Wake meandering in a model wind turbine array in a high Reynolds number turbulent boundary layer

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Abstract. Wake meandering is the dynamic shift observed in the spatial location of a wind turbine wake as it evolves downstream, considered to be caused primarily by the interaction of large flow structures in the atmospheric boundary layer with the turbulent wake. Experiments were conducted in a large boundary layer wind tunnel to investigate meandering in the wakes of both individual model wind turbines and in a wind turbine array with a large number of model turbines (19 rows x 5 columns = 95 model turbines). Unlike bluff body vortex shedding, the meandering phenomenon for a turbine wake is not characterized by a well-pronounced peak in the frequency domain, but rather by a broad spread over a low-frequencies range. For individual turbine models, both rotating three-bladed models and non-rotating porous disks, frequencies and peak energies observed in the velocity spectrum in the wake return to those of the incoming high Reynolds number turbulent boundary layer. Wake meandering also presents itself in the large wind turbine array. Here, frequencies and peak energies observed in the velocity spectrum in the wake do not return to those of the incoming turbulent boundary layer. Instead, a lower dominant frequency corresponding to the wind velocity at hub height and array spacing is observed within the array, indicating forcing of the meandering by the array itself (a type of resonance). Higher peak energies are observed in the array due to large flow structures representative of the turbine spacing.

Keywords: Wakes, Wake Meandering, Wind Turbine Arrays, Wind Tunnel Experiment

1. Introduction

Wind energy has become a significant contributor to the electric energy portfolio in many countries over the past decade. Moving forward, most countries also have plans to substantially increase wind energy contributions – both onshore and, where feasible, offshore, e.g. [1, 2]. To maximize energy conversion efficiency and minimize fatigue loading – for individual turbines, small farms and especially for large turbine arrays – wind turbine wakes and their interaction with the atmospheric boundary layer and other turbine wakes must be better understood.

The flow region downstream of a wind turbine rotor is referred to as the turbine wake — a turbulent shear flow characterized by a velocity deficit and increased levels of turbulence [3]. As the region that is affected by this turbulent wake grows outward, the velocity deficit decreases. If all terms are accounted for properly, the integral form of the momentum equation...
integrated/summed across the flow at any downstream location will yield the rotor thrust force. Given enough distance, the wake will eventually mix with the much larger momentum available in the flow outside of the wake, and the flow should return to its inflow conditions.

The dynamic shift observed in the location of the wake of a wind turbine as it evolves downstream is referred to as wake meandering. This is illustrated in Figure 1, where the red dot represents the expected location of the wake center and the black × represents the actual center of the instantaneous wake. The wake effect on downstream turbines causes a decrease in wind farm power production as well as increased rotor fatigue and failure, and wake meandering introduces significant uncertainty in predicting this. Wake meandering makes the inflow for downstream turbines more intermittent and gives it a modified turbulence structure, which can affect the reliability and stability of the power output of a wind farm.

![Figure 1: Wake meandering: large scale eddies in the the atmospheric boundary layer move the the turbine wake around. Red dot: expected wake center, black cross: actual center.](image)

The wake meandering phenomenon is not yet fully understood, however, two possible causes have been considered: One, the intrinsic instabilities from vortex shedding from a bluff body, as produced by the turbine hub or possibly the rotor “disk”, and two, large scale turbulent eddies contained in the atmospheric boundary layer. The shedding of vortices behind a bluff body is usually dominated by boundary layer separation off the sides of the object creating a shear layer and changing the wake size. For bluff bodies the shedding typically has well-defined Strouhal numbers \( (St = \frac{f_sD}{U}) \) and the wake oscillation is sinusoidal [4]. In this case the vortex shedding characteristic length scale would be either the turbine hub or turbine rotor diameter of the turbine. The shedding frequency is \( f_s \) and the characteristic velocity is \( U \). However, the turbine wake meandering phenomenon is not characterized by a well-pronounced peak in the frequency domain, but rather by a broad spread over a larger low-frequencies range, especially at turbine tip height, c.f. Barlas et al. [5]. Espana et al. [6] found that large scale turbulent eddies contained in the boundary layer only caused meandering when the length scales were larger than the rotor diameter. Bossuyt et al. [7] and Coudou et al. [8] proposed that large scale atmospheric eddies move the wakes of single turbines around. Therefore, it is presently considered that the incoming atmospheric boundary layer structures play an important, and likely dominant role in the directional advection of the wake of single turbines.

