Behavior of Immiscible Two Liquid Layers Contained in Cylindrical Vessel Suddenly Set in Rotation

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Immiscible silicone oil and water were contained in a cylindrical vessel. The ratio of their volumes, referred to as the volume ratio, was varied over a wide range. The vessel was suddenly set in rotation, and the flow velocities of the two liquids were measured with particle image velocimetry (PIV) and laser Doppler velocimetry (LDV). The flow establishment time was defined as the period from the start of rotation to the moment at which a steady state is established in the vessel. An empirical equation for the flow establishment time was proposed as a function of the volume ratio, the angular frequency of rotation, and the physical properties of the liquids. The deformation of the silicone oil-water interface was also observed to confirm the findings obtained from the velocity measurements.

KEY WORDS: flow establishment time; rotating vessel; stratified liquid layers; particle image velocimetry; laser Doppler velocimetry.

1. Introduction

In previous studies, measurements were carried out on the flow establishment time of a transient flow of a single liquid contained in a cylindrical vessel. Empirical equations for the flow establishment time in the laminar and transient flow regimes were derived as functions of the aspect ratio and the rotation Reynolds number. An equivalent diameter originally introduced by Kawashima et al. was chosen as a representative length. The measured values of the flow establishment time were satisfactorily approximated by those equations.

This paper describes the flow establishment time of immiscible two stratified liquid layers contained in a vertically placed cylindrical vessel suddenly set in rotation. Silicone oil was placed on water at different volume ratios. Laminar flows of silicone oil and water were treated. Such a flow field is a primitive model for the interaction between slag and molten steel contained in a cylindrical vessel suddenly set in rotation. As the next step, the authors are planning to carry out model experiments on the flow behavior of slag and molten metal contained in a cylindrical vessel suddenly set in eccentric rotation. Although this type of refining process does not exist at present, it may be useful for significantly increasing the slag-molten steel interfacial area provided that the kinematic viscosity of the slag is much higher than that of the molten steel.

2. Experimental Apparatus and Procedure

Figure 1 shows a schematic of the experimental apparatus. Water and one of silicone oils listed in Table 1 were contained in a cylindrical vessel made of transparent acrylic resin. Although the top of the slag layer is free surface in the real refining processes, the top is covered with a plate in this study in order to simplify the flow field. The case that the top of the slag layer is free surface will be investigated in a future study. The density of the water was adjusted by adding salt to become the same as the density of tracer particles used for PIV measurements. The interfacial tension...
between the water and any kind of silicone oil is 53 mN/m. The diameter and the height of the vessel are 46 mm and 120 mm, respectively. The origin of the cylindrical coordinate system, \((r, \theta, z)\), was placed on the center of the bottom plate. The vessel was connected to a stepping motor, and the vessel was enclosed with another transparent vessel of a square cross-section. Water was filled between the two vessels in order to decrease the parallax effect as much as possible. The cylindrical vessel was rotated suddenly at a predetermined constant rotation velocity, while the square vessel was fixed. The angular frequency of rotation, \(\omega\), was varied from 2.36 rad/s to 11.00 rad/s. The kinematic viscosity of the silicone oil, \(\nu_{so}\), was changed from 1.0 mm\(^2\)/s to 100 mm\(^2\)/s.

In general, the rotation Reynolds number is defined by:

\[
\text{Re} = R(\omega/v)^{1/2}
\]

where \(R\) is the radius of the vessel and \(v\) is the kinematic viscosity of liquid. The following two types of Reynolds numbers are introduced.

\[
\text{Re}_w = R(\omega/v_w)^{1/2}
\]
\[
\text{Re}_{so} = R(\omega/v_{so})^{1/2}
\]



where \(v_w\) is the kinematic viscosity of water. The former Reynolds number plays an essential role in this study. The measurements were carried out in the laminar flow regime.

The volume ratio of water and silicone oil is defined by:

\[
R_V = V_w/V_{so}
\]

where \(V_w\) and \(V_{so}\) are the volumes of water and silicone oil, respectively.

The tangential velocity component of water flow, \(v_{w\theta}\), was measured with particle image velocimetry (PIV)\(^1\) and the axial velocity component, \(v_{zw}\), was measured with laser Doppler velocimetry (LDV).\(^6\) This is because the axial velocity component is much smaller than the tangential velocity component, and the measurement accuracy of the LDV is higher than that of the PIV. The motions of tracer particles dispersed in the water were recorded with a CCD camera at 30 frames per second for the PIV measurement, as shown in Fig. 2. The diameter of the particles ranged from 75 to 100 \(\mu\)m. The images thus recorded with the camera were processed on a personal computer to obtain the tangential velocity component. A schematic of the experimental apparatus for the LDV measurements is shown in Fig. 3. The deformation of the interface between water and silicone oil was also observed with the camera to confirm the flow establishment time obtained from velocity measurements. Further details of the PIV and LDV measurements can be found in the previous papers.\(^3,6\)

