We review the status of models of electroweak symmetry breaking in a slice of anti–de Sitter space. These models can be thought of as dual to strongly interacting theories of the electroweak scale. After an introduction to some generic issues in bulk theories in AdS$_5$, we concentrate on the model-building of the Higgs sector.

1 The Hierarchy Problem and Strong Dynamics

As the Large Hadron Collider (LHC) gets close to start taking data, we continue to ponder what new physics could appear at the TeV scale. In the standard model (SM) the electroweak symmetry is broken by a scalar doublet. This implies the existence of an elementary Higgs boson that must be relatively light ($< 1$ TeV) to unitarize electroweak scattering amplitudes, even lighter to satisfy electroweak precision constraints. However, its mass, and with it the electroweak scale, is unstable under radiative corrections. In order to keep it below the TeV scale and close to $v \simeq 246$ GeV, the bare mass parameter (presumably controlled by ultraviolet physics) must be finely adjusted to cancel against quadratically divergent loop corrections driven by the SM states. The need for this cancellation is highly unnatural and is called the hierarchy problem. In order for naturalness to be restored, new physics must cancel the quadratic divergences at a scale not far above the TeV scale.

The solution of the hierarchy problem is likely to shed light on the origin of electroweak symmetry breaking (EWSB). We can classify scenarios of new physics beyond the SM by how they solve the hierarchy problem. For instance, in supersymmetric theories at the weak scale$^{11}$ the quadratic divergences in the Higgs mass squared are canceled by the contributions of superpartners of the SM particles. Soft SUSY breaking allows for enough contributions to $m_{h}^{2}$ to trigger EWSB radiatively.

$^{a}$Talk presented at the 43rd Rencontres de Moriond, La Thuile, March 1-8 2008.
An alternative to solve the hierarchy problem is the possibility that some sort of new strong dynamics is present at or just above the TeV scale. For instance, in Technicolor theories, the new strong interaction becomes strong enough at the TeV scale to trigger the condensation of Techni-fermions. If some of these are SU(2)_L doublets, this triggers EWSB. There is no Higgs in this type of QCD-like scenario. The hierarchy problem is solved by dimensional transmutation, i.e. just as in the strong interactions the coupling becomes strong at low energies triggering EWSB naturally. The TeV scale is encoded in the running of this new strong coupling in the same way that the typical scale of hadronic physics (~ 1 GeV) is encoded in the running of α_s.

The trouble starts when one tries to construct the operators responsible for fermion masses. For this purpose one needs to extend the gauge group in Extended Technicolor (ETC) models. If at some energy scale Λ_{ETC} the ETC gauge bosons acquire masses, they generate four-fermion operators involving both fermions and Techni-fermions:

\[ \frac{g^2_{ETC}}{M^2_{ETC}} \bar{f}_L f_R \bar{T}_R T_L \]  

(1)

At the lower TC scale Λ_{TC}, the formation of the Techni-condensate \( \langle \bar{T}_L T_R \rangle \sim \Lambda^3_{TC} \) results in a fermion mass of the order of

\[ m_f \simeq \frac{g^2_{ETC}}{M^2_{ETC}} \Lambda^3_{TC} \]  

(2)

Clearly, if one wants to explain the fermion mass hierarchy one needs either several very different ETC scales, or some non-standard type of dynamics, most probably both. Particularly troublesome for ETC models are heavier masses, say above the charm mass. To obtain the top mass the ETC mechanism fails given than the ETC scale should be right on top of the TC scale.

Several complications of the ETC/TC idea allow for the fermion mass hierarchy (tumbling, walking TC) and even for the top quark mass (top-color assisted TC). In any case, it is clear that the dynamics associated with TC/ETC models must be quite different from that of a simple scaled up QCD-type theory. In addition to their problems with the generation of fermion masses, scaled up QCD TC models tend to predict a rather large S parameter

\[ S \simeq \frac{N}{6\pi} \]  

(3)

where \( N = N_{tc} N_D \) is the product of the number of Techni-colors and the number of weak doublets.

Despite all these problems, the idea that a new strong interaction at or just above the TeV scale is responsible for EWSB (and solves the hierarchy problem) remains attractive. Perhaps the new strong interaction is quite different from QCD and we do not have yet neither the theoretical tools nor the experimental guidance needed to build it.

