Synchronization Gauges and the Principles of Special Relativity

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Abstract

The axiomatic bases of Special Relativity Theory (SRT) are thoroughly re-examined from an operational point of view, with particular emphasis on the status of Einstein synchronization in the light of the possibility of arbitrary synchronization procedures in inertial reference frames. Once correctly and explicitly phrased, the principles of SRT allow for a wide range of ‘theories’ that differ from the standard SRT only for the difference in the chosen synchronization procedures, but are wholly equivalent to SRT in predicting empirical facts. This results in the introduction, in the full background of SRT, of a suitable synchronization gauge. A complete hierarchy of synchronization gauges is introduced and elucidated, ranging from the useful Selleri synchronization gauge (which should lead, according to Selleri, to a multiplicity of theories alternative to SRT) to the more general Mansouri-Sexl synchronization gauge and, finally, to the even more general Anderson-Vetharaniam-Stedman’s synchronization gauge. It is showed that all these gauges do not challenge the SRT, as claimed by Selleri, but simply lead to a number of formalisms which leave the geometrical structure of Minkowski spacetime unchanged. Several aspects of fundamental and applied interest related to the conventional aspect of the synchronization choice are discussed, encompassing the issue of the one-way velocity of light on inertial and rotating reference frames, the GPS’s working, and the recasting of Maxwell equations in generic synchronizations. Finally, it is showed how the gauge freedom introduced in SRT can be exploited in order to give a clear explanation of the Sagnac effect for counter-propagating matter beams.
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1 Introduction

1.1 The issue of conventionality of synchronization and one-way speed of light: a bit of history

As well known, in Special Relativity Theory (SRT) it is assumed that “clocks can be adjusted in such a way that the propagation velocity of every light ray in vacuum - measured by means of these clocks - becomes everywhere equal to a universal constant c, provided that the coordinate system is not accelerated” [1] (Einstein 1907). Clocks adjusted in such a way define the so-called “Einstein synchronization”. Any inertial reference frame (IRF) turns out to be optically isotropic if, and only if, Einstein synchronization is adopted “by stipulation”.1 Anyway, Einstein points out that such a stipulation, although arbitrary on a purely logical viewpoint (as a matter of fact, we have empirical access only to the round-trip average speed of light), is not arbitrary on the physical viewpoint since it provides a symmetric and transitive relationship. As a consequence, the standard formulation of SRT gives for granted that every light ray actually propagates isotropically; consequently, simultaneity is indeed frame-dependent.

However, some authors rejected the thesis that Einstein synchronization (with all its implications, like the isotropic propagation of light) is the only choice enforced by experimental data2, and seriously considered the possibility of postulating an anisotropic propagation of light in the theoretical context of SRT. Of course, a possible anisotropic propagation of light could be accounted for only on the basis of a possible synchronization gauge consistent with all the empirical observations, provided that any synchrony choice belonging to this gauge provides a symmetric and transitive relationship among events.3 This viewpoint, leading to the thesis of “conventionality of simultaneity”, has been discussed extensively, often from a philosophical standpoint, by many authors, in particular by Reichenbach [4].

1In his popular exposition of the theory (first ed. 1916), Einstein explicitly recognized that the isotropic propagation of light is “neither a supposition nor a hypotesis about the physical nature of light, but a stipulation” [2]

2Of course this is not literally Einstein’s thesis (see the previous footnote), but a thesis firmly incorporated in the standard - traditional - formulation of SRT

3A synchronization gauge is a (very particular) group of transformations, internal to the theory, which leave the observables unchanged (“saving the phenomena”)
and Grünbaum. Although starting from different points of view, these authors agree on the idea that the only nonconventional basis for claiming that two distinct events are not simultaneous would be the possibility of a causal influence connecting the events; as a consequence any self-consistent definition of simultaneity between “spatially separated” events is, in principle, allowed in Minkowski spacetime.

Nowadays the thesis of “conventionality of simultaneity” is a talking-point among philosophers of science, but seems not particularly exciting among relativistic people, who are generally satisfied with the standard formulation of SRT. As a matter of fact, most relativistic authors sustain the standard traditional viewpoint, according to which it can be proved, on experimental grounds, that the one-way velocity of light, in any inertial reference frame (IRF) and in any direction, is an universal constant. Let us quote some examples of standard (i.e. anti-conventionalist) approaches.

In the late 1960’s Ellis and Bowman, after careful synchronization of clocks by slow transport, argue that, although consistent nonstandard synchronization does not appear totally ruled out, there are sound physical reasons for preferring standard Einstein synchronization. Their conclusion is: "the thesis of the conventionality of distant simultaneity espoused particularly by Reichenbach and Grünbaum is thus either trivialized or refuted".

Malament argues that standard synchrony is the only simultaneity relation that can be defined, relative to a given IRF, from the relation of (symmetric) causal connectibility. Let this relation be represented by $\kappa$, and let the statement that events $p$ and $q$ are simultaneous be represented by $S(p,q)$. Under suitable formal conditions (in particular $S$ should be an equivalence relation definable from $\kappa$ in any IRF), Malament’s theorem asserts that there is one and only one $S$, namely the relation of standard synchrony. More explicitly, Malament’s theorem shows that the standard simultaneity relation is the only nontrivial simultaneity relation definable in terms of the causal structure of the Minkowski spacetime of SRT.

Following Malament, Friedman (1983) claims that any non-Einstein synchrony choice entails a denial of the Minkowskian structure of spacetime (which amounts to a denial of SRT).

More recently, Bergia and Valleriani claim that the Einstein synchrony choice is not conventional at all, but is forced by some experimental evidences. In particular, they quote two astonishing evidences: the one coming from the faultless performance of the world wide

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4Let us limit ourselves to quote some of the most recent claims by Bergia, Tartaglia, Sorge.
atomic Einstein-synchronized clocks (i.e. clocks synchronized by means of radio signals), and the one coming from faultless performance of the Global Positioning System (GPS). The main argument is that, in both cases, a possible anisotropic propagation of light should cause some detectable delays, fortunately never observed, which could be more than enough to obstruct the accurate performance of these devices. A careful quantitative analysis, based on the estimated but never observed delays, shows that the possible relative spread of the speed of light is about \( \Delta c/c \sim 1/300000 \sim 3 \cdot 10^{-6} \); more than enough to rule out any alternative synchrony choice.

In the anti-conventionalist area, the issue of arbitrary synchronizations has enjoyed much consideration in order to suggest experimental tests of SRT. The approach is to use some kind of “test-theory”, i.e. a theoretical framework containing a (suitably parameterized) family of theories in competition with one other. The main characteristic of a test-theory is the presence of a particular set of parameters whose numerical values are specific of any specific theory to be tested in such a framework. If the parameters are related to some observables, then suitable experiments can single out the correct theory belonging to the test-theory. Although none of the test-theory of SRT enjoy the same status as the celebrated PPN test-theory of GRT, the test-theory approach is promising, provided that the crucial parameters are actually related to observable (and not to conventional) quantities; unfortunately, this is not always the case.

The most popular test-theories are the ones of Mansouri-Sexl, which yet contains a very important element of conventionality (see later), and the ones reviewed by Clifford Will [12], in a theoretical background which tries to test the local Lorentz symmetry by measuring a possible difference between the speed of electromagnetic radiation \( c \) and the limiting speed of material test particles, chosen to be unity via a suitable choice of units. According to such an approach, the relevant parameter is \( \delta = c^{-2} - 1 \). Possible deviations from the standard value zero would unveil a violation of Lorentz symmetry, selecting a preferred universal rest frame: presumably that of the cosmic background radiation, through which we are moving with a velocity contained in the range 300-600 km/s. Will quotes some selected tests of local Lorentz symmetry showing the bounds on the parameter \( \delta \). According to him, “recent advances in atomic spectroscopy and atomic timekeeping have made it possible to test local Lorentz symmetry by checking the isotropy of the speed of light using one-way propagation (as opposed to round-trip propagation, as in the Michelson-Morley experiment)”. Since the bounds on the parameter \( \delta \) turn out to be contained in the range \((10^{-6}, 10^{-20})\), depending on the experiment, the relative spread of the speed of light \( \Delta c/c \)
compatible with the experiments agrees with the one found by Bergia [5]. As a consequence, the one-way isotropic propagation of light turns out to be the only possibility allowed by experiments.\footnote{In a previous paper [13] Will goes into details, assuming (one version of) the Mansouri-Sexl test-theory as a starting point. Some relevant “one-way” experiments, like the two-photon-absorption (TPA) experiment, are described. It is stressed that the Mansouri-Sexl transformations from an Einstein-synchronized IRF $\Sigma$ to an arbitrary-synchronized IRF $S$ embody one vector parameter $\varepsilon$ which depends on the synchrony choice in $S$, and 3 scalar parameters $a, b, d$ which do not depend on this synchrony choice. That having said, Will shows that “the outcome of of physical experiments... is independent of synchronization”; yet “the TPA and other such one-way experiments do provide valid tests to possible violations of SRT”. However odd, this is possible because “those violations are embodied in functional forms of $a, b, d$ that could differ from those quoted above [the ones expected in SRT], not in the form of $\varepsilon$ which is arbitrary an irrelevant”. The conclusion is that “the TPA and other such one-way experiments” actually prove the isotropy of the one-way velocity of light (with relative spread severely bounded under $10^{-7}$) independently of any synchrony choice. We report here this result, but we acknowledge our incomprehension since the one-way isotropic propagation of light should imply the Einstein synchrony choice (and vice versa). We completely agree with AVS [14] who point out that all parameters - $a, b, d$ included - depend on the synchrony choice in $\Sigma$: this fact is somehow obscured by the Einstein-synchrony choice in $\Sigma$, but becomes apparent if also $\Sigma$ is arbitrarily synchronized.}

These are some of the most outstanding results found by authors who believe in the possibility of proving the isotropy of one-way propagation of light by means of actual experiments.

However, in spite of such strong and apparently conclusive claims, some underground perplexities about the possibility of reliable experimental tests of such a statement went through all along the history of SRT, because of the inescapable entanglement between remote synchronization and one-way velocity of light. After all, a careful analysis (see f.i. [14], [15, 16, 17, 18]) seems to unveil that no actual experiment, among the ones mentioned by the authors quoted before, is a genuine “one-way” experiment. As a consequence, it seems reasonable to suspect that the parameters appearing in the test-theories mentioned before could actually be beyond the reach of experimental knowledge: in other words, these parameters could be “conventional” in the sense that their numerical values have no effect on the output of any actual experiment.

Likewise, it seems reasonable to suspect that some claims mentioned before could arise from circular arguments, in the sense that the conclusions could be hidden, from the very beginning, in the hypotheses. For instance, it can be shown [19] that synchronization of clocks by slow transport is fully equivalent to Einstein synchronization, provided that the geometrical
structure of Minkowski spacetime is accepted once and for all. So that the refutation of any non-Einstein synchrony choice, starting from synchronization of clocks by slow transport, runs the risk of being tautological, unless Minkowski spacetime is embedded in a set of suitably parametrized (flat) spacetimes.

A major breakthrough occurred in 1977, when Mansouri and Sexl introduced a synchronization gauge consistent with the experimental evidence of the constancy of the two-ways velocity of light in any IRF. This is a historical cornerstone of the conventionalist viewpoint. However, in the same year, Mansouri and Sexl published three papers, and it is apparent that their philosophical approach gradually evolved from the starting position of acknowledging the conventionality of simultaneity to the opposing position that each theory has associated with it an empirically determinable synchrony choice. In particular, they came to the conclusion that the one-way speed of light could be empirically measured in the framework of their theory. In this way the Mansouri-Sexl approach becomes a test-theory; as a matter of fact, from then onwards most physicists actually used - and still uses - the Mansouri-Sexl formalism as test-theory. Most experimental tests of isotropy of the one-way velocity of light are actually performed in some background inherited by the the “Mansouri-Sexl test-theory”.

Some years later, the Mansouri-Sexl gauge was extended by Anderson, Vetharaniam and Stedman (see [14] and references therein) to such an extent that the one-way velocity of light became even more conventional, provided that it complies with a suitable synchronization gauge. Of course we are going to name this gauge, which is very wide indeed, “AVS synchronization gauge” or briefly “AVS-gauge”. Recently, simultaneity and synchronization gauges have been studied in a very interesting paper by Minguzzi.

1.2 Between conventionalism and realism: Selleri’s approach

In recent years, Franco Selleri has sided against both the conventionalist approach and the SRT, maintaining the cause of realism in a Lorentz-like background; yet he agrees with one of the main points of the conventionalist attitude, namely the statement that, in any IRF, “the constancy of the one-way speed of light is a mere convention without any

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6Let us stress that accepting the geometrical structure of Minkowski spacetime is equivalent to accepting SRT, regardless of its formal look. Therefore we completely agree with the following AVS [14] claim: “any experimental divergence between Einstein synchronization and slow clock transport would constitute an experimental violation of special relativity”.

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empirical cornerstone"\textsuperscript{7}. Unlike the AVS approach, Selleri approach is not
formal but truly physical, or to be exact conceptual, and even philosophical;
it is not surprising that such an approach gave rise to a wide and fascinating
discussion about the foundations of SRT\textsuperscript{[15,16,17,18,19,21,22,23]}. 

In particular, Selleri has undertaken a severe critical analysis of the two
principles of SRT (relativity principle and invariance of the velocity of light),
emphasizing some aspects which, after a century from their first enunciation
(1905), seem to be still somehow ‘blurred’: both concerning their formulation
in precise operational terms and concerning their connection to precise
empirical data.

Selleri bases his reconstruction of theoretical physics upon three “hard
experimental evidences”, on which the whole scientific community is in full
agreement. They are, as Selleri himself stresses, hypotheses directly supported
by the experimental data and, thus, wholly independent on any theoretical license. At variance with the two principles of SRT that, being so
loaded with theory, were formulated, in the celebrated 1905 paper, without
any reference to experimental data.\textsuperscript{8}

Suspicious of any preconceived theoretical framework, and careful only
at the experimental data, Selleri shows that there exist not only a theory, but
a whole set of theories all compliant with the ‘hard experimental evidences’,
which can be distinguished between each other by the value of a parameter $e_1$
(called “synchronization parameter”), which is \textit{à priori} unconstrained. However, Selleri’s realistic attitude cannot be satisfied with this result, which
apparently supports an unwanted operational viewpoint; in fact, this is only
the first part of Selleri’s approach. The second part, driven by a strong
realistic viewpoint, is the attempt to prove that a secret “Nature’s synchrony
choice”, although totally hidden in the class of IRF’s, actually exists, and
can be unveiled by means of suitable experiments performed in non-IRF’s.

