Gauge vectors and double beta decay

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Abstract

We discuss contributions to neutrinoless double beta (0νββ) decay involving vector bosons. The starting point is a list of all possible vector representations that may contribute to 0νββ decay via \( d = 9 \) or \( d = 11 \) operators at tree level. We then identify gauge groups which contain these vectors in the adjoint representation. Even though the complete list of vector fields that can contribute to 0νββ up to \( d = 11 \) is large (a total of 46 vectors), only a few of them can be gauge bosons of phenomenologically realistic groups. These latter cases are discussed in some more detail, and lower (upper) limits on gauge boson masses (mixing angles) are derived from the absence of 0νββ decay.

Keywords: Neutrinoless double beta decay; neutrino mass; extended gauge groups.

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1 Introduction

Neutrinoless double beta decay (0νββ decay) is the most sensitive experimental probe of lepton number violating (LNV) extensions of the Standard Model (SM) — for a review see for instance [1]. The latest experimental half-life bounds [2–4] correspond to upper limits on the effective Majorana mass of the neutrino of the order of \( \langle m_\nu \rangle \lesssim (0.1 - 0.2) \) eV, depending on nuclear matrix elements [5, 6]. However, from the theoretical point of view the mass mechanism represents only one out of many possible contributions to the 0νββ decay amplitude. In fact, quite a number of papers on 0νββ decay involving exotic (i.e. beyond SM) scalars can be found in the literature; see [7–15] for some examples.

On the other hand, there are very few publications discussing the contribution of exotic vector fields to this process. The best-known example of 0νββ decay induced by an exotic vector is perhaps the charged boson \( W_R \) exchange in the context of left-right symmetric models. To the best of our knowledge, 0νββ decay in these models was discussed for the first time in [16]. This subject has been studied in detail in many papers since then. Contributions to 0νββ decay from vector lepto-quarks were studied in [10]. A list of potential exotic vector contributions to the long-range part of the 0νββ decay amplitude [17] has been given in [18]. And, finally, there are a few papers on 0νββ decay in models based on the group \( SU(3)_C \times SU(3)_L \times U(1)_X \) (331, for short) [19, 20].

A general decomposition of \( d = 9 \) contributions to 0νββ decay has been derived in [21]. However, that paper focuses on scalars and does not discuss exotic vectors in detail. To fill this gap, in this paper we study systematically all possible exotic vectors that give tree-level contributions to 0νββ decay (associated to \( d = 9 \) and \( d = 11 \) operators, as discussed latter).

Let us start by recalling some basic (but important) aspects of 0νββ decay. At tree-level a non-zero 0νββ decay amplitude can be generated at \( d = 9 \) by only two topologies (I and II) [21] — see figure 1. The internal bosons can either be scalars or vectors; for example, in topology I one might have in the internal lines the combination vector–fermion–vector (\( V\psi V \)), vector–fermion–scalar (\( V\psi S \)) or scalar–fermion–scalar (\( S\psi S \)). Note, however, that for topology II not all combinations are equally important. This is because diagrams with \( VSS \) and \( VVV \) contain a derivative and thus, effectively these contributions are proportional to \( p_F/\Lambda_{LNV}^6 \), compared to \( 1/\Lambda_{LNV}^5 \) for the other combinations. Here \( p_F \) is the Fermi momentum of the nucleons, of the order of \( (100 - 200) \) MeV, and \( \Lambda_{LNV} \) is the scale of lepton number violation, which is also associated to the mass of the exotic particles.

All topology II diagrams contribute only to the so-called short-range amplitude of 0νββ decay [22]. This part of the amplitude involves diagrams in which all virtual particles have masses larger than \( p_F \). In topology I, however, it is possible that the internal fermion \( \psi \) is a light neutrino. In this case, one talks about a long-range contribution [17], since \( p_F \) corresponds to larger, inter-nucleon distances (typically of the order of a femtometer). This distinction between short- and long-range amplitudes is important, since the nucleon hard-core strongly suppresses the matrix elements of the short-range part of the amplitude.

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1 A table with the quantum number of vectors under \( SU(3)_C \times U(1)_{EM} \) is shown in [21].
Within the Standard Model there is only one non-renormalizable dimension 5 operator: the famous Weinberg operator $O_W = LLHH$ [23]. However, at higher order, many more $\Delta L = 2$ operators can be written down — indeed, the complete list of $\Delta L = 2$ operators with no derivatives/gauge bosons was given in [24]. Three $d = 7$, nine $d = 9$ and thirteen $d = 11$ operators contribute to $0\nu\beta\beta$ decay at tree-level.\footnote{The $d = 7$ and three of $d = 9$ $\Delta L = 2$ operators must be complemented by a SM charged-current interaction, in order to generate a $0\nu\beta\beta$ decay diagram.} From this effective operator point of view, for example the mass mechanism is simply described by $O_{47} = LLQQQHH^*H^*$. (The number “47” corresponds to the notation of [24].) If we contract this operator as $(QQ)(\overline{L})(H^*H^*)(\overline{L})(QQ)$, the internal particles of the diagram are fixed to be $V = (1,3,0) \equiv W$ and $\psi = (1,2,-1/2) \equiv L$. For the mass mechanism diagram, one just needs to select the isospin component of the external fermions to be $(\pi d)(e)(e)(\pi d)$, and connect the internal neutrinos via the Weinberg operator as shown in figure 2.

![Figure 2: Mass mechanism of neutrinoless double beta decay. In SM invariant language, this corresponds to the operator $O_{47}$ [24] contracted as $(QQ)(\overline{L})(H^*H^*)(\overline{L})(QQ)$.](image)

It is possible to find then all vectors contributing potentially to $0\nu\beta\beta$ decay by decomposing all the relevant $d = 7, 9, 11$ operators in a similar way. This is done in section 2. However, all experimentally observed fundamental vectors are gauge vectors so, in a second step, we search for gauge groups which contain one of our exotic vectors in the adjoint representation. Apart from the mass mechanism (i.e. the SM vectors), we recover the vectors of the left-right symmetric group and the 331 group mentioned above. In addition to these possibilities, we find more exotic ones, such as the vectors of the Pati-Salam group, SU(5) or more peculiar groups, such as for example $Sp(8) \times U(1)^n$ and $F_4 \times U(1)^n$. However, most of these groups are not phenomenologically interesting for $0\nu\beta\beta$ decay, either because (i) their vectors lead to proton decay or (ii) it is not possible to construct viable models with the SM fermion content. This is discussed in section 2 as well. In section 3 we then analyze the constraints on the (few) viable exotic vectors potentially contributing to $0\nu\beta\beta$ decay. We then finish with a short summary.

