Standard-like Models as Type IIB Flux Vacua

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Abstract

We construct new semi-realistic Type IIB flux vacua on $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifolds with three- and four- Standard Model (SM) families and up to three units of quantized flux. The open-string sector is comprised of magnetized D-branes and is T-dual to supersymmetric intersecting D6-brane constructions. The SM sector contains magnetized D9-branes with negative D3-brane charge contribution. There are large classes of such models and we present explicit constructions for representative ones. In addition to models with one and two units of quantized flux, we also construct the first three- and four-family Standard-like models with supersymmetric fluxes, i.e. comprising three units of quantized flux. Supergravity fluxes are due to the self-dual NS-NS and R-R three-form field strength and they fix the toroidal complex structure moduli and the dilaton. The supersymmetry conditions for the D-brane sector fix in some models all three toroidal Kähler moduli. We also provide examples where toroidal Kähler moduli are fixed by strong gauge dynamics on the “hidden sector” D7-brane. Most of the models possess Higgs doublet pairs with Yukawa couplings that can generate masses for quarks and leptons. The models have (mainly right-) chiral exotics.
I. INTRODUCTION

One of the challenging and essential problems in string theory is the construction of realistic string vacua, which can stabilize the moduli fields, generate Standard Model (SM)-like gauge structure and induce a (de Sitter) cosmological constant with supersymmetry breaking. Such constructions would provide a bridge between string theory and realistic particle physics. M-theory provides a framework where, in addition to perturbative heterotic string vacua, the physical string vacua could be probed in the perturbative Type I, Type IIA and Type IIB superstring theory. In particular, the discovery of D-brane dynamics makes it possible to construct consistent four-dimensional supersymmetric $N = 1$ chiral models with non-Abelian gauge symmetry on Type II orientifolds, by employing conformal field theory techniques in the open string sector. The first such supersymmetric models were based on $Z_2 \times Z_2$ orientifolds [1, 2]. [Non-supersymmetric constructions were given in [3, 4, 5, 6] (see also [7] and for earlier work [8, 9]).] Subsequently, a number of SM-like models, GUT models and their variations have been constructed in various orbifold backgrounds, and the associated phenomenology has been discussed. [For a partial list of non-supersymmetric constructions, see [10], further supersymmetric constructions are given in [11]-[14] and further developments in connection with the study of effective couplings and phenomenological implications, see [15]-[21] and references therein.]

[Recently important progress has been made in constructions of supersymmetric chiral solutions of Type II Gepner models; see [22, 23] and references therein. Specifically, the recent impressive results of [23] provide large classes of three-family Standard-like Models with no chiral exotics. Note however, that these exact conformal field theory models are located at the special points in the moduli space where the geometric picture is lost. In particular couplings, such as Yukawa couplings, do not possess hierarchies associated with the size of the internal spaces, such as in the case of the toroidal orbifolds with D-branes. In addition, due to the lack of geometric interpretation, the introduction of supergravity fluxes does not seem to be possible. ]

In spite of these successes, the moduli stabilization in open string and closed string sectors remained an open problem, even though in some cases some complex structure moduli (in the Type IIA picture) and dilaton fields may be stabilized due to non-perturbative gauge dynamics, associated with the gaugino condensation in the hidden sector (see, e.g., [24].) Turning on supergravity fluxes introduces a supergravity potential, which provides another way to stabilize the compactification moduli fields by lifting continuous moduli space of the string vacua in the effective four-dimensional theory (see, e.g., [25]). However, the introduction of supergravity fluxes imposes strong constraints on consistent constructions, since such fluxes modify the global Ramond-Ramond (RR) tadpole cancellation conditions.
Meanwhile, the fluxes will typically generate a back-reaction on the original geometry of the internal space, thus changing the nature of the internal space.

On the Type IIA side the supersymmetry conditions of flux compactifications are less understood. Nevertheless recent work \[26, 27\] revealed the existence of unique flux vacua for massive Type IIA string theory with SU(3) structure, whose geometry of the internal six-dimensional space is nearly-Kähler and four-dimensional space is anti-de Sitter (AdS) (for the discussion on necessary and sufficient conditions of N=1 compactifications of massive IIA supergravity to AdS(4) with SU(3) structure, see also \[28\]). One such example is the $\frac{SU(2)^3}{SU(2)} \simeq S_3 \times S_3$ coset space that has three supersymmetric three-cycles that add up to zero in homology \[27, 29\]. Therefore the total charge of the D6-branes wrapping such cycles is zero and no introduction of orientifold planes on such spaces is needed. Moreover, since the three-cycles intersect pair-wise, the massless chiral matters appear at these intersections. This construction \[27\] therefore provides an explicit example of supersymmetric flux compactifications with intersecting D6-branes. Further progress has also been made in the construction of $N = 1$ supersymmetric Type IIA flux vacua with SU(2) structures \[27\], leading to examples with the internal space conformally Calabi-Yau. However, explicit constructions of models with intersecting probe D6-branes for such flux compactifications is still awaiting further study.

On the Type IIB side the intersecting D6-brane constructions correspond to models with magnetized branes with the role of the intersecting angles played by the magnetic fluxes on the branes. The dictionary for the consistency and supersymmetry conditions between the two T-dual constructions is straightforward, see e.g., \[30, 31\]. The supersymmetric Type IIB flux compactifications are also better understood; see, e.g., \[32\]-\[35\], \[30, 31\] and references therein. In particular, examples of supersymmetric fluxes and the internal space conformally Calabi-Yau are well known. The prototype example is a self-dual combination of the Neveu-Schwarz-Neveu-Schwarz (NS-NS) $H_3$ and RR $F_3$ three-forms, corresponding to the primitive (2,1) form on Calabi-Yau space. Since the back-reaction of such flux configurations is mild, i.e., the internal space remains conformal to Calabi-Yau, these Type IIB flux compactifications are especially suitable for adding the probe magnetized D-branes in this background. However, the quantization conditions on fluxes and the modified tadpole cancellation conditions constrain the possible D-brane configurations severely. In Refs. \[30, 31\] the techniques for consistent chiral flux compactifications on orbifolds were developed, however, no explicit supersymmetric chiral SM-like models were obtained.

Most recently, by introducing magnetized D9-branes carrying negative D3-brane charges in the hidden sector, in Ref. \[36\] the first example of three-family SM-like string vacuum with one unit of quantized flux turned on was obtained, and subsequently, the first four-family SM-like string vacuum with one unit of fluxes was constructed in \[37\]. These constructions could
be T-dual to the supersymmetric models of intersecting D6-branes on $Z_2 \times Z_2$ orientifold with the $Sp(2)_L \times Sp(2)_R$ or $Sp(2f)_L \times Sp(2f)_R$ gauge symmetry in the electroweak sector, respectively [14, 18]. [Without fluxes, the first models of that type were toroidal models with intersecting D6-branes [18] where the RR tadpoles were not explicitly cancelled. (For the subsequent generalization to tilted tori see [38].) The $Z_2 \times Z_2$ orientifold construction in [14] provided the first model of that type that cancelled RR-tadpoles by introducing an additional stack of D6-branes with unitary symmetry; in the T-dual picture those are precisely the magnetized D9-branes with the negative contribution to the D3-brane charge.]

In spite of these successes, we are confronted by a number of problems:

(i) The semi-realistic SM-like string vacua with four-dimensional $N = 1$ supersymmetric fluxes have not been constructed, yet. In view of this drawback, effects of non-supersymmetric fluxes, as a key mechanism for breaking supersymmetry, has been addressed [39, 40, 41]. This analysis [39] leads to soft supersymmetry breaking masses $M_{\text{soft}} \sim \frac{M_s^2}{M_{\text{Pl}}}$ where $M_s$ and $M_{\text{Pl}}$ are the string scale and Planck scale, respectively. This result implies an intermediate string scale or one has to introduce an inhomogeneous warp factor in the internal space in order to stabilize the electroweak scale.

(ii) These flux vacua stabilize the dilaton and toroidal complex structure moduli. However, the Kähler moduli do not enter the flux-induced superpotential and hence are hard to be completely fixed. So, we are still typically faced with the vacuum degeneracy problem. [The Kähler moduli fields in Type IIB string theory are T-dual to the complex structure moduli in Type IIA string theory (intersecting D6-brane scenario). In the T-dual picture, the latter moduli can often be stabilized by employing non-perturbative dynamical mechanism, such as the gaugino and matter condensation in the hidden sector. However, these mechanisms are difficult to employ on the Type IIB side due to the additional matter content on the associated magnetized D-branes.]

(iii) For explicit Type IIB orientifolds the imaginary self-dual fluxes are quantized in rather large flux units, e.g., for $Z_2 \times Z_2$ the orientifolds elementary flux unit is 64. Therefore the constructions of semi-realistic flux vacua is very constrained; only one unit of flux is allowed for known semi-realistic three- [36] and four-family [37] models. Thus the introduction of fluxes is restricted to very few known semi-realistic examples and is not typical.

Following our previous work [37], in this paper we systematically study new constructions of three- and four-family SM-like string vacua with supergravity fluxes on Type IIB $Z_2 \times Z_2$ orientifolds. The major technical difficulty in constructions of semi-realistic flux vacua on Type IIB orientifolds is to ensure the cancellation of the large positive D3-brane charge contribution to RR tadpoles by the fluxes. Similar to the D-brane models without fluxes [14], and the subsequent work with fluxes [36, 37], the important role in tadpole cancellation is played (in the Type IIB picture) by magnetized D9-branes which carry large negative
D3-brane charges. In the past constructions ([14, 36, 37]) such D9-branes were introduced as a part of the “hidden sector”.

In this paper, we consider new types of constructions where the magnetized D9-branes with large negative D3-brane charges are introduced as a part of the SM sector. For this new setup, we find that the constructions of SM-like flux vacua are much less constrained and obtained a large class of new models. In particular, in addition to many new models with one unit of quantized flux, we obtain first three- and four-family models with two units of quantized charge, as well as the first three- and four-family examples with supersymmetric flux, i.e. three units of quantized flux. Such supersymmetric three- and four-family SM-like models have toroidal Kähler moduli fixed by supersymmetry conditions and the string scale can be close to the Planck scale. These models have (mainly right-) chiral exotics. However, most of the models have Higgs doublet pairs with Yukawa couplings to quarks and leptons and thus may generate the SM fermion mass hierarchies, explain the CKM quark mixing matrix and the PMNS neutrino mixing matrix at the tree level, and even give large masses to some of the bi-fundamental chiral exotics. Finally, with the open-string moduli fixed by the flux-induced soft masses, we are able to construct the first SM-like string vacua with strong infrared gauge dynamics on the “hidden sector” D7-branes, which leads to gaugino condensation and provides examples where the third toroidal Kähler modulus is fixed by the strong gauge dynamics à la KKLT [45].

