Non-Abelian Gauge Lepton Symmetry
as the Gateway to Dark Matter

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Abstract

Following a previous proposal, lepton number is considered as the result of a spontaneously broken non-Abelian gauge $SU(2)_N$ symmetry. New fermions are added to support this new symmetry, the spontaneous breaking of which allows these new fermions to be part of the dark sector, together with the vector gauge boson which communicates between them and the usual leptons. A byproduct is a potential significant contribution to the muon anomalous magnetic moment.
Introduction: Lepton number is an automatic global U(1) symmetry in the minimal standard model (SM) of quarks and leptons. If right-handed neutrino singlets are added, $B - L$ (baryon number minus lepton number) may then become a gauge U(1) symmetry. Many other possible variants have been studied \[1, 2, 3, 4, 5, 6, 7, 8, 9, 10\], but all involve U(1) symmetries. The first hint that leptons may be extended to a non-Abelian $SU(2)_N$ symmetry was based \[11\] on $E_6$, where only the left-handed lepton doublet is involved. It was then realized \[12, 13, 14\] that the vector gauge boson in $SU(2)_N$ and the new fermions which it links with the SM leptons could be dark matter. To go one step further, the lepton singlet $e_R$ was also proposed \[15\] to be part of a doublet under $SU(2)_N$, together with an added $\nu_R$. Thus $SU(2)_N$ acts as the gateway to a dark sector, reaffirming the notion that leptons are the key \[7, 16\] to understanding dark matter.

In this framework \[15\], the SM leptons transform under $SU(2)_N$ together with their partners $(N, E), N', E'$, which will be shown to belong to the dark sector, after spontaneous breaking of $SU(2)_N$. The residual conserved symmetry is generalized global lepton number, under which

$$\nu, e \sim 1, \quad N, E, N', E' \sim 0,$$

with the two sectors connected through the $SU(2)_N$ gauge analog (call it $X$) of the $W$ boson of the SM. This means that $X$ also has lepton number and belongs to the dark sector.

The scalar sector is very minimal, consisting only of the SM doublet, and a corresponding $SU(2)_N$ doublet. Just as the former results in heavy $W^\pm, Z$ gauge bosons, the latter yields heavy $X_{1,2,3}$ gauge bosons. The residual physical scalars are then just the SM Higgs boson $h$ and the corresponding $H$ of $SU(2)_N$.

In the following, the consequences of this new extension of the SM will be discussed, regarding to dark-matter phenomenology \[17\], as well as its contributions to the muon anomalous magnetic moment \[18, 19, 20\] as an example.
Model: Under $SU(2)_L \times U(1)_Y \times SU(2)_N$, the SM leptons $\nu, e$ and their $SU(2)_N$ partners $N, E$ and $N', E'$ transform as

$$\begin{pmatrix} \nu \\ e \\ N \end{pmatrix}_L \sim (2, -1/2; 2), \quad \begin{pmatrix} N \\ E \end{pmatrix}_R \sim (2, -1/2; 1),$$

$$\begin{pmatrix} e, E' \end{pmatrix}_R \sim (1, -1/2), \quad E'_L \sim (1, -1; 1),$$

$$\begin{pmatrix} \nu, N' \end{pmatrix}_R \sim (1, 0; 2), \quad N'_L \sim (1, 0; 1).$$

It is easy to see that this gauge extension is free of anomalies because each new left-handed fermion is balanced by an appropriate right-handed counterpart. The scalar sector is minimally simple. It has just the usual SM doublet $\Phi$ plus an $SU(2)_N$ doublet $\chi$:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \sim (2, 1/2; 1), \quad \chi = (\chi_1, \chi_2) \sim (1, 0; 2).$$