2 Building Strongly Coupled Theories in AdS

Fortunately, there is a way to build a large array of strongly coupled theories of EWSB by using the AdS/CFT correspondence. The conjecture relates a Type IIB string theory on AdS_5 × S^5 with a four-dimensional \( \mathcal{N} = 4 \) SU(N) gauge theory, which is a conformal field theory. By extension, we can assume that at low energies we can describe the string theory by a higher dimensional field theory. As long as the AdS radius \( R_{AdS} \) is much larger than the string scale \( \ell_s \), the description in the higher-dimensional theory is weakly coupled. On the other hand, this leads to \( \tilde{g}N \gg 1 \), where \( \tilde{g} \) is the gauge coupling of the Yang-Mills theory. Then, the description of a weakly coupled theory in AdS_5 corresponds to a strongly coupled four-dimensional theory. We can think of the large N limit of the 4D gauge theory in terms of planar diagrams, which in turn are reminiscent of a loop expansion in the topology of the world-sheet in string theory.
In the original AdS/CFT correspondence, the boundary of AdS space is set at infinity, and is just a Minkowski 4D boundary. The corresponding 4D theory is exactly conformal, and therefore is not suitable for building models of EWSB. For our purposes, we want to consider an ultra-violet (UV) cutoff of the 4D theory, corresponding to a UV boundary at a finite coordinate in the extra dimension. Also, in order to obtain the description of an interesting strongly coupled theory, we require that the 4D gauge theory leads to non-trivial dynamics which triggers EWSB. This infra-red (IR) physics can be mimicked in the 5D theory by the appearance of an IR boundary. Thus, if we put the UV boundary at the origin of the extra dimension, \( y = 0 \), the statement of the AdS/CFT reads

\[
\int [D\phi_0] e^{i S_{\text{UV}}[\phi_0]} \int [D\phi_{\text{CFT}}] e^{i S_{\text{CFT}}[\phi_{\text{CFT}}]} + i \int d^4x O = \int [D\phi_0] e^{i S_{\text{bulk}}[\phi]}
\]

where \( O \) is an operator in the strongly coupled 4D theory, and \( \phi_0 \equiv \phi(x, y = 0) \) is a UV boundary field which acts as a source for the 4D operator \( O \). The correspondence in Eq. (4) states the the 4D action is equivalent to the effective action for the source field \( \phi_0 \) on the UV boundary, which is obtained by integrating over the bulk degrees of freedom of \( \phi(x, y) \):

\[
e^{i S_{\text{eff}}[\phi_0]} = \int_{\phi_0} [D\phi] e^{i S_{\text{bulk}}[\phi]}
\]

Then, n-point functions of the 4D theory can be obtained by using \( S_{\text{eff}}[\phi_0] \) as a generating functional. It is in this sense that the bulk degrees of freedom are determining the dynamics of the 4D theory.

We then see that for us to build models of strongly coupled theories, for instance to explain EWSB, we must specify a weakly coupled 5D theory in AdS\(_5\). The type of strongly coupled theories that we can build this way is not completely general. For instance, it requires that it be “large N”, which should in principle translate in the presence of narrow resonances. In what follows we consider the steps to build such theories.

### 2.1 Solving the Hierarchy Problem in a slice of AdS\(_5\)

The starting point to build theories of EWSB using holography is the Randall-Sundrum setup as a solution of the hierarchy problem. We consider an extra dimension compactified on an orbifold, i.e. \( S_1/Z_2 \), with the metric

\[
ds^2 = e^{-2ky} \eta^{\mu\nu} dx_\mu dx_\nu - dy^2,
\]

where \( k \sim M_P \) is the AdS curvature. The orbifold compactification \( S_1/Z_2 \) results in a slice of AdS in the interval \([0, \pi R]\), with \( R \) the compactification radius. This metric is a solution of Einstein’s equations if we fine-tune the bulk cosmological constant to cancel the brane tensions. This choice of metric means that the graviton’s wave-function is exponentially suppressed away from the origin. In general, this metric exponentially suppresses all energy scales away from the origin. Then, if the Higgs field is localized a distance \( L = \pi R \) from the origin,

\[
S_H = \int d^4x \int_0^{\pi R} dy \sqrt{g} \delta(y - \pi R) \left[ g^{\mu\nu} \partial_\mu H \partial_\nu H - \lambda \left( |H|^2 - v_0^2 \right)^2 \right]
\]

where \( \lambda \) is the Higgs self-coupling and \( v_0 \) is its vacuum expectation value (VEV). The latter must satisfy \( v_0 \sim k \) for the theory to be technically natural. Taking into account the exponential
factors in $g^{\mu\nu}$ and $\sqrt{g}$, the renormalization of the Higgs field required to render its kinetic term canonical results in an effective four-dimensional VEV given by

$$v = e^{-k\pi R} v_0$$

Thus, in order for the weak scale $v \simeq 246$ GeV to arise at the fixed point in $y = \pi R$, we need $kR \sim (10-12)$. Then, if the Higgs is for some reason localized at or nearly at this location, this setup constitutes a solution to the hierarchy problem.

