Forecast of cross-correlation of CSST cosmic shear tomography with AliCPT-1 CMB lensing

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ABSTRACT

We present a forecast study on the cross-correlation between cosmic shear tomography from the Chinese Survey Space Telescope (CSST) and CMB lensing from Ali CMB Polarization Telescope (AliCPT-1) in Tibet. The correlated galaxy and CMB lensing signals were generated from Gaussian realizations based on inputted auto- and cross-spectra. To account for the error budget, we considered the CMB lensing reconstruction noise based on the AliCPT-1 lensing reconstruction pipeline; shape noise of the galaxylensingmeasurement; CSSTphoto-\(z\) error; photo-\(z\) bias; intrinsic alignment effect, and multiplicative bias. The AliCPT-1 CMB lensing mock data were generated according to two experimental stages, namely the “4 modules*yr” and “48 modules*yr” cases. We estimate the cross-spectra in 4 tomographic bins according to the CSST photo-\(z\) distribution in the range of \(z \in [0, 4]\).

After reconstructing the pseudo-cross-spectra from the realizations, we calculate the signal-to-noise ratio (SNR). By combining the 4 photo-\(z\) bins, the total cross-correlation SNR\(\approx 15\) (AliCPT-1 “4 modules*yr”) and SNR\(\approx 22\) (AliCPT-1 “48 modules*yr”). Finally, we study the cosmological application of this cross-correlation signal. Excluding intrinsic alignment (IA) in the template fitting would lead to roughly a 0.6\(\sigma\) increment in \(\sigma_8\) due to the negative IA contribution to the galaxy lensing data. For AliCPT-1 first and second stages, the cross-correlation of CSST cosmic shear with CMB lensing gives errors on the clustering amplitude \(\sigma_{8s} = -0.043^{+0.030}_{-0.027}\) or \(\sigma_S = \pm 0.031\) and \(\sigma_{8s} = -0.027^{+0.030}_{-0.027}\) or \(\sigma_S = \pm 0.018\), respectively.

Key words: cosmology, cosmic shear, CMB lensing

1 INTRODUCTION

Weak gravitational lensing (WL) is a powerful tool in constraining cosmology (Refregier 2003; Mandelbaum 2018; Blake et al. 2020; Jullo et al. 2019; Zhang et al. 2007), however, the so-called “\(S_8\) tension” between WL and cosmic microwave background (CMB) anisotropy observations prevent us from using their synergy (Aghanim et al. 2020c,a; Abbott et al. 2022; Heymans et al. 2021; Asgari et al. 2021; Hikage et al. 2019; Hamana et al. 2020). Whether this tension is due to new physics beyond the standard \(\Lambda\)CDM model will require us to carefully address all kinds of possible systematics (Wright et al. 2020; Yao et al. 2020, 2017; Kannawadi et al. 2019; Mead et al. 2021; Secco et al. 2022). The cross-correlations between different tracers are important tools in investigating such problems, as they are immune to many systematics and can bring extra cosmological information.

Cross-correlations of galaxy surveys with CMB lensing provide a powerful way to probe the large-scale structure (LSS) of the Universe (Robertson et al. 2021a; Zhang et al. 2021; Omori et al. 2022). CMB lensing can be reconstructed by observing temperature and polarization anisotropies, it allows us to reconstruct a map of the integrated over-the-line-of-sight overdensity of intervening matter (Hu & Okamoto 2002; Okamoto & Hu 2003). Galaxy image surveys provide a trace of LSS, WL can be detected by capturing the images of a large sample of galaxies, usually called source galaxies, and performing shape measurements that can then be analyzed statistically (Camacho et al. 2021; Mandelbaum 2018; Bartelmann & Schneider 2001a). The correlations between the CMB lensing and cosmic shear\textsuperscript{1} have been extensively measured, however with low significance, due to the limit of the current observations (Robertson et al. 2021b; Kirk et al. 2016; Harnois-Déraps et al. 2016; Omori et al. 2019; Hand et al. 2015; Marques et al. 2020; Liu & Hill 2015; Harnois-Dérap et al. 2017; Namikawa et al. 2019; Singh et al. 2017).

Several CMB experiments have reconstructed the CMB lensing signals, such as Planck (Aghanim et al. 2020c), ACT (Mallaby-Kay et al. 2020).
This paper is organized as follows. In Section 2 we will introduce the basics of CMB lensing and WL. In Section 3, we will present the method of simulating the CMB lensing $\kappa$ maps and the cosmic shear $\gamma$ maps, including both the cosmological signal and the systematic effects. In Section 4 we will present the cross-correlation pseudo-$C_\ell$ measurements method and describe the covariance matrix, computing SNR. In Section 5 we will present the cosmological parameter constraint results based on the simulated maps.

2 THEORETICAL MODELLING

2.1 CMB lensing

CMB lensing signal reconstructed from CMB temperature and polarization maps traces the integrated matter field along the line-of-sight direction from the current observer to the last scatter surface with redshift $z_L \approx 1100$

$$
k_{\text{CMB}}(\theta) = \int_0^{z_L} dz q_{\text{CMB}}(z) \delta(\chi(z)\theta, z),$$

where lensing kernel $q_{\text{CMB}}(z)$ reads

$$q_{\text{CMB}}(z) = \frac{3\Omega_m H_0^2}{2c^2 H(z)} (1+z)^2 \chi(z),$$

$\chi$ is the comoving distance, $H(z)$ is the Hubble parameter. As is well known, the maximum efficiency of lensing occurs when the lenses are positioned halfway between the source and the observer. In the case of CMB lensing, the background light originates from the last scattering surface. Consequently, the lensing efficiency gradually increases from low redshift and plateaus after $z \approx 1$ and peaks at $z \approx 2$. This plateau remains constant until high redshift values. Unlike cosmic shear, which necessitates tomographic analysis based on the distribution of source galaxies, CMB lensing signals can compress a vast majority of cosmological matter distribution information into a single sphere. For CMB lensing reconstruction, we adopt the Planck 2018 lensing paper (Aghanim et al. 2020c; Carron & Lewis 2017) formalism, which closely resembles the original Hu-Okamoto formalism (Hu & Okamoto 2002; Okamoto & Hu 2003). Since lensing creates correlations between different multipoles, the quadratic estimator utilizes these induced correlations in an almost optimal and unbiased way. The quadratic estimator spectrum captures the sought-after signal but also unavoidably includes Gaussian reconstruction noise sourced by both the CMB and instrumental noise (N0 bias) and non-primary couplings of the connected 4-point function (Kesden et al. 2003) (N1 bias). In the AliCPT-1 experimental setup (Liu et al. 2022), the former is completely dominant compared to the latter two biases. Therefore, we only consider the N0 term in the noise budget of CMB lensing in this work.

