One-loop corrections to the $Zb\bar{b}$ vertex in models with scalar doublets and singlets

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Abstract

We study the one-loop corrections to the $Zb\bar{b}$ vertex in extensions of the Standard Model with arbitrary numbers of scalar doublets, neutral scalar singlets, and charged scalar singlets. Starting with a general parameterization of theories with neutral and singly-charged scalar particles, we derive the constraints that must be obeyed by the couplings in order for the divergent contributions to cancel. Then, we show that those constraints are obeyed by the models that we are interested in, and we write down the full finite expression for the vertex in those models. We apply our results to some particular cases, highlighting the importance of diagrams with neutral scalars that are often neglected in the literature.
1 Introduction

The discovery of a scalar particle at the LHC [1, 2] urges the questions of whether there are more neutral scalars and whether there are charged scalars. Multi-scalar models have long been studied—for reviews see, for example, Refs. [3, 4, 5]. Here, we concentrate on models with \( n_d \) scalar doublets, \( n_c \) charged-scalar singlets, and \( n_n \) neutral-scalar singlets. The scalar-particle content is, thus, 2\(^n_d\) \( \equiv 2( n_d + n_c ) \) charged scalars \( H^{\pm}_a \) (\( a = 1, \ldots, n \)) and \( m \equiv 2n_d + n_n \) neutral scalars \( S^0_l \) (\( l = 1, \ldots, m \)). In our notation, \( H^+_1 = G^+ \) and \( S^0_1 = G^0 \) are, respectively, the charged and neutral would-be Goldstone bosons.

Light extra scalars may be detected directly through their production, while heavy scalars may be detected indirectly through their impact on the radiative corrections. We focus on the coupling \( Zb\bar{b} \):

\[
\mathcal{L}_{Zbb} = - \frac{g}{c_W} Z_\lambda \bar{b} \gamma^\lambda (g_L P_L + g_R P_R) b,
\]

where \( P_{L,R} \) are the projectors of chirality and, at the tree level,

\[
g_L^0 = \frac{s_W^2}{3} - \frac{1}{2}, \quad g_R^0 = \frac{s_W^2}{3}
\]

in models without extra gauge fields. As usual, \( s_W \) and \( c_W \) are the sine and the cosine, respectively, of the Weinberg angle \( \theta_W \).

Haber and Logan [7] have considered the one-loop corrections to the vertex \( Zb\bar{b} \) in models with extra scalars in any representation of the gauge group \( SU(2)_L \). We extend their analysis by considering CP-violating scalar sectors and by exploring the constraints arising from the cancellation of divergences. We write down the final results in models with singlets and doublets in a simple and usable form. This is possible due to a convenient parameterization that was introduced in Refs. [8, 9, 10], following earlier work [11]. We explicitly calculate the contributions due to diagrams with neutral scalars, which, although nominally suppressed by the bottom-quark mass \( m_b \), may be enhanced by \( ZZS^0 \) couplings proportional to ratios of vacuum expectation values (VEVs).

We present the Lagrangian and the relevant calculations in Section 2, ending with the constraints imposed by the cancellation of divergences. In Section 3, we introduce the parameterization relevant for doublets and singlets, showing that the constraints are indeed satisfied, and we simplify the final expressions. The connection with experiment is reviewed in Section 4 and then applied in Section 5 to some simple cases, looking in particular at the importance of diagrams with neutral scalars. We draw our conclusions in Section 6. An appendix summarizes the definitions of the Passarino–Veltman functions used in this paper.

2 The one-loop calculation

We use the approximation where the CKM matrix element \( V_{tb} = 1 \), requiring us to consider only the quarks bottom with mass \( m_b \) and top with mass \( m_t \). We will neglect \( m_b \) in the

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\(^1\)We use the conventions of Ref. [6], taking all the \( \eta \) signs to be positive.
propagators and loop functions, but we keep generic couplings. Indeed, enhancements
due to ratios of VEVs may arise in some couplings, and this must be properly taken into
account. One example is the tan $\beta$ enhancement that appears in some two-Higgs-doublet
models (2HDMs), which allows some diagrams proportional to $m_b \tan \beta$ to compete, in
the region of large tan $\beta$, with diagrams proportional to $m_t$. This also extends the results
in ref. [7].

2.1 Couplings

In addition to the couplings in Eqs. (1) and (2), we need

\[ \mathcal{L}_{Ztt} = - \frac{g}{c_W} Z_\lambda \bar{t} \gamma^\lambda (g_{Lt} P_L + g_{Rt} P_R) t, \] (3)
\[ \mathcal{L}_{Wtb} = - \frac{g}{\sqrt{2}} (\bar{t} \gamma^\mu P_L b W^+_{\mu} + \bar{b} \gamma^\mu P_L t W^-_{\mu}). \] (4)

In Eq. (3), at the tree level

\[ g^0_{Lt} = \frac{1}{2} - \frac{2 s^2_W}{3}, \quad g^0_{Rt} = - \frac{2 s^2_W}{3}. \] (5)

From Eqs. (2) and (5),

\[ g^0_{Rb} - g^0_{Lt} = g^0_{Lb} - g^0_{Rt} = \frac{s^2_W - c^2_W}{2}. \] (6)

The charged scalars $H_a^\pm$ and the neutral scalars $S_l^0$ interact with the quarks through

\[ \mathcal{L}_{Htb} = \sum_{a=1}^n \left[ H_a^+ \bar{t} (c_a^* P_L - d_a P_R) b + H_a^- \bar{b} (c_a P_R - d_a^* P_L) t \right], \] (7)
\[ \mathcal{L}_{Sbb} = \sum_{l=1}^m S_l^0 \bar{b} (r_l P_R + r_l^* P_L) b, \] (8)

and with the Z gauge boson through

\[ \mathcal{L}_{ZHH} = - \frac{g}{c_W} Z_\lambda \sum_{a,a'=1}^n X_{aa'} (H_a^+ i \partial^\lambda H_a^- - H_a^- i \partial^\lambda H_a^+) , \] (9)
\[ \mathcal{L}_{ZSS} = \frac{ig}{c_W} Z_\lambda \sum_{l,l'=1}^m Y_{ll'} (S_l^0 i \partial^\lambda S_{l'}^0 - S_{l'}^0 i \partial^\lambda S_l^0) , \] (10)
\[ \mathcal{L}_{ZZS} = \frac{g M_Z}{2 c_W} Z_\lambda Z^\lambda \sum_{l=1}^m y_l S_l^0 , \] (11)

where $M_Z$ is the mass of the Z. In general, the coefficients $c_a$, $d_a$, and $r_l$ in Eqs. (7)
and (8) are complex, while the $y_l$ in Eq. (11) are real. The $n \times n$ matrix $X$ in Eq. (9) is
Hermitian. The $m \times m$ matrix $Y$ in Eq. (10) is real and antisymmetric. We let $m_a$ denote
the mass of $H_a^\pm$ and $m_l$ denote the mass of $S_l^0$. 

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2.2 One-loop diagrams

At one-loop level, the diagrams contributing to the $Zb\bar{b}$ vertex are shown in Figs. 1 and 2 for charged and neutral scalars, respectively. This classification of the diagrams was proposed in Ref. [7], wherein the diagrams in Fig. 3 were also mentioned, but then neglected. The diagrams in Fig. 3 involving the charged scalars do not give new contributions beyond the Standard Model (SM) in models with only scalar singlets and doublets, because in these models there are no $ZW^\pm H_a^\mp$ couplings other than the $ZW^\pm G^\mp$ already present in the SM. The diagrams in Fig. 3 involving neutral scalars are proportional to $m_b$. This is because, if one considers (for example) two external left-handed bottom quarks, then one of them connects to a chirality-conserving $Z$ while the other one connects to a chirality-flipping scalar; this necessitates a mass insertion in the bottom-quark propagator. Since the diagrams in Fig. 3 are convergent, one may neglect them by taking $m_b = 0$, and this is what was done in Ref. [7]. Nevertheless, because $m_b$ could appear multiplied by a large coefficient (such as $\tan \beta = v_2/v_1$ in the $Z_2$-symmetric 2HDM) we will also present their calculation.

