Bridge Bounding: A Local Approach for Efficient Community Discovery in Complex Networks

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ABSTRACT
The increasing importance of Web 2.0 applications during the last years has created significant interest in tools for analyzing and describing collective user activities and emerging phenomena within the Web. Network structures have been widely employed in this context for modeling users, web resources and relations between them. However, the amount of data produced by modern web systems results in networks that are of unprecedented size and complexity, and are thus hard to interpret. To this end, community detection methods attempt to uncover natural groupings of web objects by analyzing the topology of their containing network.

There are numerous techniques adopting a global perspective to the community detection problem, i.e. they operate on the complete network structure, thus being computationally expensive and hard to apply in a streaming manner. In order to add a local perspective to the study of the problem, we present Bridge Bounding, a local methodology for community detection, which explores the local network topology around a seed node in order to identify edges that act as boundaries to the local community. The proposed method can be integrated in an efficient global community detection scheme that compares favorably to the state of the art. As a case study, we apply the method to explore the topic structure of the Lycos iQ collaborative question/answering application by detecting communities in the networks created from the collective tagging activity of users.

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Categories and Subject Descriptors
H.3.3 [Information Systems]: Information Storage and Retrieval—clustering, information filtering; G.2.2 [Discrete Mathematics]: Graph Theory

General Terms
Algorithms, Experimentation

Keywords
Graph partitioning, local community detection, tag network

1. INTRODUCTION
Network structures (also called graphs in mathematical literature) and the associated analysis methods have long emerged as a valuable tool for modeling and analyzing the relations among objects in a variety of established scientific disciplines, e.g. social sciences and biology [21]. Recent years however have witnessed a substantial adoption of network analysis techniques in the field of computer science, and more specifically, in the modeling and analysis of massive data sets produced by online information systems, such as Web 2.0 applications.

In the field of network research, the problem of community detection has lately attracted significant interest since identifying the community structure of large networks can improve our understanding of the complex relations that exist among their elements. The origins of this problem can be traced in the fields of citation study [13], bibliometrics [28] and social network analysis [25]. More recently, this problem has been restated in the context of web graphs, i.e. the networks created from mapping the web hyperlink structure to the directed network model. Two seminal web community definitions were formulated by Kumar et al. [18] and Flake et al. [11]. According to the first, a community is a dense directed bipartite subgraph of the web graph [18]. The latter definition states that a community is a vertex subset of the graph such that each of its members has at least as many edges to other members of the community as it does to
non-member vertices [11]. Although these two community
definitions are different, they both result in the formulation
of community detection as a problem of finding a partition
of a graph into subgraphs that maximizes some measure of
within-subgraph density.

Due to the extremely high complexity of providing an ex-
act solution to the community detection problem for the
complete network\textsuperscript{1}, several attempts have been made to de-
rive approximate solutions at reduced computational costs,
with some of the most efficient techniques having a com-
plexity of $O(tlog^*n)$ [22] and $O(m + n)$ [29] for networks
of $n$ nodes and $m$ edges. Despite being very efficient, most
of the existing approaches adopt a global perspective, i.e.
they operate on the full network, in order to output the de-
tected community structure. In practice, however, there is
frequently a need to explore the network structure at a local
level, e.g. in interactive network visualization [26] and infor-
mation retrieval applications [27]. Such applications impose
severe constraints on the response time of the underlying
network analysis processes, thus prohibiting the use of global
community detection methods. To date, only a few methods
have been proposed that can be used for community detec-
tion at a local level [2, 29]. However, they are either unsuit-
able for networks of scale-free topology (frequently emerging
in practice) [2] or are not local by design, thus not achieving
maximum efficiency when applied as local [29].

The situation described above motivated us to introduce a
methodology for performing community detection at a local
level; we call the proposed methodology Bridge Bounding.
Bridge Bounding initiates the community detection process
from a seed node in the network and progressively attaches
neighboring nodes to the community as long as the edges
connecting these nodes do not act as boundaries. Thus, com-
munity detection is formulated as a problem of identifying
ing edges that act as community boundaries, (which we also call
bridges, since they connect communities of the network to
each other [8, p.140]). This problem is tackled by means of
local network topology functions, i.e. functions that examine
the network structure around an edge (local network topol-
ygy) and produce a measure of the extent that these edges
act as bridges. An example of such a function is the edge
clustering coefficient [24]. In that way, we ensure that the
proposed approach has low complexity and at the same time
is capable of precisely identifying community boundaries.

