Galactic Structure From Infrared Surveys

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Abstract.

By combining the 2MASS and DENIS infrared surveys with the USNO-B proper-motion catalog, it will be possible to map the structure of the Galaxy in unprecedented detail. The key parameter that these surveys measure together, but neither measures separately, is $t_\ast = r_\ast/v_\perp$, the time it takes a star moving at transverse speed $v_\perp$ to cross its own radius $r_\ast$. This parameter cleanly separates white dwarfs, main sequence stars, and giants, and simultaneously separates disk from spheroid stars. The infrared photometry then yields photometric parallaxes for the individual stars, while the proper motions give kinematic information. Hence, it becomes possible to measure the physical and velocity distributions of the known Galactic components and to identify new ones. The analysis of such a huge data set poses a major technical challenge. I offer some initial ideas on how to meet this challenge.

1. Introduction

50 years from now, our catalogs will probably list position, color, and velocity information for $10^{10}$ stars, a large fraction of all the stars in the Galaxy and about 5 orders of magnitude more than today. But it is not clear how much more we will know about Galactic structure than we do today. For example, how much more would we know about the air in this room if we catalogued the position and velocity of its $10^{34}$ molecules, than we do just from the temperature, pressure, and composition? It is appropriate to contemplate the construction of such a gigantic star catalogue because within 2 or 3 years we will be half way there (logarithmically). It is not at all obvious how we can make use of the forthcoming vast quantities of data to learn about the structure of our Galaxy.

Traditionally, the Galaxy is broken down into components, and as more data are acquired, more components are added: disk, thick disk, spheroid, bulge/bar, spiral arms (e.g. Bahcall 1986). Each component is described by a few parameters, e.g., scale height, scale length, and luminosity function for the disk. The study of Galactic structure is basically one of identifying these components and measuring their parameters with increasing precision.

We might at first imagine taking all of our new data on the one hand, and all the global parameters describing the various Galactic components on...
the other, and dumping them both into a huge maximum likelihood program that will in turn spit out a detailed description of the Galaxy. However, such an approach would overwhelm even the most optimistic estimate of near-term computing advances.

2. Individual-Star Parameter Counting

From the standpoint of Galactic structure, the three most important quantities to learn about each star are its distance, \( d \), and its type, characterized by its radius, \( r_* \), and its temperature, \( T \). In order to learn anything about these quantities from photometry, one must simultaneously measure the extinction, \( A_V \). The two infrared surveys, 2MASS and DENIS, will measure fluxes in only three bands, \( JHK \) and \( IJK \) respectively. Simple parameter counting tells us we cannot determine four quantities from three observables. In fact, we can extract only \( A_V \), \( T \), and \( \theta_* = r_* / d \), the angular radius of the star. The deviation of the two observed colors from the black-body two-color relation yields \( A_V \), and the dereddened color gives \( T \). From the dereddened color and flux plus the Planck law one then knows \( \theta_* \). A white dwarf at 1 pc, a subdwarf at 25 pc, a disk dwarf at 40 pc, and a disk giant at 500 pc will all look approximately the same to an infrared survey if they have the same temperature. One way out might be to get a fourth color, but in fact since stars are approximately black bodies, the fourth color can already be predicted from the other three.

Thus, even to isolate the individual Galactic components, let alone measure them, requires more information than we are going to get from an infrared survey. Fortunately, before these surveys are completed, the USNO-B astrometric catalog should be available.

3. Proper Motions

From this catalog (combined with an additional epoch from 2MASS/DENIS), one should be able to achieve an astrometric precision of

\[
\sigma_\mu \sim 3 \text{ mas yr}^{-1} \sim 14 \text{ km s}^{-1} \text{ kpc}^{-1} \quad \text{(North),}
\]

and perhaps a factor 2 worse in the South (D. Monet 1998, private communication)

Photometry plus proper motions together yield a very curious quantity,

\[
t_* = \frac{r_*}{v_\perp} = \frac{r_* / d}{v_\perp / d} = \frac{\theta_*}{\mu},
\]

the time it takes a star to cross its own radius. Why is this quantity of any interest? Because, as Table [1] shows, it separates out very nicely different populations of stars.

