Degree Scale Microwave Anisotropies in NonGaussian Theories of Cosmic Structure Formation

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Abstract
The COBE satellite’s maps of the cosmic microwave background (CMB) anisotropy do not have the resolution to discriminate between theories of cosmic structure formation based on inflation, and those based on field ordering following a symmetry breaking phase transition. For this purpose it is critical to resolve the CMB anisotropy on degree scales. In this paper we report on detailed calculations of the degree scale anisotropies predicted in the global string, monopole, texture and nontopological texture scenarios of structure formation, emphasising their distinct character from those predicted by inflation, and commenting on the prospects of their detection in the near future.
I. Introduction

The COBE satellite’s detection of large angular scale anisotropy in the cosmic microwave background [1] has opened a new window on the early universe. But the picture COBE itself provides is limited by a large angular smoothing scale and a low signal to noise ratio. It falls short of discriminating between a large number of theories of structure formation, based on very different physics. For example, theories based on inflation and on cosmic field ordering are equally consistent with COBE’s findings. Both sets of theories predict an approximately scale invariant spectrum of multipole moments, and on COBE scales a very Gaussian anisotropy pattern. In particular, the distinctive non-Gaussian signatures expected in the field ordering theories (lines for strings [2], spots for textures [3]) would be smeared out by the COBE beam and instrument noise, and very hard to distinguish from the Gaussian noise pattern expected from inflation [4] [5]. For this purpose, resolution on an angular scale smaller than that subtended by the horizon at last scattering appears to be essential.

None of the preexisting theories can claim the COBE findings as an unmitigated success. Roughly speaking, the COBE result was a factor of two higher than expected in the simplest inflationary theory, and a factor of two lower than expected in the simplest field ordering theories. These theories have $\Omega = 1, \Lambda = 0$, with one free parameter determining the level of fluctuations - if this is fixed from COBE, the ‘standard’ inflation plus cold dark matter (CDM) theory predicts galaxy cluster velocity dispersions much higher than observed [6]. It is fairly easy to fix the situation by adding new adjustable parameters. In the inflationary case, considering ‘mixed’ dark matter [7], inflationary potentials which yield an important additional gravity wave component [8], [9], or adding a cosmological constant [10], all improve the situation.

The simplest field ordering theories are in principle as predictive as the simplest inflationary scenario, since the level of mass fluctuations and of microwave anisotropies depend on a single free parameter, the symmetry breaking scale $\phi_0$. An appealing aspect of these scenarios is that one naturally produces the correct amplitude of density fluctuations with $\phi_0$ of order the grand unification scale, around $10^{16}$ GeV. But unlike the simplest inflationary theory, these theories rely on nonlinear effects in an important way, and are harder to calculate with. The most detailed calculations so far suggest that if the parameter $\phi_0$ is normalised to fit the COBE $10^o$ variance, the level of mass fluctuations inferred on intermediate scales ($\sim 20h^{-1}$ Mpc) is too small to explain the observed galaxy streaming motions [5]. It has been suggested that this problem would be ameliorated in an open universe [11], by suppressing the anisotropies produced by the field ordering at late times and thus allowing a larger value for $\phi_0$. A positive cosmological constant could be expected to have the same effect.

The pattern of CMB anisotropy on degree scales offers an independent
and in principle far cleaner test, being unaffected by the complex processes involved in galaxy formation. The simplest measure of anisotropy, and the only relevant quantity in Gaussian theories, is the power spectrum of multipole moments. If the standard inflation plus CDM theory is correct, there should be a peak in the spectrum at $l \sim 200$ produced at the epoch of recombination, $Z \sim 1000$, due in part to the Doppler effect of photons scattering off moving electrons. In the theories considered here which involve topological defects ($N = 2, 3, 4$), reionisation is likely, and last scattering would be shifted forward to $Z \sim 100$. The angular scale subtended by the horizon at last scattering is (for $\Omega = 1, \Lambda = 0$) $\theta_{ls} = (1 + Z_{ls})^{-\frac{1}{2}}$, of order $2^\circ$ without reionisation and $6^\circ$ with reionisation. As will be seen, the spectrum predicted by the field ordering theories with the assumption of reionisation, is quite distinct, with much less power in higher multipoles.

