Synthesis of optimal control force for tripod manipulator drives

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Abstract. A tripod-based parallel structure manipulator is considered in the article. The movement of the working body from a known position to a given final position is realized by changing the lengths of the tripod end points. Linear links with an electric DC drive are used as end points. The algorithm was developed for the synthesis of the electric motor control voltages when moving the parallel structure manipulator working body based on the tripod from the initial position to the specified final position of the fixed travel time from the minimum criterion characterizing heating of the electric motor. The mathematical model of the manipulator controlled movements is described by linear differential equations of the second order. The static characteristic of the motor is used. The resistance force is taken proportional to the working body movement speed. The problem is solved by the methods of classical calculus of variations. The results of numerical modeling are presented, confirming the effectiveness of the proposed algorithm for determining the working body movement optimal laws.

1. Introduction

Manipulators of parallel structure are used in the assembly of microcircuits, sorting and packaging of products [1]. To mechanize these operations, a large number of various machines was created that facilitate manual labor and increase its productivity [2]. Ineffective control of manipulators increases energy costs, reduces productivity, and with manual control leads to operator fatigue and the subsequent occurrence of positioning errors. The nature of the manipulator points movements is determined by the implementation of a given technological operation. The control system must ensure the movement of the working body to a given point in the working area with the necessary accuracy. The movement of the working body from a known current position to a given final position is realized by changing the lengths of the manipulator - tripod end points. The problem is solved in two stages. First, the problem of positioning is solved. The problem is that, with a known initial configuration of the manipulator, generalized coordinates of the manipulator are found for given final coordinates of the working body [3]. At the second stage, the laws of generalized coordinates variation satisfying the
given boundary conditions are determined. This problem is most easily solved by choosing, for each generalised coordinate, one of the known laws of changing the speed lengths of the manipulator end points [4]. When performing these movements, the drive electric motors operating mode is repeated and short-lived. Losses in electric motors during the operation in this mode mainly depend on the consumed current and power. The nominal values of these parameters are set from the permissible heating of the electric motors [5]. In solving these problems, linearized models of the manipulation system dynamics are often considered, allowing synthesis of control using well-developed methods of the linear systems control theory [6]. These circumstances determine the urgency of the problem of effective control algorithms synthesis for parallel manipulators based on the tripod.

2. Materials and methods

The kinematic diagram and photo of the parallel structure manipulator are given in the Fig. 1. The manipulator consists of a spatial movable three-rod mechanism with end points of variable length $l_1$, $l_2$, $l_3$. One ends of these points are connected by means of bi-movable hinges located on a fixed base. The longitudinal axis of the three controlled hydraulic cylinders converge at one point by means of a ball hinge, which provides increased rigidity of the manipulator and small dynamic errors [3].

![Kinematic diagram and photo of the manipulator](image)

**Figure 1.** Kinematic diagram (a) and general view (b) of the manipulator - tripod on a fixed base.

As end points, linear links with an electric DC drive (actuators) are used. All actuators are equipped with a built-in analog displacement sensor. The number of manipulator freedom degrees is three. They are horizontal and vertical movement, respectively. The design of the manipulator provides holonomic connections between the coordinates of the point $M_{3d}(t)$, $y_d(t)$, $z_d(t)$ - the center of spherical hinge mass and the lengths of the end points:
\( \sqrt{x_M^2 + (y_M - OA)^2 + z_M^2} - l_1 = 0, \sqrt{(x_M - OB)^2 + y_M^2 + z_M^2} - l_2 = 0, \sqrt{(x_M + OB)^2 + y_M^2 + z_M^2} - l_3 = 0 \).

Then from the solution of the direct position equation, we obtain:

\[ x_M = \frac{l_3^2 - l_2^2}{4OB}, y_M = \frac{L}{2OA}, z_M = \left( l_2^2 - \frac{(l_2^2 - l_1^2)^2}{16OB^2} - \frac{L^2}{4OA^2} \right)^{\frac{1}{2}}, L^2 = (-l_1^2 + 0.5l_2^2 + 0.5l_3^2 - OB^2 - OA^2). \]

The manipulator-tripod has a great rigidity. Therefore, the generalized coordinates of the manipulator \( l_1(t), l_2(t), l_3(t) \) determine the Cartesian coordinates of the working body with great accuracy. The task is to determine the laws of working body movement in space, ensuring its movement from the initial position to the specified position, for a finite time with a minimum of heat losses in electric motors.

