Heavy Quark Effective Theory, Interpolating Fields and Bethe-Salpeter Amplitudes

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Abstract

We use the LSZ reduction theorem and interpolating fields, along with the heavy quark effective theory, to investigate the structure of the Bethe-Salpeter amplitude for heavy hadrons. We show how a simple form of this amplitude, used extensively in heavy hadron decay calculations, follows naturally up to $O(1/M)$ from these field theoretic considerations.
1 Introduction

In recent years the heavy quark effective theory (HQET) [1], [2] and the consequent heavy quark symmetry have emerged as useful tools in studying the decay properties of heavy hadrons. In our previous works [3] - [7] we have used the Bethe-Salpeter (B-S) formalism to derive the consequences of the heavy quark symmetry for weak transitions of hadrons of arbitrary spin.

In the heavy quark mass limit we argued [3]-[7] that the Bethe-Salpeter (B-S) amplitude (wave function) for an arbitrary heavy meson, of momentum $P$ and mass $M$, can be written in momentum space as

$$M_{\alpha \beta}(p_1, p_2) = \chi_{\alpha \delta}(v, k) A_{\delta \beta}(p_1, p_2),$$

(1)

where $\chi_{\alpha \beta}$ is a projection operator which projects out a particular spin, parity state from an unknown orbital wave function $A$. Here $p_1$ and $p_2$ are the momenta of the heavy and light quarks respectively and $v$ is the four-velocity of the heavy meson defined as $v = P/M$. The momentum $k$ is defined as $k = p_1 - p_2$.

The important observation is that $\chi$ has the Bargmann-Wigner [8] - [12] form

$$\chi(v, k) = \frac{1}{2}(1 + \gamma)\Gamma(k).$$

(2)

with $\Gamma(k)$, a Dirac matrix, depending on the spin, parity and orbital angular momentum [7]. From this simple form of the B-S amplitude follow all the dramatic results about the reduction of form factors in heavy meson decays.

We argued that this form of the B-S amplitude, eqs. (1) and (2), arises because, in the heavy quark limit, in the leading order of the heavy quark effective theory (HQET) [1], [2] the heavy quark spin is decoupled from the light degrees of freedom and as a consequence the B-S amplitude satisfies the Bargmann-Wigner (Dirac) equation on the heavy Dirac index $\alpha$. The baryons can also be represented in a similar form.

In this short note we demonstrate that this form of the B-S wave function follows naturally, up to order $1/M$, from the notions of interpolating fields and the LSZ reduction theorem, when combined with the zeroth order HQET. We will demonstrate this explicitly for the $0^-$ and $1^-$ heavy mesons and then show how it is easily generalised to arbitrary heavy meson resonances and heavy baryons. As an intermediate step we show that, in the heavy quark mass limit, only half the components of the quark fields appearing in the interpolating field contribute to the matrix element. For example, for a heavy ($Qq$) meson, in the rest frame of the meson only the quark part of the heavy quark field and only the antiquark part of the light quark field contribute.

In Section 2 we discuss the consequences of the heavy quark mass limit for interpolating fields. In Section 3 we combine the results of Section 2 with the HQET to obtain the desired form of the B-S amplitude.
2 Interpolating Fields and the Heavy Quark Mass Limit

As we mentioned in the Introduction, a variety of interpolating fields can be used to represent a particular bound state within a reduction formula [13]. In this section we will first use the equations of motion to establish relations between certain classes of interpolating fields and the normalisation constants appearing in these interpolating fields. We will then look at these relations in the heavy quark limit. Then we will study the consequences of these relations when we use these interpolating fields in reduction theorems. As a first example consider the pseudoscalar heavy meson for which we can take as interpolating fields

$$\phi_1(x) = Z_3^{1/2} \frac{1}{N_1} T \bar{\psi}_q(x) \gamma_5 \psi_Q(x)$$  \hspace{1cm} (3)

or

$$\phi_2(x) = -Z_3^{1/2} \frac{1}{N_2} \frac{i \partial_\mu}{M} T \bar{\psi}_q(x) \gamma_\mu \gamma_5 \psi_Q(x)$$ ,  \hspace{1cm} (4)

