Extracting energy from non-equilibrium fluctuations without using information

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Introduction.– Extracting energy from fluctuations challenges our fundamental understanding of thermodynamics. Maxwell first showed that energy could be extracted from thermal fluctuations by an intelligent demon that can detect the velocity of a single particle [1]. The apparent violation of the second law of thermodynamics was clarified later by noting that the energy source is information, and its erasure is inevitably dissipative [2, 3]. Recently, such an information engine has been demonstrated experimentally [4–12]. Remarkable theoretical progress has also been achieved along this line [13–17]. Another strategy of energy extraction is to exploit the temperature difference between two baths, by using a single particle. Such a stochastic heat engine has also been achieved in recent experiments [18–20]. Its energy rate and efficiency are under active investigation [14–17]. Finally, chemical engines could also be constructed, as achieved experimentally [9–12]. Remarkable theoretical progress has also been achieved along this line [4–8]. An information engine with explicit dependence on the trajectory has also been achieved [21].

Let us classify these engines based on their operation. The stochastic heat engine requires coupling two different external operations, i.e., the temperature modulation and mechanical control. So does the chemical engine. For the case of periodic driving, their thermodynamics have been studied recently [22–26]. The information engine, on the other hand, requires only one external operation, which needs to be coupled with fluctuations of the system. It is intriguing whether a blind external operation, ignorant of such fluctuations, will also work. Although such an operation is surely dissipative on passive systems, as suggested by the second law of thermodynamics, it remains unclear for active systems that are driven out of equilibrium by an internal energy source.

Here, assuming such a small operation, we find that energy extraction requires a negative response spectrum in a certain frequency region, and an external operation at these frequencies. Negative response is intrinsic to any system that adapts to a slow external operation, which could be realized through integral feedback control. These are demonstrated through a solvable model.

Extracting energy from fluctuations has been an everlasting endeavor. An important strategy is to use information-based operation, which allows energy extraction even from thermal fluctuations. However, it remains unclear whether a blind external operation, ignorant of such fluctuations, will also work. Assuming such a small operation, we find that energy extraction requires a negative response spectrum in a certain frequency region, which inevitably requires an internal power source to maintain. Energy is then extracted by operating the system at these frequencies. Negative response is intrinsic to any system that adapts to a slow external operation, which could be realized through integral feedback control. These are demonstrated through a solvable model.

General requirements.– Let us first consider how to extract energy from a fluctuating trajectory $x_t$ by applying an external force $f_t$:

$$\dot{x} = F(x, y, t) + f + \xi. \quad (1)$$

Here, $\gamma$ is the friction coefficient, $\xi$ any stationary noise, and $F(x, y, t)$ an arbitrary force that could depend on both multiple internal variables $y$ or an internal driving protocol. The assumption of an over-damped motion is not important to our main results. The Boltzmann constant is set to be 1 throughout this paper. Using $\langle \cdot \rangle_x$, to indicate long-time averaging over $t$, the energy injection rate through the external operation is given by

$$\dot{W}^{\ast}_{\text{ext}} = \langle f_t \circ \dot{x}_t \rangle_x \equiv \lim_{T \to \infty} \frac{1}{T} \int_0^T f_t \circ \dot{x}_t dt, \quad (2)$$

where $\circ$ denotes Stratonovich calculus [27]. The energy is extracted when $\dot{W}^{\ast}_{\text{ext}} < 0$, which is stored in an auxiliary system that connects with $f$.

Let us consider the following external operation:

$$f_t = (1 - \alpha) h_t - \alpha \int_{-\infty}^{\infty} G(t - \tau) \dot{x}_\tau d\tau, \quad (3)$$

which mixes a blind perturbation $h_t$, regardless of $x_t$, and an information-based operation with explicit dependence on the trajectory, supposed to be generated by Maxwell’s demon. Since the demon cannot predict the future, the Green function satisfies $G(t - \tau) = 0$ for $\tau > t$. Here, $\alpha \in [0, 1]$, which is a qualitative measure of the information contained in the external operation.

