Matrix String Models for Exact (2,2) String Theories in R-R Backgrounds

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Abstract: We formulate matrix string models on a class of exact string backgrounds with non constant RR-flux parameterized by a holomorphic prepotential function and with manifest (2,2) supersymmetry. This lifts these string theories to M-theory exposing the non perturbative string interaction which is studied by generalizing the instanton asymptotic expansion, well established in the flat background case, to this more general case. We obtain also a companion matrix model with four manifest supersymmetries in eleven dimensions.

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1 Introduction

The quantization of string theories in curved backgrounds and the study of their finiteness properties is still an open important problem. It appeared recently an interesting class of $(2,2)$ pp-wave solutions of type IIB [1] generalizing the original one studied in [2]. These string theories have been shown to admit a superconformal formulation in [3] and have been shown to be exact (finite) in [3, 4].

The corresponding $(2,2)$ $\sigma$-model has been studied in the $SU(4) \times U(1)$ formalism [5]. Within this framework, the type IIB GS action on a flat background is written in terms of four complex chiral $(2,2)$ superfields $X^+ l$, where $l = 1, \ldots, 4$, as

$$S_0 = \int d^2 z d^4 \theta X^+ l X^l$$

where the only part of the original $SO(8)$ symmetry which is left manifest is in fact a $SU(4) \times U(1)$ one. The action relative to the pp-wave backgrounds is written as the $\sigma$-model action

$$S^{IIB}_{pp} = \int d^2 z \left\{ \int d^4 \theta X^+ l X^l + \int d^2 \theta W(X^+ l) + \text{c.c.} \right\}$$

where $W$ is the holomorphic prepotential. The pp-wave metric and the RR-field curvature are parameterized by this holomorphic function. Specifically the ten dimensional metric reads

$$ds^2_{(10)} = -2 dx^+ dx^- - |\partial W|^2 (dx^+)^2 + 2 dx^l d\bar{x}^l$$

and the RR-fields $F^{(5)} \sim \partial^2 W$. In [3] these models have been shown to admit an exact superconformal formulation (by a suitable reconstruction of the light-cone degrees of freedom) and to be therefore exact string solutions. Notice that this happens for no-renormalization arguments of the central charge for $(2,2)$ linear $\sigma$-models under an arbitrary shift in the holomorphic prepotential $W$. An interesting aspect of the exact superconformal formulation is that upon a change of variables the R-NS formulation can be obtained where the string interaction is well defined, being the background non dilatonic, as the usual genus expansion. Notice that, if $W(x) = \mu \sum_i x_i^2$, then we obtain the maximal supersymmetric pp-wave with manifest $SO(4)$ symmetry and constant RR-flux $F^{(5)} \sim \mu$.

The action is given by

$$S^{IIB}_{pp} = \frac{1}{\pi} \int d^2 z \left\{ \partial_+ x^l \partial_- \bar{x}^l + \partial_+ \bar{x}^l \partial_- x^l - |\partial W|^2 + \text{fermions} \right\}$$

(1.1)

2 In principle one might consider more general $\sigma$-models by generalizing the kinetic term $X^+ l X^l \rightarrow K(X^+ l, X^- l)$, where $K$ is the Kahler potential. In this letter we will not consider such extensions.
We see immediately that in the directions in which the pp-wave effectively extends ($\partial_l W \neq 0$), the string coordinate dynamics is that of interacting scalar fields. These have in general a very much different spectrum with respect to the coordinates propagating in the directions where the pp-wave is not effectively extended ($\partial_l W = 0$) which are free massless bosons. Already in case of constant RR-flux pp-wave ($W$ quadratic) we find a free massive field theory governing the relevant string coordinates dynamics. This shows that in general the string spectrum is different in the pp-wave case versus the flat case and that in particular the zero mode spectrum will dramatically change. This can also be shown by studying the supersymmetric classical states of the theory. For the flat case, as it is well known, these are left/right handed propagating waves which are free to locate the string anywhere because of the existence of the translational zero-mode. In general, when a non zero prepotential is switched on, this zero modes tends to disappear\(^3\) and the string coordinate classically oscillates around the minima of $|\partial W|^2$. For several choices of the holomorphic prepotential $W$, there exists also solitonic solutions which carry topological charges enriching the string spectrum. Also in these cases, the string coordinate elongates around different minima of the potential $|\partial W|^2$ (periodic solitons). For this kind of backgrounds there are still some open issues because of the absence of asymptotic particle states, being the metric non asymptotically flat, and because of the irreducibility of solitonic profile strings to punctures.

