Recent Advances in Dimensionality Reduction Modeling and Multistability Reconstitution of Memristive Circuit

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1.Introduction

The intrinsic memory property [1] of the memristor makes the memristor-based nonlinear circuits and systems easily exhibit the state initial-dependent dynamical behaviors. By keeping the system parameters unchanged and changing the state initials, the trajectories of the memristive circuits and systems can asymptotically approach to different stable states, showing the state initial-dependent multistability [2, 3] or extreme multistability [4–7], i.e., coexisting multiple or infinitely many attractors. On the one hand, these coexisting multistable modes can provide more flexibility for information engineering applications [8–11]; on the other hand, it may also lead the application systems to abnormal working states [12]. These ungovernable problems pose a severe test for realizing the control of multistable modes. In addition, the dynamical behaviors of memristive circuits and systems are highly dependent on the state initials, but the state initials cannot be explicitly expressed in their state equations, which bring great obstacles in mechanical analyses of the state initial-dependent dynamical behaviors. Moreover, the memristive circuits and systems usually have line equilibrium set, plane equilibrium set, no equilibrium, or stable equilibrium, whose stabilities and induced dynamical behaviors are hard to be analyzed by using the traditional stability theory [12]. Therefore, accurate prediction, quantitative analysis, and physical control of such special phenomena have become an important research problem in the field of information science.

Traditional control strategies usually adopted nonfeedback control strategy to convert a multistable system to a mono-stable system [13–17] or adopted feedback control...
strategy to stabilize the system in a certain desired state [13, 18, 19]. But these control strategies cannot achieve the multistable control. To solve this problem, researchers proposed different dimensionality reduction modeling schemes based on the memristive circuit and system [20, 21]. In [22], the concept of dimensionality reduction modeling was proposed, which modeled the memristive circuits with two physical quantities of flux and charge as main state variables, and the dimension of the obtained flux-charge model was lower than that of the traditional voltage-current model. Bao et al. [23] built the reduce-ordered flux-charge model of a two-memristor-based circuit and analyzed its dynamical characteristics via the voltage-current and flux-charge models. Bao et al. [24] qualitatively pointed out that the flux-charge model of the memristive circuit can be equivalent to realize its dynamical behavior in the voltage-current model. However, in these early studies, the state initials of the memristive circuit were not explicitly expressed in the dimensionality reduction model [25], resulting in the information loss of state initials of the memristive circuit. Therefore, the established dimensionality reduction model could not reflect the multistability of the memristive circuit. In recent years, Corinto proposed an incremental flux-charge analysis method [26, 27] and applied it to the dimensionality reduction modeling of memristor-based cellular neural networks [28] and memristor-based oscillator array [29]. With this method, the state initials of the memristive circuit can be expressed as standalone system parameters in the flux-charge domain [12], which is conducive to the analyses and measurements of the state initial-dependent dynamical behaviors. Thereafter, this method was applied for reconstituting and analyzing extreme multistability of ideal memristor-based circuits [12, 20, 30, 31]. On this basis, the incremental integral transformation method was proposed for the analyses of memristive systems [21, 32]. Hereto, a complete set of dimensionality reduction reconstruction strategy for ideal memristor-based circuits and systems was thereby formed.

It should be noted that, in the original literature, these two methods were called the flux-charge analysis method [26, 27] and the state variable mapping method [21], respectively. But the state variables of the dimensionality reduction model are actually expressed by the incremental integral of the original memristive circuit’s and system’s state variables, whose core idea is integral transformation. Therefore, in this paper, these two methods are called the incremental flux-charge analysis method and the incremental integral transformation method, respectively. By using these two methods, on the one hand, the implicit state initials in the original memristive circuit and system can be transformed into the explicitly state initial-related system parameters appearing in the dimensionality reduction model. On the other hand, the line or plane equilibrium set in the original memristive circuit and system can be converted into the certain equilibrium, which is beneficial to the elaboration of the dynamic mechanism. In short, the state initial-dependent dynamical behaviors of the original memristive circuit and system are transformed into the parameter-dependent dynamical behaviors of the dimensionality reduction model. In addition, synchronization, as one of the basic nonlinear phenomena, has received extensive attention in the field of basic theory and engineering applications [33]. In the nonmemristor-coupled system, the state initials have significant effect on the synchronization characteristics [34–36]. Based on the above dimensionality reduction methods, in the study of the synchronization on the memristor-coupled system [37], the synchronization effect of the state initial-related system parameters can be studied quantitatively after the state initials are expressed explicitly in the state equation.

The incremental flux-charge analysis method and the incremental integral transformation method realize the mapping transformation of the state variable domain by means of integral transformation and describe and analyze the multistability of the original memristive circuit and system based on the transformed state variable domain, which provides theoretical basis for the precise prediction, quantitative analysis, and physical control of such special phenomena. In this paper, the dimensionality reduction modeling and multistability reconstruction of the memristive circuit and system are summarized to help researchers fully understand the state initial-dependent multistability dimensionality reduction reconstruction strategy of the memristive circuit and system. Then, the reconstruction strategy is applied to the synchronization research of the memristor-coupled system to quantitatively study the influence of state initials on synchronization.

2. Multistability of Memristive Circuit in the Voltage-Current Domain

2.1. Multistability and Coexisting Multiple Attractors. Multistability [13, 38–41] is an inherent phenomenon of the nonlinear dynamical system, in which multiple attractors coexist with the change of state initials under the fixed system parameters. The term “multistability” first appeared in the study of visual perception [42]. Arecchi also found the coexistence attractors’ phenomenon in electronic circuits [43] and gas lasers [44]. Later, a large number of theoretical and experimental studies have explored this special phenomenon in different systems [45–55]. In addition, in some special coupled systems [56, 57], the phenomenon of coexisting infinitely many attractors, i.e., extreme multistability [58–61], can also be observed.

