Comparative statistics and origin of triple and quadruple stars

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ABSTRACT
The statistics of catalogued quadruple stars consisting of two binaries (hierarchy 2+2) is studied in comparison with triple stars, with respective sample sizes of 81 and 724. Seven representative quadruple systems are discussed in greater detail. The main conclusions are: (i) Quadruple systems of \( \varepsilon \) Lyr type with similar masses and inner periods are common, in 42% of the sample the outer mass ratio is above 0.5 and the inner periods differ by less than 10 times. (ii) The distributions of the inner periods in triple and quadruple stars are similar and bimodal. The inner mass ratios do not correlate with the inner periods. (iii) The statistics of outer periods and mass ratios in triples and quadruples are different. The median outer mass ratio in triples is 0.39 independently of the outer period, which has a smooth distribution. In contrast, the outer periods of 25% quadruples concentrate in the narrow range from 10 yr to 100 yr, the outer mass ratios of these tight quadruples are above 0.6 and their two inner periods are similar to each other. (iv) The outer and inner mass ratios in triple and quadruple stars are not mutually correlated. In 13% of quadruples both inner mass ratios are above 0.85 (double twins). (v) The inner and outer orbital angular momenta and periods in triple and quadruple systems with inner periods above 30 d show some correlation, the ratio of outer-to-inner periods is mostly comprised between 5 and \( 10^4 \). In the systems with small period ratios the directions of the orbital spins are correlated, while in the systems with large ratios they are not. The properties of multiple stars do not correspond to the products of dynamical decay of small clusters, hence the N-body dynamics is not the dominant process of their formation. On the other hand, rotationally-driven (cascade) fragmentation possibly followed by migration of inner and/or outer orbits to shorter periods is a promising scenario to explain the origin of triple and quadruple stars.

Key words: stars:formation – stars:statistics – binaries:general – binaries:close

1 INTRODUCTION
Formation of binary and multiple stars is a subject of active research and debate, still remaining one of the major unsolved issues (Zinnecker & Mathieu 2001). Reliable data on the statistics of binary and multiple stars in different environments are essential for comparison with theory and further progress. So far, binary stars have received some attention, but the statistics of higher multiplicities is still uncertain. The aim of this paper is to review the statistics of multiple stars, focusing on quadruple systems, and to relate it to the formation scenarios, whenever possible.

Multiple stellar systems are often considered individually, hence the formation of each object can be explained by a unique chain of events. Yet, multiples are not rare “freaks” in the stellar world. At least 8% of solar-type stars have three or more components (Tokovinin 2008). The nearest star, \( \alpha \) Cen, is triple. Multiple systems are normal products of star formation, and the star-formation theory will eventually be able to model their statistical properties.

A textbook example of a quadruple system is \( \varepsilon \) Lyr – two wide visual binaries on a still wider common orbit. Two striking properties of this system are the similarity of the masses of all four components and comparable periods of the inner sub-systems, suggesting that the formation process was not completely random. We show that such quadruples with similar components and inner periods are indeed typical.

One reason why the statistics of multiple stars remains murky is the difficulty of accounting for the observational selection. Catalogued multiples result from random discoveries using a wide variety of techniques, so quantifying the observational bias seems hopeless. A better approach would be to study volume-limited samples, but the number of multiples even among nearby G-dwarfs is small, while the incompleteness of higher-order hierarchies is still significant. So, at present, the only way to look at the statistics of N > 2 stellar systems is by using catalogues. Fekel (1981) pioneered the statistical approach to triple stars. The number of known multiples has significantly increased since then, making it possible to take the first look at systems with more than three stars. About half of the known quadruples were discovered during the last 20 years as a result of systematic radial velocity programs and high-
resolution imaging, the latest ones (Shkolnik et al. 2008) are not included in the present study.

Here we use the updated Multiple Star Catalogue (Tokovinin 1997, hereafter MSC) to study the statistics of quadruple stars composed of two close pairs orbiting each other. The problem of the observational selection is partially addressed by comparing with the triple systems, assuming that the selection effects in both cases are similar. Although the exact form of this selection filter remains undetermined, we can make reasonable assumptions about its behaviour to figure out which statistical features are real. A catalogue such as MSC gives a distorted, but still useful information on the true underlying statistics of multiple stars.

We begin with a short review of multiple-star formation processes and their predictions for the statistics (Sect. 2). Some prototypical multiple stars are presented in Sect. 3 to introduce relevant terminology and to give a feeling of the data. The samples of quadruple and triple stars are described in Sect. 4 together with the selection effects. The comparative statistics is presented in Sect. 5. Section 6 summarises our findings and discusses the implications for multiple-star formation.

2 FORMATION PROCESSES

A short review of multiple-star formation scenarios is given to put the statistics in proper context. The sequence of events, from the fragmentation of pre-stellar cores to the N-body interaction and orbit migration, seems well established. Still, theoretical predictions vary widely, depending on the initial conditions, physical processes involved or modelled, and computational details. This is a rapidly evolving field.

Fragmentation is believed to be the dominant mechanism forming binary and multiple stars (Bonnell 2001; Bate, Bonnell, & Bromm 2002; Delgado-Donate et al. 2004; Goodwin, Whitworth, & Ward-Thompson 2004; Machida et al. 2008). Bodenheimer (1978) envisioned a cascade fragmentation where a rotating core first collapses into a ring or disk supported centrifugally. The ring fragments, and, if the angular momentum of the fragments is high, they undergo further rotational fragmentation, forming a 2+2 hierarchical quadruple.

Real pre-stellar cores are in the state of complex “turbulent” motions, so the cascade fragmentation gives an over-simplified picture which, nevertheless, captures the essential physics. A collapse of an isolated turbulent core of gas initially creates filaments, then clumps in the filament (or inter-sections of several filaments) form stars or sub-systems (e.g. Krumholz, Klein, & McKee 2007; Goodwin, Whitworth, & Ward-Thompson 2004). As an alternative to the rotationally-driven fragmentation, prompt fragmentation is induced by an external shock wave or by an n = 2 gravitational perturbation from another companion. The Larson’s velocity-size relation \( V \propto R^{1.5} \) means that in pre-stellar cores most of the specific angular momentum \( j \) is contained in large-scale motions, \( j \propto RV \propto R^{1.5} \). If the core is isolated, this momentum will be conserved in the orbital motion of a wide binary or an outer subsystem.

Simulations show that accretion disks can also produce binary companions by fragmentation and can even form multiple systems (cf. Fig. 4 in Bonnell 2001). Stamatellos, Hubber, & Whitworth (2007) studied disk fragmentation into several low-mass components, some of which are subsequently ejected. Unlike the first generation of stars produced directly by fragmentation, disk-generated companions form over an extended period of time, during the whole episode of collapse and accretion. Delgado-Donate et al. (2004) show, however, that disk-fragmentation companions have less chance to accrete as much matter as the first stars and are often ejected during subsequent dynamical evolution.

