NEUTRON STAR MASS MEASUREMENTS. I. RADIO PULSARS
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ABSTRACT

There are now about 50 known radio pulsars in binary systems, including at least five in double neutron star binaries. In some cases, the stellar masses can be directly determined from measurements of relativistic orbital effects. In others, only an indirect or statistical estimate of the masses is possible. We review the general problem of mass measurement in radio pulsar binaries and critically discuss all current estimates of the masses of radio pulsars and their companions. We find that significant constraints exist on the masses of 21 radio pulsars and on five neutron star companions of radio pulsars. All the measurements are consistent with a remarkably narrow underlying Gaussian mass distribution, $m = 1.35 \pm 0.04 \, M_\odot$. There is no evidence that extensive mass accretion ($\Delta m \gtrsim 0.1 \, M_\odot$) has occurred in these systems. We also show that the observed inclinations of millisecond pulsar binaries are consistent with a random distribution, and thus find no evidence for either alignment or counteralignment of millisecond pulsar magnetic fields.

Subject headings: binaries: close — pulsars: general — stars: fundamental parameters — stars: neutron

1. INTRODUCTION

Neutron stars have been the subject of considerable theoretical investigation since long before they were discovered as astronomical sources of radio and X-ray emission (Baade & Zwicky 1934; Oppenheimer & Volkoff 1939; Wheeler 1966). Their properties are determined by the interplay of all four known fundamental forces—electromagnetism, gravitation, and the strong and weak nuclear forces—but neutron stars remain sufficiently simple in their internal structure that realistic stellar modeling can be done. Measurements of their masses and radii (as well as detailed study of their cooling histories and rotational instabilities) provide a unique window on the behavior of matter at densities well above that found in atomic nuclei ($\rho > 10^{14} \, \text{g} \, \text{cm}^{-3}$). Observations of neutron stars also provide our only current probe of general relativity (GR) in the “strong field” regime, where gravitational self-energy contributes significantly to the stellar mass.

The most precisely measured physical parameter of any pulsar is its spin frequency. The frequencies of the fastest observed pulsars (PSR B1937+21 at 641.9 Hz and B1957+20 at 622.1 Hz) have already been used to set constraints on the nuclear equation of state at high densities (e.g., Friedman et al. 1988) under the assumption that these pulsars are near their maximum (breakup) spin frequency. However, the fastest observed spin frequencies may be limited by complex accretion physics rather than fundamental nuclear and gravitational physics. A quantity more directly useful for comparison with physical theories is the neutron star mass.

The basis of most neutron star mass estimates is the analysis of binary motion. Soon after the discovery of the first binary radio pulsar (Hulse & Taylor 1975), it became clear that the measurement of relativistic orbital effects allowed extremely precise mass estimates. Indeed, the measurement uncertainties in several cases now exceed in precision our knowledge of Newton’s constant $G$, requiring masses to be quoted in solar units $GM_\odot$ rather than kilograms if full accuracy is to be retained.

After several recent pulsar surveys, there are now about 50 known binary radio pulsar systems, of which five or six are thought to contain two neutron stars. It is thus possible for the first time to consider compiling a statistically significant sample of neutron star masses. It is our purpose here to provide a general, critical review of all current estimates of stellar masses in radio pulsar binaries. The resulting catalog, with a careful, uniform approach to measurement and systematic uncertainties, should be of value both to those who wish to apply mass measurements to studies of nuclear physics, GR, and stellar evolution, and as a guide to the critical observations for observational pulsar astronomers. We begin with a discussion of known methods for pulsar mass determination (§ 2), including a new statistical technique for estimating the masses of millisecond pulsars in nonrelativistic systems. In § 3 we review all known mass estimates, including new data and analysis where possible. Statistical analysis of the available pulsar mass measurements is presented in § 4. We summarize in § 5.

A second paper will consider mass estimates for neutron stars in X-ray binary systems (Chakrabarty & Thorsett 1998, hereafter Paper II). A future paper will provide detailed discussion of the implications of the combined results of this work and Paper II for studies of supernovae and neutron star formation, mass transfer in binary evolution, the nuclear equation of state, and GR.

2. METHODS OF MASS ESTIMATION

It is a familiar circumstance that estimates of astronomical masses are available only for bodies in gravitationally

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bound binary systems or clusters. Compact stars introduce the additional possibility of directly measuring the surface gravitational potential, and hence $M/R$, through the study of redshifted spectral features. Although this technique has been used with considerable success in the case of white dwarfs, and attempts have been made to fit redshifted X-ray spectra from neutron stars (Paper II), no lines have been identified in radio pulsar spectra, and other gravitational effects on the observed emission from pulsars are sufficiently complex and theory dependent that no useful limits on the neutron star properties have yet been possible. In the following we thus limit ourselves to the determination of stellar masses in binary systems.

2.1. Pulsar Timing

In any binary pulsar system, five Keplerian parameters can be very precisely measured by pulse timing techniques (Manchester & Taylor 1977): the binary period $P_b$, the projection of the pulsar’s semimajor axis on the line of sight $x \equiv a_1 \sin i/c$ (where the binary inclination $i$ is the angle between the line of sight and the orbital angular momentum vector, defined to lie in the first quadrant), the eccentricity $e$, and the time and longitude of periastron, $T_0$ and $o_{\omega}$. It is often more convenient to use the orbital angular frequency in place of the orbital period: These observational parameters are related to the pulsar and companion masses, $m_1$ and $m_2$, through the mass function

$$f = \frac{(m_2 \sin i)^3}{M^2} = n^2 x^3 \left( \frac{1}{T_0} \right) M_\odot$$

(1)

where $M = m_1 + m_2$, the masses are measured in solar units, and we introduce the constant $T_0 \equiv GM_\odot/c^3 = 4.925490947 \times 10^{-5}$ s.

Relativistic corrections to the binary equations of motion are most often parameterized in terms of one or more post-Keplerian (PK) parameters (Damour & Deruelle 1986; Taylor & Weisberg 1989; Damour & Taylor 1992). In GR, the most significant PK parameters have familiar interpretations as the advance of periastron of the orbit $\omega$, the combined effect of variations in the transverse Doppler shift and gravitational redshift around an elliptical orbit $\gamma$, the orbital decay due to emission of quadrupole gravitational radiation $\dot{P}_b$, and the “range” and “shape” parameters $r$ and $s$ that characterize the Shapiro time delay of the pulsar as it propagates through the gravitational field of its companion. In terms of measured quantities and the pulsar and companion masses (in solar units), these PK parameters are given by (Taylor 1992):

$$\dot{\omega} = 3n^{5/3}(T_0 M)^{2/3}(1 - e^2)^{-1}$$

(2)

$$\gamma = en^{-1/3}T_0^{2/3} M^{-4/3} m_1(m_1 + 2m_2)$$

(3)

$$\dot{P}_b = -\frac{192\pi}{5} n^{5/3} \left( 1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right) \times (1 - e^2)^{-7/2} T_0^{5/3} m_1 m_2 M^{-1/3}$$

(4)

$$r = T_0 m_2$$

(5)

$$s = x n^{2/3} T_0^{-1/3} M^{2/3} m_2^{-1}$$

(6)

Note that, by combining equations (1) and (6), we obtain $s = \sin i$ for GR.

The measurement of the mass function $f$ (eq. [1]) together with any two PK parameters (eqs. [2]–[6]) is sufficient in the context of GR to uniquely determine the component masses $m_1$ and $m_2$. With additional assumptions, such as a uniform prior likelihood for orbital orientations with respect to the observer (§ 2.3.4), strong statements about the posterior distribution of the masses is possible if even a single PK parameter is measured.

As an alternative to the PK formalism, which is designed for testing gravitation theory, it is sometimes advantageous to fit the timing data to a model that assumes the correctness of GR. For example, the DDGR model of Damour, Deruelle, and Taylor (Taylor 1987; Taylor & Weisberg 1989) describes the pulsar phase as a function of the five Keplerian parameters, the companion mass $m_2$, and the total mass $M$. A least-squares fit of the timing data to the DDGR model thus gives direct estimates of the uncertainties in $m_2$ and $M$, as well as the covariances between the mass estimates and estimates of other parameters.

