Anomalous Charge Dynamics in the Superconducting State of Underdoped Cuprates

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Abstract

We present fermi liquid expressions for the low temperature behavior of the superfluid stiffness, explain why they differ from those suggested recently by Lee and Wen, and discuss their applicability to data on high-$T_c$ superconductors. We find that a consistent description requires a strong, doping dependent anisotropy, which affects states near the zone corners much more strongly than those near the zone diagonals.
The work reported here was motivated by a recent paper of Lee and Wen on "Unusual Superconducting State of Underdoped Cuprates". Their point of view, which we share, is that because the superconducting state of the high-$T_c$ materials is apparently conventional and in particular supports well-defined quasiparticle excitations, it is appropriately described in terms of fermi liquid theory. Their main conclusion, that the temperature dependence of the superfluid stiffness is anomalous and implies unusual properties, is important and interesting. Their specific results must be interpreted with caution because their formulation of fermi liquid theory in the superconducting state (Eqs 1-4 of their paper) disagrees in an important way with the standard formulation. In this note we present the usual fermi liquid theory of the charge response of the superconducting state, explain why it differs from Eqs 1-4 of Lee and Wen, and apply it to penetration-depth data and discuss the underlying physics.

We begin, however, with data. We write the penetration depth $\lambda$, in terms of a superfluid stiffness per CuO$_2$ plane $\rho_s$ via $\rho_s = \hbar^2 c^2 b/16\pi e^2 \lambda^2$. Here $b$ is the mean interplane spacing. This definition yields the coefficient of the $\frac{1}{2} (\nabla \theta)^2$ term in the Ginzburg-Landau theory but differs by a factor of 4 from the quantity defined in Eq. 1. The most extensively studied family of materials is the 'YBCO' family; to avoid complications due to the CuO$_2$ chains we consider the penetration depth for current flow in the basal plane but perpendicular to the chains. In YBCO the length $b \approx 5.5\text{Å}$. The measured penetration depths are $\lambda_{aa} = 1600$ for $YBa_2Cu_3O_7$ and $\lambda_{aa} = 2000$ for $YBa_2Cu_3O_{6.6}$ implying $\rho_{saa} = 11meV$ ($O_7$) and $\rho_{saa} = 7meV$ ($O_{6.6}$). In a clean weak-coupling superconductor $\rho_s$ may be determined from the conduction band plasma frequency $\omega_p$ via $\rho_s = (\hbar \omega_p)^2 b/(16\pi e^2)$; the value calculated using band theory is $(h\omega_p)^2 = 9eV^2$ implying $\rho_{saa-band} = 70meV$. The renormalization of $\rho_s$ from the band theory prediction is large (factor of 6-10) and doping dependent.

The temperature dependence of $\rho_s$ for YBCO is given in Ref. 3. For both $O_7$ and $O_{6.6}$ $\rho_s$ is well fit by $\rho_s(T) = \rho_s(T = O)(1 - 0.5T/T_c)$. Converting the data to absolute units we find $\lim_{T \to 0} d\rho_s/dT = 0.06meV/K$ for both materials. The theoretical result is $d\rho_s/dT = (\ln 2/2\sqrt{2})(v_F/a)/d\Delta/d\theta$ where $d\Delta/d\theta$ is the angular derivative of the gap function at the
gap node and \( a = 3.8\AA \) is the lattice constant. Photoemission results\(^2\) on BSCCO show that if the gap is defined as the displacement of the midpoint of the leading edge of the photoemission spectrum from the fermi level, then \( d\Delta/d\theta \approx 40\text{meV} \approx 450\text{K} \); using the band theory \( v_F \approx 4eV - \tilde{A} \) yields \( d\rho_s/dT = 0.5\text{meV/K} \).

We now interpret these data in terms of fermi liquid theory. Fermi liquid theory is specified by the quasiparticle velocity \( v_F(\theta) \) and the Landau interaction function \( f(\theta, \theta') \); these are functions defined on the fermi surface which for the electronically two-dimensional high-\( T_c \) materials is a simple closed curve \( p_F(\theta) \) with \( \theta \) the usual polar angle (we neglect the chain bands and any bonding-antibonding splitting arising from the bilayer structure).

A basic assumption of the fermi liquid analysis is that the low-T dependence is due to thermally excited quasiparticles. Several authors have claimed\(^6,7\) that phase fluctuations of the superconducting order parameter can also lead to such a T-dependence. However, these models require a large density of normal electrons at \( T = 0 \) to screen the Coulomb interaction. In high-\( T_c \) materials there are no such electrons at \( T = 0 \); the phase fluctuations will therefore be gapped, giving an exponentially small contribution. Even if screening is included, we believe the models of refs\(^6,7\) yield a \( T^2 \) temperature dependence. Claims to the contrary seem to have arisen from Eq 7 of ref\(^8\) in which a term involving the Matsubara sum \( \sum_{n=\infty}^{n=\infty} \) was written as \( 2 \ast \sum_{n=0}^{\infty} \) so that the \( n = 0 \) term was counted twice. Evaluation of \( \sum_{n=-\infty}^{n=\infty} \) yields a \( T^2 \) temperature dependence.

