Towards a few-percent measurement of the Lense-Thirring effect with the LAGEOS and LAGEOS II satellites?

Lorenzo Iorio
Dipartimento Interateneo di Fisica dell’ Università di Bari
Via Amendola 173, 70126
Bari, Italy
e-mail: lorenzo.iorio@libero.it

Abstract

Up to now attempts to measure the general relativistic Lense-Thirring effect in the gravitational field of Earth have been performed by analyzing a suitable $J_2 - J_4$-free combination of the nodes $\Omega$ of LAGEOS and LAGEOS II and the perigee $\omega$ of LAGEOS II with the Satellite Laser Ranging technique. The claimed total accuracy is of the order of 20-30%, but, according to some scientists, it could be an optimistic estimate. The main sources of systematic errors are the mismodelling in the even zonal harmonic coefficients $J_l$ of the multipolar expansion of the gravitational potential of Earth and the non-gravitational perturbations which plague especially the perigee of LAGEOS II and whose impact on the proposed measurement is difficult to be reliably assessed. Here we present some evaluations of the accuracy which could be reached with a different $J_2$-free observable built up with the nodes of LAGEOS and LAGEOS II in view of the new preliminary 2nd-generation Earth gravity models from the GRACE mission. According to the GRACE-only based EIGEN-GRACE02S solution, a 1-sigma upper bound of 4% for the systematic error due to the even zonal harmonics can be obtained. In the near future it could be possible to perform a reliable measurement of the Lense-Thirring effect by means of the existing LAGEOS satellites with an accuracy of a few percent by adopting a time span of a few years. The choice of a not too long observational temporal interval would be helpful in reducing the impact of the secular variations of the uncancelled even zonal harmonics $\dot{J}_4$ and $\dot{J}_6$ whose impact is difficult to be reliably evaluated.

Keywords: Lense-Thirring effect, LAGEOS satellites, New Earth gravity models.
1 Introduction

The general relativistic gravitomagnetic force [1] induced by the gravitational field of a central rotating body of mass $M$ and proper angular momentum $J$ is still awaiting for a direct, unquestionable measurement. Up to now there exist some indirect evidences of its existence as predicted by the General Theory of Relativity (GTR in the following) in an astrophysical, strong-field context [2] and, in the weak-field and slow-motion approximation valid throughout the Solar System, in the fitting of the ranging data to the orbit of Moon with the Lunar Laser Ranging (LLR) technique [3]. The measurement of the gravitomagnetic Schiff precession of the spins of four spaceborne gyroscopes [4] in the gravitational field of Earth is the goal of the Stanford GP-B mission [5] which has been launched on April 2004. The obtainable accuracy should be of the order of 1% or better.

The Lense-Thirring effect on the geodesic path of a test particle freely falling in the gravitational field of a central rotating body [6] consists of tiny secular precessions of the longitude of the ascending node $\Omega$ and the argument of pericentre $\omega$

\[
\dot{\Omega}_{LT} = \frac{2GJ}{c^2a^3(1-e^2)^{3/2}}, \quad \dot{\omega}_{LT} = \frac{-6GJ\cos i}{c^2a^3(1-e^2)^{3/2}},
\]

where $a$, $e$ and $i$ are the semimajor axis, the eccentricity and the inclination, respectively, of the orbit, $c$ is the speed of light and $G$ is the Newtonian gravitational constant.

The LAGEOS III/LARES mission [7, 8] was specifically designed in order to measure such effect, but, up to now, in spite of its scientific validity and relatively low cost, it has not yet been approved by any space agency or scientific institution. Recently, a drag-free version of this project [9], in the context of the relativistic OPTIS mission [10], is currently under examination by the German Space Agency (DLR).

2 The current LAGEOS-LAGEOS II Lense-Thirring experiment

Up to now, attempts to observationally check the Lense-Thirring effect in the gravitational field of Earth have been performed by analyzing the accurately recovered orbits of the existing LAGEOS and LAGEOS II satellites with the Satellite Laser Ranging technique (SLR) [11, 12]. The adopted observable
is
\[ \delta \dot{\Omega}^L + 0.295 \delta \dot{\Omega}^L II - 0.35 \delta \dot{\omega}^L II \sim 60.2 \mu_{LT}, \quad (2) \]

where \( \delta \dot{\Omega} \) and \( \delta \dot{\omega} \) are the orbital residuals of the rates of the node and the perigee and \( \mu_{LT} \) is the solved-for least square parameter which is 0 in Newtonian mechanics and 1 in GTR. The Lense-Thirring signature, entirely adsorbed in the residuals of \( \Omega \) and \( \omega \) because the gravitomagnetic force has been purposely set equal to zero in the force models, is a linear trend with a slope of 60.2 milliarcseconds per year (mas yr\(^{-1}\)) in the following. The standard, statistical error is evaluated as 2%. The claimed total accuracy, including various sources of systematic errors, is of the order of \( 11\% \) to \( 20\% \). However, such estimate would be too optimistic according to some scientists who propose a different error budget \[13\].

