The longitudinal fivebrane and tachyon condensation in matrix theory

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ABSTRACT

We study a configuration in matrix theory carrying longitudinal fivebrane charge, i.e. a D0-D4 bound state. We calculate the one-loop effective potential between a D0-D4 bound state and a D0-anti-D4 bound state and compare our results to a supergravity calculation. Next, we identify the tachyonic fluctuations in the D0-D4 and D0-anti-D4 system. We analyse classically the action for these tachyons and find solutions to the equations of motion corresponding to tachyon condensation.

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1. Introduction

Matrix theory [1] [2] [3] is the M-theory interpretation of U(N) supersymmetric quantum mechanics which has passed many stringent tests. The brane content of matrix theory was determined in [4]. Amongst other branes, the longitudinal fivebrane was identified 3. Two types of representation for the longitudinal fivebrane were proposed. One in terms of an instanton gauge field, which was used in [8] to calculate one loop effective potentials between the D0-D4 bound state and other objects in matrix theory. Another representation was proposed in terms of two pairs of canonical conjugate variables. We use this representation to calculate one-loop effective potentials (see e.g. [6] [7] [8] [9]) between this object and a graviton, another D0-D4 bound, and a D0–anti-D4 system. Naturally, we find agreement with [8] for the cases studied there and with an extra supergravity calculation for the D0-D4 and D0-anti-D4 system.

In [10] a first step towards the understanding of Sen’s tachyon condensation mechanism [11] in matrix theory was taken, by analyzing the tachyon in the D0-D2 and D0–anti-D2 system. We concentrate on the D0-D4 and D0–anti-D4 system. We identify the tachyonic fluctuations in the D0-D4 and D0–anti-D4 background and analyse the classical action for these fluctuations in the spirit of [10]. We find solutions to the action representing condensation to a vacuum filled with D0-branes and gravitons.

The first section concentrates on a discussion of the classical solution of matrix theory corresponding to a D0-D4 bound state system. In the second section some effective potentials are calculated in detail to get acquainted with the representation of the longitudinal fivebrane in terms of canonical conjugate variables. We add a remark about the spectrum of the fluctuations around one longitudinal fivebrane. The next section deals with an analysis of the tachyonic fluctuations. Then we analyse possible solutions to the action for the tachyonic fluctuations. Finally, we add remarks on the results and open problems.

2. Preliminary discussion of the classical solution

The lagrangian of matrix theory is given by U(N) supersymmetric quantum mechanics, namely the dimensional reduction of ten-dimensional $\mathcal{N} = 1$ U(N) Super Yang-Mills theory to 0 + 1 dimensions. It reads [1]:

$$\mathcal{L} = \frac{T_0}{2} Tr \left( (D_0 X_I)^2 + \frac{1}{2} [X_I, X_J]^2 + 2 \theta^T D_0 \theta - 2 \theta^T \gamma^I [\theta, X_I] \right)$$

(2.1)

where we take $2\pi \alpha' = 1$ and $T_0 = \frac{\sqrt{2\pi}}{g}$. Furthermore we have $D_0 = \partial_t - i [A_0, .]$ and $I = 1, 2, \ldots, 9$. All fields are in the adjoint of $U(N)$. The fermions are Majorana-Weyl. The equations of motion for static configurations with trivial $A_0$ and vanishing fermions are:

$$[X_I, [X_I, X_J]] = 0.$$  

(2.2)

We study especially a background configuration ($X_I = B_I$) corresponding to a D0-D4 bound state, or longitudinal fivebrane, satisfying the following commutation rules [10]:

$$[B_1, B_2] = -i c \sigma_3 \otimes I_{\frac{4}{2} \times \frac{4}{2}}$$

The transverse fivebrane remained a puzzle [3].
\[ [B_3, B_4] = -ic \sigma_3 \otimes I_{\frac{N}{2} \times \frac{N}{2}}, \quad (2.3) \]

and the other matrices and commutators zero. Here \( \sigma_3 \) is the third Pauli matrix and \( c \) is a constant. We take the infinite background matrices to be blockdiagonal such that this configuration solves the equations of motion. We will use two representations for this solution. The first one is in terms of two 'canonical conjugate' pairs:

\[
\begin{align*}
[P_1, Q_1] &= -ic \\
[P_2, Q_2] &= -ic \\
B_1 &= \begin{pmatrix} P_1 & 0 \\ 0 & P_1 \end{pmatrix} \\
B_2 &= \begin{pmatrix} Q_1 & 0 \\ 0 & -Q_1 \end{pmatrix} \\
B_3 &= \begin{pmatrix} P_2 & 0 \\ 0 & P_2 \end{pmatrix} \\
B_4 &= \begin{pmatrix} Q_2 & 0 \\ 0 & -Q_2 \end{pmatrix} \\
\end{align*}
\quad (2.4)
\]

This representation makes it easy to interpret the brane content of the configuration. Clearly, this solution as a whole carries no membrane charge since \( q_2 = -\frac{i}{2\pi} Tr [B_I, B_J] = 0 \). It carries longitudinal fivebrane charge in the 1, 2, 3, 4 directions though:

\[
q_5 = -\frac{1}{8\pi^2} \epsilon^{IJKLM} Tr [B_I B_J B_K B_L] = N \frac{c^2}{4\pi^2} \quad (2.5)
\]

We refer to [14] for a clear and detailed analysis of the charges of the configuration, which yields the fact that the configuration you build in this way represents at least two D0-D4 bound states. That can be understood from the following reasoning.

When we focus on the left upper block, it clearly has membrane charge in directions 1, 2 and 3, 4, as well as longitudinal fivebrane charge. It represents a D0-D4-D2-D2 bound state. Zooming in on the right lower block we see a D0-D4-anti-D2-anti-D2 bound state. If we formally superimpose the two parts we find two D0-D4 bound states, the 2-brane charge cancelling out.

Thinking naively, one might be worried that this superposition is unstable, in particular, one might expect a tachyonic off-diagonal mode in the background configuration, representing a string stretching between a D2-brane and an anti-D2-brane. We will come back to this point and show that there is no such tachyonic mode. Moreover, the configuration was shown in [4] to preserve 1/4 supersymmetry, as expected from D0-D4 bound states.

