An online optimal energy management strategy for a dual-mode power-split hybrid electric vehicle based on hybrid MPC Algorithm*

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Abstract—This paper proposed a global on-line optimisation energy management strategy based on the hybrid model predictive control (HMPC) algorithm. First, the non-linear models of a dual-mode power-split hybrid electric vehicle is built. Then, the dual-mode power-split hybrid electric vehicle is considered as a hybrid system which includes continuous and discrete states, and a mixed logical dynamic (MLD) predictive model is built based on hybrid system theory. Thirdly, an HMPC energy management strategy is designed based on the MLD predictive model. The HMPC-based energy management strategy is compared with dynamic programming based and rule-based energy management strategies. The simulation results indicate that the HMPC-based energy management strategy is helpful for real-time control and improving fuel economy.

1. Introduction
A dual-mode power-split HEV is an extended version of a power-split HEV, which consists of two modes between input power-split, output power-split, and compound power-split operation [1]. An HEV could work at high efficiency, because there are two mechanical operating points in this mode. Therefore, dual-mode power-split HEVs usually include input power-split mode and compound power-split modes [2]. The GM Allison Hybrid System (AHS) is a typical dual-mode power-split system, originally developed for hybrid buses and heavy-duty vehicles[3]. Due to the characteristics of dual-mode power-split HEVs, their performance is better than that of other HEVs.

Dual-mode power-split HEV usually includes one internal combustion engine and two permanent magnet synchronous motors (PMSM). Therefore, the energy management strategy (EMS) and power distribution between engine and motors becomes very important. A good EMS can further reduce fuel consumption, and decrease exhaust gas emissions [4]. A dual-mode power-split HEV is a typical hybrid dynamic system, including continuous and discrete states, which makes the design of optimal energy management control more complex.

To exploit the fuel consumption potential of HEV, much academic research into HEVs is focused on the EMS, and can be classified into rule-based EMS and optimisation-based EMS [5]. There are many more studies concentrating on optimisation-based EMS and a variety of intelligent methods, such as the use of neural networks, particle swarm optimisation (PSO) algorithm, a genetic algorithm (GA), a simulated annealing (SA) algorithm, quadratic programming (QP), and Pontryagin’s minimum principle (PMP) have been successfully applied to design EMSs. In optimisation-based EMS, dynamic
programming (DP) [6] based EMS is the global optimisation method that serves as the benchmark for other EMSs.

A dual-mode power-split HEV includes continuous and discrete states, which is a typical hybrid dynamic system: to improve the fuel economy thereof, there are four original contributions that distinguish this work from that already published and these form the essence of this paper.

(1) The MLD predictive model of a dual-mode power-split HEV is proposed based on the hybrid system theory. (2) On the basis of the MLD predictive model, an HMPC-based energy management strategy for the dual-mode power-split HEV is proposed. (3) The continuous states and discrete states are considered in the design of the EMS based on hybrid system theory. (4) The HMPC-based energy management strategy is evaluated against a global optimal energy management strategy and a rule-based energy management strategy.

2. The Powertrain of the dual-mode power-split hybrid electric vehicle
Here, the powertrain of the dual-mode power-split HEV is as shown in Figure 1. There are two modes including in the HEV: EVT1 and EVT2. In the first mode (EVT1), the clutch (CL1) and the brake (B1) are disengaged and engaged, respectively. In the second mode (EVT2), the clutch (CL1) and the brake (B1) are engaged and disengaged, respectively. The first operating mode is usually used at low speed and high torque. And the second working mode is usually used for high-speed operation.

Figure 1. Schematics of the dual-mode power-split HEV powertrain.

3. Prediction models for a dual-mode power-split HEV
The non-linear function of the battery current on motor speed and torque is linearized using the linearisation method. The motors used in this work could operate in four quadrants and the SOC linearised model is built separately in each quadrant.

3.1 State, and output, variable update models
The equivalent fuel consumption increment and battery SOC increment in each mode is shown in Eqn (1).

\[
\Delta m_{j_1}(k) = \left(\alpha(n_1)T_1 + \beta(n_1)\right)T_1 + \frac{V_{oc}}{Q_{hl}} \cdot \eta_{eng} - \frac{V_{oc}}{Q_{hl}} \cdot \eta_{eng} \\
\Delta SOC(k) = -\frac{d_i(n_1)}{C_m} + \frac{d_i(n_1)\cdot T_1}{C_m} - \frac{a_i(n_1)}{C_m} + \frac{a_i(n_1)\cdot T_1}{C_m} \\
\]

