Experimental Investigation of Silo Stresses under Consideration of the Influence of Hopper/Feeder Interface

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Abstract

With the data gathered from an experimental silo, operating with two different kinds of bulk solid (cohesive limestone powder and free-flowing plastic pellets), it is shown to what great extent the geometry of the transition from hopper to feeder influences flow profile and thus the stress pattern within the hopper. The optimal geometry for achieving mass flow and even withdrawal of a bulk solid over the entire outlet area depends on the bulk solid’s flow properties and, in the case of the cohesive limestone powder, must be closely adhered to, as the flow pattern of this particular bulk solid is very sensitive to any deviations.

Furthermore, it is shown how the load acting on the feeder can be ascertained. While the power required of the feeder for bulk solid removal in the discharging condition can be satisfactorily calculated from Jenike’s theory, some well-known analytical methods available for the filled condition of the silo provide results which are too imprecise. The assumption of hydrostatic conditions in the hopper leads to stress values which are several times greater than those actually measured in the experimental silo. With the help of the empirically based equations, DIN 1055, Part 6, one can find acceptable solutions for the order of magnitude and for the tendency of the stresses acting on the hopper walls. How one might determine the load on the feeder from these solutions needs further looking into.

Through basic physical considerations, the dependency of the force required for removal of a bulk solid from under the outlet on the vertical load acting on the feeder, was found to be of a simple nature, from which an upper limit can be calculated. This was confirmed by measurements made on the experimental silo.

1. Introduction

In order to achieve mass flow, avoiding arching and pipe formation, a silo must be designed to suit the bulk solid it is to hold, the properties of which can be determined in shear tests1,2,3. The outlet of a silo is usually provided with a feeder which serves to regulate withdrawal of the bulk solid. It should be adapted to the outlet so as to remove the bulk solid from its entire cross-sectional area.

The form of the feeder and the geometry of the adjoining outlet play a decisive role here. This is demonstrated for the case of a belt conveyor illustrated in Fig. 1.1. A horizontally mounted belt conveyor beneath a horizontal, rectangular outlet often leads to bulk solid being removed predominantly from the rear end of the outlet (Fig. 1.1a). By inclining the lower edge of the feeder or installing a slide gate on its front side, one can effect bulk solid removal from the whole area of the outlet (Fig. 1.1b)4,5.

Fig. 1.1 Influence of feeder geometry on flow profile

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In order to remove material from under the outlet, the feeder must be in a position to move the bulk solid against the forces of stress containing it, that is, to overcome the forces of internal friction. As would be expected, these forces depend on the properties of the bulk solid itself, on the stress acting in the outlet and, consequently, on the stress in the silo as a whole. On the other hand, the feeder itself influences conditions within the silo — for example, by uneven removal of material, leading to dead zones or to regions with unequal flow. It is clear that not only the feeder is influenced by conditions in the hopper, but that these conditions themselves are influenced by the action of the feeder.

![Active and passive stress fields within the silo](image)

**Fig. 1.2 Active and passive stress fields within the silo**

In a silo, two basic stress conditions can be distinguished which lead to different loading of the feeder. In Fig. 1.2, the so called “active stress field”, present just after filling the silo, is shown on the left, while on the right the “passive stress field” developed during discharging is illustrated. Under conditions of active stress, the largest principal stress along the axis of the silo acts in a vertical direction, while the smaller principal stress acts horizontally. When the silo is discharging, due to converging in the hopper, the bulk solid is compressed in a horizontal direction, which results in the largest principal stress along the silo axis shifting to a horizontal direction. Under active stress in the hopper section, an increase or a decrease in the largest principal stress in vertical direction is possible, depending on the geometry of the hopper and on the properties of the bulk solid, whereas under conditions of passive stress, the largest principal stress is proportional to the local hopper diameter. For this reason, the vertical stress at the level of the outlet is generally less under conditions of passive stress than under conditions of active stress.

