Adaptive network approach for emergence of societal bubbles

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Abstract
Far beyond its relevance for commercial and political marketings, opinion formation and decision making processes are central for representative democracy, government functioning, and state organization. In the present report, a stochastic agent-based model is investigated. The model assumes that bounded confidence and homophily mechanisms drive both opinion dynamics and social network evolution through either rewiring or breakage of social contacts. In addition to the classical transition from global consensus to opinion polarization, our main findings are (i) a cascade of fragmentation of the social network into echo chambers (modules) holding distinct opinions and rupture of the bridges interconnecting these modules as the tolerance for opinion differences increases. There are multiple surviving opinions associated to these modules within which consensus is formed; and (ii) the adaptive social network exhibits a hysteresis-like behavior characterized by irreversible changes in its topology as the opinion tolerance cycles from radicalization towards consensus and backward to radicalization.

Keywords: Opinion formation, Polarization, Dynamical transitions, Sociophysics

1. Introduction
Throughout human history, the government was ever exerted from a minority over the majority and societies were organized through the game of oligarchies. After the rise of capitalism, the socio-political stresses were mitigated by the adoption and progressive enlargement of liberal democracy, an admirable system that creates a majority and an opposition that agrees with the politics in action [1]. Currently, the state power and power of wealth are engaged in reducing the political space [2]. Thus, dominant classes controlling mass media and the information flow at the social networks, launched out unprecedented efforts to limit the impact and to shape the opinion of the majorities. The result, visible all around the world, is the increasing of political radicalization in a networked communicational environment, catalyzing opinion polarization. Media and social networks, mainly but not exclusively, are forging pre-fabricated spaces for culture, religion, economy, law, and policy [3]. In particular, mathematical models suggest that slightly biased opinion surveys [4] and the action of agents with strategically placed opinions [5] can be determinant on the outcome of elections or campaigns.

Here, we investigate opinion polarization dynamics from the viewpoint of complex system theory. Essentially, we will be tracking the footprints of Dietrich Stauffer, a pioneer in applications of statistical physics to socially motivated problems [6, 7]. In 2004, Stauffer and Meyer-Ortmanns [8] used extensive simulation analysis to obtain a critical value $\varepsilon_c \approx 0.4$ for the tolerance threshold (see Sec. 2) for a complete consensus in the model of Deffuant \textit{et al.} [9]. In addition, they showed that the number of different opinions when no complete consensus is formed is proportional to the number of individuals. Since pioneers ignited the field, the issue of consensus versus polarization in opinion dynamics became a major topic in sociophysics and many theoretical models were designed. See Ref. [10] for a seminal review of the related physics literature. Examples include two of the most studied models in sociophysics, namely, the bounded confidence model of Deffuant \textit{et al.} [9] and the Sznajd model [11], the latter based on Ising spin variables. In contrast to the opinion polarization observed in the Deffuant model and its variants, the Sznajd model has only the complete consensus state as a dynamical attractor in one- and two-dimensional regular lattices, as well as complete graphs [10]. Furthermore, the Sznajd model exhibits bistability on the square lattice [12]: the initially majoritary opinion by chance spreads throughout the whole population. However, despite their different details, all these models reveal that even a small group of extremists or zealots sustaining long-term fixed opinions may rule the whole system dynamics and drive the majority opinion towards their own beliefs or interests [12, 13].

