Inhomogeneous multi carrier superconductivity at LaXO$_3$/SrTiO$_3$ (X = Al or Ti) oxide interfaces

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Abstract

Several experiments reveal the inhomogeneous character of the superconducting state that occurs when the carrier density of the two-dimensional electron gas formed at the LaXO$_3$/SrTiO$_3$ (X = Al or Ti) interface is tuned above a threshold value by means of gating. By reanalyzing previous measurements that highlight the presence of two kinds of carriers (low- and high-mobility), we shall provide a description of multi carrier magneto transport in an inhomogeneous two-dimensional electron gas, gaining insight into the properties of the physics of the systems under investigation. We shall then show that the measured resistance, superfluid density, and tunneling spectra result from the percolative connection of superconducting 'puddles' with randomly distributed critical temperatures embedded in a weakly localizing metallic matrix. We shall also show that this scenario is consistent with the characteristics of the superconductor-to-metal transition driven by a magnetic field. A multi carrier description of the superconducting state within a weak-coupling, BCS-like model will be also discussed.

Keywords: inhomogeneous superconductivity, gate-tunable superconductivity, oxide interfaces

(Some figures may appear in colour only in the online journal)

1. Introduction

After a two-dimensional electron gas (2DEG) was detected at the interface between two insulating oxides [1–4], an increasingly intense theoretical and experimental investigation has been devoted to these systems. The properties of this 2DEG are intriguing for several reasons. The 2DEG can be made superconducting when its carrier density is tuned by means of gate voltage (see figure 1), both in LaAlO$_3$/SrTiO$_3$ (henceforth, LAO/STO) [1, 2] and LaTiO$_3$/SrTiO$_3$ (henceforth, LTO/STO) [3, 4] interfaces, thus opening the way to voltage-driven superconducting devices. Also, it exhibits magnetic properties [5–10]; displays a strong and tunable [11, 12] Rashba spin-orbit coupling [13]; and it is extremely two-dimensional, having a lateral extension $\approx$5 nm, thereby enhancing the effects of disorder due to extrinsic and/or intrinsic [12] sources.

Magneto transport experiments reveal the presence of high- and low-mobility carriers in LTO/STO, and superconductivity seems to develop as soon as high-mobility carriers appear [4, 14], when the carrier density is tuned above a threshold value by means of gate voltage, $V_g$. When the temperature $T$ is lowered, the electrical resistance is reduced, and signatures of a superconducting fraction are seen well above the temperature at which the global zero resistance state is reached (if ever). The superconducting fraction decreases with decreasing $V_g$, although a superconducting fraction survives at values of $V_g$ such that the resistance stays finite down to the lowest measured temperatures. When $V_g$ is further reduced, the superconducting fraction eventually
disappears, and the 2DEG stays metallic at all temperatures and seems to undergo weak localization at low $T$. At yet smaller carrier densities, the system behaves as an insulator. The width of the superconducting transition is anomalously large and cannot be accounted for by reasonable superconducting fluctuations [15]. This phenomenology suggests instead that an inhomogeneous 2DEG is formed at these oxide interfaces, consisting of superconducting ‘puddles’ embedded in a weakly localizing metallic background, opening the way to a percolative superconducting transition [16]. Inhomogeneities are revealed in various magnetic experiments [5–7, 10], in tunneling spectra [17], and in piezoforce microscopy measurements [18]. It seems that inhomogeneities at the nanometric scale coexist with larger (e.g., micrometric) scale inhomogeneities, revealed by the occurrence of striped textures in the current distribution [19] and in the surface potential [20].

**Figure 1.** Scheme of an oxide interface (not in scale), in the back gating configuration. The LAO (or LTO) topmost slab has a thickness of a few nm, whereas the STO slab is ≈0.5 mm thick. The back gate voltage, $V_g$, is used to tune the carrier density of the 2DEG. The top gating configuration is also possible.

Various aspects of the phenomenology of oxide interfaces (henceforth referred to as LXO/STO interfaces, when referring to both LAO/STO and LTO/STO) have been separately discussed before [4, 12, 15, 16, 21, 22]. Here, we first provide new, compelling evidence of the inhomogeneous character of the 2DEG, extending previous multi carrier analyses of magneto transport measurements to deal with inhomogeneous systems. We then assemble the various pieces of the jigsaw puzzle into a coherent overall theoretical framework.

The outline of this paper is as follows: in section 2, we propose a model for multi carrier magneto transport in inhomogeneous systems, and show that previous analyses of the magnetoresistance and Hall resistance measurements, in terms of two different species of carriers [4], are fully consistent with the inhomogeneous character of the 2DEG at the LAO/STO and LTO/STO interfaces. In sections 3, 4, and 5, we revisit some of our previous results. Assuming inhomogeneity as an empirical evidence, we show that resistance measurements [4] and the topographic mapping of the superfluid density [10] can be accounted for within a percolative scheme. In section 6, we discuss some preliminary aspects of a theory for metal-insulator-superconductor tunneling in inhomogeneous superconductors that is apt to reproduce the measured tunneling spectra [23]. In section 7, we provide further evidence for inhomogeneous superconductivity at oxide interfaces; this evidence, which revisits the results of [21], comes from the peculiar multiple quantum critical scaling observed when superconductivity is suppressed by means of a magnetic field. In section 8, relying on the results of [22], we show that the properties of the superconducting puddles (e.g., their fractions and critical temperatures) can be extracted from experiments and used to model intra puddle, multi carrier superconductivity, gaining insight about the pairing mechanism. Although some features of the diamagnetic response are seemingly related to strong superconducting coupling [10], we show that inhomogeneities and multi carrier superconductivity fully account for the behavior of these systems within a standard, weak-coupling Bardeen–Cooper–Schrieffer (BCS) scheme. Concluding remarks are found in section 9.

