The Constrained MSSM Revisited

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Within the Constrained Minimal Supersymmetric Model (CMSSM) it is possible to predict the low energy gauge couplings and masses of the 3 generation particles from a few supergravity inspired parameters at the GUT scale. Moreover, the CMSSM predicts electroweak symmetry breaking due to large radiative corrections from the Yukawa couplings, thus relating the $Z^0$ boson mass to the top quark mass via the renormalization group equations (RGE). In addition, the cosmological constraints on the lifetime of the universe are considered in the fits. The new precise measurements of the strong coupling constant and the top mass as well as higher order calculations of the $b \to s\gamma$ rate exclude perfect fits in the CMSSM, although the discrepancies from the best fit parameters are below the 2$\sigma$ level.

1 The Constrained MSSM

The Minimal Supersymmetric Standard Model (MSSM) has become the leading candidate for a low energy theory consistent with the GUT requirements. At this conference several new results have been presented, which are crucial for the consistency checks of GUT’s. First of all, the $\alpha_s$ crisis has disappeared, since the LEP value went down and the DIS measurement as well as the value froms lattice calculations went up, and the error on the strong coupling constant has come down to an astonishing low level of about 3%. In this analysis I will use for the coupling constants $\alpha_s = 0.120 \pm 0.003$ and $\sin^2 \Theta_W = 0.2317 \pm 0.004$, which are the global fit values including the top mass from the combined data of CDF and D0, (175 $\pm$ 6 GeV) and the new higher order calculations for the important $b \to s\gamma$ rate. The latter indicate that next to leading log (QCD) corrections increase the SM value by about 10%. This can be simulated in the lowest level calculation by choosing a renormalization scale $\mu = 0.65 m_t$, which will be done in the following. Here we repeat our previous analysis with the new input values mentioned above. The input data and fitted parameters have been summarized in table I.

2 Results

The most restrictive constraints are the coupling constant unification and the requirement that the unification scale has to be above $10^{15}$ GeV from the proton lifetime limits, assuming decay via $s$-channel exchange of heavy gauge bosons. They exclude the SM as well as many other models.

Constraints from $b - \tau$ unification

The requirement of bottom-tau Yukawa coupling unification strongly restricts the possible solutions in the $m_t$ versus $\tan \beta$ plane. For $m_t = 175 \pm 6$ GeV only two regions of $\tan \beta$ give an acceptable $\chi^2$ fit, as shown in the bottom part of fig. I. The curves in the upper parts are determined by the relations between top and bottom masses and $\tan \beta$:

\[ m_t^2 = 4\pi Y_t v^2 \frac{\tan^2 \beta}{1 + \tan^2 \beta} \quad (1) \]
\[ m_b^2 = 4\pi Y_b v^2 \frac{1}{1 + \tan^2 \beta} \quad (2) \]

For increasing $\tan \beta$ $m_t^2$ reaches quickly its plateau $4\pi Y_t v^2$; for large $\tan \beta$ $Y_b$ has to compensate the
is mainly determined by the $b - \tau$ Yukawa coupling at the GUT scale and the lower part the $\Sigma$ contributions to the Higgs potential.

The middle part shows the corresponding values of the Yukawa coupling unification. The middle part shows the corresponding values of the Yukawa coupling at the GUT scale and the lower part the $\chi^2$ values. If the top constraint ($m_t = 175 \pm 6$, horizontal band) is not applied, all values of $\tan \beta$ between 1.2 and 50 are allowed (thin dotted lines at the bottom), but if the top mass is constrained to the experimental value, only the regions $1 \leq \tan \beta \leq 3$ and $20 \leq \tan \beta \leq 40$ are allowed.

$1/\tan^2 \beta$ term, so it quickly increases (see middle part). But then the (negative) $Y_b$ contributions to the running of $Y_t$ from loops involving both top and bottom cannot be neglected anymore, which decrease $Y_t$ and correspondingly the top mass for high $\tan \beta$.

**Electroweak Symmetry Breaking (EWSB)**

Radiative corrections can trigger spontaneous symmetry breaking in the electroweak sector. In this case the Higgs potential does not have its minimum for all fields equal zero, but the minimum is obtained for non-zero vacuum expectation values of the fields. Solving $M_Z$ from the minimum of the Higgs potential yields:

$$\frac{M_Z^2}{2} = \frac{m_1^2 + \Sigma_1 - (m_2^2 + \Sigma_2) \tan^2 \beta}{\tan^2 \beta - 1}, \quad (3)$$

where $m_{1,2}$ are the mass terms in the Higgs potential and $\Sigma_1$ and $\Sigma_2$ their radiative corrections.

