Optimum Techniques for the Conversion of Space Rectangular and Curvilinear Coordinates

Gafar Suara and Timothy O. Idowu

Abstract—Conversion between space rectangular (X, Y, Z) and curvilinear (φ, λ, h) coordinates is an important task in the field of Surveying, geodesy, positioning, navigation, mapping etc. Different techniques which include iterative methods, non-iterative techniques and closed form algebraic methods have been applied over the years to carry out the coordinate conversion. However, the results obtained using these techniques are deficient in one way or the other due to the inherent limitations such as inability to produce results for curvilinear coordinates when the values of X, Y and Z are subsequently or simultaneously equal to zero. Therefore, this study attempts to put forth an optimum coordinate conversion technique between space rectangular and curvilinear coordinates. The data used are coordinates of points which include the space rectangular coordinates and their equivalent curvilinear coordinates. They were observed and processed in Nigeria using Doppler 9 software by African Doppler Survey (ADOS) and they were confirmed to be of first order accuracy and hence of high quality. The data processing involved the design of the optimum techniques equations, coding of the algorithms and necessary computations to obtain results. Analyzing the results obtained, it can be inferred that the designed optimum model has successfully carried out the conversion between space rectangular and curvilinear coordinates. Therefore, the optimum technique model is recommended for use for the conversions from Space rectangular coordinates to Geocentric, Geodetic, Reduced coordinates and vice versa.

Index Terms—Coordinates Conversion, Curvilinear Coordinates, Optimum Techniques, Space Rectangular Coordinates.

I. INTRODUCTION

Coordinate conversion can be defined as the process of establishing relationship between coordinate systems in order to transform coordinates of points from one system to the other.

Coordinates and coordinate systems are ubiquitous in virtually all aspects of surveying and mapping. Importantly, they allow the users of spatial data to easily conceptualize coordinates with respect to some convenient coordinate system. As such, both coordinates and coordinate systems have had and will continue to have an essential role to play in the spatial sciences. This is because coordinate system forms a common frame of reference for the description of locations of points. Accordingly, coordinates and positions can be used interchangeably but always refer to a specific coordinate system [1]. In geodesy, two common classes of coordinate system have been used to describe positions in relation to the earth. These comprise the curvilinear coordinate system and the Cartesian coordinate system. Historically, curvilinear coordinates are used since they are conceptually more appropriate for describing positions on or near the earth’s curved surface. However, space rectangular coordinates have taken an increasing role because of the widespread use of Global Positioning Systems (GPS) and other satellite-based positioning systems [1]. The review of related literature has shown that there are several techniques such as the iterative, non-iterative and closed form methods that had been developed for the conversions of the coordinates but it has been discovered that majority of these techniques suffers one limitation or the other. While some of the techniques are good for the computations of longitude and latitude, they are not satisfactory for the computation of ellipsoidal heights. More so, it was discovered that some of these techniques can only give result for conversion to curvilinear geodetic coordinates only if the values of X-coordinate and Y-coordinate are not simultaneously equals to zero. Likewise, the existing techniques cannot compute for curvilinear coordinates when the values of space rectangular coordinates (X, Y and Z) are all zeros. With the aforementioned limitations, it has become imperative to develop a new technique that will reduce the observed limitations in the earlier techniques to the barest minimum. Therefore, it is the objectives of this study to develop an optimum coordinate conversion technique for space rectangular and curvilinear coordinates.

II. METHODOLOGY

Algorithms for the Optimum techniques of conversion between space rectangular and curvilinear coordinates were designed and used to convert space rectangular coordinates to curvilinear coordinates and vice versa.

