Abstract

Quantum “teleportation” has been formulated assuming the presence of entangled states and is interpreted as a realization of quantum non-locality. In contrast, correlation from both entanglement and disentanglement can occur and both correlations can be present in an EPR pair as it moves apart. Here it is shown that quantum “teleportation” can be formulated using an ensemble of EPR pairs. The process is then interpreted without invoking the non-local nature of quantum mechanics and without the use of instantaneous state collapse. The phenomenon is explained as a quantum state selection process from the ensemble.

Keywords: Entanglement, disentanglement, quantum correlations, quantum teleportation, quantum non-locality and locality.

1. Introduction

It has recently been shown\(^1\) that when an EPR pair separates, two types of correlation can exist. One correlation is conservation of phase that arises from the quantum interference terms. The other correlation is conservation of angular momentum (helicity). Entangled states are considered essential for quantum “teleportation” and a manifestation of quantum non-locality.

In many cases, however, as particles separate, parity, and thus entanglement, cannot be maintained. It has been demonstrated\(^2\) for a number of experiments\(^3,4,5,6,7\), which are considered to be non-local, that they do not distinguish between pure entangled states and mixed disentangled states. That is, the experimental results are described equally well by mixtures of disentangled states and further, more accurate experiments are needed to resolve the difference.

Based on the persistence of an entangled state, Bennett et al.\(^8\) presented a formulation of quantum “teleportation” that requires instantaneous collapse of the
wave function even at different space-like separated locations. Bob’s state is then found to be the same as Alice’s photon up to a unitary transformation. In this paper it is argued that the properties of Alice’s photon are not teleported to Bob and no instantaneous wave function collapse is necessary. Rather it is shown that Alice’s photon interferes with a sub-ensemble of the isotropic entangled states that have the same axis of quantization as Alice’s photon.

Although the essential difference between the two approaches lies in the interpretation of the wave function, the difference is more than philosophical because there are now experimental differences between the two. These two interpretations are essentially: the wave function describes the properties, as much as we can know, of a single quantum entity (e.g. a photon or electron) or the wave function describes a statistical ensemble of possible states available to a system. In this paper the latter view is held.

2. Entangled and disentangled density operators.

The EPR pair, introduced by Bohm and Aharonov, consists of two spins of magnitude \( \frac{1}{2} \) in an isotropic singlet state. In the notation of the basis states \( |+\rangle_{\hat{P}} \) and \( |-\rangle_{\hat{P}} \), a singlet state is represented as

\[
\Psi = \frac{1}{\sqrt{2}} \left[ |+\rangle_{\hat{P}} |-\rangle_{\hat{P}} - |-\rangle_{\hat{P}} |+\rangle_{\hat{P}} \right]
\]

(2.1)

where the numbers 2 and 3 refer to photons 2 and 3 respectively. The subscript \( \hat{P} \) defines an axis of quantization in an arbitrary coordinate frame resting on the photon pair. This leads to the isotropic EPR-density operator for the two spins as

\[
\rho_{EPR}^{23} = \frac{1}{4} \left( I^2 I^3 - \sigma^2 \cdot \sigma^3 \right)
\]

(2.2)

where the sigmas denote the Pauli spin vectors and \( \hat{I}^2 \hat{I}^3 \) is the direct product of the identity operators for spins 2 and 3. The \( \hat{P} \) component of the Pauli spin operator is an eigenfunction on these states: \( \sigma^j_{\hat{P}} | \pm \rangle_{\hat{P}} = \pm | \pm \rangle_{\hat{P}} \). According to Bennett et al., when the entangled EPR pair emerges from an EPR source it is described by the pair density operator \( \rho_{EPR}^{23} \), Eq.(2.2).
In contrast, disentanglement\(^1\) requires the pair density operator to be reduced after the two photons separate from their EPR-source leading to a mixed product state \(\rho_{23}^{PP} = \rho_p^2(+)\rho_p^3(-)\) where:

\[
\rho_p^2(+) = |+\rangle_p^2 \langle +|_p^2
\]

and

\[
\rho_p^3(-) = |-\rangle_p^3 \langle -|_p^3
\]

Likewise the single particle density operators, \(\rho_p^2(-)\) and \(\rho_p^3(+)\) are defined. Since the photon moving towards both Alice and Bob can have both left or right helicities, the two-spin disentangled density operator is written as a specific sub-ensemble with the same quantization axis,

\[
\rho_{23,PP} = \frac{1}{2} \left( \rho_p^2(+)\rho_p^3(-) + \rho_p^2(-)\rho_p^3(+) \right)
\]

In general, any quantization axis, \(\hat{P}\), can exist which in turn gives rise to an ensemble of photon pairs which are characterized by \(\hat{P}\). However, if two photons originate from the same EPR pair, then angular momentum conservation requires the quantization axes to be identical. This polarization axis is carried by each particle after separation. If Alice’s photon 1 is to form a Bell state with photon 2, the quantization axis of photons 1 and 2 must match.

