Dynamical mass matrices from moduli fields *

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Abstract

We review recent work on the structure of the fermion mass matrices in supergravity effective superstrings. They are generally given at low energy by non-trivial functions of the gauge singlet moduli fields. Interesting structures appear in particular if they are homogeneous functions of zero degree in the moduli. In this case we find Yukawa matrices very similar to the ones obtained by imposing a $U(1)$ family symmetry to reproduce the observed hierarchy of masses and mixing angles. The role of the $U(1)$ symmetry is played here by the modular symmetry. The Flavor Changing Neutral Currents effects at the Planck scale coming from the soft terms are identically zero. A dynamical scenario is discussed which allow to generate the observed hierarchies.

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1 Introduction

One of the challenges of the Standard Model and its extensions is the understanding of the fermion masses and mixings. These are usually arbitrary parameters and the large hierarchy of masses and mixing angles observed experimentally is still far to be understood.

The solutions to this problem proposed to this date fall essentially into two categories. One is a symmetry approach, first emphasized by Froggatt and Nielsen [1], which has been largely studied in the literature. It postulates a new abelian horizontal gauge symmetry spontaneously broken at a high energy scale $M_X$. The 3 families of quarks and leptons have different charges under the corresponding $U(1)_X$ group so that only a small number of the Standard Model Yukawa interactions be allowed by the symmetry $U(1)_X$. All the others appear through non-renormalisable couplings to a field whose vacuum expectation value $\langle \phi \rangle$ breaks the horizontal symmetry. In the effective theory below the scale of breaking, this typically yields Yukawa couplings of the form

$$\lambda_{ij} = \left( \frac{\langle \phi \rangle}{M_X} \right)^{n_{ij}},$$

where $n_{ij}$ depends on the $U(1)_X$ charges of the relevant fields. If $\varepsilon \equiv \langle \phi \rangle / M_X$ is a small parameter, the hierarchy of masses and mixing angles is easily obtained by assigning different charges for different fermions.

A second approach, of dynamical origin, was recently proposed [2, 3, 4, 5, 6]. The main idea is to treat Yukawa couplings as dynamical variables to be fixed by the minimization of the vacuum energy density. In this case, one can show that a large hierarchy can be naturally obtained provided that the Yukawa couplings are subject to constraints. Such a constraint could be obtained by an ad hoc imposition of the absence of quadratic divergences in the vacuum energy [2, 3] or as an approximate infrared evolution of the renormalization group equations [4]. In ref. [6], a geometric origin for these constraints was proposed, related to the properties of the moduli space in effective superstring theories. In order to illustrate the idea, we give a simple example of a model containing two moduli fields $T_1$, $T_2$ and two fermions with moduli independent Yukawa couplings $\lambda_1$, $\lambda_2$. In this simple model the low energy couplings at the Planck scale $M_P$ are simply computed to be

$$\hat{\lambda}_1 \sim \left( \frac{T_1 + T_1^+}{T_2 + T_2^+} \right)^{3/4} \lambda_1, \quad \hat{\lambda}_2 \sim \left( \frac{T_2 + T_2^+}{T_1 + T_1^+} \right)^{3/4} \lambda_2.$$
Thus \( \hat{\lambda}_1 \) and \( \hat{\lambda}_2 \) are homogeneous functions of zero degree in the moduli.

The moduli fields correspond to flat directions in the effective four-dimensional supergravity theory. If these flat directions are exact, then the couplings in (2) can be regarded as dynamical variables to be determined by the low energy physics (much in the spirit of the no-scale idea [8] used in the dynamical determination of the gravitino mass [9]). It is easily seen however that the product

\[
\hat{\lambda}_1 \hat{\lambda}_2 \sim \lambda_1 \lambda_2
\]  

should be regarded as a constraint, because the moduli dependence has disappeared in the right hand side of (3). Minimization of the vacuum energy at a low energy scale with respect to the top and bottom Yukawa couplings subject to a constraint of the type (3) was studied in detail in [6]. It was shown there that, qualitatively, the ratio of the two couplings behaves as

\[
\left( \frac{\lambda_t}{\lambda_b} \right) (\mu_0) \sim g^4(M_P) \frac{\mu}{M_{SUSY}},
\]

where \( \mu_0 \sim 1 \) TeV, \( g(M_P) \) is the gauge coupling constant at the Planck scale, \( \mu \) is the usual supersymmetric mass parameter of the MSSM and \( M_{SUSY} \) is the typical mass splitting between superpartners. For a large region of the parameter space of the MSSM, one can thus obtain \( \lambda_t/\lambda_b \sim 40 - 50 \) and easily fit the experimental masses with values of \( \tan \beta \) of order 1. Another interesting application of these dynamical ideas, related to the flavor problem in supersymmetry [10] is not discussed here.

The purpose of the present paper is twofold. First of all, we wish to show that in effective superstrings of the orbifold type [11], structures of the type (1) are naturally obtained. In this approach, the small parameter \( \varepsilon = \langle \phi \rangle /M_X \) of the \( U(1)_X \) horizontal symmetry is given here by \( \varepsilon = (T_1 + T_1^+)/T_2 + T_2^+ \) in the case of two moduli.

