Relatedness in regional development: in search of the right specification.

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Abstract

A large body of research has documented that the size and growth of an industry in a city or region depends on the local size of related industries. However, there is no consensus on how to best measure, either the relatedness between industries, or the fit of a particular industry with an entire local economy. In this paper, we perform a structured search over tens of thousands of specifications to identify optimal — in terms of out-of-sample prediction — ways to construct these quantities, using a dataset that allows us to derive relatedness from co-occurrence patterns of industries in establishments, firms, regions and countries. We find that observing industry combinations at each of these different levels of aggregation leads to different relatedness matrices, each of which can help predict the size and growth of local industries. Finally, we identify specification elements that improve predictive performance, highlight trade-offs between predictive performance and interpretability of relatedness patterns, and offer some guidance for how to use these patterns to support policy-making.

1 Introduction

The field of human geography has uncovered a number of striking empirical regularities, such as Zipf’s law for region-size distributions (Auerbach [1913] Zipf [1946]), the law of gravity for social interactions (Tinbergen [1962]), Tobler’s first law of geography for spatial dependence (Tobler [1970]) and the urban wage and productivity premiums for the exceptional role that cities play in the economy (Bettencourt et al. [2007] Rosenthal and Strange [2004]). Recently, a new regularity has been proposed: the principle of relatedness (Hidalgo et al. [2018]). According to this principle, the rate of growth of an activity in a location can be predicted from the prevalence of related activities in that same location. However, to test the principle of relatedness, researchers are left with many ad hoc choices for how to construct and then use relatedness measures. In this paper, we perform a structured search over tens of thousands of specifications of models aimed at quantifying the
strength of the principle of relatedness to provide practical guidance for empirical research in this area. At the same time, our analysis yields a number of substantive insights into the principle of relatedness and the underlying forces giving rise to it.

The principle of relatedness traces its intellectual roots to debates on how local economies reinvent themselves (e.g., Glaeser, 2005; Grabher, 1993; Jacobs, 1969; Martin and Sunley, 2006). For cities to stay relevant in a world where technologies and competitive forces constantly change, their economies need to find new growth paths. In this context, Jacobs (1969) differentiated between economic growth and economic development. The former refers to increases in efficiency: as cities become better at utilizing their existing resources, their productivity grows. The latter refers to economic renewal and diversification. According to Jacobs, cities that only raise their efficiency risk deep crises when technological paradigms change or competitive forces shift. Canonical examples can be found in the developmental histories of Detroit in the U.S., Manchester in the U.K. and the Ruhr area in Germany. The lack of renewal in such regions would later give rise to an extensive literature on path dependence and regional lock-in (e.g., Grabher, 1993; Martin and Sunley, 2006).

A common conclusion in this line of research is that, to combat decline, the successful regions of past epochs need to diversify into new activities. However, such new growth paths do not arise out of thin air. Instead, as Jacobs put it, cities grow by “adding new work to old” (Jacobs, 1969): new economic activities are often related to what a city already knows how to do.

Although the idea that local economies develop by branching into activities related to their current strengths (e.g., Frenken and Boschma, 2007) has immediate intuitive appeal, empirically validating this hypothesis initially ran into a serious obstacle: how do we decide which activities are related? It would take several decades before this issue had been resolved and Jacobs’ claims backed by quantitative evidence.

A breakthrough emerged with the insight that important information can be gained from studying the portfolios of activities in which economic entities choose to be active. Accordingly, certain combinations of economic activities generate economies of scope: there are advantages to engage in them simultaneously. Such advantages of coproduction leave a trace in the activity mix of economic entities, when myriad economic agents make micro-level portfolio decisions that aim at exploiting these economies of scope. In other words, relatedness reveals itself in the tendency of activities to co-occur in productive portfolios.1 Exploiting this insight allowed accumulating evidence in support of Jacobs’ claims across a wide variety of datasets and contexts, successfully connecting the growth of an industry (Delgado et al., 2014; Essletzbichler, 2012; Florida et al., 2012; Neffke et al., 2011; Porter, 2003), export category (Boschma et al., 2012; Zhu et al., 2017), occupation (Muneepeerakul et al., 2013), technology (Boschma et al., 2015; Petralia et al., 2017) or academic field (Guevara et al., 2016) to the local prevalence of related activities.2

These empirical studies typically proceed in three steps. First, they determine the relatedness among economic activities. Next, for each activity in a region, they calculate how prevalent related activities are in that region. Finally, they regress the growth rates of an activity on the prevalence of related activities.

Although this procedure seems straightforward, its implementation involves several ad hoc choices. In this paper, we undertake a structured exploration of tens of thousands of candidate

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1This insight was first leveraged by scholars in scientometrics (Engelsman et al., 1991) and strategic management (Teece et al., 1994). Note that, to assess which activities are related and which are not, it matters less which forces generate economies of scope than that some of these forces are shared across economic activities.

2For a recent overview, see Hidalgo et al. (2018).
specifications that aim to replicate the principle of relatedness. To do so, we create a specification grid on which we vary several aspects of the empirical model design. First, economic activities can be observed in different types of economic entities, each of which can be used to count co-occurrences of such activities. For instance, Hidalgo et al. (2007) study the co-occurrence of traded product categories in the export mix of countries, Porter (2003) and Delgado et al. (2010) of industries in regions, Teece et al. (1994) and Bryce and Winter (2009) of industries in firms, and Neffke et al. (2011) of products in manufacturing plants. Second, information on productive portfolios can be turned into relatedness matrices in different ways. Third, given a relatedness matrix, there are many ways to quantify how related one activity is to an entire economy. For each of these design aspects, we explore dozens of choices, yielding tens of thousands of candidate specifications.

Our empirical analysis relies on Dun and Bradstreet’s World Base (henceforth, “D&B data”). This dataset reports for over 100 million establishments worldwide the number of employees, geographical coordinates, headquarter-subsidiary relations and up to six industrial activities. We choose this dataset, because it allows us to observe co-occurrences of economic activities at four different levels of aggregation: the establishment, the firm, the region and the country. To test which candidate specification in our specification grid best captures the principle of relatedness, we focus on the local economies of the U.S. and aggregate the D&B data to the level of city-industry combinations. Next, we repeatedly divide the data into train and test samples and use OLS regressions to generate out-of-sample predictions for region-industry employment and employment growth patterns. This allows us to rank all candidate specifications by their out-of-sample predictive performance.

Our main findings can be summarized as follows. First, the principle of relatedness is quite robust across specifications. Many specifications corroborate the positive association between the presence of related economic activity and an industry’s local growth rate. However, that is not to say that all specifications work equally well. There is substantial heterogeneity in predictive performance across specifications, with about 10% to 20% of specifications that do not outperform a baseline specification that uses a random inter-industry relatedness matrix. Second, it proves helpful to express the prevalence of an industry in a productive unit in terms of how surprising its size is. That is two industries are related if they are often both surprisingly large in the same productive units, and an industry fits a region well if related industries are surprisingly big in the region. Third, which exact specification to use depends to some extent on the productive unit in which we observe combinations of industries and different units give rise to different relatedness matrices. Finally, good predictive performance does not guarantee that a specification is particularly useful to understand why some local industries grow faster, whereas others do not. The reason is that some of the best performing specifications yield cluttered inter-industry relatedness networks in which it is hard to disentangle clusters of related industries. However, it is possible to balance high predictive performance and the clarity of relatedness networks, in a way that makes these networks easier to use to delineate clusters of related industries.

2 Data

The D&B data are provided by Dun and Bradstreet, a business analytics firm. They contain information on over 100 million establishments across the world and offer an almost complete
variable | counts | mean | std
---|---|---|---
# employees | 165,987,254 |  |  
# establishments | 26,374,079 |  |  
# establishment (>1 SIC 3-digit industry) | 1,688,984 |  |  
# firms | 363,620 |  |  
# firm (>1 primary SIC 3-digit industry) | 83,548 |  |  
# regions | 927 |  |  
# countries | 100 |  |  
# industries | 415 |  |  
# industries per establishment | 1.08 | 0.34 |  
# industries per multi-industry establishment | 2.25 | 0.59 |  
# industries per firm | 1.66 | 2.20 |  
# industries per multi-industry firm | 3.14 | 3.42 |  
# industries per region | 288.37 | 56.11 |  
# industries per country | 316.11 | 78.72 |  
# employees per region-industry | 620.93 | 4,714.69 |  
log(# employees per region-industry) | 3.94 | 2.14 |  
Δ log(E_{ir}) | -0.08 | 0.79 |  

Table 1: Summary statistics

Multi-industry establishments are establishments with at least two distinct 3-digit SIC codes. Multi-industry firms are firms that list establishments with at least two distinct primary 3-digit SIC codes. Δ log(E_{ir}) refers to employment growth between 2011 and 2019.

census of economic establishments in the U.S.. For each establishment, the dataset reports an identifier (the so-called D.U.N.S. number). This D.U.N.S. number is unique to the establishment and remains unchanged throughout its existence, regardless of changes in ownership. Furthermore, the dataset offers for each establishment the number of employees, geographical coordinates, and the D.U.N.S. number of the parent establishment if the establishment is part of a larger corporation. Finally, each establishment can list up to six different industries in which it is active. These industries are ordered by their importance, with the primary codes identifying the establishment’s main industry. Industry codes are recorded in the SIC or NAICS classifications. In this paper, we rely on the 2011 and 2019 waves of the D&B data. Because our 2011 wave only contains SIC codes, our analysis is based on industries classified at the 3-digit level of this classification system. Table 1 provides some general statistics for the dataset.

