Finite Element Analysis of Composite Matrix Material with Micro-damage Healing Ability

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Abstract. A novel constitutive model for modelling of polymer composite matrix material is proposed. Investigated material is an advanced ethylene/methacrylic acid (E/MAA) copolymer, DuPont™ Surlyn® 8940 thermoplastic resin, which exhibits intrinsic self-healing ability. The model incorporates damage evolution and healing model, and the von Mises linear isotropic hardening plasticity model and is validated with static tensile and two-cycle tensile tests on pure Surlyn® 8940 specimens. The concept of nominal and healing configurations is used in the development and numerical implementation of the model, which is additionally streamlined by the application of the strain equivalence hypothesis. Developed constitutive model enables accurate prediction of Surlyn® 8940 behaviour under tensile loading and precise prediction of accumulated plastic strain.

1. Introduction
In composite aircraft structures, damage is almost inevitable, and it occurs at several length scales due to the heterogeneous structure of composite materials. Application of materials with self-healing ability could successfully resolve these problems.

Materials that are able to autonomously heal damage already exist for some time and [1] proposed a constitutive model for self-healing composites with extrinsic autonomic healing mechanism at a macroscale. That model was later extended to a micromechanical model by the work of [2]. Based on those two models, several models have been developed to date, and mostly for pure polymers or asphalt. In [3] a linearly elastic damage-healing model for asphalt was proposed. It was later extended with viscoelasticity and viscoelasticity by the work of [4] for the case of small deformations. Concerning polymer materials, [5] developed an isotropic elastoplastic micro-damage healing model for shape memory materials. It was later extended in [6] by adding time and strain dependencies of damage and healing variables.

This paper presents a Continuum Damage Healing Mechanics based constitutive model of a Carbon Fibre Reinforced Polymer (CFRP) matrix material, an advanced ethylene/methacrylic acid (E/MAA) copolymer, DuPont™ Surlyn® 8940 thermoplastic resin. The model is currently developed only for the matrix material, since the research is in its early phase. Model for damage and healing is taken from [3] and the parameters are adjusted to fit the experimental results. The damage-healing model of [3] is considered appropriate for modelling of Surlyn® 8940, since it exhibits material properties that are phenomenologically similar to asphalt’s, such as: viscous properties, time- and temperature dependent
intrinsic healing, rate dependent evolution of damage etc. The model is validated with static tensile and two-cycle tensile tests.

2. Constitutive model description
The developed constitutive model combines damage variable evolution equation from [7], healing variable evolution equation from [8] and a von Mises linear isotropic hardening plasticity model.

2.1. Damage and healing model
Damage and healing variables are represented as area ratios, Figure 1. The damage variable, \( \phi \), is defined as a ratio of the damaged area, \( A_d = A_{uh} + A_h \), to total area \( A \), whereas the healing variable, \( h \), is a ratio of healed area, \( A_h \), to the damaged area, \( A_d \), i.e.

\[
\phi = \frac{A_d}{A} \quad h = \frac{A_h}{A_d}.
\]

and they both range from 0 to 1. Furthermore, an effective damage variable, which couples the damage variable, \( \phi \), and the healing variable, \( h \), is defined

\[
\phi_{eff} = \phi (1 - h).
\]

It ranges from 0 to 1, where \( \phi_{eff} = 0 \) means that the material is undamaged, or all damage is healed and \( \phi_{eff} = 1 \) represents fracture.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Representation of nominal, healing and effective configurations with designated healed and unhealed areas, after [3].}
\end{figure}

