Large-momentum tail of one-dimensional Fermi gases with spin-orbit coupling

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We study the contacts, large-momentum tail, radio-frequency spectroscopy, and some other universal relations for an ultracold one-dimensional (1D) two-component Fermi gas with spin-orbit coupling (SOC). Different from previous studies, we find that the \( q^8 \) tail in the spin-mixing (off-diagonal) terms of the momentum distribution matrix is dependent on the two SOC parameters in the laboratory frame for 1D systems, where \( q \) is the relative momentum. This tail can be observed through time-of-flight measurement as a direct manifestation of the SOC effects on the many-body level. Besides the traditional 1D even-wave scattering length, we find that two new physical quantities must be introduced due to the SOC. Consequently, two new adiabatic energy relations with respect to the two SOC parameters are obtained. Furthermore, we derive the pressure relation and virial theorem at short distances for this system. To find how the SOC modifies the large-momentum behavior, we take the SOC parameters as perturbations since the strength of the SOC should be much smaller than the corresponding strength scale of the interatomic interactions. In addition, by using the operator product expansion method, we derive the asymptotic behavior of the large-momentum distribution matrix up to the \( q^{-8} \) order and find that the diagonal terms of the distribution matrix include the contact of traditional 1D even-wave scattering length as the leading term and the SOC modified terms beyond the leading term, the off-diagonal term is beyond the subleading term and is corrected by the SOC parameters. We also find that the momentum distribution matrix shows spin-dependent and anisotropic features. Furthermore, we calculate the momentum distribution matrix in the laboratory frame for the experimental implication. In addition, we calculate the high-frequency tail of the radio-frequency spectroscopy and find that the presence of the contact related to the center-of-mass momentum in the radio-frequency spectral is due to the SOC effects. This paper paves the way for exploring the profound properties of many-body quantum systems with SOC in one dimension.

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I. INTRODUCTION

In the strong-coupling regime of cold atomic gases, a series of exact universal relations was established which set up a bridge between the microscopic short-distance correlations and the macroscopic thermodynamic properties of the many-body quantum system [1–7]. These relations show that many thermodynamic properties are connected by a series of universal contact parameters which contain the information of the interaction effect in the large-momentum limit, and they are named contacts. These relations have already been successfully verified in experiments near the \( s \)-wave Feshbach resonance [8–11]. Furthermore, the universal relations were also studied in other atomic systems, such as the quantum gases with higher-partial-wave interactions [12–19], in one dimension [20–24] and two dimensions [25–32] with three-body correlations [33–39] near a Feshbach resonance [40] and with ultracold polar molecules [41]. Besides, the contact tensor was predicted in the axisymmetry-broken \( p \)-wave Fermi gases [42]. At the same time, Zhang et al. predicted the contact matrix and studied the direct connection between the contact matrix and the order parameter of a superfluid [43]. The nuclear neutron-proton contact was introduced in nuclear physics [44], and the general nuclear contact matrices were defined [45] in the context of generalized contacts for realistic interactions [46–50]. There are also other works using Coulomb [51] and realistic atom-atom interactions [52] with the generalized contact formalism.

The successful experimental realization of the synthetic coupling between atomic (pseudo) spin and momentum is another important progress in cold atomic gases [53–59]. This synthetic coupling is so-called spin-orbit coupling (SOC). In one dimension, there is only one type of SOC: The spin is only coupled to the motion of atoms along one spatial direction which is induced by two contour-propagating Raman beams [60–63]. This type of SOC has been realized in experiments for both Bose and Fermi gases [53–57]. Importantly, the SOC can strongly affect the many-body properties [64,65].

Recently, Peng et al. studied the universal relations for the Fermi gases with a three-dimensional isotropic SOC [66]. At the same time, the universal relations for the
three-dimensional ultracold gases with an arbitrary type of SOC have been investigated [67,68]. Furthermore, the universal relations for spin-orbit-coupled Fermi gases in two dimensions have been derived [69]. However, the one-dimensional (1D) Fermi gases with SOC has not been studied. Here, we investigate the 1D Fermi gases with SOC. Different from the previous studies, we derive the asymptotic behavior of the large-momentum distribution matrix up to the $q^3$ order ($q$ is the relative momentum) and find that the $q^{-3}$ tail in the spin-mixing (off-diagonal) terms of the momentum distribution matrix is dependent on the SOC parameters in the laboratory frame for 1D systems. This tail can be observed through time-of-flight measurement, and it is a direct manifestation of the SOC effects on the many-body level.

In this paper, we focus on a many-body quantum system that exhibits universal properties, i.e., the 1D two-component Fermi gas with SOC near a broad $s$-wave Feshbach resonance. Such a 1D system can be realized by applying tight transverse confinements to three-dimensional gases near $s$-wave Feshbach resonances, such as in the fermions of $^6$Li and $^{40}\text{K}$ [70–78]. First of all, we give the definition of the single-particle part of the Hamiltonian in the momentum space: $H_0 = \sum_k \psi_k^\dagger H_0 \psi_k$ with $\psi_k = (a_{k\uparrow}, a_{k\downarrow})^T$ and

$$H_0 = \begin{pmatrix} \frac{(k_{\downarrow}-k_{\uparrow})^2}{2m} & \Omega \cr \Omega & \frac{(k_{\uparrow}-k_{\downarrow})^2}{2m} \end{pmatrix},$$

where $a_{k\sigma}$ is the field operator for the fermionic atoms in the momentum space.

