A model of accelerating dark energy in decelerating gravity

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ABSTRACT

Context. The expansion of the Universe is accelerated as testified by observations of supernovae of type Ia at varying redshifts. Explanations of this acceleration are of two kinds: modifications of Einstein gravity or new forms of energy, coined dark energy. An example of modified gravity is the braneworld Dvali-Gabadadze-Porrati (DGP) model, an example of dark energy is Chaplygin gas. Both are characterized by a cross-over length scale \( r_c \) which marks the transition between physics occurring on our four-dimensional brane, and in a five-dimensional bulk space.

Aims. Assuming that the scales \( r_c \) in the two models are the same, we study Chaplygin gas dark energy in both self-accelerating and self-decelerating flat DGP geometries. The self-accelerating branch does not give a viable model, it causes too much acceleration.

Methods. We derive the Hubble function and the luminosity distance for the self-decelerating branch, and then fit a compilation of 192 SNeIa magnitudes and redshifts. This determines a confidence region in the space of the three parameters of the model.

Results. Our model with the self-decelerating branch in flat space fits the supernova data as successfully as does the \( \Lambda \)CDM model, and with only one additional parameter.

Conclusions. In contrast to the \( \Lambda \)CDM model, this model needs no fine-tuning, and it can explain the coincidence problem. It is unique in the sense that it cannot be reduced to a cosmological constant model in any other limit of the parameter space than in the distant future. If later tests with other cosmological data are successful, we have here a first indication that we live in a five-dimensional braneworld.

Key words. cosmology– dark energy

1. Introduction

The demonstration by SNeIa that the Universe is undergoing an accelerated expansion has stimulated a vigorous search of models to explain this unexpected fact. Since the dynamics of the Universe is conventionally described by the Friedmann equations which follow from the Einstein equation in four dimensions, all modifications ultimately affect the Einstein equation.

The left-hand-side of the Einstein equation encodes the geometry of the Universe in the Einstein tensor \( G_{\mu\nu} \), the right-hand-side encodes the energy content in the stress-energy tensor \( T_{\mu\nu} \). Thus modifications to \( G_{\mu\nu} \) imply some alternative geometry, modifications in \( T_{\mu\nu} \) involve new forms of energy densities that have not been observed, and which therefore are called dark energy.

A well-studied model of modified gravity is the Dvali-Gabadadze-Porrati (DGP) braneworld model (Dvali 2000, Deffayet 2001) in which our four-dimensional world is a FRW brane embedded in a five-dimensional Minkowski bulk. The model is characterized by a cross-over length scale \( r_c \) such that gravity is a four-dimensional theory at scales \( a \ll r_c \) where matter behaves as pressureless dust. In the self-accelerating DGP branch gravity "leaks out" into the bulk at scales \( a \approx r_c \) and the cosmology approaches the behavior of a cosmological constant. To explain the accelerated expansion which is of recent date (\( z \approx 0.5 \) or \( a \approx 2/3 \)), \( r_c \) must be of the order of 1. In the self-decelerating DGP branch gravity "leaks in" from the bulk at scales \( a \gg r_c \), counteracting the observed dark energy acceleration.

Another well-studied model introduces into \( T_{\mu\nu} \) the density \( \rho_x \) and pressure \( p_x \) of a fluid called Chaplygin gas (Kamenshchik 2001, Bilic 2002) following historical work in aerodynamics (Chaplygin 1904). This model is similar to the DGP model in the sense that it is also characterized by a cross-over length scale below which the gas behaves as pressureless dust, and above which it approaches the behavior of a cosmological constant. This length scale is expected to be of the same order of magnitude as the \( r_c \) scale in the DGP model.

Both the self-accelerating DGP model in flat space and the standard Chaplygin gas model have problems fitting present supernova data, as demonstrated by Davis (2007). In the standard Chaplygin gas model the Jeans instability of perturbations behaves like CDM fluctuations in the dust-dominated stage \( (a \ll r_c) \), but disappears in the acceleration stage \( (a \gg r_c) \). The combined effect of suppression of perturbations and non-zero Jeans length leads to a strong ISW effect and thus of loss of power in CMB anisotropies (Amendola 2003, Bento 2003). This has led to generalizations to higher-dimensional braneworld models which appear less motivated, and which require more parameters.

We, instead, combine the standard DGP model with the standard Chaplygin gas model.