When considering the form and capabilities of a suitable feeder, apart from the requirement of ensuring mass flow, one must find answers to the following questions regarding the two types of stress field mentioned above:

- How does the stress field in the hopper and in particular the stress of the bulk solid on the feeder itself depend on the properties of the bulk solid and on the geometry of the silo?
- How does the power required of the feeder in order to remove the material depend on the properties of the bulk solid and on the vertical stress?
- What form should the connection between hopper and feeder have in order to ensure that material is removed from the entire area of the outlet?

In the literature, one finds initial attempts at describing these relationships, whereby, however, the influence of the feeder on the conditions of stress is not taken into consideration and furthermore, they are usually only applied to cohesionless bulk solids. In order to cast more light on the matter and to seek answers to the above questions, measurements were made on an experimental silo. Using the obtained results the aforementioned attempts at description were checked for their validity and usefulness in particle.

2. Experimental Set-up

The experimental silo, which was set up specifically to answer the stated questions, was given a plane geometry (length in the direction of conveyance 800 mm, width 600 mm, height of vertical section 3700 mm) and provided with a belt conveyor suspended under the outlet slit. The silo stands on four legs, connected with diagonal struts, thus forming a stable, self-sufficient structure. The walls are made of perspex (thickness 20 mm) supported by diagonal struts to prevent bulging.

In order to vary the silo’s geometry, it was possible to alter the positions of both the vertical and hopper walls. The different combinations of adaptable walls enabled the construction of silo geometries with an outlet diameter between 100 and 300 mm and a hopper slope...
ranging from $10^\circ$ to $40^\circ$ against the vertical.

Besides these factors, those shown in Fig. 2.1 can also be varied:
- $h_{Va}$ Distance between belt and lower edge of hopper side walls (below front wall)
- $h_{Vo}$ Distance between lower edge of front wall and lower edge of hopper side walls
- $h_R$ Distance between belt and lower edge of hopper back wall
- $\alpha$ Inclination of belt to horizontal, lower edge of hopper side walls

By varying these factors — as already intimated — discharge behaviour can be influenced: according to the factors chosen, the bulk solid will be removed from across the outlet either evenly or unevenly. Furthermore, Fig. 2.1 shows the individual gauges employed in the experimental set-up. The belt conveyor is suspended from steel cables attached to force transducers for measuring the vertical force acting on the conveyor belt. Force transducers are also built into the belt conveyor for measuring the horizontal forces arising during operation.

A total of 25 stress measuring cells developed at the Institute of Mechanical Process Engineering were fitted into the silo walls. They served to record the normal and shear stresses acting on the silo walls, and were positioned mainly in the region of the hopper. Below the conveyor belt, in the area under the outlet, three further force transducers were installed for measuring the forces acting on the belt. Besides a few signal amplifiers and chart recorders, data were fed into a computer-controlled data recording unit consisting of three multi-channel digital amplifiers with a total of 130 channels and a PC.

As experimental bulk solids, cohesive limestone powder (mean particle size $x_{50} = 5.3 \, \mu m$) and plastic pellets ($4 \, mm < x < 6 \, mm$) were chosen. The bulk density, $\rho_b$, the effective angle of internal friction, $\phi_e$, and the wall friction angle, $\phi_w$, of the mentioned bulk solids are listed in Table 2.1.

![Table 2.1 Flow properties, measured in a Jenike Shear Tester at major consolidation stresses $\sigma_1 = 10 \, kPa$ (limestone powder) and $\sigma_1 = 6 \, kPa$ (plastic pellets)]

| Bulk solid              | $\rho_b$ [kg/m$^3$] | $\phi_e$ [$^\circ$] | $\phi_w$ [$^\circ$] |
|------------------------|---------------------|---------------------|---------------------|
| Limestone powder       | 1200                | 38                  | 26                  |
| Plastic pellets        | 535                 | 20                  | 10                  |

