Energetic positron propagation from pulsars: an analytical two-zone diffusion model

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Abstract. We study the cosmic rays (CR) positrons propagation from the near Earth Geminga pulsar wind nebula on the basis of a analytical model of the two-zone spherically symmetric particle diffusion from a central source. We calculate the near Earth spectral distribution of positrons originating from the pulsar. The obtained spectra are compared with the results of another authors considering the problem of the positron excess revealed by PAMELA and AMS-02 experiments.

1. Introduction

Recent measurements of the cosmic rays spectra and composition with PAMELA and AMS-02 observatories [1–3] revealed the excessive positron fluxes at energies above 10 GeV compared to the expected fluxes of secondary positrons which are thought to be produced in the reactions of the primary CR with the interstellar matter. This phenomena may be explained by the presence of a few sources accelerating positrons up to tens of TeV near the Earth. The most likely candidates for such sources are the nearby pulsars.

Modern theories and observations (see, e.g., [4–9]) show that the pulsar wind (PW) consists of the electron-positron plasma; some of the PW particles are accelerated up to tens of TeV and may carry a significant fraction of the spin-down luminosity of the pulsar [10]. A nearby pulsar with significant spin-down luminosity may be the main contributor to the observed positron fluxes. Many authors suggest the Geminga pulsar with its spectacular nebula [11] as the most appropriate candidate source [12–15].

The numerical simulations of particle propagation and their inverse Compton (IC) emission allowed Abeysekara et al. (2017) [13] to conclude that the gamma-ray maps of the Geminga pulsar vicinities obtained with HAWC imply very slow particle diffusion. They claimed that the diffusion coefficient of positrons and electrons in a few tens parsecs around the Geminga pulsar is several orders of magnitude lower than the typically assumed average interstellar coefficient [16]. They extrapolated this small positron diffusion coefficient on the entire region between the Geminga pulsar and the Earth and found that the circumterrestrial flux of positrons produced by the Geminga pulsar is much lower than the observed one. Indeed, in this case the particles with energies of hundreds GeV and below diffuse to the Earth longer than the age of the pulsar. Meanwhile, the particles with TeV energies suffer much stronger synchrotron and IC energy losses and cannot reach the Earth as well.
Fang et al. (2018) [14] assumed that the diffusion coefficient increases to the average interstellar value at a distance of several tens of parsecs from the pulsar. The authors presented the numerical solution of the diffusion equations with energy losses for a two-zone spherically symmetric diffusion model. The simulated positron spectra and gamma-ray emission from the pulsar vicinity were found consistent with AMS-02, PAMELA and HAWC measurements if the diffusion coefficient changes from a small value (two orders of magnitude smaller than the interstellar one) to an average interstellar value at about 50 pc from the Geminga pulsar.

Tang et al. (2019) [15] presented the analytical particle propagation model with two-zone particle diffusion. Using analytical solutions instead of numerical ones makes calculation of the model CR spectra much easier. In paper [17] the analytical expressions [15] are used to model the positron fluxes from a set of pulsars.

In this paper, we obtained an analytical Green function in a two-zone model of particle diffusion, taking into account the particle energy losses due to IC and synchrotron emission. Our results are compared with the results of [15]. We also show how the differences between our solutions and solutions from the work [15] can affect the modelled spectra of positrons near the Earth.

2. Analytical solution for the two-zone diffusion with energy losses

The distribution function $N(E, r, t)$ of positrons diffusing spherically symmetrically from the particle source $Q(E, t)$ at the center of coordinates obeys the equation

$$
\frac{\partial N(E, r, t)}{\partial t} - \nabla \cdot [D(E, r) \nabla N(E, r, t)] + \frac{\partial}{\partial E} [b(E) N(E, r, t)] = Q(E, t) \delta(r), \tag{1}
$$