In the original Randall-Sundrum (RS) proposal only gravity propagates in the extra dimension. However, this presents several problems, mostly associated with the fact that is not possible to sufficiently suppress higher-dimensional operators. For instance, operators mediating flavor violation might only be suppressed by the TeV scale, $k e^{-k\pi R}$. Grand Unified Theories (GUTs) might not be viable since we cannot effectively suppress proton decay. Many of these problems are solved when fermions and gauge bosons are allowed in the AdS$_5$ bulk. In fact, in order to solve the hierarchy problem, the only field that must remain localized near the $\pi R$ or TeV brane is the Higgs. In addition, allowing the standard model fields in the bulk opens up a large number of model building possibilities including viable GUTs$^{10}$ and the modeling of the origin of flavor$^{11,12}$ just to mention two very prominent cases. But most importantly, it gives us a tool to build models of EWSB that address its dynamical origin. In building bulk models of EWSB we must dynamically explain why is the Higgs localized near the TeV brane. Building such models is like constructing strongly coupled theories of EWSB but from a different perspective.

2.2 Bulk RS Theories and the Origin of Flavor

Writing a theory in the bulk require several ingredients. First, we must decide what the gauge symmetry should be. It turns out that the SM electroweak symmetry, $SU(2)_L \times U(1)_Y$, is not suitable for this since it results in a large value of the parameter $T$. The reason is that there is large explicit isospin violation in the bulk: in addition to the SM-sanctioned isospin violation proportional to $g'/g = \tan \theta_W$, the bulk adds the isospin violation of the Kaluza-Klein (KK) modes of the SM gauge fields. As a consequence, it is necessary to implement a gauge symmetry that would exhibit isospin symmetry in the bulk. A minimal extension of the SM with this feature is $SU(2)_L \times SU(2)_R \times U(1)_X$. This bulk symmetry may be broken by boundary conditions to the SM gauge group or directly to $U(1)_{\text{EM}}$. Additional discrete symmetries may be imposed to protect the $Z \rightarrow \bar{b}b$ coupling from large deviations$^{15}$.

One important consequence of writing a bulk theory, is that the KK states would start from masses of the order of $M_{KK} \sim O(1)$ TeV, independently of whether they correspond to fermions, gauge bosons or even scalars. This also means that the wave-function of the KK excitations in the extra dimension also peaks at $y = \pi R$, i.e. at the TeV brane. This is related to the fact that position in the bulk can be thought of as an energy scale, and is a generic feature independent of the model considered.

Another important issue is the localization of zero modes in the bulk, i.e. their effective 5D wave-function. The strength of couplings between zero-modes and KK modes and, since the Higgs is TeV-localized, of the zero-mode fermion Yukawas, are determined by this. Fermions propagating in the bulk can have a mass term (as long as we assume is an odd mass term). The natural order of magnitude of this fermion bulk mass is $k$, the only dimensionfull bulk parameter. We can then write the bulk fermion mass as

$$M_f = c_f k$$

where $c_f \simeq O(1)$. Expanding the 5D fermion $\Psi(x, y)$ in KK modes and solving the equation of motion for the zero mode we see that its bulk wave-function, for instance for a left-handed zero
mode, behaves like\cite{11}

\[ F^L_{ZM}(y) \sim e^{(\frac{1}{2} - c_L) ky} \]  (10)

