Charged vs. Neutral particle creation in expanding Universes: A Quantum Field Theoretic treatment.

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November 20, 2018

Abstract

A complete quantum field theoretic study of charged and neutral particle creation in a rapidly/adiabatically expanding Friedman-Robertson-Walker metric for an O(4) scalar field theory with quartic interactions (admitting a phase transition) is given. Quantization is carried out by inclusion of quantum fluctuations. We show that the quantized Hamiltonian admits an su(1,1) invariance. The squeezing transformation diagonalizes the Hamiltonian and shows that the dynamical states are squeezed states. Allowing for different forms of the expansion parameter, we show how the neutral and charged particle production rates change as the expansion is rapid or adiabatic. The effects of the expansion rate versus the symmetry restoration rate on the squeezing parameter is shown.

1 Introduction

In the cosmological context, early studies on the creation of particles in curved spacetimes have concentrated on neutral scalar fields [1], [2]. While these studies led to an understanding of relationships between particle physics in the early universe and cosmology, the inclusion of spontaneous symmetry breaking coupled with the HEP scenario for the early universe has led to the recent resurgence in

*Presented by B.Bambah at the International Congress of Mathematical Physics (ICMP2003), Lisbon Portugal, July 2003.
inflation being driven by the fluctuations in quantum fields undergoing symme-
try breaking (preheating) \[3, 4\]. The basic methodologies used for such studies
has largely been the Landau -Ginzburg model \[6\]. In this paper we ad vocate
the use of a Hamiltonian generated through an O(4) linear sigma model with
quartic interactions for such studies. The Hamiltonian is a generic Hamiltonian
applicable to many domains \[5\]. In this paper we show how it can be used to
determine the production rates of charged vs. neutral particles in an expanding
metric. We also show how squeezed states are the correct dynamical states for
the fields in expanding spacetimes. We hope to show in a later communication,
how these states may be used to study the "tachyonic" inflationary cosmological
models of current interest.

2 The Hamiltonian.

The growth of long wavelength fluctuations during breaking of symmetries has
been studied recently for the case of a single scalar field with quartic interactions
\[3\]. While they have used Lattice simulations, we advocate a model Hamiltonian
for quantum fluctuations of charged and neutral scalar fields which we shall
derive here. Our starting point is the O(4) linear sigma model with an action
given by:

\[
S = \int d^3x dt a(t)^3 \left( \frac{1}{2} \dot{\Phi}_i^2 - \frac{1}{2a^2} (\nabla \Phi_i)^2 - \frac{1}{2} m^2 \Phi_i^2 - \frac{\lambda}{4}(\Phi_i^2 - v^2)^2 \right),
\]

with

\[
\Phi_i = \begin{pmatrix}
\Phi_1 \\
\Phi_2 \\
\Phi_3 \\
\Phi_4
\end{pmatrix}
\]

where \(\Phi_i, i = 1..4\) are real scalar fields. The background spacetime is taken to
be a FRW model:

\[
ds^2 = dt^2 - a(t)^2 dx^2,
\]

where \(a(t)\) is the expansion parameter. We now use a background field analysis
to study the quantum effects. Assume \(\Phi_i\) has a background classical component
\(\phi_i\) which satisfies the classical equations of motion:

\[
\frac{\delta S}{\delta \Phi_i} |_{\Phi_i = \phi_i} = 0.
\]

Treat the quantum field \(\hat{\phi}_i\) as fluctuation around classical solution:

\[
\Phi_i \rightarrow \phi_i + \hat{\phi}_i
\]

since \(\phi_i\) satisfies the classical equations of motion,

\[
S = S[\phi_i] + \frac{1}{2} \hat{\phi}_i \frac{\delta^2 S}{\delta \Phi_i \delta \Phi_j} |_{\Phi_i = \phi_i} \hat{\phi}_j + \cdots
\]
We shall deal with a quadratic fluctuation action given simply by:

$$S_2 = \frac{1}{2} \delta \hat{\phi}_i \delta \Phi_j \big|_{\Phi = \phi \hat{\phi}_j}. \quad (7)$$

