1. INTRODUCTION

Many radio pulsars exhibit variabilities in their pulse shapes and intensities on a wide variety of timescales. These include phenomena such as subpulse drifts, nulls and bursts, and long- and short-term on/off quasiperiodic mode switching (see Cordes 2013 for a review). Until recently, these variabilities were thought to be due to changes in the radio emission process, not tied to the global properties of the pulsar magnetosphere and high-energy emission. However, Kramer et al. (2006) discovered that PSR B1931+24 shows large changes in its spin-down rate, correlated with its mode switching, suggesting that the switching is strongly tied to the global energetics of the pulsar. Later, PSR J1841-0500 (Camilo et al. 2012) and PSR J1832+0029 (Lorimer et al. 2012) were also shown to have similar behavior. A number of other pulsars display smaller changes in spin-down rates which are correlated with changes in their average pulse profiles (Lyne et al. 2010). These may correspond to a less extreme version of “mode switching.”

This connection between mode switching and radio behavior was further strengthened by the simultaneous X-ray and radio observations of PSR B0943+10 (Hermsen et al. 2013, hereafter H+13). This is a relatively old pulsar which has been known for a long time to transition from a bright, highly organized radio “on” (B) state to a quieter, more chaotic radio “off” (Q) state on timescales of a few hours (Rankin & Suleymanova 2006). In addition, its radio emission exhibits subpulse drift, which was shown by Deshpande & Rankin (2001) to likely come from 20 “sub-beams” arranged in a conical structure. Deshpande & Rankin (2001) also constrained the emission geometry of the radio beam, demonstrating that the line of sight, rotation axis, and magnetic axis are all nearly parallel, with a misalignment of only $\sim 10^\circ$. H+13 found that the radio mode switching of PSR B0943+10 is correlated with a similar switching in its X-ray emission: during the B-mode the X-ray emission is steady and non-pulsed, while during the Q-mode the X-ray flux increases by a factor of $\sim 2$ and becomes highly pulsed.

As PSR B0943+10 is a paragon of mode-switching pulsars, understanding its behavior may provide important insight into the physical origin of pulsar mode switching. In this paper, we discuss three puzzling aspects of PSR B0943+10’s correlated X-ray and radio emission (Section 2) and demonstrate that two of them can be resolved through accurate light curve modeling that accounts for the effect of a strong magnetic field (Section 3). We consider an alternative “displaced dipole” magnetic field configuration, where a magnetic dipole displaced from the center of the star produces two magnetic polar caps of different sizes and magnetic field strengths. These models are currently consistent with data in radio and X-rays and can be tested or constrained by future X-ray observations.

Key words: magnetic fields – pulsars: individual (PSR B0943+10) – stars: neutron

Online-only material: color figures
emission must have nearly 100% pulse fraction. However, since Deshpane & Rankin (2001) showed that all three axes of the pulsar (the observer line of sight, the rotation axis, and the magnetic dipole axis; see Figure 1) are aligned within \( \sim 10^\circ \), at first glance it appears unlikely that such a high pulse fraction can be achieved via the standard polar cap hot spot. In Section 3 we demonstrate that, in fact, this problem is remedied by the inclusion of magnetic beaming.

\( \text{H+13} \) proposed that the strong pulsation may be explained by periodic obscuration of the X-ray emission via electron scattering in the magnetosphere. The resonant scattering cross-section \( \sigma \) for photons with frequency \( \omega \) can be written as (e.g., Canuto et al. 1971)

\[
\sigma = \frac{4\pi^2e^2}{m_ec^3}|e_-|^2\delta(\omega - \omega_c),
\]

where \( e \) and \( m_e \) are the electron charge and mass, respectively, and \( \omega_c \equiv eB(r)/m_ec \) is the cyclotron frequency at a distance \( r \) from the neutron star (NS), and \( e_- \) specifies the circular polarization (around the magnetic field) of the photon. We characterize the magnetosphere particle (electron or positron) number density \( n(r) \) in terms of the Goldreich–Julian density, i.e., \( n(r) \approx \lambda_{GJ}n_0\Omega B(r)/(2\pi ec) \), with \( \lambda_{GJ} \) the multiplicity factor.