The fermi velocity may be determined from the dispersion of quasiparticle peaks observed in photoemission. The interpretation of photoemission data on high-\( T_c \) materials is still controversial, but there is general agreement that for momenta near the zone diagonal, well-defined peaks exist from which a dispersion can be extracted. The most detailed studies have focused on the BSCCO family of materials; the data for YBCO seem quite similar\(^9\). Fig. 3 of ref\(^10\) shows the dispersion along the zone diagonal for overdoped, optimally doped, and underdoped BSCCO samples. The data imply a \textit{doping-independent} quasiparticle velocity \( v = 1.3eV - \tilde{A} \); this is reduced from the band-structure value \( (v_F \approx 4eV) \) by about a
factor of 3, presumably because of electron-electron interaction effects. It is conventionally assumed (and has been demonstrated in some models\[^{11,12}\]) that the many-body renormalizations which change the zone-diagonal velocity by a factor of three are relatively isotropic around the fermi surface, so we might expect that the velocity everywhere to be changed by about the same factor. This, combined with the observation that the charge response should be dominated by the zone diagonals where the velocity is largest, suggests that if the Landau parameter contribution were negligible \(\rho_s\) would be renormalized by about a \textit{doping-independent} factor of three from the band theory prediction. The smaller value and especially the doping dependence \(\rho_s\) therefore suggests that the Landau interaction function plays an important role as predicted theoretically\[^{11,12}\]. We now show that while the Landau function may be important, the conventional picture makes predictions for \(d\rho_s/dT\) which are \textit{inconsistent} with the data.

The fermi liquid theory of the superconducting state was developed in\[^{2}\]. The result may be written:

\[
\rho_{sab} = \frac{\hbar^2 b}{2\pi^2} \int d\theta_1 d\theta_2 N(\theta_1)N(\theta_2)v_a(\theta_1)L(\theta_1)T_b(\theta_1, \theta_2) \tag{1}
\]

Here \(N(\theta) = 1/v(\theta)\) is the quasiparticle density of states at position \(\theta\) on the fermi surface, \(v_a\) is a component of the quasiparticle velocity and \(L(\theta) = \pi T \sum_n \Delta(\theta, \omega_n)^2/\Delta(\theta, \omega_n)^2\] is the usual angle-dependent Yoshida function. The vertex \(\vec{T}\) obeys the equation

\[
\vec{T} = \vec{v} + \langle fLT \rangle \tag{2}
\]

Here \(f\) is the Landau function defined above and products inside angle brackets are shorthand for integrals over the fermi line, weighted by density of states as in Eq. \[^{1}\]. Eq. \[^{2}\] may be solved formally and the solution used in Eq. \[^{1}\]. Noting that at \(T = 0\) \(L = 1\) and that the leading \(T\)-dependence is in \(L\) one finds:

\[
\rho_{saa}(T = 0) = <v_a(1 - f)^{-1}v_a > \tag{3}
\]

\[
\lim_{T \to 0} d\rho_{saa}/dT = <v_a(1 - f)^{-1}dL/dT(1 - f)^{-1}v_a > \tag{4}
\]
Eq. 4 differs from that written in ref 1 by the two factors of \((1 - f)^{-1}\). The authors of 1 obtained their result by analogy to Leggett’s theory of the spin susceptibility of \(^3He\); the results of 2 show the analogy is not appropriate. The physics is this: Landau parameters enter expressions for physical quantities because a perturbing field \(H\) excites quasiparticle-quasihole pairs at all points on the fermi line; i.e. it modifies the underlying fermi distribution and because of the electron-electron interaction this changes the quasiparticle energy dispersion to order \(H\), and thus affecting the number of excitations. This effect must be treated self-consistently. Now in \(^3He\) the condensate does not respond at linear order to a magnetic field (at least in the low-T “BW” phase), and because of the superconducting gap the number of quasiparticles at low T is very small; thus at low T changes to the quasiparticle dispersion may be neglected and Landau parameters drop out. By contrast a superconducting condensate does respond to a magnetic field (by producing a supercurrent); this supercurrent does correspond to a displacement of the underlying fermi distribution which does affect the superconducting quasiparticles, causing Landau parameter effects in the temperature dependence.