It is of interest to note that mass measurements from pulsar timing observations depend on the unknown relative motion of the solar system and the binary barycenter. Damour & Deruelle (1986) have shown that neglecting this velocity is equivalent to changing units of mass and time. In particular, the rest-frame mass $m$ and the barycenter frame mass $m_{ab}$ are related by $m = D m_{ab}$, with Doppler factor

$$D \equiv \frac{1 - \hat{n} \cdot v_b/c}{\sqrt{1 - v_b^2/c^2}}$$

(7)

where $\hat{n}$ is the line-of-sight unit vector and $v_b$ is the barycentric velocity of the pulsar. Although the transverse velocity of the binary system can be estimated from proper motion measurements, the radial component $\hat{n} \cdot v_b$ is unknown. For a typical velocity of $100$ km s$^{-1}$, the systematic mass error is $\sim 0.03%$—small, but in some cases much larger than other uncertainties.

2.2. Masses of Companion Stars

While timing measurements of relativistic corrections to the Keplerian orbital equations provide the most accurate and theory-independent estimates of neutron star masses, they are possible only for close, eccentric binary orbits or when the orbit is observed nearly edge-on. In the great majority of observed binaries, the mass function provides the only timing information about the component masses, and the pulsar mass can only be determined if additional constraints are found on the companion mass and binary inclination through other techniques. This section describes several ways to use observations or theoretical considerations to limit $m_2$; alternative limits on $\sin i$ are the topic of § 2.3.

2.2.1. Optical Observations of White Dwarf Companions

In recent years, over a dozen companion stars in radio pulsar binaries have been optically detected (e.g., van Kerkwijk 1996; Lundgren, Camilo, & Foster 1996). In most cases, the companions are white dwarfs; the two main-sequence exceptions are discussed in the next section. Many of the white dwarf companions are extremely faint ($m_v \sim 26$), allowing detection only with the Hubble Space Telescope and the largest ground-based telescopes.

There are a number of ways to determine white dwarf masses (e.g., Reid 1996 and references therein). Given a theoretical relation between the white dwarf mass and radius, the measurement of any combination of the mass and radius is sufficient to determine the mass. For example,
the radius can be estimated directly from estimates of the optical flux, effective temperature, and distance. Alternatively, the surface gravity, \( \log g \), can be found by fitting a model atmosphere to the observed spectrum (Bergeron, Wesemael, & Fontaine 1991) and combining the result with a temperature or luminosity estimate. In practice, difficulties arise from several sources. First, the white dwarf companions of millisecond pulsars, with typical masses \( m_2 < 0.5 \, M_\odot \), are usually believed to be helium white dwarfs that were insufficiently massive to burn to carbon; the luminosity and temperature evolution of such stars has received much less study than for more massive white dwarfs (e.g., D’Antona & Mazzitelli 1990), leading to uncertainties in the finite temperature contributions to the mass-radius relationship. Hansen & Phinney (1998a, 1998b) have recently calculated cooling curves for helium core dwarfs, using their own calculations of H and He opacities at temperatures below 6000 K, and they have applied their results to a number of radio pulsar companions. We discuss their results in more detail in § 3.

The measurement of surface gravity in cool stars is also potentially problematic. A higher helium abundance, as may occur through enhanced convective mixing in cool stars, will produce higher pressure and broader lines and hence mimic a higher white dwarf mass (Bergeron, Wesemael, & Fontaine 1991); indeed, there is some evidence that surface gravity measurements overestimate masses below about 12,000 K (Reid 1996).

2.2.2. Optical Observations of Main-Sequence Companions

Two pulsars have been found in binaries with main-sequence companions: PSR B1259−63, with a Be companion (Johnston et al. 1992), and PSR J0045−7319, with a B star companion (Kaspi et al. 1994a). Although in both cases the optical companion is quite bright and easily observed, knowledge of the companion mass \( m_2 \) and pulsar mass function \( f \) is of little use in limiting \( m_1 \) when \( m_1 \ll m_2 \). If the mass function of the companion can also be measured, then the mass ratio can be determined. This has been done in the case of J0045−7319 (§ 3.3.1).

2.2.3. The \( P_o-m_2 \) Relation

The binary millisecond pulsars are believed to be spun up to high spin frequencies through mass transfer from a companion star. In many cases, these pulsars are observed in wide, low-eccentricity binary systems with white dwarf secondaries. These characteristics indicate that the secondary must have passed through a red giant phase after the formation of the neutron star primary, during which tidal torques circularized the orbit and the giant probably filled its Roche lobe, causing the mass transfer that spun up the pulsar to millisecond periods. At the end of mass transfer, the envelope of the giant is exhausted or ejected, leaving the degenerate core as a white dwarf secondary. There is a close relation between the core mass and the radius of low-mass giants (Refsdal & Weigert 1971; Webbink, Rappaport, & Savonije 1983; Joss, Rappaport, & Lewis 1987; Rappaport et al. 1995; Rappaport & Joss 1997). Combined with the assumption that the giant filled its Roche lobe during the mass transfer, this yields a relation between the binary period at the end of mass transfer and the remnant white dwarf mass (Rappaport et al. 1995).

Following Rappaport et al. (1995) and references therein, the relation between the effective radius of the Roche lobe \( R_L \) and the binary separation \( a \) can be written

\[
R_L \approx 0.46a\left(1 + \frac{m_1}{m_2}\right)^{-1/3},
\]

where \( m_0 \) is the total mass of the giant (core and envelope). Near the end of mass transfer, the envelope mass can be neglected, so that \( m_3 \approx m_2 \) (where \( m_3 \) is the final white dwarf mass), and the giant radius \( R_g \approx R_L \). Using Kepler’s third law,

\[
P_b = 0.374R_g^{3/2}m_2^{-1/2} \text{ days},
\]

where \( R_g \) and \( m_2 \) are in solar units. We note that equation (9) is independent of the pulsar mass \( m_1 \) and relates the orbital period at the end of mass transfer to the final white dwarf mass and the giant radius \( R_g \). The radius depends, in general, on the composition and history of the giant as well as on \( m_2 \); the utility of equation (9) derives from the relatively narrow distribution of \( R_g \) for a given \( m_2 \).

Rappaport et al. (1995) have examined stellar models with a wide range of chemical compositions, varying between Population I and Population II values, as well as a variety of envelope masses and convective mixing lengths. Over a wide range of core masses greater than 0.15 \( M_\odot \), they find that the data are well described by the equation

\[
R_g = \frac{R_0 m_2^{4.5}}{1 + 4 m_2^3} + 0.5,
\]

where \( R_0 = 4950 \, R_\odot \) is the best fit to the stellar models. In all models studied by Rappaport et al. (1995), equation (10) was correct within a factor of 1.8. Over the limited range \( m_2 < 0.25 \, M_\odot \), Rappaport & Joss (1997) found that the alternative expression

\[
\log R_g = 0.31 + 1.718 m_2 + 8.04 m_2^2
\]

(11)
could be used, with the smaller uncertainty of a factor \( \sim 1.3 \). We regard these to be approximately 95% confidence regions for \( R_g \).

It is important to determine the range of applicability of equations (10) and (11). At orbital periods of less than a few days, heating of the surface of the companion by X-rays produced during accretion may cause significant bloating and modification of the core mass–radius relations (Podsiadlowski 1991; Rappaport et al. 1995). Following Rappaport et al., we trust the modeling only for binary orbits longer than 3 days. Further, the \( P_o-m_2 \) relation cannot be applied to systems like PSR J2145−0750 or J1022+10—both 16 ms pulsars that were most likely recycled in common envelopes that occurred when their companions overflowed their Roche surfaces while on the asymptotic giant branch, leaving carbon-oxygen rather than helium white dwarfs (van den Heuvel 1994)—or to pulsars like B0820+02, a slow (0.864 s), high-field \( (3 \times 10^{11} \text{ G}) \) pulsar without evidence of significant recycling. To avoid both classes of pulsar without introducing biases by cutting on apparent companion mass, we apply the \( P_o-m_2 \) relation only to pulsars with \( P < 10 \) ms, for which the assumption of an extended period of mass transfer during a low-mass X-ray binary phase seems secure.