An alternative representation of the background configuration in terms of gauge fields, discussed in detail in [14] will come in handy later on. It is given by:

\[
\begin{align*}
B^1 &= c \begin{pmatrix} -i\partial_{x_1} & 0 \\ 0 & -i\partial_{x_1} \end{pmatrix} \\
B^2 &= c \begin{pmatrix} -i\partial_{x_2} + \frac{2a_1}{c} & 0 \\ 0 & -i\partial_{x_2} - \frac{2a_1}{c} \end{pmatrix}
\end{align*}
\]
Note that we introduce the same four coordinates on the two D0-D4 bound states. This indicates our intention of treating them as a single object. Indeed, we will only analyse interaction potentials and fluctuations where the two D0-D4 bound states move as one. We define the left-upper and the right-lower part to be made up of \( \frac{N_0}{2} \) D0-branes and denote the D0-brane charge density as \( \rho_0 = \frac{N_0}{A_4} \), where \( A_4 \) represents the (possibly infinite) area of the coinciding D4-D0 bound states. Then we can derive the following relation [14]:

\[
 c^2 = \frac{A_4 N_4}{(2\pi)^2 N_0} \tag{2.7}
\]

where \( N_4 \) is the number of fourbranes and \( N_0 \) the total number of D0-branes in the bound state.

### 3. Calculating effective potentials in matrix theory at one loop

In this section we calculate some interaction potentials between the D0-D4 bound state\(^\text{4}\) and other objects explicitly. In the literature (e.g. [6] [7] [8] [9]), some of these potentials have already been calculated using the representation in terms of an instanton background gauge field [8]. But in the next section we will need a more detailed analysis of the fluctuations, when we identify the tachyonic ones. Moreover there are a few new technicalities in calculating the spectrum of the fluctuations when a single object is represented by ‘two-by-two’ matrices, which have not been discussed in the literature yet. Therefore we find it useful to first redo some of the calculations in the literature in our representation, next to treat the new case of the D0-D4 and D0–anti-D4 interaction in detail.

Because one object is sometimes represented by ‘two-by-two’ matrices, we need some new conventions and nomenclature, which we will take to be as follows. In this section, the first object will have extent \( n_0 \), the second object \( N_0 \). When one object is represented by a ‘two-by-two’ matrix, the submatrices will have half the extent of the object, e.g. \( \frac{n_0}{2} \). Moreover, suppose we have two objects in the background represented by blockdiagonal ‘two-by-two’ matrices. Then we will take the following nomenclature for the different parts of the coordinate matrices:

\[
 X_I = \begin{pmatrix}
 \text{block (1)} & 0 & \text{sector 13} & \text{sector 14} \\
 0 & \text{block (2)} & \text{sector 23} & \text{sector 24} \\
 \text{sector 13}^\dagger & \text{sector 23}^\dagger & \text{block (3)} & 0 \\
 \text{sector 14}^\dagger & \text{sector 24}^\dagger & 0 & \text{block (4)} \\
\end{pmatrix} \tag{3.1}
\]

\(^4\) Readers only interested in the tachyonic fluctuations can skip this section without much difficulty.

\(^5\) We will stop mentioning that it actually consist of two D0-D4 bound states from now on.
The off-diagonal modes have been divided up into four different sectors. Other cases
to be discussed are simpler and the nomenclature will be analogous in an obvious
way.

The technique to calculate the one-loop effective potential between two objects in
matrix theory is standard by now [6] [7]. To calculate the potential, we determine
the spectrum of the off-diagonal fluctuations corresponding to strings stretching from
one object to the other. Their mass matrix is easily determined by expanding the ac-
tion of matrix theory around the relevant background. This is slightly more involved
when objects are represented by two-by-two matrices, but the general formulae in
for instance [8] [12] can easily be adapted to our case, essentially be-cause the back-
ground matrices are block diagonal. We do not give the details of the calculation,
but summarize for each case the result.

In the following three subsections, we will discuss three different cases. For the
first object we always take the D0-D4 bound state. For the second object we take
respectively a graviton, a D0-D4 bound state, or a D0–anti-D4 bound state. The
second object will always be taken to be at a distance $b$ of the first object in some
transverse direction ("8") and it will be moving with a velocity $v$ relative to the first
object in another transverse direction ("9"). This is incorporated by choosing the
background matrices corresponding to these transverse coordinates to be:

$$
B_8 = \begin{pmatrix}
0_{n_0 \times n_0} & 0 \\
0 & b I_{N_0 \times N_0}
\end{pmatrix}
$$

$$
B_9 = \begin{pmatrix}
0_{n_0 \times n_0} & 0 \\
0 & vt I_{N_0 \times N_0}
\end{pmatrix}
$$

Finally, to make the interaction energies finite, we wrap the fourbranes on a four-
torus. This hardly influences the calculation. It is moreover convenient to take the
four-torus to have self-dual radii $R_i = \sqrt{\alpha'}$. It is straightforward to again add in
the dependence on the compactification radii in the final formulae. See for instance [8].

3.1. Interaction potential between a D0-D4 bound state system and a graviton

For the first case, namely the D0-D4 bound state interacting with a moving gravi-
ton, the non-trivial background matrices are (recall also the separation matrices $B_8$
and $B_9$ given in (3.2)):

$$
[P_1, Q_1] = -ic \\
[P_2, Q_2] = -ic
$$

$$
B_1 = \begin{pmatrix}
P_1 & 0 & 0 \\
0 & P_1 & 0 \\
0 & 0 & 0
\end{pmatrix}
$$

$$
B_2 = \begin{pmatrix}
Q_1 & 0 & 0 \\
0 & -Q_1 & 0 \\
0 & 0 & 0
\end{pmatrix}
$$

$$
B_3 = \begin{pmatrix}
P_2 & 0 & 0 \\
0 & P_2 & 0 \\
0 & 0 & 0
\end{pmatrix}
$$
\[ B_4 = \begin{pmatrix} Q_2 & 0 & 0 \\ 0 & -Q_2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \] (3.3)

For the quantum fluctuations we find identical spectra \([3]\) in the two sectors of extent \(\frac{n_0}{2} \times N_0\). We define the hamiltonian:

\[ H = P_1^2 + Q_1^2 + P_2^2 + Q_2^2 + b^2 + v^2 t^2, \] (3.4)

corresponding to two non-interacting harmonic oscillators with frequency \(c\) and a trivial extra part. After diagonalization, we find for the mass operators of the real bosons 4 : \(H \pm 2iv\); 8 : \(H \pm 2c\); 4 : \(H\) and for the fermions 8 : \(H \pm iv\); 8 : \(H \pm 2c \pm iv\), where we always state the number of fields first, and then the mass operator that corresponds to them. For instance 4 : \(H \pm 2c\) corresponds to 2 fields with mass operator \(H + 2c\) and 2 fields with mass operator \(H - 2c\). The spectrum of these mass operators is easily determined. Following \([3][7]\) it is then straightforward to calculate the phase shift due to the interactions, and to approximate the phase shift at large distances \(b^2 \gg c\) and small velocities \(v \ll b^2\):

\[ \delta = 2N \int_0^\infty ds \frac{e^{-b^2 s}}{s} \frac{1}{8 \sin vs \sinh^2 cs} \times \]
\[ (2 + 2 \cos 2vs + 4 \cosh 2cs)
- 4 \cos vs - 4 \cos vs \cosh 2cs) \]
\[ \approx \left( \frac{n_4 N_0 v}{2b^2} + \frac{v^2 n_0 N_0}{8b^2} \right) \] (3.5)