### 2.1 The error budget

The main sources of systematic errors in this experiment are the unavoidable aliasing effect due to the mismodelling in the classical secular precessions induced on \( \Omega \) and \( \omega \) by the even zonal coefficients \( J_l \) of the multipolar expansion of geopotential and the non-gravitational perturbations, which severly affect the perigee of LAGEOS II \[14, 15, 16, 17, 18\], whose impact on the proposed measurement is difficult to be reliably assessed. It turns out that the mismodelled classical precessions due to the first two even zonal harmonics of geopotential \( J_2 \) and \( J_4 \) are the most insidious source of error for the Lense–Thirring measurement with LAGEOS and LAGEOS II. The combination \( 2 \) is, by construction, insensitive just to \( J_2 \) and \( J_4 \). According to the full covariance matrix of the EGM96 gravity model \[19\], the error due to the remaining unc cancelled even zonal harmonics amounts to almost 13% \[20\] (1-sigma calculation). However, if the correlations among the even zonal harmonic coefficients are neglected and the variance matrix is used in a 1-sigma Root–Sum–Square fashion, the error due to the even zonal harmonics of geopotential amounts to 46.6% \[20\]. Such approach is considered more realistic by some authors \[13\] because nothing assures that the correlations among the even zonal harmonics of the covariance matrix of the EGM96 model, which has been obtained during a multidecadal time span, would be the same during an arbitrary past or future time span of a few years as that used in the LAGEOS–LAGEOS II Lense–Thirring experiment. A 1-sigma upper bound of almost 83% for the gravitational error can be obtained by adding the absolute values of the individual errors \[20\]. Another point to be emphasized is that the use of the perigee of LAGEOS II forces to adopt an observational time span of many years in order to view certain
long–period harmonic perturbations of gravitational and non–gravitational origin as empirically fitted quantities which can be removed from the time series without corrupting the extraction of the genuine relativistic secular trend. Indeed, it turns out that the perigee of LAGEOS II is affected by the ocean tidal perturbation $K_1, l = 3, p = 1$, which has a period of 5 years $[21]$, and by a direct solar radiation pressure harmonic with a period of 11.6 years $[14]$. According to $[14, 15]$, the non–gravitational part of the error budget would amount to 28–30% over seven years. Moreover, recent reexaminations of certain nonconservative accelerations acting upon LAGEOS II would suggest that it could be reduced down to a 13% level $[22]$ over the same time span. However, it must be pointed out that such estimates are based on certain refinements of the non–gravitational force models which were not included $[18]$ in the GEODYN II orbit processor used at the time of the analysis of $[11]$, especially as far as certain tiny non–gravitational perturbations of thermal origin $[15]$ are concerned. Moreover, it must also be recognized that the estimates of the authors of $[13]$ are different from such evaluations; indeed, it can be argued that their evaluation of the impact of the nonconservative accelerations on the measurement of the Lense–Thirring effect with the perigee of LAGEOS II reported in $[11]$ is of the order of 48–99%, if the optimistic 13% error, based on the EGM96 full covariance, is adopted.

3 The role of the new Earth gravity models from the CHAMP and GRACE missions

From the previous considerations it could be argued that, in order to have a rather precise and reliable estimate of the total systematic error in the measurement of the Lense–Thirring effect with the LAGEOS satellites it would be better to reduce the impact of geopotential in the error budget and/or discard the perigee of LAGEOS II which is very difficult to handle and is a relevant source of uncertainty due to its great sensitivity to many non–gravitational perturbations.

