Stability of frequency regulation of pumped-storage plants with multiple units sharing common penstock and busbar

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Abstract. Nowadays the frequency regulation of pumped-storage units becomes more and more important, due to the more complex structure of the grid and greater proportion of the renewable intermittent sources. The aim of this paper is to investigate the stability of pumped-storage plants (PSPs), and the subject of study is PSPs with surge tanks, shared water conduits and a common busbar. Firstly, the equations of each component in the system are deduced and simplified, to establish a linear time-invariant model by adopting the simplified transfer function (STF). A real Chinese PSP is taken as the engineering instance of this work, and simulation results of the STF model are validated with results of on-site measurements and computations based on the method of characteristics (MOC). Then, through both theoretical analysis and simulations, operating stability of the PSP system is studied under various islanded operating cases to reveal the influence of the hydraulic coupling and the electrical coupling. The results demonstrate the significant influence of the governor control modes and the parameter settings on stability of frequency regulation of the PSP. Suggestions of control strategy are also obtained.

1. Introduction

Stable and swift frequency regulation of hydropower plants (HPPs) is crucial for power systems. Especially in recent years, the amount of electricity generated by variable renewable energy sources is growing, and the regulating performance of PSPs is of great importance.

Operating stability of HPPs has been studied in the wake of vast development of hydropower industries. Stability criterion [1, 2] and eigen-analysis [3, 4] are frequently adopted for analysing stability of HPP system. While plenty of detailed models of governors, generators and loads have been proposed and implemented, studies on the complexity of hydraulic systems have also made major progress. Standard models for HPP system studies were established in [5], laying the foundation for studies involving complex hydraulic schemes. A model of two multi-machine systems with shared conduits was built and applied to study the oscillation problems during operations [6]. Three governor control modes were introduced in [7], which confirmed that control modes have very different effects on a HPP system with a surge tank. However, most of the previous works focused on single-unit HPP system, and the studies on multiple-unit system often ignored the details of hydraulic subsystem. For stability of PSPs, although considerable efforts have been made [8-10], how the hydraulic coupling affects the stability of PSPs is yet to be studied.
Hence in this paper, the object of interest is a specific type of PSP with multiple units sharing common water conduits and busbar, for analysing the influence of the hydraulic coupling and the electrical coupling. In section 2, models from former researchers are summarized and integrated to propose a STF model. In section 3, the model is validated with simulation results of MOC and on-site measurements of a real Chinese PSP. In section 4, case studies of the PSP are conducted for the stability analysis, and stability regions for various control modes with different governor settings are obtained. Finally in section 5, conclusions are condensed and suggestions for operation of PSPs are presented.

2. Model and method

Figure 1 shows a typical PSP with multiple pump-turbines installed. In this paper, a model of PSPs is established by applying STF, and equations of each component are built and presented firstly. By combining these equations, synthetic models can then be deduced. In the PSP model, two generating units share the common water conduits, indicating the hydraulic coupling; meanwhile, the two units are connected to the same busbar (sharing the same electrical load), inducing the electrical coupling. This feature of the model is a key characteristic of this paper.

![Diagram of a pump-storage plant with shared penstock and double surge tanks](image)

**Figure 1.** Layout of a pump-storage plant with shared penstock and double surge tanks

2.1. Pipeline

By neglecting minor terms and linearizing the friction term, the equation of continuity and the equation of motion can be combined and rewritten with Laplace transform [11]:

\[
\Delta H_D = \frac{\Delta H_U}{\Delta Q_U} - Z \tanh\left(\frac{l}{a} + \frac{f}{2DAa}\right)
\]

\[
\Delta Q_D = 1 - \frac{\Delta H_U}{\Delta Q_U} \frac{1}{Z} \tanh\left(\frac{l}{a} + \frac{f}{2DAa}\right)
\]

where \( Z = a/(gA) \), \( \Delta H = H - \bar{H} \), \( \Delta Q = Q - \bar{Q} \).

