Crust of accreting neutron stars within simplified reaction network

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Low-mass X-ray binaries

Properties

- \( M_{\text{comp}} \lesssim M_\odot \)
- \( t_{\text{acc}}, t_q \approx \text{years} \)
- \( T_{\text{crust}} \lesssim 5 \cdot 10^8 \text{ K} \)
- \( \gtrsim 30 \) systems known

Models of thermal evolution require

- Equation of state
- Integrated heat
- Composition (i.e. average charge, impurity parameter)

Figure: Quiescent thermal luminosities of SXTs as functions of average accretion rates \((\dot{M})\) (Potekhin et al. 2019).
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Aims of this work

Our study is directed to the crust of neutron star, namely:

- construction of a nuclear reaction network
- comparison with previous studies
- probe a sensitivity of composition to applied mass models

Figure: Cooling of quasi-persistent transient (*Meisel et al. 2018*).
Haensel & Zdunik model

Gibbs energy per Wigner-Seitz cell:

\[
G_{\text{cell}} = W_N(A, Z, n_n) + W_l(n_N, Z) + [E_e(n_e) + (1 - n_N V_N) E_n(n_n) + P]/n_N
\]

Mass model: Mackie & Baym 1977

Note: Model takes into account influence of free neutrons

\[
P_{\text{tot}} = P_e(n_e) + P_l(n_N, Z) + P_n(n_n)
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Thresholds for nuclear reactions

\[
G_{\text{cell}}(A, Z, N_n, P_{\text{thr}}) = G_{\text{cell}}(A', Z', N'_n, P_{\text{thr}})
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Feature

One-component, baryons are confined in volume

Figure: Neutron star crust structure

(Meisel et al. 2018)
Haensel & Zdunik model

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Figure: Neutron star crust structure (Meisel et al. 2018).
Our approach

Algorithm

- Raise pressure and check for available reactions (decreasing Gibbs potential)
- Reaction occur ⇒ adjust pressure to reaction threshold
- Only chunk \(10^{-4}\) of nuclei undergoes nuclear reaction (as Steiner 2012) ⇒ multicomponent composition
- Stepwise reaction scheme, governed by priority order:
  - emission of neutrons (maximum energetically allowed number)
  - 1 neutron capture
  - 2 neutrons capture
  - electron capture (with following neutron emission)
  - pycnonuclear reaction
- The order bases on timescale estimations: \(\tau_{em}^{n} \ll \tau_{ca}^{n} \ll \tau_{e}^{ca} \ll \tau_{pycn}\)
- Among electron/neutron captures firstly proceeds the most energy efficient
- Pycnonuclear reaction rate calculated following Yakovlev et al. 2006, S-factors from Afanasiev et al. 2012. Reaction threshold: \(\tau_{acc} = \tau_{pycn}\)
- Excited states of nuclei included on a qualitative basis
- Neglect \(\nu\)-losses (Gupta et al. 2007)
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Reaction scheme

→ emission of neutrons (maximum energetically allowed number)
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\[ E_{gs}(A,Z) + \mu_e \rightarrow E_{gs}(A,Z-1) + E_{exc} \rightarrow E_{gs}(A-N,Z-1) + N\cdot\mu_n \]
Comparison with *Lau et al. 2018*

**Figure:** Crust composition in *Lau et al. 2018.*

**Figure:** Crust composition in this work.

**Lau’s concept**

Full reaction network (calculation of reactions rates) driven by increasing pressure

Only allowed Gamow-Teller transitions (theoretical model of nuclei energy levels)

Mass model: Atomic Mass Evaluation (AME) 2012 + Finite-Range Droplet Macroscopic model (FRDM) 1992 \( \Rightarrow \rho \lesssim 2 \times 10^{12} \text{ g cm}^{-3} \)
Results with different mass tables

- Crust composition and reaction sequence depend on the choice of the mass tables
- FRDM12 and pure DZ31 demonstrates nearby outcome with funneling to N=50 closure shell

Figure: Crust composition in Fantina et al. 2018

Figure: Crust composition with different mass tables
Merging the mass tables

**Figure:** Crust composition in unmixed approach of merging the mass tables.

**Figure:** The same, but in joint approach.
Crust properties

**Figure:** The profiles of impurity parameter

- **HFB21+DZ31** demonstrates peculiar behaviour of impurity parameter, which is likely strongly affected by merging the mass tables.
- Results depend not only on mass tables but as well on merge method.