Accretion. Binary systems produced by fragmentation at isothermal stage of collapse are relatively wide, from \( 10^2 \) to \( 10^4 \) AU (cf. the scaling formula in Sterzik, Durisen, & Zinnecker 2003). The fragments continue to accrete surrounding gas, so the final parameters of a binary (or a sub-system) are determined by the accretion. Bate (2000) studied the evolution of the separation and mass ratio of an accreting proto-binary under the assumption that there is no angular momentum transport in the accreted gas. The results depend on the density and rotation profiles of the cloud. Typically (but not always) the mass ratio of an accreting binary increases and tends to unity, while the orbital period decreases. This trend is confirmed by numerous hydrodynamical simulations. Binaries with nearly identical components, twins (Halbwachs et al. 2003; Lucy 2004; Söderhjelm 2007), are most naturally explained by accretion. Fragmentation occurs also at adiabatic stage of the collapse and at higher densities, creating close binaries (Machida et al. 2008). However, in this case the fragments have very low masses. If these fragments do not merge, the parameters of the resulting close binaries are still settled by the accretion.

N-body dynamics plays a role in a non-hierarchical cluster of proto-stars that is formed when several fragments fall to the common mass centre of a pre-stellar core. The cluster disintegrates, ejecting some stars and leaving a stable binary or multiple system, usually with the most massive stars paired together. As the free-fall time of stars and gas to the centre is the same, accretion and dynamics proceed simultaneously. Pure N-body dynamics of small clusters (Sterzik & Durisen 1998) produces unrealistic multiples with small period ratios and large outer eccentricities (cf. Fig. 6 in Sterzik & Tokovinin 2002). The presence of gas does change the character of the process compared to the point-mass dynamics. Extensive simulations of fragmenting cores including accretion and dynamical evolution were performed by Delgado-Donate et al. (2004). They find that the chaotic dynamicalities disrupts mostly wide and/or low-mass companions with low binding energies, but has little effect on the inner sub-systems with massive components formed by fragmentation and accretion. Component masses in these surviving multiples are comparable, as in the real systems (see below). In the 10 simulated runs, three 2+2 quadruples were formed within 0.5 Myr from the start of the collapse, two of them with additional outer components. All those quadruples survived subsequent dynamical evolution of the mini-clusters.

Many authors (e.g. Sterzik, Durisen, & Zinnecker 2003; Goodwin, Whitworth, & Ward-Thompson 2004) believe that N-body interaction is a necessary step in the formation of multiple stars because it helps to reduce orbital separations by an order of magnitude compared to the fragmentation scale. We question this assertion. As large-scale motions of the core contain significant angular momentum, the fragments do not necessarily fall onto the centre and interact dynamically, but rather may end up in a stable wide binary, possibly containing inner sub-systems. This scenario is conceptually close to the cascade fragmentation and it can produce a hierarchical 2+2 quadruple system like \( \varepsilon \) Lyr with similar components’ masses and inner orbits (Delgado-Donate et al. 2004). The orbital scales of the \( \varepsilon \) Lyr system roughly match the initial fragmentation scales, so no further orbit evolution is required. The eccentricity of the outer orbit in this scenario should be moderate, the multiple system is hierarchical and dynamically stable.
Orbit migration is an accepted theory for “hot Jupiters”, where the orbit of a planet shrinks by interacting with the gaseous disk. It is necessary to evoke some kind of migration for explaining the origin of close binaries, unless they are formed directly by fragmentation at high densities. The eccentricity distributions of exo-planets and spectroscopic binaries are strikingly similar (Ribas & Miralda-Escude 2007), suggesting that interaction with disks may be important in both cases. Unlike planets, stellar components cannot migrate in low-mass debris disks, but massive accretion disks at earlier stages could act in a similar way (type III migration, Pfeilinski, Artymowicz, & Mellema 2008). Accretion onto a binary also can reduce its period, even without any angular momentum transport (e.g. Baty 2000), but it is likely that in this case massive disks are formed as well and affect the orbit of the binary.

There may be several mechanisms of orbit migration. Their common characteristic is that the orbital angular momentum is transferred to some other body, while the potential energy released from the component’s approach is dissipated. Apart from the disk migration, candidate processes are interactions with a jet (Reipurth & Aspin 2004) or with a magnetic wind. Kozai cycles with tidal friction (KCTF, Eggleton 2006) can be viewed as yet another flavor of migration where the angular momentum is deposited into a tertiary component while the orbital energy is dissipated by tides.

Statistics of close binaries help to quantify the migration observationally. It has been established (Tokovinin et al. 2006) that not all low-mass spectroscopic binaries (SBs) with periods below 30 d have tertiary components, so these SBs are produced by a migration mechanism distinct from the KCTF. The SBs with tertiary companions have shorter periods than pure binaries, but the mass ratio distributions of these two sub-populations are identical, hence the dominant migration process does not change the mass ratio. Many SBs have tertiary components too distant to cause KCTF migration, yet their periods are, statistically, shorter than the periods of pure SBs, so the migration is likely enhanced by the presence of even a distant companion.

The properties of multiple stars are reviewed with the aim to establish which formation processes are dominant.

3 TYPICAL QUADRUPLE SYSTEMS

The data for this study are taken from the current version of the Multiple Star Catalogue (MSC) Tokovinin [1997]. We begin by presenting seven typical quadruple systems to illustrate the nature of the input data and its reliability. Figure 1 depicts the hierarchy of these systems, where a hierarchical multiple system is represented as a combination of elementary binaries composed either of single stars or of closer pairs. Relevant parameters of the systems are gathered in Table 1 with notations explained in Sect. 4.1. All systems are identified by WDS codes based on the 2000 coordinates (Mason et al. 2001), with additional identifiers provided in the 2nd column. We focus only on quadruple systems composed of two close pairs on a common wide orbit (hierarchy 2+2) and leave aside quadruples of hierarchy (2+1)+1, i.e. a hierarchical triple orbited by a distant companion. Some triples and quadruples considered here contain additional components, as illustrated by Fig. 1. The terms triple and quadruple generally mean systems containing exactly 3 or 4 stars. In this paper we use them to denote objects from our samples, so some systems called here “triple” contain more than 3 stars.

Epsilon Lyrae is a visual quadruple composed of four A-type Main Sequence (MS) stars arranged in two pairs AB and CD. The component C has been resolved twice with speckle interferometry into a 0.19” binary Cab=CHARA 77, but the existence of this sub-system remains doubtful because of many negative observations. So, we ignore this uncertain couple Cab. However, some components believed now to be single stars may be resolved in the future.

Alpha Geminorum is a classical multiple star where both components A and B of a 445-yr visual binary are close spectroscopic pairs with A-type primaries and low mass ratios. The eclipsing pair of red dwarfs YY Gem (component C) at angular distance 72.5” (projected distance 1145 AU) also belongs to the system, making it sextuple. Thus, there are 2 quadruples of (2+2) hierarchy here: one with the outer 445-yr orbit AB and another with the outer orbit AB-C and the inner sub-systems Cab and AB. In the following, we ignore composite quadruples, like the outer one in α Gem, and consider only simple quadruples composed of just 4 stars (with, possibly, outer components), like the inner system AB. In the case of α Gem, we can be sure that there are only 4 stars in the AB system, but our sample of simple quadruples can contain some systems with yet undiscovered sub-structure.