The D&B data are highly representative of the US economy, but not necessarily of other economies. Therefore, we will mainly work with US data, limiting the sample to US establishments and defining firms as sets of US establishments that report to the same domestic (US-based) parent. However, to explore the industrial portfolios of countries, we aggregate information for a number of national economies that are reasonably well represented by the D&B data (see Appendix A).

Another drawback of using D&B data is that, although they provide a fairly accurate account

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3D.U.N.S is a recursive acronym for D.U.N.S. Universal Numbering System.

4Outside the U.S., most employment figures are based on estimates by D&B.
of the level of economic activity in a particular year, observed changes in economic activity tend to be quite noisy due to the fact that the information in the database is not updated in a uniform way (Crane and Decker 2019; Neumark et al. 2007). To limit such concerns, we will calculate growth rates over the longest possible time period. In spite of these shortcomings, the micro-level character of the data, the record of ownership ties and the fact that industry classifications do not change over time or across geography make the dataset uniquely suited for our purposes.

3 Setting up the specification grid

Our candidate specifications are based on three interrelated quantities. The prevalence of an economic activity describes how large an activity is in a productive unit. This prevalence is next used to determine the relatedness of pairs of industries. Finally, inter-industry relatedness and the prevalence of industries in a region are combined into a density around an industry that expresses the intensity of activity related to this industry in a local economy. Each of these quantities will be calculated in a number of different ways. Combining these different implementations yields the specification grid summarized in Figure 1.

3.1 Productive units

The D&B data allow us to identify which industries are combined in four types of productive units: establishments, firms, regions and countries. To distinguish clearly between establishment- and firm-level combinations, we only use establishments’ primary industries to determine the industrial portfolios of firms. As a regional unit, we use US metropolitan and micropolitan areas (“cities”). To assess which industries are found in the same countries, we aggregate the global dataset to the country level.

As we move from establishments to firms, regions and countries, the productive units start capturing a widening set of economies of scope. As a consequence, the notion of relatedness may change as we move to higher-order units. For instance, establishments are likely to combine activities that share similar inputs, technologies or skills. Firms can capture additional economies of scope across their establishments, by pooling managerial, marketing, sales or other organizational processes. At the level of cities, industries may coagglomerate to share pools of specialized labor and suppliers, or physical and institutional infrastructure (see, e.g., Diodato et al. 2018; Ellison et al. 2010). At the level of countries, the potential sources of economies of scope widen further to include climatic conditions and macro-level institutions such as intellectual property right regimes or sophisticated financial markets (Rajan and Zingales 1998).

However, moving to higher level productive units also comes at a cost: it increases the number of spurious or indirect relations between industries. For instance, ski-resorts exhibit few economies of scope with hydroelectric power plants. That is why we do not observe firms that specialize in both activities, let alone combine them in one and the same economic establishment. Yet, these industries do often colocate in the same regions. Such industry combinations, which are more likely to be found in higher-order productive units, confound relatedness estimates, because they are spurious from an economies-of-scope perspective. We will study this issue in more depth in section 4.3.
Figure 1: Constructing the specification grid.
3.2 Prevalence

To decide how often productive units combine certain industries, we first need to determine what we mean when we say that a productive unit is active in an industry. To do so, we introduce the notion of prevalence. Prevalence, $\nu_{iu}$, expresses how substantive an industry $i$’s presence is in productive unit $u$. The simplest way to quantify prevalence is by the size of the industry in the productive unit. In this paper, we measure size in terms of numbers of employees, possibly log-transformed to reduce distributional skew.

A downside of measuring prevalence in terms of raw size is that it doesn’t account for the fact that industries differ vastly in size, as do productive units. As a consequence, large industries and large productive units tend to dominate industry co-occurrences. To counteract this, prevalence is often expressed in relation to a benchmark. For instance, Hidalgo et al. (2007) use revealed comparative advantage (RCA, see Balassa, 1965) to determine which product categories have a significant presence in a country’s export basket. To be precise, the RCA benchmarks an industry’s relative size in a productive unit against its relative size in the overall economy:

$$\nu_{i/u}^{RCA} = RCA_{iu} = \frac{E_{iu}/E_u}{E_i/E}$$

where $E_{iu}$ is industry $i$’s employment in unit $u$ and omitted subscripts indicate summations over the corresponding dimension. To reduce distributional skew, we can map this measure onto the interval $[0, 1)$, using the following transformation (see, e.g., Neffke et al., 2017):

$$\nu_{i/u}^{RCA^*} = RCA_{iu}^* = \frac{RCA_{iu}}{RCA_{iu} + 1}$$

Alternatively, Gomez-Lievano (2018) and van Dam et al., (2020) propose taking logarithms:

$$\nu_{i/u}^{PMI} = \log RCA_{iu} = \log \frac{E_{iu}}{E_u E_i E} = \log \frac{p_{iu}}{p_i p_u}$$

where $p_{iu}$ is the probability that a randomly drawn worker works in industry $i$ and in unit $u$, $p_i$ the probability that the worker works in industry $i$ and $p_u$ that she works in unit $u$.

Eq. (3) is known as point-wise mutual information (PMI). It provides a useful information-theoretic interpretation of eq. (1): the (logarithm of the) RCA quantifies the amount of surprise in observing a local industry of size $E_{iu}$ when the nation-wide size of the industry is $E_i$ and the unit has a total employment of $E_u$. This amount of surprise may offer a more accurate signal of a productive unit’s underlying strengths than the size of the industry does.

However, the PMI’s benchmark presupposes that industries scale linearly with the sizes of the productive units in which they are found. The notion of surprise in the PMI may therefore be rather

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3 For firms, regions and countries, we only use the primary codes and attribute all employment in an establishment to its primary industry. For establishments, we impute the share of employees corresponding to each industry code, as described in the Appendix C.

6 In economic geography, this index is better known as the location quotient.

7 To avoid problems related to values of log 0, we increase all employment counts by one unit before calculating RCAs. van Dam et al. (2020) show that this amounts to Bayesian estimation of the PMI with a uniform prior.
restrictive. More generally, it may be possible to predict the size of an industry in a productive
unit from some general characteristics of the unit and the industry. This suggests to define surprise
against conditional expectations, given such characteristics. One way to form such conditional
expectations is regression analysis. We consider three such benchmarks:

\[
\log E_{iu} = \alpha + \beta_1 \log E_i + \beta_2 \log E_u + \nu_{iu}^{OLS},
\]

(4)

\[
\log E_{iu} = \delta_i + \gamma_u + \nu_{iu}^{FE},
\]

(5)

\[
P \left[ Y_{iu} = E_{iu} | X \vec{\beta} \right] = \frac{e^{-\exp \{ X_{iu} \vec{\beta} \}} \exp \{ X_{iu} \vec{\beta} \} E_{iu}}{E_{iu}}, \nu_{iu}^{POI} := \log (E_{iu} + 1) - \log (\hat{E}_{iu} + 1)
\]

(6)

Prevalence is now defined as the residual of these regressions. Different regression models
yield different residuals: eq. (4) uses an OLS regression with the size of the industry and of the
unit as explanatory variables; eq. (5) instead uses industry and unit fixed effects; and eq. (6) a
Poisson regression with the same explanatory variables as eq. (4).

Finally, many authors (e.g., Balland et al., 2019; Hidalgo et al., 2007) binarize prevalence
indices to arrive at an index of industrial presence: a dichotomous variable that marks whether or
not an industry has a significant presence in a productive unit. Although this procedure ignores
some information, it suppresses distributional skew and noise in the tails of prevalence indices. To
mimic this approach, we binarize prevalence metrics as follows:

\[
\pi_{iu}^{raw} = 1 (E_{iu} > 0),
\]

\[
\pi_{iu}^{RCA} = 1 (RCA_{iu} > 1),
\]

\[
\pi_{iu}^{OLS} = 1 (\nu_{iu}^{OLS} > 0),
\]

\[
\pi_{iu}^{FE} = 1 (\nu_{iu}^{FE} > 0),
\]

\[
\pi_{iu}^{POI} = 1 (\nu_{iu}^{POI} > 0).
\]

where 1(.) is an indicator function that evaluates to 1 if its argument is true.

3.3 Relatedness

Our grid relies on outcome-based relatedness measures, which focus on the imprint that relatedness
leaves on the behavior of economic actors. Outcome-based measures have been justified by

8The urban scaling literature has shown that industries exhibit a range of scaling coefficients. For instance, small,
complex industries often concentrate in large cities, whereas larger, more ubiquitous industries are spread across cities
of different sizes (e.g., Balland et al., 2020; Gomez-Lievano et al., 2016; Hong et al., 2020; Youn et al., 2016).

