Electroweak Constraints on Effective Theories

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Abstract. We discuss electroweak constraints on TeV scale extensions of the standard model. To obtain model-independent results, effective theory approach is adopted. Constraints are given on arbitrary linear combinations of a set of dimension-6 operators that respect the SM gauge symmetry, as well as CP, lepton and baryon number conservation. Applications of the results are also discussed.

Keywords: electroweak interaction, effective Lagrangian, little Higgs model

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INTRODUCTION

Assuming insignificant fine-tuning to the Higgs mass, we expect extensions of the standard model (SM) to appear at the TeV scale. Many TeV scale models have been extensively studied, such as supersymmetry, technicolor, extra-dimension and little Higgs models. On the other hand, the results from electroweak precision tests (EWPTs) remarkably agree with the SM predictions, and therefore tightly constrain many of the TeV scale models.

To efficiently obtain electroweak constraints, an effective theory approach [1, 2, 3, 4] is desirable. In this approach, one first integrates out all new heavy states and obtains effective operators involving only fields of the SM. From these operators, one calculates deviations from the SM and compares them with experimental data. Electroweak constraints are then obtained on the operator coefficients. Once this step is done, one can constrain any model just by calculating the coefficients of new effective operators. Here we present a study on bounds on arbitrary linear combinations of dimension-6 operators that could be relevant to TeV scale physics.

OPERATORS

We assume that just above the electroweak symmetry breaking scale, the effective theory is that of the SM with one Higgs doublet. A complete set of 80 independent dimension-6 operators consistent with the $SU(3) \times SU(2)_L \times U(1)_Y$ gauge symmetry, baryon and lepton number conservation has been presented in Ref. [1].

We are interested in constraining models of new physics pertinent to the electroweak symmetry breaking. Processes that contribute to flavor or CP violation have to be suppressed by scales much higher than the electroweak scale. Thus we impose CP conservation and $U(3)^5$ flavor symmetry on our operators. A different $U(3)$ acts on the left-handed quarks and leptons as well as on the right-handed quarks and leptons. In Ref. [4], the $U(3)^5$ symmetry is relaxed to $[U(2) \times U(1)]^5$, where the third generation is
treated differently from the first two.

We also exclude operators that are not or poorly constrained by current experiments. The remaining 21 operators are the focus of our work and listed below. The notation is standard: \( q \) and \( l \) represent the three families of the left-handed quark and lepton fields, respectively. The right handed fields are labeled \( u, d, \) and \( e \). We omit the family index which is always summed over due to the flavor \( U(3)^{3} \) symmetry.

The operators that contain only the gauge bosons and Higgs doublets are

\[
O_{WB} = (h^\dagger \sigma^a h) W^a_{\mu \nu} B^{\mu \nu}, \quad O_h = |h^\dagger D_{\mu} h|^2, \tag{1}
\]

where \( W^a_{\mu \nu} \) is the \( SU(2) \) field strength, \( B_{\mu \nu} \) the hypercharge field strength, and \( h \) represents the Higgs doublet. There are 11 four-fermion operators. These are

\[
O'_{li} = \frac{1}{2} (\overline{l} \gamma^\mu l) (\overline{\nu} \gamma^\nu \nu), \quad O'_{ll} = \frac{1}{2} (\overline{l} \gamma^\mu \sigma^a l) (\overline{\nu} \gamma^\nu \sigma^a \nu), \tag{2}
\]

\[
O'_{lq} = (\overline{l} \gamma^\mu l) (\overline{\nu} \gamma^\nu q), \quad O'_{lq} = (\overline{l} \gamma^\mu \sigma^a l) (\overline{\nu} \gamma^\nu \sigma^a q), \tag{3}
\]

\[
O_{lu} = (\overline{l} \gamma^\mu l) (\overline{u} \gamma^\nu u), \quad O_{ld} = (\overline{l} \gamma^\mu l) (\overline{d} \gamma^\nu d), \tag{4}
\]

\[
O_{ve} = \frac{1}{2} (\overline{e} \gamma^\mu l) (\overline{e} \gamma^\nu e), \quad O_{eu} = (\overline{e} \gamma^\mu l) (\overline{u} \gamma^\nu u), \quad O_{ed} = (\overline{e} \gamma^\mu l) (\overline{d} \gamma^\nu d). \tag{5}
\]

There are 7 operators containing 2 fermions that alter the couplings of fermions to the gauge bosons

\[
O'_{hl} = i (h^\dagger D_{\mu} h) (\overline{l} \gamma_{\mu} l) + \text{h.c.}, \quad O'_{hl} = i (h^\dagger \sigma^a D_{\mu} h) (\overline{l} \gamma_{\mu} \sigma^a l) + \text{h.c.}, \tag{6}
\]

\[
O'_{hq} = i (h^\dagger D_{\mu} h) (\overline{q} \gamma_{\mu} q) + \text{h.c.}, \quad O'_{hq} = i (h^\dagger \sigma^a D_{\mu} h) (\overline{q} \gamma_{\mu} \sigma^a q) + \text{h.c.}, \tag{7}
\]

\[
O_{hu} = i (h^\dagger D_{\mu} h) (\overline{u} \gamma_{\mu} u) + \text{h.c.}, \quad O_{hd} = i (h^\dagger D_{\mu} h) (\overline{d} \gamma_{\mu} d) + \text{h.c.}, \tag{8}
\]

\[
O_{he} = i (h^\dagger D_{\mu} h) (\overline{e} \gamma_{\mu} e) + \text{h.c.}. \tag{9}
\]