In recent years, the hidden attractor [62–76], as a special class of newly defined attractor, has attracted extensive attention from researchers. The attractor that we usually say is also called the self-excited attractor, which is caused by the unstable equilibrium. Unlike the self-excited attractor [67, 77], the attraction basin with the hidden attractor does not intersect any equilibrium [78], and its existence increases the uncertainty of the system. When the system has a stable equilibrium [5] or no equilibrium [3, 79, 80], the induced multistability is called hidden multistability. Note that when the system has a stable equilibrium [5] and can produce dynamical behavior other than the point attractor, it can be confirmed that the system has hidden multistability. However, if the system has no equilibrium and can produce
only one stable oscillation behavior, the system is a hidden system, but it does not have multistability [81].

It has been shown that the hidden attractor is sensitive to the state initial of the system. In the domain of stable equilibrium, the system trajectory will converge to the stable point, but under the special state initial, the system trajectory can form the stable chaotic attractor or periodic limit cycle. Figures 1(a) and 1(b), respectively, show the self-excited and hidden attractors generated by a novel Chua’s circuit [67]. Figures 1(c) and 1(d), respectively, show the local plane projection of Figures 1(a) and 1(b), where the red dots are a pair of nonzero equilibria. According to the orbit of the attractor and the position relationship of the equilibrium in the figure, it can be seen that the attraction basin of the self-excited attractor must cover the unstable equilibrium, while the attraction basin of the hidden attractor with the neighborhood of the stable equilibrium does not overlap. Therefore, the self-excited attractor and hidden attractor can be clearly distinguished according to the intersection relationship between the attraction basin and the equilibrium neighborhood in phase space.

In general, coexisting infinitely many attractors can be classified into the following four types:

(a) Different attractor types: hyperchaotic attractor, chaotic attractor, quasi-periodic limit cycle, periodic limit cycle, and stable point

(b) Different attractor topologies: the same type of attractor has completely different topologies, such as spiral and double-scroll attractors, scroll complete and incomplete attractors, and attractors with different dynamic amplitude

(c) Different number of limit cycles: limit cycles with different number of periods

(d) Different attractor positions: attractors are located in different phase space.

2.2. The Difference between Multistability and Chaotic Initial Sensitivity. Since Chua put forward the generalized concept of the memristive system [82], the circuit and system constructed by the memristor have received great attention. In the early memristive circuit [83–87], scholars found that the stability of the equilibrium was closely related to the state initial of the memristor, which meant that the coexistence of multiple attractors was easy to occur in the memristive circuit. Then, in the memristive circuit, Bao found that the state initial-dependent dynamical behavior was a special kind of multistability phenomenon, i.e., extreme multistability. And, in [88], Bao et al. explicitly proposed the extreme multistability in the memristive circuit for the first time, that is, in the memristive circuit with line equilibrium, there was a peculiar coexistence infinitely many attractor phenomenon, which relied on the internal state initial of the memristor. In particular, Jafari et al. [89] pointed out the difference between the state initial-dependent dynamical behavior (extreme multistability) of the memristive system and the chaotic initial sensitivity of the general nonlinear dynamical system. That is, for the general nonlinear dynamical system, the initial sensitivity of system trajectory was only a quantitative change, and the trajectories of the system starting from the different state initials would traversal in the corresponding attraction region along different trajectories, without changing the dynamical properties of the system. However, the extreme multistability of the memristive system was a qualitative change; the change of state initial could cause the trajectory of the memristive system to jump between the attraction domains of different dynamical behaviors. Therefore, the state initial-dependent multistability in the memristive circuit and the chaotic initial sensitivity in the general chaotic circuit are two completely different concepts.

2.3. Multistability in Memristive Circuit and System. Since physical accessibility of memristors has been reported [90], lots of investigations were carried out for various memristor-based application circuits and systems, including cellular nonlinear/neural network [91], spiking and bursting neuron circuit [92], active band-pass filter-based oscillating circuit [93], FitzHugh–Nagumo neuron circuit [94], recurrent neural network [95], hypogenetic jerk chaotic system [21, 96, 97], and hyperchaotic autonomous system [98], from which rich dynamical behaviors have been manifested by theoretical studies, numerical simulations, and experimental measurements. The results showed that the stabilities of the memristive circuit and system, especially the ideal memristor-based nonlinear circuit and system, had a great relationship with the state initial of the memristor [88, 99]. Therefore, the coexisting infinitely many attractors appeared in such memristive circuit and system [12, 20, 100]. Under the fixed system parameters, the solution trajectories of the system can be represented by diverse stable states with the varied state initials, such as point, period, quasi-period, chaos, and hyperchaos [7, 98, 101, 102]. Such a special phenomenon is mostly relevant to no equilibrium [103, 104], limited number equilibria [105], or even infinitely many equilibria [6, 106]. Particularly, when the number of coexisting attractors tends to infinite, the phenomenon is called extreme multistability [39, 56, 89, 107–109].

In principle, the coexisting infinitely many attractors caused by extreme multistability generally has a complete smooth bifurcation route with respect to the state initial, and the bifurcation trajectories are gradual [110], such as period-doubling bifurcation and Hopf bifurcation, as shown in Figure 2. It is important to emphasize that extreme multistability is not the same as coexisting infinitely many attractors. The aforesaid coexisting infinitely many attractors are commonly triggered in the memristive circuit and system with line or plane equilibrium set, entirely different from those generated from the offset-boostable flow by introducing an extra periodic signal [111–113] and also different from those generated from the attractor position offset caused by the state initial [114].

According to the definition of memristor [1, 82, 115, 116], researchers have proposed a variety of physical realizable memristor simulators with the characteristics of memristor ports [117], which can be mainly
divided into two categories: one is the ideal memristor or nonideal memristor based on the equivalent realization of operational amplifier and analog multiplier [62, 101, 118, 119]; the other is the generalized memristor with diode bridge cascade RC, RL, or LC filters [120–124]. From the essential definition of the ideal memristor [116], it can be seen that the memristor is derived from the relationship between flux and charge [115]. The ideal memristor is usually divided into the charge-controlled memristor and flux-controlled memristor. Its voltage-current relation curve has the characteristic of typical italic “8” type pinched hysteresis loop, and the main characteristics are zero crossing [82, 116, 125], double value [115], singular symmetry, tapering [116, 126], self-crossing type [127], and stability [128].