## 2 Vector contributions to $0\nu\beta\beta$ decay

### 2.1 $\Delta L = 2$ operators and vectors

Both in topology I and in topology II internal vector fields couple to a pair of external fermion lines (i.e., leptons and/or quarks), hence their gauge quantum numbers are not arbitrary. In order to explore this fact, take as a starting point the following list of dimension nine ($d = 9$) $0\nu\beta\beta$ operators:

$$O^{d=9} = LLQQQLLd\zeta,QQu'u'LL,QQu'u'\overline{L}L,QQd'd'e'Le',Qu'u'\overline{d'}Le',u'u'\overline{d'}e'e'c'. \tag{1}$$

A vector field $V_{\mu}$ must couple to a left and a right-handed fermion or, equivalently, to a combination $XY$ where both $X$ and $Y$ are left-handed fields. Given the above list of $d = 9$ operators it is straightforward to find all possible fermion bilinears coupling to vectors and identify the potentially interesting vector representations:

$$V_{\mu}^{d=9} = (8,3,0), (8,1,0), (1,3,0), (1,1,0), (6,2,\frac{1}{6}), (3,2,\frac{1}{3}), (3,3,\frac{2}{3}).$$
However, before exploring this list, it is worth reminding that the neutrino mass contribution to $0\nu\beta\beta$ decay actually comes from a $d = 11$ effective operator with two Higgs fields. Hence, in the search for new contributions to $0\nu\beta\beta$ one should not stop at the level of $d = 9$ operators (see section 1). At $d = 11$ one finds thirteen operators that contribute to $0\nu\beta\beta$: \[ O_{d=11} = u^c_u^c \bar{d} \bar{d} e^c e^c HH^*, \quad u^c_u^c \bar{d} \bar{d} f^c f^c \bar{L}\bar{L} HH, \quad u^c_u^c \bar{d} \bar{d} Q\bar{c}e^c \bar{L}\bar{H} H^*, \quad u^c_u^c \bar{d} \bar{d} Q\bar{c}e^c e^c H^* H^*, \quad u^c_u^c \bar{d} \bar{d} Q\bar{c}e^c e^c e^c H^* H^*, \]
\[ u^c_u^c \bar{d} \bar{d} Q\bar{c}e^c e^c e^c e^c H^* H^*, \quad u^c_u^c \bar{d} \bar{d} QQ\bar{c}e^c \bar{L}\bar{H} H^*, \quad u^c_u^c \bar{d} \bar{d} QQ\bar{c}e^c e^c \bar{L}\bar{H} H^*, \quad u^c_u^c \bar{d} \bar{d} QQ\bar{c}e^c e^c e^c \bar{L}\bar{H} H^*, \quad u^c_u^c \bar{d} \bar{d} QQ\bar{c}e^c e^c e^c e^c \bar{L}\bar{H} H^*. \]

(3)

Allowing for up to two external Higgs fields, the vectors can not only couple to a $X\bar{Y}$ fermion combination, but also effectively have a coupling to $X\bar{Y}H^{(*)}$ or $X\bar{Y}H^{(*)}H^{(*)}$, where $X$ and $Y$ are still left-handed fields and $H^{(*)}$ represents the Higgs field (possibly conjugated); see figure 3.

![Figure 3: Vector fields V couple directly to a fermion bilinear in the d = 9 operators (left diagram). For the d = 11 operators one must also take into account up to two additional Higgs field insertions while searching for possible vectors (center and right diagrams).](image)

Taking into account these additional Higgs insertions, one obtains the following larger list of potentially interesting vector representations:

\[ V_{dim=11}^d = ([1, 8], [1, 3], [0, 1, 2]), \quad \left(1, \frac{3}{2}, \frac{3}{2}\right), \quad \left((1, 8), [0, 1], (\frac{1}{3}, \frac{4}{1}, \frac{2}{3})\right), \quad \left(3, [\bar{3}], \frac{1}{3}, \frac{7}{6}, \frac{1}{6}, \frac{7}{6}\right), \quad \left(3, [\bar{3}], \frac{1}{3}, \frac{7}{6}, \frac{1}{6}, \frac{7}{6}\right). \]

(4)

Square brackets in this list represent independent alternatives for each quantum number — hence ([1, 8], [1, 3], [0, 1, 2]) stands for 12 different representations, for example. In total, there are 53 representations listed in equation (4).

![Figure 4: There are several dimension nine and eleven operators of the form $X_1X_2X_3X_4X_5X_6$ or $X_1\bar{X}_2\bar{X}_3\bar{X}_4\bar{X}_5\bar{X}_6$ plus Higgses. One must then ensure that each vector field V has the adequate quantum numbers to couple both to a combination $X_1\bar{X}_2$ (plus, possibly, some Higgses), and at the same time to a combination $X_3\bar{X}_4\bar{X}_5X_6$ or $X_3\bar{X}_4\bar{X}_5\bar{X}_6$ (again, plus Higgses).](image)
However, not all 53 vectors constructed in this way do actually contribute to $0\nu\beta\beta$ decay. First of all, neither $(1,1,0)$ nor $(8,1,0)$ carry electric charge and thus they can be trivially excluded. Furthermore, one actually has to ensure that the diagram is completable on the other side of the vector boson line (see figure 4). This requirement, in turn, excludes the representations $(1,4,\frac{3}{2})$, $(1,5,0)$, $(3,4,-\frac{5}{2})$, $(3,5,\frac{3}{2})$ and $(8,5,0)$. We are then left with 46 potentially interesting vector representations. All of them can play a role in either topology I or II, provided that one completes the diagrams appropriately (this might require adding two Higgs insertions). We do not provide here a list of all such valid configurations, since these can be easily constructed.

### 2.2 Gauge groups

Given the list of vectors discussed above, we now turn to the search for possible gauge groups (see also [25]). These groups must contain the SM group and for any vector to be a gauge boson of such a group, it must form a sub-representation of the adjoint representation of the larger group. There are several subtleties involved in this search, hence we shall start by making a few remarks. Firstly, a given group may break into a subgroup in more than one way. For example, $SU(3)$ contains $SU(2)$ but we may either have the branching rule $3 \to 2 + 1$ or $3 \to 3$. Mathematically both represent valid embeddings of $SU(2)$ in $SU(3)$, yet this does not mean that they are equally useful for model building. In particular, if we consider the Standard Model $SU(2)_L$ group, the embedding corresponding to the branching rule $3 \to 3$ can be ruled out since no matter which $SU(3)$ representation is chosen, it is impossible to obtain $SU(2)_L$ doublets.\(^3\)

Another important issue concerns the normalization of $U(1)$ charges. Consider a group $G$ with an $SU(3)_C \times SU(2)_L \times U(1)^n$ subgroup, $n > 0$. Without writing down the complete model based on $G$, it is not possible to determine how the $n$ $U(1)$’s combine to form the $U(1)_Y$ hypercharge group. Take, for example, the $SU(3)_C \times SU(2)_L \times SU(2)$ group: the adjoint representation contains the gluons $(8,1,0)$, the $W$’s $(1,3,0)$ and $(1,1,3)$. If we break the $SU(2)$ group to $U(1)$, this last representation branches into $(1,1,0) + (1,1,\pm 1)$, yet one is a priori free to consider that the hypercharge is a multiple of the last quantum number. Hence, without first trying to build a complete model — including all relevant fermions and scalars — one has to consider that any SM representation of the form $(1,1,y)$ is potentially contained in the adjoint of $SU(3)_C \times SU(2)_L \times SU(2)$. Furthermore, if necessary, one can add extra $U(1)$’s to the original group $G$ in order to build the SM $U(1)_Y$ hypercharge group (i.e., instead of $G$ one might consider $G \times U(1)^m$). This is indeed what happens in the well-known models based on the $SU(3)_C \times SU(3) \times U(1)$ and $SU(3)_C \times SU(2)_L \times SU(2) \times U(1)$ groups.

For these reasons, we will temporarily (a) ignore whether or not a full viable model can be made out of particular groups/embbedings, and (b) ignore $U(1)$’s altogether (since all that matters, according to the previous discussion, is whether or not the hypercharge of a vector field is non-zero). We can then try to embed the SM group in a simple group $G$, $SU(3)_C \times G$, or maybe $SU(2)_L \times G$.\(^4\) A search was performed over all such groups where $G$ is both simple and has at most 9 diagonal generators. This includes all exceptional groups, all $SU(N < 11)$, all $SO(N < 20)$, and the $Sp(2N)$ groups up to $N = 9$. Table 1 contains the results of this search.