Therefore, the inverse of the single-particle propagator matrix is given by [68,79,80]

$$G^{-1}(q_0, k) = -i\begin{pmatrix} g_0 + i\Omega + \frac{(k_{\uparrow}-k_{\downarrow})^2}{2m} & -\Omega \\
-\Omega & g_0 + i\Omega + \frac{(k_{\downarrow}-k_{\uparrow})^2}{2m} \end{pmatrix},$$

where $g_0$ is the total energy. Furthermore, we have

$$G(q_0, k) = \int_0^\infty dt e^{i\Omega t} \langle T \Psi_k(t) \Psi_k^\dagger(0) \rangle = \begin{pmatrix} G_{\uparrow\uparrow}(q_0, k) & G_{\uparrow\downarrow}(q_0, k) \\
G_{\downarrow\uparrow}(q_0, k) & G_{\downarrow\downarrow}(q_0, k) \end{pmatrix},$$

where $T$ is the time-ordered operator and the elements are

$$G_{\uparrow\uparrow}(q_0, k) = \frac{i}{q_0 - \frac{(k_{\uparrow}-k_{\downarrow})^2}{2m} + i\Omega^+ - \frac{\Omega^2}{2g_0 - \frac{(k_{\uparrow}-k_{\downarrow})^2}{2m} + i\Omega^+}},$$

$$G_{\uparrow\downarrow}(q_0, k) = \frac{i\Omega}{q_0 - \frac{(k_{\uparrow}-k_{\downarrow})^2}{2m} + i\Omega^+ - \frac{\Omega^2}{2g_0 - \frac{(k_{\uparrow}-k_{\downarrow})^2}{2m} + i\Omega^+}}.$$

To calculate the Feynman diagrams for simplicity, the interacting part of the Hamiltonian can be written as

$$H_{\text{int}} = g_{1D} \int dx \left[ \frac{1}{2} \psi(x) e^{-i\Omega^+} \psi^\dagger(x) + \frac{1}{2} \psi^T(x) e^{i\Omega} \psi(x) \right].$$
where $\epsilon = -i\sigma_y$ is the two-by-two antisymmetric matrix

$\Psi(x) = [\psi_1(x), \psi_2(x)]^T$ and we have

$$\frac{1}{2} \Psi^T(x) \epsilon \Psi(x) = \frac{1}{2} [\psi_1(x) \psi_2(x) - \psi_2(x) \psi_1(x)] = \psi_1(x) \psi_1(x).$$

We can also write the interaction part of the Hamiltonian in the momentum space [68,81–83],

$$H_{\text{int}} = g_{1D} \sum_{k,k'} \left( \frac{1}{2} \Psi_{k,k'}^+ \epsilon \Psi_{k,k'}^T \right) \left( \frac{1}{2} \Psi_{k,k'}^T \epsilon \Psi_{k,k'} \right).$$

The bare coupling constant $g_{1D}$ is given by [84]

$$\Pi_s(q_0, Q) = \int \frac{dp dp_0}{(2\pi)^2} \frac{1}{2} \text{Tr}[G^T(p_0, Q/2 + p) G(q_0 - p_0, Q/2 - p) \epsilon^\dagger]$$

as shown in Fig. 1, the two-body $T$ matrix is given by [84]

$$\Pi_s(q_0, Q) = \frac{1}{1 - (-i g_{1D}) \Pi_s(q_0, Q)},$$

where the polarization bubble is given by Refs. [68,81–83] (the derivations are given in the Appendix).

III. TWO-BODY PHYSICS

The two-body scattering process can be described by the two-body scattering amplitude or the scattering phase shift which can be determined by the two-body $T$ matrix in physics. In order to show the bare coupling constant $g_{1D}$ for the even-wave case needs to be renormalized or not, it is necessary to calculate the two-body $T$ matrix at first. Because the $T$ matrix is a physical quantity describing effective interaction in the low-energy space irrelevant to short-range physics.

We consider finite total momentum $Q$ for each two-body pairing state so that an incoming state can be set as $\langle \psi_{\uparrow} \rangle = 1$ for two fermions having momenta $\Omega/2 + k$ and $\Omega/2 - k$ to an outgoing state $\langle \psi_{\downarrow} \rangle = 1$ for two fermions having momenta $\Omega/2 + k$ and $\Omega/2 - k$. As shown in Fig. 1, the two-body $T$ matrix can be calculated as

$$\Pi_s(q_0, Q) = \int \frac{dp dp_0}{(2\pi)^2} \frac{1}{2} [G_{1\downarrow}(p_0, Q/2 + p) G_{\downarrow\uparrow}(q_0 - p_0, Q/2 - p) - G_{1\downarrow}(p_0, Q/2 + p) G_{\downarrow\uparrow}(q_0 - p_0, Q/2 - p)].$$

In the absence of the Raman coupling, the even-wave coupling constant is given by (the derivations are given in the Appendix)

$$g_{1D} = \frac{1}{m_a 1D},$$

where $m_a = m/2$ is the reduced mass, the even-wave scattering length in 1D is given by [74]

$$a_{1D} = \frac{\ell_\perp^2}{2a_{1D}} + \frac{\rho_\perp^2}{2},$$

$a_{1D}$ is the three-dimensional scattering length, $C = 1.4603$, $\ell_\perp = \sqrt{2/(m_\perp \omega_\perp)}$, and $\omega_\perp$ is the transverse trapping frequency.

Note that Eq. (15) can only be valid for either a broad Feshbach resonance or the field-free case. Furthermore, the confinement induced resonance should also be broad. Since the external confinement is never perfectly harmonic nor isotropic, the possibility of inelastic resonances is possible. For example, a splitting of confinement-induced resonances has been observed in an anisotropic quasi-1D gas of Cs atoms in experiment [73]. Later, a theoretical model of the inelastic confinement-induced resonance was presented to describe the experimental observations [78].

