Gluonic Hadrons and Charmless $B$ Decays

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Abstract

Hybrid charmonium with mass $\sim 4$ GeV could be produced via a $c\bar{c}$ color-octet component in $b \to c\bar{c}s$. These states could be narrow and could have a significant branching ratio to light hadrons, perhaps enhanced by glueballs. Decays to gluonic hadrons could make a sizable contribution to $B \to \text{no charm}$ decays. Experimental signatures and search strategies are discussed.

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1 Introduction

Intensive studies of $B$-meson decays are now underway and will soon be improved further with the emergence of $B$-factories that will lead to orders of magnitude increase in statistics. While the primary emphasis of these developments is in studying CP-violation and seeking evidence of physics beyond the standard model, we propose here that they may provide a powerful tool to search for a missing piece of the standard model, namely the predicted existence of hybrid (quark–antiquark–gluon) mesons and glueballs. We point out that there could be a sizable production of $c\bar{c}g$ hybrids (or hybrid charmonia, denoted hereafter as $\psi_g$) and other gluonic hadrons in $B$ decays which may be experimentally observable.

Our motivation is based on the following, \textit{a priori} independent, features of data and theory:

(i) CLEO has recently reported large values of $Br(\bar{B} \rightarrow K\eta')$ and $Br(\bar{B} \rightarrow \eta'X, P_{\eta'} > 2$ GeV) [1, 2].

(ii) The branching ratio $Br(b \rightarrow \text{no open charm})$ appears to be about a factor of 3 larger than expected [3].

(iii) The CKM–favored decay $b \rightarrow c\bar{c}s$ is predicted to produce the $c\bar{c}$ pair in a colour octet configuration [3] .

(iv) Decays $\psi_g \rightarrow D^{(*)}D^{(*)}$ may be suppressed by a selection rule [4, 5].

As will be discussed later, item (iii) might enhance the formation of $\psi_g$ states, some of which are predicted to be in the $4.2 \pm 0.2$ GeV region [3-10]. Furthermore, item (iv) indicates that if $\psi_g$ states occur below the $DD^{**}$ threshold ($\sim 4.3$ GeV), there is a possibility that they will cascade into conventional $c\bar{c}$ states as $\psi_g(c\bar{c}g) \rightarrow (c\bar{c})(g^g) \rightarrow (c\bar{c}) + \text{light hadrons}$ [11], or that they will directly decay via $\psi_g \rightarrow ng \rightarrow \text{light hadrons}$ (where $n \geq 2$) [3, 12] including resonant glueball enhancement. The latter process might explain the enhanced $Br(b \rightarrow \text{no open charm})$ [item (ii)], and the

\footnote{Hereafter, the notation $D^{(*)}(s)**D^{(*)}(s)**$ implies $D^{(*)(s)/**}D^{(s)**}$ or $D^{(s)**}D^{(*)(s)**}$.}
gluonic content in the final state may contribute to the \( \eta' \) enhancement, for example through rescattering of \( K^{(*)}\psi_g \) intermediate states: \( B \rightarrow K^{(*)}\psi_g \rightarrow K\eta', \eta'X \) [item (i)]. Not only \( \psi_g \) but also other gluonic hadrons may be produced in \( B \) decays at rates enhanced by non–perturbative effects over traditional expectations [13].

In the following, we will briefly summarize the spectroscopy of gluonic hadrons, discuss their production and decay in the \( B \) meson environment, and propose experimental search strategies.

2 Hybrid charmonium spectroscopy

A rich spectroscopy of hybrid charmonium is predicted by lattice gauge theory, flux–tube, bag models and QCD sum rules. These include exotic \( J^{PC} = 0^{+-}, 1^{-+}, 2^{+-} \) as well as conventional \( J^{PC} = 0^{-+}, 1^{--} \), etc. Lattice gauge theory with heavy quarks predicts \( 4.04 \pm 0.03 \) GeV for the spin-averaged masses of \( J^{PC} = 1^{--}, 1^{++}, (0, 1, 2)^{-+}, (0, 1, 2)^{+-} \) in the quenched approximation [6]. Flux–tube models inspired by the lattice predict \( 4.2 – 4.5 \) GeV [7] within an adiabatic separation of quark and flux–tube motion. More recent numerical solutions of the model find \( 4.1 – 4.2 \) GeV [8]. An adiabatic bag model calculation found \( \approx 4 \) GeV with an overall uncertainty of \( \approx 200 \) MeV [9, 10]. QCD sum rules are less clear, solutions spanning \( 4.1 – 5.3 \) GeV [14].

