Feature Selection Based on Confidence Machine

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Abstract

In machine learning and pattern recognition, feature selection has been a hot topic in the literature. Unsupervised feature selection is challenging due to the loss of labels which would supply the related information. How to define an appropriate metric is the key for feature selection. We propose a filter method for unsupervised feature selection which is based on the “Confidence Machine”. Confidence Machine offers an estimation of confidence on a feature’s “reliability”. In this paper, we provide the math model of Confidence Machine in the context of feature selection, which maximizes the relevance and minimizes the redundancy of the selected feature. We compare our method against classic feature selection methods Laplacian Score, Pearson Correlation and Principal Component Analysis on benchmark data sets. The experimental results demonstrate the efficiency and effectiveness of our method.

Keywords: Feature selection, Confidence machine, Unsupervised learning, Maximal dependency, machine learning

1. Introduction

Feature selection is a key technology to deal with high-dimensional data in machine learning [1], [9]. It is reported from 2012 the big data "size" is moving from a few dozen terabytes to many petabytes. How to deal with the huge data in case of “curse of dimensionality” is essential in real applications [22]. Feature selection, as a powerful method of dimension reduction [6], [17], has been successfully applied in pattern recognition [1], computer version [2], [5], active learning [4], and sparse coding [2], [13]. Functionally, feature selection [12], [14] is divided into three groups: filter model, wrapper model and embedded model. Filter is the most popular model in recent research, as it
has low computational cost and is robust in theoretical analysis. Depending on the class labels, feature selection is implemented in supervised fashion or unsupervised fashion. Most existing filter models are supervised. In real applications, the class labels are always scarce [3] [20]. It is meaningful to design a filter feature selection method in unsupervised fashion. The criteria of maximum dependency has been studied widely in the field of feature selection: selecting the features with highest relevance to the target class C and at the same time minimizing the redundancy with the rest of the features. This criteria is met in the fashions of mutual information or correlation. In this paper, we utilize the correlation to compute the distance among selected features, the target class and the rest of the features. Given the feature’s relevance and redundancy, which are denoted by correlation scores, we evaluate these properties and build a mathematic model of Confidence Machine as the primary part of a new unsupervised filter method for feature selection.

The main contributions of the paper are summarized as follows: a new feature selection filter model is proposed, based on the idea of Confidence Machine and correlation. Based on the model of Confidence Machine, a feature’s relevance and redundancy with the rest features and with the target class are calculated, in the way of correlation. The relevance is maximized and redundancy is minimized. The proposed method is applied to UCI [4] benchmark data sets (dual category and multiple category). A 2-D visualization case study is carried out and compared with classic filter feature selection methods (Principal Component Analysis [5], Laplacian Score [6] and Pearson Correlation [5]). Then comparison experiments of feature-based classifications are conducted to demonstrate the efficiency and effectiveness of our method.

2. Feature score based on Confidence Machine

Before introducing the idea of Confidence Machine, let’s first talk about a wide spread consensus “Max-Relevance” and “Min-Redundancy”, which was firstly introduced by Hanchuan Peng in [7]. In feature selection, it has been recognized that an optimal feature often means minimal classification error. That requires the maximal statistical dependency of the target class C on the data distribution in the subspace. This scheme is called maximal dependency. However, combinations of individually good features do not necessarily lead to good classification performance. Redundant variables within a subspace should be removed and this property is called minimal
redundancy. Usually the relevance and redundancy is characterized in terms of correlation or mutual information. In this paper, we choose to use correlation. In the following part we will discuss the definition and formal math model of confidence machine.

The algorithm of Confidence Machine describes a measure of “reliability” for every prediction made, in contrast to the algorithms that output “bare” predictions only. The estimation of prediction confidence is represented by the P-value, which is an indication of how good the selected feature is, in the context of feature selection. The general idea is that the confidence P-value corresponds to the certainty of a feature being the right choice. If a feature has a bigger P-value, then it means this feature has a greater relevance to the target class and at the same time has a smaller redundancy with other features.

Now, we give the formal mathematical model of Confidence Machine in the context of feature selection. Imagine we have a data set with \( n+1 \) dimensions \( Y=\{y_1, y_2, \ldots, y_i, \ldots, y_n, C\} \), in which the first \( n \) dimensions are data features and the last dimension is the class label. The ideal feature among \( n \) dimensions has the property of “Maximum relevance” and “Minimum redundancy”. For each feature, we need to calculate its relevance score \( pl \) and redundancy score \( ps \).

