Decision-Making under Criteria Uncertainty

V M Kureychik and I B Safronenkova

Autonomous federal state institution of higher education «Southern Federal University», Institute of computer technologies and information security, 44, Nekrasovskiy Ave., Taganrog, 347900, Russia

E-mail: safronenkova050788@yandex.ru

Abstract. Uncertainty is an essential part of a decision-making procedure. The paper deals with the problem of decision-making under criteria uncertainty. In this context, decision-making under uncertainty, types and conditions of uncertainty were examined. The decision-making problem under uncertainty was formalized. A modification of the mathematical decision support method under uncertainty via ontologies was proposed. A critical distinction of the developed method is ontology usage as its base elements. The goal of this work is a development of a decision-making method under criteria uncertainty with the use of ontologies in the area of multilayer board designing. This method is oriented to improvement of technical-economic values of the examined domain.

1. Introduction

Modern electronics engineering development depends largely on multilayer board technology progress. The fundamental problem of multilayer board technology is a layer congruence. Different alternatives of the technological complex are used to provide a required layer congruence. A lot of processes are automated intensively and high-precision testing and measuring equipment are implemented. Such factor as production efficiency warrants new approaches and methods in decision-making. One of the main problems is decision-making under uncertainty at the modern microelectronics enterprises. It takes place because of low quality or incompleteness of input data. This problem includes a finite number of decision alternatives, exactly one of which is to be chosen [1, 2].

Traditional uncertainty consideration methods (UCM) have a list of lacks. The first one is a subjective component that is in the structure of the method, for instance, estimation of weighting factors or comparison of partial criteria. It leads to a random final decision selection from a list of optimum alternatives or it needs to involve an expert group. It causes investigation extension. To provide for the best final decision in a certain sense, a mathematical decision support method is to be deprived of subjectiveness. The proposed method uses a well-known methodology that is based on some universal axioms and a consensus technique. Modification involves ontology usage as a knowledge frame within a decision-making procedure. Besides a minimal decision-maker workload, the modified method provides the integration of heterogeneous data and knowledge. Expected results of the proposed method are: 1) technical-and-economic index improvement of decision-making efficiency in domain area; 2) storage, formalization, classification and domain data and knowledge reuse.
2. Problem formalization

A decision-making problem (DMP) is specified by the set:

\[ T(G, X, Y, Q) \] (1)

where \( G \) – a goal objective (assembly of goals) that must be achieved;
\( X \) – a set of alternative situations;
\( Y \) – a set of candidate solutions;
\( Q \) – an estimation criterion (a set of criteria) of alternative situations and candidate solutions [3].

Let us consider conditions of decision making that can be classified as follows:
- determinateness condition: single solution \( y \in Y \) corresponds to each problem \( S_i \);
- conditions of risk: there is a list of all possible outcomes and assigned probabilities \( (p_{ij}) \) of various outcomes:

\[ \sum_{j=1}^{n} p_{ij} = 1 \] (2)

where \( n \) – a number of candidate solutions \( y \in Y \)

Conditions of uncertainty characterized by situations when no solution can be applied or some solutions can be applied, but correspondence probabilities of selection are unknown [4].

As has been mentioned above, uncertainty is an essential part of the procedure of decision-making. Uncertainties are classified as follows: criteria uncertainty, input data uncertainty and model uncertainty. One shall elaborate each of these types. Criteria uncertainty takes place when a candidate solution \( y \in Y \) efficiency cannot be described by one value completely and it requires to specify a value set. Real-valued functions \( f_1, f_2, ..., f_n \) compose a vector criterion:

\[ f = (f_1, f_2, ..., f_n) \] (3)

For every alternative \( x \in X \), n-dimensional vector \( f(x) = (f_1(x), f_2(x), ..., f_n(x)) \in \mathbb{R}^n \) is an image of \( x \), where \( \mathbb{R}^n \) is the n-dimensional real vector space. This space is called a criterion space. An image of vector function \( f \) is denoted by \( Y = f(X) = \{ y \in \mathbb{R}^n \mid y = f(x) \text{ for some } x \in X \} \).

The second type of uncertainty that is connected with incompleteness of knowledge about the problem is an input data uncertainty. Data absence, data corruption or data ambiguity may take place. For instance, a range of values can be given – set \( X \) that is due to data vector \( x \). This set is an uncertainty set. Taking into account criteria and input data uncertainty, candidate solution efficiency is measured by a vector criterion:

\[ f(x, y) = (f_1(x, y), f_2(x, y), ..., f_n(x, y)) \] (4)

A model uncertainty is the third type of uncertainty. It means that a rule of the cost function evaluation is rough. This type of uncertainty can be traced to the input data uncertainty with the help of correction factors.

