Moving boundary effects in Quark Gluon Plasma

J. Seixas\textsuperscript{1} and J.T. Mendonça\textsuperscript{2}

\textsuperscript{1} Centro de Física das Interacções Fundamentais, and
\textsuperscript{2} GoLP/Centro de Física de Plasmas, Instituto Superior Técnico,
1049-001 Lisboa, Portugal

Abstract

We analyze the known moving boundary effects in plasma and briefly review a mechanism for particle acceleration through a plasma front. We then show how this mechanism may affect particles crossing the boundary of an expanding quark gluon plasma.
1 Introduction

Heavy ion collisions have been studied all over the years, and their current interest lies mainly on the search for the Quark-Gluon Plasma (QGP) state \[1\]. Information on the eventual transition of the hot nuclear medium to the plasma state can be obtained from the emission of energetic particles (electrons and muons) resulting from the heavy ion collision \[2\].

Usually it is assumed that particles coming out of the plasma region have their energy equal to the one they had inside the QGP blob. However, due to the collective interactions with the surrounding plasma medium, the final particle energy can significantly change when they move across the boundary of the expanding plasma. This energy shift is due, not only to the difference in the value of the effective potential created by all the surrounding particles on each side of the boundary, but also to the velocity of the moving boundary.

This collective effect of the moving background medium has been explored by several authors in a different context, and constitutes the basis for plasma acceleration. In this respect, photon acceleration by relativistic plasma perturbations is a well defined concept \[3\], which has been tested experimentally with success \[4\]. A similar problem, suggested by Bethe \[5\], involves the neutrino plasma interactions, where the electromagnetic force is replaced by the weak force and can eventually be relevant to dense plasmas in supernovae \[6, 7\].

From the point of view of the QGP which might have, or has already, been produced in high energy heavy ion collisions, this type of collective processes can have a direct consequence, for instance, on the behaviour of dileptons produced in the plasma region. For them, QGP is a normal high temperature electromagnetic plasma and therefore one can approach the process with techniques previously developed for this kind of medium.
2 Energy states across a plasma boundary

Let us start with the dispersion relation of a particle moving in a QGP (in units $c = \hbar = 1$):

$$ (E - V)^2 = p^2 + m^2 \quad (1) $$

where $p$ is the particle momentum, $E$ its energy, $m$ its mass and $V$ is the effective potential created by the interactions with the surrounding particles of the plasma medium. From here we can obtain the wave equation for the particle normalized wave function:

$$ \left( \nabla^2 - m^2 - \frac{\partial^2}{\partial t^2} \right) \phi = 2iV \frac{\partial \phi}{\partial t} - V^2 \phi \quad (2) $$

We shall be interested in a plasma density perturbation moving with a constant velocity $\vec{v}_s$. For simplicity, we assume that the perturbation is uniform in the plane perpendicular to $\vec{v}_s$. We can then choose the $x$ coordinate parallel to that velocity. Using the d’Alembert transformations to go to the moving reference frame:

$$ \xi = x - v_c t, \quad \eta = x + v_s t \quad (3) $$

and performing a Fourier transformation in $\eta$, defined as:

$$ \phi(\xi, \eta) = \int \phi_q(\xi)e^{iq\eta}dq \quad (4) $$

we obtain:

$$ \left[ (1 - v_s^2) \left( \frac{\partial^2}{\partial \xi^2} - q^2 \right) + 2iq(1 + v_s^2) \frac{\partial}{\partial \xi} - m^2 \right] \phi_q(\xi) = \phi_q(\xi) $$

$$ = 2iqv_sV(\xi) \left( iq - \frac{\partial}{\partial \xi} \right) \phi_q(\xi) + V^2(\xi)\phi_q(\xi) \quad (5) $$

If we consider that we are in an equilibrium QGP phase, then we must accept that the lifetime of the system is larger than the relaxation times associated with the processes
occurring in the medium. In particular, we must assume that the potential $V(\xi)$ is a slowly varying function. If this is true, we can use the WKB approximation and set:

$$\phi_q(\xi) = \phi_{q0} \exp \left( i \int^\xi p(\xi')d\xi' \right)$$ (6)

Thus, we can get the local dispersion relation:

$$(1 - v_s^2)[p^2(\xi) + q^2] + 2(1 + v_s^2)q + m^2 = 2v_s V(\xi)[q - p(\xi)] + V^2(\xi)$$ (7)

We now take the usual definitions:

$$k = -i \frac{\partial}{\partial x}, \quad E = \omega = i \frac{\partial}{\partial t}$$ (8)

We can then establish the relations between the local frequency (or energy) $\omega(\xi)$, the local momentum $k(\xi)$, the local variable $p(\xi)$ and $q$:

$$2p(\xi) = k(\xi) + \frac{\omega(\xi)}{v_s}$$ (9)

$$2q = k(\xi) - \frac{\omega(\xi)}{v_s}$$ (10)

