RINGS IN FIREBALL AFTERGLOWS

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ABSTRACT

We derive the equation for the surface of equal arrival time of radiation from a thin relativistic shell interacting with an external medium, representing the afterglow of a gamma-ray burst produced by a fireball. Due to the deceleration, these surfaces become distorted ellipsoids and, at late times, most of the light comes from a ringlike region whose width depends only on age. We analyze the shape of these surfaces for different dynamic and radiative regimes and homogeneous or power-law external densities. We tabulate the most relevant parameters describing the surfaces and the source brightness distribution, both for bolometric and fixed frequency observations, which are useful for more accurate analytic estimates of the afterglow evolution.

Subject headings: gamma rays: bursts — methods: analytical

1. INTRODUCTION

The afterglows of gamma-ray bursts (GRBs) appear to be well fitted by decelerating relativistic fireball models (Tavani 1997; Vietri 1997; Waxman 1997a; Wijers, Rees, & Mészáros 1997). This picture (Mészáros & Rees 1997), in its simplest form, assumes that the bulk of the radiation comes from the external blast wave pushed ahead of the fireball with a diminishing bulk Lorentz factor, which is predicted to produce radiation at wavelengths longer than γ-rays decaying as a power law in time, in good agreement with observations. Two interesting consequences of the deceleration dynamics are that most of the late radiation comes from a narrow ring, rather than the entire visible surface (Waxman 1997b), and that the usual estimate for the transverse size of a relativistically expanding cloud underestimates the real one (as we show in § 2). This has consequences for the apparent expansion rate of the fireball, the evolution of scintillation properties of the radio-emitting remnant (Goodman 1997; Frail et al. 1997), and the probability of microflaring of GRB afterglows (Loeb & Perna 1998). The exact shape of the equal time surface depends on the dynamic regime of the remnant, as well as on the properties of the external medium. We present simple analytic expressions for the source width and its “average” longitudinal and transverse sizes, for either bolometric or fixed frequency band observations, in both homogeneous and power-law density external media.

2. EQUAL ARRIVAL TIME SURFACES

For simplicity, we assume that the radiation source can be approximated as a surface (we discuss this approximation in § 4) and take the external medium to be isotropic, but not necessarily homogeneous. Therefore, at any lab-frame time t, the fireball is spherical. The observer equal-T (detector time) surface is symmetric with respect to the line of sight (LOS) toward the center of the explosion, therefore its equation is given by two coordinates: a polar angle θ measured from this central LOS and a radial coordinate r. In the absence of deceleration, the equal-T surface is an ellipsoid (Rees 1966) with semimajor axis ΓβcT and semiminor axis ΓβT, where Γ = (1 − β2)−1/2 is the constant Lorentz factor of the freely expanding ejecta and c is the speed of light. When deceleration is present, the shape of the equal-T surface departs from that of an ellipsoid.

The Lorentz factor γsh of the shock can be approximated (Mészáros, Rees, & Wijers 1998) as a power law in r:

\[ γ_{sh} = \Gamma_{sh}(r/r_{dec})\alpha, \quad n = (3 - α)/(1 + δ) > 0, \quad (1) \]

where α < 3 characterizes the external gas density (ρ ∝ r−α), δ describes the fireball dynamics (δ = 0 for radiative and δ = 1 for adiabatic evolution), and rdec is the deceleration radius. Numerical hydrodynamic simulations of the ejecta and the external medium (Panaitescu & Mészáros 1997) show that γsh decreases slowly below rdec and that \( Γ_{sh} = (2/3)Γ_c \). Thus, equation (1) is correct only for \( r > r_{dec} \).

From equation (1), the relationship between r and t for relativistic shocks is found to be

\[ ct = r + 2(2n + 1) \Gamma_{sh}^{2n+1} r_{dec}(r/r_{dec})^{2n+1}. \quad (2) \]

The equation of the equal-T surface is \( ct - r(\cos θ = cT) \); substituting t from equation (2), this equation becomes

\[ \theta = 2 \sin^{-1}\left(\frac{1}{2\Gamma_{sh}} \sqrt{\frac{r}{a} - \frac{a^n}{2n+1}}\right), \quad (3) \]

where \( a = r/r_{dec} \) and \( \tau = T/T_{dec} \) with \( T_{dec} = (2\Gamma_{sh}^2c^{-1})r_{dec} \). At given T, the fluid moving directly toward the observer (θ = 0) is located at

\[ x_{max} = [(2n + 1)]^{(2n+1)}r_{dec}, \quad (4) \]

this being where the radius is largest and Lorentz factor is smallest on the equal-T surface: \( γ_{sh,0} = [(2n + 1)]^{(2n+1)}Γ_{sh} \). We used \( γ_{sh} ≥ 2 \) in deriving equation (2), so equation (3) is valid for τ ≤ (2n + 1)−1(Γc/2)2n+1. As an example, Figure 1 shows the equal-T surfaces at different values of τ, for \( T_0 = 500 \) (Γsh = 330), for a homogeneous external medium (α = 0). For an initial burst energy release of 1052 ergs sr−1, Tdec = 6.5 s, and the times indicated in Figure 1 correspond to 3.6 hr, 1.5 days, and 5.0 days (and 15 days, right panel), if a redshift \( z = 1 \) is assumed.

