On the Progenitors of AM CVn Stars as LISA Sources: The Evolved Donor Star Channel

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Abstract

The space gravitational wave (GW) detector Laser Interferometer Space Antenna (LISA) that is planned to be launched in the early 2030s will detect the low-frequency GW signals in the Galaxy. AM CVn stars were generally thought to be important low-frequency GW sources. Employing the MESA code, in this work we calculate the evolution of a great number of binary systems consisting of a white dwarf (WD) and a main sequence (MS) star, and diagnose whether their descendant-AM CVn stars will be visible with LISA. The simulated results show that the progenitors of these LISA sources, within a distance of 1 kpc, are WD–MS binaries with a donor star of 1.0–1.4 $M_\odot$ (for initial WD mass of 0.5 $M_\odot$) or 1.0–2.0 $M_\odot$ (for initial WD mass of 0.7 $M_\odot$), and an initial orbital period slightly smaller than the bifurcation period. Our simulations also indicate that 10 verification AM CVn sources can be reproduced by the standard magnetic braking model, and are potential LISA sources. Based on the birth rate of AM CVn stars simulated by the population synthesis, the birth rate of AM CVn-LISA sources evolving from the evolved donor star channel within a distance of 1 kpc can be estimated to be $(0.6–1.4) \times 10^{-8}$ yr$^{-1}$, and the predicted number of AM CVn-LISA sources is about 340–810. Therefore, the evolved donor star channel plays an important role in forming AM CVn-LISA sources in the Galaxy.

Unified Astronomy Thesaurus concepts: Gravitational wave sources (677); Gravitational waves (678); White dwarf stars (1799); Compact binary stars (283); Stellar evolution (1599)

1. Introduction

A new era for the study of gravitational waves (GWs) began following the detection of the double compact object merger events, GW150914 and GW170817, by the advanced LIGO and Virgo detectors (Abbott et al. 2016, 2017). The Laser Interferometer Space Antenna (LISA) is a space-based GW detector, which is planned to be launched in the early 2030s (Amaro-Seoane et al. 2017). Unlike the advanced LIGO and Virgo, the scientific aims of LISA are to detect low-frequency GW signals between the bands 0.1 mHz to 0.1 Hz (van der Sluys 2011; Amaro-Seoane et al. 2017). The potential LISA sources include ultracompact binaries in our Galaxy, supermassive black hole mergers, and extreme mass ratio inspirals (EMRIs; Amaro-Seoane et al. 2007). For the ultracompact binaries, the possible sources can be classified into two sorts: the first one is compact detached binaries including double white dwarfs (WDs), WD–neutron star (NS) binaries, double neutron stars, and binary black holes (Nelemans et al. 2001b; Liu et al. 2010; Yu & Jeffery 2015; Kremer et al. 2017a, 2018, 2019; Korol et al. 2017; Tauris et al. 2017; Lamerts et al. 2018; Tauris 2018; Andrews & Mandel 2019; Lamerts et al. 2019; Breivik et al. 2020; Lau & Mandel 2020; Li et al. 2020; Liu & Wang 2020; Sesana et al. 2020); the second one is interacting binaries including cataclysmic variables (CVs), AM CVn stars, ultracompact X-ray binaries (UCXBs; Chen et al. 2020), and compact intermediate-mass black hole X-ray binaries (Chen 2020).

AM CVn stars (hereafter AM CVns) are ultracompact binaries where a WD is accreting materials from a semidegenerate helium-rich star or a WD (Warner 1995; Nelemans 2005; Solheim 2010). In observations, it is found that the orbital periods of AM CVns lie among 5–65 minutes and the mass transfer is driven by GW radiations. Therefore, AM CVns are proposed to be ideal candidates for detecting low-frequency GW signals by LISA in the near future (Nelemans et al. 2004; Roelofs et al. 2007c).

At present, there exist three evolutionary channels toward AM CVns. The first one is the double WD model where two detached WDs evolve toward a short period system via the angular momentum loss (AML) by the GW radiation until the less massive WD starts the Roche lobe overflow (RLOF; Nather et al. 1981; Nelemans et al. 2001a). Recently, the systematic works on this channel were explored by theoretical modeling (Gokhale et al. 2007; Kremer et al. 2015; Kalomeni et al. 2016; Kremer et al. 2017a, 2017b), and observations (Kupfer et al. 2020; Rivera Sandoval et al. 2020). However, Shen (2015) proposed that these interacting WDs would probably merge due to dynamical friction within the expanding nova shell, making this formation channel for AM CVns seem impossible. The second one is the helium donor star channel, in which a nondegenerate helium star transfers matter to a WD and the system subsequently reaches a minimum orbital period of about 10 minutes (Savonije et al. 1986; Iben & Tutukov 1987). The third one is the evolved donor star channel where a main sequence (MS) donor star has lost most of its hydrogen envelope in the early mass-transfer stage (like the CV stage), and subsequently transfers the He-rich matter onto the massive WD once the orbital period is less than 1 hour and the binary system appears as an AM CVn (Tutukov et al. 1985; Podsiadlowski et al. 2003; Breedt et al. 2012; Carter et al. 2013b). Recently, Gaia14aae was identified as the first AM CVns in which the central WD was fully eclipsed by the donor star (Green et al. 2018b). Gaia14aae is proposed as an example of the evolved donor star channel for AM CVns.

