Evidence for a negative-parity spin-doublet of nucleon resonances at 1.88 GeV

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Abstract

Evidence is reported for two nucleon resonances with spin-parity $J^P = 1/2^-$ and $J^P = 3/2^-$ at a mass just below 1.9 GeV. The evidence is derived from a coupled-channel analysis of a large number of pion and photo-produced reactions. The two resonances are nearly degenerate in mass with two resonances of the same spin but positive parity. Such parity doublets are predicted in models claiming restoration of chiral symmetry in high-mass excitations of the nucleon. Further examples of spin-parity doublets are found in addition. Alternatively, the spin doublet can be interpreted as member of the 56-plet expected in the third excitation band of the nucleon. Implications for the problem of the \textit{missing resonances} are discussed.

PACS: 11.80.Et, 13.30.-a, 13.40.-f, 13.60.Le

SU(3) symmetry was the prerequisite for the interpretation of mesons and baryons \cite{1} as systems composed of quarks and antiquarks or of three quarks, respectively, and is the basis of quark models. As three-particle systems, nucleons - protons and neutrons - are expected to exhibit a rich spectrum of rotational and vibrational energy levels. The excitation levels of the nucleon are extremely short-lived and decay in a variety of different decay modes. Many states are predicted which overlap and are very difficult to resolve. Only a fraction of the expected states has been found experimentally; the absence of many states is called the problem of the \textit{missing resonances}. It is still unclear if the \textit{missing resonances} do not exist or if they escaped detection due to the limitations of experiments performed so far. Most information on the spectrum of excited nucleons is derived from pion-nucleon ($\pi N$) elastic scattering experiments which are incapable to identify resonances with weak coupling to $\pi N$. Indeed, model calculations suggest that the \textit{missing resonances} do have weak $N\pi$ coupling \cite{2}.
New experimental techniques and new data are obviously required. Photo-
production of mesons offers distinctive advantages. The use of photon beams
and inelastic reactions avoid $\pi N$ in the entrance and exit channel; polarized
photon beams, polarized hydrogen targets, and measurements of the polarization
of outgoing baryons - best accessible in the case of hyperon production -
are important to separate contributions with different quantum numbers.
Different final states are sensitive to different resonances; hence it is important
to combine different channels into a common analysis and to search for
new resonances in a variety of different reactions. In this letter we present
the results of a multichannel partial wave analysis (PWA) of a large body of
reactions, in particular of the large data base which exists on hyperon produc-
tion. From hyperon production experiments we expect a high sensitivity to
low-spin resonances above - and close to - the $\Lambda K$ and $\Sigma K$ thresholds which
range from 1610 to 1690 MeV. This is an interesting mass region since so far
all established low-spin negative-parity nucleon resonances have masses below
1700 MeV.

A large data base was fitted within the Bonn-Gatchina multichannel partial
wave analysis. The data include nearly the complete available data base
on pion-induced reactions and of photo-production off protons. In particular,
data with single pion or $\eta$ production, with hyperon production with recoiling
charged and neutral kaons, and photoproduction of $2\pi^0$ and $\pi^0\eta$ are included
in the analysis. Recent results are presented in two longer papers [3,4] where
references are given to the data included and to papers where the PWA method
is fully described. We reported two possible solutions which are both compatible
with the full data base used in the analysis. These two solutions are called
BG2011-01 and BG2011-02, respectively. Newly added here are recent data
on $\gamma p \rightarrow \Sigma^+ K^0$ [5]. In this letter we give a brief account of the experimental
findings and focus on possible interpretations of the results.

Table 1 lists the positive-parity nucleon resonances below 2.3 GeV used in the
analysis. Here, nucleon resonances are characterized by the letter $N$, by their
nominal mass from [6] or from us, by their isospin $I = \frac{1}{2}$, and by their spin
and parity $J^P$. Here, we concentrate on resonances with negative parity. A
discussion of positive-parity nucleon resonances in the 2 GeV mass range can
be found elsewhere [7].

Table 1

| Resonance | Mass (MeV) |
|-----------|------------|
| $N_{1/2}^+(1440)$ | $N_{1/2}^+(1710)$ | $N_{3/2}^+(1720)$ | $N_{5/2}^+(1680)$ |
| $N_{1/2}^+(1875)$ | $N_{3/2}^+(1900)$ | $N_{5/2}^+(1875)^*$ | $N_{7/2}^+(1990)^*$ |
| $N_{1/2}^+(2100)^*$ | $N_{5/2}^+(2200)^*$ | $N_{9/2}^+(2220)$ |
Figure 1. Differential cross section for $\gamma p \rightarrow \Sigma K^0$ [5] (left) and recoil asymmetry for $\gamma p \rightarrow n\pi^+$ (right) [8]. The full curves show our PWA solution BG2011-02, the dashed curves the best fit without resonant contributions above 1.7 GeV in the $I(J^P)=\frac{1}{2}(\frac{3}{2}^-)$ wave.

