The vacuum as a form of turbulent fluid: motivations, experiments, implications

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Abstract

Basic foundational aspects of both quantum theory and relativity might induce to represent the physical vacuum as an underlying highly turbulent fluid. By explicit numerical simulations, we show that a form of statistically isotropic and homogeneous vacuum turbulence is entirely consistent with the present ether-drift experiments. In particular, after subtracting known forms of disturbances, the observed stochastic signal requires velocity fluctuations whose absolute scale is well described by the average Earth’s motion with respect to the Cosmic Microwave Background. We emphasize that the existence of a genuine stochastic ether drift could be crucial for the emergence of forms of self-organization in matter and thus for the whole approach to complexity.
1. Introduction

According to the original Einstein view \cite{1}, the vacuum could be regarded as trivially empty since Lorentz symmetry is an exact symmetry of nature. In a Lorentzian approach \cite{2, 3, 4}, on the other hand, there is an underlying form of ether and Lorentz symmetry, rather than being postulated from scratch, should be considered as an ‘emergent’ phenomenon. In spite of these deep conceptual differences, however, it is far from obvious how to distinguish experimentally between these two points of view. This type of conclusion was, for instance, already clearly expressed by Ehrenfest in his lecture ‘On the crisis of the light ether hypothesis’ (Leyden, December 1912) as follows: “So, we see that the ether-less theory of Einstein demands exactly the same here as the ether theory of Lorentz. It is, in fact, because of this circumstance, that according to Einstein’s theory an observer must observe exactly the same contractions, changes of rate, etc. in the measuring rods, clocks, etc. moving with respect to him as in the Lorentzian theory. And let it be said here right away and in all generality. As a matter of principle, there is no experimentum crucis between the two theories”. This can be understood since, independently of all interpretative aspects, the basic quantitative ingredients, namely Lorentz transformations, are the same in both formulations.

To understand this crucial aspect, one can use a very simple argument. Suppose that the basic Lorentz transformations, rather than originating from the relative motion of a pair of observers $S'$ and $S''$, as in Einstein’s relativity, might instead be associated with their individual velocity parameters $\beta' = v'/c$ and $\beta'' = v''/c$ relatively to some preferred frame $\Sigma$ \cite{5, 6, 7}. Still, due to the fundamental group properties, the two frames $S'$ and $S''$ would also be mutually connected by a Lorentz transformation with relative velocity parameter

$$\beta_{\text{rel}} = \frac{\beta' - \beta''}{1 - \beta'\beta''} \equiv \frac{v_{\text{rel}}}{c} \quad (1)$$

(we restrict for simplicity to one-dimensional motion). This would produce a substantial quantitative equivalence with Einstein’s formulation for most standard experimental tests, where one just compares the relative measurements of a pair of observers. Hence, the importance of the ether-drift experiments where one attempts to measure an absolute velocity.

At the same time, if the velocity of light $c_\gamma$ propagating in the various interferometers coincides with the basic parameter $c$ entering Lorentz transformations, relativistic effects conspire to make undetectable the individual $\beta'$, $\beta''$, ...This means that a null result of the ether-drift experiments should not be automatically interpreted as a confirmation of Special Relativity. As stressed by Ehrenfest, the motion with respect to $\Sigma$ might remain unobservable, yet one could interpret relativity ‘à la Lorentz’. This could be crucial, for instance, to reconcile faster-than-light signals with causality \cite{8} and thus provide a different view of the apparent non-local aspects of the quantum theory \cite{9}.
However, to a closer look, is it really impossible to detect the motion with respect to Σ? This possibility, which was implicit in Lorentz’ words [4] “...it seems natural not to assume at starting that it can never make any difference whether a body moves through the ether or not...”, may induce one to re-consider the various issues and go deeper into the analysis of the ether-drift experiments.

After this general premise, the scope of this paper is threefold. First, in Sect.2, after comparing with basic foundational aspects of both quantum physics and relativity, we will argue that the physical vacuum could be represented as a random medium, similar to an underlying turbulent fluid. Second, through Sects. 3–5, we will show, by explicit numerical simulations, that a form of statistically isotropic and homogeneous vacuum turbulence is entirely consistent with the type of stochastic signal observed in the present ether-drift experiments. In particular, after subtracting known forms of disturbances, the observed signal is consistent with velocity fluctuations whose absolute scale is fixed by the average Earth’s motion with respect to the Cosmic Microwave Background. A definite confirmation (or refutation) of this result should be obtained with the next generation of cryogenic experiments. Finally, in Sect.6, in the conclusions, we will emphasize that the detection of a genuine stochastic ether drift could also be crucial to understand the emergence of forms of self-organization in matter and thus for the whole approach to complexity. In this sense, the ultimate implications of this analysis could go far beyond the mere interpretation of relativity.

2. The physical vacuum as a form of turbulent fluid

In this section, we will list several different motivations that might induce to represent the vacuum as a form of random medium which resembles a turbulent fluid.

i) One could start by recalling that at the dawn of XX century Lorentz symmetry was believed to emerge from an underlying ether represented, by Thomson, Fitzgerald and others, as an incompressible turbulent fluid (a vortex ‘sponge’) [10]. More recently, the turbulent-ether model has been re-formulated by Troshkin [11] (see also [12] and [13]) in the framework of the Navier-Stokes equation and by Saul [14] by starting from Boltzmann’s transport equation. The main point of these hydrodynamic derivations is that, due to the energy which is locally stored in the turbulent motion, on a coarse-grained scale, a fluid can start to behave as an elastic medium and thus support the propagation of transverse waves whose speed \( c_\gamma \) coincides with the average speed \( c \equiv c_{\text{turbulence}} \) of the chaotic internal motion of the elementary fluid constituents.

In this sense, the basic phenomenon of turbulence provides a conceptual transition from fluid dynamics to a different realm of physics, that of elasticity [15]. This conclusion is also

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1The origin of this concept could probably be searched into Hertz’s mechanics [15] with his idea of micro-

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supported by the formal correspondence \[16, 17\] (velocity potential vs. displacement, velocity vs. distortion, vorticity vs. density of dislocations,...) that can be established between various systems of dislocations in an elastic solid and vortex fields in a liquid. With this transition the parameter \(c\) acquires also the meaning of a limiting speed for moving dislocations. This is due to the behaviour of their elastic energy which increases proportionally to \((1 - v^2/c^2)^{-1/2}\). For this reason, dislocations have been considered as a possible model for ordinary matter, see e.g. refs.\[18]–[21].