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\(^3\)The reason is the following. Any $SU(3)$ representation will be picked up in the product of $n$ fundamental representations of $SU(3)$, for a sufficiently large $n$. As such, if the fundamental of $SU(3)$ branches into an $SU(2)_L$ triplet, an $SU(3)$ representation can only branch into $SU(2)_L$ representations contained in the product of $SU(2)_L$ triplets (i.e., $1,3,5,\ldots$ representations, but never $2,4,6,\cdots$).

\(^4\)We stress again that eventual extra $U(1)$’s, needed to correctly form the hypercharge group, are being ignored at this stage.
chirality (even if one considers extra $F$ from a model with a gauge group as small as $SU(5)$, the known fermion fields. For example, the Standard Model fermions cannot be embedded in representations of such a group. Under the relevant branching rules, we have the following breaking of $SU(3)_C \times SU(2)_L \times SU(2)_R$:

| Vector representation(s) | Minimal group(s) (without $U(1)$’s) |
|--------------------------|-------------------------------------|
| $(1, 1, y = 1, 2)$       | $SU(3)_C \times SU(2)_L \times SU(2)$ |
| $(1, 2, y = \frac{1}{3}, \frac{2}{3})$ | $SU(3)_C \times SU(3)$, $SU(3)_C \times Sp(4)$ |
| $(1, 3, y = 1, 2)$       | $SU(3)_C \times SU(2)_L$ |
| $(1, 4, y = \frac{1}{2})$ | $SU(3)_C \times SU(5)$, $SU(3)_C \times Sp(6)$, $SU(3)_C \times G_2$ |
| $(1, 5, 1)$              | $SU(3)_C \times SO(7)$, $SU(3)_C \times Sp(6)$ |
| $(3, 1, y = -\frac{1}{3}, -\frac{1}{3})$ | $SU(4)_C \times SU(2)_L$ |
| $(3, 2, y = -\frac{11}{9}, \frac{2}{9}, \frac{2}{9})$ | $SU(5), Sp(8), F_4$ |
| $(3, 3, y = -\frac{1}{3}, -\frac{1}{3}, \frac{2}{9})$ | $SU(6), SO(9)$ |
| $(3, 4, y = -\frac{1}{3}, \frac{1}{9}, \frac{2}{9})$ | $SU(7), Sp(10), E_6$ |
| $(3, 5, y = -\frac{1}{3})$ | $SU(8), SO(11)$ |
| $(6, 1, y = -\frac{1}{3}, -\frac{1}{3}, \frac{1}{3})$ | $Sp(6)_C \times SU(2)_L$ |
| $(6, 2, y = -\frac{1}{6}, -\frac{1}{6}, \frac{5}{6}, \frac{11}{6})$ | $SU(8), Sp(14), F_4$ |
| $(6, 3, y = -\frac{1}{6}, \frac{5}{6}, \frac{5}{6}, \frac{11}{6})$ | $SU(9), SO(15), Sp(12)$ |
| $(6, 4, y = -\frac{1}{6}, -\frac{1}{6}, \frac{11}{6})$ | $Sp(16)$ [*] |
| $(8, 1, y = 1, 2)$       | $SU(6)_C \times SU(2)_L$, $SO(10)$, $SU(2)_L$ |
| $(8, 2, y = \frac{1}{3}, \frac{2}{3})$ | $SU(9), SO(12), E_6$ |
| $(8, 3, y = 1, 2)$       | $SU(6), SO(11)$ |
| $(8, 4, y = \frac{1}{3})$ | $SO(14)$ [*] |
| $(8, 5, 1)$              | $SO(16)$ [*] |
|                         | $SO(18)$ [*] |

Table 1: Minimal groups (second column) containing in their adjoint representation the indicated Standard Model vector sub-representations (first column). The exact value of the hypercharge of each representation is irrelevant: it only matters whether or not it is zero (see text). The list of groups shown here is of the form $G$, $SU(3)_C \times G$, or $SU(2)_L \times G$ where $G$ is a simple group with at most 9 diagonal generators. Note, however, that it is straightforward to show that, except for those cases marked with an asterisk, the results are unchanged if this constraint is removed. Only minimal groups are displayed, hence any other group containing those listed above will obviously also work. Just the cases marked by a box lead to phenomenologically viable models, for the reasons explained in the text.

An even more important consideration is that for many of the vector representations indicated in table 1, the group listed must contain the SM gauge group in a non-conventional way. This makes it very challenging to accommodate the known fermion fields. For example, $F_4 \times U(1)^n$ is an unsuitable group: $F_4$ is an exceptional Lie group which only has real representations, hence it cannot account for the SM chirality (even if one considers extra $U(1)$’s).

As a further example, consider the $(1, 3, y = 1, 2)$ representations which can be obtained, in theory, from a model with a gauge group as small as $SU(3)_C \times Sp(4) \times U(1)^n$ for some $n$. We shall now show that the Standard Model fermions cannot be embedded in representations of such a group. Under the relevant branching rules, we have the following breaking of $Sp(4)$ representations into sub-representations of $SU(2)_L \times U(1)_X$:

\[
4 \rightarrow (2, \pm 1), \quad (5) \\
5 \rightarrow (1, \pm 2) + (3, 0), \quad (6) \\
10 \rightarrow (1, 0) + (3, 0) + (3, \pm 2), \quad (7) \\
14 \rightarrow (1, 0) + (1, \pm 4) + (3, \pm 2) + (5, 0), \quad (8) \\
16 \rightarrow (2, \pm 1) + (2, \pm 3) + (4, \pm 1), \quad (9)
\]
Note that a priori the hypercharge group $U(1)_Y$ can be a linear combination of $U(1)_X$ and the other $n$ $U(1)$ factor groups. Even so, one can rule out the possibility of embedding the SM in a $SU(3)_C \times Sp(4) \times U(1)^a$ model with the above branching rules by, for example, counting $SU(2)_L$ doublets. The argument is the following. Since all $Sp(4)$ representations are real, they must break into real and/or pairs of complex conjugated representations of $SU(2)_L \times U(1)$. In turn, this means that one will always have an even number of $SU(2)_L$ doublets, because it can be shown that there are no doublets with $X = 0$ regardless of which $Sp(4)$ representations are chosen. So any model based on this particular embedding of the SM group in $SU(3)_C \times Sp(4) \times U(1)^a$ will necessarily have an even number of $SU(2)_L$ doublets. In fact, one can be more specific: the number of doublets with a given color quantum number (singlet, triplet, anti-triplet, ...) will be even. This, however, does not lead to a satisfactory fermion sector, as we must have three quark doublets plus (eventually) pairs of vector quarks, hence the required number of fermion representations which are color triplet/anti-triplets and $SU(2)_L$ doublets must be odd.

We have argued that there are no realistic models based on the gauge group $SU(3)_C \times Sp(4) \times U(1)^a$ which contain the vector representations $(1,3,y = 1,2)$. Furthermore, a model based on $F_4 \times U(1)^a$ will not work either. Yet these are just illustrative examples of the challenge of building realistic models where the vector representations in table 1 show up. In fact, the only groups and vectors for which we were able to find viable models (and which give potentially interesting contributions to $0\nu\beta\beta$) besides the SM $W = (1, 3, 0)$ are:

- $(1, 1, 1)$ associated to $SU(3)_C \times SU(2)_L \times SU(2) \times U(1)$,
- $(1, 2, y = \frac{1}{2}, \frac{3}{2})$ associated to $SU(3)_C \times SU(3) \times U(1)$,
- $(3, 1, \frac{3}{2})$ associated to $SU(4) \times SU(2)_L \times U(1)$,

or bigger groups containing these ones. In the next section we will look carefully into these cases.

