Josephson Effect between Conventional and Rashba Superconductors

Nobuhiko Hayashi\textsuperscript{a,b}, Christian Iniotakis\textsuperscript{c},
Masahiko Machida\textsuperscript{a,b}, Manfred Sigrist\textsuperscript{c}

\textsuperscript{a} CCSE, Japan Atomic Energy Agency, 6-9-3 Higashi-Ueno, Tokyo 110-0015, Japan
\textsuperscript{b} CREST JST, 4-1-8 Honcho, Kawaguchi, Saitama 332-0012, Japan
\textsuperscript{c} Institut für Theoretische Physik, ETH-Zürich, CH-8093 Zürich, Switzerland

Abstract

We study the Josephson effect between a conventional $s$-wave superconductor and a non-centrosymmetric superconductor with Rashba spin-orbit coupling. Rashba spin-orbit coupling affects the Josephson pair tunneling in a characteristic way. The Josephson coupling can be decomposed into two parts, a ‘spin-singlet-like’ and a ‘spin-triplet-like’ component. The latter component can lead to shift of the Josephson phase by $\pi$ relative to the former coupling. This has important implications on interference effects and may explain some recent experimental results for the Al/CePt$_3$Si junction.

Key words: Non-centrosymmetric superconductor, Josephson effect, Rashba spin-orbit coupling, CePt$_3$Si
PACS: 74.50.+r, 74.20.Rp, 74.70.Tx

1 Introduction

Superconductors without inversion symmetry, the so-called non-centrosymmetric superconductors, have received much interest during recent years. The lack of an inversion center in the crystal lattice induces antisymmetric spin-orbit coupling, leading to important modifications of the superconducting phase. The spin-orbit coupling displays the Rashba form in systems such as CePt$_3$Si \cite{1} where mirror symmetry about a single plane is missing \cite{2}. Due to such an antisymmetric spin-orbit coupling, the Fermi surface is split into two sheets by the spin-degeneracy lifting and the electronic spin structure on the Fermi surfaces is modified \cite{3,4}. The specific spin structure on the Fermi surfaces due
to the Rashba coupling plays also an important role in connection with the 
Josephson effect, as we will show here. The tunneling conductance between 
a normal metal and a non-centrosymmetric superconductor [5,6,7] and the 
Josephson effect between two non-centrosymmetric superconductors [8,9,10] 
have been investigated by various groups. Here we aim at the properties 
of the Josephson effect between a conventional superconductor and a non-
centrosymmetric superconductor.

The Josephson effect between a conventional (s-wave spin-singlet) and an un-
conventional superconductor provides a possible way to probe the spin struc-
ture of the unconventional Cooper pairing state [11,12,13,14,15]. For the non-
centrosymmetric superconductor CePt$_3$Si, an experiment in this direction was 
recently performed by Sumiyama et al. [16]. In this experiment, Al/CePt$_3$Si 
junctions were prepared and their Josephson effect was investigated by ap-
plying weak magnetic fields in order to observe the interference patterns in 
the supercurrent. A Fraunhofer-shaped pattern was observed for the tunnel 
junction normal to the in-plane axis of the tetragonal crystal of CePt$_3$Si, while 
a very irregular pattern appeared for the c axis tunneling [16]. A possible expla-
nation of these findings was recently given by Leridon et al. [17] proposing 
Cooper pairing violating time-reversal symmetry. Here we introduce an alter-
native proposal which is based on the influence of Rashba spin-orbit coupling 
on the Josephson effect. First, we derive an expression for the dc-Josephson 
current between a conventional superconductor and a non-centrosymmetric 
superconductor with Rashba spin-orbit coupling. Then we analyze the spe-
cific differences between differently oriented Josephson junctions, in order to 
give an explanation for the observation in the Al/CePt$_3$Si junctions.