This perspective is similar to starting from the basic equation that determines the mutual variations of the energy \(E\) and the linear momentum \(p = Mv\) of a body

\[
dE/dt = v \cdot d(Mv)/dt
\]  
(2)

and allowing for a \(v^2\)-dependence in \(M\) (see e.g. \[22\]). This gives

\[
dE = \frac{1}{2} M dv^2 + v^2 dM
\]  
(3)

The main point is that, if ordinary matter were interpreted in terms of soliton-like excitations of an underlying turbulent ether, one now disposes of the velocity parameter \(c \equiv c_{\text{turbulence}}\). Then, by setting \(E \equiv c^2 M(v^2/c^2)\), one has \(dE/dv^2 = c^2 dM/dv^2\) and Eq. (3) becomes

\[
\frac{dM}{dv^2} (c^2 - v^2) = \frac{1}{2} M
\]  
(4)

Therefore, for \(dM/dv^2 > 0\), \(c\) plays also the role of a limiting speed and one finally obtains

\[
E = Mc^2 = \frac{M_0 c^2}{\sqrt{1 - v^2/c^2}}
\]  
(5)

On this basis, it becomes natural to introduce linear transformations of the four quantities \(E/c\) and \(p = Mv\) that preserve the quadratic combination \((E/c)^2 - p^2 = (M_0 c)^2\) and thus, by starting from a microscopic turbulent-ether scenario, Lorentz symmetry could also be understood as an emergent phenomenon. In this interpretation, its ultimate origin has to be searched in the very existence of \(c\) and thus in the deepest random fluctuations of the fluid velocity, with time at each point and between different points at the same instant, that characterize a state of fully developed turbulence and provide a kinetic basis for the observed space-time symmetry \[23\].

scopic, hidden motions whose kinetic energy is actually the source of the forms of potential energy that we observe in nature.

As an example of this proportionality relation, one can consider the case of quantum vortices (rotons) within Landau’s original quantum hydrodynamics. There, it is the squared zero-point speed \(c_{zp}^2\) of the fluid constituents to determine the proportionality relation between the energy gap \(E_{\text{roton}}\), to produce vortical excitations, and their inertial mass \(M_{\text{roton}}\) \[23\].
Notice that, once Lorentz symmetry is an emergent property, $c$ is only a limiting speed for those soliton-like, collective modes that, in an emergent interpretation, are taken as models of ordinary matter, e.g. vortices, elastic dislocations... Thus there is nothing wrong if the internal motion of the basic constituents takes place at an average speed $c$. At the same time, on the coarse grained scale which is accessible to physical rods and clocks, the basic constituents appear, so to speak, ‘frozen’ in the vacuum structure and only their collective excitations are directly observable. This means that, for the elementary ether constituents, Eq. (2) is now solved by the standard non-relativistic forms $E = \frac{1}{2}mv^2$ and $p = mv$, where $m$ is the constituent constant mass.

ii) This qualitative picture of the vacuum, as an underlying random medium, also arises from alternative views of the quantum phenomena as with stochastic electrodynamics [24]−[29] or Nelson’s mechanics [30] (see [31] for more details). The former is essentially the classical Lorentz-Dirac theory [32] with new boundary conditions where the standard vanishing field at infinity is replaced by a vacuum, random radiation field. This field, considered in a stationary state, is assumed to permeate all space and its action on the particles impresses upon them a stochastic motion with an intensity characterized by Planck’s constant. In this way, one can get insight into basic aspects of the quantum theory such as the wave-like properties of matter, indeterminacy, quantization,... For instance, in this picture, atomic stability would originate from reaching that ‘quantum regime’ [27, 29] which corresponds to a dynamic equilibrium between the radiation emitted in the orbital motions and the energy absorbed in the highly irregular motions impressed by the vacuum stochastic field. In this sense, again, Lorentz’ ether should not be thought as a stagnant fluid (for an observer at rest) or as a fluid in laminar motion (for an observer in uniform motion). Rather the ether should resemble a fluid in a chaotic state, e.g. a fluid in a state of turbulent motion. The same is true for Nelson’s mechanics. Here, the idea of a highly turbulent fluid emerges if one uses Onsager’s original result [33] that in the zero-viscosity limit, i.e. infinite Reynolds number, the fluid velocity field does not remain a differentiable function $^3$. This provides a basis to expect that “the Brownian motion in the ether will not be smooth ” [30] and thus to consider the particular form of kinematics which is at the basis of Nelson’s stochastic derivation of the Schrödinger equation.

iii) At a more elaborate level, a qualitatively similar picture is also obtained by representing relativistic particle propagation from the superposition, at very short time scales, of non-relativistic particle paths with different Newtonian mass [35]. In this formulation, particles randomly propagate (in the sense of Brownian motion) in a granular medium which thus replaces the trivial empty vacuum [36]. The essential mathematical ingredient for this

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$^3$Onsager’s argument relies on the impossibility, in the zero-viscosity limit, to satisfy the inequality $|v(x + l) − v(x)| < (\text{const.})^n$, with $n > 1/3$. Kolmogorov’s theory [34] corresponds to $n = 1/3$. 
representation is the use of ‘superstatistics’ \([37, 38]\), intended as the superposition of several statistical systems that operate at different spatio-temporal scale, which is also known to provide a very good description of fluid particle trajectories in high Reynolds-number turbulence \([39, 40]\).

iv) Finally, the idea of a fundamentally random vacuum is also motivated by quantum-gravity. According to this view, space-time, when resolved at very short distances, should exhibit quantum fluctuations and thus appear to be ‘foamy’ or ‘spongy’ in the sense of refs. \([41, 42]\). This original idea has lead to a very wide collection of ideas and intuitions including, for instance, the holographic principle (see \([43]\) for a review), possible deformations of Lorentz symmetry (Doubly Special Relativity) \([44, 45]\) or models of dark energy and dark matter \([46]\). At the same time, coupling light and matter to a fluctuating metric leads to intrinsic limitations on the measurement of lengths \([47, 48]\), to violations of the weak equivalence principle \([49]\) and to an effective decoherence of quantum systems \([50]\). These effects can be used to restrict the possible quantum gravity models by comparing with the results of modern gravity-wave detectors \([51, 52]\) or with atomic interferometry \([53]\) or with the beat signal of two ultrastable optical resonators \([54]\). What is relevant here for our purpose is that, as in the previous cases, the space-time foam of quantum gravity seems also to resemble a turbulent fluid. This idea, originally due to Wheeler \([41]\), has been more recently exploited by Ng and collaborators \([55, 56]\) who have emphasized the close analogies between holographic models of space-time foam and the limit of turbulence for infinite Reynolds number. The main conclusion of these rather formal derivations is that the metric fluctuations in the holographic model, which give rise to length fluctuations \(\Delta l \sim l^{1/3}\frac{2}{\text{planck}}\), when compared with those in moving fluids, can also be interpreted as a manifestation of Kolmogorov’s scaling law for velocity \(\Delta v \sim l^{1/3}\) \([34]\).