There is some support for these \( \approx 4 \) GeV mass predictions when one compares with the light quark sector where the flux–tube models find \( 1.8 – 1.9 \) GeV [4, 8] in accord with the \( \pi_g(1800) \) candidate [4, 15]. Lattice QCD provides some indication that the \( J^{PC} = 1^{--} \) may be the lightest of the exotic states. A candidate for a \( J^{PC} = 1^{-+} \) hybrid has been reported [16] in the predicted region of \( 2.0 \pm 0.2 \) GeV [4]. Thus the emerging hints from the light quark sector and the stability of predictions within QCD inspired models and lattice QCD suggest that \( \psi_g \) excitations could arise in a kinematically accessible region in the \( b \rightarrow c\bar{c}s \) decay.

\[ ^2 \text{Unquenching is estimated to raise the mass by 0.15 GeV.} \]
3 Hybrid production

A variety of hybrid excitations can be produced in $B$ decays. The production of $\psi_g$ may be significant since in the $b \to c\bar{c}s$ transition the $c\bar{c}$ pair is dominantly produced in colour octet $[3, 11, 12]$ which may strongly couple to the $c\bar{c}$ pair in $\psi_g$.

The direct $\psi$ production in $b$ decays is $(0.82 \pm 0.08)\%$ [20], and is not well understood theoretically. It appears to be enhanced somewhat over estimates based on the assumption [21] of color–suppressed factorization [22, 23]. The factorization assumption allows for the direct decay $b \to s\chi_{c1}$ but not for $b \to s \{\chi_{c0}, \chi_{c2}, h_c\}$. Thus, decisive observations of such modes would either necessitate a non–factorizable (such as a direct color–octet) contribution [23] or a feed-down from higher mass metastable states [11], which are expected to cascade to other charmonia observed in $B$ decays as well.

If we take the production of $\chi_{c2}$ as a measure of the colour octet production in $B$ decays, we expect from the CLEO datum $Br(B \to \chi_{c2}X) = 0.0025\pm 0.0010$ [21] that $\psi_g$ should be produced competitively at $Br \geq 0.1\%$. The sum total of $\psi_g$ for all $J^{PC}$ could be $\mathcal{O}(1\%)$, a significant contribution to the “non-charm” $B$ decays though not saturating them. If their production is to saturate these events, then their combined $Br$ should be of $\mathcal{O}(10\%)$ and their preferred decays ought to be to light hadrons. If $Br(B \to \psi_g(\text{all } J^{PC})X) \sim \mathcal{O}(1\%)$, then $Br(\psi_g \to (c\bar{c})X) = \mathcal{O}(10 - 100\%)$ is still consistent with the measured $Br(B \to (c\bar{c}) + X)$. If $\psi_g$ are produced at $\mathcal{O}(10\%)$, saturating the missing “non-charm” decays, then cascades to $(c\bar{c})$ must be a small fraction of the total. Unless some special mechanism causes $\psi_g$ to cascade into $h_c$ or other undetected conventional $(c\bar{c})$ states, the measurements on inclusive $\eta_c, \psi$ or $\chi_c$ production constrain the product of branching ratios $Br(B \to \psi_gX) \times Br(\psi_g \to (c\bar{c})X)$.

The production of hybrid $D_g$ states [$= \psi_f$] could play a non–negligible role in non–leptonic and semi–leptonic $B$–decays; this contrasts with $D$ or $D_s$ decays where $K_s$ or $\pi_s$ may mix with the charmed mesons [17]. A moderate production of $D_g$ in semi–leptonic $B$ decays $\overline{B} \to \psi_f + l\nu$ could solve the puzzle of why exclusive $\overline{B} \to (D, D^*, D^{**}) + l\nu$ transitions do not saturate the inclusive semi–leptonic branching ratio [18, 19].
We note that the $c\bar{c}$ invariant mass distribution for the quark-level $V-A$ transition $b \to c\bar{c}s$ peaks in the 3–3.7 GeV mass range [3]; such a process tends to be more inclusive at low $m_{c\bar{c}}$ and exclusive at large $m_{c\bar{c}}$, so we expect that exclusive $B \to \psi_g K^{(*)}$ will be favoured in the vicinity of 4 GeV. Quantitative estimates are model dependent and beyond the scope of this study.