The nonideal memristor-based nonlinear circuit or system usually has certain equilibria [129], and their stability is not affected by the state initials. However, under the fixed system parameters, with the varied state initials, the system will produce the coexistence steady-state mode [2], namely, multistability. And, when a memristive circuit or system has a stable equilibrium or no equilibrium [3, 5], the system will produce hidden multistability.

The ideal memristor-based nonlinear circuit or system usually has infinitely many equilibria, and their positions and stabilities are related to the internal state initials of

![Figure 1: The generated attractors from a novel Chua’s circuit (the red points are the nonzero equilibria): (a) self-excited attractor; (b) hidden attractor; (c) local plane projection of a; (d) local plane projection of b.](image1)

![Figure 2: Extreme multistability: (a) with the variation of memristive initial $x_4(0)$; (b) with the variation of memristive initial $x_5(0)$.](image2)
memristors, which indicates the extreme multistability of the memristive circuit or system. Bao et al. [88] proposed an ideal flux-controlled memristor-based Chua’s circuit with line equilibrium set and revealed the state initial-dependent extreme multistability phenomenon of the memristive circuit. In [93], the ideal flux-controlled memristor was used to replace Chua’s diode, and a memristive circuit with line equilibrium set was obtained, and the extreme multistability phenomenon of the circuit was studied. By introducing two ideal memristors into Chua’s circuit, a memristive circuit with a plane equilibrium set was obtained in [130] and further revealed the extreme multistability phenomenon. By introducing an ideal flux-controlled memristor into a threedimensional hypogeneric jerk system, the paper [96] constructed a memristive system with four line equilibria sets, which could produce the extreme multistability phenomenon dependent on the state initial of the memristor and other state initials. Yuan et al. [107] designed a memristor-based multiscroll hyperchaotic system by introducing an ideal flux-controlled memristor and revealed its extreme multistability phenomenon. By introducing a micro-perturbation into the memristive circuit, a memristive circuit with no equilibrium was constructed, which could produce the phenomenon of hidden extreme multistability [131].

3. Multistability Control Strategy

3.1. Multistability Generic Control Strategy. Multistability has been reported in different scientific fields such as physics, chemistry, biology, and economy [13]. Because of its sensitive dependence on state initial, the multistability phenomenon can induce the system to switch between different coexisting states under the fixed system parameters, which provides great flexibility for the engineering application of the multistable system [8, 9, 13, 105, 132–134]. But, at the same time, it is easy to lead the application systems to abnormal working states, which puts forward a severe test to the multistable mode control strategy. For example, in the design of equipment with certain characteristics, it is necessary to avoid multistable or to stabilize it in the desired state, which will cause a lot of inconvenience in practical application. Therefore, it is necessary to control the multistable through appropriate control strategy.

In order to convert a multistable system to a mono-stable system, nonfeedback control strategy was usually adopted. In other words, by adding external disturbance to the system, such as the introduction of short pulse [13], a specific attractor could be selected in a multistable system to achieve multistability control. By introducing pseudoperiodic driving [14, 15] or harmonic disturbance [16, 17], the undesirable attractor types could be eliminated, and then, the system could be controlled in a certain stable state. In order to stabilize the system in a certain desired state, feedback control strategy was usually adopted [18], such as periodic driving [19] and time-delay feedback [13]. Yet these control strategies cannot achieve the multistable control. However, via the special constitutive relation of the memristor, some scholars have proposed appropriate multistability dimensionality reduction reconstitution strategies for specific types of the memristive circuit and system and realized the control of multistable modes. A brief introduction is given below.

3.2. Multistability Dimensionality Reduction Reconstitution Strategy. The multistability of the memristive circuit/system can provide more flexibility for the memristive circuit/system to be applied in engineering application fields of image processing, signal encryption, and so on [8, 9, 105, 132–137]. However, due to the sensitive dependence of the multistability on the state initial, there are two main problems when the traditional analysis method is used to analyze the multistable mode of the memristive circuit/system. On the one hand, the dynamical behaviors of the multistable circuit/system are highly dependent on the state initials, but the state initials cannot be expressed explicitly in the state equation of the multistable circuit/system, which makes it impossible to quantitatively analyze the state initial-dependent dynamical behavior of the memristive circuit/system. On the other hand, since the memristive circuit/system usually has line equilibrium set, plane equilibrium set, space equilibrium set, or no equilibrium, when we use the traditional analysis method to analyze the dynamical behavior, it is very difficult to correctly judge whether the equilibrium of the system is stable or not, or cannot analyze the system equilibrium, which makes it impossible to quantitatively describe the internal mechanism of multistability. These problems make it difficult to accurately predict, quantitatively analyze, and physically control the state initial-dependent dynamical behaviors.

Therefore, in the process of analyzing the multistability of the memristive circuit and system, in order to solve these problems, researchers proposed different dimensionality reduction modeling schemes based on the memristive circuit [20] and memristive system [21]. In fact, a prototype of dimensionality reduction modeling had been developed in the earlier literature [22–24]. In [22], the concept of dimensionality reduction modeling was proposed, which modeled the memristive circuits with two physical quantities of flux and charge as main state variables, and the dimension of the obtained flux-charge model was lower than that of the traditional voltage-current model. Bao et al. [23] built the reduce-ordered flux-charge model of a two-memristor-based memristive circuit and analyzed its dynamical characteristics via the voltage-current and flux-charge models. Bao et al. [24] qualitatively pointed out that the flux-charge model of the memristive circuit could be equivalent to realize its dynamical behavior in the voltage-current model. However, in these early studies, the state initials of the memristive circuits were not explicitly expressed in the dimensionality reduction model [25], resulting in the information loss of state initials of the memristive circuit. Therefore, the established dimensionality reduction model could not reflect the original multistability of the memristive circuits and systems.

