Turbulent Heating in Solar Wind Thermodynamics

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Abstract

This paper considers the concept of wave-particle thermodynamic equilibrium in order to improve our understanding of the role of turbulent heating in the solar wind proton plasma. The thermodynamic equilibrium in plasmas requires the energy of a plasmon—the quantum of plasma fundamental oscillation—to be balanced by the proton-magnetized plasma energy, that is, the magnetic field and proton kinetic/thermal energy. This equilibrium has already been confirmed in several prior analyses, but also in this paper, by analyzing (i) multi-spacecraft data sets along the radial profile of the inner heliosphere, and (ii) representative data sets of a variety of 27 different space and astrophysical plasmas. Recently, it was shown that the slow mode of the near-Earth solar wind plasma is characterized by a missing energy source that is necessary for keeping the energy balance in the plasmon–proton-magnetized plasma. Here we show strong evidence that this missing energy is the turbulent energy heating the solar wind. In particular, we derive and compare the radial and velocity profiles of this missing energy and the turbulent energy in the inner heliosphere, also considering other minor contributions, such as the temperature of pickup protons. The connection of the missing plasmon–proton energy with the turbulent energy provides a new method for estimating and cross-examining the turbulent energy in space and astrophysical plasmas, while it confirms the universality of the involved new Planck-type constant that implies a large-scale quantization.

Unified Astronomy Thesaurus concepts: Astronomy data analysis (1858); Space plasmas (1544); Plasma astrophysics (1261); Solar wind (1534); Heliosphere (711)

1. Introduction

Space and astrophysical plasmas are a ubiquitous form of matter in the universe, nearly always found to be turbulent. The turbulence is a chaotic, stochastic process that alters the characteristics of the plasma fluctuations. The entire heliosphere is closely linked to the properties of plasma turbulence. Solar wind protons flow throughout the supersonic heliosphere under the influence of expansive cooling and two primary groups of turbulent heating sources: (i) the solar-origin large-scale energy fluctuations; and (ii) the excitation of plasma waves by newborn interstellar pickup ions (e.g., Smith et al. 2001, 2006; Adhikari et al. 2015). This paper investigates the interplay and partition of these turbulent heating sources in solar wind thermodynamics.

In thermodynamically stable space plasmas, the wave-particle thermodynamic equilibrium requires the energy of a plasmon—the quantum quasi-particle of plasma oscillations—to be balanced by the energy of the proton-magnetized plasma, that is, the field and proton average energy. This has been verified in a number of analyses in space and astrophysical plasmas (Livadiotis & McComas 2013a, 2014a, 2014b; Witze 2013; Livadiotis 2015, 2016, 2017, Ch.5; 2018a; Livadiotis & Desai 2016; Livadiotis et al. 2018). However, in the case of the expanding solar wind in the inner heliosphere, a difference between the plasmon and proton plasma energies has been observed, which decreases with the wind speed and the heliocentric distance $R$ (Livadiotis & Desai 2016). We speculate that this energy difference comprises the turbulent energy, responsible for heating the solar wind proton plasma, which was not considered in the plasmon–proton plasma energy balance. In particular, we may ask the following question:

Is there a solid connection between the solar wind turbulent energy and the plasmon–proton-magnetized plasma energy balance? If yes, then, the solar wind thermodynamic equilibrium can be used for developing a new method of estimating the turbulent heating of solar wind.

The purpose of this paper is to improve our understanding of the following subjects: (i) the nature of the missing energy, that is, the difference in the balance between plasmon and proton-magnetized plasma energies; (ii) the connection of the missing energy to the mechanisms of heliospheric turbulent heating; (iii) the partition of the energies involved in the solar wind thermodynamic equilibrium; and (iv) the thermodynamic equilibrium and the energy balance between a plasmon and a proton-magnetized plasma, interwoven with the concept of large-scale quantization constant. Thermodynamic equilibrium and large-scale quantization will be examined and further developed in Section 2. In Section 3, we investigate the velocity and radial profiles of the missing energy, while in Section 4, we examine all the components contributing to the missing energy in the plasmon–proton-magnetized plasma energy balance, focusing on the (i) turbulent energy, (ii) temperature of pick-up ions, and (iii) gravitational potential energy. In Section 5, we formulate the missing energy by assembling all the previously discussed components, and then compare the constructed missing energy with the observed turbulent energy. The results lead to rewriting the proton-magnetized plasma thermodynamic equilibrium, and provide a new method for estimating and cross-examining the turbulent energy in space and astrophysical plasmas. Finally, Section 6 summarizes the results.
2. The Plasmon–Proton Plasma Missing Energy