Figure 2. Differential cross section for $\gamma p \rightarrow \Lambda K^+$ (left) [9] and $\gamma p \rightarrow p\eta$ [10] (right). The full curves show our PWA solution BG2011-02, the dashed curves the best fit without resonant contributions above 1.7 GeV in the $I(J^P)=\frac{1}{2}(\frac{1}{2}^-)$ wave.

The $I(J^P)=\frac{1}{2}(\frac{3}{2}^-)$ and $I(J^P)=\frac{1}{2}(\frac{5}{2}^-)$ partial waves are described by two-pole K-matrices, the $I(J^P)=\frac{1}{2}(\frac{5}{2}^-)$ partial wave by one pole, with couplings to $N\pi$, $N\eta$, $\Lambda K^+$, $\Sigma K$, $N(\pi\pi)_S$-wave, $\Delta\pi$, and one unconstrained channel (parameterized as $N\rho$). Amplitudes for background contributions are included as reggeized meson exchanges in the $t$ channel and by direct couplings from initial to final states. The poles represent the well known resonances

$$N_{1/2^-}(1535) N_{3/2^-}(1520)$$
$$N_{1/2^-}(1650) N_{3/2^-}(1700) N_{5/2^-}(1675)$$

With these amplitudes, several data sets were only moderately well described unless two further resonances were introduced, called $N_{1/2^-}(1895)$ and $N_{3/2^-}(1875)$. In a first step, the new resonances were represented by coupled-channel relativistic Breit-Wigner amplitudes. With the new resonances, data and fit agreed very well. Figures 1 and 2 show a few examples, selected data on $\gamma p \rightarrow \Sigma^+ K_S^0$ [5], $\gamma p \rightarrow n\pi^+$ [8], $\gamma p \rightarrow \Lambda K^+$ [9], and $\gamma p \rightarrow p\eta$ [10]. The solid lines represent the full fit, the dashed lines in Fig. 1 our fit when $N_{3/2^-}(1875)$
Figure 3. a) Mass scan for a $N_{1/2}^-$ resonance; change of the total $\chi^2$ of the fit as a function of the assumed mass; b, c) Mass scan for a $N_{3/2}^-$ resonance; $\chi^2$ of the fit as a function of the assumed mass for an assumed width of 100 MeV. b) total $\chi^2$, c) $\chi^2$ contribution from $\gamma p \to \Sigma^+ K^0_m$ (this work).

is removed from the fit; in Fig. 2 $N_{1/2}^-$ (1895) was removed.

Introduction of $N_{3/2}^-$ (1875) improved the fit also to other data which are not shown here. Significant improvements were found in the description of the many observables in $\gamma p \to \Lambda K^+$: in the fit to differential cross sections and recoil polarization [9], to photon beam asymmetry [11], target asymmetry, and to the observables $O_{x'}$, $O_{x''}$ [12] and $C_x$, $C_z$ [13]. The latter quantities describe, respectively, the polarization transfer from linearly and circularly polarized photons to the final-state hyperons. Introduction of $N_{1/2}^-$ (1895) gave major improvements in the description of the data on $\gamma p \to \Lambda K^+$ [9,11,12,13], and for $\gamma p \to N\pi$ from different sources [3, Table 3].

The need to introduce $N_{1/2}^-$ (1895) and $N_{3/2}^-$ (1875) can be seen in mass scans. The mass of one of the two resonances was stepped through the resonance region, a new fit was made with all parameters released, except the mass of the resonance. The quality of the fit - expressed as $\chi^2$ as a function of the imposed mass - was monitored. Fig. 3a shows a mass scan for the $N_{1/2}^-$ resonance, Fig. 3b, c for $N_{3/2}^-$. The scans show very clear and highly significant

