Energy conditions and entropy density of the universe

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In the standard Friedmann-Lemaître-Robertson-Walker (FLRW) cosmological model, the energy conditions provide model-independent bounds on the behavior of the distance modulus. However, this method can not provide us the detailed information about the violation between the energy conditions and the observation. In this paper, we present an extended analysis of the energy conditions based upon the entropy density of the universe. On the one hand, we find that these conditions imply that entropy density $s$ depends on Hubble parameter $H(z)$. On the other hand, we compare the theoretical entropy density from the conservation law of energy-momentum tensor with that from the energy conditions using the observational Hubble parameter. We find that the two kinds of entropy density are in agreement, only when the present-day entropy density satisfies $0.0222 \leq s_0 \leq 0.7888$. We also obtain that the strong energy condition (SEC) accords with the first law of thermodynamics in the redshift range $z < 2.7$, the null energy condition (NEC) at $z < 3.2$, and the dominant energy condition (DEC) at $z > 2.6$. In addition, the energy conditions gives the deceleration parameter $0 \leq q(z) \leq 2$, which is in a predicament of the accelerated expansion of the universe. In particular, the NEC suggests $q(z) \geq \frac{1}{3}$.

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I. INTRODUCTION

Over the past decade many evidences for an accelerated expansion of the universe has been found with several independent cosmological probes, such as the supernova (SN) Ia observations [4-8,29], cosmic microwave background (CMB) [9-11], baryon acoustic oscillations (BAO) [12-14], integrated Sachs-Wolfe effect [15-17], galaxy clusters [18-21] and strong gravitational lensing [22]. There are various attempts to explain the acceleration, from dark energy to modified gravity. Combined analysis of the above cosmological observations support that an approximately 26% of cold dark matter (CDM) and the other part 74% dominated by an unknown exotic component with negative pressure-driving the current acceleration. In order to study the physical properties that hold for a variety of matter sources, Hawking and Ellis found the so-called energy condition [1-3,36], which are invoked in General Relativity to restrict general energy-momentum tensors. Because these conditions do not require a specific equation of state of the matter in the universe, they provide very simple and model-independent bounds on the behavior of the energy density, pressure and lookback time. Therefore, the energy conditions is one of many approaches to understand the evolution of universe.

However, what can we learn from the energy conditions? In 1997, Visser [23] found that current observations indicate that the strong energy condition is violated at some time between the epoch of galaxy formation and the present. This violation implies that no possible combination of normal matter is capable of fitting the observational data. In Ref. [24,25,28], the conditions is further investigated and found that all the energy conditions seem to be violated at lower redshift. However, this approach has also its own limitations. For example, it neither provide us the detailed information about the acceleration expansion, nor the reason of confrontation between the energy conditions and the observation.

In 1934, Tolman studied the principle of entropy increase on a periodic sequence of closed Friedmann-Robertson-Walker universes (Tolman 1934; North 1965). He found that a steady increase of entropy leads to growing of radiation pressure. As previous works have stated, energy conditions are a series of inequalities between energy density $\rho$ and pressure $p$. Indeed, it is a challenge to determine the evolution of energy density and pressure. From a theoretical point of view, it is not only important to clarify the relationship between energy conditions and entropy, but, whether the energy conditions is appropriate to describe the acceleration. This problem motivates us to study the influence of energy conditions on entropy density of the universe and the deceleration parameter $q(z)$.

This paper is organized as follows. In Sec. II, the energy conditions are briefly introduced. Next, in section III we primarily discuss the detailed evolution of entropy density on the strength of energy conditions and laws of thermodynamics, respectively; and then we give the behavior of the first-order derivative and second-order derivative of entropy density. Moreover, we compare the behavior of entropy density with Hubble parameter $H(t)$. In section IV, we present the influence of energy conditions on the deceleration parameter $q(z)$. The conclusions and discussion are given in Sec. V.