![Figure 1: Two diagrams with charged scalars contributing to the $Zb\bar{b}$ vertex.](image1)

![Figure 2: Two diagrams with neutral scalars contributing to the $Zb\bar{b}$ vertex.](image2)

The diagrams in Figs. 1 and 2 are divergent and must be renormalized. We follow the on-shell renormalization scheme of Hollik [12, 13]. Applying multiplicative renormalization, the renormalized vertex acquires some terms leading to a correction to the $Z$ propagator; these are part of the oblique parameters and were shown to be very small in Ref. [7]. Here we are looking for the terms that change the tree-level couplings, which after renormalization may be written as

$$i\hat{\Gamma}_\mu^{Zff} = -i\gamma_\mu \frac{e}{s_W c_W} \left[ (g^{0}_{Lb} + \Delta g_L) P_L + (g^{0}_{Rb} + \Delta g_R) P_R \right],$$  \hspace{1cm} (12)
where $\Delta g_{R} (\mathcal{N} = L, R)$ represent all the one-loop corrections after renormalization, including the ones involving $G^{\pm}$, $G^{0}$, and the already-observed neutral scalar with mass 125 GeV (more on this in Section [4]). To perform the renormalization one needs to evaluate the renormalization constants that are obtained from the self-energies. We therefore need to evaluate the contributions of both the charged and neutral scalars to the self-energies, shown in Fig. 4. The self-energy $\imath \Sigma (\mathcal{p})$ receives contributions proportional to $\mathcal{p}_{L}$, $\mathcal{p}_{R}$, $m_{b} P_{L}$, and $m_{b} P_{R}$. In our approximation of neglecting $m_{b}$, we write

$$\Sigma(\mathcal{p}) = \mathcal{p} \left[ \Omega_{L} (p^2) P_{L} + \Omega_{R} (p^2) P_{R} \right]. \quad (13)$$

Following Hollik’s renormalization scheme [12, 13], the self-energy produces contributions to $\Delta g_{Lb}$ and $\Delta g_{Rb}$ given by

$$\Delta g_{Lb} (c) = -g_{Lb}^{0} \Omega_{L} \left( p^2 = m_{b}^{2} \right), \quad (14a)$$

$$\Delta g_{Rb} (c) = -g_{Rb}^{0} \Omega_{R} \left( p^2 = m_{b}^{2} \right). \quad (14b)$$

Note that Ref. [7] follows an equivalent procedure, ignoring renormalization and calculating simply the reducible diagrams with self-energy corrections in the external bottom quarks, which they dub “type c) diagrams”. Although we do perform the renormalization, we will name the contributions arising from it as “type c)”, allowing for an easy comparison with Ref. [7].

Our calculations of the various diagrams have been performed analytically and then confirmed by automatic computation. In order to automatically perform the calculation we use common techniques that include FeynRules [14], QGRAF [15], and FeynCalc [16, 17].
Recently, two of us (DF and JCR) have developed the new software FeynMaster \cite{18} that handles, in an automated way, all these steps. The results involve Passarino–Veltman loop functions \cite{19}; our conventions for them coincide with those in FeynCalc and LoopTools \cite{20,21}, and are summarized in Appendix A.

We next turn to the computation of each diagram.

### 2.3 Calculating the diagrams involving charged scalars

The diagrams in Fig. 1a) lead to

\[
\Delta g_{Lb} (a) = \frac{1}{8\pi^2} \sum_{a,a'=1}^n c_a c_{a'}^* C_{00} \left( M_Z^2, 0, 0, m_a^2, m_{a'}^2, m_i^2 \right),
\]

\[
\Delta g_{Rb} (a) = \Delta g_{Lb} (a) \left( c_a \rightarrow d_a^* \right),
\]

where \( C_{00} \) is a Passarino–Veltman function defined through Eq. (106). We have set \( m_b = 0 \) inside all the Passarino–Veltman functions; however, when evaluating them numerically it is sometimes better to keep \( m_b \neq 0 \) in order to avoid numerical instabilities. We should note that the sums in Eqs. (15) start at \( a = 1 \), \( i.e. \) they include the charged Goldstone bosons \( G^\pm \). However, one may show that \( X_{1a} = X_{a1} = 0 \), and therefore the sum in Eq. (15a) may start at \( a, a' = 2 \), while the term with \( a = a' = 1 \) is separately included in the SM contribution.

The diagrams in Fig. 1b) lead, after taking into account that \( (d-2) C_{00} (\ldots) = 2 C_{00} (\ldots) - 1/2 \) \((d \text{ is the dimension of space–time})\), to

\[
\Delta g_{Lb} (b) = \frac{1}{16\pi^2} \sum_{a=1}^n |c_a|^2 \left\{ -m_i^2 g_{Lb}^0 C_0 \left( 0, M_Z^2, 0, m_a^2, m_i^2, m_i^2 \right) \\
+ g_{Rb}^0 \left[ 2 C_{00} \left( 0, M_Z^2, 0, m_a^2, m_i^2, m_i^2 \right) - \frac{1}{2} \right. \\
\left. - M_Z^2 C_{12} \left( 0, M_Z^2, 0, m_a^2, m_i^2, m_i^2 \right) \right]\right\},
\]

\[
\Delta g_{Rb} (b) = \Delta g_{Lb} (b) \left( c_a \rightarrow d_a, \ g_{Lb}^0 \leftrightarrow g_{Rb}^0 \right).
\]

The Passarino–Veltman function \( C_0 \) is defined in Eq. (104), while \( C_{12} \) is defined through Eq. (106).

As for the type c) contributions, arising through renormalization from diagram a) in Fig. 4, we find

\[
\Delta g_{Lb} (c) = \frac{g_{Lb}^0}{16\pi^2} \sum_{a=1}^n |c_a|^2 B_1 \left( 0, m_i^2, m_a^2 \right),
\]

\[
\Delta g_{Rb} (c) = \Delta g_{Lb} (c) \left( c_a \rightarrow d_a, \ g_{Lb}^0 \rightarrow g_{Rb}^0 \right).
\]

The Passarino–Veltman function \( B_1 \) is defined in Eq. (103).

In the CP-conserving limit, Eqs. (15)–(17) agree with Eqs. (4.1) of Ref. [7], and also with Ref. [22].
The functions \( B_1 \) and \( C_{00} \) are divergent; all the other Passarino–Veltman functions appearing in this paper are finite. In dimensional regularization, defining the divergent quantity
\[
\text{div} = \frac{2}{4-d} - \gamma + \ln(4\pi),
\]
one has
\[
B_1 \left( r^2, m_0^2, m_1^2 \right) = -\frac{\text{div}}{2} + \text{finite terms},
\]
\[
C_{00} \left[ r_1^2, (r_1 - r_2)^2, r_2^2, m_0^2, m_1^2, m_2^2 \right] = +\frac{\text{div}}{4} + \text{finite terms}.
\]
Therefore, the divergent terms in Eqs. (15)–(17) are
\[
\Delta g_{Lb} (a) + \Delta g_{Lb} (b) + \Delta g_{Lb} (c) = \frac{\text{div}}{32\pi^2} \left[ \sum_{a,a'=1}^n c_a X_{aa'} c_{a'}^* + \left( g_{Rt}^0 - g_{Lb}^0 \right) \sum_{a=1}^n |c_a|^2 \right] + \cdots,
\]
\[
\Delta g_{Rb} (a) + \Delta g_{Rb} (b) + \Delta g_{Rb} (c) = \frac{\text{div}}{32\pi^2} \left[ \sum_{a,a'=1}^n d_{a'}^* X_{aa'} d_{a'} + \left( g_{Lt}^0 - g_{Rb}^0 \right) \sum_{a=1}^n |d_a|^2 \right] + \cdots.
\]
We thus conclude that in any sensible theory one must have
\[
\sum_{a,a'} c_a X_{aa'} c_{a'}^* = \frac{s_W^2 - c_W^2}{2} \sum_{a} |c_a|^2,
\]
\[
\sum_{a,a'} d_{a'}^* X_{aa'} d_{a'} = \frac{s_W^2 - c_W^2}{2} \sum_{a} |d_a|^2,
\]
where we have used Eq. (6). Equations (21) are presented here for the first time. Of course, the cancellation of divergences must hold separately for \( \Delta g_{Lb} \) and \( \Delta g_{Rb} \).

2.4 Calculating the diagrams involving neutral scalars

The diagrams in Fig. 2a) lead to
\[
\Delta g_{Lb} (a) = \frac{i}{4\pi^2} \sum_{l,l'=1}^m r_l Y_{ll'} r_{l'}^* C_{00} \left( 0, M_Z^2, 0, 0, m_l^2, m_l^2 \right),
\]
\[
\Delta g_{Rb} (a) = \Delta g_{Lb} (a) \left( r_l \rightarrow r_l^* \right).
\]
The diagrams in Fig. 2b) lead to
\[
\Delta g_{Lb} (b) = \frac{g_{rb}^0}{16\pi^2} \sum_{l=1}^m |r_l|^2 \left[ 2 C_{00} \left( 0, M_Z^2, 0, m_l^2, 0, 0 \right) - \frac{1}{2} - M_Z^2 C_{12} \left( 0, M_Z^2, 0, m_l^2, 0, 0 \right) \right],
\]
\[ \Delta g_{Rb} (b) = \Delta g_{LLb} (b) \left( g_{Rb}^0 \to g_{LLb}^0 \right) . \] (23b)

As for the type c) contributions, arising through renormalization from the second Fig. 4, we find

\[ \Delta g_{LLb} (c) = \frac{g_{LLb}^0}{16\pi} \sum_{l=1}^{m} |r_l|^2 B_1 (0, 0, m_l^2) , \] (24a)

\[ \Delta g_{Rb} (c) = \Delta g_{LLb} (c) \left( g_{LLb}^0 \to g_{Rb}^0 \right) . \] (24b)

In the CP-conserving limit, Eqs. (22)–(24) agree with Eqs. (5.1) of Ref. [7].

Collecting all the divergent terms in Eqs. (22a), (23a), and (24a) we find

\[ \Delta g_{LLb} (a) + \Delta g_{LLb} (b) + \Delta g_{LLb} (c) = \text{div} \left[ \frac{32i}{32\pi^2} \sum_{l,l' = 1}^{m} r_l r_{l'}^* + \left( g_{Rb}^0 - g_{LLb}^0 \right) \sum_{l=1}^{m} |r_l|^2 \right] + \cdots . \] (25)

Since \( g_{Rb}^0 - g_{LLb}^0 = 1/2 \), a consistent theory requires

\[ \sum_{l,l' = 1}^{m} r_l r_{l'}^* = \frac{i}{4} \sum_{l} |r_l|^2 . \] (26)

This is a new constraint, which can also be obtained by collecting all the divergent terms in Eqs. (22b), (23b), and (24b).