Damage variable is defined as scalar and its evolution equation, taken from [7], is given in its rate form

\[
\dot{\phi} = \Gamma^{vd} \left( \frac{\bar{Y}}{\bar{Y}_{th}} \right)^q (1 - \phi)^2 \cdot \exp(k \cdot \bar{\varepsilon}_{eff}),
\]

where \( \Gamma^{vd} \) is the damage viscosity parameter which determines how fast does the damage variable evolve; \( \bar{Y} = \sqrt{(3/2) \sigma_{ij}' \sigma_{ij}' } \) is the damage driving force in healing configuration, where \( \sigma_{ij}' \) is the deviatoric stress tensor; \( \bar{Y}_{th} \) is the healing threshold damage force; \( q \) and \( k \) are material parameters and \( \bar{\varepsilon}_{eff} = \bar{\varepsilon}_{ij} \bar{\varepsilon}_{ij} \) is the effective (equivalent) strain in the healing configuration. Damage initiation condition is met when the healing damage driving force, \( \bar{Y} \), is greater than the healing threshold damage force, \( \bar{Y}_{th} \). Formulation of the damage variable evolution equation in Eq. (3), results in several features: (i) strain rate dependency, (ii) strain level dependency, (iii) evolution of damage both during loading and unloading.

Healing variable is also a scalar with the evolution equation in the rate form, taken from [8], such as

\[
\dot{h} = \Gamma^{h} (1 - h)^2 (1 - \phi)^2,
\]

(iii) evolution of damage both during loading and unloading.
where $T^h$ is the healing viscosity parameter which determines how fast the healing variable evolves. Healing variable evolution, defined with Eq. (4), starts when the material is not being loaded, i.e. when $\dot{\varepsilon} = 0$ and $\phi > 0$. Additionally, the healing variable can decrease due to its physical definition, according to Eq. (1) taken from [3], and the decrease is governed by

$$h^{t+\Delta t} = \frac{\phi^t}{\phi^{t+\Delta t}} \cdot h^t \quad \text{when} \quad \dot{\phi}^{t+\Delta t} \geq 0.$$  

(5)

2.2. Von Mises linear isotropic hardening plasticity model

By the definition of the von Mises linear isotropic hardening plasticity model, the yield stress, $\sigma_y$, increases linearly with each subsequent loading above the yield stress, i.e.

$$\sigma_y = \sigma_{y0} + r(p),$$

$$r(p) = \int h \cdot dp,$$

(6)

where $\sigma_{y0}$ is the initial yield stress, $r$ is the isotropic hardening variable, $h$ is the linear isotropic hardening parameter (material constant), $dp$ is the effective plastic strain increment and $p$ is the effective plastic strain. Von Mises equivalent stress is determined using

$$\sigma_e = \sqrt{\frac{3}{2} \sigma_{ij} \sigma_{ij}^\prime},$$

(7)

where $\sigma_{ij}^\prime$ is the deviatoric stress tensor.

2.3. Numerical implementation

The constitutive model is numerically implemented into Abaqus/Standard user material subroutine UMAT using the incremental approach. The subroutine is coded in FORTRAN77. Firstly, strain equivalence hypothesis is introduced which means that the strain increments in the nominal and healing configurations are equal, i.e.

$$d\bar{\varepsilon} = d\varepsilon.$$  

(8)

After the introduction of the strain equivalence hypothesis, damage, healing and plasticity models are coupled in the healing configuration. Firstly, the plasticity model is implemented, using the implicit integration scheme – a return-mapping algorithm. Subsequently, the damage and healing models are implemented and the fourth order explicit Runge-Kutta method is employed to solve differential evolution equations, Eq. (3) and Eq. (5). Finally, the nominal stress is calculated as

$$\sigma = (1 - \phi_{eff}) \cdot \bar{\sigma}.$$  

(9)

3. Model validation, results and discussion

The model is validated with two sets of experiments: (i) static tensile and (ii) two-cycle tensile tests on pure Surlyn® 8940 coupons. Laboratory tests were conducted, and the specimen models for numerical analyses were dimensioned following the guidelines from ISO 527-2.

Specimen models are discretized with linear continuum solid elements (C3D8), with four integrations points per element. Both meshed models are shown in Figure 2., where the two-cycle tensile test coupon is discretized with 256 C3D8 elements (495 nodes) and the one for the static tensile test is meshed using 384 C3D8 elements (693 nodes).
Figure 2. Meshed models for two-cycle tensile tests and static tensile tests.

