Electromagnetically induced absorption and transparency in magneto-optical resonances in elliptically polarized field

D.V. Brazhnikov, A.M. Tumaikin, and V.I. Yudin

Institute of Laser Physics, Siberian Branch of Russian Academy of Sciences, Novosibirsk 630090, Russia

A.V. Taichenachev

Novosibirsk State University, Novosibirsk 630090, Russia

Abstract

Using a $1 \rightarrow 2$ transition as an analytically tractable model, we discuss in detail magneto-optical resonances of both EIA (electromagnetically induced absorption) and EIT (electromagnetically induced transparency) types in the Hanle configuration. The analysis is made for arbitrary rate of depolarizing collisions in the excited state and arbitrary elliptical field polarization. The obtained results clearly show that the main reason for the EIA sub-natural resonance is the spontaneous transfer of anisotropy from the excited level to the ground one. In the EIA case we predict the negative structures in the absorption resonance at large field detuning. The role of the finite atom-light interaction time is briefly discussed. In addition we study non-trivial peculiarities of the resonance lineshape related to the velocity spread in a gas.
I. INTRODUCTION

Nonlinear interference effects, based on atomic coherence\cite{1}, have attracted in the past decade a growing attention. They have found numerous applications in nonlinear optics\cite{2}-\cite{10}, nonlinear high-resolution spectroscopy\cite{11}-\cite{16}, high-precision metrology (atomic clocks and magnetometers)\cite{17}-\cite{21}, laser cooling\cite{22,23}, atom optics and interferometry\cite{24}-\cite{26}, and quantum information processing\cite{27}-\cite{32}.

To date the most investigated phenomenon is electromagnetically induced transparency (EIT)\cite{33}. As a matter of fact, EIT means a broad array of nonlinear effects in which the absorption is significantly reduced due to nonlinear interference of electromagnetic waves. In particular, in certain atomic systems EIT is related to dark states and coherent population trapping (CPT)\cite{34,35}.

The opposite phenomenon of electromagnetically induced absorption (EIA), first observed by Akulshin et al. in 1998\cite{36}, is much less studied. After 1998 several groups have performed experiments on observations of EIA and related effects in different atomic systems and under different conditions\cite{37}-\cite{42}. Both two-photon resonances in a bichromatic light field and magneto-optical resonances in the Hanle configuration have been explored.

The first theoretical explanation of physical origins of EIA was given in\cite{43,44}, where, using a simple analytically tractable model of a four-level $N$-system as an example, we have shown that EIA is due to the transfer of low-frequency atomic coherence between interacting levels in the course of spontaneous radiative transitions. Subsequently, this concept has been confirmed in experiments and by numerical calculations\cite{45}, and it has been further developed in\cite{46}. Summarizing and generalizing, we can say that EIA can be observed on resonant atomic transitions of $F_g = F \rightarrow F_e = F + 1$ type with degenerate
ground state \((F > 0)\), and its physical origin is the spontaneous transfer of the light-induced anisotropy (atomic coherence or/and population difference). As the result, under two-photon resonance conditions atoms mainly populate Zeeman sub-states most coupled to the light field. If the spontaneous transfer of anisotropy is absent, i.e., the ground state is repopulated isotropically, atomic populations are distributed inversely proportional to the optical depopulation rates, which leads to the EIT-type resonances.

It is worth to note that the same physical reason (the spontaneous transfer of anisotropy) is responsible for the "anomalous" sign of nonlinear magneto-optical rotation of linearly polarized light on \(F \rightarrow F + 1\) transitions. This effect has been observed earlier in 1990 \[47\], and its theoretical explanation has been given by Kanorsky et al. \[48\] on the base of the perturbation theory method developed in \[49\]. The difference in signs of the ground-state quadrupole moments, which govern the sign of nonlinear magneto-optical rotation, for transitions \(F \rightarrow F + 1\) from one hand, and for \(F \rightarrow F\) and \(F \rightarrow F - 1\) transitions from the other hand has been discussed in \[50\].

