From infancy to middle-adolescence nonlinear physical growth in low- and middle-income countries

Senahara Korsa Wake*, Temesgen Zewotir² and Essey Kebede Muluneh³

¹College of science, Bahir Dar University, Bahir Dar, Ethiopia
²School of Mathematics, Statistics and Computer Science, University of KwaZulu-Natal, Durban, South Africa
*Correspondence: senahara2004@gmail.com

Abstract

Background: Modeling the growth curve of height has a significant role in understanding the growth trajectories over time and generated mathematical functions that depict the expected height of children at a particular age. However, modeling the mathematical growth functions for physical height is not well studied in low- and middle-income countries. Modeling and identifying nonlinear growth curves that adequately describe the growth trajectories in low- and middle-income countries were the aims of this study.

Methods: The data were obtained from the Young Lives study. Longitudinal measures of height from infancy to middle-adolescence were collected from low- and middle-income countries. A number of nonlinear growth trajectories were studied through the family of three-parameter nonlinear mixed-effects models.

Results: This study examined the performances of different growth curves for the height growth trajectories. The Logistic curve was chosen among the three-parameter nonlinear growth curves for modeling the growth trajectories from infancy to middle-adolescence. Gender and country have significant effects on the three parameters of growth curves. Males had higher asymptotic height and a lower rate of growth than females. Females reached asymptotic height earlier and shorter at asymptotic height than males. Children with low asymptotic height grow faster than those with higher asymptotic height. Compared to Ethiopian children, Indian and Peruvian children had lower asymptotic height, but Vietnamese children had higher asymptotic height. Ethiopian children approached adult height earlier than Indian children, but later than Peruvian children. However, there was no significant difference in the rate of growth between Ethiopian and Vietnamese children.
Conclusions: This study concludes that the Logistic growth curve was found to be the best growth curve to describe the height growth trajectories. Children in Ethiopia, India, Peru and Vietnam showed different growth parameters. Further enhancements may be attained with the incorporation of other plausible covariates.

Keywords: Asymptotic height, Covariance structures, Growth curves, Mixed-effect model, Rate of growth

Background
Analysis of longitudinal growth curves has a significant role in understanding and modeling the growth trajectories of children. The motivation to study the growth of children is to adequately describe the basic biological process of physical growth and monitoring their nutritional status, cognitive development, and health outcomes. Repeated measures observed on the same outcomes over time are the starting points for growth curves [1,2]. To analyze such design, special statistical approaches are required. A longitudinal study offers a more realistic view of growth patterns at individual and group levels. In longitudinal studies, individuals are observed repeatedly on the same outcome over time [3,4]. Observations measured repeatedly on the same outcome at multiple occasions tend to be inter-correlated. This correlation must be taken into account in the analysis. However, ignoring the existing correlation of longitudinal data may lead to incorrect and inefficient inferences. Thus, a key requirement for longitudinal data analysis is to appropriately model and accurately estimate the variance components so that the underlying mean and individual functions can be efficiently modeled [3, 5].

Mixed-effects models are the most widely used and flexible classes of models for correlated data that describe the dependence of the response variable on a set of covariates based on a regression paradigm. It relaxes the independence assumption of conventional analyses (regression and analysis of variance). It also takes into account a more complicated data structure in a flexible way. Additionally, random effects are introduced to incorporate the between-individual variation and within-individual correlation in the data [4, 6, 7]. This study, therefore, uses mixed-effects techniques to assess the growth trajectories of physical height. The linear growth curve is usually a sufficient model where the process under study is measured within limited time spans. However, for a long span time measurement, the process is likely to exhibit some degree of nonlinearity. In this case, a nonlinear growth curve would be applicable to handle the complexity
reflected in the individual trajectories [8]. The physical growth of children from infancy to adulthood follows nonlinear growth patterns [9, 10]. Thus, nonlinear growth curve offers convenient and flexible techniques in modeling the current growth data. Nonlinear mixed-effects model is a generalized form of the linear mixed-effects model in which the functional dependence of the mean outcome on covariate is nonlinear. As a consequence, the nonlinear model provides better predictions outside the range of observed data, and its parameters usually have natural physical interpretations. Moreover, nonlinear model uses a small number of parameters than that of the linear model [7].

The physical growth of children can be affected by countries' exposures. Socioeconomic differences in physical growth are frequently observed with shorter height in lower socioeconomic groups [11]. Low- and middle-income countries are characterized by huge socioeconomic inequality. This indicates the significant differences in physical growth between low- and middle-income countries [12]. The main focus of this study is therefore building the nonlinear growth curves for the physical growth of children from infancy to 15 years of age.

