Excitation and breaking of relativistic electron beam driven longitudinal electron-ion modes in a cold plasma

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The excitation and breaking of relativistically intense electron-ion modes in a cold plasma is studied using 1D-fluid simulation techniques. To excite the mode, we have used a relativistic rigid homogeneous electron beam propagating inside a plasma with a velocity close to the speed of light. It is observed that the wake wave excited by the electron beam is identical to the corresponding Khachatryan mode [Phys. Rev. E, 58, 61(1998)], a relativistic electron-ion mode in a cold plasma. It is also seen in the simulation that the numerical profile of the excited electron-ion mode gradually modifies with time and eventually breaks after several plasma periods exhibiting explosive behavior in the density profile. This is an well known phenomena, known as “wave breaking”. It is found that the numerical wave breaking limit of these modes lies much below than their analytical breaking limit. The discrepancy between the numerical and analytical wave breaking limit has been understood in terms of phase-mixing process of the mode. The phase mixing time (or wave breaking time) obtained from the simulations has also been scaled as a function of beam parameters and found to follow the analytical scaling.

I. INTRODUCTION

Over decades, the research of relativistically strong plasma waves (RSWs) has attracted significant amount of attention because of its active contribution to the progress of plasma physics as well as astrophysics. For instance the excitation and breaking of such waves serves a useful paradigm to illustrate the physics of plasma based acceleration schemes.1–5, fast ignition concept in the Inertial confinement fusion systems.6–8, as well as various solar and astrophysical inter-disciplinary processes.9–12. Based on the ion dynamics, the relativistic propagating modes in a cold unmagnetized plasma can be typically categorized into two types, “Akhiezer-Polovin mode”13 and “Khachatryan mode”.14 Akhiezer Polovin mode is basically a relativistic electron plasma wave when the effect of ion motion are completely ignored mostly due to their heavy mass. On the other hand Khachatryan mode refers to a relativistic electron-ion mode when both the dynamics of electrons as well as ions are important. In general, every RSWs in a cold plasma are nothing but the Khachatryan modes. Akhiezer-Polovin (AP) modes are such Khachatryan modes at which the ions are immobile. Though the contribution of the ion motion on the collective dynamics of plasmas are negligibly small in most of the laboratory and astrophysical events, but their effects are still crucial in various phenomena. For example, in most of the astrophysical systems, mostly around the pulsars, it is considered to be filled with electron-positron plasmas, namely pair plasmas (plasmas consisting of two classes of particles with opposite sign of the charge, but equal mass).15,16. In such scenarios, the excitation of relativistic electron-ion modes and their breaking are critical for understanding many astrophysical events like jet formation, Ultra-High-Cosmic-Rays (UHECRs) generation, shock acceleration process etc.9,10,16. Furthermore, Khachatryan et al.14 reported on the study of strong plasma waves in presence of ions that plasma ions (even for heavy ions) can make an essential contribution to the process of charge separation in relativistic regimes. The recent reports on plasma based particle acceleration process where RSWs are used to accelerate the charge particles into high energies indicate that the motion of ions can give rise to the transverse fields that in turn disrupt the motion of the driver beam propagation and they also can effect the energy transfer ratio from the driver beam to the accelerated particles.17–19. Therefore, the excitation and breaking of relativistic electron-ion modes is still an active area of research. To date, the excitation of relativistic electron modes (Akhiezer-Polovin (AP) modes) and their breaking have been well examined by several authors.20–22. It has been well established that AP mode can break much below its wave breaking limit due to phase mixing process if it is subjected to a arbitrary amplitude of longitudinal perturbation. The analytical scaling of phase mixing time for AP mode has also been verified with the numerical works.20–22. Such an extensive study for the excitation and breaking of relativistic electron-ion mode or Khachatryan mode is still an unexplored area of research.