Before going into details, we outline the basic idea. The Josephson current $J$ is 
expressed as $J = J_c \sin \phi_{\text{ph}}$, where $\phi_{\text{ph}}$ is the phase difference between the two 
superconductors. We suppose that we can decompose $J_c$ into two parts $J_c = 
J_1 + J_2$, where $J_1 > 0$ and $J_2 < 0$. Then we assume that, for an inhomogeneous 
interface, the relative magnitude of the two contributions $|J_1|$ and $|J_2|$ varies, 
such that regions with $J_c = J_1 + J_2 > 0$ and $< 0$ exist, i.e., regions with 0- and 
$\pi$-phase shifts $|J = -|J_c| \sin \phi_{\text{ph}} = |J_c| \sin(\phi_{\text{ph}} + \pi)]$, respectively. For such 
junctions, the interference pattern would deviate strongly from the ordinary 
Fraunhofer pattern and, in particular, the central peak in the interference 
pattern may be missing, as observed in other systems with random 0- and 
$\pi$-junctions [18,19]. In this picture, the presence of a sufficiently strong $J_2$
-component giving rise to a $\pi$ junction disturbs the Fraunhofer pattern. As 
shown later, it follows that $J_2 \neq 0$ for $\hat{n} \parallel c$ and $J_2 = 0$ for $\hat{n} \parallel a$, where $\hat{n}$ is 
a vector normal on the interface. This effect would explain the experimental 
results [16] that the Fraunhofer pattern is absent (present) for the Josephson 
junction perpendicular to the c axis (the a axis). In the following sections, 
we will derive an expression for the Josephson current composed of $J_1$ and $J_2$
parts.
2 Josephson Current

We consider a Josephson junction between two superconductors, assuming spherical Fermi surfaces for both superconductors, for simplicity. The Fermi velocities can be written as $v_F^{L(R)} = v_F^{L(R)} \hat{k}^{L(R)}$, where $\hat{k}$ is the unit vector parallel to the Fermi momentum. (We use units with $\hbar = 1$ and $k_B = 1$.) The superconductors on the left- and right-hand-side of the interface are labeled by “L” and “R”, respectively. According to Millis et al. [12], the supercurrent $J$ flowing across the interface is given by

$$J = N_F^L v_F^L \int_{\hat{n} \cdot \hat{k}^L > 0} \frac{d\Omega^L}{4\pi} \hat{n} \cdot \hat{k}^L T \sum_{\omega_n} K,$$

with

$$K = \left( \frac{i}{2\pi} \right)^{1/4} \text{Tr} \left\{ \tilde{\tau}_3 \left[ \tilde{g}^L(\hat{k}^L, r_\perp = 0^-, \omega_n), \tilde{S}^\dagger_{R,L} \tilde{g}^R(\hat{k}^R, r_\perp = 0^+, \omega_n) \tilde{S}_{R,L} \right] \right\},$$

where the interface lies perpendicular to the unit vector $\hat{n}$ located at $r_\perp = 0$ and $r_\perp$ is the coordinate perpendicular to the interface. Moreover, $N_F^L$ is the density of states on the Fermi surface in the left-hand-side superconductor, and $\omega_n = \pi T (2n + 1)$ is the Matsubara frequency with $T$ as the temperature. The commutator $[\hat{a}, \hat{b}] = \hat{a}\hat{b} - \hat{b}\hat{a}$ is defined in the usual way. The quasiclassical Green function $\tilde{g}$ is a $4 \times 4$ matrix composed of blocks in $2 \times 2$ particle-hole space and in $2 \times 2$ spin space. It can be written in the particle-hole space as

$$\tilde{g}(\hat{k}, r_\perp, i\omega_n) = -i\pi \begin{pmatrix} \hat{g} & \hat{f} \\ -i\hat{\bar{f}} & -\hat{\bar{g}} \end{pmatrix}.$$

The symbol Tr in Eq. (2) denotes the trace of the $4 \times 4$ matrix. The matrix $\tilde{\tau}_3$ is given by

$$\tilde{\tau}_3 = \begin{pmatrix} \hat{\sigma}_0 & 0 \\ 0 & -\hat{\sigma}_0 \end{pmatrix},$$

with $\hat{\sigma}_0$ as the unit matrix in the spin space. The interface is characterized by the tunneling $S$ matrix $\tilde{S}_{R,L}$ as [12]

$$\tilde{S}_{R,L}(\hat{k}_\parallel) = \begin{pmatrix} \tilde{S}_{R,L}(\hat{k}_\parallel) & 0 \\ 0 & \tilde{S}_{L,R}(\hat{k}_\parallel) \end{pmatrix}.$$
The quasiparticle momentum parallel to the interface, $k_\parallel$, is conserved in the tunneling process, i.e., $k_\parallel^L = k_\parallel^R$. We may parametrize the $S$ matrices in the spin space as \[12,15\]

\[
\hat{S}_{R,L}(k_\parallel) = s_{R,L}(k_\parallel)\hat{\sigma}_0 + m_{R,L}(k_\parallel) \cdot \hat{\sigma},
\]

\[
\hat{S}_{L,R}(-k_\parallel) = s_{L,R}(-k_\parallel)\hat{\sigma}_0 + m_{L,R}(-k_\parallel) \cdot \hat{\sigma}^{tr}.
\]

