Abstract

We calculate the $\pi\Delta\Delta$ coupling $g_{\pi^0\Delta^{++}\Delta^{++}}$ using light cone QCD sum rule. Our result is $g_{\pi^0\Delta^{++}\Delta^{++}} = (11.8 \pm 2.0)$.

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The $\pi\Delta\Delta$ coupling constant $g_1$, like the nucleon axial charge $g_A$, is a basic parameter which enters the loop calculation in all processes involved with delta resonance in chiral perturbation theory. Unfortunately it is not directly accessible experimentally. A special quartet scheme of chiral symmetry realization for even- and odd-parity baryon resonances was proposed in \cite{2}. Based on such a scheme the authors found that the parity nonchanging couplings such as $\pi\Delta^\pm N^*\mp$, $\pi\Delta^\pm\Delta^\pm$, and $\pi N^* N^*$ are forbidden at the leading order \cite{2}. Such a result is very different from quark model prediction \cite{3} and large $N_c$ argument $g_1 = \frac{9}{5}g_A$ \cite{5}.

Recently an attempt was made to extract this important coupling from the fit to the phase shift data of pion-nucleon scattering in the fourth order chiral perturbation theory analysis \cite{4}. Because this coupling only appears in the third order loop contribution, it’s very hard to pin down the value precisely. However the preliminary result was $g_1 = -0.94 \sim -2.65$ \cite{4}. These value for $g_1$ comes out very differently from the large $N_c$ prediction as noted in \cite{4}. So an independent theoretical extraction may prove useful to help clarify the present ambiguous situation concerning this coupling.

We have calculated $\pi NN$ and $\pi NN^*$ \cite{6}, $\eta NN$ \cite{7} and $\rho NN$, $\omega NN$ \cite{8} coupling constants in the framework of light cone QCD sum rule (LCQSR). The extracted values of various couplings from LCQSR are in good agreement with those used in or obtained from phenomenological analysis. In this short note we extend the same formalism to calculate the $\pi\Delta\Delta$ coupling constant.

Let’s first introduce some notations. For the $\Delta$ resonance, we use the isospurion formalism, treating the $\Delta$ field $T^i_{\mu}(x)$ as a vector spinor in both spin and isospin space with the constraint $\tau^i T^i_{\mu}(x) = 0$ \cite{1}. The components of this field are

\begin{equation}
T^3_{\mu} = -\sqrt{\frac{2}{3}} \left( \frac{\Delta^+}{\Delta^0} \right)_{\mu}, \quad T^+_{\mu} = \left( \frac{\Delta^{++}}{\Delta^+/\sqrt{3}} \right)_{\mu}, \quad T^-_{\mu} = -\left( \frac{\Delta^0/\sqrt{3}}{\Delta^-} \right)_{\mu}.
\end{equation}

The field $T^i_{\mu}$ also satisfies the constraints for the ordinary Schwinger-Rarita spin-$\frac{3}{2}$ field,

\begin{equation}
\gamma^\mu T^i_{\mu} = 0 \quad \text{and} \quad p^\mu T^i_{\mu} = 0.
\end{equation}

To be specific, for the $\pi\Delta\Delta$ and $\pi NN$ interaction we use pseudoscalalar form:

\begin{equation}
L_{PS} = g_{\pi\Delta\Delta} T^i_{\mu} i\gamma^5 \vec{T} \cdot \vec{\pi} T^i_{\mu} + g_{\pi NN} \tilde{N} i\gamma^5 \vec{T} \cdot \vec{\pi} N + \cdots
\end{equation}

The pseudoscalar coupling $g_{\pi\Delta\Delta}$ can be realated to the pseudovector one $g_1$ via the relation:

\begin{equation}
g_1 = \frac{F_\pi g_{\pi\Delta\Delta}}{m_\Delta},
\end{equation}

where $F_\pi = 92.4$ MeV is the pion decay constant.

Since the light cone QCD sum rule \cite{9} has proven useful in extracting strong coupling constants, we use it to calculate $g_{\pi\Delta\Delta}$ and consider the correlator:

\begin{equation}
i \int dx e^{ipx} < 0|T_{\mu}(x), \bar{\eta}_\nu(0)|\pi^0(q)>.
\end{equation}
with the interpolating current for $\Delta^{++}$, $\eta_{\mu}(x) = \epsilon^{abc}[u^a T \Gamma_{\mu} u^b] \sigma^{c}(x)$. We also introduce $\langle 0|\eta_{\mu}(0)|\Delta^{++}\rangle = \lambda_{\Delta^{++}}$ with $\nu_{\mu}$ a vectorial spinor for the spin $\frac{3}{2}$ delta field and $\lambda_{\Delta}$ is the overlapping amplitude. At the phenomenological side we consider the Lorentz structure $ig_{\mu\nu}(\gamma_{\nu})$ which admits contribution from resonances with $I = 3/2, J = 3/2$ only.