2.1. Plasmon–Proton Plasma Energy Balance

The wave-particle thermodynamic equilibrium in plasmas requires the energy \( E_{pl} \) of a plasmon (energy quantum) to be balanced by the proton-magnetized plasma energy \( E_p \) that is, the magnetic field and proton kinetic/thermal energy, namely:

\[
E_{pl} = E_p, \quad \text{with} \quad (1a)
\]

Plasmon energy \( (E_{pl}) \): energy of a quantum \( (\hbar \omega) \), \( (1b) \)

Proton-magnetized plasma energy \( (E_p) \): magnetic and thermal energy. \( (1c) \)

The plasmon energy \( E_{pl} \) is the energy of one quantum \( \hbar \omega \), where the frequency spectrum peaks at the fundamental plasma oscillation frequency \( \omega \sim \omega_{pl} = [n \cdot e^2 \varepsilon_0^2 (m_e^{-1} + m_p^{-1})]^{1/2} \) (e.g., Thejappa et al. 1993, 2012). Plasmons with only \( \omega \sim \omega_{pl} \) occur in the approximation of spatial scales quite larger than the Debye length. The proton plasma energy density (in the reference frame of the flow) is mainly given by the sum of its thermal energy density for a compressible flow, \( \gamma/(\gamma - 1) nk_B T \), and the magnetic energy density, \( B^2/(2\mu_0) \); note that the fraction 1/2 comes from averaging sin\(^2\)(\(\alpha\)), where \(\alpha\) is the angle between particle velocity to the magnetic field (e.g., Park et al. 2019). This proton plasma energy density, divided by the proton number density, gives the proton plasma energy per (proton) particle, which can be simply referred to as proton energy \( E_p \) (Livadiotis & McComas 2014a).

In our approximation we will consider solar wind and pickup protons; other particles, such as alphas and pickup helium, are low in density, thus they do not significantly contribute to the total plasma energy density. (Parameter symbols: \( m_e \), \( m_p \); electron and proton masses; \( e \): elementary electric charge; \( \varepsilon_0 \): permittivity; \( \mu_0 \): permeability; \( k_B \): Boltzmann constant.)

Therefore, we have the energies:

\[
E_{pl} = \hbar \omega_{pl}, \quad (2a)
\]

\[
E_p = \frac{1}{2\mu_0} B^2 / n_p + \frac{\gamma}{\gamma - 1} k_B T_p + \text{other...}, \quad (2b)
\]

where “other” means smaller energy contributions that will be examined in Section 5.

Therefore, the plasma thermodynamic equilibrium in Equation 1(a), \( E_{pl} = E_p \), is materialized by the balance of plasmon energy and proton-magnetized plasma energy, given in Equations 2(a) and (b), that is,

\[
\hbar \omega_{pl} = \frac{1}{2\mu_0} B^2 / n_p + \frac{\gamma}{\gamma - 1} k_B T_p. \quad (3)
\]

Before describing other smaller contributions to the plasma energy \( E_p \), let us examine the concept of large-scale quantization constant that is unfolded by the star subscript of Planck’s constant in Equation (3).

