String and Brane Models
with Spontaneously/Dynamically Induced Tension

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Abstract
We study in some detail the properties of a previously proposed new class of string and brane models
whose world-sheet (world-volume) actions are built with a modified reparametrization-invariant measure
of integration and which do not contain any ad hoc dimensionful parameters. The ratio of the new
and the standard Riemannian integration measure densities plays the role of a dynamically generated
string/brane tension. The latter is identified as (the magnitude of) an effective (non-Abelian) electric
field-strength on the world-sheet/world-volume obeying the standard Gauss-law constraint. As a result
a simple classical mechanism for confinement via modified-measure “color” strings is proposed where the
colorlessness of the “hadrons” is an automatic consequence of the new string dynamics.

1. Introduction: Main Ideas and Features of the Theory
One of the characteristic features of string and brane theories [1] is the introduction ad hoc from the very
beginning of a dimensionful scale – the so called string (brane) tension. On the other hand, a lot of attention
has been given to the idea that any fundamental theory of Nature should not contain any ad hoc fundamental
scales and that these scales should rather appear as a result of dynamical generation, e.g., through boundary
conditions on the classical level, spontaneous symmetry breaking and/or dimensional transmutation on the
quantum level (see, for instance, ref.[2] about spontaneous generation of Newton’s gravitational constant).

In the context of string and brane theories, the above idea has been first explored in refs.[3]. In this
Section we will briefly review, with some additional new accents, the main properties of the modified string
and brane theories of [3] in order to prepare the ground for revealing of new interesting structures inherent of
these theories. To this end let us first recall the standard Polyakov-type action for the bosonic string which
reads [4]:

\[ S_{\text{Pol}} = -T \int d^2\sigma \frac{1}{2} \sqrt{-\gamma^{ab}} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) \quad (1) \]

Here \( (\sigma^0, \sigma^1) \equiv (\tau, \sigma) \); \( a, b = 0, 1 \); \( \mu, \nu = 0, 1, \ldots, D - 1 \); \( G_{\mu\nu} \) denotes the external space-time metric; \( \gamma_{ab} \) is the metric defined on the 1 + 1-dimensional world-sheet of the string and \( \gamma = \det |\gamma_{ab}| \). \( T \) indicates the string tension – a dimensionfull quantity introduced ad hoc into the theory which defines a scale.

Now following refs.[3], instead of the standard measure of integration \( d^2\sigma \sqrt{-\gamma} \), we want to consider a new reparametrization invariant measure on the string world-sheet whose density \( \Phi \) is independent of the Riemannian metric \( \gamma_{ab} \). This approach of considering an alternative integration measure has been studied in the context of \( D = 4 \) gravitational theory, in particular, in relation with the cosmological constant problem [6] (and references therein), as well as the fermion families and long-range force problems [7].

Indeed, if we introduce two auxiliary scalar fields (scalars both from the point of view of the 1 + 1-dimensional world-sheet of the string, as well as from the point of view of the embedding \( D \)-dimensional universe) \( \varphi^i \) \( (i = 1, 2) \), we can construct the following world-sheet measure density:

\[ \Phi(\varphi) \equiv \frac{1}{2} \varepsilon_{ij} \epsilon^{ab} \partial_a \varphi^i \partial_b \varphi^j = \varepsilon_{ij} \dot{\varphi}^i \partial_\sigma \varphi^j \quad (2) \]
It is interesting to notice that \( d^2 \sigma \Phi(\varphi) = d\varphi^1 d\varphi^2 \), that is the measure of integration \( d^2 \sigma \Phi \) corresponds to integrating in the target space of the auxiliary scalar fields \( \varphi^i \) \((i = 1, 2)\).

We proceed now with the construction of a new string action that employs the integration measure \( d^2 \sigma \Phi \) instead of the usual \( d^2 \sigma \sqrt{-\gamma} \). When considering the types of actions we can have under these circumstances, the first one that comes to mind is the straightforward generalization of the Polyakov-type action (1):

\[
S_1 = -\frac{1}{2} \int d^2 \sigma \Phi(\varphi) \gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) \tag{3}
\]

Notice that multiplying \( S_1 \) by a constant, before boundary or initial conditions are specified, is a meaningless operation since such a constant can be absorbed in a redefinition of the measure fields \( \varphi^i \) \((i = 1, 2)\) that appear in \( \Phi(\varphi) \).

The form (3) is, however, not a satisfactory choice for a string action because the variation of \( S_1 \) with respect to \( \gamma^{ab} \) leads to the rather strong condition:

\[
\Phi(\varphi) \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) = 0 \tag{4}
\]

If \( \Phi \neq 0 \), it means that \( \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) = 0 \), i.e., it means that the metric induced on the string world-sheet vanishes which is clearly not an acceptable dynamics. Alternatively, if \( \Phi = 0 \), no further information is available – also an undesirable situation.

The situation may be improved by introducing external antisymmetric tensor gauge field \( B_{\mu\nu}(X) \). Then, instead of (3), we have to consider the action:

\[
S_2 = -\int d^2 \sigma \Phi(\varphi) \left[ \frac{1}{2} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) + \frac{\varepsilon^{ab}}{2 \sqrt{-\gamma}} \partial_a X^\mu \partial_b X^\nu B_{\mu\nu}(X) \right] \tag{5}
\]

where \( \varepsilon^{01} = -\varepsilon^{10} = 1 \) and \( \varepsilon^{00} = \varepsilon^{11} = 0 \). Varying (5) with respect to \( \gamma^{ab} \), we get (if \( \Phi \neq 0 \)):

\[
\partial_a X^\mu \partial_b X^\nu G_{\mu\nu} + \gamma_{ab} \frac{\varepsilon^{cd}}{4 \sqrt{-\gamma}} \partial_c X^\mu \partial_d X^\nu B_{\mu\nu} = 0 \tag{6}
\]

Contracting the latter equation with \( \gamma^{ab} \) we see that:

\[
\frac{\varepsilon^{cd}}{2 \sqrt{-\gamma}} \partial_c X^\mu \partial_d X^\nu B_{\mu\nu} = -\gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} \tag{7}
\]

Inserting relation (7) into Eq.(6) we obtain:

\[
\partial_a X^\mu \partial_b X^\nu G_{\mu\nu} - \frac{1}{2} \gamma_{ab} \gamma^{cd} \partial_c X^\mu \partial_d X^\nu G_{\mu\nu} = 0 \tag{8}
\]

which coincides with the form of the string equations of motion corresponding to the Polyakov-type action (1) when the external antisymmetric tensor gauge field \( B_{\mu\nu} \) is absent.

To make further progress and at the same time to show that one can avoid the need of incorporation of an external field, it is important to notice that terms in the action of the form:

\[
S = \int d^2 \sigma \sqrt{-\gamma} L \tag{9}
\]

which do not contribute to the equations of motion of the standard closed string, i.e., such that \( \sqrt{-\gamma} L \) is a total derivative, may yield non-trivial contributions when we consider the counter-parts of (9) of the form:

\[
S = \int d^2 \sigma \Phi(\varphi) L \tag{10}
\]

This is so because if \( \sqrt{-\gamma} L \) is a total divergence, \( \Phi L \) in general is not.
The above fact is indeed crucial. For example, let us consider the modified-measure string theory with an additional intrinsic 1 + 1-dimensional scalar curvature term:

$$S_{\text{curve}} = - \int d^2 \sigma \Phi(\varphi) R$$  \hspace{1cm} (11)$$

which now is not a topological term in contrast to $\int d^2 \sigma \sqrt{-\gamma} R$ in the ordinary string theory with the regular world-sheet integration measure. According to refs. [5], where modified-measure gravity theories in higher dimensions $D > 2$ have been explored, we know that in order to achieve physically interesting results one has to proceed in the first order formalism – employing either the affine connection or the spin connection. In the present paper we will restrict ourselves by exploring the spin connection formalism only. This means that the independent dynamical degrees of freedom are: zweibein $e^a_i$, spin connection $\omega^{\alpha}_{\beta}$, $(\alpha, \beta = 0, 1)$ are tangent “Lorentz” indexes) and the auxiliary scalar fields $\varphi^i$ entering the new integration measure density $\Phi(\varphi)$ [2].

We will use the following notations: $\gamma^{ab} = e^a_i e^b_i \tilde{\eta}^{\tilde{a}\tilde{b}}$; the scalar curvature of the spin conection is $R(\omega, e) = \varepsilon^{ab} e^{\tilde{a}b} R_{\tilde{a}b\tilde{a}b}(\omega)$ where:

$$R_{\tilde{a}b\tilde{a}b}(\omega) = \partial_a \omega_b^{\tilde{a}} + \omega_a^{\tilde{c}} \omega_{\tilde{c}b} - (a \leftrightarrow b)$$  \hspace{1cm} (12)$$

Notice now that in $D = 2$:

$$\omega_a^{\tilde{a}} = \omega_a e^{\tilde{a}b}$$  \hspace{1cm} (13)$$

where $\omega_a$ is a vector field. Therefore, we get for scalar curvature:

$$R(\omega) = \frac{\varepsilon^{ab}}{2\sqrt{-\gamma}} (\partial_a \omega_b - \partial_b \omega_a)$$  \hspace{1cm} (14)$$

We conclude that the vector field $\omega_a$, as a geometrical object associated with the spin-connection, can be treated as an abelian gauge field $A_a$ living on the world-sheet.

