The errors of metacognitive evaluation on metacognitive failure of students in mathematical problem solving

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Abstract: Metacognitive activity is very important in mathematical problems solving. Metacognitive activity consists of metacognitive awareness, metacognitive evaluation and metacognitive regulation. This study aimed to reveal the errors of metacognitive evaluation in students' metacognitive failure in solving mathematical problems. 20 students taken as research subjects were grouped into three groups: the first group was students who experienced one metacognitive failure, the second group was students who experienced two metacognitive failures and the third group was students who experienced three metacognitive failures. One person was taken from each group as the reasearch subject. The research data was collected from worksheets done using think aloud then followed by interviewing the research subjects based on the results’ of subject work. The findings in this study were students who experienced metacognitive failure in solving mathematical problems tends to miscalculate metacognitive evaluation in considering the effectiveness and limitations of their thinking and the effectiveness of their chosen strategy of completion.

1. Introduction
Problem solving is very important in mathematics learning because it can improve students' ability to choose the right resolution strategy and apply it accurately to get the right solution [3]. Realizing the importance of problem solving, Mathematics Education Department, faculty of teacher training and education, Jambi university, created a curriculum that included problem solving and set forth in the subject of Kapita Selekta I [6]. Problems of mathematical problem solving are not routine issues. Therefore, students need more complex thinking by using high-level thinking processes on this. High-level thinking processes are in dire need of students' metacognitive processes.

The metacognitive aspect is a very important aspect in mathematical problem solving because metacognitive ability is important in decision making [20]. When a person has a high metacognitive ability and knows how to apply it, there is a higher probability of problem solving to be successful [2]. Some researchers have concluded that metacognitive processes can improve the results of problem solving [1, 11, 23,4] and students who use metacognitive in problem solving often produce the correct response than those who do not use it [4].

There is a link between metacognitive and metacognition. Metacognition is thinking about thinking [16, 19, 8, 9, 15] and the metacognition component consists of metacognitive knowledge and metacognitive regulation [21, 17, 9, 5]. Metacognition refers to students' awareness of their own cognitive processes and the arrangement of those processes to achieve a particular purposes [7]. Metacognition is a person's ability to understand and monitor their own learning and how to use specific learning strategies in problem solving [13] or decision-making [18].

Although metacognitive is an important aspect of problem solving but many students who have metacognitive activities in mathematical problem solving but they still fail to do metacognition. According to [10], there are 3 metacognitive failures (metacognitive mirages, metacognitive blindness and metacognitive vandalism). Metacognitive mirage is when the student does not see any difficulties, mistakenly abandon the useful strategy, changes the wrong calculation and refuses the true answer.
Metacognitive vandalism is a metacognitive failure that occurs when a student overcomes the deadlock by taking destructive actions by means of which students can change the problem by improperly implementing conceptual structures. Metacognitive mirage is a metacognitive failure that occurs when a student fails to notice something is wrong, for example, the student survives the wrong strategy or ignores the miscalculation. As a result of all three metacognitive failures, a student experiences errors in problem solving.

Metacognitive failure can be observed from 3 metacognitive activities such as metacognitive awareness, metacognitive evaluation and metacognitive regulation [22, 14]. According to [14], metacognitive awareness relates to self-made statements or other people's mathematical thinking and shows thoughts about: (1) what one knows, (2) where the presence of someone in the process of problem solving, and (3) what needs to be done, what has been done, or what someone can do. Metacognitive evaluation deals with self-made thoughts or others’ mathematical thinking that show about: (1) the effectiveness and limitations of one's thinking, (2) the effectiveness of the chosen strategy, (3) the assessment of results, (4) (5) assessment of progress, ability and understanding of a person. Metacognitive regulation deals with self-made statements or other people's mathematical thoughts that suggest (1) problem-solving planning strategies, (2) goal setting, and (3) choosing problem-solving strategies.

Several studies related to metacognitive failure have been done. [10] found the identification of various forms of metacognitive failure by highlighting the ways in which verbal interactions could affect the thinking of each student. Furthermore, [12] states that students experience metacognitive failure in mathematical proof based on assimilation and accommodation framework.

