Analysis of Discrete Symmetries in b-Baryon Decays

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ABSTRACT: A study of the decay channels $\Lambda_b \to \Lambda V$, where $V$ is a vector meson ($1^-$), has been done by putting together kinematical and dynamical analysis. An intensive use of the helicity formalism is involved on the kinematical side, while on the dynamical side, Heavy Quark Effective Theory (HQET) is applied to calculate the hadronic matrix elements between the baryons $\Lambda_b$ and $\Lambda$. The branching ratios ($BR$) and helicity asymmetry parameters ($\alpha_{AS}$) for $\Lambda_b \to \Lambda J/\psi$, $\Lambda_b \to \Lambda \rho^0$ and $\Lambda_b \to \Lambda \omega$ have been calculated. Since both the decay products are polarized, so they offer interesting opportunities to perform tests of time reversal, CP violation and of CPT invariance. A model independent parametrization is done via spin density matrix of the angular distribution, polarizations and some of polarization correlations of the decay products. The transverse component of the polarization and two polarization correlations are sensitive to time reversal violations. Moreover several CP- and CPT-odd observables are pointed out.

KEYWORDS: Baryon Decays, Flavor Physics, CP violation, TR violation, CPT, HQET.
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1. Introduction

The quest of man to understand the nature and the phenomena occurring in the universe is as old as the human history. His curiosity led him to modern sciences to abstract the answer but he is still in search to answer the unanswered questions in the nature. Particle physics is the study of the fundamental constituents of matter and the forces governing them. It endeavors to answer the questions: What are the fundamental constituents of matter? How they interact? What is the nature of these interactions (forces)? How these constituent particles are different from each other? Why the interactions are different on different scales? Can these be unified? And several other questions essential for our understanding of the Universe.

It is now well established that the fundamental constituent particles are quarks and leptons. Quarks are of six flavors namely: up($u$), down($d$), strange($s$), charmed($c$), bottom($b$) and top($t$) with different quantum numbers associated with them. Leptons are also of six flavors, to be exact: electron($e$), muon($\mu$), tau($\tau$), electron neutrino($\nu_e$), muon neutrino($\nu_\mu$) and tau neutrino($\nu_\tau$), having different physical properties. All quarks are fractionally charged while leptons are integrally charged except neutrinos. All fundamental particles have their anti-particles with opposite quantum numbers.

These elementary particles experience the four fundamental forces of nature. These forces are the electromagnetic, strong nuclear, weak nuclear, and gravity. Electromagnetic force occurs via exchange of a photon and is experienced by charged particles. Strong nuclear force occurs by the exchange of gluons and is accountable for the stability of nuclei. Weak nuclear force is mediated by three particles known as $W^+$, $W^-$, and $Z^0$ and is responsible for the radioactive decays. Gravity is presumably mediated by a graviton and is experienced by massive particles. All of these force mediators are bosons having integer intrinsic spin. Electromagnetic and gravity have infinite range because their propagators are massless while strong and weak forces are short range as their mediators are massive. The relative strengths of strong nuclear force, electromagnetic, weak nuclear force and gravity are in the order of $1 : 10^{-2} : 10^{-7} : 10^{-40}$ respectively. The universality of these interactions implies that they are gauge forces.

All of the constituent particles of matter and forces governing them are put in a nutshell known as ‘Standard Model’ and it is the only experimentally tested model so far. The Standard Model classify all quarks and lepton into three generations. The first generation

$$\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} \nu_e \\ e \end{pmatrix}$$

is pertinent for the visible universe and the life on earth. The second and third generations

$$\begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$$

and

$$\begin{pmatrix} t \\ b \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$
do not exist naturally but can be created either in laboratory or in cosmic rays by a collision of particles of first generation.

Standard Model satisfactorily explains most of the observable phenomena in elementary particle physics. Gravity, being incredibly weak compared to the other forces is not important while studying microscopic particles and is not elucidated by the standard model. Standard Model incorporates three forces of nature including strong nuclear force, electromagnetic and weak nuclear force. It is generally believed that the Standard Model will be a part of final theory which unifies all the forces, known as Theory of Everything, which combines standard Model with Gravity.

One of the area of standard model, which is still poorly tested experimentally and has the potential to provide indications of new physics, is the physics of weak decays of heavy hadrons. The heavy hadrons contain quarks $c$ and $b$ and other lighter quarks $u$, $d$ and $s$, form lighter baryons. The $b$–baryons is a heavy hadron because it contains $b$-quark and has a mass of $5.28 \text{ GeV}/c^2$, which is more than five times the mass of the proton. The $b$–baryon would be stable if $b$–quark and companion anti-quark doesn’t have weak charge. Because they do, and because this hadron is heavier than many other hadrons, there are many channels in which it can decay. All of these decays involve the $b$-quark transforming itself into another lighter quark, which could be a $c$, $s$, $u$ or $d$ quark.

Now the question arises: why we study $b$-hadron decays? The most obvious reason is that the $b$-hadrons are the heaviest hadrons, as the top quark decays before it can hadronize. The fact that $b$-hadron is heavy has two important consequences: $b$-hadrons decays show an extremely rich phenomenology and theoretical techniques using an expansion in the heavy mass allow for model-independent predictions. The large available phase space and the the possibility for large CP-violating asymmetries in the $b$-hadron decays make it a topic of rich phenomenological study. The large CP-violating feature of $b$-hadrons is in contrast to the Standard Model expectations for the decays of $K$ and $D$ mesons. The pattern of CP violation in $K$ and $B$ system just represents the hierarchy of the CKM matrix. The $b$-hadron system offers an excellent laboratory to quantitatively test the CP-violating and time reversal violating sector of the Standard Model, determine fundamental parameters, study the interplay of strong and electroweak interactions and some of search for New Physics (NP).

In this thesis we have worked on hadronic decays of $\Lambda_b$ baryon because a huge statistics of beauty hadrons are expected to be produced at the CERN-LHC proton-proton collider started last year. Obviously this will lead to a thorough study of discrete symmetries, C, P and T in $b$-quark physics, in the framework of the Standard Model (SM) as well as beyond the Standard Model. It is also well known that the violation of CP symmetry via the Cabibbo-Kobayashi-Maskawa (CKM) mechanism is one of the cornerstone of the Standard Model of particle physics in the electroweak sector. In LHCb experiment non-leptonic and leptonic $b$-baryon decays may allow us to get information about the CKM matrix elements, analysis of the C-P-T operators may be performed and different non-perturbative aspects of QCD may also be investigated.

Looking for CP and Time Reversal (TR) violation effects in $b$-baryon decays can provide us a new field of research. Firstly, TR violation can be seen as a complementary
test of CP violation by assuming the correctness of the CPT theorem. Secondly, this can also be a path to follow in order to search for processes beyond the Standard Model. Various observables which are T-odd under time reversal operations can be measured, so that $Λ_b$-decay seems to be one of the most promising channel to reveal TR violation signal.

Although some CP violation and also a direct TR violation [2] have been detected experimentally, the nature of such symmetry violations has not yet been clarified so far. More precisely, the prediction of the size of the violation in some weak decays is strongly model dependent, which stimulates people to search for signals of new physics (NP) [3], [7], [10], [14], beyond the standard model (SM). For example, the decays involving the transition $b \to s$ present CPV parameters, like the $B^0 - \bar{B}^0$ mixing phase [11], [12] and the transverse polarization of spinning decay products of $Λ_b$ [3], which are very small in SM predictions, but are considerably enhanced in other models. In particular, recent signals of NP have been claimed in B decays: the CP violating phases of $B \to K\pi$ [11] and $B \to φJ/ψ$ [12] may be considerably greater than predicted by SM. Also $Λ_b$ decays [3], [13], [21] are suggested as new sources of CPV and TRV parameters, especially in view of the abundant production of this resonance in the forthcoming LHC accelerator.

This thesis is organized as follows: in chapters 2 and 3 we fill our toolbox with the necessary ingredients. After giving a bird eye view of the Standard Model and Discrete Symmetries in chapter 2, we present some basic tools in chapter 3, like operator product expansion, Heavy Quark Effective Theories and QCD factorization to analyze our decay. A discussion of the effective $Λ_b \to ΛV$ Hamiltonian and evolution of Form Factors are also the subjects of chapter 3.

In chapter 4 we worked out some model independent tests of TRV, CPV and CPT invariance in hadronic $Λ_b$ decays of the type $Λ_b \to ΛV$, where $V$ denoting a $J^P = 1^-$, resonance, either the $J/ψ$ or a light vector meson, like $ρ$, $ω$. Each resonance decays, in turn, to more stable particles, like, e. g., $Λ \to pπ$ and $J/ψ \to l^+l^-$. We parameterized, by means of the spin density matrix (SDM), the angular distribution and the polarizations of the decay products, without introducing any dynamic assumption at all. Then we study the behavior of these observables under CP and T, singling out those which are sensitive to T, CP and CPT violations. Our approach resembles the one proposed by Lee and Yang [4] and by Gatto [5] many years ago, to use hyperon decays for the same tests.

We derive the expressions of the spin density matrices, angular distribution and polarizations of the decay products in the above mentioned decays by using the Jacob-Wick-Jackson Helicity formalism. We also present a parametrization of the angular distribution and polarizations and point out tests for TRV, CPV and CPT. In the last chapter we have put the numerical analysis of our work and draw the conclusions.

2. Standard Model and Discrete Symmetries

The Standard Model (SM) of particle physics represents our understanding of the fundamental nature of the universe. It describes the basic constituents of which all matter is made of, which are three families of quarks and leptons, and the forces. The wish for a simple and consistent description of all observed phenomena has lead to a quantum field
theory that successfully describes many physical observables and is consistent with the Standard Model. It is a gauge quantum field theory.

Although Standard Model satisfactorily describes most of the observed phenomena of elementary Particle Physics, several questions, important for our understanding of the Universe, remain unanswered. These problems are often entitled as Physics Beyond Standard Model such as the hierarchy problem, the missing matter problem (dark matter and dark energy), phenomenon of generation, Time reversal and CP-violation and Baryogenesis.

2.1 Introduction to the Standard Model

Strong and electroweak interactions between elementary particles are best described by the Standard Model. Standard Model lagrangian is made of two parts: one for electroweak interactions known as the Glashow-Weinberg-Salam model \[25\], and the other is the Quantum Chromodynamics (QCD) for the strong interactions \[26\]. It unifies all known experimental data concerning particle interactions via the gauge group $SU_C(3) \otimes SU_L(2) \otimes U(1)$. The gauge fields of color $SU_C(3)$ are responsible for binding the quarks together, while the gauge fields of $SU_L(2) \otimes U(1)$ mediate the electromagnetic and weak interactions. Because of the low mass of the elementary particles, gravity doesn’t give effects comparable to the other forces, so the Standard Model does not include this interaction.

The symmetries that characterize the Standard Model are: $SU_L(2)$ of weak isospin $I$, $U(1)$ of hypercharge $Y$ and $SU_C(3)$ of color $C$. The $SU_L(2)$ part of the weak interaction gives rise to a triplet of vector bosons $W$ associated with the quantum number of weak isospin. To the $U(1)$ component contributes one single boson $B$ associated with the weak hypercharge $Y$, which is a combination of the electric charge $Q$ and the third component of the weak isospin $I_3$,

$$Y = 2(Q - I_3)$$

In particular, the part of the theory that describes the electro-weak interactions has to be invariant under $SU_L(2) \otimes U(1)$, while QCD has the symmetry $SU_C(3)$. Altogether, there are nineteen free parameters in the theory, suggesting it is not a complete account of particle interactions. There are three coupling constants for the groups in $SU_C(3) \otimes SU_L(2) \otimes U(1)$, two parameters in the Higgs sector, 6 quark masses, 3 mixing angles and one phase, 3 lepton masses, and the QCD vacuum angle.

2.2 Quantum Chromodynamics

Quantum chromodynamics (QCD) is a non-abelian gauge theory for strong interactions. Quark interactions can be described using a new quantum number: the color. In particular each quark can have three different colors, which generate the group $SU(3)_{\text{color}}$. Leptons do not carry color that is the reason why they do not experience strong interactions. Hadrons are bound states of quarks or quark and anti quark. Known hadrons are color singlets \textit{i.e.} Color is confined in a hadron.

$q_a$: belong to fundamental representation of $SU_c(3)$
\[ q_a \rightarrow q'_a = U^b_a q_b \quad (2.2) \]

when there is more than one type of states, e.g. \( q_a \) \((a = 1, 2, 3)\) and there exists transformations \( SU_c(3) \) between the different states, with

\[ U(x) = e^{i \frac{1}{2} \lambda_A \Lambda_A(x)} \quad (2.3) \]

where \( \lambda_A \) are Gell-Mann matrices, \( A = 1, \ldots, 8 \)

\[ UU^\dagger = 1 \quad (2.4) \]

\[ \det U = 1 \quad (2.5) \]

Here \( q_a \) for a particular quark flavor \( q \) form the fundamental representation of the color \( SU(3) \) group and \( \lambda_A \) are the eight matrix generators of the group \( SU_c(3) \).

Quarks are spin 1/2 particles. The lagrangian density for free quarks is,

\[ L = \bar{q}^a i \gamma^\mu \partial_\mu q_a - \bar{q}^a m q_a \quad (2.6) \]

where

\[ q_a = \begin{pmatrix} u_a \\ d_a \\ s_a \end{pmatrix} \quad \text{and} \quad m = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} \]

is clearly invariant under the SU(3) transformation with \( \Lambda \) constant. For the local gauge transformation Eq (2.3), with \( \Lambda(x) \) as a function of space-time, we must replace \( \partial_\mu \) by its covariant derivative \( D_\mu \):

\[ D_\mu = (\partial_\mu - i \frac{g_s}{2} \lambda_A G_{A\mu}) = (\partial_\mu - i \frac{g_s}{2} \lambda_A G_{A\mu}) \quad (2.7) \]

where \( g_s \) is a scale parameter, the coupling constant and \( G_{A\mu} \) are vector gauge fields, their number being equal to the generators of \( SU_c(3) \) group i.e. 8. Now the Lagrangian density is given by:

\[ L = \bar{q}^a i \gamma^\mu (\partial_\mu - i \frac{g_s}{2} \lambda_A G_{A\mu}) q_b - \bar{q}^a m q_a - \frac{1}{4} G^{\mu\nu} G_{A\mu\nu} \quad (2.8) \]

The eight gauge vectors bosons \( G_{A\mu} \) are called gluons. They are mediators of strong interaction between quarks just as photons are mediators of electromagnetic force between electrically charged particles. The gauge transformation given in Eq (24) is called the non-abelian gauge transformation. As non-Abelian gauge transformation was first considered by Yang Mills and gauge bosons are sometimes called Yang-Mills Fields.