A. Data Acquisition

The data used consists of coordinates of ten points which were observed and processed in Nigeria using software Doppler 9 by African Doppler Survey (ADOS) [5] as shown in Table 1.

| STN   | X(m)       | Y(m)       | Z(m)       |
|-------|------------|------------|------------|
| ANI 001 | 6 151 936.18 | 1 333 437.29 | 1 026 087.83 |
|       | N 09 19 09.938 | E 12 13 47.023 | 281.58     |
| ANI 002 | 6 293 421.52 | 715 424.95  | 748 771.60  |
|       | N 06 47 13.14  | E 06 29 07.589 | 210.96     |
| ANI 003 | 6 192 908.86  | 740 204.22  | 1 333 628.71 |
|       | N 12 08 54.54  | E 06 48 57.289 | 771.32     |
| ANI 004 | 6 285 784.60  | 921 321.54  | 565 631.10  |
|       | N 05 07 19.168 | E 08 20 18.943 | 100.54     |

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The initial value for \( T \) is approximated as \( T \) shown below [2]

\[ \lambda = \arctan\left(\frac{Y}{X}\right) \]  
\[ \lambda = \frac{\pi}{2} - 2\arctan\left(\frac{X}{\sqrt{X^2 + Y^2}}\right) + Y \]

When \( X \) or \( Y \) is equal to zero [3]

\[ \lambda = \arctan\left(\frac{1}{\sqrt{X^2 + Y^2}} - XY\right) \]

When \( X \) and \( Y \) are zero [9]

\[ \lambda = -2\arctan\left(\frac{XY}{\sqrt{X^2 + Y^2}} - XY\right) \]

When \( X \), \( Y \) and \( Z \) are zero [9]

In this technique, equations are solved iteratively as shown below [2]

\[ g(T) = PT - Z - \frac{ST}{\sqrt{1 + X^2}} \]

The initial value for \( T \) is approximated as \( T_0 \)

\[ T_0 = \left|\frac{z}{e_cN}\right|^2 \]

When \( T \) is computed as \( \tan \beta = p\alpha, P = \frac{p}{\alpha}, Z = e_c\left|\frac{z}{\alpha}\right|, E = e^2 \),

\[ e_c = \sqrt{1 - e^2} \]

\[ p = \sqrt{X^2 + Y^2} \]  
\[ \Delta r = R - r \]

\[ r = N\cos \varphi \]

\[ R = (N + h)\cos \varphi \]

The resulting iterative formula is processed as:

\[ T_n + 1 = T_n - \frac{g(T_n)}{g''(T_n) - g'(T_n)g(\varphi(T_n))/(2g'(T_n))} \]

The first derivative for \( g(T) \) is computed as:

\[ g'(T) = P - \frac{E}{\sqrt{1 + X^2}} \]

The second derivative for \( g(T) \) is computed as:

\[ g''(T) = \frac{3ET}{\sqrt{1 + X^2}} \]

Hence, the geodetic latitude (\( \varphi \)) is computed as:

\[ \varphi = \arctan\left(\frac{p}{\sqrt{1 + e^2}}\right) \]

When \( X \), \( Y \), and \( Z \) are equal to zero and \( X \) and \( Y \) are equal to zero

\[ \varphi = \arctan\left(\frac{p}{\sqrt{1 + e^2}}\right) \]

The geodetic height \( (h) \) is computed as:

\[ h = \frac{e_c|\varphi| - \frac{b\sqrt{1 + X^2}}{e_c^2}}{\sqrt{1 + X^2}} \]

When \( X \) and \( Y \) are equal to zero

\[ h = \Delta r\sqrt{1 + \left(\frac{\Delta z}{\Delta r}\right)^2} \], if \( |\Delta z| \leq |\Delta r| \), and \( h \) are simultaneously zero and when \( X \) and \( Y \) are equal to zero [6]

\[ h = |\Delta z|\left(\frac{\Delta z}{\Delta r}\right)^2 + 1 \], if \( |\Delta r| > |\Delta z| \]

Where

\[ \Delta r = R - r \]  
\[ r = N\cos \varphi \]  
\[ R = (N + h)\cos \varphi \]  
\[ \Delta z = Z - z \]  
\[ z = \left(\frac{c}{\alpha}\right)^2Ns\sin \varphi \]

The procedure for the conversion of curvilinear
coordinate to space rectangular coordinate are shown as follows [4]

\[ X = (N + h) \cos \varphi \cos \lambda \]  
\[ Y = (N + h) \cos \varphi \sin \lambda \]  
\[ Z = [N(1 - e^2) + h] \sin \varphi \]