Concerning the nature of opinion radicalization and polarization phenomena, Ramos \textit{et al.} [14] proposed a statistical predictor of the rise of radicalization in society, which can be obtained from polls. The predictor is the
onset of nonlinear behavior in the scatter plots for the fractions of individuals holding a certain extreme view versus the fraction of individuals sustaining either moderate or that extreme opinions. A radicalization regime, characterized by a wide spectrum of agents’ opinions, emerges under large controversy and strong social influence [15]. Moreover, the introduction of homophily — the preference of individuals to interact with agents holding similar views — leads to a polarized state in which the opinion distribution function is bimodal. It is worth to mention that emotions due to a conflict of opinions, certainly enhanced by controversy, may harden individuals’ and groups’ behaviors, generating a stable, conflicted social state [16]. Inflexible, extremists or zealot agents are not necessary to trigger extremism rise. According to Sobkowicz [17, 18], “Anyone can become an inflexible zealot, anyone can become an extremist”. In turn, the role of media on public opinion dynamics was considered in Ref. [19]. It was shown that, even at complete tolerance, plurality and competition within information sources allows the stable coexistence of several and distinct cultures. In contrast, an audience-oriented un-polarized media smooths the transition from polarization to consensus.

In all aforementioned works, the agents are nodes of homogeneous or heterogeneous networks with fixed structures. So, the focus was ever on the trajectories of the system states in well-defined phase spaces and opinion dynamics does not affect the static network architecture. However, there are many instances of networks whose states and topologies coevolve at similar time scales. In these adaptive networks, state transitions and topological alterations are inexorably intertwined, producing novel emergent behaviors [20]. In social phenomena, agents frequently modify their contacts depending on the states assumed by the agents with which they interact. In particular, person-to-person communication patterns and one-to many information spreading channels, such as traditional media, blogs and microblogs (like Twitter), evolve faster and faster. In the present work, we investigate a modification of the Deffuant model [9] on adaptive scale-free networks in which network topology constrains agents’ interactions and thus opinion dynamics that, in turn, induces changes in social interactions, altering the contact network. Specifically, we quest for a fragmentation transition analogous to that observed in the adaptive voter model [21, 22, 23].

The paper is organized as follows. In Section 2, the stochastic agent-based model is described. It is assumed that agent’s opinions and the social network structure coevolve in a subtle feedback loop regulated by bounded confidence and homophily. In Section 3, our major findings obtained through extensive model simulations are reported. Particular emphasis is given to the changes in topology of the adaptive social network driven by opinion dynamics. These results are further discussed in Section 4. Finally, some conclusions are drawn and possible directions for future research are pointed out in Section 5.

2. Model

The model is defined as follows. N agents are represented by the nodes of a scale-free network. Each one has a random initial opinion \( o_i(t = 0) \), uniformly distributed in the interval \([0, 1]\). The network of contacts at \( t = 0 \) is modeled using the uncorrelated configuration model (UCM) [24], in which each vertex is connected to \( k_i(t = 0) \) other agents. The initial degrees \( k_i(0), i = 1, 2, \ldots, N \) are drawn at random according a power-law degree distribution \( P(k) \sim k^{-\gamma} \) with an upper cutoff \( k_{\text{max}} = \sqrt{N} \). Connections are performed at random without self nor multiple connections. The UCM generates synthetic, undirected, and unweighted networks without degree correlations for a tunable degree exponent \( \gamma \). In the simulations reported here, the value \( \gamma = 2.7 \) was used.

We consider a discrete time dynamics. At every time step \( t > 0 \), agents’ opinions and their social contacts, thereby the network connectivity pattern, coevolve under the following rules.

**Opinion dynamics.** Every agent \( i \) compares his/her current opinion \( o_i(t) \) with the average opinion

\[
(o_i)_t = \frac{1}{k_i(t)} \sum_{j \in u_i(t)} o_j(t)
\]

of his current social neighborhood \( u_i(t) \), comprised by all the nodes \( j \) connected to \( i \) at time \( t \). Then, the difference \( \Delta_i(t) = o_i(t) - (o_i)_t \) between the agent’s opinion and that of his social group is used to update the agent’s view. The rule is:

\[
o_i(t + 1) = \begin{cases} 
    o_i(t) + \mu \Delta_i(t) & \text{if } \Delta_i(t) \leq \varepsilon \\
    o_i(t) & \text{if } \Delta_i(t) > \varepsilon,
\end{cases}
\]

