Accurate modeling of left-handed media using finite-difference time-domain method and finite-size effects of a left-handed medium slab on the image quality revisited

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The letter contains an important message regarding the numerical modeling of left-handed media (LHM) using the finite-difference time-domain (FDTD) method which remains at the moment one of the main techniques used in studies of these exotic materials. It is shown that conventional implementation of the dispersive FDTD method leads to inaccurate description of evanescent waves in the LHM. This defect can be corrected using the spatial averaging at the interfaces. However, a number of results obtained using conventional FDTD method has to be reconsidered. For instance, the accurate simulation of sub-wavelength imaging by the finite-sized slabs of left-handed media does not reveal the cavity effect reported in [Phys. Rev. Lett. 92, 107404 (2004)]. Hence the finite transverse dimension of LHM slabs does not have significant effects on the sub-wavelength image quality, in contrary to previous assertions.

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The finite-difference time-domain method (FDTD) is known as one of the powerful numerical techniques in electrodynamics. Being simple in implementation it has been proved to be very popular among researchers. This method is assumed to be extremely accurate since it involves direct numerical solution of Maxwell equations which are known as the basis of classical electrodynamics. However, the implicit belief in FDTD sometimes results in attributing certain physical properties to some electromagnetic structures based on simulation results. One typical example is a long row of works on FDTD modeling of the left-handed media (LHM), materials with negative permittivity and permeability. Such materials are not yet available experimentally and thus numerical simulations still remain one of the most common ways in exploring their properties and applications. However, as it will be shown in this letter, the conventional implementation of FDTD for modeling of LHM in the same manner as for usual dispersive dielectric materials leads to incorrect simulation results. It may seem that the conventional FDTD has been verified in the literature: the negative refraction effect which is inherent to the boundary between the free space and LHM was observed and the planar superlens behaviour has been successfully demonstrated. Actually, this only means that the LHM is correctly modeled for the case of propagating waves. As soon as the evanescent waves are considered the conventional implementation of the FDTD fails. Usually, the evanescent waves decay exponentially over the distance and thus they are concentrated in the close vicinity of sources, that is why conventional FDTD modeling of usual materials does not suffer from this trouble. In the case of LHM, the evanescent waves play key roles and have to be modeled accurately because of the perfect lens effect. A slab of LHM effectively amplifies evanescent waves which normally decay in usual materials and allows transmission of sub-wavelength details of sources to significant distances. Ideally lossless LHM slabs provide unlimited sub-wavelength resolution. However in realistic situations, the resolution is restricted by losses and the thickness of the slab, as well as the mismatch between the LHM and its surrounding medium.

In order to illustrate what we mean by incorrect description of evanescent waves provided by conventional implementation of FDTD, we have simulated propagation of plane electromagnetic waves (with TM polarization and various transverse wave vectors) through an infinite slab of LHM in the free space. We choose this problem since an analytical expression for transmission coefficient through the LHM slab is available, that allows us to check validity of our numerical results. The LHM slabs with relative permittivity and permeability \( \varepsilon = \mu = -1 - 0.001j \) and thickness \( d = \lambda/5 \), where \( \lambda \) is wavelength in the free space, are tested. The frequency dispersion of the LHM has been modeled by the Drude model with the plasma frequency \( \omega_p = \sqrt{2} \omega \) and the collision frequency \( \gamma = 0.0005 \omega \), where \( \omega \) is the operating frequency. It is implemented in FDTD using the auxiliary differential equation (ADE) method. The two-dimensional (2-D) simulation domain is bounded by perfectly matched layers (PMLs) and periodical boundary conditions (PBCs) as illustrated in Fig. 1(a). A soft current sheet source (which allows scattered waves to pass through) with phase delay corresponding to different transverse wave vectors \( k_x \) placed at the distance \( d/2 \) from the slab is used as excitation. The PBCs are also specified by the particular wave vector \( k_x \). The source is slowly and smoothly switched to its maximum value in order to avoid exciting other frequency components. The computation continues until the steady state is reached. The simulation has been done for both propagating \( (k_x < k, \text{where } k \text{ is wave vector in the free space}) \) and evanescent \( (k_x > k) \) waves. The Berenger’s original PMLs are used for absorbing propagating waves, and the modified PMLs applied when the transmission coefficient for evanescent waves is calculated. The PMLs are placed at \( \lambda/2 \) distance away from the slab. We use Yee’s square grid with periods \( \Delta x = \Delta y = \lambda/100 \).
for comparison. It is clearly visible that the numerical results from the source plane to the image plane (located at the distance $d/2$ from the other side of the slab) as function of $k_z$ is presented in Fig. 1(b) by circles. The reference curve calculated using the proposed spatial averaging at the boundaries is given by crosses. The slight disagreement between the image and plane waves ($k \leq k_\lambda$) and a small part of weakly decaying evanescent waves ($k_0 < k_x < 2k_0$). For the evanescent waves with $k_z > 2k_0$ the numerical results dramatically differ from the analytical results and the former shows resonant behavior with a strong peak at $k_x = 2.4k_0$. This effect can be explained as resonant excitation of a ‘numerical surface plasmon’ at the back interface of LHM slab. Similar phenomena can be observed in the case of metallic slabs or for unmatched LHM, but in this particular case it is purely numerical artefact. The incorrect behavior of numerical solutions remains similar if the FDTD grid period is reduced to $\lambda/200$ and $\lambda/400$, but the resonance shifts to $k_x = 2.8k_0$ and $k_x = 3.2k_0$, respectively.

