Fast localization of underground targets by magnetic gradient tensor and Gaussian-Newton algorithm with a portable transient electromagnetic system

Lijie Wang¹, Shuang Zhang¹, Shudong Chen¹, Hejun Jiang²

¹College of Electronic Science and Engineering, Jilin University, Changchun 130012, China
²Science and Technology on Near-Surface Detection Laboratory, Wuxi 214035, China

Corresponding authors: Shudong Chen (e-mail: chenshudong@jlu.edu.cn) and Hejun Jiang (e-mail: jhj68@126.com)

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ABSTRACT Differential evolution (DE) algorithm, which is a global convergence algorithm, is often used to estimate the position of underground targets detected with a portable transient electromagnetic (TEM) system. The DE algorithm is extremely time-consuming due to thousands of iterations. A new algorithm for fast localization of an underground target by magnetic gradient tensor and Gaussian-Newton algorithm with a portable TEM system is proposed. First, the gradient tensor of an underground target is constructed with the differential responses received by the portable sensor. Gradient tensor, commonly used in magnetic detection, is applied for the first time in TEM detection to estimate the target position for each measurement. Then, all the estimated positions are averaged to reduce the localization error. Taking the averaged position as the initial value, the Gaussian-Newton algorithm can complete iterations within dozens of times, which can effectively improve the speed and accuracy of the algorithm. Finally, the performance of the new method has been verified in the test-stand and field experiments. Results show that the errors of averaged positions by gradient tensor are no more than 8 cm in the horizontal direction. The errors of the estimated positions, inclination, and rotation angles by the Gaussian-Newton algorithm are no more than 4 cm, 6°, and 5°, respectively. The statistical running time of the proposed method takes approximately tens of milliseconds, accounting for about 7% of the DE algorithm. The proposed method can achieve fast and accurate localization and characterization of targets and has an important significance to the digging and recognition of underground targets.

INDEX TERMS Transient electromagnetic (TEM), Unexploded Ordnance (UXO), Magnetic gradient tensor, Gaussian-Newton algorithm.

I. INTRODUCTION

Unexploded ordnance (UXO) contamination has caused great suffering to people all over the world and hindered land use and construction [1,2]. The clearance of UXO is a longstanding economic and humanitarian problem [3]. Large amounts of metallic debris are scattered around the UXOs, which will cause some interference to the detection of UXOs. Thus, the fast and accurate detection of UXOs has important practical significance.

In recent years, various types of detection methods have been developed, such as magnetometers [4-6], electromagnetic induction (EMI) [7-10], and ground-penetrating radars [11,12]. Compared with magnetometers, EMI systems in frequency and time domains, in which operating frequency is tens to hundreds of kilohertz, are less affected by geological influences and have higher intrinsic discrimination ability [13,14]. Transient electromagnetic (TEM) detection has been proven to be an effective target...
In this work, a fast localization method of underground targets combining magnetic gradient tensor and Gaussian-Newton algorithm is proposed. Magnetic gradient tensor is applied for the first time in TEM detection to locate the target. Taking the localization result of the gradient tensor as the initial position, the Gauss-Newton algorithm can quickly and accurately complete the inversion.

The rest of the structure is organized as follows. Section II introduces the Jilin University portable metal detector (JPMD) system and the single dipole model. Section III presents magnetic gradient tensor and the Gaussian-Newton algorithm, and describes the target characterization. Section IV is the experimental design and analysis. Section V summarizes this paper.

II. SENSOR AND SINGLE DIPOLE MODEL

In this paper, all data are obtained based on the JPMD sensor. The structure and parameters of the JPMD sensor will be first introduced for data acquisition. Then, the single dipole model for data processing will be described.

A. PORTABLE SENSOR

The structure and picture of the JPMD sensor are shown in Figure 1. The sensor consists of a single-layer transmitting coil and five three-component receiving coils. The diameter, height, and number of turns of the transmitting coil are 50 cm, 7 cm, and 30, respectively. The height of the five receiving coils (R1 to R5) is 6 cm. R3 is in the center of the transmitting coil, and the distance between R3 and the other four receiving coils is 20 cm. Each receiving coil is wound in four sections, and the center tap is grounded to improve the common-mode noise. A double-layer interlaced shielding structure is also designed to suppress electrical interference. The amplitude, period, and duty-cycle of the emission current are 5 A, 80 ms, and 50%, respectively. The turn-off time of the emission current is about 40 μs.

