Approximation of the objective insensitivity regions using Hierarchic Memetic Strategy coupled with Covariance Matrix Adaptation Evolutionary Strategy *

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Abstract. One of the most challenging types of ill-posedness in global optimization is the presence of insensitivity regions in design parameter space, so the identification of their shape will be crucial, if ill-posedness is irrecoverable. Such problems may be solved using global stochastic search followed by post-processing of a local sample and a local objective approximation. We propose a new approach of this type composed of Hierarchic Memetic Strategy (HMS) powered by the Covariance Matrix Adaptation Evolutionary Strategy (CMA-ES) well-known as an effective, self-adaptable stochastic optimization algorithm and we leverage the distribution density knowledge it accumulates to better identify and separate insensitivity regions. The results of benchmarks prove that the improved HMS-CMA-ES strategy is effective in both the total computational cost and the accuracy of insensitivity region approximation. The reference data for the tests was obtained by means of a well-known effective strategy of multimodal stochastic optimization called the Niching Evolutionary Algorithm 2 (NEA2), that also uses CMA-ES as a component.

1 Introduction

The Global Optimization Problems (GOPs) are distinguished among all problems of minimizing real-valued objective function $f : D \rightarrow \mathbb{R}$ over the admissible domain $D$ embedded in some metric space, typically $\mathbb{R}^n, n \geq 1$.

$\text{argmin}_{x \in D} \{f(x)\}$. (1)

Roughly saying, GOPs can posses more than one solution, typically due to the lack of global objective’s convexity. We are focused on ill-conditioned GOPs for which there exist regions in an admissible domain on which objective takes a value close to the global minimum and exhibits a very small variability (insensitivity regions, “lowlands”, see [7] for a more formal definition).

The ill-conditioned GOPs are frequently encountered when solving Inverse Parametric Problems (IPPs) tasked to find minimizers to the misfit objective function with respect to the parameters of an investigated physical, economical, or biological model. The misfit is a distance-like function between the observed and simulated states of a phenomenon under consideration. There are many important instances of ill-conditioned GOPs studied in the literature. In particular, the ill-conditioning observed by the irradiative dryer-furnace design are described in [9], while the uncertain problem of the hydrological rainfall/runoff model calibration was studied in [3]. The ambiguity of inverse solutions is also met by the cancer tissue diagnosis [10] and by investigation of hydrocarbon layers in the magneto-telluric (MT) method [15].

The traditional approach of solving ill-conditioned GOPs exhibiting objective’s insensitivity over some regions of admissible domain is the objective regularization [1]. However, this method is strongly restricted to the problems which are conditionally regular in sense of Tikhonov [16]. Moreover, the result of such procedure is a single minimizer without information about the objective behavior in its neighborhood. An attempt at local objective regularization performed in parallel in multiple insensitivity regions combined with the advanced multi-deme memetic search HMS was proposed in [14]. Another, more general approach is to deliver an approximation of such sets [12]. The strategy followed in this paper consists in generating a random sample of points located

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in the insensitive regions, followed by assigning these points to connected components of these regions. After that, a continuous, local objective approximation is generated for each connected insensitive set component. Finally the level set of such objective approximation can be taken as an approximated representation of this component. Of course, the level at which this representation is taken must be sufficiently close to the minimum objective encountered and this distance is one of the parameters of the strategy. Some results obtained in this way were published in [12].

One of the most important factors that affects efficiency of the procedure leading to the well-approximation of insensitivity sets discussed above is to obtain high quality sample of points located in each connected component of the insensitivity set. As far as localizing such components might be performed by a stochastic search using low accuracy objective evaluation in the global phase of the strategy, this accuracy should be increased, when the final, local search is executed, which generates additional significant computational cost in the local phase. Till now, both phases were engined by the Simple Evolutionary Algorithm (SEA) with normal mutation and arithmetic crossover [13].

The main contribution of this paper is to replace SEA in the local phase by the CMA-ES algorithm, highly appreciated as an effective, self-adaptive stochastic local search in continuous domains (see e.g. [5]). We will exploit the analytical representation of the sampling measure, that is created and adapted in each CMA-ES iteration. We expect CMA-ES to quickly and cheaply stabilize the density of its sampling measure, so that the region of its large value covers the current connected insensitivity component. Then, we cover this region by a set of points suitable for the local objective approximation. The effective strategy of multimodal stochastic optimization NEA2 [8] applying also CMA-ES as a component will be used for obtaining the reference data.

