Xtra-Dimensional world(s)

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Abstract. In this simple review based on lectures given at the 2005 Corfu School, extra dimensional models are introduced from their phenomenological point of view. Models of large extra dimensions, TeV extra dimensions and Randall-Sundrum models are motivated and their main experimental signatures are shown.

1. Introduction
The idea of extra dimensions is nearly as old as general relativity [1]. In the 1920’s Kaluza and Klein tried to unified gravity with electromagnetism, the two forces known at that moment. They postulated the existence of an extra dimension where only gravity existed: upon compatification if the Einstein term is expanded in 5D, the result gives Einstein term in 4D plus the Maxwell kinetic term in 4D:

\[
G_{MN} = \begin{pmatrix} g_{\mu\nu} & A_{\mu} \\ A_{\mu} & \phi \end{pmatrix}
\]

\[
R_{5D} \rightarrow R_{4D} + \frac{1}{4} F_{\mu\nu}^2 + \ldots,
\]

where \( G \) and \( g \) represent the 5D and 4D graviton, \( A_{\mu} \) is the photon, \( R \) is the Ricci scalar in the different dimensions, and \( F_{\mu\nu} \) is the Maxwell tensor that represents the electric and magnetic fields. Thus, writing just one kinetic term accounts for two interactions thanks to the extra dimension. There are, however, problems with this theory it predicts the existence of a massless scalar \( \phi \), which has not been seen, so the theory is not phenomenologically acceptable but it works as the first model of unified theory.

Extra dimensions were then abandoned from a phenomenological point of view and regarded as mathematical tools. It was in the early 80s, when string theory [2] was invented, that extra dimensions were considered to have some importance from the physical point of view. String theory tries to address the question of merging quantum mechanics with general relativity into a consistent theory. Consistency of the theory requires that the number of space-time dimensions be 10 (or 11), which implies that 6 (or 7) of them have to be very small not to be observed in present-time experiments. In the original formulation of those string models, the size of the extra dimensions where supposed to be extremely small \( O(10^{-36} \text{ m}) \), which corresponds to the Planck length, the size at which one expects quantum gravity effects to be important. Clearly this size is much smaller than what we can prove with present or future experiments; it corresponds to a mass scale \( M_P \sim 10^{18} \text{ GeV} \), which is clearly much above any energy scale we have ever tested.
So if this is the correct description of nature, extra dimensions do exist but they have little influence in physics at the electroweak scale.

However, in 1990 [3] the possibility of extra dimensions at the weak scale was proposed and a new direction in model-building started. Since then there has been a revolution of models with extra dimensions testable in future experiments. Most of them try to solve the so-called hierarchy problem, which is the problem explaining why the electroweak and the Planck scale are so far apart. But, before entering into the details of the different models, let us review different general properties of models of extra dimensions.

We have seen previously that in the original Kaluza-Klein theory, starting with a spin-2 particle (the graviton) in 5D was sufficient to describe different particles in 4D, so we have the following correspondence for spin-2 and spin-1:

\[
G_{MN} = \begin{pmatrix} g_{\mu\nu} & A_\mu^i \\ A_{j\nu} & \phi_{ij} \end{pmatrix}
\]

\[
V_M = (V_\mu, \phi_i)
\]

where \( M = \mu, i \), i.e. \( \mu \) labels the usual 4D and \( i \) the extra compact dimensions. As can be seen through compactification different kinds of field are generated. For the case of scalars, nothing is changed, since they carry no space-time index. Fermions are more complicated, since the spinorial representation of the Poincaré group differs a lot in different dimensions; but in general the rule is that out of an extra-dimensional fermion several 4D fermions are generated.

Apart from generating different kinds of fields from a single one, there appears, for every field, a tower of massive copies. We can understand this as follows: imagining an extra dimension as the one drawn in fig. 1, the usual space-time is the surface of the cylinder and the extra dimension is compact with radius \( R \). When solving the equations of motion of the different fields in that particular geometry one can make an expansion on Fourier modes, similar to what happens in static waves or particle-on-a-box problems in quantum mechanics:

\[
\Phi(x^\mu, x^5) = \sum_{n=-\infty}^{\infty} \phi_n(x^\mu) e^{\frac{i n x^5}{R}} ,
\]

