Calculation methods of the gas flow in the impeller of a turbo-expander of ORC plant used in the heating plant

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Abstract. The calculation of a turbo-expander impeller of a thermoelectric plant operating according to organic Rankine cycle (ORC) is given. The calculation was carried out for real gas using the Peng-Robinson model. Freon R245fa was chosen as the working fluid of the plant. The approximation values of the coefficients of polynomial describing the temperature dependence of isobaric heat capacity on Freon R245fa were used in the calculation.

1. Introduction
Researches in the application field of thermoelectric plants were again resumed. The application field of these plants is in the range from 90°C to 400°C. Starting at the temperature of 250°C and lower, the ORC cycles become competitive with the classical Rankine cycles [1-4]. The paper discusses one of the approaches to the development of a turbo-expander turbine, or more exactly, its working body for a thermoelectric plant operating according to the ORC on the organic working fluid R245fa, which can be used in a hot water boiler plant. Table 1 shows the parameters of the working fluid.

Table 1. Parameters of the working fluid.

| Parameter                        | Value  |
|----------------------------------|--------|
| Molar mass, kg/kmol              | 134.05 |
| Triple point temperature, K      | 171.05 |
| Normal boiling point, K          | 288.29 |
| Critical point                   |        |
| Temperature, K                   | 427.16 |
| Pressure, MPa                    | 3.651  |
| Density, kg/m³                   | 516.08 |
| Limit of application             |        |
| Minimum temperature, R           | 171.05 |
| Maximum temperature, K           | 440.0  |
| Maximum pressure, MPa            | 200.0  |
| Maximum density, kg/m³           | 1648.8 |
Figure 1 shows the T-s diagram of the ORC installation cycle, which can be used in a hot water boiler plant. Table 2 presents the parameters of the working fluid at the cycle points. An external heat carrier transfers thermal energy to the working fluid through the evaporator. The working fluid evaporates and goes into the gas phase, which corresponds to the 1-2-3 process in the diagram. Further, the gaseous working fluid expands in the turbo-expander and does the work (points 3-4). After expansion, the working fluid cools and condenses into a liquid (process 4-5-6). The pump completes the cycle, increasing the pressure in the system up to the required 6-1.

![Figure 1. T-s diagram of the Organic Rankine cycle with R245fa working fluid.](image1)

| Point on diagram | Temperature, K | Pressure, MPa | Density, kg/m³ | Enthalpy, kJ/kg | Entropy, kJ/(kg·K) |
|-----------------|----------------|---------------|----------------|----------------|-------------------|
| 1               | 350.16         | 1.8100        | 1187.9         | 304.78         | 1.3327            |
| 2               | 390.00         | 1.8100        | 1015.7         | 367.24         | 1.5014            |
| 3               | 400.00         | 1.8100        | 100.47         | 497.85         | 1.8358            |
| 4               | 371.05         | 0.7200        | 35.810         | 484.03         | 1.8461            |
| 5               | 349.52         | 0.7200        | 40.045         | 459.32         | 1.7775            |
| 6               | 349.52         | 0.7334        | 1182.9         | 303.88         | 1.3327            |

The calculation of the turbo-expander impeller was carried out for a real gas (steam) of a given working fluid. The transfer of gas energy in the form of mechanical energy to the machine shaft occurs at uneven velocity field in the blade channels of the wheel. At first glance, it seems that this statement is in contradiction with the concept of jet flow which assumes a uniform velocity field over the cross-section. This contradiction is eliminated by the fact that we shall use the averaged one-dimensional flow as the basis for modeling of a turbomachine. The maximum flow density is achieved at a critical flow regime in a narrow nozzle section which determines the highest possible gas flow rate.

2. **Profiling the blades of the semi-open turbo-expander impeller**

Currently, several approaches to the description of the gas state are proposed. For an ideal gas, the classical equation based on the Mendeleev-Clapeyron equation is used [5-7]. The well-known state models of Redlich-Kwong [5-7], Peng-Robinson [8] and modified models of Redlich-Kwong [5-10] are used to describe the state of a real gas.

We use the most complete model of Peng-Robinson to calculate the flow of real gas through the flow channel of the impeller (figure 2) and further calculate its geometric parameters:
The Peng-Robinson equation was used for the basic model of the working fluid state. The temperature dependence function of the isobaric heat capacity was approximated by a polynomial of the third degree:

\[ c_p = \sum_{i=0}^{3} c_{pi}T^i, \]  

(2)

where \( c_{pi} \) is polynomial coefficients; \( T \) is working fluid temperature.

The relationship between the specific enthalpy (of work) and the geometry of the impeller meridional contour was found from equation [11]:

\[
\left[ 1 - a^2 \left( \frac{\partial \psi}{\partial z} \right)^2 \right] \frac{\partial^2 \psi}{\partial z^2} + \left[ 1 - a^2 \left( \frac{\partial \psi}{\partial r} \right)^2 \right] \frac{\partial^2 \psi}{\partial r^2} + \left( - \frac{2}{a^2} \frac{\partial \psi}{\partial z} \frac{\partial \psi}{\partial r} \right) \frac{1}{r} \frac{\partial \psi}{\partial r} = 0,
\]  

(3)

here \( \psi \) is specific enthalpy; \( z \) and \( r \) are axial and radial coordinates of the working contour, respectively; \( a \) is the speed of sound was determined from the equation:

\[ a = \sqrt{\kappa RZT}, \]  

(4)

where \( \kappa \) is isentropic exponent.