## 2. Multicarrier magneto transport in inhomogeneous systems

The detection of high- and low-mobility carriers by means of magneto transport measurements in LTO/STO [4] is not necessarily direct evidence of inhomogeneity, since the two species of carriers could coexist in a homogeneous manner. However, to account for the phenomenology of the superconducting state of the 2DEG at the LXO/STO interface, we propose that the 2DEG is inhomogeneous, with higher-density regions (the superconducting puddles) and lower-density regions (the metallic background) [21, 22]. Assigning a band structure to the system, one assumes that when the density is large enough to fill the bands occupied by the high-mobility carriers, the high-mobility carriers should locally coexist with low-mobility carriers occupying the lower-lying levels (see also section 8). A picture then emerges in which the low-mobility carriers alone are present in the metallic background, whereas both low- and high-mobility carriers exist in the superconducting puddles. To improve our description of LXO/STO interfaces, we must then rely on a theory for multi carrier magneto transport that is able to deal with an inhomogeneous system. One such theory has been developed in the form of an effective medium theory (EMT) for the Hall conductance of a binary medium resulting from the mixture of two phases and based on rotation transformations (see [24] for a detailed description of this method). In our description of the LXO/STO interfaces, a less dense phase hosts a species of low-mobility carriers, and the other, denser phase hosts both high- and low-mobility carriers. Following [24], we first define the conductivity tensor for each of the two coexisting phases in the presence of a magnetic field of amplitude, $B$. The diagonal elements of the
conductivity tensor are
\[
\begin{align*}
\sigma_{xx}^{(1)}(B) &= \frac{\sigma_1}{1 + \left[\beta_1(B)\right]^2}, \\
\sigma_{xx}^{(2)}(B) &= \frac{\sigma_1}{1 + \left[\beta_1(B)\right]^2} + \frac{\sigma_2}{1 + \left[\beta_2(B)\right]^2},
\end{align*}
\]
and the off-diagonal elements are
\[
\begin{align*}
\sigma_{xy}^{(1)}(B) &= \frac{\sigma_1\beta_1(B)}{1 + \left[\beta_1(B)\right]^2}, \\
\sigma_{xy}^{(2)}(B) &= \frac{\sigma_1\beta_1(B)}{1 + \left[\beta_1(B)\right]^2} + \frac{\sigma_2\beta_2(B)}{1 + \left[\beta_2(B)\right]^2}.
\end{align*}
\]

Hereafter, the superscripts (1) and (2) label the two phases, the subscripts 1 and 2 label the low- and high-mobility carriers, respectively, and we use the notations \(\beta_i(B) \equiv \mu_i(B)/e\) and \(\sigma_i \equiv e\nu_i\mu_i\), where \(n_i\) is the carrier density and \(\mu_i\) is the mobility of the \(i\)-th species of carriers. Note that the peculiar aspect of this description is that both species of carriers contribute in parallel to the conductivity tensor in phase (2), which we identify with the higher-density superconducting puddles. We use \(w \equiv \frac{1}{2} - \epsilon\) for the fraction of the system occupied by the superconducting puddles, with \(\epsilon\) being the deviation from the percolation threshold, which changes sign when the minority phase \((w < \frac{1}{2}, \epsilon > 0)\) percolates and becomes the majority phase \((w > \frac{1}{2}, \epsilon < 0)\); the threshold is \(w = \frac{1}{2}\) in two-dimensional systems.

Within EMT, one can find the diagonal element in the conductivity tensor of the inhomogeneous, two-phase system, which correctly reproduces the limiting case of a pure phase and takes the form [24]
\[
\sigma_{xx}^{\text{EMT}}(B, \epsilon) = \epsilon \left[\sigma_{xx}^{(2)}(B) - \sigma_{xx}^{(1)}(B)\right] + \sqrt{\epsilon^2 \left[\sigma_{xx}^{(1)}(B) - \sigma_{xx}^{(2)}(B)\right]^2 + \sigma_{xx}^{(1)}(B)\sigma_{xx}^{(2)}(B)}.
\] (1)

Then, exploiting duality relations that connect the various elements of the conductivity tensor when the minority and majority phases are interchanged (see [24]), one obtains the EMT expression for the off-diagonal element of the conductivity tensor of the inhomogeneous (two-phase) system
\[
\sigma_{xy}^{\text{EMT}}(B, \epsilon) = \left\{\begin{array}{l}
\sigma_{xx}^{\text{EMT}}(B, \epsilon) + \sigma_{xx}^{\text{EMT}}(B, -\epsilon) \\
\sigma_{xx}^{(1)}(B) + \sigma_{xx}^{(2)}(B) \\
\times \left[\sigma_{xy}^{(1)}(B)\sigma_{xy}^{(2)}(B) + \sigma_{xy}^{(1)}(B)\sigma_{xy}^{(2)}(B)\right] \\
- \left[\sigma_{xy}^{(1)}(B)\sigma_{xy}^{(2)}(B) - \sigma_{xy}^{(1)}(B)\sigma_{xy}^{(2)}(B)\right] \\
\times \left[\sigma_{xx}^{\text{EMT}}(B, \epsilon) - \sigma_{xx}^{\text{EMT}}(B, -\epsilon)\right] \\
\sigma_{xy}^{(1)}(B) - \sigma_{xy}^{(2)}(B)
\end{array}\right\}.
\]

Figure 2. Hall resistance as a function of the magnetic field for different gate voltages, \(V_g\), measured at \(T = 4.2\) K in an LTO/STO 15-unit-cell-thick sample (sample A of [4]). Solid lines correspond to experimental data, taken from [4], while the dashed lines obtained here fit the data in equation (2). For clarity, the curves have been shifted upwards by 50 \(\Omega\) in increasing voltage order. Inset: Sheet resistance \(R_s\), obtained via equation (1), as a function of the gate voltage.

