Quantum correlations in the 1D spin-1/2 Ising model with added Dzyaloshinskii-Moriya interaction

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We have considered the 1D spin-1/2 Ising model with added Dzyaloshinskii-Moriya (DM) interaction and presence of a uniform magnetic field. Using the mean-field fermionization approach the energy spectrum in an infinite chain is obtained. The quantum discord (QD) and concurrence between nearest neighbor (NN) spins at finite temperature are specified as a function of mean-field order parameters. A comparison between concurrence and QD is done and differences are obtained. The macroscopic thermodynamical witness is also used to detect quantum entanglement region in solids within our model. We believe our results are useful in the field of the quantum information processing.

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I. INTRODUCTION

In the last few years, the entanglement which comes from quantum information may lead to the further insight into the other areas of physics such as condensed matter and statistical mechanics\textsuperscript{1–8}. Entanglement can be of various types, e.g., bipartite, multipartite, entanglement entropy etc, for which a number of quantitative measures exist. The entanglement content of quantum states and its variations have been extensively investigated in recent years\textsuperscript{2–5}.

However, it is not the whole story and the entanglement does not show all quantum correlations among different constituents of a quantum system. The notion of quantum discord (QD) introduced by Ollivier and Zurek\textsuperscript{9} can measure the quantumness of correlations (including entanglement). The QD is considered as quantum correlation and quantum resource likewise entanglement. It is curious to ask what the characteristic of QD is at finite temperature, and what the differences are between thermal QD (TQD) and entanglement in a system. The authors pointed out that the QD, in contrast to the entanglement of formation (EOF), can detect those quantum correlations present in certain separable mixed states\textsuperscript{9}.

In the topic of quantum magnets, a lack of inversion symmetry make ones, to consider an extra exchange coupling between spins in the magnetic materials than the usual and well known the isotropic exchange $\vec{S}_i \cdot \vec{S}_j$. In this respect, Dzyaloshinskii has shown\textsuperscript{10} that an antisymmetric exchange $\vec{D}_{ij} \cdot (\vec{S}_i \times \vec{S}_j)$ should be considered in these magnetic materials. Later, Moriya has shown\textsuperscript{11} that inclusion of the spin orbit coupling on the magnetic ions in the 1st and 2nd order leads to antisymmetric and anisotropic exchange respectively.

This interaction is, however, rather difficult to handle analytically, but it is one of the agents responsible for magnetic frustration. Since this interaction may induce spiral spin arrangements in the ground state\textsuperscript{12}, it is closely involved with ferroelectricity in multiferroic spin chains\textsuperscript{13,14}. Besides, the DM interaction plays an important role in explaining the electron spin resonance experiments in some one-dimensional antiferromagnets\textsuperscript{15}. Moreover, the DM interaction modifies the dynamic properties\textsuperscript{16} and the quantum entanglement\textsuperscript{17,18} of spin chains\textsuperscript{19}. Behavior of quantum and classical correlations in the XY-spin chain with DM interaction also was discussed\textsuperscript{20}. Using Bethe Ansatz technique, authors in Ref\textsuperscript{21}, have investigated the QD in the spin-1/2 XXZ chain system.

In this article we study the thermal behavior of the pairwise QD and EOF, in an infinite spin-1/2 Ising chain with added Dzyaloshinskii-Moriya interaction subjected to a external longitudinal magnetic field. Such contributions, to the best knowledge of authors, have not been considered in previous works. Here, using the Jordan-Wigner transformations\textsuperscript{22} we diagonalize the Hamiltonian of the system and then obtain the QD and EOF as a function of order parameters. We also use macroscopic thermodynamic properties as entanglement witness to detect the presence of entanglement in solid state systems.

This paper is organized as follows. In Sect. 2 we briefly review the definition of the QD and the EOF from the information theory point of view. In Sect. 3 we introduce the spin-1/2 Ising chain with DM interaction and describe briefly the technique of Jordan-Wigner transformation and the mean-filed approximation used to diagonalize the Hamiltonian. In Sect. 4 we analyze the behavior of the QD and the EOF under the influence of the DM interaction, the longitudinal magnetic field and the temperature. In Sect. 5 we use macroscopic thermodynamic witness to detect quantum entanglement region.
in solids which is described by our model. Finally, in Sect. 6 a brief summary is given.