where the parameter \( \mu \) determines the speed of opinion convergence while the parameter \( \varepsilon \) is a tolerance threshold. So, the opinion of the agent \( i \) tends to converge to that of his social group if they differ by less than a value \( \varepsilon \). This is the Deffuant et al. [9] formulation for the bounded confidence principle: to be mutually influenced, agents (in our version, an agent and his social group) must have similar enough opinions. If the difference of opinions is too large, the communication process is impossible, and opinions do not change. In the simulations, the value \( \mu = 0.8 \) was fixed while \( \varepsilon \) was used as the control parameter of the model. All agent opinions are updated synchronously.

**Network topology dynamics.** After opinions’ update, each edge \( e_{ij}(t) \) of the social network can be broken and rewired (or not) according to the following rules: (i) if the updated opinions of the agents connected by \( e_{ij} \) differ in modulus by \( \Delta_{ij} = |o_i(t + 1) - o_j(t + 1)| > \varepsilon \), the corresponding edge \( e_{ij} \) is broken with a probability \( p = 1 - \exp(-\kappa \Delta_{ij}) \). The value \( \kappa = 3.5 \) was fixed in the present simulations. In the case of edge breakage, (ii) agents \( i \) and \( j \) rewire independently the broken half-edge to other agents \( l \) and \( l' \), respectively, of similar opinions, i.
Figure 1: (a) Order parameter $m$ at the stationary state as a function of the tolerance threshold $\varepsilon$. (b) The stationary state opinions’ variability $\chi_m = \langle o^2 \rangle_m - \langle o \rangle_m^2$ computed over the ensembles of simulations for different system sizes $N$ indicated in the legend. (c) Finite-size scaling of $\varepsilon_c$, the critical tolerance threshold. Data correspond to averages over 500 independent samples and time windows of 100 steps after relaxation to the steady state.

\[ q = \begin{cases} \exp\left(-\frac{d_{ij}}{d_0}\right) & \text{if } d_{ij} \leq d_{\text{max}} \\ 0 & \text{otherwise} \end{cases} \]

where $d_0$, a characteristic distance for rewiring, is a model parameter. A maximal distance for creation of new contacts $d_{\text{max}} = \ln N$ was adopted in the simulations. Its role is only to make the model computationally more efficient avoiding the selection of nodes with $d_{ij} \gg d_0$, for which the probability is essentially null. Unless specified the distance $d_0 = 4$ was used in all presented results. But, rewiring is ruled by homophily, since agents have preference to interact with individuals holding similar opinions. Finally, (iii) not rewired half-edges can be replaced by new contacts at later time steps according rule (ii). The limit $k_i(0)$, the agent $i$ initial degree, is imposed only for the agent who searches a contact.

The rules forbid the creation of links between agents whose opinions differ above the threshold tolerance $\varepsilon$. But in real societies very dissimilar individuals still interact and is not rare to see extremists from the Left and Right wings cooperating in politics, for example. For this reason, we have also tested a modified rule (ii): an agent can rewire a broken link to any individual of the network with similar ($\Delta_{ij} < \varepsilon$) or dissimilar ($\Delta_{ij} > \varepsilon$) opinions with fixed small probabilities $f$ and $g$, respectively. This possibility allows for the interaction between individuals within eventually distinct (isolated) clusters formed in the social network. Unless if mentioned, $f = 0.01$ and $g = 0.001$ were used for all simulations.

