Analogue Quantum Computers for Data Analysis

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Abstract

Analogue computers use continuous properties of physical system for modeling. In the paper is described possibility of modeling by analogue quantum computers for some model of data analysis. It is analogue associative memory and a formal neural network. A particularity of the models is combination of continuous internal processes with discrete set of output states. The modeling of the system by classical analogue computers was offered long times ago, but now it is not very effectively in comparison with modern digital computers. The application of quantum analogue modelling looks quite possible for modern level of technology and it may be more effective than digital one, because number of element may be about Avogadro number, \( N \sim 6 \cdot 10^{23} \).

Keywords: Analog Quantum Computer  Associative Memory  Neural Networks

1 Introduction

In the work [8] was considered possible application of analogue quantum computing to recognition of images. In present work the similar approach is developed for some particular models. For the modeling are used quantum systems with finite number of states and it is similar with current models of “digital” quantum computers [2, 4, 5]. Anyway the computations are called here “analogue” because it is extension of models supposed for realization by classical analogue computers already 20–30 years ago by different authors [11, 14].

Feynman [2] has mentioned that using of quantum systems for digital computation does not exploit all their properties. Deutsch [1, 3] has used the features for quantum parallelism, but they can be used also for some kind of analogue computations.

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2 Quantum registers

Let us consider a standard model of quantum computational network \([5]\). Some problems related with the approach is discussed in \([9]\). For description of quantum system is used finite dimensional Hilbert space \(\mathcal{H}\), i.e. complex vector space with Hermitian scalar product:

\[
(a, b) = \sum_{i=1}^{n} a_i \overline{b_i}, \quad \|a\| = \sqrt{(a, a)} \quad (1)
\]

Due to complex conjugation of second vector this form sometime is called sesquilinear. It would be bilinear, symmetrical scalar product only for vectors with all real components.

There are physical notations \([1, 3]\) \(|a\rangle\) for the vector \(a\) and \(\langle a|\) for conjugated co-vector \(a^*\), and

\[
b^*a = (a, b) \equiv \langle b | a \rangle. \quad (2)
\]

The vectors \(|\psi\rangle\) and \(\lambda|\psi\rangle\) for any nonzero complex \(\lambda\) are describe the same physical state. It means that states of quantum systems are rays in the Hilbert space (i.e. elements of complex projective space \(\mathbb{C}P^n\)). Due to the property we can consider only states with unit norm \(\|\psi\| = 1\).

A qubit \([15, 16]\) is a simple quantum system with two states, it is described by 2D complex vector space. We can choose two orthogonal vectors \(|0\rangle, |1\rangle\) as a basis of the space:

\[
|0\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |1\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (3)
\]

A quantum register is a quantum system with N states. “Binary” quantum computer uses some particular case of such q-register, n-qubit register with \(N = 2^n\), considered as compound quantum system with n qubits. For analogue quantum computation under consideration it is possible, but not necessary to use only the binary n-qubits, \(N\) could be any natural number.

A special property of quantum system is the process of measurements \([3]\). It is impossible to get full information about quantum system. The simple measurement device can be described by operator of projection

\[
P_\psi \equiv |\psi\rangle\langle\psi|; \quad P_\psi : |\varphi\rangle \rightarrow |\psi\rangle\langle\psi| |\varphi\rangle \quad (4)
\]

The \(P_\psi\) describes registration of a system \(|\varphi\rangle\) with probability \(\langle\psi | \varphi\rangle^2\). Here \(\|\psi\| = \|\varphi\| = 1\), otherwise:

\[
\text{Prob}_{\varphi\rightarrow\psi} = \frac{\langle\psi | \varphi\rangle^2}{\langle\psi | \psi\rangle\langle\varphi | \varphi\rangle} \quad (5)
\]

The probability of registration is one only if the \(|\psi\rangle\) and \(|\varphi\rangle\) describe the same physical state. The probability is zero if \(|\varphi\rangle\), \(|\psi\rangle\) are orthogonal vectors. The device has only two possibilities.
In more general case there are $N$ orthogonal output states $|\psi_i\rangle$. Let us have a quantum system $|\chi\rangle$

$$
\|\chi\| = 1, \quad \langle\psi_i | \psi_j\rangle = \delta_{ij}, \quad |\chi\rangle = \sum_{i=1}^{N} a_i |\psi_i\rangle, \quad a_i = \langle \chi | \psi_i \rangle.
$$

We have one of $|\psi_i\rangle$ due to measurement of the system $|\chi\rangle$ with probability $\Pr_i = |a_i|^2$. Here are $N$ versions of outcome instead of two in the previous example.

