Bifurcation behaviors of a high-speed bogie system with and without yaw dampers

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Abstract. In this paper the bifurcation behaviors and chaotic motions of a high-speed bogie system with and without yaw dampers are investigated. Non-smooth rail/wheel contact relation is considered in these models. And the bifurcation diagrams are constructed with gradually increasing and decreasing the speed of the bogie model under certain initial conditions. Subcritical Hopf bifurcation is found in both models, where the stationary equilibria and limit cycles coexist. Since the coexistence of multiple solutions, jumps and hysteresis caused by different levels of disturbances are found. Chaotic motions are found in the model without yaw dampers at a higher running speed.

1. Introduction

With the development of high-speed trains and the increase of the speed, the lateral dynamic behaviors of the vehicles become more and more important, which reflects how high speed the vehicle can reach. The lateral dynamics of the railway vehicle is mainly characterized by the hunting motion [1], which is a self-excited oscillation. The critical speed, which is defined as the minimum speed the periodic motions can exist, can be affected by many factors both from the vehicle and the track.

The dynamic stability of the railway vehicle depends on a variety of factors [2, 3], such as the suspension constructions, the adhesion coefficient between the wheel and the rail, the wheel/rail contact relation, and the type of the railway lines. Many researches have been carried out to study the bifurcation behaviors and chaos in the railway vehicle systems, which characterize the lateral dynamics of the systems. Kaas-Petersen and True [4] gave an excellent analysis of the lateral dynamics of the cooperrider bogie. In this model, they found chaotic motions and a symmetry-breaking bifurcation. True and his coworkers [5, 6] extended the speed of the model to unrealistically high speeds to study the mechanism of the transition to chaotic motions. Gao et al. [7-9] conducted several researches of a four-axle railway passenger car to study the symmetric/asymmetric bifurcation behaviors exist in railway vehicles. Most of the researches adopted simple wheel/rail contact relation.

Since the nonlinear wheel/rail contact relation is a key factor for the lateral dynamics of railway vehicles, many researches using complex nonlinear wheel/rail contact relation have been conducted by researchers from countries all over the world. Zeng et al. [10] used the contact table to study the lateral dynamics of a railway passenger car, and it is found hysteresis phenomena exists in the vehicle system.
Gao et al. [8] also applied the contact table in the dynamic model of a railway bogie to study the symmetric/asymmetric bifurcation behaviors, besides both of symmetry-breaking bifurcation and period-doubling process are found in this model. Xia et al. [11] applied the contact table into the dynamic analysis of a three-piece-freight truck with dry friction in the suspension system.

In this paper a realistic non-smooth wheel/rail contact relation tabulated in a contact table is used to study the lateral dynamics of a high-speed railway bogie with and without yaw dampers. It is more accurate and realistic to use the contact table to study the lateral dynamics of the railway vehicles than the simple wheel/rail contact relation. The comparison of the bifurcations between railway bogie models with and without yaw dampers helps us to have a better understanding of the role of yaw dampers in the high-speed railway vehicles. The results of this paper are appropriate in the vehicle design process. They provide a reference for further improvement of the dynamics of railway vehicles.

2. The dynamic bogie model

A two-axle high-speed railway bogie system is investigated, and the schematic model is shown in figure 1. The directions of the coordinate system are defined in figure 1. The bogie frame is connected to the wheelsets with the primary suspension, which is consist of linear elastic springs and dampers in three directions. The secondary suspension is mounted between the bogie frames and the carbody. Except for the yaw dampers consist of a linear spring and a non-smooth damper in series, all other springs and dampers in the secondary suspension have linear characteristics. The following assumptions [12] are made in this paper:

1. The carbody moves with a constant speed along the track center line.
2. The wheels run on an ideal, smooth, and straight track.
3. The vertical and pitch motions of the bogie have little influence on the lateral dynamics of the bogie model. Therefore, only lateral, yaw and roll motions of the bogie frame are considered.
4. The vertical and longitudinal motions of the wheelsets are uncoupled from the lateral motion except for the roll motion, which depends on the lateral motion.
5. All bodies are assumed to be rigid.