Diagrams c) and d) in Fig. 3 involve neutral scalars. They are not divergent and they are proportional to \( m_b \). However, we keep them because they might be enhanced when the coupling of neutral scalars to the bottom quark gets enhanced, as in the type-II 2HDM. From them we get

\[ \Delta g_{LLb} (d) = \frac{g^2 m_b}{16\pi^2 c_W^2} \sum_{l=1}^{m} y_l \text{Re} \left\{ g_{LLb}^0 \left[ C_0 \left( M_Z^2, 0, 0, M_Z^2, m_l^2, 0 \right) \right. \right. \]

\[ \left. \left. - C_1 \left( M_Z^2, 0, 0, M_Z^2, m_l^2, 0 \right) \right] + g_{Rb}^0 C_1 \left( M_Z^2, 0, 0, M_Z^2, m_l^2 \right) \right\} , \] (27a)

\[ \Delta g_{Rb} (d) = \Delta g_{LLb} (d) \left( g_{LLb}^0 \leftrightarrow g_{Rb}^0 \right) . \] (27b)

The function \( C_1 \) is defined through Eq. (105).

At this juncture we want to make a clarification. The one-loop results for \( \Delta g_{LLb} \) and \( \Delta g_{Rb} \) have imaginary parts. If there are no scalars with mass below \( M_Z/2 \), then the imaginary parts only appear through cuts of the internal bottom-quark lines of Fig. 2b), thus affecting only the contributions with neutral scalars. Although those imaginary parts may be of the same order of magnitude as the real parts, they are unimportant because the observables will depend on, for example,

\[ |g_{LLb}|^2 = |g_{LLb}^0 + \Delta g_{LLb}|^2 = |g_{LLb}^0|^2 + 2 \text{Re} \left( g_{LLb}^0 \Delta g_{LLb}^* \right) + O \left( \Delta g_{LLb}^2 \right) \]

\[ = |g_{LLb}^0|^2 + 2 g_{LLb}^0 \text{Re} \left( \Delta g_{LLb} \right) + O \left( \Delta g_{LLb}^2 \right) , \] (28)

where the last line follows from the fact that \( g_{LLb}^0 \) is real. As a result, the impact of an imaginary \( \Delta g_{LLb} \) on the observables (see the next section) effectively appears only at higher order.
2.5 Summary

A generic theory with the couplings in Eqs. (1), (3), (4), and (7)–(11) gets radiative corrections to the $Z\bar{b}b$ vertex, obtained at the one-loop level by summing our Eqs. (15a), (16a), (17a), (22a), (23a), and (24a)—and, if enhanced, (27a)—for $\Delta g_{Lb}$, and by summing our Eqs. (15b), (16b), (17b), (22b), (23b), and (24b)—and, if enhanced, (27b)—for $\Delta g_{Rb}$. The theory only makes sense if its couplings are related through Eqs. (21a), (21b), and (26), which are needed in order for the divergences to cancel.

3 Models with doublet and singlet scalars

We now focus on extensions of the SM with $n_d$ scalar doublets, $n_c$ singly-charged scalar $SU(2)_L$ singlets, and $n_n$ real scalar gauge-invariant fields. The particle content is then $2n \equiv 2(n_d + n_c)$ charged scalars $H^\pm_a$ and $m \equiv 2n_d + n_n$ neutral scalars $S^0_l$; this counting includes the Goldstone bosons $H^\pm_1 = G^\pm$ and $S^0_1 = G^0$. Without loss of generality, one may assume that the scalar with mass 125 GeV found at the LHC is $S^0_2$; generality is lost if one makes the further assumption that the masses are ordered, since there might be massive scalar(s) below 125 GeV.

The scalar doublets are

$$\Phi_k = \left( \begin{array}{c} \varphi^+_k \\ \varphi^0_k \end{array} \right), \quad \tilde{\Phi}_k \equiv i\sigma_2 \Phi^*_k = \left( \begin{array}{c} \varphi^0_k \\ -\varphi^-_k \end{array} \right).$$

(29)

The fields $\varphi^0_k$ have VEVs $v_k / \sqrt{2}$, where the $v_k$ may be complex.

Obviously, the charged and neutral $SU(2)_L$ singlets have no Yukawa couplings. The Yukawa Lagrangian is

$$L_{Yukawa} = - \left( \bar{t}_L H^+ \right) \sum_{k=1}^{n_d} \left[ f_k \left( \begin{array}{c} \varphi^+_k \\ \varphi^0_k \end{array} \right) b_R + e_k \left( \begin{array}{c} \varphi^0_k \\ -\varphi^-_k \end{array} \right) t_R \right] + \text{H.c.},$$

(30)

where the $e_k$ and the $f_k$ ($k = 1, \ldots, n_d$) are the Yukawa coupling constants.

3.1 Formalism

We use the formalism in Refs. [8, 9, 10]. We write $\varphi^+_k$ and $\varphi^0_k$ as superpositions of the physical (= eigenstate of mass) fields as

$$\varphi^+_k = \sum_{a=1}^{n} U_{ka} H^+_a,$$

(31)

$$\varphi^0_k = \frac{1}{\sqrt{2}} \left( v_k + \sum_{l=1}^{m} V_{kl} S^0_l \right).$$

(32)

The matrix $U$ is $n_d \times n$ and the matrix $V$ is $n_d \times m$.

Since $H^+_1$ and $S^0_1$ are Goldstone bosons, the first columns of $U$ and $V$ are fixed and given by

$$U_{k1} = \frac{v_k}{v}, \quad V_{k1} = \frac{iv_k}{v},$$

(33)
where \( v^2 \equiv \sum_{k=1}^{n_d} |v_k|^2 \) (\( v \) is real and positive by definition).

There is an \( n \times n \) matrix

\[
\hat{U} = \begin{pmatrix} U \\ \mathcal{T} \end{pmatrix}
\]

(34)

that is unitary, implying that

\[
UU^\dagger = \mathbb{1}_{n_d \times n_d}.
\]

(35)

The matrix \( \mathcal{T} \) in Eq. (34) only exists when the number \( n_c \) of charged scalar \( SU(2)_L \) singlets is nonzero. There is an \( m \times m \) matrix

\[
\hat{V} = \begin{pmatrix} \text{Re} \mathcal{V} \\ \text{Im} \mathcal{V} \\ \mathcal{R} \end{pmatrix}
\]

(36)

that is real and orthogonal. Therefore,

\[
\begin{align*}
\text{Re} \mathcal{V} \text{Re} \mathcal{V}^T &= \mathbb{1}_{n_d \times n_d}, \\
\text{Im} \mathcal{V} \text{Im} \mathcal{V}^T &= \mathbb{1}_{n_d \times n_d}, \\
\text{Re} \mathcal{V} \text{Im} \mathcal{V}^T &= 0_{n_d \times n_d}, \\
\text{Im} \mathcal{V} \text{Re} \mathcal{V}^T &= 0_{n_d \times n_d}.
\end{align*}
\]

(37)

The matrix \( \mathcal{R} \) in Eq. (36) only exists in models with \( n_n \neq 0 \).

One can show [9] that in this class of models

\[
X_{aa'} = s_W^2 \delta_{aa'} - \frac{(U^T U^*)_{aa'}}{2},
\]

(38)

\[
= \frac{s_W^2 - c_W^2}{2} \delta_{aa'} + \frac{(\mathcal{T}^T \mathcal{T}^*)_{aa'}}{2},
\]

(39)

\[
Y_{ll'} = -\frac{1}{4} \text{Im} (\mathcal{V}^\dagger \mathcal{V})_{ll'}. 
\]

(40)

Moreover,

\[
y_{ll} = -\text{Im} (\mathcal{V}^\dagger \mathcal{V})_{ll},
\]

(41)

leading to \( y_{l=1} = 0 \), because \( \mathcal{V}^\dagger \mathcal{V} \) is Hermitian and therefore \( \text{Im} (\mathcal{V}^\dagger \mathcal{V})_{11} = 0 \). Thus, the sum in Eq. (11) really starts at \( l = 2 \), viz. there is no vertex \( ZZG^0 \), just as there is no vertex \( ZZZ \).