For the particular scalar field action given above, assuming all fields vanish at infinity (asymptotically flat metrics),

$$\frac{\delta S}{\delta \Phi_j} = -\partial_\mu (a^3 g^{\mu\nu} \partial_\nu \Phi_j) - a^3 m^2 \Phi_j - a^3 \frac{\delta V}{\delta \Phi_j}. \quad (8)$$

Imposing the classical equations of motion, we find that

$$\partial_\mu (a^3 g^{\mu\nu} \partial_\nu \phi_i) + a^3 m^2 \phi_i + a^3 \frac{\partial V}{\partial \phi_i} = 0 \quad (9)$$

where $\frac{\partial V}{\partial \phi_i} \equiv \frac{\partial V}{\partial \Phi_i} \big|_{\phi_i}$. The equations of motion in this metric are

$$3 \frac{\dot{\hat{\phi}}_i}{a} + \phi_i^2 - \frac{1}{a^2} \sum_j \phi_j^2 + m^2 \phi_i + \frac{\partial V}{\partial \phi_i} = 0. \quad (10)$$

Since we are interested in the dynamics of the fluctuation field, we shall treat the fluctuation field in $S_2$ as a classical field and $S_2$ itself as the classical action for its dynamics. We define a Lagrangian density for studying the dynamics of the fluctuations, $\mathcal{L}$, as follows:

$$\mathcal{L} = \frac{a^3}{2} \hat{\phi}_i^2 - \frac{1}{a^2} (\sum \hat{\phi}_i)^2 - m^2 \hat{\phi}_i^2 - \hat{\phi}_i \frac{\partial^2 V}{\partial \Phi_i \partial \Phi_j} \big|_{\phi \hat{\phi}_j}. \quad (11)$$

Carrying out a Legendre transformation, it is easy to write down the Hamiltonian density

$$\mathcal{H} = \frac{1}{2a^3} \dot{p}_i^2 + \frac{a}{2} (\sum \hat{\phi}_i)^2 + \frac{a^3 m^2}{2} \hat{\phi}_i^2 + \frac{a^3}{2} (\hat{\phi}_i \frac{\partial^2 V}{\partial \Phi_i \partial \Phi_j} \big|_{\phi \hat{\phi}_j}) - \frac{\lambda a^3 v^2}{2} \phi_i^2 \quad (12)$$

where

$$\dot{p}_i = \frac{\delta L}{\delta \dot{\phi}_i} = a^3 \dot{\phi}_i. \quad (13)$$

We also have

$$\frac{\partial^2 V}{\partial \Phi_i \partial \Phi_j} \big|_\phi = 2 \lambda \phi_i \phi_j + \lambda (\phi_k^2 - v^2) \delta_{ij}. \quad (14)$$

Assume that the fluctuation field $\Phi_i$ decomposes into its constituents as:

$$\hat{\phi} = < \Phi > - \phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_0 \\ \Sigma \end{pmatrix}. \quad (15)$$
The physical fields are defined so that
\[ \phi_+ = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2); \quad \phi_- = \frac{1}{\sqrt{2}}(\phi_1 - i\phi_2). \] (16)

Analogously, we define the classical background fields as:
\[ v_+ = \frac{1}{\sqrt{2}}(v_1 + iv_2); \quad v_- = \frac{1}{\sqrt{2}}(v_1 - iv_2), \] (17)

following the identification:
\[ \phi = \begin{pmatrix} v_1 \\ v_2 \\ v_3 = v \\ \sigma \end{pmatrix} \equiv <\Phi>. \] (18)

It is easy to see that a Legendre transformation provides a Hamiltonian given as:
\[ H = H_{\text{neutral}} + H_{\text{charged}} + H_{\text{mixed}}, \] (19)

where
\[ H_{\text{neutral}} = \int d^3x \alpha^3 \left\{ \frac{p_0^2}{2a^6} + \frac{1}{2a^2}(\nabla \phi_0)^2 + \frac{1}{2}(m_\phi^2)(\phi_0)^2 + \frac{(\Omega_\phi^2 - \omega_\phi^2)}{2a^6} \phi_0^2 \\ + \frac{p_0^2}{2a^6} + \frac{1}{2a^2}(\nabla \Sigma)^2 + \frac{1}{2}(m_\Sigma^2)(\Sigma)^2 + \frac{1}{2a^6}(\Omega_\Sigma^2 - \omega_\Sigma^2)(\Sigma)^2 \right\}, \] (20)