Thus, let us consider an abelian gauge field $A_a$ defined on the world-sheet of the string, in addition to the measure-density fields $\varphi^i$ that appear in $\Phi(\varphi)$ [2], the usual Riemannian metric $\gamma_{ab}$ and the string coordinates $X^\mu$. We can then construct the following non-trivial contribution to the action of the form:

$$S_{\text{gauge}} = \frac{1}{2} \int d^2 \sigma \sqrt{-\gamma} F_{\tilde{a}b}(A) \ , \ F_{\tilde{a}b} = \partial_a A_b - \partial_b A_a$$  \hspace{1cm} (15)$$

Therefore, the total action to be considered now is $S_{\text{string}} = S_2 + S_{\text{gauge}}$ reading explicitly:

$$S_{\text{string}} = - \int d^2 \sigma \Phi(\varphi) \left[ \frac{1}{2} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} + \frac{\varepsilon^{ab}}{2\sqrt{-\gamma}} (\partial_a X^\nu \partial_b X^\nu B_{\mu\nu} - F_{\tilde{a}b}(A)) \right] \equiv - \int d^2 \sigma \Phi(\varphi) L$$  \hspace{1cm} (16)$$

The properties of this model and some of its generalizations will be studied in the following sections.

The action (14) is invariant under a set of diffeomorphisms in the target space of the measure-density fields $\varphi^i$ combined with a conformal (Weyl) transformation of the metric $\gamma_{ab}$, namely:

$$\varphi^i \rightarrow \varphi'^i = \varphi^i(\varphi) \quad \text{so that} \quad \Phi \rightarrow \Phi' = J\Phi$$  \hspace{1cm} (17)$$

where $J = \det \frac{\partial \varphi'^i}{\partial \varphi^i}$ is the Jacobian of the transformation (17), and:

$$\gamma_{ab} \rightarrow \gamma'_{ab} = J \gamma_{ab}$$  \hspace{1cm} (18)$$

In what follows we will refer to the set of transformations (17)–(18) as $\Phi$-extended two-dimensional Weyl transformations and, accordingly, to the action (14) as being $\Phi$-extended Weyl-invariant. Notice also that the spin-curvature term (Eq. (11) with $R$ as in (14)) is also $\Phi$-extended Weyl-invariant ($\Phi$-extended Weyl transformations do not affect the spin connection).

The combination $\frac{\varepsilon^{ab}}{2\sqrt{-\gamma}} F_{\tilde{a}b}$ is a genuine scalar. In two dimensions it is proportional to $\sqrt{F_{\tilde{a}b} F_{\tilde{a}b}}$. In the non-Abelian case one can consider terms in the action of the form $\Phi \sqrt{\text{Tr}(F_{\tilde{a}b} F_{\tilde{a}b})}$, the latter being
Φ-extended Weyl-invariant object \((\sqrt{Tr(F_{ab}F^{ab})})\) is also a genuine scalar. This model will be studied in Sec.5 below.

To demonstrate some general features of the theory, we will first follow the Lagrangian formalism for solution of the modified-measure string model (16) explored in refs.\([3]\). Variation of the action (16) with respect to \(\varphi^i\) yields the equations (here we set \(B_{\mu\nu} = 0\) for simplicity):

\[
\varepsilon^{ab}\partial_b \varphi_i \partial_a \left( \gamma^{cd} \partial_c X^\mu \partial_d X^\nu G_{\mu\nu}(X) - \frac{\varepsilon^{cd}}{\sqrt{-\gamma}} F_{cd} \right) = 0
\]  

(19)

If \(\det |\varepsilon_{ab}\partial_b \varphi_i| \neq 0\) meaning \(\Phi(\varphi) \neq 0\), then we conclude that all the derivatives of the quantity inside the parenthesis in Eq.(19) must vanish, i.e., such quantity must equal certain constant \(M\) which will be determined later on:

\[
\gamma^{cd} \partial_c X^\mu \partial_d X^\nu G_{\mu\nu}(X) - \frac{\varepsilon^{cd}}{\sqrt{-\gamma}} F_{cd} = M
\]  

(20)

The equations of motion of the gauge field \(A_a\) tell us about how the string tension appears as an integration constant. Indeed, these equations are:

\[
\varepsilon^{ab} \partial_b \left( \frac{\Phi(\varphi)}{\sqrt{-\gamma}} \right) = 0
\]  

(21)

which can be integrated to yield a spontaneously induced string tension:

\[
\frac{\Phi(\varphi)}{\sqrt{-\gamma}} = \text{const} \equiv T
\]  

(22)

Notice that Eq.(22) is perfectly consistent with the Φ- extended Weyl symmetry (17)–(18). Eq.(20) on the other hand is consistent with the Φ-extended Weyl symmetry only if \(M = 0\). We will see in the next paragraph that the equations of motion indeed imply that \(M = 0\). In the case of higher-dimensional \(p\)-branes, unlike the string case, the corresponding equations of motion will require a non-vanishing constant value of \(M\) (cf. Eq.(48) below).

Let us turn our attention to the equations of motion derived from the variation of (16) with respect to \(\gamma^{ab}\):

\[
\Phi(\varphi) \left( \partial_a X^\mu \partial_b X^\nu - \frac{1}{2} \gamma^{ab} \varepsilon^{cd} \frac{F_{cd}}{\sqrt{-\gamma}} \right) = 0
\]  

(23)

Solving the constraint Eq.(20) for \(\varepsilon^{ab} \frac{F_{cd}}{\sqrt{-\gamma}}\) and inserting the result back into (23) we obtain (provided \(\Phi(\varphi) \neq 0\)):

\[
\left( \partial_a X^\mu \partial_b X^\nu - \frac{1}{2} \gamma_{ab} \gamma^{cd} \partial_c X^\mu \partial_d X^\nu \right) G_{\mu\nu}(X) + \frac{1}{2} \gamma_{ab} M = 0
\]  

(24)

Multiplying the above equation by \(\gamma^{ab}\) and summing over \(a, b\), we find that \(M = 0\), i.e., Eqs.(24) with \(M = 0\) are exactly of the form of Eqs.(3) coming from the standard Polyakov-type action (1) (recall also that it is only \(M = 0\) which is consistent with the Φ-extended Weyl invariance). After Eq.(24) is used, the equations obtained from the variation of the action (16) with respect to \(X^\mu\) are seen to be exactly the same as those obtained from the usual Polyakov-type action as well.

### 2. Bosonic Strings with a Modified Measure: Canonical Approach

It is instructive to study the modified-measure string model (16) also within the framework of the canonical Hamiltonian formalism.

Before proceeding let us note that we can extend the model (16) by putting point-like charges on the string world-sheet which interact with the world-sheet gauge field \(A_a\):

\[
S = S_{\text{string}} - \sum_i e_i \int d\tau A_0(\tau, \sigma_i)
\]  

(25)

For the canonical momenta of \(\varphi^i, A_1, X^\mu\) we obtain (using the short-hand notation \(L\) from (16)):

\[
\pi_\varphi^i = -\varepsilon_{ij} \partial_i \varphi^j L \quad , \quad \pi_{A_1} = E \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}}
\]  

(26)
\[
\mathcal{P}_\mu = \Phi(\varphi) \left[ - \left( \gamma^{00} \dot{X}^\nu + \gamma^{01} \partial_\sigma X^\nu \right) G_{\mu\nu} - \frac{1}{\sqrt{-\gamma}} \partial_\nu B_{\mu\nu} \right]
\]  

(27)

Note particularly the second Eq. (23) showing that the ratio of the modified and the usual Riemannian integration-measure densities has the physical meaning of an electric field-strength on the world-sheet.

