A note on backreacting flavors from calibrated geometry

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One of the main problems in the search for string duals with backreacting, smeared flavors is the construction of a suitable source density. We review how this issue may be addressed using generalized calibrated geometry.

1. Introduction

Gauge/string duality in its original formulation\(^{[1,2]}\) relates strongly coupled \(d = 3 + 1, \mathcal{N} = 4\) super Yang-Mills to weakly coupled type IIB string theory on \(AdS_5 \times S^5\) and vice-versa. It did not take long for the duality to be generalized to gauge theories in different dimension\(^{[3]}\) or with less supersymmetry\(^{[4,5]}\). In all these cases, the dynamics of the gauge theory are captured by a closed string theory on a suitable ten-dimensional space-time.

If one adds an open string sector by the inclusion of D-branes, one introduces fields into the gauge theory that transform under the fundamental representation of the gauge group\(^{[6]}\). If these branes extend along a non-compact cycle transverse to the \(R^{1,d-1}\) associated with the gauge theory, the local \(SU(N_f)\) gauge symmetry living on these branes turns into a global flavor symmetry of the dual gauge theory – one has flavored the theory.

For many purposes, it is sufficient to work in the limit \(N_f \ll N_c\), in which the backreaction of the branes onto the geometry may be ignored. In the perturbative regime of the gauge theory this corresponds to the exclusion of flavor loops from all Feynman diagrams. It may be conceptually straightforward to go beyond this approximation and find duals for \(N_f \sim N_c\) by including the backreaction of the flavor branes onto the geometry; yet it should be no surprise that the technical challenges in doing so are often quite formidable. The issue was first addressed in\(^{[7,8]}\). One such complication is the fact that localized branes add delta-function sources to the equations of motion. The standard method of dealing with these relies on smearing the flavor branes over their transverse directions\(^{[9]}\), turning the delta-functions into smooth source distributions. As was shown in\(^{[10]}\), the construction of the source-densities involved can be helped by making use of generalized calibrated geometry\(^{[11]}\). In this note we shall review the construction of flavored supergravity duals and show what can be learned from generalized calibrated geometry\(^{[2]}\).

2. Flavored supergravity duals from calibrated geometry

Simply said, a string dual in the supergravity limit consists of the metric (\(g_{mn}\)) the dilaton (\(\Phi\)) and a set of RR (\(F(i)\)) or NS (\(H(3)\)) gauge fields, whose dynamics are captured by the relevant type IIA/B action \(S_{IIA/B}\). On an intuitive level, it is clear that the addition of backreacting sources will deform the geometry of the background, yet conserve its essential topological features. Therefore one usually begins the flavoring procedure by studying deformations of the original supergravity dual. This gives a suitable ansatz for the geometry of the flavored background. Then, in order to add backreacting Dp-brane sources to

\(^{[2]}\) For a more complete list of references to the subject see references to\(^{[9]}\) and those in\(^{[10]}\).
the system, one considers the combined action
\[ S = S_{11A/B} + S_{\text{branes}} \]  
where the brane action \( S_{\text{branes}} \) is given by the usual DBI and WZ terms. The presence of \( S_{\text{branes}} \) adds sources to the type IIA/B equations of motion. Our main interest lies in the resulting violation of the Bianchi identity. Here one considers the calibrated geometry \[ \Omega \] Subsequently, one looks for solutions of the modified equations of motion using the deformed ansatz.

As usual, supersymmetry makes things a great deal easier, because second order equations of motion can be traded for first order BPS ones. One should note that the latter are also modified by the presence of the source term. Due to a certain symmetry \[ \kappa \] of \( \Omega \) the system, one considers the combined action
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If we want to use the BPS equations as an aid towards constructing a background, the flavor brane embeddings \( X(\xi) \) have to preserve some of the supersymmetries of the background. In the flavoring literature, the standard tool for discussing supersymmetric brane embeddings is \( \kappa \)-symmetry. Let \( \epsilon \) be a SUSY spinor of the background. For a brane embedding \( X(\xi) \), one constructs the \( \kappa \)-symmetry matrix \( \Gamma_{\kappa}[X(\xi)] \), that acts on the spinors of the background. The embedding is supersymmetric if
\[ \Gamma_{\kappa}[X(\xi)]\epsilon = \epsilon \]  
This condition can be rephrased using generalized calibrated geometry [11121314]. Here one defines a \( p+1 \)-form, the calibration form, along the lines of
\[ \phi = \frac{1}{(p+1)!}(\epsilon^\dagger \Gamma_{a_0...a_p}\epsilon)e^{a_0...a_p} \]
Supersymmetry is then satisfied if the volume form induced onto the cycle defined by \( X(\xi) \) equals the pull-back of the calibration:
\[ X^*\phi = \sqrt{-\gamma_{\text{bd}}}d^{p+1}\xi \]
It is quite crucial that, for fixed \( p \), the calibration form \( \phi \) is the same for all \( Dp \)-branes, independent of their embedding, while \( \Gamma_{\kappa}[X(\xi)] \) is not. In other words, \( \phi \) knows about all SUSY embeddings of the background.

If one combines this with the modified Bianchi identity [2], one arrives at
\[ d(e^{-\Phi} \phi) = F_{(p+2)} \]
One should note, that this relation gives strong constraints on the smearing form – an issue that was first exploited in the context of flavored duals in [10] to show that the source distribution form has to respect certain symmetries of the background – yet does not fix it. The calibration depends on the vielbein \( e^a \), which again depends on the original deformed ansatz. However, as the calibration form captures the embeddings of all possible supersymmetric \( Dp \)-branes, the smearing form in its general form of knows about all possible ways of smearing them. One can constrain the original ansatz for the flavored background from the knowledge of the general structure of these two differential forms.

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