\[ H_{\text{charged}} = \int d^3x \alpha^3 \left\{ \frac{1}{a^6}((p_+ + p_-) \\ + \frac{1}{a^2}(\nabla \phi_-)(\nabla \phi_+ + (m_\phi^2)(\phi_+\phi_-) + \frac{1}{a^6}(\Omega_\phi^2 - \omega_\phi^2)(\phi_+\phi_-)) \right\}, \] (21)

\[ H_{\text{mixed}} = \int d^3x \alpha^3 \left\{ \lambda(v_+^2\phi_+^2 + v_-^2\phi_+^2) \\ + (2\lambda)(v_+v_3\phi_+\phi_0 + v_-v_3\phi_+\phi_0 + \sigma v_3\Sigma_\phi_0 + v_+\sigma\phi_-\Sigma + \sigma v_-\Sigma_\phi_0) \right\}, \] (22)

Where we have also put
\[ \frac{\Omega_\phi^2 - \omega_\phi^2}{a^6} = \lambda[2v_3^2]; \quad \frac{\Omega_\phi^2 - \omega_\phi^2}{a^6} = \lambda[2v_+v_-]; \quad \frac{\Omega_\Sigma^2 - \omega_\Sigma^2}{a^6} = \lambda[2\sigma^2]. \] (23)

and
\[ 2v_+v_- + v_3^2 + \sigma^2 = v^2. \] (24)

This is the most general Hamiltonian for the O(4) scalar field system, now broken up into a charged scalar field and two neutral scalar fields in the background.
Here we shall simplify the parametrization of the background field to two angles, \( \theta \) and \( \rho \) by letting \( \alpha = \frac{\pi}{4} \): then, \( v_1 = \frac{1}{\sqrt{2}} \cos(\rho) \sin(\theta) \), \( v_2 = \sin(\rho) \sin(\theta) \), and \( \sigma = \cos(\theta) \). Canonical quantization gives the mode decomposed Hamiltonian:

\[
H_{neutral} = \int \frac{d^3k}{(2\pi)^3} \left( \frac{\omega_\phi}{a^3} (b_k^+ b_k + c_k c_k^+) + \frac{\omega_\phi}{2a^3} (\frac{\Omega_\phi^2}{\omega_\phi} - 1) (b_k^+ b_k + c_k c_k^+) \right)
\]

\[
H_{charged} = \int \frac{d^3k}{(2\pi)^3} \left( \frac{\omega_\phi}{a^3} (b_k^+ b_k + c_k c_k^+) + \frac{\omega_\phi}{2a^3} (\frac{\Omega_\phi^2}{\omega_\phi} - 1) (b_k^+ b_k + c_k c_k^+) \right)
\]

\[
H_{mixed} = \int \frac{d^3k}{(2\pi)^3} \left( \frac{\lambda_3 v^2 \cos^2(\rho) \sin^2(\theta)}{4 \omega_\phi} (b_k a_{-k} + b_k^+ c_k^+) + \frac{\lambda_3 v^2 \sin(\rho) \sin(\theta) \cos(\theta)}{\sqrt{\omega_\phi \Omega_\Sigma}} \left( b_k a_{-k} + b_k^+ c_k^+ + d_k^+ d_k + d_k a_k + d_k^+ a_k^+ \right) \right)
\]

\[
\left( b_k a_{-k} + b_k^+ c_k^+ + d_k^+ d_k + d_k a_k + d_k^+ a_k^+ \right)
\]

where

\[
\frac{\omega_\phi^2(k)}{a^3} = \frac{\omega_\phi^2(k)}{a^3} = \frac{\omega_\phi^2(k)}{a^3} = (m_\phi^2 + \frac{k^2}{a^2}); \quad \frac{\omega_\Sigma^2(k)}{a^3} = (m_\Sigma^2 + \frac{k^2}{a^2})
\]
and
\[ \frac{\Omega^2(k)}{a^6} = \frac{k^2}{a^2} + m^2 + \lambda(v^2 + 2v_c^2); \quad \frac{\Omega^2_{\phi}(k)}{a^6} = \frac{k^2}{a^2} + m^2_{\phi} + \lambda(v^2 + 2v_+ v_-). \] (30)

