Connecting Pygmy Dipole Resonance to neutron skin

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We study the correlation between the neutron skin development and the low-energy dipole response associated with the Pygmy Dipole Resonance (PDR) in connection with the properties of symmetry energy. We perform our investigation within a microscopic transport model based on Landau-Vlasov kinetic equation by employing three different equations of state in the isovector sector. Together with Giant Dipole Resonance (GDR), for all studied systems, we identify a PDR collective mode whose energy centroid is very well described by the parametrization \(E_{PDR} = 41A^{-1/3}\). A linear correlation between the Energy Weighted Sum Rule (EWSR) associated to PDR and the neutron skin thickness is evidenced. An increase of 15\(MeV/fm^2\) of EWSR to a change of 0.1\(fm\) of neutron skin size is obtained. We conjecture that different nuclei having close neutron skin size will exhaust the same EWSR in the pygmy region. This suggests that a precise experimental estimate of total EWSR exhausted by PDR allows the determination of the neutron skin size and to constrain the slope parameter of the symmetry energy.

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The nuclear symmetry energy, which originates from both Pauli correlations and the specific features of nuclear forces, accounts for the effects which emerge as a result of the difference between the number of protons \(Z\) and neutrons \(N\) of the system. It appears in the expression of total energy per particle, \(E/N = E(\rho, I) = E\rho/A + E_{sym}(\rho)I^2\), factorizing the isospin parameter \(I = (N - Z)/A\), where \(\rho\) is the nucleons density. Several features of atomic nuclei [1, 2] and neutron stars [3] are determined by this quantity and one of the major tasks of recent experimental and theoretical investigations is to determine a consistent density parametrization of the symmetry energy which can provide a unified picture of nuclear properties below saturation as well as at large compression of asymmetric nuclear matter.

The fragmentation facilities at GANIL, GSI, MSU and RIKEN, allowing for the study of very neutron rich systems, stimulated new investigations along this direction. In this context, understanding the exotic modes of excitation [4] and the role of the neutron skin on the collective dynamics in nuclei far from stability is a challenge in modern nuclear physics [5, 7]. Indeed, several experiments performed during the last ten years reported the occurrence of an electric dipole (E1) response well below the GDR, more clear evidenced in neutron rich nuclei [8, 14], see [15, 16] for recent overviews. It manifests as a resonant-like shape exhausting few percentages of dipolar EWSR and its controversial nature attracted a considerable interest for theory too [17]. PDR was interpreted as a collective motion in phenomenological, hydrodynamic descriptions [18], in non-relativistic [13, 20] or relativistic microscopic approaches [21] and in microscopic transport models [22, 24]. Other studies, however, associate the concentration of strength to the contributions from single-particle type excitations excluding coherent, collective properties [22, 24]. It is possible that in the low-energy region the dipolar response manifests both single-particle and collective features. Moreover, a fragmentation of the E1 response is expected to determine a weakening of the collectivity [27, 28].

The goal of this Letter is to address the connection between the development of neutron skin and the emergence of a low-energy E1 response in relation with the symmetry energy dependence on density, a subject under intense debate during the last years. Because the neutron skin is an isovector indicator we employ three different parametrisations with the density of the symmetry term and perform a comparative study in a model based on the Landau theory of Fermi liquids where the dynamics of the nucleons is described by Landau-Vlasov kinetic equation. Since, as in the case of GDR, the evolution with mass of the low-energy E1 response provides an additional insight upon the nature of the mode, we shall consider the systems \(^{48}\text{Ca},^{68}\text{Ni},^{86}\text{Kr}\) and \(^{208}\text{Pb}\), as well as a chain of Sn isotopes, \(^{108,116,124,132,140}\text{Sn}\).