Continuous states and discrete states are included in a dual-mode power-split HEV, as a result, the HEV is a kind of hybrid system. To deal with the hybrid characteristics of the dual-mode power-split HEV, continuous, and discrete, states must be expressed uniformly. Therefore, the logic state variables \( \delta_1 \) and \( \delta_2 \) are defined to represent modes EVT1 and EVT2, respectively. The equivalent fuel consumption increment and battery SOC increment of a dual-mode power-split HEV at time \( k \) is as follows:

\[
\Delta m_{j_1}(k) = \delta_1(k)\Delta m_{j_1}(k) + \delta_2(k)\Delta m_{j_2}(k) \\
\Delta SOC(k) = \delta_1(k)\Delta SOC_1(k) + \delta_2(k)\Delta SOC_2(k)
\]
At time $k$, only one of modes of dual-mode power-split HEV is used, thus the logic state variables satisfy following constraint:

$$\delta_1(k) + \delta_2(k) = 1$$  \hspace{1cm} (3)

Where, $\delta_i$ represents logic state variables (the value of $\delta_i$ is either 0 or 1).

Here, battery SOC is a state variable and the equivalent fuel consumption is an output variable. The update equation of SOC and equivalent fuel consumption is as follows:

$$
\begin{align*}
\Delta m_j(k+1) & = m_j(k) + \Delta m_f(k) \\
SOC(k+1) & = SOC(k) + \Delta SOC(k)
\end{align*}
$$  \hspace{1cm} (4)

### 3.2 MLD prediction model of a dual-mode power-split HEV

The MLD prediction model of a dual-mode power-split HEV is given by:

$$
\begin{align*}
x(k+1) & = A_1x(k) + B_1u(k) + B_2\delta(k) + B_3z(k) + B_5 \\
y(k) & = C_1x(k) + D_1u(k) + D_2\delta(k) + D_3z(k) + D_5 \\
\delta(k) & \leq E_1u(k) + E_2x(k) + E_3u(k) + E_4
\end{align*}
$$  \hspace{1cm} (5)

Where, $x=[SOC]$ is a vector of continuous and binary states, $w=[T_{1}, T_{2}, T_{3}]$ are the inputs, $y$ are the outputs, $\delta=[\delta_1, \delta_2]$, $z=[z_{11}, z_{12}, z_{21}, z_{22}, z_{31}, z_{32}]$ represent auxiliary binary and continuous variables, respectively, and $A, B_1, B_2, B_3, C, D_1, D_2$ are the coefficient matrices of the state, and output, equations, where $E_{11}, E_{12}, E_{21}, E_{22}, E_{31}, E_{32}, E_{41}, E_{42}, E_{51}, E_{52}$ are the coefficient matrices of inequality constraints.

### 4. Energy management strategy for dual-mode power-split HEV

Based on the MLD prediction model of this dual-mode power-split HEV, the energy management optimal control problem can be described as:

$$
\begin{align*}
& \min_{[x(t), \delta, y, z]} \int_{x(0)}^{x(T)} \left[ J(x(t), \delta, y, z) \right] dt \\
& \text{s.t.} \quad x(k+1) = A_1x(k) + B_1u(k) + B_2\delta(k) + B_3z(k) + B_5 \\
& \quad y(k) = C_1x(k) + D_1u(k) + D_2\delta(k) + D_3z(k) + D_5 \\
& \quad \delta(k) \leq E_1u(k) + E_2x(k) + E_3u(k) + E_4 \\
& \quad u_{\min} \leq u(k) \leq u_{\max}, k = 0, 1, \ldots, T-1 \\
& \quad x_{\min} \leq x(k) \leq x_{\max}, k = 0, 1, \ldots, T-1 \\
& \quad x(T) = x_e
\end{align*}
$$  \hspace{1cm} (6)

Where, $J$ is optimisation objective function, which means the equivalent minimisation of fuel consumption, $Q_i$ ($i = 1, 2, 3$) is a weight factor, $T$ is the prediction horizon, and $u$, $x$, and $y$ are the velocity reference signal, battery SOC signal, and fuel consumption signal, respectively; $x(k)$ is the state predicted at time $(t+k)$, starting from $x(0) = x(t)$, $u_{\min}, u_{\max}$, and $x_{\min}, x_{\max}$ are hard bounds on the inputs, and states, respectively.