The forthcoming more accurate Earth gravity models from CHAMP $[23]$ and, especially, GRACE $[24]$ will yield an opportunity to realize both these goals, at least to a certain extent.
3.1 The EIGEN-GRACE02S model

In order to evaluate quantitatively the opportunities offered by the new terrestrial gravity models we have preliminarily used the recently released EIGEN-GRACE02S gravity model [25]. It is important to note that such model represents a long-term averaged GRACE-only solution (110 days); moreover, the released sigmas of the spherical harmonic coefficients of the geopotential are not the mere formal statistical errors, but are calibrated, although preliminarily. Then, guesses of the impact of the systematic error due to the geopotential on the measurement of the Lense-Thirring effect based on this solution should be rather realistic. However, caution is advised in considering the so obtained evaluations because of the uncertainties of the calibration process which affect especially the even zonal coefficients [13].

With regard to the three-elements combination (2), it turns out that the systematic error due to the even zonal harmonics of the geopotential, according to the variance matrix of EIGEN-GRACE02S up to degree $l = 70$, amounts to 0.2 mas yr$^{-1}$ (Root Sum Square calculation), yielding a 1-sigma 0.4% error in the Lense-Thirring effect. The sum of the absolute values of the individual errors yields an error of 0.4 mas yr$^{-1}$, i.e. a 1-sigma upper bound of 0.7% in the Lense-Thirring effect. Of course, even if the LAGEOS and LAGEOS II data had been reprocessed with the EIGEN-GRACE02S model, the problems posed by the correct evaluation of the impact of the non–gravitational perturbations on the perigee of LAGEOS II would still persist.

A different approach could be followed by taking the drastic decision of canceling out only the first even zonal harmonic of the geopotential by discarding at all the perigee of LAGEOS II. The hope is that the resulting gravitational error is reasonably small so to get a net gain in the error budget thanks to the fact that the nodes of LAGEOS and LAGEOS II exhibit a very good behavior with respect to the non–gravitational perturbations. Indeed, they are far less sensitive to them than the perigee of LAGEOS II. Moreover, they can be easily and accurately measured, so that also the formal, statistical error should be reduced. A possible observable is [26]

$$\delta \dot{\Omega}^L + 0.546 \delta \dot{\Omega}^{L \ II} \sim 48.2 \mu_{LT}. \quad (3)$$

A similar proposal can be found in [24], although numerical details are not released there. According to the variance matrix of EIGEN-GRACE02S up to degree $l = 70$, the residual signal due to the even zonal harmonics from $l = 4$ to $l = 70$ is 1.5 mas yr$^{-1}$ (Root Sum Square calculation), i.e. a 1-sigma 3% systematic bias in the Lense-Thirring effect. The sum of the
absolute values of the individual errors yields an upper bound of 1.9 mas yr$^{-1}$, i.e. a 1-sigma 4% systematic error. EGM96 would not allow to adopt (3) because its full covariance matrix up to degree $l = 70$ yields an error of 47.8% while the error according to its diagonal part only amounts even to 104% (1-sigma Root Sum Square calculation), with an upper bound of 177% (1-sigma sum of the absolute values of the individual errors). Note also that the combination (3) preserves one of the most important features of the combination of (2) of orbital residuals: indeed, it allows to cancel out the very insidious 18.6-year tidal perturbation which is a $l = 2, m = 0$ constituent with a period of 18.6 years due to the Moon’s node and nominal amplitudes of the order of $10^3$ mas on the nodes of LAGEOS and LAGEOS II [21]. On the other hand, the impact of the non–gravitational perturbations on the combination (3) over a time span of, say, 7 years could be quantified in 0.1 mas yr$^{-1}$, yielding a 0.3% percent error. The results of Table 2 and Table 3 in [8] have been used. It is also important to notice that, thanks to the fact that the periods of many gravitational and non–gravitational time–dependent perturbations acting on the nodes of the LAGEOS satellites are rather short, a reanalysis of the LAGEOS and LAGEOS II data over just a few years could be performed. As already pointed out, this is not so for the combination (2) because some of the gravitational [21] and non–gravitational [14] perturbations affecting the perigee of LAGEOS II have periods of many years. Then, with a little time–consuming reanalysis of the nodes only of the existing LAGEOS and LAGEOS II satellites with the EIGEN-GRACE02S data it would at once be possible to obtain a more accurate and reliable measurement of the Lense–Thirring effect, avoiding the problem of the uncertainties related to the use of the perigee of LAGEOS II.

