Identification and quantification of vortical structures in wind turbine wakes for operational wake modeling

Y Marichal¹, I De Visscher¹, P Chatelain¹,² and G Winckelmans¹,²

¹ Wake Prediction Technologies, Rue Louis de Geer 6, 1348 Louvain-la-Neuve, Belgium
² Institute of Mechanics, Materials and Civil Engineering, Université catholique de Louvain, 1348 Louvain-la-Neuve, Belgium
E-mail: yves.marichal@wapt.be

Abstract. The present paper describes a method to quantify the vortical structure characteristics from simulation results of the flow past a wind turbine, with the aim to develop an accurate, physics-based operational wake model. The wake centerline is first identified. Then, the flow characteristics are extracted by fitting a vorticity-based wake skeleton onto the velocity deficit profiles defined around the centerline and measured at several downstream distances from the rotor. The simulation results were obtained using a hybrid Vortex Particle-Mesh approach combined with an immersed Lifting Line technique to account for the blades. The characterization of the identified vortex wake structure lays a basis for the development of an operational wake model based on strong physical grounds.

1. Introduction

Currently available wake models used for the prediction of the performance of single wind turbine and wind farms have become more and more sophisticated [1, 2]. Yet, many of the operational variants among them still lack strong physical grounds [3, 4]. The present work constitutes a step towards a physics-based model capable of accurately predicting in real- or fast-time the behaviors of a wind turbine and its wake with respect to upstream conditions; including atmospheric turbulence and also turbulent wakes from upstream turbines. The model dynamic response should rely as much as possible on realistic vortex dynamics. We consider that a vorticity-based skeleton is likely the best suited candidate for such a model; see also [5]. With that target in mind, the present paper focuses on the identification and extraction of vortical structures from realistic wind turbine wake flows, and of their simplified characterization. A Vortex Particle-Mesh flow solver was used here to compute the flow past a wind turbine for various ambient conditions and operational points.

Section 2 describes the simulation tools used to compute the 3D unsteady flow field. Based on that, a wake model is presented in order to provide an accurate -yet simplified- description of the flow and of its main structures, relying on two main elements: a Vortex Sheet Tube (VST) and a Vortex Dipole Line (VDL). Results are shown in Section 3 for different simulation results. It is further argued that the procedure should be applied to short time averaged results in order to properly represent the flow structures. Finally, conclusions are drawn in Section 4 and some perspectives are suggested.
2. Methodology

We here consider the flow past a wind turbine of diameter $D$ (radius $R = D/2$) with the hub centered at $x = 0$. The upstream flow $U$ is aligned with the rotor axis $\hat{e}_z$ and it is statistically uniform. The turbulence intensity is defined by $I \triangleq w_{\text{RMS}}/U$, where $w_{\text{RMS}}$ is the root mean square of the axial velocity. The usual way to characterize the operational point of the wind turbine is given by the tip speed ratio $\lambda \triangleq \Omega R/U$, with $\Omega$ the rotor speed.

2.1. Wake flow simulation

The wake flow is here obtained through simulations of a wind turbine evolving in a turbulent wind using an immersed Lifting Line-enabled Vortex Particle-Mesh (VPM) method, as presented in Chatelain et al. Various ambient turbulence intensities and tip speed ratios are investigated. The VPM flow solver relies on the vorticity formulation of the Navier-Stokes equations. Advection is handled in a Lagrangian fashion using vorticity-carrying particles, and all the remaining spatial differential operations, such as the solution of the Poisson equation to obtain the velocity, the vortex stretching term, as well as the subgrid-scale modeling term are efficiently computed on an underlying grid, thus in an Eulerian manner (information being interpolated back and forth between the particles and the grid using high order interpolation schemes). The presence of the blades is modeled using an immersed Lifting Line approach.

The approach is well suited to capture detailed flow physics of wind turbines, from the near wake to the far wake. This is enabled by the relaxation of usual linear stability constraints (CFL criterion for advection problems) and by low numerical dispersion and numerical diffusion errors. Moreover, the computational domain is very compact, thanks to the Hockney-Eastwood algorithm that allows to enforce unbounded outer conditions on the lateral domain faces.

