Grand Unification and Physics Beyond the Standard Model

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Abstract

Recent progress in some selected areas of grand unification and physics beyond the standard model is reviewed. Topics include gauge coupling unification, $SU(5)$, $SO(10)$, symmetry breaking mechanisms, finite field theory: $SU(3)^3$, leptonic color: $SU(3)^4$, chiral color and quark-lepton nonuniversality: $SU(3)^6$.

**Talk at V-SILAFAE, Lima, Peru (July 2004).**
1 Introduction

Up to the energy scale of $10^2$ GeV, we are confident that the fundamental gauge symmetry of particle physics is that of the Standard Model (SM), i.e. $SU(3)_C \times SU(2)_L \times U(1)_Y$. New physics may appear just above this scale, but there may also be a much higher energy scale where the three gauge groups of the SM become unified into some larger symmetry. This is the notion of grand unification and depends crucially on the values of the three observed gauge couplings at the electroweak scale, as well as the particle content of the assumed theory from that scale to the unification scale.

2 Gauge Coupling Unification

The basic tool for exploring the possibility of grand unification is the renormalization-group evolution of the gauge couplings as a function of energy scale, given in one loop by

$$\alpha_i^{-1}(M_Z) = \alpha_i^{-1}(M_U) + \left(\frac{b_i}{2\pi}\right) \ln\left(\frac{M_U}{M_Z}\right),$$

with the experimentally determined values $\alpha_3(M_Z) = 0.1183(26)$, $\sin^2\theta_W(M_Z) = 0.23136(16)$, $\alpha^{-1}(M_Z) = 127.931(42)$, where $\alpha_2^{-1} = \alpha^{-1}\sin^2\theta_W$, and $\alpha_1^{-1} = (3/5)\alpha^{-1}\cos^2\theta_W$ (assuming $\sin^2\theta_W(M_U) = 3/8$). The coefficients $b_i$ are obtained from the assumed particle content of the theory between $M_Z$ and $M_U$. It is well-known that the three gauge couplings do not meet if only the particles of the SM are included. However, if the SM is extended to include supersymmetry (MSSM) thereby increasing the particle content, they do meet at around $10^{16}$ GeV.

A recent detailed analysis[1] using the more accurate two-loop analogs of Eq. (1) shows that the MSSM does allow the unification of gauge couplings but there remains a possible discrepancy, depending on the choice of inputs at the electroweak scale. In fact, this small
discrepancy is taken seriously by proponents of specific models of grand unification, and has been the subject of debate in the past two years or so.

3 \textit{SU}(5)

Consider the particle content of the MSSM. There are three copies of quark and lepton superfields:

\[
(u, d) \sim (3, 2, 1/6), \quad u^c \sim (3^*, 1, -2/3), \quad d^c \sim (3^*, 1, 1/3), \\
(v, e) \sim (1, 2, -1/2), \quad e^c \sim (1, 1, 1),
\]

and one copy of the two Higgs superfields:

\[
(\phi^0_1, \phi^-_1) \sim (1, 2, -1/2), \quad (\phi^+_2, \phi^0_2) \sim (1, 2, 1/2).
\]

The quarks and leptons can be embedded into \textit{SU}(5) as follows:

\[
5^* = (3^*, 1, 1/3) + (1, 2, -1/2), \\
10 = (3, 2, 1/6) + (3^*, 1, -2/3) + (1, 1, 1),
\]

but the Higgs superfields do not form complete multiplets: \(\Phi_1 \subset 5^*, \Phi_2 \subset 5\). Their missing partners are \((3^*, 1, 1/3), (3, 1, -1/3)\) respectively and they mediate proton decay. In the MSSM, such effective operators are dimension-five, i.e. they are suppressed by only one power of \(M_U\) in the denominator and can easily contribute to a proton decay lifetime below the experimental lower bound.

Recalling that there is a small discrepancy in the unification of gauge couplings. This can be fixed by threshold corrections due to heavy particles at \(M_U\). Using these heavy color triplet Higgs superfields to obtain exact unification, it was shown that \(^2\) their masses must lie in the range \(3.5 \times 10^{14}\) to \(3.6 \times 10^{15}\) GeV. However, the experimental lower bound on the
decay lifetime of $p \to K^+\bar{\nu}$ is $6.7 \times 10^{32}$ years, which requires this mass to be greater than $7.6 \times 10^{16}$ GeV. This contradiction is then used to rule out minimal $SU(5)$ as a candidate model of grand unification.

The above analysis assumes that the sparticle mass matrices are related to the particle mass matrices in a simple natural way. However, proton decay in the MSSM through the above-mentioned dimension-five operators depends on how sparticles turn into particles. It has been pointed out\textsuperscript{[3]} that if the most general sparticle mass matrices are used, these operators may be sufficiently suppressed to avoid any contradiction with proton decay.

