Measurements of single top quark production cross sections and $|V_{tb}|$ in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV

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We present measurements of production cross sections of single top quarks in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV in a data sample corresponding to an integrated luminosity of 5.4 fb$^{-1}$ collected by the D0 detector at the Fermilab Tevatron Collider. We select events with an isolated electron or muon, an imbalance in transverse energy, and two, three, or four jets, with one or two of them containing a bottom hadron. We obtain an inclusive cross section of $\sigma(p\bar{p} \rightarrow t + X, tbq + X) = 3.43 \pm 0.74$ pb and use it to extract the CKM matrix element $|V_{tb}| = 0.79 < |V_{tb}| < 1$ at the 95% C.L. We also measure $\sigma(p\bar{p} \rightarrow t + X) = 0.68 \pm 0.34$ pb and $\sigma(p\bar{p} \rightarrow tqb + X) = 2.86 \pm 0.69$ pb when assuming, respectively, $tqb$ and $tb$ production rates as predicted by the standard model.

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I. INTRODUCTION

Top quarks are produced at hadron colliders as $t\bar{t}$ pairs via the strong interaction or singly via the electroweak interaction $tq$. Because of the larger production rate and higher signal-to-background ratio, the production of $t\bar{t}$ pairs is better studied and indeed it was through the $t\bar{t}$ production that the existence of the top quark was established in 1995 at the Fermilab Tevatron Collider $t\bar{t}$ [5, 6]. The observation of the single top quark production, however, was possible after CDF and D0 collaborations accumulated $\approx 50$ times more integrated luminosity than what was needed for observation of top quarks in $t\bar{t}$ production $t\bar{t}$ [7, 8]. Single top quark events are produced at about half of the rate of top quark pairs and with lower jet multiplicities, and therefore their study is more susceptible to contamination from background processes.

Electroweak production of top quarks at the Tevatron proceeds mainly via the decay of a time-like virtual $W$ boson accompanied by a bottom quark in the $s$ channel $(tb = tb + b\bar{b})$ or via the exchange of a space-like virtual $W$ boson between a light quark and a bottom quark in the $t$ channel $(tq = tq + t\bar{b})$, where $q$ refers to the light quark or antiquark) $t\bar{b}$. Figure 1 shows the lowest level Feynman diagrams for $s$- and $t$-channel production $t\bar{b}$. A third process $tW$, in which the top quark is produced together with a $W$ boson, has a small cross section at the Tevatron $t\bar{b}$ and is therefore not considered in this analysis.

Single top quark events can be used to probe the
This analysis extends previous work by the D0 Collaboration \cite{1,10,17,26} and uses the same data, event selection, and modeling of signal and background as in Ref. \cite{27}. It differs however, in the assumptions used to extract the cross sections of the individual \(tb\) and \(tqb\) production modes.

The data were collected with a logical OR of many trigger conditions, which together are fully efficient for the single top quark signal. We select events containing only one isolated electron or muon with high transverse momentum \((p_T)\) and having a large imbalance in the transverse energy \((E_T)\) indicative of the presence of a neutrino. Events originating from single top quark production are expected to contain at least one \(b\) quark jet from the decay of the top quark and a second \(b\) quark jet in the \(s\) channel, or a light quark jet and a spectator \(b\) quark jet for the \(t\) channel. In both cases, gluon radiation can give rise to additional jets. Events are selected with two, three, or four jets reconstructed using a cone algorithm \cite{28} in \((y, \phi)\) space, where \(y\) is the rapidity and \(\phi\) is the azimuthal angle, and the cone radius 0.5. The jets must satisfy the following conditions: leading \(p_T > 25\) GeV, other jets with \(p_T > 15\) GeV, and with pseudorapidities of all jets \(|\eta| < 3.4\). Requirements are also placed on \(E_T\): \(20 < E_T < 200\) GeV for events with two jets, and \(25 < E_T < 200\) GeV for events with three or four jets. The maximum \(E_T\) requirement removes events that suffer from poor modeling of the high energy tail of the muon momentum resolution. We require one isolated electron with \(|\eta| < 1.1\) and \(p_T > 15\) (20) GeV for events with two (three or four) jets, or one isolated muon with \(|\eta| < 2.0\) and \(p_T > 15\) GeV.

