To further the numerical analysis we also need the numerical values of the mass and residues of these states. In Ref. [32], these values were obtained within the framework of the mass sum rules. Another of the main input parameters to be considered in studies using light-cone sum rule is the DAs of the photon. The explicit expressions of the photon DAs are given in the Appendix A.

We have determined all the parameters we need to complete the numerical analysis of magnetic moments. Using the values of the parameters we determined, we give the numerical results we obtained for the magnetic moment as follows

$$\mu_{P_{ccud}} = 1.08^{+0.38}_{-0.34} \mu_N,$$

$$\mu_{P_{ccudd}} = 0.93^{+0.32}_{-0.29} \mu_N.$$  \hspace{1cm} (25)

The light-cone sum rules calculation for magnetic moments of doubly-charmed pentaquarks also contain two arbitrary parameters, the Borel mass $M^2$ and the continuum threshold $s_0$. According to the philosophy of the method used, we should find the working intervals in which the magnetic moments are practically insensitive to variations of these parameters. Our numerical analysis indicates that the requirements of the method are fulfilled in the regions of arbitrary parameters presented as $23.0$ GeV$^2 \leq s_0 \leq 25.0$ GeV$^2$ and $5.0$ GeV$^2 \leq M^2 \leq 7.0$ GeV$^2$. In Fig. 1, we plot the dependencies of the magnetic moments of doubly-charmed pentaquark states on $M^2$ at several fixed values of the $s_0$. As can be seen from the figure, though being not completely insensitive, the magnetic moments show reasonable dependency on the arbitrary parameters, $s_0$ and $M_2$ which is acceptable in the error limits of the light-cone sum rule method.

for spin-$1/2$ doubly-charmed pentaquark states,

$$\mu_{P_{ccud}} = 4.20^{+2.00}_{-2.65} \mu_N,$$

$$\mu_{P_{ccudd}} = 2.10^{+0.98}_{-0.78} \mu_N,$$  \hspace{1cm} (26)

for spin-$3/2$ doubly-charmed pentaquark states. The errors in the results given in Eqs. (25) and (26) are due to all input parameters, extra parameters such as $s_0$ and
$M^2$, as well as the parameters on which the wave functions used in the photon distribution amplitudes depend.

When the results in Eqs. (25) and (26) are examined, it can be seen that the results acquired for the magnetic moments are of measurable size in the experiments. It is seen that the spin-1/2 doubly-charmed pentaquarks results are very close to each other, but the difference between the results of the spin-3/2 doubly-charmed pentaquark states is on the order of two. To understand the reason for this difference, we extracted the individual quark contributions to the magnetic moments. This can be done by dialing the corresponding charge factors $e_u$, $e_d$, and $e_c$. In case of spin-1/2 doubly-charmed pentaquarks, we obtained that these magnetic moments are dominantly determined by the charm-quarks. The situation is the opposite in the spin-3/2 doubly-charmed pentaquarks. In this case, the dominant contribution comes from light-quarks and the contribution of charm-quarks is negligible. A more detail investigation shows that the smallness of the charm-quarks contributions are due to an almost exact cancellation of the expressions involving the charm-quarks, though these expressions are not small themselves. It will be interesting and useful to examine the magnetic moments of these doubly-charmed pentaquark states with different theoretical approaches.

In Refs. [39, 40], the magnetic moments of the $P_{cc}(4312)$ and $P_{cc}(4380)$ hidden-charm pentaquark states have been acquired within light-cone sum rules by assuming them as diquark-diquark-antiquark and molecular configurations. In these studies, the quark content of both $P_{cc}(4312)$ and $P_{cc}(4380)$ hidden-charm pentaquark states are considered as $c\bar{c}udu$, and the obtained magnetic moments results for the diquark-diquark-antiquark picture have been given as $\mu_{P_{cc}(4312)} = 0.40 \pm 0.15 \mu_N$ and $\mu_{P_{cc}(4380)} = 1.30 \pm 0.50 \mu_N$. When the comparison is made for the states with the same quark content, it is seen that there is a significant

FIG. 1. The dependencies of magnetic moments of $P_{cc}^{1/2}$ and $P_{cc}^{3/2}$ states on $M^2$ at three different values of $s_0$; (a) and (b) for $P_{cc}^{1/2}$ states; and (c) and (d) for $P_{cc}^{3/2}$ states.
difference between the magnetic moment results obtained for the hidden-charm and double-charmed pentaquarks. Whether the results obtained in this study are consistent or not can be seen by examining the magnetic moments of these possible doubly-charmed pentaquark states with other theoretical models.