The paper is organized in the following way. In Section II, we systematize the constructions of supersymmetric string vacua with supergravity fluxes on Type IIB $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifolds. In Section III, we classify the classes of the SM-like flux vacua on Type IIB $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifolds. Subsequently, we discuss in detail explicit constructions of the representative models with one, two and three (supersymmetric) units of quantized flux, as well as a specific construction with gaugino condensation on the “hidden sector” D7-branes. In the Appendix we provide tables of all the explicitly constructed representative models. We conclude with discussions and open problems in Section IV.

II. MAGNETIZED D-BRANES AND TYPE IIB FLUX COMPACTIFICATIONS ON $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ ORIENTIFOLDS

Flux compactifications on simplest toroidal $T^6$ Type IIB orientifolds, on which most of the previous work has focused, are unlikely to provide a framework for constructions of semi-realistic flux vacua. We shall therefore focus on the simplest orbifold constructions, i.e. on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifolds.

The internal space $T^6$ is chosen to be factorized as a direct product of three two-tori, i.e.
$T^6 = T^2 \times T^2 \times T^2$, whose complex coordinates are $z_i$, $i = 1$, 2, 3 for the $i$-th two-torus, respectively. The generators $\theta$ and $\omega$ for the orbifold group $Z_2 \times Z_2$, act on the complex coordinates of $T^6$ as

$$
\theta : (z_1, z_2, z_3) \rightarrow (-z_1, -z_2, z_3) ,
\omega : (z_1, z_2, z_3) \rightarrow (z_1, -z_2, -z_3) .
$$

(1)

On $Z_2 \times Z_2$ orbifold Type IIB string theory contains in the untwisted sector the four-dimensional $N = 2$ supergravity multiplet, the dilaton hypermultiplet, $h_{11}$ hypermultiplets and $h_{21}$ vector multiplets, which are all massless. The orbifold action projects out several components of the metric of a general $T^6$ geometry and, as a result, we are left with fewer Kähler and complex structure parameters. These are encoded for the untwisted moduli in terms of the Hodge numbers, as $(h_{11}, h_{21})_{\text{unt}} = (3, 3)$. On the other hand, each of the three elements $\theta$, $\omega$ and $\theta \omega$ has a fixed-point set given by 16 $T^2$'s, and the corresponding twisted sectors also contribute to the Hodge numbers of the orbifold. For a particular choice of discrete torsion, this contribution is given by $(h_{11}, h_{21})_{\text{tw}} = (0, 3 \times 16)$. The contributions from both, the untwisted and twisted sectors hence add up to $(h_{11}, h_{21}) = (3, 51)$.

Orientifold planes are necessary for the introduction of the open-string sector, and the associated orientifold projection can be denoted by $\Omega R$, where $\Omega$ is the world-sheet parity projection and $R$ (acting on Type IIA as the holomorphic $Z_2$ involution) acts on the complex coordinates as:

$$
R : (z_1, z_2, z_3) \rightarrow (-z_1, -z_2, -z_3) .
$$

(2)

Thus, the model contains 64 $O3$-planes and 4 $O7_i$-planes, transverse to the $i^{th}$ two-torus. This orientifold action projects the above $N = 2$ spectrum to an $N = 1$ supergravity multiplet, the dilaton chiral multiplet, and 6 untwisted and 48 twisted geometrical chiral multiplets.

In order to cancel the negative RR charge contributions, due to these O-planes, we need to introduce D$(3+2n)$-branes which are filling up four-dimensional Minkowski space-time and wrapping $2n$-cycles on the compact manifold. We choose the construction with magnetized D-branes. [A detailed discussion for toroidal/orbifold compactifications with magnetized D-branes is given, e.g., in [30].] Concretely, for one stack of $N_a$ D-branes wrapped $m_a^i$ times on the $i^{th}$ two-torus $T^2_i$, we turn on $n_a^i$ units of magnetic fluxes $F_a^i$ for the center of mass $U(1)_a$ gauge factor on each $T^2_i$, such that

$$
m_a^i \frac{1}{2\pi} \int_{T^2_i} F_a^i = n_a^i .
$$

(3)

Hence, the topological information of this stack of D-branes is encoded in $N_a$-number of D-branes and the co-prime number pairs $(n_a^i, m_a^i)$. The D9-, D7-, D5- and D3-branes contain 0,
1, 2 and 3 vanishing $m_a^i$s, respectively. Introducing for the $i^{th}$ two-torus the even homology classes $[0_i]$ and $[T^2_i]$ for the point and the two-torus, respectively, the vectors of RR charges of the $a^{th}$ stack of D-branes and its image are

$$[\Pi_a] = \prod_{i=1}^{3} (n_a^i[0_i] + m_a^i[T^2_i]), \quad [\Pi'_a] = \prod_{i=1}^{3} (n_a^i[0_i] - m_a^i[T^2_i]),$$

respectively. Similarly, for the O3- and $O7_i$-planes appearing on $T^6/(Z_2 \times Z_2)$ orientifold which respectively correspond to $\Omega R$, $\Omega R\omega$, $\Omega R\theta\omega$ and $\Omega R\theta$ O-planes, we have

$$\Omega R : \ [\Pi_{O3}] = [0_1] \times [0_2] \times [0_3];$$
$$\Omega R\omega : \ [\Pi_{O7_1}] = -[0_1] \times [T^2_2] \times [T^2_3];$$
$$\Omega R\theta\omega : \ [\Pi_{O7_2}] = -[T^2_1] \times [0_2] \times [T^2_3];$$
$$\Omega R\theta : \ [\Pi_{O7_3}] = -[T^2_1] \times [T^2_2] \times [0_3].$$

The “intersection numbers”, which determine the chiral massless spectrum, are

$$I_{ab} = [\Pi_a] \cdot [\Pi_b] = \prod_{i=1}^{3} (n_a^i m_b^i - n_b^i m_a^i),$$
$$I_{a' b'} = [\Pi_a] \cdot [\Pi_{b'}] = -\prod_{i=1}^{3} (n_a^i m_{b'}^i + n_{b'}^i m_a^i),$$
$$I_{a'd'} = [\Pi_a] \cdot [\Pi_{a'}] = -8 \prod_{i=1}^{3} (n_a^i m_{a'}^i),$$
$$I_{aO} = \sum_p [\Pi_a] \cdot [\Pi_{Op}] = 8(-m_1^a m_2^a m_3^a + m_1^a n_2^a n_3^a + n_1^a m_2^a n_3^a + n_1^a n_2^a m_3^a),$$

where $[\Pi_{Op}] = [\Pi_{O3}] + [\Pi_{O7_1}] + [\Pi_{O7_2}] + [\Pi_{O7_3}]$ is the sum of O3-plane and $O7_i$-plane homology classes. Similar to the discussions in [2], the physical chiral spectrum should be invariant under the full orientifold symmetry group and is tabulated in Table I. Flux vacua on Type IIB orientifolds with four-dimensional $N = 1$ supersymmetry are primarily constrained by the RR tadpole cancellation conditions and conditions for $N = 1$ supersymmetry in four dimension, which we describe in detail in the following subsections.

A. RR Tadpole Cancellation Conditions

In the Type IIB picture the fluxes, we consider, are due to the self-dual three-from field strength, which contributes to the D3-brane field strength equation of motion, and thus modifies the D3 charge conservation (RR-tadpole cancellation) conditions on the compact
TABLE I: General spectrum for magnetized D-branes on the Type IIB $T^6/(Z_2 \times Z_2)$ orientifold. The representations in the table refer to $U(N_a/2)$, the resulting gauge symmetry due to $Z_2 \times Z_2$ orbifold projection. For supersymmetric constructions, scalars combine with fermions to form chiral supermultiplets.

| Sector | Representation |
|--------|----------------|
| $aa$   | $U(N_a/2)$ vector multiplet |
|        | 3 adjoint chiral multiplets |
| $ab + ba$ | $I_{ab} (\square_4, \square_4)$ fermions |
| $ab' + b'a$ | $I_{ab'} (\square_4, \square_4)$ fermions |
| $aa' + a'a$ | $\frac{1}{2}(I_{aa'} - \frac{1}{2}I_{a,Op}) \square_4$ fermions |
|        | $\frac{1}{2}(I_{aa'} + \frac{1}{2}I_{a,Op}) \square_4$ fermions |

orientifold. The RR charges, carried by magnetized D-branes, are classified by their associated homology classes. Explicitly, for one stack of $N_a$ D-branes with wrapping numbers $(n^i_a, m^i_a)$, it carries D3-, D5-, D7- and D9-brane RR charges

$$Q_{3a} = N_a n^1_a n^2_a n^3_a, \quad (Q_{5i})_a = N_a m^i_a n^j_a n^k_a,$$

$$\quad (Q_{7i})_a = N_a n^i_a m^j_a m^k_a, \quad Q_{9_a} = N_a m^i_a m^j_a m^k_a,$$

where $i \neq j \neq k$, and a permutation is implied for $(Q_{5i})_a$ and $(Q_{7i})_a$. So, the RR tadpole cancellation conditions can be described as

$$\sum_a N_a[\Pi_a] + \sum_a N_a[\Pi'_a] + \sum_p N_{Op}Q_{Op}[\Pi_{Op}] + N_{\text{flux}} = 0 ,$$

where the third term contribution comes from the O3- and O7-planes, with $N_{Op}$ and $Q_{Op}$ denoting their numbers and RR charges, respectively. And $N_{\text{flux}}$ is the amount of the fluxes turned on, and is quantized in units of the elementary flux.

