THE DUALIZED STANDARD MODEL AND ITS APPLICATIONS

CHAN Hong-Mo
Rutherford Appleton Laboratory, Chilton, Didcot, Oxon, OX11 0QX, United Kingdom
E-mail: chanhm@v2.rl.ac.uk

José BORDES
Dept. Fisica Teorica, Univ. de Valencia, c. Dr. Moliner 50, E-46100 Burjassot (Valencia), Spain
E-mail: jose.M.bordes@uv.es

TSOU Sheung Tsun
Mathematical Institute, University of Oxford, 24-29 St. Giles’, Oxford, OX1 3LB, United Kingdom
E-mail: tsou@maths.ox.ac.uk

The Dualized Standard Model offers a natural explanation for Higgs fields and 3 generations of fermions plus a perturbative method for calculating SM parameters. By adjusting only 3 parameters, 14 quark and lepton masses and mixing parameters (including \( \nu \) oscillations) are calculated with general good success. Further predictions are obtained in post-GZK air showers and FCNC decays.

In this article, we summarize some work which has occupied us for some years. The material has been summarized in 5 papers submitted to this conference (Papers 607, 610, 611, 613, 636), and this talk is but a summary of these summaries.

The Dualized Standard Model is a scheme which aims to answer some of the questions left open by the Standard Model (such as why Higgs fields or fermion generations should exist) and to explain the values of some of the Standard Model’s many parameters (such as fermion masses and mixing angles). In contrast to most schemes with similar aims, the DSM remains entirely within the Standard Model framework, introducing neither supersymmetry nor higher space-time dimensions. That it is able to derive results beyond the Standard Model while remaining within its framework is by exploiting a generalization of electric-magnetic duality to nonabelian Yang-Mills theory found a couple of years ago.

The concept of duality is best explained by recalling the well-known example in electromagnetism. There a dual transform (the Hodge star) is defined: \( *F_{\mu\nu} = -(1/2)\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma} \), where for both the Maxwell field \( F_{\mu\nu} \) and its dual \( *F_{\mu\nu} \), potentials \( A_\mu \) and \( \tilde{A}_\mu \) exist, so that the theory is invariant under both \( A_\mu \) and \( \tilde{A}_\mu \) gauge transformations. The theory has thus in all a \( U(1) \times \tilde{U}(1) \) gauge symmetry with \( U(1) \) corresponding to electricity and \( \tilde{U}(1) \) to magnetism. Magnetic charges are monopoles in \( U(1) \), while electric charges are monopoles of \( \tilde{U}(1) \).

The same statements do not hold for nonabelian Yang-Mills theory under the dual transform (star). However, it was shown that there exists a generalized dual transform for which similar results apply. Its exact form, in the language of loop space, need not here bother us. What matters, however, is that given this generalized transform, a potential can again be defined for both the Yang-Mills field and its dual, and that the theory is invariant under both the gauge transformations:

\[
A_\mu \rightarrow A_\mu + \partial_\mu \Lambda + ig[\Lambda, A_\mu], \quad \tilde{A}_\mu \rightarrow \tilde{A}_\mu + \partial_\mu \tilde{\Lambda} + i\tilde{g}[\tilde{\Lambda}, \tilde{A}_\mu],
\]

with \( g \) satisfying the (generalized) Dirac quantization condition \( g\tilde{g} = 4\pi \). As a result, the theory has in all the gauge symmetry \( SU(N) \times \tilde{SU}(N) \) with \( SU(N) \) corresponding to (electric) ‘colour’ and \( \tilde{SU}(N) \) to (magnetic) ‘dual colour’. And again, dual colour charges are monopoles of \( SU(N) \) while colour charges appear as monopoles in \( \tilde{SU}(N) \).

