The Glueball sector of two-flavor Color Superconductivity

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We construct the effective Lagrangian describing the light glueballs associated with the unbroken and confining $SU_c(2)$ color subgroup for the 2 flavor superconductive phase of QCD. This Lagrangian constitutes a key ingredient for understanding the non perturbative physics of 2 flavor color superconductivity. We estimate the two photon decay process of the light glueballs using the saturation of the electromagnetic trace anomaly at the effective Lagrangian level. The present results are particularly relevant to our model of Gamma Ray Bursts based on color superconductivity in Quark Stars (R. Ouyed and F. Sannino [astro-ph/0103022]).

I. INTRODUCTION

Quark matter at very high density is expected to behave as a color superconductor [3–5]. Recent work had lead to a renewed interest on the subject [6,8]. This phase is characterized by its gap energy ($\Delta$) associated to quark-quark pairing. In such a phase, the color symmetry is spontaneously broken and a hierarchy of scales, for given chemical potential, is generated. Indicating with $g_s$, the underlying coupling constant, the relevant scales are: the chemical potential $\mu$ itself, the dynamically generated gluon mass $m_{gluon} \sim g_s \mu$ and $\Delta$. Since for high $\mu$ the coupling constant $g_s$ (evaluated at a fixed scale $\mu$) is $\ll 1$, we have:

$$\Delta \ll m_{gluon} \ll \mu . \quad (1)$$

The low-energy effective Lagrangian describing the in medium fermions and the broken sector of the $SU_c(3)$ color groups for the 2 flavor color superconductor (2SC) has been constructed in Ref. [1]. The 3 flavor case (CFL) has been developed in [2]. The effective theories describing the electroweak interactions for the low-energy excitations in the 2SC and CFL case can be found in [3]. In Reference [1] it has been shown that the confining scale of the unbroken $SU_c(2)$ color subgroup is lighter than the superconductive gap $\Delta$. This is a consequence of the high dielectric constant $\epsilon$ of the 2SC medium [1]. The unbroken $SU_c(2)$ theory is still confining and the light glueball like particles are expected to be light with respect to $\Delta$ and hence play a relevant role at low energies.

In this paper we generalize the effective Lagrangian for 2 flavor by properly taking into account the confined degrees of freedom associated with the low energy unbroken $SU_c(2)$ gauge interactions. We estimate the two photon decay process of the light glueballs using the saturation of the electromagnetic trace anomaly at the effective Lagrangian level. The glueball decay was found to be crucial in our model for powering Gamma Ray Bursts [10] (involving a 2SC layer at the surface of Quark Stars).

It led to establish a link between QCD at high matter density and Gamma Ray Bursts observables (energy and duration) extracting vital information about the QCD phase diagram. In particular a value for the critical temperature (of the order of 15 MeV) above which color superconductivity cannot exist has been determined for densities few times nuclear matter density [10].

In Section II we briefly review, while setting our conventions, the low-energy effective Lagrangian for the 2SC phase of QCD [8]. The latter describes the, in medium, fermions and the broken $SU_c(3)$ gluon sector. In Section III we construct the effective Lagrangian describing the light glueballs associated with the unbroken $SU_c(2)$ color subgroup by using the information inherent to the trace anomaly and the medium effects related to a non-vanishing dielectric constant [8]. In IV we estimate the, in medium, glueball to two photon decay process which is relevant to our model of Gamma Ray Bursts Ref. [10]. We conclude in V.

II. 2SC EFFECTIVE LAGRANGIAN REVIEW

QCD with 2 flavor has gauge symmetry $SU_c(3)$ and global symmetry

$$SU_L(2) \times SU_R(2) \times U_V(1) . \quad (2)$$

At high matter density a color superconductive phase sets in and the associated diquark condensates leaves invariant the following symmetry group:

$$[SU_c(2)] \times SU_L(2) \times SU_R(2) \times \tilde{U}_V(1) , \quad (3)$$

where $[SU_c(2)]$ is the unbroken part of the gauge group. The $U_V(1)$ generator $B$ is the following linear combination of the previous $U_V(1)$ generator $\tilde{B} = \frac{1}{2} \text{diag}(1,1,1)$ and the broken diagonal generator of the $SU_c(3)$ gauge group $T^8 = \frac{1}{2\sqrt{3}} \text{diag}(1,1,-2)$.
The quarks with color 1 and 2 are neutral under $\tilde{B}$ and consequently the condensate too ($\tilde{B}$ is $\sqrt{2}S$ of Ref. [1]). The superconductive phase for $N_f = 2$ possesses the same global symmetry group of the confined Wigner-Weyl phase [11]. In Reference [11], it was shown that the low-energy spectrum, at finite density, displays the correct quantum numbers to saturate the ’t Hooft global anomalies [13]. It was also observed that QCD at finite density can be envisioned, from a global symmetry and anomaly point of view, as a chiral gauge theory [13,14]. In Reference [13] it was then seen, by using a variety of field theoretical tools, that global anomaly matching conditions hold for any cold but dense gauge theory.