An important point to note here is that while \( \Omega_{\phi}(k) \) and \( \omega_{\phi}(k) \) are momentum dependent, for ease of notation we will drop the \( k \) dependence for further calculations and revive it when necessary. Clearly, there are two interesting cases to be considered here: \( \theta = 0 \) and \( \rho = \frac{n}{2} \). For the purposes of this study, we shall only consider the first case, \( \theta = 0 \). Its easy to see that \( H \) reduces to
\[
H = \int \frac{d^3k}{(2\pi)^3} \left[ \frac{\omega_{\phi}}{2a^3} (a^\dagger_k a_k + a_k a^\dagger_k) + \frac{\Omega^2_{\phi}}{4a^3} (\frac{\Omega_{\phi}^2}{\omega_{\phi}^2} - 1) (a^\dagger_k a_k + a_k a^\dagger_k + a_{-k} a_k + a_{-k} a^\dagger_k) + \frac{\Omega_{\phi}^2}{2a^3} (d_k^\dagger d_k + d_k d_k^\dagger) + \frac{\omega_{\phi}^2}{4a^3} (\frac{\Omega_{\phi}^2}{\omega_{\phi}^2} - 1) (b_k^\dagger b_k + c_k c_k^\dagger + e_k e_k^\dagger) \right].
\] (31)

This Hamiltonian has an \( su(1,1) \) symmetry and can be diagonalized by a series of Bogolyubov transformations: In the neutral sector,
\[ A_k(t,r) = \mu(r,t) a_k + \nu(r,t) a_{-k}^\dagger = U^{-1}(r,t) a_k U(r,t). \] (32)

Similarly for the sigma field, \( D_k(t,r') \) and \( D_k^\dagger(t,r') \) are defined in analogy with the definition of \( A_k(t,r) \) and \( A_k^\dagger(t,r) \) with the \( d \)'s replacing the \( a \)'s. For the charged sector
\[ C_k(t,r) = \mu c_k + \nu b_{-k}^\dagger = U^{-1}(r,t) c_k U(r,t) \] (33)
\[ B_k(t,r) = \mu c_{-k} + \nu b_k^\dagger = U^{-1}(r,t) b_k U(r,t) \] (34)

where
\[
\mu = Cosh(r) = \sqrt{\frac{1}{2} \left[ \frac{\Omega_{\phi}}{\omega_{\phi}} + \frac{\omega_{\phi}}{\Omega_{\phi}} \right] + 1} \quad \nu = Sinh(r) = \sqrt{\frac{1}{2} \left[ (\frac{\Omega_{\phi}}{\omega_{\phi}} - \frac{\omega_{\phi}}{\Omega_{\phi}}) - 1 \right]}
\] (35)

and \( r \) is the squeezing parameter, satisfying \( \mu^2 - \nu^2 = 1 \) as required for a squeezing transformation. The complete unitary matrix for the squeezing transformation may be written down as
\[
U(r,t) = e^{\int \frac{d^3k}{(2\pi)^3} r(k,t)((a_k^\dagger a_k^\dagger - a_k a_{-k}) + (d_k^\dagger d_k^\dagger - d_k d_{-k}) + (c_k^\dagger c_k^\dagger + b_k b_{-k}) - (c_{-k}^\dagger c_{-k}^\dagger + b_{-k}^\dagger b_{-k}^\dagger))}
\] (36)
Collecting all our results for the neutral and charged sectors, the total diagonalized Hamiltonian is written in terms of various creation and annihilation operators as:

\[
H = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2a^3} \Omega \phi \left\{ (A^\dagger_k A_k + 1) + (C^\dagger_k C_k + B^\dagger_k B_k + 1) \right\} + \Omega \Sigma (D^\dagger_k D_k + \frac{1}{2}). \tag{37}
\]

