Abstract

We discuss the $e^+ e^- \to (hA) \to bbbb$ cross section in an Abelian extended SM. We work in that minimum of the scalar potential for which Higgs trilinear coupling is greater than the soft mass parameters. We find that next-to-lightest Higgs gives the essential contribution to the cross section in the small $Z - Z'$ mixing angle and leptophobic $Z'$ limit.
1 Introduction

Higgs search is one of the main goals of the present and future colliders, specially, in the case of the extensions of the SM such as MSSM or NMSSM, there are several Higgs particles whose detection in the colliders is an important issue. Usually gauge and Yukawa couplings and particle masses are unknown, and thus the predictive power of such models is limited. Thus, one has to analyze different models to find bounds as model independent as possible.

In this note we shall analyze the indications of Higgs scalars in as specific $e^+e^-\rightarrow b\bar{b}b\bar{b}$ scattering process in an Abelian extended supersymmetric SM. Implications of extra $Z$ bosons appearing in such gauge extensions of SM by an extra $U(1)$ have been widely analyzed and checked against the precision data, and in the context of the future colliders [1, 2].

In this work we shall analyze $e^+e^-\rightarrow b\bar{b}b\bar{b}$ cross section in a $U(1)$ extended supersymmetric SM. In particular, $e^+e^-\rightarrow b\bar{b}b\bar{b}$ may remind one the recent ALEPH four-jet anomaly [3]. However, Higgs interpretation is not appropriate to explain the four-jet topology there [3]. For a supersymmetrical interpretation of this event one can refer, for example, to [4].

The analysis presented here is based upon the recent work [5]. The model under concern is analyzed in detail together with the RGE analysis of the parameters of the potential in [5]. There the low energy model we discuss is obtained from a supergravity Lagrangian with appropriate non-universality in the soft masses at the String scale. Here we consider simply a low energy model and summarize some relevant results of [5] and derive the necessary quantities for the present problem. In particular, we shall work in the trilinear coupling-driven minimum of the potential which is discussed in Section 3 and Section 5 of [5].

This paper is organized as follows. In Section 2 we shall review the main results of [5] relevant for this work, and derive the necessary quantities for the problem in hand.

In Sec. 3 we shall derive $e^+e^-\rightarrow (h,A)\rightarrow b\bar{b}b\bar{b}$ cross section by using resonance approximation for the scalars. We shall base our analysis mainly on the analytical results instead of using computer codes such as PHYTIA or JETSET as it was done in [3].

We will evaluate the cross section and present the variation of the cross section against the center of mass energy and Higgs Yukawa coupling.
Finally in Section 4, a discussion on the results and their implications are given.