where the normalisation constants

$$N_1 = \langle 0 | T \bar{\psi}_q(0) \gamma_5 \psi_Q(0) | P \rangle$$  \hspace{1cm} (5)

and

$$N_2 = -\langle 0 | T \bar{\psi}_q(0) \gamma_5 \psi_Q(0) | P \rangle$$  \hspace{1cm} (6)

are chosen to ensure the normalisation condition

$$\langle 0 | \phi_i(x) | P \rangle = Z_3^{1/2} e^{-iP \cdot x}$$ ,  \hspace{1cm} (7)

for $i = 1, 2$.

One can now use the equations of motion

$$(i \not{D} - m) \psi = 0$$  \hspace{1cm} (8)

for the quark fields to show that

$$\phi_2 = \frac{m_Q + m_q}{M} N_1 \frac{N_1}{N_2} \phi_1$$  \hspace{1cm} (9)

and, further, from the normalisation condition eq. (7) we get

$$\frac{N_1}{N_2} = \frac{M}{m_Q + m_q}$$ ,  \hspace{1cm} (10)

where $m_Q$ ($m_q$) is the mass of the heavy (light) quark, leading to $\phi_1 = \phi_2$. Note that in the heavy quark limit, where $m_Q + m_q \to M$, $N_1/N_2 \to 1$. 

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The pseudoscalar case is particularly simple and perhaps not very enlightening. Much more interesting is the vector meson case which demonstrates some generic properties of interpolating fields which are not apparent in the pseudoscalar case. We will, in fact, show that the corresponding two interpolating fields in the vector, $1^-$, case are not exactly equal but differ by a term which is of $O\left(\frac{1}{M}\right)$ and can be ignored in the infinite mass limit. It will be then easy to show that such a situation always occurs for all meson resonances.

An interpolating field $\phi_\mu(x)$ for a vector particle, with momentum $P$, should satisfy

$$
\langle 0 | \phi_\mu(x) | P \rangle = Z_3^{1/2} \frac{1}{V_1} \bar{\psi}_q(x) \gamma_\mu^\perp \psi_Q(x),
$$

where the polarisation vector $\epsilon_\mu$ satisfies the transversality condition $v \cdot \epsilon = 0$ and $\epsilon^\mu \epsilon_\mu = -1$. Thus, as in the pseudoscalar case, we can consider the following two interpolating fields

$$
\phi_1^\mu(x) = Z_3^{1/2} \frac{1}{V_1} T \bar{\psi}_q(x) \gamma_\mu^\perp \psi_Q(x) \tag{12}
$$

and

$$
\phi_2^\mu(x) = -Z_3^{1/2} \frac{1}{V_2} i \partial_\nu T \bar{\psi}_q(x) \gamma^\nu \gamma_\mu^\perp \psi_Q(x) \tag{13}
$$

where $\gamma_\mu^\perp = \gamma_\mu - v \cdot \gamma_\mu$. The normalisation constants, $V_i$, with $i = 1, 2$, are given by

$$
V_1 = -\langle 0 | \bar{\psi}_q(0) \gamma^\perp \psi_Q(0) | P \rangle \tag{14}
$$

and

$$
V_2 = \langle 0 | \bar{\psi}_q(0) \gamma^\perp \psi_Q(0) | P \rangle \tag{15}
$$

Now one can use the equations of motion eq. (8) for the quark fields to obtain

$$
i \partial_\nu \bar{\psi}_q(x) \gamma^\nu \gamma_\mu^\perp \psi_Q(x) = -(m_Q + m_q) [\bar{\psi}_q(x) \gamma_\mu^\perp \psi_Q(x) - 2i \bar{\psi}_q(x) \frac{\vec{D}_\perp}{m_Q + m_q} \psi_Q(x)].
$$

where the transverse covariant derivative, $\vec{D}_\mu = \vec{D}_\mu - v_\mu v \cdot \vec{D}$, acts to the right.

