The molecular nature of the $X_0(2900)$ and its heavy quark spin partners

Mei-Wei Hu,† Xue-Yi Lao,† Pan Ling, and Qian Wang

1Guangdong Provincial Key Laboratory of Nuclear Science, Institute of Quantum Matter, South China Normal University, Guangzhou 510006, China
2Theoretical Physics Center for Science Facilities, Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China

The $X_0(2900)$ observed by the LHCb Collaboration recently in the $D^−K^+$ invariant mass of the $B^+ \to D^+ D^− K^+$ process is the first exotic candidate with four different flavors, which opens a new era for the hadron community, especially for revealing the mystery of the $D_{s2}(2460)$ and the $D_{s1}(2460)$. We have demonstrated that the $X_0(2900)$ can be well accepted as a $I(J^P) = 0(1^+)$ $D^∗K^∗$ hadronic molecule. Its observation could also give the whole heavy-quark symmetry multiplet formed by the $(D, D^*)$ doublet and the $K^+$ meson. Besides the $X_0(2900)$, there would be two additional $I(J^P) = 0(1^+)$ hadronic molecules associated with the $D^*K^*$ and $D^*K^*$ channels as well as one additional $I(J^P) = 0(2^+)$ $D^*K^*$ molecule. In the light quark limit, they are 36.66 MeV and 34.22 MeV below the $D^*K^*$ and $D^*K^*$ thresholds, respectively, which are unambiguously fixed by the mass position of the $X_0(2900)$.

Introduction. The conventional quark model [1][2], which inherits part of the properties of Quantum Chromo-Dynamics (QCD), has made a great success to understand hadrons before 2003. Quark model tells us that hadrons can be classified as either mesons made of $q\bar{q}$ or baryons made of three quarks. However, QCD tells us that any color neutral configuration (named exotics) could exist upon the two configurations mentioned above. That leaves us two questions: where to find these exotic candidates and how to understand the underlying mechanism. The observation of the first exotic candidate $X(3872)$ [3] in 2003 and the succeed tremendous experimental measurements [4][5] partly answer the first question. Among these, the observation of the first pentaquarks [6][7], the first fully heavy four quark states [8] and the first exotic candidates with four different flavors i.e. the $X_0(2900)$ [9] reported by the LHCb Collaboration recently, set milestones from experimental side. Different prescriptions from theoretical side are put forward for understanding the nature of these exotic candidates [10][11][12]. Most of the observed exotic candidates are with few MeV below nearby $S$-wave thresholds, which indicates that they could be hadronic molecules [10] as analogies of deuteron formed by proton and neutron.

However, one have to confront one problem that different configurations with the same quantum number can mix with each other and cannot be well isolated. For instance, although the $X(3872)$ is proposed as a hadronic molecule at the very beginning [13] due to its closeness to the $DD^* + c.c.$ threshold, it still could be or mix with the normal charmonium $\chi_{c1}(2P)$ [19][20]. Another typical example is the $D_{s0}(2317)$ and the $D_{s1}(2460)$ which are about 160 MeV and 70 MeV below the $J^P = 0^+$ and $J^P = 1^+$ $c\bar{s}$ charmed-strange mesons of Godfrey-Isgur quark model [27]. Meanwhile, they are about 45 MeV below the $DK$ and $D^*K^*$ thresholds, respectively, which can be explained naturally if the systems are bound states of the $DK$ and $D^*K^*$ meson pairs [28][29], respectively.

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However, because the light quark and anti-quark in the isosinglet $D^{(*)}K$ system are of the same flavor, despite of those comprehensive studies, one still cannot avoid the possibility of the mixture with the normal $c\bar{s}$ configurations [29][30][39][49]. Fortunately, the LHCb Collaboration reported a $J^P = 0^+$ narrow state $X_0(2300)$ with mass 2866±7 MeV and width $\Gamma_0 = 57\pm13$ MeV as well as another broader $J^P = 1^−$ state with mass 2904±7 MeV and width $\Gamma_1 = 110\pm12$ MeV in the $DK$ invariant mass distribution. They are the first exotic states with four different flavors, which brings us a potential ultimate solution for the problem from the following aspects:

- Both the meson-exchanged potential and the multi-gluon exchanged potential, which is flavor-blind, contribute to the $D^{(*)}K^{(*)}$ scattering. On the contrary, Only the multi-gluon exchanged potential play a role in the $D^{(*)}K^{(*)}$ scattering in the $U(3)$ flavor symmetry [7]. The comparison of the scattering of the two systems could help to distinguish the roles of these two parts.