3. Experiments

3.1 Investigation of stresses acting within the hopper and on the feeder

Measurements of filling stresses were made with hopper slopes $\Theta$ ranging from $10^\circ$ to $40^\circ$ in order to discover the relationship between the measured stresses and the hopper slope. The data obtained were compared with the results of well-known attempts to describe filling stresses in silos. In Fig. 3.1 for the examples $\Theta = 10^\circ$ and $\Theta = 20^\circ$ (limestone powder) stress patterns obtained from the well known analytical models of Walker and Walters and from the DIN 1055, Part 6 equations, which are based on empirical relationships, are shown. According to Walker and Walters, there is a certain hopper slope $\Theta$, which in the case under consideration would be lower than $10^\circ$, above which a hydrostatic increase in stress in the hopper is
One will note when comparing it to the experimental data that this model leads to vertical stresses which are too large, and that also the tendency and the order of magnitude of the normal stresses acting on the hopper walls is not correct. The overcalculated vertical stresses lead to an overestimation of the feeder load and of the power required of it to remove the bulk solid from under the outlet. This model is nevertheless recommended and used by many authors\textsuperscript{3,5,9,11}, since there is no simpler model at present which will predict the stresses acting in a hopper and because in respect of the load acting on the feeder, one wants to be on the safe side. The degree of overestimation of the feeder load decreases with increasing hopper slope angle $\Theta$.

The stress patterns calculated from the empirically based DIN 1055, Part 6 show a better correlation with the experimental data in respect of the order of magnitude and tendency. However, DIN 1055 offers no indication of how to calculate the vertical stress acting at the level of the outlet.

The course of the changes in vertical stress acting on the belt conveyor during filling is shown on the left in Fig. 3.2. One recognizes a typical course of events, whereby the stress initially rises sharply before reaching a final value which is independent of the height of the charge. In this case, the final vertical stress acting on the middle of the belt conveyor after filling was 7.7 kPa. When discharging is initiated, the vertical stress on the belt conveyor changes as shown on the right in Fig. 3.2. As soon as the belt conveyor is turned on the vertical stress drops sharply, increases somewhat, drops again, and finally settles at a constant average value. The fluctuations in stress are the result of what is going on in the hopper during discharging, eg. shear bands which descend with the bulk solid and arches which after formation immediately collapse, both resulting in a fluctuating load on the feeder.

The sharp fall in vertical stress observed immediately after beginning discharging is due to the time required for the cohesive bulk solid to change from a resting state to one of movement. Once it has gained motion, the vertical stress increases again. At the same time, the stress field within the hopper changes from that characteristic of the filling condition (active stress field) to that of the discharging condition (passive stress field), which soon leads to the vertical stress dropping again to a constant average value.

Similar experiments using plastic pellets as the experimental bulk solid show a sharp fall in vertical stress after initiating discharge, too. However, contrary to the behaviour of the limestone powder, the passive stress field here has developed and the average value of the vertical stress remains constant, i.e. there is no further increase in vertical stress after the sharp fall.

Jenike\textsuperscript{3)} developed a theory for describing conditions during discharging in terms of a so-called “passive stress field”, with which the vertical stress acting on the feeder can be calculated. Figure 3.3 shows the measured values for vertical stress and those calculated according to Jenike at the level of the outlet $\sigma_{y,e}$ for various hopper slopes $\Theta$. The values of vertical stress given are the mean values over the entire outlet cross-section. One will notice that the calculated values correspond well with the order of magnitude observed in the experiments. The fluctuation in the experimental data results from the influence of the outlet geometry, which will be described in chapter 3.3.
3. 2 Estimation of the power required by the feeder

In order to calculate the force required of a discharging device, the relationship between the vertical stress \( \sigma_v \) acting on the feeder under the outlet and the force necessary to move it must be known. Fig. 3.4 shows the measured values of the vertical force, \( F_{va,f} \) (the vertical stress multiplied by the cross-sectional area of the outlet), for various hopper geometries, and the force \( F_{h,be} \) required for withdrawal of the bulk solid at the initiation of discharging. During discharging, the bulk solid must be removed horizontally under the acting vertical stress, whereby, as in shear tests, the bulk solid is subject to shearing forces. By taking the effective angle of friction \( \varphi_e \), which describes the internal friction of the bulk solid during steady flow, to describe the relationship between the vertical force and the necessary horizontal force for removal, and assuming that the bulk solid is sheared directly beneath the lower edge of the hopper where the vertical stress \( \sigma_{va,f} \) is effective, one obtains the straight line curve shown in Fig. 3.4 (Eq. 3.1).