II. ENERGY CONDITIONS

Within the framework of the standard Friedmann-Lemaître-Robertson-Walker (FLRW) model, the energy-momentum tensor for the perfect fluid can be described by

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}. \quad (1)$$

The total energy density $\rho$ and pressure $p$ of the cosmological fluid as a function of scale factor $a(t)$ are respectively given...
by
\[ \rho = \frac{3c^2}{8\pi G} \left[ \frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} \right], \tag{2} \]

\[ p = -\frac{1}{8\pi G} \left[ 2\ddot{a} + \frac{\dot{a}^2}{a} + \frac{kc^2}{a^2} \right], \tag{3} \]

where \( k \) denotes the spatial curvature constant with \( k = +1, \) 0 and \(-1\) corresponding to a closed, flat and open universe, respectively. When we consider a FLRW universe, the null, weak, strong, and dominant energy condition can be expressed as the following forms (e.g. [1,2,3,38]):

\[ \text{NEC} \iff \rho + p \geq 0, \]
\[ \text{WEC} \iff \rho \geq 0 \quad \text{and} \quad \rho + p \geq 0, \]
\[ \text{SEC} \iff \rho + 3p \geq 0 \quad \text{and} \quad \rho + p \geq 0, \]
\[ \text{DEC} \iff \rho \geq 0 \quad \text{and} \quad -\rho \leq p \leq \rho. \]  

Thus, using Eq. (2) and (3) we can easily rewrite the energy conditions as a set of dynamical constraints on scale factor \( a(t) \)

\[ \text{NEC} : \quad \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} \geq 0, \]
\[ \text{WEC} : \quad \frac{\dot{a}^2}{a^2} \geq 0, \]
\[ \text{SEC} : \quad \frac{\ddot{a}}{a} \leq 0, \]
\[ \text{DEC} : \quad \frac{\ddot{a}}{a} + 2\left[ \frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} \right] \geq 0. \]  

Throughout this paper, we will neglect the WEC, given that this formula is always reasonable for arbitrary real \( a(t) \). Hereafter, we use the natural unity, i.e., \( h = c = G = k_B = 1. \)

### III. THE ENERGY CONDITIONS FOR ENTROPY DENSITY

In classical thermodynamics, the basic quantities are temperature \( T \), heat \( Q \), work \( W \), internal energy (actually thermal energy or simply heat) \( U \) and entropy \( S \). The classical first law is written as

\[ \Delta U = W + Q. \]  

For an inviscid fluid, the work is given by \( dW = -pdV, \) where \( V \) is the volume and \( p \) is the pressure of the fluid, so that the first law can reduce to

\[ dU = dQ - pdV. \]  

The classical second law can be read as

\[ TdS \geq dQ = dU - dW. \]  

It is clearly that energy is an element to characterize properties of entropy. Especially, it becomes quite important for an adiabatic process. Because the energy conditions renders very simple and model-independent bounds associated with energy density \( \rho \) and pressure \( p \), it does provide an essential approach which is instructive to understand the entropy of the universe.

For the perfect fluid, the conservation law of energy-momentum tensor reads

\[ T^\mu{}_{\nu\rho} = 0. \]  

In an expanding universe, it can be written as [26]

\[ a^{-3}T \frac{\partial}{\partial t} \left[ \frac{(\rho + p)a^3}{T} \right] = 0. \]  

As a result, the entropy density can be defined by

\[ s = \frac{\rho + p}{T}, \]  

which is proportional to \( a^{-3} \), namely

\[ s = s_0(a_0/a)^3, \]  

where \( s_0 \) is a constant, and the subscript 0 means that the quantity is evaluated today. Using

\[ 1 + z = \frac{a_0}{a}, \]

and combining Eq.(12) with Eq. (13), we get

\[ s = s_0(1 + z)^3. \]  

Since \( s_0 > 0 \), we easily infer that entropy density \( s \) increases with the redshift \( z \). To simplify discussion, we focus our attention only on the flat universe (\( k = 0 \)). Thereby, the energy conditions (5) can be rewritten as

\[ \text{NEC} : \quad s^{-2} \left[ \frac{1}{3} \frac{ds}{dt} \right]^2 + s^{-2} \frac{d^2s}{dt^2} \geq 0, \]
\[ \text{SEC} : \quad s^{-2} \left[ \frac{ds}{dt} \right]^2 + s^{-2} \frac{d^2s}{dt^2} \geq 0, \]
\[ \text{DEC} : \quad s^{-2} \left[ \frac{ds}{dt} \right]^2 + 3s^{-1} \frac{d^2s}{dt^2} \leq 0. \]  