3. “Teleportation” using ensembles for both entangled and disentangled states.

The essential difference between using disentanglement\(^1\) and entanglement when applied to “teleportation” lies in the difference between \(\rho_{23,PP}\), Eq.(2.5) and \(\rho_{EPR}^{23}\), Eq.(2.2). The initial photon whose properties are to be reconstructed, (Alice’s photon 1), is a general superposition state

\[
\rho^1 = \begin{pmatrix}
|a|^2 & ab^* \\
ab^*b & |b|^2
\end{pmatrix}
\]

where \(|a|^2 + |b|^2 = 1\).

It is possible to represent the spin states of photons 1 and 2 in an operator
basis at the location of Alice. This is done using the four Bell states $|\Psi^+_i\rangle$ and $|\Phi^+_i\rangle$.

Both symmetrising and antisymmetrising projection operators are given by

$$S_{12} = \frac{1}{4} \left( 3 I_i I^2 + \sigma^1 \cdot \sigma^2 \right)$$

(3.2)

and

$$A_{12} = \frac{1}{4} \left( I_i I^2 - \sigma^1 \cdot \sigma^2 \right) = |\Psi^-_{12}\rangle\langle\Psi^-_{12}|$$

(3.3)

The symmetric projection operator can be further decomposed by writing

$$S_{12} = S_{12}^S + S_{12}^A + S_{12}^Z$$

(3.4)

where

$$S_{12}^S = \frac{1}{4} \left[ I_i I^2 + \sigma^1 \cdot \sigma^2 - 2 \sigma^1_i \sigma^2_i \right] = |\Phi^-_{12}\rangle\langle\Phi^-_{12}|$$

(3.5)

$$S_{12}^A = \frac{1}{4} \left[ I_i I^2 + \sigma^1 \cdot \sigma^2 - 2 \sigma^1_i \sigma^2_i \right] = |\Phi^+_{12}\rangle\langle\Phi^+_{12}|$$

(3.6)

and

$$S_{12}^Z = \frac{1}{4} \left[ I_i I^2 + \sigma^1 \cdot \sigma^2 - 2 \sigma^1_i \sigma^2_i \right] = |\Psi^+_{12}\rangle\langle\Psi^+_{12}|$$

(3.7)

Clearly the sum obeys: $A_{12} + S_{12} = I_i I^2$.

In order to carry out the full Bell transformation, Eqs.(3.3) through (3.7) are used. The result is identical except for the form of the density operator for photon 3,

$$\begin{align*}
(\rho_i^{(3)})_{\text{diagonal}, i} &= [A_{12} \rho_i^{(3)} A_{12} + S_{12} \rho_i^{(3)} S_{12}^A + S_{12} \rho_i^{(3)} S_{12}^Z + S_{12} \rho_i^{(3)} S_{12}^Z S_{12}^Z + S_{12} \rho_i^{(3)} S_{12}^Z S_{12}^Z]
\end{align*}$$

$$= \frac{1}{4} \left[ |\Psi^-_{12}\rangle\langle\Psi^-_{12}| + |\Phi^-_{12}\rangle\langle\Phi^-_{12}| + |\sigma^1_i \rho_i^{(3)} \sigma^2_i\rangle\langle\sigma^1_i \rho_i^{(3)} \sigma^2_i| + |\Phi^+_{12}\rangle\langle\Phi^+_{12}| + |\sigma^1_i \rho_i^{(3)} \sigma^2_i\rangle\langle\sigma^1_i \rho_i^{(3)} \sigma^2_i| + |\Psi^+_{12}\rangle\langle\Psi^+_{12}| \right]$$

(3.8)

The subscript $i$ either reflects the disentangled, D, or entangled, E, photon states where the disentanglement treatment uses Eq.(2.5), and entanglement uses Eq.(2.2). Thus, entanglement is not a requirement for the form given in Eq.(3.8). The state $\rho_i^{(3)}$ can explicitly be written as
Assuming that the singlet state consists of an ensemble of spins with all possible quantization axes, Alice’s photon, 1, can only form a Bell state if its axis coincides with the sub-ensemble consisting of spins 2 with the same axis. Since photons 2 and 3 must share the same axis of quantization to entangle, and photon 3 also belongs to the same sub-ensemble, then photon’s 1 and 3 are correlated to Alice’s photon. The transformation in Eq.(3.8) expresses the way Alice’s and Bob’s photons are correlated.