The sample resulting from this selection is dominated by \(W\) bosons produced in association with jets \((W+jets)\), with smaller contributions from \(t\bar{t}\) pairs decaying into the single lepton plus jets final state or the dilepton final state when one lepton or some jets are not reconstructed. Multijet events also contribute to the background when a jet is misidentified as an isolated electron or a heavy-flavor quark decays to a muon that satisfies isolation criteria, in combination with misreconstruction of \(E_T\). Diboson \((WW, WZ, ZZ)\) and \(Z+jets\) processes contribute only marginally to the total background but are taken into account. The background from multijets is kept small \((< 6\%)\) by requiring that the total scalar sum of \((H_T)\) of the transverse momenta of the final-state objects (lepton, \(E_T\), and jets) be larger than 120/140/160 GeV for events with 2/3/4 jets, and that the \(E_T\) does not point along the transverse direction of the lepton or the leading jet. Soft-scattering processes are suppressed by requiring a minimum value for the total scalar sum of the \(p_T\) of the jets \((H_T[alljets])\) ranging from 50 to 100 GeV, depending on the number of jets in an event and the data collection period.

To enhance the signal fraction, one or two of the jets are required to contain long lived bottom hadrons (\(b\) quarks). We use a different isolation cut for the jets that are not the leading jet in each dijet invariant mass bin.

Previous measurements of single top quark production cross sections \cite{8, 10, 16, 19} included events from both the \(tb\) and \(tqb\) processes, assuming a ratio of cross sections \cite{2} for the two processes based on the SM. However, several beyond-the-SM theories predict individual \(tb\) and \(tqb\) cross sections that deviate from the SM. Examples include models with additional quark generations \cite{13}, new heavy bosons \cite{20}, flavor-changing neutral currents (FCNC) \cite{21}, or anomalous top quark couplings \cite{22, 23}. It is therefore important to also measure the individual \(tb\) and \(tqb\) production rates.

Using data corresponding to 5.4 fb\(^{-1}\) of integrated luminosity recorded with the D0 detector \cite{25}, we present an improved measurement of the production rate of \(tb+tqb\). We also present measurements of the production rates of the individual \(tb\) and \(tqb\) processes performed assuming, respectively, \(tqb\) and \(tb\) production rates as predicted by the SM. Finally, we present a new direct measurement of \(|V_{tb}|\) extracted from the measured \(tb+tqb\) cross section.
jets), as determined through a multivariate $b$-tagging algorithm \cite{30}. This algorithm uses several variables to discriminate $b$ jets from other jets such as: (i) decay length significance of the secondary vertex, (ii) the $\chi^2$ per degree of freedom of the secondary vertex fit, (iii) weighted combination of the tracks’ impact parameter significances, (iv) probability that the jet originates from the primary $pp$ interaction vertex, (v) number of tracks used to reconstruct the secondary vertex, (vi) mass of the secondary vertex, and (vii) number of secondary vertices found inside the jet. To improve sensitivity to signal, the samples are divided into six independent analysis channels, depending on the jet multiplicity (two, three, or four jets), and the number of $b$-tagged jets (one or two).

The efficiency of the event selection, including branching fraction and the $b$-tagging requirements, is $(2.9 \pm 0.4)\%$ for $t\bar{b}$ and $(2.0 \pm 0.3)\%$ for $tqb$. The $tqb$ process has a lower acceptance than the $tb$ channel because the second $b$-jet has low transverse momentum and is difficult to identify. We apply additional requirements to select two control samples used to test whether the background model reproduces the data in regions dominated by one specific type of background. The control sample dominated by $W$+jets is required to have exactly two jets, $H_T < 175$ GeV, and only one $b$-tagged jet where $W$+jets events constitute $82\%$ of this sample, and the $tt$ component is less than $2\%$. The control sample dominated by $tt$ is required to have exactly four jets, $H_T > 300$ GeV, and one or two $b$-tagged jets where $tt$ events constitute $84\%$ of the sample, and the $W$+jets component is $12\%$.