**Methods**

**Data source**

The data were obtained from the Young Lives study. The Young Lives study examines the changing nature of childhood poverty and health in Ethiopia, India, Peru and Vietnam. The study followed children from infancy to middle-adolescence in two cohort studies, the older and younger cohorts. The older cohort includes children born before the millennium development goals and the younger cohort includes children born after the millennium development goals. For this study, data only from children growing up with the promise of the millennium development goals were used. The anthropometric measurements were collected by rounds every three/four years over a time of 15 years. The first round was conducted in 2002 when children on average were one year old. Subsequently, round two was conducted in 2006, round three in 2009, round four in 2013 and round five in 2016 [13].
Nonlinear mixed-effects model and growth curves

A nonlinear mixed-effects model that follows a specified nonlinear function is used to analyze the change of outcomes over time that commonly follows a nonlinear pattern. The model allows fixed and random effects to enter a model nonlinearly [14]. Nonlinear mixed-effects model for repeated measurements has two stages. The first stage is the mean and covariance structure for a given individual and the second stage is the between-individual variations. A general form of nonlinear mixed-effects model [7] for the j-th response on the i-th individual can be expressed as:

\[ y_{ij} = f(X_{ij}, \beta_i) + \varepsilon_{ij}, \ i = 1,2, \ldots, m \ \& \ j = 1,2, \ldots, n_i \]

where, \( y_{ij} \) is the j-th repeated observation on the i-th individual, \( f(\cdot) \) is a known nonlinear function of a subject-specific parameter \( \beta_i \), \( X \) is the matrix of predictors and \( \varepsilon_{ij} \) is a normally distributed error term. From the second stage, \( d(\cdot) \) is a known function of the design matrices for \( A_i \) fixed effects, \( B_i \) random effects, the vector of fixed population parameters \( \beta \) and the vector of random effects \( b_i \). For instance, a simple linear model for \( \beta_i \) can be written as \( \beta_i = A_i \beta + B_i b_i, i = 1,2, \ldots, m \). The random effects and error terms are identically and normally distributed with mean zero and the variance-covariance, i.e., \( b_i \sim N(0, D), \varepsilon_{ij} \sim N(0, R_i) \), \( b \) and \( \varepsilon \) assumed to be independent. Equations (1) and (2) are the first and second stages of the model, respectively.

The intra-individual and inter-individual variations are separately quantified by the variance components of random effects, \( D \), and error term, \( R_i \). The covariance matrix \( D \) measures the inter-individual (between-individual) variation that is not explained by covariates whereas the covariance matrix \( R_i \) measures the within-individual variation.

The physical growth of children has an evident asymptote. Polynomial and other parsimonious linear models are unsuitable when the mean response asymptotes to an upper or lower bound. For such situations, a nonlinear model is required to fit well the data. The most popular growth curves with an asymptote are the three-parameter growth curves [15]. For this study, the logistic, Von Bertalanffy, Brody and Gompertz growth curves are considered in modeling the growth trajectories of children from infancy to middle-adolescence [16]. The nonlinear mixed-effects model for each growth curve is given as follows.
Logistic curve: \( y_{ij} = \frac{(\beta_1 + b_{1i})}{(1 + (\beta_2 + b_{2i}) \exp(-(\beta_3 + b_{3i}) t_{ij}))} + \varepsilon_{ij} \)

Brody curve: \( y_{ij} = (\beta_1 + b_{1i}) \left(1 - (\beta_2 + b_{2i}) \exp(-(\beta_3 + b_{3i}) t_{ij}))\right) + \varepsilon_{ij} \)

Gompertz curve: \( y_{ij} = (\beta_1 + b_{1i}) \exp \left(-(\beta_2 + b_{2i}) \exp(-(\beta_3 + b_{3i}) t_{ij}))\right) + \varepsilon_{ij} \)

Von Bertalanffy curve: \( y_{ij} = (\beta_1 + b_{1i}) \left(1 - (\beta_2 + b_{2i}) \exp(-(\beta_3 + b_{3i}) t_{ij}))\right)^3 + \varepsilon_{ij} \)

In all models presented, \( y \) stands for the physical height of children at age \( t \), \( \beta_1 \) stands for the asymptotic height, \( \beta_2 \) is the value predicted at \( t = 0 \) and \( \beta_3 \) is a constant related to the postnatal rate of growth that means the rate at which child growth approaches asymptotic, \( b_{1i}, b_{2i} \) and \( b_{3i} \) are the random effects associated with the three growth parameters (\( \beta_1, \beta_2 \) and \( \beta_3 \)) that assumed to be independent and identically distributed with mean zero and variance-covariance matrix \( D \) and \( \varepsilon_{ij} \) are the errors assumed to be independent and identically distributed with mean zero and variance-covariance matrix \( R_i \).