Then, if \( c_L > 1/2 \) the zero-mode fermion is localized towards the Planck brane, whereas if \( c_L < 1/2 \) it would be localized near the TeV brane. For a right-handed zero mode, localization near the Planck bran occurs if \( c_R < -1/2 \), and near the TeV brane if \( c_R > -1/2 \). If a zero mode fermion has a large wave-function near the TeV brane it has also a large overlap with the Higgs, which has to be localized there. Thus, heavier fermions must have a wave-function towards the TeV brane, whereas light fermions must be essentially Plank-brane localized in order to explain their small Yukawa couplings. We see then that fermion localization in the AdS\(_5\) bulk provides a potential explanation for the hierarchy of fermion masses in the SM. This emerging picture of flavor requires that the top quark be highly localized towards the TeV brane. Furthermore, the left-handed third-generation doublet contains the left-handed b quark, which then also has to have a substantial TeV localization. The rest of the zero-mode fermions must be localized mostly near the Plank brane. Since the zero-mode gauge bosons are flat in the extra dimension, this does not affect the universality of the zero-mode gauge couplings. However, the KK modes of gauge bosons are TeV-brane localized, and then couple stronger to the TeV-brane localized fermions, i.e. the third generation. The couplings of light fermions to KK gauge bosons are nearly universal, making low energy flavor phenomenology viable, even in the presence of tree-level flavor violation. Also since light fermions are localized near the Plank brane, the scale suppressing higher dimensional operators responsible for proton decay is again \( M_P \).

Then any sign of the characteristic tree-level flavor violation would have to appear in the interactions of the third generation quarks with the KK gauge bosons. Although these would have potentially important effects in flavor physics\cite{16,17}, a direct observation of the KK gluon decay into a single top and a jet, coming most likely from \( G^{(1)} \rightarrow t \bar{c} \), would be an unambiguous signal of this theory of flavor\cite{18}.

Finally, a word about electroweak precision constraints. Since the bulk gauge theory has an isospin symmetry built in, we need not worry about the theory generating a large \( T \) parameter. However, this class of models all share the same problem with the \( S \) parameter. They have an \( S \) parameter which is approximately\cite{13,19,20}

\[ S_{\text{tree}} \simeq 12\pi \frac{v^2}{M_{\text{KK}}^2}, \]  (11)

which results in a bound of about \( M_{\text{KK}} > 2.5 \) TeV. This is a feature we have to live with in most bulk RS constructions. We can interpret this in the dual 4D picture, as the fact that in the 4D theory we have a large \( N \), which enter in \( S_{\text{tree}} \sim \frac{N}{\pi} \), where \( N \) is typically the size of the 4D gauge group. This can in principle be made considerably smaller if the light fermions
are allowed out of the Plank brane region and have almost flat profiles in the extra dimensions. Essentially this decouples them from the KK modes and avoids $S_{tree}^{13}$. But this picture would lack a solution to the fermion mass hierarchy.

3 Electroweak Symmetry Breaking from AdS$_5$

We have arrived at a general picture of these kind of models in AdS$_5$:

- The Higgs is TeV-brane localized in order to solve the gauge hierarchy problem
- Fermion localization explains the fermion mass hierarchy: light fermions are Plank-brane localized resulting in small Yukawa couplings. Heavier fermions are TeV-brane localized ($t_R$, $t_L$ and $b_L$).
- The bulk gauge symmetry must be enlarged to protect isospin, to be at least $SU(2)^L \times SU(2)^R \times U(1)^X$.

This is already quite a rich structure with a very interesting phenomenology and lots of model building possibilities beyond EWSB and fermion masses. However, if these models are dual to a strongly coupled theory in four dimensions, as the presence of the resonances (i.e. the KK modes) appears to suggest, then we must be a bit disappointed with the Higgs sector. What keeps the Higgs localized to the TeV brane? Do we have to have a Higgs at all? In what follows we briefly discuss three possibilities for the Higgs sector in these theories.

3.1 Higgsless EWSB in AdS$_5$

In the absence of a Higgs, the bulk gauge symmetry must be broken directly to $U(1)_{EM}$ by boundary conditions (BC) for the bulk gauge fields at the fixed point of the extra dimension. On the Plank brane the BC are such that the bulk gauge symmetry breaks as

$$SU(2)^R \times U(1)^X \rightarrow U(1)^Y, \quad \text{at } y = 0$$

On the other hand, on the TeV brane

$$SU(2)^L \times SU(2)^R \rightarrow SU(2)^V, \quad \text{at } y = \pi R$$

in such a way that it it preserves custodial symmetry. Then the gauge symmetry in the bulk has a gauged version of the SM custodial symmetry. This remains as a remnant global symmetry, resulting in the correct masses for the $W$ and the $Z$.