In this paper, we have investigated thoroughly the excitation and breaking of relativistic longitudinal electron-ion modes or Khachatryan modes in a cold plasma using 1-D fluid simulation techniques. To excite the mode, we have used a rigid homogeneous pulsed electron beam which propagates inside a cold unmagnetized homogeneous plasma with a speed close to the speed of light. Here both the dynamics of plasma electrons and ions are considered for exciting the relativistic electron-ion modes. When an electron beam propels inside the plasma it expels the nearby plasma electrons and attracts the ions (in case of mobile ions) due to the space-charge force. Typically the beam displaces the plasma electron and ions in opposite
direction. In case of immobile ions, the displacement of ions will be negligibly small. They will only provide a static neutralizing background. However, as the beam propagates further inside the plasma, the displaced plasma electrons as well as ions will try to come back to their original position to nullify the charge separation. But due to their inertia, they will overshoot their original position. As a result an oscillation or a wave will be established at the wake of the beam having phase velocity equal to the velocity of the beam. In other words, a relativistic electron-ion mode will be excited at the wake of the beam propagating with a phase velocity equal to the velocity of the beam. In the present paper, With the help of three fluid description of the beam plasma medium, the excitation and spatio-temporal evolution of relativistic electron beam driven wake waves or relativistic electron-ion modes in a cold plasma has been studied using 1-D fluid simulation techniques. It is shown that the numerical profiles of the wake wave obtained from the fluid simulation show a good agreement with their corresponding analytical profiles given by Rosenzweig et al.\textsuperscript{19} for different beam density. It is also observed that the wake wave excited by the \( e^− \) beam is identical to a corresponding Khachtryan’s mode.\textsuperscript{14} Furthermore, it is seen during the space-time evolution of the excited wake in the simulation that the profile of the excited wave gradually modifies with time and deviates significantly from the analytical solution of Khachtryan as well as Rosenzweig’s solution. After several plasma periods, we see that the density profile associated with the wake becomes spiky which is a clear signature of wave breaking.\textsuperscript{21–26,29} When the wave breaks or density bursts form in the simulation, we have calculated the maximum amplitude of the electric field or wave breaking limit of the wave. It is observed that the numerical wave breaking limit lies much below than the analytical limit given by Khachtryan et al.\textsuperscript{14} This inconsistency of the analytical and numerical wave breaking limit has been understood in terms of phase mixing process.\textsuperscript{20–22,25} It has been shown that the electron-ion mode breaks much below its analytical wave breaking limit due to the gradual process of phase mixing. From the simulation, we have obtained the phase mixing time or wave breaking time and plotted it with respect to the wave parameters. It is found that the numerical curve follows the analytical scaling given by Arghya et al.\textsuperscript{29}.

In next section (Section -II), we present the basic equations governing the excitation of relativistic electron-ion mode driven by relativistic electron beam in a cold plasma. We have discussed our numerical techniques for this study in section-III. Our numerical observations and a detail discussion of these results have been covered in section-IV. A brief summary of our work presented in this paper is covered in section V.

\section*{II. GOVERNING EQUATIONS}

The basic equations governing the excitation of longitudinal relativistic electron ion mode driven by electron beam in a cold plasma are the relativistic fluid-Maxwell equations. These equations contain the continuity and momentum equations for plasma electrons, plasma ions, and also for electron beam. The Poisson’s equation have been used to calculate the electric field in the system. Therefore, the basic normalized governing equations for the study of longitudinal (\( z \)-direction) electron-ion modes in plasmas are,

\begin{align}
\frac{\partial n_e}{\partial t} + \frac{\partial(n_e v_e)}{\partial z} &= 0 \quad (1) \\
\frac{\partial p_e}{\partial t} + v_e \frac{\partial p_e}{\partial z} &= -E \quad (2) \\
\frac{\partial n_i}{\partial t} + \frac{\partial(n_i v_i)}{\partial z} &= 0 \quad (3) \\
\frac{\partial p_i}{\partial t} + v_i \frac{\partial p_i}{\partial z} &= -\mu E \quad (4) \\
\frac{\partial n_b}{\partial t} + \frac{\partial(n_b v_b)}{\partial z} &= 0 \quad (5) \\
\frac{\partial p_b}{\partial t} + v_b \frac{\partial p_b}{\partial z} &= -E \quad (6) \\
\frac{\partial E}{\partial z} &= (n_i - n_e - n_b) \quad (7)
\end{align}