Maximum likelihood and restricted maximum likelihood are the two methods for estimating the parameters in nonlinear mixed-effects model. The complex numerical issue for these estimations is the evaluation of the log-likelihood function of the longitudinal data. This could be due to the log-likelihood function comprises the evaluation of several integral that in most cases does not have a closed-form expression. To tackle the difficulty of maximizing log-likelihood in nonlinear mixed-effects model, several approximations to the log-likelihood are available [17]. Alternating approximation method [18], Laplacian approximation [19], importance sampling [20] and Gaussian quadrature [21] are some of the integral approximation methods. The parameter estimating procedures for this model is generated in SAS PROC NLMIXED by maximizing an approximate integrated likelihood. The performances of growth curves were determined based on Akaike’s information criteria (AIC) and Schwarz’s Bayesian information criteria (BIC) [22].
Results

Exploratory data analysis

The descriptive statistics of the physical height of children by gender and country are given in Table 1. The mean height is increased with age. The mean height of males is higher than females at ages 1, 5, 8, and 15, and females had a higher mean height at age 12 years in all countries. The patterns of mean height growth by gender and country are displayed in Figures 1 and 2, respectively. These figures confirmed that the growth trajectories of children in low- and middle-income countries follow nonlinear trends.

Table 1 Descriptive statistics of height for males and females

| Age (year) | Country | Gender | Mean   | SD    | Minimum | Maximum |
|------------|---------|--------|--------|-------|---------|---------|
| 1          | Ethiopia| Male   | 72.03  | 5.19  | 55.3    | 89.5    |
| 1          | Ethiopia| Female | 70.67  | 5.4   | 56      | 89.5    |
| 1          | India   | Male   | 72.86  | 4.75  | 60.4    | 89.4    |
| 1          | India   | Female | 71.36  | 4.74  | 58.4    | 86.2    |
| 1          | Peru    | Male   | 72.57  | 4.34  | 59      | 87.5    |
| 1          | Peru    | Female | 70.99  | 4.57  | 59.5    | 89.6    |
| 1          | Vietnam | Male   | 73.31  | 4.17  | 62.2    | 85.7    |
| 1          | Vietnam | Female | 71.49  | 4.02  | 57.2    | 83.7    |
| 5          | Ethiopia| Male   | 104.96 | 4.86  | 90.8    | 124.9   |
| 5          | Ethiopia| Female | 103.94 | 5.27  | 89      | 120.4   |
| 5          | India   | Male   | 104.86 | 4.46  | 84.1    | 121     |
| 5          | India   | Female | 104.41 | 4.64  | 90      | 120.3   |
| 5          | Peru    | Male   | 105.53 | 5.76  | 89.6    | 123.55  |
| 5          | Peru    | Female | 104.26 | 5.82  | 84.5    | 123.2   |
| 5          | Vietnam | Male   | 105.91 | 5.12  | 90.2    | 122.65  |
| 5          | Vietnam | Female | 104.71 | 4.55  | 90      | 120.05  |
| 8          | Ethiopia| Male   | 121.75 | 5.55  | 102     | 141     |
| 8          | Ethiopia| Female | 120.92 | 5.92  | 100     | 136.05  |
| 8          | India   | Male   | 119.77 | 5.29  | 101.8   | 138     |
| 8          | India   | Female | 119.17 | 5.69  | 102.5   | 140.4   |
| 8          | Peru    | Male   | 121.19 | 5.49  | 102.5   | 139.5   |
| 8          | Peru    | Female | 120.37 | 5.59  | 101.2   | 140.8   |
| 8          | Vietnam | Male   | 121.87 | 5.88  | 101.3   | 139     |
| 8          | Vietnam | Female | 121.2  | 5.65  | 99      | 144.5   |
| 12         | Ethiopia| Male   | 141.02 | 5.91  | 123     | 161.5   |
| 12         | Ethiopia| Female | 142.61 | 7.13  | 120     | 166     |
| Country | Male   | Female  | Male   | Female  |
|---------|--------|---------|--------|---------|
| India   | 158.94 | 155.16  | 138.5  | 171.2   |
| Peru    | 161.65 | 153.47  | 140.4  | 183.1   |
| Vietnam | 164.04 | 155.69  | 131.2  | 175.0   |