For a supersymmetric Dp/Dp'-brane system on Type IIB $T^6/(Z_2 \times Z_2)$ orientifold, only D3- and D7-branes are allowed to be wrapped along the orientifold planes. [In the following we shall refer to this type of branes as "filler branes"][8]. Given that $N_{Op}Q_{Op} = 2^{9-p}(-2^{p-4}) \equiv -32$ in Dp-brane units for $Sp$-type O-planes, the RR tadpole cancellation conditions can be further simplified as

$$- N^{(0)} - \sum_a Q_{3a} - \frac{1}{2}N_{\text{flux}} = -16 .$$

8
\[-N^{(i)} + \sum_a (Q_{7i})_a = -16, \ i \neq j \neq k, \]  
\hspace{1cm} (9)

where \(N^{(0)}\) and \(N^{(i)}\) with \(i = 1, 2\) and \(3\) respectively denote the number of filler branes, i.e. D-branes which wrap along the O3- and \(O7_i\)-planes and only contribute to one of the four kinds of D3- and D7-brane charges. As for D5- and D9-brane RR tadpoles, their cancellations are automatic since these D-branes and their \(\Omega R\) images carry the same absolute value of the corresponding charges, but with opposite sign of charges.

**B. Conditions for Four-Dimensional \(N = 1\) Supersymmetry**

Four-dimensional \(N = 1\) supersymmetric vacua from flux compactification require that 1/4 supercharges of the ten-dimensional (T-dual) Type I theory be preserved in both open string and close string sectors. We shall discuss both sectors separately.

For the closed string sector, the specific Type IIB flux solution on orientifolds comprises of self-dual three-form field strength and it has been discussed, e.g., in [32, 44]. While RR \(F_3\) and NSNS \(H_3\) three-form fluxes are turned on, the induced three-form \(G_3 = F_3 - \tau H_3\), with \(\tau = a + i/g_s\) being Type IIB axion-dilaton coupling, contributes to the D3-brane RR charges

\[N_{\text{flux}} = \frac{1}{(4\pi^2\alpha')^2} \int_{X_6} H_3 \wedge F_3 = \frac{1}{(4\pi^2\alpha')^2} \frac{i}{2\tau_I} \int_{X_6} G_3 \wedge \bar{G}_3, \]  
\hspace{1cm} (10)

where \(\tau_I\) is the imaginary part of the complex coupling \(\tau\). Dirac quantization conditions of \(F_3\) and \(H_3\) on \(T_6/(Z_2 \times Z_2)\) orientifold require that \(N_{\text{flux}}\) be a multiple of 64, and the BPS-like self-duality condition: \(*_6 G_3 = iG_3\) ensures that its contribution to the RR charges is positive. Supersymmetric configuration implies that \(G_3\) background field should be a primitive self-dual (2,1) form. A specific supersymmetric solution which is useful for our purpose is [30, 44]

\[G_3 = \frac{8}{\sqrt{3}} e^{-\pi i/6} (d\bar{z}_1dz_2dz_3 + dz_1d\bar{z}_2dz_3 + dz_1dz_2d\bar{z}_3), \]  
\hspace{1cm} (11)

where the additional factor 4 is due to the \(Z_2 \times Z_2\) orbifold symmetry. The fluxes stabilize the complex structure toroidal moduli at values

\[\tau_1 = \tau_2 = \tau_3 = \tau = e^{2\pi i/3}, \]  
\hspace{1cm} (12)

leading to the RR tadpole contribution in Eq. (9)

\[N_{\text{flux}} = 192. \]  
\hspace{1cm} (13)
This result therefore implies that in order to construct supersymmetric SM-like flux vacua, we have to introduce at least the D3 charge conservation condition, which is thus hard to achieve.

In the open-string sector, for D-branes with world-volume magnetic field \( F^i = \frac{n^i}{m^i \chi^i} \), the four-dimensional \( N = 1 \) supersymmetry is ensured if and only if \( \sum_i \theta_i = 0 \mod 2 \pi \) is satisfied [30]. Here the “angle” \( \theta_i \) with the range \( \{0, 2\pi\} \), is determined in terms of the world-volume magnetic field as \( \tan(\theta_i) \equiv (F^i)^{-1} = \frac{n^i}{m^i \chi^i} \) and \( \chi^i = R_1^i R_2^i \), the area of the \( i^{th} \) two-torus \( T_2^i \) in \( \alpha' \) units, is the Kähler modulus of the \( i^{th} \) two-torus \( T_2^i \). This supersymmetry condition can be cast in the form:

\[
\sum_i (F^i)^{-1} - (F^1 F^2 F^3)^{-1} = 0, \quad \text{along with} \quad \sum_{i<j} (F^i F^j)^{-1} - 1 < 0 \quad \text{for} \quad n_i n_j n_k > 0 \quad \text{or} \quad \sum_{i<j} (F^i F^j)^{-1} - 1 > 0 \quad \text{for} \quad n_i n_j n_k < 0,
\]

which can be rewritten in the following form:

\[
-x_A Q_9 a + x_B (Q_5)_a + x_C (Q_5_a) + x_D (Q_5)_a = 0,
\]

\[
-Q_3 a/x_A + (Q_7)_a/x_B + (Q_7_a)/x_C + (Q_7)_a/x_D < 0,
\]

(14)

where \( x_A = \lambda \), \( x_B = \lambda/\chi^2 \chi^3 \), \( x_C = \lambda/\chi^1 \chi^3 \), \( x_D = \lambda/\chi^1 \chi^2 \). The positive parameter \( \lambda \) has been introduced to put all the variables \( Q_9, Q_7_i, Q_5_i \), and \( Q_3_i \) on equal footing. These supersymmetry conditions can be easily cast in the T-dual form of the Type IIA supersymmetry constraints discussed in [2].

III. CONSTRUCTIONS OF SM-LIKE STRING VACUA FROM TYPE IIB FLUX COMPACTIFICATION

Similar to the past constructions (see, specifically those of [13]), we construct the SM-like models as descendants of the Pati-Salam model based on \( SU(4)_C \times SU(2)_L \times SU(2)_R \) gauge symmetry. The hypercharge is

\[
Q_Y = Q_{I_{3R}} + \frac{Q_{B-L}}{2},
\]

(15)

where the non-anomalous \( U(1)_{B-L} \) is obtained from the splitting of the \( U(4)_C \) branes, i.e. \( U(4) \rightarrow U(3)_C \times U(1)_{B-L} \). Similarly, the anomaly-free \( U(1)_{I_{3R}} \) gauge symmetry is from the non-Abelian \( U(2)_R \) or \( Sp(2)_R \) gauge symmetry. There are three main frameworks to realize the Pati-Salam gauge sector in the Type IIB magnetized D-brane scenario (T-dual to the Type IIA intersecting D6-brane one):

(i) The starting observable sector gauge symmetry is \( U(4)_C \times U(2)_L \times U(2)_R \) [13]. In this framework, the three “anomalous” gauge symmetries \( U(1)_C, U(1)_L \) and \( U(1)_R \) can be treated as global ones, since the associated gauge bosons obtain masses via \( B \wedge F \) Chern-Simons couplings. [Those are effective couplings that arise from the D-brane world-volume
Chern-Simons couplings and are responsible for the Abelian gauge anomaly cancellation via the Green-Schwarz mechanism.] The gauge symmetry breaking chain is of the form:

$$SU(4) \times SU(2)_L \times SU(2)_R$$

$$\rightarrow SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$\rightarrow SU(3)_C \times SU(2)_L \times U(1)_{I_{3R}} \times U(1)_{B-L}$$

$$\rightarrow SU(3)_C \times SU(2)_L \times U(1)_{Y}$$ \hspace{1cm} (16)

where the first and second step can be achieved by splitting the $U(4)_C$ branes and $U(2)_R$ branes in one two-torus direction, and the third step by giving vacuum expectation values (VEVs) to the scalar components of right-handed neutrino chiral superfields (or a scalar component of an exotic non-chiral chiral superfields) at the TeV scale. Alternatively, we may skip the second step and directly break the $SU(2)_R \times U(1)_{B-L}$ gauge symmetry down to the $U(1)_Y$ by giving VEVs to the scalar components of right-handed neutrino chiral superfields. Within this framework one typically obtains enough SM Higgs doublet pairs (from the chiral or the non-chiral massless sector) with Yukawa couplings to quarks and leptons and hence explain the mass hierarchy and mixings for the fermions in the SM sector at the tree level. However, it should also be noted that in a few cases all the SM Higgs doublet pairs have the global $U(1)$ quantum numbers that do not allow for Yukawa couplings to quarks and leptons; these models therefore face serious phenomenological obstacles.

(ii) The starting gauge symmetry is one-family $U(4) \times Sp(8)_L \times Sp(8)_R$ or two-family $U(4) \times Sp(4)_L \times Sp(4)_R$, which can be broken down to the four-family $U(4) \times U(2)_L \times U(2)_R$ or $U(4) \times SU(2)_L \times SU(2)_R$ by parallel splitting the D-branes, originally positioned on the O-planes, in three or two-tori directions, respectively. [Both the string theory and field theory aspects of the brane splittings in this framework are discussed in detail in [14] for constructions without fluxes. The flux vacua of that type (with one unit of quantized flux) was constructed in [37].] In the field theory picture (“Higgsing”) the four-families ($f = 4$) are obtained when we decompose the original chiral supermultiplets $(4, 8, 1)$ and $(\bar{4}, 1, 8)$, or $(4, 4, 1)$ and $(\bar{4}, 1, 4)$ into four copies of $(4, 2, 1)$ and $(\bar{4}, 1, 2)$ after the gauge symmetry breaking. These Higgsings, as discussed in [14], preserve the D- and F-flatness, and thus the symmetry breaking can take place at the string scale. Thus, in these cases the resulting spectrum is that of four-dimensional $N = 1$ supersymmetric four-family Pati-Salam models. The symmetry breaking chains for these two pictures are given respectively by

$$SU(4) \times Sp(8)_L \times Sp(8)_R$$

$$\rightarrow SU(4) \times U(2)_L \times U(2)_R$$

$$\rightarrow SU(3)_C \times U(2)_L \times U(2)_R \times U(1)_{B-L}$$

$$\rightarrow SU(3)_C \times U(2)_L \times U(1)_{Y}$$ \hspace{1cm} (17)
and

\[
SU(4) \times Sp(4)_L \times Sp(4)_R \\
\to SU(4) \times SU(2)_L \times SU(2)_R \\
\to SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\
\to SU(3)_C \times SU(2)_L \times U(1)_Y. \tag{18}
\]

The first and second step can be achieved by splitting the $Sp$- and $U(4)$-branes at the string scale, and the third step by giving VEVs to the scalar components of the right-handed neutrino superfields at the TeV scale. Note that for the model with the original $U(4) \times Sp(8)_L \times Sp(8)_R$ symmetry, the resulting $U(1)_L$ and $U(1)_R$ are not anomalous since they are part of the non-Abelian $Sp$ symmetries. One expects that at least the gauge boson of $U(1)_L$ will obtain a mass at the electroweak scale, which is excluded by experiments. In order to evade this problem, we allow only for the $Sp(8)_R$ gauge symmetry in the SM sector, while the $Sp(8)_L$ gauge symmetry is not. Moreover, we consider the variants of the $U(4) \times Sp(4)_L \times Sp(4)_R$ model, i.e. the models with the gauge symmetry $U(4) \times U(2)_L \times Sp(4)_R$ or $U(4) \times Sp(4)_L \times U(2)_R$. [This analysis can also be applied to three-family models with gauge symmetry $U(4) \times Sp(6)_L \times Sp(6)_R$ where the $Sp(6)_L \times Sp(6)_R$ gauge symmetry can be broken down to the $Sp(2)_L \times Sp(2)_R$ by the Higgs mechanism, however this symmetry breaking pattern breaks supersymmetry [14], and thus may only be implemented within the framework of supersymmetry breaking at scale larger than the electroweak scale.]

