Adaptive method of controlling dynamic processes in non-deterministic systems

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Abstract. This article describes a digital method for an adaptive motion control of single-degree-of-freedom object according to a given movement function on condition that external low-frequency mechanical load can be changed arbitrarily. It is proposed to simulate an external load acting on a controlled object to predict its dynamic behavior. The external load model is determined by a set of coefficients that represent the partial derivatives of the load as a function of time and spatial coordinate. These coefficients are calculated using feedback data obtained at previous moments of the movement of the controlled object. The maximum order of derivatives in a model is called the model order. The structures of zero-, first-, and second-order models are given, including the practically important case of digital measurements of the angle of rotation of the steered shaft. The article also discusses the main practical problems associated with the calculation of model coefficients and their subsequent use to control the facility.

1. Introduction

The use of adaptive control systems is due to the complexity of the tasks being solved, the impossibility of a detailed study of the processes or the need for a dynamic response to changing situations when managing complex objects that are characterized by significant disturbances.

A significant number of works are devoted to the construction of analog adaptive systems, in which, in the general case, an additional adaptation unit is used - systems with adaptation by the input signal [1-4], systems without identification of the object model [3, 5], systems with the identifier of the object model [1, 2, 6], a system with a block for predicting the output of an object [2, 7], a system with tuning to the maximum of the objective function [6], etc.

In modern conditions, the widespread use of computing and measuring digital devices creates the possibility of using adaptive digital-type control systems. This allows to remove restrictions on the complexity of the algorithms used, to implement artificial intelligence methods.

Let us consider the theoretical justification of a digital adaptive control system for an object with one degree of freedom using the example of a shaft rotational motion for which a trajectory \(\phi(t)\) with uniform accuracy \(\Delta\).

If a mechanical system performs technological and/or transport operations, changes of the external loading occur rather slowly. The low-frequency nature of the external load allows to use it for the adaptive object control [8].
Consider the application of the external load prediction to the control the rotation of an electromechanical drive shaft. The trajectory of the shaft rotation $\varphi(t)$ at a given time interval $[t_i; t_{i+1}]$ must be implemented with uniform precision $\Delta$.

2. The structure of the general model of the external load on the shaft

Consider the shaft as a single-degree-of-freedom object. Angular rotation sensor is fixed on the shaft and registers the discrete rotate angles $\varphi_i$. Time moments $t_i$ of these events are obtained by means of the control system. Pair of values $(t_i, \varphi_i)$ that specifies the point of real shaft-rotation trajectory is denoted by $\mathbf{P}_i$.

In addition, the control system determines the set of values of the engine work $A_e$, on separate time segments $[t_i; t_{i+1}]$.

Consider the simplest way to control rotation of the drive shaft, in which it is required to ensure uniform approximation of a given trajectory $\varphi(t)$ in the time interval $[t_i; t_{i+1}]$ with uniform accuracy $\Delta$.

The angle of shaft rotation is usually measured using incremental rotary encoders. This sensor produces one discrete signal after each shaft rotation with a fixed angular pitch $h$.

Let’s introduce a new integer variable

$$\psi = (\varphi - \varphi_0)/h$$

which converts the set of real incremental angles $\{\varphi_0; \varphi_1; \ldots; \varphi_n\}$ to the set of integer ones $\{\psi_0 = 0; \psi_1 = \pm 1; \ldots; \psi_n = \pm n\}$, where the upper sign corresponds to increasing values of the angle and the lower one to decreasing.

The inverse connection $\varphi(\psi)$ and dependencies between derivates of these angles in $t$ are following:

$$\varphi(\psi) = \varphi_0 \pm h\psi; \quad d\varphi/dt = (\pm 1/h)(d\psi/dt); \quad d\varphi/dt = \pm h\cdot d\psi/dt; \quad d^2\varphi/dt^2 = (1/h^2)(d^2\psi/dt^2); \quad d^2\varphi/dt^2 = h^2\cdot d^2\psi/dt^2.\quad (2)$$

We introduce the relative time $\tau = t - t_0$:

$$\tau_0 = t_0 - t_0 = 0; \quad \tau_1 = t_1 - t_0; \quad \ldots; \quad \tau_n = t_n - t_0.\quad (3)$$

Nodal points of real shaft-rotation trajectory have coordinates $\mathbf{R}_i = (\tau_i, \pm i), (i = 0, \ldots, n)$ in a system $(\tau, \psi)$. They form uniform grid with the step 1 on the angle $\psi$.

