Dynamical friction on hot bodies in gaseous, opaque media, and application to embedded protoplanets

F S Masset\(^1\), D A Velasco Romero\(^1\) and H Eklund\(^2\)

\(^1\) Instituto de Ciencias Físicas, Universidad Nacional Autónoma de México, Av. Universidad s/n, Col. Chamilpa, 62210 Cuernavaca, Mor., Mexico

\(^2\) Institute of Theoretical Astrophysics, University of Oslo, Postboks 1029 Blindern, N-0315 Oslo, Norway

E-mail: masset@icf.unam.mx

Abstract. A massive and luminous perturber moving across an opaque gas is subjected to a force different from the gravitational friction that it would experience if it were cold. The heat released by the perturber diffuses in the surrounding gas, where it gives rise to a low density region behind the perturber that exerts a force (that we call heating force) in the direction of motion, thus opposed to the standard dynamical friction. We present numerical simulations with nested meshes that confirm the analytical expression of the heating force in the limits of a low and high Mach number, respectively, and we present simulations that show that the dynamical friction exerted on a cold perturber in a gas with thermal diffusion is markedly different from that in an adiabatic gas. We then present numerical simulations of low-mass protoplanets embedded in opaque, viscous discs, that show that when these bodies have a sufficiently large luminosity their eccentricity and inclination can be excited to values comparable to the aspect ratio of the disc. We finally present numerical experiments with very high resolution that try to resolve the flow within the Bondi sphere, in an attempt to study the dependence of the heating force as a function of the ratio of the diffusive to acoustic times across the Bondi radius.

1. Introduction

While an important part of the interactions between a protoplanetary disc and a forming planet is of tidal nature (i.e. the planet’s response to the disturbance excited in the disc by its gravitational field), the planet is also subjected to a non-tidal force that arises from the disturbance created by the heat that it releases in the surrounding gas. The first investigation on this topic was about the influence of the planet’s luminosity on its own migration [1]. It was found that the disturbance created by the planet’s luminosity yields in general a positive extra torque, which may be able to counteract or reverse the inward migration of small mass planetary embryos (below a few Earth masses). The impact of heat release was subsequently investigated in the context of dynamical friction by [2]. The low-density, heated region found downstream of a hot perturber yields an extra force, called heating force, that is in the direction of the motion, and is therefore opposed to the dynamical friction. This analysis suggested that hot embryos in protoplanetary discs could have their eccentricity and inclination excited by the release of heat (a prospect that was not investigated by [1], who limited themselves to planets on fixed, circular orbits). This expectation was later confirmed by the numerical simulations of [3]. Finally, the impact of thermal diffusion, even in the absence of heat release, was investigated by [4]. It
was found, in the context of Keplerian protoplanetary discs that, for largely subsonic point-like massive perturbers, the diffusion of heat induces a perturbation of the flow with respect to the adiabatic response that is equivalent to that triggered by a massless heat sink. These proceedings are organised as follows: in section 2 we recall the basic equations of the problem at hand, in section 3 we briefly present the numerical code used in this work, in section 4 we present the analytical and numerical results about the heating force and in section 5 the analytical and numerical results about the dynamical friction on a cold body. In section 6 we summarise our results on dynamical friction, and turn to the excitation of eccentricity and inclination of embedded protoplanets in section 7. We finally present a high resolution study aimed at resolving the flow at the sub-Bondi scale in section 8, and give our conclusions in section 9.

2. Physical setup and basic equations

We consider a point-like body of mass \( M \) and luminosity \( L \) moving across a homogeneous, three-dimensional gas of density \( \rho \), thermal diffusivity \( \chi \) and sound speed \( c_s \). We denote \((u, v, w)^T\) the velocity of the gas, \( \vec{V} \) the velocity vector of the body, \( p \) and \( e \) respectively the pressure and internal energy density of the gas. As we assume it to be ideal, we have:

\[
p = (\gamma - 1)e,
\]

where \( \gamma \) is the adiabatic index. The equations that govern the response of the gas to the perturber are the continuity equation

\[
\partial_t \rho + \vec{\nabla} \cdot (\rho \vec{v}) = 0,
\]