9See, for instance, Neffke and Henning (2013) for an application of this principle.

10To avoid the problem associated with the logarithm of 0, we increase \( \hat{E}_{iu} \) by 1 before taking logarithms in these
equations.

11Note that because because RCA* and PMI are monotonous transformation of RCA, they yield the same binarized
versions. The same holds for \( E_{iu} \) and \( \log E_{iu} \).

12Neffke and Henning (2013) distinguish between resource-based and outcome-based relatedness measures.
Resource-based measures define relatedness as the extent to which industries utilize the same resources or inputs.
Examples are relatedness measures based on input-output tables (e.g., Fan and Lang, 2000), occupational employment
vectors (e.g., Farjoun, 1994) or labor flows (Neffke and Henning, 2013).
reference to the survivor principle (Teece et al., 1994). Accordingly, the fact that we observe that two industries often coincide in the same productive units means that this combination is economically viable. Outcome-based measures have two advantages. First, activities can be related in various ways, such as their human capital requirements, resource use or value chains. Outcome-based measures summarize all these linkages in a single metric that implicitly puts most weight on the linkages that weigh most heavily in the portfolio decisions of economic actors. Second, outcome-based measures do not require additional information beyond what is required to study industrial growth or diversification. For instance, in economic geography, they can be derived by data that describe the industry mix of cities. As a consequence, outcome-based relatedness measures are by far the most common in the literature.

We can summarize prevalence information for an entire economy in \((N_u \times N_i)\) prevalence matrices, \(V\), that collect industries’ prevalence across productive units. In the same way, we can use presence metrics to create \((N_u \times N_i)\) presence matrices, \(\Pi\). Outcome-based relatedness indices transform these prevalence or presence matrices into \((N_i \times N_i)\) inter-industry relatedness matrices, \(\Phi\).

Extracting relatedness matrices from prevalence matrices has a long history in the field of scientometrics (e.g., Braam et al., 1991; Leydesdorff, 2008; Leydesdorff and Vaughan, 2006). Compared to this literature, we focus on approaches that have been used in research on diversification dynamics of regional economies. In this context, the principle of relatedness furthermore provides us with a clear task in which to evaluate their predictive performance.

One set of methods operates directly on the presence or prevalence matrix. A column of such a matrix records how the employment of the corresponding industry is distributed across productive units. The similarity of different columns can now be regarded as a measure of the relatedness between the corresponding industries. We will consider two such measures. The first is the correlation between pairs of columns in a \(\Pi\) or \(V\) matrix as in Hausmann et al. (2021):

\[
\phi^{corr}_{ij} = 1 + \frac{1}{2} corr_{u \in U}(v_{ui}, v_{uj}),
\]

where \(corr_{u \in U}\) expresses the correlation across productive units \(u\), which is then rescaled to map onto the interval \([0, 1]\), and \(U\) the set of all productive units in the economy. Two industries are thus related if they exhibit correlated prevalence patterns across productive units.

As an alternative, we consider the cosine similarity between prevalence vectors:

\[
\phi^{cos}_{ij} = 1 + \frac{1}{2} \frac{\sum_{u \in U} v_{iu} v_{ju}}{\sqrt{\sum_{u \in U} v_{iu}^2 \sum_{u \in U} v_{ju}^2}}.
\]

A different set of methods first constructs a so-called co-occurrence matrix:

\[
c_{ij} = \sum_{u \in U} \pi_{iu} \pi_{ju},
\]

where \(\pi_{iu} = 1(v_{iu} > \xi)\), with \(\xi\) some threshold value. Or, in matrix form:

\[
C = \Pi' \Pi,
\]

where \(C\) is the co-occurrence matrix and \(\Pi'\) the transpose of presence matrix \(\Pi\).
Co-occurrence matrices count the number of times that two industries are present in the same productive units. Although they are typically based on binarized presence matrices, eq. (10) immediately suggests variants that use continuous prevalence information:

\[ \tilde{C} = V'V, \]  

where \( V \) represents the matrix of industry-unit prevalences and \( \tilde{C} \) the matrix of co-prevalences.

Co-occurrence matrices are typically normalized to account for the fact that some industries are present in more productive units than others. To do so, Hidalgo et al. (2007) propose calculating the minimum of two conditional probabilities:

\[ \phi_{ij}^{\text{MCP}} = \min \left( \frac{c_{ij}}{c_i}, \frac{c_{ij}}{c_j} \right), \]  

where the first fraction represents the probability of observing industry \( j \) in a productive unit, given that we had already observed industry \( i \) in the same unit, and the second term the probability of observing \( i \) given that we observed \( j \) in the unit.

We can rewrite eq. (12) as:

\[ \phi_{ij}^{\text{MCP}} = \frac{c_{ij}}{\max(c_i, c_j)}. \]  

This shows that eq. (12) effectively ignores the size of the smallest industry when creating a benchmark for the observed co-occurrences. This choice can be relaxed in a family of normalizations of the form:

\[ \phi_{ij}^{\kappa} = \frac{c_{ij}}{\min(c_i, c_j) \kappa \max(c_i, c_j)^{1-\kappa}}, \]  

where \( 0 \leq \kappa \leq 1 \). When \( \kappa = 0 \), eq. (14) reduces to the minimum conditional probability of eq. (12), whereas \( \kappa = 0.5 \) puts equal weights on the sizes of both industries.

Another intuitive normalization is analogous to the RCA transformation of eq. (1):

\[ \phi_{ij}^{\text{RCA}} = \frac{c_{ij}}{c_i c_j}, \]  

To reduce distributional skew, we can apply the same transformation as in eq. (2), casting the relatedness value between 0 and 1.

What all of these metrics have in common is that they leverage the information available in the co-prevalence, \( \tilde{c}_{ij} \), or co-occurrence, \( c_{ij} \), terms. In Appendix D, we show that this also holds for the cosine and correlation-based metrics, which turn out to be closely related to eq. (11).

Finally, some authors (e.g., Muneepeerakul et al., 2013) argue that positive relatedness (i.e., industries that coincide surprisingly often in productive units) is fundamentally different from negative relatedness or unrelatedness (i.e., industries that are combined surprisingly little). Whereas the former points to the existence of economies of scope or shared capability requirements, the latter indicates that industries generate negative externalities for, or compete with, one another. Therefore, for each relatedness matrix, we also create a version in which we truncate relatedness. That is, we set all elements that correspond to negative relatedness to the value that represents neutral relatedness, therewith ignoring negative relatedness.

\[ \text{Large } \kappa \text{ avoid high relatedness estimates that are driven by small industries, limiting the risk of spuriously high relatedness estimates.} \]

\[ \text{See, for instance, Neffke et al. (2017).} \]

\[ \text{In correlation- or cosine-based relatedness matrices, this value is equal to zero, in RCA-based relatedness matrices,} \]
3.4 Density

Relatedness matrices are typically rather sparse: whereas a small number of combinations of industries exhibit strong ties, most industries are unrelated or weakly related. In this sense, relatedness matrices describe for each industry a natural ecosystem of other industries that should help the industry thrive. By estimating the size of these ecosystems, we can get a sense of how well an industry fits the industrial mix of an entire local economy.

To do so, [Hidalgo et al. (2007)] introduce a variable that they call density. The density of industry $i$ in city $r$, $d_{ir}$, is constructed as the weighted average prevalence of all other industries in city $r$, where the weights express industry $i$'s relatedness to these other industries. Because most industries are weakly related or unrelated, some authors only use the $k$ most related neighbors of $i$ in this calculation (e.g. [Hausmann et al. (2021)]). Taken together, this yields the following expression for $d_{ir}$:

$$d_{ir} = \sum_{j \neq i} \frac{\phi_{ij}}{\sum_{k \neq i} \phi_{ik}} v_{jr}. \quad (16)$$

where $J_i$ is the set of $k$ most closely related industries to industry $i$.

3.5 Estimation

To evaluate each specification, we study its performance in two prediction tasks: out-of-sample predictions of employment levels and out-of-sample predictions of employment growth.

For the first task, we estimate the following regression model:\footnote{Note that we exclude industry $i$ from its own neighborhood in eq. \((16)\). When we predict employment levels, the contribution of the industry itself to the sum in eq. \((16)\) would render the regression analysis circular. When predicting growth, we capture the industry’s own contribution by adding a mean-reversion term.}

$$\log E_{irt} = \delta d_{irt} + \beta_1 \log E_{it} + \beta_2 \log E_{rt} + \epsilon_{irt}, \quad (17)$$

where $E_{irt}$ represents the employment of industry $i$ in city $r$ at time $t$, $E_{it}$ the employment in industry $i$ at time $t$, and $E_{rt}$ the employment in city $r$ at time $t$. $d_{irt}$ is industry $i$'s density in city $r$ at time $t$ and $\epsilon_{irt}$ a residual.\footnote{Because $d_{irt}$ has a highly skewed distribution when $v_{jr}$ is based on RCAs or on raw employment counts, we log-transform density in these specifications.}

For the second task, we estimate the following growth model:

$$\Delta \log E_{irt} = \gamma \log E_{irt} + \delta d_{irt} + \beta_1 \log E_{it} + \beta_2 \log E_{rt} + \epsilon_{irt} \text{ if } y_{irt} > 0, y_{irt+\tau} > 0; \quad (18)$$

where $\tau$ is the time-horizon over which growth is measured and $\gamma$ captures mean reversion effects. Accounting for the latter is crucial, because $E_{irt}$ and $d_{irt}$ are typically strongly and positively correlated with the value corresponding to neutral relatedness equals one. Because negative relatedness is ill-defined for the conditional-probability-based approaches, we do not pursue this truncation strategy there. Moreover, because the positive part of cosine and correlation measures already lies in the interval $[0, 1]$ we do not need to rescale and recenter these measures as in eqs \((7)\) and \((8)\).}
correlated. Therefore, failing to account for (negative) mean reversion effects will lead to an under-
estimation of the (typically positive) effect of density. Note that we only evaluate performance for
growth at the intensive margin. That is, we only look at growth rates for local industries that exist
in both 2011 and 2019. This simplifies the analysis because it avoids observations with values of
log(0) in the dependent variable or in the mean reversion term.