Based on the double WD channel and the helium star channel, Nelemans et al. (2001a) explored a thorough binary population synthesis (BPS) work on AM CVns according to whether the tidal coupling between the accretor and the orbital motion are efficient (model II) or nonefficient (model I). Their
results show that while the birth rates derived by model II are comparable for WD and helium star scenarios, model I tends to select the helium star channel as the dominant contribution for AM CVns. Considering all the cases, they proposed that the birth rate of AM CVns in the Galaxy lie between 1.1–6.8 × 10^{-3} yr^{-1} and give a space density of σ = (0.4–1.7) × 10^{-4} pc^{-3}. However, based on the large data of the Sloan Digital Sky Survey (SDSS), the later works by Roelofs et al. (2007c) and Carter et al. (2013a) proposed that the space density of AM CVns is supposed to be much lower than expected, by a factor of 2–3 (see also our discussion in Section 4). Recently, using Gaia DR2, Ramsay et al. (2018) presented the lower limit for the space density of AM CVns to be ≥3 × 10^{-7} pc^{-3}.

Adopting the evolved donor star channel, Podsiadlowski et al. (2003) carried out a detailed BPS calculation on CVs that can evolve toward AM CVns, and obtained a birth rate of ultracompact WD binaries (including AM CVns) amounting to (0.5–1.3) × 10^{-3} yr^{-1}.

Recently, Tauris (2018) investigated a systematic work on the evolution of neutron star–WD binaries as a LISA source and found that the formation and evolution of AM CVns are very similar to those of UCXBs. Obviously, the birth rate and number of AM CVns appearing as LISA sources are greater than those of UCXB-LISA sources in the Galaxy due to the initial mass function. In this paper, we attempt to explore a detailed evolution of AM CVns based on the evolved donor star channel, and evaluate the detectability of AM CVns by LISA. This paper is organized as follows. The input physics and the stellar evolution code are described in Section 2. In Section 3, we summarize the detailed simulated results. The discussion and summary are presented in Sections 4 and 5.

2. Evolutionary Code

We carried out binary evolutionary calculations for AM CVns by the Modules for Experiments in Stellar Astrophysics (MESA, version r8845; Paxton et al. 2011, 2013, 2015), with MESA SDK for Linux (version 20160129) by Townsend (2016). The primordial binary is composed of a WD (as a point mass of M_{WD}) and a MS companion star (with a mass of M_{d}). The chemical composition of the donor star is X = 0.7, Y = 0.28, Z = 0.02. Radiative opacities of the donor star are primarily from OPAL (Iglesias & Rogers 1993, 1996), with low-temperature data from Ferguson et al. (2005). The Roche lobe radius of the donor star is computed by using the expression given by Eggleton (1983). Mass transfer rates by the Roche lobe overflowing are determined following the method of Ritter (1988). The orbital AML plays a key role in influencing the formation and evolution of AM CVns. We consider three kinds of AMLs, including gravitational radiation (Landau & Lifshitz 1975), magnetic braking (Rappaport et al. 1983), and mass loss. During the evolution, once the donor star develops a convective envelope and possesses a radiative core, magnetic braking (with magnetic braking index γ = 4, see also our discussions in Section 4) will turn on. Note that many settings in the simulations are similar to Tauris (2018). In our calculations, two initial WD masses M_{WD,i} = 0.5, 0.7 M_☉ are used. In the MESA code, α = 0 and δ = 0 are set, which means that during RLOF any wind-mass loss from the donor star or formation of a circumbinary disk is neglected. The β factors, which represent the mass loss fraction from the WD vicinity, are set to be 0.8 and 0.7 for M_{WD,i} = 0.5, 0.7 M_☉, respectively.

