Generation of Magnetic Fields in Cosmology

A.D. Dolgov

INFN, sezione di Ferrara
Via Paradiso, 12 - 44100 Ferrara, Italy
and
ITEP, B. Cheremushkinskaya 25, Moscow, 117259, Russia

Abstract
Mechanisms of generation of magnetic fields in the early universe which could seed the present-day large scale galactic magnetic fields, are briefly reviewed. Three possible ways to create large scale magnetic fields are discussed: breaking of conformal invariance of electromagnetic interactions and inflationary stretching of the field wave length, first order cosmological phase transitions, and chaotic electric currents generated by turbulent flows in the primeval plasma.

1 Introduction
Astronomical observations show that there exist magnetic fields in galaxies with the field strength about 1 micro-gauss, coherent on the whole galactic size; for recent detailed reviews see [1, 2]. Though magnetic fields of individual stellar bodies can be and are much larger, an existence of coherent fields on the scales about 1 kpc or bigger presents one of the most profound mysteries of modern cosmology. There are many simple and realistic mechanisms of creation of magnetic fields in the early universe on small scales, however it is difficult to make them operate at galactic size because the latter is much larger than horizon at the epoch of field generation. Another problem is that the background cosmological model is homogeneous and isotropic and, though some primordial density perturbations should exist, they are assumed to be generated at inflationary stage and are normally of scalar (or tensor) type, i.e. vorticity free. This is also unfavorable with respect to formation of vector fields on macroscopic scales. Rather large vorticity perturbations
might be generated in the course of possible first order electroweak or QCD phase transitions when the bubbles of new phase were formed in the old one and a strong, though small scale, chaoticity is excited in the plasma. It is worth mentioning in this connection that classical solutions of equations of motion of non-abelian gauge fields and the Einstein equations demonstrate chaotic behavior, as was shown in a series of papers by S. Matinyan and collaborators [3] (and references therein). It is an interesting question if this chaoticity may be related to magnetic field generation in cosmology.

Roughly speaking there are three possible mechanisms of magnetic field creation in the early universe discussed in the literature:

1. Breaking of conformal invariance of electromagnetic interaction at inflationary stage. The latter could be realized either through new non-minimal (and possibly non gauge invariant) coupling of electromagnetic field to curvature [4], or in dilaton electrodynamics [5], or by the well known conformal anomaly in the trace of the stress tensor induced by quantum corrections to Maxwell electrodynamics [6].

2. First order phase transitions in the early universe [7] producing bubbles of new phase inside the old one. A different mechanism but also related to phase transitions is connected with topological defects, in particular, cosmic strings [8].

3. Creation of stochastic inhomogeneities in cosmological charge asymmetry, either electric [9], or e.g. leptonic [10] at large scales which produce turbulent electric currents and, in turn, magnetic fields.

In what follows I will briefly describe these three mechanisms. The literature on the subject is very rich and it is impossible to discuss all the relevant papers in this short contribution, so I will quote mostly only the original works where the idea was formulated and the latest ones where the proper list of references can be found.

2 Breaking of Conformal Invariance of Electrodynamics

It was established long ago that conformally flat gravitational field (in particular the usual cosmological Friedman-Robertson-Walker background) does not produce massless particles if the underlying theory is conformally invariant [11]. However, both gravitons [12] and minimally coupled to gravity massless scalars are not invariant and this gives rise to creation of density perturbations [13] and gravitational waves [14] of very large wave length, if
the are produced at inflationary stage. On the other hand, classical electrodynamics is known to be conformally invariant, so that photons should not be produced in cosmological background. However if one introduces a new type of interaction into the Maxwell Lagrangian, then the invariance may be broken and long wave electromagnetic fields could be generated at inflation by the same mechanism as gravitational waves. The former could be quasistatic and serve as seeds of coherent galactic magnetic fields.