The loess smooth curves presented in Figures 1 and 2 are important in understanding the functional relationship between the mean height and time (child’s age). From these plots, it can be seen that the relationship between height and time is not linear. Therefore, a nonlinear growth curve is a reasonable curve to model the growth trajectories. The initial values of the growth parameters (asymptotic height, scale parameter and rate of growth) in the models can be obtained from the visual inspection of these profile plots and from fitting the simple nonlinear curves which do not account for the nature of longitudinal measures data [8].
Figure 1 Mean profile plot of height growth by countries

Figure 2 Mean profile plot of height growth by gender
Fitted nonlinear mixed-effects growth curves

Different growth curves have been applied to describe the physical growth and age relationship for children living in different socioeconomic statuses. A growth curve is a powerful tool in modeling the growth trajectories and gives a biological meaning for parameters in the models. Four growth curves (Logistic, Gompertz, Brody and Von Bertalanffy) in the context of nonlinear mixed-effects model are fitted, and their performances were compared for the height growth from infancy to middle-adolescence. The two components of nonlinear mixed-effects growth curves, the fixed and random effects, were considered in the growth modeling process. The fixed effects signify the average height for all the subjects and the random effects denote the variability of the individual patterns around the population averages.

Before the comparison of models for growth trajectories, it is necessary to examine which growth curve parameters in the model can include a random effect. Hence, we assessed if any of the three parameters ($\beta_1$, $\beta_2$ and $\beta_3$) of the growth curves varied across the individuals. To select the proper growth curve for growth trajectories, the researchers started with the models that have three random effects and reduced the number of random effects hierarchically to the model where only one parameter is random. The presence of random effects was determined by comparing the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) for non-nested models and using the likelihood ratio tests for the nested models [23,24]. Lastly, including the random effects associated with $\beta_1$ and $\beta_3$ growth parameters in the model had improved the fitting performance of the growth curves. Table 2 summarizes the estimated parameters and the goodness of fit statistics of the three-parameter growth curves. From this Table, it can be seen that the Gompertz curve revealed the smallest values of AIC = 200681 and BIC = 200729. While Von Bertalanffy provided the highest values of AIC = 310986 and BIC = 311033 compared to the other growth curves. However, all the growth curves applied in this study provided different values of growth parameters (the asymptote, scale parameter and growth rate).
The Brody and Gompertz curves had the highest estimate of asymptotic height, on average, 237.25 and 196.00, respectively. While the lowest asymptotic height, 119.58 and 180.27, are recorded by the Von Bertalanffy and Logistic curves, respectively (Table 2). Even though the Gompertz curve exhibited the smallest information criterions of AIC and BIC compared to the other curves, the mean asymptotic height provided by this model is less practical under biological expectations and does not determine the value related to adult height properly. This indicates that less support for the Gompertz curve to be a better fit than the other curves. However, regarding the residual distribution of the Gompertz curve, a similar trend is observed with that of the Logistic curve (Figure 3). This suggests that the Logistic and Gompertz curves can achieve the growth trajectories well. In addition to using information criterions for model selection, the way to choose a suitable growth curve is to work with a theoretical framework and balance it with biologically relevant parameters [8]. We therefore prefer the Logistic curve approach in which its growth parameters have physical meaning and biologically interpretable.

The Von Bertalanffy curve is the worst model for the growth trajectories as it had the highest information criterions.

Once a proper growth curve is chosen, evaluation of the nonlinear growth trajectory with the effects of covariates is the next work. To examine the dependence of the growth parameters on gender and country, the effects of gender and country on each fixed effect were evaluated and both covariates have significant effects on the three growth parameters. The parameter estimates of the final best fit model with covariate effects added to all three parameters of the growth curve

| Parameter/fit-statistics | Logistic | Gompertz | Brody | Von Bertalanffy |
|-------------------------|----------|----------|-------|-----------------|
| Asymptote               | 180.27   | 196.00   | 237.25| 119.58          |
| Scale parameter         | 1.7182   | 1.1011   | 0.7312| -210.47         |
| Rate of growth          | 0.1609   | 0.1059   | 0.0513| 273.08          |
| AIC                     | 202542   | 200681   | 206965| 310986          |
| BIC                     | 202589   | 200729   | 207013| 311033          |
are presented in Table 3. The fitted marginal Logistic curve is described by the following function.