A promising method aiming to clarify the nature of PDR as well as the role of symmetry energy and the neutron skin is based on a systematic analysis of the influence of the neutron excess on the observables as the energy centroid or the low-energy E1 strength. Such experimental studies include isotopes of \(^{48}\text{Ca}\), \(^{68}\text{Ni}\), \(^{86}\text{Kr}\) and \(^{208}\text{Pb}\), as well as a chain of Sn isotopes, \(^{108,116,124,132,140}\text{Sn}\). From the measurements for stable Sn isotopes [34, 37] and neutron rich systems \(^{120-132}\text{Sn},^{133,134}\text{Sb}\) [19], a trend of strength increasing with the neutron-proton asymmetry \(I^2\) was reported. A threshold value of the isospin \(I\), beyond which a sizable fraction of the pygmy strength appears, was
neutron skin thickness of $^{120}\text{Sn}$ to $^{152}\text{Sn}$ where a concentration of E1 strength was evidenced between 6 and 8MeV, it was stressed that these non-collective states cannot be considered a low-energy tail of GDR, having a genuine character with a dominance of neutron excitations. The evolution of the strength distribution and energy location was closely related to the features of neutron mantle enclosing the more isospin symmetric core. Pickarewicz [40] raised the important question if a strong correlation between the neutron skin and the low-energy E1 strength can be distinguished. For $^{208}\text{Sn}$ isotopes, within a relativistic random-phase approximation (RPA), he concluded that the fraction of EWSR acquired in the energy region between 5MeV and 10MeV manifests a linear dependence with the neutron skin size up to mass $A=120$ followed by a mild anti-correlation. However, such strong correlation was questioned by Reinhard and Nazarewicz [11]. They introduced an investigation based on a co-variation analysis aimed to identify a set of good indicators that correlate very well with the isovector properties and suggested that the low-energy E1 strength is very weakly correlated with the neutron skin while the dipolar polarizability should be a much stronger indicator of isovector properties. This intriguing finding was challenged recently [42] in a relativistic Random-Phase-Approximation (RPA) approach with mixed results. It was indeed reported a strong correlation between the neutron skin thickness of $^{208}\text{Pb}$ and the dipole polarizability of $^{68}\text{Ni}$. But a strong correlation was also claimed between skin thickness of $^{208}\text{Pb}$ and low-energy E1 features, including the strength and dipole polarizability associated to pygmy mode, identified in $^{208}\text{Ni}$ as exhausting about $5\% - 8\%$ of EWSR.

Here we shall address these controversial issues proposing an investigation based on a semi-classical transport model. While the model is unable to account for effects associated with the shell structure, our self-consistent approach is suitable to describe robust quantum modes, of zero-sound type, in both nuclear matter and finite nuclei. It provides important informations about the dynamics of such collective modes, allowing for a systematic study over an extended mass and isospin domain. Such studies were inquiring already on the collective nature of PDR [22, 13] and on the role of the symmetry energy on its dynamics [23]. It was noticed that for $^{132}\text{Sn}$ the symmetry energy does not affects the PDR energy centroid but it influences the EWSR acquired by it. First we explore the properties of neutron skin and its sensitivity to the density dependence of symmetry energy. Then we determine the E1 strength function and study the mass dependence of the dipole polarizabilities and of the PDR peak. Finally we estimate the EWSR exhausted by the PDR and discuss its relation with the neutron skin thickness.

The two coupled Landau-Vlasov kinetic equations for neutrons and protons

\[
\frac{\partial f_n}{\partial t} + \frac{\mathbf{p}}{m} \frac{\partial f_n}{\partial \mathbf{r}} - \frac{\partial U_q}{\partial \mathbf{r}} \frac{\partial f_n}{\partial \mathbf{p}} = I_{\text{coll}}[f],
\]


