Can a metaphor of physics contribute to MEG neuroscience research? Intermittent turbulent eddies in brain magnetic fields

Arnold J. Mandell *

Multi Media Imaging Laboratory (MMIL), Department of Psychiatry, UCSD School of Medicine, La Jolla, CA, United States
Fetzer Franklin Fund, Kalamazoo, Michigan, United States

A R T I C L E  I N F O

Article history:
Available online 10 June 2013

A B S T R A C T

A common manifestation of nonlinear mathematical and experimental neurobiological dynamical systems in transition, intermittence, is currently being attended by concepts from physics such as turbulent eddy and the avalanche of critical systems. Do these concepts constitute an enticing poetry of dynamical universality or do these metaphors from physics generate more specific novel and relevant concepts and experiments in the neurosciences? Using six graphics and ten measures derived from the ergodic theory of dynamical systems, we study the magnetoencephalic, MEG, records of taskless, “resting” human subjects to find consistent evidence for turbulent (chaotic) dynamics marked by intermittent turbulent eddies. This brings up an apparent discrepancy via the juxtaposition of the superposition characteristics of magnetic fields and the non-superposition properties of turbulent flow. Treating this apparent inconsistency as an existent duality, we propose a physical model for how that might be the case. This leaves open the question: has the physical metaphor, turbulent eddy, contributed to a scientific understanding of the human resting MEG?

1. The context

This contribution to the group of papers dedicated to the innovative, cross-disciplinary, integrative theoretician and essayist, Gerhard Werner, was inspired, in part, by the direction his “think pieces” have taken the field of theoretical neuroscience [1–3]. We are in an era in which a growing number of theoretical physicists are turning their attention to brain and behavior. At the same time, neuroscientists are beginning to explore nonlinear mechanics, dynamical systems and quantum physics for sources of concepts and models of brain processes. The emergence of these relatively new developments has begun to yield what appear to be new neurophysiological theories, experimental designs, mathematical analyses and their interpretations.

In a 1959 lecture, the Nobel Prize winning particle physicist, Eugene Wigner [4] spoke about the unreasonable effectiveness of mathematics in the natural sciences. He used examples from particle and quantum physics. In contrast, the late mathematician and fractalist, Benoit Mandelbrot, only partially in jest, spoke about the unreasonable ineffectiveness of physics in the biological sciences. It is our opinion that this “ineffectiveness” is, in part, because the metaphors from physics [5,6], though intuitively evocative, may sometimes be awkward and even misleading as thought tools with which to address the mechanics of neurobiological observables; like liquid Legos and waterfalls composed of toy blocks. On the other hand, some new frontiers of neuroscience research have taken their lead from areas of theoretical physics. We will look at an example from the growing list of physical metaphors shared by theoretical
physics and neuroscience research: electromagnetic turbulent eddies from turbulent hydrodynamics [7,8]. Quite similar intermittent phenomena are also metaphorically labeled avalanches derived from theories of self-organized criticality [9,10]. We will mention avalanches only in passing because, thus far, using only observations limited to the electromagnetic phenomenology from MEG and EEG recordings, we cannot discriminate between the critical behavior of avalanches [10] and the intermittent eddies of magnetohydrodynamic turbulence [11,12].

As noted, one of the common manifestations of nonlinear dynamical behavior encountered in the brain sciences (and too often hidden by the inappropriate use of averaging and the misapplication of non-relevant finite variance statistics [13,14]) involves the phenomenon of intermittence. After Manneville and Pomeau [15] it can be described as fluctuations of a real quantity in time in which the quantity is relatively constant, for example zero, for large periods of time, takes large values for relatively short intervals and then decays to zero until the next burst. In the renormalized parameter space of generic intermittent systems, burst length varies as the inverse square root of the distance of the value of the parameter from that value that elicited the system’s fixed point [16]. This dynamical state, encountered frequently in a variety of mathematical bifurcation scenarios [17,18], is also a common neurophysiological manifestation in the form of: (a) Single neuron bursting with irregular inter-burst intervals and sets of neuronal discharges of varying length; (b) In local field potentials, LFP and electromagnetic fields as in EEG and MEG, intermittence presents as irregularly appearing runs of varying lengths and electromagnetic wave characteristics interspersed by their absence over varying intervals [19–21].