- Whether the short-ranged multi-gluon exchanged potential of the $D^{(*)}K^{(*)}$ system could form molecular state, in contrary with the compact tetraquark interpretation of the $X_0(2900)$ in Ref. [12], or not is still a question. The observation of the narrow $X_0(2900)$ could be a good place to check that.

In this letter, we solve the Lippmann-Schwinger Equation (LSE) with contact leading order potentials of the $D^{(*)}K$ system, in the heavy quark limit, to extract the mass position of the spin partners of the $X_0(2900)$. The

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1 In $U(3)$ flavor symmetry, the masses of $\pi, \eta$ and $\eta'$ are the same. As the result, the contributions from one-pion-exchanged, one-$\eta$-exchanged, and one-$\eta'$-exchanged diagrams are cancelled exactly. That is similar to the argument of Appendix C of Ref. [41].
As a $I(J^P) = 0(0^+)$ $\bar{D}^*K^*$ hadronic molecule, which has been indicated by the observation of the $D_{s0}(2317)$ and the $D_{s1}(2460)$. Its observation in the $\bar{D}K$ channel would be because the undetectable photons. Searching for the spin partners of the $X_0(2900)$ could help to understand the nature of both itself and the $D_{s0}(2317)/D_{s1}(2460)$. Here, $i_1 = \frac{1}{2}$, $j_2 = \frac{1}{2}$ and $j_{12} = 0, 1$ are spins of anti-charm quark $\bar{c}$, light quark $q$ and the sum of them in the $\bar{D}^{(*)}$ meson. $j_3 = 0, 1$ and $J = 0, 1, 2$ are the spins of the $K^{(*)}$ meson and the total spin of the $\bar{D}^{(*)}K^{(*)}$ system. $s_l$ on the right hand side of Eq. 1 is the light degrees of freedom of the system, which is the only relevant quantity for the dynamics in the heavy quark limit. Following Eq. 1 one can obtain the decompositions as

\[
|\bar{D}K)^{0+} = \frac{1}{2}\rangle
\]

\[
|\bar{D}K)^{1+} = \frac{1}{2}\rangle
\]

\[
|\bar{D}K^{(*)}_{1+} = \frac{1}{\sqrt{3}}|\frac{1}{2}\rangle^* + \sqrt{\frac{2}{3}}|\frac{3}{2}\rangle^*
\]

\[
|\bar{D}K^{(*)}_{0+} = -\frac{1}{2}\rangle^*
\]

\[
|\bar{D}K^{(*)}_{1+} = \sqrt{\frac{2}{3}}|\frac{1}{2}\rangle^* - \frac{1}{\sqrt{3}}|\frac{3}{2}\rangle^*
\]

\[
|\bar{D}K^{(*)}_{2+} = |\frac{3}{2}\rangle^*.
\]

Here the heavy degree of freedom is suppressed due to the same value, leaving only the light degrees of freedom $s_l$ in $|\ldots\rangle$. Although the $K$ and $K^*$ have the same quark content, the light degrees of freedoms in the first two equations and those in the last four equations can be distinguished due to the large mass difference between the $K$ and $K^*$ mesons. Analogous to those in Refs. [50], by defining the contact potential

\[
C_{2l}^{(*)} \equiv \langle l|\hat{H}_{\text{HQS}}|t\rangle^{(*)},
\]

The potential of the $\bar{D}^{(*)}K^{(*)}$ systems with different quantum numbers are

\[
V_{0+} = C_1
\]

\[
V_{1+} = C_1
\]

Framework. The heavy quark spin structure could reexpress of the hadron basis by the heavy-light basis. One could find an example for the $Z_{s0}^{(*)}$ and $Z_{s0}^{(*)}$ case with two heavy quarks in Refs. [41 44 48]. Along the same line, the $S$-wave $D^{(*)}K^{(*)}$ system with only one heavy quark can be written in terms of the heavy degree of freedom $\frac{1}{2}$ and light degree of freedom $s_l$ as the following [49]

\[
\bar{D}K^{(*)}_{12} = C_1
\]

\[
\bar{D}K^{(*)}_{13} = \left(\frac{1}{3}C_1 + \frac{2}{3}C_3\right)
\]

\[
\bar{D}K^{(*)}_{22} = C_3.
\]

