Plasmon-polaritons and diffraction on the layer of asymmetric hyperbolic metamaterial

Michael V. Davidovich

*a National Research Saratov State N.G. Chernyshevsky University, 83 Astrakanskaya, Saratov, Russia 410102
*davidovichmv @info.sgu.ru; phone 7 987 313-2161; fax 7 845 227-8529

ABSTRACT

We consider the plasmon-polaritons along a layer of hyperbolic metamaterial propagating in the plane of the anisotropy axis with an arbitrary its orientation. As a layer material, we use periodic plane-layered artificial medium – hyperbolic metamaterial of thin metal and dielectric layers and produce its homogenization. The conditions for the existence of fast, slow, leakage, gliding flowing, forward and backward plasmon-polaritons are found. The Fresnel formulas for the diffraction of a plane wave of arbitrary polarization on such a structure are obtained. The dispersion of plasmon-polaritons and plane wave diffraction are calculated. It is proposed to use a strong magnetic field to control dispersion and scattering.

Keywords: hyperbolic metamaterial, homogenization, plasmon-polaritons, dispersion equation, Fresnel equation, Fresnel formula

1. INTRODUCTION

In recent years there is increased interest in metamaterials or artificial media (AM) with hyperbolic dispersion law [1–13], called hyperbolic metamaterials (HMM), including the HMM in the frequency area where one of the components of the tensor of effective dielectric permittivity (DP) is closed to zero, or the so-called ENZ (epsilon-near-zero) AM [14].

The HMM is usually a single-axis electromagnetic AM or photonic crystal (PC), where homogenization gives a different sign of the real part of the two main diagonal components of the tensor of effective DP. It is usually made of thin conductive metal, semiconductor or graphene layers periodically embedded in the dielectric background Fig. 1. In such a uniaxial electromagnetic crystal, the axis is directed perpendicular to the layers, and two transverse to the axis components of the DP tensor may have the property \( \varepsilon^\perp < 0, \varepsilon^\perp = \varepsilon^\perp - i\varepsilon^\perp \), whereas for the longitudinal component \( \varepsilon^\parallel > 0, \varepsilon^\parallel = \varepsilon^\parallel - i\varepsilon^\parallel \) (HMM of the second type). The HMM of the first type is usually made of conductive nanocylinders (nanowires) periodically embedded in the dielectric background [1–6]. For them, a condition \( \varepsilon^\parallel = \varepsilon^\parallel < 0 \) is possible where the axis of the HMM is directed along the axis of the cylinders. We will consider the HMM of the second type as a plane-layered periodic AM or PC, but show that under some conditions it behaves as a HMM of the first type. Let the medium consist of thin metal layers of nanoscale thickness \( t_n \), periodically embedded with a period \( t_p = t_n + t_d \) in a non-dissipative dielectric with DP \( \varepsilon_d > 1 \). Here \( t_l \) is the distance between the layers of metal or the thickness of the dielectric layer Fig. 1. The DP of metal we take in the Drude-Lorentz form \( \varepsilon_m = \varepsilon_L - \omega_p^2/\left(\omega^2 - i\omega\omega_0\right) \), or \( \varepsilon_m = \varepsilon' - i\varepsilon'' \).

In reality, the DP of thin layers depends on their thickness and is determined by quantum effects. We will use approximate parameters for a massive silver sample \( \omega_p = 1.6 \times 10^{16} \) Hz, \( \omega_c = 4.5 \times 10^{13} \) Hz, \( \varepsilon_L = 9 \). Let the \( z \) axis is directed
perpendicular to the layers. The value \( \varepsilon' = \varepsilon_x - \omega_p^2 (\omega^2 + \omega_0^2) \) will be negative for frequencies \( \omega < \sqrt{\omega_p^2 / \varepsilon_L - \omega_0^2} \) or for wavelengths \( \lambda \) of about 350 nm, and the value \( \varepsilon'' = \omega_p^2 \omega (\omega^2 + \omega_0^2) \) is small. Let consider the conditions of weak dissipation. Obviously, it is \( \omega < \omega < \sqrt{\omega_p^2 / (\varepsilon_L + 1)} - \omega_0^2 = \omega_0 / \sqrt{\varepsilon_L + 1} \). This is a condition for the existence of slow plasmon-polaritones (PP) at the metal-vacuum boundary. In this case \( \varepsilon' > -1, \varepsilon'' |k| < 1 \). The waves of arbitrary directions are investigated in the HMM when the propagation direction does not coincide with the axis [13]. In the case of waveguide formation as a layer, such a HMM is called asymmetric [8]. Recently, waves in waveguides with a dielectric core and a shell of HMM have been studied [15]. In these works the dissipation was not taken into account. In this paper, we investigate a waveguide as a layer of asymmetric HMM with an arbitrary orientation of the anisotropy axis Fig.1 and when considering dissipation.