Heads and flows at the upstream or downstream end of pipe segments are labelled with subscripts “U” and “D” respectively to indicate their location. Given a constant upstream head, i.e. \( \Delta H_U \) equals zero, and normalizing change relative to rated value, equation (1) can be rewritten as:

\[
G_h(s) = \frac{h}{q} = \frac{T_w}{T_e} \tanh(T_es + F)
\]

where \( h = \Delta H/\bar{H} \), \( q = \Delta Q/\bar{Q} \), \( T_w = |\bar{Q}|/(g\bar{H}A) \), \( T_e = l/a \).

Equation (2) is the transfer function of a one-dimensional elastic pipe, but it is still too complex to be used due to its transcendental term \( \tanh(s) \). Using Taylor series expansion, a polynomial
approximation is derived with a new parameter \( \alpha \) added to fit frequency domain response to the original one [12], which here is simplified to:

\[
G_j(s) = -\frac{T_{w}}{T_{e}} \cdot \frac{T_{e} s + F}{\alpha T_{e}^2 s^2 + 2\alpha F T_{e} s + \alpha F^2 + 1}
\]

The best overall accuracy is achieved when \( \alpha \) equals 0.405, and the value is used for all pipe segments thereafter. For branching junctions, two assumptions are made: 1) the summation of inflow equals the summation of outflow; 2) a common head is shared for all pipe segments at their ends connecting to the junction.

\[
\sum \Delta Q_U \leq \sum \Delta Q_D
\]

\[\Delta H_{U} = \Delta H_{2U} = \ldots = \Delta H_{nU} = \Delta H_{1D} = \Delta H_{2D} = \ldots = \Delta H_{nD}\]

For short branch pipes adjacent to pump-turbines, (2) can be further simplified to:

\[
G_j(s) = -T_{w} s - T_{e} F / T_{e}
\]

Equation (5) is often referred to as the rigid column pipe transfer function. In this case, \( \Delta Q_U \) equals \( \Delta Q_D \).

2.2. Pump turbine

In MOC simulations, the characteristic curve (or the characteristic surface) is the most common approach to represent the operation characteristics of a pump-turbine. In the STF model, two linearized equations are utilized instead [13]:

\[
q_i = e_{w} h_i + e_{e} x_i + e_{y} y_i,
\]

\[
m_i = e_{h} h_i + e_{x} x_i + e_{y} y_i.
\]

where, \( h_i = (\Delta H_U - \Delta H_D) / \bar{H}_i \), \( q_i = \Delta Q_U / \bar{Q}_i \), \( q_i = \Delta Q_D / \bar{Q}_i \), \( x = \Delta n / \bar{n} \), \( y = \Delta Y / \bar{Y} \), \( m_i = \Delta M_i / \bar{M}_i \).

2.3. Governor and servomotor

A typical PID governor model and a first-order model of the servomotor are used in both MOC model and STF model [5]. In this work, automatic generation control (AGC) is considered “switched off”, and thus the values of \( x_c \), \( p_c \) and \( y_c \) are set to zero for all simulations. The transfer functions of the governor and servomotor under three control modes are shown below.

Frequency control:

\[
G_{gs} = -\frac{\Delta Y}{\Delta n} = -\frac{(K_d + K_p T_{id}) s^2 + (K_p + K_i T_{id}) s + K_i}{T_{id} s^3 + (T_{id} + T_j) s^2 + s}
\]

Power control:

\[G_{gp} = \frac{\Delta Y}{\Delta n} \approx \frac{\Delta Y}{m_i} = b_p \cdot G_{gs}\]

GVO control:

\[G_{gs} = 0\]

2.4. Surge tank
Surge tanks in PSPs are normally of large water inertia, which would incur simulation error if ignored. Therefore a rigid water column equation is added to form equation (11). Figure 2 shows the scheme of a surge tank and the important variables.

\[
\begin{align*}
Q_a &= Q_{n} + Q_{1} \\
Q_w &= c_v \sqrt{2g \left[H_w - H_1\right]} \\
\frac{dL_{sw}}{dt} &= \frac{Q_w}{A_w} \\
H_{sw} - z - L_{sw} - \frac{f_{sw} L_{sw}}{gD_{sw} A_{sw}^2} Q_{sw} &= \frac{2L_{sw}}{gA_{sw}} \frac{dQ_{sw}}{dt} \\
\end{align*}
\]

Equation (11) can also be rewritten through Laplace transform as:

\[
G_{st} = \frac{sA_w}{1 + sA_w (e_{sw} s + F_{sw})},
\]

where \( e_{sw} = \frac{\bar{L}_{sw}}{gA_{sw}} \).

