Progress in the determination of the $J/\psi - \pi$ cross section

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Abstract

Improving previous calculations, we compute the $J/\psi \pi \to$ charmed mesons cross section using QCD sum rules. Our sum rules for the $J/\psi \pi \to \bar{D} D^*$, $D \bar{D}^*$, $\bar{D}^* D^*$ and $\bar{D} D$ hadronic matrix elements are constructed by using vacuum-pion correlation functions, and we work up to twist-4 in the soft-pion limit. Our results suggest that, using meson exchange models is perfectly acceptable, provided that they include form factors and that they respect chiral symmetry. After doing a thermal average we get $\langle \sigma_{\pi J/\psi v} \rangle \sim 0.3$ mb at $T = 150$ MeV.
I. INTRODUCTION

For a long time charmonium suppression has been considered as one of the best signatures of quark gluon plasma (QGP) formation [1]. Recently this belief was questioned by some works. Detailed simulations [2] of a population of $c-\bar{c}$ pairs traversing the plasma suggested that, given the large number of such pairs, the recombination effect of the pairs into charmonium Coulomb bound states is non-negligible and can even lead to an enhancement of $J/\psi$ production. This conclusion received support from the calculations of [3]. Taking the existing calculations seriously, it is no longer clear that an overall suppression of the number of $J/\psi$’s will be a signature of QGP. A more complex pattern can emerge, with suppression in some regions of the phase space and enhancement in others [4, 5]. Whatever the new QGP signature (involving charm) turns out to be, it is necessary to understand better the $J/\psi$ dissociation mechanism by collisions with comoving hadrons.

Since there is no direct experimental information on $J/\psi$ absorption cross sections by hadrons, several theoretical approaches have been proposed to estimate their values. In order to elaborate a theoretical description of the phenomenon, we have first to choose the relevant degrees of freedom. Already at this point no consensus has been found. Some approaches were based on charm quark-antiquark dipoles interacting with the gluons of a larger (hadron target) dipole [6, 7, 8] or quark exchange between two (hadronic) bags [9, 10], whereas other works used the meson exchange mechanism [11, 12, 13, 14, 15, 16]. In this case it is not easy to decide in favor of quarks or hadrons because we are dealing with charm quark bound states, which are small and massive enough to make perturbation theory meaningful, but not small enough to make non-perturbative effects negligible. Charmonium is “the borderguard of the mysterious border of perturbative world of quarks and gluons and the non-perturbative world of hadrons” [17].

In principle, different approaches apply to different energy regimes and we might think that at lower energies we can use quark-interchange models [9, 10] or meson exchange models [11, 12, 13, 14, 15, 16] and, at higher energies we can use perturbative QCD [3, 7, 8]. However, even at low energies, the short distance aspects may become dominant and spoil a non-perturbative description. In a similar way, non-perturbative effects may be important even at very high energies [18].

Inspite of the difficulties, some progress has been achieved. This can be best realized
if we compare our knowledge on the subject today with what we knew a few years ago, described by B. Mueller (in 1999) as “...the state of the theory of interactions between $J/\psi$ and light hadrons is embarrassing. Only three serious calculations exist (after more than 10 years of intense discussion about this issue!) and their results differ by at least two orders of magnitude in the relevant energy range. There is a lot to do for those who would like to make a serious contribution to an important topic”. In the subsequent three years about 30 papers on this subject appeared and now the situation is much better, at least in what concerns the determination of the order of magnitude, which, as it will be discussed below, in the case of the $J/\psi$ pion interaction, is determined to be $1 < \sigma_{J/\psi - \pi} < 10 \text{ mb}$ in the energy region close to the open charm production threshold.

One of the main things that we have learned is the near-threshold behavior of $\sigma_{J/\psi - \pi}$. This is quite relevant because in a hadron gas at temperatures of $100 - 300 \text{ MeV}$ most of the $J/\psi - \pi$ interactions occur at relatively low energies, barely sufficient to dissociate the charmonium state. In some calculations a rapid growth of the cross sections with the energy was found [12, 13, 14]. This behavior was criticized and considered to be incompatible with empirical information extracted from $J/\psi$ photoproduction [20]. This criticism, however, made use of the vector meson dominance hypothesis (VDM), which, in the case of charm, is rather questionable [21]. The introduction of form factors in the effective Lagrangian approach, while reducing the order of magnitude of the cross section, did not change this fast growing trend around the threshold. Later, again in the context of meson exchange models, it was established [16] that the correct implementation of chiral symmetry prevents the cross section from rising steeply around the threshold. In a different approach, with QCD sum rules (QCDSR) [22], the behavior found in [16] was confirmed and in the present work, with improved QCDSR, we confirm again the smooth threshold behavior. We, thus, believe that this question has been answered.

Another, phenomenologically less important, but conceptually interesting issue is the energy dependence in the region far from threshold. Results obtained with the non-relativistic quark model [1] indicated a rapidly falling cross section. This behavior is due to the gaussian tail of the quark wave functions used in the quark exchange model. This result of the quark model approach could be mimicked within chiral meson Lagrangian approaches with the introduction of $\sqrt{s}$ dependent form factors [15, 23]. For $J/\psi - N$ interactions, it was found in ref. [8] that this behaviour depends ultimately on the gluon distribution in the proton at
low $x$. In the case of $J/\psi - N$, for certain parametrizations of the gluon density one could find a falling trend for the cross section \[8\], but no definite conclusion could yet be drawn.

If the $J/\psi$ is treated as an ordinary hadron, its cross section for interaction with any other ordinary hadron must increase smoothly at higher energies, in much the same way as the proton-proton or pion-proton cross sections. The underlying reason is the increasing role played by perturbative QCD dynamics and the manifestation of the partonic nature of all hadrons. Among the existing calculations, no one is strictly valid at $\sqrt{s} \simeq 20$ GeV, except the one of ref. \[18\], which is designed to work at very high energies and which gives, for the $J/\psi$-nucleon cross section the value $\sigma_{J/\psi - N} \simeq 5$ mb. This number can be considered as a guide for $J/\psi - \pi$ cross section in the high energy regime. It should, however, be pointed out, that the calculation of ref. \[18\] is based on a purely nonperturbative QCD approach. The inclusion of a perturbative contribution will add to the quoted value and will have a larger weight at higher energies. A similar conclusion was reached in \[24\]. In the traditional short distance QCD approach the cross section grows monotonically \[6, 7, 8\].

As a side-product, the theoretical effort to estimate the charmonium-hadron cross section motivated a series of calculations \[25, 26, 27\], within the framework of QCD sum rules, of form factors and coupling constants involving charmed hadrons, that may be relevant also to other problems in hadron physics.

In this work we improve the calculation done in ref. \[22\] by considering sum rules based on a three-point function with a pion. We work up to twist-4, which allows us to study the convergence of the OPE expansion. Since the method of the QCDSR uses QCD explicitly, we believe that our work brings a significant progress to this important topic.