The choice of an observational time span of just a few years would also be helpful in reducing the impact of the secular variations of the uncancelled even zonal harmonics $\dot{J}_4$ and $\dot{J}_6$. The problem of the impact of the secular variations of the even zonal harmonics on the proposed measurements of the Lense-Thirring effect has never been addressed, up to now, in a satisfactorily way. In [27] it has been claimed, perhaps too superficially, that the secular variations of the zonals would not affect the combination (3) because they can be accounted for by an effective $\dot{J}_2^{\text{eff}}$. In fact, the effective $\dot{J}_2$ means a lumped effect that it has not been possible to separate with one or two satellites. But the individual effects are still there; they just getted blurred. One can use the lumped effect to get some insight into the total error in the secular rates of the zonals, but it tells nothing about the individual
contributions. So, it is not possible to cancel them out in the combination\(^1\)\(^2\). For the topic of \(\dot{J}_i\), which has recently received great attention by the geodesists’ community in view of unexpected variations of \(J_2\), see \(^2\)\(^8\). By assuming \(\delta\dot{J}_4 = 0.6 \times 10^{-11} \text{ yr}^{-1}\) and \(\delta\dot{J}_6 = 0.5 \times 10^{-11} \text{ yr}^{-1}\) \(^2\(^8\), it turns out that the 1-sigma error on the combination \(^3\) would amount to 2.1% over one year. However, it must be pointed out that it is very difficult to have reliable evaluations of the secular variations of the higher degree even zonal harmonics of geopotential also because very long time series from the various existing SLR targets are required.

### 3.2 Alternative combinations

In \(^2\(^7\) the possibility of using a multisatellite linear combination including the the nodes of LAGEOS, LAGEOS II, Ajisai, Starlette and Stella has been investigated. It is, by construction, insensitive to the first four even zonal harmonics of the geopotential. On the other hand, the inclusion of the nodes of the other existing SLR satellites, which orbit at much lower altitudes than the LAGEOS satellites, introduces, in principle, much more noise from the higher degree even zonal harmonics which such combination would be sensitive to. The hope was that the improvements in the knowledge of just the higher degree even zonal harmonics from the new GRACE-based solutions would make such an alternative combination competitive with the two-nodes combination of \(^8\). The recent results from EIGEN-GRACE02S rule neatly out this possibility. Indeed, it turns out that, if, on the one hand, the problem of \(\dot{J}_i\) would be greatly reduced, on the other hand, the 1-sigma Root Sum Square percent error would be 13% with an upper bound of 42% from the sum of the absolute values.

More favorable and interesting, at least in principle, is the situation with another node-only combination proposed in \(^2\(^9\). It includes the nodes of LAGEOS, LAGEOS II, Ajisai and the altimeter satellite Jason-1 whose orbital parameters are similar to those of Ajisai; apart from the LAGEOS satellites, Ajisai and Jason-1 have the most interesting orbital configurations, among those of the existing accurately tracked satellites, for our purposes. The weighing coefficients of the nodes are 1 for LAGEOS, 0.347 for LAGEOS II, -0.005 for Ajisai and 0.068 for Jason-1; the gravitomagnetic slope is 49.5 mas yr\(^{-1}\). It turns out that the EIGEN-GRACE02S model yields a systematic gravitational error of 1% (1-sigma Root Sum Square calculation) and an upper bound of 2%. Moreover, \(\dot{J}_4\) and \(\dot{J}_6\) would not affect this combination.

\(^1\)I am grateful to J. Ries for helpful discussions on this problem.

\(^2\)Fortunately, any issues concerning \(J_2\) do not affect the combination \(^3\).
However, the possibility of effectively getting long time series of the node of Jason-1 should be demonstrated in reality. Finally, dealing suitably with the non-gravitational perturbations acting on it in a genuine dynamic way would be a very demanding task.

4 Conclusions

When more robust and complete terrestrial gravity models from CHAMP and GRACE will be available in the near future, the two-nodes/LAGEOS-LAGEOS II combination \(^3\) could allow for a measurement of the Lense-Thirring effect with a total systematic error, mainly due to geopotential, of the order of a few percent over a time span of some years without the uncertainties related to the evaluation of the impact of the non-gravitational perturbations acting upon the perigee of LAGEOS II. The choice of a not too long observational time span should also be helpful in keeping the systematic error due to the secular variations of the even zonal harmonics below the 10\% level.

On the other hand, the obtainable accuracy with the node-node-perigee combination \(^2\), whose error due to geopotential will remain smaller than that of \(^3\), is strongly related to improvements in the evaluation of the non-gravitational part of the error budget and to the use of time spans of many years. However, it neither seems plausible that the error due to the non-conservative forces will fall to the 1\% level nor that a reliable and undisputable assessment of it will be easily obtained.

Alternative combinations including the orbital data from the other existing SLR satellites are not competitive with the combination of \(^3\).

