Inter-relationship between joint dilatancy and frictional resistance: impact on fracture behaviour

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Abstract. Rock joints possess features that alter both the characteristics of rock formations and the fracturing process. Two important properties that govern the shear behaviour and dilatancy behaviour of discontinuities have been analysed in this paper by the discrete element method (DEM). The ability of a frictional joint to suppress fracture growth decreases as the frictional resistance increases; however, the rate and extent of fracturing increases with joint dilatancy. The influence of joint frictional resistance is more dominant at high values and in this range effects of small magnitudes of dilatancy are correspondingly insignificant. At low joint friction, the occurrence of even a small amount of dilation increases the severity of the fracturing process. This study highlights the interactions between two main joint properties with the anticipation that the concepts derived here will be useful for predicting fracture behaviour at the subsurface.

Keywords: Discrete Element Method (DEM), hydraulic fracture, rock joint, rock joint friction, rock joint dilatancy

1. Introduction

The presence of discontinuities affects the structure and behaviour of rock masses. The magnitude of the impact is very pronounced when viewed at the macro level. Thus, the macro-structure and macro-behaviour are significantly affected by the existence of discontinuities which create areas of weaknesses. Investigations to ascertain this process have taken numerous forms, carried out by a number of studies. An important subject includes the impact of discontinuities on certain aspects of the fracturing process [1-16]. For instance, Casas et al. [10] determined the effect of interfacial behaviour on fracture development by analysing the influence of varying interfacial physical and material properties, Chuprakov et al. [16] examined contributions from key variables including rock frictional resistance, fracture inclination, net pressure at the fracture/fault and differential stresses, and Blair, et al.[8] introduced an alternative method, using pressure history records, to monitor fracture interactions at interfaces.

Some earlier works carried out by Bandis et al. [19], Barton and Choubey [20], Barton [21], Ladany and Archambault [22], Lee et al. [23], and Park and Song [24] have explored issues that control the shear behaviour of joints, which include dilatancy, the impact of asperities and contact of joint planes. This work investigates the relationship between conditions that instigate hydraulic
fracturing, fracture growth and joint features. Tensile strength, shear strength and cohesion are key properties that govern the behaviour of bonded joints. Joint strength is also affected by joint roughness, stiffness of the contacting planes and the presence of infills. The characteristics and effect of infills are described in Indraratna et al. [25], Trivedi [26] and Zare et al. [27].

An association between hydraulic fracturing processes and rock joint behaviour is investigated using the Discrete Element Method (DEM) and comprises such phenomena as the intensity and pattern of fracture growth, fracture interaction, properties of joints and the characteristics of the rock mass. The inter-dependency of key joint properties (e.g. shearing resistance and dilation), their impact on the strength and displacement of joints, and the corresponding effect on fracturing behaviour is the primary focus of the paper.

The outcome of this investigation is particularly relevant in the evaluation of zones prone to landslides as well as the assessment of slope stability. This also include the evolution of natural slopes [28, 29]. The role of active fault behaviour to cause landslide processes is an emerging research area [30, 31]. The results are also applicable in scenarios such as opencast mine slopes and reservoir embankments; for instance, in Biringen et al. [32] the growth of faults has been shown to cause a reduction in strength and slope stability of a water reservoir embankment.

2. Test and calibration
The behaviour of a jointed rock mass is to a great extent influenced by the joint characteristics. The shear strength of joints therefore plays an important role. One of the major influences of the shear strength is the cohesive strength and angle of internal friction. The angle of friction is also affected by the dilatancy which, in the case of joints, is controlled by the joint roughness. The joint roughness coefficient is a measure of its smoothness [21, 33] and could be used for the assessment of non-planer joints [34]. The effective normal stress across the joint also contributes to its shear strength. Direct shear tests were carried out mainly to determine joint properties such as the cohesive strength, frictional resistance, joint wall strength (JCS) and joint roughness coefficient (JRC). The shear strength can be derived from known values of cohesive strengths and frictional resistance.