where \( \Phi \) could represent a field with or without spin. When inserting the above formula into the equations of motion, the derivative with respect of \( x^5 \) will generically create the following
term:
\[ \partial_5^2 \Phi \rightarrow \sum_{n=-\infty}^{\infty} \frac{n^2}{\pi^2 R^2} \phi_n, \] (4)

which can be interpreted as a mass term for each \( \phi_n \). So we state as general conclusion that in theories with extra dimensions there are towers of KK modes for every field that lives in those dimensions. Normally the usual Standard Model fields are considered to be the so-called zero modes, i.e. those who do not acquire mass through compactification, because the mass scale for \( 1/R \) is much larger than the mass of any ordinary particle. There are, however, exceptions to that rule in models where the extra dimension is associated to electro weak breaking [4] where the \( W^\pm \) and \( Z \) bosons are considered to be KK modes.

Having extra particles at some scale \( m \) puts some constraints on this kind of models. One can view this fact in two different ways: the first of them is that experimentally we have not found any candidate for a KK mode, on the other hand if we integrate out these particles, supposing they are too massive to be observed, this process introduces deviations from SM predictions:

\[ \mathcal{L}_{XD} \rightarrow \mathcal{L}_{SM} + \sum_{p=1}^{\infty} \frac{1}{m_p} \mathcal{O}^p(\psi_{SM}) \] (5)

so when studying these models we have to make sure that the operators \( \mathcal{O} \) do not interfere with current measurements. We can classify the types of extra dimensions by regarding the fields that live on them:

- Extra dimensions where only gravity can propagate; gravity has only been tested up to scales \( \sim 1 \mu m \) so these types of dimensions must have a size of a micron or less.
- If, on the other hand, gauge fields or matter are allowed to propagate in these XDs then collider and indirect signatures pose a bound on the scale at a few TeV.

The possibility of having models of XDs have been used to address different problems, such as the difference between the Plank and the EW scale [5], [6], supersymmetry breaking [7], or even electroweak breaking [4]. In the next sections different models will be discussed. Since the audience of these lectures were mainly experimentalists, few technical details will be included; for the advanced reader some reviews are suggested [8] (the list of references given here will be small, a larger collection can be found on the reviews).
Large Extra Dimensions

The main motivation for this kind of models is the explanation of the so-called hierarchy problem, namely the huge difference that exists between the Planck mass and the electroweak scale. Let us imagine a model of \( n \) flat extra dimensions where the SM is attached to a 4-dimensional subspace the real-world ”brane”, but gravity is free to live all over space as depicted in fig. 2.

If we write the general relativity action in \( D \) dimensions:

\[
S_D = -M_{*}^{D-2} \int d^{D}x \sqrt{-g(D)} R(D),
\]

where \( M_{*} \) is the fundamental scale, \( g(D) \) is the metric, and \( R(D) \) is the Riemann tensor in \( D \) dimensions \((D = 4 + n)\). If we compactify the \( n \) XDs, supposing for simplicity that all of them have the same size \( r \), and we look only at the zero mode, i.e. the one that has no dependence on the compact coordinates, we obtain:

\[
S_4 = -M_{*}^{D-2} r^{n} \int d^{4}x \sqrt{-g(4)} R(4)
\]

which implies \( M_{P}^{2} = M_{*}^{n+2} r^{2} \).

Setting \( M_{P} \) to its actual value and \( M_{*} \sim 1 \) TeV to explain the hierarchy problem, we can get the size of the extra dimension:

\[
r \sim 2 \times 10^{-17} \text{10}^{\frac{32}{n}} \text{ cm}.
\]

Clearly if \( n = 1 \) the model is ruled out since it predicts an extra dimension of the size of the solar system; for the case of \( n = 2 \) \((0.2 \) mm\) the model is border-line with respect to the actual experiments on deviation Newtonian potential, as can be seen in fig. 3, since this kind of models predict that the interaction between two masses will go as:

\[
V(x) = -G_N \frac{m_1 m_2}{x}(1 + e^{-x/r})
\]

where \( x \) is the distance between the masses and \( r \) is the size of the extra dimension, this corresponds in fig. 3 to \( \alpha \sim 1 \).

The previous size \( r \) can be translated into the KK masses as:

\[
m \sim N 10^{-\frac{12n-31}{n}} \text{ eV}, \ N = 1, 2, 3, \ldots
\]

Figure 3. Deviation from Newtonian potential of size \( \alpha \) for a given length \( \lambda \).
so, although these states interact very weakly (gravitationally) they can be detected at the LHC since lots of them will be produced (fig. 4) [9]. The chances are better, however, at the ILC (fig. 5).