II. QUANTUM CORRELATION

A. Quantum Discord

We first present a brief review of the QD, which is defined as the difference between two expressions of mutual information extended from the classical to the quantum system. In classical information theory, the total correlation between two systems $A$ and $B$, whose state is mathematically represented by a joint probability distribution $p(A, B)$, can be obtained by the mutual information

$$ I(A : B) = H(A) + H(B) - H(A,B), $$

(1)

where $H(\cdot) = -\sum_i p_i \log_2 p_i$ is the Shannon entropy, and $p_i$ shows the probability of an event $i$ relevant to the system $A$ or $B$ or the joint system $AB$. By using the Bayes rule, one can rewrite the mutual information as

$$ I(A : B) = H(A) - H(A|B), $$

(2)

where $H(A|B) = H(A,B) - H(B)$ denotes the classical conditional entropy and it is employed to quantify the ignorance (on average) regarding the value of $A$ when one knows $B$. In classical information theory, these two expressions are equivalent, but there is a difference between them in the quantum world. In order to generalize these expressions to the quantum world, we replace classical probability distribution by the density operator $\rho$ and the Shannon entropy by the von Neumann entropy $S(\rho) = -\text{Tr}(\rho \log_2 \rho)$. In particular, if $\rho_{AB}$ denotes the density operator of a composite bipartite system $AB$, then $\rho_A(\rho_B)$ the reduced density matrix of subsystem $A(B)$. Now one can give the quantum versions of Eqs. (1) and (2), respectively:

$$ I(\rho_A : \rho_B) = S(\rho_A) + S(\rho_B) - S(\rho_{AB}), $$

(3)

$$ \mathcal{L}(\rho_A : \rho_B) = S(\rho_A) - S(\rho_A|\rho_B), $$

(4)

where $S(\rho_A|\rho_B)$ is a quantum generalization of the conditional entropy for $A$ and $B$, and it cannot be directly obtained via the replacement of the Shannon entropy by the von Neumann entropy. To get access quantum conditional entropy, we choose projective measurements on $B$ described by a complete set of orthogonal projectors, $\Pi_i$, corresponding to outcomes labeled by $i$. Once the measurement is made, the state of the system is given by

$$ \rho_i = \frac{1}{p_i} \left( I \otimes \Pi_i^B \right) \rho_{AB} \left( I \otimes \Pi_i^B \right), $$

(5)

with

$$ p_i = \text{Tr} \left( \left( I \otimes \Pi_i^B \right) \rho_{AB} \left( I \otimes \Pi_i^B \right) \right), $$

(6)

where $I$ is the identity operator for the subsystem $A$ and $p_i$ denotes the probability for obtaining the outcome $i$. The quantum analogue of the conditional entropy is given by

$$ S(\rho_{AB}|\Pi_i^B) = \sum_i p_i S(\rho_i), $$

(7)

and then the quantum extension of classical mutual information is given by

$$ \mathcal{L}(\rho_{AB}|\Pi_i^B) = S(\rho_A) - S(\rho_{AB}|\Pi_i^B). $$

(8)

When projective measurements are made on the subsystem $B$, the non-classical correlations between the subsystems are removed. Since the value of $\mathcal{L}(\rho_{AB}|\Pi_i^B)$ depends on the choice of $\{\Pi_i^B\}$, $\mathcal{L}$ should be maximized over all $\{\Pi_i^B\}$ to ensure that it contains the whole of the classical correlations. Thus the quantity

$$ \mathcal{C}(\rho_{AB}) = \max_{\{\Pi_i^B\}} \left( \mathcal{L}(\rho_{AB}|\Pi_i^B) \right), $$