### 5. Results and discussion

Under the driving cycle, dual mode power split hybrid electric vehicle in energy management strategy based on dynamic programming (DP), energy management strategy based on rules (Rule) and energy management strategy based on hybrid model predictive control (HMPC) under the control of three algorithms get simulation results of vehicle fuel economy and battery SOC as shown in Table 1, three kinds of control strategies of the initial battery SOC are 65%. The equivalent fuel consumption per 100 kilometers is used to compare and analyze the control effects of three energy management strategies. It is generally believed that the energy management strategy based on DP can achieve the
optimal control effect. Therefore, this paper takes the energy management strategy based on DP as the benchmark and assumes that the fuel economy of the vehicle is 100%. As can be seen from the Table 1, energy management strategy based on HMPC were achieved the energy management strategy based on DP fuel economy by 80.60% control effect, and the Rule based energy management strategy under the same driving cycle conditions, respectively, to achieve energy management strategy based on DP control effect of 66.46%. Therefore, the energy management strategy based on HMPC proposed in this paper has effectively improved the vehicle fuel economy.

| Strategies | End SOC | Equivalent Fuel (L/100 km) | Percentage of DP control effect (%) |
|------------|--------|-----------------------------|-----------------------------------|
| DP         | 65.0%  | 15.6683                     | 100                               |
| HMPC       | 65.0%  | 19.4360                     | 80.60                             |
| Rule       | 64.5%  | 23.5764                     | 66.46                             |

The speed characteristic curves of the motor A and motor B under driving cycle condition are shown in Figure 2 and Figure 3 respectively. It can be seen that under the driving cycle condition, the motor A and motor B all work within the speed constraint range. Compared with the energy management strategy based on DP or Rule, under the control of the energy management strategy based on HMPC, the speed characteristics of motor A and motor B showed large fluctuation range and frequent speed changes. This is because the energy management strategy based on HMPC proposed in this paper makes full use of the decoupling characteristics of the engine and vehicle speed on dual-mode power-split hybrid vehicles. Through the fluctuation of motor A and motor B, the vehicle speed varying with the operating conditions can be compensated to the greatest extent. It shows that the energy management strategy based on HMPC is effective in fuel economy control.

The torque characteristic curves of motor under driving cycle conditions are shown in Figure 4 and Figure 5, in which the control performance of energy management strategy based on DP, Rule and HMPC is evaluated and compared. Motor A and motor B play an important role in adjusting engine torque to ensure that the engine can work efficiently and smoothly under the driving cycle condition. As can be seen from the figure, compared with the energy management strategy based on DP or Rule, under the control of the energy management strategy based on HMPC, motor A and motor B compensate the required engine torque to a certain extent, so the torque of motor A and motor B fluctuates greatly. This indicates that the energy management strategy based on HMPC proposed in this paper has good performance in dealing with multi-input, multi-output and variable constraint problems such as optimal control of dual-mode power split hybrid vehicle energy management.
Under the driving cycle condition, the battery SOC trajectories of dual-mode power-split hybrid vehicles based on three energy management strategies are shown in Figure 6. As can be seen from the figure, the energy management strategy based on HMPC can maintain the battery SOC balance at around 65% in the driving cycle condition, achieving the expected control effect of maintaining the battery SOC balance. It can also be seen from the figure that, under the control of energy management strategy based on DP, the battery SOC has a large fluctuation range. As DP algorithm is a global optimization algorithm, energy management strategy based on DP can achieve global optimal control effect under given driving cycle conditions. Therefore, energy management strategy based on DP is often used as a comparison benchmark for other energy management strategies. It indicates that the HMPC-based energy management strategy proposed in this paper is an instantaneous optimization energy management strategy with stronger attributes.

The distribution of working points of engine in driving cycle conditions is shown in Figure 7. As can be seen from the figure, compared with the control effect of energy management strategy based on HMPC and energy management strategy based on DP and Rule, the distribution of working points of engine, motor A and motor B is slightly different. Under the energy management strategy control based on HMPC, motor A and motor B give full play to the role of adjusting the engine operating points, so that most of the engine operating points are distributed near the high efficiency area, and dual-mode power shunt hybrid vehicles can obtain better fuel economy. Under the rule-based strategic control of driving cycle 1, the proportion of engine operating points in the fuel consumption rate range (0, 215g/kWh), (0, 225g/kWh) and (0, 240g/kWh) is 3.83%, 17.29% and 29.92%, respectively. Under the HMPC-based strategic control, the fuel consumption rate in the same range is increased to 17.15%, 35.38% and 46.79%. This indicates that the HMPC-based energy management strategy significantly improves fuel economy.

6. Conclusion
An energy management strategy based on a hybrid model protective control algorithm has been developed for a dual-mode power-split hybrid electric vehicle. Some conclusions are drawn from the simulation analysis: Firstly, the MLD predictive model of a dual-mode power-split HEV, which is based on hybrid system theory, offered good predictive performance. Secondly, the proposed hybrid model predictive control energy management strategy based on the MLD predictive model offered
good control performance. Finally, the control performance of the proposed HMPC-based energy management strategy is compared with that of the DP-based energy management strategy and rule-based energy management strategy. The simulation results indicated that the control performance of HMPC can achieve 80.6% of that of DP.

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