Of course, any individual star will still be ambiguous at some level. However, in an analysis of millions of stars, all that is important is that most of the stars in a given component are so identified with \( > 70\% \) probability. Maximum likelihood can then easily sort out the components statistically. Since individual components are separated in \( t_* \) by a factor 3 to 5, this condition is well satisfied.
Table 1. Stellar Radius Crossing Times

| Star Type          | $r/r_\odot$ | $v_\perp/(\text{km s}^{-1})$ | $t_*/\text{min}$ |
|--------------------|-------------|-------------------------------|------------------|
| Halo White Dwarf   | 0.02        | 200                           | 1                |
| Disk White Dwarf   | 0.02        | 40                            | 6                |
| Halo Sub-Dwarf     | 0.5         | 200                           | 30               |
| Disk Dwarf         | 0.8         | 40                            | 230              |
| Halo Giant         | 10          | 200                           | 600              |
| Disk Giant         | 10          | 40                            | 3000             |

Virtually all of the stars detected by 2MASS have counterparts on the POSS E ($R$ band) plates in the North or the equivalent ESO plates in the South, so proper motions should be available for all of them.

4. The Near Zone

At 1 kpc, a disk star has a proper motion $\mu \sim 8$ mas yr which should be detectable at least in the North. Main-sequence disk stars down to mid-G ($M_K < 4$) should be detectable in $K$ at the 10$\sigma$ level at this distance. There are about $10^7$ such stars. There are also about $10^6$ detectable thick-disk star and $10^4$ detectable halo subdwarfs. The proper motions of these latter two classes of stars would be detectable at much larger distances (2 kpc and 5 kpc respectively). However, the fraction of main-sequence stars that are detectable photometrically drops rapidly past 1 kpc. Nevertheless, of order $10^4$ halo giants will be detectable in both the photometric and proper motion surveys.

What sort of questions can be asked about the stars in this zone? First, of course, one can measure the parameters of the known components much more precisely than they have been measured before. For example, the disk is usually parameterized by a scale height $h \sim 200$ pc, and a scale length of $H \sim 3$ kpc, while the thick disk has $h \sim 600$ pc and $H \sim 3$ kpc (Gould, Bahcall, & Flynn 1997). The spheroid is parameterized by a power-law density fall-off $\nu \sim -3.1$ and a flattening $c/a = 0.8$ (Gould, Flynn, & Bahcall 1998).

However, with the vast new quantities of data that will soon be available, we should be able to ask many other questions about Galactic structure. What are the velocity distributions of the various components? Traditionally, one has tried to measure only the first and second moments, but higher moments will also be accessible. Gould & Popowski (1998) found, for example, that the kurtoses of the halo as determined from $\sim 150$ RR Lyrae stars are $\sim 2$, 3 and 4 in the three principal directions, compared to 3 for a Gaussian and 1.8 for a sharp-edged box distribution. This is undoubtedly telling us something about the origin of the stellar halo, but what? Much more precise and detailed information about all three components will be obtained from the IR and proper motion surveys. Are the scale height and scale length of the disk and thick disk functions of spectral type? Are there additional components? For example, Sommer-Larsen & Zhen (1990) and Gould et al. (1998), each some found evidence for a two-component
halo, one roughly round and the other highly flattened (but not rotating). The high vertical kurtosis found by Gould & Popowski (1998) could also be regarded as evidence for this. Or it is possible, that our current disk/thick disk dichotomy should be replaced by a continuum of components, as suggested by Norris & Ryan (1991).