determine the time evolution of the one-body distribution functions $f_n(r, p, t)$, with $q = n, p$. In the following we shall switch-off the collision integral but we have tested that the results are not strongly influenced, as expected, when it is included. For the nuclear mean-field we consider a Skyrme-like ($SKM^*$) parametrization $U_q = A \frac{\rho}{\rho_0} + B (\frac{\rho}{\rho_0})^{\alpha+1} + C (\rho) (\rho_n - \rho_p)^2 \frac{\partial C}{\partial \rho} (\rho_n - \rho_p)^2, r_n(r_p) = +1(-1)$. The saturation properties of the symmetric nuclear matter, $\rho_0 = 0.16 fm^{-3}, E/A = -16 MeV$ and a compressibility modulus $K = 200 MeV$, are reproduced if are fixed the values $A = -356 MeV$, $B = 303 MeV, \alpha = 1/6$. From the one-body distribution functions one obtains the local densities: $\rho_q(r,t) = \int \frac{2d^3p}{(2\pi\hbar)^3} f_q(r,p,t)$ as well as the quadratic radii $\left< r^2_q \right> = \frac{1}{N_q} \int r^2 \rho_q(r,t) d^3r$ and the width of the neutrons skin $\Delta R_{np} = \sqrt{\left< r^2_n \right> - \left< r^2_p \right>}$.

The symmetry energy dependence of $\Delta R_{np}$ is studied by considering in the mean-field structure different parametrizations with the density of $C(\rho)$. For the asymmetric case a linear dependence with density is adopted. Then the symmetry energy $E_{\text{sym}}/A = \frac{e_F}{3} + \frac{C(\rho)}{2} \rho / \rho_0$ at saturation takes the value $E_{\text{sym}}/A = 30 MeV$ while the slope parameter $L = 3 \rho_0 \frac{d E_{\text{sym}}/A}{d \rho}|_{\rho=\rho_0}$ is $L = 72 MeV$.

The asysuperstiff/asysoft EOS correspond to a variation faster/slower than the linear case one with the density around saturation point, with the associated slope parameters taking values $L = 96 MeV/L = 14 MeV$ [11]. The integral of the transport equations is based on the test-particle (t.p.) method, with a number of 1300 t.p. per nucleon in the case of Sn isotopes, ensuring in this way a good spanning of the phase-space.

An efficient method to extract the values of $R_n$ and $R_p$ is by observing their time evolution after a gentle perturbation. Both quantities perform small oscillations around equilibrium values and we remark that the numerical simulations keep a very good stability of the dynamics for at least 2000 fm/c. Using this procedure we obtain for the charge mean square radius of $^{208}\text{Pb}$ a value around $R_n = 5.45 fm$, to be compared with the experimental value $R_{n,\text{exp}} = 5.50 fm$. For Sn isotopes we display the mass dependence of $R_n, R_p$ in Fig. 1 (a) and of $\Delta R_{np}$ respectively in Fig. 1 (b). For the charge radii the predictions from the three asy-EOS virtually coincide and we notice a good agreement with the experimental data reported in [44]. For all adopted parametrizations the predicted values of the neutron skin thickness are within the experimental errors bars, see the data presented in [45] for the stable Sn nuclei. In the case of $^{208}\text{Pb}$ we find $\Delta R_{np} = 0.19 fm$ for asysoft, $\Delta R_{np} = 0.25 fm$ for asystiff and $\Delta R_{np} = 0.27 fm$ for asysuperstiff EOS while for $^{68}\text{Ni}$
the corresponding values are $\Delta R_{np} = 0.17, 0.19, 0.20$ fm. We see that the neutron skin thickness increases with the slope parameter $L$, an effect related to the tendency of the system to stay more isospin symmetric even at lower densities when the symmetry energy changes slowly below saturation, as is the case for asy-soft EOS.