In summary, the present model for opinion dynamics involves two fundamental processes evolving at the same time scale: node states’ changes as well as breaking and rewiring of links. Therefore, this is an adaptive network approach [20, 10] where there is a coevolution of topology adaptation and the opinion dynamics in the network, intertwined in a subtle feedback loop.
3. Results

Let us define a measure of consensus $m$ given by

$$m = \frac{1}{N} \sum_{i=1}^{N} |o_i(t) - \langle o \rangle(t)|,$$

where $\langle o \rangle(t) = \langle 1/N \rangle \sum_{i=1}^{N} o_i(t)$ is the social average opinion at time $t$. Figure 1 presents the stationary behavior of this quantity as a function of the tolerance threshold $\varepsilon$. As one can see, $m$ behaves as an order parameter, vanishing above a critical threshold $\varepsilon_c$. The variability associated to $m$ is defined as the variance of $m$ computed over the ensemble of samples for $t \to \infty$, $\chi_m = \langle o^2 \rangle_m - \langle o \rangle^2_m$. This is measurement of the collective opinions' fluctuations and shown in figure 1(b). The curves $\chi_m$ versus $\varepsilon$ exhibit a peak that can be used as an estimate of the transition point $\varepsilon_c$ in analogy with a susceptibility which gives the space-time fluctuations of a system [25]. At asymptotically large $N$, a value $\varepsilon_c \approx 0.10$ was found using a finite size scaling of the form

$$\varepsilon_c(N) = \varepsilon_c(\infty) + \text{const.} \times N^{-1/2},$$

shown in figure 1(c).

This transition corresponds to the known polarization-consensus transition typical of models for opinion dynamics [10]. Also, the critical value $\varepsilon_c \approx 0.10$ we found is significantly lower than the values $\varepsilon_c \approx 0.40$ and $\varepsilon_c \approx 0.50$ reported in references [8] and [26] for static Barabási-Albert networks [27], which are also scale-free with a degree exponent $\gamma \approx 3$. Furthermore, increasing characteristic distance $d_0$ for rewiring of social contacts promotes full consensus formation by decreasing the critical tolerance threshold (see Fig. 2). So, the transition between global consensus and multipolarized (radicalized) states is mostly governed by the opinion threshold tolerance $\varepsilon$ and the characteristic distance $d_0$ for social contacts rewiring. A phase diagram in space parameter $\varepsilon$ versus $d_0$, characterizing the polarized and consensus phases, is shown in figure 2.

In order to better illustrate the nature of the consensus-polarization transition, the stationary opinion distributions for distinct tolerance thresholds are shown in figure 3. At the full consensus regime, observed for large $\varepsilon$ values, this distribution is unimodal, since all the agents share the same opinion. In contrast, at the polarization regime observed for $\varepsilon < \varepsilon_c$, the consensus is destabilized, more opinions emerge and their distribution is bimodal. Moreover, for $\varepsilon \ll \varepsilon_c$, the number of surviving opinions at the stationary state is proportional to the number of agents in the population, as communicated by Stauffer and Meyer-Ortmanns [8].

Concerning the structure (topology) of the adaptive social network, above $\varepsilon_c$, the network is comprised by a main, well connected component and several small components with a few nodes. In turn, the coevolution of opinions and the adaptive social network leads to the formation of communities within a modular structure still in the non-fragmented network. Such modules were detected using the Louvain algorithm [28]. Slightly different leading opinions are initially present in each community. Below $\varepsilon_c$, the polarization (bimodal opinion distribution) or radicalization (multimodal opinion distribution) regimes emerge for $\varepsilon \lesssim \varepsilon_c$, forming still connected modular networks where modules are interconnected by “bridges”. As the tolerance threshold is further reduced the main component splits into two or more large components with sizes of the same order. Within these isolated subgraphs, all agents share similar opinions but as $\varepsilon$ is further reduced, these slightly different opinions lead to more module fragmentation. This process continues until very low $\varepsilon$ when the networks is formed only by small components. Such process is analogous to that observed in the adaptive voter model [21, 22, 23]. A qualitative picture of this transition is shown in figure 4.