2. METHOD OF RESEARCH

The considered AM in the approximation of the absence of spatial dispersion (SD) the homogenization is given by simple formulas [10,13]

\[
\varepsilon_\perp = \varepsilon_{xx} = \varepsilon_{yy} = \left( t_m \varepsilon_m + t_p \varepsilon_p \right) / t_p, \quad \varepsilon_\parallel = \varepsilon_{zz} = \left( t_m / t_p \right) \varepsilon_m + \left( t_d / t_p \right) \varepsilon_d \right)^{-1}.
\]

The Formulas taking into account the SD can be found in [10] and in a number of other works. Then we think \( \varepsilon_p \) of the order of 2–5. Let find the condition, when \( \varepsilon'_d < 0 \). Denoting the filling factor of the metal \( \varepsilon_d = \varepsilon_d / \varepsilon_p \), we obtain \( \omega < \sqrt{\omega_p^2 / (\varepsilon_L + (1 / K - 1) \varepsilon_d)} - \omega_0^2 \). If we neglect the dissipation and take \( K = 0.5 \), then will be \( \omega < \omega_0 / \sqrt{2} \), i.e. the wave lengths are more than 400 nm. To perform homogenization in the optical range, it is sufficient to use structures with period \( t_p < 40 \) mm, i.e. with layer thicknesses of the order of 20 mm or less. For simplicity, we put \( K = 0.5 \) and obtain the parameters of effective DP for weak dissipation. Consider a few cases. Suppose first \( \varepsilon' = - \varepsilon_d \). In this case \( \varepsilon_m = - i \varepsilon'' / \varepsilon_p \), \( \varepsilon_{zz} = 2 \varepsilon_d (1 - i \varepsilon_d / \varepsilon') \), i.e. the transverse component is small and imaginary, and the longitudinal component is highly dissipative. Let now \( \varepsilon' < - \varepsilon_d \). Then \( \varepsilon_m = \left( \varepsilon' + \varepsilon_d - i \varepsilon'' \right) / 2 \), and

\[
\varepsilon'_m = \frac{2 \varepsilon_d}{\varepsilon_d - \varepsilon'} - \frac{2 \varepsilon_d}{\varepsilon_d - \varepsilon'} \varepsilon'' / \varepsilon'' - \varepsilon_d \varepsilon'' / \varepsilon'' - \varepsilon_d \varepsilon'' / \varepsilon'' - \varepsilon_d \varepsilon'' / \varepsilon'' - \varepsilon_d .
\]

If \( - \varepsilon_d < \varepsilon' < 0 \) and \( \varepsilon_d - \varepsilon' \gg \varepsilon'' \), \( \varepsilon_m = (\varepsilon_d - \varepsilon' / 2) > 0 \), that we have

\[
\varepsilon'_m = \frac{2 \varepsilon_d}{\varepsilon_d - \varepsilon'} - \frac{2 \varepsilon_d}{\varepsilon_d - \varepsilon'} \varepsilon'' / \varepsilon'' - \varepsilon_d \varepsilon'' / \varepsilon'' - \varepsilon_d \varepsilon'' / \varepsilon'' - \varepsilon_d \varepsilon'' / \varepsilon'' - \varepsilon_d .
\]

