Onset of Turbulence in Superfluid $^3$He-B and its Dependence on Vortex Injection in Applied Flow

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Abstract. Vortex dynamics in $^3$He-B is divided by the temperature dependent damping into a high-temperature regime, where the number of vortices is conserved, and a low-temperature regime, where rapid vortex multiplication takes place in a turbulent burst. We investigate experimentally the hydrodynamic transition between these two regimes by injecting seed vortex loops into vortex-free rotating flow. The onset temperature of turbulence is dominated by the roughly exponential temperature dependence of vortex friction, but its exact value is found to depend on the injection method.

Introduction. Superfluid $^3$He-B is the best medium to study the influence of friction in vortex dynamics. The friction arises when the superfluid vortex moves with respect to the flow of the normal component. It consists of the longitudinal dissipative and the transverse reactive contributions, characterized by the mutual friction parameters $\alpha$ and $\alpha'$ in the equation for the vortex line velocity $v_L = v_n + \alpha \hat{s} \times (v_n - v_s) - \alpha' \hat{s} \times [\hat{s} \times (v_n - v_s)]$. Here $\hat{s}$ is a unit vector parallel to the vortex line element. The velocities of the normal and superfluid fractions are $v_n$ and $v_s$, while the difference $v = v_n - v_s$ is called the counterflow velocity, the hydrodynamic drive. An important feature of $^3$He-B hydrodynamics is the large viscosity of the normal component. Transient processes in the normal flow decay quickly and can be neglected. Owing to this simplification, with some modification the results for rotating flow can be carried over to other types of flow. With careful design and preparation of the sample container the energy barriers preventing vortex formation can be maintained high, so that high-velocity vortex-free flow can be achieved.

The dynamics of quantized vortex lines can be explored if one injects vortex loops in this meta-stable high-energy state of vortex-free flow. The fate of the seed loops depends on temperature: At high temperatures the number of vortices ($N$) is conserved, the injected loops expand to rectilinear lines, and the flow relaxes only partially. At low temperatures below some onset temperature, rapid vortex proliferation from the seed loops takes place in a transient turbulent burst which leads to the formation of the equilibrium number of vortices ($N_{eq} \sim 10^3$) and to a complete removal of the applied flow. At sufficiently low temperatures ($T < 0.45 T_c$) the turbulent burst always follows, even after the injection of a single vortex ring, independently of the injection method. However, in a narrow temperature regime around the onset of turbulence the situation is different: the injection may or may not lead to turbulence depending on the injection details and the velocity of the applied flow.

In this report we examine the dependence of the onset temperature on the injection properties. The motivation is the following: A number of different processes with their individual energy barriers have to be traversed sequentially before turbulence in the bulk volume becomes possible. The first is vortex nucleation. It is here avoided by the injection of the seed loops. Next follows a series of events which build up the vortex density locally somewhere in the bulk volume for turbulence to start. These processes act at the container wall. They have been explored in Ref. [2]. Compared to turbulence in viscous fluids, the path leading to turbulence in superfluids appears to be more straightforward to reconstruct.

Injection methods. In our experiment the flow is created by rotating a cylindrical sample of 110 mm length and $R = 3$ mm radius around its symmetry axis. We employ four different techniques to inject vortex loops in the rotating flow, in order to study the transient evolution from the vortex-free state to a final stable state with a central vortex cluster consisting of rectilinear vortex lines. The final state is only meta-stable, unless the cluster contains the equilibrium number $N_{eq}$ of vortex lines. With NMR techniques we measure the number of lines $N$ close to both ends of the sample. At temperatures above the onset of turbulence one observes regular ex-
pansion of the injected loops to rectilinear lines, i.e. in
the final state $N \ll N_{eq}$. At temperatures below onset the
injection evolves into a turbulent burst, which results in
a large increase in $N$, so that in the final state $N \approx N_{eq}$.

The different injection techniques are compared in
Fig. 1 and Table 1. The number of injected vortex loops,
their size, proximity, and the initial vortex density at
the injection site vary from one injection technique to
the next. We would like to answer the question whether
these properties, besides temperature and drive velocity $v = \Omega R$, influence the onset to turbulence.