where $t, r, E$ are the time, the distance from the source and the particle energy, respectively, $\delta(r)$ is the Dirac delta function, $D(E, r)$ is the diffusion coefficient, $b(E) = -\frac{4}{3} c \sigma_T \gamma^2 \sum_i \frac{U_i}{(1 + 4\pi\gamma \epsilon_i)^{3/2}} + \frac{B^2}{8\pi}$ is the equation for the energy losses due to IC and synchrotron emission, $B$ is the magnetic field, $c$ is the speed of light, $\gamma$ is the positron Lorentz factor, $\sigma_T$ is the Thomson cross-section, $U_i$ is the energy density of the radiation field in different energy channels [15], $\epsilon_i$ is the corresponding normalized photon energy. For a black body radiation with the temperature $T_i$, we have $\epsilon_i = \frac{2.8 k_B T_i}{m_e c^2}$, where $k_B$ is the Boltzmann constant and $m_e$ is the positron mass.

Further we assume that the diffusion coefficient can be factorized over the entire space, $D(E, r) = \kappa(E) D'(r)$. Following [18], introducing the variables $t' = t - \tau(E, E_0)$ and $\lambda(E, E_0)$, where $\lambda(E, E_0) = \int_{E_0}^{E_0} \frac{\kappa(E')}{|b(E')|} dE' > 0$ and $\tau(E, E_0) = \int_{E}^{E_0} \frac{1}{|b(E')|} dE' > 0$, it is easy to find the Green function of Eq. (1):

$$
G(E, E_0, r, t, t_0) = \delta(t - t_0 - \tau(E, E_0)) \frac{1}{|b(E)|} H(E, E_0, r),
$$

where $H(E, E_0, r)$ satisfies the equation

$$
\frac{\partial H(E, E_0, r)}{\partial \lambda} - \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 D'(r) \frac{\partial H(E, E_0, r)}{\partial r} \right) = \delta(\lambda) \delta(r). \tag{2}
$$

Accordingly, the solution of Eq. (1) is

$$
N(E, r, t) = \int_{-\infty}^{t} dt_0 \int_0^\infty dE_0 Q(E_0, t_0) G(E, E_0, r, t, t_0). \tag{3}
$$
Let us consider the case of the two-zone diffusion:

$$D (E, r) = \kappa (E) \begin{cases} D_1, & r < r_b \\ D_2, & r \geq r_b \end{cases}.$$  (4)

In the following, we solve Eq. (2) with the diffusion coefficient given by Eq. (4) using the separation of variables and the eigenfunction method. The right-hand side of Eq. (2) can be represented as the condition $H (\lambda, r)|_{r=0} = \delta (r)$ allowing to solve Eq. (2) with the zero right-hand side. Imposing the continuity conditions for the function $H$, $H (\lambda, r)|_{r=r_b-0} = H (\lambda, r)|_{r=r_b+0}$, and the flux, $r^2 D_1 \frac{\partial H (\lambda, r)}{\partial r} |_{r=r_b-0} = r^2 D_2 \frac{\partial H (\lambda, r)}{\partial r} |_{r=r_b+0}$, at $r = r_b$, we obtain the solution of Eq. (2):

$$H (E, E_0, r) = \int_0^\infty d\xi \frac{e^{-\xi}}{\pi^2 r_{d_1} r_{d_2}} \left( A (\xi) + B (\xi) \right) \times$$

$$\left\{ \begin{array}{l}
\sin \left( 2 \sqrt{\xi} \frac{r_{b}}{r_{d_1}} \right) \\
\sin \left( 2 \sqrt{\xi} \frac{r_{b}}{r_{d_2}} \right) + \cos \left( 2 \sqrt{\xi} \frac{r_{b}}{r_{d_1}} \right) + \cos \left( 2 \sqrt{\xi} \frac{r_{b}}{r_{d_2}} \right), \ r < r_b \\
A (\xi) \sin \left( 2 \sqrt{\xi} \frac{r_{b}}{r_{d_1}} \right) + \cos \left( 2 \sqrt{\xi} \frac{r_{b}}{r_{d_2}} \right), \ r \geq r_b
\end{array} \right.$$  (5)

where $A (\xi) = \xi \cos \left( 2 \sqrt{\xi} \frac{r_{b}}{r_{d_1}} \right) \cos \left( 2 \sqrt{\xi} \frac{r_{b}}{r_{d_2}} \right) + \sin \left( 2 \sqrt{\xi} \frac{r_{b}}{r_{d_1}} \right) \sin \left( 2 \sqrt{\xi} \frac{r_{b}}{r_{d_2}} \right)$ and

$$B (\xi) = \frac{\sin \left( 2 \sqrt{\xi} \frac{r_{b}}{r_{d_1}} \right) - A (\xi) \sin \left( 2 \sqrt{\xi} \frac{r_{b}}{r_{d_2}} \right)}{\cos \left( 2 \sqrt{\xi} \frac{r_{b}}{r_{d_2}} \right)}. \quad (6)$$