2.2. Large-scale Quantization Constant

A number of analyses have already confirmed that for space plasmas, the ratio between ion’s average energy \( E_p \) and plasma frequency \( \omega_{pl} \) is constant—as expected for the equality of \( E_{pl} = \hbar \omega_{pl} \) with \( E_p \) shown in Equation (3). Surprisingly, however, the constant value of the ratio \( E_{pl}/\omega_{pl} \) is not equal to the Planck constant \( \hbar = 1.05... \times 10^{-34} \text{J} \cdot \text{s} \); instead, it is shown that space plasmas lead to indeed a constant value, but \( \sim 12 \) orders of magnitude larger, \( (1.19 \pm 0.05) \times 10^{-22} \text{J} \cdot \text{s} \) (Livadiotis & McComas 2013a, 2014a, 2014b; Witze 2013; Livadiotis 2015, 2016, 2017, Ch.5; 2018a; Livadiotis & Desai 2016; Livadiotis et al. 2018); this large-scale analog of Planck’s constant is noted by \( \hbar \).

Figure 1 demonstrates the large variation of the representative average values and uncertainties of the plasma parameters.
of 27 space and astrophysical plasmas, while the respective values of the ratio $E_p/\omega_{pl}$ remain almost constant (see also Livadiotis & McComas 2013a, 2014a). (Note that all 27 values for each parameter are normalized to their maximum value.) Quantitative comparison of the distribution and variance of these values is shown in Figure 2. We observe that the standard deviation of the normalized values of $\log E_p/\omega_{pl}$ is 10–30 orders of magnitude smaller than the standard deviations of the other normalized parameters. (For details on the data sets used, see Table 1 and details in Appendix A.)

Having verified the small variability of the values of $\log E_p/\omega_{pl}$ for the examined 27 types of space and astrophysical plasmas, it is straightforward to apply Equation (3) to derive the value of $h_\omega$. All the values of $\log E_p/\omega_{pl}$ and their uncertainties are plotted in Figure 3(a). The corresponding value of $h_\omega$ derived from the estimates of the average values of $E_p/\omega_{pl}$ for all 27 types of space and astrophysical plasmas is $\log h_\omega \approx -21.95 \pm 0.07$ or $h_\omega \approx (1.12 \pm 0.17) \times 10^{-22}$ s, which is within the 1σ of the abovementioned known value of $h_\omega \approx (1.19 \pm 0.05) \times 10^{-22}$. The histogram of Figure 3(b) is constructed by generating 1000 normally distributed values for each of the original 27 values of $\log E_p/\omega_{pl} \pm \delta \log E_p/\omega_{pl}$, according to the technique shown by Livadiotis (2016, see their Figure 8(a)).

We have seen the constancy of the ratio $E_p/\omega_{pl}$ by examining the representative parameter values from a variety of 27 space and astrophysical plasmas. As a result, we use the actual measurements of solar wind proton plasma to again derive the ratio $E_p/\omega_{pl}$. Voyager 1 and 2 measurements of the solar wind—a largely variant plasma—reveal a quasi-fixed value of the ratio $E_p/\omega_{pl}$. Figure 4 plots the derived values of the ratio $E_p/\omega_{pl}$ against the heliocentric radial profile from 2 to 10 au. These plots, as well as the histograms on the right side confirm the constancy of the ratio $E_p/\omega_{pl}$.

3. Missing Energy

3.1. Velocity Profile of the Missing Energy

The constancy of the ratio $E_p/\omega_{pl}$, and thus, the plasmon–proton plasma thermodynamic equilibrium, has been confirmed by various space plasma measurements in previous years. Nevertheless, the thermodynamic equilibrium appears to be violated in the case of the slow and near-Earth measurements of the solar wind (e.g., Livadiotis & Desai 2016). More precisely, it has been observed that near 1 au, the ratio of the proton energy over the plasma frequency, $E_p/\omega_{pl}$, deviates from the constant value $h_\omega$ that characterizes space plasmas. This deviation is larger for smaller solar wind speed; indeed, the ratio $E_p/\omega_{pl}$ undergoes a continuous transition from the slow to the fast solar wind, tending asymptotically toward the known value of $h_\omega$ (Figure 4). The observed deviation of the ratio $E_p/\omega_{pl}$ from the constant $h_\omega$ is caused by a difference between plasmon and proton energies:

$$\text{Missing Energy} = \text{Plasmon energy} - \text{Proton energy}, \quad (4a)$$

$$\Delta E = h_\omega \cdot \omega_{pl} - \frac{1}{2} \frac{B^2}{n_p} - \frac{\gamma}{\gamma - 1} k_B T_p \quad (4b)$$