A model of this kind was proposed in ref. [4], where a non-minimal coupling of electromagnetic field to gravity was considered:

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + C_1 R A_\mu A^\mu + C_2 m^{-2} R_{\mu\nu} F^{\mu\alpha} F^{\nu}_\alpha + ... \]  

(1)

Here \( F_{\mu\nu} \) is the electromagnetic field tensor, \( R_{\mu\nu} \) is the Ricci tensor, \( R \) is the curvature scalar, \( C_j \) are dimensionless constants, and \( m \) is another constant with dimension of mass. The first term is the usual free Maxwell Lagrangian, while the others represent new hypothetical interaction which breaks conformal invariance. The second term breaks also gauge invariance and would give a non-zero mass to photons in space-time with non-vanishing curvature.

Another example of non-standard electrodynamics was discussed in ref. [5] in the dilaton electrodynamics where the free Maxwell Lagrangian was modified by the coupling to dilaton field \( S \) in the following way:

\[ \mathcal{L} = -\frac{1}{4} \exp \left( \frac{S}{\eta} \right) F_{\mu\nu} F^{\mu\nu}, \]  

(2)

where \( \eta \) is a constant parameter with dimension of mass. The dilaton model of magnetic field generation finds its natural realization in string cosmology [13]; for a review of the latter see e.g. [16]. A related idea of magnetogenesis from time variation of gauge couplings was discussed recently in ref. [7]. In a special case of dilaton theory this model corresponds to time variation of the dilaton field \( S(t) \).

On the other hand, the usual quantum electrodynamics is not conformally invariant because of quantum corrections. In particular the trace of energy-momentum tensor which should be zero in conformally invariant theory, becomes non-vanishing due to triangle diagram with the electron loop [18]. This quantum anomaly results in the following modification of the Maxwell equations [19]:

\[ \partial_\mu F^{\mu}_\nu + \kappa \frac{\partial_\mu a}{a} F^{\mu}_\nu = 0 \]  

(3)

where \( a = a(\tau) \) is the cosmological scale factor, \( \tau \) is the conformal time, the metric has the form \( ds^2 = a^2(\tau)(d\tau^2 - dr^2) \), and the contraction of the
indices is made with the metric tensor of the flat space-time. The numerical coefficient $\kappa$ in $SU(N)$-gauge theory with $N_f$ charged fermions is equal to $\kappa = \alpha/\pi \left(11N/3 - 2N_f/3\right)$. Here $\alpha$ is the fine structure constant which is to be taken at the momentum transfer $p$ equal to the Hubble parameter during inflation, $p = H$. In the asymptotically free theory one would expect $\alpha \approx 0.02$.

The additional anomalous term in eq (3) gives rise to the production of photons by conformally flat gravitational field and, thanks to inflation, to generation of large scale magnetic fields [6]. The magnitude of the effect is too small in the case of the contribution of one electron loop but in theories with many charged particles, as e.g. in grand unified theories, the effect may be significant. The masses of these particles should be small in comparison with the Hubble parameter during inflation. A detailed analysis of magnetic field generation due to quantum anomaly can be found in the recent work [20].

A way to break conformal invariance of gauge fields during inflation due to coupling to a light ($m < H$) charged scalar field, $\phi$, was suggested in ref. [4] and studied in a special model in the papers [21]. The authors argued that stochastic electric currents could be generated during inflation due to production of charged scalar particles $\phi^\pm$ by the inflaton field. However, this scenario was criticized in refs. [22], where it was shown that dissipative effects induced by the plasma conductivity, which were disregarded in the above quoted papers, would strongly diminish the currents making them too weak to seed the the galactic magnetic fields. A detailed investigation of the damping effect was done in ref. [23] in the model with $N$ charged scalar fields in large $N$ limit. The conclusion of the work was that the magnetic field may exponentially rise and though with the parameters of the model the resulting field was too weak, a much higher intensity is still not excluded.

An interesting scenario is suggested in ref. [28], which is a modification of the one described above. It is based on the breaking of conformal invariance.
by the $Z$-boson field due to the coupling of $Z$ to the electroweak Higgs field. At the end of inflation reheating restores EW symmetry and the $Z$ field is transformed into weak hypercharge field and at the EW phase transition the latter turns into magnetic field. The process could be efficient enough to seed the observed galactic fields. (The quoted paper contains also an extensive list or references.)