B. SINGLE DIPOLE MODEL

The underground target can be equivalent to a single dipole model for data processing [21]. Figure 2 is the principle of target detection based on the single dipole model.

In Figure 2, the secondary magnetic field \( B_5 \) of the dipole moment \( m \) at the receiving coil position \( r \) is written as [22]

\[
B_5 = \frac{1}{4\pi |R|} (3n - I) m = G(R)m ,
\]

where \( n \) and \( I \) are the unit vector and identity matrix, respectively. The direction of \( n \) is consistent with \( R = r - r_0 \). \(|R|\) represents the modulus of \( R \). \( G(R) \) is Green’s function related...
only to the target position. The dipole moment \( m \) is represented by the magnetic polarizability tensor \( M \) and the primary field \( B_p \) as
\[
m = MB_p(r_d),
\]
where \( M \) is a symmetric matrix, which depends upon the shape, size, orientation, permeability, and conductivity of target. According to Equations (1) and (2), the target response \( V \) is calculated as
\[
V = -G(R) \frac{dM}{dt} B_p(r_d) = G(R)L(t)B_p(r_d),
\]
where \( L(t) \) is the characteristic matrix, which is the opposite of the time derivative of \( M \).

Based on Equation (3), the target response at \( N \) different measurements is written as [21]
\[
V = \begin{bmatrix}
V_1 \\
\vdots \\
V_N
\end{bmatrix} = \begin{bmatrix}
\gamma_1(r_d) \\
\vdots \\
\gamma_N(r_d)
\end{bmatrix} p(t) = \begin{bmatrix}
\gamma_1(r_d) \\
\vdots \\
\gamma_N(r_d)
\end{bmatrix} p(t),
\]
where the coefficient matrix \( \gamma \) only depends on the target position \( r_d \), and its size is \( 15 \times 6 \). \( p(t) \) is the target characteristic vector, which is composed of six elements \( (L_{x0}, L_{y0}, L_{z0}, L_{xy}, L_{xz}, L_{yz}) \) of the characteristic matrix \( L(t) \).

Localization and characterization are key problems in response inversion. In this paper, first, the position of an underground target is estimated by gradient tensor and Gaussian-Newton algorithm. Then, the characterization is calculated on the basis of the target position.

III. INVERSION WITH MAGNETIC GRADIENT TENSOR AND GAUSSIAN-NEWTON ALGORITHM

In this section, the localization and characterization of an underground target with the JPMD sensor are discussed on the basis of the nine measurements distributed in a 3x3 grid. As shown in Figure 3, the distance between the measurements is 25 cm. Traditionally, the DE algorithm is used to estimate the localization when all the responses in nine positions are measured. The population size of the DE algorithm is generally set to 20 times the dimension of the position variable, and the generation is set from tens to hundreds. All these factors will lead to thousands of iterations, which makes the DE algorithm extremely time-consuming. In order to improve the efficiency of target detection, a new algorithm combined with gradient tensor and Gaussian-Newton is proposed in this work.

Figure 4 is the flowchart of the target localization and characterization. The parameters of underground targets, including position and the characteristic vector, can be estimated by the gradient tensor and Gaussian-Newton algorithm. First, the gradient tensor is constructed by measured data in each measurement. Then, all estimated positions are averaged as an initial value of the Gaussian-Newton algorithm to quickly and accurately estimate the position and characteristic vector of the target. The characteristic response can be obtained by performing singular value decomposition (SVD) on characteristic matrix \( L(t) \) constructed from the characteristic vector \( p(t) \).

A. MAGNETIC GRADIENT TENSOR

The relative position vector \( R \) of the sensor to target can be calculated by the magnetic field \( B_s \) and the magnetic gradient tensor \( G \) [34], which is given by
\[
R = -G^{-1}B_s,
\]
where 15 responses in each measurement are used to construct the magnetic gradient tensor \( G \).

The gradient tensor \( G \) is a symmetric matrix, which is written as
\[
G = \begin{bmatrix}
\frac{\partial B_s}{\partial x} & \frac{\partial B_s}{\partial y} & \frac{\partial B_s}{\partial z} \\
\frac{\partial B_s}{\partial x} & \frac{\partial B_s}{\partial y} & \frac{\partial B_s}{\partial z} \\
\frac{\partial B_s}{\partial x} & \frac{\partial B_s}{\partial y} & \frac{\partial B_s}{\partial z}
\end{bmatrix},
\]
where the sum of diagonal elements is zero. Hence, only five independent components need to be calculated.
Figure 5. Flowchart of the target localization.