The paper is organized as follows: in the next section we review the method of QCD sum rules, giving special emphasis to the QCD side. In section III we present some formulas for the computation of open charm production amplitudes and in section IV we give our numerical results. Finally some concluding remarks are given in section V.

II. THE METHOD

In the QCDSR approach \[28, 29\], the short range perturbative QCD is extended by an OPE expansion of the correlator, giving a series in inverse powers of the squared momentum with Wilson coefficients. The convergence at low momentum is improved by using a Borel
transform. The coefficients involve universal quark and gluon condensates. The quark-based
calculation of a given correlator is equated to the same correlator, calculated using hadronic
degrees of freedom via a dispersion relation, giving sum rules from which a hadronic quantity
can be estimated.

Let us start with the general vacuum-pion correlation function:

$$\Pi_{\mu 34} = \int d^4 x \, d^4 y \, e^{-ip_2 \cdot y} \, e^{ip_3 \cdot x} \langle 0 | \{ j_3(x) j_4(0) j_\mu^\psi(y) \} | \pi(p_1) \rangle ,$$

with the currents given by $j_\mu^\psi = c \gamma_\mu c$, $j_3 = \bar{u} \Gamma_3 c$ and $j_4 = \bar{u} \Gamma_4 d$. $p_1$, $p_2$, $p_3$ and $p_4$ are the four-momenta of the mesons $\pi$, $J/\psi$, $M_3$ and $M_4$ respectively, and $\Gamma_3$ and $\Gamma_4$ denote specific gamma matrices corresponding to the process involving the mesons $M_3$ and $M_4$. For instance, for the process $J/\psi \pi \rightarrow \bar{D} \, D^*$ we will have $\Gamma_3 = \gamma_\nu$ and $\Gamma_4 = i\gamma_5$. The advantage of this approach as compared with the 4-point calculation in ref. [22], is that we can consider more terms in the OPE expansion of the correlation function in Eq. (1) and, therefore, check the “convergence” of the OPE expansion.

Following ref. [30], we can rewrite Eq. (1) as:

$$\Pi_{\mu 34} = -\int \frac{d^4 k}{(2\pi)^4} \, Tr \{ S_{ac}(p_3 - k) \gamma_\mu S_{cb}(p_3 - p_2 - k) \Gamma_4 D_{ab}(k, p_1) \Gamma_3 \} ,$$

where

$$S_{ab}(p) = \frac{i(p + m_c)}{p^2 - m_c^2} \delta_{ab}$$

is the free $c$-quark propagator, and $D_{ab}(k, p)$ denotes the quark-antiquark component with a pion, which can be separated into three pieces depending on the Dirac matrices involved [30, 31]:

$$D_{ab}(k, p) = \delta_{ab} \left[ i\gamma_5 A + \gamma_\alpha \gamma_5 B^\alpha + \gamma_5 \sigma_{\alpha\beta} C^{\alpha\beta} \right] ,$$

with $a$, $b$ and $c$ being color indices. The three invariant functions of $k, p$ are defined by

$$A(k, p) = \frac{1}{12} \int d^4 x \, e^{ik \cdot x} \langle 0 | \bar{d}(0) i\gamma_5 u(0) | \pi(p) \rangle ,$$

$$B^\alpha(k, p) = \frac{1}{12} \int d^4 x \, e^{ik \cdot x} \langle 0 | \bar{d}(0) \gamma^\alpha u(0) | \pi(p) \rangle ,$$

$$C^{\alpha\beta}(k, p) = -\frac{1}{24} \int d^4 x \, e^{ik \cdot x} \langle 0 | \bar{d}(0) \sigma^{\alpha\beta} \gamma_5 u(0) | \pi(p) \rangle .$$

Using the soft-pion theorem, PCAC and working at the order $O(p_\mu p_\nu)$ we get up to
twist-4 [31, 32]:

$$A(k, p) = \frac{(2\pi)^4 \langle \bar{q} q \rangle}{12 f_\pi} \left[ -2 + ip_\omega \frac{\partial}{i\partial k_\omega} + \frac{1}{3} p_{\alpha_1} p_{\alpha_2} \frac{\partial}{i\partial k_{\alpha_1}} \frac{\partial}{i\partial k_{\alpha_2}} \right] \delta^{(4)}(k) ,$$

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\[ B_\alpha(k, p) = \frac{(2\pi)^4}{12} f_\pi \left[ ip_\alpha + \frac{1}{2} p_\alpha p_\alpha \frac{\partial}{i\partial k_\alpha} + \frac{i\delta^2}{36} \left( 5p_\alpha g_{\alpha\alpha} - 2p_{\alpha\alpha} g_{\alpha\alpha} \right) \frac{\partial}{i\partial k_\alpha} \frac{\partial}{i\partial k_\alpha} \right] \delta^{(4)}(k) \],
\[ C_{\alpha\beta}(k, p) = -\frac{(2\pi)^4}{24} \frac{\langle \bar{q} \bar{q} \rangle}{3 f_\pi} (p_\alpha g_{\beta\alpha} - p_\beta g_{\alpha\alpha}) \left[ i \frac{\partial}{i\partial k_\alpha} - \frac{p_{\alpha\alpha}}{2} \frac{\partial}{i\partial k_\alpha} \frac{\partial}{i\partial k_\alpha} \right] \delta^{(4)}(k) \],

where \( \delta^2 \) is defined by the matrix element \( \langle 0 | \bar{d} g \tilde{G}^{\alpha\beta} \gamma_5 u | \pi(p) \rangle = i\delta^2 f_\pi p^\alpha \), where \( \tilde{G}_{\alpha\beta} = \epsilon_{\alpha\beta\sigma\tau} G_{\sigma\tau} / 2 \) and \( G_{\alpha\beta} = t^A G_{\alpha\beta} \).

The additional contributions to the OPE comes from the diagrams where one gluon, emitted from the \( c \)-quark propagator, is combined with the quark-antiquark component. Specifically, the \( c \)-quark propagator with one gluon being attached is given by \[ \langle 3, 2 \rangle \]

\[ -\frac{g_s G_{\alpha\beta}}{2(k^2 - m_c^2)^2} [k_\alpha \gamma_\beta - k_\beta \gamma_\alpha + (\not{k} + m_c)i\sigma_{\alpha\beta}] \, , \] (7)