To quantify $\dot{W}^{\ast}$, we need to introduce the velocity correlation under the protocol $f_t$:

$$C^*_{\xi}(\tau) \equiv \langle (\dot{x}_t - v_0)(\dot{x}_{t+\tau} - v_0) \rangle_x, \quad (4)$$

with $v_0 \equiv \langle \dot{x}_t \rangle_x$, the drifting velocity. For simplicity, we take $v_0 = 0$, which excludes the trivial possibility of extracting energy with a constant load, as is
the case for Feynman’s ratchet [13]. The correlation of the perturbation protocol is also important, given by $C_{th}(\tau) \equiv \langle (h_t - h^*)(h_{t+\tau} - h^*) \rangle_\ast$, assumed to be stationary, with $h^* \equiv \langle h_t \rangle_\ast$ the average perturbative force. Assuming that $\delta h_t = h_t - h^*$ is a small perturbation to the system, the velocity response of the system is characterized by $\langle \dot{x}_t \rangle = \int_{-\infty}^{\infty} R_{ext}(t - \tau)d\tau$. This defines a velocity response function $R_{ext}(t - \tau)$, which is independent of $h_t$, but relies on the operation of the demon, which has changed the steady state. Besides, $R_{ext}$ should be zero for $t < \tau$ since any perturbation cannot affect the past response. Combining the linear response with Eq. (2) and Eq. (3), we finally obtain (See Supplemental Material for its derivation and the case with $\tau \neq 0$ [28])

$$W_{ext}^* = \int_{-\infty}^{\infty} \left[ (1 - \alpha) \tilde{R}_{ext}(\omega) \tilde{C}_{th}(\omega) - \alpha \tilde{G'}(\omega) \tilde{C}_{th}(\omega) \right] d\omega / 2\pi,$$

valid up to the second order of $h_t$. Here, the tilde denotes the Fourier transform and the prime takes the real part. On the right hand side, the first part gives the frequency-resolved energy injection rate for a blind perturbation, which generalizes a similar result in equilibrium statistical mechanics [29], while the second part is due to the information-based operation. It is reminiscent of the Harada-Sasa equality that connects the steady state dissipation with the correlation and response spectrum [30]. Although we have assumed a weak external operation, Eq. (4) is valid for any systems that are strongly driven by an external energy supply. Below, we discuss general strategies for energy extraction based on Eq. (4).

Note that $\tilde{C}_{th}(\omega) = \omega^2 |\tilde{\dot{x}}(\omega)|^2 \geq 0$ and $\tilde{C}_{th}(\omega) = |\tilde{h}(\omega)|^2 \geq 0$, with $\tilde{\dot{x}}(\omega)$ a rescaled Fourier transform of a trajectory $\dot{x}_t$, given by $\lim_{T \to \infty} \frac{1}{\sqrt{T}} \int_{0}^{T} \dot{x}_t \exp(i\omega t) dt$. $\tilde{G}'(\omega)$ could also be tuned externally to be non-negative. Therefore, when we are implementing only the information-based operation, i.e., $\alpha = 1$, we could extract energy even when the system is passive. This does not violate the second law since more energy will be dissipated to generate this information-based operation [2, 3].

On the other hand, at $\alpha = 0$, we can extract energy only when the velocity response spectrum $\tilde{R}_{ext}'(\omega) < 0$ at a certain frequency region. Negative response implies that the system pushes backwards when we apply a forward external force. This becomes precise in the frequency domain since $\langle \dot{x}'(\omega) \rangle = \tilde{R}_{ext}'(\omega) \tilde{h}(\omega)$. Given such a response spectrum that is intrinsic to the system and measurable, we can optimize the external perturbation spectrum accordingly to improve the output power.

However, the equilibrium response is always lagged behind the external operation and energy needs to be dissipated to “drag” the system forward, as suggested by the fluctuation–dissipation theorem (FDT): $\tilde{R}_{ext}'(\omega) = 1 / 2T \tilde{C}_{th}(\omega) \geq 0$, with $T$ being the temperature of the bath [31]. Here, $\tilde{C}_{th}$ is the correlation spectrum obtained at $h_t = h^*$. Alternatively, we may introduce the frequency-resolved effective temperature $T_{eff}(\omega) = C_{th}(\omega) / 2R_{ext}'(\omega)$ [32]. Therefore, a negative effective temperature is required for extracting energy, which is possible only for far-from-equilibrium systems, driven by an internal energy source.

An insensitive displacement response to low frequency perturbations, i.e., $\tilde{R}_{ext}'(\omega) \equiv \tilde{R}_{ext}'(\omega)|_{\omega \to 0} \approx 0$, is sufficient to generate negative response. To see this, we need to use the Kramers-Kröönig relation [29]: $\tilde{R}_{ext}'(\omega) = \frac{\pi}{2} \int_{0}^{\infty} \tilde{R}_{ext}(\omega_1) / \omega_1 d\omega_1$, where the double prime denotes the imaginary part and the integration is to be understood as the principle value. This relation is simply due to the causality constraint, i.e., a perturbation cannot affect responses in the past. This implies that $\tilde{R}_{ext}'(\omega) \approx 0$. $\tilde{R}_{ext}'(\omega)$ must have negative response in a certain frequency region to balance positive response in other frequencies. So does $\tilde{R}_{ext}'(\omega)$, which equals $\omega \tilde{R}_{ext}'(\omega)$.

