The inverse problem in Seismology. Seismic moment and energy of earthquakes. Seismic hyperbola

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Abstract

The inverse problem in Seismology is tackled in this paper under three particular circumstances. First, the inverse problem is defined as the determination of the seismic-moment tensor from the far-field seismic waves (P and S waves). These waves provide directly accessible (measurable) experimental data on earthquakes’ focal structure and mechanism. We use the analytical expression of the seismic waves in a homogeneous isotropic body with a seismic-moment source of tensorial forces, the source being localized both in space and time. The far-field waves provide three equations for the sixth unknown parameters of the general tensor of the seismic moment. Second, the Kostrov vectorial (dyadic) representation of the seismic moment is used. This representation relates the seismic moment to the focal displacement in the fault and the orientation of the fault (moment-displacement relation); it reduces the seismic moment to four unknown parameters. Third, the fourth missing equation is derived from the energy conservation and the covariance condition. In particular, this relation provides access to the focal volume of the fault and the near-field seismic waves. The four equations derived here are solved, and the seismic moment is determined, thus solving the inverse problem in the conditions described above. It turns out that the seismic moment is traceless, its magnitude is of the order of the elastic energy stored in the focal region (as expected), and the solution is governed by the unit quadratic form associated to the tensor (related to the magnitude of the longitudinal displacement in the P wave). It is shown that a useful picture of the seismic moment is the conic represented by the associated quadratic form, which is a hyperbola (seismic hyperbola). This hyperbola provides an image for the focal region: its asymptotics are oriented along the focal displacement and the normal to the fault. Also, it is shown that the far-field seismic waves allow an estimation of the volume of the focal region, focal strain, duration of the earthquake and earthquake energy; the later quantity
is a direct measure of the magnitude of the seismic moment. The special case of an isotropic seismic moment is presented.

Running title: Inverse Seismological Problem (or Seismic Hyperbola)

MSC: 35Q86; 35L05; 74J25

PACS: 62.30.+d; 91.10.Kg; 91.30. Ab; 91.30.Bi; 91.30.Px; 91.30.Rz

Key words: inverse problem; seismic waves; seismic moment; elasticity; seismic hyperbola

1 Introduction

The inverse problem in Seismology aims at getting information about the nature and structure of the forces acting in the earthquake’s focus from measurements of the seismic waves at distances far away from the earthquake focus (at Earth’s surface). We present here a solution to this problem by means of the seismic waves propagating in a homogeneous isotropic body with localized tensorial forces, the Kostrov vectorial representation of the seismic moment for a fault (moment-displacement relation) and the energy conservation together with the covariance condition. This relation is derived by equating the energy carried by the far-field seismic waves to the mechanical work done by forces in the focal region.

The seismic moment and seismic energy are basic concepts in the theory of earthquakes.[1]-[4] The seismic moment has emerged gradually in the first half of the 20th century, the first estimation of a seismic moment being done by Aki in 1966.[5] The relations between the seismic moment, seismic energy, the mean displacement in the focal region, the rate of the seismic slip and the earthquake magnitude are recognized today as very convenient tools for characterizing the earthquakes.[6]-[8]

The inverse (inversion) problem[9] is solved usually by determining the seismic-moment components $M_{ij}$ ($i, j = 1, 2, 3$) from information provided by far-field seismic waves at different locations and times,[10]-[14] or free oscillations of the earth, long-period surface waves, supplemented, in general, with additional relevant information (constraints; see Ref. [15] and references therein). Besides noise, the information provided by such data may reflect particularities of the structure of the focal region and the focal mechanism which are not included, usually, in equations, like the structure factor of the focal region, both spatial and temporal, or deviations from homogeneity and isotropy. In particular, waves measured at different locations (or times) may lead to overdetermined systems of equations for the unknowns $M_{ij}$, and the solutions must be "compatibilized". A proper procedure of compatibilization may lead, in fact, to redundant equations, if the covariance of the equations is not ensured. Indeed, the experimental data may often be used in a non-covariant form, which makes the results dependent on the reference frame. The covariance is understood in this paper as the invariance of the form of the
equations to translations and rotations (independence of the reference frame). We may add that the normal modes of the pure free oscillations do not imply a source of waves, while surface waves, having sources on the surface, have a very indirect connection to the body waves generated in the focal region. Surface displacement in the main shock of an earthquake is often used, which has a very indirect relevance for the earthquake source and mechanism.

We present here a direct way of determining (analytically) the seismic moment for a shear faulting (as well as for an isotropic source) by using the far-field waves generated by a time-localized tensorial point source. The waves produced by extended sources imply additional information regarding the spatial and temporal structure factors; the inverse problem in this case is a more complex problem, which remains beyond the aim of the present paper.

We consider that the available data are the displacement vectors produced by the seismic waves in the wave region. The information provided by these data is the magnitude of the longitudinal (P-wave) displacement (one parameter) and the transverse-wave displacement vector (S-wave, two parameters; we assume that the direction of the earthquake focus is known). These data provide three independent parameters, related to the components of the seismic moment by three equations. They may be viewed as a minimal set of independent data. It follows that, restricting ourselves to these data only, the seismic moment has only three independent components. On the other hand, according to Kostrov representation, the seismic moment is characterized by its magnitude and the fault orientation and the fault slip, which are two mutually perpendicular unit vectors. This information includes four independent parameters. We can see, on one hand, according to Kostrov representation, that only four out of six components of the seismic moment are independent and, on the other hand, we need a fourth equation in order to determine the four independent components of the seismic focus. We provide in this paper the fourth equation, which is the equation of energy conservation together with the covariance condition. The covariance condition reduces the four independent components of the seismic moment to three, which makes possible the determination of the seismic moment from the seismic-wave displacement. Also, we show that an image of the forces acting in the focal region and the geometry of the fault can be obtained by a so-called "seismic hyperbola".