Taking the gluon stress tensor into the quark-antiquark component, one can write down the correlation function into the form

\[ \Pi_{\mu\nu} = -4 \int \frac{d^4k}{(2\pi)^4} Tr \left[ \left( S_{\alpha\beta}(p_3 - k) \gamma_\mu S(p_3 - p_2 - k) 
+ S(p_3 - k) \gamma_\mu S_{\alpha\beta}(p_3 - p_2 - k) \right) \Gamma_4 D^{\alpha\beta}(k, p_1) \Gamma_3 \right] \],

where we have already contracted the color indices, and where we have defined

\[ S_{\alpha\beta}(k) = -\frac{1}{2(k^2 - m_c^2)^2} [k_\alpha \gamma_\beta - k_\beta \gamma_\alpha + (\not{k} + m_c)i\sigma_{\alpha\beta}] \, , \] (9)

and

\[ D^{\alpha\beta}(k, p) = \gamma_5 \sigma_{\rho\lambda} E^{\rho\lambda\alpha\beta}(k, p) + \gamma_7 \epsilon^{\alpha\beta\rho\lambda} F_{\tau\rho\lambda}(k, p) \, , \] (10)

with

\[ E^{\rho\lambda\alpha\beta}(k, p) = -\frac{1}{32} \int d^4x \, e^{ik \cdot x} \langle 0 | \bar{d}(x) \gamma_5 \sigma^{\rho\lambda} g_s G^{\alpha\beta} u | \pi(p) \rangle \, , \]
\[ F_{\tau\rho\lambda}(k, p) = \frac{1}{32} \int d^4x \, e^{ik \cdot x} \langle 0 | \bar{d}(x) \gamma_{\tau} g_s \tilde{G}^{\rho\lambda} u | \pi(p) \rangle \, . \] (11)

Up to twist-4 and at order \( \mathcal{O}(p_\mu p_\nu) \), the two functions appearing above are given by \[ \text{30, 32} \]

\[ E^{\rho\lambda\alpha\beta} = \frac{i}{32} f_{3\pi} \left( p^\alpha p^\rho g^{\lambda\beta} - p^\beta p^\rho g^{\lambda\alpha} - p^\alpha p^\lambda g^{\rho\beta} + p^\beta p^\lambda g^{\rho\alpha} \right) (2\pi)^4 \delta^{(4)}(k) \]
\[ F_{\tau\rho\lambda} = -\frac{i\delta^2 f_{3\pi}}{3 \times 32} (p_\theta g_{\tau\delta} - p_\delta g_{\tau\theta}) (2\pi)^4 \delta^{(4)}(k) \, , \] (12)

where \( f_{3\pi} \) is defined by the vacuum-pion matrix element \( \langle 0 | \bar{d} g_s \sigma_{\alpha\beta} \gamma_5 \tilde{G}^{\alpha\beta} u | \pi(p) \rangle \) \[ \text{32} \].
The phenomenological side of the correlation function, $\Pi_{\mu 34}$, is obtained by the consideration of $J/\psi$, $M_3$ and $M_4$ state contribution to the matrix element in Eq. (4). The hadronic amplitudes are defined by the matrix element:

$$i\mathcal{M} = \langle \psi(p_2, \mu) | M_3(-p_3, \nu) M_4(-p_4, \rho) \pi(p_1) \rangle$$

$$= i \mathcal{M}_{\mu 34}(p_1, p_2, p_3, p_4) e_2^
u f_3^\nu f_4^\rho,$$

where $f_i^\alpha = \epsilon_i^\alpha$ for the $D^*$ meson and $f_i^\alpha = 1$ for the $D$ meson.

The phenomenological side of the sum rule can be written as (for the part of the hadronic amplitude that will contribute to the cross section) [22]:

$$\Pi_{\mu 34}^{phen} = -\frac{m_\psi f_\psi \lambda_3 \lambda_4 \mathcal{M}_{\mu 34}}{(p_2^2 - m_\psi^2)(p_3^2 - m_\psi^2)(p_4^2 - m_\psi^2)} + \text{h. r.},$$

where h. r. means higher resonances, and where $\lambda_i$ is related with the corresponding meson decay constant: $\langle D|j_D|0 \rangle = -\lambda_D = m_D^2 f_D/m_c$ and $\langle 0|j_\alpha|D^* \rangle = \lambda_{D^*} \epsilon_\alpha = m_{D^*} f_{D^*} \epsilon_\alpha$.

III. HADRONIC AMPLITUDES FOR $J/\psi \pi \to OPEN CHARM$

The hadronic amplitudes can be written in terms of many different structures. In terms of the structures that will contribute to the cross section we can write

- for the process $J/\psi \pi \to \bar{D} D^*$:

$$\mathcal{M}_{\mu \nu} = \Lambda_1^{D^*} p_1 \mu p_1 \nu + \Lambda_2^{DD^*} p_1 \mu p_2 \nu + \Lambda_3^{DD^*} p_1 \nu p_3 \mu + \Lambda_4^{DD^*} g_{\mu \nu} + \Lambda_5^{DD^*} p_2 \nu p_3 \mu,$$

- for the process $J/\psi \pi \to \bar{D} D$:

$$\mathcal{M}_\mu = \Lambda_{DD} \epsilon_{\mu \alpha \beta \sigma} p_1^\alpha p_2^\beta p_3^\sigma,$$

- for the process $J/\psi \pi \to \bar{D}^* D^*$:

$$\mathcal{M}_{\mu \nu \rho} = \Lambda_1^{D^* D^*} H_{\mu \nu \rho} + \Lambda_2^{D^* D^*} J_{\mu \nu \rho} + \Lambda_3^{D^* D^*} g_{\mu \nu} \epsilon_{\rho \alpha \beta \gamma} p_1^\alpha p_2^\beta p_3^\gamma + \Lambda_4^{D^* D^*} \epsilon_{\rho \alpha \beta \gamma} p_3 \mu p_1^\alpha p_2^\beta p_3^\gamma + \Lambda_5^{D^* D^*} \epsilon_{\mu \alpha \beta \sigma} p_3 \mu p_1^\alpha p_2^\beta p_3^\sigma + \Lambda_6^{D^* D^*} \epsilon_{\mu \alpha \beta \sigma} p_3 \mu p_1^\alpha p_2^\beta p_3^\sigma + \Lambda_7^{D^* D^*} \epsilon_{\mu \alpha \beta \sigma} p_3 \mu p_1^\alpha p_2^\beta p_3^\sigma + \Lambda_8^{D^* D^*} \epsilon_{\mu \alpha \beta \sigma} p_3 \mu p_1^\alpha p_2^\beta p_3^\sigma + \Lambda_9^{D^* D^*} \epsilon_{\mu \alpha \beta \sigma} p_3 \mu p_1^\alpha p_2^\beta p_3^\sigma + \Lambda_{10}^{D^* D^*} \epsilon_{\mu \alpha \beta \sigma} p_3 \mu p_1^\alpha p_2^\beta p_3^\sigma + \Lambda_{11}^{D^* D^*} \epsilon_{\mu \alpha \beta \sigma} p_3 \mu p_1^\alpha p_2^\beta p_3^\sigma + \Lambda_{12}^{D^* D^*} \epsilon_{\mu \alpha \beta \sigma} p_3 \mu p_1^\alpha p_2^\beta p_3^\sigma + \Lambda_{13}^{D^* D^*} \epsilon_{\mu \alpha \beta \sigma} p_3 \mu p_1^\alpha p_2^\beta p_3^\sigma.$$
with \( H_{\mu \nu \rho} = (\epsilon_{\nu \alpha \beta \gamma} g_{\mu \rho} - \epsilon_{\rho \alpha \beta \gamma} g_{\mu \nu}) \vec{p}_1^\alpha \vec{p}_2^\beta \vec{p}_3^\gamma + \epsilon_{\mu \rho \alpha \beta} p_2^\alpha p_1^\beta p_3^\rho \) and \( J_{\mu \nu \rho} = (\epsilon_{\nu \rho \alpha \beta} p_1^\rho + \epsilon_{\mu \rho \alpha \beta} p_1^\mu + \epsilon_{\mu \rho \alpha \beta} p_2^\rho) \vec{p}_1^\alpha \vec{p}_2^\beta \vec{p}_3^\gamma + \epsilon_{\mu \nu \alpha \beta} \vec{p}_2^\rho \vec{p}_1^\alpha \vec{p}_3^\beta \).

