Effectiveness of tutorial worksheet for promoting basic vector concepts: dot product and cross product

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Abstract. In recent years, several physics education researches have reported students’ understanding and difficulty of vector concepts. Lacking of understanding about dot product and cross product might cause many serious problems when the concepts are embedded in many physics topics. Therefore, we decided to design a tutorial worksheet that guides people through the development of a conceptual framework in dot product and cross product. The worksheet was implemented with 10 and 21 volunteer science teachers. Upon using the TUV items to evaluate their understanding, the results confirmed that the tutorial worksheet had facilitated their learning. The learning normalized changes were 0.82 and 0.75 considered as high learning gain. The tutorial worksheet might be used by other physics instructors who teach this material in high schools, colleges or universities.

1. Introduction

Most physical concepts covered in introductory physics courses at both high school and university levels are represented by vectors. Therefore, a complete understanding of these physical concepts requires that students have a good grasp of basic vector concepts. In recent years, several researchers have investigated students’ understanding and difficulty of vector concepts. The research showed that students still hold misunderstanding about basic vectors although they have studied it before. Some observed that students have considerable difficulties in understanding and calculating dot product and cross product. These could be concluded from the literatures that most students have difficulties with vector concepts for both with and without physics context [1–4]. Lacking of understanding about dot product and cross product might cause many serious problems when the vector concepts are embedded in many physics topics including of torque, work, magnetic flux, electric flux, electromagnetic force and so on.

In this study, we intended to design a tutorial worksheet to enhance students’ conceptual understanding in dot product and cross product based on students’ alternative conception found in previous research. Since, tutorial worksheet was one of the most powerful tools that physics instructors have used to help their students learned about vector concepts. It is very convenient and time saving. Students had also performed well after the worksheet instruction [5-7]. Then, the designed tutorial worksheet will be studied its effectiveness for promoting dot product and cross product conceptual understanding upon using a standardized test.
2. Previous Research

2.1. Alternative Conception of Dot product

Barniol, Zavala, and Hinojosa [1] investigated students’ difficulties with the formal representation of dot product projection. They found that the most common incorrect reasoning by students was to relate the scalar nature of the dot product with the magnitude of a vector and some related the dot product with the addition of vectors. Most common incorrect reasoning was based on an improper calculation of the dot product using unit vector notation. Some students had more difficulty interpreting and calculating the dot product in the unit-vector notation than correctly applying the equation ABcosθ [8].

Zavala and Barniol [2] investigated the difficulties in calculating the dot product and the misconceptions in the interpretation of this product. The most common error (11%) was to use |A||B|Sinθ instead. Some made errors trying to multiply components or finding the components of the vectors. It seemed like all of them did not exactly understand about dot product. Only 4% among 200 students gave a completely adequate interpretation that the dot product is the projection of one vector onto a second vector multiplied by the magnitude of the second vector. Most of them interpreted dot product as mathematics equations as |A||B|cosθ and A, B, + A, B, . The most common error (23%) was to interpret the dot product as a vector.

2.2. Alternative Conception of Cross product

The main difficulty found in cross product is identifying the direction of resultant vectors example for direction of torque (τ), a vector quantity defined as τ = r×F [1]. Another difficulty is calculating its magnitude.

Zavala and Barniol [2] investigated the difficulties in calculating cross product and misconceptions in interpretation of this product. Most of the student provided correct calculation but most of them have difficulty with its direction. They showed that students have more difficulties in interpreting cross product and calculating its magnitude as ABsinθ than in correctly calculating the cross product in unit-vector notation [8].

3. Methodology

McDermott’s study [9], outlined a process for developing new instructional material. The process has three steps: 1) conduct research on student understanding, 2) use the findings to guide the development of instructional materials, and 3) carry out an effectiveness assessment of the new instructional materials, based on what the students have learned. Therefore, we collected some ideas and overview of what alternative conception the students held and what difficulties the students faced. Then, we intended to design a tutorial worksheet based on students’ alternative conception found in previous study. The worksheet then implemented to see its effectiveness.

3.1. Tutorial Worksheet

The worksheet promoted basic vector concepts on the topics of dot product and cross product. It provided experience in learning through guided inquiry and an emphasis on the constructing of the concepts. The main goal of the tutorial worksheet was to guide students in developing a conceptual framework of dot product and cross product. The tutorial worksheet posted questions and permitted a step by step description of how students develop a conceptual framework when using the worksheet. There were 3 sections in the worksheet. Section 1 interpretation of dot product: 1) guided students to drew and calculated vector projection, 2) led students to interpret the dot product as the projection of one vector onto a second vector multiplied by magnitude of the second vector, and 3) guided students to do calculation of dot product in two possible ways. Section 2 interpretation of cross product: 1) led students to recall parallelogram, area vectors, and right hand rule, 2) led students to interpret cross product as the area of parallelogram, and 3) guided students to do calculation of cross product in two possible ways. Section 3 comparison of dot product and cross product: led students to compare the differences between dot product and cross product and to also be able to form their own conclusions with regard to the calculation of these products. The designed worksheet can be found via this link: https://drive.google.com/file/d/1bFavmqcZ5mVfSIFbYqlUYBceB5qu6M-1/view?usp=sharing.
3.2. Test of Understanding of Vectors (TUV)
The test of understanding of vectors is a reliable multiple-choice vector concept test constructed by Barniol and Zavala [8]. The test, original version published in English, was translated into Thai-version by the Physics Education Network of Thailand (PENThai) members [10]. There are 20 items measuring students’ basics vector conception of direction, magnitude, component, unit vector, vector representation, addition, subtraction, scalar multiplication, dot product, and cross product. Only the 6 items related with dot product and cross product were used in this study.

