Research Article

Design of a Magnetic Interaction-Based Vibration Absorber for Continuous Beam

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Classical absorber for vibration suppression of a continuous structure is constructed as a spring-mass oscillator, which only provides coupling force to suppress the vibration of primary structure. In this study, absorber beam is introduced and coupled on the continuous primary beam with magnetic interaction. Thus, the magnetic interaction and coupling bending moment affect the responses of primary beam. Based on the model of the system and Galerkin truncation, the natural frequencies for different magnetic parameters are obtained, which demonstrates that the fundamental frequency can be reduced to zero and the vibration of primary beam can be suppressed in a wide frequency band. Considering the vibration suppression on frequency band, we propose two criteria to evaluate the vibration suppression effect: one is the width of band for vibration suppression and the other is the width for vibration absorption. The two criteria not only show the vibration reduction effect but also correspond to different vibration suppression mechanism. Due to the advantages of zero fundamental frequency induced by the proposed magnetic interaction coupling and wide vibration suppression frequency band, utilizing absorber beam in vibration suppression of continuous structure has potential applications for flexible aim in the fields of manufacturing and aerospace.

1. Introduction

Due to urgent requirements and harsh conditions in the vibration suppression techniques in the fields of aerospace [1], shipping engineering [2], and ocean platforms [3, 4], it needs the remarkable vibration suppression effectiveness in a wide frequency band. For the vibration suppression and reduction of a vibration system, tuned vibration absorber (TVA) is designed according to different requirements in practices. Usually, the TVA is a single degree-of-freedom (1-DOF) spring-damper-mass vibration oscillator designed with adjustable stiffness or damping [5, 6]. When the targeted frequency for vibration suppression of the primary system is in the region of antifrequency band induced by the vibration absorber, the vibration energy can be transmitted effectively. However, in the analysis and applications of TVA, there are mainly two limitations and issues in design and realization. First, the vibration suppression effectiveness is strongly dependent on the structural parameters of the TVA, especially the mass. For smaller mass of TVA, the vibration suppression is less sufficient or effective. But, in the practices, the mass of absorber is required lighter than 10% mass of the primary system. Second, the stiffness and damping of the vibration absorber should have tunable property to result in antiresonance at the required frequencies for the realization of optimal vibration suppression effectiveness. When the primary system has ultralow natural frequency or variable characteristics, the structural realization of TVA is required to match the characteristics of primary system.

Recent studies of vibration absorption mainly focused on the bottleneck techniques for the realization of adjustable and adaptive properties. The methods include passive vibration device, active control methodology, and hybrid control devices. Based on the remarkable tunable properties for equivalent stiffness and damping of the so-called quasi-zero stiffness system [7–9], the structure is transplanted to the absorber structure for the required vibration absorption frequencies [10]. It also discovers that the vibration performances of nonlinear tuned vibration absorber (NTVA)
may result in some beneficial dynamical phenomena in vibration suppression, such as saturation properties and internal resonances, which cannot exist for linear vibration systems [11–13]. Furthermore, for vibration suppression in a frequency band rather than just on specific frequency point, periodic structures are applied on continuous primary system and designed for wide bandgap based on local resonance theory [14, 15]. Recently, in order to extend the bandgap to low frequency band, the QZS system is utilized as the TVA [16, 17]. With the periodical assembly of oscillators with QZS property on a continuous beam, a low-frequency bandgap is realized due to the negative-stiffness structural design. But, with the reduction of bandgap to low frequency band, the width of bandgap reduces. On one hand, these studies are concentrated on the frequency tuning technologies by structural design to adapt to the targeted frequencies or frequency bands. On the other hand, the vibration amplitudes, especially at resonances, should be attended to and suppressed. Commonly, the most direct method to reduce the resonance peak is increasing the equivalent damping effect of the system. However, although increasing the equivalent damping strength can reduce the resonance peaks, the amplitudes at the nonresonance bands would be raised, inducing deterioration of vibration suppression. In order to harmonize the contradiction on the response amplitude, the most traditional method to design the TVA is the $H_{\infty}$ criterion to minimize the vibration amplitudes of the primary system around resonance band [18–21]. The optimal parameters obtained by the $H_{\infty}$ strategy work very well since the resonance peaks of the 2-DOF system are minimum and equal [19–22]. Then, for continuous structure such as beam, the TVA is also designed as a beam, connected on the continuous primary beam by spring and damper [23]. Simplifying the model as a 2-DOF system and applying the fixed-point theory, the vibration of primary beam is reduced, but the vibration suppression is dependent on the mass ratio between the primary beam and absorber beam. In [24–26], an absorber beam with/without lumped mass is fixed on a primary beam on perpendicular position. The proposed structural assembly of the proposed absorber beam can achieve more vibration reduction with same mass ratio compared with the traditional spring-damping-mass absorber.