Thus, summarizing, from the old ether view to the present quantum-gravity models, there are several independent motivations to represent the physical vacuum as an underlying turbulent fluid. One could conclude that this non-trivial degree of convergence originates from the fundamental nature of quantum gravity (e.g. from the correspondence between the metric fluctuations in the holographic model and Kolmogorov’s scaling law). However, one could also adopt the complementary point of view where instead the ubiquitous phenomenon of turbulence plays from the very beginning the most central role. In any case, it becomes natural to wonder whether this type of vacuum medium could represent the preferred reference frame of a Lorentzian approach and thus to look at the results of the modern ether-drift experiments for experimental checks. At the same time, the non-trivial interplay between large-scale and small-scale properties of turbulent flows may induce one to re-consider some assumptions adopted in the interpretation of the data. These issues will be analyzed in detail in the following three sections.
3. The ether-drift experiments and the velocity of light

As anticipated in the Introduction, the crucial issue in the context of the ether-drift experiments concerns the value of $c_\gamma$, the speed of light in the vacuum (measured on the Earth’s surface). If this coincides with the basic parameter $c$ entering Lorentz transformations relativistic effects conspire to make undetectable the individual $\beta', \beta''$, ... Therefore the only possibility is that $c_\gamma$ and $c$ do not coincide exactly, see e.g. [57]. In this case, in fact, this mismatch would show up through a tiny ether-drift effect $\delta \sim \beta^2 (c - c_\gamma)/c$.

This possibility was explored in ref.[58] within the so called emergent-gravity scenario [59, 60] where the physical vacuum is modeled as a moving fluid with a small compressibility. In this framework $(c - c_\gamma)/c$ was estimated [58] to be $O(10^{-9})$, a value which is not ruled out by the present experimental data. In fact the ether-drift, as measured from the fractional beat signal between two vacuum optical resonators [61, 62], gives $\delta \sim 10^{-15}$ and thus could indicate a value $\beta^2 \sim 10^{-6}$, or an absolute Earth’s velocity of about 300 km/s, as for most cosmic motions. This basic point can be easily checked by looking at Fig. 9(a) of ref.[63] where a typical sequence of 40 data collected at regular steps of 1 second is reported (see our Fig.2 below) [4]. As one can see, this instantaneous signal exhibits random fluctuations of about $\pm 1$ Hz and this value, for the given laser frequency $2.82 \cdot 10^{14}$ Hz, might correspond to a genuine ether-drift $\delta$ of about $\pm 3.5 \cdot 10^{-15}$. To better appreciate this point, let us resume the various aspects which are needed for the analysis of the experiments.

The basic concept in ether-drift experiments is the two-way velocity of light in the vacuum $\bar{c}_\gamma(\theta)$. This is defined in terms of the one-way velocity $c_\gamma(\theta)$ (which is not unambiguously measurable) through the relation

$$\bar{c}_\gamma(\theta) = \frac{2c_\gamma(\theta)c_\gamma(\pi + \theta)}{c_\gamma(\theta) + c_\gamma(\pi + \theta)} \quad (6)$$

and could exhibit a non-zero anisotropy

$$\frac{\Delta \bar{c}_\gamma}{c} = \frac{\bar{c}_\gamma(\pi/2 + \theta) - \bar{c}_\gamma(\theta)}{\langle \bar{c}_\gamma \rangle} \neq 0 \quad (7)$$

This theoretical concept is related to the measurable frequency shift, i.e. the beat signal, $\Delta \nu$ of two optical resonators [61, 62] through the relation

$$\delta(t) \equiv \frac{\Delta \nu(t)}{c} = \frac{\Delta \nu_{\text{phys}}(t)}{\nu_0} \quad (8)$$

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With respect to other articles, ref.[63] has the advantage to report the instantaneous raw data. The experiment also adopts a sophisticated geometrical set-up where, to minimize all possible asymmetries, the two optical cavities are obtained from the same block of ULE (Ultra Low Expansion) material. As such, the results of ref.[63] will play an important role in our analysis.
where $\nu_0$ is the reference frequency of the two optical resonators and the suffix "phys" indicates a hypothetical physical part of the frequency shift after subtraction of all spurious effects.

As a possible theoretical framework for a non-zero anisotropy, we shall concentrate on a scenario which introduces some difference with respect to standard General Relativity and has a very simple motivation: $\bar{c}_\gamma$ might differ from the basic parameter $c$ entering Lorentz transformations due to gravitational effects. To this end, as anticipated, one can consider the emergent-gravity scenario $[59, 60]$ where the space-time curvature observed in a gravitational field becomes an effective phenomenon in flat space, analogously to a hydrodynamic description of a moving fluid on length scales which are much larger than the size of its elementary constituents. In this perspective, gravity produces local modifications of the basic space-time units which are known, see e.g. $[66, 67]$, to represent an alternative way to introduce the concept of curvature $\delta$. This scenario represents the simplest modification of the standard picture which allows for a non-vanishing anisotropy and gives the correct order of magnitude $\delta \sim 10^{-15}$. As such, it will be adopted in the rest of this paper.

For the general problem of measuring the speed of light, one should start, as in ref. $[58]$, from the basic notion: the definition of speed as (distance moved)/(time taken). To this end, one has to choose some standards of distance and time and different choices can give different answers. Therefore, we shall adopt the same point of view of special relativity: the right space-time units are those for which the two-way velocity of light in the vacuum $\bar{c}_\gamma$, when measured in an inertial frame, coincides with the basic parameter $c$ entering Lorentz transformations. However, inertial frames are just an idealization. Therefore the appropriate realization is to assume local standards of distance and time such that the identification $\bar{c}_\gamma = c$ holds as an asymptotic relation in the physical conditions which are as close as possible to an inertial frame, i.e. in a freely falling frame (at least by restricting to a space-time region small enough that tidal effects of the external gravitational potential $U_{\text{ext}}(x)$ can be ignored).