### 4 Hybrid decays

An important feature of hybrid decays in at least flux–tube or bag models is that decays to two mesons with the same spatial wave function are suppressed. This selection rule [4] is expected to be broken for light flavours and less so for heavy flavours [5]. In the case of $\psi_g$, decays to $D^{(*)}D^{(*)}$ are suppressed and the sum of the widths is predicted to be $1-10$ MeV depending on $J^{PC}$ of the hybrid [3]. The decays $1^{-+} \to \pi\pi$, $\eta^{(')}\eta^{(')}$ are also suppressed [24]. However, the dissimilar nature of $\eta_c$ and $\eta^{(')}$ suggests that the decay $1^{-+} \to \eta_c\eta^{(')}$ should not be impeded sizably.

The above selection rule would be broken if the hybrid states mix with conventional excitations of $c\bar{c}$. Hybrid states with exotic $J^{PC}$ are particularly interesting as they cannot mix with excited $c\bar{c}$ conventional states and, if below 4.3 GeV in mass, will feed $B \to K^+$ “non-charm”. States with conventional $J^{PC}$ on the other hand can mix with excited states of the same $J^{PC}$ and thereby “leak” into $D^{(*)}D^{(*)}$ final states. In particular it has been suggested [10, 25] that $\psi(4040)$ and $\psi(4160)$ are strong mixtures of $\psi_{3S}(4100)$ and $\psi_g(4100)$. In addition, hybrid charmonia may mix with glueballs. Such a mixing would enhance the production of light hadrons.

For those $\psi_g$ that mix negligibly with conventional charmonia and have a mass of $< 4.3$ GeV, the prominent decays will be either by cascade $\psi_g[\equiv c\bar{c}g] \to (gg) + (\psi,\eta_c,\ldots)$ or by annihilation $\psi_g(C = +) \to (gg) \to$ light hadrons. These are at the same order in $\alpha_s$. The decay $\psi_g \to$ light hadrons is expected to be favoured at least for $C = +$ states for the following reason.

A measure of the relative importance of the cascade width compared to the annihilation width may be provided by $\Gamma(\psi' \to \psi\pi\pi) \simeq \mathcal{O}(0.1 \text{ MeV})$ versus $\Gamma(\eta_c \to$
light hadrons) \simeq \Gamma(\eta_c \to \text{light hadrons}) \times \Gamma^{ee}(\psi')/\Gamma^{ee}(\psi) \simeq \mathcal{O}(5 \text{ MeV}). The \psi' \to \psi \pi \pi gives information about \psi' \to \psi gg, while \eta'_c \to \text{light hadrons} informs about \eta'_c \to gg. Both processes are \mathcal{O}(\alpha_s^2) in rate. Those rates suggest what to expect for cascade and annihilation decays of charmed hybrids. The rates of \psi_g \to (c \bar{c})+ light hadrons and \psi_g(C = +) \to \text{light hadrons} are both down by one power in \alpha_s. Ignoring differences in wave function overlaps we roughly estimate \Gamma(\psi_g \to (c \bar{c})+ light hadrons) \sim \mathcal{O}(0.5 \text{ MeV}) and \Gamma(\psi_g(C = +) \to \text{light hadrons}) \sim \mathcal{O}(20 \text{ MeV}).

The light hadron production rate from \psi_g decays with \( C = - \) is expected to be suppressed by one power of \alpha_s with regards to \psi_g(C = +) decays. Note that the production rate of conventional charmonia \((c \bar{c})\) from either \psi_g(C = +) or \psi_g(C = -) decays is expected to be of the same order in \alpha_s and thus similar. The charge conjugation \((C = \pm)\) of the produced conventional charmonium is expected to be the same as that of the parent \psi_g in hadronic decays, since two gluons \((C = +)\) are emitted in the lowest-order process. That may prove useful in searching for and classifying \psi_g states.

Some decays are forbidden by simple conservation of quantum numbers; for example, \((J = 0) \nrightarrow (J = 0) + \gamma\) by angular momentum conservation. Similarly, the \( D^{(*)} D^{(*)}\) final states are forbidden by \( P \) and/or \( C \) conservation for the \( J^{PC} = 0^{+-}\) exotic hybrid. Thus, if the mass of the \( 0^{+-}\) is sufficiently low, it will be seen only in light hadrons or perhaps also in hidden charm decay modes (see Table 1).