where \( p_e = \gamma_e v_e \), \( p_i = \gamma_i v_i \) and \( p_b = \gamma_b v_b \) are the \( z \)-components of momentum of plasma electron, plasma ion and beam electron respectively. It has been shown that the beam can be considered to be rigid for hundred of plasma periods only if the velocity of the beam \( v_b \geq 0.99 \). In this limit, the evolution equation(6) for the beam can be ignored and the propagation of the rigid beam can be completely
depicted by the equation (5). Therefore, we finally have equations (1), (2), (3), (4), (5), and (7) which are the key equations to study the excitation and breaking of relativistic longitudinal electron-ion mode driven by relativistic electron beam in a cold plasma. Below we discuss the numerical techniques used to solve these equations ((1), (2), (3), (4), (5) and (7)).

III. FLUID SIMULATION TECHNIQUES

In this section, we present the numerical techniques used to study the excitation and breaking of relativistic electron-ion mode in a cold plasma. We have developed a 1-D fluid code using LCPFCT subroutines based on flux-corrected transport (FCT) scheme. The basic principle of FCT scheme is based on the generalization of two-step Lax-Wendroff method. LCPFCT subroutines are mainly used to solve generalized continuity like equations ((1), (2), (3), (4), (5)). To solve the Poisson equation (7), we have used successive over-relaxation (SOR) method. Coupling these schemes iteratively, we have developed the 1-D fluid code and solved the equations (1), (2), (3), (4), (5) and (7) numerically. In the simulation, the driver beam is allowed to propagate from one end to the other end of the simulation box along z-direction. For a given beam profile, we have initiated the simulation using the corresponding analytical profiles of plasma electron density, ion density, electron velocity, ion velocity, and electric field given by Rosenzweig et al. and then followed the space-time evolution of the system. The results obtained from simulation are checked repeating the simulations for different mesh sizes. The code has also bench-marked studying various standard problems. In next section, we discuss the simulation results in detail.

IV. RESULTS AND DISCUSSION

A. Excitation of the relativistic electron-ion modes in a cold plasma using electron beam

Here we present the simulation results for the excitation of relativistic electron-ion-modes in a cold plasma using electron beam. The simulations have been performed for different beam densities \( n_b \) and mass ratios \( \mu \). In all the simulations, we have kept fixed the beam velocity \( v_b = 0.9999 \) and beam length \( L_b = 4 \). Hence the phase velocity \( v_{ph} \) of the excited wake wave is fixed to 0.9999. The amplitude and the frequency of excitation is now completely determined by the values of \( n_b \) and \( \mu \). By changing the values of \( n_b \) and \( \mu \) one can excite wake waves of different amplitude and frequency. In Figs. (1) and (2), we have plotted the perturbed plasma electron density \((n_e - 1)\), ion density \((n_i - 1)\), and electric field \(E\) profiles at different times \( \omega_{pe}t \) = 0.10, and 30 for \( n_b = 0.3 \) and \( n_b = 0.5 \) respectively; where \( \mu = 1 \). To initiate the simulations of \( \omega_{pe}t = 0 \) in each cases, we have used the analytical profiles of \( n_e - 1 \), \( n_i - 1 \), \( v_e \), \( v_i \), and \( E \) by solving equations (20-23) in ref. \(^{19}\). We see that the beam excites wake wave as it passes through the plasma. It is seen that the amplitude of the wake wave increases by increasing \( n_b \) for a given value of \( \mu \). For the sake of completeness we have next plotted the analytical solutions (viz Rosenzweig’s solution) of beam driven wake wave by solving equations (20-23) from ref.\(^{19}\) on top of the corresponding numerical profile for \( n_b = 0.3 \) in Fig. (3).

We see that the simulation results shows a good agreement with the analytical results.

