D-term inflation, cosmic strings, and consistency with cosmic microwave background measurements

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Standard D-term inflation is studied in the framework of supergravity. D-term inflation produces cosmic strings, however it can still be compatible with CMB measurements without invoking any new physics. The cosmic strings contribution to the CMB data is not constant, nor dominant, contrary to some previous results. Using current CMB measurements, the free parameters (gauge and superpotential couplings, as well as the Fayet-Iliopoulos term) of D-term inflation are constrained.

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I. INTRODUCTION

The inflationary paradigm offers simple answers to the shortcomings of the standard hot big bang model. In addition, simple inflationary models offer successful candidates for the initial density fluctuations leading to the observed structure formation. One crucial question though is to answer how generic is the onset of inflation and to find consistent and natural models of inflation from the point of view of particle physics. One can argue that the initial conditions which favor successful inflationary models are the likely outcome of the quantum era before inflation. It is more difficult however to find natural ways to guarantee the flatness of the inflaton potential.

The early history of the Universe at energies below the Planck scale is described by an effective N=1 supergravity theory. Since inflation should have taken place at an energy scale $V^{1/4} \lesssim 4 \times 10^{16}$ GeV, this implies that inflationary models should be constructed in the framework of supergravity. Here is where the problem arises: it is difficult to implement slow-roll inflation within supergravity. The positive false vacuum of the inflaton field breaks spontaneously global supersymmetry, which gets restored after the end of inflation (when $V$ disappears). In supergravity theories, the supersymmetry breaking is transmitted to all fields by gravity, and thus any scalar field, which could play the role of the inflaton, gets an effective mass $\sim \sqrt{8\pi V/M_{Pl}} \sim H$, where $H$ stands for the expansion rate during inflation, and $M_{Pl}$ denotes the reduced Planck mass. This problem, known as the problem of “Hubble-induced mass”, originates from F-term interactions and thus it is resolved if we consider the vacuum energy as being dominated by non-zero D-terms of some superfields. This result led to a dramatic interest in D-term inflation, since in addition this model can easily be implemented in string theory. However, later on D-term inflation in its turn was though to be plagued with problems.

In D-term inflation, the inflationary era ends when a U(1) gauge symmetry is spontaneously broken by a vacuum expectation value of some scalar field, leading to the formation of gauge cosmic strings. As it was explicitly shown in Ref. cosmic strings are generically expected to be formed at the end of a hybrid inflation phase, in the context of supersymmetric grand unified theories. It was claimed that the cosmic strings contribution to the angular power spectrum of the Cosmic Microwave Background (CMB) temperature anisotropies is constant and dominant (75%). From the observational point of view however, strong constraints are placed on the allowed cosmic strings contribution to the CMB: it can not exceed $\sim 10\%$. Thus, standard D-term inflation was thought to be inconsistent with cosmology. To rescue D-term inflation there have been proposed different mechanisms which either consider more complicated models, or they require additional ingredients so that cosmic strings are not produced at the end of hybrid inflation. For example, one can add a nonrenormalisable term in the potential, or add an additional discrete symmetry, or consider GUT models based on non-simple groups. More recently, a new pair of charged superfields has
been introduced in D-term inflation so that cosmic strings formation is avoided.

The aim of our study is to show that standard D-term inflation leading to the production of cosmic strings is still compatible with cosmological data, and in particular CMB, without invoking any new physical mechanisms. We find that in D-term inflation the cosmic strings contribution to the CMB data depends on the free parameters, as for F-term inflation. The maximum allowed cosmic strings contribution to the CMB measurements places upper limits on the inflationary scale (which is also the cosmic string energy scale), or equivalently on the coupling of the superpotential.

We first review the results for F-term hybrid inflation, in which case the supersymmetric renormalisable superpotential reads

$$W_{\text{infl}}^F = \kappa S(\Phi_+\Phi_- - M^2) ,$$

where $S, \Phi_+, \Phi_-$ are three chiral superfields, and $\kappa, M$ are two constants. The cosmic strings contribution to the CMB is a function of the coupling $\kappa$, or equivalently of the mass scale $M$. It can be consistent with the most recent measurements, which require that it is at most equal to 9% $\xi_{\text{eff}}$, provided $\xi_{\text{eff}} \lesssim 7 \times 10^{-7}$.