### III. MODELS FOR SIGNAL AND BACKGROUND

Single top quark events are modeled for a top quark mass $m_t = 172.5$ GeV using the COMPHEP-based effective next-to-leading order (NLO) Monte Carlo (MC) event generator SINGLETOP \cite{31}, which preserves spin information in the decays of the top quark and the $W$ boson and provides event kinematics that reproduce distributions predicted by NLO calculations \cite{32, 33}. The $tt$, $W$+jets, and $Z$+jets events are simulated with the ALPGEN leading order MC generator \cite{34}. Diboson processes are modeled using PYTHIA \cite{35}. For all these MC samples, PYTHIA is also used to evolve parton showers and to model proton remnants and hadronization of all generated partons. The presence of additional $p\bar{p}$ interactions is modeled by events selected from random beam crossings matching the instantaneous luminosity profile in the data. All MC events are passed through a GEANT-based simulation \cite{36} of the D0 detector.

Differences between simulation and data in lepton and jet reconstruction efficiencies and resolutions, jet energy scale, and $b$-tagging efficiencies are corrected in the simulation by applying correction functions measured from separate data samples. Comparisons of ALPGEN with data and with other generators show small discrepancies in distributions of jet pseudorapidity and angular separations between jets \cite{37}. We therefore correct the ALPGEN $W$+jets and $Z$+jets samples by sequentially applying polynomial reweighting functions parameterized by the leading and second-leading jet $\eta$, $\Delta R = \sqrt{(\Delta \phi)^2 + (\Delta \eta)^2}$ between the two leading jets, and third-and fourth-leading jet $\eta$, if applicable. These functions are derived from the ratio between the number of $W$+jets and $Z$+jets events observed in data and the event yields predicted by MC. After these corrections, the MC description is in good agreement with our high statistics sample of events prior to the application of $b$-tagging. The multijet background is modeled using the selection discussed in Sec. 11 but choosing events that fail isolation criteria for leptons.

MC samples are scaled to the theoretical cross section at approximately NNLO \cite{1} for $tt$, and NLO \cite{38} for $Z$+jets and diboson cases. The contributions from $W$+jets and multijet are normalized by comparing the prediction for background to data before $b$-tagging. We use a procedure that relies on three distributions [lepton $p_T$, $E_T$ and $W$ reconstructed mass in the transverse plane $M_T(W)$] that have distinctive shapes for $W$+jets and multijets events and are thus sensitive to their relative contributions in the selected sample. The normalization scale factors for $W$+jets ($\lambda_{Wjets}$) and multijet ($\lambda_{multijets}$) are constrained by the following equation:

\[ N = \lambda_{Wjets}N_{Wjets} + \lambda_{multijets}N_{multijets}, \]

where $N = N_{data} - N_{non-Wjets} + N_{data} - N_{non-Wjets}$, $N_{Wjets}$, and $N_{multijets}$ are the event yields in data, non-$W$+jets MC, $W$+jets, and multijet samples, respectively. The $W$+jets sample contains events with light flavor ($Wjj$, $j = u, d, s$) and heavy flavor ($Wjc$, $Wcc$, and $Wbb$) quarks. The non-$W$+jets MC samples include single top quark, $tt$, $Z$+jets, and diboson production. The values of $\lambda_{Wjets}$ and $\lambda_{multijets}$ are varied to maximize the product of the Kolmogorov-Smirnov test values \cite{33} for the three kinematic distributions. This procedure is done separately for events with two, three, and four jets and for each lepton flavor. After the normalization, the total sum of the $W$+jets and multijets yields plus the small contributions from $tt$, single top, $Z$+jets, and diboson production equals the total data yield for each of the six analysis channels.

Without modifying the overall normalization of the $W$+jets MC sample, we apply an additional scale factor to $W$ and $Z$ boson events produced in conjunction with heavy-flavor jets ($b$ or $c$) to match NLO calculations \cite{38}: $Wbb$ and $Wcc$ by 1.47, $Zbb$ by 1.52, $Zcc$ by 1.67, and $Wcj$ by 1.32. We evaluate whether an additional normalization factor $\lambda_{HF}$ is required for the $Wbb$ and $Wcc$ samples by using events with two jets that pass the event selection described in Sec. 11 but fail the $b$-tagging requirements (zero-tag sample). The zero-tag sample has no overlap with the sample used to measure the single top quark cross section. During this study, we keep the
normalization of the $W$+jets sample fixed to the value obtained by the iterative method described above and derive $\lambda_{HF}$ with the following equation:

$$N^{(0)} = N^{(0)}_{Wlp} + \lambda_{HF}N^{(0)}_{Whp},$$

(2)

