On the symmetry improved CJT formalism in the linear sigma model

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By using the symmetry improved CJT effective formalism developed by Pilaftsis and Teresi, the chiral phase transition is reconsidered in the framework of the $O(4)$ linear sigma model in chiral limit. Our results confirm the restorations of the second-order phase transition and the Goldstone theorem in the Hartree approximation. Finally, we explicitly calculate the effective potentials via the order parameter for various temperatures.

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I. INTRODUCTION

It is widely believed that at sufficiently high temperatures and densities there is a quantum chromodynamics (QCD) phase transition between normal hadronic matter and quark-gluon plasma (QGP), where quarks and gluons are no longer confined in hadrons[1][2]. Experimentally, the study of the QCD phase transition is supported by the heavy-ion collisions in laboratories at ultrarelativistic energies, such as the Relativistic Heavy-Ion Collider (RHIC) at Brookhaven National Laboratory, the Large Hadron Collider (LHC) at CERN and future facilities. We believe that these experiments will provide us with a chance to create a hot QCD matter and study its property.

To explore the QCD phase diagram, besides the first principle calculations using lattice techniques (LQCD), there is, however, an alternative approach which respects some salient features of QCD. Following this path, it is necessary to use a variety of effective phenomenological models and QCD-like theories in order to mimic some essential characteristics intimately related to QCD. As a quite popular model, the linear sigma model 3 for the phenomenology of QCD has been composed to describe the vacuum structure with incorporating chiral symmetry and its spontaneous breaking. The model can be used to describe a restoration of the chiral symmetry at finite temperature in the Hartree-Fock or Hartree approximation 1 1 1 4 1 2 5 within the Cornwall-Jackiw-Tomboulis (CJT) formalism 11, also at finite isospin chemical potential 12 1 4 1 5. However, except the case in the larger-$N$ approximation, these kinds of studies suffer from some serious drawbacks. For example, in chiral limit, the CJT effective action violates the Goldstone theorem and gives massive pions in the spontaneous symmetry breaking phase, and the temperature-dependent order parameter will predict a first order phase transition. The prediction, of course, disagrees with the renormalization group arguments, e.g., the chiral phase transition can be of second order if the $U(1)_A$ symmetry is explicitly broken by instanton for $N_f = 2$ flavors of massless quarks 17.

Many attempts have been made to restore the Goldstone theorem within the CJT formalism in the literature, nevertheless, the existing approaches still could not provide a satisfactory solution to this long-standing problem. Very recently, Pilaftsis and Teresi have developed a novel symmetry improved CJT formalism by consistently encoding global symmetries in loop expansions or truncations of the CJT effective action 18. This formalism seems to avoid the pathologies of the existing approaches and satisfy many field-theoretic properties. Taking the $O(2)$ scalar model as simple example, they showed the phase transition is of second-order and the Goldstone boson is massless even in the Hartree-Fock approximation. Inspired by their previous work for the $O(2)$ model, in this work, we extend this formalism to more realistic model, the $O(4)$ linear sigma model, to reconsider the chiral phase transition at finite temperature in chiral limit. The advantage of this straightforward extension will become more apparent when we directly apply this discussion to study the formation and evolution of the pion string in the early universe or the heavy ion collision with thermal background 19 2 0, where the Goldstone theorem and the features of the second-order phase transition are believed to play essential role 21 2 2.

The organization of this paper is as follows. In the next section we introduce the model and fix the parameters. In Sect. III, we calculate the effective potential by using the symmetry improved CJT formalism and obtain a set of gap equations. In Sect. IV, The solution of these gap equations is presented, and the thermal effective masses and potential are calculated. In the end, we give out some summary and discussions.

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II. THE MODEL

The meson sector Lagrangian of the $SU(2)_R \times SU(2)_L$ symmetry linear sigma model has the form

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \pi \cdot \partial^\mu \pi) - U(\sigma, \pi),$$

(1)

where the potential for the $\sigma$ and $\pi$ is parameterized as

$$U(\sigma, \pi) = \frac{m^2}{2} (\sigma^2 + \pi^2) + \frac{\lambda}{24} (\sigma^2 + \pi^2)^2.$$ 

(2)

The $SU(2)_L \times SU(2)_R$ symmetry of the linear sigma model can be explicitly broken if the potential $U(\sigma, \pi)$ is made slightly asymmetric by adding a term $\epsilon \sigma$. With this addition, the vector isospin $SU(2)$ symmetry remains exact, but the axial $SU(2)$ transformation is no longer invariant. To the leading order in $\epsilon$, this shifts the minimum of the potential to $f_\pi + \frac{3}{2} \epsilon f_\pi^2$, as a result the pions acquire mass $m_\pi^2 = \frac{\epsilon}{f_\pi}$.

