Research Article

Multisensory Prediction Fusion of Nonlinear Functions of the State Vector in Discrete-Time Systems

Ha Ryong Song, 1 Il Young Song, 2 and Vladimir Shin 3

1 Flight Safety Technology Division, Korea Aerospace Research Institute, 169-84 Gwahangno, Yuseong-gu, Daejeon 305-806, Republic of Korea
2 Department of Sensor Systems, Hanwha Corporation R&D Center, 52-1 Oesam-dong, Yuseong-gu, Daejeon 305-106, Republic of Korea
3 Department of Information and Statistics, Research Institute of Natural Science, Gyeongsang National University, 501 Jinjudaero, Jinju, Gyeongsangnam-do 660-701, Republic of Korea

Correspondence should be addressed to Vladimir Shin; vishin@gnu.ac.kr

Received 25 March 2015; Revised 21 July 2015; Accepted 24 August 2015

Academic Editor: Frank Ehlers

Copyright © 2015 Ha Ryong Song et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

We propose two new multisensory fusion predictors for an arbitrary nonlinear function of the state vector in a discrete-time linear dynamic system. Nonlinear function of the state (NFS) represents a nonlinear multivariate functional of state variables, which can indicate useful information of the target system for automatic control. To estimate the NFS using multisensory information, we propose centralized and decentralized predictors. For multivariate polynomial NFS, we propose an effective closed-form computation procedure for the predictor design. For general NFS, the most popular procedure for the predictor design is based on the unscented transformation. We demonstrate the effectiveness and estimation accuracy of the fusion predictors on theoretical and numerical examples in multisensory environment.

1. Introduction

The integration of information from a combination of different types of sensors is often used in the design of high-accuracy control systems. Typical applications that benefit from usage of multiple sensors include industrial tasks, military commands, mobile robot navigation, multitarget tracking, and aircraft. One problem that arises from the use of multiple sensors is that if all local sensors observe the same target, the question then becomes how to effectively combine the corresponding local estimates. Several decentralized fusion architectures have been discussed and algorithms for estimation fusion have been developed in [1–4]. An important practical problem in the above systems and architectures is to find a fusion estimate to combine the information from various local estimates to produce a global (fusion) estimate. Optimal mean square linear fusion formulas, for an arbitrary number of local estimates with matrix and scalar weights, have been reported in [5–10].

However, because of the lack of prior information, in general, the decentralized estimation using the fusion formula is globally suboptimal compared with optimal centralized estimation [11]. Nevertheless, in this case it has advantages of lower computational requirements, efficient communication costs, parallel implementation, and fault-tolerance [11–13]. Therefore, in spite of its limitations, the decentralized estimation has been widely used and is superior to the centralized estimation in real applications. The aforementioned papers [5–13] have not focused on the prediction problem, but most of them have considered only decentralized filtering of state variables in multisensory dynamic models. The decentralized prediction of the state requires special algorithms presented in [14, 15].

Some applications require the estimation fusion of nonlinear function of the state variables, representing useful information for system control, for example, a quadratic form of a state vector, which can be interpreted as a current distance between targets or as the energy of an object [16, 17]. We refer
to the nonlinear function as the nonlinear function of the state (NFS). In [17], we have not focused on the prediction of the NFS, considering instead only filtering. To the best of our knowledge, there are no methods reported in the literature for prediction fusion of the NFS in a multisensory environment. Direct generalization of the distributed fusion filtering algorithms to the prediction problem of the NFS is impossible.

Therefore, in this paper, the prediction fusion problem of NFS is considered under a multisensory environment. The primary aim of this paper is to propose centralized and decentralized prediction fusion algorithms and analyze their statistical accuracies.

This paper is organized as follows. Section 2 presents a statement of the prediction fusion problem for NFS. In Section 3, the centralized global optimal predictor is derived. In Section 4, we propose the nonlinear decentralized prediction fusion algorithm for NFS. In Section 5, we propose effective closed-form computational procedure for prediction of multivariate polynomial functions. For prediction of a general NFS, we use the unscented transformation. In Section 6, we study the comparative analysis of the proposed fusion estimators via a theoretical example. In Section 7, the efficiency of the fusion predictors is studied for prediction of the instantaneous impact point of space launch vehicle. Finally, we conclude our results in Section 8.

2. Problem Statement

The general Kalman multisensory framework involves estimation of the state of a discrete-time linear dynamic system:

\[ x_{k+1} = F_k x_k + G_k w_k, \]

\[ y_k^{(i)} = H_k^{(i)} x_k + v_k^{(i)}, \quad i = 1, \ldots, L, \]

where \( x_k \in \mathbb{R}^n \) and \( y_k^{(i)} \in \mathbb{R}^{m_i} \) are unknown state and measurement vectors, respectively, and \( F_k \in \mathbb{R}^{n \times n}, G_k \in \mathbb{R}^{n \times q}, \) and \( H_k^{(i)} \in \mathbb{R}^{m_i \times n} \).

Assume that \( L \) sensors are used to observe the state vector simultaneously. The process noise \( w_k \in \mathbb{R}^q \sim \mathcal{N}(0, Q_k) \) and the measurement noises \( v_k^{(i)} \in \mathbb{R}^{m_i} \sim \mathcal{N}(0, R_k^{(i)}) \), \( i = 1, \ldots, L \), represent normally distributed uncorrelated random processes.

Our goal is to find a fused prediction estimate of the NFS at future time \( k+s, s \geq 0 \), where

\[ z_{k+s} = f(x_{k+s}): \mathbb{R}^n \rightarrow \mathbb{R} \]

based on overall current sensor measurements, where

\[ Y_{[1:k]} = \{y_1^{(1)}, \ldots, y_k^{(1)}\}, \]

\[ Y_{[1:k]}^{(i)} = \{y_1^{(i)}, y_2^{(i)}, \ldots, y_k^{(i)}\}, \]

\[ i = 1, \ldots, L. \]

Typical examples of such NFS may be an arbitrary quadratic form \( f(x_k) = x_k^T \Omega_k x_k \) of the state vector or magnitude of position and velocity of three-dimensional state vectors \( f(x_k) = x_{1,k}^2 + x_{2,k}^2 + x_{3,k}^2 \) and \( f(x_k) = \sqrt{x_{1,k}^2 + x_{2,k}^2 + x_{3,k}^2} \), respectively.

In general, there are two fusion estimation approaches commonly used to process the overall measured data. If a central processor receives measurements \( Y_{[1:k]} \) from all local sensors directly and processes them in real time, the corresponding result is known as centralized data processing. However, this approach has several serious drawbacks, including poor survivability and reliability, as well as heavy communication and computational burdens.

The second approach is called decentralized estimation fusion, in which every local sensor is attached to a local processor. In this approach, the processor estimates the state of a system based on its own local measurements \( Y^{(i)}_{[1:k]} \) and then transmits its local linearization or nonlinear fusion predicted estimates to the fusion center. Finally, the fusion center predicts the state (object) \( x_k \), and NFS \( z_{k+s} = f(x_{k+s}) \) based on all received local estimates. For this reason, the proposed estimation algorithm is referred to as decentralized prediction fusion algorithm. Clearly, decentralized prediction has significant practical value, because it has greater survivability in extreme situations because it can estimate objects even though the fusion center is destroyed.

We propose centralized and decentralized prediction fusion algorithms for NFS in the subsequent sections.