(6)

(7)

Here we use the Pauli matrices $\hat{\sigma} = (\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z)$ for the spin space, and $s$ and $m$ denote the spin-inactive and spin-active tunneling components through the interface, respectively.

For an interface potential which is invariant under time-reversal reflections in a plane perpendicular to the interface, the tunneling amplitudes satisfy the relations \[11,12\]

\ [
\begin{align*}
s_{R,L}(k_\parallel) &= s_{L,R}(-k_\parallel), \\
m_{R,L}(k_\parallel) &= -m_{L,R}(-k_\parallel) = c_{R,L}\hat{n} \times k_\parallel.
\end{align*}
\]

(8)

(9)

The $S$ matrix at an interface between a conventional metal and a metal with Rashba spin-orbit coupling can be calculated by applying the continuity condition for the wave function at the interface. Within this scheme, we verified the above relations in the limit that the difference between the volumes of the split Fermi surfaces is negligibly small, nevertheless taking account of the modified spin structure due to Rashba spin-orbit coupling properly \[20\]. This limit allows us to perform a simple and transparent analysis of the problem keeping the essential aspects. Actually the general expression for the $S$ matrix is rather complicated and will be discussed elsewhere \[20\].

### 3 Josephson Junction between $s$-Wave and Non-Centrosymmetric Superconductors

We now turn to the Josephson effect between a spin-singlet $s$-wave superconductor (left-hand side) and a non-centrosymmetric one (right-hand side) with Rashba spin-orbit coupling $\sim \lambda \cdot \hat{\sigma}$, where $\lambda = (-k_y^R, k_x^R, 0)$ \[21\]. The qualitative properties of such a junction can be obtained utilizing bulk Green functions as in Refs. \[11,12,14,15\]. In the spin-singlet $s$-wave superconductor with $\Psi_s$ as the order parameter, the Green functions are given as \[22\]

\[
\hat{\mathcal{G}}^L = \frac{\omega_n\hat{\sigma}_0}{\sqrt{\omega_n^2 + |\Psi_s|^2}},
\]

\[
\hat{\mathcal{G}}^L = \frac{\omega_n\hat{\sigma}_0}{\sqrt{\omega_n^2 + |\Psi_s|^2}}.
\]
On the non-centrosymmetric superconductor side, they have the form \[ \bar{k}'_z = \bar{k}_y R \pm i \bar{k}_x R, (\bar{k}'_y R)^2 + (\bar{k}'_y R)^2 = 1 \] \[ 21 \]:

\[
\begin{align*}
\hat{g}^R &= \begin{pmatrix} g_+ & -\bar{k}'_+ g_- \\ -\bar{k}'_- g_+ & g_+ \end{pmatrix}, \\
\hat{g}^I &= \begin{pmatrix} \bar{k}'_+ g_- & g_+ \\ \bar{k}'_- g_+ & \bar{k}'_+ g_- \end{pmatrix}, \\
\hat{f}^R &= \begin{pmatrix} \bar{k}'_+ f_- & f_+ \\ -f_+ & -\bar{k}'_+ f_- \end{pmatrix}, \\
\hat{f}^I &= \begin{pmatrix} \bar{k}'_- f_- & \bar{k}'_+ f_+ \\ \bar{k}'_+ f_- & -f_+ \end{pmatrix}.
\end{align*}
\] (11)