Even more distinctive is the nonGaussianity in the field ordering theories. The assumption that the anisotropy pattern is in the form of random Gaussian noise is frequently made. However, if the microwave sky is nonGaussian, there will be additional to be gleaned from anisotropy maps. In the nonGaussian theories explored here for example, details of the pattern would tell us about the pattern of symmetry breaking in particle physics at very high energies, information presently available from no other source (see e.g. [12]).

The data gathered so far from experiments at the South pole present an incomplete but nonetheless intriguing picture. If the the upper limit $(\delta T/T) < 1.4 \times 10^{-5}$ derived from one of the four channels of the Gaier et. al. experiment [13] is correct, it is enough to exclude the standard inflationary spectrum of fluctuations (normalised to COBE) with either hot or cold dark matter at the 95 % confidence level [14]. And the thirteen point scan from the Schuster et. al. experiment [15] contains an apparently nonGaussian ‘bump’. Similarly, data from the MAX experiment seem inconsistent with Gaussian statistics. Under that assumption discrepant upper and lower 95 % confidence bounds were obtained on the level of CMB anisotropy at $\frac{1}{2}^\circ$ from two different parts of the sky [16]. It is clearly of interest to determine the predictions of the nonGaussian field ordering theories on these angular scales, while the observational situation is still being resolved.

One of the sharpest distinctions between the Gaussian and nonGaussian scenarios mentioned above is the likelihood that the universe was reionised after recombination. If around $0.1-1.0$ per cent of the mass in the universe underwent nonlinear gravitational clustering and star formation at redshifts greater than of the order 100, this would likely release sufficient ionising radiation to reionise the intergalactic medium, causing the photons to rescatter. In this case the epoch of last scattering would be shifted forward to a redshift $Z_{ls} \sim 100(0.05/f\Omega_B h_{50})^2$ where $f$ is the fractional ionisation (assumed not too different from unity). In field ordering theories with topological defects like strings or textures, large local
density perturbations are produced at high redshift, and complete reionisation \( (f = 1) \) is possible [5]. But in Gaussian scale invariant theories of structure formation, reionisation is very unlikely because high amplitude fluctuations are exponentially suppressed - using the Press-Schechter [17] or number of peaks formulae [18] one deduces that for the standard CDM theory normalised to COBE, no more than of order \( 10^{-10} \) of the baryons in the universe could have undergone collapse onto dark matter lumps, and thus be bound into stars, by a redshift of 100. This is far too little to reionise the universe.

In this paper we shall make the assumption that the universe was reionised at redshifts greater than \( z_{ls} \) given above, and study the consequences for the pattern of microwave anisotropy. We shall give results for the simplest symmetry breaking theories: where an assumed \( O(N) \) global symmetry is spontaneously broken by an \( N \) component scalar field taking a GUT scale vacuum expectation value. For \( N = 2 \), this is the theory of global cosmic strings, \( N = 3 \), global monopoles, \( N = 4 \), global texture and \( N = 6 \), an example of ‘nontopological’ texture. Of these the ‘generic’ cases may be argued to be global strings, which result from the complete spontaneous breaking of any abelian continuous global symmetry, and texture, which results from the complete breaking of any nonabelian continuous global symmetry. It is important to keep in mind, however, that ‘realistic’ unified theories may include more complex symmetry breaking schemes, in which results different by a factor of two might well be obtained. For example the family symmetry scheme studied in [12] correspond to two \( N = 6 \) theories, coupled in such a way as to produce topological textures.

In previous work, Bouchet, Bennett and Stebbins [19] constructed an anisotropy map for gauged cosmic strings. They used a flat spacetime approximation to compute the anisotropy and ignored electron scattering, which in a reionised universe acts to smear out the distinctive line-like discontinuities produced by cosmic strings [20], for redshifts earlier than last scattering. Reionisation seems likely in the string scenario: in this case the strings would only start to produce clear discontinuities well after last scattering, on scales in excess of \( \theta_{ls} \sim 6^\circ \). And of course these discontinuities would be very heavily masked by the smaller angular scale fluctuations produced at earlier epochs.

In our calculation for \( N = 2 \), we do not aim at infinite resolution, but rather to produce a map smoothed on a scale comparable to the beam size adopted for degree scale measurements. We include the effects of electron scattering ignored in the earlier calculations, and we believe that this provides a more realistic picture of the anisotropy pattern one could expect from a degree scale experiment.

