Critical Ising modes in low-dimensional Kondo insulators

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We present an Ising-like intermediate phase for one-dimensional Kondo insulator systems. Resulting from a spinon splitting, its low-energy excitations are critical Ising modes, whereas the triplet sector has a spectral gap. It should occur as long as the RKKY oscillation amplitude dominates over any direct exchange between localized spins. The chiral fixed point, however, becomes unstable in the far Infra-Red limit due to prevalent fluctuations among localized spins which induce gapless triplet excitations in the spectrum. Based on previous numerical results, we obtain a paramagnetic disordered state ruled by the correlation length of the single impurity Kondo model.

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I. INTRODUCTION

The one-dimensional (1D) half-filled Kondo lattice is a simple model for a group of compounds called “the Kondo insulators” [1]. They exhibit the high-temperature behavior of usual Kondo systems, such as the Curie-Weiss-like magnetic susceptibility, but at lower temperatures evolve into a semiconducting phase with small gaps. At low-energy, a Kondo insulator is a typical realization of spin-charge separation. This aspect manifests itself in the difference in size between the charge gap and spin gap.

Since its discovery, the indirect Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction between localized magnetic impurities embedded in a host metal has played an important role in the theory of magnetism. In 1D the RKKY oscillation amplitude displays only a very slow decay $\sim 1/x$. The conduction electrons are subject to the resulting exchange field which oscillates spatially with the Fermi wavelength. It produces Kondo localization. This conclusion was obtained by an exact diagonalization study [2]. The result has been confirmed with the density matrix renormalization group method [3], and later supported by the mapping to a non-linear sigma model [4] and by the bosonization approach [5,6]. The charge gap varies linearly with the Kondo coupling $\xi$. The system should flow to a disordered spin liquid phase with a finite antiferromagnetic (AFM) length scale $\xi_{AFM}$ close to that in the single impurity Kondo model. In the following, we shall introduce the Kondo spin liquid phase as a direct consequence of the chiral fixed point instability in the presence of strong fluctuations.

The starting point is the Hamiltonian:

$$ H = -t \sum_{i,j,\alpha} c_{i,\alpha}^\dagger c_{j,\alpha} + \hbar c. + J_K \sum_{i,\alpha,\beta} \tilde{c}_{i,\alpha}^\dagger \tilde{c}_{i,\beta} \tilde{S}_i (1) $$

The first term represents the electron hopping between nearest-neighbor sites $i$ and $j$. The second term is the Kondo coupling ($J_K \ll t$) between the localized spin $\tilde{S}_i$ and the mobile electron at the same site. Adding a direct exchange $J_H > 0$ between the nearest core spins gives quite different physics. It ensures the presence of local AFM fluctuations leading to topological configurations for the localized spins. If the relation $J_H > J_{RKKY} \sim J_K^2/t$ is satisfied, the spin theory at low temperature is described in terms of an $O(3)$ nonlinear $\sigma$ model where the topological term has no contribution [3,4]. That produces disordered Kondo fluids with quite short AFM correlation lengths $\xi_{AFM} \simeq \exp(\pi \xi)$. Excitations of the $O(3)$ nonlinear $\sigma$ model are S=1 triplets.
In the extreme limit $J_H \gg J_K$, the physics becomes similar to the two-leg spin ladder [1]: spinons confine to form both $S=1$ triplet and singlet excitations with gaps $m_t$ and $m_s$ ($m_t, m_s \propto J_K^2$ and $\xi_{AFM} \sim m_t^{-1}$) [2,3].

Here, we mainly focus on the interesting case $J_H \rightarrow 0$.

II. BASIS OF OUR FORMALISM

The approach followed in this paper is based on bosonization techniques for both charge and spin degrees of freedom of the conduction electrons. For complete reviews, see refs. [4,5].

