MEASURING THE SUPERSYMMETRY LAGRANGIAN

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Abstract

The parameters of the supersymmetry Lagrangian are the place where experiment and theory will meet. We show that measuring them is harder than has been thought, particularly because of large unavoidable dependences on phases. Measurements are only guaranteed if a lepton collider with a polarized beam and sufficient energy to produce the relevant sparticles is available. Current limits on superpartner masses, WIMPs, and the supersymmetric Higgs are not general, and need re-evaluation. We also tentatively define the MRM (Minimum Reasonable Model), whose parameters may be measurable at LEP, FNAL and LHC.
1 Introduction

Supersymmetry is a complete quantum field theory. The form of the full Lagrangian is known, including the general effects of “soft” supersymmetry breaking (all terms which do not introduce power contributions from higher scales to lower scales) [1], regardless of how supersymmetry is broken. Many people have studied possible direct and indirect SUSY effects based on this Lagrangian, often by making many assumptions and studying the variation of a few parameters.

The full Lagrangian [2, 3] depends on 107 parameters beyond the 19 of the Standard Model (SM) of particle physics (including the gravitino). That may seem like a large number. But older readers may recall that at a similar stage in the development of the SM the situation was not so different; for example, if one counts by assuming that one knew there were three quark and three lepton families, allowing for Majorana neutrino masses, but that instead of knowing the interaction was V-A it could be any of S,V,T,A,P, then there are about sixty parameters in the SM. Data gathered over three decades led to the present form. Also, there are already many constraints [4] on the SUSY parameter space from the absence of flavor changing neutral processes, electric dipole moments, certain collider signals, and so on. Most processes only depend on a few parameters, so in practice using the general parameter set is not easy but is doable.

The purpose of this paper is to emphasize that once there is data on superpartners, from colliders or rare decays or WIMP signals, it is essential to measure the parameters of the Lagrangian without making any of the simplifying assumptions that have become standard. One might hope that measurements would be insensitive to ignoring some phases, or to various degeneracy assumptions. We will give examples below to show that is generally not so. Indeed, limits on on sparticle masses from colliders, or WIMPs, and the Higgs boson mass are sensitive to the assumptions and need to be re-evaluated.

It is extremely important to measure the parameters. As understanding of string theories improves models will emerge [3] that predict the 107 parameters, and the measured values will provide perhaps the best tests of theoretical predictions and ideas. Even more likely, the historical path will be followed — once the parameters are measured, their pattern will suggest how SUSY is broken and how to find the correct vacuum, just as gauge invariance was suggested by data as the SM emerged. Unfortunately, the relations connecting the data and the parameters are very non-linear, and non-physical solutions can be obtained if some parameter is assumed to have a value different from its true one. It is important to understand that in almost
all cases the physical observed particle masses are not the soft-breaking parameters that appear in the Lagrangian and have a close connection to the theory; the particle masses result from diagonalizing mass matrices. Unless a theory predicts all the relevant phases and additional soft parameters it cannot predict masses of mass eigenstates. Unless the soft parameters of the SUSY Lagrangian can be measured, the essential interplay between experiment and theory needed for progress will be threatened.

For this paper we will mainly consider the neutralino and chargino sectors. The relevant terms in the Lagrangian are

\[ \mathcal{L} = -\frac{1}{2}(M_1 \tilde{B} \tilde{B} + M_2 \tilde{W} \tilde{W} + M_3 \tilde{g} \tilde{g}) + \mu H_u H_d + c.c. \]  

(1)

Although \( \mu \) arises from the superpotential we will speak of it as soft too since its value is of the same order as the other soft terms \( M_1, M_2, M_3, \ldots \). \( M_1, M_2, M_3 \) are complex (as is \( \mu \)) but a global symmetry can be used to choose one of them real; here we will take \( M_2 \) real. From now on we write \( M_1 e^{i\varphi_1} \) and \( \mu e^{i\varphi_\mu} \) with \( M_1 \) and \( \mu \) real. The other important parameter we need is the ratio of the vacuum expectation values of the two Higgs doublets, \( \tan \beta \), which we choose to be real by setting the phase of the \( B_\mu \) soft parameter to zero using a second global symmetry. Some earlier work on relating theory and experiment once superpartner data is available, and on measuring parameters in the chargino and neutralino sector, can be found in Ref. [6].

