Assessment of Floor Micro-Vibrations Induced by Moving Vehicles in High Technology Factories Using a Fragility-Based Method

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Abstract. This study presents assessment of floor micro-vibrations induced by moving vehicles in high technology factories by using a fragility-based method. A series of time history analysis of a simplified three-span continuous beam model under moving loads simulated by a modified Kanai-Tajimi power spectral density (PSD) function considering various vehicle weights and moving speeds is performed. The best-fitting surface in terms of the maximum root mean square (RMS) velocity responses and their corresponding pair of the vehicle weight and moving speed is obtained by using the least square regression method. With the regression surface denoting the mean values and the standard deviation obtained, the fragility surfaces for each vibration criterion or limit are further constructed. The assessment of the floor micro-vibration in terms of probability of exceeding a given vibration limit for various pairs of vehicle weight and moving speed can be read accordingly from the fragility surfaces.

Keywords: Micro-vibration, High technology factory, Moving loads, Time history analysis, Fragility surfaces.

1. Introduction
Floor micro-vibrations induced by personnel walking in biotechnology labs [1] or by mechanical disturbance sources and ground-borne train excitations in high technology factories manufacturing the silicon wafers and glass panels have been extensively studied in recent years [2-4]. However, those adverse floor vibrations directly induced by interior automated material handling systems (AMHS) [5] such as automated guided vehicles (AGVs), rail guided vehicles (RGVs) and stockers on the production floor in high technology factories have rarely been considered during the design phase. Lee et al. [6-8] proposed a simplified sub-structural continuous beam model to simulate a three-span cheese floor of a single bay travelled by the AGV of a Liquid Crystal Display (LCD) factory. In these studies, the two-axle AGV was simulated as a pair of concentrated moving loads generated by a modified Kanai-Tajimi (MKT) power spectral density (PSD) function [8] considering various vehicle weights and moving speeds. Ju et al. [9] constructed a full finite element model considering the high-tech building, rails, crane and waffle slabs to assess the floor micro-vibrations induced by a moving crane.

In this study, a series of time history analysis of a target high-tech factory is performed to obtain the maximum floor micro-vibrations under various AGV weights and moving speeds. The fragility analysis that has been widely adopted in damage assessment of buildings or bridges [10-12] is then
carried out to construct the fragility surfaces corresponding to each specific vibration level or limit (VC-E to VC-A) [13] based on the relationships between the simulated maximum floor micro-vibrations and various pairs of vehicle weight and moving speed through the regression analysis. The assessment results of the floor micro-vibration in terms of probability of exceeding a given vibration limit for various vehicle weight and moving speed directly read from the developed fragility surfaces are discussed.

2. Numerical Model of the High Technology Factory

2.1. Target Building Model

A typical double-fab LCD factory with two clean rooms is illustrated in Figure 1 [6-7]. The substructure of a three-span continuous beam with the mega truss spanning 36 m equally in each bay with a width of 9.6 m subjected to AGV moving loads is considered.

The vertical displacement of a beam, \( y(x,t) \), can be represented in terms of modal contributions as

\[
y(x,t) = \sum_{i=1}^{n} q_i(t) X_i(x) \tag{1}
\]

Where \( n \) is the number of vibration modes considered when calculating the dynamic responses, \( q_i(t) \) is the \( i \)-th generalised coordinate, and \( X_i(x) \) is the \( i \)-th vibration mode. The mass and stiffness coefficients of the simplified beam model can be obtained as

\[
m_{ij} = \int_0^L \rho A(x) X_i(x) X_j(x) dx \\
k_{ij} = \int_0^L EI(x) X_i''(x) X_j''(x) dx \tag{2}
\]

Where \( \rho \) is the density, \( E \) is the Young’s modulus, \( A(x) \) is the cross-sectional area, \( I(x) \) is the moment of inertia of the cross-section. The mass per unit length is \( \rho A = 9.1505 \times 10^3 \) kg/m and the equivalent flexural rigidity is \( EI = 2.3097 \times 10^{11} \) N-m² [6-7], so that the fundamental frequency of the three-span continuous beam is around 6.1 Hz. The first five natural frequencies of the equivalent three-span continuous beam determined from eigen analysis of the mass and stiffness matrices are...
6.089, 7.804, 11.395, 24.357 and 27.759 Hz. Moreover, a 5% damping ratio is assumed for each vibration mode.