We denoted the degeneracy of the energy levels by \(N\) which in this case is given by:

\[ N = c^2 \frac{n_0}{2} N_0 \] (3.7)
\[ c^2 = \frac{n_4}{n_0} \] (3.8)

and we have used formula \((2.7)\) at self-dual radii \((A_4 = (2\pi)^2)\). Determining the degeneracy of the spectrum has been done for the equivalent problem in the Landau model – a charged particle in a magnetic field. The degeneracy for the Landau levels was determined in \([13]\). Translating the formula for the degeneracy to our problem and carefully keeping track of normalization factors, we find the following heuristic for the degeneracy in general:

\[ N = \text{Dimension fluctuation-matrix} \times \]
\[ \text{Product of frequencies of the harmonic oscillators in } H. \] (3.9)

This rule is applied in \((3.7)\), \(\frac{n_4 N_0}{2}\) being the dimension of the fluctuations and \(c^2\) being the product of the harmonic oscillator frequencies in \(H\) \((3.4)\). The end result for the phase shift matches with the supergravity calculation in the relevant regime \((A.1)\),

\footnote{Some boson contributions to the one loop effective potential are cancelled by ghost contributions. We don’t include them in the following.}
and with the result obtained in a different manner in \[8\]. The phase shift starts at order \(v\) because the background configuration preserves \(\frac{1}{4}\) supersymmetry.

3.2. D0-D4 and D0-D4 interaction

In the second case the background matrices are:

\[
\begin{align*}
[P_1, Q_1] &= -ic_1 \\
[P_2, Q_2] &= -ic_1 \\
[P_3, Q_3] &= -ic_3 \\
[P_4, Q_4] &= -ic_3
\end{align*}
\]

\[
B^1 = \begin{pmatrix}
P_1 & 0 & 0 & 0 \\
0 & P_1 & 0 & 0 \\
0 & 0 & P_3 & 0 \\
0 & 0 & 0 & P_3
\end{pmatrix},
\]

\[
B^2 = \begin{pmatrix}
Q_1 & 0 & 0 & 0 \\
0 & -Q_1 & 0 & 0 \\
0 & 0 & Q_3 & 0 \\
0 & 0 & 0 & -Q_3
\end{pmatrix},
\]

\[
B^3 = \begin{pmatrix}
P_2 & 0 & 0 & 0 \\
0 & P_2 & 0 & 0 \\
0 & 0 & P_4 & 0 \\
0 & 0 & 0 & P_4
\end{pmatrix},
\]

\[
B^4 = \begin{pmatrix}
Q_2 & 0 & 0 & 0 \\
0 & -Q_2 & 0 & 0 \\
0 & 0 & Q_4 & 0 \\
0 & 0 & 0 & -Q_4
\end{pmatrix},
\]

(3.11)

Here we find four sectors of extent \(\frac{n_0}{2} \times \frac{N_0}{2}\), two by two identical, namely sector \((13) = (24)\) and sector \((23) = (14)\). We define the two relevant hamiltonians

\[
H^{(13)} = (P_1 + P_3)^2 + (Q_1 - Q_3)^2 + (P_2 + P_4)^2 + (Q_2 - Q_4)^2 + b^2 + v^2 t^2,
\]

\[
H^{(23)} = (P_1 + P_3)^2 + (Q_1 + Q_3)^2 + (P_2 + P_4)^2 + (Q_2 + Q_4)^2 + b^2 + v^2 t^2.
\]

Each describes a system of two decoupled harmonic oscillators. The diagonalized mass operators are: in sector \((13)\) for the bosons \(4\):

\[
H^{\pm} = \frac{1}{2} iv; 8 : H^{\pm} = \frac{1}{2} (c_1 - c_3), 2 : H
\]

and for the fermions \(8\): \(H = \pm iv\); \(8 : H = \pm iv \pm 2(c_1 - c_3)\). In sector \((23)\) they read for the bosons \(4 : H = \pm 2iv; 8 : H = \pm 2(c_1 + c_3)\); \(4 : H\) and for the fermions \(8 : H = \pm iv\); \(8 : H = \pm iv \pm 2(c_1 + c_3)\). The spectrum is again easily determined, and the phase shift now gets two different contributions:

\[
\delta^{(13)+(24)} = 2N_{13} \int_0^\infty \frac{ds}{s} \frac{e^{-b^2 s}}{8 \sin vs \sinh^2 (c_1 - c_3)s} \times
\]

\[
(2 + 2 \cos 2vs + 4 \cosh 2(c_1 - c_3)s)
\]

\[
-4 \cos vs - 4 \cos vs \cosh 2(c_1 - c_3)s \]

\[
\approx N_{13} \left( \frac{v^2 + \frac{v^3}{4(c_1 - c_3)^2 b^2}}{b^2} \right)
\]

(3.12)

(3.13)
\[ \delta_{(23) + (14)} = 2N_{23} \int_0^\infty \frac{ds}{s} \frac{e^{-b^2 s}}{8 \sin vs \sinh^2 (c_1 + c_3)s} \times \\
(2 + 2 \cos 2vs + 4 \cosh 2(c_1 + c_3)s) \\
- 4 \cos vs - 4 \cos vs \cosh 2(c_1 + c_3)s \]  
(3.14)

\[ \approx N_{23} \left( \frac{v}{b^2} + \frac{v^3}{4(c_1 + c_3)^2 b^2} \right) \]  
(3.15)

giving a total phase shift

\[ \delta \approx \frac{(n_0 N_4 + N_0 n_4)v}{2b^2} + \frac{n_0 N_0 v^3}{8b^2} \]  
(3.16)

where we have used the following formulae:

\[ N_{13} = (c_1 - c_3)^2 \frac{n_0 N_0}{2} \]  
(3.17)

\[ N_{23} = (c_1 + c_3)^2 \frac{n_0 N_0}{2} \]  
(3.18)

\[ c_1^2 = \frac{n_4}{n_0} \]  
(3.19)

\[ c_3^2 = \frac{N_4}{N_0} . \]  
(3.20)

The fact that the phase shift starts at order \( v \) is due to the fact that the background configuration preserves 1/4 supersymmetry. The endresult matches with the supergravity calculation \([A.2]\) [8].