The plan of the paper is as follows. We introduce the solving strategies in Section 2. Then, we provide test results in Section 3. Finally, the conclusions are presented in Section 4.

2 Solving strategies

2.1 The global phase strategies

Hierarchic Memetic Strategy (HMS) is a complex stochastic strategy consisting of a multi-deme evolutionary algorithm and other accuracy-boosting, time-saving and knowledge-extracting techniques, such as gradient-based local optimization methods, dynamic accuracy adjustment, sample clustering and additional evolutionary components equipped with a multiwinner selection operator aimed at the discovery of insensitivity regions in the objective function landscape (see e.g. [12,15] and the references therein).

The HMS sub-populations (demes) are organized in a parent–child tree hierarchy. The number of hierarchy levels is fixed but the degree of internal nodes is not. Each deme is evolved by means of a separate single-population evolutionary engine such as SEA. In a single HMS global step (a metaepoch) each deme runs a prescribed number of local steps (genetic epochs). After each metaepoch, a change in the deme tree structure can happen: some of the demes that are not located at the maximal level of the tree can produce child demes through an operation called sprouting. It consists in sampling a set of points around the parent deme’s current best point using a prescribed probability distribution: here we use the normal distribution. The sprouting is conditional: we do not allow sprouting new children too close to other demes at the target HMS tree level. HMS typically starts with a single parent-less root deme. The maximal-level child-less demes are called leaves. The evolutionary search performed by the root population is the most chaotic and inaccurate. The search becomes more and more focused and accurate with the increasing tree level. The general idea is that the higher-level populations discover promising areas in the search domain and those areas are explored thoroughly by the child populations. It is then the leaves that find actual solutions.

The hierarchic structure of the HMS search is especially effective if the computational cost of objective evaluation strongly decreases with its accuracy, which is typically the case when solving IPPs.

The CMA-ES stochastic optimization algorithm performs a stochastic, adaptive search in a continuous domain \( D \subset \mathbb{R}^n \) for some \( n \geq 1 \). It adapts a multivariate normal distribution iteratively. Points sampled from this distribution are used to decide how to modify the mean point contained
in $\mathcal{D}$, so that it will be closer to the exact solution. The operation of the algorithm is thoroughly described in [5], we present here only its concise description.

The initial population of the cardinality $\lambda$ is sampled using the normal distribution with some starting values of a mean $m \in \mathcal{D}$ and a covariance matrix $\sigma^2 C$, where $\sigma \in \mathbb{R}_+$ is the parameter scaling the average standard deviation, $C$ is the $n \times n$ normalized covariance matrix. Typically, the initial setting of $C$ has unit entries on diagonal and zeros outside.

In each consecutive iteration $k$, a population $P_k$ is sampled from the distribution and evaluated. $P_k$ is then used to modify the representation of the sampling measure in the next iteration, i.e., new mean $m_{k+1}$, new $\sigma_{k+1}$ and new covariance matrix $C_{k+1}$ are determined.

Summing up, CMA-ES transforms the density of its sampling measure, an instance of the multivariate normal distribution parametrized by a tuple $(m_k, \sigma_k, C_k)$. At the start of each iteration $k$ it performs $\lambda$-times sampling with return according to the distribution $N(m_k, \sigma_k^2 C_k)$ obtaining the population $P_k$. The analysis of the $P_k$ evaluation (fitness values) leads to the deterministic computing of the sampling measure $(m_{k+1}, \sigma_{k+1}, C_{k+1})$ for the next iteration.

One of the possible stopping conditions of the CMA-ES algorithm is detecting stagnation of the density adaptation process, i.e. the parameters $(m_{k+1}, \sigma_{k+1}, C_{k+1})$ do not differ significantly from $(m_k, \sigma_k, C_k)$. The other, simpler condition is satisfied if at least one individual reaches fitness below the assumed $stopFitness$ value.

The strategy NEA2 introduced by Mike Preuss [8] effectively searches for the global minimizers to multimodal GOPs. It is not responsible for filling basins of attractions as well as the areas of insensitivity surrounding minimizers by the random sample. The NEA2 scheme is simple and includes two components: the Nearest Better Clustering (NBC) algorithm and the stochastic local search CMA-ES.

The NBC is a hierarchic clustering algorithm applied to the random sample of candidate solutions to the GOP under consideration. A directed graph is created, which vertices are the individuals. An edge is created for each individual pointing to its closest better neighbor, unless no such individual exists. Next, edges are cut according to two rules. Firstly, edges which are longer than $\phi$ times the mean length of the edges in the graph. Secondly, if an individual has at least 3 incoming edges and its outgoing edge is longer than $b$ times the median length of the incoming ones. $\phi$ and $b$ are the parameters of the algorithm.