From the previous studies, it can be seen that the structural design or control strategy for vibration suppression by both frequency tuning and amplitude reduction is well needed. In addition, it also requires lightweight TVA with appropriate nonlinearity. Thus, this study proposes an absorber beam with negative stiffness components, which is coupled on a primary beam. In the proposed structure, the tunable negative stiffness components are engendered by magnetic interaction. The negative stiffness components are two pairs of magnets with very small mass assembled symmetrically on the absorber beam and the free end of the primary beam, adjustable for different structural parameters. Due to the negative stiffness coupling, the stiffness and coupled stiffness of the absorber beam change cooperating with different equivalent stiffness of the primary beam. For vibration, the motion of the free end of the primary beam amplifies the coupling force in each modal, and thus the mass of the absorber beam can be very tiny. Since it has been verified that the nonlinear interaction force can suppress the vibration energy and reduce the resonance peaks [27–30], the nonlinearity introduced into the system can benefit the reduction of resonance frequency and amplitude reduction. Therefore, combining the advantages of linearity and nonlinearity, the absorber beam, with very small mass, can induce a wide nonresonance frequency band and low resonance peaks.

The paper is organized as follows. First, the model of magnetic interaction is established and its effect is discussed in Section 2. Next, the dynamical model of the nonlinear coupling system is established and all coefficients in the model are defined in Section 3. Then, based on the dynamical model, the natural frequencies and solutions are solved. Also, the evaluation criteria for effective vibration suppression are proposed in Section 4. The evaluation criteria provide conditions for mechanisms of vibration absorption and design method for structural parameters. The experimental setup and results are discussed in Section 5. The conclusion is given in the final Section.

2. Structure of System Coupled by Magnetic Interaction

2.1. Structure. Traditionally, for the vibration suppression of a continuous structure such as cantilever beam, spring-damper-mass oscillator is applied on the cantilever beam as the TVA. Considering the moment can directly induce bending deformation of continuous beam, the absorber is designed to result in both force and moment interactions on the primary system in this study. Therefore, an absorber beam, which has much smaller mass than the primary beam, is introduced and coupled on the primary beam by constraint and magnetic interaction, as shown in Figure 1. All the structural parameters in the system are listed in Table 1 in Appendix.

As shown in Figure 1(a), the primary beam is excited by an excitation force $f(t)$ and the absorber beam is coupled with the primary beam under magnetic interaction. As shown in Figure 1(b), the vibrations of the primary beam and absorber beam are $\omega_1(\xi_1, t)$ and $\omega_2(\xi_2, t)$ with coordinates $\xi_1$ and $\xi_2$, respectively. Since the motion of free end of the primary beam is $w_1(L_1, t)$ and the angle of rotation is $\theta_1(L_1, t)$, the vibration motion of absorber beam consisted of three parts as shown in Figure 1(b), written as

$$w_2(\xi_2, t) = w_1(L_1, t) + \theta_1(L_1, t)\xi_2 + \tilde{w}_2(\xi_2, t). \quad (1)$$

In equation (1), $\tilde{w}_2(\xi_2, t)$ is the deflection vibration and $w_1(L_1, t) + \theta_1(L_1, t)\xi_2$ is the rigid displacement. The values of structural parameters are fixed as given in Table 1 in Appendix. As shown in Table 1, the mass of the primary beam is $\rho_2A_2L_2 = 0.0216$ kg, which is about 120 times the mass of absorber beam equal to $\rho_2A_1L_2 = 0.00018$ kg. By coupling the absorber beam with cross section constraint and magnetic interaction, the effect and mechanism of absorber beam are estimated in this study. The model of magnetic interaction is established in the following section.
directions are established on the two magnets as \((\xi_1, \xi_2, y, z)\), where \(x\) and \(x'\) are coincided with axial deflection direction in the same direction of force on magnet-2 are equilibria in \(x\) and \(z\) directions, while the components of interaction forces in \(y\) direction are not equilibria. The interaction force of one pair of magnets in the deflection direction is in the same direction of \(y\), which is written as \(F_{\text{mag-y}}\) expressed as

\[
F_{\text{mag-y}} = \frac{\mu_0}{2\pi} \sum_{i=0}^{1} \sum_{j=0}^{1} \sum_{k=0}^{1} \sum_{l=0}^{1} \sum_{m=0}^{1} (-1)^{i+j+k+l+m} \psi_j \cdot (u_{ij}, v_{kl}, w_{pq}, r),
\]

where \(\psi_j (u, v, w, r) = \frac{1}{2} (u^2 - w^2) \ln (r - v) + uv \ln (r - u) + uw \arctan (uv/ru) + (rv/2);\) the symbols \(u_{ij}, v_{kl}, w_{pq},\) and \(r\) are written as

\[
\begin{align*}
\psi_j &= (L_2 - A - a + (-1)^j A - (-1)^j a, \\
v_{kl} &= \bar{w}_{kl} |L_2| + (-1)^j B - (-1)^j b, \\
w_{pq} &= d - c + (-1)^j C - (-1)^j c, \\
r &= \sqrt{u_{ij}^2 + v_{kl}^2 + w_{pq}^2}.
\end{align*}
\]