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2 The unit denotes for the accumulated data length.

3 the data vector refers to the time-ordered data (TOD) of CMB. The “48 modules*yr” is obtained from 4 seasons according to our current detector upgrade plan. The “4 modules*yr” is assuming 4 modules observe for 1 season.
2.2 Cosmic shear

The weak lensing effect of source galaxies due to the intervening large-scale structure is called cosmic shear. The corresponding lensing potential \( \phi(\theta) \) is defined as

\[
\phi(\theta) = -\frac{2}{c^2} \int_0^\chi_\ast \frac{d\chi}{\chi^2} \chi \Psi(\theta, \chi)
\]

where \( \Psi(\theta, \chi) \) is the 3D Weyl gravitational potential. It induces a distortion of the shape and provides a mapping from lens plane position \( \theta \) to the source plane position \( \beta \). The local properties of a gravitational lens are characterized by the Jacobian matrix \( \mathcal{A} \) of the mapping given by (Bartelmann & Schneider 2001b)

\[
\mathcal{A} = \frac{\partial \beta}{\partial \theta} = \delta_{ij} + \frac{\partial^2 \phi}{\partial \theta_i \partial \theta_j} = \begin{bmatrix}
1 - \kappa - \gamma_1 & -\gamma_2 \\
-\gamma_2 & 1 - \kappa + \gamma_1
\end{bmatrix}
\]

with

\[
\kappa = \frac{1}{2} \nabla^2 \phi,
\gamma_1 = \frac{1}{2} \left( \frac{\partial^2 \phi}{\partial \theta_1^2} - \frac{\partial^2 \phi}{\partial \theta_2^2} \right),
\gamma_2 = \frac{\partial^2 \phi}{\partial \theta_1 \partial \theta_2}.
\]

where \( \kappa \) is the lensing convergence and \( \gamma = \gamma_1 + i\gamma_2 \) is the complex lensing shear field.

Due to the intrinsic ellipticity of the source galaxies, the observed galaxy shape is actually the mixture of the cosmic shear and the intrinsic ellipticity (Bonnet & Mellier 1995). Assuming \( \kappa \ll 1 \) and \( \gamma_1, \gamma_2 \ll 1 \), up to the first order, the lensed ellipticity reads

\[
e = e_s + \gamma,
\]

where \( e_s \) is the intrinsic ellipticity of the source galaxy. Using the Poisson equation one can write the convergence in terms of the density perturbation \( \delta(\theta, z) \)

\[
\kappa_i(\theta) = \int_0^{z_i} \frac{cdz}{H(z)} q_i(z) \delta(\theta, z),
\]

where lensing efficiency \( q_i(z) \) in cosmic shear measurement is defined

\[
q_i(z) = \frac{3\Omega_m}{2} \frac{H^2(z)}{c^2} (1 + z) \chi(z) \int_z^{z_i} n_i(z') \frac{\chi(z') - \chi(z)}{\chi(z')} dz'.
\]

The source galaxies distribution in the \( i \)th redshift bin is described as a normalized distribution \( n_i(z) \). In harmonic space, we have the following relation between the convergence field, shear field, and the lensing potential field

\[
\kappa(\ell) = \frac{1}{2} \ell^2 \phi(\ell),
\gamma(\ell) = \left( \frac{\ell^2 - \ell_1^2 + 2i\ell_1 \ell_2}{|\ell|^2} \right) \kappa(\ell) = \kappa(\ell) e^{2i\beta},
\]

where \( \beta \) is the polar angle of \( \ell \) (Schneider et al. 2002). In galaxy surveys, direct measurement of the convergence field is not possible due to the inability to determine the intrinsic luminosity of source galaxies. However, we can measure the ellipticity of galaxies which contains information about the cosmic shear field \( \gamma^G \). Unfortunately, observed ellipticity is subject to contamination from both the galaxy’s intrinsic shape and intrinsic alignment induced by its local environment. Consequently, we express the measured shear as \( e = e_s + \gamma^G + \gamma^I \), where \( \gamma^I \) represents the intrinsic alignment. Furthermore, by assuming a white noise statistical property, the intrinsic shear noise power spectrum can be expressed as

\[
N_i^{\kappa\kappa}(\ell) = \frac{4\pi f_{sky}}{N_i} \sigma^2_\varepsilon,
\]

where the ellipticity dispersion \( \sigma_\varepsilon \approx 0.3 \) (Miao et al. 2022), \( f_{sky} \) is the fraction of sky used in the analysis, and \( N_i \) is total number of galaxies in the \( i \)th redshift bin.

In cosmic shear analysis, in addition to the dominant intrinsic ellipticity of galaxies, there is a second source of contamination, namely intrinsic alignment. This signal can be attributed to its correlation with either the gravitational tidal field of large-scale structures (GI term) or the intrinsic alignment of neighboring galaxies within local environments (II terms). (Troxel & Ishak 2014). Hence, the observed angular power spectrum of shear measurement is composed of four components (Bridle & King 2007)

\[
\hat{C}_{ij}^{\kappa\kappa}(\ell) = C_{ij}^{\kappa\kappa}(\ell) + C_{ij}^{\kappa I}(\ell) + C_{ij}^{GI}(\ell) + N_i^{\kappa\kappa}(\ell) \delta_{ij},
\]

where \( C_{ij}^{\kappa\kappa}(\ell) \) is the convergence power spectrum, \( C_{ij}^{\kappa I}(\ell) \) and \( C_{ij}^{GI}(\ell) \) are the Intrinsic-Intrinsic (II) and Gravitational-Intrinsic (GI) power and cross spectra, respectively.