The inflow turbulence is accounted for using vortex particles carrying the vorticity deduced from Mann’s synthetic atmospheric turbulence model, which assumes uniform shear at hub height. We here use the spectral tensor fitting the Kaimal spectra and a surface roughness adapted in order to yield the prescribed turbulence intensity.

However, the time averaged axial velocity in the inflow plane of the computational domain, which results from the discretization of Mann’s atmospheric turbulence model, is not uniform. This is undesired, as it introduces a systematic bias into the flow. Depending on the value of $I$, the time averaged flow field past the wind turbine may then present a deviation of the wake with respect to the turbine axis (the non uniform inflow velocity tends to “bend” the wake on average, just like a shear-induced deviation). A typical wake deviation can be seen in Fig. 1 with $\lambda = 7.55$ and $TI = 6.57\%$. This deviation becomes very significant, especially for higher turbulence intensities (e.g. for $TI = 15.59\%$ the distance between the wake centerline and the rotor axis is about $0.6D$ at $7D$ downstream of the rotor) and non-optimal tip speed ratios.

Since the wake model is aimed to be axisymmetric here, we wish to prevent any preferential direction in the velocity field. Therefore, some cases have been simulated with an “adjusted” Mann turbulence box. More specifically, all modes $k_z = 0$ (streamwise direction) are removed from the 3 velocity components, prior to the injection of the box into the VPM computational domain. This correction does not affect the incompressibility of the turbulent velocity field, nor does it affect the associated one-point spectra (except for the smallest wave number, which is per definition not related to a fluctuation).

2.2. Vortical structures characterization

Despite the Mann box “adjustment”, a wake deviation might subsist. This can be related to the well-known wake meandering phenomenon, consisting in a lateral fluctuation of the wake (see [1] for a detailed study). As of today, it is not clear whether this fluctuation is a direct consequence of the large eddies in the atmosphere or if it is an intrinsic wake instability (similarly to the vortex shedding occurring for bluff body flows), as was suggested in [10]. Since the VPM results
are averaged over a period $\Delta T$ corresponding to the crossing of one or two Mann box lengths through the computational domain (see results below), a significant deviation of the wake may thus still subsist if $\Delta T$ is not a multiple of the inherent wake meandering time scale.

Hence, the vortex characterization procedure starts by first identifying the wake centerline based on the time averaged velocity field, and the contribution of the wake deviation can then be readily isolated. Note that, when considering shorter time averages, this identification also allows to track the wake meandering phenomenon in an unsteady fashion, as will be shown below.

The centerline position $x_c(z)$ in any plane normal to the rotor axis is here defined as the origin of the local polar reference system leading to the “most axisymmetric” velocity deficit. More specifically, a fitting procedure is carried out by computing the least square error of the axial velocity azimuthal average with respect to the actual flow field. This can be compared to the approach from [11] where the deficit model is instead single- or double-Gaussian-shaped.

In order to gain further insight into the wake flow structure, a wake vortex model skeleton is constructed. This wake model is then calibrated using the VPM simulation results. To that end, we consider the time averaged axial velocity profiles $\overline{w}(r, z)$ at several stations $z_m$ (slices $m = 1 \ldots M$), obtained by performing an azimuthal average around $\mathbf{e}_z$ at $x_c(z_m)$ (like for the centerline identification). Note that the VPM profiles are then made axisymmetric by shifting them back onto the rotor axis, i.e. from $x_c(z_m)$ to $z_m \mathbf{e}_z$, so as to get rid of the wake deviation. The wake model as such is straight and aligned with the rotor axis. It consists of the following components (see Fig. 2):

- The near wake is represented by a regularized Vortex Sheet Tube (VST) of radius $R_{VST}(z)$, azimuthal circulation per unit length $\gamma_{VST}(z)$ and mollification parameter $\sigma_{VST}(z)$ (here using a high order algebraic regularization, see [12, 13, 14]). This element can be related to the “tangential vorticity cylinder” from Branlard and Gauñaa [5]. It is discretized using a series of circular vortex filament rings of radius $R_{VST}^k$, circulation $\Gamma_{VST}^k \triangleq \int_{F_k} \gamma_{VST} \, dz$ ($F_k$ is the region occupied by the filament $k$) and mollification parameter $\sigma_{VST}^k$. The filaments are represented using parametric cubic splines and the quadrature is based on $N$ uniformly spaced particles. The VST extends from $z_{VST}^1 = 0$ to $z_{VST}^2$.