\[ \text{Height} = \frac{A}{1 + B \exp(-Ct_{ij})} \]

Where, 

\[ A = \beta_1 + \beta_{11}\text{gender} + \beta_{1k}\text{country} \]
\[ B = \beta_2 + \beta_{21}\text{gender} + \beta_{2k}\text{country} \]
\[ C = \beta_3 + \beta_{31}\text{gender} + \beta_{3k}\text{country} \]

k is a dummy variable that represents the levels of the country (Ethiopia, India, Peru and Vietnam).

**Table 3** Estimates of the best fit mixed-effects growth curve with the effects of gender and country on the three growth parameters

| Parameter                  | Estimate | SE    | t-value | p-value | 95% CI       |
|----------------------------|----------|-------|---------|---------|--------------|
| Asymptote (\(\beta_1\))   | 171.78   | 0.2356| 728.99  | <.0001  | 171.32-172.24|
| Gender (reference category = Female) |          |       |         |         |              |
| Male asymptote (slope)     | 17.8564  | 0.5189| 34.41   | <.0001  | 16.8393-18.8736|
| Country (reference category = Ethiopia) |        |       |         |         |              |
| India asymptote (slope)    | -1.4132  | 0.1898| -7.45   | <.0001  | -1.7853-1.0412|
| Peru asymptote (slope)     | -0.4786  | 0.0829| -5.78   | <.0001  | -0.641-0.3162|
| Vietnam asymptote (slope)  | 2.5338   | 0.1975| 12.83   | <.0001  | 2.1467-2.9208|
| Scale parameter (\(\beta_2\))| 1.6662   | 0.0048| 350.21  | <.0001  | 1.6568-1.6755|
| Male scale parameter (slope) | 0.1355  | 0.0060| 22.57   | <.0001  | 0.1237-0.1473|
| India scale parameter (slope) | -0.043  | 0.0057| -7.58   | <.0001  | -0.0541-0.0319|
| Peru scale parameter (slope) | 0.0017  | 0.0056| 0.3     | 0.7631  | -0.0092-0.0125|
| Vietnam scale parameter (slope) | 0.0240  | 0.0055| 4.4     | <.0001  | 0.0133-0.0348|
| Rate of growth (\(\beta_3\))  | 0.1768   | 0.0007| 248.17  | <.0001  | 0.1754-0.1782|
| Male rate of growth (slope) | -0.0307  | 0.0009| -33.48  | <.0001  | -0.0326-0.0289|
| India rate of growth (slope) | -0.0020  | 0.0007| -3.08   | 0.0021  | -0.0033-0.0007|
| Peru rate of growth (slope) | 0.0036   | 0.0006| 5.73    | <.0001  | 0.0024-0.0048|
The growth parameters ($\beta_1, \beta_2$ and $\beta_3$) defining the growth curve are varied between gender and among countries (Table 3). To understand the difference of growth parameters among four low- and middle-income countries as well as between males and females, countries and gender are considered as fixed effects on each parameter of the growth curve. Additionally, the asymptotic height and rate of growth are varied across individuals and they are treated as random effects in modeling the growth trajectories. The parameters of the Logistic growth curve are biologically interpretable and have meaningful. The estimate of the asymptotic parameter, $\beta_1$, in the growth curve indicates the asymptotic or adult height and $\beta_2$ and $\beta_3$ are the scale parameter and rate of growth, respectively. The values of rate of growth show that how fast the children approach the adult height. Children with a higher value of the rate of growth reached asymptotic height early compared to those with a lower value of the rate of growth.

Accordingly, the estimated parameters determined by the Logistic growth curve for females are $\beta_1 = 171.78$, $\beta_2 = 1.666$ and $\beta_3 = 0.177$. The asymptotic height indicates on average the maximum adult height for females is 171.78. The value related to scale parameter $\beta_2$ represents the ratio of height gained and the value related to the parameter $\beta_3$ is the rate of change shows how fast females approach the asymptotic height. Similarly, for males $\beta_1 = 189.63$, $\beta_2 = 1.802$ and $\beta_3 = 0.146$. Male has significant positive slopes for asymptote and scale parameters but has a significant negative slope for the rate of growth. These indicate that males have higher asymptotic height and scale parameters than females. This is due to the structural and physical differences between males and females [25]. The rate of growth for males is lower than that of females, suggesting females reached adult height earlier than males.