88 Tau is a sextuple system with the same structure as α Gem. It is cited here to illustrate that such systems are not exceptional. The projected distance between the outer components A and B is 3500 AU, with A being quadruple and B binary. Recently, interferometric astrometry has permitted to effectively resolve the inner sub-systems Aa1,Aa2 and Ab1,Ab2 and to determine their orientation with respect to the 18-yr orbit Aab. Interestingly, the inclination of the more massive inner binary Aa1,Aa2 relative to the outer orbit was found to be 143° ± 2.5°, i.e. it is counter-rotating. The inclination of the second sub-system Ab1,Ab2 is either 82° or 58°, its orbit is less secure.

41+40 Dra is a well-studied quadruple with F-type MS components (Tokovinin et al. 2003). A combination of observational constraints makes the existence of any additional companions extremely unlikely, so this system is a genuine quadruple. The mass ratios of the sub-systems Aab and Bab are nearly the same, 0.93 and 0.91. The periods of Aab and Bab are different, but the orbit of Aab is highly eccentric, so the angular momenta of Aab and Bab are similar. The pairs Aab and Bab rotate in opposite directions as projected onto the sky. Although the outer orbit is still undetermined, we know that it is not co-planar with the orbit Aab, the probable relative inclination is either 60° or 160°.

GJ 225.2 was a visual triple star until recently, when a new sub-system CE has been discovered with adaptive optics.
Table 1. Selected quadruple systems

| WDS(2000) | Name        | Periods | Masses, $M_2$ | $M_1$ |
|-----------|-------------|---------|---------------|-------|
|           |             | $P_L$   | $P_{S1}$  | $P_{S2}$ | $M_1,2$ | $M_1,3$ | $M_2,3$ | $M_1,1$ |
| 18443+3938 | ε Lyr       | 340 y   | 1804 y     | 585 y   | 2.31 a  | 1.62 a  | 1.86 a  | 1.70 a  |
| 07346+3153 | α Gem       | 445 y   | 9.2 d      | 3.0 d   | 2.76 a  | 0.47 m  | 2.98 a  | 0.24 m  |
| 04356+1010 | 88 Tau      | 14 ky   | 445 y      | 0.8 d   | 3.22 s  | 3.23 s  | 0.59 *  | 0.58 *  |
| 18002+8000 | 41+40 Dra   | 10.3 ky | 1247 d     | 10.5 d  | 1.39 *  | 1.30 *  | 1.32 *  | 1.20 *  |
| 06003+3103 | GJ 225.2    | 460 y   | 67.7 y     | 23.7 y  | 0.67 *  | 0.52 *  | 0.69 *  | 0.20 *  |
| 11221−2447 | HD 98800    | 345 y   | 262 d     | 315 d   | 0.80 *  | 0.67 q  | 0.90 *  | 0.19 m  |
| 04325+1731 | GG Tau      | 45 ky   | 200 y     | 2700 y  | 0.78 *  | 0.68 *  | 0.11 *  | 0.04 *  |

Table 2. Sample of triple systems (fragment)

| WDS(2000) | log $P_L$ | log $P_S$ | $M_1$ | $M_2$ | $M_3$ | Pix |
|-----------|-----------|-----------|-------|-------|-------|-----|
|           | day       | day       | $M_\odot$ | $M_\odot$ | $M_\odot$ | mas |
| 00046−4044 | 3.63      | 3.10      | 0.06 a  | 0.06 * | 0.26 * | 76.9 |
| 00057+4548 | 7.85      | 5.74      | 0.65 a  | 0.47 a  | 0.37 a  | 85.1 |
| 00063+5826 | 4.59      | 1.68      | 0.86 *  | 0.31 *  | 0.98 *  | 49.3 |
| 00125+1434 | 3.19      | 0.27      | 0.78 m  | 0.54 a  | 0.52 q  | 24.7 |
| 00150+0849 | 6.73      | −0.08     | 1.24 a  | 1.15 *  | 1.13 *  | 12.5 |

4 STATISTICAL SAMPLES

4.1 Parameters and notation

Hierarchical multiple systems are described by a large number of parameters—component’s masses and orbital elements of each subsystem. We concentrate on masses and periods as most relevant to the formation processes and touch on other characteristics (eccentricity, mutual orbit inclination) only briefly. The semi-major axis is related to the period by the third Kepler’s law and is statistically equivalent to period, except for a weak dependence on the total mass.

Consistent notation of the parameters is required to avoid confusion. For triple stars, the outer (long) and inner (short) periods are denoted as $P_L$ and $P_S$, the masses of the inner binary companions as $M_1$ and $M_2$ and the mass of the distant tertiary as $M_3$. The inner and outer mass ratios $q_S$ and $q_L$ are defined as

$$q_S = \frac{M_2}{M_1}, \quad q_L = \frac{M_3}{M_1 + M_2},$$

although an alternative definition $q_L = M_3/M_1$ is also possible. The outer mass ratio can take values $q_L > 1$ if the tertiary is more massive than the inner binary (this happens in 5% of the triples).

The notation for quadruple stars is as follows: the period of the outer system is $P_L$, $P_{S1}$ and $P_{S2}$ are the periods of the inner sub-systems, with index 1 referring to the sub-system with the largest total mass. The masses of all 4 components are designated as $M_{1,1}$, $M_{1,2}$, $M_{2,1}$, and $M_{2,2}$.

The mass ratios of quadruple stars can be defined in various ways. We use the following definitions:

$$q_{S1} = \frac{M_{1,2}}{M_{1,1}}, \quad q_{S2} = \frac{M_{2,2}}{M_{2,1}}, \quad q_{L4} = \frac{M_{2,1} + M_{2,2}}{M_{1,1} + M_{1,2}},$$

One can argue that the outer mass ratios for triple and quadruple stars are not directly comparable because the latter compares two stellar pairs. An alternative definition of the outer mass ratio in quadruple stars can be $q_{L4}'$

$$q_{L4}' = \frac{M_{2,1}}{M_{1,1} + M_{1,2}} = \frac{q_{L4}}{1 + q_{S2}}.$$

This parameter can be compared with $q_{L4}$ more directly. For example, when all components have equal masses, we have $q_{L4} = 1$, but $q_{L4}' = q_{L4} = 0.5$. The choice of the most relevant mass ratio definition depends on the implied formation process, e.g. $q_{L4}$ and $q_{L4}$ can characterise the fragmentation of the outer sub-system.

