Virtual Top-Quark Effects on the $H \rightarrow b\bar{b}$ Decay at Next-to-Leading Order in QCD

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Abstract

By means of a heavy-top-quark effective Lagrangian, we calculate the three-loop corrections of $O(\alpha_s^2 G_F M_t^2)$ to the $H \rightarrow b\bar{b}$ partial decay width of the standard-model Higgs boson with intermediate mass $M_H \ll 2M_t$. We take advantage of a soft-Higgs theorem to construct the relevant coefficient functions. We present our result both in the MS and on-shell schemes of mass renormalization. The MS formulation turns out to be favourable with regard to the convergence behaviour. We also test a recent idea concerning the naïve non-abelianization of QCD.

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One of the central questions of elementary particle physics is whether nature makes use of the Higgs mechanism of spontaneous symmetry breaking to endow the particles with their masses. In the framework of the minimal standard model, the Higgs boson, $H$, is the missing link to be experimentally discovered in order to complete our understanding of mass generation. So far, all attempts to detect the Higgs boson on its mass shell have been in vain, with the effect that the mass range $M_H \leq 65.2$ GeV has been ruled out at the 95% confidence level (CL) [1]. However, experimental precision tests of the standard electroweak theory are sensitive to the Higgs boson via quantum corrections. A recent global fit [2] has yielded $M_H = 149^{+148}_{-82}$ GeV together with a 95% CL upper bound of 550 GeV.

A Higgs boson with $M_H \lesssim 135$ GeV decays dominantly to $b\bar{b}$ pairs. This decay mode will be crucial for Higgs-boson searches at LEP 2, the Fermilab Tevatron after the installation of the Main Injector, a next-generation $e^+e^-$ linear collider, and a future $\mu^+\mu^-$ collider.

The present knowledge of quantum corrections to the $H \to b\bar{b}$ partial decay width has recently been reviewed in Ref. [3]. At one loop, the electroweak [4] and QCD [5] corrections are known for arbitrary masses. In the limit $M_H \ll 2M_t$, in which we are interested here, the terms of $O(X_t)$, where $X_t = (G_F M_t^2 / 8\pi^2 \sqrt{2})$, tend to be dominant. They arise in part from the renormalizations of the Higgs wave function and vacuum expectation value, which are independent of the quark flavour [6]. In the case of bottom, there is an additional non-universal $O(X_t)$ contribution [4], which partly cancels the flavour-independent one. At two loops, the universal [7] and bottom-specific [8] $O(\alpha_s X_t)$ terms are available. Furthermore, the first [9] and second [10] terms of the expansion in $M_t^2 / M_H^2$ of the $O(\alpha_s^2)$ five-flavour QCD correction have been found. As for the top-quark-induced correction in $O(\alpha_s^2)$, the full $M_t$ dependence of the non-singlet (double-bubble) contribution [9] as well as the first four terms of the $M_H^2 / M_t^2$ expansion of the singlet (double-triangle) contribution [11] have been computed. At three loops, the $O(\alpha_s^3)$ non-singlet correction is known in the massless approximation [13]. Furthermore, the $H \to gg$ decay width receives a $O(\alpha_s^3)$ correction due to the $b\bar{b}g$ final state which may also be interpreted as a $O(\alpha_s^3 M_H^2 / M_t^2)$ correction to the $H \to b\bar{b}$ decay width [14].

We have taken the next step by evaluating the three-loop $O(\alpha_s^2 X_t)$ corrections to the $H \to b\bar{b}$ decay width. In this letter, we report the key results. A detailed description of our analysis together with a discussion of the $O(\alpha_s^2 X_t)$ corrections to the less important $H \to q\bar{q}$ decay widths, where $q = u, d, s, c$, will be presented elsewhere [15].

