The String Coupling Accelerates the Expansion of the Universe

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Abstract

Generic cosmological models in non-critical string theory have a time-dependent dilaton background at a late epoch. The cosmological deceleration parameter $q_0$ is given by the square of the string coupling, $g_s^2$, up to a negative sign. Hence the expansion of the Universe must accelerate eventually, and the observed value of $q_0$ corresponds to $g_s^2 \sim 0.6$. In this scenario, the string coupling is asymptotically free at large times, but its present rate of change is imperceptibly small.

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1 Introduction

One hundred years after Einstein’s ‘annus mirabilis’, theoretical physicists are struggling to come to terms with the accelerating expansion of the Universe that is indicated by recent cosmological observations [1, 2]. What Einstein once termed his ‘greatest blunder’, namely a possible cosmological constant, may actually turn out to be one of his deepest insights and a puzzle for quantum theories of gravity [3]. This vacuum energy is a second measurable quantity that, in combination with the Newton constant $G_N \equiv 1/m_P^2 : m_P \sim 10^{19}$ GeV, must be confronted with any theory of gravity. Specifically, the cosmological constant $\Lambda \sim 10^{-48}$ GeV$^4$ is a challenge for any candidate quantum theory of gravity, which must explain not only why it is non-zero, but also why it is so many orders of magnitude smaller than the apparently natural order of magnitude $\Lambda \sim m_P^4$.

This challenge is acute for string theory [4, 5], particularly in its standard paradigm as a conformal field theory on an internal world sheet, used to calculate an S-matrix for particle scattering in a static background. The central problem is that a Universe with a cosmological constant is described as a de Sitter space. This possesses an event horizon and requires a description of physics in terms of mixed quantum-mechanical states, which does not admit an S-matrix formulation of scattering [6].

One of the first attempts to transcend the standard paradigm of a static string background was a formulation of string in a time-dependent dilaton background [7]. This model can be interpreted as a non-critical string [8], in which the underlying effective field theory on the internal world sheet is no longer conformal. The deviation from conformal symmetry requires the introduction of a renormalization scale, which can be interpreted as a new scalar Liouville field in the effective world-sheet theory.

We have argued [9] that the zero mode of the Liouville field in such a non-critical string theory can be identified with time, as dictated in some explicit examples [10] by the energetics of the corresponding effective field theory in space-time. One of the miracles of standard critical string theory was to derive Lorentz invariance and hence Einstein’s Special Relativity. Conversely, one possible signature of non-critical string could be a deviation from Lorentz invariance [11], and we have suggested that distant astrophysical sources of energetic photons could provide sensitive probes of this possibility [12].

Here we further argue that any deviation from criticality in the string world-sheet theory can be regarded effectively as vacuum energy in four-dimensional space-time. This could provide
a stringy framework for discussing cosmological inflation [13, 14]. However, here we focus on another possible application of this idea, namely as a mechanism for generating the present-day vacuum energy [15]. In a wide class of models, this suggestion has a startling implication that we would like to emphasize. The cosmological deceleration parameter $q_0$, which is directly related to the vacuum energy, can be expressed in terms of the dilaton field value, which can in turn be identified with the string coupling strength. Therefore, the cosmological deceleration at asymptotically late times is a direct measure of the string coupling strength:

$$q_0 = -g_s^2.$$  \hspace{1cm} (1.1)

This remarkable equation is our central result. We do not know whether the present Universe is sufficiently asymptotic for the formula (1.1) to be directly applicable to the present cosmological data. However, we do note one fundamental implication of (1.1): because of its negative relative sign, it implies that the expansion of the Universe MUST accelerate eventually. Moreover, if we insert the present measurement of $q_0$, we estimate $g_s^2 \sim 0.6$, which is quite a acceptable value. The string coupling decreases towards zero at large time, but the present rate of change is very small.

2 Background Analysis

We now explain in more detail [15] the theoretical analysis leading to (1.1). The Ansatz of [7] for a string model of cosmology was that the dilaton field $\Phi$ could evolve linearly in the world-sheet time variable $t$:

$$\Phi = \text{constant} - Qt,$$  \hspace{1cm} (2.1)

where $Q$ is a constant whose square measures the departure of the world-sheet field theory from conformal symmetry. Since the Einstein term in the effective space-time action is conformally rescaled by a factor $e^{-\Phi}$, the cosmological time $t_E$ in the Einstein frame (in which the lowest-order curvature term in the target-space effective action has the same normalisation as the conventional Einstein scalar curvature term [4]) is related to $t$ by

$$t_E = c_1 + \frac{c_0}{Q} e^{Qt},$$  \hspace{1cm} (2.2)

and the resulting form of a spherically-symmetric four-dimensional metric is of the flat Robertson-Walker-Friedman type:

$$ds^2 = -dt_E^2 + a_E(t_E)^2 (dr^2 + r^2 d\Omega^2),$$  \hspace{1cm} (2.3)
where $a_E(t_E)$ is a time-dependent scale factor and $\Omega$ is the three-dimensional angular factor.