5. Complementarity of SDSS

If there are a continuum of components, they will be difficult to distinguish from discrete components using IR-survey plus proper-motion data alone. The argument codified in Table 1 was that discrete components could be separated based on their radius crossing times, $t_*$. However, this assumed that the components are separated by a factor two in $t_*$. To distinguish finer graduations requires a metallicity-sensitive indicator. This will not be available from 2MASS and DENIS, but will be from the Sloan Digital Sky Survey (SDSS), which has 5-color photometry out to the atmospheric cutoff. Unfortunately, in its present incarnation, SDSS will saturate at 15th magnitude, implying very little overlap with $K < 14$ surveys (and then only for stars with poor IR photometry.) Moreover, SDSS will cover only regions far from the plane.

However, from the standpoint of the statistical separation of components, it is not necessary that there be SDSS and IR photometry for the same stars. It is only necessary that the SDSS stars be close enough that they have proper motions reliable to better than a factor 2. The transverse motions of thick-disk stars are $\sim 100 \text{ km s}^{-1}$, implying factor 2 proper motions errors at a distance $\sim 3 \text{kpc}$. At this distance, the entire main sequence is unsaturated.

6. Mass Distribution of Disk

Traditionally, measurements of the local mass density have relied on stellar radial velocity and density measurements in a cone perpendicular to the Galactic plane. (e.g. Bahcall 1984; Kuijken & Gilmore 1989, 1991; Bahcall, Flynn & Gould 1992; Flynn & Fuchs 1994). Crézé et al. (1998) pioneered a radically different type of survey based on Hipparcos parallaxes and proper motions of $\sim 3000$ nearby stars. This study was by far the most sensitive to mass close to the Galactic plane. They found that the local mass density is almost completely accounted for by visible material. However, because of the Hipparcos magnitude limit ($V < 8$) they were able to probe only within 100 pc of the Sun. Moreover, since they were mainly restricted to bright (and hence young) A and F stars, it is possible that their sample was not dynamically mixed and hence subject to systematic errors.

IR surveys (plus proper motions) will provide a much more robust measurement of the density close to the plane, first because the sample will be three orders of magnitude larger than the Hipparcos sample, and second because the stars will be primarily G and later and thus older and more dynamically mixed and so less subject to systematic errors.

Of course, unlike the Hipparcos stars, these stars will not have trigonometric parallaxes. One will be forced to rely on photometric parallaxes. Will the
inevitable distance errors, including systematic errors, compromise the result. Remarkably, no. The density, $\rho$, is given by Poisson’s equation

$$\rho = -\frac{1}{4\pi G} \frac{dK_z}{dz},$$

(3)

where $K_z(z)$ is the vertical acceleration as a function of height $z$ above the plane. For simplicity, I consider an isothermal population of stars, but the argument I am about to give applies equally well to any velocity distribution. Then

$$K_z = -\bar{v}_z^2 \frac{d\ln \nu}{dz},$$

(4)

where $\bar{v}_z^2$ and $\nu(z)$ are the vertical velocity dispersion and vertical density profile of some stellar population. Now, suppose that there is a survey close to the plane (e.g. $|b| < 10^\circ$), and suppose that all photometric distances are systematically overestimated by a factor $\alpha$ relative to the true distances. Then $z \to \alpha z$ will be overestimated from the measured angular position, and $v_z \to \alpha v_z$ will be underestimated from the measured proper motion. ($\nu \to \alpha^{-3/2} \nu$ will be underestimated, but this has no effect, because only the derivative of $\ln \nu$ enters eq. (4)). Hence, from equation (3) $K_z$ will be overestimated by $\alpha^2/\alpha = \alpha$, so that from equation (3), $\rho$ will be properly estimated despite the errors. Of course, $v_z$ cannot be directly estimated from the proper motion and the distance. Rather $v_z = \mu_b d \sec b + v_\parallel \tan b$, where $v_\parallel$ is the component of motion parallel to the Galactic plane. That is, $\bar{v}_z^2 = (\mu_b d)^2 \sec^2 b - \bar{v}_\parallel^2 \tan^2 b$. To obtain $\bar{v}_z^2$, one must therefore independently estimate $\bar{v}_\parallel^2$ and subtract it. For $|b| < 10^\circ$, $\tan^2 b \lesssim 0.03$, so the systematic error in this correction is negligible. It should be possible to obtain excellent results for $z < 170$ pc (assuming sensitivity to stars at 1 kpc). For higher $b$, the systematic errors will become progressively worse. However, it will be possible to map the mass density with much greater precision than previously, especially close to the plane.