To evaluate the models of eqs (17) and (18), we first divide the dataset into a train and a test
sample. The train sample is used to construct density and control variables, as well as to fit the
model’s parameters. The models are fit using Ordinary Least Squares (OLS) models. The predictive
performance is next evaluated on the test sample. We repeat this procedure 100 times to arrive at
an average model fit for each candidate specification, expressed as the model’s out-of-sample $R^2$.
Finally, as a benchmark, we also estimate models without density terms and with density terms
that are based on (symmetric) relatedness matrices whose elements are drawn at random from a
uniform distribution.

4 Results

4.1 Grid search

Fig. 2 shows distributions of model performance across specifications. As a reference, it includes
two benchmarks. First, vertical lines mark the performance of a model without density term. Second, dashed curves show the performance when density is based on random relatedness matrices.

The majority of specifications corroborate the principle of relatedness: 88% of candidate
specifications outperform the median random benchmark in predicting employment levels and 78%
in predicting employment growth. However, there is substantial heterogeneity across specifications.
For specifications in the 99th performance percentile, adding a density term raises the $R^2$ by about
10% when predicting employment levels and by about 16% in predicting employment growth. In
the median specification, the density term raises the $R^2$ of these models by between 2 and 3%.

Which aspects of the specification grid have the largest impact on predictive performance? To
answer this question, we regress the predictive performance of a specification on its characteristics.
Fig. 3 shows how much the variation in each aspect contributes to $R^2$ of this regression. The
most relevant aspect of the model specification is how exactly we define density and, in particular,
how we define the prevalence of an industry in a region. Other specification characteristics matter
less, although for predicting growth, the choice of the productive unit also has a large impact. The
least important aspect seems to be how many neighbors we consider when constructing density

10 Different studies add different control variables. Apart from controls for the overall size of the region and of the
industry, some studies add aggregate growth rates of regions and industries – what Hausmann et al. (2021) call radial
growth. Another common specification adds industry and region dummies. Note, however, that both specifications
assume information that is not available in a forecasting exercise: radial growth explicitly assumes that aggregate future
growth rates are available and the use of region and industry fixed effects makes the same assumption implicitly.

19 To determine these contributions, we attribute fractions of the $R^2$ of a model that tries to explain performance
from specification characteristics using dominance analysis. This procedure is an application of the Shapley value: it
generates all $2^k - 1$ possible combinations of regressors and estimates the model fit for each combination. Next, for
each variable, it asks by how much the average model fit drops if the variable is removed from all covariate sets of
which it is a part. In this procedure, we code the main axes of our specification grid as dummy groups that are included
or excluded as sets and use as a dependent variable the performance in employment or employment growth predictions.
Figure 2: **Kernel density plots of out-of-sample $R^2$.**
Left: prediction of employment levels; right: prediction of employment growth. Vertical gray lines: $R^2$ of specifications of eqs (17) and (18) without density ($d_{ir_1}$) term. *Candidate specifications* shows the distribution of $R^2$ in the specification grid. *Benchmark* shows the distribution for runs in which density terms are based on random proximity matrices.
variables.

Table 2 shows the two top specifications in terms of predictive validity. The first column uses the prediction of employment levels as a criterion, the second the prediction of employment growth. Apart from relying on the same productive unit for relatedness calculations, the selected specifications have little in common. Moreover, their performance is not very robust: the specification that predicts employment levels best, fails to do well in predicting employment growth, and vice versa. This suggests that the performance criteria are too noisy to confidently select an optimal specification from our grid.

Robust performance

To draw reliable conclusions, we explore which specification choices are robustly associated with good performance. We will call a specification robust if it ranks among the best 10% of specifications in each of our two prediction tasks. Next, we count how often each choice is represented in this set of robust specifications.

Tables 3 and 4 report the results of this exercise. They display for each specification characteristic the share of specifications that yield robust performance. These shares can be interpreted as the likelihood that a randomly chosen specification that uses a given characteristic will yield robust predictive performance. Each column corresponds to a type of productive unit in which inter-industry relatedness is measured. Furthermore, the tables are split into two parts, one using binary (presence) information and one using continuous (prevalence) information to construct relatedness matrices (Table 3) or density variables (Tables 4).

The percentages in bold provide the overall shares of robust specifications. The likelihood that a random specification is robust is quite low: 2.0% for specifications that use binary co-occurrence-
based relatedness matrices and 1.2% when continuous co-prevalence-based relatedness matrices are used.

**Productive unit.** Regardless of binarization, the region level seems to be the most robust choice for relatedness calculations. The next best choice is the firm level. Country and establishment portfolios, in contrast, hardly ever result in highly robust specifications. It is important to note that the region and firm levels give rise to qualitatively different industry-unit prevalence matrices. With over 80,000 firms and only 3.14 industries per firm (see Table 1), the firm-prevalence matrix is very sparse, featuring fewer than 1% non-zero elements. In comparison, with only 927 regions, over two thirds of the region-prevalence matrix consist of nonzero elements. In section 4.3, we will show that these differences also lead to qualitative differences in the topology of the relatedness networks encoded in the relatedness matrices. In particular, firm-based matrices seem more “structured” than their region-based counterparts. This may explain, why truncation and limited neighborhoods are particularly important when using region-based relatedness matrices – which feature many noisy elements – but less so when using firm- or establishment-based matrices, in which most nonzero

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| prediction task          | baseline |
|--------------------------|----------|
|                          | log(emp.) | Δ log(emp.) |
| **RELATEDNESS**          |          |
| entity                   | Region    | Region      | n.a.          |
| prevalence resid. (OLS)  | RCA       | n.a.        |
| binarized                | yes       | n.a.        |
| metric cosine dist.      | RCA*      | n.a.        |
| κ                        | n.a.      | n.a.        |
| truncation               | yes       | yes         | n.a.         |
| **DENSITY**              |          |
| prevalence resid. (OLS)  | RCA       | n.a.        |
| binarized                | yes       | yes         | n.a.         |
| # neighbors              | 300       | 100         | n.a.         |
| **OUT-OF-SAMPLE R²**     |          |
| log(emp.)                | 0.770     | 0.710       | 0.666        |
| Δ log(emp.)              | 0.063     | 0.076       | 0.060        |

Table 2: **Top specifications.** Specifications with the highest out-of-sample $R^2$ for employment levels (first column), for employment growth regressions (second column) and without density term (third column).

20Interestingly, the performance advantage of region-based relatedness matrices is driven by their capacity to predict employment levels. If we only consider employment growth, most of the best-performing specifications rely on firm-based relatedness matrices: of all specifications that rely on firm-based relatedness measures, 19.4% rank in the top decile when it comes to predicting growth patterns. For specifications relying on region-based relatedness measures this share is 10.4%, comparable to the 10.2% for specifications relying on establishment-based relatedness measures, but much better than the 0.1% for country-based relatedness measures.
| METRIC     | Cntry | Region | Firm | Estab. | Total |
|------------|-------|--------|------|--------|-------|
| RCA*       | 0.0%  | 1.7%   | 0.0% | 0.0%   | 0.4%  |
| Pearson corr. | 0.0% | 1.3%   | 0.1% | 0.0%   | 0.3%  |
| cosine dist. | 0.0% | 0.0%   | 0.3% | 0.2%   | 0.1%  |
| $\kappa = 0.0$ | 0.0% | 1.2%   | 0.3% | 0.0%   | 0.4%  |
| ... 0.1    | 0.0%  | 0.9%   | 0.3% | 0.0%   | 0.3%  |
| ... 0.2    | 0.0%  | 0.5%   | 0.3% | 0.0%   | 0.2%  |
| ... 0.3    | 0.0%  | 0.2%   | 0.2% | 0.0%   | 0.1%  |
| ... 0.4    | 0.0%  | 0.0%   | 0.2% | 0.0%   | 0.1%  |
| ... 0.5    | 0.0%  | 0.0%   | 0.2% | 0.0%   | 0.0%  |
| Total      | 0.0%  | 5.9%   | 1.9% | 0.2%   | 2.0%  |