### 3 Phenomenologically viable cases

#### 3.1 Brief comments on non-gauge vectors

Before we turn to a discussion of the phenomenologically viable groups, we briefly comment on non-gauge vectors. One can, of course, add exotic vectors to the SM particle content without enlarging the gauge group. A classical example is electro-weak scale lepto-quarks [28, 29]. Such relatively light vectors can have interactions with the SM Higgs boson [30], which leads to a $\Delta \lambda = 2$ mixing between different lepto-quarks states after electro-weak symmetry breaking. Long-range contributions to $0\nu\beta\beta$ decay arise in such models [10].

Neutrinoless double beta decay may impose constraints on all (non-gauge) vectors in table 1 provided the corresponding models violate lepton number. We will not discuss here possible models nor give individual limits case by case. However, we do want to briefly discuss the typical limits one expects for any such model. For this estimate, one has to distinguish models leading to a contribution to the long-range $0\nu\beta\beta$ decay amplitude from those that contribute only to the short-range part.

The following subset of vectors can contribute to the long-range amplitude at the level of $d = 7$ operators: $(3, 1, 2/3), (3, 3, 2/3), (3, 2, 1/6), (1, 2, 3/2)$ and $(1, 1, 1)$. Constraints on $d = 7$ operators are always of the form $\mathcal{O}_{d=7}^{\text{long}} \propto \frac{g_{\mu\nu}^2}{\Lambda_{\text{LNV}}}^2$, where $g_{\text{eff}}$ is the geometric mean of the three couplings entering the decomposition of the relevant operator, and $\Lambda_{\text{LNV}}$ is the geometric mean of the masses of the heavy particles. Using the latest experimental half-life for $^{136}\text{Xe}$ [2] and including QCD corrections [31] in the calculation of

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5 This is true for the representations in equations (5)–(9), but also for larger ones.

6 There are $n$ vector representations with the quantum numbers $(1, 3, y = 1, 2)$ in models based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ provided that the Standard Model $SU(2)_L$ corresponds to the subgroup $[SU(2)]_{\text{diag}}$ [26]. These models have a viable fermion spectrum and indeed they have recently been explored as a possible explanation for meson anomalies (see [27] and references contained therein). However, apart from non-universal flavor effects, the interactions of the heavier $W_{\pm}$ in these models are similar to those of the SM $W_{\pm}$, hence their contribution to neutrinoless double beta decay will be subdominant.
the theoretical amplitude, lower limits in the range of $\Lambda_{LNV} \gtrsim g_{eff}(26 - 247)\text{TeV}$ are found, depending on which of the coefficients in the general expression for the $0\nu\beta\beta$ decay half-life is generated in the corresponding model. We want to stress that in some particular cases one can expect $g_{eff} \ll 1$, leading to much more relaxed bounds on $\Lambda_{LNV}$. A well-known example is the left-right symmetric model (see section 3.2). Many more vectors appear in the long-range part of $0\nu\beta\beta$ if we include operators up to $d = 9$. However, constraints are much weaker for these operators: $O_{\text{long}}^{d = 9} \propto g_{eff}^2\Lambda_{LNV}^5$. This leads to bounds of the order of $\Lambda_{LNV} \gtrsim (3.5 - 13.5) g_{eff}^4\text{TeV}$, again depending on the operator. We will discuss some examples of long-range $O_{d = 9}$ contribution (from gauged vectors) to $0\nu\beta\beta$ decay in sections 3.2 and 3.3.

In order to generate short-range contributions from exotic vectors a full model requires either an additional exotic scalar (topology II), or an exotic fermion (topology I). We will discuss a gauged example of long-range $O_{d = 9}$ contribution (from gauged vectors) to $0\nu\beta\beta$ decay in sections 3.2 and 3.3. Many more vectors appear in the long-range part of $0\nu\beta\beta$ decay via $d = 9$ operators scale as $O_{\text{short}}^{d = 9} \propto g_{eff}^4\Lambda_{LNV}^5$, assuming that the triple vector vertex has a coupling of the order $g_{eff}\Lambda_{LNV}$. Taking the general short-range decay rate, including QCD corrections, from [32, 33] and using the limit from $^{136}\text{Xe}$ [2], results in lower limits in the range $\Lambda_{LNV} \gtrsim (2.2 - 7.3) g_{eff}^{4/5}\text{TeV}$, depending on the operator.

These generic limits relating the geometric mean mass $\Lambda_{LNV}$ and the geometric mean coupling $g_{eff}$ are to be understood only as rough order of magnitude estimates. For specific models and diagrams, the masses and couplings themselves can have different values spanning several orders of magnitude, as we will discuss in the following sections. However, all numbers given in the examples below are consistent with the above estimates.

3.2 A well-known case: Left-right symmetric model

The vector representation $(1, 1, 1)$ can be identified with the charged component of a right triplet $(1, 1, 3, 0)$ of the left-right symmetric group $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. Neutrinoless double beta decay was discussed in this context first in [16]. Many more detailed calculations, with particular emphasis on the nuclear physics side of the problem, followed soon thereafter (see [34, 35] for reviews). These early calculations focused on long-range contributions, but the importance of the short-range part was stressed again in [36]. The general decay rate in left-right models, including long-range and short-range diagrams, was then given in [37, 38]. The latter reference not only calculated nuclear matrix elements but also included for the first time diagrams with a scalar $\Delta_R \equiv (1, 1, 3, 1)$ [38]. A more recent review on $0\nu\beta\beta$ decay in left-right symmetric models can be found in [39]. Our discussion below of $0\nu\beta\beta$ decay in left-right models follows essentially [18].

In the standard left-right symmetric model, the $SU(2)_R \times U(1)_{B-L}$ group is broken to $U(1)_Y$ via the vacuum expectation value (vev) of the scalar $\Delta_R$, and Majorana neutrino masses for the right-handed neutrinos are generated from the same vev. Left-handed neutrino masses are then produced after electroweak symmetry breaking through a seesaw mechanism. As noted above, in this setup both long-range and short-range contributions to $0\nu\beta\beta$ decay have to be considered — see the diagrams in figure 5.

The $d = 7$ and $d = 9$ operators associated to these diagrams are the following:

\begin{equation}
O_8 = \mathcal{L}e^c u^c \overline{d} H, \quad O_7 = \mathcal{L}e^c Q \overline{Q} H^* H^* H^*, \quad \langle m_N \rangle \approx \frac{g_H^2}{m_{W_R} (m_N)} \text{TeV}
\end{equation}

(10)

\begin{equation}
O_{-} = \mathcal{L}e^c u^c \overline{d} d^c.
\end{equation}

(11)