$V_{j+}$ and $V_{j+}^{(*)}$ are for the potentials of the $\bar{D}^{(*)}K$ and $\bar{D}^{(*)}K^*$ systems, respectively. The above decomposition and the corresponding potentials also works for the $D^{(*)}K^{(*)}$ systems, but with different values of $C_{2l}^{(*)}$. As the $D^{(*)}K^{(*)}$ system has different flavor quarks, the potential cannot be any meson exchanged potentials, but the flavor-blind multi-gluon exchanged contact potential. That is different from the $D^{(*)}K^{(*)}$ system which is associated with the property of the $D_{s0}(2317)$ and the $D_{s1}(2460)$. However, the $D_{s0}(2317)$ and the $D_{s1}(2460)$ could set a guidance that how large the $C_{1}^{(*)}$ would be.

With the above potentials, one can solve LSE

\[
T = V + VGT
\]

for each quantum number. Here two-body non-relativistic propagator is

\[
G_{\Lambda}(M) = \int \frac{d^3q}{(2\pi)^3} \frac{1}{M - m_1 - m_2 - q^2/(2\mu)}
\]

\[
\Lambda + i\frac{m_1 m_2}{2\pi(m_1 + m_2)} \sqrt{2\mu(M - m_1 - m_2)}
\]

with power divergence subtraction [51] to regularize the ultraviolet (UV) divergence. The value of $\Lambda$ should be smaller enough to preserve heavy quark symmetry, leaving the physics insensitive to the details of short-distance dynamics [50]. Here $m_1$, $m_2$ and $\mu$ are the masses of the intermediated two particles and their reduced mass, respectively. $M$ is the total energy of the system.

**Results and Discussions.** We focus on the discussion of the formation of the $\bar{D}^{(*)}K^{(*)}$ molecule instead of their
isospin breaking effect. As the result, the isospin average masses

\[ m_D = 1.867 \text{ GeV}, \quad m_{D^*} = 2.009 \text{ GeV} \quad (15) \]
\[ m_K = 0.496 \text{ GeV}, \quad m_{K^*} = 0.892 \text{ GeV} \quad (16) \]

are implemented in this letter. Due to the four-different-quark property, its leading order potential can only stem from the multi-gluon exchanged potential, denoted as \( \mathcal{V}_{\text{flavons}} \), which is flavor-blind and also the same for the \( D^{(*)}K^{(*)} \) system. For the \( D^{(*)}K^{(*)} \) system, the One-Meson-Exchanged potential (either t-channel or s-channel) also contributes, denoted as \( \mathcal{V}_{\text{OME}} \). The fact that the \( D_{s0}(2317) \) and \( D_{s1}(2460) \) can be accepted as the \( DK \) and \( D^* K^* \) molecules \[28,35\], respectively, can tell us how large the leading order contact potentials would be. As shown in Fig. 1 the \( X_0(2900) \) could be accepted as the isosinglet \( J^P = 0^+ \) \( D^* K^* \) molecule, which agrees with the result of vector-vector interaction based on hidden gauge symmetry \[52\]. The green dot-dashed and blue dashed curves are almost coincide with each other due to the fact that the \( D_{s0}(2317) \) and \( D_{s1}(2460) \) are with the same binding energy and the same potential \[28,35\]. Their deviation to the red solid curve would be balanced by the \( \mathcal{V}_{\text{OME}} \) potential which does not appear for the \( X_0(2900) \). As the \( \mathcal{V}_{\text{OME}} \) potential is considered as the next-leading order contribution, whose role would not be expected to exceed 50%. To estimate the contribution of the neglecting the role of the \( \mathcal{V}_{\text{OME}} \), we vary \( C_1 \) by 50% for the \( X_0(2900) \), as shown by brown long dashed and pink dotted curves, which are not far away from the red solid one. Using the same value of \( C_1 \) as the leading order contribution, we estimate the mass positions of the heavy quark spin partners of the \( X_0(2900) \).

The leading order, i.e. \( C_1^* = C_1 \), the \( V_{0^+}^* \), \( V_{0^+} \), \( V_{1^+} \) potentials are determined by the same parameter \( C_1 \). Once the \( V_{0^+}^* \) is determined by the \( X_0(2900) \), one can obtain the other mass positions \( m_{0^+} = 2.229 \text{ GeV}, \quad m_{0^+} = 2.376 \text{ GeV} \), which relate to the \( \bar{D}K \) and \( D^* K \) channels, respectively. They are in one hundred MeV below the corresponding thresholds, which are beyond the scope of molecular picture. They could exist only when there is a big difference between the values of the \( C_1^* \) and \( C_1 \).