We use the notation, \( g_\pm = (g_1 \pm g_\Pi)/2, f_\pm = (f_1 \pm f_\Pi)/2, \bar{f}_\pm = (\bar{f}_1 \pm \bar{f}_\Pi)/2, \)
\( g_{1\Pi} = \omega_n/B_{1\Pi}, f_{1\Pi} = \Delta_{1\Pi}/B_{1\Pi}, \bar{f}_{1\Pi} = \Delta^*_1/B_{1\Pi}, B_{1\Pi} = \sqrt{\omega_n^2 + |\Delta_{1\Pi}|^2}, \)
\( \Delta_1 = \Psi + \Delta \sin \theta_R, \text{ and } \Delta_\Pi = \Psi - \Delta \sin \theta_R. \) The superconducting order parameters \( \Delta_{1\Pi} \) are defined on the split Fermi surfaces I and II, and \( \Psi(\Delta) \) stands for the singlet (triplet) order-parameter component in the non-centrosymmetric superconductor. It is important that the relative phase between \( \Psi \) and \( \Delta \) is 0 or \( \pi \) \[ 21 \]. Furthermore, \( \theta_R \) denotes the angle relative to the \( k^R_z \) axis with \( \sin \theta_R \geq 0 \).

The above Green functions can be inserted into Eq. (2) for \( K \). Using Eqs. (8) and (9), we finally arrive at the following result,

\[
K = \frac{\pi(1 + \delta)}{D_1} \text{Im}\{w_0^* \Psi_s \Delta^*_1}\] \\
+ \frac{\pi(1 - \delta)}{D_{1\Pi}} \text{Im}\{w_0^* \Psi_s \Delta^*_{1\Pi}\} \\
+ \frac{\pi(1 + \delta)}{D_1} \text{Im}\{\lambda \cdot w^* \Psi_s \Delta^*_1\} \\
+ \frac{-\pi(1 - \delta)}{D_{1\Pi}} \text{Im}\{\bar{\lambda} \cdot w^* \Psi_s \Delta^*_{1\Pi}\},
\] (12)

where \( \lambda = (-\bar{k}_y R, \bar{k}_x R, 0) \) and \( D_{1\Pi} = 2\sqrt{\omega_n^2 + |\Psi_s|^2 \sqrt{\omega_n^2 + |\Delta_{1\Pi}|^2}}. \) The split Fermi surfaces (I and II) are taken into account also by the parameter \( \delta \) which denotes the difference of the density of states. \( \delta = (N_I - N_{1\Pi})/2N_0, \)
\( 2N_0 = N_I + N_{1\Pi} \) \[ 21 \]. The interface is described by means of the parameters, the scalar \( w_0 \) and the vector \( w = (w_x, w_y, w_z) \), given by

\[
w_0 = |s_{R,L}|^2 + |c_{R,L}|^2 |\hat{n} \times \hat{k}_\parallel|^2,
\] (13)

\[
w = 2\text{Re}\{s_{R,L} c_{R,L}^* \hat{n} \times \hat{k}_\parallel\}.
\] (14)
Within this scheme, we find that the first and second terms in Eq. (12) are similar to the Josephson coupling between spin-singlet superconductors ("singlet-like" coupling), while the third and fourth terms resemble the coupling between spin-singlet and spin-triplet superconductors ("triplet-like" coupling) [12,15]. This decomposition is independent of the explicit mixing of singlet-triplet pairing in the non-centrosymmetric superconductor, because Eq. (12) has the same form even in the "pure singlet" ($\Delta = 0$) and "pure triplet" ($\Psi = 0$) pairing cases. The origin of the factor $\vec{\lambda} \cdot \vec{w}^*$ in Eq. (12) is not the triplet pairing, but is the characteristic spin structure on the Fermi surfaces due to Rashba spin-orbit coupling $\sim \vec{\lambda} \cdot \vec{\sigma}$.