The Eddington accretion limit is set to be 2.6 × 10^{-7} M_☉ yr^{-1} and 3.6 × 10^{-7} M_☉ yr^{-1} for M_{WD,i} = 0.5, 0.7 M_☉, respectively. The excess matter is assumed to be ejected from the vicinity of the accreting WD, and carried away the specific orbital angular momentum of the WD. In our simulation, the irradiation effect of X-ray luminosities of accreting WD is not considered. The calculations will stop when the stellar age is beyond the Hubble time.

Our model assumptions are nearly consistent with Podsiadlowski et al. (2003), including the initial donor star and WD masses (the fact that M_{WD,i} = 1.0 M_☉ can be ignored due to its low weight), the initial chemical composition, the updated opacities, the mixing-length parameter, and the magnetic braking index (γ = 4). The unique difference between these two works is the β factor (the mass loss fraction from the WD vicinity), which was taken to be 1 in Podsiadlowski et al. (2003). However, the numerical calculations indicate that the change of β can hardly influence the orbital evolution of the WD–MS binaries (see also Figure 5, which illustrates the evolutionary tracks in the P_{orb}–M_{d} diagram when β = 1 and 0.7). Because our calculations are based on the evolved donor star model that is the same as Podsiadlowski et al. (2003), it is relatively reliable to employ the simulated birth rate of AM CVns given by Podsiadlowski et al. (2003), see Section 4.

Our calculations show that the descendants of the WD–MS binaries can be detected by LISA when their orbital period P_{orb} ≤ 1.5 hours, which is consistent with a GW frequency of 0.4 mHz. Adopting a mission lifetime T = 4 yr, the characteristic strain of AM CVns can be calculated by Chen (2020)

\[ h_c \approx 3.75 \times 10^{-10} \left( \frac{f_{gw}}{1 \text{ mHz}} \right)^{7/6} \left( \frac{M}{1 M_\odot} \right)^{5/3} \left( \frac{1 \text{ kpc}}{d} \right). \]

where \( f_{gw} = 2/P_{orb} \) is the GW frequency and d is the distance of the AM CVns. The chirp mass can be expressed as (Breivik et al. 2018; Tauris 2018)

\[ M = \frac{(M_{WD}M_d)^{3/5}}{(M_{WD} + M_d)^{7/5}}. \]

During the evolution, the corresponding AM CVns are thought to be LISA sources once the calculated characteristic strain is larger than the LISA sensitivity.

3. Simulated Results

Figure 1 shows the evolutionary results of the WD–MS binaries in the initial orbital period versus the donor star mass diagrams. The solid curves represent the bifurcation period (Tutukov et al. 1985; Pylyser & Rappaport 1988, 1989). The bifurcation period is defined as a critical initial orbital period of a binary, above or below which the binary will evolve toward a diverging or converging system (van der Sluys et al. 2005a, 2005b). The origin of the bifurcation period comes from the competition between orbital expansion caused by mass transfer from a less massive donor to a more massive accretor, and orbital shrinking due to AML caused by magnetic braking and gravitational radiation (e.g., see Equation (14) in Liu & Li 2017, for a relationship of the parameters). The evolution of a binary with an initial orbital period less than the bifurcation period will be dominated by the AML timescale, otherwise the evolution is dominated by the nuclear expansion timescale of the donor star (Pylyser & Rappaport 1988). Therefore, the bifurcation period plays a vital role in determining the evolutionary fates of all types of accreting
compact objects (see also Podsiadlowski et al. 2002; Ma & Li 2009; Chen & Podsiadlowski 2016; Jia & Li 2016; Liu & Li 2017). Therefore, above the bifurcation period, the WD–MS binaries would evolve toward wide orbit systems. The dashed curves denote the critical orbital periods, under which the WD–MS binaries would evolve toward AM CVns-like systems including a donor star with a hydrogen mass fraction larger than 0.4. The donor stars in four WD–MS binaries (see also solid triangles) cannot fill the Roche lobes within the Hubble time because of the low donor-star masses and long orbital periods. The solid squares in the two panels represent the WD–MS binaries that cannot evolve into AM CVns or AM CVns-like stars in the Hubble time.