In Eqs. (15), (16) and (17), \( \Lambda_i \) are the parameters that we will evaluate from the sum rules. In principle all the independent structures appearing in \( H_{\mu \nu \rho} \) and \( J_{\mu \nu \rho} \) would have independent parameters \( \Lambda_i \). However, since in our approach we get exactly the same sum rules for all of them, we decided to group them with the same parameters.

Inserting the results in Eqs. (6) and (12) into Eqs. (2) and (8) we can write a sum rule for each of the structures appearing in Eqs. (15), (16) and (17). To improve the matching between the phenomenological and theoretical sides we follow the usual procedure and make a single Borel transformation to all the external momenta taken to be equal:

\(-p_2^2 = -p_3^2 = -p_1^2 = P^2 \rightarrow M^2.\)

The problem of doing a single Borel transformation is the fact that terms associated with the pole-continuum transitions are not suppressed \[33\]. In the present case we have two kinds of these transitions: double pole-continuum and single pole-continuum. In the limit of similar meson masses it is easy to show that the Borel behaviour of the three-pole, double pole-continuum and single pole-continuum contributions are \( e^{-m_3^2/M^2}/M^4 \), \( e^{-m_2^2/M^2}/M^2 \) and \( e^{-m_1^2/M^2} \) respectively. Therefore, we can single out the three-pole contribution from the others by introducing two parameters in the phenomenological side of the sum rule, which will account for the double pole-continuum and single pole-continuum contributions. The expressions for all 19 sum rules are given in the Appendices A, B and C.

**IV. RESULTS AND DISCUSSION**

The parameter values used in all calculations are \( m_c = 1.37 \) GeV, \( m_\pi = 140 \) MeV, \( m_D = 1.87 \) GeV, \( m_{D^*} = 2.01 \) GeV, \( m_\psi = 3.097 \) GeV, \( f_\pi = 131.5 \) MeV, \( \langle \vec{p}q \rangle = -(0.23)^3 \) GeV\(^3\), \( m_0^2 = 0.8 \) GeV\(^2\), \( \delta^2 = 0.2 \) GeV\(^2\), \( f_{3\pi} = 0.0035 \) GeV\(^2\) \[32\]. For the charmed mesons decay constants we use the values from \[32\] for \( f_D \) and \( f_{D^*} \) and the experimental value for \( f_\psi \):

\[ f_\psi = 270 \) MeV, \( f_D = 170 \) MeV, \( f_{D^*} = 240 \) MeV. \[18\]

In ref. \[31\] we have analyzed the sum rule for the process \( J/\psi \pi \rightarrow \bar{D}D \). Here we choose to show the sum rule for \( \Lambda_{1D^*D^*} \) in Eq. (C1), as an example of the sum rules for the process \( J/\psi \pi \rightarrow \bar{D}^*D^* \).
FIG. 1: Sum rule for $\Lambda_1^{D^* D^*}$ related to the process $J/\psi \pi \rightarrow \bar{D}^* D^*$ as a function of the Borel mass. The dots, squares and diamonds give the twist-2, 3 and 4 contributions to the sum rule. The triangles give the result from Eq. (C1). The solid line give the fit to the QCDSR results.

In Fig. 1 we show the QCD sum rule results for $\Lambda_1^{D^* D^*} + A_1^{D^* D^*} M^2 + B_1^{D^* D^*} M^4$ as a function of $M^2$. The dots, squares and diamonds give the twist-2, 3 and 4 contributions respectively. The triangles give the final QCDSR results. We see that the twist-3 and 4 contributions are small as compared with the twist-2 contribution, following the same behaviour as the sum rule for the process $J/\psi \pi \rightarrow \bar{D} D$ given in [34]. In general all the other sum rules are similar and contain twist-2, twist-3 and twist-4 contributions corresponding to the first, second, and third terms inside the brackets in the right hand side of Eq. (C1). Only the sum rules for $\Lambda_{10}^{D^* D^*}$ up to $\Lambda_{13}^{D^* D^*}$, $\Lambda_{4}^{D^* D^*}$ and $\Lambda_{5}^{D^* D^*}$ do not get the leading twist contribution, and will be neglected in the evaluation of the cross section. It is also interesting to notice that if we consider only the leading twist contributions we recover the sum rules obtained in ref. [22]. The triangles in Fig. 1 follow almost a straight line in the Borel region $6 \leq M^2 \leq 16$ GeV$^2$. This show that the single pole-continuum transitions contribution is small. The value of the amplitude $\Lambda_1^{D^* D^*}$ is obtained by the extrapolation of the fit to $M^2 = 0$ [26, 27, 33]. Fitting the QCD sum rule results to a quadratic form we get

$$\Lambda_1^{D^* D^*} \simeq 10.5 \text{ GeV}^{-3}. \quad (19)$$

Since we worked in the soft pion limit, $\Lambda_1^{D^* D^*}$, as well as all other $\Lambda$, is just a number. All particle momenta dependence of the amplitudes is contained in the Dirac structure.
In obtaining the results shown in Fig. 1 we have used the numerical values for the meson decay constants given in Eq. (18). However, it is also possible to use the respective sum rules, as done in [22]. The two-point sum rules for the meson decay constants are given in the appendix D. The behaviour of the results for the hadronic amplitudes does not change significantly if we use the two-point sum rules for the meson decay constants instead of the numerical values, leading only to a small change in the value of the amplitudes. In Fig. 2 we show, for a comparison, both results in the case of $\Lambda_1^{D^*D^*}$.

![Graph of $\Lambda_1^{D^*D^*}$](image)

**FIG. 2:** Sum rule for $\Lambda_1^{D^*D^*}$ related to the process $J/\psi \pi \rightarrow \bar{D}^* D^*$ as a function of the Borel mass. The dots and squares give the results from Eq. (C1) when using respectively numerical values or the two-point sum rules for the meson decay constants. The solid and dot-dashed lines give the fits to the QCDSR results.

Using the respective sum rules for the meson decay constants we get

$$\Lambda_1^{D^*D^*} \simeq 13.9 \text{ GeV}^{-3}. \quad (20)$$

We will use these two procedures to estimate the errors in our calculation. It is important to mention that our results agree completely with the value obtained in [22].