2.5. Generator and load

A concise and widely used model for generators is adopted [13], with the load self-regulation coefficient set to 0. Here, the rigid electrical connection is considered, i.e., the relative change of rotational speed of each generator equals the relative change of grid frequency. The transfer function of generator-load is:

\[
\sum T_{ia} \cdot s \xi = \sum m_{ia} - m_{ia},
\]

where \( T_{ia} = GD_n^2 / (365P) \), \( m_{ia} = \Delta M_{ia} / M_{ia} \).

2.6. Overall system model

In this section, the overall system modelling is presented for STF model. The MOC model is established using the methods proposed in [11]. The block diagram of the overall system is shown in figure 3.
3. Engineering case and model validation with on-site measurements

An actual PSP with the layout shown in figure 1 is applied as the engineering case of this paper. In this section, the simulation results of the STF model are validated by on-site measurements of this PSP and results through MOC simulation. The parameters of pump turbines and surge tanks are shown in table 2 and the dimensions of pipe segments are shown in table 3. Unit No.1 and No.2 share the same model.
and operate at the same operation point, thus they have identical parameters, and subscripts indicating indexes are left out.

**Table 2. Parameters of Simulated PSP**

| Turbine parameters | Surge tank parameters | Transfer coefficients | Governor parameters |
|--------------------|-----------------------|-----------------------|---------------------|
| $H_t$              | 540.0 m               | $A_{st}$ 63.6 m$^2$   | $e_{qv}$ -0.6557    | $K_P$ 5.0           |
| $\bar{Q}_t$        | 53.4 m$^3$/s          | $e_{st}$ 0.1533 s$^2$/m$^2$ | $e_{gh}$ 0.8279    | $K_I$ 1.43          |
| $\bar{n}$          | 500 r/min             | $A_{di}$ 95.0 m$^2$   | $e_{qv}$ 0.9613     | $K_D$ 0.0           |
| $\bar{P}$          | 309.3 MW              | $e_{di}$ 0.3064 s$^2$/m$^2$ | $e_c$ -1.6896      | $b_P$ 0.04          |
| $T_a$              | 8.7 s                 | \( \backslash \)     | \( \backslash \)   | \( e_b \) 1.8984   | \( T_y \) 0.002s   |
|                    | \( \backslash \)     | \( \backslash \)     | \( \backslash \)   | \( e_t \) 1.1590   | \( \backslash \)   |

**Table 3. Pipe Parameters of Simulated PSP**

| Pipe segment name        | Subscript of variables | \( l \) (m) | \( A \) (m$^2$) | \( a \) (m/s) | \( Z \) (s/m$^2$) | \( T_e \) (s) | \( F \) (-) |
|--------------------------|------------------------|--------------|----------------|--------------|----------------|-------------|-----------|
| Upstream tunnel          | u                      | 444.23       | 30.12          | 1100         | 3.72           | 0.40        | 0.008     |
| Upstream penstock        | p                      | 865.69       | 19.95          | 1200         | 6.13           | 0.72        | 0.008     |
| Turbine No.1 penstock    | tu1                    | 117.85       | 5.29           | 1200         | 23.14          | 0.10        | 0.005     |
| Turbine No.2 penstock    | tu2                    | 108.35       | 5.17           | 1200         | 23.65          | 0.09        | 0.005     |
| Turbine No.1 draft tube  | td1                    | 155.39       | 13.81          | 1200         | 8.12           | 0.14        | 0.005     |
| Turbine No.2 draft tube  | td2                    | 165.89       | 13.89          | 1200         | 8.07           | 0.15        | 0.005     |
| Downstream tunnel        | d                      | 1080.22      | 33.95          | 1000         | 3.00           | 1.08        | 0.008     |