the Euler equation

\[
\partial_t \vec{v} + \vec{v} \cdot \vec{\nabla} \vec{v} = -\frac{\vec{\nabla} p}{\rho} - \vec{\nabla} \Phi_p,
\]

where \( \Phi_p \) is the gravitational potential of the perturber and the energy equation

\[
\partial_t e + \vec{\nabla} \cdot (e \vec{v}) = -\vec{\nabla} \cdot \vec{F}_H + L \delta (\vec{r} - \vec{r}_p),
\]

where \( \vec{r}_p \) is the position of the body and \( \vec{F}_H \) is the heat flux given by:

\[
\vec{F}_H = -\rho \chi \frac{\nabla \left( \frac{e}{\rho} \right)}{\rho}.
\]

The linearised version of Eqs. (1)-(5) have been studied by [2]. We do not reproduce here the derivation of [2], but underline its main features. We denote with a 0 subscript the unperturbed quantities and with a prime the perturbations of the quantities. We therefore have:

\[
\rho = \rho_0 + \rho',
\]

and similar relations on the velocity components, pressure and internal energy. Upon linearisation, the governing equations can be cast as:

\[
S(\vec{Q}) = M \vec{R}_1 + L \vec{R}_2,
\]

where \( \vec{Q} = (\rho', u', v', w', e')^T \) is the vector of the perturbations, \( S \) stands for the linearized system of equations and \( \vec{R}_1 \) and \( \vec{R}_2 \) are vectors of the right-hand-side that correspond to the forcing terms respectively due to the gravity and luminosity of the body:

\[
\vec{R}_1 = \frac{G}{|\vec{r} - \vec{r}_p|^3} (0, x_p - x, y_p - y, z_p - z, 0)^T
\]
and
\[ \vec{R}_2 = [0, 0, 0, 0, \delta(\vec{r} - \vec{r}_p)]^T. \]  

(9)

The linearity of the operator \( S \) implies that the solution \( \vec{Q} \) can be written \( \vec{Q} = \vec{Q}_1 + \vec{Q}_2 \), where \( \vec{Q}_1 \) and \( \vec{Q}_2 \) verify

\[
S(\vec{Q}_1) = M \vec{R}_1, \\
S(\vec{Q}_2) = L \vec{R}_2
\]

(10)

(11)

and correspond respectively to the disturbance excited in the gas by a non-luminous perturber of mass \( M \) and a massless perturber of luminosity \( L \). In the following we will consider separately the responses \( \vec{Q}_1 \) and \( \vec{Q}_2 \).

3. Numerical setup

We have performed numerical simulations to verify the analytic dependencies of [2]. We use the FARGO3D code [5], that we have modified to deal with an arbitrary number of nested meshes with a resolution doubled in each direction for each new level of refinement. This is required to make the calculations tractable: as can be seen in Fig. 1 of [2], obtaining the heating force in the subsonic regime with an accuracy of 10% requires to have a large range of distances to the perturber covered by the mesh (approximately two orders of magnitude). Obtaining this force with an even better accuracy (≈5%) requires to cover three orders of magnitude of distance to the perturber. We use a Cartesian mesh and up to 9 levels of refinement, depending on our parameters. We use a sub-cycling technique to alleviate the computational cost: for each time step performed at a given level, two smaller time steps are performed on the level with immediately finer resolution, in a recursive manner. The mesh is fixed with respect to the body, and the gas is injected at a constant speed \(-V\) at the upper \( z \)-boundary of the coarsest mesh.

4. Heating force

4.1. Analytical considerations

The force induced by the density disturbance of \( \vec{Q}_2 \) has been dubbed heating force by [2], who have worked out its analytic expression in the limits of a small and large Mach number. In the low Mach number limit, they find that the perturbation of density \( \rho' \) has the following form:

\[
\rho' = -\frac{\gamma(\gamma - 1) L}{4\pi \chi c_s^2 r} \exp \left( -\frac{\gamma V r}{\chi} \cos^2 \frac{\theta}{2} \right).
\]

(12)