Based on the researchers’ observation results at Mathematics Education study of teaching faculty in Jambi University on March 15, 2017, researchers found that 77% has done metacognitive activity in problem solving but still fails in metacognitive activities. Based on the above explanation, this study examines how students' error in metacognitive evaluation occurs in students who have: (1) one metacognitive failure, (2) two metacognitive failures and (3) three metacognitive failures.

2. Research Methods
This research was conducted on April 2017 with the students of Mathematics Education Department, faculty of teacher training and education, Jambi university, who have studied Kapita Selektia Sekolah Menengah 1. The type of study was qualitative. There were 3 subjects in this study, one subject experiencing one metacognitive failure, one subject experiencing 2 metacognitive failures and 1 subject experiencing 3 metacognitive failures. The research data was collected by using the main instrument of the researcher himself and the supporting instrument in the form of duty worksheets and interview guidelines which had been validated by the validator. Research data was analyzed by the following steps: (1) transcribing result of think aloud and result of student’s interview, (2) conducting data reduction by making abstraction, (3) encoding every thinking process done by students, (4) describing structure of students’ thinking process, (5) analyzing what happened during the study, (6) drawing conclusions.

3. The Results and Discussion
3.1. Result
This section describes the errors of metacognitive evaluation on the first subject (S1), second subject (S2) and the third subject (S3).

3.1.1. Exposure Data of first subject’s (S1)
S1 experienced metacognitive blindness in the process of thinking. It can be seen in Diagram 1.
Diagram 1. S1’s thinking process

Table 1. The terms of code on S2’s thinking process

| Code  | Terms                                                                 |
|-------|-----------------------------------------------------------------------|
| krm   | The average velocity of climbing up to hill                           |
| iwc   | Having Enough time for taking a rest                                  |
| kmb   | The average velocity of downing the hill                              |
| wdpkk | The time needed to walk from the foothills to the top of the hill and |
|       | climbing back to the foothills                                       |
| krmkrt| The average velocity of climbing plus the average velocity of going down |
|       | the hill                                                             |
| wdnt  | Time needed to climb up and go down the hill                          |
| krmjn | The average velocity of climbing is the distance per time of climbing |
| krtjnt| The average velocity of climbing is the distance per time of climbing|
| ntjs  | Climbing and going down through the same path                         |
| wnt₁, wnt₂| The time of climbing is t₁ and the time of going down time is t₂    |
| tttlct| t going down three times faster                                       |
| d     | distance                                                              |

It can be seen from diagram 1 that S1 rethought that the average velocity of going down was three times of the average velocity to climb the hill (on process 23), so the time to climb was three times going down the hill (on process 24). S1 also thought again that the time to climb the hill and went down from the hill was 6 hours (on process 25). This can be seen from the reseracher’s interview results (R) with S1.

S1: I think again because the average velocity to climb a hill is greater than the average
velocity to go down the hill, then I determine that the time to go down was 4.5 hours. The time needed to climb the hill is longer than the time to go down the hill. (Metacognition evaluation: Consider the effectiveness and limitations of thinking).

R: Next, what do you think by setting the time of going down the hill for 4.5 hours?
S1: By setting the time to go down is 4.5 hours then I get the distance from the foothills to the top of the hill is 20.25 km. I think my answer is true again (Metacognition evaluation: Considering the results).

### 3.1.2. Exposure data of second subject (S2)
S2 experienced two metacognitive failures in mathematical problem solving namely metacognitive mirage and metacognitive blindness in the process of thinking. The thinking process of S2 can be seen in Diagram 2.