where $N = N_{data} - N_{multijets} - N_{non-Wjets}$, $N_{Wlp} = N_{Wlj} + N_{Wcj}$, and $N_{Whp} = N_{Wcc} + N_{Wbb}$. The superscript $(0)$ indicates that the equation is written for the zero-tag sample defined above. The measured value of $\lambda_{HF}$ is consistent with one. Uncertainties on the assumed cross section and the $W$+jets contribution includes events 12%. There are additional sources of systematic uncertainty from modeling both the background and signal. These uncertainties usually affect the normalization and, in some cases, also the shape of the distributions. The largest uncertainties arise from the jet energy scale (3–14.6%), jet energy resolution (0.2–11.6%), corrections to $b$-tagging efficiencies (6.6–21.2%), and the correction for jet-flavor composition in $W$+jets events 12%. There are also contributions due to limited statistics of the MC samples 6.0%, the measured luminosity 6.1%, and uncertainties on the trigger modeling 5.0%.

Table I lists the numbers of expected and observed events for each process after event selection, including $b$-tagging. Figure 2 shows comparisons between data and simulation before and after applying $b$-tagging. In the same figure, the normalization and differential spectra of the two dominant backgrounds are checked using the control samples dominated by $W$+jets (e), and by $t\bar{t}$ (f) events. These plots are indicative of the adequate background modeling attained for various sample conditions in the analysis.

TABLE I: Numbers of expected and observed events in a data sample corresponding to 5.4 fb$^{-1}$ of integrated luminosity, with uncertainties including both statistical and systematic components. The $tb$ and $tqb$ contributions are normalized to their SM expectations for a top quark mass of 172.5 GeV.

| Source | 2 jets | 3 jets | 4 jets |
|--------|--------|--------|--------|
| tb     | 104 ± 16 | 44 ± 7.8 | 13 ± 3.5 |
| $tqb$  | 140 ± 13 | 72 ± 9.4 | 26 ± 6.4 |
| $tt$   | 433 ± 87 | 830 ± 133 | 860 ± 163 |
| $W$+jets | 3,560 ± 354 | 1,099 ± 169 | 284 ± 76 |
| $Z$+jets & dibosons | 400 ± 55 | 142 ± 41 | 35 ± 18 |
| Multijets | 277 ± 34 | 130 ± 17 | 43 ± 5.2 |
| Sum of above sources | 4,914 ± 558 | 2,317 ± 377 | 1,261 ± 272 |
| Data    | 4,881 | 2,307 | 1,283 |

FIG. 2: [color online] Comparisons between the data and the background model for (a) $E_T$, (b) $W$ boson transverse mass before $b$-tagging, and (c) light quark jet pseudorapidity multiplied by lepton charge, after $b$-tagging. Reconstructed top quark mass (d) after $b$-tagging, (e) in a control sample dominated by $W$+jets, and (f) in a control sample dominated by $t\bar{t}$. The hatched bands show the $\pm 1\sigma$ uncertainty on the background prediction for distributions obtained after $b$-jet identification (c–f). The $W$+jets contribution includes events from $Z$+jets and diboson sources.

IV. MULTIVARIATE ANALYSES

Since the expected single top quark contribution is smaller than the uncertainty on the background, we use multivariate analysis (MVA) methods to extract the signal. The application of these methods to the measurement of the single top quark production cross section is described in Ref. [17]. Three different MVA techniques are used in this analysis: (i) Bayesian neural networks (BNN) [10], (ii) boosted decision trees (BDT) [11], and (iii) neuroevolution of augmented topologies (NEAT) [42]. Each MVA method constructs a function that approximates the probability $Pr(S|x)$ that an event, characterized by the variables $x$, originates from the signal process, $S = \{tb, tqb, tb+tqb\}$. Therefore each method defines a discriminant $D$ that can be used to constrain the uncertainties of the background in the low-discriminant region $D \approx 0$ and extract a signal from an excess in the high-discriminant region $D \approx 1$. All three methods use the same data and model for background, performing the analyses separately on the six mutually