For $J_K = 0$, the model for the conduction band is gapless and is characterized by the separation of spin and charge. Its low-energy spin properties belong to the same universality class as the Heisenberg model. The low-temperature behavior is then described by the level-1 SU(2) Wess-Zumino-Witten (WZW) conformal field theory (CFT) [6]. The physical particles (or spinons=spin 1/2 excitations) are included through the primary fields $\Phi^{(1/2)}$ and $\Phi^{(1/2)\dagger}$ from the representation of the SU(2) group [7]. The WZW action taking into account the dynamics of the spinon objects is explicitly given by:

$$S_{WZW} = -\frac{1}{16\pi} \int d^2x Tr(\partial_\mu \Phi^{(1/2)\dagger} \partial_\mu \Phi^{(1/2)})$$

and, $A_\mu = \Phi^{(1/2)\dagger} \partial_\mu \Phi^{(1/2)}$. This description, which has its origin in the structure of the Haldane-Shastry spin chain with $1/x^2$ exchange [13], stresses the fact that the fundamental fields in this theory (or spinon fields) may be viewed as free fields apart from purely statistical (in this case: semionic [19]) interactions that may be taken into account by a rule generalizing the Pauli principle. The electronic spin density is represented as $\bar{S}_c(x) = \bar{j}_c(x) + e^{2ik_Fx} \vec{n}_c(x)$, where $\vec{j}_c = \vec{j}_{cR} + \vec{j}_{cL}$ and

$$\vec{n}_c = \frac{1}{2\pi a} Tr\{\tilde{\sigma}(\Phi^{(1/2)} + \Phi^{(1/2)\dagger})\} \cos \sqrt{2\pi} \Phi_c$$

are, respectively, the smooth and staggered parts of the magnetization. The precise relationship between the chiral spin currents and the spinon fields has been discussed in detail in ref. [13]. The lattice step $a$ defines the required short-distance cut-off.

The charge sector is similarly described in terms of a U(1) scalar field $\Phi_c$ leading to a CFT central charge $C=1$ for holons [20] as well. In the charge sector, the free action is given by:

$$S_{g,v_F} = \frac{1}{2} \int dxd\tau \left( \frac{1}{v_F g} (\partial_\mu \Phi_c)^2 - \frac{v_F}{g} (\partial_\mu \Phi_c)^2 \right)$$

Neutral excitations are 1D acoustic plasmons which propagate with velocity $v_F$ and are characterized by the Luttinger parameter $g = v_F/v_p$. For a 1D free electron gas $g$ is equal to one and $v_p = 2t \sin(k_Fa)$ is the velocity for the charge and spin degrees of freedom. In the continuum limit the Kondo interaction takes the form:

$$H_{int} = \lambda_2 (\vec{j}_{cR} + \vec{j}_{cL}) \vec{S}_j$$

$$+ \frac{\lambda_3}{2\pi a} e^{i(2k_F - \pi)x} Tr(\tilde{\sigma} \Phi^{(1/2)}) \cos \sqrt{2\pi} \Phi_c \vec{S}_j + h.c.$$

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where $\lambda_{2,3} \propto J_K$. Note that $\delta = 2k_F - \pi$ measures the deviation from half-filling. Here, we mainly consider the half-filled case $\delta \rightarrow 0$. The configuration of localized spins can be also parameterized as

$$\vec{S}_j = S(\vec{a} \vec{L}_j + (-1)^x [1 - (aL_j)^2]^{1/2} \vec{n}_j)$$

As already mentioned in the introduction, the RKKY interaction between local moments should play a crucial role, for $J_K \ll t$.

III. RKKY INTERACTION AS A STAGGERING FIELD

The standard treatment of the RKKY interaction corresponds to a calculation of the correlation function

$$C(x) = \frac{\cos(2k_F x)}{2\pi a} \langle S_0^z \text{Tr} \sigma^z \Phi^{(1/2)}(x) \cos \sqrt{2\pi} \Phi_c(x) \rangle$$

The angular brackets indicate a thermal average. This function describes the spatial correlation of the electron-spin density with the impurity spin on $j = 0$. Another impurity spin located at $j = x$ would see this correlation, and lowest-order perturbation theory in $\lambda_3$ constitutes an exact derivation of the RKKY law. For $x < \xi_T = v_F/k_BT$, this yields:

$$H_{RKKY} = \lambda_{RKKY} \sum_{(0, x)} \vec{S}_0 \vec{S}_x$$

with

$$\lambda_{RKKY} = \frac{\lambda_3}{2\pi a} C(x) = -\frac{\lambda_3^2}{2\pi v_F} \frac{\cos(2k_F x)}{x^3} a^{-1}$$