3. Centralized Multisensory Prediction Fusion: Global Optimal Predictor

In this section, the best global optimal (in the mean square error (MSE) sense) prediction algorithm for an NFS is derived. In the centralized fusion setup, a multisensory dynamic system (1) can be reformulated into a composite form:

\[ x_{k+1} = F_k x_k + G_k w_k, \quad k = 0, 1, 2, \ldots, \]

\[ Y_k = H_k x_k + v_k, \quad Y_k \in \mathbb{R}^{m}, \quad m = m_1 + \cdots + m_L, \]

where

\[ Y_k^T = \begin{bmatrix} y_1^{(1)} & \cdots & y_k^{(L)} \end{bmatrix}, \]

\[ H_k^T = \begin{bmatrix} H_k^{(1)} & \cdots & H_k^{(L)} \end{bmatrix}, \]

\[ v_k \sim \mathcal{N}(0, R_k), \quad R_k = \text{diag}\{R_k^{(1)}, \ldots, R_k^{(L)}\}. \]

The optimal centralized Kalman predictor (CKP) \( \hat{z}_{k+1|k} = E(x_{k+s} \mid Y_{[1:k]}) \) of the state vector and its error covariance
Note that the optimal Kalman predictor process the International Journal of Distributed Sensor Networks 3 (3), also represents a conditional mean; that is, centralized Kalman filter (CKF) equations for the composite matrix for the system model (4), and the initial conditions where

\[
P_{k+1}^{CKF} = F_k P_k^{CKF} F_k^T + G_k Q_k G_k^T, \quad P_0^{CKF} = P_0,
\]

\[
K_{k+1} = P_{k+1}^{CKF} H_k^T \left[ H_k P_{k+1}^{CKF} H_k^T + R_{k+1} \right]^{-1},
\]

\[
P_{k+1}^{CKF} = \left[ I_n - K_{k+1} H_{k+1} \right] P_{k+1}^{CKF},
\]

where \( I_n \) is an \( n \times n \) identity matrix, \( \Phi_{k,\ell} \) is the transition matrix for the system model (1), and the initial conditions \( \bar{x}_{k|k} = \bar{x}_k \) and \( P_{k|k} = P_k^{CKF} \) are determined by the centralized Kalman filter (CKF) equations for the composite measurement models (4) and (5):

\[
\bar{x}_{k+1} = \bar{x}_k + K_{k+1} \left[ Y_{k+1} - H_k \bar{x}_k \right],
\]

\[
\bar{x}_0 = \bar{x}_0,
\]

\[
F_k^{(i)} = F_k P_k^{CKF} F_k^{(i)} + G_k Q_k^{(i)} G_k^{(i)}^T, \quad P_0^{CKF} = P_0^{(i)}.
\]

\[
h_{k+1} = h_{k+1}^{(i)} + K_{k+1}^{(i)} R_{k+1}^{-1},
\]

\[
P_{k+1}^{CKF} = \left[ I_n - K_{k+1} H_{k+1} \right] P_{k+1}^{CKF},
\]

or

\[
\bar{x}_{k|s} = \bar{x}_{k|s-1} + F_{k|s-1} \left[ x_{k+s} - \bar{x}_{k|s-1} \right],
\]

\[
P_{k|s} = F_{k|s-1} P_{k|s-1} F_{k|s-1}^T + Q_{k+s-1},
\]

\[
\bar{Q}_{k+s} = G_{k+s} Q_{k+s-1} G_{k+s}^T, \quad s = 1, 2, \ldots,
\]

where \( L_p \) is the number of measurements.

Next, the global optimal mean square predictor of NFS \( z_{k+s} = f(x_{k+s}) \), based on the overall sensor measurements (3), also represents a conditional mean; that is,

\[
\bar{z}_{k+s|k} = \mathbb{E} \left[ z_{k+s} \mid Y_{[1:k]} \right] = \int f(x_{k+s}) p(x_{k+s} \mid Y_{[1:k]}) \, dx_{k+s}, \quad s \geq 0,
\]

where \( p(x_{k+s} \mid Y_{[1:k]}) = N(\bar{x}_{k+s|k}, P_{k+s|k}) \) is a normal probability density function with conditional mean \( \bar{z}_{k+s|k} = \mathbb{E}(x_{k+s} \mid Y_{[1:k]}) \) and covariance \( P_{k+s|k} = \text{cov}(x_{k+s} \mid x_{k+s|k}) \) determined by the centralized Kalman predictor equations (6)–(8) for a composite linear model (4), including all sensor measurements.

Thus, estimate (9) represents the optimal minimum mean square error (MMSE) centralized nonlinear predictor

\[
\bar{z}_{k+s|k}^{opt} = \mathbb{E} \left[ z_{k+s|k}^{opt} \mid x_{k+s} \right], \quad s \geq 0,
\]

which depends on the centralized Kalman filter and predictor and their error covariances.

Remark 1. There is an alternative idea to estimate the NFS. In this case, the unknown NFS \( z_{k+s} = f(x_{k+s}) \) is considered as additional state variable \( z_k \), which is determined by the nonlinear difference equation \( z_{k+1} = \Phi_k(x_k, w_k) \), \( \Phi_k = \Phi_k(F_k x_k + G_k w_k), k = 1, 2, \ldots \). Including the variable \( z_k = f(x_k) \) into the state vector of a system \( x_k \in \mathbb{R}^n \), we obtain nonlinear discrete-time system with the extended state \( x_k = [x_k^T, z_k] \in \mathbb{R}^{n+1} \). Thus, the problem of prediction the unknown NFS is reduced to the nonlinear filtering problem by replacing real state vector \( x_k \) by the respective extended state vector \( x_k^e \). And approximate nonlinear filters can be used for simultaneous prediction of the unknown state vector \( x_{k+s} \) and NFS \( z_{k+s} \), respectively. Notice that, contrary to the proposed idea (6)–(10), in this case it is impossible to separate prediction of the state \( x_{k+s} \) from prediction of the NFS \( z_{k+s} \).

Many different approximate filters have been proposed in the literature. For instance, the most common one is the extended Kalman filter (EKF), obtained by linearizing the nonlinear model equations along the state. In this case, the EKF must perform the simultaneous calculation of equations determining \( \bar{x}_k^e \) and the auxiliary matrices \( P_k^{e(i)} \) and \( K_k \).

Also computational complexity of the nonlinear approximate filters is considerably greater than complexity of the linear Kalman estimators (6)–(8).

In decentralized fusion, the fusion center tries to get the best prediction of an NFS with the processed data received from each local sensor \( Y_{(i)}^{(1:k)} = \{y_{(i)}^{(0)}, \ldots, y_{(i)}^{(k)}\}, i = 1, \ldots, L \). In Section 4, we propose decentralized multisensory prediction fusion algorithm based on the \( L \) local Kalman predictors of NFS \( z_{k+s} = f(x_{k+s}) \):

\[
\bar{z}_{k+s|k}^{(i)} = \mathbb{E} \left[ f(x_{k+s}) \mid Y_{(i)}^{(1:k)} \right], \quad i = 1, \ldots, L,
\]

which are available at the fusion center.