In order to demonstrate the benefits of our approach, we
applied it to both synthetic and real networks. As a first
step, we validated Bridge Bounding by testing its perfor-
mance on the known community structure of synthetic net-
works and comparing it with the widely cited approach of
Girvan and Newman [15]. The proposed method could suc-
cessfully detect the synthetic communities across a variety
of network generation parameters and achieved equivalent
or better performance than the competing method, while
being computationally much more efficient. Subsequently,
we employed Bridge Bounding to explore the community
structure of two tag networks, English and German, cre-
bated by tags used to annotate questions in the LYCOS IQ
tag network. Finally, Section 5 summarizes the basic contribu-
tions of the paper and delineates our future work.

2. RELATED WORK

The problem of community finding in large complex net-
works has attracted considerable research interest for some
time now. Its origins can be traced back to the first studies
of the hyperlink structure of the web, e.g. to the observa-
tion of Gibson et al. [14] that communities emerge sponta-
eaneously around authoritative web pages which are identified
by means of hub pages. Then, the works by Kumar et al.
[18] and Flake et al. [11] formally defined and systematically
tackled the problem of community detection. In the follow-
ing, we provide a list of existing methods for community
detection classified according to the approach they adopt.
A more detailed discussion of existing community detection
methods is contained in the survey by Danon et al. [19].

Subgraph enumeration. Kumar et al. [18] consider communities as dense bipartite subgraphs of the web (seen as a directed graph). A natural way to identify dense sub-
graph structures is by means of graph partition enumeration.
In order to drastically reduce the vast number of subgraphs
that are possible by complete enumeration, the authors em-
ploy a series of heuristic pruning techniques. An extension
of this definition led to the notion of $\gamma$-dense communities
[9], which can be efficiently discovered based on more so-
plicated subgraph enumeration and pruning criteria.

Maximum flow. Flake et al. [11] define communities as subsets of vertices that have more links (undirected) to
each other than to the rest of the network nodes. To detect
such communities on the web, they integrate a maximum
flow strategy with an iterative crawling process. A stricter
community definition was considered by Ino et al. [17] and
a technique was devised to detect them that was based on
both the maximum flow algorithm and an iterative graph
partitioning and contraction process.

Divisive-Agglomerative methods. According to Gir-
van and Newman [15], the community structure of a large
network should be revealed by progressively removing edges
with high edge betweenness, i.e. by following a divisive
approach. Following the same approach but with the use
of different measures, namely the edge clustering coefficient
and the bridging centrality, Radicchi et al. [24] and Hwang
et al. [16], respectively, could uncover the underlying com-
munity structure of complex networks. Later, the measure of
modularity was defined by Newman and Girvan, as a means
to quantify the quality of a network partition into
communities [23]. More specifically, modularity reflects the
extent to which a given network partition is characterized
by higher intra-community density in comparison to the one

\textsuperscript{1}The problem is believed to be NP-complete [23].

\textsuperscript{2}We collected data from both the German
(http://iq.lycos.de) and the English (http://iq.lycos.co.uk)
version of the application.
that would be observed in a random partition of the same network. Building upon this measure, the methods by Newman [22] andClauset et al. [6] describe efficient implementations of community detection by means of agglomerative strategies.

**Seed-based Flooding.** An alternative approach to assigning the nodes of a network to communities was presented by da Fontoura Costa in [7]. There, the community detection process starts from a set of hub nodes and is implemented as a parallel flooding process emanating from the hubs. Although being seed-based, the technique in [7] is not local since it requires simultaneous discovery of all communities in a network. Thus, a local method for community finding was described by Bagrow andBoltz [2]. The authors consider an expanding neighborhood around the starting node (which they call t-shell) to constitute the community around it. In order to finish the expansion process, the authors employ a criterion quantifying the change in the total emerging degree of the community [2].

**Hybrid.** A combined strategy for community detection is proposed by Du et al. [10]. The authors consider a three-step community detection process: (a) detection of maximal cliques (subgraph enumeration), (b) initial network partition by progressive expansion of the maximal cliques (flooding) and (c) adjustment of the original partition in order to maximize modularity.

Most of the methods presented above are global, meaning that they need to process the whole network in order to output the identified community structure. Even though some of these methods achieve low complexity (linear to the size of the network), their use is still prohibitive, when there are about several hundred nodes.

**3. METHODOLOGY**

In this section, we will first (Section 3.1) present the basic notations and definitions from graph theory that are necessary to formalize the problem of community detection. Then, we will introduce the Bridge Bounding community detection methodology in Section 3.2.