More problematic in these models is how to give fermions their masses. Zero-mode fermions can obtain isospin conserving masses through a TeV-brane localized mass term. Since $SU(2)^V$...
is not broken there, isospin splitting is not generated by these terms. In order to achieve the freedom to have the correct mass spectrum one must introduce the following bulk spectrum (schematically and only for the third generation case):

\[
\begin{align*}
\Psi_L &= (t_L, b_L) (2, 1)_{1/6} \\
\Psi_R &= (t_R, b_R') (1, 2)_{1/6} \\
\Psi_R' &= (t_R', b_R) (1, 2)_{1/6}
\end{align*}
\] (14)

Each of these bulk fermions contains both left and right handed components. We can choose the BCs so that the zero-modes of \(\Psi_L\) correspond to the third generation left-handed quark doublet, the zero mode of \(\Psi_R\) is the right-handed top \(t_R\), and the one corresponding to \(\Psi_R'\) is \(b_R\). Mass terms localized in the TeV brane gives rise to fermion masses. It is still true that the larger the zero-mode wave-function is at the TeV brane, the larger its mass would be. But the top mass cannot be so easily adjusted. The reason is that there is a tension between the TeV localization of the top, which if too extreme produces noticeable deviation in the \(Zb_Lb_L\) coupling, and the size of the isospin conserving TeV localized mass. The latter cannot be too large or it would induce a large mixing between \(b_R'\) and the b-quark’s zero mode through the mass term responsible for the top quark mass. This would again result in a deviation of \(Zb_Lb_L\). A way to circumvent this problem is to extend the custodial symmetry by a discrete symmetry \(P_{LR}\) that relates the two \(SU(2)\)’s. In order to protect the \(b_L\) coupling, it must be included in a bi-doublet of \(SU(2)_L \times SU(2)_R\). Then the right-handed fermions could be in singlets or triplets of the \(SU(2)\)’s. For instance, if \(t_R \sim (1, 1)_{2/3}\), then it does not contain any fermion that could mix with the left-handed b. On the other hand, the field resulting in the right-handed zero mode would have to be a full \(O(4)\) triplet \(\Psi_R \sim (3, 1)_{2/3} \oplus (1, 3)_{2/3}\) resulting in a distinct spectrum of KK fermions\(^{[22]}\).

But the most important signal of this scenario stems from the absence of the Higgs as a unitarizing field in \(VV\) scattering\(^{[21]}\), with \(V = W^\pm, Z\). The unitarization of these amplitudes, unlike in other strongly coupled theories such as (4D) Techni-color, is the result of the presence of the narrow resonances that are the KK modes of the gauge bosons. The constraint of unitarization imposes sum rules on the couplings. For instance,

\[
g_{WWW} = g_{WZW}^2 + g_{WWW}^2 + \sum_n (g_{WWV(n)})^2 = \frac{3}{4M_W^2} \left[ g_{WZW}^2 M_Z^2 + \sum_n (g_{WWV(n)})^2 M_n^2 \right]
\]

The requirement of unitarity of gauge boson scattering amplitudes means that the KK modes of gauge bosons cannot be too heavy. For example, if one wants to preserve perturbative unitarity, they must be below the TeV scale. Higgsless models in AdS\(_5\) are then characterized by relatively low mass KK excitations.