This paper is organized as follows. In Section 2 we discuss how to identify the cross-over scales in the DGP and Chaplygin gas models. This idea is motivated by the similarities in the asymptotic properties of the models, and was first presented in (Roos 2007). In Section 3 we discuss the flat-space self-accelerating basic DGP model with and without standard Chaplygin gas dark energy, and in Section 4 we turn to the self-decelerating DGP model combined with standard Chaplygin gas.
In Section 5 we summarize our results, and in Section 6 we discuss them and conclude.

2. Cross-over scales

On the four-dimensional brane in the DGP model, the action of gravity is proportional to $M_P^2$, whereas in the bulk it is proportional to the corresponding quantity in 5 dimensions, $M_5^2$. The cross-over length scale is defined as

$$r_c = \frac{M_P^2}{2M_5^2}. \tag{1}$$

It is customary to associate a density parameter with this, $\Omega_r = (2\pi r_c H_0)^{-2}$. \tag{2}

The Friedmann equation in the DGP model may be written (Deffayet 2001)

$$H^2 - \frac{k}{a^2} - \frac{1}{r_c} \sqrt{H^2 - \frac{k}{a^2}} = \kappa \rho_c, \tag{3}$$

where $a = (1 + z)^{-1}$, $\kappa = 8\pi G/3$, and $\rho$ is the total cosmic fluid energy density $\rho = \rho_m + \rho_c$. Clearly the standard FRW cosmology is recovered in the limit $r_c \to \infty$ and $\rho_c \to 0$. In the following we shall only consider $k = 0$ flat geometry. The self-accelerating branch corresponds to $\epsilon = +1$, the self-decelerating branch to $\epsilon = -1$.

Since ordinary matter does not interact with Chaplygin gas, one has separate continuity equations for the energy densities $\rho_m$ and $\rho_c$, respectively. In DGP geometry the continuity equations have the same form as in FRW geometry (Deffayet 2001).

$$\rho + 3H(\rho + p) = 0. \tag{4}$$

Pressureless dust with $p = 0$ then evolves as $\rho_m(a) \propto a^{-3}$.

The Chaplygin gas pressure is $p_c = -A/\rho_c$, where $A$ is a constant with the dimensions of energy density squared. The continuity equation for Chaplygin gas is then

$$\rho_c + 3H(\rho_c - \frac{A}{\rho_c}) = 0, \tag{5}$$

which integrates to

$$\rho_c(a) = \sqrt{A+B/a^6}, \tag{6}$$

where $B$ is an integration constant. Thus this model has two free parameters. Obviously its limiting behavior is

$$\rho_c(a) \propto \sqrt{B/a^3} \text{ for } a \ll \left(\frac{B}{A}\right)^{1/6}, \rho_c(a) \propto -p \text{ for } a \gg \left(\frac{B}{A}\right)^{1/6}. \tag{7}$$

Our first central assumption is that the DGP model cross-over scale $r_c$ and the Chaplygin gas cross-over scale $(B/A)^{1/6}$ are about the same. If we choose the proportionality

$$\left(\frac{B}{A}\right)^{1/6} = r_c H_0 = 2 \sqrt{\Omega_c}, \tag{8}$$

this permits rewriting Eq. (6) in a dimensionless form. Making use of Eq. (2), $\rho_c$ becomes

$$\rho_c(a) = H_0^{-1} \left[\Lambda a^{-3} + \int_0^a \left(1 + (4\Omega_c a^2)^{-3}\right)^{1/2} \right], \tag{9}$$

where we have replaced the energy density $\sqrt{A}$ by the dimensionless density parameter $\Omega_A = H_0^{-1} \sqrt{\Lambda}$.

The identification of the two cross-over scales evidently reduces the number of free parameters in Eq. (3) by one: they are $\Omega_c$, $\Omega_A$ and $\Omega_m = \kappa \rho_m H_0^{-2}$.

3. DGP gravities with and without Chaplygin gas

Let us now return to the Friedmann equation (3) and solve it for the expansion history $H(a)$. Substituting $\Omega_A$ from Eq. (2), $\rho_c(a)$ from Eq. (9), and $\Omega_m = \Omega_0$, it becomes

$$H(a) = \epsilon \sqrt{\Omega_r} \left[\Omega_c + \Omega_0^2 a^{-3} + \Omega_\Lambda \sqrt{1 + (4\Omega_r a^2)^{-3}}\right]^{-1/2}. \tag{10}$$

Note that $\Omega_r$ and $\Omega_\Lambda$ do not evolve with $a$. In the limit $a \ll r_c$, this equation reduces to two terms which evolve as $a^{-3}$, thus behaving as dust with density parameter $\Omega_m^0 + \Omega_\Lambda/(4\Omega_r)^{3}$. In the limit $a \gg r_c$, Eq. (10) describes a model with a cosmological constant $\Omega_\Lambda \equiv -\sqrt{\Omega_c} + \sqrt{\Omega_r} + \Omega_\Lambda$.