**Techniques**

We study the evolution of the symmetry breaking scalar fields using, for \( N > 2 \), the finite difference scheme of [5] for the nonlinear sigma model. We are able, with these techniques, to evolve the scalar fields in boxes in excess of
200^3 grid points, and for \( N > 2 \) to convincingly demonstrate that the results are independent of box size. For \( N = 2 \), we have developed a more accurate difference scheme to evolve a single ‘angular’ field. In this case we do find a systematic dependence on box size (results change by of order 20 % when we double the box size) so the results should be treated with some caution. For this case we normalised to COBE and computed the degree scale anisotropy using exactly the same physical resolution.

In this paper we extend the techniques developed in [5] for calculating the large angular scale anisotropy, including the effects due to photons scattering off electrons in the ionised medium. The effect of photon drag on the velocity of the ionised electrons (and baryons) is a strong function of redshift, the ratio of the collisional damping rate \( t_d^{-1} \) to the expansion rate \( t^{-1} \) being given by \( t/t_d \sim [(1 + Z)/200]^{7/2}h^{-1} \). Since the redshift of last scattering is somewhat smaller than 200 in the theories we are interested in, we make the approximation of ignoring photon drag, so that the velocity of the scattering electrons simply equals the local velocity of the dark matter. This simplifies the calculation enormously - we do not need to evolve the baryon-electron-photon fluid. A useful check on this approximation is that for scale invariant adiabatic initial conditions we should reproduce the results for an (unphysical) standard CDM model without recombination. We have found reasonable agreement for the spectrum of multipoles obtained using our technique and that of Crittenden et. al. [21] obtained using the Boltzmann equation [22].

Furthermore, for reasonable values of the current cosmological parameters, at the relevant epochs the universe was matter dominated and close to critical density. This means that effectively only one parameter enters our calculations - the horizon scale when optical depth was unity. One can simply rescale the angular size of our maps to obtain results for other values of the cosmological parameters \( \Omega, \Omega_B, \Lambda \), and the ionisation fraction \( f \).

As in [5], we solve the linearised Einstein equations analytically by Fourier transforming them and decomposing them into scalar, vector and tensor components. The relevant metric perturbations are obtained as analytic integrals of the appropriate pieces of the scalar field stress tensor, computed using FFT’s at each timestep. We perform a Monte Carlo calculation of the temperature anisotropy by simulating photon trajectories and computing the energy shift each photon experiences directly. One must choose a large enough number of photons so that the ‘shot noise’ due to random scatterings, particularly the Doppler term from late scatterings, is averaged out on the angular scales of interest. In practice we have found that for a few hundred photons along each line of sight was sufficient to suppress the white noise component in the multipole spectra down to angular scales corresponding to one or two grid spacings. A typical run employs a box of 100^3 for the field evolution, 50^3 for the metric perturbations and photon trajectories, with 200 photons along each line of sight.
to make a 50° map. It takes around 5 hours on the Convex C-220 at Princeton. Our technique may be readily extended to compute polarisation effects.

In synchronous gauge the temperature anisotropy experienced by photons along any path segment is given to first order by the Sachs Wolfe formula

\[
\frac{\delta T}{T}(n) = -\frac{1}{2} \int_{\eta_i}^{\eta_f} d\eta h_{ij,0}(\eta, n\eta) n^i n^j
\]

where \( n \) is the direction vector of the photon, \( h_{ij}(\eta, x) \) is the perturbation in the spatial metric, and \( \eta \) is conformal time (\( h_{0\mu} = 0 \) in synchronous gauge).

In synchronous gauge the velocity on the dark matter is zero at all times (in this linear regime of course), and there is no explicit Doppler term. However, after integration by parts the expression may be decomposed into the following pieces

\[
\frac{\delta T}{T}(n) = -\frac{1}{2} \int_{\eta_i}^{\eta_f} d\eta (h_{ij,0}^S + h_{ij,0}^V + h_{ij,0}^T)(\eta, n\eta) n^i n^j + [-n \cdot \nabla \chi(\eta, x)]_i
\]

where the surface term represents the Doppler effect of the photon scattering off moving electrons. This last term was not included in our previous work [5], as it is unimportant on angular scales larger than the horizon at last scattering. In Fourier space the expression for the velocity potential \( \chi \) is

\[
\chi(\eta, \mathbf{k}) = 8\pi G \int_0^\eta d\eta' \left( \frac{1}{45} \eta'^9 - \frac{1}{30} \eta'^8 \eta''^4 - \frac{1}{5} \eta'^6 - \frac{1}{9} \eta'^4 \eta''^2 \right) (\Theta + 2\Theta_S)(\eta', \mathbf{k})
\]

where \( \Theta \) and \( \Theta_S \) are scalar components of the source fields stress tensor. Explicit expressions for the \( h_{ij,0}^{S,V,T} \) are given in [5].

