Producing Distant Planets by Mutual Scattering of Planetary Embryos

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Abstract

It is likely that multiple bodies with masses between those of Mars and Earth (“planetary embryos”) formed in the outer planetesimal disk of the solar system. Some of these were likely scattered by the giant planets into orbits with semimajor axes of hundreds of au. Mutual torques between these embryos may lift the perihelia of some of them beyond the orbit of Neptune, where they are no longer perturbed by the giant planets, so their semimajor axes are frozen in place. We conduct N-body simulations of this process and its effect on smaller planetesimals in the region of the giant planets and the Kuiper Belt. We find that (i) there is a significant possibility that one sub-Earth mass embryo, or possibly more, is still present in the outer solar system; (ii) the orbit of the surviving embryo(s) typically has perihelion of 40–70 au, semimajor axis less than 200 au, and inclination less than 30°; (iii) it is likely that any surviving embryos could be detected by current or planned optical surveys or have a significant effect on solar system ephemerides; (iv) whether or not an embryo has survived to the present day, its dynamical influence earlier in the history of the solar system can explain the properties of the detached disk (defined in this paper as containing objects with perihelia >38 au and semimajor axes between 80 and 500 au).

Key words: Kuiper belt: general – minor planets, asteroids: general – Oort cloud – planets and satellites: dynamical evolution and stability

1. Introduction

The standard model for the formation and evolution of comets (Oort 1950) assumes that comets form in the protoplanetary disk, in the region of the giant planets, and are then excited onto highly eccentric planet-crossing orbits through planetary perturbations. The comets then undergo a random walk in energy space, taking one “step” at each perihelion passage when they interact with the giant planets. This process continues until the aphelion distance is a few times 10^6 au, at which point torques from passing stars and the tidal field of the Galaxy are sufficient to increase the perihelion beyond ~38 au in less than the energy diffusion time. Once this occurs, the comet no longer interacts gravitationally with the planetary system and its semimajor axis is frozen in place, apart from a much slower random walk due to perturbations from passing stars. This model correctly predicts many properties of the Oort comet cloud (Wiegert & Tremaine 1999; Fernández 2005; Fouchard et al. 2013), but also incorrectly predicts that there should be no objects with perihelia substantially beyond the orbit of Neptune and aphelia less than 1000 au, as there is no dynamical pathway to this part of phase space (Duncan et al. 1987). More specifically, the JPL small-body database3 contains 33 objects on orbits with semimajor axes between 80 and 500 au and perihelia well beyond the orbit of Neptune (q > 38 au). Such bodies are often said to belong to the detached disk (Gladman et al. 2002). The best known member of this group is Sedna, a body with perihelion 76 au, semimajor axis 480 au, and a radius of about 1000 km (Brown et al. 2004; Pál et al. 2012).

One model to explain the orbits of these bodies (Fernández & Brunini 2000; Brasser et al. 2006, 2012) holds that the Sun was born in a cluster of stars with density in the range 10^4–10^5 M_☉ pc^-3. Tides and close stellar encounters from this cluster exert torques on bodies with aphelia of several hundred au, raising their perihelia and thereby creating the detached disk. Another possibility is that some of the objects in the detached disk were trapped in mean-motion resonances when Neptune migrated, after which their perihelia were lifted above 40 au by resonant oscillations (Kaib & Sheppard 2016; Nesvorný et al. 2016). Alternatively, Madigan & McCourt (2016) find that collective effects allow a massive disk of planetesimals to increase their inclinations at the expense of their eccentricity, thus producing a population of detached objects. Several other models for producing the detached disk are reviewed by Morbidelli & Levison (2004).

Another idea, first suggested by Gladman et al. (2002) but not explored in detail, is that multiple Mars-sized embryos formed interior to the orbit of Neptune, and, like the comets, were excited to semimajor axes of a few hundred au via interactions with the giant planets. These embryos exerted torques on one another as well as on smaller bodies and some of their perihelia grew as a result, so their orbits were decoupled from the perturbations of the giant planets and formed a long-lived detached disk. Here, we investigate this scenario through direct N-body simulations and make predictions for the properties of the detached disk and the masses, numbers, and orbits of terrestrial-mass planets that might survive, at semimajor axes of a few hundred au. These bodies are much smaller and closer to the Sun than the hypothetical Planet IX proposed by Batygin & Brown (2016) to account for asymmetries in the orbital distribution of distant Kuiper Belt Objects (KBOs).

2. Initial Conditions

We report on 10 sets of N-body simulations containing the Sun and the 4 giant planets, as well as several embryos (0.05–2 M_⊕) initialized on nearly circular and co-planar orbits within the orbits of the giant planets. The inner planets are ignored. The giant planets are initialized at their current

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3 https://ssd.jpl.nasa.gov/sbdb_query.cgi
semimajor axes but with smaller inclinations and eccentricities (see below for details). The parameters of these simulations are summarized in the first five columns of Table 1. In each case, we represented the embryos by \(N_{\text{eb}}\) massive extra bodies (MEBs) of mass \(M_{\text{eb}}\) placed on orbits between Jupiter and Neptune. The semimajor axes of the MEBs satisfy the relation

\[
a_{i+1} = a_i + \Delta a_i^{0.5}, \quad i = 0, ..., \tag{1}
\]

where \(a_i\) is the semimajor axis of the \(i\)th extra body. If \(a_{i+1}\) lies within two Hill radii of any of the four giant planets, we assume the orbit is unstable, and we do not put a planet there. We choose \(a_0 = 5.91\) au (two Hill radii outside Jupiter’s orbit) and choose the largest value of \(\Delta\) such that the most distant body would still be within the orbit of Neptune (30 au) given that there are \(N_{\text{eb}}\) total bodies. Apart from the gaps at the giant planets, this spacing corresponds to a surface density proportional to \(R^{-1.5}\) as in Hayashi’s (1981) minimum-mass solar nebula. We ran 30 simulations in each set (except for a set of 10 simulations where we set the MEB mass to zero), differing only in the random number seeds.

