The squeezing in a nonlinear system with chaotic dynamics is considered. The model describing interaction of collection of two-level atoms with a single-mode self-consistent field and an external field is analyzed. It is shown that in semiclassical limit, in contrast to the regular behaviour, the chaotic dynamics may result in: (i) an increase in squeezing, (ii) unstable squeezing and contraction of time intervals of squeezing on large enough times. The possibility of the experimental observation of the described effects is discussed. The obtained results are rather general and do not depend on the model under consideration.

I. INTRODUCTION

In recent years a great deal of interest has been focused on the investigation of chaotic dynamics in different physical processes [1,2]. Alongside with the study of dynamical chaos in the classical systems, in the middle of 70s more and more researchers began to realize the importance of the problem of quantum chaos. The subject of quantum chaos is the study of some peculiarities in the behaviour of quantum systems which are classically chaotic [3-5]. A lot of interesting theoretical results have been obtained in the investigation of quantum chaos. One of the most important problem now is to study concrete physical systems with quantum chaos. Such systems have to meet certain requirements, i.e. they must be: a) semiclassical, b) chaotic in the classical limit, c) Hamiltonian (nondissipative). The last requirement is, apparently, not necessary but the study of the dissipative quantum chaos has only begun [6].

A very promising models in the study of quantum chaos are the models of quantum optics. First of all, these are models describing interaction between light and a collection of atoms. Really, the semiclassical conditions and those of (semi)classical chaos can be easily met for such models (e.g., see review [7]). But the majority of the optical systems with classical chaos are dissipative (lasers, bistable devices, etc.). Up to now the only exception was the BZT model [7,8] – the semiclassical Jaynes-Cummings model [9] without the assumption of the rotating-wave approximation (RWA). But the global chaos arises in the BZT model if only the RWA is violated 1, and this requires very large atom densities and values of dipole moments in a experiments (numerical estimates see in [7,8]). Only recently there appeared some modifications of Jaynes-Cummings model assuming the Hamiltonian chaos in the semiclassical description and within the framework of the RWA [10-13].

On the other hand, at present there are a number of purely quantum (nonclassical) effects arising in the interaction of light with atoms (squeezing, photon antibunching, etc.) [14-17]. As a rule, these effects are more pronounced in the semiclassical limit. However, all the nonclassical effects with light have been studied for the integrable systems with regular behaviour 2. Therefore, in order to investigate the problems of quantum chaos, as well as those of quantum optics, it is of interest to analyze the influence of the chaotic behaviour on the nonclassical optical effects.

In this paper, a squeezed light generation in a nonlinear system with chaotic behaviour is considered. The model [12] is used as a concrete example, and the method of $1/N$ expansion into modification suggested in [18] is applied. The main result of the paper is the demonstration of the fact that in the semiclassical limit chaos may increase squeezing of light. A phenomenon of the unstable squeezing is described. The unstable character of squeezing is manifested in a great change of time intervals, during which the the squeezing is possible when the initial conditions or parameters of the systems change slightly.

The paper is organized as follows. Section 2 presents a brief description of the model [12] and the procedure of $1/N$ expansion. Here also the equations are obtained which describe light squeezing in the semiclassical limit. Section 3 deals with the conditions for transition to chaos in the model discussed. Section 4 is devoted to the comparison of

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1 The RWA is violated when the coupling constant between the atoms and the field (in units of frequency) becomes of the order of transition frequency.

2 The problem of a squeezing in chaotic system was mentioned in concluding remark of the paper [19]. But no analysis of the problem was given in [19].
squeezing for regular and chaotic dynamics. In Discussion other models of quantum optics are considered permitting semiclassical chaos and as consequence a possible enhance in squeezing. The conditions for observation of chaotic squeezing in experiments are also discussed.

Preliminary results of the paper were presented in [20].