It is widely assumed that typical tectonic earthquakes originate in a localized focal region, with dimensions much shorter than the distance to the observation point (and the seismic wavelengths). The tensorial seismic force density

$$ F_i = M_{ij} \partial_j \delta(\mathbf{R} - \mathbf{R}_0) $$

(1.1)

is used for the seismic focus,[2, 4, 16] where $M_{ij}$ is the tensor of the seismic moment, $\delta$ is the Dirac delta function and $\mathbf{R}_0$ is the position of the focus (hypocentre). We assume that the position $\mathbf{R}_0$ is a known parameter. The labels $i, j$ denote the Cartesian axes and summation over repeating suffixes is assumed (throughout this paper). The seismic
tensor \( M_{ij} \) is a symmetric tensor, which, in general, has six independent components. It may be decomposed into double-couple (shear faulting) and dipole components and an isotropic component; departure from double-couple components reflects a complex shear faulting, tensile faulting, volcanic morphology, etc. \([15],[17]-[21]\) The force given by equation (1.1) is a generalization of the double-couple representation of the seismic force. Indeed, let us assume a force density \( \mathbf{F}(\mathbf{R}) = f_g(\mathbf{R}) \), where \( f \) is the force and \( g(\mathbf{R}) \) is a distribution function; a point couple associated with a force acting along the \( i \)-th direction can be represented as

\[
f_i g(x_1 + h_1, x_2 + h_2, x_3 + h_3) - f_i g(x_1, x_2, x_3) \simeq f_i h_j \partial_j g(x_1, x_2, x_3),
\]

where \( h_j, j = 1, 2, 3 \), are the components of an infinitesimal displacement \( \mathbf{h} \); \( x_i, i = 1, 2, 3 \), are the coordinates of the position \( \mathbf{R} \) and \( \partial_j \) denotes the derivative with respect to \( x_j \). The force moment (torque) \( t_{ij} = f_i h_j \) is generalized in equation (1.2) to a symmetric tensor \( M_{ij} \), which is the seismic moment entering equation (1.1); in addition, the distribution \( g(\mathbf{R}) \) can be replaced by \( \delta(\mathbf{R} - \mathbf{R}_0) \) for a spatially localized focal region. The \( \delta \)-function used in equation (1.1) is an approximation for the shape of the focal region. In equation (1.1) the focus is viewed as being localized over a distance of order \( l \) (volume of order \( l^3 \)), much shorter than the distance \( R \) to the observation point (\( l \ll R \)).

The seismic moment depends on the time \( t \); we may write \( M_{ij}(t) = M_{ij} h(t) \), where \( h(t) \) is a positive function, localized at \( t = 0 \), which includes the time dependence of the seismic moment; we assume \( \max[h(t)] = h(0) = 1 \) and denote by \( T \) the (short) duration of the seismic event; the time \( T \) is much shorter than any time of interest, such that we may view the function \( h(t) \) as being represented by \( T \delta(t) \). The particular case \( h(t) = T \delta(t) \) is called an elementary earthquake in Refs. \([16]\). (The function \( h(t) \) should not be mistaken for the magnitude of the displacement vector \( \mathbf{h} \) used above).

For a homogeneous isotropic body the seismic waves generated by the tensorial force given by equation (1.1) are governed by the equation of the elastic waves

\[
\ddot{u}_i - c_l^2 \Delta u_i - (c_l^2 - c_t^2) \partial_j \text{div} \mathbf{u} = \frac{1}{\rho} M_{ij}(t) \partial_j \delta(\mathbf{R}),
\]

where \( u_i \) are the components of the displacement vector \( \mathbf{u} \), \( c_{l,t} \) are the velocities of the longitudinal and transverse waves, respectively, \( \rho \) is the density and \( \mathbf{R} \) is the position vector drawn from the focus (taken as the origin of the reference frame) to the observation point. The solution of this equation [2] [4] [16] can be written as \( \mathbf{u} = \mathbf{u}^l + \mathbf{u}^t \), where

\[
\begin{align*}
\mathbf{u}^l_i &= -\frac{1}{4\pi \rho c_l^2} \frac{M_{ij} x_j}{R^3} h(t - R/c_l) + \\
&\quad + \frac{1}{8\pi \rho R^6} \left( M_{jj} x_j + 4M_{ij} x_j x_j - \frac{9M_{jk} x_j x_k}{R^2} \right) \cdot \left[ \frac{1}{c_t} h(t - R/c_l) - \frac{1}{c_t} h(t - R/c_l) \right]
\end{align*}
\]

(1.4)
is the near-field displacement \((R \text{ comparable with } l)\) and
\[
u_i^f = -\frac{1}{4\pi\rho c_l^2} \frac{M_{ij}x_j}{R^2} h'(t - R/c_l) - \frac{1}{4\pi\rho} \frac{M_{jk}x_i x_j x_k}{R^4} . \]
(1.5)
is the far-field displacement \((R \gg l)\). The near-field region is defined by distances \(R\) of the order \(l\), while the far-field region is defined by distances \(R\) much larger than \(l\). The short duration \(T\) of the seismic event (duration of activity of the focus) enters equations (1.4) and (1.5) through \(h(t)\) and the derivative \(h'(t)\), which is of the order \(1/T\). The displacement vectors given by equations (1.4) and (1.5) include the longitudinal wave (denoted by suffix \(l\), not to be confused with length \(l\)), propagating with velocity \(c_l\), and the transverse wave (suffix \(t\)), propagating with velocity \(c_t\); in the far-field region the displacement vectors of the longitudinal wave (\(P\) wave) and the transverse wave (\(S\) wave) are mutually orthogonal (this is not so for the \(l, t\) waves in the near-field region).