Each simulation also contains a set of 50 test particles on orbits with semimajor axes 5–50 au, with radial distribution corresponding to a \(R^{-1.5}\) surface density profile.

The initial inclinations and eccentricities of all bodies, including the giant planets, are drawn from Rayleigh distributions with \(\sigma_i = 0.001\) radians (=0°057) and \(\sigma_e = 0.002\), having probability density

\[
\rho(i) = \frac{i}{\sigma_i^2} \exp\left(-\frac{i^2}{2\sigma_i^2}\right),
\]

\[
\rho(e) = \frac{e}{\sigma_e^2} \exp\left(-\frac{e^2}{2\sigma_e^2}\right) \tag{2}
\]

We integrated the orbits for using the IAS15 package, developed by Rein & Liu (2012). Within this package, we used the IAS15 integrator (Rein & Spiegel 2015), which is designed to handle close encounters and highly eccentric orbits.

We did not include the effects of Galactic tides, tides from the Sun’s birth cluster, or passing stars. We neglect Galactic tides and passing stars because we expect these to not have a significant effect on the few hundred au scales of interest. For example, the torque from the Galactic tide would need \(\approx 10^{12}\) years to impart enough angular momentum to convert a radial orbit to a circular one at 200 au. We do not include cluster tides even though they may well have affected the evolution of the outer solar system (Fernández & Brunini 2000), as their magnitude and the time over which they act is uncertain. In any case, the goal of this paper is to focus on the effects of mutual interactions between the MEBs rather than the effects of cluster tides.

By starting the outer planets at their current semimajor axes, we have neglected migration of their orbits due to gravitational interactions with the gaseous protoplanetary disk or planetesimal disk. Neglecting the gaseous disk is justified because it probably disappeared within a few Myr of the formation of the solar system, whereas the characteristic evolution timescale in our simulations is much longer, \(\approx 100\) Myr (see Figure 5). Neglecting the planetesimal disk is more serious, because there is strong indirect evidence for migration, such as the 3:2 mean-motion resonance between Neptune and Pluto (Malhotra 1993). However, the migration history of the outer planets is uncertain, and given the preliminary nature and computational expense of this investigation, we did not investigate a variety of migration models.

We removed bodies from the simulation if they went outside a box with side length \(5 \times 10^4\) au centered on the Sun. Outside this box, our neglect of the Galactic tides is not accurate. If two particles collide, they are assumed to merge completely. In determining whether a collision occurred, we used the current radii of the four giant planets. We also removed any particle that came within 0.1 au of the Sun. Integrations were run for 4.5 Gyr, or until all of the MEBs and test particles, or one of the giant planets, were removed from the simulation.

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**Table 1**

| Sim set | \(N_{\text{sim}}\) | \(N_{\text{eb}}\) | \(M_{\text{eff}}(M_\odot)\) | \(M_{\text{eff}}(M_\odot)\) | \(f_i\) | \(\langle r \rangle\) | \(\langle N_{\text{remain}}\rangle\) | \(\langle f \rangle\) | DD with MEB | DD no MEB | Kuiper Belt | Kuiper rms Inc. |
|--------|----------------|----------------|-------------------|-------------------|--------|--------------|-----------------|------------|-------------|-------------|-------------|----------------|
| 1      | 30             | 10             | 2.0               | 6.3               | 0.67   | 4.5%         | 0.5             | 0.19       | 0.5%        | 0.5%        | 0.004%      | N/A          |
| 2      | 30             | 20             | 1.0               | 4.5               | 0.87   | 2.5%         | 0.5             | 0.26       | 0.5%        | 2.0%        | 0.05%       | 29°          |
| 3      | 30             | 40             | 0.5               | 3.2               | 0.97   | 3.2%         | 1.3             | 0.18       | 1.8%        | 0.9%        | 0.09%       | 23°          |
| 4      | 30             | 10             | 1.0               | 3.2               | 0.80   | 3.3%         | 0.3             | 0.17       | 1.0%        | 2.5%        | 0.22%       | 29°          |
| 5      | 30             | 20             | 0.5               | 2.2               | 1.0    | 3.3%         | 0.7             | 0.13       | 1.9%        | 0.8%        | 0.14%       | 23°          |
| 6      | 30             | 40             | 0.25              | 1.6               | 1.0    | 2.4%         | 1.0             | 0.12       | 1.0%        | 2.0%        | 0.12%       | 25°          |
| 7      | 30             | 20             | 0.25              | 1.1               | 1.0    | 1.2%         | 0.2             | 0.27       | 1.4%        | 0.3%        | 0.42%       | 19°          |
| 8      | 30             | 40             | 0.1               | 0.6               | 1.0    | 1.1%         | 0.4             | 0.22       | 0.2%        | 0.4%        | 1.4%        | 12°          |
| 9      | 30             | 40             | 0.05              | 0.3               | 1.0    | 1.6%         | 0.6             | 0.23       | 0.0%        | 0.0%        | 0.0%        | 4.3%         |
| 10     | 10             | 40             | 0.0               | 0.0               | 0.0    | 34.0%        | 13.6            | 1.0%       | 0.0%        | N/A         | 21%         | 0.8°         |