In the next section we consider the simplest sector, the two chargino states (linear combinations of wino and charged higgsino states). Then we examine the neutralino sector alone and in combination with charginos, including the effects relevant to the LSP as cold dark matter (CDM). Then we look at implications for the Higgs sector, and finally at the extension of these considerations to the full MSSM.

## 2 The Chargino Sector

The chargino mass matrix is

\[ \mathcal{M}_C = \begin{pmatrix} M_2 & \sqrt{2}M_W \sin \beta \\ \sqrt{2}M_W \cos \beta & \mu e^{i\varphi_\mu} \end{pmatrix}, \]

(2)

so it depends on \( M_2, \tan \beta, \mu, \varphi_\mu \). The two chargino masses \( m_1 \) and \( m_2 \) are the eigenvalues of \( \mathcal{M}_C^\dagger \mathcal{M}_C \). They are a little complicated, so consider

\[ \text{Tr}(\mathcal{M}_C^\dagger \mathcal{M}_C) = m_1^2 + m_2^2 = M_2^2 + \mu^2 + 2M_W^2, \]

(3)

\(^1\) The full Lagrangian can be seen in many places, e.g. in Ref. [2, 3].
\begin{equation}
\text{Det}(M_C^\dagger, M_C) = m_1^2 m_2^2 = M_2^2 \mu^2 + 2M_W^2 \sin^2 \beta - 2M_W^2 M_2 \mu \sin 2\beta \cos \varphi_\mu.
\end{equation}

One can immediately see from eq. 2.3 that the value of \( \tan \beta \) extracted from data on chargino masses is very sensitive to \( \cos \varphi_\mu \), so if different values for \( \varphi_\mu \) are assumed, different values for \( \tan \beta \) will result.

One might argue that \( \varphi_\mu \) is constrained by the electron or neutron electric dipole moment and may be small, but that is not clear. First, in other sectors additional phases enter and our use of \( \varphi_\mu \) should be considered mainly as an example of the role of phases, demonstrating how they can be important. Second, even the situation for \( \varphi_\mu \) itself is more complicated. No symmetry is known that implies it should be small. The analysis of the electric dipole moments has not been done with a general phase structure, and constraints will surely be smaller than when only \( \varphi_\mu \) is included. For example, Nath and Ibrahim \[8\] have included two phases and concluded that the constraint from \( d_n \) on \( \varphi_\mu \) could be rather weak. It may finally turn out that there is indeed a constraint, but that has yet to be established. Recently Garisto and Wells have examined how some of the phases evolve from high to low scales \[9\].

Thus we see that from the chargino masses two observables are available for four parameters. The phase in eq. 2.1 cannot be rotated away by rotating the wino and higgsino fields. When one adds production cross sections and asymmetries the situation does not improve since there is a sneutrino exchange diagram; then additional parameters enter and no net information is obtained (we consider the polarized beam case below). Thus measurements at LEP or FNAL or LHC of the chargino sector alone cannot determine \( \tan \beta \) or \( \mu \) or \( \varphi_\mu \) or \( M_2 \). To measure the parameters it is necessary to have data with polarization of a beam or final particles; for the latter case analyzing the polarization cannot be allowed to depend on assumptions about the relative size of various decay branching ratios, and requires very large statistics \[10\]. In particular, if charginos decay via a spinless slepton, as is somewhat likely, their polarization will not be transmitted to the final lepton. Again we stress that even if it should turn out that \( \varphi_\mu \) is not large, here we are emphasizing that dependence on many phases can be significant. Further, even if \( \varphi_\mu \) is small (or \( \pi - \varphi_\mu \) is small), there are still three parameters in the chargino mass matrix so at least three observables depending only on those parameters must be measured.
3 The Neutralino Sector

