Analysis of students’ mathematical modelling ability in solving combination problems using local instruction theory teaching materials

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Abstract. This article was written based on the evaluation result of the students’ post-learning about combinations by using Local Instruction Theory (LIT) teaching materials in a Realistic Mathematics Education. This research aimed to describe the process of students’ mathematical modelling in solving combinations problem referring to the process of horizontal and vertical mathematization. A case-study method was employed by grouping the students into three categories: high, medium, and low categories. From each category of the students’ ability, one student was selected purposively as the research subjects. The results showed that the high-ability student could do all the modelling processes well, including the process of horizontal and vertical mathematization. The medium-ability student could do the modelling process particularly on the cognitive step 2; that were understanding, simplifying, and doing the process of horizontal mathematization, albeit the process that was not yet accomplished. The low-ability student was only able to work with the first cognitive step; that were understanding, simplifying, doing an incomplete horizontal mathematization, and doing vertical mathematization, although the result was incorrect.

1. Introduction
Mathematical modeling plays an important role in the development of competence and understanding of mathematics [1]. It is important because to achieve problem-solving abilities and reasoning skills, the students are required to be able to create formulas and develop mathematical models. Such a modeling also plays a role in shaping the proceeding mathematical understanding [2] related to the real-world problems.

Modeling cannot be separated from the process of mathematization [3]. Blum and Leiss place mathematization as part of modeling [4], but there are also those who consider modeling as part of mathematization. In a simple way, mathematization is a process for deciphering a phenomenon [5], and generally mathematics in Realistic Mathematics Education (RME) involves two main processes of generalization and formalization [5]. Generalization deals with the decipherment of patterns and relationships, while formalization involves modeling, symbolizing, schematizing, and defining. De Lange divides mathematization into two branches, namely: horizontal mathematization and vertical mathematization [5]. Horizontal mathematization is related to the generalization process. The process of horizontal mathematization begins with the identification of mathematical concepts based on regularities and relations found through visualization and schematization of the problems. While,
vertical mathematization is a form of formalization process in which mathematical models obtained in horizontal mathematization become the foundation in the development of more formal mathematical concepts through a vertical mathematical process.

Some modeling processes within which there includes a mathematical process consist of six mathematical modeling processes [6]: formulating the task; selecting the relevant objects, relation etc; translating those objects and relations; using mathematical methods to achieve mathematical results and conclusions; interpreting those results and conclusions; and evaluating the validity of the model. There are seven steps of mathematical modeling according to Blum and Leiss [4], namely: understanding; simplifying/structuring; mathematizing; working mathematically; interpreting; validating; and exposing. The seven steps of that modeling are reduced to three cognitive steps, namely: understanding/mathematizing the task; working mathematically; and explaining the results [7].

The process of mathematical modeling can be trained through Realistic Mathematics Education (RME) learning involving active students and their contribution both in constructing knowledge and in interaction with the learning environment. Learning process that takes learning trajectory (learning path) into account is generally proposed in Local Instruction Theory (LIT). LIT is a theory that can guide and help a person learn a particular topic. This theory covers the process of studying a particular topic with a theory, scope, tools or learning medium in detail, gradually, and specifically. It is called local theory because it deals with the specific sphere only [8]. The LIT constructs [8,9], developed in the context of research design, demonstrate the means of outcomes of the teachers’ referencing framework for designing and involving students in a set of learning stages. The learning activities are examples that support the development of the students' mathematics focused on concepts; in this sense, the concept of combinations. LIT developed here contains characteristics of the RME, which are: using real-world context, using the model, requiring the students’ contribution, and requiring interaction and inter-relationship between the topics.

The results of which study [10] showed that students' mathematical modeling ability was still low due to the high level of their mistakes in solving the problems related combinations. This suggests that mathematical modeling is one of the most difficult topics for the students [11, 12]. It is difficult because they are not used to learning by making their own models, and generally the strategies to build mathematical models are not taught in Maths classes. Mathematics learning in the classroom tends to use a ready-to-use mathematical model [13], starting from providing the formula, followed by a sample discussion of the problem, and problem-solving exercises.

Consequently, the students are not yet accustomed to making their own models, whereas mathematical modeling can help them simplify the problems from real situations to mathematical forms. Modeling is also a regular and dynamic method that reduces the gap between mathematics and real life [14]. It is because by definition, mathematical modeling is a process of representation of real-world problems in the form of mathematics as an attempt to find the solutions [3].