- For the far wake, we use a regularized Vortex Dipole Line (VDL) of impulse per unit length $I_{VDL}(z)$ and mollification parameter $\sigma_{VDL}(z)$ (here using a Gaussian regularization), see [13]) which is discretized by vortex dipole particles of impulse $I_{VDL}^{p} \triangleq \int_{D_p} I_{VDL} \, dz$ ($D_p$ is the region occupied by the dipole particle $p$) and mollification parameter $\sigma_{VDL}^{p}$. The VDL

![Figure 1: Extraction of the wake centerline: centerline (magenta line/dot), rotor axis (black line/dot).](image-url)
starts at $z_{1 \text{VDL}}$.

- For the jet emanating from the hub, an *inner* regularized vortex dipole line (iVDL) is used. The iVDL is discretized by vortex dipole particles of impulse $p_{i \text{VDL}}$ and mollification parameter $\sigma_{i \text{VDL}}$ (here using a high order algebraic regularization) and it extends from $z_{1 \text{VDL}} = 0$ to $z_{2 \text{VDL}}$.

![Diagram of wake model components](image)

**Figure 2:** Sketch of the wake model components: axial velocity profiles at several $z = \text{const}$ stations (top), corresponding azimuthal vorticity fields (bottom).

Note that the impulse described above is vorticity-based and it is defined, in general, by $I = \frac{1}{2} \int x \times \omega \, dx$, where $\omega$ is the vorticity field.

One can further observe that only the velocity deficit is modeled here, i.e. swirl effects related to axial vorticity, such as the root vortex emanating from the hub, are ignored. As a perspective of future development, additional model components accounting for swirl might be considered, e.g. using elements similar to those of [5].

Let us further define $z_{\text{out}}$, the starting position for the outflow condition. The value of $z_{\text{out}}$ is determined by identifying the axial location where the “outer” deficit exceeds 95% of the maximum wake deficit in the VPM simulations (the “outer” deficit is the maximum deficit observed outside of the wake in the plane $z$, which typically occurs when a wind gust becomes significant compared to the wake). For $z \geq z_{\text{out}}$, the model is then simply extruded along $\mathbf{e}_z$. In the present case, the full model extends from $z = 0$ to $z = 20D$.

The wake model parameters are obtained by fitting the VPM velocity profiles at the different $z_m$ stations up to $z_{\text{out}}$. More specifically, the degrees of freedom of the different building blocks are as follows

- **VST**: $\gamma_{\text{VST}}(z_m)$, $R_{\text{VST}}(z_m)$ and $\sigma_{\text{VST}}(z_m)$.
- **VDL**: $i_{\text{VDL}}(z_m)$ and $\sigma_{\text{VDL}}(z_m)$.
- **iVDL**: $i^{\text{iVDL}}(z_m)$ and $\sigma^{\text{iVDL}}(z_m)$.
Based on the above control point values, the continuous model is obtained using a piecewise linear representation. The model is not fitted at the rotor position $z = 0$ and all properties at $z = 0$ are obtained using a constant extrapolation from $z_1$, except for $R_{\text{VST}}(0) = R$. The VST and VDL models are mutually exclusive at a given fitting section $z_m$, while the VST coexists with the iVDL near the rotor. The solution procedure is relaxed by looping over all slices and performing the fitting locally until global convergence, i.e. until the maximum velocity increment is lower than $5 \times 10^{-4} \Delta w_m$ for all sections, with $\Delta w_m$ the maximum local deficit.

