A Note on the Green-Schwarz Mechanism in Open-String Theories

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ABSTRACT

An interesting feature of some open superstring models in $D < 10$ is the simultaneous presence, in the spectrum, of gauge fields and of a number of antisymmetric tensor fields. In these cases the Green-Schwarz mechanism can (and does) take a generalized form, resulting from the combined action of all the antisymmetric tensors. These novelties are illustrated referring to some simple rational models in six dimensions, and some of their implications for the low-energy effective field theory are pointed out.
1. Introduction

Whereas the Green-Schwarz mechanism was originally discovered in the type-I superstring theory in ten dimensions \cite{1}, the construction of the heterotic string \cite{2} soon led to a widespread interest in its applications to models of oriented closed strings. These studies uncovered the deep relation that, in this case, holds between the cancellation of anomalies and the geometric property of modular invariance. A fair, if somewhat crude, way to summarize these findings is by observing that, in theories of oriented closed strings, modular invariance removes the ultraviolet region altogether, thus disposing of all anomalies at once.

The results obtained for models of oriented closed strings, however, have no direct bearing on the nature of the corresponding phenomenon in open-string theories. To wit, in this latter case modular invariance is not a property of all amplitudes. Rather, the consistency of open-string models results from delicate cancellations taking place between some of the amplitudes. In particular, the analysis of the ten-dimensional gauge anomaly of ref. [1] involves three types of contributions (fig. 1). The first two describe, respectively, the emission of six vectors from one of the boundaries of the annulus and from the single boundary of the Möbius strip. Thus, they determine the total irreducible contribution to the anomaly polynomial, proportional in this case to Tr(F^6). As pointed out in ref. [1], this contribution cancels precisely if the gauge group is SO(32). The nature of the cancellation was nicely illustrated in ref.\cite{3}: if n, the order of the SO group, differs from 32, there exists a spurious mode that, though projected out of the closed string spectrum via the Klein-bottle diagram, couples to the vacuum channel of both annulus and Möbius diagrams. The presence of non-vanishing irreducible parts of the anomaly polynomial may then be linked to a vacuum expectation value of this mode at “genus-one-half”, resulting from the contributions of the disk and the projective plane, by arguments similar to those presented in ref.\cite{4}. It should be appreciated that genus-one vacuum amplitudes capture the whole essence of the phenomenon,
since the irreducible parts of the anomalies draw their origin from the region of
coaescence of all vertices, where only the limiting behavior with respect to the
surface moduli matters. This is true for ultraviolet divergences as well, both in the
$SO(32)$ superstring\cite{5} and in the $SO(8192)$ bosonic string\cite{6}.

Out of the three diagrams in fig. 1, the third (the “non-planar” diagram)
is actually the most important one, since it is the home of the Green-Schwarz
mechanism. Indeed, being regulated by the momentum flow along the tube, it
does not contribute to the anomaly, a result that, in the limiting field theory, may
be ascribed \cite{1} to a cancellation induced by new couplings of the antisymmetric
tensor.

The purpose of the present note is to illustrate some peculiar features of the
anomaly cancellation mechanism in open-string theories. To this end, we shall refer
to one class of six-dimensional models introduced in refs.\cite{7} and \cite{8} where, as we shall
see, the Green-Schwarz mechanism is at work in its full-fledged form. This class
of open-string models is itself rather unconventional, the only available derivation
being \cite{7} \cite{8}, as far as I know, one that starts from the “parent” closed string, as
suggested in ref.\cite{9}. Thus, our original motivation was to test to a finer degree the
consistency of these models. The novelties may be related to the existence of a
number of independent sectors of the spectrum, and to the corresponding presence
of a number of antisymmetric tensors interacting with gauge fields \cite{7} \cite{8}. The very
presence of (anti)self-dual tensors coming from Ramond-Ramond sectors, noted
in refs. \cite{7} \cite{8}, is already a distinctive mark with respect to heterotic models.
Even more interesting, as we shall see, is their role in the anomaly cancellation
mechanism.
2. The Generalized Green-Schwarz Mechanism