Using \( B(r) \approx B_\rho(R_*/r)^3 \) and \( |e_-| \approx 1 \), we find that the photon optical depth \( \tau = \int n_0 \sigma dr \) is

\[
\tau \approx 10^{-3}\lambda_{GJ}\left(\frac{P}{1 \text{ s}}\right)^{-1}\left(\frac{R_*}{10 \text{ km}}\right)^{-1}\left(\frac{B_\rho}{10^{12} \text{ G}}\right)^{1/2}\left(\frac{E}{1 \text{ keV}}\right)^{-1/2},
\]

where \( P, R_*, \) and \( B_\rho \) are the period, radius, and surface magnetic field of the pulsar, and \( E = \hbar\omega \) is photon energy. For the range of photon energies of 0.2–2 keV in which the X-ray pulsation is observed, in order for sufficient obscuration to occur (\( \tau \gtrsim 1 \)), the density of the plasma must be about 1000 times the Goldreich–Julian density. While theoretically possible, it is unclear such a large particle density can be achieved in the closed magnetosphere region. Observationally, many isolated NSs (including both active radio pulsars and purely thermally emitting sources) have detectable X-ray emission from the whole stellar surface (e.g., Kaspi et al. 2006; Zavlin 2007), indicating that obscuration of surface X-rays by the magnetosphere is not significant.

The second puzzle of PSR B0943+10 is the presence of “drifting subpulses” in its radio emission (Deshpande & Rankin 2001). These drifting subpulses are thought to be a consequence of \( \mathbf{E} \times \mathbf{B} \) drifting of a pattern of “sparks” in the polar cap accelerator of the NS (Ruderman & Sutherland 1975; Ruderman & Gil 2006). This implies the presence of a “vacuum gap”—a region directly over the polar cap in which the pulsar magnetosphere has pulled away from the surface of the NS, leaving a gap in which no plasma is present.\(^5\) This gap then has a strong voltage drop across it, acting as an accelerator for charged particles and causing them to emit high-energy radiation, which later decays into highly bunched radio emission via a pair-production cascade a few tens of NS radii away from the surface. The presence of this gap is usually predicated upon the inability of ions to become dissociated from the NS surface. According to the electronic structure calculations of condensed matter in strong magnetic fields by Medin & Lai (2006), for an iron NS surface, a high surface magnetic field (\( \gtrsim 10^{14} \text{ G} \); see Figure 7 of Medin & Lai 2007) is required to keep the ions bound to the surface. The required field strength is even larger if the condensed surface is made of lighter elements (C or He). On the other hand, the surface dipole magnetic field inferred from the \( P–\dot{P} \) measurement of PSR B0943+10 is a modest \( 2 \times 10^{12} \text{ G} \), which is certainly insufficient for a gap to form. This implies that perhaps the local surface magnetic field at the polar cap of the NS is much higher than the dipole field inferred from \( P–\dot{P} \).

Finally, the fitted emission area of the thermal hot spot (M+13) is found to be much smaller (20–30 m) than the canonical polar cap size, \( R_{\text{cap}} = R_*/\sqrt{R_*\Omega}/c = 140 \text{ m} \). In Sections 3 and 4 we demonstrate that, with proper atmospheric modeling, the emission area can be consistent with the polar cap size.