The on-site measurements and simulation results of MOC and STF of unit No.1 is plotted in figure 4. Because the linearized model of pump-turbines is mainly for simulating small-disturbance cases, the STF is able to simulate the hydraulic transients of unit No.1 while not applicable for unit No.2; hence the output of unit No.2, $q_{t2}$, is substituted with the results from the MOC simulation. In this case, two units initially operating at rated point when unit No.2 suddenly disconnects from the grid and rejects all of its electrical load. The guide vane of unit No.2 closes in 40 s, and the ball valve shuts off in 45 s. The other unit, unit No.1 maintains operation with governor set to power control mode. Figure 4 illustrates that both MOC and STF approach can accurately emulate the water hammer effect during hydraulic transients induced by small signal disturbance, with an insignificant flaw reflecting the traveling wave effect in pipes. There are some noticeable offsets on peak times, due to the instability characteristics of pump turbines in non-optimal operation region, however the simulations still achieve a good overall accuracy. In short, the STF model is effective in reflecting the physical nature of hydraulic transients within the scope of small signal analysis.
(a) Pressure in spiral case and draft tube  \hphantom{a} (b) Power output and guide vane opening

Figure 4. Simulation result compared to on-site measurements of unit No.1 during a load rejection of unit No.2

4. System stability analysis

When the two units of a PSP meet the following three conditions: 1) sharing headrace and tailrace tunnels; 2) connecting to the same island grid; 3) having the symmetric layout and identical operation parameters, it can be derived that \( h_1 \) equals \( h_2 \), and equation (15) is reduced to a single equation. The two units can be treated as one “lumped unit” with doubled \( Q_r \), \( P_r \) and \( GD^2 \) as well as halved \( Z_{tu} \) and \( Z_{td} \), for which plenty of relative theories are proposed and proved effective. The discussion in [14] points out explicitly how \( T_w \) and \( T_a \) affect the stability of HPPs; more specifically, a larger \( T_w \) or a smaller \( T_a \) deteriorates the stability. It is very rare that units of an actual PSP are identical, because of the different lengths in branch pipelines, the different control modes and various parameters settings. Transfer functions of such “asymmetric unit” systems are apparently different, but the general idea that \( T_w \) and \( T_a \) are the dominant coefficients still stand. For a certain PSP, its \( T_w \) and \( T_a \) are constants, and the problem now lies in what manner do governor parameters, namely \( K_p \) and \( K_p \), exert influence on the two-unit system. To find out how exactly this affects the stability regions of these systems, multiple cases are selected and studied, as shown in table 4. It is assumed that both units are at the same operating point, and their turbine transfer coefficients are identical. For the other parameters, the default values are presented in table 2 and 3.

Table 4. Parameters and descriptions of studied cases

| Index | Description                                                                 | \( K_{p1} \) | \( K_{i1} \) | \( K_{p2} \) | \( K_{i2} \) | Other parameters |
|-------|-----------------------------------------------------------------------------|-------------|-------------|-------------|-------------|-----------------|
| 1     | Two units adopt frequency control mode with identical governor settings,     | 0.0~16.0    | 0.0~4.0     | 0.0~16.0    | 0.0~4.0     | \( T_{a1}=T_{a2}=0.002s, \)  
|       | connecting to the same island grid                                          |             |             |             |             | \( T_{i1}=T_{i2}=0.02s, \)  
|       |                                                                              |             |             |             |             |  \( b_{p1}=b_{p2}=0 \)     |
| 2     | Two units adopt frequency control mode with different \( K_p \) and \( K_i \),|
|       | connecting to the same island grid                                          | 0.0~16.0    | 0.0~4.0     | 0.0~16.0    | 0.0~4.0     | Same as case 1   |
| 3     | Unit No.1 adopts frequency control mode and No.2 adopts GVO control mode,   | 0.0~16.0    | 0.0~4.0     | \( \backslash \)     | \( \backslash \)     | Same as case 1   |
|       | connecting to the same island grid                                          |             |             |             |             |                 |
| 4     | Unit No.1 adopts frequency control mode and No.2 adopts power control mode,  | 0.0~16.0    | 0.0~4.0     | 4.0         | 1.0         | \( b_{p2}=0.04, \) Others  
|       | connecting to the same island grid                                          |             |             |             |             | same as case 1   |