2. Intermittence

Intermittence in a dynamical transition was first observed by Reynolds in the parallel flow of water through a circular pipe. His name remains on the real number
parameter, Re, representing the ratio of the forcing function to the dissipation function in pipe flow [29]. At some value of Re, what Reynolds called a turbulent flash of finite pipe length occurred and was convected downstream. The difference in the velocity of the leading and following edges of the flash increases continuously from zero such that the total length of the flash grows linearly as it goes along its laminar way [30]. The intermittency factor, the ratio of the mean length of the intermittent turbulent flashes to the total flow length goes to zero at the onset of the incipient transition.

One can regard the turbulent flashes of finite amplitude as a local solution of the time-dependent hydrodynamic flow equations that fuse imperceptibly with the spatially laminar flow both upstream and down. In the more general context of time-dependent partial differential equations (infinite dimensional) dynamics, the traveling location of the flash can be represented by $\xi = (x - vt)$. Here, $x$ is the longitudinal distance, $v$ the time, and $v$ is the relative velocity of the localized solution. Note that $\xi$ is at rest in the frame moving along $x$ with velocity $v$. It is intermittent to the non-moving observer watching the flow go by. With respect to the oscillatory fluctuations of neural membrane excitability thresholds responding to dynamical entropy (rank vector entropy) of brain electromagnetic fields related to positive entropy “up state” and negative entropy “down state” regulating burst frequency and density [31]. The Hodgkin–Huxley neural membrane equations [32] and other neuron models [33] manifest intermittent bursting in some parameter regimes. When the fixed point of a periodic trajectory (as a circle map, the first return map of a flow on the torus, $T^2$) loses its stability via the intermittency route [26], the mean interval between flashes (neuronal bursts) is relatively large.

The generality of the intermittency phenomena in transitions between dynamical states is also displayed in generic density dependent maps of the unit interval, $x \rightarrow 1 - rx^2$ in the parameter vicinity of $r \approx 7/4$ where most often the solution is an orbit of period three. However, the sudden and transient appearance of turbulent bursting can also be observed [15] in that general parameter regime as well. Turbulent intermittency was also found in the Lorenz climatology equations [34]. We have found intermittent dynamics in the “resting” state in human magnetoencephalographic, MEG, records [7]. In comparisons of the taskless MEG records from normal controls, schizophrenic proband and their unaffected siblings, intermittent bursting-like behavior in the em fields was more prominent in unmedicated siblings of schizophrenic patients, see Fig. 1 [35].

3. Invariant measures made on MEG intermittent turbulent dynamics

In the context of ergodic measure theory, intermittence affords a contrast between weak and strong mixing [36]. If $T$ is a map from a measurable space, $\Omega$, to itself and $\mu$ a $T$-invariant measure (same measure from any initial condition) on $\Omega$, let $A$ be a measurable

measurable subsets in $\Omega$, $A$ and $B$, $\mu(T^{-n} A \cap B) \rightarrow \mu(A)\mu(B)$. $T$ mixes the set $\Omega$, but once in a while, there appears large deviations from this average behavior. The probability of a large deviation is (formally) zero but they do occur in within large times. As a control parameter is changed continuously, the large deviations become more and more rare and then disappear. The unpredictable occurrence of intermittence has prevented closed form analytic solutions of the Navier–Stokes hydrodynamic equations in the relevant parameter regimes for many decades [37]. However, a pattern of multiple simultaneous invariant measures can be used to statistically characterize the intermittence condition [7,38,39].

