The Problem of Quark and Lepton Masses

C. D. Froggatt

Department of Physics and Astronomy,
University of Glasgow
Glasgow G12 8QQ, U.K.

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C. D. Froggatt
Department of Physics and Astronomy,
University of Glasgow,
Glasgow G12 8 QQ,
Scotland, U.K.

Email address: c.froggatt@physics.gla.ac.uk

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Different approaches to the fermion mass problem are reviewed. We illustrate these approaches by summarizing recent developments in models of quark and lepton mass matrices. Dynamical calculations of the top quark mass are discussed, based on (a) infrared quasi-fixed points of the renormalisation group equations, and (b) the multiple point criticality principle in the pure Standard Model. We also consider Yukawa unification and mass matrix texture. Models with approximately conserved gauged chiral flavour charges beyond the Standard Model are shown to naturally give a fermion mass hierarchy.

1 Introduction

The explanation of the fermion mass and mixing hierarchies and the three generation structure of the Standard Model (SM) constitutes the most important unresolved problem in particle physics. We shall discuss recent developments in three of the approaches to this problem:

1. The dynamical determination of the top quark mass.

2. Mass matrix ansätze and texture zeroes.

3. Chiral flavour symmetries and the fermion mass hierarchy.

Neutrino masses, if non-zero, have a different origin to those of the quarks and charged leptons; we do not have time here to discuss recent applications of the so-called see-saw mechanism, which seems the most natural way to generate neutrino masses.
2 Dynamical Top Quark Mass

There is presently a lively interest\[1,2,3,4\] in determining the top quark mass $m_t$ (or more generally third generation masses) dynamically. Most of the discussed models lead to the top quark running Yukawa coupling constant $g_t(\mu)$ being attracted to its infra-red quasi-fixed point value. We have very recently pointed out\[4\] that the top quark (and Higgs) mass can be calculated within the pure SM, assuming the multiple point criticality principle. We now discuss these two possibilities.

2.1 Top Mass as a Renormalisation Group Fixed Point

The idea that some of the properties of the quark-lepton mass spectrum might be determined dynamically as infrared fixed point values of the renormalisation group equations (RGE) is quite old\[5,6,7\]. In practice one finds an effective infrared stable quasifixed point behaviour for the SM quark running Yukawa coupling constant RGE at the scale $\mu \simeq m_t$, where the QCD gauge coupling constant $g_3(\mu)$ is slowly varying. The quasifixed point prediction of the top quark mass is based on two assumptions: (a) the perturbative SM is valid up to some high (e.g. GUT or Planck) energy scale $M_X \simeq 10^{15} - 10^{19}$ GeV, and (b) the top Yukawa coupling constant is large at the high scale $g_t(M_X) \gtrsim 1$. The nonlinearity of the RGE then strongly focuses $g_t(\mu)$ at the electroweak scale to its quasifixed point value. We note that while there is a rapid convergence to the top Yukawa coupling fixed point value from above, the approach from below is much more gradual. The RGE for the Higgs self-coupling $\lambda(\mu)$ similarly focuses $\lambda(\mu)$ towards a quasifixed point value, leading to the SM fixed point predictions\[7\] for the running top quark and Higgs masses:

$$m_t \simeq 225 \text{ GeV} \quad m_H \simeq 250 \text{ GeV} \quad (1)$$

Unfortunately these predictions are inconsistent with the CDF and D0 results\[8\], which require a running top mass $m_t \simeq 170 \pm 12$ GeV.

However the fixed point top Yukawa coupling is reduced by 15% in the Minimal Supersymmetric Standard model (MSSM), with supersymmetry breaking at the electroweak scale or TeV scale, due to the contribution of the supersymmetric partners to the RGE. Also the top quark couples to just one of the two Higgs doublets in the MSSM, which has a VEV of $v_2 = (174 \text{ GeV}) \sin \beta$, leading to the MSSM fixed point prediction for the running top quark mass\[9\]:

$$m_t(m_t) \simeq (190 \text{ GeV}) \sin \beta \quad (2)$$

which is remarkably close to the CDF and D0 results for $\tan \beta > 1$. 

