A Novel LOS Rate Estimation Method Based on Images for Strap-down Inertial Guidance

Zheng Xu 1,2,3,4,5,*, Haibo Luo 1,2,4,5, Bin Hui 1,2,4,5, and Zheng Chang 1,2,4,5

1 Shenyang Institute of Automation, Chinese Academy of Sciences, Shenyang 110016, China
2 Institutes for Robotics and Intelligent Manufacturing, Chinese Academy of Sciences, Shenyang 110016, China
3 University of Chinese Academy of Sciences, Beijing 100049, China
4 Key Laboratory of Opto-Electronic Information Processing, Chinese Academy of Science, Shenyang 110016, China
5 The Key Lab of Image Understanding and Computer Vision, Shenyang 110016, China

* Correspondence: xuzheng@sia.cn; Tel.: +86-024-2397-0757

Abstract. With the development of technology, precision guided weapon is becoming more and more important in modern war. In order to launch our recent guidance system on medium and small guided weapons, we propose a method to obtain the LOS rate by combining information from both camera and gyroscope. To be specific, we firstly calculate the body LOS angle through transforming the image pixel coordinate system into the image physical coordinate system according to camera internal parameters; then subtract the missile motion information contained in the measurement signal of the seeker and finally the LOS rate is deduced. Comparing with traditional gimbaled seekers, our strap-down seekers with camera have significantly reduced costs and influences caused by external environments on platform.

1. Introduction
Strap-down seekers have attracted more and more attention in many military affairs of different countries [1]. They have outstanding performance comparing with high-cost gimbaled seekers.

Strap-down seeker is not able to measure the line-of-sight (LOS) rate under the inertial coordinate system directly [2-3], because all measuring equipments are fixed on missile body [4]. Therefore, LOS rate is required to extracted by either subtracting the missile attitude angle from the error angle which measured by the strap-down seekers, or estimating LOS rate using Kalman filter [5-8]. A lossless Kalman filter is used to reconstruct LOS rate information in reference [9]. An indirect extraction method based on missile target angle tracking model and Kalman filter is proposed in reference [10]. Estimation of LOS rate by α-β filter is proposed in reference [11]. Reference [12] redefines a new kind of LOS in guidance system for effective tracking. However, the existing LOS rate extracting methods in above papers are complicated while our proposed method takes advantages of image information taken by our strap-down image seeker and simplifies the LOS rate decoupling operation.
2. Coordinate system and parameters

It needs several coordinate systems to describe the motion of the projectile [13]. All the parameters involved here described as \( \{X_n, Y_n, Z_n\} \) are under the ground coordinate system \( OX_nY_nZ_n \).

The three axes of this coordinate system are east, north and sky: the \( X_n \) axis points to the tangent plane of the earth's surface and points to the East, the \( Y_n \) axis points to the north, the \( Z_n \) axis under the right-hand coordinate system points to the sky, and the subscript \( n \) represents the ground coordinate system. \( \{X_b, Y_b, Z_b\} \) is the missile body coordinate system: \( Y_b \) axis points to the direction of motion along the symmetry axis or rotation axis of the missile body, \( X_b \) axis and \( Z_b \) axis point to the right and point to the top along the radial direction of the missile body, and the subscript \( b \) represents the missile body coordinate system. Figure 1 shows the Euler angle relationship between two coordinate systems.