(iii) The starting symmetry is the Pati-Salam-like $U(4)_C \times Sp(2)_L \times Sp(2)_R$. Without fluxes, the first models of that type were toroidal orientifolds with intersecting D6-branes [18] where the RR tadpoles were not explicitly cancelled. The $Z_2 \times Z_2$ orientifold construction in Ref. [14] cancelled the RR-tadpoles by introducing an additional stack of branes with unitary symmetry in the hidden sector. In the T-dual Type IIB picture those are the magnetized D9-branes with a negative D3 charge. In Ref. [36] these types of magnetized D9-branes were employed to find the MSSM-like model with one unit of quantized flux turned on. In this framework, the starting gauge symmetry is similar to the framework (i); the initial framework gauge symmetry $U(4)_C \times Sp(2)_L \times Sp(2)_R$ can be broken down to the SM gauge symmetry as

\[
SU(4) \times SU(2)_L \times SU(2)_R \\
\to SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\
\to SU(3)_C \times SU(2)_L \times U(1)_Y, \tag{19}
\]

where the first step can be achieved again, by splitting the $U(4)$ D-branes at string scale and the second step is achieved by giving VEVs to the scalar components of the right-handed
neutrino chiral superfields at the TeV scale. Note that in the present case there is no $U(1)_L$. However, in the models constructed in Ref. [18, 36], there exists only one SM Higgs doublet pair; in this case a generic problem is that only the third family can obtain the tree-level masses and it is difficult to give masses to the first two families at the quantum level [18]. In addition, the $SU(2)_R \times U(1)_{B-L}$ gauge symmetry can be broken down to the $U(1)_Y$ only by giving VEVs to the scalar components of the right-handed neutrino chiral superfields at the TeV scale in this kind of models.

The Pati-Salam-type models with only one SM Higgs pair suffer from serious phenomenological problems [48]. Even though one may be able to generate the most general Yukawa couplings via radiative corrections, the mass matrix of up-type quarks is proportional to that of down-type quarks, and the mass matrix of neutrinos is proportional to that of leptons. (Note the renormalization group equation running effects on these mass matrices are negligible.) Therefore, on the one hand, the masses for the quarks, leptons and neutrinos satisfy

$$m_u : m_c : m_t = m_d : m_s : m_b ; m_{\nu_e} : m_{\nu_\mu} : m_{\nu_\tau} = m_e : m_\mu : m_\tau.$$  

The above fermion mass relations are obviously wrong from the known experiments. On the other hand, the CKM quark mixing matrix and the PMNS neutrino mixing matrix are proportional to the identity matrix which implies that the quark and neutrino mixing angles vanish, again in contradiction with experiments. Note that these problems for the fermion masses and mixings cannot be solved by loop corrections because the $SU(2)_R \times U(1)_{B-L}$ gauge symmetry is broken at TeV scale. There is a consensus that the minimal supersymmetric Pati-Salam or Left-Right model should have at least two SM Higgs doublets [42]; therefore, the construction (without fluxes) in [14], which actually contains two SM Higgs doublets is the first one which can realize the embedding of the supersymmetric Pati-Salam model with realistic features in the Type IIA intersecting D-brane scenario, or equivalently, the T-dual Type IIB magnetized D-brane scenario.

The presence of fluxes further complicates the constructions of these types of models: in the Type IIB background, the $G_3$ fluxes give a large positive contribution to the D3-brane RR tadpoles, thus making it extremely hard to satisfy the D3 charge tadpole cancellation conditions by the magnetized D-brane sectors. In the first model with one unit of quantized flux [36], the large positive contribution to D3 charges from the flux, is cancelled by the “hidden sector” magnetized D9-branes, carrying negative D3 charges (first introduced for vacua without fluxes in [14]). Four-family models with one unit of flux and the starting SM-sector gauge symmetry (ii) were constructed by introducing a single stack of magnetized D9-branes with the negative D3 charge in [37]. These very few specific semi-realistic constructions are extremely constrained, thus implying that semi-realistic flux vacua are hard
to come by. In this paper we advance this program in a new direction, by introducing magnetized D9-branes with negative D3 charges as a part of the SM-sector within frameworks (i) and (ii). As a consequence we obtain a large number of the three- and four-family SM-like flux with as much as three units of flux turned on. In particular these constructions provide first four-dimensional $N=1$ supersymmetric SM-like string vacua (i.e. three units of flux) as well as first examples of semi-realistic SM-like string vacua with two units of flux and many new models with one unit of flux. In addition, we also obtained SM-like flux models where the “hidden sector” D7-brane gauge dynamics can induce gaugino condensation that can stabilize the third toroidal Kähler modulus (two other toroidal Kähler moduli are fixed by supersymmetry conditions in the D-brane sector); this is an explicit construction that may realize the KKLT mechanism [45].

In the following subsections we shall present explicit representative models within its class. Within each class there are typically more models and a sizable number of models within each class has been obtained by running computer code. The representative model in each class is typically chosen to have a minimal number of chiral (mainly right) exotics. In the following, we give a concise description of the representative models. (Please, refer to the Appendix for tables containing these models and a detailed explanation of the notation employed.) A concise discussion of phenomenological implications of these models will be given in [43].

A. Models with Supersymmetric Fluxes

In this subsection, we construct SM-like string vacua with the supersymmetric flux configuration, i.e. three units of quantized flux. Again, the key feature is the introduction of magnetized D9-branes with the negative D3 charge, which is a part of the SM-sector. These are the first three- and four-family SM-models within the supersymmetric flux background. The D-brane configurations of $Model - T_1 - 3$ and $Model - F_1 - 3$ are given in Table VII and Table XIV, respectively. The chiral spectrum for the three-family model ($Model - T_1 - 3$) is given in Table II. All three toroidal Kähler moduli in these models are fixed by the supersymmetry conditions for the D-brane sector [Note that the open string moduli and the Kähler moduli can form combined D-flat directions, corresponding to the brane recombination (see Ref. [2] in the context of specific orientifold compactifications). However, due to the flux back-reaction it is expected that the open string moduli could become massive [30]; in this case the supersymmetry conditions do stabilize Kähler moduli.] But, the SM Higgs doublets do not have Yukawa couplings to quarks and leptons due to the “wrong” quantum numbers under the global $U(1)$ symmetries, and one has to look for new ways to generate quark and lepton masses. We shall further discuss the masses of the SM families as well as
B. Models with Non-supersymmetric Fluxes

In this subsection, we shall consider the string vacua with non-supersymmetric fluxes, i.e. with two and one units of quantized fluxes. With fewer units of quantized flux, there is more freedom in satisfying the tadpole conditions. In the following we only present some typical three- and four-family models for each phenomenologically interesting case. Since the gauge symmetry breaking chain for each model can be easily determined from the analysis at the beginning of this section, we mainly focus on additional phenomenological aspects of these models.

1. Two Flux Units Models

We constructed the first three- and four-family SM-like string vacua with two units of quantized flux with the representative models: Model $-T_{1 - 2}$ (Table VIII), Model $-F_{1 - 2}$ (Table XV), Model $-F_{2 - 2}$ (Table XVI) and Model $-F_{3 - 2}$ (Table XVII).

For the three-family model (Model $-T_{1 - 2}$), its chiral spectrum is given in Table III. Note that there is one left-chiral exotic $(4, 2, 1, 1)$, which has Yukawa couplings to the right-chiral ones and SM Higgs doublet pairs (Higgs bidoublets), thus these exotics can obtain a mass at the electroweak scale. In addition, there are five pairs of (non-chiral) SM Higgs doublet
pairs, arising in the \(bc\) sector when the \(b\) and \(c\) stacks of D-branes are coincident on the first two-torus; these SM Higgs doublet pairs have correct global U(1) quantum numbers, allowing for the Yukawa couplings to quarks and leptons and thus the SM fermion masses and mixings can be generated at the tree level.

