A model for the location and scheduling of the operation of second-generation ethanol biorefineries

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Abstract
This document presents an economic optimization model which identifies the location, the nominal plant capacity and the operation scheduling for set of biorefineries of second-generation ethanol using the biomass obtained as waste in the sugarcane industry. The model also determines the gasoline volumes that will be mixed with ethanol in order to produce a mixed fuel. Given a planning horizon of the operation of the system, the model obtains its optimal parameters at fixed time intervals (annual) so the global optimum is obtained by minimizing the mathematical expectation of the stochastic process generated when the product demand is assumed random with known density. Partial optimization of the process is achieved using a mixed integer linear programming model. Real information obtained from the Secretariat of Energy for the management of biorefineries in the state of Veracruz of the Mexican Republic is included and numerical results are reported.

Keywords: Biorefineries location; Operations scheduling; Linear integer programming; Stochastic processes; Supply chain

1 Introduction
During the last century the world average annual temperature of the planet increased by 0.7°C, and it is expected to increase further, in the range of 0.8 to 2.6°C by the year 2050. As a consequence, there has been a decrease in the glaciers and sea ice. Today, hurricanes are of greater magnitude and frequency and previously desert-like areas receive large amounts of rain while traditionally humid areas are subject to desertification. These effects are attributable to greenhouse gas emissions [15]. Of these gases, CO2 represents more than 80% [6]. Mexico is one of the top 20 countries, with the highest production volume of CO2 emissions [14].

Worldwide, two man-made activities, the generation of energy (electrical, thermal) and transport, produced almost two thirds of global CO2 emissions. While the generation of energy comes from many sources, the transport sector depends almost exclusively on fossil fuels by 97% [13]. In Mexico gasoline imports are growing to 25 billion liters annually. For both economic and environmental reasons, it is especially important to substitute fossil fuels with biofuels [26].
Biofuels are classified as first generation and second generation or cellulose. The difference between them are the raw materials used for their manufacture. In the production of ethanol the first-generation biofuels are made from corn (Zea mays), sugar cane (Saccharum officinarum L.), sugar beet (Beta vulgaris), and grain sorghum (Sorghum spp.). In the case of biodiesel, palm oil (Elaeis guineensis), jatropha (Jatropha curcas), soybean (Glycine max), rapeseed (Brassica napus), safflower (Carthamus tinctorius) and sunflower (Helianthus annuus) can be used. The first-generation biofuels are produced using conventional technology such as fermentation (for sugars and carbohydrates), transesterification (for oils and fats), and anaerobic digestion (for organic waste). Second-generation biofuels use mainly agricultural, agroindustry and forestry wastes, and require pre-treatment before using conventional methods to obtain them [17].

First-generation ethanol is produced on a commercial scale. In the year 2000, 13 billion liters were produced and by 2016 a figure of 99 billion liters was reached, an increase of more than 750%. The main producers of ethanol worldwide are the United States and Brazil [20]. In Mexico, commercial scale production of ethanol is non-existent. However, in 2015 Petróleos Mexicanos (PEMEX) entered 10-year agreement with several Mexican companies to produce ethanol beginning in 2017. This production will be mixed 5.8% with gasoline to produce a mixed fuel. The ethanol will come from sugarcane juice, produced in San Luis Potosí and Veracruz; as well as sorghum grain, produced in Tamaulipas [18]. In addition, the Mexican government established the mandatory standard, that ethanol can be mixed up to 10% with gasoline [5].

The first-generation biofuels present some important drawbacks, such as the possibility of shortage of some crops dedicated to food, to an increase in prices and to an acceleration of deforestation. In this context, the use of cellulosic materials to produce ethanol is an alternative, which counteracts certain disadvantages of first-generation biofuels [17]. Second generation biofuels already have a promising future. In the year 2000 their production was minimal, and for the year 2014 a production of 16 billion liters was reported, led by the United States and Brazil [27]. In Mexico, several studies have been done on the annual potential of agricultural waste for the production of cellulosic ethanol. Those studies, have identified several potential sources of agricultural waste with an energy vocation, including stubble of agricultural crops: corn, sugarcane, rice, barley, sorghum and wheat; as well as some agro-industrial by products: sugarcane bagasse, corn cobbles, rice husks, wheat, coffee and sunflower. An annual potential of cellulosic ethanol in Mexico was estimated from the waste of several crops as: 3, 405 Mlt of sugarcane, 383 Mlt of sweet sorghum, 151 Mlt of coffee, 99 Mlt of wheat and 17 Mlt of rice.