2 Higgs Bosons and Vector Bosons

We will first summarize some of the results of [5], and derive the necessary quantities for the present problem. The gauge group is extended to $G = SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{Y'}$ with the respective couplings $g_3, g_2, g_Y, g_{Y'}$. We introduce the Higgs fields $\mathcal{H}_1 \sim (1, 2, -1/2, Q_1)$, $\mathcal{H}_2 \sim (1, 2, 1/2, Q_2)$, $S \sim (1, 1, 0, Q_S)$, with the indicated quantum numbers under $G$. The gauge invariance of the superpotential
\[ W \ni h_s \mathcal{H}_1 \cdot \mathcal{H}_2 \] guarantees that $Q_1 + Q_2 + Q_S = 0$. The scalar potential is given by
\[
V(H_1, H_2, S) = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + m_S^2 |S|^2 - (Ah_s \mathcal{H}_1 \cdot \mathcal{H}_2 + h.c.) + |h_s|^2 \left[ |H_1 \cdot H_2|^2 + |S|^2 (|H_1|^2 + |H_2|^2) \right] + \frac{G^2}{8} \left( |H_2|^2 - |H_1|^2 \right)^2 + \frac{g_2^2}{2} |H_1 \cdot H_2|^2 + \frac{g_{Y'}}{2} (Q_1 |H_1|^2 + Q_2 |H_2|^2 + Q_S |S|^2)^2
\] where $G = \sqrt{g_2^2 + g_{Y'}^2}$. Here the soft mass parameters $m_1^2, m_2^2, m_S^2$ can have either sign, and without loss of generality we choose $h_s A$ positive. In terms of the real fields $\phi_i, \xi_i, \psi_j$ ($i = 1, 2, 3; j = 1, 2, 3, 4$) the Higgs fields are defined by
\[
H_1 = \frac{1}{\sqrt{2}} \left( \begin{array}{c} v_1 + \phi_1 + i \xi_1 \\ \psi_1 + i \psi_2 \end{array} \right) \\
H_2 = \frac{1}{\sqrt{2}} \left( \begin{array}{c} \psi_3 + i \psi_4 \\ v_2 + \phi_2 + i \xi_2 \end{array} \right) \\
S = \frac{1}{\sqrt{2}} (v_s + \phi_3 + i \xi_3)
\] Here $\psi_i$ determines the charged Higgs sector. $v_1, v_2, v_s$ being real, there is no CP violation at the tree level so that the neutral sector of the total scalar
mass matrix is split into CP-even and CP-odd parts. In that minimum of the potential for which gauge group is completely broken down to color and electric symmetries the mass-squared matrices of CP-even and CP-odd scalars become, respectively

\[
(M_2^2)_h = \begin{pmatrix}
\frac{\kappa v_2}{v_1} + 2\lambda_1 v_1^2 & -\kappa v_s + \lambda_1 v_1 v_2 & -\kappa v_2 + \lambda_{1s} v_1 v_s \\
-\kappa v_s + \lambda_1 v_1 v_2 & \frac{\kappa v_2}{v_2} + 2\lambda_2 v_2^2 & -\kappa v_1 + \lambda_{2s} v_2 v_s \\
-\kappa v_2 + \lambda_{1s} v_1 v_s & -\kappa v_1 + \lambda_{2s} v_2 v_s & \frac{\kappa v_1}{v_s} + 2\lambda_s v_s^2
\end{pmatrix},
\]

(6)
in \((\phi_1, \phi_2, \phi_3)\) basis, and

\[
(M_2^2)_A = \begin{pmatrix}
\frac{\kappa v_2}{v_1} & \kappa v_s & \kappa v_2 \\
\kappa v_s & \frac{\kappa v_2}{v_2} & \kappa v_1 \\
\kappa v_2 & \kappa v_1 & \frac{\kappa v_1}{v_s}
\end{pmatrix},
\]

(7)
in \((\xi_1, \xi_2, \xi_3)\) basis. Here \(\kappa = h_S A/\sqrt{2}\), \(\lambda_i = \frac{G_2^2}{8} + \frac{1}{2}g_Y^2 Q_i^2\), \(\lambda_{12} = -\frac{G_2^2}{4} + g_Y^2 Q_1 Q_2 + h_s^2\), \(\lambda_{is} = g_Y^2 Q_i Q_S + h_s^2\) and \(\lambda_s = \frac{1}{2}g_Y^2 Q_S^2\). The diagonalization of \((M_2^2)_h\) yields three CP-even scalars

\[h_i = (R^{-1})_{ij} \phi_j, (i, j = 1, 2, 3).\]

(8)
The diagonalization of \((M_2^2)_A\) yields two CP-odd Goldstone bosons and a pseudoscalar boson

\[A^0 = (F^{-1})_{ij} \phi_j, (j = 1, 2, 3).\]

(9)

There are two neutral vector bosons: \(Z\) boson of \(SU(2)_L \times U(1)_Y\) and \(Z'\) boson of \(U(1)_{Y'}\) which mix through the mass-squared matrix

\[
(M_2^2)_{Z-Z'} = \begin{pmatrix}
M_Z^2 & \Delta^2 \\
\Delta^2 & M_{Z'}^2
\end{pmatrix},
\]