Applied to colour in the Standard Model, this nonabelian duality gives two new interesting features. First, dual to the \( SU(3) \) symmetry of colour, the theory possesses also an \( SU(3) \) symmetry of dual colour. Then, by a well-known result of ‘t Hooft, since colour is confined, it follows that this \( SU(3) \) of dual colour has to be broken via a Higgs mechanism. Hence, the theory already contains within itself a broken 3-fold gauge symmetry which could play the role of the ‘horizontal’ symmetry wanted to explain the existence in nature of the 3 fermion generations. Second, in the generalized dual transform, the frame vectors ('dreibeins') in the gauge group take on a dynamical role which suggests that they be promoted to physical fields. If so, then they possess the properties

*It has been shown that the duality introduced indeed satisfies the commutation relations of the order-disorder parameters used by ‘t Hooft to define his duality.
that one wants for Higgs fields for symmetry breaking (as in electroweak theory): space-time scalars belonging to the fundamental representation having classical values (v.e.’s) with finite lengths.

The basis of the Dualized Standard Model is just in making this bold assumption of identifying the dual colour SU(3) as the ‘horizontal’ generation symmetry and of the frame vectors in it as the Higgs fields for its breaking. We note that according to nonabelian duality the niches already exist in the original theory in the form of the dual symmetry and the ‘dreibeins’. One could thus claim that the DSM offers an explanation for the existence both of exactly 3 fermion generations and of Higgs fields necessary for breaking this generation symmetry.

This identification further suggests the manner in which the symmetry ought to be broken. As a result, the fermion mass matrix at tree-level takes the form

$$ m = m_T \begin{pmatrix} x \\ y \\ z \end{pmatrix} (x, y, z), $$

where $m_T$ is a normalization factor depending on the fermion-type $T$, namely whether $U$- or $D$-type quarks, charged leptons ($L$) or neutrinos ($N$), and $x, y, z$ are vacuum expectation values of Higgs fields (normalized for convenience: $x^2 + y^2 + z^2 = 1$), which are independent of the fermion-type $T$. Because $m$ is factorizable it has only one nonzero eigenvalue so that at tree-level there is only one massive generation (fermion mass hierarchy). Further, because $(x, y, z)$ is independent of the fermion-type, the state vectors of, say, the $U$- and $D$-type quarks in generation space have the same orientation, so that the CKM matrix is the unit matrix. These are already not a bad first approximation to the experimental situation.

One can go further, however. With loop corrections, it is seen that the mass matrix $m'$ remains factorizable, with the same form as (1), but the vector $(x', y', z')$, in which the relevant information of $m'$ is encoded, now rotates with the energy scale, tracing out a trajectory on the unit sphere. Hence, the lower generation fermions acquire small finite masses via ‘leakage’ from the highest generation. Furthermore, the vector $(x', y', z')$ depends now on the fermion-type, giving rise to a nontrivial CKM matrix. The result is a perturbative method for calculating fermion mass and mixing parameters.

In a 1-loop calculation it is found that out of the many diagrams only the Higgs loop diagram dominates, involving thus only a few adjustable parameters. The present score is as follows. By adjusting 3 parameters, namely a Yukawa coupling strength $\rho$ and the 2 ratios between the Higgs v.e.’s $x, y, z$, one has calculated the following 14 of the ‘fundamental’ SM parameters:

- the 3 parameters of the quark CKM matrix $|U_{rs}|$,
- $m_u, m_c, m_t$, $m_d, m_s, m_b$, $m_e, m_\mu, m_\tau$,
- the masses $m_{\nu_s}$ of the lightest and $B$ of the right-handed neutrinos,

there being no $CP$-violation at 1-loop order.

First, for the quark CKM matrix $|V_{rs}|$, where $r = u, c, t$ and $s = d, s, b$, one obtains for a sample fit

$$ |V_{rs}| = \begin{pmatrix} 0.9752 & 0.2215 & 0.0048 \\ 0.2210 & 0.9744 & 0.0401 \\ 0.0136 & 0.0381 & 0.9992 \end{pmatrix}, $$

as compared with the experimental values

$$ \begin{pmatrix} 0.9745 & 0.9760 & 0.217 - 0.224 & 0.0018 - 0.0045 \\ 0.217 - 0.224 & 0.9737 - 0.9753 & 0.036 - 0.042 \\ 0.004 - 0.013 & 0.035 - 0.042 & 0.9991 - 0.9994 \end{pmatrix}. $$

All the calculated values are seen to lie roughly within the experimental bounds.