Next consider the neutralino sector. Its mass matrix in the MSSM is $4 \times 4$, with additional parameters $M_1$ and the associated phase $\varphi_1$. First we can ask if adding masses and cross sections of neutralinos can allow a measurement of any parameters. In this case there are six parameters $(M_1, \varphi_1, M_2, \tan \beta, \mu, \varphi_\mu)$ for the combined chargino + neutralino sectors, so measurements of six masses could allow their determination, although several solutions are likely to exist since all of the relationships of parameters to data are non-linear and complicated. At LEP it is no longer possible to produce all six states, but at FNAL it may be possible when the upgraded collider takes data.

However, it is not clear that the needed masses can be measured at FNAL (or LHC) even if the states are produced. Again, cross sections are not helpful since diagrams with slepton or squark exchange can be large. Below we will see that at a lepton collider with a polarized beam (the Next Polarized Lepton Collider, NPLC) the situation can be much better. Also, there exists a set of assumptions that may permit a meaningful determination of parameters at FNAL (Section VII). Figure 1 shows that neutralino masses can depend sensitively on $\varphi_1$; they can vary even more for other values of parameters and they depend strongly on $\varphi_\mu$ as well.

4 The Higgs sector

The Higgs sector potential depends on the general form of the soft breaking parameters. As noted before, it is convenient to set the phase of the $B\mu$ soft parameter to zero. Then the tree level Higgs potential is not affected by any phases since $m_{H_1}^2$ and $m_{H_2}^2$ are real and $\mu$ enters only through $|\mu|^2$. Minimization conditions for the tree level potential in this convention do not differ from the phaseless case and $\tan \beta$ is real. The Higgs masses and the mixing angle $\alpha$ also do not change at this point and therefore the $hZ$ production cross section does not feel the presence of complex phases.

However, the phases do enter through the one-loop correction to the effective potential. As an example we consider only the contribution from the top-stop sector with the potential $V = V_{\text{tree}} + \Delta V_1$, where

$$\Delta V_1 = \frac{3}{32} \sum_{t_1, t_2} m_t^2 \left[ \log \frac{m_\tilde{q}^2}{Q^2} - \frac{3}{2} \right] - 2m_t^2 \left( \log \frac{m_t^2}{Q^2} - \frac{3}{2} \right). \tag{5}$$

The phases affect the eigenvalues of the stop mass matrix so the Higgs potential also changes.
This can lead to changes as much as 25 percent in the light Higgs mass. The effect is largest when $A_t$ and $\mu \cot \beta$ are comparable in size; the relative phase of $A_t$ and $\mu$ is what matters.

5 WIMPs

The lightest eigenvalue (LSP) of the neutralino mass matrix is a candidate to provide some or all of the cold dark matter (CDM) of the universe. Once superpartners are detected at colliders, and the soft parameters and $\tan \beta$ measured, it will be possible to calculate the contribution of the LSP to CDM. The first question will be whether the LSP CDM gives a major contribution to $\Omega h^2$. Even if superpartners are detected at LEP there may not be enough information to
answer this question quantitatively. From FNAL data it may be possible to give a tentative answer, and from NPLC a definitive answer can be provided if the energy of NPLC is large enough to produce all the charginos and neutralinos. The second stage is to ask whether $\Omega h^2$ can be calculated to a few percent accuracy; that appears to be possible from NPLC information \[1\].

These statements follow because the calculation of $\Omega h^2$ requires a knowledge of the elements of the neutralino mass matrix, all of which enter into calculating the neutralino annihilation cross section that determines the neutralino relic density. Thus the calculation can only be done reliably after $(M_1, \varphi_1, M_2, \tan \beta, \mu, \varphi_\mu)$ are measured. $\Omega h^2$ can vary by a factor of a few depending on the phases.

Similarly, it will be very exciting when a signal for a WISP (Weakly Interacting Supersymmetric Particle) is detected directly in the laboratory. Until a signal is observed it is useful to set limits on WISPs; the limits are sensitive to the phases. For example, the event rate on Ge can vary by nearly an order of magnitude as $\varphi_1$ varies. Present limits need to be re-evaluated.