The simple example of the measurements is Stern–Gerlach device [3], Fig.(1).

![Stern–Gerlach experiment with a beam of particles](image)

**Figure 1:** Stern–Gerlach experiment with a beam of particles

### 3 Data analysis

For application to data analysis let us consider more simple case of real vector space $\mathbb{R}^n$ (or projective space $\mathbb{R}P^n$) [3]. The scalar product is:

$$
(\vec{w}, \vec{v}) \equiv \sum_{i=1}^{n} w_i v_i
$$

The norm Eq.(7) is real version of complex norm used in Eqs.(1,2,4). For vectors with unit length it is just a cosine of the angle between them. The bilinear measure is often used for data analysis. An application to image recognition was described in [8].

Historically, one of possible reasons for considering such kind of models was observation of properties of usual biological neuron [11]. The formal model of neuron [10] can be used for convenient description of analogue data analysis under consideration. The formal neuron sums input signals of synapses $s_i$ with some weights $w_i$, see Eq.(8), $A = (\vec{w}, \vec{s}) = \sum_i w_i s_i$. Here $\vec{w}$ is a vector of weights and $\vec{s}$ is a vector of signals.

In models of artificial neural networks can be used some additional transformations $f : A \rightarrow f(A)$. In one of the first model, Rosenblatt’s *perceptron*, it was threshold function $f(A) = \theta(A - A_0)$, i.e. $f(A)$ is 1 if $A > A_0$ and otherwise
it is zero. An application of possible realization of similar system by quantum computer for expert systems was described shortly in [7].

In many contemporary work \( f(A) \) is nonlinear functions \([13]\), like \( s(A) = 1/(1 + e^{-cA}) \). The using of nonlinear model was related with a problem of modeling arbitrary binary functions by perceptron \([12]\). But it is not the problem for quantum networks \([5, 2]\).

Let us use algorithms without using of nonlinear functions, \([10, 14]\). The models are useful as an introduction to analogue quantum computations discussed further.

In the \([10]\) is considered method of coding a value of parameter by number of channel (CVPNC), i.e. number of (most) activated element of network corresponds to value of parameter. The work uses network of the formal neurons with equal length of weight vectors \(|\vec{w}(k)|\). Maximum of output signal \( A(k) = (\vec{w}(k), \vec{s}(k)) \) corresponds to the same direction of the vectors \( \vec{s} \) and \( \vec{w} \). Such kind of network does not distinguish the vectors \( \vec{s} \) and \( \alpha \vec{s} \) for a positive real \( \alpha \). The space of signals is similar with space of ray discussed in section 2.

A model of analogue auto-associative memory (AAAM) is presented in \([14]\). For representation of images is used vector space \( \mathbb{R}^N \). In chapter 2.3 of the book is described error correction for images by orthogonal projection to subspace, linear span of the images. As an example is used set of gray scaled pictures with \( N = n \times m \) elements. The best result is produced for orthogonal images. If number of images \( i \ll N \) then they are “almost orthogonal” because scalar product of two random vectors with unit norm is:

\[
(\vec{w}, \vec{v}) \sim N^{-1/2} N^{-\infty} 0
\]

## 4 Quantum analogue computations

Two last examples in previous section can be considered as a real basis for analogue quantum modeling because the basic operations are similar with measurement processes described by Eqs. (4,6) and discussed above in relation with the formulae.