Based on the above description, it is necessary to design LIT teaching materials and their implementation in learning, as well as analysis of the results of the implementation of the use of LIT teaching materials in achieving students' mathematical modeling abilities in terms of solving the combination problems. Analysis of students’ mathematical modeling abilities can be seen when students solve combination problems that refer to the steps of the horizontal mathematization process and steps in the vertical mathematization process.

2. Method
The subjects were 18 students of class XI IPA (Science) 2 SMAN (Senior high school) Cahaya Madani Banten Boarding School. The research was conducted in the first semester of academic year 2017/2018. The students were divided into three groups, namely: high-, medium-, and low-achiever groups based on their maths grades. The low-achiever group consists of students whose grades are below 84. The medium-achiever group consists of students whose grades are between 84 and 87. The high-achiever group consists of students whose grades are between 88 and 100. Then, the researchers selected one
student from each group to be the subjects of this study. The selection process was done on the basis that the students’ thinking process from all groups can be obtained.

The answers of each subject in solving the problems of combinations were identified based on the mathematical modeling stages proposed by Blum and Leiss. The stages proposed by [4] consist of: 1) Understanding; 2) Simplifying/Structuring; 3) Doing the mathematization; 4) Working Mathematically; 5) Interpreting; 6) Validating; and 7) Exposing.

In the ‘mathematization’ and ‘working mathematically’ steps, the students' thinking processes were identified using horizontal mathematical frameworks and vertical mathematization proposed by De Lange [5]. The researchers became the main instrument in this research equipped by some problems related to combinations, LIT teaching materials, LIT activity sheets, videos, and interview protocol. The instruments used in this research has passed the process of content validation and face validation.

The excerpt of the problem about combinations was as follows.

"In a daily test of Mathematics subject, the teacher gives seven multiple-choice questions and three short-description questions. Students may work on a problem (question) that they think is easy first. They do not have to answer all of the questions. They may choose five multiple-choice questions and two short-description questions. If question number 4 and 7 of the multiple-choice type must be done, How many ways do the students have to choose the rest of the must-do questions?"

3. Results and Discussion
Below are the results of the grouping based on the initial skill derived from the students’ grades in the tenth grade of the academic year 2016/2017, presented in Table 1.

| No. | Category of the students’ initial ability | Score range (X) | Frequency (f) | Percentage (%) |
|-----|------------------------------------------|----------------|--------------|---------------|
| 1   | High                                     | X > 88         | 5            | 25            |
| 2   | Medium                                   | 84 < X < 88    | 9            | 45            |
| 3   | Low                                      | X < 84         | 6            | 30            |

Furthermore, the achievement results of the mathematical modeling related to the solution of the combinations problems are presented in Table 2 by using the stages proposed by Blum and Leiss which are reduced to three cognitive steps [7] with the following details.

Cognitive Step 1: Understanding / Mathematizing the task (stages 1 and 2).
Cognitive Step 2: Working mathematically (stages 3 and 4).
Cognitive Step 3: Explaining the results (stages 5, 6, and 7).

| Category of the students’ initial ability | Percentage (%) of the achievements through mathematical modelling through cognitive step | Mean (%) |
|------------------------------------------|------------------------------------------|----------|
|                                          | Step 1 | Step 2 | Step 3 |               |
| High                                     | 100    | 85     | 80     | 88            |
| Medium                                   | 100    | 81     | 57     | 79            |
| Low                                      | 100    | 71     | 50     | 73            |

From Table 2 above, it appears that all students have been able to perform the initial mathematical modeling stages. Besides, in the next stage, the achievement results were still above 70%. In the last stage, only a group of students got high category of 80%. The results in Table 2 were then analyzed further by selecting one student from each category of ability. The chosen subjects were labelled: student A (with high ability), student B (with medium ability), and student C (with low ability). The analysis of the students' mathematical modeling abilities was based on the written data and interview data in solving
the combinations problems based on the seven modeling steps proposed by Blum and Leiss [4]. It includes: understanding, simplifying/arranging, mathematizing, working mathematically, interpreting, validating, and exposing.

In cognitive step 1 (the stage of understanding the problem and simplifying the problem), all students were able to identify and simplify the problem given by rewriting, “for multiple-choice questions, the students are asked to choose 3 of 5 questions, and for short-description questions, they are asked to choose 2 of 3 questions.” It can be seen from Figures 1, 2, and 3.