Magnetic field $B_s$ and gradient tensor $G$ of the target can be calculated according to the sensor. The coordinate system is established for five receiving coils shown in Figure 3(b).

In Figure 3(b), the center coil R3 of the JPMD sensor records the three components of magnetic field $B_s$. The five elements of gradient tensor $G$ are calculated on the basis of the difference of receiving coils R1, R2, R4, and R5, which can be calculated as

$$
\begin{align*}
\frac{\partial B_x}{\partial x} &= (B_{x_3} - B_{x_2})/2d \\
\frac{\partial B_y}{\partial y} &= (B_{y_2} - B_{y_3})/2d \\
\frac{\partial B_z}{\partial z} &= (B_{z_4} - B_{z_5})/2d \\
\frac{\partial B_x}{\partial y} &= -(\frac{\partial B_y}{\partial x} + \frac{\partial B_x}{\partial y})
\end{align*}
$$

(7)

Based on Equations (5) to (7), the target position vector $R$ in each measurement can be calculated by the gradient tensor $G$ and the magnetic field $B_s$. The estimated positions in all measurement will be averaged as an initial value of the Gaussian-Newton algorithm to improve the accuracy and convergence speed of the inversion.

B. GAUSSIAN-NEWTON ALGORITHM

Taking the gradient tensor localization result as the initial position, the Gaussian-Newton algorithm is applied to improve the localization accuracy and performs iteration by introducing the residual matrix $F$ and the Jacobian matrix $J$. The residual matrix includes target position and characteristic vector, constructed from the measured data and the forward model in Equation (4), which can be written as

$$
F = V_{obs} - \gamma p,
$$

(8)

where $V_{obs}$ denotes the target responses in nine measurements with five three-component receiving coils, which size is 135x1. The Jacobian matrix $J$ is related to the negative derivative of the residual matrix, which is given by

$$
J = -\frac{\partial F(r \gamma)}{\partial \gamma}.
$$

(9)

The iteration equation of the target position is written as

$$
r^{(k+1)}_t = r^{(k)}_t + \Delta r^{(k)},
$$

(10)

where the incremental $\Delta r$ is described by the residual matrix and Jacobian matrix, which can be given by

$$
\Delta r^{(k)} = (J^TJ)^{-1} J^T F(r^{(k)}).
$$

(11)

Figure 5 is a flowchart of target localization based on the Gaussian-Newton algorithm. The inversion process can be described as follows.

Step 1, input the measured data $V_{obs}$ and the averaged position calculated by magnetic gradient tensor localization.

Step 2, calculate the matrix $\gamma$ according to the target position and then use the matrix $\gamma$ and the measured data to calculate the optimal characteristic response $p(t)$ corresponding to the target position, which is calculated as

$$
p(t) = \left[\gamma(r_1)\gamma(r_2)\right]^{T} Y_{obs}.
$$

(12)

Step 3, calculate the residual matrix and Jacobian matrix according to Equations (8) and (9).

Step 4, update the target position based on Equations (10) and (11).

Finally, repeat step 2 to step 4 until the iteration stops and output the estimated target position.

C. TARGET CHARACTERIZATION

The characteristic matrix $L(t)$ constructed from the characteristic vector $p(t)$ can be expressed by SVD as

$$
L(t) = A \begin{bmatrix}
I_p(t) & 0 & 0 \\
0 & l_{i_1}(t) & 0 \\
0 & 0 & l_{i_2}(t)
\end{bmatrix} A',
$$

(13)

where $I_p(t)$ represents the primary polarizability of the long axis of the target; $l_{i_1}(t)$ and $l_{i_2}(t)$ are the secondary polarizabilities perpendicular to the long axis of the target. The secondary polarizabilities of axisymmetric targets are highly consistent. The Euler rotation tensor $A$ depended upon the angle between the target principal axis and the coordinate system, which is given by

$$
A = \begin{bmatrix}
cos(\phi)cos(\theta) & -sin(\phi) cos(\phi)sin(\theta) \\
sin(\phi) cos(\phi) & cos(\phi) sin(\phi) sin(\theta) \\
-sin(\theta) & 0 & cos(\theta)
\end{bmatrix},
$$

(14)

where $\theta$ represents the inclination angle between the axisymmetric target principal axis and the $z$-axis of the coordinate system; and $\phi$ is the rotation angle between the projection of the axisymmetric target principal axis in the XY plane and the $x$-axis of the coordinate system.