We study the E1 response considering a GDR-like initial condition [23], determined by the instantaneous excitation $V_{ext} = \eta (t - t_0) \hat{D}$ at $t = t_0$ [10]. This situation corresponds to a boost of all neutrons against all protons while keeping the Center of Mass (CM) at rest. Here $\hat{D}$ is the dipole operator. If $|\Phi_0\rangle$ is the state before perturbation then the excited state becomes $|\Phi(t_0)\rangle = e^{i\eta \hat{D}}|\Phi_0\rangle$ and the value of $\eta$ can be related to the initial expectation value of the collective dipole momentum $\hat{P}$, $(\Phi(t_0) | \hat{P} | \Phi(t_0) ) = \hbar N Z \eta / A$. Here $\hat{P}$ is canonically conjugated to the collective coordinate $\hat{X}$ which defines the distance between the CM of protons and the CM of neutrons, i.e. $[X, \hat{P}] = i \hbar$ [43]. Then the strength function $S(E) = \sum_{|n| < 0} |\langle n|\hat{D}|0\rangle|^2 \delta(E_n - E_0)$, directly related to the excitation probability in unit time, where $E_n$ are the excitation energies of the states $|n\rangle$ while $E_0$ is the energy of the ground state $|0\rangle = |\Phi_0\rangle$, is obtained from the imaginary part of the Fourier transform of the time-dependent expectation value of the dipole momentum $D(t) = \frac{NZ}{A} X(t) = (\Phi(t) | \hat{D} | \Phi(t) )$ as $S(E) = \frac{Im(D(\omega))}{\pi \hbar}$ where $D(\omega) = \int_{t_0}^{t_{max}} D(t)e^{i\omega t} dt$. We consider the initial perturbation along the z-axis and follow the dynamics of the system until $t_{max} = 1830 \text{fm}/c$ in each case. At $t = t_0 = 300 \text{fm}/c$ we extract the collective momentum and determine $\eta$. A filtering procedure, as described in [47], was applied in order to eliminate the artifacts resulting from a finite time domain analysis of the signal. A smooth cut-off function was introduced such that $D(t) \rightarrow D(t) \cos^2(\frac{t - t_{max}}{t_{max}})$. For the three asy-EOS the E1 strength functions of $^{208} \text{Pb}$ and $^{140} \text{Sn}$ are represented in Fig. 2. As a test of the quality of our method we compared the numerically estimated value of the first moment $m_1 = \int_0^\infty ES(E)dE$ with the value predicted by the Thomas-Runkle-Kuhn (TRK) sum rule $m_1 = \frac{E_{PDR}}{N Z}$. In all cases the difference was of only few percentages. The energy peak of PDR for $^{208} \text{Pb}$, see Fig. 2(a), is located around $7 - 7.5 \text{MeV}$ in good agreement with recent experimental data which indicate $E_{PDR,Pb} = 7.36 \text{MeV}$ [13]. For $^{68} \text{Ni}$ we obtain $9.8 \text{MeV}$, quite close to the recent reported data $E_{PDR,Ni} = 9.9 \text{MeV}$ [48]. We observe that GDR energy centroid is underestimated in comparison with experimental data, a fact related with the choice of the interaction which has not an effective mass [49]. In any case, a clear dependence with the slope parameter manifests as a consequence of the isovector nature of the mode. This feature shows that also the symmetry energy values below saturation are affecting the dipole oscillations of the finite systems. The Fig. 3 displays the position of the PDR energy centroid as a function of mass for all studied systems (blue circles). In addition, we represent the position of the PDR energy peaks as results from the power spectrum analysis of the pygmy dipole $D_p(t)$ after a pygmy-like initial condition, see [23] (red diamonds) and the experimental data available from the works where information about the position of the low-energy E1 centroid was reported (maroon square) [54]. The differences between the two methods are within a half of MeV. An appropriate parametrization, obtained from the fit of numerical results is $E_{PDR} = 41 A^{-1/3}$, quite close to what is expected in the harmonic oscillator shell model (HOSM) approach [43] and in agreement with some recent experimental data. While the isovector residual interaction pushes up the value of the GDR energy it seems that the PDR energy centroid is not much affected by this part of the interaction. This feature may explain the better agreement with experimental observations in comparison with the GDR case. Let us mention that for Ni, Sn and Pb isotopic chains, based on a HFB and RQRPA treatment, Paar et al. [59] studied the isotopic dependence of PDR energy and a collective mode with the energy centroid around $10 \text{MeV}$ for $^{68} \text{Ni}$, $8 \text{MeV}$ for $^{132} \text{Sn}$ and $7.5 \text{MeV}$ for $^{208} \text{Pb}$ was also predicted. A
The experimental data from Ref. \[54\].

