Development of an integrated production-inventory model for food products considering exponential perceived value loss

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Abstract. Studies on deteriorating inventory models have been done extensively from the simple EOQ to more complex joint economic lot size (JELS). The majority of studies in this area assume the quantity depletion of deteriorating inventories. In most food products, however, it is quality or value that losses over time while the quantity remains the same during a certain period. Regarding packaged food products whose expiration dates are stamped on their label, the products’ value is perceived by customers by examining their remaining life time. The customers’ willingness to pay for a product declines when they realize that the product approaches its expiration date. This phenomenon is called perceived value loss. The losses can be linear or exponential depending on products’ characteristics. In this study, the exponential perceived value loss is considered when developing an integrated production-inventory model in a single vendor single buyer supply chain system. The numerical test shows that the performance of the proposed model in terms of the total profit of the joint system is 2.08% higher compared to the benchmark model.

1. Introduction

Managing perishable inventories across a supply chain is challenging since items that are deteriorating over time are forced to wait until they are being consumed. Studies on the integrated production-inventory system for deteriorating items in a multi echelon system have been done. Most of the studies assume that some fractions of inventories vanish during storage time (some of them can be seen in [1-4]). In food products, however, it is quality which decreases over time while the quantity remains the same during a certain period. Therefore, current models need to be modified in order to develop a more representative model for managing food inventories through a multi stage system.

Some inventory studies of food products have addressed quality decrease and lifetime constraint in their models [5]. An optimization approach for managing quality degradation in bell peppers across a supply chain was introduced in [6]. As mentioned in [6], firmness and color are the quality factors observed in the peppers. The value loses over time following a kinetic model where the storage temperature affects the rate of quality loss. A supply chain strategy for fresh produce (e.g. melons and sweet corn) was proposed by [7]. In their model, three variables, namely, the picking rate, the size of transfer batch, and the transportation mode, are determined to minimize the total cost of the supply chain system. Both approaches are suitable for fresh food where physical degradation can easily be examined by customers.
In some food products, quality changes (such as microbial changes) are not easy to be detected. Therefore, shelf-life is used to justify whether the food products are still acceptable to consume. Food shelf-life in planning and scheduling of yogurt production was considered in [8]. In their studies, the benefit of selling an item depends on its remaining lifetime. The longer the remaining life time of food sold to customers, the more benefits will be obtained. The model is suitable for packaged food products where customers examine the quality of the products from its remaining lifetime. In a multi echelon system, [9] developed an integrated production-inventory model accommodating perceived value loss in food products. Customers perceived the value of finished goods as degrading once the products approach their expiration dates. Model in [9] only focuses on value loss in finished goods, while model in an integrated system also accommodates the quality loss of raw materials, which can be beneficial for reducing food waste. The model of [9] then advanced in [10] by considering quality loss in raw materials. As mentioned in [9] and [10] the perceived value loss is assumed to be linear, which is suitable for some typical food (e.g. lettuce or carrots) but not for some others (such as beef or chicken). Therefore, in this research, an integrated production-inventory model considering exponential perceived value loss in food product is proposed.

The remaining sections of this paper are organized as follows. Section 2 presents the characterization of the observed system as well as the mathematical models which represent the system. Further, the experiment and its result are discussed in Section 3. Finally, Section 4 summarizes this study.

2. System characterization and mathematical model development

In this section, the system of interest is defined and the mathematical models related to the system are developed and discussed.

2.1. System characterization

The system observed in this study is similar to [10], i.e. a single vendor-single buyer (SVSB) inventory system as presented in figure 1. In manufacturer (vendor) system, raw material is procured \( m \) times during the production period of \( T \). At the same period, the finished goods are produced and delivered \( n \) times using single setup multi delivery strategy (SSMD) introduced by [11].