The first two injection methods are the most repro-
ducible. They are based on the properties of the first order
interface between the A and B phases of superfluid $^3\text{He}$. The A phase is stabilized with a magnetic barrier field
over a short section in the middle of the long sample.
The AB interface undergoes an instability of the Kelvin-
Helmholtz type when flow is applied parallel to it. As a
result of the instability vortex loops are tossed across the
interface from the equilibrium vortex state in A phase to
the vortex-free B phase. The KH instability can be trig-
ered (1) by sweeping the rotation velocity $\Omega$ up to a
well-defined critical threshold $\Omega_c(T, P)$ or (2) by sweep-
ing at constant $\Omega > \Omega_c$, the magnetic field $H$ from a low
value up to where the A phase is suddenly nucleated with
some magnetic hysteresis at $H > H_{AB}(T, P)$.

The third injection method makes use of rapid local-
ized heating in a neutron absorption event. From the
overheated volume, which is $\sim 50 \mu m$ in diameter, one
or more vortex rings may evolve above a critical threshold $\Omega_{cn}(T, P)$. The number of rings depends on the applied
flow $v = \Omega R$ [4]: Just above $\Omega_{cn}$ only one vortex loop is
created per absorption event, but with increasing $\Omega$ the
average number of loops per injection event increases,
reaching $(N) \approx 5$ at $\Omega / \Omega_{cn} = 4$.

In the fourth method we start from an existing remnant
vortex and create the flow later. In a strict sense this is not
injection as in the three earlier cases, but in practice it
achieves the same result of placing a curved vortex loop
in applied flow. With decreasing temperature the last one
or two vortices require an ever longer time to annihilate
at $\Omega = 0$ because of the rapidly reducing vortex damp-
ing. Thus it becomes possible to catch a remnant vortex
before its annihilation in a random location at the cylin-
drical container wall. When $\Omega$ is suddenly increased to
some final stable value, we may monitor the stability of
the remnant vortex in the applied flow [2].

The variations in the injection properties prove to be
larger between the different methods than the variability
within one method from one run to the next. We there-
fore list the perturbations, which the different injection
methods generate, in an order from the strongest to the
weakest: (1) First comes the nucleation of $^3\text{He}-A$ dur-
ing a magnetic field sweep, where the injected loop num-
ber is typically in the few hundreds. Next comes (2) the

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure1.png}
\caption{Vortex loop configurations at injection in a ro-
tating cylinder: (a) Two-phase sample with roughly the equi-
librium number of vortices in A phase and vortex-free B phase.
The A phase vortices curve at the AB boundary on the interface,
forming there a surface layer of vorticity. In the KH instabil-
ity the A-phase vortices in the deepest trough of the interface
wave are tossed on the B-phase side in the region between
$R_{AB} \approx 2.6 mm$ and the cylinder wall at $R = 3 mm$. (b) When
the barrier field is slowly swept up (at constant $\Omega$, $T$, $P$) the
nucleating A-phase layer is unstable and a massive number of
vortex loops escapes to the B phase. (c) In a neutron absorption
event close to the cylinder wall vortex rings are extracted by the
applied flow from the reaction heated volume. (d) A remnant
vortex, which has not yet had sufficient time to annihilate at
$\Omega = 0$, forms a seed for new vortex formation when the applied
flow velocity $v = \Omega R$ is increased from zero.}
\end{figure}

KH instability of the AB interface, (3) followed by neu-
tron absorption at higher flow velocities. (4) Neutron ab-
sorption at lower velocities $\Omega \gtrsim \Omega_{cn} \sim 1 \text{rad/s}$ generates
only one ring. The weakest perturbation is (5) a remnant
vortex which can be studied down to low flow veloc-
ties. The absolute limit is the velocity at which a loop
with a radius of curvature $\lesssim R$ is able to expand: $v \gtrsim \kappa / (4\pi R) \ln (R / \xi)$ which corresponds to $\Omega \sim 10^{-2} \text{rad/s}$. (Here $\kappa$ is the circulation quantum and $\xi$ the coherence
length.) In practice the limit is set by our NMR detection
which at present requires $\Omega \gtrsim 0.4 \text{rad/s}$ [2].