![Figure 1. A high-speed railway bogie model.](image)

Seven degrees of freedom are considered in this paper, which are the lateral and yaw motion of each wheelset and the lateral, yaw and roll motion of the bogie frame. This paper adopted Newton-Euler method to formulate the dynamic model of a high-speed railway bogie model.
2.1. The non-smooth wheel/rail contact relation
To give a more accurate description of the wheel/rail contact relation, a non-smooth dependence of different contact parameters on the lateral displacement of the wheelset is considered in this paper. The standard wheel profiles and rail profiles, which are used in the high-speed railway vehicles, are considered in this paper. The elasticity modulus of the wheel and rail material is set as $2.1 \times 10^{11}$ Pa, and the Poisson’s ratio is set as 0.27. The wheel base distance is 1.353 m. The lateral distance from the wheel back to the measure point of the nominal rolling radius of the wheel is 0.07 m. The nominal rolling radius of the wheel is 0.46 m. The static normal force on the contact patch is 54397 N. The rail cant is 0.025 rad. The measure point of the rail gauge is 0.016 m below the rail head. The rail gauge is 1.435 m. The non-smooth kinematic relation of the wheel/rail contact is generated into a contact table. figure 2 shows non-smooth dependence of the left/right rolling radius, the left/right contact angle, the roll angle of the wheelset, the lateral distance from the mass center of the wheelset to the left/right contact point, longitudinal, lateral, lateral/spin and spin creep coefficients and the static normal forces on the lateral displacement of the wheelset. It is clear to see that the flange clearance is 4.9 mm for this combination of wheel profile and rail profile.
2.2. The dynamical equations of the bogie model

To calculate the creep forces and creep torques between the wheel and the rail, it’s necessary to calculate the creepages first. According to the definition of the creepages between the wheel and the rail [12], which are the sliding velocities in different directions between the wheel and rail normalized by the forward speed of the vehicle, the longitudinal creepage, the lateral creepage and the spin creepage of the left and right wheel of a wheelset can be expressed as [13]

\[
\begin{align*}
\xi_{x(l,r)} &= \varphi + \psi_w \dot{y}_w + r_{l(r)} (\psi_w \dot{\phi}_w - \Omega) \mp a_{l(r)} \psi_w \\
\xi_{y(l,r)} &= -v \dot{y}_w + y_w + r_{l(r)} \dot{\phi}_w \\
\xi_{y(l,r)} &= \mp (\Omega - \psi_w \dot{\phi}_w) \sin(\delta_{l(r)}) + \psi_w \cos(\delta_{l(r)}) \\
\end{align*}
\]

(1)

where the upper and lower sign of \(\mp\) apply to the left and right wheel, respectively. \(v\) is the running speed of the vehicle. \(\Omega\) is the angular velocity of the wheelset. \(\psi_w\) is the yaw angle of the wheelset.

According to Kalker’s linear creep theory [14], the wheel/rail creep forces in the contact patch can be represented by

Figure 2. Non-smooth wheel/rail contact relation.
where \( F_x, F_y \) and \( M_z \) are the longitudinal creep force, the lateral creep force and the spin creep torque, respectively.

The nonlinear Shen–Hedrick–Elkins creep model [15] is applied to revise Kalker’s linear creep theory. It combines a nonlinear model with Kalker’s linear creep theory. The resultant creep forces and creep torque are

\[
\begin{align*}
F_{x(l,r)}' &= e_{l,(l,r)} F_{x(l,r)} \\
F_{y(l,r)}' &= e_{l,(l,r)} F_{y(l,r)} \\
M_{z(l,r)}' &= e_{l,(l,r)} M_{z(l,r)}
\end{align*}
\]  

(3)

where the revision coefficient is defined as

\[
e_{l,(l,r)} = \begin{cases} 
1 - \beta_{l,(l,r)} / 3 + \beta_{l,(l,r)}^2 / 2 & \beta_{l,(l,r)} \leq 3 \\
1 / \beta_{l,(l,r)} & \beta_{l,(l,r)} > 3
\end{cases}
\]

(4)

Let \( \mu \) be the coefficient of adhesion between the wheel and the rail, and \( N_{l,(l,r)} \) are the static normal forces in the contact patch for the left and right wheel, respectively. Then \( \beta_{l,(l,r)} \) can be given by

\[
\beta_{l,(l,r)} = \sqrt{\frac{F_{x(l,r)}^2 + F_{y(l,r)}^2}{\mu N_{l,(l,r)}}}
\]