The results for all other sum rules show a similar behaviour and the amplitudes can be extracted by the extrapolation of the fit to $M^2 = 0$. The QCDSR results, evaluated using the numerical values for the meson decay constants, as well as the quadratic fits for the amplitudes associated with the process $J/\psi \pi \rightarrow \bar{D} D^*$ are shown in Fig. 3.
FIG. 3: Sum rules for $\Lambda_i^{DD^*}$ related to the process $J/\psi \pi \rightarrow \bar{D} D^*$ as a function of the Borel mass. The dots, squares, diamonds, triangles up and triangles down give the results from the QCDSR for $\Lambda_1^{DD^*}$ up to $\Lambda_5^{DD^*}$ respectively. The QCDSR results for $\Lambda_1^{DD^*}$ and $\Lambda_4^{DD^*}$ cannot be distinguished at the scale of the figure. The solid, dotted, dashed, long-dashed and dot-dashed lines give the fits to the QCDSR results.

The values for the parameters associated with the process $J/\psi \pi \rightarrow \bar{D} D^*$ are given in Table I.

| $\Lambda_i^{DD^*}$ | $\Lambda_2^{DD^*}$ | $\Lambda_3^{DD^*}$ | $\Lambda_4^{DD^*}$ | $\Lambda_5^{DD^*}$ |
|---------------------|---------------------|---------------------|---------------------|---------------------|
| 14 ± 2 GeV$^{-2}$   | −7.2 ± 0.9 GeV$^{-2}$ | −58 ± 8 GeV$^{-2}$ | 14.6 ± 2.2          | −15.6 ± 2.2 GeV$^{-2}$ |

For the process $J/\psi \pi \rightarrow \bar{D} D$ we have only one parameter which is given by \[15\]

$$\Lambda_{DD} = 13.2 \pm 1.8 \text{ GeV}^{-3}, \quad (21)$$

and the 13 parameters associated with the process $J/\psi \pi \rightarrow \bar{D}^* D^*$ are given in Table II. The errors in all parameters were estimated by the evaluation of the sum rules using the numerical values and the two-point QCDSR for the meson decay constants.

Having the QCD sum rule results for the amplitudes of the three processes $J/\psi \pi \rightarrow \bar{D} D^*$, $\bar{D} D$, $\bar{D}^* D^*$, given in Eqs. (15), (16) and (17) we can evaluate the cross sections. After including isospin factors, the differential cross section for the $J/\psi - \pi$ dissociation is
TABLE II: The best fitted values for the parameters associated with the process $J/\psi \pi \to \bar{D}^* D^*$.

| $\Lambda_{1}^{D^*D^*}$ | $\Lambda_{2}^{D^*D^*}$ | $\Lambda_{3}^{D^*D^*}$ | $\Lambda_{4}^{D^*D^*}$ | $\Lambda_{5}^{D^*D^*}$ |
|------------------------|------------------------|------------------------|------------------------|------------------------|
| 12.2 ± 1.7 GeV$^{-3}$  | −12.8 ± 1.8 GeV$^{-3}$ | 12.5 ± 1.7 GeV$^{-3}$  | −24.6 ± 3.4 GeV$^{-3}$ | 9.8 ± 1.6 GeV$^{-3}$  |

| $\Lambda_{6}^{D^*D^*}$ | $\Lambda_{7}^{D^*D^*}$ | $\Lambda_{8}^{D^*D^*}$ | $\Lambda_{9}^{D^*D^*}$ | $\Lambda_{10}^{D^*D^*}$ |
|--------------------------|------------------------|------------------------|------------------------|------------------------|
| 9.7 ± 1.6 GeV$^{-3}$     | −13.0 ± 1.8 GeV$^{-3}$ | −13.8 ± 1.8 GeV$^{-3}$ | −5.4 ± 0.9 GeV$^{-1}$  | 2.5 ± 0.2 GeV$^{-3}$  |

| $\Lambda_{11}^{D^*D^*}$ (GeV$^{-1}$) | $\Lambda_{12}^{D^*D^*}$ | $\Lambda_{13}^{D^*D^*}$ |
|--------------------------------------|------------------------|------------------------|
| (£5.5 ± 0.5)10$^{-3}$                | −0.022 ± 0.002 GeV$^{-3}$ | 0.53 ± 0.03 GeV$^{-3}$ |

given by

$$\frac{d\sigma}{dt} = \frac{1}{96\pi s p_{i,cm}^2} \sum_{\text{spin}} |\mathcal{M}|^2,$$  \hspace{1cm} (22)

where $p_{i,cm}$ is the three-momentum of $p_1$ (or $p_2$) in the center of mass frame (with $p_1$ ($p_2$) being the four-momentum of the $\pi$ ($J/\psi$)):

$$p_{i,cm}^2 = \frac{\lambda(s, m_\pi^2, m_\psi^2)}{4s},$$  \hspace{1cm} (23)

with $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$, $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$.

In Eq. (22), the sum over the spins of the amplitude squared is given by

$$\sum_{\text{spin}} |\mathcal{M}|^2 = \mathcal{M}_{\mu\nu} \mathcal{M}_{\mu'\nu'}^{*} \left( g^{\mu\mu'} - \frac{p_1^\mu p_2^\mu}{m_\psi^2} \right) \left( g^{\nu\nu'} - \frac{p_2^\nu p_2'^\nu}{m_\psi^2} \right),$$  \hspace{1cm} (24)

for $J/\psi \pi \to \bar{D} D^*$, with $p_3$ ($p_4$) being the four-momentum of $D^*$ ($D$).

$$\sum_{\text{spin}} |\mathcal{M}|^2 = \mathcal{M}_{\mu} \mathcal{M}_{\mu}^{*} \left( g^{\mu \mu'} - \frac{p_2^{\mu} p_2'^{\mu}}{m_\psi^2} \right),$$  \hspace{1cm} (25)

for $J/\psi \pi \to \bar{D} D$, and

$$\sum_{\text{spin}} |\mathcal{M}|^2 = \mathcal{M}_{\mu\alpha \gamma} \mathcal{M}_{\mu'\nu' \alpha'}^{*} \left( g^{\mu \mu'} - \frac{p_3^{\mu} p_3'^{\mu}}{m_\psi^2} \right) \left( g^{\nu \nu'} - \frac{p_3^{\nu} p_3'^{\nu}}{m_{D^*}^2} \right) \left( g^{\alpha \alpha'} - \frac{p_3^{\alpha} p_3'^{\alpha}}{m_{D^*}^2} \right),$$  \hspace{1cm} (26)

for $J/\psi \pi \to \bar{D}^* D^*$.

The structures multiplying $\Lambda_{4}^{DD^*}$ and $\Lambda_{5}^{DD^*}$ in Eq. (13), and $\Lambda_{11}^{DD^*}$ in Eq. (17) break chiral symmetry [10]. To evaluate the effect of breaking chiral symmetry in the process $J/\psi \pi \to \bar{D} D^* + \bar{D}^* D$ we show, in Fig. 4, the cross section calculated using all structures in Eq. (13) (dashed line) and neglecting $\Lambda_{4}^{DD^*}$ and $\Lambda_{5}^{DD^*}$ (solid line).
FIG. 4: $J/\psi \pi \rightarrow \bar{D} D^* + \bar{D}^* D$ cross section. The solid and dashed lines give the results which respect and break chiral symmetry respectively.