4. Decentralized Multisensory Prediction Fusion

4.1. Local Kalman Predictors for the State Vector. At first, according to (1), we have \( L \) unconnected dynamical subsystems \( i = 1, \ldots, L \) with the common state \( x_k \) and local sensor \( y_k^{(i)} \) such that

\[
x_{k+1} = F_k x_k + G_k k, \quad y_k^{(i)} = H_k^{(i)} x_k + w_k^{(i)}.
\]

where \( i \) is the index of the subsystem, \( i = 1, \ldots, L \).
Then, the optimal mean square local prediction estimate 
\( \hat{x}_{k+s|k} = E(x_{k+s} | Y_{1:k}) \), filtering estimate \( \tilde{x}_{k} = E(x_{k} | Y_{1:k}) \), and corresponding error covariances \( P_{k+s|k}^{(i)} \) and \( P_{k}^{(i)} \) are determined by recursive Kalman equations [18, 19]. We have

\[
\begin{align*}
\tilde{x}_{k+s|k} &= F_{k+s-1} \tilde{x}_{k+s-1|k}, \\
P_{k+s|k}^{(i)} &= P_{k+s-1|k}^{(i)} + \tilde{Q}_{k+s-1}, \quad P_{k}^{(i)} = P_{0}^{(i)}, \\
\tilde{Q}_{k+s-1} &= G_{k+s-1} Q_{k+s-1} G_{k+s-1}^T
\end{align*}
\]

(13)

Thus, we have \( L \) local Kalman estimates \( \tilde{x}_{k}^{(i)} \) and \( \tilde{x}_{k+s|k}^{(i)} \), and their corresponding error covariances \( P_{k+s|k}^{(i)} = \text{cov}(\tilde{e}_{k+s|k}^{(i)}, \tilde{e}_{k}^{(i)}) \) and \( P_{k}^{(i)} = \text{cov}(\tilde{e}_{k}^{(i)}, \tilde{e}_{k}^{(i)}) \) for \( i = 1, \ldots, L \).

4.2. Local Predictors for Nonlinear Function of the State Vector.

Then, the optimal local mean square prediction of NFS \( z_{k+s} = f(x_{k+s}) \), based on the local sensor measurements \( Y_{1:k|k}^{(i)} \), represents a conditional mean (11); that is,

\[
\tilde{z}_{k+s|k}^{(i)} = E \left[ f(x_{k+s}) | Y_{1:k|k}^{(i)} \right] = \int f(x_{k+s}) p(x_{k+s} | Y_{1:k|k}^{(i)}) \, dx_{k+s},
\]

(15)

where \( p(x_{k+s} | Y_{1:k|k}^{(i)}) \) is a normal conditional probability density function, \( p(x_{k+s} | Y_{1:k|k}^{(i)}) = N(\tilde{z}_{k+s|k}^{(i)}, P_{k+s|k}^{(i)}) \).

Therefore, the optimal local estimate \( \tilde{z}_{k+s|k}^{(i)} \) in (15) represents a nonlinear function of the local Kalman predictor and its error covariance; that is,

\[
\tilde{z}_{k+s|k}^{(i)} = \tilde{z}_{k+s|k}^{(i)} \left( \tilde{x}_{k+s|k}^{(i)}, P_{k+s|k}^{(i)} \right), \quad i = 1, \ldots, L.
\]

4.3. Multisensory Prediction Fusion Algorithm. Using all optimal local nonlinear predictors \( \tilde{z}_{k+s|k}^{(1)}, \ldots, \tilde{z}_{k+s|k}^{(L)} \) and fusion formula with scalar weights [8, 9], we obtain the decentralized multisensory fusion predictor for an NFS:

\[
\begin{align*}
\tilde{z}_{k+s|k}^{\text{fus}} &= \sum_{i=1}^{L} a_{k+s|k}^{(i)} \tilde{z}_{k+s|k}^{(i)}, \\
\sum_{i=1}^{L} a_{k+s|k}^{(i)} &= 1,
\end{align*}
\]

(17)

where the scalar weights \( a_{k+s|k}^{(i)} \in \mathbb{R} \) are defined as

\[
a_{k+s|k}^{(i)} = \frac{e_{z,k+s|k}^{(i)}}{\sum_{i=1}^{L} e_{z,k+s|k}^{(i)}},
\]

(18)

Remark 2. Equations (17) and (18) require additional computations of the local cross-covariances \( P_{x,z,k+s|k}^{(i)} = \text{cov}(e_{x,k+s|k}^{(i)}, e_{z,k+s|k}^{(i)}) \), which depend on the NFS \( z_{k+s} = f(x_{k+s}) \), the local Kalman covariances \( P_{x,z,k+s|k}^{(i)} = \text{cov}(e_{x,k+s|k}^{(i)}, e_{z,k+s|k}^{(i)}) \) determined by the Riccati equations (9), and the local cross-covariances \( P_{x,z,k+s|k}^{(i)} = \text{cov}(e_{x,k+s|k}^{(i)}, e_{z,k+s|k}^{(i)}) \), \( i \neq j \), which can be obtained by the Riccati-like recursive equations [14]:

\[
\begin{align*}
P_{k+s|k}^{(i)} &= F_{k+s|k} P_{k+s|k}^{(i)} F_{k+s|k}^T \\
&\quad + \sum_{h=0}^{s-1} \Phi_{k+s+k+h|k} Q_{k+s+k+h} \Phi_{k+s+k+h|k}^T, \
&\quad s \geq 0, \quad i \neq j,
\end{align*}
\]

(19)

where \( \Phi_{k+c} \) is the transition matrix and the filter gains \( K_{k}^{(i)} \) \( i = 1, \ldots, L \), are determined by the local Kalman filter equations (14).

In the following, we discuss effective computational algorithms for the evaluation of local nonlinear predictors \( \tilde{z}_{k+s|k}^{(i)} \) and cross-covariances \( P_{x,z,k+s|k}^{(i)} \) in (15) and (18), respectively, depending on the type of NFS.
5. Calculation of Fusion Predictor for Nonlinear Functions of the State

5.1. Multivariate Polynomial Functions. First, we consider an arbitrary multivariate polynomial function of the form
\[ z = f(x) = \sum_{0\leq i_1+...+i_n\leq N} D_{i_1i_2...i_n} x_1^{i_1} x_2^{i_2} \ldots x_n^{i_n}, \]
(20)
\[ i_1, \ldots, i_n \geq 0. \]

In this case, the proposed algorithm for the calculation of fusion predictors \( \hat{z}_{k+r|k} \) and \( \hat{z}_{k+s|k} \) has a closed-form because conditional expectations \( E[f(x_{k+s}) \mid Y_{[i]}] \) and \( E[f(x_{k+s}) \mid Y_{[i]}] \) depend on high-order moments \( \hat{m}_{h_{k+h_1}} = E(x_1^n x_2^n \ldots x_n^n \mid Y_{[i]} \) and \( m_{h_{k+h_1}} = E(x_1^n x_2^n \ldots x_n^n) \) of a multivariate normal distribution, which can be calculated explicitly in terms of first- and second-order moments \( \bar{x}_{k+s|k} \) and \( P_{k+s|k} \), \( i, j = 1, \ldots, L \), using recursive formulas [20–22]. The following example illustrates the idea.