Large scale coherent motions in boundary layers and other canonical flows have been studied for more than half a century. Favre et al. [9] investigated coherence in a turbulent boundary layer flow via space-time auto-correlation and cross-correlation maps, obtained by streamwise-spaced hotwires. Both streamwise and wall normal components of velocity were used to discover the large lasting features of a turbulent flow by Kovasznavy et al. [10]. These coherent, sometimes meandering structures can be quite long. Tutkun et al. [11] found that streamwise correlations in the streamwise-wall-normal plane can extend up to 7-8 boundary layer thicknesses in the log region of turbulent boundary layer flow. Hutchins and Marusic [12] identified very long streaky
structures in the log region of turbulent boundary layers extending to up to 20 boundary layer thicknesses in the streamwise direction. The strongest correlations in the streamwise direction are found within the inertial sublayer of the turbulent boundary layer, however, ramp shaped hairpin packets are found beyond the log layer and cover the entire flow domain up to boundary layer height, c.f. Adrian [13]. Large scale structures also exist in the outer region of turbulent boundary layers, and Liu et al. [14] showed they may become invariant with increasing Reynolds number. These large scale coherent boundary layer structures are part of the atmospheric boundary layer inlet flow into a wind farm and will affect how the wind farm’s internal flow structures develop.

To elucidate some of the very large scale structures that influence the production of turbulent energy inside a wind farm, the technique of proper orthogonal decomposition (POD) has been employed experimentally and through simulation. Either large eddy simulation (LES) or particle image velocimetry (PIV) in a wind tunnel experiment will provide an instantaneous snapshot of the full spatially resolved velocity field necessary to apply POD. Newman et al. [15] performed a POD analysis on the $Uu'v'$ transport term and found that 75% of the energy in the farm comes from large scale entrainment with wavelengths greater than the diameter of the turbines. Compared to a boundary layer without turbines, the influence of turbines causes energy in low modes (high energy) to decrease, meaning energy not extracted by turbines goes to turbulence production, distributing the flow energy over a wider range of scales. Larger coherent structures were found to be initially destroyed by the presence of the turbines [15]. In a wind tunnel study of micro-turbines instrumented with strain gauges, Bossuyt et al. [7] found the streamwise aligned cross-correlation to be the strongest in the array, confirming a strong correlation with presence of upstream turbines. In large eddy simulations of wind farm layouts, Andersen et al. [16] reported that structure size and energy grew with downstream distance in the array, while the coherence of structures increased.

Experiments with single model turbines and miniature wind farms have been conducted at various scales and in various arrangements, placed in artificially thickened boundary layers. Chamorro and Porté-Agel [17] showed that upstream turbines impart a signature of shed tip vortices in the velocity spectra of the wakes of downstream turbines. Wind tunnel experiments by Bossuyt et al. [7] showed that the turbine power output is temporally correlated with other turbines in the array. This unsteadiness and variability of the flow can affect wind farm power optimization. Iungo et al. [18] found from wake measurements of a three-bladed model turbine that the hub vortex oscillates sinusoidally only in the near wake, in agreement with others [19, 20]. Howard et al. [21] performed a PIV experiment with two turbines and found that near wake meandering is governed by the interaction between bluff body hub vortex shedding and higher momentum fluid entrained along the tip vortex shear layer. For a porous disk without a hub, España et al. [22] found that no defined periodic oscillations occurred in a flow with high turbulence (12% turbulence intensity) in a simulated atmospheric boundary layer. The higher turbulence forced a weaker spectral energy peak. Contiguous vortices can be shed at different circumferential locations on a disk, and this non-deterministic feature can be affected by upstream turbulence intensity and velocity gradients. This would imply that wake meandering exists, but not with a well-defined periodicity. Medici and Alfredsson [19] found that for lower tip speed ratios $\lambda$, the Strouhal number is large and the effective diameter of the turbine becomes smaller when the rotational speed is decreased (as expected, since this decreases the thrust). In a 3x3 experimental wind farm array, the Strouhal number was found to be $St = fD/U_{hub} \approx 0.20 - 0.22$ by Coudou et al. [8]. Similar peaks in Strouhal number were found by Heisel et. al [23].

The unsteady properties of a model wind turbine wake were studied by Muller et al. [24]. Using two porous disks and a hotwire rake, they looked at the ability of the wake to follow large turbulent eddies contained in the approach flow. Similarly, 2D LiDAR measurements were made in the wake of operational turbines at Denmark’s Riso campus to verify this basic conjecture in the Riso/DTU Dynamic Wake Meandering (DWM) model, cf. Bingol et al. [25, 26]. These
measurements supported that the wake deficit is advected passively by the larger than rotor size eddies in the inflow. Many researchers have had difficulty separating the effects of the large scale turbulence of the incoming flow and the local hub vortex on amplitude and intensity of wake meandering, especially in relevant turbine configurations such as the fully developed wind power plant [21]. In an array, upstream turbines can confine the wake of downstream turbines which further increases the velocity deficit. This confinement also sometimes produces acceleration between the turbines due to the blockage effect, known as jetting. Higher performance was sometimes seen from turbines that end up in these confined, non-aligned cases due to jetting, in an LES torque-controlled actuator disc simulation of the special case of the closely spaced Lillgrund offshore windfarm cf. Creech et al. [27]. More often however, the turbines far into the array experience increased turbulence, unpredictable inflows, and a further power reduction.