(9)

provides a quantitative measure of the total classical correlations. Once $\mathcal{C}$ is in hand, QD can be obtained by subtracting it from the quantum mutual information

$$ Q(\rho_{AB}) = I(\rho_A : \rho_B) - \mathcal{C}(\rho_{AB}). $$

(10)

B. Entanglement of formation

We adopt the entanglement of formation, a well known measure of the entanglement, to quantify our entanglement. All the information needed in this case is contained in the reduced density matrix $\rho_{i,j}$. Wootters has shown, for a pair of binary qubits, that the concurrence $C$, which goes from 0 to 1, can be taken as a measure of entanglement. The concurrence between sites $i$ and $j$ is defined as

$$ C(\rho) = \max \{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, $$

(11)

where the $\lambda_i$s are the eigenvalues of the Hermitian matrix $R = \sqrt{\rho \rho^* \sqrt{\rho}}$ with $\rho = (\sigma^y \otimes \sigma^y) \rho^* (\sigma^y \otimes \sigma^y)$ and $\sigma^y$ is the Pauli matrix of the spin in the $y$ direction. For a pair of qubits the entanglement can be written as

$$ E(\rho) = \epsilon \left( C(\rho) \right), $$

(12)

where $\epsilon$ is a function of the "concurrence" $C$

$$ \epsilon(\rho) = g \left( 1 - \sqrt{1-C^2} \right), $$

(13)

where $g$ is the binary entropy function

$$ g(x) = -x \log_2 x - (1-x) \log_2(1-x). $$

(14)

In this case, the entanglement of formation is given in terms of the concurrence $C$. 


III. THE MODEL

We consider the 1D spin-1/2 Ising model with added Dzyaloshinskii-Moriya interaction subjected to an external magnetic field (LF). The Hamiltonian of the model is written as

\[ H = J \sum_n S_n^z S_{n+1}^z + \sum_n D \cdot (S_n \times S_{n+1}) - h \sum_n S_n^z, \]

(15)

where \( S_n \) is the spin-1/2 operator on the \( n \)-th site, \( h \) is the longitudinal magnetic field (LF), \( J > 0 \) is the antiferromagnetic coupling constant and \( D = D\hat{z} \) denotes a uniform DM vector. In the absence of the LF, the ground state phase diagram of the model is known. In the absence of the LF, nearest neighbor (NN) spins are entangled and by increasing the LF the concurrence decreases and will be equal to zero at the critical LF, \( h_c \).

In the saturated ferromagnetic phase, NN spins are not entangled.

Theoretically, the energy spectrum is needed to investigate the thermodynamic properties of the model. In this respect, we implement the Jordan-Wigner transformation to fermionize the Hamiltonian (Eq. (15)). Using the Jordan-Wigner transformations

\[ S_n^+ = a_n^\dagger e^{i\pi \sum_{m=1}^{n-1} a_m^\dagger a_m}, \]
\[ S_n^- = e^{-i\pi \sum_{m=1}^{n-1} a_m a_m^\dagger} a_n, \]
\[ S_n^z = a_n^\dagger a_n - \frac{1}{2}, \]

(16)

Hamiltonian (15) takes the form

\[ H_F = \sum_n -\frac{iD}{2} \left( a_n^\dagger a_{n+1} + a_n a_{n+1}^\dagger \right) + \sum_n \left( a_n^\dagger a_n a_{n+1}^\dagger a_{n+1} \right) - (J + h) \sum_n a_n^\dagger a_n + \text{constant}. \]

(17)

Treating the Hamiltonian \( H_F \) in the mean-field approximation, the interacting fermionic system reduces to a 1D system of the non-interacting quasi particles\(^{30}\)

\[ H_{MF} = -\frac{iD}{2} \sum_n \left( a_n^\dagger a_{n+1} + a_n a_{n+1}^\dagger \right) - J \gamma_3 \sum_n \left( a_n a_{n+1} + a_{n+1}^\dagger a_n^\dagger \right) - J \gamma_2 \sum_n \left( a_{n+1}^\dagger a_n + a_n a_{n+1} \right) - (2\gamma_1 J - J - h) \sum_n a_n^\dagger a_n, \]