Consider an arbitrary quadratic cost function
\[ z_{k+s} = f(x_{k+s}) = x^T_{k+s} \Omega x_{k+s}, \]
\[ \Omega^T = \Omega, \quad \Omega > 0. \]
(21)

Show that optimal local nonlinear predictor (15) can be calculated explicitly in terms of a local Kalman predictor and its error covariance. Using formula \( E(x^T \Omega x) = \text{tr}(\Omega(P + mm^T)) \), \( m = E(x), \Omega = \text{cov}(x, x) \) [21], we obtain an optimal local predictor for the quadratic function
\[
\hat{z}_{k+s|k}^{(i)} = E(x^T_{k+s} \Omega x_{k+s} \mid Y_{[i]}^{(i)}), \quad (22)
\]

where the local Kalman predictor and error covariance \((\hat{z}_{k+s|k}^{(i)}, P_{k+s|k}^{(i)})\) satisfy (13) and (14).

In a special case of a polynomial NFS (20), the local cross-covariances \( P_{z_{k+s|k}^{(i)}} \) are also calculated for a multivariate normal distribution of composite random vector \( U_{k+s}^{(i)} = [x_{k+s}^{(i)}, z_{k+s|k}^{(i)}, \dot{z}_{k+s|k}^{(i)}] \) via the recursive formulas [20–22].

5.2. General Nonlinear Function and Unscented Transformation. The unscented transformation (UT) makes it much easier to calculate statistics of the transformed random variable, for example, the mean and covariance [23, 24]. The UT has become a powerful approach for designing new filtering and control algorithms for nonlinear dynamic models [23–26]. Following this, the UT procedure to calculate the best local predictor of an NFS (conditional mean)
\[
\hat{z}_{k+s|k}^{(i)} = E[f(x_{k+s}) \mid Y_{[i]}^{(i)}] \]
(23)
can be summarised as follows.

Generate the sigma points \( \{U_{h_{k+s|k}}^{(i)}\}_{h=0}^{2n} \) with corresponding weights \( \{W_{h}^{(i)}\}_{h=0}^{2n} \):

\[
U_{0, k+s}^{(i)} = x_{k+s|k}^{(i)}, \quad W_{0} = \frac{\ell}{n + \ell},
\]
\[
U_{h, k+s}^{(i)} = x_{k+s|k}^{(i)} + \sqrt{(n + \ell)} P_{k+s|k}^{(i)} h, \quad W_{h} = \frac{1}{2(n + \ell)}, \quad h = 1, \ldots, n,
\]
\[
U_{h+n, k+s}^{(i)} = x_{k+s|k}^{(i)} - \sqrt{(n + \ell)} P_{k+s|k}^{(i)} h, \quad W_{h+n} = \frac{1}{2(n + \ell)},
\]

where \( \{\sqrt{P_{k+s|k}^{(i)}} h\} \) is the \( h \)th column of the matrix square root of \( P_{k+s|k}^{(i)} \) and \( \ell \) is the scaling parameter influencing the spread of points in the state-space and, thus, the accuracy of the approximation [26]. Propagate each of these sigma points through the original nonlinear function as
\[
\hat{x}_{h, k+s}^{(i)} = f(U_{h, k+s}^{(i)}), \quad h = 0, 1, \ldots, 2n
\]
(25)
and the resulting best local estimate of the NFS is given as
\[
\hat{z}_{k+s|k}^{(i)} = \sum_{h=0}^{2n} W_{h} \hat{x}_{h, k+s}^{(i)}, \quad i = 1, \ldots, L.
\]
(26)

Similar to (23)–(26), the local cross-covariance \( P_{z_{k+s|k}^{(i)}}^{(i)} \) also can be calculated based on the UT:
\[
P_{z_{k+s|k}^{(i)}}^{(i)} = \sum_{h=0}^{2n} W_{h} [\hat{z}_{h, k+s}^{(i)} - \hat{z}_{k+s|k}^{(i)}] [\hat{z}_{h, k+s}^{(i)} - \hat{z}_{k+s|k}^{(i)}]^T.
\]
(27)
Therefore, \( \hat{z}_{k+s|k}^{(i)} \) and \( P_{z_{k+s|k}^{(i)}}^{(i)} \) are represented by the known functions of the local Kalman predictors \( \hat{x}_{k+s|k}^{(i)} \) and covariances \( P_{k+s|k}^{(i)}, i, j = 1, \ldots, L \).

5.3. Discussion

(1) The local covariances \( P_{z_{k+s|k}^{(i)}}^{(i)} \) and \( P_{z_{k+s|k}^{(i)}}^{(i)} \) and weights \( a_{k+s}^{(i)} \) can be precomputed, because they do not depend on the sensor measurements \( y_{k}^{(i)} \), \( i = 1, \ldots, L \), but only on the noise statistics \( Q_{k} \) and \( R_{k}^{(i)} \), the system matrices \( F_{k}, G_{k} \), and \( H_{k}^{(i)} \), and the initial conditions \( x_{0} \) and \( P_{0} \), which are the part of system models (1) and (2). Thus, once the measurement schedule has been settled, the real-time implementation of the fusion estimators
requires only the computation of the local predictors \( \hat{x}_{k+j}^{(i)} \) and \( \hat{z}_{k+j}^{(i)} \) and the final fusion predictor \( \hat{z}_{k+j}^{\text{ fus}} \).

(2) The implementation of the decentralized predictor consists of two stages: offline and online. The offline stage is more complex than the online stage. This is because it requires the computation of the local covariances \( P_{x,k+j}^{(i)} \) and the fusion weights \( a_{k+j}^{(i)} \) which depend on NFS \( z_{k+i} = f(x_{k+i}) \). However, it is not essential, because this stage can be precomputed. The online stage (real-time implementation) requires the computation of only the local and fusion estimates (predictors). To compute \( \hat{z}_{k+j}^{\text{ opt}} \), the centralized predictor requires all sensor measurements together at each time instant \( k = 1, 2, \ldots \), whereas the decentralized predictor computes \( \hat{z}_{k+j}^{\text{ fus}} \) sequentially.

To demonstrate the performance of the proposed centralized and decentralized predictors, they will be evaluated in the next section for the theoretical example originating from [7].

6. Theoretical Comparison of Estimation Accuracy

Consider a simple example of an application of the obtained results. We predict the quadratic function \( z = \theta^2 \) of a random constant \( \theta \sim N(0, \sigma_\theta^2) \), given two single measurements \( y^{(1)} \) and \( y^{(2)} \) of \( \theta \) corrupted by Gaussian white noises \( \epsilon^{(1)} \) and \( \epsilon^{(2)} \), respectively. The system and measurement equations describing this situation are

System: \( \theta_{s+1} = \theta_s \), \( \theta_s \equiv \theta \), \( s = 0, 1, 2, \ldots \)

Sensor 1: \( y^{(1)} = \theta + \epsilon^{(1)} \), \( \epsilon^{(1)} \sim N(0, r_1) \)

Sensor 2: \( y^{(2)} = \theta + \epsilon^{(2)} \), \( \epsilon^{(2)} \sim N(0, r_2) \).

Here, we derive precise formula for the MSE for the proposed fusion estimators and demonstrate a comparative analysis.