Since the \(\Sigma\) field decouples, we drop all terms associated with it whenever it is not essential to our arguments. The total dynamical Hamiltonian for the neutral and charged scalar fields in terms of the creation and annihilation operators \((a, a^\dagger, c, c^\dagger, b, b^\dagger)\) simplifies to:

\[
H_0 = \int \frac{d^3k}{(2\pi)^3} \frac{\Omega}{a^3} \left\{ 2(\mu^2 + \nu^2)c_k^\dagger c_k + b_k^\dagger b_k + 1 \right\} + \mu \nu \left\{ (c_k b_{-k} + b_k c_{-k}) + (c_k^\dagger b_{-k}^\dagger + b_k^\dagger c_{-k}^\dagger) \right\} + \frac{\Omega \Sigma}{a^3} (\mu^2 + \nu^2)(a_k^\dagger a_k + 1) + \nu \mu (a_{-k}^\dagger a_{-k} + a_k a_{-k}). \tag{38}
\]

We have completed the derivation of a Hamiltonian describing the dynamics of neutral and charged scalar fields with a symmetry breaking parameter in an expanding FRW metric. We shall see that these two parameters allow us to examine the amplification of charged and neutral modes as they are varied, both in a sudden quench and adiabatically. We have already seen in [5] that this Hamiltonian can be used to describe the formation and decay of the disoriented chiral condensate (DCC). The generic structure of the Hamiltonian allows us to apply its dynamics to the inflationary studies in cosmology. We also believe it provides an excellent framework for analyses of the Bose-Einstein condensate (BEC)- this is left for a later communication.

3 Evolution of the fluctuations

We note first that the sigma field decouples in this particular Hamiltonian and therefore it can be analyzed independently of the \(\phi_0, \phi_{\pm}\) fields. We write the total dynamical Hamiltonian in terms of the creation and annihilation operators \((a, a^\dagger, c, c^\dagger, b, b^\dagger)\) and the squeezing parameters as:

\[
H = \int \frac{d^3k}{(2\pi)^3} \frac{\Omega}{a^3} \left\{ 2(\mu^2 + \nu^2)c_k^\dagger c_k + b_k^\dagger b_k + 1 \right\} + \mu \nu \left\{ (c_k b_{-k} + b_k c_{-k}) + (c_k^\dagger b_{-k}^\dagger + b_k^\dagger c_{-k}^\dagger) \right\} + \frac{\Omega \Sigma}{a^3} (\mu^2 + \nu^2)(a_k^\dagger a_k + 1) + \nu \mu (a_{-k}^\dagger a_{-k} + a_k a_{-k}). \tag{39}
\]

Defining the following bilinear operators:

\[
D = a_k a_{-k} + b_k c_{-k} + c_k b_{-k} = K_1^- + K_2^- + K_3^-
\]

\[
D^\dagger = a_{-k}^\dagger a_k^\dagger + c_{-k}^\dagger b_k^\dagger + b_{-k}^\dagger c_k^\dagger = K_1^+ + K_2^+ + K_3^+
\]
\[
N = \frac{1}{2}(a_k^+a_k + a_{-k}^+a_{-k} + b_k^+b_k + b_{-k}^+b_{-k} + c_k^+c_k + c_{-k}^+c_{-k} + 3)
= K_1^0 + K_2^0 + K_3^0
\]
it is easy to see that they satisfy an \( su(1,1) \) algebra

\[
[N, D] = -D; \quad [N, D^\dagger] = D^\dagger; \quad [D^\dagger, D] = -2N. \tag{40}
\]

The \( su(1,1) \) invariant Hamiltonian for the \( \phi_0, \phi_\pm \) fields assumes the form

\[
H = \int \frac{d^3k}{(2\pi)^3} \frac{1}{a^3} 2\Omega_\phi(k,t)(A^2 + \nu^2)N + 2\Omega_\phi(k,t)\mu\nu(D + D^\dagger) \tag{41}
\]

The time dependent evolution equation is given by

\[
H(t)|\psi(t)\rangle = i\frac{d}{dt}|\psi(t)\rangle \tag{42}
\]

The particular \( su(1,1) \) structure elucidated above provides us the solution:

\[
|\psi(t)\rangle = e^{\int \frac{d^3k}{(2\pi)^3} r_k (D^\dagger - D_k)}|\psi(0)\rangle \tag{43}
\]

for the evolution of the wave function immediately. Here \( r_k \) is the squeezing parameter related to the physical variables \( \Omega_\phi(k,t) \) and \( \omega_\phi(k) \) through

\[
Tanh(2r_k) = \left( \frac{\Omega_\phi(k,t)}{\omega_\phi} \right)^2 - 1 \left( \frac{\Omega_\phi(k,t)}{\omega_\phi} \right)^2 + 1 \tag{44}
\]

Where \( \Omega_\phi(k,t \rightarrow \infty) = \omega_\phi(k) \). It is assumed that past and future infinity is flat spacetime. Thus in the evolution of the fluctuations, it is the frequency changes which bring about squeezing.

