A Study of the Duration of the Passage through the Van Allen Belts for a Spacecraft going to the Moon

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Abstract. This paper has the goal of estimating the fuel consumption and the duration of the transit in the Van Allen belts for a flight of a spacecraft going from the Earth to the Moon. This problem is very important because the region interior to the belts have a high density of energetic charged particles that can damage the satellite, so minimizing this transit time helps in protecting the equipments on board. The propulsive force is assumed to have a low magnitude and to be applied in the direction of the motion of the spacecraft to maximize the energy transferred to the space vehicle. Perturbation forces are considered in the dynamical model and they influence in both results, consumption and transit time.

1. Introduction

This paper studies maneuvers for a spacecraft going from the Earth to the Moon, in particular making estimations of the time of flight that the spacecraft stays inside the Van Allen belts and the fuel consumption to perform the maneuvers. This research is a first approach to study this problem, which depends on the propulsion system available.

The existence of the Van Allen belts was first proposed by James Van Allen, in 1958, and after that satellites launched in the same year confirmed their existence [1]. The Van Allen belts can be described as magnetic mirrors. The magnetic field of the Earth, with strong intensity on the poles and weak intensity in the equator, forms a natural magnetic mirror that confines high energetic electrons and ions [2].

There is a great importance in studying these belts, since the confined energetic particles can damage the components of the satellites and also humans travelling in space. In this way, it is important for missions that pass through these belts the minimization of the radiation exposure and the study of how much radiation the satellite can receive. In this way, the search for trajectories that minimizes the time that the satellite stays inside these belts is an important part of the mission design.

The Van Allen belts are not yet fully understood and there are still today new discoveries related to them. Two identical spacecrafts were launched in August 2012, named the Van Allen Probes, with the objective of studying the Van Allen belts. These probes have made some important discoveries about the Van Allen Belts and scientists hope they will be able to make new discoveries to understand it better in the near future [3].
2. The Spacecraft Trajectory Simulator

In order to study these maneuvers, the Spacecraft Trajectory Simulator was used. This simulator is a complex package that was developed to help mission designers to plan space missions. It is capable of simulating orbital maneuvers in a realistic environment, considering the perturbations of the Sun, the Moon, the geopotential and also the radiation pressure. It is also possible to consider errors, delays and failures in the sensors and actuators. This simulator considers a satellite in a closed-loop system with Proportional-Integral-Derivative (PID) controller. The greatest advantage of using this package in the present study is the possibility of simulating the maneuvers of the spacecraft going to the Moon in this realistic environment [4]. The spacecraft starts its motion in a circular orbit around the Earth and the goal is to send it to an orbit around the Moon. In the phase of the mission studied here, the final constraint is not that the satellite enters in orbit around the Moon or that it passes by the Moon. The constraint is just that the orbit of the spacecraft has an apogee at the altitude of the orbit of the Moon, to make an encounter possible. The perturbations considered were the\( J_2\), \( J_3\) and \( J_4\) terms of the geopotential, the third-body perturbation due to the Sun and the Moon and the solar radiation pressure. The spacecraft used in the simulations has a mass of 150 kg and the area of the object exposed to the solar radiation is 1 m\(^2\). The initial orbit has the following orbital parameters: semi-major axis = 7178 km; eccentricity = 0; inclination = 0, 45 and 90 degrees, right ascension of the ascending node = 0 degrees; mean anomaly = 0 degrees. The initial date and time of the simulation is 01/01/2010 at 05h30m00s. The reason to study different values for the inclination of the orbit of the spacecraft is that this parameter has a large impact in the transit time by the Van Allen belts, since the belts are confined in the latitudes of 65 degrees north and south, so inclinations larger than 65 degrees will have the advantage of having part of its trajectory outside the belts.

3. The Propulsion System

The propulsion systems considered in this paper were two different variations of the hall thrusters that are being under development at the UnB (Universidade de Brasilia) [5]. A Hall thruster is a kind of ion thruster that uses the electric field to accelerate the propellant. The propellant itself is composed by ions that are accelerated to high speeds. More information about the working principles of the hall thrusters can be found in reference [5]. The two types of hall thrusters used in the simulations were the PHALL I and PHALL II. These thrusters have different structures and parameters. Although this propulsion system was not used yet for real missions, many papers studying the efficiency and optimal maneuvers was already developed, like shown in reference [6], where it is studied optimal maneuvers with minimum fuel consumption with the use of these PHALLs in order to achieve a final given orbit. The parameters of both propulsion systems are given bellow [5], in Table 1.

|                | PHALL I | PHALL II |
|----------------|---------|----------|
| Average Thrust (mN) | 120.0   | 84.9     |
| Specific Impulse (s)     | 1600    | 1083     |

4. The Van Allen belts

The model of the Van Allen belts used in the present research is based in reference [7]. They are composed by two belts, the inner and the outer one. Both of them extend to 65 degrees of latitude in both north and south directions. The inner belt goes from 1000 km to 6000 km of altitude, while the outer belt extends from 13000 km to 60000 km of altitude. Figure 1 shows a sketch with their main characteristics.
5. Results

The results presented here are based on a maneuver that sends the spacecraft to an orbit with apogee of 384,400 km. The parameters of the initial orbit are given in section 2, the propulsion system is shown in section 3 and the direction of the propulsion applied to the spacecraft is tangential. The fuel consumption and the time spend in the Van Allen belt is shown in Table 1.

