Solutions in IR modified Hořava-Lifshitz Gravity

Taekyung Kim  
Department of Physics, BK21 Physics Research Division, and Institute of Basic Science,  
Sungkyunkwan University, Suwon 440-746, Korea  
pojawd@skku.edu

Chong Oh Lee  
Department of Physics and Astronomy,  
University of Waterloo, Waterloo, Ontario, N2L 3G1, Canada  
cohlee@sciborg.uwaterloo.ca

Abstract

In order to allow the asymptotically flat, we consider Hořava-Lifshitz gravity theory with a soft violation of the detailed balance condition and obtain various solutions. In particular, we find that such theory coupled to a global monopole leads to a solution representing a space with deficit solid angle, which is well matched with genuine feature of GR.
1 Introduction

The construction of the ultra-violet(UV) complete theory of gravity has been an intriguing subject of discussions for theoretical physics of the past fifty years. The discussion has been recently concentrated on the UV complete theory in space and time with an an isotropic scaling in a Lifshitz fixed point [1, 2, 3, 4, 5]. In particular, this theory is very attractive since pertubative renormalizability is realized as well as Lorentz symmetry is recovered in low energy regime in spite of being broken the Lorentz symmetry in high energy.

Horava-Lifshitz gravity (HL) has been studied in various directions, which are categorized into two. One is investigating and developing the properties of the HL theory itself [6]–[36]. The other is applying this theory to cosmological framework including the black hole solutions [38]–[49] and their thermodynamic prosperities [50]–[59].

The metric in the (3+1)-dimensional ADM decomposition can be written as

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N_i dt)(dx^j + N_j dt),$$

where $N(t, x^i)$ denotes the lapse function, $g_{ij}(t, x^i)$ is the spatial metric, and $N_i(t, x^i)$ is the shift function. Then, the Einstein-Hilbert action can be expressed as

$$S_{EH} = \frac{1}{16\pi G} \int d^4x \sqrt{g}N(K_{ij}K^{ij} - K^2 + R - 2\Lambda),$$

where $G$ is Newton’s constant and the extrinsic curvature for a spacelike hypersurface with a fixed time is

$$K_{ij} = \frac{1}{2N}(g_{ij} - \nabla_i N_j - \nabla_j N_i).$$

Here, a dot denotes a derivative with respect to $t$ and covariant derivatives defined with respect to the spatial metric $g_{ij}$.

The IR-modified HL action with asymptotically flat limit is given by [1, 39, 42]

$$S_{HL} = \int dt d^3x \sqrt{g}N(L_{IR} + L_{UV}),$$

$$L_{IR} = \frac{2}{\kappa^2} (K_{ij}K^{ij} - \lambda K^2) + \kappa^2 \mu^2 \left[ (\Lambda - \omega) R - 3\Lambda^2 \right],$$

$$L_{UV} = -\frac{\kappa^2}{2\nu^4} \left( C_{ij} - \frac{\mu\nu^2}{2} R_{ij} \right) \left( C^{ij} - \frac{\mu\nu^2}{2} R^{ij} \right) + \kappa^2 \mu^2 \frac{(1 - 4\lambda)}{32(1 - 3\lambda)} R^2,$$

where $R$ and $R_{ij}$ are three-dimensional scalar curvature and Ricci tensor, and the Cotton tensor is given by

$$C^{ij} = \frac{\epsilon^{ikl}}{\sqrt{g}} \nabla_k \left( R^l_{ij} - \frac{1}{4} R \delta^l_{ij} \right).$$
The action has parameters, $\kappa, \lambda, \nu, \mu, \Lambda$, and $\omega$. In the limit of vanishing cosmological constant $\Lambda \to 0$, one compares the IR-modified action (1.4) with the $(3+1)$-dimensional Einstein-Hilbert action (1.2) and reads the parameter $\lambda$, the speed of light $c$, Newton’s constant $G$ as

$$\lambda = 1, \quad c^2 = \frac{\kappa^4 \mu^2 \omega}{32}, \quad G = \frac{\kappa^2}{32\pi c}.$$ (1.8)

Recently, HL gravity coupled to electrostatic field of a point charge is considered and an exact solution is found, describing a space with either a surplus or deficit solid angle is found [60]. The surplus angle due to an ordinary matter with positive energy density in [60] is not well matched with known result of GR in which it can usually be materialized by the source of negative mass or energy. However, from cosmological point of view, one finds the detailed balance condition leads to obstacles [39, 61]. Furthermore by introducing a soft violation of the detailed balance condition, they show that their results are consistent with them of GR [42]. Thus one intriguing question is whether IR-modified HL theory coupled to matter field reproduces them of GR.