In Mexico, sugarcane is the most produced crop in volume and the country is an exporter of sugar; the 2015–2016 harvest produced, more than 55 million tons of sugarcane. For its harvest, in general, the system of burned cane, manual cutting and mechanized moose is used. It involves burning the ripe cane to facilitate the harvest and weeks after the first burning, a second burn eliminates the tips of cane that were left in the soil. This practice prevents the reincorporation of organic matter into the soil and degrades the physical and chemical properties. This practice is not sustainable in the long term [24]. Additionally, it does not take advantage of an important biomass volume that could serve as raw material to produce second-generation ethanol, as has already been done in Brazil [12].
2 Background

Facility location models have a long history dating back to the year 1600 with the jurist, mathematician and physicist Pierre de Fermat who is credited with creating the method of the spatial median. Today, there is a vast literature on the methods, models and results applied to the problem of location of facilities the most used being multicriteria optimization. Here, the optimality criteria are: minimizing the total setup cost, minimizing the longest distance from the existing facilities, minimizing fixed cost, minimizing total annual operating cost, maximizing service, minimizing average time/distance traveled, minimizing maximum time/distance traveled, minimizing the number of located facilities, maximizing responsiveness [9].

The variety of applications covers from the location of banks [3, 7, 10, 11, 22, 23] industrial facilities [2, 4, 8, 19, 25]. A review on location of facilities and supply chain can be found in [16]. A special contribution on the design of the bio fuels supply chain can found in [1].

This proposal addresses a real problem of planning and managing the operation of a set of refineries in a state of the Mexican Republic under optimal conditions of selection of its location, the periods of operation of these up to $T$ years, the distribution network of the ethanol produced and the determination of the quantities of gasoline and ethanol to be purchased externally.

In this document, we develop a mathematical model that addresses the problem of the production of ethanol mixed with gasoline and the amounts to be imported from various external sources to meet the demand for biofuel required in the country. Likewise, it is determined using a model of stochastic networks, the optimal trajectory of transport routes from sugarcane bagasse producing centers to the consuming centers of the final product. A clustering model initiates the proposal indicating the optimal grouping of biorefineries around the producing centers (cane mills and sugar producing fields). Subsequently, the programming of the operation of the storage centers for biofuels is modeled. For this, binary variables were used in order to establish the annual plan of operation of the whole in the area. An integer linear (binary) stochastic programming model is the model inserted in each stage of the stochastic process that shapes the decision for the global planning of the operation to $T$ years. Next our proposal is developed.

3 Description of the problem and its proposed mathematical model

In the construction of this model we assume that the biomass comes from the bagasse of the sugar cane and it is constituted by the residue of matter that remains after its juice is extracted. It is the residue (tips and leaves) that remain in the field after the harvest or when processing the sugarcane to obtain the product. The goal is to use the material to obtain ethanol. The following aspects will be considered, Figure 1:

1. The biomass can be located in two different origins: (a) the sugarcane field, (b) the sugar mills
2. The two sources of biomass produce raw material for the operation of biorefineries
3. The biorefineries send ethanol as a finished product to the storage centers
4. The storage centers also receive gasoline and ethanol imported through the corresponding customers
5. The storage centers are used as temporary stores (buffers) and as areas for mixing ethanol with gasoline
6. The total costs involved in the process (production, storage and transport) are known and deterministic.

7. The demand $D$ for ethanol is a random variable with cumulative distribution function $F_D(d)$ with finite mathematical expectation $E(D) < \infty$, and known variance $\sigma_D$.

8. The efficiency of production $\eta$ in each biorefiner is known.

9. In the decision to operate a biorefinery or not, two types of costs are included:
   (a) fixed costs $c_f$, and (b) variable costs $c_v$ in such a way that the total cost is given by $c_t = c_f + c_v$.

10. In the decision to operate or adapt a warehouse, two types of costs are included:
    (a) fixed cost $\tilde{c}_f$ and (b) variable costs $\tilde{c}_v$. Thus, the total cost would be given by $\tilde{c}_t = \tilde{c}_f + \tilde{c}_v$.

11. The model includes 5 different types of transport for the raw material, 4 for ethanol and 3 for the transportation of gasoline.

   In this model we are interested in obtaining an optimal product distribution program in the network as a function of the estimated demand from the latter year. In the set of decisions, it must be specified which refineries and warehouses should be operating in each year of project execution as well as the route of traffic of the end product.

### 3.1 Description of the model

Our proposal considers the design of an ethanol supply chain in the state of Veracruz, Mexico, using as base the biomass that is generated in the main sugar cane areas of the zone. The storage centers should be placed according to the geographical position of the suppliers of cane bagasse and local customers for the purchase of ethanol and gasoline (clustering process).