In recent years, Corinto proposed an incremental flux-charge analysis method [26], the method was based on the
Kirchhoff flux and charge law and the constitutive relation of the circuit element under the incremental flux and incremental charge. Compared with the circuit equation in the voltage-current domain, the circuit equation in the flux-charge domain established by this method had a simpler equation structure, which could simplify the complexity of dynamical analysis and clearly understand the influence of state initial. To further demonstrate the effectiveness of the method, Corinto applied the method to the analysis of Hopf bifurcation and period-doubling cascade induced by state initial [27]. Subsequently, more scholars applied it to the study of complex memristive circuits such as memristor-based cellular neural network [28] and memristor-based oscillator array [29]. Then, Chen clearly proposed that this method could represent the state initials of all dynamical elements in the circuit as standalone state initial-related system parameters [12], which was conducive to the analysis and measurement of the state initial-dependent dynamical behaviors in the memristive circuit. Moreover, this method was applied to the reconstruction and analysis of extreme multistability for the ideal memristor-based circuit [12, 20, 30, 31]. And, on this basis, the incremental integral transformation method for the memristive system was proposed [21, 32], forming a complete set of dimensionality reduction reconstruction theory for ideal memristor-based circuits and systems. That is, firstly, the integral transformations on all state variables of the original memristive circuit/system are carried out (note that all terms in the system equation must be integrable). Then, the dimensionality reduction model is implemented by using the nondynamic property [22] of the memristor in the flux-charge domain. Then, based on the dimensionality reduction model, the state initial-dependent dynamical behaviors of the original memristive circuit/system are reconstructed and analyzed.

4. Flux-Charge Constitutive Relation of Memristor

In the 1970s, Chua proposed the fourth basic circuit element, memristor, to characterize the relationship between flux and charge [115] and deduced the existence of the memristor from the symmetry of circuit variables and the characteristics of the electromagnetic field, as shown in Figure 3. As can be seen from Figure 3, there are four basic physical quantities in the circuit: current $i$, voltage $v$, charge $q$, and flux $\phi$. There are six mathematical relations among them; among which the relations between current and charge and voltage and flux are as follows:

$$q(t) = \int_{-\infty}^{t} i(\xi)d\xi, \quad (1a)$$

$$\phi(t) = \int_{-\infty}^{t} v(\xi)d\xi. \quad (1b)$$

Equations (1a) and (1b), respectively, represent that charge is the integral of current with respect to time and flux is the integral of voltage with respect to time. According to the incremental flux-charge analysis method, Corinto and Forti [26] gave the definition of incremental charge and incremental flux for any $t \geq t_0$ ($-\infty < t_0 < \infty$), i.e.,

$$q(t; t_0) = \int_{t_0}^{t} i(\xi)d\xi, \quad \phi(t; t_0) = \int_{t_0}^{t} v(\xi)d\xi,$$

and equation (1) can be further written as

$$q(t) = \int_{-\infty}^{t} i(\xi)d\xi = \int_{-\infty}^{t_0} i(\xi)d\xi + \int_{t_0}^{t} i(\xi)d\xi = q(t_0) + q(t; t_0), \quad (2a)$$

$$\phi(t) = \int_{-\infty}^{t} v(\xi)d\xi = \int_{-\infty}^{t_0} v(\xi)d\xi + \int_{t_0}^{t} v(\xi)d\xi = \phi(t_0) + \phi(t; t_0). \quad (2b)$$

It is well known that charge and flux are internal state variables of the memristor in the voltage-current domain, and the internal state initial represents the memory property of the memristor. However, its state initial cannot appear explicitly in the state equation, so it is naturally impossible to assign its value accurately. Therefore, the memory of the memristor cannot be simulated effectively in the voltage-current domain. Compared with the two basic physical quantities of voltage and current, flux and charge can better represent the basic physical properties of circuit elements [138], which provide a theoretical basis for exploring the intrinsic properties of circuit elements. To this end, it is assumed that the voltage $v(t)$ and the current $i(t)$ on the memristor adopt the associated reference direction, and two different types of memristors are selected to build their flux-charge constitutive relation.

4.1. Flux-Charge Constitutive Relation of Charge-Controlled Memristor. For the charge-controlled memristor in Figure 4(a), the voltage-current relationship between the current $i(t)$ flowing through it and the voltage $v(t)$ at both ends of it in the voltage-current domain can be described as

$$v(t) = M(q_M) i(t), \quad (3)$$

where the memristive function $M(q_M)$ is the nonlinear function about charge $q_M(t)$ and has the same dimension as resistance; the unit is ohms ($\Omega$). In this voltage-current model, $q_M(t)$ is the internal state variable of the memristor,
and its state initial \( q_{M}(t_0) \) represents the memory property of the memristor.

Compared with the voltage-current domain, we take charge and flux as state variables in the flux-charge domain; take the integral from \(-\infty\) to \(t\) for both sides of equation (3), and combine with equation (1) to obtain the flux-charge constitutive relation of the charge-controlled memristor in the flux-charge domain as follows:

\[
\phi_{M}(t) = \int_{-\infty}^{t} \nu(\xi)d\xi = \int_{-\infty}^{t} M(q_{M})i(\xi)d\xi = h(q_{M}(t)),
\]

where the function \( h(\bullet) \) is the nonlinear function about \( q_{M}(t) \).