The nature of the problem is well illustrated by referring to the first class of rational open-string models of ref. [8], whose notation we adopt. These models are chiral and supersymmetric in a six-dimensional space time. In this case a net total of four self-dual antisymmetric tensors survive the Klein bottle projection \(^\star\) of the closed string. Moreover, the corresponding open strings may have sixteen different sectors. Their sixteen charge sectors are constrained by the six tadpole conditions

\[
\sum_{i=1}^{8} n_i = \sum_{i=1}^{8} \tilde{n}_i = 16 ;
\]

\[
n_5 - n_1 + \tilde{n}_1 + \tilde{n}_6 + \tilde{n}_7 + \tilde{n}_8 = 8 ;
\]

\[
n_6 - n_2 + \tilde{n}_2 + \tilde{n}_5 + \tilde{n}_7 + \tilde{n}_8 = 8 ;
\]

\[
n_7 - n_3 + \tilde{n}_3 + \tilde{n}_5 + \tilde{n}_6 + \tilde{n}_8 = 8 ;
\]

\[
n_8 - n_4 + \tilde{n}_4 + \tilde{n}_5 + \tilde{n}_6 + \tilde{n}_7 = 8 .
\]

Still, they can accommodate non-trivial symplectic gauge groups, such as \(USp(8)^\otimes 4\), with chiral fermions in the representations \((8,8,1,1),(8,1,8,1),(1,8,1,8)\) and \((1,1,8,8)\). In addition to the scalar multiplets containing these fermions, the massless spectrum includes the supergravity multiplet, five tensor multiplets and sixteen scalar multiplets from the closed sector, as well as the gauge multiplet from the open sector. For instance, to build a model with a \(USp(8)^\otimes 4\) gauge group one may choose \(n_1 = n_2 = \tilde{n}_7 = \tilde{n}_8 = 8\), thus correcting a typographical error in ref. [8]. This class of models therefore describes the interactions of a number of antisymmetric tensors with non-trivial gauge fields and chiral matter, precisely what we are after.

\(^\star\) The sector \((1)\) contributes one antiself-dual tensor, while each of the sectors \((5), (\tilde{1}), (\tilde{6}), (\tilde{7})\) and \((8)\) contributes one self-dual tensor.
In order to proceed further, we would like to construct the anomaly polynomial for this case. Since all fermions are in fundamental or symmetric tensor representations of $Usp$ gauge groups, the polynomial may be reduced to a standard form using only a few simple trace identities. First of all, for fermions in the adjoint representation one finds

$$\text{Tr}F^2 = (n + 2) \text{tr}F^2 \quad (2.2)$$

and

$$\text{Tr}F^4 = (n + 8) \text{tr}F^4 + 3 (\text{tr}F^2)^2 \quad (2.3)$$

where, as usual, tr denotes the trace in the fundamental representation, and where the two-form $F = \frac{1}{2} F_{\mu\nu} dx^\mu dx^\nu$. On the other hand, for fermions in the $(m, n)$ representation of $G_1 \otimes G_2$ one finds

$$\text{Tr}_{(m,n)} F^2 = m \text{tr}_{(n)} F^2 + n \text{tr}_{(m)} F^2 \quad (2.4)$$

and

$$\text{Tr}_{(m,n)} F^4 = m \text{tr}_{(n)} F^4 + n \text{tr}_{(m)} F^4 + 6 \text{tr}_{(m)} F^2 \text{tr}_{(n)} F^2 \quad (2.5)$$