As shown in Figure 2, according to the directions of coordination, the motion \(\bar{w}_{2} (L_2, t)\) occurs in \(y\) direction. For the two pairs of magnets, the components of interaction force on magnet-2 are equilibria in \(x\) and \(z\) direction, while the components of interaction forces in \(y\) direction are not equilibria. The interaction force of one pair of magnets in the deflection direction is in the same direction of \(y\), which is written as \(F_{\text{mag-y}}\) expressed as

\[
F_{\text{mag-y}} = \frac{\mu_0}{2\pi} \sum_{i=0}^{1} \sum_{j=0}^{1} \sum_{k=0}^{1} \sum_{l=0}^{1} \sum_{m=0}^{1} (-1)^{i+j+k+l+m} \psi_j \cdot (u_{ij}, v_{kl}, w_{pq}, r),
\]

where \(\psi_j (u, v, w, r) = \frac{1}{2} (u^2 - w^2) \ln (r - v) + uv \ln (r - u) + uw \arctan (uv/ru) + (rv/2);\) the symbols \(u_{ij}, v_{kl}, w_{pq},\) and \(r\) are written as

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\begin{align*}
\psi_j &= (L_2 - A - a + (-1)^j A - (-1)^j a, \\
v_{kl} &= \bar{w}_{kl} |L_2| + (-1)^j B - (-1)^j b, \\
w_{pq} &= d - c + (-1)^j C - (-1)^j c, \\
r &= \sqrt{u_{ij}^2 + v_{kl}^2 + w_{pq}^2}.
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F_{\text{mag-y}} = \frac{\mu_0}{2\pi} \sum_{i=0}^{1} \sum_{j=0}^{1} \sum_{k=0}^{1} \sum_{l=0}^{1} \sum_{m=0}^{1} (-1)^{i+j+k+l+m} \psi_j \cdot (u_{ij}, v_{kl}, w_{pq}, r),
\]

where \(\psi_j (u, v, w, r) = \frac{1}{2} (u^2 - w^2) \ln (r - v) + uv \ln (r - u) + uw \arctan (uv/ru) + (rv/2);\) the symbols \(u_{ij}, v_{kl}, w_{pq},\) and \(r\) are written as

\[
\begin{align*}
\psi_j &= (L_2 - A - a + (-1)^j A - (-1)^j a, \\
v_{kl} &= \bar{w}_{kl} |L_2| + (-1)^j B - (-1)^j b, \\
w_{pq} &= d - c + (-1)^j C - (-1)^j c, \\
r &= \sqrt{u_{ij}^2 + v_{kl}^2 + w_{pq}^2}.
\end{align*}
\]
According to the property of interaction of magnets in [31, 32], the interaction force $F_{\text{mag},y}$ in equation (1) is a complex function with more than 200 terms. To simplify the magnetic interaction force $F_{\text{mag},y}$ for theoretical analysis and structural design, we utilize high-order polynomial function $F_y$ (rather than the Taylor series expansion) to fit the original expression, which is written as

$$F_y = \sum_{h=1}^{H} \alpha_{2h-1}(\bar{w}_2)|_{L_j}^{2h-1}, \tag{4}$$

where $\alpha_{2h-1} (h = 1, 2, 3, \ldots, H)$ are fitting coefficients and $H$ reflects the fitting order with sufficient accuracy. Utilizing the Least Square Method (LSM) to find the values of coefficients $\alpha_{2h-1} (h = 1, 2, 3, \ldots, H)$, the error between the original function $F_{\text{mag},y}$ and fitting polynomial function $F_y$ is defined as

$$\text{ER} = \frac{\sum_{h=1}^{H} \left( F_{\text{mag},y} - F_y \right)|_{\bar{w}_2}^2}{\sum_{h=1}^{H} (F_{\text{mag},y}|_{\bar{w}_2})^2}. \tag{5}$$

For different structural parameters $d$, the magnetic interaction force $F_{\text{mag},y}$ for one pair of magnets, the comparison between the original function $F_{\text{mag},y}$ and fitting polynomial $F_y$, and the relative error ER for various fitting orders $H$ are shown in Figure 3.

Figure 3 shows the variation of one-pair-magnetic interaction force on magnet-2 in $y$ direction for different structural parameter and the accuracy of the simplification. From the results shown in Figures 3(a) and 3(b), the interaction force varies nonlinearly around the motion equilibrium, more obvious for smaller distance $d$. The gradient of the interaction force at equilibrium is negative, which demonstrates that the magnetic interaction can induce adjustable negative-stiffness property. For the increase of distance $d$ (weaker magnetic field), the gradient becomes negative first and then tends to zero. Thus, the interaction induced by magnetic coupling occurs locally around the equilibrium with adjustable negative-stiffness property and nonlinearity. From the comparison of $F_y$ and $F_{\text{mag},y}$ in Figures 3(b) and 3(c), the fitting polynomial function $F_y$ can describe the original force function $F_{\text{mag},y}$ with high accuracy for $H > 7$. Therefore, in the following analysis, to obtain the theoretical solution of the system and propose the design approach, the magnetic interaction force function can be expressed by high-order nonlinear function $F_y$.

3. Model, Analysis, and Structural Design Criterion

3.1. Vibration Model. Separating the primary and absorber beams and analysing the forces applied on them as shown in Figure 4, the dynamical model can be obtained.