Such a response implies adaptation of the system to a slow external perturbation. To see this, let us consider a sudden jump of the external perturbation $h^* \to h^* + \Delta h$ at $t = 0$. The displacement response satisfies $\langle x_\infty \rangle - \langle x_0 \rangle = \Delta h \int_{0}^{\infty} \tilde{R}_{ext}(t) dt = \Delta h \tilde{R}_{ext}'(0) \approx 0$, i.e., $\langle x_\infty \rangle$ is recovered after a transient variation. This implies that the steady state displacement is invariant under the change of a constant external force, called adaptation.

Adaptation is widely exploited by sensory systems to recover sensitivity after a transient response to a shift of external signal [33]. Therefore, negative response should be generic in such systems. This has been revealed in a model for the sensory circuit in E. coli [34, 35], which serves to detect variation of the extracellular ligand concentration [33]. For the hair bundle from the inner ear that senses the auditory stimuli [36], negative response has been revealed experimentally [37].

A main strategy to achieve adaptation, thus negative response, is to implement integral feedback control through an internal variable $y$ [38], as illustrated in FIG. 1(a). In some sense, Maxwell’s demon is still required for extracting energy. However, it no longer controls the external operation, but rather the internal response. Given a charged particle that is trapped in such a way, we may further apply an alternating electric field with an optimized frequency spectrum to extract its energy, as illustrated in FIG. 1(b). See [3] for a similar experimental setup.

These observations are valid for more general blind external operations. Let us consider

$$\gamma \dot{x} = F(x, y, t) - \partial_x U(x, h) + \xi,$$

with $U(x, h)$ the coupling energy between $x$ and $h$. Here, we introduce the conjugate variable $B_t \equiv -\partial_x U(x_t, h^*)$, which is assumed to be bounded. The average energy injection rate through the external operation is given by $\dot{W}_{ext} = \langle -B_t \circ h_t \rangle$, or $\langle B_t \circ h_t \rangle$ due to integration by
zero-mean Gaussian white noises, satisfying \( \langle \omega \rangle < \) to external perturbations. For a step jump to the perturbation \( h \rightarrow h + 1 \). (b) The externally extracted power \(-W'_{\text{ext}}\), the internally supplied power \( W'_{\text{int}}\), and the efficiency \( \eta \) at various relative feedback speed \( \omega_y/\omega_x \), when the system is perturbed with \( h_t = 5 \cos(\omega_y t) \). Other parameters: \( \omega_x = 1, \gamma = 1, T = 1, k_1 = 1, b = 1 \) and \( k_2 = 0 \).

with \( \lambda_2 = \frac{1}{2} \omega_x (1 \pm \sqrt{1 - 4 \omega_y/\omega_x}) \). Since the real part of \( \lambda_2 \) is always positive, the elicited deviation from \( x_0 \) will eventually vanish. Therefore, this model with \( k_2 = 0 \) has achieved perfect adaptation. In the long run, the variation of \( h \) needs to be compensated by that of \( y \).

According to Eq. (8), returning to \( x_0 \) could be either oscillatory, when a relatively fast feedback is implemented, i.e., \( \omega_y/\omega_x \gg 1 \), or steady for a slow feedback. These are shown by the green curves in the insets of FIG. 2(a). Both strategies have been exploited by nature for different purposes. The hair bundle works in the fast feedback region with an oscillatory response in order to amplify an auditory signal at a specific frequency [39]. On the other hand, the sensory circuit in E. coli works in the slow feedback region without oscillation, which is suitable for detecting all fast variation of the extracellular ligand concentration [33].

Solving Eq. (7) in the Fourier space, we obtain the desired velocity response spectrum:

\[
\check{R}'(\omega) = \frac{1}{\gamma} \frac{\omega_x^2 (\omega_x^2 - \omega_y^2)}{\omega_x^2 + \omega_y^2}.
\]  

It reaches zero at \( \omega \rightarrow 0 \). Therefore, a sufficiently slow perturbation is energetically reversible. On the other hand, it approaches \( 1/\gamma \) for \( \omega \rightarrow \infty \), which implies that a super fast perturbation, to which the system is almost insensitive, is highly dissipative. This marks a sharp distinction between energetics and dynamics of a nonequilibrium system, which is quite general [33] [39]–[45].