The present paper is devoted to a detailed theoretical study of EIA/EIT magneto-optical resonances in the Hanle configuration. We explore the realistic but analytically solvable model of a \(F_g = 1 \rightarrow F_e = 2\) atomic transition. Note that this transition is the simplest transition (from the \(F \rightarrow F + 1\) class), where the light-induced quadrupole moment, precessing in a magnetic field orthogonal to the polarization ellipse plane, can appear in the ground state. Apart from the radiative relaxation, the excited-state depolarization due to collisions with a buffer gas is taken into account, which allows us to consider a gradual transition from EIA to EIT with the increase of the depolarization rate. It should be noted, in the majority of works on the magneto-optical variant of EIA/EIT resonance, except for the papers \[51\] - \[54\], the linearly polarized radiation was used or considered. Here we investigate a general
case of the resonance interaction of atoms with elliptically polarized light. We find several new features of the magneto-optical spectra connected with the light ellipticity. The treatment is carried out for atoms at rest (homogeneous broadening) as well as for atomic gas under conditions of the Doppler broadening. The role of the finite atom-light interaction time is briefly discussed.

II. PROBLEM STATEMENT

We consider the resonance interaction of atoms, having the total angular momenta \( F_g \) in the ground state and \( F_e \) in the excited state, with a monochromatic light field \( \mathbf{E}(r, t) \) in the presence of a static magnetic field \( \mathbf{B} \). More specifically, the field configuration is an elliptically polarized running wave:

\[
\mathbf{E}(r, t) = E \mathbf{e} \exp\{-i(\omega t - \mathbf{k} r)\} + \text{c.c.},
\]

where \( E \) is the complex amplitude, and \( \mathbf{e} \) is the unit polarization vector. In the coordinate frame associated with the wave (\( \mathbf{e}_z \) is directed along \( \mathbf{k} \), \( \mathbf{e}_{x,y} \) – along the polarization ellipse semiaxes) the polarization vector can be written as

\[
\mathbf{e} = \mathbf{e}_x \cos(\varepsilon) + i\mathbf{e}_y \sin(\varepsilon) = \mathbf{e}_{+1} \cos(\varepsilon - \pi/4) + \mathbf{e}_{-1} \sin(\varepsilon - \pi/4),
\]

with \( \mathbf{e}_{\pm 1} = \mp(\mathbf{e}_x \pm i\mathbf{e}_y)/\sqrt{2} \) the corresponding spherical orths, and \( \tan(\varepsilon) \) equal to the ratio of the ellipse semiaxes (see in Fig. 1.a).

The atom-light interaction Hamiltonian in the dipole and in the rotating-wave approximations has the form:

\[
\widehat{H}_{D-E} = -\widehat{d}\mathbf{E} = \hbar\kappa\mathbf{V} + \text{h.c.},
\]

where \( \kappa = -\langle F_e||\widehat{d}||F_g\rangle E/\hbar \) is the coupling constant (Rabi frequency), and, according to the
Wigner-Eckart theorem,

$$\hat{V} = \mathbf{T}^{eg} \mathbf{e},$$

with the Wigner vector operator defined through Clebsch-Gordan coefficients:

$$\hat{T}^{ab}_q = \sum_{m_a, m_b} C^{F_a, m_a}_{F_b, m_b; 1 q} |F_a, m_a\rangle \langle F_b, m_b|,$$

the indices $a$ and $b$ can take values $e, g$. The interaction with a weak static field $\mathbf{B}$ is described by the Hamiltonian

$$\hat{H}_B = -\mu \mathbf{B} = \sum_{a=e,g} \hbar \Omega_{a} \hat{F}^{(a)}_b,$$

where the operator

$$\hat{F}^{(a)}_b = \sqrt{F_a(F_a+1)} \hat{T}^{aa} \mathbf{b}$$

are the projection of the total angular momentum operator of the level $(a)$ on the direction $\mathbf{b}$; $\hbar \Omega_{a} = \mu_B g_a B$ is the Zeeman splitting between adjacent substates of the level $(a)$ with $\mu_B$ the Bohr magneton and $g_a$ the Lande $g$-factor; $\mathbf{b} = B/B$ is a unit vector directed along the magnetic field $\mathbf{B}$.