Consider a formal definition of the whole torque of the external resistance forces $M$ on a controlled shaft. Its occurrence is caused by purely external loads (vary in time $t$) and the loads created by the rotation of the shaft itself (depending on $\varphi(t)$). Therefore, the whole torque of the externa resistance forces $M$ is the composite function of time: $M(t, \varphi(t))$.

Consider the construction of the model of the external load. The structure of the model of order $k$ is proposed as a vector of average values of all partial derivatives of function $M(t, \varphi(t))$ in parameters $t$, $\varphi$ of orders 0 to $k$.

This vector for $k$-th order model of the external load is denoted as $\mathbf{M}^k$ and called a vector of the force characteristics of the model of the order $k$. Also for brevity $\mathbf{M}^k$ is called a load vector of the model.

The average values of the derivatives characterize the dynamic changes of the external load on the shaft. The maximum derivatives order, included in the $\mathbf{M}^k$, is equal to the model order.

In the general case, the load vector $\mathbf{M}^k$ contain:

0) constant component (derivative of $M$ of order 0),

1) all partial derivatives of $M$ of order 1 ($\frac{\partial M}{\partial t}, \frac{\partial M}{\partial \varphi}$),

... 

$k$) all partial derivatives of $M$ of order $k$. 


At each time moment \( t \), the value of torque of the external resistance forces \( M \) is the result of the interaction of the vector of power characteristics \( \vec{M}^k \), that defines the action of the external load, and the variable kinematic parameters of the shaft rotation. It is proposed to establish this interaction in the form of a scalar product \( M(t, \phi(t)) = (\vec{M}^k, \vec{\phi}^k(t)) \), where the vector \( \vec{\phi}^k(t) \) determines the shape of kinematic characteristics of a model of the order \( k \).

Kinematic vector \( \vec{\phi}^k(t) \) in formula \( (\vec{M}^k, \vec{\phi}^k(t)) \) contains components that depend on time \( t \) and the function of the shaft rotation \( \phi(t) \). On a trajectory already traversed, the function \( \phi(t) \) and its derivatives may be interpolated through a set of nodal points \( P_i \), obtained through feedback. At subsequent time moments the function \( \phi(t) \) is determined by the desired shaft-rotation trajectory.

The load vector \( \vec{M}^k \) can be determined from the known values of the kinematic vector \( \vec{\phi}^k(t) \) at the previous stages of \([t_i, t_{i+1}]\) rotational motion. To do this, it is proposed to use the equality of the work \( A_i \) of the resistance forces (depend on \( \vec{M}^k \)) and work of the driving forces \( A_d \) (which can be calculated by current and voltage).

3. Case \( k=0 \)
In zero order models instantaneous values of the function \( M(t, \phi(t)) \) are replaced by their averaged values. Corresponding load vector contains only a constant component of the function \( M(t, \phi(t)) \): \( \{M\}^0 = \{M_0\} \).

This part of the external load is caused by the action of the permanent components of the resistance forces (useful load, dry friction in kinematic pairs) and the average values of all varying force factors in the \( M(t, \phi(t)) \).

Consider the shaft angular displacement \([\phi_i; \phi_{i+1}]\) in the time segment \([t_i; t_{i+1}]\). Let us replace the instantaneous value of the external load \( M(t, \phi(t)) \) by its averaged value \( M = \text{const} \). Under the assumption of equality between the works \( A_d \) (engine) and \( A_i \) (resistance forces), we get:

\[
A_{ei} = \int_{\phi_i}^{\phi_{i+1}} M(d\phi) = \int_{\phi_i}^{\phi_{i+1}} d\phi = M(\phi_{i+1} - \phi_i) = M\Delta\phi_i.
\]

From this it follows:

\[
M = A_{ei}/\Delta\phi_i. \tag{4}
\]

So, calculation of the zero-order model coefficient by the simplest method (5) gives value \( M \), which is equal to integral mean value of a function \( M(t, \phi(t)) \) on the segment \([\phi_i; \phi_{i+1}]\).

Since in zero-order model \( M(t, \phi(t)) = M = M \cdot 1 \), its kinematic vector \( \vec{\phi}^0(t) \), satisfied the condition \( (\vec{M}^k, \vec{\phi}^k(t)) \), has one coefficient equal to 1:

\[
\vec{\phi}^0(t) = \{1\}.
\]

When incremental encoders of shaft rotation angle are used, we have: \( \Delta\phi = h\Delta\psi, \Delta\psi = \pm 1 \). Therefore, in this case for the segment \([\tau_i, \tau_{i+1}]\) the coefficient of zero-order model is equal to reduced work taking into account the direction of shaft rotation:

\[
M_{f} = \pm \hat{A}_{si},
\]

Let us assume that coefficient \( M \) is defined from the already passed part of the trajectory, taking into account the corresponding engine work. Then the reduced work required for the angular shaft displacement \([\psi_i, \psi_{i+1}]\) at a subsequent time segment \([\tau_i, \tau_{i+1}]\) is equal to:
4. Case \( k=1 \)

The load vector \( \vec{M}^1 \) of the first order model of external load together with the constant load factor contains all partial derivatives of function \( M(t, \phi(t)) \) of the first order.

