Comprehensive Evaluation of Weapon Equipment Systems Operational Effectiveness Based on State Space Concept

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Abstract. In recent years, there are many researches on combat test of weapon equipment systems. However, in the process of comprehensive effectiveness evaluation, there are some shortcomings in the construction of index system, such as index redundancy, experts subjective weighting, which lead to the lack of objectivity and scientificity. Focusing on the objective state of the evaluated object, this paper puts forward an idea of comprehensive evaluation in the efficiency state space (ESS), and uses intelligent data processing method based on principal component analysis to examine the correlation of the underlying indicators of the evaluation index system (EIS), and attempts to conduct a comprehensive evaluation on this basis. Finally, we give the comparison the corresponding new results with the original evaluation results in the references. The data show that comprehensive evaluation under ESS can reduce efficiency calculation dimension from 10 to 3. The final efficiency ranking is slightly different from that in the reference, but is much more rational and convincing.

1. Introduction
With the increasing complexity of weapon equipment systems and the continuous diversification of test techniques, new requirements are put forward for data processing technology and result evaluation theory in relevant identification or tests. Early tests are often carried out according to a single set or around a single performance index, whose data processing and results evaluation are relatively easy to implement manually. In recent years, the emerging equipment system-wide tests, architectonical combat tests and competitive preference tests in model development show obvious differences.

First of all, there are many levels in the effectiveness evaluation index system in combat tests, and there are a large number of indicators at all levels, especially at the bottom level[1]. The data types are different[2]. In order to operate in a unified evaluation model, it is necessary to conduct standardized pre-processing. Secondly, in the competitive preference test, the manufacturers participating in the bidding hope that the evaluation results are objective and fair, scientific and reasonable, and minimize human subjective intervention, such as human index weight assignment[3], emphasizing the single index score and ignoring the overall advantage. In addition, from the perspective of data engineering, in fact, whether there is a linear correlation between various indicators, and whether the judgment method such as the correlation coefficient method is convincing enough remains to be discussed.

Most importantly, in terms of evaluation methods, the existing comprehensive evaluation indexes mainly use analytic hierarchy process[4], grey correlation analysis[5] and fuzzy comprehensive
evaluation[6]. There main disadvantage is that the spatial distribution of the index system of the original data is ignored, which is easy to cause the superposition or weakening effect of the relevant indexes on the eigenvalues of the original state of the evaluated object in the gradual aggregation, and destroy the scientific rationality of the results.

In order to solve the above problems, this paper draws on the principal component analysis (PCA) method which is widely used in artificial intelligence and big data analysis. While reducing the influence of correlation and redundancy in effectiveness evaluation index system, it simplifies the comprehensive evaluation calculation and improves the scientificity and credibility of final results.

2. The state space of combat effectiveness

So far, almost all the domestic research on operational effectiveness evaluation follow a basic step, that is, first to establish the evaluation index system, and then calculate the underlying effectiveness index, and finally the effectiveness is aggregated upward step by step[7]. The advantages of this idea are clear logic, easy to implement, and the physical meaning of the effectiveness indicators at all levels is clear, which is convenient for the establishment of the index system. However, from the point of view of more scientific and reasonable evaluation of the evaluation object, the step by step aggregation in the above practice lacks rigorous derivation proof in mathematics, or only partial verification, which does not reflect enough comprehensiveness.

Assuming that after the completion of a combat test, the corresponding measurement values have been obtained according to the pre-set index system, then in the final comprehensive evaluation link, this paper tends to examine the overall performance of the evaluated object from the evaluation index system (EIS) to the effectiveness state space (ESS).

2.1. Effectiveness state space

The concept of state space proposed in this paper is different from the case of system stability analysis or correction in control engineering, but is a new concept used to analyse the combat effectiveness of weapon equipment systems. Theoretically speaking, no matter what kind of EIS is formulated, the measured value is the reflection of the objective state of weapon equipment system in a certain period of time (regarded as time invariant). However, to describe this objective state, there should be a set with the smallest number of variables. Each variable of the set does not necessarily have a one-to-one corresponding physical meaning, and there is no linear correlation between variables. The indicators at all levels in the artificially established evaluation index system are only the linear combination of variables in the set. Here, the space constituted by the variables in the above set is denoted as $S_{ES}$, and