For simplicity, we adopt the Limber approximation (Limber 1953) instead of the fast oscillated spherical harmonics, which is approved a good approximation with an error smaller than 2% on the scales of \( \ell > 20 \) (Kilbinger et al. 2017)

\[
C_{ij}(\ell) = \int_0^{z_i} \frac{cdz}{H(z)} q_i(z) \frac{\chi(z)}{\chi^2} P_\phi(k = \frac{\ell + 1/2}{\chi}, z),
\]

where \( P_\phi \) is the CMB power spectrum.
The Intrinsic-Intrinsic and Gravitational-Intrinsic power and cross spectra are given assuming the non-linear alignment (NLA) model (Hirata & Seljak 2004; Bridle & King 2007)

\[
C_{ij}^{II}(\ell) = \int_0^{z_\text{c}} \frac{cd\zeta}{H(z)} n_i(z) n_j(z) f_i(z) f_j(z) \delta^2(k = \frac{\ell}{\chi}, z),
\]

and

\[
C_{ij}^{GI}(\ell) = \int_0^{z_\text{c}} \frac{cd\zeta}{H(z)} n_i(z) n_j(z) q_i(z) q_j(z) \delta^2(k = \frac{\ell}{\chi}, z) + \int_0^{z_\text{c}} \frac{cd\zeta}{H(z)} n_i(z) f_i(z) q_j(z) \delta^2(k = \frac{\ell}{\chi}, z),
\]

where \(f_i(z)\) is written as

\[
f_i(z) = -A_{\text{IA}} C_1 \rho_{\text{crit}} D(z) \left( \frac{1 + z}{1 + z_0} \right)^{\eta_{\text{IA}}} \left( \frac{L_i}{L_0} \right)^{\beta_{\text{IA}}},
\]

and \(C_1 = 5 \times 10^{-14} h^{-2} M_{\odot}^{-1} \text{Mpc}^3\) as in Bridle & King (2007), \(\rho_{\text{crit}}\) is the present critical density, \(D(z)\) is the linear growth factor normalized to unity at \(z = 0\), and \(z_0 = 0.6\) and \(L_0\) are pivot redshift and luminosity, respectively. Since the change of average luminosity can be ignored, we don’t consider luminosity dependence and the fiducial values of \(A_{\text{IA}}, \eta_{\text{IA}}\) and \(\beta_{\text{IA}}\) are set to be 1, 0, 0, respectively (Joudaki et al. 2016). In Fig. 1, we show the auto- and cross-spectra of the cosmic shear and intrinsic alignment in 4 photo-z bins, which are defined in the following sections. First of all, the intrinsic alignment model we adopted is anti-correlated with cosmic shear. Second, the amplitude of the II correlation is much smaller than GG. Taking these two effects together into account, we can conclude that the \(C_{ii}^{\gamma E}\) in the Eq. (11) gets smaller once we consider the intrinsic alignment.

### 2.3 Weak lensing-CMB lensing cross-correlation

Since the convergence field and shear field are both determined by the gravitational potential \(\phi\), the CMB lensing convergence field and the cosmic shear are correlated. Furthermore, we can find a linear combinations of \(\gamma_1\) and \(\gamma_2\) to convert shear field into E and B mode (\(\gamma_E, \gamma_B\))

\[
\gamma_E = \gamma_1 \left( \frac{\ell_1^2 - \ell_2^2}{\ell_1^2 + \ell_2^2} \right) + \gamma_2 \left( \frac{2\ell_1 \ell_2}{\ell_1^2 + \ell_2^2} \right) = \kappa(\ell); \quad \gamma_B = 0.
\]

The cross-spectrum of weak lensing and CMB lensing field \(C_i^{\gamma E}(\ell)\) reads

\[
\langle \kappa_{\text{CMB}}(\ell) \gamma_E(\ell') \rangle = (2\pi)^2 \delta_D(\ell - \ell') C_i^{\gamma E}(\ell),
\]

where \(\delta_D\) is the 2-dimensional Dirac function and the sub-index \(i\) denotes for the \(i\)th redshift bin. Under the Limber approximation, the theoretical cross-spectrum reads

\[
C_i^{\gamma E}(\ell) = \int_0^{z_\text{c}} \frac{cd\zeta}{H(z)} q_{\text{CMB}}(\zeta) q_i(\zeta) \delta^2(k = \frac{\ell}{\chi}, z),
\]

where \(P_\delta(k, z)\) is the nonlinear matter power spectrum, which is calculated with pyccel (Chisari et al. 2019) and with the HALOFIT method for describing the nonlinear part (Smith et al. 2003; Takahashi et al. 2012). The cross-correlations are also affected by the IA according to Eq.
Figure 2. Lensing efficiencies that enter the cosmic shear-CMB lensing correlations as defined in Eq. (2) and Eq. (8). The vertical gray line marks the effective redshift $z \approx 0.73$ for the cosmic shear-CMB lensing cross-correlation signal, measured from the nominal CSST number density. For cosmic shear, we used the CSST source sample redshift distribution shown in Fig. 3. The combined lensing efficiency is defined in Eq. (19).

Figure 3. The mock galaxy redshift distribution in the photometric imaging survey of the CSST. The black line denotes the total redshift distribution $n(z)$, which is obtained from the COSMOS-2015 catalog (Cao et al. 2018). The blue, orange, green, and red curves are the true redshift distribution $n_i(z)$ in each of the four photo-$z$ bins. The four photo-$z$ bins are divided by the gray vertical dashed curves.

(14). As shown in Ref. Troxel & Ishak (2014); Hall & Taylor (2014), IA will suppress CMB Lensing-cosmic shear cross spectrum by roughly 15%.

In Fig. 2, we show the lensing efficiency of CMB lensing and cosmic shear$^4$ as a function of redshift, as well as the combined weight given by

$$q_{\text{combined}}(z) = \frac{H(z)}{c} D^2(z) q_{\text{CMB}}(z) q_\gamma(z),$$

(19)

where $D(z)$ is the growth function normalized to the unit at $z = 0$, accounts for the growth of matter perturbations with redshifts, and $q_\gamma(z)$ is lensing efficiency of cosmic shear with the source distribution in the entire range of redshift $n(z)$, shown in Fig. 3.