All operators are represented as matrices on the Zeeman basis of the ground and excited levels, with states $\{|F_g, m_g\rangle\}$ and $\{|F_e, m_e\rangle\}$. The atomic density matrix $\hat{\rho}$ can be separated in four matrix blocks, where the matrices $\hat{\rho}_{gg}$ and $\hat{\rho}_{ee}$ are the density submatrices for the ground and excited state, and the off-diagonal blocks $\hat{\rho}_{eg}$ and $\hat{\rho}_{ge}$ describe the optical coherences. In the rotating-wave approximation the fast time-space dependence of the kinetic equation can be removed by introducing the transformed optical coherences as

$$\hat{\rho}_{eg} = \hat{\rho}_{eg} \exp\{-i(\omega t - \mathbf{kr})\}, \quad \hat{\rho}_{ge} = \hat{\rho}_{ge} \exp\{i(\omega t - \mathbf{kr})\}.$$

Then, the generalized optical Bloch equations (GOBE) take the form

$$(\gamma_{eg} - i\delta_e)\hat{\rho}_{eg} = -i \kappa \left[ \hat{V} \hat{\rho}_{gg} - \hat{\rho}_{ee} \hat{V} \right] - i \left[ \Omega_e \hat{F}^{(e)}_b \hat{\rho}_{eg} - \hat{\rho}_{eg} \Omega_g \hat{F}^{(g)}_b \right];$$

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\[
(\gamma_{eg} + i\delta_v)\hat{\rho}_{ge} = -i\kappa \left[ \hat{V}^\dag \hat{\rho}_{ee} - \hat{\rho}_{gg} \hat{V}^\dag \right] - i \left[ \Omega_g \hat{F}^{(g)}_b \hat{\rho}_{ge} - \hat{\rho}_{ge} \Omega_e \hat{F}^{(e)}_b \right];
\]

\[
(\Gamma + \gamma_r)\hat{\rho}_{ee} + \hat{\gamma}_{coll}\{\hat{\rho}_{ee}\} = -i\kappa \left[ \hat{V} \hat{\rho}_{ge} - \hat{\rho}_{eg} \hat{V} \right] - i \Omega_e \left[ \hat{F}^{(e)}_b , \hat{\rho}_{ee} \right];
\]

\[
\Gamma \left( \hat{\rho}_{gg} - \hat{\rho}_{gg}^{(0)} \right) = \hat{G}\{\hat{\rho}_{ee}\} - i\kappa \left[ \hat{V}^\dag \hat{\rho}_{eg} - \hat{\rho}_{ge} \hat{V} \right] - i \Omega_g \left[ \hat{F}^{(g)}_b , \hat{\rho}_{gg} \right],
\]

where \([, , ]\) indicates a commutator, \(\delta_v = \omega - \omega_{eg} - kv\) is the detuning with account for the Doppler shift \(kv\) for a moving atom, \(\gamma_{eg}\) is the dephasing rate, in Eq. (11) the parameter \(\gamma_r\) is the radiative relaxation rate, the operator \(\hat{\gamma}_{coll}\{\hat{\rho}_{ee}\}\) describes the collisional depolarization in the excited state, the rate \(\Gamma\) describes relaxation of atoms to the isotropical distribution \(\hat{\rho}_{gg}^{(0)} = \hat{\Pi}_g / (2F_g + 1)\) (the operator \(\hat{\Pi}_a = \sum m_a |F_a, m_a\rangle \langle F_a, m_a|\) projects on the given energy level \((a)\) outside the light beam due to either atomic free flight through the beam or diffusion to the walls. The operator

\[
\hat{G}\{\hat{\rho}_{ee}\} = \beta \gamma_r \sum_{q=0, \pm 1} \hat{T}^{eg}_q \hat{\rho}_{ee} \hat{T}^{eg}_q
\]

in the right-hand side of Eq. (12) corresponds to the repopulation of the ground state due to the spontaneous radiative transitions. This process includes the transfer of the Zeeman-substate populations (terms with diagonal matrix elements \(\rho_{m_e m_e}\)) as well as the transfer of the Zeeman coherence (terms with off-diagonal matrix elements \(\rho^{ee}_{m_e m'_e}\) at \(m_e \neq m'_e\)). In general, it should be viewed as the spontaneous transfer of the total population and of the Zeeman anisotropy between working levels. The coefficient \(\beta \leq 1\) governs the branching ratio in the course of the spontaneous decay from the excited level \(F_e\) to the lower level \(F_g\). When \(\beta = 1\) the transition \(F_g \rightarrow F_e\) is closed, i.e. the total population is conserved \((\text{Tr}\{\hat{\rho}_{gg}\} + \text{Tr}\{\hat{\rho}_{ee}\} = 1\)\). Hereafter the symbol \(\text{Tr}\{\ldots\}\) means the trace operation over internal degrees of freedom.