II. 1/N EXPANSION AND SQUEEZING

Let a sample with a gas of \( N \) two-level systems (TLS) be placed in a ring single-mode, high-\( Q \) cavity of resonant frequency \( \omega \). Now let us consider the interaction between \( N \) two-level atoms, a self-consistent field and an external, classical, amplitude-modulated field of the form

\[
E_{\text{ext}}(t) = E_0 F(t) \cos \omega t
\]

injected into the cavity in the plane \( z = 0 \). In eq. (1) \( E_0 \) is the external field amplitude, and \( F(t) \) is a function, slowly changing in comparison with a carrier frequency \( \omega \) and defining the amplitude modulation. Using the approximation, known in the theory of optical bistability and lasers with an injected external signal as the “mean field model” [21], the Hamiltonian in the interaction representation can be written in the form

\[
H = NH_N, \quad H_N = H_{JC} + H_{\text{ext}},
\]

\[
H_{JC} = i\hbar \omega_c (a^+ J_+ - a J_-),
\]

\[
H_{\text{ext}} = i\hbar \omega_c GF(t)(a - a^+),
\]

where \( a^+, a \) are the normalized operators of creation and annihilation of photons in the cavity; \( J_\alpha = N^{-1} \sum_{i=1}^{N} \sigma_{\alpha,i} \) (\( \alpha = +, -, z \)) are collective atomic operators (\( \sigma_{\alpha,i} \) are Pauli matrices describing the \( i \)-th atom); \( N/V \) is the density of TLS, \( d \) and \( \omega_0 \) are the dipole matrix element and transition frequency, respectively; \( G = \omega_R/\omega_0^2 \), \( R \) is characteristic sample size, \( c \) is light velocity, \( \omega_R = dE_0/\hbar \) is the Rabi frequency. In (2)-(4) the case of exact resonance between atoms and field is considered \( \omega = \omega_0 \). The atom-field coupling constant in frequency units \( \omega_c = (2\pi \eta \alpha \omega_0^2/\hbar^2)^{1/2} \) is called cooperative frequency [22] (in modern literature, \( \omega_c \) is also called “collective vacuum-field Rabi frequency” [23]). It characterizes the energy exchange from the atoms to the field and back again. Such kind of oscillations have been observed experimentally in both optical and microwave domains [23-25].

One can easily see that eq. (3) represents the Hamiltonian of the Jaynes-Cummings model [9], and eq. (4) belongs to the type “external source – classical current”, according to the classification suggested in [26].

Different modifications of driven Jaynes-Cummings model were introduced in the framework of investigation of the quantum field fluctuations including squeezing [27]. But this activity was limited only by the integrable case with regular dynamics. In contrast, the model (2)-(4) demonstrates both regular and chaotic dynamics.

The quantum dynamics of our system is completely described by equation of motion for the density matrix

\[
i\hbar \dot{\rho} = [H, \rho]
\]

Now we introduce new operators

\[
X_1 = \Re a = \frac{1}{2}(a^+ + a), \quad X_2 = \Im a = \frac{i}{2}(a^+ - a)
\]

From the commutation relationships (5), we obtain the uncertainty relation

\[
\langle (\delta X_1)^2 \rangle \langle (\delta X_2)^2 \rangle \geq \frac{1}{16N}, \quad \delta X_j = X_j - \langle X_j \rangle
\]

The definition of squeezing has the form [16]

\[
S = 4N \langle (\delta X_1)^2 \rangle < 1
\]

In order to study the dynamics of squeezing in the model (2)-(4) at \( N \gg 1 \), it is natural to use the method of \( 1/N \) expansion. It has been recently mentioned in [28] that the method can be applied not only to the integrable but also
to nonintegrable systems. In accordance with this method [29], the natural states for constructing the classical limit 
\( N \to \infty \) are generalized coherent states. In our case, the state \( | \psi \rangle \) is the product of the Glauber coherent state 
\( | \alpha \rangle [14] \) and of the spin coherent state \( | \beta \rangle [30-31] \). For the model (2)-(4), the \( 1/N \) expansion is constructed in a
manner analogous to the expansion for the Jaynes-Cummings model [18]. Therefore, I’d like to dwell here only on
some principal moments.

We define a generalized \( P \)-function as

\[
\rho(t) = \int d^2 \alpha d^2 \beta \ P(\alpha, \beta, t) \ | \psi \rangle \langle \psi |,
\]

\[
\int d^2 \alpha = \int_{-\infty}^{+\infty} d(Re \alpha) \int_{-\infty}^{+\infty} d(Im \alpha)
\]