The diagonalized Hamiltonian \( H_0 \) can be converted into a Hamiltonian in terms of quantum fields corresponding to the operators \( A, B, C \) and their adjoints to obtain a purely quadratic Hamiltonian.

We can, for example, write

\[
\frac{\Omega_\phi}{a^3}(A_k^+A_k + 1) = \frac{\Omega_\phi}{a^3}(A_k^+A_k + A_k^+A_k^\dagger) = \left( \frac{\Omega_\phi}{a^3} \right)^2 \Pi^2_A(k,t) + P^2_{\Pi_A}(k,t). \tag{45}
\]

Similarly for B and C \([3]\). The Hamiltonian \( H_0 \) can then be written as:

\[
H_0(t) = \int \frac{d^3k}{(2\pi)^3} \sum_{i=A,B,C} \frac{1}{2} \left( \frac{\Omega_\phi}{a^3} \right)^2 \Pi^2_i(k,t) + P^2_{\Pi_i}(k,t) \tag{46}
\]

The Schroedinger equation for each momentum mode is simply:

\[
H_0(k,t)|\psi(k,t)\rangle = i\frac{d}{dt}|\psi(k,t)\rangle. \tag{47}
\]
If we use the $\Pi$-representation (coordinate space representation) for $\psi(k,t)$, then, the $su(1,1)$ symmetry of the Hamiltonian tells us that the solution for $\psi(k,t)$ is just a Gaussian. The equation satisfied by the wave functions for each mode are then given by:

\[
\dddot{\psi}_A(k,t) + \frac{3\dot{a}}{a}\dot{\psi}_A + \left(\frac{\Omega_\phi}{a^3}\right)^2(k,t)\psi_A(k,t) = 0. \tag{48}
\]

(similar equations hold for fields B and C) where

\[
\left(\frac{\Omega_\phi}{a^3}\right)^2(k,t) = \left(\frac{k^2}{a^2}\right) + m^2_\phi + \lambda v^2. \tag{49}
\]

The expectation values of the number operator for the neutral scalar fields for each momentum $k$ is given by:

\[
<\psi_k(t)|a_k^\dagger a_k|\psi_k(t)> = \text{Sinh}^2(r) = <\psi_k(t)|A_k^\dagger(t)A_k(t)|\psi_k(t)>. \tag{50}
\]

An identical expression holds for the charged scalar fields. We let $\d\eta = a(t)^{-1}dt$ so that the FRW metric is transformed into a conformally flat metric: $ds^2 = a(\eta)^2(\d\eta^2 - \d\xi^2)$. The equations of motion given above are then transformed into ones that resemble a harmonic oscillator with time dependent frequencies. We shall write only the generic form of the above equations: In terms of the scaled time, $\eta$, we have:

\[
\psi'' + \frac{2a'}{a}\psi' + \left(\vec{k}^2 + (m^2_\phi + \lambda<\Phi^2>-v^2)a^2\right)\psi = 0 \tag{52}
\]

where a prime denotes differential wrt $\eta$.

Lastly, scaling $\psi$: $\xi = a\psi$ so that the equation becomes:

\[
-\xi'' + V(\eta)\xi = (k^2 + m^2_\phi)\xi \tag{53}
\]

where

\[
V(\eta) = a^{-1}\frac{d^2a}{d\eta^2} + m^2_\phi(1-a^2) - \lambda<\Phi^2>-v^2). \tag{54}
\]

Thus in the symmetry broken stage $<\Phi^2> = v^2$

\[
V_b(\eta) = a^{-1}\frac{d^2a}{d\eta^2} + m^2_\phi(1-a^2) \tag{55}
\]

and in the symmetry restored stage, $<\Phi^2> = 0$

\[
V_r(\eta) = a^{-1}\frac{d^2a}{d\eta^2} + m^2_\phi(1-a^2) + \lambda a^2 v^2. \tag{56}
\]