$$S_{ES} = \{ V_1, V_2, ..., V_n \}$$

where $V_i (1 \leq i \leq m)$ denotes some state component and

$$V_i \cdot V_j = 0 \ (1 \leq i \leq m, 1 \leq j \leq m, i \neq j)$$

2.2. Relation between EIS and EES

Assuming that the combat effectiveness evaluation index system $S_{EI}$ has been established for a certain type of weapon equipment, it may be assumed that $S_{EI}$ contains n evaluation indexes, denoted as

$$S_{EI} = \{ E_1, E_2, ..., E_i, ..., E_n \}$$

where $E_i$ (1 $\leq i \leq n, m \leq n$) represents some evaluation index. In existing studies, the authors have not given the statement of satisfying $E_i \cdot E_j = 0$ (1 $\leq i \leq n, 1 \leq j \leq n, i \neq j$), that means that the evaluation indexes are not completely independent and linearly irrelevant.

Combined with the definition in 2.1 of the efficiency state space $S_{ES}$, we see that its components can be used as a set of orthogonal bases of $S_{EI}$, and then

$$E_1 = a_{11} V_1 + a_{12} V_2 + \cdots + a_{1m} V_m$$
$$E_2 = a_{21} V_1 + a_{22} V_2 + \cdots + a_{2m} V_m$$
$$\vdots$$
$$E_n = a_{n1} V_1 + a_{n2} V_2 + \cdots + a_{nm} V_m$$

(4)
where $a_{ij}$ \((1 \leq i \leq n, 1 \leq j \leq m)\) represents some constant coefficient, and the physical meaning of $E_i = a_{i1}V_1 + a_{i2}V_2 + \cdots + a_{im}V_m$ is that the measured value of the evaluation index obtained from the experiment is the projection combination of the feature vectors in $S_{ES}$.

Figure 1 shows the relation between the components in $S_{ES}$ and $S_{EI}$ when $m = 3, n = 5$, where $V_1$, $V_2$, and $V_3$ are pairwise orthogonal. Similar conclusion as above can be obtained in higher dimension.

According to the criteria of linear transformation, under the premise of non-degradation, the following formulation can be obtained through principal component transformation\cite{8}.

\[
\begin{align*}
V_1 &= b_{11}E_1 + b_{12}E_2 + \cdots + b_{1n}E_n \\
V_2 &= b_{21}E_1 + b_{22}E_2 + \cdots + b_{2n}E_n \\
&\vdots \\
V_m &= b_{m1}E_1 + b_{m2}E_2 + \cdots + b_{mn}E_n
\end{align*}
\]

where $b_{uv}$ \((1 \leq u \leq m, 1 \leq v \leq n)\) represents some constant coefficient, and $V_1, V_2, \ldots, V_m$ denote principal components. After all the measured values in $S_{EI}$ are converted to obtain all the principal component components in $S_{ES}$, the comprehensive evaluation can be carried out subsequently.

![Diagram of relation between S_{ES} and S_{EI}](image)

Figure 1. Diagram of relation between $S_{ES}$ and $S_{EI}$

3. Comprehensive evaluation in ESS

For the operational effectiveness evaluation of weapons equipment system, the underlying indicators in the index system usually have different data types. Some are from objective measurement, other are from expert subjective scoring, and the dimension units are also different. Therefore, the original data should be standardized first, then the subsequent transformation and analysis could be carried out.

3.1. Data pre-processing

In existing researches, according to specific weapon equipment system and application scene, many scholars have given the pre-processing method of some types of evaluation index values \cite{2}, most of which are normalization. However, due to a number of data types discussed in this paper, we make certain synthesis and modification of those methods.