A combination including also the nodes of the SLR Ajisai and altimeter Jason-1 satellites, together with the nodes of the two LAGEOS satellites, would be slightly better from the point of view of the reduction of the systematic error due to the geopotential, in particular with regard to the effects of the secular variations of the even zonal harmonics. However, this gain could be lost due to the difficulties of dealing with the non-gravitational perturbations affecting the nodes of Jason-1.

Acknowledgements

L. Iorio is grateful to L. Guerriero for his support to him in Bari and to the GFZ team for the public release of the EIGEN-GRACE02S gravity model. Thanks also to J. Ries for stimulating and helpful discussions.
References

[1] I. Ciufolini and J.A. Wheeler, *Gravitation and Inertia*, (Princeton University Press, Princeton, 1995)

[2] L. Stella et al., in *Nonlinear Gravitodynamics*, edited by R. Ruffini and C. Sigismondi (World Scientific, Singapore, 2003), p. 235.

[3] K. Nordvedt, in *Nonlinear gravitodynamics*, edited by R. Ruffini and C. Sigismondi (World Scientific, Singapore, 2003), p. 35.

[4] L. Schiff, Am. J. Phys. 28, 340 (1960).

[5] C.W.F. Everitt et al., in *Gyros, Clocks, Interferometers....Testing Relativistic Gravity in Space*, edited by C. Lämmerzahl, C.W.F. Everitt and F.W. Hehl (Springer, Berlin, 2001), p. 52.

[6] J. Lense and H. Thirring, Phys. Z. 19, 156 (1918), english translation by B. Mashhoon, F. W. Hehl, and D. S. Theiss *Gen. Relativ. Gravit.* 16, 711 (1984).

[7] I. Ciufolini, Phys. Rev. Lett. 56, 278 (1986).

[8] L. Iorio, D. Lucchesi and I. Ciufolini, Class. Quantum Grav. 19, 4311 (2002).

[9] L. Iorio et al., Class. Quantum Grav. 21, 2139 (2004).

[10] C. Lämmerzahl, H. Dittus, A. Peters and S. Schiller, Class. Quantum Grav. 18, 2499 (2001).

[11] I. Ciufolini et al., Science 279, 2100 (1998).

[12] I. Ciufolini, [gr-qc/0209109](https://arxiv.org/abs/gr-qc/0209109)

[13] J. C. Ries, R. J. Eanes and B. D. Tapley, in *Nonlinear Gravitodynamics*, edited by R. Ruffini and C. Sigismondi (World Scientific, Singapore, 2003), p. 201.

[14] D. Lucchesi, Pl. Space Sci. 49, 447 (2001).

[15] D. Lucchesi, Pl. Space Sci. 50, 1067 (2002).

[16] D. Lucchesi, Geophys. Res. Lett. 30, 1957 (2003).

[17] D. Lucchesi, Celest. Mech. Dyn. Astron. 88, 269 (2004).
[18] D. Lucchesi, et al., Pl. Space Sci. 52, 699 (2004).

[19] F. G. Lemoine et al., NASA/TP-1998-206861, 1998.

[20] L. Iorio, Celest. Mech. Dyn. Astron. 86, 277 (2003).

[21] L. Iorio, Celest. Mech. Dyn. Astron. 79, 201 (2001).

[22] D. Lucchesi (private communication).

[23] E. Pavlis, in Recent Developments in General Relativity, edited by R. Cianci, R. Collina, M. Francaviglia and P. Fré (Springer, Milan, 2000), p. 217.

[24] J.C. Ries et al., in Proceedings of the 13th International Laser Ranging Workshop, Washington DC, October 7-11, 2002, http://cddisa.gsfc.nasa.gov/lw13/lw_proceedings.html#science

[25] Ch. Reigber et al., J. of Geodynamics, in press, (2004).

[26] L. Iorio, in Earth Observation with CHAMP. Results from Three Years in Orbit, edited by Ch. Reigber, H. Lühr, P. Schwintzer and J. Wickert. (Springer, Berlin, 2004), p. 187.

[27] L. Iorio and A. Morea, Gen. Rel. Grav. 36, 1321 (2004).

[28] C. Cox et al., in Proceedings of the 13th International Laser Ranging Workshop, Washington DC, October 7-11, 2002, http://cddisa.gsfc.nasa.gov/lw13/lw_proceedings.html#science

[29] L. Iorio and E. Doornbos, Preprint gr-qc/0404062