New limits on the generation of magnetic field

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1 INTRODUCTION

In a variety of astrophysical systems the observed strength of magnetic field is close to an equipartition value. Generally it is believed that the present values were reached by dynamo amplification of an initially weak seed field (Widrow 2002). However, the mechanism responsible for generating the seed fields remains uncertain. Along with other possible candidates a radiation drag has been proposed. In plasma moving with respect to a radiation field the drag force acting on electrons is stronger than on protons, causing the generation of electric currents. These currents give rise to a magnetic field in the system. Mishustin & Ruzmaikin (1972) estimated that the value of magnetic fields generated in protogalaxies by radiation drag is in the range $10^{-18} - 6 \times 10^{-17}$ G. However, a subsequent analysis by Balbus (1993) implied that, independent of the radiation intensity, the maximal value of the magnetic field generated by radiation drag is $\sim m_p c \Omega / e$, where $\Omega$ is the initial angular velocity. For a typical protogalactic disk this limit is $\sim 10^{-19}$ G, which might be too small to account for the required seed field (Kulsrud 1999, but see Widrow 2002, Davis, Lilley & Tornkvist 1999). According to Balbus this limit also invalidates theoretical estimates of magnetic field in several other systems, such as bipolar jets in protostellar systems (Chloe & Jones 1993) and disks around massive compact objects (Walker 1988).

In this paper, however, we show that this limit is valid only if the plasma is fully ionized, optically thin and uncoupled to any other form of matter. If on the other hand a large fraction of the gas is neutral, condensed in optically thick clouds, or coupled to another form of matter (stars, dark matter etc) then this limit might have to be relaxed by several orders of magnitude.

2 EQUATIONS

2.1 Two-component system

Let’s assume first that plasma is fully ionized, optically thin and uncoupled to any other form of matter. For simplicity we assume that plasma is composed only of protons and electrons. In self-gravitating astrophysical ionized objects global electric forces must be similar in magnitude to the gravitational forces. This means that the large-scale deviations from neutrality $(n_p - n_e) / n_e$ are close to $G m_p^2 / e^2 \sim 10^{-38}$, the ratio between the electrical and gravitational forces. This is a very small number and therefore we assume that the plasma is locally neutral, i.e., $n_e = n_p$. Similarly, because the magnetic field of an object of size $R$ is proportional to $R (n_e V_e - n_p V_p)$, for large objects the requirement of a small magnetic field translates into nearly equal velocities for electrons and protons, i.e., $V_e = V_p$. Also for large ionized objects we can assume that electrons and protons are tied together by induction, neglecting the collisional friction. Using these assumptions we write the equations of motions for protons and electrons as follows:

$$\begin{align*}
\frac{dV_p}{dt} &= \frac{e}{m_p} (E - \frac{B \times V_e}{c}) + \nabla \phi + \frac{\nabla (n_e kT)}{n_e m_p}, \\
\frac{dV_e}{dt} &= \frac{e}{m_e} (\frac{B \times V_e}{c} - E) + \nabla \phi + \frac{\nabla (n_e kT)}{n_e m_e} + \frac{F_{\text{rad}}}{m_e},
\end{align*}$$

(1)

(2)

where $\phi$ is the gravitational potential and $F_{\text{rad}}$ is the radiation drag force on each electron. Combining both equations we find the electric field

$$E = \frac{B \times V_e}{c} + \frac{m_p - m_e}{m_p + m_e} \frac{\nabla (n_e kT)}{e n_e} + \frac{m_p}{m_p + m_e} F_{\text{rad}},$$

(3)
Taking curl of both sides of the above equation and using Faraday equation gives

$$\frac{\partial B}{\partial t} = \frac{\nabla \times V_e \times B}{c} + \frac{m_p - m_e}{m_p + m_e} \frac{\nabla(n_e kT)}{em_e} + \frac{m_p}{m_p + m_e} \frac{\nabla \times F_{rad}}{e}. \quad (4)$$

The first and the second term on the right side of (4) are respectively the dynamo amplification and the battery mechanism (Biermann 1950). Since we are interested in generation of magnetic field due to radiation drag only, we will ignore these terms. Thus we remain with

$$\frac{\partial B}{\partial t} = \frac{m_p}{m_p + m_e} \frac{\nabla \times F_{rad}}{e}. \quad (5)$$

The radiation drag will act only as long as plasma rotates with respect to photonic fluid, i.e. until it loses all its angular momentum. If we ignore possible contraction or expansion of the object during the process, then integrating (5) in time gives

$$B_{\text{max}} = \frac{m_p c}{m_p + m_e} \frac{\nabla \times (P_e + P_p)}{e} = \frac{\nabla \times (m_p V_0)}{e}, \quad (6)$$

where $V_0$ and is the initial rotational velocity of plasma and $P_e$ and $P_p$ are the initial momenta of electrons and protons. Since $\Omega = \nabla \times V$ is the angular velocity we get

$$B_{\text{max}} = \frac{m_p c \Omega_0}{e} \approx 10^{-4} \Omega_0 \text{ G} \quad (7)$$

Similar limits were previously obtained by several authors (Harrison 1970; Balbus 1993).