These equations can be interpreted in two ways allowing for simple solutions. Treating $\eta$ as a spatial variable allows us to treat them as Schroedinger
like equations with $E = \omega^2_\phi$. This then allows calculation of the reflection and transmission coefficients over the “potential barrier” provided by the $V(\eta)$ term. On the other hand, they are equations for time dependent harmonic oscillators with time dependent frequencies given by $\Omega^2_\phi$ and $\omega^2_\phi$. These two pictures allow the calculation of the squeezing parameter dependent number operator $N(k) = \text{Sinh}^2(\tau(k))$ involved in the evolution of the neutral and charged quantum fluctuations in the FRW background.

We shall now look at the scenario in the FRW metric with expansion included to show that the enhancement of the low energy modes and the squeezing parameter are dependent on the rate of the expansion mechanism by which the symmetry is restored. We have also seen that to produce substantial squeezing, we require a quenching. To show this really happens in this case, we have to compare the situation of a sudden quench with a slow adiabatic relaxation of the system from the symmetry restored stage to the symmetry broken stage.

To examine the amplification of the neutral and charged modes for the case of a smoothly expanding metric, we shall assume the expansion parameter $a(\eta)$ to be\footnote{Eqn. (57)}:

$$a(\eta)^2 = A + BTanh[bn].$$

Here $b$ measures the rate of the expansion and we choose $b$ to be in units of $m_\phi$. When viewed as a time dependent oscillator equation in an expanding metric we may write the equation for $\xi$ in the symmetry restored phase as an oscillator equation

$$\xi'' + \omega^2_\phi \xi - V_r(\eta)\xi = 0 \quad (58)$$

and in the symmetry broken stage as

$$\xi'' + \omega^2_\phi \xi - V_b(\eta)\xi = 0 \quad (59)$$

Where $V_b(\eta)$ and $V_r(\eta)$ are given by\footnote{Eqn. (56,55) with the particular choice of the expansion parameter $a(\eta)$ given by 57} Thus the change in frequency from the restored to the broken stage is given by

$$V_r(\eta) - V_b(\eta) = +\lambda v^2 a(\eta)^2 \quad (60)$$

Here, we shall choose $A$ and $B$ in eqn.\footnote{Eqn. (57) to be 1. The transmission coefficients across such a barrier are easily looked up from 11.} While the number of particles of mode $k$ is given by:

$$N(k) = \frac{(\text{Sinh}(\frac{\pi\lambda v^2}{b}))}{\text{Sinh}(\frac{\pi(k^2 + m^2_\phi)}{b})}\frac{\text{Sinh}(\frac{\pi(k^2 + m^2_\phi + 2\lambda v^2)}{b})}{\text{Sinh}(\frac{\pi(k^2 + m^2_\phi + 2\lambda v^2 + 2\lambda v^2)}{b})}$$

Here $b$ measures the duration of the quench and its effect is exhibited in figure 1.

$N(k) vs. k$ is plotted in fig. 2. We see that in the adiabatic limit of large $b$, $N(k)$ is exponentially suppressed so that there is no enhancement of low momentum modes.
Figure 1: Shows the transition from quench to adiabatic for various values of $b$.

Figure 2: Shows the variation $N(k)$ with $k$ for values of $b=0.35$ (red); 0.4 (green); 0.5 (blue).
From the above we conclude that if expansion is rapid we get the quenched limit, while, if expansion is slow, we get the adiabatic limit. Since in the squeezed state description \( N(k) = \sinh^2 r_k \), where \( r_k = r(k) \) is the squeezing parameter, we conclude that in the quenched limit the squeezing parameter is much greater than in the adiabatic limit. This demonstrates clearly the connection between the rate of expansion and squeezing and the enhancement of charged and neutral particle creation.