It is expected that identification of large coherent structures in the prevailing wind of a site and within the array can inform the passive optimization of a wind farm via placement and spacing of wind turbines. Researchers are working to integrate meandering knowledge into control systems to improve the wind farm monitoring and control and ultimately farm scale power output (Yang et al. [28]). With improved understanding of how the incoming flow and turbine wakes interact, increased array efficiency can be achieved by improving the turbulent mixing that enables wake recovery behind a turbine or series of turbines. Active optimization is also possible with knowledge of how energetic structures are distributed and behave with the influence of a wind farm.

Andersen et al. [29] simulated an infinitely long row of turbines, without free-flow turbulence or shear, and needed a high number of modes to capture half of the energy content of the flow in a POD analysis. This demonstrated that the structure of the wake is constantly morphing between highly complex shapes and is not merely a collective wake deficit advected by the incoming flow. Many high-fidelity flow simulations show persistent meandering deep into the wind farms. Large scale meandering motions are an integral part of the overall wind turbine and wind farm dynamics [5].

The experiments discussed in this paper investigated meandering downstream of individual wind turbine models, both scaled operational models and thrust-matched porous disks, and in a large array built up from the turbine models, in a large boundary layer wind tunnel. The setup of the measurement campaign, describing facility, equipment, instrumentation and experiments will be discussed first. This will be followed by a discussion of the methodology for data analysis and the results.

2. Methods

2.1. Experimental Facility

The individual model turbine wake studies and turbine array studies were conducted in the University of New Hampshire (UNH) Flow Physics Facility (FPF), shown in Figure 2. The UNH FPF is a large turbulent boundary layer wind tunnel, with sufficient flow quality to enable study of high Reynolds number turbulent boundary layer flow. The FPF test section has a width of 6 m, a nominal height of 2.7 m, and a length of 72 m. The test section height increases with downstream distance to account for boundary layer growth on all four walls and to maintain a zero pressure gradient in the core of the tunnel, and therefore a zero pressure gradient turbulent boundary layer on all walls, for an empty test section. Spanwise variation in friction velocity measured with a Preston tube is less than +/- 0.5% when outside of the side wall boundary layers. In its present state (Phase 1, open return), test section velocities of up to 14 m/s can be achieved with free stream turbulence intensities less than 0.5% at all downstream locations [30]. The FPF was designed to investigate high Reynolds number turbulent boundary layers with adequate spatial and temporal resolution. Naturally grown turbulent boundary layers on the order of 1 meter thick with the correct range of scales, and Reynolds numbers based on boundary layer
scale ratios, $\delta^+ \equiv u_\tau \delta/\nu \approx 20,000$ with $\eta = \nu/u_\tau \approx 35 - 40 \mu m$ (corresponds to $\delta = 0.7 - 0.8 m$) can be achieved towards the downstream end of the test section. Details of the FPF and qualifying boundary layer measurements were reported in Vincenti et al. [30]. The FPF is an open return wind tunnel, and changes in weather conditions such as inlet temperature, barometric pressure and humidity all affect the flow and instrumentation. This provides unique challenges, in particular for thermal flow measurement techniques, as discussed later in this section and in Turner and Wosnik [31].

![Figure 2: Section view of the UNH Flow Physics Facility, test section dimensions 6.0 m (W), 2.7 m (H) and 72 m (L), flow from left to right.](image)

![Figure 3: Side by side view of model turbine and porous disk, both with diameter $D = 0.25 m$ and hub height 0.75 $D$.](image)

Figure 2: Section view of the UNH Flow Physics Facility, test section dimensions 6.0 m (W), 2.7 m (H) and 72 m (L), flow from left to right.

### 2.2. Model Turbines

A model diameter of $D = 0.25 m$ and hub height of $H_{hub} = 0.75D (0.188 m)$ were selected to match the scale of the turbulent boundary layer in the FPF. A typical inflow condition for a real wind farm is that the atmospheric surface layer thickness is greater than three times the wind turbine hub height ($\delta/H_{hub} > 3$), which is replicated in the model studies in the FPF. Two types of turbine models were used: functional, three-bladed scaled model wind turbines and porous disks, cf. Figure 3.

#### 2.2.1. Rotating Three-Bladed Turbine Model

The 0.25 $m$ diameter model wind turbines used in this study are 1:500 scale models of turbines with a rotor diameter of 125 $m$, which is approximately equivalent to a 5 MW turbine (e.g., NREL 5 MW offshore reference wind turbine [32]). Since it is not possible to simultaneously achieve geometric, kinematic and dynamic similarity at this scale, the turbines were designed with the goal to approximate the non-dimensional performance parameters of full scale turbines, i.e., power coefficient $C_P$, drag (thrust) coefficient $C_T$ and tip speed ratio $\lambda$. The turbine rotor was designed using blade element momentum (BEM) theory [33]. The airfoil profile chosen for the blades was a NACA 2412, and blades and hub assembly were 3D printed with ABS plastic with a rapid protyping tolerance of 0.002 inches. The turbines were mounted with a hub height at 0.75 times the turbine diameter, or 0.188 $m$, which is representative of offshore wind turbine installations.