The spectrum becomes negative in the low frequency parts. Introducing the linear response function \( R_B(t - \tau) \equiv \delta(B_t)/\delta h_t \), we obtain

\[
\tilde{W}'_{\text{ext}} = \int_{-\infty}^{\infty} \tilde{R}'_B(\omega) \tilde{C}'(\omega) \frac{d\omega}{2\pi}, \tag{6}
\]

valid up to the second order of \( h \). When \( U = -x_h \), we have \( \dot{B}_t = -x_h \), which reproduces our previous result. For \( U = h x^2/2 \), which is also commonly implemented experimentally [19] [20], we have \( \dot{B}_t = -\partial_h (x_h^2/2) \).

Potential shifting model.—Below, we demonstrate our main results through the following model:

\[
\gamma \dot{x} = -\partial_y U_0(x - y) + h + \xi \tag{7a}
\]

\[
\gamma \dot{y} = -k_1(x - x_0) - k_2y + \xi \tag{7b}
\]

Here, \( \gamma \) is the friction coefficient for \( y \). Both \( \xi \) and \( \xi_1 \) are zero-mean Gaussian white noises, satisfying \( \langle \xi(\tau) \xi(\tau) \rangle = 2\gamma T \delta(t - \tau) \) and \( \langle \xi_1(\tau) \xi_1(\tau) \rangle = 2\gamma_1 T \delta(t - \tau) \). \( U_0 \) should be a single-well potential. This model may describe a colloid particle trapped by a laser, whose position \( y_0 \) is controlled dynamically with respect to that of the particle, as illustrated in FIG. 1(a). For \( U_0(x - y) = k(x - y)^2/2 \), we show in the Supplemental Material [28] that the response spectrum \( \tilde{R}'_{\text{d}}(\omega) \) becomes negative in the frequency region \( \omega < \sqrt{\omega_x \omega_y - \omega_x^2} \), where \( \omega_x \equiv k/\gamma, \omega_y \equiv bk_1/\gamma_1 \), and \( \omega_2 \equiv k_2/\gamma_1 \). This is possible only when \( \omega_2 < \sqrt{\omega_x \omega_y} \), i.e., the restoring force \(-k_2y \) should be relatively small. Below, we focus on a harmonic potential \( U_0 \), and take \( k_2 = 0 \) for simplicity. This system is characterized by a dimensionless quantity \( \omega_y/\omega_x \), which measures the relative feedback speed of the system.

Let us first study the temporal response of this system to external perturbations. For a step jump \( h \rightarrow h + 1 \) at \( t = 0 \), the system evolves as

\[
\langle x_t \rangle = x_0 + \frac{1}{\gamma} \frac{\exp(-\lambda_- t) - \exp(-\lambda_+ t)}{\lambda_+ - \lambda_-}, \tag{8}
\]
region \( \omega < \sqrt{\omega_x \omega_y} \), as shown through the red curves in FIG. 3(a), which allows energy extraction. In the lower panel of FIG. 3(a), obtained at \( \omega_y/\omega_x = 0.1 \), the negative spectrum is quite small over a broad frequency region. For the fast feedback case, the response spectrum becomes much larger within a narrow frequency window, as shown in the upper panel of FIG. 3(a). The minimum value is reached at \( \omega_x = \sqrt{\omega_x \omega_y} \), which is optimal for extracting energy using a periodic operation. The velocity correlation spectrum \( \xi_t \) is also illustrated in FIG. 3(a). The FDT is violated significantly around \( \omega = \omega_t \), and restored approximately in the high frequency region. At \( \omega_y/\omega_x = 10 \), the correlation spectrum exhibits a sharp peak at the frequency that is slightly larger than \( \omega_c \). Counterintuitively, perturbation at such a characteristic frequency is dissipative, as indicated by the vertical dashed line in the upper panel of FIG. 3(a).

Below, we analyze the energetics for this linear model under any perturbation. The averaged dissipation rate through the frictional motion of \( x \) is given by \( \dot{Q}_x^* \equiv \langle \gamma \dot{x}_t - \xi(t) \rangle \circ \dot{x}_t \). Inserting \( \gamma \dot{x}_t - \xi(t) = -k(x_t - b \dot{y}_t + h_t) \) and noting that \( \langle \dot{x}_t \circ \dot{x}_t \rangle = 0 \), we obtain \( \dot{Q}_x^* = bk \langle \dot{y}_t \circ \dot{x}_t \rangle + \dot{W}_{\text{ext}}^* \). Similarly, the averaged dissipation rate for \( y \) satisfies \( \dot{Q}_y^* \equiv \langle \gamma \dot{y}_t - \xi(t) \rangle \circ \dot{y}_t \rangle = -k_1 (\dot{x}_t \circ \dot{y}_t) \). From integration by parts, we obtain \( -\langle \dot{x}_t \circ \dot{y}_t \rangle \equiv \langle \dot{y}_t \circ \dot{x}_t \rangle \). Noting the energy conservation \( \dot{Q}_x^* + \dot{Q}_y^* = \dot{W}_{\text{ext}}^* + \dot{W}_{\text{int}}^* \), with \( \dot{W}_{\text{int}}^* \) being the power supplied by an internal energy source, we finally obtain the following relations