(18)

where \( \gamma_i (i = 1, 2, 3) \) are the mean-field order parameters,

\[ \gamma_1 = \langle a_n^\dagger a_n \rangle, \]
\[ \gamma_2 = \langle a_n a_{n+1} \rangle, \]
\[ \gamma_3 = \langle a_{n+1}^\dagger a_{n+1} \rangle. \]

(19)

By performing a Fourier transformation into the momentum space as \( a_n = \frac{1}{\sqrt{N}} \sum_{\mathbf{k}} e^{-i\mathbf{k} \cdot \mathbf{a}_n} \), the mean field Hamiltonian takes the form

\[ H_{MF} = \sum_{\mathbf{k} > 0} a(\mathbf{k}) \left( a_\mathbf{k}^\dagger a_{-\mathbf{k}} + a_{-\mathbf{k}}^\dagger a_\mathbf{k} \right) + \sum_{\mathbf{k} > 0} b(\mathbf{k}) \left( a_\mathbf{k}^\dagger a_{-\mathbf{k}} + a_{-\mathbf{k}}^\dagger a_\mathbf{k} \right) - \sum_{\mathbf{k} > 0} c(\mathbf{k}) \left( a_\mathbf{k} a_{-\mathbf{k}} - a_{-\mathbf{k}}^\dagger a_\mathbf{k}^\dagger \right), \]

(20)

where

\[ a(\mathbf{k}) = 2J \gamma_2 \cos(\mathbf{k}) + 2J \gamma_1 - J - h, \]
\[ b(\mathbf{k}) = D \sin(\mathbf{k}), \]
\[ c(\mathbf{k}) = 2J \gamma_3 \sin(\mathbf{k}). \]

(21)

Finally, by using the following Bogoliubov transformation

\[ a_\mathbf{k}^\dagger = \cos(\mathbf{k}) \beta_\mathbf{k}^\dagger + i \sin(\mathbf{k}) \beta_\mathbf{k}, \]

(22)

the diagonalized Hamiltonian is given by

\[ H_{MF} = \sum_\mathbf{k} \varepsilon_\mathbf{k} \beta_\mathbf{k}^\dagger \beta_\mathbf{k}, \]

(23)

where \( \varepsilon_\mathbf{k} = \sqrt{(a(\mathbf{k})^2 + c(\mathbf{k})^2) - b(\mathbf{k})} \) is the energy spectrum. Since, the Hamiltonian exhibits both translational and U(1) invariance, density matrix will be given by

\[ \rho_{ij,j+1} = \begin{pmatrix} \rho_{11} & 0 & 0 & 0 \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ 0 & 0 & 0 & \rho_{44} \end{pmatrix}, \]

(24)

where

\[ \rho_{11} = \langle n_j n_{j+1} \rangle \]
\[ \rho_{22} = \langle n_j (1 - n_{j+1}) \rangle, \]
\[ \rho_{33} = \langle n_{j+1} (1 - n_j) \rangle, \]
\[ \rho_{44} = \langle 1 - n_j - n_{j+1} + n_j n_{j+1} \rangle, \]
\[ \rho_{23} = \langle a_j^\dagger a_{j+1}^\dagger \rangle. \]

(25)
Therefore, the mutual information is given by

\[ \rho_{\text{concurrence}} = \frac{1}{2} \int_{-\pi}^{\pi} \frac{a(k)}{\varepsilon(k)} \left( \frac{1}{1 + e^{\beta \varepsilon(k)}} - \frac{1}{2} \right) dk, \quad (26) \]

\[ \rho_{23} = \gamma_2 \]

\[ = \frac{1}{2\pi} \int_{-\pi}^{\pi} \cos(k) \left( \frac{1}{1 + e^{\beta \varepsilon(k)}} - \frac{1}{2} \right) dk, \quad (27) \]