![Diagram 2. S2’s thinking process](image)

Table 2. The terms of code on S2’s thinking process

| Code  | Terms                                                   |
|-------|---------------------------------------------------------|
| krm   | The average velocity of climbing the hill               |
| iwc   | Having Enough Time for taking a rest                    |
| krt   | The average velocity of going down from the hill        |
| wkbpb | Time from the foothills to the top of the hill coming back to the foothills |
| jkp   | The distance from the foothills to the top of the hill  |
| nbtbe | Climbing the hill and going down from the hill are 6 hours |
| msjwt | The recent problem is the total distance and time       |
| Sdkb  | Distance from the foothills                            |
| kttkn | The velocity goes down three times from the velocity of climbing the hill |
3.1.2.1. S2 experienced metacognitive mirage
In diagram 2, S2 thought again about the relationship between the distance travelled from the hill to the top of the hill (at process 17). S2 also recalled the link between average velocity to climb and the average velocity to go down the hill at a total average of velocity. It is obtained from the average velocity of climbing the hill plus the average velocity of going down the hill (at process 18). S2 thought again that the average velocity of climbing and going down the hill with total time. The total time was the time of climbing plus time of going down the hill (at process 19). This can be seen from researcher’s interview results (R) with S2.

S2: I think again that the average total of velocity is same to the average velocity of climbing the hill plus going down the hill. I think that it is multiplied by the total time which will result in the total of distance which is the distance from the foothills to the top hill and back to the foothills. The average total of velocity is equal to the average velocity of climbing the hill plus the average velocity of going down from the hill. I think I can get a total average speed. (Metacognitive evaluation: considering the effectiveness and limitations of his thinking)

R: Next ?
S2: I think if the distance from the foothills to the top of the hill is 15 km then the time needed to climb from the foothills to the top of the hill is 10 hours. Then if the distance is 30 km then the time needed to climb is 20 hours. I think again that this way does not help me to solve this problem. (Metacognitive evaluation: considering the effectiveness of the selected strategy).

3.1.2.2. S2 experienced metacognitive blindness
In diagram 2, S2 thought again about the time needed to climb the hill and go down from the hill was 6 hours. S2 also thought again if the time to climb the hill was 2 hours and to go down from the hill was 4 hours, then S2 got that the distance to climb and go down the hill were not same. This can be seen from the researcher’s (R) interview results with S2.

S2: I think again that I have time to go down that hill. (Metacognitive evaluation: considering the effectiveness and limitations of thinking).

R: Then, what do you think for the next?
S2: I think again where I can specify that the time to climb is 2 hours and the time to go downhill is 4 hours. I think I can do it this way. (Metacognitive evaluation: considering the effectiveness of the selected strategy). I checked back the distance of climbing the hill is not the same as the distance of going down the hill. I think the information is not enough to solve this problem. (Metacognitive evaluation: considering the difficulty of the problem).

3.1.3. Data Exposure of third subject (S3)
S3 experienced metacognitive mirage, metacognitive vandalism and metacognitive blindness in the process of thinking. S3’s thinking process can be seen in Diagram 3.
Diagram 3. S3’s thinking process

Table 3. Terms of code on S3’s thinking process

| Code    | Terms                                                                 |
|---------|-----------------------------------------------------------------------|
| Jkp     | Distance from foothills to the top of the hill                        |
| Kn      | The average velocity of climbing the hill                              |
| Krt     | The average of going down from the hill                               |
| Wkp     | Time from foothills to the top of the hill                            |
| Hjwk    | The relation between distance, time and velocity                      |
| Wejdk   | The time is 6 hours from the foothills                                |
| Wkpti   | The time from foothills to the top of the hill                        |
| S       | Being questioned S                                                    |
| Jvt     | The same distance with v equals t                                     |
| ccvA    | Try from v_A                                                         |
| v_A tbd | V_A can not be determined                                              |
| dipS_A  | Distance determined from the foothills to the top of the hill         |
| Knav    | Why there are still variables                                          |
| Kkn     | The velocity goes up                                                  |
| Wnl    | The time of going up is much longer                                   |
| Wtlc   | The time of going down is faster                                      |
| Wmtlj  | The time of climbing is 3 hours                                       |
| Kmkmv  | How could that there are still the variables                          |

