Probing Gravitational Cat States in Canonical Quantum Theory vs Objective Collapse Theories

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Abstract

Using as a testbed the recently proposed “gravcat” experimental scheme in [1], we compare the properties of gravitational cat states in three descriptions: (1) canonical quantum theory (CQT) combined with the Newtonian limit of GR, (2) objective collapse theories (OCTs) extended to the regime of semiclassical Newtonian gravity, and (3) OCTs extended to incorporate quantized Newtonian gravity. For the CQT approach, we follow the treatment by Hu and Anastopoulos in [2]. For the OCTs, we consider the GRW, CSL, DP, and Karolyhazy theories, based on the semiclassical approaches of Derakhshani [3] and Tilloy-Diósi [4], respectively, and we consider the most straightforward extension of the aforementioned OCTs to the regime of quantized Newtonian gravity. We show that the gravcat scheme can, in principle, experimentally discriminate the quantum jumps in gravitational cat states predicted by the CQT approach and the quantized-gravity OCTs (which we show make effectively the same predictions as each other), from the predictions of the semiclassical-gravitational OCTs. We also show that the GRW and Karolyhazy versions of semiclassical gravity (based on Derakhshani’s approach) make distinctly different predictions from the CSL and DP versions of semiclassical gravity (based either on Derakhshani’s approach or the Tilloy-Diósi approach).

1 Introduction

Recent years have seen a flurry of papers analyzing the empirical predictions of the Schrödinger-Newton equations [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17] and canonical [1] quantum theory combined with the Newtonian limit of GR [2, 14, 1], for state-of-the-art AMO experiments designed to implement quantum superpositions of mesoscopic masses [18, 19, 20]. In parallel, a few papers [3, 21, 4] in recent years have proposed consistent extensions of well-known objective collapse theories (e.g., GRW, CSL, DP, etc.) to the regime of semiclassical Newtonian gravity. What has yet to be done is an assessment of the predictions of these semiclassical Newtonian gravity versions of objective collapse theories, as well as objective collapse theories extended to the regime of quantized Newtonian gravity, for the proposed state-of-the-art AMO experiments.

We contribute in this respect by working out the predictions of several well-known objective collapse theories (OCTs), extended to semiclassical Newtonian gravity and quantized Newtonian gravity (hereafter OCT-Newton theories), for the gravitational cat state probe setup recently proposed by Derakhshani, Anastopoulos, and Hu [1]. Additionally, we compare the predictions of these OCT-Newton theories to the predictions of canonical quantum theory within the Newtonian approximation to GR, the latter of which has been worked out by Anastopoulos & Hu in [2] and used as the theoretical basis of the grav-cat probe setup.

The paper is organized as follows. Section 2 reviews the grav-cat scenario considered by Anastopoulos & Hu as well as the grav-cat probe setup of Derakhshani, Anastopoulos, and Hu. Section 3 reviews and develops the semiclassical Newtonian gravity and quantized Newtonian gravity extensions of the GRW, CSL, DP, Tilloy-Diósi, and Karolyhazy objective collapse theories, and works out their predictions for the grav-cat

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1By “canonical”, we just mean the standard/textbook/ordinary version of quantum theory.
probe setup while pointing out where their predictions differ from (or agree with) those of canonical quantum theory; secondarily, these findings are used to swiftly assess related objective collapse theories incorporating Newtonian gravity effects [22, 23, 24, 21, 25]. Finally, section 4 summarizes and appraises our findings, and suggests future research directions.

2 GravCat states in canonical quantum theory

Here we first review the general gravitational cat state scenario examined by Anastopoulos & Hu (AH) [2], then the specific grav-cat setup proposed by Derakhshani, Anastopoulos, and Hu (hereafter DAH) [1].

2.1 General model

Consider the canonical quantum theory (CQT) description of a stationary point mass \( M \) with initial (Gaussian) wavefunction

\[
\psi_0(x) = \frac{1}{(2\pi\sigma^2)^{3/4}} e^{-\frac{x^2}{4\sigma^2}}.
\]

In the canonical formalism, this wavefunction says that the position \( x \) of the particle is a random variable with probability density \( |\psi_0(x)|^2 \). By Newton’s law, a probability density for \( x \) entails a probability distribution for the Newtonian force acting on a particle of mass \( m \) at location \( R \) as

\[
F = -\frac{GMm}{|R-x|^3} (R-x).
\]

For \( |R| \gg \sigma \), the quantum fluctuations of the Newtonian force are negligible, and one recovers (effectively) the usual deterministic Newtonian force.

Suppose then that the wavefunction of the point mass \( M \) is described by a cat state, i.e., a superposition of two identical Gaussians, each located at \( \pm \frac{1}{2}L \) and with zero mean momentum:

\[
\psi_{\text{cat}}(x) = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2\pi\sigma^2}^{3/4}} \left[ e^{-\frac{(x+L/2)^2}{4\sigma^2}} + e^{-\frac{(x-L/2)^2}{4\sigma^2}} \right].
\]

Since the force is a function of \( x \), and \( x \) is described by a quantum operator, the Newtonian force must also be an operator, and so should the corresponding gravitational potential. Thus the cat state for the point mass generates a cat state for the gravitational field. Moreover, when \( L \) is comparable to \( R \), the fluctuations of Eq. (2) are non-negligible.

Now suppose that a quantum particle of mass \( M \), confined in a symmetric potential depicted in Fig. 1, has two local minima at \( r = \pm \frac{1}{2}L \), labeled as + and −. The general cat state is then given by

\[
|\psi> = c_+|+> + c_-|->,
\]

where \(|+> \) and \(|-> \) are the state-vectors localized around the corresponding minima. The system Hamiltonian is assumed to be \( \hat{H} = \nu \hat{\sigma}_1 \), where \( \nu \) is a small tunneling rate between the minima.

To experimentally probe such a cat state, a classical probe and a quantum probe were suggested and analyzed. It was determined that the quantum probe is beyond foreseeable feasibility, so we will only review the classical probe scheme.

Consider a stationary test mass \( m \) (the probe) located near the confining potential as in Fig. 1. Assuming that the gravitational force between the probe and the quantum particle causes the cat state to collapse into one or the other of the minima, with probabilities \( |c_+|^2 \) and \( |c_-|^2 \), the force \( F \) in the horizontal direction takes only two values, \( f_0 \) and \( -f_0 \), where

\[
f_0 = \frac{GMmL}{2D^3},
\]

where \( D = \sqrt{y^2 + L^2/4} \) is the distance between the potential minimum and the location of the probe; \( y \) is shown in Fig. 1.

Then, it was shown in [2] that for an initial state \(|+>\), the quantum expectation value of \( F \) and its two-time correlation function are given by

\[
\langle F(t) \rangle = -f_0 e^{-\Gamma t},
\]

\( \Gamma = \frac{\nu}{2} \).
Figure 1: Force on a probe exerted by a massive particle in a gravitational cat state.

\[ \langle F(t')F(t) \rangle = f_0^2 e^{-\Gamma |t'-t|}, \]  \hspace{1cm} (7)

The decay constant \( \Gamma \) is defined as

\[ \Gamma = \frac{\nu^2 \tau}{2}, \]  \hspace{1cm} (8)

where \( \tau \) is the probe’s temporal resolution.

From hereon, let us refer to the above as the gravitational cat state (g-cat) scenario predicted by ‘CQT-Newton’.

Observe that in the above CQT-Newton treatment of the g-cat scenario, the test mass is assumed to be a single particle. But what if the test mass is, instead, a many-body system, such as a homogeneous sphere composed of \( N \) particles? How does CQT-Newton describe the gravitational coupling of a many-body test mass with the classical probe, in the limit that \( N \) is large? AH \[26\] have shown that CQT-Newton, when understood as the Newtonian limit of the theory of perturbatively quantized gravity, with \( N \)-body Hamiltonian (minus the renormalized mass term and assuming identical particles)

\[ \hat{H}_{\text{quant}} = \hat{T} + \hat{U}_{\text{int}} = -\sum_{i=1}^{N} \frac{\hbar^2}{2m} \nabla_i^2 - \sum_{i\neq j} \sum_{j} \frac{Gm^2}{|r_i - r_j|}, \]  \hspace{1cm} (9)

has a mean-field (Hartree) approximation given by the single-body Schrödinger-Newton (SN) equations

\[ \nabla^2 V_{\text{int}} = 4\pi G m |\chi(r,t)|^2, \]  \hspace{1cm} (10)

and

\[ i\hbar \frac{\partial \chi(r,t)}{\partial t} = \hat{H}_{\text{SN}} \chi(r,t) = \left[ -\frac{\hbar^2}{2m} \nabla^2 - G \int d\mathbf{r}' \frac{m^2 |\chi(\mathbf{r}',t)|^2}{|\mathbf{r} - \mathbf{r}'|} \right] \chi(r,t), \]  \hspace{1cm} (11)

if we assume that the \( N \)-body wavefunction \( |\Psi(t)\rangle = e^{-\langle \hat{H}_{\text{quant}} \rangle} \otimes_{i=1}^{N} |\chi_i(t)\rangle \) for \( N \) identical particles in the limit that \( N \to \infty \). It should be stressed, however, that despite formal similarities, the
physical interpretation of \((10-11)\) is different from the physical interpretation of the single-body SN equations treated as a fundamental (Newtonian) description of the coupling of a single quantum particle to gravity. Equations \((10-11)\) describe the evolution of a collective variable, \(\chi(r, t)\) describing the large \(N\) limit of a Newtonian system of \(N\) identical particles weakly interacting via the perturbatively quantized gravitational potential \(V_{\text{int}}\).

In any case, if we suppose, instead, that the test mass is a homogeneous spherical distribution of \(N\) identical particles described by the Hamiltonian \((9),\) then \((10-11)\) should be a valid description of the test mass. Moreover, if we place the \(N\)-body test mass near the double well potential illustrated in Fig. 1, we simply add \(U_{\text{well}}\) to the right side of \((11),\) and the initial state \(|\chi\rangle\) takes the cat state form \((4).\) Then, \((10-11)\) take the form

\[
\nabla^2 V_{\text{int}} = 4\pi Gm \left[|c_+|^2|\chi_+ (r, t)|^2 + |c_-|^2|\chi_- (r, t)|^2\right],
\]

and

\[
i \hbar \frac{\partial \chi(r, t)}{\partial t} = \left[-\frac{\hbar^2}{2m} \nabla^2 + U_{\text{well}} - G \int \frac{m^2 |c_+|^2|\chi_+ (r, t)|^2}{|r - r'|} - G \int \frac{m^2 |c_-|^2|\chi_- (r, t)|^2}{|r - r'|}\right] \chi(r, t),
\]

where \(|\chi(t)\rangle = e^{-\frac{i}{\hbar} H_{\text{SN}} t} |\chi\rangle, \chi_+ (r, t) = \langle r |\chi_+ (t)\rangle, \) and \(|\chi_- (r, t)\rangle = \langle r |\chi_- (t)\rangle\) (there will also be an interaction term between + and −, which we will neglect for simplicity). If we maintain the assumption of a classical probe, then \((12-13)\) say that the probe will feel a classical Newtonian gravitational force from classical mass densities localized around each minimum of the potential with proportions \(|c_+|^2\) and \(|c_-|^2\) of the total test mass \(m,\) respectively. Moreover, if we suppose that \(|c_+|^2 = |c_-|^2 = \frac{1}{2},\) then the net force on the probe will clearly be zero, in contradiction to what CQT-Newton predicts for the case of a single particle test mass. One might attempt to resolve this inconsistency by simply postulating that the classical gravitational coupling of the probe to the cat state causes \(|\chi\rangle\) to collapse into either \(|\chi_+\rangle\) or \(|\chi_-\rangle,\) with probabilities \(|c_+|^2 = |c_-|^2 = \frac{1}{2}.\) However, it has been pointed out by numerous authors \([6, 27, 3, 26]\) that \((12-13),\) whether interpreted as a fundamental theory or a mean-field theory, has no consistent Born-rule probability interpretation. So it would seem that the large \(N\) limit of CQT-Newton makes a prediction for the g-cat scenario that differs significantly from the single particle case of CQT-Newton. We might conclude from this that the mean-field approximation leading to \((12-13)\) is simply not valid for cat states, or that some other assumption in the mean-field approximation doesn’t hold for the g-cat setup. In this regard, we can make two observations: the mean-field approximation leading to \((12-13)\) is designed to be valid only when quantum fluctuations of the matter degrees of freedom are small \([28, 29, 2]\) which is clearly not the case for the cat state of the g-cat setup described in Fig. 1; and \(2)\) the ansatz that particle correlations are negligible so that \(|\Psi(t)\rangle = \otimes_{i=1}^N |\chi(t)\rangle\) will be a poor approximation if the test mass is (say) a solid spherical body of uniform density composed of \(N\) identical particles (which will indeed be the case for the experimental protocols we shall consider in subsection 2.2).