In case of quantum register we have a complex space instead of real, but it is possible to use the models. First model, CVPNC, uses property of maximum scalar product Eq.(2) for two equal vectors on hypersphere. The Hermitian norm Eq.(1) has the same property and probability of registration \( \text{Prob}(\varphi \rightarrow \psi) \) in Eq.(3) is maximum, unit, if \( |\varphi\rangle \) and \( |\psi\rangle \) are the same physical state (the same ray in Hilbert space).

It is possible to describe the quantum CVPNC model: We have set of properties. Each property described by a vector (ray). An analogue system must find number of property corresponded to input signal. It can be described also as “recognition of an image stored in associative memory of the system” (see [8]).

\(^1\)A nonlinearity is bad for realization by analogue quantum computers
If all vectors are orthogonal, it is enough to use one device described by Eq.(6) with $|\psi_i\rangle$ equal to $i$-th vector in the set of properties (images). If input signal corresponds to one of stored properties, the output channel will have desired number with certainty, but if the signal have some error the process would be probabilistic. For small errors the most probability has desired channel.

There is a problem if signal differ enough with all images. For example, if the vector of signal is composition of two images, $i$-th and $j$-th, $|s\rangle = \alpha|w(i)\rangle + \beta|w(j)\rangle$, ($|\alpha|^2 + |\beta|^2 = 1$ because norm of $|s\rangle$ is unit) then the output would be either $i$ with probability $|\alpha|^2$ or $j$ with probability $|\beta|^2 = 1 - |\alpha|^2$.

But this is not specific problem of the system under consideration, the human vision has the similar kind of behavior for an ambiguous images, see Fig.(2). For modeling such system by usual computer it would be necessary to use random number generator, but for the system under consideration such results was made “free”, without any extra modules.

The picture of a cube is an example of ambiguous image recognition. It is impossible to find with certainty that point is nearer, A or B. Sometimes the picture looks like cube with upper face is close to observer, sometimes with lower one. Sometimes it can looks like a flat figure.

![Figure 2: Ambiguous picture of a cube](image)

One of simple examples of such a system could be Stern–Gerlac h device, Fig.(1). Here an input signal is coded by state of particle in the beam [8]. A disadvantage of the device is macroscopic size. There is some problem with description of microscopic version of processes related with irreversible operators of projection $P_\psi$. On the other hand such processes are quite common and they are considered as one of main problems for quantum computers (reduction, decoherence, decay, etc.). Contrary, the processes could be useful for analogue quantum computations described above.

If vectors of images are not orthogonal it is impossible to use the simple model with one quantum system. An extra problem is because evolution of quantum system described by unitary operators. The operators by definition can not change the scalar product Eqs.(1,2). So if two vectors are not orthogonal, it is impossible to correct the problem by unitary quantum evolution.

An advantage of orthogonality was discussed in [14] for real vector space and AAAAM. There is a property of error correction by orthogonal projection to linear span of all images. For analogue quantum computation it is possible to use similar way of error correction by projection on subspace. It is similar to error correction schemes of “digital” quantum computers[17].

For complex vector spaces it is also possible to use property similar to Eq.(8), i.e. “almost orthogonality” of random rays.
The using of nonorthogonal images is possible if input signal is represented by few quantum systems in the same state instead of only one system. An example is Stern–Gerlach experiment with filtered beam of atoms as an input. It may be possible to distribute the signal between few filters \( P_{w(k)} \), see Eq.(1), for realization quantum version of CVPNC model (see also [8]).

5 Conclusion

In the paper is presented some possible applications of analogue quantum computations. For the modeling was used quantum system with finite number of states. It is shown that such kind of system could be used for development of known models like associative memory by T. Kohonen (computer science) and a model of neural network by E. Sokolov (experimental neurophysiology).

In a simplest case it is enough to use one system like atom with a few states for each element. It make possible to create media with number of elements about \( N \sim 10^{23} \). One of disadvantages of the model is using a data from associative memory instead of more advanced algorithmic data analysis. But it can be compensated for some application by such huge amount of elements. It is similar to using of reflexes [10] in standard situations instead of reasoning.

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