Mathematical model and performance calculation of a double-wishbone independent suspension

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Abstract. The paper presents a mathematical model based on which performance calculations of a double-wishbone independent suspension are performed. First, the spatial motion of a rigid body was described by coordinate transformation, which provides the theoretical basis for the hard point coordinate calculation of double-wishbone suspension mechanism. After that, a mathematical model of the wheel guide mechanism has been built, and the kinematic performance and hard point reaction load were calculated. The results obtained are useful in the design of new suspension systems.

1. Introduction

Compared with integral axle, independent suspension has excellent stability and ride comfort, the structural advantages of the double wishbone are most obvious in the field of vehicles [1]. The prominent advantages of the system is design flexibility, it can get suitable motions through change the length of the control arm and the position of guiding rod. The performance of suspension is related to many factors, so it is researched by establishing a double wishbone independent suspension model in this article, the influence rules of location parameters are also analyzed.

2. Double wishbone independent suspension physical model

![Double wishbone independent suspension](image-url)
As shown in figure 1, it is the double wishbone independent suspension, which contains the upper and
down wishbones, spring and shock absorber are installed on it. D and E are connecting point between
upper arm and car body, A and B are connecting point between down arm and car body, C and F are
kingpin axis, point G is tire center.

3. The mathematical model of independent suspension

3.1. Coordinate analysis of hard point
Firstly, the relationship between the output angle $\beta$ of the upper arm and the input angle $\alpha$ of the
down arm of the double wishbone independent suspension system is analyzed, as shown in figure 2.

![Figure 2. Rotation relation of four bar linkage.](image_url)

The knowledge of kinematics of spatial mechanism can be seen [2]:

$$R_{12}R_{23}R_{34}R_{41} = [I]$$  \hspace{1cm} (1)

In which, $R_{ab}$ is transformation matrix between coordinate system a and b, $R_{23}$ and $R_{34}$ are Euler
transformation.

So the formula (1) can be written as:

$$[R_{a}]_b[R_{a2}]_i[R_{23}]_{R_{34}}[R_{p}]_{j}[R_{qi}]_i = [I]$$  \hspace{1cm} (2)

$$P = \sum_{j=1}^{m}(h_{j}k_{j} + s_{j}k_{j}) = h_{1}k_{1} + s_{1}k_{1} + h_{2}k_{2} + s_{3}k_{3} + h_{4}k_{4} + s_{4}k_{4} = 0$$  \hspace{1cm} (3)

In which

$$i = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},
  k = \begin{bmatrix} 0 \\ 1 \end{bmatrix},
  i = \begin{bmatrix} \cos \alpha \\ \sin \alpha \end{bmatrix},
  k = \begin{bmatrix} \cos \alpha \end{bmatrix},
  k_{i} = \begin{bmatrix} R_{a2} \end{bmatrix}R_{23}k_{i}.$$  

Substituting known conditions [3]:
\[ i_4 = \begin{bmatrix} R_{-\alpha_i} \end{bmatrix} \begin{bmatrix} R_{-\beta} \end{bmatrix} i_1 = \begin{bmatrix} \cos \beta \\ -\cos \alpha_i \sin \beta \\ \sin \alpha_i \sin \beta \end{bmatrix} \]  

\[ k_4 = \begin{bmatrix} R_{-\alpha_i} \end{bmatrix} \begin{bmatrix} R_{-\beta} \end{bmatrix} k_1 = \begin{bmatrix} 0 \\ \sin \alpha_i \\ \cos \alpha_i \end{bmatrix} \]  

(4)

(5)

Simplification of the formula (3):

\[ A \sin \beta + B \cos \beta + C = 0 \]

In which,

\[ C = \frac{1}{2} \left( s_i^2 + h_i^2 + s_i^2 + h_i^2 + h_i^2 - s_i^2 \right) - s_i s_i \cos \alpha_i + h_i h_i \cos \alpha - s_i h_i \sin \alpha_i \sin \alpha; \]

\[ A = s_i h_i \sin \alpha_i - h_i h_i \sin \alpha \cos \alpha_i; \quad B = h_i h_i + h_i h_i \cos \alpha. \]

Further derived:

\[ \beta = 2 \arctan \left( \frac{A \pm \sqrt{A^2 + B^2 + C^2}}{B - C} \right) \]  

(6)

Formula (6) can describe the relationship between the \( \alpha \) and \( \beta \) when the mechanism moves.

Further, the coordinate of point \( Q_1 \) is [4]:

\[
\begin{align*}
x_{Q_1} &= x_A + L_{A01} \times \frac{(x_B - x_A)}{L_{AB}} \\
y_{Q_1} &= y_A + L_{A01} \times \frac{(y_B - y_A)}{L_{AB}} \\
z_{Q_1} &= z_A + L_{A01} \times \frac{(z_B - z_A)}{L_{AB}}
\end{align*}
\]  

(7)

The coordinate \( C_i(x, y, z)^T \) of point \( C \) rotating around the \( AB \) axis is:

\[ C_i = Q_i + \bar{a}_i \]

(8)

In which, \( u_x, u_y, u_z \) are direction cosines of the lower arm rotation axis, \( \bar{a}_i = [R_{\alpha_i}] \bar{a} \).