In the second stage, the programming of the operation of the biorefineries and the import requirements of gasoline and ethanol to cover the total fuel demand are determined. The development of both models follows.
3.2 The clustering and location of biorefineries model

For the construction of this model a distance matrix $M = m_{ij}$ was built. The matrix contains the distances by land, from the origin $i$ to the destination $j$ and it involves the biomass producing centers (mills and sugarcane fields) and local customers. With the above information, the clustering process is now done. Here, the raw material sources are grouped and the optimal places to locate the refineries are determined (first stage). Now one assigns the clusters to the customers importing ethanol and gasoline (second stage). Finally, the optimal position of the finished product storage centers is determined (third stage), Fig. 2. For the construction of the mathematical model, the following indicator sets are defined.

1. For biomass producing fields: $I = \{i: i = 1, \ldots, I\}$
2. For the sugar mills: $R = \{r: r = 1, \ldots, R\}$
3. For the biorefineries of ethanol: $J = \{j: j = 1, \ldots, J\}$
4. For customers receiving ethanol: $S = \{s: s = 1, \ldots, S\}$
5. For customers receiving gasoline: $G = \{g: g = 1, \ldots, G\}$
6. For ethanol and gasoline storage centers: $L = \{l: l = 1, \ldots, L\}$
7. For the type of transport used: $V = \{v: v = 1, \ldots, V\}$
8. For the time: $T = \{t: t = 1, \ldots, T\}$
9. For fuel delivery centers (customers): $U = \{u: u = 1, \ldots, U\}$.

Once the geographic positions of the biomass producing centers are known, the first stage consists of grouping them into representative clusters of the whole. Then, each cluster is assigned to a biorefinery according to the installed production capacity of the same. The process becomes iterative until it reaches the convergence. For the design of the clusters the method mentioned in [19] is used. Here, the use of radial distribution was used due to the transportation costs of the product and the characteristics of it. In this process, the geographical location of the import customers is considered fixed and, the distances from these to the set of clusters are amounts set automatically by the algorithm. The following algorithm was used to locate the biorefineries according to the providers (first stage).

**Clustering algorithm of the biomass producing centers (mills and cane fields).**

1. Randomly select a number of $\omega$ biomass producing centers closest to each other in a given region and locate them in a Cartesian coordinate system. Let $\Omega$ be the set of selected suppliers.
2. Determine the coordinates \((x, y)\) of the center of gravity of each cluster by solving the next non-linear program:

\[
\text{Min } z = \sum_{r=1}^{\infty} Q_r d_r \quad d_r = \left[ (x_r - x)^2 + (y_r - y)^2 \right]^{1/2},
\]

where \(Q_r\) is the average annual biomass volume from the supplier \(r\) that will be moved from this point to the refinery, and \((x_r, y_r)\) are the (known) coordinates of the biomass suppliers.

(a) If the coordinates are feasible (meaning that the place found is economically and technologically feasible to locate the refinery here), go to step 3.

(b) If the coordinates are infeasible (for example, places that are not accessible, such as lakes, rivers, buildings, or more), add or remove elements of \(S\) from the nearby clusters and apply Equation (1) while the infeasibility remains. Go to step 3.

3. Repeat the process including or eliminating suppliers in \(\Omega\) and successively apply Equation (1) in every attempt. Let \(z_1(x_1, y_1), \ldots, z_t(x_t, y_t)\) be the sequence of optimal values found in each iteration. Choose the coordinates \((x^*_\alpha, y^*_\alpha)\) of the optimum point that will represent the centroid of gravity of the polygon using the following criteria.

\[
 z^*_\alpha(x^*_\alpha, y^*_\alpha) = \min \{z_1(x_1, y_1), \ldots, z_t(x_t, y_t)\}. \tag{2}
\]

4. The point \((x^*_\alpha, y^*_\alpha)\) is an estimator of the center of gravity of the polygon formed by the suppliers and is a candidate to locate a refinery.

5. Apply the process while there are clusters without refinery assignment.

### 3.3 The clustering and location of biorefineries model

The process of assignment of customers for the purchase of ethanol is as follows (second stage). Let \(D_{js}\) be the distances by land from the biorefinery \(j\) to the customers \(s\). Then for a given biorefinery \(j\) the customers \(s\) is assigned if

\[
 D^*_js = \min\{D_{js}\}, \quad \forall s \in S. \tag{3}
\]

The process of assignment of customers for gasoline purchase is similar to the previous one.

### 3.4 The allocation of finished product storehouse model

For the construction of the model for the location of the finished product storehouse and product flow calculation the following notation is proposed (stage 3).