One sees that in this case the two interpolating fields are not exactly equal but differ by a term which can be neglected, as it is small in the heavy quark limit. Note that the term neglected, $D_\mu^\perp / M$, (as $m_Q + m_q \rightarrow M$) is exactly the kind of term which is dropped in deriving the HQET at leading order [2]. Physically this means that we consider all momenta transverse to the line of flight of the meson to be small. One can also look at this result in another way. $i \bar{\psi}_q(x) \frac{\vec{D}_\perp}{M} \psi_Q(x)$ is in fact another possible candidate for the interpolating field for the vector meson but its overlap with the physical state becomes negligible in the heavy mass limit. Note also that $i \bar{\psi}_q(x) \frac{\vec{D}_\perp}{M} \psi_Q(x)$ corresponds to a p-wave contribution to the vector, $1^-$, state coming from the anti-quark part of the heavy quark. In the zeroth order HQET recall that the heavy quark and antiquark are described by separate fields or to put it another way, the antiquark part of the
heavy quark field is suppressed and thus the overlap of this interpolating field with the physical heavy meson state becomes very small in this limit.

Hence, only in the heavy quark limit, with \( m_Q + m_q \to M \), the two interpolating fields, \( \phi_1^\mu \) and \( \phi_2^\mu \) are equal along with the important relation

\[
V_1 = V_2 ,
\]

between the normalisation constants.

The above considerations turn out to be a general feature of the heavy quark limit. Given a particular interpolating field \( N_1 \bar{T} \psi_q \Gamma \psi_Q \) then one can show that this is equal to the interpolating field \( N_2 - i \frac{\partial}{M} T \bar{\psi}_q \gamma^\mu \Gamma \psi_Q \), up to terms of \( O(\vec{D}/M) \), along with the relation between the normalisation constant \( N_1 = N_2 \), in the heavy quark mass limit. Here \( \Gamma \) is some Dirac matrix, possibly with derivatives. We shall come back to this point at the end of the next section.

We now discuss the consequences of the equality of interpolating fields for matrix elements, in particular the Bethe-Salpeter amplitude, by using them in reduction theorems. First of all we would like to address the question: what does one mean that one is free to choose equivalent interpolating fields? We will address this question in terms of the Bethe-Salpeter amplitude but the conclusions apply to any matrix element involving a heavy meson or baryon.

As an example we consider the heavy vector meson. We define the Bethe-Salpeter amplitude for the heavy vector meson as

\[
M_{\alpha}^{\beta}(x_1, x_2) = \langle 0 | \psi_Q(x_1) G(x_1, x_2) \bar{\psi}_q(x_2) | P \rangle ,
\]

(18)

In the above equations \( \psi_Q (\psi_q) \) is the heavy (light) quark field and \( |P\rangle \) represents the heavy vector meson state with a certain momentum \( P \) and mass \( M \). Here time-ordering is implicit. \( G(x_1, x_2) \) is a colour matrix chosen to make the B-S amplitude gauge invariant. However the following analysis also applies to non gauge invariant amplitudes.

One can now use the reduction theorem to write the B-S amplitude as

\[
M_{\alpha}^{\beta}(x_1, x_2) = \lim_{P^2 \to M^2} \left( P^2 - M^2 \right) \int d^4 y e^{-i P \cdot y} \langle 0 | \psi_Q(x_1) G(x_1, x_2) \bar{\psi}_q(x_2) \phi^\dagger(y) | 0 \rangle .
\]

(19)

The \( \phi^\dagger \) in (19) is \( \epsilon^\mu \phi^\dagger_\mu \) where \( \phi_\mu \) is given in eq. (12) or eq. (13). Thus one can write

\[
M_{\alpha}^{\beta}(x_1, x_2) = \lim_{P^2 \to M^2} \frac{Z_3^{1/2}(P^2 - M^2)}{V} \int d^4 y e^{-i P \cdot y} \langle 0 | T \psi_Q(x_1) G(x_1, x_2) \bar{\psi}_q(x_2) \chi_\gamma (\partial y) \bar{\psi}_Q(y) \psi_q (y) | 0 \rangle ,
\]