The following three steps are executed consecutively in the main NEA2 loop:

- The random sample is generated with the uniform probability distribution over the whole admissible domain $\mathcal{D}$.
- The sample is clustered by NBC in order to obtain clusters included in basins of attraction of separate global minimizers.
- The CMA-ES processes are started independently for individuals gathered in each cluster.

They are stopped, when the stagnation in evolution is observed.

The total strategy is stopped after the global stopping condition is satisfied, i.e. the evaluation budget is exceeded. The best individuals in each of the CMA-ES processes form the final NEA2 solution.

The setting and tuning of all NEA2 parameters including parameters of both components NBC and CMA-ES is discussed in [11, Sec. 4].

2.2 CMA-ES sampling density in insensitivity regions approximation

The synthetic representation of the sampling measure delivered by CMA-ES may be applied for the effective investigation of the sets of insensitivity and their basins of attraction. The general behavior of the CMA-ES adaptation is to increase the sampling measure in the regions of interest, i.e. regions with small objective values. It might be then conjectured, that a sufficiently low level set of such density function falls into the insensitivity region associated with a particular global minimizer.

In order to intensify finding insensitivity regions associated with distant global minimizers, we will use the HMS strategy, in which CMA-ES search engine is applied in leaf-demes. We also used NEA2 as the alternative global phase of our complex strategy. The insensitivity region identification might be performed in the following steps:
I. We run the HMS strategy for the original GOP (1), until the global stopping condition is satisfied, i.e. no new insensitivity regions can be encountered with a sufficiently large probability. The CMA-ES leaf-demes will be stopped if they encounter areas of flat fitness, when they would increase $\sigma$ value. We will utilize their sampling measure, i.e. covariance matrix, at this point.

II. We extract the final mean, $\sigma$ and covariance matrix of each CMA-ES deme, i.e. $(m, \sigma, C)_i$, where $i \in I$ denotes the deme’s index and $I$ is the set of all leaf-deme indexes. Then, we join the points generated by all the steps of every CMA-ES deme into set $Q_i$. Mahalanobis distance measures distance in terms of standard deviations from a normal distribution to a point. We consider such distance from distribution $(m, \sigma, C)_i$ to each point from $Q$. The points which are within the distance of 1 (no more than 1-sigma) from $(m, \sigma, C)_i$ are put in set $Q_i$, for each $i \in I$. Steps I, II will be called the HMS global phase of the new strategy.

III. In order to obtain the alternative for the HMS global phase we run the NEA2 for the original GOP (1). The final populations of each CMA-ES run of NEA2 and successfully stopped are accepted now as the family of sets of individuals gathered in the basins of attraction. They may replace the family $\{Q_i\}, i \in I$ in the next steps. We do not differentiate the notation for data coming from HMS and NEA2 global phases in steps IV - VIII, assuming it as generic, context dependent.

IV. We try to join the sets $Q_i, Q_j, i, j \in I, i \neq j$. The decision whether both sets of individuals should be joined is obtained on the base of a proper test, e.g. using hill-valley function, hollow-ridge version (see [17]).

V. After joining sets of individuals belonging to the common insensitivity region or the basin of attraction of such region we obtain the reduced family $\{\tilde{Q}_i\}, i \in \hat{I} \subset I$.

VI. We execute for each set of individuals $\tilde{Q}_i, i \in \hat{I}$ the so called local phase of the strategy, i.e. perform several steps of the MultiWinner Evolutionary Algorithm (MWEA) (see [12]) in order to better cover the insensitivity region with individuals. In particular we intend to spread the individuals uniformly and reduce gaps. The resulted set of individuals will be denoted by $\tilde{Q}_i, i \in \hat{I}$.

VII. The next stage consists in designing the local objective approximation for each set of individuals $\tilde{Q}_i, i \in \hat{I}$ by the methods described in [12]. Primarily, the Lagrange 1st order splines on the tetrahedral grid spanned over the $\tilde{Q}_i$ points by the Delaunay’s algorithm are prepared. Next, this function is mapped on the space of 2nd order B-splines spanned over a regular polyhedral grid using both $L^2$ and $H^1$ projections. Both types of projections result in $C^1$ smoothness of the local objective representation. The approximation of a Kriging type (see e.g. [6]) can be applied as well. Let us denote by $\tilde{f}_i$ the local approximation of the objective associated with the set of individuals $\tilde{Q}_i$ for all $i \in \hat{I}$.