We have also the following primary constraints:

\[
\pi_{A_0} = 0 \quad , \quad \pi_{\gamma^{ab}} = 0 \quad , \quad \partial_\sigma \varphi^i \pi_1^i = 0
\]

(28)

where the last constraint follows directly from the first Eq. (26). From (23) - (27) we can express the velocities in terms of the canonical coordinates and momenta as follows:

\[
\dot{X}^\mu \equiv \dot{X}^\mu(\ldots) = - \frac{G^\mu\nu}{\sqrt{-\gamma} \gamma^{00}} \left( \frac{\mathcal{P}_\nu}{E} + \partial_\nu X^\lambda B_{\nu\lambda} \right) - \frac{\gamma^{01}}{\sqrt{-\gamma}} \partial_\nu X^\nu
\]

(29)

\[
\dot{A}_1 \equiv \dot{A}_1(\ldots) = \partial_\sigma A_0 - \sqrt{-\gamma} \frac{\pi_2^\sigma}{\partial_\sigma \varphi_1} + \dot{X}^\mu(\ldots) \partial_\sigma X^\nu B_{\mu\nu} + \sqrt{-\gamma} \left( \frac{1}{2} \gamma^{00} \dot{X}^\nu(\ldots) \dot{X}^\nu(\ldots) + \gamma^{01} \dot{X}^\mu(\ldots) \partial_\sigma X^\nu + \frac{1}{2} \gamma^{11} \partial_\sigma X^\mu \partial_\sigma X^\nu \right) G_{\mu\nu}
\]

(30)

In Eq. (30) we used the short-hand notation \( \dot{X}^\mu(\ldots) \) defined in (29). Since the original Lagrangian \( \mathcal{L} \) in (10) is homogeneous of first order with respect to \( \varphi^i \) we have \( \pi_1^i \varphi^i - \mathcal{L} = 0 \) and, therefore, the canonical Hamiltonian reads:

\[
\mathcal{H} = \mathcal{P}_\mu \dot{X}^\mu(\ldots) + E \dot{A}_1(\ldots)
\]

\[
= - \frac{1}{\sqrt{-\gamma} \gamma^{00}} \frac{1}{2} \left[ \frac{G^\mu\nu}{E} \left( \mathcal{P}_\mu + E \partial_\sigma X^\mu \partial_\sigma B_{\mu\nu} \right) \left( \mathcal{P}_\nu + E \partial_\sigma X^\nu \partial_\sigma B_{\mu\nu} \right) + EG_{\mu\nu} \partial_\sigma X^\mu \partial_\sigma X^\nu \right] + \frac{\gamma^{01}}{\sqrt{-\gamma}} \partial_\nu X^\nu + \sum_i e_i \delta(\sigma - \sigma_i) A_0
\]

(31)

where we used the expressions for the velocities as functions of the canonical coordinates and momenta (23) – (30) and we also included the point-like charge interaction terms from (23). Commuting of the canonical Hamiltonian (31) with the primary constraints (28) leads to the following secondary constraints:

\[
\frac{\pi_2^\sigma}{\partial_\sigma \varphi_1} = 0 \quad , \quad \partial_\sigma E - \sum_i e_i \delta(\sigma - \sigma_i) = 0
\]

(32)

\[
\frac{G^\mu\nu}{E} \left( \mathcal{P}_\mu + E \partial_\sigma X^\mu \partial_\sigma B_{\mu\nu} \right) \left( \mathcal{P}_\nu + E \partial_\sigma X^\nu \partial_\sigma B_{\mu\nu} \right) + EG_{\mu\nu} \partial_\sigma X^\mu \partial_\sigma X^\nu = 0
\]

(33)

\[
\mathcal{P}_\mu \partial_\sigma X^\mu \equiv \left( \mathcal{P}_\mu + E \partial_\sigma X^\nu B_{\mu\nu} \right) \partial_\sigma X^\mu = 0
\]

(34)

In particular, we obtain that the canonical Hamiltonian is a linear combination of constraints only.

The Poisson algebra of the constraints can straightforwardly be computed. First, we observe that the last constraint in (28) span (centerless) Virasoro algebra:

\[
\left\{ \partial_\sigma \varphi^i \pi_1^i(\sigma) , \partial_\sigma \varphi^i \pi_1^i(\sigma') \right\} = 2 \partial_\sigma \varphi^i \pi_1^i(\sigma) \partial_\sigma \varphi^i \pi_1^i(\sigma') + \partial_\sigma \left( \partial_\sigma \varphi^i \pi_1^i \right) \delta(\sigma - \sigma')
\]

(35)

The only nontrivial commutator of the latter with the rest of the constraints is:

\[
\left\{ \partial_\sigma \varphi^i \pi_1^i(\sigma) , \frac{\pi_2^\sigma}{\partial_\sigma \varphi_1} \right\} = - \partial_\sigma \left( \frac{\pi_2^\sigma}{\partial_\sigma \varphi_1} \right) \delta(\sigma - \sigma')
\]

(36)

\(^1\)In analogy with ordinary electrodynamics/Yang-Mills theory the canonically conjugated momentum \( \pi_{A_1} \equiv E \) of the space-like gauge-field component \( A_1 \) is by definition the electric field-strength. However, unlike the ordinary case \( E \) is not proportional to \( F_{01}(A) \); see also Sect. 5 for the non-Abelian case.
Therefore, both constraints \( \partial_{\sigma}\varphi^i \partial^i \) and \( \frac{\pi^p}{\partial_{\varphi^p}} \) span a closed algebra of first-class constraints, which implies that all auxiliary scalars \( \varphi^i \) entering the modified measure \( \Phi(\varphi) \) are pure-gauge degrees of freedom.

Next, we observe that the second constraint in (32) is nothing but Gauss-law first-class constraint for the world-sheet Abelian gauge field, with \( E \) being the corresponding electric field-strength. Obviously, \( E \) is piece-wise constant (with respect to \( \sigma \)) on the world-sheet with jumps at the locations of the point-like charges:

\[
E = E_0 + \sum_i e_i (\sigma - \sigma_i)
\]

(37)

Moreover, since the canonical Hamiltonian (31) does not depend explicitly on \( A_1 \), \( E \) is conserved (world-sheet time-independent).

Finally, the constraints (33)–(34), or more properly, the linear combinations thereof:

\[
T_\pm \equiv \frac{1}{4} G^{\mu \nu} \left( \frac{P_\mu}{E} \pm (G_{\mu \kappa} \pm B_{\mu \lambda}) \partial_{\sigma} X^\kappa \right) \left( \frac{P_\nu}{E} \pm (G_{\nu \lambda} \pm B_{\nu \lambda}) \partial_{\sigma} X^\lambda \right)
\]

(38)

span the same first-class constraint algebra of two mutually commuting centerless Virasoro algebras as in the case of ordinary Polyakov-type string (in the standard case \( H_{\mu \nu \lambda}(B) \equiv 3 \partial_{\mu} [B_{\nu \lambda}] = 0 \) provided we identify the constant world-sheet electric field \( E \) with the ordinary string tension \( T \).

To summarize so far, we find that the modified-measure string model (10) (or (23)), containing no \( \text{ad hoc} \) dimensionfull parameters, produces a dynamically generated effective string tension, which is equal to the ratio of the modified and usual Riemannian integration-measure densities, and which has the physical meaning of a world-sheet electric field strength. As a result the dynamical string tension is (piece-wise) constant along the string with possible jumps at the locations of attached point-like charges (see Sect.5.3 for explicit examples).

3. Bosonic Branes with a Modified Measure

The action of bosonic \( p \)-branes with a modified world-volume integration measure reads (cf. 3):

\[
S_{p\text{-brane}} = - \int d^{p+1}\sigma \Phi(\varphi) \left[ \frac{1}{2} \epsilon^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu \nu} + \frac{\epsilon^{a_1 \ldots a_{p+1}}}{(p+1)\sqrt{-\gamma}} \left( \partial_a X^{\mu_1} \ldots \partial_{a_{p+1}} X^{\mu_{p+1}} F_{\mu_1 \ldots \mu_{p+1}} - F_{a_1 \ldots a_{p+1}}(A) \right) \right] - \int d^{p+1}\sigma \Phi(\varphi)L
\]

(39)

\[
\Phi(\varphi) \equiv \frac{1}{(p+1)!} \epsilon^{i_1 \ldots i_p+1} \epsilon^{a_1 \ldots a_{p+1}} \partial_{a_1} \varphi^{i_1} \ldots \partial_{a_{p+1}} \varphi^{i_{p+1}} = \frac{1}{p!} \epsilon^{ij} \epsilon_{j}^p \epsilon^{a_1 \ldots a_p} \varphi^i \partial_{a_1} \varphi^{i_j} \ldots \partial_{a_p} \varphi^{i_j}
\]

(40)

Here the following notations are used:

\[
\sigma \equiv (\sigma^a) \equiv (\sigma^b \equiv \tau, \sigma^\alpha) \equiv (\tau, \sigma) ; \quad F_{a_1 \ldots a_{p+1}}(A) = (p+1)\partial_{a_1} A_{a_2 \ldots a_{p+1}}
\]

(41)

where \( a, b = 0, \ldots, p; \alpha, \beta = 1, \ldots, p; i, j = 1, \ldots, p+1; \mu, \nu = 0, 1, \ldots, D-1; G_{\mu \nu} \) and \( B_{\mu_1 \ldots \mu_{p+1}} \) denote space-time metric and antisymmetric \( p+1 \)-rank tensor external fields, respectively. Also, it is convenient within the Hamiltonian formalism to introduce the following notations:

\[
\epsilon^{a_1 \ldots a_p} F_{a_1 \ldots a_p}(A) = \dot{A} - \partial_{a} \mathcal{A}_a
\]