\( Pl_i \) is defined as the correlation between the current feature \( y_i \) and the target class \( C \). According to the definition of Pearson’s correlation, which is the most familiar measure of dependence between two quantities, the relevance value of feature \( y_i \) is defined as:

\[
pl_i = \frac{cov(y_i, C)}{\sigma_y \sigma_C} = \frac{E[(y_i - \mu_{y_i})(C - \mu_C)]}{\sigma_y \sigma_C}
\]

\( Ps_i \) is defined as the correlation between the selected feature \( y_i \) and other features. It is calculated as:

\[
Ps_i = \sum (|Ps_{i1}|, |Ps_{i2}|, \ldots, |Ps_{in}|)
\]

And \( Ps_{ij}(j=1,2,\ldots,n) \) is the correlation value between feature \( i \) and \( j \):

\[
ps_{ij} = \frac{cov(y_i, y_j)}{\sigma_{y_i} \sigma_{y_j}} = \frac{E[(y_i - \mu_{y_i})(y_j - \mu_{y_j})]}{\sigma_{y_i} \sigma_{y_j}}
\]

For each feature \( y_i \), relevance \( Pl_i \) is the value needs to be maximized and redundancy \( Ps_i \) is the value needs to be minimized, if \( y_i \) is an optimal
choice. Then non-conforming score $\alpha$ is introduced. This measure directly defines the relevance and redundancy of the current feature in relation to the rest features and the class label. In our case the non-conforming score for a feature $i$ is defined as:

$$\alpha_i = \frac{P_i}{P_{\text{si}}}$$

$P_{\text{si}}$ is safe as a denominator, since it will always greater than 0. And that is the reason why $P_{\text{si}}$ is calculated as the sum of absolute value of $Psij (j=1,2,\ldots,n)$. This is a natural measure to use, as the non-conformity of a feature increases when the distance from the class becomes bigger or when the distance from the other features becomes smaller.

Provided with the definition of non-conforming score, we will use the following formula to compute the p-value:

$$P(\alpha_{\text{now}}) = \frac{\# \{i: \alpha_i > \alpha_{\text{now}}\}}{n}$$

In the above equation, $\#$ denotes the cardinality of the set, which is computed as the number of elements in finite set. $\alpha_i$ is the non-conforming score of the test feature. From the above discussion, we can see that the algorithm will output a sequence $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$, and based on these non-conforming scores a sequence of confidence value $\{P_1, P_2, \ldots, P_n\}$ will be produced. Every P-value denotes the reliability of a feature and is counted as the score of that feature. At the end of our algorithm, those features with high confidence (P-value) will be chosen.

3. Experimental evaluation

In this section, the empirical experiments are conducted on ten data sets from UCI Repository [4] to demonstrate the effectiveness of our method. There are six binary data sets and four multiple categorical data sets. The detailed information for the data sets are listed in Table 1. In the experiment, each data set is randomly separated into two equal parts. One part is training data and the rest of the parts are testing data. We used the training data to build the model for feature selection. Four filter feature selection models are utilized for comparison experiments: Our proposed unsupervised
Filter model via Confidence Machine, Pearson correlation, Laplacian score and Principal Component Analysis. We use Liu_Corr, PER, LAP and PCA as abbreviations to denote these four methods in the experiment.

3.1. Case study of 2-D visualization

A simple case study for dataset wine is shown. In total, there are 13 features for wine, such as “Alcohol”, “Magnesium” and “Proline”. We use four filter methods based on training data (with size 89) and apply on the testing data. Each method chooses two features for 2-D visualization on testing data. The results are shown in Fig. 1. Two features are selected by 4 different methods and plotted in each sub figure. It can be observed that the feature “Flavanoids” and feature “Color intensity” selected by Liu_Corr method are crucial for discrimination.