**TABLE III:** The chiral spectrum in the open string sector of \(Model - T_1 - 2\).

| Model \(-T_1 - 2\) | \(U(4)_C \times U(2)_L \times U(2)_R \times S^p(4)\) | \(Q_4\) | \(Q_{2L}\) | \(Q_{2R}\) | \(Q_{em}\) | \(B - L\) | Field |
|---------------------|---------------------------------|------|--------|--------|--------|------|------|
| \(ab\)             | \(3 \times (\overline{3},1)\)   | 1   | -1     | 0      | \(-\frac{1}{3}, \frac{2}{3}, -1, 0\) | \(1, -1\) | \(Q_L, Q_L\) |
| \(ab'\)            | \(1 \times (\overline{3},1)\)   | 1   | -1     | 0      | \(\frac{1}{3}, -\frac{2}{3}, 1, 0\)    | \(-\frac{1}{3}, 1\) | \(Q_R, Q_R\) |
| \(ac\)             | \(8 \times (1,1,2)\)            | -1  | 0      | 1      | \(\frac{1}{3}, -\frac{2}{3}, 1, 0\)    | \(-\frac{1}{3}, 1\) | \(Q_R, Q_R\) |
| \(ac'\)            | \(8 \times (1,1,2)\)            | 1   | 0      | 1      | \(-\frac{1}{3}, \frac{2}{3}, -1, 0\)   | \(\frac{1}{3}, -1\) | \(H\) |
| \(bc\) (Non-chiral)| \((1,2,2,1) + (\overline{3},\overline{3},1)\) | -1  | ±1     | ±1     | -      | -    | -    |
| \(bc'\)            | \(4 \times (1,2,2)\)            | 0   | 1      | 1      | -1, 0, 0, 1, 0 | 0    | 0    |
| \(a(D7)_2\)        | \(1 \times (1,1,1,4)\)          | -1  | 0      | 0      | \(\frac{1}{6}, -\frac{1}{2}\)          | \(\frac{1}{3}, -1\) | 0    |
| \(b(D7)_2\)        | \(2 \times (1,2,1,4)\)          | 0   | 1      | 0      | \(\pm\frac{1}{2}\)                     | 0    | 0    |
| \(c(D7)_2\)        | \(6 \times (1,1,2,4)\)          | 0   | 0      | 1      | \(\pm\frac{1}{2}\)                     | 0    | 0    |
| \(c\)              | \(32 \times (1,1,3)\)           | 0   | 0      | 2      | 0, ±1 | 0    | 0    |
| \(c\)              | \(112 \times (1,1,1)\)          | 0   | 0      | 2      | 0     | 0    | 0    |

For the four-family models, there do not exist any left chiral exotics. Similar to the \(Model - T_1 - 2\), the suitable SM fermion masses and mixings can be obtained due to the Yukawa couplings of the SM Higgs doublet pairs in \(Model - F_3 - 2\). However, the SM fermion masses and mixings can not be generated at the tree level in the \(Model - F_1 - 2\) and \(Model - F_2 - 2\) because the SM Higgs doublet pairs have wrong quantum numbers under the U(1) global symmetries and thus no Yukawa couplings to quarks and leptons.

### 2. One Flux Unit Models with \(U(2)_{L,R}\) Negative D3 Charge Branes

When one unit of flux is turned on, there is a wealth of models and these constructions of three- or four-family SM-like flux vacua can be classified in the following way:

1. \(Model - T_1 - 1\) (Table IX) and \(Model - F_1 - 1\) (Table XVIII). Except for some symmetric and anti-symmetric representations, these two models do not contain any bifundamental chiral exotics in the observable sector. Their chiral spectra are given in Table IV and Table V, respectively. In particular, even though for the four-family model \(Model - F_1 - 1\) there is an anomaly-free \(U(1)_R\) gauge symmetry, it is broken at the “right-handed” scale, when
the $SU(2)_R \times U(1)_{B-L}$ gauge symmetry is broken down to the $U(1)_Y$ by giving VEVs to the scalar components of right-handed neutrino chiral superfields. However, in these two models, the SM Higgs doublet pairs do not have Yukawa couplings to quarks and leptons, due to the wrong global $U(1)$ quantum numbers.

**TABLE IV:** The chiral spectrum in the open string sector of $Model - T_1 - 1$.

| Model $- T_1 - 1$ | $U(4)_C \times Sp(2)_L \times U(2)_R \times Sp(4)$ | $Q_4$ | $Q_{2R}$ | $Q_{3m}$ | $B - L$ | Field |
|-------------------|-----------------------------------------------|------|----------|----------|--------|-------|
| $ab$              | $3 \times (4, \overline{2}, 1, 1)$             | 1    | 0        | $-\frac{1}{3}, \frac{2}{3}, -1, 0$ | $\frac{1}{3}, -1$ | $Q_L, L_L$ |
| $ac$              | $3 \times (\overline{4}, 1, 2, 1)$             | -1   | 1        | $\frac{1}{3}, -\frac{2}{3}, 1, 0$ | $-\frac{1}{3}, 1$ | $Q_R, L_R$ |
| $bc$              | $8 \times (1, \overline{2}, 2, 1)$             | 0    | 1        | $-1, 0, 0, 1$ | 0      | $H$    |
| $c(D3)$           | $1 \times (1, 1, \overline{2}, 4)$             | 0    | -1       | $\pm \frac{1}{3}$ | 0      |        |
| $q$               | $23 \times (1, 1, 3, 1)$                        | 0    | 2        | 0, $\pm 1$ | 0      |        |
| $q$               | $73 \times (1, 1, 1, 1)$                        | 0    | 2        | 0, 0      | 0      |        |

**TABLE V:** The chiral spectrum in the open string sector of $Model - F_1 - 1$.

| Model $- F_1 - 1$ | $U(4)_C \times U(2)_L \times Sp(8)_R \times Sp(8)$ | $Q_4$ | $Q_{2L}$ | $Q_{3m}$ | $B - L$ | Field |
|-------------------|-----------------------------------------------|------|----------|----------|--------|-------|
| $ab$              | $4 \times (4, \overline{2}, 1, 1)$             | 1    | -1       | $-\frac{1}{3}, \frac{2}{3}, -1, 0$ | $\frac{1}{3}, -1$ | $Q_L, L_L$ |
| $ac$              | $1 \times (\overline{3}, 1, 8, 1)$             | -1   | 0        | $\frac{1}{3}, -\frac{2}{3}, 1, 0$ | $-\frac{1}{3}, 1$ | $Q_R, L_R$ |
| $bc$              | $4 \times (1, \overline{2}, 8, 1)$             | 0    | -1       | $-1, 0, 0, 1$ | 0      | $H$    |
| $a(D7)_2$         | $2 \times (4, 1, 1, 8)$                        | 1    | 0        | $\frac{1}{3}, -\frac{2}{3}$ | $\frac{1}{3}, -1$ |       |
| $b(D7)_2$         | $4 \times (1, \overline{2}, 1, 8)$             | 0    | -1       | $\pm \frac{1}{2}$ | 0      |       |
| $a$               | $2 \times (10, 1, 1, 1)$                        | -2   | 0        | $\frac{1}{3}, -1$ | $\frac{2}{3}, -2$ |       |
| $a$               | $2 \times (6, 1, 1, 1)$                        | 2    | 0        | $\frac{1}{3}, -1$ | $\frac{2}{3}, -2$ |       |
| $b$               | $10 \times (1, \overline{3}, 1, 1)$            | 0    | -2       | 0, $\pm 1$ | 0      |       |
| $b$               | $54 \times (1, \overline{1}, 1, 1)$            | 0    | -2       | 0, 0      | 0      |       |

(2) In the $Model - T_2 - 1$ (Table X), $Model - T_3 - 1$ (Table XI) and $Model - F_3 - 1$ (Table XX), there are four or five pairs of non-chiral Higgs bidoublets arising from the $bc$ sector which can couple via Yukawa couplings to quarks and leptons, which can generate fermion mass hierarchies and mixings. Some of the additional chiral exotics can obtain large masses by coupling to these non-chiral Higgs bidoublets.
Even though these models employ non-chiral Higgs bidoublets to give masses to fermions, we can easily find models with chiral Higgs bidoublets with appropriate Yukawa couplings to quarks and leptons, see, e.g., Model $- F_1 - 2$ (Table XV).

Another interesting four-family model is Model $- F_4 - 1$ (Table XXI); it does not contain any chiral exotics in the observable sector. The chiral Higgs bidoublets in the $bc$ sector allow for the Yukawa couplings to quarks and leptons and thus the fermion masses and mixings can be generated at the tree level. Note also that the $U(4)_C$ symmetry emerges by $Sp(16)$ D7-brane splitting at the string scale and thus $U(1)_C$ gauge symmetry is non-anomalous and is broken at the “right-handed” scale by the VEVs of the scalar components of the right-handed neutrino chiral superfields.

3. One Flux Unit Models with $U(4)_C$ Negative D3 Charge Branes

In this construction the $U(4)_C$ is due to the negative D3 charge magnetized D9-branes. Model $- T_4 - 1$ (Table XII) and Model $- F_5 - 1$ (Table XXII) are such three- and four-family SM-like models. In both models, the $SU(2)_R$ gauge symmetry is generated by D7-brane splitting, which yields eight and four copies of right-chiral representations, respectively. In particular, the four-family model (Model $- F_5 - 1$), whose chiral spectrum is given in Table VI, have several nice phenomenological features:

(i) No additional bifundamental chiral exotics in the observable sector;

(ii) The suitable fermion masses and mixings can be generated by tree level Yukawa couplings to the SM Higgs doublet pairs;

(iii) No additional electroweak scale $U(1)$ symmetry.

| $Model - F_5 - 1$ | $U(4)_C \times U(2)_L \times Sp(8)_R \times USp(4)$ | $Q_4$ | $Q_{2L}$ | $Q_{em}$ | $B - L$ | Field |
|-----------------|-----------------------------------------------|-------|---------|---------|--------|-------|
| $ab$            | $4 \times (4, \overline{2}, 1, 1)$            | 1     | 0       | $-\frac{1}{3}, \frac{2}{3}, -1, 0$ | $\frac{1}{3}, 1$ | $Q_L, L_L$ |
| $ac$            | $1 \times (4, 1, 8, 1)$                        | $-1$  | 1       | $\frac{1}{3}, -\frac{2}{3}, 1, 0$ | $-\frac{1}{3}, 1$ | $Q_R, L_R$ |
| $bc$ (Non-chiral)| $(1, 2, 8, 1) + (1, \overline{2}, 8, 1)$      | $\pm 1$|         |         |        |       |
| $a(D7)_1$       | $4 \times (4, 1, 1, 4)$                        | 1     | 0       | $\frac{1}{6}, -\frac{1}{2}$ | $\frac{1}{3}, -1$ |        |
| $a$             | $2 \times (10, 1, 1, 1)$                       | 2     | 0       | $\frac{1}{3}, -1$ | $\frac{2}{3}, -2$ |        |
| $b$             | $30 \times (6, 1, 1, 1)$                       | 2     | 0       | $\frac{1}{3}, -1$ | $\frac{2}{3}, -2$ |        |
| $l$             | $2 \times (1, 3, 1, 1)$                        | 0     | 2       | 0, $\pm 1$ | 0       |        |
| $l$             | $2 \times (1, \overline{1}, 1, 1)$            | 0     | $-2$    | 0       | 0       |        |
C. Models with Kähler Moduli Stabilized by D7-brane Gauge Dynamics

In this subsection, we consider the possibility of realizing a stabilization of the toroidal Kähler moduli stabilization à la KKLT mechanism [45]. In the original paper, the KKLT vacua are achieved via three steps:

1. Turning on self-dual three-form fluxes on Type IIB Calabi-Yau manifold, the flux-induced superpotential will fix the dilaton and all the complex structure moduli;
2. Introducing a Kähler-moduli dependent non-perturbative superpotential, due to D7-brane strong infrared gauge dynamics or Euclidean D3-brane instanton effect. The vacuum is a supersymmetric anti-de Sitter one with the Kähler moduli fixed;
3. Adding a set of anti-D3-branes to lift the anti-de Sitter vacuum to a de Sitter one.