where \( \beta = \frac{1}{k_B T} \) and the Boltzmann constant is considered equal to one (\( k_B = 1 \)). One should note that the Fermi distribution function is \( f(k) = \frac{1}{1 + e^{\beta \varepsilon(k)}} \). Using the solution of the retarded Green’s function, \( \rho_{11} \) approximately is obtained as \( \langle n_j n_{j+1} \rangle = \gamma_1^2 - \gamma_2^2 \). Thus the concurrence as a function of the mean-field order parameters is given as

\[ C_j = \max \{0, 2(\rho_{23} - \sqrt{\rho_{11} \rho_{44}})\}, \]

\[ = \max \{0, 2(\gamma_2 - \sqrt{(\gamma_1^2 - \gamma_2^2)(1 - \gamma_1)^2 - \gamma_2^2)}\}. \]

(28)

At zero temperature, the magnetization in the absence of the LF is zero, therefore \( \gamma_1 = 1/2 \). In addition, there is no spin correlation in the \( xy \) plane and \( \gamma_2 = 0 \). This leads to unentangled state in the pure Ising model. On the other hand, in the pure DM interaction model, there is not any interaction along the \( z \) direction and magnetization is zero (\( \gamma_1 = 1/2 \)). In this case concurrence will be equal to \( \max \{0, 2(\gamma_2 - |\gamma_1/2 - \gamma_2|)\} \). One can show that the mean-field order parameter \( \gamma_2 \) only in the resonating valence bond (RVB) state takes the maximum value \( 1/2 \). Thus, the concurrence gets the maximum value, \( C_j^{\text{max}} = 0.5 \).

Due to the induced quantum fluctuations in the 1D spin-1/2 Ising model with added DM interaction, the concurrence should be less than 0.5. Now for studying the QD we follow the prescription which is introduced by M. Sarandy\textsuperscript{32}. By defining new variables

\[ c_1 = 2\rho_{23} = 2\gamma_2, \]

\[ c_2 = \rho_{11} + \rho_{44} - 2\rho_{22} = 1 - 4\gamma_1 + 4(\gamma_1^2 - \gamma_2^2), \]

\[ c_3 = \rho_{11} - \rho_{44} = 2\gamma_1 - 1. \]

(29)

The eigenvalues of \( \rho_{i,i+1} \) can be read as

\[ \lambda_0 = \gamma_1^2 - \gamma_2^2, \]

\[ \lambda_1 = 1 + \gamma_1^2 - \gamma_2^2 - 2\gamma_1, \]

\[ \lambda_2 = -\gamma_1^2 + \gamma_2^2 + \gamma_1 + \gamma_2, \]

\[ \lambda_3 = -\gamma_1^2 + \gamma_2^2 + \gamma_1 - \gamma_2. \]

(30)

Therefore, the mutual information is given by

\[ I(\rho_{i,i+1}) = S(\rho_i) + S(\rho_{i+1}) + \sum_{\alpha=1}^{3} \lambda_\alpha \log \lambda_\alpha, \]

\[ S(\rho) = \sum_{\{n_i\}} \rho_{n_i} \log \rho_{n_i} \quad \text{(31)} \]

where

\[ S(\rho_i) = S(\rho_{i+1}) = \left[ \gamma_1 \log(\gamma_1) + (1 - \gamma_1) \log(1 - \gamma_1) \right]. \]

Classical correlations are obtained by following procedure. We first introduce a set of projectors for a local measurement on the part \( (i + 1) = B \) given by \( \{B_k = V \Pi_k V^\dagger\} \) where \( \{\Pi_k = |k\rangle\langle k| : k = 0, 1\} \) is the set of projectors on the computational basis \( |0\rangle \equiv |\uparrow\rangle \) and \( |1\rangle \equiv |\downarrow\rangle \) and \( V \in U(2) \). Note that the projectors \( B_k \) represent an arbitrary local measurement on \( B \). We parameterize \( V \) as

\[ V = \begin{pmatrix} \cos \theta e^{i\phi} & \sin \theta e^{-i\phi} \\ \sin \theta e^{i\phi} & -\cos \theta \end{pmatrix}, \]