The initial conditions in these theories are taken to be zero temperature distortion and zero metric perturbation. Each of the physical quantities of interest in the present calculation - the path dependent gravitational redshift term, the Doppler term, and the ‘intrinsic’ anisotropy term, can be shown to be dominated at the appropriate horizon crossing epoch. Thus the calculation is insensitive to the early behaviour of the source fields, and issues of ‘compensation’ [23] do not arise. To construct photon trajectories we evolve each photon backwards in time along paths starting at the position of the observer. We compute the probability of a scattering event occurring during each timestep, and then decide whether one actually occurred by calling a random number. The information needed to reconstruct the entire photon trajectory (initial position and direction, plus an appropriate random number seed) are stored and later used to step along the photon paths during a simulation.

For the earliest part of its trajectory, a photon executes a random walk with step length much less than the horizon scale. During this early phase, under the assumption that the metric perturbation is constant over the scales traversed by the photon, it can be seen from (1) that the photon acquires an energy shift of just \(-\frac{1}{6} h\). We use this as an initial condition at the start of each photon trajectory, at the time when the photon mean free path equals one grid unit. We compute the scalar ‘velocity potential’ \( \chi \) and the metric perturbations \( h_{ij,0}^{S,V,T} \) in (2) in real space as the fields evolve, and use them to
determine the energy shift experienced along each photon path segment, and at each scattering event.

**Results**

Maps of the CMB anisotropy produced by global strings ($N = 2$), monopoles ($N = 3$), textures ($N = 4$) and a representative case of ‘nontopological texture’ ($N = 6$) are shown in Figure 1 a-d. Figure 1e shows an equivalent map for the standard CDM theory [24]. The maps have been smoothed with a Gaussian window of FWHM 1.5°, and the angular scale subtended by the box side is $\Theta = 30^\circ$ and $20^\circ$ for the field ordering theories and standard CDM respectively. These angular scales are for $h = 0.5, \Omega_B = 0.0125, f = 1, \Omega = 1, \Lambda = 0$. For other values of the cosmological parameters $\Omega, h, \Omega_B, \Lambda$, and the ionisation fraction $f$ the maps can be simply rescaled, for example if $\Omega = 1, \Lambda = 0$ the angular sizes scale as $\theta \propto (\Omega_Bfh_{50}/0.05)^{1/3}$.

In Figure 2, we show the multipole spectra for $N = 2, 3, 4$ and 6, computed by averaging over the six maps obtained by observers looking at each face of a cube. At small $l$ the results are consistent with previous all sky simulations [5]. There is significant scatter (‘cosmic variance’), particularly at lower $l$, which probably accounts for all the apparent ‘features’ on the curves - none appear statistically significant. There is a striking suppression of power at higher $l$, most significant in the texture and nontopological texture theories. At $l \sim 100$, there is ten times less power in the texture theory than in standard CDM, if both are normalised to COBE. Note however that this holds only if $\Omega = 1, \Lambda = 0$. In an open universe, or in a $\Lambda$ dominated flat universe, there would be suppression of the anisotropy on COBE scales, requiring a larger value for $\epsilon$, and increasing the power on degree scales.

Is the suppression entirely a result of reionisation? We have compared these spectra with those produced by standard CDM with complete reionisation assumed, calculated both using our techniques and also from the Boltzmann equation [21]. For that case, $l^2c_l$ is approximately flat, but with a small Doppler bump, up to around $l \sim 70$, before falling off to very small values for $l > 100$. The lack of high $l$ power in the texture and nontopological texture cases seems to be a result both of reionisation, and the fact that the anisotropy comes mainly from a time dependent potential with a large coherence length.

In spite of the strong suppression of higher $l$ moments, there is some power up to very large $l$. Some of this high $l$ power is undoubtedly attributable to the localised defects themselves, but the best way to look for defects is certainly not in the power spectrum.
NonGaussianity

The simplest measures of nonGaussianity are the third and fourth irreducible moments of the temperature distribution, known in dimensionless form as the skewness and kurtosis. Using the six maps produced for each theory, we have calculated the mean skewness and kurtosis. None of these appeared to be statistically significant - i.e. the mean never deviated by many standard deviations from zero.