### 3.2 Gauge-Higgs Unification

A remarkable mechanism to obtain a Higgs field naturally localized near the TeV, naturally light and suitable for EWSB is that in which the Higgs comes from a gauge field in 5D. In general, a 5D gauge field \(A_M(x, y)\), \(M = 0, 1, 2, 3, 5\), can be decomposed in a vector \(A_\mu(x, y)\) and a scalar component \(A_5(x, y)\). If we want to extract the Higgs \(SU(2)_L\) doublet from a gauge field in 5D, then the gauge symmetry in 5D has to be enlarged beyond the SM gauge symmetry. In order to illustrate how this works let us take a simple example, an \(SU(3)\) bulk gauge theory\(^{[23]}\). We can use BCs to break this gauge symmetry as \(SU(3) \rightarrow SU(2)_L \times U(1)_Y\). By choosing the BCs
appropriately, the gauge fields are
\[
\begin{align*}
& t^a A_\mu^a : \\ & \begin{pmatrix}
(\pm, \pm) & (\pm, \pm) & (-, -) & (\pm, \pm) \\
(\pm, \pm) & (\pm, \pm) & (-, -) & (\pm, \pm) \\
(-, -) & (-, -) & (\pm, \pm) & (-, -) \\
(\pm, \pm) & (\pm, \pm) & (-, -) & (\pm, \pm)
\end{pmatrix}
\end{align*}
\]
where \( a = 1, 2, 3 \) is the \( SU(3) \) adjoint index and \( t^a \) are the \( SU(3) \) generators, and we use the fact that the BCs for the \( A_\mu^a(x, y) \) are always opposite from those for \( A_\mu^a(x, y) \). The signs correspond to the BCs in the Plank and TeV branes respectively. Thus, we see that the spectrum of zero-mode gauge bosons corresponds to \( SU(2) \times U(1) \) as the gauge symmetry. The symmetry has been reduced or broken by this choice of BCs. Furthermore, we see that the \( A_\mu^a(x, y) \)'s corresponding to the “broken” generators, i.e. the generators for which \( A_\mu^a(x, y) \) does not have a zero mode, have zero modes (i.e. \((+, +)\) BCs). In fact, these constitute four real degrees of freedom that can be seen to be a doublet of \( SU(2) \) and its adjoint. Then, we can identify this \( SU(2) \) doublet with the Higgs. In the case of an AdS\( _5 \) metric, if we impose the unitary gauge, this results in
\[
\partial_y (ke^{-k y} A_5(x, y)) = 0,
\]
which results in a scalar doublet with a profile localized towards the TeV brane. Thus, we achieved Higgs TeV-brane localization and extracted the Higgs from a gauge field in the bulk. These models are clearly related to Little Higgs theories, where the Higgs is a (pseudo-)Nambu-Goldstone boson (pNGB): the Higgs is associated to the broken generators of a symmetry, which from the 4D interpretation would be a global one. It cannot have a potential since shift symmetry (the remnant gauge symmetry for the \( A_5^a \) zero-modes) forbids it. Thus, these models should be dual to 4D theories of a composite Higgs, where the Higgs is a pNGB.

This simple \( SU(3) \) model of Gauge-Higgs unification has a lot of the features that we want. However, it does not have custodial symmetry in the bulk. The way to cure this is to simply enlarge the bulk gauge symmetry. The minimal realistic model of Gauge-Higgs unification in AdS\( _5 \) requires that we start with \( SO(5) \times U(1)_X \) broken down to \( SO(4) \times U(1)_X \) on the TeV brane, whereas it is reduced to \( SU(2)_L \times U(1)_Y \) on the Plank brane\(^24,21\). Just as in the previous case, in the unitary gauge only the \( A_5^a(x, y) \)'s associated with the broken generators, i.e. transforming in the coset space \( SO(5)/SO(4) \), have zero modes. These are arranged in a \( 4 \) of \( SO(4) \), or a bi-doublet of \( SU(2)_L \times SU(2)_R \).

Fermion masses are not very problematic and can be obtained by localization just as in the generic models with a TeV-brane localized Higgs. Regarding electroweak precision constraints, these models can evade them more efficiently. In particular, the \( S \) parameter can be made in agreement with experiment for gauge KK masses above 2 TeV. On the other hand, KK fermions could be quite a bit lighter, especially once the additional discrete custodial symmetry is introduced in order to keep the \( Z \to b_L b_L \) in check. For instance, possible embeddings are\(^24\)
\[
5_{2/3} = (2, 2) \oplus (1, 1)
\]
or a
\[
10_{2/3} = (2, 2) \oplus (1, 3) \oplus (3, 1)
\]
With it, KK fermions can be as light as 500 GeV, and in some cases would have exotic charge assignments. The main reason for some KK fermions to be this light has to do with the need to get the top mass correctly. There are striking signals at the LHC, for instance from the pair production of charge \( 5/3 \) KK fermions, a distinct signature for the presence of the extended custodial symmetry\(^25,26\).
3.3 Higgs from Fermion Condensation

Another alternative to dynamically generate the Higgs sector of the bulk Randall-Sundrum scenario is the condensation of zero-mode fermions. Since the localization of fermions near the TeV brane implies they must have strong couplings to the KK excitations of gauge bosons, it is possible that the induced four-fermion interaction is strong enough to result in fermion condensation, thus triggering EWSB. Among the SM fermions, the candidate would be the top quark: it has the strongest localization toward the TeV brane, therefore the largest coupling to KK gauge bosons, particularly the first excitation of the KK gluon. The four-fermion interaction would result in a top-condensation scenario. But we already know that this does not work if the scale of the underlying interaction is $O(1)$ TeV, as we expect the KK gluon mass to be. The problem is that the mass of the condensing fermion should be about 600 GeV in order for the condensation to result in the correct weak scale, $v \approx 246$ GeV. We can then consider the possibility of a fourth generation, one that is highly localized near the TeV brane, more than the top quark. This results in an effective four-fermion interaction -mostly mediated by KK gluons- that is attractive enough to trigger the condensation of at least one of the fourth-generation quarks. For instance, the up-type fourth-generation quark $U$ has a four-fermion interaction given by

$$-rac{g_{01}^L g_{01}^R}{M_{KK}^2} \left( \bar{U}_L \gamma_\mu t^A U_L \right) \left( \bar{U}_R \gamma^\mu t^A U_R \right),$$