6.1. Centralized Optimal Estimator for Quadratic Function. In this case, \( L = 2, K = 1, \) and \( s = 1, 2, \ldots \). Using (22) at \( \Omega = 1 \), the optimal centralized estimator \( \hat{z}_{k+1}^{\text{ opt}} = \hat{z}_{k+1}^{\text{ opt}(i)} \) of the quadratic function \( z = \theta^2 \equiv \theta^2 \) takes the form

\[
\hat{z}_{k+1}^{\text{ opt}} = \mathcal{E} (\theta^2 | y^{(1)}, y^{(2)}) = \int \theta^2 \mathcal{N} (\hat{\theta}^{\text{ CKP}}, P_s^{\text{ CKP}}) \, d\theta_s
\]

\[
= P_s^{\text{ CKP}} + (\hat{\theta}^{\text{ CKP}})^2, \quad s \geq 1, \tag{29}
\]

where \( \hat{\theta}^{\text{ CKP}} = \mathcal{E} (\theta | y^{(1)}, y^{(2)}) \), \( s \geq 1 \), is the centralized Kalman estimator (the best global MMSE estimate) of an unknown constant state \( \theta_s \equiv \theta \) based on the two single sensor measurements \( Y^T = [y^{(1)}, y^{(2)}] \) and \( P_{s}^{\text{ CKP}} = \mathcal{E} [(\theta_s - \hat{\theta}^{(i)}_s)^2] \) is its centralized error variance such that [7]

\[
\hat{\theta}^{\text{ CKP}} = \frac{r_2 \sigma_\theta^2}{r_1 r_2 + (r_1 + r_2) \sigma_\theta^2} y^{(1)} + \frac{r_1 \sigma_\theta^2}{r_1 r_2 + (r_1 + r_2) \sigma_\theta^2} y^{(2)}, \quad s \geq 1, \tag{30}
\]

Thus, (28) and (29) represent the exact formulas for the centralized fusion estimator of the quadratic function \( z_s = \theta^2 \). The error between the unknown \( \theta \) and the centralized estimator \( \hat{z}_{k+1}^{\text{ opt}} = P_s^{\text{ CKP}} + (\hat{\theta}^{\text{ CKP}})^2 \) can be measured in terms of the MSE \( P_s^{\text{ opt}} = \mathcal{E} [(\theta^2 - \hat{z}_{k+1}^{\text{ opt}})^2] \), \( s \geq 1 \). Using the result that the 4th-order moment \( \mu_4 = \mathcal{E} (\theta^4) \) of a normal random variable \( \theta \sim N(0, \sigma_\theta^2) \) is equal to \( \mu_4 = 3 \sigma_\theta^4 \), we obtain

\[
P_s^{\text{ opt}} = 3 \left( 1 - \rho^2 \right)^2 \sigma_\theta^4 + (P_s^{\text{ CKP}})^2 + 2 \left( 3 \rho^2 - 1 \right) \sigma_\theta^2 d + 3d^2
\]

\[
- 2 \left( 1 - \rho^2 \right) P_s^{\text{ CKP}} \sigma_\theta^2 + 2P_s^{\text{ CKP}} d, \quad s \geq 1, \tag{31}
\]

where \( \rho = W_1 + W_2, \)

\[
d = W_1 r_1 + W_2 r_2,
\]

\[
W_1 = \frac{r_1 \sigma_\theta^2}{[r_1 r_2 + (r_1 + r_2) \sigma_\theta^2]},
\]

\[
W_2 = \frac{r_2 \sigma_\theta^2}{[r_1 r_2 + (r_1 + r_2) \sigma_\theta^2]}.
\]

Together with the centralized estimator (29), we apply the decentralized fusion estimator developed in Section 4.

6.2. Decentralized Fusion Estimator for Quadratic Function. The local MMSE estimator \( \hat{\theta}_s^{(i)} = \mathcal{E} (\theta_s | y^{(i)}) \), \( s \geq 1 \), of the unknown \( \theta_s \equiv \theta \) based on the single measurement \( y^{(i)} \), the corresponding error variances \( P_{s}^{(i)} = \mathcal{E} [(\theta_s - \hat{\theta}_s^{(i)})^2] \), and
the cross-covariance \( P_{s}^{(12)} = \mathbb{E}[(\theta - \tilde{\theta}_s)(\theta - \tilde{\theta}_s')] \) take the following forms [7]:

\[
\tilde{\theta}_s^{(i)} = \frac{\sigma^2_{\theta}}{r_i + \sigma^2_{\theta}} y_s^{(i)}, \\
P_{s}^{(ii)} = \frac{r_i \sigma^2_{\theta}}{r_i + \sigma^2_{\theta}}, \\
P_{s}^{(12)} = \frac{r_i r_s \sigma^2_{\theta}}{(r_i + \sigma^2_{\theta})(r_s + \sigma^2_{\theta})},
\]

(32)

\[
\tilde{\theta}_s^{(i)} = \tilde{\theta}_s^{(i)}, \quad P_{s}^{(ii)} = P_{s}^{(ii)}, \quad P_{s}^{(12)} = P_{s}^{(12)}, \quad i = 1, 2, \quad s > 1.
\]

Next the first stage gives two optimal local nonlinear estimators of the quadratic function \( z_s = \theta^2 \):

\[
z_s^{(i)} = \mathbb{E}(\theta^2 | y_s^{(i)}) = \int \theta^2 \mathcal{N} \left( \tilde{\theta}_s^{(i)}, P_{s}^{(ii)} \right) d\theta = P_{s}^{(ii)} + \left( \tilde{\theta}_s^{(i)} \right)^2, \quad i = 1, 2; \quad s \geq 1,
\]

(33)

where \( \tilde{\theta}_s^{(i)} \) and \( P_{s}^{(ii)} \) are calculated by (32).

In the second stage, using fusion formula (17), we obtain

\[
z_s^{\text{fus}} = a_s^{(1)} z_s^{(1)} + a_s^{(2)} z_s^{(2)}, \quad a_s^{(1)} + a_s^{(2)} = 1, \quad s = 1, 2, \ldots,
\]

where

\[
a_s^{(1)} = \frac{P_{z,s}^{(11)} - P_{z,s}^{(12)}}{P_{z,s}^{(11)} - 2P_{z,s}^{(12)} + P_{z,s}^{(22)}}, \quad a_s^{(2)} = \frac{P_{z,s}^{(11)} - P_{z,s}^{(12)}}{P_{z,s}^{(11)} - 2P_{z,s}^{(12)} + P_{z,s}^{(22)}},
\]

(34)

\[
P_{z,s}^{(jj)} = \mathbb{E} \left[ \left( z_s - z_s^{(j)} \right) \left( z_s - z_s^{(j)} \right) \right], \quad z_s = \theta^2, \quad i, j = 1, 2.
\]

6.3. Comparative Analysis of Estimators. The MSE is an important value that can be used to reflect the accuracy of state estimation. Table 1 illustrates the exact values of the MSEs for the centralized \( P_{s}^{\text{opt}} = P_{s}^{\text{fus}}, \ s \geq 1 \) (see (31)), and decentralized \( P_{s}^{\text{fus}} = P_{s}^{\text{fus}}, \ s \geq 1 \), estimators (see (32)–(36)) for the different model parameters \( \sigma^2_{\theta}, r_1, \) and \( r_2 \). Table 1 shows the relative errors for the decentralized estimator with respect to the global optimal centralized estimator, \( \Delta_s = |(P_{s}^{\text{fus}} - P_{s}^{\text{opt}})/P_{s}^{\text{opt}}| \times 100\% \).