The presence of ‘numerical surface plasmons’ provides evidence that the boundaries between the LHM and the free space have not been modeled accurately. If at the boundaries the mean value of permittivity of LHM and the free space is used for updating the tangential component of electric field (which is equivalent to the spatial averaging suggested in [11]) then the spurious ‘numerical surface plasmons’ disappear and the modeling happens to be extremely accurate. The transmission coefficient calculated using the proposed spatial averaging at the boundaries are presented in Fig. 1(b) by crosses. It is clear that the result repeats the estimated analytical values with very good accuracy for the whole spatial spectrum of waves. The calculation has been performed for $\Delta x = \Delta y = \lambda/100$, and it remains accurate even for larger grid periods. The above simple test allows us to conclude, that the conventional FDTD method fails to describe the propagation of high-order evanescent waves in the LHM if no corrections at the boundaries of the LHM are made. As a result, a number of previously obtained results using the conventional FDTD have to be reconsidered. This especially concerns the modeling of sub-wavelength imaging by LHM slabs, which involves operation with evanescent waves. Note, that the simulations where only propagating waves are involved (e.g. demonstration of negative refraction for an obliquely incident plane wave) are not affected by this problem.

The numerical transmission coefficient for LHM slabs reported in [12] is a typical example of using the FDTD method without averaging. The study can actually be treated as an investigation of the ‘numerical surface plasmons’, their sensitivity to losses and efficiency of their excitation for various thicknesses of the slab. Unfortunately, these results have no relations with the properties of actual LHM slabs. The performance of the LHM slab as a sub-wavelength imaging device indeed depends on the losses and the thickness of the slab as it is shown in [13], but it is completely different dependence as compared to results reported in [12].

One of the recent most puzzling results related to the quality of imaging provided by LHM slabs is reported in [13]. It is claimed that the operation of the finite-sized structures is significantly affected by their transverse dimensions. Having the FDTD code with spatial averaging at the interfaces which has been proven to be accurate, we decide to check this statement. The finite-sized slabs of LHM excited by magnetic current sources are modeled for three different transverse dimensions: $L = \lambda$, $2\lambda$ and $4\lambda$, as illustrated by the sketch in Fig. 2. The parameters of the LHM slab ($\varepsilon = \mu = -1 - 0.001j$, $d = \lambda/5$) and distance between the source and the front interface equal to $d/2$ are kept the same for all simulations. The simulation domain is truncated by PMLs located at $\lambda/2$ distance away from the LHM slab and both source and image planes. The same grid periods $\Delta x = \Delta y = \lambda/100$ and the time step $\Delta t = \Delta x/\sqrt{2}c$ are used as for previous plane-wave simulations. The source is switched slowly and smoothly in order to avoid contributions from undesired frequency components and the simulations last until the steady state is reached. The intensity distributions in the image planes for all three cases of different transverse dimensions are plotted in Fig. 2. It is clear that the image quality is practically unaffected by the transverse size of the slab. The distributions repeat the source distribution, which is plotted in the same figure, with good sub-wavelength resolution. We do not observe any distortion of images caused by the finiteness of the transmission device, in contrast to the conclusions made in [12].
the source is due to the finite resolution of lenses caused by the losses in LHM. We suppose that the resonant effects and image distortions related to transverse dimensions reported in [13] can be interpreted as excitation of ‘numerical surface plasmons’ at the interfaces of the slab and can be observed only in inaccurate FDTD simulations without spatial averaging at the boundaries. Thus, these effects are purely numerical and have no relation to the properties of actual LHM slabs. In reality, the imaging performance of finite-sized LHM slabs is unaffected by their transverse dimensions. This statement also has been confirmed by full-wave electromagnetic simulation using Ansoft HFSS™ package.