(10)
where

\[
M_Z^2 = \frac{1}{4}G^2(v_1^2 + v_2^2),
\]

(11)
\[
M_{Z'}^2 = g_Y^2(v_1^2 Q_1^2 + v_2^2 Q_2^2 + v_s^2 Q_S^2),
\]

(12)
\[
\Delta^2 = \frac{1}{2}g_Y G(v_1^2 Q_1 - v_2^2 Q_2).
\]

(13)
\[ M_{Z_1,Z_2}^2 = \frac{1}{2}\left[M_Z^2 + M_{Z'}^2 \pm \sqrt{(M_Z^2 - M_{Z'}^2)^2 + 4\Delta^4}\right], \]  

(14)

The \( Z - Z' \) mixing angle \( \alpha \) is given by
\[
\alpha = \frac{1}{2} \arctan \left( \frac{2\Delta^2}{M_{Z'}^2 - M_Z^2} \right)
\]

(15)

The coupling of neutral vector bosons to \( h_i \) and \( A^0 \) can be calculated straightforwardly:
\[
K_{\mu}^{Z_1,A^0h_i} = \frac{i}{2} \{(G \cos \alpha - 2g_Y Q_1 \sin \alpha) R_{1i} F_{1A} \\ - (G \cos \alpha + 2g_Y Q_2 \sin \alpha) R_{2i} F_{2A} \\ - 2g_Y Q_S \sin \alpha R_{3i} F_{3A}\}(p_A - p_{h_i})_\mu
\]

(16)

\[
K_{\mu}^{Z_2,A^0h_i} = \frac{i}{2} \{(G \sin \alpha + 2g_Y Q_1 \cos \alpha) R_{1i} F_{1A} \\ - (G \sin \alpha - 2g_Y Q_2 \cos \alpha) R_{2i} F_{2A} \\ + 2g_Y Q_S \cos \alpha R_{3i} F_{3A}\}(p_A - p_{h_i})_\mu
\]

(17)

Moreover, the \( b\bar{b}h_i \) and \( b\bar{b}A^0 \) vertices are given by
\[
K^{h_i,bb} = \frac{m_b}{v_1} R_{1i}
\]

(18)

\[
K^{A^0,bb} = \frac{m_b}{v_1} i\gamma_5 F_{1A}
\]

(19)

3 \( e^+e^- \rightarrow (h_iA) \rightarrow b\bar{b}b\bar{b} \) cross section

We shall calculate the cross section for each possible CP-even neutral particle. The scattering process under concern involves four particles in the final state. Thus, the phase space integration is too complicated to be carried out analytically. We shall calculate the total cross section by replacing the \( h_i \) and \( A^0 \) lines with resonances:
\[
\frac{1}{|p_A^2 - m A^2 + i m A \Gamma_A|^2} \rightarrow \frac{\pi}{m A \Gamma_A} \delta(p_A^2 - m_A^2)
\]

(20)

\[
\frac{1}{|p_{h_i}^2 - m_{h_i}^2 + i m_{h_i} \Gamma_{h_i}|^2} \rightarrow \frac{\pi}{m_{h_i} \Gamma_{h_i}} \delta(p_{h_i}^2 - m_{h_i}^2)
\]

(21)
where $\Gamma_A$ and $\Gamma_{h_i}$ are the total widths of $A^0$ and $h_i$. Under this approximation one can now calculate the total cross section analitycally,

$$\sigma^i = \frac{1}{48\pi} s(c_V^{(i)} + c_A^{(i)})^2 \lambda^3(1, m_A^2/s, m_{h_i}^2/s)BR(h_i \rightarrow b\bar{b})BR(A^0 \rightarrow b\bar{b})$$