$^4$ calculated using the CSST source mock redshift distribution (solid black curve in Fig. 3)
3 SIMULATION

In this section, we present our simulations to generate correlated signals between CMB lensing and cosmic shear. The sky coverage of the CSST wide field is 17,000 deg$^2$ (Zhan 2021), while the AliCPT-1 deep patch’s sky coverage is approximately 4,000 deg$^2$ in the high latitude of the northern hemisphere (Liu et al. 2022). Therefore, the entire AliCPT-1 patch is covered by CSST. In our signal generation, we adopt the flat sky approximation for simplicity. To match this approximation, we choose a 30x30 deg$^2$ sky area for the cross-correlation computation. However, it should be noted that considering only a sky overlap of 900 deg$^2$ instead of the full 4000 deg$^2$ is a significant limitation of this work. While using the full field of AliCPT-1 would yield more realistic results, it would also require a more complicated spherical harmonics transformation than the simple Fourier transformation used in the flat field. As such, we have adopted the flat-sky approximation in this paper. To the best of our knowledge, this approximation becomes increasingly inaccurate for $\ell < 10$ (Lemos et al. 2017), which corresponds to an angular scale greater than roughly 30 degrees.

In order to generate the correlated signals between CMB lensing and cosmic shear, the most straightforward way is to measure these two quantities in the same mock catalog. However, due to the unmatched lensing kernel between WL and CMB lensing, it will be very expensive to do a large sky coverage ray tracing through N-body simulation until the redshift range where CMB lensing efficiency is enough high. To evade this obstacle, we generate the correlated cosmic shear and CMB lensing signals from the Gaussian realizations based on the inputted auto- and cross-spectra. We generate both CMB lensing $\kappa$ map and cosmic shear with healpy (Górski et al. 2005; Zonca et al. 2019a). The resolution is chosen as $N_{\text{side}} = 512$, corresponding to a pixel size of about $7'$.

We choose the same Gaussian window function for both the CMB lensing convergence and WL shear maps, which equals $\ell_\kappa = 800, W_\kappa = \exp(-\ell^2/\ell_\kappa^2)$. We neglect the baryonic effect since it’s only sensitive on scales smaller than $\ell = 1500$ (Joachimi et al. 2021). To avoid the leakage from the sharp edges due to the Fourier transformation, we apodize the maps with a cosine function and apodization scale of $2^\circ$. In order to generate a set of maps that satisfy the desired auto- and cross-spectrum of CMB lensing and cosmic shear, we extend Kamionkowski’s method (Kamionkowski et al. 1997) to an arbitrary number of maps. The maps we are going to simulate in harmonic space are written as

$$M_i(\ell) = \zeta_i(\ell) s_0 + \zeta_i(\ell) s_1 + \ldots + \zeta_i(\ell) s_n \quad (i = 0, 1, 2, ..., n)$$

$s$ has the following recursive form

$$s_{ij} = C_{ij} - \sum_{k=0}^{i-1} s_{ki} s_{kj},$$

$$C_{ij} = \left(C_{ii} - \sum_{k=0}^{i-1} s_{ki}^2\right)^{1/2},$$

where $\zeta_i(\ell)$s are two independent complex numbers drawn from a Gaussian distribution with zero mean and unit variance. $C_{ij}$ denotes the auto- and cross-spectrum, for $i, j = 0$ we have $s_0 = C_{00}$. We would like to emphasize that the method we presented here is not novel. There are pioneer works that have already implemented this algorithm into CMB (including integrated Sachs–Wolfe effect) and galaxy number count cross-correlations, such as Giannantonio et al. (2008). Besides, several commonly used software in the cosmology community have also implemented this algorithm, such as healpy (Zonca et al. 2019b), HEALPix (Górski et al. 2005) and NaMaster (Alonso et al. 2019).

3.1 CMB lensing map

In CMB lensing, we generate the convergence signal spectrum using pyccl, while the noise spectrum is obtained from AliCPT mocks using both temperature and polarization (Liu et al. 2022; LIU JinYi 2022). Our analysis employs an observed patch that covers about 12% of the sky (Li et al. 2018). We investigate two noise levels in this study, which are depicted by the orange dashed and green dotted curves in Fig. 4 for both temperature and polarization (Liu et al. 2022; LIU JinYi 2022). Our analysis employs an observed patch that covers about 12% of the sky.

In order to generate a set of maps that satisfy the desired auto- and cross-spectrum of CMB lensing and cosmic shear, we extend Kamionkowski’s method (Kamionkowski et al. 1997) to an arbitrary number of maps. The maps we are going to simulate in harmonic space are written as

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In NaMaster algorithm, the mode-coupling due to the partial sky effect is mitigated by inverting the mode-coupling matrix in the calculation of the pseudo-$C_\ell$. Hence, the apodization is not a necessary step anymore. We have verified that whether or not to perform apodization on the mask does not affect the measurement of pseudo-$C_\ell$. We appreciate the anonymous referee for pointing this out.

A similar expression is also derived in Giannantonio et al. (2008)
3.2 Galaxy samples and cosmic shear

In this section, we describe the methodology used to generate our cosmic shear maps. To create the simulated convergence and shear maps, we utilized Eq. (12)-(14). The IA signals were implemented in the power spectra following Eq. (11). To validate these maps, we compared the reconstructed auto-spectrum with the theoretical ones. We considered three types of errors or biases: photometric redshift, galaxy shape error, and intrinsic alignment. In this work, we assumed that these three errors are independent. While we did not model the intrinsic alignment at the map level, we treated it as part of the shear signal and modeled it in the angular power spectrum in Eq. (11). Two types of shear power spectrum models were considered: one with only cosmic shear signals (model I) and another with both cosmic shear signals and intrinsic alignment (model II). As previously demonstrated, since there is an anti-correlation between cosmic shear and intrinsic alignment, the amplitude in model II is smaller than in model I. The total galaxy number density is assumed to be 20 arcmin$^{-2}$. We divide the source galaxy samples into 4 bins as it is shown. We use this galaxy redshift distribution to calculate the cosmic shear signal according to Eq. (8). We generate the harmonic coefficients of the convergence map according to Eq. (20) with $C_{ij} = C^{\gamma E \gamma E}_{ij}(\ell)$

$$k^\text{sig}_i(\ell) = \sum_k \zeta_k(\ell) s_{ki}.$$  

Here, we emphasize that Eq. (20) is only applicable for the spin-0 field, namely the convergence field. To get the shear field (spin-2 field), we need to go through Eq. (9), where $\beta$ is the polar angle of the vector $\ell$. The shape noise is generated according to its power spectrum given by Eq. (10). The noise level is shown in Fig. 1 and is jointly determined by the galaxy number density ($n_i$) and the ellipticity dispersion of a single galaxy ($\sigma_\epsilon$).