Onset of turbulence. Let us examine with our different
injection methods the probability of turbulence within
the transition regime. We first consider the situation at a
higher temperature of $0.53 T_c$: (1) Here the nucleation
of $^3\text{He}-A$ with magnetic field always gives the equilibrium
vortex state. (2) Also KH injection has a high probability $p_{AB} = 0.96$ to start turbulence ($P = 29 \text{bar}$, $\Omega = 0.8–
1.6 \text{rad/s}$) [5]. In contrast, (3) for neutron absorption at
$\Omega = 2.32 \Omega_{cn}$ the probability is $p_n = 0.09$ (Fig. 3). (4) No turbulent bursts have been observed with neutron in-
projection, if \( \Omega < 2\Omega_{cn} \). (5) Similarly vortex multiplication is not triggered by a remnant vortex at this temperature (at \( P = 29 \) or 10 bar). The different injection processes thus yield different onset temperatures for turbulence.

When temperature and mutual friction damping decrease, the magnitude of the flow perturbation, which is needed to start turbulence, decreases as well. At 0.45\( T_c \) (\( P = 29 \) bar), vortex injection via (1) the nucleation of the A phase or (2) the KH instability always result in a turbulent burst. (3) Similarly neutron absorption at high flow \( \Omega > 2\Omega_{cn} \) also has unit probability to initiate turbulence. (4) Even at lower flow velocity \( \Omega \gtrsim \Omega_{cn} \), where only a single vortex ring can be injected from the neutron bubble, the probability of obtaining turbulence is \( p_n = 0.9 \) – 1. (5) A remnant vortex gives a probability of about 0.9 (\( P = 10 \) bar) [2]. We concede that at low temperatures even the smallest perturbation, a single quantized vortex loop, will evolve to a turbulent tangle.

As seen from Table 1, the various injection methods differ in a variety of ways. The intensity of the perturbation, which they present to vortex-free flow, arises from a combination of different properties. A most important characteristic is the number of injected loops. This is illustrated as a probability distribution in Fig. 2 for two particular cases of injection, namely via KH instability and neutron absorption. Clearly the above examples from injection experiments at 0.53 and 0.45\( T_c \) support the simple notion that the more vortex loops are initially injected, the larger is the probability of turbulence. To explain these observations, we have to assume that at 0.53\( T_c \) one needs to inject 4 – 5 loops to achieve turbulence, while at 0.45\( T_c \) a single loop suffices. A characterization of the injection methods in Table 1 in terms of the number of injected loops would seem like a gross oversimplification which ignores other differences, like the size of the volume where the initial perturbation is localized. Still, in the light of our results it looks conceivable that the number of injected loops is a reasonable first measure of the intensity of the flow perturbation.

**Discussion.** With each injection method the transition from regular vortex dynamics at high temperatures to turbulent vortex dynamics at low temperatures occurs in a temperature interval of certain width. We attribute this width to the variability in the injection from one time to the next, since each injection method is characterized by a certain distribution of configurations of injected loops. Only some of these configurations result in turbulence at higher temperatures. With decreasing temperature more configurations become effective and eventually at low enough temperatures all configurations produced by a given injection method result in turbulence. The typical full width for a given injection method is \( \sim 0.06T_c \) [2]. This width is generally smaller than the difference between the average onset temperatures measured with different injection methods.

Stability considerations of steady-state turbulence lead to the conclusion that on average the transition between regular and turbulent vortex motion is given by a condition on the ratio of the mutual friction coefficients:

\[
q = \frac{\alpha}{(1 - \alpha')} - 1 \quad (1, 3)
\]

Our measurements with different injection methods and at different external conditions of \( T, P, \Omega \) show that the onset temperature is approximately predicted by this criterion, but the exact value of temperature does not correspond to any universal critical value of \( q \). Whether \( q \) may still have a universal critical value in the particular case of sustained homogeneous well-developed turbulence is unclear, since no such measurements exist in the relevant range \( q \sim 1 \).