Selleri’s approach can be briefly summarized by the following statements:

(i) in the ensemble of the theories consistent with the “hard experimental
evidences”, the synchronization parameter $e_1$ cannot be fixed by any
experiment performed in IRF’s;

(ii) the SRT belongs to this ensemble and it corresponds to a definite

\textsuperscript{7}Translation by the authors.

\textsuperscript{8}Actually, as well known, Einstein’s aim was basically to recover the ‘unity of physics’
which, at the beginning of the twentieth century, seemed to have been lost because of
the existence of two distinct invariance groups: one pertaining to Newton’s mechanics
and the other one valid for Maxwell-Lorentz’s electromagnetism. Einstein thus aimed at
rebuilding the theoretical physics so that all its branches share for the same invariance
group.
value of the parameter $e_1$, namely $e_1 = -\beta\gamma/c$;

(iii) all the theories in which $e_1 \neq -\beta\gamma/c$ are inconsistent with SRT;

(iv) even though statement (i) entails that the synchronization parameter cannot be fixed by any experiment performed in unaccelerated reference frames, such a parameter can be fixed by suitable experiments performed in accelerated, in particular rotating, reference frames. Selleri \cite{17} actually, considering some of such experiments (primarily the Sagnac effect), finds that Nature forces the synchrony choice $e_1 = 0$, which is different from the relativistic one $e_1 = -\beta\gamma/c$.

The choice $e_1 = 0$ turns out to be consistent with the relativity of space and time, but not with the relativity of simultaneity. As a consequence “absolute simultaneity” can be re-introduced in physics, against the “relative simultaneity” (which is a typical feature of SRT) and in agreement with the “realistic” (Lorentz-like) ideological assumption that a privileged IRF, namely the IRF in which the ether is at rest, actually exists.

1.3 Plan of the paper

For sake of clarity, we split this paper into two parts. In the first one, far and away the most extensive (Secs. 2-5), only IRF’s will take into account; in the second part (Sec. 6) we will briefly consider also rotating reference frames. It is ultimately in this second part that a disagreement with Selleri’s approach and conclusions will emerge: in particular, conclusion (iv) will be disproved. But all along the first part, i.e. until accelerations are neglected, a basic agreement on Selleri’s conclusions, in particular conclusions (i), (ii), will appear. Let us emphasize that we shall give a mathematical proof of Selleri’s basic idea, namely statement (i). Actually, such a statement is a mere conjecture, expressed in Ref. \cite{16} in the following “weak” form: *in the class of IRFs, no physical experiment can discriminate the case $e_1 = 0$ from the case $e_1 = -\beta\gamma/c$. As a matter of fact, Selleri does not prove this statement, but only disproves specific attempts to discriminate the case $e_1 = 0$ from the case $e_1 = -\beta\gamma/c$ by means of some empirical evidence. Selleri concludes: "I’m convinced that with a bit of work this theorem can be [proved and] extended to the set of all possible values of $e_1"^9."

A masterly suggestion. This theorem will be proved for the set of all possible values of $e_1$ even twice: first in Sec. 4 following Selleri’s approach and showing its compatibility with SRT; then in Sec. 5 in the full theoretical background of SRT.

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9Translation by the authors.
Until accelerated frames will be neglected, the only thing we do not agree
is statement (iii): however, we strongly suspect that this disagreement sim-
ply depends on an improper use of the expression “SRT”, which is, all over
relativistic literature, commonly identified with the expression “standard
formulation of the SRT”. In our opinion, until the SRT is identified with
its standard formulation Selleri is perfectly right, since statement (iii) is an
unavoidable consequence of the standard formulation of the axiomatic basis
of SRT. As a consequence, the incorrect statement (iii) imperiously shows
the need of recasting the axiomatic basis of SRT in a more appropriate way,
overriding - once and for all - the ambiguities that have been passed down
over almost a century. This way Selleri’s approach turns out to be very
useful towards a deeper understanding of SRT, not only with regard to the
points (i), (ii) with which we agree, but also with regard to the point (iii),
with which we don’t agree.

In this paper the axiomatic basis of SRT will be re-examined in a very
thorough way from an operational point of view, with particular emphasis
on some crucial details concerning the status of Einstein synchronization
(Sec. 2). In this reassessment, a central role will be played by the so-called
‘round-trip axiom’ (Sec. 2.3): the only possible ‘synchrony-independent’ for-
mulation, strongly supported by empirical evidence, of the principle of in-
variance of velocity of light. We will then show (Sec. 4) that, once correctly
and explicitly phrased, the principles of SRT are, at the kinematical level,
fully equivalent to the three ‘hard experimental evidences’: tuning the values
of the parameter $e_1$ with respect to the one corresponding to SRT, one gets
theories which, at variance with Selleri’s claims, are not at all alternative
to SRT, but constitute a simple rewriting of Special Relativity. In other
words, one obtains ‘theories’ that differ from SRT for the difference in the
chosen synchronization procedures but that are wholly equivalent to SRT
in predicting empirical facts. Technically speaking, this will result in the
introduction, in the full background of SRT, of what we are going to name
“Selleri synchronization gauge” (Sec. 3). Such a gauge, which turns out to
be physically meaningful (unlike the AVS-gauge, wider but rather formal)
and leaving all the observables unchanged, enables us to face the relation-
ships between Einstein’s relativity and the so-called “alternative theories”
of Selleri, usually considered to be on pretty bad terms with one other. This
direct comparison will be properly formalized, thus allowing to reconcile
Selleri and Einstein, on the ground of a careful re-examination of the con-
ceptual bases of SRT. In our opinion, this is a very important contribution
to the clarification of SRT, coming from Selleri’s severe critique.
As a conclusion, we have proved that Selleri synchronization gauge does not lead to an alternative theory with respect to SRT, but must be incorporated in the formalism of SRT. According to a scholastic dialectic scheme, if SRT is the thesis and Selleri “alternative theory” the antithesis, a suitable reworking of SRT, starting from the “hard experimental evidences” and incorporating the Selleri synchronization gauge, should be the synthesis. This is the scheme of the first part of the paper (Secs. 2, 3, 4). The main part of the paper (Sec. 5) incorporates the Selleri synchronization gauge in the theoretical background of SRT in a fully formal way, namely as a suitable sub-gauge of the Cattaneo gauge, which is the set of all the possible parametrization of a given physical reference frame; in particular, Selleri’s “absolute synchronization” - which could sound somewhat ‘heretical’ to ‘orthodox’ ears - emerges naturally from Minkowski spacetime of SRT as a simple parametrization effect, involving a frame-invariant foliation of spacetime.

The paper ends (Sec. 6) with a brief analysis of Selleri’s statement (iv), which maintains that the “hard experimental evidences” should force the synchronization parameter $e_1$ to take the value zero when rotating reference frames are taken into account. Conversely our analysis, carried out in the theoretical background of SRT incorporating Selleri synchronization gauge, shows that: (i) contrary to Selleri’s claim, $e_1$ is a free parameter in any case$^{10}$; (ii) the gauge freedom introduced in SRT is not a marginal detail or a philosophical nicety, but a very useful tool which allows a clear explanation of the Sagnac effect, in the full background of SRT, for counter-propagating matter beams.

2 Einstein’s approach: postulates of SRT reviewed (thesis)

In this section we aim to thoroughly analyze the operational meaning of the two postulates of SRT, with an especial care for their connections to Einstein synchronization procedure.

2.1 Traditional formulation of the axiomatic basis of SRT

As well known, the standard expression of the two postulates is the following:

$^{10}$It could be said, partly for fun and partly for real, that we take Selleri synchronization gauge more seriously than Selleri himself.
(α) **Relativity principle**: all physical laws are the same in any IRF. No inertial reference frame is ‘privileged’, i.e. distinguishable from the other IRF’s by means of ‘internal’ empirical evidences.

(β) **Invariancy of the velocity of light**: the velocity of light in empty space is the same in any IRF. Its value is given by the universal constant \( c \).

Let us notice that, in all the customary textbook treatments of the theory \[24, 25, 26\], the problems of distant simultaneity and of the definition of the synchronization procedure are dealt with after the introduction of the postulates (α) and (β). Nonetheless, assuming that proposition (β) retain a precise meaning without the need of a previous definition of a synchronization procedure can result – and, as we will see, does result – into misleading interpretations of the postulate itself. According to a strict operational approach, it is therefore convenient to specify the definition of Einstein synchronization procedure *independently from postulate (β)*.

### 2.2 Einstein synchronization

Einstein synchronization procedure is defined, without any reference to postulate (β), by the following sequence of operations (cfr. \[27\]):

1. An arbitrary spatial origin of the reference frame, which will be called \( O \), is chosen. A standard clock, together with a light emitter, is lodged in \( O \). In any other point of space, to which we will generically refer as \( A \), an identical standard clock, together with a mirror, is lodged.

2. At time \( t_0 \) the light emitter sends a pulse from \( O \). Such a pulse reaches \( A \), is reflected and reaches back \( O \) at time \( t_0 + \Delta t_0 \). Thus, if \( l_{OA} \) is the spatial length of the segment \( OA \), then the mean velocity of the light pulse along the closed trip \( OAO \) is *empirically given* by \( c_{OAO} \equiv 2l_{OA}/\Delta t_0 \).

3. At time \( t_1 \) a second pulse is emitted from \( O \) towards \( A \). At the reception of the pulse in \( A \), the clock here located is set to the time \( t_A \equiv t_1 + \Delta t_0/2 \). Thus the *one-way velocity* of light, from \( O \) to \( A \), is *conventionally defined* by \( c_{OA} \equiv c_{OAO} = 2l_{OA}/\Delta t_0 \). In other words the set time \( t_A \) can be expressed in terms of the one-way velocity: \( t_A \equiv t_0 + l_{OA}/c_{OA} \).
We mention that such a synchronization procedure, fully conventional as it is, does not guarantee, by itself, neither the property of *optical isotropy*, nor the crucial requirement of the *transitivity* of the synchronization procedure.\(^\text{11}\) Only the introduction of a proper empirical hypothesis about the propagation of light, namely the round-trip axiom (see Sec. 2.3 below), can lead to such properties. In this context (in which the round-trip axiom is going to play a key role) Einstein synchronization, while maintaining its conventional character, will reveal all its theoretical usefulness and its remarkable physical and heuristic meaning. However, we shall see in Sec. 3.3 that the widespread statement that Einstein synchronization is the unique transitive synchronization (that is to say the unique self-consistent synchronization) is an untenable prejudice.

2.3 Operational formulation of the axiomatic basis of SRT

We move now to investigating the link connecting postulates (\(\alpha\)) and (\(\beta\)) to Einstein synchronization, in order to unveil the operational contents of the postulates. This analysis will lead to a strict formulation of the kinematical consequences of (\(\alpha\)) and to an utter recasting of (\(\beta\)).

First of all, let us translate proposition (\(\alpha\)) as suggested by Bergia and Valleriani in ref. \(^\text{11}\): “the same experiments performed under the same conditions in different inertial systems yield the same results”. Provided that the (rather slippery) expression “under the same conditions” is properly defined, this statement unveils the physical content of the Relativity principle. Now, let us stress an obvious but crucial consequence of this statement at a purely kinematical level (which we are going to name “Kinematical Relativity principle”):

\((\alpha_1)\) **Kinematical Relativity principle**: once Einstein synchronization has been performed in any IRF, the space-time coordinate transformations between any two IRF’s have to be symmetric and dependent on the relative velocity of the two frames alone.

This is a formal expression of the Relativity principle at kinematical level\(^\text{12}\), which might appear pedantic and even boring. However, let us

\(^{11}\)Let us point out that only “*transitivity* is actually a vital requirement for any self-consistent definition of simultaneity. On the contrary, “*optical isotropy*” is just a lovely feature, which - in case - could be dropped without any problem

\(^{12}\)The formal expression of the Relativity principle - in its full generality - consists in the requirement that *all the physical laws must be covariant with respect to the group of symmetric coordinate transformations among Einstein-synchronized IRF’s.*
mention that a less specific formulation of the Relativity principle has lead Franco Selleri to a, in our opinion unjustified, refusal of the principle itself.

Proposition $(\beta)$ admits a strict operational meaning if and only if it is interpreted as follows:

$$(\beta_1) \textbf{Invariancy of the velocity of light: } \text{in any IRF, once Einstein synchronization has been performed, the velocity of light is } c \text{ along any path.}$$

The absence of an explicit reference to Einstein synchronization in the usual formulation of the principle of invariancy of the velocity of light has brought many authors to claim that proposition $(\beta)$ cannot be empirically tested [28, 3, 20, 14, 18]. While such a point of view is formally correct, we stress that the principle, if (and only if) rigorously recast as $(\beta_1)$, holds a strict physical and operational meaning. Actually, an empirically testable formulation of the principle, fully equivalent to $(\beta_1)$ and avoiding any reference to the synchronization procedure (a single clock being involved), can be promptly given. Such a formulation, the so called ‘round-trip axiom’, was introduced by Reichenbach [28], and reads.\textsuperscript{14}

$$(\beta_2) \textbf{Round-trip axiom: } \text{The velocity of light is a universal constant } c \text{ in any IRF along any closed path.}$$

\textbf{Theorem 1:} The round-trip axiom $(\beta_2)$ is equivalent to the principle of invariancy of the velocity of light, provided it is formulated in the operationally meaningful form $(\beta_1)$:

$$(\beta_2) \iff (\beta_1)$$

$\Leftarrow$ The demonstration is immediate.

$\Rightarrow$ Let us consider an inertial frame $S$, with spatial origin $O$, in which Einstein synchronization has been performed. Let $AB$ be a generic

\textsuperscript{13}The formulation of the principle adopted by Selleri lacks the crucial “once Einstein synchronization has been performed in any IRF”.

\textsuperscript{14}To be precise, we mention that Reichenbach’s original formulation [28, sec. 12] slightly differs from proposition $(\beta_2)$. However, in deference to the original, we keep the appellation ‘round-trip axiom’.

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path (see Fig. 4) of length $l_{AB}$. Let a light pulse be emitted from $A$ at time $t_1$ and reach $B$ along $\overline{AB}$ at time $t_2$. Our goal is showing that $t_2 = l_{AB}/c$ exploiting ($\beta_2$).