The crosses in Figure 1 represent the WD–MS binaries experiencing an unstable mass transfer. The stability of mass transfer is determined by two exponents of donor-star (or Roche lobe) radius to mass, \( \zeta_{\text{ad}} = \left( \frac{d \ln R_{\text{ad}}}{d \ln M_{\text{d},i}} \right) \) and \( \zeta_{\text{RL}} = \left( \frac{d \ln R_{\text{RL},d}}{d \ln M_{\text{d},i}} \right) \) (Soberman et al. 1997; Tout et al. 1997), where \( \zeta_{\text{ad}} \) is the adiabatic response of the donor star to mass loss, and \( R_{\text{d},i} \) and \( R_{\text{RL},d} \) are the radius of the donor star and its Roche lobe, respectively. If \( \zeta_{\text{RL}} > \zeta_{\text{ad}} \), the radius of the donor star that is obviously larger than its Roche lobe radius cannot respond quickly enough to the orbital angular momentum changes, and the donor star deviates from hydrostatic equilibrium. If the initial donor star mass is equal to 1.6 \( M_\odot \) (\( M_{\text{WD},i} = 0.5 M_\odot \)) or 2.2 \( M_\odot \) (\( M_{\text{WD},i} = 0.7 M_\odot \)), the WD–MS binaries would experience an unstable mass transfer due to a high mass ratio, and it would be impossible to form AM CVns. Furthermore, the unstable mass transfer also occurs if the initial orbital period is too small (the separation of the binary will continuously shrink) or too large (the donor develops a deep convective envelope prior to the mass transfer). As a result, the mass transfer proceeds on a dynamical timescale and becomes unstable (Tauris & van den Heuvel 2006; Paxton et al. 2015). This runaway mass transfer event (the mass transfer rates are approximately in the range of \( 10^{-4} – 10^{-3} M_\odot \text{ yr}^{-1} \)) causes the timestep of the calculation to quickly decrease below the limit setting in the MESA, and the calculation ceases. Subsequently, the systems are thought to enter a common envelope stage. Therefore, the initial donor star masses of WD–MS binaries that can evolve toward the AM CVn-LISA source within a distance of 1 kpc or 0.3 kpc would experience an unstable mass transfer event due to a high binary mass ratio, and it would be impossible to form AM CVns.

Figure 1 shows the final evolution of WD–MS binaries with different initial donor-star masses and initial orbital periods in the characteristic strain versus GW frequency diagrams. Considering the possible differences of the initial WD masses, they are assumed to be \( M_{\text{WD},i} = 0.5 \) and 0.7 \( M_\odot \) in the top and bottom panels, respectively. Even if \( d = 10 \) kpc, all our simulated pre-AM CVns and AM CVns can be visible as LISA sources (see also the gray curves in the top and bottom panels), implying AM CVns are important LISA sources in the Galaxy. Table 1 lists the observed and derived parameters of 11 verification AM CVns (Kupfer et al. 2018). We then obtain the characteristic strain of the 10 AM CVns according to Equation (1). The distances of sources with a low-mass WD, AM CVn and HP Lib are 0.299 and 0.276 kpc, respectively. According to the observed distances of these two sources, we plot the evolutionary tracks with the distance \( d = 0.3 \) kpc by the blue curves (the solid and dashed lines are for \( M_{\text{donor},i} = 1.0 M_\odot, P_{\text{orb},i} = 3.58 \) days and \( M_{\text{donor},i} = 1.2 M_\odot, P_{\text{orb},i} = 3.65 \) days respectively) in the top panel, which can fit sources AM CVn and HP Lib very well. It appears difficult for our model to reproduce the verification source HM Cnc due to a relatively low primary mass (\( \approx 0.55 M_\odot \)) and long distance (5 kpc). The remaining sources with relatively high-mass WD can be reproduced by WD–MS binaries with high initial mass WD in a distance of 1 kpc or 0.3 kpc (the blue short dashed line for \( M_{\text{donor},i} = 2.0 M_\odot, P_{\text{orb},i} = 3.58 \) days).

To compare our simulated results with the observations, we plot the evolutionary tracks of WD–MS binaries with different WD and donor star masses in the orbital period versus the donor star mass plane in Figure 3. The open circles denote the 11 known verification AM CVns sources in Table 1. The curves plotted by the black, red, blue, green, violet, and orange colors

Figure 1. Distribution of WD–MS binaries with different evolutionary fates in the initial orbital period vs. initial donor-star mass diagram. The initial WD masses are \( M_{\text{WD},i} = 0.5 \) and 0.7 \( M_\odot \) in the top and bottom panels, respectively. The solid circles represent the WD–MS binaries that can evolve toward AM CVns and can be detected by LISA (hereafter defined as AM CVn-LISA sources) within a distance of \( d = 1 \) kpc. The crosses denote the systems experiencing unstable mass transfer. The open triangles indicate the binaries that evolve toward systems with long orbital periods. The open squares denote the binaries that can evolve toward AM CVns that are invisible to LISA. Note that the dashed curve denotes the boundary line, under which AM CVns cannot be formed because the core hydrogen mass fraction is still larger than 0.4 (we define them as AM CVn-LISA stars).
correspond to donor star masses $M_{d,i} = 1.0, 1.2, 1.4, 1.6, 1.8,$ and $2.0 \, M_e$, respectively. Different curves with the same color represent the evolutionary tracks with different initial orbital periods and the same donor star mass. Nine sources among 11 verification AM CVn-type sources can be well reproduced by the standard magnetic braking description given by Rappaport et al. (1983), apart from HM Cnc and V407 Vul. Note that the two dashed curves represent the same systems in the top panel in Figure 2, which can successfully fit the observations of two sources AM CVn and HP Lib with low-mass WDs.