3.3. D0-D4 and D0-anti-D4 interaction

In the third case the background matrices are:

\[
\begin{align*}
\{P_1, Q_1\} &= -ic_1 \\
\{P_2, Q_2\} &= -ic_1 \\
\{P_3, Q_3\} &= -ic_3 \\
\{P_4, Q_4\} &= -ic_3 \\
B^1 &= \begin{pmatrix}
P_1 & 0 & 0 & 0 \\
0 & P_1 & 0 & 0 \\
0 & 0 & -P_3 & 0 \\
0 & 0 & 0 & -P_3 \\
\end{pmatrix} \\
B^2 &= \begin{pmatrix}
Q_1 & 0 & 0 & 0 \\
0 & -Q_1 & 0 & 0 \\
0 & 0 & Q_3 & 0 \\
0 & 0 & 0 & -Q_3 \\
\end{pmatrix} \\
B^3 &= \begin{pmatrix}
P_2 & 0 & 0 & 0 \\
0 & P_2 & 0 & 0 \\
0 & 0 & P_4 & 0 \\
0 & 0 & 0 & P_4 \\
\end{pmatrix}
\end{align*}
\]
Note the partial sign change in the first background matrix, turning the second object into a D0–anti-D4 bound state. We find four sectors of extent \( \frac{n_0}{2} \times \frac{N_0}{2} \), all with identical spectra, when we ignore the origin in terms of the different coordinates \(^7\). The relevant hamiltonian is:

\[
H^{(13)} = (P_1 - P_3)^2 + (Q_1 - Q_3)^2 + (P_2 + P_4)^2 + (Q_2 - Q_4)^2 + b^2 + v^2 t^2,
\]
corresponding to a system of two harmonic oscillators. We will always suppose that \( c_1 - c_3 \) is positive, the other case being fully equivalent. The mass operators \( s \) are for each sector for the bosons:

\[
H_{\pm} = 2\sqrt{\pi} \int_0^{\infty} ds \left( e^{-b s} \left( 2 + 2 \cos vs + 2 \cosh 2(c_1 - c_3) s + 2 \cosh 2(c_1 + c_3) s \right) - 8 \cos vs \cosh (c_1 + c_3) s \cosh (c_1 - c_3) s \right)
\]

\[
\approx \frac{n_4 N_4}{b^3} + \frac{(n_0 N_4 + N_0 n_4) v^2}{4b^3} + \frac{n_0 N_0 v^4}{16b^3}
\]

Compared to the previous case \( (3.16) \), there is an extra interaction between the D4-brane and the anti-D4-brane. The interaction potential is non-trivial at zero velocity and the background fully breaks supersymmetry. The end result is reproduced by our supergravity calculation \( (A.3) \) in the appendix. Clearly, the formula for the potential breaks down at small distances \( b^2 \leq 2c_3 \). Then there is a tachyon in the spectrum of the bosons since the lowest energy mode has mass:

\[
E = (c_1 - c_3) + (c_1 + c_3) + b^2 - 2(c_1 + c_3) = b^2 - 2c_3.
\]

We will treat the system at short distances in section 5.

3.4. Summary

The conclusions we draw from these calculations are the following. At the level we are probing the system, the representation of the D0-D4 system that we use is equivalent to the instanton gauge field representation used in \( [8] \). We found full agreement when we compared the long range one loop potentials with supergravity results, also for the case of the D0-D4 and the D0–anti-D4 system, as expected. Moreover, we showed that it makes perfect sense to divide the off-diagonal modes into different sectors and treat them separately, which will be important in the second part of our paper.

4. Remark on the fluctuations around one longitudinal fivebrane

We refer to \( [14] \) for an analysis of the effective action for the fluctuations around the D0-D4 bound state system, but we add a remark that fits well into the context of our

\(^7\)We can do so for calculating the effective potential, but in section 5 we need the precise origin of the tachyonic modes in terms of the coordinate matrices. We return there to this point.
paper. As we mentioned in section 2, you might expect a tachyonic off-diagonal mode in the coordinate matrices spanning the fivebranes (2.4), since they could correspond to strings stretching from a D2-brane to an anti-D2-brane. That this does not happen is shown by a small calculation. The relevant mass matrix for these modes is, for instance for the fluctuations in the coordinates $X_1$ and $X_2$:

$$M_{12} = \begin{pmatrix} H & -2ic \\ 2ic & H \end{pmatrix}$$

(4.1)

where

$$H = P_1^2 + Q_1^2 + P_2^2 + Q_2^2.$$  

(4.2)

Diagonalizing the mass matrix and determining the spectrum yields two kinds of fluctuations with the following energies:

$$E = c(2n + 1) + c(2m + 1) + 2c$$

(4.3)

$$E' = c(2n' + 1) + c(2m' + 1) - 2c$$

(4.4)

Note that for the last kind of fluctuation, we find a massless mode, and not a tachyonic one. This is due to the quantum mechanical zero point energies coming from the object spanning in the 1,2 as well as the 3,4 direction. For a membrane–anti-membrane system this mode would be tachyonic [6] [10].

5. The action for the tachyonic fluctuations

5.1. The tachyonic fluctuations

From now on, we will consider the D0-D4 system and the D0–anti-D4 system to lie on top of each other, so we put the background matrices $B_8$ and $B_9$ (3.2) to zero. Then, when we compute the mass matrix for the fluctuations in the coordinate matrices $X_1$ and $X_2$, we find the following matrix for sector 13:

$$M_{12}^{(13)} = \begin{pmatrix} H^{(13)} & -2i(c_1 + c_3) \\ 2i(c_1 + c_3) & H^{(13)} \end{pmatrix}$$

(5.1)

where

$$H^{(13)} = (P_1 - P_3)^2 + (Q_1 - Q_3)^2 + (P_2 + P_4)^2 + (Q_2 - Q_4)^2.$$  

(5.2)

We diagonalize the mass matrix and determine the spectrum for the diagonal fluctuations [8].

$$E = (c_1 + c_3)(2n + 1) + (c_1 - c_3)(2m + 1) + 2(c_1 + c_3)$$

(5.3)

$$E' = (c_1 + c_3)(2n' + 1) + (c_1 - c_3)(2m' + 1) - 2(c_1 + c_3)$$

(5.4)