Let \( X \) be a set of uncertainties, then cost function \( f(x, y) \) is defined by the product of sets \( X \times Y \). A decision-making problem under uncertainty can be governed by triad without loss of generality:

\[ \langle X, Y, f(x, y) \rangle \] (5)

where \( X \) — a set of uncertainties;
\( Y \) — a set of candidate solutions,
\( f(x, y): X \times Y \rightarrow R \) — a cost function that can also be called a generalized loss function. Under this work, the loss is an effectiveness measure that should be minimized [5-7].

3. Uncertainty consideration method analysis

To analyse the traditional uncertainty consideration methods (UCM), let us introduce the following definitions. UCM is a rule that puts in correspondence function \( f(x, y) \) over \( X \), value \( F(X, y) \) - for fixed \( y \) \( \in Y \). If criterion \( f(x, y) \) is called a generalized loss function, then \( F(X, y) \) is called an uncertainty generalized loss function (U-generalized loss function). In other words, it is a generalized loss function with regard to uncertainty. Function \( f(x, y) \) is normalized [8].
The paper introduces that U-generalized loss function is calculated by the formulas:

\[ F(X, y) = G^{-1}\left(\frac{1}{N} \sum_{i=1}^{N} G(f(x_i, y))\right), \]

\[ F(X, y) = G^{-1}\left(\frac{1}{S_x(x \in X)} \int_{X} G(f(x, y))dx\right), \]

where \( X \) – a finite-dimensional Cartesian space, \( X = \{x_i\}, i=1,...,N \);
\( S_x \) – an X region measure;
\( G(t) \) – a generating function for each UCM. The generating function is an analogue and strictly monotone function; besides, \( G(0) = 0, G(1) = 1 \)

So, a set of UCM \( S \) is a class of analogue and strictly monotone functions \( G(t) \) in a traditional method of decision-making. Set \( S \) contains the infinite number of elements. So this set is needed to be substituted with finite set \( T \) in a most complete way.

Let us briefly outline the main optimality principles of the decision-making process under uncertainty. Pareto optimal strategy does not provide a complete decision-making problem solution because a decision-maker has to choose the best solution from a Pareto set.

Pascal principle provides the good results in the problems with a summation effect. Here, the effects of some factors can be suppressed or reinforced by others. In case if a partial criterion vanishes, the contribution of the other criteria is completely eliminated. In some areas, Pascal principle may be applied. For instance, when talking about reliability or safe criteria in aerospace manufacturing industry. In such domain, no other criterion improvement is able to make compensation for the above listed criteria.

A decision-maker has to choose weighting factors for partial criteria in linear convolution according to Laplace principle.

As is evident from the foregoing, the optimality principles usage for the decision-making procedure does not excuse from uncertainty. More importantly, a decision-maker has to carry out informal work such as weighting factors determination and selection of satisfactory principle for a concrete problem. Traditional algorithms (MAUT – Multi-Attribute Utility Theory, ELECTRE – Elimination Et Choix Traduisant la Realite, AHP – Analytic Hierarchy Process) for the decision making procedure have a variety of limitations in uncertainty. Among these, there is inconvenience, procedure artificiality and loss of use for criteria multiplicity [10-13].

4. Ontology-based approach for decision-making procedure

According to [14], a principle of decision-making under the uncertainty procedure is as follows:
1. describe an obtainable set of uncertainty consideration methods (UCM);
2. select the most representative UCM that will be used for decision-making;
3. form a set an iterative procedure for solution total evaluator under uncertainty on the basis of the UCM.

According to this approach, a decision-maker does not have to make a decision. Without limiting the foregoing, it gives an opportunity to present the decision-maker’s preferences by referring the partial criteria to different significance groups.

Let us examine the following example to demonstrate clearly a problem complexity of the multicriterion decision-making procedure under uncertainty in the microelectronics domain.

Set up of the problem: develop a decision-making method under criteria uncertainty with the help of ontologies in the context of solving an optimization problem of interconnection thickening in multilayer printed boards.

The main components of the decision-making procedure are:
- a set of candidate solutions \( Y \);
- a set of uncertainty \( X \);
- a set of uncertainty consideration methods \( S \) described by the generating functions: \( G(t) \): \( S = \{G(t), G(0)=0, G(1)=1\} \);
– generalized loss function $f(x,y)$, where $0 \leq f(x,y) \leq 1$.