Using equations (9)–(11), the observed orbital period in a millisecond pulsar binary can be used to limit the range of \( m_2 \). When combined with the mass function (eq. [1]) and the restriction \( \sin i < 1 \), an upper limit can be placed on the pulsar mass. The resulting limits are discussed in § 3.2.
a random distribution of orbits is assumed for the set of observed millisecond pulsars, then statistical arguments lead to limits on the masses of millisecond pulsars as a class. These arguments are described in §4.

2.3. Alternative Methods of Estimating the Orbital Inclination

If the companion mass can be estimated and bounds can be found on the orbital inclination, then the mass of the pulsar can be found using the Keplerian mass function. In this section, we discuss measurements that may allow direct estimates of $i$.

2.3.1. Polarization Measurements

In the standard model of millisecond pulsar formation, the pulsar is spun up by mass transfer from a companion star. After spin-up, the pulsar spin axis will be aligned with the orbital angular momentum, so a measurement of the angle $\zeta$ between the pulsar spin axis and the line of sight also determines the orbital inclination: $i = \zeta$, or $i = \pi - \zeta$ for $\zeta$ in the second quadrant. (Although this is also true at an early stage of the formation of double neutron star binaries, an asymmetry in the second supernova explosion may leave the spin and orbit misaligned.)

In the rotating vector model of pulsar emission, the radio signal is elliptically polarized, with the major axis of the polarization ellipse aligned with the plane of curvature of the dipolar magnetic field lines. The position angle of the observed linear polarization $\psi$ can be expressed as a function of the rotational phase $\phi$ of the pulsar:

$$\tan \left[ \psi(\phi) - \psi_0 \right] = \frac{\sin \alpha \sin \phi}{\sin \zeta \cos \alpha - \cos \zeta \sin \alpha \cos \phi},$$

where $\alpha$ is the angle between the pulsar spin axis and the magnetic pole. In practice, the difference $\zeta - \alpha$ can often be estimated quite well from equation (12), but accurate estimates of $\zeta$ or $\alpha$ are possible only if polarized emission can be detected over a broad range of pulse phase $\phi$.

Application of equation (12) to millisecond pulsars, in which the light cylinder bounds the magnetosphere at only a few stellar radii, may be complicated by multipolar field geometries, aberration, and other deviations from the simple rotating vector model. Indeed, attempts to fit equation (12) to millisecond pulsar data have met with few unqualified successes (Thorsett & Stinebring 1990; Navarro et al. 1997).

2.3.2. Interstellar Scintillation

Observed pulsar signal strengths vary in both frequency and time because of scintillation in the interstellar medium. A phase-changing screen along the line of sight produces an interference pattern across which the pulsar moves; the characteristic scintillation decorrelation timescale $\tau_{\text{iss}}$ is inversely proportional to the transverse component of the pulsar velocity. Observations of scintillation rates have been widely used to estimate the proper motions of isolated pulsars (e.g., Cordes 1986). In binary systems, the transverse velocity is modulated by the orbital motion, with a small amplitude of modulation for orbits viewed face-on and a large amplitude for orbits viewed on edge. Scintillation measurements can thus provide an alternative path to estimating the inclination angle $i$ (Lyne 1984).

The real physical situation is more complex. The measured scintillation velocity $v_{\text{iss}}$ of a pulsar depends not only on its proper motion $v_{\text{pm}}$ and orbital motion $v_{\text{orb}}(t)$, but also on the Earth’s motion $v_\odot(t)$, the mean motion $v_{\text{pm}}$ of the scattering medium and the ratio of the effective distance to the scattering screen to the pulsar distance, $f$:

$$v_{\text{iss}}(t) = (1 - f)[v_{\text{orb}}(t) + v_{\text{pm}}] + f v_\odot(t) - v_{\text{iss}}.$$

If the proper motion is known, from timing measurements or interferometry, then the annual modulation of $v_{\text{iss}}$ and the orbital modulation of $v_{\text{orb}}$ can in principle be used to find not only the orbital inclination $i$ but also $f$, the pulsar distance $d$, and the position angle on the sky of the orbital ascending node $\Omega$.

Only a few attempts to determine orbital inclinations from scintillation observations have been published. In no case was $v_{\text{pm}}$ known, nor has the annual modulation due to $v_\odot$ been measured (although the latter has been observed in the isolated millisecond pulsar B1937+21 by Ryba 1991). Lyne (1984) found that the inclination of PSR B0655+64 is either $62^\circ$ or $84^\circ$ (the discrete ambiguity could be broken through observations at another time of year, using variations in $v_\odot$). Jones & Lyne (1988) later expressed reservations about this measurement. In the most convincing success of the scintillation technique, Dewey et al. (1988) found a limit on the inclination of PSR B1855+09: $i > 0.94$. Shapiro time delay measurements have since shown that $i \approx 0.9993$ (§3.2.7).

The problem of estimating scintillation parameters from data is considered by Cordes (1986). The dominant error source in estimating scintillation parameters is the finite number $N$ of scintillation features sampled: $\sigma_i/v_{\text{iss}} \approx 0.6/N^{1/2}$. Scintillation intensity fluctuations are exponentially distributed; for observing bandwidth $B$ and time $T$, we have roughly $N \approx 10^{-2}BT/(\Delta v_{\text{iss}} \tau_{\text{iss}})$. Considering a case relevant to many recently discovered pulsar binaries, a pulsar at a dispersion measure $DM = 20$ will have typical scintillation parameters at 430 MHz $\tau_{\text{iss}} \approx 10$ minutes and $\Delta v_{\text{iss}} \approx 100$ kHz (Rickett 1988). With $B = 10$ MHz and $T = 1$ hr, $\sigma_i/v_{\text{iss}} \approx 25\%$, so a study of orbital dependence of scintillation parameters requires a substantial amount of observing time. Furthermore, to use a bandwidth of 10 MHz requires a spectrometer with $\approx 100$ frequency channels. Such observations will be more easily done with the new generation of flexible, all-digital pulsar data recorders (Shráuer et al. 1996; Jenet et al. 1997). It is possible that long-term variability in $f$ or $v_{\text{iss}}$ will limit the use of the annual variation in $v_\odot$ to estimate $f$.

2.3.3. Secular Variation of $x = a_1 \sin i$

Proper motion of the binary system across the sky leads to a secular change in the projected semimajor axis $x$. If $\Omega$ is the position angle of the ascending node and $\mu_x$ and $\mu_y$ are the components of the proper motion $\mu$ in right ascension and declination, then

$$\frac{\dot{x}}{x} = \cot \dot{i} - \mu_x \sin \Omega + \mu_y \cos \Omega. \tag{14}$$

The angle $\Omega$ is generally unknown (though it is accessible to scintillation measurements, §2.3.2), but equation (14) can be rewritten as the limit

$$\tan \dot{i} < \frac{x}{\dot{x}} \left| \frac{\mu}{\dot{x}} \right|. \tag{15}$$

A similar expression can be found relating proper motion to a purely geometric advance of periastron $\dot{\omega}$—in principle,
measurement of both $x$ and $\omega$ would determine $i$—but the effect is too small to have yet been seen in any pulsar binary.

### 2.3.4. Random Distribution of Orbital Inclinations

When no other information is available on the orbital inclination, it is sometimes useful in statistical calculations to assume that binary orbits are randomly oriented with respect to the line of sight. The differential distribution of inclinations is then proportional to $\sin i$ (i.e., the most likely orbital viewing angle is edge-on). Values of $\cos i$ should then be uniformly distributed between 0 and 1.

In binary systems where the directions of the pulsar spin axis and the orbital angular momentum vector are expected to be correlated, such as millisecond pulsar systems in which significant mass transfer has occurred since the last supernova, nonrandom orientations of the pulsar magnetic field axis with respect to the spin axis may lead to a pulsar discovery bias that skews the observed orbital orientation distribution. Some models of pulsar evolution predict magnetic field alignment, counteralignment, or both (e.g., Ruderman 1991).

