Analysis of nonlinear oscillation of four-quadrant converter based on discrete describing function approach

Fei Lin, Shihui Liu, Zhongping Yang, and Hu Sun
School of Electrical Engineering, Beijing Jiaotong University.
No. 3 ShangyuanCun, Haidian District, Beijing, China
a) flin@bjtu.edu.cn

Abstract: The mechanism and characteristic of current oscillation of four-quadrant converter with predictive current controller caused by nonlinear characteristic of the system is analyzed in the paper. With the discrete model considering the digital control delay of the single-phase four-quadrant converter, the stability condition is derived. Using the describing function of saturation characteristic, the frequency and amplitude of the nonlinear oscillation is obtained. The simulation and experiment results verify the analyses.

Keywords: four-quadrant converter, nonlinear oscillation, current control, describing function

Classification: Electron devices, circuits and modules

References

[1] T. Kerekes, et al.: “A new high-efficiency single-phase transformerless PV inverter topology,” IEEE Trans. Ind. Electron. 58 (2011) 184 (DOI: 10.1109/TIE.2009.2024092).
[2] G. W. Chang, et al.: “Modeling characteristics of harmonic currents generated by high-speed railway traction drive converters,” IEEE Trans. Power Deliv. 19 (2004) 766 (DOI: 10.1109/TPWRD.2003.822950).
[3] M. Mitzenmacher: “Presentation of a four-quadrant converter based system in traction applications — reference to modeling, simulation and analysis,” EPE 2007 (2008) 397 (DOI: 10.1109/EPE.2007.4417646).
[4] E. Möllerstedt and B. Bernhardsson: “Out of control because of harmonics—an analysis of the harmonic response of an inverter locomotive,” IEEE Control Systems 20 (2000) 70 (DOI: 10.1109/37.856180).
[5] J. Gao, et al.: “Improved deadbeat current controller with a repetitive-control-based observer for PWM rectifiers,” J. Power Electron. 11 (2011) 64 (DOI: 10.6113/JPE.2011.11.1.064).
[6] J. Gao, et al.: “Improved predictive current controller for four-quadrant converters,” IPEMC (2009) 1719 (DOI: 10.1109/IPEMC.2009.5157670).
[7] M. Brema, et al.: “New stability analysis for Tuning PI controller of power converters in railway application,” IEEE Trans. Ind. Electron. 58 (2011) 533 (DOI: 10.1109/TIE.2010.2047823).
[8] O. Kukrer and H. Komurcugil: “Control strategy for single-phase PWM rectifiers,” Electron. Lett. 33 (1997) 1745 (DOI: 10.1049/el:19971188).
1 Introduction

The four-quadrant converter (4QC) has the advantages of high power factor, low harmonic current and high regenerative braking performance, which has caused wide attention [1, 2, 3]. It is widely used in the field of unit factor rectification, industrial DC power supply, AC drives and so on. Especially, the 4QC is the standard rectifier for high-speed trains and locomotives.

However, there are some actual operation accidents caused by nonlinear oscillation in railway application. In April 1995, Zurich, Switzerland, harmonic current (165 Hz) triggered multi-train protection devices, thus trains had to stop running [4]; in 2007, in Jixian South traction substation of the Beijing-Harbin line power supply section, China, a strong oscillation (accident 850∼1050 Hz) also underwent; in December 2009, in the power supply section between Hefei to Longcheng in the Hefei-Wuhan line, China, harmonic current (850 Hz) amplification phenomenon appeared. Therefore, this kind of events must be paid attention to, and be studied through theory.

It’s shown in [5], [6] and [7] that these oscillations are related to the instability of the system. Four quadrant converter often uses double loop control, in which the inner loop is current loop [8]. The relationship between the control parameters and the stability of the 4QC is studied. These papers, however, don’t reveal the mechanism of sustained oscillation of the system, and there is no research on the frequency of oscillation.

This paper builds discrete mathematic models of the 4QC. In view of the commonly used predictive current control, the current loop stability and its
influencing factors are analyzed. This article applies the describing function method to research the saturation characteristic of the 4QC, analyzes system oscillation mechanism and its frequency with the Nyquist graph. Simulation and experimental results validate the correctness of the theory.