(22)

where

$$\lambda(x, y, z) = \sqrt{(x - y - z)^2 - 4yz}$$

(23)

is the phase space factor coming from $Z_{1,2} \rightarrow h_i A^0$ decay. The vector and axial couplings are defined by

$$c_V^{(i)} = f_1^{(i)} v_1 + f_2^{(i)} v_2$$

(24)

$$c_A^{(i)} = f_1^{(i)} a_1 + f_2^{(i)} a_2.$$  

(25)

Here the coefficients $f_j^{(i)}$ include the vector boson propagators and couplings of the vector bosons to scalars

$$f_j^{(i)} = Q_{Zj}^{A_0 h_i} / (s - M_{Zj}^2) \quad (i = 1, 2, 3), (j = 1, 2)$$

(26)

where, using (16) and (17) we defined $Q_{Zj}^{A_0 h_i}$ via

$$K_{Zj}^{A_0 h_i} = Q_{Zj}^{A_0 h_i} (p_A - p_{h_i})_{\mu}$$

(27)

$v_i$ and $a_i$ in (24) and (25) are given by

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} \cos \alpha & \sin \alpha \\ \sin \alpha & -\cos \alpha \end{pmatrix} \begin{pmatrix} v_e \\ v'_e \end{pmatrix}$$

(28)

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} \cos \alpha & \sin \alpha \\ \sin \alpha & -\cos \alpha \end{pmatrix} \begin{pmatrix} a_e \\ a'_e \end{pmatrix}$$

(29)

where

$$v_e = \frac{g_2}{4 \cos \theta_W} (1 - 4 \sin^2 \theta_W)$$

(30)

$$a_e = \frac{g_2}{4 \cos \theta_W}$$

(31)

$$v'_e = \frac{g_{Y'}}{2} (Q_L + Q_E)$$

(32)

$$a'_e = \frac{g_{Y'}}{2} (Q_L - Q_E)$$

(33)
with \( Q_L \) and \( Q_E \) being the \( U(1)_{Y'} \) charges of lepton doublet \( L \) and lepton singlet \( E^c \).

The minimization of the potential in (3) should be done such that the mixing angle (15) should be below the phenomenological bound \( \sim 10^{-3} \). Next, if \( Z' \) boson is to have an effect on this scattering process its mass should be under LEP2 or LHC reach. These two constraints can be met under the following conditions:

- If the trilinear coupling \( h_S A \) is large compared to the soft mass parameters then potential is minimized for
  \[
  v_1 \sim v_2 \sim v_s \sim 174 \text{GeV} 
  \tag{34}
  \]

- and, if \( U(1)_{Y'} \) charges of Higgs doublets satisfy
  \[
  Q_1 \sim Q_2 
  \tag{35}
  \]

so that \( \Delta^2 \), consequently \( Z - Z' \) mixing angle becomes small without a large \( v_s \). This yields a vanishingly small \( Z - Z' \) mixing angle and a relatively light \( Z_2 \).

In using this procedure we assume that the absolute minimum of the potential does not occur in the sfermion sector, which otherwise breaks the color and charge symmetries. In this large trilinear coupling limit one has definite predictions for the scalar and vector boson masses independent of the sign and magnitude of the soft masses in (3):

\[
\begin{align*}
m_A & \sim \sqrt{\frac{3}{2}} h_S v \\
m_{h_1} & \sim \frac{h_S}{\sqrt{2}} v \\
m_{h_2} & \sim \frac{1}{2} \sqrt{G^2 + 2 h_S^2 v} \\
m_{h_3} & \sim \sqrt{3 g_{Y'}^2 Q_1^2 + \frac{h_S^2}{2}} v \\
M_{Z_1} & \sim M_{Z_0} \\
M_{Z_2} & \sim \sqrt{3 g_{Y'} Q_1 v} 
\end{align*}
\tag{36-41}
\]
where \( v = 246 \text{ GeV} \), and the mass spectrum obeys the ordering \( m_{h_3} > M_{Z_2} > m_{h_2} > m_A > m_{h_1} \), for \( g^2, Q^2 \sim G^2 \).