There is actually a set of similar theories also for type IIA strings. These can be obtained as subcases of the ones we reviewed above if a T-duality can be performed along a spacelike isometric direction. Because of $(2,2)$ manifest supersymmetry, this has to be a complex direction. Therefore, to have it manifest, we have to split the four chiral superfields $X^{+l}$ as $\tau$ and $\phi^i$, where $i = 1, 2, 3$, and consider only prepotentials independent on $\tau$, i.e. $\partial_\tau W = 0$ that is $W = W(\Phi^i)$. This is essentially the same condition of existence of at least one transverse spectator flat direction considered in \cite{4}. Notice that this choice rules out \(^4\) the maximal supersymmetric case $W = \mu \sum_i x_i^2$.

Now we are allowed to perform a T-duality transformation (along one of the flat directions corresponding to $\tau$). The effect of this T-duality is to transform the chiral superfield $\tau$ in a

\(^3\)It is a very well known fact that massive and massless quantum field theories are inequivalent – i.e. the relative realizations of the CCRs are mutually irreducible – exactly for this reason.

\(^4\)In the superconformal formulation \cite{3} of this class of theories the light-cone coordinate $X^+$ is not a fundamental field, but it is obtained by a partial on shell procedure. This enables one to perform straightforwardly changes of variables involving light-cone directions to try to render manifest other spacelike isometric directions.
twisted chiral one that we call Σ.

The action for type IIA is then given by

$$S_{\text{IIA}}^{\text{pp}} = \int d^2 z \left\{ \int d^4 \theta \left\{ -\frac{1}{4} \Sigma \bar{\Sigma} + \phi^i \bar{\phi}^i \right\} + \int d^2 \theta W(\phi^i) + \text{c.c.} \right\} \quad (1.2)$$

Notice that, in particular, if $W = 0$, this action reproduces the type IIA GS action on a flat background.

In principle we could add by hand to (1.2) also a twisted prepotential term

$$\int d^2 z d\theta^+ d\bar{\theta}^- U(\Sigma)|_{\theta^-=\theta^+=0} + \text{h.c.}$$

for an arbitrary holomorphic function $U$ in the twisted chiral field. This corresponds to switch on an additional background RR 2-form flux along the directions $+$ and one within the complex directions spanned by the scalar components of $\Sigma$. Allowing such couplings one can easily reproduce, by choosing quadratic prepotentials, a whole set of manifest (2,2) type IIA strings on pp-wave backgrounds with constant RR-fluxes. We will not study these additional couplings here and just comment about their effect in the concluding section.

At this point the possibility of lifting to M-theory these backgrounds can be posed clearly. Matrix String Theory (MST) links perturbative string theory and M(atrix)-theory by explaining how the theory of strings is included in the latter. In particular, this realizes type IIA string theory on flat background as the superconformal fixed point of the (8,8) Super-Yang-Mills theory with gauge group U(N) at the strong Yang-Mills coupling limit and in the large $N$ regime. The interacting structure of perturbative string theory, namely the genus expansion, is recovered as an asymptotic expansion of the gauge theory partition sum around the conformal fixed point in the inverse gauge coupling which is then, accordingly with S-duality, interpreted as the string coupling. This asymptotic expansion is concretely built as a WKB expansion around certain supersymmetric instanton configurations whose spectral data encode the relative string Mandelstam diagrams. Notwithstanding its not competing power in evaluating perturbative string amplitudes (although it solves positively the old problem with genus proliferation in certain high energy regimes of string amplitudes and enables the exact evaluation of protected couplings and supersymmetric indices allowing interesting duality checks), since it models string interactions non-perturbatively, MST has a strong conceptual importance. It is therefore crucial, once some string background is studied,

\[5\text{See [8] for details. For definitions, properties and notation here and in the subsequent part of this letter, we refer the reader to [9].}\]
to obtain its Matrix String Theory counterpart to properly embed that perturbative string theory in M-theory. This has been done for pp-wave backgrounds with constant R-R flux in [12] and in [13]. Here we will discuss this issue for a wide set of (2,2) backgrounds with non constant R-R flux.

