Conformal anomaly of (2,0) tensor multiplet in six dimensions and AdS/CFT correspondence

F. Bastianelli\textsuperscript{a}, S. Frolov\textsuperscript{b,*} and A.A. Tseytlin\textsuperscript{c,†}

\textsuperscript{a}Dipartimento di Fisica, Università di Bologna,
V. Irnerio 46, I-40126 Bologna
and
INFN, Sezione di Bologna, Italy

\textsuperscript{b}Department of Physics and Astronomy
University of Alabama, Box 870324
Tuscaloosa, Alabama 35487-0324, USA

\textsuperscript{c}Department of Physics
The Ohio State University
Columbus, OH 43210-1106, USA

Abstract

We compute the conformal anomaly in the free $d = 6$ superconformal (2,0) tensor multiplet theory on generic curved background. Up to a trivial covariant total-derivative term, it is given by the sum of the type A part proportional to the 6-d Euler density, and the type B part containing three independent conformal invariants: two $\mathcal{C}\mathcal{C}\mathcal{C}$ contractions of Weyl tensors and a $\mathcal{C}\nabla^2\mathcal{C} + \ldots$ term. Multiplied by the factor $4N^3$, the latter Weyl-invariant part of the anomaly reproduces exactly the corresponding part of the conformal anomaly of large $N$ multiple M5-brane (2,0) theory as predicted (hep-th/9806087) by AdS\textsubscript{7} supergravity on the basis of AdS/CFT correspondence. The coefficients of the type A anomaly differ by the factor $\frac{4}{7} \times 4N^3$, so that the free tensor multiplet anomaly does not vanish on a Ricci-flat background. The coefficient $4N^3$ is the same as found (hep-th/9703040) in the comparison of the tensor multiplet theory and the $d = 11$ supergravity predictions for the absorption cross-sections of gravitons by M5 branes, and in the comparison (hep-th/9911135) of 2- and 3-point stress tensor correlators of the free tensor multiplet with the AdS\textsubscript{7} supergravity predictions. The reason for this coincidence is that the three Weyl-invariant terms in the anomaly are related to the $h^2$ and $h^3$ terms in the near flat-space expansion of the corresponding non-local effective action, and thus to the 2-point and 3-point stress tensor correlators in flat background. At the same time, the type A anomaly is related to the $h^4$ term in the non-local part of the effective action, i.e. to a certain structure in the 4-point correlation function of the stress tensors. It should thus capture some non-trivial dynamics of the interacting theory. This is different from what happens in the $d = 4$ SYM case where the type B and type A anomalies are related to the 2-point and 3-point stress tensor correlators.

\textsuperscript{*}Also at Steklov Mathematical Institute, Moscow.

\textsuperscript{†}Also at Lebedev Physics Institute, Moscow and Imperial College, London.
1 Introduction and summary

While the low energy dynamics of a single M5 brane is described by the free $d = 6$, $\mathcal{N} = (2, 0)$ tensor multiplet, the low energy theory describing $N$ coincident M5 branes remains rather mysterious. One of the key predictions of the supergravity description of multiple M5 branes is that the entropy [1] and the 2-point stress tensor correlators [2, 3] of the large $N$ theory should scale as $N^3$. Further quantitative information about this interacting $(2,0)$ conformal theory can be obtained using the AdS/CFT correspondence [4, 5, 6]. In the large $N$ limit this leads directly to the analysis of $d = 11$ supergravity compactified on $AdS_7 \times S^4$. In particular, spectrum of the chiral operators, some of their 2- and 3-point functions and the structure of the anomalies have been studied [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20].

In spite of the lack of a useful field-theoretic description of the large $N$ $(2,0)$ theory, it is interesting to compare its properties to those of a $d = 6$ free conformal theory of a number $\sim N^3$ of tensor multiplets (after all, the free $(2,0)$ tensor multiplet theory is the only $d = 6$ superconformal theory with the right symmetry properties which is known explicitly). The idea is to try to follow the pattern which worked in the case of the D3 brane theory where certain features of the strong coupling large $N$ $d = 4$, $\mathcal{N} = 4$ SYM theory as described by $AdS_5 \times S^5$ supergravity can be reproduced by a free theory of $N^2$ vector multiplets.

In a previous paper [21], we have found that the 2- and 3-point correlation functions of the stress tensor of $(2,0)$ theory as predicted by the $AdS_7 \times S^4$ supergravity [22, 15] are exactly the same as in the theory of $4N^3$ free tensor multiplets. The remarkable coefficient $4N^3$ is the same as found earlier in [2] in the comparison of the M5 brane world volume theory and the $d = 11$ supergravity expressions for the absorption cross-sections of longitudinally polarized gravitons by $N$ M5 branes. This is not surprising since the ratios of the predictions for the 2-point stress tensor correlators and the absorption cross sections should be the same on the basis of unitarity [3, 1]. That the same coefficient appears also in the ratio of the 3-point correlators (which in general have a complicated structure parametrized by 3 independent constants [23]) is quite surprising and is likely to be a consequence of the extended $d = 6$ supersymmetry of the theory in question.

Here we extend such a comparison to conformal anomalies in external $d = 6$ metric. On the supergravity side of the AdS/CFT correspondence the conformal anomaly of the large $N$ M5 brane theory was already found in [11] (see also [24]). Below we compute the conformal anomaly of a free $d = 6$, $\mathcal{N} = (2, 0)$ tensor multiplet which contains 5 scalars, 2 Weyl fermions and a chiral two-form.

In general, the trace anomaly in the stress tensor of a classically Weyl-invariant theory in
$d = 2k$ dimensions has the following structure \cite{25,26,27,28,29}: $< T > = A + B + D$, where 
$A = aE_d$ is proportional to the Euler density in $d$ dimensions (i.e. is a total derivative of a non-covariant expression), 
$B = \sum_n c_n I_n$ is a sum of independent Weyl invariants, i.e. Weyl tensor contractions with extra conformal derivative operators, $(C_{...})^k$, ..., $C_{...}(\nabla^{k-2} + ...)C_{...}$, and 
$D = \nabla_i J^i$ is a total derivative of a covariant expression. Only type A and type B anomalies are genuine (with the latter determining the UV related scale anomaly), while the type D one is ambiguous (renormalization scheme dependent) as it can be changed by adding local covariant but not Weyl-invariant counterterms to the effective action. In 6 dimensions \cite{27,28}

\begin{align}
< T > &= aE_6 + (c_1 I_1 + c_2 I_2 + c_3 I_3) + \nabla_i J^i , \quad (1.1)
\end{align}

where

\begin{align}
E_6 &= -\epsilon_6 \epsilon_6 RRR , \\
I_1 &= C_{amnb} C^{mnij} C_{ij}^a b , \\
I_2 &= C_{ab}^{mn} C_{mn}^{ij} C_{ij}^a b , \\
I_3 &= C_{mabc} \left( \nabla^2 \delta^m_n + 4R^m_n - \frac{6}{5} R \delta^m_n \right) C_{mabc} + \text{total derivative} . \quad (1.3)
\end{align}

Computing the conformal anomalies of the fields in the free tensor multiplet and comparing the resulting coefficients to the supergravity prediction \cite{11} for the anomaly of the (2,0) theory we have found that

\begin{align}
a^{(2,0)} &= \frac{16}{7} N^3 a^{\text{(tens.)}} , \\
c_n^{(2,0)} &= 4N^3 c_n^{\text{(tens.)}} . \quad (1.4)
\end{align}

Once again, the set of $4N^3$ tensor multiplets reproduces exactly the type B or scale anomaly of the (2,0) theory! However, the coefficients of the type A anomaly then differ.