For completeness, we have also acquired higher multipole moments, quadrupole (Q) and octupole (O), of the \( P_{c\bar{c}}^{3/2} \) pentaquark states as

\[
P_{cud\bar{q}} \text{ state: } Q = 0.048^{+0.016}_{-0.014} \text{ fm}^2, \\
O = 0.0022^{+0.0008}_{-0.0006} \text{ fm}^3, \\
\]

\[
P_{cud\bar{d}} \text{ state: } Q = 0.024^{+0.008}_{-0.008} \text{ fm}^2, \\
O = 0.0011^{+0.0005}_{-0.0004} \text{ fm}^3. \\
\]

APPENDIX A: DISTRIBUTION AMPLITUDES OF THE PHOTON

In the present appendix, the matrix elements \( \langle \gamma(q) | \bar{q}(x) \Gamma_i q(0) \rangle \) and \( \langle \gamma(q) | \bar{q}(x) \Gamma_i G_{\mu\nu} q(0) \rangle \) associated with the photon DAs are presented as follows [44],

\[
\langle \gamma(q) | \bar{q}(x) \gamma_\mu q(0) \rangle = e_q f_{3}\gamma \left( \varepsilon_\mu - q_\mu \frac{\varepsilon_x}{qx} \right) \int_0^1 du e^{i\bar{u}q x} \psi^\dagger(u) \\
\langle \gamma(q) | \bar{q}(x) \gamma_\mu \gamma_\nu q(0) \rangle = \frac{1}{4} e_q f_{3}\gamma \epsilon_{\mu\nu\alpha\beta} \varepsilon_\nu q^\dagger x^\beta \int_0^1 du e^{i\bar{u}q x} \psi^\dagger(u) \\
\langle \gamma(q) | \bar{q}(x) \sigma_\mu q(0) \rangle = -ie_q \langle \bar{q} q \rangle \varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu \int_0^1 du e^{i\bar{u}q x} \chi_\nu \varphi_\mu(u) + x^2 \frac{A_\mu}{16}(u) \\
- \frac{i}{2(qx)} e_q \bar{q} q \left[ x_\nu \left( \varepsilon_\mu - q_\mu \frac{\varepsilon_x}{qx} \right) - x_\mu \left( \varepsilon_\nu - q_\nu \frac{\varepsilon_x}{qx} \right) \right] \int_0^1 du e^{i\bar{u}q x} h_\gamma(u) \\
\langle \gamma(q) | \bar{q}(x) g_\lambda G_{\mu\nu}(vx) q(0) \rangle = -ie_q \langle \bar{q} q \rangle \varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu \int D\alpha_i e^{i(\alpha_\lambda + \nu \alpha_\mu)q x} S(\alpha_i) \\
\langle \gamma(q) | \bar{q}(x) g_\lambda \bar{G}_{\mu\nu}(vx) \gamma_\alpha q(0) \rangle = -ie_q \langle \bar{q} q \rangle \varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu \int D\alpha_i e^{i(\alpha_\lambda + \nu \alpha_\mu)q x} A(\alpha_i) \\
\langle \gamma(q) | \bar{q}(x) g_\lambda \bar{G}_{\mu\nu}(vx) \gamma_\alpha q(0) \rangle = e_q f_3\gamma q_\alpha \varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu \int D\alpha_i e^{i(\alpha_\lambda + \nu \alpha_\mu)q x} \gamma(\alpha_i) \\
\langle \gamma(q) | \bar{q}(x) G_{\mu\nu}(vx) \gamma_\alpha q(0) \rangle = e_q f_3\gamma q_\alpha \varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu \int D\alpha_i e^{i(\alpha_\lambda + \nu \alpha_\mu)q x} \gamma(\alpha_i) \\
- \left( \varepsilon_\mu - q_\mu \frac{\varepsilon_x}{qx} \right) \left( g_\beta q_\nu - \frac{1}{qx} (q_\beta x_\nu + q_\nu x_\beta) \right) q_\alpha - \left( \varepsilon_\nu - q_\nu \frac{\varepsilon_x}{qx} \right) \left( g_\alpha q_\mu - \frac{1}{qx} (q_\alpha x_\mu + q_\mu x_\alpha) \right) q_\beta
\]