This is essential to obtain an operative definition of the otherwise unknown parameter $c$. With these premises, light propagation for an observer $S'$ sitting on the Earth’s surface can be described with increasing degrees of approximations $[58, 31]$:

1) In a zeroth-order approximation, $S'$ is considered a freely falling frame. This amounts to assume $c_\gamma = c$ so that, given two events which, in terms of the local space-time units of $S'$, differ by $(dx, dy, dz, dt)$, light propagation is described by the condition (ff='free-fall')

$$ (ds^2)_\text{ff} = c^2 dt^2 - (dx^2 + dy^2 + dz^2) = 0 $$

This point of view has been vividly represented by Thorne in one of his books $[68]$: "Is space-time really curved? Isn’t it conceivable that space-time is actually flat, but clocks and rulers with which we measure it, and which we regard as perfect, are actually rubbery? Might not even the most perfect of clocks slow down or speed up and the most perfect of rulers shrink or expand, as we move them from point to point and change their orientations? Would not such distortions of our clocks and rulers make a truly flat space-time appear to be curved? Yes".

$\delta$
ii) However, is really the Earth a freely-falling frame? To a closer look, in fact, an observer \( S' \) placed on the Earth’s surface can only be considered as a freely-falling observer up to the presence of the Earth’s gravitational field. Its inclusion leads to tiny deviations from the standard Eq. (9). These can be estimated by considering \( S' \) as a freely-falling observer (in the same external gravitational field described by \( U_{\text{ext}}(x) \)) that however is also carrying on board a heavy object of mass \( M \) (the Earth’s mass itself) that affects the effective local space-time structure, see Fig.1 of ref. [3]. To derive the required correction, let us again denote by \((dx, dy, dz, dt)\) the local space-time units of the freely-falling observer \( S' \) in the limit \( M = 0 \) and by \( \delta U \) the extra Newtonian potential produced by the heavy mass \( M \) at the experimental set up where one wants to describe light propagation. In a flat-space interpretation, light propagation for the \( S' \) observer can then be described by the condition

\[
(ds^2)_{\delta U} = \frac{c^2 d\hat{t}^2}{N^2} - (d\hat{x}^2 + d\hat{y}^2 + d\hat{z}^2) = 0 \tag{10}
\]

where, to first order in \( \delta U \), the space-time units \((d\hat{x}, d\hat{y}, d\hat{z}, d\hat{t})\) are related to the corresponding ones \((dx, dy, dz, dt)\) for \( \delta U = 0 \) through an overall re-scaling factor

\[
\lambda = 1 + \frac{|\delta U|}{c^2} \tag{11}
\]

and we have also introduced a vacuum refractive index \(^6\)

\[
N = 1 + 2\frac{|\delta U|}{c^2} \tag{12}
\]

Therefore, to this order, light is formally described as in General Relativity where one finds the weak-field, isotropic form of the metric

\[
(ds^2)_{\text{GR}} = c^2 d\tau^2 (1 - 2\frac{|U_N|}{c^2}) - (dX^2 + dY^2 + dZ^2)(1 + 2\frac{|U_N|}{c^2}) \equiv c^2 d\tau^2 - dl^2 \tag{13}
\]

In Eq. (13) \( U_N \) denotes the Newtonian potential and \((dT, dX, dY, dZ)\) arbitrary coordinates defined for \( U_N = 0 \). Finally, \( d\tau \) and \( dl \) denote the elements of proper time and proper length in terms of which, in General Relativity, one would again deduce from \( ds^2 = 0 \) the same universal value \( c = \frac{dl}{d\tau} \). This is the basic difference with Eqs. (10)-(12) where the physical unit of length is \( \sqrt{d\hat{x}^2 + d\hat{y}^2 + d\hat{z}^2} \), the physical unit of time is \( d\hat{t} \) and instead a non-trivial refractive index \( N \) is introduced. For an observer placed on the Earth’s surface, its value is

\[
N - 1 \sim \frac{2G_N M}{c^2 R} \sim 1.4 \cdot 10^{-9} \tag{14}
\]

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\(^6\) A general isotropic metric \((A, -B, -B, -B)\) depends on two functions which, in a flat-space picture, can be interpreted in terms of an overall re-scaling of the space-time units and of a refractive index. Since physical units of time scale as inverse frequencies, and the measured frequencies \( \hat{\omega} \) for \( \delta U \neq 0 \) are red-shifted when compared to the corresponding value \( \omega \) for \( \delta U = 0 \), this fixes the value of \( \lambda \). Furthermore, independently of the specific underlying mechanisms, the two functions \( A \) and \( B \) can be related through the general requirement \( AB = 1 \) which expresses the basic property of light of being, at the same time, a corpuscular and undulatory phenomenon [69]. This fixes the value of \( N \).
where $G_N$ is Newton’s constant and $M$ and $R$ are respectively the Earth’s mass and radius.

iii) Differently from General Relativity, in a flat-space interpretation with re-scaled units ($d\hat{x}, d\hat{y}, d\hat{z}, d\hat{t}$) and $\mathcal{N} \neq 1$, the speed of light in the vacuum $c_{\gamma}$ no longer coincides with the parameter $c$ entering Lorentz transformations. Therefore, as a general consequence of Lorentz transformations, an isotropic propagation as in Eq.(10) can only be valid for a special state of motion of the Earth’s laboratory. This provides the operative definition of a preferred reference frame $\Sigma$ while for a non-zero relative velocity $V$ one expects off diagonal elements $g_{0i} \neq 0$ in the effective metric and a tiny light anisotropy. As shown in Ref.[58], to first order in both $(\mathcal{N} - 1)$ and $V/c$ one finds

$$g_{0i} \sim 2(\mathcal{N} - 1) \frac{V_i}{c}$$

These off diagonal elements can be imagined as being due to a directional polarization of the vacuum induced by the now moving Earth’s gravitational field and express the general property [70] that any metric, locally, can always be brought into diagonal form by suitable rotations and boosts. In this way, by introducing $\beta = V/c$, $\kappa = (\mathcal{N}^2 - 1)$ and the angle $\theta$ between $V$ and the direction of light propagation, one finds, to $O(\kappa)$ and $O(\beta^2)$, the one-way velocity [58]

$$c_{\gamma}(\theta) = \frac{c}{\mathcal{N}} \left[ 1 - \kappa \beta \cos \theta - \frac{\kappa}{2} \beta^2 (1 + \cos^2 \theta) \right]$$

and a two-way velocity of light

$$\bar{c}_{\gamma}(\theta) = \frac{2c_{\gamma}(\theta) c_{\gamma}(\pi + \theta)}{c_{\gamma}(\theta) + c_{\gamma}(\pi + \theta)}$$

$$\sim \frac{c}{\mathcal{N}} \left[ 1 - \beta^2 \left( \kappa - \frac{\kappa}{2} \sin^2 \theta \right) \right]$$