![Figure 1. Rotation order of Euler angle between the ground coordinate system and the missile body coordinate system](image)

The conversion between the ground system and the missile system is to rotate the ground coordinate system around the \( Z_n \) axis at yaw angle \( \psi_n^b \) firstly, then rotate it around the new \( X \) axis at the pitch angle \( \theta_n^b \), and finally rotate it around the new \( Y \) axis at the roll angle \( \phi_n^b \). In this way, the relationship between the two coordinate systems can be expressed by the direction cosine matrix (DCM) \( C_n^b \), which is the expression of three Euler angles \( \psi_n^b, \theta_n^b \) and \( \phi_n^b \):

\[
C_n^b = \begin{bmatrix}
\cos \phi \cos \psi + \sin \phi \sin \psi \sin \theta & -\cos \phi \sin \psi + \sin \phi \cos \psi \sin \theta & -\sin \phi \cos \theta \\
\sin \phi \cos \theta & \cos \psi \cos \theta & \sin \theta \\
\sin \phi \cos \psi - \cos \phi \sin \psi \sin \theta & -\sin \phi \sin \psi - \cos \phi \cos \psi \sin \theta & \cos \phi \cos \theta
\end{bmatrix}
\]  

(1)

3. Extraction and calculation of LOS rate

3.1 Extraction of body line of sight angle

The body LOS angle represents the relationship between the target and the missile body, which can be represented by two azimuth angles as shown in Figure 2.

Body LOS yaw angle \( q_\phi \): the angle between the missile and the target line and the plane of \( OX_bY_b \) in the missile body coordinate system

Body LOS pitch angle \( q_\theta \): the angle between the projection of the missile and the target on the plane of \( OX_bY_b \) in the missile body coordinate system and the \( OX_b \) axis

The transformation matrix \( L(q_\phi, q_\theta) \) from body LOS coordinate system to the body coordinate system can be expressed as following:
Using the method of rotation matrix, we can get the transformation from the body LOS coordinate system to the body coordinate system:

\[
L(q_\beta, q_\alpha) = \begin{bmatrix}
\cos q_\alpha \cos q_\beta & -\sin q_\alpha \cos q_\beta & \sin q_\beta \\
\sin q_\alpha & \cos q_\alpha & 0 \\
-\sin q_\beta & \sin q_\alpha \sin q_\beta & \cos q_\alpha \sin q_\beta \cos q_\beta
\end{bmatrix}
\] (3)

Body LOS angles can be obtained by imaging the target in the camera. As shown in Figure 2, the three coordinate axes of the missile body coordinate system and the camera imaging coordinate system are parallel to each other. By transforming the image pixel coordinate system into the image physical coordinate system according to camera internal parameters and combining with the geometric relationship, the body LOS angle \([q_\beta, q_\alpha]^T\) can be calculated.

\[\begin{align*}
\alpha &= \tan^{-1}\left(\frac{x-x_0}{f}\right) = \tan^{-1}\left(\frac{(u-u_0)d_x}{f}\right) \\
\beta &= \tan^{-1}\left(\frac{y-y_0}{f}\right) = \tan^{-1}\left(\frac{(v-v_0)d_y}{f}\right)
\end{align*}\] (4)

3.2 Calculation of LOS angle

The definitions of LOS angles are defined below:

LOS yaw angle \(q_\alpha\): the angle between the missile and the target line and the plane of \(OX_nY_n\) in the ground coordinate system

LOS pitch angle \(q_\beta\): the angle between the projection of the missile and the target on the plane of \(OX_nY_n\) in the ground coordinate system and the \(OX_n\) axis

The same as transformation matrix \(L(q_\beta, q_\alpha)\), transformation matrix \(L(q_\lambda, q_\gamma)\) which represents from LOS coordinate system to the ground coordinate system is
Because the seeker can only measure the body LOS angle of the missile body. In the real guidance process, the missile body is moving relative to the inertial space (earth space). Therefore, there are two parts of the measured LOS angle: one is the LOS angle of the target relative to the inertial space, which is also needed in the actual guidance process; the other is the movement information of the missile body, which cannot be obtained directly. In order to achieve the LOS rate, the missile motion information contained in the measurement signal of the seeker needs to be subtracted. The following is the derivation of the required LOS angle based on the relationship between the coordinate systems defined above. The coordinates of the target in the LOS coordinate system and the body LOS coordinate system are the same. If it is set to \( (x, 0, 0) \), the coordinates of the target in the body coordinate system and the ground coordinate system can be expressed as:

\[
\begin{pmatrix}
X_n \\
Y_n \\
Z_n
\end{pmatrix} = L(q_\alpha, q_\gamma) \begin{pmatrix}
x \\
y \\
z
\end{pmatrix} = \begin{pmatrix}
x \cos q_\alpha \cos q_\gamma \\
x \sin q_\alpha \\
-x \sin q_\alpha \sin q_\gamma \\
\end{pmatrix}
\]