The above limit was obtained in the context of SO(10) gauge group. Upper limits of the same order of magnitude are found for other gauge groups $\xi_{\text{eff}}$.

This result implies that F-term inflation leading to the production of cosmic strings of the GUT scale can be compatible with measurements, provided the coupling is sufficiently small. Thus, hybrid supersymmetric inflation losses some of its appeal since it is required some amount of fine tuning of its free parameter, $\kappa$ should be of the order of $10^{-6}$ or smaller. This constraint on $\kappa$ is in agreement with the one given in Ref. $\xi_{\text{eff}}$. The parameter $\kappa$ is also subject to the gravitino constraint which imposes an upper limit to the reheating temperature, to avoid gravitino overproduction. Within supersymmetric GUTs, and assuming a see-saw mechanism to give rise to massive neutrinos, the inflaton field will decay during reheating into pairs of right-handed neutrinos. Using the constraints on the see-saw mechanism it is possible $\xi_{\text{eff}}$ to convert the constraint on the reheating temperature to a constraint on the coupling parameter $\kappa$, namely $\kappa \lesssim 8 \times 10^{-3}$, which is clearly a weaker constraint.

The superpotential coupling $\kappa$ is allowed to get higher values, namely it can approach the upper limit permitted by the gravitino constraint, if one employs the curvaton mechanism $\xi_{\text{eff}}$. Such a mechanism can be easily accommodated within supersymmetric theories, where one expects to have a number of scalar fields. For fixed $\kappa$, the cosmic strings contribution decreases rapidly as the initial value of the curvaton field, $\psi_{\text{init}}$, decreases. Thus, the WMAP measurements lead to an upper limit on $\psi_{\text{init}}$, namely $\psi_{\text{init}} \lesssim 5 \times 10^{13}(\kappa/10^{-2})$ GeV $\xi_{\text{eff}}$. This limit holds for $\kappa$ in the range $[5 \times 10^{-5}, 1]$; for lower values of $\kappa$, the cosmic strings contribution is always suppressed and thus lower than the WMAP limit.

The above results hold also if one includes supergravity corrections. This is expected since the value of the inflaton field is several orders of magnitude below the Planck scale.

### II. D-TERM INFLATION

D-term inflation is derived from the superpotential

$$W_{\text{infl}}^D = \lambda S\Phi_+\Phi_-, \quad (3)$$

where $S, \Phi_-, \Phi_+$ are three chiral superfields and $\lambda$ is the superpotential coupling. D-term inflation requires the existence of a nonzero Fayet-Iliopoulos term $\xi$, permitted only if an extra U(1) symmetry beyond the GUT framework, is introduced. In the context of supersymmetry we calculate the radiative corrections leading to the effective potential,

$$V_{\text{eff}}^{D-\text{SUSY}}(|S|) = \frac{g^2\xi^2}{2} \left\{ 1 + \frac{g^2}{16\pi^2} \left[ 2 \ln \frac{|S|^2\lambda^2}{A^2} + (z + 1)^2 \ln(1 + z^{-1}) + (z - 1)^2 \ln(1 - z^{-1}) \right] \right\} , \quad (4)$$

where $z = \lambda^2|S|^2/(g^2\xi)$, with $g$ the gauge coupling of the U(1) symmetry and $\xi$ the Fayet-Iliopoulos term, chosen to be positive; $A$ stands for a renormalisation scale. In the absence of the curvaton mechanism, the quadrupole anisotropy is the sum of the inflaton field (scalar and tensor parts) and cosmic strings contributions and we normalise it to the COBE data.

We compute the mass scale of the symmetry breaking, given by $\sqrt{\xi}$, and we find that it increases with $\lambda$. We then calculate the cosmic strings contribution to the temperature anisotropies. We find that within supersymmetry D-term inflation is consistent with CMB data provided the superpotential coupling $\lambda$ is quite small, namely $\lambda \lesssim 3 \times 10^{-5}$. 