At tree level and zero temperature the parameters of the Lagrangian are fixed in a way that these masses agree with the observed value of pion masses and the most commonly accepted value for sigma mass. Unlike the pions, unfortunately, the mass of the sigma meson is still a poorly known number, but the most recent compilation of the Particle Data Group considers that $m_\sigma$ can vary from 400 MeV to 550 MeV with full width $400 - 700$ MeV[23]. So that we take $m_\sigma = 500$ MeV and $f_\pi = 93$ MeV as typical values in this work. The coupling constant $\lambda$ of the model then can be related to zero temperature properties of the pions and sigma through the expression

$$\lambda = \frac{3(m^2_\sigma - m^2_\pi)}{f^2_\pi}.$$ 

(3)

The negative mass parameter $m^2$ is introduced in order to ensure spontaneous breaking of symmetry and its value is chosen to be

$$- m^2 = (m^2_\sigma - 3m^2_\pi)/2 > 0.$$ 

(4)

In the chiral limit $\epsilon = 0$, the parameters $\lambda$ and $-m^2$ appearing in the Lagrangian can be further simplified as

$$\lambda = \frac{3m^2_\sigma}{f^2_\pi}, \quad -2m^2 = m^2_\sigma > 0.$$ 

(5)

In the following discussion, we will focus on the case of chiral limit and investigate the natural properties of chiral phase transition within the symmetry improved CJT formalism in detail.

III. THE SYMMETRY IMPROVED CJT FORMALISM IN HARTREE APPROXIMATION

In this section we derive the symmetry improved CJT effective potential for the linear sigma model at finite temperature. The starting point is the Lagrangian given in Eq. (1) with the choice of the symmetric potential given in Eq. (2). By shifting the sigma field as $\sigma \rightarrow \sigma + \phi$, the classical potential takes the form

$$U(\phi) = \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{24} \phi^4.$$ 

(6)

The tree-level sigma and pion propagators corresponding to the above Lagrangian have the form

$$D^{-1}_\sigma = \omega_n^2 + \vec{k}^2 + m_\sigma^2 + \frac{\lambda}{2} \phi^2,$$

$$D^{-1}_\pi = \omega_n^2 + \vec{k}^2 + m^2 + \frac{\lambda}{6} \phi^2.$$ 

(7a)

(7b)

We use the imaginary-time formalism to compute quantities at nonzero temperature, our notation is

$$\int \frac{d^4 k}{(2\pi)^4} f(k) \rightarrow \frac{1}{\beta} \sum_n \int \frac{d^3 \vec{k}}{(2\pi)^3} f(i\omega_n, \vec{k}) \equiv \int f(i\omega_n, \vec{k}).$$
where $\beta$ is the inverse temperature, $\beta = \frac{1}{k_B T}$, and as usual Boltzmann’s constant is taken as $k_B = 1$, and $\omega_n = 2\pi n T$, $n = 0, \pm 1, \pm 2, \pm 3, \cdots$. For simplicity we have introduced a subscript $\beta$ to denote integration and summation over the Matsubara frequency sums.

Evaluating the effective potential in Hartree approximation means that one needs to take into account the “$\infty$” type diagram only. In the case of the linear sigma model, we can derive the following expression for the effective potential at finite temperature\cite{11}:

$$V(\phi, G) = U(\phi) + \frac{1}{2} \int_{\beta} \ln G^{-1}_\sigma(\phi; k) + \frac{3}{2} \int_{\beta} \ln G^{-1}_\pi(\phi; k) + \frac{1}{2} \int_{\beta} [D^{-1}_\sigma(\phi; k)G_\sigma(\phi; k) - 1] + \frac{3}{2} \int_{\beta} [D^{-1}_\pi(\phi; k)G_\pi(\phi; k) - 3] + V_2(\phi, G_\sigma, G_\pi).$$

