Stability analysis of constrained L1 inversion of gravity data for estimating discontinuous basement relief

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Abstract. L1 inversion of gravity data is widely used to estimate the discontinuous relief of basement. We used the L1 inversion method to recover a complex model with faults, a smooth feature, and a V-shaped interface. The noise of data, the constraints with errors and the incorrect density contrast are considered in the stability analysis of inversion. The results show that the constrained L1 inversion of gravity data is stable and efficient for complex discontinuous relief of basement.

1. Introduction

The gravity data are sensitive to the horizontal variation of density and the relief of density interface in the crust. In general, the basement or the bedrock is discontinuous in the shallow depth. The changing relief will cause gravity anomaly due to the variation of density interface between the upper layer of soft sediment and the lower layer of hard rocks. Moreover, the discontinuous basement relief is related to the faults, tectonics and geological structure. Delineating the basement relief is vital in predicting the hydrocarbon potential of a sedimentary basin and to guide exploration work \cite{1-2}. Some faults may be active in the near-surface and would be potential hazards for our human-being in urbanization. Therefore, obtaining the depth and the discontinuities of basement is crucial for near-surface geophysics.

To date, the discontinuous basement relief is recovered by gravity inversion, such as L1 inversion \cite{3} and Lp inversion \cite{4}. Otherwise, the V-shaped density interface is an estimate based on Lp-norm regularization \cite{5}. However, the other methods of Parker-Oldenburg and L2 inversion of gravity data only yield smooth relief of basement. We used the L1 inversion proposed by \cite{3} to solve the non-smooth problem. The algorithm was tested via a complex relief model in our study instead of the simple model in \cite{3}. We further introduced constraints of depth and locations of the faults and analyzed the stability of the inversion.

2. L1 inversion of gravity data

2.1. Algorithm

We used the algorithm of gravity inversion proposed by \cite{3}. The algorithm adopted L1-norm regularization to recover the discontinuous basement relief. We first obtain the initial approximation \( p^I \) through the solution of the linear system in the L1-norm:

\[
\begin{bmatrix}
B \\
\mu R
\end{bmatrix} p^I = \begin{bmatrix}
g^0 \\
0
\end{bmatrix}
\]

(1)
where \( \mathbf{g}_0 \) is the N-dimensional vector of gravity observations, \( \mathbf{R} \) is the matrix of discrete first derivatives along x- and y-direction, and \( \mu \) is the regularization parameter. The method of obtaining \( \mu \) value depends on the experiment is inefficient. \( \mathbf{B} \) is an \( N \times N \) matrix defined as:

\[
\mathbf{B} = \{ b_{ij} \} = \gamma \eta \Omega_i[d(x_i, y_j)]
\]

where \( \gamma \) is the gravitational constant, \( \rho \) is an estimate of the density contrast, \( \eta \) is a multiplicative factor to be determined by numerical tests. \( \Omega_i[d(x_i, y_j)] \) is the solid angle that includes the area of the jth ribbon when viewed from the position of the ith observation (Santos et al., 2015).

\[
\Omega_{ABCD} = \Omega_{ABC} + \Omega_{CDA}
\]

\[
\begin{aligned}
&= \frac{2 \arctan}{\frac{r_A r_B r_C}{r_A}} \\
&+ \frac{2 \arctan}{\frac{r_C D r_A + r_D D r_A + r_D C + r_A C}{r_A}}
\end{aligned}
\]

In equation 2, the depth \( d(x_i, y_j) \) is determined by \( \mathbf{g}_0 \):

\[
d(x_i, y_j) = \frac{\mathbf{g}^0(x_i, y_j)}{2 \pi \gamma \rho}
\]

L1-norm regularization is a particular way to solve equation 1. The algorithm finds the minimum sum of the absolute values of residuals between the observed value and the fitted value and gives the interface fluctuation as much as possible in the modulus. The regularization matrix \( \mathbf{R} \) is set to make the algorithm get a more stable solution.

2.2. Setting of regularization parameters and constraints

2.2.1. Setting of regularization parameters. Figure 1a shows a simulated basin with a faulted basement using 264 prisms and observations \((N_x=22\) and \(N_y=12\)) and density contrast between sediments and basement is \((0.18 g/cm^3)\). Figure 1b shows the model's Bouguer gravity anomaly.
The setting of regularization parameter $\mu$

The regularization matrix $R$ makes the initial approximation $P_1$ more stable. $\mu$ controls the effect of $R$ in the estimation process. We take the middle position of the model and take a longitudinal section. We choose different $\mu$ values and make the corresponding initial approximation $P_1$. We note that the value of $\mu=0.2$ (Figure 2d) is the best one.