The equation of motion of the beam system under \( N \) moving loads can be further elaborated in a matrix formation as

\[
\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{C}\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{E}\mathbf{w}(t)
\]  

(3)

Where \( \mathbf{M} = [m_{ij}] \) is the \( n \times n \) mass matrix with the coefficients shown in Eq. (2), \( \mathbf{K} = [k_{ij}] \) is the \( n \times n \) stiffness matrix with the coefficients shown in Eq. (2), \( \mathbf{C} \) is the \( n \times n \) damping matrix determined by the damping ratios assigned to each vibration mode considered in the analysis and by the eigenvectors obtained from the \( \mathbf{M} \) and \( \mathbf{K} \) matrices, \( \mathbf{q}(t) = \begin{bmatrix} q_1(t) \\ q_2(t) \\ \vdots \\ q_n(t) \end{bmatrix} \) is the \( n \times 1 \) modal displacement vector, \( \mathbf{E} = \begin{bmatrix} X_1[x_1(t)] & X_1[x_2(t)] & \cdots & X_1[x_n(t)] \\ X_2[x_1(t)] & X_2[x_2(t)] & \cdots & X_2[x_n(t)] \\ \vdots & \vdots & \ddots & \vdots \\ X_n[x_1(t)] & X_n[x_2(t)] & \cdots & X_n[x_n(t)] \end{bmatrix} \) is the \( n \times N \) location matrix of the moving loads on the beam (at \( x_{pi}(t), \ i = 1, 2, \ldots, N \)), and \( \mathbf{w}(t) = \begin{bmatrix} P_1(t) \\ P_2(t) \\ \vdots \\ P_n(t) \end{bmatrix} \) is the \( N \times 1 \) moving load vector considering the constant self-weight of the vehicle and the random dynamic engine force. The analytical solution to Eq. (3) can be further obtained by a recursive difference state-space equation [7]. Once the modal displacement vector is obtained, the vertical floor vibration responses can be computed from using Eq. (1).

2.2. Simulation of Moving Loads

The maximum weight of the AGV considered is \( W = 2200 \) kgf with a distance between the two axles of \( l_w = 1.2 \) m. The AGV is simulated as a pair of concentrated moving loads of \( w_1(t) = w_2(t) = W/2 + F(t)/2 \) in which \( F(t) \) is the dynamic engine force time history. A modified Kanai-Tajimi power spectral density (PSD) function [8] is adopted to simulate the dynamic engine force in a frequency-domain as

\[
F(f) = \frac{1 + 4\xi_2^2(f/f_2)^2(f/f_1)^2S_0^2}{\left[1 - (f/f_1)^2\right]^2 + 4\xi_1^2(f/f_1)^2\left[1 - (f/f_2)^2\right]^2 + 4\xi_2^2(f/f_2)^2} 
\]

(4)

Where \( f_1, \ \xi_1, \ f_2, \ \text{and} \ \xi_2 \) are the constant parameters (where \( f_1 \) and \( f_2 \) control the upper bound and lower bound of the predominant bandwidth of the dynamic engine force, respectively), and \( S_0 \) is the force intensity. Figure 2 illustrates the PSD functions of the dynamic engine forces for various AGV weights and moving speeds. The intensity and the predominant frequency content of AGV engine forces increase with the AGV weight and moving speed. In order to assess AGV-induced floor micro-vibrations under various AGV weights and moving speeds in the target LCD factory, this study takes into account the total static weight of the AGV with \( W_5 = 1.0W = 21582 \) N (mass=2200 kg), \( W_4 = 0.8W = 17266 \) N (mass=1760 kg), \( W_3 = 0.6W = 12949 \) N (mass=1320 kg), \( W_2 = 0.4W = 8633 \) N (mass=880 kg) and \( W_1 = 0.2W = 4316 \) N (mass=440 kg), and the moving speed of the AGV with \( v = 2.0 \) m/s, 1.5 m/s (0.75\( v \)), 1.0 m/s (0.5\( v \)), and 0.5 m/s (0.25\( v \)) when performing the dynamic
analysis. Figure 3 shows the corresponding AGV engine force time histories for various AGV weights under $v=2.0$ m/s. It should be noted that the maximum engine force of each axle was scaled to be $2453$ N (equivalent to mass=250 kg).