This form is fundamentally the \( r^{-1} \) law that corresponds to the diffusion from a point-like source in a medium at rest, with a cut-off that depends on the angle \( \theta \) with respect to the direction of the velocity \( \vec{V} \). In the particular case \( \theta = \pi \), which corresponds to the downstream flow in the body’s frame, there is no cut-off. This perturbation of density exerts the force

\[
\vec{F}_H = \frac{\gamma(\gamma - 1) G M L}{2 \chi c_s^2} \cdot \frac{\vec{V}}{V}.
\]

(13)

This force has the same direction as the velocity \( \vec{V} \) of the perturber. It is therefore opposed to the dynamical friction, and has a motor role on the perturber’s motion. Remarkably, its magnitude \( \gamma(\gamma - 1) G M L/2 \chi c_s^2 \) does not depend on the perturber’s velocity. This can be understood from the form of \( \rho' \) given at Eq. (12). Close to the perturber, the distribution of density is nearly symmetric. The asymmetry between the front and rear distribution of material occurs at a length scale \( \lambda \sim \chi/V \). The volume of the corresponding shell is \( V \sim \lambda^3 \), and it corresponds to front and rear masses that are proportional to \( V/\lambda \), since the front coefficient of Eq. (12) is
inversely proportional to the radius. The masses involved are thus proportional to \( \lambda^2 \), and the forces they exert on the body do not depend on \( \lambda \), hence on \( V \).

In the large Mach number limit, \[2\] find that the heating force has the approximate following form:

\[
\vec{F}_H = \frac{(\gamma - 1)GML}{\chi V^3} \left[ -1.96 - \log \left( \frac{r_{\text{min}}V}{4\chi} \right) \right] \vec{V},
\]

where \( r_{\text{min}} \) is a minimal distance of integration of the force, which can amount to the body’s physical radius, or to the mean free path of photons (whichever is largest). The magnitude of the force scales as \( V^{-2} \), as does the standard dynamical friction. As in the subsonic regime, the force is along the direction of motion and has a motor role on the body.

4.2. Numerical results
We have performed various series of runs covering a range of Mach numbers from 0.02 to 10, in geometric sequence. The coarsest mesh has \( x, y, z_{\text{min, max}} = \pm 408.13 \) and 323 cells. The parameters are: \( c_s = 1 \), \( \chi = 0.5 \), \( L = 10^{-5} \), \( G = 1 \) and \( \gamma = 1.4 \). We take \( \rho_0 = 1 \) (although the value of \( \rho_0 \) does not feature in the heating force expression, if it is too small, the perturbation can become non-linear even for the low luminosity considered here). We have two series of runs: one with 8 and another one with 6 levels of refinement. We plot in Fig. 1 the results of these runs. We find that the asymptotic value of the force at low Mach number is \( \sim 10\% \) below the analytic expectation for the low resolution case, and \( \sim 5\% \) below for the high resolution case. We also find that the force does exhibit a \( V^{-2} \) dependence in the supersonic case, and that the force depends strongly on the resolution in this regime, as expected from the logarithmic divergence of the heating force on \( r_{\text{min}} \). We can further investigate the supersonic regime by evaluating the heating force for different resolutions. We neither apply a truncation of the force summation nor soften the body’s potential. The effective minimal radius of Eq. (14) therefore scales with the resolution, and is divided by a factor of 2 every time a new level of refinement is added. We therefore expect the specific force to increase by the following amount every time the resolution is doubled:

\[
\Delta F = \frac{(\gamma - 1)GL}{\chi V^2} \log 2
\]

Fig. 2 shows that this is indeed the case. We note that Eq. (14) is only an approximation, which is valid in the regime where \( r_{\text{min}}V \ll 4\chi \). When this relation breaks down (which occurs for the lowest resolutions in Fig. 2), the spacing between successive values of the force is no longer \( \Delta F \).