4.1. Case 1 - Both units adopt frequency control mode with identical governor settings, connecting to the same island grid

When connected to the same island grid, \( x_1 \) and \( x_2 \) can be replaced by \( x \). The total load torque \( m_g \) is the summation of \( m_{g1} \) and \( m_{g2} \). With equation (15), the system transfer function of \( x \) with respect to \( m_g \) can be expressed as:

\[
G_{ss}(s) = \frac{x}{m_g} = \frac{1}{I_1 + I_2 - J_1 h_1 + J_2 h_2 - (T_{a1} + T_{a2}) s} \tag{16}
\]

The high-order transfer function of the system described by (16) has 15 zeros and 16 poles and cannot be solved analytically. Alternatively, all variables must be substituted with actual values in order to obtain a numerical solution. To reflect the influence of governor parameters \( K_p \) and \( K_i \) on system stability, a mesh grid of the two parameters is applied in time domain and root locus analysis. Poles and zeros of the system are shown in table 5 as a reference point (conjugate roots are recorded only once). All time domain simulations are conducted under a load step disturbance (\( m_g=0.2 \)).
In Table 5, poles 1st–6th represent a series of high frequency fluctuations that relate to the traveling wave of elastic water column and other components with small time-constants. These poles are of little influence on the waveform of fluctuations due to their high frequency and damping rate. The pole 7th and 8th account for fluctuations associated with \( K_p \) and \( K_i \). As presented in Table 5, it is obvious that pole 7th has no matching zero nearby and is comparatively closer to the imaginary axis, thus it is the dominant pole of this system. The periods of fluctuations of the pole 9th and 10th match the natural period of the upstream and downstream surge tanks, and it is clear that these two poles determine the very low frequency fluctuations. In summary, poles 7th, 9th and 10th are the influential poles to this system.

### Table 5. Roots of case 1 system with referenced governor parameters

| Index | Pole (-) | Natural period (s) | Damping (-) | Nearest zero** (-) |
|-------|----------|--------------------|-------------|--------------------|
| 1     | -501.8371 | 0.0125             | 1.0000      | -500.0000          |
| 2     | -48.4940  | 0.1296             | 1.0000      | -50.0000           |
| 3     | -0.1572±13.8485 | 0.4537 | 0.0114 | -0.1463±13.8384 |
| 4     | -0.4128±5.3677  | 1.1671 | 0.0767 | -0.322±5.3372 |
| 5     | -0.7141±3.6678  | 1.6815 | 0.1911 | -0.6751±3.3942 |
| 6     | -3.1536     | 1.9924             | 1.0000      | -3.1537           |
| 7     | -0.0463±0.6418 | 9.7641 | 0.0719 | /                 |
| 8     | -0.2481     | 25.3246            | 1.0000      | /                 |
| 9     | -0.0081±0.0966 | 64.7930 | 0.0834 | -0.0105±0.0972 |
| 10    | -0.0027±0.0542 | 115.6911 | 0.0503 | -0.0043±0.0544 |

*The values of governor parameters \( K_p \) and \( K_i \) are 4.0 and 1.0 respectively.

**The term ‘Nearest’ means that the distance between this zero and its related pole is considerably smaller than any other. A ‘/’ indicates that there is no such zero in proximity, as are the case of pole 7th and pole 8th. There are also unpaired zeros, which are -0.9213 and 0.0000.

The influence of \( K_p \) and \( K_i \) on system response is clearly shown in figure 5. In cases with higher values of \( K_p \) and \( K_i \), the system becomes less stable and leads to a much longer settling time. In contrast, smaller values of \( K_p \) and \( K_i \) decreases the regulation rapidity, resulting in a large frequency overshoot; fluctuations in surge tanks are also aggravated, increasing the settling time. To gain a better understanding of the effect of \( K_p \) and \( K_i \) on influential poles, the root loci are plotted in figure 6 (a).