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**Note:** The text is a scientific description of a study involving multivariate analyses for top quark signal extraction. The table and figure references indicate that the data and analysis methods are discussed in detail elsewhere, and the methods used are described in the references provided. The mathematical equation and table data are presented to illustrate the numerical and conceptual framework of the analysis. The section on multivariate analyses outlines the use of different methods to extract the signal from the background, emphasizing the consistency and precision of the results achieved. The figures are omitted due to the nature of the text, which focuses on the methodology rather than visual presentation.
exclusive subsamples defined before. All three methods also consider the same sources of systematic uncertainty, and are trained using variables for discriminating signal from background chosen from a common set of well-modeled variables. These variables can be classified in five categories: single object kinematics, global event kinematics, jet reconstruction, top quark reconstruction, and angular correlations. The BNN uses four-vectors of the lepton and jets and a two-vector for $E_T$ to build the discriminant. The BNN performance is improved by adding variables containing the lepton charge and b-tagging information, resulting in 14, 18, and 22 variables for events with 2, 3, and 4 jets. The BDT ranks and selects the best fifty variables for all the analysis channels, while NEAT uses the TMVA implementation of the “RuleFit” algorithm to select the best thirty variables in each channel.

Each MVA method is trained separately for the two single top quark production channels: (i) for the $tb$ discriminants, with $tb$ considered signal and $tqb$ treated as a part of the background, and (ii) for $tqb$ discriminants, with $tqb$ considered signal and $tb$ treated as a part of the background.

Using ensembles of datasets containing contributions from background and SM signal, we infer that the correlation among the outputs of the individual MVA methods is $\approx 70\%$. An increase in sensitivity can therefore be obtained by combining these methods to form a new discriminant. To achieve the maximum sensitivity, a second BNN is used to construct a combined discriminant for each channel, for $tb$, $tqb$, and $tb+tqb$ events, defined as $B_{tb}$, $B_{tqb}$ and $B_{tb+tqb}$. The $B_{tb}$ and $B_{tqb}$ discriminants take as inputs the three discriminant outputs of BDT, BNN, and NEAT, and they are trained by assuming $tb$ or $tqb$ as signals, respectively. The combined $tb + tqb$ discriminant ($B_{tb+tqb}$) takes as input the six discriminant outputs of BDT, BNN, and NEAT that are trained separately for the $tb$ and the $tqb$ signal. The training for $B_{tb+tqb}$ treats the combined $tb+tqb$ contribution as signal with relative production rates predicted by SM.

Figure 3 shows the outputs of the $B_{tb}$, $B_{tqb}$, and $B_{tb+tqb}$ discriminants, where good agreement is observed over the entire range. In these plots, the bins are sorted and merged (“ranked”) as a function of the expected signal-to-background ratio (S:B) such that S:B increases monotonically within the range of the discriminant. For the $tqb$ and $tb+tqb$ discriminants, presence of signal is significant in the plots. For the $tb$ discriminant, the signal presence is not as significant.

V. MEASURING SIGNAL CROSS SECTIONS

A. Bayesian approach

We use a Bayesian approach to extract the production cross sections. The method consists of forming a binned likelihood as a product of all six analysis channels (2, 3, or 4 jets with 1 or 2 b-tags) and bins using the full discriminant outputs. We assume a Poisson distribution for the number of events in each bin and uniform prior probabilities for non-negative values of the signal cross sections ($tb$, $tqb$ and $tb + tqb$ correspondingly). Systematic uncertainties and their correlations are taken into account by integrating over signal acceptances, background yields, and integrated luminosity, assuming a Gaussian prior for each source of systematic uncertainty. A posterior probability density as a function of the single top quark cross section is constructed, with the position of the maximum defining the value of the cross section and the width of the distribution in the region that encompasses 68% of the entire area corresponding to the uncertainty (statistical and systematic components combined). The expected cross sections are obtained by setting the number of data events in each channel equal to the value given by the prediction of signal plus background.
B. Ensemble testing

The methods used for extracting the cross sections are validated by studies performed using ensembles of pseudo-experiments that are generated taking into account all systematic uncertainties and their correlations. These ensembles of events are processed through each MVA method for each single top quark production mode and through the same analysis chain as used for the data. Five arbitrary signal cross sections (including the SM prediction) are used to calibrate the \( tb \), \( tqb \), and \( tb + tqb \) cross section extraction procedure. Means and standard deviations are determined by fitting Gaussian function to the distributions of extracted values of the measured cross sections in each ensemble. Figure 6 shows the resulting distributions and Gaussian fits for SM ensembles for \( tb \), \( tqb \), and \( t+b+tqb \) processes. Straight-line fits of the extracted mean cross sections to the input values are shown in Fig. 6, where the shaded bands reflect the standard deviations of the extracted cross sections in each ensemble.