Second, for the lepton CKM matrix $|U_{rs}|$, one obtains with the same 3 input parameters:

$$ |U_{rs}| = \begin{pmatrix} 0.97 & 0.24 & 0.07 \\ 0.22 & 0.71 & 0.66 \\ 0.11 & 0.66 & 0.74 \end{pmatrix}, $$

where $r = e, \mu, \tau$ and $s = 1, 2, 3$ label the physical states of the neutrinos. The empirical values of $|U_{rs}|$ for leptons are much less well-known. Collecting all the information so far available from neutrino oscillation experiments, one arrives at the following tentative arrangement:

$$ |U_{rs}| = \begin{pmatrix} \ast & 0.4 - 0.7 & 0.0 - 0.15 \\ \ast & \ast & 0.45 - 0.85 \\ \ast & \ast & \ast \end{pmatrix}. $$

which is seen to be in very good agreement with the prediction for $U_{e3}$ and $U_{\mu 3}$, but not for $U_{e2}$.

Lastly, from the same calculation with the same 3 parameters, one obtains the fermion masses listed in Table 1: The second generation masses agree very well with experiment. Those of the lowest generation were obtained

| Calculation | Experiment |
|-------------|-------------|
| $m_c$ | 1.327 GeV | 1.0 - 1.6 GeV |
| $m_s$ | 0.173 GeV | 100 - 300 MeV |
| $m_\mu$ | 0.106 GeV | 105.7 MeV |
| $m_t$ | 235 MeV | 2 - 8 MeV |
| $m_d$ | 17 MeV | 5 - 15 MeV |
| $m_s$ | 7 MeV | 0.511 MeV |
| $m_{\nu_e}$ | $10^{-2}$ eV | < 10 eV |
| $B$ | 400 TeV | ? |
by extrapolation on a logarithmic scale and should be regarded as reasonable if of roughly the right magnitude. As for the 2 neutrino masses, the experimental bounds are so weak that there is essentially no check.

In summary, out of the 14 quantities calculated, 8 are good (\(|V_{rs}|, |U_{\mu 3}|, |U_{e 3}|, m_c, m_s, m_\mu\)), 2 are reasonable (\(m_d, m_e\)), 2 are unsatisfactory (\(|U_{e 2}|, m_\alpha\)), and 2 are untested, which is not a bad score for a first-order calculation with only 3 parameters.

One interesting feature for the calculation outlined above is that the trajectories traced out by the vector \((x’, y’, z’)\) for the 4 different fermion-types \(U, D, L, N\) all coincide to a very good approximation, only with the 12 physical fermion states at different locations (Figure 1). The points \((1, 0, 0)\) and \(\frac{1}{\sqrt{3}}(1, 1, 1)\) are fixed points so that the rate of flow is slower near the ends of the trajectory than in the middle. For this reason, the states \(t\) and \(b\) are close together in spite of their big mass difference. This observation will be of relevance later.

Since neutrino oscillations\(^\text{12}\) are of particular interest at this conference, let us take a closer look\(^\text{13}\). The element \(U_{\mu 3}\) of the lepton CKM matrix giving the mixing between the muon neutrino \(\nu_\mu\) and the heaviest neutrino \(\nu_3\) is constrained mainly by the data on atmospheric neutrinos. From the old Kamiokande data,\(^\text{14}\) an analysis by Giunti et al\(^\text{15}\) gives the bounds on \(U_{\mu 3}\) shown in Figure 2. In the DSM scheme, with parameters already fixed by the fit to the quark sector,\(^\text{16}\) the elements of \(|U_{rs}|\) are calculable given the masses of \(\nu_3\) and \(\nu_2\). Then, with \(m_{\nu 2}\) taken in the range \(10^{-11}\text{eV}^2 < m_{\nu 2}^2 < 10^{-10}\text{eV}^2\) as suggested by the Long Wave-Length Oscillation (LWO) (or the ‘vacuum’ or ‘just-so’) solution to the solar neutrino problem\(^\text{17}\),\(^\text{18}\), the predicted band of values of \(|U_{\mu 3}|\) for a range of input values of \(m_{\nu 3}\) is shown in Figure 3 passing right through the middle of the allowed region. No similar detailed analysis of the new SuperKamiokande data\(^\text{18}\) has yet been performed, but the predicted band can be seen to remain well within the allowed region: \(.53 < U_{\mu 3} < .85, 5 \times 10^{-4} < m_{\nu 3}^2 < 6 \times 10^{-3}\text{eV}^2\).