The allowed Yukawa couplings are

$$\mathcal{L}_Y = f_e [\bar{e}_R (\nu_L \phi^- + e_L \phi^0) + \bar{E}'_R (N_L \phi^- + E_L \phi^0)] + f_\nu [\nu_R (\nu_L \phi^0 - e_L \phi^+)] + \bar{N}'_R (N_L \phi^0 - E_L \phi^+)$$

$$+ f_0 [\bar{N}_R (N_L \chi_1 - \nu_L \chi_2) + \bar{E}_R (E_L \chi_1 - e_L \chi_2)] + f_E E'_L (E'_R \chi_1 - e_R \chi_2)$$

$$+ f_N \bar{N}'_L (N'_R \chi_1 - \nu_R \chi_2) + f'_E \bar{E}'_L (N_R \phi^- + E_R \phi^0) + f'_N \bar{N}'_L (N_R \phi^0 - E_R \phi^+) + H.c.$$ (6)

Let $\langle \phi^0 \rangle = v$ and $\langle \chi_1 \rangle = u$, then

$$m_e = f_e v, \quad m_\nu = f_\nu v,$$ (7)

whereas the $2 \times 2$ mass matrices for $(E, E')$ and $(N, N')$ are

$$\mathcal{M}_{E,E'} = \begin{pmatrix} f_0 u & m_e \\ f'_E v & f_E u \end{pmatrix}, \quad \mathcal{M}_{N,N'} = \begin{pmatrix} f_0 u & m_\nu \\ f'_N v & f_N u \end{pmatrix}.$$ (8)

These two fermion sectors are clearly separated. They are however connected through the $SU(2)_N$ gauge boson which takes $\chi_1$ to $\chi_2$, as well as $e$ to $E, E'$ and $\nu$ to $N, N'$. If $e, \nu$ are assigned residual lepton number $L = 1$, then the $L$ assignment of $E, N, E', N'$ could be set equal to $n \neq 1$, and that of $\chi_2$ to $n - 1$. A convenient choice is $n = 0$, then the dark
fermions have $L = 0$ and the dark gauge boson has $L = -1$. Equivalently, a dark charge may be defined, under which $E, N, E', N'$ have $D = 0$ together with all other SM particles as well as $X_3 = Z'$ and the one Higgs boson $H$ from $SU(2)_N$ breaking, whereas the $(X_1 - i X_2)/\sqrt{2} = X$ gauge boson has $D = 1$. This is then analogous to the notion of electric charge in the case of $SU(2)_L \times U(1)_{Y}$ with $(W_1 - iW_2)/\sqrt{2} = W$ and $W_3$ mixing with the $U(1)_{Y}$ gauge boson to form the neutral $Z$ and photon.

**Gauge and Scalar Sector**: The $SU(2)_L \times U(1)_{Y}$ gauge sector is as in the SM. The $SU(2)_N$ gauge sector is separate but is analogous to just $SU(2)_L$ alone. The $X_{1,2,3}$ gauge bosons obtain masses from $\langle \chi_1 \rangle = u$, resulting in

$$m_{X_{1,2,3}}^2 = \frac{1}{2} g_N^2 u^2.$$  \hspace{1cm} (9)

The simple Higgs potential is

$$V = \mu_1^2 \Phi^\dagger \Phi + \mu_2^2 \chi^\dagger \chi + \frac{1}{2} \lambda_1 (\Phi^\dagger \Phi)^2 + \frac{1}{2} \lambda_2 (\chi^\dagger \chi)^2 + \lambda_3 (\Phi^\dagger \Phi)(\chi^\dagger \chi).$$  \hspace{1cm} (10)

In terms of the physical Higgs bosons $h = \sqrt{2}[Re(\phi^0) - v]$ and $H = \sqrt{2}[Re(\chi_1) - u]$, it becomes

$$V = \lambda_1 v^2 h^2 + \lambda_2 u^2 H^2 + 2 \lambda_3 v u h H + \frac{1}{\sqrt{2}} \lambda_1 v h^3 + \frac{1}{8} \lambda_1 h^4$$

$$+ \frac{1}{\sqrt{2}} \lambda_2 u H^3 + \frac{1}{8} \lambda_2 H^4 + \frac{1}{\sqrt{2}} \lambda_3 v h H^2 + \frac{1}{\sqrt{2}} \lambda_3 u H h^2 + \frac{1}{4} \lambda_3 h^2 H^2.$$  \hspace{1cm} (11)

Assuming that $\lambda_3$ is small, then $h$ and $H$ are almost mass eigenstates with $m_h^2 = 2\lambda_1 v^2$ and $m_H^2 = 2\lambda_2 u^2$.