Massless excitations are protected by the aforementioned constraints and dominate physical processes. The low-energy theorems governing their interactions can be usefully encoded in effective Lagrangians (like for cold and dilute QCD [16]). It is possible to order the effective Lagrangian terms describing the Goldstone boson self interactions in number of derivatives. The resulting theory for dilute QCD is named Chiral Perturbation Theory [16]. Unfortunately this well defined scheme is not sufficient for a complete description of hadron dynamics since new massive hadronic resonances appear at relatively low energies. For instance, the $\sigma$ or the vector $\rho$ and new effective Lagrangian models of the type described in [17] are needed.

The dynamics of the Goldstone bosons can be efficiently encoded in a non-linear realization framework. Here, see [1], the relevant coset space is $G/H$ with $G = SU_c(3) \times \tilde{U}_V(1)$ and $H = SU_c(2) \times \tilde{U}_V(1)$ is parameterized by:

$$\mathcal{V} = \exp(i\xi^i X^i) ,$$  

(5)

where $\{X^i\} i = 1, \cdots, 5$ belong to the coset space $G/H$ and are taken to be $X^i = T_+^{i+3}$ for $i = 1, \cdots, 4$ while $X^5 = B + \frac{\sqrt{3}}{3} T^5 = \text{diag}(\frac{1}{2} \frac{1}{2} 0) .$$  

(6)

$T^a$ are the standard generators of $SU(3)$. The coordinates

$$\xi^i = \frac{f}{\bar{f}} , \quad i = 1, 2, 3, 4 , \quad \xi^5 = \frac{\bar{f}}{f} ,$$  

(7)

via $\Pi$ describe the Goldstone bosons. The vevs $f$ and $\bar{f}$ are expected, when considering asymptotically high densities [13], to be proportional to $\mu$.

$\mathcal{V}$ transforms non linearly:

$$\mathcal{V}(\xi) \to u_{\nu} g \mathcal{V}(\xi) h^\dagger(\xi, g, u) h V(\xi, g, u) ,$$  

(8)

with $u_{\nu} \in U_V(1), g \in SU_c(3), h(\xi, g, u) \in SU_c(2)$ and $h V(\xi, g, u) \in \tilde{U}_V(1)$.

It is convenient to define:

$$\omega_\mu = i \mathcal{V}^\dagger D_\mu \mathcal{V} \quad \text{with} \quad D_\mu \mathcal{V} = (\partial_\mu - ig_\mu G_\mu) \mathcal{V} ,$$  

(9)

with gluon fields $G_\mu = G^m T^m$ while $\omega$ transforms as:

$$\omega_\mu \to h(\xi, g, u) \omega_\mu h^\dagger(\xi, g, u) + i h(\xi, g, u) \partial_\mu h^\dagger(\xi, g, u) + i h V(\xi, g, u) \partial_\mu h V(\xi, g, u) .$$  

(10)

Following [16] we decompose $\omega_\mu$ into

$$\omega_\mu^{\|} = 2 S^a \text{Tr} [S^a \omega_\mu] \quad \text{and} \quad \omega_\mu^{\perp} = 2 X^i \text{Tr} [X^i \omega_\mu] ,$$  

(11)

where $S^a$ are the unbroken generators of $H$ with $S^{1,2,3} = T^{1,2,3}, S = B / \sqrt{2}$. Summation over repeated indices is assumed.

To be able to include the fermions in the picture we define:

$$\bar{\psi} = \mathcal{V}^\dagger \psi ,$$  

(12)

transforming as $\bar{\psi} \to h V(\xi, g, u) h(\xi, g, u) \bar{\psi}$ and $\psi$ possesses an ordinary quark transformations (as Dirac spinor).