Since the amplification is large for the low momentum modes, let us look at the charged and neutral number particle distributions for zero momentum. We shall add a subscript 0 to denote zero momentum modes so that the distributions are given by:

\[
P_{n_0, n_+, n_-} = |\langle n_0, n_+, n_- | \psi \rangle|^2
= |\langle n_0 | e^{r_0 (a_0^\dagger)^2 - r_0 a_0^2} | 0 \rangle \langle 0 | e^{2r_0 (b_0^\dagger c_0^\dagger - b_0 c_0)} | n_+, n_- \rangle|^2
\]

defining \( S(r_0) \) as the one mode squeezing operator

\[
S(r_0) = \langle n_0 | e^{r_0 ((a_0^\dagger)^2 - a_0^2)} | 0 \rangle = S_{n_0, 0}.
\]

\( S_{n_+ n_-, 0} \) is then the two mode squeezing operator

\[
< n_+, n_- | e^{r_0 (b_0^\dagger c_0^\dagger - b_0 c_0)} | 0 \rangle = S_{n_+ n_-, 0}.
\]

The neutral and charged particle number distribution is:

\[
P_{n_0, n_+} = \langle S_{n_0}^2 > > S_{n_+ n_-, 0}^2 >
\]

Writing \( n_+ = n_0 = n_- \), we get the distribution of charged particles to be

\[
P_{n_+} = \sum_{n_0} P_{n_0, n_+} = \frac{(\tanh(r_0))^{2n_+}}{(\cosh(r_0))^2}
\]

and the distribution of neutral particles to be

\[
P_{n_0} = \sum_{n_+} P_{n_0, n_+} = \frac{n_0! (\tanh(r_0))^{n_0}}{((\frac{n_0}{2})!)^2 (\cosh(r_0))^{2n_0}}
\]

Thus the neutral and charged particle number distributions are significantly different as the two types of squeezed states appearing in the expressions above have different properties. We now illustrate the effect of squeezing in these two distributions. Figs. 3, and 4 show the difference in the charged and neutral particle number distributions as we vary the squeezing parameter from a low value to a high value.

We now illustrate the effect of quenching versus adiabaticity on these two distributions. Figs. 5 and 6 show the difference in the charged and neutral particle number distributions as we vary from the adiabatic limit where the difference is negligible to the quenched limit where the difference is significant.
Figure 3: Shows the variation of $P_{nc}$ with $n$ for the $r_0 = 3, 3.5$ and 4

Figure 4: Shows the variation of $P_{nc}$ with $n$ for the $r_0 = 3, 3.5$ and 4

Figure 5: Shows the variation of $P_{n0}$ (solid line) and $P_{nc}$ (dashed line) with $n$ for the adiabatic limit ($r_0 = 2$)
Figure 6: Shows the variation of $P_{n0}$ (solid line) and $P_{nc}$ (dashed line) with $n$ for the quenched limit ($r_0 = 4$)

4 Conclusion.

To conclude, in this paper we have constructed an effective Hamiltonian for the study of the dynamics of charged and neutral scalar particles in an expanding background FRW metric. Starting from an O(4) sigma model through the inclusion of second order quantum fluctuations we have shown that the Hamiltonian is quite generic and in addition to applications to the formation of the DCC \[5\], can also be used to study the enhancement of low momentum charged and neutral particle modes in an FRW metric suggesting an application to recent ideas of "preheating" in an inflationary cosmological scenario. We have seen the appearance of SU(1,1) symmetries leading to the presence of squeezed states in their dynamics. We find that in the quenched limit (fast expansion) the low momentum modes are enhanced significantly, whereas in the adiabatic (slow expansion) limit, no such enhancement occurs. This has been shown to be directly related to the value of the squeezing parameter. The manifestation of this difference shows up directly in the total neutral and charged number distributions at zero momentum.

5 Acknowledgements

BB would like to thank the DST (India) and INSA (India) for partial travel support for attending the International Congress of Mathematical Physics (ICMP 2003) where this work was presented. She would also like to thank Prof. Jean-Claude Zambrini of the University of Lisbon for assistance and hospitality in Lisbon. CM would like to thank Prof. K.N. Pathak, Vice-Chancellor, Panjab University for supporting his research. His research was actively hindered by the University Grants Commission (India), under its "Research Scientists" scheme. Hence this work was financed through his salary as UGC Research Scientist "C" (Professor level!!) graciously released by Panjab University.
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