Thus, for the description of a solid homogeneous spherical test mass composed of \(N\) identical particles and placed in the cat state illustrated in Fig. 1, we should stick to the exact quantum description corresponding to the Hamiltonian \((9),\) but with the addition of an interaction potential \(U_{\text{int}}(\mathbf{r}_i - \mathbf{r}_j)\) reflecting the non-gravitational (e.g., electrostatic) binding forces between the particles. Then we should rewrite the Hamiltonian in terms of the center of mass (CM) coordinate of the test mass (which means we can drop \(\hat{U}_{\text{int}}\) and \(\hat{U}_{\text{ele}}\) since they only contribute to the relative mass Hamiltonian), add \(U_{\text{well}},\) and add a gravitational potential \(-\frac{GMm_{\text{tot}}}{D}\) describing the g-coupling of the CM of the test mass with the CM of the classical probe. Putting it all together, we have the CM Schrödinger equation

\[
i \hbar \frac{\partial \psi(\mathbf{r}_{\text{cm}}, t)}{\partial t} = \left[-\frac{\hbar^2}{2m_{\text{tot}}} \nabla^2_{\text{cm}} - \frac{GMm_{\text{tot}}}{D} + U_{\text{well}}\right] \psi(\mathbf{r}_{\text{cm}}, t),
\]

where \(m_{\text{tot}} = Nm, \) \(D = \sqrt{y_{\text{cm}}^2 + \hat{x}_{\text{cm}}^2}, \) \(\hat{x}_{\text{cm}}\) has eigenvalues \(-L/2\) and \(+L/2,\) and \(\mathbf{y}_{\text{cm}}\) is the fixed (c-number) y-displacement of the CM of the test mass from the CM of the probe. With this description in hand, we can take \((4)\) as the initial state of the CM wavefunction in \((14)\) and straightforwardly apply AH’s single particle analysis, thereby reaching their same general conclusions.

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\(^2\)However, in a forthcoming paper, we will show that even if we incorporate the back-reaction of the quantum fluctuations of \(\chi(r, t)\) on \(V_{\text{int}}\) via the Newtonian limit of Hu and Verduguer’s stochastic gravity theory \([28],\) this does not yield a prediction for the g-cat setup that’s in better agreement with the exact CQT-Newton description.
Let us now revisit the CQT-Newton assumption that the gravitational force interaction with the classical probe ‘causes’ the quantum particle’s cat state to collapse into a definite position eigenstate with Born-rule probabilities. It might ask why this should be so, as opposed to the classical gravitational field of the probe acting as a mere external field that weakly perturbs the Hamiltonian of the quantum particle. Moreover, if the gravitational force measurement by the probe does act as a projective measurement, how can experimentally fashioned cat states of any mass remain stable at all (which they evidently can [30]), given the presence of other (and much more massive) gravitating bodies such as the Earth? (The usual assumption in the application of the canonical quantum measurement postulates to experiments is that the physical coupling of the system to the pointer variable is the strongest coupling in the experiment, which is clearly not the case here.)

These questions run right into the well-known quantum measurement problem that afflicts CQT, insofar as CQT is intrinsically vague about exactly what kinds of physical interactions in nature constitute projective measurements [1] exactly when projective measurements occur in or outside experiments, exactly where they occur in the (so-called) von-Neumann chain of an experiment, and exactly what dynamical laws govern the state-vector reduction process (as opposed to unitary evolution) [31, 32, 33, 34]. Nevertheless, if we view canonical quantum theory as a convenient operationalist formalism (i.e., a formalism about agents and how they can extract information from the microscopic physical world in experiments), then we might say the following: CQT requires, in order for a projective measurement to occur, that there exists a well-defined macroscopic pointer variable that agents can use to extract information from microscopic systems they experimentally couple to the pointer variable. Insofar as the classical probe is designed to be such a macroscopic pointer, it is reasonable to predict that the gravitational force measurement will indeed play the role of a projective measurement. By contrast, for other massive gravitating bodies in nature (e.g., Earth), it is difficult to see what could play the role of an appropriate pointer variable, so we have no justification (within operationalist CQT) for expecting that gravitational force interactions between the quantum particle and (say) the Earth will collapse the latter’s cat state.

Of course, this raises the question what ‘information extraction’ and ‘macroscopic’ mean, exactly. While we concur that these notions needs further elaboration, for the purposes of this paper, we will not pursue the issue. Rather, we will take it as a working assumption that the gravitational force interaction with the classical probe (and only the classical probe) plays the role of a projective measurement, in accordance with the usual measurement postulates of CQT.

2.2 Experimental setup

Within the framework of CQT-Newton, DAH proposed an experimental scheme to actually measure the gravitational force between a classical probe [1] and a massive quantum particle in a cat state. For preparing the g-cat state, they primarily considered Romero-Isart et al.’s [19] proposed experimental protocol involving the use of a superconducting lead (Pb) microsphere (the quantum particle) of $M \sim 10^{14}$ amu and radius $R = 2 \mu$m, which is first trapped (via Meissner effect) in a harmonic potential created by a magnetic quadrupole, then parametrically coupled to a qubit circuit to put the microsphere into a spatial superposition of $|L \sim 1 \mu m\rangle$. For the role of the classical probe, it was decided that the most promising experimental implementation is Reinhardt et al.’s [35] trampoline resonator made from $Si_3N_4$, with effective mass $m = 4.0 ng$, width $100 \mu m$, and projected force sensitivity of $\sim 14 \times 10^{-30} N$ at cryogenic temperatures (14mK).

While the resonator is a square-like membrane rather than a point particle, Eq. (5) can be used for an order of magnitude estimate of the force. For a resonator of mass $m = 4.0 ng$, a microsphere of mass $M = 0.38 ng$, $L = 1 \mu m$ and $D = 3 \mu m$ (or $1 \mu m$ larger than the radius of the Pb microsphere), we obtain

$$f_0 = \frac{GmML}{2D^3} \sim 2 \times 10^{-30} N,$$

which is around ten orders of magnitude beyond the reach of the projected force sensitivity range of the

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3To quote John Bell, “It would seem that the theory [quantum mechanics] is exclusively concerned about ‘results of measurement’, and has nothing to say about anything else. What exactly qualifies some physical systems to play the role of ‘measurer’? Was the wavefunction of the world waiting to jump for thousands of millions of years until a single-celled living creature appeared? Or did it have to wait a little longer, for some better qualified system ... with a Ph.D.? If the theory is to apply to anything but highly idealized laboratory operations, are we not obliged to admit that more or less ‘measurement-like’ processes are going on more or less all the time, more or less everywhere. Do we not have jumping then all the time?” [31].

4This perspective was suggested by Charis Anastopoulos (private communication).

5We shall forego analysis of a quantum probe since DAH found that the use of a quantum probe makes it far too difficult to measure the gravitational force.
To examine the optimal means by which to enhance the resonator–microsphere gravitational interaction, DAH write \( D = R + a \), where \( R \) is the radius of the microsphere and \( a \) is a fixed distance between the surface of the sphere and the resonator (we will consider \( a \approx 1 \mu m \)). Then

\[
 f_0 \simeq (2) \frac{G \rho_{\text{lead}} ML}{(1 + a/R)^3},
\]

where \( \rho_{\text{lead}} = M/ \left( \frac{4}{3} \pi R^3 \right) \) is the density of the microsphere. From this we can see that the most important parameter to increase the force is the cat state size \( L \), followed by the radius \( R \), then the density \( \rho_{\text{lead}} \) of the microsphere (consideration of Casimir forces \[36\] puts a practical lower bound on \( a \geq 1 \mu m \)). With these considerations, DAH showed that if we can increase the size of the cat \( L \) by one order of magnitude, use a Tantalum microsphere of density \( \rho_{\text{tantalum}} = 16.7 \frac{g}{cm^3} \), and assume that \( R = 5 \mu m \) is feasible for a Tantalum microsphere, we obtain

\[
 f_0 = 0.6 \times 10^{-28} N,
\]

or still about eight orders of magnitude from the peak sensitivity of the resonator.

However, DAH suggested other possibilities for upping the sphere-resonator force, such as increasing the mass of the resonator (though the importance of the gravitational self-energy of the probe would then have to be assessed). Another is to use a different protocol for preparing a microsphere in a cat state, since further increases in the \( R \) (hence \( M \)) of the microsphere in Romero-Isart et al.’s protocol are limited by decoherence from trap fluctuations \[19, 37\]; in particular, Pino et al.’s \[20\] recently proposed protocol involving free expansion in a magnetic skatepark potential (see Figure 3 therein), which makes possible a microsphere mass of \( M \sim 10^{-13} \text{amu} \) \( (R \gtrsim 1 \mu m) \) with \( L \sim 500 \text{nm} \) or more (since trap fluctuations are significantly lessened by the free expansion). With this value for \( L \) the above force estimates would increase by five orders of magnitude or more, i.e., \( \sim 10^{-25} N \) for the initial assessment, and \( \sim 10^{-23} N \) for the second assessment. Thus DAH concluded that “the quantum effects of a matter source manifested through its gravitational field interactions could become measurable in the next (or next-next) generation of experiments” \[1\].

We should note that an experimental implementation of the double well potential was not discussed by DAH, which limits our analysis to only an estimation of the probe-sphere gravitational force interaction when the sphere is initially prepared in the cat state \( 4 \).

Given the well-known conceptual ambiguities associated with the nature of measurement in CQT (the measurement problem), and the speculative nature of extrapolating the standard quantum measurement postulates to the gravitational field, it is natural to ask how alternative quantum theories that unambiguously solve the measurement problem might change the predicted outcomes of the above g-cat setup. Since, to date, the only alternative quantum theories that have been consistently extended to the regime of semiclassical Newtonian gravity are objective collapse theories \[39, 40, 41, 42, 43, 44, 22, 44, 23, 3, 21, 25, 4\], we will consider them specifically.

### 3 GravCat states in objective collapse theories

Here we analyze and compare the predictions of the most well-known and well-defined objective collapse theories that have been extended to semiclassical Newtonian gravity, and compare their predictions to those of CQT-Newton, for the g-cat setup considered in the previous section. Then we do the same for objective collapse theories extended to incorporate quantized Newtonian gravity.