The simplest non-linearly realized effective Lagrangian describing in medium fermions, the five gluons and their self interactions, up to two derivatives and quadratic in the fermion fields is:

$$\begin{align*}
\mathcal{L} = & \ f^2 a_1 \text{Tr} \left[ \omega_\nu^\dagger \omega_\nu - \alpha_1 \omega_\mu^\dagger \omega_\mu \right] \\
& + f^2 a_2 \left[ \text{Tr} [\omega_\nu^\dagger] \text{Tr} [\omega_\nu^\dagger] - \alpha_2 \text{Tr} [\bar{\omega}^\dagger] \text{Tr} [\bar{\omega}^\dagger] \right] \\
& + b_1 \bar{\psi} \left[ \gamma^0 (\partial_0 - i \omega_0^\dagger) + \beta_1 \gamma^i (\bar{\nabla} - i \bar{\omega}^\dagger) \right] \bar{\psi} \\
& + b_2 \bar{\psi} \left[ \gamma^0 \omega_0^\dagger + \beta_2 \gamma^i \bar{\omega}^\dagger \right] \bar{\psi} \\
& + m_M \bar{\psi} C \gamma^5 (i T^2)^j \bar{\psi} + \text{h.c.} ,
\end{align*}$$  

(13)

where $\bar{\psi} C = i \gamma^2 \bar{\psi}^*$, $i, j = 1, 2$ are flavor indices and

$$T^2 = S^2 = \frac{1}{2} \begin{pmatrix} a^2 & 0 \\ 0 & 0 \end{pmatrix} ,$$  

(14)

$a_1, a_2, b_1$ and $b_2$ are real coefficients while $m_M$ is complex. The breaking of Lorentz invariance, following [16] to the $O(3)$ subgroup has been taken into account by providing different coefficients to the temporal and spatial indices of the Lagrangian, and it is encoded in the coefficients $\alpha$s and $\beta$s. For simplicity, the flavor indices are omitted. From the last two terms, representing a Majorana mass term for the quarks, we deduce that the massless degrees of freedom are the $\psi_{\mu = 3,i}$ which possess the correct quantum numbers to match the ’t Hooft anomaly conditions [11]. The generalization to the electroweak processes relevant for the cooling history of compact stars has been investigated in [16].

The Lagrangian in Eq. (13) together with the one describing the relevant $SU_c(2)$ degree of freedom which we are about to construct will be used elsewhere to derive the 2SC equation of state.
III. SUc(2) GLUEBALL EFFECTIVE LAGRANGIAN

The SUc(2) gauge symmetry does not break spontaneously and it is expected to confine. If the new confining scale is lighter than the superconductive quark-quark gap the associated confined degrees of freedom (light glue-balls) can play, together with the true massless quarks, as shown in [10] a relevant role for the physics of Quark Stars featuring a 2SC superconductive surface layer.

One would expect that below the scale $\Delta$, the heavy degrees of freedom decouple and the low-energy theory is simply an SUc(2) Yang Mills theory (together with the ungapped quarks); with the new running coupling constant matched with the original SUc(3) at the scale $\Delta$. However this is a misleading argument. Indeed QCD at high chemical potential develops multiple scales making it difficult to define a simple matching procedure.

Since the ungapped fermions are neutral with respect to the SUc(2) - together with the diquarks built out of the quarks carrying non trivial charge under SUc(2) - it would seem natural for the medium to be transparent with respect to the associated gluons. However, according to the findings in [1], the medium does still lead to partial SUc(2) screening. In other words the medium is polarizable, i.e., acquires a dielectric constant $\epsilon$ different from unity (in fact $\epsilon \gg 1$ in the 2SC case [3]) leading to an effectively reduced gauge coupling constant.

In general, a medium possesses a dielectric constant and a magnetic permeability $\lambda \neq 1$. (Here, we note, that in the approximations of [3] $\lambda$ is still unity.) By assuming locality the SUc(2) effective action takes the form [3]:

$$ S_{\text{eff}} = \int d^4x \left[ \frac{\epsilon}{2} \bar{E}^a \cdot \partial \bar{E}^a - \frac{1}{2\lambda} \bar{B}^a \cdot \partial \bar{B}^a \right] $$

(15)

with $a = 1, 2, 3$ and $E_\mu^a \equiv F_\mu^a / \sqrt{\epsilon}$ and $B_\mu^a \equiv \frac{1}{2} \epsilon_{ijk} F_{\mu j}^a$. Here one assumes an expansion in powers of the fields and derivatives. The gluon speed in this regime is $v = 1/\sqrt{\epsilon \lambda}$.