The same calculation gives the prediction shown in Figure 3 for the element \(U_{e 3}\) representing the mixing between the electron neutrino \(\nu_e\) and \(\nu_3\), which is constrained mainly by the reactor data from CHOOZ\(^\text{19}\) and Bugey\(^\text{20}\). If \(m_{\nu 3}^2\) is higher than \(2 \times 10^{-3}\text{eV}^2\), as favoured by the old Kamiokande data\(^\text{21}\),\(^\text{22}\) and the new data from Soudan reported in this conference,\(^\text{23}\) then the negative result from CHOOZ restricts \(U_{e 3}\) to quite small values, as indicated in Figure 3 and quoted in\(^\text{18}\). The new SuperKamiokande data\(^\text{18}\) gives a best-fit value for \(m_{\nu 3}^2\) of \(2.2 \times 10^{-3}\), still implying by CHOOZ a small value for \(U_{e 3}\), but do not exclude lower values of \(m_{\nu 3}\) and hence much larger values of \(U_{e 3}\). In any case, as seen in Figure 3, the band of values predicted by the DSM calculation falls always comfortably within the allowed region.

The DSM results summarized above for neutrino oscillations were obtained assuming \(m_{\nu 2}^2\) of order \(10^{-11}\) to \(10^{-10}\text{eV}^2\), as suggested by the LWO solution\(^\text{17},\text{18}\). The alternative MSW\(^\text{24}\) solutions for solar neutrinos require \(m_{\nu 3}^2 \sim 10^{-5}\text{eV}^2\), for which no sensible DSM solution was found\(^\text{18}\). It is thus intriguing to hear in this conference that the new SuperKamiokande data on the day-night variation and energy spectrum of solar neutrinos\(^\text{25}\) also

Figure 1: The trajectory of \((x’, y’, z’)\) as the energy scale varies.

Figure 2: 90 % CL limits on \(U_{\mu 3}\) compared with DSM calculation.

Figure 3: 90 % CL limits on \(U_{e 3}\) compared with DSM calculation.
favours the LWO solution.

Further, generation being identified with dual colour in DSM, one expects only 3 generations of neutrinos. Thus, the result from Karmen reported in this conference against the existence of another neutrino with mass of order eV, as previously suggested by the LSND experiment, is also in the DSM’s favour.

It is particularly instructive to compare the CKM matrices for leptons and quarks. Both the empirical (4), (5) and the calculated (3), (6) matrices show the following salient features:

- that the 23 element for leptons is much larger than that for quarks,
- that the 13 elements for both quarks and leptons are much smaller than the rest,
- that the 12 element is largish and comparable in magnitude for quarks and leptons.

These features, all so crucial for interpreting existing data, not only are all correctly reproduced by DSM calculation, but also can be understood within the scheme using some classical differential geometry as follows.

First, it turns out that in the limit when the separation between the top 2 generations is small on the trajectory traced out by \((x', y', z')\), which is the case for all 4 fermion-types as seen in Figure 1, then the vectors for the 3 generations form a Darboux triad composed of (i) the radial vector \((x', y', z')\) for the heaviest generation, (ii) the tangent vector to the trajectory for the second generation, and (iii) the vector normal to both the above for the lightest generation. The CKM matrix is thus just the matrix which gives the relative orientation between the Darboux triads for the two fermion-types concerned. Secondly, by the Serret–Frenet–Darboux formulae, it follows that the CKM matrix can be written, to first order in the separation \(\Delta s\) on the trajectory between \(t\) and \(b\) for quarks and between \(\tau\) and \(\nu_3\) for leptons, as

\[
CKM \sim \begin{pmatrix}
1 & \kappa_2 \Delta s & -\tau_2 \Delta s \\
\kappa_3 \Delta s & 1 & \kappa_0 \Delta s \\
\tau_3 \Delta s & -\kappa_n \Delta s & 1
\end{pmatrix}, \tag{7}
\]

where \(\kappa_n\) and \(\kappa_\tau\) are respectively the normal and geodesic curvature and \(\tau_\tau\) is the geodesic torsion of the trajectory. Lastly, for our case of a curve on the unit sphere, \(\kappa_n = 1\) and \(\tau_\tau = 0\), from which it follows that:

- the 23 element equals roughly \(\Delta s\),
- the 13 element is of second order in \(\Delta s\),
- the 12 element depends on the details of the curve.

In Figure 1, \(\Delta s\) between \(\tau\) and \(\nu_3\) is much larger than that between \(t\) and \(b\), hence also the 23 element of the CKM matrix. Indeed, measuring the actual separations in Figure 1, one obtains already values very close to the actual CKM matrix elements in (3) and (5) or in (4) and (6). The 13 elements should be small in both cases, as already noted. As for the 12 elements, they depend on both the locations and details of the curve, which explains why they need not differ much between quarks and leptons in spite of the difference in separation, and also why the DSM prediction in (5) is not as successful for this element as for the others.

To test DSM further, one seeks predictions outside the Standard Model framework. These are not hard to come by. Identifying generation to dual colour, which is a local gauge symmetry, makes it imperative that any particle carrying a generation index can interact via the exchange of the dual colour gauge bosons, leading to flavour-changing neutral current (FCNC) effects. Given the calculations on the CKM matrices outlined above, all low energy FCNC effects can now be calculated in terms of a single mass parameter \(\zeta\) related to the vev’s of the dual colour Higgs fields. Inputting the mass difference \(K_L - K_S\) which happens to give the tightest bound on \(\zeta \sim 400\text{TeV}\), one obtains bounds on the branching ratios of various FCNC decays. In the following paragraph, an argument will be given which converts these bounds into actual order-of-magnitude estimates. In particular, the mode \(K_L \rightarrow \mu^+\mu^-\) has a predicted branching ratio of around \(10^{-13}\), less than 2 orders away from the new BNL bound of \(5.1 \times 10^{-12}\) reported in this conference.

Since neutrinos carry a generation index, it follows that they will also acquire a new interaction through the exchange of dual colour bosons. At low energy, this interaction will be very weak, being suppressed by the large mass parameter \(\zeta\). However, at C.M. energy above \(\zeta\), this new interaction will become strong. With an estimate of at least \(400\text{TeV}\), the predicted new interaction is not observable in laboratory experiments at present or in the foreseeable future, but it may be accessible in cosmic rays. For a neutrino colliding with a nucleon at rest in our atmosphere, \(400\text{TeV}\) in the centre of mass corresponds to an incoming energy of about \(10^{20}\text{eV}\). Above this energy, neutrinos could thus in principle acquire a strong interaction and produce air showers in the atmosphere. Now air showers at and above this energy have been observed. They have long been a puzzle to cosmic ray physicists since they cannot be due to proton or nuclear primaries which would be quickly degraded from these energies by interaction with the 2.7 K microwave background. Indeed, the GZK cutoff for protons is at around \(5 \times 10^{19}\text{eV}\). Neutrinos, on the other hand, are not so affected by the microwave background. Hence, if they can indeed produce air showers via the new interaction predicted by the DSM, they can give a very neat explanation for the old puzzle of air showers beyond the GZK cutoff. Further tests for the proposal have been suggested. The proposal also gives a rough upper bound on the mass parameter \(\zeta\) governing FCNC effects which is close to the lower bound obtained in the preceding paragraph. It was on the basis of this coinci-
dence that the above FCNC bounds were converted into actual order-of-magnitude estimates.

The conclusions are summarized in Figure 4.

It is a pleasure for us to acknowledge our profitable and most enjoyable collaboration with Jacqueline Fari-
dani and Jakov Pfaudler. TST also thanks the Royal Society for a travel grant to Vancouver.