(5)

Since the above calculation is performed under the contact coordinate systems, a transformation is applied to transform the creep forces and creep torque from the contact coordinate systems into the track coordinate system. The resultant creep forces and creep torque under the track coordinate system [12] are

\[
\begin{align*}
F_{x(l,r)} &= F_{x(l,r)}' \cos(\psi_{w}) - F_{y(l,r)}' \cos(\delta_{l,(l,r)} \pm \phi_{w}) \sin(\psi_{w}) \\
F_{y(l,r)} &= F_{x(l,r)}' \sin(\psi_{w}) + F_{y(l,r)}' \cos(\delta_{l,(l,r)} \pm \phi_{w}) \cos(\psi_{w}) \\
M_{z(l,r)} &= M_{z(l,r)}' \cos(\delta_{l,(l,r)} \pm \phi_{w})
\end{align*}
\]

(6)

where the upper and lower sign of \( \pm \) apply to the left and right wheel, respectively.

The lateral components of the normal contact forces are given by

\[
N_{y(l,r)} = \mp N_{l,(l,r)} \sin(\delta_{l,(l,r)} \pm \phi_{w})
\]

(7)

where the upper and lower sign of \( \pm \) and \( \mp \) apply to the left and right wheel, respectively.

The primary suspension forces in the three directions (longitudinal, lateral and vertical direction) are given by

\[
\begin{align*}
F_{x(l,r)} &= 2K_{p_{x}} d_{w}(\psi_{w i} - \psi_{i}) + 2C_{p_{x}} d_{w}(\dot{\psi}_{w i} - \dot{\psi}_{i}) \\
F_{y(l,r)} &= 2K_{p_{y}} (y_{w i} - y_{i} \mp l_{w} \psi_{i} - h_{w} \phi_{i}) + 2C_{p_{y}} (\dot{y}_{w i} - \dot{y}_{i} \mp l_{w} \dot{\psi}_{i} - h_{w} \dot{\phi}_{i}) \\
F_{z(l,r)} &= 2K_{p_{z}} d_{w}(\phi_{w i} - \phi_{i}) + 2C_{p_{z}} d_{w}(\dot{\phi}_{w i} - \dot{\phi}_{i})
\end{align*}
\]

(8)

where \( i = 1, 2 \) represent the front and rear wheelset, respectively. And the secondary suspension forces in the three directions are
The yaw damper force is calculated by
\[ F_i = K_{s}(d_{s}\psi_i - s) \]
\[ F_i = 2K_{w}d_{s}\psi_i + 2C_{w}d_{i}\psi_i \]
\[ F_i = 2K_{w}(y_i - h_{w}\phi_i) + 2C_{w}(y_i - h_{w}\phi_i) \]
\[ F_i = 2K_{w}d_{s}\phi_i + 2C_{w}d_{i}\phi_i \]  \hspace{1cm} (9)

The yaw damper force is calculated by
\[ F_{i} = K_{s}(d_{s}\psi_{i} - s) \]
\[ F_{i} = \begin{cases} 
173.8 \times 10^3 s & \text{if } \dot{s} \leq 3.09 \text{mm/s} \\
(537.042 + 207.9 \times 10^4 \times |s| - 3.09 \times 10^3) \text{sign}(s) & \text{if } 3.09 \text{mm/s} < |\dot{s}| \leq 6.10 \text{mm/s} \\
(1162.821 + 855.9 \times 10^4 \times |s| - 6.10 \times 10^3) \text{sign}(s) & \text{if } 6.10 \text{mm/s} < |\dot{s}| \leq 20 \text{mm/s} \\
(13059.83 + 855.9 \times 10^4 \times |s| - 20 \times 10^3) \text{sign}(s) & \text{if } |\dot{s}| > 20 \text{mm/s} 
\end{cases} \]
\[ F_{i} = F_{b} = 0 \] \hspace{1cm} (10)

where \( s \) and \( s \) are the longitudinal displacement and velocity between the spring and the damper of the yaw damper system.