It is customary in analytic derivations to consider that, at a given time T, the emitting surface is located at \( r = 2γ^2(T)cT \), and that the disk seen by the observer has a radius \( R = [2γ(T)]^{-1}r = γ(T)cT \), as it would be in the absence of deceleration (i.e., an ellipsoid), and to calculate the properties...
of the received radiation using the physical parameters (magnetic field, electron and flow Lorentz factor, etc.) of the fluid at \( (x = r, y = 0) \), the center of the projected surface. When deceleration is present, the radial coordinate \( x_{\text{max}} \) of the center of the equal-\( T \) surface can be related to \( T \) by integrating \( dT = (2 \gamma_0 c)^{-3} dr \), using equation (1):

\[
x_{\text{max}} = 4(2n + 1) \gamma_0^2 c T,
\]

where the flow \( \gamma_0 \equiv \gamma(x_{\text{max}}) \) was instead used instead of the shock \( \gamma_{\text{sh,0}} = 2^{1/2} \gamma_0 \). Therefore, \( x_{\text{max}} \) is larger by a factor \( 2(2n + 1) \) than the typically used value of \( 2 \gamma_0 c T \). For \( n = 1.5 \) (adiabatic remnant and homogeneous external gas) one obtains in equation (5) a factor 16 (Sari 1997), but this factor could be as large as 28 for a radiative remnant \( (n = 3) \). The inappropriate use of the geometry of an ellipsoid in a decelerating fireball would lead to a transverse source size \( y_{\text{max}}^2 = x_{\text{max}}(2 \gamma_0) = 2(2n + 1) \gamma_0 c T \). The numerical factor in this equation is 8 for \( n = 1.5 \). Waxman (1997b) argued that such a large transverse size is incompatible with observations and that the correct transverse size is smaller by a factor 4. Since the transverse size is important for self-absorption considerations, for the timescale and amplitude of afterglow radio scintillations and for gravitational microlensing, it is worth calculating accurately the above coefficient. From equation (3), the maximum value of \( y = r \sin \theta \) is

\[
y_{\text{max}} = 2(2n + 1) \left( \sqrt{2}(n + 1)^{n+1} \right) \gamma_0 c T,
\]

where \( \gamma_0 \) is a function of \( T \) (see previous equation for \( \gamma_{\text{sh,0}} \)). Thus, the ellipsoid approximation overestimates the transverse size by a factor 2.3 for a radiative remnant, and by 1.9 for an adiabatic one, in a homogeneous external medium.

### 3. Bolometric and Band Brightness Distribution

The properties of the observed radiation are determined by integrating over the equal-\( T \) surfaces the emission from different parts of the shocked fluid, taking into account relativistic effects and the fact that each ring \( [\theta, \theta + d\theta] \) is characterized by different physical parameters (magnetic field, electron density, electron Lorentz factor, flow Lorentz factor). We assume that the electrons cool only through synchrotron radiation (our numerical simulations show that this is a good approximation) and that they are either in the radiative or adiabatic regime. In the former case the remnant as a whole can be either radiative or adiabatic, depending on the strength of the coupling between electrons, protons, and magnetic fields (for details on the radiative regime and dynamics see Mészáros et al. 1998), while in the latter case the remnant can be only adiabatic.