In this case, provided \( \varepsilon_d - \varepsilon' \gg \varepsilon'' / \varepsilon_d \), we obtain \( \varepsilon_m > 0 \) and \( \varepsilon' < 0 \), i.e. the AM becomes a HMM of the first kind. Let, finally \( \varepsilon' \approx 0 \). In this region at finite dissipation \( \varepsilon_m = 2 \varepsilon'' / (\varepsilon_d - \varepsilon'_d) \), \( \varepsilon_m = (\varepsilon_d - i \varepsilon' / 2) \), i.e. the longitudinal component is small and strongly dissipative. This is the so-called ENZ region [14]. We will further be interested in the case \( \varepsilon' = - \varepsilon_d \). This equation is easily solved. If \( \varepsilon' < - \varepsilon_d \), then the condition \( \varepsilon'_m = - \varepsilon'_m \) is met with the small term \( \varepsilon''^2 / (2 \varepsilon''^2) \) if \( \varepsilon_d = \frac{3 - \sqrt{3}}{2} \) or \( \varepsilon' = - \varepsilon_d \left( \frac{3 + \sqrt{3}}{2} \right) \). In this case we have \( \varepsilon'_m = \frac{2 \varepsilon''^2 / (2 \varepsilon''^2)}{\varepsilon''^2 / \varepsilon''^2 / (2 \varepsilon''^2)} \), \( \varepsilon''^2 = \frac{\sqrt{3} - \sqrt{3} + \sqrt{3}}{2} = 0.086 \varepsilon' \). If \( - \varepsilon_d < \varepsilon' < 0 \), then the condition \( \varepsilon'_m'' = - \varepsilon'_m'' \) leads to the solution \( \varepsilon_d = \frac{3 + \sqrt{3}}{2} \). In this case one can again find \( \varepsilon'_m = \frac{2 \varepsilon''^2 / (2 \varepsilon''^2)}{\varepsilon''^2 / \varepsilon''^2 / (2 \varepsilon''^2)} \), and \( \varepsilon''^2 = \frac{\sqrt{3} + \sqrt{3} + \sqrt{3} + \sqrt{3}}{2} = 5.32 \varepsilon' \), i.e. the dissipation here is higher. Obviously, when the frequency is shifted, the condition \( \varepsilon'_m'' = - \varepsilon'_m'' \) can be fulfilled accurately, and the dissipation will somewhat change.

In an infinite medium of HMM, we consider an electromagnetic wave of the form \( E(x, z, t) = E_0 \exp(i(\omega t - k_x x - k_z z)) \), \( H(x, z, t) = H_0 \exp(i(\omega t - k_x x - k_z z)) \). This medium is described by a homogenized effective permittivity tensor

\[
\hat{\varepsilon} = \begin{bmatrix} \varepsilon_m & 0 & 0 \\ 0 & \varepsilon_m & 0 \\ 0 & 0 & \varepsilon_m \end{bmatrix}
\]

(1)
in which for HMM of periodic plane-layered structures with conductive films, the normal to which is oriented along the z axis, we have \( e_{ox} = e_{oy} = e_{oz} - i e_{oz} \), and the value \( e_{oz} \) can be negative. Consider the matrix of rotation of the structure around the y axis by an angle \( \alpha \):

\[
\mathbf{T}(\alpha) = \begin{bmatrix}
\cos(\alpha) & 0 & -\sin(\alpha) \\
0 & 1 & 0 \\
\sin(\alpha) & 0 & \cos(\alpha)
\end{bmatrix}.
\tag{2}
\]

Acting on vector \( \mathbf{E} \), it gives: \( E_x' = E_x \cos(\alpha) - E_y \sin(\alpha) \), \( E_y' = E_x \sin(\alpha) + E_y \cos(\alpha) \). i.e. there is a counterclockwise rotation. The matrix (1) will take the form \( \mathbf{z} = \mathbf{T}(\alpha) \mathbf{z} = \mathbf{z} \mathbf{T}(-\alpha) \mathbf{T}(\alpha) \) or

\[
\mathbf{z} = \begin{bmatrix}
\cos^2(\alpha) e_{ox} + \sin^2(\alpha) e_{oz} & 0 & \sin(\alpha) \cos(\alpha) (e_{oy} - e_{oz}) \\
0 & 1 & 0 \\
\sin(\alpha) \cos(\alpha) (e_{oy} - e_{oz}) & 0 & \cos^2(\alpha) e_{oz} + \sin^2(\alpha) e_{oz}
\end{bmatrix}.
\tag{3}
\]

We write the homogeneous Maxwell equations in such an AM as \( \mathbf{\nabla} \times \mathbf{H} = i \omega e_0 \mathbf{E} \), \( \mathbf{\nabla} \times \mathbf{E} = -i \omega \mu_0 \mathbf{H} \). Painting them by components, we have

\[
\begin{align*}
\partial_y H_z - \partial_z H_y &= -\partial_x H_z = i \omega e_0 \left( \mathbf{z} \times \mathbf{E}_y + \mathbf{E}_z \right), \\
\partial_x H_y - \partial_y H_z &= i \omega e_0 \left( \mathbf{E}_x + \mathbf{z} \times \mathbf{E}_y \right), \\
\partial_z H_x - \partial_x H_y &= i \omega e_0 \left( \mathbf{z} \times \mathbf{E}_x + \mathbf{E}_z \right), \\
\partial_x E_z - \partial_z E_x &= -i \omega \mu_0 \mathbf{H}_x, \\
\partial_y E_x - \partial_y E_y &= -i \omega \mu_0 \mathbf{H}_y, \\
\partial_z E_y - \partial_z E_y &= -i \omega \mu_0 \mathbf{H}_z.
\end{align*}
\]