Backer (1998) has claimed, based on the distribution of observed mass functions in 21 pulsar–white dwarf binaries, that millisecond pulsars have a nonrandom distribution of observed binary inclinations. He suggests that an apparent preference for high inclination orbits may arise because the pulsars’ magnetic fields are preferentially oriented perpendicular to the spin axis. However, his analysis makes several assumptions and approximations that prove unwarranted. In particular, he compares the observed distribution of minimum companion masses $m_{2,\text{min}}$ with the predicted distribution given the observed mass functions, random inclinations, and a single fixed value for the true companion masses $m_2$ (i.e., a delta function distribution in $m_2$), finding poor agreement. However, the predicted distribution for $m_{2,\text{min}}$ actually depends quite sensitively upon the assumed distribution for $m_2$; the delta function distribution is an unreasonable assumption. Indeed, the $P_0$-$m_2$ relation suggests that the observed $m_2$ values should vary by a factor of 3. Knowing the precise limits and distribution of these masses is crucial for analyzing the distribution of $m_{2,\text{min}}$.

We repeated Backer’s analysis with different assumptions about the distribution of $m_2$ values. If we assume that the true values $m_2$ fall uniformly in the range $0.15$–$0.35 M_\odot$, then the predicted and observed distributions for $m_{2,\text{min}}$ agree at an 81% confidence level according to a Kolmogorov-Smirnov (KS) test (e.g., Eadie et al. 1971). However, assuming a $0.1$–$0.4 M_\odot$ range reduces the agreement to the 33% confidence level. The discrepancy is primarily due to a deficit of observed systems with small values of $m_{2,\text{min}}$ as noted by Backer. Unfortunately, our knowledge of the true underlying distribution of $m_2$ in these binaries is sufficiently uncertain that no conclusion regarding the distribution of inclinations is possible from this line of analysis. However, we argue below (§ 4) that for systems with orbital periods longer than 3 days, where the $P_0$-$m_2$ relation can be applied to estimate the $m_2$ distribution, the observed mass functions are consistent with a random distribution of inclinations.

### 3. RADIO PULSAR MASSES

The parameters of 48 radio pulsar binary systems are listed in Table 1. At least five of these pulsars have neutron star companions, so a total of 53 neutron stars are known in

#### TABLE 1

| PSR Name | $P_0$ (s) | $P_0$ (d) | $e$ | Notes |
|----------|----------|----------|-----|-------|
| J1518 + 4904 | 0.049035 | 8.634 | 0.24948 |        |
| B1534 + 12 | 0.037904 | 0.421 | 0.27368 |        |
| B1913 + 16 | 0.05903 | 0.323 | 0.61713 |        |
| B2127 + 11C | 0.030529 | 0.335 | 0.68141 | 1      |
| B2303 + 46 | 1.066371 | 12.34 | 0.65837 | 1      |
| B0201 - 72E | 0.003536 | 2.257 | 0.000 | 1      |
| B0201 - 72I | 0.003485 | 0.226 | 0.000 | 1      |
| B0201 - 72J | 0.002101 | 0.121 | 0.000 | 1      |
| J0034 - 0534 | 0.001877 | 1.589 | 0.000 |        |
| J0218 + 4232 | 0.002323 | 2.029 | 0.00000 |        |
| J0437 - 4715 | 0.005757 | 5.741 | 0.00000 |        |
| J0613 - 0200 | 0.003062 | 1.199 | 0.00000 |        |
| B0655 + 64 | 0.195671 | 1.029 | 0.00000 |        |
| J0751 + 1807 | 0.003479 | 0.263 | 0.00000 |        |
| B0820 + 02 | 0.864873 | 1232.47 | 0.01187 |        |
| J1012 + 5307 | 0.002526 | 0.605 | 0.00000 |        |
| J1022 + 00 | 0.016453 | 7.805 | 0.00001 |        |
| J1045 - 4509 | 0.007474 | 4.084 | 0.00000 |        |
| B1330 + 18 | 0.031163 | 255.8 | 0.002 | 1      |
| J1455 - 3330 | 0.007987 | 76.175 | 0.00017 |        |
| J1603 - 7202 | 0.014842 | 6.309 | 0.00000 | 2      |
| B1620 - 26 | 0.011076 | 191.443 | 0.02531 | 1, 4  |
| J1640 + 2224 | 0.003163 | 175.461 | 0.00008 |        |
| B1639 + 36B | 0.003528 | 1.259 | 0.005 | 1      |
| J1643 - 1224 | 0.004622 | 147.017 | 0.00051 |        |
| J1713 + 0747 | 0.004570 | 67.825 | 0.00007 |        |
| B1718 - 19 | 1.004037 | 0.258 | 0.000 | 1      |
| B1744 - 24A | 0.011563 | 0.076 | 0.00000 |        |
| B1800 - 27 | 0.334415 | 406.781 | 0.00051 |        |
| B1802 - 07 | 0.023101 | 2.617 | 0.212 | 1      |
| J1804 - 2717 | 0.009343 | 11.129 | 0.00004 | 2      |
| B1918 - 11 | 0.279828 | 357.762 | 0.79462 | 3      |
| B1831 - 00 | 0.520954 | 1.811 | 0.000 |        |
| B1855 + 09 | 0.005362 | 12.327 | 0.00002 |        |
| J1910 + 0004 | 0.003619 | 0.141 | 0.000 | 1      |
| J1911 - 1114 | 0.003626 | 2.717 | 0.00000 | 2      |
| B1953 + 29 | 0.006133 | 117.349 | 0.00033 |        |
| B1957 + 20 | 0.001607 | 0.382 | 0.00000 |        |
| J2019 + 2425 | 0.003935 | 76.512 | 0.00011 |        |
| J2033 + 17 | 0.005949 | 56.2 | 0.000 |        |
| J2051 - 0827 | 0.004509 | 0.099 | 0.000 | 5      |
| J2129 - 5271 | 0.003726 | 6.625 | 0.00000 | 2      |
| J2145 - 0750 | 0.016052 | 6.839 | 0.00002 |        |
| J2229 + 2643 | 0.002978 | 93.016 | 0.00026 |        |
| J2317 + 1439 | 0.003445 | 2.459 | 0.00000 |        |

#### Notes

1 Believed to be a globular cluster member; 2 Lorimer et al. (1996); 3 may be a double neutron star system; 4 probably a triple system (Thorsett, Arzoumanian, & Taylor 1996); 5 Stappers et al. 1996.

* Unless otherwise indicated, all data are from the 1995 revision by Taylor et al. of the Princeton Pulsar Catalog, available at ftp://pulsar.princeton.edu/pub/catalog.
3.1. Double Neutron Star Binaries

3.1.1. PSR J1518 + 4904

PSR J1518 + 4904 is in a moderately relativistic binary system; one PK parameter has been measured, the relativistic advance of periastron \( \Omega = 0.01111(2)^\circ \) yr\(^{-1} \), yielding a total system mass of 2.62(7) \( M_\odot \) (Nice, Sayer, & Taylor 1996). Given the mass function \( f = 0.115988 \ M_\odot \), the lower limit on the companion mass (using \( \sin i < 1 \)) is \( m_2 > (f M_1)^{1/3} = 0.93 \ M_\odot \), and the lower limit on the inclination (given \( m_1 > 0 \)) is \( \sin i > (f/M_1)^{1/3} = 0.35 \) (or \( i > 20^\circ \)). Using a uniform prior distribution in \( \sin i \) over the interval 0.35 < \( \sin i < 1 \), we find the central 68% confidence intervals \( m_1 = 1.56+0.13-0.44 \) and \( m_2 = 1.05+0.41-0.14 \) and central 95% confidence intervals \( m_1 = 1.56+0.20-1.20 \) and \( m_2 = 1.05+0.21-0.14 \).

The identification of the companion as a neutron star is compelling, although it is not possible to completely rule out a low-mass black hole. Optical observations (van Kerkwijk, quoted in Sayer 1996) show no source at the pulsar position to \( m_h \sim 24.5 \), excluding a main-sequence companion, and a white dwarf in an eccentric orbit is not expected on evolutionary grounds.