The ratio $4N^3$ of the $c_n$ coefficients is, in fact, in direct correspondence with the result for the ratio of the 2- and 3-point correlators of the stress tensor found in \cite{21}. At the same time, the coefficient $a$ of the Euler density term in the anomaly turns out to be related to a coefficient of a certain structure in the 4-point correlation function of the stress tensor and should thus reflect some of the non-trivial dynamics of the interacting theory.\footnote{Note that the type A anomaly coefficient $a$ (in any dimension) plays a special role from the point of view of the supergravity analysis \cite{21}.}

To appreciate this novel feature of the $d = 6$ theory it is useful to compare the above results with what happens in the $d = 4$ case – the $\mathcal{N} = 4$ super Yang–Mills theory. In $d = 4$ the type B anomaly contains just one independent term proportional to the square of the Weyl tensor whose coefficient is directly related to the one in the 2-point function $< TT >$ of the stress tensor. The type A (Euler) anomaly is instead related to the 3-point correlation function $< TTT >$. Thus known non-renormalization theorems for the 2- and 3-point functions of the stress tensor multiplet guarantee that the trace anomaly of $N^2$ free
$N = 4$ vector multiplets should reproduce that of the full interacting non-abelian theory (see [30, 4]).

In $d = 6$ the coefficient $a$ is related to the 4-point function and thus there is no reason to expect that it should not be renormalized.\footnote{If the $d = 6$ free and interacting CFT’s discussed above could be linked by a renormalization group flow preserving maximal supersymmetry, then our results would suggest that only the coefficient $a$ of type A anomaly can flow. However, it is difficult to see how such picture could be realized since the interacting theory at large $N$ does not have suitable scalar operators of dimensions $\Delta \leq 6$ which could be used to deform the theory (the only candidates are charged under the R symmetry and would break maximal supersymmetry). Similarly, the cohomological analysis of [31] indicates that a theory containing a free chiral two-form field cannot be continuously deformed in a non-trivial manner.}

It would be interesting to see if the $R^4$ correction to the $d = 11$ supergravity action generates an order $N$ correction to the coefficient $a$ in the supergravity expression for the conformal anomaly, like it does in the entropy of multiple M5 branes [32].

The above mentioned correspondence between particular terms in the conformal anomaly and correlation functions of stress tensor on flat background can be understood by studying the relation between the type A and type B conformal anomalies and corresponding terms in the effective action following [33, 28, 29, 34]. In a general even dimension $d = 2k$ the Weyl-invariant terms in type B part of the conformal anomaly $(C_{\ldots})^k, \ldots, C_{\ldots}(\nabla^{k-2} + \ldots)C_{\ldots}$ can be obtained by the Weyl variation from the non-local scale-dependent terms in the effective action like \( \int (C_{\ldots})^{k-1} \ln(\mu^{-d} \Delta_d)C_{\ldots}, \ldots, \int C_{\ldots} \Delta_d \ln(\mu^{-d} \Delta_d)C_{\ldots} \). Here $\Delta_d = \nabla^d + \ldots$ is an appropriate ‘Weyl-covariant’ operator acting on Weyl tensor $[33]$. Expanded near flat space, $g_{mn} = \delta_{mn} + h_{mn}$, these terms start with $h^k, \ldots, h^2$, respectively, i.e. correspond to particular structures in the $k-, \ldots, 2-$ point correlators of the stress tensor in flat background, respectively.

In the case of $d = 4$, i.e. $k = 2$, the coefficient of the type B anomaly $(C^2)$ is thus correlated with the coefficient in the 2-point function $< TT >$. In the case of $d = 6$, i.e. $k = 3$, the coefficient $c_3$ of the $I_3$ term in (1.1) corresponds to the one in the 2-point function, while the two other coefficients $c_1, c_2$ should be directly related to the two remaining independent coefficients in the generic $d = 6$ CFT correlator $< TTT >$.\footnote{In a generic $d = 6$ CFT the 3-point stress tensor correlator depends on 3 arbitrary parameters but one combination of them is related by Ward identity to the coefficient in the 2-point function [23].}

As for the type A anomaly, the corresponding term in the effective action can be constructed by integrating the conformal anomaly like it was done in 2 [35] and 4 [36, 37] dimensions. One can introduce the modified Euler density [34] by combining the type A anomaly with a particular type D anomaly, $\tilde{E}_d = E_d + \nabla_i \tilde{J}^i$, where $\tilde{J}^i$ is a covariant expression such that the Weyl variation $\delta g_{mn} = \phi g_{mn}$ of $\tilde{E}_d$ is proportional to $\Delta_4 \phi$ where
$\Delta_d = \nabla^d + ...$ is the Weyl-invariant operator acting on scalars (see, e.g., [38] and refs. there). Then the corresponding term in the effective action is \[ \int \tilde{E}_d \frac{1}{\Delta_d} \tilde{E}_d. \] Expanding this term near flat space and discarding local terms (which correspond to contact terms in the stress tensor correlators) it is possible to argue that the leading non-local structure with single $\partial^{-2}$ pole contains $k + 1$ graviton factors. Thus the type A anomaly term is related to the $\frac{d}{2} + 1$ point correlator of stress tensors, i.e. to 3-point correlator in $d = 4$, 4-point correlator in $d = 6$, etc.

Coming back to (1.4), the disagreement of the total expressions for the conformal anomalies of the free tensor multiplet and interacting (2,0) CFT can be easily seen using the results which already existed in the literature. Choosing a Ricci-flat $d = 6$ background one finds that (2,0) theory anomaly found in [11] vanishes, but the combined anomaly of the fields in the tensor multiplet is non-zero. For $R_{mn} = 0$ the $d = 6$ anomaly (1.1) depends (modulo a covariant total derivative term) only on 2 coefficients and can be written in the form \[ \int \tilde{E}_6 \frac{1}{\Delta_6} \tilde{E}_6. \] The coefficient $s_2$ is proportional to the graded number of degrees of freedom and thus vanishes (5 + 3 − 2 × 4 = 0) for the tensor multiplet. Computing the coefficient $s_1$ by combining the known results for a $d = 6$ scalar [11, 12], spinor [43] and the antisymmetric tensor [39] (using that the conformal anomaly of the chiral 2-form field is half of the anomaly of a non-chiral 2-form) one finds that (see eqs. (4.21),(4.39) in [39]) $s_1 = -\frac{1}{32} \times \frac{1}{12}$. Thus, in contrast to the $d = 4$ case where the conformal anomaly of the $N = 4$ vector multiplet vanishes on the Ricci flat background, there does not seem to exist a free $d = 6$ conformal matter theory which shares the same property.

Our aim below in Section 2 will be to find the scalar, spinor and 2-form anomalies without assuming $R_{mn} = 0$ and thus to be able to compare the tensor multiplet anomaly to the supergravity prediction [11] for the anomaly of the (2,0) theory. Our starting point will

---

4For example, in $d = 4$ \[ \Delta_4 \sim \partial^4 + \partial R_0 \partial + ... , \] so that \[ \int \tilde{E}_4 \frac{1}{\Delta_4} \tilde{E}_4 \sim \int \tilde{R_0} \partial^{-2} \partial R_0 \partial^2 R_0 + ... . \] In $d = 6$ $\tilde{E}_6$ has the structure (see [34]) $\tilde{E}_6 = \partial^{-2} \partial R_0 \partial^2 R_0 + ...$. The coefficient $s_2$ is proportional to the graded number of degrees of freedom and thus vanishes (5 + 3 − 2 × 4 = 0) for the tensor multiplet. Computing the coefficient $s_1$ by combining the known results for a $d = 6$ scalar [11, 12], spinor [43] and the antisymmetric tensor [39] (using that the conformal anomaly of the chiral 2-form field is half of the anomaly of a non-chiral 2-form) one finds that (see eqs. (4.21),(4.39) in [39]) $s_1 = -\frac{1}{32} \times \frac{1}{12}$. Thus, in contrast to the $d = 4$ case where the conformal anomaly of the $N = 4$ vector multiplet vanishes on the Ricci flat background, there does not seem to exist a free $d = 6$ conformal matter theory which shares the same property.

5In the notation of $\tilde{R}_6 = -32 E$ with the Euler number being $\chi = \frac{1}{(4\pi)^2} \int d^8x \sqrt{\gamma} \frac{1}{3} E$ and the invariants called $I_1, I_2$ in [38] are $A_{16}, A_{17}$, or, for $R_{mn} = 0, I_2, -I_1$ in the present paper.