2 Four-quadrant converter control and stability analysis

2.1 Predictive current control of four-quadrant converter

Fig. 1 is the main circuit structure of four-quadrant converter. Where, $S_1$-$S_4$ are semiconductor switching devices; $L$ is ac-side inductor; $C_d$ is dc support capacitor; $u_g$ is grid voltage, $u_r$ is converter ac-side voltage; $i_g$ and $i_n$ are respectively the grid current and load current; $u_{dc}$ and $i_{dc}$ are respectively the dc output voltage and dc current.

According to Kirchhoff Laws, the ac-side voltage equation of 4QC can be got as Eq. (1).

$$u_r = u_g - L \frac{di_g}{dt}$$  \hspace{1cm} (1)

Assuming that the 4QC switching period is a constant value $T_s$ and the current sampling time is $kT_s$, the discrete model of Eq. (1) can be written as Eq. (2) [9, 10, 11].

$$u_r^{av}(k | k + 1) = u_g^{av}(k | k + 1) - \frac{L}{T_s} [i_g(k + 1) - i_g(k)]$$  \hspace{1cm} (2)

In Eq. (2), $u_r^{av}(k | k + 1)$ and $u_g^{av}(k | k + 1)$ denote the average value of the converter ac-side voltage and the grid voltage over the switching period $[kT_s, (k + 1)T_s]$, respectively. $i_g(k)$ and $i_g(k + 1)$ denote the instantaneous value of the grid current at the sampling points $kT_s$ and $(k + 1)T_s$, respectively.

Assuming that the current peak instructions from dc outer voltage-loop is $I_m^{*}$ and phase angle of grid voltage in $kT_s$ moments is $\theta_k$, the aim of a current controller is to make the grid current $i_g^{*}(k + 1)$ at the sampling point $(k + 1)T_s$ is equal to $I_m^{*} sin\theta_{k+1}$. According to the basic principle of the conventional predictive current control strategy [12], the converter side reference voltage in this period can be written as Eq. (3).

$$u_r^{*}(k) = \hat{u}_g^{av}(k | k + 1) - \frac{L}{T_s} [i_g^{*}(k + 1) - i_g(k)]$$  \hspace{1cm} (3)

In Eq. (3), $\hat{u}_g^{av}(k | k + 1)$ denotes the average value of the grid voltage in the period $[kT_s, (k + 1)T_s]$ and it can be estimated from the previously measured values using
linear extrapolation. If \( u_g \) is equal to \( \hat{u}_g \), by Eq. (2) and Eq. (3), single cycle current control target that instantaneous current value \( i_g(k+1) \) is equal to instructions value \( i_g^*(k+1) \) is achieved. The control block diagram is shown in Fig. 2.

\[ C(z) = \frac{L^*}{T_s} \]  

The ratio between \( L^* \) and the actual inductance value \( L \) is \( k_L \), called inductance coefficient, written as Eq. (5).

\[ k_L = \frac{L^*}{L} \]

Current loop controlled object, \( G_p(s) \), can be written as Eq. (6). After discretization by ZOH, it can be written as Eq. (7).

\[ G_p(z) = \left( 1 - z^{-1} \right) \cdot Z \left[ \frac{G_p(s)}{s} \right] = \frac{T_s}{L} \cdot \frac{1}{z - 1} \]

\[ \text{2.2 Four quadrant converter stability analysis} \]

Without considering the saturation loop, the closed loop pulse transfer function of the current loop is written as Eq. (8).

\[ G_{C}(z) = \frac{zG(z)}{1 + G(z)} = \frac{C(z)G_p(z)}{1 + C(z)z^{-1}G_p(z)} = \frac{zk_L}{z^2 - z + k_L} \]

By calculating the characteristic equation, the current loop stable condition is \( 0 < k_L < 1 \). All the analysis below is based on this situation. In order to obtain good control performance of current loop, the accuracy of inductance coefficient \( k_L \) must be ensured when using predictive current control [13, 14, 15].

According to the theoretical analysis, when the inductance coefficient in the stable range, the current is stable; over the stable range, the current diverges.

\[ \text{3 Non-linear oscillation analyses with saturation characteristic} \]

In fact, the system can maintain the DC-link voltage near the reference value when the inductance coefficient is far beyond the stable range calculated above. Although not divergent, too large inductance coefficient will result in the voltage and current
oscillation, cause train-grid resonance and other issues. This phenomenon is explained by the describing function method of nonlinear system, and the stability of the system is further analyzed in the following.