In Reference [3] the $\epsilon$ and $\lambda$ were obtained studying the polarization tensor at asymptotically high densities of the SUc(2) gluons and by finally expanding it in powers of the momenta in order to get a local effective action. Their results are:

$$ \epsilon = 1 + \frac{g^2 a^2 \mu^2}{18\pi^2 \Delta^2}, \quad \lambda = 1. $$

(16)

Now, at asymptotically high densities, the gap $\Delta$ is exponentially suppressed compared to the chemical potential $\mu$ [19]. Indeed $\Delta \propto \mu g_\ast^2 e^{-c/\mu}$ with $c = 3\pi^2 / \sqrt{2}$, while $g_\ast$ is the SUc(3) coupling constant evaluated at $\mu$. Equation (16) thus suggests that a 2SC color superconductor can have a large positive dielectric constant. This implies that the Coulomb potential between SUc(2) color charges is reduced in the 2SC medium. The fact that the medium easily polarizes can be intuitively understood by recalling that Cooper pairs have a typical size of the order of $1/\Delta$.

There seems to be no effect on the magnetic permeability and in the asymptotic high chemical potential regime.

Clearly Eq. (16) is relevant to understand what happens to the low-energy SUc(2) gluons. Unfortunately, although formally correct and valuable, the perturbative results are very limited when considering phenomenological applications since, according to [20], the results are quantitatively valid only for $\mu \gg 10^9$ MeV. Besides, the theory is believed to still confine and hence SUc(2) glue-balls like particles are expected to emerge. These particles are light with respect to $\Delta$ and are shown to play a relevant role in Quark Stars featuring a superconductive 2SC surface layer [10]. So, the low-energy SUc(2) theory should be well represented by the effective Lagrangian describing its hadronic low lying states: the light glue-balls. This Lagrangian has to be added to the one of Eq. (13) [3] and will be derived below.

A straightforward way to tackle the problem is to build the SUc(2) energy stress tensor $\theta^{\mu\nu}$ whose trace is related to a dilatation anomaly. We consider the theory at scales lower than the gap.

The first step is to rescale the coordinates and the SUc(2) fields as follows:

$$ x^0 = \frac{x^0}{\sqrt{\lambda \epsilon}}, \quad \hat{g} = g_\ast \left( \frac{\lambda}{\epsilon} \right)^{\frac{1}{4}}, \quad \hat{A}_0^a = \lambda^{\frac{1}{4}} \epsilon^{\frac{1}{4}} A_0^a, \quad \hat{A}_i^a = \lambda^{\frac{1}{4}} \epsilon^{\frac{1}{4}} A_i^a. $$

(17)

In the limit $\lambda \to 1$ we recover the rescaling used in [3]. Here we consider a more general rescaling, by not assuming $\lambda = 1$, since for not too large chemical potentials there is no guarantee that a small magnetic permeability might not arise. If this were the case than note that the coupling constant is sensitive to the ratio $\lambda/\epsilon$ which is nevertheless much less than one (according to the perturbative regime calculations).

Using the rescaled variables, the SUc(2) action becomes:

$$ S_{\text{SU(2)}} = -\frac{1}{2} \int d^4x \text{Tr} \left[ \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} \right], $$

(18)

and $\hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu + i \hat{g} \left[ \hat{A}_\mu, \hat{A}_\nu \right]$ with $\hat{A}_\mu = \hat{A}_\mu^a T^a$ and $a = 1, 2, 3$.

The low-energy effective 3 gluon dynamics in the color superconductor medium (with non-vanishing dielectric constant and magnetic permeability) is similar to the in vacuum theory. The expansion parameter is:

$$ \hat{\alpha} = \frac{\hat{g}^2}{4\pi} = \frac{g^2}{4\pi} \sqrt{\frac{\lambda}{\epsilon}}. $$

(19)

Notice that $g_\ast$ is the SUc(3) coupling constant evaluated at the scale $\mu$ while we now, following Ref. [3], interpret $\hat{g}$ as the SUc(2) coupling at $\Delta$. The matching of the scales is encoded in $\sqrt{\lambda/\epsilon}$. Below $\Delta$ we use the action of Eq. (18) to investigate the SUc(2) properties.
Now we are ready to construct the glueball effective potential valid to all orders in the loop expansion. This was achieved in Ref. [22] (in the vacuum case), by using the information of the full, rather than just the one loop, beta function appearing in the trace anomaly saturation procedure [23]. The explicit dependence on the full beta function of the theory allowed [24] to investigate theories with large number of flavors, relative to the number of colors, with nearby infrared fix points.