Apply the Newton-Euler equations for the railway vehicle bogie system. Then the seven coupled non-smooth ordinary differential equations (ODEs) for the dynamic model can be derived as
\[ m_{w} \ddot{y}_{w} + F_{y} - F_{y,i} - F_{y,fr} - N_{y,i} - N_{y,fr} = 0 \]
\[ I_{w} \psi_{w} + d_{w} F_{y} - a_{w}(F_{y,i} - F_{y,fr}) - a_{w}\psi_{w}(F_{y,i} + N_{y,i} - F_{y,fr} - N_{y,fr}) \]
\[ -M_{r,i} - M_{r,fr} + I_{w} \phi_{w} = 0 \]
\[ m_{w} \ddot{y}_{i} - F_{s,f1} - F_{s,f2} = 0 \]
\[ I_{w} \psi_{i} - F_{s,f1} - F_{s,f2} - d_{w}(F_{s,f1} + F_{s,f2}) + d_{i} F_{s} + 2d_{w} F_{s} = 0 \]
\[ I_{w} \phi_{i} - h_{w}(F_{s,f1} + F_{s,f2}) - d_{w}(F_{s,f1} + F_{s,f2}) - h_{w} F_{s} + d_{i} F_{s} = 0 \] \hspace{1cm} (11)

where the dot above the symbols indicates differentiation with respect to the time \( t \).

3. The investigation methods

Let \( X = [y_{w1}, y_{w2}, \psi_{w1}, \psi_{w2}, y_{i}, \psi_{i}, \phi_{w}, \psi_{w1}, \psi_{w2}, \psi_{w1}, \psi_{w2}, \psi_{w1}, \psi_{w2}, y_{i}, \psi_{i}, \phi_{i}]^T \), then the second-order ODEs can be reduced to first-order ODEs. Therefore, the dynamical model system can be formulated as an initial value problem for the autonomous dynamical system with time \( t \) as the only independent variable and the forward speed \( \nu \) as the control parameter. Given suitable initial conditions, the ODEs can be integrated, and the solution vector is obtained after a certain length of time.

Since the dynamical system under investigation is a non-smooth and high DOFs (degrees of freedom) system, a numerical investigation is necessary. For a clear understanding of the transitions among different kinds of solutions, a Poincaré section \( \Pi = \{(X, \nu) \in R^{14} \times R^+ \mid y_{w1} = 0, y_{w2} \geq 0\} \) (without yaw dampers) or \( \Pi = \{(X, \nu) \in R^{15} \times R^+ \mid y_{i} = 0, y_{i} \geq 0\} \) (with yaw damper) is selected. The bifurcation diagram of the bogie system both with and without yaw damper is constructed using the Poincaré section selected with gradually increasing and decreasing the forward speed of the bogie.
model. Different kinds of solutions (stationary equilibrium, periodic and aperiodic solutions, and chaotic motions) can be found from the bifurcation diagram.

To determine the non-smooth wheel/rail contact parameters at an arbitrary lateral displacement of the wheelset used in this paper, a linear interpolation with a refined contact table is applied. For the integration method for the ODEs, the standard solver ode15s in MATLAB with a variable step size is used. In this paper, both the absolute and relative errors are set as $10^{-7}$.

4. Numerical results and discussions

In this section, the results of the numerical calculation for the dynamic analysis of the high-speed bogie system without yaw dampers in the speed range from 60m/s to 100m/s and with yaw dampers from 120m/s to 200m/s are presented. The parameter values of high-speed bogie system are listed in table 1. In the following the numerical results for high-speed bogie model without and with yaw dampers are given in subsection 4.1 and 4.2, respectively.

4.1. Definition of Hopf bifurcation

Figure 3 shows the general forms of Hopf bifurcations can happen in railway vehicle systems. The first one is called subcritical Hopf bifurcation, and the rest two are called supercritical Hopf bifurcation. It can be seen if the railway vehicle system undergoes a subcritical Hopf bifurcation, the equilibrium of the system loses stability at speed $A$, and an unstable limit cycle bifurcates to the left side of the Hopf bifurcation point. When the railway vehicle runs at a speed higher than $A$, the system jumps from an equilibrium to a limit cycle of a big amplitude, which is dangerous for the railway vehicle system. When the speed gradually reduces until point $B$, the limit cycle dies out to stable equilibrium position through a saddle-node bifurcation. Therefore, hysteresis phenomenon can happen to the railway vehicle system if it undergoes a subcritical Hopf bifurcation. As for supercritical Hopf bifurcations, the stable equilibrium loses stability at speed $A$, and a stable limit cycle of a small amplitude bifurcates to the right side of the Hopf bifurcation point. In figure 3(b), with the increase of the running speed the railway vehicle system jumps from a stable limit cycle of a small amplitude to a stable limit cycle of a big amplitude at speed $D$. The stable limit cycle oscillation jumps back to a stable equilibrium as the speed gradually reduces until speed $B$. In this condition, hysteresis phenomenon can also happen to the railway system. As for figure 3(c), when the railway vehicle runs at speed higher than speed $A$, the amplitude of limit cycle increases gradually with the increase of the running speed, and no hysteresis phenomenon happens.