Falk, Olive, and Srednicki showed \[7\] that whenever sfermion mixing was large (therefore, at least for annihilations involving the stop sector), and even more so if the the phase of $A_t$ were large, WISP limits had to be re-evaluated. That is an example of our general argument, though the main results we discuss here from the chargino and neutralino sector occur in addition to this effect and are not related to it; our effects are large even for WISPs for which sfermion mixing has a negligible effect on annihilation or scattering.

6 The Next Polarized Lepton Collider (NPLC)

At NPLC the key technique is to produce charginos with a polarized beam. Then the sneutrino exchange diagram is absent for a right-handed polarized electron beam, and one can add cross sections and forward-backward asymmetries as observables for the two chargino states $\tilde{C}_{1,2}$. There are then twelve possible chargino and neutralino observables, the two chargino masses, the four neutralino masses, the cross sections for production of $\tilde{C}_1 + \tilde{C}_1$, $\tilde{C}_1 + \tilde{C}_2$, $\tilde{C}_2 + \tilde{C}_2$ ($\sigma_{11}$, $\sigma_{12}$, $\sigma_{22}$), and the associated asymmetries $A_{11}^{FB}$, $A_{12}^{FB}$, $A_{22}^{FB}$. Measurement of seven or eight should suffice to determine the six parameters $M_1$, $\varphi_1$, $M_2$, $\mu$, $\varphi_\mu$, $\tan \beta$. Thus $\tan \beta$ and other parameters can be measured at NPLC. If all twelve can be measured the resulting accuracy may be quite good. (We are presently studying models to understand how well the parameters can be measured.) Once this is done, the LSP relic abundance can be calculated to interesting
accuracy \( [1] \). The values of these parameters will be important inputs to measuring most of the soft parameters from FNAL and LHC data. The polarization effects will also help separate slepton soft parameters, including phases. These results imply that a lepton collider with a polarized beam and sufficient energy is a necessary condition for a complete measurement of the parameters of the supersymmetry Lagrangian, a stronger result than has been previously available.

7 Sleptons, Squarks and the MRM

Since it will be a long time before there is sufficient data to measure all of the MSSM parameters, it is desirable to make some assumptions that reduce the number. But it is important to do so in a way that retains sufficient generality to have a good chance of correctly describing nature. From the gaugino sector we have a minimal set \( M_1, \varphi_1, M_2, M_3, \varphi_3, \mu, \varphi_\mu, \tan \beta \).

If flavor mixing is small, a reasonable assumption, then we can take the slepton and squark mass matrices as diagonal. From hermiticity it follows that the diagonal elements are real. Similarly we can assume the soft Yukawa coefficients \( A_k \) are flavor-diagonal; the \( A \)'s will still be complex in general. At this stage the slepton parameters are \( m_{\tilde{e}_L}^2, m_{\tilde{\mu}_L}^2, m_{\tilde{\tau}_L}^2, m_{\tilde{e}_R}^2, m_{\tilde{\mu}_R}^2, m_{\tilde{\tau}_R}^2, A_e, A_\mu, A_\tau, \varphi A_e, \varphi A_\mu, \varphi A_\tau \). We can go to a minimal reasonable set by taking \( m_{\tilde{e}_L}^2 = m_{\tilde{\mu}_L}^2 = m_{\tilde{\tau}_L}^2 \), and \( m_{\tilde{e}_R}^2 = m_{\tilde{\mu}_R}^2 = m_{\tilde{\tau}_R}^2 \), and \( A_e = A_\mu = 0 \) (they actually need not be zero but we assume their effect to be proportional to the \( e \) and \( \mu \) masses so they can be neglected). The assumption of universality for the soft slepton mass parameters is somewhat ad hoc at this point since there is no known physical reason why it should be the case in nature but for the sake of keeping the number of parameters under control we include this assumption. Thus a minimal set is \( m_{\tilde{e}_L}^2, m_{\tilde{e}_R}^2, A_\tau, \varphi A_\tau \).