In previous studies, the performance of the model turbines was characterized, in a free stream and inside the boundary layer in the FPF. The model wind turbine achieved a coefficient of performance of $C_{P,max} \approx 0.3$ and a thrust coefficient of $C_T \approx 0.73$ at a tip speed ratio of $\lambda = 3.5$, in the free stream, with $C_T$ increasing as $\lambda$ is increased. Details of the turbine design and results from various performance tests are given in [34, 35].

#### 2.2.2. Porous Disks

The porous disks were approximately drag (thrust) matched to the BEM theory designed rotating model turbines, and their wakes compared for the for the purpose of
building up larger experimental arrays, c.f. Charmanski et al. [36]. The porous disks have 0.1875" (4.76 mm) diameter holes with a 0.25" (6.35 mm) center to center spacing, with a 60 degree staggered center hole arrangement, resulting in an open area of 51%. They are made of Type 304 stainless steel, with a thickness of 0.0480" (1.22 mm) (McNichols Perforated Metal). The porous disk hub height is also located at 0.75D from the floor. The porous disks do not rotate. The drag (thrust) coefficient of the porous disks was measured as $C_T = 0.92$ in a uniform inflow, at Reynolds numbers comparable to what the porous disk model would encounter at hub height at the first row of the array. This value is consistent with recent measurements and predictions from Hultmark and Steiros [37].

2.3. Instrumentation

Velocity measurements were performed with a custom made, single wire constant temperature anemometer hot-wire (CTA-HW) probe, made of tungsten wire 5 µm in diameter with an exposed active length of 1 mm. The hot-wire was attached to a sting, positioning it 0.5 m upstream of a vertical traversing system, and connected to an AN-1003 AA Labs anemometry system. Each point was sampled for 60 seconds at high enough temporal resolution (10 kHz) to capture the spectral density of the velocity time series. The hot wire was calibrated in the free stream against a reference pitot-static tube, before and after each velocity profile. Temperature was measured during calibration and throughout each profile. The hot wire calibration applied in post-processing was a linear interpolation between the before and after calibration, corresponding to the actual variation in temperature. If the temperature varied more than 2°C between the before and after calibration, the profile was rejected and repeated. Typically, a velocity profile with calibration before and after will take over 2 hours to complete, so temperature variations are a limitation in experiment design. The vertical traverse was streamlined using 3-D printed ABS plastic airfoils, and controlled using a stepper motor and a NI MID-7604 controller.

A Pitot static tube was used to measure reference velocity in the free stream at a height of 1.7 m from the floor (about 60% of wind tunnel height), with a MKS Baratron 698A Differential Pressure Transducer. A k-type thermocouple at the same height was used to measure temperature. The pressure transducer data was corrected in post-processing for temperature, barometric pressure and humidity [38]. These corrections are required due to the open return configuration of the FPF.

2.4. Individual Model Wind Turbine Wake Measurements

Wake velocity profiles were measured and compared for the rotating model turbine and the porous disk in the boundary layer in the UNH FPF. The turbine models were positioned at $x = 62$ m from the test section inlet. The boundary layer thickness at this location was approximately 0.8 m, which fully immerses the models in a shear flow and puts the hub height of the models in the bottom 1/3rd of the boundary layer. The FPF was operated at a free stream velocity of nominally 0.7 m/s (600 RPM). Vertical profiles consisting of 67 points each were obtained with a custom traversing system at the spanwise centerline of the models, at 8 different locations downstream of the disk and turbine at $x/D = 1, 2, 4, 6, 8, 10, 14, \text{ and } 20$. The experimental setup with a single turbine model (here: porous disk) is shown in Figure 4, and the incoming boundary layer profile is shown in Figure 5 (with the same vertical spacing as for the wake velocity profiles).

2.5. Disk and Turbine Array

To study wake meandering in the wind turbine array boundary layer, hot wire measurements were performed in an array built up from both model wind turbines and porous disks. A total of 9 model turbines and 86 porous disks were used for this study. The disks and turbines were set up in arrays in the UNH FPF of 5 columns and 19 rows, for a total of 95 model turbines.
The large test section of the FPF allowed for variable stream wise spacing and array size. The turbines and disks were carefully aligned to within ±1 mm which corresponds to ±0.5 m full scale. Details on layout and turbine placement are reported by Turner [39]. Velocity measurements were taken within the array using the same hot wire and traversing system as for the single wake comparisons.