The second goal is to show that one of the cases leading to these Froggatt-Nielsen structures corresponds precisely to Yukawa couplings being homogeneous functions of the moduli. We can then apply the above-mentioned dynamical mechanism and determine by minimization the whole structure of the fermion matrices. The hierarchy translates into different vacuum expectation values of the moduli fields and different modular weights of the fermions with respect to these moduli.

\[1\] From now on, we will call Froggatt-Nielsen structures mass matrices for which the order of magnitude of all entries are rational powers of a small common quantity \( \varepsilon \).
Section 2 presents all the cases corresponding to Froggatt-Nielsen structures in orbifold-like effective models. In some instances, they appear when some – but not all – moduli fields are fixed to their self-dual values $T_\alpha = 1$. An appealing situation is the case where the theory possesses a “diagonal” modular symmetry; then the Yukawa couplings are homogeneous functions of zero degree in the moduli and Froggatt-Nielsen structures appear even if all $T_\alpha$ are different from 1. Remarkably enough, there are no contributions at all to the FCNC effects at the Planck scale coming from the soft terms. This is to be contrasted to the $U(1)_X$ horizontal symmetry approach, where departures from universality are difficult to avoid. Moreover, the predictions we find for the soft terms are a weaker form of the conditions obtained by imposing the 2-loop ultraviolet finiteness of the theory, recently discussed in the literature.

Section 3 analyses, in analogy with the $U(1)_X$ approach, the relation between the mass matrices and the one-loop modular anomalies. It is shown that if there are no string threshold corrections in the gauge coupling constants the anomalies can be eliminated only by the Green-Schwarz mechanism \[12\] which uses the Kalb-Ramond antisymmetric tensor field present in superstring theories. In the case relevant for the dynamical approach, the modular anomalies can be cancelled by this mechanism only if there exists at least two moduli with modular anomalies cancelled by the Green-Schwarz mechanism. If the threshold corrections are present, they can account for a part of the modular anomalies and can provide a correct gauge coupling unification scenario.

Section 4 deals with the dynamical determination of the mass matrices at low energy along the lines of ref. \[6\]. Two additional constraints on the modular weights are needed for the mechanism to be effective.

Some conclusions are presented at the end, together with open questions that remain to be investigated.

2 Low-energy mass matrices.

The low energy limit of the superstring models relevant for the phenomenology is the $N = 1$ supergravity described by the Kähler function $K$, the superpotential $W$ and the gauge kinetic function $f$ \[13\]. The generic fields present in the zero-mass string spectrum contain an universal dilaton-like field $S$, moduli fields generically denoted by $T_\alpha$ (which can contain the radii-
type moduli $T_\alpha$ and the complex structure moduli $U_\beta$) and some matter chiral fields $\phi^i$, containing the standard model particles. The Kähler potential and the superpotential read

$$K = K_0 + \sum_i \prod_\alpha t^{(\alpha)}_\alpha |\phi^i|^2 + \cdots,$$

$$K_0 = \hat{K}_0(T_\alpha, T_\alpha^+) - \ln(S + S^+), \quad (5)$$

$$W = \frac{1}{3} \lambda_{ijk} \phi^i \phi^j \phi^k + \cdots,$$

where the dots stand for higher-order terms in the fields $\phi^i$. In (5), $t_\alpha = \text{Re}T_\alpha$ are the real parts of the moduli and $n^{(\alpha)}_i$ are called the modular weights of the fields $\phi^i$ with respect to the modulus $T_\alpha$. The $\lambda_{ijk}$ are the Yukawa couplings which may depend nonperturbatively on $S$ and $T_\alpha$. We define the diagonal modular weight of the field $\phi^i$ as $n_i = \sum_\alpha n^{(\alpha)}_i$. An important role in the following discussion will be played by the target-space modular symmetries $SL(2,\mathbb{Z})$ associated with the moduli fields $T_\alpha$, acting as

$$T_\alpha \rightarrow a_\alpha T_\alpha - ib_\alpha, \quad a_\alpha d_\alpha - b_\alpha c_\alpha = 1, \quad a_\alpha \cdots d_\alpha \in \mathbb{Z}. \quad (6)$$

In effective string theories of the orbifold type [11], the matter fields $\phi^i$ transform under (6) as

$$\phi^i \rightarrow (ic_\alpha T_\alpha + d_\alpha)^{n^{(\alpha)}_i} \phi^i \quad (7)$$

in order for the Kähler metric $K^{ij}_i = \partial K/\partial \phi^i \partial \phi^j$ to be invariant.

A typical example is a model with $n$ moduli fields and Kähler potential

$$K_0 = -\sum_{\alpha=1}^n p_\alpha \ln(T_\alpha + T_\alpha^+) - \ln(S + S^+). \quad (8)$$

Under (8), it transforms as

$$K \rightarrow K + p_\alpha \ln |ic_\alpha T_\alpha + d_\alpha|^2 \quad (9)$$

Associating a modular weight $n^{(\alpha)}_{ijk}$ with the trilinear couplings in eq.(8), the transformation of $W$ gives

$$n^{(\alpha)}_i + n^{(\alpha)}_j + n^{(\alpha)}_k + n^{(\alpha)}_{ijk} = -p_\alpha. \quad (10)$$
Taking the sum of all such relations for the moduli fields, we find
\[ n_i + n_j + n_k + n_{ijk} = -p. \] (11)

Eq. (11) is a weaker form of eqs. (10), expressing the invariance of the theory under the diagonal modular transformations with respect to all moduli:
\[ \phi^i \rightarrow \prod_{\alpha} (i c_\alpha T_\alpha + d_\alpha) c^{(\alpha)} \phi^i. \] (12)

The difference between the individual modular transformations and the less restrictive diagonal one will be essential in the following.

The low energy spontaneously broken theory contains the canonically normalized field \( \hat{\phi}^i \) defined by \( \phi^i = (K^{-1/2})^l_i \hat{\phi}^i \) and the Yukawas \( \hat{\lambda}_{ijk} \) which give the physical masses. The matching condition at the Planck scale \( M_p \) relating the low energy and the original Yukawa couplings is
\[ \hat{\lambda}_{ijk} = e^{\frac{K_0}{2}} (K^{-1/2})^l_i (K^{-1/2})^l_j (K^{-1/2})^l_k \lambda_{i'j'k'}. \] (13)

From eq. (13) we see that the \( \hat{\lambda}_{ijk} \) are functions of the moduli through the Kähler potential \( K \) and eventually the \( \lambda_{i'j'k'} \). The possible dependence of \( \lambda_{ijk} \) on the moduli fields in connection with the fermion mass matrices was analyzed in detail in the literature [14]. In most of the following considerations we consider only the case \( \lambda_{ijk} = \text{cst} + O(e^{-T}) \), in the limit where the moduli dependence can be neglected (for example, large compactification radius limit). The Yukawa matrices in fermionic string constructions were studied in detail, too (see, e.g. [15]).