Furthermore, it is to be noted that the equations (1), (2), (3), (4), (5), and (7) with the term \( n_b = 0 \) are nothing but the relativistic fluid-Maxwell equations in 1-D. The wave-frame solutions of these equations are the well known solutions of Khachatryan mode (see sec II in ref.\(^{14}\)) which is a relativistic electron-ion mode in a cold plasma. The beam density vanishes at the wake of the beam i.e. \( n_b = 0 \). Therefore we expect that the wake wave excited by the beam must be a corresponding Khachatryan mode. A Khachatryan mode can be parameterized in terms of \( E_{max}, v_{ph}, \) and \( \mu \); where \( E_{max} \) represents the maximum amplitude of the electric field of the excitation. For a given values of \( E_{max}, v_{ph}, \) and \( \mu \), one can easily show the corresponding structure of a Khachatryan mode by solving equation (4-9) of ref.\(^{14}\). Using the numerical values of \( E_{max}, v_{ph}, \) and \( \mu \) of the excited wake wave from the simulation for \( n_b = 0.3 \), we have solved equations (4-9) of ref.\(^{14}\) and plotted the corresponding Khachatryan mode on top of the wake wave in Fig. (4). It is seen that the wake wave is nothing but a corresponding Khachatryan mode propagating with a phase velocity equal to the velocity of the beam. To further emphasize the fact that the wake wave excited by the relativistic electron beam is a corresponding Khachatryan mode, we have plotted for different values of \( n_b = 0.5 \) and also \( \mu = 1/2000 \) in Fig. (5). We see that Khachatryan mode converts to Akhiezer-Polovin mode for low \( \mu \).

B. Breaking of relativistic electron-ion modes in a cold plasma

Next, we have also observed several other interesting features by following the space-time evolution of the excited wave in the simulation for a long time. In Figs. (6) and (7), we have plotted the profiles of the perturbed electron density \((n_e - 1)\), ion density \((n_i - 1)\), and electric field \(E\) for \( n_b = 0.5 \) at \( \omega_{pe}t = 105 \) and \( n_b = 1 \) at \( \omega_{pe}t = 48 \) respectively. We see that the numerical profiles of electron density, ion density and electric field get modified with time and deviate significantly from their analytical profiles after several plasma periods. It is also seen that the amplitude of the electron density and ion density gradually increases with time and eventually acquire spiky structure at later times. After a certain time (e.g. \( \omega_{pe}t = 90 \) for \( n_b = 0.5 \) (see Fig. 6) and \( \omega_{pe}t = 38 \) for \( n_b = 1 \) (see Fig. 7)), the amplitude of the density profile (electron and ions) becomes maximum and suddenly decreases afterwards. The electric field amplitude also gets suppressed after this critical time. This is a clear signature of wave breaking. The time at which the amplitude of the density spikes get maximized and then goes down is known as wave breaking time. Typically in a medium, a wave
breaks when the amplitude of the wave reaches to its maximum value, known as wave breaking limit\textsuperscript{20,23}. If the amplitude of the excitation crosses that limit, the wave can not be sustained by the medium and the wave finally breaks. The wave breaking limit for a plasma wave can be defined by the amplitude of its electric field ($E_{WB}$). For Khachatryan mode or a relativistic electron ion mode, the analytical expression of the wave breaking limit is given by Khachatryan et al.\textsuperscript{14} as, $E_{WB} = \sqrt{2} \gamma_{ph}^2 [1 + (1 - \xi_1^2) \xi_2^2] / \mu$; where $\xi_1 = 1 + \mu$, $\xi_2 = 1 + [\mu (\gamma_{ph} - 1) / (\gamma_{ph} + 1)]$ and $\gamma_{ph} = (1 - v_{ph}^2)^{-\frac{1}{2}}$. When the wake wave or Khachatryan mode breaks in the simulation for $n_b = 0.6$ and $n_b = 1$, we have determined the maximum amplitude of the electric field for different values of $\mu$ and plotted in Fig. (8) as a function of $\mu$ along with their corresponding analytical values. We have seen that the numerical wave breaking limit lies much below the analytical wave breaking limit. This indicates that the numerically excited electron-ion mode breaks before it touches to its analytical wave breaking limit.