#### 3.1 Collapse theories with semiclassical gravity

##### 3.1.1 GRW

Among objective collapse theories the mathematically simplest one is the GRW theory \[39, 41\], based as it is on the Poisson process. Likewise, among existing objective collapse theories that have been extended to the regime of semiclassical Newtonian gravity, the mathematically and conceptually simplest one appears to be the GRW-Newton (hereafter GRWmN) theory of Derakhshani \[9\]. Let us briefly review this approach.

\[6\] The state-of-the-art method for experimentally fashioning double well potentials appears to be the use of optical tweezers, which can produce double-well minima spacings as small as 600 nm \[35\]. It seems implausible that optical tweezers (or any other method) could produce well-defined minima spacing of 1 pm. But it seems not far off to produce minima spacings of 500 nm, which is relevant for the Pino et al. protocol.
For a single-body system, we postulate the existence of an ontic matter density field in space-time,

\[ m(x, t) = m|\psi(x, t)|^2, \]

which is used as a source in the Newton-Poisson equation,

\[ \nabla^2 V(x, t) = 4\pi Gm(x, t), \]

where

\[ V(x, t) = -G \int \frac{m(x', t)}{|x - x'|} d^3x'. \]

This gravitational ‘self-potential’ couples back to the wavefunction via the Schrödinger-Newton (SN) equation,

\[ i\hbar \partial_t \psi(x, t) = -\frac{\hbar^2}{2m} \nabla^2 \psi(x, t) - Gm \int \frac{m(x', t)}{|x - x'|} \psi(x, t), \]

but now the wavefunction undergoes discrete and instantaneous intermittent collapses according to the GRW collapse law. That is, the collapse time \( T \) occurs randomly with constant rate per system of \( N\lambda_{GRW} = \lambda_{GRW} = 10^{-16}\frac{1}{s} \), where the post-collapse wavefunction \( \psi_{T+} = \lim_{t \rightarrow T} \psi_t \) is obtained from the pre-collapse wavefunction \( \psi_{T-} = \lim_{t \rightarrow T} \psi_t \) through multiplication by a Gaussian function,

\[ \psi_{T+}(x) = \frac{1}{C} g(x - X)^{1/2} \psi_{T-}(x), \]

where

\[ g(x) = \frac{1}{(2\pi\sigma^2)^{3/2}} e^{-\frac{x^2}{2\sigma^2}} \]

is the 3-D Gaussian function of width \( \sigma_{GRW} = 10^{-7}m \), and

\[ C = C(X) = \left( \int d^3x g(x - X)|\psi_{T-}(x)|^2 \right)^{1/2} \]

is the normalization factor. The collapse center \( X \) is chosen randomly with probability density \( \rho(x) = C(x)^2 \), and the space-time locations of the collapses are given by the ordered pair \( (X_k, T_k) \). Between collapses, the wavefunction evolves by (18-21).

The generalization to an \( N \)-body system is as follows. We have \( N \) matter density fields in 3-space,

\[ m(x, t) = \sum_{i=1}^{N} \int d\mathbf{y}_1...d\mathbf{y}_N |\psi(y_1, ..., y_N, t)|^2 m_i \delta^{(3)}(x - y_i), \]

which act as the mass density source in the Newton-Poisson equation,

\[ \nabla^2 V(x, t) = 4\pi G \sum_{i=1}^{N} \int d\mathbf{y}_1...d\mathbf{y}_N |\psi(y_1, ..., y_N, t)|^2 m_i \delta^{(3)}(x - y_i). \]

The solution of (26) couples back to the \( N \)-body wavefunction via

\[ i\hbar \partial_t \psi(x_1...x_N, t) = -\sum_{i=1}^{N} \frac{\hbar^2}{2m_i} \nabla_i^2 \psi(x_1...x_N, t) - G \sum_{i,j=1}^{N} \int \frac{m_i m_j (x_j', t)}{|x_i - x_j'|} dx'_i...dx'_N, \]

and the solution of (27) undergoes collapse according to

\[ \psi_{T+}(x_1, ..., x_N) = \frac{1}{C} g(x_i - X)^{1/2} \psi_{T-}(x_1, ..., x_N), \]

with probability density.
\[ \rho(X) = C(X)^2 = \int dx_1 \cdots dx_N g(x_i - X) |\psi_{T-}(x_1, \ldots, x_N)|^2, \]

where \( i \) is chosen randomly from \( 1, \ldots, N \).

The equations of \( N \)-body GRWmN say the following: the wavefunction propagates on configuration space \( \mathbb{R}^{3N} \), evolves by the many-body SN equations, \((26-27)\), and undergoes the collapse process in \((28-29)\); this wavefunction drives the dynamical evolution of \( N \) matter density fields in 3-space via \((25)\) so that when the wavefunction collapses, it localizes the matter density fields around randomly chosen (non-overlapping) points in 3-space, each of width \( 10^{-7} \) meters, with rate \( N\lambda_{GRW} \), and with probability density given by \((29)\). As before, each of these matter density fields acts as a source for a classical Newtonian gravitational potential in 3-space that couples back to the \( N \)-body wavefunction via \((26-27)\), which in turn alters the evolution of the matter density fields via \((25)\) again. As shown in \((30)\), this dynamics suppresses macroscopic gravitational cat states and has a consistent single-particle probabilistic interpretation, in contrast to the SN equations alone \((15) \leftarrow (26) \leftarrow (20) \leftarrow (19)\).

Insofar as GRWmN is based on the SN equations, the nonlinearity of the theory makes it possible, in principle, to do superluminal signaling using (for example) spin-1/2 particles passing through a Stern-Gerlach apparatus. However, the signaling effect is so tiny that it is well-beyond present experimental capabilities to detect \((31)\). Thus, while it might be regarded by some as a philosophically undesirable feature of the theory (and certainly one in inherent conflict with special relativity), it doesn’t seem to entail empirical inadequacies at the present time \(\footnote{Bahrami et al. \cite{37} point out that for state-of-the-art experiments, which can achieve quantum interference with \( m \sim 10^9 amu \) and Stern-Gerlach detectors with a spatial separation of \( d \sim 1 \mu m \), the minimum distance on which a Stern-Gerlach experiment would need to be carried out to do superluminal signaling is 1 light-year.}

We now apply GRWmN to the g-cat setup. To do this, we model the center of mass of the microsphere cat state with the single-body GRWmN equations, and we consider the case of the classical force probe (see again Fig. 1). In this case, for the initial cat state

\[ \psi_{cat}(x) = \frac{1}{\sqrt{2}} \left( e^{-\frac{(x+L/2)^2}{4\sigma^2}} + e^{-\frac{(x-L/2)^2}{4\sigma^2}} \right), \]

which we assume is confined to a symmetric double-well potential with the two local minima located at \( r = \pm L/2 \), measurement probabilities \( |\psi_+|^2 = |\psi_-|^2 = \frac{1}{2} \), and such that the overlap between the summands in \((30)\) is negligible (which is reasonable if we make the barrier potential sufficiently large), we then have the cat state matter density

\[ M_{cat}(x) = M|\psi_{cat}(x)|^2 = \frac{M}{2} \left( e^{-\frac{(x+L/2)^2}{4\sigma^2}} + e^{-\frac{(x-L/2)^2}{4\sigma^2}} \right) = \frac{M_+(x)}{2} + \frac{M_-(x)}{2}. \]

Here \( M \) is taken to be the mass of the microsphere cat state, and the terms \( \frac{1}{2} M_+(x) \) and \( \frac{1}{2} M_-(x) \) describe lumps of halved center-of-mass microsphere matter densities localized around the position eigenvalues of the left and right minima of the potential, respectively. Thus, for a classical probe of mass \( m \) located at a distance \( D = \sqrt{y^2 + L^2/4} \) from the two minima, the horizontal force that the probe feels from the two matter densities is

\[ f = \frac{GMmL}{4D^3} - \frac{GMmL}{4D^3} = 0. \]

That is, at every instant in time, the probe feels no net gravitational force from the cat state.

Note that this prediction will only hold when either (i) each collapse event negligibly changes the cat state wavefunction, or (ii) the number of particles composing the microsphere does not imply an appreciable collapse rate, and the gravitational coupling of the probe to the cat state does not drive up the collapse rate of the cat state.

Are these conditions satisfied by GRWmN applied to the g-cat setup?

In the case of (i), recall that in the Romero-Isart et al. protocol the microsphere cat size \( L = 10^{-12} m \). By comparison, \( \sigma_{GRW} = 10^{-7} m \). Thus, GRW collapses will leave the microsphere cat state unchanged, if the cat state is prepared with the Romero-Isart et al. protocol. In the Pino et al. protocol, by contrast, the

\footnote{If one is worried about superluminal signals creating causal paradoxes in different Lorentz frames \((38) \leftarrow (46)\), this can be eliminated through the introduction of a preferred foliation of spacetime \((39)\) such as the foliations already used in relativistic flat-space extensions of ordinary GRW and CSL \((50) \leftarrow (51)\).}
microsphere cat size $L \gtrsim 5 \times 10^{-7} m$; so in this case, GRW collapses will appreciably localize the microsphere cat state.

In the case of (ii), recall that, in the GRW formalism, the rate of collapse for a many-body system scales (under the simplest assumption) as $N\lambda_{GRW}$. Now, what determines that a given system is ‘comprised of $N$ particles’ is that the particles are interacting strongly enough that their interaction Hamiltonian is non-negligible and implies a non-separable many-body wavefunction $\psi(x_1,\ldots,x_N)$. Certainly the microspheres used in the Romero-Isart et al. and Pino et al. protocols satisfy this condition. So for a microsphere mass of $M \sim 10^{18} amu$, we have $N\lambda_{GRW} \sim 10^{14}$, $10^{-10} \frac{s}{t} = 10^{-2} \frac{s}{t}$, or one collapse event every 100 seconds. Since 100 seconds is a timescale well beyond any feasible timescale of the g-cat experiment, whether using the Romero-Isart et al. protocol or the Pino et al. protocol for preparing the cat state, we can ignore this collapse rate for microspheres of said mass. Furthermore, it is clear that the semiclassical gravitational coupling (as described by the SN equations) between the microsphere matter density and the probe’s matter density will not drive up the microsphere’s collapse rate, simply because the semiclassical gravitational coupling does not lead to an entanglement between the many-body wavefunction of the microsphere cat state and the many-body wavefunction of the probe; that is, the semiclassical gravitational coupling implies that the probe’s wavefunction and associated matter density do not evolve into a superposition of orthogonal pointer states that are correlated with the position states of the microsphere cat state (i.e., the two minima of the double well potential)\(^9\). Instead, all that the semiclassical gravitational coupling does is introduce a slight phase shift in the many-body wavefunction of the microsphere. Moreover, the same will be true of semiclassical gravitational coupling between the microsphere and any other matter density present in the experiment (e.g., the Earth, the sun, etc.).

Additionally, it is easy to see that the gravitational self-energy of the (un-collapsed) cat state matter density is negligible in both protocols since the interaction energy between the two lumps of matter in the two minima is given by $U_{\text{self}} = -\frac{GM^2}{4L}$, where $U_{\text{self}}|_{L=1 pm} \sim 10^{-35} J$ and $U_{\text{self}}|_{L=500 nm} \sim 10^{-41} J$. By contrast, a potential barrier between the minima of just $1 eV \approx 10^{-19} J$. In other words, the cat state matter density will remain stably confined in the double well potential.

As noted in section 2, the Pino et al. protocol\(^9\) affords us the possibility of even larger microsphere masses (hence larger collapse rates) than does the Romero-Isart et al. protocol. Suppose then that we assume the Pino et al. protocol for preparing our microsphere cat state and that it allows us a sphere mass as large as $\sim 10^{18} amu$. (We will forego a discussion of the practical details of how to experimentally implement the double-well potential, but the general idea is that each minimum of the potential would be located around one of the slits in the double-slit barrier, so that when the microsphere matter density emerges from the slits as two distinct lumps of half-mass matter densities, each lump will be trapped in one of the minima, as in Fig. 1.) Then the intrinsic collapse rate of the microsphere will be $10^{-10} \frac{s}{t}$, or 1 collapse event every millisecond. By comparison, in the Pino et al. protocol, the total time between preparing and detecting a coherent microsphere cat state of $10^{18} amu$ with $L = 0.5 \mu m$ is the sum over the time intervals for steps 2-4 in the protocol, or $\sim 500 ms$. For a microsphere of $\sim 10^{18} amu$, it will presumably take at least this long to form a coherent cat state in the protocol. In this case, the coherence time of the microsphere will in fact exceed the inverse collapse rate of the microsphere. More precisely, the coherent microsphere would undergo dozens of collapse events before being put into a cat state via diffraction through the two-slit ‘barrier’, and dozens of collapse events thereafter. Note that, in the latter case, each collapse event would correspond to multiplying the cat state wavefunction, which essentially takes the form (30) with the peaks of the Gaussians separated by $0.5 \mu m$, by the Gaussian function (22), giving

\(^{9}\)This can be seen by noting that since the initial matter densities imply no net gravitational deflection of the probe in the x-direction, time-evolving the sphere-probe system with the Hamiltonian (27) will not evolve the probe’s wavefunction into effectively orthogonal pointer states.