(20)

where \( \chi(\partial y) \) is either \( \gamma \) or \( \frac{i \phi}{M} \gamma \). Correspondingly, \( V \) in eq. (20) is either \( V_1 \) or \( V_2 \).
The statement that one can use any one of the alternative interpolating fields means that they should give the same answers in the reduction theorems. To see what this means consider the second interpolating field, i.e. $\chi(\partial_y) = i\frac{\partial}{\partial \xi}$ in the matrix element (21). Integrating by parts we can shift the $y$ derivative in $\chi$ onto $\exp(-iP \cdot y)$ to obtain

$$M_{\alpha \beta}(x_1, x_2) = \lim_{P^2 \to M^2} Z_{3/2} \frac{(P^2 - M^2)}{V_2} . \int d^4y e^{-iP \cdot y} \langle 0| T\psi_{Q\alpha}(x_1)G(x_1, x_2)\bar{\psi}_q(\gamma)(\phi_q^\dagger(\gamma)\psi_{q\beta}(y)|0) . \right) \right) .$$

(21)

With the other choice of the interpolating field we have instead

$$M_{\alpha \beta}(x_1, x_2) = \lim_{P^2 \to M^2} Z_{3/2} \frac{(P^2 - M^2)}{V_1} . \int d^4y e^{-iP \cdot y} \langle 0| T\psi_{Q\alpha}(x_1)G(x_1, x_2)\bar{\psi}_q(\gamma)(\phi_q^\dagger(\gamma)\psi_{q\beta}(y)|0) . \right) \right) .$$

(22)

Thus comparing we see, that if the two expressions, (21) and (22), are to be equal, then $\gamma = \frac{V_2}{V_1}$, as an operator inside the matrix element, in the mass shell limit. In general this is not much of a restriction, as $V_1$ and $V_2$ are functions of $\nu$. However, we have shown that, in the heavy quark limit, the ratio of the normalisation constants is unity leading to $\gamma = 1$. In fact we see from (21) and (22) that this condition means that only certain components of the quark fields entering in the interpolating fields contribute to the matrix element, i.e those satisfying

$$\bar{\psi}_Q(y)\gamma = \bar{\psi}_Q(y)$$

$$\gamma\psi(y) = -\psi(y) .$$

(23)

within the reduction formula, in the mass shell limit. In other words, in the rest frame of the on-shell meson, only the quark part of the heavy quark field and surprisingly also only the antiquark part of the light quark field contributes to the matrix element. The same relation holds for the pseudoscalar case. Also the same relation will hold in any matrix element involving a heavy meson state when we use the interpolating fields to reduce the state.

It is obvious that this physical requirement will be enforced by taking the interpolating field for the heavy vector meson, in the heavy quark limit, to be

$$\phi_\mu(x) = \frac{1}{2}(\phi_1^\mu + \phi_2^\mu)$$

$$= Z_{3/2} \frac{1}{2V_1} [T\bar{\psi}_q(x)\gamma_\mu^\dagger \psi_{Q}(x) - i\frac{\partial}{\partial \xi} T\bar{\psi}_q(x)\gamma_\mu^\dagger \psi_{Q}(x)] .$$

(24)
In fact even if we start with an arbitrary linear combination of the two fields, only the sum survives in the heavy quark limit.

With this choice the B-S amplitude picks up the correct projection operator $\frac{1+\gamma^\mu}{2}$ to enforce the above condition and can be written in the form

$$M_{\alpha\beta}(x_1, x_2) = \chi_\gamma^\delta(v)A^{\gamma\delta}_{\alpha\beta}(x_1, x_2),$$

(25)

with

$$\chi_\gamma^\delta(v) = \frac{1}{2}[(1 + \gamma^\mu)\not\!v]^\delta_\gamma$$

(26)

and

$$A^{\gamma\delta}_{\alpha\beta}(x_1, x_2) = \lim_{P^2 \to M^2} Z_3^{1/2} \frac{(P^2 - M^2)}{V_1} \int d^4y e^{-iP\cdot y} \langle 0 | T\psi_{Q\alpha}(x_1)G(x_1, x_2)\bar{\psi}^\beta_q(x_2)\bar{\psi}^\gamma_{Q\nu}(y)\psi_q(\gamma) | 0 \rangle.$$ 

(27)

The B-S amplitude for the heavy pseudoscalar particle is also of the above form with $\not\!v$ replaced by $\gamma_5$ and $V_1$ by $N_1$. It is not very useful as it stands because of the unknown matrix function $A^{\gamma\delta}_{\alpha\beta}$. However in the next section we see that when combined with the HQET it leads to the previously proposed [3] - [7] form of the heavy meson B-S amplitude.