6It is easy to check that this anomaly cannot be cancelled by adding, e.g., a non-dynamical 5-form field which carries no degrees of freedom ($s_2 = 0$) but does produce a non-trivial conformal anomaly \[ \int \tilde{E}_5 \frac{1}{\Delta_5} \tilde{E}_5, \] $s_1 = -\frac{1}{32} \times \frac{1}{2}$. Duality rotation of scalars into 4-form fields also does not help ($s_1(4\text{-form}) - s_1(0\text{-form}) = \frac{1}{32} \times \frac{1}{4}$), and, in any case, the duality transformation is not consistent with conformal invariance.

7Note, however, that the combined Seeley coefficient $b_6$ of the fields of the $d = 11$ supergravity or its reduction to 6 dimensions does vanish [39].

8Possible importance of theories in which the coefficients $a$ and $c_i$ are related in such a way that the anomaly vanishes for $R_{mn} = 0$ was advocated in [43].
be the general expression \cite{41} for the corresponding Seeley coefficient $b_6$ (sometimes called also $a_3$) of the second order Laplacian. While the scalar field anomaly was computed in the past \cite{41,5,7,18} (though was not correctly put into the required conformal basis form), our explicit expressions for the spinor and the 2-form anomalies are new.

Appendix A contains conventions and definitions of the basic curvature invariants in $d = 6$. In Appendix B we present the full expression for the Seeley coefficient $b_6$ taken from \cite{41} but written in a slightly different form. In Appendix C we give the results for the $b_6$ coefficients for the 1-form and 2-form fields in a space of general dimension $d$.

\section{Conformal anomaly of $d = 6$ (2,0) tensor multiplet}

The low-energy effective theory of a single M5-brane is described by the (2,0) tensor multiplet consisting of 5 scalars $X^a$, an antisymmetric tensor $B_{ij}$ with (anti)selfdual strength and 2 Weyl fermions $\psi^I_L$. It is sufficient for the purposes of computing the conformal anomaly to consider the non-chiral (2,2) conformal model described by the following action

$$S = \int d^6x \sqrt{g} \left( -\frac{1}{12} H_{ijk}^2 - \frac{1}{2} \nabla_i X^a \nabla^i X^a - \frac{1}{10} X^a X^a R + i \bar{\psi}^I \gamma^m \nabla_m \psi^I \right),$$

(2.5)

where $H_{ijk} = \partial_i B_{jk} + \partial_j B_{ki} + \partial_k B_{ij}$, $i,j,k = 1,...,6$, $\alpha = 1,...,10$, and $I = 1,2$. The trace anomaly of the (2,0) tensor multiplet is then equal to $1/2$ of the trace anomaly of the (2,2) multiplet. Indeed, we can consistently disregard the gravitational anomalies of the (2,0) multiplet related to the imaginary part of the chiral 2-form and Weyl spinor determinants and focus only on their real part leading to the trace anomalies (see \cite{49,21}). The anomaly of the (2,2) theory is given by the sum of trace anomalies of one non-chiral 2-form, 10 conformal scalar fields and 2 Dirac fermions which we shall compute separately below.

\subsection{Conformal anomaly as Seeley-DeWitt coefficient}

We begin by recalling the relation of free-theory conformal anomaly to the Seeley-DeWitt coefficients. Consider a one-loop approximation to a model of a bosonic field $\phi$ taking values in a smooth vector bundle $V$ over a compact smooth Riemannian manifold $M$ of dimension $d$. The partition function and the effective action of the model are given by

$$Z = \int d\phi \, e^{-\frac{1}{2} \int_M \phi \Delta \phi}, \quad \Gamma = \frac{1}{2} \log \det \Delta,$$

$$\Delta = -\nabla^2 - E,$$

(2.6)

\footnote{While the definition of the partition function of a chiral p-form theory on generic manifolds is subtle (see \cite{50,61}) the coefficients in the local trace anomaly are universal (cannot depend on details of space-time topology) and can be determined, e.g., by a Feynman diagram calculation near flat space-time.}
where $\nabla$ is a covariant derivative on $V$ and the matrix function $E$ is an endomorphism of $V$. It is well-known that the trace anomaly is related to the logarithmically divergent part of the effective action. Using the Seeley-DeWitt asymptotic expansion \[52, 53\]

$$\text{Tr} e^{-s\Delta} \sim \sum_{p=0}^{d} s^{\frac{d(p-d)}{2}} \int_M d^d x \sqrt{g} b_p$$

we get the logarithmically divergent term in the effective action

$$\Gamma_{\infty} = -\frac{1}{2} \log \frac{L^2}{\mu^2} \int_M d^d x \sqrt{g} b_d ,$$

where $L \to \infty$ is an UV cut-off. The trace anomaly of the stress tensor is then equal to $b_d$:

$$\langle T^m_m(x) \rangle = b_d(x) . \quad (2.7)$$

Thus to find the conformal anomaly of the $d = 6$ tensor multiplet we need to know the coefficients $b_{d}$ for various second order Laplace operators corresponding to the fields in (2.7).

The coefficient $b_{6}$ was explicitly computed for any operator of the form (2.6) in \[41\], and can be written as follows

$$b_{6}(\Delta) = \frac{1}{(4\pi)^3} \text{tr}_V \left[ 18 A_1 + 17 A_2 - 2 A_3 - 4 A_4 + 9 A_5 
+ 28 A_6 - 8 A_7 + 24 A_8 + 12 A_9 + \frac{35}{9} A_{10} - \frac{14}{3} A_{11} + \frac{14}{3} A_{12}
- \frac{208}{9} A_{13} + \frac{64}{3} A_{14} - \frac{16}{3} A_{15} + \frac{44}{9} A_{16} + \frac{80}{9} A_{17}
+ 14 \left( 8 V_1 + 2 V_2 + 12 V_3 - 12 V_4 + 6 V_5 - 4 V_6 + 5 V_7 
+ 6 V_8 + 60 V_9 + 30 V_{10} + 60 V_{11} + 30 V_{12} + 10 V_{13} + 4 V_{14}
+ 12 V_{15} + 30 V_{16} + 12 V_{17} + 5 V_{18} - 2 V_{19} + 2 V_{20} \right) \right], \quad (2.8)$$

where the invariants $A_s$ and $V_p$ (depending on the metric tensor, the connection curvature tensor and the endomorphism $E$) are listed in the Appendix A. An explicit expression for $b_{6}$ can be found in Appendix B.

As already discussed in the Introduction, the conformal anomaly in a classically Weyl-invariant $d = 6$ theory, or the coefficient $b_{6}$ for a conformally invariant kinetic operator, must have the form \[27, 28, 29\]

$$b_{6} = a E_{6} + c_1 I_1 + c_2 I_2 + c_3 I_3 + \nabla_i J^i . \quad (2.9)$$

Here the first term is the type A anomaly proportional to the Euler density polynomial

$$E_{6} = -\epsilon_{m_1 n_1 m_2 n_2 m_3 n_3} \epsilon^{a_1 b_1 a_2 b_2 a_3 b_3} R_{a_1 b_1}^{m_1 n_1} R_{a_2 b_2}^{m_2 n_2} R_{a_3 b_3}^{m_3 n_3}$$
while the next three terms represent the type B anomalies that are combinations of the following Weyl invariants

\[
I_1 = \frac{19}{800} A_{10} - \frac{57}{160} A_{11} + \frac{3}{40} A_{12} + \frac{7}{16} A_{13} + \frac{9}{8} A_{14} - \frac{3}{4} A_{15} - A_{17},
\]

\[
I_2 = \frac{9}{200} A_{10} - \frac{27}{40} A_{11} + \frac{3}{10} A_{12} + \frac{5}{4} A_{13} + \frac{3}{2} A_{14} - 3 A_{15} + A_{16},
\]

\[
I_3 = \frac{11}{50} A_{10} + \frac{27}{10} A_{11} - \frac{6}{5} A_{12} - 3 A_{13} - 4 A_{14} + 4 A_{15} + \frac{1}{10} A_6 - A_7 + A_9 + \nabla_i J^i,
\]

where

\[
C_{abcd} = R_{abcd} - \frac{1}{4} (g_{ac} R_{bd} + g_{bd} R_{ac} - g_{ad} R_{bc} - g_{bc} R_{ad}) + \frac{1}{20} (g_{ac} g_{bd} - g_{ad} g_{bc}) R
\]

is the Weyl tensor in 6 dimensions,

\[
\nabla_i J^i = 5 C_5 - 8 C_7 = 3 A_3 - 6 A_4 + 3 A_5 + \frac{1}{2} A_6 - 5 A_7 + 5 A_9 + 2 A_{13} - 2 A_{14} - 4 A_{15} + 2 A_{16} + 8 A_{17},
\]

and the invariants \(C_k\) are defined in Appendix A. The invariant \(I_3\) was defined up to the total derivative term \(\nabla_i J^i\) in [34] (similar invariants in [27, 28, 38] are linear combinations of \(I_3\) with the other invariants), and is related to the invariant \(\Omega_6\) used in [17] as \(I_3 = 3 \Omega_6 + 16 I_1 - 4 I_2\). Finally, the last term in eq. (2.9) is a total derivative of a covariant expression which can be cancelled by the Weyl variation of a finite local covariant counterterm. Thus, only the coefficients of the first four terms in (2.9) have unambiguous (scheme-independent) meaning and will be of our main interest below.