The country has a significant effect on the three parameters of the growth curve, indicating that growth trajectories differ among low- and middle-income countries. For instance, the estimated asymptote slopes reported for India, Peru and Vietnam are -1.413, -0.479 and 2.534, respectively. These values are the estimated asymptotes difference between the asymptote of the reference group (Ethiopia) and the corresponding countries. India and Peru have significant negative slopes for the asymptote, while Vietnam has a significant positive slope for the asymptote. The negative asymptote slopes for India and Peru show that children in India and
Peru have a lower asymptotic height compared with the asymptotic height of children in Ethiopia. The positive asymptote slope for Vietnam shows children in Vietnam have higher asymptotic height compared with the asymptotic height of children in Ethiopia. Regarding the rate of growth, both Peru and Vietnam have positive slopes, while India has a negative slope. The mean rate of change for Ethiopian children is higher than that of Indian children and lower than that of Peruvian children. However, for Vietnam, the slope is not statistically significant. Children in Ethiopia approached adult height earlier than children in India, but later than children in Peru.

**Figure 3** Trends of residual distributions for Logistic (the left panel) and Gompertz (the right panel) growth curves

The random effects associated with the growth parameters were selected by fitting distinct models and comparing their information criterion statistics or using likelihood ratio tests. Children are varied across their asymptotic height and rate of change. The estimated variance-covariance matrix is given as follows.

\[
\hat{D} = \begin{bmatrix}
0.0555 & -0.0001 \\
-0.0001 & 5.08E-7
\end{bmatrix}
\]

The variance related to the asymptotic height \((\sigma^2_{b1} = 0.0555)\) implies that children showed variations in their asymptotic height level. Furthermore, the variance related to the parameter of the rate of growth is \((\sigma^2_{b3} = 5.08E-7)\), showing that the variation of rate of growth between-individual growth trajectories. The negative estimate of the covariance between asymptotic
height and rate of growth \( (\sigma_{u_1u_3} = -0.0001) \) indicates that children with lower asymptotic height grow faster than those with higher asymptotic height. The biological correlation between asymptote and the rate of growth is most important in the growth curve [25].

**Discussion**

This study was investigated various growth curve approaches for modeling the growth trajectories from infancy to middle-adolescence in low- and middle-income countries. Our concern is to fit the three-parameter growth curve that adequately describes the growth trajectories. The three-parameter growth curve is a flexible curve for comparing and summarizing physical growth. However, growth curves with many parameters may lead to model fitting challenges [26]. A nonlinear growth curve attempt to estimates between-individual variations in within-individual change. It is more sophisticated for longitudinal data that follow nonlinear time trends [27]. Therefore, the families of three-parameter nonlinear growth curves in the context of the mixed-effects model were fitted to analyze the growth trajectories. The profile plots presented in Figures 1 and 2 confirmed the trends of height growth are curvilinear. Hence, nonlinear function is a reasonable function to model the growth trajectories for the current data.

The three-parameter growth curves such as Brody, Von Bertalanffy, Gompertz and Logistic curves have been widely and frequently used for modeling animal and forest growth [22, 23, 27, 28]. In this study, we introduced these growth curves for modeling the physical growth from infancy to middle-adolescence. Comparisons of models’ goodness of fit and selection procedures were carried out according to the goodness of fit indicators, residual distribution of the models and biologically meaningful growth parameters. Besides comparing the growth curves that best fit the height growth using the goodness of fit indicators, considering the biological expectation of the growth parameters is also helpful [8]. The plots of residuals against age for both the Gompertz and Logistic curves have similar trends with no strong association over time (Figure 3). This indicates both curves can achieve the growth trajectories well. However, the Gompertz curve overestimated the asymptotic height. The mean adult height provided by the Logistic curve is biological the expected mean adult height. Therefore, for the current data, the Logistic growth curve was preferred to model the growth trajectories. Lampl M., [30] noted that the Logistic and Gompertz curves are the most common mathematical functions used to model human growth as a function of age.
The Von Bertalanffy curve is the worst function for the current data. Ahmadi and colleagues [31] were compared three growth curves, the Jenss, the Reed and the Gompertz curve, to the height of children from birth to age six years and reported the Gompertz curve did not fit well for both males and females. However, they did not include the other three-parameter growth curves rather than the Gompertz curve.