Our goal is to analyze the general structure of the mass matrices for the quarks and leptons as a function of the moduli fields. They are described by the superpotential \( \hat{W} \) of the Minimal Supersymmetric Standard Model (MSSM) which we take to be the minimal model obtained in the low-energy limit of the superstring models, plus eventually some extra matter singlet under the Standard Model gauge group. \( \hat{W} \) contains the Yukawa interactions
\[ \hat{W} \supset \hat{\lambda}_{ij} Q^i U^c_j H_1 + \hat{\lambda}_{i'j'} Q^i D^c_j H_2 + \hat{\lambda}_{ij} L^i E^c_j H_1, \] (14)

where \( H_1 \) and \( H_2 \) are the two Higgs doublets of MSSM, \( Q^i \), \( L^i \) are the \( SU(2) \) quark and lepton doublets and \( U^c_j \), \( D^c_j \), \( E^c_j \) are the right-handed \( SU(2) \) singlets.
Consider the case of two moduli $T_1$ and $T_2$. Using eqs. (5), (13) and (14), a $U$-quark coupling reads

$$\hat{\lambda}_{ij} = e^{\frac{K_0}{2}} t_2^{-\frac{n_{Q_i} + n_{U_j} + n_{H_2}}{2}} \left( \frac{t_1}{t_2} \right)^{\frac{n_{Q_i} + n_{U_j} + n_{H_2}}{2}} \lambda_{ij}^U$$

or equivalently,

$$\hat{\lambda}_{ij} = e^{\frac{K_0}{2}} t_1^{-\frac{n_{Q_i} + n_{U_j} + n_{H_2}}{2}} \left( \frac{t_2}{t_1} \right)^{\frac{n_{Q_i} + n_{U_j} + n_{H_2}}{2}} \lambda_{ij}^U .$$

Suppose that one of the two moduli-dependent factors in (15) (or equivalently (16)) happens to be family blind. Then the structure obtained for the Yukawa matrix turns out to be very similar to the one that would be derived from an horizontal $U(1)$ symmetry of the Froggatt-Nielsen type [1]. Modular weights play the role of the $U(1)$ charges. Such a situation may arise in the following three cases of interest:

i) $t_1 = t_2 = t \neq 1$. Then

$$\hat{\lambda}_{ij} = e^{\frac{K_0}{2}} t^{-\frac{n_{Q_i} + n_{U_j} + n_{H_2}}{2}} \lambda_{ij}^U ,$$

where $n_{Q_i}$, etc are the diagonal modular weights. For $t << 1$ this could produce hierarchical Yukawa couplings. This case is disfavoured in the case where the relation (11) holds with $n_{ijk} = 0$.

ii) $t_2 = 1$, $\frac{t_1}{t_2} = \varepsilon << 1$ or vice versa $t_1 \leftrightarrow t_2$. Then using eq.(15) we get

$$\hat{\lambda}_{ij} \sim \varepsilon^{-\frac{n_{Q_i} + n_{U_j} + n_{H_2}}{2}} \lambda_{ij}^U .$$

where we dropped the universal $e^{\frac{K_0}{2}}$ factor, irrelevant here. Hierarchical structures are obtained if the dynamics imposes $\varepsilon = \frac{t_1}{t_2}$ small (typically of the order of the Cabibbo angle to some power). Remark that the relevant modular weights correspond to the modulus whose ground state falls away from the self-dual points, $t_i \neq 1$ (for an example of such a situation, see Ref.[16]).

iii) one has the condition

$$n_{Q_i} + n_{U_j} + n_{H_2} = \text{ independent of } i \text{ and } j .$$

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and \( \frac{t_1}{t_2} = \varepsilon \ll 1 \) or vice-versa. This obviously implies that \( n_{Q_i} = n_Q \) and \( n_{U_i} = n_U \) for any \( i = 1, 2, 3 \).

For example, in the case when the diagonal modular symmetry holds, the constant \( \lambda \) is equal to \(-p\) and the Yukawa couplings can be written as

\[
\hat{\lambda}_{ij} = e^{-\frac{K_0}{2} (t_1 t_2)^2} \left( \frac{t_1}{t_2} \right)^{\frac{n_{Q_i}(1) + n_{U_j}(1) + \frac{3}{2}}{2}} \lambda_{ij} = e^{-\frac{K_0}{2} t_1 t_2} \left( \frac{t_2}{t_1} \right)^{\frac{n_{Q_i}(2) + n_{U_j}(2) + \frac{3}{2}}{2}} \lambda_{ij}
\]

or

\[
\hat{\lambda}_{ij} = e^{-\frac{K_0}{2} (t_1 t_2)^2} \left( \frac{t_1}{t_2} \right)^{\frac{n_{Q_i}(12) + n_{U_j}(12) + \frac{3}{2}}{4}} \lambda_{ij}
\]

where \( n_{Q_i}^{(12)} = n_{Q_i}^{(1)} - n_{Q_i}^{(2)} \), etc. The last form \( (21) \) is particularly useful in that it relates the Froggatt-Nielsen-like structures with the asymmetry between the modular weights corresponding to the two moduli fields. Notice that the structures survive in the large radius limit \( T_\alpha \to \infty \), because only ratios of moduli fields appear in \( (21) \).