The data on the periods and masses are taken from the MSC. The periods are either known more or less exactly from the spectroscopic or visual orbits or, otherwise, estimated roughly from the projected separations in the case of wide binaries. The components’
Table 3. Sample of quadruple systems (fragment)

| WDS(2000) | log $P_L$ | log $P_{S1}$ | log $P_{S2}$ | $M_{1,1}$ | $M_{1,2}$ | $M_{2,1}$ | $M_{2,2}$ | Plx |
|-----------|-----------|--------------|--------------|-----------|-----------|-----------|-----------|-----|
| 00134+2659 | 7.20 | 5.13 | 0.78 | 2.50 * | 1.54 a | 0.92 * | 0.88 * | 8.1 |
| 00247-2709 | 4.16 | 2.30 | 3.41 | 0.10 * | 0.08 * | 0.08 * | 0.08 * | 132.8 |
| 00316-6258 | 6.68 | 5.26 | 4.21 | 3.84 a | 0.40 v | 1.94 a | 0.92 * | 23.4 |
| 00345-0433 | 7.18 | 5.56 | 1.91 | 1.70 * | 1.14 * | 0.10 m | 1.94 a | 23.4 |
| 00364-4908 | 8.57 | 0.57 | 1.34 | 0.83 a | 0.78 a | 0.31 m | 8.1 |

Notes:
00134+2659 = HR 40. VB (368y G0III) and SB (6d G7V) at 18′′.
00247-2709 = GJ 2005, visual quadruple composed of M5 dwarfs, 0.27′′ and 0.05′′ at 1.07′′ from each other.
00316-6258 = HR 126+127+125, Beta Tuc, 6 components. The inner quadruple contains 2 VBs (2.4′′ B9V and 44.7y A2V) at 27′′. The distant pair at 544′′ is itself a 0.1′′ VB with a short, but yet unknown period.
00345-0433 = HIP 2713. VB (1.9′′ G8II) and SB1 (81d G0V) at 19.5′′.
00364-4908 = HIP 2888 = GJ 24AB. Two SBs (3.7d G3V and 22d K0V) at 329′′, CPM.

masses are estimated by various techniques, as reflected by the 1-character codes. The most common mass codes are a (mass from the spectral type), * (mass estimates from orbits, isochrone fitting, etc.), q (secondary of an SB2), m (minimum mass of an SB1 secondary), v (estimate from the magnitude difference in a visual binary), and : (uncertain mass, e.g. in an interferometric binary without relative photometry where a magnitude difference of 2 is assumed). The mass code s means that the “component” consists of several stars and the sum of their masses is given. The reader is referred to the MSC for further details. Obviously, the mass estimates in the MSC are quite crude, so the mass ratios discussed below are only indicative of statistical trends, rather than well-measured.

4.2 Triple and quadruple systems

The data on triple and quadruple stars are listed in the Tables 2 and 3, respectively. These tables are available in full electronically, their fragments are given here. Each system is identified by its WDS (2000) code. Additional identifiers are given in the Notes to Table 3. More information on each system can be obtained from the on-line MSC.

We selected all “simple” triple systems composed of an inner pair and a distant single tertiary, with possibly some other more distant components. The systems with uncertain masses (code :) are omitted, leaving $N = 724$ triples listed in Table 2.

The sample of $2+2$ quadruples from the MSC contains 135 entries. We exclude from the statistical analysis composite quadruples, systems where some periods or masses are not known (e.g. astrometric sub-systems without computed orbits), leaving 81 simple quadruples. Periods, masses, and mass codes are given in the electronic Table 3, together with the notes on individual systems. We keep in the sample 3 systems (04226+2538, 11395−6524, 17146+1423) where one of the 4 masses is poorly determined (code :), but exclude some PMS multiples with yet uncertain masses like those recently discovered by Correia et al. (2006). Three quadruple systems from Table 3 (SZ Cam, µ Ori, QZ Car) belong to open clusters.

Both samples are quite heterogeneous with respect to masses, ages, and reliability of the data. The most massive is the 40 $M_{\odot}$ primary component of the quadruple QZ Car (10444−6000), while the red dwarf primary in the quadruple GJ 2005 (00247−2709) has the smallest mass of 0.1 $M_{\odot}$. Figure 2 shows the cumulative distributions of the primary-component masses in the samples of triple and quadruple stars. In both cases about 80% of primaries have masses between 0.8 and 5 $M_{\odot}$, with the median mass of 1.7 $M_{\odot}$. We tried to split the quadruples into low- and high-mass parts around this median and found the statistics to be similar.

1 http://www.ctio.noao.edu/˜atokovin/stars/index.php
In a reasonably complete nearby sample the number of objects within a given distance \( d \) would be proportional to \( d^3 \). The number of triple and quadruple stars in our samples is rather proportional to \( d^3 \), because low-mass objects are located closer than high-mass ones (Fig 3 bottom). Such weak dependence on \( d \) is expected for magnitude-limited samples. We deduce from the ratio of the coefficients in the \( d^3 \) fits that triples are 5 times more frequent than 2+2 quadruples. We recover the \( N \propto d^3 \) law out to \( d < 40 \) pc if only primaries more massive than 1 \( M_\odot \) are retained. The median distances to triples and quadruples are 100 pc and 70 pc, respectively.

### 4.3 Selection effects

Multiple stars are discovered by different techniques or their combinations, each with its own biases. Moreover, the choice of the surveyed objects is random rather than systematic. As a result, the current knowledge of multiplicity is incomplete even in the solar neighbourhood. Here we study the statistics of the catalogue, not the statistics of real multiple stars in the sky.

It is standard practise to adopt some models of observational selection for deriving true statistics from catalogues or biased samples. This approach is justified for some well-defined samples (e.g. Tokovinin et al. 2006), but it is problematic for compilative catalogues resulting from random discoveries, such as the MSC. A selection function \( f \) is the ratio of objects in the sample to their true number in the same stellar population. Here we intentionally leave the selection function undefined and do not attempt to study true distributions of periods and mass ratios. However, we investigate the correlations between the parameters of the catalogued systems and try to figure out the influence of selection effects qualitatively.

Two reasonable assumptions are made. (i) The discovery of a subsystem depends mostly on its period \( P \) and mass ratio \( q \), so, to the first order, the selection function has the form \( f(P,q) \). (ii) The selection function \( f(P,q) \) is smooth, without features favouring or disfavouring particular periods or mass ratios. The selection function acts as a filter superposed on the observed distribution. If there are correlations between the parameters of real multiple stars, they are likely to be seen in the catalogue as well. Sharp features in the real distributions will be recovered in the catalogue, too.

The integral of \( f(P,q) \) gives the completeness of the sample \( P_0 \). The MSC certainly misses many low-mass companions. However, a bold assumption that we see only the “tip of the iceberg” and that stars are surrounded by swarms of yet-undiscovered low-mass companions meets with problems, at least for nearby dwarfs. For example, Eggenberger et al. (2007) find that only 15% of stars previously considered as single and surveyed with adaptive optics have low-mass companions at separations above 7 AU. Precise radial velocities of 93% of “single” dwarfs are stable (Nidever et al. 2002). It is therefore reasonable to assume that the completeness of the MSC for nearby dwarfs is of the order \( P_0 \sim 0.5 \) or higher.

The same techniques are used for discovering inner subsystems in quadruple and triple stars, so the selection functions \( f(P_s,q_s) \) for triples and \( f(P_{s1},q_{s1}) \) for quadruples should be similar. Unfortunately, we cannot assume that the selection functions \( f(P_L,q_L) \) for triples and \( f(P_{L1},q_{L1}) \) for quadruples are the same. Discovering a faint tertiary can be easier than finding that this tertiary is in fact double (cf. the system GJ 225.2). A systematic study of tertiary components has converted many of them into additional pairs, promoting triples to quadruples (Tokovinin & Smekhov 2002). It has been shown that 30% of tertiary companions believed to be single are in fact binary.