- \(q_i\) Volume of biomass collected from the cane field \(i\) in the year \(t\) (in m\(^3\))
- \(q_{rt}\) Volume of biomass collected in the sugar mill \(r\) in the year \(t\) (in m\(^3\))
- \(c_{it}\) Cost of lifting biomass in the sugarcane field \(i\) (include loading the truck and packaging) in the year \(t\) (in USD/m\(^3\))
- \(\tilde{c}_{rt}\) Cost of collecting biomass in the sugar mill \(r\) in the year \(t\) (in USD/m\(^3\))
- \(s_{st}\) Cost of importing ethanol for customers \(s\) in the year \(t\) (in USD/m\(^3\))
- \(\theta_{gt}\) Cost of importing gasoline for customers \(g\) in the year \(t\) (in USD/m\(^3\))
- \(\tilde{c}_{jt}\) Fixed cost of biorefinery operation \(j\) in the year \(t\) (in USD/year)
\[ \gamma_{jt} \equiv \text{Variable cost of biorefinery operation } j \text{ in the year } t \text{ (in USD/m}^3\text{-year)} \]

\[ \psi_{ijtv} \equiv \text{Costs of transport of the biomass generated in the field } i \text{ to the, biorefinery } j \text{ in the year } t \text{ using the transport } v \text{ (in USD/m}^3\text{)} \]

\[ \check{\psi}_{rjtv} \equiv \text{Transportation costs of the biomass generated in the mill } r \text{ to the, biorefinery } j \text{ in the year } t \text{ using the transport } v \text{ (in USD/m}^3\text{)} \]

\[ x_{ijtv} \equiv \text{Volume of biomass transported from the field } i \text{ to the biorefinery } j \text{ in the year } t \text{ using the transport } v \text{ (in m}^3\text{)} \]

\[ \check{x}_{rjtv} \equiv \text{Volume of biomass transported from the mill } r \text{ to the biorefinery } j \text{ in the year } t \text{ using the transport } v \text{ (in m}^3\text{)} \]

\[ \check{\varepsilon}_{jltv} \equiv \text{Transportation costs of ethanol generated in the biorefinery } j \text{ towards the plant of storage and mixing } l \text{ in the year } t \text{ using the transport } v \text{ (in USD/m}^3\text{)} \]

\[ \check{y}_{dltv} \equiv \text{Volume of ethanol transported from the biorefinery } j \text{ to the plant of storage and mixing } l \text{ in the year } t \text{ using the transport } v \text{ (in m}^3\text{)} \]

\[ \check{\eta}_{rltv} \equiv \text{Volume of ethanol purchased (imported) by customers } s \text{ during the year } t \text{ (in m}^3\text{)} \]

\[ \check{\zeta}_{rjtv} \equiv \text{Volume of ethanol produced in the biorefinery } j \text{ during the year } t \text{ (in m}^3\text{)} \]

\[ \check{\zeta}_{st} \equiv \text{Volume of ethanol purchased (imported) by customers } s \text{ during the year } t \text{ (in m}^3\text{)} \]

\[ \check{\varrho} \equiv \text{Unit cost of mixing ethanol with gasoline and its storage in the store } l \text{ in the year } t \text{ (in USD/m}^3\text{)} \]

\[ \kappa_{jt} \equiv \text{Installed production capacity of the biorefinery } j \text{ in the year } t \text{ (in m}^3\text{/year)} \]

\[ i_l \equiv \text{Inventory of final product (ethanol mixed with gasoline) that must be in the warehouse } l \text{ at the end of year } t \text{ (in m}^3\text{)} \]

\[ \theta_{lt} \equiv \text{Cost of storing a cubic meter of final product in the warehouse } l \text{ during the year } t \text{ (in USD/m}^3\text{-year)} \]

\[ \delta_{lt} \equiv \text{Warehouse storage capacity } l \text{ in the year } t \text{ (in m}^3\text{)} \]

\[ E_{imt} \equiv \text{Volume of ethanol to import in the year } t \text{ (in m}^3\text{)} \]

\[ G_{imt} \equiv \text{Volume of gas to import in the year } t \text{ (in m}^3\text{)} \]

The strategy to optimize this model consists in obtaining the minimum mathematical expectation of the process in each year of operation of the system. So, for each \( t \in T \) we have, the objective function and the model is constructed as follows:

1. Elements of the objective function

   a. The (fixed) costs of lifting the biomass in the sugar cane \( i \) field during the year \( t \), plus the costs of acquiring the biomass in the mill \( r \) during the year \( t \), are given by the function

   \[
   Q(q_i, \check{q}_r) = \sum_{i \in I} \sum_{j \in J} c_i q_{ijt} + \sum_{r \in R} \sum_{j \in J} \check{c}_r \check{q}_{rjt} = \sum_{i \in I} \sum_{j \in J} c_i q_{ijt} + \sum_{r \in R} \check{c}_r \check{q}_{rjt} \tag{4}
   \]

   b. The costs of transporting biomass from the sugar cane field \( i \) towards the biorefinery \( j \) in the year \( t \), using the transport \( v \); plus, the costs of transporting the biomass from the mill \( r \) towards the biorefinery \( j \) in the year \( t \), using the transport \( v \)