In this paper, we address this question. We consider IR-modified HL in presence of the global monopole, and find a spherically symmetric solution describing a space with deficit solid angle.

The paper is organized as follows. In section 2, vacuum solutions are discussed under spherical symmetry. In section 3, we obtain the deficit solid angle due to the solution of IR modified HL gravity with the global monopole. Finally, we give a conclusion.

## 2 Vacuum Solutions under Spherical Symmetry

Let us investigate a spherically symmetric solution with the static metric ansatz

$$ds^2 = -\mathcal{F}(r)e^{2\rho(r)}dt^2 + \frac{dr^2}{\mathcal{F}(r)} + r^2(d\theta^2 + \sin^2 \theta d\varphi^2).$$ (2.1)

Since all the components of Cotton tensor vanish under this metric, the action (1.4) reduces to

$$S_{HL} = 4\pi \int_{-\infty}^{\infty} dt \int_0^\infty dr r^2 e^\rho \left\{ -\frac{\kappa^2 \mu^2}{8} \left[ \left( \mathcal{F}' \right)^2 + \frac{2}{r^4 \left( 1 - F - r F' \right)^2} \right] \right. \right.$$

$$+ \left. \frac{\kappa^2 \mu^2}{8(1 - 3\lambda)} \left[ \frac{1 - 4\lambda}{r^4} (1 - F - r F')^2 + \frac{2(\Lambda - \omega)}{r^2} (1 - F - r F') - 3\Lambda^2 \right] \right\}$$

$$= \frac{\pi \kappa^2 \mu^2}{2(3\lambda - 1)} \int dt \int dr e^\rho \times$$

$$\left\{ (1 - 3\lambda) \left[ \mathcal{F}^2 + 2\left( \frac{\tilde{F}}{r} + \frac{\tilde{F}'}{2} \right)^2 \right] - (1 - 4\lambda) \left( \frac{\tilde{F}}{r} + \frac{\tilde{F}'}{2} \right)^2 + 2(\Lambda - \omega) r \left( \frac{\tilde{F}}{r} + \mathcal{F}' \right) + 3\Lambda^2 r^2 \right\}.$$ (2.2)
where \( \tilde{F} = \mathcal{F} - 1 \). Then, the equations of motion are obtained as

\[
\left[ (\lambda - 1)\tilde{F}' - \frac{2\lambda}{r}\tilde{F} - 2(\Lambda - \omega)r \right] \rho' + (\lambda - 1)\tilde{F}'' - \frac{2(\lambda - 1)}{r^2}\tilde{F} = 0, \tag{2.3}
\]

\[
(1 - 3\lambda) \left[ \tilde{F}'^2 + 2 \left( \frac{\tilde{F}}{r} + \frac{\tilde{F}'}{2} \right)^2 \right] - (1 - 4\lambda) \left( \frac{\tilde{F}}{r} + \tilde{F}' \right)^2 + 2(\Lambda - \omega)r \left( \frac{\tilde{F}}{r} + \tilde{F}' \right) + 3\Lambda^2 r^2 = 0. \tag{2.4}
\]

We start by giving a brief discussion of the asymptotic behaviors of the solutions to Eqs. (2.3) and (2.4). In the low energy regime, taking the \( \lambda = 1 \) and neglecting the quadratic terms in the metric functions, the equations (2.3)–(2.4) reduce to the Einstein equations, which reproduce Schwarzschild solution in the limit \( \Lambda \to 0 \) as we expect

\[
\begin{align*}
\frac{dr}{d\rho} &= 0, \quad \rightarrow \quad \rho(r) = \rho_0 = 0, \quad (2.5) \\
\frac{d}{dr} (r\mathcal{F}) &= 1, \quad \rightarrow \quad \mathcal{F}(r) = 1 - \frac{M}{r}, \quad (2.6)
\end{align*}
\]

where \( M \) is an integration constant.

For sufficiently large \( r \) at asymptotic region, it is assumed that the divergence of \( \mathcal{F}(r) \) arises as a power behavior. A straightforward calculation with Eq. (2.4) leads to

\[
\mathcal{F}(r) \approx \begin{cases} 
(I) & (\omega - \Lambda) r^2 - \sqrt{\omega(\omega - 2\Lambda)} r^2 & \text{for arbitrary } \lambda \\
(II) & \mathcal{F}_{\text{IR}} r^p & \text{for } \lambda > 1
\end{cases}, \tag{2.7}
\]

where the coefficient \( \mathcal{F}_{\text{IR}} \) is an undetermined constant and

\[
p = \frac{2\lambda + \sqrt{2(3\lambda - 1)}}{\lambda - 1}. \tag{2.8}
\]

It is shown that the behavior of the long distance in (I) without a cosmological constant agrees with that of the leading IR behavior in (2.4). The long distance behavior in (II) seems to imply a new possible solution which comes from higher derivative terms.