| Table 2 |
|---|
| Masses and widths of selected negative-parity resonances. The second column gives the PDG [6] star rating, ranging from 4-star (established) to 1-star (poor evidence). |
| $N_{1/2}^-$ (1895): $M_{BW} = 1895 \pm 15$ $\Gamma_{BW} = 90^{30}_{-15}$ [MeV] |
| $N_{3/2}^-$ (1875): $M_{BW} = 1880 \pm 20$ $\Gamma_{BW} = 200 \pm 25$ [MeV] |
| $N_{1/2}^-$ (2090): 1* no evidence |
| $N_{3/2}^-$ (2150): 2* $M_{BW} = 2150 \pm 60$ $\Gamma_{BW} = 330 \pm 45$ [MeV] |
| $N_{5/2}^-$ (2060): 2* $M_{BW} = 2060 \pm 15$ $\Gamma_{BW} = 375 \pm 25$ [MeV] |
| $N_{7/2}^-$ (2190): 4* $M_{BW} = 2180 \pm 20$ $\Gamma_{BW} = 335 \pm 40$ [MeV] |
| $N_{9/2}^-$ (2250): 4* $M_{BW} = 2280 \pm 40$ $\Gamma_{BW} = 520 \pm 50$ [MeV] |
minima. Formally, the statistical significance for $N_{1/2}^-(1895)$ corresponds to 25 standard deviations, the significance for $N_{3/2}^-(1875)$ is even higher. The widths of the minima in Fig. 3 reflects the natural width of the resonance. We believe that the minima in Fig. 3 constitute solid evidence for the existence of these two resonances.

In a second scan, the new resonances were included as third K-matrix poles in the two partial waves and a search was made for higher-mass resonances. In the $N_{3/2}^-$ wave, a clear minimum was observed at 2125 MeV which we identify with the known two-star $N_{3/2}^-(2200)$ [6]. A scan for a further $N_{1/2}^-$ resonance - known as one-star $N_{1/2}^-(2090)$ [6] - showed no significant additional minimum. We searched for other high-spin nucleon resonances; the results, summarized in Table 2, confirm established particles.

Evidence for the two resonances $N_{1/2}^-(1895)$ and $N_{3/2}^-(1875)$ has been reported before. From $\pi N$ scattering, Höhler et al. [14] gave Breit-Wigner parameters of $M = 1880 \pm 20$, $\Gamma = 95 \pm 30$ MeV for a pole in the $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ wave. Manley et al. [15] found a broad state, $M = 1928 \pm 59$, $\Gamma = 414 \pm 157$ MeV which is possibly related to the resonance discussed here. Vrana et al. [16] reported a pole at $M_{\text{pole}} = 1795$, $\Gamma_{\text{pole}} = 220$ MeV. A third and a forth pole in the $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ wave was suggested in [17]. The third pole was given with mass and width of $M_{\text{pole}} = 1733$ MeV; $\Gamma_{\text{pole}} = 180$ MeV, and in [18] with $M_{\text{pole}} = 1745 \pm 80$; $\Gamma_{\text{pole}} = 220 \pm 95$ MeV. A forth pole in this partial wave may have been seen by Cutkosky et al. [19] at $M_{\text{pole}} = 2150 \pm 70$, $\Gamma_{\text{pole}} = 350 \pm 100$ MeV and confirmed by Tiator et al. [17].

In the $\frac{1}{2}(\frac{3}{2}^-)$ wave, Cutkosky et al. [19] reported two resonances, the lower mass state at $M_{\text{pole}} = 1880 \pm 100$, $\Gamma_{\text{pole}} = 160 \pm 80$ MeV, the higher mass pole at $M_{\text{pole}} = 2050 \pm 70$, $\Gamma_{\text{pole}} = 200 \pm 80$ MeV. A few further suggestions exist, partly supporting the lower mass, partly the higher mass [6]. Based on SAPHIR data on $\gamma p \rightarrow \Lambda K^+$ [20], Mart and Bennhold claimed evidence for a $\frac{1}{2}(\frac{3}{2}^-)$ resonance at 1895 MeV [21] which was confirmed by us on a richer data base in [22,23], with mass and width of 1875 $\pm$ 25 and 80 $\pm$ 20 MeV, respectively. The high-mass $N_{3/2}^-$ was also seen in [22,23] with $M = 2166^{+25}_{-50}$, $\Gamma = 300 \pm 65$ MeV and in [24] with of $M = 2100 \pm 20$ MeV and $\Gamma = 200 \pm 50$ MeV.

We now discuss possible interpretations. Hadron resonances often appear in Table 3

Nucleon resonances as parity doublets. For an easier comparison we give our mass values and not the nominal values from PDG [6]. A star * denotes values which are not uniquely defined, a second solution exists with a different mass.

| $N_{1/2}^-$ (1895) | $N_{3/2}^-$ (1875) | $N_{5/2}^-$ (2060) | $N_{7/2}^-$ (2190) | $N_{9/2}^-$ (2250) |
|------------------|------------------|------------------|------------------|------------------|
| $N_{1/2}^+$ (1875) | $N_{3/2}^+$ (1900) | $N_{5/2}^+$ (2095)* | $N_{7/2}^+$ (2110)* | $N_{9/2}^+$ (2200) |
Table 4
Nucleon and ∆ resonances assigned to the third excitation shell. The masses of nucleon resonances are from our work, most ∆ resonances from [6], one* from [27].

