Multi-Antenna Gaussian Broadcast Channels with Confidential Messages

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Abstract—In wireless data networks, communication is particularly susceptible to eavesdropping due to its broadcast nature. Security and privacy systems have become critical for wireless providers and enterprise networks. This paper considers the problem of secret communication over a Gaussian broadcast channel, where a multi-antenna transmitter sends independent confidential messages to two users with information-theoretic secrecy. That is, each user would like to obtain its own confidential message in a reliable and safe manner. This communication model is referred to as the multi-antenna Gaussian broadcast channel with confidential messages (MGBC-CM). Under this communication scenario, a secret dirty-paper coding scheme and the corresponding achievable secrecy rate region are first developed based on Gaussian codebooks. Next, a computable Sato-type outer bound on the secrecy capacity region is provided for the MGBC-CM. Furthermore, the Sato-type outer bound proves to be consistent with the boundary of the secret dirty-paper coding achievable rate region, and hence, the secrecy capacity region of the MGBC-CM is established. Finally, a numerical example demonstrates that both users can achieve positive rates simultaneously under the information-theoretic secrecy requirement.

I. INTRODUCTION

The demand for efficient, reliable, and secret data communication over wireless networks has become increasingly critical in recent years. Due to its broadcast nature, wireless communication is particularly susceptible to eavesdropping. The inherent nature of wireless networks exposes not only vulnerabilities that a malicious user can exploit to severely compromise the network, but also multiplies information confidentiality concerns with respect to in-network terminals. Hence, security and privacy systems have become critical for wireless providers and enterprise networks.

In this work, we consider multiple antenna secret broadcast in wireless networks. This research is inspired by the seminal paper [1], in which Wyner introduced the so-called wiretap channel and proposed an information theoretic approach to secret communication schemes. Under the assumption that the channel to the eavesdropper is a degraded version of that to the desired receiver, Wyner characterized the capacity-secrecy tradeoff for the discrete memoryless wiretap channel and showed that secret communication is possible without sharing a secret key. Later, the result was extended by Csiszár and Körner who determined the secrecy capacity for the non-degraded broadcast channel (BC) with a single confidential message intended for one of the users [2].

In more general wireless network scenarios, secret communication may involve multiple users and multiple antennas. Consequently, a significant recent research effort has been invested in the study of the information-theoretic limits of secret communication in different wireless network environments including multi-user communication with confidential messages [3]–[8], secret wireless communication on fading channels [9]–[11], and the Gaussian multiple-input single-output (MISO) and multiple-input multiple-output (MIMO) wiretap channels [12]–[16].

These issues and results motivate us to study the multi-antenna Gaussian BC with confidential messages (MGBC-CM), in which independent confidential messages from a multi-antenna transmitter are to be communicated to two users. The corresponding broadcast communication model is shown in Fig. 1. Each user would like to obtain its own message reliably and confidentially.

To give insight into this problem, we first consider a single-antenna Gaussian BC. Note that this channel is degraded [17], which means that if a message can be successfully decoded by the inferior user, then the superior user is also ensured of decoding it. Hence, the secrecy rate of the inferior user is zero and this problem is reduced to the scalar Gaussian wiretap channel problem [18] whose secrecy capacity is now the maximum rate achievable by the superior user. This analysis gives rise to the question: can the transmitter, in fact, communicate with both users confidentially at nonzero rate under some other conditions? Roughly speaking, the answer is in the affirmative. In particular, the transmitter can communicate when equipped

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with sufficiently separated multiple antennas.

We here have two goals motivated directly by questions arising in practice. The first is to determine conditions under which both users can obtain their own confidential messages in a reliable and safe manner. This is equivalent to evaluating the secrecy capacity region for the MGBC-CM. The second is to show how the transmitter should broadcast confidentially, which is equivalent to designing an achievable secret coding scheme. To this end, we first describe a secret dirty-paper coding (DPC) scheme and derive the corresponding achievable rate region based on Gaussian codebooks. The secret DPC is based on double-binning [6] which enables both joint encoding and preserving confidentiality. Next, a computable Sato-type outer bound on the secrecy capacity region is developed for the MGBC-CM. The second is motivated directly by questions raised by the execution of the former scheme and preserving confidentiality. Next, a computable Sato-type outer bound on the secrecy capacity region is developed for the MGBC-CM. The second is motivated directly by questions raised by the execution of the former scheme.