\(^{10}\)The protocol involves seven steps\(^{20}\): (1) **Cooling.** Cooling the center-of-mass (CM) motion of the superconducting microsphere in a harmonic trap of frequency $\omega_1$ for a time $t_1$ to a definite phonon occupation number; (2) **Boost.** Evolving the CM in an inverted harmonic potential of frequency $\omega_2$ for a time $t_2$ in order to boost the sphere’s kinetic energy; (3) **Free.** Free evolution for a time $t_3$ to delocalize the CM over long distances; (4) **Split.** Continuous-time measurement of the position-squared for a time $t_4$ to implement diffraction through a double-slit ‘barrier’ (where the measurement outcome determines the slit separation, and the strength of the measurement determines the width of the slits) and prepare a quantum spatial superposition state; (5) **Rotation.** Short evolution for a time $t_5$ in a harmonic trap of frequency $\omega_5$ to give opposite momenta to the wavepackets in the superposition; (6) **Inflation.** Evolution in an inverted quadratic potential of frequency $\omega_\rho$ for a time $t_6$ to exponentially generate interference fringes; and (7) **Measurement.** A measurement of the position-squared for a time $t_7$ to unveil the interference pattern.
\[\psi_{T+}(x) = \frac{1}{C}g(x - X)^{1/2}\psi_{\text{cat}}(x),\]  

which is a wavepacket of width \(\sigma = 0.1\mu m\) with collapse center probability

\[C = C(X) = \left(\iint d^3x g(x - X)|\psi_{\text{cat}}(x)|^2\right)^{1/2},\]

where \(X = \{+\frac{1}{2}, -\frac{1}{2}\}\).

Here we should elaborate on what happens to the microsphere wavepacket and matter density immediately after a GRW collapse event. As we’ve noted, in between collapse events, the wavepacket resumes its evolution via the \(N\)-body SN equations, where the gravitational interaction potential clearly causes the \(N\)-body wavepacket to self-gravitate. Will this self-gravitation be sufficiently strong to keep the collapsed wavepacket from dispersing again? This was studied numerically by Giulini & Grossardt [12], who found that for a homogeneous sphere of mass \(M\) and initial radius \(R = 0.5\mu m\), the density of the corresponding spherically symmetric center-of-mass (CM) wavepacket begins to undergo “gravitational collapse” at a mass as low as \(\sim 5 \times 10^9\text{amu}\), and reaches a minimum radius in a time of \(\sim 20,000s\). For \(R = 1\mu m\), the critical mass for gravitational collapse to set in is \(\sim 8 \times 10^9\text{amu}\). And for the maximum simulated mass of \(\sim 10^{11}\text{amu}\) \((R = 0.5\mu m)\), the shortest gravitational collapse duration was observed to be \(\sim 2,200s\). Giulini & Grossardt were unable to simulate larger masses due to numerical limitations, so it is not possible to say by how much more the gravitational collapse time would be reduced for \(\sim 10^{18}\text{amu}\) and \(R = 1\mu m\). But it seems implausible that it would be reduced to a timescale of \(\sim 500\text{ms}\). In any case, these results make it clear that the SN self-interaction is strong enough to override the quantum mechanical wavepacket dispersion, for a microsphere with \(R = 1\mu m\) or less.

So when a GRW collapse occurs and localizes the microsphere CM wavepacket to the width \(\sigma_{GRW} = 0.1\mu m\), we can be sure that the SN self-interaction will prevent the wavepacket from delocalizing again. Moreover, any subsequent GRW collapse event will make no change to the CM wavepacket width. Thus, when a GRW collapse event occurs before the microsphere is split into two lumps via the double-slit (step 4 in the protocol), we can predict that when the microsphere does finally interact with the double-slit (where the slit width \(w = 10.61\text{nm}\) and slit separation \(d = 0.5\mu m\)), it will either get reflected or pass through one or the other of the slits, emerging on the other side as a full-mass lump trapped within one or the other of the double-well minima. In other words, the GRW collapse coupled with the SN self-interaction will actually make it unfeasible to put the microsphere into a cat state with the Pino et al. protocol. Consequently, the gravitational force continuously detected by the probe will come from only one of the minima and remain so, as the SN self-interaction and GRW collapses will inhibit tunneling between the minima.

By contrast, CQT-Newton predicts that even for \(M \sim 10^{18}\text{amu}\) and \(R = 1\mu m\), it should be possible to form the microsphere cat state with the Pino et al. protocol, and the gravitational force interaction with the probe will instantaneously collapse the cat state CM wavefunction into one of the minima, with probabilities \(|c_+| = |c_-| = \frac{1}{2}\). Additionally, continuous monitoring of the gravitational field by the probe will still find the force undergoing quantum jumps between the minima, as described by Eqs. (20-22).

We conclude then that it is necessary to adopt the Pino et al. protocol, in order to have a chance of increasing the microsphere mass to a level where (i) the probe-microsphere force becomes detectable, and (ii) the probe-microsphere net force predicted by GRWmN becomes non-zero and has correlation functions that distinctly differ from those of CQT-Newton [13].

---

[13] We should note that our conclusions change nontrivially if we consider GRWmN in place of GRWmN [3]. In GRWmN, each “flash” (i.e., space-time collapse center \(\{x_k, T_k\}\)) is accompanied by the sudden appearance of a point mass at the space-time location of the flash, and in between collapse events the wavefunction evolves by the usual linear Schrödinger equation. Thus, when no flashes are present in space-time, no matter density is present in space-time, and no gravitational interactions can be present. So then, for the g-cat setup, if the collapse rate is too low for the timescale of the experiment, the probe will detect no gravitational force from the microsphere cat state because there will be no mass density in space-time associated with the cat state. On the other hand, if the collapse rate is sufficiently high that dozens of collapse events can occur on the timescale of the experiment, then we can predict the following for the Pino et al. protocol: the GRW collapses of the microsphere wavepacket that occur before it reaches the double-slit barrier will be immediately followed by delocalization of the wavepacket due to Schrödinger evolution. If a collapse occurs just before the wavepacket interacts with the double-slit barrier, then the wavepacket will be sufficiently narrow that it can only pass through one of the slits (whichever one it is localized near), thus emerging into only one of the minima (say the + one) on the other side. From thereon, the probe will detect an instantaneous force from the + minimum only when a GRW collapse event happens, followed by no force until another GRW collapse happens. Since, in between GRW collapses, the wavepacket evolves by the usual Schrödinger equation, it is possible in these intermittent times that the
3.1.2 CSL

As is well-known, the CSL theory is a quantum field theoretic generalization of the GRW theory [43, 52, 44] based (in its simplest formulation) on a continuous-time Markov process, namely, the Wiener process. For our first approach, we consider a straightforward semiclassical gravitational generalization of CSL analogous to GRWmN (hereafter CSLmN). In particular, we adopt from the mass-proportional version of non-relativistic CSL the stochastic Schrödinger equation for the \( N \) -particle sector of Fock space:

\[
\frac{\partial}{\partial t} |\psi_t\rangle = \left[-\frac{i}{\hbar} \hat{H} + \frac{\sqrt{\gamma}}{m} \int d^3x \left( \hat{m}(x) - \langle \hat{m}(x) >_t \right) dW_t(x) \right. \\
\left. - \frac{\gamma}{2m^2} \int d^3x d^3y g(x-y) \left( \hat{m}(x) - \langle \hat{m}(x) >_t \right) \left( \hat{m}(y) - \langle \hat{m}(y) >_t \right) \right] |\psi_t\rangle,
\]

(35)

where

\[
\hat{H} = \hat{H}_0 + \hat{U}_{g-int}
\]

(36)

is the system Hamiltonian, a sum of the usual kinetic energy operator and gravitational interaction energy operator. The term

\[
\hat{m}(y) = \frac{1}{m_{nuc}} \int d^3y g(x-y) \sum_s m_s a^\dagger_s(y)a_s(y)
\]

(37)

is the smeared spatial mass density operator, defined in terms of the number density operator \( a^\dagger_s(y)a_s(y) \), where the sum is over particle species \( s \) of mass \( m_s \). The parameter \( m_{nuc} \) is the nucleon mass, and \( g(x-y) \) is a spatial correlation function chosen equal to the 3-D Gaussian

\[
g(x) = \frac{1}{(2\pi r_c^2)^{3/2}} e^{-\frac{x^2}{2r_c^2}}.
\]

(38)

Now, we define the semiclassical gravitational potential via the Poisson equation

\[
\nabla^2 V_{g-int}(x, t) = 4\pi G \langle \hat{m}(x) >_t
\]

(39)

where it is readily confirmed that (40) is equivalent to the \( N \)-body mass density field (25), making the \( V_{g-int}(x, t) \) of (39) equivalent to the \( V(x, t) \) of (26). Then the semiclassical gravitational interaction energy (which includes self-interaction) is given by

\[
\hat{U}_{g-int} = \int d^3x V_{g-int}(x, t) \hat{m}(x) = -G \int d^3x \int d^3y \frac{\hat{m}(x) < \hat{m}(y) >_t}{|x-y|},
\]

(41)

The noise term \( dW_t(x)/dt = w(x, t) \) satisfies

\[
E[w(x, t)] = 0,
\]

(42)

and

\[
E[w(x, t_1)w(y, t_2)] = g(x-y)\delta(t_2 - t_1),
\]

(43)

where \( E[... \] \) is the stochastic average. Thus the noise field is independent of the gravitational self-interaction, affording us a consistent probabilistic interpretation of the spontaneous localization of of the wavefunction. We also note that the fundamental parameter \( \gamma = 10^{-36} m^3 s^{-1} \) [44], and the collapse rate \( \lambda_{CSL} \) is related to \( \gamma \) by

\[
\lambda_{CSL} = \frac{\gamma}{8\pi^{3/2}r_c^3} \approx 10^{-17} s^{-1},
\]

(44)

wavepacket tunnels from the + minimum to the − minimum. So when the next GRW collapse event happens, the probe might feel an instantaneous force coming from the - minimum, followed by no force again until the next GRW collapse (which might still come from the − minimum or change back to the + minimum). On the other hand, if the wavepacket is still delocalized as it is interacting with the double-slit barrier, then the cat state wavefunction will form on the other side and remain that way until a GRW collapse occurs. From hereon, the gravitational force changes in time as already described. In either case, the predictions differ noticeably from both CQT-Newton and GRWmN.
choosing the correlation length \( r_c = 10^{-7} m \). In CSL, spatial superpositions separated by more than \( r_c \) are localized with effective rate \( \gamma = \lambda_{CSL} N^2 k_0 \),
\[
\gamma = \lambda_{CSL} N^2 k_0,  \tag{45}
\]
where \( N \) is the number of particles within a distance \( r_c \) and \( k_0 \) is the number of such clusters of particles. (This dependence on \( N^2 \) turns out to depend on the identity of the particles \[14\].) Accordingly, in a molecule or microsphere where the inter-particle distances are much smaller than \( r_c \), the collapse only affects the center of mass motion, implying \( \Gamma_{cm} = \lambda_{CSL} N^2 = \lambda_{CSL} \left( \frac{m_{\text{nuc}}}{m} \right) \), where \( m \) is the total mass and \( m_{\text{nuc}} \) is the mass of a nucleon.

Applying CSLmN to the g-cat setup, it follows again that the probe-microsphere gravitational interaction, defined here by
\[
\hat{U}_g^{\text{probe-sphere}} = -\frac{G}{2} \int d^3x \int d^3y \frac{m_{\text{probe}}(x) < \hat{m}_{\text{sphere}}(y) >_t}{|x - y|},  \tag{46}
\]
does not drive up the collapse rate of the microsphere, only the number of nucleons composing the microsphere do so. So for \( m_{\text{sphere}} \sim 10^{14} \text{amu} \) we have \( N \sim 10^{14} \) nucleons, giving an effective localization rate on the center of mass motion of \( \Gamma_{cm} = \lambda N^2 = 10^{11.5} \). As with GRW/GRWmN, this collapse rate will have no effect on the cat state formed by the Romero-Isart et al. protocol; but for the Pino et al. protocol, this collapse rate means that for CSL/CSLmN, it will not be possible, in practice, to experimentally prepare a coherent and stable microsphere cat state in the form of (4) or (30) using Pino et al.’s free expansion protocol (in contrast to GRW/GRWmN and CQT/CQT-Newton). More precisely, the microsphere cat state formed by the Pino et al. protocol will very quickly be suppressed by the first collapse event that happens, and the SN self-interaction of the collapsed microsphere matter density will ensure that the microsphere stays localized in the minimum in which it got localized. So, were force probe sensitive enough to measure the gravitational force from \( m_{\text{sphere}} \sim 10^{18} \text{amu} \), it would only detect a force from a full-mass microsphere matter density that’s continuously localized around one of the two minima in the g-cat setup, with zero probability of tunneling between the minima in between collapse events (because of the appreciable SN self-interaction). Clearly this prediction for the gravitational force interaction would be experimentally indistinguishable from what GRWmN predicts (when a collapse event happens in GRWmN before the microsphere passes through the double-slit and forms into a cat state), but sharply differs from what CQT-Newton predicts. Finally, we note that these findings about CSLmN also apply to the dissipative generalization of CSL (hence dissipative CSLmN) proposed by Smirne & Bassi \[64\], insofar as the dissipative terms they incorporate don’t alter the CSL collapse rate.