In order to ascertain as well as calibrate the joint properties, a joint with specified values of properties was created in samples of the rock mass and direct shear tests conducted under varying effective normal stress conditions (ranging between 1e6 MPa to 5e6 MPa). For the first series of tests the micro-property representing the joint friction angle was varied with each successive test but the joint cohesive strength was kept constant. The values of the microscopic properties of the synthetic rock sample and assigned joint properties are given in Table 1 and Table 2.

| Parameter Description                                | Values          |
|------------------------------------------------------|-----------------|
| Contact-bond normal strength (mean)                  | 5.0 MPa         |
| Contact-bond normal strength (std deviation)         | 1.25 MPa        |
| Contact-bond shear strength                          | 5.0 MPa         |
| Contact-bond normal strength (std deviation)         | 1.25 MPa        |
| Particle size (radius)                               | 0.002 m – 0.004 m|
| Particle friction coefficient                        | 1.0             |
| Particle normal stiffness, $K_n$                     | 29.0 MPa        |
| Particle shear stiffness, $K_s$                       | 10.36 MPa       |
| Particle density                                     | 2650 kg/ m$^3$  |
| Porosity                                             | 0.16            |
| Particle-particle contact modulus                     | 14.5 GPa        |
| Particle stiffness ratio                             | 2.8             |
| Parameter Description          | Values                      |
|--------------------------------|-----------------------------|
| Normal stiffness, $k^n$        | $1.583 \times 10^{12}$ Pa/m |
| Shear stiffness, $k^s$         | $0.565 \times 10^{12}$ Pa/m |
| Friction coefficient, $\phi$  | Varied accordingly (btw 0.0 & 1.0) |
| Dilation angle, $\psi$        | Varied accordingly (btw 0.0 & 40) |
| Cohesive strength, $c$        | 0.0                         |

**Table 2.** Rock mechanical properties.

| Parameter Description          | Values          |
|--------------------------------|-----------------|
| Contact-bond normal strength, $\tilde{Q}$ | 11.7 MPa        |
| Elastic modulus, $E$            | 9.7 GPa         |
| Poisson ratio                  | 0.19            |

Using samples with dimensions of height=0.3 m and width 0.6 m, a single planar longitudinal joint was created along the centre of the intact sample (Figure 1). The joints were made to be smooth and as such have negligible joint roughness coefficients ($JCR \approx 0$). Barton and Choubey [20] provides standard curves that state values of $JCR$ corresponding to the roughness of joints. A value of $JCR$ ranging between 0 and 2 is suggested for smooth planar joints. Particles bordering the two planar surfaces of the joint that meet the criterion for selection for the smooth-joint contact were identified and a zero bond strength allocated to those contacts (Figure 2b) dividing the two joint surfaces.

![a) Position and orientation of joint](image1)

![b) Collection of smooth joint contacts that form the joint](image2)

**Figure 2.** Joint configuration and position of contacts

The layout of the shear tests is shown in Figure 3. Vertical stresses representing effective normal stresses were applied via the horizontal walls. The bottom left wall was fixed in the horizontal direction but particles in contact with it were allowed to slide vertically. Similarly, particles were able to slide laterally along the top and bottom horizontal walls. To apply a steady load, a constant horizontal velocity was applied to the top left vertical wall in the W-E direction. The horizontal velocity was set to 0.003 m/s to guarantee quasi-static equilibrium during the test.
Figures 4 and 5 depict the shearing behaviour of different frictional joints under different normal loading conditions. The shear stress reaches a peak strength value before gradually reducing to a residual strength.

**Figure 3** Configuration of shear test and boundary conditions.

(a) Schematic of test configuration showing boundary conditions.

(b) Alignment of walls with respect to the synthetic sample.

**Figure 4.** Shear behaviour of the joint during the test.
Comparison of shear behaviours of a joint with a friction coefficient of 0.2.

Comparison of shear behaviours of a joint with a friction coefficient of 0.5.

Figure 5. Shear behaviour under different normal stress conditions.

To complete the calibration of the joint friction property failure envelopes were constructed for various assigned friction coefficients, using peak shear strength values and effective stresses normal to the joint plane as shown in Figures 6. In the figure, the limit of the shear strength when a friction coefficient of 0.2 is specified is shown. The equation describing the curve in terms of the peak shear stress and effective normal stress is

\[
\tau = 0.0045 + 0.1993\sigma_n
\]  

(4a)

From Equation 5 the cohesive strength \( c \) is 0.0045 MPa, which is relatively negligible \( c \approx 0 \). The friction coefficient \( \phi \) is 0.1993 (\( \approx 0.2 \)) and the corresponding friction angle is given as

\[
\phi = \tan^{-1}(0.1993) = 11.27^\circ
\]

(4b)