The model turbines were arranged in a 3×3 array within the array of porous disks. All velocity profiles were measured downstream of the last (third) row of model turbines at span wise center line. This 3×3 model turbine array is used to reintroduce the rotating wake components of the flow and helps correct for any turbulence differences between the porous disk and the turbine wake. A schematic of this setup for the full 19 rows is shown in Figure 6. Note that two dummy rows of turbines (porous disks) were placed downstream of the measurement region to establish the proper flow resistance and prevent the flow from encountering an adverse pressure gradient and decelerating in the measurement region. This was confirmed by measurements of the streamwise pressure gradient in the free stream. A spanwise spacing of 4 diameters ($S_y = 4D$, or 1.0 m) was used for all array studies. This spanwise spacing allowed for the maximum amount of spanwise tunnel space to be used without major influence from tunnel wall effects. Two streamwise spacings were investigated, 8 and 10 diameters ($S_x = 8D, 10D$, or 2.0 m and 2.5 m).
Four different array sizes were used: 19×5, 14×5, 9×5, and 5×5, with velocity profile measured at center line downstream of the 17th, 12th, 7th and 3rd rows, respectively. Again, the FPF was operated at a free stream velocity of nominally 6.7 m/s (600 RPM). Vertical profiles consisting of 74 points each were obtained: the first 21 points were spaced 0.04D apart, beginning at turbine hub height \( (z = 0.188 \, m) \) to 0.3D above turbine top tip height \( (z = 0.388 \, m) \), and the next 53 points were spaced 0.08D apart (up to \( z = 1.448 \, m \)). Note that the goal of this study was to capture the flow physics and wind turbine array boundary layer dynamics near the top tip height of the turbines and above the array and therefore near-wall points were not included.

3. Analysis and Results

3.1. Analysis in the Frequency Domain

The analysis of the velocity time series is summarized here first, before providing more detail with example data and figures. First, the spectrum (power spectral density) of the velocity time series is calculated, using a Fast-Fourier Transform (FFT) algorithm. Then, the spectrum is multiplied by frequency to produce the energy spectrum. This pre-multiplied spectrum is useful for visualizing energetic peaks. A broad spectral energy peak is found using a carefully designed and bounded regression algorithm, and the corresponding frequency and energy content are obtained from the peak value of the regression. It is assumed that the broad spectral energy peak represents the meandering of the wake. These peak values are then plotted at different downstream locations from the model turbine or within the array at different wall normal heights.

3.1.1. Power Spectral Density

Figure 7 shows the variance conserved one-sided power spectral density \( S(f) \) at turbine top tip height (TT, \( z = 0.313 \, m \)) for both the incoming boundary layer flow and in the wake 6D downstream of a porous disk turbine model. The spectra were computed with 50 degrees of freedom (DoF), and a line representing an \( f^{-5/3} \) slope is shown for reference. In can be seen that the wake of the disk has increased energy in the spectrum at all frequencies. The disk wake generates turbulence which increases variance in the frequency domain, while decreasing momentum in the wake.

![Figure 7: Power spectra of inlet boundary layer flow and disk wake at x/D = 6, both at turbine top tip (TT) height. Shown for reference is a f^{-5/3} slope.](image)

The power spectra in Figure 7 do not drop off at lower frequencies, likely because some external influence on the flow is not being resolved due to the length of the velocity time series. This could be a longer period temperature variation, or a resonance of the wind tunnel test.
section, which, for the FPF test section length of 72 m and a nominal free stream velocity of 6.7 m/s would occur at just below 0.1 Hz, (cf. [30]), which is not resolved in the spectrum. The resolution could be increased by obtaining a longer time series, however, concerns about temperature variations during a profile and overall profile duration constrained this. For a more detailed discussion of spectrum processing and temporal resolution cf. [39]. The focus for the discussion here will be on the 1-10 Hz range, where we expect to find peaks of wake meandering.

3.1.2. Peak finding in the premultiplied spectrum  To find the most energetic structures in the velocity time series, the spectrum is premultiplied by frequency to obtain the energy spectrum $f \cdot S(f)$. This method was also used in a wake meandering study of wind tunnel and field measurements by Heisel et. al [23]. The spectrum was averaged with 10 bands over 1 ensemble (20 DoF) and a Hanning window was applied in an attempt to minimize the spectral leakage and help clarify the peak. The meandering phenomenon is not an exact periodic display of vortex shedding, but a dynamic shift in the wake over time; therefore we expect a small section of the spectrum to be raised in the 1 Hz – 10 Hz region instead of a well-defined single sharp peak. The peak energy content becomes more visible when multiplying by frequency and acts as a dynamical interpretation of the flow visualized by $S(f)$.

The peak of this energy spectrum is then located by means of polynomial regression for every time series. To create a polynomial fit that is appropriate, the logarithmic spacing of binned frequencies in the spectrum was accounted for. The regression range was discretized and interpolated to normalize the weights of the logarithmically-spaced binned frequencies. Figure 8 shows an example of a 5th order polynomial regression fit from 0.5 Hz – 100 Hz on the energy spectrum of a single porous disk in the boundary layer with time series measured at a wall-normal top tip height at 6 D downstream. The overlaid red line represents the regression. The peak of this regression is used as the corresponding maximum frequency which is used as that object’s meandering frequency at a given wall-normal location. This regression method was used to calculate the spectral peaks for the time series at all wall-normal locations at each downstream position.