The, in medium, anomaly-induced effective potential is based on the trace anomaly arising from the rescaled theory written in Eq. (18):

\[ \hat{\beta}_\mu = \frac{\beta(\hat{\theta})}{2\hat{\theta}} \hat{F}_{\alpha}^{\mu \nu} \hat{F}_{\mu \nu \alpha} = \frac{2b}{v} H , \]  

(20)

with \( a = 1, 2, 3 \) and we have defined \( \beta(\hat{\theta}) = -b\hat{\theta}^2/16\pi^2 \).

At one loop \( b = \frac{11}{3} N_c \) with \( N_c = 2 \) the color number. \( H \) is the composite field describing, upon quantization, the scalar glueball [22] in medium and possesses mass-scale dimensions 4. The specific velocity dependence is introduced to properly account for the velocity factors.

The general non-derivative effective potential saturating the trace anomaly is a solution of [22, 23]:

\[ \hat{\beta}_\mu = 4H \frac{\delta V}{\delta H} - 4\hat{V} , \]  

(21)

and is

\[ \hat{V} = \frac{b}{2v} H \log \left[ \frac{H}{\Lambda^4} \right] , \]  

(22)

where \( \Lambda \) is some intrinsic scale associated with the theory.

To the potential \( \hat{V} \) one has still the freedom to add a non derivative term proportional to \( H \). Since this term does not affect any of our conclusions it can be safely omitted. It is worth mentioning that a similar type of potential was derived in [23] when breaking the \( N = 1 \) Super Yang-Mills theory to ordinary Yang-Mills at the effective Lagrangian level.

In order to estimate \( \Lambda \) we consider the following one loop relation:

\[ \hat{\Lambda} = \Delta \exp \left[ -\frac{8\pi^2}{b_0 \hat{\theta}^2(\Delta)} \right] \]  

\[ = \Delta \exp \left[ -\frac{8\pi^2}{b_0 \hat{\theta}^2(\mu)} \sqrt{\frac{\epsilon(\mu/\Delta)}{\lambda(\mu/\Delta)}} \right] \]  

\[ \approx \Delta \exp \left[ -\frac{2\sqrt{2\pi}}{11} \frac{\mu}{g_s(\mu/\Delta)} \right] , \]  

(23)

with \( b_0 = 22/3 \) for \( SU_c(2) \) and in the last step we considered the asymptotic solution of Ref. [10], for convenience reported in Eq. (10).

By using \( \Lambda_{QCD} \approx 300 \text{ MeV}, \mu \approx 500 \text{ MeV} \) and a gap value of about 30 MeV (roughly 2 times [23] the superconductive critical temperature \( \sim 15 \text{ MeV} \) estimated in [10]) one gets \( \hat{\Lambda} \approx 1 \text{ MeV} \). We recall that this estimate relies on the one loop running and a reasonable value of \( \Lambda_{QCD} \) while we used the phenomenological value for \( \Delta \) [10]. This small value of \( \hat{\Lambda} \) reinforces the need for therelativistic light glueball effective Lagrangian as a crucial ingredient of the low-energy effective action. Our estimate is consistent with the one presented in [9] while further constraining \( \hat{\Lambda} \) values.

The glueballs are light (with respect to the gap) and might barely interact with the ungapped fermions. They are stable with respect to the strong interactions unlike ordinary glueballs.

The potential in Eq. (24) can be considered a zeroth order model [22, 24, 25] for a Yang-Mills theory, in medium, in which the glueballs are the associated hadronic particles. The potential has a minimum at

\[ \langle H \rangle = \frac{\hat{\Lambda}^4}{\epsilon} , \]  

(24)

at which point

\[ \langle \hat{V} \rangle = -\frac{b}{2v\epsilon} \hat{\Lambda}^4 \]  

(25)

From Eq. (24) this is seen to correspond to a magnetic-type condensation of the glueball field \( H \). The negative sign of \( \langle \hat{V} \rangle \) is consistent with the bag model [29] in which a “bubble” with \( \langle \hat{V} \rangle = 0 \) is stabilized against collapse by the zero point motion of the particles within. For the zero density case a number of phenomenological questions have been discussed using toy models based on Eq. (22, 23, 25).