3.1.3.1. S3 Experienced the metacognitive mirage

In diagram 3 it can be seen that S3 thought again that the average velocity could be calculated by Δv, distance was calculated by ΔS and time was calculated by Δt. (at process 15). The distance was the average velocity multiplied by Δt (at process 16) and the distance was also same with the sum of the average velocity of climbing the hill and going down from the hill multiplied by the time (at process 17). Then S3 determined that the distance from the foothills to the top of the hill was 36 km (at process 18, 19, 20 and 21). This can be seen from the researcher’s (R) interview results with S3.
S3: I remember when I was learning physics, mam. I think again that the average velocity is the distance change divided by the time change. Distance is the average velocity multiplied by the time change then I can get the distance from the foothills to the top of the hill. (Metacognitive evaluation: considering the effectiveness and limitations of their own thoughts or thoughts of others)

R: Then, what do you think for the next?

S3: I think again that the distance can be obtained from the average total of velocity multiplied by the time. It is obtained from the average velocity of climbing the hill plus the average velocity going down the hill. I can do this way and my answer is still correct (Metacognitive evaluation: considering the results).

3.1.3.2. S3 experienced metacognitive vandalism.

It can be seen from diagram 3, S3 has obtained the distance’s formula of climbing the hill and going down the hill. S3 thought again that the time to climb could be substituted with the time to go down the hill. S3 could not continue the completion and got stuck. S3 thought again that the distance from the foothills to the top of the hill and went back to the foothills was obtained from combined average velocity multiplied by the total time (process 61). S3 thought again that the distance from the foothills to the top of the hill was the overall distance divided by two (at process 62, 63, 64 and 65). This can be seen from the researcher’s (R) interview results with S3.

S3: I think that the distance from the foothills to the top of the hill is 9 minus 1.5 multiplied by the time of climbing the hill. The distance from the top of the hill to the foothills is 27 minus 4.5 multiplied by the time of going down the hill. Why is there still a variable? I think this way does not help me to solve the problem (Metacognitive evaluation: considering the effectiveness of the selected strategy) and I also think that there is no information to solve this problem (Metacognitive evaluation: considering the difficulty of the problem)

R: What do you think?

S3: I think about it for a long time, but I do not find the solution. I think again and change the way of the solution. It comes to my mind that the distance from the foothills to the top of the hill is the same with the distance from the top of the hill to the foothills. Therefore, I divide two of the overall distance and I get the distance from foothills to the top of the hill which is 18 km. I also think again that my answer is correct (Metacognitive evaluation: considering results).

3.1.3.3. S3 experienced metacognitive blindness

It can be seen from diagram 3, S3 has already understood that the time needed to climb the hill is longer than the time to go down the hill. It needed 6 hours to climb and go down the hill (at process 70 and 71). S3 thought again that 4 hours was the time to climb (at process 72) and 2 hours was the time to go down the hill (at process 73). S3 does not think about the relationship among the specified time, velocity and distance, so that S3 obtained the distance of climbing the hill was different with going down the hill (at process 74 and 75). This can be seen from researcher’s interview results (R) with S3.

S3: I think again that the time of climbing the hill is longer than the time of going down the hill which is 6 hours. ........ (S3 kept silent for a moment)

R: What are you thinking when you are silent?

S3: I think again that I set the time to climb is 4 hours and the time to go down is 2 hours because the time of climbing is much longer than going down the hill and the specified time needed to do both is 6 hours. I think I can solve it (Metacognitive evaluation: considering the strategy chosen) and I think my answer is correct. (Metacognitive evaluation: considering results)
3.2. Discussions

The activity of metacognitive evaluation occurs to subjects who experienced the metacognitive failure in problem solving. It is explored through the results of think aloud, the researcher’s interview results with the subjects based on the duty worksheets.

Subjects who experienced metacognitive blindness at first has thought correctly. The subject thought if the average velocity of climbing the hill was 3 times of going down the hill, so that the time to climb the hill was same which was 3 times of going down the hill. The overall time required to climb and go down the hill was 6 hours, then the subject thought that the time needed to go down the hill was only 4.5 hours. The subject has a failure of considering the effectiveness and limitations of his thinking. Furthermore, by setting the time to go down the hill, 4.5 hours, the subject got 20.25 km as the distance from the foothills to the top of the hill. In this case, the subject experienced an error of metacognitive evaluation’s in considering the outcome.