Other effects of this kind of models are related to the modification of gravity; for example, the appearance of such an amount of gravitons in high energy reactions can affect the cooling of supernovae. The quantum-gravity effect at a TeV scale will, in general, modify the usual predictions of cosmology for the evolution of the Universe and can also lead to more drastic effects such as black-hole production in colliders. Some of the effect just described can be the smoking gun of the models of large extra dimensions.

3. TeV flat extra dimensions
As was said in the introduction, when other fields apart from gravity are allowed to propagate in extra dimensions there are constraints coming from collider and indirect tests that force this kind of extra dimensions to be at the TeV scale or more. Hence the name of TeV extra dimensions.

The main motivation for these models is also the hierarchy problem, and addressing the question of electroweak breaking. In this section the most relevant applications will be discussed.

The first aspect of theories where matter or gauge fields live in extra dimensions is that normally one extra dimension is preferred, as depicted in fig. 6. The reason for this is that on 5D the sums over KK modes are finite:
Figure 6. Scheme for a flat extra dimension

\[ A_k, \bar{\psi} \]

\[ x^0 \]

\[ x^5 \]

\[ \phi = 0 \]

\[ x_5 = 0 \]

\[ \phi = \pi \]

\[ x_5 = \pi R \]

---

Figure 7. Example of loops leading to a finite result

\[ \sum_{n=0}^{\infty} \frac{1}{p^2 - \frac{n^2}{R^2}} = \frac{1}{2p^2} + \frac{\pi R \cot(p\pi R)}{2p}, \]  

which then makes diagrams as the one of fig. 7 finite and therefore makes any prediction independent of the cut-off of the theory. This interesting feature is not shared in theories with a larger number of extra dimensions, so in the rest of this section we will only talk about theories with one extra dimension. For the more advanced reader there are the reviews cited in the references.

One important issue when allowing matter fields to live in an extra dimension is that, as we explained in the introduction, this necessarily implies the appearance of new fields, not only as KK modes, but as part of the same multiplet (see Eq. (2)). For fermions, what happens, is that, in 5D, there is no chirality so every fermion is a Dirac fermion, whereas in the SM fermions are chiral or Weyl. The usual way of getting a chiral spectrum from a 5D theory is to compactify not on a normal circle but on a circle with a parity, an orbifold, as depicted in fig. 8.

Every field (or component) will then have an associated parity \( P \), which is either plus or minus: this will mean that the field (or component) will either be even or odd. If we write a general Dirac fermion in 5D as:

\[ \Psi = \begin{pmatrix} \Psi_L \\ \Psi_R \end{pmatrix} \]  

where \( L, R \) represents left- or right-handed chirality. We can associate \( P(\Psi_L) = 1 \) and
Figure 8. Example of an orbifold

![Orbifold Diagram]

Figure 9. Unification of the couplings in a theory with one extra dimension

\[ P(\Psi_R) = -1, \]

which means that the left-handed chirality will be even and the right one odd. If we now expand the field \( \Psi \) in KK modes:

\[
\Psi(x_\mu, x_5) = \sum_n \left( \cos\left(\frac{n x_5}{R}\right) \psi_{L_n}(x_\mu) + \sin\left(\frac{n x_5}{R}\right) \psi_{R_n}(x_\mu) \right)
\]

(13)

we can see that only the left-handed field has a massless \( n = 0 \) mode, which corresponds, as usual, with the mode with no dependence on the extra dimension. In this same way some of the components of Eq. (2) can be modded out of the massless spectrum with a suitable choice of parity; for example, in the case of a vector boson \( V_M \), only the \( V_\mu \) will be even and the scalar \( V_5 \) odd (or vice versa).

Let us discuss different applications of models with one extra dimension. Having lots of KK states with quantum numbers of the SM will affect the evolution or running of gauge couplings, and it turns out that unification is still possible, but in an accelerated way, as can be seen in fig. 9 [10].

Having an extra dimension can provide an explanation for fermion masses [11]; in a model as the one of fig. 10 one can try to explain the origin of fermion masses as the overlaps of wave functions in the extra dimension, the heavier being the one with the larger overlap with the Higgs.

