Improved whale optimization algorithm based on the tent chaotic mapping and nonlinear convergence factor

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Abstract. To overcome the shortcomings of whale optimization algorithm (WOA) such as the slow convergence rate, and low convergence accuracy, and being easy to fall into the local optimum, an improved whale optimization algorithm based on the tent chaotic mapping and nonlinear convergence factor (TNWOA) is proposed. Firstly, in this algorithm, tent chaotic mapping, which enhances the diversity of the initialization population, is introduced into the initialization of population, therefore, the search space can be searched more thoroughly; Secondly, trigonometric function and the beta distribution are introduced in the convergence factor ‘a’, which balance the global search ability as well as local optimization ability and speed up the convergence speed of the algorithm. Simulation experiments on the four kinds of common test functions on CEC2017 show that under the same experimental conditions, the improved whale optimization algorithm improves the solution accuracy and convergence speed significantly, and its performance is obviously better than other smart optimization algorithms and other improved WOA algorithms.

1. Introduction

Optimization has always been a hot topic in many fields such as computer science, artificial intelligence and engineering practice. Researchers are inspired by the intelligent behavior (foraging methods, migration routes, mating choices, and information sharing mechanisms) of biological swarms in nature, and propose a variety of intelligent optimization algorithms [2]. Such algorithms mainly include Particle Swarm Algorithm (PSO) [3], Fruit fly Optimization algorithm (FOA) [4], Fireflies Algorithm (FA) [5], Bat Algorithm (BA) [6], Grey Wolf Optimization (GWO) [7], etc. In recent years, these optimization algorithms have played an important role in the optimization of complex functions and fully proved the powerful optimization performance.

In 2016, Mirjalili [1] was inspired by observing the foraging behavior of whales, and proposed a new swarm intelligence optimization algorithm—Whale Optimization Algorithm (WOA). The main idea of this algorithm is to solve the target problem by simulating the behavior of whale predation. It relies on simple concepts and contains fewer operators, so it has attracted the attention of many scholars.

At present, relevant researches on the WOA algorithm have been carried out successively. Khashan et al. [8] balanced the local and global search by using nonlinear and random "a" parameter changes and using the inertial weight strategy to update the parameter "c". [9] proposed a whale optimization
algorithm based on adaptive adjustment of weights and search strategies, which improved the convergence speed of the algorithm and the ability to break away from the local optimum; Mohammed [10] paved a way to present a new technique by hybridizing both WOA and BAT algorithms. The BAT algorithm is used for the exploration phase, whereas the WOA algorithm is used for the exploitation phase. Statistical results obtained from WOA-BAT are very competitive and better than WOA; Sun [11] introduced chaos into the initialization process of WOA, and proposed a chaos method based on the concept of chaos, which used the characteristics of chaos to improve the diversity of searching objects and the self-centeredness of search objects. However, due to the late proposal of WOA, its theoretical analysis is still in its infancy. In the high-dimensional and multi-extremum complex optimization problem, it also has problems such as low convergence accuracy and easy to fall into local optimality.

The aim of the paper is to introduce whale optimization algorithm based on tent chaotic mapping and convergence factor (TNWOA). When the population is initialized, in order to ensure the convergence speed of the algorithm, the tent chaotic map is introduced, which enhances the diversity of the initialized population and can exploit the search space more comprehensively. Secondly, the introduction of trigonometric functions and beta distribution into the convergence factor balances the algorithm's global search capabilities and local optimization capabilities, and accelerates the algorithm's convergence speed; Finally, The TNWOA has been tested on the four types of test functions commonly used in CEC2017. Under the same experimental conditions, TNWOA significantly improves the solution accuracy and convergence speed, and its performance is better than other intelligent algorithms and other improved whale optimization algorithms.

The rest of the paper is organized as follows. In Section 2, review of the mathematical model of WOA is presented. Improved whale optimization algorithm based on the tent chaotic mapping and convergence factor is proposed in Section 3. In Section 4, simulation experiments and result analysis have been described. Finally, the last section summaries our work.

2. Mathematical model of Whale Optimization Algorithm

WOA is to imitate the spiral bubble network strategy of humpback whales and forage through three mechanisms: spiral predation, shrinking enclosing mechanism and random predation [1]. WOA's mathematical model is described in the following sections.