Such time-dependent cosmological backgrounds are in general described by non-critical string theories [8, 9], in which conformal symmetry is restored by dressing the field operators with the Liouville field, which characterizes the overall size of the string and acts as a renormalization scale. The identification of the zero mode of this Liouville field with physical time has been checked by many calculations from two-dimensional black holes [9] to effective potentials between D-branes [16, 14].

Non-critical string models generically relax at large cosmic times to equilibrium points in the string ‘theory space’, which are conformal field theories describing locally-Minkowski spaces with a linear dilaton background. One example was a ten-dimensional Type-0 string theory [17] compactified on a space with five flat dimensions and a non-trivial flux parallel to the remaining dimension [18]. In this example, the sizes of the extra dimensions rapidly froze to fixed values, while the positive central-charge deficit $Q^2 > 0$ relaxes to a constant value $Q_0^2$ at large times. Other examples are provided by colliding brane worlds [19, 10].

In such models, the scale factor takes the following form at large cosmic times $t_E$: 

$$a_E(t_E) \simeq \frac{\beta Q_0}{\gamma} \sqrt{1 + \gamma^2 t_E^2}.$$  

(2.4)

where $\beta$ and $\gamma$ are numerical constants characteristic of the specific model under consideration. For instance, for the type-0 model of [18], we have $\gamma \equiv (\beta^2 Q_0^2)/(\alpha A)$, where $\alpha = \sqrt{[11 + \sqrt{17}]/[2(3 + \sqrt{17})]}$, $\beta = 2/(1 + \sqrt{17})$, and $A$ is the flux in the large extra dimension.

We see from (2.4) that $a(t_E)$ scales linearly with $t_E$ at very large values of this Einstein-frame cosmological time [18]. Hence the cosmic horizon expands logarithmically, allowing for the proper definition of asymptotic states and thus a scattering matrix. At large $t_E$, the Hubble parameter becomes

$$H(t_E) \simeq \frac{\gamma^2 t_E}{1 + \gamma^2 t_E^2},$$

(2.5)

and the effective four-dimensional vacuum energy density is [18]:

$$\Lambda_E(t_E) \simeq \frac{\gamma^2}{\beta^2(1 + \gamma^2 t_E^2)},$$

(2.6)

where we use the fact that [18] the central-charge deficit approaches its equilibrium value $Q_0$ for large $t_E$. Thus, the dark energy density relaxes to zero for $t_E \to \infty$ in such non-critical string cosmologies [18, 15]. Finally, during this epoch the deceleration parameter becomes [15]:
\[ q(t_E) = -\frac{(d^2a_E/dt^2_E) a_E}{(da_E/dt_E)^2} \approx -\frac{1}{\gamma^2 t^2_E}. \]  
\( (2.7) \)

Therefore, up to proportionality constant factors which by convention are normalized to unity, it can be identified with the square of the string coupling:

\[ q(t_E) = -\exp[2(\Phi - \text{const})] = -g_s^2. \]  
\( (2.8) \)

which is our central result announced earlier \(^1\). Because of the minus sign in \( (2.8) \), this non-critical string theory predicts that the expansion of the Universe must accelerate asymptotically.

3 Time for Discussion

The important ingredient in this approach is the treatment of time as a dynamical world-sheet renormalisation-group scale \(^9\), which flows irreversibly between fixed points in string theory space that correspond to equilibrium theories. The irreversible evolution of this world-sheet scale is due to information loss associated with world-sheet modes whose two-dimensional momentum scales pass beyond the ultraviolet cutoff, leading in turn to microscopic irreversibility of time. Deviations from such fixed points arise from relevant perturbations that might be due to catastrophic cosmic events such as the collision of two brane worlds, or simple quantum fluctuations. During the irreversible flow to some final fixed point in the string landscape, the Universe expands and may pass through various transitions such as inflation and reheating.

As we have just showed, in this scenario the expansion of the Universe must accelerate, and its rate of acceleration is equal to the current value of the string coupling. This non-critical string approach predicts a new type of asymptotic freedom, as the string coupling decreases with increasing cosmic time.

We close by recalling that this approach to target time in non-critical string theory yields a number of important, physically falsifiable predictions. These include potential violations of Lorentz symmetry and the principle of equivalence, associated with the microscopic curvature of space-time \(^9\)\(^{14}\), and the possibility that microscopic quantum mechanics may be modified.

One can also use non-critical strings to discuss supersymmetry breaking in brane worlds \(^10\) as well as inflation and reheating \(^13\). The relative separations and velocities of recoiling branes in

\(^1\)Consistency with perturbation theory requires \( g_s < 1 \), which is easily satisfied in phenomenologically realistic string models \(^18\)\(^{15}\). We note also that the present rate of change of \( g_s \) is unobservably slow.
some ekpyrotic models of inflation [13] can be constrained by available and future astrophysical data, such as Cosmic Microwave background fluctuation measurements. Thus, a century after Einstein’s Special Theory of Relativity, one scenario for its quantum-gravitational counterpart is close to experimental test - and possible disproof. *Time will tell.*

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