   \[
   \sum_{i \in I} \sum_{j \in J} \sum_{t \in T} \sum_{v \in V} \psi_{ijtv} x_{ijtv} + \sum_{r \in R} \sum_{j \in J} \sum_{t \in T} \sum_{v \in V} \check{\psi}_{rjtv} \check{x}_{rjtv}
   \]
\[= \sum_{j \in J} \sum_{t \in T} \sum_{v \in V} \left[ \sum_{i \in I} \psi_{ijtv}x_{ijtv} + \sum_{r \in R} \tilde{\psi}_{rjtv}x_{rjtv} \right] \] (5)

c. The fixed operating costs of the biorefinery \( j \) in the year \( t \) plus the variable costs of ethanol production in that year

\[\sum_{j \in J} \sum_{t \in T} \left[ \tilde{\psi}_{jt} + \gamma_{jt} \tilde{\psi}_{jt} \right] \] (6)

d. The operation costs of mixing ethanol with gasoline and its corresponding stock are given by

\[\sum_{l \in L} \sum_{t \in T} \sum_{v \in V} \rho \left[ \sum_{j \in J} \xi_{ltv}y_{jltv} + \sum_{s \in S} \tilde{\xi}_{sltv}y_{sltv} \right] \] (7)

e. The costs of acquiring ethanol through customers \( s \) in the year \( t \)

\[\sum_{s \in S} \sum_{t \in T} \zeta_{st} \] (8)

f. The costs of transporting ethanol from biorefineries \( j \) towards the storage centers \( l \), using the transportation \( v \) in the year \( t \) plus the costs of transporting ethanol from customers \( s \) to the \( l \) storage centers using the transportation \( v \) in the year \( t \) plus the costs of transporting gas from customers \( g \) to the \( l \) storage centers using the transportation \( v \) in the year \( t \)

\[= \sum_{l \in L} \sum_{t \in T} \sum_{v \in V} \left[ \sum_{j \in J} \xi_{ltv}y_{jltv} + \sum_{s \in S} \tilde{\xi}_{sltv}y_{sltv} + \sum_{g \in G} \tilde{\xi}_{gtlv}y_{gtlv} \right] \] (9)

g. The costs of transporting the final product (ethanol mixed with gasoline) from storage centers \( l \) to the consumer centers \( u \) using the transportation \( v \) in the year \( t \)

\[\sum_{l \in L} \sum_{u \in U} \sum_{t \in T} \sum_{v \in V} \phi_{lutv}z_{lutv} \] (10)

h. Cost of storing the final product in the warehouse \( l \) during the year \( t \) (in USD/m\(^3\)-year)

\[\sum_{l \in L} \sum_{t \in T} \theta_{lt} I_{lt} \] (11)

2. Constitutive elements of the constraints (restrictive set)

a. Product offer equations

\[\sum_{j \in J} \sum_{v \in V} x_{jtv} \leq q_{u}, \quad \forall i \in I, t \in T \] (12)
\[ \sum_{j \in J} \sum_{v \in V} \hat{x}_{rjt} \leq \hat{q}_{rt}, \quad \forall r \in R, t \in T \]  

(13)

b. Let \( k_1 = 0.35 \) y \( k_2 = 0.120 \) be, the standardization constants defined as the cubic meters of ethanol produced by each cubic meter of biomass in the producing centers 1 (sugarcane field) y 2 (sugar mill). Then, the flow balance equations from the biomass supply areas to the biorefineries considering an efficiency \( \eta = 0.9 \) of the process are

\[ \eta \left[ \sum_{i \in I} \sum_{j \in J} \sum_{v \in V} k_1 x_{ijtv} + \sum_{r \in R} \sum_{j \in J} \sum_{v \in V} k_2 \hat{x}_{rjt} \right] = \sum_{j \in J} \sum_{l \in L} \sum_{v \in V} y_{jltv}, \quad \forall t \in T \]  

(14)

c. The flow balance equations from the biorefineries to the storage and mixing zones considering the inventory \( I \) that there must be of ethanol at the end of the year \( t \)

\[ \sum_{j \in J} \sum_{v \in V} y_{jltv} + \sum_{s \in S} \sum_{v \in V} \hat{y}_{sltv} - \sum_{u \in U} \sum_{v \in V} z_{lutv} = 0, \quad \forall l \in L, \forall t \in T \]  

(15)

d. Ethanol demand equations in consumer centers is

\[ \sum_{v \in V} \sum_{l \in L} z_{lutv} \geq D_{ut}, \quad \forall u \in U, t \in T \]  

\( D_{ut} \) is a random variable with known density \( f_{D}(d) \), for \( D > 0, \forall t \in T \).