To this end, we will first formulate Matrix String Theory on flat backgrounds in terms of (2,2) superfields in order to reach a comfortable homogeneous framework. Then we will show how MST generalizes to the description of the above pp-wave backgrounds. We study the quantum stability of the strong coupling limit and sketch an asymptotic expansion of the partition sum in such a regime. As a byproduct of our analysis we obtain also a novel class of matrix models on eleven dimensional pp-wave backgrounds with non constant 4-form flux and four manifest supersymmetries. Comments and open questions are contained in the last section.

2 Matrix String Theory in (2,2) superfields

Type IIA Matrix String Theory [8,10] can be obtained by reducing the $\mathcal{N} = 1$ D=10 SYM theory with gauge group $U(N)$ down to two dimensions [14]. In order to write down the MST action in the (2,2) superfield formalism it is more useful to perform an intermediate dimensional reduction to $\mathcal{N} = 4$ D=4 SYM which can be written in $\mathcal{N} = 1$ superfields. In these terms its spectrum is given by a vector multiplet $\mathcal{A}$ and three chiral hypermultiplets $\Xi^i$ in the adjoint representation of the $U(N)$ gauge group. The only additional datum which specifies the action is the prepotential $L = g \text{Tr} \Xi^1 [\Xi^2, \Xi^3]$, where $g$ is the gauge theory coupling constant.

Under dimensional reduction $\mathcal{N} = 1$ susy in D=4 reduces to (2,2) susy in D=2 and in particular we have the following reduction map

$$\begin{align*}
\text{Vector} [\mathcal{A}] & \rightarrow \text{Twisted Chiral} [\Sigma] \\
\text{Chiral} [\Xi^i] & \rightarrow \text{Chiral} [\Phi^i]
\end{align*}$$

By the above map we obtain the MST action in (2,2) superfields as

$$\int d^2z d^4\theta \left( -\frac{1}{4g^2} \Sigma \bar{\Sigma} + \Phi^i \bar{\Phi}^i \right) + \int d^2z \left[ d^2 \theta L(\Phi^i) + c.c. \right] \quad (2.3)$$

with the prepotential $L = g \text{Tr} \Phi^1 [\Phi^2, \Phi^3]$. Obviously, the three chiral superfields $\Phi^i$ are still in the adjoint representation of the gauge group $U(N)$ and the trace over the gauge indices is understood.
Notice that in the $(2,2)$ superfields formalism only a $U(1) \times SU(3)$ subgroup of the original $SO(8)$ R-symmetry is manifest (exactly as it was in the previous section for the action (1.2) in the flat case.).

The strong gauge coupling expansion of MST can be performed in this formalism directly only for the chiral superfield degrees of freedom. The chiral superfields extremal values are dictated by the vanishing of the first derivative of the prepotential $\partial_{\Phi^i} L = \frac{1}{2} \epsilon_{ijk} [\Phi^j, \Phi^k] = 0$. This selects them to lay on a common Cartan subalgebra $t$. As far as $\Sigma$ is concerned, one has to enter its elementary fields content to find the analogous commutators (see [7] to check explicit formulas) which select the twisted chiral superfield to lay on the same Cartan subalgebra $t$.

The strong coupling limit action is then, after the rescaling of the gauge field to reabsorb the gauge coupling in the covariant derivatives,\footnote{The quantum exactness of this limiting procedure is guaranteed by supersymmetry at one loop where the non-Cartan sector is effectively integrated out. For more details see [10] and next section.}

$$S = \int d^2 z \int d^4 \theta \left\{ -\frac{1}{4} \Sigma^t \bar{\Sigma}^t + \Phi^t \bar{\Phi}^t \right\}$$

where the suffix $t$ projects the fields on the Cartan component along $t$. This is the MST version of (1.2) on a flat background i.e. the $S_N$ symmetric superposition of $N$ copies of the type IIA tree level string theory.

### 3 MST on $(2,2)$ pp-wave geometries

The point we are addressing in this letter is an extension of MST, as given in the previous section, to the exact $(2,2)$ pp-wave geometries that we review in the introduction.