According to Figure 5, the missing energy $\Delta E$ is larger for low solar wind speeds and smaller for high solar wind speeds, and is actually negligible for speeds higher than $V_{sw} > 550$ km s$^{-1}$. This dependence of the missing energy $\Delta E$ on solar wind speed is similar to the behavior of the turbulent energy in the interplanetary space. Indeed, turbulence is more intense in the slow rather than the fast solar wind (Hadid et al. 2017).

3.2. Radial Profile of the Missing Energy

We examine in detail the plasmon–proton plasma thermodynamic equilibrium, as well as its violation observed in the slow and near-Earth solar wind. Using Equation 4(b) we calculate the missing energy, $\log \Delta E/m_p$, and illustrate it as a function of solar wind speed $V_{sw}$ and the heliocentric distance $R$ (Figure 6). In panel (a), the logarithm of the missing energy, $\Delta E$ per proton mass, as formulated in Equation 4(b), is depicted as a function of the solar wind speed $V_{sw}$, for each radial bin of the heliocentric distances $R$ from 0.29 to 5.41 au; each radial bin is color-coded.

In particular, for each radial bin, we perform a second binning among the values of the solar wind speed $V_{sw}$ (with constant width of bins $\Delta V_{sw} = 10$ km s$^{-1}$). Then, for each $V_{sw}$–bin we estimate the mean value and standard error of $\log \Delta E/m_p$. The central value and half-width of each $V_{sw}$–bin determine the mean value and error of $V_{sw}$, respectively. Furthermore, we perform a linear fitting of the points $(V_{sw} \pm \delta V_{sw}, \log \Delta E/m_p \pm \delta \log \Delta E/m_p)$ within each radial bin. The intercept and slope—and their errors—derived from these fits are plotted in panels (b) and (c), respectively, as a
function of $R$ (again, the central value and half-width of each radial bin determine the mean value and error of $R$ in these panels). We observe that on average both the intercept and slope decrease when $R$ increases.

4. Components of the Plasmon–Proton Plasma Missing Energy

4.1. Turbulent Energy

There are three primary sources of turbulence in the heliosphere: (1) turbulence driven by shear due to the interaction between fast and slow solar wind streams (Coleman 1968; Roberts et al. 1992), (2) compressional sources of turbulence due to stream–stream interactions and shock waves (Whang 1991), and (3) turbulence due to pickup ions created by charge exchange between solar wind protons and interstellar neutral hydrogen (Williams & Zank 1994). The sources can be divided into two groups: (1) solar-origin large-scale energy fluctuations (stream shears and shock waves) driven turbulence, and (2) interstellar pickup ion driven turbulence (e.g., Smith et al. 2001, 2006; Adhikari et al. 2015). Both of these groups of sources contribute to the solar wind heating, but (1) is dominant in the inner heliosphere and (2) is dominant in the outer heliosphere.

The turbulent energy, developed along the solar wind radial expansion, is given by:

$$\frac{E_i}{\omega_{pl}} = \frac{\sigma_{\mathbf{v}}^2}{m_p},$$

that is, the variance of the Elsässer vector variable $Z_+$. The Elsässer variables are defined by $Z_+ \equiv \mathbf{V}_w \pm \mathbf{v}_s$ (Tu & Marsch 1995), where $\mathbf{v}_s = B/\sqrt{\mu \rho}$ denotes the Alfvén velocity, $\rho \approx m_p \cdot n$ is the mass density, and $m_p$ is the proton mass.
The Elsässer vector variable $Z$ corresponds to Alfvénic modes with an outward radial direction of propagation (in the solar wind frame). The outward propagating turbulent energy radial profile in the inner heliosphere, $E_t^+(R)$ for $R < 5.5$ au was derived for Helios 1 and 2 and Ulysses data sets (from 0.29 to $\sim$5.4 au) by Bavvasano et al. (2000) and later by Adhikari et al. (2015). There is some difference in the results of these two analyses, caused by the different lengths of data intervals (hour versus days), thus we use their weighted average. This was performed by (i) binning both the radius $R$ and energy $E_t^+$—on log–log scales, and then, (ii) averaging at each bin the results of the two papers. The results are shown in Figure 7.