The original measured value vector of an index in $S_{EI}$ is denoted as

\[
\bar{E}_i=\begin{bmatrix} e_{i1}, e_{i2}, \cdots, e_{ij}, \cdots, e_{iJ} \end{bmatrix}, 1 \leq i \leq n, 1 \leq j \leq J
\]

(6)

After normalization, the corresponding vector of $\bar{E}$ can be denoted as

\[
\tilde{E}_i=\begin{bmatrix} \bar{e}_{i1}, \bar{e}_{i2}, \cdots, \bar{e}_{ij}, \cdots, \bar{e}_{iJ} \end{bmatrix}, 1 \leq i \leq n, 1 \leq j \leq J
\]

(7)

Specific calculations for different types of indicators are as follows:

For benefit-type indicators

\[
\bar{e}_{ij} = \frac{e_{ij} - \min(e_{ij})}{\max(e_{ij}) - \min(e_{ij})}, \quad \min(e_{ij}) < e_{ij} < \max(e_{ij})
\]

(8)

For cost-type indicators
For optimum-type indicators
\[
\bar{e}_{ij} = \frac{\max(e_{ij}) - e_{ij}}{\max(e_{ij}) - \min(e_{ij})}, \quad \min(e_{ij}) < e_{ij} < \max(e_{ij})
\] (9)

For interval-type indicators
\[
\bar{e}_{ij} = \begin{cases} 
1 - \frac{q_{ij} - e_{ij}}{d_j}, & e_{ij} < q_{1j} \\
1, & q_{1j} < e_{ij} < q_{2j} \\
1 - \frac{q_{ij} - e_{ij}}{d_j}, & e_{ij} > q_{2j}
\end{cases}
\] (10)

where \(d_j = \max\{q_{ij} - \min(e_{ij}), \max(e_{ij}) - q_{ij}\}\) (12)

3.2. Calculation procedure

After the normalization of the measured values of the underlying indicators in EIS is completed, it can be transformed into ESS by principal component transformation.

3.2.1 Calculation of covariance matrix \(C\). According to basic mathematical statistics theory, we have

\[
C = (c_{ij})_{n \times n} = \begin{bmatrix}
c_{11} & c_{12} & \cdots & c_{1n} \\
c_{21} & c_{22} & \cdots & c_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
c_{n1} & c_{n2} & \cdots & c_{nn}
\end{bmatrix}, \quad i, j = 1, 2, \cdots, n
\] (13)

where \(c_{ij} = \text{Cov}(\bar{E}_i, \bar{E}_j)\) denotes the covariance of normalized evaluation index vectors \(\bar{E}_i\) and \(\bar{E}_j\).

3.2.2 Finding eigenvalues and eigenvectors of \(C\). From the characteristic equations \(|\lambda I - k_j| = 0\), we can obtain the characteristic roots denoted as \(\lambda_i\) here \((i = 1, 2, \cdots, k)\). Assuming that the roots sorted by value can be expressed as \(\lambda_1 > \lambda_2 > \cdots > \lambda_k > 0\), and the corresponding eigenvector as \(I_1, I_2, \cdots, I_k\), where

\[
I_i = [\xi_{i1}, \xi_{i2}, \cdots, \xi_{in}]
\] (14)

The characteristic roots are actually the variances of principal components, reflecting their respective degree of importance in final synthetic evaluation.

3.2.3 Determine the number of principal components. One of the purposes of transferring data from EIS to ESS is to reduce data dimension and facilitate evaluation operation. This can be achieved by calculating the number of principal components. Denoting it as \(p\), the calculation method adopted widely is to make \(p\) satisfy:

\[
\frac{\sum_{i=1}^{p} \lambda_i}{\sum_{i=1}^{k} \lambda_i} \geq 85\%
\] (15)

3.2.4 Comprehensive evaluation based on principal components. The calculation method of the first \(p\) principal components is

\[
V_i = \xi_{i1}E_1 + \xi_{i2}E_2 + \cdots + \xi_{in}E_n = [v_1, v_2, \cdots, v_i, \cdots, v_p], (i = 1, 2, \cdots, p)
\]

The total efficiency in the effectiveness state space can be determined by the first \(p\) eigenvalues and the above principal components:

\[
E = \sum_{i=1}^{p} \frac{\lambda_i}{\sum_{j=1}^{k} \lambda_j} v_i
\] (16)
4. Example and comparison
In Reference[9], six types of antiaircraft gun weapon systems (denoted as $S_i$, $1 \leq i \leq 6$) were evaluated comprehensively for their ability to attack air targets, military manoeuvre, self-protection and intelligence support for acquiring air target information (denoted as $D_i$, $1 \leq i \leq 10$). The original data are shown in Table 1.