Let us consider external load function \( M(t, \phi(t)) \) having continuous derivatives up to the second order. Also \( M(t, \phi(t)) \) has essential constant component \( M \) and first derivative \( dM/dt \), but small second and others higher derivatives.

We denote the initial time by \( t_0 \). Considered above load function \( M(t, \phi(t)) \) will approximate at \( t = t_0 + \Delta t \) fairly accurately by Taylor series:

\[
M(t, \phi(t)) = M(t_0, \phi(t_0)) + \frac{dM}{dt}(t_0, \phi(t_0))\Delta t + R_2(\Delta t).
\]

Remainder \( R_2(\Delta t) \to 0 \) as \( \Delta t^2 \to 0 \).

For separate accounting of the influence of parameters \( t \) and \( \phi \) on the value of \( M(t, \phi(t)) \) it is necessary to express the total derivative \( dM/dt \) of (6) through the partial derivatives in \( t \) and \( \phi \):

\[
\frac{dM}{dt} = \frac{\partial M}{\partial t} + \frac{d\phi}{dt} \cdot \frac{dM}{d\phi}.
\]

Let us consider small increment \( \Delta t \). After substitution of the expression (6) into (5) and the replacement of precise value of the function and its partial derivatives into their average values on the segment \([t_0, t_0 + \Delta t]\), we obtain the following approximate equation

\[
M(t_0 + \Delta t, \phi(t_0 + \Delta t)) \approx M + \left( \frac{\partial M}{\partial t} + \frac{d\phi}{d\phi} \frac{dM}{dt} \right) \Delta t.
\]

In (7) the approximate value of \( M((t_0 + \Delta t, \phi((t_0 + \Delta t)) \) is expressed through three coefficients \( M, \frac{\partial M}{\partial t}, \frac{d\phi}{d\phi} \cdot \frac{dM}{dt} \), characterizing the fixed and variable components of the external load. Also (7) includes the increment time \( \Delta t \), which is calculated from initial instant of time \( t_0 \). In order to simplify the formulas we introduce the relative time: \( \tau = \Delta t = t - t_0 \).

Considering the formula (7) as the scalar product (4) we obtain that load and kinematic vectors in the first-order load model are defined as follows:

\[
\vec{M}^1 = \left[ M, \frac{\partial M}{\partial t}, \frac{\partial M}{\partial \phi} \right]; \ \vec{\varphi}^1(t) = \{1; \tau; \phi(\tau) \cdot \tau\}.
\]

If we use an incremental measurement of the shaft rotation angle, then vectors of the first order load models in the coordinate system \((\tau, \psi)\) have a similar form:

\[
\vec{M}^1 = \left[ M, \frac{\partial M}{\partial t}, \frac{\partial M}{\partial \psi} \right]; \ \vec{\psi}^1(t) = \{1; \tau; \psi(\tau) \cdot \tau\}.
\]
5. Case k=2

The second derivative \( \frac{\partial^2 M}{\partial \varphi^2} \) determines the change of shaft torque, depending on its angular acceleration \( \frac{\partial^2 \varphi}{\partial t^2} \). The main contribution to the coefficient \( \frac{\partial^2 M}{\partial \varphi^2} \) is made by inertial loads. They may be comparable and even exceed the others load components.

Similar to (5), three-term expansion of function \( M(t, \varphi(t)) \) at \( t = t_0 + \Delta t \) in a Taylor series relatively \( t_0 \) has the form:

\[
M(t, \varphi(t)) = M(t_0, \varphi(t_0)) + \frac{dM}{dt}(t_0, \varphi(t_0))\Delta t + \frac{1}{2} \frac{d^2 M}{dt^2}(t_0, \varphi(t_0))\Delta t^2 + R_3(\Delta t)
\]