Then, a decision-making method can be described by the following set:

$$(X,Y,S,f(x,y)),$$ \hspace{1cm} (8)

Let us propose modifying the decision-making method via ontology inclusion as a key element. If a set of elements is considered as some data, then set (10) can be represented in terms of ontology. Such approach solves a problem of data integration. A model of data representation plays a key role in this case [15]. In the context of artificial intelligence, the paper defines ontology as an "explicit specification of a conceptualization," which is, in turn, "the objects, concepts, and other entities that are presumed to exist in some area of interest and the relationships that are preserved among them" [16]. Ontology presents the domain formally and links a variety of such descriptions with each other [17]. In terms of ontology, set (10) is as follows (Fig.1).

**Figure 1.** “Meta-ontology” of the decision-making method

A class “DMUT” (Decision-making Uncertainty Technique) has four subclasses: “Candidate_solution”, “Uncertainty”, “UCM” and “Cost_function”. Subclasses of proposed ontology are the sets from formula (10). Developed ontology handles common concepts that do not depend on the domain area. Let us denote this ontology as “meta-ontology”. Meta-ontology is general and may be used for the decision-making problem under uncertainty over any domain area.

Now let us turn attention to the domain area of “multilayer board designing”. According to the authors’ task, it is necessary to develop a domain ontology. Formally ontology is a non-empty set of classes $C_i$: $C_i = \{c_{i1}, c_{i2}, ..., c_{in}\}$, where $C_i = \{N_i, D_i, C_i\}$, $i = 1, n$.

- $N_i$ – a class name;
- $D_i$ – a class volume;
- $C_i$ – a class content or class characteristic (a set of attributes with different names) [18-20].

The domain ontology is a part of the “meta-ontology” and it consists of concepts, relations and axioms related to the domain area.

The first step is to define a set of candidate solutions – $Y$. The state of the art of multilayer board manufacturing technology has shown three main directions that provide the interconnection density increase in multilayer boards:

- connection point size decrease;
- semiconductor width decrease;
- layer increase;
- multilayer wiring usage;
- 3D-structure usage.

In terms of ontology, a set of candidate solutions is as follows (Fig. 2). The subclass “Candidate_solution” has a set of individuals that describes decision alternatives.
The second step is to define a set of criteria uncertainty – X:
– wiring path increase;
– path length decrease;
– preproduction low cost;
– production low cost;
– output reliability.

In terms of ontology, a set of interconnection of density increase criteria uncertainty is as follows (Fig. 3). A class “Uncertainty” has three subclasses: “Criteria”, “Model” and “Input_data”. Individuals for subclass “Criteria” correspond to a set of criteria uncertainty- X that was mentioned above [21].

As was mentioned above, the UCM is unknown for a decision-maker. It is therefore necessary for UCM infinite set S to be represented by an optimum finite typical set that will represent the whole set sufficiently. Table 1 gives one of potential sets that contains 7 elements.
Table 1. UCM set.

| Name       | Generating function               |
|------------|-----------------------------------|
| The best   | \( t^k, k \to -\infty \)         |
| The worst  | \( t^k, k \to \infty \)          |
| Middle     | \( t \)                           |
| Optimistic | \( \sqrt{t} \)                   |
| Careful    | \( t^4 \)                         |
| Relay-type | \( 0.5(1 + \frac{1}{2\sqrt{t-1}}) \) |
| Leveling   | \( 0.5(1 + (2t-1)^3) \)           |

The next step of the decision-making procedure is a forming a solution total evaluator in a context of uncertainty. This appears as correlation of expert evaluations. Each UCM from the typical set is a mathematical model of some expert \( S_i \). This expert uses formulas (8), (9), considers uncertainty \( x \in X \) and makes its own evaluation \( F_{m}(y) \) for each solution \( y \in Y \) [14].

In terms of ontology, each UCM is an individual of subclass “UCM” (Fig. 4).

According to the application task, the generalized loss function is a performance function for the optimization problem of increased interconnection density in the multilayer board.

In this paper, “meta-ontology” and domain ontology were developed. These ontologies perform the functions of storage, classification and formalization of heterogeneous data. Ontologies as frames of knowledge are intended to be used for intelligent decision support systems.