Note that the other Elsässer vector $\mathbf{Z}$ corresponds to Alfvénic modes with an inward radial propagation direction. The corresponding turbulent energy $E_t^-/m_p = \sigma_t^2$ is quite smaller than the energy of the outward propagation, $E_t^+ \ll E_t^+$, in the inner heliosphere (see Figure 1 in Adhikari et al. 2015), thus it was ignored by the presented analysis; however, the two energies have similar values in the outer heliosphere (see Figure 2 in Zank et al. 2018).

4.2. Temperature of Pickup Ions

Pickup ions (PUIs) play an essential role in the thermodynamic energy balance of the solar wind. The internal particle energy of the solar wind is dominated by PUIs beyond $\sim$20 au from the Sun (McComas et al. 2017), and PUIs are responsible for the majority of the energy dissipation at quasi-perpendicular shocks in the outer heliosphere (e.g., Zank et al. 1996; Kumar et al. 2018; Zirnstein et al. 2018).
The average energy of a proton must take into account the energy of a solar wind proton as well as the energy of the pickup proton. Thus, before we compare the energy missing energy of a solar wind proton as well as the energy of the plasma energies by the proton mass, where we substitute \( \Phi(R) = \Phi(R) \) in Equation 10(a), and the gravitational potential energy given by Equation 9, in order to improve the proton energy \( E_p \) in Equation 2(b).

5. Plasmon–Proton Plasma Missing Energy versus Turbulent Energy

5.1. Formulation of the Missing Energy

Having estimated the minor contributions of the PUI energy and the gravitational potential energy, the total proton plasma energy in Equation 2(b) becomes

\[
E_p = E_{\text{non-t}} + \text{turbulent energy},
\]

where the non-turbulent part of the proton plasma energy is

\[
E_{\text{non-t}} = \frac{1}{2\mu} B^2 / n_p + \frac{\gamma}{\gamma - 1} k_B T_p + \Phi(R) + E_{\text{pui}}(R).
\]

Then, we (i) substitute \( E_{\text{pui}}(R) \) and \( \Phi(R) \) in Equation 10(b), taken from Equations 8(b) and 9, respectively, (ii) construct the difference \( \Delta E = E_p - E_{\text{non-t}} \), and (iii) divide all involved energies by the proton mass, where we find the missing energy, \( \Delta E / m_p \):

\[
\begin{align*}
\Delta E / m_p & = 4183.5 \sqrt{n} / \text{cm}^3 \\
& - 237.88 \cdot (B/[\text{nT}])^2 / (n/\text{cm}^3) \\
& - 10318 \cdot (T/[\text{MK}]) + 887.13 \cdot (R/[\text{au}]) \\
& - 5.045 \cdot (R/[\text{au}])^{1.9}.
\end{align*}
\]

As shown in Equation 10(a), the missing energy is to be compared to the turbulent energy.
turbulent energies

In addition, the last three minor terms (potential, PUL, and turbulent energies) may be approximated by a radial profile model $f(R)$, namely

$$h* \cdot \omega_{pl} = \frac{1}{2\mu} B^2 / n_p + \frac{\gamma}{\gamma - 1} k_B T_p + \Phi + E_{\text{PUL}} + E_t.$$  \hspace{1cm} (13)