### 3.1.3 DP

The DP theory \[41, 42, 53, 55, 56\] is structurally equivalent to mass-proportional CSL in that the equation of motion for \( |\psi_t\rangle \) is given by Eq. (35), just with the replacements \( \sqrt{\gamma/m} \to 1 \) and \( \gamma/2m^2 \to 1/2 \). The key physical difference of DP is in the choice of spatial correlation function:
\[
g(x) = \frac{G}{\hbar} \frac{1}{|x|},  \tag{47}
\]
In the density matrix formulation of DP, one then has
\[
\frac{\partial}{\partial t} \hat{\rho}(t) = -\frac{i}{\hbar} \left[ \hat{H}, \hat{\rho}(t) \right] + \frac{G}{\hbar} \int \int \frac{d^3x d^3y}{|x - y|} \left[ \hat{m}(x) \hat{\rho}(t) \hat{m}(y) - \frac{1}{2} \{ \hat{m}(x) \hat{m}(y), \hat{\rho}(t) \} \right].  \tag{48}
\]

The different choice of spatial correlation function in DP entails divergences in (48) which Diósi proposed to remedy with a length-scale cut-off \( R_0 \) \[41, 42, 56\]; however, even with the cut-off, the energy of a system of particles increases monotonically, and one also has the problem of overheating \[44, 56\]. As shown by Bahrami et al. \[56\], this overheating problem can be dealt with by adding dissipative terms to (48) (including the cut-off), but the overheating is still appreciable unless one also requires an upper limit on the temperature of the noise field in DP; in particular, Bahrami et al. deduce that the dissipative terms lead to an asymptotic value of the noise field energy corresponding to a temperature
\[
T = \frac{\hbar^2}{8k_B m_r R_0^2} = \frac{10^{-19}}{m_r R_0^2}.  \tag{49}
\]
Here $m_r$ corresponds to the center of mass consistent with the asymptotic value of $T$. Diósi proposed the choice of cut-off $R_0 = 10^{-15}m$ (corresponding to the Compton wavelength of a nucleon), which for Bahrami et al.’s (physically reasonable) choice of $T = 1K$ gives $m_r \sim 10^{11}amu$. For $m \ll m_r$, Bahrami et al. [56] find that the dissipation mechanism becomes too strong, leading them to conclude that the dissipative DP theory is valid only as an effective theory for masses comparable to or larger than $m_r$.

The regularized version of Eq. (48) can be readily solved in the single-particle case for a two-state spatial superposition. Consider the density matrix $\langle x | \hat{\rho}(t) | x' \rangle$, where $x$ and $x'$ are the two distinct locations in the superposition. For short timescales, one can neglect the pure Schrödinger contribution in (48) and solve to obtain

$$\rho(x, x', t) = \exp \left( -\frac{t}{\tau(x, x')} \right) \rho(x, x', 0),$$

(50)

where for $|x - x'| \gg R_0$, one finds that the characteristic damping time

$$\tau(x, x') = \frac{\hbar}{\Delta E} \approx \frac{1}{\Lambda_{DP}} = \frac{\sqrt{\pi} \hbar R_0}{Gm^2},$$

(51)

where $\Delta E$ is the Newtonian gravitational self-energy of the massive particle at superposed locations $x$ and $x'$, and $\Lambda_{DP}$ is the gravitational decoherence rate (the rate at which the spatial superposition decays).

Thus, for Romero-Isart et al.’s protocol involving a Pb microsphere of $m \sim 10^{14}amu \sim 10^{-13}kg$, placed into a center of mass spatial superposition of $|x_m - x_m'| \sim 10^{-12}m$ via parametric coupling to a qubit, and using Diósi’s choice of $R_0 = 10^{-15}m$, Eq. (51) gives

$$\tau_{sphere}(x, x') \sim 10^{-13}s.$$  

(52)

So the dissipative DP theory of Bahrami et al. predicts suppression of the superposition on a timescale several orders of magnitude smaller than any feasible timescale for the experimental setups we’ve considered. A straightforward conclusion to draw, then, is that the dissipative DP theory predicts that, for a microsphere with $m \sim 10^{14}amu$, a cat state of CM position states formed by either parametric coupling to a qubit (Romero-Isart et al. protocol) or by free expansion in a magnetic skatepark potential (Pino et al. protocol) will be rapidly suppressed to a width of $10^{-15}m$, on the timescale given by (52).

If we assume a semiclassical gravitational extension of the dissipative DP theory via the SN approach, i.e. (dissipative) DPmN, then for the g-cat setup, continuous monitoring of the gravitational field of the microsphere (confined to a double-well potential) by a classical probe will result only in a force from the microsphere located in one of the two minima of the potential for all times, as one would expect classically. Insofar as this prediction holds for the g-cat setup involving a microsphere cat state prepared by the Romero-Isart et al. protocol, this prediction of dissipative DPmN is indistinguishable from CSLmN; and for the g-cat setup involving a microsphere cat state prepared by the Pino et al. protocol, the prediction is the same as both CSLmN and GRWmN.

It is interesting to compare this result with Pino et al.’s analysis of the DP theory using their experimental proposal [29]. They consider the original DP theory with only the cutoff $R_0$ and calculate a gravitational decoherence timescale of $\sim 10^{-2}s$ for a microsphere of radius $R_0 = 1\mu m$ and center of mass $m \sim 10^{13}amu$. Clearly, then, the dissipative DP/DPmN theory is even more easily falsifiable than the original DP theory.

### 3.1.4 Tilloy-Diósi

Because of its nonlinear dynamics, CSLmN implies superluminal signaling just as GRWmN. Again, however, the effect is too small to measure with state-of-the-art technology but might nevertheless be considered philosophically unpalatable. Motivated by this philosophical dissatisfaction, Tilloy & Diósi [4] developed a semiclassical Newtonian gravitational extension of CSL that eliminates the nonlinearity that implies superluminal signaling. In contrast to CSLmN (and GRWmN), their theory implies only inter-particle gravitational potentials and no single-particle self-interaction that depends on the wavefunction. In addition, their theory contains as special cases the CSL theory and the Diósi-Penrose theory, each amended with the inclusion of inter-particle gravitational potentials. It is therefore worthwhile to also assess the predictions of the Tilloy-Diósi approach for the experimental setups considered here.

In order to circumvent the superluminal signaling entailed by using $< \hat{m}(x) >_t$ as the source of the Newtonian gravitational potential, Tilloy and Diósi (TD) propose to use a mass density source defined from a fictitious model of hidden (and possibly entangled) detectors of spatial resolution $\sigma$ that continuously monitor
the mass density operator \( \hat{m}(\mathbf{x}) \), the latter defined as in Eq. (37). That is, they use the continuous equivalent of a von-Neumann measurement result, i.e., the “signal” defined by

\[
m_t(\mathbf{x}) = \langle \hat{m}(\mathbf{x}) \rangle >_t + \delta m_t(\mathbf{x}),
\]

where \( \delta m_t(\mathbf{x}) \) is a spatially correlated white noise field defined by

\[
E[\delta m_t(\mathbf{x}) \delta m_t(\mathbf{y})] = \gamma_{xy}^{-1} \delta(t_2 - t_1),
\]

where \( \gamma_{xy} \) is a semi-positive definite kernel encoding correlations between the fictitious detectors at positions \( \mathbf{x} \) and \( \mathbf{y} \). In order to implement these assumptions into a continuous stochastic localization theory, TD suppose that, for an \( N \)-body system, the dynamics of the \( N \)-body density matrix \( \hat{\rho} \) is defined by the stochastic master equation (SME)

\[
\frac{d\hat{\rho}}{dt} = -i [\hat{H}, \hat{\rho}] - \int d^3x d^3y \gamma_{xy} \left[ \hat{m}_\sigma(\mathbf{x}), [\hat{m}_\sigma(\mathbf{y}), \hat{\rho}] \right] + \int d^3x d^3y \gamma_{xy} \hat{H} [\hat{m}_\sigma(\mathbf{x})] \hat{\rho} \delta m(y),
\]

where \( \hat{H} [\hat{m}_\sigma(\mathbf{x})] (\hat{\rho}) = \{ \hat{m}_\sigma(\mathbf{x}) - \langle \hat{m}_\sigma(\mathbf{x}) \rangle, \hat{\rho} \} \) and we set \( \hbar = 1 \). The deterministic term involving the double-commutator describes the decoherence induced by the coupling with the fictitious detectors and diagonalizes the density matrix in the position basis for large-mass objects. The stochastic term induces localization of the density matrix into one of its diagonal components, as a result of the conditioning on the signal.

Now, in order to implement the back-action of quantized matter on the classical gravitational field, TD define the Poisson equation for the mass density signal, \( m_t(\mathbf{x}) \):

\[
\nabla^2 V_{g-int}(\mathbf{x}) = 4\pi G m_t(\mathbf{x}).
\]

Accordingly, the stochastic semiclassical gravitational self-interaction energy associated with the signal is given by

\[
\hat{U}_{g-int} = \int d^3x V_{g-int}(\mathbf{x})\hat{m}_\sigma(\mathbf{x}) = \int d^3x \hat{m}(\mathbf{x}) V_{g-int(\sigma)}(\mathbf{x})
\]

where the subscript \( \sigma \) denotes an optional convolution with the smearing function, \( g_{\sigma} \). The feedback from Eq. (57) to \( \hat{\rho} \) is introduced by having the self-interaction energy act an infinitesimal amount of time after the free-evolution given by Eq. (55), i.e.,

\[
\hat{\rho} + d\hat{\rho} = e^{-i\hat{U}_{g-int} dt} (\hat{\rho} + d\hat{\rho}^{free}) e^{i\hat{U}_{g-int} dt}.
\]

Then, expanding the exponential in (58) up to second order, TD obtain the SME

\[
\frac{d\hat{\rho}}{dt} = -i \left[ \hat{H} + \hat{U}_{g,\sigma} + \int d^3x \delta m(\mathbf{x}) \hat{V}_\sigma(\mathbf{x}), \hat{\rho} \right]
\]

\[
- \int d^3x d^3y \frac{\gamma_{xy}}{8} \left[ \hat{m}_\sigma(\mathbf{x}), [\hat{m}_\sigma(\mathbf{y}), \hat{\rho}] \right] + \frac{\gamma_{xy}^{-1}}{2} \left[ \hat{V}_\sigma(\mathbf{x}), [\hat{V}_\sigma(\mathbf{y}), \hat{\rho}] \right]
\]

\[
+ \int d^3x d^3y \gamma_{xy} \hat{H} [\hat{m}_\sigma(\mathbf{x})] \hat{\rho} \delta m(y),
\]

where

\[
\hat{U}_{g,\sigma} = \frac{1}{2} \int d^3x \hat{m}_\sigma(\mathbf{x}) \hat{V}_\sigma(\mathbf{x}) = -G \int d^3x d^3y \frac{\hat{m}_\sigma(\mathbf{x}) \hat{m}_\sigma(\mathbf{y})}{|\mathbf{x} - \mathbf{y}|}
\]

is the Newtonian gravitational pair-potential up to \( \sigma \)-smearing of the mass density around the point-like constituents of the localization events. Here we can see how TD’s proposal avoids nonlinearity: the self-interaction of each signal only shifts the \( N \)-body system Hamiltonian by finite amounts and have no dynamical consequences. We also see that the gravitational back-action induces an additional local decoherence term that depends on \( \hat{V}_\sigma(\mathbf{x}) \). Finally, we note that the stochastic term that drives the localization of the density matrix remains the same as in the free-evolution case.