7. The Far Zone

Paczyński & Stanek (1998) have recently suggested $I$-band luminosities of clump giants (metal rich core-helium burning stars) as standard candles. Whether clump giants eventually turn out to have the same $I$-band luminosities independent of age, metallicity, and other environmental factors remains open to question. However, the bulge clump giants can reasonably be expected to have a common set of environmental characteristics and therefore act at least as relative standard candles for the bulge. That is, they may or may not tell us about the galactocentric distance, but they can certainly trace the density structure of the bulge. With $K_0 \sim 13$ and $I_0 \sim 14$, they will be visible in the bulge in all regions with $A_V < 9$. They can be dereddened in the standard way. Hence, they will provide a three dimensional map of the bulge. The typical proper motions (from internal dispersion) are only $\sim 3$ mas yr$^{-1}$. Since these are the same order as the errors, it may be possible to obtain statistical information on the velocity dispersion as a function of position, although systematics will be a major concern.
Spiral arms contain large numbers of early type stars, probably of common metallicity, which should provide good relative distances. However, they will yield almost no proper-motion information.

It will also be possible to map the three dimensional distribution of dust using early type stars and clump giants.

8. Likelihood

We can imagine, then, maximizing the likelihood of a given model represented by say 100 parameters, given the observation of say 400,000,000 data values. The parameters being disk scale height, scale length, luminosity function, thick disk parameters, spheroid parameters, velocity ellipsoids, kurtoses, bar parameters, spiral arm densities and pitch angles, etc. The data are JHK photometry and proper motions for the stars, plus other colors when available from various sources. In principle, one could just dump all this in a giant black-box likelihood program and wait for it to spit out the answer. A big job, perhaps, but one can always hope that computers will improve sufficiently by then. Good luck!

In fact, new computational methods will be required. The evaluation of the likelihood function requires the prediction of the probability of hundreds of millions of observations for a huge ensemble of models spanning a vast parameter space. Here I outline one possible approach based on the one used by Gould et al. (1998).

Consider a bin in observation space \( J, H, K, \alpha, \delta, \mu_\alpha, \mu_\delta \). The probability of observing \( n \) stars in this bin is

\[
P_{J,H,K,\alpha,\delta,\mu_\alpha,\mu_\delta} = \frac{1}{n!} \exp(-\tau_{J,H,K,\alpha,\delta,\mu_\alpha,\mu_\delta})
\]

where \( \tau \) is the expected density of stars times the volume of the parameter-space bin. Then, by definition, \( L = \prod_\beta P_\beta \) where \( \beta = (J, H, K, \alpha, \delta, \mu_\alpha, \mu_\delta) \). Hence,

\[
\ln L = \sum_\beta n_\beta \ln \tau_\beta - \sum_\beta \tau_\beta - \sum_\beta \ln n_\beta!.
\]

In the limit of very small bins \( n \to 0, 1 \), which implies that \( \ln n_\beta! = 0 \). Hence,

\[
\ln L = \sum_{\beta, \text{detections}} \ln \tau_\beta - N_{\text{exp}}
\]

where \( N_{\text{exp}} \) is the total number of detections predicted by the model.