The states in other channels are also relevant with the parameter \( C_1^* \). To get a guidance, how the poles move with the variation of the parameter \( C_1^* \), we take the two parameter sets \( \Lambda = 0.05 \text{ GeV}, \quad C_1^* = 33.56 \text{ GeV}^{-2} \) and \( \Lambda = 0.03 \text{ GeV}, \quad C_1^* = 102.09 \text{ GeV}^{-2} \) as an illustration, see Fig. 2. The blue triangle and green square curves show the pole trajectory of the bound state and resonance in the \( 1^+ \) channel. One can see that, with \( C_1^* \) variation of the same order of the \( C_1^* \), one bound state and one resonance emerge with tens of MeV below the \( \bar{D}K^* \) and \( D^* K^* \) thresholds, respectively. The variation of the bound state in the \( 2^+ \) channel is large. If one would expect that light quark spin symmetry also works here as that for the two \( Z_8 \) states \[53\], i.e. \( C_1^* = C_1 \), we find the pole position of the above three states are

\[ m_{1^+} = 2.666 \text{ GeV}, \quad m_{1^+} = 2.722 \text{ GeV}, \quad m_{1^+} = 2.866 \text{ GeV} \quad (17) \]

The vanishing imaginary part of the higher \( 1^+ \) state is because of the degenerate of the two \( 1^+ \) states. Searching for these heavy quark spin partners would help to reveal the nature of the \( X_0(2900) \) and deepen our understanding of the \( D_{s0}(2317) \) and \( D_{s1}(2460) \).

Summary and Outlook. The existence of the \( D_{s0}(2317) \) and the \( D_{s1}(2460) \) indicates analogies in the \( \bar{D}K^* \) channel. One of them has been reported by the LHCb Collaboration recently, i.e. the \( X_0(2900) \) as a \( I(J^P) = 0(0^+) \) \( \bar{D}K^* \) hadronic molecule. In the hadronic picture, we extract the mass positions of its heavy quark spin partners, i.e. \( 2.722 \text{ GeV} \) and \( 2.866 \text{ GeV} \) for \( 1^+ \) state, and \( 2.866 \text{ GeV} \) for \( 2^+ \) state, with respecting light quark spin symmetry. Searching for those states would help to shed light on the nature of the \( X_0(2900) \). Furthermore, a comparison of the exotic states in the \( \bar{D}K^* \) channels with four different flavors with those in the \( D^{(*)}K^{(*)} \) channels provide a potential way to isolate the role of the multi-gluon exchanged potential, which still needs to be quantified in further works. During the update of this manuscript, several works \[54,62\] appear to discuss the relevant topics.

The discussions with Tim Burns, M.L. Du, Li-Sheng Geng, Ming-Zhu Liu, Eulogio Oset, Jun-Jun Xie are appreciated. This work is supported in part by the National Natural Science Foundation of China (NSFC) and the Deutsche Forschungsgemeinschaft (DFG) through the funds provided to the Sino-German Collaborative Research Center “Symmetries and the Emergence of
FIG. 2. The pole trajectories of the $1^+$ and $2^+$ channels in terms of the parameter $C_3^*$ with $\Lambda = 0.05$ GeV, $C_3^* = 33.56$ GeV$^{-2}$ (left panel) and $\Lambda = 0.03$ GeV, $C_3^* = 102.09$ GeV$^{-2}$ (right panel) are shown in the figures. The blue triangles and green squares are bound states and resonances for the $1^+$ channels, associated with the $D^*K^*$ and $D^{*}K^*$ channels. The red circles are bound states related to the $D^*K^*$ channel. The black arrows indicate the direction of the increasing $C_3^*$. The black stars are the mass positions in the light quark spin symmetry.

Structure in QCD$^*$ (NSFC Grant No. 11621131001 and DFG Grant No. TRR110), the research startup funding at SCNU, Guangdong Provincial with No.2019QN01X172, and and Science and Technology Program of Guangzhou (No. 2019050001). MWH and XYL are also supported by Entrepreneurship competition for College Students of SCNU.

* qianwang@m.scnu.edu.cn
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