\( \phi \approx 11.3^\circ \) and \( c \approx 0.0 \) MPa

Likewise, when a friction coefficient of 0.5 is set as an input parameter value to define the joint characteristics, the shear envelope derived is describe by:

\[
\tau = 0.0261 + 0.4729\sigma_n
\]

(5a)

The cohesive strength \( c \) is 0.0261 MPa, which is also relatively negligible. The friction coefficient \( \phi \) is 0.4729 (\( \approx 0.5 \)) and the corresponding friction angle is given as

\[
\phi = \tan^{-1}(0.4729) = 25.31^\circ
\]

(5b)

\( \phi \approx 25.3^\circ \) and \( c \approx 0.0 \) MPa
From Equations 4-5, the numerical experimentally derived joint strength properties match the values of the input micro-parameters (e.g. friction angle and cohesive strength). Shear tests using other values of inputted friction coefficient and cohesive strength show matching results. Table 3 shows a comparison of some of the results.

**Table 3.** Comparison between inputted and derived joint properties.

| Parameters          | Batch | Micro-property | Derived value |
|---------------------|-------|----------------|---------------|
|                     |       | Coefficient   | Angle         | Coefficient | Angle |
| Friction            | Test 1| 0.20           | 11.30 °C      | 0.199       | 11.27 °C |
|                     | Test 2| 0.50           | 26.57 °C      | 0.473       | 25.30 °C |
| Cohesive strength   | Test 1| 0.0            |               | 0.0261      |       |
| (MPa)               | Test 2| 0.0            |               | 0.0045      |       |

**Figure 6.** Joint failure envelope for an assigned friction coefficient of 0.2.

The contact force distribution for joints with friction coefficients of 0.2 is shown in Figure 7. Localised concentration of contact forces occur at the joint surface and the distribution of such spots along the joint increases with the normal stress. This is attributed to the increase in contact area between the two planes of each joint as the stress acting normally to it increases. Higher concentrations of contact forces also exist at the top left and bottom right sections of the rock mass, mainly because of the pressure/loading on the top left wall as well as the lateral restrictive support of the bottom right wall. Concentration and distribution of contact forces increases with the normal stress as localisation is more distinct at lower normal stresses. At higher normal stresses, dilation of the joint is restrained. Figure 8 shows the micro tensile and shear cracks, mostly initiated near the joint planes. The prevalence of micro cracks increases with normal stress and joint frictional resistance.
Figure 7. Contact force distribution at various normal stress conditions (Friction coefficient = 0.2) [Tensile: red, Compressive: black]
3. Simulation methodology

3.1. Features of the model domain

Samples of synthetic rock materials with properties similar to that tested (e.g. using biaxial and direct shear tests) were adopted. The synthetic rock material is representative of sandstone. Whereas the micro properties and mechanical properties are given in Table 1-2, the sample dimensions as shown in Figure 9 is width = 2 m and height = 1 m. Also the rock domain is deliberated jointed within by placing joints at strategic locations. The number, positions and spacing of the joints make up for different scenarios of the model set-up. The effect of the various layouts as well as the features of the network of joints is considered. The joint network comprises the size, position, spacing between each joint and connectivity if applicable. The first round of tests was conducted on a rock mass consisting of two parallel through joints inserted at locations of equal distance (0.15 m) from the centre of the domain. Both joints are lateral spanning in the XY direction in 2D (Figure 9-10). It is expected that in a 3D domain the joints will cut across the out of plane direction (Z plane).
3.2. Boundary conditions and loading

Top and bottom vertical stresses, representing an overburden effect, were applied in addition to lateral confining stresses. The combination of these generates in-situ stress conditions. The maximum principal stress ($\sigma_1$) and minimum principal stresses $\sigma_3$ act in the vertical and horizontal directions respectively, where $\sigma_1 = 2.5$ MPa and $\sigma_3 = 2.0$ MPa. Fluid is introduced at the centre of the rock mass (Figure 10) at a final injection pressure of 35 MPa, maintained for the entire duration of the simulation. The loading is intended to cause a perturbation of fluid pressure as a result of the flow of fluid from a remote and singular location (e.g. an injection well). The main fluid properties are presented in Table 4.

