Dynamical phases and hysteresis in a simple one-lane traffic model

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A two parameter model for single lane car-following is introduced and its equilibrium and non-equilibrium properties are studied. Despite its simplicity, this model exhibits a rich phenomenology, analogous to that observed in real traffic, like transitions between different dynamical regimes and hysteresis in the fundamental flux-density diagram. We show that traffic jams can spontaneously appear in clustered-like structures. In the jammed phase, we observe a slow relaxation phenomenon ruled by the outgoing car flux that determines the hysteretic dependence of the fundamental flux-density diagram. Coexisting phase regimes are also evidenced so as propagating or stationary density waves. The model can be easily calibrated to reproduce experimental observations.

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The occurrence of traffic jams or small traffic congestions without obvious reasons are common effects that almost every driver has once experienced. These effects are typical signatures of the complex behavior of traffic flows and their study is of practical interest in the context of traffic control. The evolution of such spontaneous time-space structures has long attracted attention in the understanding of non-equilibrium properties of externally driven many-body systems. Sophisticated concepts were in particular developed to study critical phenomena and successfully applied to the description of phase transitions in traffic flow models [6,12].

Real traffic exhibits a very rich variety of phenomena. For this reason, the most adapted techniques for efficient traffic control are still debated. Experimental investigations on highways revealed that traffic can exhibit well identified dynamical regimes that depend on the external car flux and on the car density [6,13]. Phase transitions occur in traffic flows when the vehicle density exceeds a critical threshold. Below this threshold, traffic is free. Beyond it, vehicles either briefly slow down due to high density traffic or stop in a jam. Jams can appear without obvious reasons, they can merge and extend to large scales. The outcome of experimental measurements were moreover shown to be strongly influenced by the traffic behavior near the measurement site. Near ramps for example, peculiar behaviors such as avalanches or oscillations are known to take place [1]. Traffic flow results are thus far from universal [6].

Planning and optimizing real traffic flow has motivated many theoretical approaches, ranging from cellular automata models [1] to coupled maps [2] and from hydrodynamics [6,10] to kinetic theory [11,13]. Theoretical results depend on the drivers behavior. In deterministic models for instance non-linearities induce the jamming transition and the jammed phase persists in time [10] while for stochastic models a jamming transition occurs due to the intrinsic noise and no jam persists for ever [7]. Most of the theoretical models are based on heuristic arguments and their parameters have to be calibrated using experimental outputs.

With the aim of retrieving experimental properties of traffic, we focus on a simple 2-parameters car following model. We consider a one lane street of length $L=10$ km. Vehicles on this street are all identical with size $d_{\text{car}}=5$ meters [13] and indexed such that car number $i+1$ precedes car $i$. They cannot overtake and, due to periodic boundary conditions the vehicle ahead car 1 is $N$. When approaching a slower car the driver slows down. The velocity of the front car remains unaffected.

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The prefactor 25 in the first equation is then a consequence of the empirical driving rule. Indeed, in order to proceed safely, a driver at velocity $v_{max}$ has to slow down when the headway distance becomes smaller than 125 m = $25d_{car}$. Although very simple, we show herein that simulations of Eqs. (3) reproduce spatio-temporal observations of experimental traffic flows. For the time evolution of the position of each car we use a synchronous updating based on the Euler scheme $x_i(t + \Delta t) = x_i(t) + v_i(t)\Delta t + O(\Delta t^2)$ with time step $\Delta t = 0.1 \text{s}$. Due to this time discretization and in order to avoid artificial car crash we have to constraint numerically $v(i) = 0$ when $d(i) < d_{car}$. This introduces a small dynamical noise that slightly perturbs our simulations. We checked however that the main results presented herein still hold for $\Delta t = 0.01 \text{s}$.

We study model (1) under driven external car flux, i.e. we allow the average car density on the street $\rho$ to vary in time due to in-going (resp. out-going) cars through an on-ramp (resp. off-ramp) located at position $x = 0 \text{ km}$ (resp. $x = L = 10 \text{ km}$). To check the validity of this model, we first pay attention to the experimental observations of reference [3]. For given position $x$ on the street, we consider the one minute averaged velocity $v_x$ of the cars crossing $x$ and $\rho_x$ the one minute averaged density of a 1 km band centered on $x$. We deal with one realization of the system and we model the external car flux by a simple superposition of three Maxwellian functions. For this, we constraint the time evolution of the car density on the whole street to be $\rho(t) = \rho_0 \sum_{i=1,2,3} \exp[-((t - t_i)^2)/2]$, with $t_1 = 7:00$, $t_2 = 12:00$ and $t_3 = 17:00$ and where $t$ is expressed in hours. Using the experimental results of [3] we set $\rho_0 \sim 50 \text{ cars/km}$ and $t_i$’s are supposed to be rush periods. Practically, we estimate the density $\rho(t)$ every 10 seconds and, according to its value, either keep the total number of cars to the same value or add/remove a single car from the street. This means that the maximal car flow is 360 cars/h which is consistent with constant external flux models [6] but below experimental observations [3].

