A method for collective excitation of Bose-Einstein condensate

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It is shown that by an appropriate modification of the trapping potential one may create collective excitation in cold atom Bose-Einstein condensate. The proposed method is complementary to earlier suggestions. It seems to be feasible experimentally — it requires only a proper change in time of the potential in atomic traps, as realized in laboratories already.

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Spectacular experimental realizations of Bose-Einstein condensate (BEC) in cooled and trapped atomic gases stimulated intensive investigations of possible modifications, control and manipulations of this new state of matter. Here a macroscopic sample of atoms is in a well defined quantum state. Thus several typical quantum mechanical phenomena may now be investigated on a macroscopic level.

As an example of manipulation of the condensate one may consider the splitting of the condensate into two parts, well separated in space and yet coherent with each other. The latter property may be tested by superimposing, at some later time, the two parts and observation of the interference fringes. Another example is the leakage of atoms from the condensate that may be used to prepare an “atom laser”.

Another fascinating possibilities are revealed when one considers possible collective excitations of the condensate. Several schemes have been proposed to create either solitary waves or vortices in the condensate. Both these types of excitations are the solutions of the time-dependent Gross-Pitaevskiy equation (GPE) as appropriate for the mean field, effective single particle description of the gas of weakly interacting bosons in the limit of vanishing temperature (for reviews see ). In analogy to nonlinear optics one may consider bright solitons (bell shaped structures propagating without dispersion), dark solitons (with a node in the middle — an analog of the first excited state in the noninteracting particles picture) or the intermediate grey solitons.

The early propositions for creation of solitons in BEC utilized collisions between spatially separated condensates. Soon it was realized that less violent approaches are also possible. Typical for atomic laser control — resonant Raman excitation scheme — to excite vortex states has been proposed. This approach relies on the resonance condition which is, however, modified during the transfer process due to the nonlinearity of GPE. Another possibility which takes the nonlinearity fully into account is the adiabatic scheme. It utilizes effectively internal atomic transitions combined with appropriate states of the condensate for a controlled laser induced adiabatic transfer, populating solitonic or vortex solutions of GPE, depending on the details of the process. The latter approach seems more robust against typical experimental uncertainties. A yet different approach produces a phase shift between two parts of the condensate — such a phase imprinting method, originally proposed in , has been actually utilized to create dark solitons both in cigar shaped BEC and in the spherically symmetric condensate. The same method has been successfully applied to create vortices . The latter have been also demonstrated experimentally using laser stirring approach .

The aim of this communication is to propose yet another scheme for effective collective excitation of the BEC. The method is in some sense similar, in another sense opposite, to the adiabatic passage of . In the approach of one slowly tunes the laser frequency following adiabatically the levels. The transfer of population between two internal atomic states is accompanied by an appropriate change of the condensate wavefunction into a dark soliton, two-soliton or vortex solution of the GPE. In our proposition, discussed below, we consider a single internal state and sweep the laser across the trap modifying in this way the trapping potential.

For explanation of the effect we assume first that the condensate consists of non-interacting particles. While such a condensate is not realized in nature, it may provide a good starting point for an analysis of weakly interacting Bose gas. We show later that the picture remains valid for interacting particles by considering the numerical example with attractive atom-atom interactions.

To excite collectively a condensate we are going to modify trapping harmonic potential along one of the independent directions only. Therefore, as the non-interacting particles system is separable, it is enough to consider atomic motion restricted to one dimension (the generalization to a three dimensional case is simple). Originally the condensate occupies the ground state of the trap. Since we consider non-interacting particles it is sufficient to consider a single particle picture. By imposing a laser beam, being appropriately tuned off (but close to) the resonance with respect to an internal
atomic transition we may modify the trapping potential by adding a gaussian-shaped local well
\[
V(x) = \frac{x^2}{2} + U_0 \arctan(x_0) \exp\left(\frac{-(x-x_0)^2}{2\sigma^2}\right).
\]
(1)

In the following, as above, we use the trapping harmonic oscillator units, i.e. \(\omega t\) for time and \(\sqrt{\hbar/m\omega}\) for length, where \(\omega\) is harmonic oscillator frequency while \(m\) stands for atomic mass. Similar modification of the potential has been used to split the condensate into two parts [4] — there instead of a local well, a potential barrier has been created. We suggest here to produce such a well on the very edge of the harmonic potential (thus not affecting the condensate). Then we slowly sweep the well across the potential (by moving the laser beam) simultaneously decreasing the depth of the well (by adjusting the intensity of the beam) — it corresponds, for \(U_0 > 0\), to a change of \(x_0\) from some negative value to zero, see Fig. 1.

