Polarisabilities from Compton Scattering on $^3$He

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Abstract. This executive summary of recent theory progress in Compton scattering off $^3$He focuses on determining neutron polarisabilities; see ref. [2] and references therein for details and a better bibliography.

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Two plenaries discuss the large-scale international effort, gains and goals of a new generation of high-precision facilities to extract nucleon polarisabilities from Compton scattering experiments, and show that determining them by experiments takes years of planning, execution and analysis — and commensurate theory support. Others highlight the importance of electromagnetic polarisabilities in many contexts. We thus refer to all these contributions [1] for motivation and context, and concentrate on theory progress for one target nucleus: $^3$He.

Setting the Stage

Low-energy Compton scattering $\gamma X \rightarrow \gamma X$ probes a target’s internal degrees of freedom in the electric and magnetic fields of a real photon. These fields induce radiation multipoles by displacing the target constituents. The angular and energy dependence of the emitted radiation encodes information from the symmetries and strengths which govern the interactions of the constituents with each other and with photons. After subtracting the “Born contributions” (known from one-photon data like form factors), its multipoles parametrise the stiffness of a nucleon $N (\text{spin } \ell)$ against transitions $X \rightarrow Y$ at frequency $\omega (l’ = l \pm \{0; 1\}; X, Y = E, M; T_{ij} = \frac{1}{2}(N_j T_i + N_i T_j); T = E, B)$:

$$2\pi N^\dagger [\alpha_{E1}(\omega)\mathbf{E}^2 + \beta_{M1}(\omega)\mathbf{B}^2 + \gamma_{E1E1}(\omega)\mathbf{E} \cdot \mathbf{E} + \gamma_{M1M1}(\omega)\mathbf{E} \cdot \mathbf{B} + \gamma_{E1M2}(\omega)\mathbf{B} \cdot \mathbf{E} + \beta_{M1E2}(\omega)\mathbf{B} \cdot \mathbf{B} + \gamma_{M1E2}(\omega)\mathbf{E} \cdot \mathbf{E} + \gamma_{M1M1}(\omega)\mathbf{B} \cdot \mathbf{B} + (\text{higher multipoles})] N.$$

Six two-photon response functions suffice up to about 400 MeV: two scalar polarisabilities $\alpha_{E1}(\omega)$ and $\beta_{M1}(\omega)$ for electric and magnetic dipole transitions; and the four dipole spin-polarisabilities $\gamma_{E1E1}(\omega), \gamma_{M1M1}(\omega), \gamma_{E1M2}(\omega), \gamma_{M1E2}(\omega)$. These test the nucleon-spin structure and complement information from Jefferson Lab’s spin programme. Intuitively, the electromagnetic field of the spin degrees causes bi-refringence in the nucleon, like in the classical Faraday-effect. The static values, $\alpha_{E1} \equiv \alpha_{E1}(\omega = 0)$ etc., are often just called “the” polarisabilities and condense the rich information on the pion cloud, on the $\Delta (1232)$

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excitation, and on the interplay between chiral symmetry breaking and short-distance interactions. These fundamental quantities provide stringent tests for theoretical descriptions of hadron structure. Moreover, they are ingredients to the neutron-proton mass difference, the proton charge-radius puzzle, and the Lamb shift of muonic hydrogen. To extract them, one must reliably extrapolate from data to $\omega = 0$. Since pure neutron targets are unfeasible, nuclear binding and meson-exchange effects must also be subtracted with reliable theory uncertainties. Fortunately, Chiral Effective Field Theory ($\chi$EFT) provides model-independent estimates of higher-order corrections and encodes the correct low-energy dynamics of QCD. For few-nucleon systems, it consistently incorporates hadronic and nuclear currents, rescattering effects and wave functions. The photon’s interaction with the charged pion-exchange between nucleons also probes few-nucleon binding. Even if scattering on a free neutron were feasible, cross sections and signals for coherent scattering from nuclei are markedly larger. Elastic Compton scattering from $^3$He is a promising means to access neutron polarisabilities. In ref. [3] and subsequent publications, Shukla et al. showed that the differential cross section between 50 and 120 MeV is sensitive to the electric and magnetic dipole polarisabilities of the neutron, $\alpha_{E1}^{(n)}$ and $\beta_{M1}^{(n)}$, and that scattering on polarised $^3$He provides good sensitivity to the neutron spin polarisabilities. This triggered several approved proposals at MAMI and HI$\gamma$S.

