Violation of SUSY equivalence in triple gauge boson 
and gaugino couplings

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Abstract

Supersymmetry implies that the trilinear couplings between gauge bosons and gauginos are equal to all orders. However if SUSY is broken and some of the superpartners have large SUSY breaking masses then they can induce non-decoupling deviations in this relation through radiative corrections. In this work we show that the deviation in the ratio \( \frac{\tilde{g}_w x}{g_{\gamma w w}} \) from unity can be around 2.3% (3.3%) in heavy QCD models (2-1 models) for a point in the gaugino region. Precision measurement of triple gaugino couplings at future high energy colliders can therefore probe such violations and the existence of heavy superparticles in the multi Tev region which are otherwise inaccessible.
Introduction

In recent years SUSY has emerged as a leading candidate for the origin of electroweak symmetry breaking. If SUSY is to provide a cure for the hierarchy problem and the quadratic divergence problem then the superparticles must lie between 100 Gev and a few Tev range otherwise various fine tunings [1] are required to reconcile it with low energy constraints. Most of the future colliders are therefore being designed with a view to produce these sparticles which would lead to the discovery of SUSY. After their discovery we need to verify whether these particles are really the SUSY partners of ordinary particles. SUSY implies that the couplings of superparticles be equal to the corresponding couplings of ordinary particles. In particular if $g_i$ are the SM gauge couplings and $\tilde{g}_i$ are the corresponding superpartner couplings, then exact SUSY implies that $g_i = \tilde{g}_i$ to all orders. However if SUSY is broken and some of the superpartners have large SUSY breaking mass splittings then they can induce violations to the above relation through radiative corrections. These violations are similar to the violations in custodial $SU(2)$ symmetry [2] due to a split $SU(2)$ multiplet. In fact if some of the sparticles have masses between 1-10 Tev then they will be inaccessible at the planned high energy colliders. Although they decouple from most low energy processes their contribution to the difference $\frac{\tilde{g}_i}{g_i} - 1$ grows logarithmically with the heavy superpartner mass scale. Hence they induce a non-decoupling effect in low energy processes involving light sparticles and ordinary particles. Precision measurements of the couplings of light sparticles at future high energy colliders can therefore be used to probe such deviations from exact supersymmetric relations and also the heavy superpartner mass scale.

Violation of SUSY equivalence between gauge and gaugino couplings

In Ref. 3 Cheng et al considered the deviation of the gaugino couplings to $f - \tilde{f}$ from the corresponding SM gauge coupling. They chose to parameterize these deviations in terms of three superoblique parametrs $\tilde{U}_i = \frac{\tilde{g}_i}{g_i} - 1$ where the subscript $i$ denotes the
relevant SM gauge group. These authors considered precision measurements in chargino pair production at $e^+e^-$ collider and selectron pair production at $e^+e^-$ and $e^-e^-$ colliders. Since the cross-section in these processes measure the gaugino couplings to $f - \tilde{f}$ pair, the deviations in these couplings from the corresponding SM gauge couplings are the relevant ones to consider. They showed that $\tilde{U}_1 \approx 0.35\% (0.29\%) \times \ln R$ and $\tilde{U}_2 \approx 0.71\% (0.80\%) \times \ln R$ for 2-1 models (heavy QCD models). Here $R = \frac{\tilde{M}}{\tilde{m}}$ is the ratio of heavy to light sparticle mass scales. The light sparticle mass scale corresponding to electroweak gauginos is typically of the order of 100 Gev. The heavy superpartner mass scale $\tilde{M}$ is usually around 1 Tev for heavy QCD models like gauge mediated SUSY breaking models [4] where strongly interacting sparticles get large masses through QCD interactions. There is another class of models known as 2-1 models [5] where some of the sparticles can be very heavy. In these models the sfermions belonging to the first two generations lie between 10-50 Tev but the third generation sfermions are near the weak scale. The estimates for $\tilde{U}_1$ and $\tilde{U}_2$ considered by Cheng et al. therefore lie between 1.61\%-3.27\% (0.67\%-1.84\%) for 2-1 models (heavy QCD models) depending upon the relevant EW gauge group.

