Three regimes of QCD.

L. Ya. Glozman

Institut für Physik, FB Theoretische Physik, Universität Graz

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Milestones of the quark-gluon plasma

Hagedorn, 1965 - Hadron gas has a limited temperature (Hagedorn temperature)

Cabibbo-Parisi, 1975 - Divergency of the partition function $\rightarrow$ hadronic matter to quark-gluon matter phase transition

Collins-Perry, 1975 - Asymptotic freedom predicts at high T free quarks and gluons.

Shuryak, 1978 - Debye screening at high T; a system of free quarks and gluons at high T was named the Quark-Gluon Plasma

Polyakov, 1978 - a Criterium for deconfinement was formulated

Linde, 1980 - there cannot be completely free quarks and gluons because of the infrared divergences

The latter was mostly ignored and a 40-years long hunt of the QGP phase began
Milestones of the quark-gluon plasma

RHIC, 2000 -2005 - Eliptic flow with low viscosity and jet quenching

lattice, 2000 - 2005 - Equation of state and pseudocritical temperature of the hadron gas → QGP crossover

2004 - 2005 - Declaration of a discovery of the QGP

Is it true?

Nowadays - the Polyakov loop as a criterium of deconfinment disappeared. A lattice wisdom - only the color-singlet states can propagate a macroscopical distance at any temperature.
The Dirac Lagrangian in the chiral limit

\[ \mathcal{L} = i \bar{\Psi} \gamma_\mu \partial^\mu \Psi = i \bar{\Psi}_L \gamma_\mu \partial^\mu \Psi_L + i \bar{\Psi}_R \gamma_\mu \partial^\mu \Psi_R, \]

where

\[ \psi_R = \frac{1}{2} (1 + \gamma_5) \psi, \quad \psi_L = \frac{1}{2} (1 - \gamma_5) \psi, \]

is chirally symmetric

\[ SU(N_F)_L \times SU(N_F)_R \times U(1)_A \times U(1)_V. \]

The fermion charge (Lorentz-invariant)

\[ Q = \int d^3x \bar{\Psi}(x) \gamma_0 \Psi(x) = \int d^3x \bar{\Psi}^\dagger(x) \Psi(x) \]

is invariant with respect to any unitary transformation. So far known unitary transformations are those which leave the Dirac Lagrangian invariant:

\[ SU(N_c), SU(N_F), U(N_F)_L \times U(N_F)_R \]
The $SU(2)_{CS}$ chiralspin transformations and generators:

$$\psi \rightarrow \psi' = \exp \left( i \frac{\epsilon_n \Sigma^n}{2} \right) \psi,$$

$$\Sigma^n = \{ \gamma_k, -i \gamma_5 \gamma_k, \gamma_5 \},$$

$n = 1, 2, 3$. $\gamma_k$ is any Hermitian Euclidean gamma-matrix:

$$\gamma_i \gamma_j + \gamma_j \gamma_i = 2 \delta^{ij}; \quad \gamma_5 = \gamma_1 \gamma_2 \gamma_3 \gamma_4.$$

The $su(2)$ algebra $[\Sigma^a, \Sigma^b] = 2i \epsilon^{abc} \Sigma^c$ is satisfied with any $k = 1, 2, 3, 4$.

The free massless Dirac Lagrangian does not have this symmetry. However, it is a symmetry of the fermion charge

The fermion charge has a larger symmetry than the Dirac eq.
The chiralspin transformation and generators can be presented in an equivalent form. With $k = 4$ they are

$$\Sigma^n = \{1 \otimes \sigma^1, 1 \otimes \sigma^2, 1 \otimes \sigma^3\}. $$

Here the Pauli matrices $\sigma^i$ act in the space of spinors

$$\Psi = \begin{pmatrix} R \\ L \end{pmatrix}, \quad (1)$$

where $R$ and $L$ represent the upper and lower components of the right- and left-handed Dirac bispinors.

The $SU(2)_{CS}, \ k = 4$ transformation can then be rewritten as

$$\Psi \rightarrow \Psi' = \exp \left( i \frac{\varepsilon^n \Sigma^n}{2} \right) \Psi = \exp \left( i \frac{\varepsilon^n \sigma^n}{2} \right) \begin{pmatrix} R \\ L \end{pmatrix}. \quad (2)$$

A fundamental irreducible representation of $SU(2)_{CS}$ is two-dimensional and the $SU(2)_{CS}$ transformations mix the $R$ and $L$ components of fermions.
An extension of the direct $SU(2)_{CS} \times SU(N_F)$ product leads to a $SU(2N_F)$ group. This group contains the chiral symmetry of QCD $SU(N_F)_L \times SU(N_F)_R \times U(1)_A$ as a subgroup.