The target characteristic response $I(t)$ can be expressed as

$$
I(t) = \begin{bmatrix}
l_p(t) \\
l_{i_1}(t) \\
l_{i_2}(t)
\end{bmatrix}.
$$

(15)

IV. EXPERIMENTAL RESULTS

In the experiments, the performance of the method proposed in this work will be verified. Besides, we introduce the traditional DE algorithm and compared it with the proposed method. The population size of the DE algorithm is set to 60, and the generation is set to 50.

Figure 6 shows the experimental targets, and the detailed parameters are illustrated in Table I. In Figure 6, eight UXOs and four harmless objects are numbered from U1 to U8 and O1 to O4, respectively. In Table I, the length of UXOs U1 to U8 is 18 cm to 51 cm, and the outer diameter is 37 mm to 100 mm. Two pipes, namely, O1 and O2, with a diameter of 75 mm are 30 and 20 cm in length. O3 with a length of 12.5 cm is a three-way connector, and O4 with a diameter of 64 mm is a steel ball.

A. TEST-STAND EXPERIMENT
The inclination and rotation angles estimated as the initial value of the Gaussian decrease with the increase of depth. The electromagnetic characteristics of the target can be estimated by the gradient tensor. When the depth increases to 70 cm, the SNR of the response fluctuates. Meanwhile, the errors of horizontal direction and depth are reduced to less than 3 and 4 cm, respectively. The position and posture difference between the estimated and the true values can be negligible.

As shown in Table II, the maximum localization error of the average position of the gradient tensor in depth is 28 cm when the target depth is 30 cm, which is less than the baseline distance. The maximum localization error in depth is only 17 cm when the target depth is 70 cm. The maximum horizontal error of the gradient tensor is 8 cm. Results show that when the target depth is shallow, the magnetic gradient tensor constructed from the measured data in the TEM detection has a large error, resulting in a large localization error.

With further optimization by the Gaussian-Newton algorithm, the localization errors of horizontal direction and depth are reduced to less than 3 and 4 cm, respectively. Meanwhile, the errors of the inclination and rotation angles are no more than 6°. The position and posture difference between the estimated and the true values can be negligible.

In Table II, the average time of the DE algorithm takes approximately 1.12 s, and the average time of the proposed algorithm takes approximately 0.072 s, which is only about 7% of the DE algorithm.

The electromagnetic characteristics of the target can be determined according to the target position. The calibrated electromagnetic characteristics are applied for comparison [36].

For the convenience of discussion, the characteristic responses are expressed in Arb. In Figures 10 and 11, the target response is sampled from 60 μs to 20 ms.

As shown in Figure 10 and 11, the characteristic responses \( b(t), l_{x1}(t), \) and \( l_{z2}(t) \) are highly consistent with the calibrated results, except that the target is located at a depth of 30 cm in
the vertical orientation. In Figure 10(a), the target head is closer to the sensor. In this case, the characteristic response $l_p(t)$ mainly reflects the response of the target head. Therefore, the inverted characteristic response $l_p(t)$ is smaller than the calibrated result. The amplitudes of the electromagnetic characteristics at different depths are highly consistent and will not change with the decrease or increase in depth. In the late time, the signal will fluctuate with the depth increase. In summary, the results show that the method proposed in this work can quickly and accurately locate and characterize the underground target.

**B. FIELD EXPERIMENT**

In the field experiment, first, use the survey mode to roughly determine the horizontal position of the buried target according to the abnormal response amplitude, and then lay a 3×3 grid directly above this position for measurement. The measurement area is 50 cm×50 cm for each target, and the measurement interval is 25 cm. An underground target is

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**FIGURE 8.** Localization results of the gradient tensor for an 82 mm mortar shell with θ=0° at different depths.