### 2.2 Conformal anomaly of a scalar field

In the simplest case of a \(d = 6\) conformal scalar field the Laplace operator \(\Delta\) is given by

\[
\Delta_S = -\nabla^2 + \frac{1}{5} R,
\]

where the connection in \(\nabla\) is trivial (\(F_{ij} = 0\)), and the endomorphism \(E\) is

\[
E = -\frac{1}{5} R.
\]
A straightforward calculation based on (2.8) gives the trace anomaly of the conformal scalar as

\[ A_S = \langle T_m^{Sm}(x) \rangle = b_6^S(x) = \]
\[ = \frac{1}{(4\pi)^3 7!} \left( \frac{6}{5} A_1 + \frac{1}{5} A_2 - 2 A_3 - 4 A_4 + 9 A_5 \right. \]
\[ - 8 A_7 + \frac{8}{5} A_8 + 12 A_9 - \frac{7}{225} A_{10} + \frac{14}{15} A_{11} - \frac{14}{15} A_{12} - \frac{32}{45} A_{13} \]
\[ \left. - \frac{16}{15} A_{14} - \frac{16}{3} A_{15} + \frac{44}{9} A_{16} + \frac{80}{9} A_{17} \right) . \]  

This formula can be rewritten in the form (2.9) by using the identities from Appendix A:

\[ A_S = \frac{1}{(4\pi)^3 7!} \left( -\frac{5}{72} E_6 - \frac{28}{3} I_1 + \frac{5}{3} I_2 + 2 I_3 + \nabla_i J^i \right) , \]  

where

\[ \nabla_i J^i = \frac{6}{5} C_1 - \frac{2}{5} C_2 + 4 C_3 + \frac{12}{5} C_4 + \frac{17}{5} C_6 + 12 C_7 . \]

Note that our expression (2.17) differs from the one derived in [47].

2.3 Conformal anomaly of a Dirac fermion

The square of the Dirac operator gives the following second order differential operator \( \Delta_F \)

\[ \Delta_F = -(\nabla)^2 = -\nabla^2 + \frac{1}{4} \mathbf{R} \cdot 1 . \]  

(2.18)

The calculation of the corresponding \( b_6 \) coefficient (2.8) gives the following expression for the trace anomaly of a Dirac fermion (we account for the Fermi statistics by reversing the sign of \( b_6 \))

\[ A_F = \langle T_m^{Fm}(x) \rangle = -b_6^F(x) = \]
\[ = -\frac{8}{(4\pi)^3 7!} \left( -3 A_1 + \frac{5}{4} A_2 - 9 A_3 + 3 A_4 - 5 A_5 + \frac{7}{2} A_6 \right. \]
\[ - 8 A_7 - 4 A_8 - 9 A_9 - \frac{35}{72} A_{10} + \frac{7}{3} A_{11} + \frac{49}{24} A_{12} + \frac{44}{9} A_{13} \]
\[ \left. - \frac{20}{3} A_{14} + \frac{5}{3} A_{15} - \frac{101}{18} A_{16} - \frac{109}{9} A_{17} \right) . \]  

(2.19)
By using the identities from Appendix A we can rewrite this in the form (2.9)

\[ \mathcal{A}_F = \frac{1}{(4\pi)^3} \frac{1}{7!} \left( -\frac{191}{72} E_6 - \frac{896}{3} I_1 - 32 I_2 + 40 I_3 + \nabla_i J^i \right), \tag{2.20} \]

where

\[ \nabla_i J^i = 24C_1 - \frac{148}{15} C_2 + 136C_3 + 48C_4 - 168C_5 + 96C_6 + 352C_7. \]

### 2.4 Conformal anomaly of a 2-form field

To find the conformal anomaly of an antisymmetric tensor field we use a covariant gauge fixing with the standard triangle-like ghost structure [54]. This leads to the following representation for the partition function

\[ Z^{(2)} = (\det \Delta^{(2)})^{-\frac{1}{2}} \det \Delta^{(1)} (\det \Delta^{(0)})^{-\frac{3}{2}}, \tag{2.21} \]

where the Hodge-DeRham operators \( \Delta^{(p)} \) are defined as

\[
\begin{align*}
(\Delta^{(2)})_{mn}^{ab} &= -\nabla^2 \delta_{mn}^{ab} + 2 R_{[m}^{[a} \delta_{n]}^{b]} - R_{mn}^{ab}, \\
(\Delta^{(1)})_m^n &= -\nabla^2 \delta_m^n + R_m^n, \\
\Delta^{(0)} &= -\nabla^2.
\end{align*}
\]

As follows from (2.21), the conformal anomaly of a 2-form field \( B_{mn} \) is given by

\[ \mathcal{A}_B = b_6^{(2)} - 2b_6^{(1)} + 3b_6^{(0)}, \tag{2.22} \]

where \( b_6^{(p)} \) are the Seeley-DeWitt coefficients of the operators \( \Delta^{(p)} \).

The coefficient \( b_6^{(0)} \) is obtained from (2.8) by dropping out all the invariants \( V_p \) (in this case of \( \Delta^{(0)} \) the connection and \( E \) are trivial)

\[
\begin{align*}
b_6^{(0)} &= \frac{1}{(4\pi)^3} \frac{1}{7!} \left( 18A_1 + 17A_2 - 2A_3 - 4A_4 + 9A_5 \\
&\quad + 28A_6 - 8A_7 + 24A_8 + 4A_9 + 35 \frac{1}{9} A_{10} - 14 \frac{1}{3} A_{11} + 14 \frac{1}{3} A_{12} \\
&\quad - 208 \frac{1}{9} A_{13} + 64 \frac{1}{3} A_{14} - 16 \frac{1}{3} A_{15} + 44 \frac{1}{9} A_{16} + 80 \frac{8}{9} A_{17} \right). \tag{2.23}
\end{align*}
\]

The coefficient \( b_6^{(1)} \) of the Hodge-DeRham operator \( \Delta^{(1)} \) acting on 1-forms is found by taking into account that the connection is defined by the Christoffel symbols so that

\[ (F_{ij})_a^b = R_{ij}^a_0 b, \]
while the endomorphism \( E \) is
\[
E_a^b = -R_a^b.
\]

Computing the invariants \( V_p \) and expressing them in terms of \( A_s \), we get for \( d = 6 \)
\[
b_6^{(1)}(x) = \frac{1}{(4\pi)^3 7!} \left(24A_1 - 66A_2 + 352A_3 + 32A_4 - 58A_5 - 140A_6 + 792A_7 + 32A_8 - 96A_9 - \frac{140}{3}A_{10} + 420A_{11} - 70A_{12} - \frac{2600}{3}A_{13} + 16A_{14} + 444A_{15} - \frac{164}{3}A_{16} - \frac{344}{3}A_{17}\right).
\] (2.24)

To compute the coefficient \( b_6^{(2)} \) of the operator \( \Delta^{(2)} \) acting on 2-forms, we note that the curvature tensor of the connection and the endomorphism here are
\[
(F_{ij})^{cd}_{ab} = 2R_{ij[a}^{[c|d]b]} , \\
E^{cd}_{ab} = -2R_{[a}^{[c|d]b] + R_{ab}^{cd} .
\]