\[
\dot{Q}_x^* = \frac{bk}{k_1} \dot{Q}_y^* + \dot{W}_{\text{ext}}^*, \quad \dot{W}_{\text{int}}^* = \frac{bk + k_1}{k_1} \dot{Q}_y^*, \tag{10}
\]

valid for any \( h_t \). Therefore, the efficiency is given by

\[
\eta \equiv \frac{-\dot{W}_{\text{ext}}^*}{\dot{W}_{\text{int}}^*} = \frac{bk}{bk + k_1} - \frac{k_1}{bk + k_1} \frac{\dot{Q}_y^*}{\dot{Q}_x^*} \leq \frac{1}{1 + (k_1/bk)}, \tag{11}
\]

where we have used the constraint that dissipation is always positive, i.e., \( \dot{Q}_x^* \geq 0 \) and \( \dot{Q}_y^* \geq 0 \), since \( h_t \) contains no information about this system. Eq. (11) gives the upper bound of the efficiency, determined only by \( k_1/bk \), the coupling asymmetry between \( x \) and \( y \), which remains valid even when \( k_2 \neq 0 \).

Now, we consider perturbation at the optimal frequency \( \omega_x \), i.e., \( h_t = h_0 \cos(\omega_x t) \). In this case, \( \xi_t^*(\omega) = \frac{h_0}{2} \frac{\omega_y}{\omega_x} \delta(\omega - \omega_x) \). Combined with Eq. (9) and Eq. (4) and noting that \( \alpha = 0 \), we obtain

\[
-\dot{W}_{\text{ext}}^* = \frac{h_0^2}{2 \gamma} \frac{\omega_y/\omega_x}{1 + 2 \sqrt{\omega_y/\omega_x}}. \tag{12}
\]

In the Supplemental Material [28], we calculate the rate of internal energy supply:

\[
\dot{W}_{\text{int}}^* = \frac{h_0^2 \omega_y}{2 \gamma \omega_x} (1 + \frac{k_1}{bk} \theta + T \omega_y (2 + \frac{bk}{k_1} + \frac{k_1}{bk})), \tag{13}
\]

where \( \theta = (\sqrt{\omega_y/\omega_x} + 1)/(2 \sqrt{\omega_y/\omega_x} + 1) \), ranging between [0.5, 1]. At \( h_0 = 0 \), \( \dot{W}_{\text{int}}^* \geq 4 \omega_y T \), with the equality achieved at symmetric coupling: \( bk = k_1 \). This irreducible energetic cost is needed to maintain negative response. Although a larger perturbation amplitude increases the output power, it also requires a larger internal energy supply. Interestingly, both the output power and efficiency can be increased simultaneously by a larger \( h_0 \).

In FIG. 3(b), we illustrate how the extracted power and the internal energy supply depend on the feedback speed \( \omega_y \), rescaled by \( 1/\omega_x \). The extracted power increases linearly with \( \omega_y \) when the feedback is relatively slow, but scales with \( \sqrt{\omega_y} \) when the feedback becomes faster. On the other hand, the internal energy supply always grows with \( \omega_y \). Therefore, the efficiency \( \eta \propto 1/\sqrt{\omega_y} \) in the fast feedback region, as shown in the inset in FIG. 3(b).

In the slow feedback region, the efficiency is maximized, bounded from above by \( bk/(bk + k_1) \), as shown by the brown dashed line in FIG. 3(b). This upper bound could be achieved in the limit \( h_0 \to \infty \) and \( \omega_y/\omega_x \to 0 \). In general, the output power and efficiency cannot be optimized simultaneously, as in the case of heat engines.

**Concluding Remarks.** Given a system that is powered out of equilibrium by an internal energy source, it seems possible to extract its energy that would otherwise be dissipated. Here, considering a blindly applied external operation, we find that negative response in a certain frequency region is necessary for energy extraction, and the spectrum of the external operation must be localized at this region. Negative response could be achieved with an internal variable that implements integral feedback control. Remarkably, energy extraction has already been demonstrated with hair bundles in the inner ear [46]. With the physics clarified here, we expect simpler experimental demonstration in the near future.