To show the evolutionary history of WD–MS binaries, Figure 4 plots their evolution in the mass transfer rate versus the stellar age diagram. The top panel illustrates the evolutionary examples of a WD–MS binary with $M_{WD,i} = 0.5 \, M_\odot$ and $M_{d,i} = 1.2 \, M_\odot$ when the initial orbital period $P_{\text{orb},i} = 3.96$ days, the WD–MS binary will experience three stages including CV, pre-AM CVn (or post-CV), and AM CVn stages (see also Table 2). After 5.174 Gyr of nuclear evolution, the donor star fills its Roche lobe and begins a mass transfer, and the accreting WD appears as a CV. At the

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**Table 1**

| Source     | $d$ (pc) | $P_{\text{orb}}$ (s) | $M_{\text{WD}}$ ($M_\odot$) | $M_d$ ($M_\odot$) | Refs. |
|------------|----------|-----------------------|------------------------------|-------------------|-------|
| HM Cnc     | 5000     | 321.53                | 0.55                         | 0.27              | 1,2   |
| V407 Vul   | 1786     | 569.39                | 0.8                          | 0.177             | 3     |
| ES Cet     | 1584     | 620.21                | 0.8                          | 0.161             | 4     |
| SDSS J1351-0643 | 1317 | 943.84                | 0.8                          | 0.100             | 5     |
| AM CVn     | 299      | 1028.73               | 0.68                         | 0.125             | 6.7   |
| SDSS J1908+3940 | 1044 | 1085.73               | 0.8                          | 0.085             | 8.9   |
| HP Lib     | 276      | 1102.70               | 0.64                         | 0.068             | 10.11 |
| PTFJ J1919+4815 | 1338 | 1347.35               | 0.8                          | 0.066             | 12    |
| CXOJ1B J1751-2940 | 971  | 1375.0                | 0.8                          | 0.064             | 13    |
| CR Boo     | 337      | 1471.3                | 0.88                         | 0.066             | 11.14 |
| V803 Cen   | 347      | 1596.4                | 0.98                         | 0.084             | 11.15 |

**References.** [1] Strohmayer (2005), [2] Roelofs et al. (2010), [3] Ramsay et al. (2002), [4] Espaillat et al. (2005), [5] Green et al. (2018a), [6] Skillman et al. (1999), [7] Roelofs et al. (2006), [8] Fontaine et al. (2011), [9] Kupfer et al. (2015), [10] Patterson (2002), [11] Roelofs et al. (2007a), [12] Levitan et al. (2014), [13] Wevers et al. (2016), [14] Provencal et al. (1997), [15] Roelofs et al. (2007b).
stellar age $t = 6.89$ Gyr, the CV evolves into a detached pre-AM CVn with $M_d = 0.163 \, M_\odot$ (the donor star remains a low-mass He core) and $P_{\text{orb}} = 0.287$ day due to the Roche lobe decoupling. Subsequently, a low-mass He WD is first formed after $\sim 2$ Gyr contraction stage, and then starts a cooling phase (Istrate et al. 2014b). With the spiraling due to the AML driven by GW radiation, low-frequency GW signals are emitted from the pre-AM CVns (Tauris 2018). At $t = 10.59$ Gyr, the low-mass He WD fills its Roche lobe, and triggers the second mass transfer stage when $P_{\text{orb}} = 13.98$ minutes. In this stage, the compact WD binary is observed as an AM CVn. Finally, the donor star evolves into a planet-like donor star with a mass of $7.56 \times 10^{-3} \, M_\odot$ (Tauris 2018). Similar to Chen et al. (2020), it is very sensitive to the initial orbital period on whether the WD–MS binaries can evolve into a detached pre-AM CVns. When the initial orbital period is $3.65$ days, the system can still form AM CVns, while it always experiences mass transfer (see also the blue curve in the top panel of Figure 4) without a Roche lobe decoupling stage. If the initial orbital periods are obviously less than the bifurcation periods, the WD–MS binaries would directly evolve into AM CVns without experiencing a detached pre-AM CVns stage (Chen et al. 2020).