It would be interesting to know what is the deviation of gaugino coupling from the gauge coupling in the pure gauge sector namely the triple gauge boson couplings. The supersymmetric version of Slavnov Taylor identity implies that the gaugino coupling $g_{\tilde{\chi}f\tilde{f}}$ to $f - \tilde{f}$ is equal to $g_{\tilde{\chi}\tilde{\gamma}}$ to all orders if SUSY is unbroken. However if SUSY is broken then the Slavnov-Taylor identity in the sparticle sector also breaks down. Therefore it is not apriori clear if the violation of SUSY equivalence in $g_{\tilde{\chi}\tilde{\gamma}}$ would be of the same order as that in $g_{\tilde{\chi}f\tilde{f}}$. Further in unbroken SUSY the Slavnov-Taylor identity only equates the leading logarithm corrections in $g_{\tilde{\chi}\tilde{\gamma}}$ and $g_{\tilde{\chi}f\tilde{f}}$, but not the finite terms which are quite important in our case since the leading log terms are not very large. In this work we shall consider the violation of SUSY equivalence between trilinear couplings of gauge bosons and gauginos. We shall show that in heavy QCD models where $R \approx 10$ the correction is of the order of 2.3\%. On the other hand in 2-1 models where $R \approx 100$ the correction can
be as large as 3.3%. Therefore if the trilinear gaugino Yukawa coupling can be measured with an accuracy of about 1% from chargino decay at future high energy colliders then not only we can probe such deviations but also the heavy sparticle mass scale.

**Specification of the model and assumptions**

We shall evaluate the radiative corrections in two distinct scenarios. Under the first scenario we shall consider heavy QCD models where all strongly interacting sparticles are very heavy and lie in the few Tev range. The sparticles which have only EW quantum numbers will be assumed to be light and in the 100 Gev range. Under the second scenario we shall consider the 2-1 models which were proposed to solve the SUSY flavor problem and CP problem without requiring degeneracy, alignment or small CP phases in the squark mass matrices. In these models all sparticles belonging to the first two generations are very heavy and they decouple from the remaining sparticles comprising the squarks and sleptons of the third generation and gauginos. Bounds from flavor changing neutral currents imply that the heavy superpartner mass scale in 2-1 models can be as large as 10-50 Tev [5].

We shall evaluate the relevant one loop diagrams using the \( \bar{MS} \) scheme. More accurately we should use the SUSY preserving \( \bar{DR} \) scheme. But since the loop diagrams in our case involves only fermions and sfermions the \( \bar{DR} \) scheme give the same result as the \( \bar{MS} \) scheme. We shall set the arbitrary renormalization mass scale \( \mu \) equal to the light superpartner mass scale \( \tilde{m} \). This is the superpartner mass scale that will be accessible at planned future colliders. To simplify our calculations we shall assume that the lightest chargino is dominantly a gaugino and the lightest neutralino is a photino. If the lightest chargino and neutralino have significant higgsino components then there will be additional contributions from the Yukawa couplings of higgsino to top quark or top squark that cannot be neglected. However our results for 2-1 models where the third generation sfermions are light will be almost unaffected as we move from the gaugino dominated region to the higgsino region.
Radiative corrections in heavy QCD models

In this section we shall calculate the radiative corrections to the couplings $\tilde{g}_w\tilde{\chi}\tilde{\gamma}$, $g_\gamma\tilde{\chi}\tilde{\chi}$ and $g_\gamma\ww$ in heavy QCD models. Unbroken SUSY implies that $\tilde{g}_w\tilde{\chi}\tilde{\gamma} = g_\gamma\tilde{\chi}\tilde{\chi} = g_\gamma\ww$ to all orders. However if SUSY is broken then there will be small deviations from it induced by heavy superpartners. We shall assume that the external momenta of any loop diagram satisfy the condition $p_i^2 \approx \tilde{m}^2 \ll \tilde{M}^2$. In evaluating the renormalization constants from the loop diagrams we shall retain both heavy and light particles. However loop diagrams that do not contain any heavy particles do not give rise to large logarithms of the form $\ln \frac{\tilde{M}^2}{\mu^2}$ and can be neglected. In our case the leading log terms are not very large. Therefore besides the leading logs we shall keep all the finite terms that do not vanish in the limit $\frac{\tilde{m}^2}{\tilde{M}^2} \to 0$.

The renormalization of the coupling constant $\tilde{g}_w\tilde{\chi}\tilde{\gamma}$ is given by $\tilde{g}_w\tilde{\chi}\tilde{\gamma}(\tilde{M}) = \frac{Z_{\tilde{\chi}\tilde{\gamma}W}}{\sqrt{Z_{\tilde{\chi}}\sqrt{Z_{\tilde{\gamma}}\sqrt{Z_w}}}}$ where $Z_{\tilde{\chi}\tilde{\gamma}W}$ is the vertex renormalization constant. $\sqrt{Z_{\tilde{\chi}}}$, $\sqrt{Z_{\tilde{\gamma}}}$, $\sqrt{Z_w}$ are the wavefunction renormalization constants associated with chargino, photino and W boson fields respectively. Due to the Ward identity associated with the unbroken $U(1)_q$ gauge symmetry the renormalization of $g_\gamma\tilde{\chi}\tilde{\chi}$ and $g_\gamma\ww$ are given by $\frac{g_\gamma\tilde{\chi}\tilde{\chi}(\tilde{M})}{g_\gamma\tilde{\chi}\tilde{\chi}(\mu)} = \frac{g_\gamma\ww(\tilde{M})}{g_\gamma\ww(\mu)} = \frac{1}{\sqrt{Z_\gamma}}$. In heavy QCD models ($N_f = N_c = 3$) we get