In these equations, we took into account that the fields are independent of \( y \). These equations are divided into two systems of equations: \( E_y \neq 0 \), \( H_z \neq 0 \) and \( H_x \neq 0 \), \( E_z \neq 0 \). The first has the form:

\[
\begin{align*}
Z_0 (k H_z - k H_x) e_{ox} + k H_y &= -Z_0 k H_x e_{ox}, \quad k, E_y = Z_0 k H_x e_{ox}, \\
H_z &= E_x = 0.
\end{align*}
\]

This equation gives Fresnel’s dispersion equation (DE)

\[
\Delta(k, k x, z) = \sqrt{k_0^2 - k_x^2} + \sqrt{k_0^2 - k_z^2} = 0.
\]

The second system of equations gives the H-wave with respect to \( z \)-axis, and the second is E-wave. Consider first the latter. Equality to zero of its determinant gives

\[
\begin{align*}
\Delta(k, k x, z) &= \sqrt{k_0^2 - k_x^2} + \sqrt{k_0^2 - k_z^2} = 0, \\
\mathbf{E} &= \frac{Z_0 H_z + \mathbf{E}_x}{k_0}.
\end{align*}
\]

Substituting them in the third, we have the DE, which we write in the form

\[
k_0^2 - k_x^2 = \frac{\Delta(k, k x, z)}{k_0^2},
\]

Here it’s marked \( \Delta(k, k x, z) = \sqrt{k_0^2 - k_x^2} + \sqrt{k_0^2 - k_z^2} \). From this Fresnel equation for the extraordinary wave the two values are defined

\[
k_x^2 = -k_0^2 \frac{k_0^2 + k_z^2 \pm \sqrt{k_0^2 + 2 k_0 k_z + k_z^2}}{k_0^2 - k_z^2}.
\]

These two values correspond to opposite waves along \( \pm k \). The second equation in (4) allows us to find the impedance:

\[
Z^* = E_x / H_z = \frac{Z_0 \Delta(k, k x, z)}{k_0} = \frac{\pm Z_0 \sqrt{k_0^2 - k_z^2}}{\Delta}.
\]

It depends on the direction: \( Z^* = \pm Z_0 \rho \). \( \rho = \sqrt{[1 - k_z^2 / k_0^2]} \). Equating it to the impedance of an E-wave in a vacuum \( Z_0 \rho = \sqrt{[1 - k_z^2 / k_0^2]} \) propagating along the \( z \)-axis, we obtain the dispersion equation (DE) for the E-plasmon-polariton (EPP) along the surface in \( z \) direction

\[
\rho = \sqrt{[1 - k_z^2 / k_0^2]}.
\]

It defines two waves along each direction: \( k_0 z = \pm k_0 \sqrt{(\Delta - \Delta_e) / (\Delta - 1)} \). In the case of symmetry \( \Delta_e = 0 \) we have the solution \( k_0 = \pm k_0 \sqrt{(\Delta - 1) / (\Delta - 1)} \). This PP very slow, if \( \Delta_e = 0 \), this is the Zenneck DE [13,17–21]. In the layered structure this equality is impossible. Such a layer must be homogeneous, either dielectric or metallic. In the latter, the maximum deceleration will be at \( \Delta = -1 \), i.e. at the frequency of the plasmon resonance.

In the case of a wave in the plate, the solution is

\[
H_z = \exp(-ik_z z) \left[ A^+ \exp(-ik_z' x) + A^- \exp(-ik_z'' x) \right],
\]

the components of the electric field are determined by the formulas (4). We’ll need a component

\[
E_x = -c e_0 k_0 \left[ A^+ \rho \exp(-ik_z' x) - A^- \rho \exp(-ik_z'' x) \right].
\]

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In vacuum, we also need the solutions of the wave equation in the form:
\[ H_x = B \exp(-ikz) \exp(-ik_0(x-d)) \quad \text{and} \quad E_z = -BZ_0(k_0/k) \exp(-ikz) \exp(-ik_0(x-d)), \]
\[ H_x = C \exp(-ikz) \exp(ik_0x) \quad \text{and} \quad E_z = CZ_0(k_0/k) \exp(-ikz) \exp(ik_0x). \]

They are written for areas \( x > d \) and \( x < 0 \) respectively. Here we have \( k_0^2 + k^2 = k^2 \). Moreover, the direction of energy motion of the fast wave (\( \Re(k^2) < k_0^2 \)) is taken from the plate into vacuum (leakage). For a symmetric structure \( k^* = -k^2 \).