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\[1,2,3,4\]\[5,6,7\]\[8\]\[9\]
For large $\tan \beta$ it is possible to have a bottom quark Yukawa coupling satisfying $g_b(M_X) \gtrsim 1$, which then approaches an infrared quasifixed point and is no longer negligible in the RGE for $g_b(\mu)$. Indeed with $\tan \beta \simeq m_t(m_t)/m_b(m_t) \simeq 60$ we can trade the mystery of the top to bottom quark mass ratio for that of a hierarchy of vacuum expectation values, $v_2/v_1 \simeq m_t(m_t)/m_b(m_t)$, and have all the third generation Yukawa coupling constants large:

$$g_t(M_X) \gtrsim 1 \quad g_b(M_X) \gtrsim 1 \quad g_\tau(M_X) \gtrsim 1 \quad (3)$$

Then $m_t$, $m_b$ and $R = m_b/m_\tau$ all approach infrared quasifixed point values compatible with experiment [10]. This large $\tan \beta$ scenario is consistent with the idea of Yukawa unification [11]:

$$g_t(M_X) = g_b(M_X) = g_\tau(M_X) = g_G (4)$$
as occurs in the SO(10) SUSY-GUT model with the two MSSM Higgs doublets in a single 10 irreducible representation and $g_G \gtrsim 1$ ensures fixed point behaviour. However it should be noted that the equality in Eq. (4) is not necessary, since the weaker assumption of large third generation Yukawa couplings, Eq. (3), is sufficient for the fixed point dynamics to predict [10] the running masses $m_t \simeq 180$ GeV, $m_b \simeq 4.1$ GeV and $m_\tau \simeq 1.8$ GeV in the large $\tan \beta$ scenario. Also the lightest Higgs particle mass is predicted to be $m_{h^0} \simeq 120$ GeV (for a top squark mass of order 1 TeV).

The origin of the large value of $\tan \beta$ is of course a puzzle, which must be solved before the large $\tan \beta$ scenario can be said to explain the large $m_t/m_b$ ratio. It is possible to introduce approximate symmetries [12, 13] of the Higgs potential which ensure a hierarchy of vacuum expectation values - a Peccei-Quinn symmetry and a continuous $\mathcal{R}$ symmetry have been used. However these symmetries are inconsistent with the popular scenario of universal soft SUSY breaking mass parameters at the unification scale and radiative electroweak symmetry breaking [14]. Also, in the large $\tan \beta$ scenario, SUSY radiative corrections to $m_b$ are generically large: the bottom quark mass gets a contribution proportional to $v_2$ from some one-loop diagrams with internal superpartners, such as top squark-charged Higgsino exchange, whereas its tree level mass is proportional to $v_1 = v_2/\tan \beta$. Consequently these loop diagrams give a fractional correction $\delta m_b/m_b$ to the bottom quark mass proportional to $\tan \beta$ and generically of order unity [13, 14]. The presence of the above-mentioned Peccei-Quinn and $\mathcal{R}$ symmetries and the associated hierarchical SUSY spectrum (with the squarks much heavier than the gauginos and Higgsinos) would protect $m_b$ from large radiative corrections, by providing a suppression factor in the loop diagrams and giving $\delta m_b/m_b \ll 1$. However, in the absence of experimental information on the superpartner spectrum, the predictions of the third generation quark-lepton masses in the large $\tan \beta$ scenario must, unfortunately, be considered unreliable.

3
2.2 Criticality and the Standard Model

Here we consider the idea \[15\] that Nature should choose coupling constant values such that several “phases” can coexist, in a very similar way to the stable coexistence of ice, water and vapour (in a thermos flask for example) in a mixture with fixed energy and number of molecules. The application of this so-called multiple point criticality principle to the determination of the top quark Yukawa coupling constant requires the SM (renormalisation group improved) effective Higgs potential to have coexisting vacua, which means degenerate minima: \( V_{\text{eff}}(\phi_{\text{min}}) = V_{\text{eff}}(\phi_{\text{min}}') \). The important point for us, in the analogy of the ice, water and vapour system, is that the choice of the fixed extensive variables, such as energy, the number of moles and the volume, can very easily be such that a mixture must occur. In that case then the temperature and pressure (i.e. the intensive quantities) take very specific values, namely the values at the triple point, without any finetuning. We stress that this phenomenon of thus getting specific intensive quantities is only likely to happen for strongly first order phase transitions, for which the interval of values for the extensive variables that can only be realised as an inhomogeneous mixture of phases is rather large.

In the SM, the top quark Yukawa coupling and the Higgs self coupling correspond to intensive quantities like temperature and pressure. If these couplings are to be determined by the criticality condition, the two phases corresponding to the two effective Higgs field potential minima should have some “extensive quantity”, such as \( \int d^4x |\phi(x)|^2 \), deviating “strongly” from phase to phase. If, as we shall assume, Planck units reflect the fundamental physics it would be natural to interpret this strongly first order transition requirement to mean that, in Planck units, the extensive variable densities \( \int d^4x |\phi(x)|^2 = < |\phi|^2 > \) for the two vacua should differ by a quantity of order unity. Phenomenologically we know that for the vacuum 1 in which we live, \( < \phi >_{\text{vacuum 1}} = 246 \text{ GeV} \) and thus we should really expect \( < \phi >_{\text{vacuum 2}} \) in the other phase just to be of Planck order of magnitude. In vacuum 2 the \( \phi^4 \) term will a priori strongly dominate the \( \phi^2 \) term. So we basically get the degeneracy to mean that, at the vacuum 2 minimum, the effective coefficient \( \lambda(\phi_{\text{vacuum 2}}) \) must be zero with high accuracy. At the same \( \phi \)-value the derivative of the renormalisation group improved effective potential \( V_{\text{eff}}(\phi) \) should be zero because it has a minimum there. Thus at the second minimum the beta-function \( \beta_\lambda \) vanishes as well as \( \lambda(\phi) \).