However, the dependence of $z_Q$ (the index $Q$ denotes the scale corresponding to the quadrupole anisotropy) on the superpotential coupling $\lambda$ results to values of the inflaton field $S_Q$ above the Planck mass. This implies that the correct analysis has to be done in the framework of supergravity. For small values of the gauge coupling $g$, the study in the context of supergravity becomes just the analysis within supersymmetry. Some previous studies [5, 13] found in the literature kept only the first term of the radiative corrections. We find that it is necessary to perform the analysis using the full effective potential, which we calculated for minimal supergravity. More precisely, using a minimal Kähler potential ($K = |\phi_-|^2 + |\phi_+|^2 + |S|^2$) and a minimal gauge kinetic function ($f(\Phi_i) = 1$), the scalar potential reads:

$$V^\text{eff-SUGRA} = \frac{g^2\xi^2}{2} \left\{ 1 + \frac{g^2}{16\pi^2} \left[ 2 \ln \frac{|S|^2\lambda^2}{\Lambda^2} \exp \left( \frac{|S|^2}{M_{Pl}^2} \right) + (z + 1)^2 \ln(1 + z^{-1}) + (z - 1)^2 \ln(1 - z^{-1}) \right] \right\},$$

where $z = [\lambda^2|S|^2/(g^2\xi)] \exp(|S|^2/M_{Pl}^2)$. The number of e-foldings is

$$N_Q = \frac{2\pi^2}{g^2} \int_1^{z_Q} \frac{W(c\tilde{z})}{z^2 f(z)[1 + W(c\tilde{z})]} \, d\tilde{z},$$

where $W(x)$ denotes the “W-Lambert function” defined by $W(x) \exp[W(x)] = x$, and $c \equiv (g^2\xi)/(\lambda^2M_{Pl}^2)$. The number of e-foldings $N_Q$ is thus a function of $c$ and $z_Q$, for $g$ fixed. Setting $N_Q = 60$ we obtain a numerical relation between $c$ and $z_Q$ which allows us to construct a function $z_Q(\xi)$ and express the three contributions to the CMB only as a function of $\xi$. The total $(\delta T/T)$ is given by

$$\left[ \left( \frac{\delta T}{T} \right)^2_{Q-\text{tot}} \right]^{1/2} \sim \left( \frac{\xi}{M_{Pl}} \right)^2 \left\{ \frac{\pi^2}{90g^2} z_Q^{-2} f^{-2}(z_Q) \frac{W(cz_Q)}{1 + W(cz_Q)^2} + \left( \frac{0.77g}{8\sqrt{2}\pi} \right)^2 + \left( \frac{9\pi}{4} \right)^2 \right\},$$

where the three contributions come from the scalar and tensor parts of the inflaton field, and the cosmic strings, respectively. We normalise the l.h.s. of Eq. 1 to the COBE data, i.e., $(\delta T/T)^\text{COBE}_Q \sim 6.3 \times 10^{-6}$, and we solve it numerically to obtain $\xi$, and thus, the three contributions for given values of $g$ and $\lambda$.

The cosmic strings contribution to the CMB data, is found to be an increasing function of the mass scale $\sqrt{\xi}$, as shown in Fig. 1 below.

![Fig. 1: The cosmic strings contribution to the CMB data, as a function of the mass scale $\sqrt{\xi}$ in units of $10^{15}$ GeV.](image)

Our results, summarized in Fig. 2 differ from the results obtained in the framework of supersymmetry unless $\lambda \gtrsim 10^{-3}$ or $g \lesssim 10^{-4}$. The cosmic strings contribution to the CMB turns out to be dependent on the free parameters, with however the robust result that the cosmic strings contribution is not constant, nor is it always dominant, in contradiction to Ref. 13. This implies that contrary to what is often assumed, the simplest D-term inflation is still an open possibility and one does not need to consider more complicated models. Our analysis shows that $g \gtrsim 1$ necessitates multiple-stage inflation, since otherwise we cannot have sufficient e-foldings to resolve the horizon problem of standard cosmology, while $g \gtrsim 2 \times 10^{-2}$ is incompatible with the WMAP measurements. For $g \lesssim 2 \times 10^{-2}$, we can also constrain the superpotential coupling $\lambda$ and get $\lambda \lesssim 3 \times 10^{-5}$. This limit was already found in the framework of supersymmetry 13 and it is in agreement with the constraint $\lambda \lesssim O(10^{-4} - 10^{-5})$ of Ref. 13. Supergravity corrections impose in addition a lower limit to the coupling $\lambda$. If for example $g = 10^{-2}$, the cosmic strings contribution imposes $10^{-8} \lesssim \lambda \lesssim 3 \times 10^{-5}$. The constraint induced by CMB measurements is expressed as a single constraint on the Fayet-Iliopoulos term $\xi$, namely $\sqrt{\xi} \lesssim 2 \times 10^{15}$ GeV.