It should be observed that the VST and VDL models become equivalent when $R_{\text{VST}} / \sigma_{\text{VST}} \rightarrow 0$ and $i_{\text{VDL}} = \pi(R_{\text{VST}})^2 \gamma_{\text{VST}}$ (impulse per unit length of a VST section) is held constant. The rationale behind using a VDL rather than a VST stems from the fact that it is far less computationally expensive to evaluate, since it is discretized using a single particle per line segment, as opposed to $N$ particles per VST filament ring. This difference will be significant for the development of an operational wake model aiming at real-time capabilities. In the far wake, the VDL provides a satisfactory approximation of the velocity profile and therefore replace the VST.

The location of the transition between the VST and VDL components is computed dynamically by first testing whether the VST model is such that $R_{\text{VST}}(z_m)/\sigma_{\text{VST}}(z_m) < 1$. In that case, the VDL model $L_2$ error is compared to the VST error and if it is smaller than 150% of the VST error, the VDL model is adopted for $z \geq z_m$. Moreover, ensuring the continuity of the model requires using a blending region (overlapping) between adjacent sections $z_m$ (i.e. $z_2^\text{VDL}$) and $z_{m+1}$ delimiting the VST and VDL models. Their respective strengths ($\gamma_{\text{VST}}$ and $i_{\text{VDL}}$) go to zero but their geometric parameters ($R_{\text{VST}}, \sigma_{\text{VST}}$ and $\sigma_{\text{VDL}}$) are linearly extrapolated across the transition.

The distance between adjacent particles is such that we satisfy a kernel overlapping condition, i.e. we here ensure that $\sigma/d \geq 1.5$ with $\sigma$ the local mollification parameter (VST, VDL or iVDL) and $d$ the distance.

The iVDL model extent $z_2^\text{VDL}$ is pre-computed based on the VPM profiles. The model fitting is enabled as long as there is a significant hub jet velocity at the wake center, i.e. as long as $(w_{hj} - \min_r \overline{w}) > 0.05 (U - \min_r \overline{w})$, where $w_{hj}$ is the hub jet velocity (maximal velocity observed near the wake center). Then we set $z_2^\text{VDL} = z_{M*} + D/2$ ($z_{M*}$ is the last section where the iVDL is fitted), $\sigma^\text{VDL}(z_2^\text{VDL}) = \sigma^\text{VDL}(z_{M*})$ and $i_{\text{VDL}}(z_2^\text{VDL}) = 0$.

Next to the wake deviation, the presence of the gusts have another side effect that affects the quality of the model calibration. In some cases, the velocity profiles do not tend to $U$ outside of the wake, as one would expect. This can be related to the fact that some gusts persist even after the temporal and azimuthal averaging. As a consequence, the wake characteristics have to be defined with respect to a different reference value $\tilde{U}$. The present approach accounts for this by adapting the “upstream” velocity $\tilde{U}(z_m)$ locally. If there is a significant velocity overshoot around the wake at $z_m$, $\tilde{U}(z_m)$ is set to the average of the 5 last points of $\overline{w}$; otherwise $\tilde{U}(z_m)$ becomes a new degree of freedom for the local fitting procedure.

3. Results

The present study focuses on the flow past the NREL-5MW offshore wind turbine [15] and the different setups are provided in Table 1 (the optimal tip speed ratio for this turbine is $\lambda = 7.55$ and I is measured at hub height, i.e. at 90m above the ground). The computational domain extends from $z = -1D$ to $z = 10D$ with $D/h = 64$ ($h$ is the mesh spacing) and the Mann box has an axial size of $16D$ and a lateral width of $3D$. The cases $R_6^0$ and $R_6^{15}$ were not run using an “adjusted” Mann box turbulent inflow. Indeed, at an optimal tip speed ratio, the wake deviation is less pronounced since the wake deficit is predominant compared to the gust amplitude, at least in the domain of interest.