Not surprisingly, Table 1 illustrates that the centralized estimator \( P_{s}^{\text{opt}} \) exhibits a performance that is completely superior to the decentralized estimator \( P_{s}^{\text{fus}} \). Thus, the theoretical relation \( P_{s}^{\text{opt}} < P_{s}^{\text{fus}} \) between the two estimators \( (z_s^{\text{opt}}) \) and \( (z_s^{\text{fus}}) \) is obvious.

| Model parameters | \( P_{s}^{\text{fus}} \) | \( P_{s}^{\text{opt}} \) | Relative error |
|------------------|------------------------|------------------------|-----------------|
| \( \sigma^2_{\theta} = 4; r_1 = 1; r_2 = 1 \) | 7.833 | 6.716 | 16.6% |
| \( \sigma^2_{\theta} = 10; r_1 = 1; r_2 = 1 \) | 20.36 | 18.59 | 9.5% |
| \( \sigma^2_{\theta} = 4; r_1 = 0.1; r_2 = 0.3 \) | 1.214 | 1.167 | 4.0% |
| \( \sigma^2_{\theta} = 4; r_1 = 0.1; r_2 = 1.0 \) | 1.460 | 1.406 | 3.8% |
| \( \sigma^2_{\theta} = 1; r_1 = 1; r_2 = 1 \) | 1.312 | 1.111 | 18% |
| \( \sigma^2_{\theta} = 1; r_1 = 2; r_2 = 4 \) | 1.760 | 1.632 | 7.8% |

Then, the overall MSE \( P_{s}^{\text{fus}} = \mathbb{E}[(\theta^2 - \hat{z}_s^{\text{fus}})^2] \) can be evaluated as

\[
P_{z,s}^{\text{fus}} = (a_s^{(1)})^2 P_{z,s}^{(11)} + (a_s^{(2)})^2 P_{z,s}^{(22)}
\]

(35)
7. Experimental Analysis of Fusion Predictors

A comparative experimental analysis of the proposed predictors is considered in an example of the prediction of the instantaneous impact point (IIP) of a space launch vehicle (SLV). In the space rocket launch, a precise and real-time prediction of the IIP plays an important role for the range safety operations. Hence, online IIP prediction is carried out during rocket launch to follow the expected touch down point for a rocket body.

The dynamic model of a SLV, in general, varies from linear model to nonlinear model. Typical nonlinear model considers comprehensive factors such as thrust, gravity, drag coefficient, March number, and air density [27]. Although the nonlinear model precisely describes a motion of SLV, it needs complex prior information concerning a SLV flight environment. In case of the linear dynamic model, on the other hand, a constant acceleration (CA) model with multiple hypotheses which takes advantage of Singer’s model [28] is introduced in [29]. In [29], to describe motion of a sounding rocket using the CA model, the rocket motion is separated into two parts, propelled flight and free fall flight phase by utilizing empirically tuned, independent probability density function. This multiple model approach is suitable for the sounding rocket which has relatively short propelled flight phase with large free fall flight phase. In contrast, most of the SLV flight phases fall into the propelled flight. Therefore, for simplicity, we can reduce the dynamic model of the SLV to a CA model. To apply the proposed algorithm for prediction of IIP, a discretized CA model for motion of SLV takes the following form:

\[
X_{k+1} = F_k X_k + w_k,
\]

\[
X_k = [x_{p,k} \ x_{v,k} \ x_{a,k} \ y_{p,k} \ y_{v,k} \ y_{a,k} \ z_{p,k} \ z_{v,k} \ z_{a,k}]^T,
\]

\[
w_k = [w_{p,k} \ w_{v,k} \ w_{a,k} \ w_{p,k} \ w_{v,k} \ w_{a,k} \ w_{p,k} \ w_{v,k} \ w_{a,k}]^T,
\]

where the state vector \( X_k \in \mathbb{R}^9 \) consists of the position, velocity, and acceleration components along the \( x \)-axis, \( y \)-axis, and \( z \)-axis, respectively, \( F_k \in \mathbb{R}^{9 \times 9} \) is the system matrix, and \( w_k \in \mathbb{R}^9 \) is the white Gaussian noise, \( w_k \sim N(0, Q_k) \):

\[
Q_k = \sigma^2 \begin{bmatrix} Q_x & 0_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & Q_y & 0_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} & Q_z \end{bmatrix},
\]

\[
Q_x = Q_y = Q_z = \begin{bmatrix} \Delta t^2 & \Delta t^4 & \Delta t^6 \\ \Delta t^4 & \Delta t^6 & \Delta t^2 \\ \Delta t^6 & \Delta t^2 & 2 \end{bmatrix},
\]

where \( \Delta t \) is the discretization interval.

Next radar sensor observes the range \((\rho), \) azimuth \((\varphi), \) and elevation \((\theta)\) of the SLV. In general, due to reliability reason of the SLV, tracking system uses multiple radar to cover large trajectory. Let us assume that \( L \) radars simultaneously observe trajectory of the SLV within joint radar coverage. Then, the nonlinear multisensory measurement equations are given by

\[
Z_k^{(i)} = h(X_k) + v_k^{(i)},
\]

\[
Z_k^{(i)} \in \mathbb{R}^3, \ v_k^{(i)} \in \mathbb{R}^3, \ i = 1, \ldots, L,
\]

or

\[
Z_k^{(i)} = \begin{bmatrix} r_{m,k}^{(i)} \\ \theta_{m,k}^{(i)} \\ \rho_{m,k}^{(i)} \end{bmatrix} + \begin{bmatrix} r_{p}^{(i)} \\ \varphi^{(i)} \\ \theta^{(i)} \end{bmatrix},
\]

where \( r_{m,k}^{(i)} = N(0, R_{m,k}^{(i)}), \) \( R_{m,k}^{(i)} = \text{diag}([\sigma_p^2, \sigma_{\varphi}^2, \sigma_\theta^2]), \) and \( i \) is index of radar \( (i = 1, \ldots, L)\).

Using 3D debiased converted measurement form [30], we can transform the original nonlinear equations (39) into linear form as

\[
Z_k^{(i)} = H_k X_k + v_k^{(i)}, \ v_k^{(i)} \in \mathbb{R}^3, \ i = 1, \ldots, L,
\]

or

\[
Z_k^{(i)} = \begin{bmatrix} x_{m,k}^{(i)} \\ y_{m,k}^{(i)} \\ z_{m,k}^{(i)} \end{bmatrix} + \begin{bmatrix} r_{x,k}^{(i)} \\ \rho_{x,k}^{(i)} \end{bmatrix},
\]

where

\[
Z_k^{(i)} = \begin{bmatrix} 1 & 0_{1 \times 5} & 0_{1 \times 3} \\ 0_{1 \times 3} & 1 & 0_{1 \times 5} \\ 0_{1 \times 5} & 1 & 0_{1 \times 2} \end{bmatrix} X_k + \begin{bmatrix} r_{x,k}^{(i)} \\ \rho_{x,k}^{(i)} \end{bmatrix}.
\]
Here $v_{c,k}^{(i)}$ is the converted measurement noise expressed in terms of Cartesian coordinates; that is, $v_{c,k}^{(i)} \sim \mathcal{N}(0,R_{c,k}^{(i)})$, 

$$
R_{c,k}^{(i)} = \begin{bmatrix}
R_{c,xx}^{(i)} & R_{c,xy}^{(i)} & R_{c,xz}^{(i)} \\
R_{c,xy}^{(i)} & R_{c,yy}^{(i)} & R_{c,yz}^{(i)} \\
R_{c,xz}^{(i)} & R_{c,yz}^{(i)} & R_{c,zz}^{(i)}
\end{bmatrix}, \quad i = 1, \ldots, L.
$$