Its transformations are given by

$$\Psi \rightarrow \Psi' = \exp \left( i \frac{\epsilon^m T^m}{2} \right) \Psi,$$

where $m = 1, 2, \ldots, (2N_F)^2 - 1$ and the set of $(2N_F)^2 - 1$ generators is

$$\{(\tau^a \otimes 1_D), (1_F \otimes \Sigma^n), (\tau^a \otimes \Sigma^n)\} \quad (3)$$

whith $\tau$ being the flavour generators with the flavour index $a$ and $n = 1, 2, 3$ is the $SU(2)_{CS}$ index.

$SU(2N_F)$ is also a symmetry of the fermion charge, while not a symmetry of the Dirac eq.
Symmetries of the QCD action

Interaction of quarks with the gluon field in Minkowski space-time:

$$\bar{\Psi} \gamma^\mu D_\mu \Psi = \bar{\Psi} \gamma^0 D_0 \Psi + \bar{\Psi} \gamma^i D_i \Psi.$$ 

The temporal term includes the interaction of the color-octet charge density

$$\bar{\Psi}(x) \gamma^0 \frac{t^c}{2} \Psi(x) = \Psi(x)^\dagger \frac{t^c}{2} \Psi(x)$$

with the chromo-electric part of the gluonic field. It is invariant under $SU(2)_{CS}$ and $SU(2N_F)$.

The spatial part contains the quark kinetic term and the interaction with the chromo-magnetic field. It breaks $SU(2)_{CS}$ and $SU(2N_F)$.

The quark chemical potential term $\mu \Psi(x)^\dagger \Psi(x)$

$$S = \int_0^\beta d\tau \int d^3x \bar{\Psi} \left[ \gamma_\mu D_\mu + \mu \gamma_4 + m \right] \Psi,$$

is $SU(2)_{CS}$ and $SU(2N_F)$ invariant.
Low mode truncation

Banks-Casher:

\[ < \bar{q}q > = -\pi \rho(0). \]

What we do (initiated by C.B. Lang, M. Schroeck, 2011):

\[ S = S_{\text{Full}} - \sum_{i=1}^{k} \frac{1}{\lambda_i} |\lambda_i\rangle\langle\lambda_i|. \]

Chiral symmetry expectations for \( J = 1 \) mesons:

| (0, 0)         | \( f_1(0,1++) \) | \( \omega(0,1^-) \) |
|----------------|------------------|-------------------|
| (1/2, 1/2)\text{a} | \( b_1(1,1^-) \) | \( \omega(0,1^-) \) |
| \( U(1)\text{A} \) | \( \Psi(\tau^a \otimes \gamma^0 \gamma^k)\Psi \) | \( \Psi(1_F \otimes \gamma^0 \gamma^k)\Psi \) |
| (1/2, 1/2)\text{b} | \( \rho(1,1^-) \) | \( h_1(0,1^+) \) |
| \( \Psi(\tau^a \otimes \gamma^0 \gamma^k)\Psi \) | \( \Psi(1_F \otimes \gamma^5 \gamma^0 \gamma^k)\Psi \) |
| (1, 0) \oplus (0, 1) | \( \rho(1,1^-) \) | \( a_1(1,1^+) \) |
| \( \Psi(\tau^a \otimes \gamma^k)\Psi \) | \( \Psi(\tau^a \otimes \gamma^5 \gamma^k)\Psi \) |
We use $N_f = 2$ JLQCD overlap gauge configurations.

We clearly see a larger degeneracy than the $SU(2)_L \times SU(2)_R \times U(1)_A$ symmetry of the QCD Lagrangian.
Approximate $SU(2)_{CS}$ and $SU(4)$ symmetries at high $T$.

Three regimes of QCD.
Conclusions to part I and prediction for high T

Observed on the lattice $SU(2)_{CS}$ and $SU(2N_F)$ symmetries of hadrons upon elimination of the near-zero modes are symmetries of electric confinement in QCD.

The chromo-magnetic interaction in QCD contributes only to the near-zero modes. It breaks explicitly both $SU(2)_{CS}$ and $SU(2N_F)$ symmetries of confinement.

The hadron spectra observed in the real world can be viewed as a result of a splitting of the primary energy levels with the $SU(4)$ symmetry by means of dynamics associated with the near-zero modes that includes magnetic effects.