Another crude test is the level of maxima and minima in the maps compared to the r.m.s. value. The monopole maps show strong positive and negative peaks at a level of ±3.5σ. The texture maps also appear to be significantly nonGaussian - the cold spot in Figure 1c is a 3.7σ event, and other maps showed similar hot and cold spots. It is worth emphasising that strong cold spots with peak level around $(\delta T/T) \sim 5 \times 10^{-5}$ are expected in the texture theory, and are hard to mimic with astrophysical sources. The $N = 6$ maps appear more Gaussian, in line with expectation, and have maxima and minima at around the 2.5σ level, similar to the standard CDM map. These numbers are suggestive, but a more detailed assessment of the level of nonGaussianity requires many more realisations. We have applied an optimised statistic for detecting nonGaussianity [25] to scans simulating the Schuster et. al. experiment on our maps, and do find the nonGaussianity to be detectable with signal to noise ratios as low as 1.5. It is clear however that detecting the nonGaussianity requires a sophisticated statistical treatment, as well as very clean experimental data.

Conclusions

We have developed the techniques required for a full relativistic computation of the degree scale CMB anisotropy in nonGaussian theories of structure formation based on symmetry breaking and phase ordering. It is now straightforward to construct a statistical ensemble of microwave sky maps from which results for any experiment may be predicted, and confidence levels deduced. Our main conclusion is that the goal of distinguishing theories based on field ordering from Gaussian theories based on inflation should be readily achievable from microwave anisotropy data available in the near future. One striking difference is in the spectra of multipoles, caused partly by the reionisation expected in theories with topological defects. But even more distinctive is the nonGaussianity of the anisotropy pattern - which may be detectable by the degree scale experiments currently in progress.

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Table 1. Standard Deviation on 1.5° scale

| Defect            | N | $\delta T/T_{1.5^\circ}$ |
|-------------------|---|-------------------------|
| Strings           | 2 | 1.8 ± 1.0 × 10^{-5}     |
| Monopoles         | 3 | 1.7 ± 0.4 × 10^{-5}     |
| Textures          | 4 | 1.5 ± 0.3 × 10^{-5}     |
| N.T. Textures     | 6 | 1.0 ± 0.2 × 10^{-5}     |
| Standard CDM      | 3 | 3.0 × 10^{-5}           |

Table 1. The standard deviation of the anisotropy maps after smoothing with $1.5^\circ$. All theories have been normalised to fit the standard deviation on the 10° scale reported by COBE, $(\delta T/T)_{10^\circ} = 1.1 ± 0.18 × 10^{-5}$. The errors quoted on the field ordering theories are estimates of our (one sigma equivalent) systematic errors. Statistical errors are of order 10%. The number for standard CDM is the value for the map shown in Figure 1e [24].

Figure Captions

Figure 1: Temperature anisotropy maps in the global string, monopole, texture, nontopological texture and standard CDM scenarios. The side of the box subtends an angular scale of 30° for the field ordering cases, 20° for standard CDM. $\delta T/T$ is given in units of $\epsilon = 8\pi G\phi^2/20$ for the field ordering theories for $N > 2$, where $\phi_0$ is the symmetry breaking scale, and $8\pi G\mu$ for $N = 2$, where $\mu$ is the string tension. Normalising to COBE, $\epsilon = 5.9 ± 1.1 × 10^{-5}$ for $N = 3$, 4 and 6 respectively, while $8\pi G\mu = 5.8 ± 2.2 × 10^{-5}$. All the maps have been smoothed with a Gaussian of FWHM 1.5°, similar to the beam for the South Pole experiments of refs. [13],[15].

Figure 2: Power spectra for the degree scale temperature anisotropy in the global string, monopole, texture, nontopological texture and standard CDM scenarios. If the temperature anisotropy is expanded as a sum of spherical harmonics, $(\delta T/T) = \sum_{l,m} a_{lm} Y_{lm}(\theta, \phi)$, then $c_l$ is defined as the ensemble average $\langle |a_{lm}|^2 \rangle$. The power per logarithmic interval in $l$ is then $l^2 c_l$, constant for a scale invariant pattern.