For any value of the Yukawa coupling \( h_S, m_A = \sqrt{3}m_{h_1} \). This sets an ever-existing gap between the masses of the lightest Higgs \( h_1 \) and the pseudoscalar \( A^0 \).

In the minimum of the potential under concern, the matrices \( R \) in (3) and \( F \) in (3) become

\[
R = \begin{pmatrix}
h_1 \\
1/\sqrt{3} & 1/\sqrt{2} & -1/\sqrt{6} \\
1/\sqrt{3} & 1/\sqrt{2} & -1/\sqrt{6} \\
1/\sqrt{3} & 0 & 2/3
\end{pmatrix}
\]  

(42)

\[
F = \begin{pmatrix}
A^2 \\
-1/\sqrt{2} & -1/\sqrt{6} & 1/\sqrt{3} \\
0 & 2/3 & 1/\sqrt{3} \\
1/\sqrt{2} & -2/3 & 1/\sqrt{3}
\end{pmatrix}
\]  

(43)

where \( A^2 \) and \( A^1 \) are the would-be pseudoscalar Goldstone bosons.

Since \( F_{iA} \) and \( R_{ih_1} \) are identical, one has from (16) and (17),

\[
K_{Z_1 A^0 h_1} = 0
\]

(44)

\[
K_{Z_2 A^0 h_1} = 0
\]

(45)

Using \( R_{ih_2} \) and \( F_{iA} \) one gets

\[
K_{Z_1 A^0 h_2} = -\frac{1}{\sqrt{6}} G \cos \alpha (p_A - p_{h_3})_\mu
\]

(46)

\[
K_{Z_2 A^0 h_2} = 0
\]

(47)

where in demonstrating that \( K_{Z_2 A^0 h_2} \) vanishes, we used the equality of the \( U(1)_{Y'} \) charges of the doublets.

Finally, using \( R_{ih_3} \) and \( F_{iA} \) one gets

\[
K_{Z_1 A^0 h_3} = -\frac{1}{\sqrt{2}} g_{Y'} Q_S \sin \alpha (p_A - p_{h_3})_\mu
\]

(48)

\[
K_{Z_2 A^0 h_3} = \frac{1}{\sqrt{2}} g_{Y'} Q_S \cos \alpha (p_A - p_{h_3})_\mu
\]

(49)
which reflect completely the extended nature of the model.

Now we shall discuss the implications of the vector boson - Higgs couplings in (44)-(49). As is seen from (44) and (45) there is no coupling of vector bosons to the lightest Higgs scalar and the pseudoscalar boson. These two equations guarantee that the $U(1)_Y$, extended model under concern behaves very much like the SM in the large trilinear coupling driven minimum. In particular, (44) is a reminder of the SM where there is no pseudoscalar boson at all. As dictated by (45), in this minimum of the potential $Z_2$ is also similar to $Z$ in this respect. This vanishing of the coupling constants is important in that $h_1$ does not contribute to the total cross section.

The coupling of the vector bosons to next-to-lightest Higgs $h_2$ and $A^0$ are given by (46) and (47). As is seen from (46) $Z_1$ now feels $h_2$ and $A^0$ by a non-zero coupling constant $-\frac{1}{\sqrt{6}} G \cos \alpha$. As we require the mixing angle be small ($\cos \alpha \sim 1$) this coupling constant is no way negligible in the minimum of the potential under concern. The nature of the coupling is essentially weak, since extended nature of the model enters only by the $Z - Z'$ mixing angle.

The coupling of the vector bosons to heaviest Higgs $h_3$ and pseudoscalar $A^0$ are given by (48) and (49). In this case both $Z_1$ and $Z_2$ couple to scalars. It is for $h_3$ case that the couplings carry the seeds of the extended nature of the model. Strengths of the couplings are proportional to the $U(1)_{Y'}$ coupling constant times the $U(1)_{Y'}$ charge of the singlet $S$. Unless $g_{Y'} Q_S$ is unnaturally large as compared to the moderate choice $g_{Y'} Q_S \sim G$, in the small $Z - Z'$ mixing angle limit $Z_1$ essentially decouples from scalars leaving room only for $Z_2$.