### 3.1 Basic notation and definitions

We consider undirected graphs $G = (V, E)$, where $V$ is the set of vertices and $E \subseteq V \times V$ is the set of edges connecting the vertices. An edge connecting nodes $i, j \in V$ is denoted as $e_{ij}$. For a vertex $s$ of the graph, we consider its neighborhood $N(s)$ consisting of all vertices which are directly connected to $s$, i.e. $\forall n \in N(s) : e_{sn} \in E$. We define the degree of vertex $v$ as $d(v) = |N(v)|$. In a similar way, the neighborhood of an edge $e_{st}$ consists of all edges that share at least one endpoint with $e_{st}$. $N(e_{st}) = \{e_{xy} \mid \{x, y\} \cap \{s, t\} \neq \emptyset\}$.

Global community detection algorithms process a graph $G$ in order to partition the graph into a set of communities, $P \equiv \{C_0, C_1, ..., C_K\}$, where $C_i \subseteq V$. When the communities produced by a method are mutually exclusive, then $C_i \cap C_j = \emptyset, \forall i, j \in \{1, 2, ..., K\}$, with $i \neq j$. During the community detection process, we consider the set of nodes $C_U \in P$ comprising all nodes that have not been assigned to any community until that moment. For convenience, we also employ the mapping $\gamma_C : V \rightarrow P$, which returns the community a vertex is assigned to (or $C_U$ if the vertex has not been assigned to any community yet).

Local methods for community detection adopt a seed-based approach, i.e. given $G$ and a node $s$ in the graph, a local method will produce a community $C_s$ around the node. It is possible to induce a global community detection method based on a local one by repeatedly applying the local community detection method to randomly selected nodes from $C_U$ until this set is empty (i.e. all nodes of the graph have been assigned to some community). In the context of our evaluation (Section 4), we are going to induce such a global community detection scheme by employing the local Bridge Bounding method, which we describe in Section 3.2. We will refer to this scheme as progressive community detection.

### 3.2 Community detection by Bridge Bounding

Bridge Bounding is based on a simple strategy in order to identify the community $C_s$ surrounding a seed node $s$. A formal description of this strategy is presented below, in Algorithm 1. Starting from $s$, each node $n$ belonging to the neighborhood of $s$ is considered a member of $C_s$ as long as it meets two conditions (line 8 of the algorithm): (a) it is not already member of another community and (b) the edge connecting it to $s$ is not a community boundary, i.e. not a bridge (in the sense of [8, p.140]). Then, all neighbors of the newly assigned nodes (the frontier set $F$) are checked against the same conditions and are attached to $C_s$ (line 9, lines 5-6) if they meet them. This process is repeated un-

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3Community detection is frequently termed clustering in the respective literature.
Bridge Bounding is equivalent to a flooding process, similar to the one described in [7], which stops when all nodes belonging to its frontier are adjacent to a bridge (community boundary).

### Algorithm 1 LocalCommunityDetection

**Require:** Seed node $s \in G = (V, E)$

**Require:** Community mapping $gc : V \rightarrow P$

**Require:** Bridge function $b : E \rightarrow [0.0, 1.0]$

1. $C_s = \emptyset$
2. Frontier set $F = \{s\}$
3. while $|F| > 0$ do 
   4. $c \leftarrow F$.pop()
   5. $C_s \leftarrow C_s \cup \{c\}$
   6. $C_U \leftarrow C_U \setminus \{c\}$
   7. for all $n \in N(c)$ such that $e_{cn} = (c, n) \in E$ do
      8. if $gc(n) = C_U$ and $b(e_{cn}) \leq B_L$ then
         9. $F$.push($n$)
   10. end if
11. end for
12. end while
13. $P \leftarrow P \cup C_s$

The quality of the community structure output by Bridge Bounding is entwined with the success of quantifying the bridging behavior of edges. Let us consider the function $b : E \rightarrow [0.0, 1.0]$, which maps edges to real numbers in the given interval, to quantify the extent to which they act as bridges. In order for Bridge Bounding to make a binary decision on whether an edge $e$ is a bridge or not (in order to stop or continue the community flooding process along this edge), the output of the bridging function, $b(e)$, is compared against some threshold $B_L$ (which can be derived by analysis of the distribution of $b(e)$ values as will be shown later).

The problem of quantifying the bridging behavior of edges on a graph has been already studied and several measures based on graph topology have been developed with the goal of capturing the extent to which an edge acts as a bridge between different communities. One of the first attempts to define $b(e)$ was by means of its *betweenness centrality* as described in [23]. For a given edge $e$, its betweenness centrality is defined as the fraction of shortest paths running along the edge, $\sigma_{st}(e)$ to the number of all possible shortest paths $\sigma_{st}$ between $s$ and $t$.

$$b_{b}(e_{st}) = \Phi(e_{st}) = \sum_{s \neq t \in V} \frac{\sigma_{st}(e)}{\sigma_{st}}$$

An extension to this measure, called *bridging centrality*, appeared in [16]. Bridging centrality was defined as the rank product of the edge betweenness (Equation 1) and the edge bridging coefficient, which made use of the local network topology to quantify the extent to which an edge acts as a bridge.