In addition, the last three minor terms (potential, PUL, and turbulent energies) may be approximated by a radial profile model $f(R)$, namely

$$4183.5 \cdot \sqrt{n/[\text{cm}^{-3}]} = 237.88 \cdot (B/[\text{nT}])^2 / n/[\text{cm}^{-3}])$$

$$+ \quad 10318 \cdot (T/[\text{MK}]) + f(R),$$  \hspace{1cm} (14a)

with

$$f(R) = -887.13/(R/[\text{au}]) + 5.0450 \cdot (R/[\text{au}])^{1.9}$$

$$+ \quad 3020.0 \cdot (R/[\text{au}])^{-1.43}.$$  \hspace{1cm} (14b)

Note that the radial model $f(R)$ is minimized near $R \sim 3.3$ au, reaching the value $f_{\text{min}} = 230.72$. Equation 14(b) can be used in future analyses to derive missing measurements of density, temperature, or magnetic field strength (e.g., see Livadiotis (2015)).

5.2. Comparison between the Constructed Missing Energy and the Observed Turbulent Energy

Next, we compare the missing energy $\Delta E/m_p$ with the turbulent energy. We have already plotted the radial profile of turbulent energy in Figure 7. Therefore, we need to calculate the radial profile of the missing energy. Then, we will compare the two radial profiles. For this, we calculate the missing energy $\Delta E/m_p$ using daily averages of the solar wind and interplanetary magnetic field data taken from Helios 1 and 2, Wind, and Ulysses S/C, for the heliocentric distance from 0.29 to 5.41 au. Then, we construct the radial profile of the missing energy $\Delta E$, and compare this result with the radial profile of the turbulent energy shown in Figure 7. Finally, the two radial profiles are shown in Figure 8.

The missing energy $\Delta E$, derived from Equation (11), and the turbulent energy, derived by Bavvasano et al. (2000), Adhikari et al. (2015), and averaged as shown in Figure 7, are coplotted in Figure 8 on a (a) semi-log, and (b) log–log scale; the linear fits in (b) correspond to similar power laws with average energy:

$$\bar{E}(R)/m_p = 10^{3.48 \pm 0.04} \cdot R^{-1.43 \pm 0.07}. \hspace{1cm} (12)$$

The $p$-value of the statistical hypothesis that the two data sets describe the same statistical population is very high ($\sim 0.4$), thus the hypothesis is statistically confident.

5.3. Rewriting the Proton-magnetized Plasma Thermodynamic Equilibrium

We have shown that the energy balance between the plasmon and the proton plasma magnetized energy is written as

$$E_t / m_p = \frac{1}{2\mu} B^2 / n_p + \frac{\gamma}{\gamma - 1} k_B T_p + \Phi + E_{\text{PUL}} + E_t.$$  \hspace{1cm} (13)

In addition, the last three minor terms (potential, PUL, and turbulent energies) may be approximated by a radial profile model $f(R)$, namely

$$4183.5 \cdot \sqrt{n/[\text{cm}^{-3}]} = 237.88 \cdot (B/[\text{nT}])^2 / n/[\text{cm}^{-3}])$$

$$+ \quad 10318 \cdot (T/[\text{MK}]) + f(R),$$  \hspace{1cm} (14a)

with

$$f(R) = -887.13/(R/[\text{au}]) + 5.0450 \cdot (R/[\text{au}])^{1.9}$$

$$+ \quad 3020.0 \cdot (R/[\text{au}])^{-1.43}.$$  \hspace{1cm} (14b)

Note that the radial model $f(R)$ is minimized near $R \sim 5.3$ au, reaching the value $f_{\text{min}} = 230.72$. Equation 14(b) can be used in future analyses to derive missing measurements of density, temperature, or magnetic field strength (e.g., see Livadiotis (2015)).

6. Conclusions

This paper considered the concept of thermodynamic equilibrium between plasmons and proton-magnetized plasma and determined their energy balance in order to quantify the contribution of the turbulent energy. This equilibrium was shown and confirmed in several prior publications, but also in this paper, by analyzing (i) multi-spacecraft data sets along the radial profile of the inner heliosphere ($R < 10$ au), and (ii) representative data sets of a variety of 27 different space and astrophysical plasmas.