Using the commutation relations between the Bose operators and projectors [14] and the spin operators and projectors
[31]

\[
a^+ | \alpha \rangle \langle \alpha | = U_{a^+} | \alpha \rangle \langle \alpha |,
\]

\[
a | \alpha \rangle \langle \alpha | = U_a | \alpha \rangle \langle \alpha |,
\]

\[
| \alpha \rangle \langle \alpha | a^+ = U_{a^+}^* | \alpha \rangle \langle \alpha |,
\]

\[
| \alpha \rangle \langle \alpha | a = U_a^* | \alpha \rangle \langle \alpha |,
\]

\[
U_{a^+} = \alpha^* + \frac{1}{N} \frac{\partial}{\partial \alpha}, \quad U_a = \alpha
\]

\[
J_+ | \beta \rangle \langle \beta | = U_{J_+} | \beta \rangle \langle \beta |,
\]

\[
J_- | \beta \rangle \langle \beta | = U_{J_-} | \beta \rangle \langle \beta |,
\]

\[
| \beta \rangle \langle \beta | J_+ = U_{J_+}^* | \beta \rangle \langle \beta |,
\]

\[
| \beta \rangle \langle \beta | J_- = U_{J_-}^* | \beta \rangle \langle \beta |,
\]

from eq.(6), we have the equation of motion for the function \( P \)

\[
\frac{\partial P}{\partial t} = \omega_c \left[ -\frac{\partial}{\partial \alpha} (V_\alpha P) - \frac{\partial}{\partial \beta} (V_\beta P) + \frac{\partial^2}{\partial \alpha \partial \beta} (W_{\alpha \beta} P) \right] + c.c.
\]

\[
V_\alpha = \frac{\beta}{1+|\beta|^2} + \frac{1}{N} \frac{\partial}{\partial \beta}, \quad V_\beta = -\frac{\beta^2}{N} \frac{\partial}{\partial \beta}
\]

\[
W_{\alpha \beta} = \frac{\beta^2}{N}
\]

It follows from (13) and (14) that the variables \( \alpha \) and \( \beta \) can be considered to be real. From eq. (13) one can obtain
the following equations of motion for averages [18]

\[
\frac{d}{dt} \langle \varphi \rangle = \omega_c (V_\varphi),
\]

\[
\frac{d}{dt} \langle \delta \varphi \delta \varphi \rangle = \omega_c (V_\varphi \delta \varphi) + \omega_c (\delta \varphi V_\varphi) + \omega_c (W_{\varphi \varphi}),
\]

where \( \delta \varphi = \varphi - \langle \varphi \rangle \), and \( \varphi \) is one of the variables \( \alpha \) or \( \beta \). However, the equations (15) and (16) are not closed. So,
let us use the Taylor’s expansion of the function \( V_\varphi \)

\[
V_\varphi = \langle V_\varphi \rangle + \sum_{\varphi'} \left( \frac{\partial V_\varphi}{\partial \varphi'} \right) \langle \delta \varphi \rangle + \cdots
\]
and the analogous equation for $W_{\varphi \varphi}$. Due to the uncertainty relation of the type (8), it is clear that $\delta \varphi(t = 0) \simeq N^{-1/2}$. Therefore, substituting eq. (17) into (15) and (16) one can obtain a closed system of equations for observed values in any order over $1/N$. In a zero order, the equation of motion takes the form

$$\frac{d}{dt}\langle \varphi \rangle = \omega_c \langle V_{\varphi} \rangle_{\langle \varphi \rangle} + O(1/N)$$

(18)

These equations are equivalent to the equations obtained from a coupled Maxwell-Bloch system in [12]. After replacing the variables $\beta = \tan(x/2)$ [30], eq. (18) can be written as follows

$$\ddot{x} + \omega_c^2 \sin x = -2G\omega_c^2 F(t),$$

(19)

where

$$\alpha = p/2, \quad p = -\frac{1}{\omega_c^2} \dot{x}, \quad \langle J_+ \rangle = -\frac{1}{2} \sin x, \quad \langle J_z \rangle = \frac{1}{2} \cos x$$

(20)

It is known already [1,2,12] that eq. (19) assumes both regular and chaotic behaviour. The conditions for the transition to chaos in (18),(19) will be discussed in the next Section.