Table 3: Definition of relatedness.
elements are very large, leaving little doubt about whether or not two industries are related.\footnote{Similarly, the implicit recommendation of Table 3 of using small values of $\kappa$ reduces noisy links created by small industries. This recommendation is more forceful for region-based than for firm-based measures.}

**Proximity metric.** Regardless of the productive unit, the general approach of Hidalgo et al. (2007) to calculating relatedness works remarkably well. Here, the relatedness matrix relies on binarized co-occurrence information, where an industry is considered to be present in a unit when its $RCA > 1$ and relatedness is defined using minimum conditional probabilities.\footnote{This generalizes to the less restrictive versions of this principle introduced in eq. (13), as long as $\kappa$ is well below 0.5 to ensure that most emphasis is put on the probability that conditions on the presence of the larger of the two industries in each pair. In Appendix E, we analyze the choice of $\kappa$ more extensively. In general, $\kappa$s should be kept relatively close to 0. However, when predicting growth rates, higher values, of around 0.2 or 0.3, also often yield high performance.} Only when relatedness is based on regional colocation patterns, defining proximity as the RCA of co-occurrences may perform even better. If, instead, continuous coprevalence information is used, matters are less clear-cut. For regional coprevalences, using RCAs, provided that they are suitably transformed to reduce skew (as in $RCA^{*}$ or PMI), suffices. However, in firm portfolios, an industry’s prevalence is best determined as the residual from the OLS regression of eq. (4). This procedure yields good results across productive units. Moreover, further experiments (not shown) that use counties instead of MSAs as a regional unit also suggest that OLS regressions generate the most useful benchmark to determine an industry’s prevalence in a productive unit.

**Density.** The aspect that has the largest impact on a specification’s performance is how we assess the prevalence of an industry in a region when calculating densities. Table 4 shows that robustness is highest when density variables are based on continuous prevalence measures. In particular, comparing an industry’s size in a city to its predicted size based on the OLS models of eq. (4) works well. Furthermore, it is helpful to focus on positive relatedness and neglect negative relatedness, supporting the argumentation in Muneepeerakul et al. (2013). Finally, the number of closely-related “neighboring” industries is not terribly important, but ideally kept small, at between 10 and 20\% of the total number of industries in the economy.

**Recommendations.** The findings in Tables 3 and 4 do not yield a single best specification for quantifying the principle of relatedness. However, we can identify a number of specification choices that should be avoided, because they generally perform poorly:

- *When constructing inter-industry relatedness matrices*

  - don’t define an industry’s prevalence in a productive unit in terms of its raw size, but evaluate this size against a benchmark;
  
  - don’t use complicated models (i.e., with unit and industry fixed effects or Poisson models) to generate these benchmarks: simple OLS specifications typically work well;
  
  - don’t use prevalence variables with highly skewed distributions such as the RCA, but instead transform these variables using, for instance, eq. (2);
### (BINARY) PRESENCE

| PREVALENCE | Cntry | Region | Firm | Estab. | Total |
|------------|-------|--------|------|--------|-------|
| RCA/RCA*/PMI | 0.0%  | 2.1%   | 0.1% | 0.4%   | 0.7%  |
| resid. (POI)| 0.0%  | 2.1%   | 0.1% | 0.4%   | 0.7%  |
| resid. (OLS)| 0.0%  | 0.0%   | 0.4% | 0.0%   | 0.1%  |
| Total       | 0.0%  | 4.3%   | 0.7% | 0.8%   | 1.4%  |

### TRUNCATION

| TRUNCATION | Cntry | Region | Firm | Estab. | Total |
|------------|-------|--------|------|--------|-------|
| positive   | 0.0%  | 1.7%   | 0.3% | 0.0%   | 0.5%  |
| all        | 0.0%  | 0.9%   | 0.3% | 0.8%   | 0.5%  |
| n.a.       | 0.0%  | 1.6%   | 0.1% | 0.0%   | 0.4%  |
| Total       | 0.0%  | 4.3%   | 0.7% | 0.8%   | 1.4%  |

### #NEIGHBORS

| #NEIGHBORS | Cntry | Region | Firm | Estab. | Total |
|------------|-------|--------|------|--------|-------|
| 50         | 0.0%  | 1.3%   | 0.3% | 0.5%   | 0.5%  |
| 100        | 0.0%  | 1.2%   | 0.1% | 0.3%   | 0.4%  |
| 200        | 0.0%  | 0.8%   | 0.1% | 0.0%   | 0.2%  |
| 300        | 0.0%  | 0.5%   | 0.1% | 0.0%   | 0.1%  |
| 415        | 0.0%  | 0.5%   | 0.1% | 0.0%   | 0.1%  |
| Total       | 0.0%  | 4.3%   | 0.7% | 0.8%   | 1.4%  |

### (CONTINUOUS) PREVALENCE

| PREVALENCE | Cntry | Region | Firm | Estab. | Total |
|------------|-------|--------|------|--------|-------|
| resid. (OLS)| 0.0%  | 1.3%   | 2.3% | 0.0%   | 0.9%  |
| RCA*       | 0.0%  | 1.1%   | 0.1% | 0.0%   | 0.3%  |
| resid. (POI)| 0.0%  | 0.9%   | 0.0% | 0.0%   | 0.2%  |
| PMI        | 0.0%  | 0.9%   | 0.0% | 0.0%   | 0.2%  |
| log(emp.)  | 0.0%  | 0.7%   | 0.1% | 0.0%   | 0.2%  |
| emp.       | 0.0%  | 0.6%   | 0.0% | 0.0%   | 0.2%  |
| Total       | 0.0%  | 5.5%   | 2.5% | 0.0%   | 2.0%  |

### TRUNCATION

| TRUNCATION | Cntry | Region | Firm | Estab. | Total |
|------------|-------|--------|------|--------|-------|
| positive   | 0.0%  | 2.4%   | 0.4% | 0.0%   | 0.7%  |
| all        | 0.0%  | 0.9%   | 0.2% | 0.0%   | 0.3%  |
| n.a.       | 0.0%  | 2.3%   | 1.8% | 0.0%   | 1.0%  |
| Total       | 0.0%  | 5.5%   | 2.5% | 0.0%   | 2.0%  |

### #NEIGHBORS

| #NEIGHBORS | Cntry | Region | Firm | Estab. | Total |
|------------|-------|--------|------|--------|-------|
| 100        | 0.0%  | 1.4%   | 0.5% | 0.0%   | 0.5%  |
| 50         | 0.0%  | 1.3%   | 0.4% | 0.0%   | 0.4%  |
| 200        | 0.0%  | 1.1%   | 0.6% | 0.0%   | 0.4%  |
| 415        | 0.0%  | 0.9%   | 0.5% | 0.0%   | 0.3%  |
| 300        | 0.0%  | 0.8%   | 0.5% | 0.0%   | 0.3%  |
| Total       | 0.0%  | 5.5%   | 2.5% | 0.0%   | 2.0%  |

Table 4: **Definition of density.**
• when using the generalized conditional probability approach for binary co-occurrence matrices, don’t use values of $\kappa$ over 0.3;

• don’t use country-level or establishment-level co-occurrences, but if available, co-occurrences at intermediate levels of aggregation.

*When constructing density variables*

• don’t use binarized prevalences;

• don’t use the raw size of an industry in a city, but compare this size against a benchmark;

• don’t use highly skewed prevalences;

• don’t use (negative) relatedness, but only focus on industries that are (positively) related to the focal industry;

Based on these recommendations, poor specifications account for about 85% of the grid. Figure 4 compares the performance of these poor specifications to the remaining specifications. Red lines show histograms over the performance percentiles of poor specifications, green lines of all remaining specifications. The figure corroborates the recommendations formulated above: the remaining specifications are much more likely to perform well in either prediction task than the ones we expect to perform poorly.

**Preferred specifications.** Because there are still many suitable specifications to choose from, we also analyze two specifications that, in light of Tables 3 and 4, are *a priori* expected to work particularly well. The first specification uses a relatedness matrix based on region-level binarized co-occurrence information. The second uses a relatedness matrix based on firm-level continuous co-prevalence information. As points of reference, we also consider the specification in [Hidalgo et al. 2007], once based on country-level co-occurrences as used in Hidalgo et al.’s original study and once based on the region-level co-occurrences that have subsequently been frequently used in economic geography (Boschma et al., 2013; Montresor and Quatraro, 2017; Zhu et al., 2017).

Table 5 shows that both preferred specifications work remarkably well, ranking among the top 5% of specifications in terms of predicting employment and employment growth. [Hidalgo et al. specification performs only marginally worse, as long as it uses region colocation information to calculate relatedness.]

### 4.2 Sector-specific estimations

So far, we have studied the principle of relatedness in the economy as a whole. However, for some industries, economies-of-scope and capability-based reasoning may not explain location patterns well. For instance, the location of industries that rely on natural resources, such as mining, agriculture or fishing, depends on geological conditions. Similarly, many nontraded services, such as restaurants and shops, require easy access to large markets, and the presence and size of public

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23We cannot exclude that the poor performance of the country-level specification is an artefact of using the D&B data, which may not be representative for all countries.
services, such as health care and education, depend on government policies. Many authors therefore restrict their analysis to industries in the private sector that produce tradable products and are not based on natural resources.