We now outline our procedure. We construct a $n_f = 5$ effective Yukawa Lagrangian, $\mathcal{L}_{Y}^{\text{eff}}$, by integrating out the top quark. This Lagrangian is a linear combination of dimension-four operators acting in QCD with $n_f = 5$ quark flavours, while all $M_t$ dependence is contained in the coefficient functions. We then renormalize this Lagrangian and, by exploiting the renormalization-group (RG) invariance of the energy-momentum tensor, rearrange it in such a way that the renormalized operators and coefficient functions are separately independent of the renormalization scale, $\mu$. The final result for $\mathcal{L}_{Y}^{\text{eff}}$...
exhibits the following structure:

\[
\mathcal{L}_V^{\text{eff}} = -2^{1/4} G_F^{1/2} H \left( 1 + \delta_u \right) \left[ C_1 [O_1'] + \sum_q \left( C_{2q} [O_{2q}'] + C_{3q} [O_{3q}'] \right) \right], 
\]

where \( q \) is runs over \( u, d, s, c, b \) and the primed objects refer to QCD with \( n_f = 5 \). Here, \( \delta_u \) is the universal correction resulting from the renormalizations of the Higgs wave function and vacuum expectation value. The square brackets denote the renormalized counterparts of the bare \( n_f = 5 \) operators

\[
O_1' = \left( G_{a\mu\nu}^{0q} \right)^2, \quad O_{2q}' = m_q^0 q^0 q^0, \quad O_{3q}' = \bar{q}^0 \left[ \frac{i}{2} \left( \not{D} - \not{F} \right) - m_q^0 \right] q^0, 
\]

where \( G_{a\mu\nu} \) is the colour field strength, \( D_\mu \) is the covariant derivative, and the superscript 0 labels bare fields and parameters. \( C_1, C_{2q}, \) and \( C_{3q} \) are the respective renormalized coefficient functions. Notice that \( [O_{3q}'] \) vanishes by the fermionic equation of motion, so that \( C_{3q} \) will not appear in our final result. Nevertheless, the inclusion of \( O_{3q}' \) is indispensable for the determination of \( C_1 \) and \( C_{2q} \). The RG-improved formulation thus obtained provides a natural separation of the \( n_f = 5 \) QCD corrections at scale \( \mu = M_H \) and the top-quark-induced \( n_f = 6 \) corrections at scale \( \mu = M_t \), in the sense that the final result for the \( H \to b\bar{b} \) decay width will not contain logarithms of the type \( \ln(M_H^2/M_t^2) \) if the \( n_f = 5 \) and \( n_f = 6 \) corrections are expanded in \( \alpha_s^{(5)}(M_H) \) and \( \alpha_s^{(6)}(M_t) \), respectively.

In contrast to the two-loop \( \mathcal{O}(\alpha_s X_t) \) case \([8]\), where it was sufficient to consider just one term in \( \mathcal{L}_V^{\text{eff}} \), we need to take into account three types of operators and to allow for them to mix under renormalization. The mixing terms are related to the \( \mathcal{O}(\alpha_s^2 X_t) \) double-triangle contribution considered in Ref. \([12]\) and extend the latter to \( \mathcal{O}(\alpha_s^2 X_t) \).

Similarly to Ref. \([3]\), we can take advantage of the higher-order formulation \([16]\) of a well-known soft-Higgs theorem \([17]\) to simplify the calculation of the coefficient functions. This allows us to relate a huge number of three-loop three-point diagrams to a manageable number of three-loop two-point diagrams. Specifically, we have to compute 24 irreducible three-loop two-point diagrams for \( q \neq b \) and, in addition, 54 ones for \( q = b \). Typical examples are depicted in Fig. \([1]\). Such a theorem is not available for the gauge interactions, which might explain why three-loop \( \mathcal{O}(\alpha_s^2 X_t) \) corrections have not yet been calculated for the \( Z \to q\bar{q} \) decay widths, including the important case of \( Z \to b\bar{b} \), which has recently attracted much attention in connection with the so-called \( R_b \) anomaly.