By using the formulas for the traces from Appendix C, we find in \( d = 6 \)
\[
b_6^{(2)}(x) = \frac{1}{(4\pi)^3 7!} \left(-66A_1 + 3A_2 - 254A_3 + 164A_4 + 107A_5 + 28A_6 - 120A_7 - 88A_8 + 348A_9 + \frac{595}{3}A_{10} - 2478A_{11} + 518A_{12} + \frac{10384}{3}A_{13} + 4912A_{14} - 4896A_{15} + \frac{2992}{3}A_{16} - \frac{1616}{3}A_{17}\right). 
\] (2.25)

Then from (2.22) we obtain the conformal anomaly of the 2-form field \( B_{ij} \)
\[
\mathcal{A}_B = \frac{1}{(4\pi)^3 7!} \left(-60A_1 + 186A_2 - 964A_3 + 88A_4 + 250A_5 + 392A_6 - 1728A_7 - 80A_8 + 576A_9 + \frac{910}{3}A_{10} - 3332A_{11} + 672A_{12} + \frac{15376}{3}A_{13} + 4944A_{14} - 5800A_{15} + \frac{3364}{3}A_{16} - \frac{848}{3}A_{17}\right). 
\] (2.26)

One can easily check that on a Ricci flat manifold the anomaly coincides with the one found in \[39\].

The identities from Appendix A allow to represent (2.26) in the required form (2.9)
\[
\mathcal{A}_B = \frac{1}{(4\pi)^3 7!} \left( -\frac{221}{4}E_6 - \frac{8008}{3}I_1 - \frac{2378}{3}I_2 + 180I_3 + \nabla_i J^i \right),
\] (2.27)

where
\[
\nabla_i J^i = -60C_1 + \frac{2036}{15}C_2 - 1152C_3 - 120C_4 - 504C_5 - 646C_6 + 856C_7 .
\]

\[10\] We present the coefficients \( b_6^{(1)} \) and \( b_6^{(2)} \) for generic dimension \( d \) of the manifold in Appendix C.
Finally, all is prepared to write down the expression for the conformal anomaly of the chiral (2,0) tensor multiplet

\[ A_{\text{tens.}} = \frac{1}{2} (A_B + 10 A_S + 2 A_F) \]

\[ = \frac{1}{(4\pi)^3 7!} \left( 84A_2 - 420A_3 + 210A_5 + 168A_6 - 840A_7 + 420A_9 + \frac{777}{5}A_{10} \right. \]

\[ - \left. 1680A_{11} + 315A_{12} + 2520A_{13} + 2520A_{14} - 2940A_{15} + 630A_{16} \right) , \] (2.28)

or, in the form (2.9),

\[ A_{\text{tens.}} = \frac{1}{(4\pi)^3 7!} \left( -\frac{245}{8} E_6 - 1680I_1 - 420I_2 + 140I_3 + \nabla_i J^i \right) , \] (2.29)

where

\[ \nabla_i J^i = 56C_2 - 420C_3 - 420C_5 - 210C_6 + 840C_7 . \]

It is easy to see using the identities in Appendix A that for \( R_{mn} = 0 \) this expression agrees with the expression following from [39] which was already mentioned in the Introduction, i.e. \( A_{\text{tens.}} = -\frac{1}{(4\pi)^3 32 \times 12} E_6 \), up to a covariant total derivative term (~ \( C_5 = \frac{1}{2} \nabla^2 (C_{mnkl}) \)).

Let us now compare the result (2.29) with the conformal anomaly of the interacting (2,0) theory describing large number \( N \) of coincident M5 branes as predicted on the basis of AdS/CFT correspondence in [11]. In terms of the invariants we are using here the expression obtained in [11] takes the form (note that it vanishes for \( R_{mn} = 0 \) as it should)

\[ A_{(2,0)} = \frac{4N^3}{(4\pi)^3 7!} \left( -\frac{35}{2} E_6 - 1680I_1 - 420I_2 + 140I_3 + \nabla_i J^i \right) , \] (2.30)

where

\[ \nabla_i J^i = 420C_3 - 504C_4 - 840C_5 - 84C_6 + 1680C_7 . \]

Comparing (2.29) and (2.30) we conclude that up to the common factor \( 4N^3 \) only the coefficient in front of the Euler polynomial is different (the difference in coefficients of total derivative terms is not important since they are scheme-dependent). The interpretation of this result was already discussed in the Introduction.

\[ ^{11} \text{Note that for } R_{mn} = 0 \text{ and ignoring the total derivative term one has the following relations } I_3 = -I_2 + 4I_1, \ E_6 = -32I_2 - 64I_1. \]
Appendix A: Conventions, invariants and identities

We use the following conventions for the curvature tensors:

\[
[\nabla_a, \nabla_b]V^c = R_{ab}{}^c{}_d V^d, \quad R_{ab} = R_{ca}{}^c{}_b, \quad R = R^a_a, \quad [\nabla_a, \nabla_b] \phi = F_{ab} \phi.
\]

The basis of metric invariants is\(^{12}\)

\[
A_1 = \nabla^4 R, \quad A_2 = (\nabla_a R)^2, \quad A_3 = (\nabla_a R_{mn})^2, \quad A_4 = \nabla_a R_{bm} \nabla^b R_{am}, \quad A_5 = (\nabla_a R_{mnij})^2, \\
A_6 = R \nabla^2 R, \quad A_7 = R_{ab} \nabla^2 R^{ab}, \quad A_8 = R_{ab} \nabla_m \nabla^b R_{am}, \quad A_9 = R_{abmn} \nabla^2 R^{abmn}, \quad A_{10} = R^3, \\
A_{11} = R R_{ab}, \quad A_{12} = R R_{abmn}, \quad A_{13} = R_{a}^m R_{m}^i R_{i}^a, \quad A_{14} = R_{ab} R_{mn} R_{amn}, \\
A_{15} = R_{ab} R_{amnl} R_{bml}, \quad A_{16} = R_{ab} R_{mn} R_{ij} R_{ij}^{ab}, \quad A_{17} = R_{ambn} R_{aibj} R_{ij}^{mn}.
\]

Another convenient basis of the metric invariants is obtained by replacing the Ricci tensor \(R_{ij}\) by its traceless part

\[
B_{ij} = R_{ij} - \frac{1}{d} R g_{ij},
\]

and the Riemann tensor – by the Weyl tensor

\[
C_{ijkl} = R_{ijkl} - \frac{1}{d - 2} (g_{jl} B_{ik} - g_{jk} B_{il} + g_{ik} B_{jl} - g_{il} B_{jk}) - \frac{R}{d(d - 1)} (g_{kj} g_{ik} - g_{jk} g_{il}).
\]

Then we get the following 17 invariants \(B_s\)

\[
B_1 = \nabla^4 R, \quad B_2 = (\nabla_a R)^2, \quad B_3 = (\nabla_a B_{mn})^2, \quad B_4 = \nabla_a B_{bm} \nabla^b B^{am}, \quad B_5 = (\nabla_a C_{mnij})^2, \\
B_6 = R \nabla^2 R, \quad B_7 = B_{ab} \nabla^2 B^{ab}, \quad B_8 = B_{ab} \nabla_m \nabla^b B^{am}, \quad B_9 = C_{abmn} \nabla^2 C^{abmn}, \quad B_{10} = R^3, \\
B_{11} = R B_{ab}, \quad B_{12} = R C_{abmn}, \quad B_{13} = B_{a}^m B_{m}^i B_{i}^a, \quad B_{14} = B_{ab} B_{mn} C^{abmn}, \\
B_{15} = B_{ab} C^{amnl} C^{bml}, \quad B_{16} = C_{ab} C_{mn} C_{ij} C_{ij}^{ab}, \quad B_{17} = C_{ab} C_{mn} C_{ab} C_{mn}.
\]