(42)

with:

\[
\mathcal{A} \equiv \epsilon^{a_1 \ldots a_p} A_{a_1 \ldots a_p} , \quad \mathcal{A}_a \equiv \epsilon^{a \beta_1 \ldots \beta_{p-1}} A_{0 \beta_1 \ldots \beta_{p-1}}
\]

(43)

In analogy with the string case we can put (closed) \( (p-1) \)-branes on the world-volume of the modified-measure \( p \)-brane (39) coupled to the latter via the auxiliary world-volume \( p \)-form gauge field \( A_{a_1 \ldots a_p} \) giving rise to the following additional term in the action (39):

\[
S = S_{p\text{-brane}} + S_{(p-1)\text{-brane}}
\]

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\[ S_{(p-1)-brane} = \sum_i e_i \int d^{p+1} \sigma A_{a_1 \ldots a_p}(\sigma) \int d^p u \frac{1}{p!} \varepsilon_{a_1 \ldots a_p} \frac{\partial}\partial u_{a_1} \ldots \frac{\partial}\partial u_{a_p} \delta^{(p+1)}(\sigma - \sigma_i(u)) \]
\[ = \sum_i e_i \int d^{p+1} \sigma A_0^\sigma(\sigma) \int d^{p-1} u \frac{1}{(p-1)!} \varepsilon_{\alpha_1 \ldots \alpha_{p-1}} \frac{\partial\sigma_1^\alpha}{\partial u_{\alpha_1}} \ldots \frac{\partial\sigma_{p-1}^\alpha}{\partial u_{\alpha_{p-1}}} \delta^{(p)}(\sigma - \sigma_i(u)) \]

Here \( u \equiv (u^0 = \tau, u^m) \equiv (\tau, \vec{u}) \) with \( m = 1, \ldots, p-1 \) are the world-volume parameters of the pertinent \((p-1)\)-branes embedded in the world-volume of the original \( p \)-brane via the parameter equations \( \sigma = \sigma(u) \) (and we have chosen the static gauge \( \sigma^0 = \tau = u^0 \) for all of them). Also, in the second equality \[44\] we have used the notations from \[43\].

The Lagrangian formalism analysis of the modified-measure \( p \)-brane model (without attached lower-dimensional branes) \[39\] has been performed in \[13\]. It parallels the analysis of the modified-string model (cf. Sect.1) where the analogues of Eqs. \[20\] - \[23\] now read (taking for simplicity \( B_{\mu_1 \ldots \mu_{p+1}} = 0 \)):

\[ \frac{1}{2} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} - \frac{\varepsilon^{a_1 \ldots a_{p+1}}}{(p+1)\sqrt{-\gamma}} F_{a_1 \ldots a_{p+1}}(A) = M \]  
\[ \varepsilon^{a_1 \ldots a_p} \partial_a \left( \frac{\Phi(\varphi)}{\sqrt{-\gamma}} \right) = 0 \quad \rightarrow \quad \frac{\Phi(\varphi)}{\sqrt{-\gamma}} = \text{const} \equiv T \]  
\[ \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} - \frac{\gamma^{ab}}{(p+1)\sqrt{-\gamma}} \varepsilon^{a_1 \ldots a_{p+1}} F_{a_1 \ldots a_{p+1}}(A) = 0 \]

In Eq. \[43\], \( M \) denotes arbitrary integration constant which enters the relation between the intrinsic and the induced metrics on the \( p \)-brane world-volume which follows from \[45\] and \[46\]:

\[ \gamma_{ab} = \frac{p-1}{2M} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} \]

Also we have:

\[ \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} - \frac{\gamma_{ab}}{p+1} \gamma^{cd} \partial_a X^\mu \partial_c X^\nu G_{\mu\nu} = 0 \]

The arbitrariness of \( M \) is due to the manifest invariance of the modified-measure \( p \)-brane action \[43\] under the following global scale symmetry \[13\]:

\[ \varphi^i \rightarrow \lambda_i \varphi^i \quad , \quad \gamma_{ab} \rightarrow \left( \prod_i \lambda_i \right) \gamma_{ab} \quad , \quad A_{a_1 \ldots a_p} \rightarrow \left( \prod_i \lambda_i \right) A_{a_1 \ldots a_p} \]

which can be used to fix the value of \( M \), e.g., \( M = \frac{1}{4}(p-1) \). Note that the “boundary” term \[44\] is not invariant under the scale symmetry \[39\], unless we simultaneously rescale the “charge” coupling constants \( e_i \). Moreover, unlike the string case there is no analogue of the \( \Phi \)-extended Weyl symmetry \[47\] - \[48\] for the modified-measure \( p \)-brane model \[39\]. The reason is due to the fact that for \( p \geq 2 \) the standard measure density \( \sqrt{-\gamma} \) transforms differently \( \sqrt{-\gamma} \rightarrow (J(\varphi))^{\frac{p-1}{2}} \sqrt{-\gamma} \) than the modified measure density \( \Phi(\varphi) \) \[51\] (cf. refs. \[13\]).

The canonical Hamiltonian treatment of the \( p \)-brane model \[39\] with attached \((p-1)\)-branes on its world-volume similarly follows the same steps as the canonical treatment of the modified-measure string model in the previous Section. For the canonical momenta of \( \varphi^i, A, X^\mu \) we have (using the short-hand notation \( L \) from \[39\]):

\[ \pi_i^\varphi = -\varepsilon_{ij_1 \ldots j_p} \varepsilon^{a_1 \ldots a_p} \partial_{a_1} \varphi^{j_1} \ldots \partial_{a_p} \varphi^{j_p} L \quad , \quad \pi_A \equiv E = \frac{\Phi(\varphi)}{\sqrt{-\gamma}} \]

\[ \mathcal{P}_\mu = \Phi(\varphi) \left[ -\left( \gamma^{00} X^\nu + \gamma^{0a} \partial_a X^\nu \right) G_{\mu\nu} - \frac{\varepsilon^{a_1 \ldots a_p}}{\sqrt{-\gamma}} \partial_{a_1} X^{\alpha_1} \ldots \partial_{a_p} X^{\alpha_p} B_{\mu\nu_1 \ldots \nu_p} \right] \]

Also, similarly to \[23\] we have the following primary constraints:

\[ \pi_{A_0} = 0 \quad , \quad \pi_{\varphi^i} = 0 \quad , \quad \partial_\alpha \varphi^i \pi_i^\varphi = 0 \]

\[2\] In what follows we assume that the \((p-1)\)-branes do not intersect each other on the original \( p \)-brane world-volume.
where the last Virasoro-like constraints follow directly from the first Eq.\((51)\).

At this point it is convenient to reexpress the world-volume Riemannian metric \(\gamma_{ab}\) in terms of its purely space-like part \(\gamma_{\alpha\beta}\) and the associated shift vector \(\vec{\sigma}\) and lapse function \(N\) (see e.g. \([8]\)) :

\[
\gamma_{00} = -N^2 \bar{\gamma} + \bar{\gamma}_{\alpha\beta} N^\alpha N^\beta , \quad \gamma_{0\alpha} = \bar{\gamma}_{\alpha\beta} N^\beta , \quad \bar{\gamma}_{\alpha\beta} = \gamma_{\alpha\beta}
\]

where \(\bar{\gamma} = \det |\gamma_{\alpha\beta}|\). In particular, \(\sqrt{-\gamma} = N\bar{\gamma}\).

Using Eqs.\((51)\)–\((52)\) and the notations \((54)\) we find the following canonical Hamiltonian (cf. Eq.\((31)\)) :

\[
\mathcal{H} = \frac{N}{2} \left( \frac{G^{\mu\nu}}{E} \tilde{P}_\mu \tilde{P}_\nu + \mathcal{E} \bar{\gamma} \gamma^{\alpha\beta} G_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu + 4M \bar{\gamma} \mathcal{E} \right) + \mathcal{E} N \mathcal{F}(\pi^\varphi, \partial \varphi) + -N^\alpha \tilde{P}_\mu \partial_\alpha X^\mu + \mathcal{E} \partial_\alpha \mathcal{A}_{\alpha}^0 + \left( (p-1) - \text{brane terms} \right)
\]

with the short-hand notations:

\[
\tilde{P}_\mu = \mathcal{P}_\mu + \mathcal{E} \varepsilon^{\alpha_1 \ldots \alpha_p} \partial_{\alpha_1} X^{\mu_1} \ldots \partial_{\alpha_p} X^{\mu_p} B_{\mu_1 \ldots \mu_p}
\]

and where the last terms in \((55)\) come from \([14]\).