3.1.3. Repeatability/uncertainty of peak finding  The ability of the data analysis and regression to find the peak in the spectrum in a repeatable manner was examined carefully, via a parametric...
study varying: frequency bounds, order of regression, DoF in spectrum and length of time series processed. The first three parameters modified the resultant peak frequencies only by a few percent. The length of the time series used, however, had a significant, order of magnitude larger effect on the results.

The regression frequency bounds were maintained at 0.5 Hz to 100 Hz, which centers the regression around 10 Hz on a log scale. The length of the time series was modified by truncating shorter sections (e.g. two 30 second time series, six 10 second time series, for a one minute data set). Shortening the time series generally increases the variance between truncated sections. The single one minute time series is truncated into smaller time intervals and the peak frequencies are calculated with the algorithm in Figure 9. A dashed red line indicates the peak frequency with the full one minute time series. It can be seen that the variation of peak frequencies decreases with increased time series length. Longer time series increase the resolution of the low frequencies in the premultiplied spectra and help calculate a more reliable estimate of the larger scales. At

![Figure 9: Spectral peak frequencies for varying time series lengths. The red dashed line indicates the peak frequency for the full one minute time series.](image)

![Figure 10: Variance of the spectral peak frequency for varying time series length at different vertical positions](image)

a single location, if we consider Taylor’s frozen flow hypothesis, we are able to see or measure the scale of passing eddies based on their measurement at a stationary location. For instance, if we are to view a passing eddy for 2 seconds and the flow speed is 4 m/s, we would be able to resolve an eddy of scale 8 meters, or a frequency down to 0.5 Hz. However, this would only be one single event, which would decrease the reliability of this event. If we increase the amount of time to view the flow to 10 seconds, we could view an event of 8 meter scale five times, increasing our reliability of seeing that type of event. In Figure 9, we are capturing the increased reliability of finding that peak frequency with increased length of time series.

For each time truncation, a variance of peak frequencies is seen visually in Figure 9. This variance is calculated with respect to the different time truncations and displayed with other vertical locations at the same downstream distance ($\alpha/D = 6$) from the porous disk in the boundary layer in Figure 10. The vertical positions are bottom tip height $H_{BT}$ ($= H_{Hab} - 0.5D$), hub height $H_{Hab}$, top tip height $H_{TT}$ ($= H_{Hab} + 0.5D$), one diameter above hub height $H_{Hab+1D}$, and one diameter above top tip height $H_{TT+1D}$ ($= H_{Hab} + 1.5D$). The variance of the peak frequency is obviously large for small time series and decreases with increased time series length. (Note that the variances computed in Figure 10 are not actually 0 at $t=60$ s; this is just a consequence of the time series being 60 s long.) The peak frequencies obtained for one minute
long time series are therefore expected to be reasonable estimates for wake meandering, but do carry appreciable uncertainty. The plots presented here are representative of the entire measurement campaign. A longer discussion about the balance of length of time series and data acquisition accuracy is found in [31, 39].

3.2. Individual Model Turbines

3.2.1. Individual model turbine wakes

Figures 11b and 11a show the wake velocity profiles in the boundary layer downstream of the turbine and the porous disk at various downstream positions normalized by free stream velocity and turbine diameter ($y/D = 0$ is turbine hub height). The solid black line represents the inlet turbulent boundary layer profile, also shown in Figure 5.

In Figures 11b and 11a both wakes show a large initial velocity deficit in the near-wake at low $x/D$, and then recover towards the undisturbed, incoming boundary layer profile at higher $x/D$. Both also produce a small overshoot of the velocity profile in the near wake at the edge of the rotor disk: At the upper edge for the model turbine, and at the upper and lower edge for the porous disk. At 20 diameters downstream, about a 5-10% difference to the incoming boundary profile can still be observed at hub height for the two cases, but the momentum overall has recovered significantly (The turbine model wake appears to recover slightly faster, due to a slightly smaller $c_T$, and possible due the influence of wake rotation). For the turbine (Figure 11b) evidence of hub and tip vortices can be seen in the near-wake, whereas the porous disk near-wake (Figure 11a) more closely resembles that of a bluff body. Progressing further downstream, say 10 to 14 diameters, the velocity deficit is seen to mostly recover and the velocity profiles in the wake become increasingly similar. It was determined that the wakes of the porous disks and model turbines become sufficiently similar in this setup at a downstream distance of about $6D$, to build up an array of both objects for the array experiments. The array, using a combination of porous disks and model turbines, is spaced in the stream wise direction at $8D$ and $10D$.