Application to the results\(^5,6,7\) of experiments designed to study quantum “teleportation” requires knowledge of the relevant state that is measured. It is most common to measure a coincidence between Alice’s two photons and Bob’s one. Denoting the specific state that is measured in such experiments by\(|\Phi_{123}\rangle\), the expectation value is then,

\[
\langle \Phi_{123} | \rho_{123}^{EPR} | \Phi_{123} \rangle = \langle \Phi_{123} | \rho_1 \rho_{EPR}^{23} | \Phi_{123} \rangle
\] (3.11)

Equation (3.11) gives the final results if it is assumed that photons 2 and 3 remain entangled. If, however, the photons disentangle, then it is necessary to perform an average over the ensemble of different quantization axes, \(\hat{P}\). Denoting the ensemble average by a bar, the result is

\[
\langle \Phi_{123} | \rho_{123}^{D,P} | \Phi_{123} \rangle = \langle \Phi_{123} | \rho_1 \rho_{EPR}^{23,D} \hat{P} | \Phi_{123} \rangle
\] (3.12)

Here the disentangled density operator is given by Eq.(2.5), given explicitly in terms of the spherical angles that orient \(\hat{P}\), (\(\theta \phi\)),

\[
\rho_3^D = \begin{pmatrix} |a|^2 & 0 \\ 0 & |b|^2 \end{pmatrix}
\] (3.9)

\[
\rho_3^E = \begin{pmatrix} |a|^2 & ab^* \\ a^*b & |b|^2 \end{pmatrix}
\] (3.10)
\[ \rho_{D,\hat{p}}^{23} = \frac{1}{2} \left( \begin{pmatrix} \cos^2 \theta & \cos \theta \sin \theta e^{-i\phi_2} \\ \cos \theta \sin \theta e^{i\phi_2} & \sin^2 \theta \end{pmatrix} \otimes \begin{pmatrix} \cos^2 \theta & \cos \theta \sin \theta e^{-i\phi_1} \\ \cos \theta \sin \theta e^{i\phi_1} & \sin^2 \theta \end{pmatrix} \right) \]

\[ + \left( \begin{pmatrix} \sin^2 \theta & -\cos \theta \sin \theta e^{-i\phi_2} \\ -\cos \theta \sin \theta e^{i\phi_2} & \cos^2 \theta \end{pmatrix} \otimes \begin{pmatrix} \sin^2 \theta & -\cos \theta \sin \theta e^{-i\phi_1} \\ -\cos \theta \sin \theta e^{i\phi_1} & \cos^2 \theta \end{pmatrix} \right) \]

(3.13)

where the superscripts denote photons 2 and 3. In this expression we have retained the distinction between the azimuthal angle for photons 2 and 3, respectively \( \phi_2 \) and \( \phi_3 \). Since disentanglement does not conserve parity in general, \( \phi_2 \neq \phi_3 \). That is, as seen from Eq.(3.13), an ensemble of many different phases exists between photons moving left and right from the same EPR pairs. Only those photons that have the correct phase relationship between EPR pairs lead to non-zero results after ensemble averaging. For example, in some experiments\(^7\) the use of non-linear crystals causes state changes, e.g., \( |+\rangle \leftrightarrow |-\rangle \) which lead to the relationship \( \phi_2 \rightarrow \phi_2 + \pi \). The polar angle, \( \theta \), remains unchanged as photons pass through non-linear crystals. However, successful state matching between photons 2 and 3 requires \( \phi_2 = \phi_3 \) (or in the case of the above example of using non-linear crystals, \( \phi_3 = \phi_2 + \pi \)). Whereas all entangled photons satisfy these phase conditions, few disentangled photons do. This is one reason for the low detection rate observed in such experiments.

Ensemble averaging of the disentangled case is performed only in the plane perpendicular to the photon beam direction. This follows because for photons the direction of the linear momentum defines the axis about which the helicity is quantized. It is necessary to first change the polar coordinates to cylindrical, \( z, r \) and \( \chi \). Defining the direction of propagation as the \( z \) axis, then \( x = y = 0 \) for an isotropic distribution in the plane perpendicular to the photon propagation. This fixes the average radial angle at \( \bar{\theta} = 45^\circ \) and the average value of \( \bar{r} = 1 \). Ensemble averaging over \( z \) is not required and the angle \( \chi \) is integrated from 0 to \( 2\pi \). From this emerges the classical view of the helicity of a photon with angular momentum of unity.

More details and application to some experiments is found in reference 2.
4. Conclusions

It is shown in a recent paper\(^2\) that many of the “teleportation” experiments, which assume entangled states persist after separation, agree equally well with the calculations where the photons do not remain entangled. The relevant expectation values are measured in coincidence experiments involving both Alice and Bob’s photons, Eqs.(3.11) or (3.12).

Quantum “teleportation” has been interpreted here as “quantum state selection” where Alice’s photon can only entangle with an appropriate sum-ensemble of the entangled pair, 23. If entangled photons can be prepared and maintained over large separations, the advantage of using entangled photons over disentangled photons lies in the number of states available to photons 2 and 3: only four in the former case and a possible infinite number in the latter case. In the disentangled case, only a few of the photons in the ensemble have the correct phase properties to permit successful state selection with Alice’s photon.

Selecting out the appropriate sub-ensemble is a local phenomenon. Space-like instantaneous action-at-a-distance is not a requirement for the ensemble interpretation of the singlet state. More details of the experimental applications can be found in reference 2.

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