[8] Recall that we chose $c_1 \geq c_3$. 
From the second line, we find a tachyonic mode, as expected, with mass \(-2c_3\). Note that for \(c_1 < 2c_3\) you find several tachyonic modes. When you follow the simple diagonalization procedure in detail, you find that the tachyonic fluctuation \(^9\) is:

\[
\phi = \frac{y^{(13)}_2 - i y^{(13)}_1}{\sqrt{2}} \tag{5.5}
\]

where \(y^{(mn)}_I\) denotes the fluctuation in sector \((mn)\) and coordinate matrix \(X_I\). The fluctuation

\[
\bar{\phi} = \frac{y^{(13)}_2 + i y^{(13)}_1}{\sqrt{2}} \tag{5.6}
\]

corresponds to (5.3) and is never tachyonic. In the other sectors the computation goes analogously for a total of four tachyonic fields that correspond to strings stretching between the two D4 branes and the two anti-D4 branes (in the presence of the D0-branes). They are given by:

\[
\phi = \frac{y^{(13)}_2 - i y^{(13)}_1}{\sqrt{2}}
\]

\[
\phi' = \frac{y^{(24)}_2 + i y^{(24)}_1}{\sqrt{2}}
\]

\[
\chi = \frac{y^{(14)}_4 - i y^{(14)}_3}{\sqrt{2}}
\]

\[
\chi' = \frac{y^{(23)}_4 + i y^{(23)}_3}{\sqrt{2}} \tag{5.7}
\]

5.2. The action

Next we turn to the analysis of the action for the tachyonic fluctuations in the spirit of \([10]\). We expand the classical action around the D0-D4 and D0-anti-D4 background, only keeping track of the tachyonic fluctuations and the gauge fields of the unbroken gauge group \(U(1)^4\) under which the tachyons are charged. All fields we believe to be irrelevant, we put to zero, for instance (5.6). For simplicity, we take the number of D0-D4 bound states and D0-anti-D4 bound states to be equal, i.e. \(c_1 = c_3 = c\). Now the second representation introduced in section 2 comes in handy. Under the preceding assumptions, the coordinate matrices are given by:

\[
X^1 = c\begin{pmatrix}
-i\partial_{x_1} + A^{(1)}_{x_1} + a^{(1)}_{x_1} & 0 & \frac{i\phi}{\sqrt{2c}} & 0 \\
0 & -i\partial_{x_1} + A^{(2)}_{x_1} + a^{(2)}_{x_1} & 0 & -i\frac{\phi'}{\sqrt{2c}} \\
-i\frac{\phi^*}{\sqrt{2c}} & 0 & -i\partial_{y_1} + A^{(3)}_{y_1} + a^{(3)}_{y_1} & 0 \\
0 & i\frac{\phi'^*}{\sqrt{2c}} & 0 & -i\partial_{y_1} + A^{(4)}_{y_1} + a^{(4)}_{y_1}
\end{pmatrix}
\]

\(^9\)By abuse of language, we take 'tachyonic fluctuation' to mean that the field includes a tachyonic mode.
\[ X^2 = c \begin{pmatrix} -i \partial_{x_2} + A^{(1)}_{x_2} + a^{(1)}_{x_2} & 0 & \frac{\phi}{\sqrt{2}c} & 0 \\ 0 & -i \partial_{x_2} + A^{(2)}_{x_2} + a^{(2)}_{x_2} & 0 & \frac{\phi'}{\sqrt{2}c} \\ \phi^* \sqrt{2}c & 0 & -i \partial_{y_2} + A^{(3)}_{y_2} + a^{(3)}_{y_2} & 0 \\ 0 & \phi'^* \sqrt{2}c & 0 & -i \partial_{y_2} + A^{(4)}_{y_2} + a^{(4)}_{y_2} \end{pmatrix} \]

\[ X^3 = c \begin{pmatrix} -i \partial_{x_3} + A^{(1)}_{x_3} + a^{(1)}_{x_3} & 0 & 0 & 0 \\ 0 & -i \partial_{x_3} + A^{(2)}_{x_3} + a^{(2)}_{x_3} & 0 & \frac{\chi}{\sqrt{2}c} \\ 0 & i \frac{\chi}{2c} & -i \partial_{y_3} + A^{(3)}_{y_3} + a^{(3)}_{y_3} & 0 \\ -i \frac{\chi^*}{\sqrt{2}c} & 0 & -i \partial_{y_3} + A^{(4)}_{y_3} + a^{(4)}_{y_3} & 0 \end{pmatrix} \]

\[ X^4 = c \begin{pmatrix} -i \partial_{x_4} + A^{(1)}_{x_4} + a^{(1)}_{x_4} & 0 & 0 & 0 \\ 0 & -i \partial_{x_4} + A^{(2)}_{x_4} + a^{(2)}_{x_4} & 0 & \frac{\chi'}{\sqrt{2}c} \\ 0 & \frac{\chi'^*}{\sqrt{2}c} & -i \partial_{y_4} + A^{(3)}_{y_4} + a^{(3)}_{y_4} & 0 \\ -i \frac{\chi^*}{\sqrt{2}c} & 0 & -i \partial_{y_4} + A^{(4)}_{y_4} + a^{(4)}_{y_4} & 0 \end{pmatrix} \]

where \( A \) is the background gauge field and \( a \) the gauge field fluctuation. The background is invariant under \( U(1)^4 \), each \( U(1) \) has its own upper index. We choose the background gauge fields such that the appropriate commutation relations between the background matrices are satisfied:

\[
A^{(1)}_{x_2} = -A^{(2)}_{x_2} = \frac{x_1}{c}, \quad A^{(1)}_{x_4} = -A^{(2)}_{x_4} = \frac{x_3}{c}, \quad A^{(3)}_{y_2} = -A^{(4)}_{y_2} = \frac{y_1}{c}, \quad A^{(3)}_{y_4} = -A^{(4)}_{y_4} = \frac{y_3}{c}, \tag{5.8}
\]

and the rest zero. Each tachyonic mode is charged under two of the abelian gauge symmetries, with opposite charges, as can easily be seen by looking at the transformation properties of the full coordinate matrix.