Finally, there is an operator that modifies the triple gauge boson interactions

\[
O_W = \varepsilon^{abc} W^a_{\mu} W^b_{\nu} W^c_{\lambda}. \tag{10}
\]

Eqs. (1) through (10) define our basis of the 21 operators. Adding these operators to the SM, we have the effective Lagrangian as

\[
\mathcal{L} = \mathcal{L}_{SM} + \sum_i a_i O_i \tag{11}
\]

\[
= \mathcal{L}_{SM} + a_{WB} O_{WB} + a_h O_h + \ldots + a_W O_W, \tag{12}
\]

where we have denoted the coefficients \( a_i \) using the same indices as the corresponding operators.
CONSTRAINTS

We include in our analysis all relevant electroweak precision observables (EWPOs). The three most precisely measured ones, $\alpha$, $G_F$, and $M_Z$ are taken to be the input parameters, from which the SM gauge couplings and the Higgs vev are inferred. The other observables include the $W$ boson mass, observables from atomic parity violation, deep inelastic scattering and $Z$-pole experiments, and fermion and $W$ boson pair production data from LEP 2.

Starting from the Lagrangian (12), we calculate deviations from the SM as functions of the coefficients $a_i$. Due to the excellent agreement between the experiments and the SM, we only need to work to linear order in $a_i$. For a given observable $X$, we have

$$X_{th} = X_{SM} + \sum_i a_i X_i,$$

where $X_{th}$ is the prediction in the presence of additional operators, $X_{SM}$ is the SM prediction, which is well known, and $\sum_i a_i X_i$ are corrections from our new operators.

We then compare the theoretical predictions with the experimental values $X_{exp}$, and calculate the total $\chi^2$ distribution. For non-correlated measurements,

$$\chi^2(a_i) = \sum_X \frac{(X_{th}(a_i) - X_{exp})^2}{\sigma^2_X},$$

where $X_{exp}$ is the experimental value for observable $X$ and $\sigma_X$ is the total error both experimental and theoretical.

Since that each $X_{th}$ is linear in $a_i$, the total $\chi^2$ is quadratic:

$$\chi^2 = \chi^2_{SM} + a_i \hat{v}_i + a_i \mathcal{M}_{ij} a_j.$$

The numerical values for the matrix $\mathcal{M}_{ij}$ and the vector $\hat{v}_i$ are our main results. It can be obtained from the author. Given the $\chi^2$, what is remaining to be done to constrain a model is only calculating the operator coefficients $a_i$.

APPLICATIONS

Our analysis is a systematic generalization of other model-independent analyses of electroweak constraints. For example, the operator $O_{WB}$ and $O_h$ correspond to the oblique $S$ and $T$ parameters [5]. It is straightforward to reproduce the $S$ and $T$ fit in Ref. [6] using the 2 by 2 submatrix of $\mathcal{M}$ and the first two components of the vector $\hat{v}_i$.

Our results can be easily used to constrain many models with also non-oblique corrections. As an illustration, we apply the results to the littlest Higgs model [7]. More examples can be found in Ref. [3, 4, 8].

In the littlest Higgs model, there exist new heavy gauge bosons, quarks and scalars. After integrating out the new particles, we obtain the following operator coefficients:

$$a_h = \frac{-5(c'^2 - s'^2)^2}{2F^2} + \frac{2\lambda^2}{M_\phi^4},$$
\[ a_{hq}^d = a_{hl}^d = \frac{-(c^2 - s^2)c^2}{2F^2}, \]
\[ a_{hf}^d = \frac{5s'c'(c^2 - s^2)}{F^2} \left( Y_2^{f'} s' - Y_1^{f'} s' \right), \]
\[ a_{lq}^d = a_{ll}^d = -\frac{c^4}{F^2}, \]
\[ a_{ff'}^s = \frac{-20s'^2c'^2}{F^2} \left( Y_2^{f'} c' - Y_1^{f'} c' \right) \left( Y_2^{f'} s' - Y_1^{f'} s' \right). \]  

(17)

The details of the littlest Higgs model and how to obtain Eqs. (17) are described elsewhere [7, 8]. We emphasize here that one can obtain bounds on this model by simply substituting in Eq. (16) the above coefficients, without detailed knowledge of EWPTs. It turns out that there exist significant parameter space where the bounds on the heavy particles’ masses are mild enough to solve the fine-tuning problem, as long as the fermions are charged equally under the two $U(1)$ gauge groups in the model.

**CONCLUSION**

We have identified a set of dimension-6 operators that are relevant to TeV scale extensions of the SM and can be tightly constrained by EWPTs. We have obtained constraints on arbitrary linear combinations of the operators, using all precisely measured EWPOs. The results can be easily applied to constrain many TeV scale models, especially those with heavy particles contributing to EWPOs at tree level.

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