We also find that adaptation to a slow external operation necessitates negative response in a certain frequency region. Therefore, adaptation of sensory systems must be powered by an internal energy supply even without signal variation, as has been conjectured previously in a case study for the sensory circuit in E. coli [17, 48]. Our argument is based on causality, and the assumption that the external signal only affects the output directly, as valid for both the sensory circuit in E. coli and the hair bundles. It remains to be explored whether other response behavior will also cost energy.

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which gives the first and second order contribution explicitly. Inserting Eq. (S2) and Eq. (S4) back into Eq. (S1), we finally obtain

\[
\bar{W}_{ext}^* = \alpha h^* \langle \dot{x}_{ss} \rangle - (1 - \alpha) \bar{G}'(0)v_0 + \int_{-\infty}^{\infty} \left( \alpha \bar{R}_s^*(\omega)C_h^*(\omega) - (1 - \alpha) \bar{G}'(\omega)C_h^*(\omega) \right) \frac{d\omega}{2\pi}.
\]

In the equation without drifting motion, i.e., \( \langle \dot{x} \rangle_{ss} = \langle \dot{x} \rangle_s = 0 \), we obtain Eq. (4) in the Main Text.

Below, we discuss the property of the correlation spectrum. Firstly,

\[
\bar{C}_h^*(\omega) = \int_{-\infty}^{\infty} \bar{C}_h(s) \exp(i\omega s) ds = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \delta h_t \delta h_{t+s} \exp(i\omega s) ds dt,
\]

which gives

\[
\bar{C}_h^*(\omega) = \delta h^*(\omega) \delta h^*(\omega) = |\delta h^*(\omega)|^2 \geq 0.
\]

At \( \omega \neq 0 \), it becomes \( \bar{C}_h^*(\omega) = |\delta h^*(\omega)|^2 \geq 0 \) and at \( \omega = 0 \), \( \bar{C}_h^*(0) = 0 \).

Similarly, for \( \omega \neq 0 \), the velocity correlation spectrum satisfies

\[
\bar{C}_x^*(\omega) = \langle \dot{x}^*(\omega) \dot{x}^*(\omega) \rangle = \omega^2 (|\bar{x}(\omega)|^2) \geq 0,
\]

where ensemble average is needed to deal with the stochastic nature of the trajectory \( \dot{x}_i \). To arrive at the second equality, we have used \( \dot{x}(\omega) = i\omega \bar{x}(\omega) \). At \( \omega = 0 \), we have

\[
\bar{C}_x^*(0) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \langle \dot{x}_{t+s} - v_0 \rangle (\dot{x}_t - v_0) ds = 2D \geq 0,
\]

with \( D \) being the effective diffusion coefficient in the large time limit. This is due to the Green-Kubo relation [31]. If there is no drifting velocity, i.e., \( v_0 = 0 \), we have \( D = 0 \).

Generating negative response in a 2-d system

Consider a linear version of Eq. (7) in the Main Text:

\[
\gamma \ddot{\bar{x}} = -k \bar{x} + b \bar{y} + h + \xi, \quad \gamma_1 \ddot{\bar{y}} = -k_1 \bar{x} - k_2 \bar{y} + \xi_1.
\]

SUPPLEMENTAL MATERIAL

Derivation of Eq. (4) in the Main Text

To apply an external force \( f_t \) given by Eq. (3) in the Main Text, we need to dissipate energy at the rate

\[
\bar{W}_{ext}^* = (1 - \alpha)\bar{W}_h^* + \alpha \bar{W}_{f_b}^*.
\]

where \( \bar{W}_h^* = \langle h_t \circ \dot{x}_t \rangle_s \) is the contribution from blind perturbation while \( \bar{W}_{f_b}^* = -\left( \int_{-\infty}^{\infty} \tilde{G}(t - \tau) \dot{x}_\tau \circ \dot{x}_\tau \right) \) is from the information-based operation.