Fig. 1 summarizes the results of the application of a suite of six graphics and ten ergodic (invariant) measures [8] made on seven consecutive 10 s intervals from central, C16, electrode pair differences (symmetric sensor difference series, ssds (7.8)) in the “resting” MEG record from a normal control (left) and an unaffected sibling of a schizophrenic proband (right) obtained from the archives of the NIMH Intramural Core MEG Facility [40]. In Fig. 1, reconstruction of the sequentially lagged orbits into three dimensional phase space portraits [41] established the setting for the study of the systems’ ergodic and related invariant measures [42]. These included the power spectral scaling exponent, $\chi$, [43] the leading Lyapounov exponent, $\Lambda$ [44,45], the topological entropy, $h_T$ [46,47], the metric entropy, $h_M$ [48,49] the capacity dimension, $D_c$ [50] an aggregate dynamical entropy measure, the measureable entropy manifold volume, $\text{memv}= (\lambda^* h_T')$, the dynamical non-uniformity descriptor, $U_n = [h_T - h_3] (11, 51)$, the unwinding number, $U_T$ (the number of lags to reach an asymptotic value for the correlation dimension, $D_2$) and the distribution function’s high moments, skew, $\lambda^3$, and kurtosis, $\lambda^4$. Values of these measures were averaged for each of these two subject over seven consecutive 10 s epochs and then the means of the values of each epoch were averaged for each subject. Mean and median of seven consecutive ten seconds of multiple approximate measure-theoretic characterizations of the phase space attractors of each subject using these invariant measures. Table 1 is a list of intuitive descriptions of the important invariant measures in our analyses.

An important Ornstein theorem offers proof that statistical invariant (ergodic) measures made on complicated (chaotic, turbulent) dynamical systems are entropy equivalent [70]. In the absence of a singular algorithm for its computation, we choose to use multiple “imperfect” measures that capture a variety of expressions of the attractor’s measure-theoretic entropy. Its for this reason that Fig. 1 is composed of six graphics and ten numerical measures under conditions of less (left side) and more (right side) intermittent turbulence. To see the leading dynamical pattern more clearly, the time series was transformed into an autocovariance matrix, its leading eigenvector computed and composed with the original sequence yielding (the displayed) eigenvector weighted time series as the Broomhead-King autocovariance eigenfunctions [52]. They both look very similar. Their wavelet plots, however, discriminate them. Note that the abscissa = time, ordinate = mother wavelet wavelength, vertically from bottom, short to top, long and color,
from blue to red, encodes the amplitude squared, power of the leading autocovariance eigenfunctions. A comparison of the wavelet plots demonstrate differences in the patterns of the cross-scale, short to long wavelength upward vertical cascades with “more” intermittency seen in the record on the right. The region to the left that is above both the eigenfunctions represent the log–log power (frequency) spectral transformations of the original data series. These manifest (the slope of the middle third plot) mean scaling exponents of $\alpha \approx 1.5$ which is similar to a Kolmogorov scaling exponent for intermittent hydrodynamic turbulence [27]. Unlike the Kolmogorov cascades, however, the wavelength of the power ranges from bottom, short wave lengths, to top, long wavelengths. The approximate $\alpha \approx 1.5$ is also the universal power spectral exponent of critical systems’ such as the sand pile-like avalanches observed in vitro and in vivo in multi-electrode studies of aggregates of neocortical neurons [9]. The right region above the graph of the eigenfunction is a box-like, aggregate, multiplicative volume entropy measure, MEMV, measurable entropy manifold volume, the leading Lyapunov exponent $x$ topological entropy. The dark space within each box indicative of the relative occupancy of the total possible entropy is smaller in the MEMV system (on the right) with greater intermittency. Increasing intermittence decreases Fig. 1’s multiple reflections of measure theoretic entropy.

In Fig. 1, above the log–log power spectra and the MEMV box, are the tables of the values of the ergodic invariant measures for comparison of the two states of less (left) and more (right) intermittence. The differences are consistent across measures indicating lower entropy-equivalent measures in the greater intermittence condition. Greater intermittence is associated with a decrease in the leading Lyapunov exponent, greater slope $\alpha$, decrease in topological and metric entropies and their (absolute difference–non uniformity), decrease in capacity dimension and MEMV. Also observed were larger unwinding numbers and larger third and fourth moments. All measures indicate an increase in turbulent intermittency, each measure reflecting differing dynamical expressions of the accompanying lower entropy.

Viewing the graphics below the wavelet plots in Fig. 1, we see that in the comparison of less (left side) and more (right side) intermittency, the phase space orbit (left) inscribes semi-uniform chaotic cyclic magnetic field line orbits whereas the more intermittent record (right) demonstrates more non-uniform magnetic field line orbits and a “knot” characteristic of intermittent bursting. On right in the graphics below the wavelet plots, the recurrence plots (return map plotted as a point as a function of how close–as arbitrarily defined–the current orbit is from the previous one [53]) demonstrate a relatively smooth, plane of time one against time two planar plots. On the right, dense aggregates of points indicate sequential closeness in time of the components of the bursts of intermittent dynamics. The difference in intermittence in these transformed resting MEG recordings are visualized more directly in the wavelet plots [7].