We will only focus on the second step, i.e. the generation of the non-perturbative superpotential, since it may help us stabilize all the toroidal moduli fields.

In our framework, the Type IIB SM-like flux vacua typically require a stack of (filler) D3-branes, sitting on the O3-planes, in order to cancel the D3 charge tadpoles due to redundant negative D3 charges introduced by the magnetized D9-branes. This stack of D3-branes has typically a negative beta function; it thus possesses a non-perturbative gauge dynamics that results in a non-perturbative superpotential, due to gaugino condensation. However, this superpotential depends only on the dilaton-axion field, and is independent Kähler-moduli.

Thus, in order to generate the non-perturbative superpotential that depends on the toroidal Kähler moduli, the strong gauge dynamics has to arise due to D7-branes [45].

Recently, it was suggested [39] that the flux-induced soft mass terms may help decouple the open-string moduli on D7-branes, leaving an infrared gauge theory with strong dynamics on the world-volume of this stack of D7-branes (in the hidden sector). However, in the concrete constructions of SM-like flux vacua a stack of hidden sector D7-branes typically do not have a negative beta function; this is due to the fact that the magnetized D9-branes with negative D3 charge have large “intersecting numbers” with the D7-branes and thus a large number of chiral matter, charged under D7-brane gauge symmetry. Additional chiral matter typically drastically modifies the infrared gauge dynamics. [Light chiral matter can influence the (supersymmetric) gauge dynamics in two key aspects (see, e.g., [46] and references therein): (i) the chiral matter contributes to the beta function and thus affect the infrared dynamics (phase structure); (ii) it may induce matter condensation and contribute to the non-perturbative superpotential.]

In this paper, we present the first SM-like flux vacua with strong gauge dynamics, resulting in gaugino condensation, on D7-branes of the hidden sector: Model $-T_5 - 1$ (Table XIII) and Model $-F_6 - 1$ (Table XXIII). These models have three- and four-family fermions (and additional chiral exotics), respectively. In these models the two (out of three) toroidal
Kähler moduli have been fixed by supersymmetry conditions. For the four-family model (Model $- F_6 - 1$), its hidden sector is composed of two stacks of D7-branes denoted by $(D7)_1$ and $(D7)_2$, respectively. Both of them carry $Sp(4)$ gauge symmetry and the associated beta functions are $-3(0)$ and $-5(-2)$, respectively. Here the beta functions in the brackets include the one-loop contribution from the open-string moduli. These open-string moduli are expected to become massive due to the flux back-reaction (see, e.g.,[30]), and in this case the strong gauge dynamics can in principle induce the non-perturbative superpotential. Note, however, that gauge dynamics of the $(D7)_1$-branes results in the superconformal or Coulomb phase regime and the non-perturbative superpotential cannot be dynamically generated. As a result, only $(D7)_2$-branes can generate a Veneziano-Yankielowicz-type superpotential, induced by gaugino and matter condensations. A similar analysis can be applied to the three-family model (Model $- T_5 - 1$): in the case that open string moduli are decoupled (the beta function is $-9(0)$), the Veneziano-Yankielowicz superpotential is generated due to the gaugino and matter condensates on the D7-brane.

IV. DISCUSSIONS AND CONCLUSIONS

In this paper we have advanced a program for explicit constructions of supersymmetric SM-like string vacua with supergravity fluxes turned on. In particular we obtained large classes of such Type IIB models on $Z_2 \times Z_2$ orientifolds with one, two, and three units of quantized flux turned on. These models provide an important stepping stone toward broader classes of realistic constructions that not only contain the three- (or four-) family SM sector, but also stabilize some of the moduli (typically all toroidal moduli can be fixed).

Before our work, techniques for constructions of the chiral D-brane sectors flux vacua on Type IIB orientifolds were developed in [30, 31], however, the semi-realistic constructions remained elusive until recently. The technical reason for such difficulties are large positive three-form flux contributions to the D3 charge in the internal space (D3-branes RR-tadpole), which makes the D3 charge conservation constraint hard to satisfy. In the first examples of SM flux vacua on $Z_2 \times Z_2$ orientifolds this constraint has been satisfied [36, 37] by the introduction of the hidden sector magnetized D9-branes which carry negative D3 charges. [This type of D-branes were first introduced in the Type IIA context with intersecting D6-branes without fluxes in [14].] Within this framework one three-family [36] and one four-family [37] SM-like models with one unit of quantized flux were obtained. However, one remains to be confronted by serious problems:

(i) There remains the outstanding problem of constructing semi-realistic SM-like string vacua with supersymmetric fluxes, that for $Z_2 \times Z_2$ orientifolds correspond to three units of quantized flux. The focus therefore shifted to the study of non-supersymmetric flux
effects as the key to break supersymmetry, and a detailed study of soft supersymmetry breaking masses due to fluxes [39, 40]. The flux-induced supersymmetry breaking, however, leads typically to soft masses $M_{\text{soft}} \sim \frac{M_s^2}{M_{\text{Pl}}}$, which implies an intermediate string scale or inhomogeneous warp factor in the internal space in order to stabilize the electroweak scale;

(ii) In spite of the successes in stabilizing the dilaton and toroidal complex structure moduli, the toroidal Kähler moduli are not completely fixed, thus one remains faced with the large vacuum degeneracy problem;

(iii) The fact that (supersymmetric) supergravity fluxes are abundant, makes it imperative to extend the constructions to semi-realistic models with more than one unit of quantized flux.

In this paper, we have made important progress in addressing the above issues. The key ingredient in the new SM-like flux constructions is the introduction of the negative D3 charge magnetized D9-brane as a part of the SM sector. These constructions turned out to be less constraining and resulted in three- and four-family SM-like string vacua with up to three units of quantized flux, thus leading to the first fully supersymmetric SM-like flux vacua. In addition to toroidal complex structure moduli and the dilaton being fixed by the flux, the supersymmetry conditions in the D-brane sector typically fix all the toroidal Kähler moduli, and the string scale is close to the Planck scale. We also constructed the first three- and four-family SM-like models with two units of the quantized flux and many new models with one unit of the quantized flux. Typically the representative models have (mainly right-) chiral exotics in the SM sector; however, we have also presented a few three- and four-family models with no SM-sector chiral exotics. (Note, the models do have additional tensor fields, i.e. chiral superfields in the symmetric and/or anti-symmetric representation of the unitary gauge symmetry, typically associated with the negative D3 charge magnetized D9-brane.)

We have also been able to construct SM-like flux vacua with strong infrared gauge dynamics on the hidden sector D7-branes, which leads to non-perturbative superpotential that can fix a remaining toroidal Kähler modulus.

The SM Higgs doublet pairs appear at the intersections of the $SU(2)_L$ and $SU(2)_R$. For most models, the SM fermion masses and mixings can be generated due to the tree-level Yukawa couplings of such SM Higgs doublet pairs to quarks and leptons. However, in a few cases, most notably for the three-family SM-like model with the supersymmetric flux, all such Higgs fields have the wrong global $U(1)$ quantum numbers that prevent them from coupling to quarks and leptons via Yukawa couplings; these models therefore face serious phenomenological difficulties. Note also that typically in these models some of the SM chiral exotics can obtain masses at least at the electroweak scale, due to the Yukawa couplings of the SM Higgs doublet pairs to such exotics.

In spite of a number of advances made with these new constructions, there are open
problems which deserve further study. In particular, further discussion of the specific Yukawa
couplings to quarks, leptons and chiral exotics, as well as the subsequent further implications
for the masses and mixings of the SM chiral fermions are needed [43]. For SM-like models
with supersymmetric fluxes, one has to address the supersymmetry breaking mechanism.
In principle the supersymmetry breaking could be due to the hidden D-brane strong gauge
dynamics; however, the present models do not possess such a sector. Complete stabilization
of all the moduli (and not only the toroidal closed sector one) remains an open problem.
We postpone these issues for future research.

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Appendix: D-brane Configurations and Intersection Numbers for SM-like Flux Vacua

In this Appendix, we tabulate D-brane configurations and intersection numbers for the representative three- and four-family models within our new setup. In the first column of each table, $a$, $b$ and $c$ denote the $U(4)$ ($Sp(16)$), $U(2)_L$ ($Sp(2)_L$ or $Sp(4)_L$), and $U(2)_R$ ($Sp(4)_R$ or $Sp(8)_R$) stacks of branes, respectively. $D3$, $(D7)_1$, $(D7)_2$, and $(D7)_3$ represent the filler branes along respective $\Omega R$, $\Omega R\omega$, $\Omega R\theta\omega$ and $\Omega R\theta$ orientifold planes, resulting in $Sp(N)$ gauge groups. $N$, in the second column, corresponds to the number of filler D-branes in each stack. The third column depicts the wrapping numbers of the various D-branes. The intersection numbers between the various D-brane stacks are given in the remaining right columns where $b'$ and $c'$ are respectively the $\Omega R$ images of $b$ and $c$. In addition, the number of symmetric and antisymmetric chiral superfield representations for specific D-brane configurations is given. For convenience, we also tabulate the relations among the three toroidal Kähler moduli parameters $\chi_i$, imposed by the supersymmetry conditions. The model labels “Model $- (T, F)_i$ $- n$” appearing in the tables denote the “Model $- (\text{three, four family})_i$ $- (n \text{ units of fluxes})$”. [Since all the models have an even number of chiral supermultiplets in the fundamental representation of the $Sp(N)$ gauge groups, these models are automatically free of discrete global gauge anomalies [47].] Finally, we emphasize that in this paper we do not fix the convention between the chirality and the sign of the intersection number. Instead, we consider the $SU(2)$ D-branes that carry the more realistic chiral spectrum (typically only three- or four- families) as the $SU(2)_L$ D-branes. These representative models therefore possess (mainly) right-chiral exotics.