In the first order in $1/N$, one can obtain from eq. (16) with an account of (17)

$$\frac{d}{dt}\langle (\Delta p)^2 \rangle = 2\omega_c \cos x \langle \Delta p \Delta x \rangle,$$

$$\frac{d}{dt}\langle (\Delta x)^2 \rangle = -2\omega_c \langle \Delta p \Delta x \rangle,$$

$$\frac{d}{dt}\langle \Delta p \Delta x \rangle = \omega_c \cos x \langle (\Delta x)^2 \rangle - \omega_c \langle (\Delta p)^2 \rangle,$$

(21)

where

$$\langle (\Delta p)^2 \rangle = 4\langle (\delta \alpha)^2 \rangle + 2/N, \quad \langle (\Delta x)^2 \rangle = 4R^2 \langle (\delta \beta)^2 \rangle + 2/N,$$

$$\langle \Delta p \Delta x \rangle = 4R \langle \delta \alpha \delta \beta \rangle, \quad R = (1 + \beta^2)^{-1}$$

(22)

In the present paper, we shall focus our attention at the conditions when initially field is in the coherent state and the atoms are in the ground state ($J_z = -1/2$) corresponding to $x(0) = 0$ and $p(0) = p_0$. The condition for squeezing in variables (22) takes the form

$$S = N\langle (\Delta p)^2 \rangle < 3$$

(23)

Before we start investigating the nonlinear dynamics of the systems (19) and (21), let us discuss the condition for validity of the semiclassical approach to squeezing.

The time-scale for validity of semiclassical approach and $1/N$ expansion in cooperative optical system were studied systematically in [32]. For regular dynamics, this time-scale has a power dependence on $N$. In contrast, it is proportional to $\log N$ for a chaotic dynamics [32].

Here we only estimate a required number of TLS, when the semiclassical description (18),(19) and (21) can be used to describe the dynamics of squeezing. Introduce the “convergence radius”

$$d(t) = \left[ \langle (\delta \alpha)^2 \rangle + \langle (\delta \beta)^2 \rangle \right]^{1/2} \simeq \left[ \langle (\Delta p)^2 \rangle + \langle (\Delta x)^2 \rangle \right]^{1/2} \ll 1,$$

(24)

where $d(0) \simeq N^{-1/2}$. If $d(t) \ll 1$, then $1/N$ expansion and consequently equations (18),(19) and (21) are correct. The behaviour of $d(t)$ differs considerably for regular and chaotic dynamics. Equations (21), in fact, coincide

\[ ^{3}\text{The difference is in the fact that in the definition of Lyapunov exponent only the linearization near } x(t), p(t) \text{ is considered and not the behaviour of the linear fluctuations as in (21). However, in this case the difference is insignificant.} \]
where \( \lambda > 0 \) is the maximal Lyapunov exponent. The dependence (25) is related to the presence of the strong local instability of the chaotic motion. We will consider the effective squeezing to occur for the time \( t_1 \), being of the order of several \( \omega_c^{-1} \), and \( 1/N \) expansion is correct, if \( d \lesssim 0.01 \). With this assumption, from (25) it follows that one need to have \( N \simeq 10^5 \div 10^6 \) for correct semiclassical description of squeezing at chaos conditions. This simple estimation is in a good agreement with the results of numerical simulation [20].

III. CHAOS

In this section, following [12], we will briefly discuss the conditions for transition to chaos in the model (18), (19). We will need this information below.

The motion of a pendulum without perturbation \((G = 0)\) is periodic, and in phase plane it has two types of the fixed points: elliptic, with coordinates \( p = 0, x = 2\pi n \) \((n = 0, \pm 1, \pm 2, \ldots)\) corresponding (see (20)) to the initial populations of de-excited levels of TLS, and hyperbolic, with coordinates \( p = 0, x = \pi(2n + 1) \) \((n = 0, \pm 1, \ldots)\) corresponding to the complete filling of the upper levels of TLS. The pendulum separatrix (a special trajectory in the phase plane, separating the vibrational and rotational motion and going through the hyperbolic points) corresponds to the complete energy transformation from the atoms to the field and back again.

The dynamics of (18),(19) at \( G \neq 0 \) depends considerably on the number of harmonics in the spectrum of \( F(t) \).

Let \( F(t) = \sin \Omega t \). The criteria for the transition to chaos obtained by means of the Chirikov’s resonance overlapping method [1-4] are different in two limiting cases: \( \kappa > 1 \) and \( \kappa \ll 1 \), \( \kappa \equiv 2G\omega_c^2/\Omega^2 \).

1) If \( \kappa \ll 1 \) and for \( \Omega > \omega_c \), a narrow stochastic layer is found in the vicinity of the separatrix; the rest of the phase space is filled by periodic trajectories. At \( \Omega \lesssim \omega_c \) in the vicinity of the separatrix there appears a broad stochastic layer which occupies the major part of the phase space, except the area in the vicinity of the elliptic point.