VIII. Finally, the level set of $\tilde{f}_i$, taken at a sufficiently low level with respect to the local minimum encountered, will be taken as the approximation of the insensitivity set component, associated with the set of individuals $\tilde{Q}_i, i \in \hat{I}$

$$\mathcal{I}\mathcal{S}^i_e \overset{\text{def}}{=} \{ x \in \text{dom}(\tilde{f}_i) \subset D; \tilde{f}_i(x) \leq \min_{y \in \tilde{Q}_i} \{ \tilde{f}_i(y) \} + \varepsilon \}, \forall i \in \hat{I} \quad (2)$$

where $\varepsilon$ is the tolerance parameter, that determines the maximum variability of the local objective approximation $\tilde{f}_i$ which shouldn’t be handled as an essential one.

3 Experimental verification

We will show three benchmark cases. The first will use a fitness function, which contains 4 non-convex regions of insensitivity. We will show how particular stages of the algorithm up to the local approximation (VII) work on this example. The second will use a fitness function with similar features, but with more regions of insensitivity — we present a comparison based on metrics for this case. The third one uses a 4D Rastrigin function to show how higher-dimensional cases are handled by the strategy. In all these benchmarks both insensitivity regions and their approximations are constructed as sublevel sets with cutoff 0.1, i.e. subsets of the domain where the objective function (or its approximation) assumes values less than 0.1.
3.1 First case - 4 regions of insensitivity

The fitness function is shown in Figure 1 and is defined as follows. We build the benchmark problem by using inverted Gaussian functions defined like that:

\[ g_{r_0}(x) = 1 - \exp \left( -\ln(2)(x - x^0)^T S(x - x^0) \right), \]

where \( r \in \mathbb{R}_+^N \) and \( S \) is a diagonal matrix with \( S_{i,i} = 1/r_i^2 \), \( S_{i,j} = 0, i \neq j \). We multiply it thrice to obtain a C-shaped valley:

\[ c(x) = g_{[0.5,1]} \cdot g_{[1.0,5]} \cdot g_{[0.5,1]} \]

We then define a rotated version of \( c \):

\[ c^\phi(x) = c((x_1 \cos \phi - x_2 \sin \phi, x_1 \sin \phi + x_2 \cos \phi)) \]

And then

\[ h_1(x) = \prod_{i=0}^{1} \prod_{j=0}^{1} c^\theta(x - (2 + 4i, 2 + 4j)), \text{ where } \theta = \frac{\pi}{2}(i + j \mod 4) , \]

\[ \text{flat}_T^T : x \rightarrow \max \left( \frac{f(x) - T}{1 - T}, 0 \right) , T \in \mathbb{R} , \]

and we build the first benchmark function

\[ f_1 : [0,6]^2 \ni (x_1, x_2) \rightarrow \text{flat}_T^{h_1} \in [0,1) . \]

Fig. 1: The plots of the first and the second benchmark functions in their domains.

We compare two algorithms: NEA2 and HMS-CMA-ES, each with attached local phase. The global phase was stopped when it exceeded the budget of 500 evaluations.

HMS has two levels and a metaepoch length of 3. The root level was SEA with population of 40 individuals, crossover probability 0.1, normal mutation with probability 0.5 and standard deviation 2. Sprout was allowed if no other deme or other sprout seed were found within Euclidean distance of 1 and its fitness was below 0.5. The second level was CMA-ES with initial sigma 0.5 which was stopped in case the average objective didn’t change more than 0.01 in 3 epochs. Moreover, CMA-ES demes were also terminated if they began to increase the sigma value. CMA-ES instances used in NEA2 also had initial sigma 0.5.

The hollow-ridge cluster merging method used 3 intermediate points. The reduced clusters’ sizes were used as the MWEA population size if they were inside the range [10,100]. Otherwise, the population was either randomly sampled to reduce its size to 100 or the missing individuals were generated during the first epoch of MWEA.
The MWEA demes formed from the clusters were stopped after 3 epochs. The mutation standard deviation was set to \(1/2\) of the initial population’s diameter.

The results of the run are shown in the Figure 2. The first row shows the clusters which are produced by the global phase, from left, NEA2 and HMS-CMA-ES. The second row depicts clusters after they are reduced by the hill-valley method and the last one shows the points accumulated during the MWEA operation.