In fermi liquid theory one defines the Landau parameters \(A_{1s}\), \(F_{1s}\) via
\[
<v_a(1 - f)^{-1}v_a> = <v_a^2> / (1 - A_{1s}/2) = <v_a^2> (1 + F_{1s}/2).
\]
If the renormalization of the velocity is isotropic over the fermi surface we may evaluate \(<v_a^2>\) by dividing the band theory \(\rho_s\) by three (recall the density of states factor in the definition of \(<>\)) to obtain \(<v_a^2> = 23 meV\). By comparing this to the data we conclude that \((1 + F_{1s}/2) \approx 1/2 (O_7)\) and \((1 + F_{1s}/2) \approx 1/4 (O_{6,6})\). Now in a Galilean-invariant fermi liquid \(v_F = p_F/m^*\) and \(<v_a^2> \approx n/m^*.\) In a two dimensional Galilean-invariant system the Landau parameter obeys \(1 + F_{1s}/2 = m^*/m\) so the Landau parameter cancels the mass renormalization. In non-Galilean-invariant systems such as the high-\(T_c\) superconductors there is no a-priori relation between \(F_{1s}\) and \(m^*/m\). As we see, in the actual materials the observed \(\rho_s\) is smaller than one would expect from the observed \(v\); thus \(F_{1s}\) acts in the same direction as \(m^*\), instead of compensating it.

In many fermi liquids the angular dependence of the interaction is not too strong, so one expands the operator \(f\) in appropriate harmonics and retains only the one most closely
corresponding to $\vec{v}_v$; this amounts to replacing the operator $(1 - f)^{-1}$ by the scalar $1 + F_{1s}/2$ in both eqs. 3 and 4. We refer to this as the “conventional fermi liquid” approximation. A particular realization of such a fermi liquid is found in the slave boson calculations of\textsuperscript{11,12}. To compute $d\rho_s/dT$ in this conventional fermi liquid approach we take the ”band theory” value of $0.5 meV/K$ obtained earlier, divide by a factor of three for the velocity renormalization and a further factor of 4 for $YBa_2Cu_3O_7$ and 16 for $YBa_2Cu_3O_{6.63}$, yielding $d\rho_s/dT = 0.04 meV/K$ for $O_7$ and 0.01 $meV/K$ for $O_{6.6}$. Although the estimate of $d\rho_s/dT$ for $O_7$ is within a factor of two of the measured value of $0.06 meV/K$, the estimate for $O_{6.6}$ is badly off. Similarly, ref\textsuperscript{11} found $\rho_s(T) = \langle vL(T)v \rangle / (1 + c < vL(T)v >$ with $c \approx -A_{1s} \sim 1/x$, so $d\rho_s/dT \sim \rho_s(T = 0) < vdL/dTv > /c \sim 1/x^2$. Thus both the conventional fermi liquid approach and explicit calculations predict a strong doping dependence where none exists experimentally. In ref\textsuperscript{1} the disagreement of the result of ref\textsuperscript{11} with the observed $d\rho_s/dT$ was argued to indicate the inadequacy of the $U(1)$ gauge theory formalism used in\textsuperscript{11}. The present analysis makes it clear that the difficulty is more general.

The doubtful point of the discussion given above is the assumption of weak angular dependence around the fermi surface. This enters the argument in two places: first, that the factor-of-three velocity renormalization observed for quasiparticles along the zone diagonal applies all over the fermi surface, and second that the Landau interaction operator $(1 - f)^{-1}$ could be replaced by the Landau parameter $1 + F_{1s}/2$. These two issues are clearly related: an interaction giving a relatively larger velocity renormalization at the zone corners than at the zone diagonals will lead to a Landau function with a stronger angular dependence.

If the velocity renormalization increases rapidly as one moves away from the zone diagonal then the estimate of the renormalization of $\rho_s$ relative to band theory increases and the value of the Landau parameter decreases. There is evidence of anomalous flatness of bands and shortness of lifetimes in the vicinity of the zone corners\textsuperscript{14}. In order to explain the data the size of the ”non-flat” regions must be small and x-dependent. Evidence on this point is not conclusive.

Consider now the possibility of strong anisotropy in the Landau function. We see from
Eq. 4 that because the quasiparticle excitations are concentrated in the nodes, at low \( T \) the temperature-dependent quantity \( dL/dT \) is non-zero only near the zone diagonal. If the Landau function \( f(\theta_1, \theta_2) \) were anomalously small for either \( \theta_1 \) or \( \theta_2 \) near this point then the effect of the Landau function on \( d\rho_s/dT \) would be greatly reduced in agreement with data. The range of angles over which this occurs must be small and \( f \) must be \( x \)-dependent.

To summarize: the temperature dependence of the superfluid stiffness strongly suggests that some process acting in the high-\( T_c \) superconductors strongly suppresses the contribution of the particles away from the zone diagonal to the current.

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