Consider first the short-range diagram generated by $O_{-}$. This diagram is proportional to $\frac{g_H^2}{m_{W_R} (m_N)}$, where $\langle m_N \rangle$ is defined as $1/\langle m_N \rangle = \sum_{j=1}^{6} V_{ej}^2 / m_j$ and in turn $V_{ej}$ is the right-handed neutrino mixing matrix in the notation of [35] and the index $j = 4, 5, 6$ runs over the heavy neutrino mass eigenstates. Similar to the case of the well-known mass mechanism, there can be cancellations among terms in the sum over $j$, due to Majorana phases in the right-handed neutrino sector. However, $\sum_{j=1}^{6} V_{ej}^2 \approx 1$ from unitarity. Updating [38] with the latest half-life bound from [2] results in a lower limit on the $W_R$ boson mass, roughly given by $m_{W_R} \gtrsim 1.9 (g_{R}/g_{L}) (\frac{1}{\langle m_N \rangle})^{1/2}$ TeV.
For the long-range part of the amplitude, one finds two different contributions, which are shown in figure 5. One can identify these diagrams with the $\lambda$ and $\eta$ terms in the notation of [35]:

\[
\mathcal{O}_8 \rightarrow \langle \lambda \rangle = \sum_{j=1}^{3} U_{ej} V_{ej} \lambda = \sum_{j=1}^{3} U_{ej} V_{ej} \left( \frac{m_{WL}}{m_{W_R}} \right)^2, \tag{12}
\]

\[
\mathcal{O}_7 \rightarrow \langle \eta \rangle = \sum_{j=1}^{3} U_{ej} V_{ej} \eta = \sum_{j=1}^{3} U_{ej} V_{ej} \tan \zeta. \tag{13}
\]

Here, $\zeta$ is the mixing angle between the $W_L$ and $W_R$ states, typically also roughly of the order of $\left( \frac{m_{WL}}{m_{W_R}} \right)^2$. $U_{aj}$ is the left-handed neutrino mixing matrix and orthogonality of $U_{ej}$ and $V_{ej}$ implies $\sum_{j=1}^{6} U_{ej} V_{ej} \equiv 0$. However, the sums in equations (12) and (13) run only over the light neutrinos and thus, are incomplete. Typical expectations for this heavy-light neutrino mixing, $\sum_{j=1}^{3} U_{ej} V_{ej}$, in models with an ordinary type-I seesaw are of the order of $\frac{m_D}{M_M} \sim \sqrt{m_\nu/M_M}$, where $m_D, M_M$ and $m_\nu$ are the Dirac mass term, the Majorana mass term for the right-handed neutrinos and the light neutrino mass, respectively. Using again the limit from $^{136}$Xe [2] results in upper limits for $\langle \lambda \rangle < \sim 2 \times 10^{-7}$ and $\langle \eta \rangle < \sim 1.1 \times 10^{-9}$ using the nuclear matrix elements of either [5] or [40].

For $\sum_{j=1}^{3} U_{ej} V_{ej} > \sim 10^{-4}$ the limit on $\langle \lambda \rangle$ would become a more stringent limit on $m_{W_R}$ than the one derived from the short range diagram. However, for $\langle m_N \rangle \approx 1$ TeV one would expect $\sum_{j=1}^{3} U_{ej} V_{ej} \sim 10^{-6}$. It is interesting to note [18], that the constraint on $\mathcal{O}_7$ is actually more stringent than the one on $\mathcal{O}_8$, despite being formally of higher order, due to the contribution of the nuclear recoil matrix element to the former [35].

### 3.3 Models based on the 331 group

In this section, we discuss examples of exotic contributions based on the $SU(3)_C \times SU(3)_L \times U(1)_X$ group. It turns out that there is a rich variety of such models, and the most important distinguishing factor among them is the way the SM hypercharge is embedded in $SU(3)_L \times U(1)_X$: $Y = \beta T_3 + X$. From purely group theoretical considerations, $\beta$ is a continuous parameter. However, perturbativity of the gauge interactions requires that $\beta \lesssim 1.8$ (see for example [41, 42]). Furthermore, if one additionally demands that there are no fractionally charged color singlet fermions in the model, $\beta$ may take only the values $\pm 1/\sqrt{3}, \pm \sqrt{3}$.

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7 It is amusing to note that using the nuclear matrix elements of either [5] or [40] results in nearly identical numerical limits.
In 331 models, usually all three generations of left-handed leptons are assigned to triplets. Anomaly cancellation then requires that two left-handed quark generations are assigned to anti-triplets, while the third one must be a triplet.\(^8\) The prototype 331 model for this kind of construction (with \(\beta = -1/\sqrt{3}\)) was proposed in [44] and [45]. The only difference between the variants [44] and [45] is that the former assumes the presence of additional right-handed neutral singlets \(\nu_R\), while the latter explicitly excludes such states from the fermion spectrum.

For the values \(\beta = \pm 1/\sqrt{3}\) and \(\beta = \pm \sqrt{3}\), some of the gauge bosons transform as \((1, 2, \frac{1}{2})\) and \((1, 2, \frac{3}{2})\), respectively, under the SM gauge group, and both these representations were identified earlier as potentially important for neutrinoless double beta decay. However, even though it is possible to build one or more models for each of these values of \(\beta\), from the point of view of \(0\nu\beta\beta\) we find that \(\beta = -\sqrt{3}\) is the most interesting choice, given that the third element of the lepton \(SU(3)_L\) triplets corresponds to the SM right-handed charged leptons \(\ell^c_\alpha\). The significance of this fact will be discussed shortly.

We will then concentrate on the 331 model based on the choice \(\beta = -\sqrt{3}\) and the fields shown in table 2. This variant was first proposed in [46, 47], and therefore we will call it the Pisano-Pleitez-Frampton (PPF) model.

| Field \(\psi_{\ell,\alpha}\) | # flavours | 331 representation | \(G_{SM}\) decomposition | Components | Lepton number |
|---|---|---|---|---|---|
| \(\psi_{\ell,\alpha}\) | 3 | \((1, 3, 0)\) | \(\left(\frac{1}{2}, \frac{1}{2}\right) + \left(1, \frac{1}{3}\right)\) | \((\nu_\alpha, \ell_\alpha, \ell^c_\alpha)^T\) | \((1, 1, -1)^T\) |
| \(Q_{\alpha=1,2}\) | 2 | \((3, 3, -\frac{1}{3})\) | \(\left(\frac{2}{3}, \frac{1}{6}\right) + \left(3, \frac{1}{3}\right)\) | \((d_\alpha, -u_\alpha, J'_\alpha)^T\) | \((0, 0, 2)^T\) |
| \(Q_3\) | 1 | \((3, 3, \frac{2}{3})\) | \(\left(\frac{2}{3}, \frac{1}{2}\right) + \left(3, \frac{2}{3}\right)\) | \((t, b, J^T_3\) | \((0, 0, -2)^T\) |
| \(u^c_\alpha\) | 3 | \((3, 1, -\frac{2}{3})\) | \(\left(\frac{1}{2}, -\frac{1}{3}\right)\) | \(u^c_\alpha\) | 0 |
| \(d^c_\alpha\) | 3 | \((3, 1, \frac{2}{3})\) | \(\left(\frac{1}{2}, \frac{1}{3}\right)\) | \(d^c_\alpha\) | 0 |
| \(J_{\alpha=1,2}\) | 2 | \((3, 1, \frac{4}{3})\) | \(\left(\frac{1}{2}, \frac{4}{3}\right)\) | \(J_\alpha\) | -2 |
| \(J_3\) | 1 | \((3, 1, -\frac{2}{3})\) | \(\left(\frac{1}{2}, -\frac{2}{3}\right)\) | \(J_3\) | 2 |
| \(\phi_1\) | 1 | \((1, 3, 1)\) | \(\left(\frac{1}{2}, \frac{1}{2}\right) + \left(1, \frac{1}{3}\right)\) | \((\phi_1^+, \phi_1^0, \tilde{\phi}_1^{++})^T\) | \((0, 0, -2)^T\) |
| \(\phi_2\) | 1 | \((1, 3, -1)\) | \(\left(\frac{1}{2}, -\frac{3}{2}\right) + \left(1, \frac{1}{3}\right)\) | \((\phi_2^-, \phi_2^0, \tilde{\phi}_2^-)^T\) | \((2, 2, 0)^T\) |
| \(\phi_3\) | 1 | \((1, 3, 0)\) | \(\left(\frac{1}{2}, -\frac{1}{2}\right) + \left(1, \frac{1}{3}\right)\) | \((\phi_3^0, \phi_3^-, \tilde{\phi}_3^+)\) | \((0, 0, -2)^T\) |

Table 2: Fields in the Pisano-Pleitez-Frampton (PPF) model [46, 47], which is based on the \(SU(3)_C \times SU(3)_L \times U(1)_X\) gauge group.