As long as the function \(h(t)\) may be viewed as a localized function, the magnitude of the displacement vectors varies as \(1/R^2\) for the near-field wave and \(1/R\) for the far-field waves. Their direction is determined by the tensor of the seismic moment \(M_{ij}\) (in particular the vector with components \(M_{ij}x_j\)). The far-field waves given in equation (1.5) are shell spherical waves with a thickness of the order \(\Delta R \simeq c_{l,t}T\). A superposition of forces given by equation (1.1), localized at different positions \(\mathbf{R}_0\) and different times, corresponds to a structured focus, and the elementary displacement given by equations (1.4) and (1.5) gives access to the structure factor of the focal region.[16]

2 Far-field seismic waves

It is convenient to introduce the notations
\[
M_i = M_{ij}n_j \, , \quad M_0 = M_{ii} \, , \quad M_4 = M_{ij}n_in_j \, ,
\]
(2.1)
where \(\mathbf{n}\) is the unit vector along the radius drawn from the focus to the observation point (observation radius), \(x_i = Rn_i\), and \(h_{l,t} = h(t - R/c_{l,t})\); henceforth we consider the unit vector \(\mathbf{n}\) a known vector. \(M_0\) is the trace of the seismic-moment tensor and \(M_4\) is the quadratic form associated to the seismic-moment tensor, constructed with the unit vector \(\mathbf{n}\); we call it the unit quadratic form of the tensor. The vector \(\mathbf{M}\) can be called the "projection" of the tensor along the focus-observation point direction (observation direction).

Making use of these notations, the seismic waves given by equations (1.4) and (1.5) can be decomposed into \(l\)- and \(t\)-waves, written as \(\mathbf{u}^n = \mathbf{u}^n_l + \mathbf{u}^n_t\),
\[
\mathbf{u}^n_l = -\frac{h_l}{8\pi\rho c_l^2 R^6} [(M_0 - 9M_4)\mathbf{n} + 4\mathbf{M}] \, ,
\]
\[
\mathbf{u}^n_t = -\frac{h_t}{8\pi\rho c_t^2 R^6} [(M_0 - 9M_4)\mathbf{n} + 6\mathbf{M}] \, ,
\]
(2.2)
and \( u' = u'_f + u'_t \),

\[
\begin{align*}
u'_f &= -\frac{h'_f}{4\pi \rho c^2 t R} M_4 n, & u'_t &= \frac{h'_t}{4\pi \rho c^2 t R} (M_4 n - M) .
\end{align*}
\]

(2.3)

For numerical purposes we take the "maximum deviation" of the near-field displacement \( u''_{n,t} \) (with its sign) for \( t = R/c_{l,t} \), i.e. we take \( h_{l,t}(0) = 1 \). Equally well, we can take the average values of the vectors \( u''_{n,t} \) over the support \( T \) of the functions \( h_{l,t} \), or \( \Delta R \), which is of the order \( c_{l,t} T \). Henceforth, \( h_{l,t} \) in equations (2.2) are understood as \( h_{l,t}(0) = 1 \).

The functions \( h'_{l,t} \) are scissor-like functions ("double-shock" functions), with two sides with opposite signs (corresponding to \( t > 0 \) or \( t < 0 \)), extending over \( T \), or the distance \( \Delta R \); their "maximum deviations" are of the order \( \pm 1/T \); for numerical estimations it is convenient to introduce the notations \( v_{l,t} = u''_{l,t}/T h'_{l,t} \) and take the "maximum deviation" of these functions (with their sign), on any side of the functions \( h'_{l,t} \), the same side for \( v_l \) and \( v_t \) (\( v_{l,t} \) may depend on the side of the functions \( h'_{l,t} \), since the functions \( h_{l,t}(t) \) are not necessarily symmetric with respect to \( t = 0 \)). Similarly, we can take the average values of \( v_{l,t} \) over any side of the functions \( h'_{l,t} \) (the same for \( v_l \) and \( v_t \)). The displacement vectors \( v_{l,t} \) are directly accessible experimentally. We consider them as data for our problem. Making use of these notations, equations (2.3) become

\[
\begin{align*}
v_l &= -\frac{1}{4\pi \rho T c^3 l R} M_4 n , & v_t &= \frac{1}{4\pi \rho T c^3 t R} (M_4 n - M) .
\end{align*}
\]

(2.4)

We note that the vectors \( R^2 u''_{n,t} \) and \( R v_{l,t} \) depend on the density \( \rho \), the duration \( T \), the seismic moment and the elastic coefficients of the body (velocities of the elastic waves); if local deviations from this pattern are observed, the body is not locally homogeneous and isotropic (or the focus is not localized).

The displacement in the far-field waves is determined by three independent parameters: the magnitude of the vectors \( v_{l,t} \) (two parameters) and the direction of the transverse vector \( v_t \) (one parameter). Consequently, we may view the equations

\[
M = -4\pi \rho TR (c^3_l v_l + c^3_t v_t) ,
\]

(2.5)

derived from equations (2.3), as three independent equations for the six unknown components \( M_{ij} \) of the seismic moment; by multiplying by \( n_i \) and summing over \( i \), we get the first equation (2.3),

\[
M_4 = M_{ij} n_i n_j = -4\pi \rho TR c^3_l (v_l n) = -4\pi \rho TR c^3_t v_t ,
\]

(2.6)

which is not independent of the three equations written above. We view \( v_{l,t} \) as (known) quantities measured experimentally, and \( \rho, R, c_{l,t} \) as known parameters; duration \( T \) will be determined shortly. A simple observation would show that for given displacements \( v_{l,t} \) and given \( T \) we may solve equations (2.5) and get the three independent components of the seismic moment \( M_{ij} \). Unfortunately, leaving aside that the other
three components are left as free parameters by such a procedure, the measurement of the duration $T$ from $\Delta r/c_{l,t}$, where $\Delta r$ is the projection of $\Delta R$ on Earth’s surface, is dependent on the local frame, and, consequently, would not provide a suitable input data for covariant equations.