2.1. Shrinking and enclosing mechanism

During predation, whales need to locate their prey the location of their prey in order to surround and catch it, but the position in the search space is usually unknown. WOA assumes that the optimal position in the current population is a prey, and all other whale individuals update their positions based on the current optimal whale position. This process can be expressed by a mathematical model as:

$$\vec{D} = \left| \vec{C} \times \vec{X}_p(t) - \vec{X}(t) \right|$$

$$\vec{X}(t+1) = \vec{X}_p(t) - A \cdot \vec{D}$$

(1)

(2)

where $t$ is iteration, $\vec{X}_p(t)$ is the best position in the current population, $\vec{A}$ and $\vec{C}$ are the coefficient vector used to control the movement effect of the search agents, which can be described as:

$$\vec{A} = 2\vec{\alpha} \times \vec{r} - \vec{\alpha}$$

$$\vec{C} = 2 \cdot \vec{r}$$

(3)

(4)

Among them, $\vec{r}$ is random numbers in the range (0,1). $\vec{\alpha}$ represents the convergence factor, which gradually decreases from 2 to 0 as the iteration progresses:

$$\vec{\alpha} = 2(1 - t/T_{max})$$

(5)

where $T_{max}$ is the maximum number of iterations. With the decrease of $\vec{\alpha}$, the shrinking enclosing mechanism can realize the continuous contraction during whale predation.
2.2.  spiral predation
Whales, when hunting, spiral upward toward their prey. In WOA, the mathematical model of the spiral updating mechanism is described as follows:
\[ \tilde{X}(t+1) = \tilde{D}\cdot e^{it}\cdot \cos(2\pi t) + \tilde{X}_p(t) \]  \hfill (6)

Among them, \( b \) is a constant defining the spiral shape; \( t \) is a random number between \([-1,1]\); \( \tilde{D} \) denotes the distance between the optimal individual before the update and the optimal position:
\[ \tilde{D} = \left| \tilde{X}_p(t) - \tilde{X}(t) \right| \]  \hfill (7)

Whales move toward their prey along a spiral path while shrinking around. This behavior is the predation of whales. In WOA, when \(| A |<1\), the whale searches for the best in the encircled circle and performs a local optimal search. To implement the mathematical model, the algorithm first generates a random number \( p \) in \([0,1]\), assuming that the use probabilities of the two models are equal. If \( p<0.5 \), select the shrinking enclosing mechanism. If \( p\geq0.5 \), the spiral update mechanism is selected. The mathematical model of location update is as follows:
\[
\begin{align*}
\tilde{X}(t+1) &= \begin{cases} 
\tilde{X}_p(t) - \tilde{A} \cdot \tilde{D}, & p < 0.5 \\
\tilde{D}\cdot e^{it}\cdot \cos(2\pi t) + \tilde{X}_p(t), & p \geq 0.5
\end{cases}
\end{align*}
\]  \hfill (8)

2.3.  random predation
In fact, in addition to shrink and enclose the prey, when \(| A |\geq1\), the algorithm randomly selects an individual whale in the current whale swarm as the global optimal solution, and other whales flock to it. The mathematical model of random predation is described as:
\[ \tilde{X}(t+1) = \tilde{X}_{\text{rand}}(t) - \tilde{A} \cdot \tilde{C} \cdot \tilde{X}_{\text{rand}}(t) - \tilde{X}(t) \]  \hfill (9)

where, \( \tilde{A} \) has the same meaning as Eq (3), \( \tilde{X}_{\text{rand}}(t) \) indicates a randomly selected position vector from the search agent. It can be seen from Eq (9) that the search agent can realize the process of whale randomly finding prey by selecting random vectors.

3.  Improved whale optimization algorithm based on the tent chaotic mapping and convergence factor

3.1.  Tent chaotic mapping
Before the basic WOA is iterated, the randomly generated initial population cannot effectively ensure that the search agent is uniformly distributed in the search space, resulting in a decrease in search efficiency during the iteration process [12].

Chaotic systems have the following dynamic characteristics: sensitivity to initial conditions, ergodicity and randomness. Based on these characteristics of chaotic systems, population diversity can be enhanced, so as to avoid falling into the local optimal state and improve the quality of searching for global optimality. In summary, in order to make the initial population individuals make the best use of the information of the solution space, this paper introduces the tent mapping from chaos theory into the population initialization of the improved WOA algorithm. The mathematical model of tent chaotic mapping is as follows:
\[
\begin{align*}
x_{i+1} &= \frac{x_i}{\mu}, & 0 < x_i < \mu \\
x_{i+1} &= \frac{(1-x_i)}{(1-\mu)}, & \mu \leq x_i < 1
\end{align*}
\]  \hfill (10)