### 3.2 Cancellation of the divergences

It follows from Eqs. (7), (8), and (30)–(32) that

\[
c_a = \sum_{k=1}^{n_d} \mathcal{U}_{ka} c_k
\]

(42a)

\[
= (U^T E)_a,
\]

(42b)
\[ d_a = \sum_{k=1}^{n_d} U_{ka} f_k \]
\[ = (U^T F)_a, \quad (43a) \]
\[ r_l = -\frac{1}{\sqrt{2}} \sum_{k=1}^{n_d} V_{kl} f_k \]
\[ = -\frac{1}{\sqrt{2}} (V^T F)_l, \quad (44a) \]

where we have defined the \( n_d \times 1 \) vectors

\[
E = \begin{pmatrix} e_1 \\ e_2 \\ \vdots \\ e_{n_d} \end{pmatrix}, \quad F = \begin{pmatrix} f_1 \\ f_2 \\ \vdots \\ f_{n_d} \end{pmatrix}. \quad (45)
\]

From Eqs. (33) and (42)–(44),

\[
|c_1| = \left| \sum_{k=1}^{n_d} \frac{v^*_k v}{v} e_k \right| = \sqrt{2} m_t \frac{v}{v} = 0, \quad (46a)
\]
\[
|d_1| = \left| \sum_{k=1}^{n_d} \frac{v_k v}{v} f_k \right| = \sqrt{2} m_b \frac{v}{v} = 0, \quad (46b)
\]
\[
|r_1| = \left| \frac{1}{\sqrt{2}} \frac{v}{v} f_k \right| = m_b \frac{v}{v} = 0. \quad (46c)
\]

We further define the \( m \times 1 \) column vector

\[
R = \begin{pmatrix} r_1 \\ r_2 \\ \vdots \\ r_m \end{pmatrix}. \quad (47)
\]

It then follows from Eq. (44) that

\[
\sum_{l=1}^{m} |r_l|^2 = \frac{1}{2} F^T \mathcal{V} \mathcal{V}^T F^* \]
\[
\begin{align*}
F^T (\text{Re} \mathcal{V} + i \text{Im} \mathcal{V}) (\text{Re} \mathcal{V}^T - i \text{Im} \mathcal{V}^T) F^* & = \frac{1}{2} F^T (\text{Re} \mathcal{V} + i \text{Im} \mathcal{V}) (\text{Re} \mathcal{V}^T - i \text{Im} \mathcal{V}^T) F^* \\
& = \frac{1}{2} F^T (\text{Re} \mathcal{V} \text{Re} \mathcal{V}^T + \text{Im} \mathcal{V} \text{Im} \mathcal{V}^T + i \text{Im} \mathcal{V} \text{Re} \mathcal{V}^T - i \text{Re} \mathcal{V} \text{Im} \mathcal{V}^T) F^*. \quad (48)
\end{align*}
\]

We now use Eqs. (37) to obtain
\[
\sum_{l=1}^{m} |r_l|^2 = \frac{1}{2} F^T (\mathbb{1}_{n_d \times n_d} + \mathbb{1}_{n_d \times n_d} + i \times 0_{n_d \times n_d} - i \times 0_{n_d \times n_d}) F^* \\
= F^T F^* \\
= \sum_{k=1}^{n_d} |f_k|^2. \quad (49)
\]

From Eqs. (42) and (35),
\[
\sum_{a=1}^{n} |c_a|^2 = E^T U^T E = E^T E = \sum_{k=1}^{n_d} |e_k|^2. \quad (50)
\]

Notice that the two sums in Eq. (50) run over different spaces (up to \(n\) and \(n_d\), respectively). Similarly,
\[
\sum_{a=1}^{n} |d_a|^2 = \sum_{k=1}^{n_d} |f_k|^2. \quad (51)
\]

From Eqs. (38), (42), and (35),
\[
\sum_{a,a' = 1}^{n} c_a X_{aa'} c_{a'}^* = s_W^2 E^T U^* U^T E^* - \frac{E^T U^* U^T U^* U^T E^*}{2} \\
= s_W^2 E^T E^* - \frac{E^T E^*}{2} \\
= \frac{s_W^2 - c_W^2}{2} \sum_{k=1}^{n_d} |e_k|^2 \\
= \frac{s_W^2 - c_W^2}{2} \sum_{a=1}^{n} |c_a|^2, \quad (52)
\]

where the last equality follows from Eq. (50). This proves that this class of models obeys the consistency Eq. (21a). Similarly, one can show that Eq. (21b) is also obeyed, confirming within this class of models the cancellation of the divergences of the contributions from charged scalars.

Next we compute
\[
\sum_{l,t=1}^{m} r_l \operatorname{Im}(\mathcal{V}^T \mathcal{V})_{ll'} r_{l'}^* = \frac{1}{2} F^T \mathcal{V} \operatorname{Im} \left[ ((\text{Re} \mathcal{V}^T - i \text{Im} \mathcal{V}^T) (\text{Re} \mathcal{V} + i \text{Im} \mathcal{V})) \mathcal{V}^T F^* \right] \\
= \frac{1}{2} F^T (\text{Re} \mathcal{V} + i \text{Im} \mathcal{V}) (\text{Re} \mathcal{V}^T \text{Im} \mathcal{V} - \text{Im} \mathcal{V}^T \text{Re} \mathcal{V})
\]

\[ \times (\text{Re} \nu^T - i \text{Im} \nu^T) \ F^*. \] (53)

We use once again Eqs. (37) to obtain
\[
\sum_{l,l'=1}^m r_l \text{Im}(\nu^\dagger \nu)_{ll'} r_{l'}^* = \frac{1}{2} F^T (\text{Im} \nu - i \text{Re} \nu) (\text{Re} \nu^T - i \text{Im} \nu^T) F^*
\]
\[= \frac{1}{2} F^T (-2i \times 1_{n_d \times n_d}) F^* \]
\[= -i F^T F^* \]
\[= -i \sum_{k=1}^{n_d} |f_k|^2 \]
\[= -i \sum_{l=1}^m |r_l|^2, \] (54)

where in the last step we have used Eq. (49). Taking into account Eq. (40), we conclude that in this class of models the consistency Eq. (26) also holds.

3.3 Simplification of the charged-scalars contribution

In this class of models, from Eqs. (39) and (6),
\[
X_{a a'} = (g_{Lb}^0 - g_{Rb}^0) \delta_{a a'} + \frac{(T^T T^*)_{aa'}}{2} (g_{Rb}^0 - g_{Lb}^0) \delta_{a a'} + \frac{(T^T T^*)_{aa'}}{2}. \] (55)

Therefore, one may write the charged-scalars contribution as
\[
(16\pi^2) \Delta g_{Lb} = \sum_{a=1}^n |c_a|^2 \left\{ -g_{Ll}^0 m_l^2 C_0 (0, M_Z^2, 0, m_a^2, m_t^2, m_t^2) + g_{Rt}^0 \left[ 2 C_{00} (0, M_Z^2, 0, m_a^2, m_t^2, m_t^2) - \frac{1}{2} - 2 C_{00} (0, M_Z^2, 0, m_t^2, m_a^2, m_a^2) \right. \right.
\]
\[\left. \left. - M_Z^2 C_{12} (0, M_Z^2, 0, m_a^2, m_t^2, m_t^2) \right] \right. \]
\[\left. + g_{Lb}^0 \left[ B_1 (0, m_t^2, m_a^2) + 2 C_{00} (0, M_Z^2, 0, m_t^2, m_a^2, m_a^2) \right] \right\} \]
\[+ \sum_{a,a'=1}^n (T^T T^*)_{aa'} c_a c_{a'}^* C_{00} (0, M_Z^2, 0, m_t^2, m_a^2, m_a^2). \] (56)

The first column of the matrix \( T \) is zero, because \( \sum_k |U_{k1}|^2 = \sum_k |v_k|^2 / v^2 = 1 \). Thus, \( (T^T T^*)_{1a} = (T^T T^*)_{a1} = 0 \) and the charged Goldstone boson does not contribute to the sum in the last line of Eq. (56). On the other hand, the Goldstone boson does contribute to the sum over \( a \) in the first four lines, but \( |c_1| \) has the same value as in the SM, cf. Eq. (46a); therefore, the contribution of the charged Goldstone boson is always the same and should be subtracted out. The simplified expression for the charged-scalar contributions to \( \Delta g_{Rb} \) is obtained from Eq. (56) through the changes \( c_a \to d_a^* \) and \( L \leftrightarrow R \).
Suppose a model with no charged $SU(2)_L$ singlets. Then the matrix $\mathcal{T}$ does not exist. If one furthermore makes the approximation $M_Z = 0$, then the contribution of the charged scalars in Eq. (56) becomes

$$\left(16\pi^2\right) \Delta g_{L_b} = \sum_{a=1}^{n} |c_a|^2 \left\{ -g_{Lt}^0 m_t^2 C_0 \left(0, 0, 0, m_a^2, m_t^2, m_t^2\right) \\
+ 2g_{Rt}^0 \left[ C_{00} \left(0, 0, 0, m_a^2, m_t^2, m_t^2\right) - C_{00} \left(0, 0, 0, m_t^2, m_a^2, m_a^2\right) - \frac{1}{4}\right] \\
+ g_{Lb}^0 \left[ B_1 \left(0, m_t^2, m_a^2\right) + 2C_{00} \left(0, 0, 0, m_t^2, m_a^2, m_a^2\right)\right]\right\} , \quad (57)$$

and similarly for $\Delta g_{R_b}$, with $c_a \rightarrow d_a$ and $L \leftrightarrow R$. One easily finds that

$$B_1 \left(0, m_t^2, m_a^2\right) + 2C_{00} \left(0, 0, 0, m_t^2, m_a^2, m_a^2\right) = 0, \quad (58)$$

and that

$$C_{00} \left(0, 0, 0, m_a^2, m_t^2, m_t^2\right) - C_{00} \left(0, 0, 0, m_t^2, m_a^2, m_a^2\right) - \frac{1}{4} = \frac{m_t^2}{2} C_0 \left(0, 0, 0, m_a^2, m_t^2, m_t^2\right). \quad (59)$$