$$Z_{\tilde{\chi}\tilde{\gamma}W} = 1 - \frac{g^2}{32\pi^2} N_f N_c (\ln R^2 - \frac{1}{2}).$$  \hspace{1cm} (1)

$$\frac{1}{\sqrt{Z_{\tilde{\chi}}}} = 1 + \frac{g^2}{64\pi^2} N_f N_c (\ln R^2 - \frac{1}{2}).$$ \hspace{1cm} (2)

$$\frac{1}{\sqrt{Z_{\tilde{\gamma}}}} = 1 + \frac{e^2}{32\pi^2} \frac{5}{9} N_f N_c (\ln R^2 - \frac{1}{2}).$$ \hspace{1cm} (3)

$$\frac{1}{\sqrt{Z_w}} = 1 + \frac{g^2}{64\pi^2} N_f N_c \ln R^2.$$ \hspace{1cm} (4)

and

5
\[
\frac{1}{\sqrt{Z_{\gamma}}} = 1 + \frac{e^2}{32\pi^2} \frac{5}{9} N_f N_c \ln R^2. \tag{5}
\]

Here \( R = \frac{\tilde{M}}{\mu} \). We expect SUSY to be restored above the heavy superpartner mass scale \( \tilde{M} \). Using the boundary condition \( \tilde{g}_{w\tilde{\chi}\tilde{\gamma}}(\tilde{M}) = g_{\gamma\tilde{\chi}\tilde{\chi}}(\tilde{M}) = g_{\gamma\gamma w}(\tilde{M}) \) we get

\[
\frac{\bar{g}_{w\tilde{\chi}\tilde{\gamma}}(\mu)}{g_{\gamma\tilde{\chi}\tilde{\chi}}(\mu)} = \frac{\bar{g}_{w\tilde{\chi}\tilde{\gamma}}(\mu)}{g_{\gamma\gamma w}(\mu)} = \frac{1}{\sqrt{Z_w}} = 1 + \frac{g^2}{64\pi^2} N_f N_c \ln R^2 = \frac{1}{\sqrt{Z_{\tilde{\chi}}}}. \tag{6}
\]

Note that when SUSY is broken and squarks get a large mass, quark loops do not contribute to large logarithms and hence their contribution to \( Z_w \) and \( Z_{\gamma} \) can be neglected. However in the SUSY limit the contribution of quark loops has to be added. When that is done we get

\[
\frac{1}{\sqrt{Z_w}} = 1 + \frac{g^2}{64\pi^2} N_f N_c \ln R^2 = \frac{1}{\sqrt{Z_{\tilde{\chi}}}}. \tag{7}
\]

\[
\frac{1}{\sqrt{Z_{\gamma}}} = 1 + \frac{e^2}{32\pi^2} \frac{5}{9} N_f N_c \ln R^2 = \frac{1}{\sqrt{Z_{\tilde{\chi}}}}. \tag{8}
\]

The renormalization of \( \tilde{g}_{w\tilde{\chi}\tilde{\gamma}} \) in the SUSY limit is therefore given by

\[
\frac{\bar{g}_{w\tilde{\chi}\tilde{\gamma}}(\tilde{M})}{\bar{g}_{w\tilde{\chi}\tilde{\gamma}}(\mu)} = \frac{1}{\sqrt{Z_{\gamma}}} = \frac{1}{\sqrt{Z_{\tilde{\chi}}}} = \frac{g_{\gamma\gamma w}(\tilde{M})}{g_{\gamma\gamma w}(\mu)}. \tag{9}
\]

This is in accord with the SUSY version of of Slavnov-Taylor identity which demands that in the SUSY limit \( \tilde{g}_{w\tilde{\chi}\tilde{\gamma}} \) should renormalize exactly like \( g_{\gamma\gamma w} \). However if SUSY is broken then Slavnov-Taylor identity breaks down for \( \tilde{g}_{w\tilde{\chi}\tilde{\gamma}} \). The Wavefunction renormalization constants of \( W \) and \( \chi \) no longer cancel the vertex renormalization constant \( Z_{\tilde{\chi}\tilde{\gamma} W} \).
completely. As a result $\tilde{g}_{w\tilde{\chi}\tilde{\gamma}}$ receives its dominant renormalization from asymptotically free $SU(2)_w$ interactions.