A number of authors have explained a possible excess in $R_b$ by a lepto-phobic $Z'$ [6]. The most recent LEP data [7] weakened the possibility of an $R_b$ excess. So $Z'$, if exists, would probably be hadrophobic. Thus we are to take $Z'$ as leptophobic as possible as required by the present data.

We can summarize the situation concerning the process under consideration (under the resonance approximation) as follows. $h_1$ is excluded from the process by (44) and (45) and, consequently it brings no constraint on Yukawa and gauge couplings. As we see from (46)-(49), $Z_2$ does not contribute to the scattering process until $s$ exceeds the kinematical threshold $(m_A + m_{h_3})^2 > (m_A + M_{Z_2})^2$. $m_{h_3}$ is bounded by $M_{Z_2}$ from below, and the latter depends on the $U(1)_{Y'}$ coupling constant and $U(1)_{Y'}$ charge of the singlet $S$. If $M_{Z_2}$ is beyond the LEP2 reach so does $m_{h_3}$.  


The axial and vector couplings of the vector bosons are given in (24) and (25). From these equations, and from (46) and (47), it follows that

\[
c_{v}^{(2)} = -\frac{1}{\sqrt{6}}G \cos \alpha \frac{1}{s - M_{Z_{1}}^{2}}(\cos \alpha v_{e} + \sin \alpha v'_{e}) \tag{50}
\]

\[
c_{A}^{(2)} = -\frac{1}{\sqrt{6}}G \cos \alpha \frac{1}{s - M_{Z_{1}}^{2}}(\cos \alpha a_{e} + \sin \alpha a'_{e}) \tag{51}
\]

In the case of small $Z - Z'$ mixing angle, which is really the case in the minimum of the potential, $U(1)_{Y'}$ contribution to vector and axial couplings of leptons is suppressed by $\sin \alpha$. Therefore, the vector and axial couplings in (50) and (51) practically do not get any significant contribution from leptonic $U(1)_{Y'}$ charges;

\[
c_{v}^{(2)} \sim -\frac{G}{\sqrt{6} s - M_{Z_{1}}^{2}} v_{e} \tag{52}
\]

\[
c_{A}^{(2)} \sim -\frac{G}{\sqrt{6} s - M_{Z_{1}}^{2}} a_{e}. \tag{53}
\]

For moderate values of $g_{Y'}Q_{S}$, $(g_{Y'}Q_{S} \sim G)$, in the small $Z - Z'$ mixing angle limit, one can neglect coupling of $Z_{1}$ to $h_{3}$ and $A^{0}$ in (48). Then $Z_{2}$ couples to scalars by a non negligible coupling constant $\sim \frac{1}{\sqrt{2}}g_{Y'}Q_{S}$, as is seen from (49). Under this approximation, the leptonic couplings are given by

\[
c_{v}^{(3)} \sim -\frac{1}{\sqrt{2}}g_{Y'}Q_{S} \frac{1}{s - M_{Z_{2}}^{2}} v'_{e} \tag{54}
\]

\[
c_{v}^{(3)} \sim -\frac{1}{\sqrt{2}}g_{Y'}Q_{S} \frac{1}{s - M_{Z_{2}}^{2}} a'_{e} \tag{55}
\]

which can be large enough to make $Z_{2}$ effects be seen in the present-day experiments. Although it was not the case in the $h_{1}$ and $h_{2}$ couplings, here one has to choose $Z'$ as leptophobic as required by the experiment. From the form of the $v'_{e}$ and $a'_{e}$ given in (32) and (33), we conclude that $U(1)_{Y'}$ charges of both $L$ and $E^{c}$ must be chosen small. This makes $Z_{2}$ to be hardly observable.