This allows to define the RMS [71, 72] anisotropy parameter $\mathcal{B}$ through the relation

$$\frac{\Delta \bar{c}_\theta}{c} = \frac{\bar{c}_{\gamma}(\pi/2 + \theta) - \bar{c}_{\gamma}(\theta)}{\langle \bar{c}_{\gamma} \rangle} \sim \mathcal{B} \frac{V^2}{c^2} \cos(2\theta)$$

with

$$|\mathcal{B}| \sim \frac{\kappa}{2} \sim \mathcal{N} - 1$$

From the previous analysis, by replacing the value of the refractive index Eq.(14) and adopting, as a rough order of magnitude, the typical value of most cosmic motions $V \sim 300$ km/s, one expects a tiny fractional anisotropy

$$\frac{\langle \Delta \bar{c}_\theta \rangle}{c} \sim |\mathcal{B}| \frac{V^2}{c^2} = \mathcal{O}(10^{-15})$$

that could finally be detected in the present, precise ether-drift experiments.
4. The experiments in more details

Let us now consider in more detail the experimental aspects. To increase the statistics, the present experiments exhibit rotating optical resonators. In this case, the relative frequency shift for a symmetric setup can be expressed as

\[
\frac{\Delta \nu_{\text{phys}}(t)}{\nu_0} = 2S(t) \sin 2\omega_{\text{rot}} t + 2C(t) \cos 2\omega_{\text{rot}} t
\]

where \(\omega_{\text{rot}}\) is the rotation frequency of the apparatus. The overall factor of two on the right hand side of the above equation is needed to correctly normalize the measured shifts in terms of the functions \(S(t)\) and \(C(t)\) extracted from the non-symmetric apparatus of ref.\[73\]. Notice also that in some articles the function \(S(t)\) is denoted as \(B(t)\).

In this framework, the existence of possible time modulations of the signal that might be synchronous with the Earth’s rotation has always represented a crucial ingredient for the analysis of the data. This expectation derives from a model where one assumes a fixed preferred frame \(\Sigma\). Then, for short-time observations of 1-2 days, the time dependence of a hypothetical physical signal can only be due to (the variations of the projection of the Earth’s velocity \(V\) in the interferometer’s plane caused by) the Earth’s rotation. In this case, the two functions \(S(t)\) and \(C(t)\) admit the simplest Fourier expansion \[73\] (\(t' = \omega_{\text{sid}} t\) is the sidereal time of the observation in degrees)

\[
S(t) = S_0 + S_1 \sin t' + S_{c1} \cos t' + S_{s2} \sin(2t') + S_{c2} \cos(2t')
\]

\[
C(t) = C_0 + C_1 \sin t' + C_{c1} \cos t' + C_{s2} \sin(2t') + C_{c2} \cos(2t')
\]

with time-independent \(C_k\) and \(S_k\) Fourier coefficients.

This theoretical framework, accepted so far by all experimental groups, leads to average the various \(C_k\) and \(S_k\) obtained from fits performed during a 1-2 day observation period. By further averaging over many short-period experimental sessions, the data support the general conclusion \[74, 75, 76\] that, although the typical instantaneous \(S(t)\) and \(C(t)\) are indeed \(\mathcal{O}(10^{-15})\), the global averages \((C_k)^{\text{avg}}\) and \((S_k)^{\text{avg}}\) for the Fourier coefficients are much smaller, at the level \(\mathcal{O}(10^{-17})\), and, with them, the derived parameters entering the phenomenological SME \[77, 78\] and RMS models.

However, there might be different types of ether-drift where the straightforward parameterizations Eqs.(22), (23) and the associated averaging procedures are not allowed. In fact, before assuming any definite theoretical scenario, one should first ask: if light were really

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7In the non-symmetric apparatus of ref.\[73\] one measures the combination \(\bar{c}_\gamma(0) - \bar{c}_\gamma(\theta)\). On the other hand, in a fully symmetric apparatus one measures the other combination \(\bar{c}_\gamma(\pi/2 + \theta) - \bar{c}_\gamma(\theta)\). Apart from a constant offset, by using Eq.(17), the angular dependence of the two expressions differs by a relative factor of two.
propagating in a physical medium, an ether, and not in a trivial empty vacuum, how should
the motion of (or in) this medium be described? Namely, could this relative motion exhibit
variations that are not only due to known effects as the Earth’s rotation and orbital re-
volution? The point is that, by representing the physical vacuum as a fluid, the standard
assumption of smooth sinusoidal variations of the signal, associated with the Earth’s rotation
(and its orbital revolution), corresponds to assume the conditions of a pure laminar flow
associated with simple regular motions. Instead, by adopting the model of the vacuum as an
underlying turbulent fluid, there might be other forms of time modulations. In this alterna-
tive scenario, the same basic experimental data might admit a different interpretation and a
definite instantaneous signal \( \Delta \nu(t) \neq 0 \) could become consistent with \((C_k)_{\text{avg}} \sim (S_k)_{\text{avg}} \sim 0\).

To discuss this alternative scenario, it is convenient to first re-write Eq.(21) as

\[
\frac{\Delta \nu^{\text{phys}}(t)}{\nu_0} = 2A(t) \cos(2\omega_{\text{rot}} t - 2\theta_0(t))
\]

where

\[
C(t) = A(t) \cos 2\theta_0(t) \quad S(t) = A(t) \sin 2\theta_0(t)
\]

so that

\[
A(t) = \sqrt{S^2(t) + C^2(t)}
\]

Here \( \theta_0(t) \) represents the instantaneous direction of a hypothetical ether-drift effect in the x-y
plane of the interferometer (counted by convention from North through East so that North
is \( \theta_0 = 0 \) and East is \( \theta_0 = \pi/2 \)). By also introducing the magnitude \( v = v(t) \) of the projection
of the full \( \mathbf{V} \), such that

\[
v_x(t) = v(t) \cos \theta_0(t) \quad v_y(t) = v(t) \sin \theta_0(t)
\]

we obtain the theoretical relations \[58\]

\[
A(t) = \frac{1}{2} |\mathcal{B}| \frac{v^2(t)}{c^2}
\]

and

\[
C(t) = \frac{1}{2} \mathcal{B} \frac{v_x^2(t) - v_y^2(t)}{c^2} \quad S(t) = \frac{1}{2} \mathcal{B} \frac{2v_x(t)v_y(t)}{c^2}
\]

where \( \mathcal{B} \) is the anisotropy parameter Eq.(18). In the forthcoming section we shall produce
a numerical simulation by assuming a model of turbulent flow for the velocity components
\( v_x(t) \) and \( v_y(t) \) and computing \( |\mathcal{B}| \) through Eqs.(19) and (14).

