Generation of terahertz radiation in the interaction of coherent electromagnetic wave with a resonant medium in a constant external electric field

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Abstract. The paper shows that it is sufficient to place a resonance medium in an external dc electric field for the generation of low-frequency radiation at the resonant interaction of the medium and a coherent wave with carrier frequency of the optical range. Under certain conditions the carrier frequency of the generated radiation equals the Rabi frequency of the resonant interaction.

1. Introduction
Practical applications have encouraged the study of different ways to generate low-frequency radiation, whose carrier frequencies are in the terahertz range. For practical purposes different sources of terahertz radiation are needed - intensive and not really intensive but having the possibility of tuning the carrier frequency. In resonant processes environments with a permanent dipole moment are actively investigated [1–6]. This report shows that low-frequency radiation of low intensity but of easily tunable carrier frequency is not difficult to obtain in any resonance process in environments with no permanent dipole moment when a resonant medium is additionally irradiated with a pulse of a quasipermanent electric field, for example, by placing a resonant medium in the condenser. The generation of low-frequency radiation near the leading edge of the resonant excitation pulse of the optical range is considered and values for the intensity of the generated field and its carrier frequency are obtained. The results follow from consideration of the resonant interaction in view of an external electric field on the basis of the algebraic theory of perturbations.

2. Formulation of the task and basic equations
Assume that a coherent electromagnetic wave propagates in the direction of the axis \( Z \), with the intensity \( E \) of the electric field being represented in the form of a slowly varying amplitude \( \mathcal{E} \) and a rapidly changing phase defined by the wave vector \( \vec{k} \parallel Z \) and the carrier frequency \( \omega \):

\[
E = \mathcal{E} \exp[i(kz - \omega t)] + c.c. \equiv \mathcal{E} \exp[-i\Phi] + c.c. \tag{1}
\]

The area of space \( 0 \leq Z \leq L \) is filled with the environment consisting of particles of the wave resonant (1) energy levels \( |g\rangle \) and \( |e\rangle \), with the level \( |e\rangle \) being treated as the excited one.
Frequency \( \omega_{eg} \) of transition \( |e\rangle \rightarrow |g\rangle \) is close to frequency \( \omega \), \( |\omega_{eg} - \omega| \ll \omega \), and the very transition is optically permitted, i.e. in states \( |g\rangle \) and \( |e\rangle \) the dipole matrix element is equal to zero \( d_{ge} = d_{eg} = 0 \).

If one neglects the interaction between particles and relaxation, then the dipole moment of the environment volume unit \( P \) can be represented as

\[
P = n \langle \Psi | d | \Psi \rangle,
\]

where \( n \) is the density of resonant particles in the volume unit, \( d = \sum_{kj} d_{kj} |E_k \rangle < E_j | \) is the dipole moment operator of a resonant particle, \( |E_k \rangle \) is a quantum state of a resonant particle with energy \( E_k \), and \( |\Psi \rangle \) is its statevector. The polarization of the environment appears in the Maxwell equation, which is to be written in the unidirectional approximation \[7\]

\[
\frac{\partial}{\partial t} + \frac{1}{c} \frac{\partial}{\partial z} E = - \frac{2 \pi}{c} \frac{\partial}{\partial t} P.
\]

Neglecting relaxation and spontaneous emission of particles in the electric dipole approximation of the interaction of particles with a coherent field, the Schrödinger equation for the state vector of a resonant particle is conventional

\[
\frac{i \hbar}{d} \frac{d}{dt} |\Psi\rangle = (H_0 - E_d - E_c d) |\Psi\rangle,
\]

where \( H_0 \) is the Hamiltonian of an isolated particle, \( E_c \) is the intensity of dc field which is additionally applied to the environment.

Standard unitary transformation \[8\]

\[
|\tilde{\Psi}\rangle = e^{-iQ} |\Psi\rangle, \quad Q = Q^*
\]

allows substantiating resonant approximation and defining corrections for the environment polarization with respect to the permanent electric field. The Schrödinger equation for the transformed state vector of resonant atoms: \( i \hbar \frac{d}{dt} |\tilde{\Psi}\rangle = \tilde{H}^{(1)} |\tilde{\Psi}\rangle \) is defined by the effective Hamiltonian (\( \sigma \) are the Pauli matrices)

\[
\tilde{H}^{(1,0)} = \frac{\omega_{eg}}{2} \sigma_3 - \sigma_\sigma^* \exp[-i\Phi] d_{bd} \sigma_+ - \sigma_\sigma^* \exp[i\Phi] d_{bd} \sigma_-
\]

and concurs with the equation for the wave function of a two-level particle in the approximation of a rotating wave \[8\], and environment polarization (2) at low frequencies is defined by the transformed operator of the atomic dipole moment

\[
\tilde{d} = e^{-iQ} d e^{iQ} = d - i[Q, d] - \frac{1}{2} [Q, [Q, d]] + ... = \sum_{kj} \tilde{d}_{kj} |E_k \rangle < E_j |.
\]

Use is made of the expansion of the transformed Hamiltonian \( \tilde{H} \) in powers of constants of interaction of a resonant particle of quantum system with coherent and permanent electric fields whose order is marked by upper indices (the left index is the interaction with a coherent field) \[8\]:

\[
Q(t) = Q^{(1,0)}(t) + Q^{(0,1)}(t) + ... \quad \tilde{H} = \tilde{H}^{(1,0)}(t) + \tilde{H}^{(0,1)}(t) + ...
\]
Low-frequency radiation is defined by the operator \( \omega_{jk} = (E_j - E_k) / \hbar \)

\[
Q^{(0,1)} = E_i \sum_{jk} \frac{d_{jk}}{\hbar \omega_{jk}} |E_j| \sim |E_k|, 
\]

in consequence of which an atom receives permanent dipolar moments in an electric field

\[
\tilde{d}_{ee} = -E_c \sum_j \frac{|d_{ej}|^2}{\hbar \omega_{ej}}, \quad \tilde{d}_{gg} = -E_c \sum_j \frac{|d_{gg}|^2}{\hbar \omega_{gg}}.
\]

(5)

Recall that the states are characterized by a certain parity, that is why in summations (5) \( d_{ee} = d_{gg} = 0 \) so much as the corresponding terms are equal to zero.