The first term $U(\phi)$ is the classical potential and the last term $V_2(\phi, G_\sigma, G_\pi)$ denotes the contribution from the “$\infty$” diagrams, which is equivalent to the Hartree approximation. Explicitly,

$$V_2(\phi, G_\sigma, G_\pi) = \frac{3\lambda}{24} \left[ \int_{\beta} G_\sigma(\phi; k) \right]^2 + 6\frac{\lambda}{24} \left[ \int_{\beta} G_\sigma(\phi; k) \right] \left[ \int_{\beta} G_\pi(\phi; k) \right] + 15\frac{\lambda}{24} \left[ \int_{\beta} G_\pi(\phi; k) \right]^2.$$  

Minimizing the effective potential with respect to full propagators we obtain the following system of nonlinear gap equations:

$$G^{-1}_\sigma = D^{-1}_\sigma + \frac{\lambda}{2} \int_{\beta} G_\sigma(\phi; k) + \frac{\lambda}{2} \int_{\beta} G_\pi(\phi; k),$$ \hspace{1cm} (10a)  

$$G^{-1}_\pi = D^{-1}_\pi + \frac{\lambda}{6} \int_{\beta} G_\sigma(\phi; k) + \frac{5\lambda}{6} \int_{\beta} G_\pi(\phi; k).$$ \hspace{1cm} (10b)

The bare propagators $D_\sigma$ and $D_\pi$ are given above. Since the self-energies in Eqs.\cite{10} are independent of momentum in the Hartree approximation, the full propagators assume the simple form

$$G^{-1}_\sigma = \omega_n^2 + k^2 + M^2_\sigma,$$

$$G^{-1}_\pi = \omega_n^2 + k^2 + M^2_\pi,$$

where $M_\sigma$ and $M_\pi$ are the effective masses of $\sigma$ meson and $\pi$ dressed by interaction contributions from the “$\infty$” diagrams. Then the sigma and pion thermal effective masses are given by

$$M^2_\sigma = m^2 + \frac{\lambda}{2} \phi^2 + \frac{\lambda}{2} F(M_\sigma) + \frac{\lambda}{2} F(M_\pi),$$ \hspace{1cm} (11a)  

$$M^2_\pi = m^2 + \frac{\lambda}{6} \phi^2 + \frac{\lambda}{6} F(M_\sigma) + \frac{5\lambda}{6} F(M_\pi).$$ \hspace{1cm} (11b)

Here we have used a shorthand notation and introduced the function

$$F(M) = \int_{\beta} \frac{1}{\omega_n^2 + k^2 + M^2}.$$ \hspace{1cm} (12)

At the level of the Hartree approximation, the thermal effective masses are independent of momentum and are functions of the order parameter $\phi$ and temperature $T$. The loop integral in equation \cite{12} requires renormalization\cite{24,27}. Renormalisation in many-body approximation schemes is a nontrivial procedure, but in the case of linear sigma model it has been shown by Rischke and Lenaghan\cite{28}, that at least in the Hartree approximation the results are not affected qualitatively. We therefore simply drop the divergent terms and keep only the finite temperature part of the integrals. A similar procedure has been adopted in a number of other investigations using the linear sigma model\cite{4,10}.

In general, the effective potential of the CJT formalism can be written down in terms of the solutions of the gap equations\cite{11}. By minimizing the potential with respect to the expectation value for the sigma field $\phi$, we obtain one more the stationarity equation

$$m^2 + \frac{\lambda}{6} \phi^2 + \frac{\lambda}{2} F(M_\sigma) + \frac{\lambda}{2} F(M_\pi) = 0.$$ \hspace{1cm} (13)
By solving the system of the three equations (11) and (13), the effective masses as functions of temperature can be calculated. However, as we know this kind of “standard” procedure will produce the “wrong” results, such as the first-order chiral phase transition and the massive Goldstone boson in the model in chiral limit. These difficulties are mostly because that in the loop expansion of the CJT effective action the global symmetries are not exactly maintained at a given loop order of the expansion. In our case, the truncation of the CJT effective action in Hartree approximation violates the Goldstone theorem by higher order terms, which in turn gives rise to massive pions in the spontaneous symmetry breaking phase. In analogy to the standard One-Particle-Irreducible effective action, an improved formalism for the CJT effective action has been proposed in Ref.[18] to fix these problems. Based on their improved scheme, the stationarity equation (13) is replaced by the constraint:

\[ \phi M_\pi^2 = 0. \]  
(14)