In fact, in terms of the Hubble parameter

\[ H = \frac{\dot{a}}{a} = -\frac{1}{3} s^{-1} \frac{ds}{dt}, \]  

the energy conditions can further be expressed as

\[ \text{NEC} : \quad H(s) \geq H_0 s^{-13/9}, \]
\[ \text{SEC} : \quad H(s) \geq H_0 s^{-2}, \]
\[ \text{DEC} : \quad H(s) \leq H_0 s^{-4/3}, \]  

where \( H_0 = \dot{a}(t)/a(t) \bigg|_{t=0} = 100h \text{ km sec}^{-1} \text{ Mpc}^{-1} \) is the Hubble parameter today. For a flat universe, the WMAP 7-year data give \( H_0 = 71.0 \pm 2.5 \text{ km sec}^{-1} \text{ Mpc}^{-1} \) [39].
Using Eq. (17), we can obtain

\[ H(z) = \frac{\Omega_m}{H_0} (1 + z)^{3/2} + \Omega_{\Lambda}(1 + z)^{3/2} + \Omega_k(1 + z)^2, \]

(19)

which strongly and explicitly depends upon the four cosmological parameters, non-relativistic matter energy density parameter \( \Omega_m = 8\pi G \rho_m / (3 H_0^2) \), relativistic matter energy density parameter \( \Omega_r = 8\pi G \rho_r / (3 H_0^2) \), vacuum energy density parameter \( \Omega_{\Lambda} = 8\pi G \rho_{\Lambda} / (3 H_0^2) \), spatial curvature parameter \( \Omega_k \equiv -k / (a_0^2 H_0^2) \) and redshift \( z \). Based on the observational Hubble parameter \( H(z) \) data, we get that constraint from NEC and SEC on entropy density \( s \) becomes stronger with the increasing redshift \( z \). However, in contrast, the constraint from DEC becomes weaker (see Fig. 1). We also find that the theoretical value of entropy density given by Eq. (14) is in agreement with the the energy conditions, only when the present-day entropy density satisfies \( 0.0222 \leq s_0 \leq 0.7888 \).

A. Entropy density

Here, we focus in the constraints on the entropy density. Using Eq. (17), we can obtain

\[ NEC : \quad s \leq \left( \frac{H(z)}{H_0} \right)^{-9/13}, \]

\[ SEC : \quad s \leq \left( \frac{H(z)}{H_0} \right)^{-1/2}, \]

\[ DEC : \quad s \geq \left( \frac{H(z)}{H_0} \right)^{-3/4}. \]

(18)

To get the entropy density with observational data, we use the observational Hubble parameter \( H(z) \) and \( H_0 \), and substitute them into the Eq. (18). The observational Hubble parameter \( H(z) \) are given in Table 1, and the \( H_0 \) is given in Ref. [39].

![FIG. 1: Entropy density as a function of the redshift. The dots represent the entropy density which are derived from Eq. (18), using the observational Hubble parameter data (OHD). The data are given in Table 1. The dashed curve, solid curve, respectively, corresponding to \( s_0 = 0.7888, 0.0222 \), are theoretical prediction based on Eq. (14). More detailed description is in the main body of the text.](image1)

![FIG. 2: Comparison of entropy density \( s \) for two different cases. The dotted line, dash-dotted line, dashed line are speculated by NEC, SEC, DEC, respectively. The solid line corresponds to the first law of thermodynamics (FLT). It is evident that the constraints on the entropy density posted by the energy conditions and FLT have very different evolutional traits.](image2)
day entropy density satisfies 0.0222 ≤ s₀ ≤ 0.7888. The cosmological density parameter of the basic ΛCDM model given by the WMAP 7-year data [39, 40] are, respectively, the physical baryon density Ω_bh² = 0.02258±0.00057², the physical cold dark matter density Ω_c d² = 0.1109 ± 0.0056, and the dark energy density Ω₂ = 0.734 ± 0.029. Therefore, the expansion rate is E(z) = \sqrt{0.2648(1 + z)^3 + 0.734}. Eventually, we have

NEC : s ≤ [0.2648(1 + z)^3 + 0.734]^{9/26},
SEC : s ≤ [0.2648(1 + z)^3 + 0.734]^{-1/4},
DEC : s ≥ [0.2648(1 + z)^3 + 0.734]^{-3/8}. \hspace{1cm} (20)

The Eq. (20) is illustrated in Figure 2. The dotted line, dash-dotted line, dashed line are speculated by NEC, SEC, DEC, respectively. We note that the constraint from NEC and SEC become stronger with the increasing redshift z, which is the same as Figure 1. At present, it reaches the same maximum value 1. That is to say, it is not easy to distinguish these conditions now. As the second law of thermodynamics described, the entropy S always inevitably increases. On the other hand, the universe is expanding all the time. How does the entropy density evolve? Next we will study the energy conditions from the thermodynamics in §A.1 and compare the change rate of entropy density with expand rate-Hubble parameter in §B.1.