As shown in Figure 4, for the primary beam, it is a cantilever beam which is fixed on the base; for the absorber beam, it is also a cantilever beam fixed on the free end of the primary beam, which is a vibration point rather than a fixed point. According to D’Alembert’s principle of a small segment from the primary beam, the dynamical equation of the primary beam is written as

$$V_1 + \frac{\partial V_1}{\partial \xi_1} \, d\xi_1 - V_1 - \left( \rho_1 A_1 \frac{\partial^2 w_1}{\partial t^2} + c_1 \frac{\partial w_1}{\partial t} \right) d\xi_1 = f(t) g(\xi_1), \tag{6a}$$

while, for the absorber beam, the dynamical equation of small segment is written as

$$V_2 + \frac{\partial V_2}{\partial \xi_2} \, d\xi_2 - V_2 - \left( \rho_2 A_2 \frac{\partial^2 w_2}{\partial t^2} + c_2 \frac{\partial w_2}{\partial t} \right) d\xi_2 = 0. \tag{6b}$$

The shear force and the bending moment on each section have the general relation as $V = (\partial M/\partial \xi) = (\partial/\partial \xi)(-EI(\partial^2 w/\partial \xi^2))$. Substituting this relation and equation (1) into the dynamical equation of absorber beam and rearranging the two equations, the model is written as
\[
\begin{align*}
\rho & A_1 \frac{\partial^2 w_1(\xi_1, t)}{\partial t^2} + E_1 I_1 \frac{\partial^4 w_1(\xi_1, t)}{\partial \xi_1^4} + c_1 \frac{\partial w_1(\xi_1, t)}{\partial t} = f(t) g(\xi_1), \\
\rho & A_2 \frac{\partial^2 \bar{w}_2(\xi_2, t)}{\partial t^2} + E_2 I_2 \frac{\partial^4 \bar{w}_2(\xi_2, t)}{\partial \xi_2^4} + c_2 \frac{\partial \bar{w}_2(\xi_2, t)}{\partial t} = -c_2 \left( \frac{d w_1(L_1, t)}{dt} + \frac{d \theta_1(L_1, t)}{dt} \right) - \rho A_2 \left[ \frac{d^2 w_1(L_1, t)}{dt^2} + \frac{d^2 \theta_1(L_1, t)}{dt^2} \right] \xi_2.
\end{align*}
\]

According to the forces analysis on the beams shown in Figure 4, at the free end of the primary beam, the interaction force of magnets \(F_y\) is applied on magnet-1; similarly, for the absorber beam, the interaction force of magnet \(F_y\) is applied on magnet-2 at its free end. At the connection section \(\xi_1 = L_1 (\xi_2 = 0)\), the forces \(V_1(L_1)\) and \(V_1'(0)\) are one pair of reaction forces, which are equal and in the opposite direction, as similarly as the bending moments \(M_1(L_1)\) and \(M_1'(0)\). Thus, we can obtain the boundary conditions at \(\xi_1 = 0\) and \(\xi_2 = L_2\), respectively, and the compatibility conditions of the connection section at \(\xi_1 = L_1 (\xi_2 = 0)\) as

\[
\begin{align*}
\end{align*}
\]
Since the primary beam and absorber beam are both cantilever beams, the mode shape functions $\phi_{1i}$ and $\phi_{2i}$ are set as

\[
\phi_{1i}(\xi_1) = B_{1i} \left\{ \cos \left( \frac{\lambda_1 \xi_1}{L_1} \right) - \cosh \left( \frac{\lambda_1 \xi_1}{L_1} \right) \right\} + r_{1i} \left\{ \sin \left( \frac{\lambda_1 \xi_1}{L_1} \right) - \sinh \left( \frac{\lambda_1 \xi_1}{L_1} \right) \right\},
\]

\[
\phi_{2j}(\xi_2) = B_{2j} \left\{ \cos \left( \frac{\lambda_2 \xi_2}{L_2} \right) - \cosh \left( \frac{\lambda_2 \xi_2}{L_2} \right) \right\} + r_{2j} \left\{ \sin \left( \frac{\lambda_2 \xi_2}{L_2} \right) - \sinh \left( \frac{\lambda_2 \xi_2}{L_2} \right) \right\},
\]

where $r_{1i} = -(\cos \lambda_i + \cos \lambda_j)/(\sin \lambda_i + \sin \lambda_j)$ and $r_{2j} = -(\cos \lambda_j + \cos \lambda_j)/(\sin \lambda_i + \sin \lambda_j)$. The functions $\phi_{1i}$ and $\phi_{2j}$ should satisfy the orthogonality and normalization principle, written as

\[
\int_{0}^{L_1} \rho_1 A_1 \phi_{1i}(\xi_1) \phi_{1k}(\xi_1) d\xi_1 = \delta_{ik},
\]

\[
\int_{0}^{L_2} \rho_2 A_2 \phi_{2j}(\xi_2) \phi_{2n}(\xi_2) d\xi_2 = \delta_{jn},
\]

\[
E_1 I_1 \int_{0}^{L_1} \phi_{1i}''(\xi_1) \phi_{1k}(\xi_1) d\xi_1 = \Omega_{1i k},
\]

\[
E_2 I_2 \int_{0}^{L_2} \phi_{2j}''(\xi_2) \phi_{2n}(\xi_2) d\xi_2 = \omega_{2n}^2.
\]
Truncating the vibration responses of primary system for $K^{th}$ order and absorber beam for $N^{th}$ order as