At present, when $a = 1$ and $H = H_0$, we solve it for $\Omega^0_m$:

$$\Omega^0_m = 1 - 2\epsilon \sqrt{\Omega_r} - \Omega_\Lambda \sqrt{1 + (4\Omega_r a^2)^{-3}}. \tag{11}$$

In the well-studied standard self-accelerating DGP model, $\epsilon = +1$ and $\Omega_\Lambda = 0$, so that

$$\Omega^0_m = 1 - 2\sqrt{\Omega_r}. \tag{12}$$

This equation represents the condition for flatness, and corresponds to the linear relation $\Omega^0_m + \Omega_\Lambda = 1$ in the $\Lambda$CDM model. Here, however, Eq. (12) is nonlinear, causing $\Omega^0_m$ always to be smaller than in the $\Lambda$CDM model. This is the reason why the standard self-accelerating DGP model is a worse fit to SNeIa data than the $\Lambda$CDM model (cf. eg. Davis 2007, Rydenbey 2007). The failure has led to studies of various generalized DGP models implying higher-dimensional bulk spaces and additional free parameters that detract from its original simplicity and elegance.

The inclusion of the Chaplygin gas term with $\Omega_\Lambda > 0$ in Eq. (11) leads a further reduction in the value of $\Omega^0_m$, and thus to an even worse fit to SNeIa data. DGP self-acceleration and Chaplygin gas dark energy simply yield too much acceleration, separately as well as in combination. In the next Section we therefore turn to what represents our second central assumption, self-decelerating DGP gravity with $\epsilon = -1$. Its expansion history is still given by Eq. (10) and its flat-space condition by Eq. (11). The physics then changes, as one can see best in Eq. (11), where the two last terms get opposite signs. (The case with $\Omega_r = 0$ is not interesting here, because it does not lead to any acceleration.)

4. Data and method of analysis

The data we use to test this model are the same 192 SNeIa as in the compilation used by Davis 2007, which is a combination of the "passed" set in Table 9 of Wood-Vasey 2007 and the "Gold" set in Table 6 of Riess 2007.

We are sceptical about using CMB and BAO power spectra, because they have been derived in FRW geometry. SNeIa data are, however, robust for our analysis, since the distance moduli are derived from light curve shapes and fluxes, that do not depend on the choice of cosmological models. In one of our fits we nevertheless include a value for $\Omega^0_m$ as a Gaussian prior which we take from Table 2 of Tegmark 2006, who has obtained it in a multi-parameter fit to WMAP and SDSS LRG data. Tegmark’s value is $\Omega^0_m = 0.239 + 0.018 / - 0.017$, but we do not use these 1σ errors which have been obtained by marginalizing over all other parameters, and which would constrain our fit too strongly.

We take the $\Omega_m$ prior to have a large error, $\Delta\Omega^0_m = 0.09$, in order not to bias the conclusions from the SNeIa data set. We do
not marginalize, but quote full three-dimensional confidence regions: a 1σ error then corresponds to a contour at $\chi^2_{best} + 3.54$ around the best value $\chi^2_{best}$.

The Davis’ compilation lists magnitudes $\mu_i$, magnitude errors $\Delta\mu_i$ for SNeIa at redshifts $z_i$, $i = 1, 192$. We compute model magnitudes

$$\mu(z_i, \Omega_m, \Omega_\Lambda, \Omega_A) = 5 \log[d_L(z_i, \Omega_m, \Omega_\Lambda, \Omega_A)] + 25,$$

where the luminosity distance in Mpc at redshift $z_i$ is

$$d_L(z_i, \Omega_m, \Omega_\Lambda, \Omega_A) = (1 + z_i) \int_0^{\infty} \frac{dz}{H(z)},$$

where $H(z)$ is given by Eq. (10) with $\epsilon = -1$.

We then search in the parameter space for a minimum of the $\chi^2$ sum

$$\chi^2 = \sum_{i=1}^{192} \left( \frac{\mu_i - \mu(z_i, \Omega_m, \Omega_\Lambda, \Omega_A)}{\Delta\mu_i} \right)^2 + \frac{0.24 - \Omega_m}{\Delta\Omega_m}^2.$$  

Occasionally we do not include the last term.

The calculations are done with the classical CERN program MINUIT (James and Roos 1975) which delivers $\chi^2$ per parameter, error contours and parameter correlations.