The deviation between the analytical and the numerical breaking limit of relativistic propagating electron-ion modes has been understood in terms of phase mixing process\textsuperscript{21,22,26}. Normally phase mixing process occurs when the frequency of the wave becomes space dependent. As a result the neighboring fluid elements or particles sustaining the wave will oscillate with different frequencies. Hence the profile of the wave gets modified with time gradually. Eventually a time will occur when the two neighboring elements will oscil late with out of phase. As a consequence they will cross each other and the wave will break exhibiting sharp spikes in the density profile at the crossing point. After the breaking, the wave will loose its coherency and the energy of the wave goes to random particle motion. The effect of phase mixing process on the wave-breaking phenomena of relativistic electron plasma waves or Akhiezer-Polovin modes has already been extensively studied by several authors\textsuperscript{21,22,26,29}. They have shown that AP mode can break much below its wave breaking limit due to phase mixing process. In one of our earlier works\textsuperscript{22} for immobile ions, we have shown that the wake wave excited by relativistic beam is nothing but a corresponding Akhiezer-Polovin (AP) mode\textsuperscript{13} and breaks much below the analytical wave breaking limit. Normally a pure Akhiezer-Polovin mode never breaks as its wave breaking limit $E_{WB} = \sqrt{2} (\gamma_{ph} - 1)$. For relativistic AP modes, $E_{WB} \to \infty$ as $\gamma_{ph} \to \infty$. In 2007, Prabal et al.\textsuperscript{20} showed that AP mode can break below the wave breaking limit if it is subjected to a longitudinal perturbation of an arbitrary amplitude. Due to the longitudinal perturbations, the frequency of the wave becomes space dependent. As a result, the wave breaks via phase mixing process before it reaches to its wave breaking limit. In the simulation for immobile ions, the beam excites pure AP mode at its wake. The inherent numerical fluctuation then gets superimposed with this wave and acts as a perturbation on it. As a results the wake wave or AP mode breaks via phase mixing process.

The similar observations have also been made in the present scenario but for the mobile ions. Due to the presence of the ion, though the basic characteristics of the wave has changed from the AP mode to Khachtryan mode, but the wave breaking mechanism is found to be similar. The beam excites a pure Khachtryan mode in which the frequency becomes independent of space and time. As time goes, the original solution of Khachtryan mode excited by the beam gets perturbed by the gradual accumulation of inherent numerical fluctuations. As a result, the frequency of the pure Khachtryan mode becomes space dependent and breaks via phase mixing process much below its wave breaking limit. Following the same methodology given in ref.\textsuperscript{21} Arghya et al.\textsuperscript{29} has shown the analytical wave breaking time for a Khachtryan mode in terms of the wave parameters as,

$$\tau_{mix} = \frac{2 \pi v_{ph}}{\delta^2} \left[ \frac{1}{u_{me}^2 + \mu^2 u_{mi}^2} \right]$$

Where $u_{me}$, $u_{mi}$, and $\delta$ are the maximum amplitude of plasma electron velocity, ion velocity and the amplitude of the perturbation respectively. In our simulations for a given $\mu$, $u_{me}$ and $u_{mi}$ are decided by the electron beam density $n_b$. Typically, the value of $u_{me}$ and $u_{mi}$ increases with $n_b$ for a fixed $\mu$ (see figures (1)-2). For different beam density ranging from 0.3 to 1 we have performed our simulations for $\mu = 1$ and obtained the corresponding wave breaking time $\tau_{mix}$, $u_{me}$ and $u_{mi}$. In Fig. (9), we have plotted the $\omega_{pe}\tau_{mix}$ as a function of $u_m^2 = u_{me}^2 + u_{mi}^2$ along with the analytical values from equation (8). It is to be noted that the value of $u_{me}$ and $u_{mi}$ are equal for $\mu = 1$. For the analytical plotting in Fig. (9), we have used the first data point ($n_b = 0.3$, $u_{me} = u_{mi} = 0.32$ and $\omega_{pe} \tau_{mix} = 180$) from the simulation to calculate the factor $\frac{2 \pi v_{ph}}{\delta^2}$. Using this factor, we then draw the curve of $\omega_{pe} \tau_{mix}$ for different $u_m$ from equation 8. We see that the numerical values follow the same pattern and match with the analytical values.