**FIGURE 9.** Localization results of the gradient tensor for an 82 mm mortar shell with θ=90° at different depths.

**TABLE II**

| Truth orientation (θ, φ) (°) | Estimated angles (°) | Angle errors (°) | True positions (cm) | Gradient tensor localization (cm) | Gaussian-Newton localization (cm) | Localization errors (cm) | Proposed algorithm time (s) | DE algorithm time (s) |
|-----------------------------|----------------------|------------------|---------------------|-------------------------------|-------------------------------|------------------------|--------------------------|-------------------------|
| (0, 0)                      | (0, 0)               | 0, 0, 0          | (25, 25, 0)         | (20, 0, 0)                    | (0, 0, 0)                     | 0, 0, 0                | 0.004                    | 0.006                   |
| (0, π)                      | (0, 0)               | 0, 0, 0          | (25, 25, 0)         | (20, 0, 0)                    | (0, 0, 0)                     | 0, 0, 0                | 0.004                    | 0.006                   |
| (π, 0)                      | (0, 0)               | 0, 0, 0          | (25, 25, 0)         | (20, 0, 0)                    | (0, 0, 0)                     | 0, 0, 0                | 0.004                    | 0.006                   |
| (π, π)                      | (0, 0)               | 0, 0, 0          | (25, 25, 0)         | (20, 0, 0)                    | (0, 0, 0)                     | 0, 0, 0                | 0.004                    | 0.006                   |
| (0, 0)                      | (0, 0)               | 0, 0, 0          | (25, 25, 0)         | (20, 0, 0)                    | (0, 0, 0)                     | 0, 0, 0                | 0.004                    | 0.006                   |
| (0, π)                      | (0, 0)               | 0, 0, 0          | (25, 25, 0)         | (20, 0, 0)                    | (0, 0, 0)                     | 0, 0, 0                | 0.004                    | 0.006                   |
| (π, 0)                      | (0, 0)               | 0, 0, 0          | (25, 25, 0)         | (20, 0, 0)                    | (0, 0, 0)                     | 0, 0, 0                | 0.004                    | 0.006                   |
| (π, π)                      | (0, 0)               | 0, 0, 0          | (25, 25, 0)         | (20, 0, 0)                    | (0, 0, 0)                     | 0, 0, 0                | 0.004                    | 0.006                   |
excited nine times with the JPMD system. The test site is on the campus of Jilin University, and all targets used in the field experiment are shown in Figure 6. The results of the target localization are shown in Table III.

In Table III, the errors of inclination and rotation angles are no more than 4°, and the localization errors of horizontal position and depth are no more than 4 and 3 cm, respectively. The estimated target positions are highly close to the true positions. Meanwhile, with the averaged position of gradient tensor-based localization as the initial value, the Gaussian-Newton algorithm only takes tens of milliseconds.

In the field experiment, the average time of the proposed method is approximately 0.057 s, which is only 5% of the DE algorithm. The electromagnetic characteristics of the target can be calculated based on the target position. As shown in Figure 12, the signal is sampled from 60 μs to 20 ms. The characteristic responses $l_0(t)$ and $l_2(t)$ perpendicular to the target principal axis are highly consistent for most axisymmetric targets. The characteristic response $l_0(t)$ decays slower than $l_1(t)$ and $l_2(t)$ and is higher than $l_1(t)$ and $l_2(t)$.

For the three-way connector, the characteristic responses $l_0(t)$, $l_1(t)$, and $l_2(t)$ of the 64 mm steel ball are highly fitted. The amplitudes of the characteristic responses of UXOs and harmless targets are slightly different in the early stages. Characteristic response $l_2(t)$ of UXO decays slower than that of harmless targets. The duration with high SNR of the characteristic response for UXOs is about 20 ms, while the duration with high SNR of characteristic response for harmless targets is about 2-3 ms. Based on late response, UXOs and harmless targets can be effectively classified.

In general, the proposed method can achieve fast and accurate localization and characterization of underground targets, which considerably improves detection efficiency.

FIGURE 10. Inverted characteristic response of the 82 mm mortar shell with an inclination of 0° at different depths: (a) 30 cm, (b) 50 cm, and (c) 70 cm.

FIGURE 11. Inverted characteristic response of the 82 mm mortar shell with an inclination of 90° at different depths: (a) 30 cm, (b) 50 cm, and (c) 70 cm.