Graphs of the respective log–log frequency (power) spectra are seen above the BK eigenfunctions in which the slope of the red line delimited portion was computed as exponent $\alpha$ [12]. The qualitative results of the two lists of comparative invariant measures are summarized in the legend at the top of Fig. 1 and are characteristic of differences in intermittence in human brain MEG fields [7]. For comparison, Table 2 represents the statistical expectancies for these measures as determined from central, C16, electrode locations as carried out in 171 subject records obtained from the Archives of the NIMH Intramural Core MEG Facility [40].

## Table 1
Qualitative descriptions of invariant measures.

| a | Leading Lyapunov exponent, $A$, (divergence, expansivity and mixing). |
| b | Power spectral scaling exponent, $\alpha$, (slope $\leq 0$ of log-log power (frequency) spectrum). |
| c | Topological entropy $h_T$ (rate of new em field-line-loop formation, “orbital reconnections”). |
| d | Metric entropy, $h_M$, the distribution of statistical weights on the loops. |
| e | Capacity dimension, $D_c$, of manifold of support on which the dynamics take place. |
| f | Nonuniformity, $U_N = |h_T - h_M|$ of stretching, folding, (re)connection of magnetic field lines (Nonuniform hyperbolicity). |
| g | Measurable entropy manifold volume, $MEMV = (A \times h_T \times D_c)$ entropy as a multiplicative volume. |
| h | Unwinding number, $U_W$ (number of delays to “unwind” solenoidal spiral flow, lags to asymptotic correlation dimension). |
| i | Skew, $X^3$ (asymmetry of the probability distribution). |
| j | Kurtosis, $X^4$ (peakedness and heaviness of the tail of the probability distribution). |

## Table 2
Qualitative and quantitative descriptions of the properties indexed by the invariant measures.

| Invariant Measure | Phase Space | Computational Methods | Mean/SD $n = 171$ |
|------------------|-------------|----------------------|-------------------|
| Lyapunov, $A$    | Divergence of neighboring trajectories | Exponential rate of separation of nearby initial conditions, $A = 1 \rightarrow$ doubling in time between sample points | $0.4865 \pm 0.11$ |
| Topological entropy, $h(T)$ | Emergence of new orbits | Growth rate of the trace of exponentiated transition-incidence matrix | $0.8042 \pm 0.10$ |
| Capacity dimension, $D_c$ | Complexity of manifold of support | Slope log of fraction of hypercubes occupied/log linear size of hypercubes | $2.254 \pm 0.192$ |
| Spectral scaling exponent, $a (f^{-a})$ | Relative power of frequency modes | Slope of least squares fit of log frequency-log power plot of frequency (power) spectra | $1.50 + 0.34$ |
Of great interest is the many measure indication and confirmation of magnetic field turbulence including intermittent turbulent eddies in the human MEG [12]. Using the Robinson MEG beamformer localization technique [54] and a new entropic complexity measure, Rank Vector Entropy, RVD, signs of turbulence systematically localized in the expected, function-related manner [31,55].

4. Does the measure-consistent, physical metaphor of turbulent eddies in brain magnetic fields make a contribution to MEG neuroscience research?

These larger n findings (Table 2) confront us both with instructive commonalities as well as inconsistencies in this instance of the combination of theoretical physics and MEG-related, neuroscience theory and research [56]. With multiple measures consistent with that observed in intermittent turbulent flow in physical and chemical systems, it’s as though our implicit claim is that the Hodgkin–Huxley membrane [32,57] and the Navier–Stokes hydrodynamic partial differential equations [27,37,58] can be made dynamically equivalent. A recent successful effort at a formal synthesis [59] has serious limitation with respect to real world applications: it required unphysical linearization.

Neuroscience’s interest in and use of electromagnetic, em, signals almost always involve questions of localization of their source. In the brain and behavioral neurosciences, a neurological location has often served as a brain mechanism. This effort at inverse modeling involves locating the magnetic source using two general strategies: (a) Find some corresponding neuroanatomical source as a function of time (b) Use reversible transformations, decompositions and filters to isolate components of the complicated brain signals that correspond as a function of time.