We note in equations (2.5) and (2.6) the consistency (compatibility) relation $M^2 > M_4^2$, derived from $v_{l,t}^2 > 0$ ($v_{l,t}$ denote the magnitudes of the vectors $v_{l,t}$). The inverse problem discussed in this paper consists in determining the tensor $M_{ij}$ from the displacement $v_{l,t}$ in the far-field waves, making use of additional, model-related, information. The model we use is provided by the fault geometry of the focal zone. We can see that only three components of the seismic moment $M_{ij}$ are independent. We determine the seismic-moment tensor by means of the vectors $M$ and $n$ (experimentally accessible). The special case of an isotropic moment is presented. We note that equations (2.4) are manifestly covariant. Also, we note that having known $M$ and $M_4$ we can have access to the near-field displacement given by equations (2.2), provided we know $M_0$.

3 Energy of earthquakes

If we multiply equation (1.3) by $\dot{u_i}$ and sum over the suffix $i$, we get the law of energy conservation

$$\frac{\partial}{\partial t} \left[ \frac{1}{2} \rho \dot{u_i}^2 + \frac{1}{2} \rho c_l^2 (\partial_j u_i)^2 + \frac{1}{2} \rho (c_l^2 - c_t^2) (\partial_i u_j)^2 \right] - \rho c_l^2 \partial_j (\dot{u_i} \partial_j u_i) - \rho (c_l^2 - c_t^2) \partial_j (\dot{u_j} \partial_i u_i) = \dot{u_i} M_{ij} \partial_j \delta(R) .$$

(3.1)

According to this equation, the external force performs a mechanical work in the focus ($\dot{u_i} M_{ij} \partial_j \delta(R)$ per unit volume and unit time). The corresponding energy is transferred to the waves (the term in the square brackets in equation (3.1)), which carry it through the space (the term including the $\text{div}$ in equation (3.1)). It is worth noting that outside the focal region the force is vanishing. Also, the waves do not exist inside the focal region. Therefore, limiting ourselves to the displacement vector of the waves, we have not access to the mechanical work done by the external force in the focal region. This circumstance arises from the localized character of the focus.

In the far-field region we can use the decomposition $u = u_l + u_t$ in longitudinal and transverse waves, where $\text{curl} u_t = 0$ and $\text{div} u_t = 0$; this decomposition leads to

$$\frac{\partial e_{l,t}}{\partial t} + c_{l,t} \text{div}s_{l,t} = 0 \ ,$$

(3.2)

where

$$e_{l,t} = \frac{1}{2} \rho \left( \dot{u_{l,t}}^2 \right) + \frac{1}{2} \rho c_{l,t}^2 \left( \partial_i u_{l,tj}^f \right)^2$$

(3.3)

is the energy density and

$$s_{l,tj} = -\rho c_{l,tj} \dot{u_{l,tj}}^f \partial_i u_{l,tj}^f$$

(3.4)
are the components of the energy flux densities per unit time (the flow vectors). From equation (3.2) we can see that the energy is transported with velocities \( c_{l,t} \) (as it is well known). The volume energy \( E = \int dR (e_l + e_t) \) is equal to the total energy flux \( \Phi = -\int dt dR (c_l \text{div} s_l + c_t \text{div} s_t) = -\int dt \oint dS (c_l s_l + c_t s_t) \). (3.5)

Making use of equations (2.3) and taking \( h'' = -\frac{1}{T^2} \) as an order-of-magnitude estimate, we get

\[
E = \Phi = 4\pi \rho T R^2 (c^2_l v^2_l + c^2_t v^2_t) ;
\]

(3.6)

this relation gives the energy released by the earthquake in terms of the displacement measured in the far-field region and the (short) duration of the earthquake. From equations (2.4) we get the relation

\[
E = \frac{1}{4\pi \rho c^2 T^3} [M^2 - (1 - c^5_l/c^5_t) M^2_4] \]

(3.7)

between energy and the seismic moment.

### 4 Geometry of the focal region

Let us consider a point torque \( t_{ij} = f_i h_j \), where \( h_j \) are viewed as infinitesimal distances and \( f_i \) denote the components of a force \( f \); the force \( f \) originates in a volume force density \( \partial_j \sigma_{ij} \), where \( \sigma_{ij} \) is the stress tensor; the latter can be expressed as \( \sigma_{ij} = 2\mu u_{ij} + \lambda u_{kk} \delta_{ij} \), where \( \mu \) and \( \lambda \) are the Lame coefficients \( c^2_l = (2\mu + \lambda)/\rho, c^2_t = \mu/\rho \), \( u_{ij} = \frac{1}{2}(\partial_j u_i + \partial_i u_j) \) is the strain tensor and \( u \), with components \( u_i \), is the displacement vector.\[22\] We can write

\[
t_{ij} = f_i h_j = \int d\mathbf{r} \partial_k \sigma_{ik} \cdot h_j = \\
= \mu \int d\mathbf{r} \partial^2_k u_i \cdot h_j + (\mu + \lambda) \int d\mathbf{r} \partial_k u_k \cdot h_j = \\
= \mu \oint dS \cdot s_k \partial_k u_i \cdot h_j + (\mu + \lambda) \oint dS \cdot s_k \partial_i u_k \cdot h_j ,
\]

(4.1)

where the \( r \)-integration is performed over the focal volume surrounded by the surface \( S \) and \( s \) is the unit vector normal to this surface. We may write \( \partial_k u_k \simeq \Delta u_k/\Delta x_i \) for the derivatives of \( u_k \) and use \( \Delta u_k = \Delta u_k \delta_{ij} = u_k \delta_{ij} \), where \( u_k \) is the displacement on the surface. These equalities follow from the point-like nature of the torque. We note that \( u \) here is the focal displacement, which is distinct from the displacement in the waves. It follows

\[
t_{ij} = \mu S \cdot \overline{s_j u_i} + (\mu + \lambda) S \cdot \overline{s_k u_k} \delta_{ij} ,
\]