Analyzed by the describing function method, nonlinear system is simplified into a typical structure connected by a nonlinear link and a linear closed loop system. As shown in Fig. 2, the control block diagram contains a nonlinear link considering saturation nonlinear characteristics. The describing function of the nonlinear link can be equivalent to a proportional element with complex variables gain. In this way, the nonlinear system is equivalent to a linear system, so frequency domain stability criterion of linear system can be used to analyze the stability of the nonlinear system. When the nonlinear characteristic is approximated by the describing function, the characteristic equation of the closed-loop system is written as Eq. (9).

\[
1 + N(X)G(Z) = 1 + N(X)C(z)z^{-1}G_p(z) = 1 + N(X) \frac{k_L}{z(z - 1)} = 0
\]

\(-1/N(X)\) is the negative-inverse description function of nonlinear link. According to the Nyquist stability criterion \([16]\), system stability can be judged by the relative position between \(N(X)G(Z)\) image and critical point \((-1 + j0)\). The intersection of the \(G(Z)\) curve and \(-1/N(X)\) curve shows the operating point of system self-sustained oscillation. Amplitude and frequency of the oscillation is determined by the intersection parameters. In nonlinear system, the self-sustained oscillation is also known as a limit cycle.

The nonlinear link named saturation block is shown in Fig. 2. The describing function of the saturation nonlinear characteristics is written as Eq. (10).

\[
N(X) = \begin{cases} 2k \left[ \arcsin \frac{S}{X} + \frac{S}{X} \sqrt{1 - \left(\frac{S}{X}\right)^2} \right], & (X \geq S) \\ k, & (X < S) \end{cases}
\]

In Fig. 2 and Eq. (10), \(S\) and \(M\) are the amplitude of input signal when the output is just saturated and the maximum instantaneous value of the output signal, respectively. \(k\) is the proportional coefficient in the unsaturated zone. In this problem, \(k\) is equal to one.

In before analysis, the current loop is stable when the inductance coefficient, \(k_L\), in the range of 0 to 1. Taking into account the nonlinear link, the stability of the system is reanalyzed. We have derived the describing function of the saturation nonlinear characteristics. Because the describing function of the saturation nonlinear characteristics has zero phase angle, \(-1/N(X)\) lies strictly on the negative real axis of the Nyquist plot to the left from \(-1\). Nyquist diagram of the nonlinear link and other part of the system are drew as follows.

In Fig. 3, \(k_L\) is 0.9. It’s less than previously determined stability boundary. There is no intersection. The system is stable by the previous analysis.

In Fig. 4, \(k_L\) is 1.1. It’s larger than previously determined stability boundary. The two curves intersect in \((-1.1, 0)\), this value reflects the oscillation amplitude. The closed-loop system is in critical stable state and system exists self-sustained oscillations internally.
Even if inductance coefficient $k_L$ larger than previously determined stability boundary, system will not diverge but keep critical stable with self-sustained oscillation. This explains the reasons why the system does not diverge, and also explains the mechanism of the system oscillation. The intersection of the two curves is the operating point of system self-sustained oscillation, amplitude and frequency of the oscillation are determined by the intersection parameters. The intersection shows the oscillation frequency is about $1330 \text{ rad/sec}$, when the sampling frequency of the 4QC is 1250 Hz in Fig. 4(a); the oscillation frequency is about $3650 \text{ rad/sec}$, when the sampling frequency is 3500 Hz in Fig. 4(b). The oscillation frequency is proportional to the switching frequency.

### 4 The time domain simulation

The simulation parameters are shown in Table I:

| Parameters                        | Value | Parameters                        | Value   |
|-----------------------------------|-------|-----------------------------------|---------|
| Input voltage (V)                 | 150   | Load resistance ($\Omega$)        | 200     |
| Output voltage (V)                | 300   | Ac inductance (H)                 | 7e-3    |
| Dc capacitor (F)                  | 3.3e-3| Switching frequency (Hz)         | 3500/1250 |

Fig. 3. Nyquist diagram of the nonlinear link and other part, $f = 3500 \text{ Hz}$, $k_L = 0.9$.