![Figure 1. Inventory of raw materials and finished goods in a SVSB system [10].](image)

The raw material is a deteriorating item whose quality loses over time with deteriorating rate \( k \) following the kinetic model. Meanwhile, the finished goods are perishable packaged products with expiration dates stamped on their label. The price of the product is not constant during the selling period or \( C_t \). It is because the customer’s willingness to pay (WTP) for a product decreases when the product approaching its expiration dates or during the period of \( T_{start} - T_{SL} \) as seen in figure 2. \( T_{SL} \) is
product shelf-life time while $T_{start}$ is the time when a customer perceives that the value of the aging product has decreased and feels reluctant to buy it at the same price. Consequently, the product demand rate becomes slower (as seen in figure 2, below section). Without any action, such as discounting product price, the perceived value loss (PVL) potentially causes more outdated items. In [10] the PVL follows the linear pattern (figure 1, line A). However, in this study the exponential decrease (referring to line B in figure 1) is used. This exponential pattern is more suitable for representing perceived value loss in some food products such as beef and chicken [12].

Figure 2. Decreasing customer’s WTP of finished goods in retailer.

Further, as the customer’s WTP decreases during the PVL period, retailer will offer discounted products to maintain demand and avoid outdated items. There will be three regions of price offered during $Ct$ (as seen in figure 3). During the first region, the products will be sold with normal price or $p_{max}$, while during the second region, a PVL occurs; hence, the price of the products is discounted. Unlike (10), the decrease in product price will be exponential. Finally, in the last region, i.e. the region in which the products have been expired, the price of expired products is set as $p_{min}$. Selling products with $p_{min}$ occurs if $CompTime$ is longer than $T_{sl}$. ($CompTime$ is time for the last product in the batch sold to customers or $CompTime=E_i+Ct$). Different from [10], the age of batch entering retailer or $E_i$ is set to be younger than $T_{start}$. This is to ensure that retailer will have fresh products with a good price ($p_{max}$).

Figure 3. Shelf-life based price function.
2.2. Mathematical model development
The proposed model is developed based on the following assumptions:

a. The production rate and demand rate are constant.

b. Once the customer’s WTP starts decreasing (i.e. the product approaches its expiration dates), its price will decline exponentially.

c. All delivering batches enter the retailer’s warehouse before the customer’s WTP declines.

d. Lead time is neglected and no shortage is allowed.

Further, the following notations are used in the mathematical model.

For the manufacturer:

- \( P \) : production rate (unit product/unit time)
- \( D \) : demand rate from retailer (unit product/unit time)
- \( K \) : deterioration rate (unit quality/unit time)
- \( c_{\text{raw}} \) : purchasing cost of raw material ($/unit product)
- \( A_{\text{raw}} \) : delivering cost of raw material ($/shipment)
- \( q_{\text{raw}} \) : ordering size of raw material (unit product)
- \( H_{\text{raw}} \) : holding cost of raw material ($/unit product/unit time)
- \( c_{\text{loss}} \) : cost of quality loss ($/unit quality/unit time)
- \( TC_{\text{raw}}(m,T) \) : total cost of raw material system ($/unit time)
- \( c_{\text{mfc}} \) : manufacturing cost ($/unit product)
- \( A_{\text{mfc}} \) : production setup cost ($/setup)
- \( H_{\text{mfc}} \) : holding cost of finished goods ($/unit product/unit time)
- \( TC_{\text{mfc}}(m,T) \) : total cost of finished goods system ($/unit time)
- \( TP_{\text{mfc}}(m,n,T) \) : total profit of the manufacturer ($/unit time)

For the retailer:

- \( c_{\text{ret}} \) : product selling price from manufacturer to retailer ($/unit product)
- \( H_{\text{ret}} \) : holding cost of finished goods at retailer ($/unit product/unit time)
- \( A_{\text{ret}} \) : transportation cost for delivering product to retailer ($/shipment)
- \( \theta \) : shape parameter of the exponential distribution
- \( T_{\text{SL}} \) : product shelf-life (unit time)
- \( T_{\text{start}} \) : time when customer’s WTP starts decreasing (unit time)
- \( p_{\text{max}} \) : maximum product price ($/unit product)
- \( p(t) \) : price of product at age \( t \) ($/unit product) or the shelf-life based price function
- \( E_{i} \) : age of batch \( i \) when entering retailer (unit time)
- \( C_l \) : finished goods delivery cycle (unit time)
- \( CompTime \) : completion time of the last product in the transferred batch (unit time)
- \( TC_{\text{ret}} \) : total cost of the retailer ($/unit time)
- \( R_{\text{ret}} \) : revenue of the batch ($/unit time)
- \( TP_{\text{ret}}(n,T) \) : total profit of the retailer ($/unit time)
- \( TP(m,n,T) \) : total profit of the integrated supply chain system ($/unit time)