Another popular use of extra dimensions is the possibility of breaking extra symmetries with parities (such as the one used to create chiral models) to address problems such as supersymmetry or electroweak breaking. In the first case there are models of hidden brane breaking as the one shown in fig. 11. One can suppose that SUSY breaking takes place on a hidden brane, physically separated from where matter lives; it is communicated through the bulk of the extra dimension by gauge interactions that are flavour blind, that is they do not generate dangerous flavour,
changing neutral currents (FCNCs), which are heavily constrained.

Using extra parities to break symmetries opens a new window for model building. In fig. 12 we have a model with two parities, the one we explained in fig. 8 and a second one. This type of orbifold can be used to break gauge symmetry in a unification model, which solves the doublet–triplet splitting, or can be used to break SUSY through boundary conditions. We said before that a field (or component) should be either even or odd, but having an extra parity allows us to further classify fields (or components) with respect to the two parities that we now have, leading to even–even, even–odd, odd–even and odd–odd fields, depending on how they transform with respect to both parities. Only even–even fields will have a zero mode and if we arrange these parities to break the gauge structure or SUSY we can have a new breaking pattern. This kind of models can even be used to break the electroweak symmetry without a Higgs [4]. A broader description of different models can be found in the reviews of ref. [8].

Before closing this section on flat TeV extra dimensions, let us point out the possible collider signatures of these models and constraints coming from EW observables. As before, these KK modes will be produced at colliders, but in this case, instead of having a continuum of states, there can be a possibility to see different resonances as depicted in fig. 13 [12]. The exact signals will depend a lot on the details of the models, but it is in general expected that some of those KK modes be visible at the LHC.

The second issue will concern the bounds on the compactification scale that EW observables put. As we explain before, having new particles the KK modes will, in general, induce deviations
on observables such as the left–right asymmetries of the $Z$ boson, the decay width, or even in flavour physics. Since these quantities have been measured to a great accuracy at LE, any contribution coming from new physics has to be sufficiently small not to contradict what has been measured. When this is taken into account we find the following bounds:

- If in the model there is universality in fermions couplings, i.e. if all fermions couple in the same way to KK modes, then the bounds are $M_{KK} \sim 3\text{--}4$ TeV.
- On the other hand if there is some kind of non-universal couplings, specially between the first two families, then the bounds can be as large as $M_{KK} \sim 1000$ TeV.

For further reading on these models, ref. [8] can be very useful.

4. Warped extra dimensions

Up to now we have just considered models of flat extra-dimensions, or models where the extra dimensional structure does not mix with the 4D space-time. But there are other types of extra dimensions which has attracted a lot lof attention, those of warped models [6].

Imagine that there is an extra dimension with constant curvature\(^1\), where the boundaries have the tension tuned in order to cancel the total curvature, and the four-dimensional space is Minkowski. The solution for that kind of space is:

\(^1\) This is called *(Ant) De Sitter* space in general relativity, depending on the sign of the curvature.
Figure 14. Example of a warped extra dimension

\[ ds^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2 \]  

(14)

where \( k \) represents the curvature of the extra dimension and its size is set by the fundamental scale. We can try to plot that solution in Fig. 14, where we have the extra dimension and the two boundaries (branes).

The first effect of these kinds of models is that mass scales depend on the position of the extra dimension:

\[ S = \int d^5x \sqrt{-g} \mathcal{L} \rightarrow \Lambda(y) = e^{-ky} M_*, \]

(15)

where \( \Lambda \) is the maximum scale in each point and \( M_* \) is the fundamental scale in the \( y = 0 \) brane.

So if we suppose that EW physics only takes place in the \( y = \pi R \) brane then:

\[ \Lambda(\pi R) = 1 \text{ TeV for } M_* = M_P \text{ and } k\pi R = 30, \]

(16)

So the hierarchy problem can be solved if we located the Higgs field on the \( y = \pi R \) brane. Hence in this kind of models the \( y = 0 \) brane is called the Planck brane because the typical energy scale of the theory is Planckian, whereas the \( y = \pi R \) is called the TeV brane and is where EW breaking takes place. Therefore this a solution to the hierarchy problem without invoking supersymmetry.