### Table 1. Physical properties.

| Liquid   | Kinematic viscosity (mm\(^2\)/s) | Density (kg/m\(^3\)) |
|----------|----------------------------------|----------------------|
| Water    | 1.0                              | 1030                 |
| Silicone oil1 | 1.0                              | 818                  |
| Silicone oil5 | 5.0                              | 915                  |
| Silicone oil10 | 10                               | 935                  |
| Silicone oil100 | 100                              | 965                  |

3. Experimental Results and Discussion

#### 3.1. Definition of Flow Establishment Time

The histories of the tangential velocity component of water flow for three different volume ratios are shown in Figs. 4 through 6, where \(\phi\) is the dimensionless tangential velocity, \(\tau\) is the dimensionless time, and \(\xi\) is the dimensionless radial distance expressed by:

\[
\text{Fig. 2. Schematic of experimental apparatus for PIV measurement.}
\]

\[
\text{Fig. 3. Schematic of experimental apparatus for LDV measurement.}
\]

\[
\text{Fig. 4. Tangential velocity profiles.} (v_{w\theta}=10 \text{ mm}^2/\text{s}, \text{Re}_w=35.6, \frac{V_w}{V_{so}}=0.5, 1.0, 2.0, \xi=0.62).
\]
where \( t \) is time and \( r \) is the radial distance.

The three curves were drawn so as to pass through the mean of the measured values at \( z = H/2 \) under three experimental conditions. This is intended to avoid crowding in the figure. In every case the measured values were distributed around the curve within a scatter of approximately \( \pm 7\% \). It is clear that the development of the velocity distribution becomes fast as the volume ratio, \( R_v \), decreases. The reason lies in the fact that the apparent kinematic viscosity defined in a later section is higher for a mixture of water and silicone oil of lower volume ratio. The flow establishment time was determined based on these distributions, as the water flow reached a steady state after the silicone oil flow did.

The flow establishment time, \( T_s \), is defined as schematically shown in Fig. 7. In Fig. 7(a) \( \tau_\xi \) denotes the dimensionless time at which the dimensionless tangential velocity reaches 0.95 times as large as its steady value. The dimensionless flow establishment time \( \tau_s = \frac{V_w T_s}{R^2} \), is evaluated on the vessel axis, as shown in Fig. 7(b). Further details of the definition of \( T_s \) should be referred to the previous papers.\(^{2,3}\)

Figure 8 demonstrates that the experimentally measured dimensionless flow establishment time for a volume ratio of unity increases with a decrease in the kinematic viscosity of silicone oil. Figure 9 shows that the dimensionless flow es-
establishment time for \( v_{so} = 10 \text{ mm}^2/\text{s} \) increases as the volume ratio increases.

### 3.2. Flow Establishment Time for Volume Ratio of Unity

Measurements of the flow establishment time were carried out for a volume ratio of unity. The flow establishment time increased at every rotation Reynolds number with a decrease in the kinematic viscosity of silicone oil, as shown in Fig. 10. The measured values of \( T_s \) for silicone oil of \( n_{so} = 10 \text{ mm}^2/\text{s} \) are collectively shown in Fig. 11. It is evident that the flow establishment time increases with an increase in the volume ratio regardless of the rotation Reynolds number, \( Re_w \). The measured values of \( T_s (= \tau R^2/v_w) \) are compared with the following empirical equation in Fig. 12.

\[
\Pi_2 = \Pi_1^{max} \times 10^{0.749} \quad \text{(8)}
\]

\[
\Pi_2 = T_w \quad \text{(9)}
\]

\[
\Pi_1^{max} = v_{max} / (\alpha L^2) \quad \text{(10)}
\]

\[
1/v_{max} = (1/v_{so} + 1/v_w) / 2 \quad \text{(11)}
\]

\[
L = V/S \quad \text{(12)}
\]

\[
V = \pi D^2 H / 4 \quad \text{(13)}
\]

\[
V = V_w + V_{so} \quad \text{(14)}
\]

\[
S = \pi D^2 / 2 + \pi DH \quad \text{(15)}
\]

where \( V \) is the total volume of water and silicone oil and \( S \) is the wetted area.

Equation (8) was proposed here by referring to an empirical equation derived by Kawashima et al. for a single-liquid contained in an annular passage suddenly set in rotation. The functional relationship of Eq. (8) is the same as that proposed by Kawashima et al. Originality of the present authors for Eq. (8) lies in the introduction of an apparent kinematic viscosity represented by \( v_{max} \).

Equation (8) somewhat underestimates the flow establishment time measured in this study. The following empirical equation therefore was newly proposed by changing the index of \( \Pi_1^{max} \) in Eq. (8).

\[
\Pi_2 = \Pi_1^{0.670} \times 10^{0.749} \quad \text{(16)}
\]

This equation can approximate the measured values for \( R_v = 1.0 \) within a scatter of \( \pm 20\% \).

