Study on the Theoretical Method of Determining Reasonable Water Injection Intensity

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Abstract: Currently, water flooding is the main method to maintain reservoir pressure and improve oil production in many oilfields. A calculation procedure is developed based on the Buckley-Leverett theory ($\phi$ function) to determine the reasonable water injection rate, which directly improve the effectiveness of field development. $\phi$ function (the derivative of water fractional flow with respect to water saturation) is the reciprocal of PV number, which relates to the swept area, effective reservoir thickness, water cut and cumulative water injection. Considering all these factors, the proposed method can ensure the rationality of water injection rate.

1. Introduction
Water flooding optimization via injection rate control is one of the most-widely used reservoir management tools to date[1]. It can effectively increase the oil displacement efficiency and maintain the reservoir pressure for a long time [2-3]. In China, most oilfields use water injection to improve oil recovery from reservoirs. The optimum injection rate is essential in operational and economical decisions for reservoir management.

Following the fractional flow theory proposed by Buckley and Leverett in 1942, numerous models had been developed to simulate the process of multiphase flow in porous media[4]. To date, the theory has been widely used to predict the performance of waterflooding and enhanced oil recovery (EOR) processes[5-8]. The fractional flow approach considers concurrent flow of the two phases - oil and water, by describing separately the flowing behavior of each phase. In this study, we use the Buckley-Leverett theory to determine the optimum water injection rate for maximum oil production.

2. $\phi$ function mechanism
The Buckley-Leverett equation, shown below, is valid for one dimensional unidirectional flow, as depicted in Figure 1.

$$ x - x_0 = \frac{W_i(t)}{\phi A} \phi(S_w) $$

(1)

Where:
- $x$ -- the position of a certain cross-section plane, m;
- $t$ --the start time of water injection, s;
- $W_i(t)$ --cumulative water injection volume, V;
- $S_w$ -- water saturation of the reservoir cross-section plane at $x$;
Figure 1 One dimensional unidirectional percolation flow model

For steady flow, \( f_w \) can be expressed as:

\[
 f_w = \frac{K_{rw}}{\mu_w} \left| \frac{K_{rw}}{\mu_w} + \frac{K_{ro}}{\mu_o} \right|
\]

Therefore:

\[
 \phi(S_w) = \frac{d f_w}{d S_w}
\]

Where:

- \( x_o \) -- Injection well point position;
- \( \phi \) -- Reservoir porosity;
- \( A \) -- Cross-section area of the reservoir;
- \( \phi(S_w) \) -- Derivative of water fractional flow \( f_w \) to water saturation \( S_w \);
- \( \mu_w \) -- Water phase viscosity;
- \( K_{rw} \) -- Water phase relative permeability;
- \( \mu_o \) -- Oil phase viscosity;
- \( K_{ro} \) -- Oil phase relative permeability;

At time \( t \), the relationship between water saturation and cumulative water injection is obtained from equations:

\[
 x_1 - x_o = \frac{W_i(t)}{\phi A} \phi(S_{w1})
\]

\[
 x_2 - x_o = \frac{W_i(t)}{\phi A} \phi(S_{w2})
\]

Combining equation (4) and equation (5):

\[
 x_2 - x_1 = \frac{W_i(t)}{\phi A} \left[ \phi(S_{w2}) - \phi(S_{w1}) \right]
\]

Define \( I_{pv} \) as:

\[
 I_{pv} = \frac{W_i(t)}{\phi A (x_2 - x_1)}
\]

Under the assumption of incompressible steady flow, the cumulative water injection, the cumulative liquid production and the cumulative liquid passing through any cross-section in the reservoir are all equal. Thus, \( I_{pv} \) is the multiple of cumulative influent liquid into the reservoir unit between \( x_2 - x_1 \) to the pore volume. Combining equation (6) and (7):
\[ \varphi(S_{w2}) = \varphi(S_{w1}) + \frac{1}{P_{ pv}} \]

Equation (8) indicates that \( \varphi \) function of a certain reservoir unit is equal to the sum of the \( \varphi \) function of the upstream reservoir unit and the reciprocal of multiple of cumulative influent fluid into this unit to pore volume. Equation (1) can be simplified into:

\[ \varphi(S_{w}) = \frac{\phi A(x-x_{0})}{W_{i}(t)} \]

Therefore, \( \varphi(S_{w}) \) equals to the cumulative pore volume of water injection from the injection end to position \( x \). While at the production end, \( \varphi(S_{w}) \) is the reciprocal of pore volume of cumulative water injection into reservoir, namely the reciprocal of pore volume of cumulative liquid production.

3. Water injection rate determination

Ten relative permeability curves from Daqing oilfield are analyzed, and the result shows there is a parabolic relation between the \( \varphi \) function and the water cut, as shown in the following equation:

\[ \varphi(S_{w}) = \frac{V_{p}}{W_{i}(t)} = af_{w}^{2} + bf_{w} + c \]

2 relative permeability curves are randomly chosen to verify the parabolic relation, and the comparison results are presented in Figure 2, 3:

![Figure 2 Typical relative permeability curve 1 of Daqing oilfield](image1)

![Figure 3 Relation between \( \varphi \) function and water cut according to relative permeability curve](image2)

Therefore, from equation (10), an equation group can be established with the pore volume, water cut and cumulative water injection of each well group, from which the coefficients of \( a, b, c \) can be determined. The equation group is formulated as:

\[
\begin{align*}
\varphi_1 &= \frac{V_{p}}{W_{i1}(t)} = af_{w1}^{2} + bf_{w1} + c \\
\varphi_2 &= \frac{V_{p}}{W_{i2}(t)} = af_{w2}^{2} + bf_{w2} + c \\
&\vdots \\
\varphi_n &= \frac{V_{p}}{W_{in}(t)} = af_{wn}^{2} + bf_{wn} + c
\end{align*}
\]
4. Calculation of the Optimum Water Injection Rate in an Actual Block.