Let us therefore suppose that a second pulse goes from $O$ to $A$ along the segment $OA$, reaching $A$ at time $t_1$ (exactly when the first pulse is going off), and that a third pulse goes from $B$ to $O$ along the segment $BO$, starting from $B$ at time $t_2$ (exactly when the first pulse is coming). Let $l_{OA}$ and $l_{OB}$ be, respectively, the length of the segments $OA$ and $OB$. Now, it is easily verified that the round-trip axiom ($\beta_2$) and Einstein synchronization together ensure that the velocity of light is $c$ along any straight line passing by $O$, in both directions. Therefore, the sequence of events characterizing the global space-time path of the pulses along the closed spatial path $OABO$ is the following

\[(O, t_1 - l_{OA}/c) \leadsto (A, t_1) \leadsto (B, t_2) \leadsto (O, t_2 + l_{OB}/c) . \]  

(1)

Straightforwardly applying ($\beta_2$) then gives

\[ \frac{l_{OA} + l_{AB} + l_{OB}}{\frac{l_{OA}}{c} + t_2 + \frac{l_{OB}}{c}} = c , \]  

(2)

from which one gets $t_2 = l_{AB}/c$, which completes the proof. □

Summarizing, we have shown that ($\beta_1$) and ($\beta_2$) are two equivalent hypotheses, each of them being empirically testable.

### 3 Selleri’s approach: from hard experimental evidences to ”inertial transformations” (antithesis)

After a review of the basic hypotheses of SRT according to the canonical Einstein’s approach, we want to outline here the hypotheses which Selleri’s alternative approach is based on. As we shall see, his approach is both original and self consistent, and straightforwardly leads to a generalization of relativistic kinematics which encompasses a multiplicity of synchronization procedures; among them, Einstein’s procedure is nothing but a particular case.

---

15 It is enough to note that a two-ways trip is a round-trip, so that ($\beta_2$) ensures that $c_{OBO} = c$ for any point $B$. But according to Einstein synchronization $c_{OB} = c$; as a consequence $c_{BO} = c$ (for any $B$)
3.1 The “hard experimental evidences”

Selleri \[16\] aims at finding the most general coordinate transformations among IRFs on the ground of the following three hypotheses:

(i) - There exists at least one IRF, let us call it $S_0$, where the velocity of light is $c$ at each point and in every direction.

(ii) - The two ways velocity of light is the same in every direction, in each IRF.

(iii) - The ticking of clocks moving with respect to $S_0$ with velocity $v$, is slowed down by the usual factor $\sqrt{1 - \beta^2}$ where $\beta = \frac{v}{c}$ (“retardation of clocks”).

These hypotheses are supported by the experimental results to such a high degree that they appear to be practically independent of any theoretical speculation. Indeed, Bergia \[5\] considers them the “hard core of the experimental knowledge” pertaining to relativistic theories: following him, we shall call them hard experimental evidences.

Statements (i) and (iii) requires some clarifications which take into account the operational details outlined in Sec. 2.3.

Statement (i) asserts that in the IRF $S_0$, after that an Einstein synchronization has been performed, the velocity of light is $c$ along any path. Notice that (i) does not rule out the possibility of having more than one IRF where the propagation of light is isotropic.

Statement (iii) asserts that if $\delta \tau$ is the time interval between two events which occur at the same place in an IRF $S$, moving with a velocity $v$ with respect to $S_0$, and $\delta t_0$ is the time interval between the two events as measured by two clocks Einstein-synchronized in $S_0$, then

$$\delta \tau = \delta t_0 \sqrt{1 - \beta^2}. \quad (3)$$

The asymmetry of eq. (3) reflects the different operational meaning of the time intervals $\delta \tau$ and $\delta t_0$.\[16\]

\[16\] $\delta \tau$ is measured by a single clock at rest in $S$; $\delta t_0$ is measured by a couple of clocks at rest in different locations in $S_0$, provided that they are Einstein-synchronized in such a frame.
3.2 Selleri general coordinate transformations

That being stated, Selleri\textsuperscript{17} obtains the most general coordinate transformations from the optically isotropic IRF $S_0$ to a generic IRF $S$, in motion with respect to $S_0$ with a dimensionless velocity $\beta \equiv v/c$, which are in agreement with (i), (ii), (iii). If the $x, y, z$ directions of the two frames are parallel, and the velocity $\beta$ is along the $x$ direction, the coordinate transformations turn out to be:

\[
\begin{align*}
  t &= \gamma^{-1}t_0 + e_1(x_0 - \beta ct_0) \\
  x &= \gamma(x_0 - \beta ct_0) \\
  y &= y_0 \\
  z &= z_0
\end{align*}
\]

where $\gamma \equiv 1/\sqrt{1 - \beta^2}$ and $e_1$ is an arbitrary function of $\beta$, whose dimensions are [velocity]^{-1}.

As a consequence, the most general coordinate transformations consistent with the hard experimental evidences, in the kinematical conditions outlined above, turn out to be a family of transformations ("Selleri synchronization gauge") parameterized by the function $e_1(\beta)$. Every $e_1(\beta)$ is admissible, and each of them corresponds to a different synchronization procedure in each IRF moving with dimensionless velocity $\beta$ with respect to $S_0$ (hence, $e_1$ is called "synchronization parameter")\textsuperscript{17} and, according to Selleri, "a different theory."\textsuperscript{18}

From (4) it is possible to obtain the velocity of light in $S$ as a function of the angle $\vartheta$ between the propagation direction and $\beta$:

\[
\tilde{c}(\vartheta) = \frac{c}{1 + \Gamma \cos \vartheta},
\]

where

\[
\Gamma \equiv \beta + e_1(\beta)c\gamma^{-1}
\]

\textsuperscript{17}Of course all possible functions $e_1(\beta)$ must satisfy the limiting condition

\[
\lim_{\beta \to 0} e_1(\beta) = 0
\]

so that eqns. \textsuperscript{17} reduce to identity for vanishing $\beta$. Provided this obvious constraint is satisfied, the IRF $S_0$ turns out to be Einstein-synchronized for any choice of the synchronization parameter $e_1$.

\textsuperscript{18}The latter terminology expresses Selleri’s conviction that the synchronization parameter can be determined on the basis of the observational results in accelerated frames. This means that, in Selleri’s opinion, the “hard experimental evidences” should lead to a test-theory, provided that accelerated reference frames are taken into account.
is a function of the of the dimensionless velocity $\beta$ (in absolute value) and depends on the synchronization parameter $e_1$, i.e. on the synchrony choice inside the Selleri gauge.\textsuperscript{19}

**Does Selleri synchronization gauge challenge the Relativity principle?**

As extensively pointed out by Selleri, when $\Gamma = 0$, i.e. when $e_1(\beta) = -\beta \gamma / c$, each IRF becomes optically isotropic, and reduces to the special Lorentz transformation along the $x$ axis. In this synchrony choice ("Einstein synchrony choice"), and only in this synchrony choice, the coordinate transformations (4) take a lovely symmetric form, clearly reflecting the Relativity principle into the formalism. But how about non-Einstein synchrony choices ($e_1(\beta) \neq -\beta \gamma / c$)?

The widespread but, as we have seen, ambiguous formulation ($\alpha$) of the Relativity principle leads Selleri to utterly reject the Relativity principle for any value of $e_1$ different from $-\beta \gamma / c$. In any non-Einstein synchrony choices, the asymmetry of transformations (4), relating the optically isotropic IRF $S_0$ to the optically anisotropic IRF $S$, leads Selleri to the following claim \textsuperscript{17}: "transformations (4) violate the Relativity principle for any $e_1$, except for the relativistic value $-\beta \gamma / c$"; and coherently concludes: "Varying $e_1$ one obtains different theories, all equivalent to SRT as far as the explanation of the most known experimental results is concerned. However, any theory different from SRT is not compatible with the principle of Relativity."\textsuperscript{20}

This critical attitude towards the Relativity principle is based, as we have already clarified, on a wrong formulation of the principle on the kinematical level. In fact one cannot require the symmetry of the transformations without imposing the fundamental condition "once the same synchronization procedure has been adopted in any IRF". The crucial point here is that, in the IRF $S$, the synchronization procedures resulting from values of $e_1$ different from the relativistic one depend on the relative velocity $\beta$ of $S$ with respect to the 'privileged' (i.e. Einstein-synchronized) IRF $S_0$. Therefore the resulting transformations’ asymmetry, far from violating the Relativity principle, simply reflects into the formalism the difference between the synchronization procedures adopted in $S$ and $S_0$.

In other words, the asymmetry of the transformations (4) does not accord any kind of privilege, on the physical ground, to the IRF $S_0$ in which

\textsuperscript{19}Notice that $\Gamma$, as well as $e_1(\beta)$, is independent of the space-time coordinates of the IRF, for each synchrony choice.

\textsuperscript{20}Translation by the authors.
the adoption of Einstein synchronization has been stipulated, but constitutes
the formal expression of the asymmetry of the synchronization procedures
adopted in \( S \) and \( S_0 \) respectively.

On the other hand, it would be a striking surprise if, adopting in \( S \)
a non-Einstein synchronization procedure, depending on \( \beta \) by definition,
whereas an Einstein synchronization procedure is adopted in \( S_0 \), we could
obtain transformations relating the two frames that are symmetric and not
dependent on \( \beta \)!

Summarizing, the ‘privileged role’ played by \( S_0 \) in Selleri’s theory is
a mere artificial element, \( S_0 \) being just the IRF in which, \textit{by stipulation},
Einstein synchronization has been performed: as a matter of fact, \textit{any IRF}
\( S \) can play the role of \( S_0 \).

### 3.3 Selleri synchronization

In operational terms, each synchronization procedure belonging to Selleri
gauge can be obtained, in the inertial frame \( S \) moving with respect to \( S_0 \)
with (dimensionless) velocity \( \beta \), by means of the following operations, at
each point \( A \in S \):

a First of all, let us choose an arbitrary origin of the spatial coordinates,
that we shall call \( O \). Let us suppose that a standard clock and a source
of light signals are lodged in \( O \). Identical clocks and mirrors are lodged
in all points of space; let \( A \) be one of these points.

b At \( t_0 \) the light source in \( O \) emits a signal. This signal reaches \( A \), is re-
flected by the mirror and comes back to \( O \), where it arrives at \( t_0 + \Delta t_0 \).
Consequently, if \( l_{(OA)} \) is the spatial length of \( OA \), the mean velocity
of the signal along the closed path \( OAO \) is \( c_{(OAO)} \equiv 2l_{(OA)}/\Delta t_0 \).

\[ \text{[21]} \]

\[ t_A \equiv t_1 + \frac{l_{(OA)}}{c_{(OA)}} = t_1 + l_{(OA)} \left(1 + \Gamma \cos \vartheta_{OA}\right)/c \quad (7) \]

\[ \text{[22] This is a \textit{conventional} velocity, because the dimensionless parameter} \Gamma \text{is an arbitrary}
\text{function of} \beta. \]

\[ \text{[5]} \]

This is an \textit{empirical} velocity.
An obvious consequence is that, in general, \( c_{OA} \neq c_{OAO} \): this expresses the fact that a generic inertial frame \( S \), different from \( S_0 \), is not optically isotropic for any value of \( \Gamma \) different from the relativistic value \( \Gamma = 0 \).

As we shall show in Sec. 4 (Theorem 3), the velocity of light \( \tilde{c}(\partial OA) \) given by (5) implies that the observable quantity “time of flight of a light pulse along a generic closed path” must be in agreement with the round-trip axiom for any synchrony choice, i.e. for each value of \( \Gamma \). \(^{23}\)

That having been said, the crucial question is the self-consistency, namely the transitivity, of Selleri synchronizations. This is not a matter of conventions but a matter of facts, because the transitivity of a synchronization procedure is a fact empirically testable, just like the time of flight of a light pulse along a closed path.

**Theorem 2:** the transitivity of Selleri synchronization procedures, for any value of the synchronization parameter \( e_1(\beta) \), is fully equivalent to the round-trip axiom.

The proof of this important theorem takes almost one page; so it is boxed up in Appendix.

### 3.4 Selleri “inertial transformations”

According to Selleri\[^{16}\], the “gauge choice” \( e_1 = 0 \) is the only one that allows to get rid of the inconsistencies between the weakly accelerated systems and the inertial frames: we shall go deeper into the details of such a matter below, in Sec. 6. Let us just point out here that all gauge choices are equally legitimate, including of course \( e_1 = 0 \).

If we set \( e_1 = 0 \) in (4), we obtain the following “inertial transformations” (according to Selleri’s terminology):

\[
\begin{align*}
t & = \gamma^{-1}t_0 \\
x & = \gamma(x_0 - \beta ct_0) \\
y & = y_0 \\
z & = z_0
\end{align*}
\]

\(^{(8)}\)

\[^{23}\text{In particular, the observable quantity } \Delta t_o, \text{ expressing the time of flight of the signal along the there and back path } OAO, \text{ turns out to be in agreement with the round-trip axiom for each value of } \Gamma.\]
The first peculiar consequence of eqns. (8) is the anisotropic propagation of light in any IRF different from $S_0$, in such a synchrony choice. In fact, with this choice of $e_1$ the velocity of light in $S$ becomes

$$\tilde{c}(\vartheta) = \frac{c}{1 + \beta \cos \vartheta}. \quad (9)$$

Another consequence, even more peculiar, is the occurrence of an “absolute synchronization”. In fact, eq. (8) expresses the relativity of time but not the relativity of simultaneity, since $\Delta t = 0 \iff \Delta t_0 = 0$: this means that the notion of simultaneity between events occurring at distinct points of space, in this synchrony choice, turns out to be independent of the IRF that one considers. This could be puzzling for relativistic people, accustomed to the relativity of synchronization: the conundrum is that Selleri’s absolute synchronization, which at first sight could look like a weirdness in the light of a ‘traditional’ relativistic approach, turns out to be perfectly legitimate also in the full context of SRT. In fact: (i) on the operational viewpoint, it can be obtained by means of actual operations (see Sec. 3.3); (ii) on the formal viewpoint, in the full context of SRT, it is an unavoidable consequence of a perfectly legitimate synchrony choice (see Sec. 5). This issue will be further discussed in Sec. 5.3.

4 Einstein and Selleri reconciled (synthesis)

It is generally given for granted, both in Selleri’s papers and in the ones of Selleri’s opponents, that the “hard experimental evidences”, from which the general coordinate transformations (1) follow, are more general than the postulates of SRT. More explicitly, it is widely given for granted that the postulates of SRT directly imply Lorentz transformations, which in turn emerge from the general coordinate transformation (1) when $e_1(\beta) = -\beta \gamma / c$, namely in the Einstein synchrony choice: the only one allowed by the postulates. Contrary to this widespread belief, we are going to prove a theorem which shows that the hard experimental evidences are not more general than the postulates of SRT but are completely equivalent to them, provided that the operational formulation of the postulates of SRT (see Sec. 2.3) is adopted.