Using the scaling relationships for the magnetic field \( B' \propto \gamma r^{-\alpha/2} \), the comoving electron density \( n'_e \propto \gamma r^{-\beta} \), synchrotron cooling timescale \( t'_c \propto \gamma^{-\alpha} r^{-\beta/2} \), expansion timescale \( t'_\text{exp} \propto \gamma^{-\alpha} r^{-\beta/2} \), synchrotron power \( P'_e \propto \gamma^\alpha r^{-\alpha/2} \), and peak of synchrotron spectrum \( \nu'_p \propto \gamma^{\alpha+1} r^{-\alpha/2} \), where \( \gamma \) is the flow Lorentz factor, the comoving spectral intensity at the synchrotron peak \( I'_e \propto n'_e P'_e \nu'_p \min \{ t'_c, t'_\text{exp} \} \) is \( I'_e \propto \gamma^{-\alpha-1} r^{-\alpha/2} \) for radiative electrons and \( I'_e \propto \gamma^{-1} r^{-\alpha/2} \) for adiabatic electrons. The observed spectral intensity at the detected peak of the synchrotron spectrum \( \nu = [\gamma(1 - \beta \cos \theta)]^{-1/2} \nu'_p \) is \( I_e = (\nu'_p)^{-1} I'_e \); therefore, \( I_e \propto d^{-2(2n+1)/3} \left( 1 + \gamma^2 \beta^2 \right)^{-3} \) if \( t'_c < t'_\text{exp} \), and \( I_e \propto d^{-2(2n+1)/3} \left( 1 + \gamma^2 \beta^2 \right)^{-3} \) if \( t'_c > t'_\text{exp} \). The bolometric comoving intensity is \( I' \sim I'_p \nu'_p \) and the observed bolometric intensity is \( I \sim (\nu'_p)^{-1} I' \). The synchrotron spectrum is approximated as a broken power law: \( I_e = (\nu'_p)^{-1} I' \), below the peak \( (\nu < \nu_p) \) and \( I_e = (\nu'_p)^{-1} I'_p \), above the peak \( (\nu > \nu_p) \). We considered \( \epsilon = 1\/2 \) and \( e = \mu/2 \) for radiative electrons and \( \epsilon = 1/3 \) and \( e = (p - 1)/2 \) for adiabatic electrons, \( p \) being the index of the electron power law (here \( p = 2.5 \)).

The bolometric brightness distribution on the equal-\( T \) surface is plotted in Figure 1, which shows where most of the radiation comes from: the upper half-highlighted zone radiates 50% of the total energy; 25% of it is emitted by the cap extending from \( \theta = 0 \) up to the indicated region and the other 25% is radiated by the area extending toward the origin. Similarly, the lower half-highlighted part radiates 80% of the energy detected. Note that the observer does not receive most of the flux from the central LOS of the deformed ellipsoid and that there is a significant difference between the average radial coordinates of the regions highlighted in Figure 1 and that of the fluid on the LOS toward the center (keeping also in mind that all relevant radiation parameters are power laws in \( \gamma \), which is a power law in \( r \)). Therefore, more accurate calculations of the afterglow radiation can be obtained by using a brightness-weighted average longitudinal coordinate \( \bar{x} \). The factor \( (x_{\text{max}} \bar{y})^\gamma \) estimates the difference between an average \( \bar{\gamma} \) that should be used instead of \( \gamma_0 \) (the egg-shaped equal-\( T \) surface is elongated and \( r \sim x \) is a good approximation). It is also useful to calculate a brightness-weighted average transverse coordinate \( \bar{y} \), so that \( \bar{y}/y_{\text{max}} \) is a first-order measure of the brightness distribution on the equal-\( T \) surface. These averages are given
in Table 1, as well as the width \( w \) of the outer ring of the source projection on the plane perpendicular to the central LOS containing 50% of the entire flux received at the detector. Also in Table 1 are the ratios between the brightness-averaged synchrotron peak frequency \( \nu_p \) over the equal-\( T \) surface, and \( \nu_{0} \equiv \nu_{p}(\theta = 0) \).

Observations at a fixed frequency band are shown in Figure 2. The upper graphs give the transverse distribution of the observed synchrotron peak frequency \( \nu_p \) for various constant-\( T \) surfaces. The lower graphs show the bolometric luminosity of the disk of radius \( r \) as a function of \( \gamma \). The ring appears as a steep rise in the integrated luminosity, where \( \nu_p \) varies by approximately 1 order of magnitude around \( \nu_p(y_{max}) \), the peak frequency from the region seen tangentially by the observer. If observations are made at energies \( \lesssim 10^{-1} \nu_p(y_{max}) \) (e.g., in radio, for the times in Fig. 2), then the observer practically sees only the low-energy part of the synchrotron spectrum and the entire disk appears actually almost equally bright. However, if observations are made at energies \( \gtrsim 10 \nu_p(y_{max}) \) (optical or X-ray for Fig. 2), then the observer sees mainly the high-energy tail of the synchrotron spectrum from the power-law distribution of electrons, and the visible region reduces to a ring. For a given observed frequency band, as the shocked fluid is decelerated, \( \nu_p(y_{max}) \) crosses the observed band and the region radiating in that band shrinks from the full disk to a ring with outer edge at \( y_{max} \), the edge of the radiating surface. At energies far above or below \( \nu_p(y_{max}) \), these quantities and the width of the “visible” zone are approximately constant in time. Table 2 gives the asymptotic range of the same coefficients as Table 1, for observations at a given frequency. The first number in each column gives the value of the coefficient when the source is seen as a disk \([ \nu < \nu_{p}(y_{max}) \), larger width \( w \)], and the last number gives the asymptotic value of the coefficient when the source has reduced to a ring \([ \nu \gg \nu_{p}(y_{max}) \), smaller \( w \)]. The coefficients have the same range for all frequencies. The particular frequency of the band only determines the time of the transition between the two asymptotic values, earlier in X-rays (few hours) than in optical (~1 day) or radio (>10 days). It can be seen that the radiative remnant gives narrower rings, and that the ring is wider for expansion into a decreasing density medium (e.g., \( \alpha = 2 \)) than into a homogeneous medium (\( \alpha = 0 \)).