In addition to the number of surviving opinions in the polarization/radicalization regimes, it is also relevant to
know the size of the social groups holding those opinions. Since opinion consensus (homogenization) involves agents in the same community or module, we determined the size of the largest connected component $S_1$. Below $\varepsilon_c$, the fraction $s = S_1/N$ scales very well as $s - s_0 \approx (\varepsilon - \varepsilon_c)^\delta$, where $s_0 \approx 0.00135$ corresponds to the fractional size of the largest component (of order $S = 1$) as $\varepsilon \to 0$, and $\delta \approx 1.7$; see figure 5. The approach to the critical tolerance threshold for $\varepsilon > \varepsilon_c$ is also consistent with a scaling law in the form $s - s_0 \approx (\varepsilon - \varepsilon_c)^\delta$ where $\delta' \approx 0.21$. Here, it worths to note that for $\varepsilon > 0.5$, we obtained $s = 1$ meaning that disconnected components are absent, in agreement with the transition found in Ref. [26] for the Deffuant model.

Both polarization and radicalization in opinion dynamics affect the adaptive network topology through the breakage and rewiring of edges (social connections). A result is the emergence of a significant number of agents with small degrees and a few nodes with degrees larger than the UCM cut-off $k_{\text{max}} = N^{1/2}$ [24]. In consequence, the network degree distribution deviates from the initial UCM power-law degree distribution only at their tails (small and large degrees), except for $\varepsilon \ll \varepsilon_c$ when the degree distribution becomes exponential in the highly fragmented regime; see figure 6(a). Due to homophily, individuals tend to form ties with other individuals that also interact. This can be quantified by the average clustering coefficient $\langle C \rangle$ [29] of the network that gives the average fraction of contacts of an individual which are themselves also connected. Figure 6(b) presents the clustering coefficient found for contact networks in the stationary state as function of the tolerance threshold. As expected, the initial null value of the clustering coefficient $\langle C \rangle \sim 1/N$ of the original UCM network [24] becomes finite when the adaptive dynamics is at work. The maximum value of $\langle C \rangle$ occurs for $\varepsilon < \varepsilon_c$, whereas $\langle C \rangle$ tends to zero in the limits $\varepsilon \to 0$ and $\varepsilon \gg \varepsilon_c$. Changes of topology can be detected by the average shortest path length $\langle D \rangle$ [29] of the largest connected component as illustrated in Fig. 6(c). The average distances present local maxima and minima as $\varepsilon$ is increased from zero. The former occurs when the fragmentation of a module into two large components is imminent, while the latter corresponds to the emergence of groups with distinct opinions and community formation within a connected fragment. Examples of network structures for minima and maxima shown in figure 6 illustrate these structural changes.

The coevolution of opinions and the underlying adaptive social network exhibits a hysteresis-like loop and, therefore, is irreversible below or across the polarization transition. The $\varepsilon$-loop starts from an initial condition (opinions randomly distributed on a UCM social network), which evolves up to its stationary state at fixed $\varepsilon_0 \approx 0$. Then, the stationary state (state variables and network topology) becomes the initial state of the system, which now evolves at fixed $\varepsilon_1 = \varepsilon_0 + \delta \varepsilon$ ($\delta \varepsilon \ll 1$). This process is iterated $m$ times.
until $\varepsilon_m = \varepsilon_0 + m\delta\varepsilon$. So, the loop in $\varepsilon$ is closed decreasing $\varepsilon$ by $\delta\varepsilon$ at each iteration, up to $\varepsilon_0$. As shown in figure 7, the network topology after the hysteresis loop is very different from that at the starting of the loop since the system does not return to its initial configuration. In the first part of the $\varepsilon$-loop, modules or communities interconnected by bridges become progressively more connected. On the way back, once consensus was formed, those connected components do not fragment as before, and we return to a system (opinion distributions and network topology) certainly distinct from the initial one. The impacts of such a phenomena in real societies can be huge.