### 1. The first law of thermodynamics

Now we turn our attention to the thermodynamics feature of the entropy density. Before the development of the kinetic theory of heat, thermodynamics was applied under the assumption that matter is a continuum [37]. A major step to the understanding of entropy and of the second law of thermodynamics was made following Boltzmann’s statistical interpretation about entropy. It is a notable relationship between entropy and the total number of microstates of a system. Macroscopically, the system is represented by volume, number of particles and a given energy. In addition, the second law of thermodynamics is deemed as an absolute law, i.e., entropy always increases in a spontaneous process in an isolated system-the so called principle of entropy increase. This is not different from Newton’s laws which are always obeyed-no exception. To study relationship between entropy and the total energy, many works were made.

In a closed physical system, e.g., a sphere in three spatial dimensions, the Bekenstein bound [30], the ratio of the total entropy S to total energy E, can be expressed as

\[ \frac{S}{2\pi R^2} \leq 1, \hspace{1cm} (21) \]

where R denotes the radius of the sphere. This idea has recently been further elaborated in Ref. [31,32].

For a FLRW universe, we take the standard form of the first law of thermodynamics [34]

\[ TdS = dE + pdV, \hspace{1cm} (22) \]

where the total entropy S = sV, the total energy E = ρV, the volume of universe V = \frac{4}{3}πR³, respectively. The Hubble horizon for a flat universe is given by

\[ \bar{r}_H = \frac{c}{H} \hspace{1cm} \text{(23)} \]

Substituting S, E, V into Eq.(22), we obtain

\[ Td(sV) = d(ρV) + pdV. \hspace{1cm} (24) \]

It can be further reduced to

\[ (Td s - dρ)V = (ρ + p - T s)dV = 0, \hspace{1cm} (25) \]

where the Eq. (11) is used. For arbitrary volume, we can obtain

\[ Td s - dρ = 0. \hspace{1cm} (26) \]
Eqs. (15)-(17), the energy conditions can be expressed in the
form of entropy density itself. Unfortunately, it is difficult to present
the threshold of $d^2s/dt^2$ at dominant energy condition (DEC),
because $s$ is coupled with the square of $ds/dt$.

The first-order evolution and second-order evolution of en-
tropy density are presented in Figure 3 and Figure 4, respec-
tively. They both show that their values are negative. Because
the entropy density value is positive, the energy conditions
predicts that the entropy density decreases in the expanding
universe. As described in Figure 3, the constraint from NEC
is stronger than other two cases. However, these curves inter-
sect at the point $s = 1$.

As in Figure 4, if we take the range of present-day entropy
density $s_0 \in (0.0222, 0.7888)$, we can get the corresponding
redshift $z \in (0.8507, 6.6657)$, $s \in (5, 10)$. NEC implies that
$d^2s/dt^2$ tends to zero. For SEC, the $d^2s/dt^2$ tends to zero
when $z \in (0.9667, 6.6657)$. In addition, when $s > 10$, i.e.,
the redshift $z > 6.6657$, $d^2s/dt^2$ has no value. But when $s < 5$,
i.e., the redshift $z < 0.9667$, entropy density is in accelerated
decrease.

1. Ratio of derivative to Hubble parameter

In Boltzmann’s interpretation of entropy, the correlation of
entropy with disorder is perhaps the earliest, and has its own
roots. Increase in entropy can be associated with increase in
order. That is to say, entropy characterize the degree of dis-
order of a system. Higher entropy indicates the system is in
“confusion” and “scatter”, while lower entropy indicates the
system is in “order” and “gather”. In a free expansion pro-
cess, particles change from gather to scatter. The process of
work converted into heat energy, is from order to disorder, i.e.,
a process of entropy increase. In a local region, we compare
the evolution rate of entropy density with Hubble parameter.
In terms of Eq. (16) and Eq. (31), the energy conditions can be expressed as

\[
\begin{align*}
\text{NEC:} & \quad \frac{\dot{s}}{a} \leq -3, \\
\text{SEC:} & \quad \frac{\dot{s}}{a} \leq -3, \\
\text{DEC:} & \quad \frac{\dot{s}}{a} \geq -3,
\end{align*}
\]

It shows that the ratio is a constant, independent of redshift
$z$. Eventually, the ratio divides the energy conditions into two parts.