In the symmetry breaking phase of the model, the constrain (14) implies \( M_\pi^2 = 0 \), yielding

\[ M_\sigma^2 = m^2 + \frac{\lambda}{2} \phi^2 + \frac{\lambda}{2} F(M_\sigma) + \frac{\lambda}{2} F(0), \]  
(15a)

\[ 0 = m^2 + \frac{\lambda}{6} \phi^2 + \frac{\lambda}{6} F(M_\sigma) + \frac{5\lambda}{6} F(0). \]  
(15b)

Now the massless pions are naturally realized whenever the order parameter is not vanished. On the contrary, in the symmetric phase, we have \( \phi = 0 \) and the constraint is automatically satisfied. As a consequence, the two mass-gap equations become degenerate, the particles have the same mass with \( M_\sigma = M_\pi \equiv M \), and

\[ M^2 = m^2 + \lambda F(M). \]  
(16)

It is worth to point out that the constraint proposed in Eq.(14) can be obtained without any assumption in the large-\( N \) approximation in the framework of the usual CJT formalism, therefore, the large-\( N \) approximation implies that the pions should be massless in the symmetry broken phase also. In other words, once we take the constraint(14), the well-known results in the large-\( N \) approximation could be copied.

Using the definition in the framework of the symmetry improved CJT formalism, we can rewrite the finite temperature effective potential \( V_{eff} \) as a function of \( \phi \) in the thermal Hartree approximation by extending the mass-gap equations (11) from \( \phi \to \varphi \) while

\[ M_\pi^2(\varphi) = \frac{1}{\varphi} \frac{dV_{eff}(\varphi)}{d\varphi}. \]  
(17)

Now the Goldstone boson masslessness condition can be naturally implemented in the symmetry breaking phase, and the integration of the effective potential \( V_{eff} \) with respect to the field \( \varphi \) really create a typical potential possessing the second order phase transition. As mentioned in above, a explicit symmetry breaking term \( \epsilon \sigma \) can be include into the original Lagrangian to generate the pion observed masses. Since the \( \epsilon \) only appears in the classical potential, rather than in the Gap equations, therefore the introduction of this term in our discussions would be rather trivial by adding additional term \( \epsilon \varphi \) in the effective potential \( V_{eff} \) in Eq.(17). Accordingly, the constraint is rewritten as

\[ \varphi M_\pi^2 = \epsilon. \]  
(18)

Then at low temperature the pions appear with the observed masse since the global symmetry has already been explicitly broken.

IV. NUMERICAL RESULTS

In this section, we discuss the numerical results at non-zero temperature in chiral limit. We solve the system of mass-gap Eqs.(11) with the constraint (14) using a numerical method based on the Newton-Raphson method of solving nonlinear equations. In this way, we are able to determine the effective masses \( M_\sigma \) and \( M_\pi \) and the order parameter \( \phi \) as function of temperature \( T \). For comparison with the previous studies, we also solve the gap equations with the stationarity equation (13) according to the usual CJT scenario.

The sigma and pion masses for various temperature \( T \) are shown in Fig.1 within the symmetry improved CJT formalism and the usual CJT formalism. In both formalisms the sigma mass first smoothly decreases and then rebounds and grows again at high \( T \). At large \( T \) all mesons masses grow linearly with increasing \( T \). For the pion masses, at low temperature, in the case of the symmetry improved CJT formalism, they appear as massless Goldstone bosons.
FIG. 1: Solution of the system of gap equations in the case of chiral limit. (a) The thermal effective masses of the sigma and pions are given as function of temperature within the symmetry improved CJT formalism. (b) The thermal effective masses of the sigma and pions are given as function of temperature within the traditional CJT formalism.

bosons until the temperature $T$ approaches some critical temperature $T_c$ around 131.5 MeV. After that, the thermal contribution to the effective masses make them degenerate with the sigma signaling a restoration of chiral symmetry. From the right panel in Fig 1, the thermal effective masses of sigma and of pions just show the typical behaviors of the first-order phase transition within the usual CJT formalism.