$$\omega_i(\xi_1, t) = \sum_{j=1}^{K} \phi_{ij}(\xi_1)q_{ji}(t),$$

and substituting equation (9) into the dynamical equation (7) and applying partial integration, the following equations can be obtained:

$$E_1 I_1 \omega''_1(\xi_1, t)\phi_{1k}(\xi_1)|_{\xi_1=0}^{L_1} - E_1 I_1 \omega''_1(\xi_1, t)\phi_{1k}(\xi_1)|_{\xi_1=L_1}^{L_1} + E_1 I_1 \sum_i \int_0^{L_1} \phi_{ij}''(\xi_1)\phi_{1k}(\xi_1)\,d\xi_1\,q_{ji}(t)$$

$$+ \rho_1 A_1 \sum_i \int_0^{L_1} \phi_{ij}(\xi_1)\phi_{1k}(\xi_1)\,d\xi_1\,q_{ji}(t) + c_1 \sum_i \int_0^{L_1} \phi_{ij}(\xi_1)\phi_{1k}(\xi_1)\,d\xi_1\,q_{ji}(t)$$

$$= \int_0^{L_1} f(t)\delta(\xi_1 - \xi_1)\phi_{1k}(\xi_1)\,d\xi_1,$$

$$E_2 I_2 \ddot{w}_2(\xi_2, t)\phi_{2n}(\xi_2)|_{\xi_2=0}^{L_2} - E_2 I_2 \ddot{w}_2(\xi_2, t)\phi_{2n}(\xi_2)|_{\xi_2=L_2}^{L_2} + E_2 I_2 \sum_j \int_0^{L_2} \phi_{2j}''(\xi_2)\phi_{2n}(\xi_2)\,d\xi_2\,q_{j2}(t)$$

$$+ \rho_2 A_2 \sum_j \int_0^{L_2} \phi_{2j}(\xi_2)\phi_{2n}(\xi_2)\,d\xi_2\,q_{j2}(t) + c_2 \sum_j \int_0^{L_2} \phi_{2j}(\xi_2)\phi_{2n}(\xi_2)\,d\xi_2\,q_{j2}(t)$$

$$= -c_2 \sum_j \int_0^{L_2} [\phi_{1j}(L_1) + \phi_{1j}(L_1)]\phi_{2n}(\xi_2)\,d\xi_2\,q_{j2}(t) - \rho_2 A_2 \sum_j \int_0^{L_2} [\phi_{1j}(L_1) + \phi_{1j}(L_1)]\phi_{2n}(\xi_2)\,d\xi_2\,q_{j2}(t).$$

In equation (12), by considering the boundary conditions and compatibility conditions as equation (8), the terms by definite integration are
After simplification, the discretization equation of the system can be written as

\[
\ddot{q}_{1k}(t) + \Omega_{1k}^2 q_{1k}(t) + C_{1k} \dot{q}_{1k}(t) = f(t) \phi_{1k}(l_1) - 2F_y \phi_{1k}(L_1) - 2m_{mag1} \left[ \sum_i \phi_{ii}(L_1) \dot{q}_{1i}(t) \right] \phi_{1k}(L_1) \\
- E_2 I_2 \left[ \sum_j \dot{\phi}_{2j}(0) q_{2j}(t) \right] \phi_{1k}(L_1) + E_2 I_2 \left[ \sum_j \dot{\phi}_{2j}(0) q_{2j}(t) \right] \phi_{1k}(L_1) \\
+ C_{2n} q_{2n}(t) + \omega_{2n}^2 q_{2n}(t) + C_{2n} \dot{q}_{2n}(t)
\]

where

\[
C_{1k} = c_1 \int_0^{L_1} \phi_{ii}(\xi_1) \phi_{1k}(\xi_1) d\xi_1 = c_1 \delta_{ik}, \quad C_{2n} = c_2 \int_0^{L_2} \phi_{2j}(\xi_2) \phi_{2n}(\xi_2) d\xi_2 = c_2 \delta_{jn}, \quad C_{3n} = c_2 \int_0^{L_2} \phi_{2n}(\xi_2) d\xi_2, \quad m_{cn} = \rho_2 A_2 \int_0^{L_2} \phi_{2n}(\xi_2) \xi_2 d\xi_2.
\]

According to equation (14) and the definitions of coefficients (15), for \( K^{th} \) order and \( N^{th} \) order truncation for the primary beam and absorber beam, respectively, the discretization equation is written as

\[
M \ddot{q}(t) + Kq(t) + C \dot{q}(t) = F(t) + G(q),
\]

where \( M \) is the mass matrix, \( K \) is the linear stiffness matrix, \( C \) is the linear damping matrix, \( F \) is the excitation vector, and \( G \) is the vector for nonlinear terms, expressed as follows. The mass matrix is

\[
M = \begin{bmatrix} M_{ik} & 0 \\ M_{ji} & M_{jn} \end{bmatrix},
\]

\[
m_{ik} = \begin{cases} 
1 + 2m_{mag1} \phi_{ii}(L_1) \phi_{1k}(L_1), & i = k, \\
2m_{mag1} \phi_{ii}(L_1) \phi_{1k}(L_1), & i \neq k,
\end{cases}
\]

\[
m_{ji} = \begin{cases} 
1 + 2m_{mag2} \phi_{2j}(L_2) \phi_{2n}(L_2), & j = n, \\
2m_{mag2} \phi_{2j}(L_2) \phi_{2n}(L_2), & j \neq n.
\end{cases}
\]