From the above relations, the expression for the Hall resistance of the inhomogeneous system can finally be derived as
\[
R_{H}^{\text{EMT}}(B, \epsilon) = \frac{\sigma_{xy}^{\text{EMT}}(B, \epsilon)}{\left[\sigma_{xx}^{\text{EMT}}(B, \epsilon)\right]^2 + \left[\sigma_{yy}^{\text{EMT}}(B, \epsilon)\right]^2},
\] (2)

which we use to accurately fit the experimental Hall resistivity data of LTO/STO [4] under a strong magnetic field and at different gate voltages (see figure 2). This procedure allows us to extract the values of both the mobilities, \(\mu_i\), and the densities, \(n_i\), of the two species of carriers, as well as the fraction of the system occupied by the superconducting puddles, \(w\). These values are reported in figure 3, while the inset of figure 2 displays the evolution of the normal-state sheet resistance, \(R_s\), of phase (2), as \(R = [\sigma_{xx}^{\text{EMT}}(B = 0, \epsilon)]^{-1}\) with changing \(V_g\).

Our present findings cast a new perspective on the analysis carried out in [4], where the appearance of more mobile carriers around \(V_g = 0\) V was found. On the one hand, we fully confirm that result, but on the other hand, we find here that the magneto transport data are well described, assuming that these carriers do not appear uniformly in the whole system. Rather, high- and low-mobility carriers coexist in a high-mobility metallic phase, identified here with phase (2), which is inhomogeneously distributed at the interface and spatially separated from the less metallic phase (1), where only one species of less-mobile carriers is present. In this framework, the enhanced conductivity around \(V_g = 0\) V and the changes in the slope of the Hall resistance at a high magnetic field occur because a finite fraction, \(w\), of phase (2), which is characterized by an overall higher carrier density and
hosts high-mobility carriers, appears around this gate voltage (see figure 3(a)). In agreement with the conclusions of [4], we also find that the more mobile carriers have a lower-density than the low-mobility carriers (figure 3(b)) but, in contrast with the results obtained within a homogeneous description, the mobilities of the two species stay rather constant over the whole range of gate voltages in the present inhomogeneous scheme (see figure 3(c)).

3. Percolative superconductivity

After assessing the occurrence of inhomogeneous multi carrier magneto transport at LXO/STO interfaces, we discuss the superconducting transition that is driven by tuning the gate voltage, $V_g$ (i.e., the carrier density), above a threshold value. It has been shown that the superconducting transition that occurs in inhomogeneous systems is well described within the EMT [15]. EMT is a mean-field-like theory used to describe a random resistor network (RRN) that lacks spatial correlations. The EMT equations are obtained by embedding one given random resistance, $R_i$, in an effective medium of constant resistance, $R$. The latter is chosen so the same current that flows through $R_i$ flows in the RRN. The EMT resistance can be shown [15] to be larger than the parallel of random resistances, and smaller than the series of random resistances with the two limiting values being reached in infinite and one dimensions, respectively.

The resistance of the LTO/STO interface exhibits a marked suppression due to incipient superconductivity, which is accurately fitted [22], assuming that the superconducting puddles occupy a fraction, $w < 1$, of the sample, and that each puddle is characterized by a random local critical temperature, $T_c$. For the sake of definiteness, we adopt a Gaussian distribution of critical temperatures, $W(T_c)$, parametrized by its mean value, $T_r$, and its width, $\gamma$. The remaining $1 - w$ fraction of the sample is occupied by the metallic background. The resistance at temperature $T$ is found within EMT to be [15, 16, 22]

$$R(T) = R_\infty \left[ (1 - w) + w \operatorname{erf}\left(\frac{T - T_r}{\gamma\sqrt{2}}\right) \right].$$

and the results form the metallic background (the first term inside the square brackets) and also form not-yet-superconducting puddles (i.e., those puddles with $T_c < T$), the second term inside the square brackets, with erf being the error function); the remaining puddles (i.e., those with $T_c > T$), have become superconducting and do not contribute to the resistance. The high-temperature resistance $R_\infty$, $w$, $T_r$, and $\gamma$ are used as fitting parameters. The global zero resistance state is reached at the percolative transition temperature, $T_p \leq T_r$, such that

$$\operatorname{erf}\left(\frac{T_p - T_r}{\gamma\sqrt{2}}\right) = \frac{w - 1}{w}. \quad (3)$$

A solution for $T_p$ only exists if the superconducting fraction of the 2DEG can percolate in the two-dimensional system (i.e., for $w \geq \frac{1}{2}$). When $T_p < 0$, or when it is not at all defined (for $w < \frac{1}{2}$), the resistance remains finite down to $T = 0$, although the presence of a sizable (yet not percolating) superconducting fraction is mirrored by a sizable suppression of $R(T)$, as sketched in figure 4(a).

The resistances measured in an LTO/STO sample as an function of $T$ for various values of the gate voltages, $V_g$, [4] are shown in figure 5(a), along with the fitting EMT curves (symbols and solid lines, respectively). Figure 5 is inspired by a similar figure in [22], with new elements included to make explicit connections with the overview contained in this paper. The set of parameters $R_\infty$, $w$, $T_r$, $\gamma$ (i.e., the distribution of the puddles), changes when the carrier density changes by means of gating.

We are thus able to extract from the fits both the mean intra puddle critical temperature, $T_r$ (i.e., the temperature at which $R(T)$ changes curvature within EMT), which will be analyzed in section 8, and the overall superconducting fraction $w$, which is tracked by the solid line with filled squares in figure 5(c). Note that EMT disregards spatial correlations, so that the presence of pronounced tails in the resistance, in the presence of a percolating superconducting cluster, forces the overall superconducting fraction to be [15] $w \approx \frac{1}{2}$, as shown in figure 5(c). The width, $\gamma$, of the Gaussian distribution of $T_r$ is plotted as a solid line with filled squares in figure 5(d). It increases as the fraction, $w$, of the puddles goes to zero. This is rather natural, because a reduction of the carrier density is expected to emphasize the effects of disorder, so that fluctuations of the local superconducting critical temperature should increase, leading to a broadening of the $T_r$ distribution.

