Reversibility of continuous-variable quantum cloning

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We analyze a reversibility of optimal Gaussian cloning of a coherent state using only local operations on the clones and classical communication between them and propose a feasible experimental test of this feature. Performing Bell-type homodyne measurement on one clone and anti-clone, an arbitrary unknown input state (not only a coherent state) can be restored in the other clone by applying appropriate local unitary displacement operation. We generalize this concept to a partial LOCC reversal of the cloning and we show that this procedure converts the symmetric cloner to an asymmetric cloner. Further, we discuss a distributed LOCC reversal in optimal cloning of coherent states which transforms it to optimal cloning for $M' < M$. Assuming the quantum cloning as a possible eavesdropping attack on quantum communication link, the reversibility can be utilized to improve the security of the link even after the attack.

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I. INTRODUCTION

The linearity of quantum mechanics imposes a fundamental constraint on the processing of quantum information that has no classical counterpart. Perhaps the most famous example is the no-cloning theorem which states that an unknown quantum state cannot be copied. However, although exact copying is forbidden, one can still perform an approximate cloning. A quantum cloning machine is a device that with the help of an ancilla produces two or more approximate copies of an unknown quantum state. The problem of designing the optimal cloning transformations that maximize the fidelity of the cloned has attracted a considerable amount of attention during recent years and optimal universal cloning machines for qubits and qudits have been found. Experimentally, quantum cloning of polarization state of a photon was accomplished by means of stimulated parametric down-conversion and also by projecting two photonic qubits on a symmetric subspace with the help of a balanced beam splitter. Recently, the concept of cloning has been also extended to the domain of continuous variables where Gaussian cloning machine for coherent states has been proposed.

The cloning operation can be represented by a unitary transformation on the total system consisting of the original state, blank copies and ancillas. This operation can be trivially reversed by applying inverse unitary transformation to the total system. In contrast, in this paper, we analyze a different LOCC reversiblity of cloning which uses only local operations on the clones and classical communication between them. The LOCC reversibility of universal cloning was theoretically discussed by Bruss et al. who shown that the reversal can be accomplished by local operations on the two clones and classical communication by two different methods: (a) probabilistically, using separate measurements on ancilla and clone with unit fidelity but only with maximal success rate 1/3 or (b) deterministically, using complete Bell-state analysis on one clone and ancilla followed by an appropriate unitary operation on the other clone, similarly as in the quantum teleportation.

In this paper, we extend the concept of LOCC reversibility to continuous-variable (CV) quantum cloning of coherent states where a feasible experimental test of deterministic LOCC reversibility of the cloning can be envisaged. Note that a different kind of deterministic LOCC reversibility was recently analyzed in the context of CV quantum non-demolition measurements in Ref. Here, we shall demonstrate that the Braunstein-Kimble teleportation scheme realizes deterministic LOCC reversal of the optimal Gaussian symmetric and even asymmetric cloning of coherent states. The method is general and works for an arbitrary initial state (not only for coherent states). We point out the differences between the LOCC reversibility of the qubit and CV quantum cloning. Then we will show that it is possible to perform only a partial reversal of the 1 → 2 cloning by means of LOCC operations. Finally, we extend the LOCC reversibility to the 1 → M cloning.

Besides being of fundamental interest, the full and partial reversal of the cloning via LOCC operations may also find practical applications in quantum communication with continuous variables. For instance, the asymmetric cloning of coherent states represents the optimal individual Gaussian eavesdropping attack on the quantum key distribution protocol with coherent states, homodyne detection and direct reconciliation and it also saturates the information exclusion principle when used as an eavesdropping attack on quantum key distribution protocol based on squeezed states proposed in Ref. Imagine that the eavesdropper Eve keeps one clone and the anti-clone and sends the other clone to Bob, see Fig. 1. This attack may significantly reduce the rate of the se-
performs a suitable measurement after the attack via a cloning machine, the eavesdropper Eve does not learn anything about the secret information from Eve. Therefore, if Eve subsequently agrees to cooperate with Bob (perhaps for some fee) she can supply to Bob the data that will allow him to correct the outcomes of his homodyne measurements such that Alice and Bob will be able to distill secret key from the corrected data. As we shall see, the data transmitted during the reversal procedure does not carry any information about the signal, so in principle Eve does not learn anything about the secret key by performing the reversal operation.