The natural model to study is given by a gauging of the type IIA action (1.2) as

$$S = \int d^2 z \left\{ \int d^4 \theta \left\{ -\frac{1}{4g^2} \Sigma \Sigma + \Phi^i \bar{\Phi}^i \right\} + \int d^2 \theta \left( L(\Phi^i) + \bar{W}(\Phi^i) \right) + c.c. \right\}$$

(3.4)

where now the superfields are relative to the $U(N)$ gauge theory (we keep the notation of the previous section where the trace over the gauge group indices is understood). $\Sigma$ and $\Phi^i$ are the same $U(N)$ superfields we considered in the previous section for the flat case. As far as the definitions of the matrix function $\bar{W}$ the natural requirement is that once evaluated on Cartan fields it reproduces the prepotential in (1.2) as $\text{Tr} \bar{W}(\Phi^i) = \sum_m W(\Phi^i_m)$. in an
orthonormal basis of $t$. This requirement specifies these structure function to be given by $W(\Phi)$ up to the prescription of the relevant matrix ordering.

The natural ordering is of course the total symmetrization. In the related context of D-geometry this problem has been clarified [13] by showing that if the background satisfies the string equations, then the total symmetric ordering is the correct one in order to reproduce the correct open string masses assignments. Since the string backgrounds we are considering are exact, TS-duality with the type IIB D-string picture justifies this ordering. Notice moreover that these D-geometry arguments have been already applied in a similar context [16] for matrix string theory on Kahler backgrounds. In principle one could also consider more general models by allowing a non quadratic Kahler potential in (3.4). We consider the above arguments enough to justify the total symmetric ordering.

For completeness, we give the bosonic part of the action (3.4) that is

$$S_b = \frac{1}{2} \int d^2z \left\{ D_z \phi^i D_{\bar{z}} \bar{\phi}^\bar{} + D_{\bar{z}} \phi^i D_z \bar{\phi}^\bar{} + D_z \sigma D_{\bar{z}} \bar{\sigma} + D_{\bar{z}} \sigma D_z \bar{\sigma} + \bar{F}_i F^i + \frac{1}{2g^2} |F_{z\bar{z}}|^2 - \frac{g^2}{2} [\sigma, \bar{\sigma}]^2 - \frac{g^2}{2} [\phi^i, \bar{\phi}^\bar{i}]^2 + g^2 [\sigma, \phi^i][\bar{\sigma}, \bar{\phi}^\bar{i}] + g^2 [\phi^i, \bar{\phi}^\bar{i}][\sigma, \bar{\phi}^\bar{i}] + g^2 [\sigma, \phi^i][\bar{\sigma}, \bar{\phi}^\bar{i}] \right\}$$

where the $F$-term is fixed to $F^i = \frac{\partial (W + L)}{\partial \Phi^i}(\phi)$.

Notice that the model we are considering is the dimensional reduction from four to two dimensions of a generalized Leigh-Strassler deformation [17] of $\mathcal{N} = 4$, D=4 super Yang-Mills.

### 3.1 The strong gauge coupling limit

We assume that the moduli in the prepotential (namely the pp-wave mass) are finite with respect to the rescaling gauge coupling constant. This means that, as far as the strong gauge coupling analysis of the classical potential, we can ignore the subleading effects due to the presence of a non null $W$. Therefore the strong gauge coupling limit still implies that in that regime only Cartan valued fields survives, the complement being suppressed.

Therefore the strong gauge coupling limit of (3.4) is the expected $S_N$ symmetric superposition of copies of [12], that is

$$S^\infty = \int d^2z \left\{ \int d^4\theta \left\{ -\frac{1}{4} \Sigma^i \Sigma^i + \phi^i \bar{\phi}^{\bar{i}} \right\} + \int d^2\theta W(\phi^i) + c.c. \right\} \quad (3.5)$$