For sufficiently small \( r \) at the UV regime, assuming the divergence of \( B(r) \) follows as power behavior

\[
\mathcal{F}(r) \sim \frac{\beta}{r^l}, \quad (\beta = \text{constant}, \ l > 0), \tag{2.9}
\]

the leading term in Eq. (2.4) is proportional to \( 1/r^{2l+2} \). The contribution to the correction term due to the soft violation of the detailed balance condition in Eq. (2.4) can be neglected since such contribution is proportional to \( 1/r^l \). Thus, the leading UV behavior in IR modified HL theory is exactly the same as that in HL theory. The allowed powers for various \( \lambda \) are given as
\[ F(r) \approx \begin{cases} 
(A) & 1 \text{ for arbitrary } \lambda \\
(B) & b \text{ for } \lambda = \frac{1}{2} \\
(C) & F_{\text{UV}+r^p} \text{ or } F_{\text{UV}-r^q} \text{ for } \frac{1}{3} \leq \lambda < \frac{1}{2} \\
(D) & F_{\text{UV}+r^p} \text{ for } \frac{1}{2} < \lambda < 1 
\end{cases}, \quad (2.10) \]

where \( b \) denotes an integration constant, \( B_{\text{UV}±} \) are undetermined constants, \( p \) is given in (2.7), and \( q \) is

\[ q = \frac{2\lambda - \sqrt{2(3\lambda - 1)}}{\lambda - 1}. \quad (2.11) \]

We show that we find new exact vacuum solutions and discuss how they connect to asymptotes with various values of \( \lambda \). For arbitrary \( \lambda \), a solution to the equations (2.3)–(2.4) obtained as

\[ F = 1 + (\omega - \Lambda)r^2 - \sqrt{\omega(\omega - 2\Lambda)}r^2, \quad \rho = \rho_0 = 0, \quad (2.12) \]

which connects (I) and (A). For \( \lambda = 1/3 \), another static exact solution is

\[ F = 1 + (\omega - \Lambda)r^2 - \sqrt{\omega(\omega - 2\Lambda)}r^2 - \frac{M}{r}, \quad \rho = \rho_0 = 0, \quad (2.13) \]

which reproduces AdS Schwarzschild black hole solution with twice cosmological constant for \( \omega = 0 \). This result in IR modified HL theory agrees with that in HL \[38, 60\]. For \( \lambda = 1 \), the known exact solution is obtained by \[18\]

\[ F = 1 + (\omega - \Lambda)r^2 - \sqrt{\omega(\omega - 2\Lambda)}r^4 + cr, \quad \rho = \rho_0 = 0, \quad (2.14) \]

where \( c \) is an integration constant. This solution also connects (I) and (A).

In contrast to the exact vacuum solutions in HL theory \[38, 60\], it is not clear how they have connection between (2.7) and (2.10) since it seems that there are not other exact solutions in IR modified HL theory except previous exact solutions (2.12)–(2.14), i.e., there do not exist exact solutions with covering all range of \( \lambda \) for \( \lambda \geq 1/3 \). It presumably implies that all the vacuum solutions in IR modified HL theory do not always follows as power behavior.

Horizons and singularities in HL gravity have been discussed in the previous work \[60\]. However, we do not deal with them since HL theory does not have full diffeomorphism invariance and both of the previous concepts are not easy to discern \[47\].
3 Global Monopole Solution

In the presence of matter field, it is described by action

\[ S_m = \int dt d^3x \sqrt{g} N \mathcal{L}_m(N, N_i, g_{ij}) \]

\[ = 4\pi \int_{-\infty}^{\infty} dt \int_0^\infty dr r^2 e^\rho \mathcal{L}_m(\mathcal{F}, \rho). \] (3.15)

Then, the equations of motion are given by

\[
\begin{align*}
&\left[(\lambda - 1) \tilde{F} - \frac{2\lambda}{r} \tilde{F} - 2(\Lambda - \omega)r\right] \rho' + (\lambda - 1) \tilde{F}'' - \frac{2(\lambda - 1)}{r^2} \tilde{F} = \frac{8(1 - 3\lambda)}{\kappa^2 \mu^2} \frac{\partial \mathcal{L}_m}{\partial \mathcal{F}},