The paper is organized as follows. In Section II we discuss the full LOCC reversal of the asymmetric $1 \to 2$ cloning of coherent states. In Section III we extend this procedure to only a partial LOCC reversal that effectively converts symmetric cloning machine to the asymmetric one. A generalized reversal procedure for $1 \to M$ cloning machines is devised in Section IV. Finally, the conclusions are drawn in Section V.

II. TOTAL REVERSAL OF ASYMMETRIC CLONING

The proposed setup for the complete LOCC reversal of asymmetric cloning of coherent states is schematically depicted in Fig. 2. The setup can be divided into several blocks including the cloning, the Bell measurement on one clone and the anti-clone, displacement of the state of the other clone and, finally, the analysis of the state of this clone by means of homodyne detection in order to confirm the success of the reversal. In what follows, we discuss each of these operations in detail.

The optimal and universal (in the sense that all coherent states are cloned with the same fidelity) Gaussian $1 \to 2$ asymmetric quantum cloning of coherent states is based on phase-insensitive amplification of the input single-mode optical field $S$. The phase-insensitive amplification of coherent states was demonstrated with a good fidelity in the back-action evading measurements $[9, 10]$. For this reason, we propose an implementation of the asymmetric cloning using this available experimental technique.

A scheme of the cloning device is situated in the upper left part of Fig. 2. First we split a fraction of the input coherent state in linearly polarized mode $S$ to the orthogonal polarization mode $S'$ with the use of a half-wave plate WP1 aligned at angle $\theta_1$ with respect to the optical axis. The polarization mode $S'$ is subsequently deflected to different spatial mode by a polarizing beam splitter PBS1. The mode $S$ is then amplified with controllable amplitude gain $G$ in a type-II phase-matched nonlinear crystal (KTP). The signal $S$ and idler $I$ modes are two orthogonally polarized beams co-propagating through the crystal. The pump $P$ for the crystal is prepared by frequency doubler $2\omega$ from the master laser source $L$ and it is injected and extracted by two dichroic mirrors DM1 and DM2. The intensity of pumping controls the gain $G$ of the amplification. The idler beam containing the anti-clone (phase-conjugated state) after amplification is subtracted by polarizing beam splitter PBS2 and replaced by the mode $S'$ having the same polarization. The two orthogonally polarized modes $S, S'$ are combined on half-wave plate WP2 aligned at angle $\theta_2$ and finally $S$ and $S'$ are separated to different spatial modes at polarizing beam splitter PBS3. The optimal Gaussian asymmetric cloning machine for coherent states is obtained by setting the angles $\theta_1, \theta_2$ and the gain $G$ as follows $[9]$

$$
\tan \theta_1 = \sqrt{2}\sinh(\gamma),
G = \sqrt{2}\cosh(\gamma),
\tan \theta_2 = -\exp(2\gamma),
$$

where $\gamma$ characterizes the asymmetry of the cloner.

It is most convenient to perform the calculations in the Heisenberg picture, where the optimal Gaussian $1 \to 2$ asymmetric cloning of coherent states is described by the following linear canonical transformation of the annihila-
The annihilation operators $A_i = (X_i + iP_i)/\sqrt{2}$, $i = S, S', I$ satisfy the commutation relations $[A_i, A_i^\dagger] = 1$. The fidelity $F_j$ between the input coherent state and an output state of the $j$th clone can be expressed as

$$F_j = \frac{1}{1 + \bar{n}_{ch,j}},$$

where $\bar{n}_{ch,j}$ is the mean number of chaotic photons in $j$th clone. Specifically, the asymmetric cloning produces two replicas with the fidelities $\bar{F}_S = \frac{2}{\exp(-2\gamma) + 2}, \bar{F}_{S'} = \frac{2}{\exp(2\gamma) + 2}.$

The optimal symmetric Gaussian cloner with $\gamma = 0$ yields two equivalent clones with fidelity $2/3$ with respect to the initial coherent state. In case of symmetric cloning $\theta_1 = 0$ and the proposed setup can be simplified because we can remove WP1 and also PBS1. Interestingly, the asymmetric cloning machine can be obtained from the symmetric cloner by feeding the input ports $S'$ and $I$ with two-mode squeezed vacuum.