(18)

where $g_{01}^{L,R}$ are the couplings of the left and right-handed $U$ quarks to the first KK mode of the gluon of mass $M_{KK}$, and $t^A$ are the QCD generators. After Fierz rearrangement, we can re-write this interaction as

$$\frac{g_U^2}{M_{KK}^2} \left\{ \bar{U}_L^a U_R^b \bar{U}_R^b U_L^a - \frac{1}{N_c} \bar{U}_L^a U_R^b \bar{U}_R^b U_L^a \right\},$$

(19)

where $a, b$ are $SU(3)_c$ indices, and we have defined

$$g_U^2 \equiv g_{01}^L g_{01}^R.$$

(20)

The color singlet term in (19) is attractive, whereas the color octet is repulsive, as well as suppressed by $1/N_c$. There is a critical value of $g_U^2$ above which there forms a condensate $\langle \bar{U}_L U_R \rangle$ leading to electroweak symmetry breaking and dynamical masses for the condensing fermions. This is

$$g_U^2 > \frac{8\pi^2}{N_c}.$$  

(21)

One can also write an effective theory in terms of a scalar doublet which becomes dynamical at low energies. So this theory gets a composite Higgs that is heavy and made of the already mostly-composite fourth-generation up quarks. The $U$ quark gets a large dynamical mass. All other zero-mode fermions, including the SM fermions and the other fourth-generation zero-modes, get masses through four-fermion interactions with the $U$ quark. These operators come from bulk higher dimensional operators such as

$$\int dy \sqrt{g} \frac{C^{ijkl}}{M_P^2} \bar{\Psi}_L(x, y) \Psi_R^j(x, y) \bar{\Psi}_R^k(x, y) \Psi_L^\ell(x, y),$$

(22)

where $C^{ijkl}$ are generic coefficients, with $i, j, k, \ell$ standing for generation indices as well as other indices such as isospin, and the $\Psi(x, y)$’s can be bulk quarks or leptons. Upon condensation of the $U$ quarks these result in fermion masses that have the desired pattern as long as we choose the localization parameters appropriately.

These models have roughly the same $S$ parameter problem as the generic ones. But to the tree-level $S$, now we must add also the loop contributions coming from a heavy Higgs as well as from the fourth-generation. These are, however, not as large as $S_{\text{tree}}$. 
The masses of the fourth-generation zero modes could be as small as 300 GeV due to mixing with KK modes, and as large as $\sim 600$ GeV. The Higgs is typically rather heavy, in the (700 – 900) GeV range.

The phenomenology of these models is quite different from the three-generation RS models due to the fact that the fourth generation is the one that couples strongest to the KK gauge bosons, i.e. to the new physics in the s channel\textsuperscript{28}. For instance the branching ratio of a KK gauge boson to $U(1)\bar{U}(1)$ is likely to be 5 – 10 times larger than that for the $t\bar{t}$ channel, which is the one always dominating in three-generation RS models. Also the KK gluon tends to be very broad and can only be seen as an excess in the production of the fourth-generation, which is dominated by QCD. In general, all KK gauge bosons will be considerably broader, although electroweak KK gauge bosons might have more manageable widths.

4 Conclusions

Model building in a slice of AdS$_5$ extends the Randall-Sundrum solution to the hierarchy problem, and opens up the possibility of addressing dynamically the origin of EWSB and fermion masses, among other things. Through the AdS/CFT correspondence we can see that these weakly coupled 5D theories are dual to strongly coupled 4D gauge theories. Thus, building bulk RS theories of EWSB corresponds, through holography, to certain strongly coupled electroweak sectors. Although the type of strongly coupled theory is not completely generic, this procedure gives us access to a large variety of strongly coupled theories of the electroweak sector.