For a WD–MS binary with $M_{\text{WD,i}} = 0.7 \, M_\odot$ and $M_d = 1.6 \, M_\odot$, the evolutionary tracks display a similar tendency (see also the bottom panel of Figure 4).

Table 2 lists some main evolutionary parameters of WD–MS binaries that can evolve into detached pre-AM CVns that would appear as LISA sources within a distance $d = 1$ kpc. Although the accreting objects of AM CVns and UCXBs are different, the whole evolutionary process is very similar because of the same donor stars. The He WD masses in the pre-AM CVns lie within a narrow range of $0.155–0.167 \, M_\odot$, which is similar to the results of AM CVns and UCXBs given by Tauris (2018) and Chen et al. (2020). It is closely related to the accretion efficiency and magnetic braking index $\gamma$ on whether a WD–MS binary would evolve toward a detached pre-AM CVn. In UCXBs, there exists a relatively wide range of initial orbital periods for NS-MS binaries that can evolve toward detached pre-UCXBs if a high magnetic braking index $\gamma = 5$ were adopted (Istrate et al. 2014a; Sengar et al. 2017). It seems that the timescale in which AM CVns appear as LISA sources are one order of magnitude higher than those for UCXBs in Chen et al. (2020). However, the timescales in which UCXBs appear as LISA sources are based on a detection distance of 15 kpc.

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4. Discussion

Based on BPS simulations with a grid of 120 binary evolution sequences performed by detailed stellar evolution models, Podsiadlowski et al. (2003) obtained Galactic birth rates of AM CVns (including AM CVn-like stars) evolving from the evolved donor star channel as $R_{\text{AM}} = (0.5–1.3) \times 10^{-3} \, \text{yr}^{-1}$, which depends on the efficiency of magnetic braking and the common envelope efficiency parameters (see also their Table 2). In our parameter space, all AM CVns (see also solid circles and open squares in Figure 1) have probabilities $P \approx 40\%$ $(M_{\text{WD,i}} = 0.5 \, M_\odot)$ and $P \approx 10\%$ $(M_{\text{WD,i}} = 0.7 \, M_\odot)$ to evolve into AM CVn-LISA sources (see also solid circles in Figure 1) within a distance of 1 kpc.

If the AM CVns satisfy a uniform distribution in the Galactic disk, the birth rate of AM CVn-LISA sources within a distance between 0 kpc to 1 kpc can be written as

$$R_{0.1} = R_{\text{AM}} P / 225,$$

here the radius and the scale height of the Galaxy are assumed to be 15 and 1 kpc.

Adopting a mean probability $P = 25\%$ and $R_{\text{AM}} = (0.5–1.3) \times 10^{-3}$ (Podsiadlowski et al. 2003), $R_{0.1} \approx (0.6–1.4) \times 10^{-6} \, \text{yr}^{-1}$. This birth rate is approximately one order of magnitude higher than that of UCXB-LISA sources within a distance of 1 kpc (Chen et al. 2020). Taking a rough timescale that AM CVns appear as LISA sources $\Delta_{\text{LISA}} \approx 560$ Myr, the number of AM CVn-LISA sources within a distance of 1 kpc $N_{0.1} = R_{0.1} / \Delta_{\text{LISA}} = 340–810$. If the space distribution of AM CVns is the same as UCXBs, the birth rate of AM CVn-LISA sources from 0 kpc to 15 kpc $R_{0.15} \approx 6.5 R_{0.1} \approx (0.4–1) \times 10^{-5} \, \text{yr}^{-1}$ (see also Equation (12) in Chen et al. 2020). Therefore, the number of AM CVn-LISA sources evolving from the evolved donor star channel is approximately 2200–5200 in the Galaxy.

Based on population synthesis simulations, Nelemans et al. (2001a) proposed a space density $\sigma = (0.4–1.7) \times 10^{-4} \, \text{pc}^{-3}$ for AM CVns, while this estimation was very uncertain because of dependence on the model parameters. Adopting the large scale SDSS data, Roelofs et al. (2007c) obtained a space density $\sigma = (1–3) \times 10^{-5} \, \text{pc}^{-3}$. Subsequently, Carter et al. (2013a) derived the most reliable estimation of space density $\sigma = (5 \pm 3) \times 10^{-7} \, \text{pc}^{-3}$ from a significantly expanded SDSS sample. Therefore, we can estimate the number of AM CVn-LISA sources in the Galaxy.
CVNs within a distance of 1 kpc to be $N = 2\pi R^2 \sigma = 3140 \pm 1880$. Assuming that the probability of AM CVNs that can be detectable by LISA is similar to our simulation ($P = 25\%$), the total number (including the double WDs, the He star, and the evolved donor star channels) of AM CVn-LISA source within a distance of 1 kpc in the Galaxy is $N_{\text{AM, LISA}} = 790 \pm 470$. Therefore, our estimated number (340–810) of AM CVn-LISA sources within a distance of 1 kpc occupies a fraction of $\sim 1/4$–$2/3$, implying that the evolved donor star channel donates an important contribution on the formation of AM CVn-LISA sources.