Replacing \( \tau \) by \( s + t \), we have

\[
\bar{W}_{f_b}^* = -\int_{-\infty}^{\infty} \bar{G}(-s) \langle \dot{x}_{s+t} \circ \dot{x}_t \rangle_s ds
\]

\[
= -\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \bar{G}(-s)[C_h^*(s) + 2\pi v_0 \delta(s)] ds
\]

\[
= -\bar{G}'(0)v_0 - \int_{-\infty}^{\infty} \bar{G}'(\omega)\tilde{C}_h^*(\omega) \frac{d\omega}{2\pi}.
\]

The last equality is obtained by replacing \( G(-s) \) and \( C_h^*(s) \) by their corresponding Fourier transform, i.e.,

\[
\bar{G}(-s) = \int_{-\infty}^{\infty} \bar{G}(\omega) \exp(-i\omega s) d\omega / 2\pi \quad \text{and} \quad C_h^*(s) = \int_{-\infty}^{\infty} \tilde{C}_h^*(\omega) \exp(-i\omega s) d\omega / 2\pi,
\]

and integrating over \( s \) and \( \omega \). The imaginary part of \( \tilde{G}(\omega) \), denoted as \( \tilde{G}'(\omega) \), does not appear since it is an odd function while \( \tilde{C}_h^*(\omega) \) is an even function. As mentioned in the Main Text, \( \tilde{C}_h \) depends on how the system is perturbed, which is different from \( \tilde{C}_x \), obtained with \( h = 0 \).

On the other hand, treating \( \delta h_t = h_t - h^* \) as a small perturbation to the system, the leading order response is given by

\[
\langle \dot{x}_t \rangle = \langle \dot{x} \rangle_{ss} + \int_{-\infty}^{\infty} \tilde{R}_s^*(t - \tau) \delta h_t \delta h_t d\tau.
\]

Here, \( \langle \dot{x} \rangle_{ss} \) is the steady-state mean velocity before applying perturbation, which may be different from \( v_0 \), and \( A \) characterizes the amplitude of the perturbative force \( h \). Therefore,

\[
\bar{W}_h^* = \langle \dot{x} \rangle_{ss} h^* + (\delta h_t) \int_{-\infty}^{\infty} \tilde{R}_s^*(t - \tau) \delta h_t \delta h_t d\tau
\]

\[
= \langle \dot{x} \rangle_{ss} h^* + \int_{-\infty}^{\infty} \tilde{R}_s^*(\omega) \tilde{C}_h^*(\omega) \frac{d\omega}{2\pi}.
\]
Fourier transforming these equations give
\[ -i\omega\gamma\ddot{x} = -k\ddot{x} + bk\dot{y} + \dot{h} + \dot{\xi}, \quad -i\omega\gamma_1\ddot{y} = -k_1\ddot{x} + k_2\dot{y} + \ddot{\xi}. \]

With \( \omega_x \equiv k / \gamma, \omega_y \equiv bk_1 / \gamma_1 \) and \( \omega_2 \equiv k_2 / \gamma_1 \), we have
\[ \ddot{x} = \frac{1}{\gamma} \frac{(i\omega - \omega_2)(\dot{h} + \ddot{\xi}) - \ddot{\xi} bk / \gamma_1}{\omega^2 - \omega_2\omega_x - \omega_2\omega_y + i\omega(\omega_e + \omega_2)}, \tag{S10} \]

The velocity response spectrum is then given by
\[ \dot{R}_x(\omega) = -i\omega \left( \frac{\delta\dot{x}(\omega)}{\delta h(\omega)} \right) \]
\[ = \frac{1}{\gamma} \frac{\omega^2 + i\omega\omega_2}{\omega^2 - \omega_2\omega_x - \omega_2\omega_y + i\omega(\omega_e + \omega_2)}, \tag{S11} \]
and its real part is
\[ \dot{R}_x'(\omega) = \frac{1}{\gamma} \frac{\omega^2(\omega^2 - \omega_2\omega_x + \omega_2\omega_y)}{\omega^2 - \omega_2\omega_x - \omega_2\omega_y + i\omega(\omega_e + \omega_2)^2}. \tag{S12} \]

Therefore, the response becomes negative in the frequency region
\[ \omega < \sqrt{\omega_2\omega_y - \omega_2^2}, \tag{S13} \]
which requires that \( \omega_2 < \sqrt{\omega_2\omega_y} \), i.e., a relatively small restoring force.