(6)

\[
\begin{pmatrix}
X_b \\
Y_b \\
Z_b
\end{pmatrix} = L(q_\beta, q_\delta) \begin{pmatrix}
x \\
y \\
z
\end{pmatrix} = \begin{pmatrix}
x \cos q_\beta \cos q_\delta \\
x \sin q_\beta \\
-x \sin q_\beta \cos q_\delta
\end{pmatrix}
\]

(7)

According to the transformation rule between body coordinate system and the ground coordinate system that deduced before in (1), we can get equation:

\[
\begin{pmatrix}
X_n \\
Y_n \\
Z_n
\end{pmatrix} = C_n^b \begin{pmatrix}
X_b \\
Y_b \\
Z_b
\end{pmatrix}
\]

(8)

After solving the above equation, LOS angles \( q_\alpha \) and \( q_\gamma \) are:

\[
q_\alpha = \tan^{-1} \frac{M}{N}
\]

\[
M = \cos q_\beta \left( \cos \phi \cos \psi \sin \theta - \cos \phi \sin \psi - \sin q_\beta \cos \phi \cos \theta \right)
\]

\[
N = \cos q_\beta \left( \cos \phi \cos \psi + \sin \phi \sin \psi \sin \theta \right) - \tan q_\alpha \left( \sin \phi \sin \psi + \cos \phi \cos \psi \sin \theta \right)
\]

(9)

4. Method validation

4.1 Method feasibility analysis

It is to be noticed that \( q_\alpha \) and \( q_\beta \) are deduced before in (4), three missile attitude angles \( \psi, \theta \) and \( \phi \) are able to be calculated through integral operation of data from gyroscope over time \( t \).

\[
\begin{align*}
\psi &= \int_0^t \omega_\psi dt + \psi_0 \\
\theta &= \int_0^t \omega_\theta dt + \theta_0 \\
\phi &= \int_0^t \omega_\phi dt + \phi_0
\end{align*}
\]

(10)

In equation (10), \( \omega_\psi, \omega_\theta \) and \( \omega_\phi \) are the real-time angular velocity provided by gyroscope. Finally, under the condition that \( q_\alpha, q_\beta, \psi, \theta \) and \( \phi \) are known, the LOS angles \( q_\alpha \) and \( q_\gamma \) can be obtained from (9) and then the LOS rate \( q_\alpha^\prime \) and \( q_\gamma^\prime \) can be obtained by differential formula. However, the instantaneous field of view of strap-down imaging seeker is larger than that of traditional seeker, which has more background noise and thermal noise. The differential algorithm can amplify the
measurement noise, which is not desirable in practice. Therefore, the unscented Kalman filter (UKF) is applied to the above model to estimate the LOS rate.

4.2 Method error analysis

![Tracking Results on images in datasets taken by the Defence Advanced Research Projects Agency (DARPA): red boxes indicate tracking results](image)

Figure 3. Tracking Results on images in datasets taken by the Defence Advanced Research Projects Agency (DARPA): red boxes indicate tracking results

Red boxes in figures above indicate the tracking results using our latest deep tracking network. The source image resolution is 640*480. We calibrated and recorded the deviation of results below in Figure 4.