II. SYSTEM MODEL AND DEFINITIONS

A. Channel Model

We consider the communication of confidential messages to two users over a Gaussian BC via $t \geq 2$ transmit-antennas. Each user is equipped with a single receive-antenna. As shown in Fig. 1, the transmitter sends independent confidential messages $W_1$ and $W_2$ in $n$ channel uses with $nR_1$ and $nR_2$ bits, respectively. The message $W_1$ is destined for user 1 and eavesdropped upon by user 2, whereas the message $W_2$ is destined for user 2 and eavesdropped upon by user 1. This communication scenario is referred to as the multi-antenna Gaussian BC with confidential messages. The Gaussian BC is an additive noise channel and the received symbols at user 1 and user 2 can be represented as follows:

$$y_{1,i} = h^H x_i + z_{1,i}, \quad y_{2,i} = g^H x_i + z_{2,i}, \quad i = 1, \ldots, n$$ (1)

where $x_i \in \mathbb{C}^t$ is a complex input vector at time $i$, $\{z_{1,i}\}$ and $\{z_{2,i}\}$ correspond to two independent, zero-mean, unit-variance, complex Gaussian noise sequences, and $h, g \in \mathbb{C}^t$ are fixed, complex channel attenuation vectors imposed on user 1 and user 2, respectively. The channel input is constrained by $\text{tr}(KX) \leq P$, where $KX$ is the channel input covariance matrix and $P$ is the average total power limitation at the transmitter. We also assume that both the transmitter and users are aware of the attenuation vectors.

B. Important Channel Parameters for the MGBC-CM

For the MGBC-CM, we are interested in the following important parameters, which are related to the generalized eigenvalue problem. Let $\lambda_1$ and $e_1$ denote the largest generalized eigenvalue and the corresponding normalized eigenvector of the pencil $(I + Phh^H, I + Pgg^H)$ so that $e_1^H e_1 = 1$ and

$$(I + Phh^H)e_1 = \lambda_1(I + Pgg^H)e_1.$$ (2)

Similarly, we define $\lambda_2$ and $e_2$ as the largest generalized eigenvalue and the corresponding normalized eigenvector of the pencil $(I + Pgg^H, I + Phh^H)$ so that $e_2^H e_2 = 1$ and

$$(I + Pgg^H)e_2 = \lambda_2(I + Phh^H)e_2.$$ (3)

A useful property of $\lambda_1$ and $\lambda_2$ is described as follows.

Lemma 1: For any channel attenuation vector pair $h$ and $g$, the largest generalized eigenvalues of the pencil $(I + Phh^H, I + Pgg^H)$ and the pencil $(I + Pgg^H, I + Phh^H)$ satisfy $\lambda_1 \geq 1$ and $\lambda_2 \geq 1$. Moreover, if $h$ and $g$ are linearly independent, then both $\lambda_1$ and $\lambda_2$ are strictly greater than 1.

C. Definitions

We now define the secret codebook, the probability of error, the secrecy level, and the secrecy capacity region for the MGBC-CM as follows.

An $(2^{nR_1}, 2^{nR_2}, n)$ secret codebook for the MGBC-CM consists of the following:

1) Two message sets: $W_k = \{1, \ldots, 2^{nR_k}\}$, for $k = 1, 2$.
2) A stochastic encoding function specified by a conditional probability density $p(x^n|w_1, w_2)$, where $x^n = [x_1, \ldots, x_n] \in \mathbb{C}^{t \times n}$, $w_k \in W_k$ for $k = 1, 2$, and

$$\int p(x^n|w_1, w_2) = 1.$$ (3)

3) Decoding functions $\phi_1$ and $\phi_2$. The decoding function at user $k$ is a deterministic mapping $\phi_k : Y_k^n \rightarrow W_k$.

At the receiver ends, the error performance and the secrecy level are evaluated by the following performance measures.

1) The reliability is measured by the maximum error probability

$$P_e(n) \triangleq \max \{P_{e,1}^{(n)}, P_{e,2}^{(n)}\}$$

where $P_{e,k}^{(n)}$ is the error probability for user $k$.

2) The secrecy levels with respect to confidential messages $W_1$ and $W_2$ are measured, respectively, at user 2 and user 1 with respect to the equivocation rates $\frac{1}{n}H(W_1|Y_2^n)$ and $\frac{1}{n}H(W_2|Y_1^n)$.