References

1. Chan Hong-Mo and Tsou Sheung Tsun, Phys. Rev. D57, 2507, (1998).
2. Chan Hong-Mo, Jacqueline Faridani, and Tsou Sheung Tsun, Phys. Rev. D53, 7293, (1996).
3. CH Gu and CN Yang, Sci. Sin. 28, 483, (1975); TT Wu and CN Yang, Phys. Rev. D12, 3843 (1975).
4. A.M. Polyakov, Nucl. Phys. 164, 171, (1980).
5. Chan Hong-Mo and Tsou Sheung Tsun, Some Elementary Gauge Theory Concepts (World Scientific, Singapore, 1993).
6. Chan Hong-Mo and Tsou Sheung Tsun, Phys. Rev. D56, 3646, (1997).
7. Chan Hong-Mo, Jacqueline Faridani, and Tsou Sheung Tsun, Phys. Rev. D51, 7040 (1995).
8. G. ’t Hooft, Nucl. Phys. B138, 1, (1978); Acta Phys. Austriaca. Suppl. 22, 531, (1980).
9. J Bordes, Chan H-M, J Faridani, J Pfaudler, Tsou ST, Phys. Rev. D58, 013004, (1998)
10. José Bordes, Chan Hong-Mo, Jakov Pfaudler, Tsou Sheung Tsun, Phys. Rev. D58, 053003 (1998).
11. Chan Hong-Mo and Tsou Sheung Tsun, Acta Phys. Polonica, B28, 3041, (1997)
12. Particle Physics Booklet, (1996), RM Barnett et al., Phys.Rev.D54, 1, (1996); web updates.
13. SuperKamiokande data, presented by C McGrew, M Vagins, M Takita at ICHEP’98.
14. K.S. Hirata et al., Phys. Letters, B205, 416, (1988); B280, 146, (1992); Y. Fukuda et al. Phys. Letter B335, 237, (1994).
15. C Giunti, CW Kim, M Monteno, hep-ph/9709439.
16. V. Barger, R.J.N. Phillips, and K. Whisnant, Phys. Rev. Letters, 69, 3135, (1992).
17. PI Krastev, ST Petcov, PRL 72, 1960, (1994).
18. CHOOZ collaboration, M. Apollonio et al., Phys. Lett. B420, 397, (1997).
19. B. Ackar et al., Nucl. Phys. 434, 503, (1995).
20. Soudan II, presented by H Gallacher at ICHEP’98.
21. L Wolfenstein, Phys.Rev. D17, 2369, (1978); SP Mikheyev, AYu Smirnov, Nuo. Cim. 9C, 17 (1986).
22. For example see G.L. Fogli, E. Lisi, and K. Whisnant, Phys. Rev. D54, 2048 (1995).
23. Karmen, presented by J Kleinfellner at ICHEP’98.
24. C Athanassopoulous et al, PRL 75, 2650 (1995).
NONABELIAN DUALITY
Chan–Faridani–Tsou

STANDARD MODEL

' T HOOFT THEOREM

DUALIZED STANDARD MODEL
Chan–Tsou

HIGGS FIELDS
(AS FRAME VECTORS)

3 GENERATIONS
(AS DUAL COLOUR)

SYMMETRY
BREAKING PATTERN

NEUTRINOS
INTERACT
STRONGLY AT
HIGH ENERGY

FERMION MASS HIERARCHY:
e.g. $m_t \gg m_c \gg m_u$
FERMION MASSES AND MIXINGS
CALCULABLE PERTURBATIVELY

(1-LOOP, 1ST ORDER)
QUARK AND LEPTON MASSES
QUARK CKM MATRIX
Bordes–Chan–Faridani–Pfaudler–Tsou

'EXPLANATION' FOR PUZZLE OF
AIR SHOWERS
AT $E > 10^{20}$eV
Bordes–Chan–Faridani–Pfaudler–Tsou

PREDICTIONS
OF B.R. FOR
FCNC DECAYS
Bordes–Chan–Faridani–Pfaudler–Tsou

(EXTENDED TO NEUTRINOS)
NEUTRINO OSCILLATIONS
Bordes–Chan–Pfaudler–Tsou

Figure 4: Summary flow-chart.