**Note.** Column 2: number of simulations in this set. Column 3: number of massive extra bodies (MEBs). Column 4: mass of each MEB. Column 5: \(M_{\text{eff}} \equiv M_{\text{eb}} \sqrt{N_{\text{eb}}}\) is proportional to the magnitude of the torques experienced by the MEBs if they are randomly distributed. Column 6: fraction of systems in which all four giant planets would be expected to escape dynamical detection and be fainter than a \(V\)-band magnitude of 19 (see Section 5 for details). Column 7: fraction of test particles that end up in the detached disk \((q > 38 \text{ au and } 80 \text{ au} < a < 500 \text{ au})\) for simulations with at least one remaining MEB. Column 8: fraction of test particles that end up in the Kuiper Belt \((30 \text{ au} < R < 50 \text{ au})\). Column 13: rms inclination of particles in Kuiper Belt \((30 \text{ au} < R < 50 \text{ au})\).
Simulation sets 1–3 and 4–6 have total masses of $\mathcal{M}_2$ and $\mathcal{M}_0$ respectively in MEBs, with varying masses for the individual MEBs between 0.25 and 2 $\mathcal{M}_0$. Simulation sets 7–9 explore smaller total masses of MEBs. Set 10 is a control set where the mass of the MEBs is set to zero. We also ran more simulations to explore parameter space; these informed our conclusions but will not be reported on explicitly.

### 3. Simulation Results

When reporting orbital elements, we use Jacobi coordinates. Inclinations are reported relative to the fixed reference plane near which the bodies were initialized.

Figure 1 shows the interdecile range (10th to 90th percentile) of $\sqrt{\epsilon_N^2 + i_N^2}$, where $\epsilon_N$ and $i_N$ are the eccentricity and inclination of Neptune, respectively as a function of the effective mass of the MEB population, given by

$$M_{\text{eff}} = M_{\text{eb}} \sqrt{\mathcal{N}_{\text{eb}}}.$$  \hspace{1cm} (3)

If the MEBs are randomly distributed azimuthally, then $M_{\text{eff}}$ is proportional to the net torque that the MEBs exert on each other and smaller planetesimals. The figure shows that $M_{\text{eff}}$ is a good predictor of the eccentricity and inclination excitation of Neptune. Similar results apply to Uranus.

All but the smallest values of $M_{\text{eff}}$ considered in our simulations excite the eccentricity and inclination of Neptune to values that exceed those in the real solar system. It seems we require $M_{\text{eff}} \lesssim M_\oplus$ in order for the eccentricity and inclination of Neptune to be roughly compatible with the current observations. The constraints from Uranus are much weaker due to its higher current eccentricity of 0.046. Only simulations 8 and 9 have the current value of $\sqrt{\epsilon_N^2 + i_N^2}$ within their interdecile range, but the current value of $\sqrt{\epsilon_N^2 + i_N^2}$ is not far outside the range for simulations 6, 7, and 10. Subsequent damping of the eccentricities and inclinations (see Section 6) could relax this limit.

Column 6 of Table 1 shows the fraction of systems that survive in each simulation set. We define a simulation as “surviving” if all four giant planets remain on bound orbits at the end of the simulation. A significant fraction of simulations in the sets with the highest values of $M_{\text{eff}}$ did not survive. As Figure 1 indicates that the degree of eccentricity and inclination excitation is roughly proportional to $M_{\text{eff}}$, we conclude that if $M_{\text{eff}} \gtrsim 5 M_\oplus$, there is a significant chance that one of the giant planets will be ejected or collide with the Sun or another planet (unless the eccentricities and inclinations are subsequently damped, see Section 6). In the figures and discussion that follow, we only report results from surviving simulations.

Figure 2 is a scatter plot of perihelion versus semimajor axis for MEBs remaining in the solar system. Points are color-coded according to the value of $M_{\text{eff}}$ for the corresponding simulation. The black squares correspond to our control simulation with massless extra bodies (MEBs) if they are randomly distributed, is a good predictor of the typical value of $\sqrt{\epsilon_N^2 + i_N^2}$.

![Figure 1. Excitation of Neptune’s eccentricity and inclination ($\sqrt{\epsilon_N^2 + i_N^2}$) vs. $M_{\text{eff}} = M_{\text{eb}} \sqrt{\mathcal{N}_{\text{eb}}}$]{figure1.png}
$q = 38$ au is roughly the distance outside which non-resonant perturbations from Neptune are unimportant over the age of the solar system (Gladman et al. 2002). It is expected that after several Gyr, there will be few non-resonant bodies inside this line, as most would have been ejected by Neptune. Finally, we have drawn a line where the aphelion $Q$ is equal to 140 au. This is a rough lower limit to the distance at which an MEB of $1 M_\odot$ could have escaped detection (see Section 5).