3.3. Individual turbine peak frequencies/wake meandering

The regression method discussed in Section 3.1 is now applied to the data sets obtained downstream of the model turbines placed in the high Reynolds number boundary layer. Both peak frequency and energy are extracted from the regression on the energy spectra. The evolution
of peak frequencies was investigated here. The peak frequency regression was applied to the energy spectra for all measured downstream locations for a porous disk (left) and a turbine (right) in the boundary layer at all vertical locations, and plotted as a colormap in Figure 12. Vertical locations are represented in Figure 12 as a percentage of diameter of the object, where $y/D = 0$ is hub height. The color bar was cut off at a frequency of 10 Hz to more easily detect variances in the peak frequencies in the region of interest (saturates at full scale for higher frequencies). Near wake structures in the near wake of the models have higher peak frequencies, and these peaks decay with downstream position. Further, the turbine model has higher peak frequencies in the near wake than the porous disk due to the rotation of the blades. This type of investigation into the energy containing structures in the wake provides another view of the similarities of a drag matched porous disk and a rotating turbine in boundary layer flow at as close as 6 diameters downstream in the current configuration. Additional spurious high frequency peaks are seen at some of the highest vertical measurement positions, where the peak finding regression encounters overly flat spectra. The FPF has a free stream turbulence intensity of $\leq 0.5\%$. The low free stream turbulence results in a low variance in the spectrum and therefore a flatter energy spectral density. There is no discernible peak in the energy spectrum at these free stream locations. In this region, premultiplying the spectrum is essentially just an amplification of noise. The spectra far away from any object have particularly low energy in the lower frequencies. This type of lower energy at larger length scales is indicative of small scale turbulence with no large structures, as would be expected in the free stream.

In Figure 13a the hub height and top tip height measurements from the disk and the rotating turbine wake are extracted and plotted with respect to downstream distance. The hub height and top tip height locations are thought to be the most influenced by the non periodic movement, or meandering, of the velocity deficit downstream of the object. Also included are the maximum energy frequencies obtained from the incoming high Reynolds number boundary layer at hub height and top tip height for reference. Initially the turbine peak frequency from the energy spectrum is very high at both hub height and top tip height due to the blade passage frequency.
which affects the near wake. Beyond the near wake in the $x/D = 6$ downstream regime the Strouhal number is calculated to be $\approx 0.25$, similar to results reported by Coudou et al. [8]. Further downstream, the peaks trend towards lower frequencies and eventually return to that of incoming boundary layer peak frequencies at hub and tip locations very far downstream. It is noted that the near wake of the disk has significantly lower peak frequencies than the turbine model at both vertical locations.

![Figure 13: Peak frequency and maximum energy vs downstream position for hub height and top tip height wake velocity data for individual turbine models in boundary layer.](image)

The peak energy content obtained from the premultiplied spectra of the objects at hub height and top tip height is shown in Figure 13b. The peak energy for both objects starts high in the near wake and with progression downstream decays towards the incoming energy levels. In the near wake the energy is higher at top tip height than at hub height for both models, indicating that the energy is higher in the shear layer. The wakes of a porous disk and a rotating turbine in a mean shear have been validated as similar in the literature, e.g. Aubrun et al. [40], including for these models, cf. Charmanski et al. [36]. It is seen here that the wakes for both models also become very similar in peak frequency and energy in spectra with increased downstream distance. These results could be used as another confirmation of the similarities of the wakes of porous disks and rotating model turbines in shear flows after a certain downstream distance.

### 3.4. Model Turbine Array

The model turbine array was built up mostly with porous disks, but included a 3×3 array of rotating model turbines just upstream of the measurement location to reintroduce a rotating flow component, i.e. tip vortices and wake swirl. Single hot wire profiles were acquired at centerline of the array, downstream of the 3rd row of rotating turbines, as described in section 2.5, at locations of $x/D = 2$ and $x/D = 4$. The array consisted of 5 columns, with up to 19 rows. Two streamwise spacings were used, $8D$ and $10D$, while the spanwise spacing was kept at $4D$. Two “dummy rows” of turbine models were put in place behind the measurement location as to not affect the pressure gradient due to a sudden decrease in blockage.

#### 3.4.1. Array wake meandering

The peak frequency regression is again applied to the energy spectra for all velocity data. Peak frequency vs downstream position in the array of porous disks and turbines for two array spacings is shown in Figure 14. The labels on the abscissa, e.g. ‘R3,x/D = 2’, contain two parts of information: The first, ‘R3’ is the turbine row number where
Figure 14: Maximum frequency per downstream position for an array of turbines in a boundary layer at two spacings. 8D (left) and 10D (right)

the measurement takes place (i.e., downstream of row 3), and ‘x/D =2’ is the measurement location downstream of the turbine model, either 2 or 4 diameters (0.5 m or 1 m respectively). The measurements were taken for two streamwise array spacings, 8D and 10D. The wall-normal position (ordinate) is is normalized with turbine model diameter, y/D, where 0 indicates hub height. Through the extent of the array, higher frequencies are observed in a streak moving up and towards the right which follow the internal wind turbine array boundary layer growth.