We use the renormalisation group to relate the couplings at the scale of vacuum 2, i.e. at \( \mu = \phi_{\text{vacuum 2}} \), to their values at the scale of the masses themselves, or roughly at the electroweak scale \( \mu \approx \phi_{\text{vacuum 1}} \). Figure 1 shows the running \( \lambda(\phi) \) as a function of \( \log(\phi) \) computed for two values of \( \phi_{\text{vacuum 2}} \) (where we impose the conditions \( \beta_\lambda = \lambda = 0 \)) near the Planck scale \( M_{\text{Planck}} \approx 2 \times 10^{19} \text{ GeV} \).
Figure 1: Plot of $\lambda$ as a function of the scale of the Higgs field $\phi$ for degenerate vacua with the second Higgs VEV at the scale (a) $\phi_{\text{vacuum}}^2 = 10^{20}$ GeV and (b) $\phi_{\text{vacuum}}^2 = 10^{19}$ GeV. We formally apply the second order SM renormalisation group equations up to a scale of $10^{25}$ GeV.

Combining the uncertainty from the Planck scale only being known in order of magnitude and the $\alpha_{QCD}(M_Z) = 0.117 \pm 0.006$ uncertainty with the calculational uncertainty, we get our predicted combination of top and Higgs pole masses:

$$M_t = 173 \pm 4 \text{ GeV} \quad M_H = 135 \pm 9 \text{ GeV}.$$ (5)

3 Ansätze and Mass Matrix Texture

The best known ansatz for the quark mass matrices is due to Fritzsch [16]:

$$M_U = \begin{pmatrix} 0 & C & 0 \\ C' & 0 & B \\ 0 & B' & A \end{pmatrix}$$  

$$M_D = \begin{pmatrix} 0 & C' & 0 \\ C' & 0 & B' \\ 0 & B' & A' \end{pmatrix}$$ (6)

where it is necessary to assume: $|A| \gg |B| \gg |C|$, $|A'| \gg |B'| \gg |C'|$ in order to obtain a good fermion mass hierarchy. However, in addition to predicting a generalised version of the relation $\theta_c \approx \sqrt{m_t/m_s}$ for the Cabibbo angle, which originally motivated the ansatz, it predicts the relationship:

$$|V_{cb}| \approx \sqrt{\frac{m_s}{m_b}} - e^{-i\phi_2} \sqrt{\frac{m_c}{m_t}}$$ (7)
which cannot be satisfied with a top quark mass \( m_t > 100 \text{ GeV} \). Consistency with experiment can be restored by, for example, introducing a non-zero 22 mass matrix element \[18\]. In fact a systematic analysis \[19\] of symmetric quark mass matrices with 5 or 6 “texture” zeros at the SUSY-GUT scale has been made, yielding 5 ansätze consistent with experiment. Recently ansätze incorporating the Georgi-Jarlskog \[20\] SUSY-GUT mass relations between leptons and quarks, 
\[
m_b(M_X) = m_\tau(M_X), \quad m_s(M_X) = m_\mu(M_X)/3 \quad \text{and} \quad m_d(M_X) = 3m_e(M_X),
\]
have been studied. In particular a systematic analysis of fermion mass matrices in SO(10) SUSY-GUT models \[12, 21\] has been made in terms of 4 effective operators. A scan of millions of operators leads to just 9 solutions consistent with experiment of the form:

\[
Y_u = \begin{pmatrix}
0 & \frac{1}{27}C & 0 \\
-\frac{1}{27}C & 0 & x_u'B \\
0 & x_uB & A
\end{pmatrix} \quad Y_d = \begin{pmatrix}
0 & C & 0 \\
C & E e^{i\phi} & x_d'B \\
0 & x_dB & A
\end{pmatrix} \quad Y_l = \begin{pmatrix}
0 & C & 0 \\
C & 3E e^{i\phi} & x_l'B \\
0 & x_lB & A
\end{pmatrix}
\]

For each of the 9 models the Clebsch \( x_i \) and \( x'_i \) have fixed values and the Yukawa coupling matrices \( Y_i \) depend on 6 free parameters: \( A, B, C, E, \phi \) and \( \tan\beta \). Each solution has Yukawa unification and gives 8 predictions consistent with the data.