Subjects who experienced metacognitive mirage and metacognitive blindness, at first, thought that the problem was the distance and time needed to climb and go down the hill. The subject thought that that the average total of velocity can be obtained from the average velocity of climbing the hill plus the average velocity of going down the hill. In this case the subject has been failure in considering the effectiveness and limitations of his thinking. Next, the subject thought again if the distance from foothills and the top of the hill was 15 km, then the time to climb the hill was 10 hours, likewise if the distance from foothills to the top of the hill was 30 km then time needed to climb the hill was 20 hours. This does not help the subject to solve the problem, so that the subject experienced the error of metacognitive evaluation in considering the effectiveness of the selected strategy.

The subject thought that the average velocity of going down the hill was 3 times of the average velocity of climbing the hill and the time to climb and go down the hill was 6 hours, then the time to climb the hill was 6 divided by 3 which was 2 hours and 4 hours to go down. In this case, the subject failed to consider the effectiveness of the chosen strategy. Furthermore, the subject thought that there was not enough information to solve these problems. Thus, it can be said that the subject experienced an error of metacognitive evaluation in considering the problems’ difficulties.

Subjects who experienced the metacognitive mirage, metacognitive vandalism and metacognitive blindness. At first, the subject thought that distance was the average velocity multiplied by time. Then he thought again that the distance was the distance multiplied by the Δt. Next, the subject also thought again that the distance was the sum of the average velocity of climbing and going down the hill multiplied by the time. The subject thought again that he could get the distance from foothills to the top of the hill. In this case, he experienced the error of metacognitive evaluation in considering the effectiveness and limitations of his thinking.

The subject also thought that he could get distance from the time and the average velocity of climbing and going down the hill. In this case, he experienced an error of metacognitive evaluation in considering the effectiveness of the new strategy and the error of metacognitive failure in considering the outcome.

The subject thought again that there were still variables if the subject calculated the distance from the foothills to the top and this way did not help him to solve his problem. In this case, he failed in considering the effectiveness of the chosen strategy. Next the subject thought again that there was no information to solve this problem. This could be said that the subject experienced the error of metacognitive evaluation in considering the difficulty of the problem.

The subject thought again that the distance from the foothills to the top was same with the distance from the top to the foothills, then the overall distance was divided by 2. The subject thought again that the answer was true. In this case, the subject experienced an error of metacognitive evaluation in considering the outcome.

The subject thought that the time needed to climb and go down the hill was 6 hours. He also thought that the time to climb the hill was longer than go down the hill. Next, he thought that the time to climb the hill was 4 hours and the time to go down the hill was 2 hours. Then he also thought that
he could solve the problem and the subject also thought that his answer was correct. In this case the subject experienced an error of *metacognitive evaluation* in considering the chosen strategy and the subject also experienced an error of *metacognitive evaluation* in considering the outcome.

4. Conclusion and Suggestions

4.1. Conclusion

The conclusions of this study are:

1. Subjects experiencing a *metacognitive failure* have an error of *metacognitive evaluation* in considering: the effectiveness and limitations of their thinking and the results obtained.
2. Subjects experiencing two *metacognitive failures* have the errors of *metacognitive evaluation* in considering: the effectiveness and limitations of their thinking, the effectiveness of the chosen strategy and the difficulty of the problems encountered.
3. Subjects experiencing three *metacognitive failures* have the errors of *metacognitive evaluation* in considering: the effectiveness and limitations of their thinking, the effectiveness of new strategies and strategies chosen, the difficulty of the problems encountered and the results obtained.

4.2. Suggestions

Based on the results of the study about the error of *metacognitive evaluation* towards the *metacognitive failure* of students in solving mathematical problems, researchers give suggestion to other researchers to investigate the error of metacognitive evaluation at every step of problem solving.

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