The fields on both sides have identical dependencies, and can have either an exponential decay in the vacuum side (surface gliding wave), or an exponential growth (antisurface or leakage wave). Therefore, it is enough to enter one constant in (11) with matching the fields on one surface. The gliding means the movement of energy from the vacuum on both sides and absorption in it.

At weak energy inflow the surface wave can be weakly dissipative. Leakage means the emission of stored energy from the plate into the vacuum. Strong leakage even with weak dissipation is accompanied by large radiation losses. Signs in (11) are chosen according to the conditions of emission, i.e. outflow of energy. Leakage can be replaced by gliding with increasing frequency. Gliding from one side and leakage from the other side (or in the other direction) for the structure under consideration are not possible. For the symmetric case, the proof is simple. Its DE is obtained by equating the input impedance on the one hand to the impedance of the wave in vacuum [17–21], i.e. imposing the condition \( R = 0 \). By transforming the impedance of a wave in a vacuum and equating it to the same impedance, we obtain \( \tan\theta(k,d) = 0 \). This is not a DE, but the matching condition in which the reflection coefficient is zero. For the transparent layer, this is the condition of the matching energy output for the transmission line at half-wave thickness of the dielectric layer. Transformation means unidirectional energy transfer. The DE is produced, if we change the sign of one of the impedances. For the asymmetric case the different leakage/gliding conditions lead to a change in the sign of the impedance and \( k_0 \) in one of the equations (11).

Such a system of equations has no solution. Determining \( k_0 = \pm \sqrt{k_0^2 - k^2} \), it should be taken into account that the sign should be chosen so that the slow wave in the dissipative structure of the HMM from the vacuum was gliding with energy flowing (\( k_0^2 < 0 \)), i.e. the energy from the vacuum should flow into the plate [17–21]. In this sense, taking \( k_0 = k_0^* - ik^2 \), we should require the implementation \( k_0^* < 0 \) and \( k_0^* > 0 \) for gliding wave and \( k_0^* < 0 \), \( k_0^* > 0 \) for leakage one. One can see that the gliding wave is surface (decreasing from the surface towards vacuum) and the leakage wave is antisurface (exponentially increasing). Let take \( k_0^* = k^2 - ik^2 \). At weak dissipation in a slow wave \( k_0 > k^2 \) we have

\[ k_0 = \pm \sqrt{k_0^2 - k^2} = \pm i \sqrt{k_0^2 - k^2} = k^2 - ik^2 \quad \text{and} \quad k_0^* = -k^2 + ik^2. \]

Therefore \( k_0^* > 0 \), \( k_0^* < 0 \) and \( k_0^* = 0 \) for leakage one. We determine as the direction of motion of energy, i.e. for a positive we take such, when \( k_0^* > 0 \). Therefore it is the dependence \( \exp(-k_0^2z) \), i.e. attenuation in the direction of motion of energy. If at the same time \( k_0^* > 0 \), such a wave is forward. If, however \( k_0^* < 0 \), wave is backward. In it the phase and the energy move oppositely. We see that in the backward wave with weak dissipation, the energy flow is replaced by the outflow and vice versa. However, with strong dissipation, the root extraction may not lead to such an effect, i.e. all modes are possible.

Matching tangential components, we get

\[ B = A^* \exp(-ik_0'd) + A \exp(-ikd), \quad C = A^* + A, \]

\[ B \rho_0 = \rho \left[ A^* \exp(-ik_0'd) - A \exp(-ikd) \right], \quad C \rho_0 / \rho = A^* - A. \]

Here \( A^* = (1 + \rho_0 / \rho) \). We divide the third equation by the first and get DE in the form

\[ \rho_0 (\rho - \rho_0) \exp(-ikd) - (\rho + \rho_0) \exp(-ik_0'd), \quad \rho \exp(-ikd) + (\rho + \rho_0) \exp(-ik_0'd). \]

Here \( A^* = (1 + \rho_0 / \rho) \). We divide the third equation by the first and get DE in the form

\[ B \rho_0 = \rho \left[ A^* \exp(-ik_0'd) - A \exp(-ikd) \right], \quad C \rho_0 / \rho = A^* - A. \]

Here \( A^* = (1 + \rho_0 / \rho) \). We divide the third equation by the first and get DE in the form

\[ \rho_0 (\rho - \rho_0) \exp(-ikd) - (\rho + \rho_0) \exp(-ik_0'd), \quad \rho \exp(-ikd) + (\rho + \rho_0) \exp(-ik_0'd). \]
In the case of symmetry it takes the form \( \rho_{hh} = -\rho_{\rho} + i\rho \tan(kz)\). The upper sign corresponds to the electric wall in the center, and the lower one corresponds to the magnetic wall. In generally

\[
2\rho\rho_{h\alpha} = \rho^2 + \rho_{h\alpha}^2, \quad \alpha = \frac{\exp(-ikz\rho) + \exp(-ikz\rho)}{\exp(-ikz\rho) - \exp(-ikz\rho)}.
\]