One way to get the condition \( (19) \) is to search for models where the \( \hat{\lambda}_{ijk} \) are homogeneous functions of the moduli \( T_\alpha \), i.e. \( \sum_\alpha t_\alpha \partial \hat{\lambda}_{ijk} / \partial t_\alpha = 0 \). In this case, using the relation \( \sum_\alpha t_\alpha \partial K^j_\alpha / \partial t_\alpha = n_i K^j_\alpha \) and the matching condition \( (13) \), we arrive at an equation for the original couplings \( \lambda_{ijk} \)

\[
\left( \frac{1}{2} t_\alpha K^\alpha - \frac{n_i + n_j + n_k}{2} + t_\alpha \frac{\partial}{\partial t_\alpha} \right) \lambda_{ijk} = 0.
\]

If \( t_\alpha K^\alpha = -p \) and the \( \lambda_{ijk} \) are pure numbers we recover eq. \( (11) \). In such a case \( (n_{ijk} = 0) \), the relation \( (22) \) can be derived from assuming the diagonal modular symmetry discussed above in \( (12) \). This approach was used in \( [2] \), \( [3] \) in a dynamical approach to the fermion mass problem proposed in \( [4] \), \( [5] \) and studied in \( [1] \) and \( [4] \). We will return to it in section 4.

The experimental data on the fermion and the mixing angles can be summarized as follows. Defining \( \lambda = \sin \theta_c \sim 0.22 \) where \( \theta_c \) is the Cabibbo angle, the mass ratios and the Kobayashi-Maskawa matrix elements at a high scale \( M_X \sim M_P \) have the values

\[
\frac{m_u}{m_t} \sim \lambda^7 \text{ to } \lambda^8, \quad \frac{m_d}{m_t} \sim \lambda^4, \quad \frac{m_s}{m_t} \sim \lambda^2, \quad \frac{m_e}{m_t} \sim \lambda^4, \quad \frac{m_\mu}{m_t} \sim \lambda^2, \quad |V_{us}| \sim \lambda, \quad |V_{cb}| \sim \lambda^2, \quad |V_{ub}| \sim \lambda^3 \text{ to } \lambda^4.
\]

\( \text{(23)} \)
Taking as a small parameter \( \varepsilon = \lambda^2 \sim \frac{1}{20} \), these values are perfectly accommodated by the following modular weight assignment

\[
\begin{align*}
 n_{Q_3}^{(1)} - n_{Q_1}^{(1)} &= 3, \quad n_{Q_3}^{(1)} - n_{Q_2}^{(1)} = 2 \\
n_{s}^{(1)} - n_{b}^{(1)} &= 0, \quad n_{b}^{(1)} - n_{d}^{(1)} = 1 \\
n_{t}^{(1)} - n_{c}^{(1)} &= 2, \quad n_{t}^{(1)} - n_{u}^{(1)} = 4
\end{align*}
\]

(24)

corresponding to the mass matrices

\[
\hat{\lambda}_U = \lambda^{-x} \begin{pmatrix}
\lambda^7 & \lambda^5 & \lambda^3 \\
\lambda^6 & \lambda^4 & \lambda^2 \\
\lambda^4 & \lambda^2 & 1
\end{pmatrix}, \quad \hat{\lambda}_D = \lambda^y \begin{pmatrix}
\lambda^4 & \lambda^3 & \lambda^3 \\
\lambda^3 & \lambda^2 & \lambda^2 \\
\lambda & 1 & 1
\end{pmatrix}.
\]

(25)

In (25) \( x \geq 0 \) (the top coupling should be at least of order one at a high scale) and \( y \geq 0 \) (the bottom coupling should correspondingly be smaller or equal to one). Remark that the negative power of \( \lambda \) in \( \hat{\lambda}_U \) is impossible to obtain in a horizontal symmetry approach because of the analyticity of the superpotential.

An interesting aspect of the case iii) discussed above should be stressed which concerns the sfermion masses \((M^2_0)_{ij}\). Non-diagonal sfermion mass matrices give potentially dangerous contributions to flavor changing neutral current processes like \( b \to s \gamma \) or \( \mu \to e \gamma \) \([18]\). The general expression in supergravity is

\[
(M^2_0)_{ij} = (G_{ij} - G_\alpha R^\alpha_{i\beta j\gamma} G^\beta) m^2_{3/2},
\]

(26)

where \( G = K + \ln |W|^2 \) and \( G_{ij} = \frac{\partial^2 G}{\partial \phi_i \partial \phi_{\bar{j}}} \) is the metric on the Kähler space. The indices \( \alpha, \beta \) correspond to moduli fields which contribute to supersymmetry breaking \(< G_\alpha \neq 0 \) and we assume \(< G_\alpha G^\alpha > + < G_S G^S > = 3 \) \([19]\). \( R_{i\beta j\gamma}^\alpha \) is the Riemann tensor of the Kähler space and \( m^2_{3/2} \) the gravitino mass. The trilinear soft breaking terms \( A_{ijk} \) associated with the Yukawa couplings \( W_{ijk} \) are given by \([20]\)

\[
A_{ijk} = \left[(3 + G^\alpha D_\alpha + G^S D^S) \frac{W_{ijk}}{W}\right] m^2_{3/2}.
\]

(27)

If the superpotential does not depend on the moduli fields, \(< G_\alpha G^\alpha > = p \). In this case, a straightforward supergravity computation for the soft
terms gives (M is the universal gaugino mass)

\[ \tilde{m}_Q^2 = m_{3/2}^2 (1 + n_Q) \mathbb{I}, \]
\[ A^U_{ij} = -M + (n_Q + n_U + n_{H_2} + p)m_{3/2}, \]
\[ M^2 = (3 - p)m_{3/2}^2, \]