In the presence of the Raman fields, the polarization bubble with zero total momentum ($Q = 0$) can be calculated as

$$\Pi_s(E_0, 0) = \frac{m^3 \Omega^2}{2(m_0 k_0^2 - k_0^2 + m^2 \Omega^2) \sqrt{m E_0 + i0^+ - k_0^2}} + \frac{m k_0^2}{4} \left[ \sqrt{m E_0 + k_0^2 + 2 \sqrt{m E_0 k_0^2 + m^2 \Omega^2}} \right. $$

$$+ \sqrt{m E_0 + i0^+ + k_0^2 - 2 \sqrt{m E_0 k_0^2 + m^2 \Omega^2}}} \left. + \sqrt{m E_0 + k_0^2 + 2 \sqrt{m E_0 k_0^2 + m^2 \Omega^2}} \right],$$

where $E_0 = k_0^2/m$ and the $+i0^+$ terms are needed when $E_0 < k_0^2/m$ and $m E_0 + k_0^2 < 2 \sqrt{m E_0 k_0^2 + m^2 \Omega^2}$, such as $\sqrt{m E_0 + i0^+ - k_0^2} = \sqrt{m E_0 + k_0^2 + 2 \sqrt{m E_0 k_0^2 + m^2 \Omega^2}}$.
the density of the Hamiltonian (1),
the contact
adiabatic relation (20),
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FIG. 2. Feynman diagrams for the matrix elements of the two-atom local operator
Ψ_i(R)Ψ_j(R)|i\hat{n}_i + \hat{a}_k^+(4m)|Ψ_i(R)Ψ_j(R),
Ψ_i(R)Ψ_j(R)(-\imath\hat{d}_k)|Ψ_i(R)Ψ_j(R),
and
Ψ_i(R)Ψ_j(R)(\imath\hat{a}_k)(-\imath\hat{d}_k)|Ψ_i(R)Ψ_j(R)
with
j, n = 1, 2, ..., 7. The open dots represent the operators.

\begin{align}
&i \sqrt{k_0^2 - mE_0} + \sqrt{mE_0 + i0^+ + k_0^2} - 2\sqrt{mE_0 k_0 + m^2 \Omega^2} = \\
&i \sqrt{2\sqrt{mE_0 k_0^2 + m^2 \Omega^2} - (mE_0 + k_0^2)}.
\end{align}

Note that the bubble (16) does not diverge and we do not need to have renormalization relation for $g_{1D}$.

The even-wave scattering amplitude in 1D can be written as [20,74] $f_{1D}(k) = -1/(1 + \imath \cot \delta_k)$, where $\delta_k$ is the scattering phase shift. With $T_k(R) = ik f_{1D}(k)/m_r$ and Eq. (16), one can get

\begin{equation}
\cot \delta_k = \frac{k}{m_r} - \frac{1}{g_{1D}} - \int \left[ \Pi_\nu(E_0, 0) \right] \left( \frac{m_r}{k} \right).
\end{equation}

IV. EVEN-WAVE CONTACT

To define the 1D even-wave contact, we need to give the adiabatic relation [20],

\begin{equation}
\frac{C_\lambda}{2m} = \frac{\partial E}{\partial a_{1D}} = \int dR \left( \frac{\partial \mathcal{H}}{\partial a_{1D}} \right),
\end{equation}

where $E$ is the total energy of the many-body system, $\mathcal{H}$ is the density of the Hamiltonian (1), $C_\lambda$ is the 1D even-wave contact, and we have used the following relations:

\begin{equation}
\frac{\partial \mathcal{H}}{\partial a_{1D}} = \left( \frac{\partial \mathcal{H}}{\partial g_{1D}} \right) \frac{\partial g_{1D}}{\partial a_{1D}} = m \frac{2 \lambda_{1D}(\Psi_i(R)\Psi_j(R)\Psi_\eta(R))}{2 m \Omega l_{1D}}.
\end{equation}

Substituting Eq. (19) into (18), we get the expression for the contact $C_\lambda$ as

\begin{equation}
C_\lambda = \frac{m^2}{g_{1D}} \int dR (\Psi_i(R)\Psi_j(R)\Psi_\eta(R)),
\end{equation}

where it is indicated that $C_\lambda$ refers to the two-atom operator.

Furthermore, we calculate the expectation values of the two-atom operator $\langle O_j | \Psi_i(R)\Psi_j(R)\Psi_\eta(R) | I_k \rangle$ as shown in Fig. 2 [17],

\begin{equation}
\langle O_j | \Psi_i(R)\Psi_j(R)\Psi_\eta(R) | I_k \rangle = \sum_{j=a,b,c,d} \langle O_j | \Psi_i(R)\Psi_j(R)\Psi_\eta(R) | I_k \rangle.
\end{equation}

Substituting Eq. (12) into (21), we have

\begin{equation}
\langle O_j | \Psi_i(R)\Psi_j(R)\Psi_\eta(R) | I_k \rangle = \frac{(T_{q0, Q})^2}{g_{1D}^2}.
\end{equation}

Substituting Eq. (22) into (20), we get the even-wave contact as

\begin{equation}
C_\xi = m^2 \int dR (\Psi_i(R)\Psi_j(R)) = \frac{\partial \mathcal{H}}{\partial a_{1D}}.
\end{equation}

V. UNIVERSAL RELATIONS

A. Adiabatic relations

It is known that a contact can be defined to characterize the variation of energy as shown in Eq. (18). When SOC is present, there are two new physical quantities $C_\xi$ and $C_\eta$ refer to single-particle properties, i.e., they are obviously nonzero even for a single-particle or a noninteracting Fermi gas. Therefore, one cannot call them contacts, and we call them new physical quantities here.

B. Pressure relation

For a uniform gas, the pressure relation can be derived following the expression of the Helmholtz free-energy density $F = F/L$ which can be expressed in terms of [5,20,21]

\begin{equation}
F(T, n, n, a_{1D}, k_0, \Omega)
= \frac{k_F^3}{2m} f \left( 2mT, n, n, a_{1D}, k_0, 2m \Omega \right),
\end{equation}

where $L$ is the length along the $x$ direction, $f$ is a dimensionless function, $T$ is the temperature, $n = n + n = k_F/\pi$ is the Fermi particle number density, and $k_F$ is the Fermi wave vector.