Let us begin by restricting our attention to a model with four types of quantum numbers. In this case, after imposing the tadpole conditions, the anomaly polynomial is

$$A = \frac{1}{8} \left\{ (\text{tr}F_1^2)^2 + (\text{tr}F_2^2)^2 + (\text{tr}F_7^2)^2 + (\text{tr}F_8^2)^2 \right\}$$

$$+ \frac{1}{16} \left\{ \text{tr}F_1^2 + \text{tr}F_2^2 + \text{tr}F_7^2 + \text{tr}F_8^2 \right\} \text{tr}R^2$$

$$- \frac{1}{4} \left\{ \text{tr}F_1^2 \text{tr}F_7^2 + \text{tr}F_1^2 \text{tr}F_8^2 + \text{tr}F_2^2 \text{tr}F_7^2 + \text{tr}F_2^2 \text{tr}F_8^2 \right\}$$

$$- \frac{1}{32} (\text{tr}R^2)^2 \quad (2.6)$$

where the two-form $R^{ab} = \frac{1}{2} R_{\mu\nu}^{\ ab} dx^\mu dx^\nu$.  

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This result seems rather disappointing at first sight, since the residual anomaly polynomial does not factorize. Thus, the prospects for this model (and for the whole construction of refs. [7] [8] ) would appear rather bleak, since the conventional Green-Schwarz mechanism does not apply in this case. On the other hand, the model contains a number of antisymmetric tensors, and one may wonder whether they could realize a more general type of Green-Schwarz mechanism by acting in a combined fashion. The answer to this question may be deduced from the anomaly polynomial, provided one regards it as a quadratic form in the field traces and turns it into its diagonal form. This is rather simple to do in this case, and the result is

\[
A = -\frac{1}{32} \left\{ \text{tr} F_1^2 + \text{tr} F_2^2 + \text{tr} F_7^2 + \text{tr} F_8^2 - \text{tr} R^2 \right\}^2 \\
+ \frac{3}{32} \left\{ \text{tr} F_1^2 + \text{tr} F_2^2 - \text{tr} F_7^2 - \text{tr} F_8^2 \right\}^2 \\
+ \frac{1}{32} \left\{ \text{tr} F_1^2 - \text{tr} F_2^2 + \text{tr} F_7^2 - \text{tr} F_8^2 \right\}^2 \\
+ \frac{1}{32} \left\{ \text{tr} F_1^2 - \text{tr} F_2^2 - \text{tr} F_7^2 + \text{tr} F_8^2 \right\}^2.
\]

(2.7)

This expression displays the expected phenomenon: disposing of the anomaly requires the combined action of a number of antisymmetric tensors. It should be appreciated that eq. (2.7) contains precisely six contributions, as many as the antisymmetric tensors in the model. Moreover, the antiself-dual tensor belonging to the supergravity multiplet is the only one that couples to the gravitational Chern-Simons form, in analogy with the standard case. Since all other tensor couplings involve only combinations of Yang-Mills Chern-Simons forms, one may investigate their nature using the low-energy supergravity. We shall return to this issue in the next section.

Eq. (2.7) actually suggests the proper way to extend the result to models with sixteen charge sectors. The key observations are as follows. First of all, the gravitational Chern-Simons form should couple only to the antisymmetric tensor in the supergravity multiplet. Moreover, the antisymmetric tensors that are supposed
to be at work in the cancellation mechanism belong to the sectors contributing the massless tadpoles. Since the $S$ matrix of the conformal theory determines the vacuum channel, one should be able to read from it all the proper combinations of Yang-Mills Chern-Simons forms corresponding to the non-planar diagrams. Thus, the proper generalizations of the last three lines in eq. (2.7) should contain combinations of field traces weighted according to suitable lines of the $S$ matrix. This is actually the case for the model with sixteen charge sectors, where one may verify that the diagonal form of the anomaly polynomial is

$$A = -\frac{1}{2} \left\{ \sum_m S_{1m} \, \text{tr} F_m^2 - 4 \, \text{tr} R^2 \right\}^2 + \frac{1}{2} \sum_k \left\{ \sum_m S_{km} \, \text{tr} F_m^2 \right\}^2,$$

(2.8)

where $m$ runs over the range $(1-8)$ and $(\bar{1}-\bar{8})$, and where $k$ runs over the “tadpole” sectors $(5)$, $(\bar{1})$, $(\bar{6})$, $(\bar{7})$ and $(\bar{8})$.