Let us now examine the CSL case of TD’s theory. This corresponds to the spatial correlator
\( \gamma_{xy} = \gamma \delta(x - y), \)  

(61)

along with the values \( \sigma = 10^{-7} m \) and \( \gamma = 10^{-24} m^3 s^{-1} \) (though other values for these parameters are possible). Additionally, since in CSL it is possible to define (57) in terms of the sharp mass density without getting infinities, this is done by TD too (which amounts to dropping the \( \sigma \) subscripts in \( \hat{U}_{g-int} \)). The resulting SME takes the form

\[
\frac{d\hat{\rho}}{dt} = -i \left[ \hat{H} + \hat{U}_{g, \sigma} + \int d^3x \delta m(x) \hat{V}, \hat{\rho} \right] \\
- \int d^3x \left( \frac{\gamma}{8} [\tilde{m}_x(x), [\tilde{m}_x(x), \hat{\rho}]] + \frac{1}{2\gamma} \left[ \hat{V}(x), \left[ \hat{V}(x), \hat{\rho} \right] \right] \right) \\
+ \int d^3x \frac{\gamma}{2} \hat{H} [\tilde{m}_x(x)] \hat{\rho} \delta m(y). \tag{62}
\]

We can now apply TD’s version of CSL (TD-CSL) to the g-cat setup. The term \( \hat{U}_{g, \sigma} \) in (62) describes the gravitational interaction energy between the probe and the microsphere; treating the probe as a classical mass density, \( m_{probe}(x) \), we have

\[
\hat{U}_{g, \sigma}^{probe-sphere} = -\frac{G}{2} \int d^3x d^3y \frac{m_{probe}(x) \tilde{m}_{sphere}(y)}{|x - y|}, \tag{63}
\]

which will just contribute a phase shift to \( \hat{\rho} \) as an external field, but otherwise will not change the CSL collapse rate \( \Gamma \). As in CSLmN, the collapse rate will only depend on the number of particles composing the microsphere and so will also yield \( \Gamma_{cm} = \lambda N^2 = 10^{11} \) for \( m_{sphere} \sim 10^{14}amu \). We can also calculate the effect of the decoherence term due to gravitational back-action, in the special case of a single particle of mass \( m \) and density matrix \( \hat{\rho}(x_1, x_2) \), namely,

\[
\hat{D}_{g}[\hat{\rho}] = -\int d^3x \frac{1}{2\gamma} \left[ \hat{V}(x), \left[ \hat{V}(x), \hat{\rho} \right] \right] \\
= -\frac{G^2 m^2}{8\gamma} \int d^4r \left( \frac{1}{|r - x_1|} - \frac{1}{|r - x_2|} \right)^2 \rho(x_1, x_2) \tag{64}
\]

This expression tells us that the back-action decoherence term damps the phases of the density matrix in proportion to the distance \( |x_1 - x_2| \) separating the position \( x_1 \) and \( x_2 \) corresponding to the two possible locations of the signal. If we take \( |x_1 - x_2| \sim 10^{-6}m \), which is applicable to the Pino et al. protocol, we obtain

\[
\hat{D}_{g}[\hat{\rho}] \sim -10^{-29} \rho(x_1, x_2), \tag{65}
\]

indicating extremely slow phase damping. For \( |x_1 - x_2| \sim 10^{-12}m \), which is applicable to the Romero-Isart et al. protocol, it is obvious the phase damping rate is even slower (but we can ignore this case since, as we know, the CSL collapses will not destroy a cat state with this small a distance separating \( x_1 \) and \( x_2 \)). So we can conclude that TD-CSL also predicts that it should not be possible to experimentally prepare stable microspheres with (centers of) mass \( \sim 10^{14}amu \) in coherent spatial superpositions using Pino et al.’s protocol. Moreover, like CSLmN, TD-CSL predicts that for the g-cat setup, a classical probe will detect a virtually constant force from a microsphere (prepared with the Pino et al. protocol) that’s virtually continuously localized in one of the two minima of the double-well potential. We say “virtually” because, unlike CSLmN, no SN self-interaction is present, so there is still a small probability of each component of the cat state tunneling between the minima between collapse events, but it seems unlikely to be observable on realistic timescales of the g-cat experiment.

For the DP case of TD’s theory (TD-DP), the correlator is less trivial:

\[
\gamma_{xy} = \kappa G \frac{1}{|x - y|}, \tag{66}
\]
where the constant $\kappa$ is a dimensionless parameter fixed to 2 for certain physical reasons. For the mass density signal, we have the smeared form

$$m_{\sigma,t}(x) = \langle \hat{m}_{\sigma}(x) \rangle_t + \delta m_t(x),$$

(67)

where $\delta m_t(x)$ now satisfies

$$E[\delta m_{t_2}(x)\delta m_{t_1}(y)] = -\frac{1}{4\pi\kappa G}\delta(t_2 - t_1)\nabla^2 \delta(x - y).$$

(68)

To avoid obvious divergences, TD use the smeared density in $\dot{U}_{g,\sigma}$, resulting in the SME:

$$\frac{d\hat{\rho}}{dt} = -i \left[ \hat{H} + \dot{U}_{g,\sigma} + \int d^3x \delta m(x)\dot{V}_\sigma, \hat{\rho} \right]$$

$$- \frac{\kappa G}{8} \int d^3x d^3y \langle \hat{m}_{\sigma}(x), [\hat{m}_{\sigma}(x), \hat{\rho}] \rangle$$

$$- \frac{1}{8\pi\kappa G} \int d^3x \left[ \nabla\dot{V}_\sigma(x), \left[ \nabla\dot{V}_\sigma(x), \hat{\rho} \right] \right]$$

$$+ \frac{\kappa G}{2} \int d^3x d^3y |H| [\hat{m}_{\sigma}(x)] \hat{\rho} \delta m(x),$$

(69)

where the gravitational back-action

$$\dot{U}_{g,\sigma} = \int d^3x \dot{V}_\sigma(x)\hat{m}_{\sigma}(x) = -\frac{G}{2} \int d^3x d^3y \frac{\hat{m}_{\sigma}(x)\hat{m}_{\sigma}(y)}{|x - y|}. \quad (70)$$

By combining the two decoherence terms in (66) and setting $\kappa = 2$ on the requirement that decoherence be minimal, they obtain the local SME:

$$\frac{d\hat{\rho}}{dt} = -i \left[ \hat{H} + \dot{U}_{g,\sigma} + \int d^3x \delta m(x)\dot{V}_\sigma, \hat{\rho} \right]$$

$$- \frac{1}{8\pi G} \int d^3x \left[ \nabla\dot{V}_\sigma(x), \left[ \nabla\dot{V}_\sigma(x), \hat{\rho} \right] \right]$$

$$- \int d^3x |H| [\dot{V}_\sigma(x)] \hat{\rho} \delta m(x).$$

(71)

The back-action term thereby doubles the decoherence term present in the original DP master equation (48). TD take their cut-off $\sigma = 10^{-15}m$ to remedy the divergence problem, but the overheating problem remains. Hence, it is necessary again to introduce dissipative terms as done by Bahrami et al. While TD don’t incorporate dissipative terms in their equations, it is clear that doing so will lead us to Bahrami et al.’s constraint (49), in turn leading us to the conclusion that TD-DP should also be regarded as an effective theory valid only for masses comparable to or greater than $10^{11}amu$.

So the dissipative generalization of the TD-DP theory should coincide with the dissipative DPmN theory on the following prediction: for a microsphere with $m \sim 10^{14}amu$, a cat state of CM position states formed by either parametric coupling to a qubit (Romero-Isart et al. protocol) or by free expansion in a magnetic skatepark potential (Pino et al. protocol) will be rapidly suppressed to a width of $10^{-15}m$, on the timescale given by (52). So in the g-cat setup, the classical probe will detect a force from a full-mass microsphere located in one of the two potential well minima for virtually all times, as in TD-CSL.

### 3.1.5 K-model

The collapse model of Karolyhazy (K-model) posits that intrinsic spacetime fluctuations couple to quantum systems and induce a discrete-time state-vector reduction process similar to the GRW process [40, 44]. In particular, the spacetime fluctuations are encoded in a family of perturbed metrics $\{g_{\mu\nu}^\beta\}$ very close to the Minkowski metric; these metrics modify the $N$-body Schrödinger equation for free particles to the form

$$i\hbar \partial_t \psi_\beta = \left( -\sum_{i=1}^{N} \frac{\hbar^2}{2m_i} \nabla_i^2 + U_\beta \right) \psi_\beta, \quad (72)$$
where $U_\beta$ encodes small perturbations given by
\[ U_\beta(x, t) = \sum_i m_i c^2 \gamma_\beta(x_i, t), \] (73)
and where $\gamma_\beta(x, t)$ encodes the spacetime fluctuations that induce the state-vector reduction.

Note that Eq. (72) has a straightforward SN analogue: we simply adopt Eqs. (25-27) in subsection 3.1, make the replacement $\psi \rightarrow \psi_\beta$, and add to the Hamiltonian of (27) the $U_\beta$ term. Then we have an $N$-body K-model with matter density ontology that includes $N$-body gravitational pair interactions along with $N$ single-body gravitational self-interactions of SN type. Such an extended K-model (which we will call the KmN-model) can then be used to describe the g-cat setup.

For a single elementary particle, the K-model gives the “critical width” for a wavepacket as
\[ a_c \approx \left( \frac{L}{L_p} \right)^2 L, \] (74)
where $L \approx \hbar/mc$, and the “critical time” of reduction
\[ \tau_c \approx \frac{ma_c^2}{\hbar}. \] (75)

If we compare the K-model to GRW, $a_c$ is analogous to $\sigma_{GRW}$ and $\tau_c$ is analogous to $\lambda_{GRW}^{-1}$. For a macroscopic body with center of mass $m_{tot} = \sum_{i=1}^{N} m_i$ and size $R$, it can be shown that Eq. (74) becomes
\[ a_c \approx \left( \frac{R}{L_p} \right)^{2/3} L, \] (76)
where now $L \approx \hbar/m_{tot}c$. Hence, for a microsphere of $R \sim 1 \mu m$ and mass $m_{tot} \sim 10^{14} amu \sim 10^{-13} kg$, Eqs. (76) and (75) give\[ a_c^{sphere} \sim 10^{-11} m, \] (77)
and
\[ \tau_c^{sphere} \approx \frac{m_{tot} (a_c^{sphere})^2}{\hbar} \sim 0.1 s. \] (78)
So the microsphere would undergo ten collapses in one second; and like GRWmN and CSLmN, in the KmN-model, after the first collapse, the SN self-interaction will prevent delocalization of the wavepacket, making subsequent collapse events ineffectual to the subsequent width of the wavepacket.

The timescale (78) is out of the range of the Romero-Isart et al protocol for preparing the coherent microsphere, and the critical width is $a_c > 10^{-12} m$, implying that a collapse event won’t destroy the cat state, entailing no net force on the probe in the g-cat setup. On the other hand, the timescale (78) is well within the timescale of Pino et al.’s protocol (which we recall has a coherence time on the order of one second) [20]. So we can predict that if one collapse event happens before the microsphere interacts with the double-slit barrier, the microsphere matter density can only then pass through one of the slits (recall that the slit width $w = 10.61 nm$, or three orders of magnitude larger than $a_c^{sphere}$) and end up in only one of the minima of the double-well potential on the other side of the barrier. Moreover, like in GRWmN and CSLmN, in the KmN-model the SN self-interaction (and any subsequent collapse events) will inhibit tunneling of the microsphere matter density between the two minima.

For a microsphere mass $m_{tot} \sim 10^{18} amu$ (keeping $R$ still $\sim 1$ micron), we note that $\tau_c^{sphere}$ for a microsphere prepared with the Pino et al.’s protocol would be the same as $\lambda_{GRW}$. So the KmN-model’s prediction for the g-cat setup is the same as that of GRWmN.