### 2.3 Spontaneous Symmetry Breaking

The Higgs field which is associated with the Higgs particle interacts with the quarks, leptons, and weak bosons to give them masses. The coupling of the the Higgs is the only thing that differentiates the three generations of quarks and leptons. The Higgs is the only particle in the Standard Model which has not yet been observed experimentally.
The striking inconsistency of the masses of the gauge bosons with gauge invariance seeks for a satisfying mechanism to explain these properties observed in experiments. In the SM, this is provided by the so-called Higgs mechanism. The Higgs mechanism is a theory which explains the masses of particles. The particles acquire mass as they move through the Higgs field. This is an essential part of the standard model as without it, the theory suggests all particles would be massless (For more details on can see for example [18]). To prove this mechanism, experiments are trying to detect the Higgs boson a quantum of the Higgs field. In this model, the mass is generated by the interaction of particles with the Higgs complex scalar field $\phi$,

$$\phi = (\phi^+, \phi^0)$$  \hspace{1cm} (2.9)

To illustrate the idea consider a $U(1)$ group and a complex scalar field:

$$\mathcal{L}_{\text{Higgs}} = \partial^\mu \bar{\phi} \partial_\mu \phi - U(\phi)$$  \hspace{1cm} (2.10)

with

$$U(\phi) = \mu^2 \bar{\phi} \phi + \lambda (\bar{\phi} \phi)^2$$  \hspace{1cm} (2.11)

The potential $U(\phi)$ has rotational symmetry and has its minimum on a circle at

$$|\phi|^2 = \frac{-\mu^2}{2\lambda}$$  \hspace{1cm} (2.12)

This means that, in principle, any state with

$$|\phi|^2 = \frac{v^2}{2}, \text{ with } v^2 = \frac{-\mu^2}{\lambda}$$  \hspace{1cm} (2.13)

could be the ground-state in this potential. This is a classical approximation to the vacuum expectation of $\phi$, i.e the ground state,

$$\langle 0 | \phi | 0 \rangle = \sqrt{\frac{-\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}}$$  \hspace{1cm} (2.14)

breaks the symmetry. In other words Lagrangian is invariant but the Hamiltonian is not.

![Figure 1: The Higgs Potential $U(\phi)$](image)

Regarding a small excitation of this ground state,

$$\phi = \frac{1}{\sqrt{2}} (v + H + i\eta)$$  \hspace{1cm} (2.15)
where
\[ \langle 0|H|0 \rangle = 0 \] (2.16)
\[ U(H) = -\frac{1}{2} \lambda v^2[(v + H)^2 + \eta^2] + \frac{1}{4} \lambda [(v + H)^2 + \eta^2]^2 \] (2.17)
and putting in the covariant derivative here, yields the Lagrangian density for Higgs field,
\[ \mathcal{L} = \frac{1}{2} (\partial^\mu H)(\partial_\mu H) + \frac{1}{2} (\partial^\mu \eta)(\partial_\mu \eta) - \frac{1}{2} \lambda v^2[(v + H)^2 + \eta^2] + \frac{1}{4} \lambda [(v + H)^2 + \eta^2]^2 \] (2.18)
\[ m_H^2 = \lambda v^2, \quad m_\eta^2 = 0. \]
Lagrangian is invariant under global gauge transformation
\[ \phi'(x) = U^{-1}(x)\phi(x)U = e^{i\Lambda(\phi(x))} \] (2.19)
but not under the local gauge transformation when \( U \) is a function of \( x \). Gauge invariance requires a vector field \( B_\mu \)
\[ B_\mu \rightarrow B_\mu - \frac{1}{g} \partial_\mu \Lambda \] (2.20)
Gauge invariant Lagrangian can be obtained by replacing \( \partial_\mu \) with \( D_\mu \),
\[ \partial_\mu \rightarrow D_\mu = \partial_\mu - igB_\mu \] (2.21)
\[ \mathcal{L} = -\frac{1}{4} B^{\mu\nu} B_{\mu\nu} + (\partial_\mu + igB_\mu) \bar{\phi} (\partial_\mu - igB_\mu) \phi - U(\phi) \] (2.22)
where \( B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \). The unwanted zero mass mode due to spontaneous symmetry breaking can be eliminated by means of field dependent gauge transformation
\[ \phi(x) \rightarrow \frac{1}{\sqrt{2}} [v + H(x)] e^{i\eta(x)} \] (2.23)
\[ B_\mu \rightarrow B_\mu - \frac{1}{vg} \partial_\mu \eta(x) \] (2.24)
\[ \mathcal{L} = -\frac{1}{4} B^{\mu\nu} B_{\mu\nu} + \frac{1}{2} (\partial^\mu H)(\partial_\mu H) + \frac{1}{2} g^2 (v^2 + 2vH + H^2) B^{\mu\nu} B_{\mu\nu} + \frac{1}{2} v^2 \lambda (H + v)^2 - \frac{\lambda}{4} (H + v)^4 \] (2.25)
\[ m_B^2 = \frac{1}{2} g^2 v^2, \quad m_H^2 = v^2 \lambda \]
The vector boson becomes massive, the Goldstone field \( \eta(x) \) has been transformed away, it has been eaten away by \( B_\mu \) to give it a longitudinal component.

### 2.4 Cabibo-Kobayashi-Masakawa Matrix (CKM)

The mechanism of quark mixing is a fundamental pillar of the Standard Model. This formalism successfully describes transitions between the quark families. It makes use of the Cabibbo-Kobayashi-Maskawa Matrix, a \( 3 \times 3 \) unitary matrix, which can be parameterized by four independent parameters. A precise determination of the Standard Model parameters allows to test predictions derived from these input numbers.
The Higgs couplings not only give masses to the quarks and leptons, but also allows transitions between generations since the mass eigenstates are not equal to the weak eigenstates. The advantage of the mass basis is that the quark states can be identified experimentally by their masses. Because the weak couplings of the three generations are all identical, linear combinations of the three weak eigenstates can be constructed so that the up type quarks \((u, c, t)\) are both weak and mass eigenstates. The weak eigenstates of the down type quarks are denoted \(d', s', b'\) and the mass eigenstates are denoted \(d, s, b\).

These two bases are related by the Cabibbo-Kobayashi-Maskawa (CKM) matrix,

\[
\begin{pmatrix}
  d' \\
  s' \\
  b'
\end{pmatrix} =
\begin{pmatrix}
  V_{ud} & V_{us} & V_{ub} \\
  V_{cd} & V_{cs} & V_{cb} \\
  V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix}
\]

Thus the weak eigenstates can be written in terms of the mass eigenstates as

\[
|b'\rangle = V_{td}|d\rangle + V_{ts}|s\rangle + V_{tb}|b\rangle
\]

In the lepton sector there is an analogous matrix called Maki-Nakagawa-Sakata (MNS) matrix.

The weak interaction only produces transitions between the weak eigenstates of the same generation: \((u \rightarrow d')\), \((c \rightarrow s')\), and \((t \rightarrow b')\). The mass eigenstates are the states observed experimentally, because the quarks are identified by their masses. Because \(|V_{tb}|\) is much greater than \(|V_{us}|\) and \(|V_{ud}|\), the bottom quark mass eigenstate is mostly \(b'\) and very little \(s'\) and \(d'\). Therefore the decays of the top quark to the bottom quark (emitting a \(W^+\)) are much more common than decays of the top quark to the strange and down quarks. So, the probability of each transition is governed by the overlaps of the mass and weak eigenstates which is described by the CKM matrix.

### 2.4.1 Parametrization of CKM Matrix

With three generations of fundamental fermions, the CKM matrix can be parameterized with four parameters: three Euler angles and one phase. Wolfenstein \[24\] noticed that \(|V_{cb}|^2 \simeq |V_{us}|^2\) and proposed to use \(|V_{us}| = \lambda \simeq 0.22\) as an expansion parameter for the elements of the CKM matrix after the experimental observation that the \(b\) quark decays predominantly to the charm \((|V_{cb}| >> |V_{ub}|)\). Because the CKM matrix is a basis transformation, it must be unitary. This constraint reduces the number of free parameters in the CKM matrix. Without changing the Lagrangian and hence any observables, the phases of quarks in the standard Model Lagrangian can be changed. This can be used to remove another five free parameters from the CKM matrix (these are the five relative phases of the quark fields). The Wolfenstein parametrization \[23\] of the CKM matrix exploits the smallness of the off-diagonal elements to construct a representation in which
the relationships between the elements are manifest:

\[
V_{CKM} = \begin{pmatrix}
1 - \frac{\lambda^2}{2} - \frac{\lambda^4}{8} & \lambda & \lambda A\lambda^3 (\rho - i\eta) \\
-\lambda + \frac{1}{2} A^2 \lambda^5 (1 - 2 (\rho + i\eta)) & 1 - \frac{\lambda^2}{2} - \frac{1}{8} \lambda^4 (1 + 4 A^2) & A\lambda^2 (1 + \frac{1}{2} A) \\
A\lambda^3 (1 - \bar{\rho} - i\bar{\eta}) & -A\lambda^2 + \frac{1}{2} A\lambda^4 (1 - 2 (\rho + i\eta)) & 1 - \frac{A^2 \lambda^4}{2}
\end{pmatrix}
\]

(2.28)

where

\[
\bar{\rho} = \rho \left( 1 - \frac{\lambda^2}{2} \right) \quad \text{and} \quad \bar{\eta} = \eta \left( 1 - \frac{\lambda^2}{2} \right)
\]

(2.29)

and \( \eta \) plays the well-known role of the CP-violating phase in the Standard Model framework. From the CKM matrix, expressed in terms of the Wolfenstein parameters and constrained with several experimental data, we will take in our numerical applications,

\[
0.076 < \rho < 0.380 \quad \text{and} \quad 0.280 < \eta < 0.455.
\]

The values for \( A \) and \( \lambda \) are assumed to be well determined experimentally:

\[
\lambda = 0.2265 \quad \text{and} \quad A = 0.801
\]

2.5 Discrete Symmetries

Most of the symmetries in elementary-particle physics are continuous. A typical example is the symmetry generated by rotations around an axis, where the angle of rotation can assume any value between zero and \( 2\pi \). In addition to continuous symmetries, there are also discrete symmetries, for which the possible states assume discrete values classified with the help of a few integers. For instance, snowflakes exhibit the discrete symmetry of rotations under \( 60^\circ \), and crystals exhibit various types of discrete symmetries. In elementary-particle physics there are three discrete symmetries of basic importance: parity, charge conjugation and time-reversal.

Parity is the reflection of space coordinates and will be denoted by \( P \). Under parity there are two states, the object and its space reflection. Parity is familiar from quantum mechanics, where the eigenstates of Hamiltonian are classified according to their properties under space reflection. For spherically symmetric potentials the wave functions are proportional to the spherical harmonics \( Y_{l}^m(\theta, \phi) \) whose parity is \((-1)^l\). For a long time it was assumed that the fundamental interactions respect \( P \), but in 1956 a critical review of experimental evidence led two theoreticians, T. D. Lee and C. N. Yang, to suggest that parity may be violated by the weak interactions \[20\]. One year later, an experiment led by C. S. Wu brought the proof that the \( P \) symmetry is indeed violated by weak interactions.

The symmetry of charge conjugation, to be denoted by \( C \), exchanges particles with antiparticles. One can imagine building an antiworld by replacing all particles by antiparticles. In the antiworld the three interactions gravity, the strong force, and electromagnetism are the same, but the weak interactions are different. For example in the antiworld muon-type antineutrinos are right-handed and produce \( \mu^+ \) which are also right-handed. In comparison neutrinos are left-handed and always produce, in high-energy reactions, left-handed \( \mu^- \). In the weak interactions the \( C \) symmetry is broken. However, it was assumed,
at that time, that the observed processes do respect the combined CP transformation, the one obtained by applying both C and P transformations.

There is a fundamental reason why CP symmetry plays a crucial role. It is intimately linked to the time-reversal transformation (T). This transformation consists of “looking” at an experiment running backward in time. Although, at the macroscopic level, one can distinguish the real sequence of events from the time-reversed one in terms of large-scale phenomena such as entropy or the expansion of the Universe. This is not a priori evident for microscopic interactions, i.e. it is not a priori evident that the amplitudes for reactions and for the time-reversed reactions are equal.

The analysis of CP violation is facilitated by an important theorem known as CPT theorem. It states that any local field theory based on special relativity and quantum mechanics is invariant under the combined action of C, P, and T. A consequence of the theorem is that CP symmetry implies T symmetry, because any CP violation should be compensated by T violation. Until 1964 the decays and interactions of particles showed that the CP symmetry was conserved; this created the belief that microscopic phenomena also obey the T symmetry. In 1964 CP violation was observed in an experiment dedicated to the study of $K^0$ and $\bar{K}^0$ mesons. Since then it has become an active topic of research, with CP violation having been observed so far in the K and the B mesons.

3. Physics of Heavy Quarks and Beauty(b)-Hadrons

In 1964, G. Zweig and M. Gell-Mann independently proposed that hadrons are made up of constituents, called quarks by Gell-Mann. The particle that experience strong interaction are called Hadrons. They are the bound states of quarks and are color singlets. We can divide the hadrons into two large classes, Meson(integer spin) and Baryon (half integer spin). Mesons are made up of quark-antiquark ($q\bar{q}$) system where as baryons are made up of quark-quark-quark ($qqq$) system. This scheme extends the isospin internal symmetry, which is based on the group SU(2) to SU(3), a larger unitary group. We immediately stress that the SU(3) symmetry has two very different roles in Particle Physics:

1. The classification of the hadrons, or rather the hadrons with up (u), down (d) and strange (s) quarks.

2. The symmetry of the charges of one of the fundamental forces, the strong force.

For quarks we have representation 3 and for antiquark we have $\bar{3}$ in SU(3). Mesons are the members of the multiplets belonging to the product of $3 \otimes \bar{3}$. Group theory tells us that we can write them in irreducible representation of

$$3 \otimes \bar{3} = 8 \oplus 1$$

(3.1)

While baryons have classification in SU(3) multiplets is $3 \otimes 3 \otimes 3$ and their irreducible representations are given by

$$3 \otimes 3 \otimes 3 = 10 \oplus 8' \oplus 8 \oplus 1$$

(3.2)
3.1 Heavy Quark Physics

For many reasons the strong interactions of hadrons containing heavy quarks are easier to understand than those of hadrons containing only light quarks. The first is asymptotic freedom, the fact that the effective coupling constant of QCD becomes weak in processes with large momentum transfer, corresponding to interactions at short distance scales. At large distances, on the other hand, the coupling becomes strong, leading to nonperturbative phenomena such as the confinement of quarks and gluons on a length scale $R_{\text{had}} \sim 1/\Lambda_{\text{QCD}} \sim 1\text{ fm}$, which determines the size of hadrons. Roughly speaking, $\Lambda_{\text{QCD}} \sim 0.2\text{ GeV}$ is the energy scale that separates the regions of large and small coupling constant. When the mass of a quark $Q$ is much larger than this scale, $m_Q \gg \Lambda_{\text{QCD}}$, it is called a heavy quark. The quarks of the Standard Model fall naturally into two classes: up, down and strange are light quarks, whereas charm, bottom and top are heavy quarks. For heavy quarks, the effective coupling constant $\alpha_s(m_Q)$ is small, implying that the strong interactions are perturbative and much like the electromagnetic interactions.