4 Directional dependence of Josephson current

The Josephson current is obtained by integrating Eq. (12) over the Fermi surface ($\sim \int_{\hat{n} \cdot \hat{k} > 0} d\Omega_\hat{k} (\hat{n} \cdot \hat{k}) K = \int_{\hat{n} \cdot \hat{k} > 0} d\phi d\theta \sin \theta (\hat{n} \cdot \hat{k}) K$). In the first and second terms of Eq. (12) (singlet-like coupling), $w_0$ is always positive according to Eq. (13), and therefore these terms correspond to $J_1 \sin \phi_{ph}$ ($J_1 > 0$). Note that $w_0$ is real [Eq. (13)] and the factor $\text{Im}\{w_0^* \Psi_s \Delta_{I,II}^s\}$ in Eq. (12) leads to $w_0 |\Psi_s| |\Delta_{I,II}| \sin \phi_{ph}$. On the other hand, the third and fourth terms of Eq. (12) (triplet-like coupling) can be negative because of the coefficient $c_{R,L}$ of the spin-active tunneling [Eq. (9)], the sign of which depends on an interface potential formed at the junction between two different materials [20]. The actual interface potential formed between Al and CePt$_3$Si is unknown. However, we assume here that the interface potential gives rise to a negative sign for $c_{R,L}$. Under this assumption, the third and fourth terms of Eq. (12) can correspond to $J_2 \sin \phi_{ph}$ ($J_2 < 0$). Note that from Eq. (14) it follows that $\vec{w}$ is real, and the factor $\text{Im}\{\vec{\lambda} \cdot \vec{w}^* \Psi_s \Delta_{I,II}^s\}$ in Eq. (12) leads to $\vec{\lambda} \cdot \vec{w} |\Psi_s| |\Delta_{I,II}| \sin \phi_{ph}$. Now, we will demonstrate that $J_2$, namely the integration of the third and fourth terms in Eq. (12), becomes zero for a certain direction of the interface.

The interface ($\hat{n} \parallel a$) normal to the $x$ axis ($a$ axis) yields $\vec{w} \sim \hat{n} \times \hat{k} \parallel \sim (0, -\cos \theta, \sin \phi \sin \theta)$, where we use spherical coordinates according to $\hat{k} = (\cos \phi \sin \theta, \sin \phi \sin \theta, \cos \theta)$ and $(\hat{k}_x^R, \hat{k}_y^R) = (\cos \phi, \sin \phi)$. Hence we obtain the factor $\vec{\lambda} \cdot \vec{w} = (-\hat{k}_y^R)w_x + \hat{k}_x^R w_y \sim -\cos \phi \cos \theta$, which is an odd function with respect to $\theta' (= \theta - \pi/2)$. All other factors, $D_{I,II}$ and $\Delta_{I,II}$ in Eq. (12), keep their sign with respect to $\theta$ ($0 \leq \theta \leq \pi$). From these, we conclude that $\int_{\hat{n} \cdot \hat{k} > 0} d\phi d\theta \sin \theta (\hat{n} \cdot \hat{k})$ [$\{3\text{rd and 4th terms of } K\} = 0$ for $\hat{n} \parallel a$, because of the factor $\cos \theta$ originating from $\vec{\lambda} \cdot \vec{w}$, where the integral range for $\theta$ is $0 \leq \theta \leq \pi$.

In contrast, for the interface ($\hat{n} \parallel c$) normal to the $z$ axis ($c$ axis), $\vec{w} \sim \hat{n} \times \hat{k} \parallel \sim (-\sin \phi \sin \theta, \cos \phi \sin \theta, 0)$, and we get $\vec{\lambda} \cdot \vec{w} = (-\hat{k}_y)w_x + \hat{k}_x w_y \sim (\sin^2 \phi + \cos^2 \phi) \sin \theta = \sin \theta$. This factor remains positive over the integral range $0 \leq \theta \leq \pi/2$. The other factors such as $D_{I,II}$ and $\Delta_{I,II}$ in Eq. (12)
show no sign change in the whole range of $\theta$ ($0 \leq \theta \leq \pi/2$). Thus we find
\[
\int_{\hat{n} \cdot \hat{k} > 0} d\phi \ d\theta \sin \theta (\hat{n} \cdot \hat{k}) \{3\text{rd and 4th terms of } K\} \neq 0 \text{ for } \hat{n} \parallel c, \text{ because the integrand does not change its sign over the integral range.}
\]

Consequently, the results $J_2 \neq 0$ for $\hat{n} \parallel c$ and $J_2 = 0$ for $\hat{n} \parallel a$, have been obtained. They explain the experimental results for the Al/CePt$_3$Si junctions [16] as discussed in Introduction. From a symmetry point of view, our discussion would not be changed qualitatively for the case of anisotropic Fermi surfaces. Symmetries such as the mirror symmetry about the $k_xk_y$ plane remain valid, even under the influence of antisymmetric (Rashba) spin-orbit coupling [23]. For this reason, the above argumentation on the signs of the integrands must hold even for anisotropic Fermi surfaces.