In this particular case, HMS-CMA-ES manages to obtain better localized samples, which separate basins of attraction better than NEA2. NEA2 generated clusters with more individuals, one of which spanned more than one basin of attraction. The resulting MWEA demes' in NEA2 variant as well separate the basins of attraction worse.

### 3.2 Second case - 25 regions of insensitivity

The second benchmark function is shown in Figure 1. We build it using (5) and (7):

\[
h_2(x) = \prod_{i=0}^{4} \prod_{j=0}^{4} c^\theta(x - (2 + 4i, 2 + 4j)), \text{ where } \theta = \frac{\pi}{2} (i + j \mod 4),
\]

(9)

\[
f_2 : [0, 20]^2 \rightarrow \flat h_2 \in [0, 1).
\]

(10)

The configurations of the algorithms stay the same, apart from the flexible budget in this test. The budget will vary from 2000 to 10000 evaluations. For each budget, each variant is run 10 times to collect statistics of metrics.

We assess the algorithm variant performance by analyzing several factors. The effect of running a global phase (I-III) is a set of clusters \(Q_i\). After reducing them (IV-V) to \(\hat{Q}_i\) their number should be of the order of the number of insensitive regions — then it is possible to have a single cluster cover a single region.

After running MWEA (VI) we obtain sets \(\tilde{Q}_i\). We assess their quality using the metric of the ratio of covered regions. A region is said to be covered by a cluster \(\tilde{Q}_i\) if every ellipsoid of the region contains at least a single point from \(\tilde{Q}_i\). An ellipsoid has axes of length 1 and 0.5. The ratio is calculated as a fraction of the covered regions to the total number of regions, i.e. 25.

The graphs of the number of clusters after the global phase and after reduction, and the ratio of covered regions are shown in Figure 3. The two first plots show how the algorithms behaved directly after the global phase. In global phase, HMS was much more conservative as the number of clusters go — which is understandable because of the limitation on sprout distances. This is valuable because of the ability to cover a larger region with a similar budget.

The issue with NEA2 clusters is that they don’t separate the basins of attraction well enough. This leads the hill-valley reduction method to join all the clusters into a single one. NEA2 was designed with a focus on identification of localized minimizers in mind and not as a global phase of insensitivity regions approximation strategy. However, its interests are close enough to our approach to make sense to compare it to our solution.

The regions covered ratio is shown in the third graph. The results of HMS-CMA-ES variant increase with budget, while NEA2 variant stagnates.

The level sets for the points from MWEA demes are shown in Figure 4. Since there are many disjoint insensitivity regions with the same shape, approximation of only one of these is presented. Approximation based on points generated by HMS-CMA-ES is significantly better than in case of NEA2.

This is confirmed by the insensitivity region approximation quality data presented in Table 1. For each run and each of the 25 regions we computed Hausdorff distance between the exact insensitivity region and its approximations, except for cases where the method failed to discover a region. The table presents average distances for NEA2 and HMS-CMA-ES with different budgets and for various approximation strategies. While for \(H_1\)-projections the differences are minor, when using \(L_2\)-projections, and especially the Kriging method, HMS-CMA-ES approach is clearly superior.
Fig. 2: The strategy steps visualised for NEA2 and HMS-CMA-ES global phase. Steps are presented in consecutive rows: clusters after the global phase (I-II/III), clusters after reduction (IV-V) and points obtained by MWEA (VI). The clusters from the first row are not exhaustive to keep the plots readable. Each cluster is shown with a different mark type and the solid lines are 0.1 isolines of the fitness function.
Fig. 3: Metric values for NEA2 and HMS-CMA-ES variants in the second case. The data points are shown for budgets ranging from 2000 to 10000 evaluations. Each point was obtained by running the target configuration 10 times.

Fig. 4: Each graph presents contours of the insensitivity region generated by approximation methods compared to the exact contour. $L^2$, $H^1$ and Kriging methods are shown.