e. Decision on the biorefineries that must operate in the year \( t \)

\[ \sum_{l \in L} y_{jltv} \leq \alpha_{jt}, \quad \forall j \in J, t \in T, v \in V \]  

(17)

Where

\[ \alpha_{lt} = \begin{cases} 1 & \text{if the warehouse } l \text{ works in the year } t, \\ 0 & \text{elsewhere}. \end{cases} \]  

(18)

f. Decision on the stores that must operate in the year \( t \)

\[ \sum_{v \in V} \left[ \sum_{j \in J} y_{jltv} + \sum_{s \in S} \hat{y}_{sltv} \right] \leq \beta_{lt}, \quad \forall l \in L, \forall t \in T \]  

(19)

where

\[ \beta_{lt} = \begin{cases} 1 & \text{if the warehouse } l \text{ works in the year } t, \\ 0 & \text{elsewhere}. \end{cases} \]  

(20)

g. The optimum number of biorefineries that will operate during the planning horizon is given by

\[ \mu = \sum_{j \in J} \sum_{l \in L} \alpha_{lt}, \]  

(21)
h. The total amount of warehouses required to fulfill the customer’s demand

\[
\bar{\mu} = \sum_{l \in L} \sum_{t \in T} \beta_{lt},
\]

(22)

i. The amount of ethanol to be imported is given by

\[
E_{imt} = \sum_{s \in S} \sum_{v \in V} \tilde{y}_{slvt}, \quad \forall l \in L, t \in T,
\]

(23)

j. The amount of gas to be imported is

\[
G_{imt} = \sum_{g \in G} \sum_{v \in V} \tilde{y}_{gvt}, \quad \forall l \in L, t \in T.
\]

(24)

k. The equation of continuity of inventories

\[
I_{lt} = I_{l,t-1} + \sum_{j \in J} \sum_{v \in V} \eta_{jltv} + \sum_{s \in S} \sum_{v \in V} \tilde{y}_{slvt} - \sum_{l \in L} \sum_{v \in V} z_{lvt} \leq \delta_{lt}, \quad \forall l \in L, t \in T.
\]

(25)

l. Capacity constraints for inventories

\[
I_{lt} \leq \delta_{lt}, \quad \forall l \in L, t \in T
\]

(26)

m. Capacity constraints for biorefineries

\[
\sum_{l \in L} \sum_{j \in J} \sum_{v \in V} \eta_{1jtv} + \sum_{r \in R} \sum_{j \in J} \sum_{v \in V} \eta_{2jtv} - \sum_{l \in L} \sum_{v \in V} \tilde{y}_{jltv} \leq k_j \quad \forall t \in T.
\]

(27)

For each \( t \in T \), let \( D_t' = \sum_{u \in U} [\delta D_u + (1 - \delta) \tilde{D}_u] \) be the total demand for mixed fuel (ethanol and gasoline), with \( \delta \in (0,1) \) in the centers of consumption in the year \( t \). Note that, for each random variable \( D_t' \) we have a set of values \( \nu_t = (x, \tilde{x}, y, \tilde{y}, z, \tilde{z}, I, \mu, \tilde{\mu}) \). That, we are interested in to obtain.

\[
\sum_{t \in T} \{ \min \mathbb{E}[g(\nu_t)] \mid D_t' \} = g^*(\nu^*),
\]

(28)

where the components of \( \nu \) satisfy completely the restrictive set, and \( g \) represents the objective function, \( \mathbb{E} \) is the mathematical expectation operator, and by virtue of Equation (28), \( g(\theta_t) \) depends on \( g(\theta_{t-1}) \).

4 Numerical example
Clusterization and biorefineries location yields a total of 5. Geographical locations provide the following information (Table 1):

In the construction of this instance, 4 cane fields and 3 sugar mills are considered, and its production covers the years 2014 to 2016 (3 years). The average quantities of bagasse collected during the three years were (Table 2).
Table 1  Geographical locations of biorefineries

| Biorefinery number | Longitude  | Latitude |
|--------------------|------------|----------|
| 1                  | -98.28     | 21.91    |
| 2                  | -96.43     | 19.39    |
| 3                  | -96.71     | 18.94    |
| 4                  | -96.61     | 18.70    |
| 5                  | -96.28     | 18.61    |

Table 2  Amounts of bagasse collected per year of planning

| t    | 2014 (1) | 2015 (2) | 2016 (3) |
|------|----------|----------|----------|
| q_{1t}| 8741     | 9174     | 10,569   |
| q_{2t}| 24,142   | 25,339   | 29,192   |
| q_{3t}| 3450     | 3621     | 4172     |
| q_{4t}| 19,166   | 17,610   | 20,426   |
| q_{5t}| 22,674.12| 20,777.76| 20,595.00|
| q_{1t}’| 18,870.41| 17,486.04| 17,332.00|
| q_{2t}’| 22,674.12| 20,777.76| 20,595.00|
| q_{3t}’| 22,782.73| 18,668.10| 18,504.00|