3.1.2. PSR B1534 + 12

PSR B1534 + 12 is in a highly relativistic binary system, presumably with a second neutron star companion. All five of the PK parameters defined by equations (2)-(6) have been measured, three with better than 1% precision (Stairs et al. 1998). Using the DDGR timing model, the total system mass is found to be \( M = 2.67838(8) \ M_\odot \), while the individual component masses \( m_1 \) and \( m_2 \) are both 1.339(3) \( M_\odot \). (The quoted errors are 68% confidence regions; the 95% confidence regions are about twice as large.) It is remarkable that the pulsar and companion masses agree to better than 0.4%; the assumption that the companion is a second neutron star seems secure. The uncertainty on the individual masses is expected to decrease significantly in the next year, when observations are again possible with the Arecibo telescope.

3.1.3. PSR B1913 + 16

PSR B1913 + 16 was the first binary pulsar discovered, and with observations stretching over two decades, it remains one of the best studied. PSR B1913 is a highly relativistic system: the PK parameters \( \dot{\Omega} \), \( \gamma \), and \( P_b \) have all been measured precisely. A fit to the DDGR timing model yields the total system mass \( M = 2.82843(2) \ M_\odot \) and component masses \( m_1 = 1.4411(7) \) and \( m_2 = 1.3874(7) \ M_\odot \) (Taylor 1992). (The quoted errors are 68% confidence regions; the 95% confidence regions are about twice as large.) The uncertainties are approaching the level where they will be dominated by kinematic effects (eq. [7]).

3.1.4. PSR B2127 + 11C

PSR B2127 + 11C is a relativistic binary in the globular cluster M15. Precise measurements have been made of the PK parameters \( \dot{\Omega} \) and \( \gamma \), resulting in mass estimates for the pulsar and companion of 1.349(40) and 1.363(40) \( M_\odot \), respectively, and a total system mass \( M = 2.7121(6) \ M_\odot \) (Deich & Kulkarni 1996). (The quoted errors are 68% confidence regions; the 95% confidence regions are approximately twice as large.)

3.1.5. PSR B2303 + 46

PSR B2303 + 46 is a pulsar in a moderately relativistic orbit with, most likely, a neutron star companion. One PK parameter, the relativistic advance of periastron, has been measured. A timing analysis was published by Thorsett et al. (1993a) and updated by Arzoumanian (1995). Using data from early 1985 to late 1994, we find an improved value \( \dot{\Omega} = 0.01019(13) \) yr\(^{-1} \), yielding a total system mass \( M = 2.64 \pm 0.05 \ M_\odot \) and the constraints \( m_1 < 1.44 \ M_\odot \) and \( m_2 > 1.20 \ M_\odot \). Using a uniform prior distribution in \( \sin i \), we find the central 68% confidence intervals \( m_1 = 1.30+0.13-0.46 \) and \( m_2 = 1.34+0.47-0.13 \) and central 95% confidence intervals \( m_1 = 1.30+0.18-0.10 \) and \( m_2 = 1.34+0.10-0.15 \).

3.2. Neutron Star/White Dwarf Binaries

3.2.1. PSR J0437 – 4715

PSR J0437 – 4715 is the brightest and closest known millisecond pulsar and therefore one of the best studied. It has a mass function \( f = 1.243 \times 10^{-3} \ M_\odot \) and a white dwarf companion that has been detected optically (Johnston et al. 1993; Bailyn 1993; Bell, Bailes, & Bessell 1993; Danziger, Baade, & Della Valle 1993); and both thermal and nonthermal X-ray emission from the neutron star have been observed (Becker & Trümper 1993; Halpern, Martin, & Marshall 1996). The distance is known, \( d = 178 \pm 26 \) pc, from the effects of parallax on the pulsar timing signal (Sandhu et al. 1997). Using the optical data of Danziger et al., Hansen & Phinney (1998b) found an effective temperature of the companion \( T_{\text{eff}} = 4600 \pm 200 \) K. Combining the \( T_{\text{eff}} \) and distance estimates with their cooling curves, they find consistent companion models for all masses \( 0.15 < m_2 < 0.375 \ M_\odot \). This range encompasses the mass limits derived from the \( P_b m_2 \) relation (§2.2.3), \( 0.16 < m_2 < 0.23 \ M_\odot \).

The binary orbit is nearly circular, and despite the very high timing precision achieved, no PK timing parameters have been measured. However, the proximity of the pulsar leads to a high proper motion \( (\mu = 141 \ \text{mas yr}^{-1}) \), changing the projected orbital size (§2.3.3) at a rate \( \dot{x} = 2.43(12) \times 10^{-14} \) s\(^{-1} \) (Sandhu et al. 1997). The implied limit on the inclination angle is \( i < 43^\circ \), or \( \sin i < 0.682 \).

Attempts to measure the system geometry using the polarization of the radio beam (§2.3.1) have proved difficult because of the very complex emission pattern. Well-calibrated intensity and polarization data have been reported by Manchester & Johnston (1995), who have modeled the sweep of the linear polarization position angle across the profile in terms of the rotating vector model. They find that an impact parameter of the line of sight on the magnetic pole \( \beta = -5^\circ \) and an angle between the line of sight and the spin axis \( \zeta = 140^\circ \) produced reasonable agreement with the data, but there are strong systematic deviations from the simple rotating vector model. Using the same data, Gil & Krawczyk (1997) model the multicomponent profile and find a comparable impact parameter, \( \beta = -4^\circ \), but a very different \( \zeta = 16^\circ \). They show that this geometry also explains the observed polarization sweep over much of the pulse period. With the assumption that the pulsar spin axis is aligned with the orbital axis, the implied orbital inclination is \( \sin i = 0.64 \) (Manchester & Johnston model) or
The timing limit on \( \sin i \), together with the mass function and the upper limit on \( m_2 \) from the core mass–orbital period relation, gives an upper limit to the pulsar mass of \( m_1 < 1.51 \) \( M_\odot \), while the optical upper limit on \( m_2 \) gives only the much weaker limit \( m_1 < 3.29 \) \( M_\odot \). The Manchester & Johnston inclination together with the \( P_m \)-\( m_2 \) mass range gives \( \sin i = 0.28 \) (Gil & Krawczyk model). Either is consistent with the timing data.

Improvements of the parallax measurement and the optical photometry and spectroscopy are needed to further constrain the companion mass. An independent mass determination would test the core mass–orbital period relation as well as improve the limit on \( m_1 \). Measurement of the position angle of the ascending node (perhaps through scintillation studies), combined with the measurement of \( \dot{s}/x \), would give the inclination of the orbit.

### 3.2.2. PSR J1012+5307

PSR J1012+5307 has a hot, bright white dwarf companion that has been extensively studied. Van Kerkwijk, Bergeron, & Kulkarni (1996) have used the models of Bergeron, Wesemael, & Fontaine (1991) to determine the effective temperature \( T_{\text{eff}} = 8550 \pm 25 \) K and surface gravity \( \log g = 6.75 \pm 0.07 \) of the companion. The latter value is in disagreement with an unpublished value of Callanan & Koester, \( \log g = 6.4 \pm 0.2 \), which is quoted by Hansen & Phinney (1998b). Using the optical observations and their cooling models for low-mass helium white dwarfs, Hansen & Phinney find a companion mass \( 0.165 < m_1 < 0.215 \) \( M_\odot \) for the van Kerkwijk et al. (1996) gravity measurement and \( 0.13 < m_1 < 0.18 \) for the Callanan & Koester value.

The radial velocity of the companion has been measured, making J1012+5307 a double-line spectroscopic pulsar binary. Van Kerkwijk et al. have found \( m_1/m_2 = 9.5 \pm 0.5 \) (van Kerkwijk 1998, private communication; their earlier published value \( m_1/m_2 = 13.5 \pm 0.7 \) was corrupted by a calibration problem). Depending on the gravity measurement, the resulting (1 e) pulsar mass limit is \( 1.5 < m_1 < 2.2 \) \( M_\odot \) or \( 1.2 < m_1 < 1.8 \) \( M_\odot \). Clearly resolution of the discrepant gravity measurements must be a high priority: the radial velocity measurements now contribute negligibly to the total error on \( m_2 \). For now, we adopt Hansen & Phinney's (1998b) conservative conclusion that \( 1.2 < m_1 < 2.2 \) \( M_\odot \), and we regard the error range as roughly a 68% confidence interval.