Hence,

$$\left(16\pi^2\right) \Delta g_{L_b} = \sum_{a=1}^{n} |c_a|^2 \left( g_{Lt}^0 - g_{Lt}^0 \right) m_t^2 C_0 \left(0, 0, 0, m_a^2, m_t^2, m_t^2\right)$$

$$= -\sum_{a=1}^{n} \frac{|c_a|^2}{2} m_t^2 C_0 \left(0, 0, 0, m_a^2, m_t^2, m_t^2\right) , \quad (60a)$$

$$\left(16\pi^2\right) \Delta g_{R_b} = \sum_{a=1}^{n} |d_a|^2 \left( g_{Lt}^0 - g_{Lt}^0 \right) m_t^2 C_0 \left(0, 0, 0, m_a^2, m_t^2, m_t^2\right)$$

$$= +\sum_{a=1}^{n} \frac{|d_a|^2}{2} m_t^2 C_0 \left(0, 0, 0, m_a^2, m_t^2, m_t^2\right) . \quad (60b)$$

The dependence on $\theta_W$ disappeared! This must indeed happen because, in the limit $M_Z = 0$, the $Z$ gauge boson is indistinguishable from the photon—since they are both massless—, and therefore the Weinberg angle loses its meaning and must disappear from any physically meaningful quantity. The function

$$m_t^2 C_0 \left(0, 0, 0, m_a^2, m_t^2, m_t^2\right) = \frac{x}{1 - x} \left( 1 + \frac{\ln x}{1 - x} \right) \quad \text{with} \quad x = \frac{m_t^2}{m_a^2} \quad (61)$$

has been given in Eq. (4.5) of Ref. [7] and has been used in all the subsequent analyses, by many authors, of models with extra doublets (and possibly neutral singlets). In our more general result (56), though, we keep CP violation, we allow for charged singlets, and do not make $M_Z = 0$.

As a consequence of Eqs. (60), in a 2HDM, where there is only one physical charged scalar,

$$\frac{\Delta g_{L_b}}{|c_2|^2} = -\frac{\Delta g_{R_b}}{|d_2|^2}. \quad (62)$$
In general, as long as there are no charged singlets and the approximation $M_Z \approx 0$ is good, $\Delta g_{Lb}$ and $\Delta g_{Rb}$ have opposite signs when the contribution of the neutral scalars is not taken into account.

4 Connection with experiment

The couplings $g_{Lb}$ and $g_{Rb}$ in Eq. (1) may be determined experimentally from

1. The rate

$$R_b = \frac{\Gamma (Z \rightarrow b\bar{b})}{\Gamma (Z \rightarrow \text{hadrons})}. \tag{63}$$

2. Several asymmetries, including

(a) the $Z$-pole forward–backward asymmetry measured at LEP1

$$A_{FB}^{(0,f)} = \frac{\sigma (e^- \rightarrow b_F) - \sigma (e^- \rightarrow b_B)}{\sigma (e^- \rightarrow b_F) + \sigma (e^- \rightarrow b_B)} = \frac{3}{4} A_e A_b, \tag{64}$$

where $b_F$ ($b_B$) stands for final-state bottom quarks moving in the forward (backward) direction with respect to the direction of the initial-state electron;

(b) the left–right forward–backward asymmetry measured by the SLD Collaboration

$$A_{LR}^{FB} = \frac{\sigma (e_L^- \rightarrow b_F) - \sigma (e_L^- \rightarrow b_B) - \sigma (e_R^- \rightarrow b_F) + \sigma (e_R^- \rightarrow b_B)}{\sigma (e_L^- \rightarrow b_F) + \sigma (e_L^- \rightarrow b_B) + \sigma (e_R^- \rightarrow b_F) + \sigma (e_R^- \rightarrow b_B)} = \frac{3}{4} A_b, \tag{65}$$

where $e_L^-$ ($e_R^-$) are initial-state left-handed (right-handed) electrons.

Introducing the vector- and axial-vector bottom-quark couplings

$$v_b = g_{Lb} + g_{Rb}, \quad a_b = g_{Lb} - g_{Rb}, \quad \text{and} \quad r_b = \frac{v_b}{a_b}, \tag{66}$$

one has

$$R_b = \left(1 + \frac{\sum s_b \eta_{\text{QCD}}}{s_b \eta_{\text{QCD}}} \eta_{\text{QED}} \right)^{-1}, \tag{67}$$

$$A_b = \frac{2 r_b \sqrt{1 - 4 \mu_b}}{1 - 4 \mu_b + (1 + 2 \mu_b) r_b^2}. \tag{68}$$

In Eq. (67), $\eta_{\text{QCD}} = 0.9953$ and $\eta_{\text{QED}} = 0.99975$ are QCD and QED corrections, respectively. Moreover,

$$\mu_b = \frac{m_b (M_Z)^2}{M_Z^2}, \tag{69}$$

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2See the discussion by Erler and Freitas in Ref. [23].
\[ s_b = (1 - 6\mu_b) a^2_b + v^2_b, \quad (70) \]

\[ \Sigma = \sum_{q=u,d,s,c} (a^2_q + v^2_q). \quad (71) \]

Neglecting \( \mu_b \approx 10^{-3} \) and setting the QCD and QED corrections to unity, one gets

\[ R_b \approx \frac{2 (g^2_{Lb} + g^2_{Rb})}{2 (g^2_{Lb} + g^2_{Rb}) + \Sigma}, \quad (72) \]

\[ A_b \approx \frac{g^2_{Lb} - g^2_{Rb}}{g^2_{Lb} + g^2_{Rb}}. \quad (73) \]

Equation (73) with \( b \rightarrow e \) defines the \( A_e \) appearing in Eq. (64), which has also been determined experimentally.

The recent fit to the electroweak data by Erler and Freitas in Ref. [23] finds

\[ R_b^{\text{fit}} = 0.21629 \pm 0.00066, \quad (74a) \]

\[ A_b^{\text{fit}} = 0.923 \pm 0.020, \quad (74b) \]

to be compared with the SM values \( R_b^{\text{SM}} = 0.21582 \pm 0.00002 \) and \( A_b^{\text{SM}} = 0.9347 \). Thus, the experimental \( R_b \) is about 0.7\( \sigma \) above the SM value, while \( A_b \) is about 0.6\( \sigma \) below the SM value. However, this good agreement only applies to the overall fit of many observables producing Eqs. (74).

The measured values of the bottom-quark asymmetries by themselves alone reveal a much larger discrepancy; as pointed out in Ref. [23], extracting \( A_b \) from \( A_{FB}^{(0,b)} \) and using \( A_e = 0.1501 \pm 0.0016 \) leads to a result which is 3.1\( \sigma \) below the SM (the precise value of \( A_b \) depends on which observables \( A_e \) is extracted from), while combining \( A_{FB}^{(0,b)} \) with \( A_{FB}^{LR} \) leads to \( A_b = 0.899 \pm 0.013 \), which deviates from the SM value by 2.8\( \sigma \).

There are, thus, two possible approaches. The first one consists in taking as good the values (74) obtained from the SM fit and using \( R_b^{\text{fit}} \) and \( A_b^{\text{fit}} \) as constraints on New Physics (NP). The second one is seeking NP that might explain an \( R_b \) just slightly above the SM, together with an \( A_b \) that undershoots the SM by 2.8\( \sigma \).

It is convenient to switch from the parameterization in Eq. (12), which splits the couplings \( g_{\ell b} \) as \( g_{\ell b}^0 + \Delta g_{\ell b} \), where \( g_{\ell b}^0 \) is the tree-level piece and \( \Delta g_{\ell b} \) is the one-loop piece, to the alternative parameterization

\[ g_{\ell b} = g_{\ell b}^{\text{SM}} + \delta g_{\ell b}, \quad (75) \]

which splits them into the SM piece \( g_{\ell b}^{\text{SM}} \) (which includes the SM loop correction) and the NP piece \( \delta g_{\ell b} \). A simple rule of thumb can be obtained by expanding to first order in the deviations; one finds [7]

\[ \delta R_b = -0.7785 \delta g_{Lb} + 0.1409 \delta g_{Rb}, \quad (76a) \]

\[ \delta A_b = -0.2984 \delta g_{Lb} - 1.6234 \delta g_{Rb}. \quad (76b) \]

This shows that, assuming (rather arbitrarily) \( \delta g_{Rb} \approx -\delta g_{Lb} \), \( \delta R_b \) is pulled down (up) and \( \delta A_b \) is pulled up (down) by a positive (negative) \( \delta g_{Lb} \). Inverting Eqs. (76) [7],

\[ \delta g_{Lb} = -1.2433 \delta R_b - 0.1079 \delta A_b, \quad (77a) \]
\[ \delta g_{Rb} = 0.2286 \delta R_b - 0.5962 \delta A_b. \] (77b)

If one wishes to follow the second approach above, viz. using NP to keep \( R_b \) close to its SM value while reducing \( A_b \) significantly, then one needs to get a small \( \delta g_{Lb} \) together with a significant positive \( \delta g_{Rb} \).