5. Dynamical friction on a cold body
5.1. Analytic considerations
We now turn to the force exerted by the perturbation corresponding to \( \vec{Q}_1 \) (see section 2). This is the perturbation imparted by a massive, non-luminous body on the gas. This perturbation and the resulting force have been worked out by [6] for an adiabatic gas. Here the situation is different, as thermal diffusion is present and the gas is not adiabatic. [2] have speculated that the force would correspond to the force expression worked out by [6], in which the speed sound to be used should have a value intermediate between the adiabatic and isothermal sound speeds, depending on the amount of thermal diffusion. The situation is actually different from this early expectation. In the low Mach number limit, the effect of thermal diffusion is adding to the perturbation of the adiabatic case the perturbation that would be imparted by a point-like heat sink with luminosity \(-L_c\), with \( L_c = 4\pi GM\chi \rho_0/\gamma \). The derivation will be presented elsewhere (Velasco Romero and Masset, in prep., see also [4]). This implies that the dynamical friction has the following form, using the expression of \( L_c \), Eq. (13) and the expression given by [6] for
the subsonic case:

\[ \vec{F} = -\frac{2\pi (GM)^2 \rho_0}{c_s^2} \frac{V}{\gamma - 1 + \frac{2}{3} \mathcal{M}} \left( \frac{\gamma - 1}{\gamma - 1 + \frac{2}{3} \mathcal{M}} \right) \hat{V}, \]

(16)

where \( \mathcal{M} = V/c_s \) is the Mach number. Eq. (16) is an expansion of the dynamical friction to first order in \( \mathcal{M} \). There is no first order term in \( \mathcal{M} \) in the expansion of the new force component arising from the finite thermal diffusivity (Velasco Romero and Masset, in prep.). This new component therefore accounts for the constant term in Eq. (16), while the first order term in \( \mathcal{M} \) comes from the expansion of Ostriker’s formula.

The dynamical friction in the supersonic regime with thermal diffusion is presently under study and will be presented elsewhere.

5.2. Numerical results

We show in Fig. 3 the results of preliminary numerical simulations of the dynamical friction in a medium with thermal diffusion, in the subsonic regime.

The blue curve (in the electronic version) represents the results obtained in an adiabatic gas. They match satisfactorily the analytic expectation of [6] (see also [7]).

The set of the other three solid curves are obtained with three different values of the thermal diffusivity. They clearly confirm that the force does not tend to zero when the body’s velocity does, and that the value of the friction is nearly independent of the value of the thermal diffusivity in the limit of a low Mach number. We also see that the friction measured in the simulations falls short of the analytic expectation by \( \sim 10 - 20 \% \). This is along the lines of what we noted for the heating force in section 4.2, and can be traced back to the restricted range of length scales accessible to the numerical simulations. Higher resolution simulations with more levels of refinement and larger coarsest boxes are under way and will be presented elsewhere.
Figure 3. Dynamical friction in an adiabatic disc (blue dots) and in discs with thermal diffusion [respectively $\chi = 0.05$ (green), $\chi = 0.1$ (red) and $\chi = 0.2$ (cyan)]. The dynamical friction is normalized to $2\pi(GM)^2\rho_0/c_s^2$. We use seven levels of refinement (including the coarsest mesh). Each refined level has $16^3$ cells.

As the dynamical friction is independent of the velocity in the low Mach number limit, it may be regarded as “dry” or “solid” friction in that limit. For this statement to be true, however, the stopping time of the body $MV/F$ should be much shorter than the response time of the force (which in this limit merely amounts to the response time of the cold, dense trail $\chi/V^2$, see [2]), or equivalently the stopping distance $MV^2/(2F)$ should be much shorter than the trail’s size $\chi/V$. This property is verified in the subsonic regime if $L_{grav} \ll L_c$, the gravitational luminosity being $L_{grav} = c_s^3/G$.

6. Putting together the results on dynamical friction
The net force experienced by the perturber is the sum of that exerted by the perturbation of density associated to $\vec{Q}_1$ and that associated to $\vec{Q}_2$ (see section 2). In the subsonic regime, a first-order expansion of the force in the Mach number, using Eqs. (13) and (16), is:

\[
\vec{F} = \left[ \frac{\gamma(\gamma - 1)GM}{2\chi c_s^2} - \frac{2\pi(GM)^2\rho_0}{c_s^2} \left( \gamma - 1 + \frac{2}{3}\mathcal{M} \right) \right] \frac{\vec{V}}{V},
\]