3.1. Static tensile tests on pure Surlyn® 8940 coupons
Results of the analysis are compared with experimental results for three specimens in a nominal stress-strain diagram, Figure 3. Model parameters are set to best fit the experimental results and they are given in Table 1. Healing model parameters are omitted here because no healing is expected during this test.

The diagram in Figure 3. shows good agreement of analysis results with experimental ones. Figure 4. shows numerical prediction of the evolution of the damage variable and the effective plastic strain with dependence on total strain. Normally, it would be expected for the effective plastic strain to exhibit linear behaviour for the von Mises linear isotropic hardening plasticity model. Here it is nonlinear due to the presence of damage, and it is a proof of good coupling between the damage and plasticity model.

| Parameter | Value |
|-----------|-------|
| $E$       | 500 MPa |
| $\nu$     | 0.3   |
| Damage model | $r^{vd}$ | $7.4 \times 10^{-4}$ s$^{-1}$ |
|           | $Y_{th}$ | 0.01 MPa |
|           | $k$     | 1.8    |
|           | $q$     | 0.659  |
| Plasticity model | $\sigma_y$ | 8 MPa |
|           | $h$     | 1 800 MPa |

| Parameter | Value |
|-----------|-------|
| $E$       | 700 MPa |
| $\nu$     | 0.3   |
| Damage model | $r^{vd}$ | $5.51 \times 10^{-3}$ s$^{-1}$ |
|           | $Y_{th}$ | 0.02 MPa |
|           | $k$     | 0.1    |
|           | $q$     | 0.7    |
| Healing model | $r^h$ | 0.85 s$^{-1}$ |
|           | $m$     | 2      |
| Plasticity model | $\sigma_y$ | 3 MPa |
|           | $h$     | 2 600 MPa |
Figure 3. Nominal stress-strain diagram of the static tensile test. A comparison of experimental results and numerical analysis results.

Figure 4. Evolution of the damage variable (left diagram) and effective plastic strain (right diagram) vs total strain for different values of the damage viscosity parameter $\Gamma^{vd}$, during the static tensile test.

3.2. Two-cycle tensile tests on pure Surlyn® 8940 coupons

During these tests, specimens were loaded up to 220 N and unloaded, in the first cycle. Afterwards, they were left to heal for 5 s and then, in the second cycle, they were loaded up to 260 N and unloaded, Figure 5. These tests are used to validate the plasticity and healing parts of the constitutive model. Parameters used during these tests are given in Table 2, and they differ from the ones given in Table 1. This is caused by the fact that the consistency of specimens’ thermal history is not assured. Analysis results of a two-cycle tensile test are compared with the experimental results for three specimens in a stress-strain diagram, Figure 6. Good agreement of analysis results with experimental ones is achieved with accurate prediction of plastic strain. However, some differences between numerical and experimental results are present in the second cycle. Those are caused by the arrangement of the equipment which is necessary for measurements of displacements during the second cycle.
Effective damage variable, Figure 7., and the healing variable, Figure 8., are plotted vs time, not total strain, because total strain oscillates from zero to maximum value in a two-cycle tensile test.

4. Conclusions
A novel constitutive model for modelling of polymer composite matrix material, an advanced ethylene/methacrylic acid (E/MAA) copolymer, DuPont™ Surlyn® 8940 thermoplastic resin is proposed and validated using static and two-cycle tensile tests. Combining damage evolution model from [7] and the healing model from [8] with the common von Mises linear isotropic hardening plasticity model resulted in a constitutive model which enables accurate prediction of Surlyn® 8940 behaviour under tensile loading conditions and precise prediction of accumulated plastic strain. The most important, it can simulate the healing phenomenon which enables tangent stiffness recovery between
the cycles. Further research is focused on the development of a constitutive model of a carbon fibre reinforced Surlyn® 8940.

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