The critical temperature $T_c$ can be calculated in high temperature limit where we can set $\phi = 0$ and $M_\pi = M_\sigma$, then we only need to solve just one equation (16). If we further decrease the temperature but keep the $\phi = 0$, we can reach some point where the mass of the particles vanishes, then this equation reduces to a well known result

$$0 = m^2 + \lambda \frac{T^2 \pi^2}{2}\frac{\pi^2}{6}. \quad (19)$$

This relationship exactly defines the transition temperature $T_c = \sqrt{2} f_\pi \approx 131.5$ MeV in the model within both formalisms, in which all particle masses become zero. Can this point be also realized if we go from the low temperature to high temperature too? For the case of the usual CJT formalism, by combining the gap equations (11) with the stationarity equation (13), we arrive at

$$M^2_\sigma = \frac{1}{3} \lambda \phi^2. \quad (20)$$

This equation guarantees the sigma mass varies proportionally to the order parameter, and for $\phi = 0$, we always observe $M_\sigma = 0$. Similarly, for another case, we can eliminate $\phi$ in the equations (15) to end up with the following equation

$$M^2_\sigma = -2 \left( m^2 + \lambda F(0) \right) = \frac{\lambda}{6} \left( T^2_c - T^2 \right). \quad (21)$$

The last equation leads to an obvious output $M_\sigma = M_\pi = 0$ for $T = T_c$.

The order parameter $\phi$ vanishes continuously in the symmetry improved CJT formalism case shown in Fig 2, this could be taken as a corroborator to show that there is a second-order phase transition in the Hartree approximation within the symmetry improved CJT formalism. To display this feature more explicitly, we can calculate the effective potential $V_{eff}$ as a function of the temperature and the order parameter by integrating the effective potential $V_{eff}$ with respect to the field $\phi$. Fig 3 presents the numerical solution of the equation (17) at three different temperatures, $T = 0$ MeV, $T = 110$ MeV and $T = 131.5$ MeV. The shape of the potential tells that a second-order phase transition takes place in the model.
FIG. 2: Evolution of the order parameter $\phi$ as a function of $T$ in the Hartree approximation with the symmetry improved CJT formalism (SICJT) and the traditional CJT formalism (CJT) in chiral limit.

FIG. 3: Symmetry improved CJT effective potential $V_{\text{eff}}$ in Hartree approximation as a function of the order parameter $\phi$ for several temperatures: $T = 0$ MeV, $T = 110$ MeV and $T = 131.5$ MeV.

V. SUMMARY AND DISCUSSION

We have discussed the effective masses of the mesons and the effective potential at finite temperature in the framework of the $O(4)$ linear sigma model in chiral limit by adopting the symmetry improved CJT formalism. Unlike the traditional CJT formalism, in the Hartree approximation, we find a second-order phase transition, this observation seems to be in agreement with the universality argument: the chiral phase transition in $N_f = 2$ QCD is likely to be of second order at $\mu_B = 0$ [17]. Moreover, we have shown that the Goldstone bosons resulting from spontaneous symmetry breaking of the $O(4)$ symmetry are massless, and a naive truncation of the symmetry improved CJT effective action does not violate the Goldstone theorem in the Hartree approximation. To illustrate the more apparent nature of the second-order phase transition, we also have constructed the thermal effective potential for several temperatures. From the comparison with the results in the usual CJT formalism, we conclude that the symmetry improved CJT formalism really quite improves the original one in the $O(4)$ linear sigma model.
Of course, in the present study, we have only simply considered the Hartree approximation and ignore the renormalization of the model. Any attempts to go beyond the usual Hartree approximation are worthy to do in order to identify their effects in this novel CJT formalism, e.g. the sunset-type diagrams\cite{29} or more higher loops. Such an extension is straightforward but technically complicated. Finally, the O(4) linear sigma model with two quarks could be combined with the Polyakov loop which allows to investigate both the chiral and the deconfinement phase transition\cite{30}, and recently in the two-particle irreducible (2PI) approximation, the QCD phase diagram has been already investigated in Ref\cite{31}. Since here we have the gauge fields and the spontaneous breaking via the quark condensate, it would be more interesting to explore how the symmetry improved CJT formalism works in the Polyakov-loop extended quark meson model with two or three flavors by directly comparing with the Lattice data or the other effective models. All these studies will make us getting closer to the real QCD world.

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