The invariants \(A_s\) are related to \(B_s\) as follows

\[
A_1 = B_1, \quad A_2 = B_2, \quad A_3 = B_3 + \frac{1}{d} B_2, \quad A_4 = B_4 + \frac{d - 1}{d^2} B_2, \\
A_5 = B_5 + \frac{4}{d - 2} B_3 + \frac{2}{d(d - 1)} B_2, \quad A_6 = B_6, \quad A_7 = B_7 + \frac{1}{d} B_6, \\
A_8 = \frac{d}{d - 2} B_8 + \frac{1}{2d} B_6 + \frac{2}{d - 2} B_{14} - \frac{2d}{(d - 2)^2} B_{13} - \frac{2}{(d - 1)(d - 2)} B_{11}, \\
A_9 = B_9 + \frac{4}{d - 2} B_7 + \frac{2}{d(d - 1)} B_6, \quad A_{10} = B_{10}, \\
A_{11} = B_{11} + \frac{1}{d} B_{10}, \quad A_{12} = B_{12} + \frac{4}{d - 2} B_{11} + \frac{2}{d(d - 1)} B_{10}.
\]

\(^{12}\)We use the same notation as in [47]. Note, however, that there are a number of misprints in that paper.
One can show that the following linear combinations

\[
A_{13} = B_{13} + \frac{3}{d} B_{11} + \frac{1}{d^2} B_{10} \\
A_{14} = B_{14} - \frac{2}{d - 2} B_{13} + \frac{2d - 3}{d(d - 1)} B_{11} + \frac{1}{d^2} B_{10} \\
A_{15} = B_{15} + \frac{4}{d - 2} B_{14} + \frac{2(d - 4)}{(d - 2)^2} B_{13} + \frac{1}{d} B_{12} + \frac{4(2d - 3)}{d(d - 1)(d - 2)} B_{11} + \frac{2}{d^2(d - 1)} B_{10} \\
A_{16} = B_{16} + \frac{12}{d - 2} B_{15} + \frac{24}{(d - 2)^2} B_{14} + \frac{8(d - 4)}{(d - 2)^3} B_{13} \\
+ \frac{6}{d(d - 1)} B_{12} + \frac{24}{d(d - 1)(d - 2)} B_{11} + \frac{4}{d^2(d - 1)^2} B_{10} \\
A_{17} = B_{17} - \frac{3}{2d} B_{12} + \frac{3(d - 4)}{2d(d - 1)} B_{11} + \frac{d - 2}{2d^2(d - 1)^2} B_{10}. 
\]

are total derivatives. These are the important identities used in the main text.

The basis of invariants \( V_p \) depending on the curvature \( F_{ij} \) and the endomorphism \( E \) is

\[
V_1 = \nabla_k F_{ij} \nabla^k F^{ij}, \quad V_2 = \nabla_j F_{ij} \nabla^k F^{ik}, \quad V_3 = F_{ij} \nabla^2 F^{ij}, \quad V_4 = F_{ij} F^{jk} F_{k}^i, \\
V_5 = R_{mnij} F^{mn} F_{ij}, \quad V_6 = R_{ijk} F^{in} F_{nk}, \quad V_7 = R F_{ij} F^{ij}, \quad V_8 = \nabla^4 E, \quad V_9 = E \nabla^2 E, \\
V_{10} = \nabla_k E \nabla^k E, \quad V_{11} = V^3, \quad V_{12} = E R^{ij}, \quad V_{13} = R \nabla^2 E, \quad V_{14} = R_{ij} \nabla^i \nabla^j E, \\
V_{15} = \nabla_k R \nabla^k E, \quad V_{16} = E E R, \quad V_{17} = E \nabla^2 R, \quad V_{18} = E R^2, \\
V_{19} = E R_{ij}^2, \quad V_{20} = E R_{ijkl}^2.
\]

**Appendix B: Heat kernel expansion and \( b_6 \) coefficient**

The heat kernel coefficients for a general Laplace operator of the form \( \Delta = -\nabla^2 - E \) with connection of curvature \( F_{ab} \) as defined in appendix A and matrix potential \( E \), were computed, up to and including \( b_6 \), in [41]. For convenience, we present the explicit form of these leading terms in the heat kernel expansion below and use this opportunity to cast this expansion into a form that may be advantageous for certain computational purposes. In principle, one can compute various terms of the heat kernel expansion by using the standard perturbation theory for a quantum mechanical path integral [53, 56]. The latter naturally
separates connected and disconnected particle theory diagrams and suggests the following representation of the heat kernel expansion

\[ \text{Tr}[\sigma(x)e^{-s\Delta}] = \frac{1}{(4\pi s)^d} \text{Tr}[\sigma(x)\sum_{n=0}^{\infty} a_{2n}s^n] = \frac{1}{(4\pi s)^d} \text{Tr}[\sigma(x)\exp(\sum_{n=1}^{\infty} a_{2n}s^n)] . \]

The standard \( b_{2n} \) coefficients are then

\[ b_{2n} = \frac{1}{(4\pi)^d} a_{2n} , \quad a_{2n} = \alpha_{2n} + \beta_{2n} , \]

where \( \alpha_{2n} \) and \( \beta_{2n} \) indicate the parts coming from connected and disconnected quantum mechanical diagrams, respectively. In the above expression \( \sigma(x) \) is an arbitrary function and \( \text{Tr}(\ldots) \equiv \int_M d^dx \sqrt{g} \text{tr} V(\ldots) \). Using the cyclicity of the trace one finds

\[ \beta_0 = 0, \quad \beta_2 = 0, \quad \beta_4 = \frac{1}{2} \alpha_2^2, \quad \beta_6 = \frac{1}{6} \alpha_2^3 + \alpha_2 \alpha_4 . \]

Then the formulas of [41] for \( b_0, b_2, b_4, b_6 \) imply that

\[ \begin{align*}
\alpha_0 & = 1 \\
\alpha_2 & = E + \frac{1}{6} R \\
\alpha_4 & = \frac{1}{6} \nabla^2 \left( E + \frac{1}{5} R \right) + \frac{1}{180} (R_{abmn}^2 - R_{ab}^2) + \frac{1}{12} F_{ab}^2 \\
\alpha_6 & = \frac{1}{7!} \left[ 18 \nabla^4 R + 17 (\nabla_a R)^2 - 2 (\nabla_a R_{mn})^2 - 4 \nabla_a R_{bm} \nabla^b R_{am} \\
& + 9 (\nabla_a R_{mnij})^2 - 8 R_{ab} \nabla^2 R_{ab} + 12 R_{ab} \nabla^a \nabla^b R + 12 R_{abmn} \nabla^2 R_{abmn} \\
& + 8 R_{a}^{m} R_{m}^{i} R_{i}^{a} + 8 R_{ab} R_{mn} \nabla^a \nabla^b R + 16 R_{ab} R_{mn} R_{abmn} \\
& + 8 R_{abmn} R_{mnij} R_{ij}^{ab} - \frac{80}{9} R_{abij} R_{mnij} R_{mn}^{ab} \\
& + \frac{1}{2} \left[ 8 (\nabla_a F_{mn})^2 + 2 (\nabla^a F_{mn})^2 + 12 F_{ab} \nabla^2 F_{ab} - 12 F_{a}^{m} F_{m}^{i} F_{i}^{a} \\
& + 6 R_{abmn} F_{ab} F_{mn} - 4 R_{ab} F_{am} F_{m}^{b} + 6 \nabla^4 E + 30 (\nabla_a E)^2 \\
& + 4 R_{ab} \nabla^a \nabla^b E + 12 \nabla_a R \nabla^a E \right] . \end{align*} \]

In terms of the \( A_s \) and \( V_p \) invariants of Appendix A the expression for \( b_6 \) reads as in eq. (2.8).