![Figure 3. AGV engine force histories.](image)

**Figure 3.** AGV engine force histories.

**Figure 4.** Central floor vibrations of each span.

### 3. Simulation Results

**3.1. Maximum RMS Floor Responses**

Figure 4 illustrates the central acceleration responses of each span under the maximum AGV weight moving at a constant speed of 2.0 m/s. Figure 5 shows the maximum RMS floor velocities (summarized from the one-third octave-band spectral analysis [8]) at various locations (6 m uniformly-spaced, except at the supports) along the AGV moving bay of the three-span continuous beam subjected to AGV moving loads. Generally speaking, the floor micro-vibrations increase with the AGV weight and moving speed. The micro-vibrations may fall within the VC-E (for smaller
weight, $w_1$) and VC-D (for larger weight, $w_5$) levels when the AGV moves at a lower speed of 0.5 m/s, and increase to be within the VC-C (for $w_1$) and VC-B (for $w_5$) levels when the AGV moves at a higher speed of 2.0 m/s.

Figure 5. Maximum RMS floor velocities at various locations along the AGV moving bay.

3.2. Fragility Analysis

The maximum RMS floor velocities ($v_{rms}$) of the middle span under various AGV velocity ratio (VR) and AGV weight ratio (WR) are shown in Figure 6. Moreover, the relationships between $\ln(v_{rms})$ and ($\ln(VR), \ln(WR)$) pairs are represented in the form as [11]

$$\ln(v_{rms}) = m_X \ln(VR, WR) = a + b \ln(VR) + c \ln(WR)$$

(5)

Where $m_X$ is the mean value of $\ln(v_{rms})$ assumed to be normally distributed around the best-fitting regression plane (Figure 6) with the parameters, $a=5.6960$, $b=0.5372$ and $c=0.9420$, determined from the least-squares regression method. Furthermore, the standard deviation of the $n$ simulated data points for the regression plane can be obtained by using the following equation as [11]

$$\sigma_X = \sigma_{\ln(v_{rms})} = \sqrt{\frac{1}{n-3} \sum_{i=1}^{n} \left( \ln(v_{rms_i}) - \left[ a + b \ln(VR_i) + c \ln(WR_i) \right] \right)^2}$$

(6)
Figure 6. Simulated maximum RMS velocities and the corresponding best-fitting regression surface.

For the lognormally distributed random variable $v_{rms}$, the fragility function ($P_f$), giving the probability that the RMS floor velocity ($v_{rms}$) will exceed a certain vibration criterion (VC) or limit, conditional on a given pair of (VR, WR) can be obtained as

$$P_f = P(v_{rms} > VC | (VR, WR)) = 1 - \Phi\left(\frac{\ln(VC) - \mu_X (VR, WR)}{\sigma_X}\right) \tag{7}$$

Where $\Phi$ is the cumulative distribution function of a standard normal variable, with a mean of 0 and a standard variation of 1.

