Research on Application of Finite Element Method in Static Analysis

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Abstract. The finite element method (FEM) is a numerical calculation method for solving the approximate solution of the boundary value problem of partial differential equations. Because of its simple principle and strong problem-solving ability, finite element analysis has been widely used in the fields of mechanics, electromagnetics, and heat conduction. The advantages are obvious when dealing with complex nonlinear boundary conditions. This article is based on the basic theory of elasticity and combined with the finite element method to design a statics simulation system. The calculation results are based on ANSYS.

Key words: FEM, Statics, ANSYS

1. Introduction
In actual scenarios, the solution of partial differential equations is often very difficult, or even impossible. Therefore, approximate solution has become a new way to solve this kind of problems. The finite element method is a numerical calculation method for approximately solving partial differential equations. Its core idea is to divide a model into a finite set of elements of the same type (Fig. 1 shows the use of tetrahedral elements to divide a three-dimensional model). By using basis functions to simulate the displacement characteristics of each node, combining the principle of virtual displacement and the principle of minimum potential energy to establish a mechanical equilibrium equation, the boundary value problem is transformed into an equation problem, and the equation solving can be further transformed into a matrix problem. Matrix analysis is very suitable for computer processing. Therefore, many scholars in related fields have invested in the research of commercial software based on finite element analysis, and mature finite element commercial software such as ANSYS and ABAQUS have appeared.
2. Second order basis function

Relevant research results show that the accuracy of finite element solution is related to the number of meshes and the selection of basis functions. The principle of finite element convergence shows that as the density of the grid increases, the finite element solution results will infinitely become more accurate. Similarly, the improvement of the accuracy of the basis function can effectively improve the accuracy of the finite element solution. However, the increase in the number of grids and the increase in the accuracy of the basis function will bring serious computational burden to the computer. Therefore, in the design of commercial software, second-order basis functions are usually used to simulate the displacement characteristics of each node[1],[2].

On the basis of the conventional tetrahedral element, add a node at the midpoint of each edge of the tetrahedron to form a ten-node tetrahedral element. In three-dimensional problems, the following displacement functions can be used to characterize the displacement characteristics of the nodes.

\[
\begin{align*}
    u &= \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 z + \alpha_5 x^2 + \alpha_6 y^2 + \alpha_7 z^2 + \alpha_8 xy + \alpha_9 yz + \alpha_{10} zx \\
    v &= \alpha_{11} + \alpha_{12} x + \alpha_{13} y + \alpha_{14} z + \alpha_{15} x^2 + \alpha_{16} y^2 + \alpha_{17} z^2 + \alpha_{18} xy + \alpha_{19} yz + \alpha_{20} zx \\
    w &= \alpha_{21} + \alpha_{22} x + \alpha_{23} y + \alpha_{24} z + \alpha_{25} x^2 + \alpha_{26} y^2 + \alpha_{27} z^2 + \alpha_{28} xy + \alpha_{29} yz + \alpha_{30} zx
\end{align*}
\]

(1)

It can be seen that there are 30 undetermined coefficients in the displacement function of the element, and 30 parameters are needed to fully determine the coefficients. There are ten nodes in a ten-node tetrahedral element, and each node has three parameters, which can just determine the displacement. The coefficient in the function[3].

In the ten-node tetrahedral element, in order to simplify the calculation formula, we introduce volume coordinates.

In Fig. 3., the position of any point P in the tetrahedron can be determined by the following four ratios.

\[
L_1 = \frac{V_1}{V}, \quad L_2 = \frac{V_2}{V}, \quad L_3 = \frac{V_3}{V}, \quad L_4 = \frac{V_4}{V}
\]

(2)

These four ratios are called the volume coordinates of point P. Since \(V_1+V_2+V_3+V_4=1\), \(L_1+L_2+L_3+L_4=1\). In the formula derivation process of the ten-node tetrahedral element, the most used one is the integral of the volume coordinate[4].
\[ \iiint \limits_{\epsilon} L^a L^b L^c L^d dx dy dz = 6V \frac{a!b!c!d!}{(a+b+c+d+3)!} \quad (3) \]
\[ N_1 = L_1, N_2 = L_2, N_3 = L_3, N_4 = L_4 \quad (4) \]

3. Case Analysis

![Fig. 3 Using tetrahedral mesh for model division](image)

In the actual situation, the bottom of the part is fixed, that is, the zero displacement boundary condition is applied, and the top ring position is subjected to downward force. The specific data is that the elastic modulus of the part material is \( E=1e \times 10 \), the Poisson's ratio \( \nu=0.25 \), the zero displacement boundary condition is applied to the bottom surface, and the surface load of 20Mpa is applied to the top surface. Use our program to calculate the deformation under this condition.

![Fig. 4 Displacement field map distribution. (a) The result of our program. (b) The result of ANSYS.](image)

We continue to refine the mesh of the model to study whether the calculation program can effectively converge for different mesh densities.

![Fig. 5 Convergence of ANSYS, first-order basis function and second-order basis function](image)
Through the analysis results, we know that our program can effectively converge when using the second-order basis function, and the calculation results are in good agreement with the ANSYS simulation results. The first-order basis function has a certain error.

4. Conclusion
Through experiments, it can be found that when using the second-order basis function to simulate the displacement characteristics of the nodes, good results are obtained, which are almost consistent with the simulation results of ANSYS, and can converge quickly. Due to the poor accuracy of the first-order basis function, its convergence speed is relatively slow. The second-order basis functions can simulate nodal displacements well and take into account calculation performance, so they are widely used in related commercial software.

References
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