Equation (26) implies the scaling behavior of the Helmholtz free-energy density as follows:

\begin{equation}
\tilde{F}(T, n, n, a_{1D}, k_0, \Omega)
= \frac{k_F^3}{2m} f \left( \tilde{T}, \tilde{n}, \tilde{n}, \tilde{n}, \tilde{k}, \tilde{\Omega} \right),
\end{equation}

where $\tilde{x}$ is a dimensionless and arbitrary parameter.
Taking the derivative of Eq. (27) with respect to \( \tilde{\lambda} \) at \( \tilde{\lambda} = 1 \), we have

\[
3\mathcal{F} = \left( 2T \frac{\partial}{\partial T} + n_1 \frac{\partial}{\partial n_1} + n_1 \frac{\partial}{\partial n_1} - a_{1D} \frac{\partial}{\partial a_{1D}} + k_0 \frac{\partial}{\partial k_0} + 2\Omega \frac{\partial}{\partial \Omega} \right) F. \tag{28}
\]

Replacing the free-energy density \( \mathcal{F} \) on the left side of Eq. (28) by \( \mu \) and substituting \( S = -\partial F/\partial T \) and \( \mu = \partial F/\partial n_\sigma \) into Eq. (28), one gets

\[
3(\mu \_1 + n_1 \mu _1 - \mathcal{P}) = -2TS + n_1 \mu _1 + n_1 \mu _1 - a_{1D} \frac{\partial F}{\partial a_{1D}} + k_0 \frac{\partial F}{\partial k_0} + 2\Omega \frac{\partial F}{\partial \Omega}, \tag{29}
\]

where \( \mathcal{P} \) is the pressure density, \( S \) is the entropy, and \( \mu _\sigma \) is the chemical potential with spin \( \sigma \).

Using the adiabatic relations (18), (24), and (25), we can get the pressure relation as

\[
\mathcal{P} = 2E + \frac{a_{1D} a_\sigma}{2mL} - \frac{k_0 \lambda}{L} - \frac{2\Omega C_\Omega}{L}, \tag{30}
\]

where \( E = E/L \) is the energy density and we use \( E = F + TS \).

C. Virial theorem

For an atomic gas in a harmonic potential \( V_T = m/4 \lambda^2 x^2/2 \) with the trapping frequency \( \omega \), the free energy can be expressed in terms of [5,20,21]

\[
F(T, \omega, a_{1D}, k_0, \Omega, N_1, N_1) = \omega \tilde{f}(T, \omega, \lambda, a_{1D}/a_\sigma, k_0 a_\sigma, \Omega, N_1, N_1), \tag{31}
\]

where \( N_1 + N_1 \) is the particle number, \( a_\sigma = \sqrt{2/(m\omega)} \) is the harmonic-oscillator length and the dimensionless function \( f \) is dependent on the dimensionless ratios \( T/\omega, a_{1D}/a_\sigma, k_0 a_\sigma, \Omega/\omega, \) and particle numbers \( N_1 \) and \( N_1 \).

With Eq. (31), we can get the scaling law,

\[
\tilde{\lambda}F(T, \omega, a_{1D}, k_0, \Omega, N_1, N_1) = F\left(\tilde{T}, \tilde{\omega}, \tilde{\lambda}^{-1/2} a_{1D}, \tilde{\lambda}^{1/2} k_0, \tilde{\lambda} \Omega, N_1, N_1\right), \tag{32}
\]

where \( \tilde{\lambda} \) is a dimensionless and arbitrary parameter.

The derivative of Eq. (32) with respect to \( \tilde{\lambda} \) at \( \tilde{\lambda} = 1 \) gives

\[
F = \left( T \frac{\partial}{\partial T} + \omega \frac{\partial}{\partial \omega} \right) F. \tag{33}
\]

VI. LARGE-MOMENTUM TAIL

In this section, we derive the tail of the momentum distribution for 1D fermions with SOC near a broad Feshbach resonance using the OPE method [4,5,21,22].

OPE is an ideal tool to explore the short-range physics. One can expand the product of two operators as

\[
\left( \psi_\sigma \Psi \right)\left( R + \frac{x}{2} \right) \left( \psi_\sigma \Psi \right)\left( R - \frac{x}{2} \right) = \sum_\sigma C_\sigma(x) O_\sigma(R), \tag{36}
\]

where \( O_\sigma(R) \) are the local operators and \( C_\sigma(x) \) are the short-distance coefficients. \( C_\sigma(x) \) can be determined by calculating the matrix elements of the operators on both sides of Eq. (36) in the two-body state \( |Q/2 + k, \sigma; Q/2 - k, \sigma' \rangle \).

By using the Fourier transformation on both sides of Eq. (36), we have the expression of momentum distribution as [5]

\[
n_{\sigma}(q) = \int \frac{dR}{L} \int dx e^{-iqx} \left| \langle \psi_\sigma | (R + \frac{x}{2}) \psi_\sigma | (R - \frac{x}{2}) \rangle \right|^2, \tag{37}
\]

where \( q \) is the relative momentum. There are four types of diagrams which can be used to denote the operators on the left-hand side of the OPE equation (36). However, the only nonanalyticity comes from the diagram as shown in Fig. 3. Therefore, one can evaluate the diagram in Fig. 3 as (the derivations are given in the Appendix)

\[
\begin{align*}
\langle 0 | \langle \psi_{\sigma_1} (R + \frac{x}{2}) \psi_{\sigma_1} (R - \frac{x}{2}) | L_{\sigma_1} \rangle | \langle \psi_{\sigma_2} (R + \frac{x}{2}) \psi_{\sigma_2} (R - \frac{x}{2}) | L_{\sigma_2} \rangle \\
\langle 0 | \langle \psi_{\sigma_1} (R + \frac{x}{2}) \psi_{\sigma_1} (R - \frac{x}{2}) | L_{\sigma_1} \rangle | \langle \psi_{\sigma_2} (R + \frac{x}{2}) \psi_{\sigma_2} (R - \frac{x}{2}) | L_{\sigma_2} \rangle \\
= \left[ -i \Gamma \left( q_0, Q \right) \right]^2 \int \frac{dp_0 dp_1}{(2\pi)^2} G(p_0, p) e^{i G \left( q_0 - p_0, Q - p \right)} eG(p_0, p) e^{-ipx}. \tag{38}
\end{align*}
\]
where $q_0 = Q^2/(4m) + k^2/m$ is the total energy, $Q$ is the center-of-mass momentum, and we use the Feynman rules for the operator vertices [5],

$$
\Psi(R + \frac{x}{2}) \Psi(R - \frac{x}{2}) \sim \exp \left[ -ip(R + \frac{x}{2}) \right] \exp \left[ ip(R - \frac{x}{2}) \right].
$$

(39)

the incoming and outgoing momenta are $p$ and $p'$ with $p = p'$.