Manipulator geometrical parameters: length of end points \( l_k(t) \); angles \( \phi_k \) and \( \delta_k \) (k=1÷3), defining the orientation of the points in space; the Cartesian capture coordinates \( x_M(t), y_M(t), z_M(t) \) relative to the fixed coordinate system \( Oxyz \) uniquely describe the manipulator configuration at the current time.

Three coordinates are independent [3]. To synthesize the algorithm for controlling the manipulator-tripod efforts in [7], the manipulator dynamics mathematical model was used taking into account the masses of end points. The analysis of the manipulator points movement differential equations shows that when studying the capture movements of end points significantly shorter lengths, the physical meaning of the Lagrange multipliers included in these equations consists in the equality of the sum of the control forces projections on the fixed coordinate system corresponding axis [7]. In this case, the equations of the drive can be written in the form of material point differential equations. In normal format, these equations have the form:

\[
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= \frac{F_1}{m} + k_1 g - \mu X_2, \\
\dot{x}_3 &= x_4, \\
\dot{x}_4 &= \frac{F_2}{m} + k_3 g - \mu X_4, \\
\dot{x}_5 &= x_6, \\
\dot{x}_6 &= \frac{F_3}{m} + k_5 g - \mu X_6,
\end{align*}
\]

where \( x_1 = l_1, x_3 = l_2, x_5 = l_3; F_i(t), k=1÷3 \) are efforts in the manipulator points (control functions); \( m \) is the mass of a spherical assembly with a portable load; \( \mu \) is the relative coefficient of viscous resistance forces; \( k_1, k_3, k_5 \) are constant coefficients that take into account the magnitude of gravity acting on the \( k \) actuator.

The static characteristic of an independent direct current drive DC motor is [8]:

\[
F_k = ru_k - s_i_k, (k = 1 - 3),
\]

where \( r, s \) are coefficients depending on the parameters of the engine and mechanical transmission, which converts the electric motor rotor rotational movement into the translational movement of the actuator rod; \( u_k \) is the control voltage.

The equation of determining the functions \( x_{2i}(t), i=1,2,3 \) is formulated, which moves the manipulator working body from the initial position to the final during the time \( T \) and delivering a minimum to the functional [9]:

\[
J = 0.5 \int_0^T \left( F_1^2 + F_2^2 + F_3^2 \right) dt.
\]
The required functions must satisfy the following boundary conditions:
\[ x_{2i-1}(0) = x_{2i-1,0}, \quad x_{2i-1}(T) = x_{2i-1,T} \] and \[ x_2(0) = x_2(T) = 0. \] (4)

The control functions are the ones applied to the armature of the electric motor voltage \( u_k(t) \).

The necessary optimality conditions [10] are written in the form:
\[ \frac{\partial H(\mathbf{x}, \mathbf{u}, \mathbf{\lambda})}{\partial u_k} = 0. \] (5)

where the Hamiltonian \( H \) in accordance with (1-3) has the form:
\[ H(t) = 0.5(F_1^2 + F_2^2 + F_3^2) + \dot{\lambda}_1 x_1 + \lambda_2 \dot{x}_2 + \lambda_3 x_3 + \lambda_4 \dot{x}_4 + \lambda_5 x_5 + \lambda_6 \dot{x}_6 \] (6)

Based on the equation (1, 6), and following (5), we obtain:
\[ \frac{\partial^2 H(\mathbf{x}, \mathbf{u}, \mathbf{\lambda})}{\partial u_k \partial \mathbf{u}} > 0, \] the necessary condition (5) is sufficient.