The tent mapping is a chaotic map within its parameter range, but when \( \mu = 0.5 \), the system presents a short period state, so in this article \( \mu \neq 0.5 \).
3.2. Nonlinear convergence factor
The parameter $\bar{A}$ in the original WOA algorithm is used to adjust the global exploration and local development capabilities of the algorithm. The convergence factor $\bar{a}$ decreases linearly during the iteration process, which easily makes the algorithm convergence too slow and unable to adapt to the actual situation. When $\bar{a}$ increases, the global exploration capability of the algorithm is enhanced; when $\bar{a}$ decreases, the local optimization capability of the algorithm is stronger. Trigonometric function and the beta distribution are introduced in the convergence factor, which balance the global search ability as well as local optimization ability and rise the convergence speed of the algorithm.

The beta distribution is a density function that satisfies the binomial distribution and the Bernoulli distribution [13], and its distribution is between [0,1]. The beta function formula and probability distribution function are:

$$B(h_1, h_2) = \int_{0}^{1} t^{h_1-1}(1-t)^{h_2-1} dt, h_1 > 0, h_2 > 0$$ (11)

$$f(x) = \frac{x^{h_1-1}(1-x)^{h_2-1}}{B(h_1, h_2)}, 0 < x < 1$$ (12)

In summary, this article proposes an adjustment strategy for the convergence factor:

$$a = 2 - 2\cos(2\pi \cdot \frac{t}{T_{max}}) + 0.2B(h_1, h_2)$$ (13)

3.3. Pseudo code and discussion of TNWOA
In summary, the pseudo code of the proposed TNWOA approach summarized as follows:

Initialize the algorithm parameters

Initialize population using the proposed chaotic mapping $\{X_i, i=1,2,..., N\}$ according to Eq (10)

Calculate the fitness of each search agent $\{F(X_i), i=1,2,..., N\}$

$S$=the best solution

while $(t<T_{max})$

for $i=1$ to $N$

Calculate the value of convergence factor $a$ according to Eq (13)

Update $A$, $C$, $l$ and $p$

if1 $(p<0.5)$ do

if2 $(|A|<1)$ do

Update the position of the current search agent according to Eq (8)

else if2 $(|A|\geq1)$

Select a random search agent

Update the position of the current search agent according to Eq (9)

end if2

else if1 $(p\geq0.5)$

Update the position of the current search agent according to Eq (8)

end if1

end for

Updates the position of the current search agent

$t = t + 1$

end while

return $S$

4. Simulation experiments and result analysis

4.1. Benchmark functions
To evaluate TNWOA performance a suite of six benchmark functions from CEC2017 have been chosen to carry on the simulation experiment. The average value of the optimal value of each test function is selected to evaluate the optimization performance of the algorithm. All details of these functions are demonstrated in Table 1.

| Number | Function type                  | Name                     | Domain     | Optimal solution |
|--------|--------------------------------|--------------------------|------------|------------------|
| f1     | Unimodal Functions             | Bent Cigar               | [-100,100] | 100              |
| f2     | Simple Multimodal Functions    | Non-Continuous Rastrigin’s Function | [-100,100] | 800              |
| f3     | Hybrid Functions               | Hybrid Function 4 (N = 4) | [-100,100] | 1400             |
| f4     | Hybrid Functions               | Hybrid Function 10 (N = 6) | [-100,100] | 2000             |
| f5     | Composition Functions          | Composition Function 5 (N=5) | [-100,100] | 2500             |
| f6     | Composition Functions          | Composition Function 10 (N=3) | [-100,100] | 2900             |

4.2. Parameter setting
In this study, the population of all comparison algorithms is set as 30. Furthermore, in order to increase the difficulty of searching for optimization, the dimension is set as 50 and the maximum number of iterations is set as 1500. According to [15], the parameters of beta distribution in TNWOA algorithm are set as $h_1 = 1, h_2 = 2$.