### 3.2.3. PSR J1045–4509

PSR J1045–4509 is in a 4.08 day orbit with mass function \( 1.765 \times 10^{-3} \) \( M_\odot \). The \( P_m \)-\( m_2 \) relation gives an upper limit to the companion mass of 0.168 \( M_\odot \), leading to a limit on the pulsar mass of \( m_1 < 1.48 \) \( M_\odot \).

### 3.2.4. PSR J1713+0747

PSR J1713+0747 is a bright pulsar in a nearly circular, 68 day orbit with a white dwarf companion. At the timing precision reached (about 500 ns for 1.4 GHz observations), Camilo (1995) found that the Shapiro PK parameters \( r \) and \( s \) were required to adequately model the data, but strong covariances between the range parameter \( r \) and other orbital parameters (especially \( x \)) prevented him from setting interesting limits on the pulsar or companion mass.

The \( P_m \)-\( m_2 \) relation predicts \( 0.26 < m_2 < 0.35 \) \( M_\odot \). Companion mass estimates are also possible by combining unpublished optical observations of the white dwarf by Lundgren et al. (1996) with the parallax from pulsar timing (Camilo, Foster, & Wolszczan 1994). The resulting limits are \( 0.15 < m_2 < 0.31 \) \( M_\odot \) if the white dwarf has a thick H envelope and \( m_2 < 0.27 \) \( M_\odot \) if it has a thin H envelope (Hansen & Phinney 1998b).

If the observed mass function \( f = 7.896 \times 10^{-3} \) \( M_\odot \) is combined with the \( P_m \)-\( m_2 \) limit on \( m_2 \) and the restriction \( \sin i < 1 \), an upper limit to the pulsar mass is found: \( m_1 < 1.94 \) \( M_\odot \). The optical observations yield a tighter limit, \( m_1 < 1.63 \) \( M_\odot \), as was noted by Hansen & Phinney (1998b). More interesting limits can be obtained by combining the Shapiro measurements with the limits on \( m_2 \) (recalling that \( m_2 = r \)). For example, if \( m_2 = 0.299 \) \( M_\odot \), the timing data of Camilo (1995) are consistent with only the restricted range \( \sin i = 0.963(4) \). By letting \( m_1 \) vary over the range allowed by the \( P_m \)-\( m_2 \) relation, we find the allowed pulsar mass \( m_1 = 1.45 \pm 0.31 \), where the uncertainties are dominated by the systematic uncertainties in the \( P_m \)-\( m_2 \) relation. If the companion mass is limited to the intersection of the regions allowed by \( P_m \)-\( m_2 \) and the optical measurements, \( 0.26 < m_2 < 0.31 \) \( M_\odot \), then we find \( m_1 = 1.34 \pm 0.20 \) \( M_\odot \).

### 3.2.5. PSR B1802–07

PSR B1802–07 is in the globular cluster NGC 6539. Its companion is most likely a white dwarf; the system's large eccentricity can be understood as the result of gravitational perturbations of the system by close stellar encounters in the dense cluster.

The relativistic advance of periastron \( \dot{\omega} \) was measured by Thorsett et al. (1993a), and an improved measurement was published by Arzoumanian (1995). Using the same analysis techniques, with data extending through 1997 October, we find a slightly improved value \( \dot{\omega} = 0.0578(16) \), implying a total system mass \( M = 1.62(7) \) \( M_\odot \). The lower bound on the companion mass from the requirement that \( \sin i < 1 \) is \( (J/M^2)^{1/3} = 0.29 \) \( M_\odot \), and the lower bound on the inclination from the requirement \( m_2 > 0 \) \( \sin i > (J/M^2)^{1/3} = 0.18 \) (or \( i > 10^\circ \)). Using a uniform prior for \( \sin i \) in the range \( 0.18 < \sin i < 1 \), we find a 68% confidence bound on the pulsar mass \( m_1 = 1.26^{+0.08}_{-0.06} \) \( M_\odot \) and a 95% confidence bound \( m_1 = 1.26^{+0.15}_{-0.06} \) \( M_\odot \).

### 3.2.6. PSR J1804–2718

PSR J1804–2718 is in an 11.1 day orbit. The \( P_m \)-\( m_2 \) relation limits its companion mass to \( 0.185 < m_2 < 0.253 \) \( M_\odot \); the resulting upper limit on the pulsar mass is \( m_1 < 1.73 \) \( M_\odot \).

### 3.2.7. PSR B1855+09

PSR B1855+09 is a 5.4 ms pulsar in a 12.3 day circular orbit \( (e = 2 \times 10^{-5}) \) with a white dwarf companion. Because it is a bright pulsar for which high-precision timing measurements can be made and because the orbital inclination is high, measurements have been made of the PK parameters \( r \) and \( s \).

We have reanalyzed all of the available data, extending from 1986 January to 1994 January. (For the data collected after mid-1989, taken with the Princeton Mark III timing system [Stinebring et al. 1992], a new algorithm was used for identifying times and frequencies where the signal strength was enhanced by interstellar scintillation and for weighting these data in subsequent analysis.) The details and complete timing solution will be published elsewhere.
The Shapiro parameters are \( r = \frac{Gm_2/c^3}{3} = 0.248(11) \, M_\odot \) and \( s = \sin \, i = 0.9993(2) \). The resulting limits on the pulsar mass are \( m_1 = 1.41(10) \, M_\odot \) (68% confidence; the 95% confidence region will be about twice as large). The new measurement is in good agreement with previous values: \( m_1 = 1.27^{+0.23}_{-0.13} \, M_\odot \) (Ryba & Taylor 1991) and \( m_1 = 1.50^{+0.16}_{-0.12} \) (Kaspi, Taylor, & Ryba 1994).  

The \( P_b-m_2 \) relation predicts \( 0.19 < m_2 < 0.26 \, M_\odot \), in good agreement with the timing measurement. The corresponding upper limit to the pulsar mass is \( m_1 < 1.51 \, M_\odot \). As noted above, scintillation measurements have also been used to limit \( \sin \, i \geq 0.94 \) (Dewey et al. 1988).

3.2.8. PSR J2019+2425

PSR J2019+2425 is in a 76.5 day orbit. The \( P_b-m_2 \) relation gives a limit on the companion mass \( 0.264 < m_2 < 0.354 \, M_\odot \); the resulting upper limit on the pulsar mass is \( m_1 < 1.68 \, M_\odot \).

3.3. Neutron Star/Main-Sequence Binaries

The pulsar J0045−7319 is the only known pulsar in the Small Magellanic Cloud. It is in a binary orbit, with mass function \( f = 2.17 \, M_\odot \) (Kaspi et al. 1994a). The companion has been identified as a B1 V star. The radial velocity of the companion has been measured, giving a mass ratio \( q = m_2/m_1 = 6.5 \pm 1.2 \) (Bell et al. 1995). We have compared the observed optical luminosity \( L = 1.2 \times 10^4 \, L_\odot \) and temperature \( T_{\text{eff}} = 2.4(1) \times 10^4 \, \text{K} \) to the grids of stellar models calculated by Schaller et al. (1992) for the low metallicity \( (Z = 0.001) \) appropriate for the SMC and estimate the companion mass to be \( 10 \pm 1 \, M_\odot \). The pulsar mass is then \( m_1 = m_2/q = 1.58 \pm 0.34 \, M_\odot \) (68% confidence), where the uncertainty is dominated by the uncertainty in the amplitude of the companion’s radial velocity curve.

4. DISCUSSION

For a dozen neutron stars, useful mass constraints are available with no assumptions beyond the applicability of the general relativistic equations of orbital motion to binary pulsar systems. Ten of these stars are members of double neutron star binaries. With the possible exception of PSR B2127+11C, in the globular cluster M15, the pulsar in each system is believed to have undergone a short period of mass accretion during a high-mass X-ray binary phase \( (\Delta m \sim 10^{-3} \, M_\odot) \) (Taam & van den Heuvel 1986). The companion stars have not undergone accretion; their masses most directly preserve information about the initial mass function of neutron stars.