To represent the action in terms of an integral over the worldvolume of the branes, we use the rules of \[\ref{eq:4}\], improved in \[\ref{eq:14}\] and elaborated upon in \[\ref{eq:10}\]. We rally some of the technical details to appendix B. The following definitions come in handy in writing down the endresult. The non-center-of-mass coordinates are:

\[ u_i = \frac{x_i + y_i}{2}. \tag{5.9} \]

Covariant derivatives and field strengths are defined as (Upper indices label the gauge symmetries, lower indices \( w_i = (x_i, y_i) \) label coordinates.) :

\[
\nabla^{(\pm m)}_{w_i} = \partial_{w_i} \pm i A^{(m)}_{w_i} \pm i a^{(m)}_{w_i}, \quad F^{(m)}_{w_i w_j} = i \left[ \nabla^{(m)}_{w_i} , \nabla^{(m)}_{w_j} \right],
\]

\[
\nabla^{(m, \pm n)}_{u_i} = \nabla^{(m)}_{u_i} \pm \nabla^{(\pm n)}_{y_i}, \quad F^{(m, \pm n)}_{u_i u_j} = i \left[ \nabla^{(m, \pm n)}_{u_i} , \nabla^{(m, \pm n)}_{u_j} \right] = F^{(m)}_{x_i x_j} \pm F^{(n)}_{y_i y_j} \tag{5.10}
\]
By a small $f$ we will denote the field strength $F$ without the background gauge fields contribution. The relevant part of the action for the fluctuations that we consider is then given by $S = \int d^4 u \mathcal{L}$, and the lagrangian by (up to an overall factor):

$$-\mathcal{L} = \left(\frac{c^2}{2} f^{(1,-3)} u_{1u_2} + c + |\phi|^2 + \frac{c^2}{2} f^{(1,-4)} u_{1u_4} + c + |\chi|^2\right)^2$$

$$+ \left(\frac{c^2}{2} f^{(2,-4)} u_{1u_2} + c - |\phi'|^2 + \frac{c^2}{2} f^{(2,-3)} u_{1u_4} + c - |\chi'|^2\right)^2$$

$$+ \frac{c^2}{2} \left((\nabla^{(1,-3)}_{u_2} + i \nabla^{(1,-3)}_{u_1}) \phi|^2 + 2 |\nabla^{(1,-3)}_{u_3} \phi|^2 + 2 |\nabla^{(1,-3)}_{u_4} \phi|^2 \right)$$

$$+ \left((\nabla^{(1,-4)}_{u_4} + i \nabla^{(1,-4)}_{u_3}) \chi|^2 + 2 |\nabla^{(1,-4)}_{u_3} \chi|^2 + 2 |\nabla^{(1,-4)}_{u_4} \chi|^2 \right)$$

$$+ \left((\nabla^{(2,-4)}_{u_3} - i \nabla^{(2,-4)}_{u_4}) \phi'|^2 + 2 |\nabla^{(2,-4)}_{u_3} \phi'|^2 + 2 |\nabla^{(2,-4)}_{u_4} \phi'|^2 \right)$$

$$+ \left((\nabla^{(2,-3)}_{u_4} - i \nabla^{(2,-3)}_{u_3}) \chi'|^2 + 2 |\nabla^{(2,-3)}_{u_3} \chi'|^2 + 2 |\nabla^{(2,-3)}_{u_4} \chi'|^2 \right)$$

$$+ \frac{c^4}{4} \left(f^{(1,3)}_{u_1 u_3} + f^{(1,-3)}_{u_1 u_4} + f^{(2,4)}_{u_1 u_3} + f^{(2,-4)}_{u_1 u_4} + f^{(2,4)}_{u_1 u_3} + f^{(2,4)}_{u_1 u_4} + f^{(2,-4)}_{u_1 u_4} + f^{(2,-4)}_{u_1 u_3}\right)$$

$$+ \left(f^{(1,3)}_{u_1 u_3} + f^{(1,-3)}_{u_1 u_4} + f^{(2,4)}_{u_1 u_3} + f^{(2,-4)}_{u_1 u_4} + f^{(2,4)}_{u_1 u_3} + f^{(2,4)}_{u_1 u_4} + f^{(2,-4)}_{u_1 u_4} + f^{(2,-4)}_{u_1 u_3}\right)$$

$$+ \left(f^{(1,3)}_{u_1 u_3} + f^{(1,-3)}_{u_1 u_4} + f^{(2,4)}_{u_1 u_3} + f^{(2,-4)}_{u_1 u_4} + f^{(2,4)}_{u_1 u_3} + f^{(2,4)}_{u_1 u_4} + f^{(2,-4)}_{u_1 u_4} + f^{(2,-4)}_{u_1 u_3}\right)$$

$$+ \left((\phi \chi^* - \chi \phi^*)^2 + (|\phi \chi^* - \chi \phi^*|^2 \right)$$

where all fields only depend on the non-center-of-mass coordinates. Note that it is the Lagrangian you expect, with the usual kinetic terms for the gauge fields, the appropriate covariant derivatives hitting the tachyons and a Higgs potential for the tachyons. There are some interactions between the tachyons and the gauge fields $u_{1,2,3,4}$ and an interaction potential between the different tachyons.

5.3. Boundary conditions

The background gauge fields corresponding to the diagonal U(1)’s (5.8) can be rewritten as follows:

$$A_{u_1} = 0$$

$$A_{u_2} = \begin{pmatrix} A^{(1)}_{u_2} & 0 & 0 & 0 \\ 0 & A^{(2)}_{u_2} & 0 & 0 \\ 0 & 0 & A^{(3)}_{u_2} & 0 \\ 0 & 0 & 0 & A^{(4)}_{u_2} \end{pmatrix}$$

$$= \frac{u_1}{c} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A_{u_3} = 0$$

$$A_{u_4} = \frac{u_3}{c} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

The non-zero background gauge fields appearing in the covariant derivatives in the
kinetic terms for the tachyons are:

\[
\mathcal{A}^{(1,-3)}_{u_2} = \frac{2u_1}{c} \\
\mathcal{A}^{(2,-4)}_{u_2} = -\frac{2u_1}{c} \\
\mathcal{A}^{(2,-3)}_{u_4} = -\frac{2u_3}{c} \\
\mathcal{A}^{(1,-4)}_{u_4} = \frac{2u_3}{c}
\]

(5.13)

Taking the background gauge fields to live on a four-torus with radii \( R_{u_i} \), they satisfy 't Hooft's twisted boundary conditions [17]. They read in direction \( u_1 \):

\[
\mathcal{A}_{u_i}(R_{u_1}, u_2, u_3, u_4) = -i\Omega_{u_i} \partial_{u_i} \Omega_{u_1}^{-1} + \Omega_{u_i}(0, u_2, u_3, u_4)\Omega_{u_1}^{-1}
\]

(5.14)

and analogous for the other directions, where \( \Omega_{u_i} \) are the transition functions. The transition functions can be choosen to be:

\[
\Omega_{u_1} = \exp \left[ -iu_2 \frac{R_{u_1}}{c} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right] \\
\Omega_{u_2} = 1 \\
\Omega_{u_3} = \exp \left[ -iu_4 \frac{R_{u_3}}{c} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \right] \\
\Omega_{u_4} = 1
\]

(5.15)

These boundary conditions are due to the presence of the background field, i.e. due to the magnetic field made up of the D0-branes, representing the background objects. For the full background matrix this implies:

\[
B_I(R_{u_1}, u_2, u_3, u_4) = \Omega_{u_1} B_I(0, u_2, u_3, u_4)\Omega_{u_1}^{-1}
\]

(5.16)

and analogously for the other directions.