**Theorem 3.** The three “hard experimental evidences” (i)-(iii), on which Selleri’s theory rests, are equivalent to postulates $(\alpha_1)$-$(\beta_1)$ of Einstein’s SRT:

$$(i) \land (ii) \land (iii) \iff (\alpha_1) \land (\beta_1) \iff (\alpha_1) \land (\beta_2).$$
The demonstration of this direction is immediate: it is enough to notice that (i), (ii) and (iii) are manifestly compliant with SRT.

Consider a generic IRF $S$ moving with dimensionless velocity $\beta$ with respect to $S_0$. Let $\sigma$ be a generic closed spatial path, at rest in $S$, $dl$ the line element of $\sigma$, $\Delta l = \int_{\sigma} |dl|$ the length of $\sigma$ and $\Delta t$ the time interval, measured by a single clock at rest on $S$, taken by the light to perform a round trip of $\sigma$. The observable quantity $\Delta t$ is given by

$$\Delta t = \int_{\sigma} \frac{|dl|}{\tilde{c}(\vartheta)} = \int_{\sigma} \frac{|dl|}{c} + \int_{\sigma} \frac{\Gamma \cos \vartheta}{c} |dl|, \quad (10)$$

where eq. (5) has been applied. The last term of eq. (10) is the line integral of a constant vector field on a closed path, and obviously vanishes. Therefore

$$\Delta t = \int_{\sigma} \frac{|dl|}{c} = \frac{\Delta l}{c} . \quad (11)$$

Eq. (11) clearly shows that any velocity of light of the form (5) - whatever the choice of the synchronization parameter $\epsilon_1$ - complies with the round-trip axiom ($\beta_2$). This is an alternative form of Theorem 3.

On the other hand, we have shown in Sec. 2.3 (Theorem 1) that the round-trip axiom ($\beta_2$) is equivalent to the principle of invariance of the velocity of light, provided that it is formulated in the operationally meaningful form ($\beta_1$); briefly, $(\beta_2) \iff (\beta_1)$. As a consequence, the validity of the observable relation (11) legitimates the adoption of Einstein synchronization not only in the "formally privileged" IRF $S_0$, but also in the generic IRF $S$; that is to say in any IRF. The adoption of such a synchronization in any IRF can thus be seen as a useful convention, fully allowed by the observable data expressed by the round-trip axiom ($\beta_2$).

As a conclusion, we have come to the crucial conclusion that, if one assumes, as Selleri does, the hypotheses (i), (ii) and (iii), then Einstein synchronization is perfectly legitimate in any IRF, and its adoption does not imply any loss of generality.

**Once this synchronization procedure is conventionally adopted, any IRF turns out to be optically isotropic**\(^{24}\); then the spacetime transformations between two IRF’s get symmetric and depend only on the relative velocity between the two frames\(^{25}\). Namely the kinematical relativity principle ($\alpha_1$)

\(^{24}\)Recall that proposition ($\beta_1$) states that any IRF is isotropic with respect to Einstein synchronization, so that $\tilde{c}(\vartheta) = c \iff \Gamma = 0$.

\(^{25}\)Recall also that adopting Einstein convention reduces the general Selleri’s transformation (\(\beta_1\)) to the usual Lorentz transformation, according to the standard formulation of SRT.
holds and \( S_0 \) loses its formally privileged status. This completes the proof. \( \square \)

The above theorem straightforwardly shows that no empirical evidence can discriminate between different values of the parameter \( e_1 \) of Selleri’s theory. It is worth stressing that this impossibility is fundamental and not due to accidental reasons (like, e.g., limitations in experimental technologies), being the consequence of the full equivalence between Selleri’s and SRT’s axiomatic foundations.

Theories with the same axiomatic foundations are indistinguishable by observations: they are said to be equivalent. In our opinion, addressing to them as to different physical theories is inappropriate and somewhat misleading: they look rather as alternative formalisms of a unique physical theory. All the possible choices of the parameter \( e_1(\beta) \) correspond to different formalisms of the same theory, operatively indistinguishable from SRT, the difference among them being merely a different choice of the synchronization procedure.

So, how can we pick out a well-founded synchrony choice?

Summing up, each formalism ensuing from Transformations (4) stems from a synchrony choice; we have called Selleri synchronization gauge the set of all possible synchrony choices, related to different choices of the synchronization parameter \( e_1 \). SRT, requiring the one-way optical isotropy of all inertial systems, corresponds to a particular gauge choice, namely \( e_1 = -\beta \gamma / c \). We have tried to stress that such an optical isotropy is not an unavoidable choice forced by empirical evidences, but the combined result of the principle \( (\beta_2) \), supported by a terrific mass of empirical observations, and of the Einstein choice of the synchronization procedure, which is not supported by any empirical evidence, being fully conventional (actually, on a formal viewpoint, it is just one of the infinite choices belonging to Selleri gauge).

In compliance with the vast majority of the scientific community we accord our preference to Einstein synchrony choice, since it is definitely the simplest, the most elegant and the most fruitful one. It allows for a drastic simplification of physical laws and of their symmetry properties (see the Appendix, where the form of Maxwell equations is outlined in an arbitrary synchrony choice); it conforms to slow clocks transport; it allows for a simple mathematical treatment of the causal structure of spacetime (through the light-cone structure); it agrees with Minkowski-orthogonality
of 3-dimensional space with respect to the wordlines of the test-particles of a given physical reference frame and so on.

However, we do not agree with the standard approach to the matter of the scientific community, who is used to assuming Einstein’s choice as “the right one” and Selleri’s choice as “the wrong one”; nor we agree with Selleri’s opposite approach, which simply overturns this statement. A ”right choice”, simply, does not exist.

5 Selleri’s synchronization gauge in the full context of SRT

In the previous sections we have tried to prove the compatibility of Selleri approach with the Einstein approach of SRT starting from Selleri’s “hard experimental evidences”; namely, on the formal ground, starting from the Selleri general transformations \( \text{[1]} \). In this section we plan to get the same result, in a quite formal way, starting from the very beginning in the full context of SRT - and sticking to such a context in all that follows.

In particular we shall show how: (i) any formalism belonging to Selleri gauge may be recovered by a fully orthodox relativistic approach; (ii) any IRF may be reparametrized in order to pass from relative (Einstein’s) simultaneity to absolute (Selleri’s) simultaneity.

5.1 Parametrizing a physical reference frame in Minkowskian spacetime

The Minkowskian spacetime of SRT is an affine pseudo-Euclidean manifold \( \mathcal{M}^4 \), with signature \((1, -1, -1, -1)\). A Physical Reference Frame (PRF) is a time-like congruence \( \Gamma \) in \( \mathcal{M}^4 \) made up by the set of world lines of the test-particles constituting the “reference fluid”.\(^{26}\) The congruence \( \Gamma \) is identified by the field of unit vectors tangent to its world lines. Briefly speaking, the congruence is the (history of the) physical reference frame.

Let \( \{x^\mu\} = \{x^0, x^1, x^2, x^3\} \) be a system of coordinates in a suitable neighborhood \( U_p \) of a point \( p \in \mathcal{M}^4 \); these coordinates are said to be admissible (with respect to the congruence \( \Gamma \)) when\(^{27}\)

\[
g_{00} > 0 \quad g_{ij} dx^i dx^j < 0 \tag{12}\]

\(^{26}\)The concept of ‘congruence’ refers to a set of word lines filling the manifold, or some part of it, smoothly, continuously and without intersecting.

\(^{27}\)Greek indices run from 0 to 3, Latin indices run from 1 to 3.
Thus the coordinate lines \( x^0 = \text{var} \) can be seen as describing the world lines of the \( \infty^3 \) particles of the reference fluid, while the label coordinates \( \{x\} = \{x^1, x^2, x^3\} \) can be seen as the name of any particle of the reference fluid.

When a PRF has been chosen, together with a set of admissible coordinates, the most general coordinate transformation which does not change the physical frame (i.e. the congruence \( \Gamma \)) has the form

\[
\begin{align*}
  x'^0 &= x^0(x^0, x^1, x^2, x^3) \\
  x'^i &= x'^i(x^1, x^2, x^3)
\end{align*}
\]

(13)

with the additional condition \( \partial x'^0 / \partial x^0 > 0 \), which ensures that the change of time parameterization does not change the arrow of time. The coordinate transformation \( 13 \) is said to be ‘internal’ to the PRF \( \Gamma \) or, more simply, an internal gauge transformation. In particular, Eq. \( 13_2 \) just changes the names of the reference fluid’s particles, while Eq. \( 13_1 \) changes the “coordinate clocks” of the particles. In the following we will refer to this gauge as to the Cattaneo gauge.

**Observables as frame-dependent, but coordinate-independent, physical quantities.**

Every relativist knows the famous, enlightening statement by Minkowski: “Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality”. This “kind of union” is, of course, Minkowski spacetime. This is not the proper place to face the controversial issue of the ontologic status of Minkowski spacetime: the only physical (or maybe metaphysical) reality, or just a theoretical framework able to interrelate observable events? However, this is the proper place to point out that, from the operational point of view, we have empirical access only to Minkowski’s “shadows”; in fact, most observable physical quantities (space, time, energy, momentum, and so on) are frame-dependent. So, almost all of our empirical knowledge is knowledge of shadows; but we should be quite clear about which kind of shadows we are speaking of. “Observables” are not elusive coordinate-dependent shadows of some 4-dimensional metaphysical reality, but rather “hard shadows” which, according to the physical meaning of observables, can depend on the PRF but not on its parametrization - which is of course fully conventional. This means that, **once a PRF is given, any observable must be gauge-invariant with respect to the gauge transformation \( 13 \) internal to the given PRF.**
5.2 From AVS to Selleri’s synchronization gauge

Inside Cattaneo gauge, which is the set of all the possible parameterizations of a given PRF, the transformation

\[
\begin{align*}
\begin{cases}
x'^0 &= x^0(x^0, x^1, x^2, x^3) \\
x'^i &= x^i
\end{cases}
\end{align*}
\]

(14)

defines a sub-gauge (the synchronization gauge) which describes the set of all the possible synchronizations of the PRF.\(^{28}\)

Within the synchronization gauge (14), Einstein synchronization is the only one which does not discriminate points and directions, i.e. which is homogeneous and isotropic.\(^{29}\)

In particular, starting from Einstein synchronization, any IRF can be resynchronized according to the following transformation:

\[
\begin{align*}
\begin{cases}
\tilde{t} &= \tilde{t}(t, x^1, x^2, x^3) \\
\tilde{x}^i &= x^i
\end{cases}
\end{align*}
\]

(15)

where \(t\) is the Einstein coordinate time of the IRF. According to eqns. (15), the wordlines \(x = const\) of the test-particles by which the IRF is made up turn out to be parametrized by the resynchronized time \(\tilde{t}\) instead of the Einstein time \(t\).

The most interesting synchronization gauge is the one in which a linear dependence of the resynchronized time \(\tilde{t}\) with respect to the Einstein time \(t\) is imposed. In particular, Anderson, Vetharaniam and Stedman consider, in a long series of papers which converge on the extensive monograph \(^{14}\) (1998), the synchronization gauge

\[
\begin{align*}
\begin{cases}
\tilde{t} &= t - \tilde{k} \cdot x \\
\tilde{x} &= x
\end{cases}
\end{align*}
\]

(16)

where \(\tilde{k} = \tilde{k}(x)\) is an arbitrary smooth vector field (let us call it synchronization field) only depending on the space variable \(x\). The set of all the possible synchronizations defined by eqns. (16) defines the Anderson-Vetharaniam-Stedman synchronization gauge, the “AVS-gauge” in brief. The most interesting feature of this gauge is that it is the only synchronization gauge which

\(^{28}\)That is to say “the set of all the possible ways to spread time over space” in the given PRF.

\(^{29}\)A synchronization is called isotropic if it implies, in a generic IRF, a one-way speed of light which (at a given point) does not depend on the direction.

A synchronization is called homogeneous if it implies, in a generic IRF, a one-way speed of light which (along a given direction) does not depend on the point.
is formally consistent with the round-trip axiom. In fact, if $\Delta t$ is the time taken by a light signal for a generic round-trip, eq. (16) straightforwardly shows that the resynchronized time $\Delta \tilde{t}$ is again the same: $\Delta \tilde{t} = \Delta t$.

A non-null synchronization field $\tilde{\mathbf{k}}$ shatters the isotropy of Einstein synchronization; moreover, any possible dependence of $\tilde{\mathbf{k}}$ on the space variable $\mathbf{x}$ breaks the homogeneity of Einstein synchronization.

Since eqns. (16) imply $\tilde{\mathbf{k}} \cdot \mathbf{x} = const$ along any wordline $\mathbf{x} = const$, the AVS-gauge can be interpreted as a change in the origin of Einstein coordinate time for any point (or test-particle wordline) of the IRF, generally variable from point to point of the given IRF. The simplest particular case is $\tilde{\mathbf{k}} \cdot \mathbf{x} = const$ everywhere in the whole IRF: in this instance, eq. (16) reduces to a trivial change in the origin of Einstein coordinate time in the whole IRF.

The re-synchronization (16) redefines the simultaneity hypersurfaces, that are now described by $\tilde{t} = const$. Therefore, the set of these hypersurfaces defines a foliation of spacetime which depends not only on the IRF, as well known in Einstein synchronization (“relativity of synchronization”), but also on the synchronization field $\tilde{\mathbf{k}}(\mathbf{x})$.

A simple but absolutely non trivial case, namely the instance $\tilde{\mathbf{k}} = const$, was considered by Mansouri and Sexl [20] in the 1977; as far as we know, this is the first attempt in which a non trivial synchronization gauge is brought to attention.