**Table 1** – Results for different maneuvers

| Propulsion System | Thrust capacity (N) | Inclination of the orbit (degrees) | Time spend in the Van Allen belts (days) | Fuel Consumption (Kg) |
|-------------------|---------------------|-----------------------------------|----------------------------------------|----------------------|
| Phall I (8 thrusters) | 1 N | 0 | 5.50 | 46.39 |
| Phall I (8 thrusters) | 1 N | 45 | 5.63 | 47.33 |
| Phall I (8 thrusters) | 1 N | 90 | 3.96 | 47.21 |
| Phall I (4 thrusters) | 0.5 N | 0 | 10.96 | 47.23 |
| Phall I (4 thrusters) | 0.5 N | 45 | 11.04 | 48.20 |
| Phall I (4 thrusters) | 0.5 N | 90 | 8.09 | 48.06 |
| Phall II (8 thrusters) | 0.68 N | 0 | 7.59 | 63.52 |
| Phall II (8 thrusters) | 0.68 N | 45 | 7.61 | 64.7 |
| Phall II (8 thrusters) | 0.68 N | 90 | 4.42 | 64.52 |
| Phall II (4 thrusters) | 0.34 N | 0 | 15.15 | 64.47 |
| Phall II (4 thrusters) | 0.34 N | 45 | 15.18 | 65.65 |
| Phall II (4 thrusters) | 0.34 N | 90 | 10.90 | 65.50 |

From Table 1 it is possible to see several aspects of each maneuver. The maneuvers with inclination equal to 45 degrees were the worst maneuvers to consider for the mission. They spend more time in the Van Allen belts, due to the geometry of this orbit. The fuel consumption is also larger, if compared to the other maneuvers. The reason for this larger fuel consumption is the effects

![Fig. 1 - Geometry of the Van Allen belts.](image-url)
of the perturbation due to $J_2$, which is considered during the propelled phase of the maneuver. This perturbation dominates the system for a long time of the maneuver and it is stronger for orbits with 45 degrees of inclination, been smaller for equatorial orbits. Maneuvers with zero degrees of inclination (equatorial orbits) were the second ones in terms of the time spend on the Van Allen belts, but the fuel consumption were smaller than the others. For maneuvers with 90 degrees of inclination, the times spend on the Van Allen belts were minimum. This occurs because of the symmetry of the belts, which extends to 65 degrees north and south in latitude. The amount of fuel consumption for these maneuvers is between the values of the others.

Phall I is more efficient than Phal II, because of the higher specific impulse and also the higher magnitude of the thrust. The times spend in the Van Allen belts when using Phall I were smaller with respect to the same maneuvers made using Phall II, and also the fuel consumption were smaller. The largest time obtained in the Van Allen belts were slightly more than 15 days, and it occurred for the maneuver realized with 4 Phall II thrusters and considering an inclination equal to 45 degrees.

Next, it is shown some figures for the maneuver realized with 8 Phall I thrusters, shown in the first line of Table I. The inclination of this maneuver is zero, so a planar view of the trajectory is shown here, in an inertial reference frame with the Earth in the center and the reference plane passing by the equator.

Fig. 2 – Maneuver realized with 8 Phall I thrusters as the propulsion system.
From Figure 2 it is possible to see the orbit of the satellite gaining velocity until it achieves the final orbital condition. Figure 3 shows the thrust applied in each axis of the inertial frame and the total thrust applied, which in this case is one Newton. This figure shows how the control system works to perform the maneuver, by specifying each component of the thrust force applied to the spacecraft, so given the solution of the problem in terms of the force vector. There is no component of the force in the z-axis, since the maneuver is equatorial. Figure 4 shows the fuel consumption as a function of the time. It is possible to note that the fuel consumption increases linearly as the maneuver goes by.

![Thrust applied as a function of the time.](image1)

**Fig. 3** – Thrust applied as a function of the time.

![Fuel consumption as a function of the time.](image2)

**Fig. 4** – Fuel consumption as a function of the time.

### 6. Conclusions

This work studied the fuel consumption of maneuvers that send a spacecraft from a low altitude orbit around the Earth to an orbit that has apogee in the orbit of the Moon, as well as the time the spacecraft spend in the Van Allen Belts, for different kinds of propulsion systems.

The propulsion systems considered were two hall thrusters available under study at the Universidade de Brasilia. The study of orbital maneuvers considered several perturbation forces in the
dynamical model, such that the $J_2$, $J_3$ and $J_4$ terms of the geopotential, the third-body perturbation due to the Sun and the Moon and the solar radiation pressure.

The model proposed for the Van Allen belts is simple, but provides a good estimative for the purposes of the present study. The belts are constantly changing and the density of the ionized particles varies constantly. In this way, the approach used here was to estimate an area for the Van Allen belts based on the average of the real data available.

The orbits with inclination equal to 90 degrees were the ones where the spacecraft spends less time inside the belts, because the geometry of these orbits makes the spacecraft to pass by latitudes below and above the limits of the belts, 65 degrees, so reducing the exposure of the spacecraft to the radiation. The 45 degrees of inclination and the equatorial orbits (zero inclination) have very similar results, since they both stay all the time inside the limits of the belts. The equatorial orbits implies in slight larger transit times, because these orbits are more perturbed and so it is necessary to apply the thrust for longer times in order to achieve the desired apogee distance.

The maneuvers to an orbit with the same apogee of the orbit of the Moon can have several purposes, such that sending a spacecraft to perform a Swing-By with the Moon to gain energy. This idea is being considered for the Aster mission, which is under study in Brazil, and has the goal of sending a spacecraft to an asteroid after gaining energy from the passage by the Moon.

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