A rate pair $(R_1, R_2)$ is said to be achievable for the MGBC-CM if, for any $\epsilon > 0$, there exists an $(2^{nR_1}, 2^{nR_2}, n)$ code that satisfies $P_e^{(n)} \leq \epsilon$, and the information-theoretic secrecy requirement

$$nR_1 - H(W_1|Y_2^n) \leq n\epsilon \quad \text{and} \quad nR_2 - H(W_2|Y_1^n) \leq n\epsilon.$$ (4)

The secrecy capacity region $C_s^M$ of the MGBC-CM is the closure of the set of all achievable rate pairs $(R_1, R_2)$.

III. MAIN RESULT

The two-user Gaussian BC with multiple transmit-antennas is non-degraded. For this channel, we have the following closed-from result on the secrecy capacity region under the information-theoretic secrecy requirement.
Theorem 1: Consider an MGBC-CM modeled as in (1). Let
\[ \gamma_1(\alpha) = \frac{1 + \alpha P|g^H e_1|^2}{1 + \alpha P|g^H e_1|^2}, \]
\[ \gamma_2(\alpha) \] be the largest generalized eigenvalue of the pencil
\[ \left( I + \frac{(1 - \alpha)Pgg^H}{1 + \alpha P|g^H e_1|^2}, I + \frac{(1 - \alpha)Phh^H}{1 + \alpha P|h^H e_1|^2} \right), \] (5)
and \( R^{MG}(\alpha) \) denote the union of all \((R_1, R_2)\) satisfying
\[ 0 \leq R_1 \leq \log_2 \gamma_1(\alpha) \]
and \[ 0 \leq R_2 \leq \log_2 \gamma_2(\alpha). \]

Then the secrecy capacity region of the MGBC-CM is
\[ C_s^{MG} = co \left\{ \bigcup_{0 \leq \alpha \leq 1} R^{MG}(\alpha) \right\} \]
where \( co\{S\} \) denotes the convex hull of the set \( S \).

Proof: The achievability part of Theorem 1 is based on secret dirty-paper coding outer bound in Sec. [V]. The converse part is based on Sato-type outer bound in Sec. [V]. We provide the complete proof in [14].

Corollary 1: For the MGBC-CM, the maximum secrecy rate of user 1 is given by
\[ R_{1,\text{max}} = \max_{0 \leq \alpha \leq 1} \log_2 \gamma_1(\alpha) = \log_2 \lambda_1 \]
where \( \lambda_1 \) is defined in (2).

Example: (MISO wiretap channels) A special case of the MGBC-CM model is the Gaussian MISO wiretap channel studied in [12], [19], [20], where the transmitter sends confidential information to only one user and treats another user as an eavesdropper. Let us consider a Gaussian MISO wiretap channel modeled in (1), where user 1 is the legitimate receiver and user 2 is the eavesdropper. Corollary 1 implies that the secrecy capacity of the Gaussian MISO wiretap channel corresponds to the corner point of \( C_s^{MG} \). Hence, the secrecy capacity of the Gaussian MISO wiretap channel is given by
\[ C_s^{MISO} = \log_2 \lambda_1, \]
which coincides with the result of [19].

For the MGBC-CM, the actions of user 1 and user 2 are symmetric to each other, i.e., each user decodes its own message and eavesdrops upon the confidential information belonging to the other user. Based on symmetry of this two-user BC model, we can express the secrecy capacity region \( C_s^{MG} \) in an alternative way.

Corollary 2: For an MGBC-CM modeled in as (1), the secrecy capacity region can be written as
\[ C_s^{MG} = co \left\{ \bigcup_{0 \leq \beta \leq 1} R^{MG-2}(\beta) \right\} \]
where \( R^{MG-2}(\beta) \) denotes the union of all \((R_1, R_2)\) satisfying
\[ 0 \leq R_1 \leq \log_2 \xi_1(\beta) \]
and \[ 0 \leq R_2 \leq \log_2 \xi_2(\beta), \]
\( \xi_1(\beta) \) is the largest generalized eigenvalue of the pencil
\[ \left( I + \frac{(1 - \beta)Pgg^H}{1 + \beta P|g^H e_2|^2}, I + \frac{(1 - \beta)Phh^H}{1 + \beta P|h^H e_2|^2} \right) \]
and
\[ \xi_2(\beta) = \frac{1 + \beta P|g^H e_2|^2}{1 + \beta P|h^H e_2|^2}. \]

Remark 1: Theorem 1 and Corollary 2 imply that if \( \alpha \) and \( \beta \) satisfy the implicit function \( \gamma_1(\alpha) = \xi_1(\beta) \), then
\[ R^{MG}(\alpha) = R^{MG-2}(\beta). \]

For example, it is easy to check \( R^{MG}(1) = R^{MG-2}(0) \).