5. Numerical simulation of a physical, stochastic component

Before trying to simulate a physical stochastic component of the signal, to obtain the correct
normalization, we should first subtract from the existing data the known spurious effects. To
obtain a precise statistical indicator we shall consider the root square of the Allan variance (RAV) for an integration time \( \tau \sim 1 \) second which we’ll take as our definition of instantaneous signal. In fact, for the considered laser frequency \( \nu_0 \sim 2.82 \cdot 10^{14} \) Hz, our model predicts typical frequency shifts \( \Delta \nu \lesssim 1 \) Hz so that, when looking for a beat signal, it only makes sense to compare with sequences of data collected at time steps of 1 second or larger.

The RAV describes the time dependence of an arbitrary function \( z = z(t) \) which can be sampled over time intervals of length \( \tau \). In this case, by defining

\[
\overline{z}(t_i; \tau) = \frac{1}{\tau} \int_{t_i}^{t_i + \tau} dt \, z(t) \equiv \overline{z}_i
\]  

one generates a \( \tau \)–dependent distribution of \( \overline{z}_i \) values. In a large time interval \( \Lambda = M\tau \), the RAV is then defined as

\[
\text{RAV}(\tau) = \sqrt{\sigma^2(z, \tau)}
\]  

where

\[
\sigma^2(z, \tau) = \frac{1}{2M} \sum_{i=1}^{M} (\overline{z}_i - \overline{z}_{i+1})^2
\]

Now, for the non-rotating set up, the RAV of the frequency shift for \( \tau \sim 1 \) second was determined \[63\] to be 0.8 Hz (\( 2.8 \cdot 10^{-15} \) in dimensionless units) \[8\] and found much larger than the corresponding disturbances in the individual resonators (typically about 0.02-0.03 Hz). The only exception is the possible effect of thermal disturbances in the mirrors and the spacers of the optical resonators. This particular component should be independent of the integration time and, for ULE optical resonators, on the basis of the results of ref.\[79\], was estimated in ref.\[80\] to be about \( 1.15 \cdot 10^{-15} \) in dimensionless units. Therefore, for a laser frequency \( \nu_0 = 2.82 \cdot 10^{14} \) Hz, we would expect \( \text{RAV(thermal-noise)} \sim 0.32 \) Hz. It is questionable how to subtract this effect from the full measured value 0.8 Hz. One might argue that, if the physical signal has also a stochastic nature, one should subtract quadratically. This would give

\[
\text{RAV(physical,} \, \tau \sim 1 \, \text{second)} = \sqrt{(0.8)^2 - (0.32)^2} \sim 0.73 \text{ Hz}
\]  

Instead, we shall adopt the more conservative attitude of subtracting linearly, i.e.

\[
\text{RAV(physical,} \, \tau \sim 1 \, \text{second)} = 0.8 \text{ Hz} - 0.32 \text{ Hz} = 0.48 \text{ Hz}
\]  

or \( 1.7 \cdot 10^{-15} \) in dimensionless units. Since for a symmetric non-rotating set-up the physical signal is simply \( 2C(t)\nu_0 \), we conclude that there is a potentially important contribution to

\[\text{We tried to obtain an analogous indication from the other experiment of ref.}\[75\]. \text{However, for} \, \tau \sim 1 \, \text{second, it is not so easy to determine the value of the RAV. In fact, by inspection of their figure 2, in the narrow range from} \, \tau = 0.8 \, \text{seconds to} \, \tau = 1 \, \text{second, the data for the non-rotating set-up (the red dots) exhibit a very steep, sizeable decrease from about} \, 2.8 \cdot 10^{-15} \, \text{down to} \, 1.4 \cdot 10^{-15} \, \text{Hz.}\]
C(t) which corresponds to a stochastic signal with an Allan variance of about $8.5 \cdot 10^{-16}$ for $\tau \sim 1$ second. This value will be the basic input for our simulation.

Let us now return to Eqs. (29) and assume for the velocity components $v_x(t)$ and $v_y(t)$ a model of turbulent flow. This could be done in many different ways. Here we shall restrict to the simplest case of a turbulence which, in a wide range of scales, appear statistically isotropic and homogeneous. To describe the temporal pattern of the signal, we shall follow ref. [81] where velocity flows, in statistically isotropic and homogeneous 3-dimensional turbulence, are generated by unsteady random Fourier series. The perspective is that of an observer moving in the turbulent fluid who wants to simulate the two components of the velocity in his x-y plane at a given fixed location in his laboratory. This leads to the general expressions

$$v_x(t) = \sum_{n=1}^{\infty} [x_n(1) \cos \omega_n t + x_n(2) \sin \omega_n t]$$ (35)

$$v_y(t) = \sum_{n=1}^{\infty} [y_n(1) \cos \omega_n t + y_n(2) \sin \omega_n t]$$ (36)

where $\omega_n = 2n\pi/T$, $T$ being a time scale which represents a common period of all stochastic components. In our simulation we have fixed the typical value $T = T_{\text{day}} = 24$ hours. However, we have also checked with a few runs that the statistical distributions of the various quantities do not change substantially if we vary $T$ in the rather wide range $0.1T_{\text{day}} \leq T \leq 10T_{\text{day}}$.

The coefficients $x_n(i = 1, 2)$ and $y_n(i = 1, 2)$ are random variables with zero mean. They have the physical dimension of a velocity and we shall denote by $[-\tilde{v}, \tilde{v}]$ the relevant interval of these parameters. In terms of $\tilde{v}$ the quadratic mean values can be expressed as

$$\langle x_n^2(i = 1, 2) \rangle = \langle y_n^2(i = 1, 2) \rangle = \frac{\tilde{v}^2}{3n^2\eta}$$ (37)

for the uniform probability model (within the interval $[-\tilde{v}, \tilde{v}]$) which we have chosen for our simulations. Finally, the exponent $\eta$ controls the power spectrum of the fluctuating components. For our simulation, between the two values $\eta = 5/6$ and $\eta = 1$ reported in ref. [81], we have chosen $\eta = 1$ which corresponds to the point of view of an observer moving in the fluid.