By the Keplerian motion \[31\], the IIP can be represented as nonlinear function of the unit vectors of the position ($e_{p,k}$) and velocity ($e_{v,k}$). Therefore, in this case, the IIP prediction problem is reduced to estimation of the following unit vectors:

$$
e_{p,k} = [e_{p,k} \ e_{p,k} \ e_{p,k}]^T, $$
$$
e_{v,k} = [e_{v,k} \ e_{v,k} \ e_{v,k}]^T, $$
$$
e_{p,k} = \frac{x_{p,k}}{\sqrt{x_{p,k}^2 + y_{p,k}^2 + z_{p,k}^2}}, $$
$$
e_{p,k} = \frac{y_{p,k}}{\sqrt{x_{p,k}^2 + y_{p,k}^2 + z_{p,k}^2}}, $$
$$
e_{p,k} = \frac{z_{p,k}}{\sqrt{x_{p,k}^2 + y_{p,k}^2 + z_{p,k}^2}}, $$
$$
e_{v,k} = \frac{x_{v,k}}{\sqrt{x_{v,k}^2 + y_{v,k}^2 + z_{v,k}^2}}, $$
$$
e_{v,k} = \frac{y_{v,k}}{\sqrt{x_{v,k}^2 + y_{v,k}^2 + z_{v,k}^2}}, $$
$$
e_{v,k} = \frac{z_{v,k}}{\sqrt{x_{v,k}^2 + y_{v,k}^2 + z_{v,k}^2}}, $$

where $x_{p,k}, \ldots, z_{v,k}$ are the position and velocity components of the state vector $X_k \in \mathbb{R}^3$ described by (37) and (38).

As the unit vectors in (42) represent an NFS, we apply the UT (Section 5.2) to calculate the local predicted estimates

$$
\hat{e}_{p,k+1|k}^{opt} = \mathbb{E}[e_{p,k+1|k} | e_{p,k+1|k}], \quad \hat{e}_{v,k+1|k}^{opt} = \mathbb{E}[e_{v,k+1|k} | e_{v,k+1|k}],
$$

their local error covariances, and the final centralized ($\hat{e}_{p,k+1|k}^{opt}$, $\hat{e}_{v,k+1|k}^{opt}$) and decentralized ($\hat{e}_{p,k+1|k}^{fus}$, $\hat{e}_{v,k+1|k}^{fus}$) fusion predictors. Since the fusion predictors of the unit vectors are not normalized (i.e., $|\hat{e}_{p,k+1|k}^{opt}| \neq 1$, $|\hat{e}_{v,k+1|k}^{opt}| \neq 1$ for $k = 1, 2, \ldots$), we correct the obtained predictors

$$
\hat{e}_{p,k+1|k}^{opt} = \left[\hat{e}_{p,k+1|k}^{opt} \hat{e}_{p,k+1|k}^{opt} \hat{e}_{p,k+1|k}^{opt}\right], $$
$$
\hat{e}_{v,k+1|k}^{opt} = \left[\hat{e}_{v,k+1|k}^{opt} \hat{e}_{v,k+1|k}^{opt} \hat{e}_{v,k+1|k}^{opt}\right], $$

(43)

To compare the MSEs for $x$-component, $y$-component and $z$-component of the centralized and decentralized fusion predictors ($\hat{e}_{p,k+1|k}^{opt}$, $\hat{e}_{v,k+1|k}^{opt}$) and ($\hat{e}_{p,k+1|k}^{fus}$, $\hat{e}_{v,k+1|k}^{fus}$), the Monte Carlo method with 1000 runs was performed.

The SLV model parameters, noise statistics, initial conditions, and prediction lead were taken to

$$
\Delta t = 0.1; $$
$$
k_T = 20 \text{ s}; $$
$$
L = 2; $$
$$
\sigma = 0.1; $$
$$
\sigma_p = 10 \text{ m}; $$
$$
\sigma_q = 0.01; $$
$$
\sigma_\theta = 0.01; $$
$$
s = 5; $$

\[32]$$
Figures 1 and 2 show the global optimal and fusion prediction MSEs:

\[
\begin{align*}
\bar{x}_0 &= \left[ 1000 \text{ m}; 0.1 \text{ m/s}; 0 \text{ m/s}^2; 1000 \text{ m}; \right. \\
&\quad \left. - 0.1 \text{ m/s}; 0 \text{ m/s}^2; 100 \text{ m}; 1 \text{ m/s}; 0 \text{ m/s}^2 \right]^T, \\
P_0 &= \text{diag} [100; 1; 0.01; 100; 1; 0.01; 100; 1; 0.01].
\end{align*}
\] (44)

In addition, Figure 7 illustrates the relative errors between the optimal and fusion prediction MSEs:

\[
\begin{align*}
\Delta_{p_x,k+j|k} &= \left[ \frac{p_{fus}^p - p_{opt}^p}{p_{opt}^p} \right] \times 100\%, \\
\Delta_{p_y,k+j|k} &= \left[ \frac{p_{fus}^p - p_{opt}^p}{p_{opt}^p} \right] \times 100\%, \\
\Delta_{p_z,k+j|k} &= \left[ \frac{p_{fus}^p - p_{opt}^p}{p_{opt}^p} \right] \times 100\%.
\end{align*}
\] (46)

for position along x-axis, y-axis, and z-axis, respectively. Analogously, Figure 8 illustrates the relative errors \(\Delta_{v_x,k+j|k}\), \(\Delta_{v_y,k+j|k}\), and \(\Delta_{v_z,k+j|k}\) for velocity components.

Figure 7 shows that the relative errors \(\Delta_{p_x,k+j|k}\) and \(\Delta_{p_y,k+j|k}\) are very close and their values are ranging from 0.1% to 18% at \(k > 1\). The relative error \(\Delta_{p_z,k+j|k}\) along z-axis varies from 3% to 9%. However, all relative errors for position display a tendency around 5.5% at \(k > 15\). Figure 8 illustrates similar results for velocity components. The values \(\Delta_{v_x,k+j|k}\) and \(\Delta_{v_y,k+j|k}\) are ranging from 0.1% to 7.3%, and \(\Delta_{v_z,k+j|k}\) varies from 0.1% to 5.8%. At \(k > 15\), the relative errors for velocity change around 5.5–6.2%. So the results in Figures 7 and 8 demonstrate that for our example the application of the proposed decentralized predictor can produce good results for a long time period.

8. Conclusion

In some control problems, nonlinear functionals of state variables are interpreted as cost functions, which denote useful
Centralized fusion predictor
Decentralized fusion predictor

Figure 3: Centralized and decentralized MSEs of fusion predictors for $e_{p,y,k+s}$.

Centralized fusion predictor
Decentralized fusion predictor

Figure 4: Centralized and decentralized MSEs of fusion predictors for $e_{v,y,k+s}$.

Centralized fusion predictor
Decentralized fusion predictor

Figure 5: Centralized and decentralized MSEs of fusion predictors for $e_{p,z,k+s}$.