3.3. Data Collection
During 2019 academic year, both TUV and the tutorial worksheet of dot product and cross product were implemented to two groups of 10 and 21 volunteer science in-service teachers. The 6 multiple-choice item test of Thai-version TUV was applied as a pre-test. The teachers spent approximately 5-7 minutes to complete. The implementation of the worksheet was carried out by an experienced instructor without lecturing or giving answers, but rather help learners to arrive at their own answers by posing questions that guided them through the necessary reasoning. The responses to their questions were not directly answers but questions to help them arrive at their own answers. The science teachers spent approximately 90 minutes through the worksheet step by step. After completing the worksheet, the 6 multiple-choice item test of Thai-version TUV was applied again as a post-test. Then, their learning gains had been verified. The effectiveness of the worksheet was calculated by comparing the differences between percentages of their correct responses during pre- and post-test to the maximum possible gain.

3.4. Data Analysis
The science teachers’ pre- and post-test scores were compared. The effectiveness of tutorial worksheet is the differences between pre- and post-test scores before and after conducting the tutorial worksheet. To see the progression in each problem, the normalized gains were then calculated separately for each problem by comparing the differences of each correct response between pre- and post-test to the maximum possible gain of that problem [11]. For one who had full or zero scores for both pre-test and post-test, their scores would be removed from the data set because the student performance was beyond the scope of the measurement instrument [12]. The learning average normalized gains were divided into three regions; high gain is where \(<g>\) is greater than 0.7 (<g>high> 0.7), medium gain with \(<g>\) between 0.3 and 0.7 (0.3 ≤ <g>medium < 0.7) and low gain with \(<g>\) less than 0.3 (<g>low< 0.3) [11].

4. Results
Upon using the TUV to evaluate science teachers’ understanding, we found that the tutorial worksheet had facilitated the science teachers ‘learning.

| Table 1. Percentages of 10 science teachers’ responses to the TUV items and their normalized changes of dot product and cross product |
|---------------------------------|---------|---------|---------|---------|
| items                          | % pre-test | % post-test | gain   | \(<g>\) |
| Dot product                    |          |          |        |         |
| 3                              | 20       | 100      | 1.00   | 0.88    |
| 6                              | 80       | 100      | 1.00   |         |
| 8                              | 40       | 80       | 0.67   |         |
| Cross product                  |          |          |        |         |
| 12                             | 60       | 100      | 1.00   | 0.78    |
| 18                             | 40       | 90       | 0.83   |         |
| 15                             | 20       | 70       | 0.63   |         |
| Averages                       | 43.3     | 90.0     | 0.82   |         |

There were significant differences in the distribution of the answers for both the 10 and 21 science teacher groups. Table 1 shows percentages of 10 science teachers’ responses to the TUV items and their normalized changes of dot product and cross product. The average percentages of their pre- and post-test were 43.3% and 90.0%. The overall learning gain was 0.82. Focused on each dot product and cross product concepts, their learning gains were 0.88 and 0.78 respectively.
Table 2. Percentages of 21 science teachers’ responses to the TUV items and their normalized changes of dot product and cross product

| items       | % pre-test | % post-test | gain | <g> |
|-------------|------------|-------------|------|-----|
| Dot product | 3          | 30          | 91   | 0.86| 0.72|
|             | 6          | 75          | 100  | 1.00|     |
|             | 8          | 25          | 62   | 0.49|     |
| Cross product | 12      | 55          | 76   | 0.47| 0.78|
|             | 18         | 45          | 95   | 0.91|     |
|             | 15         | 45          | 95   | 0.91|     |
| averages    |            | 45.8        | 86.5 | 0.75|     |

Table 2 shows percentages of 21 science teachers’ responses to the TUV items and their normalized changes of dot product and cross product. The average percentages of pre- and post-test were 45.8% and 86.5%. Their overall learning gain was 0.75. Focused on each dot product and cross product concepts, their learning gains were 0.72 and 0.78 respectively.

5. Discussion and Conclusion
The conceptual learning gains of those two groups of volunteer science teachers were greater than 0.7 considered as high gains [11] that might be rarely reached by conducting classes even with active learning activities. Since, the worksheet guided the teachers to interpret dot product and cross product step by step until they got their own conclusions. The worksheet also permitted them to calculate the products in both unit-vector notation and graphical representation, instead of providing only equations to get numerical answers. Therefore, the worksheet could be served as an effective teaching strategy that enhances conceptual understanding in interpretation of dot product and cross product.

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