Now, let us stress a very interesting case, which belongs to the Mansouri-Sexl instance. Let $S_0$ be an inertial reference frame (IF) in which an Einstein synchronization procedure is adopted by stipulation; and let $S$ be an IRF travelling along the $x$-axis (of unit vector $\mathbf{e}_1$) with constant dimensionless velocity $\beta \equiv v/c$. If both $S_0$ and $S$ are Einstein synchronized, the standard Lorentz transformation follows

$$\begin{aligned}
&t = \gamma(t_0 - \frac{\beta}{c}x_0) \\
&x = \gamma(x_0 - \beta c t_0) \\
y = y_0 \\
z = z_0
\end{aligned}$$

(17)

where $\gamma \equiv (1 - \beta^2)^{-1/2}$ is the Lorentz factor. Now, let us re-synchronize $S$ according to transformation (16), in which the synchronization field $\tilde{\mathbf{k}}$ is chosen as follows:

$$\tilde{\mathbf{k}} = \frac{-\Gamma(\beta)}{c} \mathbf{\hat{x}}$$

(18)
\( \mathbf{x} \) being the unit vector in direction \( x \) and \( \Gamma(\beta) \) being an arbitrary function of \( \beta \).\(^{30}\) We get:

\[
\begin{align*}
\tilde{t} &= t + \frac{\Gamma(\beta)}{e} x = \gamma(t_0 - \frac{\beta}{e} x_0) + \frac{\Gamma(\beta)}{e} \gamma(x_0 - \beta c t_0) \\
\tilde{x} &= x = \gamma(x_0 - \beta c t_0) \\
\tilde{y} &= y = y_0 \\
\tilde{z} &= z = z_0
\end{align*}
\]

(19)

If the function \( \Gamma(\beta) \) is written as follows:

\[
\Gamma(\beta) \equiv \beta + e_1(\beta) c \gamma^{-1}
\]

(20)

where \( e_1(\beta) \) is an arbitrary function of \( \beta \), then Eqns. (19) take the form

\[
\begin{align*}
\tilde{t} &= \gamma^{-1} t_0 + e_1(\beta)(x_0 - \beta c t_0) \\
\tilde{x} &= x = \gamma(x_0 - \beta c t_0) \\
\tilde{y} &= y = y_0 \\
\tilde{z} &= z = z_0
\end{align*}
\]

(21)

which exactly coincides with Selleri general coordinate transformations (\ref{eqn:10}).

Eqns. (21) define the “\textit{Selleri synchronization gauge}”. Such a gauge can be interpreted as the set of all the possible synchronizations of a given IRF which comply with the “hard experimental evidences”; in particular, it complies with the standard Einstein expression of time dilation with respect to the IRF \( S_0 \), according to Selleri’s statement (iii).\(^{31}\)

The arbitrary function \( e_1(\beta) \) is nothing but Selleri’s “synchronization parameter”.

All this shows that the set of all the “theories” belonging to Selleri gauge can be interpreted, \textit{in the theoretical background of SRT}, as a set of parameterizations (in particular synchronizations) of a given IRF. Different parameterizations give rise only to different formalisms of the SRT, not to different physical theories. Selleri’s “alternative theories” are merely “alternative writings” of a unique physical theory, which is nothing but the SRT. This completely agrees with the results found, \textit{in the theoretical background of Selleri’s approach}, in Sec. 4.\(^{14}\)

\(^{30}\)Note that the synchronization field \( \mathbf{k} \) defined by eq. (13) does not depend on \( x \): this means that the case under consideration actually belongs to the Mansouri-Sexl instance.

\(^{31}\)Of course in the AVS-gauge, which includes the Selleri gauge, the time dilatation - as an observable quantity - does not change; however, on the formal point of view it takes a more complicated shape depending on the vector field \( \mathbf{k} \) (as well as on the dimensionless velocity \( \beta \)), see Ref. 14.
5.3 Selleri “absolute simultaneity” in the full context of SRT

The synchrony choice \( e_1(\beta) = -\beta \gamma/c \) (i.e. \( \Gamma(\beta) = 0 \)) in Eqns. (21) gives the standard Einstein synchronization, whereas the synchrony choice \( e_1(\beta) = 0 \) gives Selleri synchronization. In the AVS formalism, the synchrony choice \( e_1(\beta) = 0 \) is equivalent to the choice

\[
\tilde{k} = -\frac{\beta}{c} \hat{x}
\]

(22)

for the synchronization field \( \tilde{k}(x) \). In such a synchrony choice, eqns. (21) take the form

\[
\begin{align*}
\tilde{t} &= \gamma^{-1} t_0 \\
\tilde{x} &= x = \gamma(x_0 - \beta c t_0) \\
\tilde{y} &= y = y_0 \\
\tilde{z} &= z = z_0
\end{align*}
\]

(23)

which are nothing but the “inertial transformations” advocated by Selleri. Now, let us consider, in the full context of SRT, the "puzzling consequence" mentioned in Sec. 3.4 of Eqns. (23): “absolute simultaneity”.

As widely pointed out by Selleri, the time transformation expressed by \( \tilde{t} \) does not contain the term which is responsible for the relativity of simultaneity\(^{32}\); consequently, the notion of simultaneity between “spatially separated” events, in this synchrony choice, turns out to be independent of the IRF that one considers. This can be expressed by saying that the synchronization procedure described by the transformations \( \tilde{t} \), is, in Selleri’s terms, “absolute”\(^ {33}\).

Absolute synchronization is interpreted by Selleri \(^ {33}\) on the ground of a paradigm based on the actual existence of physically privileged IRF (at kinematical level), in agreement with the hypothesis of the stationary ether, proper of the classical paradigm shared by Lorentz e Poincaré (see, for instance, \(^ {33}, \text{34} \)). In particular, according to Selleri “time is no more an infinite series of subjective viewpoints, all of them equally legitimate, but gets a solid objectiveness, similar to the one of pre-relativistic physics” \(^ {17}\).

\(^{32}\)In fact, the re-synchronization defined by the choice \( e_1(\beta) = 0 \) cancels out the term \(-\gamma \frac{\beta}{c} x_0 \) appearing in Lorentz time transformation and responsible for the relativity of simultaneity \(^ {22}, \text{19} \).

\(^{33}\)From an operational viewpoint, the absolute synchronization of an inertial frame \( S \) is obtained by setting the reading of a clock in \( S \) equal to \( \gamma^{-1} t_0 \) when its spatial position coincides with the one of a clock in \( S_0 \), whose reading is \( t_0 \) (obviously, this event is unique in the biography of a clock of \( S \)).
We are going to discuss the issue of “absolute synchronization” keeping carefully apart (i) the actual possibility of “absolute synchronization” in SRT and (ii) its Lorentz-like interpretation, as outlined by Selleri.

(i) Although the expression “absolute synchronization” sounds rather eccentric in a relativistic framework, we remind from Sec. 3.3 that, from the operational viewpoint, it can be obtained by means of actual operations (see Sec. 3.3). This would be enough to legitimate the “absolute synchronization”, but in the previous section we have found an even more stringent justification: from a formal viewpoint in the full context of SRT, “absolute synchronization” is an unavoidable consequence of a perfectly legitimate synchrony choice. This means that the synchronization gauge, once incorporated in the formalism of SRT, should allow a sort of peaceful coexistence of Einstein relative simultaneity and Selleri “absolute” simultaneity; to be exact, it should allow to introduce in SRT an “absolute simultaneity” without affecting neither the logical structure, nor the predictions of the theory. This is possible if the geometrical structure of Minkowski spacetime is so democratic as to accommodate, in its texture, both Einstein relative simultaneity and Selleri “absolute” simultaneity. Is it indeed possible?

According to the “realistic” Selleri’s interpretation of absolute simultaneity, this is simply impossible: Einstein relative simultaneity and Selleri “absolute” simultaneity are in contention, only one of them is able to fit the physical world. This rigid viewpoint is shared by many “orthodox” relativists on the basis of the claim (on which we agree) that the geometrical structure of Minkowski spacetime reflects the physical world without any ambiguity; according to them, since the relativity of simultaneity is a fundamental feature - embedded in the geometrical structure of Minkowski spacetime - of the physical world. There is no room for conventions about simultaneity.

Both viewpoints agree that one definition of simultaneity is right and one is wrong, and they just disagree on which is right and which is wrong.

According to us instead, this “peaceful coexistence” is possible without any difficulty because Selleri “absolute” simultaneity turns out to be a simple parametrization effect in Minkowski spacetime. Let us clarify this claim.

Our viewpoint is very plain: all the kinematical content of SRT is encoded in the geometrical structure of Minkowski spacetime, which actually reflects the physical world, at the kinematical level, without any ambiguity; yet the way of parameterizing the Minkowski spacetime is indeed a matter of convention. To put it in terms of observables: observable effects are determined uniquely by the geometrical structure of Minkowski spacetime,
which is physically meaningful, not by the way it is parameterized, which is physically meaningless.

The choice $e_1 = \beta \gamma / c$, corresponding to Einstein synchronization, involves a frame-dependent foliation of Minkowski spacetime. Specifically, such a foliation is realized by the hypersurfaces that are Minkowski-orthogonal with respect to the world lines of the test-particles by which the reference frame is made up.

On the other hand, the choice $e_1 = 0$ involves a frame-invariant foliation, which is nothing but the Einstein foliation of the IRF $S_0$, assumed to be optically isotropic by stipulation.

As a conclusion, since the way of foliating the Minkowski spacetime is a matter of convention, both Einstein relative simultaneity and Selleri "absolute" simultaneity can peacefully coexist in Minkowski spacetime as different conventions, see fig. 2.

Einstein showed the extraordinary power of explanation of the relative simultaneity convention, as well as its heuristic potential; as a matter of fact, the relativity of simultaneity - provided that it is operationally well defined, and incorporated in Minkowski spacetime as a well defined family of frame-dependent foliations - is the deepest conceptual root of the theory, in particular from the heuristic viewpoint.

On the other hand, Selleri has showed that, in some particular cases, the "absolute" synchronization - provided that the "privileged" IRF $S_0$ is properly chosen - can actually lead to a simpler description of facts. Let us mention some examples proposed by Selleri: (i) the case of two twins living on two rocket-ships moving along two congruent worldlines, shifted along a spatial direction, who share the same experience and of course also the same proper time, but share the same age only in a "privileged" IRF; (ii) the problem of a causally consistent description of spatially separated events who get in contact through a superluminal (tachionic) interaction; (iii) the issue of synchronizing clocks on the Earth, by means of electromagnetic signals travelling between the locations of the clocks via a geostationary satellite [35].

The point is that, while classical physics only allows for an absolute synchronization, relativistic physics also allows for relative simultaneity. This is enough to lead to a completely different description of the physical world; in philosophical terms, to a completely different paradigmatic world-view.

(ii) The main difference between our point of view and Selleri’s lies in a different paradigmatic background. Which, in our opinion, does not properly
belong to physics, but rather to the interpretation of physics. Selleri’s work develops on the paradigmatic background of the ether hypothesis, which we consider as an unnecessary and misleading superstructure - at least on the kinematical level - that can be rejected as an ideological fossil, empty of any operational meaning, without rejecting the possibility that an absolute synchronization can be actually performed: this is definitely our position.

This position does not exclude the existence of a “local ether”, defined as the optically isotropic IRF of the cosmic background radiation of the region spanned by the Solar System. In fact, it is well known that such an inertial frame exists; as a matter of fact, it is privileged for the description of some astrophysical phenomena. What we reject, on the basis of the Relativity principle (“equal experiments performed in the same conditions lead to the same results”), is the idea of the existence of an IRF where some physical laws hold, but the same laws do not hold in other IRF’s (we think, in particular, of the observable properties of the propagation of light).

6 Synchronization gauge and rotating reference frames

As widely seen in the previous sections, synchronization is a matter of convention as far as IRF’s are concerned. This can be formalized through a suitable synchronization gauge. Starting from the so-called “hard experimental evidences”, Selleri suggest the synchronization gauge (4), in which any allowed synchrony choice is fixed by the “synchronization parameter” $e_1$, and claims that such a parameter cannot be fixed by any experiment performed in IRF’s. However Selleri claims (in our opinion inconsistently, but consistently with his own “realistic” Lorentz-like approach) that, as soon as rotating frames are considered, synchronization cannot be conventional any longer: when rotation is taken into account, the synchronization parameter $e_1$ is forced to take the value zero, and the “absolute synchronization” turns out to be the only legitimate synchronization [17, 18, 35]. In his words, “the famous synchronization problem is solved by Nature itself: it is not true that the synchronization procedure can be chosen freely, because all conventions but the absolute one lead to an unacceptable discontinuity in the physical theory” [17, 35].

The discontinuity to which Selleri refers descends from the comparison of the global velocity of light on a round-trip along the rim of a rotating platform, as measured by a single clock at rest on the rim, and the local velocity of light, as measured by two very near clocks at rest in a local
comoving inertial frame (LCIF) $S$ on the rim. If $R$ is the radius of the circular rim of the platform and $\Omega$ its angular velocity as measured in the central IRF $S_0$, such a discontinuity persists even when we perform the limit $\Omega \to 0, R \to \infty$ in such a way that the peripheral velocity $\Omega R$ is kept fixed (see [17] for details). This is the reason why Selleri properly regards this discontinuity as an “unacceptable” one.

Elsewhere, see Ref. [19], we have given a simple solution of this alleged ‘paradox’. Summing up, the discontinuity is uniquely originated by the fact that, in Selleri “absolute” synchrony choice (we prefer to say in Einstein synchrony choice of the central IRF $S_0$), the synchronization procedure is different in the central IRF $S_0$ and in the LCIF $S$; to be exact, an isotropic synchronization is chosen in $S_0$ and an anisotropic synchronization is chosen, according to Eqns. [8], [9], in any LCIF on the rim. This means that the discontinuity found by Selleri is not a physical discontinuity, but merely expresses different synchrony choices in the involved IRF’s: as a matter of fact, this discontinuity disappears if the same synchronization procedure is adopted in any (local or global) IRF, according to a proper formulation of the Relativity principle (see Sec. 2.3).

Let us stress that the issue which leads Selleri to mistake different synchrony choices in different IRF’s for an “unacceptable physical discontinuity” is nothing but the standard formulation of the Relativity principle; as a matter of fact, the optical isotropy of an IRF is misleadingly considered, in the standard textbooks of SRT, as a physical property of the IRF itself, rather than a combined consequence of observable physical properties of light propagation (the round-trip axiom) and of a “suitable” (conventional) synchrony gauge choice in the considered IRF. Towards the discontinuity found by Selleri we have just two possibility: rejecting the SRT, as Selleri does, or reformulating the Relativity principle, as we have done in Sec. 2.3.

In our opinion, Selleri’s discontinuity is a lightening tool, which unveils the inadequacy not of SRT, but of the standard formulation of SRT: a noteworthy contribution to the understanding of the theory, coming from an alleged antirelativist.

In particular, disagreeing both with Selleri and with many relativistic authors, who claim that the choice of the synchronization parameter $e_1$ is forced by suitable empirical evidences (just disagreeing on the actual value of this parameter), we claim that no empirical evidence is able to force the choice of $e_1$. Neither the GPS evidence (see below), nor the empirical evidence of Sagnac interference fringes of two light or matter beams counter-propagating along the rim of the platform [36], nor the empirical evidence of the difference in the ages of two slow travelling twins, after a complete
round-trip in opposite directions [19], nor any other experimental evidence. For a thorough, critical discussion of synchronization problems in rotating (and, more generally ‘non time-orthogonal’) reference frames we cross-refer to Ref. [37].