In cognitive step 2 (the stage of mathematization and mathematical work), student A and C were able to relate the problems encountered. It is the determination of $C_3^5$ to select 3 multiple-choice questions from the remaining 5 questions and $C_2^3$ to select 2 short-description questions from 3 given questions. While, student B erroneously solved the problem by choosing 2 questions from the selection of 2 short-description questions of the 3 given questions. From the interview results, it was found that student B doubted that it was similar to the selection of multiple-choice questions. Besides, considering the cognitive step 1 of the student B’s answer, it seemed incomplete in writing the simple form of the given problem, so that this gave impact to the next cognitive step.

Interestingly from the answer of this cognitive step, students A, B, and C did not use the $C_r^n$ symbol but directly solved it mathematically. It was because during the learning by using LIT teaching materials, the students were required to construct their own knowledge, not using ready-made formulas. Thus, they were accustomed to solving the problems without standard formula, where these meaningless formulas can contribute to the existence of their fear of mathematics.

In the cognitive step 3 (the stage of interpreting, validating, and exposing), student A (see Figure 1) was able to apply the concept of combinations and multiplication rules. In determining the final answer, subject A actually applied the multiplication rules, i.e., $C_3^5 \cdot C_2^3 = 10 \cdot 3 = 30$. Student A performed the validation of the calculated results obtained by matching the command on the problem. It showed that in validating and exposing the results of the problem solving, student A only related the concept or experience to the answer obtained from the problem with a simple command on the problem being solved. From the interview results, student A explained that the rules of multiplication were used with the reason that the problem of choosing the multiple-choice questions and the short-description ones was solved simultaneously, and the formed choice was a case of recurring sum or multiplication.
In cognitive step 3, student B (see Figure 2) was able to apply the concept of combinations and multiplication rules. In determining the final answer, student B applied the multiplication rules because the answer to the cognitive step 2 was incorrect, so the final result of the student B’s answer was incorrect. Student B performed the validation of the calculated results obtained by matching the command on the problem solved. It showed that in validating and exposing the results of the problem solving, student B only related the concept or experience that belongs to the answer obtained from the problem to a simple command on the problem solved.

Meanwhile, student C mistakenly applied the concept of combinations and the multiplication rules (see Figure 3). In determining the final answer, student C applied the addition rules, so the final result of the student C’s answer was incorrect. Student C did not validate the calculation results obtained by matching the command on the problem solved. The interview results with student C showed that student C has not been able to distinguish a multiplications or addition case in the form of a story. When asked whether the multiple-choice questions and the short-description questions were done at the same time or by choice (working on the multiple-choice questions or short-description only)? Student C also replied that they were done at the same time. However, student C still doubted the understanding achieved.

The process of horizontal mathematization manifested in the cognitive step 1 to the cognitive step 2, and the process of vertical mathematization manifested in the cognitive step 2 to the cognitive step 3. Thus, from the examples taken from the students, student A was able to achieve the process of mathematization well, while students B and C still encountered obstacles in the process of vertical mathematization. On the other hand, viewed from their ability category, the medium-achiever student achieved a better process of vertical mathematization than the low-achiever student.

The preliminary research done by the researchers [10], it was obtained that the level of students’ errors in completing the combinations problem in the cognitive steps 2 and 3 reached 58%. This means that the achievement of the students’ mathematical modeling ability has not reached 50%, while the results obtained in this study showed that they have reached up to 79% in the cognitive step 2 and 62% in the cognitive step 3. From these results, it can be seen that the students who learnt by using LIT teaching materials have developed their mathematical modeling ability better; from the early cognitive stage to the advanced cognitive stage.

The combinatorial LIT teaching materials used in this learning are built on the elements of "guided reinvention" and "mathematizing" [15,16]. Guided reinvention is a process of first allowing students to develop informal strategies and ways to model these approaches for solving problems, and then, by critically examining both these strategies and models, encouraging students to develop more sophisticated, formal, conventional and abstract strategies and algorithms [17].

4. Conclusion
Overall, the students achieved all the cognitive steps in the modeling process above 79%. The results mean that both horizontal and vertical mathematization processes have been achieved well. However, the low-ability student’s achievement of modeling ability was still small; especially in the final cognitive step. This means that the low-ability student has done horizontal and vertical mathematization, although both were not completely done, and the results were still incorrect.
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