The expression of \( K \) is
4. Isolation Effectiveness

4.1. Resonance Frequency Bands. First, the natural frequencies of the system are obtained by the linearization model of equation (14) to show the resonance frequency bands after the assembly of absorber beam with magnetic interaction. According to the defined matrix $M$ and $K$ as (19) and (20), the natural frequencies for linearization can be solved by

$$\det\left(\mathbf{M}\bar{\Omega}_k^2 - \mathbf{K}\right) = 0,$$

where $\bar{\Omega}_k$ ($\beta = K + N$) are the linearization natural frequencies for the system. Then, the comparison of the natural frequencies $\bar{\Omega}_k$ of the system with/without magnetic interaction coupling is shown in Figure 5.

The nonlinearity terms vector $G$ is written as

$$G = \left[ g_{11}, g_{12}, \ldots, g_{1K}, g_{21}, g_{22}, \ldots, g_{2N} \right]^T,$$

where

$$g_{1k} = -2F_y \phi_{1k}(L_1) = -2 \sum_{l=1}^{L_2} \left[ \sum_{j=1}^{N} \phi_{2j}(L_2) q_j(t) \right]^{2h-1} \phi_{1k}(L_1),$$

$$g_{2n} = 2F_y \phi_{2n}(L_2) = 2 \sum_{l=1}^{L_2} \left[ \sum_{j=1}^{N} \phi_{2j}(L_2) q_j(t) \right]^{2h-1} \phi_{2n}(L_2).$$

According to the dynamical model (16), coefficients matrix, and nonlinearity terms in (17)–(20), the natural frequencies and responses can be obtained for different structural parameters under different excitations.
natural frequency increased to maximum. Therefore, the magnetic interaction further optimized the effective absorption band, allowing it to start from approximately zero. Without the absorber, frequency ranges within 22∼66 rad/s. With the absorber, the frequency range is widened to 0∼150 rad/s. This is the novelty of the structure design. Moreover, there is no need to install or uninstall the magnets while working. If the magnetic interaction is not needed, increasing the magnetic distance $d$ to a certain value can eliminate the influence of the magnets and allow the structure to turn back to its original state. The system can be easily optimized by adjusting $d$ value.

4.2. Solutions on Frequency Band. Based on the discretization equation (16), without considering the internal resonance, the vibration solutions of the primary beam $q_1(t)$ and the absorber beam $q_2(t)$ for different orders of modal are written as

\[
\begin{align*}
\Omega_1 &= 22.1 \\
\Omega_2 &= 66.46 \\
\Omega_3 &= 190.6 \\
\Omega_4 &= 169.4 \\
\Omega_5 &= 27.03 \\
\Omega_6 &= 100.0 \\
\Omega_7 &= 145.0 \\
\Omega_8 &= 190.6 \\
\Omega_9 &= 215.0 \\
\Omega_{10} &= 250.0 \\
\Omega_{11} &= 300.0
\end{align*}
\]

Table 2: Natural frequencies for different truncations $K$ and $N$ for $a_1 = -100$ ($d = 23$ mm).

| $\Omega_j$ (rad/s) | $K = 6$, $N = 4$ | $K = 6$, $N = 3$ | $K = 5$, $N = 4$ | $K = 5$, $N = 3$ | $K = 4$, $N = 4$ | $K = 4$, $N = 3$ |
|-------------------|----------------|----------------|----------------|----------------|----------------|----------------|
| $\Omega_1$        | 5.79           | 5.79           | 5.79           | 5.79           | 5.77           | 5.87           |
| $\Omega_2$        | 119.57         | 119.59         | 119.79         | 119.82         | 120.2          | 119.65         |
| $\Omega_3$        | 261.97         | 262.12         | 262.19         | 262.47         | 262.5          | 262.14         |
| $\Omega_4$        | 565.72         | 566.04         | 565.93         | 566.25         | 566.6          | 570.9          |
| $\Omega_5$        | 1110.3         | 1110.87        | 1113.35        | 1113.89        | 1121.47        | 1126.2         |
| $\Omega_6$        | 1691.28        | 1692.22        | 1698.46        | 1703.2         | 1719.28        | 1692.1         |
| $\Omega_7$        | 1904.95        | 1903.98        | 1911.77        | 1910.93        |                |                |
| $\Omega_8$        | 2862.24        | 2861.98        |                |                |                |                |
| $\Omega_9$        | 5533.08        | 5619.92        | 5540.21        | 5627.01        | 5547.1         | 5634.28        |
| $\Omega_{10}$     | 11718.2        | 11717.9        | 11717.6        |                |                |                |

Figure 5: (a) The natural frequencies for different structural parameters $d$; (b) the first three natural frequencies and the comparison cases with/without the absorber beam and with/without the magnetic interaction.
Substituting the solution in (22) into equation (16) and choosing the same order of harmonic components \(m\pi t\) and \(n\pi t\), we obtain \(2 \times m \times \beta = 2 \times m \times (K + N)\) algebraic equations for \(a_{1m}, b_{1m}, a_{2m}, b_{2m}\). In the following analysis of amplitude-frequency curves, single harmonic components for primary resonances are considered and the super/subharmonic components are ignored; then, there are \(2 \times \beta\) algebraic equations for solving the \(2 \times \beta\) amplitudes.