**Figure 5.** Frequency response to a step load with different governor parameter settings of case 1
With the increase of the value of $K_i$, the dominant pole moves closer to the imaginary axis, reducing the damping rate of waveform dictated by the governor. The effect of $K_p$ on the real part of the dominant pole is not monotonic as it grows when $K_p$ increases from 1.6 to 3.0 and decreases afterwards. In figure 6, the black crosses “x” mark the dominant poles when $K_p$ equals 4.0 and $K_i$ equals 1.0, and the two rectangle frames delimit the trajectories of pole 9th and 10th. It is clear that pole 9th and 10th vary slightly with the change of $K_p$ and $K_i$ at the magnitude of $10^{-3}$. This reveals that although these two parameters can be optimized to improve quality of regulation as seen in figure 5, they cannot significantly enhance the fundamental stability of oscillation in surge tanks.

4.2. Case 2, 3, 4 - Two units sharing load using different governor settings

In this section, the influence of two units in different control settings is studied. The same procedure for case 1 is executed on case 2, 3 and 4. For these three cases, equation (16) is still valid but transfer functions of governors need to be substituted according to the control mode. Trajectories of the dominant pole are plotted in figure 6 (b) and figure 7.

As illustrated in figure 6 (b) and figure 7, the black crosses “x” mark the location of the reference dominant pole. Each red cross indicates the dominant pole of the corresponding case given $K_p$ equals 4.0 and $K_i$ equals 1.0 for unit(s) presented in table 4. For case 2, when $K_p$ equals 4.0 and $K_i$ equals 1.0, the system is exactly the same as case 1, and the red cross coincides with the black one. Comparing (a) and (b) in figure 6, the root loci plot of case 2 appears to shrink to approximately half the size of case 1. More precisely, the distances of points on the trajectories to the reference are halved. This shrinking effect indicates that a case in which two units with different $K_p$ and $K_i$ is similar to one of two units with average $K_p$, $K_i$. This is also reflected in root loci of case 3, as shown in figure 7 (a). As the black ‘x’ lies where $K_p$ equals 7.7 and $K_i$ equals 1.8, approximately twice the value than in case 1 and case 2, while $K_{p2}$ and $K_{i2}$ can all be ignored as if they were 0 in GVO control mode. In other words, the effects of $K_p$ and $K_i$ of two units are linear. Case 4 is slightly different from case 1, 2 and 3, because the transfer function of the system has 17 zeros and 18 poles. Nevertheless, these new poles and zeros are close to each other and their effects on the system is diminished, hence the dominant pole remains unchanged. Between root loci of case 3 and 4, there is no major difference except that the latter case is more unstable with higher values of $K_p$ and $K_i$. It is demonstrated that the effect of GVO control is similar with that of power control, and this is understandable that the feedback through the permanent droop has some influences on dynamic behaviour of the system. The corresponding time domain simulations are plotted in figure 8, validating the above statements.
4.3. Overall stability analysis

The stability of a linear time invariant model is determined by its dominant pole, and based on figure 6 and 7, the stability region can be presented on the same plot. As shown in figure 9, the stability boundaries of case 1, 3 and 4 are plotted wrapping the regions, and the dashed curve is created by doubling the values of $K_p$ and $K_i$ of case 1 to demonstrate the “shrinking” effect mentioned in Section 4.2.
5. Conclusions

In this paper, stability of PSPs with multiple units sharing common penstock and busbar is studied, and the influence of the hydraulic coupling and the electrical coupling is investigated. The conclusions of this paper are condensed as follows.

The proposed STF model is a reliable way for simulating PSPs with two coupled units, either hydraulically or electrically, and it is also suitable for theoretical analysis compared to the MOC. It can be further utilized to study small signal stability problems in PSPs with more than two units.

The stability region of the two-unit system has a similar shape to the one for one unit in parameterized plane of $K_p$ and $K_i$. With larger values of $K_p$ and $K_i$, the magnitude of governor control output increases, reducing maximum overshoot. However, an excessively high setting of these parameters is adverse to system stability. The stability of two units with both hydraulic coupling and electrical coupling under different values of $K_p$ and $K_i$ resembles that of two units with averaged values of $K_p$ and $K_i$. One of the units adopting power control mode will slightly deteriorate the system stability, comparing to the case in which the unit applies GVO control mode.