Yet, we are going to conclude this section facing two experimental evidences, performed in a rotating reference frame, in which both the Einstein and the Selleri synchrony choices can be profitably used and confronted.

6.1 A case where Selleri synchrony choice is fitting: a simplified description of the GPS working

In this section we shall limit ourselves to consider a very peculiar rotating reference frame: the Earth. The aim of this section is to account for the good working of the global positioning system (GPS) in Selleri gauge in a suitable IRF, namely the IRF \( S_T \) in which the terrestrial axis is at rest (called by Ashby [38] “Earth-Centered IRF”). Notice that the good working of the GPS is indeed the most relevant testground for relativistic kinematical (as well as gravitational) effects in rotating systems [38]. But is it, in particular, a testground of isotropic propagation of light, as often claimed by several authors?

It should be clear from the previous sections that we have direct empirical access only to the roundtrip speed of light, not to the one-way speed of light; in particular, we know from Sec. 5.1 that, in a given PRF, any observable must be invariant with respect to any synchronization gauge [34]. Such a matter should be thus closed. However, we cannot simply ignore so many authoritative opposite opinions, often supported by lots of (alleged) experimental evidences [39], [5], [40]. For instance, Van Flandern [39] claims that “the system has shown that the speed of radio signals (identical to the ‘speed of light’) is the same from all satellites to all ground stations at all times of day and in all directions to within \( \pm 12 \) meters per second (m/s). The same numerical value for the speed of light works equally well at any season of the year”; “our result here merely points out that the measured speed [of light] does not change as a function of time of day or direction of the satellite in its orbit when the clock synchronization correction is kept unchanged over one day”. This extract is cited by Bergia [5], [11] who comments: “it is hard to think that the intricate network of cross controls would not cause anomalies if the isotropy were not a real data” [35], and concludes: “very compelling limits on a parameter related to \( e_1 \) were obtained by [40].

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[34] Which is a sub-gauge of the gauge transformation internal to the PRF.
[35] Translation by the authors.
This means that Selleri gauge is considered by many authors as a test-theory, and the GPS was used as a tool aimed at determining, within compelling limits, the synchronization parameter $e_1$. The result is a strong evidence for isotropic propagation of light, basically “because the system is too effective. In other words, the localization based on the hypothesis that the speed of light is independent on the direction is too accurate. Something that sounds like a verification - very accurate indeed - of such a hypothesis”.

Facing with so many claims which support the idea that the (terrifically good) working of the GPS should be considered as an empirical evidence for the isotropic propagation of light in the Earth-Centered IRF $S_T$, we are forced to study this issue in detail.

**Preliminary approach in Einstein synchrony choice.**

First of all, let us explain how the GPS works by means of a basic bidimensional example and relying on Einstein synchronization in a suitable IRF $S_T$ (see below), as a conventional but useful framework (this is what Selleri would call “absolute synchronization”). Recall that the aim is to determine the position $\mathbf{r}_I$ of an object on which a GPS device $I$ is installed. The device $I$ disposes of a clock which has not yet been synchronized with other clocks. Let us consider the IRF $S_T$ in which the terrestrial axis is at rest and let us synchronize the GPS satellites $A$, $B$ and $C$ according to Einstein procedure in $S_T$ (starting from the ‘central station’ $O$ fixed in $S_T$). At a given time of its clock, $I$ receives from the three satellites $A$, $B$ and $C$ the following information: the emission times $t_A$, $t_B$ and $t_C$ of the signals and the positions $\mathbf{r}_A$, $\mathbf{r}_B$ and $\mathbf{r}_C$ of $A$, $B$ and $C$ at the times $t_A$, $t_B$ and $t_C$ respectively. For simplicity, let us assume that $O$, $I$, $A$, $B$ and $C$ belong to the same plane. The position $\mathbf{r}_I = (x_I, y_I)$ is then determined by the following system, in terms of the cartesian coordinate $x$, $y$ of the plane

$$
\begin{align*}
\sqrt{(x_I - x_A)^2 + (y_I - y_A)^2} &= c(t_I - t_A) \\
\sqrt{(x_I - x_B)^2 + (y_I - y_B)^2} &= c(t_I - t_B) \\
\sqrt{(x_I - x_C)^2 + (y_I - y_C)^2} &= c(t_I - t_C)
\end{align*}
$$

(24)

whose solution corresponds to the intersection point of three circumferences. The three dimensional extension obviously requires another satellite. Notice however that actual GPS devices exploit the data of about ten satellites at a time.

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36 Translation by the authors.

37 We assume, for simplicity, a simultaneous reception of the signals. This results in more simple expressions, without any loss of generality.
If the clock in $I$ had already been Einstein synchronized (in $S_T$) with the other clocks, then the system (24) would be ‘overdetermined’. But this is not the case, since $t_I$ is an additional unknown of the system (apart from $x_I$ and $y_I$). Now, obtaining $t_I$ as a solution of system (24) is equivalent to stipulate that the one-way velocity of light between any one of the satellites and $I$ takes the value $c$. In other words, the clock in $I$ is so implicitly Einstein synchronized with all the other clocks of $S_T$. This can be rephrased stating that one of the three equations of system (24) synchronizes à la Einstein the clock in $I$, while the other two determine the position $r_I$ as the intersection of two circumferences.\footnote{Such circumferences represent the points that the two signal reaches “simultaneously” at the instant $t_I$ (where “simultaneously” means “simultaneously for Einstein synchronization”).\footnote{Actually the synchronization refers to clocks at rest in $S_T$. However the clock of $I$ moves with dimensionless velocity $\beta_I$ with respect to $S_T$ and gets thus desynchronized by a factor $\gamma^{-1} = \sqrt{1 - (\beta_I)^2}$; the same remark applies to the satellites’ clocks. This makes no difference on the conceptual level. The time dilations effects are automatically corrected by the GPS software (cfr., e.g.,Ref. \[11\]). Another correction implemented by the GPS software is needed because of the gravitational redshift, experienced by the satellites which are placed at different heights in the Earth’s potential. These corrections will be understood in the following.}}

**General approach in Selleri gauge.**

We now aim at generalizing the previous argument, showing that the effectiveness of the GPS can be accounted for on the basis of any synchronization procedure belonging to Selleri gauge. We recall that the adoption of a synchronization procedure defined by a certain choice of $e_1(\beta)$ is equivalent to the reparameterization of the IRF (in this instance of $S_T$) by the resynchronization (16) with the choice (18). Let $x_I$, $y_I$ and $t_I$ be the solutions of the system (24), that is to say the values of space and time provided by the GPS in Einstein synchrony choice ($\Gamma = 0$). To explain the proper working of the GPS under any choice of $e_1$ one has to show the following theorem.

**Theorem 4.** Let $t_I, x_I, y_I$ the coordinates of an event in the Earth-Centered Einstein synchronized IRF $S_T$, solutions of the system (24); and let $\tilde{t}_I, \tilde{x}_I, \tilde{y}_I$ the resynchronized coordinates defined, in agreement with Eqs. (16), by

$$\tilde{t}_I = t_I + \frac{\Gamma}{c} x_I$$
\[ \tilde{x}_I = x_I \]  
\[ \tilde{y}_I = y_I \]  

Then the coordinates \( \tilde{t}, \tilde{x}, \tilde{y}, \tilde{z} \) are solutions of the system (24) recast in the resynchronized chart \( \{ \tilde{t}, \tilde{x}, \tilde{y}, \tilde{z} \} \). In particular, the spatial position of the device I is the same as in the Einstein chart \( \{ t, x, y, z \} \).

Proof. We aim at rewriting Eqs. (24) in the resynchronized chart \( \{ \tilde{t}, \tilde{x}, \tilde{y}, \tilde{z} \} \). Since the instant \( t_A \) of Einstein chart corresponds, in the resynchronized chart, to the instant

\[ \tilde{t}_A = t_A + \frac{\Gamma}{c} x_A, \]  

(26)

the time interval \( \tilde{t}_I - \tilde{t}_A \) needed by the signal to go from \( A \) to \( I \), over a space distance \( l_{(AI)} \), is given - following Eq. (5) - by

\[ \tilde{t}_I - \tilde{t}_A = \int_{A}^{I} \frac{|dl|}{c(d\theta)} = \int_{A}^{I} \frac{|dl|}{c} + \int_{A}^{I} \frac{\Gamma \cos \theta}{c} |dl| = \frac{l_{(AI)}}{c} + \Gamma \int_{A}^{I} \frac{dx}{c} = \frac{l_{(AI)}}{c} + \frac{\Gamma (x_I - x_A)}{c}. \]  

(27)

where \( \theta \) is the angle between \( \beta \) and \( AI \) while \( \cos \theta |dl| = dx \). Analogous expressions hold for the time intervals \( \tilde{t}_I - \tilde{t}_B \) and \( \tilde{t}_I - \tilde{t}_C \) needed by the signal to cover the straight paths \( BI \) and \( CI \). Therefore the system of equations which describes the propagation of the signals along \( AI, BI \) and \( CI \), can be written in the resynchronized chart \( \{ \tilde{t}, \tilde{x}, \tilde{y}, \tilde{z} \} \) as

\[
\begin{align*}
\sqrt{(x_I - x_A)^2 + (y_I - y_A)^2 + \Gamma(x_I - x_A)} &= c \left( \tilde{t}_I - \tilde{t}_A \right) \\
\sqrt{(x_I - x_B)^2 + (y_I - y_B)^2 + \Gamma(x_I - x_B)} &= c \left( \tilde{t}_I - \tilde{t}_B \right) \\
\sqrt{(x_I - x_C)^2 + (y_I - y_C)^2 + \Gamma(x_I - x_C)} &= c \left( \tilde{t}_I - \tilde{t}_C \right)
\end{align*}
\]  

(28)

Exploiting Eqs. (25) and (26) one can reduce the first equation of the system (28) to

\[
\sqrt{(x_I - x_A)^2 + (y_I - y_A)^2 + \Gamma(x_I - x_A)} = c \left( t_I + \frac{\Gamma}{c} x_I - t_A - \frac{\Gamma}{c} x_A \right). \]  

(29)

All the terms containing \( \Gamma \), which is, exactly as \( \epsilon_1 \), a free parameter describing the synchronization choice, cancel out; the same occurs, clearly, for the other two equations. That is, the system (28) exactly reduces to (24).
This implies that, if the values $x_I$, $y_I$ and $t_I$ solve the system (24), then the values $\tilde{x}_I$, $\tilde{y}_I$ and $\tilde{t}_I = t_I + \frac{\Gamma}{c}x_I$ solve the system (28), which completes the proof. □

Summing up, what has been formally demonstrated in this theorem is that the observable of interest for the GPS, i.e. the spatial position of the device $I$, does not depend on the way the Earth-Centered IRF is synchronized, provided that every synchronization belongs to Selleri synchronization gauge; more explicitly, such an observable is the same both in the Einstein chart $\{t, x, y, z\}$ and in the resynchronized chart $\{\tilde{t}, \tilde{x}, \tilde{y}, \tilde{z}\}$.

As a conclusion, the formalism introduced by Selleri turns out to be in full compliance with the observational data provided by the GPS in the rotating frame of the Earth - and in particular with the good working of such a system. However, this example also shows how Einstein synchrony choice allows for a drastic simplification of the description of GPS’s working. As a consequence, only the belief in a physically privileged IRF, namely the IRF in which the cosmic background radiation is isotropic (assumed as the ether rest frame), could lead to a different synchrony choice: in this case the natural choice should be the resynchronized time (26) with $\Gamma = \beta \sim 10^{-3}$, corresponding to the velocity of the Earth-Centered IRF $S_T$ with respect to the ether rest frame ($\sim300$ km/sec). Strangely enough, Selleri assumes the Earth-Centered IRF $S_T$ as a convenient ether rest frame: this does not agree - at least in our opinion - with Selleri ideologic assumption, but agrees very well with our relativistic viewpoint, according to which any IRF can play the role of “ether rest frame”.

**Remark.** There is something more to be pointed out. We have widely seen that the time $t$ adopted in synchronizing the rotating GPS satellites is not the time resulting from an Einstein synchronization in the rotating system of the Earth (which, as well known, is allowed only locally); but, rather, the time resulting from Einstein synchronization in the Earth-Centered IRF $S_T$ which plays, in this instance, the role of $S_0$ [41]. The importance of the time $t$ of the IRF $S_T$ depends on the fact that such a ‘central inertial time’

---

40We recognize this is a substantial variance of Selleri’s approach. In fact, the masterly calculation performed in Ref. 35 aims at a different goal: synchronizing clocks on the Earth by means of electromagnetic signals travelling between the locations of the clocks via a geostationary satellite. It is noteworthy that Selleri synchrony choice $e_1 (\beta) = 0$ takes automatically into account the Sagnac shift due to Earth rotation; we consider such a calculation one of the most relevant results found by Selleri. However, it should be noted that such a calculation is performed along an idealized parallel, and no altitude variations are taken into account.
(which Selleri would call “absolute”), although being incompatible with slow transport synchronization, is the only procedure (up to resynchronizations belonging to the Selleri gauge) allowing a global synchronization of clocks on the platform.\textsuperscript{41} This is a well known fact: as far as we know, any relativistic approach to the study of rotating reference frames actually uses such a central inertial time in order to synchronize clocks in the rotating frame. This explains, in particular, the surprising ‘Galilei-like’ coordinate transformation between the central IRF and the rotating reference frame \textsuperscript{32}, \textsuperscript{37}.

Yet, does the central time $t$ really come from the Selleri synchrony choice $e_1(\beta) = 0$? To be precise, the time coming from Selleri synchrony choice $e_1(\beta) = 0$ is not exactly the time $t$ of the Earth-Centered IRF $S_T$,\textsuperscript{42} but the time $t$ rescaled by a factor $\gamma^{-1}$, see eq. [8]. Along a parallel (a very idealized circular path) the Lorentz factor is constant: so it has the role of an innocuous scale factor, and Selleri’s approach is sound. Yet along a more general path the Lorentz factor turns out to be dependent both on the latitude and on the altitude above sea level: so we prefer to avoid such a variable scale factor, and simply synchronize clocks everywhere, on the Earth and in the sky, by means of the time $t$ of the Earth-Centered IRF. We suppose this actually is Selleri’s “absolute” synchronization, beyond the arguable niceties of the formalism.\textsuperscript{43}

\textsuperscript{41}It could be said that the central inertial time $t$ automatically accounts for the desynchronization effects (responsible of the Sagnac effect) suffered by Einstein synchronization in rotating systems.

\textsuperscript{42}I.e. the time $t$ read on the clock of $S_0$ by which an arbitrary clock on the platform $K$ passes at a given instant.