4. Discussion

Our coevolution model for the intertwined opinion and social network dynamics revealed a very rich phenomenology. Indeed, four qualitatively dynamical regimes, controlled mainly by the tolerance threshold $\varepsilon$, were found. This parameter is crucial to generate a heterogeneous opinion distribution. Starting from a random distribution of opinions, larger values yield full consensus, whereas $\varepsilon \rightarrow 0$ yields an extreme heterogeneity of opinions at the stationary state characterized by a uniform opinion distribution function. The regimes and transitions/crossovers we have observed exhibit the following traits.

Full consensus. At large tolerance threshold values ($\varepsilon > 0.50$) a full consensus state, in which all agents hold the same opinion is reached. In this regime, the social network is a graph without isolated nodes or clusters and, therefore, the chemical distance $d_{ij}$ between any pair $(i, j)$ of agents is finite. The network does not have a modular structure and its topology (degree distribution, clustering coefficient, average distance, etc.) is very similar to that.
of the initial UCM network; see figure 4(a). The onset of full consensus at $\varepsilon \sim 0.50$ is consistent with the universal tolerance threshold for complete consensus obtained in reference [26] for the Deffuant et al. model in non-adaptive (fixed) networks.

Majoritary consensus. For intermediate values of the tolerance threshold $0.10 < \varepsilon < 0.5$, an increasing, but small number of agents disconnect from the network. These agents stay isolated or eventually form minute groups which keep their opinion forever. So, the dynamics is ruled by a single and giant component of size hugely larger than that of all other disconnected present on the network; see figure 4(b). The topology of the giant component is still similar to that of the initial UCM network, but larger clustering coefficients are observed (see figure 6(b)) and the degree distribution deviates from the UCM counterpart only at very small and large degree values. The full and majoritary consensus are separated by a smooth crossover.

Polarization. For tolerance thresholds in the range $0.08 < \varepsilon < 0.10$ the largest component becomes organized in a modular structure containing two main communities. Further decreasing $\varepsilon$ makes these two modules progressively more evident and less interconnected. Slightly below the tolerance threshold the bridges linking two loosely interconnected modules are broken (see figure 6(c)). The emergence of a modular architecture renders the network very distinct from the initial UCM graph. The transition between majoritary consensus and polarization at $\varepsilon_c \approx 0.10$ is characterized by a non-divergent peak at the opinions’ variability, figure 1(b), a local maximum in the average distance between nodes due to the split of the population into two weakly connected modules, figure 6(c), and a crossover in the scaling behavior of the largest module size $s_1$; see figure 5.

Radicalization. At the tolerance threshold $\varepsilon \approx 0.08$, the two large network components characterizing the polarization regime fragment into three weakly interconnected modules; see figures 4(d) and 6(c). Further decrease of $\varepsilon$ leads to a cascade of fragmentation and rupture of the bridges interconnecting these modules. There are multiple surviving opinions associated to these modules within which consensus is formed. The polarization-radicalization crossover at is marked by a peak in the size of the second largest component which is of same order as the largest one (data not shown). In the range $0.008 < \varepsilon < 0.087$, there are several isolated and loosely interconnected communities, significant in size, and have large clustering coefficients; see figure 6(b). The independent dynamics of these components is the cause of the polarization-radicalization transition. Such a scenario is supported by references [30, 31] in which extended control parameter regions with non-universal power-law decays of activity in time were found in infinite dimensional, loosely coupled network of modules. These ingredients indicate the existence of Griffiths phases [32]. For $\varepsilon < 0.008$, the social network is pulverized in a “dust” of isolated agents or very minute groups (the largest group size is $S_1 < 5$ for $\varepsilon = 0$), and the average distance between nodes decays suddenly to $d \approx 1$ (see figure 6(c)). Also, the opinion distribution function is almost continuous and its dynamics becomes effectively frozen.