As shown in table 2, the standard model lepton doublets are in \(SU(3)_L\) triplets, \(\psi_{\ell,\alpha}\), with \(\alpha = e, \mu, \tau\), which also contain a singlet charged lepton, \(\ell^c_\alpha\). Note that our conventions are that all fields are left-handed, meaning that \(\ell^c_\alpha\) corresponds to the charged-conjugate of the right-handed leptons. It is of utmost importance to correctly identify the source or sources of lepton number violation if we are to discuss neutrinoless double beta decay and, as explained in detail in [42], the original PPF model contains a single lepton number violating interaction:

\[ \mathcal{L}^{LNV} = \lambda_7 \phi_1 \phi_2 \phi_3 \phi_4^* + \text{h.c.} \]  \hspace{1cm} (14)

The original PPF model predicts that the charged lepton masses are \((-m, 0, m)\), in gross violation of the experimental data, and thus has to be modified. There are two simple possibilities to generate a realistic charged lepton spectrum. One of them is to introduce a vector-like pair of singlet charged fermions [48, 49]. We will call this version of the model PPF-E. The other possibility involves adding a scalar \(SU(3)_L\) sextet with no \(U(1)_X\) charge. This variant was first proposed in [50], and we will call it PPF-S.

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\(^8\) Alternatively one can put all left-handed quark generations into anti-triplets. In this “flipped” scenario, two left-handed lepton generations are assigned to triplets, while the remaining one is part of an \(SU(3)_L\) sextet [43].
Consider first the addition of a vector-like lepton singlet $E/E^c$, which is allowed to have the following interactions:

$$\mathcal{L}_{EE^c} = h_{E^c,\alpha} \psi_{\ell,\alpha} E^c \phi_1^+ + h_{E,\alpha} \psi_{\ell,\alpha} E \phi_2^- + m_{EE^c} E E^c. \quad \text{(15)}$$

Once $\phi_{1,2}$ obtain vevs, the SM charged leptons mix with the vector-like state and the charged lepton masses can be made to agree with the experimental data. As discussed in [42], correctly fitting the $\tau$ mass requires $|h_{E^c,\tau} h_{E,\tau}| \sim \mathcal{O}(10^{-2})$ for $m_{EE^c} \sim \text{TeV}$.

The PPF-E model also makes it possible to fit neutrino data, since the interactions shown in equations (14) and (15) can be used to generate the 1-loop neutrino mass diagram shown in figure 6. A simple estimate of this loop gives [42]

$$\langle m_\nu \rangle_{\alpha\beta} \sim \left( \frac{m_{EE^c}}{\text{TeV}} \right) \left( \frac{\text{100 GeV}}{m_{\phi_{1,2}}} \right)^2 \left( \frac{|h_{E^c,\alpha} h_{E,\beta} + h_{E^c,\beta} h_{E,\alpha}|}{10^{-2}} \right) \left( \frac{\lambda_7}{10^{-7}} \right) 10^{-1} \text{eV}. \quad \text{(16)}$$

Note that since $\langle m_\nu \rangle = \sum_j U^2_{ej} m_j \equiv \langle m_\nu \rangle_{ee}$, neutrinoless double beta decay is sensitive only to $\langle m_\nu \rangle_{ee}$, i.e. to the entry of the neutrino mass where $\alpha = \beta = e$.

Figure 6: One loop neutrino mass diagram in the PPF-E model.

The second variation of PPF model mentioned above involves adding a scalar sextet, $S = (1,6,0)$:

$$S = \begin{pmatrix}
\Delta^0 \\
\Delta^- \\
\frac{1}{\sqrt{2}} \Delta^0 \\
\frac{1}{\sqrt{2}} \Delta^- \\
\frac{1}{\sqrt{2}} H^+_S \\
\frac{1}{\sqrt{2}} H^-_S
\end{pmatrix} \quad \text{(17)}$$

$S$ decomposes into a triplet, a doublet, and a singlet of $SU(2)_L$, hence we have named the corresponding components with the names $\Delta$, $H_S$, and $\sigma$, respectively. The interaction term of the sextet with the leptons decomposes as follows:

$$y_S \psi_{\ell,\alpha} S^* \psi_{\ell,\beta} = y_S \left[ \nu_\alpha \nu_\beta \Delta^{0*} \frac{1}{\sqrt{2}} (\ell_\alpha \ell^c_\beta + \ell^c_\alpha \ell_\beta) H^{0*} + \cdots \right], \quad \text{(18)}$$

and from this expression it is obvious that a non-zero $\Delta^0$ vev will give rise to a type-II seesaw. This vev necessarily violates lepton number by two units ($\Delta L = 2$). The charged leptons now receive two contributions to their mass: $m_\ell = y_\ell \langle \phi^0_3 \rangle + y_S \langle H^0_S \rangle$. To generate a large enough $m_\tau$, this requires that either $\langle y_S \rangle_{\nu_\tau} \langle H^0_S \rangle$ and/or $\langle y_S \rangle_{\tau\tau} \langle H^0_S \rangle$ are of the order of $m_\tau$, which puts a lower limit of roughly $\mathcal{O}(10^{-2})$ on at least one of these Yukawa couplings.

This is important since the same Yukawa couplings contribute to the Majorana neutrino mass matrix and thus one must have $\langle \Delta^0 \rangle / \langle H^0_S \rangle \lesssim 10^{-10}$ in order not to violate upper bounds on the neutrino mass scale. Given this seemingly strong constraint, one might wonder whether it is possible to find a symmetry that produces an exactly vanishing triplet vev, $\langle \Delta^0 \rangle = 0$, as was done in the original work [50]. The answer is that such symmetry would also eliminate all lepton number violating terms from the PPF-S model [42].

\textsuperscript{9}Note that it is also possible to write down new scalar interactions involving $S$ which break explicitly lepton number by two units.
This is thus not an acceptable solution, since neutrinos would be massless. On the other hand, it is possible to arrange a small but non-zero ratio of vevs $\langle \Delta^0 \rangle / \langle H_0^0 \rangle$ with a proper choice of scalar potential parameters; some of them would have to be small though (for example the coefficient of the trilinear interaction $\phi_3 \phi_5 S^*$). In any case, we will assume a small $\langle \Delta^0 \rangle \neq 0$ in the following discussion.