How relevant are these restrictions? To answer this question, we reanalyze our specification grid for four (mutually exclusive) sectors: public-sector industries, resource-based industries, non-traded services, and all remaining industries, to which we will refer as traded industries. Note that we only refit the regression models used to study out-of-sample performance. Relatedness matrices and density variables themselves do not change.

Figs 5a and 5b show scatter plots of the out-of-sample performance in these four different sectors against the out-of-sample performance in the overall economy. Predictive performance is highly correlated across sectors: specifications that perform well in the overall sample also tend to do so in each subsector. This consistency is reassuring: it suggests that our analysis is robust, yielding similar preferred specifications in different subsamples.

Fig. 6 explores whether predictive performance differs by sector. It shows by which percentage our preferred specifications, as well as specifications in the 90th and 95th performance percentile,

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24See Appendix B for a definition of each sector.
Table 5: Preferred specification.
Left panel: specifications for binary and continuous data. Right panel: specifications in Hidalgo et al. (2007) using industry combinations observed within countries and regions.

| RELATEDNESS          | Preferred specifications | Hidalgo et al. (2007) |
|----------------------|--------------------------|------------------------|
|                      | binary                   | continuous             | country       | region       |
| entity               | Region                   | Firm                   | Country       | Region       |
| prevalence           | RCA                      | resid. (OLS)           | RCA           | RCA          |
| binarized            | yes                      | no                     | yes           | yes          |
| metric               | RCA*                     | cosine dist.           | cond. prob.   | cond. prob.  |
| 𝜅                    | n.a.                     | n.a.                   | 0.0           | 0.0          |
| truncation           | yes                      | yes                    | n.a.          | n.a.         |
| DENSITY              |                          |                        | RCA           | RCA          |
| prevalence           | resid. (OLS)             | resid. (OLS)           | RCA           | RCA          |
| binarized            | no                       | no                     | yes           | yes          |
| # neighbors          | 100                      | 100                    | 415           | 415          |

| OUT-OF-SAMPLE PERFORMANCE |
|---------------------------|
| $R^2$ - levels (percentile) | 0.707 (0.950) 0.730 (0.989) | 0.685 (0.623) 0.704 (0.929) |
| $R^2$ - growth (percentile) | 0.069 (0.974) 0.068 (0.955) | 0.062 (0.508) 0.067 (0.926) |

Out-of-sample performance is indeed high in the traded sector and low for nontraded services and public-sector industries. Nevertheless, density still helps predict location patterns in all sectors. Moreover, it is particularly powerful in predicting the location patterns of resource-based industries and, provided that we base relatedness on regional colocation patterns in predicting the location of public-sector activities. Overall, this suggests that restricting the sample to traded industries may be helpful, but not necessary. Moreover, it casts doubt on the typical narrative to explain the principle of relatedness, namely, that it reflects the sharing of capabilities across industries.

### 4.3 Industry spaces

An important aspect of each empirical specification is the relatedness matrix. These matrices are often visualized as networks, or *industry spaces* (e.g., Hidalgo et al. [2007]) and can be used to identify clusters of related industries in the economy. In fact, how to delineate industrial clusters from relatedness matrices is an active field of research (Delgado et al. [2016]; O’Clery et al. [2019]). Here, we explore how well each matrix lends itself to this task.

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25Because growth predictions are relatively noisy, we focus here on the performance in predicting employment levels.

26The former may reflect that different resource-based activities are attracted by the same geological conditions or that their presence is betrayed by the presence of downstream industries. The latter may reflect that government services locate in predictable locations such as regional capitals or following central place theory (Christaller [1933]).
Figure 5: Out-of-sample performance by sector.
Plots show $\frac{R^2_s - R^2_b}{R^2_b}$, with $R^2_s$ the out-of-sample $R^2$s of specification $s$ and and $R^2_b$ of the baseline specification. Each observation in the scatterplots represents a specification in our specification grid, with its performance in the overall sample on the horizontal axis and in a specific subsample on the vertical axis. Panel 5a refers to predictive performance in employment levels, panel 5b to predictive performance in employment growth.

Relatedness matrices

Figure 7 first shows how relatedness matrices differ across specifications. It does so by plotting the correlations between all different relatedness matrices that were created in the specification grid. Specifications are ordered along both axes first by the productive unit in which relatedness was measured and then by other aspects of the specification.

The large diagonal blocks show that different productive units yield different relatedness matrices, suggesting that they identify different types of economies of scope.$^{27}$ Furthermore, the exact details of the specification matter less and less as we move from establishments to higher-order productive units. At the country level, relatedness matrices become very similar regardless of the specification used.

$^{27}$Within a productive unit, there are three pronounced smaller blocks. The first two correspond to binarized specifications, the third to continuous specifications. Moreover, the block in the middle is quite distinct from the other two blocks. This block contains binarized specifications, based on residuals from first fixed effects and then a simple OLS regression.
Figure 6: Increase in out-of-sample performance by sector for predicting employment levels. Bars show $\frac{R^2_s - R^2_b}{R^2_b}$, where $R^2_s$ and $R^2_b$ are the out-of-sample $R^2$s of specification $s$ and of the baseline specification respectively for the two preferred specifications (upper panels) or for the 90th- and 95th-percentile specifications (lower panels) in different sectors and using different relatedness matrices.

Modularity

Appendix F fuses our preferred binary and continuous specifications to plot two industry spaces for each productive unit. Some industry spaces exhibit more structure than others. Although deciding how “structured” a network is is somewhat subjective, we can quantify some aspect of this by determining how easy it is to identify communities in the network.

To do so, we calculate the modularity and the effective number of communities for each industry space in Figs. F.1 and F.2. The results are shown in Fig. 8. Modularity measures the extent to which a network consists of easily separable communities. It is defined as the fraction of ties that form between industries belonging to the same community, minus the fraction of such ties that we would have expected, had links formed at random. A high modularity score thus means that most links
Figure 7: **Pairwise Pearson correlation of relatedness matrices**

Observation \((i, j)\) represents the correlation between two vectors that stack the columns of the relatedness matrices associated with specification \(i\) and \(j\). Relatedness matrices are first grouped by the productive entities in which industry combinations are observed. Within these groupings, specifications are collected by the definition of industry prevalence, the metric used to convert coprevalence or co-occurrence information into a relatedness matrix and finally the exact parameter settings used therein.

The analysis reveals an interesting tension. Earlier, we saw that relatedness matrices based on regional colocation patterns yield high predictive performance. However, Fig. 8 shows that their industry spaces are comparatively cluttered and therewith less helpful in delineating clusters of industries. Firm-level relatedness matrices, in contrast, combine solid predictive performance with clear community structures in their industry spaces. The firm-based relatedness may therefore offer a better trade-off between predictive validity and the interpretability of results.

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Because relatedness matrices are too dense for network analysis, we use the networks shown in Figs F.1 and F.2 not of the full relatedness networks. To calculate modularity, we rely on the Louvain algorithm (Blondel et al., 2008) to compute the best community division and then calculate the corresponding modularity score. The random benchmark used in these calculations preserves node degrees, but randomly rewires connections.
Figure 8: Modularity and effective number of communities.
Vertical axis depicts the modularity and the horizontal axis the effective number of communities – defined as \( e \) entropy community size distribution – for different relatedness matrices. Left panel uses our preferred specification for binary presence data to construct the relatedness matrices, right panel for continuous prevalence data.

Conformance with classification hierarchy

Finally, we ask how closely relatedness matrices follow the hierarchical structure of the SIC system. To do so, we define the SIC relatedness between two industries as the number of leading digits their SIC codes shares. Given that we work with 3-digit industries, SIC relatedness can be 0, 1 or 2.

Fig. 9 shows the average relatedness for industry pairs at different levels of SIC relatedness. The binary and continuous specifications are quite similar in terms of their conformity to the classification hierarchy. However, there are pronounced differences across productive units. The closest agreement with the classification system is achieved by relatedness matrices that use establishment-level portfolio information. The next closest agreements relies on firm information. In contrast, the industrial portfolios of regions or countries yield relatedness matrices that are much less aligned with the classification hierarchy. As a consequence, the establishment- and firm-colocation based relatedness matrices will yield clusters that conform relatively closely to the higher level sectors in the industrial classification system. However, this conformity is mostly absent from relatedness matrices based on industries’ city- or country-level colocation patterns. This reiterates the trade-off between interpretability and predictiveness when using regional colocation-based relatedness.

5 Discussion and conclusion

Overall, the principle of relatedness is robust across the majority of specifications that we considered. However, we document also substantial differences in predictive performance as well as interpretability across specifications. Moreover, our findings allow us to draw some tentative conclusions about the mechanisms that underpin the principle of relatedness.
Figure 9: **Agreement with the industrial classification system.**
The vertical axis depicts the average relatedness and its 95% confidence interval between two industries based on our preferred binary (left) or continuous (right) specification. To ensure comparability, links are normalized by the sum total of elements in each relatedness matrix, multiplied by 100,000 for graphical convenience. The horizontal axis depicts industries’ SIC relatedness, where 0 means that they belong to different sectors.