From Eq. \([1]\) we may derive an expression for the \( H \to b\bar{b} \) decay width, appropriate for \( M_H \ll 2M_t \), which accommodates all presently known corrections, including the new ones of \( \mathcal{O}(\alpha_s^2 X_t) \). It reads

\[
\Gamma \left( H \to b\bar{b} \right) = \Gamma_{bb}^{\text{Born}} \left( 1 + \Delta_b^{\text{QED}} \right) \left( 1 + \Delta_b^{\text{weak}} \right) \left( 1 + \Delta_b^{\text{QCD}} \right) \left( 1 + \Delta_b^t \right) + \Xi_b^{\text{QCD}} \Xi_b^t. 
\]

Here,

\[
\Gamma_{bb}^{\text{Born}} = \frac{3G_F M_H m_b^2}{4\pi \sqrt{2}} \left( 1 - \frac{4m_b^2}{M_H^2} \right)^{3/2} 
\]

(3)
Figure 1: Typical diagrams generating universal and non-universal $\mathcal{O}(\alpha_s^2 X_t)$ corrections to $\Gamma(H \to b\bar{b})$. Bold-faced (dashed) lines represent top quarks (Higgs or Goldstone bosons).

is the Born result including the full mass dependence. As is well known [5], we may avoid the appearance of large logarithms of the type $\ln(M_H^2/m_b^2)$ in the QCD correction, $\Delta_b^{QCD}$, by taking $m_b$ in Eq. (4) to be the $\overline{\text{MS}}$ mass evaluated with $n_f = 5$ quark flavours at scale $\mu = M_H$, $m_b^{(5)}(M_H)$. Consequently, we may put $m_b = 0$ in $\Delta_b^{QCD}$. We may proceed similarly with the QED correction, $\Delta_b^{QED}$, which then takes the form

$$\Delta_b^{QED} = \frac{17}{36} \alpha(M_H) \pi,$$

(5)

where $\alpha(\mu)$ is the $\overline{\text{MS}}$ fine-structure constant. In turn, $m_b^{(5)}(M_H)$ is then also shifted by a QED correction [18] from the pole mass, $M_b$, $\Delta_b^{\text{weak}}|_{X_t=0}$ denotes the weak correction with the leading $\mathcal{O}(X_t)$ term stripped off. If we put $m_b = 0$ and consider the limit $M_H \ll 2M_W$, $\Delta_b^{\text{weak}}|_{X_t=0}$ simplifies to [4]

$$\Delta_b^{\text{weak}}|_{X_t=0} = \frac{G_F M_Z^2}{8\pi^2 \sqrt{2}} \left( \frac{1}{6} - \frac{7}{3} c_w^2 - \frac{16}{3} c_w^4 + 3 \frac{c_w^2}{s_w^2} \ln c_w^2 \right),$$

(6)

where $c_w^2 = 1 - s_w^2 = M_W^2/M_Z^2$ and $M_Z$ ($M_W$) is the $Z$-boson ($W$-boson) mass. $\Delta_b^{QCD}$ is the well-known QCD correction for $n_f = 5$ [3],

$$\Delta_b^{QCD} = \frac{17}{3} a_5 + a_5^2 \left( \frac{8851}{144} - \frac{47}{6} \zeta(2) - \frac{97}{6} \zeta(3) \right),$$

(7)

where $a_5 = \alpha_s^{(5)}(M_H)/\pi$ and $\zeta$ is Riemann’s zeta function, with values $\zeta(2) = \pi^2/6$ and $\zeta(3) \approx 1.202$. This correction originates from the class of diagrams where the Higgs boson directly couples to the final-state $b\bar{b}$ pair. As is well known [12], starting at $\mathcal{O}(\alpha_s^2)$, $\Gamma(H \to b\bar{b})$ also receives leading contributions from the $b\bar{b}$ and $bbg$ cuts of the double-triangle diagrams where the top quark circulates in one of the triangles. In the language
of Eq. (\text{[15]}), where the top quark only appears in the coefficient functions, this class of contributions is generated by the interference diagram of the operators \([O'_1]\) and \([O'_{2b}]\). The absorptive part of this diagram also includes a contribution from the \(gg\) cut, which is well known and must be subtracted. This leads to

\[
\Xi_b^{QCD} = a_5 \left( -\frac{76}{3} + 8\zeta(2) - \frac{4}{3} \ln^2 \frac{m_t^2}{M_H^2} \right).
\]

(8)

The would-be mass singularity proportional to \(\ln^2(m_b^2/M_H^2)\) cancels if the \(b\bar{b}(g)\) and \(gg\) decay channels are combined \([12]\).