In order to capture between- and within-individual growth patterns, the fixed and random effects were considered in the Logistic growth curve. The fixed effects in the model evaluate the mean height growth of all subjects, whereas the random effects in the model evaluate the variation between the individual trajectories. Children are varied across their asymptotic height and rate of growth. The effects of gender and country on the three parameters of the Logistic curve were analyzed. The model provided that the inclusion of covariates in the growth modeling process considerably reduced the values of fit statistics. Both covariates have significant effects on all growth parameters. The parameters in the growth curves are biologically interpretable [22,32]. The estimate of the asymptotic height parameter in the models indicates the mean adult height. Children with a higher value of $\beta_3$ parameter reached asymptotic height early compared to those with a lower value of $\beta_3$ parameter.

Males had significantly higher asymptotic height and scale parameters but had a lower rate of growth than females. Females reached the asymptotic height faster than males. The mean height of children at the end of the growth stage in Ethiopia, India, Peru and Vietnam is 171.78, 170.37, 171.30 and 174.31, respectively. India and Peru have significantly negative slopes for asymptote, while Vietnam has a positive slope. The mean asymptotic height of children in Ethiopia is higher than the mean asymptotic height of children in India and Peru, but lower than the mean asymptotic height of children in Vietnam. Children in Ethiopia reached the adult height faster than children in India, but lower than children in Peru. The differences in the growth trajectories among these countries could be due to their socioeconomic differences [33].

**Conclusions**

The study examined the performances of different three-parameter growth curves for the growth trajectories of children in Ethiopia, India, Peru and Vietnam. The nonlinear Logistic curve was a better fitting curve for modeling the growth trajectories. Gender and country have significant
effects on the three-parameter of the curve. Children in Ethiopia, India, Peru and Vietnam have different values of growth parameters. Further enhancements may be attained with the inclusion of other potential covariates.

**Acknowledgments**

We, thank Young Lives study for giving us access to the data files.

**Availability of data and materials**

The datasets analyzed during the current study are available in the Young Lives study repository, [http://www.younglives.org.uk/](http://www.younglives.org.uk/).

**Authors’ contributions**

SKW analyzed and drafted this manuscript. TZ, as a principal advisor for SKW, designed, advised and supervised the start, the analysis, and the write-up of the manuscript. EKM, advised and supervised the analysis and the write-up of the manuscript. All authors have read and approved the final manuscript.

**Funding**

None

**Ethics approval and consent to participant**

Not applicable

**Consent for publication**

Not applicable

**Competing interests**

The authors declare that they have no competing interests.

**Authors’ information**

1College of Science, Bahir Dar University, Bahir Dar, Ethiopia

2School of Mathematics, Statistics and Computer Science, University of KwaZulu-Natal, Durban, South Africa

**Reference**
1. Bollen K, Curran P. Latent curve models: A structural equation perspective. John Wiley Sons. 2006;
2. Hauspie RC, Cameron N, Molinari L (Eds). Methods in Human Growth Research. Cambridge University Press.; 2004.
3. Diggle PJ, Heagerty P, Liang KY, Leger SL. Analysis of Longitudinal Data. 2nd ed. Oxford University Press; 2002.
4. Hedeker D, Gibbons, RD. Longitudinal data analysis. John Wiley & Sons; 2006.
5. Wu H, Zhang JT. Nonparametric regression methods for longitudinal data analysis: mixed-effects modeling approaches. John Wiley & Sons; 2006.
6. Helen B, Robin P. Applied Mixed Models in Medicine. Second Edi. John Wiley & Sons. 2006. 551–659 p.
7. Wu L. Mixed effects models for complex data. CRC Press; 2009.
8. Grimm KJ, Ram N, Estabrook R. Growth Modeling: Structural Equation and Multilevel Modeling Approaches. Methodology in the Social Sciences. Guilford Publications; 2016.
9. Cossio-Bolaños M, Campos RG, Andruske CL, Flores AV, Luarte-Rocha C, Olivares PR, et al. Physical growth, biological age, and nutritional transitions of adolescents living at moderate altitudes in Peru. Int J Environ Res Public Health. 2015;12(10):12082–94.
10. Zong X, Li H. Physical growth of children and adolescents in China over the past 35 years. Bull World Health Organ. 2014;92:555–64.
11. Bann D, Johnson W, Li L, Kuh D, Hardy R. Socioeconomic inequalities in childhood and adolescent body-mass index, weight, and height from 1953 to 2015: an analysis of four longitudinal, observational, British birth cohort studies. Lancet Public Heal. 2018;3(4):e194–203.
12. Mansukoski L, Johnson W, Brooke-wavell K, Galvez-sobral JA, Furlán L, Cole TJ, et al. Four decades of socio-economic inequality and secular change in the physical growth of Guatemalans. Public Health Nutr. 2020;23(8):1381–91.
13. Jeffery K, Chatterjee I, Lavin T, Li IW. Young lives and wealthy minds: The nexus between household consumption capacity and childhood cognitive ability. Econ Anal Policy. 2020;65:89–104.
14. Smith N, Blozis S. Options in Estimating Nonlinear Mixed Models : Quadrature Points and Approximation Methods. West Users SAS Softw. 2015;
15. Zimmerman D, Núñez-Antón V, Gregoire T, Schabenberger O, Hart J, Kenward M, et al. Parametric modelling of growth curve data: An overview. 2001;10(1):1–73.