Above $T_c$ $SU(2)_L \times SU(2)_R$ gets restored and the near-zero modes of the Dirac operator are suppressed. Then we expect emergence of $SU(2)_{CS}$ and $SU(4)$ - no deconfinement. L.Ya.G. 1610.00275
Spatial correlators at high T. Full QCD, no truncation.

C. Rohrhofer, Y. Aoki, G. Cossu, H. Fukaya, L.Ya.G., S. Hashimoto, C. B. Lang, S. Prelovsek, PRD 96 (2017) 094501

C. Rohrhofer, Y. Aoki, G. Cossu, H. Fukaya, C. Gattringer, L.Ya.G., S. Hashimoto, C. B. Lang, S. Prelovsek, PRD 100 (2019) 014502

$N_f=2$ QCD with the chirally symmetric Dirac operator.

$$C_{\Gamma}(n_z) = \sum_{n_x, n_y, n_t} \langle O_{\Gamma}(n_x, n_y, n_z, n_t) O_{\Gamma}(0, 0)^\dagger \rangle.$$  

where $O_{\Gamma}(x) = \bar{q}(x) \Gamma \frac{T}{2} q(x)$ are all possible $J=0$ and $J=1$ local operators:

| Name               | Dirac Structure | Abbreviation | Abbreviation |
|-------------------|-----------------|--------------|--------------|
| Pseudoscalar      | $\gamma^5$     | $PS$         | $U(1)_A$     |
| Scalar            | $1$             | $S$          |              |
| Axial-vector      | $\gamma_k \gamma^5$ | $A$         | $SU(2)_A$    |
| Vector            | $\gamma_k$      | $V$          |              |
| Tensor-vector     | $\gamma_k \gamma_3$ | $T$         | $U(1)_A$     |
| Axial-tensor-vector | $\gamma_k \gamma_3 \gamma^5$ | $X$         |              |

Three regimes of QCD.
Spatial correlators at high $T$. Full QCD, no truncation.

\[(A_x, T_t, X_t) - SU(2)_{CS}; \quad (V_x, A_x, T_t, X_t) - SU(4)\]
Spatial correlators at high T. Full QCD, no truncation.

\[ \kappa = \frac{|C_{Ax} - C_{Tt}|}{|C_{Ax} - C_S|}. \]

If \( \kappa \ll 1 \), then we can declare an approximate or — if zero — an exact symmetry. If \( \kappa \sim 1 \), the symmetry is absent.
Time correlators → spectral density

C. Rohrhofer, Y. Aoki, L.Ya.G., S. Hashimoto, 1.2 \( T_c \)

Chiral-Parity group for spin-1 mesons

\[
(0,0) \\
(1/2,1/2)\_a \\
(1/2,1/2)\_b \\
(1,0) \oplus (0,1)
\]

\[
U(1)\_A \\
SU(2)\_A \\
SU(2)\_B
\]

Assume \( SU(2)_{\text{CS}} \) and \( SU(4) \) symmetries at high \( T \).

Conclusions to part I

Approximate \( SU(2)_{\text{CS}} \) and \( SU(4) \) symmetries at high \( T \).

Conclusions to part II

Three regimes of QCD.

C. Rohrhofer, Y. Aoki, L.Ya.G., S. Hashimoto, 1.2 \( T_c \)
Three regimes of QCD

- **Quark-gluon plasma**
- **“Stringy fluid”** $SU(2)_{cs}$
- **Hadron gas** $\langle \bar{q}q \rangle \neq 0$
Will a non-zero chemical potential break this symmetry?

\[ S = \int_0^\beta d\tau \int d^3x \overline{\Psi} [\gamma_\mu D_\mu + \mu \gamma_4 + m] \Psi, \]  

(4)

The quark chemical potential term \( \mu \overline{\psi}^\dagger \psi \) is invariant under \( SU(2)_{CS} \) and \( SU(2N_F) \).

L.Ya.G., EPJA 54 (2018) 117
We observe the emergence of the approximate $SU(2)_{CS}$ and $SU(4)$ symmetries with increasing temperature.

The emergence of $SU(2)_{CS}$ and $SU(4)$ symmetries indicates that the chromo-magnetic interaction is suppressed while the confining chromo-electric interaction is still active.

These symmetries are incompatible with the scenario of a plasma of asymptotically free, deconfined quarks and gluons.

Elementary objects: chiral quarks connected by chromo-electric field: ”Strings”

Three regimes of QCD: $0 - T_c$ - Hadron Gas; $T_c - 3T_c$ - Stringy Fluid; $T > 5T_c$ - QGP