### 3.3. Flow Establishment Time for Different Volume Ratios

Equation (11) was modified to consider the effect of the volume ratio as follows:

\[
1/v_{max} = (V_w/V) / v_w + (V_{so}/V) / v_{so} \quad \text{(17)}
\]

The kinematic viscosity is a measure for the rate of momentum transfer. That is, the momentum transfer is enhanced as the kinematic viscosity is increased. In this sense, the inverse of the kinematic viscosity may be regarded as a kind of the resistance in electrical circuits. The momentum however is transferred in the three directions, and, hence, \( v_{max} \) was expressed using the volumes of the water and silicone oil, as shown in Eq. (17). Equation (17) reduces to Eq. (11) for \( R_v = 1.0 \).

Equation (16) was compared with all the measured values in Fig. 13. The measured values of the flow establishment time can be predicted by Eq. (16) within a scatter of \( \pm 20\% \).

### 3.4. Deformation of Interface between Water and Silicone Oil Layers

The deformation of the interface between the water and the silicone oil layers was observed. The interface returned near its original horizontal plane after a steady state was es-
established. Figure 14 shows an example of the deformation of the interface. As the kinematic viscosity of the silicone oil is higher than that of the water, momentum is transferred from the vessel wall to the silicone oil faster than to the water. As a result, the silicone oil becomes to move earlier than the water. Since the silicone oil is subjected to larger centrifugal force, it moves outward and then descends along the side wall of the vessel, as shown in the photograph at $t=5$ s. The interfacial area between silicone oil and water at $t=5$ s is approximately two times as large as its initial value ($\pi D^2/4$). As time elapses, the water also becomes to rotate, and finally, the interface returns near its original horizontal plane (see photograph at $t=90$ s).

Figure 15 shows the vertical displacement, $h$, of the interface on the centerline of the vessel. The displacement, $h$, was measured from the initial interfacial plane. For a rotation Reynolds number, $Re_e$, of 35.6, the displacement, $h$, is very small for the three different kinematic viscosities. The displacement, $h$, becomes large with an increase in the rotation Reynolds number, as shown in Fig. 15. The measured value of the flow establishment time for $Re_e=74.9$ and $v_{so}=100$ mm$^2$/s was approximately 32 s. It is evident that $h$ approaches its final value at around $t=35$ s. The final value of $h$ depends on the centrifugal force and the interfacial force. Further discussion on the final value also must be left for a future study.

3.5. Axial Velocity Component of Water Flow on the Centerline of Vessel

The axial velocity component of water flow near the initial silicone oil–water interface was measured on the centerline of the vessel with the LDV. A representative result is shown in Fig. 17. Each symbol represents the mean value averaged over 1 s. The large scatter of the measured values for $t>100$ s is attributable to the fact that the number of velocity signals decreases as the flow approaches the steady state. It should be noted that the axial velocity component, $v_{zw}$, is very small. The axial velocity component, $v_{zw}$, also departs from zero and then returns to zero again just like the displacement of the interface, $h$, shown in Fig. 16. It is evident that $v_{zw}$ for $v_{so}=100$ mm$^2$/s returns to zero at around $t=30$ s, which is approximately equal to the flow establishment time indicated by an arrow in Fig. 16.
4. Conclusions

Measurements were carried out to derive an empirical equation for the flow establishment time of immiscible two liquid layers contained in a vertically placed cylindrical vessel suddenly set in rotation. This system is a primitive model for a promising refining system using a rotating vessel. The measured values of the flow establishment time in the laminar flow regime were approximated by Eq. (16) within a scatter of ±20%.

Nomenclature

- $D$: Vessel diameter (mm)
- $H$: Vessel height (mm)
- $H_w$: Height of water layer (mm)
- $H_{so}$: Height of silicone oil layer (mm)
- $h$: Vertical displacement of interface (mm)
- $R$: Radius of vessel (mm)
- $Re_w$: Reynolds number (–)
- $R_v$: Volume ratio (–) $= V_w / V_{so}$
- $(r, \theta, z)$: Cylindrical coordinate system
- $T_s$: Flow establishment time (s)
- $t$: Time (s)
- $V$: Volume of vessel (mm$^3$) $= V_w + V_{so}$
- $V_w$: Volume of water (mm$^3$)
- $V_{so}$: Volume of silicone oil (mm$^3$)
- $v_{zw}$: Vertical velocity component of water flow (mm/s)
- $v_{qw}$: Tangential velocity component of water flow (mm/s)
- $n_{so}$: Kinematic viscosity of silicone oil (mm$^2$/s)
- $n_w$: Kinematic viscosity of water (mm$^2$/s)
- $\omega$: Angular frequency of rotation (rad/s)

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