The influence functions \( \lambda_i(t), \quad i = 1, 6 \) are determined by the Euler–Lagrange equations [10]:
\[ \dot{\lambda}_i(t) = -\frac{\partial H(\mathbf{x}, \mathbf{u}, \mathbf{\lambda})}{\partial \mathbf{x}}^T, \]
\[ \dot{\lambda}_1(t) = 0, \] (10)
\[ \dot{\lambda}_2(t) = (ru_1 - sx_2)r + \dot{\lambda}_2 \left( \frac{s}{m} + \mu \right), \] (11)
\[ \dot{\lambda}_3(t) = 0, \] (12)
\[ \dot{\lambda}_4(t) = (ru_2 - sx_4)r + \dot{\lambda}_4 \left( \frac{s}{m} + \mu \right), \] (13)
\[ \dot{\lambda}_5(t) = 0, \] (14)
\[ \dot{\lambda}_6(t) = (ru_3 - sx_6)r + \dot{\lambda}_6 \left( \frac{s}{m} + \mu \right). \] (15)

From (10, 12, 14) it follows:
\[ \dot{\lambda}_1(t) = \text{const.}, \dot{\lambda}_3(t) = \text{const.}, \dot{\lambda}_5(t) = \text{const.}, \] and from (7, 8, 9):
\[ \frac{ru_1 - sx_2}{m} = -\frac{\dot{\lambda}_2}{m}, \quad \frac{ru_2 - sx_4}{m} = -\frac{\dot{\lambda}_4}{m}, \quad \frac{ru_3 - sx_6}{m} = -\frac{\dot{\lambda}_6}{m}. \] (16)

Then, taking into account the equations of system (1) and correlations (16), we find differential equations for extremals of functional (3):
\[ \ddot{x}_{2i} - \mu^2 x_{2i} = \frac{\lambda_{2i-1}}{m^2} - \mu k_{2i-1} g. \] (17)
The general solution of equations (17), written by taking into account the boundary conditions (4) is:

\[ x_{2i} = \frac{B_{2i-1}}{\mu^2} \left( -1 + \frac{1 - e^{-\mu t}}{e^\mu - e^{-\mu t}} e^{\mu t} - \frac{1 - e^{\mu t}}{e^\mu - e^{-\mu t}} e^{-\mu t} \right), \quad (18) \]

The notation is introduced here:

\[ \lambda_{2i-1} = \frac{\mu^2 (x_{2i-1,0} - x_{2i-1,T})}{\mu T + 2 (2 - e^{\mu T} - e^{-\mu T})}. \]

By integrating (18) we find:

\[ x_{2i-1}(t) = x_{2i-1,0} - \frac{B_{2i-1}}{\mu^2} t + \frac{B_{2i-1}}{\mu^3} \left( \frac{1 - e^{-\mu t}}{e^\mu - e^{-\mu t}} (e^{\mu t} - 1) + \frac{1 - e^{\mu t}}{e^\mu - e^{-\mu t}} (e^{-\mu t} - 1) \right). \quad (19) \]

The boundary conditions (4) can be satisfied for:

\[ \lambda_{2i-1} = m^2 (B_{2i-1} + \mu k_{2i-1,0}). \]

Thus, the optimal laws of end points lengths changing are found. Based on the laws of generalized coordinates variation (18, 19) and control voltages (16), it is possible to form the structure of a control system with feedbacks. However, the equation (18) shows that the motion along extremals is unstable. Methods for ensuring parametric stability of closed-loop control systems in the synthesis of controller algorithms are described in [11].

3. Conclusion

The value of the optimized functional depends on the parameters of the mathematical model \( \mu, m, T \) and does not depend on the parameters of the electric motor and mechanical transmission. To evaluate the effectiveness of the found laws of working body movement change, its values \( \eta = \left( J_{\mu} - J_{\mu_{opt}} \right) / J_{\mu_{opt}} \) were compared for the found optimal laws and for the widespread triangular law of speed change for various values of the mathematical model parameters (table 1):

| №  | \( \mu \) | \( m, \text{kg} \) | \( T, \text{c} \) | \( \eta \) |
|----|-----------|-----------------|--------------|------|
| 1  | 0.145     | 3               | 3            | 0.091|
| 2  | 0.145     | 5               | 3            | 0.091|
| 3  | 0.072     | 5               | 6            | 0.374|
| 4  | 0.072     | 3               | 6            | 0.374|

Thus, the calculation results show that without comparing the values of the quality criterion with optimal control and with other types of control - simpler, it is impossible to draw a conclusion about the appropriateness of applying optimal control.

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