In Fig. 1 we display the local flux-density $(q_x, \rho_x)$ fundamental diagram and the time evolution of velocity $v_x$. We consider two cross sections: The first is situated at position $x = 5 \text{ km}$ and the second near the off-ramp at $x = 9 \text{ km}$. The $(q_x, \rho_x)$ representation of Figs. 2 look very similar to the one obtained in [6]. The numerical points tend however to accumulate in the neighborhood of $\rho = 30 \text{ cars/km}$. This effect is a consequence of the simplicity of the time dependence of $\rho(t)$ that tends to leave the system longer near $\rho = 25 \text{ cars/km}$. It can be removed by adding small random fluctuations to $\rho(t)$.

When comparing $v_5$ and $v_9$, we observe larger fluctuations in the time behavior of $v_9$ (see full line in Fig. 2(b) and (d)). They appear in time periods where the variations of $\rho(t)$ are fast (i.e. when the external flux is large). These fluctuations are of particular interest near on-ramps where they can be seen as signature of avalanche-like effects that take place when cars arrive in a yet “busy” highway.

The effect of ramps was investigated both experimentally [4] and theoretically [2][3][4]. This local breakdown effect triggers jams and remains even if fluctuations in the external flow are negligible. Time evolution of $v_1$ and $q_1$ is shown in Fig. 2 between time 4:00 and 13:00. Both figures exhibit five different dynamical regimes: stable free traffic (SFT for $t \in [4:00,5:00]$), local break down traffic ($LB DT$ for $t \in [5:00,6:30]$), clustered traffic (CT for $t \in [6:30,8:00]$), synchronized traffic (ST for $t \in [8:00,9:00]$) and unstable free traffic (UFT for $t \in [9:00,11:30]$). In order to identify these regimes, we applied a noise to $\rho(t)$ and observed that all the regimes but UFT were not significantly modified. UFT turns out to be sensitive to perturbations and disappears for too large noise amplitudes [12]. LB DT on the other hand appears when the density grows beyond $\rho_c \sim 15 \text{ cars/km}$ and is characterized by an avalanche-like decreasing velocity $v_1$ for increasing density [1]. This regime is located in the vicinity of the ramps and triggers complex non-homogeneous traffic structures that evolve over the whole street (see Fig. 1). CT consists in small jammed islands that appear away from ramps, persist in time and can be compared to droplets that form in supercooled gas. ST phase appears in highway measurements when a similar average car velocity is observed on all the lanes of the highway and is mostly localized in the neighborhood of the ramps. One line descriptions were however successfully used to study this regime [3][4]. ST is characterized by a low average velocity but a rather high value of the local car flux $q_1$ when compared to the one of the jammed traffic phase (JT). This latter phase does not appear in the previous
figures since $p_0$ is too small. JT is a stable phase (any perturbation fade as time evolves) and is characterized by densely packed queues of cars that can extend in a single jam over the whole street.

![Image of a street with a jam]

FIG. 2. Detail of the time dependence of the average velocity $v_1$ (a), flux $q_1$ and density $p_1$ (b) at cross section $x=1$ km. Legends and simulation conditions are the same than in Fig. 1. The vertical dot-dashed line in (a) indicates the moment where the phase transition between free traffic flow and congested flow with critical density $p_{c1} \sim 15$ cars/km. The dotted line in (a) corresponds to the time dependence of $p_1$. In (b) we show the succession between $SFT$, $LBDT$, $CT$, $ST$ and $UFT$.

Many features of traffic flows can be compared to fluid dynamics. For instance JT can be seen as a compressible liquid without coexisting vapor whereas $SFT$ can be trivially compared to a dilute gas. Consequently, a criterion for model (1) to be realistic constraints the transition between JT and SFT to be hysteric as the liquid-gas transition. This prompts us to rapidly focus on the ensemble averaged total flux $q$ and total density $p$. For each realization of the system, we perform a simulation that starts with $p = 1$ car/km, with a zero out flow $\phi_{out} = 0$ of cars and a constant in flow $\phi_{in} = \phi$ (in-flow period). Cars are introduced on the street one by one, to keep the system as close as possible to its stationary state, and $q$ is averaged over the time running between two introductions. When density $p_{max} \sim 160$ cars/km is reached, the street is uniformly jammed. At this density, in-flow is interrupted and we start to remove cars one by one setting $\phi_{in} = 0$ and $\phi_{out} = \phi$. We proceed this way down to density $p = 1$ car/km (out-flow period). This procedure is repeated over 100 realizations. Fig. 3 displays the ensemble averaged $(q,p)$ fundamental diagram for several values of $\phi$. The SFT and the UFT regimes clearly show up for $p < p_{c2}$ and $p \in [p_{c3}, p_{c2}]$ respectively, with $p_{c2} \sim 20$ cars/km. Both regimes do not strongly depend on the value of $\phi$. For $p > p_{c3}$ we also retrieve the characteristic hysteresis loop of traffic behavior. The lower branch corresponds to in-flow periods. It is independent of the in-flow rate $\phi_{in}$. This is clearly not the case for out-flow periods as shown by the existence of several upper branches in Fig. 3. A direct consequence is that JT occurs beyond a threshold density $p_{c2}$ that itself depends on the out-flow rate $\phi_{out}$, where $p_{c3}$ is defined as the density $p > p_{c2}$ for which the two branches of the hysteresis loop differ less than 10%. Hence, the global stability of model (1) is only determined by the out-going car flow $\phi_{out}$. This property remains true even when modifying the model [1].