We see that most of the remaining MEBs are on orbits with perihelia less than 80 au and semimajor axes less than a few hundred au. In contrast, the test particles in the model of Brasser et al. (2012), where the torque was applied by tides and stellar flybys in the birth cluster, typically have semimajor axes greater than 1000 au.

Figure 3 is a scatter plot of orbital inclination against semimajor axis. There are no remaining MEBs with inclination greater than 60°. The median inclinations of the surviving MEBs for our different simulation sets (not including set 10) range from 7° to 18°, although much of this scatter is due to small-number statistics. This is very different from the inclination distribution of detached objects that are formed in models involving cluster tides and flybys (Brasser et al. 2012) — even in the innermost part of their cloud, the median inclination is between 45° and 55°. Thus, the inclination distribution in the detached disk provides a clean test to distinguish our model from cluster tides.

Figure 4 shows the density of the MEBs in semimajor axis, eccentricity, inclination, and perihelion. The curves for semimajor axis and perihelion were smoothed with a log-normal kernel, with a dispersion of 0.2 in $\ln a$ or $\ln q$. To determine the curves for eccentricity, we smoothed the eccentricity vector with a two-dimensional Gaussian with $\sigma_e = 0.1$, and then integrated the resulting probability distribution over angle to recover the distribution of scalar eccentricity. This procedure ensures that $\rho(e) \sim e$ for small values of $e$ — consistent with a non-singular distribution in phase space around $e = 0$. We applied the same method to the inclination using $\sigma_i = 5°$. Median eccentricities range from 0.19 to 0.40 (excluding simulation set 10), although as can be seen from the figure, the distributions are quite broad. Values of eccentricity near unity are uncommon because particles are mostly limited to $<< q_{\text{crit}}$, where $q_{\text{crit}} \approx 38$ au is the perihelion below which a planet will be ejected by perturbations from Neptune. Systems with higher values of $M_{\text{eff}}$ have higher mean values of the perihelion. This can be understood qualitatively as being due to the larger typical torques experienced by particles in these simulations.

Figure 5 shows the evolution of the number of MEBs over time. The top panel shows the total fraction of remaining MEBs. Most of them are removed over $\sim 10^7$ year timescales. In contrast, about one-third of the MEBs survive for the duration of the simulation if their masses are negligible (black line, simulation set 10). This difference arises because the stability of test particles on near-circular orbits in the region of the giant planets (5–30 au) depends strongly on the eccentricities of the planets. In our simulations, the planets start on nearly circular orbits (Equation (2)) but MEBs with non-zero
masses rapidly excite the eccentricities, while those with zero masses do not. The bottom panel shows the fraction of MEBs on orbits with $q > 38$ au. This number grows for a few times $10^8$ years and slowly declines thereafter. As expected, there are no bodies with $q > 38$ au if the MEBs have zero mass (i.e., there is no black line in the bottom panel of Figure 5). The approximate timescale for the creation of bodies with perihelion greater than 38 au can be derived simply. Assuming a typical specific torque of $GM_{\text{eff}}/\langle R \rangle$, where $\langle R \rangle$ is the average orbital separation, then the timescale to produce orbits with perihelion greater than $q_{\text{crit}}$ can be approximated as

$$\tau = \frac{\sqrt{2GM_{\text{eff}}q_{\text{crit}}}}{GM_{\text{eff}}/\langle R \rangle} = 7 \times 10^7 \frac{M_{\text{eff}}}{M_\oplus} \langle R \rangle^{-1} \text{ years},$$

where we have used $q_{\text{crit}} = 38$ au in the numerical estimate. Equation (4) agrees fairly well with the results shown in Figure 5. In particular, higher values of $M_{\text{eff}}$ lead to more rapid emplacement of bodies in large-perihelion orbits. On the other hand, after a group of MEBs is placed on large-perihelion orbits, higher values of $M_{\text{eff}}$ also lead to more rapid depletion over the remaining age of the solar system. Because the MEB population evolves on such long timescales, we expect the results of this model to not depend strongly on the details of the initial conditions and early evolution of the solar system.

Figure 3 shows a histogram of the number of remaining MEBs with $a > 38$ au in the different runs of each simulation set. A larger fraction of the MEBs remains to the end of the simulations in systems with intermediate-mass MEBs. There are few systems with more than one remaining MEB in which $M_{\text{eb}}$ is greater than $0.5 M_\oplus$, presumably because the mutual torques between MEBs of larger masses are sufficiently strong to lower the perihelia to the point that MEBs are ejected by one of the giant planets, until only one MEB is left.

To summarize, our simulations show that a population of MEBs with effective mass $M_{\text{eff}} = \sqrt{N_{\text{eb}}M_{\text{eb}}}$ exceeding $\sim 1 M_\oplus$ will excite the eccentricities and inclinations of Uranus and Neptune to values larger than observed—thus either there has been subsequent damping, or $M_{\text{eff}} < M_\oplus$. In the latter case, for $M_{\text{eb}} \gtrsim 0.05 M_\oplus$, we find a probability of up to $\sim 50\%$ that one or occasionally more MEBs survive in bound orbits until the present time. Their orbits typically have perihelia of 40–70 au, semimajor axes less than 200 au, and inclinations $\lesssim 30\degree$.