To be able to deduce further dynamical properties one needs a kinetic term which does not affect the trace anomaly. A viable two derivative trace invariant term (in the rescaled coordinates) is [23]:

\[ \hat{\mathcal{L}}_{kin} = \frac{v}{2} \sqrt{b} H^{-\frac{1}{2}} \hat{\partial}_\mu H \hat{\partial}^\mu H . \]  

(26)

Here \( c \) is a positive dimensionless constant and the factor \( \sqrt{b} \) has been conveniently introduced. The previous 2-derivative term has scale dimensions four and hence does not affect the trace anomaly equation. The complete simplest light glueball action in the unrescaled coordinates for the, in medium, Yang-Mills theory is:

\[ S_{G-ball} = \int d^4x \left\{ \frac{c}{2} \sqrt{b} H^{-\frac{1}{2}} \left[ \hat{\partial}^\mu H \hat{\partial}_\mu H - v^2 \hat{\partial}^2 H \hat{\partial}^2 H \right] \right\} - \frac{b}{2} H \log \left[ \frac{H}{\Lambda^4} \right] . \]  

(27)

Hence the glueballs move with the same velocity as the underlying gluons in the 2SC color superconductor.

We define the mass-dimension one glueball field \( h \) via:

\[ H = \langle H \rangle e^{\frac{h}{\sqrt{b}}} . \]  

(28)
By requiring a canonically normalized kinetic term for \( h \) one finds:

\[
F_h^2 = \frac{c}{\sqrt{2}} \sqrt{2b(H)}. \tag{29}
\]

The glueball mass term is (obtained by expanding the Lagrangian up to the second order in \( h \)):

\[
M_h^2 = \frac{\sqrt{b}}{2c} \sqrt{\langle H \rangle} = \frac{\sqrt{b}}{2c} \sqrt{\Lambda^2}, \tag{30}
\]

which is clearly of the order of \( \hat{\Lambda} \) (estimated in the MeV range from Eq. (23)) since \( c \) is a positive constant of order unity. At large \( N_c \), and for zero matter density Yang-Mills theories one has \( \langle \hat{\theta}^\mu \rangle \sim O \left( N_c^2 \right) \) which fixes \( c \sim O \left( N_c \right) \) while \( F_h^2 \sim O \left( N_c^2 \right) \) which fixes \( b \sim O \left( N_c \right) \) and hence \( F_h^2 \sim O \left( N_c^2 \right) \). Clearly, for large \( N_c \) the glueball self interactions are suppressed. However for theories at finite matter densities \( N_c \) cannot be changed at will. Indeed a 2SC superconductive phase might be used to constraint the coefficients of the effective Lagrangian which can then be used for extracting dynamical results.

This completes the effective action for the \( SU_c(2) \) glueball in medium.

**IV. \( H \rightarrow \gamma \gamma \) PROCESS IN THE 2SC MEDIUM**

Once created, the light \( SU_c(2) \) glueballs are stable against strong interactions but not with respect to electromagnetic processes. Indeed, in analogy to the case of the \( \pi^0 \) decay into two photons at zero and high matter density for the 3 flavor color superconductive case (CFL) \([35a]\), the glueballs couple to two photons via virtual quark loops. More specifically, the \( \pi^0 \rightarrow \gamma \gamma \) process is a direct consequence of the gauging of the global anomalies which are seen to hold at finite matter density \([11,15]\), and the explicit electromagnetic gauging is provided, for the CFL case, in \([8]\). On the other side the two-photon coupling of any object which dominates the energy-momentum tensor at low energies is based on the electromagnetic trace anomaly \([35]\). This has been formulated at zero density by constructing a suitable effective Lagrangian \([31]\) describing the ordinary glueball decay into two photons. Mimicking the zero density case we modify the trace-anomaly induced potential term as follows:

\[
V = \frac{1}{4} \left[ \left( 2bH + \tilde{H}_{em} \right) \log \left( \frac{H}{\Lambda^4} \right) \right], \tag{31}
\]

where

\[
\tilde{H}_{em} = -\frac{\bar{c}^2}{24\pi^2} \left[ \sum_{\text{quarks}} \bar{Q}^2_{\text{quarks}} \right] \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu}, \tag{32}
\]

is the electromagnetic contribution to the trace anomaly, with \( \tilde{F}_{\mu\nu} = \partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu \). Here \( \tilde{A}_\mu \) is the in medium photon field corresponding to the following massless linear combination of the old photon and the eighth gluon \([33]\):

\[
\tilde{A}_\mu = \cos \theta_Q A_\mu - \sin \theta_Q G^8_\mu, \tag{33}
\]

with \( \tan \theta_Q = e/(\sqrt{3}g_\mu) \). The new electric constant is related to the in vacuum one via:

\[
\bar{e} = e \cos \theta_Q. \tag{34}
\]