### 4. DISCUSSION

The main conclusions from our calculations are the following: (1) For the afterglow of a fireball, the equal arrival time

![Figure 2](image-url)  
**Fig. 2:** Distribution of peak of synchrotron spectrum (upper) and bolometric luminosity (lower) on the equal-\( T \) surface. Left panels are for a radiative remnant \((\alpha = 3)\), right panels for an adiabatic one \((\alpha = 1.5)\). (lower graphs) The ring is shown by the steep rise in integrated bolometric luminosity. The brightness distribution as observed in two fixed frequency bands (2 eV and 10 GHz), at \( T = 5 \) days, are also shown. (Lower right graph) the effect of the electron radiative regime is illustrated is that adiabatic electrons (solid curve) lead to a narrower ring than radiative ones (dot-dashed curve).
surfaces are distorted ellipsoids whose shape depends on the dynamical regime of the remnant and the density distribution of the external medium. Equation (6) should be used to determine the source size evolution (this is of relevance for the scintillation of the afterglow in radio—Goodman 1997; Frail et al. 1997). (2) Afterglow spectrum and brightness estimates should use the gas parameters at the averaged coordinates (Tables 1 and 2) rather than those of the LOS to the center. For given dynamic and radiative regimes, the ratio between the brightness-weighted peak frequency and that arriving from the top of the equal-$T$ surface is constant in time, so the power-law time dependence of fluxes predicted by fireball afterglow models (Mészáros & Rees 1997; Mészáros et al. 1998) are unchanged. (3) For narrow energy band measurements, the observed shape (ring or disk) of the source depends on the frequency. In any band, the observer should see the source increasing in size and changing its shape from a full disk to a relatively narrow ring, at least while the expansion is relativistic. This is important for the possible gravitational microlensing of afterglows (Loeb & Perna 1998). If bolometric observations are obtained by piecing together band observations spanning many orders of magnitude, then most of the energy of the afterglow should be seen coming from a relatively narrow ring, at any time.

The thickness of the zone radiating most of the energy determines the spread $\delta T$ in the arrival time of photons emitted at lab-frame $t$. In the radiative case, electrons cool on a timescale much shorter than the expansion time, only a very thin zone located behind the blast wave front releases significant energy, and the thickness of the radiating fluid can be neglected. The effect of the shell thickness is important only in the adiabatic case and was taken in consideration by Waxman (1997b).

We have employed here the equal arrival time surfaces using the kinematics of the blast wave. Since the radiation emitted by the fluid behind the shock is received at a later time and the source size is increasing in time, it results that a finite thickness leads to wider rings and lower values of the averages $\overline{x}$ and $\overline{y}$ given in Tables 1 and 2. Our estimates of the ring’s width for an adiabatic remnant are larger by a factor up to 2.3 than calculated by Waxman (1997b). Other departures include allowance for different radiative regimes, and for external density variations.

Additional complications arise when different regions on the equal-$T$ surface are in different dynamic and/or radiative efficiency regimes. As the fireball decelerates, the cooling timescale of electrons increases and they eventually become adiabatic. If there is a strong coupling between electrons and protons + magnetic field, the remnant and electrons evolve together from the radiative to the adiabatic regime. If this coupling is weak, then the remnant becomes adiabatic early in its evolution. A more complex case occurs when the power-law distributed electrons are in different radiative regimes: low energy electrons may be adiabatic, while higher energy electrons may be radiative. Our discussion assumed a spherically symmetric fireball. However, if the ejecta is emitted in a jet of half-angle $\theta_{\text{jet}}$ then the energy requirements are reduced by a factor $(1 - \cos \theta_{\text{jet}})/2$, without changing our results as long as $\gamma \gtrsim \omega_{\text{p}}^{-1}$. In the opposite case, the observer will see the jet’s edge and the source size and width will be below our estimates; this could serve as a test for whether the outflow is jetlike.

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