(4.2)

where the overbar denotes the average over the surface with area \( S \). This relation acquires a useful form for a localized (plane) fault. We assume that the fault focal
region includes two plane-parallel surfaces, each with (small) area $S$, separated by a (small) distance $d$, sliding against one another. The focal area is determined by two lengths $l_{1,2}$, $S = l_1 l_2$. In general, the lengths $l_1, l_2, d$ are distinct; in order to ensure the compatibility with the localization provided by the $\delta$-function (used in deriving the waves), we assume $l_1 = l_2 = d = l$. For such a model of localized fault the product $S_j u_i$ may be replaced by $2s_j u_i$, where the vector $s$ is the unit vector normal to the fault (we note that the integration over the surfaces perpendicular to the fault is zero, due to the opposing (sliding) displacements). In view of the small extension of the focal region, we may drop the average bar over $u_i$. In addition, this model of fault-slip implies $s_k u_k = 0$, i.e. the normal to the fault $s$ and the focal displacement (fault slip) $u$ are mutually orthogonal vectors. In order to distinguish the focal displacement from the displacement in the seismic waves, we attach the superscript $0$ to the focal displacement. The seismic moment is obtained by symmetrizing the expression given by equation (4.2); we get

$$M_{ij} = 2\mu S \left( s_i u_j^0 + s_j u_i^0 \right) = 2\mu S u^0 \left( s_i a_j + a_i s_j \right)$$  \hspace{1cm} (4.3)$$

where we introduce the unit vector $a$ along the direction of the focal displacement; we write $u_i = u^0 a_i$, where $u^0$ is the magnitude of the focal displacement and $a_i^2 = 1$. We can see that the seismic moment is represented in equation (4.3) by two orthogonal vectors ($a s = 0$): the unit vector $a$ along the focal displacement $u^0$ and the unit vector $s$, which gives the orientation of the fault. This is the moment-displacement relation derived by Kostrov\[7, 8\] for the slip along a (point-like) fault surface (see also Refs. [2, 4]); it can be called a vectorial, or dyadic, representation of the seismic moment. We note the invariant $M_0 = M_{ii} = 0$, which tells that the seismic moment in this representation is a traceless tensor. This particularity gives access to the near-field waves (equations (2.2)), which become

$$u^n_i = \frac{h_t}{8\pi \rho c^2 R^2} (4M - 9M_4 n) \hspace{1cm} u^n_t = -\frac{3h_t}{8\pi \rho c^2 R^2} (2M - 3M_4 n)$$  \hspace{1cm} (4.4)$$

($M$ and $M_4$ are given by equations (2.5) and (2.6)). In addition, we note the relations $M_0^0 = M_{ij} s_i s_j = 0$ and $M_4^0 = M_{ij} s_j = 2\mu S u^0 a_i$; the former relation shows that the quadratic form associated to the seismic moment in the focal region is degenerate (it is represented by a conic), while the latter relation shows that the "force" in the focal region is directed along the focal displacement; both relations are expected from the Kostrov construction of the tensor of the fault seismic moment (Fig. 4.1). The relations $M_0 = 0$ and $M_4^0 = 0$ reduce the number of independent parameters of the tensor $M_{ij}$ from six to four.

It is worth noting an uncertainty (indeterminacy) of the dyadic construction of the seismic-moment tensor. We can see from equation (4.3) that the seismic moment is invariant under the inter-change $s \leftrightarrow a$. This means that from the knowledge of the seismic moment $M_{ij}$ we cannot distinguish between the two orthogonal vectors $s$ and $a$ (fault direction and fault slip). Another symmetry of the seismic moment given by
Figure 4.1: A fault focal cross-section with area $S$ (dimension $l$, focus $F$); $s$ is the unit vector normal to the fault and $a$ is the unit vector of the focal displacement (in the plane of the fault); the seismic-moment tensor $M_{ij}$ is represented by the rectangular hyperbola with the axes along the vectors $s$ and $a$.

Equation (4.3) is $s \leftrightarrow -a$ (and $s \leftrightarrow -s$, $a \leftrightarrow -a$), which means that we cannot distinguish between the signs of the vectors $s$ and $a$ (as expected from the construction of the seismic moment in equation (4.3)); this uncertainty is shown in Fig. 4.2.

In equation (4.3), the seismic moment is determined by four parameters: three components of the displacement vector $u^0$ and one component of the (transverse) unit vector $s$. By using this vectorial representation, the number of independent parameters of the seismic moment is reduced from six to four. We have, up to this moment, only the three equations (2.5) for these unknown parameters. The considerations made above for the vectorial representation of the seismic moment provides a fourth equation, relating the mechanical work $W$ done in the focal region to the magnitude of the focal displacement.

Indeed, from equation (3.1) the mechanical work in the focal region is given by

$$W = \int dt \int dR \dot{u}_i^0(t) M_{ij}(t) \partial_j \delta(R) ;$$

(4.5)

we may assume $\dot{u}_i^0(t) = \dot{h}(t) u_i^0$, and, since $M_{ij}(t) = M_{ij} h(t)$, we get

$$W = \frac{1}{2} \int dR u_i^0 M_{ij} \partial_j \delta(R) .$$

(4.6)

In this equation we may view the function $\delta(R)$ as corresponding to the shape of the focal surface, such that we may replace $\partial_j \delta(R)$ by $s_j/l^4$; using $V = l^3$ for the focal...
Figure 4.2: Two couples of sliding displacements ($u^0$) and two orthogonal orientations ($s$) in a fault focal region, illustrating the indeterminacy in the Kostrov construction of the seismic moment; $F$ denote the forces which give the torque.

volume, we get $W \simeq \frac{1}{2} u^0_i M_{ij} s^j$. Here, we may take approximately $u^0$ for $l$, which leads to $W \simeq \frac{1}{2} a_i M_{ij} s^j$. Therefore, making use of equation (4.3), we get $W \simeq \mu S u^0 \mu V$; we can see that the mechanical work done in the focal region is of the order of the elastic energy stored in the focal region, as expected. By equating $W$ with energy $E$ (and $\Phi$) given by equation (3.6), the fourth equation

$$
\mu V = \frac{4\pi \rho}{T} R^2 \left( c_l v_l^2 + c_t v_t^2 \right)
$$

(4.7)

is obtained; it can also be written as

$$
V = \frac{4\pi}{c_l^2 T} R^2 \left( c_l v_l^2 + c_t v_t^2 \right).
$$

(4.8)

This equation gives the volume of the focal region in terms of the displacement in the far-field seismic waves (provided duration $T$ is known); the seismic moment given by equation (4.3) can be written as

$$
M_{ij} = 2\mu V (s_i a_j + a_i s_j),
$$

(4.9)

where $V$ can be inserted from equation (4.8). It remains to determine the vectors $a$ and $s$ by using equations (2.5) and the covariance condition, in order to solve completely the inverse problem. We note that the elaborations done in equations (4.1) are, in fact, not necessary, since the torque can be immediately inferred from $t_{ij} = f_i h_j$ by $f_i \simeq 2\mu S u^0_i / l$ and $h_j \simeq l s_j$; we get $t_{ij} \simeq 2\mu V a_i s_j$.