Another interesting feature of these models is that gravity is localized towards the Planck brane. Even in the case when there is no TeV brane and the extra dimension is semi-infinite: there exists a normalizable massless mode for the graviton that represents 4D gravity. In other words, this would be a model for a semi-infinite warped extra dimension, where matter would be located in the brane and gravity would propagate on the bulk but 4D gravity would exist. We can understand this by studying the equation of motion for gravitons in this background. This equation can be written as:

\[ [-\partial_z^2 + V(z)]\psi = m^2 \psi, \]

(17)

where \( m \) represents the mass of the graviton and \( V(z) \) is the "vulcano" potential drawn in fig. 15. It can be proved that there is a solution to the previous equation when \( m = 0 \) with a wave function localized in the well of the potential. This only applies to gravity, and if we had
written the equation of motion for a gauge field in this semi-infinite space we would have found no massless mode to describe 4D gauge theory. So photons are not trapped.

Coming back to the model with two branes, in this case the KK modes are described with Bessel functions instead of the usual trigonometric functions. They are always localized towards the TeV brane and have TeV masses. The localization of the zero mode depends on the spin of the particle:

- For spin 2 (graviton) we have seen that it is localized towards the Planck brane.
- For spin 1 (gauge fields) gauge invariance forces them to be flat.
- For spin 1/2 (matter fields) there is no restriction and, depending on a free parameter, they can be localized toward either brane or even be flat.

These KK modes will be produced in future colliders. In fig. 16 we can see the production cross section for gravitons at the ILC [13]. Similar plots for different KK modes can be found in the literature.

This setup also predicts a deviation from Newton’s potential for gravity:

\[
V(r) = G_N \frac{M_1 M_2}{r} \left(1 + \frac{3}{2k^2 r^2}\right),
\]

which can be tested in the next generation of experiments.

Let us finish this section with a classification of the models in warped extra dimensions:

- If gravity is the only field allowed to propagate in the extra dimension, the hierarchy problem is solved but there is no answer to questions about unification or flavour issues.
• Allowing gauge or matter fields to propagate in the bulk can help understanding flavour, or one can construct models for unification.
• One can even try to address the source of electroweak breaking with this kind of models.

In any case the lowest KK masses have to be around 3 TeV. More details on these models can be found in the references.

5. Conclusions
Extra dimensions are a prediction of most theories of quantum gravity. Traditionally these extra dimensions were supposed to be so small that they were impossible to detect. It this short lecture, models where these extra dimensions are large enough to be tested were presented. They open different possibilities to solve problems that the SM does not address. These include an explanation for the weakness of gravity, solutions to the hierarchy problem, new ideas for flavour or unification, models for electroweak breaking, or even models for a solution to the cosmological constant problems.

Although there are many models and only few of them were described in this short lecture, all models share the following properties:
• Appearance of extra particles as KK modes
• Effects on the electroweak observables
• Effects on cosmology or astrophysics

In any case, if these models are to address questions regarding TeV physics, they should be discovered at LHC, and if they have something to do with actual problems of cosmology they will show up in the next generation of experiments and satellites.

References
[1] Kaluza T 1921 Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys.) 1921 966, Klein O 1926 Z. Phys. 37 895
[2] Green M B, Schwarz J H and Witten E 1988 Superstring theory (Cambridge University Press)
[3] Antoniadis A 1990 Phys. Lett. B 246 377
[4] Csaki C, Grojean C and Murayama H 2003 Phys. Rev. D 67 085012
[5] Arkani-Hamed N, Dimopoulos S and Dvali G R 1998 Phys. Lett. B 429 263
[6] Randall L and Sundrum R 1999 Phys. Rev. Lett. 83 3370 and 4960
[7] Delgado A, Pomarol A and Quirós M 1999 Phys. Rev. D 60 095008
[8] Csaki C 2004 hep-ph/0404096, Sundrum R 2005 hep-ph/0508134, Kribs G D 2006 hep-ph/0605325
[9] Giudice G F, Rattazzi R and Wells J D 1999 Nucl. Phys. B 544 3
[10] Dienes K R, Dudas E and Gherghetta T 1998 Phys. Lett. B 436 55
[11] Kaplan D E and Tait T M P 2000 JHEP 0006 020
[12] Antoniadis A, Benakli K and Quirós M 1999 Phys. Lett. B 460 176
[13] Davoudiasl H, Hewett J L and Rizzo T G 2000 Phys. Rev. Lett. 84 2080