Depending on the leptophobicity of $Z'$ to suppress the vector and axial couplings in (54) and (55), one concludes that only $h_{2}$ gives a significant contribution to the total cross section through (46). This is an important
result in that one can single out the next-to-lightest Higgs among others by measuring the cross section. Reading $m_A$ and $m_{h_2}$ from (36) and (38), and using (52) and (53) we can rewrite the total cross section as follows

$$
\sigma_{\text{tot}} \simeq \sigma^{(2)} = \frac{1}{72 \pi v^2} \left( v_e^2 + a_e^2 \right) \frac{r_Z f(r_Z, r_S)}{(1 - r_Z)^2} BR(h_2 \to \bar{b}b) BR(A^0 \to \bar{b}b) \quad (56)
$$

where we introduced the definitions

$$
r_Z = \frac{M_{Z_1}^2}{s} \simeq \frac{M_{Z_0}^2}{s} \quad (57)
$$

$$
r_S = \frac{h_S^2}{G^2} = \frac{\mu^2}{M_{Z_0}^2} \quad (58)
$$

$$
f(r_Z, r_S) = \left\{1 + (4r_S - 1)^2 r_Z^2 - 2(8r_S + 1)r_Z\right\}^{3/2} \quad (59)
$$

and effective $\mu$ parameter is defined by $\mu = h_S v_s/\sqrt{2}$. Thus, cross section depends on two variables only: $r_Z$ and $r_S$, in particular, it does not depend upon the $U(1)_{Y'}$ coupling constant and $U(1)_{Y'}$ charges of leptons and Higgs fields.

In Fig. 1 we present the dependence of the cross section $\sigma$ on the $r_S$ and $r_Z$ for particular values of the branching ratios $BR(h_2 \to \bar{b}b) \sim 0.8$ and $BR(A^0 \to \bar{b}b) \sim 0.8$. When the scalars are heavy enough the dominant decay mode is $\bar{b}b$ (including the gluonic final states). Under the present lower bounds on the scalar masses (which are mostly model dependent) $\bar{b}b$ dominance is guaranteed so that small $r_S$ portion of Fig. 1 is irrelevant.

In Fig. 1, $r_Z$ is allowed to vary from 0.15 ($\sqrt{s} \sim 240\,GeV$) to 0.5 ($\sqrt{s} \sim 130\,GeV$). At the latter end, cross section is bigger due to the closeness of this end to the $Z_0$ pole. As $s$ grows to larger values the cross section falls gradually and becomes numerically $\sim 0.2\,pb$ around ($\sqrt{s} \sim 240\,GeV$). When $s$ is below the kinematical threshold of $(m_A + m_{h_2})^2$, $Z_i \to A^0h_2$ is forbidden, and we plot in these regions $\sigma = 0$ surface to form a reference level. In an exact treatment of the process, this region would be smaller since the resonance approximation puts the restriction of reality on the scalars whereby narrowing the available phase space. As we observe from the figure for higher values of $s$ the region of 'non-zero' cross section becomes wider, making observability possible. Actually, the phenomenological bounds on the pseudoscalar mass must have been taken into account, however as these bounds might change, we present the plot in entire $h_S$ range to show the variation of cross section with $h_S$ and $s$.  

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Let us note that, in the above discussion, we have concentrated on the coupling of the vector bosons to the CP-even and CP-odd scalars to analyze the scattering process under concern. Although this discussion is sufficient for the aim of the work, one could analyze other modes of Higgs production, such as the Higgs strahlung process \[8\], \(e^+e^- \rightarrow h_jZ_k\) in the model under consideration. To give an idea, we shall list the \(Z_iZ_jh_k\) coupling when \(g_{Y'}Q_S \sim G\) and mixing angle is negligibly small.