7See the conclusions for further arguments related to the preservation of the background symmetries.
where we denote by the index $t$ the Cartan component. From a full quantum field theory viewpoint, the above discussion has to be implemented by performing the path integration over the non Cartan direction fields (as in [10]). The strong coupling expansion of the gauged $\sigma$-model action reads

$$S = S^\infty + \int d^2 z \left\{ \int d^4 \theta \left\{ -\frac{1}{4} \Sigma^n \Sigma^n + \Phi^n i \bar{\Phi}^n i \right\} + \int d^2 \theta \Phi^n [\Phi^t, \Phi^n] + c.c. \right\} + O\left( \frac{1}{\sqrt{g}} \right)$$

where the superscript $n$ indicates the projection on the complement of the Cartan algebra $t$ in the matrix field space and the proper superfields dressing with $e^{iV^t} [ ] e^{-iV^t}$, where $V^t$ is the Cartan component of the full vector superfield $V$ from which $\Sigma$ is built as the superfield curvature. This leads, at lowest order in the inverse gauge coupling, to a supersymmetric (trivial) Gaussian path integral, fully justifying (3.5) in the quantum theory.

The MST interpretation and the world-sheet reconstruction from string bits apply then to our model exactly as in the flat case.

Notice that we assumed that the background moduli do not interfere with the strong gauge coupling limit. It would be interesting to study in detail how the above picture changes if some pp-wave mass scales with the gauge coupling. In general, it is easy to see that in a background of this type, one can freeze string oscillation along some transverse directions just by switching on an high enough mass parameters. From the MST model we gave above, string interaction do not spoil this picture, but the strong gauge coupling limit is possibly interfered by the scaling pp-wave moduli. It would be very interesting to formulate a picture for these phases of the theory.

### 3.2 String Interactions from matrix string instantons

An interesting property of the (2,2) supersymmetric models that we are studying is that the supersymmetric stringy instantons encoding the string interaction diagrams as spectral curves [10] are still manifest. This is because of the existence of the two flat directions relative to the scalars in the twisted multiplet. Due to the fact that these scalars do not enter the F-terms, it is in fact still possible to polarize matrix string diagram in those direction. The relevant instanton is modeled, as is the flat case, by the Hitchin system. We will find that if we try to polarize instanton matrix string diagrams in the directions in which the pp-wave extends (namely the $\phi^i$ directions), we fail due to a general argument which implies

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8See also [18], where a constructive method is systematically applied to the relevant Hitchin system.
the staticity of supersymmetric saturated solutions involving these fields as active. In this section we sketch a picture of the reconstruction of the genus expansion of the partition sum for type IIA strings in the geometries that we discussed so far.

We consider field configurations saturating some of the manifest supersymmetry of the model at hand. As usual we set the fermion fields to zero and study the stability of this condition under supersymmetry transformation. This results in a set of first order equations for the bosonic fields.

As far as the three chiral multiplets are concerned, we have

\[ \delta \psi^i_+ = \sqrt{2} \epsilon_+ F^i - 2 \epsilon_+ [\tilde{\sigma}, \phi^i] + i \epsilon_- \sqrt{2} (D_0 + D_1) \tilde{\phi}^i \]

\[ \delta \psi^i_- = \sqrt{2} \epsilon_- F^i + 2 \epsilon_- [\sigma, \phi^i] - i \epsilon_+ \sqrt{2} (D_0 - D_1) \phi^i \]

where \( \Phi^i = \phi^i + \theta \cdot \psi^i + \theta^2 F^i \), \( \sigma \) is the complex scalar in the twisted multiplet and the relevant \( F \)-term is \( \tilde{F}^i = \frac{\partial (\tilde{W} + L)}{\partial \phi^i} (\phi) \).

As far as the twisted multiplet is concerned, the supersymmetric variations of its fermions \( \lambda_\pm \) are

\[ \delta \lambda_+ = (i D - F_{01} - [\sigma, \tilde{\sigma}]) \epsilon_+ + \sqrt{2} (D_0 + D_1) \tilde{\sigma} \epsilon_- \]

\[ \delta \lambda_- = (i D + F_{01} + [\sigma, \tilde{\sigma}]) \epsilon_- + \sqrt{2} (D_0 - D_1) \sigma \epsilon_+ \]

(and analogous for \( \delta \tilde{\lambda}_\pm \)) where the relevant \( D \)-term is \( D = -i \sum_i [\phi^i, \tilde{\phi}^i] \).