The near-Earth solar wind plasma, observed in the slow wind mode, is characterized by a small deviation from the plasmon–proton-magnetized plasma energy balance (Livadiotis & Desai 2016). This is expressed as a missing energy that prevents the plasmon–proton-magnetized plasma energy balance. The paper performed theoretical and space plasma data analyses in order to improve our understanding of the origin and nature of the missing energy. In particular, we investigated the velocity and radial profiles of the missing energy along the inner heliosphere. We also examined the interplay and partition of the turbulent heating sources in solar wind thermodynamics, and showed that radial profiles of the missing energy coincide with the radial profile of the turbulent energy.

In addition, the thermodynamic equilibrium and the energy balance between a plasmon and a proton-magnetized plasma are interwoven with the concept of large-scale quantization constant. Recently, strong evidence has shown that space and astrophysical plasmas are linked to a Planck-like constant, but ~12 orders of magnitude larger. The plasmon–proton energy balance is described confirming the universality of this large-scale quantization constant.
The connection of the missing plasmon–proton energy with the turbulent energy provides a new method for estimating and cross-examining the turbulent energy in space and astrophysical plasmas. Specifically, in stable and stationary plasmas—where the thermodynamic equilibrium would have made sense, the plasmon energy, that is, a single quantum of energy, equals the energy of the proton-magnetized plasma, that is, summing all the applied energy sources in the proton plasma including the turbulent energy.

In summary, the paper results are outlined as follows:

1. Verified the concept of plasmon–proton-magnetized plasma thermodynamic equilibrium, and the corresponding energy balance.
2. Showed the partition of the proton-magnetized plasma energy into the magnetic field energy, the proton thermal energy, and the turbulent energy, as well as the minor contributions of pickup ion thermal energy and gravitational potential energy.
3. Resolved the plasmon–proton energy balance deviation that characterizes the case of the slow solar wind plasma in the inner heliosphere.
4. Improved understanding of the interplay and partition of the sources of proton turbulent heating in the expanding solar wind in the inner heliosphere.
5. Verified the concept of large-scale quantization constant for space and astrophysical plasmas.
6. Developed a new method for estimating the turbulent energy in space and astrophysical plasmas.

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Appendix A
Data Sets of Representative Values of Space and Astrophysical Plasmas

Information about the plasmas’ density, temperature, and magnetic field, as well as their variability, which is represented here by the uncertainty, is taken from the cited references. It is important to note that all data were carefully selected in order to be (1) representative of the majority of bibliographic sources, (2) cross-referenced with various sources, and (3) reliable, with priority, from higher to lower, given to books, referred papers, theses, and other isolated sources/analyses. Notes:

(1) Abbreviations of examined space and astrophysical plasmas (in order of appearance in Table 1): CIRs (cr); coronal loops (cl); AGN (ag); LISM (li); planetary nebula (pn); CMEs (cm); solar wind—Helios (wh); solar wind—ACE (wa); solar wind—Ulysses (wu); solar wind—I au average (wa); ionosphere (io); aurora (au); plasma sheet (ps); plasmasphere (pl); sunspot plume (sp); shock example by Burlaga & King (1979) (sb); shock example with CME by Gopalswamy & Yashiro (2011) (sg); magnetosheath (ms); inner heliosheath (ih); magnetosphere—average (ma); magnetosphere—Cluster (mc); outer corona (oc); inner corona (ic); coronal holes (ch); Van Allen belts (va); Jovian magnetosphere—average (jm); termination shock (ts).