We now propose an experimentally feasible total reversal of the asymmetric cloning. Suppose that Eve possesses one clone in mode $S'$ and also the idler mode $I$ (i.e. the anti-clone) while Bob has the other clone in mode $S$, see Fig. 1. They would like to restore in Bob’s mode $S$ the original state sent by Alice only by means of LOCC operations. It follows from Eq. (2) that the noise arising in the signal mode $S$ can be fully eliminated by the coherent displacement on mode $S$

$$A''_S = A'_S - y,$$

which finishes the reversal. In the experiment, a suitable displacement can be accomplished by overlapping the mode $S$ with a coherent state modulated in amplitude and phase by electro-optical modulators AM and PM on a strongly unbalanced (e.g. 99 : 1) beam splitter BS9, as illustrated in Fig. 2. To achieve the required displacement, the photo-currents are pre-amplified in $g_x$ and $g_p$ in a proper way. Note that the reversal operation is essentially the quantum teleportation of continuous-variable state. Finally, a tomography of reversed state can be performed in the state analyzer.

Since $A''_S = A_S$ the LOCC reversibility of CV asymmetric cloning can be performed deterministically for arbitrary input state. Moreover, if a thermal state with a high temperature is injected in the auxiliary inputs we are able to precisely restore any input state of mode $S$ although both clones exhibit only a negligible fidelity and all the information about the input state is stored in the intermodal correlations. In addition, this reversal can be performed even a-posteriori, after the homodyne measurement on Bob’s clone, simply by data manipulation corresponding to the displacement. This is important for the quantum cryptographic applications, as discussed in the Introduction.

The reversibility of quantum cloning for qubit and CV coherent states seems to be analogical. However, there are important differences between the reversibility of qubit and CV quantum cloning arising from a different distribution of the discrete and continuous variable quantum information. Mainly, it was shown in [11] that we can reversibly restore the initial state with unit fidelity (albeit with non-unit success rate) only by separate local measurements on one clone and ancilla, which is impossible for the CV cloning. Further, it was shown in Ref. [11] that sufficient information to conditionally restore the input state in the anti-clone can be gained by measurement performed on the two clones. The same is clearly not true in the present case because in order to recover the original state of the signal mode $S$ in the idler mode $I$ we would have to measure the “noise” operator $Z = \sqrt{2}(A_I \cosh \gamma - A_{S'} \sinh \gamma) + A_S$. However, the Hermitian and anti-Hermitian parts of $Z$ do not commute hence it is impossible to measure $Z$.

### III. PARTIAL REVERSAL OF ASYMMETRIC CLONING

So far, we have considered only full reversal of the cloning transformation that perfectly restores the original state on the receiver’s side. In this section, we ex-
FIG. 3: Logical quantum network schematic of the LOCC partial reversal of 1 → 2 cloning of coherent states. The two clones are contained in modes S and S’, respectively. I is the anti-clone and A and B denote auxiliary modes. The lines connecting the modes indicate CV QND operations with control mode indicated by full circle and target mode by empty circle, respectively. The quadratures X_A and P_B are measured by two balanced homodyne detectors (BHD) and the quadratures of the Bob’s mode S are displaced accordingly. See text for details.

tend this procedure to a partial optimal reversal. We shall show that by means of LOCC operations it is possible to convert the output of the asymmetric cloning machine (2) to the output of another optimal Gaussian asymmetric cloning machine with a different asymmetry parameter $\gamma'$.