Thus, within this simple model for the stochastic signal, $\tilde{v}$ is our only free parameter and will be fixed by imposing that the generated $C-$values give a RAV of $8.5 \cdot 10^{-16}$ for integration time $\tau = 1$ second. By taking into account the typical variation of the results, due to both the truncation of the Fourier modes and the dependence on the random sequence, this constraint gives a range $\tilde{v} \sim (332 \pm 10)$ km/s which, remarkably, has a definite counterpart in the known Earth’s motion with respect to the Cosmic Microwave Background (CMB).

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9This picture reflects the basic Kohnogorov theory [34] of a fluid with vanishingly small viscosity.
Figure 1: A simulation of the instantaneous values of the $C$ and $S$ functions, in units $10^{-15}$, as obtained from a typical sequence of 40 seconds. The combination $2C(t)\nu_0$ gives the frequency shift for a symmetric non-rotating set up. Its general trend should be compared with Fig.2 below from ref.[63]. In our simulation the effect of thermal noise has been preliminarily subtracted out.

In fact, it coincides exactly with the daily average of the projection $\sqrt{\langle v^2 \rangle} \sim 332$ km/s in the interferometer’s plane for an apparatus at the latitude of the laboratories in Berlin-Düsseldorf. This can be checked by using the relation [58]

$$\langle v^2 \rangle = V^2 \left( 1 - \sin^2 \gamma \cos^2 \chi - \frac{1}{2} \cos^2 \gamma \sin^2 \chi \right)$$

and setting $V = 370$ km/s, angular declination $\gamma \sim -6$ degrees and co-latitude $\chi \sim 38$ degrees.

After these preliminaries, the results of our numerical simulation can be illustrated by starting from the building blocks of our scheme, namely the instantaneous values $C_i = C(t_i)$ and $S_i = S(t_i)$ of the $C$ and $S$—functions that determine the frequency shift Eq. (21). In Fig.1 we report a typical sequence of 40 values of these functions. In particular the combination $2C(t)\nu_0$ gives the frequency shift for a symmetric non-rotating set up. The resulting general trend should be compared with the experimental signal from ref.[63] reported in our Fig.2. The experimental frequency shifts are somewhat larger due to the effect of thermal noise which has been preliminarily subtracted out in our simulation.

In terms of these basic quantities, one can construct a first type of averages over a time
Figure 2: The experimental signal, for a symmetric non-rotating set up, reported in Fig. 9(a) of ref. [63] (courtesy Optics Communications). For the given laser frequency \( \nu_0 = 2.82 \cdot 10^{14} \) Hz a frequency shift \( \Delta \nu = \pm 1 \) Hz corresponds to a dimensionless ratio \( \Delta \nu / \nu_0 \) of about \( \pm 3.5 \cdot 10^{-15} \).

scale \( \tau \equiv N \) seconds

\[
\overline{C}(t_i; N) = \frac{1}{N} \sum_{n=i}^{i+N-1} C_n \quad \text{and} \quad \overline{S}(t_i; N) = \frac{1}{N} \sum_{n=i}^{i+N-1} S_n
\]  

(39)

so that \( \overline{C}(t_i; 1) = C_i \) and \( \overline{S}(t_i; 1) = S_i \). This first type of averaging is essential to compare with experiments where the \( C \) and \( S \)–functions are always determined after averaging the basic instantaneous data over times \( \tau \) in the typical range 40-400 seconds. With these auxiliary quantities, collected during a large time scale \( \Lambda = M \tau \), one can form a statistical distribution and determine mean values

\[
\langle C \rangle_{\tau} = \frac{1}{M} \sum_{i=1}^{M} \overline{C}(t_i; \tau) \quad \text{and} \quad \langle S \rangle_{\tau} = \frac{1}{M} \sum_{i=1}^{M} \overline{S}(t_i; \tau)
\]  

(40)

and variances

\[
\sigma^2_C(\tau) = \sum_{i=1}^{M} \frac{(\overline{C}(t_i; \tau) - \langle C \rangle_{\tau})^2}{M-1} \quad \text{and} \quad \sigma^2_S(\tau) = \sum_{i=1}^{M} \frac{(\overline{S}(t_i; \tau) - \langle S \rangle_{\tau})^2}{M-1}
\]  

(41)

We report in Fig. 3, for \( \tau = 1 \) second, the distribution functions of the simulated \( \overline{C} \) and \( \overline{S} \) values (panels (a) and (b)). Notice that these distributions are clearly very different from a Gaussian shape. This kind of behavior is known to characterize probability distributions in turbulent flow at small time scales (see e.g. [90, 40]).
Figure 3: We show, see (a) and (b), the histograms $W$ of the simulated $C$ and $S$ values, in units $10^{-15}$, for $\tau = 1$ second. The vertical normalization is to a unit area. The mean values are $\langle C \rangle_\tau = -1.1 \cdot 10^{-18}$, $\langle S \rangle_\tau = -1.9 \cdot 10^{-18}$ and the standard deviations $\sigma_C(\tau) = 8.5 \cdot 10^{-16}$, $\sigma_S(\tau) = 9.4 \cdot 10^{-16}$. The total statistics correspond to a time $\Lambda = M\tau = 86400$ seconds.

By starting to average the instantaneous values, the statistical distributions of the simulated $C$ and $S$ tend to assume a gaussian shape. This is already evident from about $\tau = 5 - 6$ seconds. In Fig. 4 we show the two distributions for $\tau = 40$ seconds.

As it might be expected, for all $\tau$ the statistical averages $\langle C \rangle_\tau$ and $\langle S \rangle_\tau$ are vanishingly small in units of the typical instantaneous signal $O(10^{-15})$ and any non-zero value has to be considered as statistical fluctuation. The standard deviations, on the other hand, have definite values and exhibit a clear $1/\sqrt{\tau}$ trend so that, to good approximation, one can express

$$
\sigma_C(\tau) \sim \frac{8.5 \cdot 10^{-16}}{\sqrt{\tau (\text{sec})}} \quad \sigma_S(\tau) \sim \frac{9.4 \cdot 10^{-16}}{\sqrt{\tau (\text{sec})}}
$$

By keeping $\tilde{v}$ fixed at 332 km/s, the above two values for $\tau = 1$ second have an uncertainty of about 5% which reflects the typical variation of the results due to both the truncation of the Fourier modes and the dependence on the random sequence.