5 Simple particular cases

5.1 The inert doublet model

This is a 2HDM characterized by

\[ \Phi_1 = \left( v + S_2^0 + iG^0 \right)/\sqrt{2}, \quad \Phi_2 = \left( S_3^0 + iS_4^0 \right)/\sqrt{2}. \] (78)

Thus, all the VEV is in the first doublet and the matrices

\[ U = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad V = \begin{bmatrix} i & 1 & 0 & 0 \\ 0 & 0 & 1 & i \end{bmatrix}. \] (79)

We have

\[ c_1 = \frac{\sqrt{2}m_t}{v}, \quad d_1 = \frac{\sqrt{2}m_b}{v}, \] (80)

which may be taken to be real and positive. The free parameters of NP are: the mass \( M_{H^+} \) of the charged scalar, the masses \( M_3 \) and \( M_4 \) of the two new neutral scalars \( S_3^0 \) and \( S_4^0 \), respectively, and the couplings \( c_2 = e_2 \) and \( d_2 = f_2 \). Since there are no charged singlets,

\[ X_{aa'} = \frac{s_W^2 - c_W^2}{2} \delta_{aa'}, \] (81)

while, from the matrix \( V \) in Eq. (79),

\[ \text{Im}(V^t V) = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}, \quad R = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix} = -\frac{1}{\sqrt{2}} \begin{bmatrix} i d_1 \\ d_1 \\ d_2 \\ i d_2 \end{bmatrix}. \] (82)

5.1.1 Charged-scalar contribution

The contribution of the charged Goldstone boson can be separated and included in the SM. The genuine new contribution is

\[ \delta g_{Lb} = \frac{|c_2|^2}{16\pi^2} \left[ (s_W^2 - c_W^2) C_{00} (0, M_Z^2, 0, m_t^2, M_{H^+}, M_{H^+}^2) \\
- g_{L_t}^0 m_t^2 C_0 (0, M_Z^2, 0, M_{H^+}, m_t^2, m_t^2) \\
+ g_{R_t}^0 \left[ 2 C_{00} (0, M_Z^2, 0, M_{H^+}, m_t^2, m_t^2) - \frac{1}{2} \right] \right]. \]
\[-M_Z^2 C_{12}(0, M_Z^2, 0, M_{H^+}^2, m_t^2, m_t^2)\]
\[+ g_{Lb}^0 B_1(0, m_t^2, M_{H^+}^2)\],

\[
\delta g_{Rb} = \frac{|d_2|^2}{16\pi^2} \left[ \left( s_W^2 - c_W^2 \right) C_{00}(0, M_Z^2, 0, m_t^2, M_{H^+}^2, M_{H^+}^2) \right. \\
- g_{Rb}^0 m_t^2 C_0(0, M_Z^2, 0, M_{H^+}^2, m_t^2, m_t^2) \right. \\
+ g_{Lb}^0 \left[ 2 C_{00}(0, M_Z^2, 0, M_{H^+}^2, m_t^2, m_t^2) - \frac{1}{2} \right. \\
\left. \left. - M_Z^2 C_{12}(0, M_Z^2, 0, M_{H^+}^2, m_t^2, m_t^2) \right] \right. \\
+ g_{Rb}^0 B_1(0, m_t^2, M_{H^+}^2) \right].
\]

If we plot \(\delta g_{Lb}/|c_2|^2\) and \(\delta g_{Rb}/|d_2|^2\), we get general results for any 2HDM. We have used LoopTools [20] to perform the numerical integrations contained in the Passarino–Veltman functions. The results are shown in Fig. 5. One sees that \(0 < \delta g_{Lb} \lesssim 0.002 |c_2|^2\) and \(0 < \delta g_{Rb} \lesssim 0.002 |d_2|^2\).

Figure 5: Contribution of the charged scalar to \(\delta g_{Lb}\) (red curve) and to \(-\delta g_{Rb}\) (blue curve) in a general 2HDM.

that Eq. (62) holds to an excellent approximation; this indicates that the approximation \(M_Z = 0\) is in fact very good. This is vindicated by Fig. 6 which displays the asymmetries \(R_{g_{L,R}}\) between the values of \(\delta g_{Lb}/|c_2|^2\) and \(\delta g_{Rb}/|d_2|^2\) computed with \(M_Z \neq 0\) and with \(M_Z = 0\)

\[ R_{gL} = \frac{\delta g_{Lb}(M_Z)/|c_2|^2 - \delta g_{Lb}(0)/|c_2|^2}{\delta g_{Lb}(M_Z)/|c_2|^2 + \delta g_{Lb}(0)/|c_2|^2} \]
\[ R_{gR} = R_{gL} (L \rightarrow R, c_2 \rightarrow d_2) . \]  (84)

One observes that both asymmetries are at most of order 1%.

To have an idea of the impact of these contributions on the experimental constraints on \(R_b\) and \(A_b\) we display the latter in Fig. 7. One sees that \(A_b\) does not impose a relevant
constraint, but $R_b$ does. The result is dominated by the variation in $|c_2|$, although there is also dependence on the charged Higgs mass as can be seen in Fig. 5. In order to have agreement with the data $|c_2|$ tends to be small, approaching the SM point, although larger values are possible if the charged Higgs mass is also large.

5.1.2 Neutral-scalar contribution

Taking into account Eq. (40), in our class of models Eq. (22a) reads

$$\Delta g_{Lb} (a) = \frac{-i}{16\pi^2} \sum_{l,l'} r_l \text{Im} (V^\dagger V)_{ll'} r_{l'}^* C_{00} (0, M_Z^2, 0, 0, m_{l'}^2, m_l^2).$$

(85)

Since $C_{00} (0, M_Z^2, 0, 0, m_{l'}^2, m_l^2) = C_{00} (0, M_Z^2, 0, 0, m_l^2, m_{l'}^2)$, Eq. (85) may be simplified to

$$\Delta g_{Lb} (a) = \frac{1}{8\pi^2} \sum_{l=1}^{m-1} \sum_{l'=l+1}^{m} \text{Im} (V^\dagger V)_{ll'} \text{Im} (r_{l'} r_{l'}^* C_{00} (0, M_Z^2, 0, 0, m_{l'}^2, m_l^2).$$

(86)

In the case of the inert-doublet model, because of Eq. (82), Eq. (86) reads

$$\Delta g_{Lb} (a) = \frac{1}{16\pi^2} \left[ |d_1|^2 C_{00} (0, M_Z^2, 0, 0, m_{l'}^2, m_l^2) - |d_2|^2 C_{00} (0, M_Z^2, 0, 0, m_{l'}^2, m_l^2) \right].$$

(87)

Since $S^0_1 = G^0$ is the neutral Goldstone boson and $S^0_2$ is the Higgs particle of the SM, the first term in the right-hand side of Eq. (87) is an SM contribution that we are uninterested in; we just care about the NP contributions, which are

$$\delta g_{Lb} = \frac{|d_2|^2}{16\pi^2} \left( - C_{00} (0, M_Z^2, 0, 0, M_3^2, M_4^2) \right)$$
Figure 7: Values for $A_{b}$ and $R_{b}$ due to the charged-scalar contributions to $\delta g_{L,R_{b}}$. The blue lines are the 2$\sigma$ limits on $R_{b}$ and $A_{b}$ given by Eqs. (74). The star marks the SM result. The red line corresponds to $|c_{2}| = |d_{2}| = 1$, while $M_{H^{\pm}}$ is allowed to vary between 100 GeV (far from the SM point) and 1 TeV (close to the SM point). The light-green points have $|c_{2}|, |d_{2}| \in [0.1, 1]$; the dark-green points have $|c_{2}|, |d_{2}| \in [1, 1.5]$. The right figure is a close-up of the left one.

\[ \frac{g_{R_{b}}^{0}}{2} \left\{ 2 C_{00} \left( 0, M_{Z}^{2}, 0, M_{3}^{2}, 0, 0 \right) - \frac{1}{2} - M_{Z}^{2} C_{12} \left( 0, M_{Z}^{2}, 0, M_{3}^{2}, 0, 0 \right) \right\} + \frac{g_{L_{b}}^{0}}{2} \left\{ B_{1} \left( 0, 0, M_{3}^{2} \right) + B_{1} \left( 0, 0, M_{4}^{2} \right) \right\}, \]  

(88a)

\[ \delta g_{R_{b}} = \frac{|d_{2}|^{2}}{16 \pi^{2}} \left\{ - C_{00} \left( 0, M_{Z}^{2}, 0, 0, M_{3}^{2}, M_{4}^{2} \right) \right. \]

\[ + \frac{g_{R_{b}}^{0}}{2} \left\{ 2 C_{00} \left( 0, M_{Z}^{2}, 0, M_{3}^{2}, 0, 0 \right) - \frac{1}{2} - M_{Z}^{2} C_{12} \left( 0, M_{Z}^{2}, 0, M_{3}^{2}, 0, 0 \right) \right\} \]

\[ + 2 C_{00} \left( 0, M_{Z}^{2}, 0, M_{3}^{2}, 0, 0 \right) - \frac{1}{2} - M_{Z}^{2} C_{12} \left( 0, M_{Z}^{2}, 0, M_{4}^{2}, 0, 0 \right) \]

\[ + \left. \frac{g_{R_{b}}^{0}}{2} \left\{ B_{1} \left( 0, 0, M_{3}^{2} \right) + B_{1} \left( 0, 0, M_{4}^{2} \right) \right\} \right\}. \]  

(88b)

We have evaluated these contributions by using LoopTools. We have checked in the numerical simulation that the divergences indeed cancel, by verifying that the results are independent of the $\Delta$ parameter of LoopTools. Without loss of generality, we have required that $M_{4} > M_{3}$. The results are shown in Fig. 8. It is seen that $\delta g_{L_{b}} > 0$ but $\delta g_{R_{b}} < 0$ (recall that a negative $\delta g_{R_{b}}$ goes in the wrong direction if one wishes to explain $A_{b}$ below the SM value); both are typically $O \left( 10^{-4} \right) |d_{2}|^{2}$ unless $M_{3} \sim 200$ GeV and $M_{4} \sim 1$ TeV, in which case they may reach $O \left( 10^{-3} \right) |d_{2}|^{2}$.

\[ ^{3}\text{It is convenient to substitute the zeros in many arguments of the Passarino–Veltman functions by some small nonzero squared masses, lest LoopTools is driven to spurious numerical instabilities.} \]
Figure 8: $\delta g_{lb}$ and $\delta g_{Rb}$ in Eqs. (88) as a function of $M_3$.