There is an interesting possibility if light hadrons such as \( \eta' \) or \omega contain \( c \bar{c} \) in their Fock states \[26\]. Charmed hybrids that do not mix could decay into \((\psi, \eta_c, \chi_c, h_c, \ldots) + (\eta', \omega)\) via a Zweig-allowed 4-charmed intermediate state. That amplitude would of course interfere with the traditional amplitude governing hidden charmonia production.

5 Experimental signatures and search strategies

Some information useful in the search of \psi_g is listed in Table \[4\]. Not only the exotic \psi_g with \( J^{PC} = 0^{+-}, 1^{-+}, 2^{+-} \) will involve unique characteristics, even the
non–exotic \( \psi_g \) that mix negligibly with conventional charmonia will have striking signatures (corresponding tables could be produced easily). There are three major categories of decay: (a) open charm, (b) hidden charm, and (c) light hadrons. Light hadronic modes that involve one or more \( K \overline{K} \) pairs (more generally \( s\overline{s} \) pairs) could also be searched for. In general, we suggest that a dedicated study of \( B \rightarrow \psi_g X_s \), where \( X_s \) is light hadron(s) with total strangeness = +1, be made as follows:

(i) \( \psi_g \rightarrow D^{(\ast,\ast)}D^{(\ast,\ast)} \). In addition to a search for \( \psi_g \), the \( D^{(\ast,\ast)}D^{(\ast,\ast)} \) system should be studied to seek evidence of \( \psi(4040; 4160) \), and other excited \( (c\overline{c}) \) states (see section 4). On general grounds we advocate measuring the \( J^P C \) dependence of charm pair production by these channels. Note that such channels feed a “wrong sign” \( \overline{B}[b\overline{q}] \rightarrow \overline{D}[c\overline{q}'] + \ldots \) charm production that has been observed by several experiments [27], and so relevant data may already be at hand.

(ii) \( \psi_g \rightarrow (c\overline{c}) + (\text{light hadron(s)}, \gamma) \), where \( (c\overline{c}) \) is a conventional charmonium \( \psi, \eta_c, \chi_c, h_c, \ldots \). The light hadronic system is likely to have \( C = + \) and zero isospin. Other quantum numbers, however, should also be studied (see Table 1).

(iii) \( \psi_g \rightarrow \text{light hadrons} \). For examples, consult the last column of Table 1.

When \( X_s = K^{(\ast)} \), it has definite momentum in the \( B \) rest frame. Thus careful studies of \( K^{(\ast)} \) momentum spectra may establish excesses beyond what is expected from other sources.

We recommend not only to search for \( \psi_g \) production, but also for other gluonic hadrons in \( B \) decay. Significant yields of light hadrons in \( B \) decays could feed through light gluonic hadrons. In that scenario the \( b \rightarrow c\overline{c}s \) transition is followed by non–perturbative \( c\overline{c} \) annihilation, such as multiple gluon exchange between the spectator quark and the closed charm line [28]. The resulting intermediate state is rich in soft gluons and light quarks, which can arrange themselves into glueballs, light hybrids and glue–rich mesons like \( \eta' \). The resulting final-states have lost their
charm content. Some decay modes of glueballs into light hadrons are summarized in the last column of Table 1. Predicted $J^{PC}$ and masses of glueballs can be found in Table 2.

There is also the possibility for the $K$ system to resonate as $K_g$. The lightest of these states is predicted to occur $\sim 2$ GeV in mass [7, 17] which leaves $\lesssim 3$ GeV available for the mass of the $c\bar{c}$ system. Excitation of $K_g$ is therefore likely only with low mass charmonia (such as $\psi, \eta_c$) or with $(\eta^{(')}, \omega, \rho, \ldots)$ if they contain $c\bar{c}$ [26, 29], or with light hadrons. A search for $B \rightarrow K_g \eta^{(')}$ could be interesting, in light of the large $B \rightarrow K \eta'$.

We note that vertex detectors can utilize the long lifetime of $B$ and $D$ hadrons to reduce backgrounds, and the excellent $p/K/\pi$ separation capabilities at $B$–facilities will further improve the sensitivities. Full exploration of multibody decays of $b$-hadrons will require the ability to detect $\pi^0, \eta^{(')}, \gamma$ as well.

6 Conclusions

$B$ decays are a fertile ground for searching and discovering gluonic hadrons, including hybrid charmonia which may be copiously produced in the process $b \rightarrow c\bar{c}s$. Some of them may significantly decay to light hadrons contributing to $B$ decays to final states without charm. We have studied the patterns of production and decay of such hybrids, and proposed experimental search strategies.