Instead of adjusting the color triplet masses to obtain exact unification, a new and popular way is to invoke extra space dimensions. For example, in a five-dimensional theory, if Higgs fields exist in the bulk, then there can be finite threshold corrections from summing over Kaluza-Klein modes. A specific successful $SU(5)$ model\textsuperscript{[4]} was proposed using the Kawamura mechanism of symmetry breaking by boundary conditions.

4 $SO(10)$

The power of $SO(10)$ is historically well-known. A single spinor representation, i.e. 16, contains the 5* and 10 of $SU(5)$ as well as a singlet $N$, which may be identified as the right-handed neutrino. The existence of three heavy singlets allows the three known neutrinos to acquire naturally small Majorana masses through the famous seesaw mechanism, and the decay of the lightest of them may also generate a lepton asymmetry in the early Universe which gets converted by sphalerons during the electroweak phase transition to the present observed baryon asymmetry of the Universe.

What is new in the past two years is the realization of the importance of the electroweak Higgs triplet contained in the 126 of $SO(10)$. Whereas the Higgs triplet under $SU(2)_R$
provides \( N \) with a heavy Majorana mass, the Higgs triplet under \( SU(2)_L \) provides \( \nu \) with a small Majorana mass. This latter mechanism is also seesaw in character and may in fact be the dominant contribution to the observed neutrino mass. For a more complete discussion of this and other important recent developments in \( SO(10) \), see the talk by Alejandra Melfo in these proceedings.

5 Symmetry Breaking Mechanisms

The breaking of a gauge symmetry through the nonzero vacuum expectation value of a scalar field is the canonical method to obtain a renormalizable field theory. If fermions have interactions which allow them to pair up to form a condensate with \( \langle \bar{f}f \rangle \neq 0 \), then the symmetry is also broken, but now dynamically. With extra dimensions, a recent discovery is that it is possible in some cases for a theory without Higgs fields (in the bulk or on our brane) to be recast into one with dynamical symmetry breaking on our brane. It is of course known for a long time that the components of gauge fields in extra dimensions may also be integrated over the nontrivial compactified manifold so that

\[
\int A_i dx^i \neq 0, \tag{7}
\]

thereby breaking the gauge symmetry. More recently, bulk scalar field boundary conditions in a compact fifth dimension, using \( S^1/Z_2 \times Z'_2 \) for example, have become the mechanism of choice for breaking \( SU(5) \) and other grand unified groups to the MSSM. This method can also be applied to breaking supersymmetry itself.

6 Finite Field Theory: \( SU(3)^3 \)

If \( \beta_i = 0 \) and \( \gamma_i = 0 \) in an \( N = 1 \) supersymmetric field theory, then it is also finite to all orders in perturbation theory if an isolated solution exists for the unique reduction of
all couplings.\[9\] This is an attractive possibility for a grand unified theory between the unification scale and the Planck scale. The conditions for finiteness are then boundary conditions on all the couplings of the theory at the unification scale where the symmetry is broken, and the renormalization-group running of these couplings down to the electroweak scale will make predictions which can be compared to experimental data. In particular, the mass of the top quark and that of the Higgs boson may be derived. Successful examples using $SU(5)$ already exist.\[10, 11\] Recently, an $SU(3)^3$ example has also been obtained.\[12\]

Consider the product group $SU(N)_1 \times \ldots \times SU(N)_k$ with $n_f$ copies of matter superfields $(N, N^*, 1, \ldots, 1) + \ldots + (N^*, 1, 1, \ldots, N)$ in a “moose” chain. Assume $Z_k$ cyclic symmetry on this chain, then

$$b = \left( -\frac{11}{3} + \frac{2}{3} \right) N + n_f \left( \frac{2}{3} + \frac{1}{3} \right) \left( \frac{1}{2} \right) N = -3N + n_f N.$$  

(8)

Therefore, $b = 0$ if $n_f = 3$ independent of $N$ and $k$.

Choose $N = 3, k = 3$, then we have the trinification model,\[13\] i.e. $SU(3)^3$ which is the maximal subgroup of $E_6$. The quarks and leptons are given by $q \sim (3, 3^*, 1), q^c \sim (3^*, 1, 3)$, and $\lambda \sim (1, 3, 3^*)$, denoted in matrix notation respectively as

\[
\begin{pmatrix}
d & u & h \\
d & u & h \\
d & u & h 
\end{pmatrix},
\quad
\begin{pmatrix}
d^c & u^c & h^c \\
d^c & u^c & h^c \\
d^c & u^c & h^c 
\end{pmatrix},
\quad
\begin{pmatrix}
N & E^c & \nu \\
E & N^c & e \\
\nu^c & e^c & S
\end{pmatrix}.
\]