In addition to solving the hierarchy problem, RS bulk models provide a framework to understand the hierarchy of fermion masses. This implies the presence of tree-level flavor violation with KK gauge bosons. Rigorous compatibility with low energy data may require some level of flavor symmetry in the bulk. However, this is not surprising since fermion localization already signaled some amount of flavor breaking in the UV. The central point is that the metric in AdS$_5$ provides the necessary large scale separation between the light and the heavy fermions.

Finally, there are several alternatives for the Higgs sector. Higgsless models are viable, although require some measure of fine-tuning to cancel contributions to $Z \rightarrow b_L \bar{b}_L$. Gauge-Higgs unification models are very promising and in best agreement with electroweak precision constraints. They provide a dynamical origin for the localization of the Higgs near the TeV brane, and in the Holographic dual correspond to a composite pNGB Higgs. Finally, another alternative to localize a composite Higgs near the TeV brane, is the condensation of fourth-generation quarks via the attractive four-fermion interaction mediated mostly by the KK gluon. This is a distinct possibility, a realization of the fourth-generation condensation proposed long ago by Bardeen, Hill and Lindner\textsuperscript{29}. These three Higgs sector possibilities have very different phenomenology and should be distinguishable at the LHC.

Acknowledgements

The author acknowledges the support of the State of São Paulo Research Foundation (FAPESP), and the Brazilian National Counsel for Technological and Scientific Development (CNPq).

References

1. S. P. Martin, [arXiv:hep-ph/9709356].
2. C. T. Hill and E. H. Simmons, *Phys. Rept.* 381, 235 (2003), [Erratum-ibid. 390, 553 (2004)].
3. E. Farhi and L. Susskind, *Phys. Rev.* D 20, 3404 (1979).
4. E. Eichten and K. D. Lane, *Phys. Lett.* B 90, 125 (1980).
5. Y. Kikukawa and N. Kitazawa, *Phys. Rev. D* **46**, 3117 (1992).
6. B. Holdom, *Phys. Rev. D* **24**, 1441 (1981).
7. C. T. Hill, *Phys. Lett. B* **345**, 483 (1995).
8. J. M. Maldacena, *Adv. Theor. Math. Phys.* **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1999)].
9. L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83**, 3370 (1999).
10. A. Pomarol, *Phys. Rev. Lett.* **85**, 4004 (2000) [arXiv:hep-ph/0005293].
11. T. Gherghetta and A. Pomarol, *Nucl. Phys. B* **586**, 141 (2000).
12. Y. Grossman and M. Neubert, *Phys. Lett. B* **474**, 361 (2000).
13. K. Agashe, A. Delgado, M. J. May and R. Sundrum, *JHEP* **0308**, 050 (2003).
14. C. Csaki, C. Grojean, L. Pilo and J. Terning, *Phys. Rev. Lett.* **92**, 101802 (2004).
15. K. Agashe, R. Contino, L. Da Rold and A. Pomarol, *Phys. Lett. B* **641**, 62 (2006).
16. G. Burdman, *Phys. Rev. D* **66**, 076003 (2002).
17. G. Burdman, *Phys. Lett. B* **590**, 86 (2004).
18. P. M. Aquino, G. Burdman and O. J. P. Eboli, *Phys. Rev. Lett.* **98**, 131601 (2007).
19. G. Burdman and Y. Nomura, *Phys. Rev. D* **69**, 115013 (2004).
20. G. Cacciapaglia, C. Csaki, C. Grojean and J. Terning, *Phys. Rev. D* **70**, 075014 (2004).
21. A. Birkedal, K. Matchev and M. Perelstein, *Phys. Rev. Lett.* **94**, 191803 (2005).
22. G. Cacciapaglia, C. Csaki, G. Marandella and J. Terning, *Phys. Rev. D* **75**, 015003 (2007).
23. R. Contino, Y. Nomura and A. Pomarol, *Nucl. Phys. B* **671**, 148 (2003).
24. K. Agashe, R. Contino and A. Pomarol, *Nucl. Phys. B* **719**, 165 (2005).
25. R. Contino, L. Da Rold and A. Pomarol, *Phys. Rev. D* **75**, 055014 (2007).
26. M. Carena, A. D. Medina, B. Panes, N. R. Shah and C. E. M. Wagner, arXiv:0712.0095 [hep-ph].
27. G. Burdman and L. Da Rold, *JHEP* **0712**, 086 (2007).
28. G. Burdman, L. Da Rold, O. J. P. Eboli and R. Matheus, in preparation.
29. W. A. Bardeen, C. T. Hill and M. Lindner, *Phys. Rev. D* **41**, 1647 (1990).