Commuting the canonical Hamiltonian \((54)\) with the primary constraints \((53)\) (where upon using the notations \((54)\) we have \(\pi_N = 0, \pi_{N^\alpha} = 0, \pi_{\gamma_{\alpha\beta}} = 0\) instead of \(\pi_{\gamma_{ab}} = 0\)) we obtain a set of secondary constraints. Using the Poisson-bracket relation:

\[
\left\{ \partial_{\alpha} \varphi^i (\vec{\sigma}), \mathcal{F}(\pi^\varphi, \partial \varphi)(\vec{\sigma}) \right\} = -\delta(\vec{\sigma} - \vec{\sigma}^\prime) \partial_\alpha \mathcal{F}(\pi^\varphi, \partial \varphi)(\vec{\sigma})
\]

we get the following secondary constraint:

\[
\partial_\alpha \mathcal{F}(\pi^\varphi, \partial \varphi) = 0 \rightarrow \mathcal{F}(\pi^\varphi, \partial \varphi) = -2M \equiv \text{const}
\]

where \(M\) is arbitrary constant (it is the Hamiltonian counterpart of the arbitrary integration constant \(M\) appearing within the Lagrangian treatment, cf. Eq.\((13)\)). Once again, as in the string case, we find that the Virasoro-like constraints \(\partial_\alpha \varphi^i (\vec{\sigma})\) together with \(\mathcal{F}(\pi^\varphi, \partial \varphi) + 2M = 0\) (the latter being defined in \((14)\)) form a closed algebra of first class constraints implying that the auxiliary scalar fields \(\varphi^i\) are pure-gauge degrees of freedom.

Next, commuting \((55)\) with \(\pi_{\mathcal{A}_{\alpha}^0}\) yields:

\[
\partial_\alpha \mathcal{E}(\varphi) + \sum_i e_i \int d^{p-1}u \frac{1}{(p-1)!} \varepsilon_{\alpha_1 \ldots \alpha_{p-1}} \frac{1}{(p-1)!} \varepsilon^{m_1 \ldots m_{p-1}} \frac{\partial \sigma_{\alpha_1}^{m_1}}{\partial u_{m_1}} \ldots \frac{\partial \sigma_{\alpha_{p-1}}^{m_{p-1}}}{\partial u_{m_{p-1}}} \delta(\vec{\sigma} - \vec{\sigma}_i(\vec{u})) = 0
\]

which is the \(p\)-brane analog of the “Gauss” law constraint in the string case (second Eq.\((32)\)). Further, since the canonical Hamiltonian \((53)\) does not depend explicitly on \(\mathcal{A}\) (canonically conjugate to \(\mathcal{E}\), the \(p\)-brane “electric” field strength \(\mathcal{E}\) is conserved (world-volume time-independent) and as long as it obeys the generalized “Gauss law” on the world-volume Eq.\((59)\), \(\mathcal{E}\) is also world-volume piece-wise constant field with jumps along the normals equal to the “charge” \(e_i\) when crossing the world-hypersurface of the \(i\)-th \((p-1)\)-brane.

The rest of the secondary constraints reads:

\[
\frac{G^{\mu\nu}}{E} \tilde{P}_\mu \tilde{P}_\nu + \mathcal{E} \bar{\gamma} \gamma^{\alpha\beta} G_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu - 4M \bar{\gamma} \mathcal{E} = 0
\]

\[
\tilde{P}_\mu \partial_\alpha X^\mu = 0 , \quad \partial_\alpha X^\mu \partial_\beta X^\nu G_{\mu\nu} - 2M \frac{1}{p-1} \bar{\gamma}_{\alpha\beta} = 0
\]

We now observe that for the special choice \(M = \frac{1}{2}(p-1)\), and provided we identify the “electric” field strength \(\mathcal{E}\) as a dynamical brane tension \(T\), the constraints \((56)\)–\((63)\) coincide with the secondary constraints.
within the Hamiltonian treatment of the usual Polyakov-like $p$-brane (the latter together with the primary constraints form a mixture of first-class and second-class constraints).

Thus, we conclude that the modified-measure $p$-brane model (59) possesses, apart from the same brane degrees of freedom as the standard Polyakov-like $p$-brane, an additional brane degree of freedom $E$ – an world-volume “electric” field strength, which can be identified as a dynamical brane tension and which, according to Eq. (72), may be variable in general.

4. Superstrings with a Modified Measure

We consider the following Green-Schwarz-type of superstring action with a modified world-sheet integration measure:

$$ S_{\text{superstring}} = \int d^2\sigma \Phi(\varphi) \left[ -\frac{1}{2} \gamma^{ab} \Pi_a \Pi_b + \frac{2}{\sqrt{-\gamma}} \left( \Pi_a (\theta \sigma_\mu \partial_\mu \theta) + \frac{1}{2} F_{ab}(A) \right) \right] = - \int d^2\sigma \Phi(\varphi)L $$

with the same notations as in (14) and (2) (for simplicity we take now $G_{\mu\nu} = \eta_{\mu\nu}, B_{\mu\nu} = 0$) and where:

$$ \Pi_a \equiv \partial_a X^\mu + i \theta \sigma^\mu \partial_a \theta $$

Here $\theta \equiv (\theta^\alpha)$ ($\alpha = 1, \ldots, 16$) denotes 16-dimensional Majorana-Weyl spinor in the embedding $D = 10$ space-time, whereas $\sigma^\mu \equiv (\sigma^\mu)_{\alpha\beta}$ indicate the upper diagonal $16 \times 16$ blocks of the $32 \times 32$ matrices $C^{-1} \Gamma^\mu$ with $\Gamma^\mu$ and $C$ obeying the $D = 10$ Dirac and charge-conjugation matrices, respectively.

The Lagrangian in (62) is explicitly invariant under space-time supersymmetry transformations:

$$ \delta_\epsilon \theta = \epsilon \theta, \quad \delta_\epsilon X^\mu = -i (\epsilon \sigma^\mu \theta) \quad \delta_\epsilon A_a = i (\epsilon \sigma^\mu \theta) \left( \partial_a X^\mu + \frac{2}{3} \theta \sigma^\mu \partial_a \theta \right) $$

In particular, the algebra of supersymmetry transformations (62) closes on $A_a$ up to a gauge transformation:

$$ \{ \delta_\epsilon_1, \delta_\epsilon_2 \} A_a = \partial_a \left( -\frac{2}{3} (\epsilon_1 \sigma^\mu \theta)(\epsilon_2 \sigma^\mu \theta) \right) $$

Let us note that the action (62) bears resemblance to the modified Green-Schwarz superstring action proposed by Siegel [9] provided we replace the modified integration measure density $\Phi(\varphi)$ with the ordinary one $\sqrt{-\gamma}$ and provided we redefine the auxiliary gauge field $A_a$ as fermionic bilinear composite $A_a = -i \theta_a \partial_a \psi^a$ (cf. second ref. [3]) with $\phi$ indicating Siegel's auxiliary fermionic world-sheet field which is a space-time spinor similar to $\theta$. However, let us emphasize that our present approach to the modified-measure superstring model (32) is consistently based on a fundamental (non-composite) gauge field $A_a$.

For the canonical momenta of $\varphi^i, A_1, X_\mu, \theta$ we have (using the short-hand notation $L$ from 62 and (63)):

$$ \pi^\varphi_i = - \epsilon_\varphi \partial_\sigma \varphi^i L, \quad \pi_{A_1} \equiv E = \frac{\Phi(\varphi)}{\sqrt{-\gamma}} $$

$$ \mathcal{P}_\mu = \Phi(\varphi) - (\gamma^{00} \Pi_{0\mu} - \gamma^{01} \Pi_{1\mu} + \frac{i}{\sqrt{-\gamma}} (\theta \sigma_\mu \theta')) $$

$$ \mathcal{P}_\theta = \Phi(\varphi) - (\gamma^{00} \Pi_{\theta\mu} - \gamma^{01} \Pi_{1\mu} - \frac{i}{\sqrt{-\gamma}} \gamma X_\mu^\varphi \theta \sigma^\mu ) $$

where the prime now indicates the derivative $\partial_\gamma$. From (67)–(68) and taking into account the second Eq. (38) we obtain the fermionic primary constraint:

$$ i \mathcal{D} \equiv \mathcal{P}_\theta - \left( \mathcal{P}_\mu - E \Pi_{1\mu} \right) i \theta \sigma^\mu = 0 $$

Therefore, we have the following set of primary constraints:

$$ \pi_{A_0} = 0, \quad \pi_{X_\mu} = 0, \quad \partial_\sigma \varphi^i \pi^{\varphi}_i = 0, \quad \mathcal{D} = 0 $$
Now, for the velocities as functions of the canonical coordinate and momenta we get:

$$\dot{X}^\mu + i\theta\sigma^\mu\dot{\theta} \equiv \Pi_0^\mu(\ldots) = \frac{1}{\sqrt{-\gamma^{00}}}\left(-\mathcal{P}^\mu + i\theta\sigma^\mu\theta'\right) - \frac{1}{\gamma^{00}}\Pi_1^\mu$$  \hspace{1cm} (71)

$$\dot{A}_1 - i\Pi_1^\mu(\theta\sigma_\mu\dot{\theta}) \equiv \dot{A}_1(\ldots) = \partial_\sigma A_0 - \sqrt{-\gamma}\frac{\pi_2^\sigma}{\partial_\sigma\varphi_1} + \sqrt{-\gamma}\frac{1}{2}(\mathcal{P}_{00}\Pi_0^\mu(\ldots) \Pi_0^\mu(\ldots) + \gamma^{01}\Pi_0^\mu(\ldots)\Pi_1^m + \frac{1}{2}\gamma^{11}\Pi_1^0\Pi_1^m) - i(\theta\sigma_\mu\theta')\Pi_0^\mu(\ldots)$$  \hspace{1cm} (72)