Defining polynomials $F^{2(i)}$ corresponding to the various lines of eq. (2.8), one may write

$$A = -\frac{1}{2} \sum_{ij} \eta_{ij} \, F^{2(i)} \, F^{2(j)}$$

(2.9)

where $\eta$ is the Minkowski metric with signature $(1-n)$. If, following standard practice, the eight-form in eq. (2.9) is converted into the Green-Schwarz counter-term

$$\Delta L = +\frac{1}{2} \sum_{ij} \eta_{ij} \, F^{(i)} B^{(j)}$$

(2.10)

the modified field strengths for the antisymmetric tensors are

$$H_{(i)} = dB_{(i)} + \omega_{(i)}$$

(2.11)

where $\omega_{(i)}$ denote the combinations of (Yang-Mills and gravitational) Chern-Simons forms corresponding to the various field traces in eq. (2.8). We have repeated
the analysis for several of the models in ref. [8], reaching identical conclusions. Namely, in general the Green-Schwarz mechanism is the result of the combined action of several antisymmetric tensors. In four-dimensional models, one may envisage interesting applications of this generalized mechanism to models with a number of anomalous $U(1)$ factors in their gauge groups.

It should be appreciated that the structure of the residual anomaly polynomial of eq. (2.10) is suggestive of an $SO(1,5)$ symmetry relating the antisymmetric tensors, broken only by the explicit form of the Chern-Simons couplings. In the next section we shall see that, in general, an $SO(1,n)$ symmetry of this kind is precisely in the spirit of the limiting supergravity theory for this class of models.

3. Field Equations of Six-Dimensional $N = 2b$
Supergravity Coupled to Vector and Tensor Multiplets

We would like to construct the field equations of a class of supergravity models related to the string spectra of the preceding section. The restriction to field equations is natural in all cases where (anti)self-dual tensors are present. These models describe the coupling of chiral $N = 2b$ supergravity in six dimensions to a number of tensor multiplets, as well as to vector multiplets. This extends the work of ref. [12], where the coupling to a number of tensor multiplets was discussed, and the work of ref. [13], where the coupling to a single tensor multiplet and to arbitrary matter was constructed. We adopt notation and spinor conventions of ref. [13]. As in refs. [11] and [12], we confine our attention to terms of lowest order in the fermions.

* In this case the antiself-dual tensor in the supergravity multiplet may be combined with the self-dual tensor in the tensor multiplet to give an ordinary antisymmetric tensor with no (anti)self-duality, and one may write an action using standard techniques.
Let us begin by recalling the results of ref. [12], while rephrasing them in a slightly different fashion. In addition to the supergravity multiplet, that contains a graviton, a left-handed gravitino and an antiself-dual tensor, we consider \( n \) tensor multiplets, each containing a self-dual tensor, a right-handed spinor and a scalar. Following ref. [12], we let the \( n \) scalar fields parametrize the coset space \( SO(1,n)/SO(n) \), and we introduce the \( SO(1,n) \) matrix

\[
V = \begin{pmatrix} v_0 & v_M \\ x^{m}_0 & x^{m}_M \end{pmatrix}.
\]

Since \( V \) is a (pseudo)orthogonal matrix, its elements satisfy the relations (\( r = 0, \ldots, M \))

\[
\tilde{v}^r v_s + \tilde{x}^r m x^m s = \delta^r s, \quad v_r \tilde{v}^r = 1, \quad \tilde{x}^m r \tilde{x}^n _r = \delta^m n, \quad \tag{3.2}
\]

where \( \tilde{v}^r = \eta^{rs} v_s \) and \( \tilde{x}^r_m = -\eta^{rs} x^m s \), with \( \eta \) the Minkowski metric with signature \( (1-n) \). From these expressions one may derive the composite \( SO(n) \) connection

\[
S_{\mu} [mn] = (\partial_{\mu} x^m r ) \tilde{x}^r n, \quad \tag{3.3}
\]

antisymmetric in \((m,n)\) because of eq. (3.2). The scalar kinetic term is then built out of

\[
P^m_{\mu} = \sqrt{\frac{1}{2}} (\partial_{\mu} v_r ) \tilde{x}^r m, \quad \tag{3.4}
\]

where \( P \) satisfies \( D_{[\mu} P_{m]} = 0 \).