A study of parametric resonance amplification of the generation of magnetic fields during preheating and inflation was performed in ref. [26] for several different models of breaking of conformal invariance. A possible relevance of parametric resonance to the problem of primordial magnetic field was noticed in the paper [27].

Another idea to break conformal invariance and as a result to amplify vector fluctuations during inflation was suggested in the paper [29]. In supergravity theories the photon field can be mixed with the so called graviphoton, i.e. the massive vector component $V_\mu$ of the gravitational supermultiplet:

$$L = -F_{\mu\nu}F^{\mu\nu}/4 - G_{\mu\nu}G^{\mu\nu}/4 + \zeta F_{\mu\nu}G^{\mu\nu} + m^2 V_\mu V^\mu,$$

(5)

where $G_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$, and $\zeta$ is a dimensionless constant. Earlier a model of two-photon mixing was proposed in the paper [30] in a different connection.

All models mentioned in this section could easily give large scale fields because they are operative at inflationary stage and the characteristic wave length of the field could be exponentially huge, but the amplitude of the field is usually smaller than the necessary value so that galactic dynamo [31] should be invoked to amplify these seed field by more than 10 orders of magnitude. However, the dynamo may have problems in producing large scale magnetic fields because of fast saturation, see e.g. the review [32]. Moreover, it seems impossible to explain strong magnetic fields observed in clusters of galaxies by the dynamo amplification [33].

3 Cosmological Phase Transitions.

The idea that phase transitions in the early universe might have produced seed magnetic fields was pioneered in ref. [4]. Expanding bubble walls between two phases could generate electric currents which in turn produce magnetic fields. Bubble collisions or hydrodynamical instability on the bubble walls could create turbulence flows and give rise to magneto-hydrodynamic (MHD) amplification of the magnetic fields. Such models were studied in series of papers both for electro-weak (EW) [34] and QCD phase transitions [35].
(see more references in the review [2]). However the existing experimental data indicating that the Higgs boson is heavy makes first order electro-weak phase transition rather improbable and there are also some doubts if the QCD phase transition is first order. Furthermore, even if phase transitions are first order, still in such a scenario is hardly possible to create large scale magnetic fields. Indeed, the comoving coherence length of the magnetic field cannot be larger than the Hubble horizon at the phase transition, which is much smaller than the galactic size. Though the coherence length may grow due to MHD effects, this happens at the expense of the magnetic field strength. According to ref. [36] neither EW or QCD phase transition could create sufficiently large magnetic fields on the galactic scales.

An attempt to overcome the large scale problem was done in the papers [37] under assumption that there could exist cosmological domain walls and that the fermions living on the wall could develop spontaneous magnetization orthogonal to the wall. This idea was further pursued in the recent works [38]. However, as was argued in refs. [39], magnetization is either absent or, even if it is non-zero, a magnetized domain wall cannot produce a correlated on large scale cosmologically interesting magnetic field.

More efficient could be generation of magnetic fields by cosmic, possibly superconducting, strings [3, 40]. String motion would create large scale vorticity perturbations inside their wakes and according to ref. [42] vorticity should produce electric currents and magnetic fields. According to ref. [41], superconduction strings could produce magnetic fields even more effectively because they might carry very large electric currents generated by their motion through cosmological plasma. On the other hand, this scenario is strongly restricted by the recent measurements of the angular fluctuation of cosmic microwave background radiation (see e.g. the reviews [43]) that put very restrictive limits on the cosmological density of cosmic strings.

4 Chaotic Electric Currents at Large Scales.

Chaotic electric currents could be generated in the universe if an inhomogeneous charge asymmetry between particles and antiparticles had been created at an earlier stage. When the wave length of the inhomogeneity becomes smaller than the horizon, some turbulent flows would evolve and give rise to non-vanishing electric currents. As is argued in ref. [4], if at an early stage of the evolution of the Universe the gauge invariance of electromagnetism was spontaneously broken, an electric charge asymmetry would develop through the same mechanism as cosmological baryon asymmetry. Electric asymmetry
could be inhomogeneous if the mechanisms similar to creation of isocurvature perturbation in the baryon sector were operative (see e.g. the review [44]). Gauge invariance must be restored at low temperatures and after its restoration the asymmetry should disappear so that the net electric charge density must vanish. The compensating charge could be produced from the Higgs vacuum in the form of heavy charged particles. Since the electric asymmetry was inhomogeneous, the number density of these particles would be inhomogeneous as well. Correspondingly energetic products of their decay would create an electric current and a local charge asymmetry.