Now, by applying Corollary 2 and setting \( \beta = 1 \), we can show that the rate pair \((0, \log_2 \lambda_2)\) is the corner point corresponding to the maximum achievable rate of user 2 in the capacity region \( C_s^{MG} \).

Corollary 3: For the MGBC-CM, the maximum secrecy rate of user 2 is given by
\[ R_{2,\text{max}} = \log_2 \lambda_2 \]
where \( \lambda_2 \) is defined in (3).

Corollaries 1 and 3 imply that for the MGBC-CM, both users can achieve positive rates with information-theoretic secrecy if and only if \( \lambda_1 > 1 \) and \( \lambda_2 > 1 \). Furthermore, Lemma 1 illustrates that this condition can be ensured when the attenuation vectors \( h \) and \( g \) are linearly independent.

IV. ACHIEVABILITY: SECRET DPC SCHEME

A. Double-Binning Inner bound for the BC-CM

An achievable rate region for the broadcast channel with confidential messages (BC-CM) has been established in [6] based on a double-binning scheme that enables both joint encoding at the transmitter by using Gel’fand-Pinsker binning and preserving confidentiality by using random binning.

Lemma 2: ([6, Theorem 3]) Let \( V_1 \) and \( V_2 \) be auxiliary random variables, \( \Omega \) denote the class of joint probability densities \( p(v_1, v_2, x, y_1, y_2) \) that factor as
\[ p(v_1, v_2)p(x|v_1, v_2)p(y_1, y_2|x), \]
and \( R_4(\pi) \) denote the union of all \((R_1, R_2)\) satisfying
\[ 0 \leq R_1 \leq I(V_1; Y_1) - I(V_1; Y_2, V_2) \]
and \[ 0 \leq R_2 \leq I(V_2; Y_2) - I(V_2; Y_1, V_1) \]
for a given joint probability density \( \pi \in \Omega \). For the BC-CM, any rate pair
\[ (R_1, R_2) \in co \left\{ \bigcup_{\pi \in \Omega} R_1(\pi) \right\} \] (6)
is achievable.
B. Secret DPC Scheme for the MGBC-CM

The achievable strategy in Lemma 2 introduces a double-binning coding scheme. However, when the rate region (6) is used as a constructive technique, it is not clear how to choose the auxiliary random variables \( \mathbf{V}_1 \) and \( \mathbf{V}_2 \) to implement the double-binning codebook, and hence, one has to “guess” the density of \( p(\mathbf{v}_1, \mathbf{v}_2, \mathbf{x}) \). Here, we employ the DPC technique with the double-binning code structure to develop the secret DPC (S-DPC) achievable rate region.

For the MGBC-CM, we consider a secret dirty-paper encoder with Gaussian codebooks. Based on Lemma 2, we obtain a S-DPC rate region for the MGBC-CM as follows.

**Lemma 3:** [S-DPC region] Let \( \mathcal{R}^{S-DPC}(K_{U_1}, K_{U_2}) \) denote the union of all rate pairs \((R_1, R_2)\) satisfying

\[
0 \leq R_1 \leq \log \left( \frac{1 + h^H K_{U_1} h}{1 + g^H K_{U_1} g} \right) \tag{7}
\]

and

\[
0 \leq R_2 \leq \log \left( \frac{1 + h^H (K_{U_1} + K_{U_2}) g}{1 + h^H (K_{U_1} + K_{U_2}) h} \cdot \frac{1 + h^H K_{U_1} h}{1 + g^H K_{U_1} g} \right) \tag{8}
\]

Then, any rate pair

\[
(R_1, R_2) \in \mathcal{R}^{S-DPC}(K_{U_1}, K_{U_2})
\]

is achievable for the MGBC-CM.