&\left(1 - 3\lambda\right) \left[\tilde{F}'^2 + 2\left(\frac{\tilde{F}'}{r} + \frac{\tilde{F}}{2}\right)^2\right] - (1 - 4\lambda) \left(\frac{\tilde{F}'}{r} + \tilde{F}'\right)^2 + 2(\Lambda - \omega)r \left(\frac{\tilde{F}}{r} + \tilde{F}'\right) + 3\Lambda^2 r^2

&= \frac{8(1 - 3\lambda)}{\kappa^2 \mu^2} \left(\mathcal{L}_m + \frac{\partial \mathcal{L}_m}{\partial \rho}\right).
\end{align*}
\] (3.17)

(3.18)

When we consider a global monopole of O(3) linear sigma model and magnetic monopole of U(1) gauge theory in the HL type field theory, the long distance behavior of the Lagrangian density in IR regime must be proportional to $1/r^n$ irrespective of the value of $z$ (see Ref. [60] for more details)

\[
\frac{\partial \mathcal{L}_m}{\partial \mathcal{F}} \approx 0, \quad \mathcal{L}_m + \frac{\partial \mathcal{L}_m}{\partial \rho} \approx -\frac{\gamma}{r^n}, \quad (n = 0, 1, 2, ...),
\] (3.19)

where a constant $\gamma$ is determined by the explicit Lagrangian form and the monopole configurations of interest. Positive $\gamma$ can be read off from the energy momentum tensor of matter fields and $n$ must be a positive integer in order to get a finite energy. A straightforward calculation with Eqs. (3.17) and (3.18) leads to

\[
\mathcal{F} = 1 + \left[(\omega - \Lambda) \pm \sqrt{\omega(\omega - 2\Lambda)}\right] r^2 + \frac{8(n - 3)\gamma}{n^2 \kappa^2 \mu^2 \sqrt{\omega(\omega - 2\Lambda)}} r^{2-n},
\] (3.20)

\[
\rho = (2n - 3) \ln(r/r_0) + \left(\frac{3}{n} - 2\right) \ln \left[\frac{8\gamma(n - 3)^2}{\kappa^2 \mu^2} - \omega(\omega - 2\Lambda)n^3 r^n\right],
\] (3.21)

for $n \neq 3$ and $\lambda = (n^2 - 4n + 6)/n^2$. In particular, in the case of $n = 3$, there exists solution only by taking $\lambda = 1/3$. Then, matter contributions vanish in (3.17) and (3.18) when $\lambda = 1/3$. Therefore, such solution exactly goes back to the vacuum solution (2.13). One also finds special
solution for $\lambda = 1$,

$$
\mathcal{F} = 1 + (\omega - \Lambda)r^2 \pm \sqrt{\omega(\omega - 2\Lambda)r^4 + f r + \frac{16\gamma}{(3-n)\kappa^2\mu^2}r^{4-n}}, \quad \rho = \rho_0 = 0, \quad (n \neq 3)
$$

$$
\mathcal{F} = 1 + (\omega - \Lambda)r^2 \pm \sqrt{\omega(\omega - 2\Lambda)r^4 + f r + \frac{16\gamma}{\kappa^2\mu^2}r\ln r}, \quad \rho = \rho_0 = 0, \quad (n = 3)
$$

with an integration constant $f$.

Let us study the details of the global monopole solution. The O(3) sigma model action is presumably taken as

$$
S_{O(3)} = \int d^4x \sqrt{-g} \left( -\frac{g^{00}}{2} \partial_0 \psi^a \partial_0 \psi^a - V \right),
$$

where, $\psi^a (a = 1, 2, 3)$ denote a scalar fields and $g^{00} = 1/N^2$. For simplicity, we assume an ordinary quadratic spatial derivatives and of a quartic order self-interactions,

$$
V(\psi^a, \partial^i \psi^a, \ldots) = -\frac{g^{ij}}{2} \partial_i \psi^a \partial_j \psi^a - \frac{\lambda_m}{4} (\psi^2 - v^2)^2, \quad \psi^2 \equiv \psi^a \psi^a,
$$

(3.25)

For anisotropic scaling $z = 1$ ($n = 2$), the IR action (3.24) is

$$
S_{O(3)} = 4\pi \int_{-\infty}^{\infty} dt \int_0^{\infty} dr r^2 e^\rho \left[ -\frac{\mathcal{F}}{2} \psi'^2 - \frac{\psi^2}{r^2} - \frac{\lambda_m}{4} (\psi^2 - v^2)^2 \right],
$$

(3.26)

and, under a hedgehog ansatz

$$
\psi^a = \hat{r}^a \psi(r) = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta) \psi(r),
$$