The fragility surfaces corresponding to (VR, WR) pairs constructed by using Eq. (7) are illustrated in Figure 7 from which the probability of exceeding a specific vibration level (VC-E, VC-D, VC-C, VC-B and VC-A, or $3.175 \times 10^{-2}$ cm/sec, $6.35 \times 10^{-2}$ cm/sec, $1.3 \times 10^{-3}$ cm/sec, $2.5 \times 10^{-3}$ cm/sec and $5.1 \times 10^{-3}$ cm/sec) can be directly read and evaluated. Taking the VC-C limit as an example, the median AGV moving speed ratio (with 50% fragility or 50% exceedance) was found to be 0.36 (or 0.72 m/sec) under the maximum AGV weight ratio of 1.0, and 0.90 (or 1.8 m/sec) under the minimum AGV weight ratio of 0.25. Moreover, the probabilities of exceeding the more stringent vibration levels (e.g. VC-E and VC-D) may reach 100%, while the values reduce to 0 for the VC-A level. The determinations of the proper AGV weight and moving speed can be evaluated from the constructed fragility surfaces based on the required vibration limits for the production areas.
4. Concluding Remarks
This study adopted a fragility-based method to assess the floor micro-vibrations induced by AGV moving in the high technology factories. By using the constructed fragility surfaces, the probability of exceeding a given required vibration limit (VC-E to VC-A) for the floor travelled by an AGV with various weights and moving speeds can be read directly. The developed fragility surfaces may be a promising tool for assessing the floor micro-vibrations in the high technology factories subjected to random moving loads and can be adopted to determine the proper AGV weight and moving speed, or to design the floor systems based on the required vibration limit during the conceptual design phase of a new factory building.

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References
[1] T.C. Pan, A. Mita, L. Li. Evaluation of Floor Vibration in a Biotechnology Laboratory Caused by Human Walking. Journal of Performance of Constructed Facilities, ASCE. 2008, 22 (3):122 -130.
[2] S. Kim, S.W. Choi. Experimental Evaluation for the Microvibration Performance of a Segmented PC Method Based High Technology Industrial Facility Using 1/2 Scale Test Models. Shock and Vibration. 2017: 1- 9.
[3] Y. L. Xu, X. J. Hong. Stochastic Modelling of Traffic-induced Building Vibration. Journal of Sound and Vibration. 2008: 313(1-2): 149 -170.
[4] S. H. Ju. Finite Element Investigation of Traffic Induced Vibrations. Journal of Sound and Vibration. 2009: 321(3-5): 837-853.
[5] Y. J. Jang, G.H. Choi. Introduction to Automated Material Handling Systems in LCD Panel Production Lines, Proceeding of the 2006 IEEE, International Conference on Automation Science and Engineering, Shanghai, 2006: 223-229.
[6] C. L. Lee, Y.P. Wang, R.K.L. Su. A study on AGV-induced Floor Micro-vibration in TFT-LCD
[7] C. L. Lee, Y.P. Wang, R.K.L. Su. Assessment of Vibrations Induced in Factories by Automated Guided Vehicles. Proceedings of the Institute of Civil Engineers-Structures and Buildings, 2013, 166(SB4): 182-196.

[8] C.L. Lee, R. K. L. Su, Y.P. Wang. AGV-induced Floor Micro-Vibration Assessment in LCD Factories by Using a Regressional Modified Kanai-Tajimi Moving Force Model. Structural Engineering and Mechanics. 2013, 45(4): 543-568.

[9] S.H. Ju, H.H. Yu, S.W. Yu. Comparison of Crane Induced Vibration on Steel Structural Levels in High-tech Factories Using FEA and Experiments. In Bian X, Chen Y, Ye X (eds), Environmental Vibrations and Transportation Geodynamics. ISEV 2016, Spring, 2016: 447-453.

[10] C.L. Lee, R.K.L. Su. Fragility Analysis of Low-rise Masonry In-filled Reinforced Concrete Buildings by A Coefficient-based Spectral Acceleration Method. Earthquake engineering and Structural Dynamics, 2012, 41(4): 697-713.

[11] Y. Pan, A.K. Agrawal, M. Ghosn, S. Alampalli. Seismic Fragility of Multispan Simply Supported Steel Highway Bridges in New York State. II: Fragility Analysis, Fragility Curves, and Fragility Surfaces. Journal of Bridge Engineering, 2010, 15(5): 462-472.

[12] D.M. Seyedi, P. Gehl, J. Douglas, L. Davenne, N. Mezher, S. Ghavamian. Development of Seismic Fragility Surfaces for Reinforced Concrete Buildings by Means of Nonlinear Time-history Analysis. Earthquake Engineering and Structural Dynamics, 2010, 39(1): 91-108.

[13] C.G. Gordon. Generic Criteria for Vibration Sensitive Equipment, Proceedings of International Society for Optical Engineering (SPIE), 1991, 1619: 71-85.