**Implications for economic theory**

First, industrial portfolios of regions and firms provide particularly useful information for estimating relatedness between industries. One explanation for this is that these productive units are at an intermediate level of aggregation. Therefore, they may offer a reasonable trade-off between the benefit of capturing a wide range of economies of scope and the cost of adding spurious connections between industries.

Second, when it comes to measuring density, it is interesting that continuous versions perform somewhat better than binarized ones. Density measures start from a description of the industry mix. When we use continuous prevalence information to characterize a city’s industrial portfolio, density quantifies the size or mass of related industries, reminiscent of measures of Marshallian externalities. Note that by doing so, we implicitly treat related industries as perfect substitutes: if we shift employment between two (equally related) industries, density does not change. In contrast, if we use binarized presences, the density variable becomes a proximity-weighted count of related industries in the region and can therefore be regarded as a measure of related variety (Frenken and Boschma, 2007). This approach comes closer to measures of Jacobs externalities. Moreover, in the binary approaches, shifting employment from an existing to an equally related, yet nonexisting local industry may drastically change density. From this perspective, the fact that binarized approaches perform marginally worse than continuous approaches suggests that the mass of related activity is (slightly) more relevant than its variety.

Third, the fact that the principle of relatedness is a robust feature of economic development provides further support for an expanding body of research that aims to understand path dependence in regional growth. This literature typically relies on conceptual frameworks that emphasize shared
capability requirements. Accordingly, industries collocate and spur each other’s growth because they draw on the same parts of the region’s capability base. These capabilities may reside in the workforce, in strong ties with local suppliers, in shared knowledge infrastructure, such as universities or R&D facilities and so on. In this light, it is reassuring that the principle of relatedness finds particularly strong support for the location patterns of traded industries. However, inter-industry relatedness also helps predict location patterns for other types of industries, in particular for resource-based industries. This means that the presence of resource-based industries must be correlated with the presence of other industries. For instance, different mining activities may collocate because the minerals they mine collocate. Alternatively, resource-based industries may collocate with downstream industries that process raw materials before shipping products further. This may be particularly important when this processing leads to significant weight reduction or otherwise large savings in transportation costs.

Although none of this invalidates the principle of relatedness, it suggests that relatedness patterns should be interpreted somewhat cautiously: not all inter-industry relatedness patterns that are extracted from coproduction or colocation patterns necessarily reflect shared capabilities. Indeed, understanding why industries collocate is an important emerging area of research (e.g., Diodato et al., 2018; Ellison et al., 2010).

Limitations

Furthermore, it is important to note certain limitations in our study. First, our results rely on a single database that was not originally intended to offer a representative depiction of the world economy. Further analysis in countries other than the U.S., using, for instance, administrative datasets, could help corroborate or falsify the findings in this study. Another promising approach in this direction would rely on patent data, where co-occurrences of technologies can be studied using patents, individuals, firms, regions and countries as productive units.

Second, we have focused on the principle of relatedness in economic geography. That is, we studied model performance in predicting the presence and growth of industries in cities. This choice may explain the exceptional performance of regional colocation-based relatedness matrices. Future research could analyze specifications and predictive performance of the principle of relatedness applied to growth and diversification of establishments, firms and countries.

Third, in spite of covering important specifications, there are further specifications that could be tested. For instance, we did not consider set-based distance metrics, such as the Jaccard distance. Similarly, when residualizing industry-unit size data, we only considered industry and unit size, leaving aside other characteristics, such as diversity or productivity.

Policy implications

Finally, our results offer some insights for how to use relatedness matrices to support policy-making. First, the fact that we can corroborate the principle of relatedness shows that the growth paths of a region are predictable. However, as discussed above, this predictability may not be solely due to the exploitation of shared capabilities. Maximizing predictive power may therefore not necessarily lead to a better understanding of why certain growth paths are available. In fact, it is likely that machine learning offers more effective tools than relatedness matrices to predict future economic
development (Tacchella et al., 2021). However, these tools may not offer the same interpretability in terms of which industries benefit from each other’s presence.

In this light, our results offer a second important result. Contributing to recent research on cluster delineation (Delgado et al., 2016; O’Clery et al., 2019), we show that not all predictive relatedness matrices yield industry spaces with economically meaningful clusters of industries. Because clusters of industries are helpful concepts to organize and focus actions of multiple stakeholders, researchers face the challenge of constructing relatedness matrices that not only offer good predictive power in regional development, but also offer an intuitive, actionable map of the economy. Relatedness metrics differ in the extent to which they are confounded by indirect connections between industries. Moreover, the industrial co-occurrences shed light on different economies of scope depending on which unit of observation is chosen.

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A Representivity of industry-country aggregates

We use the 2011 wave of the D&B database to construct relatedness measures at the country level. To do so, we aggregate the number of employees to 3-digit SIC industries for all 235 countries presented in the database. Next, we select countries for which the D&B data offer a reasonably accurate depiction of the industrial composition of the national economy.

To do so, we compare the industrial employment composition for each country in the D&B data to data provided by the International Labor Organization (ILO) that provide estimates of country-level employment in 14 broad sectors. After harmonizing industry classifications, we calculate Pearson correlation coefficients for industrial employment shares in both data sources. We furthermore calculate these correlations after excluding the agriculture sector, which may be dominated by self-employed workers, especially in developing economies and therefore be poorly covered by the D&B data.

Next, we retain countries if:

1. the country has at least 100,000 employees in the D&B data,
2. the correlation of employment structure is at least 0.4 for all sectors, or at least 0.5 after excluding the agriculture sector,

This selection results in the following list of included countries:

AGO, ARE, ARG, AUS, AUT, BEL, BGD, BGR, BHR, BIH, BLR, BOL, BRA, CAN, CHE, CHL, CHN, CMR, COL, CRI, CYP, CZE, DEU, DNK, DOM, ECU, ESP, EST, ETH, FIN, FRA, GBR, GRC, GTM, HKG, HND, HRV, HTI, HUN, IDN, IND, IRN, ISL, ISR, ITA, JAM, JOR, JPN, KAZ, KHM, KOR, LBN, LKA, LTU, LUX, LVA, MAR, MDG, MEX, MKD, MLI, MMR, MOZ, MUS, MYA, NIC, NLD, NOR, NPL, NZL, PAK, PAN, PER, PHL, POL, PRT, PRY, QAT, ROU, RUS, SAU, SDN, SGP, SLV, SVK, SVN, SWE, SYR, THA, TUN, TUR, TWN, UAR, URY, USA, VEN, VNM, YEM, ZAF, ZMB

Note that data from thee countries is only used to calculate country-level relatedness matrices. All other analyses are run on the US segment of the D&B data only.

B Definition of four groups of industries

In this research, we split the 415 3-digit SIC codes into 4 groups to understand their predictability:

The resource-based industries are mainly agriculture, forestry, fishing and mining related activities, which depend on the existence of local resources. The list of 3-digit SIC codes are:

011, 013, 016, 017, 018, 019, 021, 024, 025, 027, 029, 071, 072, 074, 075, 076, 078, 081, 083, 085, 091, 092, 097, 101, 102, 103, 104, 106, 108, 109, 122, 123, 124, 131, 132, 138, 141, 142, 144, 145, 147, 148, 149

The non-traded services industries include construction, transportation, various wholesale and retail trades, etc. The list of 3-digit SIC codes are:
The public-sector industries are mainly social and governmental activities. The list of 3-digit SIC codes are:

801, 802, 804, 805, 808, 809, 821, 823, 829, 832, 833, 835, 836, 839, 861, 862, 863, 864, 865, 866, 869, 881, 911, 912, 913, 919, 921, 922, 931, 941, 943, 944, 945, 951, 953, 961, 962, 963, 964, 965, 966, 971, 972

All remaining industries were classified as traded industries, which are mainly manufacturing industries and the tradable services that could serve beyond the boundaries. The list of 3-digit SIC codes are:

201, 202, 203, 204, 205, 206, 207, 208, 209, 211, 212, 213, 214, 221, 222, 223, 224, 225, 226, 227, 228, 229, 231, 232, 233, 234, 235, 236, 237, 238, 239, 241, 242, 243, 244, 245, 249, 251, 252, 253, 254, 259, 261, 262, 263, 265, 267, 271, 272, 273, 274, 275, 276, 277, 278, 279, 281, 282, 283, 284, 285, 286, 287, 289, 291, 295, 299, 301, 302, 305, 306, 308, 311, 313, 314, 315, 316, 317, 319, 321, 322, 323, 324, 325, 326, 327, 328, 329, 331, 332, 333, 334, 335, 336, 339, 341, 342, 343, 344, 345, 346, 347, 348, 349, 351, 352, 353, 354, 355, 356, 357, 358, 359, 361, 362, 363, 364, 365, 366, 367, 369, 371, 372, 373, 374, 375, 376, 379, 381, 382, 384, 385, 386, 387, 391, 393, 394, 395, 396, 399, 481, 482, 483, 484, 489, 501, 502, 503, 504, 505, 506, 507, 508, 509, 511, 512, 513, 514, 515, 516, 517, 518, 519, 601, 602, 603, 606, 608, 609, 611, 614, 615, 616, 621, 622, 623, 628, 631, 632, 633, 635, 636, 637, 639, 671, 672, 673, 679, 701, 702, 703, 704, 731, 732, 733, 734, 735, 736, 737, 738, 781, 782, 783, 784, 791, 792, 793, 794, 799, 803, 806, 807, 811, 822, 824, 841, 842, 871, 872, 873, 874

C Distribution of employees across industries in the same establishment

The D&B database includes up to six different industry codes for each establishment that are listed in decreasing order of importance. However, the database does not offer an estimate of the number of employees associated with each industry. To impute such an estimate, we proceed as follows.