According to the relation between the vibration response \(w\) and vibration deflection \(\ddot{w}\) in equation (1), the vibration response \(w_2 (\xi_2, t)\) is written as

\[
q_{1t}(t) = \sum_m \left[ a_{1m} \cos(m \Omega t) + b_{1m} \sin(m \Omega t) \right],
\]

\[
q_{2t}(t) = \sum_m \left[ a_{2m} \cos(m \Omega t) + b_{2m} \sin(m \Omega t) \right].
\]

(22)

\[
w_2 (\xi_2, t) = \sum_{j=1}^{K} \phi_{1j}(L_1) q_{1j}(t) + \sum_{j=1}^{K} \phi_{2j}(L_2) q_{2j}(t) + \sum_{j=1}^{N} \phi_{2j}(L_2) q_{2j}(t) \tag{23}
\]

Thus, the maximum vibration response of the primary beam \(w_1 (L_1, t)\), relative motion \(\ddot{w}_2 (L_2, t)\), and the maximum vibration response of the absorber beam \(w_2 (L_1, t)\) are, respectively, written as

\[
w_1 (L_1, t) = \sqrt{\left( \sum_i \phi_{1i}(L_1) a_{1i1} \right)^2 + \left( \sum_i \phi_{1i}(L_1) b_{1i1} \right)^2},
\]

\[
= \sqrt{\phi_{11}(L_1) a_{11} + \phi_{12}(L_1) a_{12} + L^2 + \left( \phi_{11}(L_1) b_{11} + \phi_{12}(L_1) b_{12} + L \right)^2},
\]

(24)

\[
\ddot{w}_2 (L_2, t) = \sqrt{\left( \sum_j \phi_{2j}(L_2) a_{2j1} \right)^2 + \left( \sum_j \phi_{2j}(L_2) a_{2j2} \right)^2},
\]

\[
= \sqrt{\phi_{21}(L_2) a_{21} + \phi_{22}(L_2) a_{22} + L^2 + \left( \phi_{21}(L_2) b_{21} + \phi_{22}(L_2) b_{22} + L \right)^2},
\]

\[
w_2 (L_2, t) = \sqrt{\left( \sum_i \phi_{1i}(L_1) + \phi_{1i}'(L_1) \right) a_{1i1} + \sum_j \phi_{2j}(L_2) a_{2j1}}^2 + \left( \sum_i \phi_{1i}(L_1) + \phi_{1i}'(L_1) \right) b_{1i1} + \sum_j \phi_{2j}(L_2) b_{2j1}}^2.
\]

The vibration of the free end of the primary system is the point with largest amplitude, which is the response point to describe the vibration suppression effectiveness. Figure 6 shows the comparison of amplitudes of deflections of the primary beam and absorber beam on frequency band for different structural parameters of magnetic interaction.

From Figures 6(a)–6(e), it can be seen that not only are the resonance bands and nonresonance changed, but also the amplitudes dependent on the magnetic field are changed. For the increase of distance \(d\) (reduction of magnetic field), the first nonresonance frequency band increases and then reduces, which conforms to the results for the variations of natural frequencies in Figure 5. From the amplitude-frequency curves of the primary beam shown in Figure 6, the response at the first resonance frequency displays linearity, while nonlinear phenomenon occurs at higher-order resonance frequency, which is induced by the magnetic interaction. Compared to the case without magnetic interaction in Figure 6(f), the resonance peaks for the cases with magnetic interaction are much lower. In addition, for different values of \(d\), the system displays hard-spring or soft-spring properties at the high-order resonance frequencies. This phenomenon reveals that the vibration of the primary beam is effectively suppressed because of the nonlinearity coupling of magnetic interaction, especially at resonance frequency band.

4.3 Width of Frequency Band-Based Design and Evaluation Criteria. To design the structure and magnetic field, the evaluation criteria are proposed according to the effect and mechanism of vibration suppression of the primary system. Here, we define two criteria to evaluate the vibration suppression effect based on the width of frequency band for vibration suppression.

Firstly, the absorber beam is considered effective when the response amplitude of the primary beam is lower than the case for nonabsorber. Similarly, the magnetic interaction is considered useful when the response amplitude of the primary beam is lower than the case for nonmagnetic connection. Then, the width of frequency bands where the vibration amplitude of primary beam is smaller than the case
Figure 6: The amplitude-frequency curves when fixing the force amplitude $f_0 = 0.1$ N for different structural parameters of magnets as (a) $d = 5$ mm, (b) $d = 10$ mm, (c) $d = 20$ mm, (d) $d = 30$ mm, (e) $d = 40$ mm, and (f) the case without magnets.
without absorber beam or magnetic interaction is expressed as

\[ I_1 = \left\{ R_1 \mid \| w_1 \| \leq \| w_0 \| \right\}, \]  

(25)

where \( w_0 \) represents the vibration response of the primary beam for the case without absorber beam or magnetic interaction. The frequency bands obtained by the above criterion (25) show the band for effective vibration absorption.