A (blue) and 42

\[\alpha\]
calculate the nuclear dipole polarizability between the two theoretical approaches. The comparison with our results shows a good concordance and asystiff (black, circles) EOS. All systems mentioned of mass for asysoft (red, triangles), asystiff (blue, squares).

The dipole polarizability is between 4

\[\text{fm}^3\]

The variation with the slope parameter \(L\) seems to be the greater value of the dipole polarizability is obtained. a given system, the larger is the neutron skin thickness

\[m_{1,y}(\text{fm})\]

The dependence of the moment \(m_{1,y}\) on the neutron skin thickness is shown in Fig. 5 where the informations concerning all mentioned systems for the three asy-EOS were included. While below 0.15 \(fm\) the EWSR acquired by PDR manifests a saturation tendency, above this value a linear correlation clearly manifests. For the same system, when we pass from asysuperstiff to asysoft parametrisation, the neutron-skin shrinks and correspondingly the value of \(m_{1,y}\) decreases. This behavior is in agreement with the results reported in \[50\] in a self-consistent RPA approximation based on a relativistic energy density functionals. Moreover we notice that the variation rate appears to be system independent, obtaining an increase of 15 \(MeV fm^2\) of the exhausted EWSR to a change of 0.1 \(fm\) of the neutron skin width. Such features suggest that the acquired EWSR should not differ too much even for different nuclei if they have close values of neutrons skin thickness. We would like to mention that these findings look qualitatively in agreement with those of Inakura \[51\], based on systematic calculations within a RPA with Skyrme functional SkM* treatment, where a linear correlation of the fraction of EWSR exhausted in the low-energy region was evidenced for several isotopic chains.

However some differences are also worthwhile to be mentioned. As in \[40\] \[52\] a mild anti-correlation for very neutron rich systems, i.e. thick neutron skins, was observed, a feature which is missing in our model. At variance, we remark a continuous rise of \(m_{1,y}\) with the neutron skin size, in concordance with other studies based on microscopic treatments \[48\] \[54\]. One can relate these differences to some shell and angular momentum effects but further investigations are required for a definite answer.

In conclusion, we addressed some open questions raised recently \[16\] regarding the nature of PDR. By performing a systematic analysis which includes several nuclear systems we obtained new results aiming to contribute to

\[m_{1,y} = \int_{PDR} ES(E)dE\]  

(1)
a more complete picture of PDR dynamics. In a microscopic transport approach, a low-energy dipole collective mode was evidenced as an ubiquitous feature of all investigated nuclei. The analysis leads us to a dependence of the PRD energy centroid with mass described by $E_{PDR} = 41A^{-1/3}$, in agreement with several recent experimental informations. This indicates a close connection with the characteristic frequency of the HOSM, $\omega_0 = 41A^{-1/3}$, and a weak influence of the residual interaction in the isovector sector. Such behavior can be related to the isoscalar-like nature of this mode. From our calculations an universal, linear correlation of the EWSR exhausted by PDR with the neutron skin thickness occurs. It appears as a very specific signature, showing that the neutrons which belong to the skin play an essential role in shaping the E1 response in the PDR region.

However this fact should not lead to an oversimplified picture of PDR as corresponding to the oscillations of excess neutrons against the isospin symmetric core. Within the same transport model the dynamical simulations show a more complex structure of PDR. We consider that the new findings presented here can be useful for further, systematic experiments searching for this, quite elusive, mode. A precise estimate of EWSR acquired by PDR can provide indications about the neutron skin size which in turn will add more constraints on the slope parameter $L$ of the symmetry energy.

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