IV. SUMMARY

Motivated by the latest discovery of doubly-charmed tetraquark \( T_{cc}^+ \) by LHCb collaboration, we have been studied the magnetic moments of the possible doubly-charmed pentaquark states with quantum numbers \( J^P = 1/2^- \) and \( J^P = 3/2^- \) in the framework of the light-cone sum rules method. In the analysis, these possible pentaquark states are considered in diquark-diquark-tetraquark structure. We are also calculated quadrupole and octupole moments of the spin-3/2 doubly-charmed pentaquark states. The magnetic moments of hadrons encode helpful details about the distributions of the charge and magnetization inside the hadrons, which help us to figure out their geometric configurations. The discovery of the first doubly-charmed tetraquark state gave a new platform for hadron physics. More theoretical and experimental attempts are required for to figure out its fundamental structure and non-perturbative nature of QCD dynamics in this region. It would be exciting to predict future experimental attempts that will search possible doubly-charmed pentaquark states and test the obtainment from the present analysis.
where \( \varphi_\gamma(u) \) is the DA of leading twist-2, \( \psi^v(u), \psi^r(u), \mathcal{A}(\alpha_i) \) and \( \mathcal{V}(\alpha_i) \), are the twist-3 amplitudes, and \( h_\gamma(u), \mathcal{A}(\alpha_i), \mathcal{S}(\alpha_i), \mathcal{T}_1(\alpha_i), \mathcal{T}_2(\alpha_i), \mathcal{T}_3(\alpha_i) \) and \( \mathcal{T}_4(\alpha_i) \) are the twist-4 photon DAs. The measure \( \mathcal{D}\alpha_i \) is defined as

\[
\int \mathcal{D}\alpha_i = \int_0^1 d\alpha_\bar{q} \int_0^1 d\alpha_q \int_0^1 d\alpha_g \delta(1 - \alpha_\bar{q} - \alpha_q - \alpha_g).
\]

The expressions of the DAs that entering into the matrix elements above are described as follows:

\[
\varphi_\gamma(u) = 6u\bar{u} \left( 1 + \varphi_2(\mu)C_2^3(u - \bar{u}) \right),
\]

\[
\psi^v(u) = 3 \left( 3(2u - 1)^2 - 1 \right) + \frac{3}{64} \left( 15w^V_\gamma - 5w^A_\gamma \right) \left( 3 - 30(2u - 1)^2 + 35(2u - 1)^4 \right),
\]

\[
\psi^r(u) = \left( 1 - (2u - 1)^2 \right) \left( 5(2u - 1)^2 - 1 \right) \frac{5}{2} \left( 1 + \frac{9}{16}w^V_\gamma - \frac{3}{16}w^A_\gamma \right),
\]

\[
h_\gamma(u) = -10 \left( 1 + 2\kappa^+ \right) C_2^\frac{v}{u}(u - \bar{u}),
\]

\[
\mathcal{A}(\alpha_i) = 360\alpha_q\alpha_\bar{q}\alpha_g^2 \left( 1 + w^A_\gamma \frac{1}{2}(7\alpha_g - 3) \right),
\]

\[
\mathcal{V}(\alpha_i) = 540w^V_\gamma (\alpha_q - \alpha_\bar{q})\alpha_q\alpha_\bar{q}^2,
\]

\[
\mathcal{T}_1(\alpha_i) = -120(3\zeta_2 + \zeta_2^+)\alpha_q\alpha_\bar{q}^2\alpha_g,
\]

\[
\mathcal{T}_2(\alpha_i) = 30\alpha_\bar{q}^2(\alpha_q - \alpha_\bar{q}) \left( (\kappa + \kappa^+) + (\zeta_1 - \zeta_1^+)\right)(1 - 2\alpha_g) + \zeta_2(3 - 4\alpha_g),
\]

\[
\mathcal{T}_3(\alpha_i) = -120(3\zeta_2 - \zeta_2^+)\alpha_q\alpha_\bar{q}\alpha_g^2,
\]

\[
\mathcal{T}_4(\alpha_i) = 30\alpha_\bar{q}^2(\alpha_q - \alpha_\bar{q}) \left( (\kappa + \kappa^+) + (\zeta_1 + \zeta_1^+)\right)(1 - 2\alpha_g) + \zeta_2(3 - 4\alpha_g),
\]

\[
\mathcal{S}(\alpha_i) = 30\alpha_\bar{q}^2\left( (\kappa + \kappa^+)\alpha_q\alpha_\bar{q} + (\zeta_1 + \zeta_1^+)(1 - 2\alpha_g) + \zeta_23(\alpha_q - \alpha_\bar{q}) - \alpha_g(1 - 4\alpha_g)\right),
\]

\[
\mathcal{S}(\alpha_i) = -30\alpha_\bar{q}^2\left( (\kappa + \kappa^+)\alpha_q\alpha_\bar{q} + (\zeta_1 + \zeta_1^+)(1 - 2\alpha_g) + \zeta_23(\alpha_q - \alpha_\bar{q})^2 - \alpha_g(1 - 4\alpha_g)\right).
\]