From Fig. 4 we see that the cross section obtained with the amplitude that breaks chiral symmetry grows very fast near the threshold. Since this is the energy region where this kind of process is probable more likely to happen, it is very important to use models that respect chiral symmetry when evaluating the $J/\psi - \pi$ cross section. As mentioned before, the sum

rules for $\Lambda^{D\bar{D}^*}, \Lambda^{D^*D^*}$ and $\Lambda^{D_1^*D_1^*}$ up to $\Lambda^{D_3^*D_3^*}$, do not get the leading order contribution and will be neglected when evaluating the cross sections. It is important to keep in mind that, since our sum rules were derived in the limit $p_1 \rightarrow 0$, we can not extend our results to large
values of $\sqrt{s}$. For this reason we will limit our calculation to $\sqrt{s} \leq 4.5$ GeV.

In Fig. 5 we show separately the contributions for each one of the process. Our first conclusion is that our results show that, for values of $\sqrt{s}$ far from the $J/\psi \pi \rightarrow \bar{D}^* D^*$ threshold, 
$$\sigma_{J/\psi \pi \rightarrow \bar{D}^* D^*} \geq \sigma_{J/\psi \pi \rightarrow \bar{D}D^* + DD^*} \geq \sigma_{J/\psi \pi \rightarrow \bar{D}D},$$
in agreement with the model calculations presented in [12] but in disagreement with the results obtained with the nonrelativistic quark model of [9], which show that the state $\bar{D}^* D$ has a larger production cross section than $\bar{D}^* D^*$. Furthermore, our curves indicate that the cross section grows monotonically with the c.m.s. energy but not as fast, near the thresholds, as it does in the calculations in Refs. [12, 13, 14, 15, 16]. Again, this behavior is in opposition to [9], where a peak just after the threshold followed by continuous decrease in the cross section was found.

![Graph](image)

FIG. 6: $J/\psi - \pi$ dissociation cross sections from meson exchange model [12] (dot-dashed line), quark exchange model [9] (dashed line), short distance QCD [7, 8] (dotted line) and QCD sum rules (solid lines). The shaded area give an evaluation of the uncertainties in our calculation.

In Fig. 6 we show, for comparison, our result for the total cross section for the $J/\psi \pi$ dissociation (solid lines) and the results from meson exchange model [12] obtained with a cut-off $\Lambda = 1$ GeV (dot-dashed line), quark exchange model [9] (dashed line) and short distance QCD [7, 8] (dotted line). The shaded area in our results give an evaluation of the uncertainties in our calculation obtained with the two procedures described above. It is very
interesting to notice that below the \(DD^*\) threshold, our result and the results from meson exchange and quark exchange models are in a very good agreement. However, as soon as the \(DD^*\) channel is open the cross section obtained with the meson exchange and quark exchange models show a very fast grown, as a function of \(\sqrt{s}\), as compared with our result. As discussed above, this is due to the fact that chiral symmetry is broken in these two model calculations.

The momentum distribution of thermal pions in a hadron gas depends on the effective temperature \(T\) with an approximate Bose-Einsten distribution. Therefore, in a hadron gas, pions collide with the \(J/\psi\) at different energies, and the relevant quantity is not the value of the cross section at a given energy, but the thermal average of the cross section. The thermal average of the cross section is defined by the product of the dissociation cross section and the relative velocity of initial state particles, averaged over the energies of the pions: \(\langle \sigma^\pi_{J/\psi} v \rangle\), and is given by [14]

\[
\langle \sigma^\pi_{J/\psi} v \rangle = \frac{\int_{z_0}^{\infty} dz [z^2 - (\alpha_1 + \alpha_2)^2][z^2 - (\alpha_1 - \alpha_2)^2]K_1(z)\sigma^\pi_{J/\psi}(s = z^2T^2)}{4\alpha_1^2 K_2(\alpha_1)\alpha_2^2 K_2(\alpha_2)},
\]

(27)

where \(\alpha_i = m_i/T \ (i = 1 \text{ to } 4)\), \(z_0 = \max(\alpha_1 + \alpha_2, \alpha_3 + \alpha_4)\) and \(K_i\) is the modified Bessel function.

FIG. 7: Thermal average of \(J/\psi\) dissociation cross section by pions as a function of temperature \(T\). The shaded area give an evaluation of the uncertainties in our calculation.

As shown in Fig. 7, \(\langle \sigma^\pi_{J/\psi} v \rangle\) increases with the temperature. Since the \(J/\psi\) dissociation by a pion requires energetic pions to overcome the energy threshold, it has a small thermal
average at low temperatures. The magnitude of our thermal average cross section is of the same order as the meson exchange model calculation in ref. [14] with a cut-off $\Lambda = 1$ GeV. The shaded area in Fig. 7 give an evaluation of the uncertainties in our calculation due to the two procedures used to extract the hadronic amplitudes.

V. SUMMARY AND CONCLUSIONS

We have evaluated the hadronic amplitudes for the $J/\psi$ dissociation by pions using the QCD sum rules based on a three-point function using vacuum-pion correlation functions. We have considered the OPE expansion up to twist-4 and we have worked in the soft-pion limit. Our work improves the former QCDSR calculation, done with a four-point function at the pion pole [22], since we have included more terms in the OPE expansion. We have shown that the twist-3 and twist-4 contributions to the sum rules are small when compared with the leading order contribution, showing a good “convergence” of the OPE expansion. We have checked that, taking the appropriated limit, we recover the previous result of [22].

From the theoretical point of view, the use of QCDSR in this problem was responsible for real progress, being a step beyond models and beyond the previous leading twist calculations [3, 6, 8, 22, 33]. This is specially true in the low energy region, close to the open charm production threshold. At higher energies our treatment is less reliable due to the approximations employed.

Although a more sophisticated analysis of our uncertainties is still to be done, the shaded area in Fig. 6 shows that we can make some unambiguous statement concerning the behavior of $\sigma_{\pi J/\psi}$ with the energy $\sqrt{s}$. Our cross section grows monotonically with the c.m.s. energy but not as fast, near the thresholds, as it does in the calculations using meson exchange models [12, 13, 14, 15, 16]. We have also shown the importance of respecting chiral symmetry, since the increase of the cross section near the threshold is strongly intensified when chiral symmetry is broken. In other words, our results suggest that, using meson exchange models is perfectly acceptable, provided that they include form factors and that they respect chiral symmetry. With these precautions, they can be a good tool to make predictions at somewhat higher energies.

We have also evaluated the thermal average of the $J/\psi - \pi$ dissociation cross section. It increases with the temperature and at $T = 150$ MeV we get $\langle \sigma_{\pi J/\psi v} \rangle \sim 0.2 - 0.4$ mb which
is compatible with the values presented in Fig. 5 of ref. [14], i.e., in a meson exchange model with monopole form factor with cut-off Λ = 1 GeV. The use of this information will reduce the uncertainties in the calculations of the hadronic lifetimes of $J/\psi$, which are needed in simulations like those of ref. [3].