![Figure 9. Layout of rock mass including two parallel lateral joints.](image)

![Figure 10. Fluid injected at the centre of the rock mass in between two parallel joints](image)

| Parameter Description | Values     |
|-----------------------|------------|
| Viscosity, $\mu$      | $3.95 \times 10^2$ Pa-s |
| Density, $\rho_f$     | $479$ Kg/m$^3$ |
| Bulk modulus, $K_f$   | $0.035$ GN/m$^2$ |
4. Results and discussion
The key properties controlling joint behaviour include the shearing resistance, dilatancy, surface roughness and joint wall compressive strength. The direct impact of these properties on the overall joint performance and the corresponding role of the affected joints in association with subsurface events are of interest. Joint shearing resistance is described by its friction angle (or friction coefficient) which contributes to the shear strength and is in fact considered a measure of the joint shear strength. Variations in the joint friction angle will therefore affect its responses to natural and induced phenomena as well as alter the way in which the joint impacts on surrounding activities. In previous studies the extent of dependence of subsurface events on the frictional behaviour of joints within the proximity of such occurrences was determined. A typical case of a subsurface activity is the perturbation caused by fluid pressure (hydrostatic or via fluid flow) and the resulting onset and proliferation of fractures. Propagation and interconnectivity of fractures is an important phenomenon that could have an enormous impact. As a product of fluid pressure perturbation, the pattern and intensity of fractures as a function of both changes in the joint frictional resistance and changes in dilatancy is appraised. The friction coefficient was varied according to the following values: 0.0, 0.2, 0.5, 0.7 and 1.0, which correspond to friction angles of 0.0°, 11.3°, 26.6°, 35.0° and 45.0° respectively.

4.1. The relationship between joint dilatancy and frictional behaviour
The effect of the friction resistance of non-dilatant ($\varphi = 0$) rock joints on the fracturing process has been illustrated in a previous study. A similar investigation to determine the influence of frictional resistance on dilatant ($\varphi \neq 0$) rock joints invariably requires that its dilatant behaviour be considered. Consequently, a combination of two properties is essential: rock joint friction angle and dilation angle. The joint frictional resistance is a dependent variable that is also influenced by its dilation. In order to establish a link between the properties with respect to their joint effect on the fracturing process the two properties are simultaneously varied. The following presentation examines the characteristics of dilatant rock joints in an attempt to highlight the conformity of certain fracturing processes to peculiar combinations of frictional and dilatant behaviour. A summary of values representing some combinations of the joint dilatancy and frictional resistance is presented in Table 5.

| Case 1 | Case 2 | Case 3 | Case 4 |
|--------|--------|--------|--------|
| Friction Dilatancy | Friction Dilatancy | Friction Dilatancy | Friction Dilatancy |
| 2.5° | 5.0° | 0.2 (11.3°) | 5.0° |
| 5.0° | 2.5° | 10.0° | 0.5 (26.6°) |
| 0.2 (11.3°) | 10.0° | 5.0° | 20.0° |
| 20.0° | 0.7 (35.0°) | 10.0° | 30.0° |
| 35.0° | 1.0 (45.0°) | 20.0° | 40.0° |

The range of friction coefficient is 0.2 – 1.0 while the range of dilation angle is 2.5° – 40° and for some cases the frictional resistance is fixed whereas the degree of dilatancy is adjusted. For the combination of a low friction coefficient and a very low dilation angle (0.2, 2.5°), fracture propagation is constrained between the joints with only a few isolated cracks occurring at the outer regions away from the perimeter of the joint plane. Although, the intensity of fracturing is similar to cases involving non frictional non-dilatant rock joints and low frictional non-dilatant rock joints, the pattern of fracturing is somewhat different. The fracturing due to the presence of non-dilatant non-frictional/low frictional joints is more widely spread. For the same non/low frictional rock joint, a small increment in
the dilatancy; for instance, an increase to $\varphi = 5.0^\circ$, results in an upward and perpendicular propagation of fractures across the top joint and the generation of pockets of crack initiation at several remote locations along the plane of the top and bottom joints (Figure 11a). The fracture growth at the outer section becomes relatively rapid once it crosses the joint plane. The low frictional resistance of the joints means that the joints are more sensitive and as such the impact of dilation is more pronounced causing a greater severity in the movement and shearing of the joint planes.