The relationship between geometric and thermodynamic parameters is determined by the equation:

\[ p = \frac{RT}{v - b} \frac{a(T)}{v^2 - 2vb + b^2}, \]  

(1)

where \( p \) is absolute gas pressure; \( v \) is specific volume; \( T \) is absolute temperature; \( R \) is gas constant; \( a, b \) are coefficients from [8].

Figure 2. Meridional contour of the impeller flow channel:

\( d_1 \) – diameter of the impeller main disk; \( ds \) – diameter of the impeller exit funnel; \( d_d \) – diameter of the impeller bushing; \( b_1 \) – blades width at the impeller inlet; \( 0, 1, 2 \) – points of the gas state at the outlet of the nozzle diaphragm, at the impeller inlet and outlet, respectively.
where $u$ is gas circumferential velocity; $Y$ is specific energy, which was determined by the equation for a compressible medium:

$$Y = \left(\frac{k-1}{\kappa}\right) c_p T_s,$$

where $c_p$ is isobaric heat capacity (table 3); $\pi_t$ is the ratio of gas pressure at the impeller inlet and any point in the impeller flow channel.

| Section      | Temperature, K | Pressure, MPa | Density, kg/m$^3$ | Enthalpy, J/kg | Entropy, J/(kg·K) | Isochoric heat capacity, J/(kg·K) | Isobaric heat capacity, J/(kg·K) | Compressibility factor |
|--------------|----------------|---------------|-------------------|----------------|------------------|-----------------------------------|-----------------------------------|-----------------------|
| impeller inlet | 382.17         | 1.0292        | 52.48             | 490.34         | 1842.7           | 955.81                            | 1089.1                           | 0.82734               |
| impeller outlet | 371.05         | 0.7200        | 35.90             | 483.09         | 1842.3           | 934.11                            | 1041.7                           | 0.87144               |

Figure 3 presents the 3D-model of the turbo-expander flow channel calculated by the developed technique.

As a result of profiling using a gas-dynamic model, we determine the following:
- impeller outer diameter;
- impeller width;
- exit funnel diameter (exit funnel relative diameter);
- bushing diameter (bushing relative diameter);
- blade angle at the impeller inlet;
- average blade angle at the impeller outlet;
- blade-exit angel distribution;
axial impeller size.

The impeller profile is made using the design programs of 3D-modeling (SolidWorks, Kompas) (figure 4).

Figure 4. Example of impeller calculated using thermo- and gas-dynamic models profiled in the SolidWorks software package.

The model is semi-empirical and uses the parameters obtained by the graph-analytical method, which is quite easy to use in practice and correlates well with the temporary complex models, suitable for calculation only in productive software systems (ANSYS, Flowvision). Table 4 gives the comparison of the calculation results by the presented method and the simulation results in the CFD module of ANSYS.

Table 4. The values of the impeller design parameters at different stages of calculation and the error.

| Parameter                              | Calculation of geometry by the method | CFD-calculation and refinement [8] | Error, % |
|----------------------------------------|--------------------------------------|-----------------------------------|----------|
| Impeller diameter at the inlet d1, mm | 296.5                                | 300.0                             | 1.2      |
| Impeller bushing diameter db, mm       | 107.4                                | 107.5                             | 0        |
| Outlet funnel diameter ds, mm          | 207.5                                | 211.5                             | 1.9      |
| Blades width at the impeller inlet b1, mm | 16.8                               | 15.5                              | 9.2      |
| Impeller inlet angle, degree           | 92.5                                 | 90                                | 2.7      |
| Impeller outlet angle, degree          | 37                                   | Linear distribution in the range from 25.9 to 44.8 | –        |
| Adiabatic efficiency                   | 0.77                                 | 0.80                              | 3.9      |
In these complexes, similar models include very complex systems of differential equations with a large number of boundary conditions and additional functions, but in the end the solution comes down to the final simplification of the system similar to those described and obtaining a clear picture of gas movement in the blade channel (figure 3-4).

3. Conclusions
The method based on the mathematical model of gas-dynamic processes occurring in a turbo-expander of a thermoelectric plant is developed. It describes the processes of gas flowing through the working volume of the impeller. The method includes the dependences of the physical parameters of the working fluid on the flow velocity and profile geometry of the impeller blades, and it also allows determining the balance of the optimal pressure drop and efficiency coefficient.

Acknowledgments
The work related to the modeling of the blades’ profile of the semi-open impeller of a turbo-expander of the thermoelectric plant was carried out with the financial support of the Russian Foundation for Basic Research and the Government of the Republic of Tatarstan, grant No. 18-48-160033.

The work related to the verification of the gas flow model in the impeller and the refinement of the structural parameters of the geometry was performed as part of the state assignment for the basic scientific research cycle on the topic No. 0217-2018-0006.

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