\textsuperscript{43}Selleri’s synchrony choice results in the choice of a coordinate time $\hat{t}$ on the platform that coincides with the proper time of the clocks at rest on the platform:

$$\hat{t} = 1/t = 1/t \cdot (\gamma \tau) = \tau$$

The global simultaneity criterium is given by the spacetime foliation $t = \text{const}$, that implies $\hat{t} = \text{const}$ on the rim $r = R$, but not on the whole platform:

$$t = \text{const} \Rightarrow \hat{t} = 1/t = 1/t \cdot \frac{\Omega R}{c} = \text{const}$$

As a consequence, the time coordinate allowing a global synchronization on the whole platform (i.e. for $0 \leq r \leq R$) is actually the time $t$ of the central inertial frame $S_0$. 

38
6.2 A case where Selleri synchrony choice is not fitting: Sagnac effect for matter beams

As shown in Sec. 5.3, all observable effects depend on the geometrical structure of Minkowski spacetime, which unambiguously reflects the physical world at the kinematical level, but they do not depend on the way such a space is parameterized.

This sort of obviousness, unfortunately clouded by the standard formulation of SRT, allows to embody the physical contents of SRT in a multiplicity of formalisms. In particular, it allows us to take a very pragmatic view: both Selleri “absolute” synchronization and Einstein relative synchronization can be used, depending on the aims and circumstances. As a matter of fact, we known that a global Einstein synchronization is not allowed in the reference frame of a rotating platform; so the possibility of using different synchronization conventions for different aims seems to be attractive.

If we look for a global synchronization, we are forced to use the Einstein simultaneity criterium (i.e. the Einstein foliation) of a suitable IRF, that is to say a simultaneity criterium borrowed from a suitable IRF. Basically, this is Selleri “absolute” synchronization, although some formal difference actually exists, as showed in the previous section. As already pointed out, the “suitable IRF” is nothing but “the more convenient” one. Let us mention some examples actually considered in Selleri’s papers: if the rotating reference frame is the Earth, the “suitable IRF” is the Earth-Centered IRF $S_T$; if the rotating reference frame is a beam of relativistic muons in a storage ring, the “suitable IRF” is the laboratory frame, at rest on the Earth (notice that the hypothesis of dragging of the ether is ruled out by experiments performed by Werner et al. [43]); more generally, if the rotating reference frame is a rotating platform, the “suitable IRF” is the central IRF. Last but not least, it seems surprising that, in all these examples, the only serious candidate to the role of ether rest frame, namely the IRF in which the cosmic background radiation is isotropic, keeps playing no role at all. So it should be realized (or at least suspected) that the “ether rest frame” is nothing but a misleading expression which can be used for every useful IRF, contrary to a Lorentz-like approach and according to a relativistic approach.

On the other hand, if we look for a plain kinematical relationship between local velocities, the local Einstein synchronization, not the global “absolute” synchronization, is required in any LCIF [19], [44]. If the synchronization is a matter of convention, the choice of an opportune synchronization only
depends on what we may call “descriptive simplicity”: an opportune synchronization is the one which leads to a simpler description of a physical phenomenon.

That being said, let us outline the advantages of the local Einstein synchronization on a rotating platform with respect to the “absolute” synchronization.

First of all, the velocity of light has the invariant value \( c \) in every LCIF, both in co-rotating and counter-rotating direction, if and only if the LCIFs are Einstein-synchronized.\(^{44}\) We are aware that this statement sounds arbitrary or even wrong to some authors\(^{15, 16, 17}\), who claim that only absolute synchronization in the LCIFs is allowed in order to get the same value for the local and the global (round-trip) relative velocity of the light beams. So we try to suggest a more stringent argument. As we showed in \(^{36}\) (see in particular Sec. 3), the well known Sagnac time difference

\[
\Delta \tau = \frac{4 \pi R^2 \Omega}{c^2} \left(1 - \frac{\Omega^2 R^2}{c^2}\right)^{-1/2}
\]

holds for two light or matter beams travelling - according to some kinematical condition - in opposite directions along the rim of a turnable of radius \( R \), uniformly rotating with angular velocity \( \Omega \).

Selleri deals with light beams, but light beams are not discriminating at all, since they allow a multiplicity of sound explanations: local isotropy, leading to an Einstein synchrony choice in every LCIF, seems a sound requirement, but the identity of local and global (round-trip) relative velocity of every light beam, leading to an “absolute” synchrony choice in every LCIF (\(^{15, 16, 17}\)), is a sound requirement too. Therefore, elegance being a too indefinite and subjective criterium, it is impossible to single out the simpler description of Sagnac effect for counter-propagating light beams: the unpleasant but unavoidable conclusion is that the local synchrony choice for counter-propagating light beams is a matter of taste.

Things go differently for counter-propagating matter beams. As showed in Ref. \(^{36}\), the Sagnac time difference \(^{37}\) for matter beams holds under the kinematical condition

\[
\beta'_+ = -\beta'_-\]

\(^{44}\)If a LCIF is Einstein synchronized, light propagates isotropically by definition; if the LCIF is “absolute” synchronized (i.e. if it borrows its synchronization from the central IRF), light propagates anisotropically. Let us recall \(^{14}\) that the local isotropy or anisotropy of the velocity of light in a LCIF is not a fact, with a well defined ontological meaning, but a convention which depends on the synchronization chosen in the LCIF.
where $\beta'_+ \pm \beta'_-$ are the dimensionless relative velocities, with respect to any LCIF along the rim, of the co-propagating and counter-propagating beam, provided that any LCIF is Einstein-synchronized. So, condition (31) means “equal relative velocity in opposite directions”: this is a plain and meaningful condition which explicitly requires that every LCIF is Einstein-synchronized.\footnote{Recall that the “relative velocity” $\beta'_+, \beta'_-$ of each travelling beam is not an intrinsic property of the beam, but depends on the local synchrony choice, i.e. on the synchronization of any LCIF along the rim. As a consequence, the condition “equal relative velocity in opposite directions” singles out a very clear synchrony choice in any LCIF. Calculations performed in Ref. \cite{36} show that such a choice is Einstein synchrony choice.}

Of course such a condition can be easily translated also into Selleri “absolute” synchronization, namely in the Einstein synchronization of the central IRF. Yet it would result in a very artificial and convoluted requirement, expressed by

$$\beta^r_- = -\beta^r_+ \frac{1 - \beta^2}{1 - 2 \beta^r_+ - \beta^2} \tag{32}$$

where $\beta^r_\pm$ are the dimensionless velocities of the matter beams, with respect to the absolute-synchronized LCIF, and $\beta$ is the dimensionless velocity of the rim of the turntable.

Comparing eq. (32) with eq. (31), it is apparent that only Einstein synchronization allows the clear and meaningful requirement: “equal relative velocity in opposite directions”.

Summarizing, “absolute” synchronization avoids inconsistencies pertaining to the issue of synchronizing clocks \textit{globally} in a rotating frame; however \textit{local} Einstein synchronization is by far more useful if the issue is explaining the Sagnac time delay for counter-propagating matter beams in a simple and not artificial way.

7 Synchronization gauges in SRT: some conclusive remarks

We know from Sec. 5.2 that the synchronization of an IRF is not “given by God”, as often both relativistic and anti-relativistic authors assume, but can be arbitrary chosen within the synchronization gauge (14). However, such an extremely wide gauge is of very poor utility in order to label events expediently, since it allows the behavior of every single clock to be irregular and even completely random; as a consequence, it allows a time of flight of
a light pulse along the closed path, as measured by a too wild clock, to be completely random, so shattering the round-trip axiom.

Within the synchronization gauge (14), Einstein synchronization is the only one which does not discriminate points and directions, i.e. which is homogeneous and isotropic [32]. Starting from Einstein synchronization, any IRF can be resynchronized in several ways, according to several conventions and constraints. Of course the formal validity of the round-trip axiom is the most obvious constraint in order to pick out a set of synchronizations of some utility to describe the physical world according to the SRT.

The most general sub-gauge which is formally consistent with the round-trip axiom is the AVS synchronization gauge (16), individualized by a “synchronization field” \( \tilde{k}(x;\beta) \) only dependent on the space variable \( x \) and, in case it could be of some advantage, on some constant parameter \( \beta \). Of course, for an arbitrary \( \tilde{k}(x,\beta) \) no optical homogeneity and isotropy is in general expected.

It could be expedient to choose an Einstein synchronization in a given IRF \( S_0 \) and to synchronize any other IRF, moving with dimensionless velocity \( \beta \) with respect to \( S_0 \), by means of a synchronization field \( \tilde{k}(x;\beta) \) depending on the absolute value of the (constant) velocity \( \beta \), provided that such a field vanishes for vanishing \( \beta \). In this case, for any non null synchronization field we find that every IRF different from \( S_0 \) is not optically isotropic. This way \( S_0 \) enjoys the peculiarity of being the only optically isotropic IRF: a merely formal privileged status.

According to this conventional approach, it could be reasonable to require optical isotropy in the planes orthogonal to \( \beta \). If this condition is explicitly required, the synchronization field takes the value (18) and the Selleri synchronization gauge (21) is obtained. In order to get a useful description of the physical world, it is interesting to stress that the Selleri gauge (21) is the most general gauge which complies with the “hard experimental evidences” pointed out by Selleri himself: in particular, it is the most general gauge which complies with both the round-trip axiom and the standard Einstein expression of time dilation with respect to the IRF \( S_0 \).

All synchrony choices belonging to the Selleri gauge can be individualized by a suitable constant synchronization field \( \tilde{k}(\beta) \), depending on \( \beta \) only, which is related to the so-called “synchronization parameter” \( e_1(\beta) \). In particular, the synchrony choice \( e_1(\beta) = -\beta\gamma/c \) gives the standard Einstein synchronization, which is “relative”, i.e. frame-dependent. In this synchrony choice Eqns. (21) end up in the (symmetric) Lorentz transformations. Conversely, the synchrony choice \( e_1(\beta) = 0 \) gives the Selleri synchronization, which is ”absolute”, i.e. frame-independent; in this synchrony choice eqns. (21) end
According to Selleri, absolute synchronization is interpreted on the ground of a paradigm based on the actual existence of a physically privileged IRF (at the kinematical level), in agreement with the hypothesis of the stationary ether. In order to credit the stationary ether with some kind of physical meaning, it is necessary to reinterpret the Selleri synchronization gauge as a test-theory, and to look for some empirical evidence able to fix the synchronization parameter. Rotating reference frames are the tool used by Selleri in order to find out the secret synchrony choice of Nature.

According to SRT, provided the synchronization gauge is incorporated into the formalism, an absolute synchronization springs from a frame-invariant foliation of Minkowski spacetime, which is nothing but the Einstein foliation of the IRF $S_0$ assumed to be optically isotropic by stipulation. The geometrical structure of Minkowski spacetime is physically meaningful, but the way of foliating such a spacetime is a matter of convention: as a consequence, both Einstein’s relative simultaneity and Selleri’s “absolute” simultaneity can peacefully coexist in Minkowski spacetime as different conventions. There is no Nature’s synchrony choice, since any synchrony choice belonging to the Selleri gauge is consistent with any observable effect.

However, the criterium of “descriptive simplicity” singles out Einstein synchronization as the privileged one for a number of reasons in a great variety of circumstances. In fact, such a synchronization is the only one which permits to directly read physical properties of the physical world in the geometrical structure of Minkowskian spacetime, which allows for a drastic simplification of physical laws and of their symmetry properties (see Appendix A.2), which conforms to slow clocks transport, and so on. Moreover, if we are not led astray by some Lorentz-like ideological assumption, it is hard to deny that a synchrony choice which ensures optical isotropy is simpler than a synchrony choice which ensures optical anisotropy.

On the other hand, optical isotropy or anisotropy is not a physical property of a given IRF, but the combined result of observable physical properties of light propagation (the round-trip axiom) and of a conventional synchrony choice inside Selleri gauge. So we can look to anisotropies in the vacuum space (to be understood as dependences of the light velocity on the direction) as theoretical artifacts depending on the synchrony choice, which can be cancelled out by a proper resynchronization.

This does not mean at all that non-Einstein synchrony choices are a priori inconvenient. As a matter of fact, the most suitable formalism for
any specific problem is usually suggested by the problem itself. The fact that the standard formalism of SRT be only one of the possible legitimate choices, allows for a freedom which may be prove useful in many instances\textsuperscript{46}. On the other hand, in the relativistic literature one quite often encounters formalisms which do belong to Selleri gauge, sometimes without even a full awareness of the author himself. This has spurred some longstanding annoying consequences, namely ambiguities related to some concepts defined in not completely clear or satisfactory ways. An historical example is the issue of the velocity of light along the rim of a rotating platform\textsuperscript{47}, which has entailed ambiguous related consequences, especially concerning the theoretical interpretation of the Sagnac effect for counter-propagating light beams.

As pointed out by many authors, see \cite{37} and references therein, the global (round-trip) and the local speed of light along the rim do not agree in Einstein synchrony choice. This obvious fact, often unnoticed in the standard formalism of SRT, induces some authors \cite{15}, \cite{17} to the harebrained and stubborn belief that the SRT is incapable of plainly explaining the effect, unless it is not rigged with some proper ‘ad hoc corrections’: such a belief is a child of the previously mentioned ambiguities, namely of a rigid relativistic formalism which does not admit any synchrony choice different from the Einstein one. The advantage of the relativistic approach, associated with a suitable synchronization gauge, becomes apparent when not only light beams, but also matter beams are taken into account: in fact, in this case the extremely plain condition “equal relative velocity in opposite directions” unambiguously singles out the Einstein synchrony choice in any LCIF.

Last, but maybe not least, just a few words to defend ourselves from the charge of “anti-realistic conventionalism”. In this paper, as well as in some other papers \cite{19}, \cite{36}, we do not propose to cloud the hard reality of the physical world with a conventionalist fog; in particular, we do not agree with the extreme positivistic viewpoint according to which only what is measurable does exist.

Without getting involved in an ontological debate which would be, however, inappropriate in this context, we restrict to some remarks clarifying

\textsuperscript{46}Some situations susceptible of being effectively described by Selleri’s absolute simultaneity are pointed out in Ref. \cite{17} and in other Selleri’s papers.