(17)

\[
\vec{F} = \left[ \frac{\gamma(\gamma - 1)GM(L - L_c)}{2\chi c_s^2} - \frac{4\pi(GM)^2\rho_0}{3c_s^2}\mathcal{M} \right] \frac{\vec{V}}{V}.
\]

(18)

This expression slightly differs from that of [2], in which the term in $L_c$ is absent. As a consequence, a body at rest is unstable and will eventually acquire a finite velocity with respect to the medium if and only if $L > L_c$. When its terminal speed is largely subsonic, it scales with $L - L_c$. We note that the luminosity above which the gas response becomes non-linear over the extent of the hot region, given by [2] as

\[
L_{NL} = \frac{4\pi\chi^2 c_s^2\rho_0}{\gamma^2(\gamma - 1)|V|}
\]

(19)
is much larger than $L_c$ when $GM/(\chi c_s) \ll M^{-1}$. The ratio of the left-hand-side appears as the ratio of the mass to critical mass $\chi c_s/G$ above which one expects a cut-off of the heating force (see [2, 3, 4] and section 8 below). One therefore expects sub-critical mass bodies with $L > L_c$ to experience a net positive force and therefore to be accelerated.

7. Application to planets embedded in protoplanetary discs
The above analysis suggests that planetary embryos heated by planetesimal or pebble accretion in opaque, gaseous protoplanetary discs may be subjected to a force directed along their velocity with respect to the gas. This can have important consequences on the evolution of their eccentricity and inclination. However, the results obtained in the context of dynamical friction can only serve as a proxy for the more complex case of hot embryos in Keplerian disc, where the shear leads to a more complex response than in a uniform medium at rest. The shear can only be neglected when the response time of the wake is shorter than the time scale of the shear, $\Omega_p^{-1}$, where $\Omega_p$ is the planet’s orbital frequency. For the tidally excited wake, this condition is equivalent to $e \gg h$, where $e$ is the eccentricity and $h$ the disc’s aspect ratio [8, 9]. For the response excited by the heat release, the condition on time scales is equivalent to $e \gg \sqrt{\chi/\Omega_p/a}$ where $a$ is the planet’s semi-major axis [3]. This last condition is less stringent than that on the tidal wake [3]. In order to study the time behaviour of the evolution of embedded, hot embryos, we have resorted to numerical simulations using the public hydrocode FARGO3D which we have coupled to a module of radiative transfer in the flux limited diffusion limit (see [1] for details). We find that hot embryos, with accretion rates typical of those found in the literature, experience a growth of eccentricity and inclination [3]. We present in Fig. 4 the time behaviour of the eccentricity $e$ and inclination $i$ of an Earth-mass embryo which has initially either $e = 0.01$ or $i = 0.01$. The embryo has a mass doubling time $\tau = 10^5$ yr due to accretion of solids, which corresponds to a luminosity:

$$L = \frac{GM^2}{R_p \tau},$$

(20)

where $R_p$, the embryo’s physical radius, is estimated assuming the embryo to have a mean density $\rho_p = 3$ g cm$^{-3}$. It is located at 5.2 au from the central star, which has a solar mass. The disc in which it is embedded has a surface density $\Sigma = 200$ g cm$^{-2}$, an effective viscosity $\nu = 10^{15}$ cm$^2$ s$^{-1}$, and a Rosseland opacity $\kappa = 1$ cm$^2$ g$^{-1}$. It is relaxed, prior to the simulations, toward hydrostatic and thermal equilibrium, and tends to $H/r = 0.043$, where $H$ is the pressure length scale and $r$ the radius. We note that the critical luminosity $L_c$ is $\sim L/5$, so that it is nearly negligible compared to $L$.

We find that the eccentricity and inclination grow respectively to $e_c = 0.031$ and $i_c = 0.028$ for these fiducial calculations. These values are comparable in order of magnitude to the aspect ratio of the disc. Similar results have been obtained for the eccentricity in two-dimensional calculations by [10]. These results are in sharp contrast with the standard view that the interaction with the disc damps the eccentricity and inclination to negligible values over time scales much shorter than the migration time scale.