According to equations (2a) and (2b), equation (4) can be further rewritten as

\[
\phi_{M}(t; t_0) = h(q_{M}(t; t_0) + q_{M0}) - \phi_{M0},
\]

where \( \phi_{M0} = \phi_{M}(t_0) \) and \( q_{M0} = q_{M}(t_0) \); equation (5) describes the memristor as a special nonlinear element whose memory is explicitly shown by the internal state initial \( q_{M0} \) [27], i.e., the internal state initial of the memristor can be explicitly expressed in the flux-charge domain, which is conducive to the quantitative analysis of state initial-dependent dynamical behavior. By comparing equation (5) with equation (3), it is not difficult to conclude that the state variable in the flux-charge domain is expressed by the incremental integral of the state variable in the voltage-current domain, and its state initial is zero, i.e., when \( t = t_0, \phi_{M}(t_0; t_0) = q_{M}(t_0; t_0) = 0 \). Figure 4 visually shows the transformation of the charge-controlled memristor from voltage-current constitutive relation to flux-charge constitutive relation.

4.2. Flux-Charge Constitutive Relation of Flux-Controlled Memristor. For the flux-controlled memristor in Figure 5(a), the voltage-current relationship between the current flowing through it and the voltage at both ends of it in the voltage-current domain can be described as

\[
i(t) = W(\phi_{W})\nu(t),
\]

where \( \phi_{W} \) is the internal state variable of the memristor and \( W(\phi_{W}) \) is the memductance function.

Similar to the charge-controlled memristor, charge and flux are used as state variables in the flux-charge domain. The integral of both sides of equation (6) from \(-\infty\) to \(t\) is taken, and the flux-charge constitutive relation of the flux-controlled memristor in the flux-charge domain is obtained by combining with equation (1) as follows:

\[
q_{W}(t) = \int_{-\infty}^{t} i(\xi)d\xi = \int_{-\infty}^{t} W(\phi_{W})\nu(\xi)d\xi = f(\phi_{W}(t)),
\]

where the function \( f(\bullet) \) is the nonlinear function about \( \phi_{W}(t) \). Further, equation (7) can be written as

\[
q_{W}(t; t_0) = f(\phi_{W}(t; t_0) + \phi_{W0}) - \phi_{W0},
\]

where \( \phi_{W0} = \phi_{W}(t_0) \) and \( q_{W0} = q_{W}(t_0) \). Similarly, the flux-controlled memristor can also be expressed as a special nonlinear element in the flux-charge domain, and its memory is reflected by the internal state initial \( \phi_{W0} \) and, when \( t = t_0, \phi_{W}(t_0; t_0) = q_{W}(t_0; t_0) = 0 \). Figure 5(b) shows the flux-charge constitutive relation of the flux-controlled memristor.

5. Incremental Flux-Charge Analysis

Method for Memristive Circuit

Bao et al. [22] pointed out that, in the voltage-current model, the memristor was a dynamic element, resulting in an increase in the order of the circuit equation. In the flux-charge model, the memristor was a nondynamic element, so the order of the circuit remains the same. Therefore, for the memristive circuit, when flux and charge are taken as state variables rather than voltage and current [25, 139], the memristor is described as a nondynamic element, which can reduce the dimension of the established mathematical model [12, 20, 30], from which the term "dimensionality reduction" is derived. It should be noted that the flux-charge model and the voltage-current model are different from each other in their algebraic equations, but they are equivalent representations in nonlinear dynamical behaviors. And, the implicit state initials of all dynamic components in the voltage-current model can be expressed as the explicit initial-related system parameters in the flux-charge model, which is convenient to realize the mechanism explanation of state initial-dependent dynamical behavior in the
Figure 5: Flux-controlled memristor: the constitutive relation transformation from the voltage-current domain to the flux-charge domain; (a) voltage-current constitutive relation; (b) flux-charge constitutive relation.

memristive circuit, namely, to realize the multistability re-
constitution [30]. In addition, dimensionality reduction modeling can reduce the complexity of quantitative analysis and numerical simulation, which has certain theoretical significance and engineering application value.

Bao et al. [130] proposed a two-memristor-based Chua’s circuit, as shown in Figure 6, and revealed its state initial-dependent extreme multistability phenomenon. Based on the memristive circuit, Chen adopted the incremental flux-
charge analysis method [31] and obtained the dimension-
ality reduction model in the flux-charge domain, which not only solved the special dynamic characteristics problem of

the circuit which could not be quantitatively explained in the voltage-current domain but also made the system model simpler and more conducive to the analysis of its dynamical formation mechanism.

To be specific, this article mainly solved the following five problems:

(a) The 5-order dynamic circuit of the original system was described by a 3-dimensional system model, and the dimensionality reduction modeling was realized.

The two-memristor-based Chua’s circuit [130] in the voltage-current domain was

\[
\begin{align*}
\frac{dV_1}{dt} &= -\frac{1}{RC_1}(V_1 - V_2) + \frac{1}{R_0C_1}(1 - g_1V_2^2)V_1, \\
\frac{dV_2}{dt} &= -\frac{k}{RC_2}(V_1 - V_2) + \frac{k}{R_0C_2}(1 - g_2V_2^2)V_2 - \frac{2k + 1}{(k + 1)R_1C_2}V_3, \\
\frac{dV_3}{dt} &= -\frac{k + 1}{RC_3}(V_1 - V_2) + \frac{k + 1}{R_0C_3}(1 - g_3V_3^2)V_2 - \frac{2}{R_1C_3}V_3, \\
\frac{dV_4}{dt} &= -\frac{1}{R_4C_4}V_1, \\
\frac{dV_5}{dt} &= -\frac{1}{R_5C_5}V_2.
\end{align*}
\]