The measures of betweenness and bridging centrality are global bridging measures, i.e. they are computed by processing the whole graph. To reduce the computational requirements, one may consider local bridging measures, e.g. the *edge-clustering coefficient* [24]:

$$C^{(3)}_{st} = \frac{z^{(3)}_{st}}{\min[(d(s) - 1), (d(t) - 1)]}$$

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The exact probability distribution function of $b_L$ for a given graph is available only after computing the local bridging function for each edge of the graph, introducing in that way a global graph processing step in the Bridge Bounding methodology. However, this step does not impose severe restrictions on the computational process. First, according to Equation 3, the computation of $b_L$ can be carried out in a streaming fashion, since only the neighborhoods of the two endpoints of each edge are required during the computation. To further reduce the computational requirements, it is possible to derive an approximation of the $b_L$ probability distribution by computing the local bridging values of a small random subset of the network edges. Finally, one

![Graph](image)

(a) Graph $G \equiv (V, E)$

(b) $b_L(e)$ distribution, $e \in E$

Figure 1: Relation of edge position in the graph and local bridging $b_L$ probability distribution function (pdf). Edges drawn with dashed lines on the network of Figure 1(a) are also the ones with the highest local bridging values (the part of the distribution in Figure 1(b) plotted in dashed line).

where $z^{(3)}_{st}$ is the number of triangles containing that edge. Note that the larger the clustering coefficient is, the less the edge acts like a bridge. Hence, we define the local bridging of an edge as:

$$b_L(e_{st}) = 1 - C^{(3)}_{st} = 1 - \frac{|N(s) \cap N(t)|}{\min[(d(s) - 1), (d(t) - 1) \}}$$

In order for $b_L(e)$ to have a low value, the two endpoints of $e$ need to have a lot of common neighbors (relative to their degree). Effectively, this means that in order to move from one of the endpoints to the other, one has multiple options in addition to $e$. Thus, $e$ is considered as an intra-(or within-) community edge. In the opposite case, when the two endpoints of a bridge have very few or no neighbors in common, then this edge is crucial for the connection between its endpoints. For that reason, we consider in the latter case, where $b_L(e)$ has a high value, that $e$ is an inter-community edge or bridge.

In order to derive a decision threshold $B_L$ for identifying the bridge edges of the graph (see line 8 of Algorithm 1), one needs to inspect the distribution of $b_L$ values among the edges of the graph. Figure 1 illustrates how the position of edges on a graph with community structure affects their local bridging values. The graph of Figure 1(a) was generated to comprise a synthetic four-community structure. Edges drawn with dashed lines on the network of Figure 1(b) are also the ones with the highest local bridging values (the part of the distribution in Figure 1(b) plotted in dashed line).

![Graph](image)

(a) Graph $G \equiv (V, E)$

(b) $b_L(e)$ distribution, $e \in E$
could even completely skip the distribution estimation step if it has been already performed for a graph of similar nature in the past (in which case one could reuse the previously estimated threshold \(B_L\)).

The simple measure of local bridging (Equation 3) employed by Bridge Bounding is ideal for networks with very clear community structure (such as the one of Figure 1(a)). However, the measure is often not well-suited for detecting communities in real networks. In particular, when a network is characterized by scale-free topology, the distribution of \(b_L\) values will have a spiky shape, similar to the one in Figure 2(a), where the depicted \(b_L\) distribution comes from the English Lycos iQ tag network of Section 4.2. In such cases, it is hard to differentiate between bridge and non-bridge edges. For instance, according to Figure 2(a), 8% of the network edges have local bridging \(b_L = 0\), thus \(\forall B_L \geq 0\), Algorithm 1 will always consider 8% of the network edges as non-bridges. In networks with scale-free topology (which commonly emerge in practice), such a decision would cause Bridge Bounding to detect a community structure that consists of one large community and many singleton communities, i.e. communities comprising just one member. The reason for such an outcome is that scale-free networks maintain a large connected component even when a large fraction of their edges are removed\(^4\) [1]. Figure 3 illustrates the output of Bridge Bounding on a scale-free graph generated by the preferential attachment model of Barabási-Albert [3].