5. Conclusion
A decision-making problem under uncertainty was observed in this work. The problem formalization was given. Uncertainty consideration methods were presented and examined. A methodological foundation for a new method of decision-making under uncertainty is a method based on some universal axioms and the consensus technique. Method modification involves ontology usage as a knowledge frame that is a core of information support in intelligent decision support systems. For this purpose, “metaontology” and domain ontology were developed. “Metaontology” is a universal for a
decision-making problem under uncertainty. And the second one performs the functions of storage, classification and formalization of heterogeneous data of the domain area. As an illustration of problem complexity of the multicriterion decision-making procedure under uncertainty, the domain “multilayer board designing” was examined. The analysis of alternatives and criteria of the performance function was held. The ontology-based approach for decision-making procedure is intended to be an upcoming trend for intelligent decision-support systems development. It increases the technique-economic index in the decision-making procedure and provides highly competitive enterprises.

6. Acknowledgments
The reported study was funded by the Russian Foundation for Basic Research No. 18-07-00050, project No. 25537.2017-6.7.

References
[1] Lahin O I 2017 Principles of building the knowledge management system for rocket and space enterprises Ontology of designing 3(25) 270–283
[2] Trahtengerc Je A 1998 Computer support of decision making in CAD (Moscow: Avtomatizacija proektirovaniya)
[3] Rozen V V, Bessonov L V 2008 Decision-making mathematical models in economy (Saratov: UC “Novye tehnologii v obrazovanii”)
[4] Tihonov A N, Cvetkov V Ja 2001 Methods and Systems of Decision Making (Moscow: MAKS Press)
[5] Bashlykov A A 2017 Principle of design of intelligence decision-making systems (Moscow: INFRA-Mb)
[6] Shirjaev V I, Shirjaev E V 2016 Decision making: Mathematical foundation. Static problems (Moscow: Knizhnyj dom «LIBROKOM»)
[7] Karpenko A P 2014 Modern algorithms of search optimization. Bioinspired algorithms (Moscow: Izdatel'stvo MGU im. N.Je. Baumana)
[8] Pijavskij S A 2004 Optimization and decision-making methods (Samara: SGASU)
[9] Kononjuk A E 2011 General theory of consulting (Kiev: Osvita Ukrainy)
[10] Malyshiev V V, Pijavskij B S and Pijavskij S A 2010 A decision-making method under conditions of diversity of means of reducing uncertainty Journal of Computer and Systems Sciences International 1 44–58
[11] Sarin R K 2001 Multi-attribute utility theory (USA: Springer)
[12] Figueira J R, Greco S, Roy B and Słowiński R 2010 Electre Methods: Main Features and Recent Developments Handbook of Multicriteria Analysis. Applied Optimization 103 51-89
[13] Broekhuizen H, Groothuis-Oudshoorn C G M and van Til J A 2015 A Review and Classification of Approaches for Dealing with Uncertainty in Multi-Criteria Decision Analysis for Healthcare Decisions Pharmacoeconomics 33 445–455
[14] Djellali C 2013 A New Data Mining System for Ontology Learning Using Dynamic Time Warping Alignment as a Case The 4th International conference on emerging ubiquitous systems and pervasive networks (EUSPN-2013 vol.21) ed Elhadi M. Shakshuk (Canada: Elsevier B.V) pp 75-82
[15] Chistjakova I S 2014 Role of ontologies in semantic data integration in the Web Software design 17 57-60
[16] Noy N, McGuinness D 2001 Ontology development 101: a guide to creating your first ontology Retrieved from: https://protege.stanford.edu/publications/ontology_development/ontology101.pdf
[17] Lebedev S V 2017 Ontology-driven approach to medical data fusion Ontology of designing 2(24) 145–159
[18] Gavrilova T A, Kudryavtsev D V, Muromtsev D I 2016 Knowledge Engineering: Models and Methods (St. Petersburg: Lan’)
[19] Kureichik V, Safronenkova I 2017 Integrated algorithm of the domain ontology development. Artificial Intelligence Trends in Intelligent Systems (AISC vol. 573) ed Silhavy R, Senkerik R, Kominkova O Z, Prokopova Z, Silhavy P (Cham: Springer)
[20] Antoniou G, van Harmelen F, Hoekstra R 2016 Semantic Web (Moscow: DMK Press)
[21] Mylov G V, Medvedev A M, Semenov P V and Konstantinov P N 2016 Scientific bases of printed board designing (Moscow: Telekom)
[22] Pijavskij S A 2014 A simple and universal method of decision making within the scope of criteria of «cost and efficiency» Ontology of designing 13 89–102