TABLE VII: D-brane configurations and intersection numbers for Model $- T_1 - 3$.

| Model $- T_1 - 3$ | $[U(4)_C \times U(2)_L \times U(2)_R]_{\text{Observable}}$ | $\chi_3 = \chi_2 = 2\chi_1$ | $\chi_3 = 2\sqrt{10}$ |
|-------------------|-------------------------------------------------|-------------------------------|-------------------------------|
| $j$ | $N(n^1, m^1)(n^2, m^2)(n^3, m^3)$ | $n$ | $b$ | $b'$ | $c$ | $c'$ | $\chi_3 = \chi_2 = 2\chi_1$ | $\chi_3 = 2\sqrt{10}$ |
| $a$ | 8 | (1, 0)(1, 1)(1, -1) | 0 | 0 | -3 | 1 | 2 | -10 | $\chi_3 = \chi_2 = 2\chi_1$ | $\chi_3 = 2\sqrt{10}$ |
| $b$ | 4 | (1, 1)(2, -1)(1, 0) | -2 | 2 | - | - | 6 | -6 | $\chi_3 = \chi_2 = 2\chi_1$ | $\chi_3 = 2\sqrt{10}$ |
| $c$ | 4 | (-2, -1)(4, 1)(3, 1) | -46 | -146 | - | - | - | - | $\chi_3 = \chi_2 = 2\chi_1$ | $\chi_3 = 2\sqrt{10}$ |
TABLE VIII: D-brane configurations and intersection numbers for Model $- T_1 - 2$.

| Model $- T_1 - 2$ | $[U(4)_C \times U(2)_L \times U(2)_R]_{\text{Observable}} \times [Sp(4)]_{\text{Hidden}}$ |
|-------------------|----------------------------------------------------------------------------------|
| $j$    | $N$ | $(n^1, m^1)(n^2, m^2)(n^3, m^3)$ | $n$ | $n'$ | $b$ | $b'$ | $c$ | $c'$ |
| $a$    | 8   | $(1, 0)(1, 1)(1, -1)$ | 0   | 0   | -3  | 1   | 8   | -8  |
| $b$    | 4   | $(2, 1)(-1, 1)(1, 0)$ | 0   | 0   | -   | 0   | -   | -4  |
| $c$    | 4   | $(-2, -1)(3, 1)(3, 1)$ | -32 | -112| -   | -   | -   | -   |
| $(D7)_2$ | 4   | $(0, 1)(1, 0)(0, -1)$ | $\chi_3 = \chi_2 = \chi_1 = \sqrt{\frac{21}{2}}$ |

TABLE IX: D-brane configurations and intersection numbers for Model $- T_1 - 1$.

| Model $- T_1 - 1$ | $[U(4)_C \times Sp(2)_L \times U(2)_R]_{\text{Observable}} \times [Sp(4)]_{\text{Hidden}}$ |
|-------------------|----------------------------------------------------------------------------------|
| $j$    | $N$ | $(n^1, m^1)(n^2, m^2)(n^3, \tilde{m}^3)$ | $n$ | $n'$ | $b$ | $c$ | $c'$ |
| $a$    | 8   | $(1, 0)(3, 1)(3, -1/2)$ | 0   | 0   | -3  | 3   | 0   |
| $b$    | 2   | $(0, 1)(0, -1)(2, 0)$ | 0   | 0   | -   | 8   | -   |
| $c$    | 4   | $(-2, -1)(4, 1)(3, 1/2)$ | -23 | -73  | -   | -   | -   |
| $D3$   | 4   | $(1, 0)(1, 0)(2, 0)$ | $\chi_2 = \chi_3, \frac{12}{\chi_2} + \frac{14}{\chi_3\chi_2} = 1$ |

TABLE X: D-brane configurations and intersection numbers for Model $- T_2 - 1$.

| Model $- T_2 - 1$ | $[U(4)_C \times U(2)_L \times U(2)_R]_{\text{Observable}} \times [Sp(4) \times Sp(2)]_{\text{Hidden}}$ |
|-------------------|----------------------------------------------------------------------------------|
| $j$    | $N$ | $(n^1, m^1)(n^2, m^2)(n^3, \tilde{m}^3)$ | $n$ | $n'$ | $b$ | $b'$ | $c$ | $c'$ |
| $a$    | 8   | $(1, 0)(1, 1)(1, -1/2)$ | 0   | 0   | 3   | -2  | -4  | 4   |
| $b$    | 4   | $(2, -1)(1, 0)(5, 1/2)$ | 3   | -3  | -   | -   | 0   | -4  |
| $c$    | 4   | $(-2, 1)(3, -1)(3, -1/2)$ | 16  | 56  | -   | -   | -   | -   |
| $D3$   | 4   | $(1, 0)(1, 0)(2, 0)$ | $\chi_3 = \chi_2 = \frac{5}{2} \chi_1 = \sqrt{39}$ |
| $(D7)_2$ | 2   | $(0, 1)(0, -1)(2, 0)$ | | | | | |
TABLE XI: D-brane configurations and intersection numbers for Model $- T_3 - 1$.

| Model $- T_3 - 1$ | $[U(4)_C \times U(2)_L \times U(2)_R]_{\text{Observable}} \times [Sp(8)]_{\text{Hidden}}$ | j | N | $(n^1, m^1)(n^2, m^2)(n^3, m^3)$ | $n$ | $n'$ | $b$ | $b'$ | c | $c'$ |
|-------------------|-----------------------------------------------------------------|---|---|---------------------------------|-----|-----|-----|-----|---|-----|
| a                 | 8                  | (1,0)(1,1)(1,−1) | 0  | 0 | 3  | 1  | -3 | 3               |
| b                 | 4                  | (2,−1)(1,0)(1,2) | -6 | 6 | -  | 0  | 12  |
| c                 | 4                  | (−2,1)(2,−1)(2,−1) | 10 | 54 | -  | -  | -   |
| $(D7)_3$          | 8                  | (0,1)(0,−1)(1,0) | $\chi_3 = \chi_2 = \frac{1}{2} \chi_1 = \sqrt{6}$ |

TABLE XII: D-brane configurations and intersection numbers for Model $- T_4 - 1$.

| Model $- T_4 - 1$ | $[U(4)_C \times U(2)_L \times Sp(4)_R]_{\text{Observable}} \times [Sp(4)]_{\text{Hidden}}$ | j | N | $(n^1, m^1)(n^2, m^2)(n^3, m^3)$ | $n$ | $n'$ | $b$ | $b'$ | c |
|-------------------|-----------------------------------------------------------------|---|---|---------------------------------|-----|-----|-----|-----|---|
| a                 | 8                  | (−1,−1)(2,1)(2,1) | -2 | -30 | 3  | -5 | -4   |
| b                 | 4                  | (1,0)(3,1)(1,−1) | 4  | -4 | -  | -  | 0    |
| c                 | 4                  | (1,0)(0,1)(0,−1) | 0  | 0  | -  | -  | -    |
| $D3$              | 4                  | (1,0)(1,0)(1,0) | $3 \chi_3 = \chi_2, \frac{12}{\chi_2^2} + \frac{8}{\chi_1 \chi_2} = 1$ |

TABLE XIII: D-brane configurations and intersection numbers for Model $- T_5 - 1$, here $\beta^g$ are beta functions for the associated gauge symmetries in the hidden sector.

| Model $- T_5 - 1$ | $[U(4)_C \times U(2)_L \times Sp(8)_R]_{\text{Observable}} \times [Sp(8) \times Sp(8)]_{\text{Hidden}}$ | j | N | $(n^1, m^1)(n^2, m^2)(n^3, m^3)$ | $n$ | $n'$ | $b$ | $b'$ | c |
|-------------------|-----------------------------------------------------------------|---|---|---------------------------------|-----|-----|-----|-----|---|
| a                 | 8                  | (1,0)(1,1)(1,−1) | 0  | 0 | 3  | -3 | -1   |
| b                 | 4                  | (−2,−1)(2,1)(2,1) | -10 | -54 | -  | -  | -4  |
| c                 | 8                  | (0,1)(0,−1)(1,0) | 0  | 0  | -  | -  | -   |
| $D3$              | 8                  | (1,0)(1,0)(1,0) | $\chi_2 = \chi_3, \frac{1}{\chi_2^2} + \frac{8}{\chi_1 \chi_2} = 1$ |
| $(D7)_2$          | 8                  | (0,1)(1,0)(0,−1) | $\beta^g_{D3} = -14(-5), \beta^g_{(D7)_2} = -9(0)$ |
TABLE XIV: D-brane configurations and intersection numbers for $Model - F_1 - 3$.

| $Model - F_1 - 3$ | $[U(4)_C \times Sp(4)_L \times U(2)_R]_{\text{Observable}}$ | | | | | | Kähler moduli |
|---|---|---|---|---|---|---|---|
| j | $N$ | $(n^1, m^1)(n^2, m^2)(n^3, m^3)$ | $\nu$ | $n$ | $b$ | $c$ | $c'$ |
| a | 8 | $(1, 0)(2, 1)(1, -1)$ | 2 | -2 | 8 | -12 | $\chi_2 = 2\chi_3$ |
| b | 4 | $(0, 1)(0, -1)(1, 0)$ | 0 | 0 | -8 | - | $\frac{24}{\chi_2} + \frac{20}{\chi_1\chi_2} = 1$ |
| c | 4 | $(-2, -1)(4, 1)(3, 1)$ | -46 | -146 | - | - | |

TABLE XV: D-brane configurations and intersection numbers for $Model - F_1 - 2$.