|      | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ | $S_6$ | $S_7$ |
|------|-------|-------|-------|-------|-------|-------|-------|
| $D_1$ | 3000  | 1650  | 4000  | 2500  | 2000  | 3300  | 3000  |
| $D_2$ | 90    | 60    | 125   | 70    | 120   | 45    | 90    |
| $D_3$ | 1600  | 3000  | 1200  | 3700  | 3400  | 1300  | 1600  |
| $D_4$ | 1040  | 1030  | 1175  | 970   | 1100  | 1080  | 1040  |
| $D_5$ | 80    | 68    | 120   | 44    | 64    | 48    | 80    |
| $D_6$ | 12.7  | 12.8  | 13.8  | 10    | 12.4  | 10    | 12.7  |
| $D_7$ | 410   | 406   | 475   | 400   | 430   | 400   | 410   |
| $D_8$ | 100   | 800   | 1000  | 1000  | 800   | 1100  | 100   |
| $D_9$ | 60    | 60    | 45    | 30    | 60    | 30    | 60    |
| $D_{10}$ | 100  | 60    | 80    | 100   | 80    | 60    | 100   |

After data preprocessing and covariance matrix calculation, the eigenvalues and eigenvectors are obtained respectively as follows:

- $\lambda_1 = 4.71$, $\xi_1 = [0.26 0.37 -0.26 0.40 0.44 0.39 0.43 -0.08 0.18 0.02]$
- $\lambda_2 = 2.48$, $\xi_2 = [-0.42 0.19 0.32 -0.21 0.01 0.28 -0.09 -0.46 0.53 0.25]$
- $\lambda_3 = 1.42$, $\xi_3 = [0.41 0.06 -0.20 -0.22 0.03 -0.18 -0.07 -0.37 -0.24 0.71]$
- $\lambda_4 = 1.15$, $\xi_4 = [-0.10 0.44 0.57 0.01 -0.03 -0.10 0.31 0.45 -0.24 0.34]$
- $\lambda_5 = 0.25$, $\xi_5 = [0.00 -0.43 0.12 -0.53 0.53 0.40 0.12 0.19 -0.18 0.04]$

And the projection value of original measurement value in each characteristic direction of the index system can be calculated (shown in Table 2).

|      | $\xi_1$ | $\xi_2$ | $\xi_3$ | $\xi_4$ | $\xi_5$ |
|------|---------|---------|---------|---------|---------|
| $S_1$ | 0.67    | 1.54    | 1.60    | -1.21   | -0.10   |
| $S_2$ | -0.87   | 1.04    | -1.63   | -0.68   | 0.57    |
| $S_3$ | 3.71    | -1.22   | 0.06    | 0.51    | 0.32    |
| $S_4$ | -2.54   | -0.18   | 1.05    | 1.34    | 0.33    |
| $S_5$ | 0.43    | 1.25    | -0.86   | 0.96    | -0.73   |
| $S_6$ | -1.40   | -2.42   | -0.23   | -0.92   | -0.39   |

Since $\lambda_1 + \lambda_2 + \lambda_3 > 85\%$, $\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 > 95\%$, according to general practice in PCA field, the principal components are extracted according to the 85% standard, and the eigenvalues are used as the weights to calculate the comprehensive scores (denoted as $S_i$) of various types of weapon systems, and we have

$$S_1 = 0.67 \times \frac{\lambda_1}{\sum_{i=1}^{5} \lambda_i} + 1.54 \times \frac{\lambda_2}{\sum_{i=1}^{5} \lambda_i} + 1.60 \times \frac{\lambda_3}{\sum_{i=1}^{5} \lambda_i} = 0.93$$
$$S_2 = -0.87 \times \frac{\lambda_1}{\sum_{i=1}^{5} \lambda_i} + 1.04 \times \frac{\lambda_2}{\sum_{i=1}^{5} \lambda_i} - 1.63 \times \frac{\lambda_3}{\sum_{i=1}^{5} \lambda_i} = -0.38$$
\[ S_3 = 3.71 \frac{\lambda_1}{\sum_{i=1}^{5} \lambda_i} - 1.22 \frac{\lambda_2}{\sum_{i=1}^{5} \lambda_i} + 0.06 \frac{\lambda_3}{\sum_{i=1}^{5} \lambda_i} = 1.45 \]