(32)

where \( 0 \leq \theta \leq \pi \) and \( 0 \leq \phi < 2\pi \). Note that \( \theta \) and \( \phi \) can be interpreted as the azimuthal and polar angles, respectively, of a qubit over the Bloch sphere. By using Eq. (26). One can show that the state of the system after measurement \( B_k \) will change to one of the states

\[ \rho_0 = \left( \frac{I}{2} + \sum_{j=1}^{3} q_{0j} S_j^I \right) \otimes (V \Pi_0 V^\dagger), \]

(34)

\[ \rho_1 = \left( \frac{I}{2} + \sum_{j=1}^{3} q_{1j} S_j^I \right) \otimes (V \Pi_1 V^\dagger), \]

(35)

where

\[ q_{k1} = (-1)^k c_1 \left[ \frac{\sin \theta \cos \phi}{1 + (-1)^k c_3 \cos \theta} \right], \]

\[ q_{k2} = \tan \phi q_{k1}, \]

\[ q_{k3} = (-1)^k \left[ \frac{c_2 \cos \theta + (-1)^k c_3}{1 + (-1)^k c_3 \cos \theta} \right]. \]

(36)

Then, by evaluating the von Neumann entropy from Eqs. (31) and (33) and using \( S(V \Pi_0 V^\dagger) = 0 \), we obtain

\[ S(\rho_k) = -\left( 1 + \frac{\theta_k}{2} \right) \log \left( 1 + \frac{\theta_k}{2} \right) + \left( 1 - \frac{\theta_k}{2} \right) \log \left( 1 - \frac{\theta_k}{2} \right), \]

(37)

with

\[ \theta_k = \sqrt{\sum_{j=1}^{3} q_{kj}^2}. \]

(38)

Therefore, the classical correlation for the spin pair is given by

\[ C(\rho) = \max_{\{n_i\}} \left( S(\rho_i) - S(\rho_0) - S(\rho_1) - c_3 \cos \theta S(\rho_0) - S(\rho_1) \right), \]

(39)

In general, \( C(\rho) \) has to be numerically evaluated by optimizing over the angles \( \theta \) and \( \phi \). Once the classical correlation is obtained, insertion of Eq. (31) and Eq. (33) into Eq. (40) can be used to determine the quantum discord.
are entangled in the quantum LL phase. It can be clearly seen that at zero temperature, NN spins concurrence (Fig. 1(a), (b)) and the QD (Fig. 1(c), (d)).

Theoretical formulation introduced in the previous sections. One can derive the equation

\[ \beta = \frac{1}{2} \int \frac{dk}{\pi} \cos(k) \left( \frac{\epsilon(k)}{\epsilon(k)} \right) \left( \frac{1}{1 + e^{\beta \epsilon(k)}} - 1/2 \right) dk. \]

Moreover, for the LF more than the quantum critical point, NN spins are unentangled at zero temperature (see the inset of Fig. 1). By increasing the temperature from zero, NN spins remain unentangled up to a critical temperature, \( T_c \). The existence of this critical temperature can be related to the fact that the system is gapped in the saturated ferromagnetic phase. It shows that in the gapped saturated ferromagnetic phase, the quantum correlations appear only at a finite value of the temperature proportional to the spin gap. As soon as the temperature increases from \( T_{c_1} \), both the concurrence and QD retrieve and take a maximum value. It should be noted that the amount of the \( (QD)_{max} \) is almost three times bigger than the maximum value of the concurrence. More increasing the temperature, the concurrence and QD decrease and concurrence reaches to zero at a second critical temperature \( T_{c_2} \), while QD again shows asymptotic behavior. This can be considered as an important difference between the QD and concurrence. The existence of the second critical temperature shows that thermal fluctuations destroy a part of quantum correlations related to the concurrence.