The calculation is routine. We first make operator product expansion and express Eq. (5) with pion light cone wave functions [9]. After finishing Fourier transformation we make double Borel transformation twice to extract the double spectral density $\rho(s_1, s_2)$. Finally we subtract the continuum contribution to Eq. (5). The present sum rule is symmetric with $s_1, s_2$ which enables a clean subtraction of the continuum. Similar and detailed calculation steps can be found in [6]. We present final sum rule directly.

\[
m_{\Delta}^{2} g_{\pi^{a} \Delta^{++} \Delta^{++}} e^{-\frac{m_{\Delta}^{2}}{\bar{M}^{2}}} = \frac{e_{\pi}}{2} \left\{ \frac{M^{6} f_{2}(\frac{s_{\Delta}}{\bar{M}^{2}})}{48} \right\} - \frac{3}{2} \left[ g_{1}(\frac{1}{2}) + G_{2}(\frac{1}{2}) \right] M^{4} f_{1}(\frac{s_{\Delta}}{\bar{M}^{2}}) + \frac{1}{18} a_{\mu} \sigma_{\mu} \phi_{\sigma}(\frac{1}{2}) M^{2} f_{0}(\frac{s_{\Delta}}{\bar{M}^{2}}) \right\} + \cdots
\]

where $M^2$ is the Borel parameter, $a = -4\pi^2(qq) = 0.55$ GeV$^3$ is the quark condensate, $\mu_\pi = 1.65$ GeV at the scale of 1 GeV, $s_\Delta$ is the continuum threshold of delta mass sum rule, $\phi_\pi(\frac{1}{2})$ etc are the values of various pion wave functions at the point $\frac{1}{2}$. Their values are $\phi_\pi(\frac{1}{2}) = 1.5 \pm 0.2, g_1(\frac{1}{2}) + G_2(\frac{1}{2}) = 0.042, \phi_\sigma(\frac{1}{2}) = 1.47$ at the scale 1 GeV [3]. Functions $f_n(x) = 1 - e^{-x} \sum_{k=0}^{n} \frac{x^k}{k!}$ are used to subtract the continuum contribution. We need delta mass sum rule [10]:

\[
(2\pi)^{4} \phi_{N}^{2} e^{-\frac{m_{\pi}^{2}}{M^{2}}} = \frac{1}{5} M^{6} f_{2}(\frac{s_{N}}{M^{2}}) - \frac{5}{72} b M^{2} f_{0}(\frac{s_{N}}{M^{2}}) + \frac{4}{3} a^{2} = \frac{7}{9} a^{2} M^{2}
\]

where $b = \langle g_{G}^{2} G^{2} \rangle = 0.48$ GeV$^4$ is the gluon condensate, $m_{0}^{2} = \langle \frac{g_{\pi} \sigma \cdot G_{0}}{q q} \rangle = 0.8$ GeV$^2$.

For comparison we collect the sum rule for $g_{\pi^{a} PP}$ and nucleon mass sum rule in literature below.

\[
m_{N}^{2} \phi_{N}^{2} e^{-\frac{m_{N}^{2}}{M^{2}}} = \frac{e_{\pi}}{2} \left\{ \frac{M^{6} f_{2}(\frac{s_{N}}{M^{2}})}{48} \right\} - \frac{3}{2} \left[ g_{1}(\frac{1}{2}) + G_{2}(\frac{1}{2}) \right] M^{4} f_{1}(\frac{s_{N}}{M^{2}}) + \frac{1}{18} a_{\mu} \sigma_{\mu} \phi_{\sigma}(\frac{1}{2}) M^{2} f_{0}(\frac{s_{N}}{M^{2}}) \right\} + \cdots
\]

\[
(2\pi)^{4} \phi_{N}^{2} e^{-\frac{m_{N}^{2}}{M^{2}}} = \frac{1}{2} M^{6} f_{2}(\frac{s_{N}}{M^{2}}) + \frac{1}{8} b M^{2} f_{0}(\frac{s_{N}}{M^{2}}) + \frac{2}{3} a^{2} = \frac{1}{6} a^{2} M^{2}
\]
Numerically we get

\[ g_{\pi^0\Delta^{++}\Delta^{++}} = (11.8 \pm 2.0) \]
\[ g_{\pi^0pp} = (13.2 \pm 1.5) \] (10)

The central value corresponds to \( M^2 = 1.4 \text{ GeV}^2 \), \( s_N = 2.25 \text{ GeV}^2 \), \( s_\Delta = 3.5 \text{ GeV}^2 \). The errors arise from the variation with the continuum threshold and Borel parameter in the working region of the sum rules only. Our calculation shows that the \( \pi\Delta\Delta \) coupling is large although it is only half of the quark model [3], SU(6) [11], U(12) [12] and especially, large \( N_c \) prediction [3]: \( g_{\pi^0\Delta^{++}\Delta^{++}} = \frac{9}{\pi} g_{\pi^0pp} \). It’s interesting to note that our result is consistent with the phenomenological value extracted from an old isobar production experiment in \( \pi^- p \to \pi^+ \pi^- n \) near threshold [13].
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Figure Captions

Fig 1. The variation of $g_{\pi N}^{\Delta++\Delta++} [g_{\pi P P}]$ with the Borel parameter $M^2$ and the continuum threshold $s_\Delta [s_N]$. The upper and lower three curves are for $g_{\pi P P}$ and $g_{\pi N}^{\Delta++\Delta++}$ respectively. From top to bottom $s_N = 2.35, 2.25, 2.15 \text{ GeV}^2$ and $s_\Delta = 3.6, 3.5, 3.4 \text{ GeV}^2$ respectively.
\[ \ell_{\pi}^0 \Delta^{++} \Delta^{++} \]

\[ g_{\pi}^0 \Delta^{++} \Delta^{++} \]

\[ s_N = 2.35 \text{ GeV}^2 \]
\[ s_N = 2.25 \text{ GeV}^2 \]
\[ s_N = 2.15 \text{ GeV}^2 \]
\[ s_\Delta = 3.4 \text{ GeV}^2 \]
\[ s_\Delta = 3.5 \text{ GeV}^2 \]
\[ s_\Delta = 3.6 \text{ GeV}^2 \]