2) If \( \kappa \gg 1 \), the criterion for the transition to chaos

\[
K > 1, \quad K = \text{const} \frac{\omega_c}{\Omega \kappa^{1/4}}, \quad \text{const} \simeq 10 
\]

can be easily satisfied. The oscillation amplitude in this case may be rather large

\[
|p_{max}| \simeq \frac{\kappa \Omega}{\omega_c} = \frac{2G\omega_c}{\Omega} \quad (27)
\]

According to eq. (26) at \( \Omega \to 0 \) chaos is always present, but in this case the diffusion rate falls in proportional to \( \Omega \), and the chaotic motion in the system is slow (adiabatic chaos). Of great importance is the fact that in the case under consideration the transition to chaos is possible, if only the lower levels of TLS are initially occupied.

A numerical analysis shows that at \( \kappa \simeq 1 \) the dynamics of the system is qualitatively analogous to the case of \( \kappa \gg 1 \).

When the number of harmonics for \( F(t) \) is increased, the chaotic properties of the system become strongly.

Fig. 1 (a,b) and 2 show examples of chaotic, regular oscillations and adiabatic chaos, respectively \((\tau = \omega_c^{-1} t)\). Fig 3 demonstrates the presence of the local instability in the chaotic motion (a) and its absence in the regular motion (b).

Dissipative analog of such model was considered in [19] (both atomic and field dumping were included). In that case chaos is transient.

IV. SQUEEZING AND CHAOS

Let us consider now the dynamics of light squeezing. As the system (18),(19) is Hamiltonian, the Liouville theorem is always true. However, the change in the shape of phase volume for regular and chaotic dynamics is different. It is known [1,2] that for chaos the change of the phase volume shape is the strongest and fastest. This is related to the fact that in the Hamiltonian systems the mechanisms for the onset of chaos are the phase contour stretching in one direction (and squeezing in another) and its further folding. This procedure of squeezing and folding is repeated several times. Eventually, the structure of the phase volume contour becomes complicated, and its perimeter length increases exponentially with time. The time intervals, when full squeezing is possible, become very short and alternate irregularly due to the chaotic motion. The dynamics of phase volume contour for chaos is very sensitive to any changes.
in the initial conditions and the system’s parameters. This peculiarity of the chaotic system is naturally manifested in the behaviour of the system (21) determining the dispersion behaviour.

Though in the evolution process the phase volume shape of a nonlinear system with regular dynamics also undergoes considerable deformation, the stretching is less and rather regular. The increase in length of the phase volume contour has a power dependence. Therefore, the time intervals of squeezing have to be long even for large times. The regular character of motion has also to result in a weak dependence of the intervals on the parameters and initial conditions.

These simple qualitative considerations have been verified numerically. The squeezing (23) has been determined by solving the system (19) and (21) when \( F(t) = \sin \Omega t \). The Runge-Kutta method of the fourth order has been applied with an accuracy up to \( 10^{-9} \div 10^{-13} \). The precision of these calculations has been monitored by checking that the time-invariant of motion

\[
L = p/2 - \cos x + 2xG \sin \psi + \Omega I,
\]

\[
\frac{dx}{d\tau} = \frac{\partial L}{\partial p}, \quad \frac{dp}{d\tau} = -\frac{\partial L}{\partial x}, \quad \frac{d\psi}{d\tau} = \Omega, \quad \frac{dI}{d\tau} = -\frac{\partial L}{\partial \psi}, \quad I(0) = 0, \quad \tau = \omega_c t
\]

is satisfied. Fig 4a shows the largest squeezing \( S_{\text{min}} \) for the time \( \tau = 10\omega_c^{-1} \) as a function of the external perturbation \( G \), and Fig 5a shows the same as a function of the external field frequency \( \Omega \). The increase of \( d \) during the same time characterizes a degree of the dynamical instability (Figs. 4b and 5b). The conclusion about the character of oscillations in the system (regular (R), chaotic (C), adiabatic chaos (AC)) have been made on the basis of a phase portrait and the behaviour of \( d(t) \) for times \( \tau = 200 \). A boundary in the space of parameters between chaos and adiabatic chaos is, naturally, quite relative. It can be easily seen from figures that the degree of squeezing is larger for chaos. The value of squeezing in this case makes up \( 10^{-2} \div 10^{-3} \).