Assume that a particle is originally in the ground state of the harmonic potential. For a sufficiently slow sweep the levels in the “time-dependent” potential may be followed adiabatically except in the vicinity of avoided crossings. By appropriately choosing \(U_0\) and \(\sigma\) in Eq. (1) we may arrange the situation in which a narrow (with respect to a mean level spacing) avoided crossing between the ground and the first excited state of the potential occurs when the local well sweeps the trap, see Fig. 2. If the avoided crossing is narrow enough it may be passed diabatically and when the local potential well disappears, the particle is left with a high probability in the excited state. This is nothing else than the Landau-Zener transition. The Landau-Zener effect has been explored in BEC but for the transition of internal (not external) atomic degrees of freedom [8].

\[
\begin{align*}
    &|\Psi(x,t)|^2 = (1-p)|\psi_0(x)|^2 + p|\psi_1(x)|^2 + 2\sqrt{p(1-p)}\cos(\omega t)|\psi_0(x)\psi_1(x)|,
\end{align*}
\]
(2)

where \(\psi_0(x)\) and \(\psi_1(x)\) are harmonic oscillator ground and excited states (in the real representation), respectively.

To check whether it is possible to realize an efficient transfer using the method proposed we have simulated the situation numerically. Choosing, without any special optimization attempt, the parameters of the potential \(U_0 = 6.4, \sigma = 0.5\) as \(x_0\) from \(-5\) to \(0\) with the velocity 0.1 we get \(p = 0.97\). The final single particle reduced probability density of the condensate is then depicted in Fig. 3 at various times of its periodic behavior.

As a specific example we propose in this communication to excite the condensate by sweeping the trapping potential using the local potential well. However, the
excitation may be realized in different ways — the key point is to arrange, in the level dynamics, a narrow isolated avoided crossing between the ground and excited states.

One may argue that the proposed model of non-interacting particles is very simple. The role of the interactions may be subtle. They clearly modify the energy levels of the system. Such a modification will be felt mostly close to avoided crossings that may be shifted and broadened. Will this spoil completely the proposed scheme? In our believe it will not, although an adjustment of the laser beam intensity and other parameters may be necessary to optimize the transfer of population.

As a test of this assumption we consider the excitation of the BEC with attractive atom-atom interaction as realized for Li atoms [3]. While the one-dimensional approach for interacting atoms is not exact (nonlinearity couples different degrees of freedom) a one-dimensional approach based on GPE is often used and may be justified for asymmetric traps [22-24].

We integrate time-dependent GPE,

$$i \frac{\partial \psi}{\partial t} = -\frac{1}{2} \frac{\partial^2 \psi}{\partial x^2} + V(x)\psi + g |\psi|^2 \psi$$  \hspace{1cm} (3)

with $g = -5$ [25], starting with the condensate in the ground state of the harmonic trap. By adjusting the parameters of the potential (to compensate the influence of the interaction) taking $U_0 = 10$, $\sigma = 0.3$ and changing $x_0$ from $-5$ to $0$ with the velocity 0.05, we were able to get a 97.5% transfer of population into a collective state corresponding to the first excited state in the independent particle model. The final wavefunction obtained via integration of the time-dependent GPE is depicted in Fig. 4.

It is interesting to compare the present “diabatic” approach with the adiabatic scheme considered in [13] (as the closest in spirit among the techniques proposed). We must admit that the method of [13] may be more robust and flexible, in particular it is adaptable to excitation of vortices and multiple solitons. The latter may be realized also by our method, one needs simply to apply the sweeping potential twice or more times (this as well as application to repulsive atom-atom interactions will be considered in future). While the method of [13] uses two internal states (two component condensate) which is the common trend also in other treatments of collective excitations, our approach considers a single internal state. This may be advantageous in some applications. Importantly also the adiabatic scheme [13] takes necessarily much longer time for an effective transfer (of the order of 200 or more periods of the harmonic trap) than our diabatic approach (here a typical transfer time is 20 periods). While such comparisons may be quite encouraging the best way of verifying our scheme would be a laboratory test. Experimental setup requires only slight modifications of the present atomic traps, thus, such an experiment can be realized immediately.

To summarize we have proposed a simple scheme which enables us to create a collective excitation of the Bose-Einstein condensate. The proposed scheme may serve, we hope, as an alternative to other proposed and experimentally used already methods.

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