5. Results

Our first fit to determine the region in $\Omega_m^0, \Omega_\Lambda, \Omega_A$-space, required by the SNeIa data, keeps all the parameters free, except that $\Omega_\Lambda^0$ is restricted to positive values and an upper limit is imposed on $\Omega_A$. We do not include the last term in Eq. (15). We find a solution at

$$\Omega_m^0 = 0.40^{+0.13}_{-0.12}, \Omega_\Lambda^0 = 1.22^{+0.26}_{-0.71}, \Omega_A^0 = 3.7^{+1.0}_{-1.0}$$

with $\chi^2 = 195.1$. We have checked that the standard $\Lambda$CDM model fits the same data with the (insignificantly) higher value $\chi^2 = 195.6$ (as was also found by Davis 2007).

Since the confidence region exceeds the limits fixed for $\Omega_m^0$ and $\Omega_\Lambda$, obviously $\chi^2$ is very insensitive to values of these parameters near their limits. Moreover, the $|\Omega_m^0, \Omega_\Lambda^0|$, correlation coefficient is 0.97, thus one of those parameters is almost superfluous. In that sense, our model appears almost as a two-parameter model. One could actually fix $\Omega_A$ at an arbitrary value, but that would be an assumption ad hoc.

To cure these fitting problems, we proceed instead to include the last term in Eq. (15) as a weak prior, choosing $\Delta\Omega_m$ as large as possible, in this case $\Delta\Omega_m = 0.09$. This has the effect of neatly putting the error contours well inside all imposed (and now unnecessary) limits, and reducing the correlation coefficient, $|\Omega_m^0, \Omega_\Lambda^0|$ to 0.87. We now find as best solution the parameter values

$$\Omega_m^0 = 0.26 \pm 0.16, \Omega_\Lambda^0 = 0.82^{+0.69}_{-0.22}, \Omega_A^0 = 2.21^{+0.50}_{-0.22}$$

with $\chi^2 = 195.5$.

In Fig.1 we plot the best fit confidence region in the $(\Omega_m^0, \Omega_\Lambda^0, \Omega_A^0)$-plane, a banana-shaped closed contour. A cross in the Figure marks the point of best fit.

6. Discussion and conclusions

To learn how well our three-parameter model fits the confidence region determined by the data, we turn to the flat-space condition, Eq. (11). Here this condition is a surface in the $\Omega_m^0, \Omega_\Lambda, \Omega_A$-space which, if our model is successful, should cut the banana-shaped confidence region in Fig.1. Unfortunately the exact value of $\Omega_A$ is not known, so we must draw Eq. (11) for several values. Obviously the model is a good fit to the observational data when $\Omega_A$ is within the 1σ range quoted in Eq. (17).

Substituting the solution from Eq. (17) into Eq. (2), the value of the cross-over scale is $r_c = 0.55/\sqrt{H_0}$. The relation Eq. (8) was a conjecture that could well have been different by some numerical factor. Then the fitted values of the free parameters would have changed, but a good fit could still have been obtained. Thus we do not consider that the relation Eq. (8) is a fine-tuning. In contrast to the $\Lambda$CDM model, to the Quintessence model, and to many other models, the present model does not imply any fine-tuning.

To compare models by using Information Criteria BIC (Schwarz 1978) or AIC (Akaike 1974) as do Davis 2007 we consider extremely crude. The reason is that no information on parameter correlations is included. If one parameter pair has a correlation coefficient near 0.99, it should be counted as a single parameter.

It is easy to explain the coincidence problem in this model. It is caused merely by the ratio of the scales of the action, the Planck scale $M_P$ on our brane and the bulk scale $M_5$. These constants just happen to have particular time-independent values which determine the DGP cross-over scale $r_c$ by the definition (1).

Our model should still be tested against other cosmological data, but this has to wait until CMB and BAO power spectra have been derived for five-dimensional braneworld cosmology. As for ISW data, the problems encountered in the simplest Chaplygin gas model are alleviated if not eliminated by the presence of DGP self-deceleration.

If this model meets all criteria, we have here a first indication that we live in a five-dimensional braneworld.

Note added in print. Recently cosmologies embedding the generalized Chaplygin model in self-decelerating DGP gravity have been studied elsewhere (Bouhmadi-López & Lazkoz 2007).

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Fig. 1. The closed contour is the confidence region in the $(\Omega^0_m, \Omega^0_r)$-plane from a fit to SNeIa data. The point of best fit is marked by a cross. The curves crossing from left to right correspond to the flat-space condition Eq. (11) for the upper 1σ value (top), the central value (middle) and the lower 1σ value (bottom) of the parameter $\Omega_A$.

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