Fig. 4. Nyquist diagram of the nonlinear link and other part, $k_L = 1.1$, (a) $f = 1250 \text{ Hz}$, (b) $f = 3500 \text{ Hz}$.
When switching frequency is 3500 Hz, comparison of current waveforms and FFT analysis diagrams with different inductance coefficient, $k_L$, is shown in Fig. 5. With the increase of the inductance coefficient, current waveform keeps sinusoidal but the current distortion degree is increased. Total harmonic distortion (THD) increases from 11.27% to 118.69%. The amplitude value of fundamental wave is basically unchanged, but the amplitude of harmonic around 560 Hz is increasing rapidly. Current oscillates in this frequency. Once the switching frequency is determined, the oscillation frequency is also determined. The bigger the inductance coefficient, $k_L$, the more the harmonic content.

![Fig. 5. Current waveforms and FFT analysis diagrams when $f = 3500$ Hz, (a) $k_L = 0.9$, (b) $k_L = 1.1$.](image)

When inductance coefficient, $k_L$, is 1.1, comparison of current waveforms and FFT analysis diagrams with different switching frequency is shown in Fig. 6. When the switching frequency is 3500 Hz, current oscillation frequency is around 560 Hz; when the switching frequency is 1250 Hz, current oscillation frequency is around 200 Hz. It is consistent with the previous theoretical analysis, the oscillation

![Fig. 6. Current waveforms and FFT analysis diagrams when $k_L = 1.1$, (a) $f = 3500$ Hz, (b) $f = 1250$ Hz.](image)
frequency is proportional to the switching frequency when other conditions are all the same.

Combining simulation results shown in Table II and Nyquist diagrams above, the conclusion is as follows: With bigger inductance coefficient, $k_L$, the harmonic content is bigger; the oscillation frequency is proportional to the switching frequency when the other conditions are the same. Once inductance coefficient, $k_L$, is more than the critical stable value, system exists self-sustained oscillations.

5 The experimental results

In order to verify the analyses of current oscillations and the proposed characteristics of oscillating current, the experimental platform of four-quadrant converter is built. The experimental parameters are the same with simulation parameters, as shown in Table I. Switching frequency is 3500 Hz.

When the inductance is set to 3.5 mH in procedures, namely inductance coefficient $k_L$ is 0.5, the current waveform is stable as shown in Fig. 7. There are burrs on current waveform, but the current waveform basically keeps sine waveform.

When the inductance is set to 7 mH in the procedures, namely inductance coefficient $k_L$ is 1.0, the current waveform and FFT analysis of the waveform are shown in Fig. 8. Compared with the previous smooth waveform, the waveform appears obvious distortion.

It can be seen from Fig. 8 that harmonic amplitude significantly increased than before, odd harmonics exist obviously, the cursor is located in 550 Hz where harmonic amplitude is the biggest. The oscillation frequency is close to simulation

| Switching frequency, $f$, is 3500 Hz | Inductance coefficient, $k_L$, is 1.1 |
|--------------------------------------|---------------------------------------|
| Inductance coefficient | THD (%) | Switching frequency (Hz) | Oscillation frequency (Hz) |
|-------------------------|---------|-------------------------|-------------------------|
| 0.9                     | 11.27   | 3500                    | 560                     |
| 1.1                     | 118.69  | 1250                    | 200                     |

Fig. 7. When $f = 3500$ Hz, $k_L = 0.5$, current experimental waveform.
results. It can be prove that, with the increase of inductance coefficient, $k_L$, harmonic amplitude and THD will increase. The oscillation frequency is proportional to the switching frequency. Experimental results verify the previous analysis.

**6 Conclusion**

In this paper, the current control strategy and nonlinear characteristics of four quadrant converter are studied.

Based on the discrete system model of 4QC, the stability of the current loop is mainly related to inductance coefficient, $k_L$. Considering the saturation nonlinear characteristics of four-quadrant converter, the describing function method is applied to analyse the current loop oscillation. Once inductance coefficient, $k_L$, is more than the critical stable value determined by the system parameters and control strategy, system exists self-sustained oscillations. The oscillation frequency is proportional to the switching frequency. Simulation and experiment verify the conclusion.

**Acknowledgments**

This work was supported by the National Natural Science Foundation of China under Grant 51577010.