Several comparatively low values of squeezing (\( S_{\text{min}} \simeq 0.1 \)) observed under chaos for some parameter values have been due to the following reasons:

1) Weak statistical properties of the chaotic oscillations (e.g., point 1 in figures 4 and 5) which are, as rule, related to the fact that these parameters are in the vicinity of the area of parameters corresponding to the regular dynamics.
2) Adiabatic chaotic dynamics. A characteristic period of the oscillations in this case (see Fig. 2) is compared with the time during which the squeezing was determined (\( \tau_1 = 10 \)).
3) Anomalously strong instability of motion for \( \tau \leq 10 \) (see point 2 in figure 4a,b). The analysis of the behaviour \( S(t) \) at this parameter value shows that after a short fall \( S(t) \) quickly increases during the interval (\( \tau_1 = 10 \)). This is due to fast and strong deformation of the phase volume. However, strong squeezing would be also possible further, e.g., for \( \tau_1 = 20 \) \( S_{\text{min}} \simeq 2.6 \cdot 10^{-3} \).

Fig.6 shows temporary intervals of squeezing (\( S < 3 \)), when the dynamics of the system is regular or chaotic. It is seen that the intervals are quite long and regular even at \( \tau > 10 \) for the regular dynamics (a) and short and irregular for the chaotic behaviour (d). It is noteworthy that at \( \tau \gtrsim 30 \) the intervals of squeezing become very short and their finding in numerical calculation makes difficulties. The time interval of squeezing for the chaotic motion is more sensitive to slight changes of the initial conditions than for the regular one (compare (b) and (e), (c) and (f)).

Thus, the chaotic dynamics leads to the short, irregular and unstable squeezing at \( \tau > 10 \). It should be noted that for \( N \gg 1 \) the value of the maximal squeezing does not, in fact, depend on \( N \).

V. DISCUSSION

Thus, the main result of the paper is somewhat paradoxical: a nonlinear system, which is in fact a generator of noise, may be more effective in suppressing quantum noise. The maximal squeezing for the chaotic motion may be by a factor of \( 10^3 \) more than for the regular dynamics.

We believe that the effect discussed may be observed in a modification of the experiments on squeezing, when the light interacts with an ensemble of two-level atoms [33]. The amplitude modulation of the external field leading to chaos may be not only sinusoidal but may also represent a sequence of short light pulses. Such a sequence could be generated by a mode locked laser.

It should be also noted that in this paper a ring cavity is considered. But it can be shown that the transition to chaos is also possible for other cavity configurations. For example, in the case of a Fabry-Perot cavity eq. (19) has to be replaced by the pendulum type equation with a Bessel function instead of sinus (see the second reference in [12]). Such an equation has also chaotic solutions, but Hamiltonian chaos is transient [12].
Besides our model, an analogous scheme can be applied to other models of cooperative optical system with chaotic dynamics in the semiclassical limit:

1) The BZT model [8] and also its generalization for the multi-level systems [34]. In these models the global chaos is possible if only the RWA is violated.

2) The Shepelyansky model [10]: an ensemble of 3-level systems interacting with two modes of self-consistent field. The global chaos is possible within the framework of the RWA when the dipole moments of one- and two photon transitions are commensurable.

3) The ensemble of TLS interacting with the self-consistent field and the resonance external field of constant amplitude [11,13]. The transition to chaos is also possible within the framework of the RWA. According to classification of [26] this model belongs to the type “external source – classical field”.

Consequently, in all the cases one should expect an increase in squeezing when the conditions for chaos are satisfied.

Another class of models assuming the $1/N$ expansion and chaotic dynamics are those models which describe the parametric interaction of light waves. This can be easily understood if to take into account the fact that any cooperative optic process may correspond to a parametric process (e.g., see [35]). At present time, the several situations are known when multiple parametric interaction of both scalar light waves [36] and waves of different polarizations [37] appears at chaotic spatial evolution.

The consideration of squeezing at chaotic light waves evolution will be the subject of our future publication.

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Fig. 3. (a) Local instability for chaotic behaviour shown in Fig. 1a; (b) the absence of local instability for regular motion. The parameter values are the same as in Fig. 1b. In both cases \( N = 10^6 \).
Fig. 4. Maximal squeezing $S_{\text{max}}$ and the value of dynamical instability $\delta$ for the time $\tau = 10$ as a function of the external perturbation $G$. The initial conditions: $x(0) = p(0) = 0$; $N = 10^{6}$, $\omega / \omega_c = 1$. Point 1 corresponds to $G = 0.1$, and point 2 to $G = 1.1$. 
