Mathematical model of free-flying space manipulation robot when approaching a non-cooperative spacecraft

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Abstract. The paper investigates the service satellite control system when approaching a non-cooperative spacecraft in the state of uncontrollable motion. The service satellite control system is considered as a two-level multichannel control system, and multicriteria optimization methods with cross-links are developed. The mathematical model of the control system for a free-flying space manipulation robot at the guidance and stabilization phase is proposed. The efficiency criteria are presented.

1. Introduction
A service spacecraft design is one of the promising concepts for the development of space technology. Service maintenance of out-of-order satellites, relocation of such satellites, and on-orbit assembly will allow extending the lifetime of the existing satellites.

Nowadays many research centers such as NASA [1], DARPA [2], State Space Corporation ROSCOSMOS [3], DLR [4], Northrop Grumman [5], Effective Space [6] are working in the field of space service satellites.

On-orbit satellite servicing tasks include replacement of failed blocks, in-orbit assembly, repair work, refueling of a serviced spacecraft, correction of the client-satellite orbit, or putting it into a disposal orbit [7].

A service satellite is proposed to be a free-flying spacecraft with one or more manipulators mounted on it. The serviced space vehicle is a non-cooperative one, that, in general case, may move along an unknown and rather complex trajectory.

The control strategy of a service satellite can be divided into the following stages:
- Long-range guidance – a service satellite approaching a target spacecraft at a safe distance.
- Close-range navigation – maneuvering in close proximity with service satellite.
- Stabilization occurs simultaneously with the manipulation system deployment.
- Capture is performed with the manipulator. Capture can be carried out both in automatic mode and in remote mode by controlling from the Earth or aboard a spacecraft.
- Stabilization of the service satellite and passive target spacecraft as one object.
- Service and installation work – repair of the serviced satellite, failed units’ replacement, refueling, etc. can be fulfilled at this stage.
- Service satellite separation from the served one or their further joint movement.
- Launching a serviced spacecraft into a disposal orbit.
One of the most critical tasks of any service mission is guidance, navigation, and control (GNC) [8]. The task becomes much more complicated when we deal with a passive non-cooperative tumbling target satellite.

In paper [9] some methods for guidance and control of a space robot-manipulator at its approach to a non-cooperative space vehicle on a sun-synchronous orbit are developed in conditions of uncertainty and incompleteness of measurements.

In [10] a dynamic compensation filter (DCF) is proposed to estimate the relative state and the unknown maneuver of the non-cooperative target. The trajectory tracking and prediction of space non-cooperative target can also be used for target satellite parameter estimation [11].

The proximity operations and the guidance for achieving rendezvous problems using the control based on a linear quadratic regulator approach (LQR) are addressed in paper [12]. The fly-by approach to guidance of a possibly rotating passive satellite was investigated in [13]. In paper [14] the Pontryagin minimum principle is used to solve the problem of minimum-time and minimum-energy optimal trajectories of rendezvous of a powered chaser and a passive tumbling target.

In this paper, a multichannel multicriteria system is proposed to be used for rendezvous trajectory optimization. The proposed control is based on a Coordinated Stable-Effective Compromises Methodology [15] and uses Pareto-Nash-Stackelberg game principles [16].

This paper is devoted to the control of a service satellite at the stage of guidance and stabilization when approaching a non-cooperative spacecraft. The mathematical model of a two-level multichannel hierarchical control system of the satellite while approaching a non-cooperative spacecraft that is under uncontrollable movements is proposed. The optimized trajectory of the service satellite approaching a target spacecraft at a safe distance is formed at this stage.

2. Mathematical Model
When solving the problem of relative navigation, we use a coordinate system rigidly fixed with the service satellite (Fig. 1).

![Fig. 1. Used coordinate systems](image)

The following coordinate systems are used: \( O_e X_e Y_e Z_e \) — inertial coordinate system related to the center of the earth, \( O_0 X_0 Y_0 Z_0 \) — coordinate system related to the service satellite, \( O_t X_t Y_t Z_t \) — coordinate system related to the target satellite, \( O_0 X_v Y_v Z_v \) — velocity coordinate system.
The state vector of the service satellite movement consists of 12 elements and includes coordinates of the service satellite, its angular and linear velocity, rotation angles of the related coordinate system, and the inclination and rotation angles of a service satellite trajectory.

\[
X^T = [x, y, z, v, \omega_x, \omega_y, \omega_z, \varphi, \theta, \psi, \Phi, \Psi]
\]

where \(x, y, z\) – coordinates of the service satellite center of mass in the inertial coordinate system, \(v\) – the service satellite velocity, \(\omega_x, \omega_y, \omega_z\) – the service satellite angular velocities, \(\varphi, \theta, \psi\) – rotation angles of related coordinate system in inertial coordinate system, \(\Phi\) – the inclination angle of the service satellite trajectory, \(\Psi\) – the rotation angle of the service satellite trajectory, \(\chi\) – the inclination angle of the target sight line, \(\chi\) – the rotation angle of the target sight line.