Equation (\text{[3]}) is arranged in such a way that the leading \(M_t\) dependence is carried by

\[
\Delta_b^t = \left( 1 + \delta_u \right)^2 (C_{2b})^2 - 1, \quad \Xi_b^t = \left( 1 + \delta_u \right)^2 C_1 C_{2b},
\]

(9)

where \(\delta_u, C_1,\) and \(C_{2b}\) are defined in the context of Eq. (\text{[3]}). Since we use \(m_b^{(5)}(M_H)\) in the \(n_f = 5\) segments of Eq. (\text{[3]}), it appears natural to also renormalize the top-quark mass in the \(n_f = 6\) terms according to the MS prescription. The scale-independent definition \(\mu_t = m_t^{(6)}(\mu_t)\) is singled out, since it eliminates the RG logarithms of the type \(\ln(\mu^2/m_t^2)\). By analogy to \(X_t\), we define \(x_t = \left( G_F \mu_t^2 / 8\pi^2 \sqrt{2} \right)\). We shall return to the \(M_t\) formulation later on. From Ref. (\text{[19]}), we know that

\[
\delta_u = x_t \left[ \frac{7}{2} + a_6 \left( \frac{19}{3} - 2\zeta(2) \right) - 9.598 a_6^2 \right],
\]

(10)

where \(a_6 = \alpha_s^{(6)}(\mu_t)/\pi\). An analytic expression for \(\delta_u\), valid for \(\mu\) and \(N_c\) arbitrary, may be found in Ref. (\text{[19]}). As mentioned above, \(C_1\) and \(C_{2b}\) are RG invariant per construction. Owing to the separation of the \(n_f = 5\) physics at scale \(\mu = M_H\) and the \(n_f = 6\) physics at scale \(\mu = \mu_t\), they are given by \([13]\)

\[
C_1 = \frac{\alpha_s^{(5)}(\mu_t) \beta^{(5)}(M_H)}{\alpha_s^{(5)}(M_H) \beta^{(5)}(\mu_t)} C_1, \quad C_{2b} = \frac{4\alpha_s^{(5)}(\mu_t)}{\pi \beta^{(5)}(\mu_t)} \left( \gamma_m^{(5)}(\mu_t) - \gamma_m^{(5)}(M_H) \right) C_1 + C_{2b},
\]

(11)

where \(\beta^{(n_f)}(\mu) = \beta \left[ \alpha_s^{(n_f)}(\mu) \right]\) is the Callan-Symanzik beta function, \(\gamma_m^{(n_f)}(\mu) = \gamma_m \left[ \alpha_s^{(n_f)}(\mu) \right]\) is the quark-mass anomalous dimension, and \([13]\)

\[
C_1 = a_6 \left( -\frac{1}{12} + \frac{1}{4} x_t \right),
\]

(12)

\[
C_{2b} = 1 + \frac{5}{18} a_6^2 + x_t \left\{ -3 - 7a_6 + a_6^2 \left[ -\frac{4201}{72} + 38\zeta(2) - \frac{5}{2} \zeta(3) - n_f \left( \frac{241}{72} - \frac{4}{3} \zeta(2) \right) \right] \right\}.
\]

Here, we have displayed the coefficient of \(n_f = 6\), for reasons which will become clear later on. General expressions for \(C_1\) and \(C_{2b}\), valid for \(\mu\) and \(N_c\) arbitrary, will be provided in Ref. (\text{[19]}). If we expand Eq. (\text{[11]}), and substitute Eq. (\text{[12]}), Eq. (\text{[9]}) becomes

\[
\Delta_b^t = a_6^2 \left[ \frac{5}{9} + \frac{2}{3} \ln \frac{\mu^2}{M_H^2} \right] x_t \left[ 1 + a_6 \left( -\frac{4}{3} - 4\zeta(2) \right) + a_6^2 \left( -59.163 + \frac{2}{3} \ln \frac{\mu^2}{M_H^2} \right) \right],
\]

\[
\Xi_b^t = a_6 \left( -\frac{1}{12} - \frac{1}{12} x_t \right).
\]

(13)
The appearance of the logarithms \( \ln(\mu^2/M_H^2) \) witnesses the loss of the RG improvement inherent in Eq. (11).