Its dimensionality reduction model [31] in the flux-
charge domain was

\[
\begin{align*}
C_1\frac{d\phi_1(t; t_0)}{dt} &= \frac{1}{R}(-\phi_1(t; t_0) + \phi_2(t; t_0)) - \phi_4(t; t_0) + C_1V_1(t_0), \\
C_2\frac{d\phi_2(t; t_0)}{dt} &= \frac{k}{R}(-\phi_1(t; t_0) + \phi_2(t; t_0)) + k\phi_5(t; t_0) - \frac{2k + 1}{(k + 1)R_1}\phi_3(t; t_0) + C_2V_2(t_0), \\
C_3\frac{d\phi_3(t; t_0)}{dt} &= \frac{k + 1}{R}(-\phi_1(t; t_0) + \phi_2(t; t_0)) + (k + 1)\phi_5(t; t_0) - \frac{2}{R_1}\phi_3(t; t_0) + C_3V_3(t_0),
\end{align*}
\]
where

\[
\begin{align*}
q_4(t; t_0) &= \frac{\varphi_1(t; t_0)}{R_b} \left[ 1 - \frac{g_1 \varphi_1^2(t; t_0)}{3 (R_b C_4)^2} \right] - \frac{\varphi_1(t; t_0)}{R_b} \left[ -g_1 V_4^2(t_0) + \frac{g_1 V_4(t_0)}{R_b C_4} \varphi_1(t; t_0) \right], \\
q_5(t; t_0) &= \frac{\varphi_2(t; t_0)}{R_d} \left[ 1 - \frac{g_2 \varphi_2^2(t; t_0)}{3 (R_d C_5)^2} \right] + \frac{\varphi_2(t; t_0)}{R_d} \left[ -g_2 V_5^2(t_0) + \frac{g_2 V_5(t_0)}{R_d C_5} \varphi_2(t; t_0) \right].
\end{align*}
\]

(b) Converted the plane equilibrium set of the original memristive circuit to three or five determined equilibria.

Original memristive circuit (9) had a plane equilibrium set \( P = \{(V_1, V_2, V_3, V_4, V_5)\} \) \[ V_1 = V_2 = V_3 = 0 \text{ V}, V_4 = \mu \text{ V}, V_5 = \eta \text{ V} \] which led to two critical stable zero eigenvalues at the equilibrium set. Therefore, it was impossible to accurately determine the stability of equilibrium set, resulting in local inconsistency between the stability interval divided by the nonzero eigenvalues and the actual observed dynamical behavior. In flux-charge dimensionality reduction model (10), the plane equilibrium set was transformed into three or five determine equilibria which were related to the initial-related system parameter \( V_i(t_0) \) \( (i = 1, 2, 3, 4, \text{ and } 5) \), which eliminated the ill-posed zero eigenvalues of the original memristive circuit. According to the evolution characteristics of the determined equilibria with \( V_i(t_0) \), the theoretical explanation of the inconsistency between the stability interval of the equilibrium set and the dynamical behavior of the original memristive circuit was given, and the state initial-dependent dynamical mechanism of the original memristive circuit was quantitatively expounded.

(c) Reasonable reasons for the significant change of dynamical behavior under the change of small state initial were expounded.

The state initial \( V_i(t_0) \) of original memristive circuit (9) was fine-tuned from \( 10^{-9} \) to \( -10^{-9} \) (that is, the initial-related system parameter of dimensionality reduction model (10) was fine-tuned from \( 10^{-9} \) to \( -10^{-9} \)), and the dynamical behavior changed greatly. This phenomenon could not be reasonably explained in the voltage-current domain, but after dimensionality reduction modeling, it could be explained in the flux-charge domain according to the symmetry of dimensionality reduction model (10).

(d) Extreme multistability reconstitution was implemented.

The implicit state initial \( V_i(t_0) \) of system (9) was explicitly expressed in dimensionality reduction model (10) as the initial-related system parameter. When the state initial of system (10) was set as \( (0, 0, 0, 0) \), the kinetic map shown in [31] had the same dynamical behavior as the attraction basin shown in [130], intuitively illustrated dimensionality reduction model (10), perfectly reconstructed the state initial-dependent dynamical behavior of original memristive circuit (9), and realized the extreme multistability reconstitution.

(e) In the hardware circuit of the flux-charge model, the multistable mode control of the memristive circuit was realized by changing the initial-related system parameters.

6. Incremental Integral Transformation Method for Memristive System

On the basis of the incremental flux-charge analysis method, for the ideal memristor-based system, the incremental
The state initial-dependent extreme multistability of the original memristive system was studied quantitatively, so as to realize the reconstitution of the extreme multistability, and then, the theoretical basis of the dimensionality reduction reconstitution of the memristive system was given [110]. However, it should be noted that this method only applies to the simple ideal memristor-based system with only memristor nonlinear terms. But for the complex memristive system with other nonlinear terms besides the memristor nonlinear term [32], because it is difficult to obtain an explicit expression of the time integral of complex nonlinear terms, it is necessary to find appropriate intermediate variables and variable substitution to achieve the purpose of equivalent dimensionality reduction modeling. For this reason, the hybrid incremental integral transformation method was proposed in [32].

6.1. Incremental Integral Transformation Method for Simple Memristor-Based System. For the simple ideal memristor-based system with only the memristor nonlinear term, the incremental integral transformation method is used to realize dimensionality reduction modeling and multistability reconstitution. Taking the memristive hyperjerks system as an example, the system has only one memristor nonlinear term with smooth hyperbolic tangent memductance [140], and its mathematical model is

\[
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= x_3, \\
\dot{x}_3 &= x_4, \\
\dot{x}_4 &= \tanh(x_1)x_2 - x_3 - 0.5x_4.
\end{align*}
\]

(12)

According to the incremental integral transformation method [21], the dimensionality reduction model can be obtained as

\[
\begin{align*}
\dot{X}_2 &= X_3 + \delta_2, \\
\dot{X}_3 &= X_4 + \delta_3, \\
\dot{X}_4 &= -X_3 - 0.5X_4 + \ln \cosh(X_2 + \delta_1) - \ln \cosh(\delta_1) + \delta_4.
\end{align*}
\]

(13)

The corresponding equilibrium is transformed from the line equilibrium set to two determined equilibria, and the state variables between the dimensionality reduction model and the original system have the following corresponding relationship:

\[
\begin{align*}
x_1 &= X_2 + \delta_1, \\
x_2 &= X_3 + \delta_2, \\
x_3 &= X_4 + \delta_3, \\
x_4 &= \dot{X}_4.
\end{align*}
\]

(14)

It should be noted that the system parameter \(\delta_j \quad (i = 1, 2, 3, \text{ and } 4)\) of the dimensionality reduction model represents the state initial \(x_j(0) \quad (i = 1, 2, 3, \text{ and } 4)\) of the original memristive system. Similar to the above incremental flux-charge analysis method, when the initial is set to \((0, 0, 0)\), based on the dimensionality reduction model, the state initial-dependent extreme multistability reconstitution of the original memristive system can be realized.