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**APPENDIX A: SUM RULES FOR THE PROCESS $J/\psi \pi \to \bar{D} D^*$**

Using $\Gamma_3 = \gamma_\nu$ and $\Gamma_4 = i\gamma_5$ in Eqs. (2) and (8), we obtain the following sum rules for the structures in Eq. (15):

$$
\Lambda_1^{D D^*} + A_1^{D D^*} M^2 + B_1^{D D^*} M^4 = \frac{1}{C_{D D^*} f_{D D^*} (M^2)} \left[ -f_\pi m_c + f_\pi \left( 3 + \frac{m_c^2}{2M^2} \right) e^{-\frac{m_c^2}{M^2}} \right],
$$

(A1)

$$
\Lambda_2^{D D^*} + A_2^{D D^*} M^2 + B_2^{D D^*} M^4 = \frac{1}{C_{D D^*} f_{D D^*} (M^2)} \left[ f_\pi m_c + \frac{4 \langle \bar{q}q \rangle}{3 f_\pi} \right]
$$

$$
- \frac{m_c}{6M^2} \left( \frac{4 \langle \bar{q}q \rangle m_c}{f_\pi} + \frac{\delta^2 f_\pi}{3} \left( 19 + \frac{5m_c^2}{M^2} \right) \right) e^{-\frac{m_c^2}{M^2}} M^2,
$$

(A2)

$$
\Lambda_3^{D D^*} + A_3^{D D^*} M^2 + B_3^{D D^*} M^4 = \frac{1}{C_{D D^*} f_{D D^*} (M^2)} \left[ 2f_\pi m_c - \frac{8 \langle \bar{q}q \rangle}{3 f_\pi} \right]
$$

$$
- \frac{m_c}{6M^2} \left( \frac{8 \langle \bar{q}q \rangle m_c}{f_\pi} + \frac{\delta^2 f_\pi}{3} \left( -2 + \frac{10m_c^2}{M^2} \right) \right) e^{-\frac{m_c^2}{M^2}} M^2,
$$

(A3)

$$
\Lambda_4^{D D^*} + A_4^{D D^*} M^2 + B_4^{D D^*} M^4 = \frac{m_c^2}{C_{D D^*} f_{D D^*} (M^2)} \langle \bar{q}q \rangle e^{-\frac{m_c^2}{M^2}} / M^2,
$$

(A4)
\[ \Lambda_{5DD^*}^D + A_{5DD^*}^D M^2 + B_{5DD^*}^D M^4 = -\frac{2}{C_{DD^*} f_{DD^*}(M^2)} \frac{\langle \bar{q}q \rangle}{f_\pi} e^{-m_c^2/M^2}, \]  

(A5)

where

\[ f_{DD^*}(M^2) = \frac{1}{m_\psi^2 - m_{D^*}^2} \left[ \frac{e^{-m_b^2/M^2} - e^{-m_c^2/M^2}}{m_\psi^2 - m_D^2} - \frac{e^{-m_b^2/M^2} - e^{-m_c^2/M^2}}{m_{D^*}^2 - m_D^2} \right], \]  

(A6)

and

\[ C_{DD^*} = \frac{m_c}{m_D^2 m_{D^*} m_\psi f_{D^*} f_\psi}. \]  

(A7)

\( A_{iDD^*}^D \) and \( B_{iDD^*}^D \) are the parameters introduced to account for double pole-continuum and single pole-continuum transitions respectively.

**APPENDIX B: SUM RULES FOR THE PROCESS \( J/\psi \pi \to \bar{D} D \)**

Using \( \Gamma_3 = i\gamma_5 \) and \( \Gamma_4 = i\gamma_5 \) in Eqs. (16) and (18), we obtain the following sum rule for the structure in Eq. (16):

\[ \frac{\Lambda_{DD} + A_{DD} M^2 + B_{DD} M^4}{m_\psi^2 - m_D^2} \left[ \frac{e^{-m_b^2/M^2} - e^{-m_c^2/M^2}}{M^2} - \frac{e^{-m_b^2/M^2} - e^{-m_c^2/M^2}}{m_\psi^2 - m_D^2} \right] \]

\[ = \frac{m_c^2}{m_D^2 m_\psi f_{D^*} f_\psi} \frac{e^{-m_c^2/M^2}}{M^2} \left[ f_\pi - \frac{2m_c \langle \bar{q}q \rangle}{3f_\pi M^2} - \frac{f_\pi \delta^2}{18M^2} \left( 17 + \frac{5m_c^2}{M^2} \right) \right]. \]  

(B1)

**APPENDIX C: SUM RULES FOR THE PROCESS \( J/\psi \pi \to \bar{D^*} D^* \)**

Using \( \Gamma_3 = \gamma_\nu \) and \( \Gamma_4 = \gamma_\rho \) in Eqs. (2) and (8), we obtain the following sum rules for the structures in Eq. (17):

\[ \Lambda_{1DD^*}^{D^*} + A_{1DD^*}^{D^*} M^2 + B_{1DD^*}^{D^*} M^4 = \frac{1}{C_{D^*D^*} f_{D^*D^*}(M^2)} \left[ -f_\pi + \frac{2m_c \langle \bar{q}q \rangle}{3f_\pi M^2} \right] \frac{e^{-m_c^2/M^2}}{M^2}, \]  

(C1)

\[ \Lambda_{2DD^*}^{D^*} + A_{2DD^*}^{D^*} M^2 + B_{2DD^*}^{D^*} M^4 = \frac{1}{C_{D^*D^*} f_{D^*D^*}(M^2)} \left[ f_\pi - \frac{2m_c \langle \bar{q}q \rangle}{3f_\pi M^2} \right] \frac{e^{-m_c^2/M^2}}{M^2}, \]  