Comparing Figs. 5 and 8, one sees that, unless the masses of the two NP neutral scalars are close to each other, there is in general no rationale for neglecting the neutral-scalar contribution as compared to the charged-scalar one, as most authors do in their analyses.

5.2 The complex 2HDM

The complex 2HDM (C2HDM) is a two-Higgs-doublet model with a softly broken $\mathbb{Z}_2$ symmetry. The scalar potential is

$$V_H = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - m_{12}^2 \Phi_1^\dagger \Phi_2 - m_{12}^2 \Phi_2^\dagger \Phi_1 + \frac{\lambda_1}{2} \Phi_1^\dagger \Phi_1 \Phi_1^\dagger \Phi_1 + \frac{\lambda_2}{2} \Phi_2^\dagger \Phi_2 \Phi_2^\dagger \Phi_2 + \lambda_3 \Phi_1^\dagger \Phi_1 \Phi_1^\dagger \Phi_1 + \lambda_4 \Phi_1^\dagger \Phi_2 \Phi_2^\dagger \Phi_1 + \frac{\lambda_5}{2} \left( \Phi_1^\dagger \Phi_2 \right)^2 + \frac{\lambda_5^*}{2} \left( \Phi_2^\dagger \Phi_1 \right)^2,$$

(89)

where all the parameters, except $m_{12}^2$ and $\lambda_5$, are real. In general, $\text{Im} \left[ (m_{12}^2)^2 \lambda_5^* \right]$ is allowed to be nonzero. By rephasing $\Phi_1$ and $\Phi_2$, we go to a basis where the VEVs are real and positive: $\langle 0 | \varphi_k^0 | 0 \rangle = v_k / \sqrt{2}$ for $k = 1, 2$. We write

$$v_1 = v c_\beta, \quad v_2 = v s_\beta,$$

(90)

where $v = 246$ GeV. Thenceforth, $c_\theta$, $s_\theta$, and $t_\theta$ represent the cosine, sine, and tangent, respectively, of whatever angle $\theta$ is in the subindex. We write the scalar doublets as

$$\Phi_k = \begin{pmatrix} \varphi_k^+ \\ (v_k + \eta_k + i \chi_k) / \sqrt{2} \end{pmatrix} \quad (k = 1, 2).$$

(91)

We transform the fields into the so-called Higgs basis through

$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix},$$

(92)
Then $H_2$ does not have a VEV:

$$
H_1 = \left( \frac{G^+}{(v + H^0 + i C^0) / \sqrt{2}} \right), \quad H_2 = \left( \frac{H^+}{(R_2 + i I_2) / \sqrt{2}} \right). \quad (93)
$$

$G^+$ and $G^0$ are the Goldstone bosons. There is a charged pair $H^\pm$ with mass $m_{H^\pm}$.

In a standard C2HDM notation, $\eta_3 := I_2$ and the neutral mass eigenstates are obtained from the three neutral components as

$$
\begin{pmatrix}
S_2^0 \\
S_3^0 \\
S_4^0
\end{pmatrix}
= R
\begin{pmatrix}
\eta_1 \\
\eta_2 \\
\eta_3
\end{pmatrix}. \quad (94)
$$

The orthogonal matrix $R$ diagonalizes the neutral mass matrix

$$
(M^2)_{ij} = \frac{\partial^2 V_H}{\partial \eta_i \partial \eta_j}, \quad (95)
$$

through

$$
R M^2 R^T = \text{diag} \left( m_2^2, m_3^2, m_4^2 \right), \quad (96)
$$

where $m_2 = 125 \text{ GeV} \leq m_3 \leq m_4$ are the masses of the neutral scalars ($m_1$ is the unphysical mass of the Goldstone boson $S_1^0 = G^0$). We parameterize the orthogonal matrix $R$ as [26]

$$
R = \begin{pmatrix}
  c_{\alpha_1} c_{\alpha_2} & s_{\alpha_1} c_{\alpha_2} & s_{\alpha_2} \\
  -s_{\alpha_1} c_{\alpha_3} - c_{\alpha_1} s_{\alpha_2} c_{\alpha_3} & c_{\alpha_1} c_{\alpha_3} - s_{\alpha_1} s_{\alpha_2} s_{\alpha_3} & c_{\alpha_2} s_{\alpha_3} \\
  s_{\alpha_1} s_{\alpha_3} - c_{\alpha_1} s_{\alpha_2} c_{\alpha_3} & -c_{\alpha_1} s_{\alpha_3} - s_{\alpha_1} s_{\alpha_2} c_{\alpha_3} & c_{\alpha_2} c_{\alpha_3}
\end{pmatrix}. \quad (97)
$$

Without loss of generality, the angles may be restricted to [26]

$$
-\pi/2 < \alpha_1 \leq \pi/2, \quad -\pi/2 < \alpha_2 \leq \pi/2, \quad 0 \leq \alpha_3 \leq \pi/2. \quad (98)
$$

Taking the limit $\alpha_2, \alpha_3 \to 0$ one recovers a 2HDM with softly broken $Z_2$ symmetry and no CP violation; this is the ‘real 2HDM’, in which $S_4^0 = A$ is the massive CP-odd scalar.

Comparing with Eqs. (31) and (32), we find

$$
U = \begin{pmatrix}
  c_{\beta} & -s_{\beta} \\
  s_{\beta} & c_{\beta}
\end{pmatrix}, \quad V = \begin{pmatrix}
  i c_{\beta} & R_{11} - i s_{\beta} R_{13} & R_{21} - i s_{\beta} R_{23} & R_{31} - i s_{\beta} R_{33} \\
  i s_{\beta} & R_{12} + i c_{\beta} R_{13} & R_{22} + i c_{\beta} R_{23} & R_{32} + i c_{\beta} R_{33}
\end{pmatrix}. \quad (99)
$$

Equation [81] still holds and

$$
\text{Im} \left( V^\dagger V \right) = \begin{pmatrix}
  0 & -c_{\beta} R_{11} - s_{\beta} R_{12} & -c_{\beta} R_{21} - s_{\beta} R_{22} & -c_{\beta} R_{31} - s_{\beta} R_{32} \\
  c_{\beta} R_{11} + s_{\beta} R_{12} & 0 & -c_{\beta} R_{31} - s_{\beta} R_{32} & -c_{\beta} R_{21} - s_{\beta} R_{22} \\
  c_{\beta} R_{21} + s_{\beta} R_{22} & c_{\beta} R_{31} + s_{\beta} R_{32} & 0 & -c_{\beta} R_{11} - s_{\beta} R_{12} \\
  c_{\beta} R_{31} + s_{\beta} R_{32} & c_{\beta} R_{21} + s_{\beta} R_{22} & -c_{\beta} R_{11} - s_{\beta} R_{12} & 0
\end{pmatrix}. \quad (100)
$$

---

4In this subsection we assume that the observed particle with mass 125 GeV is the lightest neutral scalar.
Assuming the Yukawa couplings to follow the type-II 2HDM pattern, viz. \( c_2 = f_1 = 0 \) and

\[
e_1 = -\frac{\sqrt{2} m_t}{v_1}, \quad f_2 = -\frac{\sqrt{2} m_b}{v_2},
\]

we have

\[
c_2 = \frac{\sqrt{2} m_t}{v} \tan \beta, \quad d_2 = -\frac{\sqrt{2} m_b}{v} \cot \beta.
\] (102)

Note that, contrary to the assumptions in the previous subsection, here \(|c_2|\) and \(|d_2|\) usually are of vastly different orders of magnitude; in particular, \(|d_2| \ll |c_2|\) for large values of \(\tan \beta\).

This model was studied in detail in Ref. [27], which introduced the code \texttt{C2HDM\_HDECAY} implementing the C2HDM in \texttt{HDECAY} [28, 29]. For illustrative purposes, we take points from that fit, where, invoking constraints from Flavour Physics, \(\tan \beta\) was taken below 20. We denote this by the “low \(\tan \beta\) set”. We also show results for a new dedicated run with very large \(\tan \beta > 20\), which we dub the “large \(\tan \beta\) set”. Such high values of \(\tan \beta\) may be in contradiction with certain Flavour Physics observables, notably (as we will now show) \(Z \rightarrow b\bar{b}\).

Nevertheless, we will consider such extreme values since we wish to stress that the details of such a bound may require both charged and neutral contributions. As shown in Fig. 8 of Ref. [30], very large \(\tan \beta\) is only consistent with current measurements at LHC if \(\alpha_1\) lies in a very restricted range, which we impose in this run ab initio.