The main idea behind the partial reversal of the cloning is to perform a partial quantum non-demolition (QND) measurement of the two commuting noise operators $X'_j$ and $P'_j$. This allows us to reduce to some extent the noise in Bob’s clone while the states of modes S’ and I are also partially preserved although some noise is added due to the back-action of the QND measurement. The whole procedure is schematically depicted in terms of the quantum network in Fig. 3. Two auxiliary modes initially prepared in the vacuum state and labeled A and B are used on Eve’s side. The information on the measured quadratures is transferred onto the quadratures X_A and P_B, respectively, via a sequence of four QND interactions

$$X'_j = X_j, \quad P'_j = P_j - \kappa P_k, \quad X'_k = X_k + \kappa X_j, \quad P'_k = P_k,$$  \hspace{1cm} (6)

where j and k are labels of the two interacting modes. We say that j is the control mode (denoted by full dot in Fig. 3) and k is the target mode (denoted by open circle with cross in Fig. 3). The inverse of (6) is obtained by change of the sign of the coupling constant, $\kappa \rightarrow -\kappa$, which is indicated in Fig. 3 by a dagger. The value of $\kappa$ determines the strength of the measurement of the noise quadratures, i.e. the asymmetry of the two clones after the partial reversal. Recently, the QND interaction has been realized between two polarization modes of optical field (18, 19) and also between polarization of bright beam and collective-spin degree of freedom of a macroscopic atomic ensemble (21, 22, 23).

Since the QND interaction is phase-sensitive it is most convenient to work with quadrature operators. After the sequence of the four QND interactions depicted in Fig. 3, we have

$$X''_S = X_S - \frac{\exp(\gamma)}{\sqrt{2}} (X_{S'} - X_I) - \kappa X_B,$$
$$P''_S = P_S - \frac{\exp(\gamma)}{\sqrt{2}} (P_{S'} + P_I) + \kappa P_A,$$
$$X''_A = \sqrt{2} \cosh(\gamma) X_I - \sqrt{2} \sinh(\gamma) X_{S'} + X_S - \kappa X_B,$$
$$P''_A = \frac{\exp(\gamma)}{\sqrt{2}} (P_{S'} + P_I) + \frac{\kappa}{\sqrt{2}} (P_{S'} - P_S - \kappa P_A),$$
$$X''_B = X_A + \frac{\kappa \exp(-\gamma)}{\sqrt{2}} (X_{S'} + X_I),$$
$$P''_B = P_B + \frac{\kappa \exp(-\gamma)}{\sqrt{2}} (P_I - P_{S'}).$$  \hspace{1cm} (7)

The operators $X''_A$ and $P''_B$ are measured by means of two balanced homodyne detectors and the measurement results $x_A$ and $p_B$ are communicated to Bob who displaces the quadratures $X''_A$ and $P''_B$ of his clone as follows:

$$X''_S = X''_S - g x_A, \quad P''_S = P''_S + gp_B.$$  \hspace{1cm} (8)

The “teleportation gain” $g$ should be chosen such as to minimize the fluctuations of the output quadratures $X''_S$ and $P''_B$. In the case of optimal Gaussian asymmetric cloning of coherent states we find that the optimal gain reads

$$g = \frac{\kappa}{\exp(2\gamma) + \kappa^2}$$  \hspace{1cm} (9)

and we effectively get

$$X''_S = X_S + \frac{e^\gamma}{e^{2\gamma} + \kappa^2} \frac{1}{\sqrt{2}} (X_{S'} + X_I) - \frac{\kappa}{e^{2\gamma} + \kappa^2} X_A,$$
$$P''_S = P_S + \frac{e^\gamma}{e^{2\gamma} + \kappa^2} \frac{1}{\sqrt{2}} (P_{S'} - P_I) + \frac{\kappa}{e^{2\gamma} + \kappa^2} P_B.$$  \hspace{1cm} (10)

We claim that with the gain (9) the partial reversal procedure produces two optimal Gaussian asymmetric clones in modes S and S’. To prove this we calculate the fidelities of the clones in modes S and S’ after partial reversal, assuming that the mode S was initially in coherent state while $S'$ and I were in vacuum state. With the help of formula (3) we obtain after a simple algebra

$$F_S = \frac{2[\kappa^2 + \exp(2\gamma)]}{2[\kappa^2 + \exp(2\gamma)] + 1},$$
$$F_{S'} = \frac{2}{2 + \exp(2\gamma) + \kappa^2}.$$  \hspace{1cm} (10)

These are exactly the fidelities (4) with an effective asymmetry parameter $\gamma'$ given by

$$\gamma' = \frac{1}{2} \ln[\exp(2\gamma) + \kappa^2].$$  \hspace{1cm} (11)

This confirms that our procedure yields the same output as the optimal Gaussian asymmetric cloner (2) with $\gamma = \gamma'$.
have been already reported \[19\], this experimental implementation of the partial Bell type measurement for the continuous variables appears to be feasible.