**Figure:** Equations of state.
Heating

- Integrated heat depends on the model, however stand between curves presented in Fantina et al. 2018 and Lau et al. 2018.
- The reason why these models can be used as upper and lower benchmark consists in consideration of nuclear transition.

**Figure:** The profiles of accumulated heat release.
Violating of diffusion equilibrium

• Force balance equations with no diffusion (see e.g. Beznogov & Yakovlev 2013):

\[
\begin{cases}
    e \nabla \phi + \mu_e g - \nabla \mu_e = 0 \\
    -eZ \nabla \phi + m_i g - \sum_{\alpha} \frac{n_{\alpha}}{n_i} \nabla \mu_{\alpha} = 0 \\
    m_n g - \nabla \mu_n = 0
\end{cases}
\]

as \( m_e \ll m_i, \nabla P = \rho g \), nuclei are not degenerate: \( n_\alpha \nabla \mu_\alpha = 0 \) and using quasineutrality condition \( n_e = Z n_i \), lead to:

\[
\left. \frac{\partial \mu_e}{\partial P} \right|_{\text{NoDiff}} = \frac{m_i}{Z \rho} \approx \frac{Am_U}{Z \rho}
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• We have regions with absence of diffusion equilibrium
Unavoidable diffusion

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Violation of diffusion equilibrium

Real problem with traditional approach?

\[ (-Z \nabla \mu_e) + (m_i g) = f_i \]

- Superfluid neutrons ⇒ such crust can not exist
- Not superfluid, estimation of currents:

\[ J_n \approx \frac{n_n}{n_i \sigma_{ni} v_n} \frac{f_{ni}}{m_n} \approx 3 \times 10^{28} \text{ cm}^{-2} \text{ s}^{-1} \]

\[ J^E_{acc} = \frac{\dot{m}^E}{m_U} \approx 6 \times 10^{28} \text{ cm}^{-2} \text{ s}^{-1} \]

Both cases: fast redistribution of neutrons in the inner crust
Real problem with traditional approach?

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Conclusion

Our simplified approach can reproduce main properties of more detailed *Lau et al. 2018* model applicable up to $\rho \lesssim 2 \times 10^{12}$ g cm$^{-3}$.

We used our method to simulate the evolution of matter under compression with various mass tables and figured out, that crust properties are sensitive to selection of the mass model. However, variation of $Q_{\text{imp}}$ can be constrained as $\lesssim 15$ after outer/inner crust transition, and matter tends to purification to magic $N=50$ for simulation frame.

Integrated heat in our model locates between curves presented in *Fantina et al. 2018* and *Lau et al. 2018* ($1.3$ MeV/nucleon $\sim 1.8$ MeV/nucleon, at $\rho \approx 3 \times 10^{12}$ g cm$^{-3}$).

**Results.** Crust composition: *MNRAS 2019 490, 3454-3463*  
Composition with Mackie & Baym mass model: *J. of Phys. conf. ser. 2019, 1400, 022016*  
Violation of diffusion equilibrium - to be published.

We reveal an inconsistency of the standard approach: it considers the nuclear evolution of matter element on course of compression, but in fact assumes that it is contained in a box with impermeable walls. The problem is, that in NSs the walls are absent – the diffusion crucialy affects the nuclear composition (*Gusakov M. E. & Chugunov A. I., 2020, to be submitted*).
Physics of Neutron Stars 2020

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- Alice K. Harding (TBC)
- Jason W. T. Hessels
- James M. Lattimer
- Sandro Mereghetti
- Samaya M. Nissanke
- Evan P. O'Connor
- Alessandro Papitto
- Emily Petroff
- Alexander A. Philippov
- Bettina Posselt
- Sergey S. Tsygankov
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