Due to the errors in determining photometric redshift, the real galaxy redshift distribution in the $i$th photo-$z$ bin are conventionally expressed as eg. (Ma et al. 2006)

$$n_i(z) = \int_{z_{P,\text{min}}}^{z_{P,\text{max}}} dz^P n(z) p(z^P|z),$$  

where $z^P$ is photo-$z$, $n(z)$ is the total redshift distribution (shown in Fig. 3), which is calculated from COSMOS-2015 catalog by applying the observational limits from the CSST bands (Liu et al. 2023; Cao et al. 2018). $p(z^P|z)$ is the photo-$z$ distribution function given the real redshift $z$

$$p(z^P|z) = \frac{1}{\sqrt{2\pi}\sigma_z(1+z)} \exp \left[ -\frac{(z - z^P - \Delta_z)^2}{2(\sigma_z(1+z))^2} \right].$$  

where $\Delta_z$ and $\sigma_z$ are the redshift bias and scatter, respectively. In this work, we adopt the typical values in the 4th generation surveys as $\Delta_z = 0.005$ and $\sigma_z = 0.05$. The photo-$z$ errors in the maps arise via redistribute the true galaxy redshift according to the above probability.
Figure 5. CMB lensing convergence map and cosmic shear maps in 4 redshift bins. The map in the first row is the CMB convergence. The second and third rows, from left to right, are the real and imaginary parts of the plural cosmic shear from the first to the fourth redshift bins, respectively.

distribution. For the pixelized representation of cosmic shear catalogs, we construct re-weighted tomographic maps as

\[
\hat{\gamma}_i(p) = \frac{\sum_j w_{j\rightarrow i}(p)(\gamma^{\text{sig}}_j(p) + \kappa_j(p))}{\sum_j w_{j\rightarrow i}(p)},
\]

(26)

where \( p \) denotes pixel index, \( w_j \) are the probability of a shear signal leaking from \( j \)th redshift bin to \( i \)th photo-\( z \) bin. The weights are drawn by multigaussian distribution \( w_{j\rightarrow i} \sim M\{N_{\text{bins}}, [p_{j\rightarrow i}]\} \)

\[
p_{j\rightarrow i} = \int_{z_{i,\text{min}}}^{z_{i,\text{max}}} dz^P p(z^P | z_j) = \frac{1}{2} \left[ \text{erf}(x_{j\rightarrow i}^\text{max}) - \text{erf}(x_{j\rightarrow i}^\text{min}) \right],
\]

(27)

with

\[
x_{j\rightarrow i}^{\text{max/min}} = \left[ \frac{(z_j - z_{i,\text{max/min}} - \Delta z_i^j)^2}{2(\sigma_z(1 + z_j))^2} \right],
\]

(28)

where \( \text{erf}(x) \) is error function. The resulting shear maps are shown in the second row and third row in Fig. 5.
Table 1. Signal-to-noise ratio of pseudo-C$_\ell$s measured from 4 photo-z bins and the combined one. Different CMB lensing noise, galaxy photo-z bias as well as intrinsic alignment are considered.

| S/N | Bin1 | Bin2 | Bin3 | Bin4 | Total |
|-----|------|------|------|------|-------|
| N04 | 6.35 | 7.83 | 10.50 | 12.91 | 15.01 |
| N04 with bias-ζ | 6.38 | 7.87 | 10.53 | 12.94 | 15.04 |
| N04 with IA | 5.60 | 7.00 | 10.08 | 12.65 | 14.70 |
| N048 | 7.42 | 11.53 | 16.00 | 21.15 | 22.63 |
| N048 with bias-ζ | 7.49 | 11.60 | 16.05 | 21.16 | 22.66 |
| N048 with IA | 6.21 | 10.26 | 15.41 | 20.77 | 22.20 |

4 PSEUDO POWER SPECTRA MEASUREMENT

We generated 600 simulated maps of the CMB convergence field and the cosmic shear field using the same method but with different random seeds. The angular cross spectra between the CMB and shear maps were computed using a pseudo-C$_\ell$ estimator based on the NaMaster algorithm (Alonso et al. 2019) employing Eq. (17). To accelerate the covariance computation and invert the mode coupling matrix in the pseudo-C$_\ell$ calculation, we binned the cross-spectrum in $\ell$. We utilized 19 linearly spaced multipole bins with a width of $\Delta \ell = 40$ in the range $20 \leq \ell \leq 800$, resulting in estimated band powers denoted as $\tilde{C}^{\text{GEMB}_\ell}$ (L), where L denotes the multipole bin and i for the photo-z bin. To simplify the index notations, we merged the multipole index L and photo-z bin index i into a single index $\nu = [L^{(1)}, L^{(2)}, L^{(3)}, L^{(4)}]$, where we sequentially list the banded multipoles in each photo-z bin. In order to highlight the higher multipoles visually, we use the multipole-weighted spectrum $D_\nu = L C_{\nu} B_\nu^{-1}$, where $C_{\nu} = [\tilde{C}^{\text{GEMB}_\ell}(L), \tilde{C}^{\text{GEMB}_\ell}(L), \tilde{C}^{\text{GEMB}_\ell}(L), \tilde{C}^{\text{GEMB}_\ell}(L)]$ and $B_{\nu}$ is the beam function in the band power. In the spectrum estimation, we de-convolved the beam. Finally, we de-convolved the beam in the spectrum estimation to obtain the covariance matrix, which reads as follows:

$$
\mathbb{C}_{\nu\nu'} = \frac{1}{N_{\text{sim}} - 1} \sum_{\alpha=1}^{N_{\text{sim}}} \left[ \hat{D}_\nu^{XY, \alpha} - \bar{D}_\nu^{XY} \right] \left[ \hat{D}_{\nu'}^{XY, \alpha} - \bar{D}_{\nu'}^{XY} \right],
$$

(29)

with $X, Y = \{\kappa_{\text{CMB}}, \gamma_E\}$ and $\alpha$ denotes the number of the simulation. $\hat{D}_\nu^{XY, \alpha}$ is the estimated cross spectrum from the $\alpha$th simulation and $\bar{D}_\nu^{XY}$ is the mean over 600 simulations

$$
\bar{D}_\nu^{XY} = \frac{\sum_{\alpha=1}^{N_{\text{sim}}} \hat{D}_\nu^{XY, \alpha}}{N_{\text{sim}}}. 
$$

(30)