A comment is now in order. When dealing with superconducting puddles embedded in a metallic background, we
expect the proximity effect \cite{25} to play an important role in L XO/STO interfaces. Within EMT, this effect certainly entails the temperature dependence of the superconducting fraction, \( w \). However, when fitting the resistance curves, \( R(T) \), the introduction of a temperature-dependent \( w(T) \) is not viable, because it would make a good fit by definition. Nonetheless, the role of the proximity effect can be analyzed when discussing tunneling spectra. We shall come back to this point in section 6.

4. Superfluid density in inhomogeneous superconductors

The inhomogeneous character of the 2DEG at the LXO/STO interfaces raises the question of the description of the superfluid properties in a mixture of two phases. The superfluid density, \( n_s \), was measured \cite{10} by means of a Superconducting Quantum Interference Device (SQUID) in LAO/STO interfaces. The measurements average over micrometric scales and are therefore not sensitive to submicrometric inhomogeneities. Nonetheless, the idea that the 2DEG at these interfaces is inhomogeneous is still supported by the evidence of variations of \( n_s \) on the micrometric scale within a given sample. Encouraged by the marked similarity of the resistance curves in LAO/STO and in LTO/STO, we also apply EMT to describe the measured local \( n_s \) as an average over an inhomogeneous state of submicrometric puddles. We point out that recent experiments in LTO/STO \cite{21} led us to estimate the typical size of the puddles to be \( \approx 100 \) nm, thereby supporting this idea.

We proceeded to extend EMT to a small frequency, \( \omega \) \cite{22}, and assigned a Drude-like complex conductivity, \( \sigma_M(\omega) = \Lambda (\Gamma + i\omega)^{-1} \), to the metallic background and assigned a purely reactive conductivity, \( \sigma_S(\omega) = \Lambda (i\omega)^{-1} \), to the superconducting puddles. We then defined the resistivity, \( \rho_S(\omega) \equiv \sigma_S^{-1}(\omega) \) and \( \rho_M(\omega) \equiv \sigma_M^{-1}(\omega) = \rho_0 + \rho_S(\omega) \), with \( \rho_0 \equiv \Gamma/\Lambda \). At high \( T \), the system is metallic and \( \rho(\omega) = \rho_M(\omega) \). However, when the temperature is lowered, the static resistivity vanishes within each individual puddle as soon as \( T = T_c \) equals the local \( T_c \). Although the full expression of the complex resistivity can be found within EMT, aiming at describing the static diamagnetic response, we give only the expression up to terms \(-\omega^2\); that is, \( \rho(\omega) \approx \rho_0(w_M - w_S) \delta(w_M - w_S) + i w_M - w_S^1 \rho_S(\omega) \), where \( \delta \) is the Heaviside function, \( w_M \) is the fraction of puddles that have become superconducting at a given temperature, and \( w_M = 1 - w_S \) is the non-superconducting fraction resulting both from puddles that have not yet become superconducting and from the metallic background. Evidently, when \( w_M > w_S \), the conductivity is Drude-like. However, below the percolation temperature \( T_p \) (whenever defined), \( w_M < w_S \), and the conductivity is purely reactive: \( \sigma(\omega) = \Gamma (w_S - w_M)(i\omega)^{-1} \). Therefore, using equation (3), we find that the superfluid density of the percolating two-
The behavior of the superconducting fraction and (c) width of the structure of the superconducting cluster at oxide interfaces.

Figure 6. (a) Superfluid density (symbols) as a function of $T$ and EMT fits, according to equation (4) (solid lines). (b) Weight of the superconducting fraction and (c) width of the $T_c$ distribution extracted from the EMT fits, as a function of $V_r$. The lines are guides for the eye.

\[
n_s \propto W_s - W_M = w \left[ \text{erf}\left(\frac{T_p - T_c}{\gamma \sqrt{2}}\right) - \text{erf}\left(\frac{T - T_c}{\gamma \sqrt{2}}\right) \right].
\]

In figure 6(a), this average superfluid density and our EMT fits (equation (4)) are reported for different values in $V_r$. Figure 6 is inspired by a similar figure in [22], again suitably modified to have explicit relevance to the present discussion. The behavior of $n_s(T)$, although qualitatively resembling the BCS prediction, may quantitatively differ from it. The slope at $T = T_p$, for instance, is controlled by the width, $\gamma$, of the distribution $W(T_c)$. Thus, the deviations from the standard BCS prediction measured in [10] and attributed to a tendency to strong coupling are alternatively explained here by claiming that the 2DEG is in a weak-coupling BCS regime (see section 8), but that the superfluid density is intrinsically inhomogeneous at the submicrometric scale, and is averaged at the micron scale by the SQUID pick-up loop used in [10].

We point out that our mean field approach is justified by the observation that the channels down to 500 nm wide share properties similar to those of the larger samples [26]. Our fits yield the fraction, $w$, of the volume occupied by the puddles, extracted from measurements of the superfluid density and reported in figure 6(b). It ranges from $\frac{1}{7}$ to 1, and is always larger than the fraction obtained from transport measurements. This is not surprising, because transport mainly probes the long-range connectivity of the percolating path, regardless of dead ends and disconnected superconducting regions [27], whereas diamagnetic screening measurements are sensitive to all sufficiently large superconducting loops, even when they are not connected to the backbone. In this case, the diamagnetic fraction can be large and the long-distance connectivity can be small if many puddles or loops are disconnected (see section 5). Figure 6(c) reports the behavior of the width, $\gamma$, of the distribution $W(T_c)$, inferred from the superfluid density. We point out that, despite the fact that we are dealing with different materials and physical quantities, the behavior of $\gamma$ resembles that obtained from transport in LTO/STO, as seen in figure 5(d), and is of comparable magnitude.