The mathematical model of the service satellite motion consists of the classical rotational equation, kinematic equation, and equation of translation motion.

\[
\begin{align*}
\dot{\omega} &= J^{-1}(-\omega \times J\omega + M) \\
2\dot{\lambda} &= \Lambda^* \omega \\
\dot{\psi} &= -\frac{\mu r}{|r|^3} + a_e + a_m + a_s \\
\dot{r} &= \psi
\end{align*}
\]

where \(M = M_u + M_m + M_a + M_g + M_c\) – the sum of the control, magnetic, aerodynamic, gravitational moments, and moments of light pressure forces, \(J\) – the satellite inertia tensor in related coordinate system, where \(\Lambda = [\lambda_0, \lambda_1, \lambda_2, \lambda_3]\) – inertial coordinate system rotation quaternion in related coordinate system, \(\mu\) – gravitational parameter, \(r\) – distance vector from the center of the Earth to the service satellite, \(a_e\) – acceleration vector caused by the shape of the Earth influence and the uneven distribution of its mass, \(a_m, a_s\) – acceleration vectors caused by the gravity of the Moon and the Sun, respectively.

Three-axis rotational equation takes the form:

\[
\begin{align*}
J_x \ddot{\omega}_x + (J_y - J_z)\omega_y \omega_z &= M_x \\
J_y \ddot{\omega}_y + (J_z - J_x)\omega_x \omega_z &= M_y \\
J_z \ddot{\omega}_z + (J_x - J_y)\omega_x \omega_y &= M_z
\end{align*}
\]

Three-axis kinematic equation takes the form:

\[
\begin{align*}
2\dot{\lambda}_0 &= -[\lambda_1 \omega_x + \lambda_2 \omega_y + \lambda_3 \omega_z] \\
2\dot{\lambda}_1 &= \lambda_0 \omega_x + \lambda_2 \omega_z - \lambda_3 \omega_y \\
2\dot{\lambda}_2 &= \lambda_0 \omega_y + \lambda_3 \omega_x - \lambda_1 \omega_z \\
2\dot{\lambda}_3 &= \lambda_0 \omega_z + \lambda_1 \omega_y - \lambda_2 \omega_x
\end{align*}
\]

Rotation angles \(\varphi, \theta, \psi\) can be found using the equations:

\[
\begin{align*}
\varphi &= \arctan \frac{2(\lambda_1 \lambda_0 + \lambda_3 \lambda_2)}{1 - 2(\lambda_1^2 + \lambda_3^2)} \\
\theta &= \arcsin(2(\lambda_3 \lambda_0 - \lambda_2 \lambda_1)) \\
\psi &= \arctan \frac{2(\lambda_2 \lambda_0 + \lambda_1 \lambda_3)}{1 - 2(\lambda_1^2 + \lambda_3^2)}
\end{align*}
\]

The inclination angle of the service satellite trajectory \(\Phi\) and the rotation angle of the service satellite trajectory \(\Psi\) can be found using the following equations:
Here $\alpha$ and $\beta$ – the angle of attack and the angle of slide, respectively. Those angles are depicted in Figure 1.

The inclination angle of the target sight line $\gamma = \arcsin \frac{\sqrt{z^2 - y^2}}{|r|}$ and the rotation angle of the target sight line $\chi = \arccos \frac{z - z_0}{x_1 - x}$ are the setpoints of the control system.

It is necessary to find the optimal control $\sigma_g$ and $\sigma_s$ at the levels of guidance and stabilization, respectively, that provides the service satellite movement from the initial point to a safe distance with a target non-cooperative spacecraft. The strategy to find those controls is discussed below.

3. **Multilevel structure of control system**

The control system of the service satellite is presented as a two-level multichannel control system. Control is carried out by using multicriteria optimization methods for multilevel multichannel systems [16]. Three-axis control at the guidance and stabilization stages is realized.

The general scheme of a two-level multi-channel control system is shown in Figure 2.

![Fig. 2. The general scheme of a two-level multi-channel control system](image)

The most effective control at the guidance and stabilization levels is formed for optimal control. After that, the optimization of the overall system with interlevel balancing is carried out.

The algorithm for multicriteria optimization of a two-level control system is as follows.

1. Choosing the structure of the control system.
2. Level balancing according to efficiency criteria.
3. Multicriteria optimization of the balanced levels.
4. Optimization of inter-level coordination based on hierarchical balancing to obtain coordinated, stable-effective compromises.

Each of the stages is discussed below.

3.1. **Structure of the control system**

The control system has a hierarchical structure and consists of a two-channel guidance level and a three-channel stabilization level.

At the guidance stage, two angles are controlled - the inclination angle of the target sight line and the rotation angle of the target sight line.

The structure of the control system is shown in Figure 3.
3.2. Level balancing

At the second stage the target efficiency criteria are formed and level balancing according to the efficiency criteria is carried out based on the Nash equilibrium.