As mentioned above, only in few multiple systems we can be sure that all components are actually known. When a new component is found in the future at some intermediate level (e.g. orbiting a close pair with a more distant and already known tertiary), our current knowledge of the system’s structure is obviously wrong, as the values of periods and masses in the Tables 2 and 3 correspond to some other hierarchical levels. When a new component is found in a close orbit around some star, a simple triple or quadruple is converted into a complex one. Hopefully, such incompleteness affects only a fraction of the systems and the distortion of the statistics caused by these missing levels is tolerably small. It can be reduced by restricting the samples to nearby, better-studied systems.

### 5 COMPARISON BETWEEN TRIPLES AND QUADRUPLES

Periods and masses of the companions are determined by the formation and early evolution, therefore we study the statistics of these parameters. Nevertheless, a triple is characterised by 5 numbers (2 periods and 3 masses), a quadruple – by 7 numbers. Looking for correlations in this multi-parameter space is not easy, so we give below only some plots which appear to be most relevant. We begin by looking at the periods in the inner and outer sub-systems. Then the inner and outer mass ratios are compared. Finally, we explore correlations of the inner and outer mass ratios with respective periods, the correlations between the orbital angular momenta, and the properties of quadruple systems.

#### 5.1 Outer and inner periods

Figure 3 plots the triples and quadruples in the period-period diagram. Throughout the paper, the periods are measured in days and plotted on the logarithmic scale. Hierarchical systems are dynamically stable when \( P_L/P_S > 5 \) (e.g. Eggleton 2006), this limit is shown by the dashed lines. As the periods of wide systems are estimated only crudely, some points can fall slightly below this line. The distributions of inner and outer periods in 185 triple stars within 50 pc and in all quadruple stars are plotted in Fig. 4.

The inner periods in both triples and quadruples are distributed in the same way, clustered in two groups: close (log \( P_S < 1.5 \)) and wide (log \( P_S > 4 \)), with a partially filled gap in-between. As the discovery techniques for the close sub-systems are predominantly spectroscopic and for the wide sub-system mostly visual, the gap could be attributed to the selection effect, but it could also be real. Note that the inner and outer periods in triple stars show some correlation for \( P_S > 30 \) d. Most points are located in the band \( 5 < P_L/P_S < 10^4 \) (see also Tokovinin 2004), while the space above this band is almost empty, contributing to the gap in the distributions of the inner periods. This correlation must be genuine because the inner and outer sub-systems are discovered independently of each other. The correlation does not hold for short inner periods \( P_S < 30 \) d.

Tidal interaction in multiple stars (KCTF) shortens the inner periods, leading to the excess of sub-systems with periods of few days (Fabrycky & Tremaine 2007). There is a strong peak at log \( P_S = 0.25...0.75 \) in the distribution of inner periods in the complete sample of triple stars. However, it can be a selection effect produced by the eclipsing systems which are discovered at long distances, because this peak almost disappears for nearby triples within 50 pc (Fig. 4). The distribution of the inner periods in the homogeneous radial-velocity survey of triple stars (Tokovinin & Smekhov 2002) is nearly flat out to \( P_S \sim 60 \) d.
Statistics of triple and quadruple stars

Figure 3. Comparison of the outer and inner periods in triple (top) and quadruple (bottom) stars. All periods are in days. For quadruples, $P_{S1}$ are plotted as diamonds, $P_{S2}$ as crosses, the points belonging to the same system are connected by dotted lines. The diagonal lines delineate the approximate dynamical stability limit $P_L/P_S > 5$.

KCTF can produce only a mild depletion at $P_S \sim 10$ d but cannot explain the whole gap.

Note that the period distribution in the volume-limited samples of dwarf binaries is a rising function in the $\log P$ interval from 0 to 3, with a possible dip near $\log P \sim 2$ (Fig. 10 in Halbwachs et al. 2003). The rise is even steeper for non-triple (pure) binaries, because all SBs with $P < 3$ d are triple, while the proportion of triples decreases at longer periods (Tokovinin et al. 2006). The obvious conclusion is that the migration mechanism produces shorter orbital periods in triple stars than in pure binaries. It is then plausible that the gap in the inner-period distribution in triple and quadruple stars is real and is caused by migration to shorter periods, rather than by the observational selection.

The distributions of the outer periods of quadruples and triples are different. All quadruples except two have $\log P_L > 3.8$ ($P_L > 17$ y). Moreover, there is a concentration of points near the lower limit $\log P_L \sim 4$, not seen in the triple systems where the distribution of $P_L$ is smooth and extends to shorter periods. It is shown below that tight quadruples with $\log P_L < 4.5$ are also distinguished by larger outer mass ratios $q_{L4}$ and more similar inner periods, so the reality of this special group of objects is based on more than just outer periods. They are further studied in Appendix A.

The quadruple system with the shortest outer period of 355 d is VW LMi, it combines a 0.5-day eclipsing pair with a 8-day binary, all composed of dwarf stars (Pribulla et al. 2006). The triple star with the shortest outer period is $\lambda$ Tau: $P_L = 33$ d, $P_S = 4$ d, $M_1 = 7.2 M_\odot$.

5.2 Outer and inner mass ratios

It appears that there is no correlation between inner and outer mass ratios neither in triples nor in quadruples (Fig. 5). The distributions of $q_S$ in triples and $q_{S1}, q_{S2}$ in quadruples look quite similar. Both show some excess of sub-systems with nearly identical masses, twins (Halbwachs et al. 2003; Lucy 2006; Söderhjelm 2007).

In triple systems where the mass of the outer companion equals the mass of an inner companion, $M_3 = M_1$ or $M_3 = M_2$, there is a relation between inner and outer mass ratios: $q_{L3} = 1/(1 + q_S)$ and $q_{L3} = q_S/(1 + q_S)$, respectively (dotted lines in Fig. 5 top). There seems to be a concentration of points along these lines, suggesting that the phenomenon of twins may extend to the outer sub-systems of triple stars in this strange and un-explained way.

The fraction of triple systems where the outer companion has the smallest mass, $M_3 < M_2$, is 332/724=46% for the whole sample and 71/184=39% for triples within 50 pc. The median outer
The mass ratio is \( q_{L3} = 0.39 \), 81% of triples have \( q_{L3} > 0.2 \). Thus, tertiary companions tend to have masses comparable with the components of the inner binary. Triple systems resulting from the N-body decay, on the contrary, have only low \( q_{L3} \) (e.g. Fig. 4 in Delgado-Donate, Clarke, & Bate 2003) and do not match real triples in this respect. Although the observational selection does favor multiples with comparable masses, the difference with the simulations is too strong to be explained entirely by the selection. The survey of PMS stars by Correia et al. (2006) confirms that low-mass tertiaries are indeed rare, not just missed.

The distributions of the outer mass ratios in quadruples and triples, \( q_{L4} \) and \( q_{L3} \), are different. A strong tendency to large \( q_{L4} \) is obvious, 97% of quadruples having \( q_{L4} > 0.2 \). The concentration of points in the upper right corner of Fig. 5 corresponds to the quadruples where all component masses are similar. Simulations of multiple stars with components selected independently are presented in Appendix B to show how different they are from the real systems.