The results of these pseudo-experiments show that the biases on the cross sections are negligible compared to the standard deviations of the extracted values. We therefore do not apply corrections to the measured values of the cross sections in data.

C. \( tb \), \( tqb \) and \( t+b+tqb \) channel cross sections

To measure the individual \( tb \) (\( tqb \)) production cross section, we construct a one-dimensional (1D) posterior probability density function with the \( tqb \) (\( tb \)) contribution normalized with a Gaussian prior centered on the predicted SM cross section and treated as a part of the background. This is implemented for each individual MVA method and also for their combination. To measure the total single top quark production cross section of \( t+b+tqb \), we construct a 1D posterior probability density function assuming the production ratio of \( tb \) and \( tqb \) predicted by the SM.

Figure 6 shows the expected and observed posterior density distributions for \( tb \), \( tqb \), and \( t+b+tqb \) using the combined discriminants \( B_{tb} \), \( B_{tqb} \), and \( B_{tb+tqb} \), respectively. Table II lists the expected and measured cross sections for the individual MVA analyses. All of the results are consistent with SM predictions, and the measured \( t+b+tqb \) production cross section is the most precise current measurement, with a precision comparable to the world average. All results assume a top quark mass of 172.5 GeV and have a small correction for events with more than four jets based on the SM. The dependence of the measured cross section on \( m_t \) is summarized in Table III. The assumed top quark mass affects the yield and differential properties for the signal acceptance and the modeling of \( tt \) events, which constitute the second largest background. The interplay between these two effects can cause the measured cross section to vary substantially (as observed in the \( tb \) channel) or in a way that is not monotonic with the assumed top quark mass (as observed in the \( tqb \) channel).

### Table II: Expected and observed cross sections in pb for \( tb \), \( tqb \), and \( t+b+tqb \) production

| Discriminant | Expected | Observed |
|--------------|----------|----------|
| \( tb \) production |          |          |
| BNN          | 1.08\+0.52\-0.50 | 0.79\+0.44\-0.43 |
| BDT          | 1.07\+0.47\-0.43 | 0.66\+0.41\-0.39 |
| NEAT         | 1.06\+0.74\-0.50 | 0.17\+0.41\-0.17 |
| \( B_{tb} \)  | 1.12\+0.45\-0.43 | 0.68\+0.38\-0.35 |
| \( tqb \) production |          |          |
| BNN          | 2.49\+0.76\-0.67 | 2.93\+0.87\-0.73 |
| BDT          | 2.40\+0.71\-0.66 | 3.04\+0.78\-0.66 |
| NEAT         | 2.36\+0.77\-0.77 | 2.73\+0.75\-0.63 |
| \( B_{tqb} \)  | 2.43\+0.67\-0.61 | 2.86\+0.69\-0.63 |
| \( t+b+tqb \) production |          |          |
| BNN          | 3.46\+0.84\-0.78 | 3.11\+0.77\-0.71 |
| BDT          | 3.41\+0.82\-0.74 | 3.04\+0.80\-0.75 |
| NEAT         | 3.33\+0.94\-0.80 | 3.59\+0.96\-0.80 |
| \( B_{tb+tqb} \)  | 3.49\+0.77\-0.73 | 3.43\+0.73\-0.74 |

### Table III: Dependence on \( m_t \) of the measured cross sections in pb for \( tb \), \( tqb \), and \( t+b+tqb \) production, using the combined discriminants for the assumed top quark masses.

| \( m_t \) GeV | 170 GeV | 172.5 GeV | 175 GeV |
|---------------|---------|-----------|---------|
| \( tb \)      | 1.20\+0.62\-0.56 | 0.68\+0.38\-0.35 | 0.53\+0.34\-0.30 |
| SM            | 1.12\+0.04\-0.04 | 1.04\+0.04\-0.04 | 0.98\+0.04\-0.04 |
| \( tqb \)     | 2.65\+0.65\-0.59 | 2.86\+0.69\-0.63 | 2.45\+0.57\-0.47 |
| SM            | 2.34\+0.12\-0.12 | 2.26\+0.12\-0.12 | 2.16\+0.12\-0.12 |
| \( t+b+tqb \) | 3.70\+0.78\-0.80 | 3.43\+0.73\-0.74 | 2.56\+0.69\-0.61 |
| SM            | 3.46\+0.16\-0.16 | 3.30\+0.16\-0.16 | 3.14\+0.16\-0.16 |