**1/2 BPS matrix string configurations** From the equations above we see immediately that we can saturate diagonal supersymmetry (i.e. preserve lets say arbitrary \( \epsilon_\pm \)) if

\[ F_{01} + [\sigma, \tilde{\sigma}] = 0 \quad (D_0 + D_1) \tilde{\sigma} = 0 \quad (D_0 - D_1) \sigma = 0 \quad (3.6) \]

while the \( \phi^i = x_i \times 1_N \) are constant diagonal matrices whose eigenvalues satisfy the equation \( \partial_i W(x) = 0 \). Upon the relevant Wick rotation, equations (3.6) become the Hitchin system \[19\]

\[ F_{zz} + [\sigma, \tilde{\sigma}] = 0 \quad \text{and} \quad D_z \sigma = 0 \quad (3.7) \]

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\[ \text{The complete set of BPS equations include } \sum_i [\phi^i, \tilde{\phi}^i] = 0, [\sigma, \phi^i] = 0, [\tilde{\sigma}, \phi^i] = 0, D_\pm \phi^i = 0 \text{ and } \frac{\partial (\tilde{W} + L)}{\partial \phi^i} = 0. \]

Assuming \( \sigma \) truly active, i.e. assuming that its spectrum has not to be algebraically constrained, we find the condition above on \( \phi \). For disconnected matrix string configurations, this has to be true just block by block.
The spectral data classifying its solutions is given by the moduli space of plane curves in two (complex) dimensions, i.e. the light-cone and $\sigma$, of order $N$. The generic curve $S$ is defined by the $\sigma$-spectral equation

$$0 = \det(\sigma(z) - \sigma 1_N).$$

The way in which these spectral curves represent the Mandelstam diagrams is discussed in detail in [10].

1/2 BPS solitonic string configurations

We can otherwise saturate the supersymmetry as $\bar{\epsilon}_\pm = \epsilon_\pm$. This embeds in the gauged sigma model the usual static BPS solutions (solitons for appropriate choice of $W$). We find that $\sigma$ is forced to be passive (i.e. $\sigma \propto 1_N$) and the gauge connection to be flat $F_{01} = 0$ (still not trivial on the cylinder). We are left then with the equations

$$D_0 \phi^i = 0 \quad , \quad \sum_i [\bar{\phi}^i, \phi^i] = 0 \quad \text{and} \quad iD_1 \phi^i + F^i = 0 \quad (3.8)$$

Solutions to these equations are long static (solitonic) strings in the $\phi^i$ directions whose bits are modeled on the $S_N$-twisted (soliton) spectrum of the associated ungauged $\sigma$-model.

From the supersymmetric variations $\delta \lambda$ and $\delta \psi$ it is not possible to saturate supersymmetry in other ways (up to phase redefinitions of the preserved supersymmetries). Namely, it is not possible to obtain instantonic configurations with active $\phi^i$s since only static solutions saturate the Bogomolny bound formula for $F$-terms.

As in the flat case, in the strong coupling limit, the partition sum can be calculated as a WKB expansion around the Hitchin system solutions. The Cartan field content, which is left as the effective field spectrum at strong coupling, lifts [3] to the spectral curve $S$ of the relevant instanton configuration and the action $S^\infty$ can be rewritten as

$$S^\infty = \int_S d^2z \left\{ \int d^4\theta \left\{ -\frac{1}{4} \hat{\Sigma} \hat{\Sigma} + \hat{\phi}^i \hat{\phi}_i \right\} + \int d^2\theta W(\hat{\phi}^i) + c.c. \right\} \quad (3.9)$$

where we denoted by a hat the lifted fields. This is the GS type IIA string action on the matrix string worldsheet plus a decoupled $U(1)$ Maxwell field where the worldsheet metric is the Mandelstam one. The integration over the $U(1)$ gauge field (see again [10] for details) produces – because of the rescaling of fields in the calculation of the strong coupling limit – a factor of $g^{\chi_S}$, where $\chi_S$ is the Euler characteristic of $S$. This gives the correct perturbative weight to the string amplitude and identifies the genus counting parameter (i.e. the string

\[\text{There is an ambiguity regarding the lift of the fermions because of the need of choosing their spin structure. By mediating over all the possible inequivalent lifts we reproduce the spin structure sum.}\]
coupling) as $g_s = g^{-1}l_s^{-1}$. All this happens as a consequence of the $(2,2)$ supersymmetry of
the model which preserves the full structure of the gauge supermultiplet fields.