Systems composed of a heavy quark and other light constituents are like Hydrogen atom but it is more complicated because it also involves the gluonic interactions with quarks and themselves. The size of such systems is determined by $R_{\text{had}}$, and the typical momenta exchanged between the heavy and light constituents are of order $\Lambda_{QCD}$. The heavy quark is surrounded by a complicated, strongly interacting cloud of light quarks, antiquarks and gluons. As in the case of charm and bottom quarks, the masses are $\sim 1.5\text{ GeV}$ and $\sim 4.9\text{ GeV}$, respectively, and $\Lambda_{QCD}$ is $\sim 0.2\text{ GeV}$. In such systems the heavy quark is almost on-shell; its momentum fluctuates around the mass shell by an amount of order $\Lambda_{QCD}$. The corresponding fluctuations in the velocity of the heavy quark vanish as $\Lambda_{QCD}/m_Q \to 0$. The velocity becomes a conserved quantity and is no longer a dynamical degree of freedom. Because the velocity does not depend on the heavy quark mass, different heavy quarks interact identically in the heavy quark mass limit. This is known as flavor symmetry. The heavy quark spin also decouples from the strong interaction. The decoupling of the spin in the heavy quark limit leads to the spin symmetry.

Therefore, the light degrees of freedom are blind to the flavor (mass) and spin orientation of the heavy quark. They experience only its color field, which extends over large distances because of confinement. In the rest frame of the heavy quark, it is in fact only the electric color field that is important; relativistic effects such as color magnetism vanish as $m_Q \to \infty$. Since the heavy-quark spin participates in interactions only through such relativistic effects, it decouples. These two symmetries have important consequences, especially for the decays of beauty hadrons to lighter hadrons. These symmetries are only true in the heavy quark limit and are violated at order $\Lambda_{QCD}/m_Q$. These observations can be formalized by writing the Standard Model Lagrangian as an expansion in $1/m_Q$.

Heavy-quark symmetry is an approximate symmetry, and corrections arise since the quark masses are not infinite. In many respects, it is complementary to chiral symmetry, which arises in the opposite limit of small quark masses. There is an important distinction, however, whereas chiral symmetry is a symmetry of the QCD Lagrangian in the limit of vanishing quark masses, heavy-quark symmetry is not a symmetry of the Lagrangian (not...
even an approximate one), but rather a symmetry of an effective theory that is a good approximation to QCD in a certain kinematic region. Nevertheless, results derived on the basis of heavy-quark symmetry are model-independent consequences of QCD in a well-defined limit. The symmetry-breaking corrections can be studied in a systematic way. To this end, it is however necessary to cast the QCD Lagrangian for a heavy quark,

\[ \mathcal{L} = \bar{\Psi}_Q (iD_\mu \gamma^\mu - m_Q) \Psi_Q \]  

into a form suitable for taking the limit \( m_Q \to \infty \).

### 3.2 Heavy Quark Effective Theory

The QCD Lagrangian does not explicitly contain heavy quark spin-flavor symmetry as \( m_Q \to \infty \). It is often helpful to use an effective field theory for QCD in which these symmetries are apparent. The effective field theory is constructed so that only inverse powers of \( m_Q \) appear in the effective Lagrangian. The QCD lagrangian describing a quark \( Q \) of mass \( m_Q \) and its interactions with the gluons is given by

\[ \mathcal{L} = \bar{\Psi}_Q (iD_\mu \gamma^\mu - m_Q) \Psi_Q \]  

with

\[ D_\mu = \partial_\mu - ig_s T^a A_\mu \]

This effective field theory is known as heavy quark effective theory (HQET) and is described the dynamics of hadrons containing single heavy quark.

The heavy-quark effective theory (HQET) is constructed to provide a simplified description of processes where a heavy quark interacts predominantly by the exchange of soft gluons. At short distances, i.e. for energy scales larger than the heavy-quark mass, the physics is perturbative and is described by perturbative QCD. For mass scales much below the heavy-quark mass, the physics is complicated and non-perturbative because of confinement. Our goal is to obtain a simplified description in this region using an effective field theory. To separate short- and long-distance effects, we introduce a separation scale \( \mu \) such that \( \Lambda_{QCD} \ll \mu \ll m_Q \). The HQET will be constructed in such a way that it is equivalent to QCD in the long-distance region, i.e. for scales below \( \mu \).

In the short-distance region, the effective theory is incomplete, since some high-momentum modes have been integrated out from the full theory. The fact that the physics must be independent of the arbitrary scale, allows us to derive renormalization-group equations, which can be employed to deal with the short-distance effects in an efficient way.

Compared with most effective theories, in which the degrees of freedom of a heavy particle are removed completely from the low-energy theory, the HQET is special in that its purpose is to describe the properties and decays of hadrons which do contain a heavy quark. In the heavy quark limit \( (m_Q \to \infty) \), the conserved velocity \( v_\mu \) of the heavy quark and its four momentum may be decomposed as:

\[ p_\mu = m_Q v_\mu + k_\mu \]
with $v^2 = 1$, where $m_Q v_\mu$ and $k_\mu$ are on-shell and off-shell parts respectively. The components of residual momentum $k$ are much smaller than $m_Q$ and are changed by interactions of the heavy quark with light degrees of freedom by $\Delta k \sim \Lambda_{QCD}$.

We can separate out the large and small components of the heavy quark field as

$$h_v(x) \equiv e^{imv \cdot x} \frac{1 + ik}{2} \Psi_Q(x)$$

and

$$H_v(x) \equiv e^{imv \cdot x} \frac{1 - ik}{2} \Psi_Q(x)$$

with the properties $\not\!v h_v = h_v$ and $\not\!v H_v = -H_v$, respectively. The heavy quark field in terms of the new fields can be expressed as

$$\Psi_Q(x) = e^{-imv \cdot x} (h(x) + H_v(x))$$

One may split the covariant derivative $D$ into 'longitudinal' and 'transverse' parts as:

$$D_\perp = D^\mu - v^\mu v \cdot D$$

with $v \cdot D_\perp = 0$, $\{ D_\perp, \not\!v \} = 0$.

Using relations as $\bar{h}_v H_v = 0$ and $\bar{h}_v D_\perp h_v = 0$, the lagrangian takes the form

$$L_{eff} = \bar{h}_v i(v \cdot D)h_v - \bar{H}_v (iv \cdot D + 2m_Q)H_v + \bar{h}_v i D_\perp H_v + \bar{H}_v i D_\perp h_v$$

Thus equation of motion for $\bar{H}_v$ becomes

$$H_v(x) = \frac{1}{2m_Q + iv \cdot D} i D_\perp h_v$$

This allows us, on a classical level, to eliminate out the heavy degree of freedom $H_v$ from the lagrangian:

$$L_{eff} = \bar{h}_v i(v \cdot D)h_v + \bar{h}_v i D_\perp \frac{1}{2m_Q + iv \cdot D} i D_\perp h_v$$

$$= \bar{h}_v i(v \cdot D)h_v + \frac{1}{2m_Q} \sum_{n=0}^{\infty} \bar{h}_v i D_\perp \left( \frac{-iv \cdot D}{2m_Q} \right)^n i D_\perp h_v$$

The above equation can be written as:

$$L_{eff} = \bar{h}_v i(v \cdot D)h_v + \frac{1}{2m_Q} \bar{h}_v (i D_\perp)^2 h_v + \frac{g_s}{4m_Q} \bar{h}_v \sigma_{\mu \nu} G^{\mu \nu} h_v + O(1/m_Q)^2$$

where $G_{\mu \nu}$ is the gluon field strength tensor and is defined as $G_{\mu \nu} = [i D_\mu, i D_\nu] = ig_A t^a G_a^{\mu \nu}$ and $\sigma_{\mu \nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu]$.

In limit $m_Q \to \infty$, only term

$$L_\infty = \bar{h}_v i(v \cdot D)h_v$$
survives. There appears neither Dirac matrices nor quark masses in this equation. For $m_Q \to \infty$, the interaction of heavy quarks and gluons become independent of the spin of the quark. Furthermore, when extending the theory to more than one heavy quark moving at the same velocity, the lagrangian $L_\infty$ is symmetric under rotations in the flavor space. This is the heavy quark flavor symmetry. The spin-flavor symmetry leads to many interesting relations between the properties, especially the spectroscopy, of hadrons containing a heavy quark. In the following sections we will use the HQET to evaluate the hadronic form factors which appear in the transition matrix.

3.3 The Physics of Beauty(b)-Hadrons

The hadrons which contain one beauty(bottom) b-quarks as an ingredient with lighter quarks like, $u$, $d$, $s$, or $c$, are called b-hadrons. As b-quark is the heaviest quark which can be hadronized so the b-hadron can give us rich phenomenology to understand the nature of fundamental interactions by studying these hadrons.

The B meson is the hydrogen atom of quantum chromodynamics (QCD), the simplest non-trivial hadron. In the leading approximation, the b-quark in it just sits at rest at the origin and creates a chromoelectric field. Light constituents (gluons, light quarks, and antiquarks) move in this external field. Their motion is relativistic; the number of gluons and light quark–antiquark pairs in this light cloud is undetermined and varying. Similarly, the $\Lambda_b$ baryon is the simplest b-baryon, its quark contents are $bud$. Both, B-meson and $\Lambda_b$-baryon, have a light cloud with a variable number of relativistic particles. The size of this cloud is the confinement radius $1/\Lambda_{QCD}$; its properties are determined by large-distance nonperturbative QCD.

In this work, we will consider the analysis of simplest b-baryon $i.e.$ $\Lambda_b$ decay. More specifically we will consider the hadronic decay of the type $\Lambda_b \to \Lambda V (1^-)$, as $J/\psi$, $\rho$ or $\omega$. The effective Hamiltonian for the decay can be given as:

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{qb} V_{qs}^* \sum_{i=1}^{10} C_i (m_b) O_i (m_b)$$ (3.15)

where $C_i (m_b)$ are the Wilson Coefficients and the operators, $O_i (m_b)$ can be understood as local operators which govern the weak interaction of quarks in the given decay.

By using the Factorization assumption we can get the helicity amplitude for the decay $\Lambda_b \to \Lambda V (1^-)$ as

$$A_{(\lambda,\lambda')} = \frac{G_F}{\sqrt{2}} f_V E_V \langle \Lambda(p', s') | \bar{s} \gamma_\mu (1 - \gamma_5) b | \Lambda_b(p, s) \rangle_{(\lambda,\lambda')} \left\{ V_{CKM}^{T_i} C_i^{T} - V_{CKM}^{P_i} C_i^{P} \right\}$$ (3.16)

where $f_V$ and $E_V$ are the decay constant and energy of Vector meson. $V_{CKM}^{T,P_i}$ are the CKM matrix elements for the tree and penguin diagrams while $C_i^{T,P}$ are Wilson Coefficients. The baryonic matrix element $M^{ab}_{(\lambda,\lambda')}$ is calculated by using the Heavy Quark Effective Theory(HQET), in the preceding sections.
3.4 Operator Product Expansion

The Operator Product Expansion (OPE) [27] is used to separate the calculation of a baryonic decay amplitude, into two distinct physical regimes, as discussed above. One is called hard or short-distance physics, represented by Wilson Coefficients and the other is called soft or long-distance physics. This part is described by $O_i(\mu)$, and is derived by using a nonperturbative approach. The operators, $O_i$’s, entering from the Operator Product Expansion (OPE) to reproduce the weak interaction of quarks, can be understood as local operators which govern a given decay. They can be written, in a generic form, as,

$$O_i = (\bar{q}_a \Gamma_i q_\beta)(\bar{q}_\mu \Gamma_i q_\nu)$$  \hspace{1cm} (3.17)

where $\Gamma_{ij}$ denotes the gamma matrices. They should respect the Dirac structure, the color structure and the type of quark relevant for the decay being studied. Two kinds of topology contributing to the decay can be defined: there is the tree diagram of which the operators structure and the type of quark relevant for the decay being studied. Two kinds of topology mentioned previously are the following,

$$O_1 = \bar{q}_a \gamma_\mu (1 - \gamma_5) u_\beta \bar{u}_\beta \gamma^\mu (1 - \gamma_5) b_\alpha,$$

$$O_2 = \bar{q}_a \gamma_\mu (1 - \gamma_5) u_\beta \bar{u}_\beta \gamma^\mu (1 - \gamma_5) b_\alpha,$$

$$O_3 = \bar{q} \gamma_\mu (1 - \gamma_5) b \sum_{q'} \bar{q}' \gamma^{\mu 1 + \gamma_5} q',$$

$$O_4 = \bar{q}_a \gamma_\mu (1 - \gamma_5) b_\beta \sum_{q'} \bar{q}'_\beta \gamma^{\mu 1 + \gamma_5} q'_\alpha,$$

$$O_5 = \bar{q} \gamma_\mu (1 - \gamma_5) b \sum_{q'} \bar{q}' \gamma^{\mu 1 + \gamma_5} q',$$

$$O_6 = \bar{q}_a \gamma_\mu (1 - \gamma_5) b_\beta \sum_{q'} \bar{q}'_\beta \gamma^{\mu 1 + \gamma_5} q'_\alpha,$$

$$O_7 = \frac{3}{2} \bar{q} \gamma_\mu (1 - \gamma_5) b \sum_{q'} e_q \bar{q}' \gamma^{\mu 1 + \gamma_5} q',$$

$$O_8 = \frac{3}{2} \bar{q}_a \gamma_\mu (1 - \gamma_5) b_\beta \sum_{q'} e_q \bar{q}'_\beta \gamma^{\mu 1 + \gamma_5} q'_\alpha,$$

$$O_9 = \frac{3}{2} \bar{q} \gamma_\mu (1 - \gamma_5) b \sum_{q'} e_q \bar{q}' \gamma^{\mu 1 + \gamma_5} q',$$

$$O_{10} = \frac{3}{2} \bar{q}_a \gamma_\mu (1 - \gamma_5) b_\beta \sum_{q'} e_q \bar{q}'_\beta \gamma^{\mu 1 - \gamma_5} q'_\alpha,$$

In the above expressions, $\alpha$ and $\beta$ are the color indices. $e_q$ denotes the quark electric charge and $q'$, is for the quarks $u$, $d$, $c$, $s$, which may contribute in the penguin loop.