**Appendix C: Some \( b_6 \) coefficients in arbitrary \( d \)**

The coefficient \( b_6^{(1)} \) of the Hodge-DeRham operator \( \Delta^{(1)} \) acting on 1-forms in a \( d \)-dimensional manifold is given by

\[ b_6^{(1)} = \frac{1}{(4\pi)^{d+1}} \left( (18d - 84)A_1 + (17d - 168)A_2 + (364 - 2d)A_3 + (56 - 4d)A_4 \right) . \]
\[ + (9d - 112)A_5 + (28d - 308)A_6 + (840 - 8d)A_7 + (24d - 112)A_8 + (12d - 168)A_9 \\
+ \left( -\frac{35d}{9} - 70 \right)A_{10} + (448 - \frac{14d}{3})A_{11} + \left( \frac{14d}{3} - 98 \right)A_{12} + \left( -\frac{208d}{9} - 728 \right)A_{13} \\
+ \left( \frac{64d}{3} - 112 \right)A_{14} + (476 - \frac{16d}{3})A_{15} + \left( \frac{46d}{9} - 84 \right)A_{16} + \left( \frac{80d}{9} - 168 \right)A_{17} \right). \]

To compute the coefficient \( b_6^{(2)} \) of the Hodge-DeRham operator \( \Delta^{(2)} \) acting on 2-forms, we need several relations involving the curvature tensor of the connection and the endomorphism

\[
(F_{ij})_{ab}^c = 2R_{[ij}^{[a} \delta_{b]}^c, \\
E_{ab}^{cd} = -2R_{[a}^{[c} \delta_{b]}^{d]} + R_{ab}^{cd}.
\]

One can show that

\[
\begin{align*}
\text{tr} \, (F_{ij}F_{kl}) &= (2 - d)R_{ijab}R_{klab} \\
\text{tr} \, (F_{ij}F_{kl}F_{mn}) &= (d - 2)R_{ijab}R_{klbc}R_{ca} \\
\text{tr} \, E^2 &= R_{ijab}R_{ijab} + (d - 6)R_{ab}R_{ab} + R^2 \\
\text{tr} \, E^3 &= R_{ijab}R_{abkl}R_{klij} - 6R_{iabc}R_{jabc}R_{ij} - 6R_{ijab}R_{jabc}R_{ib} \\
&+ \left( 10 - d \right)R_{ab}R_{bc}R_{ca} - 3RR_{ab}R_{ab} \\
\text{tr} \, (E F_{ij}F_{ij}) &= R_{ijab}R_{abkl}R_{klij} + (d - 6)R_{iabc}R_{jabc}R_{ij} + RR_{abcd}R_{abcd}.
\end{align*}
\]

Using these relations, we obtain \( b_6^{(2)} \) for an arbitrary \( d \)-dimensional manifold

\[
b_6^{(2)} = \frac{1}{(4\pi)^2} \frac{1}{7!} \left( (9d^2 - 93d + 168)A_1 + \left( \frac{17d^2}{2} - \frac{353d}{2} + 756 \right)A_2 \\
+ (-d^2 + 365d - 2408)A_3 + (-2d^2 + 58d - 112)A_4 + \left( \frac{9d^2}{2} - \frac{233d}{2} + 644 \right)A_5 \\
+ (14d^2 - 322d + 1456)A_6 + (-4d^2 + 844d - 5040)A_7 + (12d^2 - 124d + 224)A_8 \\
+ (6d^2 - 174d + 1176)A_9 + \left( \frac{35d^2}{3} - \frac{1295d}{3} + 560 \right)A_{10} + \left( \frac{7d^2}{3} - \frac{1351d}{3} - 5096 \right)A_{11} \\
+ (\frac{7d^2}{3} - \frac{301d}{3} + 1036)A_{12} + \left( \frac{104d^2}{9} - \frac{6448d}{9} + 8176 \right)A_{13} + \left( \frac{32d^2}{3} - \frac{368d}{3} + 5264 \right)A_{14} \\
+ (-\frac{8d^2}{3} + \frac{1436d}{3} - 7672)A_{15} + \left( \frac{22d^2}{9} - \frac{778d}{9} + 1428 \right)A_{16} + \left( \frac{40d^2}{9} - \frac{1552d}{9} + 336 \right)A_{17} \right). \]

Acknowledgements

F.B. would like to thank D. Anselmi for useful discussions. A.A.T. is grateful to M. Henningson, Yu. Obukhov and K. Skenderis for helpful discussions and correspondence. We are also grateful to H. Osborn for an important comment on the first version of the paper. The work of S.F. was supported by the U.S. Department of Energy under grant No. DE-FG02-96ER40967. The work of A.T. was supported in part by the DOE grant No. DOE/ER/01545-783, the EC TMR programme grant ERBFMRX-CT96-0045, INTAS grant No.96-538 and NATO grant PST.CLG 974965.
References

[1] I.R. Klebanov and A.A. Tseytlin, “Entropy of Near-Extremal Black p-branes,” Nucl. Phys. B475, 164 (1996), hep-th/9604089.

[2] S.S. Gubser, I.R. Klebanov and A.A. Tseytlin, “String theory and classical absorption by three-branes,” Nucl. Phys. B499, 217 (1997), hep-th/9703040.

[3] S.S. Gubser and I.R. Klebanov, “Absorption by branes and Schwinger terms in the world volume theory,” Phys. Lett. B413, 41 (1997), hep-th/9708005.

[4] J. Maldacena, “The large N limit of superconformal field theories and supergravity,” Adv. Theor. Math. Phys. 2, 231 (1998), hep-th/9711200.

[5] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, “Gauge theory correlators from non-critical string theory,” Phys. Lett. B428, 105 (1998), hep-th/9802109.

[6] E. Witten, “Anti-de Sitter space and holography,” Adv. Theor. Math. Phys. 2, 253 (1998), hep-th/9802150.

[7] O. Aharony, Y. Oz and Z. Yin, “M-theory on AdS_p × S^{11-p} and superconformal field theories,” Phys. Lett. B430, 87 (1998), hep-th/9803051.

[8] S. Minwalla, “Particles on AdS(4/7) and primary operators on M(2/5) brane worldvolumes,” JHEP 10, 002 (1998), hep-th/9803053.

[9] R.G. Leigh and M. Rozali, “The large N limit of the (2,0) superconformal field theory,” Phys. Lett. B431, 311 (1998), hep-th/9803068.

[10] E. Halyo, “Supergravity on AdS(4/7) × S(7/4) and M branes,” JHEP 04, 011 (1998), hep-th/9803077.

[11] M. Henningson and K. Skenderis, “The holographic Weyl anomaly,” JHEP 07, 023 (1998), hep-th/9806087.

[12] J.A. Harvey, R. Minasian and G. Moore, “Non-abelian tensor-multiplet anomalies,” JHEP 09, 004 (1998), hep-th/9808060.

[13] H. Awata and S. Hirano, “AdS(7)/CFT(6) correspondence and matrix models of M5-branes,” Adv. Theor. Math. Phys. 3, 147 (1999), hep-th/9812218.

[14] C.R. Graham and E. Witten, “Conformal anomaly of submanifold observables in AdS/CFT correspondence,” Nucl. Phys. B546, 52 (1999), hep-th/9901021; M. Henningson and K. Skenderis, “Weyl anomaly for Wilson surfaces,” JHEP 06, 012 (1999), hep-th/9905163.

[15] G. Arutyunov and S. Frolov, “Three-point Green function of the stress-energy tensor in the AdS/CFT correspondence,” Phys. Rev. D60, 026004 (1999), hep-th/9901121.
[16] R. Corrado, B. Florea and R. McNees, “Correlation functions of operators and Wilson surfaces in the $d = 6$, $(2,0)$ theory in the large $N$ limit,” Phys. Rev. D60, 085011 (1999), hep-th/9902153.

[17] F. Bastianelli and R. Zucchini, “Bosonic quadratic actions for 11D supergravity on $AdS_7/4 \times S_4/7$,” Class. Quant. Grav. 16, 3673 (1999), hep-th/9903161; “Three point functions of chiral primary operators in $d = 3$, $N = 8$ and $d = 6$, $N = (2,0)$ SCFT at large $N$,” Phys. Lett. B467, 61 (1999), hep-th/9907047; “Three point functions for a class of chiral operators in maximally supersymmetric CFT at large $N$,” hep-th/9909179.

[18] H. Nastase, D. Vaman and P. van Nieuwenhuizen, “Consistent nonlinear KK reduction of 11d supergravity on $AdS_7 \times S_4$ and self-duality in odd dimensions,” Phys. Lett. B469, 96 (1999), hep-th/9905075; “Consistency of the $AdS_7 \times S_4$ reduction and the origin of self-duality in odd dimensions,” hep-th/9911238.

[19] M. Nishimura and Y. Tanii, “Local symmetries in the AdS(7)/CFT(6) correspondence,” hep-th/9910192.

[20] C. Imbimbo, A. Schwimmer, S. Theisen and S. Yankelowicz, “Diffeomorphisms and Holographic Anomalies,” hep-th/9910267.