In the range of temperature $x < \xi_T$, the RKKY oscillation amplitude displays only a very slow algebraic decay. In the noninteracting case $g = 1$, the usual $x^{-1}$ decay is recovered. The RKKY law, prominent at high temperatures, should then favor the following configuration for the localized spins: $\vec{n}_j = \frac{1}{a} \vec{n}$ where $\vec{n}$ is a fixed unit vector and $1 - (aL_j)^2 \sim 1$. Using the Hamiltonian [6], we
find that it generates a static staggering magnetic field \( h_s = S\lambda_3 \) on conduction electrons:

\[
\mathcal{H}_{int} = h_s \vec{n}_c \cdot \vec{n}
\]

\[
= \frac{h_s}{2\pi a^2} \cos \sqrt{2\pi \Phi} \text{Tr}(\hat{\sigma} \Phi^{1/2}) \cdot \vec{n} + h.c.
\]

At high temperatures, the conduction electrons are subject to a perfectly static potential as if there was a finite staggered spin moment.

Now, we expand the partition function \( Z = Z_0 + \delta Z[h_s \neq 0] \) to the second order in \( h_s \). We find that \( \delta Z/Z_0 \) is equal to:

\[
\left( \frac{h_s}{2\pi} \right)^2 \int d\tau_1 d\tau_2 dx_1 dx_2 \frac{\left( (x_1 - x_2)^2 + (\tau_1 - \tau_2)^2 \right)}{a^4} \frac{\sqrt{\pi}}{2\pi \Phi}
\]

\[
\cdot \sqrt{2\pi \Phi} \text{Tr}(\hat{\sigma} \Phi^{1/2}) \cdot \vec{n} + h.c.
\]

\[
\delta Z/Z_0 = \left( \frac{h_s}{2\pi} \right)^2 \cdot \sqrt{2\pi \Phi} \text{Tr}(\hat{\sigma} \Phi^{1/2}) \cdot \vec{n} + h.c.
\]

\[
Z_0 \text{ is the partition function of the free system with } J_K = 0. \text{ To make the (total) partition function invariant under the cut-off transformation } a \rightarrow a' = ae^{d\ln L}, \text{ } h_s \text{ has to obey:}
\]

\[
\frac{dh_s}{d\ln L} = (2 - \frac{1}{2} - g) h_s
\]

Note that \( Z_0 \) is not affected by such a rescaling. Starting with a free electron gas (\( g \rightarrow 1 \)), we confirm that it produces the localization of conduction electrons at an energy scale \( \Delta_c \propto h_s = J_K/2 \) [14], defined such as \( h_s(\Delta_c) \sim 1 \). It should be noted that spin flip events do not contribute to the perturbative result [14], to this order. Of course, the exponent \( g \) is also affected by the Kondo interaction. We find:

\[
\frac{dg}{d\ln L} = -2\pi g^2 \lambda_3^2 J_o(\delta(L)a)
\]

\( J_o \) is the Bessel function. Since we restrict our arguments to the case of half-filling, we put \( \delta = 0 \) and \( J_o(0) = 1 \). Correlations of \( \cos \sqrt{2\pi \Phi} \) show a long-range order at zero temperature because the renormalized exponent \( g^* \) goes to zero. We can average \( \langle \cos \sqrt{2\pi \Phi} \rangle \sim \frac{1}{\sqrt{\Delta_c}} \).

The charge motion is frozen and the leading order parameter in the charge sector is the so-called 4\( k_F \) charge density wave (CDW): \( \rho_{4k_F} \propto \cos \sqrt{8\pi \Phi} \langle x \rangle \). As for the occurrence of the Mott-Hubbard gap due to Umklapps [14], the (Mott)-Kondo insulating state occurs due to the pinning of the 4\( k_F \) CDW [22]:

\[
\langle \rho_{4k_F} \rangle \propto x^{-4g^*} \sim \text{constant}
\]

The charge field, on the other hand, could also be affected by the disorder. As shown in ref. [1], a strongly disordered 1D Kondo array will crossover to an Anderson localization state. Thus, we now study spin properties of such an insulating (Mott)-Kondo state with very small randomness.