The two resonances $N_{5/2}^- (2060)$ and $N_{7/2}^- (2190)$ could belong to the $S=3/2$ quartet or to a $S=1/2$. Two resonances are missing. $N_{5/2}^- (2060)$ and $N_{7/2}^- (2190)$ may both consist of two unresolved resonances, one belonging to the quartet, the other one to the doublet.

| $L$ | $N$ | $S$ | $N_{1/2}^-(1895)$ | $N_{3/2}^-(1875)$ | $N_{3/2}^-(1900)$ | $N_{5/2}^-(1940)$ | $N_{5/2}^-(1930)$ |
|-----|-----|-----|------------------|------------------|------------------|------------------|------------------|
| 1   | 1   | $\frac{1}{2}$ | $N_{1/2}^-(1895)$ | $N_{3/2}^-(1875)$ |
|     |     | $\frac{3}{2}$ | $N_{3/2}^-(1890)$ | $N_{5/2}^-(1900)$ | $N_{7/2}^-(2190)$ |

parity doublets [25]. Table 3 shows a striking consistency with this conjecture. In particular, the new negative-parity spin-doublet $N_{1/2}^-(1895)$, $N_{3/2}^-(1875)$ is mass degenerate with $N_{1/2}^+(1875)$ and $N_{3/2}^+(1900)$. Also the masses of the higher-spin resonances of opposite parity are consistent. There is, however, one caveat. $N_{5/2}^+(2095)$, with 2000 MeV nominal mass [6], is not well defined. The $I(J^P) = 1/2^+$ wave can be described by one resonance above $N_{5/2}^+(1680)$. In this case, mass and width are determined to $M, \Gamma = (2090\pm 20), (450\pm 40)$ MeV. (In Table 3 we list this resonance as $N_{5/2}^+(2095)$ to avoid confusion with $N_{3/2}^-(2090)$.) If the wave is described by three poles, the (Breit-Wigner) mass and width of the highest pole is found at $(2190\pm 40), (550\pm 100)$ MeV, and a further pole shows up. At present, its position cannot be defined precisely; any mass between 1800 and 1950 MeV gives a good description of the data. Also for $N_{7/2}^+(1990)$ there are two solutions. In one solution, its mass and width are determined to $(2100\pm 15), (260\pm 25)$ MeV, a mass which is consistent with parity doubling. Solution BG2011-01 yields $(1990\pm 10), (180\pm 25)$ MeV. $N_{3/2}^- (2150)$ stands alone, so far with no parity partner.

In quark models, baryon resonances are organized in SU(6) multiplets combining spin and flavor according to the decomposition $6 \otimes 6 \otimes 6 = 56_S \oplus 70_M \oplus 70_M^+ \oplus 20_A$ [26]. The multiplets are characterized by the SU(6) dimensionality $D$, the leading orbital angular momentum $L$, the shell number $N$ and the parity $P$ in the form $(D, L_N^P)$. Instead of $N$, we often use the radial excitation quantum number $N$, with $N = L + 2N$. Restricted to non-strange baryons, the 56-plet decomposes into a spin-doublet of nucleon resonances and a spin quartet of ∆ resonances, $56 = ^1_{-1}10 \oplus ^28$. A 70-plet is formed by a spin quartet and a spin doublet of nucleon resonances and a spin doublet of ∆ resonances.

The two resonances $N_{1/2}^- (1895)$ and $N_{3/2}^- (1875)$ could form a spin doublet like eq. (1) or be members of a spin triplet like eq. (2). In the latter case, a close-by resonance with $I(J^P) = 1/2^-$ should be expected. A scan gives a minimum - with a gain in $\chi^2$ of 2500 units - at 2075 MeV, seemingly unrelated.
to $N_{1/2}^{-}(1895)$ and $N_{3/2}^{-}(1875)$. Hence we interpret these two resonances as spin doublet. The spin doublet is not accompanied by a close-by spin quartet (degenerate into a triplet like in eq. (2); $L = 1$ and $S = 3/2$ are combined to $J^P = \frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^-$. Hence the doublet must belong to a 56-plet. The expected spin quartet of $\Delta$ resonances is degenerate to a triplet. Indeed, such a triplet seems to exist. The Particle Data Group [6] lists $\Delta_{1/2}^{-}(1900)$, $\Delta_{3/2}^{-}(1940)$, $\Delta_{5/2}^{-}(1930)$. These five states and their quantum number assignment are listed in Table 4. They can be assigned naturally to a 56-plet, and exhaust the non-strange sector of this multiplet. Note that a 56-plet is symmetric in its spin-flavor wave function. Hence the spatial wave function must be symmetric, too, in spite of the odd angular momentum. In three-particle systems, odd angular momenta with a symmetric spatial wave function can indeed be constructed, except for $L = 1$ and $N = 0$. With $N = 2$, the resonances would belong to the fifth excitation shell; due to their low mass, they have very likely $N = 1$. The five resonances belong to the $(56, 1^-)$ multiplet.