The Wilson coefficients [27], $C_i(\mu)$, represent the physical contributions from scales higher than $\mu$ (of the order of $O(m_b)$ in b-quark decay) and since QCD has the property of asymptotic freedom, they can be calculated in perturbation theory. we taken the Wilson coefficients from [28] for $q^2/m_b^2 = 0.5$ and their values are summarized as:

| $C_1$ | $-0.3125$ | $C_2$ | $1.1502$ |
|-------|-----------|-------|-----------|
| $C_3$ | $2.12 \times 10^{-2} + i2.174 \times 10^{-3}$ | $C_4$ | $-4.869 \times 10^{-2} - i1.552 \times 10^{-2}$ |
| $C_5$ | $1.42 \times 10^{-2} + i5.174 \times 10^{-3}$ | $C_6$ | $-5.729 \times 10^{-2} - i1.552 \times 10^{-2}$ |
| $C_7$ | $-8.34 \times 10^{-5} - i9.94 \times 10^{-5}$ | $C_8$ | $3.84 \times 10^{-4}$ |
| $C_9$ | $-1.02 \times 10^{-2} - i9.94 \times 10^{-5}$ | $C_{10}$ | $1.96 \times 10^{-3}$ |

Finally, in the following one lists the tree and penguin amplitudes which appear in the given transition:
for the decay $\Lambda_b \to J/\psi$,  
\begin{align}
A_{J/\psi}^T(a_1, a_2) &= a_1 \tag{3.18} \\
A_{J/\psi}^P(a_3, ..., a_{10}) &= a_3 + a_5 + a_7 + a_9 \tag{3.19}
\end{align}

for the decay $\Lambda_b \to \rho^0$,  
\begin{align}
A_{\rho}^T(a_1, a_2) &= \frac{a_1}{\sqrt{2}} \tag{3.20} \\
A_{\rho}^P(a_3, ..., a_{10}) &= \frac{3}{2\sqrt{2}}(4(a_3 + a_5) + a_7 + a_9) \tag{3.21}
\end{align}

for the decay $\Lambda_b \to \omega$,  
\begin{align}
A_{\omega}^T(a_1, a_2) &= \frac{a_1}{2\sqrt{2}} \tag{3.22} \\
A_{\omega}^P(a_3, ..., a_{10}) &= \frac{3}{2\sqrt{2}}(a_7 + a_9) \tag{3.23}
\end{align}

where $a_i = C_i + C_j / N_c$ with $i, j = 1, 2, ..., 10$ and $N_c$ is the number of colors.

### 3.5 Evolution of Baryonic Form Factors in HQET

In this section, the Heavy Quark Effective Theory (HQET) formalism is used to evaluate the hadronic form factors involved in $\Lambda_b$-decay. Weak transitions including heavy quarks can be safely described when the mass of a heavy quark is large enough compared to the QCD scale, $\Lambda_{QCD}$. Properties such as flavor and spin symmetries can be exploited in such way that corrections of the order of $1/m_Q$ are systematically calculated within an effective field theory.

#### 3.5.1 Transition Form Factors

The decay, $\Lambda_b \to \Lambda V$, involves the hadronic transition matrix $\langle \Lambda | \bar{s} \gamma_\mu (1 - \gamma_5) b | \Lambda_b \rangle$. Based on Lorentz decomposition, the hadronic matrix element can be written as,

\begin{equation}
\langle \Lambda(p', s') | \bar{s} \gamma_\mu (1 - \gamma_5) b | \Lambda_b(p, s) \rangle = \bar{u}_\Lambda(p', s') \left\{ \begin{array}{c}
(f_1(q^2) \gamma_\mu + i f_2(q^2) \sigma_{\mu\nu} q^\nu + f_3(q^2) q^\mu) \\
- (g_1(q^2) \gamma_\mu + i g_2(q^2) \sigma_{\mu\nu} q^\nu + g_3(q^2) q^\mu) \gamma_5
\end{array} \right\} u_{\Lambda_b}(p, s) \tag{3.24}
\end{equation}

where $\bar{u}_\Lambda(p', s')$ and $u_{\Lambda_b}(p, s)$ are the spinners of $\Lambda$ and $\Lambda_b$ respectively, while $p', s'$ and $p, s$ are their momentum and spin. The square of momentum transfer in the hadronic transition is given by

\[ q^2 = (p - q')^2 \]

Here $f_i(q^2)$ and $g_i(q^2)$ are the form factors corresponding to the vector and axial vector parts of the transition matrix, respectively.

Another way of parameterizing the electroweak amplitude in decays of baryons is the following:

\begin{equation}
\langle \Lambda(p', s') | \bar{s} \gamma_\mu (1 - \gamma_5) b | \Lambda_b(p, s) \rangle = \bar{u}_\Lambda(p', s') \left\{ \begin{array}{c}
\left( F_1(q^2) \gamma_\mu + F_2(q^2) v_\mu \Lambda_b + F_3(q^2) \frac{p'}{m} \right) \\
- \left( G_1(q^2) \gamma_\mu + G_2(q^2) v_\mu \Lambda_b + G_3(q^2) \frac{p'}{m} \right) \gamma_5
\end{array} \right\} u_{\Lambda_b}(p, s) \tag{3.25}
\end{equation}
By comparing the two sets of form factors given in Eqs. (3.24) and (3.25), we get the following relations between the \( f_1(q^2)'s \) and \( g_1(q^2)'s \) and \( F_1(q^2)'s \) and \( g_1(q^2)'s \):

\[
f_1(q^2) = F_1(q^2) + (m + m') \left[ \frac{F_2(q^2)}{2m} + \frac{F_3(q^2)}{2m'} \right],
\]

\[
f_2(q^2) = \frac{F_2(q^2)}{2m} + \frac{F_3(q^2)}{2m'},
\]

\[
f_3(q^2) = \frac{F_2(q^2)}{2m} - \frac{F_3(q^2)}{2m'}
\]

and

\[
g_1(q^2) = G_1(q^2) - (m - m') \left[ \frac{G_2(q^2)}{2m} + \frac{G_3(q^2)}{2m'} \right],
\]

\[
g_2(q^2) = \frac{G_2(q^2)}{2m} + \frac{G_3(q^2)}{2m'},
\]

\[
g_3(q^2) = \frac{G_2(q^2)}{2m} - \frac{G_3(q^2)}{2m'}
\]

In case of working in the HQET formalism, the matrix element of the weak transition, \( \Lambda_b \rightarrow \Lambda \), takes the following form,

\[
\langle \Lambda(p', s') | \bar{s}_\gamma \mu (1 - \gamma_5) b | \Lambda_b(p, s) \rangle = \bar{u}_\Lambda(p', s') \left[ \theta_1(q^2) + \theta_2(q^2) \gamma_\mu \Lambda_b \right] u_{\Lambda_b}(p, s) \tag{3.32}
\]

In Eq. (3.32), \( v_{\Lambda_b} \), defines the velocity of the baryon \( \Lambda_b \). Writing the momentum, \( p \), of the heavy baryon, \( \Lambda_b \) as,

\[ p = m_b v_{\Lambda_b} + k, \]

where \( k \) is the residual momentum, the velocity of heavy quark is almost that of the heavy baryon. Since \( m_b \gg \Lambda_{QCD} \), the parametrization of the hadronic matrix element in term of velocity, \( v_{\Lambda_b} \), gives us a reasonable picture where we can consider only corrections of \( 1/m_b \) expansion.

Since we know that in heavy hadrons the spectator quark retains its original momentum and spin state before final hadronization, the energy carried by the spectator quark is equal to that of the spectator in the rest frame of the final state particle and the relevant \( b \)-quark space momenta are much smaller than the \( b \) quark mass: indeed, it is assumed to be of the order of the confinement scale, \( \Lambda_{QCD} \). This approach firstly used in the meson case by Stech but can be generalized to a heavy baryon considered as a bound state of a \( b \) quark and a scalar diquark as considered in [1]. Thus in the baryon case hadronic matrix can be written in terms of components of Dirac Spinors as, \( \bar{u}_s(p', m_s) \gamma_\mu (1 - \gamma_3) u_b(p = 0, m_b) \) leads to the following expressions for the form factors, \( \theta_1 \) and \( \theta_2 \), when the \( m_b \rightarrow \infty \):

\[
\theta_1 = (E_\Lambda + m' + m_s) \frac{1}{(E_\Lambda + m_s)} \sqrt{\frac{(E_\Lambda + m_s)m'}{(E_\Lambda + m')m_s}}
\]

\[
\theta_2 = (m_s - m') \frac{1}{2(E_\Lambda + m_s)} \sqrt{\frac{(E_\Lambda + m_s)m'}{(E_\Lambda + m')m_s}}
\]
where $E_\Lambda$, is the energy of $\Lambda$ in the rest frame of $\Lambda_b$, and is given by:

$$E_\Lambda = \frac{m^2 + m'^2 - q^2}{2m} \tag{3.35}$$

Here $m$ and $m'$ are the masses of $\Lambda_b$ and $\Lambda$, respectively, with $q^2$ as described above. It is convenient to define the invariant velocity transfer, $\omega(q^2)$, as

$$\omega(q^2) = v \cdot v' = \frac{m^2 + m'^2 - q^2}{2mm'} \tag{3.36}$$

where $v$ and $v'$ are the four velocities of $\Lambda_b$ and $\Lambda$. The minimum and maximum values of $\omega(q^2)$ are obtained corresponding to $q^2 = (m - m')^2$ and $q^2 = 0$ as

$$\omega_{\min}(q^2) = 1, \quad \omega_{\max}(q^2) = \frac{m^2 + m'^2}{2mm'}$$

The zeroth order form factors $F^0_i$’s and $G^0_i$’s in terms of $\theta_1$ and $\theta_2$ are given as:

$$F^0_1 = \theta_1 - \theta_2, \quad F^0_2 = 2\theta_2, \quad F^0_3 = 0, \tag{3.37}$$

and

$$G^0_1 = \theta_1 + \theta_2, \quad G^0_2 = 2\theta_2, \quad G^0_3 = 0. \tag{3.38}$$

These zeroth order form factors lead to the following relations,

$$G^0_1 = F^0_1 + F^0_2; \quad G^0_2 = F^0_2; \quad G^0_3 = F^0_3 = 0 \tag{3.39}$$

or equivalently,

$$g_1 = f_1; \quad g_2 = f_2; \quad g_3 = f_3 = -f_2 \tag{3.40}$$

The radiative corrections will not be taken into account since they are not relevant in our analysis whereas the corrections proportional to $\Lambda_{QCD}/m_b$ will be systematically calculated. These latter nonperturbative corrections are computed in the next section. In the following, all the form factors will be defined as a function of the invariant velocity transfer, $\omega(q^2)$, instead of the momentum transfer, $q^2$.

### 3.5.2 $1/m_b$ Corrections to the Form Factors

Since the effective lagrangian in HQET is given by

$$\mathcal{L}_\infty = \bar{h}_v i(v \cdot D) h_v \tag{3.41}$$

where $h_v$ is the quark field as defined in the previous section, it corresponds to $b$-quark in our case. Including the corrections of $1/m_b$ the effective Lagrangian has the form,

$$\mathcal{L}_{\text{eff}} = \bar{h}_v i(v \cdot D) h_v + \frac{1}{2mQ} \bar{h}_v (iD_{\perp})^2 h_v + \frac{g_s}{4mQ} \bar{h}_v \sigma_{\mu\nu} G^{\mu\nu} h_v + O(1/mQ)^2 \tag{3.42}$$

In case of heavy to light quarks mass transition, the weak current will have the following general structure, up to the $1/m_b$ corrections;

$$\bar{q} \Gamma \psi(b) \rightarrow \bar{q} \Gamma h_v + \frac{1}{m_b} \bar{q} \gamma^\mu \gamma_5 D h_v + O(1/mQ)^2 \tag{3.43}$$
where $\Gamma$ can have values $\gamma_\mu$ or $\gamma_\mu \gamma_5$.

By including the covariant derivative, $D$, as well as the corrections at the order of $1/m_b$ to the effective Lagrangian, it leads, respectively, to the local correction given by,

$$\delta L_{\text{lo},1} = \frac{1}{2m_b} \bar{q} \Gamma_i \, D_h v$$

and the non-local corrections given by,

$$\delta L_{\text{nlo},2} = \frac{1}{2m_b} \bar{h}_v (i v \cdot D) v^2 h_v,$$

$$\delta L_{\text{nlo},3} = \frac{1}{2m_b} \bar{h}_v (i D) v^2 h_v,$$

$$\delta L_{\text{nlo},4} = \frac{1}{2m_b} \bar{h}_v \sigma_{\mu\nu} G^{\mu\nu} h_v,$$

where $q$ stands for the light quarks $u$, $d$ or $s$.

Let us start with the local term correction, $\delta L_{\text{lo},1}$, to the effective lagrangian. The matrix element usually takes the form,

$$\langle \Lambda(p', s') | \bar{q} \Gamma_i D_h v | \Lambda_b(p, s) \rangle = \bar{u}_\Lambda(p', s') \phi^\mu(\omega) \Gamma_\mu u_{\Lambda_b}(p, s)$$ (3.48)

where the form of the R.H.S of the above equation follow from the spin symmetry. The most general form of $\phi^\mu$ is,

$$\phi^\mu = (\phi_{11} v^\mu + \phi_{12} v'^\mu + \phi_{13} \gamma^\mu) + \phi'(\phi_{21} v^\mu + \phi_{22} v'^\mu + \phi_{23} \gamma^\mu)$$ (3.49)

On the other hand, the equation of motion for heavy quark is,

$$v \cdot D_h v = 0$$ (3.50)

When it is applied on eq.(3.48), we get,

$$v \cdot \langle \Lambda(p', s') | \bar{q} \Gamma_i D_h v | \Lambda_b(p, s) \rangle = 0$$ (3.51)

and it leads therefore to the following constraints, for $\Gamma = 1$ and $\Gamma = \gamma_5$, respectively,

$$\bar{u}_\Lambda(p', s') [v \cdot \phi(\omega)] u_{\Lambda_b}(p, s) = 0$$ (3.52)

$$\bar{u}_\Lambda(p', s') [v \cdot \phi(\omega) \gamma_5] u_{\Lambda_b}(p, s) = 0$$ (3.53)

Thus, two relations between the $\phi_{ij}$'s can be obtained from the above constraints as,

$$\phi_{11} + \omega \phi_{12} = -\phi_{23}$$ (3.54)

$$\phi_{21} + \omega \phi_{22} = -\phi_{13}$$ (3.55)

On the other hand, the momentum conservation also implies that,

$$\langle \Lambda(p', s') | i \partial_\mu (\bar{q} \Gamma_i D_h v) | \Lambda_b(p, s) \rangle = \langle \Lambda(p', s') | i D_\mu \bar{q} \Gamma_h v | \Lambda_b(p, s) \rangle + \langle \Lambda(p', s') | \bar{q} \Gamma_i D_\mu h_v | \Lambda_b(p, s) \rangle$$

$$= \{ (m - m') v_\mu - m' v'_\mu \} \langle \Lambda(p', s') | \bar{q} \Gamma_i D_h v | \Lambda_b(p, s) \rangle$$ (3.56)
where the equation of motion for light quark, \((i D - m_q)q = 0\) has been used. Changing \(\Gamma\) by \(\gamma_\mu \Gamma\), so that \(\Gamma\) is limited to 1 and \(\gamma_5\) for vector and axial vector currents. We get,

\[
\langle \Lambda(p', s') | \bar{q} \gamma^\mu \Gamma iD_{\mu}h_v | \Lambda_b(p, s) \rangle = \{(m - m')v_\mu - (m' - m_q)v'_\mu\} \langle \Lambda(p', s') | \bar{q} \gamma^\mu \Gamma iDh_v | \Lambda_b(p, s) \rangle
\]

For \(\Gamma = 1\) and \(\Gamma = \gamma_5\), from the above equation we get, respectively,

\[
\left[\frac{\omega - 1}{\omega}\right] (f_{11} - f_{12}) - \left[\frac{2\omega + 1}{\omega}\right] f_{13} + \left[\frac{4\omega - 1}{\omega}\right] f_{23} = (m - m_b)(F_1^0 + F_2^0) - (m' - m_q)(F_1^0 + \omega F_2^0),
\]

and

\[
\left[\frac{\omega + 1}{\omega}\right] (f_{11} - f_{12}) - \left[\frac{2\omega - 1}{\omega}\right] f_{13} + \left[\frac{4\omega - 1}{\omega}\right] f_{23} = (m - m_b)(G_1^0 - G_2^0) - (m' - m_q)(G_1^0 + \omega G_2^0),
\]

In the above equations, \(F_i^0\)'s and \(G_i^0\)'s are the zeroth order form factors as given by eqs. (3.37), (3.38).