We note here the representation

$$
u^0_{ij} = \frac{1}{2} (s_i a_j + a_i s_j) = \frac{1}{4\mu V} M_{ij}
$$

(4.10)
for the focal strain, which follows immediately from the considerations made above on
the geometry of the focal region. This equation relates the focal strain to the seismic
moment; it may be used for assessing the accumulation rate of the seismic moment
from measurements of the surface strain rate. [23, 24]

It is worth noting that the estimations made above are affected by an order-of-magnitude
error in the numerical factors; this error is related to the parameters $T$, $l$, the estima-
tion of the derivatives $\partial \delta$, the assumption $l_1 = l_2 = d = l$, the volume $V = P^3$, etc.
These errors affect mainly the volume $V$ in equations (4.8) and (4.9). The errors in
the seismic-moment parameters, especially those related to noise, have been analzyed
recently in Ref. [25].

5 Solution of the inverse problem

Making use of the reduced moment $m_{ij} = M_{ij}/2\mu V$ and $m_i = M_i/2\mu V = M_{ij}n_j/2\mu V$, equation (4.9) leads to

$$s_i(na) + a_i(us) = m_i ; \quad (5.1)$$

using equations (2.5) and (4.7) the components $m_i$ of the reduced moment are given by

$$m_i = -\frac{T^2}{2R} \cdot \frac{c_l^3 v_i + c_t^3 v_i}{c_l v_i^2 + c_t v_i^2}. \quad (5.2)$$

We solve here the equations (5.1) for the unit vectors $a$ and $s$, subject to the conditions

$$s_i^2 = a_i^2 = 1, \quad s_i a_i = 0. \quad (5.3)$$

Since $M_0 = 0$ and $M^2 > M_4^2$, we have $m_0 = m_{ii} = 0$ and $m^2 > m_4^2$ (where $m_4 = m_{ij}n_i n_j$
and $m^2 = m_2^2$). From equation (5.2) we have $m_i < 0$. The compatibility condition
$m^2 > m_4^2$ can be checked immediately from equation (5.2) (it arises from $v_i^2 > 0$). We
write equations (5.1) as

$$\alpha s + \beta a = m, \quad (5.4)$$

where we introduce two new notations $\alpha = (na)$ and $\beta = (ns)$. We assume that the
vectors $s$, $a$ and $n$ lie in the same plane, i.e.

$$\beta s + \alpha a = n. \quad (5.5)$$

This condition determines the system of equations and ensures the covariance of the
solution; it is the covariance condition. From equations (5.4) and (5.5) we get

$$2\alpha \beta = m_4, \quad \alpha^2 + \beta^2 = m^2 = 1. \quad (5.6)$$

The equality $m^2 = 1$ (covariance condition) has important consequences; it implies
$M^2 = (2\mu V)^2$, such that we can write the seismic moment from equation (4.9) as

$$M_{ij} = M (s_i a_j + a_i s_j). \quad (5.7)$$

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it follows the magnitude of the seismic moment \((M_{ij}^2)^{1/2} = \sqrt{2}M\); \(M\) is the magnitude of the "projection" of the seismic-moment tensor along the observation radius. In addition, from \(E = W = \mu V\) (equation (4.6)) we have \(E = M/2 = (M_{ij}^2)^{1/2}/2\sqrt{2}.

The magnitude \((M_{ij}^2)^{1/2} = \sqrt{2}M = 2\sqrt{2}E\) may be used in the Gutenberg-Richter relation \(\log(M_{ij}^2)^{1/2} = 1.5M_w + 16.1\), which defines the magnitude \(M_w\) of the earthquake; in terms of the earthquake energy this relation becomes \(\log E = 1.5(M_w - \log 2) + 16.1\) (where \(\log 2 \approx 0.3\)). We note that an error of an order of magnitude in the seismic moment \((M, E, (M_{ij}^2)^{1/2})\) induces an error \(\approx 0.3\) in the magnitude \(M_w\).

Further, from equation (5.2), the equality \(m^2 = 1\) can be written as

\[
\frac{T^4}{4R^2} \cdot \frac{c_i^6 v_i^2 + c_i^6 v_i^2}{(c_i^6 v_i^2 + c_i^6 v_i^2)^2} = 1 ,
\]

(5.8)

which gives the duration \(T\) in terms of the displacements \(v_{l,t}\) measured at distance \(R\). Inserting \(T\) in equation (4.8), we get

\[
V^2 = \frac{8\pi^2 R^3}{c_i^2} (c_i v_i^2 + c_i v_i^2) \left( c_i^6 v_i^2 + c_i^6 v_i^2 \right)^{1/2}
\]

(5.9)

and the magnitude of the seismic moment and the energy of the earthquake

\[
M = 2E = 2\mu V = 2\pi \rho (2R)^{3/2} \left( c_i v_i^2 + c_i v_i^2 \right)^{1/2} \left( c_i^6 v_i^2 + c_i^6 v_i^2 \right)^{1/4}
\]