In the mass range from 2000 to 2300 MeV, four further nucleon and two further $\Delta$ resonances are known which have negative parity and which, in the harmonic oscillator approximation, can be assigned to the third excitation shell. These are listed in the lower part of Table 4. The $N_{9/2}^{-}(2280)$ resonance must have $L = 3, S = 3/2$ coupling to $J^P = \frac{9}{2}^-$ as dominant angular momentum configuration; there could be a small $L = 5$ component in the wave function but resonances with $L = 5$ as leading orbital angular momentum are expected at much higher masses. With $L = 3, S = 3/2$ coupling to $\frac{9}{2}^-$ as anchor, we expect a full quartet with $J^P = \frac{3}{2}^-, \frac{5}{2}^-, \frac{7}{2}^-, \frac{9}{2}^-$. SU(6) symmetry then demands the existence of an additional $J^P = \frac{5}{2}^-, \frac{7}{2}^-$ doublet, i.e. six states in total. Instead of six resonances, only four are observed here. Possibly, the two expected resonances with $\frac{5}{2}^-$ are unresolved, and both hide in the one observed $N_{5/2}^{-}(2060)$. Likewise, two resonances may hide within $N_{7/2}^{-}(2190)$. Thus only four resonances instead of six are observed or observable. In the $\Delta$ sector, the Particle Data Group lists one negative-parity resonance in this mass range, $\Delta_{7/2}^{-}(2200)$, and SAID finds $\Delta_{5/2}^{-}(2223)^*$. These two resonances form a natural spin doublet with $L = 3, S = 1/2$ coupling to $J^P = \frac{5}{2}^-, \frac{7}{2}^-$. In SU(6), this group of nucleon and $\Delta$ resonances can all be assigned to one multiplet $(70, 3^-)$. Apart from the problem that two pairs nucleon resonances may hide in one observed spin doublet, the $(70, 3^-)$ is completely filled.

Quark models predict six further multiplets which are completely empty, $(56, 3^-)$, $(20, 3^-)$, $(70, 2^-)$, $(70, 1^-)$, $(70, 1^-)$, $(20, 1^-)$. There is not one additional resonance which may hint at the possibility that one of these multiplets may be required. Some of these resonances would have noticeable features. From the $(56, 3^-)$ multiplet, a $\Delta$ resonance with $J^P = \frac{9}{2}^-$ is expected. A resonance with these quantum numbers is observed, but at 2400 MeV [6], too high in mass to fall into the third excitation shell. We speculate that it may
have an additional unit of radial excitation and may belong to the \((56, 3^5_2)\) multiplet. The \((70, 2^-_3)\) multiplet predicts a quartet of states with \(J^P = \frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^-, \frac{7}{2}^-\), all with even angular momentum and odd parity. These are just absent in the spectrum. Out of eight multiplets, six are completely empty, two are fully equipped.

This is a remarkable observation: in two of the eight expected SU(6) multiplets, all members seem to be identified experimentally. In contrast, the other six multiplets remain completely empty. At present, one thus should have to conclude that missing resonances are not just voids which might be filled when new data become available. It seems, instead, that whole multiplets are unobserved and are possibly unobservable. If this conjecture should be confirmed in future experiments and analyses, there must be a dynamical reason which prohibits formation of certain SU(6) multiplets.

In summary, we have reported evidence for a spin doublet of nucleon resonances, \(N_{1/2^-}(1895)\) and \(N_{3/2^-}(1875)\). The spectrum of negative parity resonances in this mass range shows remarkable features. The resonances can be grouped, jointly with positive parity states, into parity doublets. Within a quark-model classification, the negative parity resonances around 2.1 GeV can be assigned to two multiplets while six multiplets remain completely empty. It will be important to see whether indeed entire multiplets are missing as opposed to individual states within multiplets. This observation may hint to new features of intra-baryon dynamics.

We would like to thank the members of SFB/TR16 for continuous encouragement. We acknowledge support from the Deutsche Forschungsgemeinschaft (DFG) within the SFB/ TR16 and from the Forschungszentrum Jülich within the FFE program.

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