We can get the expressions for \(f_{ij}\)'s from the above eqs. (3.58) and (3.59) in terms of zeroth order form factors as:

\[
\begin{align*}
\phi_{11}(\omega) &= \frac{\omega(\omega + 1)}{2(\omega^2 - 1)} [(m - m_b)(F_1^0 + F_2^0) - (m' - m_q)(F_1^0 + \omega F_2^0)] \\
&\quad + \frac{\omega(\omega - 1)}{2(\omega^2 - 1)} [(m - m_b)(G_1^0 - G_2^0) + (m' - m_q)(G_1^0 + \omega G_2^0)] - \frac{7\omega - 1}{\omega^2 - 1} \phi_{123}(\omega) \\
\phi_{12}(\omega) &= \frac{\omega - 1}{2(\omega^2 - 1)} [- (m - m_b)(F_1^0 + F_2^0) + (m' - m_q)(F_1^0 + \omega F_2^0)] \\
&\quad + \frac{\omega}{2(\omega^2 - 1)} [- (m - m_b)(G_1^0 - G_2^0) - (m' - m_q)(G_1^0 + \omega G_2^0)] - \frac{\omega - 7}{\omega^2 - 1} \phi_{123}(\omega) \\
\phi_{21}(\omega) &= \frac{\omega(\omega + 1)}{2(\omega^2 - 1)} [- (m - m_b)(F_1^0 + F_2^0) + (m' - m_q)(F_1^0 + \omega F_2^0)] \\
&\quad + \frac{\omega(\omega - 1)}{2(\omega^2 - 1)} [(m - m_b)(G_1^0 - G_2^0) + (m' - m_q)(G_1^0 + \omega G_2^0)] + \frac{6\omega^2 - \omega + 1}{\omega^2 - 1} \phi_{123}(\omega) \\
\phi_{22}(\omega) &= \frac{\omega + 1}{2(\omega^2 - 1)} [- (m - m_b)(F_1^0 + F_2^0) + (m' - m_q)(F_1^0 + \omega F_2^0)] \\
&\quad + \frac{\omega - 1}{2(\omega^2 - 1)} [(m - m_b)(G_1^0 - G_2^0) + (m' - m_q)(G_1^0 + \omega G_2^0)] + \frac{1 - 7\omega}{\omega^2 - 1} \phi_{123}(\omega)
\end{align*}
\]

In the above equations we have used the assumption of \(\phi_{13}(\omega) \approx \phi_{23}(\omega) \equiv \phi_{123}(\omega)\) since they are equal at zeroth order and are negligible at the first order corrections, which is of the order of \(1/m_b\), as discussed below.
The basic assumption involved in such an analysis is the following: in HQET on the scale of the heavy quark mass the light degrees of freedom have small momentum spread about their central equal velocity value. For strange baryon or meson this is not true. However, it is possible that the smearing of the momentum of the light degrees averages out effectively. In the limit of equal hadron masses we would then have the normalization condition at $\omega = 1$,

$$ F_1 + F_2 + F_3 = 1 $$  \hspace{1cm} (3.64)

which implies,

$$ F_1^0 + F_2^0 = 1 $$  \hspace{1cm} (3.65)

We therefore get the condition on corrections to the form factors as

$$ \delta F_1 + \delta F_2 + \delta F_3 = 0 $$  \hspace{1cm} (3.66)

in the limit of equal hadron masses.

In this work we do not assume the validity of an $1/m_s$ expansion but we make the assumption that the eq.(3.66) is valid up to the order we are working in even for unequal hadron masses or at most the R.H.S of eq.(3.66) $\sim \epsilon/2m_b$ for unequal hadron masses. This is indeed the case in heavy to heavy transitions where for example both eqns.(3.66) and (3.65) are true for $\Lambda_b \rightarrow \Lambda_c$ up to $1/m_Q^2$ for unequal hadron masses and it is a consequence of Luke’s theorem. So in our case we have unequal masses of hadron so we have,

$$ \delta F_1^{lo,1} + \delta F_2^{lo,1} + \delta F_3^{lo,1} = \frac{\epsilon}{2m_b} $$  \hspace{1cm} (3.67)

This allows us to derive the expression of $\phi_{123}(\omega)$ as,

$$ \phi_{123}(\omega) = \frac{\omega + 1}{16(\omega - 1)} \left[ \epsilon + (m - m_b)(F_1^0 + F_2^0) - (m' - m_q)(F_1^0 + \omega F_2^0) \right] $$

$$ + \frac{1}{8} \left[ -(m - m_b)(G_1^0 - G_2^0) - (m' - m_q)(G_1^0 + \omega G_2^0) + \frac{\epsilon(\omega + 1)}{2(\omega - 1)} \right] $$  \hspace{1cm} (3.68)

It is now obvious to calculate the local corrections to the form factors as;

$$ \delta F_1^{lo,1}(\omega) = -\frac{1}{2m_b} [\phi_{11}(\omega) + (2\omega + 1)\phi_{12}(\omega) - \phi_{21}(\omega) + \phi_{22}(\omega)] $$  \hspace{1cm} (3.69)

$$ \delta F_2^{lo,1}(\omega) = \frac{1}{m_b} [2\phi_{11}(\omega) + 2\omega \phi_{12}(\omega) + \phi_{21}(\omega) + \phi_{22}(\omega)] $$  \hspace{1cm} (3.70)

$$ \delta F_3^{lo,1}(\omega) = \frac{1}{m_b} [\phi_{12}(\omega) + \phi_{21}(\omega)] $$  \hspace{1cm} (3.71)

and

$$ \delta G_1^{lo,1}(\omega) = \frac{1}{2m_b} [\phi_{11}(\omega) + (2\omega - 1)\phi_{12}(\omega) - \phi_{21}(\omega) + \phi_{22}(\omega)] $$  \hspace{1cm} (3.72)

$$ \delta G_2^{lo,1}(\omega) = \frac{1}{m_b} [2\phi_{11}(\omega) + 2\omega \phi_{12}(\omega) - \phi_{21}(\omega) + \phi_{22}(\omega)] $$  \hspace{1cm} (3.73)

$$ \delta G_3^{lo,1}(\omega) = \frac{1}{m_b} [\phi_{12}(\omega) - \phi_{22}(\omega)] $$  \hspace{1cm} (3.74)
We can safely neglect the non-local corrections to the form factors because \( \delta L_{\text{nlo},2} = \frac{1}{2m_b} \bar{h}_v (iv \cdot D)^2 h_v \), \( \delta L_{\text{nlo},3} = \frac{1}{2m_b} \bar{h}_v (iD)^2 h_v \), and \( \delta L_{\text{nlo},4} = \frac{1}{2m_b} \bar{h}_v \sigma_{\mu\nu} G^{\mu\nu} h_v \) will only appear at the order of \( 1/m_b^2 \) and such these have a negligible contributions.

Thus the full form factors after incorporating the \( 1/m_b \) corrections, are

\[
F_i(\omega) = F_i^0 + \delta F_i \quad (3.75)
\]
\[
G_i(\omega) = G_i^0 + \delta G_i \quad (3.76)
\]

Explicitly, we can write the expressions of the form factors as

\[
F_1(\omega) = F_1^0(\omega) - \frac{1}{2m_b} [\phi_{11}(\omega) + (2\omega + 1)\phi_{12}(\omega) - \phi_{21}(\omega) + \phi_{22}(\omega)] \quad (3.77)
\]
\[
F_2(\omega) = F_2^0(\omega) + \frac{1}{m_b} [2\phi_{11}(\omega) + 2\omega\phi_{12}(\omega) + \phi_{21}(\omega) + \phi_{22}(\omega)] \quad (3.78)
\]
\[
F_3(\omega) = F_3^0(\omega) + \frac{1}{m_b} [\phi_{11}(\omega) + \phi_{21}(\omega)] \quad (3.79)
\]

and,

\[
G_1(\omega) = G_1^0(\omega) + \frac{1}{2m_b} [\phi_{11}(\omega) + (2\omega - 1)\phi_{12}(\omega) - \phi_{21}(\omega) + \phi_{22}(\omega)] \quad (3.80)
\]
\[
G_2(\omega) = G_2^0(\omega) + \frac{1}{m_b} [2\phi_{11}(\omega) + 2\omega\phi_{12}(\omega) - \phi_{21}(\omega) + \phi_{22}(\omega)] \quad (3.81)
\]
\[
G_3(\omega) = G_3^0(\omega) + \frac{1}{m_b} [\phi_{12}(\omega) - \phi_{22}(\omega)] \quad (3.82)
\]

The computational work and the evolution of the form factors verses the invariant velocity transfer, \( \omega \), are done in last chapter.

4. Analysis of Discrete Symmetries in \( \Lambda_b \to \Lambda V(1^-) \)

Looking for discrete symmetry violation effects, in \( b \)-baryon decays, can provide us a new field of research. Especially time-reversal (TR) violation effects can be of great interest. Firstly, TR can be seen as a complementary test of CP violation by assuming the correctness of the CPT theorem. Secondly, this can also be a path to follow in order to search for processes beyond the Standard Model. So that \( \Lambda_b \)-decay seems to be one of the most promising channel to reveal TR violation and CP violation signal.

A general formulation based on the M. Jacob- G.C. Wick-J.D. Jackson (JWJ) helicity formalism has been set for studying the decay process \( \Lambda_b \to \Lambda V(1^-) \). Emphasis is put on the importance of the initial \( \Lambda_b \) polarization as well as the correlations among the angular distributions of the final decay products. On the dynamical side, the Hadronic Matrix Elements (HME) appearing in the decay amplitude were computed, at the tree level approximation, in the framework of the factorization ansatze for two-body non-leptonic weak decay of heavy quark.
4.1 Kinematical properties of $\Lambda_b \rightarrow \Lambda V$ decays

The hyperons produced in proton-proton collisions as well as in other hadron collisions are usually polarized in the transverse direction. The average value of the hyperon spin being non equal to zero and, owing to Parity conservation in strong interaction, the spin direction is orthogonal to the production plane defined by the incident beam momentum, $\vec{P}_p$, and the hyperon momentum, $\vec{P}_h$. Usually, the degree of polarization depend on the centre of mass energy and the hyperon transverse momentum. We define $\vec{e}_z$ as the normal vector to the production plane:

$$\vec{e}_z = \vec{n} = \frac{\vec{p}_p \times \vec{p}_h}{|\vec{p}_p \times \vec{p}_h|}$$

Here $\vec{p}_p$ and $\vec{p}_h$ are, respectively, the proton momentum and the hyperon momentum.

Let $(\Lambda_b,XYZ)$ be the rest frame (See Fig 4.1) of the $\Lambda_b$ particle. The quantization axis $\vec{n}$ is chosen to be parallel to $\vec{e}_z$. The other orthogonal axis $\vec{e}_x$ and $\vec{e}_y$ are arbitrary in the production plane.

![Figure 2: $\Lambda_b$ decay in its transversity frame](image)

We study the kinematical properties of decays $\Lambda_b \rightarrow \Lambda V$ by the Jacob-Wick-Jackson helicity formalism, since helicity formalism has some advantages:

1. Helicity, $\lambda = \frac{\vec{\sigma} \cdot \vec{p}}{|\vec{p}|}$, depends on spin $\vec{s}$ and momentum $\vec{p}$ of the particle and does not depend on its orbital angular momentum $\vec{l}$, so that it is rotationally invariant.

2. We can work easily in the rest frame of resonances in this formalism.

It is more convenient to define a frame of three mutually orthogonal unit vectors

$$\vec{e}_z = \vec{n} = \frac{\vec{p}_p \times \vec{p}_h}{|\vec{p}_p \times \vec{p}_h|}; \quad \vec{e}_x = \frac{\vec{p}_p}{|\vec{p}_p|}; \quad \vec{e}_y = \vec{e}_z \times \vec{e}_x$$

If $\Lambda_b$ produced by means of strong interactions then it is polarized along $\vec{n}$. Therefore we choose the quantization axis along $\vec{e}_z = \vec{n}$. $\Lambda_b$ being transversally polarized, its polarization value is given by $\vec{P}^{\Lambda_b} = \langle \vec{S}_{\Lambda_b} \cdot \vec{e}_z \rangle$. Let $M_i$ be the $\Lambda_b$ spin projection along $\vec{e}_z$ axis.
We define the Spin Density Matrix (SDM) for $\Lambda_b$ as:

$$\rho^{\Lambda_b} = \frac{1}{2} \left( 1 + \vec{\sigma}^{\Lambda_b} \cdot \vec{\sigma} \right)$$  \hspace{1cm} (4.1)$$

where $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli spin matrices.