\(^5\) It is well known that a full vacuum gap potential would lead to too fast drifting subpulses, so a partially screened gap may be required (e.g., Gil et al. 2003; van Leeuwen & Timokhin 2012). See also Jones (2014) for an alternative model for drifting subpulses in pulsars.
profile of PSR B0943+10. We employ a magnetized partially ionized hydrogen atmosphere model (Ho et al. 2008) to generate X-ray spectra and pulse profiles with a procedure that accounts for relativistic effects (Ho 2007). We take a magnetic field of $2 \times 10^{12}$ G (the dipole field inferred from $P$ and $P$) that is directed perpendicular to the surface normal. We denote by $\beta$ the angle between the line of sight and the rotation axis, and by $\theta_B$ the angle between the magnetic and rotation axes (see Figure 1). We assume $\beta = 7^\circ-9^\circ$ and $\theta_B = 11^\circ-15^\circ$, as constrained by Deshpande & Rankin (2001), and NS mass and radius $M = 1.2 M_\odot$ and $R_N = 12$ km, so that the gravitational redshift is $1 + z = 1.19$. Whether we include or neglect an antipodal hot spot has no effect on our results since this second hot spot is not visible for the assumed viewing geometry and gravitational redshift. Here we describe our approximate fits, leaving detailed fits for future work with higher-quality data.

We find that an atmosphere with $T_{\text{eff}} \approx (1.4-1.5) \times 10^6$ K and emission radius $R_{\text{em}} \approx 85$ m matches qualitatively the radio-quiet X-ray spectrum of PSR B0943+43 from H+13. Previous blackbody fits find $T_{\text{BB}} = 3 \times 10^6$ K and $R_{\text{BB}} \approx 20-30$ m (H+13, M+13); note that the lower temperature and larger emitting area are typical characteristics of atmosphere emission compared to blackbody emission (see, e.g., Romani 1987; Shibano et al. 1992). The atmosphere emission radius (85 m) is much closer to that expected for a centered dipole (140 m) than the blackbody emission radius. Figures 2 and 3 show examples of our atmosphere model results for the pulse profile and pulse fraction compared to the measurements of M+13 (errors are calculated from the square root of the number of counts; see Gehrels 1986). We see that the atmosphere model shows promise, as long as the low-energy pulse fraction is indeed <100%.

4. LIGHT CURVES AND THE DISPLACED DIPOLE GEOMETRY

In the previous section, we allowed for the low-energy pulse fraction of the Q-mode emission to be relatively low. If future observations of PSR B0943+10 show that the pulse fraction is high across all energy bands, then the “canonical” model of Section 3 cannot work. We thus explore an alternative model for the emission geometry, in which the magnetic dipole axis is displaced by a vector from the center, such that $\theta_B$ is unchanged and the radio emission geometry and polarization profile are not altered (see Figure 1), assuming that the radio emission originates beyond $\sim 10 R_N$ away from the surface.

Displaced magnetic dipoles have been invoked before to explain various phenomena exhibited by pulsars. For example, Arons (2000) demonstrated that the position of the pulsar death line can be altered by displacing the magnetic dipole, and that to explain all observed active pulsars displacements of as much as $(0.7-0.8) R_N$ are needed. Recently, Burnett & Melatos (2014) showed that a displaced magnetic dipole may explain anomalies exhibited by the $Q-U$ phase portraits of some radio pulsars.

We consider a $M = 1.4 M_\odot$, $R_N = 10$ km NS with a dipole surface magnetic field $B_d \approx 2 \times 10^{12}$ G and propose the following scenario: (1) all or most of the X-ray emission from PSR B0943+10 is thermal in nature, which is, to date, consistent with the analyses of the X-ray spectra carried out by H+13 and M+13. (2) The magnetic dipole axis is transposed such that both polar cap hot spots are potentially visible, as shown in Figure 1. The larger (far) spot has a lower magnetic field, while the smaller (near) spot has a larger magnetic field. In the Q-mode, the large spot comes close to being invisible (note that the strong light bending effects enable the surface hot spots to be observed up to $135^\circ$ from the line of sight), and thus produces a nearly 100% pulsation. The observed Q-mode X-ray light curve is then composed of the nearly constant emission.
from the near spot, and the strongly pulsed emission from the far spot. (3) The transition from the \(Q\)-mode to the \(B\)-mode involves a small shift in the magnetosphere or the dipole axis, although the physical mechanism for such a shift is unknown at present. As a result, the large spot—which is already close to being unseen—becomes completely invisible, leaving only the nearly constant X-ray light curve of the small spot, and our line of sight now cuts through a slightly different portion of the small spot’s radio cone, thereby altering the observed radio emission as well.