We can now write \( \tau_\beta \) as a vector product of two types of quantities, one of which depends on the choice of model parameters, and the other of which is independent. For example, assuming disk model parameters scale height \( h \), scale length \( H \), and \( K \)-band luminosity function with 1-mag averaged bin densities \( \Phi_l \), one can write,

\[
\tau_{i,j,k,p} = \sum_{l,m} \Phi_l \nu_{k,m}(h,H)\text{cmd}_{i(m),j(m),k,l,m}\text{pm}_{k,p,m}(v_1,v_2,v_3,\sigma_1,\sigma_2,\sigma_3).
\]
Here \( i, j, k, p, q \) are observational bins, \( i = (H - K)_0 \), \( j = K_0 \), \( k = (\alpha, \delta) \), and \( p = (\mu_\alpha, \mu_\delta) \). The model-dependent density is given by \( \nu_{k,m}(h,H) \) as a function of distance modulus \( m \), position vector \( k \), and model parameters \( h \) and \( H \). The model-dependent observed proper motion distribution, \( \text{pm}_{k,p,m} \), is given as a function of \( k, m \), and proper-motion vector \( p \), together with the model parameters \( v_1, v_2, v_3 \) bulk velocity and \( \sigma_1, \sigma_2, \sigma_3 \) dispersions. However, the color-mag diagram, which requires a cumbersome numerical integration over observational errors, is given once and for all.

Similarly, one can write \( N_{\text{exp}} \) as

\[
N_{\text{exp}} = \sum_{k,l,m} \phi_l \nu_{k,m}(h,H) \text{cmd}_{\text{tot},k,l,m},
\]

where

\[
\text{cmd}_{\text{tot},k,l,m} = \sum_{i,j} \text{cmd}_{i,j,k,l,m}.
\]

Again \( \text{cmd}_{\text{tot},k,l,m} \), which involves an enormous numerical integration, needs to be calculated only once. Equation (10) (and its derivatives with respect to all parameters) can be calculated quite easily. If one were to really try to evaluate equation (8) for each of \( 10^7 \) stars for each iteration, it would still be a cumbersome project. Just storing the matrix elements for the model-independent function \( \text{cmd}_{i,j,k,l,m} \) might prove unwieldy. However, likelihood space can be searched on progressively larger subsets of the data, say first using \( 10^5 \) stars to locate the approximate solution, and then increasing the sample to get more precise results. The last iterations would be time consuming, but there would be few of them.

9. Conclusions

The completion of the 2MASS and DENIS surveys, together with the USNO-B astrometric catalog will revolutionize our knowledge of Galactic structure. The astrometric input is required not only to obtain kinematic information, but also to separate out the various components of the Galaxy, which otherwise would be almost totally degenerate. The computational facilities required to interpret these data sets appear at first sight daunting. I have tried to give some initial suggestions on how the analysis can be simplified.

Acknowledgments. I thank Martin Weinberg for several lengthy discussions that were critical to the preparation of this paper. This work was supported in part by grant AST 97-27520 from the NSF and in part by grant NAG5-3111 from NASA.

References

Bahcall, J. N. 1984, ApJ, 276, 169
Bahcall, J. N. 1986, ARA&A, 24, 577
Bahcall, J. N., Flynn, C., & Gould, A. 1992, ApJ, 389, 234
Bienaymé, O., Robin, A., & Crézé, M. 1987, A&A, 180, 94
Crézé, M., Chereul, E., Bienaymé, O., & Pichon, C. 1998, A&A, 329, 920
Flynn, C. & Fuchs, B. 1994, MNRAS, 270, 471
Gould, A., Bahcall, J. N., & Flynn, C. 1997, ApJ, 482, 913
Gould, A., Flynn, C., & Bahcall, J. N. 1997, ApJ, 508, 000 [astro-ph 9711263]
Gould, A. & Popowski, P. 1998, ApJ, 508, 000 [astro-ph 9805176]
Kuijken, K. & Gilmore, G. 1989, MNRAS, 239, 605
Kuijken, K. & Gilmore, G. 1991, ApJ, 367, L9
Norris, J. E. & Ryan, S. G. 1991, ApJ, 380, 403
Paczyński, B. & Stanek, K.Z. 1998, ApJ, 494, L219
Sommer-Larsen, J. & Zhen, C. 1990, MNRAS, 242, 10