![Figure 3. General forms of Hopf bifurcations in railway vehicle systems.](image-url)
Table 1. System parameters for the high-speed bogie model.

| Parameter | Description                                           | Value   | Unit   |
|-----------|-------------------------------------------------------|---------|--------|
| $M_t$     | Mass of the bogie frame                              | 2056    | kg     |
| $I_{tx}$  | Roll moment of inertia of the bogie frame            | 1390    | kg·m$^2$ |
| $I_{tz}$  | Yaw moment of inertia of the bogie frame             | 3800    | kg·m$^2$ |
| $M_w$     | Mass of the wheelset                                 | 1627    | kg     |
| $I_{wy}$  | Pitch moment of inertia of the wheelset              | 132     | kg·m$^2$ |
| $I_{wz}$  | Yaw moment of inertia of the wheelset                | 830     | kg·m$^2$ |
| $K_{px}$  | Primary longitudinal stiffness (per axle box)        | 120.9198| MN·m$^{-1}$ |
| $K_{py}$  | Primary lateral stiffness (per axle box)             | 13.4198 | MN·m$^{-1}$ |
| $K_{pz}$  | Primary vertical stiffness (per axle box)            | 120.9198| MN·m$^{-1}$ |
| $K_{sx}$  | Secondary longitudinal stiffness (per side of the bogie frame) | 0.133  | MN·m$^{-1}$ |
| $K_{sy}$  | Secondary lateral stiffness (per side of the bogie frame)  | 0.133   | MN·m$^{-1}$ |
| $K_{sz}$  | Secondary vertical stiffness (per side of the bogie frame)  | 0.203   | MN·m$^{-1}$ |
| $K_{sxx}$ | Joint stiffness of the yaw damper                    | 35      | MN·m$^{-1}$ |
| $C_{px}$  | Primary longitudinal damper (per axle box)           | 0       | N·s·m$^{-1}$ |
| $C_{py}$  | Primary lateral damper (per axle box)                | 0       | N·s·m$^{-1}$ |
| $C_{pz}$  | Primary vertical damper (per axle box)               | 10000   | N·s·m$^{-1}$ |
| $C_{sx}$  | Secondary longitudinal damper (per side of the bogie frame)  | 0       | N·s·m$^{-1}$ |
| $C_{sy}$  | Secondary lateral damper (per side of the bogie frame)  | 15000   | N·s·m$^{-1}$ |
| $C_{sz}$  | Secondary vertical damper (per side of the bogie frame)  | 60000   | N·s·m$^{-1}$ |
| $d_w$     | Half lateral distance of the primary suspension       | 1.0     | m      |
| $d_s$     | Half lateral distance of the secondary suspension     | 0.95    | m      |
| $d_{sc}$  | Half lateral distance of the yaw damper              | 1.325   | m      |
| $l_t$     | Half of the axle distance                            | 1.25    | m      |
| $a_0$     | Half of the track gauge                              | 0.7465  | m      |
| $r_0$     | Nominal rolling radius of the wheel                  | 0.46    | m      |
| $h_{bt}$  | Height of the bogie frame mass center to the secondary suspension | -0.06  | m      |
| $h_{bw}$  | Height of the bogie frame mass center to the wheelset mass center     | 0.06    | m      |
| $W$       | Axle load                                            | 108794  | N      |
| $\mu$     | Coefficient of adhesion                              | 0.15    |        |
4.2. High-speed bogie model without yaw damper

Using the method mentioned in section 3, the bifurcation diagrams for this model with gradually increasing and decreasing speed are shown in figure 4. From figure 4 it can be seen that a subcritical Hopf bifurcation happens at the speed of \( v = 94.6 \text{ m/s} \), where the asymptotically stable stationary solution loses its stability as shown in figure 5, and a fold bifurcation occurs when the speed decreases to \( v = 63.1 \text{ m/s} \), which is the lowest speed the stable periodic solution can exist as shown in figure 5. Several small jumps are found with decreasing the speed, which are believed to be related to the non-smoothness of the wheel/rail contact relation referred to section 2.