(C2)
\[
\Lambda^{D^*D^*}_3 + A_3^{D^*D^*} M^2 + B_3^{D^*D^*} M^4 = \frac{1}{C_{D^*D^*} f_{D^*D^*} (M^2)} \left[ -f_\pi + \frac{2m_c \langle \bar{q}q \rangle}{3f_\pi M^2} + \frac{f_\pi \delta^2}{6M^2} \left( \frac{19}{3} + \frac{5m_c^2}{3M^2} \right) \right] e^{-m_c^2/M^2} + \frac{5m_c^2}{3M^2} e^{-m_c^2/M^2}, \tag{C3}
\]

\[
\Lambda^{D^*D^*}_4 + A_4^{D^*D^*} M^2 + B_4^{D^*D^*} M^4 = \frac{1}{C_{D^*D^*} f_{D^*D^*} (M^2)} \left[ 2f_\pi - \frac{4m_c \langle \bar{q}q \rangle}{3f_\pi M^2} - \frac{f_\pi \delta^2}{3M^2} \left( \frac{23}{3} + \frac{5m_c^2}{3M^2} \right) \right] e^{-m_c^2/M^2}, \tag{C4}
\]

\[
\Lambda^{D^*D^*}_5 + A_5^{D^*D^*} M^2 + B_5^{D^*D^*} M^4 = \frac{1}{C_{D^*D^*} f_{D^*D^*} (M^2)} \left[ -f_\pi + \frac{23f_\pi \delta^2}{18M^2} \right] e^{-m_c^2/M^2}, \tag{C5}
\]

\[
\Lambda^{D^*D^*}_6 + A_6^{D^*D^*} M^2 + B_6^{D^*D^*} M^4 = \frac{1}{C_{D^*D^*} f_{D^*D^*} (M^2)} \left[ -f_\pi + \frac{25f_\pi \delta^2}{18M^2} \right] e^{-m_c^2/M^2}, \tag{C6}
\]

\[
\Lambda^{D^*D^*}_7 + A_7^{D^*D^*} M^2 + B_7^{D^*D^*} M^4 = \frac{1}{C_{D^*D^*} f_{D^*D^*} (M^2)} \left[ f_\pi - \frac{2m_c \langle \bar{q}q \rangle}{3f_\pi M^2} - \frac{3m_c f_{3\pi}}{M^2} \right] e^{-m_c^2/M^2}, \tag{C7}
\]

\[
\Lambda^{D^*D^*}_8 + A_8^{D^*D^*} M^2 + B_8^{D^*D^*} M^4 = \frac{1}{C_{D^*D^*} f_{D^*D^*} (M^2)} \left[ f_\pi - \frac{2m_c \langle \bar{q}q \rangle}{3f_\pi M^2} + \frac{2m_c f_{3\pi}}{M^2} \right] e^{-m_c^2/M^2}, \tag{C8}
\]

\[
\Lambda^{D^*D^*}_9 + A_9^{D^*D^*} M^2 + B_9^{D^*D^*} M^4 = \frac{1}{C_{D^*D^*} f_{D^*D^*} (M^2)} \left[ -\frac{m_c^2 f_\pi}{2} + \frac{m_c \langle \bar{q}q \rangle}{2f_\pi} + \frac{f_\pi \delta^2}{3} \left( \frac{53}{6} - \frac{13m_c^2}{6M^2} + \frac{5m_c^4}{12M^2} \right) \right] e^{-m_c^2/M^2}, \tag{C9}
\]

\[
\Lambda^{D^*D^*}_{10} + A_{10}^{D^*D^*} M^2 + B_{10}^{D^*D^*} M^4 = \frac{1}{C_{D^*D^*} f_{D^*D^*} (M^2)} \left[ 2m_c \langle \bar{q}q \rangle - \frac{m_c f_{3\pi}}{3} \right] e^{-m_c^2/M^2}, \tag{C10}
\]

\[
\Lambda^{D^*D^*}_{11} + A_{11}^{D^*D^*} M^2 + B_{11}^{D^*D^*} M^4 = \frac{2}{C_{D^*D^*} f_{D^*D^*} (M^2)} \frac{m_c \langle \bar{q}q \rangle}{f_\pi} \frac{e^{-m_c^2/M^2}}{M^2} \tag{C11}
\]

\[
\Lambda^{D^*D^*}_{12} + A_{12}^{D^*D^*} M^2 + B_{12}^{D^*D^*} M^4 = \frac{1}{C_{D^*D^*} f_{D^*D^*} (M^2)} \left[ 2m_c f_{3\pi} - \frac{f_\pi \delta^2}{3} \right] e^{-m_c^2/M^2}. \tag{C12}
\]
\[
\Lambda_{13}^{D^*D^*} + A_{13}^{D^*D^*} M^2 + B_{13}^{D^*D^*} M^4 = - \frac{2}{C_{D^*D^*} f_{D^*D^*}(M^2)} \frac{f_\pi g^2 e^{-m_\pi^2/M^2}}{3 M^4},
\]
(C13)

where
\[
f_{D^*D^*}(M^2) = \frac{1}{m_{D^*}^2 - m_{\psi}^2} \left[ \frac{e^{-m_{D^*}^2/M^2}}{M^2} - \frac{e^{-m_{\psi}^2/M^2}}{m_{\psi}^2 - m_{D^*}^2} \right].
\]
(C14)

and
\[
C_{D^*D^*} = \frac{1}{m_{D^*}^2 m_{\psi} f_{D^*}^2 f_\psi^2}.
\]
(C15)

APPENDIX D: SUM RULES FOR THE MESON DECAY CONSTANTS

For consistency we use in our analysis the QCDSR expressions for the decay constants of the \( J/\psi,\ D^* \) and \( D \) mesons up to dimension four in lowest order of \( \alpha_s \):

\[
f_D^2 = \frac{3m_c^2}{8\pi^2 m_D^4} \int_{m_D^2}^{s_D} ds \left( \frac{s - m_c^2}{s} \right)^2 e^{(m_D^2 - s)/M^2} - \frac{m_c^3}{m_D^4} \langle \bar{q}q \rangle e^{(m_D^2 - m_c^2)/M^2},
\]
(D1)

\[
f_{D^*}^2 = \frac{1}{8\pi^2 m_{D^*}^4} \int_{m_D^2}^{s_{D^*}} ds \left[ \frac{(s - m_c^2)^2}{s} \left( 2 + \frac{m_c^2}{s} \right) \right. \\
\left. \times e^{(m_{D^*}^2 - s)/M^2} \right] - \frac{m_c^3}{m_{D^*}^4} \langle \bar{q}q \rangle e^{(m_{D^*}^2 - m_c^2)/M^2},
\]
(D2)

\[
f_{\psi}^2 = \frac{1}{4\pi^2} \int_{4m_c^2}^{s_\psi} ds \left( \frac{s + 2m_c^2}{s} \right)^2 \sqrt{s - 4m_c^2} e^{(m_\psi^2 - s)/M^2},
\]
(D3)

where \( s_M \) stands for the continuum threshold of the meson \( M \), which we parametrize as \( s_M = (m_M + \Delta_s)^2 \). The values of \( s_M \) are, in general, extracted from the two-point function sum rules for \( f_D \) and \( f_{D^*} \) and \( f_{\psi} \) in Eqs. (D1), (D2) and (D3). Using the Borel region \( 3 \leq M_H^2 \leq 6 \text{ GeV}^2 \) for the \( D^* \) and \( D \) mesons and \( 6 \leq M_H^2 \leq 12 \text{ GeV}^2 \) for the \( J/\psi \), we found good stability for \( f_D, f_{D^*} \) and \( f_{\psi} \) with \( \Delta_s \sim 0.6 \text{ GeV} \). We obtained \( f_D = 160 \pm 5 \text{ MeV}, f_{D^*} = 220 \pm 10 \text{ MeV} \) and \( f_{\psi} = 280 \pm 10 \text{ MeV} \), which are compatible with the numerical values in Eq. (18).

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