(9)

With three families, there are 11 invariant $f$ couplings of the form $\lambda q^c q$ and 10 invariant $f'$ couplings of the form $\det q + \det q^c + \det \lambda$. An isolated solution of $\gamma_i = 0$ is

$$f^2_{iii} = \frac{16}{9} g^2,$$

(10)

and all other couplings = 0. Assuming that $SU(3)^3$ breaks down to the MSSM at $M_U$, this predicts $m_t \sim 183$ GeV, in good agreement with the present experimental value of $178.0 \pm 2.7 \pm 3.3$ GeV.
7 Leptonic Color: $SU(3)^4$

Because of the empirical evidence of gauge coupling unification, almost all models of grand unification have the same low-energy particle content of the MSSM, including all models discussed so far. However, this does not rule out the possibility of new physics (beyond the MSSM) at the TeV energy scale, without spoiling unification. I discuss two recent examples. The first\(^{14}\) is nonsupersymmetric $SU(3)^4$ and the second\(^{15}\) is supersymmetric $SU(3)^6$.

In trinification, quarks and leptons are assigned asymmetrically in Eq. (9). To restore complete quark-lepton interchangeability at high energy, an $SU(3)_l$ model of quartification\(^{14}\) has been proposed. The idea is to add leptonic color\(^{16}\) $SU(3)_l$ which breaks down to $SU(2)_l \times U(1)_{Yl}$, with the charge operator given by

\[
Q = I_{3L} + I_{3R} - \frac{1}{2} Y_L - \frac{1}{2} Y_R - \frac{1}{2} Y_l. \tag{11}
\]

The leptons are now $(3, 3^*)$ under $SU(3)_L \times SU(3)_l$ and $(3, 3^*)$ under $SU(3)_l \times SU(3)_R$, i.e.

\[
l \sim \begin{pmatrix}
x_1 & x_2 & \nu \\
y_1 & y_2 & e \\
z_1 & z_2 & N
\end{pmatrix}, \quad l^c \sim \begin{pmatrix}
x_1^c & y_1^c & z_1^c \\
x_2^c & y_2^c & z_2^c \\
\nu^c & e^c & N^c
\end{pmatrix}. \tag{12}
\]

The exotic particles $x, y, z$ and $x^c, y^c, z^c$ have half-integral charges: $Q_x = Q_z = Q_{y^c} = 1/2$ and $Q_{x^c} = Q_{z^c} = Q_y = -1/2$, hence they are called “hemions”. They are confined by the $SU(2)_l$ “stickons” to form integrally charged particles, just as the fractionally charged quarks are confined by the $SU(3)_q$ gluons to form integrally charged hadrons.

The particle content of $SU(3)^4$ immediately tells us that if unification occurs, then

\[
\sin^2 \theta_W(M_U) = \frac{\sum I_{3L}^2}{\sum Q^2} = 1/3
\]

instead of the canonical $3/8$ in $SU(5)$, $SU(3)^3$, etc. This means that it cannot be that of the MSSM at low energy. Instead the SM is extended to include 3 copies of hemions at the TeV scale:

\[
(x, y) \sim (1, 2, 0, 2), \quad x^c \sim (1, 1, -1/2, 2), \quad y^c \sim (1, 1, 1/2, 2), \tag{13}
\]
under $SU(3)_C \times SU(2)_L \times U(1)_Y \times SU(2)_I$, without supersymmetry. In that case, it was shown\cite{14} that the gauge couplings do meet, but at a much lower unification scale $M_U \sim 4 \times 10^{11}$ GeV. However, proton decay is suppressed by effective higher-dimensional Yukawa couplings with $\tau_p \sim 10^{35}$ years. Also, the exotic hemions at the TeV scale have $SU(2)_L \times U(1)_Y$ invariant masses such as $x_1 y_2 - y_1 x_2$, so that their contributions to the $S, T, U$ oblique parameters are suppressed and do not spoil the agreement of the SM with precision electroweak measurements.

8 Chiral Color and Quark-Lepton Nonuniversality: $SU(3)^6$

Each of the $SU(3)$ factors in supersymmetric unification may be extended:

$$SU(3)_C \rightarrow SU(3)_{CL} \times SU(3)_{CR},$$

which is the notion of chiral color;\cite{17}

$$SU(3)_L \rightarrow SU(3)_{qL} \times SU(3)_{lL},$$

which is the notion of quark-lepton nonuniversality;\cite{18, 19} and

$$SU(3)_R \rightarrow SU(3)_{qR} \times SU(3)_{lR},$$

which is needed to preserve left-right symmetry. Quarks and leptons are now $(3, 3^*)$ under $SU(3)_{CL} \times SU(3)_{qL}$, $SU(3)_{qR} \times SU(3)_{CR}$, and $SU(3)_{lL} \times SU(3)_{lR}$. The three extra $(3, 3^*)$ multiplets $x, x^c, \eta$ transform under $SU(3)_{qL} \times SU(3)_{lL}$, $SU(3)_{lR} \times SU(3)_{qR}$, $SU(3)_{CR} \times SU(3)_{CL}$ respectively, with $x, x^c$ having the same charges as $\lambda$ and zero charge for $\eta$. With this assignment, $\sin^2 \theta_W(M_U) = 3/8$.