The effect of the LF on the quantum correlations is studied in Fig. 2. In this figure, the concurrence (Fig. 2(a) and (b)) and the QD (Fig. 2(c) and (d)) are plotted as a function of the LF. At zero temperature, NN spins are entangled in the absence of the LF. When a LF applies the NN spins remain entangled up to the critical LF \( (h_c) \). In the saturated ferromagnetic field the system becomes unentangled. It is worth to note that at zero temperature the QD also show a critical field in which it is zero in the saturated ferromagnetic phase. At
finite temperature, both concurrence and QD decrease with increasing the LF and vanishes in a LF which depends on the temperature. Here we point out the DM interaction effects on quantum correlations. Concurrence and QD show minor decreasing trend versus temperature for higher DM interaction. It is obvious how the higher DM interaction in the system can persist over quantum fluctuations and preserve the quantum correlations. This future is more profound in the concurrence. Indeed, in Fig. 2 we have only chosen two DM interactions $D = 0.5, 1.5$ to verify this fact. In what follows, we will focus on this issue in details.

Fig. 3 shows the behavior of the concurrence and the QD versus the DM interaction. When DM interaction is very small, the ground state of the system is in the Neel phase in the region $h < h_c$. In the Neel phase the NN spins are entangled. On the other hand, in the saturated ferromagnetic phase, $h > h_c$, NN spins are unentangled at zero temperature. It is obvious that by applying the DM interaction, concurrence and QD increase and reach to a saturation value. In addition, by increasing temperature all quantum correlations reduce, but only quantum correlations of the concurrence will be destroyed at a critical temperature. To this reason, a zero-plateau is observed in the curve of the concurrence (Fig. 3(a)). The existence of the zero-plateau in Fig. 3(b) is rooted to the quantum fluctuations in the saturated ferromagnetic phase. The lake of zero-plateau in the curve of the QD confirms this fact that the quantum correlations of QD are very robust in comparison with concurrence.

![FIG. 4: (Color online.) The parameter region of critical temperature $T$ and (a) the DM interaction ($h = 0.5$), (b) the LF ($D = 0.5$).](image)

![FIG. 3: (Color online.) The concurrence ((a) and (b)) and QD ((c) and (d)) between NN spins as a function of the DM interaction for different values of the temperature and LF.](image)

V. ENTANGLEMENT WITNESS

The realization that entanglement can also affect macroscopic properties of bulk solid-state systems, has increased the interest in characterizations of entanglement in terms of macroscopic thermodynamical observables. The entanglement witness is called an observable which can distinguish between entangled and separable states in the quantum physics. As a result of these studies, entanglement witnesses have been obtained in terms of expectation values of thermodynamic observables such as internal energy, magnetization and magnetic susceptibility. As we have shown, the spin-orbit coupling introduces off-diagonal terms in the Ising Hamiltonian. In this case, results concerning Hamiltonian based on diagonal exchange...
interactions do not apply. In a recent work, an appropriately generalized entanglement witness is constructed where will be used in following. Let $U = \langle H \rangle$ and $M = \sum_{n=1}^{N} S_{n}^{z}$ be the internal energy and the magnetization along field respectively. Then Eq. (15) yields

$$\frac{U + hM}{N} = \frac{1}{N} \sum_{n} \left[ \langle D(S_{n}^{x} S_{n+1}^{x} - S_{n}^{y} S_{n+1}^{y}) + JS_{n}^{z} S_{n+1}^{z} \rangle \right]_{\text{Max}} =$$

The right-hand of this equation is an entanglement witness. For any classical mixture of products states (or separable state), the density matrix is written as

$$\rho = \sum_{k} \omega_{k} \rho_{k}^{1} \otimes \rho_{k}^{2} \otimes \ldots \rho_{k}^{N}, \quad \text{(43)}$$

where $\sum_{k} \omega_{k} = 1$ and $\omega_{k} \geq 0$. Using such density matrix, easily one can find: $\langle S_{n}^{x} S_{n+1}^{x} \rangle = \langle S_{n}^{y} \rangle \langle S_{n+1}^{y} \rangle$ and $\sum_{n=x,y,z} \langle S_{n}^{z} \rangle^2 \geq \frac{1}{4}$. Thus, the entanglement witness in the separable state has the upper bound as

$$\left| \frac{1}{N} \sum_{n} \left[ \langle D(S_{n}^{x} S_{n+1}^{x} - S_{n}^{y} S_{n+1}^{y}) + JS_{n}^{z} S_{n+1}^{z} \rangle \right] \right|_{\text{Max}} =$$

$$\left| ND + \frac{J - D}{N} \sum_{n} \langle S_{n}^{z} S_{n+1}^{z} \rangle \right|. \quad \text{(44)}$$