![Centering Error - Average](image)

Figure 4. Statistical results of SiamFC and our tracking method in DARPA VIVID datasets

Figure 4 shows the statistical results for 5 video sequences (over 3000 frames) and the average of centering error in pixels. The red curve indicates centering error in our tracking method. In most frames our method has a centering error less than 10 pixels. This deviation is tolerable as long as the detected target center is still at the real target position. \( d_x, d_y \) and \( f \) in (4) can be calculated and acquired either by calibration or from camera parameters. For instance, our camera has the following parameters: \( d_x = d_y = 1.5 \mu m \) and \( f = 4.15 \text{ mm} \). The maximum pixel error is 20 pixels, which leads to a Maximum angle error of 0.414°. In most frames our centering error is around 5 pixels which leads to an angle error of 0.104°. The influence of the errors of LOS yaw angle and LOS pitch angle on the final LOS angle is related to the attitude angle of the projectile. Generally, when LOS angle error is 0.414°, the errors of LOS yaw angle and LOS pitch angle are smaller than 0.414°. The algorithm proposed in this paper provides a new idea for the existing target guidance and detection system, which makes the system simpler and lighter without platforms.

5. Conclusion

In this paper, a method of combining information from both camera and gyroscope to extract LOS rate in the target tracking system is proposed. Pixel coordinates of the sequence image are utilized to get
the body line of view angle of the target, and the method of real-time angular velocity integral is utilized to get the attitude angle of the missile body. Then, the calculation of LOS rate is verified by method feasibility analysis. Method error analysis shows that the error of LOS angle is less than 0.414°.

References
[1] Captain Thomas, Callen R. Guidance law design for tactical weapons with strapdown seekers[R]. AIAA-79-1732.
[2] Zhou Ruiqing, Liu Xinhua, Shi Shouxia, et al. Strap –down seeker stability and track technology [M]. Beijing: National Defence Industry Press, 2010: 8.
[3] Yao Yu, Zhang Guojiang. Discussion on strapdown imaging guidance system[J]. Infrared and Laser Engineering, 2006, 35(1): 1-6.
[4] Qian Xingfang, Lin Ruixiong, Zhao Yanan. Missile Flight Mechanics [M]. Beijing : Beijing Institute of Technology Press, 2000: 28-77.
[5] Zhang Yue, Chu Hairong. Strapdown optical seeker characteristics and multi -dimension optimal guidance law [J]. Infrared and Laser Engineering, 2013, 42 (11): 2967- 2973.
[6] Zhang Xin, Du Zhiyuan, Qiao Yanzeng, et al. Study on linear field of strapdown semi -active laser seeker [J]. Chinese Optics, 2015, 8(3): 415-421.
[7] Zhang Yue, Chu Hairong. Technical characteristics of strapdown image seeker guidance [J]. Optics and Precision Engineering, 2014, 22(10): 2825-2831.
[8] Zhang Yue, Liu Bo, Yin Shengli. Strapdown optical seeker: stabilization,tracking principle and system simulation [J]. Optics and Precision Engineering, 2008, 16 (10): 1942 - 1948.
[9] Song Jianmei, Kong Lixia, Fan Jianhua. The guidance information reconstruction of semi-strapdown imaging seeker guidance system [J]. Acta Armamentarii, 2010, 31 (12): 1573-1579.
[10] Jia Xiaoyuan, Zhao Chao. New stabilization control and guidance information extract approach with a semi-strapdown structure [J]. Infrared and Laser Engineering, 2011,
[11] Li Fugui, Xia Qunli. Study on air-to-ground missile with strapdown imaging infrared seeker against moving target[C].Proceedings of the 32nd Chinese Control Conference, 2013: 5149-5152. 40(12): 2474-2479.
[12] Qiao Pengpeng, Li Xiaobing, Yang Yuzhe. One calculational methods for the target-missile LOS angle[J]. Opto-Electronic Engineering, 2012, 39(7): 75-80.
[13] T. Harkins, “Understanding body-fixed sensor ouptuts from projectile flight experiments,”U.S. Army Research Laboratory, ARL-TR-3029, September 2003.