As in the inert doublet model discussed previously, the contribution due to the charged Goldstone bosons decouples, it is included in the SM and subtracted out, and the result is still given by Eqs. (83) and Fig. 5. Note that \(\delta g_{Lb}\) is positive while \(\delta g_{Rb}\) is negative. Recall that \(\delta g_{Lb}\) tends to make \(R_b\) smaller and from there comes a bound in the \(m_{H^\pm} - \tan \beta\) plane. The correction of \(\delta g_{Rb}\) is too small to have an impact on \(R_b\). On the other hand, \(\delta g_{Rb}\) has a substantial impact on \(A_b\), but it goes in the wrong direction when compared with the direct experimental measurements.

We are particularly interested in the corrections arising from neutral scalars, because they are usually discarded. Firstly, we would like to know under which circumstances can contributions from the neutral scalars be large. Secondly, we are interested in studying the specific neutral scalar contributions of “type d)”, in Fig. 3. Recall that the latter are proportional to \(m_b\) and they are always neglected, despite the fact that they could possibly be enhanced by \(\tan \beta\).

Let us denote by a superscript \(c\) (\(n\)) the new physics contributions coming from the charged (neutral) scalars. We separated the data from our scans in three different sets,

- Small \(\tan \beta \in [0, 10]\), blue in the plots
- Intermediate \(\tan \beta \in [10, 30]\), green in the plots
- Large \(\tan \beta > 30\), red in the plots

In the left panel of Fig. 9 we show \(\delta g_{Lb}^c\) versus \(\delta g_{Lb}^n\) for all three sets. In the right panel of Fig. 9 we show \(\delta g_{Rb}^c\) versus \(\delta g_{Rb}^n\) (because both \(\delta g_{Rb}^c\) and \(\delta g_{Rb}^n\) are negative). We see that, contrary to popular belief, \(|\delta g_{Rb}^c|\) is of order but usually larger than \(|\delta g_{Rb}^n|\). They

---

Moreover, extremely high or extremely low values of \(\tan \beta\) will also violate perturbativity.
approach each other for moderate values of $\tan \beta$ and eventually both become quite large for very large values of $\tan \beta$. In contrast, for low $\tan \beta \sim 1$, $\delta g_{Lb}^n$ is of order $10^{-1}$ and much larger than $\delta g_{Lb}^c$. For intermediate values of $\tan \beta$, though, $\delta g_{Lb}^c \sim \delta g_{Lb}^n \sim 10^{-5}$ and, for larger $\tan \beta$, the result (although small) is dominated by the contributions from the neutral scalars. The sums are shown as functions of $\tan \beta$ in Fig. 10. We see that a significant impact on $A_b$ and $R_b$ can only occur, either for very low or very high values of $\tan \beta$.

Figure 9: Comparison of $\delta g_{Lb}^n$ with $\delta g_{Lb}^c$ (left plot) and of $\delta g_{Rb}^n$ with $\delta g_{Rb}^c$ (right plot).

Figure 10: Total contribution of the neutral and charged scalars to $\delta g_{Lb}$ and $\delta g_{Rb}$.

The impact on $A_b$ and $R_b$ is shown in Fig. 11 for the low $\tan \beta$ set. The points in Fig. 11 only stray from the $2\sigma R_b$ bounds for $\tan \beta < 0.8$. This is why only points above $\tan \beta = 0.8$ were taken in Ref. [27]. In the low $\tan \beta$ region, the new contributions to $\delta g_b$ are dominated by the diagrams with charged scalars, as can seen from Fig. 9.

We are now interested in studying what would happen in the large $\tan \beta$ region, especially in what concerns the contributions from neutral scalars. In Fig. 12a) we see that, for $\delta g_L$, the neutral scalar contributions (in dark blue) are typically larger than the charged scalar contributions (in green) by orders of magnitude, when $\tan \beta > 30$. This
Figure 11: $A_b$ versus $R_b$ in the C2HDM for low $\tan \beta$ ($\tan \beta < 10$). In this regime the charged scalars contribution is dominant.

is an important conclusion that we draw, as we had already seen in Fig. 9. One cannot neglect the effect from neutral scalars to $\delta g_{Lb}$. We expect this to be even more important in models with more than two Higgs doublets and/or extra singlets. It is interesting to inquire about the importance of the type d) neutral scalar contributions (in red). They are nominally of “order $m_b$”, and were discarded in Ref. [7]. We see in Fig. 12a) they dominate up to $\tan \beta \sim 160$, when the other neutral scalar diagrams (in purple) take over. But, because they have the opposite sign, we see a pronounced dip in $\delta g_{Lb}^n$ around $\tan \beta \sim 160$.

Fig. 12b) shows the neutral scalar contributions to $\delta g_{Rb}$ from type d) diagrams in red. We see that these diagrams are basically irrelevant for $\delta g_{Rb}$. However, the overall contribution from neutral scalars (in green) is a relevant percentage of the contribution from charged scalars (in blue). The curve in purple shows the sum of all contributions.

The effect of the total contribution to $A_b$ and $R_b$ in the C2HDM, for all values of $\tan \beta$, is shown in Fig. 13. The contribution of the charged scalars is shown in red for $\tan \beta < 0.8$, in orange for $0.8 < \tan \beta < 250$, and in brown for $\tan \beta > 250$. For low values of $\tan \beta$ (red curve) the contribution from the charged scalars tends to make $R_b$ small and no effect on $A_b$ (see Fig. 11). However, for the large $\tan \beta$ regime, the charged scalars (brown curve) would tend to make $R_b$ small and $A_b$ large. The contribution from neutral scalars is shown in cyan for $\tan \beta < 350$ and in blue for $\tan \beta > 350$. We see that, in contrast with the charged scalars, the neutral scalars contributions, for large values of $\tan \beta$, tend to make $R_b$ large and $A_b$ small. And these are precisely the dominant contributions for very large $\tan \beta$, as we see from the total (green) curve.

We conclude that, when studying the impact of $Z \to b\bar{b}$ on multi scalar models at very large couplings (very large $\tan \beta$ in our example of the C2HDM) the neutral scalar contributions must be taken into account. Of course, we will need to include in any model all theoretical and experimental constraints, which may curtail a large part of such extreme couplings. This must be evaluated in a case by case basis.
Figure 12: Contribution of the neutral scalars to $\delta g_b$, for all values of $\tan \beta$. See text for details.

6 Conclusions

We have studied the one-loop contributions to $Z \to b\bar{b}$ in models with extra scalars. We have started by deriving the conditions on generic couplings that must hold for the divergences to cancel. We have then concentrated on models with extra $SU(2)_L$ doublets and singlets, either neutral, as in Ref. [7], or charged. The final expressions are greatly simplified, due to the parametrization in Refs. [8] [9] [10] [11]. We also extend the analysis in Ref. [7] to models with CP violation in the scalar sector. We highlight the importance of the neutral-scalar contributions. In particular, Fig. 13 shows that, for extreme values of $\tan \beta$, the bounds coming from $A_b$ and $R_b$ are mostly due to the neutral scalars and not the charged scalars. This highlights the need to consider both contributions in generic models with extra singlet and doublet scalars.

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A Passarino–Veltman functions

In this appendix we expose our definition of the Passarino–Veltman functions, which coincides with that of FeynCalc [16] [17] and LoopTools [20] [21] used in the algebraic and numerical calculations [18]. We use dimensional regularization; the Feynman integrals are performed in a space–time of dimension $d = 4 - \epsilon$. Then,

$$
\mu' \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 - m_0^2} \frac{1}{(k + r)^2 - m_1^2} k^\lambda = \frac{i}{16\pi^2} r^\lambda B_1 \left( r^2, m_0^2, m_1^2 \right).
$$

(103)
Figure 13: $A_b$ versus $R_b$ in the C2HDM for all values of $\tan\beta$. The contributions from the charged (neutral) scalars are shown, for different ranges of $\tan\beta$, in red, orange and brown (cyan and blue) and the total in green (see text for details).

Moreover,

$$
\mu^e \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 - m_0^2} \frac{1}{(k + r_1)^2 - m_1^2} \frac{1}{(k + r_2)^2 - m_2^2} = \frac{i}{16\pi^2} C_0 \left[ r_1^2, (r_1 - r_2)^2, r_2^2, m_0^2, m_1^2, m_2^2 \right].
$$

(104)

Also,

$$
\mu^e \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 - m_0^2} \frac{1}{(k + r_1)^2 - m_1^2} \frac{1}{(k + r_2)^2 - m_2^2} k^\lambda = \frac{i}{16\pi^2} \left[ r_1^\lambda C_1 + r_2^\lambda C_2 \right] \left[ r_1^2, (r_1 - r_2)^2, r_2^2, m_0^2, m_1^2, m_2^2 \right].
$$

(105)

Finally,

$$
\mu^e \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 - m_0^2} \frac{1}{(k + r_1)^2 - m_1^2} \frac{1}{(k + r_2)^2 - m_2^2} k^\lambda k^\nu = \frac{i}{16\pi^2} \left[ g^\lambda_\nu C_{00} + r_1^\lambda r_2^\nu C_{11} + r_2^\lambda r_2^\nu C_{22} + (r_1^\lambda r_2^\nu + r_2^\lambda r_1^\nu) C_{12} \right] \left[ r_1^2, (r_1 - r_2)^2, r_2^2, m_0^2, m_1^2, m_2^2 \right].
$$

(106)
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