Notice that our model predicts a monotonic decrease of the dispersion of the data by increasing the averaging time $\tau$ and, therefore, does not reproduce the linear increase of the Allan variance which is seen, in all present room temperature experiments, above about $\tau = 100$ seconds. This is usually believed to be a spurious thermal effect which, by the way, was also found in the classical ether-drift experiments.\textsuperscript{10} For this reason, the present

\textsuperscript{10}To this end, one can look at the original Michelson-Morley data, Am. J. Sci. 34 (1887) 333. As explained in Miller’s review article (see D. C. Miller, Rev. Mod. Phys. 5 (1933) 203) the fringe shifts were obtained after
Figure 4: The histograms $W$ of the simulated $C$ and $S$ values, in units $10^{-15}$, and the corresponding gaussian fits for $\tau = 40$ seconds. The vertical normalization is to a unit area. The mean values are $\langle C \rangle_\tau = -9 \cdot 10^{-19}$, $\langle S \rangle_\tau = -5 \cdot 10^{-19}$ and the standard deviations $\sigma_C(\tau) = 1.34 \cdot 10^{-16}$, $\sigma_S(\tau) = 1.48 \cdot 10^{-16}$. The total statistics correspond to a time $\Lambda = M\tau = 864000$ seconds.

limits on Lorentz invariance refer crucially to the short-term stability of the resonators. This thermal interpretation is also in agreement with the cryogenic experiment of ref.\cite{82} where the Allan variance (in the quiet phase between two refillings of the tank of liquid helium) was found to exhibit a monotonic decrease up to about $\tau = 250$ seconds. It remains to be seen how far this decreasing trend will be extended by the forthcoming generation of experiments with cryogenic sapphire resonators \cite{65} that are expected to have a short-time stability of a few $10^{-18}$. Thus it will be possible to obtain a precise check of our predictions. In particular, the typical instantaneous signal should be about 100 times larger than the experimental sensitivity and the distributions of the $C(t_i; \tau)$ and $S(t_i; \tau)$, for $\tau = 100$ seconds, should extend up to values which are still 10 times larger.

6. Summary and outlook

The ether-drift experiments play a fundamental role for our understanding of relativity. In fact, so far, they are the only known experiments which, in principle, can distinguish Einstein’s interpretation from the Lorentzian point of view with a preferred reference frame $\Sigma$. Up to now, the interpretation of the data has been based on a theoretical model where all type of first correcting the data for the observed linear thermal drift. This was producing a difference between the first reading and the final reading obtained after a complete rotation of the interferometer. If this correction were not implemented, no meaningful interpretation of the classical ether-drift experiments can be obtained.
signals that are not synchronous with the Earth's rotation tend to be considered as spurious instrumental noise and no particular effort is made to understand if there could be genuine physical effects which do not fit within the adopted scheme.

However, there is a logical gap which has been missed so far. Even though the relevant Earth's cosmic motion corresponds to that indicated by the anisotropy of the CMB \((V \sim 370 \text{ km/s}, \text{angular declination } \gamma \sim -6 \text{ degrees}, \text{and right ascension } \alpha \sim 168 \text{ degrees})\) it might be difficult to detect these parameters in microscopic measurements of the speed of light performed in a laboratory. The link between the two concepts depends on the adopted model for the vacuum. The point of view adopted so far corresponds to consider the vacuum as some kind of fluid in a state of regular, laminar motion. In these conditions global and local properties of the flow coincide.

We believe that, without fully understanding the nature of that substratum that we call physical vacuum, one should instead keep a more open mind. As discussed in Sect.2, the physical vacuum might be similar to a form of turbulent ether, an idea which is deep rooted in basic foundational aspects of both quantum theory and relativity and finds additional motivations in those representations of the vacuum as a form of 'space-time foam' which indeed resembles a turbulent fluid. In this case, global and local velocity fields might be very different and there could be forms of random signals that have a genuine physical origin. For instance, by combining the point of view of ref.\[58\], where gravity is considered a long-wavelength phenomenon which emerges from a space-time which is fundamentally flat at very short distances, with the idea of a turbulent ether, one arrives to an instantaneous stochastic signal of typical magnitude \(10^{-15}\) which could fit very well with the present experimental data.

For this reason, after reviewing in Sects.3–4 the general theoretical framework and the basics of the modern ether-drift experiments, we have presented in Sect.5 a numerical simulation of the possible effects that one might expect in a simple model where, at small scales, vacuum turbulence appears statistically isotropic and homogeneous. After subtracting the known forms of disturbances, we have found that the observed distribution of the instantaneous data requires a value \(\bar{v} \sim 332 \text{ km/s}\) of the scalar velocity parameter which characterizes the fluctuations. Remarkably, this has a definite counterpart in the known Earth's cosmic motion with respect to the CMB. In fact, it corresponds exactly to the average projection of the Earth’s velocity in the interferometer’s plane for an apparatus placed at the latitude of the laboratories in Berlin-Düsseldorf. However, by the very nature of the model, this correspondence with the global Earth’s motion is only valid at the level of statistical distributions and is not detectable from the naive time dependence of the data. We have also found that, differently from trivial thermal noise, the stochastic signal of an underlying turbulent vacuum should exhibit a transition from non-gaussian to gaussian distributions of the
data by increasing the averaging time, in agreement with analogous phenomena observed in turbulent flows. Furthermore, the typical instantaneous signal should be about 100 times larger than the short-term stability, a few \(10^{-18}\), which is expected with the forthcoming generation of cryogenic experiments [65]. A confirmation of these predictions would represent compelling evidence for an unconventional form of ether-drift with non-trivial implications for our understanding of both gravity and relativity.

We emphasize that the existence of a genuine ether drift could have other non-trivial consequences. In fact, in agreement with the intuitive notion of an ether wind, it would mean that all physical systems are exposed to a tiny energy flux, an effect that, in principle, can induce forms of spontaneous self-organization in matter [83]. In slightly different terms, the detection of a stochastic drift implies that not all possible effects of the underlying vacuum state get re-absorbed into the basic parameters of the physical theory but there remains a weak, residual form of ‘noise’. In principle, this fundamental noise, intrinsic to natural phenomena (‘objective noise’ [84]), could be crucial. In fact it has becoming more and more evident that, thanks to the presence of noise, many classical and quantum systems can increase their efficiency and evolve toward a more ordered behaviour compared to the fictitious situation where spatial and/or temporal randomness were absent [85] (see e.g. photosynthesis in sulphur bacteria [86], protein crystallization [87], noise enhanced stability [88] or stochastic resonance [89]).

In this sense, the outcome of ether-drift experiments could determine a new framework where long-range correlations, complexity and even life, might be thought as ultimately emerging, at higher physical levels, from underlying dynamical processes. Specifically, the idea of a turbulent ether introduces a peculiar element of statistical physics, namely those ‘fat-tailed’ Probability Density Functions, characteristic of turbulent flows at short time scales [40, 90], that also characterize many complex systems (see e.g. [37, 38, 91, 92]).
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