**Radiative corrections in 2-1 models**

In this section we shall present the results for radiative corrections to $\tilde{g}_{w\tilde{\chi}\tilde{\gamma}}$, $g_{\gamma\tilde{\chi}\tilde{\chi}}$ and $g_{\gamma w w}$ in 2-1 models. Evaluating the one loop diagrams we get

\[ Z_{\tilde{\chi}\tilde{\gamma}} = 1 - \frac{g^2}{4\pi^2}(\ln R^2 - \frac{1}{2}). \]  
\[ (10) \]

\[ \frac{1}{\sqrt{Z_{\tilde{\chi}}}} = 1 + \frac{g^2}{8\pi^2}(\ln R^2 - \frac{1}{2}). \]  
\[ (11) \]

\[ \frac{1}{\sqrt{Z_{\tilde{\gamma}}}} = 1 + \frac{e^2}{6\pi^2}(\ln R^2 - \frac{1}{2}). \]  
\[ (12) \]

\[ \frac{1}{\sqrt{Z_w}} = 1 + \frac{g^2}{24\pi^2} \ln R^2. \]  
\[ (13) \]

and

\[ \frac{1}{\sqrt{Z_{\gamma}}} = 1 + \frac{e^2}{18\pi^2} \ln R^2. \]  
\[ (14) \]

The effect of supersymmetry breaking on the trilinear gauge couplings in 2-1 models is given by

\[ \frac{\tilde{g}_{w\tilde{\chi}\tilde{\gamma}}(\mu)}{g_{\gamma\tilde{\chi}\tilde{\chi}}(\mu)} = \frac{\tilde{g}_{w\tilde{\chi}\tilde{\gamma}}(\mu)}{g_{\gamma w w}(\mu)} \]

\[ = [1 + \frac{g^2}{4\pi^2}\left\{ \frac{1}{3}(1 - \frac{4}{3}s_w^2) \ln R^2 - \frac{1}{2}\left( \frac{1}{2} - \frac{2}{3}s_w^2 \right) \right\}] \]

\[ \approx 1 + .033. \]  
\[ (15) \]

The same results as above can also be obtained by decoupling the heavy fields at $\tilde{M}$ and evolving the couplings down to $\mu$ according to the low energy theory of light degrees.
Comparing our result with that of Cheng et al we can conclude that the violation of SUSY equivalence in trilinear gaugino coupling is of the same order as that in $\tilde{g}_{\tilde{\chi}ff}$. Hence the violation of Slavnov-Taylor identity that relates the renormalization of $\tilde{g}_{w\tilde{\chi}\tilde{\gamma}}$ to that of $\tilde{g}_{\tilde{\chi}ff}$ is either very small or its effect on the quantity under study is negligible.

**Experimental prospects**

The results obtained in the previous sections imply that if the gaugino coupling $\tilde{g}_{w\tilde{\chi}\tilde{\gamma}}$ can be measured with an accuracy of around 1% then we can probe such violations in supersymmetric relations and also the heavy superpartner mass scale. If the decay $\tilde{\chi} \to \tilde{\gamma}w$ is kinematically allowed then it would provide the best source for measuring the coupling $\tilde{g}_{w\tilde{\chi}\tilde{\gamma}}$. In fact in some models like the gauge mediated SUSY breaking the LH slepton doublet is heavier than the lightest chargino. This makes $\tilde{\chi} \to \tilde{\gamma}w$ the dominant decay mode with an appreciable branching ratio. At a 500 Gev $e^+e^-$ collider the chargino pair production cross-section for LH incoming $e^-$ beam, $M_\tilde{\chi} = 170$ Gev and $m_{\tilde{\nu}_e} = 200$ Gev is about 600 fb. With a design luminosity of 50 fb$^{-1}$/yr, 60000 charginos will be produced. If the decay $\tilde{\chi} \to \tilde{\gamma}w$ has a branching fraction of .8 then 48000 charginos will decay in this mode. This would yield a statistical error of .46% in the chargino decay width via this mode and .23% in the coupling $\tilde{g}_{w\tilde{\chi}\tilde{\gamma}}$. However we also need to calculate the theoretical systematic error arising out of uncertainties in $M_\tilde{\chi}$, $M_{\tilde{\gamma}}$ and $m_{\tilde{\nu}_e}$ which would affect the determination of $\tilde{g}_{w\tilde{\chi}\tilde{\gamma}}$. Further all the relevant backgrounds have to be computed and subtracted from the signal to reduce the total uncertainty in the decay width. With a bit of optimism it may be fair to say that although it may not be possible to measure $\tilde{g}_{w\tilde{\chi}\tilde{\gamma}}$ with the same degree of accuracy as $\tilde{g}_{\tilde{\gamma}ee}$ or $\tilde{g}_{\tilde{\chi}\nu e}$ the prospects for measuring it with a somewhat reduced degree of accuracy seem quite bright.

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