(28)

where \( \tilde{m}_Q^2 \) is the left-left squark squared mass and similar relations with obvious replacements hold for all the other fermions. In (28) \( n_Q \) and \( n_U \) are the diagonal modular weights which in the case iii) discussed above are the same for the three generations. Using the modular invariance conditions (11) with \( n_{ijk} = 0 \), we find the soft terms predictions at the Planck scale

\[ A^U_{ij} = A^D_{ij} = A^L_{ij} = M, \]
\[ \tilde{m}_Q^2 + \tilde{m}_U^2 + \tilde{m}_{H_2}^2 = M^2, \]
\[ \tilde{m}_Q^2 + \tilde{m}_D^2 + \tilde{m}_{H_1}^2 = M^2, \]
\[ \tilde{m}_L^2 + \tilde{m}_E^2 + \tilde{m}_{H_1}^2 = M^2. \]

(29)

Notice that the relations (29) are a weaker form of the ultraviolet finiteness conditions for the soft terms in finite theories, widely studied in the last years [22], [23] (they can be also obtained as infrared fixed point predictions of an underlying theory [24]). This surprising connection between finiteness and our predictions can be technically traced back to the modular invariance of the theory and the assumption that supersymmetry breaking is saturated by the moduli and dilaton fields.

The important aspect of (28) is that the sfermion soft-breaking mass matrix and the trilinear soft matrices are proportional to the identity. Going to the basis where the quark mass matrices are diagonal, we see that the soft squark masses are still proportional to the unit matrix. Consequently, there are no flavor changing neutral currents induced at the supergravity level. It is well known that usually models of fermion masses based on horizontal symmetries have problems with FCNC processes due to the soft terms [25], [26]. As far as I know, the models discussed here are the only known examples with fermion mass matrices of Froggatt-Nielsen type [1] and exact flavor independence of soft terms at the Planck scale. Even if we know by now [13] that SUGRA - induced flavor changing neutral currents are not as severe as thought several years ago, it is still worth emphasizing the virtue of the case iii), corresponding to considering Yukawa as being homogeneous functions of the moduli.
3 Modular anomalies and moduli mass textures.

In the context of horizontal abelian symmetries used to explain fermion mass hierarchies, an interesting connection has been established [27, 28] between anomalies associated with such symmetries and mass hierarchies as given in (23).

Gauge anomalies in usual field theories must be absent in order to define a consistent quantum theory. This requirement imposes non-trivial constraints on the particle spectrum in chiral theories. Applied to the ten-dimensional superstrings, this led to the famous Green-Schwarz anomaly cancellation mechanism [12]. This mechanism has a counterpart in 4 dimensions which allows to fix the value of $\sin^2 \theta_W$ at the string scale without advocating a grand unified symmetry [29]. It was shown in [28] and generalized in [17, 30] that, using this mechanism, it is possible to infer from the observed hierarchies (23) in the mass matrices the standard value of $3/8$ for $\sin^2 \theta_W$.

Effective string models also have another type of anomalies, named $\sigma$-model anomalies [31]. They appear in triangle diagrams with two gauge bosons and one modulus. We will show in this section that the cancellation of these anomalies plays a role very similar to the one of mixed gauge anomalies in the abelian horizontal $U(1)_X$ symmetry approach.

The cases of interest to be analyzed in this paper are orbifold compactifications. Consider the diagonal Kähler moduli for which $K_0(T_\beta, T_\beta^\dagger) = -\ln(T_\beta + T_\beta^\dagger)$ (and possibly the complex structure moduli). The gauge group is $G = \prod_a G_a$ and there are matter fields in different representations $R_a$ of $G_a$. The anomalous triangle diagrams give a non-local contribution to the one-loop effective lagrangian [32] which reads

$$L_{nl} = \frac{1}{8\pi^2} \sum_a \int d^4 \theta (W^a W_a) \frac{D^2}{\Box} \sum_{\beta} b_a^{(\beta)} \ln(T_\beta + T_\beta^\dagger) + h.c. \quad (30)$$

In eq. (30) where superfield notations are used, $W^a$ is the Yang-Mills field strength superfield and $b_a^{(\beta)}$ are the anomaly coefficients.

The change of $L_{nl}$ under the modular transformations (6) is given by the local expression

$$\delta L_{nl} = \frac{1}{2\pi} \sum_a \int d^2 \theta (W^a W_a) \sum_{\beta} b_a^{(\beta)} \ln(i c_\beta T_\beta + d_\beta) + h.c. \quad (31)$$
There are two ways of compensating this anomaly. The first, which is particularly interesting in our case is reminiscent of the Green-Schwarz mechanism. It requires the non-invariance of the dilaton field under the modular transformations $S \to S - \frac{1}{8\pi\tau} \sum_\beta \delta_{GS}^{(\beta)} \ln(i\tau T_\beta + d_\beta)$. The factor $\delta_{GS}^{(\beta)}$ is the gauge group independent Green-Schwarz coefficient and induces a mixing between the dilaton field $S$ and the moduli fields $T_\beta$. This mechanism can completely cancel the anomalies only if the anomaly coefficients $b_\alpha^{(\beta)}$ satisfy the equalities $\delta_{GS}^{(\beta)} = \frac{b_\alpha^{(\beta)}}{k_a} = \frac{b_\alpha^{(\beta)}}{k_b} = \cdots$ for all the group factors of the gauge group $G = \prod_a G_a$.