We assume the property

\[
\text{Tr}\{\hat{\gamma}_{coll}\{\hat{\rho}_{ee}\}\} = 0.
\]
If collisions are negligible, we have

\[ \hat{\gamma}_{\text{coll}} = 0 , \quad \gamma_{eg} = \gamma_r / 2 + \Gamma . \]  

(15)

The collisional relaxation term \( \hat{\gamma}_{\text{coll}} \{ \hat{\rho}_{ee} \} \) has the simplest form

\[ \hat{\gamma}_{\text{coll}} \{ \hat{\rho}_{ee} \} = \gamma_1 \left[ \hat{\rho}_{ee} - \hat{\Pi}_e \text{Tr} \{ \hat{\rho}_{ee} \} / (2F_e + 1) \right] \]

(16)

in the model case, when all the multipole moments relax due to collisions with the same rate \( \gamma_1 \) (apart from the total population, which is conserved according to Eq.(14)).

More specifically, we will study just one closed (\( \beta = 1 \)) transition \( F_g = 1 \rightarrow F_e = 2 \). This transition is realistic and simultaneously sufficiently simple, allowing analytical description of the EIA/EIT effects. Let the static magnetic field is directed orthogonal to the polarization ellipse, i.e. along the \( z \) axis (see in Fig. 1a). The corresponding scheme of the light-induced transitions is shown in Fig. 1b. As a spectroscopic signal we consider the total excited-state population as a function of the magnetic field amplitude \( B \) (Hanle-type spectroscopy)

\[ \pi_e = \text{Tr} \{ \hat{\rho}_{ee} \} . \]

(17)

This signal is proportional to the total fluorescence and to the total light absorption in optically thin media. We investigate the influence of the radiative relaxation operator (13) in combination with the collisional depolarization operator \( \hat{\gamma}_{\text{coll}} \{ \hat{\rho}_{ee} \} \) on the lineshape of the resonance described by Eq.(17).

We will be interested in sub-natural width structures (\( \Omega_{e,g} \ll \gamma_r \)), which appear in nonlinear spectra in the low-saturation limit, when the light field is sufficiently weak:

\[ S = \frac{|\kappa|^2}{\gamma_{eg}^2 + \delta_v^2} \ll 1 . \]

(18)

With these approximations, eliminating the optical coherences, we arrive at the following
closed set of equations for the excited-state and ground-state density matrices:

\[
(\Gamma + \gamma_r)\hat{\rho}_{ee} + \gamma_1 \left[ \hat{\rho}_{ee} - \hat{H}_e \text{Tr} \{ \hat{\rho}_{ee} \} / (2F_e + 1) \right] = 2\gamma_{eg} S \hat{V} \hat{\rho}_{gg} \hat{V}^\dagger,
\]

\[
\Gamma \left( \hat{\rho}_{gg} - \hat{\rho}_{gg}^{(0)} \right) + i\Omega_g \hat{F}_b^{(g)}, \hat{\rho}_{gg} = - \left\{ (\gamma_{eg} + i\delta_v) S \hat{V}^\dagger \hat{\rho}_{gg} + h.c. \right\} + \gamma_r \sum_{q=0,\pm 1} T_{eg q}^\dagger \hat{\rho}_{ee} T_{eg q},
\]

where the collision relaxation operator is taken in the form (16).