6.2. Hybrid Incremental Integral Transformation Method for Complex Memristive System. For the ideal memristor-based system with other nonlinear terms besides the memristor nonlinear term, the hybrid incremental integral transformation method was proposed in [32], which successfully solved the dimensionality reduction modeling and multistability reconstitution problems of the memristive system with complex nonmemristor cubic nonlinear terms.

To be specific, this article mainly solved the following three problems:

(a) By introducing a new intermediate variable, the problem that the nonmemristor cubic nonlinear term could not be expressed by a simple relation was eliminated.

The mathematical model of a four-dimensional complex memristive system with a nonmemristor cubic nonlinear term [32] is described as

\[
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= (1 - x_4)x_3, \\
\dot{x}_3 &= x_1 - ax_2 - x_3 - bx_1^3, \\
\dot{x}_4 &= -x_3.
\end{align*}
\]

(15)

Incremental integral transformation method [21] was adopted to carry out integral transformation on the system. By introducing an intermediate variable \(W = b \int_0^t x_3^3 dt\), the problem that this integral term could not be expressed as a simple relational expression was eliminated, and a four-dimensional intermediate transformation system with the same dimension as the original system was obtained, that is,

\[
\begin{align*}
\dot{X}_1 &= X_2 + \delta_1, \\
\dot{X}_2 &= 0.5X_3^2 + (1 - \delta_1)X_3 + \delta_2, \\
\dot{X}_3 &= X_1 - aX_2 - X_3 - W + \delta_3, \\
\dot{W} &= b( X_2 + \delta_4 )^3.
\end{align*}
\]

(16)

(b) The variable substitution method was used to eliminate the divergence of state variables in the intermediate transformation system, and then, the dimensionality reduction modeling was realized.

Through variable substitution \(Y_1 = X_1 - W, \quad Y_2 = X_2, \quad \text{and} \quad Y_3 = X_3\), the divergence problem of state variables \(X_1\) and \(W\) was eliminated, and the equivalent three-dimensional dimensionality reduction model of the system was obtained, i.e.,

\[
\begin{align*}
\dot{Y}_1 &= Y_2 + \delta_1 - b(Y_2 + \delta_4)^3, \\
\dot{Y}_2 &= 0.5Y_3^2 + (1 - \delta_1)Y_3 + \delta_2, \\
\dot{Y}_3 &= Y_1 - aY_2 - Y_3 + \delta_3.
\end{align*}
\]

(17)
(c) The extreme multistability reconstitution of the memristive system with other nonlinear terms besides memristor nonlinear terms was realized. The three line equilibria sets of the original system were transformed into six determined equilibria, and the ill zero eigenvalue of the original system was eliminated. And, the state initial $x_i(t_0)$ ($i = 1, 2, 3,$ and 4), as the initial-related system parameter $\delta_i$ ($i = 1, 2, 3,$ and 4), was explicitly expressed in the dimensionality reduction model. When $Y_1(0), Y_2(0),$ and $Y_3(0)$ were set to 0, dimensionality reduction system (17) could reconstruct the extreme multistability of original system (15).

6.3. Hidden Extreme Multistability Reconstitution. Different from the memristive system mentioned above, the nonautonomous FitzHugh–Nagumo (FHN) neuronal circuit was used to solve the problem of critical stability (i.e., hidden attractors) of the system [141]. By using dimensionality reduction modeling, it was proved that the attractors generated by the system were indeed hidden [70, 141]. This article mainly solved the following four problems:

\[
\begin{aligned}
\dot{X}_1 &= X_2 + 0.5X_1 + 0.5 \ln \cosh(X_1 - \delta_3) + 1.8 \sin(\tau) + \delta_1 - 0.5 \ln \cosh(\delta_3), \\
\dot{X}_2 &= -X_1 - X_2 + \delta_2.
\end{aligned}
\]

(a) It made up for the gap that the nonautonomous memristive circuit produced extreme multistability. A memristor with a smooth hyperbolic tangent nonlinear memductance was used to replace the nonsmooth piecewise linear memductance in the FHN neuron circuit in [94]; a nonautonomous memristive FHN neuron model that could produce extreme multistability was obtained.

(b) By using the incremental integral transformation method, the original 3-dimensional system was transformed into 2-dimensional dimensionality reduction. The nonautonomous memristive FHN neuron model is described as

\[
\begin{aligned}
\dot{x}_1 &= x_2 + 0.5(1 - \tanh x_3)x_1 + 1.8 \cos(\tau), \\
\dot{x}_2 &= -x_1 - x_2, \\
\dot{x}_3 &= -x_1.
\end{aligned}
\]

After the incremental integral transformation, the model of dimensionality reduction was obtained:

(c) The hidden extreme multistability reconstitution of the nonautonomous memristive system was realized.

(d) The critical stability of the original system was transformed into the deterministic stability of the dimensionality reduction model. The original system contained the nonautonomous term, which caused the system’s equilibrium to change alternately between stable line equilibrium set and no equilibrium with time. The attractor generated by no equilibrium was hidden. However, due to the existence of zero eigenvalue, the line equilibrium set had critical stability, so it was impossible to determine whether the system produced hidden attractor. After dimensionality reduction modeling, no equilibrium and zero eigenvalue were eliminated, and the dimensionality reduction model only had certain equilibria which changed with time and were always stable, thus the equilibria had certain stability; it was proved that the attractors generated by the original system were indeed hidden.