Alternatively, such an asymmetry could be created even if the electric current was always conserved but a cosmological asymmetry in another non-conserved charge existed. The characteristic length of the inhomogeneities could be astronomically large if the conditions for their creation were cooked during inflation. The primary currents which created the asymmetry as well as those damping it via plasma discharge could generate chaotic magnetic fields on astronomically interesting scales. These fields might be large enough to seed the observed magnetic fields in galaxies via a protogalactic dynamo.

Another model of similar type was recently suggested [10], which practically does not demand any new physics and, in contrast to all discussed above models, operates at very low energies in MeV range. In the simplest version of the scenario the only necessary assumption is that of light sterile neutrino, $\nu_s$, weakly mixed with the ordinary active ones, i.e. $\nu_e$, $\nu_\mu$, or $\nu_\tau$. Neutrino oscillations in the early universe could create a very large lepton asymmetry in the active neutrino sector, $(n_\nu - \bar{n}_\nu)/(n_\nu + \bar{n}_\nu) > 0.1$ if MSW resonance condition is fulfilled, i.e. if sterile neutrinos are lighter than active ones [15] (more references where this effect is discussed can be found in recent papers [46]). Moreover, as was found in ref. [47] this large lepton asymmetry should be strongly inhomogeneous on superhorizon scale, if there were very small inhomogeneities in the primordial lepton or baryon asymmetries. Another possible mechanism [18, 44] of generation of inhomogeneous and large lepton asymmetry is based on Afleck and Dine scenario of lepto/baryogenesis [49].

If inhomogeneous cosmological lepton asymmetry was indeed generated by one or other mechanism mentioned above, neutrino currents should be developed along the density gradients when the neutrino mean free path $\ell_\nu(T)$ grew and became comparable to the wave length of the inhomogeneity, $\lambda$. Elastic scattering of the diffusing neutrinos on electrons and positrons in the primeval plasma would be able to accelerate the electron-photon fluid producing vorticity in the plasma. Depending on the amplitude and wavelength of the fluctuations of the charge asymmetric difference, $n_{\nu_a}(x) - n_{\bar{\nu}_a}(x)$
(a = e, \mu, \tau), this period could be sufficient for the MHD engine to generate magnetic field in equipartition with the turbulent kinetic energy. The seed field required to initiate the process arises naturally as a consequence of the difference between the \(\nu_a e^-\) and \(\nu_a e^+\) cross sections and of the neutrino-antineutrino local asymmetry.

Starting from the Boltzmann equation one can obtain the equation describing evolution of the average flux of \(i\)-th component of neutrino momentum

\[ \frac{\partial}{\partial t} K_i(x, t) + 4H K_i(x, t) + \frac{\partial}{\partial x^j} K_{ij}(x, t) = -\tau_w^{-1} K_i \]  

where

\[ K_i = \int k_i f_\nu(E, k) \frac{d^3k}{(2\pi)^3}, \quad K_{ij} = \int \frac{k_i k_j}{E} f_\nu(E, k) \frac{d^3k}{(2\pi)^3} \]

In the above \(E\) and \(k\) are respectively the neutrino energy and spatial momentum, \(H\) is the universe expansion rate, \(\tau_w\) is the effective weak interaction time, and \(f_\nu(E, k, t, x)\) is the neutrino distribution function. The source of magnetic field is proportional to curl of electric current, \(\nabla \times J\), which in turn is proportional to the local vorticity of the source term \(\partial_j K_{ij}\) in kinetic equations (6). The latter is nonvanishing for an anisotropic random initial distribution of neutrino leptonic charge and is numerically close to \(\partial_j K_{ij}\) divided by \(\lambda\).