As a by-product, we may derive from Eq. (1) a formula for the \( H \to gg \) decay width which includes the \( O(x_t) \) correction. The result is

\[
\Gamma(H \to gg) = \frac{2\sqrt{2}G_FN_H^3}{\pi}(1 + \delta_u)^2(C_1)^2 = \frac{G_FN_H^3a_6^2}{36\pi\sqrt{2}}(1 + x_t),
\]

which is in agreement with Ref. [20].

If we express Eq. (12) in terms of \( M_t \), we obtain

\[
C_{2b}^{OS} = 1 + \frac{5}{18}A_6^2 + X_t \left\{ -3 + A_6 + A_6^2 \left[ -\frac{65}{3} + 52\zeta(2) + 4\zeta(2)\ln 2 - \frac{7}{2}\zeta(3) \right] \right. \\
+ \left. n_f \left( \frac{7}{18} - \frac{10}{3}\zeta(2) \right) \right\},
\]

where \( A_6 = \alpha_s^\text{(6)}(M_t)/\pi \), while \( C_1^{OS} \) emerges from \( C_1 \) by merely replacing \( a_6 \) and \( x_t \) with \( A_6 \) and \( X_t \), respectively. From Eq. (13) we read off that the leading \( O(X_t) \) term receives the QCD correction factor \( (1 - 3.333A_6 - 11.219A_6^2) \). We thus recover a pattern similar to the electroweak parameter \( \Delta \rho \) [21] and the corrections \( \delta_u, \delta_{WWH}, \) and \( \delta_{ZZH} \) [19] to the \( l^+l^-H, W^+W^-H, \) and \( ZZH \) vertices, respectively. In fact, the corresponding QCD expansions in \( A_6 \) of these four observables all have negative coefficients which dramatically increase in magnitude as one passes from two to three loops [19]. On the other hand, if the top-quark mass is renormalized in the \( \overline{\text{MS}} \) scheme at scale \( \mu = \mu_t \), then the respective QCD expansions in \( a_6 \) are found to have coefficients which have variant signs and nicely group themselves around zero [19]. We note in passing that the study of infrared renormalons [22] offers a possible theoretical explanation for this observation. In the case of \( C_{2b} \), the QCD correction factor reads \( (1 - 2.333a_6 + 7.032a_6^2) \). We conclude that, also in the case of the \( bbH \) interaction, the QCD expansion in the on-shell scheme exhibits a worse convergence behaviour than the one in the \( \overline{\text{MS}} \) scheme. However, the difference is less striking than in the previous four cases.

Finally, we would like to test Broadhurst’s rule concerning the naïve non-abelianization of QCD [23]. Guided by the observation that the \( n_f \)-independent term of \( \beta_0 \) emerges from the coefficient of \( n_f \) by multiplication with \(-33/2 \), Broadhurst conjectured that this very relation between the \( n_f \)-independent term and the coefficient of \( n_f \) approximately holds for any observable at next-to-leading order in QCD. In Ref. [19], this rule was applied to \( \Delta \rho, \delta_u, \delta_{WWH}, \) and \( \delta_{ZZH} \), and it was found that, in all four cases, the signs and orders of magnitude of the \( n_f \)-independent terms are correctly predicted. Except for \( \delta_{ZZH} \), these predictions come, in fact, very close to the true values. If we multiply the coefficients of \( n_f \) in Eqs. (12) and (15) with \(-33/2 \), we obtain \(-19.041 \) and \( 84.055 \), which has to be compared with the respective \( n_f \)-independent terms, \(-28.018 \) and \( 64.223 \). Once again, the signs and orders of magnitude of the \( n_f \)-independent terms, which are usually much harder to compute than the coefficients of \( n_f \), come out correctly.
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