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Nomenclatures

| $A$ | Sectional area of pipe/surge tank |
| $a$ | Pressure propagation speed in pipe |
| $b_p$ | Governor parameters: for permanent droop |
| $c_{st}$ | Throttling factor of surge tank |
| $D$ | Diameter of pipe/surge tank |
| $D_1$ | Turbine diameter |
| $e_{gh}$ | Partial derivative of turbine flow to turbine head |
| $e_{gq}$ | Partial derivative of turbine flow to turbine speed |
| $e_{gy}$ | Partial derivative of turbine flow to turbine GVO |
| $h$ | Head change relative to nominal head |
| $i$ | Imaginary unit |
| $K_p$ | Governor parameters: for proportional term |
| $K_i$ | Governor parameters: for integral term |
| $K_D$ | Governor parameters: for differential term |
| $l$ | Length of pipe |
| $M$ | Torque |
| $L$ | Water level of surge tank |
| $\bar{L}$ | Nominal water level of surge tank |

Figure 9. Stability boundaries of the PSP in different cases
\( e_h \) Partial derivative of turbine torque to turbine head
\( e_s \) Partial derivative of turbine torque to turbine speed
\( e_y \) Partial derivative of turbine torque to turbine GVO
\( e_{st} \) Hydraulic inductance of surge tank
\( F \) Lumped friction factor of pipe/surge tank
\( f \) Darcy-Weisbach friction factor of pipe/surge tank
\( G_h \) Elastic pipe transfer function
\( G_f \) Simplified elastic pipe transfer function
\( G_r \) Rigid pipe transfer function
\( G_{gr} \) Governor transfer function in frequency control mode
\( G_{gg} \) Governor transfer function in GVO control mode
\( G_{gp} \) Governor transfer function in power control mode
\( G_u \) Transfer function of upstream shared tunnel
\( G_p \) Transfer function of upstream shared penstock
\( G_{mu} \) Transfer function of upstream branch pipe
\( G_{md} \) Transfer function of downstream branch pipe
\( G_d \) Transfer function of downstream shared tunnel
\( G_{ust} \) Transfer function of upstream surge tank
\( G_{dst} \) Transfer function of downstream surge tank
\( G_{ss} \) System transfer function for island grid condition
\( GD^2 \) Unit inertia torque
\( g \) Gravity acceleration
\( H \) Head
\( \Delta H \) Head change
\( \bar{H} \) Nominal head
\( \Delta M \) Torque change
\( m \) Torque change relative to nominal torque
\( n \) Rotational speed
\( \Delta n \) Rotational speed change
\( P \) Power
\( \Delta Q \) Flow change
\( Q \) Flow
\( \bar{Q} \) Nominal flow
\( q \) Flow change relative to nominal flow
\( s \) Laplace operator
\( T_a \) Machine starting time constant
\( T_w \) Water starting time constant
\( T_e \) Time constant of water column elasticity
\( T_{1d} \) Differential filter time constant
\( T_y \) Time constant of guide vane servomotor
\( t \) Time
\( X \) Frequency
\( x \) Frequency change relative to nominal value
\( Y \) Guide vane opening
\( \Delta Y \) Guide vane opening change
\( Z \) Characteristic impedance of pipe
\( z \) Bottom level of surge tank
\( \beta \) Inclination angle of pipe

Subscripts

i) Indexes
\( i \) The \( i \)-th turbine/branch pipe

ii) Abbreviations of elements
\( u \) Upstream tunnel
\( p \) Upstream penstock
\( tu \) Upstream branch pipe
\( td \) Downstream branch pipe
\( d \) Downstream tunnel

iii) Position on pipes
\( U \) Upstream end of pipe
\( D \) Downstream end of pipe

iv) Pipe segment abbreviations
\( st \) Variables of surge tank
\( t \) Variables of pump-turbine

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