In order to alleviate this problem, we consider the 2\(^{nd}\) order local bridging of an edge \(e\), \(b_L^2(e)\), by computing the weighted sum (with a mixing parameter \(\alpha\)) of its local bridging, \(b_L(e)\) and the mean local bridging of the edges constituting its neighborhood:

\[
b_L^2(e_{st}) = \alpha \cdot b_L(e_{st}) + (1 - \alpha) \frac{1}{|N(e_{st})|} \sum_{e \in N(e_{st})} b_L(e) \tag{4}
\]

By applying Equation 4, we carry out a smoothing of the local bridging function by taking into account the values of the function in the neighborhood around a given edge. The \(\alpha\) parameter defines the extent to which the values of the neighboring edges are taken into account in the computation of \(b_L^2\). Figure 2(b) illustrates the distribution of \(b_L^2\) (using \(\alpha = 0.7\)) for the Lycos iQ English tag network of Section 4.2. Low \(b_L^2\) values are distributed more evenly in comparison to the \(b_L\) ones. Hence, it is possible to select a value for \(B_L\) such that only a very-low fraction of edges are considered as intra-community (\(\approx 1%\) in this example).

Effectively, the computation of 2\(^{nd}\) order local bridging makes use of topological information from a wider neighborhood around a given edge in comparison to local bridging. Following this, one could consider the \(\nu^{\text{th}}\) order local bridging, \(b_L^{(\nu)}\), which for sufficiently high values of \(\nu\), would utilize topological information from the whole graph. Obviously, since the computation of \(b_L^{(\nu)}\) is carried out in an iterative manner, the complexity of computing the measure increases with its order \(\nu\).

In terms of complexity, a progressive global community detection scheme based on Bridge Bounding is decomposed in two steps: (a) local network topology function computation and (b) community detection. Computing the basic local bridging measure for a graph of \(n\) nodes and \(m\) edges with average node degree \(d\) has a complexity of \(O(d \cdot m)\) since for each edge, we need to find the intersection of two sets of average size \(d\)\(^5\). The community detection step has a complexity of \(O(d \cdot n)\), when Algorithm 1 is used in the global community detection scheme described in 3.1, since for each node of the network \(d\) candidate nodes are considered as candidates for assignment to the community that is currently being created. Thus, in total, Bridge Bounding scales with \(O(d^2 \cdot m + d \cdot n)\).
comparing the performance of progressive global community detection based on Bridge Bounding with the one achieved by the community detection method of Girvan-Newman [15]. This comparison is carried out on synthetic networks with known (predefined) community structure, thus giving the possibility of objective measurement of the method performance. In the second part of the experiments, we aim at gaining insights into real-world complex networks. Therefore, we apply our community detection technique on two networks created from the user tagging activities in the English and German version of the Lycos iQ question/answering application. Since there is no ground truth concerning the community structure of the Lycos iQ tag network, we use our subjective judgement in order to draw conclusions on the performance of the proposed method.

4.1 Synthetic networks

We created a parameterized community mixture generator following the strategy described in [23] and [20]. According to this, the generation process results in a network with $N$ nodes which consists of $K$ communities. We control the average degree $z_{tot}$ of the network nodes, as well as the probability $p_{out}$ that a node’s edge will connect to a node of a different community. Thus, out of the $z_{tot}$ edges of each node (on average), $z_{out} = p_{out} \cdot z_{tot}$ edges connect the node to nodes of different communities. Obviously, higher values of $p_{out}$ will lead to networks with less profound community structure. Figure 4 depicts the difference in the conspicuity of community structure in relation to the fraction $p_{out}$ of inter-community edges. This network generation process can be described by a four-element parameter set comprising $N$, $K$, $z_{tot}$, and $p_{out}$. We also consider a fifth parameter, namely the community size variation $s_{var}$, which is calculated as the ratio of the biggest community size to the size of the smallest one. In this case, each community $C_i$ will have a different average node degree $z_{tot}^i$ and therefore we define $z_{tot} = \frac{1}{K} \cdot \sum_{i=1}^{K} z_{tot}^i$. In the end, we consider the five-element parameter set:

$$SPAR = \{N, K, z_{tot}, p_{out}, s_{var}\} \quad (5)$$

Two widely used measures to evaluate the effectiveness of data partitioning methods, e.g. community detection, when the true partition structure is known (which is the case when testing with synthetic networks) are (a) the fraction $F_C$ of correctly classified instances [23] and (b) the Normalized Mutual Information (NMI) introduced in [12] and applied for the evaluation of community detection in [20]. Consider two partitions of the $n$-node graph, $P^a = \{C^a_0, C^a_1, ..., C^a_K\}$ (true community structure) and $P^b = \{C^b_0, C^b_1, ..., C^b_K\}$ (community structure found by algorithm). The fraction $F_C$ of correctly classified instances is straightforward to compute only when $K_a = K_b = K$. When the true number of communities $K_a$ differs from the number of communities $K_b$ found by the algorithm, we need to first identify a subset of the found communities $P^b \subseteq P^a$ that can be matched to a subset of the true communities, $P^a \subseteq P^b$. We consider two communities as matching if they present overlap of more than 50%. Then, assuming that community $C^a_i \in P^a$ is the matching community of $C^b_i \in P^b$, $F_C$ is computed by the following equation:

$$F_C = \frac{1}{n} \cdot \sum_{C^a_i \in P^a} |C^a_i \cap C^b_i| \quad (6)$$

The Normalized Mutual Information between the true partition, $P^a$, and the one found by the algorithm, $P^b$, quantifies the extent to which they are similar to each other from an information-theoretic point of view [12].

$$\text{NMI}(P^a, P^b) = \frac{-2 \cdot \sum_{i=1}^{K_a} \sum_{j=1}^{K_b} n_{ij} \log \left( \frac{n_{ij}}{n_{ii}n_{jj}} \right) \cdot \sum_{i=1}^{K_a} n_{ii} \log \left( \frac{n_{ii}}{n} \right) + \sum_{i=1}^{K_a} n_{ii} \log \left( \frac{n_{ii}}{n_a} \right) - \sum_{i=1}^{K_a} n_{ii} \log \left( \frac{n_{ii}}{n} \right)}{\sum_{i=1}^{K_a} n_{ii} \log \left( \frac{n_{ii}}{n} \right) + \sum_{i=1}^{K_a} n_{ii} \log \left( \frac{n_{ii}}{n_a} \right)} \quad (7)$$

In Equation 7, $n_{ij}$ and $n_{ii}$ denote the number of nodes in communities $C^a_i$ and $C^b_j$ respectively, and $n_{ij}$ denotes the number of shared nodes between communities $C^a_i \in P^a$ and $C^b_j \in P^b$. In general, NMI is preferred to the simplistic $F_C$ measure, since it handles gracefully the cases where $K_a \neq K_b$. $F_C$ is presented here together with NMI mainly due to the ease in its interpretation.

To demonstrate the effectiveness of Bridge Bounding in detecting the underlying community structure of networks, we compare the performance of the progressive global community detection scheme (see Section 3.1) based on Bridge Bounding in terms of both $F_C$ and NMI to the performance of the community detection method by Girvan and Newman (GN) [15] on a multitude of synthetic networks. Since the GN method employs a divisive approach, it results in a hierarchical community structure, which contains multiple graph partitions to communities. Therefore, we needed to select a single partition from the hierarchy, which we would use to evaluate the performance of the method. The strategy used by Newman and Girvan in [23] to make this selection is to calculate the modularity $Q$ of each partition and select the partition which maximizes it.

The modularity of a network partition into $K$ communities is calculated from the $K \times K$ symmetric matrix $e$ whose element $e_{ij}$ is the fraction of all edges in the network that link vertices in community $i$ to vertices in community $j$. Further, we define the row (or column) sums $\alpha_i = \sum_j e_{ij}$ which represent the fraction of edges that connect to vertices in community $i$. Based on the above definitions, the measure of modularity is defined as:

$$Q = \sum_i (\epsilon_{ii} - \alpha_i^2) \quad (8)$$

This quantity measures the fraction of edges in the net-
Table 1: Comparison of performance between a global scheme based on Bridge Bounding with local bridging (BB), Bridge Bounding with 2nd order local bridging (BB') and the method of Girvan and Newman (GN) [15]. The performance is measured on synthetic networks generated using the set $S_{PAR}^{1} = \{200, 40, p_{\text{out}}, 1.0\}$ of parameters, with $p_{\text{out}}$ being the free parameter.

| $p_{\text{out}}$ | $P_{C}$ | NMI |
|----------------|--------|-----|
| 0.01           | 100    | 1.0 | 1.0 |
| 0.05           | 100    | 1.0 | 1.0 |
| 0.1            | 100    | 1.0 | 0.86|
| 0.15           | 99     | 0.98| 0.86|
| 0.20           | 99     | 0.98| 0.86|
| 0.25           | 24     | 0.54| 0.56|

Table 2: Similar comparison of performance as in Table 1, but on synthetic networks that were generated using the set $S_{PAR}^{2} = \{200, 40, 0.01, s_{\text{var}}\}$ of parameters, with $s_{\text{var}}$ being the free parameter.