Then the solid state system is in an entangled state if

$$W = \frac{U + hM}{ND} \geq \frac{1}{4}. \quad \text{(45)}$$

Applying the fermionized operators, the entanglement witness is obtained as

$$W = |\gamma_{1}^{2} - \gamma_{2}^{2} - \gamma_{1} + \gamma_{2} + \frac{1}{4}|. \quad \text{(46)}$$

Using this equation we have determined the parameter regions where entanglement can be detected in the solid state systems. Results are presented in Fig. 3 (a) and (b). In the presence of the magnetic field less than the quantum critical point (Fig. 4(a)), by adding DM interaction, the critical temperature increases almost linearly in complete agreement with our results on the concurrence. On the other hand, when the ground state has the ferromagnetic long-range order, there is not any quantum correlation at zero temperature. Increasing the temperature from zero, the NN spins remains unentangled up to a critical temperature which is rooted in the spin gap. More increasing the temperature, the concurrence and the QD regain and take a maximum value. The amount of the $(\langle QD \rangle)_{\text{max}}$ is almost three times larger than the maximum value of the concurrence. After this maximum, concurrence and QD decrease and concurrence reaches zero at a critical temperature while QD never goes zero. The existence of the second critical temperature shows that thermal fluctuations will destroy a part of quantum correlations related to the concurrence but are not enough strong to destroy all quantum correlations.

VI. CONCLUSION

To summarize, we have studied the thermal quantum correlations in the 1D spin-1/2 Ising model with added Dzyaloshinskii-Moriya (DM) interaction in the presence of a uniform longitudinal magnetic field (LF). First, using the Jordan-Wigner transformation the model is transformed to a 1D fermionized model. Then, using the mean-field approach the energy spectrum in an infinite chain is obtained.

By using the mean-field order parameters, we have determined the quantum discord (QD) and the concurrence between nearest neighbor (NN) spins at finite temperature. In principle, our approach is applicable to all cases contain the interacting fermions. A complete comparison between concurrence and the QD is done. At zero temperature, NN spins are entangled in the Neel phase $(h < h_{c})$ and unentangled in the saturated ferromagnetic phase $(h > h_{c})$. When system is in the Neel phase, the thermal fluctuations decrease all quantum correlations, but only destroy the quantum correlations of the concurrence. The quantum correlations of the QD exist even at finite temperatures which shows that the quantum correlations of QD are very stronger than the concurrence. On the other hand, when the ground state has the ferromagnetic long-range order, there is not any quantum correlation at zero temperature. Increasing the temperature from zero, the NN spins remains unentangled up to a critical temperature which is rooted in the spin gap. More increasing the temperature, the concurrence and the QD regain and take a maximum value. The amount of the $(\langle QD \rangle)_{\text{max}}$ is almost three times larger than the maximum value of the concurrence. After this maximum, concurrence and QD decrease and concurrence reaches zero at a critical temperature while QD never goes zero. The existence of the second critical temperature shows that thermal fluctuations will destroy a part of quantum correlations related to the concurrence but are not enough strong to destroy all quantum correlations.

Using macroscopic thermodynamic witnesses we have investigated quantum entanglement region in solids within our model. We have found that in the presence of the magnetic field less than the quantum critical point by adding DM interaction, the critical temperature increases almost linearly. The effect of the LF on the critical temperature is also shown that the critical temperature decreases with increasing the LF and will be zero for values of the LF $h(D = 0.5) \geq 1.2$.

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