\[
F_{h,be} = F_{va,f} \tan (\varphi_e)
\]  

This relationship is valid on condition that the bulk solid is actually removed by the feeder, which will be the case with a rough belt or a plate reclaimer, when the angle of friction between the conveyor belt and the bulk solid is about as great as the effective angle of friction. In the experimental set-up, the angle of friction between the conveyor belt and the bulk solid \( \varphi_{x,ag} \) was about 34° and somewhat smaller than the effective angle of friction \( \varphi_e \), which was 38°. In this case, the maximum possible force with which a bulk solid can be removed is equal to the maximum force of friction which the conveyor belt can exert on the bulk solid, giving a maximum horizontal force of removal of:

\[
F_{h,be} = F_{va,f} \tan (\varphi_{x,ag})
\]  

The vertical force \( F_{va,f} \) effective on the conveyor belt is greater by the weight of the bulk solid between the lower edge of the hopper and the conveyor belt than the vertical force \( F_{va,f} \) at the level of the outlet. Therefore, any curve will cut the abscissa not at the origin but at a point shifted to the left by an amount proportional to the weight of this bulk solid. This curve represents an upper limit for all the measured values.

In the literature, one finds various relationships applied to the upper limit of the force \( F_{h,be} \) required of a feeder at the initiation of discharging, which are included in Fig. 3.4. These relationships are as follows: Johansen\(^{12}\) and McLean and Arnold\(^{13}\):

\[
F_{h,be} = F_{va,f} \sin (\varphi_e)
\]  

Rademacher\(^{14}\) and Manjunath and Roberts\(^{5}\):

\[
F_{h,be} = F_{va,f} \cdot 0.8 \sin (\varphi_e)
\]  

Reiner\(^{15}\) and Bruff\(^{16}\):

\[
F_{h,be} = F_{va,f} \cdot 0.4
\]  

These relationships were established empirically and, in the case under consideration, sometimes led to too low an estimation of the power required of the feeder.

As already shown, the vertical stress \( \sigma_{va,f} \) effective after filling drops sharply at first, before finally taking on the passive stress field. For this condition, a similar diagram can be drawn showing the relationship between the horizontal force and the vertical force. For the relationship between these two forces the same is valid as shown for initiation of discharging.

3. 3 Influence of the hopper/feeder interface

As shown in principle in Fig. 1.1, the geometry of the hopper/feeder interface influences the velocity field inside the silo. To investigate this interrelationship, the particle velocity at the silo walls during discharge, was measured by marking the position of single particles on
the outside of the perspex silo walls at regular time intervals. For the experiments, plastic pellets with a particle size between 4 mm and 6 mm were used. With the data gathered from those experiments, the velocity distribution over the silo walls was calculated by use of an interpolation method. Fig. 3.5 shows the measured velocity vectors at one of the hopper side walls during discharge for two different geometries of the hopper/feeder interface. The length and direction of each vector line represent the magnitude and the direction of the velocity at that point. The geometry (a), shown on the left of Fig. 3.5, yields a relatively even velocity distribution, the geometry (b) on the right side a very uneven one. The stress distributions corresponding to the shown velocity vectors are presented in Fig. 3.6.a and b. On the left in the figure is the back wall, on the right is the front wall, and in the middle is a hopper side wall, all represented schematically. The circles represent the normal stresses acting perpendicularly to the wall at these points, which are proportional to the lengths of the radii and correspond to the scale shown in the top left hand corner. In the case of the geometry producing the more uneven velocity distribution (b), the stresses acting on the hopper side, back and front walls are also more uneven than those associated with the more even velocity distribution. This observation agrees with similar measurements and calculations made on silos characterized by funnel flow, where it was also found that the stresses in regions of flow were less than in the dead zones \(^{17,18}\). Apparently, stresses in the slowly downward moving regions increase, while in faster flowing regions they are decreased. The uneven stress distribution can be explained qualitatively by a simple model. The cause of the change in wall stress is the shear stress between the two regions flowing at different velocities. The shear stress acts on the slower moving region in the direction of gravity and on the faster moving region in the opposite direction. Thus there is an extra downward force acting on the slower flow region which must be taken up by the adjacent hopper side walls, leading to an increase in hopper wall stress. The faster moving region, on the other hand, is supported by the slower moving region, leading to a reduction of stress in the hopper wall adjacent to the faster moving region.