We denote by \(\zeta\) the angle between the rotation axis and the large polar cap (see Figure 1). Quite naturally, the only way to produce a thermal light curve with a \(100\%\) pulse fraction across all energy bands is through geometry: a hot spot must be placed such that it is close to becoming invisible, in the range \(\zeta \approx (125^\circ, 135^\circ)\). Then the possible magnetic field axis orientations are constrained to a conical shell of width \(2\theta_B\) centered on the hot spot. However, since the observed peaks of X-ray and radio emissions from PSR B0943+10 are in phase \((H+13)\), the magnetic axis must be oriented as shown in Figure 1. It follows that the dipole must be displaced by a distance \(s \approx (0.85-0.9)R_*\), though its precise position, as well as the effective temperatures of each hot spot, remain unknown. We do not attempt to explore this parameter space. Rather, we present one test case to demonstrate that, in principle, all the X-ray observations of PSR B0943+10 can be explained by this model.

Although in principle both magnetic poles may produce radio emission cones, only the cone from the smaller “near” spot can sweep through our line of sight and be observed. As discussed in Section 2, the presence of drifting subpulses in the observed radio emission likely requires a strong magnetic field; therefore, we place the magnetic dipole very close to the surface of the small spot.

We use the method (“Temperature Template with Full Transport,” or TTFT) developed in Shabaltas & Lai (2012) to compute synthetic light curves for our chosen hot spot configuration. The TTFT method uses temperature profiles (as a function of depth) of magnetic NS atmospheres to efficiently compute the observed surface thermal radiation, taking into account the effects of magnetic atmosphere opacities, beam pattern, vacuum polarization, and gravitational light bending. For this work, we have used realistic ionized hydrogen magnetic NS atmosphere models (Ho & Lai 2001, 2003) to calibrate our temperature profiles for the relevant effective temperature and surface magnetic field values.

As the blackbody fits of both the \(B\)-mode and the \(Q\)-mode spectra are very similar, we fix the two hot spots to have equal temperature, \(T_{\text{eff}} = 1.5 \times 10^6 \text{ K}\). We fix the magnetic field strengths to be \(2 \times 10^{14}\text{G}\) and \(2.7 \times 10^{12}\text{G}\) for the small and large spots, respectively; it follows that the hot spot radii must be \(\approx 20 \text{ m}\) and \(\approx 100 \text{ m}\). Figure 4 presents our results.

Satisfactory agreement with the \(Q\)-mode observation is achieved. However, the \(B\)-mode light curve in this particular case is not entirely constant; it has a pulse fraction of \(31\%\), coming mostly from the lower-energy part of the emission. We note that this pulse fraction is within the \(3\sigma\) upper limit (i.e., \(\leq 50\%\)) derived by M+13 for the \(B\)-mode pulse fraction. Figure 5 shows that the \(B\)-mode theoretical light curve is consistent with the most up-to-date observations. We speculate that a careful search in the available phase space can reduce this pulsation, though it is not likely to disappear entirely.

5. DISCUSSION AND CONCLUSION

The simultaneous radio and X-ray observations of the model-switching pulsar PSR B0943+10 provide a unique opportunity for understanding this class of objects. In this paper, we have examined several puzzles presented by PSR B0943+10. Our conclusions can be summarized as follows.

We show that the large X-ray pulse fraction of the additional thermal component observed in the radio \(Q\)-mode can be adequately reproduced by using the canonical emission geometry of PSR B0943+10, as constrained by radio observations, and taking into account beaming due to a strong surface magnetic field. We find that the low-energy pulse fraction produced by this model is small (\(\sim 50\%\)).