Numerical values of parameters used in DAs are: \( \varphi_2(1 \text{ GeV}) = 0 \), \( w^V_\gamma = 3.8 \pm 1.8 \), \( w^A_\gamma = -2.1 \pm 1.0 \), \( \kappa = 0.2 \), \( \kappa^+ = 0 \), \( \zeta_1 = 0.4 \), \( \zeta_2 = 0.3 \).
APPENDIX B: THE EXPLICIT EXPRESSION OF $\Delta^{QCD}$ FUNCTION

In here, we present the explicit expression for the function $\Delta^{QCD}$ acquired from the light-cone sum rule in subsection II A. It is acquired by selecting the $\not{p}$ structure as follows

$$\Delta^{QCD} = \frac{P_2}{849346560 \pi^3} \left\{ 10 P_1 (e_d + e_u) \left[ 4m^2_c 3m_0^2 \left( I[0, 2, 1, 0] - 2I[0, 2, 1, 1] + I[0, 2, 1, 2] - 2I[0, 2, 2, 0] 
  + 2I[0, 2, 2, 1] + I[0, 2, 3, 0] - 2I[1, 1, 1, 0] - 2I[1, 1, 1, 1] + I[1, 1, 1, 2] - 2I[1, 1, 2, 0] + 2I[1, 1, 2, 1] + I[1, 1, 3, 0] \right) 
  + 4 \left( I[0, 3, 1, 0] - 3I[0, 3, 1, 1] + 3I[0, 3, 1, 2] - I[0, 3, 1, 3] - 2I[0, 3, 2, 0] + 4I[0, 3, 2, 1] 
  + I[0, 3, 3, 0] - 3I[0, 3, 3, 1] + 3I[1, 2, 1, 1] - 2I[1, 2, 1, 2] + I[1, 2, 2, 1] + 2I[1, 2, 2, 2] 
  - 2I[0, 3, 2, 2] + I[1, 2, 3, 1] \right) \right] + 3 \left( I[0, 4, 2, 0] - 3I[0, 4, 2, 1] + 3I[0, 4, 2, 2] - I[0, 4, 2, 3] - 3I[0, 4, 3, 0] 
  + 6I[0, 4, 3, 1] - 3I[0, 4, 3, 2] + 3I[0, 4, 4, 0] - 3I[0, 4, 4, 1] - I[0, 4, 5, 0] + 4 \left( 3I[1, 3, 2, 1] - 3I[1, 3, 2, 2] + I[1, 3, 4, 1] \right) \right] \right\}$

$$+ 9e_c \left[ 16m^2_c \left( P_1 \left( I[0, 3, 1, 1] + 3I[0, 3, 1, 2] - 2I[0, 3, 2, 0] + 2I[0, 3, 2, 1] + I[0, 3, 3, 0] \right) 
  - 45m_0^2 \left( I[0, 4, 1, 1] - 2I[0, 4, 1, 2] + I[0, 4, 1, 3] - 2I[0, 4, 2, 1] + 2I[0, 4, 2, 2] + I[0, 4, 3, 1] \right) 
  + 18 \left( I[0, 5, 1, 2] - 2I[0, 5, 1, 3] + I[0, 5, 1, 4] - 2I[0, 5, 2, 2] + 2I[0, 5, 2, 3] + I[0, 5, 3, 2] \right) \right] + 3 \left( I[0, 4, 2, 3] - 3I[0, 4, 3, 0] + 6I[0, 4, 3, 1] - 3I[0, 4, 3, 2] + 3I[0, 4, 4, 0] - 3I[0, 4, 4, 1] - I[0, 4, 5, 0] 
  + 4 \left( 3I[1, 3, 2, 1] - 3I[1, 3, 2, 2] + I[1, 3, 3, 3] \right) \right] \right\}$