The \((n+1)\) tensor fields \( A^r_{\mu\nu} \) are then taken to transform in the fundamental representation of \( SO(1,n) \), and out of them one defines the composite field strengths

\[
H_{\mu\nu\rho} = v_r F^r_{\mu\nu\rho}, \quad K^m_{\mu\nu\rho} = x^m r F^r_{\mu\nu\rho}. \quad \tag{3.5}
\]

To lowest order in the spinor fields, the tensor equations are then defined to be the
conditions that $H$ be self-dual and that $K^m$ be antiself-dual:

\[
H_{\mu\nu\rho} = \tilde{H}_{\mu\nu\rho} \\
K^m_{\mu\nu\rho} = - \tilde{K}^m_{\mu\nu\rho} .
\] (3.6)

These equations and the Bianchi identities then imply the second-order equations

\[
D_\mu H^{\mu\nu\rho} = -\sqrt{2} P^m_{\mu} K^{m\mu\nu\rho} \\
D_\mu K^{m\mu\nu\rho} = -\sqrt{2} P^m_{\mu} H^{\mu\nu\rho} .
\] (3.7)

For convenience, all Weyl fermions satisfy an $Sp(2)$ Majorana condition. A number of identities familiar from the four-dimensional case then apply, for instance [13]

\[
\bar{\chi} \gamma^{r_1 \ldots r_n} \lambda = (-1)^n \bar{\lambda} \gamma^{r_n \ldots r_1} \chi .
\] (3.8)

With all these ingredients one may then show that, to lowest order in the fermions, the field equations of the spinor fields

\[
\gamma^{\mu\nu\rho} D_\nu \psi_\rho + H^{\mu\nu\rho} \gamma_\nu \psi_\rho - \frac{i}{2} K^{m\mu\nu\rho} \gamma_\rho \chi^m - \frac{i}{\sqrt{2}} P^m_{\mu} \gamma^\nu \gamma^\rho \chi^m = 0 \\
\gamma^\mu D_\mu \chi^m - \frac{1}{12} H_{\mu\nu\rho} \gamma^{\mu\nu\rho} \chi^m - \frac{i}{2} K^{m\mu\nu\rho} \gamma_\mu \psi_\nu - \frac{i}{\sqrt{2}} P^m_{\nu} \gamma^\mu \gamma^\nu \psi_\mu = 0
\] (3.9)

transform into

\[
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - H_{\mu\rho\sigma} H^{\rho\sigma} - K^m_{\mu\rho\sigma} K^m_{\nu} \rho\sigma - 2 P^m_{\mu} P^m_{\nu} + g_{\mu\nu} P^m_{\rho} P^{m\rho} = 0 \\
D_\mu P^{m\mu} - \frac{\sqrt{2}}{3} H^{\mu\nu\rho} K^m_{\mu\nu\rho} = 0 ,
\] (3.10)
as well as into eqs (3.6), under the local supersymmetry transformations

\[
\begin{align*}
\delta \epsilon^m_\mu &= -i \bar{\epsilon} \gamma^{m} \psi_\mu \\
\delta \psi_\mu &= D_\mu \epsilon + \frac{1}{4} H_{\mu \nu \rho} \gamma^{\nu \rho} \epsilon \\
\delta A^r_{\mu \nu} &= i \bar{\psi} \gamma^r \psi_\mu \gamma^\nu \epsilon - \frac{1}{2} \bar{x}_m r^{m} \chi^m \gamma_{\mu \nu} \epsilon \\
\delta \chi^m &= - \frac{i}{\sqrt{2}} \gamma^\mu P^m_\mu \epsilon + \frac{i}{12} K_{\mu \nu \rho} m \gamma_{\mu \nu \rho} \epsilon \\
\delta \nu_r &= x^m r \bar{\epsilon} \chi^m .
\end{align*}
\tag{3.11}
\]