| $s_{\text{var}}$ | $P_{C}$ | NMI |
|----------------|--------|-----|
| 1.1            | 100    | 1.0 | 1.0 |
| 1.5            | 100    | 1.0 | 1.0 |
| 1.6            | 99.5   | 0.99| 1.0 |
| 1.7            | 88     | 0.82| 0.96|
| 1.8            | 85.5   | 0.79| 0.95|
| 1.9            | 58.5   | 0.68| 0.82|
| 2.0            | 12.5   | 0.45| 0.73|
| 2.5            | 62     | 0.45| 0.63|

work that connect vertices of the same community (i.e. intra-community edges) minus the expected value of the same quantity in a network with the same community partition but random connections between the vertices. If the number of intra-community edges is no better than random, we would get $Q = 0$. For perfect separation to communities (i.e. communities that are completely disconnected from each other on the graph), we get $Q = 1$. In practice, modularity values in the range from 0.3 to 0.7 indicate significant community structure.

We created two sets of networks containing synthetic communities. The first set of such networks was generated holding the four network generation parameters of Equation 5 constant and varying $p_{\text{out}}$. This is a widely adopted test process [23, 24, 20] to test the performance of a community detection method as the communities of the synthetic graph gradually become less well-separated. Table 1 presents the comparison between the performance of Bridge Bounding (by use of both first- and second-order local bridging) and the GN method [15]. Both Bridge Bounding methods present equally good or better performance than GN across the range of $p_{\text{out}}$ values that were used for testing.

A further test involved the generation of an additional set of networks by varying the $s_{\text{var}}$ parameter in order to end up with networks comprising communities of unequal sizes. Table 2 provides an overview of the results obtained from the three methods of our study. Apparently, the use of the local bridging function (Equation 3) becomes problematic for Bridge Bounding as soon as the size variation among the underlying communities exceeds a certain value (e.g. for $s_{\text{var}} \geq 2$, we measured NMI(BB') < 0.5). In contrast, Bridge Bounding with the use of 2nd order local bridging as well as the GN method yielded consistently better results in this series of tests. Hence, it becomes clear that the use of more sophisticated local topology measures, such as the 2nd order local bridging, could be crucial for the success of the proposed method.

4.2 Lycos iQ Tag Network

LYCOS iQ is a collaborative question/answering application where people ask and answer questions on any topic. The application is available in six languages, German, English, French, Danish, Swedish and Dutch with German attracting the largest community of users. In order to support the users’ efforts of searching for relevant questions, the application incorporates a tagging functionality, similar to the one used in typical social tagging systems such as delicious\(^6\) and flickr\(^7\). There are no static categories and tags are not predefined by the system, but the users’ inputs are checked against tags existing in the system database to prevent duplicates.

Question submitters have the possibility of attaching more than one tag to each of their questions. Therefore, it is possible to create a tag network from the collaborative tagging activities of users. In this network, the vertex set comprises the tags chosen by users to tag their questions and the edge set contains the co-occurrences between tags in the users’ questions. When a question is tagged with more than two tags, then all possible pairwise co-occurrences are added to the network. For each tag of the network, its frequency ($t_f$) is available. Further, the co-occurrence frequency ($cf$) between each pair of tags is available.

Figure 5 illustrates the rank plots of tag and cooccurrence frequencies and node degrees for the German and English LYCOS iQ tag networks.

\(^6\)http://delicious.com
\(^7\)http://flickr.com
Table 3: Tag networks used in this case study. For each network $G = (V, E)$, it is $|V| =$ tags and $|E| =$ tag-pairs.

| Language | Tags | Tag-Pairs | Questions |
|----------|------|-----------|-----------|
| English (UK) | 9,517 | 77,243 | 62,497 |
| German (DE) | 78,138 | 896,486 | 942,405 |

in the German and English Lycos iQ tag networks. A highly skewed behavior is obvious in the tagging activities of users, e.g. in the English dataset, a small set of tags is used very frequently (hundreds of times), while the majority of them is used less than 10 times. The frequency of cooccurrence between tags follows a similar pattern, with less than a thousand tag pairs occurring together in more than a few questions. Finally, the node degrees follow a long-tail distribution, indicating that the tag networks are characterized by scale-free topology. A considerable number of tags are even disconnected from the rest of the network meaning that they were used in isolation. To reduce the amount of noisy tags in the network, we filtered out tags that were either disconnected or appeared less than twice in the dataset.

Table 3 provides a summary of the two tag networks that we obtained after the aforementioned filtering step. The tag network induced from the tagging activity on the German version of Lycos iQ, is far larger than the one created from the English version. Nevertheless, we preferred to present community snapshots and examples only from the English tag network to ensure that even readers who are not familiar with German can understand them. Since the proposed community detection method relies only on information regarding the network topology, the outcome of the method is language independent. We could confirm this intuition by inspecting the community detection results on both tag networks.