The setting of the multiplication factor $\eta$

The function of the multiplication factor $\eta$ is to make the initial approximation $P_1$ closer to the real model. By comparison of initial approximation $P_1$ for different $\eta$ values, we note that the value of $\eta=1.2$ (Figure 3d) is the one that best fits the model depth.

In this model, when the regularization parameter $\mu$ takes 0.2 and the multiplication factor $\eta$ takes 1.2, the Initial approximation $P_1$ are closest to the real model.

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**Figure 2.** Initial approximation $P_1$ for different $\mu$ values($\eta=1.2$)

**Figure 3.** Initial approximation $P_1$ for different $\eta$ values($\mu=0.2$)
However, setting constraints during the iteration process is different for special terrains. For the sloping terrain, we don't need to add any constraints. The data error function can achieve the desired effect. For the V-shaped terrain, we make the adjusted step size $b^k$ at the horizontal position of the tip greater. The increment need to be obtained through experiments. For the U-shaped terrain, we don't need to add any constraints. Even though we mistake the U-shape model for a V-shape model and increase the adjusted step size, the solution will also fit the theoretical model. For the discontinuities basement relief, we propose two effective constraints: 1) Increase the adjusted step size $b^k$ at the fracture; 2) Make the prisms close to the fracture in the same depth as the surrounding prisms.

We built a model that included all the types of terrain mentioned above. The solution obtained by adding constraints is almost in agreement with the model (Fig. 4).

![Figure 4. Inversion process of the model mentioned above](image)

**3. Stability analysis of inversion**

We used several models to analyze the stability of inversion. The noisy data and the artifacts of location can lower the accuracy of recovered depth. However, the error in density contrast is one of the key issues that would overestimate or underestimate the depth of bedrock.

![Figure 5. Noise effects of the initial approximation $P^1$ and the solution](image)
3.1. Noise effects
We here analyze the noise of gravity observations of the discontinuous basement relief and the V-shaped terrain. We note that the smaller the noise of data, the smaller the standard deviation of solutions are obtained. As the depth of the bottom interface increases, the remaining standard deviation of the depth of the solution will increase. Numerically, the standard deviation of the Bouguer gravity anomaly corresponding to the solution is about one-third of the standard deviation of the observations with noise. We illustrate that the method itself has a recovery effect depends on the noise (Fig. 5).

3.2. Location effects
Without constraints, the solution of the discontinuous basement relief appears unstable at the locations of faults. We use two constraint methods to analyze the location effects. The results show that this iterative method has the certain controlling ability for the final solution. Only if the constraint is added to the boundary position of the bottom of the model, it will produce an unstable and incorrect solution (Figures 6a and 6b).

![Figure 6. Location effects on the solution of the discontinuous basement relief](image)

For V-shaped terrain, we comprehend that if the constraint position is input incorrectly, the solution of the bottom of the V-shape cannot fit the theoretical model. As the location where the constraint added erroneously away from the bottom of the model, the solution will approach the solution without constraints (Fig. 7).

![Figure 7. Location effects on the solution of the V-shaped terrain](image)
3.3. Density effects

The difference in depth between the solution and the real model depth increases with the difference between the model density and the true density. If the model density is greater than the true density, the solution is shallower than the true depth. Conversely, the solution is deeper than the true depth. For both the discontinuous basement reliefs, the effects of the density are greater at the bottom than that at the sides and greater at the bottom sides than at the middle of the bottom. For V-shaped terrain, the effect of density on the position of the V-tip is greater than in other positions (Fig. 8).

![Figure 8. Density effects of the discontinuous basement relief and the V-shaped terrain](image)

4. Conclusions

The L1 inversion of gravity data with prior information can be used to estimate the basement relief in the near-surface geophysical investigation. The method of Santos et al. (2015) is easily applied to a complex model with faults, smooth feature, and V-shaped interface. We have analyzed the effects of the noise of data, the constraints with errors and the incorrect density contrast in the inversion, showing that the constrained L1 inversion of gravity data is stable and efficient for the complex discontinuous relief of basement.

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