The boundary conditions for the tachyons that are trivial with respect to the background can be read off from (5.16):

\[
\phi(u_1 = R_1) = \phi(u_1 = 0)e^{-2iu_2R_1/c} \\
\phi'(u_1 = R_1) = \phi'(u_1 = 0)e^{2iu_2R_1/c} \\
\chi(u_3 = R_3) = \chi(u_3 = 0)e^{-2iu_4R_3/c} \\
\chi'(u_3 = R_3) = \chi'(u_3 = 0)e^{2iu_4R_3/c}
\]

(5.17)

and the other background boundary conditions are trivial.

6. A solution to the equations of motion
First we look for a solution to the equations of motion where the total Lagrangian \((5.11)\) vanishes and the background boundary conditions are satisfied. We make the following ansatz:

\[
\begin{align*}
\phi &= \phi^{\prime\ast}(u_1, u_2) \\
\chi &= \chi^{\prime\ast}(u_3, u_4).
\end{align*}
\] (6.1)

Then we find we can take:

\[
\begin{align*}
a^{(1)}_{u_1, u_2} &= -a^{(2)}_{u_1, u_2} = -a^{(3)}_{u_1, u_2} = a^{(4)}_{u_1, u_2} \\
a^{(1)}_{u_3, u_4} &= -a^{(2)}_{u_3, u_4} = a^{(3)}_{u_3, u_4} = -a^{(4)}_{u_3, u_4}.
\end{align*}
\] (6.2)

The remaining non-trivial equations are:

\[
\begin{align*}
\frac{c^2}{2} f_{u_1 u_2}^{(1, -3)} - c + |\phi|^2 &= 0 \\
(\nabla_{u_2}^{(1, -3)} + i\nabla_{u_1}^{(1, -3)}) \phi &= 0 \\
\frac{c^2}{2} f_{u_3 u_4}^{(1, -4)} - c + |\chi|^2 &= 0 \\
(\nabla_{u_4}^{(1, -4)} + i\nabla_{u_3}^{(1, -4)}) \chi &= 0
\end{align*}
\] (6.3)

Under the assumption (6.1), we get two copies of the Bogomolny equations. These have been studied in the context of Chern-Simons theory in detail \([18] [19]\) and we only summarize some main features. We can find magnetic soliton solutions to these equations with the background boundary conditions (5.17). Since the spatial world-volume of the D4-brane is fourdimensional, and the tachyons have non-trivial winding number around a circle at infinity, the magnetic solitons are twodimensional. The boundary conditions are treated in detail in \([10]\). Using the solutions, we calculate the D0-brane charge from the worldvolume action of the D4-branes:

\[
N = \frac{1}{8\pi^2} \int d^4u \left( F^{(1)} F^{(1)} + F^{(2)} F^{(2)} - F^{(3)} F^{(3)} - F^{(4)} F^{(4)} \right)
\] (6.4)

\[
= \frac{1}{4\pi^2} \int d^4u F^{(1, -3)}_{u_1 u_2} F^{(1, -4)}_{u_3 u_4}
\]

\[
= \frac{A_4}{c^2\pi^2},
\] (6.5)

which is the original D0-brane charge. The D0 charge is concentrated at the intersections of the orthogonal twodimensional solitons. Moreover, from (6.2) we find that the D2-brane charge cancels. This is consistent with the fact that we find, from the commutators (5.3) and (5.4), and the supersymmetry variations

\[
\delta \theta = \frac{1}{2} \left( D_0 X^I \gamma_I + \frac{1}{2} [X^I, X^{I'}] \gamma_{I,I'} \right) \epsilon + \epsilon'
\] (6.6)

that the tachyon condensation restores all dynamical supersymmetry. We conclude that the end products after tachyon condensation are the original D0-branes, and extra gravitons as argued in \([10]\).
7. Remarks and conclusion

In the previous section, we considered tachyon condensation where the tachyons had trivial boundary conditions relative to the background. We can consider more general possibilities, where the tachyons satisfy different boundary conditions. In the case of a membrane–anti-membrane configuration, this amounts to the following. By choosing the topological sector of the tachyon on the D2-brane anti-D2-brane to be non-trivial, one can add or subtract D0-brane charge. After condensation, this gives an arbitrary number of D0-branes. Technically, this is a trivial extension of [11]. In particular, the approximate solution to the equations of motion in [11] remains practically unchanged. In the case of the D0-D4 and D0-anti-D4, we have more possibilities. For instance, by changing the topological sectors of the four tachyons simultaneously, we can modify the amount of D0-brane charge in the end product in a fairly obvious manner (keeping the condition (6.1)). It is clear that for a more general choice of topological sectors, the end product will have D2-brane charge. It would be interesting to study such condensation in detail.

In this paper we have studied the interactions between a D0-D4 bound state and a D0-anti-D4 bound state in matrix theory. First, we calculated the interaction potential at large distances and successfully compared the result to an equivalent supergravity calculation. Next, we looked at a coinciding D0-D4 and D0-anti-D4 bound state system and identified the tachyonic fluctuations. We derived the classical action for these tachyonic fluctuations and found solutions to the equations of motion corresponding to tachyon condensation to D0-branes.