Only two “millisecond” pulsars, the end products of extended mass transfer in low-mass X-ray binaries, have interesting mass estimates based on GR alone: PSR B1802−07 and PSR B1855+09. Because such pulsars must accrete \( \sim 0.1 \, M_\odot \) to reach millisecond periods (Taam & van den Heuvel 1986), and much more \( \sim 0.7 \, M_\odot \) in some field decay models (e.g., van den Heuvel & Bitzaraki 1995), obtaining additional mass measurements of millisecond pulsars is of particular interest in testing evolutionary models and in locating the maximum neutron star mass.

As noted in § 2.2.3, the \( P_b-m_2 \) relation can be used to estimate the companion mass in recycled binary systems with circular orbits and orbital periods \( P_b \gtrsim 3 \) days. There are now 13 such millisecond pulsars known, excluding those in globular clusters (where gravitational interactions may have significantly perturbed the orbital parameters since spin-up). In each case, the measured mass function and the inferred companion mass, together with the requirement that \( \sin \, i < 1 \), yields an upper limit on the mass of the pulsar itself. A number of systems in which this upper limit is particularly constraining have been mentioned in § 3.2.

Additional constraints on the neutron star mass in these systems can be derived using statistical arguments, given a prior assumption about the distribution of binary inclinations. The simplest such assumption is that the binaries are randomly oriented on the sky, though biases toward high or low inclinations are possible in some models (§ 2.3.4).

However, as discussed below, we believe there is currently no evidence for such a bias, so for the remainder of this discussion we assume random orbital orientations.

For an individual system, we are interested in the probability distribution \( p(m_1; f, P_b) \) for the neutron star mass \( m_1 \) given the measured mass function \( f \) and binary period \( P_b \). We can neglect the measurement uncertainty in \( f \) and \( P_b \). Then the probability distribution for \( m_1 \) can be written schematically as

\[
p(m_1; f, P_b) = \int_{0}^{1} d(cos \, i) \times \int_{m_2_{\text{max}}(P_b)}^{m_2_{\text{min}}(P_b)} dm_2 \, p(m_2; P_b)(\cos \, i)p(m_1 | m_2, \cos \, i; f),
\]

where \( m_f = f^{-1/2}(m_2 \sin \, i)^{3/2} - m_2 \) is restricted to positive values. We have evaluated this numerically for each system, assuming that \( p(\cos \, i) \) is uniform between zero and unity and that \( p(m_2; P_b) \) is uniformly distributed within the appropriate factor (see § 2.2.3) of the implied by equations (9)–(11). Not surprisingly, the width of the distribution \( p(m_1; f, P_b) \) is dominated by the range of allowed \( \cos \, i \) rather than the uncertainty in \( m_2 \) for a given \( P_b \). For each of the 13 binaries, we have plotted the cumulative distribution \( CDF(m_1) = \int_{0}^{m_1} p(m_1') \, dm_1 \) in Figure 1.

The median and 68% and 95% confidence regions for each pulsar mass are given in Table 2. Although several of the pulsars have, under the assumptions made, most likely masses well above \( 2 \, M_\odot \), some such results are expected even if all the masses are quite low. In fact, in only one case of the 13 pulsars does \( 1.35 \, M_\odot \) lie outside the 95% central confidence region (J1045−4509), and in six cases of 13 is \( 1.35 \, M_\odot \) excluded at 68% confidence, consistent with chance.

It is interesting to ask whether a single, simple distribution of neutron star masses is consistent with all of our observational constraints. We considered two models for this question: a Gaussian distribution of masses with mean \( \bar{m} \) and standard deviation \( \sigma \), and a uniform distribution of masses between \( m_1 \) and \( m_n \) (cf. Finn 1994). A maximum likelihood analysis was used to estimate the parameters \( \bar{m}, \sigma, m_1, \) and \( m_n \) (assuming a uniform prior distribution for all four parameters). The resulting 68% and 95% joint con-

\[\text{Note that both Ryba & Taylor (1991) and Kaspi et al. (1994b) incorrectly (over)estimate the error on } m_1 \text{ by calculating the joint 68% confidence bound on } r = T_c/m_1 \text{ and } s = \sin \, i \text{ and then interpreting this as a 68% confidence bound on each of } m_2 \text{ and } \sin \, i.\]
Fig. 1.—Cumulative probabilities \( \{ dm, p m_1, f, P_b \} \) for the 13 pulsars in Table 2, as described in the text. A vertical line is shown at 1.35 \( M_\odot \).

Confidence limits on \( \bar{m} \) and \( \sigma \) are shown in Figure 2, and on \( m_1 \) and \( m_2 \) in Figure 3. In each model, the distribution of neutron star masses is remarkably narrow: the maximum likelihood solutions are \( \bar{m} = 1.35 \ M_\odot \) and \( \sigma = 0.04 \ M_\odot \), and \( m_1 = 1.26 \ M_\odot \) and \( m_2 = 1.45 \ M_\odot \).

Of course, any model (even a poor one) will yield maximum likelihood parameters for a given data set. However, it is obvious by inspection that both the Gaussian and uniform distributions for the neutron star mass are good fits to the extremely narrow observed range of neutron star masses in the double neutron star binaries. While it is difficult to quantify the goodness-of-fit for the entire data set, because of the diverse assumptions made in the various mass estimates and the sometimes highly non-Gaussian error estimates, we can easily test some neutron star subsamples against the maximum likelihood Gaussian model \( m_1 = 1.35 \pm 0.04 \ M_\odot \). For the 13 neutron star–white dwarf binaries, we used a Monte Carlo technique to evaluate the fit quality. For each binary (with its measured \( P_b \) and \( f \)), we simulated a large number of Monte Carlo trials where the neutron star mass was drawn from the maximum likelihood model, was drawn from the appropriate uniform distribution implied by \( \bar{m} \) and \( \sigma \), and \( \cos i \) was drawn from a uniform distribution. The Monte Carlo trials were then used to construct the probability distribution for the mass function, and this distribution was used to compute the cumulative probability for the measured mass function, \( p(f' < f) \). If the model and the associated assumptions are correct, then the cumulative probabilities for the listed parameters are expected to be close to those listed in Table 2.

**Table 2**

**Mass Estimates from the \( P_b-m_2 \) Relation**

| PULSAR       | \( P_b \) (days) | \( f \) \( (10^{-3} \ M_\odot) \) | \( m_1^a \) \( (M_\odot) \) | \( m_2^a \) \( (M_\odot) \) | Predicted \(^c\) |
|--------------|------------------|-------------------|------------------|------------------|-------------------|
| J0437−4715   | 5.741            | 1.243             | 0.164            | 0.106            | 0.472             | 1.044             | 1.573             | 1.971             | 0.87              | 3.5               |
| J1045−4509   | 4.083            | 1.765             | 0.132            | 0.047            | 0.234             | 0.557             | 0.965             | 1.273             | 0.97              | 6.2               |
| J1455−3330   | 76.174           | 6.272             | 0.305            | 0.098            | 0.491             | 1.144             | 1.681             | 2.058             | 0.85              | 5.9               |
| J1640+2224   | 175.460          | 5.907             | 0.351            | 0.135            | 0.651             | 1.495             | 2.190             | 2.681             | 0.73              | 4.6               |
| J1643−1224   | 147.017          | 0.783             | 0.341            | 0.553            | 2.106             | 4.439             | 6.335             | 7.674             | 0.38              | 1.6               |
| J1713+0747   | 67.825           | 7.896             | 0.299            | 0.078            | 0.405             | 0.960             | 1.419             | 1.741             | 0.91              | 7.6               |
| J1804−2718   | 11.128           | 4.137             | 0.212            | 0.073            | 0.369             | 0.856             | 1.265             | 1.558             | 0.94              | 6.7               |
| B1855+09     | 12.327           | 5.557             | 0.219            | 0.060            | 0.315             | 0.745             | 1.101             | 1.351             | 0.97              | 9.0               |
| B1953+29     | 117.349          | 2.417             | 0.328            | 0.236            | 1.024             | 2.251             | 3.251             | 3.955             | 0.58              | 2.8               |
| J2019+2425   | 76.511           | 10.686            | 0.305            | 0.062            | 0.338             | 0.818             | 1.218             | 1.500             | 0.95              | 10.0              |
| J2033+17     | 56.2             | 2.75              | 0.290            | 0.172            | 0.722             | 1.720             | 2.489             | 3.029             | 0.67              | 3.2               |
| J2129−5721   | 6.625            | 1.049             | 0.176            | 0.138            | 0.591             | 1.293             | 1.900             | 2.348             | 0.80              | 2.7               |
| J2229+2643   | 93.015           | 0.839             | 0.315            | 0.463            | 1.789             | 3.787             | 5.409             | 6.552             | 0.42              | 1.7               |

\(^a\) Central value from \( P_b-m_2 \) relation, eq. [9] and eqs. [10] and [11].