There are two DEs: \( \rho_{h\alpha} = \rho_{h\beta} \pm \sqrt{\rho^2 - 1} \). Let denote \( \beta = \alpha \pm \sqrt{\rho^2 - 1} \). Then for the square of the deceleration we find

\[
\mu^2 = \left( e_{\alpha} e_{\alpha} - e_{\beta} e_{\beta} \right) / \left( e_{\alpha} e_{\alpha} - e_{\beta} e_{\beta} \right).
\]

Large deceleration is possible if the denominator is small or the numerator is large. Let consider the DE for H-PP. Substituting the components of the magnetic field in the first equation, we have the DE

\[
\mu^2 + k^2 = k^2 e_{\alpha}.
\]

This is the Fresnel equation for ordinary wave. For this wave the impedance along the x-axis has the form \( Z = E_z H_z = Z_{\alpha} k_0 / \sqrt{k^2 e_{\alpha} - k^2} \). It cannot be matched with the corresponding impedance in vacuum, so H-PP along the boundary plane of infinite HMM sample does not exist. However, it occurs if there is a finite layer of HMM. In this case, as well as for a single metal layer, we have two solutions:

\[
\sqrt{k^2 e_{\alpha} - k^2} = \sqrt{k^2 - k_0^2} \left[ \tan \left( d \sqrt{k^2 - k_0^2} / 2 \right) \right]^{1/2}.
\]

Figure 1. The layer of HMM with thickness \( d \) consisting of conductive metal sheets periodically embedded in the dielectric (from above), and the same layers obtained by cuts the HMM to the crystallographic axis at angle \( \alpha = \pi/4 \) (in center) and angle \( \alpha = \pi/2 \) (below)

The solution with the sign “plus” corresponds to the electric wall, and with the sign “minus” – to the magnetic wall in the center of the layer. These equations are the same as for the metal layer with replacement \( e_{\alpha} \rightarrow e_{\beta} \) [17]. Since slow PPs \( k^2 > k_0^2 \) are possible, it is convenient the DE (14) to convert:

\[
\sqrt{k^2 - k_0^2} e_{\alpha} = \sqrt{k^2 - k_0^2} \left[ \tanh \left( d \sqrt{k^2 - k_0^2} / 2 \right) \right]^{1/2}.
\]

Denote the hyperbolic tangent as \( T \), square equation (15) and assume that the plasmon is sufficiently slow. Then we have \( k^2 = k_0^2 (1 - T) / (1 + T) \). If dissipation can be neglected, then \( k^2 = k_0^2 (1 + T^2) / (1 - T^2) \). In this case, for a slow PP the value \( T \) must be less than one and close to it. We see that it is possible the solution \( k^2 = k_0^2 (1 + T^2) / (1 - T^2) \) with slow PP, and the plasmon \( k^2 \) cannot be very slow and weakly dissipative.
3. THE FRESNEL’S FORMULAS

The HMM structure is equivalent to a set of plane-parallel waveguides or lattices rotated at an angle and is able to effectively control the diffraction of a plane wave, especially if optically pumped semiconductor layers or graphene sheets are used. Tensor conductivity of graphene from THz to UV ranges was obtained in a number of studies [22,23]. In the first approximation in the Kubo-Greenwood model it can be considered scalar [22,23]. At low frequencies it is inductive, but taking into account interband transitions, it can have a capacitive region and even becomes negative with external pumping [23]. Possible homogenization, taking into account the tensor character of the conductivity \( \sigma_{xx} \neq \sigma_{yy} \), \( \sigma_{xx} = \sigma_{yy} \), and scalar conductivity [13,22]. However, it should be taken into account that the tensor conductivity of graphene does not divide into E-waves and H-waves, so it is necessary to cross-link all four tangent to the boundaries field components or use the 4x4 transmission matrix [13]. This complicates the DE and Fresnel’s formulas. Metal tapes with a small thickness \( l \) can be considered as surface current density \( J = \omega \varepsilon_0 \varepsilon \mathbf{E} \). P-polarization excites only densities \( J_x \) and \( J_z \) or E-wave, s-polarization leads to \( J_y \) or H-wave. The tensor conductivity connects all three components of the current to the field. In a normal fall on the plate with \( \alpha=0 \) the wave with polarization of the electric vector is normal to the layers passes with much lower losses than the wave with orthogonal polarization. In the asymmetric case, this works for p-polarized and for s-polarized waves incident at a certain angle.