In the rest frame of $\Lambda_b$ the components of its polarization vector are

$$P^\Lambda_b = \frac{1}{2} \left( \rho^{\Lambda_b} - \rho^{\Lambda_b}_- \right), \quad P^\Lambda_b = \Re \left( \rho^{\Lambda_b}_+ \right), \quad P^\Lambda_b = -\Im \left( \rho^{\Lambda_b}_+ \right)$$

$\rho^{\Lambda_b}_{MM'}$ are the matrix elements of $\rho^{\Lambda_b}$; $M, M' = \pm$ denoting the values of the third component of the $\Lambda_b$ spin along the quantization axis $\vec{e}_z$. $\rho^{\Lambda_b}$ verifies the normalization condition

$$Tr \left( \rho^{\Lambda_b} \right) = \left( \rho^{\Lambda_b}_{++} + \rho^{\Lambda_b}_{--} \right) = 1$$

In the framework of the JWJ helicity formalism the decay amplitude, $A_0(M_i)$, for $\Lambda_b(M_i) \to \Lambda(\lambda_1)V(\lambda_2)$ is obtained by applying the Wigner-Eckart theorem to the $S$-matrix element:

$$A_0(M_i) = \langle p, \theta, \phi; \lambda_1, \lambda_2 | S^0 | 1/2, M_i \rangle = A_{(\lambda_1, \lambda_2)} D^{1/2*}_{M_i M_f} (\phi, \theta, 0)$$  \hspace{1cm} (4.2)$$

where $\vec{p} = (p, \theta, \phi)$ is the momentum of the hyperon $\Lambda$ in the $\Lambda_b$ frame (Fig 4.1) and the Wigner matrix is given by

$$D^{1/2*}_{M_i M_f} (\phi, \theta, 0) = d^{ij}_{M_i M_f} (\theta) \exp(-i M_i \phi)$$

$\lambda_1$ and $\lambda_2$ are the respective helicities of $\Lambda$ and $V$ with the possible value $\lambda_1 = \pm 1/2$ and $\lambda_2 = -1, 0, +1$. If $M_i$ is the helicity of $\Lambda_b$ and has the values $\pm 1/2$ then by conservation of angular momentum we have four possible values for the pair $(\lambda_1, \lambda_2) = (1/2, 0), (1/2, 1), (-1/2, -1), (-1/2, 0)$.

The differential cross-section can be written as

$$d\sigma \sim \sum_{M_i, M_i'} \sum_{\lambda_1, \lambda_2} \rho^{\Lambda_b}_{M_i M_i'} A_{(\lambda_1, \lambda_2)} (\Lambda_b \to \Lambda V)^2 d^{1/2}_{M_i M_f} d^{1/2}_{M_f \Lambda} \exp(i M_i \phi)$$  \hspace{1cm} (4.3)$$

where we have taken into account the initial state helicity and have summed over the final state helicities. The total angular momentum along the helicity axis, $\lambda = M_f = \lambda_1 - \lambda_2$, being fixed. As parity is not conserved in the weak interactions therefore

$$A_{(\lambda_1, \lambda_2)} (\Lambda_b \to \Lambda V) \neq A_{(-\lambda_1, -\lambda_2)} (\Lambda_b \to \Lambda V)$$

It is worthwhile to introduce the helicity asymmetry parameter, $\alpha_{AS}$, for $\Lambda_b$ as:

$$\alpha_{AS} = \left| \frac{A_{\frac{1}{2} 0}^2 + A_{-\frac{1}{2}, -1}^2 - A_{-\frac{1}{2}, 0}^2 - A_{\frac{1}{2}, 1}^2}{A_{\frac{1}{2} 0}^2 + A_{-\frac{1}{2}, 1}^2 + A_{-\frac{1}{2}, 0}^2 + A_{\frac{1}{2}, -1}^2} \right|^2$$  \hspace{1cm} (4.4)$$

The differential decay rate can be expressed in-terms of asymmetry parameter as

$$\frac{d\sigma}{d\Omega} \propto 1 + \alpha_{AS} \rho^{\Lambda_b} \cos \theta + 2 \alpha_{AS} \Re(\rho^{\Lambda_b}_+ \exp i \phi) \sin \theta$$  \hspace{1cm} (4.5)$$
Then, by averaging over the azimuthal angle, $\phi$, a standard relation is obtained for the polar angular distribution:

$$\frac{d\sigma}{d\cos \theta} \propto 1 + \alpha_{AS} P_{\Lambda_b} \cos \theta$$

(4.6)

where it can be noticed that polar angular dissymmetries are intimately related to the initial polarization of the $\Lambda_b$ resonance.

4.2 The Helicity Amplitude

On the dynamical side, both tree and penguin diagrams are involved in the evaluation of the Hadronic Matrix Elements (HME). Heavy Quark effective theory is extensively used for the calculation of HME.

In tree approximation, the effective interaction Hamiltonian, $H^{eff}$ is,

$$H^{eff} = \frac{G_F}{\sqrt{2}} V_{q_6} V_{qs}^{\ast} \sum_{i=1}^{10} C_i(m_b) O_i(m_b)$$

(4.7)

where $C_i(m_b)$ are the Wilson Coefficients and the operators, $O_i(m_b)$ can be understood as local operators which govern the weak interaction of quarks in the given decay. They can be written as

$$O_i = (\bar{q}_a \Gamma_{i1} q_\beta) (\bar{q}_\mu \Gamma_{i2} q_\nu)$$

where $\Gamma_{ij}$ denotes the gamma matrices.

By using the Factorization assumption one can get the helicity amplitude for the decay $\Lambda_b \to \Lambda V (1^-)$ as

$$A_{(\lambda,\lambda')} = \frac{G_F}{\sqrt{2}} f_V E_V \langle \Lambda(p', s') | \bar{s} \gamma_\mu (1 - \gamma_5) b | \Lambda_b(p, s) \rangle_{(\lambda,\lambda')} \{ V_{CKM}^{T} C_i^T - V_{CKM}^{P} C_i^P \}$$

(4.8)

where $f_V$ and $E_V$ are the decay constant and energy of Vector meson. $V_{CKM}^{T,P}$ are the CKM matrix elements for the tree and penguin diagrams while $C_i^{T,P}$ are Wilson Coefficients. The baryonic matrix element $M_{\lambda,\lambda'} = \langle \Lambda(p', s') | \bar{s} \gamma_\mu (1 - \gamma_5) b | \Lambda_b(p, s) \rangle_{(\lambda,\lambda')}$ is calculated by using the Heavy Quark Effective Theory (HQET), and read as

$$M_{\lambda,\lambda'} = \frac{1}{\sqrt{2}} \left( \frac{|\bar{P}_V|}{E_A + m' \xi^- (\omega) + \xi^+ (\omega)} \right)$$

(4.9)

$$M_{\lambda,\lambda'} = \frac{1}{\sqrt{2}} \left( \frac{|\bar{P}_V|}{E_A + m' \xi^- (\omega) - \xi^+ (\omega)} \right)$$

(4.10)

$$M_{\lambda,\lambda'} = \frac{1}{\sqrt{2}} \left( \frac{|\bar{P}_V|}{E_A + m' \xi^- (\omega) + \xi^+ (\omega)} \right)$$

(4.11)

$$M_{\lambda,\lambda'} = \frac{1}{\sqrt{2}} \left( \frac{|\bar{P}_V|}{E_A + m' \xi^- (\omega) - \xi^+ (\omega)} \right)$$

(4.12)
where $|\vec{P}_V|$ and $E_V$ are the momentum and energy of vector meson in the rest frame of $\Lambda_b$, are given as

$$|\vec{P}_V| = \sqrt{\left[ m^2 - (m_V + m'_V)^2 \right] \left[ m^2 - (m_V - m'_V)^2 \right]}$$

(4.13)

$$E_V = \frac{m^2 + m_V^2 - m'_{V}^2}{2m}, \quad \text{and} \quad E_{\Lambda} = \frac{m^2 + m_V^2 - m_{\Lambda_b}^2}{2m}$$

(4.14)

and the form factors $\xi^\pm(\omega) = \xi_1(\omega) \pm \xi_2(\omega)$ are defined for convenience. While the form factors $\xi_{1,2}(\omega)$ are evaluated in terms of heavy quark effective form factors $F_i$ as

$$\xi_1(\omega) = \frac{1}{2} \left[ 2F_1(\omega) + F_2(\omega) + F_3(\omega) \left( 1 + \frac{m_{\Lambda_b}}{m_{\Lambda}} \right) \right]$$

(4.15)

$$\xi_2(\omega) = \frac{1}{2} F_2(\omega)$$

(4.16)

### 4.3 Polarizations and Angular Distributions

Parity violation in $\Lambda_b$ weak decays into $\Lambda, V$ necessarily leads to a polarization process of the two intermediate resonances $\Lambda$ and $V$. In order to determine the vector-polarization of each resonance, a new set of axis is defined as

$$\vec{e}_L = \frac{\vec{p}}{|\vec{p}|}; \quad \vec{e}_z = \vec{n} = \frac{\vec{p} \times \vec{p}_b}{|\vec{p} \times \vec{p}_b|}; \quad \vec{e}_N = \vec{e}_z \times \vec{e}_L; \quad \vec{e}_T = \vec{e}_L \times \vec{e}_N$$

where $\vec{p}$ is the momentum of $\Lambda$ and $\vec{n}$ is the quantization plane as defined in previous section.

In this new frame, the vector-polarization of any resonance defined in the original $\Lambda_b$ frame can be written as:

$$\vec{P}^i = P_L \vec{e}_L + P_T \vec{e}_T + P_N \vec{e}_N$$

where $i = \Lambda$ or $V$ and $P_L, P_N, P_T$ are longitudinal, normal and transverse polarizations of the decay resonance.

It is worth noticing that the basis vectors $\vec{e}_L, \vec{e}_N$ and $\vec{e}_T$ have the following properties according to parity and TR: $P$-odd, $T$-odd; $P$-odd, $T$-odd and $P$-even, $T$-even respectively, while the polarization-vector $\vec{P}$ is $P$-even and $T$-odd. So using these properties we can get $P_L = P - odd, T - even, P_N = P - odd, T - even$ and $P_T = P - even, T - odd$. As $P_T$ is $T$-odd so any non-zero value of this polarization will be a clear signature of Time Reversal violation.

### 4.4 Polarization of final state resonances

Intermediate resonance states, $\Lambda$ and $V$, can be described by a density-matrix named $\rho^f$ whose analytic expression is given by standard quantum-mechanical relations:

$$\rho^f = \mathcal{T}^\dagger \rho^{i\nu} \mathcal{T}$$

(4.17)

where $\mathcal{T}$ is the transition-matrix related to the S-matrix by $S = 1 + i\mathcal{T}$. The matrix elements of the SDM $\rho^f$ are obtained from (4.17) by projecting the operators involved in
that expression onto the initial and final states. The latter ones are characterized by a
given three-momentum in the $\Lambda_b$ center-of-mass system and by a pair of helicities, $\lambda_1$ and
$\lambda_2$, corresponding to each resonance $\Lambda$ and $V$. Therefore the SDM of this two-particle
system is endowed with two pairs of indices, as:

$$\rho_{\lambda_1\lambda_2}^{f} = \sum_{M,M'} F_{\lambda_1\lambda_2}^{JM}(\theta,\phi) \rho_{\lambda_1\lambda_2}^{\Lambda_b} \rho_{\lambda_1\lambda_2}^{JM}(\theta,\phi)$$  (4.18)

where $\theta$ and $\phi$ are the polar and azimuthal angles of the momentum of $\Lambda$ resonance in the
$\Lambda_b$ rest frame respectively as shown in fig (4.1).

Taking into account the angular momentum conservation, we can write

$$\chi = \lambda_1 - \lambda_2 \quad \chi' = \lambda'_1 - \lambda'_2$$

where $\chi, \chi' = \pm 1/2$. So one can write the eq.(4.18) of the SDM as

$$\rho_{\lambda_1\lambda_2}^{f} = \sum_{M,M'} F_{\lambda_1\lambda_2}^{JM} = \sum_{M,M'} F_{\lambda_1\lambda_2}^{JM} \rho_{\lambda_1\lambda_2}^{\Lambda_b} \rho_{\lambda_1\lambda_2}^{JM}(\theta,\phi)$$  (4.20)

Jacob-Wick helicity formalism gives

$$F_{\lambda_1\lambda_2}^{JM}(\theta,\phi) = N_{J} A_{\lambda_1\lambda_2}^{I} D_{\lambda_1\lambda_2}(\theta,\phi,0)$$  (4.21)

where

$$A_{\lambda_1\lambda_2}^{I} = 4\pi \left( \frac{m_b}{|\vec{p}|} \right) \langle J, M; \lambda_1, \lambda_2 | T | J, M \rangle$$  (4.22)

is the helicity amplitude, which is evaluated in the previous section by using the HQET.

After summing over the initial polarizations of $M, M'$ of $\Lambda_b$, and taking into account
the angular momentum conservation and properties of Wigner matrices, the SDM of the
final states will be

$$\rho_{\lambda_1\lambda_2}^{f} = \frac{1}{4\pi} \left\{ A_{\lambda,\lambda'} \chi A_{\lambda',\lambda} (1 + 4\chi P_1^{\Lambda_b}) \delta_{\lambda,\lambda'} + 2 A_{\lambda,\lambda'} \chi A_{\lambda',\lambda} (P_2^{\Lambda_b} + 2i\chi P_3^{\Lambda_b}) \delta_{\lambda,\lambda'} \right\}$$  (4.23)

where we have draped the index $J$ from the amplitude and the index 1 from helicities. By
angular momentum conservation, $\chi$'s have the form,

$$\chi = \lambda_1 - \lambda_2 = \pm \frac{1}{2}; \quad \chi' = \lambda'_1 - \lambda'_2 = \pm \frac{1}{2}$$

Moreover we have set

$$P_1^{\Lambda_b} = \vec{P}^{\Lambda_b} \cdot \hat{p}, \quad P_2^{\Lambda_b} = \vec{P}^{\Lambda_b} \cdot \vec{e}_N, \quad P_3^{\Lambda_b} = \vec{P}^{\Lambda_b} \cdot \hat{r},$$  (4.24)

where $\hat{p}$ is the unit vector of the $\Lambda$ momentum and

$$\vec{e}_N = \vec{e}_T \times \hat{p}, \quad \vec{e}_T = \frac{\vec{n} \times \hat{p}}{|\vec{n} \times \hat{p}|}, \quad \hat{r} = \vec{e}_T \times \vec{n}.$$
The angular distribution of the decay products, $W(\theta, \phi)$, can be deduced from the SDM, according to the formulae

$$W(\theta, \phi) = \text{Tr} \left( \rho_{\Lambda \lambda'}^{f} \right)$$

Taking into account $\rho_{\Lambda \lambda'}^{f}$, we get

$$W(\theta, \phi) = \frac{1}{4\pi} \left( G_{W} + \Delta G_{W} P_{1}^{\Lambda b} \right)$$

where

$$G_{W} = \left| A_{1/2,0} \right|^{2} + \left| A_{-1/2,-1} \right|^{2} + \left| A_{-1/2,0} \right|^{2} + \left| A_{1/2,1} \right|^{2} \quad (4.25)$$

$$\Delta G_{W} = 2 \left( \left| A_{1/2,0} \right|^{2} + \left| A_{-1/2,-1} \right|^{2} - \left| A_{-1/2,0} \right|^{2} - \left| A_{1/2,1} \right|^{2} \right) \quad (4.26)$$

The polarization vectors of the resonance states can be evaluated as

$$\vec{P}^{i} = \frac{\text{Tr} \left( \rho_{\Lambda \lambda'}^{f} \cdot \vec{s} \right)}{\text{Tr} \left( \rho_{\Lambda \lambda'}^{f} \right)} = \frac{\text{Tr} \left( \rho_{\Lambda \lambda'}^{f} \cdot \vec{s} \right)}{W(\theta, \phi)} \quad (4.27)$$

so that,

$$W(\theta, \phi) \vec{P}^{i} = \text{Tr} \left( \rho_{\Lambda \lambda'}^{f} \cdot \vec{s} \right) \quad (4.28)$$

where $\vec{s} \equiv (s_{x}, s_{y}, s_{z})$ denotes the spin vector operator of the resonance state.