Since \( \lambda_3 \) violates the separation of charge and spin one can expect massive triplet modes with a spectral gap \( J_K S \). At quite short distance, one can treat \( \vec{n} \) as a constant vector. Writing \( \Theta = i\vec{\sigma} \cdot \vec{n} \), the result is a uniaxial Kondo effect, leading to an Ising-like solution \( \Phi^{1/2} = \Phi^{(cl)} \). Using properties of Pauli matrices [10,25]:

\[
\text{Tr}[\hat{\sigma} \Phi^{(cl)}] \text{Tr}[\hat{\Theta}] + h.c. = \text{Tr}[\Phi^{(1)}], \quad \Phi^{(1)} = \Phi^{(cl)} \Theta
\]

we check that spins of conduction and localized electrons confine to form triplets (or magnons represented by the ‘composite’ field \( \Phi^{(1)} \)) with a mass \( m_t = \Delta_c \). Such a classical solution is ruled by:

\[
\vec{n}_c = \text{Tr}[\hat{\sigma} \Phi^{(cl)}] = -\text{Tr}[\hat{\Theta}] = -\vec{n}
\]

When \( x \simeq L_{loc} = (J_K S)^{-1} \), the so-called spin density glass state aims to arise [3]: conduction electrons feel a staggered potential within this scale and open a quasiparticle gap due to Bragg scattering. However, starting with an isotropic system, it cannot be longer stabilized for temperatures much lower than \( \Delta_c \). Indeed, from Eq. (10), we find the following recursion law for the RKKY exchange in the delocalized phase:

\[
\frac{d\lambda_{RKKY}}{d\ln L} = \frac{1}{2\pi v_F} \lambda_3^2
\]

It cannot provide an energy scale more relevant than \( \Delta_c \) since it involves processes in \( \lambda_3^2 \) (the electron gas localization occurs due to scattering events \( \propto \lambda_3 \)). Then, the conduction electrons effectively ‘screen away’ the internal field before a true magnetic transition (with breaking SU(2) symmetry) can occur.

**IV. SU(2) SYMMETRY NOT BROKEN: CONSEQUENCES**

The uniaxial solution \( \Theta(x, \tau) = \Theta \) and \( \Phi^{1/2} = \Phi^{(cl)} \) is not available at long distances: there are certainly states in the gap.

For \( J_H \gg J_{RKKY} \), it is important to include a topological term (for the local moments) first derived by Haldane [8]:

\[
S_{top} = \frac{i}{2\pi} \int dx d\tau \epsilon_{\mu\nu} (\vec{n} \cdot [\delta_\mu \vec{n} \times \partial_\nu \vec{n}])
\]

Then, the model becomes solvable in the semi-classical limit (‘large-S expansion’), by including the Berry phase and integrating out both conduction electron variables and fast modes \( L \). The result is an 0(3) nonlinear \( \sigma \) model built out the field \( \vec{n}(x, \tau) \) (with triplet excitations). The

*To simplify the expression, we have taken \( v_p \sim v_F = 1 \).
resulting topological term has no contribution and then the gapless ordered state of the isotropic sigma model is marginally unstable (due to the implicit breaking of conformal invariance) and it opens a gap (‘the so-called Haldane gap’) \[ \Delta \approx 0 \].

\[ m \sim J K S \exp -\pi S \] (20)

which is not so far from \( \Delta \). The spin gap \( m \) has a topological origin, and excitations in the interval between \( m \) and \( J K S \) are known to be massive spin polarons formed due to an interaction between electrons and kinks of the unit vector \( \vec{n}(x, \tau) \) [\(^8\)]. To my knowledge, such Kondo insulator (ruled by two different energy scales in charge and spin sectors) was the first realization of so-called spin-charge separation. On the other hand, a direct exchange between localized moments is crucial for the occurrence of massive spin polarons [\(^7\)] and such a topological description is not expected to remain available when \( J_H = 0 \). Numerical results \[ [4, 9] \] predict in contrast a pure Kondo ground state with a very large AFM length \( \xi_{AFM} \) driven by spin flip processes or the Kondo term \( \lambda_2 \), which has not been taken into account in the semi-classical approach of ref. \[ [3, 4] \]. To obtain precisely low-energy spin excitations in this limit we adopt the following scheme.