With the investigated plastic pellets, the uneven distribution of stress, starting from a stress pattern of the fully charged silo, took form shortly after initiating discharge. In Fig. 3.7, the course of the non-dimensional wall stress \(S\) at various positions on the hopper wall (A to F), as well as on the conveyor belt (G), over the non-dimensional time \(T\) are presented for the geometry shown in Fig. 3.5.b and 3.6.b. The non-dimensional stress \(S\) and the non-dimensional time \(T\) are defined as follows:

\[
S = \frac{\sigma_w}{(\rho g B)} \tag{3.6}
\]

\[
T = \frac{t}{(V_T/Q)} \tag{3.7}
\]
where:

- \( \sigma_w \): wall normal stress [Pa]
- \( \rho_b \): bulk density [kg/m\(^3\)]
- \( B \): width of the vertical section of the experimental silo, \( B = 0.6 \) m [m]
- \( g \): acceleration due to gravity [m/s\(^2\)]
- \( t \): time [s]
- \( V_{Tr} \): hopper volume [m\(^3\)]
- \( Q \): volume flow rate [m\(^3\)/s]

The non-dimensional time \( T = 1 \) is the time needed to discharge a volume of bulk solid equal to the hopper volume \( V_{Tr} \). From the stress-time curves presented in Fig. 3.7, it can be seen that all of the shown stresses fluctuate around a stationary mean value. The fluctuations are typical for hopper stresses during discharge. They result from shear bands moving downwards with the bulk solid. The mean values of the stresses \( S \) remain almost constant with time after a short initial period of \( T = 0.1 \) to \( T = 0.2 \), where the stress field is changing from the active to the passive state.

Figure 3.8 shows stress-time curves for the limestone powder. In this particular case, the geometry of the hopper/feeder interface was such, by choosing a small height \( h_{Vo} \), that the bulk solid in the rear part of the outlet slit flowed more quickly than that in the front part,
which resulted in very different stresses on the rear and front walls. In contrast to the curves measured with plastic pellets, stresses were found to vary with time over longer periods; some of them still change at $T > 2$. Qualitative velocity measurements carried out by inserting thin rods through holes in the hopper walls, showed a decrease in velocity near the front wall with time until the velocity at the front wall was zero ($T \approx 2 \ldots 3$). Velocity equal to zero at the front wall is equivalent to the formation of a dead zone in this region.

Comparing the stress-time curves gathered from measurements with plastic pellets and limestone powder, the following points become clear:

- Uneven removal of a bulk solid causes an uneven velocity field and, hence, an uneven stress distribution.
- The uneven velocity field and stress distribution develop after a short period of time ($T = 0.1 \ldots 0.2$) in the case of plastic pellets, a virtually incompressible bulk solid, and remain almost constant. In the case of the compressible limestone powder, the velocities and stresses change over evidently longer periods of time ($T > 2$).

The geometry depicted in Fig. 3.8 with $h_{v_0} = 0$ yields an uneven outflow of the bulk solid and an uneven stress distribution. By choosing greater outlet heights $h_{v_0}$ and/or $h_{v_0}$, one can achieve an even outflow velocity over the entire area of the outlet, which in turn evens out the stresses acting on the hopper walls. A further increase in $h_{v_0}$ leads to more bulk solid passing through the front part of the outlet, whereby the stresses measured in the front part of the silo are less than in the rear part. The stresses acting on the hopper side walls follow the same tendency as those acting on the vertical hopper walls under conditions of different outlet heights.