### 4.4.1 Polarization Vector of $\Lambda$

The components of the polarization vector of $\Lambda$ can be evaluated from the above relations by summing over the helicity states. The three components of the $\mathcal{P}^{\Lambda}$ have the following form:

$$W(\theta, \phi) P_{L}^{\Lambda}(\theta, \phi) \propto \gamma(+1/2) \left( \left| A_{1/2,0} \right|^{2} - \left| A_{-1/2,-1} \right|^{2} \right) + \gamma(-1/2) \left( \left| A_{1/2,1} \right|^{2} - \left| A_{-1/2,0} \right|^{2} \right)$$

$$W(\theta, \phi) P_{N}^{\Lambda}(\theta, \phi) \propto \Re \left( A_{1/2,0} A_{-1/2,0}^{*} - \mathcal{P}^{\Lambda b} \sin \theta + 2\Re(e^{i\phi} \rho_{\Lambda b}^{L}) \cos \theta + 2i\Im(e^{i\phi} \rho_{\Lambda b}^{L}) \right)$$

$$W(\theta, \phi) P_{T}^{\Lambda}(\theta, \phi) \propto -3 \left( A_{1/2,0} A_{-1/2,-1}^{*} - \mathcal{P}^{\Lambda b} \sin \theta + 2\Re(e^{i\phi} \rho_{\Lambda b}^{L}) \cos \theta + 2i\Im(e^{i\phi} \rho_{\Lambda b}^{L}) \right)$$

where

$$\gamma(\pm1/2) = \frac{1}{2} \left( 1 \pm \mathcal{P}^{\Lambda b} \cos \theta \pm 2\Re(e^{i\phi} \rho_{\Lambda b}^{L}) \sin \theta \right)$$

One can get the explicit relations for the components of polarization vector of intermediate states, which only depends on the helicity amplitude, as:

$$W(\theta, \phi) P_{L}^{\Lambda}(\theta, \phi) = \frac{1}{4\pi} \left( G_{L}^{\Lambda} + \Delta G_{L}^{\Lambda} P_{1}^{\Lambda b} \right) \quad (4.29)$$

$$W(\theta, \phi) P_{T}^{\Lambda}(\theta, \phi) = \frac{1}{4\pi} \left( G_{T}^{\Lambda} P_{2}^{\Lambda b} + \Delta G_{T}^{\Lambda} P_{3}^{\Lambda b} \right) \quad (4.30)$$

$$W(\theta, \phi) P_{N}^{\Lambda}(\theta, \phi) = \frac{1}{4\pi} \left( G_{N}^{\Lambda} P_{2}^{\Lambda b} + \Delta G_{N}^{\Lambda} P_{3}^{\Lambda b} \right) \quad (4.31)$$
where

\[ G_L^A = \frac{1}{2} \left( |A_{\frac{1}{2},0}|^2 - |A_{-\frac{1}{2},-1}|^2 + |A_{\frac{1}{2},1}|^2 \right) \]  
(4.32)

\[ \Delta G_L^A = |A_{\frac{1}{2},0}|^2 - \left| A_{-\frac{1}{2},-1} \right|^2 - \left| A_{\frac{1}{2},1} \right|^2 \]  
(4.33)

\[ G_T^A = -23 \left( A_{\frac{1}{2},0}A^+_{-\frac{1}{2},0} + A_{\frac{1}{2},1}A^+_{-\frac{1}{2},-1} \right) \]  
(4.34)

\[ \Delta G_T^A = 2R \left( A_{\frac{1}{2},0}A^+_{-\frac{1}{2},0} - A_{\frac{1}{2},1}A^+_{-\frac{1}{2},-1} \right) \]  
(4.35)

\[ G_N^A = 2R \left( A_{\frac{1}{2},0}A^+_{-\frac{1}{2},0} + A_{\frac{1}{2},1}A^+_{-\frac{1}{2},-1} \right) \]  
(4.36)

\[ \Delta G_N^A = -23 \left( A_{\frac{1}{2},0}A^+_{-\frac{1}{2},0} - A_{\frac{1}{2},1}A^+_{-\frac{1}{2},-1} \right) \]  
(4.37)

### 4.4.2 Polarization Vector of $V(1^-)$

To calculate the components of the polarization of Vector meson we have to take into account its spin $\vec{s}$ vector and corresponding helicity states, one can get:

\[ W(\theta, \phi) P_L^{AV}(\theta, \phi) = \frac{1}{4\pi} \left( G_L^V + \Delta G_L^V P^{Ab}_1 \right) \]  
(4.38)

\[ W(\theta, \phi) P_T^{V}(\theta, \phi) = \frac{1}{4\pi} \left( G_T^V P^{Ab}_2 + \Delta G_T^V P^{Ab}_3 \right) \]  
(4.39)

\[ W(\theta, \phi) P_N^{V}(\theta, \phi) = \frac{1}{4\pi} \left( \Delta G_T^V P^{Ab}_2 - \Delta G_T^V P^{Ab}_3 \right) \]  
(4.40)

where

\[ G_L^A = -2 \left( \left| A_{-\frac{1}{2},-1} \right|^2 + \left| A_{\frac{1}{2},1} \right|^2 \right) \]  
(4.41)

\[ \Delta G_L^A = \left| A_{-\frac{1}{2},-1} \right|^2 - \left| A_{\frac{1}{2},1} \right|^2 \]  
(4.42)

\[ G_T^A = -2\sqrt{23} \left( A_{\frac{1}{2},1}A^+_{\frac{1}{2},0} - A_{\frac{1}{2},-1}A^+_{\frac{1}{2},0} \right) \]  
(4.43)

\[ \Delta G_T^A = 2\sqrt{23} \left( A_{\frac{1}{2},1}A^+_{\frac{1}{2},0} + A_{\frac{1}{2},-1}A^+_{\frac{1}{2},0} \right) \]  
(4.44)

### 4.4.3 Polarization Correlations

We now define four polarization correlations, similar to those defined by Chiang and Wolfenstein [13]:

\[ W(\theta, \phi) P_{TT(NN)}(\theta, \phi) = \frac{1}{2} Tr \left[ \rho^{f} \sigma_{y(x)}^{A} s_{y(x)}^{V} \right] \]  
(4.45)

\[ W(\theta, \phi) P_{TN(NT)}(\theta, \phi) = \frac{1}{2} Tr \left[ \rho^{f} \sigma_{y(x)}^{A} s_{x(y)}^{V} \right] \]  
(4.46)

These observables are related to the angular distributions of the decay products of resonances $\Lambda$ and $V$, similar to those considered in ref. [3].
Making use of eq. (4.23) in the above equation, we get

\[ W(\theta, \phi) P_{TT}(\theta \phi) = \frac{1}{4\pi} \left( G_{TT} + \Delta G_{TT} P_{1}^{A_{b}} \right) \]  

(4.47)

\[ W(\theta, \phi) P_{NT}(\theta \phi) = \frac{1}{4\pi} \left( \Delta G_{TN} + G_{TN} P_{2}^{A_{b}} \right) \]  

(4.48)

\[ W(\theta, \phi) P_{TN}(\theta \phi) = \frac{1}{4\pi} \left( G_{TN} + \Delta G_{TN} P_{1}^{A_{b}} \right) \]  

(4.49)

\[ W(\theta, \phi) P_{NN}(\theta \phi) = -\frac{1}{4\pi} \left( G_{TT} + \Delta G_{TT} P_{1}^{A_{b}} \right) \]  

(4.50)

with

\[ G_{TT} = -\frac{1}{\sqrt{2}} \Re \left( A_{-\frac{1}{2}}, 0 + A_{\frac{1}{2}}, 0 \right), \]

\[ \Delta G_{TT} = -\sqrt{2} \Re \left( A_{-\frac{1}{2}}, 0 - A_{\frac{1}{2}}, 0 \right), \]

\[ G_{TN} = \sqrt{2} \Im \left( A_{-\frac{1}{2}}, 0 + A_{\frac{1}{2}}, 0 \right), \]

\[ \Delta G_{TN} = \frac{1}{\sqrt{2}} \Im \left( A_{-\frac{1}{2}}, 0 - A_{\frac{1}{2}}, 0 \right). \]

### 4.5 Parametrization of Observables

We can write a model independent parametrization, based on the previous formulae, of the angular distribution, of the polarization of Λ, V and polarization correlations. Our purpose for parameterizations is to describe the observables in terms of a minimum number of independent parameters.

The formulae of the angular distribution and of the polarization of Λ can be rewritten as

\[ W(\theta, \phi) = \frac{1}{4\pi} G_{W} \left( 1 + 2\alpha_{W} P_{1}^{A_{b}} \right) \]  

(4.51)

\[ \vec{P}^{A_{b}} = \frac{1}{1 + 2\alpha_{W} P_{1}^{A_{b}}} (C_{L} \vec{e}_{L} + C_{T} \vec{e}_{T} + C_{N} \vec{e}_{N}) \]  

(4.52)

with

\[ \alpha_{W} = \frac{\Delta G_{W}}{2G_{W}} \]  

(4.53)

\[ C_{L} = B_{L} \left( 1 + 2\alpha_{L} P_{1}^{A_{b}} \right) \]  

(4.54)

\[ C_{T} = B_{T} \left( P_{2}^{A_{b}} + 2\alpha_{T} P_{3}^{A_{b}} \right) \]  

(4.55)

\[ C_{N} = B_{N} \left( P_{2}^{A_{b}} + 2\alpha_{N} P_{3}^{A_{b}} \right) \]  

(4.56)

where

\[ B_{L} = \frac{G_{L}^{A}}{G_{W}}; \quad B_{T} = \frac{G_{T}^{A}}{G_{W}}; \quad B_{N} = \frac{G_{N}^{A}}{G_{W}}; \]  

(4.57)

\[ \alpha_{L} = \frac{\Delta G_{L}^{A}}{2G_{L}^{A}}; \quad \alpha_{T} = \frac{\Delta G_{T}^{A}}{2G_{T}^{A}}; \quad \alpha_{N} = \frac{\Delta G_{N}^{A}}{2G_{N}^{A}} \]  

(4.58)
One can get similar correlations for Vector meson. Where as the polarization correlations can be reshaped as:

\[ P_{TT} = \frac{1}{1 + 2\alpha_W P^\Lambda_1} B_{TT} \left( 1 + 2P^\Lambda_1 \alpha_{TT} \right) \]  
(4.59)

\[ P_{TN} = \frac{1}{1 + 2\alpha_W P^\Lambda_1} B_{TN} \left( 1 + 2P^\Lambda_1 \alpha_{TN} \right) \]  
(4.60)

where,

\[ B_{TT} = \frac{G_{TT}}{G_W}; \quad B_{TN} = \frac{G_{TN}}{G_W}; \]  
(4.61)

\[ \alpha_{TT} = \frac{\Delta G_{TT}}{2G_{TT}}; \quad \alpha_{TN} = \frac{\Delta G_{TN}}{2G_{TN}}. \]  
(4.62)

4.6 TRV, CP-V and CPT Tests

Now we illustrate properties of the above observables under discrete transformations and indicate the violation of parameters introduced in the previous section under discrete transformations.

4.6.1 Time reversal violation

Since the helicity is invariant under the time reversal (TR) operation but the rotationally invariant helicity amplitudes transform under time TR in such a way that

\[ A_{\lambda,\lambda'} A_{-\lambda,-\lambda'}^* \rightarrow A_{\lambda,\lambda'}^* A_{-\lambda,-\lambda'} \]  
(4.63)

This follows from the anti-unitarity character of TR. As the parameters \( G_T^\Lambda, \Delta G_T^\Lambda, G_T^V, \Delta G_T^V, G_{TN} \) and \( \Delta G_{TN} \) reverse sign in TR operation, and as such these parameters along with eq. (1.24) suggest that the transverse polarizations \( P^\Lambda_T(\theta, \phi), \quad P^V_T(\theta, \phi) \) and the polarization correlations \( P_{NT}(\theta, \phi) \) and \( P_{TN}(\theta, \phi) \) are T-odd under this transformation. So non-zero value of any of these observables will be a clear signature of direct TRV. Our numerical results are discussed in the next section for TRV. These observable are also promising for possible effects of New Physics as discussed in [7] and [8].

4.6.2 CP-Violation

The CP transformation causes, according to the usual phase conventions [7]

\[ A_{\lambda,\lambda'} \rightarrow -\bar{A}_{-\lambda,-\lambda'} \]  
(4.64)

where the barred amplitude refers to the \( \bar{\Lambda}_b \) decay. For detecting possible CP violation we define the following asymmetry parameters, which depend on the observables defined in
Any non-zero value of the above ratios would be a CPV asymmetry parameter. The numerical values corresponding to these observables are discussed in the next chapter. It is interesting to see that all the above ratios are even under time reversal, therefore they can be applied to test for the CPT theorem and possibly the signature of New Physics.

5. Physical Results and Conclusions

In this chapter we will put our numerical results of the observables, which we have analytically formulated in the previous chapters.

5.1 Transition form factors and Branching Ratios

The constituent quark masses are used in order to calculate the electroweak form factor transitions between baryons and our used values are:

| Table 2: Quark masses in GeV |
|-----------------------------|
| $m_u$ | $m_d$ | $m_s$ | $m_c$ | $m_b$ |
| 0.350 | 0.350 | 0.500 | 1.300 | 4.900 |

For hadron masses, we shall use the following values:

| Table 3: Hadron masses in GeV |
|-----------------------------|
| $m_{\Lambda_b}$ | $m_{\Lambda}$ | $m_{J/\psi}$ | $m_{\rho}$ | $m_{\omega}$ |
| 5.624 | 1.115 | 3.096 | 0.769 | 0.782 |

The baryon heavy-to-light form factors, $F_i(\omega)$ and $G_i(\omega)$, depending on the inner structure of the hadrons have been calculated in Chapter 3. The decay constants for vector mesons, $f_V$, do not suffer from uncertainties as large as those for form factors since they are well determined experimentally from leptonic and semi-leptonic decays. Let us first recall the usual definition for a vector meson,

$$c\langle V(q)|q_1\gamma_\mu q_2|0\rangle = f_V m_V \epsilon_\mu$$  (5.1)
where $m_V$ and $\epsilon_\mu$ are respectively the mass and polarization 4-vector of the vector meson, and $c$ is a constant depending on the given meson for example: $c = \sqrt{2}$ for the $\rho^0$ and $\omega^0$ and $c = 1$ for $J/\psi$. Numerically, in our calculations, for the decay constants we take (in MeV), $f_\rho = 209$, $f_\omega = 187$, $f_{J/\psi} = 400$. Finally, for the total $\Lambda_b$ decay width, $\Gamma_{\Lambda_b} = \frac{1}{\tau_{\Lambda_b}}$, we use $\tau_{\Lambda_b} = 1.229 \pm 0.080\,\text{ps}$.