\( \tilde{Q} \) is the new electric charge operator associated with the field \( \tilde{A}_\mu \):

\[
\tilde{Q} = r^3 \times 1 + \frac{\bar{B} - L}{2} = Q \times 1 - \frac{1}{\sqrt{3}}1 \times T^8, \tag{35}
\]

where \( L = 0 \) is the lepton number, \( r^3 \) the standard Pauli’s matrix, \( Q \) the quark matrix, the new baryon number \( \bar{B} \) is defined in Eq. (8) and following the notation of Ref. \([8]\) we have flavor \( 2 \times 2 \) \times color \( 3 \times 3 \). The quarks that acquire a mass term (i.e. the ones in the color direction one and two) have half integer charges under \( \tilde{Q} \) while the massless quarks (the ones in direction three of color) have the ordinary proton and neutron charges in units of \( \bar{e} \). Hence:

\[
\sum_{\text{quarks}} Q^2_{\text{quarks}} = \text{Tr} \left[ Q^2 \right] = N_c \text{Tr} Q^2 + \frac{1}{3} = 2, \tag{36}
\]

with \( N_c = 3 \) the underlying number of colors. Differently from the \( \pi^0 \rightarrow \gamma \gamma \) case, a part from a modified electron coupling, one also finds an electromagnetic trace anomaly coefficient which differs from the in vacuum case (corresponding to the first term in Eq. (33)). This, once again, shows the special role played by the chiral anomalies.

The relevant Lagrangian term is:

\[
\mathcal{L}_{\gamma \gamma} = \frac{\bar{e}^2}{48\pi^2} \frac{M_h}{\sqrt{2b(H)}} \left[ \sum_{\text{quarks}} Q^2_{\text{quarks}} \right] h \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu}, \tag{37}
\]

leading to the following decay width of the glueballs into two photons in medium:

\[
\Gamma \left[ h \rightarrow \gamma \gamma \right] = \frac{\alpha^2}{576\pi^3} \cos^{\theta_Q^2} \left[ \sum_{\text{quarks}} Q^2_{\text{quarks}} \right] \frac{M_h^5}{2b(H)} \approx 1.2 \times 10^{-2} \cos^{\theta_Q^2} \left[ \frac{M_h}{1 \text{ MeV}} \right]^5 eV, \tag{38}
\]

where \( \alpha = e^2/4\pi \approx 1/137 \). For illustration purposes we consider a glueball mass of the order of 1 MeV which leads to a decay time \( \tau \sim 5.5 \times 10^{-14} \). We used \( \cos \theta_Q \sim 1 \) since \( \theta_Q \sim 2.5^\circ \) when assuming for \( \Lambda_{QCD} \) and...
the chemical potential the values adopted in the previous section. While we are aware of the possible contribution from other hadrons to the saturation of the electromagnetic trace anomaly\[31\], here we assume it to be dominated by the $SU_c(2)$ glueballs. In any case, it is hard to imagine the photon decay process to be completely switched-off.

V. CONCLUSION

We constructed the, in medium, effective Lagrangian describing the light glueballs associated with the unbroken and still confining $SU_c(2)$ color subgroup. This Lagrangian has to be added to the one presented in\[6\] and constitutes a key ingredient for understanding the non perturbative physics of 2 flavor color superconductivity. We have shown that the light glueballs are unstable to photon decay and estimated the, in medium, two photon decay rate.

This work shows that a consistent portion of the glue (3/8 or 37.5%) filling the 2SC medium is very rapidly and efficiently converted into electromagnetic radiation. The relevance of the above in Quark Stars has been demonstrated in\[10\]. In particular we showed that, if 2SC develops at the surface of such stars (at the very early stage of their cooling history), glueball formation and subsequent two-photon decay process provide the fuel to power Gamma Ray Bursts. We discovered a plausible link between Color Superconductivity and Gamma Ray Bursts with important consequences to astrophysics and QCD\[10\].

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