Note that $\int_0^\infty H (\lambda, r) 4\pi r^2 dr = 1$.

The following solution of equation (2) for the two-zone diffusion was obtained in the paper [15]

$$H_{TP} (E, E_0, r) = \frac{\xi (\xi + 1)}{\pi^{3/2} r_{d_1}^3 \left[ 2 \xi^2 \text{erf} \left( r_b/r_{d_1} \right) - \xi (\xi - 1) \text{erf} \left( 2r_b/r_{d_1} \right) + 2 \text{erfc} \left( r_b/r_{d_1} \right) \right]} \times$$

$$\left\{ \begin{array}{l}
e^{-r^2/r_{d_1}^2} + \left( \frac{\xi - 1}{\xi + 1} \right) \left( \frac{2r_b}{r_{d_1}^2} \right) e^{-r^2/2r_{d_1}^2}, \ r < r_b \\
\left( \frac{2r_b}{r_{d_1}^2} \right) \left[ \frac{r_b^2 + \xi (1 - \frac{r_b}{r_{d_1}^2})}{\xi^2} \right] e^{-r^2/2r_{d_1}^2}, \ r \geq r_b
\end{array} \right.$$  (8)

where $\text{erf} (x)$ is the error function, $\text{erfc} (x) = 1 - \text{erf} (x)$ and $\int_0^\infty H_{TP} (\lambda, r) 4\pi r^2 dr = 1$.

3. Discussion

Solutions (5) and (8) significantly differ for the same parameters. The reason is that Tang et al. (2019) [15] used the standard function replacement $F (\lambda, r) = rH (\lambda, r)$ leading to the equation
for $F(\lambda, r)$ (see Appendix A [15] eq. (A1) and (A13)) which should be solved further only if the diffusion coefficient does not depend on $r$.

The last condition is obviously violated in the two-zone theory at $r = r_b$. As a result, in [15] (see Appendix A [15] eq. (A8), (A16) and (A17)) the flux continuity conditions were used for the function $F(\lambda, r)$ instead of the function $H(\lambda, r)$, which resulted in the incorrect solution of the problem. One of the consequences of the incorrect solution (8) for $\xi < 1$ is that the function $H_{TP}(E, E_0, r)$ has negative values for small $r$ (see the left panel of Fig. 1), which is impossible, since the function $H(E, E_0, r)$ determines the number of particles per unit volume.

When $\xi = 1$ ($D' = D_2 = D_1$), both solutions (5) and (8) give correct one-zone expression (the integral in Eq. (5) could be taken analytically), $H_1(E, E_0, r) = \frac{e^{-r^2/r_d^2}}{\pi^{3/2}r_d^3}$, where $r_d(E, E_0) = 2\sqrt{D'\lambda(E, E_0)}$.

Figure 1 compares the solutions (5), (8) for different parameters. The right panel of Figure 1 shows the dependency on $\lambda$, since it is convenient to switch to the variable $\lambda$ when integrating over $E_0$ in Eq. (3).

Figure 2 compares the positron fluxes $J(E, r) = cN(E, r) / 4\pi$ produced by the Geminga pulsar near the Earth calculated using the expressions (5) and (8) for the same parameters of the particle source and diffusion taken from [15].