Where \( N = \frac{a}{(1 - e^2 \sin^2 \beta)^{1/2}} \)

\[ e^2 = a^2 - b^2 \]

\[ e^2 = 2f - f^2 \]

1. Transition within Geocentric (ψ), Geodetic (φ) and Reduced Latitude (β)

\[ \tan \psi = (1 - e^2) \tan \varphi = \frac{b}{a} \tan \beta \]  

(26)

### III. Presentation of Results

The results obtained include results of conversions from space rectangular coordinates to curvilinear coordinates when the values of \( X, Y \) and \( Z \) are greater than and subsequently or simultaneously equals to zero. Also, it includes the results of reverse conversion from curvilinear coordinates to space rectangular coordinates.

### TABLE II: Conversion from Space Rectangular Coordinates to Curvilinear Coordinates

| STN | Input (X, Y, Z) | Converted \((\varphi, \lambda, h)\)| Standard \((\varphi, \lambda, h)\)| Differences |
|-----|----------------|---------------------------------|---------------------------------|-------------|
| ANI 001 | X=6151936.18, Y=1333437.29, Z=1026087.83 | \(\varphi=90\ 9.938\), \(\lambda=12\ 13.702\), \(h=281.5763\) | \(\varphi=90\ 9.938\), \(\lambda=12\ 13.702\), \(h=281.5763\) | \(\varphi=90\ 9.938\), \(\lambda=12\ 13.702\), \(h=281.5763\) |

### TABLE III: Conversion from Curvilinear Coordinates to Space Rectangular Coordinates

| STN | Input \((\varphi, \lambda, h)\) | Converted \((X, Y, Z)\) | Standard \((X, Y, Z)\) | Differences |
|-----|-----------------------------|------------------------|------------------------|-------------|
| ANI 001 | \(\varphi=90\ \lambda=12\ h=281.576\) | X=6151936.18, Y=1333437.29, Z=1026087.83 | X=6151936.18, Y=1333437.29, Z=1026087.83 | \(\varphi=90\ \lambda=12\ h=281.576\) |

### TABLE IV: Converted Curvilinear Coordinates When the Values of \( X, Y \) and \( Z \) are Subsequently and Simultaneously Zero

| STN | Input \((X, Y, Z)\) | Converted \((\varphi, \lambda, h)\) |
|-----|------------------|-------------------------------|
| ACS 01 | X=0 | \(\varphi=90\ \lambda=12\ h=281.576\) |
The results obtained after the conversion from curvilinear coordinates to space rectangular coordinates and vice versa are analyzed as shown in the following steps:

The aim here is to test whether the converted coordinates (μ) satisfactorily represent the standard coordinates (μ₀) of the same points. That is, the following hypothesis is tested:

\[ \text{H₀: } \mu = \mu_0 \]
\[ \text{H₁: } \mu > \mu_0 \]

At significance level \( \alpha = 0.05 \), the computed statistic is given as:

\[ t = \frac{\bar{\mu} - \mu_0}{\sqrt{\frac{S^2}{n}}} \]

Where \( t \) is referred to as student-\( t \) distribution statistic, \( \mu \) is the mean of converted coordinates, \( \mu_0 \) is the mean of the standard coordinates, \( S \) is the standard deviation (SD) of the converted coordinates and \( n \) is the total number of data.

Reject \( H₀ \), if \( t > t_{0.05, \alpha} \)
Accept \( H₀ \), if \( t < t_{0.05, \alpha} \)

Draw appropriate conclusion based on the acceptance or rejection of Null Hypothesis.

A. Statistical evaluation for the converted Space Rectangular Coordinates

1) Test for Easting Coordinates (X)

Mean of standard Eastings, \( \mu_0 = 6232287.691 \)
Mean of converted Eastings, \( \mu = 6232287.68971 \)
Standard Deviation, SD = 75910.6747647
\( t = 0.980368.5374 \)
Degree of freedom = 10-1 = 9
Accept \( H₀ \) if \( t < t_{0.05, 9} \)
\( t_{0.05, 9} = 1.833 \) (from table)
Since \( t = 0.00000003004 < 1.833 \), the null hypothesis is accepted.