\textsuperscript{47}cfr. f.i. Ref. \cite{48}, where the usual clarity in the presentation conceals an underlying ambiguity: the velocity of light is in fact evaluated locally in Einstein’s gauge choice but globally in Selleri’s gauge choice. A wholly legitimate twofold choice: but, maybe, somehw misleading, not being explicitly declared and especially evident at a first reading.
our point of view on the subject. The SRT is a beautiful axiomatic system, characterized by a remarkable conceptual simplicity, from which some observable facts can be logically derived. Of course the mathematical building incorporates a lot of conventional features, but the observables must be invariant with respect to the class of all the possible conventions allowed by the theory. This directly leads to the concept of gauge. This paper deals, in particular, with the synchronization gauges - with particular emphasis on the Selleri synchronization gauge - in order to separate what pertains to the physical world from what pertains to the conventional synchrony choice used to describe the world.

We point out that the actual measurement of whatever physical quantity depends on the setting of the experimental apparatus: different settings lead to different measurements, although the quantity to be measured is still the same. Of course this does not mean that the quantity under consideration is conventional: this simply means that the result of its measurement depends on some conventional assumptions fixing the setting\textsuperscript{48}. In particular, we do not think that the one-way speed of light is a meaningless concept because it is not measurable; we simply think that the lack of observability allows a multiplicity of conventional assumptions, encapsulated in some synchronization gauge, which are consistent with any possible experimental evidence.

As a matter of fact, all our empirical knowledge of the physical world is knowledge of observables; however, we find somehow naive to believe that the observables do exhaust the physical world. Yet, observables only define the horizon of the events to which we can have, at least in principle, empirical access. In such a context, the synchronization gauges, which have been the main characters of this work, are effective mathematical warnings, delimiting the objective bounds of our knowledge as experimental knowledge.

A Appendix

A.1 Transitivity of Selleri synchronizations: proof of Theorem 2

Theorem 2: the transitivity of Selleri synchronization procedures, for any value of the synchronization parameter $e_1(\beta)$, is fully equivalent to the round-
trip axiom:

transitivity of Selleri synchronizations ⇐⇒ round-trip axiom

Proof.

⇒ Taking into account Theorem 3, this side of the proof is immediate. Let us consider three clocks, lodged in points A, B and C, at rest in an IRF. Choosing an arbitrary synchronization parameter $e_1$, let us Selleri-synchronize, according to the operations outlined before, the clock in A with the clock in B and the clock in B with the clock in C. If such a synchronization procedure is transitive, then the clock in A should be synchronized with the clock in C, with the same synchronization parameter $e_1$. That is to say, along any segment of the triangle $ABC$, the velocity of light read by the three clocks will take the value $\bar{c}(\theta)$, given by (5). But, as we have shown in Sec. 4 (Theorem 3), the velocity of light $\bar{c}(\theta_{OA})$ given by (5) requires that the time of flight of a light pulse along the closed path $ABC$ must be $\tau_{ABC} = \frac{l_{ABC}}{c}$, according to the round-trip axiom: transitivity of Selleri’s synchronizations implies therefore the round-trip axiom.

⇐ Let us now assume the validity of the round-trip axiom, that is let us assume that the light take a time interval $\tau_{ABC} = \frac{l_{ABC}}{c}$ to perform a round-trip of the triangle $ABC$ of length $l_{ABC}$. Let us then synchronize, choosing an arbitrary $e_1$, the clock in A with the clock in B and the clock in B with the clock in C: we want to show that, as a result, the clock in C gets synchronized with the clock in A, that is to say that the performed synchronization procedure is transitive. The two synchronization operations performed stipulate, to adopt a term preferred by Selleri himself, that the one-way velocity of light along the paths $AB$ and $BC$ takes the form (5). The round-trip axiom itself provides us with the time of flight of the signal along $ABC$. Therefore, we can easily compute the time of flight read by the clocks in C and A, during the propagation of light along the (one-way) path $CA$:

$$t_{CA} = \frac{l_{ABC}}{c} - \frac{l_{AB}(1 + \Gamma(e_1) \cos \theta_{AB})}{c} - \frac{l_{BC}(1 + \Gamma(e_1) \cos \theta_{BC})}{c} = \frac{l_{AB}}{c} + \frac{l_{BC}}{c} + \frac{l_{CA}}{c} - \frac{l_{AB}(1 + \Gamma(e_1) \cos \theta_{AB})}{c} - \frac{l_{BC}(1 + \Gamma(e_1) \cos \theta_{BC})}{c} = \frac{l_{CA} - l_{AB}\Gamma(e_1) \cos \theta_{AB} - l_{BC}\Gamma(e_1) \cos \theta_{BC}}{c} = \frac{l_{CA}(1 + \Gamma(e_1) \cos \theta_{CA})}{c}.$$  (33)
This time of flight, as it is plain to see, is just the one predicted by Eq. (5), so that the clock in $C$ is indeed synchronized with the clock in $A$, with synchronization parameter $e_1$. □

### A.2 Electromagnetism in Selleri gauge

To better illustrate Selleri’s formalism and to clarify the description of the optical properties of inertial frames in such a formalism, we provide here the expression of Maxwell equations in a generic IRF $S$ moving with respect to $S_0$ with velocity $v \equiv \beta \hat{v}$ (where $\hat{v} \equiv v/v$), under a generic synchrony choice inside the Selleri synchronization gauge (4). Furthermore, we solve the wave equation and find a generally anisotropic one-way propagation velocity of light, which turns out to be in full agreement with eq. (5).

We will skip the details of the derivation, which can be promptly verified by invoking the covariance of Maxwell equations under synchronization changes. For the sake of simplicity we introduce the dimensionless vector field $\tilde{\kappa}$ which, in terms of the vector field $\tilde{k}$ of Eq. (18), reads $\tilde{\kappa} = \nabla (c \tilde{k} \cdot r)$ (where $r$ stands for the three-dimensional position vector). In terms of the parameter $\Gamma$, one has $\tilde{\kappa} = -\Gamma (\beta) \hat{v}$. Of course this includes, as particular cases, both the Einstein vector field $\tilde{\kappa}_E = 0$ and the Selleri vector field $\tilde{\kappa}_S = -\beta \hat{v}$.

The current density 4-vector $J^\mu$ is transformed as follows under resynchronization

$$J^\mu = (\rho, J) \mapsto \tilde{J}^\mu = (\rho + \tilde{\kappa} \cdot j, j) \equiv (\tilde{\rho}, \tilde{j}). \quad (34)$$

This equation deserves a comment: what is seen, in Einstein synchronization, as a current density with null charge density (occurring whenever moving charges balance the stationary ones) appears, in a more general synchronization, as a current density with non null charge density

$$\tilde{\rho} = \tilde{\kappa} \cdot j \quad (35)$$

This somehow counterintuitive feature is due to the fact that spatially separated time measurements, necessary to properly collect the moving charges in a given space, are crucially synchrony dependent.

Considering the transformation of the electromagnetic tensor $F^{\mu\nu}$ one gets the transformation of the electric and magnetic fields

$$\tilde{E} = E + c \tilde{\kappa} \times B, \quad \tilde{B} = B. \quad (36)$$

47
in whose terms we write Maxwell equations

\[ \tilde{\nabla} \cdot \tilde{\mathbf{E}} = \tilde{\rho}/\varepsilon_0 , \]  
(37)

\[ \tilde{\nabla} \times \tilde{\mathbf{B}} = \mu_0 \tilde{\mathbf{J}} + \frac{1}{c^2} \frac{\partial \tilde{\mathbf{E}}}{\partial \tilde{t}} , \]  
(38)

\[ \tilde{\nabla} \cdot \tilde{\mathbf{B}} - \frac{1}{c} \tilde{\mathbf{\kappa}} \cdot \frac{\partial \tilde{\mathbf{B}}}{\partial \tilde{t}} = 0 , \]  
(39)

\[ \tilde{\nabla} \times \tilde{\mathbf{E}} - \frac{1}{c} \tilde{\mathbf{\kappa}} \times \frac{\partial \tilde{\mathbf{E}}}{\partial \tilde{t}} = (|\tilde{\mathbf{\kappa}}|^2 - 1) \frac{\partial \tilde{\mathbf{B}}}{\partial \tilde{t}} - c(\tilde{\mathbf{\kappa}} \cdot \tilde{\nabla}) \tilde{\mathbf{B}} . \]  
(40)

Above, the operator \( \tilde{\nabla} \) stands for derivation with respect to the resynchronized variables \( \tilde{t} = t - \tilde{\mathbf{\kappa}} \cdot \mathbf{r} \), \( \tilde{x} = x \), \( \tilde{y} = y \) and \( \tilde{z} = z \), with \( \mathbf{r} \equiv (x, y, z) \).

We find now a wave solution in the vacuum of Maxwell equations (37-40), explicitly showing that they yield a velocity of electromagnetic signals in agreement with (5).

Computing \( \tilde{\nabla} \times \tilde{\nabla} \times \tilde{\mathbf{E}} \) and exploiting (37), (38), (40) and \( \tilde{\rho} = 0 \), one gets the following equation for \( \tilde{\mathbf{E}} \):\(^{49}\)

\[ \tilde{\nabla}^2 \tilde{\mathbf{E}} - \frac{1}{c^2} \frac{\partial^2 \tilde{\mathbf{E}}}{\partial \tilde{t}^2} - \frac{2}{c} (\tilde{\mathbf{\kappa}} \cdot \tilde{\nabla}) \frac{\partial \tilde{\mathbf{E}}}{\partial \tilde{t}} + \frac{|\tilde{\mathbf{\kappa}}|^2}{c^2} \frac{\partial^2 \tilde{\mathbf{E}}}{\partial \tilde{t}^2} = 0 . \]  
(41)

Let \( \phi \) be the Fourier transform of \( \tilde{\mathbf{E}} \)

\[ \tilde{\mathbf{E}} = \int \phi(\tilde{\omega}, \tilde{\mathbf{k}}) e^{i(\tilde{\mathbf{k}} \cdot \tilde{x} - \tilde{\omega} \tilde{t})} d\tilde{\omega} d\tilde{x} . \]  
(42)

Applying Eq. (41) to \( \phi \) allows to recover the dispersion relation in Selleri gauge

\[ |\mathbf{k}|^2 - \frac{\tilde{\omega}^2}{c^2} + 2 \frac{\tilde{\omega}}{c} \tilde{\mathbf{\kappa}} \cdot \mathbf{k} + \frac{|\tilde{\mathbf{\kappa}}|^2 \tilde{\omega}^2}{c^2} = 0 . \]  
(43)

By performing the change of variables \( \omega = \tilde{\omega} \) and \( \mathbf{k} = \tilde{\mathbf{k}} + \tilde{\omega} \tilde{\mathbf{\kappa}}/c \)\(^{43}\) turns into the well known

\[ \omega = |\mathbf{k}|c . \]  
(44)

Actually, \( \mathbf{k} \) and \( \omega \) are just the wave vector and the frequency in Einstein synchronization, for which the usual dispersion relation in vacuum holds.

\(^{49}\)The same equation is achieved, in absence of currents, for \( \tilde{\mathbf{B}} \) exploiting (38), (39), (40).
Let us now consider a monocromatic pulse propagating in \( S \) along the spatial direction \( \hat{n} \). One has \( k = |k| \hat{n} \). Moreover, Eq. (42) clearly shows that the resynchronized velocity \( \tilde{c} (\hat{n}) \) along the direction \( \hat{n} \) is given by \( \tilde{c} = \tilde{\omega} / k \cdot \hat{n} \). Thus, recalling Eq. (44) and the previous change of variables one obtains

\[
\tilde{c} = \frac{\tilde{\omega}}{k \cdot \hat{n}} = \frac{\omega}{k \cdot n - \omega \kappa \cdot n/c} = \frac{c}{1 - \kappa \cdot \hat{n}} = \frac{c}{1 + \Gamma \cos \vartheta},
\]

(45)

where we have employed the definition of \( \kappa \) and \( \vartheta \) (the latter being the angle between the velocity \( v \) of \( S \) with respect to \( S_0 \) and the direction of propagation of the signal \( \hat{n} \)).

A few comments are in order.

(i) As one should expect, the generally anisotropic speed of light (5) is recovered; in particular, we recover the isotropic propagation of light in Einstein synchrony choice \( (\Gamma = 0) \) and the anisotropic propagation of light in Selleri’s synchrony choice \( (\Gamma = \beta) \).

(ii) The Maxwell equations (37), (38), (39), (40) take their beautiful standard symmetric form in Einstein synchrony choice \( (\Gamma = 0 \Rightarrow \kappa = 0) \); on the contrary, in Selleri’s synchrony choice they maintain the unpleasant asymmetric form (37), (38), (39), (40) with \( \kappa = \beta \). Briefly, Maxwell equations are covariant under synchronization changes, but optical anisotropy breaks their standard symmetric form: another unavoidable consequence of Selleri’s inertial transformations.

Acknowledgments

According to T. W. Adorno what ultimately matters in a piece of work, either artistic, philosophic or scientific, are not the subjective intentions of the author, but rather the “objectivity” which the work itself is capable of achieve. Such an objectivity makes the work speak by its own content, often at variance with the particular aims and intentions which gave birth to it. This is especially true, in our opinion, for the scientific work of Franco Selleri. Through a free and bright intellectual path, Selleri reaches a new theory which, according to his subjective purposes, appears to be alternative to SRT. In our opinion the result is not the one expected by the author, but we believe it is not of least importance because new independent light is thrown on SRT. As a consequence, a deeper understanding of SRT is allowed, and a suitable reformulation of the theory is required. We would like to thank Franco Selleri for this important, stimulating contribution, while wishing him all the best for his seventieth birthday.
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Figure 1: Round-trip of the pulse.
Figure 2: The Einstein synchrony choice $e_1 = -\beta \gamma/c$ entails a foliation of Minkowski spacetime depending on the considered IRF, and made of "Minkowski-orthogonal" hypersurfaces (straight lines) $t = \text{const}$ with respect to the wordlines of the test-particles at rest in the IRF (straight lines) $x = \text{const}$. On the contrary, the re-synchronization of the IRF with the Selleri synchrony choice $e_1 = 0$ entails a frame-invariant foliation (straight lines) $\tilde{t} = \text{const}$, i.e. $t_o = \text{const}$. 
Figure 3: Three satellites GPS system.
Figure 4: Round-trip of the pulse.

Figure 5: The Einstein synchrony choice $e_1 = -\beta \gamma / c$ entails a foliation of Minkowski spacetime depending on the considered IRF, and made of "Minkowski-orthogonal" hypersurfaces (straight lines) $t = \text{const}$ with respect to the wordlines of the test-particles at rest in the IRF (straight lines) $x = \text{const}$. On the contrary, the re-synchronization of the IRF with the Selleri synchrony choice $e_1 = 0$ entails a frame-invariant foliation (straight lines) $\tilde{t} = \text{const}$, i.e. $t_o = \text{const}$.

Figure 6: Three satellites GPS system.