4.3. Comparative Experiment
In order to verify the effectiveness of TNWOA, it is compared with several well-established original algorithms: PSO [3], WOA [1]. In the meantime, TNWOA is further compared with some advanced algorithms, including MSWOA [14], WOABAT [10]. In order to eliminate the random influence of the algorithm, the algorithm was run independently for 30 times. Then the mean and the variance over these 30 runs presented in Table 2, where the best solution has been marked in bold and * mark. As can be seen from Table 2, TNWOA is superior to other competitors in these benchmark functions.

| Function | PSO | WOA | MSWOA | WOABAT | TNWOA |
|----------|-----|-----|-------|--------|-------|
| f1       | 9.77E+10 | 1.03E+20 | 3.73E+09 | 1.66E+18 | 3.31E+10 | 2.46E+19 | 2.08E+10 | 4.23E+19 | 9.59E+08* | 5.56E+16* |
| f2       | 1.44E+03 | 1.59E+03* | 1.32E+03 | 1.03E+04 | 1.35E+03 | 1.74E+03 | 1.32E+03 | 1.93E+03 | 1.25E+03* | 4.91E+03 |
| f3       | 1.36E+08 | 3.94E+12 | 2.55E+06 | 3.09E+12* | 1.98E+07 | 2.47E+14 | 5.32E+06 | 4.31E+13 | 2.21E+06* | 3.94E+12 |
| f4       | 1.44E+08 | 3.41E+04 | 3.82E+03 | 1.78E+05 | 3.73E+03* | 1.23E+05 | 4.17E+03 | 1.63E+05 | 3.80E+03 | 1.14E+05* |
| f5       | 1.33E+04 | 2.70E+06 | 3.78E+03 | 4.11E+04 | 6.46E+03 | 4.12E+05 | 5.50E+03 | 3.72E+05 | 3.48E+03* | 6.33E+03* |
| f6       | 5.28E+04 | 4.64E+09 | 8.51E+03 | 1.57E+06 | 1.38E+04 | 1.23E+07 | 1.16E+04 | 1.16E+07 | 7.89E+03* | 1.18E+06* |

The mean value reflects the convergence of the algorithm. In Table 2, we can see that except the effect on f4 is not as good as MSWOA, the search results of TNWOA are stronger than original WOA, PSO, MSWOA, WOABAT on other benchmark functions. TNWOA’s optimization accuracy for proposed functions is several orders of magnitude higher than other algorithms. From this we can know TNWOA has better convergence and higher convergence accuracy. The variance reflects the deviation degree between the results of 30 runs of algorithm and the mean. A small variance indicates that the degree of dispersion of the experimental results is low and the experimental results are more stable.
From the results of function calculations, the variance of TNWOA is slightly inferior in f2 and f3. In other functions, the TNWOA’s calculation results of other functions are stronger than that of the above algorithms, which shows that compared with other algorithms, TNWOA’s experimental results are concentrated in a smaller range and the algorithm is more stable.

Based on the results, TNWOA enhanced the optimization search in term of speed and accuracy. Figure 1 (a) ~ (f) show the convergence process of five different intelligent optimization algorithms for solving functions. f1 is a unimodal function. From (a), it can be seen that the TNWOA algorithm has better convergence than the other four algorithms, and the search performance is better in the later stage, so the final optimization result is better; f2 is a multimodal function. It can be seen from (b) that its convergence effect is better, it does not fall into the local optimum and jumps out of the premature situation; f3 and f4 are Hybrid functions. As shown in (c) and(d), TNWOA converges fast in the later stage of the optimization process of the function, jumps out of the local optimum, and has higher optimization accuracy; f5 and f6 are composition function, it can be seen that TNWOA has higher optimization accuracy in this function and faster convergence speed.

Fig1. Convergence trends of TNWOA and other selected algorithms
In general, from the above experimental results and analysis, TNWOA has better convergence and higher solution accuracy in solving four different kinds of functions. The main reason is that proposed RDWOA adopts tent chaotic mapping initialization and improvement of nonlinear convergence factor, which effectively improves the overall exploration ability and exploitation capability of the original WOA.

5. Conclusion
This article presented an improved whale optimization algorithm based on tent chaotic mapping and convergence factor (TNWOA). When the population is initialized, in order to ensure the convergence speed of the algorithm, the tent chaotic map is introduced, which enhances the diversity of the initialized population and can exploit the search space more comprehensively. Secondly, the introduction of trigonometric functions and beta distribution into the convergence factor balances the algorithm's global search capabilities and local optimization capabilities, and accelerates the algorithm's convergence speed.

TNWOA has been tested on the four types of test functions commonly used in CEC2017. Under the same experimental conditions, the TNWOA is significantly improved in solving accuracy and convergence speed, and its performance is significantly better than other intelligent algorithms and other improved whale optimization algorithms.

In the next step, the stability and robustness of TNWOA should be further improved. TNWOA performance will be further improved and applied to practical optimization problems.

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