Note that the radiative corrections are needed, since unification at the GUT scale with $m_1 = m_2$ would lead to $M_Z < 0$. In order to obtain $M_Z > 0$ one needs to have which happens at low energy since $\Sigma_2 (\Sigma_1)$ contains large negative corrections proportional to $Y_t (Y_b)$ and $Y_t \gg Y_b$. Electroweak symmetry breaking for the large $\tan \beta$ scenario is not so easy, since eq. (3) can be rewritten as:

$$\tan^2 \beta = \frac{m_1^2 + \Sigma_1 + \frac{1}{2} M_Z^2}{m_2^2 + \Sigma_2 + \frac{1}{2} M_Z^2}. \quad (4)$$

For large $\tan \beta$ $Y_t \approx Y_b$, so $\Sigma_1 \approx \Sigma_2$. Eq. (4) then requires the starting values of $m_1$ and $m_2$ to be different in order to obtain a large value of $\tan \beta$, which could happen if the symmetry group above the GUT scale has a larger rank than the SM, like e.g. $SO(10)$. In this case the quartic interaction (D-) terms in the Higgs potential can generate quadratic mass terms, if the Higgs fields develop non-zero v.e.v’s after spontaneous symmetry breaking.

Alternatively, one has to assume the simplest GUT group $SU(5)$, which has the same rank as the SM, so no additional groups are needed to break $SU(5)$ and consequently no D-terms are generated. In this case EWSB can only be generated, if $Y_b$ is sufficiently below $Y_t$, in which case the splitting

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**Figure 1:** The top quark mass as function of $\tan \beta$ (top) for values of $m_0, m_{1/2} \approx 1$ TeV. The dependence shown is mainly determined by the $b - \tau$ Yukawa coupling unification. The middle part shows the corresponding values of the Yukawa coupling at the GUT scale and the lower part the $\chi^2$ values. If the top constraint ($m_t = 175 \pm 6$, horizontal band) is not applied, all values of $\tan \beta$ between 1.2 and 50 are allowed (thin dotted lines at the bottom), but if the top mass is constrained to the experimental value, only the regions $1 \leq \tan \beta \leq 3$ and $20 \leq \tan \beta \leq 40$ are allowed.

**Figure 2:** Contours of the $\chi^2$-distribution for the low and high $\tan \beta$ solutions. The different shades indicate steps of $\Delta \chi^2 = 1$, so basically only the light shaded region is allowed. The stars indicate the optimum solution. Contours enclose domains excluded by the particular constraints used in the analysis.
between $m_1$ and $m_2$ at low energies is sufficient to generate EWSB. The resulting SUSY mass spectrum is not very sensitive to the two alternatives for obtaining $m_1^2 + \Sigma_1 > m_2^2 + \Sigma_2$: either through a splitting between $m_1$ and $m_2$ already at the GUT scale via D-terms or by generating a difference via the radiative corrections.

**Discussion of the remaining constraints**

In fig. 3 the total $\chi^2$ distribution is shown as a function of $m_0$ and $m_{1/2}$ for the two values of tan $\beta$ determined above. One observes minima at $m_0, m_{1/2}$ around (200, 270) and (800, 90), as indicated by the stars. These curves were still produced with the data from last year. With the new coupling constants one finds slightly different minima, as given in table 3. In this case the minimum $\chi^2$ is not as good, since the fit wants $\alpha_s \approx 0.125$, i.e. about 1.6$\sigma$ above the measured LEP value and the calculated $b \rightarrow s\gamma$ rate is above the experimental value too, if one takes as renormalization scale $\mu \approx 0.65m_0$. At this scale the next higher order corrections, as calculated by 4, are minimal.

The contours in fig. 3 show the regions excluded by different constraints used in the analysis:

**LSP Constraint:** The requirement that the LSP is neutral excludes the regions with small $m_0$ and relatively large $m_{1/2}$, since in this case one of the scalar staus becomes the LSP after mixing via the off-diagonal elements in the mass matrix. The LSP constraint is especially effective at the high tan $\beta$ region, since the off-diagonal element in the stau mass matrix is proportional to $A_t m_0 - \mu$ tan $\beta$.