### 5.3 Inner mass ratios vs. inner periods

Figure 6 presents a comparison of mass ratios with periods for the inner sub-systems. The statistics of triple and quadruple stars appear to be similar. We note the relative paucity of inner periods between \( 3 \times 10^4 \) and \( 1 \times 10^6 \) days (the period gap, see above) and the concentration of points towards \( q_S \sim 1 \) (twins) on both sides of the gap. In both samples the sub-systems with \( \log P_S > 4 \) tend to have more similar masses, \( q_S > 0.5 \). This trend may be caused by the selection (sub-systems with low \( q \) are not discovered by visual techniques), as illustrated by the GJ 225.2 system. We checked that the distributions of the inner and outer mass ratios \( q_S \) and \( q_{L3} \) in nearby triple stars with inner (resp. outer) periods in the same range, between \( 10^4 \) and \( 10^6 \) days, are statistically indistinguishable. On the other hand, sub-systems with short \( P_S \) and low \( q_S \) could be formed by an alternative mechanism such as disk fragmentation.

### 5.4 Outer mass ratios vs. outer periods

Figure 7 shows the correlation of outer mass ratios with outer periods for triples and quadruples. These plots are strikingly different. In triple stars, we see no dependence of median \( q_{L3} \) (diamonds) on the outer period, although the dispersion of \( q_{L3} \) seems to increase at larger periods. Delgado-Donate, Clarke, & Bate (2003) predicted that close inner binaries with low-mass tertiaries should be common, but there is no correlation between \( q_{L3} \) and \( P_S \) to support this prediction.

Note the complete absence of quadruples in the lower left corner with \( q_{L4} < 0.6 \) and \( \log P_L < 5.4 \) and the relative abundance of
triples with similar parameters. This difference could be caused by the observational selection. Quadruples with $q_{L4} < 0.6$ can escape detection because the low-mass pair is much fainter than the main pair. One such system with estimated $q_{L4} = 0.63$ is χ Tau B where the light of the second binary is not detected at all, while its binarity is inferred from the total mass and infrared colour (Torres 2006). Missed tight quadruples with $q_{L4} < 0.6$ and $P_L < 100$ y would be considered today as triples with $q_{L3} < 0.6$. We can establish how many of such low-$q_{L3}$ triples are actually quadruple by monitoring the radial velocity of faint tertiary components with the detection technique of D'Angelo, van Kerkwijk, & Rucinski (2006).

5.5 Orbital angular momenta

Orbital angular momentum $J$ of a binary star equals

$$J = \sqrt{a(1-e^2)} M_1 M_2 \sqrt{G/M},$$

where $M_1$ and $M_2$ are the component’s masses, $M = M_1 + M_2$ is the total mass, $e$ is the eccentricity, $a$ is the semi-major axis, and $G$ is the gravitational constant. In the following we neglect the eccentricity because for a typical $e = 0.5$ the factor $(1-e^2)^{1/2} = 0.87$ is insignificant in comparison with the large scatter of $J$ (there are notable exceptions like 41 Dra). We calculate $a$ from periods and masses using the Kepler’s law. The specific angular momentum $j = J/M$ is estimated for each sub-system.

Angular momentum is strongly related to orbital period, so the plots in Fig. 8 resemble the plots in Fig. 3. The correlation between inner and outer periods translates into the correlation between the momenta, a tighter correlation for wide multiples. Considering that $J \propto a^{1/2} \propto P^{3/5}$, the typical ratio $J_L/J_S \sim 10$ translates to $P_L/P_S \sim 10^3$.

Relative orientation of the outer and inner orbits gives additional clues to the formation and dynamical evolution of multiple stars. For triples, it has been established that the angular momentum vectors of the inner and outer orbits do show some correlation, the average angle between them, $\Phi$, being about $50^\circ$ instead of $90^\circ$ for un-correlated spins (Sterzik & Tokovinin 2002). The correlation of orbital spins is stronger at low $P_L/P_S$ ratios (close to the dynamical stability limit) and disappears at large $P_L/P_S$.

There are two simple quadruple stars, 88 Tau and μ Ori, with complete elements of both inner and outer orbits known. In 88 Tau, the inner orbits are not co-planar to the outer orbit, with $\Phi > 90^\circ$ (counter-rotation) for at least one sub-system (Lane et al. 2007). Similarly, the Aab sub-system in μ Ori is counter-rotating, $\Phi = 137^\circ \pm 8^\circ$, while the orbit of the Bab sub-system is nearly perpendicular to the orbit of AB (Muterspaugh et al. 2008). There are other cases where the apparent rotation of the resolved inner

**Figure 7.** Mass ratios vs. periods in the outer sub-systems of triple (top) and quadruple (bottom) stars. The diamonds show median mass ratio for triples in each decade of the outer periods.

**Figure 8.** Specific orbital angular momentum in the outer sub-system $j_L$ vs. specific momentum in the inner sub-system(s) $j_S$ in triple (top) and quadruple (bottom) stars. The solid lines denote equality of the momenta, the dotted lines show $j_L/j_S = 2.5$.
and outer sub-systems is opposite, suggesting non-aligned spins. On the other hand, some wide quadruples are close to alignment (e.g. ε Lyr, GG Tau, HD 98800). Although this evidence remains circumstantial, it hints on the same trend as in the triples, i.e. the alignment of orbital spins for low $P_L/P_S$ and mis-alignment for large period ratios. A dedicated interferometric survey of inner orbits is obviously needed to increase the number of multiple systems with known sense of relative rotation.

### 5.6 Quadruple systems

In this sub-section, we focus only on the quadruple stars and look for correlations between mass ratios and periods. There is no correlation between the inner mass ratios $q_{S1}$ and $q_{S2}$ (Fig. 9). However, there is a concentration of points towards $q_{S2} \approx 1$, and a less obvious preference of sub-systems with $q_{S1} \approx 1$. There is a cluster of 11 points in the upper right corner, $q_{S1} > 0.85, q_{S2} > 0.85$ marked by the dotted line in Fig. 9. If the points were distributed uniformly in the $(q_{S1}, q_{S2})$ plane between 0 and 1, we would expect only 2.2% in this corner, while the actual number is 11/83=13%. The family of quadruples where both sub-systems are twins (double twins like 41+40 Dra) thus appears to be distinct from the majority of other quadruples. BD−22° 5866 is yet another low-mass quadruple composed of two twins (Shkolnik et al. 2008). The “definition” of twins $q > 0.85$ is adopted here arbitrarily, guided by Fig. 9.

Sub-systems with $q_{S1} > 0.85$ or $q_{S2} > 0.85$ are found at all inner periods (cf. Fig. 6 bottom). Interestingly, the proportion of twins in the sub-systems of low mass seems to be higher than in the high-mass ones. Similarly, there are more twins among the secondary (less massive) sub-systems. These trends need a confirmation with larger samples. The origin of twin binaries is not yet understood. They could result from the preferential accretion onto the secondary that drives $q$ towards one and, at the same time, shortens the orbital period (Bate 2000). However, the inner mass ratios do not correlate with the inner periods (Fig. 6). Figure 9 demonstrates the lack of correlation between the periods and mass ratios in the two inner sub-systems belonging to the same quadruple. Therefore, the inner periods and mass ratios are determined by different processes.