This allows us to write the dispersion relation (7) in the form:

$$k^2(\xi) + m^2 - \omega^2(\xi) = -2\omega(\xi)V(\xi) + V^2(\xi)$$ (11)

And, solving for $\omega(\xi)$, we get:

$$\omega(\xi) = V(\xi) \pm \sqrt{k^2(\xi) + m^2}$$ (12)
Now, by definition, \( q \) is a constant parameter, which for a given quantum state is invariant inside and outside the plasma region. Thus, we can quite easily relate the initial values of the energy and momentum, \( \omega_i = \omega(\xi_i) \) and \( k_i = k(\xi_i) \), for a particle created inside the plasma, with the final values \( \omega_f \) and \( k_f \), observed outside:

\[
2q v_s = k_i v_s - \omega_i = k_f v_s - \omega_f \tag{13}
\]

The energy shift of the particle will then be:

\[
\Delta \omega = \omega_f - \omega_i = (k_f - k_i)v_s \tag{14}
\]

But the initial and final momenta, \( k_i \) and \( k_f \) can also be expressed in terms of the energies, according to equation (11). Assuming that the effective potential is equal to zero outside the plasma, \( V_f = 0 \), because there are no interacting particles, we get:

\[
k_i = \omega_i \sqrt{(1 - V_i/\omega_i)^2 - m^2/\omega_i^2} \\
k_f = \omega_f \sqrt{1 - m^2/\omega_f^2} \tag{15}
\]

Replacing this in equation (13), we obtain:

\[
\omega_f = \omega_i \frac{1 - v_s \sqrt{(1 - V_i/\omega_i)^2 - m^2/\omega_i^2}}{1 - v_s \sqrt{1 - m^2/\omega_f^2}} \tag{16}
\]

In order to get an explicit result for the final energy, let us assume that the rest mass of the particle \( m \) is negligible, which is certainly true for the electrons. The results is then:

\[
\omega_f \approx \omega_i \left[ \frac{1 - v_s(1 - V_i/\omega_i)}{1 - v_s} \right] \tag{17}
\]

Which means that the energy shift (14) is simply given by:
\[
\Delta \omega \simeq \omega_i \frac{v_s}{1 - v_s}
\]

This means that, when \( v_s \) tends to 1, the energy shift can be considerable, even if the initial effective potential inside the plasma is a small correction to the energy of the ejected particle, \( V_i \ll \omega_i \).

Notice that if \( v_s < 0 \), that is, if we have an imploding surface, leptons traversing it will lose energy. For \( v_s \sim 1 \) leptons can change their energy by as much as half their value inside the plasma region.
3 Particle dynamics

A few remarks should be made concerning the above result. First, equations (16 - 18) are only valid for \( V_f = 0 \), which means that the particle has to leave the plasma region before it spreads away. Second, the exactly resonant case, corresponding to \( v_s = 1 \) is not determined by these equations because in this case the particle will never cross the boundary and, as a result, its energy remains constant: \( \Delta \omega = 0 \). Finally, the above model of a constant velocity \( v_s \) can only be relevant if the changes in the plasma main parameters, such as density and temperature, are negligible during the interaction time with the moving boundary.

These, and other, aspects of the problem can be elucidated by using the classic equations of motion for the emerging particle. They can be used in the same spirit of the WKB approximation. These equations can be stated as:

\[
\frac{d\vec{r}}{dt} = \frac{\partial \omega}{\partial \vec{k}}, \quad \frac{d\vec{k}}{dt} = -\frac{\partial \omega}{\partial \vec{r}} \quad (19)
\]

Here, \( \vec{r}, \vec{k} \) and \( \omega \) are the mean values of the position, momentum and energy of the particle. In a one dimensional model, we can write:

\[
\frac{dx}{dt} = \frac{\partial \omega}{\partial k} = \frac{2k}{\sqrt{k^2 + m^2}} \quad (20)
\]

\[
\frac{dk}{dt} = -\frac{\partial \omega}{\partial x} = -\frac{\partial V}{\partial x} \quad (21)
\]

Notice that the energy \( \omega \) will change when the particle moves across the boundary, according to:

\[
\frac{d\omega}{dt} = \frac{\partial \omega}{\partial t} = \frac{\partial V}{\partial t} \quad (22)
\]
For a boundary moving without deformation with velocity $v_s$, we can use $V(x, t) = V(x - v_s t)$, which leads to:

$$\frac{\partial V}{\partial x} = -\frac{1}{v_s} \frac{\partial V}{\partial t} \quad (23)$$

Comparing with equations (21, 22), we conclude again that $(\omega - kv_s)$ is a constant of motion, as considered before.