For \( J_H = 0 \), there is no mathematical justification to include a topological term of the form of Eq. (20) for local moments since the model is not integrable. Then we do not introduce it. In consequence, we start with a spin array which is described by usual spin waves with spin \( \Delta S^2 = \pm 1 \). Since a true magnetic transition does not arise, ferromagnetic local fluctuations may occur in the far IR limit leading to free triplet particles in the spectrum. The forward Kondo scattering term should then play a crucial role at very long distances. The effect of fast variables \( L_j \) will be considered later when \( x \sim \xi_{AFM} \) (i.e. when the presumed quasi long-range order is completely destroyed). Using the so-called operator product expansions of ref. [21], we can check that \( \lambda_3 \) is not renormalized by a term like \( g^*\lambda_3^2/2\pi u_F \) due to the insulating nature of the ground sate \( (g^* \rightarrow 0) \). Fermionic fluctuations are well decoupled if we neglect the Berry phase (which is only reponsible for the correct quantization of local spins).

Before, let us show that spinons in the electron gas cannot \textit{vanish} (confine) completely when the spin array develops only a \textit{quasi-order} (with isotropic correlation functions): the spin density glass state becomes less stable and critical Ising modes can survive in the spectrum.

**A. Refermionization and critical Ising modes**

Let us start with a spin array which satisfies the following quasi-order condition:

\[ \langle n^z(x)n^z(0) \rangle = \langle n^+(x)n^-(0) \rangle \propto x^{-2\alpha} \] (21)

It means that the local magnetization operator \( n^i(x, \tau) \) with \( i = x, y, z \) has the scaling dimension \( \alpha \). Since the RKKY interaction between local moments yields only a very slow algebraic decay in 1D, we deduce: \( \alpha \ll 1/2 \), \( 1/2 \) being the scaling dimension of the staggered magnetization operator in the Heisenberg chain with nearest neighbor exchange [\(^{13}\)]. To solve the problem at intermediate distances (when \( L_{loc} \ll x \ll \xi_{AFM} \)), we assume that the parameter \( \alpha \) is too small that we can replace in first approximation local moment operators by effective expectation values \( \langle n^i(x, \tau) \rangle \approx \langle n^+(x, \tau) \rangle = \gamma \neq 0 \), obtained by averaging in time and space slow fluctuations in the spin array. The electron gas is submitted to an SU(2)-invariant \textit{quasi-static} staggering potential. Then, it is not difficult to integrate out local spin degrees of freedom. To treat correctly the electron gas, it is now convenient to use the Abelian representation [\(^4\)].

\[ \tilde{n}_c \sim (\cos 2\pi \Theta_s, -\sin 2\pi \Theta_s, \sin 2\pi \Phi_s) \] (22)

where \( \Theta_s \) is the field dual to \( \Phi_s \). Then, at long distances, the term \( \lambda_3 \) can be symmetrized as [\(^4\)]:

\[ \mathcal{H}_{int} \sim \frac{1}{2\pi a} \Delta^3 c^3 2\pi (\sin 2\pi \Phi_s(x) + \sin 2\pi \Theta_s(x)) \] (23)

We have replaced the charge operator by its expectation value and used the definition of \( \Delta = J K S \). In view to respect conventional notations we have changed \( \Phi_s \rightarrow \Phi_s + \sqrt{4}/4 \) and \( \Theta_s \rightarrow \Theta_s + \sqrt{4}/4 \) in the second equation. In ref. [\(^4\)], we did not solve the Hamiltonian in the isotropic case. To solve it, we require an effective fermionic theory similar to Hubbard [\(^2\)] or spin ladder [\(^{23}\)] models, and carbon nanotube problems [\(^{23}\)]. The refermionization technique is defined, as follows.