Table 3  Fixed and variable costs used in this instance

| Bio | Fixed cost | Variable costs |
|-----|------------|----------------|
|     | f_1 | f_2 | f_3 | f_1 | f_2 | f_3 |
| 1   | 4500| 3900| 5200| 0.51| 0.51| 0.56 |
| 2   | 4500| 3950| 5200| 0.52| 0.52| 0.54 |
| 3   | 4510| 3905| 5200| 0.51| 0.51| 0.55 |
| 4   | 4560| 4200| 5600| 0.50| 0.50| 0.54 |
| 5   | 6160| 7100| 8700| 0.49| 0.35| 0.54 |

Table 4  The shipping costs \( \xi_{jltv} \) and \( \tilde{\xi}_{jltv} \) for all \( t \in T \)

| Bio (\( \xi \)) | v=1 | v=2 | v=3 |
|-----------------|-----|-----|-----|
| Sc1  Sc2  Sc3  |     |     |     |
| 1   24  50  45  | 34  45  80  | 32  60  64  |
| 2   34  58  69  | 34  72  93  | 97  76  84  |
| 3   93  75  32  | 37  59  85  | 65  65  39  |
| 4   72  86  99  | 110 98  105 | 110 115 120 |
| 5   76  94  110 | 105 96  98  | 97 108 120 |

| Bio (\( \tilde{\xi} \)) | v=1 | v=2 | v=3 |
|-----------------|-----|-----|-----|
| Cust (\( \tilde{\xi} \)) |     |     |     |
| 1   124 130 145 | 134 150 180 | 132 160 164 |
| 2   164 100 119 | 139 190 931 | 171 191 194 |

Here, the following values will be used \( c_{lt} = 7 \) USD/m³ and \( \tilde{c}_{lt} = 3.5 \) USD/m³. \( \forall i \in I, r \in R, t \in T \) Equation (5) is completely defined with the following values \( \psi_{ijt1} = 3.5, \psi_{ijt2} = 4.5, \psi_{ijt3} = 5. \)

For Equation (6), the following costs were considered (Table 3)

Analogously, for Equation (7) we use \( g = 30 \) USD/m³ and for Equation (8) we used \( \xi_{1t} = 690 \) and \( \xi_{2t} = 950 \) for \( t = 1, 2, 3 \). In this model, 3 storage centers (\( Sc \)) were evaluated (prefixed in advance). The shipping costs used are as follows (Table 4), Equation (9). The value used for \( \xi_{jltv} \) were \( \xi_{jltv} = 200, \tilde{\xi}_{jltv} = 250 \) for all \( l, t, v \).

The distribution of the product is done from the storage centers to 4 consumption centers (\( C_{C} \)). Table 5 shows the transportation costs, Equation (10).
Table 5 Costs of transporting the final product to the consumption centers for all $t \in T$

| $\nu$ | $\nu = 1$ | $\nu = 2$ | $\nu = 3$ |
|-------|-----------|-----------|-----------|
| $x_{11}$ | 300 | 300 | 340 |
| $x_{12}$ | 290 | 330 | 400 |
| $x_{13}$ | 330 | 300 | 350 |
| $x_{14}$ | 390 | 340 | 410 |
| $x_{21}$ | 320 | 380 | 30 |
| $x_{22}$ | 300 | 280 | 350 |
| $x_{23}$ | 350 | 300 | 410 |
| $x_{24}$ | 290 | 180 | 370 |
| $x_{31}$ | 290 | 300 | 350 |
| $x_{32}$ | 350 | 420 | 370 |
| $x_{33}$ | 410 | 370 | 350 |
| $x_{34}$ | 390 | 300 | 295 |

Table 6 Values used for demand sampling

| $t$ | $r = 1$ | $r = 2$ | $r = 3$ |
|-----|--------|--------|--------|
| $x_{11}$ | $N \sim (8120, 3480)$ | $N \sim (9220, 2180)$ | $N \sim (10,500, 2900)$ |
| $x_{12}$ | $N \sim (4770, 6032)$ | $N \sim (7270, 1900)$ | $N \sim (13,500, 1900)$ |
| $x_{13}$ | $N \sim (24,142)$ | $N \sim (35,450)$ | $N \sim (50,000)$ |
| $x_{14}$ | $N \sim (19,166)$ | $N \sim (21,82)$ | $N \sim (9182)$ |
| $x_{21}$ | $N \sim (2850)$ | $N \sim (2850)$ | $N \sim (2850)$ |
| $x_{22}$ | $N \sim (7864)$ | $N \sim (8770)$ | $N \sim (2850)$ |
| $x_{23}$ | $N \sim (3291)$ | $N \sim (4113)$ | $N \sim (4113)$ |
| $x_{24}$ | $N \sim (6228, 95)$ | $N \sim (6228, 95)$ | $N \sim (6228, 95)$ |

The storage costs of the final product are given by $\theta_{1t} = 1500, \theta_{2t} = 1800, \theta_{3t} = 1450, \theta_{4t} = 1550$. Finally, the demand for the final product was sampled from the Box–Muller Method [21]. The values used were, Table 6.