We also consider another, more extreme, magnetic field geometry that is consistent with various observations, including the possibility that the extra thermal component has high pulse fractions in all energy bands. We show that by displacing the
magnetic dipole by a large amount ($\sim 0.85 - 0.9 R_\star$) from the center of the star, both the $B$- and the $Q$-mode X-ray observations can be explained as thermal emission coming from the magnetic polar caps, one of which is small in size, has a large surface magnetic field, and is responsible for the radio emission, while the other is large, has a lower magnetic field, and acts as the highly pulsed thermal component observed in the $Q$-mode. We speculate that, when the pulsar is radio bright, the larger spot becomes invisible, leaving only the smaller spot’s emission to be observed.

Overall, the models explored in this paper can be tested by future X-ray observations (e.g., the “extreme” model requires that most of the observed X-rays are thermal in nature and predicts a $\sim 30\%$ pulse fraction even in the $B$-mode). These will be valuable for constraining the magnetosphere plasma density and the magnetic field strength and geometry of PSR B0943+10, and will thereby further our understanding of the enigmatic behavior of pulsar mode switching.

W.C.G.H. appreciates the use of the computer facilities at KIPAC. This work has been supported in part by NSF grants AST-1008245, AST-1211061 and NASA grant NNX12AF85G.

REFERENCES

Arons, J. 2000, in ASP Conf. Ser. 202, IAU Colloq. 177, Pulsar Astronomy 2000 and Beyond, ed. M. Kramer, N. Wex, & R. Wielebinski (San Francisco, CA: ASP), 449

Burnett, C. R., & Melatos, A. 2014, MNRAS, 440, 2519

Camilo, F., Ransom, S. M., Chatterjee, S., Johnston, S., & Demorest, P. 2012, ApJ, 746, 63

Canuto, V., Lodenquai, J., & Ruderman, M. 1971, PhRvD, 3, 2303

CorDES, J. M. 2013, ApJ, 775, 47

Deshpande, A. A., & Rankin, J. M. 2001, MNRAS, 322, 438

Gehrels, N. 1986, ApJ, 303, 336

Gil, J., Melikidze, G. I., & Geppert, U. 2003, A&A, 407, 315

Hermsen, W., Hessels, J. W. T., Kuiper, L., et al. 2013, Sci, 339, 436

Ho, W. C. G. 2007, MNRAS, 380, 71

Ho, W. C. G., & Lai, D. 2001, MNRAS, 327, 1081

Ho, W. C. G., & Lai, D. 2003, MNRAS, 338, 233

Ho, W. C. G., Potehkin, A. Y., & Chabrier, G. 2008, ApJS, 178, 102

Jones, P. B. 2014, MNRAS, 437, 4027

Kaspi, V. M., Roberts, M. S. E., & Harding, A. K. 2006, in Compact Stellar X-ray Sources, ed. W. H. G. Lewin & M. van der Klis (Cambridge: Cambridge Univ. Press), 279

Kramer, M., Lyne, A. G., O’Brien, J. T., Jordan, C. A., & Lorimer, D. R. 2006, Sci, 312, 549

Lorimer, D. R., Lyne, A. G., McLaughlin, M. A., et al. 2012, ApJ, 758, 141

Lyne, A., Hobbs, G., Kramer, M., Stairs, I., & Stappers, B. W. 2010, Sci, 329, 408

Medin, Z., & Lai, D. 2006, PhRvA, 74, 062508

Medin, Z., & Lai, D. 2007, MNRAS, 382, 1833

Mereghetti, S., Tiengo, A., Esposito, P., & Turolla, R. 2013, MNRAS, 435, 2568

Rankin, J. M., & Suleymanova, S. A. 2006, A&A, 453, 679

Romani, R. 1987, ApJ, 313, 718

Ruderman, M., & Gil, J. 2006, A&A, 460, L31

Ruderman, M. A., & Sutherland, P. G. 1975, ApJ, 196, 51

Shabaltas, N., & Lai, D. 2012, ApJ, 748, 148

Shibanov, I. A., Zavlin, V. E., Pavlov, G. G., & Ventura, J. 1992, A&A, 266, 313

van Leeuwen, J., & Timokhin, A. N. 2012, ApJ, 752, 155

Zavlin, V. E. 2007, arXiv:astro-ph/0702426