4. Fate of the Test-particle Population

As described in Section 2, in each set of simulations we also included 50 test particles that were originally in dynamically cold orbits (initial eccentricity and inclination less than 1%) between 5 and 50 au, with surface density $\propto R^{-1.5}$. These are included to monitor the effect of the MEBs on the Kuiper Belt and to study the efficiency of injection of particles into the detached disk (defined in this paper as containing any body with perihelion greater than 38 au and semimajor axis between 80 and 500 au) and the Oort comet cloud. Gladman & Chan (2006) showed that massive bodies exterior to Neptune can lift test particles to large perihelion distances, thus providing a natural mechanism to create a disk of detached objects such as Sedna (Brown et al. 2004) or 2012VP113 (Trujillo & Sheppard 2014). We make predictions for the mass and orbital distribution of a detached disk produced in this way. Many of our results are similar to those of Gladman & Chan (2006).
The Kuiper Belt exhibits a rich dynamical structure. In addition to the detached disk, there are Plutinos in the 3:2 resonance with Neptune, objects in other mean-motion resonances (1:2, 3:5, 4:7, etc.), a “scattered disk” consisting of objects with perihelion \( q \lesssim 38 \) au and eccentricity \( e \gtrsim 0.2 \), and “classical” KBOs, which have low eccentricities and semimajor axes in the range 40–50 au. The classical population is generally divided into “cold” and “hot” components, with rms inclinations of about 3° and 10–20°, each containing about half of the total population (Brown 2001; Gulbis et al. 2010). Because our simulations have far fewer test particles than the number of known KBOs, we cannot compare our model predictions to the detailed structure of the Kuiper Belt. Thus, we restrict ourselves to a few brief, crude comparisons.

Figure 7 shows scatter plots of perihelion and inclination versus semimajor axis for all test particles still present at the end of our simulations. We divide the results into top and bottom panels based on whether any of the MEBs are still present at the end of the simulation to see if there are clues in the distributions as to whether or not the solar system still has an undiscovered MEB. As expected, almost all of the test particles with perihelion \( q \lesssim 38 \) au have been ejected. The orbits of the remaining test particles are similar to those of the remaining MEBs (Figure 2) and seem not to depend strongly on whether any of the MEBs remain. The inclination distribution of the test particles is slightly broader than that of the MEBs. The inclination and semimajor axis distributions are consistent with observations of the detached disk. Our results also imply that few bodies will be found in the detached disk with semimajor axes \( \gtrsim 500 \) au.

Figure 4. Smoothed estimates of the density of the MEB population in semimajor axis, eccentricity, inclination, and perihelion. The line color corresponds to \( M_{\text{eff}} \), and the legend refers to the simulation numbers in column 1 of Table 1. The drop of all of the curves \( \rho(e), \rho(i) \) to zero at \( e = 0 \) and \( i = 0 \) is due to our smoothing kernel and is expected for any smooth distribution in phase space (see the text). The semimajor axis distributions peak between 50 and 150 au. Higher values of \( M_{\text{eff}} \) lead to higher mean values of the perihelion \( q \).

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Column 10 in Table 1 shows the fraction of test particles that end up in the detached disk for simulations in which at least one MEB remains. Column 11 shows this same fraction but for simulations in which no MEBs remain. We find typically that around 1% of the initial population is transferred to the detached disk, with a variation of a factor of two or so among the simulations.

For comparison, the efficiency of transferring planetesimals to the Oort cloud is a few per cent (Kaib & Quinn 2008; Brasser et al. 2010; Dones et al. 2015; Shannon et al. 2015), with the precise number depending on the initial semimajor axis distribution of the planetesimals, the migration history of the outer planets, the definition of the minimum semimajor axis of the Oort cloud, and the history of the Sun’s environment. In our model, the transfer efficiency to the detached disk is about 1%, so we predict that the detached disk should be a factor of a few less massive than the Oort cloud. The mass of the Oort cloud is estimated to be \( \sim 3 M_{\oplus} \), with large uncertainties (e.g., Morbidelli 2008), so we arrive at an estimate of \( \sim 1 M_{\oplus} \) for the scattered disk. This estimate may be
unrealistically large because of the well-known problem that the estimates of Oort cloud formation efficiency and mass used in this estimate also imply an unrealistically massive initial planetesimal disk (e.g., Morbidelli & Levison 2004).

We may also directly estimate the mass in the detached disk predicted by our model. The test particles in our simulations are distributed with surface density proportional to $R^{-1.5}$ from 5 to 50 au (Hayashi 1981). Following Hahn & Malhotra (1999) and Gomes et al. (2004), we assume that about $40 M_{\oplus}$ of solid planetesimals were left in the disk between 20 and 50 au after formation of the planets (larger masses cause Neptune to migrate rapidly to the outer edge of the disk, rather than stopping at its current position). With this assumption, the total mass of our test-particle disk is $\sim 70 M_{\oplus}$. If 1% of the test particles are transferred to the detached disk, then the detached disk should contain $\sim 0.5-1 M_{\oplus}$. For comparison, Brown et al. (2004) make a rough estimate that there is $\sim 5 M_{\oplus}$ in bodies with Sedna-like orbits. This estimate assumed that the Sedna-like population was isotropically distributed, so for the flattened inclination distribution observed in the detached disk, a better estimate is probably $\sim 1 M_{\oplus}$. This result was based on only one body (Sedna), so it is subject to considerable uncertainty. Gladman et al. (2002) estimate that there are $10^6$ bodies with diameter greater than 100 km in the detached disk. This corresponds to a minimum mass of $0.2 M_{\oplus}$, assuming a density of $2 \text{ g cm}^{-3}$. For a plausible size distribution, the actual mass could be larger by a factor of five or more. We conclude that the mass of the detached disk estimated by our model is consistent with the mass estimated by observers, although with substantial uncertainties in both approaches.