(2) Data sources of the examined space and astrophysical plasmas (in alphabetical order): active galactic nuclei (ag) (Liu et al. 2003; Sutter et al. 2012); aurora (au) (Berthelier & Sturges 1967; Chasten et al. 1999); corona holes (ch) (Doschek et al. 1997; Curtain et al. 2007); corona, inner (ic) (Kivelson & Russell 1997; Gary & Keller 2004); coronal mass ejections (cm) (Mitsakou & Moussas 2014); corona, outer (oc) (Kivelson & Russell 1997; Gary & Keller 2004); inner heliosheath (ih) (Livadiotis & McComas 2014a); ionosphere (io) (Daglis et al. 1999; Baumjohann & Treumann 2006; Sibanda & McKinell 2011; Huba 2013); Jovian magnetosphere—average (jm) (Dessler 1983; Divine & Garrett 1983); local interstellar medium (li) (Livadiotis & McComas 2014a); magnetosphere—average (ma) (Palermo et al. 2010); magnetosphere—cluster (mc) (Gurnett & Bhattacharjee 2005; Livadiotis & McComas 2014b); magnetosheath (ms) (Sanders et al. 1981; Gosling et al. 1991); planetary nebula (pn) (Zhang et al. 2004; Washimi et al. 2006; Sabin 2009); plasma sheet (ps) (Baumjohann & Treumann 2006); plasmasphere (pl) (Gannon et al. 2005; Baumjohann & Treumann 2006); shock example (sb) (Burlaga & King 1979); shock example with CME (sg) (Gopalswamy & Yashiro 2011); sunspot plume (sp) (Doyle & Majdarska 2003; Solanki 2003); solar wind—ACE (wa),—Helios (wh), and—Ulysses (wu) (Livadiotis & McComas 2014a); near 1 au, average (wa) (Foukal 2004); termination shock (ts) (Richardson et al. 2008); Van Allen belts (va) (typical averaged values in Chen 1984, p.14).

(3) Polytropic index γ: it is taken as γ ~ 5/3 (adiabatic; Nicolaou et al. 2014), except for the cases of planetary/heliospheric sheaths where γ ~ 0 (isobaric) (Livadiotis & McComas 2013b).

(4) Temperature liability: in case the temperature is measured either by fitting the energy distribution or by calculating the statistical moments, then the latter is preferred to avoid misestimations (Nicolaou & Livadiotis 2016).
Table B
Uncertainties Estimation

Table 2 includes the uncertainties formulation for quantities mentioned in the paper, such as the proton plasma energy $E_p$, the plasm energy $E_p$, and their difference that gives the missing energy, $\Delta E$. The propagation uncertainty $\Delta E$ is derived with respect to four parameters $X$: $n$, $T$, $\nu = 1 + 1/\gamma$, $B$, with $\Delta E = \sum (\partial E_p/\partial X_i)^2$. The uncertainty of $E_p$ is the propagation of $\delta n$, $\delta \omega_p$; the uncertainty of $\Delta E$ is the propagation of $\delta E_p$, $\delta \omega_p$.

Table 2
Uncertainties of Derived Quantities

| X     | $\Delta X$                                                                 |
|-------|---------------------------------------------------------------------------|
| $\gamma$ | $\sqrt{\frac{\sigma_{\text{rms}}^2}{N(N-2)} \cdot \sigma_{\text{rms}}^2}$ |
| $E_p$  | $\left[ \mu_0 n_i^2 B_i^2 \left[ \ln (B_i) \right]^2 + \frac{1}{4} \left( \sigma \ln n_i \right)^2 \right]$ |
| $T_{p,\text{int}}$ | $\sqrt{\left[ \left( \sigma \ln n_i \right)^2 \left( T_{p,\text{int}} \right)^2 + \left( \sigma \ln n_i \right)^2 \left( T_{p,\text{int}} - T_{p,\text{int}} \right)^2 \right]}$ |
| $\omega_p$ | $\frac{1}{2} \sigma \ln n_i$                                               |
| $\Delta E$ | $\sqrt{\left( \sigma \ln n_i \right)^2 \left( \Delta E \right)^2 + \left( \sigma \ln n_i \right)^2 \left( \Delta E \right)^2}$ |
| $\log (E_p/\omega_p)$ | $\sqrt{\left( \sigma \ln n_i \right)^2 \left( \Delta E \right)^2 + \left( \sigma \ln n_i \right)^2 \left( \Delta E \right)^2}$ |

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