We recently extended these $\chi$EFT predictions by one order to N$^3$LO [$O(e^2\delta^3)$] by adding a dynamical Delta degree of freedom, and provided results for photon lab energies between 50 and 120 MeV for the differential cross section, for the beam asymmetry $\Sigma_3$, and for the two double asymmetries with circularly polarised photons and transversely or longitudinally polarised targets, $\Sigma_{2x}$ and $\Sigma_{2z}$. These are the only non-zero observables below pion-production threshold in our formulation. We also found that the pioneering results were obtained from a computer code which contained mistakes, triggering an erratum to ref. [3].

At such energies, the complete photonuclear operator at N$^3$LO [$O(e^2\delta^3)$] is: the Thomson and other minimal-substitution terms; magnetic-moment cou-
pllings; dynamical single-nucleon effects such as virtual pion loops and the Delta excitation; and couplings of photons to the charged-pion exchange. All terms are evaluated with $^3$He wave functions found from the same $\chi$EFT expansion.

**Results** The dynamical Delta effects are obvious in all observables for $\omega_{lab} \gtrsim 100$ MeV; see fig. 1. They markedly invert the fore-aft asymmetry of the cross section and increase the magnitude of double asymmetries and their sensitivity to spin polarisabilities, echoing similar findings for the deuteron. The chiral expansion converges in this energy range quite well; see e.g. fig. 1. The dependence on the choice of the $^3$He wave function is small and can usually be distinguished from the effects of polarisabilities by a different angular dependence.

We found that $\alpha^{(n)}_{E1} - \beta^{(n)}_{M1}$ can be extracted from the cross section; $\Sigma_{2x}$ has a non-degenerate sensitivity to $\gamma^{(n)}_{E1M1}$ around $90^\circ$; and $\Sigma_{2x}$ to $\gamma^{(n)}_{E1E1}$ and $\gamma^{(n)}_{E1M2}$; see fig. 2. The beam asymmetry $\Sigma_3$ is dominated by the single-nucleon Thomson term and not very useful to directly determine polarisabilities. Ultimately, the most accurate polarisabilities will be inferred from data of all four observables. For the spin polarisabilities, data at $\omega_{lab} \gtrsim 100$ MeV will be crucial.

This exploration is part of an ongoing dialogue with our experimental colleagues on the best kinematics and observables to extract neutron polarisabilities. An interactive *Mathematica* notebook is available from hgris@wvu.edu. Results are quite robust. Varying the single-nucleon amplitudes of complementary approaches like dispersion relations will lead to sensitivities which are hardly discernible from ours. Once data exist, a polarisability extraction will of course need to address residual uncertainties with more diligence; see e.g. ref. [4].

**Nuclear Binding** $\chi$EFT also quantifies the angle- and energy-dependent corrective to the naïve $^3$He picture as the sum of two protons with antiparallel spins and one neutron. Sensitivity to the scalar polarisabilities enters indeed approximately via $2\alpha^{(p)}_{E1} + \alpha^{(n)}_{E1}$ and $2\beta^{(p)}_{M1} + \beta^{(n)}_{M1}$, and the double-asymmetries are 10-to-20 times more sensitive to the spin polarisabilities of the neutron than of the proton.

![Fig. 2.](image-url)  
*Fig. 2.* The sensitivity of $\Sigma_{2x}$ on the two spin polarisabilities with the biggest impact.
However, fig. 3 confirms that there is no energy where polarised $^3$He simply acts as a “free neutron-spin target”. The sensitivities to neutron spin polarisabilities closely mimic those of free-neutron observables. But their magnitudes do not.

![Graph of double asymmetries](image)

**Fig. 3.** The double asymmetries with circularly polarised beam and transversely (left) or longitudinally (right) polarised target, for the proton, neutron and $^3$He.

An impulse approximation would thus omit a key mechanism: charged pion-exchange currents. Without their large interference with the polarisabilities, results are severely distorted. The $\chi$EFT expansion provides quantitative predictions of the two-body currents, with reliable theory uncertainties. Detailed checks of the convergence of the expansion for exchange currents and for the other pieces of the $^3$He-Compton amplitude by performing a N$^4$LO [$O(e^2\delta^4)$] calculation and extending the applicable energy range are under way. They will allow for even more accurate extractions of polarisabilities from upcoming data.

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