5 Conclusion

We investigated the behavior of a Josephson junction between a singlet $s$-wave superconductor and a non-centrosymmetric superconductor with Rashba spin-orbit coupling. The expression for the Josephson current-phase relation was derived for such a junction. This allowed us to give a possible explanation for the recent experimental results [16] for the Al/CePt$_3$Si junction. Furthermore, we anticipate that in the absence of an external magnetic field, spontaneous magnetic fluxes could appear along the interface normal to the $c$ axis of CePt$_3$Si owing to random $\pi$- and 0-junctions, which can be observed experimentally, in principle, by scanning SQUID microscopes as in the case presented in Ref. [19]. Moreover, Andreev bound states can be formed at surfaces of certain orientations in non-centrosymmetric superconductors [6,10]. The influences of such bound states have been neglected here for the qualitative discussions as in Refs. [11,12,14,15]. A more detailed analysis taking these aspects into account is left for future studies.

ACKNOWLEDGMENTS

We are grateful to A. Sumiyama, D. F. Agterberg, P. A. Frigeri, S. Fujimoto and K. Wakabayashi for helpful discussions. We also acknowledge financial support from the Swiss Nationalfonds and the NCCR MaNEP.
References

[1] E. Bauer, G. Hilscher, H. Michor, Ch. Paul, E. W. Scheidt, A. Gribanov, Yu. Seropegin, H. Noel, M. Sigrist, P. Rogl, Phys. Rev. Lett. 92 (2004) 027003.

[2] P. A. Frigeri, D. F. Agterberg, A. Koga, and M. Sigrist, Phys. Rev. Lett. 92 (2004) 097001; and references therein.

[3] S. S. Saxena, P. Monthoux, Nature 427 (2004) 799.

[4] S. Fujimoto, J. Phys. Soc. Jpn. 76 (2007) 051008, cond-mat/0702585.

[5] T. Yokoyama, Y. Tanaka, J. Inoue, Phys. Rev. B 72 (2005) 220504; Phys. Rev. B 72 (2005) 035318.

[6] C. Iniotakis, N. Hayashi, Y. Sawa, T. Yokoyama, U. May, Y. Tanaka, M. Sigrist, Phys. Rev. B 76 (2007) 012501.

[7] J. Linder, A. Sudbø, Phys. Rev. B 76 (2007) 054511.

[8] K. Børkje, A. Sudbø, Phys. Rev. B 74 (2006) 054506.

[9] S. S. Mandal, S. P. Mukherjee, J. Phys.: Condens. Matter 18 (2006) L593.

[10] K. Børkje, Phys. Rev. B 76 (2007) 184513.

[11] V. B. Geshkenbein, A. I. Larkin, JETP Lett 43 (1986) 395; J. A. Sauls, Z. Zou, P. W. Anderson, unpublished.

[12] A. Millis, D. Rainer, J. A. Sauls, Phys. Rev. B 38 (1988) 4504.

[13] S.-K. Yip, O. F. De Alcantara Bonfim, P. Kumar, Phys. Rev. B 41 (1990) 11214.

[14] Y. Hasegawa, J. Phys. Soc. Jpn. 67 (1998) 3699.

[15] M. Sigrist, K. Ueda, Rev. Mod. Phys. 63 (1991) 239, Sec. IV B.

[16] A. Sumiyama, K. Nakatsuji, Y. Tsuji, Y. Oda, T. Yasuda, R. Settai, Y. Onuki, J. Phys. Soc. Jpn. 74 (2005) 3041.

[17] B. Leridon, T.-K. Ng, C. M. Varma, Phys. Rev. Lett. 99 (2007) 027002.

[18] H. Hilgenkamp, J. Mannhart, B. Mayer, Phys. Rev. B 53 (1996) 14586.

[19] J. Mannhart, H. Hilgenkamp, B. Mayer, Ch. Gerber, J. R. Kirtley, K. A. Moler, M. Sigrist, Phys. Rev. Lett. 77 (1996) 2782.

[20] N. Hayashi, et al., unpublished.

[21] N. Hayashi, K. Wakabayashi, P. A. Frigeri, M. Sigrist, Phys. Rev. B 73 (2006) 024504; Phys. Rev. B 73 (2006) 092508.

[22] U. Klein, J. Low Temp. Phys. 69 (1987) 1.

[23] P. A. Frigeri, D. F. Agterberg, M. Sigrist, New J. Phys. 6 (2004) 115.