Using the rule of differentiating composite functions and taking into account the \( \frac{\partial}{\partial \varphi}\left(\frac{dp}{dt}\right) = \left(\frac{d^2 p}{dt^2}\right)^2 \left(\frac{dp}{dt}\right) \) we express the total second derivative terms of the partial derivatives in \( t \) and \( \varphi \):

\[
\frac{d^2 M}{dt^2} = \frac{d}{dt} \left( \frac{\partial M}{\partial t} + \frac{\partial M}{\partial \varphi} \frac{dp}{dt} \right) = \frac{\partial}{\partial t} \left( \frac{\partial M}{\partial t} + \frac{\partial M}{\partial \varphi} \frac{dp}{dt} \right) + \frac{\partial}{\partial \varphi} \left( \frac{\partial M}{\partial t} + \frac{\partial M}{\partial \varphi} \frac{dp}{dt} \right) \frac{dp}{dt} = \\
\frac{\partial^2 M}{\partial t^2} + 2 \frac{\partial^2 M}{\partial t \partial \varphi} \frac{dp}{dt} + \frac{\partial^2 M}{\partial \varphi^2} \left( \frac{dp}{dt} \right)^2 + 2 \frac{\partial M}{\partial \varphi} \frac{d^2 p}{dt^2}.
\]

Denoted shaft angular velocity \( \frac{dp}{dt} \) and acceleration \( \frac{d^2 p}{dt^2} \) by \( \varphi', \varphi'' \). After substitution of expressions (6) and (9) in (8) we obtain follow approximate three-term expansion of function \( M(t, \varphi(t)) \):

\[
M(t_0 + \Delta t, \varphi(t_0 + \Delta t)) = M(t, \varphi(t)) + \left( \frac{\partial M}{\partial t} + \frac{\partial M}{\partial \varphi} \varphi \right) \Delta t + \\
+ \frac{1}{2} \left[ \frac{\partial^2 M}{\partial t^2} + 2 \frac{\partial^2 M}{\partial t \partial \varphi} \varphi' + \frac{\partial^2 M}{\partial \varphi^2} \left( \varphi' \right)^2 + 2 \frac{\partial M}{\partial \varphi} \varphi'' \right] \Delta t^2.
\]

After introducing the relative time \( \tau = \Delta t = t - t_0 \), from the second order expansion (10) of the external load we obtain follow load and kinematic vectors:

\[
\vec{M} = \left\{ M, \frac{\partial M}{\partial t}, \frac{\partial M}{\partial \varphi}, \frac{\partial^2 M}{\partial t^2}, \frac{\partial^2 M}{\partial t \partial \varphi}, \frac{\partial^2 M}{\partial \varphi^2} \right\}; \quad \vec{\varphi}(t) = \left\{ 1, \tau, \varphi', \varphi'', \frac{1}{2} \varphi'^2, \frac{1}{2} \varphi''^2 \right\}.
\]

Load and the kinematic vectors have a similar form in the coordinate system \( \left( \tau, \varphi \right) \).

After calculating the coefficients of the load vector \( \vec{M} \), magnitude of reduced work which is necessary to the shaft rotation angle \( [\psi_i, \psi_{i+1}] \) to the subsequent time segment \( [\tau_i, \tau_{i+1}] \) according to the function \( \psi(\tau) \) may be represented in such form:

\[
\vec{A}(t) = \int_{\tau_i}^{\tau_{i+1}} \left( \vec{M}, \vec{\psi}(\tau) \right) \psi'(\tau) d\tau = M(\psi_{i+1} - \psi_i) + \frac{\partial M}{\partial t} \left[ \psi'(\tau) \right]_{\tau_i}^{\tau_{i+1}} d\tau + \frac{\partial M}{\partial \psi} \left[ \left( \psi(\tau) \right)^2 \right]_{\tau_i}^{\tau_{i+1}} d\tau + \int_{\tau_i}^{\tau_{i+1}} \psi'(\tau) \psi''(\tau) d\tau.
\]
\[
+ \frac{1}{2} \frac{\partial^2 M}{\partial \tau^2} \int_{\gamma} \psi'(\tau) \tau^2 \, d\tau + \frac{1}{2} \frac{\partial^2 M}{\partial \phi^2} \int_{\gamma} (\psi'(\tau))^2 \tau^2 \, d\tau + \frac{1}{2} \frac{1}{\partial \phi^2} \int_{\gamma} (\psi'(\tau))^2 \tau^2 \, d\tau.
\]

6. Conclusion
Application of external load prediction enables to control effectively the processes and objects in which low-frequency external load from the controlling object can change freely. Naturally, the possibilities of this method are limited by the power of the drive.
Adapting of drive control system parameters to external load allows to simplify radically general structure of control systems for complex devices with multiple degree of freedom, such as robots, transport systems and others.

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