Let us consider the case where $V(x, t) \neq V(x - v_s t)$, and this constant of motion is not valid. As a first example we can use:

$$V(x, t) = V_0(t) \frac{1}{2} \{1 + \tanh [k_s(x - x_0(t))]\} \quad (24)$$

where $k_s$ defines the slope of the boundary, and $x_0(t) = x_0 + v_s t$ is its mid position. Here, we can also define the amplitude of the potential $V_0(t)$ as nearly proportional to the mean plasma density. Actually, there is also a small dependence on the plasma temperature, associated with Debye screening. If the plasma is initially inside a sphere of radius $r_0$, and it is expanding along the axis $x$, we can then approximately write:

$$V_0(t) = V_0 \frac{1}{1 + (v_s/r_0)t} \quad (25)$$

The energy shift of the particle moving across the boundary along the $x$ axis can be determined by:

$$\omega(x, t) = V(x, t) + \sqrt{k^2(t) + m^2} \quad (26)$$

where the particle position $x(t)$ and momentum $k(t)$ are determined by the above equations of motion.

A more realistic example is provided by the expanding fireball model [8] for which we assume a cylindrical expanding plasma region such that the volume evolves like

$$V_0(t) = 2\pi \left\{x_0 + v_s t + \frac{1}{2} a_s t^2 \right\} (\pi_0 + \frac{1}{2} a_{\perp} t^2)^2 \quad (27)$$
where the parameters are fixed using flow data. Typical values are $\tau_0 \sim 4.6$ fm and a total freezout time of $t_f \sim 10 - 12$ fm in the longitudinal $x$ direction and $3 - 4$ fm along the perpendicular direction, which implies a final velocity of the front along the $x$ axis, $v_x \sim 0.75c$ and $v_\perp \sim 0.55c$. In this case, for two particles with the same energy inside the plasma $\omega_i$, but crossing different plasma boundaries we have the relation

$$\frac{\omega_{f\perp}}{\omega_{f||}} \sim \frac{1 - v_{s\parallel}}{1 - v_{s\perp}} \frac{1 - v_{s\perp}(1 - V_i/\omega_i)}{1 - v_{s\parallel}(1 - V_i/\omega_i)}$$

In order to illustrate the relevance of the collective plasma effects and to point out the main aspects of the problem of determining the initial energy of the observed particles, we present numerical examples resulting from the integration of the equations of motion. In figure 1, we show the energy shift for two different values of the front velocity $v_s$, and a constant plasma potential. We notice that we can obtain a non-negligible energy shift $\Delta \omega \gg V$, which increases when $v_s$ approaches one. This illustrates the importance of the collective effects associated with a rapidly moving potential on the particle dynamics. In figure 2, we show that such energy shifts can be reduced if the maximum potential amplitude decreases during the interaction time with the boundary. Even so, there is still a large range of parameters for which the energy shifts can be significant.
4 Conclusions

We have shown that the energy of a particle, for instance, an electron or a muon, measured during high energy ion collisions and emerging from the dense nuclear matter gas or quark gluon plasma can be significantly different than its initial energy. This results from the influence of the effective potential acting on that particle as a consequence of its interaction with the background plasma particles. Even if this effective potential remains negligible with respect to the initial particle energy, its influence can be important due to the fact that it is changing on very fast space and time scales. In particular, the potential time variation create a force which attains its maximum value at the expanding plasma boundary. Even if this force is weak, it can act for a long time if the particle moves along with the boundary. The results are valid for electromagnetic and for strong interactions with the background particles. They suggests that the mean field acceleration processes already known in other other areas of plasma physics can also be relevant to high energy physics.

This work shows that the resulting energy shifts are very sensitive to the space and time variations of the expanding plasma, and in particular to the velocity of its boundary.

In this paper we have used the WKB approximation, which is not valid if the background plasma potential changes on scales comparable to the wavelength of the emerging particles. However, it can easily be realized that a full quantum model will lead to the same energy shifts. Typical quantum processes not described by the present model, such as reflection of particles from the boundary and coupling between different helicity states will be discussed in a future work.
References

[1] L.P. Csernai, *Introduction to Relativistic Heavy ion collisions*, John Wiley and Sons, New York (1994).

[2] M. Abreu et al, *Phys. Lett. B*, 410, 337 (1997).

[3] J.T. Mendonça, *Theory of Photon Acceleration*, Institute of Physics Publishing, Bristol, (2001).

[4] J.M. Dias et al., *Phys. Rev. Lett.*, 78, 4773 (1997).

[5] H. Bethe, *Phys. Rev. Lett.*, 56, 1305 (1986).

[6] L.O. Silva et al., *Phys. Rev. Lett.*, 83, 2703 (1999).

[7] J.T. Mendonça et al., *J. Plasma Phys.*, 64, 97 (2000).

[8] R. Rapp, G. Chanfray and J. Wambach, Nucl. Phys. A617 (1997) 472.

FIGURE CAPTIONS

Figure 1 - Particle energy as a function of time, for $m^2 = 0.1$, for a tangent hyperbolic front with $k_s = 1$, and a constant potential $V_i = 1$. The front velocities are: (a) $v_s = 0.98$, and (b) $v_s = 0.99$.

Figure 2 - The same as in Figure 1, but for a time dependent potential, as defined by equation (25), with $v_s/r_0 = 0.1$. 