### 2.2 Three-component system

We now introduce a third component into the system, which interacts with electrons and protons but not with radiation. The relevant equations of motion are now

$$m_p \frac{dV_e}{dt} = eE + K_p (V_e - V_3), \quad (8)$$

$$m_e \frac{dV_e}{dt} = -eE + K_e (V_e - V_3) + F_{rad}, \quad (9)$$

$$(m_p + m_e) \frac{dV_3}{dt} = \chi (K_p + K_e) (V_e - V_3), \quad (10)$$

where $\chi$ is the mass density ratio of the plasma to the third component, $V_3$ is the velocity of the third component, and $K_e$ and $K_p$ are, respectively, coupling constants of the electrons and protons to the third component. Combining the equations we obtain

$$eE \left( \frac{1}{m_e} + \frac{1}{m_p} \right) = \frac{m_p + m_e}{\chi (K_p + K_e)} \left( \frac{K_p}{m_p} - \frac{K_e}{m_e} \right) \frac{dV_e}{dt} + \frac{F_{rad}}{m_e}. \quad (11)$$

If there is a strong coupling between the plasma and the third component, or equivalently the timescale of the drag $m_p V_e / F_{rad}$ is much larger than $m_p / (K_p + K_e)$, then the velocity of the third component is close to the velocity of plasma, i.e., $V_3 \approx V_e$. Then

$$(1 + \chi^{-1}) \left( m_p + m_e \right) \frac{dV_e}{dt} = F_{rad} \quad (12)$$

Combining eqs. 11 and 12 we obtain

$$eE = \left( m_p + \frac{K_p}{K_p + K_e} \frac{m_p + m_e}{\chi} \right) \frac{dV_e}{dt}. \quad (13)$$

Using Faraday’s law and integrating in time we derive the following upper limit on the magnetic field

$$B_{\text{max}} = \left( 1 + \frac{1 + m_e/m_p}{\chi (K_e + K_p)} \right) \frac{m_p c \Omega_0}{e}. \quad (14)$$

If $K_p \gg K_e$, as is the case for coupling with neutral gas, then this limit reduces to

$$B_{\text{max}} \approx (1 + \chi^{-1}) \frac{m_p c \Omega_0}{e} \quad (15)$$

### 3 CONCLUSIONS

The new limit is therefore larger by a factor of $(1 + \chi^{-1})$. One should not conclude, however, that the magnetic field becomes unbounded when $\chi$ goes to zero. For an object of a fixed mass and angular momentum decreasing $\chi$ equivalent to decreasing $n_e$, so for too small $\chi$ induction becomes weaker than friction, which contradicts our initial assumption. In this case, according to eq. 9 the maximal velocity of the electrons relative to the protons is $F_{rad} / K_e$ and the upper limit on magnetic field is obtained from

$$\nabla \times B_{\text{max}} = \frac{4\pi}{c} j = \frac{4\pi c e}{K_e} \frac{F_{rad}}{K_e} \quad (16)$$

For an object with a radius $R$ this yields

$$B_{\text{max}} = \frac{4\pi e}{c} \frac{F_{rad}}{K_e} R \quad (17)$$

Since $K_e$ is proportional to the density of the third component then $B_{\text{max}} \propto n_e / K_e \propto \chi$.

As the upper limit on $B$ varies non-monotonically with $\chi$, the maximal magnetic field is reached for some intermediate value of $\chi$, which depends on the physical parameters of the system (temperature, radiation strength, size of the object etc). Still in large astrophysical systems $\chi$ needs to be very small before eq.15 becomes invalid and thus the new limit would be much larger than the previous one. Taking protogalaxies in the post-recombination era as an example we find that since the fraction of ionized gas is of order $10^{-4}$ the new upper limit is $\sim 10^{-15}$G, which is consistent with Mishustin & Ruzmaikin results.

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