The source term $Q(E, t)$ depends on the time since the birth of the Geminga pulsar $t$ due to temporal evolution of the pulsar spin-down luminosity $L(t) = \dot{E}_{spin} \left( \frac{1 + \frac{t_{age}}{\tau_0}}{1 + \frac{t}{\tau_0}} \right)^2 \left( \frac{1 + \frac{t}{\tau_0}}{1 + \frac{t_{age}}{\tau_0}} \right)^{-2}$ [19]. Here $t_{age} = \tau_c \left[ 1 - P_0^2 / P^2 \right]$ is the current age of the Geminga pulsar, $\tau_c = 330$ kyr is

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Comparison of solutions $H(\lambda, r)$ of Eq. (2) for $D_2 = 100D_1$. The green curves show the two-zone solutions (5) obtained in the present paper, while the blue curves present the two-zone solutions given by Eq. (8), which were obtained in [15]. The one-zone solutions for the diffusion coefficient $D_1$ and $D_2$ are shown by black and red curves, respectively. In the left panel $r_b = 50$ pc; the solid curves correspond to $r_{d1} = r_b$, the dashed curves – to $r_{d1} = 2r_b$. In the right panel $r = 250$ pc; for the two-zone solutions: the dashed curves are for $r_b = 20$ pc, the solid – for $r_b = 50$ pc, the dot-dashed – for $r_b = 80$ pc.}
\end{figure}
This paper

The characteristic age [4], \( \tau_0 = \tau_c (P_0/P)^2 \) is the initial spin-down time-scale, \( P_0 = 0.14 \) s is the initial period, \( P = 0.237 \) s is the observed period [20, 21], \( \dot{E}_{\text{spin}} = 3.4 \times 10^{34} \) erg s\(^{-1}\) is the observed spin-down power [4], \( r = 250 \) pc is the distance between the Earth and the pulsar [22]. We used \( D_1 \kappa(E) = 4.5 \times 10^{27} \) cm\(^{-2}\) s\(^{-1}\) \((E_{100 \text{ TeV}}) \)\(^{1/3}\) [13], \( D_2 = 100D_1 \), and \( r_b = 60 \) pc.

We chose \( Q(E,t) = q(t) E^{-\alpha} \) with \( \alpha = 2.34 \). The kinetic modelling of particle acceleration in the pulsar wind nebulae with bow shocks predicts hard particle spectrum with \( \alpha < 2 \) at low energies [23–25], however, the positron fluxes obtained in [15] for a chosen set of parameters are quite similar for a power-law with \( \alpha = 2.34 \) and a broken power-law with \( \alpha = 1.5 \) below 30 TeV and 2.34 above. Following [15], we assumed that the efficiency of conversion of the spin-down luminosity in the electron-positron pulsar wind \( \eta = 2L^{-1}(t) \int_{E_{\text{min}}}^{E_{\text{max}}} E Q(E,t) dE = 0.4 \), where \( E_{\text{min}} = 1 \) GeV and \( E_{\text{max}} = 500 \) TeV are the minimum and maximum energies of the accelerated positrons in the PW, respectively. When calculating the energy losses \( b(E) \), cosmic microwave background (\( T_{\text{CMB}} = 2.7 \) K, \( U_{\text{CMB}} = 0.26 \) eV cm\(^{-3}\)), infrared (\( T_{\text{IR}} = 20 \) K, \( U_{\text{IR}} = 0.3 \) eV cm\(^{-3}\)) and optical (\( T_{\text{optic}} = 5000 \) K, \( U_{\text{optic}} = 0.3 \) eV cm\(^{-3}\)) photon fields are taken into account. The interstellar magnetic field \( B \) is assumed to be equal to 3 \( \mu \)G.

The positron fluxes at energies of about 1 TeV are several times smaller than those observed in the AMS-02 experiment for both our model and the model of Tang & Piran [15] (for any parameters of the source and the particle diffusion in the interstellar medium considered in their paper). The entire positron flux can be explained by a set of pulsars, including PSR J0437–4715, the Geminga pulsar, PSR B0656+14 and possibly a few others [10]. In the same time, as one can see from Figures 1 and 2, the difference in the expressions (5) and (8) leads to a noticeable difference in the simulated positron fluxes from the pulsar. The differences in the fluxes can be even more significant for other model parameters. Accordingly, for the correctness of the simulation results one should use the expression (5).
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