Because all the fermions are arranged in a moose chain, this model is automatically free of anomalies, in contrast to the case of chiral color by itself or quark-lepton nonuniversality
by itself, where anomalies exist and have to be canceled in some ad hoc way. At the TeV scale, the gauge group is assumed to be $SU(3)_{CL} \times SU(3)_{CR} \times SU(2)_{qL} \times SU(2)_{lL} \times U(1)_Y$ with the following 3 copies of new supermultiplets:

$$h \sim (3, 1, 1, 1, -1/3), \quad h^c \sim (1, 3^*, 1, 1, 1/3), \quad \eta \sim (3^*, 3, 1, 1, 0); \quad (17)$$

$$\left(\nu_x, e_x\right) \sim (1, 1, 2, 1, -1/2), \quad \left(e_x^c, \nu_x^c\right) \sim (1, 1, 1, 2, 1/2); \quad (18)$$

$$\begin{pmatrix} N_x & E_x^c \\ E_x & N_x^c \end{pmatrix} \sim (1, 1, 2, 2, 0). \quad (19)$$

Again they all have $SU(2)_L \times U(1)_Y$ invariant masses. With this particle content, it was shown\cite{15} that unification indeed occurs at around $10^{16}$ GeV. What sets this model apart from the MSSM is the rich new physics populating the TeV landscape. In addition to the particles and sparticles listed above, the heavy gauge bosons and fermions corresponding to the breaking of chiral color to QCD as well as quark-lepton nonuniversality to the usual $SU(2)_L$ should also be manifest, with unmistakable experimental signatures.

The consequences of $SU(2)_{qL} \times SU(2)_{lL} \to SU(2)_L$ have been discussed\cite{19} in some detail. They include the prediction $(G_F)_{lq} < (G_F)_{ll}$, which may be interpreted as the apparent violation of unitarity in the quark mixing matrix, i.e. $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 < 1$, as well as effective sin$^2 \theta_W$ corrections in processes such as $\nu q \to \nu q$, polarized $e^- e^- \to e^- e^-$, and the weak charge of the proton, etc. However, the constraints from $Z^0$ data imply that these effects are very small and not likely to be measurable within the context of this model. On the hand, since the new particles of this model are required to be present at the TeV scale, they should be observable at the Large Hadron Collider (LHC) when it becomes operational in a few years.
9 Conclusion

Assuming a grand desert from just above the electroweak scale to $10^{16}$ GeV, the particle content of the MSSM allows the unification of the three known gauge couplings. If studied closely, taking into account proton decay and neutrino masses, etc., this appears to favor $SO(10)$ as the grand unified symmetry over $SU(5)$ but the latter is still viable, especially if a fifth dimension is invoked for example.

Instead of a single simple group, the product $SU(N)^k$ supplemented by a cyclic $Z_k$ discrete symmetry is an interesting alternative. Using a moose chain in assigning the particle content of such a supersymmetric theory, a necessary condition for it to be finite is to have 3 copies of this chain, i.e. 3 families of quarks and leptons. A realistic example has been obtained\cite{12} using $N = k = 3$.

For $N = 3$, $k = 4$ without supersymmetry, the notion of leptonic color which has a residual unbroken $SU(2)_l$ gauge group can be implemented in a model of $SU(3)^4$ quartification\cite{14}. This model allows unification at $10^{11}$ GeV without conflicting with proton decay, and predicts new half-integrally charged particles (hemions) at the TeV scale.

For $N = 3$, $k = 6$ with supersymmetry, the notions of chiral color and quark-lepton nonuniversality can be implemented\cite{15}, which cooperate to make the theory anomaly-free and be observable at the TeV scale, without spoiling unification.

In a few years, data from the LHC will tell us if the MSSM is correct [as predicted for example by $SU(5)$ and $SO(10)$], or perhaps that supersymmetry is not present but other new particles exist [as predicted for example by $SU(3)^4$], or that there are particles beyond those of the MSSM as well [as predicted for example by $SU(3)^6$]. Excitement awaits.
Acknowledgments

I thank Javier Solano and all the other organizers of V-SILAFAE for their great hospitality and a stimulating meeting in Peru. This work was supported in part by the U. S. Department of Energy under Grant No. DE-FG03-94ER40837.

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