Where $N \sim (\mu, \sigma)$ means normally distributed with mean $\mu$ and standard deviation $\sigma$.

5 Results obtained

In the analysis of this instance for obtaining results the following initial values were used.

$I_{10} = 0, I_{20} = 0, I_{30} = 0$. The capacities of refineries were $k_{11} = 90,000, k_{21} = 80,000, k_{31} = 60,000, k_{41} = 70,000, k_{51} = 73,000$. The storage capacities of the storage of the final product were $\delta_{11} = 2,500,000, \delta_{21} = 2,800,000, \delta_{31} = 5,400,000$. The volumes of harvested bagasse were $q_{11} = 8741, q_{21} = 24,142, q_{31} = 3450, q_{41} = 19,166, q_{11} = 2182, q_{21} = 9182, q_{31} = 9783$.

The results obtained for year 1 were: $x_{4311} = 19,166, x_{4412} = 477,603.2, x_{5312} = 6228.95, y_{411} = 138,971, y_{411} = 5696, y_{411} = 8770, y_{411} = 2850, y_{411} = 11,724, I_{11} = 116,160, I_{11} = 1, I_{12} = 1, I_{12} = 1, I_{12} = 1. $ The value of the objective function was $201,657,400$ USD.

The initial conditions of year 2 were $I_{21} = 116,160, I_{21} = 0, I_{21} = 0$. The optimal solution for year 2 is as follows: $x_{2412} = 2435, x_{4412} = 557.308, x_{4111} = 1, y_{4111} = 2435, y_{1111} = 116,160, y_{1111} = 7864, y_{2121} = 62,994, y_{3121} = 12,005, y_{4121} = 9578, y_{12} = 26,154, y_{12} = 116,160$. $I_{11} = 116,160, I_{11} = 1, I_{12} = 1, I_{12} = 1, I_{12} = 1, I_{12} = 1. $ The value of the objective function was $92,839,550$ USD.

The previous result constitutes the entry conditions of year 3 through inventory in warehouses, i.e., $I_{12} = 26,154, I_{12} = 0, I_{12} = 0$. The optimal solution for year 3 was as follows: $x_{4311} = 19,166, x_{4311} = 1, y_{3111} = 6228.95, y_{1111} = 128,905.1, y_{4112} = 22,750, y_{1111} = 26,154, y_{12} = 10,500, y_{23} = 2131 = 16,900, y_{33} = 11,300, y_{43} = 13,800$. In this year, the objective function had a value of $245,450,500$ USD.

6 Conclusions

In this document, a mixed integer linear programming model (MILP) has been developed to determine the levels of operation of a group of refineries that process ethanol. The objective of the global model is to determine which plants should operate and at what levels of their capacity. The same situation is presented for storage and mixing warehouses. The model also allows obtaining the quantities of biogas and gasoline that must be imposed on the system so that it works correctly. The first part of this model required using...
a clustering model to conveniently group the biorefineries that must operate around the producing centers (sugar mills and sugarcane fields).

Although originally the problem is posed as a stochastic model, the solution strategy was to obtain 500 runs of the three years (one after the other) and evaluate the inventory parameters resulting from year 1 and 2 to feed the model in years 2 and 3 respectively. From this, the minimum value of the sequence of the three years is obtained.

Special mention should be made for the result of year 2. Here, there was no need to operate the global system because the inventory for year 1 was sufficient to supply the demand for year 2 with the reserves saved in year 1. Note that the model requires production and operation of the system again because reserves are depleted in year 2, requiring their activity again for year 3.

The stochastic nature of the original model deserves to be approached from another perspective using heuristic methods. The size of the original instance was reduced to evaluate the efficiency of the model. However, this can easily be scaled to the desired size. Future extensions to the proposed model may consider the transport capabilities involved, the levels of customer service and even other, more complicated, network topologies. In any case, the resulting instance grows disproportionately as a function of the number of variables involved, as well as its number of constraints.

The exercise presented here shows a wide utility for modeling georeferenced localization systems as well as for the design of the distribution network of its products. Special attention is given to the design of networks for the distribution of perishable products.
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