Morbidelli & Valsecchi (1997) and Petit et al. (1999) suggest that gravitational interactions with large planetesimals scattered outward by the giant planets could explain why the Kuiper Belt is much less massive and more excited than it is thought to have been primordially. This suggestion is qualitatively consistent with our finding that the MEBs remove most of the test particles in our simulations. We now make a more quantitative comparison. Gladman et al. (2001) estimate a current mass of $0.1 M_{\oplus}$ for the Kuiper Belt, which they define to include all bodies (except for Neptune) in the distance range 30–50 au from the Sun. If we assume, as we did above, that the planetesimal population between 5 and 50 au had a mass of $70 M_{\oplus}$, then about 0.14% of this population is found in today’s Kuiper Belt. To compare this result to our simulations, we added the fractional times that each surviving test particle spent between 30 and 50 au and divided this sum by the total initial number of test particles. The results are shown in Column 12 of Table 1. By this metric, it appears that simulations 2–7 do a reasonable job of reproducing the 0.14% target.4

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4 We also considered a more restricted definition of the Kuiper Belt that only includes particles with semimajor axes between 30 and 60 au and weights them by the fraction of the time they spend between 30 and 50 au. This reduces contamination of the “Kuiper Belt” by objects in the scattered or detached disks. This definition causes modest reductions in many of the Kuiper Belt populations but does not change the qualitative picture. In this definition, fractions (0%, 0.02%, 0.02%, 0.1%, 0.1%, 0.05%, 0.4%, 1.3%, 4.3%, 21%) survive (cf. column 12 of Table 1) for simulations 1–10. That being said, in most cases we only have 1500 test particles per simulation set, so results smaller than $\sim 0.5\%$ are uncertain because of small-number statistics.

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**Figure 5.** The top panel shows the fraction of remaining MEBs as a function of time. The bottom panel shows the mean fraction of MEBs in the detached disk (perihelion greater than 38 au and aphelion in the range 80 au $\leq a \leq 500$ au) as a function of time. The detached disk forms over a few times $10^8$ years and decays slowly afterwards.
An important concern is whether or not our model can reproduce the division of the classical Kuiper Belt into cold and hot components. It is commonly argued (e.g., Shannon et al. 2016) that the cold component formed in situ between $40 - 50$ au and has remained undisturbed since then, and this view is difficult to reconcile with a late stage of inclination excitation by MEBs. We note that the division into distinct cold and hot components is somewhat artificial, as it relies on the assumption that this inclination distribution of each component must have the Rayleigh form $\mu + 2 \sigma^2/3$. This assumption is often justified by the central limit theorem, but this theorem applies only if the inclination excitation occurs through a large number of weak perturbations. In contrast, if the excitation is dominated by a few strong perturbations, the distribution will have fatter tails than a Gaussian. For example, Collins et al. (2007) show that the eccentricity distribution excited by shear-dominated encounters has the non-Rayleigh form $dN \propto e^{-e_c^2 + e^2}/e^{3/2}$, where $e_c(t)$ is a time-dependent scaling factor. Similarly, the inclination distribution excited by MEBs may be dominated by a few close encounters and could perhaps produce the observed shape of the inclination distribution from a single population. Testing this hypothesis would require dedicated simulations in which the Kuiper Belt is represented by a much larger test-particle population than we have in our runs. Existing simulations of the effects of MEBs on the Kuiper Belt (Fernández 1980; Muñoz-Gutiérrez et al. 2015) are mostly for MEBs on low-eccentricity, low-inclination orbits and hence are not directly applicable.

Our simulations contain an insufficient number of remaining Kuiper Belt particles for us to determine the functional form of the inclination distribution. Column 13 of Table 1 shows the rms inclinations of the remaining particles with $a > 30$ au, weighted by the time the particle spends interior to 60 au. Unless $M_{\text{eff}} \lesssim M_m$, the particles have somewhat higher inclination than even the hot component in the parameterizations of the inclination distribution of the classical Kuiper Belt given in Brown (2001) and Gulbis et al. (2010).