Secondly, we only consider the requirement for the vibration amplitude of the primary beam rather than the comparison of the primary beam and absorber beam. Thus, we define the effective vibration suppression band when the vibration amplitude of the primary system is less than an acceptable value \( A_0 \) which is required in engineering practices. The width of frequency band for response with amplitude in the predefined value gives the guidance for the design of magnetic interaction. The width of frequency band is expressed as

\[ I_2 = \left\{ R_2 \mid \| w_1 \| \leq A_0 \right\}, \]  

(26)

where \( A_0 \) is fixed as \( A_0 = 0.5 \text{ mm} \) in the following case study.
and also has significant effect on the vibration suppression, especially for low-frequency resonance.

4.4. Discussions on the Superharmonic Resonance Induced by Nonlinearity. Since the nonlinearity induced by the magnetic interaction is obvious at the equilibrium, it would bring high-order harmonic components, which would not occur in linear system. Therefore, we consider the high-order harmonic components of responses as

\[
q_{i_1}(t) = a_{i_1} \cos(Ωt) + b_{i_1} \sin(Ωt) + a_{i_2} \cos(3Ωt) + b_{i_2} \sin(3Ωt), \\
q_{j_1}(t) = a_{j_1} \cos(Ωt) + b_{j_1} \sin(Ωt) + a_{j_2} \cos(3Ωt) + b_{j_2} \sin(3Ωt).
\]  

(27)

By substituting the solution into dynamical equation and separating harmonic components for \(\cos(Ωt), \sin(Ωt), \cos(3Ωt), \text{and} \sin(3Ωt),\) there are \(2 \times 2 \times (K + N)\) algebraic equations. The amplitudes of superharmonic components for different structural parameters and the comparison of amplitude-frequency curves between the cases considering superharmonic components are shown in Figure 9.

From Figure 9, the superharmonic components would occur due to the nonlinearity from magnetic interaction. Also, with the increase of excitation amplitude, the superharmonic components are growing up. But, with the comparison of the amplitudes between the primary- and superharmonic components, the values of superharmonic amplitudes are much smaller than the primary-harmonic ones. The primary-harmonic amplitudes for the case considering the superharmonic components have little error compared to the case without considering superharmonic components. Thus, the structural parameters of absorber beam and magnetic field can be designed according to the two proposed criteria (25) and (26) for vibration suppression requirements and the superharmonic vibrations can be ignored in practice.

4.5. Experimental Setup and Results. The corresponding experimental prototype of the magnetic interaction based vibration absorber for continuous beam is shown in Figure 10. The experimental setup is shown in Figure 10(a). The magnetic field is shown in Figure 10(b). The absorber beam with different strengths of magnetic coupling is shown in Figure 10(c). The structures are mounted on a vibration table (APS 400). A signal generator (RIGOL DG1022U) controls the structures. An oscilloscope (Tektronix TDS 2002C) measures and stores the output voltage signals. An absorber beam, which has much smaller mass than the primary beam, is introduced and coupled on the primary beam by constraint and magnetic interaction. Two pairs of magnets are assembled on the primary and absorber beams.

The experiment’s results are shown in Figure 11. The left and right parts of the figure show the vibration responses and output voltage for the main structure, respectively.
Figure 9: The amplitude curves of primary system considering the first-harmonic and second-harmonic components.

Figure 10: (a) The experimental setup; (b) the magnetic field; (c) the absorber beam with different strengths of magnetic coupling.
Figures 11(a) and 11(b) correspond to weak magnetic interactions, 11(c) and 11(d) correspond to middle magnetic interactions, and 11(e) and 11(f) correspond to strong magnetic interactions. The strength of magnetic interaction can be adjusted by changing the magnetic parameters in the connection region. The frequency-sweep tests with a linear sweep rate of 0.1 Hz/s in the frequency range between 1 Hz and 5 Hz are carried out with upward and downward sweeping.
Discussions:

In conclusion, for the proposed absorber beam with the magnetic coupling and assembly, remarkable vibration suppression can be obtained. The responses and design principle of the absorber beam with magnetic interaction are obtained based on the modeling of system and magnetic interaction. The proposed vibration suppression method for resonance vibration in wide frequency band has significant applications as flexible aim for continuous structure in the fields of manufacturing and aerospace.

Appendix

This appendix presents the structural parameters and coefficients in the theoretical analysis and simulation and natural frequencies for different truncation orders.

Assuming the primary beam and absorber beam are both acrylic materials, the structural parameters are fixed and shown in Table 1.

Nomenclature

\( u_1 \): Vibration of the primary system
\( \bar{w}_2 \): Deflection vibration of the absorber beam
\( f(t) \): Force excitation
\( \bar{\xi}_1 \): Coordinate for the primary beam
\( \Omega_h \): Natural frequencies of the system
\( \omega_{2m} \): Natural frequencies of absorber beam
\( \bar{w}_2 \): Displacement of the absorber beam
\( \theta_1 \): Angle of rotation of primary beam
\( f \): Force excitation amplitude
\( \bar{\xi}_2 \): Coordinate for absorber beam
\( \Omega_{1k} \): Natural frequencies of primary beam
\( \Omega \): Excitation frequency.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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