IV. DECELERATION PARAMETER

The cosmic accelerated expansion was primarily inferred
from observations of distant type Ia supernovae (SNe Ia; Riess
et al. 1998; Perlmutter et al. 1999). It indicates that the
unexpected gravitational physics, which a kind of energy with
negative pressure-so called dark energy, plays an important role
in the evolution of the universe.

The dark energy model with the Equation of State (con-
stant) $w < -1/3$ makes a positive contribution to the accel-
eration of the universe, while the model with $w = -1/3$ has ef-
fect on neither the acceleration nor deceleration. On the other
hand, because there still exists difficulty in the accurate treat-
ment of the defect of cosmological models, these models have
not been very thoroughly studied (Spergel & Pen 1997). Due to this motivation, many works were made to test these cosmic defect models. For example, the properties of $\Omega_{DE} - \Omega_{M}$ plane was used to test them in Ref. [35]. It is interesting to investigate the behavior of the universe from the energy conditions. The generalized epoch-dependent deceleration parameter can be defined as $q(z) = -a\dot{a}(aH^2(z))$. As seen from this definition, the accelerated growth of the cosmic scale factor $a(t)$ means $\ddot{a}(t) > 0$ which corresponds to $q(z) < 0$, while the decelerated growth of its $\ddot{a}(t) < 0$ corresponding to $q(z) > 0$. From Eq. (12) and (16), we obtain

$$q = 3[1 + s\left(\frac{ds}{dt}\right)^2 \frac{d^2s}{dt^2}].$$

(33)

Substituting Eq. (15) into Eq. (33), the energy conditions give

- **NEC**: $q(z) > \frac{5}{3}$,
- **SEC**: $q(z) \geq 0$,
- **DEC**: $q(z) \leq 2$.

(34)

From Eq. (34), we can obviously obtain $0 \leq q(z) \leq 2$. That is to say, the universe was undergoing deceleration expansion, which is not in agreement with several independent cosmological probes, such as SN Ia data, CMB, BAO [4-14].

V. CONCLUSIONS AND DISCUSSION

Although some previous works have pointed out some energy conditions violate the observational data, it did not provide us the detailed information. In this paper, we present an extended analysis of the energy conditions on entropy density. It is provided that entropy density increases with the increasing redshift $z$. Meanwhile, they give the bound of entropy density, respectively. Comparing with the observational Hubble parameter, we estimate that the present-day entropy density may be $0.0222 \leq s_0 \leq 0.7888$. Considering the first law of thermodynamics, we have found that $s$ was proportional to Hubble parameter $H(z)$. Generally speaking, entropy stands for the degree of disorder of the system, while Hubble parameter stands for the expansion rate of universe. Based on these physics, we also investigate the ratio of the evolution rate of entropy density to the Hubble parameter of universe. It shows that the ratio is a constant, independent of redshift $z$. Eventually, the ratio divides the energy conditions into two parts. At the transition redshift $z_T$, the universe reaches $\ddot{a}(z_T) = 0$ or $q(z_T) = 0$ and evolves from decelerated to accelerated expansion. In Ref. [5,35], they point out the transition redshift $z_T = 0.46 \pm 0.13$. Finally, it is worth emphasizing that we have applied the energy conditions to deceleration parameter $q(z)$. They indicated that $0 \leq q(z) \leq 2$, which is independent of redshift $z$. In summary, the energy conditions indicate that: when $0.9667 < z < 6.6657$, the universe was constantly expanding; and when $z \approx 0.9667$, the universe is in accelerated expansion. On the other hand, discussion above reveal that the null, strong, and dominant energy conditions only can be applied to the redshift range $z < 6.6657$.