Centralized fusion predictor
Decentralized fusion predictor

Figure 6: Centralized and decentralized MSEs of fusion predictors for $e_{v,z,k+s}$.

information of the target systems for control. To predict an NFS under a multisensory environment, prediction fusion algorithms are proposed and their estimation accuracies are discussed. In general, the centralized fusion algorithm is considered the most accurate. However, owing to the inherent drawbacks of centralized processing, here the decentralized algorithm is found to be the best between fusion prediction algorithms. To show performance of the fusion predictors for NFS by practical application, multisensory fusion prediction of unit vector of position and velocity under constant acceleration motion of SLV is considered. In the example part, the comparative analysis and simulation results show that the proposed decentralized fusion predictor for NFS has competitive performance in an aspect of MSE and relative errors.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.
Figure 7: Relative errors for position components.

Relative error (%)

Time (k)

- Relative error (x-pos)
- Relative error (y-pos)
- Relative error (z-pos)

Figure 8: Relative errors for velocity components.

Relative error (%)

Time (k)

- Relative error (x-vel)
- Relative error (y-vel)
- Relative error (z-vel)

References

[1] Y. Zhu, J. Zhou, X. Shen, E. Song, and Y. Luo, Networked Multisensor Decision and Estimation Fusion, CRC Press, Boca Raton, Fla, USA, 2013.

[2] H. B. Mitchell, Multi-Sensor Data Fusion: An Introduction, Springer, 2010.

[3] M. E. Liggins, D. L. Hall, and J. Llinas, Eds., Handbook of Multisensor Data Fusion: Theory and Practice, The Electrical Engineering and Applied Signal Processing Series, CRC Press, Boca Raton, Fla, USA, 2nd edition, 2009.

[4] Y. M. Zhu, Multisensor Decision and Estimation Fusion, Kluwer Academic Publishers, Boston, Mass, USA, 2002.

[5] Y. Bar-Shalom and L. Campo, “The effect of the common process noise on the two-sensor fused-track covariance,” IEEE Transactions on Aerospace and Electronic Systems, vol. 22, pp. 803–805, 1986.

[6] X. R. Li, Y. M. Zhu, J. Wang, and C. Z. Han, “Optimal linear estimation fusion—part I: unified fusion rules,” IEEE Transactions on Information Theory, vol. 49, no. 9, pp. 2192–2208, 2003.

[7] V. Shin, Y. Lee, and T.-S. Choi, “Generalized Millman’s formula and its application for estimation problems,” Signal Processing, vol. 86, no. 2, pp. 257–266, 2006.

[8] V. Shin, G. Shevlyakov, and K. Kim, “A new fusion formula and its application to continuous-time linear systems with multisensor environment,” Computational Statistics and Data Analysis, vol. 52, no. 2, pp. 840–854, 2008.

[9] S.-L. Sun and Z.-L. Deng, “Multi-sensor information fusion kalman filter weighted by scalars for systems with colored measurement noises,” Journal of Dynamic Systems, Measurement and Control, vol. 127, no. 4, pp. 663–667, 2005.

[10] J. Zhou, Y. Zhu, Z. You, and E. Song, “An efficient algorithm for optimal linear estimation fusion in distributed multisensor systems,” IEEE Transactions on Systems, Man, and Cybernetics, vol. 36, no. 5, pp. 1000–1009, 2006.

[11] K. C. Chang, R. K. Saha, and Y. Bar-Shalom, “On optimal track-to-track fusion,” IEEE Transactions on Aerospace and Electronic Systems, vol. 33, no. 4, pp. 1271–1276, 1997.

[12] K. C. Chang, T. Zhi, and R. K. Saha, “Performance evaluation of track fusion with information matrix filter,” IEEE Transactions on Aerospace and Electronic Systems, vol. 38, no. 2, pp. 455–466, 2002.

[13] J. A. Roekker and C. D. McGillem, “Comparison of two-sensor tracking methods based on state vector fusion and measurement fusion,” IEEE Transactions on Aerospace and Electronic Systems, vol. 24, no. 4, pp. 447–449, 1988.

[14] H. R. Song, M. Jeon, Y. S. Lee, T.-S. Choi, and V. Shin, “Two fusion predictors for multisensor discrete-time linear system,” International Journal of Robotics and Automation, vol. 24, no. 4, pp. 338–345, 2009.

[15] H.-R. Song, M.-G. Jeon, and V. Shin, “Distributed fusion prediction for mixed continuous-discrete linear systems,” in Discrete Time Systems, chapter 3, pp. 39–52, InTech, Rijeka, Croatia, 2011.

[16] A. G. O. Mutambara, Decentralized Estimation and Control for Multisensor Systems, CRC Press, Boca Raton, Fla, USA, 1998.

[17] I. Y. Song, V. Shin, S. Lee, and W. Choi, “Multisensor estimation fusion of nonlinear cost functions in mixed continuous-discrete stochastic systems,” Mathematical Problems in Engineering, vol. 2014, Article ID 218381, 12 pages, 2014.

[18] Y. Bar-Shalom, X. R. Li, and T. Kirubarajan, Estimation with Applications to Tracking and Navigation, John Wiley & Sons, New York, NY, USA, 2001.

[19] F. L. Lewis, Optimal Estimation: With An Introduction to Stochastic Control Theory, John Wiley & Sons, New York, NY, USA, 1986.

[20] C. D. Karlgaard and H. Shen, “Robust state estimation using desensitized divided difference filter,” ISA Transactions, vol. 52, no. 5, pp. 629–637, 2013.

[21] V. S. Pugachev and I. N. Sinitsyn, Stochastic Differential Systems. Analysis and Filtering, John Wiley & Sons, Chichester, UK, 1987.

[22] R. Kan, “From moments of sum to moments of product,” Journal of Multivariate Analysis, vol. 99, no. 3, pp. 542–554, 2008.
[23] S. Julier, J. Uhlmann, and H. F. Durrant-Whyte, "A new method for the nonlinear transformation of means and covariances in filters and estimators," *IEEE Transactions on Automatic Control*, vol. 45, no. 3, pp. 477–482, 2000.

[24] S. J. Julier and J. K. Uhlmann, "Unscented filtering and nonlinear estimation," *Proceedings of the IEEE*, vol. 92, no. 3, pp. 401–422, 2004.

[25] D.-J. Lee, "Nonlinear estimation and multiple sensor fusion using unscented information filtering," *IEEE Signal Processing Letters*, vol. 15, pp. 861–864, 2008.

[26] L. Chang, B. Hu, A. Li, and F. Qin, "Transformed unscented Kalman filter," *IEEE Transactions on Automatic Control*, vol. 58, no. 1, pp. 252–257, 2013.

[27] T. Yuan, Y. Bar-Shalom, P. Willett, E. Mozeson, S. Pollak, and D. Hardiman, "A multiple IMM estimation approach with unbiased mixing for thrusting projectiles," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 48, no. 4, pp. 3250–3267, 2012.

[28] R. Singer, "Estimating optimal tracking filter performance for manned maneuvering targets," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 6, no. 4, pp. 473–483, 1970.

[29] J. Cesar Bolzani de Campos Ferreira and J. Waldmann, "Covariance intersection-based sensor fusion for sounding rocket tracking and impact area prediction," *Control Engineering Practice*, vol. 15, no. 4, pp. 389–409, 2007.

[30] P. Suchomski, "Explicit expressions for debiased statistics of 3D converted measurements," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 35, no. 1, pp. 368–370, 1999.

[31] F. J. Regan and S. M. Anandakrishnan, *Dynamics of Atmospheric Re-Entry*, AIAA Education Series, AIAA, 1993.