In Eq. (72) we used the short-hand notation $\Pi_0^\mu(\ldots)$ defined in (71). The canonical Hamiltonian reads:

$$\mathcal{H} = \mathcal{P}_\mu \dot{X}^\mu(\ldots) + \mathcal{P}_0 \dot{\theta}(\ldots) + E\dot{A}_1(\ldots) + i\Lambda D^a = \mathcal{P}_\mu \Pi_0^\mu(\ldots) + E\dot{A}_1(\ldots) + iD\left(\dot{\theta}(\ldots) - \Lambda\right)$$  \hspace{1cm} (73)

Here $\ldots$ indicate that all velocities as considered as functions of the canonical coordinate and momenta according to (71)–(72). $D$ is the fermionic primary constraint (70) and $\Lambda$ is the corresponding fermionic Lagrange multiplier which is determined from the requirement of the preservation of the constraint $D$ under the Hamiltonian dynamics by (70). Inserting in (73) the expressions (71)–(72) we obtain:

$$\mathcal{H} = -\frac{1}{\sqrt{-\gamma}}\frac{1}{2}\left|\mathcal{P}^\mu - iE(\theta\sigma^\mu\theta')\right| \left(\mathcal{P}_\mu - iE(\theta\sigma_\mu\theta')\right) + \mathcal{E}\Pi_0^\mu\Pi_1^\mu$$

$$+ \frac{\gamma^{01}}{\gamma^{00}}\left(\mathcal{P}_\mu - iE(\theta\sigma_\mu\theta')\right)\Pi_0^\mu + i\Lambda D + E\partial_\sigma A_0 - E\sqrt{-\gamma}\frac{\pi_2^\sigma}{\partial_\sigma\varphi_1}$$  \hspace{1cm} (74)

Commuting of the canonical Hamiltonian (74) with the primary constraints (70) leads to the following secondary constraints:

$$\frac{\pi_2}{\partial_\sigma\varphi_1} = 0 \; , \; \partial_\sigma E = 0 \; \left(\text{“Gauss law”}\right)$$  \hspace{1cm} (75)

$$\mathcal{T}_+ \equiv \frac{1}{4}\left[\mathcal{P} + E\left(X' - 2i\theta\sigma\theta'\right)\right]^2 - i\theta'\mathcal{D} = 0 \; , \; \mathcal{T}_- \equiv \frac{1}{4}\left(\mathcal{P} - E\mathcal{X}'\right)^2$$  \hspace{1cm} (76)

Therefore, as in the purely bosonic case we conclude that the canonical Hamiltonian is a linear combination of constraints only.

As in the bosonic case, the constraints involving the auxiliary scalar fields $\varphi^i$ span the same Poisson-bracket algebra (35)–(36) and, therefore, the auxiliary scalars are again pure-gauge degrees of freedom. The rest of the constraint algebra is the same as in the case of the standard Green-Schwarz formulation provided (in full analogy with the purely bosonic case) we identify the world-sheet “electric” field strength $E$ as dynamically generated string tension $T$.

5. Strings with “$\Phi$-Extended Weyl Invariant” Action for Non-Abelian World-Sheet Gauge Field

5.1 The Regular-Measure Version of the Theory

As it is well known, in four space-time dimensions the standard gauge field action $\propto \int \sqrt{-g}d^4x Tr(F_{\mu\nu}F^{\mu\nu})$ is invariant under transformations $g_{\mu\nu} \rightarrow \Omega^2(x)g_{\mu\nu}$, i.e., it is conformally invariant. In $D = 2$, the appropriate conformally invariant action, provided we use the standard measure $\sqrt{-\gamma}$, would be:

$$\int d^2\sigma \sqrt{-\gamma} \frac{1}{2}Tr(F_{ab}(A)F_{cd}(A))\gamma^{abcd} = \int d^2\sigma \sqrt{Tr(F_{01}(A)F_{01}(A))}$$  \hspace{1cm} (77)

3Let us recall that the fermionic spinor constraint $D$ (68) contains a Lorentz non-covariant mixture of first-class (“kappa”-symmetry) and second-class constraints. To solve the problem of super-Poincare covariant quantization of the standard Green-Schwarz superstring a new reformulation of the latter has been proposed in refs. involving a special set of auxiliary bosonic pure-gauge world-sheet scalar fields (“harmonic” variables). For recent developments on this subject, see and references therein.
where:
\[ F_{ab}(A) = \partial_a A_b - \partial_b A_a + i [A_a, A_b] \] (78)
is a non-Abelian world-sheet gauge field-strength and we have used \( F_{ab}(A) = \varepsilon_{ab} F_{01}(A) \). As we see, the action (77) is not only independent of the conformal factor in the metric, but also it is totally metric independent, i.e., the \( D = 2 \) “square-root Yang-Mills” model (77) is topological in the same sense as, e.g., the \( D = 3 \) Chern-Simons model. Due to this fact the string and gauge degrees of freedom turn out to be decoupled.

To see that such theory does not lead to a well defined dynamics and instead a modified-measure version of (77) is necessary, we consider first the equations of motion that result from (77). Variation with respect to gauge fields \( A_a \) yields:
\[ \nabla_a \left( \frac{F_{01}}{\sqrt{\text{Tr}(F_{01} F_{01})}} \right) = 0 \] (79)
or, equivalently:
\[ \nabla_a F_{01} - F_{01} \frac{\text{Tr}(F_{01} \nabla_a F_{01})}{\text{Tr}(F_{01} F_{01})} = 0 \] (80)
which in turn are equivalent to the equations:
\[ \nabla_a F_{01} = \partial_a f F_{01} \] (81)
with \( f \equiv f(\tau, \sigma) \) being an arbitrary colorless world-sheet scalar field. The general solution of (81) reads:
\[ F_{01} = G^{-1} e^{\int f(\tau, \sigma) \mu_0 G} \] (82)
\[ A_0 = G^{-1} \left( - \mu_0 \int d\sigma' e^{\int f(\tau, \sigma') \frac{G}{G'} - i G^{-1} \partial_\tau G} \right), \quad A_1 = -i G^{-1} \partial_\sigma G \] (83)
where \( G \) is arbitrary \( (\tau, \sigma) \)-dependent element of the gauge group (reflecting the gauge freedom) whereas \( \mu_0 \) is arbitrary constant element of the corresponding Lie algebra.

Thus, we see that in the \( D = 2 \) “square-root Yang-Mills” action (77) there is an additional freedom in equations of motion (beyond the usual non-Abelian gauge symmetry) which is manifested in the appearance of the arbitrary (not determined by the dynamics) world-sheet scalar field \( f(\tau, \sigma) \) in (81)–(83).

This can be equivalently understood from the canonical Hamiltonian point of view. Namely, one can show that the canonical Hamiltonian of the \( D = 2 \) “square-root Yang-Mills” model (77) is a linear combination of first-class constraints only in contrast to the ordinary Yang-Mills case:
\[ \mathcal{H} = \text{Tr} \left( E \left( \partial_\tau A_0 + i \left[ A_1, A_0 \right] \right) \right) + \Lambda_0 \pi_{A_0} + \frac{\Lambda}{2} \left( \text{Tr} E^2 - 1 \right) \] (84)
where \( \pi_{A_0} \) and \( E = \frac{F_{01}}{\sqrt{\text{Tr}(F_{01} F_{01})}} \) are the canonical momenta of \( A_0 \) and \( A_1 \), respectively, and where \( \Lambda_0, \Lambda \) are the corresponding Lagrange multipliers. Notice the appearance of the third first-class constraint term in (84) instead of the standard non-constraint term \( \frac{1}{2} \text{Tr} E^2 \). Moreover, the total number of first-class constraints in (84) exceeds the number of the underlying degrees of freedom.

### 5.2 Modified-Measure Version – The Case of Closed Strings without Charges

We will now see that the modified-measure version of Non-Abelian world-sheet gauge fields has a well defined dynamics (in contrast to the regular measure case of the previous subsection) provided that the theory possesses the \( \Phi \)-extended Weyl symmetry. We consider the following non-Abelian generalization of the original bosonic string action with a modified measure (14) (now we take for simplicity \( G_{\mu \nu} = \eta_{\mu \nu} \) and \( B_{\mu \nu} = 0 \)):
\[
S = -\int d^2 \sigma \Phi(\varphi) \left[ \frac{1}{2} \gamma^{ab} \partial_\sigma X^\mu \partial_\sigma X_\mu - \frac{1}{2} \text{Tr}(F_{ab}(A) F_{cd}(A)) \gamma^{ac} \gamma^{bd} \right]
\]
\[= -\int d^2 \sigma \Phi(\varphi) \left[ \frac{1}{2} \gamma^{ab} \partial_\sigma X^\mu \partial_\sigma X_\mu - \frac{1}{2} \sqrt{\text{Tr}(F_{01}(A) F_{01}(A))} \right] \equiv -\int d^2 \sigma \Phi(\varphi) L \] (85)
where $F_{ab}(A)$ is the non-Abelian world-sheet gauge field-strength as in (78).