We obtain Fresnel formulas when p-polarized and s-polarized wave falling from below at the angle \( \phi \) on a structure with an arbitrary angle \( \alpha \). We use of the impedance approach, considering the movement along the x-axis. In the bottom we have the wave \( E_y = \exp(-ik_y x) + R_y \exp(i\kappa_x x), Z_0 H_x = \exp(-ik_y x) - R_y \exp(i\kappa_x x) \rho_0 \). From above we have \( E_y = T_y \exp(-ik_y x), Z_0 H_x = T_y \exp(-ik_y x) \rho_0 \). In structure we write \( E_y = \frac{A_y^s \exp(-ik_y x) + A_y^p \exp(-ik_x x)}{\rho_0 \rho_{0y}} \), \( Z_0 H_x = \rho_{0y} \exp(-ik_x x) / \rho_0 - A_y^s \exp(-ik_y x) / \rho_0 \). Here for \( q=p \) we have \( E_y = -E_x, H_y = H_x, \rho_{0y} = \sqrt{k_x^2 + k_y^2}, \rho_0 = k_0 \sqrt{k_x^2 + k_y^2} \). When \( q=s \) we have \( E_y = E_x, H_y = H_x, \rho_{0y} = 1/\sqrt{1-k_x^2/k_y^2}, \rho_0 = k_0 \sqrt{k_x^2 - k_y^2} \). By matching the field components, one can obtain the solution:

\[
T_y = \frac{4\rho_0 \rho_{0y}}{\exp(ik_x^* d) \rho_0 + \rho_{0y} + \exp(ik_y^* d) \rho_0^* - \rho_{0y}^* },
\]

where \( A_y^s = \exp(ik_y^* d) \gamma_y (1 + \rho_0 / \rho_{0y})/2, A_y^p = T_y \exp(ik_y^* d) (1 - \rho_0 / \rho_{0y})/2 \). Using this one can find the impedance

\[
Z_y = (1 + R_y)/(1 - R_y) = \frac{A_y^s + A_y^p}{(\rho_{0y} / \rho_0)(A_y^s - A_y^p)},
\]

\[ 0.00 \quad 0.25 \quad 0.50 \quad 0.75 \quad 1.00 \]
\[ 0.0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1.0 \quad 1.2 \quad 1.4 \quad 1.6 \]
\[ \phi \]

Figure 2. The dependence of the reflectance \( R \) (solid curves) and transmission \( T \) (dashed curves) of the angle of incidence \( \phi \) for the structure of HMM with \( d=420 \text{ nm} \), \( t_x=t_y=200 \text{ nm}, \varepsilon_0 = 3 \) at different values of angle \( \alpha \): \( \alpha=0 \) (curve 1), \( \alpha=\pi/12 \) (2), \( \alpha=\pi/8 \) (3), \( \alpha=\pi/4 \) (4), \( \alpha=\pi/3 \) (5).
and the reflection coefficient in the form \( R_q = (Z_q - 1)/(Z_q + 1) \) or as \( R_q = A_q^+ + A_q^- - 1 \). It should be borne in mind that in these relations the magnitude \( k_1 < k_0 \) and the real, and the angle of incidence is defined as \( \phi = \arctan[k_1/\sqrt{k_0^2 - k_1^2}] \). In the case of symmetry, the equation is simplified and takes the form

\[
T_q = \frac{4\rho_1\rho_0\exp(-ik_d)}{(|\rho_0| + \rho_1)^2 + \exp(-2ik_d)|\rho_0 - \rho_1|^2}.
\]  

(17)

4. NUMERICAL RESULTS AND DISCUSSION

Figure 2 shows the calculation results \( R = R_q \) and \( T = T_q \) depending on the angle of incidence for wave length \( \lambda = 500 \text{ nm} \). Note that in the theory of diffraction on lattices, such problems for infinitely thin perfectly conducting strips are reduced to integral equations and have been solved. It is possible to take into account the final impedance of metal tapes. However, accounting for the dielectric layer in this approach is complex and requires the introduction of combined volume-surface integral equations. The results of calculation of dispersion and losses for PP at \( \alpha = 0 \) and \( \alpha = \pi/2 \) are shown in Fig. 3. Parts of the curves in the left region separated by line \( \eta_1 = 1 \) correspond to fast gliding waves, and to the right region describe the slow ones. The waves in the lower plane-layered structure \((\alpha = \pi/2)\) are slower, and in the region above the frequency of plasmon resonance are inverse. This distinguishes the plane-layered AM from the metal one, for which there are no backward waves. For HMM with \( \alpha = 0 \) there are no backward waves, and the maximum deceleration corresponds to higher frequencies for which \( \eta' > 0 \). For convenience the curves are constructed so that \( k'_1 > 0 \), therefore, the backward waves correspond to a kind of negative loss. In a vacuum energy is always transferred along the motion of the phase. The presence of backward waves is an integral effect associated with the fact that in metal structures the component of the Poynting vector can change the sign if \( \eta' < 0 \).