![Flux-density fundamental diagram](image)

FIG. 3. Flux-density fundamental diagram averaged over 100 realizations for several values of the in/out car flow $\phi$. $p_{c2} \sim 20$ cars/km determines the threshold beyond which UFT appears. The lower branch (resp. upper branches) of the hysteresis loop is (resp. are) generated during the in-flow (resp. out-flow) period. $p_{c3} \sim 40$, $p_{c4} \sim 70$ and $p_{c5} \sim 110$ cars/km are defined in the text and refer to simulations with $\phi = 360$ cars/h (full lines). For this value of $\phi$ congested traffic appears when $p \in [p_{c2}, p_{c5}]$ whereas JT appears for $p > p_{c5}$ cars/km.

For SFT, UFT and JT regimes the corresponding spatial distribution of the cars is uniform over the whole street. Since we focus on ensemble averaged quantities, only limited informations can be obtained from Fig. 3 on the complex space-time patterns of congested traffic. It is however interesting to note that for $\rho > p_{c3} \sim 40$ cars/km the in-flow branch is almost linear. This behavior is similar to the coexisting regime in fluid dynamics with the fluid and its gas in thermal equilibrium. Fig. 4 suggests that such a picture also holds for traffic flows although these systems are far from equilibrium. The explanation is that in this density range the system decomposes in two phases: A low density one ($p_{c2} < p < p_{c3}$) that we identify as the CT and a high density one ($p > p_{c3}$) corresponding to JT. This is illustrated in Fig. 4 where we show the space-time density contour plot. Clustered structures of CT show up in Fig. 4 when $p_{c3} < p < p_{c5}$. As the density inside clusters is close to the critical value $p_{c3}$ this situation can be compared to the small droplets that form in a gas near condensation. As new cars arrive on the road, the JT first appears at time $t=1:7$ near the ramp and coexists with CT. As time evolves, due to increasing $p$, JT extends backwards and finally overcomes CT (at time 3:00 where $p \sim p_{c5}$). Forward propagating density waves are evident in Fig. 4. The contour plot for $p > p_{c5}$ in Fig. 4 shows complex space time structure and an almost uniform JT phase that extends over the complete street when $p \sim p_{max}$.

Nucleated structures of CT also show up in the out-
flow period when $t \in [6:7]$ ($\rho_{c4} < \rho < \rho_{c5}$). But in this
time range, it coexists with two quiet well separated JT
and SFT phases. For longer times, when $\rho_{c5} < \rho < \rho_{c4}$, a
new separated phase between JT and SFT appears whith
two density fronts. As $\rho$ decreases with time, the in-flow
of cars is no more sufficient to maintain JT that progress-
sively disappears. In the down stream direction the front
is standing while in the upstream one, the front pro-
propagates due to the 'evaporation' of the JT phase. From
these results, $\rho_{c4}$ turns out to play the role of a physical-
threshold between a triphasic JT-CT-SFT situation
and a situation where JT and SFT coexist in a simple
space-time shrinking structure.

![Space-time density contour plot](image)

FIG. 4. Time evolution of the total density $\rho$ (left) and
space-time density contour plot (right) for the run of Fig. 3
with $\phi = 360$ cars/h, during in flow $t < 4:30$ and out-flow
$t > 4:30$ periods. In these figures we restricted ourselves to
the time period where $\rho \geq \rho_{c2}$. In the left graph, the hor-
izontal lines indicate the time at which the critical density
$\rho_{c1}$, $i = 3, 4, 5$, is reached. The street is divided in 20 sections
of 500 meters and in each of them the density $\rho$ is evaluated
every minute. The legend of colors are given in cars/km.

We considered herein a simple 2-parameters micro-
scopic traffic model. When driven far from equi-
librium by an external car flux, this model reproduces
the main characteristics of experimental fundamental di-
agram. Near ramps, we retrieve avalanche-like effects
so as synchronized traffic. The ensemble averaged funda-
mental diagram exhibits the traditional histeresis loop of
traffic models. This a signature of coexisting phases and
we showed that among these phases two regimes have to
be distinguished: a clustered phase where the system de-
velops complex space time patterns and a regime where
phases are separated by well identified density fronts.

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conversion of $v_i$ in km/h is $v_i = (d_i/T) (3600/1000) = d_i$.
[21] Similar hysteresis curves are obtained when considering
a spread-out car-size distribution that follows a uniform
random law with mean value $d_{car} = 5$ m.