For the squarks we can make similar assumptions so a minimal set is \( m_Q^2, m_u^2, m_d^2, A_b, A_t, \varphi A_b, \varphi A_t \). To these we add from the Higgs sector \( m_{h^0} \) (we have already counted \( \tan \beta \)), and the gravitino mass and phase \( m_{\tilde{G}} \) and \( \varphi_{\tilde{G}} \). Thus the MRM (minimal reasonable model) has 22 parameters. It could well happen that this is sufficient to describe the full Lagrangian. Small flavor changing effects could eventually be studied in rare decays or at colliders, but may not affect significantly the non-flavor-changing processes. This set of 22 MRM parameters is also self-contained (and separates into self-contained subsets) under renormalization group running. The MRM set is sensible to use for analyzing data in order to measure the Lagrangian soft parameters, and therefore in detector design studies as well. Once there is data it is of course
also sensible to check simpler hypotheses as well but there is a danger that unphysical results will be obtained because the equations relating the soft parameters to the masses are non-linear and errors on masses and cross sections will not be small. Our MRM set allows for effects of D-terms and extra neutralinos from additional $U(1)$ symmetries and Planck scale operators, while a smaller parameter set does not. Since any given process depends on a few of the parameters the MRM set is not as large as it seems. We are not arguing that the full soft parameter set reduces to the MRM one, but that the MRM one is a reasonable place to start analyses that might be general. It could well be that different approaches to the flavor problem (such as horizontal symmetries) would lead to useful alternative MRM2, MRM3, etc.

In the MRM, one could then analyze LEP chargino data with only one extra parameter, the sneutrino mass. However, the normal destructive interference of $\gamma + Z$ and $\tilde{\nu}$ exchanges can be modified by the phase structure at the gauge boson-chargino vertices. More generally, it may be possible to determine enough observables at LHC (possibly even at FNAL) to measure the full set of MRM parameters.

8 Concluding Remarks

Once candidates for fundamental theory, e.g. string theories, are well enough understood to make contact with experiment some of their main predictions will be the 107 parameters of the Supersymmetry Lagrangian. Similarly, any theory of supersymmetry breaking will make such predictions. Measuring the parameters of the Lagrangian will be essential to test these predictions. If history is any guide, measurement of the Lagrangian parameters may be essential to lead the theorists to the correct theory of supersymmetry breaking or the correct string vacuum.

As supersymmetric theories were studied, it was appropriate to initially make many simplifying assumptions to gain intuition about the phenomena the theory predicted. Some of the usual assumptions may be true, some not. We have shown in this paper that observables depend more strongly on assumptions, particularly about phases, than has generally been realized. We have illustrated the effects of phases partly with the phase $\varphi_\mu$ of the $\mu$ parameter; $\varphi_\mu$ may or may not be large, but it should be interpreted here as a typical one of a number of phases, most of which are only weakly constrained if at all.

In particular we have demonstrated that LEP and FNAL may have to be content with discovering the first superpartners (and a Higgs boson) and measuring some of their masses.
Rigorous measurement of the Lagrangian parameters and $\tan \beta$ is not possible at these colliders (data on the masses cannot be uniquely converted into Lagrangian parameters). Before saying “so what”, keep in mind that this would be somewhat analogous to a knowledge of the Standard Model with a few events of $Z$ production but no $W$ production and no measurement of $\sin^2 \theta_W$ in different processes, no top quark, no study of $Z$ decays or of QCD jets, and no tests of the Higgs sector, not only no Higgs boson but no $\rho$ parameter either. We have also shown that a sufficiently energetic lepton collider with a polarized beam will be able to make the essential measurements needed to relate experiment to theory, and together with hadron colliders may be able to determine all of the Lagrangian parameters. We have defined a minimal set of parameters that allows for general effects (MRM); the MRM parameters may be measurable at LHC or possibly even FNAL.

Because of the sensitivities to parameters, limits on superpartner masses may be weaker than has been reported and need to be re-evaluated. This is also true for the Higgs mass limits, where the effect can be large.

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