8. Cut-off at larger mass
The linear analysis performed by [2] requires that the heat released by the central object diffuses to the region where the flow’s perturbation is linear. A ratio that features prominently in that analysis is $GM/\chi c_s$. This ratio is also the ratio of the diffusion to acoustic time scales across the Bondi radius $r_B = GM/c_s^2$. This radius corresponds to the distance from the perturber below which the flow is non-linear when $L < L_c$. When the heat diffusion across the Bondi radius is shorter than the acoustic time (i.e. $M < \chi c_s/G$), most of the heat released by the central body diffuses to the region where the perturbation is linear, and the perturbation of density
Figure 4. Time behaviour of the eccentricity (red curve in the electronic version) and inclination (orange curve in the electronic version). These curves represent the results of two different calculations: one in which the planet is coplanar with the disc ($i = 0$) and has a finite eccentricity, and another one in which the planet has a circular orbit ($e = 0$) and a finite inclination. The details of the setup are given in the text.

Figure 5. Heating force $F_H$ measured in numerical simulations (obtained by subtracting the force measured in a run with a luminous body and a run with a non-luminous body), normalised to the heating force expected from the linear analysis in the limit of a vanishing Mach number [given by Eq. (13)], as a function of the body’s mass, normalised to the critical mass $M_c = \chi c_s / G$.

in that region takes therefore the form given by Eq. (12). In the opposite situation (that of a mass larger than the critical mass $\chi c_s / G$) the energy deposited near the body may not emerge out of the Bondi sphere as an excess of temperature but can also be carried away by acoustic waves and the density perturbation may differ from that given by Eq. (12). As an investigation of this effect requires to resolve the Bondi sphere, where the flow is highly non-linear, we have undertaken numerical simulations with nested meshes, using FARGO3D, to study the behaviour of the heating force when the body’s mass varies over a range including the critical mass $M_c = \chi c_s / G$.

We have run for this purpose two series of runs: one in which the resolution is fixed, so that the Bondi sphere of large mass bodies is well resolved while it is barely resolved for the lowest mass considered, and one in which the resolution adapts to the setup and is one sixth of the Bondi radius (on the finest level of refinement), regardless of the mass. These calculations have been run with $M = 0.4$, and the size of the mesh at each level was $48^3$. In addition, we have also performed a series of runs with a fixed resolution and $M = 0.2$ (such calculations are more expensive as the size of the hot trail is twice larger than in the former, and the time required for convergence is a factor of four larger). The calculations becomes increasingly expensive at smaller mass for the adapted series because of the higher resolution requirements and because by definition, for sub-critical masses, the timestep is not limited by the sound speed but by the thermal diffusion. The results are presented in Fig. 5. We clearly see that the force approximately tends toward the analytic expectation when the mass is small compared to the critical mass, whereas it decays to zero when the mass is much larger. The dispersion of the measurements at larger mass is due to the fact that the force measured in that case is not stationary and may
oscillate. The ratio at smaller mass can be larger than one: this is because the force is measured at \( M = 0.4 \) and not for \( M \to 0 \), which results in a heating force larger than its asymptotic value at low Mach number (see Fig. 1).

9. Conclusions

Thermal diffusion strongly alters the response of the gas in the vicinity of a massive perturber. The dynamical friction is enhanced, and behaves much differently than in the adiabatic case at low Mach number. If, in addition, the body releases heat in the surrounding gas, it triggers a response that has a form similar to that of the perturbation due to thermal diffusion, but of opposite sign. In particular, the extra force arising from the heat release is along the motion and may result, if the luminosity is large enough, in a net force that accelerates the body, rather than slowing it down. In protoplanetary discs, the release of heat can lead to an excitation of the eccentricity and inclination. This has potentially a very important effect on scenarios of planetary growth. However, there is a cut-off of these effects that occurs for masses larger than the critical mass \( \chi c_s / G \). In the planet forming regions of protoplanetary discs, this is of the order of an Earth mass. We have presented here a study of the heat release for masses in excess of the critical mass by means of high resolution calculations with nested meshes that resolve the flow within the Bondi sphere (such as those of [11]). As the sub-Bondi flow is only marginally resolved in this study, its results should be regarded as preliminary. A complete understanding of the high mass regime is required to get reliable estimates of the values of eccentricity and inclination that a hot protoplanet can ultimately attain.

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