III. SOLUTION FOR ATOM AT REST

Consider first the total excited-state population \(\pi_e\) of an atom at rest \((v = 0)\), when the detuning \(\delta = \omega - \omega_{eg}\). The solution for a moving atom with the given velocity \(v\) can be easily derived from the expressions below by the substitution \(\delta \rightarrow \delta_v\). As is seen from Eq.(20) and Fig.1.b, the coherence between just two Zeeman substates \((|F_g = 1, m_g = \pm 1\rangle)\) is sensitive to the magnetic field. Thus, as it has been shown in [55], \(\pi_e\) (as well as any spectroscopic signal) is a quotient of polynomials of second order in \(\Omega_g\):

\[
\frac{\pi_e}{\pi_e^{(0)}} = \frac{\sum_{i=0}^{2} N_i(\Delta, \tilde{\gamma}_1, \tilde{\Gamma}, \varepsilon) \Omega^i}{\sum_{k=0}^{2} D_k(\Delta, \tilde{\gamma}_1, \tilde{\Gamma}, \varepsilon) \Omega^k},
\]

where \(\pi_e^{(0)} = 2\gamma_{eg} S / (\gamma_r + \Gamma)\) corresponds to the linear absorption of unpolarized atoms, \(\Omega = \Omega_g / (\gamma_{eg} S), \Delta = \delta / \gamma_{eg}, \tilde{\gamma}_1 = \gamma_1 / \gamma_r, \tilde{\Gamma} = \Gamma / (\gamma_{eg} S), \) and \(\varepsilon\) is the ellipticity parameter of the light wave. The numerator and denominator can be expanded in \(\Delta\) powers:

\[
N_i = \sum_{j=0}^{2} N_{ij}(\tilde{\gamma}_1, \tilde{\Gamma}, \varepsilon) \Delta^j,
\]

\[
D_k = \sum_{l=0}^{2} D_{kl}(\tilde{\gamma}_1, \tilde{\Gamma}, \varepsilon) \Delta^l.
\]

The coefficients \(N_{ij}\) and \(D_{kl}\) with \((i + j)\) and \((k + l)\) odd numbers are equal to zero due to symmetry reasons. For the sake of brevity, here we give explicit analytical expressions
for nonzero coefficients \( \mathcal{N}_{ij} \) and \( \mathcal{D}_{kl} \) in the limiting case, when the power broadening \( \gamma_{eg} \) dominates over the transit-time broadening \( \Gamma \), i.e. at \( \Gamma = 0 \):

\[
\mathcal{N}_{00} = (5 + 7 \tilde{\gamma}_1) \left[ 25 + 115 \tilde{\gamma}_1 + 172 \tilde{\gamma}_1^2 + 84 \tilde{\gamma}_1^3 - c^2 \left( 15 + 9 \tilde{\gamma}_1 - 76 \tilde{\gamma}_1^2 - 84 \tilde{\gamma}_1^3 \right) - 4 c^4 \tilde{\gamma}_1 \right]
\]

\[
\mathcal{N}_{02} = -4(1 + \tilde{\gamma}_1)^2 \left[ -25 \left( 5 + 16 \tilde{\gamma}_1 + 12 \tilde{\gamma}_1^2 \right) + c^2 \left( 175 + 320 \tilde{\gamma}_1 - 12 \tilde{\gamma}_1^2 \right) - 12 c^4 \left( 5 - 4 \tilde{\gamma}_1 - 24 \tilde{\gamma}_1^2 \right) \right]
\]

\[
\mathcal{N}_{11} = 160 s (1 + \tilde{\gamma}_1)^2 \left[ 15 + 48 \tilde{\gamma}_1 + 36 \tilde{\gamma}_1^2 - c^2 \left( 8 - 6 \tilde{\gamma}_1 - 36 \tilde{\gamma}_1^2 \right) \right]
\]

\[
\mathcal{N}_{20} = 48 (1 + \tilde{\gamma}_1)^2 \left[ 12 \left( 1 + 2 \tilde{\gamma}_1 \right) \left( 5 + 6 \tilde{\gamma}_1 \right) - 5 c^2 \left( 7 - 5 \tilde{\gamma}_1 - 30 \tilde{\gamma}_1^2 \right) \right],
\]

\[\text{(24)}\]

\[
\mathcal{D}_{00} = (5 + 7 \tilde{\gamma}_1) \left[ (5 + 7 \tilde{\gamma}_1) \left( 5 + 32 \tilde{\gamma}_1 + 36 \tilde{\gamma}_1^2 \right) - 4 c^2 \left( 2 + 5 \tilde{\gamma}_1 - 8 \tilde{\gamma}_1^2 - 14 \tilde{\gamma}_1^3 \right) \right]
\]