To calculate the inverse covariance, we adopt the unbiased estimator used in Hartlap et al. 2007, which is given by

$$
\hat{\mathbb{C}}_{\nu\nu'}^{-1} = \frac{N_{\text{sim}} - N_{\text{bin}} - 2}{N_{\text{sim}} - 1} \mathbb{C}_{\nu\nu'}^{-1}, 
$$

(31)

where $N_{\text{sim}} = 600$ and $N_{\text{bin}} = 4 \times 19$ is the number of data points and $\mathbb{C}_{\nu\nu'}^{-1}$ is the normal inverse of $\mathbb{C}_{\nu\nu'}$. In order to include the error propagation from the error in the covariance matrix into the fitting parameters (Percival et al. 2014) we rescale the covariance matrix,

$$
\hat{\mathbb{C}}_{\nu\nu'}^{-1} = \frac{1 + B(N_{\text{bin}} - N_p)}{1 + A + B(N_p + 1)} \hat{\mathbb{C}}_{\nu\nu'}^{-1}, 
$$

(32)

here $N_p$ is the number of the fitting parameters, and

$$
A = \frac{2}{(N_{\text{sim}} - N_{\text{bin}} - 1)(N_{\text{sim}} - N_{\text{bin}} - 4)} 
$$

(33)

$$
B = \frac{N_{\text{sim}} - N_{\text{bin}} - 2}{(N_{\text{sim}} - N_{\text{bin}} - 1)(N_{\text{sim}} - N_{\text{bin}} - 4)} 
$$

(34)

We present Fig. 6, which displays Pseudo-C$_\ell$s associated with different noise types. The yellow and blue boxes represent the error bars originating from AliCPT-1’s “4 modules*yr” and “48 modules*yr” experimental setups, respectively, whose noise spectra correspond to the orange dashed and green dotted curves in Fig. 4. The first row in Fig. 6 is calculated without any biases, while the green solid curve represents the theoretical cross-spectra. In the second row, a photo-z bias $\Delta z = 0.005$ is added. By visual inspection, the red solid curve (with the bias) and the green solid curve (without the bias) appear indistinguishable. In the third row, intrinsic alignment effects are incorporated into the simulations, where the green solid curve still represents the theoretical shear-CMB cross-correlation, while the red solid curve assumes an intrinsic alignment amplitude $A_{\text{IA}} = 1$. Notably, the intrinsic alignment can significantly reduce the signal at lower redshifts.

We report the signal-to-noise ratio (SNR), which is computed as

$$
\text{SNR} = \sqrt{\sum_{\nu\nu'} \hat{D}_\nu^{XY} \hat{\mathbb{C}}_{\nu\nu'}^{-1} \hat{D}_{\nu'}^{XY}},
$$

(35)

where $\hat{D}_\nu^{XY}$ is generated by a new simulation that is independent of the other 600 ones. The normalized covariance matrices of cross-correlation
alignment is included but not in the step of spectrum estimation. Here estimation procedures, the intrinsic alignment is consistently concluded. The one "with IA/G" denotes the case that only in the simulation step the intrinsic 

Figure 6. The estimation of the shear-CMB cross-spectrum in four $\gamma$-bins. The yellow and blue boxes are the binned error bars in multipoles, with two different CMB lensing reconstruction $N0$ noises. The first row is the pseudo-$C_\ell$ estimation without any biases. The second row is the one with redshift bias $\Delta_z = 0.005$. In the third row are the results with intrinsic alignment amplitude $A_{1A} = 1$. The solid curves are the theoretical predictions. The spectrum covariances are estimated from 600 mocks with $N0-4$ and $N0-48$ noise setups.

| $\chi^2/N_{bin}$ | Bin1 | Bin2 | Bin3 | Bin4 | Total |
|------------------|------|------|------|------|-------|
| N04              | 1.20 | 1.10 | 1.24 | 1.19 | 1.06  |
| N04 with bias-\(\gamma\) | 1.20 | 1.10 | 1.24 | 1.19 | 1.06  |
| N04 with IA/G    | 1.00 | 1.26 | 1.31 | 1.28 | 1.03  |
| N04 with IA/G+I  | 1.11 | 1.12 | 1.25 | 1.23 | 1.05  |
| N048             | 1.18 | 1.09 | 1.37 | 1.30 | 1.09  |
| N048 with bias-\(\gamma\) | 1.18 | 1.09 | 1.37 | 1.30 | 1.09  |
| N048 with IA/G   | 1.02 | 1.42 | 1.54 | 1.43 | 1.11  |
| N048 with IA/G+I | 1.10 | 1.14 | 1.39 | 1.33 | 1.09  |

Table 2. Reduced $\chi^2$ measured from 4 photo-$\gamma$ bins and the combined one. The name "with IA/G+I" denotes the case that both in the simulation and spectrum estimation procedures, the intrinsic alignment is consistently concluded. The one "with IA/G" denotes the case that only in the simulation step the intrinsic alignment is included but not in the step of spectrum estimation. Here $N_{bin} = 4 \times 19$ is our total data number and $N_{bin} = 19$ for each bin.
are shown in Fig. 7. One can see that the correlations of the same multipoles between different photo-z bins are obvious. This reflects that the lensing has a broader kernel in the redshift dimension. We summarise the SNRs in Tab. 1 and the reduced $\chi^2$ in Tab. 2. The total SNR ≈ 15 and 22 in the “4 modules*yr” and “48 modules*yr” cases, respectively. The reduced $\chi^2 ≈ 1$ suggests no significant deviation between the model and the data and validates that our spectrum estimation binning choices are appropriate. Finally, we summarise our simulation and spectrum estimation pipeline in the cartoon picture, Fig. 8.