**Proof:** A detailed proof can be found in [14].

The S-DPC achievable rate region requires optimization of the covariance matrices \( K_{U_1} \) and \( K_{U_2} \). In order to achieve the boundary of \( C_s^{MG} \), we choose \( K_{U_1} \) and \( K_{U_2} \) as follows:

\[
K_{U_1} = \alpha P e_1 e_1^H \\
K_{U_2} = (1 - \alpha) P c_2(\alpha) c_2^H(\alpha), \quad 0 \leq \alpha \leq 1 \tag{9}
\]

where \( e_1 \) is defined in (2) and \( c_2(\alpha) \) is a normalized eigenvector of the pencil (5) corresponding to \( \gamma_2(\alpha) \). Next, inserting (9) into (7) and (8), we obtain

\[
\frac{1 + h^H K_{U_1} h}{1 + g^H K_{U_1} g} = \gamma_1(\alpha)
\]

and

\[
\frac{1 + g^H (K_{U_1} + K_{U_2}) g}{1 + h^H (K_{U_1} + K_{U_2}) h} \cdot \frac{1 + h^H K_{U_1} h}{1 + g^H K_{U_1} g} = \gamma_2(\alpha). \tag{10}
\]

Now, by substituting (10) into Lemma 3, we obtain the desired achievable result.

V. CONVERSE: Sato-TYPE OUTER BOUND

A. Sato-Type Outer Bound

We consider an important property for the BC-CM in the following lemma.

**Lemma 4:** Let \( \mathcal{P} \) denote the set of channels \( p_{\tilde{Y}_1, \tilde{Y}_2|x} \) whose marginal distributions satisfy

\[
p_{\tilde{Y}_1|x}(y_1|x) = p_{Y_1|x}(y_1|x)
\]

and

\[
p_{\tilde{Y}_2|x}(y_2|x) = p_{Y_1|x}(y_2|x)
\]

Then, for all \( y_1, y_2 \) and \( x \). The secrecy capacity region \( C_s^{MG} \) is the same for the channels \( p_{\tilde{Y}_1, \tilde{Y}_2|x} \in \mathcal{P} \).

We note that \( \mathcal{P} \) is the set of channels \( p_{\tilde{Y}_1, \tilde{Y}_2|x} \) that have the same marginal distributions as the original channel transition density \( p_{Y_1, Y_2|x} \). Lemma 4 implies that the secrecy capacity region \( C_s^{MG} \) depends only on marginal distributions.

**Theorem 2:** Let \( \mathcal{R}_O(P_{Y_1, Y_2|x}, P_X) \) denote the union of all rate pairs \((R_1, R_2)\) satisfying

\[
R_1 \leq I(X; \tilde{Y}_1 | Y_2)
\]

and

\[
R_2 \leq I(X; \tilde{Y}_2 | Y_1)
\]

for given distributions \( P_X \) and \( P_{Y_1, Y_2|x} \). The secrecy capacity region \( C_s^{MG} \) of the BC-CM satisfies

\[
C_s^{MG} \subseteq \bigcap_{p_{\tilde{Y}_1, \tilde{Y}_2}|x} \left\{ \mathcal{R}_O(P_{\tilde{Y}_1, \tilde{Y}_2|x}, P_X) \right\}. \tag{11}
\]

**Proof:** A detailed proof can be found in [14].

**Remark 2:** The outer bound (11) follows by evaluating the secrecy level at each receiver end in an individual manner, while letting the users decode their messages in a cooperative manner. In this sense, we refer to this bound as “Sato-type” outer bound.

For example, we consider the confidential message \( W_1 \) that is destined for user 1 (corresponding to \( \tilde{Y}_1 \)) and eavesdropped upon by user 2 (corresponding to \( \tilde{Y}_2 \)). We assume that a genie gives user 1 the signal \( Y_2 \) as side information for decoding \( W_1 \). Note that the eavesdropped upon signal \( Y_2 \) at user 2 is always a degraded version of the entire received signal \( (\tilde{Y}_1, \tilde{Y}_2) \). This permits the use of the wiretap channel result of [1].

**Remark 3:** Although Theorem 2 is based on a degraded argument, the outer bound (11) can be applied to the general broadcast channel with confidential messages.

B. Sato-Type Outer Bound for the MGBC-CM

For the Gaussian BC, the family \( \mathcal{P} \) is the set of channels

\[
\tilde{y}_1 = h^H x + \tilde{z}_1 \\
\tilde{y}_2 = g^H x + \tilde{z}_2
\]

where \( \tilde{z}_1 \) and \( \tilde{z}_2 \) correspond to arbitrarily correlated, zero-mean, unit-variance, complex Gaussian random variables. Let \( \rho \) denote the covariance between \( \tilde{Z}_1 \) and \( \tilde{Z}_2 \), i.e.,

\[
\text{Cov}(\tilde{Z}_1, \tilde{Z}_2) = \rho \quad \text{and} \quad |\rho|^2 \leq 1.
\]

Now, the rate region \( \mathcal{R}_O(P_{\tilde{Y}_1, \tilde{Y}_2|x}, P_X) \) is a function of the noise covariance \( \rho \) and the input covariance matrix \( K_X \). We consider a computable Sato-type outer bound for the MGBC-CM in the following lemma.