The development dynamic and static data of from a block in Daqing oilfield is used to solve equation group (11). The solving process should satisfy the condition: \( \phi = 0 \) when \( f_w = 1 \).

The solution of the equation (11) is \( a = -115, \ b = 206, \ c = -91 \). The \( \phi \) function and cumulative water injection, when the planning water cut of the whole block is 92.71\%, can be calculated by substituting this solution in equation (10).

\[
\phi_{92.71} = a \cdot 0.9271^2 + b \cdot 0.9271 + c
\]

The values of \( a, b, c \) are substituted in equation (12), and \( \phi_{92.71} \) can be calculated as:

\[ \phi_{92.71} = 1.145 \]

When water cut of the block reaches a value of 92.71\%, the cumulative water cut can be calculated as:

\[ W_{92.71} = \frac{V_p}{\phi_{92.71}} \]

\( \phi_{92.71} = 1.145 \) is substituted into equation (13):

\[ W_{92.71} = \frac{V_p}{\phi_{92.71}} = 89837.26 \times 10^3 \text{ m}^3 \]

Therefore, the block cumulative water injection in 2011 is calculated as:

\[ Q_{2011} = W_{92.71} - W_{2010} = 3699.8 \times 10^3 \text{ m}^3 \]

Where:

\( \phi_{92.71} \) -- \( \phi \) function when the water cut takes a value of 92.71\%;

\( W_{92.71} \) -- Cumulative water injection when the water cut takes a value of 92.71\% (10^4 t);

\( Q_{2011} \) -- Cumulative water injection of the block in 2011 (10^4 t);

Statistical data from a block in Daqing oilfield shows that the cumulative water injection in the east part of the block is 1964 thousand cubic meters in 2010 and the cumulative water injection of the whole block is 3789.4 thousand cubic meters. Therefore, the cumulative water injection in east part of the block in 2011 can be calculated as:

\[ Q_{2011} = \frac{Q_{2011}}{Q_{2010}} \cdot Q_{2010} = 196.4 \cdot 378.94 = 1910 \times 10^3 \text{ m}^3 \]

Where:

\( Q_{2011} \) -- Cumulative water injection of the east part of the block in 2011, \( \times 10^3 \text{ m}^3 \)

5. Calculation of Optimum Water Injection for Well Group.

5.1. Determination of injection time

The injection period from 2010 to the time when the water cut increases to 98 percent can be calculated with the assumption that the annual liquid production in the period from 2011 to the time when the water cut increases to 98 percent remains the same value as the of planning liquid production in 2011.

After the water injection period from 2010 to the time the water cut increases to 98 percent is solved, the annual water injection of each well group can be calculated according to \( \phi \) function.
5.2. Calculation of Water Injection for Well Group

With the pore volume, water cut and cumulative water injection data of each well group, equation group (14) can be established from equation (10) based on $\phi$ function method.

$$
\begin{align*}
\phi_1 &= \frac{V_p}{W_1(t)} = af_{w1}^2 + bf_{w1} + c \\
\phi_2 &= \frac{V_p}{W_2(t)} = af_{w2}^2 + bf_{w2} + c \\
&\quad\quad \ldots\ldots \\
\phi_n &= \frac{V_p}{W_n(t)} = af_{wn}^2 + bf_{wn} + c
\end{align*}
$$

(14)

Therefore, the values the coefficients of $a$, $b$, $c$ in the relation between $\phi$ function and water cut for each well group can be determined. With the water cut of each well group, the coefficients of $a$, $b$, $c$ for each well group can be obtained. These values can be substituted in equation (10) to determine the $\phi$ function and cumulative injection when the water cut increases to 98 percent.

The cumulative water injection of well group $i$ in 2011 can be presented as:

$$
Q_{2011} = \frac{W_{i98} - W_{i2010}}{T}
$$

(15)

Where,

- $\phi_{98}$ -- $\phi$ function when water cut increases to 98 percent;
- $W_{i98}$ -- Cumulative water injection of an injection well when water cut increases to 98 percent;
- $Q_{2011}$ -- Annual water injection of an injection well in 2011;
- $T$ -- Water injection period;

6. Development Efficiency Prediction on Optimized Water Injection

Based on the controlled water injection, the numerical model of the block is used to predict the development efficiency of the block in the period from 2012 to 2013. In order to show the development efficiency, the data before and after water injection adjustment are compared. The change of block water cut in 2012 and 2013 for original and adjusted water injection schemes is shown in Figure 4.

![Figure 4](image-url)

Figure 4 Comparative result of oil production of two schemes predicted based on numerical model

Figure 4 shows that, from 2011 to 2013, block water cut in adjusted water injection scheme is lower than that of the original scheme by a value of 0.25 percent, indicating that water injection adjustment has a good effect.
7. Conclusions
(1) Based on the Buckley-Leverett theory, a water injection optimization algorithm suitable for high water cut condition has been developed. The algorithm comprehensively considers porosity, oil-bearing area, effective thickness, water cut, water cut rising rate, cumulative water injection, etc.
(2) The algorithm can be used to predict well group optimum water injection. First, time period it takes for the water cut to increase to 98 percent is determined. Then, cumulative water injection volume of each injection well up to 2010 and when water cut increases to 98 percent are calculated, so as to predict the optimum water injection volume of well group. Water injection upper calculated is controlled according to pressure differential method and adjusted according to the block water injection of \(1900 \times 10^3 \text{ m}^3\).
(3) After the adjustment of water injection of block and well group according to the water injection optimization algorithm is proposed, the whole block oil production is improved by 0.7 percentage points and the water cut reduced by 0.2 percentage points.

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