### 3 Bethe-Salpeter Amplitudes and HQET

As is well known by now [2], one can go to the zeroth order HQET by simply replacing the QCD heavy fields in eq. (27) by the zeroth order heavy quark effective fields. Thus we shall now simply consider the heavy quark fields, $\psi_Q$, appearing above to be the corresponding fields appearing in the effective theory. One can now decouple these heavy fields through the transformations [3]

$$\psi_Q(x) = W \left[ x \atop v \right] Q(x)$$

(28)

and

$$\bar{\psi}_Q(x) = \bar{Q}(x)W \left[ x \atop v \right]^{-1},$$

(29)

where

$$W \left[ x \atop v \right] = Pexp \left[ ig \int_{-\infty}^x ds A \cdot v \right]$$

(30)

is a path-ordered exponential, wherein the path is a straight line from $-\infty$ to $x$ along the $v$-direction.

\footnotetext[1]{Although this form of the B-S amplitude has been forced on us in the heavy quark limit, we could as well use it for the light meson as after all we are free to choose any linear combination of the two fields with the appropriate normalisation.}

6
Because the transformed fields, $Q$ and $\bar{Q}$, are now decoupled from the gluons, the right hand side of eq. (24) factorises and we can now use the heavy quark propagator

$$\langle 0 | Q^\alpha(x_1) \bar{Q}^\gamma(y) | 0 \rangle = \int d^4 p \left[ \frac{e^{-ip \cdot (x_1 - y)}}{\not{p} \cdot v - m_Q} \right]_\alpha$$

(31)

to write the B-S amplitude as

$$M^\beta_{\alpha}(x_1, x_2) = \lim_{p^2 \to M^2} Z^{1/2}_3 \frac{(P^2 - M^2)}{V_1} \cdot \int d^4 y d^4 p e^{-iP \cdot y} \langle 0 | \bar{\psi}^\beta_q(x_2) W \left[ \frac{x_1}{v} \right] G(x_1, x_2) W \left[ \frac{y}{v} \right]^{-1} \frac{e^{-ip \cdot (x_1 - y)}}{\not{p} \cdot v - m_Q} \chi^\gamma(v) \bar{\psi}^\delta_q(y) | 0 \rangle .$$

(32)

The projection operator $\chi(v)$ now gets rid of the $\not{v}$ in the heavy quark propagator to give

$$M^\beta_{\alpha}(x_1, x_2) = \chi^\alpha_\gamma(v) A^\beta_{\delta}(x_1, x_2) ,$$

(33)

with

$$A^\beta_{\delta}(x_1, x_2) = \lim_{p^2 \to M^2} Z^{1/2}_3 \frac{(P^2 - M^2)}{V_1} \cdot \int d^4 y d^4 p e^{-iP \cdot y} \frac{e^{-ip \cdot (x_1 - y)}}{\not{p} \cdot v - m_Q} \langle 0 | \bar{\psi}^\beta_q(x_2) W \left[ \frac{x_1}{v} \right] G(x_1, x_2) W \left[ \frac{y}{v} \right]^{-1} \bar{\psi}^\delta_q(y) | 0 \rangle .$$

(34)

Eq. (33) is essentially the result we have been looking for. We can now transform this result to momentum space by defining the Fourier transform

$$M^\beta_{\alpha}(p_1, p_2) = \chi^\alpha(v) A^\beta_{\delta}(p_1, p_2) ,$$

(35)

with

$$A^\beta_{\delta}(p_1, p_2) = \int d^4 x_1 d^4 x_2 e^{ip_1 \cdot x_1} e^{ip_2 \cdot x_2} A^\beta_{\delta}(x_1, x_2) .$$

(36)

The B-S amplitude in this form now satisfies the Bargmann-Wigner equation on the 'heavy' label $\alpha$, leading to all the nice results of the heavy quark symmetry.