[21] F. Bastianelli, S. Frolov and A. A. Tseytlin, “Three-point correlators of stress tensors in maximally-supersymmetric conformal theories in $d = 3$ and $d = 6$,” hep-th/9911135.

[22] H. Liu and A.A. Tseytlin, “D = 4 super Yang-Mills, D = 5 gauged supergravity, and D = 4 conformal supergravity,” Nucl. Phys. B533, 88 (1998), hep-th/9804083.

[23] H. Osborn and A.C. Petkou, “Implications of conformal invariance in field theories for general dimensions,” Ann. Phys. 231, 311 (1994), hep-th/9307010; J. Erdmenger and H. Osborn, “Conserved currents and the energy-momentum tensor in conformally invariant theories for general dimensions,” Nucl. Phys. B483, 431 (1997), hep-th/9605009.

[24] R. Graham, “Volume and area renormalizations for conformally compact Einstein metrics”, math.DG/9909042.

[25] M. J. Duff, “Observations On Conformal Anomalies,” Nucl. Phys. B125, 334 (1977); “Twenty years of the Weyl anomaly,” Class. Quant. Grav. 11, 1387 (1994), hep-th/9308075.

[26] S. Deser, M. J. Duff and C. J. Isham, “Nonlocal Conformal Anomalies,” Nucl. Phys. B111, 45 (1976).

[27] L. Bonora, P. Pasti and M. Bregola, “Weyl Cocycles,” Class. Quant. Grav. 3, 635 (1986).

[28] S. Deser and A. Schwimmer, “Geometric classification of conformal anomalies in arbitrary dimensions, Phys. Lett. B309, 279 (1993), hep-th/9302047.
[29] D. R. Karakhanian, R. P. Manvelian and R. L. Mkrtchian, “Trace anomalies and cocycles of Weyl and diffeomorphism groups,” Mod. Phys. Lett. A11, 409 (1996), hep-th/9411068. T. Arakelian, D. R. Karakhanian, R. P. Manvelian and R. L. Mkrtchian, “Trace anomalies and cocycles of the Weyl group,” Phys. Lett. B353, 52 (1995); S. D. Odintsov and A. Romeo, “Conformal sector in \( D = 6 \) quantum gravity,” Mod. Phys. Lett. A9, 3373 (1994), hep-th/9410191.

[30] D. Anselmi, D. Z. Freedman, M. T. Grisaru and A. A. Johansen, “Nonperturbative formulas for central functions of supersymmetric gauge theories,” Nucl. Phys. B526, 543 (1998), hep-th/9708047. “Universality of the operator product expansions of SCFT(4),” Phys. Lett. B394, 329 (1997), hep-th/9608123. P.S. Howe, E. Sokatchev and P. C. West, ”3-Point Functions of Chiral Operators in D=4, N=4 SYM at Large N”, Phys. Lett. B444, 341 (1998), hep-th/9808162.

[31] X. Bekaert, M. Henneaux and A. Sevrin, “Deformations of chiral two-forms in six dimensions,” hep-th/9909094. X. Bekaert, “Interactions of chiral two-forms,” hep-th/9911109.

[32] S.S. Gubser, I.R. Klebanov and A.A. Tseytlin, “Coupling constant dependence in the thermodynamics of \( N = 4 \) supersymmetric Yang-Mills theory,” Nucl. Phys. B534, 202 (1998), hep-th/9805156.

[33] S. Deser, “Closed form effective conformal anomaly actions in \( D \geq 4 \),” hep-th/9911129.

[34] D. Anselmi, “Towards the classification of conformal field theories in arbitrary dimension,” hep-th/9908014.

[35] A. M. Polyakov, “Quantum geometry of bosonic strings,” Phys. Lett. B103, 207 (1981); “Quantum Gravity In Two-Dimensions,” Mod. Phys. Lett. A2, 893 (1987).

[36] R.J. Riegert, “A Nonlocal Action For The Trace Anomaly,” Phys. Lett. B134, 56 (1984).

[37] E.S. Fradkin and A.A. Tseytlin, “Conformal Anomaly In Weyl Theory And Anomaly Free Superconformal Theories,” Phys. Lett. B134, 187 (1984).

[38] J. Erdmenger, “Conformally covariant differential operators: Properties and applications,” Class. Quant. Grav. 14, 2061 (1997), hep-th/9704108. J. Erdmenger and H. Osborne, “Conformally covariant differential operators: Symmetric tensor fields,” Class. Quant. Grav. 15, 273 (1998), gr-qc/9708040.

[39] E.S. Fradkin and A.A. Tseytlin, “Quantum Properties Of Higher Dimensional And Dimensionally Reduced Supersymmetric Theories,” Nucl. Phys. B227, 252 (1983).

[40] T. Sakai, “On eigenvalues of Laplacian and curvature of Riemannian manifold”, Tohoku Math. J. 23, 589 (1971).

[41] P.B. Gilkey, “Spectral geometry of a Riemann manifold”, J. Diff. Geom. 10, 601 (1975).

[42] J.S. Dowker, “Single-loop divergences in six dimensions”, J. Phys. A10, L63 (1977).
[43] R. Critchley, “The Trace Anomaly: Results For Spinor Fields In Six-Dimensions,” J. Phys. A A11, 1113 (1978).

[44] M. J. Duff and P. van Nieuwenhuizen, “Quantum Inequivalence Of Different Field Representations,” Phys. Lett. B94, 179 (1980).

[45] D. J. Toms, “Renormalization Of Interacting Scalar Field Theories In Curved Space-Time,” Phys. Rev. D26, 2713 (1982).

[46] D. Anselmi, “Quantum irreversibility in arbitrary dimension,” hep-th/9905003.

[47] T. Parker and S. Rosenberg, “Invariants of conformal Laplacians”, J. Diff. Geom. 25, 199 (1987).

[48] S. Ichinose and N. Ikeda, “Weyl anomaly in higher dimensions and Feynman rules in coordinate space,” J. Math. Phys. 40, 2259 (1999), hep-th/9810256.

[49] L. Alvarez-Gaume and E. Witten, “Gravitational Anomalies,” Nucl. Phys. B234, 269 (1984).

[50] E. Witten, “Five-brane effective action in M-theory,” J. Geom. Phys. 22, 103 (1997), hep-th/9610234; L. Dolan and C. R. Nappi, “A modular invariant partition function for the fivebrane,” Nucl. Phys. B530, 683 (1998), hep-th/9806016.

[51] M. Henningson, B. E. Nilsson and P. Salomonson, “Holomorphic factorization of correlation functions in (4k+2)-dimensional (2k)-form gauge theory,” JHEP 9909, 008 (1999), hep-th/9908107.

[52] B.S. De Witt, Dynamical theory of groups and fields (Gordon and Breach, N.Y., 1965).

[53] R.T. Seeley, “Complex powers of an elliptic operator”, Proc. Symp. Pure. Appl. Math. 10, 288 (1967).

[54] A.S. Schwarz, “The Partition Function Of Degenerate Quadratic Functional And Ray-Singer Invariants,” Lett. Math. Phys. 2, 247 (1978); W. Siegel, “Hidden Ghosts,” Phys. Lett. B93, 170 (1980), J. Thierry-Mieg, “BRS Structure Of The Antisymmetric Tensor Gauge Theories,” Nucl. Phys. B335, 334 (1990); E. Sezgin and P. van Nieuwenhuizen, “Renormalizability Properties Of Antisymmetric Tensor Fields Coupled To Gravity,” Phys. Rev. D22, 301 (1980); Y.N. Obukhov, “The Geometrical Approach To Antisymmetric Tensor Field Theory,” Phys. Lett. B109, 195 (1982); A.S. Schwarz and Y. S. Tyupkin, “Quantization Of Antisymmetric Tensors And Ray-Singer Torsion,” Nucl. Phys. B242, 436 (1984).

[55] F. Bastianelli, “The Path integral for a particle in curved spaces and Weyl anomalies,” Nucl. Phys. B376, 113 (1992), hep-th/9112033.

[56] F. Bastianelli and P. van Nieuwenhuizen, “Trace anomalies from quantum mechanics,” Nucl. Phys. B389, 53 (1993), hep-th/9208059.