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APPENDIX

A. The probe-background calculation

The standard technique for calculating the interaction potential (or corresponding phase shift) between two objects from the Born-Infeld action and supergravity approach is the following. You treat one object as the background and take the corresponding solution of the supergravity equations of motion. Next, you consider the worldvolume action of the other object in this background and calculate the potential it feels due to the background. This has been done for many situations in the literature (see for instance [8] [9]). We state the results of these calculations for comparison with the results obtained for matrix theory in the body of the paper. We take the conventions of [8] (2πα' = 1) and we work at self-dual radius (R_i = √α') for the compactified directions. We moreover approximate the potential at large distances and small relative velocities between the two objects. For the interaction between a D0-brane bound state and a D0-D4 bound state we find the following phase shift δ [8]:

\[ \delta \approx \frac{1}{2b^2} N_0 \left[ n_4 v + \frac{1}{4} (n_0 + 2n_4) v^3 \right] + O(\frac{1}{b^5}, v^5). \]  

(A.1)

For the interaction between two D0-D4 bound states, we find [8]:

\[ \delta \approx \frac{1}{2b^2} \left[ (n_0 N_4 + N_0 n_4) v + \frac{1}{4} (n_0 N_0 + n_4 N_4 + 2n_0 N_4 + 2n_4 N_0) v^3 \right] + O(\frac{1}{b^5}, v^5). \]  

(A.2)

For the interaction between a D0-D4 bound state and a D0-anti-D4 bound state we generalize the calculation in [8], to find the potential:

\[ \mathcal{V} \approx \frac{1}{4b^3} \left[ 4n_4 N_4 + (n_0 N_4 + N_0 n_4) v^2 + \frac{1}{4} (n_0 N_0 + n_4 N_4 + 2n_0 N_4 + 2N_0 n_4) v^4 \right] + O(\frac{1}{b^5}, v^6). \]  

(A.3)

The results agree with the matrix theory calculation at large N_0 and n_0. Note that the results for potentials (or corresponding phase shifts (δ = \int dt \mathcal{V}(\sqrt{b^2 + (vt)^2})) in matrix theory can also be compared directly to string theory calculations [7] [15].

B. Technical details

Some of the technical details for determining the action (5.11) are assembled here. We refer to [14] and [10] for the rules to convert matrices into functions and traces into integrals. We only keep the relevant terms and consider static configurations only. First we define the non-center-of-mass coordinates – the center of mass coordinates just describe overall movements of the system in which we are not interested –:

\[ u_i = \frac{x_i + y_i}{2}. \]  

(B.1)
Using the definitions given in the body of the text (5.10), we can write down the
commutators of the coordinate fields relevant to the problem in a reasonably compact
form:

\[
\begin{pmatrix}
0 & -\frac{\sqrt{2}}{2} (\nabla_{u_2}^{(1, -3)} + i \nabla_{u_1}^{(1, -3)}) \phi & 0 \\
\frac{1}{2} (\nabla_{u_2}^{(1, -3)} + i \nabla_{u_1}^{(1, -3)}) \phi & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
\]

\[
\begin{pmatrix}
0 & -\frac{\sqrt{2}}{2} (-\nabla_{u_2}^{(2, -4)} + i \nabla_{u_1}^{(2, -4)}) \phi' & 0 \\
\frac{1}{2} (-\nabla_{u_2}^{(2, -4)} + i \nabla_{u_1}^{(2, -4)}) \phi' & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
\]

The other relevant commutators are all analogous to the following one:

\[
\begin{pmatrix}
0 & -\frac{\sqrt{2}}{2} (\nabla_{u_2}^{(1, -3)} + i \nabla_{u_1}^{(1, -3)}) \phi & 0 \\
\frac{1}{2} (\nabla_{u_2}^{(1, -3)} + i \nabla_{u_1}^{(1, -3)}) \phi & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
\]

\[
\begin{pmatrix}
0 & -\frac{\sqrt{2}}{2} (-\nabla_{u_2}^{(2, -4)} + i \nabla_{u_1}^{(2, -4)}) \phi' & 0 \\
\frac{1}{2} (-\nabla_{u_2}^{(2, -4)} + i \nabla_{u_1}^{(2, -4)}) \phi' & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
\]

The commutators are antihermitian. We then simplify the action by concentrating
on the non-center-of-mass fluctuations of the tachyon (compare [110]):

\[
\phi(x_1, y_1) = \phi(u_i) \sqrt{\delta(x_1 - y_1) \delta(x_2 - y_2) \delta(x_3 - y_3) \delta(x_4 - y_4)}
\]

\[
\chi(x_1, y_1) = \chi(u_i) \sqrt{\delta(x_1 - y_1) \delta(x_2 - y_2) \delta(x_3 - y_3) \delta(x_4 - y_4)}
\]

Then the action reduces to (5.11), the integration running over four variables only.

---

10We leave out the factors of the zero brane density \( \rho_0 \) to avoid cluttering the formulas even more. They can easily be added in [14] [10]
References

[1] T. Banks, W. Fischler, S. Shenker and L. Susskind, Phys. Rev. D55 (1997) 5112, hep-th/9610043
[2] L. Susskind, hep-th/9704080
[3] N. Seiberg, Phys. Rev. Lett. 79 (1997) 3577, hep-th/9710009 ; A. Sen, Adv. Theor. Math. Phys. 2 (1998) 51, hep-th/9709220
[4] T. Banks, N. Seiberg and S. Shenker, Nucl. Phys. B490 (1997) 91, hep-th/9612157
[5] G. Lifschytz, Phys. Lett. B409 (1997) 124, hep-th/9703201; E. Halyo, hep-th/9704086; M. Berkooz and M. Douglas Phys. Lett. B395 (1997) 196, hep-th/9610236
[6] O. Aharony and M. Berkooz, Nucl. Phys. B491 (1997) 184, hep-th/9611213; G. Lyfschitz and S. Mathur, Nucl. Phys. B507 (1997) 621, hep-th/9612087
[7] G. Lyfschitz, Nucl. Phys. B520 (1998) 105, hep-th/9612223
[8] I. Chepelev and A. Tseytlin, Phys. Rev. D56 (1997) 3672, hep-th/9704127
[9] I. Chepelev and A. Tseytlin, Nucl. Phys. B515 (1998) 73, hep-th/9709087; E. Keski-Vakkuri and P. Kraus, Nucl. Phys. B518 (1998) 212, hep-th/9709122
[10] H. Awata, S. Hirano and Y. Hyakutake, hep-th/9902158 (v3)
[11] A. Sen, hep-th/9904207 and references therein.
[12] D. Kabat and W. Taylor, Adv. Theor. Math. Phys. 2 (1998) 181, hep-th/9711078
[13] Y. Aharonov and A. Casher, Phys. Rev. A 19 (1979) 2461
[14] E. Keski-Vakkuri and P. Kraus, Nucl. Phys. B510 (1998) 199, hep-th/9706196
[15] C. Bachas, Phys. Lett. B374 (1996) 37, hep-th/9511043 ; G. Lifschytz, Phys. Lett. B388 (1996) 720, hep-th/9604156
[16] O. Ganor, S. Ramgoolam and W. Taylor, Nucl. Phys. B492 (1997) 191, hep-th/9611202
[17] G. ’t Hooft, Commun. Math. Phys. 81 (1981) 267
[18] R. Jackiw and S-Y. Pi, Prog. Theor. Phys. Suppl. 107 (1992) 1
[19] P. Olesen Phys. Lett. B 265 (1991) 361