The fermion interactions with $X$ and $Z'$ are easily read off from Eqs. (2)-(4), i.e.

$$\mathcal{L}_{int} = \frac{1}{\sqrt{2}} g_N X_{\mu} [\tilde{N}_L \gamma^\mu \nu_L + \tilde{E}_L \gamma^\mu e_L + \tilde{E}'_L \gamma^\mu e_R + \tilde{N}'_R \gamma^\mu \nu_R] + H.c.$$  

$$+ \frac{1}{2} g_N Z'_{\mu} [\tilde{N}_L \gamma^\mu N_L + \tilde{E}_L \gamma^\mu E_L + \tilde{E}'_L \gamma^\mu E_R' + \tilde{N}'_R \gamma^\mu N'_R - \tilde{\nu} \gamma^\mu \nu + \tilde{e} \gamma^\mu e].$$  \hspace{1cm} (12)
**Dark Sector**: Because of the structure of the particles under the $SU(2)_N$ gauge symmetry, a dark U(1) symmetry remains after its spontaneous breaking. The SM particles have $D = 0$. Of the new particles, the $N, E, N', E'$ fermions and the vector boson $X$ have $D = 1$, whereas the Higgs boson $H$ and the vector boson $Z'$ have $D = 0$. The natural dark-matter candidate is $X$. [The $N'$ fermion may also be considered [15], but its mixing in Eq. (8) to $N$ must be suppressed so that it has negligible coupling to the SM $Z$ boson to avoid direct-search constraints.] Since the coupling of $X \bar{X}$ to $Z'$ vanishes for $X, \bar{X}$ at rest, whereas the $X \bar{X} H$ coupling does not, its relic abundance comes from $X \bar{X} \to HH$ annihilation [21], assuming $H$ to be lighter than $X$, as shown in Fig. 1. The sum of these diagrams (with a factor of 2 in the last one) for $X \bar{X}$ annihilation at rest is

\[
A = \frac{g_N^2}{2} \left[ 1 + \frac{3m^2_H}{4m^2_X - m^2_H} - \frac{4m^2_X}{2m^2_X - m^2_H} \right] \bar{\epsilon}_1 \cdot \bar{\epsilon}_2 + \frac{2g_N^2}{2m^2_X - m^2_H} (\bar{\epsilon}_1 \cdot \bar{k})(\bar{\epsilon}_2 \cdot \bar{k}), \tag{13}
\]

where $\bar{\epsilon}_{1,2}$ are the polarization vectors of $X, \bar{X}$ and $\bar{k}$ is the momentum of $H$.

The annihilation cross section times relative velocity is

\[
\sigma_{\text{ann}} \times v_{\text{rel}} = \frac{g_N^4 \sqrt{1 - r}}{576 \pi m^2_X} \left[ 3A^2 + 2AB(1 - r) + B^2(1 - r)^2 \right], \tag{14}
\]

where $r = m^2_H/m^2_X$ and

\[
A = \frac{1}{2} \left[ 1 + \frac{3r}{4 - r} - \frac{4}{2 - r} \right], \quad B = \frac{2}{2 - r}. \tag{15}
\]
Once produced, $H$ decays through its very small mixing with $h$ to SM particles. For $m_H/m_X = 0.8$, the typical value of $3 \times 10^{-26}$ cm$^3$/s for the correct relic abundance is obtained for $m_X/g_N^2 = 352.5$ GeV. Assuming $m_X = 210$ GeV to be above the highest energy of the LEP II $e^+e^-$ collider, $g_N = 0.77$ is obtained.