In order to illustrate this statement we have performed the simulation for a LHM slab with transverse size $L = \lambda$ excited by two magnetic line sources placed at $\lambda/8$ distance between each other using the FDTD method with spatial averaging at the boundaries. The rest of parameters is the same as before. The distance between the sources is larger than the resolution of the lens which should be better than $\lambda/12$ based on the fact that the transfer function plotted in Fig. 1 is close to unity for $k_x/k < 6$. This allows us to expect two well-resolved maxima in the image plane. The distribution of magnetic field intensity in a sub-domain near the LHM slab (the actual FDTD domain is larger) in the steady state is presented in Fig. 3(a) is close to unity for $k_x/k < 6$. This allows us to expect two well-resolved maxima in the image plane. The distribution of magnetic field intensity in a sub-domain near the LHM slab (the actual FDTD domain is larger) in the steady state is presented in Fig. 3(a) together with the source. Two maxima at the distance of $\lambda/8$ are clearly visible in the image plane. This confirms sub-wavelength imaging capability of the LHM lens with only one wavelength width. Thus, we do not observe any limitations on the functionality of the LHM slabs as sub-wavelength imaging devices due to their finite transverse dimensions. However, if the same system is modeled by FDTD method without corrections at the boundaries then completely different distribution of the field around the slab is observed, see Fig. 3(b). The distribution of the field along the slab interface is smooth which once again confirms that high-order evanescent waves are not correctly modeled. As a result, the sub-wavelength details of the source are not resolved in the image plane. Instead of expected two closely located maxima the intensity distribution in the image plane has only one wide maximum, see Fig. 3(c).

We suppose that the presented comparison between FDTD models with and without spatial averaging at the boundaries clearly demonstrates the limitation of the conventional FDTD method for modeling of LHM lenses. The significant discrepancies appear only in the cases when evanescent waves are involved. However, we encourage to use the model with corrected updating equations at the boundaries (with spatial averaging of permittivity as we propose or with averaging of the current as proposed in [11] which are equivalent) in all cases in order to avoid numerical artefacts. The spatial averaging is known as a second-order correction in the case of boundaries between dielectrics with positive permittivity [14], but in the case of evanescent waves in LHM it
transforms into essential and mandatory correction.

In addition to the corrections at the boundaries we would like to stress a few other aspects which are important to accurate FDTD modeling of LHM. The numerical dispersion is usually assumed to have very little influence on the quality of FDTD simulation. However, the properties of LHM are known to be extremely sensitive to minor changes of material parameters. A typical example is the degradation of resolution due to small variations of material parameters from \( \varepsilon = \mu = -1 \). This means that even a tiny amount of numerical dispersion may cause crucial discrepancy between simulated and theoretical results. Fortunately, the numerical dispersion can be easily analyzed analytically and an estimation for the difference between effective numerical material parameters and the ‘real’ ones are known. This allows to adjust parameters of the FDTD simulation to ensure that the effective numerical material parameters correspond to required values. We suggest using this adjustment in the cases of large time steps and small losses.

It is well known that the switching time considerably influences the oscillation of images created by LHM lenses. The switching time equal to thirty periods was used in [3, 12]. However, perhaps this is the reason why no stable images could be obtained in [3, 12]. Recently, it was reported in [13] that a switching time equal to one hundred periods is required to obtain stable images. We have used such a switching time in all our simulations and we recommend to pay attention to the issue of switching time in all FDTD simulations of LHM.

The high-order evanescent waves travel very slowly in the LHM slabs and the procedure of the amplification of evanescent waves requires very long time to reach the steady-state. That is why, in addition to smooth and slow switching of the source we recommend to add losses into the simulations in order to limit the spatial spectrum of evanescent waves which are involved in operation. Otherwise, in the lossless case the transient processes may last extremely long.

The diffraction on the wedges and corners also may provide certain problems because of the singularity effects reported in [14]. However, in the case of LHM with \( \varepsilon = \mu \approx -1 \) the singular behavior disappears and the waves operate mainly as retroreflectors. That is why, in the simulations presented in this letter we have not observed any singularities at the wedges.

In conclusion, we have demonstrated in the letter that the conventional FDTD method for modeling of LHM leads to inaccurate description of high-order evanescent waves and does not simulate sub-wavelength imaging correctly. The simulations suffer from the artefact of ‘numerical surface plasmons’ and are completely numerical. These effects are absent in real structures and there are no restrictions on functionality of LHM sub-wavelength lenses due to their transverse dimensions. Our FDTD simulations with spatial averaging at the boundaries demonstrate the imaging of two line sources located at sub-wavelength distance between each other by a LHM slab with only one-wavelength width. Keeping in mind that simulations using the FDTD method remain one of the most popular approaches in the studies of LHM, the general recommendations on how to accurately model LHM using the FDTD method are also provided.

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