Similar to what we have seen in Sec.1, the variation with respect to the measure $\Phi$ degrees of freedom $\varphi^i$ leads to the equation (provided that $\Phi \neq 0$):

$$\frac{1}{2} \gamma^{ab} \partial_\mu X^a \partial_\mu X^b - \frac{1}{\sqrt{-\gamma}} \sqrt{\text{Tr}(F_{01}F_{01})} = M$$  \hspace{1cm} (86)

Varying the action (83) with respect to $\gamma^{ab}$ we get:

$$\partial_\mu X^a \partial_\mu X^b - \frac{1}{\sqrt{-\gamma}} \gamma^{ab} \sqrt{\text{Tr}(F_{01}F_{01})} = 0$$  \hspace{1cm} (87)

Contracting this equation with $\gamma^{ab}$ and comparing with (80) we conclude that again, similar to what has been shown in the simpler model of Section 1, $M = 0$ and we obtain finally:

$$\frac{1}{2} \sqrt{-\gamma} \gamma^{ab} \partial_\mu X^a \partial_\mu X^b = \sqrt{\text{Tr}(F_{01}F_{01})}$$  \hspace{1cm} (88)

$$\partial_\mu X^a \partial_\mu X^b - \gamma^{ab} \frac{1}{2} \gamma^{cd} \partial_\mu X^c \partial_\mu X^d = 0$$  \hspace{1cm} (89)

Varying the action (83) with respect to $A_a$ we obtain:

$$\nabla_a \mathcal{E} \equiv \partial_a \mathcal{E} + i [A_a, \mathcal{E}] = 0 \quad , \quad \mathcal{E} \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}} \frac{F_{01}}{\sqrt{\text{Tr}(F_{01}F_{01})}}$$  \hspace{1cm} (90)

with $\mathcal{E}$ being the non-Abelian electric field-strength – the canonically conjugated momentum of $A_1$. Accordingly, Eq.(90) for $a = 1$ represents the non-Abelian Gauss law on the world-sheet. Using Eqs.(90) one can easily show:

$$0 = \text{Tr} \left( \mathcal{E} \nabla_a \mathcal{E} \right) = \frac{1}{2} \partial_a \left( \text{Tr} \mathcal{E}^2 \right) = \frac{1}{2} \partial_a \left( \frac{\Phi(\varphi)}{\sqrt{-\gamma}} \right)^2$$  \hspace{1cm} (91)

i.e., the ratio of the measure densities (the magnitude of the non-Abelian electric field-strength), which plays the role of a dynamically generated string tension, is again constant: $|\mathcal{E}| \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}} = \text{const}$. The equations of motion (91), upon using this fact, coincide with the equations (80) (Eqs.(81)–(83) similarly hold). However, in contrast to the regular measure version of the theory, now in the context of the modified-measure model (83) we have the equation (88) which upon substituting the solution (82) in $\sqrt{\text{Tr}(F_{01}F_{01})} = e^{f(\tau,\sigma)}\sqrt{\text{Tr} \mathcal{M}^2}$ determines completely the function $f(\tau,\sigma)$ in terms of the string solution.

### 5.3 Charges, Strings and Classical Mechanism for Confinement

Classical treatment of strings in the context of the Polyakov approach (with the regular measure density $\sqrt{-\gamma}$) allows two possibilities for the string topology: the first one is a closed string where the string tension is a constant all over the string; the second possibility is an open string with end-points (and/or $ad$ hoc with point-like charges at the end-points).

In the modified-measure string theory there are more possibilities due to the dynamical mechanism of tension generation. In fact, for both cases, i.e., for closed and open strings, one can study models where one or more point-like charges $C_i$, in general $\text{non-Abelian}$ “color” ones, are located inside the string\footnote{Generically one can consider smooth charge or current distributions along the string. Such more general cases we will study elsewhere; see also Appendix.}. A simple model describing this situation consists of adding to the action (83) the following interaction term:

$$S_{\text{int}} = -\sum_i \int \frac{d\sigma^a}{d\tau_i} \text{Tr}(C_i A_a) d\tau_i$$  \hspace{1cm} (92)

where $\tau_i$ indicate the corresponding proper times. In the simplest case of $\text{static}$ “color” charges $C_i$ localized at the points $\sigma_i$, ($i = 1, 2, ...$), Eq.(92) reads:

$$S_{\text{int,static}} = -\sum_i \text{Tr} C_i \int d\tau A_0(\tau, \sigma_i)$$  \hspace{1cm} (93)
The only changes in the equations of motion, comparing to the equations of the previous subsection, occur in Eq. (90) which in the axial gauge \((A_1 = 0)\) take the form:

\[
\partial_\sigma \mathcal{E} - \sum_i C_i \delta(\sigma - \sigma_i) = 0
\]  

(94)

with \(\mathcal{E}\) as defined in the second Eq. (51).

Let us first consider the solution of the “Gauss law” Eq. (94) in the case with two static point-like (color) charges \(C_1\) and \(C_2\) localized at the points \(\sigma_1\) and \(\sigma_2\) with \(\sigma_1 < \sigma_2\). To get this solution we perform integration in (94) over \(\sigma\) from some \(\sigma < \sigma_1\) up to some \(\sigma > \sigma_2\). Then we obtain:

\[
\mathcal{E}(\sigma) = \begin{cases} 
\mathcal{E}_1 & \text{for } \sigma < \sigma_1 \\
\mathcal{E}_2 & \text{for } \sigma_1 < \sigma < \sigma_2 \\
\mathcal{E}_3 & \text{for } \sigma > \sigma_2 
\end{cases} \quad \text{and} \quad \mathcal{E}_2 - \mathcal{E}_1 = C_1, \quad \mathcal{E}_3 - \mathcal{E}_2 = C_2
\]  

(95)

To realize the physical case of such an open string (no periodic boundary conditions in \(\sigma\) are assumed) with finite energy we have to consider a finite string which is possible only if \(\mathcal{E}_1 \equiv \mathcal{E}_3 \equiv 0\). Then the charges \(C_1\) and \(C_2\) appears to be the end-points and it follows from (95) that:

\[
C_1 + C_2 = 0 \quad \text{and} \quad \mathcal{E}_2 = C_1
\]  

(96)

Therefore, Eq. (96) becomes the statement for color confinement of the two point-like charges \(C_i\) (“quarks”) in a colorless “meson-like state” as a result of the variable dynamical tension of the string connecting them.

In a similar way one can construct a classical string model for baryons. Let us consider a closed string parametrized by \(\sigma\) \((0 \leq \sigma \leq 2\pi)\) with three static point-like color charges \(C_1, C_2, C_3\) localized at the points \(\sigma_1, \sigma_2, \sigma_3\), respectively. Then solving Eq. (94) we obtain for the “chromoelectric” field, i.e., the dynamical string tension (90):

\[
\mathcal{E}(\sigma) = \begin{cases} 
\mathcal{E}_{12} & \text{for } \sigma_1 = 0 < \sigma < \sigma_2 \\
\mathcal{E}_{23} & \text{for } \sigma_2 < \sigma < \sigma_3 \\
\mathcal{E}_{31} & \text{for } \sigma_3 < \sigma < 2\pi
\end{cases}
\]  

(97)

where \(\mathcal{E}_{12}, \mathcal{E}_{23}, \mathcal{E}_{31}\) are constants, which implies:

\[
\mathcal{E}_{12} - \mathcal{E}_{31} = C_1, \quad \mathcal{E}_{23} - \mathcal{E}_{12} = C_2, \quad \mathcal{E}_{31} - \mathcal{E}_{23} = C_3
\]  

(98)

Summing Eqs. (98) we get:

\[
C_1 + C_2 + C_3 = 0
\]  

(99)

which means that color confinement appears again, now in the case of a “baryon-like” configuration.

Notice that not only the orientations of \(\mathcal{E}_{12}, \mathcal{E}_{23}, \mathcal{E}_{31}\) in color space, but also their magnitude are different in general. The last statement follows from the fact that Eq. (71) does not hold in the points where the charges are located. This means that the charges can be sources of discontinuities of the tension (notice that the second equation in (90) still holds). This is possible here precisely due to the identification of the string tension with the ratio of the measure densities \(\frac{2\phi(\sigma)}{\gamma(\sigma)}\) (second Eq. (90)) being also the magnitude of the pertinent world-sheet “chromoelectric” field strength. Due to these properties we may call the above modified-measure string model with a \(\Phi\)-extended Weyl-invariant non-Abelian world-sheet gauge field action (51) a “color” string model.