### 3.2 Collapse theories with quantized gravity

Here we compare the predictions of the semiclassical gravitational OCTs to the predictions we would obtain from treating the Newtonian gravitational potential as a quantized field. To do this, we need only analyze in detail the GRW case, since the results therein will be readily applicable to the other OCTs.
To start off, we consider the GRW theory with no primitive ontology (i.e., no matter density in space-time and no flashes), which we will call “GRW0” \[57\]. For GRW0-Newton, we will treat the gravitational potential sourced by matter as an operator-valued field satisfying the Poisson equation

$$
\nabla^2 \tilde{V}_g = 4\pi G \sum_{i=1}^{N} m_i \delta^{(3)}(x - \hat{R}_i),
$$

(79)

where the right hand side is a sum over all the (first quantized) mass density operators associated to each particle. Because we are treating the gravitational potential as operator-valued, the Schrödinger evolution in GRW0-Newton is linear in \(\psi\), in contrast to the SN evolution in GRWmN. Moreover, the classical gravitational potential associated to \(\tilde{V}_g\) is obtained by taking the quantum expectation value of both sides of (79):

$$
\langle \psi | \nabla^2 \tilde{V}_g | \psi \rangle = \langle \psi | \sum_{i=1}^{N} m_i \delta^{(3)}(x - \hat{R}_i) | \psi \rangle = 4\pi G \sum_{i=1}^{N} \int d\mathbf{r}_1...d\mathbf{r}_N |\psi(\mathbf{r}_1, ..., \mathbf{r}_N, t)|^2 m_i \delta^{(3)}(x - \mathbf{r}_i),
$$

(80)

where \(\Psi = \Psi(\mathbf{r}_1, ..., \mathbf{r}_N, t)\) and the position operators \(\hat{R}_i\) give \(\hat{R}_i\Psi = \mathbf{r}_i\Psi\). Accordingly, for an N-body system of identical particles with Newtonian gravitational interactions, the N-body Schrödinger equation of GRW0-Newton is given by

$$
i\hbar \partial_t \Psi(\mathbf{r}_1, ..., \mathbf{r}_N, t) = \left[ -\sum_{i=1}^{N} \frac{\hbar^2}{2m} \nabla^2_i - \sum_{i\neq j} \frac{G m^2}{|\hat{R}_i - \hat{R}_j|} \right] \Psi(\mathbf{r}_1, ..., \mathbf{r}_N, t).
$$

(81)

And, of course, we have the GRW process wherein the solution of (81) undergoes intermittent, discontinuous collapses of the form

$$
\Psi_{T^+}(\mathbf{r}_1, ..., \mathbf{r}_N) = \frac{1}{\sqrt{g}} \rho(\mathbf{r}_i - \mathbf{X})^{1/2} \Psi_{T^-}(\mathbf{r}_1, ..., \mathbf{r}_N),
$$

(82)

with collapse width \(\sigma_{GRW}\), collapse rate \(N\lambda_{GRW}\), and probability density

$$
\rho(\mathbf{X}) = C(\mathbf{X})^2 = \int d\mathbf{r}_1...d\mathbf{r}_N g(\mathbf{r}_i - \mathbf{X})|\Psi_{T^-}(\mathbf{r}_1, ..., \mathbf{r}_N)|^2,
$$

(83)

where \(i\) is chosen randomly from \(1, ..., N\).

It might be noticed that (81) is also the Schrödinger equation of CQT-Newton. Thus, if we assume that the particles are weakly interacting, then by imposing \(|\Psi(t)\rangle = \lim_{N \to \infty} e^{-\frac{i}{\hbar} H_{\text{quant}} t} \otimes_{i=1}^{N} |\psi\rangle = \otimes_{i=1}^{N} |\chi(t)\rangle\), we recover the mean-field equations

$$
\nabla^2 \tilde{V}_g = 4\pi G m |\chi(\mathbf{r}, t)|^2,
$$

(84)

and

$$
i\hbar \partial_t \chi(\mathbf{r}, t) = \left[ -\frac{\hbar^2}{2m} \nabla^2 - G \int d\mathbf{r}' \frac{m^2 |\chi(\mathbf{r}', t)|^2}{|\mathbf{r} - \mathbf{r}'|} \right] \chi(\mathbf{r}, t),
$$

(85)

for the collective variable \(\chi(\mathbf{r}, t)\). Notice that since this approximation assumes that the many-body wavefunction can be factorized as \(|\Psi\rangle = \otimes_{i=1}^{N} |\chi\rangle\), the collapse rate for \(\chi(\mathbf{r}, t)\) is just \(\lambda_{GRW}\). So the mean-field description of GRW0-Newton is effectively indistinguishable from the mean-field description of CQT-Newton. And, like CQT-Newton, the mean-field description leading to (84-85) is inadequate for modeling our g-cat setup (because the factorization ansatz is a poor approximation for microspheres).

Instead, we must consider the microsphere CM Schrödinger equation given by (14):

$$
i\hbar \partial_t \Psi(\mathbf{x}_{cm}, t) = \hat{H}_{cm} \Psi(\mathbf{x}_{cm}, t) = \left[ -\frac{\hbar^2}{2m_{tot}} \nabla^2_{cm} - \frac{G m_{tot}}{D} \right] \Psi(\mathbf{x}_{cm}, t),
$$

(86)

where \(m_{tot} = Nm\), \(M\) is the probe mass, \(\hat{D} = \sqrt{\frac{\hbar^2}{m_{cm}} + [\hat{x}_{cm}]^2}\), \(\hat{x}_{cm}\) has eigenvalues \(-L/2\) and \(+L/2\), and \(y_{cm}\) is the fixed (c-number) y-displacement of the CM of the test mass from the CM of the probe. Then we can follow AH in introducing the initial cat state

$$
|\Psi> = c_+|+ > + c_- |-- >,
$$

(87)
which for a microsphere mass of $\sim 10^{14}$amu will have a collapse rate of $10^{-2}s^{-1}$, or $10^{2}s^{-1}$ for a microsphere mass of $\sim 10^{18}$amu. As before, for the microsphere cat state produced by the Romero-Isart et al. protocol, the collapses will be ineffectual since $L = 1\mu m \ll \sigma_{GRW} = 0.1\mu m$. But for the microsphere cat state produced by the Pino et al. protocol, the collapses will appreciably change the width of the cat state since now $L = 0.5\mu m$.

Recall how AH [2] assumed that the Newtonian gravitational interaction between the classical probe and the microsphere acts as a projective measurement according to the usual quantum measurement postulates; but because the usual quantum measurement postulates are based on ambiguous notions like “information extraction” and “macroscopic”, this assumption was difficult to rigorously justify. For GRW0-Newton, the Newtonian gravitational interaction between the classical probe and the microsphere will indeed act as a projective measurement in the sense that the probe-sphere gravitational coupling will drive up the collapse rate of the microsphere cat state, and thus lead to predictions in agreement with CQT-Newton as described by AH (apart from minute differences in statistics due to the GRW process).

To show this we must, however, describe the probe within the context of GRW0-Newton as well [2]. In particular, we must attribute to the probe a CM wavefunction in the ‘ready state’ $\Phi_0(r_{cm})$. Projecting the cat state (87) onto the CM coordinate space gives $\Psi(r_{cm}) = c_+\Psi_+(r_{cm}) + c_-\Psi_-(r_{cm})$, indicating that $\Psi(r_{cm})$ is not an eigenstate of the CM position operator $R_{cm}$. Then the interaction Hamiltonian in (86) implies that $\hat{U}_{\text{probe-sphere}} (\Psi_+ \otimes \Phi_0) = \Psi_+ \otimes \Phi_+$ and $\hat{U}_{\text{probe-sphere}} (\Psi_- \otimes \Phi_0) = \Psi_+ \otimes \Phi_-$, where $\Phi_+$ denotes the probe wavefunction ‘deflected’ (correlated) towards the position of the $+$ minimum and $\Phi_-$ denotes the probe wavefunction deflected towards the position of the $-$ minimum. By the linearity of (86), we then have the entangled state

$$\hat{U}_{\text{probe-sphere}} (\Psi \otimes \Phi_0) = c_+\Psi_+ \otimes \Phi_+ + c_-\Psi_- \otimes \Phi_-, \quad (88)$$

since the probe-pointer states are orthogonal, i.e., $\Phi_+ \cdot \Phi_- \approx 0$ (this follows from the assumption that the probe is sensitive enough to the gravitational force from the cat state that its two possible CM positional deflections are macroscopically distinct, i.e. separated by a distance greater than $10^{-7}m$). Moreover, because (88) is an entangled state, if one of the probe particles undergoes a GRW hit described by (82), then the entire state (88) will collapse as well. Thus the collapse rate of the probe-sphere CM wavefunction will be $(N_{\text{probe}} + N_{\text{sphere}}) \lambda_{GRW}$, and the probability density for collapse into either $\Psi_+ \otimes \Phi_+$ or $\Psi_- \otimes \Phi_-$ will be given by

$$C(X)^2 = \int d^3r_{cm} g(r_{cm} - X)|c_+\Psi_+ \otimes \Phi_+ + c_-\Psi_- \otimes \Phi_-|^2, \quad (89)$$

where $X = \{+\frac{1}{2}, -\frac{1}{2}\}$. We stress that these conclusions will apply to the g-cat setup using either the Romero-Isart et al. protocol or the Pino et al. protocol, in agreement with CQT-Newton.

Since the probe is assumed to be a macroscopic device composed of a much larger number of particles than the microsphere (e.g., $N_{\text{probe}} \sim 10^{23}$), we should expect the collapse of (88) to be frequent enough that macroscopic observers (such as experimentalists, who themselves will also correspond to many-body wavefunctions evolving by the GRW0-Newton laws of motion, and entangled with the probe) will ‘perceive’ (through the macroscopic deflections of the probe) the microsphere as undergoing seemingly instantaneous quantum jumps between the $+$ and $-$ minima. Let us estimate the rate of collapse, based on the assumption that the probe is described by the Sankey et al. trampoline resonator [35]. The actual trampoline (i.e., the part of the probe that plays the role of the pointer) has a mass of only $4.0mg \sim 10^{15}$amu, but it is tethered to a much larger and more massive (square shaped) silicon wafer. The wafer has thickness $675\mu m$ and width $3mm$, and solid silicon has density $\rho_{Si} = 2.33\frac{g}{cm^3}$. From these values we can calculate that the wafer is composed of $\sim 10^{20}$ nucleons. Since the wavefunction of the trampoline resonator and the wavefunction of the wafer are strongly entangled, their joint many-body wavefunction $\Phi_0$ (however complicated it looks) therefore has a collapse rate of $\sim 10^{20} \cdot \lambda_{GRW} = 10^4s^{-1}$, or around 10,000 collapses per second. (We neglect further increases in the collapse rate due to entanglement with the thermal environment since we assume that the resonator operates at a temperature of $14mK$ or lower, where its force sensitivity is at peak value.) So when the wafer-resonator system (i.e., the probe) gets entangled with the microsphere through gravitational coupling, this collapse rate will also apply to (88) (the microsphere adds only $\sim 10^{14}$ nucleons, which negligibly increases the collapse rate).

Using the above analysis, we can also show without ambiguity why the sphere’s g-coupling to the Earth’s field (and the field of any other massive body in the environment) doesn’t collapse the sphere’s cat state.