Related issues are (i) the presence of correlations in the classical Kuiper Belt population between inclination and physical properties—color (Tegler & Romanishin 2000; Trujillo & Brown 2002; Peixinho et al. 2008; Terai et al. 2017), size (Fraser et al. 2014), albedo (Brucker et al. 2009), and binarity (Noll et al. 2008). These could reflect either different physical properties in the cold and hot populations or inclination-dependent
environmental effects on a single population; (ii) many KBOs are in mean-motion resonances with Neptune, and the number and mass of MEBs affects the fraction of resonant KBOs and their libration amplitudes (Nesvorný & Vokrouhlický 2016). To summarize, our simulations show that MEBs can excite planetesimals into orbits with typical perihelia of 40–80 au, semimajor axes as large as a few hundred au, and inclinations 0°–40°, consistent with the observed properties and (within large uncertainties) total mass of the observed detached disk. The MEBs may also be responsible for destroying most of the Kuiper Belt, which is much less massive than expected from extrapolating the solid content of the minimum solar nebula. These conclusions hold whether or not one or more MEBs is eventually found in the outer solar system, as the properties of the detached disk and Kuiper Belt are mostly independent of whether or not any MEBs survive. Our simulations do not have enough test particles to determine whether interactions with the MEBs produce a residual Kuiper Belt that is fully consistent with the observed dynamical structure of the Kuiper Belt.

5. Observational Constraints

MEBs can be detected dynamically or photometrically. We first consider dynamical detection. Fienga et al. (2016) find that a hypothetical planet of 10 $M_\oplus$ would be dynamically detectable, mostly from its perturbations to Saturn, if it were currently closer to the Sun than about 370 au. This is a model-dependent limit, as they were considering a planet on a particular orbit proposed by Batygin & Brown (2016), but is probably the best one available for our purposes. To good approximation, a planet at that distance only interacts with the known planets via a stationary tidal potential over the interval of modern observations. Thus, we may assume that the detectability limit scales as $M/R^3$, where $M$ is the hypothetical planet’s mass and $R$ its current distance. Therefore, an MEB

![Figure 7. Orbital properties of the test-particle population. The black squares represent known members of the detached disk ($q > 38$ au, $80$ au $< a < 500$ au); we did not plot observed objects with $a < 80$ au, as they are quite numerous and do not fall under our definition of detached disk objects. Otherwise, the point properties described in the legend are the same as for Figure 2, except that simulation 10 is not plotted. The top panels are for surviving runs in which at least one of the MEBs remains at the end of the simulation. The bottom panels are for surviving runs in which all of the MEBs were ejected. There is an observational selection bias that favors the discovery of objects with small semimajor axes and perihelia.](image-url)
should be dynamically detectable if its current heliocentric distance is less than about

\[ R_{\text{crit}} = 170 \left( \frac{M}{M_\odot} \right)^{1/3} \text{ au.} \]  

We next consider photometric detection. The most extensive systematic survey is the Palomar Distant Solar System Survey (Schwamb et al. 2010), which covered 30% of the sky to a limiting r-band magnitude of 21.3. Using serendipitous discoveries from the Catalina Sky Survey and the Siding Spring Survey, Brown et al. (2015) estimate that there is a 30% chance that the solar system contains an undetected KBO brighter than a V-band magnitude of 19. In an abstract, Holman et al. (2016) report that Pan-STARRS has completed a search for slow-moving objects brighter than an r-band magnitude of 22.5 over the entire sky north of \(-30^\circ\) in declination (75% of the sky). This survey area includes most, but not all, of our bodies. Assuming a geometric albedo of 0.04 and a constant density of 2.0 g cm\(^{-3}\) (appropriate for long-period comets; Lamy et al. 2004), an r-band limit of 19.0 (22.5) corresponds to a critical detection distance of 140 (313) \((M/M_\odot)^{1/6}\) au. This increases to 235 (526) \((M/M_\odot)^{1/6}\) au if we take the albedo to be 0.32, appropriate for Sedna (Pál et al. 2012). Thus, the Pan-STARRS survey has a good chance of detecting MEBs, if any are still present.

Column 9 in Table 1 shows the probability that none of the MEBs remaining at the end of a given simulation are either dynamically detectable or brighter than magnitude 19 assuming an albedo of 0.04. The probability is only calculated for simulations in which at least one MEB remains. To compute this probability, we use the smaller of 170 \((M/M_\odot)^{1/3}\) au (Equation (5)) and 140 \((M/M_\odot)^{1/6}\) au to calculate \(R_{\text{crit}}\) for each remaining MEB. Then, given the orbital parameters of the MEB at the end of the simulation, we calculated the fraction of the orbital period that would be spent beyond \(R_{\text{crit}}\). This is the probability that a given MEB would be undetected. Then, to find the probability that all of the planets are undetectable, we multiply all of the individual probabilities. This is not completely correct, even assuming the bodies’ phases to be uncorrelated, as their dynamical effect on the known planets could add constructively or destructively depending on their current locations. These probabilities are generally a few tens of per cent, so if MEBs formed this way were still present in the solar system, it is likely but far from certain that they would have been detected.

Volk & Malhotra (2017) find that there is a statistically significant (at the 97% level) warp in the mean plane of the Kuiper Belt by comparing the orbits of non-resonant bodies with semimajor axis in the range from 42 to 48 au to those with semimajor axes from 50 to 80 au. They comment that this warp could be caused by an unseen Mars-mass body orbiting at 65–80 au, consistent with the properties of the remaining MEBs in our simulations.