Given the large number of ergodic (invariant) MEG measures consistent with the dynamics of intermittent turbulent flow (Tables 1 and 2 and Fig. 1) [7,8,12,60,61], we find time-dependent patterns that are not theoretically linearly additive nor with the justifiable assumptions of their superposition [26,62]. At the same time, the source estimations using forward and inverse modeling of [63] the magnetic signals of the MEG assume that the B, (B = scalar magnetic) and E (E = vectorial electrical) fields accompanying the impressed and return currents of pyramidal cell dendritic fields are the results of the linear superposition of a multiplicity of sources [54,64–66]. Often, the MEG analyses are in terms of theoretical magnetic dipoles as the limit of closed loops of cortical pyramidal dendritic (impressed and recurrent) electrical current loops. The resolution of this physics-neuroscience inconsistency remains to be achieved. How could turbulent dynamics occur in magnetic fields that require signal superposition.

Figs. 2 and 3 are graphical representations of a conjectured global and local dynamic for the generation of intermittent turbulent magnetic fields using a model derived from nonlinear magnetohydrodynamics dynamo theory [11,51,58,67–69].
The idealized emergence of an idealized turbulent magnetic dynamo. The brain magnetic field(s) in spherical coordinates and decomposed into orthogonal poloidal and toroidal planes, the sequence illustrates classical chaos engendering, shear-induced stretching and folding, magnetic field disconnections and reconnections and finally the time-dependent structure of scale free turbulence.

5. Conclusions

1. Very weak human neocortical magnetic fields ($10^{-15}$–$10^{-11}$ T) manifest phase space, power spectral, recurrence plot, wavelet space, symbolic dynamic and multiple ergodic (invariant) measures including leading Lypounov exponent, capacity dimension, topological and metric entropy, higher statistical moments and power scaling exponent $x$ that are consistent with intermittent, magnetohydrodynamics, MHD, turbulence.

2. The composition of the weak, scalar magnetic field implicates the property of superposition. The magnetic field as a turbulent dynamical system implies non-superposition. This apparent inconsistency in observables-analyses relations awaits resolution.

3. An Ornstein theorem [70] proved that any effective measure of a chaotic dynamical system was measure theoretically equivalent to its entropy. Not knowing how to definitively calculate that, we instead seek consistency in 14 only partially adequately variously equivalent statistical measures. Literature since 1987 [71] has suggested that responsive biological systems optimize their entropy in “readiness” for function and manifest lost entropy with pathological reduction in regulatory responsiveness.

4. The nonlinear dynamical characteristics of the very weak neocortical in-origin magnetic field suggest the possibility of their sensitivity to parametrically optimized driving. It is speculated that transcranial magnetic stimulation many orders of magnitude less than is currently being used in treatment refractory depression [72] may be effective in increasing the rate of turbulent generation of entropy in the human brain scalar magnetic fields.

Acknowledgements

My appreciation goes to the Fetzer Franklin Trust, Jan Walleczek and Bruce Fetzer whose encouragement and support made this work possible. To Eric Halgren and Anders Dale who gave me an academic home in the Multime-dia Imaging Laboratory, MML, at UCSD. To Richard Coppola who earlier graced me with a temporary new home in his laboratory at NIMH. The most pleasurable thanks go to my ever stimulating intellectual playmate, Stephen Robinson of the NIMH MEG group.