The extension to higher orders is then tedious but possible in principle, for instance using superspace techniques, as in ref.\[14\]. In particular, if the field strengths in the fermionic transformations are extended to their supercovariant forms, one may verify that the supersymmetry algebra closes on the bosons in terms of all local symmetries.

Our next step will be extending the construction to the case when a number of vector multiplets are coupled to the model. To this end, we begin by including in the tensor field strengths suitable Chern-Simons forms\[15\]. Let us recall that, under a gauge transformation, the Chern-Simons form

\[
\omega = \text{tr}(A dA - \frac{2ig}{3} A^3 )
\tag{3.12}
\]

varies according to

\[
\delta \omega = d\text{tr}(A dA ) .
\tag{3.13}
\]

If one modifies the tensor field strengths according to

\[
F^r = dA^r - c^{rz} \omega_z ,
\tag{3.14}
\]

where \(z\) labels the various factors of the gauge group, and where \(c^{rz}\) is a matrix of constants, related to the elements of the \(S\) matrix of the conformal theory, the
invariance of $F^r$ under the vector gauge transformations demands that

$$
\delta A^r = e^{r z} tr_z( \Lambda \ dA ) \ .
$$

The Bianchi identity is also modified, and eqs. (3.7) become

$$
D_\mu H^{\mu \nu \rho} = - \sqrt{2} \ P^m_\mu \ K^{m \mu \nu \rho} - \frac{1}{8e} \ e^{\mu \nu \rho \alpha \beta \gamma} v_r \ c^{rz} \ tr_z(F_{\mu \alpha} \ F_{\beta \gamma}),
$$

$$
D_\mu K^{m \mu \nu \rho} = - \sqrt{2} \ P^m_\mu \ H^{\mu \nu \rho} + \frac{1}{8e} \ e^{\mu \nu \rho \alpha \beta \gamma} x^m \ c^{rz} \ tr_z(F_{\mu \alpha} \ F_{\beta \gamma}),
$$

where $e$ denotes the determinant of the vielbein.

The field equations of the spinor fields now include additional terms,

$$
\gamma^{\mu \nu \rho \delta} D_\nu \psi_\rho + H^{\mu \nu \rho} \gamma_\nu \psi_\rho - i \frac{1}{2} K^{m \mu \nu \rho} \gamma_\nu \chi^m - i \frac{1}{\sqrt{2}} \ P^m_\nu \ \gamma^\nu \gamma^\mu \chi^m
$$

$$
- \frac{1}{2 \sqrt{2}} \ \gamma^{\mu \nu \rho \delta} \gamma_\nu \psi_\rho \ c^{rz} \ tr_z(F^{\sigma \tau} \lambda) = 0
$$

$$
\gamma^\mu D_\mu \chi^m - \frac{1}{12} H_{\mu \rho \sigma} \gamma^{\mu \rho \sigma} \chi^m - i \frac{1}{2} K^{m \mu \nu \rho} \gamma_\nu \psi_\rho + i \frac{1}{\sqrt{2}} \ P^m_\nu \ \gamma^\nu \gamma^\mu \psi_\mu
$$

$$
- \frac{i}{2 \sqrt{2}} \ x^m \ c^{rz} \ tr_z(\gamma^{\mu \nu} \lambda \ F_{\mu \nu}) = 0 \ .
$$