(3.27)

it leads to

$$
\frac{\partial \mathcal{L}_m}{\partial \mathcal{F}} = -\frac{1}{2} \psi'^2,
$$

(3.28)

$$
\mathcal{L}_m + \frac{\partial \mathcal{L}_m}{\partial \rho} = -\frac{\mathcal{F}}{2} \psi'^2 - \frac{\psi^2}{r^2} - \frac{\lambda_m}{4} (\psi^2 - v^2)^2.
$$

(3.29)

Two boundary conditions of the above equations are imposed by requiring single-valuedness of the field at the monopole position and finite energy at spatial infinity

$$
\psi(0) = 0, \quad \psi(\infty) = v.
$$

(3.30)

From the boundary conditions, one can take the following configuration

$$
\psi(r) = \begin{cases} 
0, & \text{for } r \leq \frac{1}{v\sqrt{\lambda_m}}, \\
\frac{1}{v\sqrt{\lambda_m}}, & \text{for } r > \frac{1}{v\sqrt{\lambda_m}},
\end{cases}
$$

(3.31)
which means the scalar field $\psi(r)$ has vacuum expectation value zero in the region inside the monopole core and $v$ outside, respectively. Therefore, the field equations (3.28)–(3.29) near the vacuum reduce to

$$\frac{\partial L_m}{\partial F} \approx 0, \quad L_m + \frac{\partial L_m}{\partial \rho} \approx -\frac{v^2}{r^2}. \quad (3.32)$$

In particular, $\gamma$ in (3.19) is given as $v^2$ for $n = 2$. Then, the metric function $F(r)$ is obtained by

$$F = 1 + \left[ (\omega - \Lambda) \pm \sqrt{\omega(\omega - 2\Lambda)} \right] r^2 - \frac{2v^2}{\kappa^2 \mu^2 \sqrt{\omega(\omega - 2\Lambda)}}, \quad (3.33)$$

$$\rho = \ln(r/r_0) - \frac{1}{2} \ln \left[ \frac{v^2}{\kappa^2 \mu^2} - \omega(\omega - 2\Lambda) \right], \quad (3.34)$$

which leads to

$$ds^2 = -\frac{1}{\kappa^2 \mu^2} \left[ r^2 \right] + \frac{dr^2}{1 + \left[ (\omega - \Lambda) \pm \sqrt{\omega(\omega - 2\Lambda)} \right] r^2} + r^2 \left( 1 - \frac{2v^2}{\kappa^2 \mu^2 \sqrt{\omega(\omega - 2\Lambda)}} \right) (d\theta^2 + \sin^2 \theta d\varphi^2), \quad (3.35)$$

after rescaling the coordinates,

$$dt \to \left( 1 - \frac{2v^2}{\kappa^2 \mu^2 \sqrt{\omega(\omega - 2\Lambda)}} \right)^{-1} r_0 dt, \quad dr \to \sqrt{1 - \frac{2v^2}{\kappa^2 \mu^2 \sqrt{\omega(\omega - 2\Lambda)}}} dr. \quad (3.36)$$

The metric (3.35) describes a space with a deficit solid angle $4\pi \Delta = \frac{8\pi v^2}{\kappa^2 \mu^2 \sqrt{\omega(\omega - 2\Lambda)}}$, for $0 < \frac{2v^2}{\kappa^2 \mu^2 \sqrt{\omega(\omega - 2\Lambda)}} < 1. \quad (3.37)$

In (3.35) a black hole horizon is formed at

$$r_H = \frac{2v^2}{\kappa^2 \mu^2 \sqrt{\omega(\omega - 2\Lambda)}} - 1 \quad \text{for} \quad \frac{2v^2}{\kappa^2 \mu^2 \sqrt{\omega(\omega - 2\Lambda)}} \geq 1. \quad (3.38)$$

These results show two genuine features of GR; there does not exist a surplus but deficit solid angle and a source which gives rise to deficit angle is not an electric field but a scalar field.

In this section, we concentrate on investigating a solid angle in low energy limit. One can also examine other issues such as a potential in the UV action and energy configurations near the Lifshitz fixed point as in [60].

8
4 Conclusion

We introduce HL gravity theory with a soft violation of the detailed balance condition with/without matter fields of power-law behaviors as $1/r^n$ and find various solutions. The IR-modified HL theory coupled to matter field for $n = 2$ is of particular interest since such theory has only the deficit solid angle and source giving rise to deficit angle is the scalar field, which agree with well known results of GR. It seems to imply the detailed balance condition should be violated if one applies HL theory as cosmological fame works and wants to obtain the realistic cosmological results in our universe.

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