Using equation of motion of electron-positron fluid disturbed by the neutrino flux one can estimate the magnitude of electric current induced by neutrinos as

\[ J_{\text{ext}} = 4 \cdot 10^{-20} e T^3 \left( \frac{T}{\text{MeV}} \right)^3 \left( \frac{\delta n_\nu}{n_\nu} \right), \]

which creates the seed field with the strength \(B^\text{seed}_\lambda \approx 10^{-22} (T/\text{MeV})^2\) at the time \(t/\lambda \sim 1\). In the equation above \(e\) is the charge of electron, \(T\) is the plasma temperature, and \(\delta n_\nu\) is the magnitude of the fluctuation in neutrino number density.

The evolution of magnetic field is given by the equation

\[ \partial_t B + 2HB = \nabla \times (v \times B) + \gamma^{-1} \nabla \times J_{\text{ext}} \]

where \(\gamma\) is (large) electric conductivity of relativistic cosmological plasma. Numerical solution of this equation showed that magnetic field quickly enough approaches energy equipartition with the fluid motion.
Such mechanism could give rise to magnetic fields of intensity $B_0 \sim x10^{-5}$ Gauss at the present time with a coherence length $\lambda_0 = \lambda_d r_H(t_d) (T_d/T_0) \simeq 10^5 \lambda_d$ pc. Galactic magnetic fields are observed with characteristic strength of the order of $1 \mu G$ extending over scales $\sim 1$ kpc \[1\]. Taking into account flux conservation during the protogalaxy collapse, the primordial origin of galactic fields would require a protogalactic field with the strength $\sim 10^{-10}$ Gauss and the coherence length of $0.1$ Mpc \[1, 2\]. Although this scale is much larger than the coherence length predicted by the discussed model, it is natural to expect that some homogenization could take place during galaxy formation. Since the field orientation is random over scales larger than $\lambda_0$, the predicted mean field on the protogalactic scale will be obtained by a suitable volume average \[6\]

$$B(0.1 \text{ Mpc}) \simeq B_0 \left(\frac{\lambda_0}{0.1 \text{ Mpc}}\right)^{3/2} \simeq 10^{-10} \lambda_d^{3/2} \text{ G.} \quad (9)$$

One can conclude from this expression that galactic magnetic fields may be produced by the neutrino number fluctuations with the relative amplitude $\sim 1$ extending over scales comparable to the Hubble horizon at neutrino decoupling \[10\].

5 Conclusion

Thus, it seems that the models that invoke inflation for creation of large scale magnetic fields are in a better shape than others. Possibly the existence of galactic magnetic field might be considered as an additional indication to inflation. On the other hand, all the concrete scenarios based on inflationary stretching of the magnetic fields, generated in the early universe, are heavily based on new physics, they demand an introduction of new fields or interactions and their predictive power is rather poor, simply because there is a plethora of the models and it is difficult to judge which one is the real, without knowledge the physics at very high energies far beyond the reach of the present day accelerators.

The scenarios using non-equilibrium phase transitions or topological defects (especially domain walls) encounter serious difficulties and at the present day look as outsiders. Moreover, astronomical data are rather against abundant cosmic topological defects and particle physics indicates that EW and QCD phase transitions are most probably second order. The conclusion that the mechanisms based on phase transitions are unlikely is supported by an astrophysical analysis performed in ref. \[50\].
It would be very nice if a mechanism of creation of seed magnetic fields at a later stage of cosmological evolution is found. It would make a problem of the coherence length less severe because of a larger Hubble horizon and the underlying physics could be accessible to direct tests. The nearest to this request is the model of ref. [10], according to which magnetic field is generated at temperatures of a fraction of MeV and the only essential assumption of the model is an existence of light sterile neutrino weakly mixed with one or several usual neutrino flavors. Still the problem of the large scale is present in this model as well, though at a much weaker level. Possibly the most efficient mechanism would be based on the assumption of a new light and very long-lived particles whose decays could induce electric current on astronomically big scales and in turn generate the seed magnetic field. However it is difficult (if possible) to satisfy existing cosmological and astrophysical constraints on the properties of such particles.

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