References

[1] Werner G. Metastability, criticality and phase transitions in brain and its models. Biosystems 2007;90:496–508.
[2] Werner G. Consciousness related neural events viewed as brain state space transitions. Cognit Neurodyn 2009;3:83–95.
[3] Werner G. Fractals in the nervous system: conceptual implications for the theoretical neurosciences. Front Physiol 2010;1:1–18.
[4] Wigner E. The unreasonable effectiveness of mathematics in the natural sciences (Courant Lecture, NYU, May, 1959). Commun Pure Appl Math 1959;13:1–14.
[5] Bak P. How nature works: the science of self-organized criticality. NY: Copernicus; 1996.
[6] Chialvo D. Emergent complex neural dynamics. Nat Phys 2010;6:744–50.
[7] Mandell A, Selz K, Holroyd T, Rutter L, Coppola R. Intermittent vorticity, power spectral scaling and dynamical measures on resting brain magnetic field fluctuations. In: Ding M, Glanzman D, editors. The dynamic brain. Oxford: Oxford University Press; 2011. p. 306–337.
[8] Mandell AJ, Selz KA, Aven J, Holroyd T, Coppola R. Daydreaming, thought blocking and strudels in the taskless, resting human brain’s magnetic fields. In: Longhini P, Palacios A, editors. International conference on applications in nonlinear dynamics (ICAN II), V. Lake Louise, Alberta, Canada: American Institute of Physics; 2011. p. 7–24.
[9] Beggs J, Plenz D. Neuronal avalanches are diverse and precise activity patterns that are stable for many hours in cortical slices. J Neurosci 2002;24:5216–519.
[10] Petermann T, Thiagarajana TM, Bebedev M, Miguel A, Nicolesil A, Chialvo D, et al. Spontaneous cortical activity in awake monkeys composed of neuronal avalanches. Proc Natl Acad Sci 2009;106:15921–6.
[11] Ott E, Du Y, Sreenivasan KR, Juneya A, Suri AK. Sing-singular measures: fast magnetic dynamos and high Reynolds-number fluid turbulence. Phys Rev Lett 1992;60.
[12] Mandell AJ, Selz K, Schrader C, Holroyd T, Coppola R. The turbulent human brain. In: Plenz D, Niebur E, Schuster P, editors. Criticality in neural systems. NY, Bethesda, MD: Wiley; 2013.
[13] Montroll E, Shlesinger M. On the wonderful world of random walks. In: Lebowitz J, Montroll E, editors. Non-equilibrium phenomena II. From stochastics to hydrodynamics. Amsterdam: North-Holland; 1984. p. 1–121.
[14] Shlesinger M. Fractal time in condensed matter. Ann Rev Phys Chem 1989;39:269–90.
[15] Mannevile P, Pomeau Y. Different ways to turbulence in dissipative dynamical systems. Physica D 1980;1:219–32.
[16] Pomeau Y. In: Approche de la Turbulence In Conference at Clermont Ferrand. Editions de Physique. Orsay-Courtaboeuf; 1981.
[17] Hirsch JE, Huberman BA, Scalapino D. Theory of intermittency. Phys Rev A 1982;25:519–32.
[18] Berge P, Pomeau Y, Vital C. Order and chaos chapter IX. New York/Paris: IntermittencyJohn Wiley/Hermann; 1984.
[19] Mandell AJ, Selz K, Rutter L, Holroyd T, Coppola R. Intermittent magnetic fields. In: Ding M, Glanzman D, editors. The dynamic brain. Oxford: Oxford University Press; 2011. p. 296–337.
[20] Chialvo D. Emergent complex neural dynamics. Nat Phys 2009;106:15921–6.
[21] Belousov EV, Frisch U. Turbulence the legacy of A.N. Kolmogorov. Cambridge, U.K.: Cambridge University Press; 1995.
[22] Bak P. How nature works: the science of self-organized criticality. NY: Copernicus; 1996.
[23] Bak P. How nature works: the science of self-organized criticality. NY: Copernicus; 1996.
[24] Field R, Burger M. Oscillations and traveling waves in chemical reactions. In: Lebowitz J, Montroll E, editors. Non-equilibrium phenomena II. From stochastics to hydrodynamics. Amsterdam: North-Holland; 1984. p. 1–121.
[25] Shlesinger M. Fractal time in condensed matter. Ann Rev Phys Chem 1988;39:269–90.
[26] Mannevile P, Pomeau Y. Different ways to turbulence in dissipative dynamical systems. Physica D 1980;1:219–32.
[27] Pomeau Y. In: Approche de la Turbulence In Conference at Clermont Ferrand. Editions de Physique. Orsay-Courtaboeuf; 1981.
[28] Hirsch JE, Huberman BA, Scalapino D. Theory of intermittency. Phys Rev A 1982;25:519–32.
[29] Berge P, Pomeau Y, Vital C. Order and chaos chapter IX. New York/Paris: IntermittencyJohn Wiley/Hermann; 1984.
[30] Mandell AJ, Selz K, Rutter L, Holroyd T, Coppola R. Intermittent magnetic fields. In: Ding M, Glanzman D, editors. The dynamic brain. Oxford: Oxford University Press; 2011. p. 296–337.