In the region of plasmon resonance \( k'_1 \approx k'_0 \approx 1/\sqrt{\varepsilon} \), i.e. to obtain large decelerations, dissipation should be reduced. The estimation for the first structure gives in the resonance region \( k_1 = k_0(1-\varepsilon/2)/\varepsilon \). For the second structure, there are two resonances \( k_1 = k_0(1-\varepsilon)/\varepsilon \) and \( k_2 = k_0(1+\varepsilon)/\varepsilon \), respectively, a low-frequency at \( \varepsilon = -\varepsilon_0 \) and a high-frequency at \( \varepsilon = 1/\varepsilon_0 \). It is immediately seen that the latter corresponds to the backward PP. All investigated PP are gliding, since the leakage from the dissipative half-space is impossible.

In general case we should iteratively solve the equation (13), presenting it, for example, in the form of

\[
k'_1 = \pm k_1 \left( \frac{\varepsilon_0 f(k_0,k_1)/\Delta - 1}{f(k_0,k_1)/\Delta - 1} \right)^{1/2}.
\]  

(18)

Here the even function of \( k_1 \) is denoted:

\[
f(k_0,k_1) = \left[ (\rho - \rho_0)\exp(-ik_1d) - (\rho + \rho_0)\exp(-ik_0d) \right]^2.
\]

Indeed, it can be represented as

\[
f(k_0,k_1) = \left[ (\rho - \rho_0)\exp(-i\psi) - (\rho + \rho_0)\exp(i\psi) \right]^2,
\]

where \( \psi = \pm d \sqrt{k_0^2 - k_1^2} / \lambda \). Two branches in (18) \( k_1^+ \) and \( k_1^- = -k_1^+ \) are related with \( k_1^\pm(k_0) \) from (6), so that \( k_1^+(k_0') = -k_1^-(k_0') \). Thus, the DE (18) defines two waves with opposite directions of phase velocities for mutual structure.

Two values of \( \psi \) give two dispersive branch of waves for any direction, one of which more slow then another. So, the total number of waves of both directions is four. Fig. 4 shows the results of the iterative solution of DE (18). Maximum deceleration increases with \( \alpha \) increasing. The result are normalized to plasmonic wavenumber \( k_p \). For symmetrical cases \( \alpha = 0 \) and \( \alpha = \pm \pi/2 \) also there are two branches: symmetric and antisymmetric (with electric and magnetic walls in the center) which described by the DEs \( \rho_0 = \rho[\tan(k_p d/2)]^{11} \). We must propose, that \( t_0 << d \) to use the effective media approximation for the case \( \alpha = \pm \pi/2 \). This limits the thickness of the structure from below and, accordingly, the maximum deceleration of slower plasmon (with a magnetic wall) compared to the plasmon along a thin metal film [13].

Proc. of SPIE Vol. 11458 1145813-7
5. CONCLUSIONS

In this paper, using the simplest homogenization, exact solutions are obtained for PP along a layer in the general case of asymmetric HMM in the form of a plane-layered periodic metal-dielectric structure. An additional degree of freedom can be introduced into the equations by varying the fill factor, or by using multilayer structures in the period. Taking into account SD, i.e. dependence $\varepsilon(k_x,k_y)$, leads to complex nonlinear DE and Fresnel equations, which can only be analyzed numerically. Both SD and dissipation distort the hyperbolic law of dispersion and limit the modulus of the wave vector components, i.e. close the isofrequency surface. The conditions for the existence of slow and fast, gliding and leakage, as well as forward and backward PPs are found. The asymmetric layer of HMM is interesting in that it...
supports PP with different conditions of gliding/leakage on both sides. The backward PPs were found along the plane-layered structure of the half-space, but they are absent along the metallic half-space. To reduce the frequency $\omega_p$ and plasmonic resonance frequency one can use the semiconducting layers instead metallic, and to reduce a losses and increase the wave deceleration the low temperature of graphene optical pumping HMMs are needed [23].

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