\[
K_{h_1Z_1\mu Z_1\nu} = \frac{v}{\sqrt{24}} G^2 g_{j\mu\nu}, \quad K_{h_3Z_1\mu Z_1\nu} = \frac{v}{\sqrt{48}} G^2 g_{j\mu\nu}, \quad K_{h_1Z_2\mu Z_2\nu} = \frac{v}{\sqrt{24}} 3g_{Y'}^2 Q_S^2 g_{j\mu\nu},
\]

\[
K_{h_3Z_2\mu Z_2\nu} = \frac{v}{\sqrt{48}} 3g_{Y'}^2 Q_S^2 g_{j\mu\nu}, \quad K_{h_2Z_1\mu Z_2\nu} = \frac{v}{2} G g_{Y'} Q_S^2 g_{j\mu\nu}
\]

(60)

The remaining couplings are proportional to \(\sin\alpha\), and thus small compared to those in (60), \(g_{Y'}Q_S \sim G\). As a result, coupling of \(Z_iZ_j\) to \(h_2\) is negligibly small compared to others. There is again a profound difference between \(h_2\) and others. Namely, unlike \(h_{1,3}\), \(h_2\) shifts \(Z_1\) to \(Z_2\) and vice versa, which allows one to single out \(h_2\) among others. Higgs search through Higgs-strahlung channel will be discussed elsewhere.

4 Conclusions and Discussions

The smallness of the mixing angle is an essential phenomenological restriction on such gauge extensions of MSSM \[1, 2\]. Furthermore, leptophobility is an indispensable requirement according to LEP results. Under these two requirements, we have analyzed the \(e^+e^- \rightarrow (h_iA) \rightarrow bbbb\) scattering in an Abelian extended SM.

In the large trilinear coupling limit, model yields interesting results in that only next-to-lightest Higgs contribute to the process. The \(Z'\) boson is essentially unobservable as far as the leptonic current is considered. In this sense model is similar to NMSSM \[10\] where one extends the Higgs sector with a singlet. However, determination of the parameters requires a different analysis which is outside the scope of this work.

We have analyzed the present model at the tree-level. At the loop level, behaviour of the potential, and thus, all the physical parameters derived here, including the mixing angle itself, would naturally change \[11\]. Strength of the variations in the potential parameters depends on the \(U(1)_{Y'}\) charges of the
particle spectrum, Yukawa couplings (especially the top Yukawa coupling),
gauge couplings and soft parameters. Here we limited our work to tree level
analysis, leaving the consideration of the radiative corrections to another
work.

In near future, LEP II, LHC or NLC may catch the signals of $Z'$ boson
after which the predictions or the assumptions of this work can be tested
against the experimental results. For example, after a two year run, LEP II
will be able to count up to $\sim 150$ events depending on the lower bound on the
scalar masses.

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References

[1] M. Cvetic, P. Langacker, Mod. Phys. Lett. A 11 (1996) 1247.

[2] M. Cvetic, S. Godfrey, hep-ph/9504216.

[3] ALEPH Collab. D. Buskulic et. al., Z. Phys. C 71 (1996) 197.

[4] M. Carena, G. F. Guidice, S. Lola, C. E. M. Wagner, Phys. Lett. B 395
(1996) 225.

[5] M. Cvetic, D. A. Demir, J. R. Espinosa, L. Everett, P. Langacker, hep-
ph/9703317.

[6] See, for example, V. Barger, K. Cheung, P. Langacker, Phys. Lett. B
381 (1996) 226.

[7] LEP Electroweak Working Group and SLD Heavy Flavour Group,
CERN-PPE/96-183.

[8] See, for example, F. de Campos et. al. Phys. Lett. B 336 (1994) 446.

[9] D. A. Demir, N. K. Pak (work in progress)

[10] See, for example, S. F. King, P. L. White, Phys. Rev. D 52 (1995)
4183.

[11] See, for example, M. Quiros, hep-ph/9703412.
Figure 1: Dependence of $\sigma$ on $r_S$ and $r_Z$