In addition to the mass mechanism, the PPF model and its variants may produce other long-range contributions to $0\nu\beta\beta$ decay, which are associated to the diagram shown in figure 7. To understand the physics behind this diagram, we first have to discuss charged-current interactions. Apart from the ordinary contributions to $0\nu\beta\beta$ decay, which are associated to the diagram shown in figure 7, a $W^\pm$ boson which couples to left-handed doublets, the PPF model contains exotic heavy charged bosons which, following [46], we will denote as $V^\pm$ and $U^{\pm\pm}$. The charged current interactions then contain the terms:

$$\mathcal{L}_{cc}^{V^\pm} = - \frac{g_L}{\sqrt{2}} \sum_{\alpha=1}^{3} \left( \pi_{\alpha}(\gamma^\mu \gamma^5) \nu_\alpha V^\mu_\mu + \bar{c}_\alpha(\gamma^\mu \gamma^5) \ell_\alpha U^{\mu\mu} + \text{h.c.} \right), \quad (19)$$

$$\mathcal{L}_{Q_{\alpha=1,2}}^{cc} = - \frac{g_L}{\sqrt{2}} \sum_{\alpha=1}^{2} \left( \pi_{\alpha}(\gamma^\mu \gamma^5) \nu_\alpha V^\mu_\mu - \bar{d}_\alpha(\gamma^\mu \gamma^5) J^\mu_\alpha V^+_\mu + \text{h.c.} \right), \quad (20)$$

$$\mathcal{L}_{Q_3}^{cc} = - \frac{g_L}{\sqrt{2}} \left( \bar{c}(\gamma^\mu \gamma^5) b U^{++}_\mu + \text{h.c.} \right). \quad (21)$$

It is important that $V^\pm$ couples only “off-diagonally” to quarks, i.e. it always connects one SM quark with one of the exotic states $J^\mu_3$. (The part of the (Majorana) neutrino propagator does not violate lepton number.) Similarly to the situation encountered in the left-right symmetric model, such a $\Delta L = 2$ term can be generated by $V^\pm - W^\pm$ mixing. As shown in figure 8, there are different possibilities to generate gauge boson mixing in the PPF 331. We will briefly discuss each of these possibilities in turn.

In the PPF-E model, $V^\pm - W^\pm$ mixing is generated through a loop which is shown in figure 7. On top of the quartic interaction in equation (14), this loop requires one of the following terms in the scalar potential:

$$\mathcal{L}_S = \mu \phi_1 \phi_2 \phi_3 + \lambda_4 |\phi_1|^2 |\phi_2|^2 + \lambda_5 |\phi_1|^2 |\phi_3|^2 + \lambda_6 |\phi_2|^2 |\phi_3|^2 + \cdots . \quad (23)$$

Note that while only one possible diagram contributing to $V^+ - W^+$ mixing is shown in the figure, permutations of this loop using different $\lambda_k$, $k = 4, 5, 6$ or two terms proportional to $\mu$ all contribute. One can make a rough estimate of the size of the mixing angle:

$$\theta_{VVW} \sim \frac{g_L^2}{16\pi^2} \lambda_7 \lambda_k \left( \frac{v^2}{n} \right)^3 . \quad (24)$$
Here, $k = 4, 5, 6$ and we assume that all the doublet vevs $\langle \phi_{1,3}^0 \rangle$ are of the order of the SM vev, $v \simeq 174$ GeV, while the SM singlet vev $\langle \phi_0^3 \rangle$ is of the order of the 331 scale, denoted as $n$. For $n$ of the order of one TeV, and using the upper limit of $C_3^L = 1.9 \times 10^{-9}$ [31], one gets a bound $\lambda n \lambda_3 \lesssim 10^{-4}$ which is to be compared with the limit on $\lambda_3$ obtained from the absolute neutrino mass scale — see equation (16). This later constraint seems more important, even though it depends on extra parameters such as $h_E$ and $h_{F}$.

For the PPF-S model (see figure 8) the estimate of the mixing angle is simply $\theta_{VW} \sim g_L^2 (\Delta^0) (H_3^0)/n^2$. As discussed above, the triplet vev must be tiny due to the upper limit on the neutrino masses and the simultaneous need to obtain the correct charged lepton masses. Thus, from a non-zero $\langle \Delta^0 \rangle$ we get $\theta_{VW} \lesssim 10^{-13}$ in the PPF-S model and long-range contributions to $\theta/\beta$ decay due to this vev are completely negligible, contrary to the claims made in [19, 20].

Finally, we would like to briefly comment that $V^\pm - W^\pm$ mixing can easily be made much larger in non-minimal models. For example, let us consider adding a $10$-plet of $SU(3)_L$ to the PPF model. Under the SM group this field decomposes as:

$$\begin{align*}
(1, 10, 0) &\rightarrow (1, 4, -\frac{3}{2}) + (1, 3, 0) + (1, 2, \frac{3}{2}) + (1, 1, 3) = K + \Omega + \cdots.
\end{align*}$$

Both $K$ and the $\Omega$ contain neutral components, which may acquire vevs; $\langle K^0 \rangle$ and $\langle \Omega^0 \rangle$. Non-zero values for $\langle K^0 \rangle$ and $\langle \Omega^0 \rangle$ generate $V^\pm - W^\pm$ mixing, as indicated in figure 8.

Before estimating the constraints on $V^\pm - W^\pm$ mixing, let us briefly comment on the $\rho$ parameter. Including the $10$-plet, in the limit $n \gg v_{SM}$, the $W^\pm$ and $Z^0$ masses are given by

$$\begin{align*}
m_{W^\pm}^2 &= \frac{1}{2} g_L^2 \left( \langle \phi_1^0 \rangle^2 + \langle \phi_3^0 \rangle^2 + 3 \langle K^0 \rangle^2 + 4 \langle \Omega^0 \rangle^2 \right),
m_{Z^0}^2 &= \frac{1}{2} \left( g_L^2 + g_Y^2 \right) \left( \langle \phi_1^0 \rangle^2 + \langle \phi_3^0 \rangle^2 + 9 \langle K^0 \rangle^2 \right).
\end{align*}$$

Taking the value $\rho = 1.00040 \pm 0.00024$ from [51], these formulas imply that one has the $2\sigma$ limits $\langle K^0 \rangle \leq 1.3$ GeV for $\langle \Omega^0 \rangle \equiv 0$, and $\langle \Omega^0 \rangle \leq 2.9$ GeV for $\langle K^0 \rangle \equiv 0$. Note that there is a special direction, $\langle \Omega^0 \rangle = 3/2\langle K^0 \rangle$, for which no constraint on the vevs can be derived from $\rho$.

Absence of $\nu/\beta$ decay puts an upper bound on $\theta_{VW}$ also in this setup. From $C_3^L = 1.9 \times 10^{-9}$ [31] one can estimate $\langle \Omega^0 \rangle \langle K^0 \rangle \lesssim 0.0019$ GeV$^2$ for $n = 1$ TeV. This constraint is much more stringent than the one derived from the $\rho$ parameter.

Before closing this section, we will also discuss possible short-range contributions to $\nu/\beta$ decay in 331 models, which are shown in figure 9. Consider first the diagram to the left: it exists in any variant of the PPF model in which $V^+$ and $W^+$ mix. One can estimate the constraint on the mixing angle $\theta_{VW}$ imposed by this diagram:

$$\left( \frac{g_1^L}{8 m_W^2} \frac{p_F}{m_{W^+}} \theta_{VW} \right) / \left( \frac{G_3^L}{2m_F} \right) = C_3^{LL} \lesssim 1.1 \times 10^{-8},$$

Figure 8: Different realizations of gauge boson mixing in PPF 331 models. From left to right: PPF-E, PPF-S, and a PPF variant with a scalar field transforming as a $10$ under $SU(3)_L$. This decuplet decomposes into a quadruplet $K$, a triplet $\Omega$, and other representations of $SU(2)_L$. 

\[ \Phi \to V^\pm, W^\pm, K^0, \Omega^0 \]
where the numerical constraint is taken from [32] using the limit from $^{136}$Xe [2]. Taking $m_{U^{++}} = 1$ TeV, this results in $\theta_{VW} \lesssim 1.6 \times 10^{-2}$, several orders of magnitude weaker than the constraint derived from the long-range diagram. This result is not surprising, since the short-range diagram suffers from two suppression factors: (a) as mentioned already in the introduction, short-range matrix elements are smaller than long-range ones; and (b) relative to the long-range part there is an additional suppression factor of $p_F m_W / m_{U^{++}}$. Similar arguments also render unimportant those diagrams in which the $U^{++}$ is replaced by the doubly-charged scalar $\tilde{\phi}^{++}$ [20]. (In this case, the suppression factor $p_F m_W / m_{U^{++}}$ has to be replaced by a factor of roughly $m_e / m_{\tilde{\phi}^{++}}$, leading to the same conclusions.)