Figure 7. Sketch of the RRN with superpuddles (green regions) connected by filaments of puddles (purple circles). The large shaded region, which does not contribute to superconducting transport, contributes instead to the diamagnetic response in the case of a zero-field-cooling measurement, since the superconducting currents confined in the superpuddles and loop structures encircled by the puddles are able to screen a magnetic field. In the case of a field-cooling measurement, one expects instead that the magnetic flux is frozen within the loop structures encircled by the puddles.

5. Space correlations within the superconducting cluster

A possible explanation for the discrepancy between the superconducting fraction, $w$, observed in both transport and diamagnetism measurements may rest upon the filamentary structure of the superconducting cluster at oxide interfaces.

\[
\text{erf}(x) \approx \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt.
\]
So far, we have made use of the mean-field-like EMT, which completely neglects spatial correlations. To investigate the mechanisms determining the superconducting fraction observed in transport, \( w \approx \frac{1}{2} \), we solved an RRN where the superconducting puddles form a spatially correlated cluster embedded in a metallic matrix. Preliminary results [16] indicated that a superconducting cluster, which is dense at short distances and filamentary at larger distances, is necessary to reproduce the observed tails of the resistance curves near percolation.

To this purpose, we generated a fractal-like cluster that has small long-scale connectivity and percolates only when almost all bonds have become superconducting. We point out that the fractality of the clusters is an artifact to produce spatially correlated networks that are at the same time dense at short distances and filamentary over long distances. For a Gaussian \( W(T_c) \), the low temperature tail of the distribution must be reached, and a correspondingly pronounced tail in the resistance is obtained. In the absence of filamentary structures over long distances, compact clusters fail to reproduce the tails in the resistance. Our systematic investigation showed that the presence of loosely connected filaments is necessary for the reproduction of the behavior of \( R(T) \) near percolation. On the other hand, a purely filamentary structure, no matter how dense at short distances, is too loose and is not apt to describe the behavior of \( R(T) \) at higher temperatures. In order to tune the density of the superconducting cluster without significantly changing the long-distance connectivity, we decorate the filaments with randomly distributed super-puddles, which we assume are circular for simplicity’s sake, whose number and size are chosen to produce weights, \( w \), ranging from 0.3 to 0.7 (see the sketch in figure 7). Super-puddles may be produced either by large tails in the distribution of the puddle sizes or by extrinsic pinning centers, promoting the nucleation of much larger puddles. We systematically investigated the effect of the size and density of the super-puddles. A smaller fraction of larger super-puddles or a larger fraction of smaller super-puddles are essentially equivalent, as long as the super-puddles do not overlap to form percolating clusters.

In figure 5(b), we show our fits of the resistance obtained within our RRNs. Noticeably, even though the superconducting fraction, \( w \), is no longer forced to \( \frac{1}{2} \), as it was in EMT, the resistance displays pronounced tails only if 0.50 \( \leq w \leq 0.65 \). The lower bound is imposed by the high slope at intermediate temperatures, whereas the upper bound is due to the pronounced tail near percolation. In figures 5(c) and (d), we show the fraction, \( w \), occupied by the superconducting cluster and the width, \( \gamma \), of the Gaussian distribution \( W(T_c) \) that is obtained within the RRN (open circles): \( \gamma \) qualitatively resembles the width obtained within EMT, and increases upon lowering \( V_g \). Thus, EMT and RRN models lead to similar results about the distribution of \( T_c \) and its variation with \( V_g \), provided the RRN is dense at short distances and filamentary over long distances.

### 6. Tunneling spectra of inhomogeneous superconductors

Another possibility for examining the inhomogeneous character of the superconducting state at LXO/STO interfaces is provided by tunneling experiments. Recent metal-insulator-superconductor tunnel spectroscopy measurements on LAO/STO [23] reveal the occurrence of a state with finite resistance, but with superconducting-like density of states (DOS). The measurements are performed by depositing a metallic Au electrode on top of the insulating LAO layer and applying a bias voltage, \( V \), to drive a tunnel current, \( I \), between the electrode and the 2DEG. The electrode measures several hundred \( \mu m \) across, thus being orders of magnitude larger than the nanoscale inhomogeneities [21]. At the lowest measured temperature, \( T = 30 \) mK, the spectra reveal a gap in the DOS at the Fermi energy over the entire range of explored gate voltages, \( V_g \in [-300, 300] \) V, accompanied by more or less broadened coherence peaks above the gap, pointing to superconducting coherence and pairing as the origin of DOS suppression (see figure 4(h)). In the carrier-depleted regime (\( V_g < 0 \)), the suppression is found even in the absence of global superconductivity, again highlighting the inhomogeneous character of the state formed by superconducting puddles embedded in a metallic matrix. At very low carrier concentration, \( V_g = 200 \) V, the 2DEG displays a superconducting gap and coherence peaks that decrease with increasing temperature and vanish around 300 mK, which agrees with the critical temperature of bulk STO reported in [29]. At \( V_g \leq 0 \), the gap closes and the coherence peaks vanish at temperatures much higher than the global \( T_c \).

Since, as we observed above, the tunneling spectra are taken over a spot measuring several hundred \( \mu m \) across, we proposed [30] that the observed pseudo-gap results from an average over superconducting regions (the superconducting puddles, with a DOS described by standard BCS theory) and metallic regions (composed of both the metallic background and not-yet- superconducting puddles) with constant DOS, \( N_0 \).