Let us define new effective fermion operators for the spin channel. Right- and left-moving components \( \psi_R, \psi_L \) can be written in terms of the bosonic phase field,

\[ \psi_{sq}(x) = \frac{\eta_q}{\sqrt{2\pi a}} \exp \{-i\sqrt{\pi}(q\Theta_s + \Phi_s)(x)\} \] (24)

Then we have

\[ \frac{1}{\pi a} \cos 4\pi \Phi_s = \eta R \eta_L (\psi_{sL}^\dagger \psi_{sL} - \psi_{sR}^\dagger \psi_{sR}) \] (25)

\[ \frac{1}{\pi a} \cos 4\pi \Theta_s = -\eta R \eta_L (\psi_{sL}^\dagger \psi_{sL} - \psi_{sR}^\dagger \psi_{sR}) \]

Klein factors have been chosen as \( \eta R \eta_L = i \). Apart from the usual mass bilinear term (which favors the pinning of the spin density wave towards the easy-axis), the Hamiltonian \( \mathcal{H}_{int} \) also contains a ‘Cooper-pairing’ term originating from the cosine of the dual field, which guarantees the presence of quantum fluctuations at zero temperature. Now, we introduce two Majorana fields...
and by using \((25)\) we find \((25)\) \[ \cos \frac{\sqrt{4\pi} \Phi_s}{4\pi a} = \frac{1}{2}(\xi s - \xi s^*), \] \[ \sin \frac{\sqrt{4\pi} \Theta_s}{4\pi a} = \frac{1}{2}(\xi s^* - \xi s). \] (27)

Refermionization of the spin sector then yields

\[ \mathcal{H}(s) = \frac{-i v_F}{2} \sum_{j=1}^{2} \int dx \left( \xi_j R \partial_x \xi_j R - \xi_j L \partial_x \xi_j L \right) \]

\[ + i \Delta_c \int dx \xi_2R \xi_2L \] (28)

Of course, for \( m_2 = -\Delta \), the model flows to strong couplings rendering the Majorana field \( \xi_2 \) massive. The Majorana fermion \( \xi_1 \) remains massless.

The Hamiltonian \( \mathcal{H}(s) \) shows explicitly that the bosonic mode \( \Phi_s \) (or \( \Phi^{(1/2)} \)) in the non-Abelian language) decouples into two modes of real (Majorana) fermions (or half-spinons) having different spectra. We have a splitting of the spinon field and it should lead to \( \langle \text{Tr} \sigma^\Phi(1/2) \rangle \neq 0 \). The spin-singlet real fermions \( \xi_1 \) remain free. Let us stress that it contributes to a new chiral fixed point. The specific heat is still linear at low temperatures, \( T \ll \Delta_c \), and comes from gapless spin excitations. Using the general formula \( C_V = \beta CT/3v^2 \) \([27]\), we find \( C_V = \pi T/6v_F \). A free Hamiltonian of Majorana fermions is ruled by a central charge \( C = 1/2 \) \([10]\).

### B. Remnant of electronic spin fluctuations

To compute spin correlation functions, it is accurate to exploit the well-known correspondence between the 2D Ising model and 1D Majorana fermions. Both are described by \( C = 1/2 \). We can define two decoupled Ising models as follows \([23]\):

\[ \cos \frac{\sqrt{4\pi} \Phi_s}{4\pi a} = \mu_1 \mu_2, \quad \sin \frac{\sqrt{4\pi} \Phi_s}{4\pi a} = \sigma_1 \sigma_2 \] \[ \cos \frac{\sqrt{4\pi} \Theta_s}{4\pi a} = \sigma_1 \mu_2, \quad \sin \frac{\sqrt{4\pi} \Theta_s}{4\pi a} = \mu_1 \sigma_2 \] (29)