7. Synchronization Application of Memristor-Coupled System

Because of the nano-sized property, memristors are used to mimic biological neuronal synapses [142–145], which play important roles in the process of information transmission among the coupled neurons [146–149]. And, various memristor-coupled systems are studied, such as memristor-coupled Hindmarsh–Rose neurons [150] and memristor-coupled Hopfield neural network [151, 152].

It is all known that abundant collective behaviors appear in the actual neural system due to the interactions in neurons [153, 154]; among them, synchronization is the outstanding collective features in neuroscience [155–157], which is regarded as one of the mechanisms to propagate and to code information in brain [158, 159]. However, there are different kinds of brain disorder diseases, such as Alzheimer’s, epilepsy, Parkinson’s, and schizophrenia, which are involved with the abnormal activities of synchronization [160]. Therefore, neuron synchrony is a fundamental topic in neuroscience.

Different from the traditional nonlinear elements, the memristor is a special nonlinear element with internal state variables [1]. Therefore, using the memristor to couple the nonlinear system can easily generate special synchronization behaviors that depend on the initials of the memristor, which is completely different from the general nonlinear coupling system [161, 162]. In the general nonlinear coupled system, as long as the coupling strength is large enough, the master system and slave system starting from any state initials will always asymptotically achieve complete synchronization [163]. Naturally, in the nonmemristor-coupled system, some scholars have analyzed the initial influence on synchronization from the qualitative point of view and...
found that the synchronization stability depended on the state initial setting to some extent [35]. For memristor-coupled systems, various synchronization research studies have attracted important attention, and different influencing factors on synchronization were proposed. For examples, in [164], the effect of coupling strength on synchronization transition was investigated. In [165], the influence of coupling intensity and induction coefficient on phase synchronization was discussed. In [166], the effect of electromagnetic parameters on synchronization was studied. In [94], the effect of the coupling memristor parameter on synchronization was given. In [147, 167, 168], the robust analysis approach to asymptotic finite-time synchronization and interval matrix method of global exponential synchronization were proposed for investigations of the delayed memristive neural networks.

However, the dynamical effects of the state initials on synchronization in the memristor-coupled systems were rarely concerned in the published literatures [37, 169, 170]. The first reason is that the state initials are implicit parameters and cannot be expressed explicitly in the state equations. Secondly, such a result makes many researchers question it, because according to our previous understanding of synchronization, state initials are irrelevant to synchronization behavior [161, 162, 171]; however, now, it is said that state initials have influence on synchronization behavior, and their influence cannot be ignored; most scholars are doubtful about this conclusion. At the same time, if the conclusion is presented only by numerical analysis and other qualitative means, its credibility is undoubtedly not enough.

Interestingly, these problems can be solved by simplifying the mathematical models via using appropriate state variables or applying reasonable approximation and simplification [21, 37, 172, 173]. In [173], the initial effects on synchronization for the memristor-coupled system were quantitatively analyzed by the incremental flux-charge analysis method. Due to the inherent state initial mismatches between the two identical coupling systems, the two systems could not achieve complete synchronization under a large coupling strength [173], but synchronous motion with parallel offset could be realized, as shown in Figure 7. Based on the above dimensionality reduction reconstitution method, in the study of the synchronization of the memristor-coupled system, the inherent state initial mismatches between two identically coupled systems can be expressed as the initials-related parameter mismatches between two nonidentically coupled dimensionality reduction systems, and then, the quantitative theoretical research on the influence of the state initial on synchronization can be easily realized.

8. Summary and Prospect

The inherent memory property of the memristor makes the memristor-based circuit and system easy to produce the state initial-dependent dynamical behavior. Especially, the state initial-dependent extreme multistability phenomenon has been paid more and more attention by scholars, and abundant results have been obtained. Most of the existing literatures verify this special phenomenon through numerical simulation or circuit simulation, or capture different attractors randomly by closing and disconnecting the power supply in hardware experiments. The dimensionality reduction analysis method proposed in the literature theoretically realizes the precise prediction, quantitative analysis, and physical control of extreme multistability. For the ideal memristor-based circuit and system, the incremental flux-charge analysis method and incremental integral transformation method can effectively realize dimensionality reduction modeling and extreme multistability reconstitution of memristive circuits and systems, and then, physical control and mechanism exposition of extreme multistability can be realized through quantitative analysis. It can be seen from the existing research contents and results that although great progress has been made in the study of the state initial-dependent dynamical behavior of memristive circuits and systems, there are still many problems to be studied, mainly focusing on the following seven aspects: (a) prediction and control of the nonideal memristor-based circuit and system by state initial; (b) how to model the dimensionality reduction of the memristive circuit and system with high order or complex nonlinear terms; (c) study on the influence of state initial on the dynamical behavior of the memristor-coupled circuit and system and neural electrical networks; (d) for different types of complex memristive systems (such
as time-delay memristive system and fractional-order memristive system), how to carry out equivalent transformation and dimensionality reduction modeling, so as to realize the reconstitution of its state initial-dependent dynamical behaviors; (e) the multistability of the original memristive circuit and system can be reconstructed from the dimensionality reduction model constructed by the incremental flux-charge analysis method and incremental integral transformation method, only when the state initial is set as the origin. However, the dimensionality reduction model is usually a nonlinear system, and its state initial will have a great influence on the system. Therefore, when the state initial is set to nonzero, how to predict and control the multistability of the original memristive circuit and system; (f) at present, the dimensionality reduction methods are used to study continuous memristive systems, so how to study the multistability of discrete memristive systems is an urgent scientific problem to be solved; (g) it is also a scientific problem to be solved whether the extreme multistability system can be built with real memristor devices and tested experimentally to make the research method more practical.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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