IV. DISTRIBUTED REVERSAL OF 1→(M,M−1) CV CLONING

Let us now consider an extension of the LOCC reversal procedure to the symmetric 1 → (M, M − 1) cloning machine that produces M clones and M − 1 anticlones from a single replica \[11, 12, 22\]. We assume that M − 1 clones and M − 1 anti-clones are distributed among M − 1 eavesdroppers such that each eavesdropper possesses one clone and one anticlone. The remaining single clone is sent to Bob, as is depicted in Fig. 5. After the distribution, a part L ≤ M − 1 of eavesdroppers decide to collaborate with Bob who would like to increase the fidelity of his clone. We show that it is sufficient if each collaborating eavesdropper performs local Bell type measurement on her clone and anti-clone and sends the measurement results to Bob via classical communication channel. As a result, Bob can obtain a clone with fidelity corresponding to optimal Gaussian 1 → M − L cloning only using LOCC operations.

The symmetric 1 → (M, M − 1) cloner is based on the phase-insensitive amplification, similarly as the 1 → 2 cloner considered in Sec. II. After the amplification with gain G = \[
\sqrt{M},
\]
the signal mode is split into M spatially separated modes by M × M balanced coupler that can be described by the following transformation of annihilation operators

\[
A_{S,j}^{\prime} = \frac{A_S^\dag}{\sqrt{M}} + \sum_{k=1}^{M-1} u_{jk} A_k,
\]

(12)

where \(j = 1, \ldots, M\) and \(A_{S,j}^\dag\) is annihilation operator of signal mode leaving the amplifier which is sent to the \(M\)th input port of the coupler. The modes in the remaining \(M-1\) input ports characterized by annihilation operators \(A_k\) are assumed to be in vacuo state. We define \(u_{jM} = 1/\sqrt{M}\) and the matrix \(u_{jk}\) is unitary. Similarly, the idler mode is also split into \(M-1\) separate modes by the same technique. The symmetric 1 → (M, M − 1) cloner can be thus described by the following transformation of annihilation operators

\[
A_{S,j}^{\prime} = A_S + \sqrt{1-M^{-1}} A_j^\dag + \sum_{k=1}^{M-1} u_{jk} A_k,
\]

\[
A_{I,l}^{\prime} = A_S^\dag + \sqrt{M^{-1}} A_l + \sum_{k=2}^{M-1} v_{lk} B_k,
\]

(13)

where \(l = 1, \ldots, M-1\), \(B_k\) are annihilation operators of auxiliary modes which are assumed to be in vacuo states. The matrix \(v_{lk}\) is unitary if we define \(v_{11} = 1/\sqrt{M-1}\). Each clone in the \(M\) signal modes

![Diagram](image-url)

FIG. 4: Partial CV Bell-state measurement. KTP is a type-II phase-match nonlinear crystal, BS are balanced beam splitters, DM is a dichroic mirror, WP1, WP1', WP2, WP2' are half-wave plates aligned at evasion angle, PBS is polarizing beam splitter, BHD are balanced homodyne detectors. See text for more details.
plex numbers y reveals that results are available to Bob, he can displace his mode hermitian operator be also conveniently interpreted as measuring the non-commuting two-mode quadratures on every pair, which can take into account that all modes S after the displacement (15) can be easily calculated if we noting that 1 Pers Eve willing to collaborate with Bob. The partial reversal of the link using only a classical information received from Eve1,...,EveL.

\[ A_{S_j}' \] produced by this cloner exhibits the same fidelity
\[ F_{1\rightarrow M} = \frac{M}{2M-1}. \]  