| Algorithm  | Budget | $L^2$-projection | $H^1$-projection | Kriging |
|------------|--------|------------------|------------------|---------|
| NEA2       | 2,000  | 1.372 ± 0.13     | 1.910 ± 0.35     | 1.313 ± 0.07 |
|            | 4,000  | 1.369 ± 0.12     | 1.766 ± 0.18     | 1.315 ± 0.11 |
|            | 6,000  | 1.390 ± 0.11     | 1.707 ± 0.18     | 1.315 ± 0.10 |
|            | 8,000  | 1.395 ± 0.08     | 1.797 ± 0.18     | 1.309 ± 0.06 |
|            | 10,000 | 1.319 ± 0.09     | 1.746 ± 0.21     | 1.322 ± 0.07 |
| HMS-CMA-ES | 2,000  | 1.382 ± 0.16     | 1.761 ± 0.19     | 1.056 ± 0.11 |
|            | 4,000  | 1.062 ± 0.09     | 1.746 ± 0.15     | 0.797 ± 0.05 |
|            | 6,000  | 0.939 ± 0.14     | 1.781 ± 0.11     | 0.664 ± 0.07 |
|            | 8,000  | 0.907 ± 0.13     | 1.700 ± 0.16     | 0.610 ± 0.08 |
|            | 10,000 | 0.940 ± 0.15     | 1.770 ± 0.20     | 0.640 ± 0.09 |

Table 1: Average Hausdorff distances between the exact sets of insensitivity and their approximations.
3.3 Third case - 4D Rastrigin

The fitness function in this case is defined as follows:

\[ f_3: [-5, 5] \times [-2, 2]^3 \ni (x_1, \ldots, x_4) \rightarrow 2 - \frac{1}{2} \left( \cos \frac{\pi x_i}{5} + \sum_{i=2}^{4} \cos \pi x_i \right) \in [0, 4]. \]  

The minima are placed on a regular grid with step size 10 in the first dimension and distance 2 in the remaining ones. There are 27 global minima in this case.

The settings of the algorithms remain the same as in the previous tests, apart from the budget of the global phase, which is set to 50000 evaluations. At such budget the performance of the methods approximately stagnates. We ran each algorithm 10 times to gather statistics.

The ratio of covered minima in this case is 44.1%±0.6% for NEA2 and 59%±4% for HMS-CMA-ES. A minimum is considered to be covered by a deme if the deme has generated an individual closer than 0.4 from the minimum.

Insensitivity region approximation results are presented in Table 2. As in the previous benchmarks, for each of the 27 insensitivity regions we compare the exact region with its approximation constructed as a level set of objective function approximation based on points generated by NEA2 and HMS-CMA-ES. For both algorithms the best results are obtained using Kriging method and in this case once again, HMS-CMA-ES proves clearly superior. It is worth to note that \( H^1 \)-projection method performs relatively worse compared to other approximation strategies, the difference being significantly more pronounced than in the previous benchmark.

| Algorithm    | \( L^2 \)-projection | \( H^1 \)-projection | Kriging     |
|--------------|-----------------------|-----------------------|-------------|
| NEA2         | 1.426 ± 0.08          | 1.722 ± 0.62          | 1.406 ± 0.09|
| HMS-CMA-ES   | 1.181 ± 0.01          | 2.618 ± 0.68          | 0.722 ± 0.05|

Table 2: Average Hausdorff distances between the exact sets of insensitivity and their approximations for 4D Rastrigin benchmark.

4 Conclusions and remarks

We have presented a new strategy of determining shapes of insensitivity areas which surround global minimizers, being a subsequent stage of our research [12]. The first phase of the strategy is global search, which aim is to identify basins of attraction, a process which should make as few objective evaluations as possible while maintaining the global search capabilities. HMS invented by the authors was applied in this phase. We have also used NEA2, which is well known to efficiently identify multiple minima, as a reference algorithm. The random sample gathered after the global phase is then transformed in local phase to better fill the regions of insensitivity and separate their connected components.

Instead of using SEA in leaves of HMS, we substituted it by CMA-ES. We exploit probability density information gathered during its convergence to better localize the area of each separate, connected insensitivity region. It allows us to abandon a more complex to configure density-clustering method.

We demonstrate the performance of our strategy on three benchmarks. The first one shows the random sample transformation in its consecutive stages. The second one allows to compare HMS-CMA-ES results with baseline, i.e. NEA2 for an objective function with 25 separate regions of insensitivity. The last one shows the efficiency of the strategy by solving a 4D problem with 27 minimizers. Generally, the strategy using HMS-CMA-ES fills and separates the connected components of insensitivity regions slightly better than the NEA2 one (see Figures 2, 3, 4, Table 1, 2).

We plan to further improve the strategy to be able to handle multi-objective problems. It can be achieved by using MOEA algorithm with NSGA selection (see e.g. [2]), or a modified selection
based on domination ranks (see [4]). The sets to be found in this case are associated with the connected components of a Pareto set.

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