For establishments with only one industry code, we associate the total number of employees in the establishment with that industry code. For establishments with multiple industry codes, we need to estimate a function \( r_i = f(N_{industry}, rank_i) \) that maps the number of industries \( N_{industry} \) and the rank of an industry code \( rank_i \) to a ratio \( r_i \), such that \( \sum_i r_i = 1 \). For example, for an establishment with a total number of employee \( E \) and three industry codes, the mapping would require that we estimate \( E_1 = Ef(3, 1), E_2 = Ef(3, 2) \) and \( E_3 = Ef(3, 3) \).
To complete this task, we assume that the distribution of employment across industries at the establishment level is similar to that at the firm level. That is, we estimate the function above from the shares of employees for primary industry codes in US firms. Take \( f(3, 1) \) as an example, the ratio is estimated as the normalized geometric mean of the largest employment shares in firms with three different primary industry codes. The results of this estimation are shown in Table C.1.

### Table C.1: Estimation of industrial employment shares within establishments level

| \( N_{industry} \) | \( rank_i \) | \( r_i \) |
|---------------------|-------------|---------|
| 1                   | 1           | 1.000   |
| 2                   | 1           | 0.803   |
| 2                   | 2           | 0.197   |
| 3                   | 1           | 0.735   |
| 3                   | 2           | 0.202   |
| 3                   | 3           | 0.063   |
| 4                   | 1           | 0.686   |
| 4                   | 2           | 0.203   |
| 4                   | 3           | 0.082   |
| 4                   | 4           | 0.029   |
| 5                   | 1           | 0.647   |
| 5                   | 2           | 0.205   |
| 5                   | 3           | 0.089   |
| 5                   | 4           | 0.042   |
| 5                   | 5           | 0.017   |
| 6                   | 1           | 0.610   |
| 6                   | 2           | 0.205   |
| 6                   | 3           | 0.098   |
| 6                   | 4           | 0.050   |
| 6                   | 5           | 0.025   |
| 6                   | 6           | 0.012   |

The conversion of \((N_u \times N_i)\) industrial prevalence or industrial presence matrices into \((N_i \times N_i)\) inter-industry relatedness matrices relies on rescaling the intensity of co-prevalences or co-presences of industries across productive units. Measures may differ in the way they rescale such intensities, although they share commonalities at a more fundamental level. Here we illustrate this by comparing relatedness measures based on continuous prevalence information to their discrete counterparts. First, note that \( \phi_{ij}^{corr} \) in eq. (7) can be written as:

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\[ \phi_{ij}^{corr} = \frac{\frac{1}{N_u} \sum_{u \in U} v_{iu}v_{ju} - \left( \frac{1}{N_u} \sum_{u \in U} v_{iu} \right) \left( \frac{1}{N_u} \sum_{u \in U} v_{ju} \right)}{\sqrt{\frac{1}{N_u} \sum_{u \in U} v_{iu}^2 - \left( \frac{1}{N_u} \sum_{u \in U} v_{iu} \right)^2} \sqrt{\frac{1}{N_u} \sum_{u \in U} v_{ju}^2 - \left( \frac{1}{N_u} \sum_{u \in U} v_{ju} \right)^2}}, \]  

(D.1)

where \( N_u \) denotes the number of different productive units in the economy.

Also note that \( \phi_{ij}^{cos} \) in eq. (8):

\[ \phi_{ij}^{cos} = \frac{\sum_{u \in U} v_{iu}v_{ju}}{\sqrt{\sum_{u \in U} v_{iu}^2 \sum_{u \in U} v_{ju}^2}}, \]  

(D.2)

is essentially equivalent to \( \phi_{ij}^{corr} \) in eq. (D.1), except for the fact that \( \phi_{ij}^{cos} \) does not mean-center prevalences.

Now compare these continuous co-prevalence metrics to the following expression for the binary co-occurrence approach of \( \phi_{ij}^{RCA} \) in eq. (15):

\[ \phi_{ij}^{RCA} = \frac{\sum_{u \in U} v_{iu}v_{ju}}{\sum_{u \in U} \sum_{k \in J} v_{iu}v_{ku} \sum_{u \in U} \sum_{l \in J} v_{ju}v_{lu} \sum_{u \in U} \sum_{l \in J} v_{lu}v_{mu}}, \]  

(D.3)

where \( J \) represents the set of all industries in the economy.

The two expressions are very similar. Like the correlation approach, the main signal of relatedness is derived from the term \( \sum_{u \in U} v_{iu}v_{ju} \), which expresses the co-prevalence or co-occurrence of two industries \( i \) and \( j \). However, the cosine metric normalizes this co-prevalence by the raw second moments of the prevalence distributions of \( i \) and \( j \), whereas the correlation metric normalizes by their centered second moments, i.e., their variances.

The \( \phi_{ij}^{RCA} \) metric, in contrast, normalizes \( \sum_{u \in U} v_{iu}v_{ju} \) by the frequency with which industry \( i \) and \( j \) participate in co-occurrences.\(^{29}\) Therefore, it penalizes industries more heavily that tend to find themselves in large productive units, i.e., in productive units that host many industries.

In fact, if we also feed dichotomous data into eq. (8) and if each productive unit holds exactly two industries – such that the number of co-occurrences in which an industry participates equals the number of productive units that host it – we arrive at almost the same rescaling factor as in eq. (15). To see this, note that we can write the denominator of eq. (15) as:

\[ \sum_{u \in U} \sum_{k \in J} v_{iu}v_{ku} \sum_{u \in U} \sum_{l \in J} v_{ju}v_{lu} = S_iS_j, \]  

(D.4)

where \( S_a \) denotes the number of productive units in which industry \( i \) is found, and of eq. (8) as:

\[ \sqrt{\sum_{u \in U} v_{iu}^2 \sum_{u \in U} v_{ju}^2} = \sqrt{S_iS_j}. \]  

(D.5)

\(^{29}\)Note that the final term, \( \sum_{u \in U, l \in J, m \in J} v_{lu}v_{mu} \), just rescales the metric for all industry pairs to ensure that a neutral level of co-occurrence coincides with a value of 1.
Figure E.1: Conditional probability.
The graphs show the out-of-sample $R^2$ for specification that rely on eq. (14) to estimated relatedness for different values of $\kappa$, where $\kappa = 0$ corresponds to the minimum conditional probability approach in Hidalgo et al. (2007). The boxes display the interquartile range, the whiskers extend from the 10th to the 90th percentile. The gray circles mark the $R^2$ of the five best specifications.

Therefore, differences between relatedness metrics reflect how we penalize large industries. Do we want to penalize industries with a large overall presence or prevalence, as in the correlation or cosine metric? This penalization can then be done in a scale-invariant way as in the correlation metric, which mean-centers the correction terms, or not, as in the cosine metric. Or do we want to penalize industries with large co-prevalences or many co-occurrences as in the $\phi_{i,j}^{RCA}$? In this case, we account for the fact that some industries tend to be overrepresented in large productive units, whereas others are overrepresented in smaller productive units. Or do we want the penalization to be asymmetric as in $\phi_{i,j}^\kappa$?

E Grid search figures

In this section, we plot distribution of performance statistics for different choices of parameter $\kappa$ in the conditional probability metric and for different numbers of neighbors in the construction of the density measure.
Figure E.2: Number of neighbors.
The graphs show the out-of-sample $R^2$ for different numbers of neighbors considered in the calculation of density, $d_{i,t}$. The boxes display the interquartile range, the whiskers extend from the 10th to the 90th percentile. The gray circles mark the $R^2$ of the five best specifications.

F Industry spaces

In this appendix, we visualize industry spaces for each entity type using relatedness matrices from our preferred binary and continuous specifications. To do so, we follow the approach of Hidalgo et al. (2007) to create a network visualization from the dense relatedness matrix. That is, we first calculate the maximum spanning tree to extract the backbone of the relatedness matrix, and then add links with high relatedness until the number of edges equals three times the number of nodes. Nodes in the networks represent industries, ties strong relatedness connections. Nodes are furthermore colored by the high level sectors to which their industries belong.
Figure F.1: Industry spaces - binary.
| Establishment | Firm |
|---------------|------|
| Region        | Country |

Agriculture, forestry, and fishing  
Mining  
Construction  
Manufacturing  
Transportation, communications & utilities  
Wholesale trade  
Retail trade  
Finance, insurance & real estate  
Services  
Public administration

Figure F.2: Industry spaces - continuous.