Figure 3.9 shows the stress $\sigma_w$ on the hopper walls for various outlet heights $h_{v_0}$ ($h_{v_0}$ and $h_{v_R}$ are constant) during discharge (limestone powder, hopper slope $\Theta = 30^\circ$, outlet width $b = 200$ mm). In the figure, the back wall is on the left, the front wall is on the right, and in the middle is a hopper side wall, all represented schematically. The circles represent the normal stresses acting perpendicularly to the wall at these points, which are proportional to the length of the radii and correspond to the scale shown in the bottom left hand corner. It is evident that even a relatively small change in the outlet height $h_{v_0}$ has a very strong influence on stress formation in the hopper. Where there is a strong flow of bulk solid, the stress is less, while where flow is weak or has come to a standstill, stress increases. The stress pattern which arises when a bulk solid is withdrawn unequally from across the outlet deviates, particularly at the front and back wall of the silo, from that predicted by Jenike's theory, so that one must question its validity under such conditions.

In this particular case, the optimal outlet height $h_{v_0}$, which serves to evenly withdraw the material, seems to be in the range of $h_{v_0} = 44$ mm to $h_{v_0} = 38$ mm, this is a total outlet height ($h_{v_0} + h_{v_R}$) of between 96 mm and 99 mm. Deviations of a few mm from this optimum cause an unequal velocity and stress distribution (see Fig. 3.9).

Optimal geometry depends largely on the flow properties of the bulk solid. The optimal geometry found for limestone powder causes an uneven velocity and stress distribution in the case of plastic pellets, and vice versa.

Taking a look at the stresses acting on the hopper walls during discharging (see Fig 3.8), one notices that it is only after a certain amount of time — in this case after the hopper content has been discharged about 2 to 3 times — a stable pattern of stress is established. Only now is it possible to say whether or not the chosen silo geometry effects an even discharge across the outlet, and thus an even pattern of stress on the silo walls. Dead zones in the front
and rear part of the silo sometimes only arise after a considerable time, which seems to be dependent on the ‘difference’ between the chosen geometry of the hopper/feeder interface and the optimum geometry. In Fig. 3.10, the stress distribution on the hopper side wall at various non-dimensional times $T$ is shown. The geometry is very close to the optimal geometry; $h_{V0}$ is only about 6 mm greater than the optimal value of $h_{V0}$. The non-dimensional time $T=0$ corresponds to the filling stresses (active stress field). After switching on the feeder, the passive stress field develops. From Fig. 3.10, it can be seen that the hopper stresses remain quite even until $T=0.5$ (i.e. discharge of a volume equal to 50% of the hopper volume). With greater values of $T$ (e.g. $T=1.725$ and $T=3.467$, Fig. 3.10), stress distribution becomes more and more uneven. In contrast to these results, the hopper stresses shown in Fig. 3.8, due to a geometry with a great divergence from the optimal geometry, are uneven from the very beginning of discharge (e.g. non-dimensional stress $S$ at points $E$ and $F$, Fig. 3.8).

In this respect, a cohesive bulk solid like limestone powder behaves differently from a non-cohesive one such as plastic pellets, where a stable stress pattern is established very soon after discharge initiation. The cause of the limestone powder’s behaviour is its compressibility: locally differing stresses lead to the limestone powder being variously compressed, which in turn influences the stress pattern, which again influences the compression, and so on.

The results of the measurements in respect to achieving an even withdrawal of a bulk solid can be summarized as follows.

- Optimal geometry is dependent on the bulk solid’s flow properties.
- The range of geometric values which ensure even withdrawal is narrow.
- In the case of cohesive limestone powder, the velocities and stresses in the hopper vary during discharge over relatively long periods of time. Therefore, a geometry similar to, but not identical with the optimal geometry causes uneven withdrawal only after discharge of a greater amount of bulk solid. A geometry very different from the optimum causes uneven withdrawal after a relatively short period of time.