The points in Fig. 9 (bottom) concentrate to the coordinate origin, showing that in many quadruples the inner mass ratios and inner periods are similar, as in the ε Lyr. The top plot of Fig. 10 shows that the period disparity between the inner sub-systems in quadruple stars is less when $P_L$ is short. This is expected because inner periods are comprised between few hours (contact binaries) and the dynamical stability limit, $-0.7 < \log P_S < \log P_L - 0.7$. At shorter $P_L$ this range is smaller, hence the inner periods are forced to be more similar, $|\log(P_{S1}/P_{S2})| < \log P_L$. This range,
shown in Fig. 10 (top) by the dotted lines, is wider than the distribution of the points by roughly 2 dex. Therefore the similarity of the inner periods is genuine and not caused by the trivial dynamical constraints.

The numbers of quadruple systems with positive and negative \( |\log(P_{S1}/P_{S2})| \) are statistically similar, 38 and 43 respectively, so the periods of the more and less massive inner sub-systems do not differ systematically. The migration process that determined these periods should not depend on the mass. The inner periods are similar to within 1 dex in 42 systems, i.e. in \( 52 \pm 8 \% \) of the quadruples. The relation between the ratio of the inner periods and the outer mass ratio is further explored in Fig. 10 (bottom). As a rule, sub-systems with similar inner periods also have similar total masses (larger \( q_{L4} \)). There are 34 quadruples (42% of the sample) with \( |\log(P_{S1}/P_{S2})| < 1 \) and \( q_{L4} > 0.5 \), i.e. resembling ε Lyr. Conversely, in the systems with dissimilar masses (low \( q_{L4} \)), the ratio of inner periods \( \log(P_{S1}/P_{S2}) \) can take large positive or negative values with equal probability.

6 SUMMARY AND DISCUSSION

The properties of catalogued triple and quadruple systems are:

(i) The distributions of periods in the inner sub-systems of quadruple and triple stars are similar and bimodal. They are different from the period distribution of pure binaries. There is no correlation between inner mass ratios and inner periods.

(ii) The distributions of periods and mass ratios in the outer sub-systems of quadruple and triple stars are different. The median outer mass ratio in triples is 0.39. It does not depend on the outer period, which has a smooth distribution. In contrast, the outer periods of 25% quadruples concentrate in the narrow range from 10 yr to 100 yr, all these tight quadruples have \( q_{L4} > 0.6 \) (possibly a selection effect), while the periods of their two inner binaries are similar.

(iii) Quadruple systems of ε Lyr type with comparable inner periods and component masses are common. In 42% of the quadruples the outer mass ratios are above 0.5 and the inner periods differ by less than 1 dex.

(iv) The outer and inner mass ratios in triple and quadruple stars are not mutually correlated. In 13% of quadruples both inner mass ratios are above 0.85 (double twins).

(v) The values of the inner and outer orbital angular momenta (or periods) in wide (\( P_S > 30 \) d) triple and quadruple systems show some correlation, the ratio of outer-to-inner periods is mostly comprised between 5 and \( 10^4 \). In the systems with small period ratios (low hierarchy) the directions of the orbital spins are correlated, while in the systems with large ratios they are not.

Statistical properties of multiple stars lead to several conclusions regarding their formation mechanisms.

1. N-body dynamics is not the dominant process of multiple-star formation. Triple stars produced in N-body decay (with or without accretion) have distinctive properties: a narrow period distribution (width of 1–2 dex), moderate period ratios \( P_L/P_S \sim 10 \) (not too far from the dynamical stability limit), high eccentricities of outer orbits distributed as \( f(e) = 2e \) or steeper [cf. Fig. 6 in Delegra-Donate, Clarke, & Bate (2003) and Fig. 6 in Sterzik & Tokovinin (2002)], and low outer mass ratios \( q_{L4} < 0.2 \). Quadruple stars with 2+2 hierarchy are produced by this process only exceptionally. In contrast, in real multiple stars the period ratios can be large, the outer eccentricities are moderate (Shatsky 2001, Tokovinin 2004, see also Table X1), and the outer companions are often massive (81% have \( q_{L3} > 0.2 \)). Dynamical disruption of small-N clusters leaves too many single stars and too few binaries and multiples (Goodwin & Kroupa 2005).

The existence of outer weakly bound components or sub-systems, as in the cases of α Gem and 88 Tau (at projected distances 1145 and 3500 AU, respectively), speaks against their dynamical origin. If these multiples were parts of larger clusters, the stellar density must be low enough for the survival of the outer components, hence much too low to explain the dynamical formation of the close inner sub-systems where larger binding energies are involved.

2. The rotationally-driven (cascade) fragmentation scenario is promising. It naturally produces quadruple stars of 2+2 hierarchy. As the motions in the pre-stellar cores are predominantly large-scale, the net angular momentum is sufficiently high, preventing disruptive N-body interactions between sub-systems. The net angular momentum is conserved in the orbital motion of the outer sub-system. The eccentricity of the outer orbit in this scenario should be moderate, the system is hierarchical and dynamically stable.

Suppose that two stellar pairs are formed from the fragments of one core. The initial separation between the components in each pair must be of the order of \( 10^2 \) to \( 10^4 \) AU. The proto-binaries interact with the surrounding gas, create spiral waves and transfer the orbital angular momentum outwards, becoming tighter or even merging into single stars (disk migration, Krumholz, Klein, & McKee 2007). The merger must not happen in the case of quadruples, however. Each sub-system evolves independently of the other system until its closest reservoir of gas is exhausted. As the directions of the net angular momentum in each sub-condensation can differ, the orbital spins of the sub-systems are not necessarily co-aligned with each other.

Accretion of the more distant envelope proceeds on a longer time scale, when the sub-systems already “fall” onto each other and end up forming a quadruple. The matter from the outer envelope settles in a circum-quadruple disk. The result resembles a quadruple system GG Tau, where the circum-quadruple disk slowly feeds the circum-binary disk around the most massive sub-system Aab. Some components may form as fragments in accretion disks around other, more massive stars. The delayed formation of such components means that less mass is available for accretion, possibly creating binaries with low q. The AB quadruple in α Gem could be produced by such a process.

3. Migration shortens both inner and outer orbits. It is likely that there are several different types of migration. Accretion onto a binary and associated braking by the massive circum-binary disk is one such process confirmed by hydrodynamical simulations (Bate, Bonnell, & Bromm 2002). Accretion usually increases the mass ratio and can produce twins with q \( \sim 1 \). However, we find no correlation between mass ratios and periods and infer that the dominant migration mechanism producing close binaries is not accompanied by the accretion of substantial mass. An interaction with circum-binary disk is a promising candidate for such alternative migration mechanism. Such interaction also involves accretion, but possibly at a much reduced level not affecting the mass ratio. It is not clear whether migration can lead to a merger. If this is possi-
ble, massive tertiary components in triple stars could be mergers in former 2+2 quadruples.