The dependence of the onset on the intensity of the flow perturbation resembles that observed in recent measurements [3] on the flow of a classical viscous liquid in a circular pipe. Here a perturbation of finite magnitude \( \varepsilon \) needs to be injected in the flow, to turn it from laminar to turbulent. A scaling law connects the smallest possible perturbation \( \varepsilon \) and the Reynolds number: \( \varepsilon \propto Re^{-1} \), i.e. the minimum perturbation decreases with increasing flow velocity. In superfluids the analog of the Reynolds number Re is \( q^{-1} = (1 - \alpha')/\alpha \) [1, 3]. Its magnitude increases monotonically with decreasing temperature (but does not depend on flow velocity).

The hydrodynamic transition from laminar (regular) to turbulent flow in viscous liquids and in superfluids dif-

| Method                        | Trigger                        | Number of loops | Length scale                        | Location in sample container                      |
|-------------------------------|--------------------------------|-----------------|-------------------------------------|--------------------------------------------------|
| Kelvin-Helmholtz instability of AB interface [3] | Slow \( \Omega \) sweep across \( \Omega_{c}(T, P, H) \) \sim 0.8 – 1.6 rad/s | smooth distribution \( N \sim 3 – 30 \) (peak \sim 8, long tail up to 30) | \( 0.1 – 1 \) mm, a wavelength of ripplon | At AB interface close to cylindrical wall at \( \Delta R \sim 2.6 \) mm \(< R \) |
| Nucleation of \(^3\)He-A in magnetic field from \(^3\)He-B | Slow sweep of barrier field up to \( H_{AB}(T, P) \) | \( 1 \ll N < N_{eq} \) | Circumference of AB interface along cylindrical wall | At newly forming AB interface |
| Neutron absorption [4, 2] | Neutron absorption event | \( N \sim 1 – 5 \) depends on \( \Omega/\Omega_{cn} \) | \( \sim 100 \mu m \) (diameter of largest vortex ring) | Random location close to cylindrical wall |
| Remnant vortex [2] | Rapid increase of \( \Omega \) from zero | \( N \sim 1 \) | \( \sim R \), size of remnant vortex loop at \( \Omega = 0 \) | Random and distributed along sample |
fers from a usual first order phase transition, such as e.g. the transition from supercooled meta-stable A phase to the stable equilibrium B state of superfluid $^3$He. A single energy barrier, the creation of a sufficiently large seed bubble of B phase with a critical radius of order $\sim 1$ $\mu$m, prevents the A$\rightarrow$B phase transition. If such a seed bubble is injected, then the stable B phase is irreversibly created in all of the available volume. If the seed is too small, then it will shrink and disappear. In viscous liquid flow along a circular pipe [8], an injected perturbation creates turbulent “puffs” and “slugs” in the laminar flow, which are limited in space and do not extend over the whole length of flow. In our superfluid experiments an analogous case appears if vortex multiplication in a turbulent burst stops before the vortex number has reached $N_{eq}$ and large applied flow remains. Such intermediate events are rare, but have been observed in the onset regime: In the middle of the transition region their proportion among the well-characterized events (i.e. those which conserve the number of injected vortices and those where the meta-stable flow is completely removed) is at most a few percent. The existence of such intermediate behavior is a further demonstration that in hydrodynamic transitions often there is no single well-defined energy barrier which one should overcome to cause a spontaneous change from a meta-stable to a stable flow pattern [2].

**Conclusion.** The onset temperature of turbulence after the injection of vortex loops in meta-stable vortex-free flow depends on several variables, but the decisive factor appears to be the number of injected loops. When the number of loops increases they are more likely to become unstable towards turbulence. As a result the onset temperature of turbulence increases and the value of the friction parameter $q$ at onset decreases. This connection between the amplitude of the flow perturbation and the “Reynolds number” $q$ at onset resembles the scaling law of classical turbulence [8].

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