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[1] S.W. Hawing and G.F.R. Ellis, *The Large Scale Structure of Spacetime* (Cambridge University Press, England, 1973).
[2] M. Visser, *Lorentzian Wormholes* (AIP Press, New York, 1996).
[3] S. Carrol, *Spacetime and Geometry: An Introduction to General Relativity* (Addison Wesley, New York, 2004).
[4] A.G. Riess et al. (High-z Supernova Search Team), The Astronomical J. 116, 3 (1998).
[5] A.G. Riess et al. (High-z Supernova Search Team), The Astrophys. J. 607, 665 (2004).
[6] S. Perlmutter et al. The Astrophys. J. 517, 2 (1999).
[7] M. Kowalski et al. The Astrophys. J. 686, 2 (2008).
[8] M. Hicken, Wood-Vasey, W. M. Blondin, et al. ApJ 700, 1097 (2009).
[9] P. de Bernardis, Ade, P. A. R., Bock, J. J., et al. Nature 404, 955 (2000).
[10] D. N. Spergel, L. Verde, H. V. Peiris, et al. ApJS 148, 175 (2003).
[11] E. Komatsu, J. Dunkley, M. R. Nolta, et al. ApJS 180, 330 (2009).
[12] D. J. Eisenstein, I. Zehavi, D. W. Hogg, et al. ApJ 633, 560 (2005).
[13] W. J. Percival, S. Cole, D. J. Eisenstein, et al. MNRAS 381, 1053 (2007).
[14] W. J. Percival, B. A. Reid, D. J. Eisenstein, et al. MNRAS 401, 2148C2168 (2010).
[15] T. Giannantonio, R. Scranton, R. G. Crittenden, et al. Phys. Rev. D 77, 123520 (2008).
[16] B. R. Granett, M. C. Neyrinck, and I. Szapudi, ApJ 683, L99 (2008).
[17] S. Ho, C. Hirata, N. Padmanabhan, U. Seljak, and N. Bahcall, Phys. Rev. D 78, 043519 (2008).
[18] S. W. Allen, D. A. Rapetti, R. W. Schmidt, et al. MNRAS 383, 879 (2008).
[19] A. Mantz, S. W. Allen, H. Ebeling, and D. Rapetti, MNRAS 387, 1179 (2008).
[20] A. Mantz, S. W. Allen, D. Rapetti, and H. Ebeling, MNRAS submitted (also arXiv:0909.3009) (2009).
[21] A. Vikhlinin, A. V. Kravtsov, R. A. Burenin, et al. ApJ 692, 1060 (2009).
[22] Xin-Juan Yang and Da-Ming Chen, Mon. Not. R. Astron. Soc. 394, 1449C1458 (2009).
[23] M. Visser, Science 276, 88 (1997); Phys. Rev. D 56, 7578 (1997).
[24] J. Santos, J.S. Alcaniz and M.J. Reboucas, Phys. Rev. D 74, 067301 (2006); J. Santos, J.S. Alcaniz, N. Pires and M.J. Reboucas, ibid. 75, 083523 (2007).
[25] Yungui Gong and Anzhong Wang, Phys. Lett B 652, 63-68 (2007).
[26] Scott Dodelson, in Modern Cosmology (Academic Press, World Publishing Company Ltd, 2008), p. 37-40.
[27] R. Jimenez, L. Verde, T. Treu, and D. Stern, ApJ, 593, 622-629 (2003), arXiv:astro-ph/0302560.
[28] M.P. Lima, S.D.P. Vitenti, M.J. Reboucas, Phys. Lett. B 668, 83-86 (2008).
[29] P. Astier et al. (SNLS collaboration), Astron. and Astrophys. 447, 31 (2006).
[30] J.D. Bekenstein, Phys. Rev. D 23, 287 (1981).
[31] D. Klemm, A.C. Petkoua, G. Siopsis, Nuclear Physics B 601, 380C394 (2001).
[32] W.G. Unruh, and R. M. Wald, Phys. Rev. D 25, 942 (1982).
[33] S.W. Wei, Y.X. Liu, and Y.Q. Wang, arXiv:1001.5238v2.
[34] M. Akbar and Rong-Gen Cai, Phys. Rev. D 75, 084003 (2007).
[35] Yuan Qiang and Tong-jie Zhang, Modern Physics Letters A 21, 75-87 (2006).
[36] R. M. Wald, General Relativity (University of Chicago Press, Chicago, 1984).
[37] Ben-Naim Arieh, Entropy Demystified, (World Scientific Publishing Co. Pte. Ltd, Singapore, 2008).
[38] C. Cattoën, M. Visser, Class. Quantum Grav. 25, 165013 (2008).
[39] D. Larson, et al., arXiv:1001.4635v1.
[40] E. Komatsu, et al., arXiv:1001.4538v2.
[41] D. Stern, R. Jimenez, L. Verde, M. Kamionkowski, and S. A. Stanford, Journal of Cosmology and Astro-Particle Physics 2, 8 (2010), arXiv:0907.3149.
[42] E. Gaztañaga, A. Cabrér, and L. Hui, MNRAS 399, 1663-1680 (2009), arXiv:0807.3551.