Bosonic exponents are expressed in terms of the order \( (\sigma) \) and disorder \( (\mu) \) parameters of two Ising models. On the other hand, a theory of a massive Majorana fermion field describes long-distance properties of the two-dimensional Ising model, the fermionic mass being proportional to \( m \sim (T_0 - T)/T_0 \). We conclude that \( \mathcal{H}(s) \) is equivalent to two decoupled 2D Ising models. The Ising model \( (\sigma_1, \mu_1) \) related to \( \xi_1 \) will be critical \( (T = T_c) \), while the Ising model \( (\sigma_2, \mu_2) \) related to \( \xi_2 \) will be under criticality (since \( m_2 = -\Delta_c \ll 0 \) we have \( T < T_c \)). To perform the complete correspondence, using Eq. \((29)\) one obtains: \( \xi_1 \sim \cos \sqrt{\pi} (\Phi_s + \Theta_s) \sim \sigma_1 \mu_1 \) and \( \xi_2 \sim \sin \sqrt{\pi} (\Phi_s + \Theta_s) \sim \sigma_2 \mu_2 \).

Since the second Ising model is under criticality, the average of the disorder operator is zero, \( \langle \mu_2 \rangle = 0 \). The order operator \( (\sigma_2) \) has then a finite value, and correlation functions of \( \cos \sqrt{\pi} \Phi_s \) and \( \cos \sqrt{\pi} \Theta_s \) decay exponentially. From the exact solution of the 2D Ising model one immediately obtains \([23]\):

\[ \langle \sin \sqrt{\pi} \Phi_s(x) \sin \sqrt{\pi} \Theta_s(x') \rangle \sim |x - x'|^{-1/4} \] (30)

with the same result for the \( \sin \sqrt{\pi} \Theta_s \) operator. The above proves that the fields \( \Phi_s \) and \( \Theta_s \) are not pinned: values of \( \sin \sqrt{\pi} \Phi_s \) and \( \sin \sqrt{\pi} \Theta_s \) are not fixed to 1. By virtue of the Heisenberg uncertainty relation, we have simply shown that it is impossible to completely pin a self-dual field. Note that for \( \lambda_3 = 0 \), scaling dimensions of all these bosonic operators is \( 1/4 \). For \( \lambda_3 \neq 0 \), it rescales \( 1/4 \rightarrow 1/8 \). More generally, the operators \( \sin \sqrt{2\pi} \Phi_s \) and \( \sin \sqrt{2\pi} \Theta_s \) acquire a ‘halved’ scaling dimension: \( \eta = 3^2/8\pi \), with \( \beta = \sqrt{2\pi} \) (in respect to the ground state \( \cos \sqrt{2\pi} \Theta_s \) (\( \cos \sqrt{2\pi} \Phi_s \) tends to zero).

The uniform part of the correlation functions decays exponentially. The nature of this new chiral fixed point mainly manifests itself in staggered fusion rules between spins of conduction electrons:

\[ \text{Tr} \sigma^a \Phi^{(1/2)}(x) \text{Tr} \sigma^b \Phi^{(1/2)}(0) \sim \frac{\delta_{ab}}{z^{1/2}} + e^{abc} \zeta z^{1/2} J_c (31) \]

with \( x = (x, \tau), 0 = (0, 0) \) and \( z, \zeta = x \pm iv_F \). The occurrence of the factor \( (z/\zeta)^{1/2} \) is usual and confirms that braiding properties of spinons are those of semions \([9]\).

On the other hand, we emphasize that the halved scaled exponent \( 1/2 \) in the first term characterizes a new universality class for the \( S=1/2 \) Heisenberg chain which is finally equivalent to a \( C=1/2 \) critical theory. In this new chiral fixed point half of the spinon field still fluctuates. Regarding simply a spinon (spin \( 1/2 \) topological object) as a domain wall \( \cdots \uparrow \downarrow \downarrow \uparrow \downarrow \cdots \), the fact that the electronic \( \xi_2 \) modes acquire a mass (in our language) means physically that the other half participates in bound states with local moments. Taking into account local moment degrees of freedom, observable massive excitations are triplets only. Then, we have:

\[ \Phi^{(1)} + \xi_1 = \Phi^{(1/2)} \Theta(x, \tau) \] (32)