A second mechanism for the cancellation of the term $\delta_{GS}^{(\beta)}$ uses the one-loop threshold corrections to the gauge coupling constants, which can be different for different gauge group factors. If the modular symmetry group is $(SL(2,\mathbb{Z}))^3$, the one-loop running gauge coupling constants at a scale $\mu$ reads

$$\frac{1}{g_5^2(\mu)} = \frac{k_a}{g_5^2} + \frac{b_a}{16\pi^2} \ln \frac{M_s^2}{\mu^2} + \frac{1}{16\pi^2} \sum_{\alpha=1}^{3} \left( b_\alpha^{(\alpha)} - k_a \delta_{GS}^{(\alpha)} \right) \ln \left[ (T_\alpha + T_\alpha^+) |\eta(T_\alpha)|^4 \right] \quad (32)$$

In $(32)$, $g_5$ is the string coupling constant, $M_s$ is the string scale and $b_a$ are the RG $\beta$-function coefficients ($a = 1, 2, 3$) for $U(1)_Y$, $SU(2)_L$ and $SU(3)$ respectively. The Dedekind function is defined by $\eta(T) = \exp(-\pi T/12) \prod_{n=1}^\infty \left[ 1 - \exp(2\pi nT) \right]$, which transforms under $(1)$ as $\eta(T_\alpha) \to \eta(T_\alpha)(ic_\alpha T_\alpha + d_\alpha)^{1/2}$. The unification scale $M_U$ is computed to be

$$M_U = M_s \prod_{\alpha=1}^3 \left[ (T_\alpha + T_\alpha^+) |\eta(T_\alpha)|^4 \right] \frac{s_{\alpha}^{(\alpha)k_a - t_{\alpha}^{(\alpha)k_b}}}{\prod_{a}^{3} b_{\alpha}^{(\alpha)k_a - b_{\alpha}^{(\alpha)k_b}}} \quad (33)$$

with $a \neq b \in \{1, 2, 3\}$. In the simplest approximation of neglecting all the threshold corrections, a one-loop RG analysis for $g_5$ and $\sin^2\theta_W$ gives a good agreement with the experimental data if $M_U \simeq M_s/50$ 3).

Consider now a minimal orbifold model with the particle content of the MSSM (respectively $Q_i, U_i, D_i, L_i$, $i$ being a family index, and the two Higgs supermultiplets $H_1$ and $H_2$), plus possibly extra Standard Model singlet fields. The mixed Kähler $SU(3) \times SU(2) \times U(1)_Y$ triangle anomalies are described by the coefficients 34)

$$b_1^{(\beta)} = 11 + \sum_{i=1}^{3} \left( \frac{1}{3} n_{Q_i}^{(\beta)} + \frac{8}{3} n_{U_i}^{(\beta)} + \frac{2}{3} n_{D_i}^{(\beta)} + n_{E_i}^{(\beta)} + 2n_{E_i}^{(\beta)} \right) + n_{H_1}^{(\beta)} + n_{H_2}^{(\beta)} \quad (34)$$
\[ b_2^{(\beta)} = 5 + \sum_{i=1}^{3} (3n_{Q_i}^{(\beta)} + n_{L_i}^{(\beta)}) + n_{H_1}^{(\beta)} + n_{H_2}^{(\beta)}, \]

\[ b_3^{(\beta)} = 3 + \sum_{i=1}^{3} (2n_{Q_i}^{(\beta)} + n_{U_i}^{(\beta)} + n_{D_i}^{(\beta)}). \]  

Apart from the modular weight independent piece, these coefficients are identical to the ones encountered for the mixed \( U(1)_X \) \(-\) \( G_a \) gauge group anomalies in the abelian horizontal \( U(1)_X \) gauge symmetry approach. Again, the role of the \( U(1)_X \) charges is played here by the modular weights of the different fields. Consequently we will closely follow the analysis performed in \([27]\), \([28]\) and \([17]\).

We place ourselves in the case (iii) of the preceding section. Starting from the relation (21), we obtain:

\[ \text{Det} \hat{\lambda}_U \left( \text{Det} \hat{\lambda}_L \right)^3 \left( \text{Det} \hat{\lambda}_D \right)^{-2} \sim \varepsilon^{-\frac{3}{8} b_1^{(12)} + b_2^{(12)} - 2b_3^{(12)}} \]

\[ \frac{\text{Det} \hat{\lambda}_L}{\text{Det} \hat{\lambda}_D} \sim \varepsilon^{-\frac{3}{8} b_1^{(12)} + b_2^{(12)} - 4b_3^{(12)} - 2(n_{H_1}^{(12)} + n_{H_2}^{(12)})} \]  

The first of eqs. (35) is very useful to discuss anomaly cancellation conditions. Taking as an example \( \varepsilon \sim \lambda^m \), it requires

\[ b_1^{(1)} + b_2^{(1)} - 2b_3^{(1)} = b_1^{(2)} + b_2^{(2)} - 2b_3^{(2)} - \frac{48}{m}. \]  

As shown in \([28]\) for the case of an horizontal symmetry, the second of eqs. (35) has the following interesting solution, which automatically gives the value 3/8 for \( \sin^2 \theta_W \) at unification:

\[ n_{H_1}^{(1)} + n_{H_2}^{(1)} = n_{H_1}^{(2)} + n_{H_2}^{(2)}, \]

\[ b_1^{(1)} + b_2^{(1)} - \frac{8}{3} b_3^{(1)} = b_1^{(2)} + b_2^{(2)} - \frac{8}{3} b_3^{(2)}. \]  