Let 1 ≤ L ≤ M – 1 be the number of eavesdroppers willing to collaborate with Bob. The partial reversal of the 1 → (M, M – 1) cloning can be achieved by L-fold eight-port homodyne measurement separately performed on pairs of modes Sj, Ij, where j = 1, ..., L, see Fig. 5. Similarly as before, it is a measurement of two commuting two-mode quadratures on every pair, which can be also conveniently interpreted as measuring the non-hermitian operator Yj = A_{Sj}' – A_{Ij}'†, yielding a set of complex numbers yj = xj + ipj. If all these measurement results are available to Bob, he can displace his mode Sm

\[ A_{S_m}'' = A_{S_m}' + g \sum_{j=1}^{L} y_j, \]  

where g is electronic gain adjusted properly so as to maximally enhance the fidelity of the Bob’s clone. A simple algebra reveals that

\[ Y_j = -\frac{A_j}{\sqrt{M(M-1)}} + \sum_{k=1}^{M-1} u_{jk} A_k - \sum_{k=2}^{M-1} v_{jk}^* B_k. \]  

The mean number of chaotic photons in the clone SM after the displacement (15) can be easily calculated if we take into account that all modes Ak and Bk are initially in vacuum or coherent state. One obtains

\[ \bar{n}_{ch} = \frac{M-1}{M} - \frac{2gL}{M} + \frac{g^2L^2}{M(M-1)} + g^2 \sum_{j,j'=1}^{L} \sum_{k=2}^{M-1} v_{jk}^* v_{j'k}. \]  

The triple summation in Eq. (17) can be evaluated by noting that v_{jk} is unitary and v_{j1} = 1/√{M-1}. The sum over k reads \( \sum_{k=2}^{M-1} v_{jk}^* v_{j'k} = \delta_{j,j'} - 1/(M-1) \). On inserting this back into Eq. (17) and summing over j and j’ we finally obtain

\[ \bar{n}_{ch} = \frac{M-1}{M} - \frac{2gL}{M} + \frac{g^2L^2}{M(M-1)} + g^2 \left( L - \frac{L^2}{M-1} \right). \]

The maximum fidelity is achieved by minimizing the noise \( \bar{n}_{ch} \) and the optimal gain is given by

\[ g_{opt} = \frac{1}{M-L}. \]

For this gain we get \( \bar{n}_{ch} = (M-L-1)/(M-L) \). The corresponding fidelity of coherent-state clone after the reversal

\[ F_S'' = \frac{M-L}{2(M-L)-1}, \]  

is exactly the fidelity of optimal symmetric Gaussian 1 → M – L cloning of coherent states. A complete restoration of the original occurs only for L = M – 1, i.e. when all eavesdroppers collaborate with Bob.

V. CONCLUSIONS

In summary, we have analyzed the LOCC reversibility of quantum cloning of coherent states and proposed a feasible setup for an experimental test. We have proved that the Braunstein-Kimble teleportation scheme for continuous variables [14] can be used for deterministic LOCC reversal of the asymmetric 1 → 2 cloning machine [8, 9] for arbitrary input state. Furthermore, it is possible to perform only a partial LOCC reversal based on a partial Bell type measurement which effectively converts the symmetric cloner to asymmetric one. This principle can be extended also to reversal of 1 → (M, M – 1) cloning machines.

The prospects for the experimental demonstration of the total LOCC reversal of the symmetric 1 → 2 cloning seem to be very good. First, a high-quality amplification of the coherent signal was experimentally demonstrated [18]. Second, in the CV teleportation experiment [15], the achieved fidelity (F ≈ 0.64) is close to the maximal value corresponding to the finite amount of entanglement shared between Alice and Bob. These spectacular experimental achievements strongly suggest that with the present day technology it should be possible to demonstrate the increase of cloning fidelity from 2/3 to the value near unity via the LOCC reversal procedure.

Quantum cloning machines represent frequently considered individual coherent eavesdropping attacks on quantum key distribution protocols [16, 17, 20]. An eavesdropper Eve uses two imperfect clones of the state sent by Alice. Eve transmits one clone to Bob, and measures the other when the measurement basis is revealed during the reconciliation procedure. Due to the reversibility of quantum cloning, it is possible in principle
to recover a full security of the quantum communication link only by local operations on clones and classical communication between Eve and Bob in the case that Eve has decided to collaborate with Bob in a time after her attack on the link and before the measurement on her clone and anti-clone.

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