The interactions of $X$ with the SM leptons are shown in Eq. (12), and through the $Z'$ gauge boson. In underground direct-search experiments using nuclear recoil, only the Higgs exchange is applicable, which occurs through $h - H$ mixing. The spin-independent cross section for elastic scattering off a xenon nucleus is

$$\sigma_0 = \frac{1}{\pi} \left( \frac{m_X m_{Xe}}{m_X + m_{Xe}} \right)^2 \left| \frac{54 f_p + 77 f_n}{131} \right|^2,$$

where

$$\frac{f_p}{m_p} = \left[ 0.075 + \frac{2}{27} (1 - 0.075) \right] \frac{2 \lambda_3 m_X}{m_H^2},$$

$$\frac{f_n}{m_n} = \left[ 0.078 + \frac{2}{27} (1 - 0.078) \right] \frac{2 \lambda_3 m_X}{m_H^2}.$$  

For $m_X = 210$ GeV, and $m_{Xe} = 122.3$ GeV, the upper limit on $\sigma_0$ is $2 \times 10^{-46}$ cm$^2$. Using $m_h = 125$ GeV and $m_H = 168$ GeV, the $h - H$ mixing parameter $\lambda_3$ is then less than $1.27 \times 10^{-4}$.

Muon Anomalous Magnetic Moment: Since $Z'$ and $X$ couple to leptons as shown in Eq. (12), there are contributions to the muon anomalous magnetic moment. The $Z'$ contribution is

$$\Delta a_\mu(Z') = \frac{(g_N/\sqrt{2})^2 m_\mu^2}{12\pi^2 m_{Z'}^2} = \frac{m_\mu^2}{24\pi^2 u^2}.$$  

If $E$ and $E'$ do not mix, the $X$ contribution is

$$\Delta a_\mu(X) = \frac{2(g_N/\sqrt{2})^2 m_\mu^2}{32\pi^2 m_X^2} = \frac{m_\mu^2}{16\pi^2 u^2}.$$  

where $m_E = m_{E'} = m_X$ has been assumed for convenience. Using $g_N = 0.77$ and $m_X =$
\[ m_{Z'} = 210 \text{ GeV} \] which imply \[ u = 385.7 \text{ GeV} \], their sum is \[ 7.9 \times 10^{-10} \]. This is of the right sign, but not large enough to account for the observed discrepancy \[ 18, 19 \] of \[ 25.1 \pm 5.9 \times 10^{-10} \].

Consider now \( E - E' \) mixing for the muon. Neglecting \( m_\mu \) in the \( E_L E'_R \) term, the \( 2 \times 2 \) mass matrix shown in Eq. (8) linking \( (E, E')_L \) to \( (E, E')_R \) is of the form

\[
M_{E,E'} = \begin{pmatrix} m_0 & 0 \\ m' & m_E \end{pmatrix}.
\] (21)

This is diagonalized by two unitary matrices, one on the left and one on the right. For illustration and simplicity, let \( m_0^2 = m_E^2 + m'^2 \), then the mass-squared eigenvalues are

\[
m_1^2 = m_0^2 + m'm_0, \quad m_2^2 = m_0^2 - m'm_0,
\] (22)
corresponding to the eigenstates \( E_{1,2} \)

\[
E_{1L} = \frac{1}{\sqrt{2}}(E_L + E'_L), \quad E_{2L} = \frac{1}{\sqrt{2}}(-E_L + E'_L),
\] (23)
\[
E_{1R} = (c_R E_R + s_R E'_R), \quad E_{2R} = (-s_R E_R + c_R E'_R),
\] (24)
where

\[
s_R = \frac{m_2}{\sqrt{2}m_0} = \sqrt{\frac{1 - r'}{2}}, \quad c_R = \frac{m_1}{\sqrt{2}m_0} = \sqrt{\frac{1 + r'}{2}},
\] (25)
with \( r' = m'/m_0 \). Whereas the \( X \) couplings to \( E(E') \) are purely left(right)-handed, the fact that they are no longer mass eigenstates results in an additional contribution to \( \Delta a_\mu \) which is enhanced \[ 20, 24 \] by \( m_{E_{1,2}}/m_\mu \). However, the effect must be proportional to \( r' \) because \( m' = 0 \) is the limit of no mixing.