5 COSMOLOGICAL CONSTRAINTS FROM COSMIC SHEAR-CMB LENSING CROSS-CORRELATION

The final step is to study the cosmological implications of the shear-CMB cross-correlation signal. We estimate the cosmological parameter constraint ability by using the Markov Chain Monte Carlo (MCMC) method. We use the Emcee code (Foreman-Mackey et al. 2013), a public implementation of the affine invariant MCMC ensemble sampler (Goodman & Weare 2010). We assume a Gaussian likelihood functions for the constraint ability by using the MCMC method. We use the cross-spectrum implementation of the affine invariant MCMC ensemble sampler (Goodman & Weare 2010). We assume a Gaussian likelihood functions for

\[ \chi^2 = \sum_{\nu} \left( \hat{D}_{\nu}^{XY} - D_{\nu}^{XY} (\theta) \right)^T \bar{C}_{\nu}^{-1} \left( \hat{D}_{\nu}^{XY} - D_{\nu}^{XY} (\theta) \right), \]  \hspace{1cm} (36)

where $\theta$ represents the set of parameters, including the cosmological as well as the nuisance parameters. $\bar{C}$ is the covariance matrix. $\hat{D}_{\nu}^{XY} (\theta)$ and $D_{\nu}^{XY} (\theta)$ are the measured bandpower and weighted theoretical spectra, $D_{\nu}^{XY} (\theta)$ is defined as

\[ D_{\nu}^{XY} (\theta) = \sum_{i=1}^{\Delta L} w(\nu, \ell_i) D(\ell_i, \theta) \]  \hspace{1cm} (37)

where $w(\nu, \ell_i)$ are the weights of bandpower $L$, we have chosen equal weights and normalized the weights for all bandpowers $\sum_{i=1}^{\Delta L} w(\nu, \ell_i) = 1$. The priors are summarized in Tab. 3. The posterior on the model parameters is then given by

\[ P(\theta|\hat{D}^{XY}) = L(\hat{D}^{XY}|\theta)P(\theta), \]  \hspace{1cm} (38)

where $P(\theta)$ are the priors.

In this work, we are interested in the $\Omega_m$ and $\sigma_8$ constraints. Moreover, we convert the $\sigma_8$ constraint into $S_8$. For $S_8$, we use the following definition

\[ S_8 = \sigma_8 \left( \frac{\Omega_m}{0.3} \right)^{\alpha}, \]  \hspace{1cm} (39)

where, the power law index $\alpha$ is calculated via the PCA method (Abdi & Williams 2010).

For the analysis, we considered four types of data and two types of theoretical models. The data comprises various kinds of noises and biases, while the model difference lies in whether the intrinsic alignment is included or not. We summarized our data vectors and model templates in

\[ \text{We do not present here the constraints by adding the redshift bias since we find it has negligible effect.} \]
Table 4. MCMC chains were run by combining the aforementioned data and model templates. Our findings indicate that the typical $1\sigma$ errors on $\sigma_8$ are approximately 0.038 with the AliCPT-1 “4 modules*yr” setup and approximately 0.027 with the AliCPT-1 “48 modules*yr” setup. Additionally, we discovered that if the IA is included in the data but not in the fitting template, it can introduce some level of bias in estimating $\sigma_8$ estimation. Our main results for parameter estimation are summarized in Table 5 and Figure 9. Notably, the most significant shift in Figure 9 is the contour from Data-IV with Model-I (solid yellow), emphasizing the importance of adequately modeling intrinsic alignment.

As for the power index in $S_8$, we compute it from the posterior directly. In detail, the power law index $\alpha$ was obtained from the correlation

Figure 8. Sketch picture of photo-z error, pseudo-$C_\ell$ measurement and covariance matrix computing method.
where gives the eigenvectors and eigenvalues for the normalized variables by using matrix of Table 6.

Table 5. The parameter estimation results of and fix. The errors are shown in Tab. 6. The results of are derived from and parameters as fixed variables.

Table 6. The figure-of-merit of each data vector and model template.
6 CONCLUSION

In this work, we explore the cosmological constraints for the future CSST × AliCPT. We construct simulated maps for cosmic shear and CMB lensing based on the experimental nominal setup. In order to evade the complicated ray tracing technique, in this paper, we developed a simulation pipeline based on the Gaussian realization of the given signal and noise spectra. We forecast the S/N and the constraints on the cosmological parameters for the cross-correlation, considering statistical error from the two observations. We study the impact of the most important lensing systematics, photo-z error, photo-z bias, intrinsic alignment, and multiplicative bias, on the predicted cosmological parameters.

More specifically, we simulate the maps (Fig. 5) according to CSST and AliCPT-1 nominal parameter setup. We consider standard shape noise for CSST cosmic shear (Fig. 1), and the N0 noise from the disconnected primary CMB for AliCPT (Fig. 4). As the map-building method (Eq. (21) and Kamionkowski et al. (1997)) is based on Gaussian random fields, the covariance of their cross-correlation will contain the contribution for the above noises and the cosmic variance. We note that for AliCPT CMB lensing, the noise varies for the “4 modules*yr” and “48 modules*yr” stages. We perform the standard Pseudo-$C_\ell$ spectrum estimation and find the shear-CMB cross-correlation can reach the SNR≈ 15 for the “4 modules*yr” case, and the SNR≈ 22 for the “48 modules*yr” case, as shown in Fig. 6. We investigate the cosmological implication of these cross-correlated signals in Fig. 9. We find that for the “4 modules*yr” case, the typical 1σ errors on $\sigma_8$ is about

\[
\sigma_8 \approx 0.7
\]

\[
\Omega_m \approx 0.3
\]

\[
S_8 \approx 0.8
\]

\[
A_{IA} \approx 2
\]
Figure 10. The constraint results of $\sigma_8$ vs. $\Omega_m$ with N048 noise in different photo-\(z\) bins. 1\(\sigma\)(68.3\%) C.L. are shown. The left panels show the results without properly considering the intrinsic alignment in the template fitting; while the right panels show the correct one. The input parameter $\sigma_8$ and $\Omega_m$ is marked by gray dashed lines.

0.038 – 0.043; for the “48 modules*yr” case, the typical 1\(\sigma\) errors on $\sigma_8$ is about 0.027 – 0.030, which is promising in investigating the current $S_8$ tension.