\( \Phi^{(1)} \) being defined in Eq.\((16)\). This is the main result of the present work \([9]\).
It can lead to interesting experimental predictions. A related quantity of direct experimental relevance is the NMR relaxation rate $T_1$. For large $J_K$, the RKKY regime fails ($C(x) \to 0$) and the main contribution comes from the staggered spin-spin correlation functions in the electron gas. The corresponding susceptibility has the temperature dependence

$$\chi(2k_F) \sim T^{2\eta-2} \text{ with } 2\eta = 1/2 \quad (33)$$

Then, we predict an NMR relaxation rate, $1/T_1 \propto T^{-1/2}$. This $T$-dependence is quite unusual because the underlying magnetic order between electrons changes the effective field seen by the nuclei. But a related behavior has also been reported for the so-called dimerized, frustrated ladder models when the magnetic field reaches the critical field $H_c$ (typically the spin gap) [30]. The resulting $T_1$ is different from that in the single impurity case. There, Friedel oscillations in the unitary limit lead to an NMR rate which in contrast increases with the distance $31,32$.

V. FIXED POINT AND CONCLUSION

Now, let us turn to the stability of the chiral phase when one moves away from $(aL)^2 = 0$ in the far IR limit, where strong fluctuations are permitted. There, $1 - (aL)^2 \sim \gamma^2 \to 0$: the $\lambda_3$ Kondo exchange becomes $2k_F$ oscillating, leading to free interchain S=1 spins in the spectrum ($m_3^2 \to 0$). However, the charge sector should not be affected because the localization length is sufficiently large, $\xi_{loc} \gg a$. Integrating out the remnant of massive $\xi_2$ modes, the leading far IR behavior of the $q=0$ electronic spin component coincides again with the one in the effective S=1/2 spin chain. At small enough $\lambda_2$, the forward Kondo exchange can be viewed as a weak perturbation to the chiral fixed point. Starting from order $\lambda_2^2$ on, spin flips contribute:

$$\frac{d\lambda_2}{d\ln L} = \frac{1}{2\pi v_F} \lambda_2^2 \quad (34)$$

The system flows to a purely Kondo state at an energy scale:

$$m \sim J_K S \exp -2\pi v_F / \lambda_2 \ll \Delta_c \quad (35)$$

typically the single-site Kondo temperature. Both $\xi_1$ and $\xi_2$ acquire a mass. We do not predict a particular lattice enhancement effect [4]. A similar conclusion has been obtained in ref. [23] (but there is an important mistake in the recursion law for the Kondo coupling $\lambda_3$). All the spin correlation functions now decay exponentially. Such a Kondo liquid is another theoretical example of spin-charge separation (occurring in the 1D Kondo lattice model) in agreement with experiments on three dimensional Kondo insulators [1] and previous numerical results [6]. This can be shown by the absence of a field-induced metal-insulator transition [33]. The system remains insulating even when a finite magnetization is induced by the external field because $m \ll \Delta_c$. The most typical case is the large-U limit of the 1D Hubbard model. The charge gap is of order U while magnetic excitations are gapless. During the magnetization process (occurring for a small field $\sim t^2 / U$), it does not change.

In summary, there exists an intermediate but still low-energy region where the chiral fixed point presented here is dominant. It brings new physics in Kondo ladder systems. Indeed, a spin liquid with both massive triplet and critical Ising modes takes place. It should be noted that similar spin spectra have been occurred in other coupled spin chain systems, like the so-called Kagomé lattice antiferromagnet stripped to its basis [8] or the two-leg spin ladder with a dimer four-spin interaction [34]. On the other hand, strong fluctuations among localized spins in the far IR produce gapless triplet excitations so that the chiral phase becomes unstable. Driven by the dynamics of localized spins, the system flows to a typical Kondo fluid. Based on previous numerical results, the gap value is similar to the single-impurity Kondo temperature.

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Let us define the holon as the excitation associated with adding or removing a charge $\pm e$ in the system.

We have not included the Umklapp operator because it is ‘only’ marginal. The Kondo term $\lambda_3$ is more relevant and produces the localization of the electron gas at a higher energy scale.

When $\vec{n}$ is assumed to be a constant vector, the same study implies that both $\xi_1$ and $\xi_2$ become massive. The $S=1/2$ classical magnetization of the electron gas has been completely pinned by the perfect uniaxial staggering potential created by the spin array and physical excitations are massive triplets only [Eq.(16)].