Further, from translation invariance one can write

$$\langle 0 | \bar{\psi}^\beta_q(x_2) W \left[ \frac{x_1}{v} \right] G(x_1, x_2) W \left[ \frac{y}{v} \right]^{-1} \bar{\psi}^\delta_q(y) | 0 \rangle = \int d^4 q d^4 k e^{-iq \cdot (x_2 - x_1)} e^{-i(k \cdot x_1 - y)} \Delta^\beta_{\delta}(q, k) ,$$

(37)

leading to

$$A^\beta_{\delta}(p_1, p_2) = \lim_{p^2 \to M^2} Z^{1/2}_3 \frac{(P^2 - M^2)}{V_1} \delta^4(P - p_1 - p_2) \int d^4 k \frac{\Delta^\beta_{\delta}(p_2, k)}{v \cdot (P - k) - m_Q} .$$

(38)
One way to generate the physical pole at $P^2 = M^2$ is for the Green’s function
$\Delta(p^2, k)$ to be peaked at $v \cdot k = \Lambda$, where $\Lambda = (M - m_Q)$. In other words
the “longitudinal” mass of the light degrees of freedom is $\Lambda$. This is of course
trivially true in the weak binding or equal velocity approximation, where there
is no transverse momentum and $m_q = \Lambda$.

It is now easy to generalise to arbitrary orbital resonances. One can take as
interpolating fields, for orbital angular momentum $L$, either

$$\phi^1_L(x) = Z_3^{1/2} \frac{1}{L_1} T\bar{\psi}_q(x)\Gamma \psi_Q(x)$$

or

$$\phi^2_L(x) = -Z_3^{1/2} \frac{1}{L_2} i \frac{\partial}{M} T\bar{\psi}_q(x)\gamma^\nu \Gamma \psi_Q(x)$$

where the $\Gamma$ are listed in the table for the four possible spin-parity states. We
have omitted Lorentz indices and the $L_i$ are normalisation constants. Again
using the equations of motion it is easy to show that the difference between
these two possible interpolating fields is always either of the form

$$\bar{\psi}_q \cdots \frac{i}{m_Q + m_q} \frac{\partial}{M} \bar{\psi}_q$$

or

$$\bar{\psi}_q \cdots \frac{i}{m_Q + m_q} \bar{\psi}_q.$$  As usual these are small in the heavy quark limit. (Here
$\frac{i}{M} = \frac{i}{m_Q} - \frac{i}{m_q}.$) Hence in the heavy quark mass limit we find $\phi^1_L(x) = \phi^2_L(x)$ and
$L_1 = L_2$. Thus the rest of the analysis follows in general. One can see that it is
also, in principle, quite easy to generalise to heavy baryons.

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Table Caption.

Lowest gamma structure appearing in interpolating fields for mesonic L-wave states.
\[
\begin{array}{ccc}
\text{state} & J^\text{PC} & \Gamma \\

3L_{L-1} & (L-1)^{(-)^{L+1}}(-)^{L+1} & \not\rightarrow \not\rightarrow \not\rightarrow \not\rightarrow \not\rightarrow \not\rightarrow \not\rightarrow \not\rightarrow \not\rightarrow \not\rightarrow \\
3L_{L} & L^{(-)^{L+1}}(-)^{L+1} & \gamma_5[\not\phi, \gamma_{\mu_1} ] \not\rightarrow \not\rightarrow \not\rightarrow \not\rightarrow \not\rightarrow \not\rightarrow \\
3L_{L+1} & (L+1)^{(-)^{L+1}}(-)^{L+1} & \gamma_{\mu_1} \not\rightarrow \not\rightarrow \not\rightarrow \not\rightarrow \not\rightarrow \not\rightarrow \\
1L_{L} & L^{(-)^{L+1}}(-)^{L} & \gamma_5 \not\rightarrow \not\rightarrow \not\rightarrow \not\rightarrow \not\rightarrow \not\rightarrow \\
\end{array}
\]