Figure 6 provides a high-level view of the most prominent topics coming up through the users' questions in the Lycos iQ application. The tags depicted in this view were selected based on their degree in the network. Although the resulting network is very densely connected, one can already see that all tags (apart from the pair “IQ”-“GENERAL INTEREST”) belong to different communities (since the dashed edges have been found to be inter-community edges, after thresholding based on the $b'_L$ distribution of the network in Figure 7).

In order to explore the topic structure of the tag network in more depth, we selected some of the top-level tags as seed nodes and inspected the resulting communities. Figures 8 and 9 present the communities around tags “computers” and “history”. In both figures it is apparent that most edges are considered intra-community. Also, note that while the “computers” community is densely connected, the “history” community resembles a star-shaped graph: it remains connected through its central tag (“history”).

Four additional tag communities are depicted in Figure 10. The complexity of their structure depends on the topic of the respective community. For instance, the community formed around the tag “music” (Figure 10(a)) has a much simpler structure than the one created using “science” as the seed tag (Figure 10(b)). There are two possible reasons for this: (a) science is a more general topic than music, containing sub-topics such as physics, medicine, biology and astronomy (these correspond to the four large nodes of Fig.
Figure 9: Community around tag “history”.

Further, a noteworthy observation regarding the structure of the communities around “film” (Figure 10(c)) and “animals” (Figure 10(d)) is the existence of small cliques (between 3 and 5 members) within them. Those correspond to tags related to particular films in the “film” community (e.g. “batman”-“Christian Bale”-“comic”) or tags related to groups of animals (e.g. “leopards”-“panthers”-“mammals”) in the “animals” community. This indicates the existence of semantic hierarchies within topics (e.g. “mammals” are a subclass of “animals”; “leopards”, “panthers” are a subclass of “mammals”), which could be further validated by means of machine learning techniques [30].

As stated earlier, detecting the topic communities within a tag network, similar to the one created from LYCOS iQ application (nowadays, there are plenty of Web 2.0 applications incorporating collaborative tagging characteristics), can be beneficial for both the users and the administrators of the application. Users can be provided with a community view of the tags that are related to their context. For instance, when a LYCOS iQ user submits a question to the system, the text of her question can be parsed and matched against the tags already available in the system. Then, by identifying the community (or communities) that her question belongs to, it is possible to recommend relevant tags for use as descriptors of the question or relevant questions that have been tagged with tags belonging to the respective community. Further, administrators of such applications could use community detection in the context of a content monitoring and trend tracking framework for supporting the operation of important administrative tasks, e.g. online ad targeting or content moderation (which is most frequently synonymous to spam detection).

5. CONCLUSIONS

We introduced Bridge Bounding, a local methodology for community detection in large networks. The methodology is based on the notion of local network topology functions to quantify the extent to which edges act as community boundaries, i.e. bridges. We showed that use of local bridging, a topology function based on the widely used edge clustering coefficient, resulted in successful discovery of existing community structure in synthetic networks, but failed to do so in networks of scale-free topology. For that reason, we employed the second- and higher-order local bridging functions to derive smoother estimates of the bridging properties of edges. The proposed methodology is extremely efficient, scaling with $O(d^2 \cdot m + d \cdot n)$ for networks of $n$ nodes and $m$ edges with average node degree $d$.

A series of tests on synthetic networks with controlled community structure provides evidence that the Bridge Bounding method (with use of the 2nd order local bridging function) performs equally well or better than the widely used method of Girvan and Newman. Moreover, application of our method on two large tag networks coming from the LYCOS iQ question/answering application proved beneficial in studying the underlying topic structure and can benefit both users and administrators of Web 2.0 applications with social tagging features.

In the future, we plan to carry out more thorough evaluation tests on the tag communities produced by Bridge Bounding. Specifically, we plan to conduct a user study among selected LYCOS iQ users in order to derive manual judgements on the quality of the detected communities. Subsequently, we are going to consider the potential of new edge bridging functions and of more sophisticated strategies for community detection based on Bridge Bounding. Instead of the currently employed fixed-threshold strategy for deciding whether an edge is intra- or inter-community, we will test the potential of adaptive threshold strategies. Finally, we intend to look into extensions that will endow the method with capabilities for uncovering hierarchical relations within the community structure.
6. ACKNOWLEDGMENTS

This work was supported by the WeKnowIt project, partially funded by the European Commission, under contract number FP7-215453. We would also like to acknowledge the use of the JUNG\textsuperscript{8} framework for parts of our implementation. Finally, we would like to acknowledge the use of the English and the German tag data sets from the Lycos iQ application operated by Lycos Europe.

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\textsuperscript{8}http://jung.sourceforge.net