The key factor is

\[
\frac{sr_{m_1}}{1 - x + (m_1^2/m_X^2)x} - \frac{cr_{m_2}}{1 - x + (m_2^2/m_X^2)x} = -\frac{\sqrt{2}m_1m_2r'x}{m_X},
\] (26)
where \( x \) is an integration variable in the formula for \( \Delta a_\mu \), and \( r' \ll 1 \) is assumed with \( m_X \approx m_0 \). The contribution from \( E - E' \) mixing is then

\[
\Delta a_\mu(E_{1,2}) = \frac{(g_N/\sqrt{2})^2 m_\mu r'}{24\pi^2 m_X^2} = \frac{m_\mu^2}{24\pi^2 u^2} \left( \frac{m'}{m_\mu} \right).
\] (27)
For $m'/m_\mu = 5.43$, this addition would make the above equal to $17.2 \times 10^{-10}$, thus explaining fully the muon $g - 2$ discrepancy.

**Concluding Remarks**: A simple $SU(2)_N$ gauge symmetry is studied, under which leptons transform. Together with new fermions $(N, E), N', E'$ and an $SU(2)_N$ Higgs doublet which renders the $X_{1,2,3}$ gauge bosons of $SU(2)_N$ heavy, this model results in a dark sector, so that $X = (X_1 - iX_2)/\sqrt{2}$ becomes vector dark matter. A numerical example is given with $m_X = 210$ GeV, for which the various contributions to the muon anomalous magnetic moment may add up to the observed discrepancy.

**Acknowledgement**: This work was supported in part by the U. S. Department of Energy Grant No. DE-SC0008541.

**References**

[1] X.-G. He, G. C. Joshi, H. Lew, and R. R. Volkas, Phys. Rev. **D43**, 22 (1991).

[2] E. Ma, Phys. Lett. **B433**, 74 (1998).

[3] E. Ma, D. P. Roy, and S. Roy, Phys. Lett. **B525**, 101 (2002).

[4] P. Fileviez Perez and M. Wise, Phys. Rev. **D82**, 011901 (2010).

[5] M. Duerr, P. Fileviez Perez, and M. Wise, Phys. Rev. Lett. **110**, 231801 (2013).

[6] C. Kownacki, E. Ma, N. Pollard, and M. Zakeri, Phys. Lett. **B766**, 149 (2017).

[7] E. Ma, Phys. Lett. **B809**, 135736 (2020).

[8] E. Ma, Phys. Lett. **B813**, 136066 (2021).

[9] J.-Y. Cen, Y. Cheng, X.-G. He, and J. Sun, arXiv:2104.05006 [hep-ph].
[10] E. Ma, arXiv:2104.10324 [hep-ph].

[11] D. London and J. L. Rosner, Phys. Rev. D34, 1530 (1986).

[12] J. L. Diaz-Cruz and E. Ma, Phys. Lett. B695, 264 (2011).

[13] S. Bhattacharya, J. L. Diaz-Cruz, E. Ma, and D. Wegman, Phys. Rev. D85, 055008 (2012).

[14] E. Ma and J. Wudka, Phys. Lett. B712, 391 (2012).

[15] B. Fornal, Y. Shirman, T. M. P. Tait, and J. R. West, Phys. Rev. D96, 035001 (2017).

[16] E. Ma, Phys. Rev. Lett. 115, 011801 (2015).

[17] For a review, see for example G. Bertone and D. Hooper, Rev. Mod. Phys. 90, 045002 (2018).

[18] B. Abi et al. (Muon g-2 Collaboration), Phys. Rev. Lett. 126, 141801 (2021).

[19] T. Aoyama et al., Phys. Rep. 887, 1 (2020).

[20] F. S. Queiroz and W. Shepherd, Phys. Rev. D89, 095024 (2014).

[21] E. Ma, Phys. Lett. B772, 442 (2017).

[22] J. Hisano, K. Ishiwata, N. Nagata, and T. Takesako, JHEP 1107, 005 (2011).

[23] E. Aprile et al. (XENON Collaboration), Phys. Rev. Lett. 121, 111302 (2018).

[24] T. Hambye, K. Kannike, E. Ma, and M. Raidal, Phys. Rev. D75, 095003 (2007).