16. Koya PR, Goshu AT. Generalized Mathematical Model for Biological Growths. Open J Model Simul. 2013;01(04):42–53.

17. Pinheiro J, Bates D. Approximations to the log-likelihood function in the nonlinear mixed-effects model. J Comput Graph Stat. 1995;4(1):12–35.

18. Lindstrom M, Bates D. Nonlinear Mixed Effects Models for Repeated Measures Data. Biometrics. 1990;46:673–87.

19. Tierney L, Kadane JB. Accurate approximations for posterior moments and marginal densities. J Am Stat Assoc. 1986;81(393):82–6.

20. Geweke BYJ. Bayesian Inference in Econometric Models Using Monte Carlo Integration. Econom Soc. 1989;57:1317–39.

21. Davidian M, Gallant AR. Smooth nonparametric maximum likelihood estimation for population pharmacokinetics, with application to quinidine. J Pharmacokinet Biopharm. 1992;20(5):529–56.

22. Sariyel V, Aygun A, Keskin I. Comparison of growth curve models in partridge. Poult Sci. 2017;96(6):1635–40.

23. Melesse SF, Zewotir T. Fitting three parameter growth curves using a nonlinear mixed effects modelling approach. South African Stat J. 2015;49(2):233–40.

24. Do DN, Miar Y. Evaluation of Growth Curve Models for Body Weight in American Mink. Animals. 2020;10(1):22.

25. da Silva LSA, Fraga AB, da Silva F de L, Guimarães Beelen PM, de Oliveira Silva RM, Tonhati H, et al. Growth curve in Santa Inês sheep. Small Rumin Res. 2012;105(1–3):182–5.

26. Gossett J, Simpson P, Casey P, Whiteside-Mansell L, Bradley R, Jo CH. Growing growth curves using PROC MIXED and PROC NLMIXED. University of Arkansas for Medical Sciences, Little Rock, Arkansas. 2007.

27. Curran PJ, Obeidat K, Losardo D. Twelve frequently asked questions about growth curve modeling. J Cogn Dev. 2010;11(2):121–36.

28. WİRADARYA T, PUTRA W, HARAHAP A, SUSKA A. The growth curve of body weight in Kacang goats managed by smallholders at Tambang District of Indonesia. Int J
29. Menchetti L, Padalino B, Brasileiro Fernandes F, Nanni Costa L. Comparison of nonlinear growth models and factors affecting body weight at different ages in Toy Poodles. Ital J Anim Sci. 2020;19(1):792–802.

30. Lampl M. Perspectives on modelling human growth: Mathematical models and growth biology. 2012;39(June):342–51.

31. Ahmadi S, Bodeau-Livinec F, Zoumenou R, Garcia A, Courtin D, Alao J, et al. Comparison of growth models to describe growth from birth to 6 years in a Beninese cohort of children with repeated measurements. BMJ Open. 2020;10(9):e035785.

32. Teleken JT, Galvão AC, Robazza W da S. Avaliação comparativa de modelos matemáticos não lineares para descrever o crescimento animal. Acta Sci - Anim Sci. 2017;39(1):73–81.

33. Bann D, Johnson W, Li L, Kuh D, Hardy R. Socioeconomic inequalities in childhood and adolescent body-mass index, weight, and height from 1953 to 2015: an analysis of four longitudinal, observational, British birth cohort studies. Lancet Public Heal. 2018;3(4):e194–203.