Due to the lateral movement of the joints the occurrence of cracks initiated along the joint plane is preponderantly shear induced. Nevertheless, propagation of the resulting fractures is mainly caused by tensile failure of the rock material. The increment in dilatancy not only creates additional spots of crack initiation, it intensifies propagation in directions away from the joint plane. With a further increase in joint dilatancy ($\varphi = 20.0^\circ$), the intensity of fracturing increases correspondingly (Figure 11b). The fractures created along the joint plane, as shown in this case, propagate away from the plane as well as in directions parallel and adjacent to the joint plane. There is a preference for fractures to grow along the joint surface than away from it. Isolated spots of fractures initially created are readily linked to form a coalescence of cracks before further proliferation away from the joint plane. A more intense state of fracturing occurs when the dilatancy is within the high range ($\varphi = 40.0^\circ$) (Figure 11c). The fractures spread along many sections of the joint planes and then grow extensively into other regions outside the enclosure of the two joints.

Another effect of joint dilatancy is the possibility of cavity initiation along the joint surfaces. This phenomenon is more noticeable when the dilatancy is in the medium to high range with the number and sizes of cavities increasing as dilatancy increases. To illustrate, a greater number of cavities are created and a greater number grow into considerably sizes when the joint dilation angle is $40^\circ$ (Figure 11c) as compared with a dilation angle of $20^\circ$ (Figure 11b). Apart from the initial location above the point of fluid injection where the fracture propagating directly from the point of fluid injection crosses the upper joint, the location of individual cavities is inconsistent with the degree of dilatancy. A comparison between joints of dilation angle $10^\circ$, $20^\circ$, and $40^\circ$ indicates that besides the cavity created just above the point of fluid injection the number and location of other cavities is different for dissimilar joint dilatancies.

(a) Fracture development (dilation angle: 5.0).
The rate of generation and population of cracks indicates an intensity of shear induced cracks significantly higher than tensile induced cracks (Figure 12); the disparity in the rate and number of cracks increases progressively with time such that at the later stages the extent of shear cracks is so much greater than tensile cracks. The preponderance of shear cracks is attributed to the sliding performance of the joint surfaces which drives its shearing behaviour. The less frictional a joint is the greater the tendency for the surfaces to slide against each other that may result in the onset and propagation of cracks caused by shear stresses. Highly frictional joints allow relatively fewer cases of shearing events.

The effect of rock joint dilatancy on the fracturing process becomes apparent when results for the different magnitudes of dilatancies are juxtaposed. The intensity of total fracturing increases with the dilation angle (Figure 13a). The rate of increment between the two variables is disproportionate especially within the range of high dilation angles (Figure 14) and a similar trend occurs throughout the elapse period. For instance, at a timestep of 2.48e2, the total number of cracks is as follows: 22, 51, 100, 254 and 641, for a joint dilation angle of 2.5°, 5.0°, 10.0°, 20.0° and 40.0°, respectively. The same pattern is observed for both shear and tensile cracks (Figure 13 b-c).
(a) Fracture development (dilation angle: 2.5).

(b) Fracture development (dilation angle: 40.0).

**Figure 12.** Tensile and shear fracture development at various joint dilatancies (Friction coefficient: 0.2).

(a) Tensile fracture development

(b) Shear fracture development.

**Figure 13.** Effect of dilatancy on fracture development (Friction coefficient: 0.2)

**Figure 14.** Magnitude of total fracture development at different rock joint dilatancies (Friction coefficient: 0.2).
6. Conclusions
The existence of discontinuities (joints) in rock masses affects its strength and response to both in-situ and external forcings. They major elements that determine the attributes of discontinuities are often contained in their mechanical and geo-chemical properties, which makes the study of their characteristics essential, especially if the joint interface is bonded. The behaviour of bonded discontinuities is determined by the nature of the fill, which may be permeable or impermeable; on the other hand, the behaviour of unbounded discontinuities is mainly governed by its mechanical and physical properties. The compressive strength (JCS), shear strength and dilatancy of the interface are some prevailing factors. This is especially so in cases where they dominate the over-burden pressure and in-situ stresses (e.g. at shallow depths).