| $Model - F_1 - 2$ | $[U(4)_C \times U(2)_L \times U(2)_R]_{\text{Observable}} \times [Sp(4)]_{\text{Hidden}}$ | | | | | | Kähler moduli |
|---|---|---|---|---|---|---|---|
| j | $N$ | $(n^1, m^1)(n^2, m^2)(n^3, m^3)$ | $\nu$ | $n$ | $b$ | $c$ | $c'$ |
| a | 8 | $(1, 0)(2, 1)(1, -1)$ | 2 | -2 | -4 | 0 | 4 | -10 |
| b | 4 | $(1, 1)(2, -1)(1, 0)$ | -2 | 2 | - | - | 5 | -3 |
| c | 4 | $(-2, -1)(3, 1)(3, 1)$ | -32 | -112 | - | - | - | |
| $(D7)_2$ | 4 | $(0, 1)(1, 0)(0, -1)$ | $\chi_2 = 2\chi_3 = 2\chi_1 = 3\sqrt{6}$ | |

TABLE XVI: D-brane configurations and intersection numbers for $Model - F_2 - 2$.

| $Model - F_2 - 2$ | $[U(4)_C \times Sp(4)_L \times U(2)_R]_{\text{Observable}} \times [Sp(8) \times Sp(4)]_{\text{Hidden}}$ | | | | | |
|---|---|---|---|---|---|
| j | $N$ | $(n^1, m^1)(n^2, m^2)(n^3, m^3)$ | $\nu$ | $n$ | $b$ | $c$ | $c'$ |
| a | 8 | $(1, 0)(2, 1)(1, -1)$ | 2 | -2 | -2 | 4 | -10 |
| b | 4 | $(0, 1)(0, -1)(1, 0)$ | 0 | 0 | -6 | - | - |
| c | 4 | $(-2, -1)(3, 1)(3, 1)$ | -32 | -112 | - | - | - |
| $(D7)_2$ | 8 | $(1, 0)(1, 0)(1, 0)$ | $2\chi_3 = \chi_2$ | |
| $(D7)_2$ | 4 | $(0, 1)(1, 0)(0, -1)$ | $\frac{18}{\chi_2} + \frac{18}{\chi_1\chi_2} = 1$ | |

28
TABLE XVII: D-brane configurations and intersection numbers for Model $- F_3 - 2$.

| Model $- F_3 - 2$ | $[U(4)_C \times U(2)_L \times U(2)_R]_{\text{Observable}} \times [Sp(4)]_{\text{Hidden}}$ |
|-------------------|-----------------------------------------------------------------|
| $j$               | $N$                  | $(n^1, m^1)(n^2, m^2)(n^3, m^3)$ | $m$ | $n$ | $b$ | $b'$ | $c$ | $c'$ |
| $a$               | 8                    | $(1, 0)(2, 1)(1, -1)$ | 2   | -2  | -3  | -1  | 4   | -10  |
| $b$               | 4                    | $(2, 1)(1, -1)(1, 0)$ | 2   | -2  | -   | -0  | -   | -8   |
| $c$               | 4                    | $(-2, -1)(3, 1)(3, 1)$ | -32 | -112| -   | -   | -   | -    |
| $(D_7)_2$         | 4                    | $(0, 1)(1, 0)(0, -1)$ | $\chi_2 = 2\chi_3 = \frac{1}{2}\chi_1 = 3\sqrt{3}$ |

TABLE XVIII: D-brane configurations and intersection numbers for Model $- F_1 - 1$.

| Model $- F_1 - 1$ | $[U(4)_C \times U(2)_L \times Sp(8)_R]_{\text{Observable}} \times [Sp(8)]_{\text{Hidden}}$ |
|-------------------|--------------------------------------------------------------------------------|
| $j$               | $N$                  | $(n^1, m^1)(n^2, m^2)(n^3, m^3)$ | $m$ | $n$ | $b'$ | $c$ |
| $a$               | 8                    | $(1, 0)(1, 1)(2, -1)$ | -2  | 2   | 4   | 0   | -1  |
| $b$               | 4                    | $(-2, -1)(2, 1)(2, 1)$ | -10 | -54 | -   | -4  | -   |
| $c$               | 8                    | $(0, 1)(0, -1)(1, 0)$ | 0   | 0   | -   | -   | -   |
| $(D_7)_2$         | 8                    | $(0, 1)(1, 0)(0, -1)$ | $\chi_3 = 2\chi_2, \frac{2}{\chi_2^2} + \frac{6}{\chi_1\chi_2} = 1$ |

TABLE XIX: D-brane configurations and intersection numbers for Model $- F_2 - 1$.

| Model $- F_2 - 1$ | $[U(4)_C \times U(2)_L \times U(2)_R]_{\text{Observable}} \times [Sp(16) \times Sp(4)]_{\text{Hidden}}$ |
|-------------------|--------------------------------------------------------------------------------------------------|
| $j$               | $N$                  | $(n^1, m^1)(n^2, m^2)(n^3, m^3)$ | $m$ | $n$ | $b'$ | $c$ | $c'$ |
| $a$               | 8                    | $(1, 0)(1, 1)(2, -1)$ | -2  | 2   | 4   | -4  | 10  |
| $b$               | 4                    | $(3, -1)(1, 0)(2, 1)$ | -2  | -   | -   | 5   | 5   |
| $c$               | 4                    | $(-2, 1)(3, -1)(3, -1)$ | 32  | 112 | -   | -   | -   |
| $D_3$             | 16                   | $(1, 0)(1, 0)(1, 0)$ | $\chi_3 = 2\chi_2 = \frac{2}{3}\chi_1 = \sqrt{30}$ |
| $(D_7)_3$         | 4                    | $(0, 1)(0, -1)(1, 0)$ | $\chi_3 = 2\chi_2 = \frac{2}{3}\chi_1 = \sqrt{30}$ |
TABLE XX: D-brane configurations and intersection numbers for Model $- F_3 - 1$.

| $Model - F_3 - 1$ | $[U(4)_C \times U(2)_L \times U(2)_R]_{\text{Observable}} \times [Sp(8)]_{\text{Hidden}}$ |
|-------------------|--------------------------------------------------------------------------------|
| $j$ | $N$ | $(n^1, m^1)(n^2, m^2)(n^3, m^3)$ | $p$ | $b$ | $b'$ | $c$ | $c'$ |
| $a$ | 8 | $(1, 0)(1, 1)(1, -1)$ | 0 | 0 | -4 | 2 | 4 | -6 |
| $b$ | 4 | $(2, 1)(3, -1)(1, 0)$ | -2 | 2 | - | 0 | 4 |
| $c$ | 4 | $(-2, -1)(2, 1)(3, 1)$ | -18 | -78 | - | - | - |
| $(D7)_2$ | 8 | $(0, 1)(1, 0)(0, -1)$ | $\chi_3 = \chi_2 = \frac{3}{2} \chi_1 = \sqrt{21}$ |

TABLE XXI: D-brane configurations and intersection numbers for Model $- F_4 - 1$.

| $Model - F_4 - 1$ | $[Sp(16)_C \times U(2)_L \times U(2)_R]_{\text{Observable}}$ |
|-------------------|--------------------------------------------------------------------------------|
| $j$ | $N$ | $(n^1, m^1)(n^2, m^2)(n^3, m^3)$ | $p$ | $b$ | $b'$ | $c$ | $c'$ | Kähler moduli |
| $a$ | 16 | $(1, 0)(1, 0)(1, 0)$ | 0 | 0 | 1 | -1 | - | $\chi_1 \chi_3 = 6$ |
| $b$ | 4 | $(2, -1)(0, 1)(3, -1)$ | -10 | 10 | - | 64 | 0 | $\frac{\chi_1}{\chi_2} + \frac{12}{\chi_1 \chi_2} = 1$ |
| $c$ | 4 | $(-2, -1)(4, 1)(1, 1)$ | -6 | -58 | - | - | - |

TABLE XXII: D-brane configurations and intersection numbers for Model $- F_5 - 1$.

| $Model - F_5 - 1$ | $[U(4)_C \times U(2)_L \times Sp(8)_R]_{\text{Observable}} \times [Sp(4)]_{\text{Hidden}}$ |
|-------------------|--------------------------------------------------------------------------------|
| $j$ | $N$ | $(n^1, m^1)(n^2, m^2)(n^3, m^3)$ | $p$ | $b$ | $b'$ | $c$ |
| $a$ | 8 | $(-1, -1)(2, 1)(2, 1)$ | -2 | -30 | -4 | 0 | 1 |
| $b$ | 4 | $(1, 0)(1, 1)(2, -1)$ | -2 | 2 | - | - | 0 |
| $c$ | 8 | $(1, 0)(1, 0)(1, 0)$ | 0 | 0 | - | - | - |
| $(D7)_3$ | 4 | $(1, 0)(0, 1)(0, -1)$ | $\chi_3 = 2 \chi_2, \frac{2}{\chi_2} + \frac{3}{\chi_1 \chi_2} = 1$ |
TABLE XXIII: D-brane configurations and intersection numbers for Model $- F_6 - 1$, here $\beta^g$ are beta functions for the associated gauge symmetries in the hidden sector.

| $Model - F_6 - 1$ | $[U(4)_C \times Sp(8)_L \times U(2)_R]_{Observable} \times [Sp(4) \times Sp(4)]_{Hidden}$ |
|-------------------|----------------------------------------------------------------------------------|
| $j$ | $N$ | $(n^1, m^1)(n^2, m^2)(n^3, m^3)$ | $n$ | $b$ | $c$ | $c'$ |
|-----|-----|---------------------------------|-----|-----|-----|-----|
| $a$ | 8   | $(1, 0)(1, 1)(1, -1)$            | 0   | 0   | -1  | 6   | -4  |
| $b$ | 8   | $(0, 1)(0, -1)(1, 0)$            | 0   | 0   | -3  | -   | -   |
| $c$ | 4   | $(−1, −1)(3, 1)(2, 1)$          | -4  | -44 | -   | -   | -   |
| $(D7)_1$ | 4 | $(1, 0)(0, 1)(0, -1)$          | $\chi_2 = \chi_3, \frac{4}{\chi_2} + \frac{5}{\chi_1\chi_3} = 1$ |
| $(D7)_2$ | 4 | $(0, 1)(1, 0)(0, -1)$          | $\beta^g_{(D7)_1} = -3(0), \beta^g_{(D7)_2} = -5(-2)$ |