\[
\mathcal{D}_{02} = -4(1 + \tilde{\gamma}_1)^2 \left[ -25 \left( 5 + 32 \tilde{\gamma}_1 + 36 \tilde{\gamma}_1^2 \right) + 4 c^2 \left( 35 + 194 \tilde{\gamma}_1 + 166 \tilde{\gamma}_1^2 \right) - 32 c^4 \left( 1 + \tilde{\gamma}_1 - 6 \tilde{\gamma}_1^2 \right) \right]
\]

\[
\mathcal{D}_{11} = 160 s (1 + \tilde{\gamma}_1)^2 \left[ 3 \left( 5 + 32 \tilde{\gamma}_1 + 36 \tilde{\gamma}_1^2 \right) - c^2 \left( 4 + 2 \tilde{\gamma}_1 - 24 \tilde{\gamma}_1^2 \right) \right]
\]

\[
\mathcal{D}_{20} = 192 (1 + \tilde{\gamma}_1)^2 \left[ 3 \left( 5 + 32 \tilde{\gamma}_1 + 36 \tilde{\gamma}_1^2 \right) - c^2 \left( 4 - 25 \tilde{\gamma}_1^2 \right) \right],
\]

\[\text{(25)}\]

where \( c = \cos(2\varepsilon) \) and \( s = \sin(2\varepsilon) \).

It is convenient for analysis to present the lineshape (21) in the form of generalized Lorentzian \[55, 56\] :

\[
\pi_e = A \frac{w^2}{(\Omega_g - \Omega_0)^2 + w^2} + B \frac{w (\Omega_g - \Omega_0)}{(\Omega_g - \Omega_0)^2 + w^2} + C,
\]

\[\text{(26)}\]

where all the parameters are expressed through the coefficients \( \mathcal{N}_i \) and \( \mathcal{D}_k \) (see equations (22,25)) in the following way. The amplitude of the symmetric part

\[
\frac{A}{\pi_e^{(0)}} = \frac{2 (2N_0 D_2^2 + N_2 D_1^2 - N_1 D_1 D_2 - 2 N_2 D_0 D_2)}{D_2 (4 D_0 D_2 - D_1^2)},
\]

the amplitude of antisymmetric part

\[
\frac{B}{\pi_e^{(0)}} = \frac{2 (N_1 D_2 - N_2 D_1)}{D_2 \sqrt{4 D_0 D_2 - D_1^2}},
\]

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the background

\[ \frac{C}{\pi e^{(0)}} = \frac{\mathcal{N}_2}{\mathcal{D}_2}, \]

the resonance position

\[ \frac{\Omega_0}{\gamma_{eg} S} = -\frac{\mathcal{D}_1}{2\mathcal{D}_2}, \]

and the resonance width

\[ \frac{w}{\gamma_{eg} S} = \frac{\sqrt{4\mathcal{D}_0\mathcal{D}_2 - \mathcal{D}_1^2}}{2|\mathcal{D}_2|}. \]

It is important that \( B \) and \( \Omega_0 \) are proportional to the detuning \( \Delta \) and to the degree of circular polarization \( s = \sin(2\varepsilon) \). In other words, the spectroscopic signal is symmetrical with respect to zero of the magnetic field either at zero detuning or in the case of linear polarization. In these symmetric cases the sign of \( A \) depends on the depolarization rate \( \tilde{\gamma}_1 \). For instance, when \( \varepsilon = 0 \)

\[ \frac{A}{\pi e^{(0)}} = \frac{3(5 - 2\tilde{\gamma}_1)(1 + 2\tilde{\gamma}_1)}{4(187 + 565\tilde{\gamma}_1 + 418\tilde{\gamma}_1^3)} \]

and the sign of the resonance is changed, EIA is transformed into EIT, at \( \tilde{\gamma}_1 = 5/2 \) independently of the detuning. In the other case \( \Delta = 0 \) the sign-reversal point weakly depends on the ellipticity \( \varepsilon \) and it lies between \( \tilde{\gamma}_1 = 2 \) for \( \varepsilon \to \pm \pi/4 \) and \( \tilde{\gamma}_1 = 5/2 \) for linearly polarized light.