Let us notice that we assumed that the strong gauge coupling limit is not interfered by
the pp-wave mass scale $\mu$ – inserting dimensionfull parameters $W = \mu w(\phi/l_s)$, where $l_s$ is
the string scale. This translates to the condition $\mu g_s << l_s^{-1}$ which is the range of validity
of our analysis.

4 A related set of matrix models

The above model admits a lift to an eleven dimension matrix model on the lifted background
geometry. This can be obtained just by dimensional reduction of the gauged $\sigma$-model that
we considered so far to a 1+0 matrix quantum mechanics with four manifest supersymme-
tries. This matrix model corresponds to M-theory (strongly coupled type IIA) in a pp-wave
background. This eleven dimensional background can be calculated as

$$ds_{(11)}^2 = -2dx^+dx^- - |\partial_t W|^2dx^+dx^- + d\phi^i d\bar{\phi}^i + (d\sigma^a)^2$$

$$F_4 = dx^+ \wedge \omega^{(3)}$$

where $a = 1, 2, 3$. Here $\omega^{(3)}$ is the harmonic three form satisfying the supergravity equations,
the only non trivial equation being $R_{++} \propto |F_4|^2$. This amounts to

$$\left( \partial^a_t + \partial_j \partial_j \right) |\partial_t W|^2 = \partial_j \partial_j |\partial_t W|^2 \propto |\omega^{(3)}|^2$$

Which is solved by the choice $\omega^{ijk} = c\epsilon^{ijk} \partial_\phi \bar{\partial}_\phi W$, where $c$ is a normalization numerical constant, and all other components vanish but the complex conjugate. This is in fact the 11
dimensional lift of the R-R background field we considered so far. Notice the appearing of
an explicit $SO(3)$ isometry which rotates the $\{\sigma^a\}$ directions in (4.10).

The matrix model, obtained by dimensional reduction to one dimension, therefore gen-
eralizes to non constant fluxes the matrix models elaborated in [20, 12, 21]. The action can
be written in components as $S = S_b + S_f$, where

$$S_b = \int dt \left\{ (\partial_t \sigma^a)^2 + |\partial_t \phi^i|^2 - \frac{g^2}{4} \left[ \sigma^a, \sigma^b \right]^2 - \frac{g^2}{4} \left[ \sigma^a, \phi^i \right] \left[ \sigma^a, \bar{\phi}^i \right] - \frac{g^2}{2} \left[ \phi^i, \bar{\phi}^i \right]^2 + \frac{g}{2} \epsilon_{ijk} [\phi^j, \phi^k] + \bar{\partial}_t W \right\}$$

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and

\[
S_f = \int dt \left\{ \frac{i}{2} \bar{\psi} \partial_t \psi - \frac{g}{2} \bar{\psi} \Gamma^a [\sigma^a, \psi] - g \bar{\psi} \Gamma^i [\phi^i, \psi] + c.c. - c \bar{\psi} \Gamma^{ijk} \epsilon_{ij} \partial_i \partial_j \tilde{W} \psi + c.c. \right\}
\]

The study of the rich BPS spectrum of this set of matrix models deserves a deep analysis.

5 Conclusions and open questions

In this paper we have worked out a type IIA analog of the (2,2) exact pp-wave backgrounds with non constant RR-flux studied in [1]. Then we have generalized to this set of pp-wave backgrounds the Matrix String Theory picture. This has been tested by showing the quantum stability of the strong coupling regimes in which the long strings are generated. String interaction has also been recovered thanks to the existence of a flat complex direction. In the last section, we also obtained a set of new matrix theories on eleven dimensional pp-wave backgrounds with non constant 4-form flux.