Once this general scenario was built, we will try to correlate it with recent phenomena observed in real societies. On line communication networks, as Twitter and Facebook, changed the way people behave, decide, opine and make choices [33]. These social networks lead naturally to opinion polarization in communities with distinct views, ultimately creating echo-chambers in which users reinforce their believes discussing with each other within a closed, almost impermeable group. Social polarization and echo chambers were detected in several contexts, namely, majoritary elections [34], political crisis [35], street protests [36, 37], and online spreading of misinformation [38]. This is exactly what happens in the polarization and transition regimes of our model driven by homophily and operating at low tolerance thresholds. In these regimes, the isolated or loosely interconnected modules correspond to the echo-chambers, because within such modules consensus is reached and their unanimous opinions can never meet each other. As in real societies, the surviving opinions in the radicalization regime can not be rebutted in a democratic debate. The challenge for liberal democracy is significant: A wrong politic can be widely accepted if riding in one of the unanimity flows.

Furthermore, the hysteresis phenomenon reported here can have huge impacts on democracy. Indeed, in democratic societies the government is under daily pressures imposed by the multiplicity and heterogeneity of the organized interests of the social groups. These demands are the source of distinct political agendas, as well as ignite numer-
able and simultaneous self-controlled combats. Growing radicalization (decreasing $\varepsilon$) increases the social demands associated to the emergence of new group interests. This means not only enhanced stresses on mass democracies, but also an increasing number of potential discontented with the decisions and policies implemented. Such reverse effect of democracy — the number of upset interests tends to overcome the number of served interests — foster further radicalization and repudiation to the results of the democratic practice. However, our results indicate that a reversal of radicalization trend does not rescue the original, less fragmented (or radicalized) social network. According to the results, it is legitimate to speculate if the increase and differentiation of social demands triggered by the outcomes of public policies and the reverse effects of democratic competition are unavoidable in advanced capitalist societies grounded in universalized political rights.

5. Conclusions

In this work we analyzed how opinion dynamics ruled by bounded confidence and homophily can lead to social fragmentation and generate a society in bubbles. To achieve this goal, extensive computer simulations of a stochastic agent-based model, in which social contacts are broken and rewired with probabilities dependent on the current opinion difference between two agents were performed. The coevolution of opinion and social network topology driven by bounded confidence and homophily revealed a very rich phenomenology characterized by several dynamical regimes and continuous crossovers between them. We found that, starting with a large tolerance threshold to opinion differences, the adaptive social network progressively fragments in bubbles and the number of surviving opinions increases as the tolerance threshold decreases. The bubbles are echo chambers whose size distributions change from exponential with peaks at the tail (large modules) to power-law and back to exponential, as the tolerance threshold decreases. Within each bubble, consensus is formed. At the full consensus regime (a unanimous opinion) the original network itself is the bubble. At the majoritarian consensus a single giant bubble rules the dynamics. This giant bubble organizes into two modules that progressively disconnect from each other at the polarization regime (two dominant opinions). At the radicalization regime (multiple surviving opinions) the network is fragmented in several modules (bubbles). We also found that the adaptive social network exhibits a hysteresis-like behavior characterized by irreversible changes in its topology as the opinion tolerance cycles from radicalization consensus towards majoritarian and backward to radicalization consensus. The fragmentation in bubbles and its associated irreversibility can have deep impact in majority formation and democratic decision processes, as seems to be observed in contemporary real societies.

A forthcoming extension of the present model is to include the coevolution dynamics of the tolerance threshold, the major control parameter of the model. In addition, the effects of the rewiring characteristic distance and probabilities ($d_0$, $f$, and $g$) calls for additional investigation. Finally, extended versions of the present model, in which media and strategic agents are included, can be investigated to tackle the central issue of exogenous influences on the opinion dynamics.

Acknowledgments

Authors acknowledge the financial support of Conselho Nacional de Desenvolvimento Científico e Tecnológico - CNPq (Grants no. 430768/2018-4 and 311183/2019-0) and Fundação de Amparo à Pesquisa do Estado de Minas Gerais - FAPEMIG (Grant no. APQ-02393-18).

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