**Derivation of Eq. (13) in the Main Text**

Here, we consider the model introduced in Eq. (S9) at \( k_2 = 0 \). Similarly as Eq. (S10), we obtain
\[ \ddot{x}^+(\omega) = \frac{1}{\gamma} \frac{i\omega(\ddot{h}^+ + \ddot{\xi}^+) - \ddot{\xi}^+ bk / \gamma_1}{\omega^2 - \omega_2\omega_y + i\omega(\omega_e + \omega_2)}, \tag{S14} \]
where \( \ddot{x}^+(\omega) \) is the rescaled Fourier transform of \( x_t \). Its velocity correlation spectrum at \( h = 0 \) is given by
\[ \tilde{C}_x(\omega) = \omega^2 \langle |\ddot{x}^+(\omega)|^2 \rangle \]
\[ = \frac{2}{\gamma} \frac{\omega^2(\omega^2 + \omega\omega_e k / \gamma_1)}{(\omega^2 - \omega_2\omega_y)^2 + \omega^2\omega_e^2}, \tag{S15} \]
where we have used \( \langle |\ddot{\xi}^+(\omega)|^2 \rangle = 2T\gamma \) and \( \langle |\ddot{\xi}_1^+(\omega)|^2 \rangle = 2T\gamma_1 \).

Now, we proceed to calculate \( \dot{W}_{int}^+ \), the rate of internal energy supply. Firstly, the dissipation rate through frictional motion of \( y_t \) is given by
\[ \dot{Q}_y = -k_1 \langle x_t \circ \dot{y}_t \rangle \]
\[ = \frac{k_1^2}{\gamma_1} \langle x_t^2 \rangle \tag{S16} \]
In the second equality we have replaced \( \dot{y} \) by \( (-k_1 x + \xi_1) / \gamma_1 \) and used \( \langle x_t \circ \xi_1(t) \rangle = 0 \). Combined with Eq. (10) in the Main Text, we obtain the rate of internal energy supply:
\[ \dot{W}_{int}^+ = \omega_y (k + k_1 / b) \langle x_t^2 \rangle \tag{S17} \]
The long-time averaged variance \( \langle x_t^2 \rangle \) is given by
\[ \langle x_t^2 \rangle = \int_{-\infty}^{\infty} \langle \ddot{x}^+(\omega)\ddot{x}^+(\omega) \rangle \frac{d\omega}{2\pi}, \tag{S18} \]
Inserting \( \ddot{x}^+(\omega) \) from Eq. (S14), we have
\[ \langle x_t^2 \rangle = \int_{-\infty}^{\infty} \frac{1}{\gamma^2} \frac{\omega^2 \tilde{C}_x^+(\omega) + 2T\gamma + 2T\gamma_1(bk / \gamma_1)^2}{(\omega^2 - \omega_2\omega_y)^2 + \omega^2\omega_e^2} \frac{d\omega}{2\pi} \]
\[ = \int_{-\infty}^{\infty} \frac{1}{\gamma^2} \frac{\omega^2 \tilde{C}_x^+(\omega) + 2T\gamma + 2T\gamma_1(bk / \gamma_1)^2}{(\omega^2 - \omega_2\omega_y)^2 + \omega^2\omega_e^2} \frac{d\omega}{2\pi} + \frac{T}{k} + \frac{k_1}{k}, \tag{S19} \]
where we have used the following integrals
\[ \int_{-\infty}^{\infty} \frac{\omega^2}{(\omega^2 - \omega_2\omega_y)^2 + \omega^2\omega_e^2} \frac{d\omega}{2\pi} = \frac{1}{\omega_2}, \tag{S20} \]
\[ \int_{-\infty}^{\infty} \frac{1}{(\omega^2 - \omega_2\omega_y)^2 + \omega^2\omega_e^2} \frac{d\omega}{2\pi} = \frac{1}{2\omega_2^2\omega_y}. \tag{S21} \]
To proceed, we need to specify the perturbation spectrum. Let us consider a periodic perturbation \( h_t = h_0 \cos(\omega_h t) \). Its correlation spectrum is
\[ \tilde{C}_h(\omega) = \frac{\pi h_0^2}{2} \delta(\omega + \omega_h) + \delta(\omega - \omega_h). \tag{S22} \]
Plugging it into Eq. (S19), we obtain
\[ \langle x_t^2 \rangle = \frac{h_0^2}{2\gamma^2} \frac{\omega^2}{(\omega^2 - \omega_2\omega_y)^2 + \omega^2\omega_e^2} + \frac{T}{k} + \frac{k_1}{k}, \tag{S23} \]
Therefore, according to Eq. (S17), we have
\[ \dot{W}_{int}^+ = \omega_y (k + k_1 / b) \frac{h_0^2}{2\gamma^2} \frac{\omega^2}{(\omega^2 - \omega_2\omega_y)^2 + \omega^2\omega_e^2} + \omega^2\omega_e^2 \tag{S24} \]
At the optimal frequency \( \omega_c = \sqrt{\omega_2\omega_y / (1 + \sqrt{\omega_2 / \omega_y})} \), we finally obtain Eq. (13) in the Main Text.