### VI. SIGNAL DOMINATED DISTRIBUTIONS

Figure 7 shows a comparison of the distributions of four kinematic variables with large discriminating power, for single top quark production in a data sample selected with S:B > 0.24 based on the \( B_{tb+tqb} \) discriminant. Variables shown are: leading b-tagged jets \( p_T \), W boson transverse mass, centrality, defined as the ratio of the scalar sum of the \( p_T \) of the jets to the scalar sum of the energy of the jets in the event, and reconstructed \( m_t \).
The presence of the single top quark signal is needed to ensure a good description of the data.

VII. $|V_{tb}|$ MEASUREMENT

The single top quark production cross section is directly proportional to the square of the CKM matrix element $|V_{tb}|^2$, enabling us to measure $|V_{tb}|$ directly without any assumption on the number of quark families or the unitarity of the CKM matrix [17]. We assume that SM sources for single top quark production and that top quarks decay exclusively to $Wb$. We also assume that the $Wtb$ interaction is CP-conserving and of the $V-A$ type, but maintain the possibility for an anomalous strength of the left-handed $Wtb$ coupling ($f_L^L$), which could rescale the single top quark cross section [17]. Therefore, we are measuring the strength of the $V-A$ coupling, i.e., $|V_{tb}f_L^L|$, which can be $>1$.

We form a Bayesian posterior $|V_{tb}f_L^L|^2$ with a flat prior based on the $B_{tb+tqb}$ discriminant. Additional theoretical uncertainties are considered for the $tb$ and $tqb$ cross sections [20]. Using the measured $tb+tqb$ cross section, we obtain $|V_{tb}f_L^L| = 1.02^{+0.10}_{-0.11}$. If we restrict the prior to the SM region $[0,1]$ and assume $f_L^L = 1$, we extract a limit of $|V_{tb}| > 0.79$ at the 95% C.L. Figure 8 shows the posterior density functions for $|V_{tb}f_L^L|^2$ and for $|V_{tb}|^2$, assuming $f_L^L = 1$ and $0 \leq |V_{tb}|^2 \leq 1$.

VIII. SUMMARY

In summary, we have measured the single top quark production cross section using 5.4 fb$^{-1}$ of data collected by the D0 experiment at the Fermilab Tevatron Collider. For $m_t = 172.5$ GeV, we measure the cross sections for...
FIG. 7: [color online] Distributions for data in the regions of large value for signal discrimination: (a) leading $b$-tagged jet $p_T$, (b) $W$ boson transverse mass, (c) centrality, defined as the ratio of the scalar sum of the $p_T$ of the jets to the scalar sum of the energy of the jets in the event, and (d) reconstructed $m_t$. The contributions from signal have been normalized to the measured $tb$+$tqb$ cross section.

FIG. 8: The posterior density functions for (a) $|V_{tb}|f^2_1$ and (b) $|V_{tb}|^2$. The shaded (dark shaded) band indicates regions of 68% (95%) C.L. relative to the peak values.

tb and tqb production to be

$$\sigma(p\bar{p} \to tb + X) = 0.68^{+0.38}_{-0.35} \text{ pb}$$

$$\sigma(p\bar{p} \to tqb + X) = 2.86^{+0.69}_{-0.63} \text{ pb}$$

assuming, respectively, $tqb$ and $tb$ production rates as predicted by the SM. The tqb cross section is consistent with the value $\sigma(p\bar{p} \to tqb + X) = 2.90 \pm 0.59$ pb measured in Ref. [21], where we use the same dataset and discriminant but extract the cross section without any assumption on the $tb$ production rate. The total cross section $tb + tqb$ is found to be

$$\sigma(p\bar{p} \to tb + tqb + X) = 3.43^{+0.74}_{-0.74} \text{ pb}$$

assuming the SM ratio between $tb$ and $tqb$ production. All measurements are consistent with the SM predictions for a top quark mass of 172.5 GeV. Finally, we derive a direct limit on the CKM matrix element $|V_{tb}| > 0.79$ at the 95% C.L. assuming a flat prior within $0 \leq |V_{tb}|^2 \leq 1$.

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