The following criteria has been formed.

1. The optimization criteria at the guidance level:
   - accuracy $J_{g11} = (y_t - Y(t))^2 \rightarrow \min, J_{g12} = (x_t - X(t))^2 + (z_t - Z(t))^2 \rightarrow \min$
   - velocity $J_{g21} = (v_y - v_{ty})^2 \rightarrow \min, J_{g22} = (v_x - v_{tx})^2 + (v_z - v_{tz})^2 \rightarrow \min$

2. The optimization criteria at the stabilization level:
   - energy efficiency $J_{s11} = \int_{t_0}^{t_f} |\sigma_s| \, dt, J_{s12} = \int_{t_0}^{t_f} |\sigma_s^2| \, dt, J_{s13} = \int_{t_0}^{t_f} |\sigma_s^3| \, dt$
   - static accuracy $J_{s21} = (\varepsilon_s - \varepsilon_{s req})^2 \rightarrow \min; J_{s22} = (\varepsilon_s - \varepsilon_{s req})^2 \rightarrow \min; J_{s23} = (\varepsilon_s - \varepsilon_{s req})^2 \rightarrow \min$
   - oscillation $J_{s3i} = (\sigma_s - \sigma_{s req})^2 + (\eta_s - \eta_{s req})^2 \rightarrow \min, \quad i = \{\omega_x, \omega_y, \omega_z\}$
   - response time $J_{s4i} = (t - t_{req})^2 \rightarrow \min, i = \{\omega_x, \omega_y, \omega_z\}$

The solution is the Nash equilibrium if the following inequality system is true:

$$\begin{align*}
(J_{g1}(q_1, q_2^r) & \leq J_{g1}(q_1^r, q_2^r)) \\
(J_{g2}(q_1^r, q_2) & \leq J_{g2}(q_1^r, q_2^r))
\end{align*}$$

where $q^r = \{q_1^r, q_2^r\}$ – Nash equilibrium, $J_{g1} = J_{g11} + J_{g12}, J_{g2} = J_{g21} + J_{g22}$.

The deviation of the solutions from the equilibrium decreases the values of the efficiency criteria subvector.

So, a channel deviated from the equilibrium loses its efficiency. The Nash equilibrium solution at the stabilization level is found in the same way.
3.3. Multicriteria optimization
Then it is necessary to move from the equilibrium solution to the solution that provides the maximum efficiency of each of the channels. This solution corresponds to the point closest to the equilibrium solution on the Pareto border (Fig. 4).

![Pareto optimal solution](image)

In Figure 4 the point $O_n$ – the Nash equilibrium, $O_p$ – the Pareto optimal solution. The Nash arbitration method is used to find the Pareto optimal solution:

$$
\left[ J_{g1}(q) - J_{g1}(q^*) \right] \left[ J_{g2}(q) - J_{g2}(q^*) \right] \rightarrow \max_{q \in Q}
$$

3.4. Optimization of inter-level coordination
After optimization at the guidance and stabilization levels, inter-level coordination is carried out based on the hierarchical Stackelberg balancing.

The algorithm of inter-level coordination consists of three steps. At the first step the guidance subsystem informs the stabilization subsystem of its coordination $\sigma_g(t,X) \in U$. At the second step at the stabilization level, the corresponding mapping is formed. The mapping $R: U \rightarrow V$ at the stabilization level is such as for each fixed $\sigma_g(t,X) \in U$:

$$
\text{extr}_{\sigma_g} \varphi_s \left( J_{s1}(\sigma_g,\sigma_s), J_{s2}(\sigma_g,\sigma_s), J_{s3}(\sigma_g,\sigma_s) \right) = \varphi_s \left( J_{s1}(\sigma_g, R\sigma_g), J_{s2}(\sigma_g, R\sigma_g), J_{s3}(\sigma_s, R\sigma_s) \right)
$$

At the third stage, the guidance subsystem chooses a solution considering the mapping obtained at the previous step:

$$
\text{extr}_{\sigma_g} \varphi_g \left( J_{g1}(\sigma_g, R\sigma_g), J_{g2}(\sigma_g, R\sigma_g) \right) = \varphi_g \left( J_{g1}(\sigma^0_g, R\sigma_g), J_{g2}(\sigma^0_g, R\sigma_g) \right)
$$

where $\sigma^0_g, R\sigma_g$ – hierarchical Stackelberg equilibrium.

4. Conclusion
In this paper, the guidance and control method for rendezvous of a service satellite and a non-cooperative passive spacecraft has been proposed. The method allows one to get an efficient solution for multicriteria optimization task.

The hierarchical structure of a control system makes it possible to consider a cross-linking of channels and to provide hierarchy levels with inter-level balancing according to the efficiency criteria.

The proposed method can significantly simplify the control of a service satellite during rendezvous with a non-cooperative passive spacecraft being in the state of unknown uncontrolled motion.

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