As a next step we examine whether there is a mechanism to allow more natural values of the couplings. Assuming the existence of a scalar field, that is subdominant during inflation as well as at the beginning of the radiation
dominated era, such a field (the curvaton) gives an additional contribution to the temperature anisotropies, which we calculate below for the case of D-term inflation. The curvaton contribution, in terms of the metric perturbation, reads \[ \left( \frac{\delta T}{T} \right)_{\text{curv}} = \frac{4 \, \delta \psi_{\text{init}}}{9 \, \psi_{\text{init}}} \, . \] (8)

The initial quantum fluctuations of the curvaton field are given by \( \delta \psi_{\text{init}} = H_{\text{infl}} / (2\pi) \). The expansion rate during inflation, \( H_{\text{infl}} \), is a function of the inflaton field and it is given by the Friedmann equation: \( H_{\text{infl}}^2(\varphi) = (8\pi/3)V(\varphi) \). Thus, for the D-term tree-level effective potential, the additional curvaton contribution to the total temperature anisotropies is given by \[ \left( \frac{\delta T}{T} \right)_{\text{curv}}^2 = \frac{1}{6} \left( \frac{2}{27\pi} \right)^2 \left( \frac{g \xi}{M_{\text{Pl}} \psi_{\text{init}}} \right)^2 \, . \] (9)

Normalising the total \( \left( \frac{\delta T}{T} \right) \) to COBE we then obtain the contributions of the different sources (inflaton field splitted into scalar and tensor parts, cosmic strings, curvaton field) to the CMB as a function of one of the three parameters \( \psi_{\text{init}}, \lambda, g \), keeping the other two fixed. We show in Fig. 3 the three contributions as a function of \( \psi_{\text{init}} \), for \( \lambda = 10^{-1} \) and \( g = 10^{-1} \). Clearly, there are values of \( \psi_{\text{init}} \) which allow bigger values of the superpotential coupling \( \lambda \) and of the gauge coupling \( g \), than the upper bounds obtained in the absence of a curvaton field.

More explicitly, the fine tuning on the couplings can be avoided provided

\[ \psi_{\text{init}} \lesssim 3 \times 10^{14} \left( \frac{g}{10^{-2}} \right) \text{ GeV} \quad \text{for} \quad \lambda \in [10^{-1}, 10^{-4}] \, . \] (10)

Clearly, for smaller values of \( \lambda \), the curvaton mechanism is not necessary.
We would like to bring to the attention of the reader that in the above study we have neglected the quantum gravitational effects, which would lead to a contribution to the effective potential, even though $S_Q \sim O(10M_{Pl})$. Our analysis is however still valid, since the effective potential given in Eq. 5 satisfies the conditions $V(|S|) \ll M_{Pl}^4$ and $m_S^2 = d^2V/dS^2 \ll M_{Pl}^2$, and thus the quantum gravitational corrections $[\Delta V(|S|)]_{QG}$ are negligible when compared to the effective potential $V_{\text{eff-SUGRA}}$.

### III. CONCLUSIONS

D-term inflation gained a lot of interest since it was shown that it avoids the problem of “Hubble-induced mass”, but it was later thought to be plagued with an inconsistency with the data. Standard D-term inflation ends with the formation of cosmic strings which was claimed to lead to a constant and dominant contribution to the CMB data, much higher than the one allowed by measurements. In this study, we show that this is not the case, and therefore, standard D-term hybrid inflation can still be compatible with cosmological data.

We consider standard D-term inflation in its simplest form and without any additional ingredients. We perform our analysis in the framework of supergravity, since we reach scales above the Planck scale, and we consider all one-loop radiative corrections. We show that the cosmic strings produced at the end of D-term inflation can lead to a contribution to the CMB data which is allowed by the measurements. The price to be paid is that the couplings must be small. However, this constraint can be less severe if one invokes the curvaton mechanism.

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