V. CONCLUSION

In summary, we have studied the excitation and breaking of relativistic electron-ion mode in a cold plasma using 1-D fluid simulation techniques. For the excitation of the mode, we have used an external rigid homogeneous electron beam into the plasma. As the beam propagates through the plasma it creates an wake wave having phase velocity equal to the velocity of the beam. It is observed that the excited wake wave is identical to the corresponding Khachtryan mode or a relativistic propagating electron-ion mode in a cold plasma. It is also found in the simulation that the numerical profile of the excited Khachtryan mode or wave wake gradually modifies with time and eventually breaks after several plasma periods. We have seen that the numerical wave breaking limit of this wave lies much below the analytically estimated limit given by Khachtryan et al.\textsuperscript{14}. The difference between the numerical and analytical limit has been understood in terms of phase-mixing process of this mode. We have also scaled the phase mixing time (or wave breaking time) obtained from the simulations as a function of beam parameters and found to follow...
the well known existing analytical scaling given by Arghya et al.\textsuperscript{29}.

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FIG. 1. Plot of normalized perturbed electron density \( (n_e - 1) \), ion density \( (n_i - 1) \), and electric field \( (E) \) profiles at different times for the normalized beam density \( (n_b) = 0.3 \), beam length \( (l_b) = 4 \), beam velocity \( (v_b) = 0.99999 \) and mass ratio \( (\mu) = 1 \).

FIG. 2. Plot of normalized perturbed electron density \( (n_e - 1) \), ion density \( (n_i - 1) \), and electric field \( (E) \) profiles at different times for the normalized beam density \( (n_b) = 0.5 \), beam length \( (l_b) = 4 \), beam velocity \( (v_b) = 0.99999 \) and mass ratio \( (\mu) = 1 \).
FIG. 3. Plot of numerical and analytical normalized perturbed electron density \((n_e - 1)\) and ion density \((n_i - 1)\) profiles at \(\omega_{pe} t = 20\) for the normalized beam density \((n_b) = 0.3\), beam length \((l_b) = 4\), beam velocity \((v_b) = 0.9999\) and mass ratio \((\mu) = 1\).

FIG. 4. Plot of normalized perturbed electron density \((n_e - 1)\) and electric field \((E)\) profiles obtained from simulation and corresponding Khachatryan mode (analytical) at \(\omega_{pe} t = 15\) for the normalized beam density \((n_b) = 0.3\), beam length \((l_b) = 4\), beam velocity \((v_b) = 0.9999\) and mass ratio \((\mu) = 1\).
FIG. 5. Plot of numerical and analytical profiles of normalized electric field ($E$) for the mass ratio ($\mu$)=$1/2000$ at $\omega_{pe}t = 40$ for the normalized beam density ($n_b$)=0.5, beam length ($l_b$)=4, beam velocity ($v_b$) =0.9999.

FIG. 6. Plot of numerical profile of normalized perturbed electron density ($n_e - 1$), ion density ($n_i - 1$), and electric field ($E$) at $\omega_{pe}t = 125$ for the normalized beam density ($n_b$)=0.3, beam length ($l_b$)=4, beam velocity ($v_b$) =0.9999 and mass ratio ($\mu$) =1.
FIG. 7. Plot of numerical profile of normalized perturbed electron density ($n_e - 1$), ion density ($n_i - 1$), and electric field ($E$) at $\omega_{pe} t = 125$ for the normalized beam density ($n_b$)=1, beam length ($l_b$)=4, beam velocity ($v_b$)=0.9999 and mass ratio ($\mu$)=1.

FIG. 8. Plot of theoretical and numerical wave breaking electric field ($E_{WB}$) vs. mass ratios ($\mu$) for normalized beam density $n_b = 0.6$ and $n_b = 1$; where beam length ($l_b$)=4, beam velocity ($v_b$)=0.9999
FIG. 9. Plot of theoretical and numerical scaling of phase mixing time ($\omega_{pe} \tau_{mix}$) vs. amplitude of the waves ($u_m$).