With the Fourier transforms, we get the momentum distribution matrix as

$$
n(q) = \left[ -iT_q(q_0, Q) \right]^2 \int_{-\infty}^{\infty} \frac{d p_0}{2\pi} G(p_0, q) e^{i q \cdot \hat{q}} G^* (q_0 - p_0, Q - q) e G(p_0, q) = \begin{pmatrix} n_{\uparrow \uparrow}(q) & n_{\uparrow \downarrow}(q) \\ n_{\downarrow \uparrow}(q) & n_{\downarrow \downarrow}(q) \end{pmatrix},
$$

(40)

where the analytical expressions for the elements of the matrix are shown in the Appendix.

Matching Eq. (40) with Eq. (22), we get the momentum distribution matrix in the large-$q$ limit up to the $q^{-8}$ order,

$$
n(q) = \frac{C_a}{q^5 L} + \frac{2\hat{q} \cdot C_{Q1} - 4k_0 C_a \sigma_z}{q^5 L} + \frac{2C_{R1} + 10k_0^2 C_a - 10k_0 \hat{q} \cdot C_{Q1} \sigma_z + 5C_{Q2} / 2}{q^5 L} \cdot
$$

$$
\left. \right. + \frac{-12k_0 C_{R1} \sigma_z - 20k_0^2 C_a \sigma_z + 6\hat{q} \cdot C_{11} + 30k_0 \hat{q} \cdot C_{Q1} - 15k_0 C_{Q2} \sigma_z + 5 \hat{q} \cdot C_{Q3} / 2}{q^5 L} \cdot
$$

$$
\left. \right. + \frac{3C_{R2} + 42k_0^2 C_{R1} + 35k_0^2 C_a - 42k_0 \hat{q} \cdot C_{11} \sigma_z - 70k_0 \hat{q} \cdot C_{Q1} \sigma_z + 21C_{12} \sigma_z + 105k_0 \hat{q} \cdot C_{Q3} \sigma_z / 2 + 35C_{Q4} / 16}{q^8 L}
\right).
$$

(41)

where we take the SOC parameters as perturbations since the strength of the SOC should be much smaller than the corresponding strength scale of the interatomic interactions, $\hat{q}$ is the unit vector, and the new contacts are defined as

$$
C_{Rj} = m^{2+j} \frac{\hat{q}^2}{2} \int_{D} dR \langle \psi^\dagger_\uparrow(R) \psi^\dagger_\upsilon(R) \rangle \cdot
$$

$$
\times \left. \right. \left( i\partial_R + \frac{\partial^2_R}{4m} \right)^j \psi_\upsilon(R) \psi_\upsilon(R),
$$

(42)

$$
C_{Qn} = m^2 \frac{\hat{q}^2}{2} \int_{D} dR \langle \psi^\dagger_\uparrow(R) \psi^\dagger_\upsilon(R) \rangle \cdot
$$

$$
\times \left. \right. \left( i\partial_R + \frac{\partial^2_R}{4m} \right)^n \langle \psi_\upsilon(R) \psi_\upsilon(R) \rangle, \quad (43)
$$

$$
C_{jn} = m^{2+j} \frac{\hat{q}^2}{2} \int_{D} dR \langle \psi^\dagger_\uparrow(R) \psi^\dagger_\upsilon(R) \rangle \cdot
$$

$$
\times \left. \right. \left( i\partial_R + \frac{\partial^2_R}{4m} \right)^n \langle -i\partial_R \rangle^n \psi_\upsilon(R) \psi_\upsilon(R),
$$

(44)

\(j, n = 1, 2, \ldots \). Note that, if $n$ is an odd number, the corresponding contact is a vector.

Different from $C_j$ and $C_{Q2}$, $C_{Rj}$ refers to the effective range (as well as higher-order terms in the cotangent of the phase-shift expansion) which we do not discuss in our single-channel model Hamiltonian (1), $C_{Qn}$ refers to the center-of-mass momentum, and $C_{jn}$ refers to both of the effective range (as well as higher-order terms in the cotangent of the phase shift expansion) and center-of-mass momentum.

In the laboratory frame, the single-particle Hamiltonian is given by [62,63]

$$
\mathcal{H}_{lab} = \begin{pmatrix} \frac{k^2}{2m} & \Omega e^{i 2k_0 x} \\
\Omega e^{-i 2k_0 x} & \frac{k^2}{2m} \end{pmatrix}.
$$

(45)

By transforming $n_{\uparrow \uparrow}(q)$ to $n_{\uparrow \downarrow}(q - k_0)$ and $n_{\downarrow \uparrow}(q)$ to $n_{\downarrow \downarrow}(q + k_0)$, the momentum distribution can go back to the laboratory frame as discussed in Ref. [68]. Furthermore, the diagonal elements of the momentum distribution matrix in the laboratory frame are calculated as

$$
n_{\text{lab}}(q) = \frac{C_a}{q^5 L} + \frac{2\hat{q} \cdot C_{Q1}}{q^5 L} + \frac{4C_{R1} + 5C_{Q2}}{2q^5 L} \cdot
$$

$$
\left. \right. + \frac{12\hat{q} \cdot C_{11} + 5\hat{q} \cdot C_{Q3} + 48C_{R2} + 168C_{12} + 35C_{Q4}}{16q^8 L}.
$$

(46)

It shows an anisotropic behavior at the $q^{-5}$ and $q^{-7}$ tails due to the center-of-mass momentum.