Prior investigations conducted by, for instance, Hanson et al. [4], Hanson et al. [39] and Anderson [40] are limited in scope. Unlike previous studies [4, 39, 40], a more detailed analysis of the fracturing phenomenon which elaborately highlights the role of the key properties that control the behaviour of unbonded interfaces and the interaction between approaching fractures, the interface and fluid pressure perturbation has been presented in this research. The interplay between selected interface properties was also discussed and the numerical methodology adopted provided the means for a more rigorous investigation of the processes at the inter-particle level as well as enabled the flexible control of certain features of the interface. Two important and related phenomena were investigated: the shear behaviour of interfaces represented by the frictional resistance (the cohesive strength is negligible for unbounded interfaces), interface dilatancy and the interplay between the two properties. Although the numerical experiments involved the interface between rock in formations (also referred herein as joints), the concepts applied and results obtained are applicable to analogous forms of discontinuities. The following outlines the outcome of this research:

The dilatancy of rock joints contribute to its performance and impacts on its frictional behaviour. At low joint friction the occurrence of even a small amount of dilation increases the severity of the fracturing process. In addition to facilitating extra zones of fracture initiation at the joint plane, increments in dilatancy intensify fracture propagation in directions away from the joint plane, albeit there is a preference for fractures to grow along the joint plane. Dilation enhances the tendency for cavity initiation along the joint plane and the number and size of cavity increases as dilation increases.

The onset of cracks at joint planes is predominantly instigated by shear failure and tensile failure of the rock material induces propagation of the resulting fractures. The intensity of shear induced fractures is considerably greater than tensile induced fractures and the predominance of shear fractures is attributed to the sliding performance of the joint planes. Generally, the rate and magnitude of shear and tensile fracturing increases with joint dilatancy. For a constant magnitude of joint dilatancy the intensity of fracturing reduces appreciably as the joint frictional resistance increases. The effect of the frictional resistance becomes more dominant as it tends towards high values and within this range the effect of small magnitudes of dilatancy is trivial. At the vicinity of highly frictional joints, fracture development at low joint dilatancy is greater than fracture development at medium joint dilatancy. Unless joint dilatancy is low, the rate and magnitude of fracturing decreases as joint friction increases. Further studies are essential to include the impact of variations in overburden pressure conditions as well as differences in rock material properties due to layering.

Nomenclature

\[ c \quad \text{Cohesive strength} \]
\[ E \quad \text{Young’s Modulus} \]
\[ E^* \quad \text{Young’s Modulus in plain strain} \]
\( e_y \) Axial strain
\( e_x \) Lateral strain
\( \varepsilon_v \) Volumetric strain
\( \varepsilon_1 \) Major principal strain
\( \varepsilon_3 \) Minor principal strain
\( JCR \) Joint roughness coefficient
\( JCS \) Joint wall compressive strength
\( k_n \) Particle normal stiffness
\( k_s \) Particle shear stiffness
\( k^n \) Normal stiffness
\( k^s \) Shear stiffness
\( K_f \) Bulk modulus
\( \hat{n}_j \) Unit normal vector defining the joint plane
\( N_{tc} \) Estimated rate of development of tensile cracks
\( N_{sc} \) Estimated rate of development of shear cracks
\( \bar{q} \) Compressive strength
\( q_u \) Unconfined compressive strength
\( T \) Tensile strength
\( t \) Elapsed time
\( \tau_p \) Peak shear strength
\( \tau_r \) Residual value of shear strength
\( \tau_\theta \) The shear stress required to overcome the volumetric expansion
\( \rho_f \) Density
\( \nu \) Poisson’s ratio
\( \nu^* \) Poisson’s ratio in plain strain
\( \gamma \) Plastic shear strain
\( \gamma_{max} \) Maximum plastic shear strain
\( \theta \) Dip angle
\( \theta_d \) Average angle of deviation of the joint plane/ joint surface particles from the direction of applied shear stress
\( \vartheta \) Dip direction
\( \sigma_D \) Differential stress
\( \sigma_n \) Normal stress
\( \sigma_y \) Axial stress
\( \sigma_x \) Lateral stress
\( \sigma_1 \) Major effective principal stress
\( \sigma_3 \) Minor effective principal stress
\( \phi \) Angle of internal friction (friction angle)
\( \phi_r \) Residual value of friction angle  
\( \phi_b \) Basic friction angle  
\( \phi_{crit} \) Critical friction angle  
\( \phi_f \) Interparticle friction angle corrected for work done or energy dissipated due to expansion  
\( \phi_t \) The true angle of friction between the mineral surfaces of the particles  
\( \phi_{cv} \) Angle of friction under constant volume  
\( \varphi \) Dilation of a material, joint or discontinuity  
\( \varphi_p \) Peak dilation, which is the same as the maximum dilation  
\( \mu \) Viscosity  

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