\(^b\) Median and central 68% and 95% confidence bounds.

\(^c\) Mean value of \( \sin i \) and Shapiro delay amplitude \( \Delta \tau = 2m_1 T_c \log (1 - \sin i) \), assuming a Gaussian underlying neutron star mass distribution \( m_1 = 1.35 \pm 0.04 \) and uniform \( m_2 \) in the range allowed by the \( P_b-m_2 \) relation (§5).
FIG. 3.—Maximum likelihood estimate of the minimum and maximum neutron star mass, $m_l$ and $m_u$, assuming masses are uniformly distributed between the upper and lower bounds. The maximum likelihood solution is marked with a cross, and contours indicate 68% and 95% confidence regions.

13 measured mass functions should be consistent with a uniform distribution between zero and unity (cf. a classical $V/V_{\text{max}}$ test; Schmidt 1968). A KS test of the distribution shows consistency with a uniform distribution at the improbably good 99% level (Fig. 4).

For the 10 stars for which Gaussian error estimates $\sigma_e$ are available (both stars in the relativistic binaries B1534+12, B1913+16, and B2127+11C, as well as J1012+5307, J1713+0747, B1855+09, and J0045−7319), we can calculate a $\chi^2$ statistic, $\sum (m - \bar{m})^2/\sigma^2 = 7.5$, consistent with expectations for a $\chi^2$ distribution with 10 − 2 degrees of freedom.

We conclude, therefore, that at least in the radio pulsar systems, there is no evidence for neutron star masses above about 1.45 $M_\odot$. Indeed, the data appear very well modeled by very narrow distributions centered around 1.35 $M_\odot$.

5. SUMMARY

There are now 26 neutron stars in binary radio pulsar systems for which useful mass constraints can be derived. Of these, about half are neutron star–white dwarf binaries in which the mass determination depends on the validity of the $P_b$-$m_2$ relation and the isotropy of the binary orbits with respect to the line of sight, as discussed in the previous section. All other mass constraints are listed in Table 3 and shown in Figure 5.

Although we defer a full discussion of the underlying neutron star mass distribution and its implications for neutron star formation and evolution until after analysis of the X-ray binary systems (Paper II), we note here a few points of particular interest about the radio pulsar binaries. Figure 5 is striking primarily for the very small variations in the masses of well-measured stars. This has, of course, been noted before (e.g., Thorsett et al. 1993a), but it remains surprising that no new mass measurements differ greatly from 1.4 $M_\odot$. In the five double neutron star binaries for which a relativistic periastron advance yields an accurate

FIG. 4.—Cumulative distribution of $p(f' < f)$ (solid line, § 4) and of $\cos i$ (dotted line, § 5) for the 13 millisecond pulsars with $P_b > 3$ days and $P < 10$ ms, assuming a pulsar mass distribution $m_p = 1.35 \pm 0.04 M_\odot$ and companion mass distribution as predicted by the $P_b$-$m_2$ relation (see text). Each distribution is consistent with uniform (dashed line; KS probability of 99% and 81%, respectively), as expected if orbital inclinations are randomly distributed and the $P_b$-$m_2$ relation correctly predicts the companion mass distribution.

FIG. 5.—Neutron star masses from observations of radio pulsar systems. All error bars indicate central 68% confidence limits, except upper limits are one-sided 95% confidence limits. Five double neutron star systems are shown at the top of the diagram. In two cases, the average neutron star mass in a system is known with much better accuracy than the individual masses; these average masses are indicated with open circles. Eight neutron star–white dwarf binaries are shown in the center of the diagram, and one neutron star–main-sequence star binary is shown at bottom. Vertical lines are drawn at $m = 1.35 \pm 0.04 M_\odot$. 

Fig. 5. Neutron star masses from observations of radio pulsar systems.
The most surprising implication of the current results is that there is little evidence for mass transfer of 0.1 \( M_\odot \) or more in the millisecond pulsar systems. It is important, then, to reiterate the assumptions upon which this conclusion rests: (1) the \( P_b-m_2 \) is correct, at least within the (modest) claimed precision, and (2) binary orbits are randomly oriented with respect to the line of sight. There are good prospects for testing both assumptions.

The reliability of the \( P_b-m_2 \) relation depends principally upon the core-mass–radius relation for red giants (eqs. [10] and [11]). The latter relation can be tested observationally by careful study of nearby red giants. Precise measurements of bolometric flux and angular size (through optical/ infrared photometry and interferometry; see, for example, Dyck et al. 1996; Perrin et al. 1998) together with accurate distance measurements (e.g., using \textit{Hipparcos}) can be used to probe the relationship between luminosity and radius. Since the core-mass–luminosity relation for red giants should be inherently more precise than the core-mass–radius relation (Rappaport et al. 1995 and references therein), such observations would provide an effective test of the core-mass–radius (and hence \( P_b-m_2 \)) relation. Further, we hope that the power of the \( P_b-m_2 \) relation as a statistical tool will encourage more detailed theoretical investigations of, in particular, the extension of the relation to orbital periods below about 3 days, where X-ray heating and bloating of the companion star become important.

Although any technique that limits \( \sin i \) (§ 2.3; e.g., polarization or scintillation studies) can be used to test the hypothesis that orbits are randomly inclined, the most precise measurements will come from pulsar timing and Shapiro time-delay measurements. In Table 2 we list the mean predicted amplitude of the Shapiro delay signal \( \Delta T_s = 2m_2 T_\odot \log (1 - \sin i) \) for each of the 13 pulsars discussed in § 4, under the assumption that the neutron stars are all 1.4 \( M_\odot \) and the companion masses are the central value predicted by the \( P_b-m_2 \) relation. Of these systems, only B1855+09 has a well-measured signal (Raia & Taylor 1991; Kaspi, Taylor, & Ryba 1994b); it and the less studied J2019+2425 have the largest predicted signals. (It is also interesting to note that of the pulsars in Table 2, the only two that have been clearly shown to have “classical” inter-
pulse emission separated from the main radio emission peak by ~180° are B1855+09 and J1804–2718. These are two of the four pulsars that the $P_{\text{b}}-m_{\text{b}}$ relation suggests are observed on lines of sight that are most nearly equatorial, as would be expected for systems in which both magnetic poles are seen.)

An assumed pulsar mass distribution and the $P_{\text{b}}-m_{\text{b}}$ relation allow us to test our assumption that the orbital inclinations are randomly distributed: i.e., $\cos i$ is uniformly distributed. In Figure 4 we show the cumulative distribution of $\cos i$ for the 13 pulsar–white dwarf binaries discussed above, using $\sin i$ values from Table 2 except for B1855+09, for which a better estimate is available from timing. We find that the measured values are consistent with uniform at the 81% level. Because the pulsar spin and orbital angular momenta are most likely aligned, we further conclude that there is no evidence for either alignment or counteralignment of pulsar magnetic fields in millisecond pulsars, contrary to some predictions (e.g., Ruderman 1991).

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Note added in proof.—Recently, M. H. van Kerkwijk and S. R. Kulkarni (in preparation, 1999) have detected a faint blue object at the position of PSR B2303+46. Identification of the object with the binary system would imply that the companion is a massive white dwarf rather than a second neutron star as assumed in § 3.1.5 and Table 3. Assuming the companion is below the Chadrashar mass limit allows a tighter limit to be placed on the pulsar mass: $m_1 = 1.38_{-0.10}^{+0.06} M_\odot$ (68% confidence) or $m_1 = 1.38_{-0.10}^{+0.06} M_\odot$ (95% confidence).