TABLE III

| Numbers | True angles (°) | Estimated angles (°) | Angle errors (°) | True positions (cm) | Estimated positions (cm) | Positions errors (cm) | Proposed algorithm time (s) | DE algorithm time (s) |
|---------|----------------|---------------------|-----------------|--------------------|-------------------------|------------------------|---------------------------|----------------------|
| U1      | (90, 0)        | (88, 0)             | (~2, 0)         | (25, 25, −33)      | (26, 27, −36)           | (1, 2, −3)             | 0.059                     | 1.172                |
| U2      | (60, 0)        | (63, 0)             | (3, 0)          | (25, 25, −33)      | (29, 26, −36)           | (4, 1, −3)             | 0.079                     | 1.171                |
| U3      | (60,0)         | (62,0)              | (2, 0)          | (25, 25, −37)      | (26, 26, −38)           | (1, 1, −1)             | 0.058                     | 1.164                |
| U4      | (0,0)          | (0,0)               | (0, 0)          | (25, 25, −39)      | (27, 26, −37)           | (2, 1, 2)              | 0.057                     | 1.165                |
| U5      | (90, 0)        | (88, 0)             | (~2, 0)         | (25, 25, −35)      | (28, 26, −38)           | (3, 1, −3)             | 0.066                     | 1.176                |
| U6      | (90, 0)        | (86, 0)             | (~4, 0)         | (25, 25, −35)      | (26, 26, −37)           | (1, 1, −2)             | 0.045                     | 1.182                |
| U7      | (45,0)         | (46,1)              | (1, 1)          | (25, 25, −40)      | (22, 26, −37)           | (3, 1, 3)              | 0.056                     | 1.172                |
| U8      | (90, 0)        | (87,0)              | (~3, 0)         | (25, 25, −35)      | (24, 26, −37)           | (~1, 1, −2)            | 0.071                     | 1.186                |
| O1      | (0,1)          | (3,1)               | (3, 1)          | (25, 25, −40)      | (24, 23, −39)           | (~1, −2, 1)            | 0.048                     | 1.171                |
| O2      | (90,90)        | (88,89)             | (~2, −1)        | (25, 25, −35)      | (24, 26, −36)           | (~1, −1)               | 0.049                     | 1.169                |
| O3      | (0,1)          | (0,0)               | (0, 0)          | (25, 25, −38)      | (24, 22, −37)           | (~1, −3)               | 0.047                     | 1.161                |
| O4      | (0,1)          | /                   | /               | (25, 25, −40)      | (26, 26, −42)           | (1, 1, −2)             | 0.052                     | 1.152                |
FIGURE 12. Inverted target characteristic responses in the field experiment.

V. CONCLUSIONS

A fast localization method for underground targets by magnetic gradient tensor and Gaussian-Newton algorithm based on the JPMD system is proposed in this work.

Magnetic gradient tensor has been applied for the first time in UXO detection based on TEM systems, which can locate the underground target by a single measurement in space. The test-stand experimental results show that the estimated target positions by gradient tensor fluctuate tens of centimeters in horizontal direction and depth. The error of the averaged positions with nine measurements is much smaller. The maximum horizontal error is no more than 8 cm. The depth error is 17 cm when the target is 70 cm in depth and 28 cm when the target is 30 cm in depth. Results show that the localization accuracy obtained by the Gauss-Newton algorithm is much higher than the localization based on the magnetic gradient tensor. Therefore, the averaged position of the gradient tensor can be used as the initial value of the Gaussian-Newton algorithm, which can ensure that the Gaussian-Newton algorithm quickly and accurately converges to the true position, so that the magnetic gradient tensor can be used for target localization in TEM detection.

The test-stand experimental results show that the localization errors of horizontal direction and depth are no more than 4 cm, and the errors of angles are no more than 6°.
The localization accuracy is not affected by the depth and posture of the target. The characteristic response \( I_p(t) \) of the 82 mm mortar shell under different conditions is highly consistent with the calibrated results. The field experimental results show that the localization errors of horizontal direction and depth are no more than 4 cm, and the errors of inclination and rotation angles are no more than 4\(^\circ\). The maximum running time is no more than 80 ms, which is only about 7\% of the traditional method. The estimated characteristic responses \( I_p(t) \) and \( I_c(t) \) of the most axisymmetric targets are highly consistent.

In summary, magnetic gradient tensor, which has been widely used in magnetic detection, can also be used to detect UXOs based on the TEM system. The method proposed in this work can quickly and accurately locate and characterize the underground target, which effectively improves detection efficiency and provides the possibility for real-time localization and characterization of underground targets.

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