In the general case, when \( \varepsilon \neq 0 \) and \( \Delta \neq 0 \), the signal is asymmetric and its position is shifted with respect to the zero magnetic field point \((\Omega_g = 0)\). At \( \varepsilon = \pi/8 \) the dependence of the lineshape parameters \( A, B, w, \) and \( \Omega_0 \) on the detuning \( \Delta \) is shown in Fig. 2 and Fig. 3 in two opposite cases. Fig. 2 corresponds to the pure radiative relaxation \( \gamma_1 = 0 \), when the transfer of anisotropy is maximal, while at \( \gamma_1 = 10\gamma_r \) used in Fig. 3 the collisional depolarization of the excited state is almost complete. As is seen from these figures, the amplitudes of symmetric \( A \) and antisymmetric \( B \) parts are comparable. It should be also
noted that the amplitude $A$ changes sign when the detuning $\Delta$ increases. Such a behaviour in the EIT case was well-known in the simplest model of a three-level $\Lambda$ system \cite{57}. This effect is usually related to the Raman absorption peak in the probe field spectra \cite{57} - \cite{59}. In the EIA case at large detunings we see a negative structure in the absorption resonance (Fig. 4(b)), which can be also attributed to the Raman scattering. This structure is asymmetric and significantly narrower than the EIA resonance at $\Delta = 0$ (Fig. 4(a)). Such a Raman transparency resonance has not been discussed previously to the best of our knowledge.

It is instructive to consider the influence of the finite atom-light interaction time on the lineshape parameters. We find that this influence becomes substantial starting from $\Gamma \approx \gamma_{eg} S$ and it manifests mainly in the suppression of the resonance amplitudes $A$ and $B$ especially in the wings $|\Delta| > \gamma_{eg}$. At sufficiently large $\Gamma$ the amplitude $A$ becomes of fixed sign as shown in Fig. 5 for $\Gamma = 0.005\gamma_{eg}$ in the EIA case. In a pure gas cell typically $\Gamma \approx 0.001 - 0.01\gamma_{eg}$ and in order to observe, say, the Raman absorption peak at the EIT condition one needs a buffer gas cell, where the ratio of the ground-state relaxation rate $\Gamma$ to the optical linewidth $\gamma_{eg}$ is the more favorable $\Gamma \approx 10^{-5} - 10^{-6}\gamma_{eg}$. Note, the closely related results have been recently reported in \cite{54, 60}. Another strategy, more suitable for the observation of the Raman transparency dip in the EIA case, is the use of more intense light fields.
IV. RESONANCE LINESHAPE IN A GAS

In a gas with the Maxwell velocity distribution the absorption signal is proportional to the average excited-state population:

$$\langle \pi_e \rangle_v = (\sqrt{\pi} \bar{v})^{-1} \int_{-\infty}^{+\infty} \pi_e \exp\{-v/\bar{v}\} \, dv,$$  \hspace{1cm} (27)

where the parameter $\bar{v} = (2k_B T/M)^{1/2}$, with $k_B$ the Boltzmann constant, $T$ temperature, and $M$ mass of an atom. This signal is a superposition of contributions of atoms with different detunings $\delta_v = \omega - \omega_{eg} - kv$ (due to the Doppler shift $kv$). As is shown in the previous section, the resonance lineshape $\pi_e(\Omega_g)$ is significantly deformed and shifted at given nonzero detuning (see in Fig. 4b). As a result, the resonance lineshape in a gas can acquire non-trivial peculiarities. For example, in Fig. 6 we show the line shapes in the EIA case ($\hat{\gamma}_1 = 0$) for different values of the ellipticity parameter $\varepsilon$ and the average velocity $\bar{v}$. One can see that for elliptically polarized light the resonance width is significantly less than in the case of linear polarization. This effect is an analog of the Doppler narrowing of the two-photon resonance discussed in [61] - [63]. Note, in our problem statement the circular polarization components $e_{\pm 1}$ with different amplitudes at $\varepsilon \neq 0$ play the roles of “strong” control and “weak” probe fields, that is one of the conditions for the narrowing [61] - [63]. With the increase of $\varepsilon$ the resonance lineshape becomes complicated – the narrow EIA peak inside of the wider dip. This can be viewed as a consequence of the sign reversal of the resonance amplitude $A$ at nonzero detunings discussed above. The sharp structures in the line center should also be noted. In general, a detailed study of lineshape of the magneto-optical resonances in elliptically polarized fields in a gas is of great interest and it can be the subject of a separate publication. Our results in this direction will be presented elsewhere.
V. CONCLUSION