The question is how can even withdrawal of bulk solid be ensured. The results of the measurements show the difficulties in determining the optimal geometry, especially in the case of the cohesive limestone powder. Furthermore, as one can imagine, optimal geometry is influenced by changes in the flow properties, e.g. due to changes in particle size or humidity.

One possible solution is to force the bulk solid to flow out evenly. For this purpose, a
hopper/feeder interface geometry was chosen which caused withdrawal from the rear end of the outlet. In addition, an insert was fixed in the outlet slit perpendicular to the conveying direction half way between the front and back wall (Fig. 3.11). The distance $h_M$ between the lower edge of the insert and the belt was adjusted so that the volume flow rate $Q_M$ below the insert was half as much as the flow rate $Q_V$ below the front wall. Due to the insert, the volume flow rate in the front half of the silo was equal to that in the rear half, i.e. $Q_M$.

Limestone powder (discharging)

| $\phi$ | $b$ |
| 15 deg | 200 mm |

![Diagram](a) $h_v = 50$ mm $h_v = 0$ mm $h_v = 13$ mm (with insert)

![Diagram](b) $h_v = 50$ mm $h_v = 0$ mm $h_v = 13$ mm (without insert)

Fig. 3.12 Hopper wall stresses when using an insert (a) and without an insert (b) (limestone powder)

Measurements carried out with a well adjusted insert show an even stress distribution and no occurrence of dead zones, even after discharge over long periods of time (Fig. 3.12.a). To show the influence of the insert, Fig. 3.12.b shows the wall normal stresses for the same hopper/feeder interface geometry without the insert. The measured stresses show a large change in magnitude in horizontal direction. By use of the insert, two outlet slits are simulated, both with a length to width ratio of about 2. Obviously, it is easier to achieve even withdrawal with smaller length to width ratios. This easily understandable relationship agrees with Rademachers' recommendations to choose a length to width ratio for the outlet slit of no more than 2 or 3. But, following Jenike's method for silo design, the length of an outlet slit should be at least 3 times the width, and, in many practical applications, slit lengths of more than 3 times the width are necessary. Therefore, the use of inserts to ensure uniform withdrawal seems to be a reasonable solution. When using such inserts, it has to be ensured that they do not cause flow obstructions like dead zones or stable arches.

Nomenclature

- $b$ : outlet width [m]
- $B$ : width of the vertical section of the experimental silo [m]
- $F_h$ : required horizontal force of the feeder [N]
- $F_{va}$ : vertical force, acting at the hopper outlet [N]
- $F_{vg}$ : vertical force, acting on the feeder [N]
- $g$ : acceleration due to gravity [$m/s^2$]
- $h$ : height [m]
- $h_R$ : distance between belt and lower edge of hopper back wall [m]
- $h_{vo}$ : distance between belt and lower edge of hopper side walls (below front wall) [m]
- $h_{vo}$ : distance between lower edge of front wall and lower edge of hopper side walls [m]
- $Q$ : volume flow rate [$m^3/s$]
- $S$ : non-dimensional stress [-]
- $t$ : time [s]
- $T$ : non-dimensional time [-]
- $V_T$ : hopper volume [$m^3$]
- $\alpha$ : inclination of belt to horizontal, lower edge of hopper side walls [-]
- $\rho_p$ : bulk density [$kg/m^3$]
- $\sigma_h$ : horizontal stress [Pa]
- $\sigma_v$ : vertical stress [Pa]
- $\sigma_{va}$ : vertical stress at the hopper outlet [Pa]
- $\sigma_{vg}$ : vertical stress acting on the feeder [Pa]
- $\sigma_w$ : wall normal stress [Pa]
- $\beta$ : effective angle of internal friction [$^\circ$]
- $\varphi$ : wall friction angle [$^\circ$]
- $\varphi_x$ : wall friction angle (conveyor belt) [$^\circ$]
- $\theta$ : hopper slope angle measured against the vertical [$^\circ$]

Indices:

- $f$ : filling
- $d$ : discharging
- $be$ : beginning discharging

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