The peak frequencies and maximum energies were extracted from the surface plots at hub height and top tip height for the 8D and 10D spacings for a single downstream measurement location, either x/D = 2 or x/D = 4, which is shown in Figures 15 and 16, respectively. The red dashed lines indicate peak frequency or maximum energy in the spectra of the incoming boundary layer flow at hub and tip locations.

The maximum frequencies occur at a downstream location of x/D = 2 for the 8D spacing at top tip height. It is expected for higher energy to exist in the tip region due to the tip vortex shedding phenomenon. However, this does not explain the lower maximum frequency values for the 10D spacing arrangement. Moving downstream and increasing the number of turbines in the array, a downward trend in maximum frequencies is seen. Also plotted on Figure 15 is the the data for hub and tip locations of the incoming boundary layer. It is shown that with an increased number of turbines, the most energetic structures in the flow are no longer the maximum frequency peaks in the incoming boundary layer. Moving further downstream in the turbine array modulates the incoming large scale eddies and slower, larger structures begin to dominate the flow. These larger structures represent a forcing by the array spacing itself (a type of resonance).

With regards to the observed frequencies, a few different characteristic velocities can be investigated.

- The wind tunnel free stream speed of 6.7 m/s.
- The incoming boundary layer velocity at hub height, which is about 5.7 m/s.
Figure 15: Peak frequency and maximum energy at hub and top tip heights at different positions in the array of model turbines in a boundary layer, measured at a distance of $x/D = 2$ downstream of the indicated turbine row.

Figure 16: Peak frequency and maximum energy at hub and top tip heights at different positions in the array of model turbines in a boundary layer, measured at a distance of $x/D = 4$ downstream of the indicated turbine row.

- The velocity at hub height, deep in the array, at 2 diameters upstream of each turbine, which is 3.8 m/s on average.

With these velocities and an array spacing of either 8D or 10D (corresponding to 2.0 m or 2.5 m,
respectively), the expected frequencies that could be caused by this spacing would range from $3.35 \ Hz$ to $1.52 \ Hz$. In Figures 15 and 16, peak frequencies of about $1 - 2 \ Hz$ were observed far downstream in the array, indicating that the array-internal velocity at hub height is most appropriate to calculate array forcing/resonance. By 17 rows downstream, the meandering has arranged itself in a very large scale motion with a dominant frequency lower than the large scale atmospheric turbulence in the approaching flow. The fully developed flow has a peak frequency dominated by array spacing.

Unlike for the individual model turbines in Figure 13b, the maximum energy in the array is seen to increase with downstream distance. This shows that there is an increase in energy associated with the structures formed by the turbine spacing.

It should be noted that due to the large extent of the array, the peak frequency of the incoming boundary layer flow would also be expected to change over the extent of the array as the boundary layer continues to grow. The boundary layer data of Vincenti et al. [30] was analyzed to see how peak frequencies along the test section of the tunnel compared to those measured in the turbine array. Although peak frequencies in the boundary layer decrease with downstream distance, when comparing the peak frequencies at the first location — after only three rows of turbines in Figures 15 and 16 with the boundary layer data — it is found that the maximum frequencies are different from those found in the incoming flow, as the wind turbine array creates a large disturbance. It is argued here that the array creates a large enough disturbance in the flow that any modification in frequency over the extent of the array is due to the influence of the structures modifying the flow and not the natural development of the boundary layer.

4. Summary and Conclusions

Experiments were conducted to investigate meandering in the wakes of both individual model wind turbines and in a wind turbine array with a large number of model turbines, placed in the boundary layer of a large boundary layer wind tunnel, using single point statistics. The meandering phenomenon for a turbine wake does not exhibit a well-defined peak in the frequency domain, but exhibits a broad spread of increased energy in the pre-multiplied spectrum over a low-frequencies range. For individual turbine models, both rotating three-bladed models and non-rotating porous disks, frequencies and peak energies observed in the velocity spectrum in the wake return to those of the incoming high Reynolds number turbulent boundary layer. Wake meandering also presents itself in the large wind turbine array. Here, frequencies and peak energies observed in the velocity spectrum in the wake do not return to those of the incoming turbulent boundary layer. Instead, a lower dominant frequency corresponding to the wind velocity at hub height and array spacing is observed deep within the array, indicating forcing of the meandering by the array itself (a type of resonance). Higher peak energies are observed in the array due to large flow structures representative of the turbine spacing.

A scale decomposition of the velocity signal was performed using the Hilbert transform to separate large and small scale motions. It was shown that large scale oscillations amplitude modulate the small scale energy in the wake. The small scale motions were then low pass filtered to uncover modulation from low frequency large scale structures in the flow. A correlation coefficient was applied to the relation between large and small scales. It was found that within the wake of the model turbines the correlation was high, while outside the wake the coefficient decreases.

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