Using simple theoretical model of $1 \rightarrow 2$ closed atomic transition, the influence of the spontaneous transfer of anisotropy on the magneto-optical absorption resonances in an elliptically polarized field is studied. The analytical expression for the absorption for atoms with given velocity is obtained. In the low-saturation limit, when the resonance width is much less than the natural width, the lineshape is a generalized Lorentzian. It is shown that in the case of linearly polarized filed ($\varepsilon = 0$) or in the exact resonance ($\delta = 0$) the lineshape is symmetric, and the resonance sign is governed by the rate of the excited-state collisional depolarization $\gamma_1$. With an increase of $\gamma_1$ one can see a gradual transition from the EIA type to the EIT type. In the general case of elliptical polarization $\varepsilon \neq 0$ and $\delta \neq 0$ the absorption signal is asymmetric and shifted with respect to the zero magnetic field point. The sign-reversal of the symmetric part of the resonance is detected at large detuning. This effect related to the Raman scattering takes place in EIA as well as in EIT cases. Some peculiarities of the lineshape in a gas are investigated. In particular, the Doppler narrowing and the bimodal structure of the resonance are observed.

Acknowledgments

We thank A. Sidorov, A. Akulshin and V. Velichansky for helpful discussions. This work was partially supported by Russian Foundation for Basic Research (grants #04-02-16488,
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Fig. 1. (a) Mutual orientation of the polarization ellipse and the magnetic field, and (b) the scheme of light-induced transitions.

Fig. 2. The lineshape parameters versus $\Delta$. (a) $A$ and (b) $B$ in arbitrary units, and (c) $w$ and (d) $\Omega_0$ in $\gamma_{eg}$ units. The case of the pure radiative relaxation $\gamma_1 = 0$. Other parameters $\varepsilon = \pi/8$, $\Gamma = 0$, $\kappa = 0.1 \gamma_{eg}$.

Fig. 3. The lineshape parameters versus $\Delta$. (a) $A$ and (b) $B$ in arbitrary units, and (c) $w$ and (d) $\Omega_0$ in $\gamma_{eg}$ units. The case of the total excited-state depolarization $\gamma_1 = 10$. Other parameters $\varepsilon = \pi/8$, $\Gamma = 0$, $\kappa = 0.1 \gamma_{eg}$.

Fig. 4. The absorption resonance lineshapes in the EIA case $\gamma_1 = 0$. (a) The total excited-state population in arbitrary units versus the ground-state Zeeman shift in $\gamma_{eg}$ units at $\Delta = 0$; (b) the same but $\Delta = 5 \gamma_{eg}$. Other parameters $\varepsilon = \pi/8$, $\Gamma = 0$, $\kappa = 0.1 \gamma_{eg}$.

Fig. 5. The lineshape parameters versus $\Delta$. (a) $A$ (solid line) and $B$ (dashed line) in arbitrary units, and (b) $w$ (solid line) and $\Omega_0$ (dashed line) in $\gamma_{eg}$ units. The case of the pure radiative relaxation $\gamma_1 = 0$. Other parameters $\Gamma = 0.005 \gamma_{eg}$, $\kappa = 0.1 \gamma_{eg}$, and $\varepsilon = \pi/8$.

Fig. 6. The resonance lineshapes in a gas. The case of the pure radiative relaxation $\gamma_1 = 0$ and $\gamma_{eg} = \gamma_r/2 + \Gamma$. The total excited-state population averaged over Maxwell velocity distribution $\langle \pi_e \rangle_v$ versus the ground-state Zeeman shift $\Omega_g$ in $\gamma_r$ units at different ellipticity parameters (a) $\varepsilon = 0$, (b) $\varepsilon = \pi/10$, and (c) $\varepsilon = \pi/5$. The Doppler width varies in each panel $k\bar{v} = 0$ (dotted line), $k\bar{v} = \gamma_r$ (dashed line), and $k\bar{v} = 20 \gamma_r$ (solid line). Other parameters $\delta = 0$, $\Gamma = 0.001 \gamma_r$, and $\kappa = 0.2 \gamma_r$. For all curves the background is subtracted and all curves are normalized to the absorption in the center $\Omega_g = 0$. 

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