**$b \rightarrow s\gamma$ Rate:** At low tan $\beta$ the $b \rightarrow s\gamma$ rate is close to its SM value for most of the plane. The charginos and/or the charged Higgses are only light enough at small values of $m_0$ and $m_{1/2}$ to contribute significantly. The trilinear couplings were found to play a negligible role for low tan $\beta$. However, for large tan $\beta$ the trilinear coupling needs to be left free, since it is difficult to fit simultaneously $b \rightarrow s\gamma$, $m_b$ and $m_\tau$. The reason is that the corrections to $m_b$ are large for large values of tan $\beta$ due to the large contributions from $\tilde{g} - \tilde{q}$ and $\tilde{\chi}^\pm - \tilde{t}$ loops proportional to $\mu$ tan $\beta$. They become of the order of 10-20%. In order to obtain $m_b (M_Z)$ as low as 2.84 GeV, these corrections have to be negative, thus requiring $\mu$ to be negative. The $b \rightarrow s\gamma$ rate is too large in most of the parameter region for large tan $\beta$, because of the dominant chargino contribution, which is proportional to $A_t \mu$. For positive (negative) values of $A_t \mu$ this leads to a larger (smaller) branching ratio $BR (b \rightarrow s\gamma)$ than for the Standard Model with two Higgs doublets. In order to reduce this rate one needs $A_t (M_Z) > 0$ for $\mu < 0$. Since for large tan $\beta$ $A_t$ does not show a fix point behaviour, this is possible.

**Relic Density:** The long lifetime of the universe requires a mass density below the critical density, else the overclosed universe would have collapsed long ago. This requires that the contribution from the LSP to the relic density has to be below the critical density, which can be achieved if the annihilation rate is high enough. Annihilation into electron-positron pairs proceeds either through t-channel selectron exchange or through s-channel $Z^0$ exchange with a strength given by the Higgsino component of the lightest neutralino. For the low tan $\beta$ scenario the value of $\mu$ from EWSB is large. In this case there is little mixing between the higgsino- and gaugino-type neutralinos as is apparent from the neutralino mass matrix: for $|\mu| \gg M_1 \approx 0.4m_{1/2}$ the mass of the LSP is simply $0.4m_{1/2}$ and the “bino” purity is 99%. For the high tan $\beta$ scenario $\mu$ is much smaller and the Higgsino admixture becomes larger. This leads to an enhancement of $\tilde{\chi}^0 - \tilde{\chi}^0$ annihilation via the s-channel Z boson exchange, thus reducing the relic density. As a result, in the large tan $\beta$ case the constraint $\Omega h^2 < 1$ is almost always satisfied unlike in the case of low tan $\beta$. 

| Fitted SUSY parameters and masses in GeV |
|-----------------------------------------|
| Symbol | $m_0$, $m_{1/2}$ | $\mu (M_Z)$, tan $\beta$ | $Y_t(m_0)$, $A_t (M_Z)$ | $\tilde{\chi}^0$, $\tilde{\chi}^0$ | $\tilde{\chi}^0$, $\tilde{\chi}^0$ | $\tilde{\chi}^0$, $\tilde{\chi}^0$ | $g$, $\tilde{g}$, $\tilde{t}$ | $h$, $H$ | $A$, $H^\pm$ |
|-------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|       | 230, 225        | -880, 17        | 0.008, -370     | 96, 194         | 509, 519        | 194, 518        | 558, 545, 563   | 74, 673         | 680, 684        |
|       | 850, 115        | -190, 30        | 0.006, 86       | 47, 92          | 414, 417        | 92, 422         | 300, 885, 854   | 109, 624        | 624, 630        |

Table 2: Values of the fitted SUSY parameters (upper part) and corresponding susy masses (lower part) for low and high tan $\beta$ solutions using the new input data discussed in the text.
value of $\tan\beta$ gives the main contribution. The upper scale indicates the prediction from the third generation only, which apparently is below the predicted values, at least for the SM 

For high $\tan\beta$ the gluino-neutralino loop can decrease $b \rightarrow s\gamma$ somewhat.

The lightest particles preferred by these fits are charginos and higgses. The latter has a mass below 90 GeV for a top mass below 180 GeV in the low $\tan\beta$ scenario, which is within reach of LEP II.

It should be noted that recent speculation about evidence for SUSY from the $ee\gamma\gamma$ event observed by the CDF collaboration and the ALEPH 4-jet events all pointed to a SUSY parameter space inconsistent with the CMSSM, since they require very light sparticles (selectron, stop, chargino and/or neutralino). However, the $R_b$ anomaly has practically disappeared and the ALEPH 4-jet events observed at 135 GeV have not been confirmed at 161 GeV.

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