Considering two formation routes of AM CVNs, including a double WD channel and a He donor star channel, Nelemans et al. (2004) found that several thousand AM CVNs are potential LISA sources in the Galaxy. Recently, Kremer et al. (2017a) explored the long-term evolution of interacting double WDs involving both direct-impact and disc accretion. By a galactic population synthesis, they predicted that the distribution of double WDs which can be detected by LISA with signal-to-noise ratios $\langle S/N \rangle > 5$ in the Galaxy. Meanwhile, Brown et al. (2020) found that the merger rate of the observed He + CO WD binaries exceeds the birth rate of AM CVNs by a factor of 25, i.e., the majority of He + CO WD binaries would experience unstable mass transfer and merge, and are difficult to evolve toward AM CVNs. Therefore, the maximum number of AM CVn-LISA sources in the Galaxy evolving from the double WDs channel should be several thousand. As a result, the predicted number (2200–5200) by the evolved donor star channel is indeed comparable with the double WDs channel, and provides a considerable contribution on the AM CVn-LISA sources in the Galaxy.

There exist some uncertainties in our estimations for the birth rate and number of AM CVn-LISA sources evolving from the evolved donor star channel. The first uncertainty is the AML mechanism by the magnetic braking, which plays an important role in determining the evolutionary fates of WD–MS binaries. In the CV or pre-AM CVn stage with orbital periods greater than 3 hr, the AML is dominated by the magnetic braking. Compared with the magnetic braking, the influence of the mass loss can be ignored (see also Figure 5). However, the gravitational radiation dominates the orbital evolution of the CVs or pre-AM CVNs with orbital periods less than 2 hr. Therefore, the standard magnetic braking model is successful to account for the period gap (2–3 hr) of CVs (Rappaport et al. 1983). The numerical simulations by the MESA indicate that a low magnetic braking index (e.g., $\gamma = 2, 3$) would induce a relatively small minimum orbital period (see also Figure 5), and cause the initial parameter space in Figure 1 to move downward slightly. As a result, the birth rate and number of AM CVn-LISA sources would alter accordingly. However, it is successful that the standard magnetic braking model with $\gamma = 4$ accounts for the verification AM CVn sources. Therefore, our estimation for the birth rate and number of AM CVn-LISA sources remains fairly reliable.

The second uncertainty is the space density of AM CVNs, which would determine the contribution of the evolved donor star channel on the AM CVn-LISA sources. The earliest space density estimation for the AM CVNs in the Galaxy from the observations is $\sigma = 3 \times 10^{-6} \text{ pc}^{-3}$ (Warner 1995). Using the He emission dominated spectra, six new AM CVNs were discovered in the SDSS spectroscopic database (Anderson et al. 2005, 2008; Roelofs et al. 2005). By calibrating the simulations from BPS, a local space density $\sigma = (1–3) \times 10^{-7} \text{ pc}^{-3}$ was obtained (Roelofs et al. 2007c). However, the reliability of this result was limited due to a small sample size. Using the latest photometric database of SDSS, Carter et al. (2013a) explored 2000 candidates, which should include most AM CVNs ($\sim 50$) in the SDSS, to derive an observed space density of $\sigma = (5 \pm 3) \times 10^{-7} \text{ pc}^{-3}$. The accurate distances are very important in modeling the spatial distribution and space density of the AM CVNs. Using the distances from Gaia Data Release 2 to the known AM CVNs, Ramsay et al. (2018) presented a lower limit on the space density to be $\geq 3 \times 10^{-7} \text{ pc}^{-3}$. Obviously, this space density limit is compatible with that given by Carter et al. (2013a).