Meanwhile, the spin-dependent momentum distribution matrix in the laboratory frame can be written as

$$
n_{\text{lab, spin-dep}}(q) = \begin{pmatrix} -\frac{16k_0^2 \Omega^2 C_a}{q^5 L} & -\frac{56k_0^3 \Omega^2 \hat{q} \cdot C_{Q1}}{q^5 L} \\
-\frac{16k_0^2 \Omega^2 C_a}{q^5 L} & -\frac{16k_0^2 \Omega^2 C_a \sigma_z}{q^5 L} \end{pmatrix}.
$$

(47)

The diagonal elements of the momentum distribution matrix provide the expectation value of the atomic number with either spin $\uparrow$ or spin $\downarrow$ with a certain relative momentum $q$. The off-diagonal elements indicate the mixing of different spins. From the above equation (47), it is found that the $q^{-8}$ tail in the off-diagonal terms of the momentum distribution matrix is dependent on the SOC parameters in the laboratory frame. This tail can be observed through time-of-flight measurement as a direct manifestation of the SOC effects on the many-body level.

VII. RADIO-FREQUENCY SPECTROSCOPY

The radio-frequency spectroscopy can be used as an important experimental tool in cold atom systems, and it can be derived from the Raman spectroscopy with zero relative momentum between two Raman lasers. In order to get the
radio-frequency spectroscopy, we should derive the Raman spectroscopy first as follows. When the transfer momentum and frequency is larger compared to the many-body scale, the Raman spectroscopy can be related to the contacts.

We apply a Raman coupling with frequency $\omega$ and momentum $k$ which is applied to transfers fermions from the internal spin state $|\sigma\rangle$ ($\sigma = \uparrow, \downarrow$) into a third spin state $|3\rangle$. The Hamiltonian reads

$$H_c = \sum_{\sigma} \Omega_{\sigma} \int dx e^{i(kx-\omega t)} O_{3\sigma}(x, t) + H.c.,$$

(48)

where $O_{3\sigma}(x, t) \equiv \psi_{3\sigma}^\dagger(x, t)\psi_{\sigma}(x, t)$, $\Omega_{\sigma}$ is the radio-frequency Rabi frequency determined by the strength of the radio-frequency signal and we assume $\omega > 0$.

The transition rate function $R(\omega, k)$ is given by the Fermi golden rule, which is related to the imaginary part of the time-ordered two-point correlation function [85,86],

$$\Gamma_{\sigma\sigma'}^{R}(\omega, k) = \frac{1}{\pi} \text{Im} \int dR \int dt e^{i\omega t} \int dx e^{-ikx}$$

$$\times i(\mathcal{T}O_{3\sigma}(R + x, t) O_{3\sigma'}^\dagger(R, 0)).$$

(49)

Explicitly, we have the transition rate function $R(\omega, k)$,

$$R(\omega, k) = 2\pi \sum_{\sigma, \sigma'} \Omega_{\sigma} \Omega_{\sigma'}^* \Gamma_{\sigma\sigma'}^{R}(\omega, k).$$

(50)

Furthermore, we can evaluate the diagram in Fig. 4 as

$$\Gamma_{\sigma\sigma'}^{R}(\omega, k)$$

$$= \frac{1}{\pi} \text{Im} i[ -i T_0 (E_0, Q)]^2$$

$$\times \int \frac{dp d\rho_0}{(2\pi)^3} G_0(p_0 + \omega, \rho + k)$$

$$\times [G(p_0, p)e^\Gamma G^\dagger(E_0 - p_0, Q - \rho - p)eG(p_0, p)]_{\sigma\sigma'},$$

(51)

where $E_0 = Q^2/(4m) + q^2/m$ is the total energy, $Q$ is the total momentum, and we have defined $G_0(p_0, p) = i/[p_0 - p^2/(2m) + i0^+]$.

FIG. 4. Feynman diagram for the matrix element of $\int dt e^{i\omega t} \int dx e^{-ikx} \mathcal{T}O_{3\sigma}(R + x, t) O_{3\sigma}^\dagger(R, 0)\langle \sigma = \uparrow, \downarrow \rangle$.

Matching Eq. (49) with Eq. (22) and taking the limit of $k = 0$, we have the radio-frequency spectral $\Gamma_{\sigma\sigma'}^{R}(\omega) = \Gamma_{\sigma\sigma'}^{R}(\omega, 0)$ in the high-frequency limit,

$$\Gamma^{R}(\omega) = \frac{m}{2\pi} \left[ \frac{C_{\sigma}}{(m\omega)^{3/2}} + \frac{25k_0^2C_{\sigma} - 4C_{R1} - 10k_0 \hat{q} \cdot C_{Q1}\sigma_1}{8(m\omega)^{7/2}} \right]$$

$$- \frac{5m^2\Omega_{\sigma}}{4\pi(m\omega)^{7/2}} \sigma_1.$$  

(52)

As shown in Eq. (52), it is found that the high-frequency radio-frequency spectrum contains $C_{\sigma}$, $C_{R1}$, and $C_{Q1}$ as expected. Especially, the presence of $C_{Q1}$ in the radio-frequency spectrum is due to the SOC effects.

VIII. SUMMARY

We study the 1D two-component Fermi gases with SOC using the single-channel model and discuss some universal behaviors near a broad Feshbach resonance. Through the variation of energy with respect to the SOC parameters, two new physical quantities are defined, the pressure relation, and the viral theorem are derived in terms of the new SOC physical quantities. Utilizing the technique of OPE, the tail of the momentum distribution matrix is obtained. The momentum distribution matrix shows a spin-dependent behavior due to the SOC, and it shows an anisotropic behavior at the center-of-mass momentum. In the laboratory frame, the $q^{-8}$ tail in the momentum distribution can be observed through time-of-flight measurement as a direct manifestation of the SOC effects on the many-body level. In addition, the presence of the contact related to the center-of-mass momentum in the radio-frequency spectral is also due to the SOC effects.