3.2. Feature based classification

When selected features are more than two, we used the feature based classification to compare the feature selection methods. The experiment is conducted five times and mean outputs are obtained. The target selected features size is from one to around 80% of whole feature size to give comprehensive comparison. In order to show the classification performances, we use

| Name               | Features | Training size | Testing size | Class |
|--------------------|----------|---------------|--------------|-------|
| car                | 6        | 864           | 864          | 2     |
| pima               | 8        | 384           | 384          | 2     |
| tic-tac-toe        | 9        | 479           | 479          | 2     |
| yeast              | 7        | 742           | 742          | 2     |
| hill valley        | 100      | 303           | 303          | 2     |
| vehicle silhouettes| 18       | 473           | 473          | 2     |
| wine               | 13       | 89            | 89           | 3     |
| image segmentation | 19       | 1165          | 1165         | 7     |
| wine quality       | 11       | 2449          | 2449         | 6     |
| libras             | 90       | 180           | 180          | 15    |
Figure 1: Data wine plotted in 2-D with selected features. 4 methods have selected different 2 features.
Figure 2: Comparison of feature based classification accuracies for data set wine quality

three classic classifiers: k nearest neighbor (k = 5 in the experiment) and Lib-SVM. For brevity, we only plot one data set (a multi-category data set) result. We abbreviate the classifiers as LibSVM and KNN in the figures. Fig. 2 shows the comparison results for data winequality. When the selected features size is greater than 4, our method results rank first with LibSVM classifier. And our method ranks second when the selected feature size is smaller than 4. In the case of classifier KNN in Fig. 3, the performances of Liu_Corr rank first when we select 6 features, and rank second with other feature sizes. It is important to note our method is competitive to Pearson’s correlation in most feature sizes. In order to give intensive comparison of different feature selection methods on multiple data sets, the mean accuracy in low dimension (from feature size one to around 40% of whole feature sizes) are calculated based on each data set and each classifier. Table 2 shows the detail mean outputs and the comparison results. The highest accuracy for each classifier is highlighted. It can be observed that our filter method won 6 and 4 times of 10 data sets with classifiers LibSVM and KNN separately.

4. Conclusion and future work

We present a new filter feature selection method in the unsupervised fashion. Our approach aims to use correlation to evaluate the distance between current feature with the target class, and between current feature with other features. Then by calculating the prediction confidence of every feature, we select the features which have the highest relevance with class label and have the lowest redundancy with other features. Experimental comparisons with related filter methods have demonstrated that our method is effective
Table 2: Mean accuracy in low dimension (in %)

| Data set       | LibSVM | KNN |
|----------------|--------|-----|
|                | Liu_Corr | PER | LAP | PCA | Liu_Corr | PER | LAP | PCA |
| cor            | 73.7    | 69.1 | 73.1 | 73.1 | 70.7    | 65.1 | 60.7 | 63.5 |
| pima           | 71.6    | 64.9 | 65.3 | 67.4 | 68.6    | 61.4 | 64.1 | 65.8 |
| tic-tac-toe    | 55.9    | 50.8 | 47.9 | 50.9 | 62.9    | 61   | 66.6 | 68.9 |
| yeast          | 72.7    | 71.7 | 72.8 | 71   | 70.7    | 66.7 | 69.6 | 67.4 |
| hill valley    | 55.9    | 50.8 | 47.9 | 50.9 | 52.3    | 53.8 | 50.6 | 49.4 |
| vehicle        | 46.7    | 55.7 | 51.3 | 48.9 | 83.6    | 69.3 | 68.7 | 73.4 |
| wine           | 65.4    | 63.2 | 68   | 60.1 | 48.8    | 52.9 | 49.3 | 49.7 |
| image seg      | 71.3    | 72.6 | 66.8 | 57.5 | 51.7    | 55.8 | 51.3 | 51.2 |
| wine quality   | 57.2    | 55.9 | 52   | 51.8 | 52.8    | 53.6 | 49.1 | 48.4 |
| libras         | 78.8    | 78.7 | 78.4 | 79.4 | 74.4    | 56.6 | 60.2 | 68.2 |
| Mean Wins      | 6       | 2    | 1    | 1    | 5       | 4    | 0    | 1    |

in terms of visualization and classification. Future research work will focus on increasing dimension of the data set, statistical analysis among different filter models and improving the theoretical framework of Confidence Machine based filter for feature selection. Also, we plan to apply the our method to human group recognition [21], social networks analysis [19], [20] and sparse representations [21], [23].

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