More important short-range contributions can exist in extended 331 models; the diagram on the right of figure 9 is one such example. The idea behind this extension is to generate lepton number violation via a Yukawa coupling of $\phi_3$ to an exotic vector-like quark $X$. In this case $X = (\mathbf{3}, \mathbf{2}, 1 - 5/6)$ is an exotic vector-like doublet. Its component with electric charge $-1/3$ can mix with the down-type quarks of the standard model, as indicated in the figure. At the $SU(3)_L$ level the $X$ can be embedded into the representation $\tilde{X} = (\mathbf{3}, \mathbf{3}, 1/3)$, in which case the following new Lagrangian terms are allowed by the symmetries of the model:

$$\mathcal{L}_X = Y_{QX} Q_{1,2} \tilde{X} \phi_3^* + Y_{dX} \tilde{X} c^d \phi_3^* + m_X \tilde{X} c \tilde{X} + \text{h.c.},$$

(29)

assuming that the vector partner of $\tilde{X}$, $\tilde{X} c$, is also present. This construction makes it possible to generate the $d = 11$ operator $O_{54} = QQ\bar{d}^cL\pi^cHH$. The following is a rough estimate of the size of this diagram:

$$\left(\frac{g_{L}^4}{8 m_W m_{U^{++}} m_{\phi_3} m_X} Y_{QX} Y_{dX}\right) / \left(\frac{G_F^2}{2 m_P}\right) = C_{54}^{LL} \lesssim 9.5 \times 10^{-9},$$

(30)

where the numerical constraint is taken again from [32]. For $m_{U^{++}} = m_{\phi_3} = m_X \equiv \Lambda$ we estimate $\Lambda \gtrsim 2.7(Y_{QX} Y_{dX})^{1/5}$ TeV. Note that this limit is of the order of what is expected for a $d = 9$ operator, despite being a $d = 11$ formally. This can be understood easily, since the diagram contains an ordinary $W_L$ boson.

### 3.4 Models based on the $SU(4) \times SU(2)_L \times SU(2)_R$ group

The exotic vector representation $T \equiv (\mathbf{3}, \mathbf{1}, \mathbf{2})$ was earlier found, in section 2, to be potentially relevant for neutrinoless double beta decay. And it turns out that this field is contained in the adjoint representation of the $SU(4) \times SU(2)_L$ group. However, realistic models incorporating the Standard Model fermions require an extra $U(1)_R$, which can be seen as a remnant subgroup of an $SU(2)_R$ (the full group then becomes the
so-called Pati-Salam group). We shall consider here this last scenario where on top of the $T$ vector field, the $W_R$ gauge bosons are also present. The discussion will be brief for two reasons: (a) the phenomenology is very similar to the one of the left-right symmetric model discussed above, and (b) compared to other lower limits on the Pati-Salam scale, $0\nu\beta\beta$ decay provides very weak constraints.

The main features of models based on the $SU(4) \times SU(2)_L \times SU(2)_R$ group are the following. Left-handed quarks and leptons form a $(4, 2, 1)$ representation, while the standard model right-handed fermions are part of a $(4, 1, 2)$. Note that this last representation also contains right-handed neutrinos $(\nu_R)^c$. Breaking of the Pati-Salam group to the SM one can be done either directly or via an intermediate $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry group. In the latter case, the discussion of $0\nu\beta\beta$ decay becomes very similar to the one presented in section 3.2. In particular, lepton number violation is generated by the spontaneous breaking of $SU(2)_R \times U(1)_{B-L}$, which we assume is achieved with a non-zero vacuum expectation value of a $\Delta_R$ scalar field.

Figure 10: Some neutrinoless double beta decay diagrams involving vector fields which are present in models based on the Pati-Salam group. The diagram on top shows a long-range contribution, while the two on the bottom illustrate short-range contributions.

Figure 10 shows $0\nu\beta\beta$ decay diagrams containing the vector $T$. A comparison with figure 5 reveals that these diagrams are obtained by replacing the $W_R$’s with $T$’s, with the matching replacements of lepton and quark fields. (Of course, the coupling constant $g_R$ has to be replaced by $g_4$ as well.) In all cases, the source of lepton number violation is the Majorana mass term for $(\nu_R)^c$. The diagram at the top of the figure is long-range and it is proportional to $\frac{g_R^2 g_4^2}{m_{W_R}^2 m_N^2}$ and/or $\frac{g_4^4}{m_N^4}$. As in section 3.2, the numerical limits will again be of the order of $1$ – $2$ TeV.

Since the gauge bosons couple to all fermion generations with the same strength in the Pati-Salam model, very stringent lower limits on the scale of the Pati-Salam group breaking can be derived from lepton flavor violating meson decays. Naive limits are of the order of several 100’s of TeV, but fermion mixing makes it possible to suppress certain decays, resulting in much reduced bounds on the Pati-Salam scale. Note that, because leptons and quarks are members of the same multiplet of the Pati-Salam group, besides the CKM and PMNS matrices which regulate interactions of the $W^\pm$ boson with fermions, there are also analogous matrices describing fermion mixing under $T$ interactions. However, as discussed in [52], not all meson decays can be simultaneously suppressed by mixing, hence an unavoidable bound of $m_T \gtrsim 38$ TeV can be derived, which is significantly more stringent than the bounds from $0\nu\beta\beta$ decay.
4 Conclusions

We have systematically studied contributions of vector fields to neutrinoless double beta decay. First, we have identified all possible exotic vector representations which can participate in $d = 9$ and $d = 11 \, 0\nu\beta\beta$ decay operators at tree-level. It turns out that there are 46 possibilities. Then, in a second step, we have searched for the minimal gauge groups for which the vectors in our list are contained in the adjoint representation of these extended gauge groups. Nevertheless, such a search does not take into account two important facts. Firstly, some vectors cause proton decay and thus require a very high scale of symmetry breaking, which in turn suppresses the $0\nu\beta\beta$ decay rate to negligible values. But more importantly, most of the relevant vector representations require non-standard embeddings of the standard model gauge group, making it impossible to construct viable models with the correct fermion spectrum.

The phenomenology associated to the few remaining vectors and groups was then discussed in more detail. The valid groups are the left-right symmetric group, the $SU(3)_C \times SU(3)_L \times U(1)_X$ (331) group and the Pati-Salam group. The first one has been known to give potentially important contributions to $0\nu\beta\beta$ decay for a long time, so we only briefly discussed this case. For 331-based models, we have entered in some more detail, discussing possible long- and short-range contributions to $0\nu\beta\beta$ decay. We have placed a strong emphasis on the identification of lepton number (and its violation) in these models, in order to be sure that the correct interpretation of $0\nu\beta\beta$ decay bounds was found. In the case of the Pati-Salam group, we identified some new contributions to $0\nu\beta\beta$ decay which are expected to be sub-dominant. In other words, other constraints on the scale of the Pati-Salam group are much more stringent than the ones derived from $0\nu\beta\beta$ decay. Finally we remarked that constraints on the mass and interactions of any vector in our list which is not a gauge boson can, of course, also be derived from $0\nu\beta\beta$ decay.

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