FIG. 5. Feynman diagram for the polarization bubble $\Pi_1$. Here, $X = (x_1, t_1)$ and $Y = (x_2, t_2)$.

FIG. 6. Feynman diagram for the matrix of the operator $\psi_{\uparrow}^T(Z)\psi_{\uparrow}^T(Z')$. Here, $X = (x_1, t_1)$, $Y = (x_2, t_2)$, $Z = (x_3, t_3)$, and $Z' = (x_4, t_4)$. 

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APPENDIX

1. Derivations of Eq. (13)

The polarization bubble can be calculated by the diagram in Fig. 5 as

\[ \Pi_s(q_0, Q) = \int dX \: dY \left\{ \frac{1}{2} \Psi_f^T(X) e \Psi_f(X) \right\} \left\{ \frac{1}{2} \Psi_f^T(Y) e \Psi_f^T(Y) \right\} \]

\[ = \frac{1}{4} \int dX \: dY \left\{ [T \Psi_f^T(X) e \Psi_f(X) \Psi_f^T(Y) e \Psi_f^T(Y)] + [T \Psi_f^T(X) e \Psi_f(X) \Psi_f^T(Y) e \Psi_f^T(Y)] \right\} \]

\[ = \frac{2}{4} \int dX \: dY \left\{ [T \Psi_f^T(X) e \Psi_f(X) \Psi_f^T(Y) e \Psi_f^T(Y)] \right\} \text{(two equivalent contractions),} \]

\[ = \frac{1}{2} \int dX \: dY \left\{ [T \Psi_f^T(X) \Psi_f(X) \Psi_f^T(Y) \Psi_f^T(Y)] \right\}_{ab}(\epsilon)_{ab}[\Psi_f(X) \Psi_f(Y)]_{mn}(\epsilon^+)_{mn} \]

\[ = \frac{1}{2} \int dX \: dY \left\{ [T \Psi_f^T(X) \Psi_f(X) \Psi_f^T(Y) \Psi_f^T(Y)] \right\}_{ab}(\epsilon)_{ab}[G(X - Y)]_{mn}(\epsilon^+)_{mn} \]

\[ = \frac{1}{2} \int dX \: dY \left\{ [G^T(X - Y)]_{mn}(\epsilon)_{ab}[G(X - Y)]_{mn}(\epsilon^+)_{mn} \right\} \]

\[ = \int dX \: dY \left\{ \frac{1}{2} \text{Tr}[G^T(X - Y) e G(X - Y) e^+] \right\} \] \hspace{1cm} (A1)

where \( \int dX \: dY = \frac{1}{2} \int d(X + Y) d(X - Y) \), we label the field operator \( \Psi = \Psi_s + \Psi_f \) for outline as \( \Psi_s \) and inner lines as \( \Psi_f \), \( \Psi_f = (\psi_f, \psi_f^T) \), \( X = (x_1, t_1) \), \( Y = (x_2, t_2) \), \( q_0 = Q^2/(4m) + p^2/m \), and we use the definition \( G(X - Y) = \langle T \Psi_f(X) \Psi_f^T(Y) \rangle \).

2. Derivations of Eq. (14)

In the absence of the Raman coupling, the polarization bubble is given by

\[ \Pi_s(q_0, Q) = \int \frac{dp \: dq \: d\phi}{(2\pi)^2} \frac{i}{p_0 - (\Omega/2 + p^2 / 2m) + i0^+} \frac{i}{q_0 - p_0 - (\Omega/2 - p^2 / 2m) + i0^+} = \frac{m}{2k} \] \hspace{1cm} (A2)

where \( q_0 = Q^2/(4m) + k^2/m \).

The even-wave scattering amplitude can be written as \[ f_{1D}(k) = -1/(1 + i \cot \delta_k) \approx -1/(1 + i k a_{1D}) \], where \( \delta_k \) is the scattering phase shift. With \( T_s(k) = i k f_{1D}(k)/m \), one can derive Eq. (14) in the main text.

3. Derivations of Eq. (38)

As shown in Fig. 6, the vacuum expectation of the operator \( \Psi^T(Z) \Psi^T(Z') \) is calculated as follows:

\[ \langle O_s | \Psi^T(Z) \Psi^T(Z') | I_s \rangle_d \]

\[ = \left( \langle O_s | \psi_f^T(Z) \psi_f(Z') | I_s \rangle_d \langle O_s | \psi_f^T(Z) \psi_f(Z') | I_s \rangle_d \right) \]

\[ = \left( [-iT_s(q_0, Q)]^2 \int dX \: dY \left\{ T \Psi_f^T(X) \left\{ \frac{1}{2} \Psi_f^T(X)e \Psi_f(X) \right\} \left\{ \frac{1}{2} \Psi_f^T(Y)e \Psi_f^T(Y) \right\} \right\} \right) \]
\[
\begin{align*}
&= \frac{-iT_s(q_0, Q)}{4} \int dX \; dY \left[ \langle \Psi^T_1(Z) \Psi_1^T(X) \rangle \langle \Psi_1^T(Y) \rangle \right] \\
&+ \langle \langle \Psi^T_1(Z) \rangle \langle \Psi_1^T(X) \rangle \rangle \langle \langle \Psi_1^T(Y) \rangle \rangle + \langle \langle \Psi^T_1(Z) \rangle \langle \Psi_1^T(X) \rangle \rangle \langle \langle \Psi_1^T(Y) \rangle \rangle \\
&+ \langle \langle \Psi^T_1(Z) \rangle \langle \Psi_1^T(X) \rangle \rangle \langle \langle \Psi_1^T(Y) \rangle \rangle \\n&= \frac{-iT_s(q_0, Q)}{4} \int dX \; dY \left[ \langle \Psi^T_1(Z) \rangle \langle \Psi_1^T(X) \rangle \langle \langle \Psi_1^T(Y) \rangle \rangle \right] \quad \text{(four equivalent contractions),}
\end{align*}
\]