Therefore, it can be concluded that there is no significant difference between the standard and the converted easting coordinates.

2) Test for Northing Coordinates (Y)

Mean of standard Northings, \( \mu_0 = 807873.091 \)
Mean of converted Northings, \( \mu = 807873.093774 \)
Standard Deviation, SD = 400.41683151
\( t = 1.833 \) (from table)
Since \( t = 0.0000000257 < 1.833 \), the null hypothesis is accepted.

Therefore, it can be concluded that there is no significant difference between the standard and the converted northing coordinates.

3) Test for Ellipsoidal Heights (Z)

Mean of standard Ellipsoidal heights, \( \mu_0 = 386.952298 \)
Mean of converted Ellipsoidal heights, \( \mu = 386.951 \)
Standard Deviation, SD = 400.41683151
\( t = 0.00010251 \)
Degree of freedom = 10-1 = 9
Accept \( H₀ \) if \( t < t_{0.05, 9} \)
\( t_{0.05, 9} = 1.833 \) (from table)
Since \( t = 0.000000003004 < 1.833 \), the null hypothesis is accepted.

Therefore, it can be concluded that there is no significant difference between the standard and the converted ellipsoidal coordinates.
t0.95, 9 = 1.833 (from table)
Since t = 0.000010251 < 1.833, the null hypothesis is accepted.
Therefore, it can be concluded that there is no significant difference between the standard and the converted ellipsoidal height coordinates.

V. CONCLUSION
This paper has attempted to develop a new coordinate conversion techniques known as the ‘Optimum Coordinate Conversion Model’ for the transformation of space rectangular coordinates to curvilinear coordinates and vice versa. Based on the results obtained and the statistical analysis carried out at 95% confidence level, it can be concluded that the optimum coordinate conversion techniques developed in this study has successfully and accurately produced optimum results for the conversion of space rectangular coordinate to curvilinear coordinates and vice versa. Also, this has removed the inherent limitations observed in other techniques of coordinate conversion. Therefore, it is recommended that the optimum technique should be adopted for the conversions between space rectangular and curvilinear coordinates which are very vital in geodetic positioning, mapping and navigations all other aspects of Geodesy.

REFERENCES
[1] W. E. Featherstone and P. Vaníček, “The role of coordinate systems, coordinates and heights in horizontal datum transformations”, The Australian Surveyor, 44, 143–150, 1999.
[2] T. Fukushima, “Transformation from Spatial to Geographic coordinates Accelerated by Halley’s Method”, Journal of Geodesy, 79 (12): 689-693, 2006.
[3] W. Heiskanen and H. Moritz, Physical Geodesy, W.H. Freeman and Company, San Francisco, CA, 1967.
[4] B. Hofman-Wellenhof and H. Moritz, Physical Geodesy, 2nd Edition, Springer Wien, New York, 2006.
[5] T. O. Idowu, E. G. Ayodele and I. D. Dodo, “A comparison of Methods of Computing Geodetic Coordinates from Cartesian Coordinates”, National Journal of Space Research 6: 1-20, 2009.
[6] G. Panou, “Cartesian to Geodetic coordinates conversion on an oblate spheroid using the bisection method”, Researchgate.net/publication, 2019.
[7] R. H. Rapp, Geometric Geodesy Part I. Lecture Notes, Department of Geodetic Science and Surveying, Ohio State University, Columbus, Ohio, USA pp 24-26,1991.
[8] H. Vermeille, “Direct transformation from geocentric to geodetic coordinates”, J. Geodesy, 76, 451-454, doi:10.1007/s00190-002-0273-6, 2002.
[9] G. Suara, “Development of a computational tool for the optimum conversion of space rectangular and curvilinear coordinates”, M.Tech Dissertation, Department of Surveying & Geoinformatics, Federal University of Technology, Akure, Ondo Nigeria, 2019.

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