Moreover, the Einstein equation becomes

$$
R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R - H_{\mu \rho \sigma} H^{\rho \sigma} - K^{m \mu \rho \sigma} K^{m \nu \rho \sigma} - 2 P^m_\mu P^m_\nu + g_{\mu \nu} P^m_\rho P^{m \rho} + 2 v_r \ c^{rz} \ tr_z(F_{\lambda \mu} F^{\lambda \nu} - \frac{1}{4} g_{\mu \nu} F^2) = 0 \ ,
$$

while the equation of the scalar fields becomes

$$
D_\mu P^{m \mu} - \frac{\sqrt{2}}{3} H^{\mu \nu \rho} K^{m \mu \nu \rho} + \frac{1}{2 \sqrt{2}} \ x^m \ c^{rz} \ tr_z(F_{\alpha \beta} F^{\alpha \beta}) = 0 \ .
$$

To these we should add the supersymmetry transformations for the vector multi-
plets,

\[ \delta \lambda = - \frac{1}{2\sqrt{2}} F_{\mu \nu} \gamma^{\mu \nu} \epsilon \]
\[ \delta A_{\mu} = - \frac{i}{\sqrt{2}} (\bar{\epsilon} \gamma_{\mu} \lambda) \]  

and the corresponding field equations

\[ (v_r c^{rz}) \gamma^\mu D_\mu \lambda + \frac{1}{\sqrt{2}} P^m_{\mu}(x^m_r c^{rz}) \gamma^\mu \lambda + \frac{1}{2\sqrt{2}} (v_r c^{rz}) F_{\lambda \tau} \gamma^{\mu} \gamma^{\lambda \tau} \psi_\mu \]
\[ + \frac{i}{2\sqrt{2}} (x^m_r c^{rz}) \gamma^{\mu \nu} \bar{\chi}^m F_{\mu \nu} = 0 \]  

and

\[ (v_r c^{rz}) D^\mu F_{\mu \nu} + \sqrt{2} (x^m_r c^{rz}) P^{m \mu} F_{\mu \nu} - (v_r c^{rz}) F_{\rho \sigma} H_{\rho \sigma} \]
\[ - (x^m_r c^{rz}) F^{\rho \sigma} K^{m}_{\nu \rho \sigma} = 0 \]  

Finally, the supersymmetry transformation of the antisymmetric tensors acquires an additional contribution,

\[ \delta A^{r}_{\mu \nu} = - c^{rz} \text{tr}_z(A_{[\mu} \delta A_{\nu]}), \]  

necessary in order that the commutator algebra give rise to the proper vector gauge transformation. Again, with proper supercovariantizations, the supersymmetry algebra on the bosons closes on all local symmetries.

It should be appreciated that the vector couplings change under SO(1, n) transformations, since the \( A^r \) tensors are coupled to different combinations of Chern-Simons forms. As a result, from eqs. (3.21) and (3.22) one may see that, for instance, the scalar fields should be restricted to the region where

\[ v_r c^{rz} > 0 \]  

since at the boundaries of this region the coupling constants of the vector fields become infinite. These restrictions have in fact a number of conventional analogues
in supergravity models. For instance, with a single tensor multiplet, and in the absence of vector couplings, one might conclude that the only physical scalar should live on the hyperbola

$$(v_0)^2 - (v_1)^2 = 1.$$  \hspace{1cm} (3.25)

On the other hand when, according to common practice, the scalar matrix is parametrized in terms of the “dilaton” field $\phi$ [13], writing

$$V = \begin{pmatrix} \cosh(\sqrt{2}\phi) & \sinh(\sqrt{2}\phi) \\ \sinh(\sqrt{2}\phi) & \cosh(\sqrt{2}\phi) \end{pmatrix},$$ \hspace{1cm} (3.26)

one is implicitly covering only one branch of the curve. Moving to the other branch would require that one continue the dilaton according to $\sqrt{2}\phi \rightarrow \sqrt{2}\phi + i\pi$ but, in open-string theories, this would alter the sign of the vector kinetic term, that comes from “genus-one-half”.

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**Figure Captions**

**Figure 1**

Contributions to the gauge anomaly in open-string theories: (a) planar diagram; (b) non-orientable diagram; (c) non planar diagram.
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