Therefore, we have

\[
\begin{align*}
&\left( \langle O_1 \Psi^T_1(Z) \Psi_1^T(Z') \rangle | L_1 \rangle_d \langle O_1 \Psi^T_1(Z) \Psi_1^T(Z') \rangle | L_1 \rangle_d \right)^T \\
&= \left[ \frac{-iT_s(q_0, Q)}{4} \right]^2 \int dX \; dY \; G(X - Z) e^{iG} G^T(Y) e^{iG}(Z' - Y). 
\end{align*}
\]

**4. Elements of the momentum distribution matrix Eq. (40)**

\[
n_{q\downarrow}(q) = \frac{-iT_s(q_0, Q)}{4} \int_{-\infty}^{\infty} \frac{dp_0}{2\pi} \left[ G^2_{q\downarrow}(p_0, q) G_{q\downarrow}(q - p_0, Q - q) + G_{q\downarrow}(q - p_0, Q - q) G_{q\downarrow}(p_0, q) G_{q\downarrow}(q - p_0, Q - q) \right],
\]

\[
n_{q\downarrow}(q) = \frac{-iT_s(q_0, Q)}{4} \int_{-\infty}^{\infty} \frac{dp_0}{2\pi} \left[ G^2_{q\downarrow}(p_0, q) G_{q\downarrow}(q - p_0, Q - q) G_{q\downarrow}(p_0, q) \right] \\
+ G_{q\downarrow}(q - p_0, Q - q) G_{q\downarrow}(p_0, q) G_{q\downarrow}(p_0, q) \\
- G_{q\downarrow}(p_0, q) G_{q\downarrow}(p_0, q) G_{q\downarrow}(q - p_0, Q - q) - G^2_{q\downarrow}(p_0, q) G_{q\downarrow}(q - p_0, Q - q),
\]

\[
n_{q\downarrow}(q) = \frac{-iT_s(q_0, Q)}{4} \int_{-\infty}^{\infty} \frac{dp_0}{2\pi} \left[ G^2_{q\downarrow}(p_0, q) G_{q\downarrow}(q - p_0, Q - q) G_{q\downarrow}(p_0, q) \right] \\
+ G_{q\downarrow}(q - p_0, Q - q) G_{q\downarrow}(p_0, q) G_{q\downarrow}(p_0, q) \\
- G_{q\downarrow}(p_0, q) G_{q\downarrow}(p_0, q) G_{q\downarrow}(q - p_0, Q - q) - G^2_{q\downarrow}(p_0, q) G_{q\downarrow}(q - p_0, Q - q),
\]

\[
n_{q\downarrow}(q) = \frac{-iT_s(q_0, Q)}{4} \int_{-\infty}^{\infty} \frac{dp_0}{2\pi} \left[ G^2_{q\downarrow}(p_0, q) G_{q\downarrow}(q - p_0, Q - q) G_{q\downarrow}(p_0, q) \right] \\
+ G_{q\downarrow}(q - p_0, Q - q) G_{q\downarrow}(p_0, q) G_{q\downarrow}(p_0, q) \\
- G_{q\downarrow}(p_0, q) G_{q\downarrow}(p_0, q) G_{q\downarrow}(q - p_0, Q - q) - G^2_{q\downarrow}(p_0, q) G_{q\downarrow}(q - p_0, Q - q),
\]

\[
n_{q\downarrow}(q) = \frac{-iT_s(q_0, Q)}{4} \int_{-\infty}^{\infty} \frac{dp_0}{2\pi} \left[ G^2_{q\downarrow}(p_0, q) G_{q\downarrow}(q - p_0, Q - q) G_{q\downarrow}(p_0, q) \right] \\
+ G_{q\downarrow}(q - p_0, Q - q) G_{q\downarrow}(p_0, q) G_{q\downarrow}(p_0, q) \\
- G_{q\downarrow}(p_0, q) G_{q\downarrow}(p_0, q) G_{q\downarrow}(q - p_0, Q - q) - G^2_{q\downarrow}(p_0, q) G_{q\downarrow}(q - p_0, Q - q),
\]
5. Perturbations of the elements of the single-particle propagator matrix

We use the expansions with a small $\Omega$ as a perturbation,

$$G_{\uparrow\downarrow}(p_0, q) \approx \frac{i}{p_0 - \frac{(q-k_0)^2}{2m} + i0^+} \left[ 1 + \frac{\Omega^2}{\left[p_0 - \frac{(q-k_0)^2}{2m} + i0^+\right]} \right],$$  \hspace{1cm} (A9)

$$G_{\downarrow\uparrow}(p_0, q) = G_{\uparrow\downarrow}(p_0, q) \approx \frac{i\Omega}{\left[p_0 - \frac{(q-k_0)^2}{2m} + i0^+\right]} \left[ 1 + \frac{\Omega^2}{\left[p_0 - \frac{(q-k_0)^2}{2m} + i0^+\right]} \right],$$  \hspace{1cm} (A10)

$$G_{\downarrow\downarrow}(p_0, q) \approx \frac{i}{p_0 - \frac{(q-k_0)^2}{2m} + i0^+} \left[ 1 + \frac{\Omega^2}{\left[p_0 - \frac{(q-k_0)^2}{2m} + i0^+\right]} \right].$$  \hspace{1cm} (A11)

6. One-dimensional Fourier transforms

The 1D Fourier transforms are [20,87]

$$\int dx x^\alpha e^{-iqx} = -2\alpha! \sin\left(\frac{\pi \alpha}{2}\right) \frac{1}{|q|^{\alpha+1}},$$  \hspace{1cm} (A12)

with $\alpha$ as an odd number, and

$$\int dx \text{sgn}(x) x^\beta e^{-iqx} = -2i\beta! \cos\left(\frac{\pi \beta}{2}\right) \text{sgn}(q) \frac{1}{|q|^{\beta+1}},$$  \hspace{1cm} (A13)

with $\beta$ as an even number and $\text{sgn}(x)$ as the sign function.

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