$$+ 36m_0^2 \left( I[0, 5, 2, 0] - 4I[0, 5, 2, 1] + 6I[0, 5, 2, 2] - 4I[0, 5, 2, 3] + I[0, 5, 2, 4] - 3I[0, 5, 3, 0] + 9I[0, 5, 3, 1] 
  - 9I[0, 5, 3, 2] + 3I[0, 5, 3, 3] + 3I[0, 5, 4, 0] - 6I[0, 5, 4, 1] + 3I[0, 5, 4, 2] - I[0, 5, 5, 0] + 3I[0, 5, 5, 1] 
  + 5 \left( I[1, 4, 2, 1] - 3I[1, 4, 2, 2] + 3I[1, 4, 2, 3] - I[1, 4, 2, 4] - 3I[1, 4, 3, 1] - 2I[1, 4, 3, 2] + I[1, 4, 4, 1] 
  - I[1, 4, 4, 1] + I[1, 4, 4, 2] - I[1, 4, 5, 1] \right) \right] \right\}$

$$- 3I[1, 5, 5, 2] - I[1, 5, 5, 3] - I[1, 5, 5, 4] - I[1, 5, 5, 2] - I[1, 5, 5, 2] \right) \right\} \right] \right\}$

$$+ \frac{P_2}{(13589544960 \pi^3)} \left\{ 2560(e_d - e_u) f_{3\gamma} m_c^2 P_1 \pi^2 (2I_2[A]I[0, 2, 2, 0] - I_5[I_5[I[0, 2, 3, 0]]]) 
  - 80m_c^2 (144(2e_d + e_u) f_{3\gamma} m_c^2 \pi^2 I_2[V] + e_q P_1 \left( - 44I_1[S] - 33I_1[T_2] - 184I_3[T_1] + 138I_3[T_2] + 93I_3[T_3] \right) \right) \right\}$

$$\times \left( 32m_0^2 I[0, 4, 3, 0] + 3m_0^2 I[0, 4, 5, 0] \right) + 864 (e_q m_c^2 (4I_3[S] - 3I_3[T_1] - 3I_3[T_2]) I[0, 5, 3, 0] 
  + (2e_d + e_u) f_{3\gamma} \pi^2 I_2[V] I[0, 5, 5, 0] \right) \right\}$

$$- 432e_q \left( 4I_1[S] - 3I_1[T_2] + 2(-8I_3[S] + 6I_3[T_1] + 8I_3[T_2] + 3I_3[T_3]) \right) \right\}$

$$\times I[0, 6, 5, 0] \right\}$
\[
+ 2560(e_d - e_u)f_3\pi^2 \left( I[0, 2, 1, 0] - 2I[0, 2, 1, 1] + I[0, 2, 1, 2] - 2I[0, 2, 2, 0] + 2I[0, 2, 2, 1] + I[0, 2, 3, 0] \right) \\
\times \psi_u[u_0] \right) \right],
\]

where \( P_1 = (q^2 G)^2 \) is gluon condensate, \( P_2 = (\bar{q}q) \) stands for u/d quark condensate. We should also remark that in the Eq.\((29)\), for simplicity we have only given the terms that give significant contributions to the numerical values of the magnetic moments and neglected to give many higher dimensional operators though they have been taken into account in the numerical analyses. The \( I[n, m, l, k], \ I_1[I], \ I_2[I], \ I_3[I], \ I_4[I], \ I_5[I], \ I_6[I] \) functions are defined as:

\[
I[n, m, l, k] = \int_{4m_c^2}^{s_0} ds \int_0^1 dt \int_0^1 dw \ e^{-s/M^2} s^n (s - 4 m_c^2)^m t^l w^k,
\]

\[
I_1[I] = \int D\alpha_1 \int_0^1 dv \ A(\alpha, \alpha, \alpha) \delta' (\alpha + \bar{v} \alpha - u_0),
\]

\[
I_2[I] = \int D\alpha_1 \int_0^1 dv \ A(\alpha, \alpha, \alpha) \delta' (\alpha + v \alpha - u_0),
\]

\[
I_3[I] = \int D\alpha_1 \int_0^1 dv \ A(\alpha, \alpha, \alpha) \delta (\alpha + \bar{v} \alpha - u_0),
\]

\[
I_4[I] = \int D\alpha_1 \int_0^1 dv \ A(\alpha, \alpha, \alpha) \delta (\alpha + v \alpha - u_0),
\]

\[
I_5[I] = \int_0^1 du \ A(u) \delta' (u - u_0),
\]

\[
I_6[I] = \int_0^1 du \ A(u),
\]

where \( \mathcal{F} \) denotes the corresponding photon DAs.

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