**Lemma 5:** Let \( \mathcal{R}_O^{MC}(\rho, K_X) \) denote the union of all rate pairs \((R_1, R_2)\) satisfying

\[
0 \leq R_1 \leq f_1(\rho, K_X)
\]

and

\[
0 \leq R_2 \leq f_2(\rho, K_X)
\]
where

\[
f_1(\rho, K_X) = \min_{\nu \in \mathcal{C}} \log_2 \frac{(h - \nu g)^H K_X (h - \nu g) + \psi_1(\nu, \rho)}{(1 - |\rho|^2)}
\]

\[
f_2(\rho, K_X) = \min_{\mu \in \mathcal{C}} \log_2 \frac{(g - \mu h)^H K_X (g - \mu h) + \psi_2(\mu, \rho)}{(1 - |\rho|^2)}
\]

\[
\psi_1(\nu, \rho) = 1 + |\nu|^2 - \nu^* \rho - \rho^* \nu
\]

and

\[
\psi_2(\mu, \rho) = 1 + |\mu|^2 - \mu^* \rho - \rho^* \mu.
\]

For the MGBC-CM, the secrecy capacity region \( \mathcal{C}_s^{MG} \) satisfies

\[
\mathcal{C}_s^{MG} \subseteq \bigcup_{\text{tr}(K_X) \leq P} \mathcal{R}_O(\rho, K_X)
\]

for any \( 0 \leq |\rho| \leq 1 \).

**Remark 4:** Lemma 5 is based on the fact that Gaussian input distributions maximize \( \mathcal{R}_O(P_{Y_1, Y_2} | X \), \( F_X \) for Gaussian broadcast channel. To illustrate this point, we consider

\[
I(X; Y_1 | Y_2) = h(\tilde{Y}_1 | \tilde{Y}_2) - \log_2(2\pi e)(1 - |\rho|^2)
\]

\[
\leq h(\tilde{Y}_1 - \nu Y_2) - \log_2(2\pi e)(1 - |\rho|^2).
\]

Moreover, the maximum-entropy theorem [17] implies that \( h(\tilde{Y}_1 - \nu Y_2) \) is maximized by Gaussian input distributions.

Finally, we prove that the Sato-type outer bound of Lemma 5 coincides with the secrecy capacity region \( \mathcal{C}_s^{MG} \) by choosing the parameter \( \rho = (g^H e_1) / (h^H e_1) \). A detail proof can be found in [14].

### VI. Numerical Examples

In this section, we study a numerical example to illustrate the secrecy capacity region of the MGBC-CM. For simplicity, we assume that the Gaussian BC has real input and output alphabets and the channel attenuation vectors \( h \) and \( g \) are also real. Under these conditions, all calculated rate values are divided by 2.

![Fig. 2. Secrecy capacity region vs. time-sharing secrecy rate region for the example MGBC-CM in (12).](image)

In particular, we consider the following MGBC-CM

\[
\begin{bmatrix}
y_1 \\
y_2
\end{bmatrix} = \begin{bmatrix} 1.5 & 0 \\ 1.801 & 0.871 \end{bmatrix} \begin{bmatrix} x_1 \\
x_2
\end{bmatrix} + \begin{bmatrix} z_1 \\
z_2
\end{bmatrix}
\]

(12)

where \( h = [1.5, 0]^T \), \( g = [1.801, 0.872]^T \), and the total power constraint is set to \( P = 10 \). Fig. 2 illustrates the secrecy capacity region for the channel (12). We observe that even though each component of the attenuation vector \( h \) (imposed on user 1) is strictly less than the corresponding component of \( g \) (imposed on user 2), both users can achieve positive rates simultaneously under the information-theoretic secrecy requirement. Moreover, we compare the secrecy capacity region with the secrecy rate region achieved by the time-sharing scheme (indicated by the dash-dot line). Fig. 2 demonstrates that the time-sharing scheme is strictly suboptimal for providing the secrecy capacity region.

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