We have calculated the transition form factors for the decay $\Lambda_b \to \Lambda V$, by using the heavy quark symmetry. The baryonic form factors involved in evaluation of transition matrix $\mathcal{M}_{\Lambda_b}^{\Lambda} \equiv \langle \Lambda(p', s') | \bar{s} \gamma_\mu (1 - \gamma_5) b | \Lambda_b(p, s) \rangle_{\lambda, \lambda'}$ and calculated in chapter 3 are plotted versus the invariant velocity for $\omega$ in Figures [5.1], [5.2] and [5.3].

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure3.png}
\caption{Form Factors $F_1$ solid curve, $F_2$ short-dashed curve and $F_3$ long-dashed curve versus invariant velocity transfer, $\omega$}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure4.png}
\caption{Form Factors $G_1$ solid curve, $G_2$ short-dashed curve and $G_3$ long-dashed curve versus invariant velocity transfer, $\omega$}
\end{figure}

The kinematical analysis and the factorization procedure developed in chapters 3 and 4, allows us to compute the branching ratios of $\Lambda_b \to \Lambda J/\psi$, $\Lambda_b \to \Lambda \rho^0$ and $\Lambda_b \to \Lambda \omega$. We have not taken into account the non-factorizable effects coming from the color octet contribution, calculations have been performed by restricting the number of colors, $N_c$ takes the value 3.
The decay width of any process like $\Lambda_b \to \Lambda V$ is given by the following formula [30],

$$\Gamma(\Lambda_b \to \Lambda V) = \left(\frac{E_\Lambda + m_\Lambda}{m_{\Lambda_b}}\right) \frac{P_V}{16\pi^2} \int_\Omega |\sum_{\lambda_\Lambda, \lambda_V} A_{\lambda_\Lambda, \lambda_V} (\Lambda_b \to \Lambda V)|^2 d\Omega \quad (5.2)$$

where in the $\Lambda_b$ rest frame, momentum of vector meson is,

$$|\vec{P}_V| = \sqrt{\left[m^2 - (m_V + m_r)^2\right] \left[m^2 - (m_V - m_r)^2\right]} \quad (5.3)$$

In eq.(5.2), $E_\Lambda$ is the energy of $\Lambda$ baryon and $\Omega$ is the decay solid angle. The helicity amplitude $A_{\lambda_\Lambda, \lambda_V} (\Lambda_b \to \Lambda V)$ is calculated in chapter 4. Branching ratios, $\mathcal{BR}$, have been calculated for the above three decays their values are,

$$\mathcal{BR}(\Lambda_b \to \Lambda J/\psi) = 6.3 \times 10^{-4} \quad (5.4)$$
$$\mathcal{BR}(\Lambda_b \to \Lambda \rho^0) = 3.8 \times 10^{-6} \quad (5.5)$$
$$\mathcal{BR}(\Lambda_b \to \Lambda \omega) = 1.6 \times 10^{-6} \quad (5.6)$$

It is worth noticing that the experimental branching ratio for $\Lambda_b \to \Lambda J/\psi$ is $(4.7 \pm 2.8) \times 10^{-4}$. So our calculated $\mathcal{BR}$ is within the experimental error. As for as the branching ratio of $\Lambda_b \to \Lambda \rho^0$ and $\Lambda_b \to \Lambda \omega$ are concerned, we have no experimental data for these channels, so its hard to make any solid conclusion.

The numerical results for the helicity asymmetry parameter, $\alpha_{AS}$, as defined in chapter 3, are summarized in Table [5.3], which can lead to a complete determination of the polar angular distribution of the $\Lambda$ hyperon in the $\Lambda_b$ rest-frame.

| $\Lambda_b \to \Lambda J/\psi$ | $\Lambda_b \to \Lambda \rho^0$ | $\Lambda_b \to \Lambda \omega$ |
|-----------------------------|-----------------------------|-----------------------------|
| $\alpha_{AS}$               | 46%                         | 43%                         | 43%                         |

Figure 5: Farm Factors $\xi_1$ solid curve, $\xi_2$ dashed curve verses invariant velocity transfer, $\omega$
5.2 Physical Observables for TR and CP violations

As discussed in the previous chapter, we have several time reversal violating parameters. Their numerical values corresponding to the decays $\Lambda_b \rightarrow \Lambda J/\psi$, $\Lambda_b \rightarrow \Lambda \rho^0$ and $\Lambda_b \rightarrow \Lambda \omega$ are summarized in Table [5.4]:

| $V (1^-)$ | $G_T^\Lambda$ | $\Delta G_T^\Lambda$ | $G_{TN}$ | $\Delta G_{TN}$ | $B_T$ | $B_{TN}$ |
|-----------|---------------|----------------------|-----------|-----------------|-------|---------|
| $J/\psi$  | $2.9 \times 10^{-27}$ | $-1.1 \times 10^{-8}$ | $-4.9 \times 10^{-26}$ | $-1.1 \times 10^{-26}$ | $1.0 \times 10^{-19}$ | $-1.7 \times 10^{-18}$ |
| $\rho^0$  | $-5.4 \times 10^{-27}$ | $-8.6 \times 10^{-10}$ | $-2.0 \times 10^{-29}$ | $5.5 \times 10^{-27}$ | $-4.1 \times 10^{-18}$ | $-1.5 \times 10^{-20}$ |
| $\omega$  | $-4.9 \times 10^{-27}$ | $-3.5 \times 10^{-10}$ | $5.2 \times 10^{-27}$ | $1.9 \times 10^{-27}$ | $-9.1 \times 10^{-18}$ | $9.6 \times 10^{-18}$ |

The numerical values of CP-violating ratios, as defined in the previous chapter, corresponding to the decays $\Lambda_b \rightarrow \Lambda J/\psi$, $\Lambda_b \rightarrow \Lambda \rho^0$ and $\Lambda_b \rightarrow \Lambda \omega$, are summarized in Table [5.5] as:

| CPV ratio | $J/\psi$ | $\rho^0$ | $\omega$ |
|-----------|---------|---------|---------|
| $R_W$     | $-0.03$ | $0.02$  | $0.62$  |
| $R_L$     | $7.02 \times 10^{-16}$ | $9.24 \times 10^{-16}$ | $4.99 \times 10^{-16}$ |
| $R_T$     | $-1.02$ | $1.05$  | $-0.48$ |
| $R_N$     | $0$     | $-1.99 \times 10^{-16}$ | $0$     |
| $R_{TT}$  | $-3.01 \times 10^{-16}$ | $-1.90 \times 10^{-16}$ | $0$     |
| $R_{TN}$  | $-9.36 \times 10^{-16}$ | $-8.30 \times 10^{-16}$ | $-4.52 \times 10^{-16}$ |
| $\gamma_W$ | $0.98$  | $-0.95$ | $2.06$  |
| $\gamma_L$ | $-0.99$ | $0.44$  | $-2.27$ |
| $\gamma_T$ | $-8.56 \times 10^{-17}$ | $-2.09 \times 10^{-16}$ | $-1.05 \times 10^{-16}$ |
| $\gamma_N$ | $14.23$ | $-0.99$ | $0.03$  |
| $\gamma_{TT}$ | $9.42 \times 10^{-17}$ | $7.59 \times 10^{-17}$ | $7.59 \times 10^{-17}$ |
| $\gamma_{TN}$ | $-0.70$ | $1.01$  | $-0.12$ |

It interesting to note that non-zero value of the CP-violating ratios in Table[5.5] are a clear signature of CP-violation in the baryonic sector. We hope that in the forthcoming LHCb experiment these CP-asymmetric ratios will be seen.

6. Conclusions

We have studied the decay process $\Lambda_b \rightarrow \Lambda V (1^-)$ where we considered the vector meson $V$ as either $J/\psi$, $\rho$ or $\omega$. In our analysis, we have investigated the branching ratios $BR$, polarizations of the decay products and helicity symmetry violating parameters for the same channels. We have also signaled out direct Time Reversal TRV and CP violating observables in a model independent analysis.
Thanks to the Jacob-Wick-Jackson helicity formalism, rigorous and detailed calculations of the $\Lambda_b$-decays into one baryon and one vector meson have been carried out completely. This helicity formalism allows us to clearly separate the kinematical and dynamical contributions in the computation of the amplitude corresponding to $\Lambda_b \rightarrow \Lambda V (1^-)$ decay. The cascade-type analysis is indeed very useful for analysing polarization properties and Time Reversal effects since the analysis of every decay in the decay chain can be performed in its respective rest frame. In order to apply our formalism, all the numerical calculations are done in Mathematica – FeynCalc. We also dealt at length with the uncertainties coming from the input parameters. In particular, these include the Cabibbo-Kobayashi-Maskawa matrix element parameters, $\rho$ and $\eta$, etc. Moreover, the heavy quark effective theory has been applied in order to estimate the various form factors, $F_i(q^2)$ and $G_i(q^2)$, which usually describe dynamics of the electroweak transition between two baryons. Corrections at the order of $O(1/mb)$ have been included when the form factors were computed.

In the calculation of b-baryon decays, we need the Wilson coefficients, $C(mb)$, for the tree and penguin operators at the scale $m_b$. One of the major uncertainties is that the hadronic matrix elements for both tree and penguin operators involve nonperturbative QCD. We have worked in the factorization approximation, with $N_c = 3$. Although one must have some doubts about factorization, it has been pointed out that it may be quite reliable in energetic weak $b$-decays.

As regards theoretical results for the branching ratios $\Lambda_b \rightarrow \Lambda J/\psi$, $\Lambda_b \rightarrow \Lambda \rho^0$ and $\Lambda_b \rightarrow \Lambda \omega$, we made a comparison with PDG for $\Lambda_b \rightarrow \Lambda J/\psi$ where an agreement is found as $BR^{exp}(\Lambda_b \rightarrow \Lambda J/\psi) = 4.7 \pm 2.8 \times 10^{-4}$ and our $BR^{th}(\Lambda_b \rightarrow \Lambda J/\psi) = 6.3 \times 10^{-4}$. So this provide some justification for the theoretical calculations we made in this thesis. For $\Lambda_b \rightarrow \Lambda \rho$ and $\Lambda_b \rightarrow \Lambda \omega$, the lack of experimental results does not allow us to draw any solid conclusions. However, we made their theoretical branching ratio predictions, as $3.8 \times 10^{-6}$ and $1.6 \times 10^{-6}$, respectively. In this work we have not taken into account the final state interactions as well as non-factorizable effects, which may have some important consequences on these decays.

The determination of the helicity asymmetry parameter, $\alpha_{AS}$, for $\Lambda_b \rightarrow \Lambda J/\psi$, and $\Lambda_b \rightarrow \Lambda \rho(\omega)$, has allowed us a complete determination of the polar angular distribution of the $\Lambda$ hyperon in the $\Lambda_b$ rest-frame. In fact, the knowledge of the $\Lambda$ polarization, $P^\Lambda$, which depends on the nature of the vector meson produced, in addition to that of the SDM elements, $\rho^\Lambda_{ij}$, may be useful to calculate the polar and azimuthal angular distributions of the proton (and pion) in the $\Lambda$ rest frame, resulting from decay $\Lambda \rightarrow p\pi$. In a similar way, the polar and azimuthal angular distributions of leptons and pseudo-scalar mesons in the vector meson rest-frame can also be computed. From weak decays analysis, one knows that vector-polarizations of outgoing resonances (or some of their components in appropriate frames) are T-odd observables.

In our work, we have shown that some new observables can be measured: by studying angular distributions of the transverse polarization vectors in the decay planes of the intermediate resonances in the $\Lambda_b$ rest-frame. We found that the magnitude of their effects is directly related to the $\Lambda_b$ polarization density matrix (PDM) and more precisely to the
non-diagonal elements, $\rho_{ij}^{\Lambda_b}$, appearing in the interference terms of the decay amplitude.

Our Time reversal violating parameters, summarized in Table[5.4], have very small values as compared to unity because we have used the Standard Model for our numerical analysis. The values for these TR and CP violating parameters may be enhanced in other models. These TR and CP violating parameters may be experimentally observable at the forthcoming LHCb run.

We conclude this note with some remarks about the method considered for the polarizations, angular distributions and C-P-T violating parameters.

1. Our analysis is completely model independent and is also independent of spurious effects [3], [7], [28] caused by final state interactions [29], which we have neglected in our analysis. In particular, we stress that our tests for TRV do not rely on any assumptions. Our calculation can be used as an input for calculating the model predictions of the observables considered here [22].

2. It is important to note that the TRV tests based on $\Lambda_b$ polarization are similar to those proposed for hyperon decays [5], [10]. However in our case we may also consider the polarization correlations [19], which provide a TRV test independent of the polarization of the parent resonance. Decays of the type $\Lambda_b \to \Lambda V$ are very suitable for detecting possible TRV, as also pointed out by other authors in studying CP violations [7], [8].

3. The observables considered in the present work are very sensitive to NP, since they are rid of unpleasant effects of Wilson’s coefficients. These quantities have been considered even more convenient than $B^0 - \bar{B}^0$ mixing phases [7].

4. Reactions similar to those studied here have been proposed also by other authors[41, 42] in a different context, for LHC forthcoming experiment. Then it appears not unrealistic to suggest to measure also some of the observables considered in the present note, that is, the angular distribution and the polarization of at least one of the decay products.

In our opinion, new fields of research like direct CP violation and T-odd observables indicating a possible non-conservation of Time Reversal symmetry can be investigated in the sector of beauty baryons and especially the $\Lambda_b$-bayons which can be copiously produced in the future hadronic machine like LHC. In order to reach this aim, all uncertainties in our calculations still have to be decreased, for example non-factorizable effects have to be evaluated with more accuracy and final state effects should be taken into consideration. Moreover, we strongly need more numerous and accurate experimental data in $\Lambda_b$-decays, especially the $\Lambda_b$ polarization. We expect that our predictions should provide useful guidance for future investigations and urge our experimental colleagues to measure all the observables related to $\Lambda_b$ baryon decays, if we want to understand direct CP violation and Time Reversal symmetry better.
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