![Bifurcation diagrams for the high-speed bogie model without yaw dampers.](image1)

**Figure 4.** Bifurcation diagrams for the high-speed bogie model without yaw dampers.

![Time series of the model.](image2)

**Figure 5.** Time series of the model.

Since the bifurcation phenomena in the range \( v = 94.6 \sim 100 \text{ m/s} \) is more complicated, a detailed analysis of this range is given in the following, where the mechanism from periodic solutions to chaos is explained. From the bifurcation diagram shown in figure 6, where the upper and lower branch indicate decreasing and increasing speed respectively, it can be seen the system entered the first chaotic region from periodic solutions through a period-doubling cascade with slowly increasing the speed (the lower branch in figure 6). Then the chaotic motions disappear through a blue-sky bifurcation, where a period-3 solution appears. Through another period-doubling cascade the system enters chaotic motions again. Finally, it jumps to a high amplitude of chaotic motions. While slowly decrease the speed from 100 m/s (the upper branch in figure 6), the system changes from chaotic motions to period-3 solutions through a reverse period-doubling cascade. Then it enters chaotic motions again through a blue-sky bifurcation. Finally, the system enters periodic oscillations through another reverse period-doubling cascade.
4.3. High-speed bogie model with yaw damper
The bifurcation diagrams for this model with gradually increasing and decreasing speed are shown in figure 7, from which it can be seen the Hopf bifurcation point is at $v = 156.5 \text{ m/s}$, and the fold bifurcation point is at $v = 126.5 \text{ m/s}$. The critical speed of the bogie with yaw damper is much higher than that of the one without yaw damper. Comparing figure 5 with figure 7, it can be seen when the stable stationary solution loses its stability, the wheels of the bogie without yaw dampers will have flange contact with the rails, while the amplitude of the oscillation of the bogie with yaw dampers is much smaller. Chaotic motions usually happen around or above the speed when the flange contact happens. Since the yaw dampers can reduce the amplitude of the wheelset to such a small value that no flange contact will happen for a quite high speed, no chaotic motions will happen in the bogie model with yaw dampers.

Figure 6. Bifurcation diagram in the speed range $v = 94.6 \sim 100 \text{ m/s}$.

Figure 7. Bifurcation diagram for the bogie system with yaw dampers.
5. Conclusions
As for the bogie system without yaw dampers, with gradually changing the forward speed of the bogie system, different kinds of solutions (stationary equilibrium, periodic and aperiodic solutions, and chaotic motions) can be found. When the bogie system runs under a sufficiently low speed, only unique and asymptotically stable stationary equilibrium, which is the trivial solution of the system. When the speed reaches the subcritical Hopf bifurcation point, the stable stationary equilibrium loses stability, and an unstable periodic solution bifurcates from the left side. The unstable periodic solution changes stability at the fold bifurcation point, and a stable periodic solution is found. With the speed increasing to a higher value, the system runs into chaotic motions through a periodic doubling process. As for the bogie system with yaw dampers, the transition is much simpler, where only subcritical Hopf bifurcation and fold bifurcation happen. The critical speed of the bogie with yaw damper is much higher than that of the one without yaw damper. At the speed when the stable stationary solution loses its stability, the wheels of the bogie without yaw dampers will have flange contact with the rails, while the amplitude of the oscillation of the bogie with yaw dampers is much smaller. Chaotic motions usually happen around or above the speed when the flange contact happens. Since the yaw dampers can reduce the amplitude of the wheelset traverse to a such small value that no flange contact will happen for a quite high speed, no chaotic motions will happen in the bogie model with yaw dampers. Since the chaotic motions, which are related to the flange contact, are sources for the uneven wear of the wheels and rails, the unevenness of wear is reduced in the bogie with yaw dampers.

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