For the VST component, $N = 128$ is set in order to ensure a proper kernel overlapping. The
Table 1: Description of the simulation setups (the subscript “o” indicates an optimal tip speed ratio, whereas “n” indicates a non-optimal tip speed ratio); the term “adjusted” refers to the procedure of removing the modes \( k_z = 0 \) from the Mann turbulent velocity field.

| Label, \( \lambda \), I | \( \lambda \) | I | adjusted |
|-------------------------|--------|-----|----------|
| \( R^6 \)               | 7.55   | 6.57 | no       |
| \( R^{15}_o \)          | 7.55   | 15.59| no       |
| \( R^{15}_n \)          | 5.70   | 15.47| yes      |

size of the different model components and the averaging period \( \Delta T \) are provided in Table 2; the main wake model parameters are shown in Fig. 3 for the three cases with a long time average. Fig. 4 also shows the associated axial velocity and azimuthal vorticity profiles at some stations \( z_m \) resulting, on the one hand, from the VPM simulations and, on the other hand, from the calibrated wake model. Note that all quantities have been non-dimensionalized with respect to the local \( \tilde{U} \) instead of the upstream \( U \).

Table 2: Parameters related to the fitting of the VPM results.

| VPM run | \( tU/D \) | \( \Delta T \, U/D \) | \( z_{out}/D \) | \( z_2^{VDL}/D \) | \( z_1^{VDL}/D \) |
|---------|---------|----------------|----------------|----------------|----------------|
| 1. \( R^6 \) | -       | 31.86          | 10.0           | 2.5            | 8.0            |
| 2. \( R^{15}_o \) | -       | 31.80          | 7.0            | 1.5            | 3.0            |
| 3. \( R^{15}_n \) | -       | 16.00          | 3.5            | 1.5            | 3.0            |
| 4. \( R^6 \) | 20.37   | 2.10           | 8.0            | 3.0            | 4.0            |
| 5. \( R^6 \) | 30.43   | 2.09           | 8.0            | 3.0            | 4.5            |

The wake model is able to represent quite well both profiles in all three cases (recall that the vorticity profiles are not taken into account during the fitting procedure, though). One may observe that, near the rotor, the axial velocity overshoot in the vicinity of the blade tips is not as well captured. To the authors’ opinion, this might be related to a missing potential flow contribution which comes from the blockage effect induced by the wind turbine on average.

As expected, the deficit recovery is faster when the turbulence intensity increases. Similarly, the hub jet breakdown occurs earlier for \( R^{15}_o \) compared to \( R^6 \). Despite an optimal tip speed ratio for \( R^6 \) and \( R^{15}_o \) (and hence a similar energy extraction), a significant difference in the velocity profile is already visible one diameter behind the rotor. At constant I, the deficit is lower for \( R^{15}_n \) than for \( R^{15}_o \) which is due to the fact that the tip speed ratio is non-optimal, again as expected.

Even though the model provides accurate velocity profiles in the VST-VDL transition region, some fluctuations of the fitting parameters across that region are present. Yet, the main overall trends can still be identified.

The “strength” of the wake can be related to the impulse per unit length and it is clearly decreasing in the wake along \( z \) (top left in Fig. 3). On the contrary, the mollification parameter \( \sigma \) is seen to be steadily increasing, due to the deficit spreading (top right in Fig. 3). One should note that the meaning of this parameter is not the same in the VST and VDL regions since the first component is smoothed using a high order algebraic kernel, whereas the second one relies on a Gaussian mollification (as discussed in Section 2.2). In the near wake VST region, the
value of $\sigma$ is very similar for $R_{15}^o$ and $R_{15}^n$ (same $I$) and slightly smaller for $R_6^o$. The associated growth rate is approximately the same for all three cases. The impulse per unit length seems to be dependent on both $I$ and $\lambda$, except at the rotor where the values for $R_6^o$ and $R_{15}^o$ seem to be quite close to each other. Indeed, the onset of the wake mostly depends on the tip speed ratio.

The VST radius remains nearly constant for the optimal tip speed ratio cases ($R_6^o$ and $R_{15}^o$), while it is slightly decreasing for the degraded extraction case $R_{15}^n$. We stress that this radius does not correspond to the classical wake radius definition, which is typically expanding downstream (the wake radius definition is then based on the radial distance where the velocity deficit is a certain fraction of that on the centerline). The expansion of the wake is here also related to the mollification parameter $\sigma$.