Moreover, using the conditions (11) in the case \( n_{ijk} = 0 \) and the expressions (34), we obtain

\[ b_1' + b_2' - 2b_3' = 8, \]

\[ b_1' + b_2' - \frac{8}{3} b_3' = 2(8 + n_{H_1} + n_{H_2}), \]  

where \( b_1' = b_1^{(1)} + b_1^{(2)} \), etc. Eqs. (36) and (38) clearly express the fact that the theory has one-loop modular anomalies.
An analysis of all the possibilities for the anomalies related to the two moduli leads to the conclusion that, without threshold corrections, the mixed case with zero anomalies for one modulus and Green-Schwarz mechanism for the other modulus is physically uninteresting (it requires $\varepsilon \sim \lambda^{\pm 6}$). In the case of anomalies cancelled by the Green-Schwarz mechanism for both moduli, we obtain $n_{H_1}^{(1)} + n_{H_2}^{(1)} = n_{H_1}^{(2)} + n_{H_2}^{(2)} = -4$ and $y^{(i)} = 6(1 \pm 6/m)$ for $i = 2, 1$. The only other allowed case is when threshold corrections are present for both moduli. In this case, we obtain $n_{H_1} + n_{H_2} + 8 = -\frac{60\lambda^m}{\pi} \ln(M_2^2/M_1^2)$. A realistic value for $M_U$ requires $m \geq 2$ and is obtained for example for $m = 2$, $n_{H_1} + n_{H_2} = -14$. Let us note that $\sin^2 \theta_W$ can still be found equal to $3/8$ at unification scale, irrespective of the choices made in order to obtain the desired value for $M_U$.

4 Dynamical determination of couplings.

The duality symmetries imply the existence of flat directions in the corresponding moduli fields. If they are respected to all orders in the supergravity interactions, then the only way to lift them is by breaking supersymmetry. Given the scale expected for this breaking, one may expect the low energy sector to play an important role in the determination of the moduli ground state. Under these conditions, the low energy minimization with respect to the moduli fields is presumably equivalent to the minimization with respect to the Yukawa couplings, through their non-trivial dependence on the moduli. This was the attitude taken in Refs. [4, 5, 6] to dynamically determine the top/bottom Yukawa couplings. A very important point in this program is the existence of constraints between Yukawas, of a type which is typical of the approach based on moduli dynamics. It was shown in Ref. [6] that this can be enforced if the Yukawa couplings are homogeneous functions of the moduli. In what follows, we will therefore place ourselves in the case iii) of section 2 and analyze how the two approaches can be merged, leading to a dynamical determination of the fermion mass hierarchies and mixing angles.

We start by reviewing the results of Ref. [3]. To compute the vacuum energy at the low-energy scale $\mu_0 \sim M_{susy}$ we proceed in the usual way. Using boundary values compatible with the constraints at the Planck scale $M_P$ (identified here with the unification scale), we evolve the running parame-
ters down to the scale $\mu_0$ using the RG equations and adopt the effective potential approach [35]. The one-loop effective potential has two pieces

$$V_1(\mu_0) = V_0(\mu_0) + \Delta V_1(\mu_0),$$

(39)

where $V_0(\mu_0)$ is the renormalization group improved tree-level potential and $\Delta V_1(\mu_0)$ summarizes the quantum corrections given by the formula

$$\Delta V_1(\mu_0) = \frac{1}{64\pi^2} Str M^4 \ln \frac{M^2}{\mu_0^2} - \frac{3}{2}.$$

(40)

In (40) $M$ is the field-dependent mass matrix, $Str M^\alpha = \sum_J (-1)^J (2J + 1) Tr M^J_J$ is the ponderated trace of the mass matrix for particles of spin $J$ and all the parameters are computed at the scale $\mu_0$. The vacuum state is determined by the equation $\partial V_1/\partial \phi_i = 0$, where $\phi_i$ denotes collectively all the fields of the theory. The vacuum energy is simply the value of the effective potential computed at the minimum.

As expected there is no Yukawa coupling dependence at the tree level. At the one-loop level it appears through

$$\frac{1}{3} Str M^4 = A_U Tr \lambda_U^2 + A_D Tr (\lambda_D^2 + \frac{1}{3} \lambda_L^2) + 8\mu Tr (\lambda_U A_U + \lambda_D A_D + \frac{1}{3} \lambda_L A_L)v_1 v_2,$$

(41)

where $v_1$ and $v_2$ are the vacuum expectation values of the two Higgs doublets. In (41) $A_U$, $A_D$ and $A_L$ are trilinear soft breaking terms and the trace is in the family space. $A_U$ and $A_D$ are given by the expressions

$$A_U = 2 \left[ 2\mu^2/tg^2 \beta + 4M^2 - M_Z^2 \right] v_2^2,$$

$$A_D = 2 \left[ 2\mu^2/tg^2 \beta + 4M^2 - M_Z^2 \right] v_1^2,$$

where $g_1, g_2$ are the $U(1), SU(2)$ gauge couplings, $M$ is a universal squark soft mass and $M_Z^2 = \frac{1}{4}(g_1^2 + g_2^2)(v_1^2 + v_2^2)$ is the $Z$ mass. In order to show that $A_U, A_D > 0$, one may use the phenomenological inequality

$$(Str M^2)_{\text{quarks + squarks}} = 4M^2 > M_Z^2.$$

(43)

The vacuum energy (39) has roughly the Nambu form [2] with an additional linear term which does not change the shape of the vacuum energy as a function of the Yukawas, but which plays an essential role in the minimization process.

\footnote{In a first approximation, if the moduli masses are larger than the average superpartner mass $\bar{m}$, the factor $\ln M^2/\mu_0^2$ in (40) can be replaced by $\ln \bar{m}^2/\mu_0^2 < 0$. [5, 6]}
The positivity of $A_U, A_D$ is a consequence of supersymmetry in the sense that it is due to the Yukawa dependent bosonic contributions in (41). In the non-supersymmetric Standard Model the sign is negative and the present considerations do not apply. Using eq.(39) and eq.(40), we obtain the vacuum energy as a function of the matrices $\lambda_U$ and $\lambda_D$, which is a paraboloid unbounded from below. If the minimization is freely performed, then they are driven to the maximally allowed values and no hierarchy is generated.