6. Damping

We found (Section 3) that the eccentricity and inclination of Neptune were excited above their observed values when \(M_{\text{eff}} \gtrsim M_\oplus\). This constraint on \(M_{\text{eff}}\) could be relaxed if the eccentricity and inclination were subsequently damped. Multiple groups (e.g., Kokubo & Ida 1995; Tsiganis et al. 2005) have found that the presence of many small bodies in a disk can damp the eccentricity of larger planetary-mass objects through dynamical friction. Directly modeling this process in our simulations over the lifetime of the solar system was impractical, even ignoring interactions between planetesimals. Instead, we experimented with implementing a drag force

\[ a_{r,i} = -\frac{v_{r,i}}{\tau_{\text{damp}}} - \frac{v_{r,i}}{\tau_{\text{damp}}}, \]  

where \(a_{r,i}\), \(v_{r,i}\) and \(a_{r,i}, v_{r,i}\) are the radial and vertical accelerations and velocities of the \(i\)th giant planet. We assumed the damping time \(\tau_{\text{damp}}\) to be given by

\[ \tau_{\text{damp}}(t) = \tau_1 \exp(t/\tau_2). \]  

We take \(\tau_1 = 10^5\) years and \(\tau_2 = 10^7\) years. \(\tau_1\) is consistent with the time over which small bodies are removed from the system (see Figure 5). The damping force does \textit{not} act upon the MEBs, as they are presumed to be insufficiently massive to be affected by dynamical friction with a planetesimal disk.

This formalism does produce some damping of the semimajor axes as well, but provided that the eccentricities and inclinations are much less than unity, one can show that under the influence of damping,

\[ \frac{d \ln(a)}{dt} = \frac{d}{dt} \left( e^2 + i^2 \right). \]  

Therefore, if the eccentricities and inclinations remain low, they are damped much faster than the semimajor axis.

We ran simulations analogous to our simulation sets 1–3 using the damping scheme discussed above and found little qualitative difference from the results presented in this paper. Even with \(\tau_1 = 10^5\) years, large MEBs \((M_{eb} \geq 2 M_\oplus)\) could still eject one or more of the giant planets. As can be seen in the second panel of Figure 5, the MEB population still interacts with the giant planets after more than \(10^5\) years, at which point one would expect most of the planetesimal disk to be gone and the damping to therefore be negligible.

7. Conclusions

It is highly likely that multiple “planetary embryos” or MEBs of up to a few Earth masses form among the giant planets. These are scattered outwards by gravitational perturbations from the giant planets. The MEBs exert torques on one another before they can be ejected from the solar system, thus increasing their perihelia beyond the gravitational reach of the giant planets. They can therefore remain in the solar system for 4.5 Gyr, outside the orbit of Neptune but far inside the Oort cloud. Additionally, these bodies exert torques on smaller planetesimals, thereby naturally creating a detached disk containing objects with orbits similar to Sedna and 2012VP\(_{13}\).

The evolution of the population of MEBs is characterized by their effective mass \(M_{\text{eff}} = M_{eb} \sqrt{N_{eb}}\). If \(M_{\text{eff}}\) exceeds about an Earth mass, then the eccentricity and inclination of Neptune are usually excited above the observed values, which rules out this range of parameter space unless some process subsequently damp them. However, significant effects occur for smaller values of the effective mass. For example, if 20 bodies of 0.25 Earth masses were present (Simulation 7; \(M_{\text{eff}} = 1.1 M_\oplus\)), then in 23% of cases, one MEB will remain in a moderate
inclination orbit with semimajor axis between 45 and 130 au, and on average, 0.5% of the mass in the planetesimal belt between 5 and 50 au is transferred to the detached disk. In many of our simulations with remaining extra bodies, at least one of them would be expected to have been detected either dynamically or photometrically, although this result depends strongly on studies done for other purposes or surveys that are still in progress. In a few tens of per cent of simulations with remaining MEBs, the bodies were remote enough to escape dynamical and photometric detection so far.

The extra bodies that we discuss in this paper are much smaller and closer than the Planet IX proposed by Batygin & Brown (2016): ~0.1–0.5 M⊕ compared to 5–20 M⊕ for Planet IX, and ≤200 au compared to 400–1000 au for Planet IX. The motivation for our model is also different from the motivation for Planet IX: ours is motivated by simple considerations arising from the standard model for the formation of the outer planets, while the Planet IX hypothesis is motivated by asymmetries in the orbital element distribution of a selected set of KBOs.

MEBs also transfer a few per cent of the mass of the initial planetesimal belt into a detached disk composed of bodies on moderately inclined orbits with perihelia greater than 38 au and semimajor axes between 80 and 500 au. Like the observed sample of 33 objects in our definition of the detached disk, the population of test particles in our simulations are on moderately inclined orbits. The amount of material in the detached disk does not depend strongly on whether any of the MEBs remain in the system; thus, the viability of this mechanism for producing the detached disk does not require the discovery of new terrestrial-mass planets in the outer solar system.

The original motivation for this model was to explain the properties of the detached disk, which is not predicted by the standard model of the formation and evolution of the Oort comet cloud. There are other tensions between observations and the standard model. These include a predicted mass for the Oort cloud that is too low given the likely mass in the planetesimal disk, a predicted mass for the Oort cloud of comets that is too high, and a predicted inclination distribution for long-period comets with too many retrograde orbits (e.g., Wiegert & Tremaine 1999; Morbidelli 2008). It is possible that the presence of MEBs in the outer planetesimal disk will alleviate or resolve some of these tensions, but studying this possibility is beyond the ambition of the current paper.

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