The differential conductance is usually written as

\[
\frac{dI}{dV}(V) = G_0 - G_1 \int_{-\infty}^{\infty} f'(E + eV)N(E) dE,
\]

where \( N(E) \) is the DOS, \( f'(E) \) is the derivative of the Fermi distribution function, the positive constant \( G_0 \) customarily accounts for effects such as leakage currents, and \( G_1 \) is a dimensional constant. We recall that in our model, superconducting pairing occurs within each puddle below a local critical temperature, \( T_c \), randomly distributed according to a probability distribution, \( W(T_c) \). The DOS of the 2DEG

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probed in the tunneling spectra has three distinct contributions,

\[ N(E) = (1 - w)N_0 + wN_0 \int_{-\infty}^{T_c} dT_c \, W(T_c) \]
\[ + w \int_{T_c}^{\infty} dT_c \, W(T_c)N_{\Delta(T_c,T)}(E). \tag{6} \]

The first two terms correspond to the metallic background and to puddles where pairing has not taken place yet, respectively, and fully account for the zero bias background observed in the tunneling spectra, allowing us to take \( G_0 = 0 \) in equation (5). The third term corresponds to puddles that developed a finite pairing gap, \( \Delta \). The DOS within these puddles is taken as

\[ N_{\Delta}(E) = \left[ (1 - x)\frac{|E|}{\sqrt{E^2 - \Delta^2}} + x \right] N_0 \Theta(|E| - \Delta). \]

The first term is the standard BCS expression and describes coherent pairing occurring within a \((1 - x)\) fraction of the gapped part. The second term describes puddles that have a gap, but are too small to exhibit phase coherence. We define \( w_{\text{pair}} \) as the total fraction of the system that can display pairing down to \( T = 0 \), and the coherently and incoherently paired fractions \( w_{\text{coh}} = (1 - x)w_{\text{pair}} \) and \( w_{\text{inc}} = x w_{\text{pair}} \). The latter term is necessary because the experimental spectra are gapped but display no coherence peaks when \( V_g \ll 0 \). \[ \text{[23]} \]

The gap is assumed to take the BCS expression

\[ \Delta(T_c, T) = 1.76 T_c \tanh \left( \frac{\pi}{1.76} \sqrt{\frac{T_c - T}{T_c}} \right). \]

The value of \( N_0 \) is readily determined by the high-bias part of the spectra, while \( w, x, \gamma \), and the parameters of the distribution of critical temperatures, \( T_c, \gamma \), are used as fitting parameters.

At low temperatures, we obtained accurate fits of the spectra and of their evolution as a function of the gate voltage, \( V_g \) [30]. Remarkably, the width of the distribution of critical temperatures, \( \gamma \), turned out to be very weakly dependent on \( V_g \), indicating that the distribution of \( T_c \) in the sample is essentially related to structural properties like the local amount of disorder.

When fitting the temperature dependence of the spectra at fixed \( V_g \), the attempt to use the same temperature-independent set of parameters captures the main features of the spectra, but yielded fits that are definitely less convincing than the fits at low temperature. However, by relaxing the severe constraint of temperature-independent parameters, very good fits are obtained by letting \( w \) and \( x \) vary, while the parameters of the distribution of critical temperatures, \( T_c, \gamma \), are kept fixed. The variation of \( w \) and \( x \) with the temperature is more clearly understood when expressed through the quantities \( w_{\text{coh}} \) and \( w_{\text{inc}} \) defined above. It turns out that the fraction of the sample occupied by the superconducting cluster increases with decreasing temperature, and saturates at low \( T \); likely indicating that a sizable part of the metallic background is gradually driven superconducting by the proximity effect [30].

\[ \text{Figure 8. Sketch of the procedure adopted to extract the quantum critical behavior when the superconducting phase is suppressed by means of a magnetic field perpendicular to the interface. (a) When the resistance, } R, \text{ is plotted as a function of temperature, } T, \text{ three behaviors are observed over a certain temperature interval, exemplified by the curves corresponding to the three magnetic fields, } B_1 < B_2 < B_3; \text{ superconducting, critical, and insulating. (b) To better identify the critical value of the magnetic field, } B_c \text{, or } B_{\text{inc}}, \text{ is plotted as a function of } B, \text{ exemplified by the three curves corresponding to the temperatures } T_1 < T_2 < T_3; \text{ the crossing point of the isotherms corresponds to } B_0 (B_0 \text{ or } B_{\text{inc}}), \text{ and the corresponding characteristic value } R \text{ is obtained on the resistance axis.} \]

7. Multiple quantum critical behavior in inhomogeneous superconductors

The recent analysis of the quantum critical behavior observed when the superconducting phase is suppressed by a magnetic field perpendicular to the interface [21] provides further support in favor of our scenario for inhomogeneous superconductivity at oxide interfaces. When the resistance, \( R \), of LTO/STO, rescaled by a characteristic value \( \overline{R} \), is plotted as a function of the variable \((B - \overline{B})/T^{1/\gamma}\), where \( \overline{B} \) is a characteristic magnetic field (see figure 8), \( \gamma \) is the dynamical exponent converting a critical length scale in a time scale, and \( \nu \) is the critical exponent of the correlation length as a function of temperature, two quantum critical scaling regimes are found in different temperature ranges. These two scaling regimes, separated by a crossover at intermediate temperatures, are in correspondence with two different values of the characteristic magnetic field \( B = B_c, B_s \). The smaller field, \( B_{\text{inc}} \), is related to the scaling at higher \( T \), with \( \nu v \approx \frac{1}{2} \), whereas the (slightly) larger field, \( B_c \), corresponds to the scaling at lower \( T \), with \( \nu v \approx \frac{2}{3} \). Note that the characteristic fields \( B_{\text{inc}} \) and \( B_s \) coalesce at low carrier density (i.e., low gate voltage, \( V_g \)). When \( V_g \) is increased, \( B_{\text{inc}} \) saturates to a constant, whereas \( B_s \) closely tracks the superconducting critical temperature, \( T_c \), with a conversion factor of 1 T corresponding to 1 K.