A first comment about string interactions and symmetries is in order. As matrix string theory gives a nonperturbative (although involved) definition of interacting string theory, it is interesting to consider the relation between symmetries of the back-ground and interactions. Our type IIA background is explicitly invariant under \( SO(2) \) (acting on the \( \sigma \) complex plane) and the \( SU(3) \) transformations under which \( W \) is invariant (up to additional constants). Let us notice that, since the additional prepotential \( L = g \text{Tr} \Phi^1 [\Phi^2, \Phi^3] \) is fully \( SU(3) \) invariant and since the (trace of the) total symmetrization preserves all possible \( W \) invariances (up to the same constant) in \( SU(3) \), we obtain a picture in which the interacting theory preserves naturally all the background symmetries. Let us notice that this agrees with the issue raised in [22] where the symmetry of the background pp-wave metric is taken as a guiding principle for the construction of a well defined string perturbation theory.

In section 3 we have not added by hand to the action a twisted chiral prepotential of the type already discussed in the introduction. The type IIA background in this case would result

\[ \tilde{W}(\gamma \Phi) = \tilde{W}(\gamma(\Phi)) = \tilde{W}(\Phi) + k_1 1_N \]

Notice that another ordering choice, not commuting with linear transformations, would produce an interacting model which does not preserve background symmetries.
from a compactification on a circle of a generalization of the eleven dimensional background given in the last section. The addition of such a twisted prepotential term would not change at all the strong coupling analysis of section 3.1 and would generate additional dielectric couplings generalizing the ones already discussed in [12]. These additional couplings have anyway a drawback consisting of a shift in the D-term which changes the BPS equations and possibly the instanton equations studied in section 3.2 (which were given by the flat transverse direction). This seems to imply the need of a further refinement of our model (or of our analysis) for those cases. Actually, the exactness of the pp-wave backgrounds is proven as far as the absence of $\alpha'$ corrections is concerned, but nothing is known about possible string higher genus corrections due to a still too weak control on perturbative string interaction on such kind of backgrounds. The implementation of corrections of this kind would significantly modify our picture, because of the relation between the gauge coupling and the string coupling, in the structure of the gauge sector in the finite gauge coupling regime. Moreover, the MST realization of the model studied in [23] should be recovered. We do not study this very interesting issues here and we leave them for future researches.

Let us notice that, as well explained in [24], working in the light-cone gauge is extremely hard as far as the concrete string amplitudes calculations are concerned. Despite that, it would be very interesting to develop a string bits model as an effective theory for the matrix string bits in these backgrounds in order to use it as a possible calculational tool for a comparison between states in the interacting string theory on pp-waves and possible supersymmetric gauge theory duals along the lines of [25]. Notice that in the flat case, the construction of the DVV vertex is fixed by the $SO(8)$ R-symmetry and conformal dimension, while in the generic (2,2) pp-wave background we have much less R-symmetry and therefore, in principle, more possible candidates.

The matrix string models that we have formulated here can be generalized to include also real Killing potentials if holomorphic transverse isometries are gauged and subsequently frozen. Moreover, our model can be generalized to the case in which the transverse three complex dimensional space is a non compact orbifold $\mathbb{C}^3/\Gamma$, where $\Gamma$ is a discrete subgroup of $SU(3)$ whose action is a symmetry (up to an additional constant) of the prepotential $W$. It would be very interesting to generalize the $\mathcal{N} = 2$ Landau-Ginsburg techniques elaborated in [7] to study then blow-ups of these orbifold singularities and geometric transitions in general. This leads directly (see also [26] for further motivations) to the issue concerning if Matrix String Theory can effectively improve our understanding of gauge/string dualities
and eventually make it deeper. A tempting conjectural picture arises by considering the Dijkgraaf-Vafa [27] prescription relating the evaluation of exact prepotentials in $\mathcal{N} = 1$ four dimensional gauge theories via matrix models (see also [28]). In these terms, gauge/string duality seems related to an extension of the IKKT [29] matrix description of the type IIB string theory in terms of D-instantons. This, upon double dimensional oxidation induced by two T-dualities, can be related (exactly in the case in which a spectator complex flat direction is present) to the matrix string picture which is TS–dual to type IIB D-strings. Therefore, it arises a conjectural picture in which matrix string theory would be an effective non perturbative link between gauge four dimensional theories and string theories. This kind of path is not unexpected [30]. After all, gauge/string correspondence is a manifestation of the open/closed string duality and the effective duality chain advocated in [27] relays on their topological versions at planar/tree level respectively.

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