The above simple picture of point-like charge confinement via “color” strings can be straightforwardly generalized to the case of higher-dimensional branes. Namely, let us consider \(N\) non-intersecting “charged” closed \((p - 1)\)-branes living on a closed \(p\)-brane whose dynamics is governed by the modified-measure brane action (57) and (61). Let us also recall that the dynamically generated brane tension \(\mathcal{E}\) (cf. second Eq. (71)) obeys the brane “Gauss law” constraint Eq. (59). Denoting by \(\mathcal{E}_i\) the constant value of \(\mathcal{E}\) in the strip on the fixed-time world-hypersurface of the \(p\)-brane situated between the \((i - 1)\)-th and the \(i\)-th “charged” \((p - 1)\)-branes we find from (59):

\[
\mathcal{E}_{i+1} = \mathcal{E}_i + e_i, \quad i = 0, 1, \ldots, N \quad \text{with} \quad \mathcal{E}_0 \equiv \mathcal{E}_N, \quad e_0 \equiv e_N
\]  

(100)
Summing up Eqs. (100), we find similarly to the string case that the only possible configuration of static “charged” closed \((p-1)\)-branes coupled pair-wise via modified-measure \(p\)-branes \((39)\) is the zero-charge one.

6. Discussion and Conclusions

We have seen above how modifying the world-sheet (world-volume) measure of integration can significantly affect the implications of string and brane dynamics. First of all, it turns out that to get an acceptable dynamics, the corresponding string and brane theories need the introduction of auxiliary world-sheet gauge field (world-volume \(p\)-form tensor gauge field). Furthermore, the tension of the string or brane is not any more a fundamental parameter \(i.e., a\ given \text{ ad hoc}\) scale: it is dynamically determined as the magnitude of the pertinent gauge field strength and it is proportional to the ratio of the measure densities \(\Phi/\sqrt{−γ}\). If no charges exist on the world-sheet (world-volume) then for closed strings (branes) the standard Polyakov type equations are obtained and the Poisson-bracket algebra of the relevant Hamiltonian constraints is the same as that of the standard string (brane) theory. The same result holds also for the modified-measure superstring model.

The string tension is identified as the canonically conjugate momentum of the spatial component of the auxiliary world-sheet gauge potential, therefore, it assumes the role of an “electric” field strength. The latter is shown to obey the “Gauss law” equation. Thus, in the presence of world-sheet charges, the string tension can change dynamically. The latter becomes possible since the tension, \(i.e., the\ “electric”\ field strength\) is proportional to the ratio of the measure densities \(\Phi/\sqrt{−γ}\). In particular, point-like charges living on the string can be responsible for discontinuous changes of the string tension. The special case, when the string tension changes from a finite value to zero, can be regarded as the formation of an “edge” on the string or, equivalently, as a new way of formulating open strings. We have shown that similar results hold also for modified-measure theories of \(p\)-branes. Namely, \(p\)-form (tensor gauge) charges living on the \(p\)-brane, in particular, lower-dimensional “charged” \((p-1)\)-branes lead to a dynamically changing brane tension.

Finally, we studied a conformally (Weyl-) invariant modified-measure string theory with non-Abelian gauge (“square-root Yang-Mills”) field living on the string world-sheet called “color” string. As a result, a simple classical mechanism for “color” confinement of point-like “color” charges via “color” strings is proposed with the colorlessness of the corresponding composite “hadrons” automatically emerging due to the new dynamics inherent in the modified-measure string model. Similar picture of confinement and colorlessness arises also for systems of “charged” \((p-1)\)-branes coupled via modified-measure \(p\)-branes.

As a byproduct, it is found that a nice geometrical meaning can be given for the auxiliary string world-sheet gauge fields: if these are of the abelian type, they can represent the world-sheet spin-connection associated with the (Abelian in 1 + 1 dimensions) Lorentz group (see Eq. (14) above).

Notice that world-sheet gauge fields have also been considered in the very interesting work \([12]\). In the latter case, however, a Nambu-Goto approach is employed so that the issue of conformal invariance peculiar to the Polyakov formulation is lost.

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Appendix. Strings with a Modified Measure Coupled to World-Sheet Currents

Let us briefly discuss the case of bosonic strings with a modified world-sheet integration measure coupled to an external space-time dilaton field. The pertinent action reads:

\[
S = −\int d^2σ \Phi(ϕ) \left[ \frac{1}{2} γ^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) − R(ω) U(X) \right] \equiv −\int d^2σ \Phi(ϕ)L \tag{101}
\]

with \(R(ω)\) being the scalar curvature of the \(D=2\) spin connection \(ω_a\) defined in Eq. (14). Varying (101) with respect to \(ω_a\) we obtain once again dynamically generated string tension as:

\[
E \equiv πω_1 = \frac{Φ(ϕ)}{\sqrt{−γ}} U(X) = \text{const} \equiv T \tag{102}
\]
with $\pi_{\omega_1}$ being the canonically conjugated momentum of $\omega_1$, which brings the action (101) to the form:

$$S = -T \int d^2 \sigma \left( \frac{1}{2} \sqrt{-\gamma} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu \frac{G_{\mu\nu}(X)}{U(X)} \right)$$  

(103)

i.e., an action describing string motion in a conformally modified external space-time background with $G'_{\mu\nu}(X) = G_{\mu\nu}(X)/U(X)$. Thus, the model (101) differs significantly from the ordinary Polyakov-type string coupled to a dilaton:

$$S = -T \int d^2 \sigma \left( \frac{1}{2} \sqrt{-\gamma} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu + R(\gamma) U(X) \right)$$  

(104)

Now, let us consider a generalization of the string model (16) describing the coupling of the bosonic modified-measure string through the auxiliary gauge field $A_a$ to a conserved world-sheet current $\varepsilon^{ab} \partial_b v$ where $v$ is a world-sheet scalar field:

$$S = - \int d^2 \sigma \Phi(\varphi) \left( \frac{1}{2} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu + \frac{1}{2} \gamma^{ab} \partial_a v \partial_b v - \frac{\varepsilon^{ab}}{2 \sqrt{-\gamma}} F_{ab}(A) \right) + \epsilon \int d^2 \sigma A_a \varepsilon^{ab} \partial_b v$$  

(105)

Notice that the last term in Eq.(105) can be rewritten in the form:

$$\epsilon \int \sqrt{-\gamma} d^2 \sigma A_a \frac{\varepsilon^{ab}}{\sqrt{-\gamma}} \partial_b v$$  

(106)

which means that by including this term we study a model which belongs to the class of Two Measures Theories (TMT).

The equations of motion with respect to $A_a$ read:

$$\varepsilon^{ab} \partial_b \left( \frac{\Phi(\varphi)}{\sqrt{-\gamma}} + \epsilon v \right) = 0 \quad \text{i.e.} \quad \frac{\Phi(\varphi)}{\sqrt{-\gamma}} = C - \epsilon v$$  

(108)

where $C$ is a dynamically generated constant scale. The canonical Hamiltonian treatment of (105) is completely analogous to the simpler case of (16) in Sect.2. In particular, for the auxiliary “electric” field strength we obtain:

$$\pi_{A_1} \equiv E = \frac{\Phi(\varphi)}{\sqrt{-\gamma}} \quad \rightarrow \quad E + \epsilon v = C$$  

(109)

(cf. Eq.(108)) and the canonical Hamiltonian becomes:

$$H = - \frac{1}{\sqrt{-\gamma} \gamma^{00}} \frac{1}{2} \int \left[ \frac{1}{E} P^2 + E (\partial_\sigma X)^2 + \frac{1}{2} (\pi_v + \epsilon A_1)^2 + E (\partial_\sigma v)^2 \right] + \frac{\gamma^{01}}{\gamma^{00}} \left[ P_\mu \partial_\sigma X^\mu + (\pi_v + \epsilon A_1) \partial_\sigma v \right]$$  

(110)

We have skipped in (110) the linear combination of the rest of the primary (28) and secondary (32) constraints which remain unaltered by the presence of the new field $v$ except for the “Gauss law” constraint which now reads (cf. (109)):

$$\partial_\sigma (E + \epsilon v) = 0$$  

(111)

One can check that the basic constraints entering (110) span again a closed Poisson-bracket algebra which this time involves also the “Gauss law” constraint (111) and the following variable string tension equal to the world-sheet “electric” field (109):

$$T \equiv E = C - \epsilon v$$  

(112)

5In D-dimensional space-time the action has generically the form:

$$S = \int d^D x \Phi(\varphi) L_1 + \int d^D x \sqrt{-\gamma} L_2$$  

(107)

where the Lagrangian densities $L_1$ and $L_2$ are independent of the degrees of freedom $\varphi^i$ building up $\Phi$. 

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