\[ S_4 = -2.54 \frac{\lambda_1}{\sum_{i=1}^{5} \lambda_i} - 0.18 \frac{\lambda_2}{\sum_{i=1}^{5} \lambda_i} + 1.05 \frac{\lambda_3}{\sum_{i=1}^{5} \lambda_i} = -1.09 \]

\[ S_5 = 0.43 \frac{\lambda_1}{\sum_{i=1}^{5} \lambda_i} + 1.25 \frac{\lambda_2}{\sum_{i=1}^{5} \lambda_i} - 0.86 \frac{\lambda_3}{\sum_{i=1}^{5} \lambda_i} = 0.39 \]

\[ S_6 = -1.40 \frac{\lambda_1}{\sum_{i=1}^{5} \lambda_i} - 2.42 \frac{\lambda_2}{\sum_{i=1}^{5} \lambda_i} - 0.23 \frac{\lambda_3}{\sum_{i=1}^{5} \lambda_i} = -1.29 \]

Then the comprehensive operational effectiveness of the six types of weapon systems is ranked as \( S_3 > S_1 > S_5 > S_2 > S_4 > S_6 \) while the ranking from traditional evaluation in reference[9] is given as \( S_3 > S_5 > S_1 > S_4 > S_2 > S_6 \). After comparison, it can be seen that \( S_1 \) has still the best efficiency and \( S_6 \) still the worst. But the ranking of the other four weapon systems has changed. If \( S_1 \) and \( S_5 \) belong to the second level, \( S_2 \) and \( S_4 \) belong to the third level, then the ranking of the four levels does not change. However, in the second and third level, the evaluation results in this paper and reference[14] are opposite. This is due to the correlation effect in the original evaluation index system.

5. Conclusion

In effectiveness evaluation of many previous weapon equipment system combat tests, index redundancy due to correlation lead to inadequate scientificity and persuasiveness of final result. This paper puts forward the concept based on evaluation state space, aiming at eliminating the influence of correlation between the indexes in EIS on the scientificity and objectivity of evaluation results, and reducing the reference data dimension in comprehensive evaluation stage. Based on PCA method, a case in existing researches is investigated. The results show that large redundancy exists in its index system, and the evaluation result obtained using the proposed method is quite different from that obtained through the original method. In addition, the research in this paper provide ideas and methods for the comprehensive evaluation of competitive test in weapon equipment procurement or military exercise training. In later research, we will find more evaluation index data from various kinds of operational test to verify the method proposed in this paper. Furthermore, find practical and effective ways to simplify the index system for as many types of effectiveness evaluation tests.

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References

[1] Wu X., Wang T.H., Gao Z.H. (2020) Research on establishment and optimization method of evaluation index system for weapons equipment operational test. Fire Control & Command Control, 45(3): 75-80.

[2] Li K., Peng J., Song S. (2018) Evaluation Index System of Communication Equipment Operation Test. Communications Technology, 51(7): 1649-1655.

[3] Wang X., Zhang K. (2018) Research on Evaluation Method of Selection for General Support Equipment Based on Competitive Test. Advances in Aeronautical Science and Engineering, 9(3): 428-433.

[4] Sun S.H., Liao X.J. (2018) Research on Comprehensive Evaluation of Equipment Operational Suitability. Journal of Information Engineering University, 19(2): 248-252.

[5] Wei D.T., Liu X.D., Shan Z.F. (2020) Fuzzy Cluster Analysis of Weapon System of Systems Based on Entropy Weight and Grey Relational Degree. Journal of Information Engineering University, 21(5): 626-630.
[6] Li A.A., Liao X.J. (2016) Application of Fuzzy Comprehensive Evaluation in Qualitative Evaluation of Weapon Equipment Operational Test. Journal of Ordnance Equipment Engineering, 37(5): 15-24.

[7] Guo Q.S., Zhang L. (2013) Research Summary of Weapons Equipment Systems Effectiveness Evaluation Methods. Computer Simulation, 30(8): 1-18.

[8] Sun Y.K., Fang Z.G. (2021) Multi-Time Threat Assessment Based on Dynamic Grey Principal Component Analysis. Systems Engineering and Electronics, 43(3): 740-746.

[9] Ji X.Y., Wang J. (2019) Evaluation of Integrated Capability of Antiaircraft Gun Weapon Based on Entropy Weight. Fire Control & Command Control, 35(7): 112-115.