A possible explanation for this multiple critical behavior relies on the assumption that superconductivity within an isolated puddle would be suppressed by the smaller critical field, \( B_c \). However, the puddles are coupled, since they are embedded in a common metallic background [31]. When inter puddle coupling eventually intervenes, superconductivity is strengthened and survives up to the (slightly) larger critical field, \( B_s \). If we borrow the value of the dynamical exponent \( z = 1 \), which is commonly adopted in similar situations [32],
the intra puddle criticality is described by the XY model [33] in the clean limit, where $\nu \approx \frac{2}{3}$. However, when the temperature decreases, the coherence length increases, eventually exceeding the puddle size, and inter puddle superconductivity establishes in the inhomogeneous landscape of puddles embedded in the metallic background. In this case, the exponent $\nu$ must obey the Harris criterion [34] for disordered systems, and indeed we find $\nu \approx \frac{3}{4} > 1$.

8. Multicarrier BCS model

According to the discussion developed so far, the inhomogeneous character of the 2DEG at the LXO/STO oxide interfaces induces a distribution $W(T_f)$, of local critical temperatures, whose mean value, $T_c$, depends on the overall carrier density (i.e., on $V_z$). This dependence is obtained by fitting the resistance data in figure 5(a) within EMT, shedding light on the intra puddle pairing mechanism. As discussed in section 1, detailed magneto transport measurements [4] highlighted the coexistence of a sizable amount of low-mobility carriers and a smaller amount of high-mobility carriers in LTO/STO. Here, as well as in LAO/STO [35], superconductivity seems to develop as soon as the high-mobility carriers appear.

Accordingly, we propose [22] that superconducting pairing within the 2DEG formed at the oxide interface may be described by a multi band [36–40] BCS-like Hamiltonian

$$H_{BCS} = \sum_{\mathbf{k},\ell} \xi_{\mathbf{k},\ell} \left( a_{\mathbf{k},\ell}^\dagger a_{\mathbf{k},\ell} + \text{h.c.} + a_{\mathbf{k},\ell}^\dagger a_{\mathbf{k},\ell} + \text{h.c.} \right)$$

$$+ \sum_{\mathbf{k},\ell} \frac{\Delta_{\mathbf{k}}}{N} a_{\mathbf{k},\ell}^\dagger a_{\mathbf{k},\ell} + a_{\mathbf{k},\ell}^\dagger a_{\mathbf{k},\ell} + \text{h.c.}$$

where $a_{\mathbf{k},\ell}^\dagger (a_{\mathbf{k},\ell})$ creates (annihilates) an electron with the two-dimensional wave vector $\mathbf{k} = (k_x, k_y)$, parallel to the plane of the interface, and spin projection $\sigma = \uparrow, \downarrow$, belonging to the $l$-th sub-band, with dispersion law

$$\xi_{\mathbf{k},\ell} = \tilde{\xi}_{\ell} + \frac{\hbar^2 k_x^2}{2m_{\ell,x}} + \frac{\hbar^2 k_y^2}{2m_{\ell,y}} - \mu,$$

where $m_{\ell,x(y)}$ are the (possibly anisotropic) effective masses of the charge carriers, $\mu$ is the chemical potential, $R_{\ell}$ are the intraband (for $\ell = \ell'$) or interband (for $\ell \neq \ell'$) pairing amplitudes, and $N$ is the number of $\mathbf{k}$ points within the first Brillouin zone. The sub-bands can originate, for example, from the multiband structure of STO [12, 41] and/or from the quantum confinement within the self-consistent potential well at the interface [4, 42]. In our schematic description, we represent the whole set of low-lying bands with one sub-band ($\ell = 1$) accommodating the non superconducting low-mobility carriers, while the high-mobility carriers in the sub-band $\ell = 2$ are paired and give rise to a finite $T_c$ (see figure 9, which shows a sketch of a sub-band structure originating from the quantum confinement adapted from a similar sketch in [22]).

Thus, according to our interpretation, the superconducting puddles are regions where the $\ell = 2$ sub-band is locally filled, whereas the (weakly localizing) metallic background corresponds to regions where the $\ell = 2$ sub-band is empty. The phenomenology of the superconducting phase at the oxide interfaces is reproduced assuming that the pairing amplitudes are such that $g_{11} \ll g_{12}, g_{21} \ll g_{22}$. This condition is also consistent with the analysis of a two-band model in [43]. For simplicity, to reduce the number of free parameters to a minimum, in the following we assume that $g_{11} = g_{12} = g_{21} = 0$. According to the standard BCS approach, the pairing amplitudes are only effective in a window $|\xi_{\mathbf{k},\ell}|, |\xi_{\mathbf{k},\ell'}| \leq \hbar \omega_0$, where $\omega_0$ is a characteristic cut off frequency. The prime superscript attached to the last sum in equation (7) implies this restriction. We assume that the bottoms of the two sub-bands are well separated, $\tilde{\xi}_{2} - \tilde{\xi}_{1} \gg \hbar \omega_0$, and take henceforth $\tilde{\xi}_{2} = 0$.

In principle, when deducing a BCS-like Hamiltonian, one should take care of vertex corrections, which are expected to be relevant when Migdal’s condition is violated. However, the task of deducing a BCS-like Hamiltonian in a multi band model, where electrons with large and small Fermi energies coexist, is overwhelmingly difficult. Instead, we take equation (7) as a phenomenological low-energy effective Hamiltonian, imagining that the various high-energy effects have been accounted for by a suitable dressing of the bare physical parameters. Thus, the identification of $\omega_0$ with the characteristic frequency of the pairing mediator is expected to hold only indicatively.