![Figure 3: Wake model parameters: $R_6^o$ (blue), $R_{15}^o$ (red) and $R_{15}^n$ (green); VST (——) and VDL (- - - -); fitting stations $z_m$ (○).](image)

Keep in mind that the above results are obtained by fitting a long time averaged velocity field. This has strong implications on the deficit measured in the far wake. As a matter of fact, a significant smearing occurs due to the wake meandering phenomenon. Fig. 5 provides an overview of the effect of different time averaging periods $\Delta T$, by showing the associated axial velocity field, also at two different stages. The wake meandering time scale is expected to be of the order of $D/U$ and the fluctuating lateral wake deviation can be clearly identified for $\Delta T U/D = 2.10$.

The average available power as would be seen by a second turbine downstream of the first one scales like $\overline{w^3}$ which is lower than $\overline{\overline{w^3}}$ due to the turbulent fluctuations, and thus also to the slow wake meandering effect. Moreover, these time fluctuations also affect the blade structures through increased fatigue loading. For a wake that would not fluctuate laterally (i.e. without
wake meandering, yet with the same decay properties of the deficit along the centerline), the available power would be even smaller and, in that sense, the wake meandering can be considered beneficial [16], though. Therefore, it is important to account for (low frequency) unsteady phenomena when designing an operational wake model aiming at the prediction of the available power and of the associated fluctuations in space and time; as was also shown in [1].

Applying the same wake characterization procedure to a velocity field averaged over a shorter period of time $\Delta T U/D = 2.10$ confirms the above assertion about the higher deficit compared to a long time average, especially in the far wake, as can be seen in Fig. 6.

The fitting of the short time averaged wake further reveals that the VDL mollification parameter in the far wake remains smaller in the short time average case, hence with a slightly

Figure 4: Axial velocity profiles (left) and azimuthal vorticity profiles (right): $R_6^0$ (blue), $R_{15}^0$ (red) and $R_{n}^{15}$ (green); VPM (dark ——) and fitted (light - - - -).
smaller expansion rate; see Fig. 7 (as a side note, we mention that the value for $z_{\text{out}} = 8D$ has been fixed here). As a consequence, the associated spreading is also less pronounced. Note that, near the rotor, the fitting parameters are very close for both the long and short time average cases: this is as expected, as the wake meandering starts further downstream with an associated fluctuation amplitude that grows with $z$. From $z = 2D$ on, one may observe for the short time averaged cases a slight time variability in the fitting parameters which is also reflected in the velocity and vorticity profiles from Fig. 6. This can be related to the strength of the incoming gusts and, therefore, larger fluctuations of the deficit can be expected for higher values of $I$.

### 4. Conclusion and perspectives

Based on realistic simulation results obtained using a Vortex Particle-Mesh method, an approach has been presented that allows the identification and extraction of the wake flow properties behind a wind turbine. The described method can be used to deduce evolution laws for the main parameters determining the wake skeleton. To that end, the main overall trends of these parameters have been presented. It has been further shown that it is necessary to account for low frequency unsteady phenomena, such as the wake meandering phenomenon, in order to provide an accurate description of the wake flow and of the power that is available on average behind a wind turbine. Based on these results, the probability density function of the wake deviation could also be quantified as a function of $\lambda$ and $I$.
Figure 6: Axial velocity profiles (left) and azimuthal vorticity profiles (right) for $R_o^6$: $\Delta T U/D = 31.86$ (blue) and $\Delta T U/D = 2.10$ with $tU/D = 20.37$ (orange) and $tU/D = 30.43$ (magenta); VPM (dark ——) and fitted (light - - - -).

Figure 7: Wake model parameters for $R_o^6$: $\Delta T U/D = 31.86$ (blue) and $\Delta T U/D = 2.10$ with $tU/D = 20.37$ (orange) and $tU/D = 30.43$ (magenta); VST (——) and VDL (- - - -); fitting stations $z_m (\bigcirc )$.

As a perspective of application, the lessons learned from this procedure will be exploited for developing a robust, accurate and physics-based model that will account for different flow (wind turbulence, wind shear, wake rotation effects, misalignment effects, other wake, meandering) and operational conditions (tip speed ratio, pitch angle).
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