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Table 1: Some possible experimentally accessible final states of $J^{PC}$ exotic charmed hybrids and glueballs below $D^{**}D$ threshold. Note that open charm modes of $\psi_g$ may be suppressed by a selection rule \[4\]. For hidden charm modes, the charmonia tend to have the same $C$ as that of the parent $\psi_g$. The light hadron modes are expected to be enhanced for $\psi_g$ with $C = +$. See the main text for details. Decays to $p\bar{p}\{\pi, \eta, \omega, \rho, \phi\}$ are allowed for all states listed.

| $J^{PC}$ | Open charm | Hidden charm | Light hadrons |
|----------|------------|--------------|---------------|
| 0$^{-+}$ | Quantum numbers forbid $D^{(*)}D^{(*)}$ | $J/\psi\{f_{0,1,2}, (\pi\pi)_S\}$, $h_c\eta$, $\eta_c h_1$, $\chi_c\omega$, $\chi_{c(1,2)}\{\omega, h_1, \gamma\}$ | $a_{0,1,2}\rho$, $a_{1,2}\{b_1, \gamma\}$, $b_1\pi$, $h_1\eta^{(*)}$, $(\pi\pi)_S$, $f_0\{\omega, \phi\}$, $f_{1,2}\{\omega, h_1, \phi, \gamma\}$ |
| 0$^{--}$ | $D^*D$ | $h_c(\pi\pi)_S$, $J/\psi\{f_{1,2}, \eta^{(*)}\}$, $\chi_c h_1$, $\eta_c\{\omega, \phi\}$, $\chi_{c(1,2)}\{\omega, h_1, \gamma\}$ | $a_{0,1,2}b_1$, $a_{1,2}\{\rho, \gamma\}$, $\rho\pi$, $f_0h_1$, $h_1^{(*)}\{\omega, \phi\}$, $f_{1,2}\{\omega, h_1, \phi, \gamma\}$ |
| 1$^{-+}$ | $D^*D$, $D^*D^*$ | $\chi_{c(0,1,2)}(\pi\pi)_S$, $\eta_c\{f_{1,2}, \eta^{(*)}\}$, $\chi_{c(1,2)}\eta$, $\{h_c, J/\psi\}\{\omega, h_1, \phi, \gamma\}$ | $a_{0,1,2}a_{0,1,2}$, $a_{1,2}\pi$, $f_{0,1,2}f_{0,1,2}$, $f_{1,2}\eta^{(*)}$, $\rho, \gamma\{\rho, b_1\}$, $b_1b_1$, $\{\omega, h_1, \phi, \gamma\}\{\omega, h_1, \phi, \gamma\}$ |
| 2$^{+-}$ | $D^*D$, $D^*D^*$ | $\{h_c, J/\psi\}\{f_{0,1,2}, (\pi\pi)_S\}$, $\{h_c, J/\psi\}\eta^{(*)}$, $\{\eta_c, \chi_{c(0,1,2)}\}\{\omega, h_1, \phi, \gamma\}$ | $a_{0,1,2}\{\rho, b_1, \gamma\}$, $\{\rho, \gamma, b_1\}\pi$, $\{\eta^{(*)}, f_{0,1,2}\}\{\omega, h_1, \phi, \gamma\}$ |
Table 2: Glueball masses in GeV in the 3 – 4.5 GeV mass range accessed by $B \to K^{(*)}+\text{glueball}$, according to lattice gauge theory [30]. The $0^{-+}$ glueball mass is poorly determined. No $J^{PC}$ exotic glueballs are expected below 3 GeV.

| $J^{PC}$ | $1^{-+}$ | $2^{-+}$ | $3^{++}$ | $1^{++}$ | $2^{--}$ | $1^{--}$ |
|---------|----------|----------|----------|----------|----------|----------|
| Mass    | $2.9 \pm 0.3$ | $3.0 \pm 0.2$ | $3.9 \pm 0.5$ | $4.0 \pm 0.3$ | $4.0 \pm 0.4$ | $4.6 \pm 0.5$ |
| $J^{PC}$ | $1^{++}$ | $0^{+-}$ | $2^{+-}$ | $1^{--}$ | $4.1$ | $3.7$ |
| Mass    | $\lesssim 4.1$ | $\lesssim 3.7$ | $3.9 \pm 0.7$ | $4.6 \pm 0.5$ | $4.6 \pm 0.5$ | $4.6 \pm 0.5$ |