For $\mu \leq 0$, the sub-band hosting the high-mobility carriers is empty, and $T_c = 0$. For $0 < \mu < \hbar \omega_0$, pairing occurs at the critical temperature

$$T_c \approx 1.14 (\hbar \omega_0 \mu)^{\frac{1}{2}} e^{-1/\mu},$$

where $\lambda \equiv g_{22}^{HMC} N_0$ is the dimensionless superconducting coupling and $N_0^{HMC}$ is the DOS of the sub-band filled by the
high-mobility carriers. For $\mu \geq \hbar \omega_0$, $T_c$ saturates to the standard BCS value

$$ T_c^{BCS} \approx 1.14 \hbar \omega_0 e^{-1/\lambda}. $$

The previous results can be cast into a single expression

$$ T_c(\mu) = T_c^{BCS} \delta(\mu) \min\left(\frac{\mu}{\hbar \omega_0}, 1\right). \tag{8} $$

Remarkably, the amplitude, $\Delta \mu$, of the interval, in correspondence of which $T_c$ is an increasing function of $\mu$, provides a direct measure of the cut off energy scale, $\hbar \omega_0 = \Delta \mu$, which, with the caveat recalled above, we interpret as a possible crude estimate of the characteristic energy scale of the pairing mediator.

The fit of the curve $T_c(V_g)$, extracted from the experimental data in figure 5(a) within EMT, with the curve $T_c(\mu)$ in equation (8) exploiting the approximately linear relation between $V_g$ and $\mu$ [4] yields the result illustrated in figure 10 (orange line with diamonds and empty squares, respectively). Figure 10 is inspired by a similar figure in [22], redrawn here in a modified fashion to make explicit connections with the overview contained in this paper. The various band masses are all taken to be equal to 0.7 electron masses, corresponding to a scenario where the sub-band structure originates from the quantum confinement of the lowest $t_{2g}$ band of bulk STO near the interface. We then obtain the dimensionless coupling constant $\lambda \approx 0.125$ (therefore, consistent with our assumption that the system falls in the weak coupling regime) and a cut off energy scale, $\hbar \omega_0 \approx 23$ meV, which is compatible with a typical phonon energy in STO [29].

9. Concluding remarks

In conclusion, we first analyzed magneto transport experiments in LTO/STO oxide interfaces [4] within a multi carrier EMT model, confirming the occurrence of two kind of carriers, with high and low mobility, within an inhomogeneous landscape. We proposed that the system consists of regions of higher carrier density, where both carriers coexist, and regions of lower carrier density, where only the low-mobility carriers exist. We also confirm that the high-mobility carriers have a lower-density than the low-mobility ones. The mobilities of the two species turn out to be almost independent of the gate voltage. Thus, the enhancement of conductivity observed around $V_g = 0$ V and the change in the slope of the Hall resistance at high magnetic fields occur because a finite fraction, $w$, of regions with higher carrier density, hosting high-mobility carriers, appears around this gate voltage.

We then described superconductivity in LAO/STO and LTO/STO within a scenario in which superconducting puddles (the regions with coexisting high- and low-mobility carriers) are embedded in a metallic background (the regions with low-mobility carriers only), and form a percolating network. In this framework, the sheet resistance of LTO/STO interfaces is very well described by EMT or by an RRN for an inhomogeneous 2DEG with a substantial filamentary character. Fitting the experiments, we were able to extract the random distribution of $T_c$ at various $V_g$ (i.e., various carrier densities). A similar approach was adopted to fit the microscopically averaged superfluid stiffness [10] and the pseudogap in the tunneling spectra [23] of LAO/STO. In particular, we showed that our model accounts well for the metal-insulator-superconductor tunneling spectra measured in LAO/STO, and allows us to conclude that the fraction of the sample occupied by the superconducting puddles increases when the temperature is reduced, and saturates at low temperature, likely indicating that a sizable fraction of the metallic background gradually becomes superconducting by the proximity effect.

Assuming an effective two-band model with superconductivity triggered by the presence of few high-mobility carriers locally filling the highest-energy band, we account for the density dependence of the intra-puddle $T_c$ within a simple BCS weak coupling scheme. As an important by-product, we find that the range of variation in $V_g$ of the average intra-puddle $T_c$ is directly related (via the chemical potential $\mu$) to the cut off energy scale, $\hbar \omega_0$. Taking this value as a crude estimate of the typical energy scale of the pairing mediator, we find that this is compatible with phonon-mediated superconductivity.

Although it was not the main focus of this paper, a few words are now in order to discuss the origin of electron inhomogeneity at LXO/STO interfaces. Extrinsic mechanisms [44], like impurities and growth defects, are always accountable for rendering an interface inhomogeneous. However, experimental data show that inhomogeneity is never reduced below a sizable extent, even in the best samples. Furthermore,
the observation of negative electron compressibility in a low-filling regime [45] suggests that intrinsic mechanisms (e.g., in the form of effective electron-electron attractions) are present, which may render the 2DEG formed at the LZO/STO interface inhomogeneous by phase separation, even in a perfectly clean and expectedly homogeneous system. These facts led us to look for intrinsic mechanisms of inhomogeneity. A possible intrinsic mechanism for the formation of an inhomogeneous 2DEG was proposed [12, 41] that relies on the strong density-dependent Rashba spin-orbit coupling (RSOC) inferred at these interfaces [11] and yields a non-rigid band structure, with the possibility that the chemical potential of the carriers is a non-monotonic function of the carrier density for reasonable values of the model parameters, giving rise to thermodynamic instability. The resulting RSOC-driven electronic phase separation provides a natural framework for the occurrence of regions with higher and lower carrier densities (the superconducting puddles and the metallic background, respectively), accounting for the inhomogeneous character of the superconducting phase at the oxide interfaces. Of course, whenever electronic phase separation comes about, one should worry about the Coulomb energy cost that must be paid for the segregation of charge carriers. Thanks to the large dielectric constant of STO, we showed [41] that the size of the domains can grow as large as several tens of nanometers. An alternative route to intrinsic inhomogeneity might be provided by the pronounced density-dependence of the self-consistent potential well, V(\zeta), confining the 2DEG at the interface (see figure 9). Calculations within a Schrödinger-Poisson self-consistent scheme [42] also show that this mechanism is apt to yield a non monotonic chemical potential, again resulting in electronic phase separation.

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