The gap in the distribution of the inner periods can be tentatively related to two distinct migration mechanisms. Presumably, accretion-induced migration acts at large separations, shortening the inner (and sometimes outer) periods to $\sim 30$ y (separation $\sim 10$ AU) and creating a broad maximum in the period distribution at $\log P_S \sim 5$. The second migration mechanism shortens inner periods even further (mostly below 30 d) and produces the gap, but it does not affect the mass ratio. The second migration must be less efficient in pure binaries, so their periods are longer compared to the inner sub-systems of multiple stars (Tokovinin et al. 2006). This explanation of the period gap is still highly hypothetical.

In the case of tight quadruples, a significant shrinking of both inner and outer orbits must have happened. Most likely, the outer orbit shrinks by accretion from the envelope, increasing the outer mass ratio. This could explain high outer mass ratios of tight quadruples (Fig. 7) and matches the simulations of Delgado-Donate et al. (2004). The gas accreted by the inner binaries inherits its angular momentum from the outer orbit, so the similarity of the inner periods in tight quadruples is a natural consequence of such process. We also expect the alignment of inner and outer angular momentum vectors, which is the case in some, but not all triple and quadruple stars. Accretion of even a small amount of gas can efficiently brake the binary if the angular momentum of the gas is grossly mis-aligned with respect to the binary, as must happen in quadruple and triple stars with mis-aligned (e.g. counter-rotating) inner and outer systems. A tight quadruple with mis-aligned inner orbits, such as 88 Tau, could result from this process. It is noteworthy that both triples and quadruples with large $P_L/P_S$ ratios show a tendency to mis-aligned orbital spins.

To preserve dynamical stability, migration in the inner sub-systems must happen before or proceed faster than in the outer orbits. In the hydrodynamical simulations of Bate, Bonnell, & Bromm (2002) some multiple systems become dynamically unstable because their outer orbits shrink too fast due to accretion and disk braking. We may be witnessing such situation in GG Tau which interacts with its circum-quadruple disk and is now close to the stability limit (Beust & Dufrey 2006). A fraction of present-day triple stars may have been produced by dynamical decay of 2+2 quadruples. The triple star 4 Tau with $P_L = 33$ d and $P_L/P_S = 8$ could result from the outer-orbit migration which stopped just before the dynamical breakup.

4. Kozai cycles with tidal friction (KCTF) is not the dominant process in producing close sub-systems. For long initial $P_S$, this mechanism “turns on” in a very narrow range of relative inclinations near 90°. Moreover, at large $P_L/P_S$ ratios as commonly found in triple systems the Kozai effect becomes too weak and is perturbed by other factors such as relativistic precession. Hence, a population of inner systems with sufficiently short $P_S$ must be created by some other process before the KCTF can further shorten some periods to few days (Fabrycky & Tremaine 2007). This said, inner binaries with periods of $\sim 1$ d must be produced late, when the components have already settled on the MS, either by KCTF or by magnetic braking.

Figure 11 illustrates the scenario of quadruple-star formation with a $P_L - P_S$ diagram where some individual systems are located. The area of long inner and outer periods can be identified with the systems that did not evolve significantly after fragmentation. Otherwise, the points migrate to the lower left in this plot, faster in $P_S$ than in $P_L$ to stay always above the dynamical stability limit.

Figure 11. Schematic evolution of quadruple stars. The grey-scale background shows the smoothed version of the $P_L - P_S$ plot for quadruple systems (Fig. 4).

Cascade fragmentation with subsequent orbit evolution is a promising explanation for the origin of multiple stars, including tight quadruples. This scenario remains very sketchy, however, as many details are still not clear, many alternatives remain to be explored. From the theoretical side, simulations of accretion onto a multiple system with non-coplanar orbits are needed. Observationally, establishing the unbiased statistics of triple and quadruple stars in well-defined samples, such as nearby dwarfs, will be a crucial step in understanding the formation on these objects. We hope that the present catalogue-based study will stimulate such projects.

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Figure 11. Schematic evolution of quadruple stars. The grey-scale background shows the smoothed version of the $P_L - P_S$ plot for quadruple systems (Fig. 4).
APPENDIX A: TIGHT QUADRUPLES

A group of tight quadruples with log $P_L < 4.5$ is distinguished in the period-period plot (Fig. 3). They also tend to have comparable masses and periods of the inner sub-systems. These properties are already obvious at log $P_L < 5.4$, so the exact upper limit $P_L$ to distinguish tight quadruples from the rest is not known. We set this limit at $P_L = 100 \text{ yr}$ (log $P_L < 4.56$) and study the properties of such quadruples in more detail.

Table A1 lists all 20 known quadruples with $P_L < 100 \text{ yr}$: the WDS codes, alternate identifications, outer periods, apparent semi-major axes (or separations) of the outer orbits $a_L$, their eccentricities $e_L$, and primary masses $M_{1,1}$. Only 3 out of 20 quadruples still have no computed outer orbits (missing eccentricities in Table A1). In 7 systems there are 5 or 6 known components, like in 88 Tau. This information is complementary to the main Table 3.

The separations in the outer orbits are sub-arcsecond in 18 systems, so the study of the inner orbits did not rely on spatially resolving the outer pairs, but rather on the spectroscopy of components in the combined light. Short outer periods can be easily discovered by spectroscopy, therefore the sharp drop in the number of quadruples at log $P_L < 3$ must be genuine, not a selection effect. T. Pribulla (private communication) confirms that in a large sample of triple stars discovered spectroscopically the outer periods are mostly long and the tertiary companions are mostly single, so the quadruple VW LMi with $P_L = 355 \text{ d}$ stands out as a unique system.

APPENDIX B: MASS RATIOS WITH INDEPENDENT PAIRING

Is is established that the mass ratios in binary stars do not correspond to the random selection of component masses from the initial mass function (IMF) (e.g. Halbwachs et al. 2003, Kouwenhoven et al. 2007), the component masses rather tend to be comparable. The same tendency is expected in the multiple systems. Nevertheless, it is instructive to simulate mass ratio distributions resulting from the independent pairing and to compare them with the data. We used the numerical recipe of Fisher (2004) to generate the masses. Only triples with $M_1 > M_{\text{min}}$ and quadruples...
Figure B1. Mass ratios in 865 simulated triples (top) and 463 simulated quadruples (bottom) with independent component masses selected from the IMF. Compare with Fig. 5.

with $M_{1,1} > M_{\text{min}}$, with $M_{\text{min}} = 0.8 M_\odot$, were retained from a larger pool of simulated systems dominated by small masses. This is done to mimic our samples, although the distribution of simulated primary masses is more strongly concentrated towards $M_{\text{min}}$ than the actual masses (Fig. 2). The plots of the simulated mass ratios are shown in Fig. B1. The difference with the actual distributions is too strong to be attributed solely to selection. If the masses in the real multiple stars were indeed independent and we were missing a huge number of multiples with low mass ratios, it would mean that the actual frequency of multiple systems is many times higher than observed now. Systems with similar components are rare in the simulated multiples. The median simulated mass ratios are $q_{1,3} = 0.13, q_{1,4} = 0.29$. A loose correlation between the small values of $q_3$ and $q_{1,3}$ seen in the Figure is explained by the threshold imposed on the primary mass, while the other components most frequently have similarly low masses near the IMF peak.

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