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12We are grateful to Dennis Dieks for suggesting the general outlines of the ensuing argument.
wavefunction, despite the magnitude of the Earth-sphere g-coupling being nearly seventeen orders of magnitude greater than the probe-sphere g-coupling. Suppose we take the CM of the Earth as the ‘pointer variable’ which correlates to the sphere states. We represent the CM of the Earth by the ready state \( \Phi_0^{\text{Earth}}(r_{cm}) \), and replace the probe-sphere interaction term in (86) with the Earth-sphere interaction term \(-m_{tot}g\hat{z}, \) where \( \hat{z} \) is the operator-valued vertical displacement of the CM of the sphere from the ground-level of the lab. Then, following through the same argument leading to (88), we have

\[
\hat{U}_{\text{Earth-sphere}} (\Psi \otimes \Phi_0^{\text{Earth}}) = c_+\Psi_+ \otimes \Phi_+^{\text{Earth}} + c_-\Psi_- \otimes \Phi_-^{\text{Earth}}.
\]

This time, however, because of the huge mass of the Earth relative to the sphere, the CM states \( \Phi_+^{\text{Earth}} \) and \( \Phi_-^{\text{Earth}} \) have considerable overlap and therefore are not orthogonal. Indeed, the huge mass of the Earth implies that the relative separation between \( \Phi_+^{\text{Earth}} \) and \( \Phi_-^{\text{Earth}} \) will be much less than \( \sigma_{\text{GRW}} \). That means \( \Phi_+^{\text{Earth}} \approx \Phi_-^{\text{Earth}} \approx \Phi_0^{\text{Earth}} \), and we can well approximate (90) as

\[
\hat{U}_{\text{Earth-sphere}} (\Psi \otimes \Phi_0^{\text{Earth}}) \approx (c_+\Psi_+ + c_-\Psi_-) \otimes \Phi_0^{\text{Earth}}.
\]

Accordingly, when one of the Earth particles undergoes a GRW hit, no change is entailed for the sphere’s cat state. So the only particles that are physically relevant to the collapse of the cat state are the particles composing the sphere, and even then only if the relative separation of the cat state components is greater than \( \sigma_{\text{GRW}} \) (e.g., as in the Pino et al. protocol).

An objection that might be raised towards GRW0-Newton is that it is an empirically incoherent theory because it predicts no space-time and no matter in space-time to which experiments, observers, and the perceptions of observers correspond (in stark contrast to our very definite perceptions of living in a 3D-space with matter evolving in it) [25]. Indeed, the fundamental ontology of GRW0-Newton is just an N-body wavefunction on configuration space. So when we say that “the probe wavefunction deflected towards the – minimum”, what we really mean is that the probe-sphere wavefunction collapses (effectively) to the sub-space of the probe-sphere Hilbert space corresponding to the state \( \Psi_- \otimes \Phi_- \). Two possible answers to this objection are as follows: (i) Albert’s (philosophical) functionalist analysis of the GRW wavefunction [22] can be employed to deduce 3D-space and a matter density (or flash) ontology within 3D-space as emergent ontological variables; (ii) we can still postulate, in addition to an ontic wavefunction on configuration space, the existence of a 3D-space and a matter density (or flash) ontology within it, but with the understanding that these primitive ontological variables are causally inert in space-time (i.e., the matter density fields in space-time don’t physically interact with each other through classical forces, but only indirectly through the evolution of the wavefunction in configuration space, and the flash events don’t get accompanied by point masses at the flash locations). It is debatable which of these two options is more plausible than the other (or, for that matter, if either option is plausible in its own right), but for the purposes of this paper, we simply note that they are both logically possible solutions to the ‘empirical incoherence’ objection.

By analogy with GRW0-Newton, it is straightforward to construct CSL0-Newton, DP0-Newton, and K0-Newton. (Note that there is no TD0-Newton, since the TD theory is specifically designed to treat the gravitational field as classically sourced by the flash ontology in the setting of CSL dynamics). Apart from minute differences in experiment statistics resulting from the different intrinsic collapse rates predicted by the CSL, DP, and K-model processes, it is straightforward to show, using the same arguments as above, that these three variants of GRW0-Newton will predict the same outcomes as GRW0-Newton for the g-cat setup.

### 3.3 Related collapse theories

While we have left out certain objective collapse theories from our analyses above [22, 23, 24, 21, 25], our findings up to this point allow us to quickly assess these other ones.

The stochastic extension of the SN equations given by Nimmrichter and Hornberger (NH) [21] results in cancellation of any gravitational self-interaction or pair interaction. Thus the NH theory predicts no gravitational coupling between probe and microsphere, regardless of whether or not the microsphere can be put into a coherent and stable spatial superposition.

The theory of Kafri et al. [24] is mathematically and conceptually equivalent to the original DP theory, which makes our analysis of the DP theory applicable to their theory as well.

The theory of Bera et al. [25] is formally equivalent to the K-model in that it predicts the critical widths (74) and (76), as well as the collapse timescale (75) and (78). (Although these timescales are associated with a gravitationally-induced decoherence process rather than a state-vector reduction process.) Thus our
predictions for the K-model apply as well to Bera et al.’s theory (modulo the conceptual difference between a
decoherence process and a state-vector reduction process).

Finally, the Trace Dynamics theory of Stephen Adler [22] results in an effective stochastic master equation
that can (under certain assumptions) be put into a form equivalent to that of the CSL master equation. As
such, our conclusions about CSL with semiclassical gravitational pair interactions, whether in the form of
CSLmN or TD-CSL, will presumably also apply to Adler’s semiclassical gravitational generalization of Trace
Dynamics [23] (when one considers the Newtonian limit).

4 Conclusion

We have appraised the most well-known and well-developed objective collapse theories in light of DAH’s
proposed g-cat setup [1], including an extension of the g-cat setup to incorporate Pino et al.’s protocol [20],
and compared the predictions of said collapse theories to the predictions of CQT-Newton. In particular, we
assessed the predictions of GRW, CSL, DP, and the K-model, in the context of two cases: (i) extended to
include semiclassical gravitational interactions in the approach of Derakhshani in [3] and/or the approach
of Tilloy-Diósi in [4]; and (ii) extended to include quantized Newtonian gravitational interactions between
particles. We then used these results to assess other (closely related) objective collapse theories in the recent
literature, namely, the theories of Nimmrichter & Hornberger [21], Kafri et al. [24], Bera et al. [25], and
Adler’s Trace Dynamics [22].

The results of our primary analyses can be summarized as follows:

1. GRWmN: (i) The probe-microsphere (or even Earth-microsphere) semiclassical gravitational coupling
for the g-cat setup will not drive up the collapse rate of the microsphere; (ii) the number of nucleons
composing the microsphere is too few to bring its collapse rate within the coherence time of the g-cat
setup using the Romero-Isart et al. protocol, and in any case the relative separation of $L = 1 \mu m$ is
much smaller than $\sigma_{GRW}$, thereby implying that any collapse event will make no physical change to
the sphere’s cat state wavefunction and associated matter density (so the probe will just feel net-zero
gravitational force from an uncollapsed cat state matter density); (iii) for the g-cat setup using the Pino
et al. protocol (which makes possible $L \gtrsim 0.5 \mu m$), the microsphere mass can potentially be increased by
as much as four orders of magnitude, thereby bringing the sphere’s collapse rate well within the coherence
time of the protocol and making it possible that the force probe could measure a GRW-type quantum
jump of the gravitational force from the microsphere cat state, or else just a continuous gravitational
force from the microsphere being localized to one of the minima of the double-well potential illustrated
in Fig. 1 (with no probability of tunneling between the minima, due to SN self-interaction).

2. CSLmN: (i) The probe-microsphere g-coupling for the g-cat setup will not drive up the collapse rate of
the microsphere; (ii) however, the rate of effective localization on the center-of-mass motion of the
microsphere will be so high that a microsphere cat state prepared using Pino et al.’s protocol will be
quickly suppressed and remain suppressed thereafter (due to SN self-interaction), while for a microsphere
cat state prepared with the Romero-Isart et al. protocol the cat state will remain stable for the coherence
time of the experiment; (iii) thus, if the g-cat setup were experimentally implemented using the Pino
et al. protocol, the probe would measure a continuous gravitational force from a full-mass microsphere
matter density that sits in one of the minima of a double-well potential for all times, while use of the
Romero-Isart et al. protocol would entail net-zero gravitational force on the probe from an uncollapsed
cat state matter density.

3. DPmN: Corrected with a cut-off and the inclusion of dissipative terms to prevent overheating, DPmN
predicts: (i) that the probe-microsphere g-coupling for the g-cat setup will not drive up the collapse rate of
the microsphere; (ii) for the microsphere, a characteristic damping time (i.e., state-vector reduction rate)
even more rapid than that of CSL (two orders of magnitude more, to be exact); (iii) rapid collapse of the
microsphere cat state in both the Romero-Isart et al. protocol (because the spatial cut-off $R_0 = 10^{-15} m$
vs. $L = 1 \mu m$) and the Pino et al. protocol; (iv) thus, for both the Romero-Isart et al. protocol and the
Pino et al. protocol, the probe in the g-cat setup would measure a continuous gravitational force from a
full-mass microsphere matter density that sits in one of the minima of the double-well potential for all
times (due to SN self-interaction).
4. **TD-CSL/DP:** The CSL case predicts: (i) semiclassical g-coupling to the probe or any other massive body in the environment doesn’t drive up the collapse rate of the microsphere; (i) extremely slow phase damping (decoherence) of the microsphere density matrix due to gravitational back-action; (ii) the same collapse rate and collapse width for the microsphere as in CSLmN (and ordinary CSL), which means collapse events make a physical difference for the microsphere prepared by the Pino et al. protocol and no physical difference for the Romero-Isart et al. protocol. But because there is no SN self-interaction, there is a small probability of the cat state tunneling between the minima in between collapse events. However, this tunneling rate is so low that a tunneling event is unlikely to be observed on the timescales of the g-cat experiment. So TD-CSL makes effectively the same prediction for the g-cat setup as CSLmN and DPmN, for both the Pino et al. protocol and the Romero-Isart et al. protocol. Similarly, the DP case predicts: (i) same as TD-CSL; (ii) doubling of the decoherence term in the original DP master equation; and (iii) when corrected with the appropriate length-scale cut-off and dissipative terms, the same characteristic damping time as the original dissipative DP(mN) theory. And like TD-CSL, the tunneling rate in between collapse events is negligible for the g-cat experiment. So TD-DP makes effectively the same predictions for the g-cat setup as DPmN.

5. **KmN-model:** (i) semiclassical g-coupling to the probe or any other massive body in the environment doesn’t drive up the collapse rate of the microsphere; (ii) the predicted microsphere collapse rate is on the order of a tenth of a second (for $m \sim 10^{14}$ amu), which is out of the range of the coherence time of the microsphere for the Romero-Isart et al. protocol, and in any case a collapse event will yield no physical change to the sphere’s cat state wavefunction and associated matter density since $a_{\text{sphere}} \sim 10^{-11} m$, so no net force on the probe in the corresponding g-cat setup; (iii) however, the collapse rate falls within the coherence timescale of the Pino et al. protocol, and for that rate makes essentially the same predictions as GRWmN for the g-cat setup.

6. **GRW0-Newton:** (i) The probe-microsphere gravitational coupling for the g-cat setup will drive up the collapse rate of the microsphere, and thereby result in the same predictions as CQT-Newton for the case of a classical probe continuously monitoring the g-field of the microsphere cat state, for both microsphere-preparation protocols (apart from minute differences in the g-cat experiment statistics entailed by the GRW process); (ii) but the g-coupling of the sphere to other massive bodies in the environment, such as the Earth, will not drive up the collapse rate of the sphere, unless those other massive bodies satisfy the physical conditions required to play the role of a pointer variable (as in the case of the probe); and (iii) apart from small differences entailed by the different intrinsic collapse rates predicted by the CSL, DP, and K-model processes, CSL0-Newton, DP0-Newton, and K0-Newton will predict the same outcomes as GRW0-Newton for the g-cat setup.

We therefore conclude that the g-cat setup is, in principle, capable of: (i) experimentally discriminating between the predictions of the aforementioned semiclassical gravitational OCTs, to the extent that some of these OCTs make predictions that differ from each other for the g-cat setup; and (ii) experimentally discriminating between some or all of the predictions of the aforementioned semiclassical gravitational OCTs versus the predictions of CQT-Newton and GRW0/CSL0-DP0-K0-Newton, to the extent that the analyzed semiclassical gravitational OCTs make different predictions by virtue of treating the gravitational field semiclassically instead of (perturbatively) quantized.

For the purpose of solidifying the theoretical foundations of the OCT theories analyzed here, it seems prudent to extend the semiclassical gravitational OCTs to the regime of semiclassical Einstein gravity (if possible!), take the Newtonian limit, and compare the resulting predictions for the g-cat setup to the predictions obtained in this paper. Likewise, to extend GRW0, CSL0, DP0, and K0-model to the regime of relativistic perturbatively quantized gravity, take the non-relativistic limit, and compare the predictions for the g-cat setup to the predictions obtained in this paper. These are tasks for future work.

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