As an extension, we also explore the impact of photo-\(z\) bias, multiplicative bias and intrinsic alignment, which are the main sources of systematics in weak lensing. In the generated mock data, we shift the mean redshift to represent the photo-\(z\) bias and input an intrinsic alignment signal following the NLA model (Eq. (15) and Fig. 1). We note that the true IA signal could potentially deviate from the assumed NLA model, for example, the TATT model (Blazek et al. 2019; Samuroff et al. 2021; Hoffmann et al. 2022; Blazek et al. 2015; Samuroff et al. 2019, 2022) or the halo model (Fortuna et al. 2021). We leave those alternatives to future studies, as they are more dominant at smaller scales, while in this work the limit from CMB lensing noise (see Fig. 4) reduce their impact. Using additional observables to self-calibrate the impact from IA is an alternative solution (Yao et al. 2023). We show the contamination of photo-\(z\) and intrinsic alignment in the observed power spectra this work the limit from CMB lensing noise (see Fig. 4) reduce their impact. Using additional observables to self-calibrate the impact from IA is an alternative solution (Yao et al. 2023). We show the contamination of photo-\(z\) and intrinsic alignment in the observed power spectra in Fig. 6. We find that for the required photo-\(z\) precision for CSST with $\Delta_z = 0.005$, the bias in the power spectrum is negligible, while the IA contamination with $A_{IA} = 1$ is more significant. We find that if we do not consider the intrinsic alignment in the spectrum modeling, this will introduce about 0.6\(\sigma\) shift in $\sigma_8$ but an almost negligible effect on $S_8$ (Fig. 9). By including the correct IA model while introducing more nuisance parameters, the figure-of-merit in the $\sigma_8$ – $\Omega_m$ space will be reduced from $\approx 2404$ to $\approx 1942$ (Fig. 10), representing the loss in the cosmological constraining power to the IA parameter. As for the multiplicative bias, we investigate its impact with the Fisher matrix method. With the typical value ($\sigma_m = 0.01$) of the stage-III survey, the CMB lensing-cosmic shear cross-correlations are very insensitive to the multiplicative bias.

Interestingly, the map-making method of this paper provides not only an alternative check to the conventional Fisher matrix method but can also quickly generate correlated maps. A similar method has already been applied to the DES Year 3 joint analysis of galaxy clustering and weak lensing Krause et al. (2021). This technique is essential in discussing systematic contaminations when combined with future simulations, as we can directly use maps from simulations rather than assume a model for the power spectrum, especially when sometimes the simulation and the model deviate at some level (Jagvaral et al. 2022; Schneider et al. 2019). However, there are some caveats that shall be properly addressed. First of all, the result presented here is based on the flat-sky and limber approximation, which obstructs us to use the full overlapped area of the two surveys. Hence, it limits us to reveal the full power of the cross-correlation. To do so, we need to use the curved sky expression and spherical harmonic transformation instead of the Fourier transformation. Second, we only vary three major parameters, namely $\sigma_8$, $\Omega_m$, and $A_{IA}$ due to the limited SNR. Compared with the cosmic shear auto-correlations, the cross-correlation is still only playing a sub-leading role in the cosmology implication. But as we show in the validation section, it is immune to some systematic auto-correlation, such as the multiplicative bias. Besides, we note that it is also important to include the impact of non-Gaussian covariance and other sources of systematics, such as the baryonic effect, but they are beyond the scope of this work and we leave them for future studies.
DATA AVAILABILITY
The data underlying this article will be shared on reasonable request to the corresponding author.

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APPENDIX A: VALIDATION

In order to check the algorithm and test the constraint of the parameters, we use the Fisher method to make a purely theoretical cosmological constraint, and we compute the theoretical covariance. The angular power spectra are computed within the multipole range \((0 < \ell < 800)\) and binned \(\Delta \ell = 40\), the Gaussian covariance is calculated by

\[
\begin{align*}
\mathcal{C}_{\ell \ell'}^{\nu \nu'} & = \frac{\delta_{\ell \ell'}}{(2L + 1)\Delta L_{\text{sky}}} \left[ C^{\nu \nu'}_{\ell} C^{\nu \nu'}_{\ell'} + (C^{XX} + N^{XX})(C^{YY}_{\ell} + \delta_{\ell \ell'} N^{YY}_{\ell}) \right], \\
\mathbb{F}_{pp'} & = \frac{\partial C^{\nu \nu'}_{\ell}}{\partial p} \left\{ C^{-1}_{\nu \nu', \text{theo}} \left( \frac{\partial C^{\nu \nu'}_{\ell}}{\partial p'} \right)^T \right\},
\end{align*}
\]

where \(\delta_{\ell \ell'}\) is the Kronecker delta function, \(C^{XX}_{\ell}\) and \(C^{YY}_{\ell}\) are theoretical power spectra. Fisher matrix is written as follows

\[
\mathbb{C}_{pp'} = \mathbb{F}_{pp'}^{-1},
\]

and the covariance matrix of parameters

\[
\gamma_{\text{obs}} = (1 + m)\gamma_{\text{true}} + c,
\]

where \(p \in \{\Omega_m, \sigma_8, A_{\text{IA}}\}\). For comparison, the theoretical error bars were obtained from the Fisher matrix. They are in generally good agreement with the MCMC error estimates, which are shown in Fig. A1, however, are slightly larger (by up to \(\sim 20\%\)).

We consider the bias from galaxy shape measurement in different photo-z bins and it can be conventionally described approximately as a linear model with an additive bias \(c\) and a multiplicative bias \(m\) (Miller et al. 2013; Mandelbaum et al. 2015).

\[
\gamma_{\text{obs}} = (1 + m)\gamma_{\text{true}} + c,
\]

where we only consider the effect of multiplicative bias \(m\), parameter \(p \in \{\Omega_m, \sigma_8, A_{\text{IA}}, m_1, m_2, m_3, m_4\}\) and adopt \(c = 0\), and the corresponding theoretical cross spectrum should be written as

\[
\tilde{C}^{\nu \nu'} = (1 + m_{\nu})C^{\nu \nu'},
\]

the subscript \(\nu\) of \(m\) denotes multiplicative bias in different photo-z bins. We adopt a Gaussian prior with null mean and standard deviation.
\[ \sigma_m = 0.01 \] of multiplicative bias in our validation, the results are shown in Fig. A2. And we also consider the effect of photo-\( z \) bias in different photo-\( z \) bins, parameter \( p \in \{ \Omega_m, \sigma_8, A_{1A}, \Delta_{z1}, \Delta_{z2}, \Delta_{z3}, \Delta_{z4} \} \), we adopt a Gaussian prior with null mean and standard deviation \( \sigma_{\Delta z} = 0.01 \), and the results are shown in Fig. A3. We note that both multiplicative bias and photo-\( z \) bias have a small impact on the constraints of cosmological parameters.

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Figure A2. Validation with multiplicative bias. Red contours denote constraint results from Fisher’s methods without multiplicative bias, and the yellow contours are from Fisher’s methods with multiplicative bias.
Figure A3. Validation with photo-$z$ bias. Red contours denote constraint results from Fisher’s methods without photo-$z$ bias, and the yellow contours are from Fisher’s methods with photo-$z$ bias.