4 Chiral Flavour Symmetry and the Mass Hierarchy

It is natural \[5\] to interpret the fermion mass hierarchy in terms of partially conserved chiral quantum numbers beyond those of the SM gauge group. Mass matrix elements are then suppressed by powers of a symmetry breaking parameter, which may be thought of as the ratio of the new chiral symmetry breaking scale to the fundamental scale of the theory. The degree of forbiddenness of a mass matrix element is then determined by the quantum number difference between the left- and right-handed SM Weyl states under consideration and the assumed superheavy fermion spectrum. For example the four effective operators in the ansatz of Eq. \[5\] can each be associated with a unique tree diagram, by assigning an approximately conserved global \( U(1)_f \) flavour charge appropriately to the quarks, leptons and the superheavy states, which are presumed to belong to vector-like SO(10) \( 16 + \overline{16} \) representations. The required parameter hierarchy \( A \gg B, E \gg C \) is naturally obtained in this way and, in particular, the texture zeros reflect the assumed absence of superheavy fermion states which could mediate the transition between the corresponding Weyl states.

We now turn to models in which the chiral flavour charges are part of the extended gauge group. The values of the chiral charges are then strongly constrained by the anomaly conditions for the gauge theory. It will also be assumed that any superheavy state needed to mediate a symmetry breaking transition ex-
ists, so that the results are insensitive to the details of the superheavy spectrum. The aim in these models is to reproduce all quark-lepton masses and mixing angles within a factor of 2 or 3.

Ibanez and Ross \[22\] have constructed an anomaly free $MSSM \times U(1)_f$ model. The $U(1)_f$ charges assigned to the quarks and leptons generate Yukawa matrices of the following form:

$$
Y_u \simeq \begin{pmatrix}
\epsilon^8 & \epsilon^3 & \epsilon^4 \\
\epsilon^3 & \epsilon^2 & \epsilon \\
\epsilon^4 & \epsilon & 1
\end{pmatrix} \quad
Y_d \simeq \begin{pmatrix}
\bar{\epsilon}^8 & \bar{\epsilon}^3 & \bar{\epsilon}^4 \\
\bar{\epsilon}^3 & \bar{\epsilon}^2 & \bar{\epsilon} \\
\bar{\epsilon}^4 & \bar{\epsilon} & 1
\end{pmatrix} \quad
Y_l \simeq \begin{pmatrix}
\bar{\epsilon}^5 & \bar{\epsilon}^3 & 0 \\
\bar{\epsilon}^3 & \bar{\epsilon} & 0 \\
0 & 0 & 1
\end{pmatrix}
$$

which are symmetric up to factors of order unity. The correct order of magnitude for all the masses and mixing angles are obtained by fitting $\epsilon$, $\bar{\epsilon}$ and $\tan \beta$. This is a large $\tan \beta \simeq m_t/m_b$ model, but not necessarily having exact Yukawa unification.

The $U(1)_f$ symmetry is spontaneously broken by two Higgs singlets, $\theta$ and $\bar{\theta}$, having $U(1)_f$ charges $+1$ and $-1$ respectively and equal vacuum expectation values. The $U(1)_f^3$ $U(1)_Y$ gauge anomaly vanishes. The $U(1)_f^3$ anomaly and the mixed $U(1)_f$ gravitational anomaly are cancelled against unspecified spectator particles neutral under the SM group. However cancellation of the mixed $SU(3)_c^2U(1)_f$, $SU(2)_L^2U(1)_f$ and $U(1)_R^2U(1)_f$ anomalies is only possible in the context of superstring theories via the Green-Schwarz mechanism \[23\] with $\sin^2 \theta_W = 3/8$. Consequently the $U(1)_f$ symmetry is spontaneously broken slightly below the string scale.

A number of generalisations of this model has been considered during the last year. By using non-symmetric mass matrices an anomaly free model has been constructed \[24\] without the need for the Green-Schwarz mechanism. Models have also been considered \[24, 25\], in which the $U(1)_f$ symmetry is broken by just one chiral singlet field $\theta$ having a $U(1)_f$ charge, say, $-1$. It then follows, from the holomorphicity of the superpotential, that only positive $U(1)_f$ charge differences between left and right handed Weyl states can be balanced by $\theta$ tadpoles. Consequently mass matrix elements corresponding to negative $U(1)_f$ charge differences have texture zeros \[26\]. Furthermore if the two Higgs doublet fields carry $U(1)_f$ charges that do not add up to zero, the $\mu H_1H_2$ term is forbidden in the superpotential \[27\]. Finally we remark that in effective superstring theories the role of the $U(1)_f$ symmetry can be played by modular symmetry \[2\], with the $U(1)_f$ charges replaced by the modular weights of the fermion fields.

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