\[
[R_{\alpha_i}] = \begin{bmatrix}
    u_x (1 - \cos \alpha) + \cos \alpha & u_x (1 - \cos \alpha) - u_z \sin \alpha & u_y (1 - \cos \alpha) + u_z \sin \alpha \\
    u_x (1 - \cos \alpha) + u_z \sin \alpha & u_z (1 - \cos \alpha) - \sin \alpha & u_y (1 - \cos \alpha) + u_z \sin \alpha \\
    u_x (1 - \cos \alpha) - u_z \sin \alpha & u_y (1 - \cos \alpha) + u_z \sin \alpha & u_z (1 - \cos \alpha) + \sin \alpha
\end{bmatrix}
\]

Similarly,

\[ F_i = Q_2 + \bar{b}_i \]
In which, $\bar{b}_i = \left[R_\beta \right] \bar{b}$, $\bar{b} = Q_x F$.

$$\left[R_\beta \right] = \begin{bmatrix}
u_1^2(1 - \cos \beta) + \cos \beta & u_1 u_y (1 - \cos \beta) - u_1 \sin \beta & u_1 u_z (1 - \cos \beta) + u_1 \sin \beta \\
u_1 u_1 (1 - \cos \beta) + u_1 \sin \beta & u_1^2 (1 - \cos \beta) + \cos \beta & u_1 u_y (1 - \cos \beta) - u_1 \sin \beta \\
u_1 u_1 (1 - \cos \beta) - u_1 \sin \beta & u_1 u_y (1 - \cos \beta) - u_1 \sin \beta & u_1^2 (1 - \cos \beta) + \cos \beta \\
\end{bmatrix}$$

Coordinate of knuckle arm pin point $L_4$ [5]:

$$\sqrt{(x_t - x_s)^2 + (y_t - y_s)^2 + (z_t - z_s)^2} = d_1$$
$$\sqrt{(x_t - x_c)^2 + (y_t - y_c)^2 + (z_t - z_c)^2} = d_2$$
$$\sqrt{(x_t - x_f)^2 + (y_t - y_f)^2 + (z_t - z_f)^2} = d_3$$

(9)

In which, $d_1$ is distance between L and S, $d_2$ is distance between L and C, $d_3$ is distance between L and F.

The coordinate of M which is tire to ground contact point is:

$$\begin{bmatrix}
x = x_G - \frac{w_1 w_3 R}{w_1^2 + w_2^2} \sqrt{\frac{\left(w_1^2 + w_2^2\right)^2}{w_1^2 w_3^2 + w_2^2 w_3^2 + \left(w_1^2 + w_2^2\right)^2}} \\
y = y_G - \frac{w_2 w_3 R}{w_1^2 + w_2^2} \sqrt{\frac{\left(w_1^2 + w_2^2\right)^2}{w_1^2 w_3^2 + w_2^2 w_3^2 + \left(w_1^2 + w_2^2\right)^2}} \\
z = \sqrt{\frac{\left(w_1^2 + w_2^2\right)^2}{w_1^2 w_3^2 + w_2^2 w_3^2 + \left(w_1^2 + w_2^2\right)^2}} R \\
\end{bmatrix}$$

(10)

In which, $x = x_M - x_G$, $y = y_M - y_G$, $z = z_M - z_G$, $w_1 = x_G - x_H$, $w_2 = y_G - y_H$, $w_3 = z_G - z_H$.

Through the above derivation, the main hard point coordinates are determined, it can be used to describe the parameters of suspension alignment.

Caster angle is:

$$\text{ANG}_{\text{caster}} = \arctan \frac{x_F - x_C}{z_F - z_C}$$

(11)

Toe angle is:

$$\text{ANG}_{\text{toe}} = \arctan \frac{x_H - x_G}{y_G - y_H}$$

(12)

Wheel camber angle is:
\[ \text{ANG}_{\text{camber}} = \arctan \frac{y_M - y_G}{z_G - z_M} \] (13)

Kingpin inclination angle is:

\[ \text{ANG}_{\text{inc}} = \arctan \frac{y_F - y_C}{z_F - z_C} \] (14)

3.2. Force analysis of hard point

The force of the down wishbone is the most complicated, as shown in figure 3:

![Figure 3. Space force of the down wishbone.](image)

Through the force balance in the direction of X can be obtained:

\[ F_{LX} + F_{PX} + F_{RX} + F_{FX} = 0 \] (15)

Through the force balance in the direction of Y can be obtained:

\[ F_{LY} + F_{PY} + F_{RY} + F_{FY} = 0 \] (16)

Through the force balance in the direction of Z can be obtained:

\[ F_{LZ} + F_{PZ} + F_{RZ} + F_{FZ} = 0 \] (17)

Simultaneous above equations, the force of the A and B can be calculated by the coordinate of the points.

4. Performance calculation of independent suspension

Programming based on MATLAB software, the double wishbone points of static position are: A(414, -209.3), B(922.8, -274.8), C(467.9, -579.8), D(1014.9, -673.8), E(762.9, 90.2), F(762.9, -619.8), H(1265.5, -1054). The different parameters of suspension following the displacement can be solved through the above formulas as follows:
As shown in figure 4-9, the mathematical model of double wishbone independent suspension can be calculated through Matlab, it can be seen that the caster angle is slightly larger follow the wheel
jumps upward, which could be used to offset the smaller inclination of caster angle when the vehicle brakes nods. In addition, the small change of the toe in angle ensures the lack of steering, so researchers can analyze the influence of important parameters of suspension on the vehicle handling stability, the conclusion could be used as a reference for design of the device.

5. Conclusions
A mathematical model of the independent double-wishbone suspension has been presented. This model served to evaluate several important performance parameters (caster angle, toe-in angle, kingpin inclination angle and track change with the wheel bounce and rebound), as well as the forces occurring at the suspension hard points. The results presented are useful in the design refinement of new suspension systems of automobiles.

References
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