(5.10)

in terms of the displacements \(v_{l,t}\) measured at distance \(R\). In addition, eliminating \(R^2\) between equations (4.8) and (5.8) we can express the focal volume as

\[
V = \frac{\pi T^3}{c_i^2} \cdot \frac{c_i^6 v_i^2 + c_i^6 v_i^2}{c_i v_i^2 + c_i v_i^2} .
\]

(5.11)

The solutions of the system of equations (5.6) are given by

\[
\alpha = \sqrt{\frac{1 + \sqrt{1 - m_i^2}}{2}} , \quad \beta = \text{sgn}(m_4) \sqrt{\frac{1 - \sqrt{1 - m_4^2}}{2}}
\]

(5.12)

and \(\alpha \leftrightarrow \pm \beta, \alpha, \beta \leftrightarrow -\alpha, -\beta\). Making use of equations (5.2) and (5.8), the parameters \(m_i\) and \(m_4\) are given by

\[
m_i = -\frac{c_i^3 v_i + c_i^3 v_i}{(c_i^6 v_i^2 + c_i^6 v_i^2)^{1/2}} , \quad m_4 = -\frac{c_i^3 (v_i n)}{(c_i^6 v_i^2 + c_i^6 v_i^2)^{1/2}} .
\]

(5.13)

Finally, we get the vectors

\[
s = \frac{\alpha}{\alpha^2 - \beta^2} m - \frac{\beta}{\alpha^2 - \beta^2} n ,
\]

\[
a = -\frac{\beta}{\alpha^2 - \beta^2} m + \frac{\alpha}{\alpha^2 - \beta^2} n ;
\]

(5.14)
from equations (5.4) and (5.5); these solutions are symmetric under the operations
$s \leftrightarrow a$ ($\alpha \leftrightarrow -\beta$) and $s \leftrightarrow -a$ ($\alpha \leftrightarrow \beta$, or $\alpha, \beta \leftrightarrow -\alpha, -\beta$). The seismic
moment given by equation (5.7) is determined up to these symmetry operations. We
can see that the seismic-moment tensor given by equation (5.7) is determined by
$M$ (equation (5.10)) and the vectors $s$ and $a$ given by equations (5.14), with the coefficients
$\alpha, \beta$ given by equations (5.12); the vector $n$ is known and the vector $m$ and the scalar $m_4$
are given by the experimental data (equations (5.13)). Equations (5.14) are manifestly
covariant.

The eigenvalues of the seismic moment given by equation (5.7) are
$\pm M$ (we leave
aside the eigenvalue zero); the corresponding eigenvectors $w$ are given by $aw = \pm sw$,
which imply $mw = \pm nw; \text{the vectors } w \text{ are directed along the bisectrices of the}
angles made by $s$ and $a$, or $m$ and $n$ ($w \sim s \pm a$). The associated quadratic form
$M_{ij}x_ix_j = \text{const}$ is a rectangular hyperbola in the reference frame defined by the vectors
$s$ and $a$; by using the coordinates $u = sx$ and $v = ax$ in equation (5.7), the equation
of this hyperbola is $uv = \text{const}/2M$. Actually, in the local frame (coordinates $x_i$), the
quadratic form $M_{ij}x_ix_j = \text{const}$ is a degenerate hyperboloid, consisting of a family of
parallel hyperbolas displaced along the third axis (perpendicular to the $u$- and $v$-axes).
Making use of equations (5.7) and (5.14), this quadratic form can also be written as
\[
2\xi\eta - m_4 (\xi^2 + \eta^2) = \text{const},\tag{5.15}
\]
where the coordinates $\xi = m_ix_i$ and $\eta = n_ix_i$ are directed along the vectors $m$ and $n$,
respectively. The asymptotics of this hyperbola are $\xi = m_4\eta/\left(1 + \sqrt{1 - m_4^2}\right)$ and $\eta = 
m_4\xi/\left(1 + \sqrt{1 - m_4^2}\right)$ (corresponding to the asymptotics $u = (\alpha\xi - \beta\eta)/(\alpha^2 - \beta^2) = 0$
and \( v = (-\beta \xi + \alpha \eta)/(\alpha^2 - \beta^2) = 0 \). (Fig. 5.1)

Finally, by making use of equations (5.14) in equation (5.7) we get the solution for the seismic moment

\[
M_{ij} = \frac{M}{1 - m_4^2} [m_i n_j + m_j n_i - m_4 (m_i m_j + n_i n_j)] ,
\]

(5.16)

where \( M \) is given by equation (5.10) and \( m_i, m_4 \) are given by equations (5.13); the focal strain is \( u_{ij}^0 = M_{ij}/2M \) (equation (4.10)). In equation (5.16) there are only three independent components of the seismic tensor, according to the equations \( m_{ij} n_j = m_i (m_{ij} = M_{ij}/M) \): the vectors \( n \) and \( m \) are known (equation (5.13)) from experimental data, such that these equations can be viewed as three conditions imposed upon the six components \( M_{ij} \). Also, we can see that there exist only three independent components of the seismic tensor \( M_{ij} \) from the conditions \( M_0 = M_{ii} = 0, M_{ij} s_j s_i = 0 \) (or \( M_{ij} a_i a_j = 0 \)) and \( m_i^2 = 1 \). The later equality arises from the covariance condition, which, together with the energy conservation, determines the duration \( T \) of the earthquake, the volume \( V \) of the focal region and the magnitude parameter \( M \) of the seismic moment.