Both permanent electric and resonant coherent fields define the Stark shift of resonant energy levels. Its magnitude is defined by the terms \( \tilde{H}^{(2,0)} \) and \( \tilde{H}^{(1,2)} \) in the effective Hamiltonian, which, however, should be neglected as they give only corrections for the investigated effect whose emergence is fully defined by the obtained corrections (4), (5). They, in their turn, determine environment polarizations at a low frequency

\[
P = n < \tilde{\Psi} | \tilde{d} | \tilde{\Psi} >
\]

which provides for low-frequency generation in the process of resonant interaction [5,6].

3. Frequency control of the generated low-frequency radiation

It is well known that near the leading edge of the rectangular resonant pulse, as it propagates in the resonant medium, there occur nutational oscillations of pulse intensity due to re-emission of photons in the resonant wavelength. This phenomenon is known as optical nutation and it was repeatedly investigated earlier [8]. At the same time, populations of resonant levels are also subjected to nutational oscillations defined by the Rabi frequency

\[
\Lambda = 2 |ad_{eg}| / \hbar ,
\]

which is the interaction energy of a coherent wave with a resonant particle.

Emergence of the dipole moment of the particles (4), (5) causes oscillations in the polarization of the medium at the Rabi frequency in resonance conditions

\[
P_{\Lambda} = \frac{1}{2} nE_i \left( \sum_j \frac{|d_{ej}|^2}{\hbar \omega_{ej}} - \sum_j \frac{|d_{gg}|^2}{\hbar \omega_{gg}} \right) \cos \Lambda t .
\]

In the case of optically thin media of the length \( L \), this leads to the following expression for the electric field intensity of the radiation generated at the Rabi frequency beyond the resonant medium

\[
E_{\Lambda}(t,z) = \frac{\pi n \Lambda L}{c} \left( \sum_j \frac{|d_{ej}|^2}{\hbar \omega_{ej}} - \sum_j \frac{|d_{gg}|^2}{\hbar \omega_{gg}} \right) \sin \Lambda (t - \frac{z}{c}),
\]

(6)

from which the equality of carrier frequency of low-frequency radiation and the Rabi frequency \( \Lambda \) is evident. In order to generate low-frequency radiation in the terahertz range \( \Lambda \sim 10^{12} \) \( s^{-1} \) exciting resonant fields with the intensity of the order of \( I \sim 10^8 \) W/cm² are needed for typical atomic and molecular media. At the same time, an increase in intensity of this radiation can be achieved in ensembles of quantum dots and other artificial objects in which the values of dipole moments are significant, and the frequency of transitions can be reduced during production.
Typically, frequencies of resonance transitions are scattered near some average transition frequency $\omega_0$. Then it is not difficult to generalize (6)

$$E_\Lambda(t,z) = \frac{\pi nL}{c} \left( \sum_j \frac{|d_{ij}|^2}{\hbar \omega_{ij}} - \sum_j \frac{|d_{ij}|^2}{\hbar \omega_{glj}} \right) \langle \frac{\Lambda}{\Omega} \sin \Omega(t - \frac{z}{c}) \rangle.$$

Here the angle brackets denote averaging over the specified frequency spread with some distribution function $F(\Delta)$, $\Omega = \sqrt{\Lambda^2 + \Delta^2}$.

In general, the carrier frequency of the generated radiation will not be strictly equal to the Rabi frequency and the frequency spectrum occurs near the Rabi frequency. However, importance of this spectrum will be determined by the Rabi frequency quite unambiguously. For example, in the case of symmetrical spread of the Gaussian form, intensity of electric field of low-frequency generation is determined by the Bessel function of order zero:

$$E_\Lambda(t,z) = \frac{\pi^{3/2} nL \Lambda^2 T_0}{c} \left( \sum_j \frac{|d_{ij}|^2}{\hbar \omega_{ij}} - \sum_j \frac{|d_{ij}|^2}{\hbar \omega_{glj}} \right) (J_0(\Lambda(t - \frac{z}{c})) + \Phi_{\text{Error}}((t - \frac{z}{c})/T_0) - 1).$$

Here $1/T_0$ is the frequency width of spread which is supposed to be much larger than that of the Rabi frequency, $\Phi_{\text{Error}}(t)$ is the error function integral.

To experimentally investigate the proposed mechanism of generation of low-frequency radiation, the method of Stark pulse was previously proposed [9,10]. In addition, resonant frequency spread of particles leads to damping of low-frequency radiation despite the fact that the radiation exciting it continues to exert on the resonance environment. Here the signals of low-frequency radiation will emerge near the front and near the edges of the Stark pulses of a permanent electric field. Emergence of a nutational echo signal [10] at low frequency is also possible, which can simultaneously serve as a source of a variety of spectroscopic information. Another way of providing effective resonant interaction of medium with an exciting field of allegedly rectangular shape is the method of tuning the exciting radiation frequency [11,12].

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