Firstly, the proposed model is developed by formulating the joint total cost \( JTC \) as can be seen in equation (1). Detailed steps in constructing equation (1) can be found in [10].

\[
JTC(m,T,n) = C_{\text{mfc}}D + \frac{A_{\text{mfc}}}{T} + H_{\text{mfc}} \frac{DT}{2n} \left( \frac{D}{P} (2-n) + (n-1) \right) + C_{\text{raw}}D + A_{\text{raw}} \frac{m}{T} + H_{\text{raw}} \frac{D^2T}{2mp} + L(m,T) + \frac{O_{\text{ret}}}{T} + \frac{A_{\text{ret}}n}{T} + H_{\text{ret}} \frac{DT}{2n}
\]

(1)
The next step is formulating joint total revenue (JTR) which is obtained by adding revenue in manufacturer ($R_{\text{manu}}$) with revenue in retailer ($R_{\text{ret}}$). Equation 2 represents the total revenue in manufacturer system where $C_{\text{ret}}$ denotes selling price from manufacturer to retailer and $D$ is demand.

$$R_{\text{manu}} = C_{\text{ret}} D$$

(2)

As previously mentioned, the price of products from retailer to end customers is varied during the selling period because of the PVL. For representing this, an exponential shelf-life based price function is developed as represented by equation (3) where $p(t)$ is the price of product at time $t$.

$$p(t) = \begin{cases} \frac{p_{\text{max}}}{p_{\text{min}}} (e^{-\lambda(t-T_{\text{start}})} - 1) & 0 \leq t < T_{\text{start}} \\ p_{\text{min}} & T_{\text{start}} \leq t < T_{sd} \\ 0 & t \geq T_{sd} \end{cases}$$

(3)

Further, equation (4) represents a formula for calculating the age of batch entering retailer ($E_i$) where $i$ denotes $i^{th}$ batch or the transferred batch sequence ($i = 1 \ldots n$). The detailed procedure for modelling the age of batch can be seen in [10].

$$E_i = (i-1)T - (i-2)DT - np$$

(4)

The information of $E_i$ will be used to compute the completion time of the batch ($\text{CompTime}_i$) in retailer i.e. $\text{CompTime}_i = E_i + Ct$. Hence, the general procedure to calculate revenue in retailer ($R_{\text{ret}}$) is represented by equations (5), (6) and (7).

**Case 1:** If $\text{CompTime}_i < T_{\text{start}}$ then the revenue of the batch ($R_{\text{ret}}$) is maximized as all products are sold with $p_{\text{max}}$ as defined by:

$$R_{\text{ret}}(n,T) = \frac{dp_{\text{max}}Ct}{T}$$

(5)

**Case 2:** If $T_{\text{start}} \leq \text{CompTime}_i$ then $R_{\text{ret}}$ is defined by:

$$R_{\text{ret}}(n,T) = \frac{dt}{T} \left[ p_{\text{max}}(T_{\text{start}} - E_i) + \int_{T_{\text{start}}}^{E_i + Ct} p(t)dt \right]$$

(6)

**Case 3:** If $T_{sd} < \text{CompTime}_i$ then $R_{\text{ret}}$ is represented by:

$$R_{\text{ret}}(n,T) = \frac{dt}{T} \left[ p_{\text{max}}(T_{\text{start}} - E_i) + \int_{T_{\text{start}}}^{T_{sd}} p(t)dt + p_{\text{min}}(E_i + Ct - T_{sl}) \right]$$

(7)

Finally, the joint total revenue (JTR) in the SVSB system is represented by equation (8).

$$JTR(n,T) = \sum_{i=1}^{n} R_{\text{ret}}(n,T) + R_{\text{manu}}$$

(8)

Hence, the joint total profit of the system (JTP) is shown by equations (9), (10), (11) and (12), where equation (11) is a constraint to ensure that the age of batch received by retailer is always younger than $T_{\text{start}}$.