### Table 2

Selected Evolutionary Properties for AM CVNs and Their Progenitors for Different Initial Donor Star Masses and Initial Orbital Periods

| $M_{\text{d,i}}$ ($M_\odot$) | $M_{\text{WD,i}}$ ($M_\odot$) | $P_{\text{orb,i}}$ (days) | $t_{\text{RLOF}}$ (Gyr) | $t_{\text{det}}$ (Gyr) | $P_{\text{AM CVn}}$ (minutes) | $P_{\text{LISA}}$ (minutes) | $\delta_{\text{LISA}}$ (mHz) | $\Delta \delta_{\text{LISA}}$ (Myr) |
|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|
| 1.2                     | 0.5                     | 3.96                    | 5.174                   | 6.890                   | 0.287                   | 0.163                   | 10.59                   | 13.98                   | 5.86                   | 0.40                   | 559.5                   |
| 1.4                     | 0.5                     | 4.11                    | 2.796                   | 4.472                   | 0.329                   | 0.166                   | 11.45                   | 10.52                   | 5.38                   | 0.39                   | 572.8                   |
| 1.2                     | 0.7                     | 3.54                    | 5.064                   | 7.606                   | 0.248                   | 0.162                   | 9.08                    | 31.48                   | 6.68                   | 0.37                   | 569.5                   |
| 1.4                     | 0.7                     | 3.64                    | 2.711                   | 5.382                   | 0.326                   | 0.167                   | 10.90                   | 12.05                   | 5.49                   | 0.37                   | 573.9                   |
| 1.6                     | 0.7                     | 3.58                    | 1.561                   | 4.375                   | 0.264                   | 0.155                   | 7.02                    | 14.40                   | 6.34                   | 0.37                   | 589.4                   |
| 1.8                     | 0.7                     | 3.62                    | 1.162                   | 2.083                   | 0.274                   | 0.164                   | 8.44                    | 7.34                    | 5.16                   | 0.36                   | 579.1                   |

Note. Column name list (in order): the initial donor star mass, the initial WD mass, the initial orbital period, the stellar age at the onset of RLOF, the stellar age and the orbital period including the donor star mass when the system becomes detached, the stellar age and the orbital period when the system appears as an AM CVn, the minimum orbital period, the initial GW frequency when the system starts to be detected by LISA within a distance of 1 kpc, and the timescale that the binary appears as a LISA source.
Meanwhile, Ramsay et al. (2018) found that the mass transfer rate in most sources among 15 AM CVns is greater than predicted by standard tracks, implying the majority of donor stars in the AM CVn population are not fully degenerate. This evidence also indicates that the evolved donor star channel cannot be negligible in forming AM CVns population.

5. Summary

In this work, we investigate the formation and evolution of AM CVns produced by the WD–MS evolutionary channel, and diagnose the detectability of these AM CVns as LISA sources.\(^3\)

Our main conclusions are as follows:

1. The initial donor star masses and initial orbital periods of WD–MS binaries that can evolve toward AM CVn-LISA sources within a distance of 1 kpc are in the range of 1.0–1.4 M\(_{\odot}\) and 3.1–4.1 days, and 1.0–2.0 M\(_{\odot}\) and 3.0–3.6 days when the initial WD masses are 0.5 and 0.7 M\(_{\odot}\), respectively. In our investigated parameter space, a fraction of 40% and 10% of AM CVns can evolve toward AM CVn-LISA sources that can be visible within a distance of 1 kpc for \(M_{\text{WD}_{\odot}} = 0.5\) and 0.7 M\(_{\odot}\), respectively.

2. The progenitors of all AM CVn-LISA sources should have an initial orbital period slightly smaller than the bifurcation period. If the initial orbital periods are much smaller than the bifurcation periods, the relevant AM CVns would be invisible to LISA. If the initial orbital periods are approximately equal to the bifurcation periods, the WD–MS binaries would experience three stages including CVs, the detached WD-He WD binaries, and AM CVns. In this case, the AM CVns would emit relatively high frequency GW signals, which can be detected by LISA even if for a long distance of 1 kpc.

3. In the detached pre-AM CVns, the He WD masses are in a narrow range of 0.155–0.167 M\(_{\odot}\), which can be used to constrain the primary WD masses (Tauris 2018).

4. The standard magnetic braking model given by Rappaport et al. (1983) can reproduce the orbital periods and the derived donor star masses of 10 verificiation AM CVns sources.

5. Based on a birth rate of AM CVns given by the BPS simulations (Polsisadlowski et al. 2003), the birth rate of AM CVn-LISA sources evolving from the evolved donor star channel within a distance of 1 kpc is estimated to be \((0.6–1.4) \times 10^{-6}\) yr\(^{-1}\), and the relevant number of AM CVn-LISA sources is about 340–810. Compared with the derived space density by significantly expanded SDSS samples, the evolved donor star channel contributes a considerable fraction of AM CVn-LISA sources.

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\(^3\) Actually, the GW signals emitted by these sources can also be detected by the detectors including Taiji (Ruan et al. 2020) and TianQin (Luo et al. 2016; Huang et al. 2020).