Consider now the mass matrices (25) with $\lambda \sim (t_1/t_2)^{-1/2}$ a dynamical parameter to be determined by the minimization. We discussed in Ref.[6] two types of constraints: (a) the proportionality constraint where one of the couplings is proportional to another (to some positive power) $\lambda_1 = \text{cst} \cdot \lambda_2^n$, $n > 0$, (b) a multiplicative constraint where the product of two couplings (or positive powers of them) is fixed to be a moduli independent constant: $\lambda_1 \lambda_2^n = \text{cst}$, $n > 0$. Only the second constraint leads to dynamical hierarchy of couplings. Fortunately for $x, y > 0$ in (25) we get the second type of constraints, for example $(\hat{\lambda}^{33}_U)^y (\hat{\lambda}^{33}_D)^x = \text{cst}$. In this case if $\hat{\lambda}^{33}_U$ for example is big, the constraint (valid at $M_P$) forces $\hat{\lambda}^{33}_D$ to be small and we naturally obtain small numbers.

For the case of two moduli, the conditions to have $x > 0$, $y > 0$ read

$$n_{Q_3} + n_{U_3} + n_{H_2} > -3/2, n_{Q_3} + n_{D_3} + n_{H_1} < -3/2$$

and they should be fulfilled in order to obtain multiplicative-type constraints. An interesting case (treated in detail in [6], where we kept only $\lambda^{33}_U$ and $\lambda^{33}_D$ in the computations) is $x = y$. The relevant constraints are then symmetric in the up and down quarks.

The low energy effective potential is to be minimized with respect to $\lambda$. For this the RG equations are used in order to translate the structures (25) from $M_P$ to $\mu$. The analysis is essentially the same as in [6], the whole structure of the mass matrices does not change qualitatively the results. There are essentially two conditions for the top quark to be the heaviest fermion. The first is (for $g_1 = 0$)

$$tg^2 \beta > \frac{2M^2 + m_1^2}{2M^2 + m_2^2}.$$

where $m_1, m_2$ are the supersymmetric mass terms for the two Higgs. The second is a rather involved lower bound for the dilaton vacuum expectation value, so that the underlying string theory must be in a perturbative regime.
We therefore need a minimal critical value for $tg\beta$ of order one, which depends on the soft masses, in order to have a heavy top quark. Under these two assumptions, there is no need of fine tuning to obtain a value of $\lambda$ of order 0.2 which allows to understand the hierarchy between the top quark and the other fermions.

5 Concluding remarks.

In this paper we analyzed the structure of the fermion mass matrices in the effective superstring theories. It is found that, in some cases of phenomenological interest, they are similar to the structures obtained by imposing abelian horizontal symmetries. The analog of the abelian charges are the modular weights of the matter fields; the small expansion parameters are provided by the vev’s of some moduli fields away from their self-dual values. It is known that by imposing a horizontal $U(1)$ symmetry in effective orbifold models we obtain a relation between modular weights and the $U(1)$ charges of different families [26]. We found here [7] that even without a horizontal symmetry, the fermion mass matrices can present similar structures, if the theory have a diagonal modular symmetry (12). Hierarchical structures for the mass matrices are obtained by assigning different modular weights for the three families of quarks and leptons with respect to some moduli fields. A particular case of interest is when the Yukawas are homogeneous functions of the moduli, which can be viewed as a consequence of a ‘diagonal’ modular symmetry of the theory, in the case where the original string couplings are pure numbers. An interesting consequence is that the squark and slepton mass matrices and the trilinear soft terms are proportional to the identity matrix. Consequently they give no contributions to the FCNC processes like $b \to s\gamma$ or $\mu \to e\gamma$.

We stressed an intriguing connection between the mass matrices and the modular anomalies, similar to the one between mass matrices and mixed gauge anomalies in the horizontal symmetry approach recently discussed in the literature. This is probably an additional argument in favor of a close relationship between horizontal symmetries and modular symmetries in effective string models. A phenomenologically relevant mass spectrum requires one-loop modular anomalies, which can be cancelled in two ways. The first one is the Green-Schwarz mechanism of superstrings. In this context, if the Yukawa couplings are homogeneous functions of moduli and if the sum of the modular weights of the two Higgs doublets of the MSSM
is symmetric in the moduli, then a correct mass pattern asks for a Green-Schwarz mechanism with $k_1 = \frac{5}{3}$ and the Weinberg angle is predicted to be $\sin^2 \theta_W = \frac{3}{8}$. The second way uses the moduli dependent threshold corrections to the gauge coupling constants. In this case we obtain a relation between the fermion masses, modular weights and the unification scale $M_U$. Our analysis shows that we can accommodate a low value $M_U \sim M_\text{s}/50$ provided the Higgs modular weights satisfy a constraint which is allowed at Kac-Moody level two or three in abelian orbifolds. Hence we have the possibility of a successful unification scheme.

We have also investigated a dynamical mechanism for understanding the fermion masses as a low-energy minimization process, previously restricted to the top and bottom couplings. We show that the mechanism is easily generalized to account for the whole structure of the mass matrices, provided two inequalities on the modular weights hold.

We have given in [7] orbifold examples where the hierarchies of the type that we propose are allowed. There are no examples at Kac-Moody level one due to the limited range of the allowed modular weights, but we give examples at level two and three.

There are, of course, many open questions and problems which deserve further investigations. First of all the vev’s of the moduli fields should be fixed by the dynamics, which usually prefers the self-dual points. In the dynamical approach, it would be also interesting to view the determination of the Yukawa couplings directly from the point of view of the moduli fields; in particular why the corresponding flat directions remain unlifted down to low energies.

Finally it would be interesting to construct explicit orbifold models with hierarchical mass matrices along these lines and to investigate their phenomenological virtues.

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