Maximise:

$$JTP(m,T,n) = JTR(n,T) - JTC(m,T,n)$$

(9)

Subject to:

$$P \in D;$$

(10)

$$E_i < T_{\text{start}}; \text{ for } i = 1, 2, \ldots, n$$

(11)

$$T > 0;$$

(12)

The proposed model will be evaluated and compared to the benchmark model in the numerical test section.
3. Numerical test
In this experiment, parameters in [9] are adopted with some adjustment, as shown in table 1.

Table 1. Parameters in numerical test

| Manufacturer | \( P \) | \( D \) | \( k \) | \( \epsilon_{\text{raw}} \) | \( \Lambda_{\text{raw}} \) | \( H_{\text{raw}} \) | \( \epsilon_{\text{loss}} \) | \( \chi_{\text{mfc}} \) | \( \Lambda_{\text{mfc}} \) | \( H_{\text{mfc}} \) |
|--------------|-------|-------|------|----------------|----------------|-------------|----------------|---------------|---------------|---------------|
| Value        | 19,200| 4,800 | 0.01 | 5              | 1000           | 1           | 10             | 5             | 600           | 6             |
| Retailer     | \( \epsilon_{\text{ret}} \) | \( H_{\text{ret}} \) | \( \Lambda_{\text{ret}} \) | \( O_{\text{ret}} \) | \( T_{\text{ret}} \) | \( T_{\text{start}} \) | \( \lambda \) |
| Value        | 30    | 7     | 25   | 50             | 0.208          | 0.167       | 200            |

To determine the decision variables \( m \), \( T \) and \( n \), the genetic algorithm (GA) tool in Matlab 2009 is utilized. The GA’s parameters used in this test are the same with those in [10]. The result can be seen in table 2.

Table 2. Total cost, revenue and total profit of the SVSB system

| Benchmark Model [10] | Decision Variables | SVSB |
|----------------------|--------------------|------|
|                      | \( m=2 \)          | \( n=5 \) | \( T=0.1486 \) |
| Total Cost           | Manufacturer       | Retailer | SVSB     |
|                      | 132,672.81         | 146,350.29 | 279,023.10 |
| Revenue              | 144,000.00         | 308,133.90 | 452,133.90 |
| Total Profit         | 11,327.19          | 161,783.61 | 173,110.80 |

| The Proposed Model   | Decision Variables | SVSB |
|----------------------|--------------------|------|
|                      | \( m=2 \)          | \( n=5 \) | \( T=0.1448 \) |
| Total Cost           | Manufacturer       | Retailer | SVSB     |
|                      | 132,901.64         | 146,385.84 | 279,287.48 |
| Revenue              | 144,000.00         | 311,997.10 | 455,997.10 |
| Total Profit         | 11,098.36          | 165,611.26 | 176,709.62 |

As seen in table 2, the linear perceived value loss model leads to \( m = 2, \ n = 5 \) and \( T = 0.1486 \). These decision variables result in the total profit of $173,110.80. Meanwhile, the values of the decision variables in the exponential model are \( m=2, \ n=5 \) and \( T=0.1449 \) leading to $176,709.62 total profit (±$3,598 higher than the linear model). It means that using linear approach while managing product with exponential perceived value loss characteristic may lead to profit reduction (losing a chance to improve the profit until 2.08%).

Further, still from table 2 in the manufacturer column, it can be seen that the total cost of manufacturer increases around $228. Reducing the length of production cycle or \( T \) (±2.53%) can be one of the causes since it affects the setup cost. On the other hand, although the total cost in retailer also increases (around $35), the shorter \( T \) gives a significant benefit to retailer i.e. the total revenue increases up to $3,863.21. It seems that, in this case, using the shorter \( T \) when dealing with exponential perceived value products gives more advantages to retailer.

4. Conclusion
In this research, an integrated production-inventory model considering an exponential perceived value loss of finished goods has been addressed. The numerical test shows that the exponential model is potential to increase the profit if it is used to manage a product where the customer’s WTP for the product nonlinearly decreases once the product approaches its expiration dates. For the given parameters, the performance of the exponential model is better than the benchmark model i.e. the joint profit of the proposed model significantly increases. However, the proposed strategy seems to give...
more benefit to retailer as the total cost of manufacturer slightly increases. Therefore, a benefit sharing scenario is worth to be proposed to encourage the manufacturer which may be resistant to adopt this strategy since it disadvantages them.

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