### 6 Isotropic seismic moment

An isotropic seismic moment \( M_{ij} = -M \delta_{ij} \) is an interesting particular case, since it can be associated with seismic events caused by explosions. In this case the transverse displacement is vanishing (\( u_{ij}^{nf} = 0 \)), \( M = -M n, M_4 = -M \) and \( v_l = (R/c_l T) u_l \) (equations (2.2) and (2.4)); from equations (2.5) and (3.6) we get

\[
M = -4\pi \rho R^3 c_l^2 v_l , \quad E = \frac{4\pi \rho R^2}{T} c_l v_l^2
\]

(6.1)

we can see that \( v_l n > 0 \) corresponds to \( M > 0 \) (explosion), while the case \( v_l n < 0 \) corresponds to an implosion. The focal zone is a sphere with radius of the order \( l \), and the vectors \( s \) and \( a \) are equal (\( s = a \)) and depend on the point on the focal surface; the magnitude of the focal displacement is \( u^0 = l \). The considerations made above for the geometry of the focal region lead to the representation

\[
M_{ij} = -2V (2\mu + \lambda) \delta_{ij} = -2\rho c_l^2 V \delta_{ij} ,
\]

(6.2)

where \( V = Sl \) denotes the focal volume and \( S \) is the area of the focal region (we note that \( t_{ij} \) changes sign in equation (4.2)). Similarly, the energy is \( E = W = \frac{1}{2} M (M > 0) \), such that, making use of equations (6.1), we get \( c_l T = \sqrt{2Rv_l} \),

\[
M = 2\pi \rho c_l^2 (2Rv_l)^{3/2} = 2\rho c_l^2 V ,
\]

(6.3)

and the focal volume \( V = \pi (2Rv_l)^{3/2} \). These equations determine the seismic moment and the volume of the focal region from the displacement \( v_l \) measured at distance \( R \). A superposition of shear faulting and isotropic focal mechanisms cannot be resolved, because the longitudinal displacement \( v_l \) includes indiscriminately contributions from both mechanisms.
7 Discussion and concluding remarks

We can summarize the results as follows. Making use of the longitudinal displacement $v_l$ and the transverse displacement $v_t$, measured at the Earth’s surface, we compute the magnitude parameter $M$ from equation (5.10) and the vector $m$ and the scalar $m_4$ from equation (5.13); then, from equation (5.16) we get the seismic moment $M_{ij}$. The energy released by the earthquake is $E = M/2$ and an estimate of the focal volume is given by $V = M/2\rho c^2$ (equations (4.9) and (5.7)). An estimation of the duration $T$ of the earthquake is provided by equation (5.8). The focal slip is of the order $V^{1/3}$ and the focal strain is of the order $M_{ij}/2M$ (equation (1.10)). From the magnitude $(M_{ij}^2)^{1/2} = \sqrt{2M}$ of the seismic moment we may estimate the magnitude $M_w$ of the earthquake by means of the Gutenberg-Richter relation. A similar procedure holds for an isotropic seismic moment (preceding section).

Making use of $m$ and $m_4$ in equations (5.14) we compute the normal $s$ to the fault plane and the unit slip vector $a$ in the fault plane; the quadratic form associated to the seismic moment is a degenerate hyperboloid which reduces to a hyperbola in the $(s, a)$-plane with asymptotics along the vectors $s$ and $a$. This hyperbola is tighter (closer to the origin) for higher $M$.

It is convenient to have an estimation of the order of magnitude of the various quantities introduced in this paper. To this end we use a generic velocity $c$ for the seismic waves and a generic vector $v$ for the displacement in the far-field seismic waves. Equation (5.8) (which is $m^2 = 1$) gives $cT \simeq \sqrt{2Rv}$, which provides an estimate of the duration of the earthquake in terms of the displacement measured at distance $R$. The focal volume can be estimated from equation (4.8) as $V \simeq \pi (2Rv)^{3/2} \simeq \pi (cT)^3$, as expected (dimension $l$ of the focal region of the order $cT$; the rate of the focal slip is $l/T \simeq c$). Also, from equation (5.10) we have the energy $E \simeq \mu V \simeq M/2 \simeq 2\rho c^2 V$, where $M$ is related to the magnitude $(M_{ij}^2)^{1/2} = \sqrt{2M}$ of the seismic moment (and the magnitude of the vector $M_{ij}n_j$). From equation (1.10) we get a focal strain of the order unity, as expected.

In conclusion, it is shown in this paper that the displacement in the far-field seismic waves provides information about the structure of the focal region; in particular, this displacement can be employed to determine the seismic-moment tensor for a fault slip, localized both in space and time (the inverse problem in Seismology). In this case the vectorial (Kostrov) representation of the seismic moment (dyadic representation) is written with four (unknown) parameters; one is the magnitude of the focal displacement, while the other three define the spatial orientation of the seismic tensor (orientation of the fault and the displacement direction). These unknown parameters are determined from the three equations relating the far-field displacement to the seismic tensor and the equation which relates the energy released in the earthquake (and carried by the seismic waves) to the focal displacement (and the fault focal volume), via the mechanical work done in the focal region, together with the covariance condition.
The solution of the resulting system of equations makes the graphical representation of the quadratic form associated to the seismic-moment tensor, which is a hyperbola, to offer a (three-dimensional) image of the focal region. The asymptotics of the hyperbola give the direction of the focal displacement and the orientation of the fault (seismic hyperbola). Besides solving the inverse problem in Seismology for a localized fault slip, the geometry of the fault focal region (which leads to Kostrov representation) and the displacement in the far-field seismic waves provide reasonable estimations of the fault focal volume, focal strain, duration and energy of the earthquake and magnitude of the seismic moment. Also, the special case of an isotropic seismic moment is presented. More complex situations, like a superposition of point-like faults, or a combination of point-like faults and isotropic and dipole components imply more than four unknowns in the seismic tensor; since we have only four equations, the inverse problem in such cases is undetermined, within the present procedure. The procedure presented in this work makes use of manifestly covariant expressions of the data for determining the seismic moment.

Finally, we note that a similar deduction of the seismic-moment tensor can be done by using the (quasi)-static displacement at Earth’s surface, derived in Ref. [28]; since it implies a specific treatment, its presentation is deferred to a forthcoming publication.

Acknowledgments. The author is indebted to his colleagues in the Department of Engineering Seismology, Institute of Earth’s Physics, Magurele-Bucharest, for many enlightening discussions, and to the members of the Laboratory of Theoretical Physics at Magurele-Bucharest for many useful discussions and a throughout checking of this work. This work was partially supported by the Romanian Government Research Grant #PN16-35-01-07/11.03.2016.

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