Time response of frequency of the hydro-turbine governing system under the coupled action of surge tank and power grid

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Abstract. For grid connected hydropower plants with surge tanks, the time response of frequency of the hydro-turbine governing system under the coupled action of surge tank and power grid with load disturbance have been examined. On the basis of a mathematical model of a hydro-turbine governing system operating in an isolated grid with a surge tank, a power grid model of the hydro-turbine governing system connected to a grid has been developed. Using Laplace transformation, a comprehensive transfer function, taking the disturbance of the hydro-turbine as the input signal and the speed of the hydro-turbine as the output signal, has been developed for the mathematical model. Finally, by using MATLAB-Simulink to numerically simulate the time response of frequency of the system under the coupled action of the grid at different scales and the surge tank with different sectional areas, the mechanism of the coupled action of the surge tank and the power grid and the effect of grid scale on time response of frequency of system has been analysed. It is concluded that surge tank only affects the tail wave of the time response of frequency, and for surge tanks with large sectional area, the fluctuations in the tail wave are gentler. Hence, the system is easier to become steady. The power grid has an inhibiting effect, which becomes greater for larger grid scale, on both the head wave and the tail wave of the time response of frequency in which the head wave is separated into several wavelets and the tail wave become gentler.

1. Introduction

In modern hydropower system, hydropower plant due to its flexible operation characteristics undertakes the primary task of peak and frequency modulation. In the hydropower system, the turbine regulating system is the core component of load frequency control (LFC) [1]. The hydro-turbine generating unit can operate in a grid-connected mode or in an isolated network operation mode. The former is the main operation mode while the latter is an accidental and temporary operation mode. Hence, it is of vital importance to study the time response of frequency of the hydro-turbine regulating system under the grid-connected operation mode [2-5].

Researchers have carried out a lot of work in this field [2-5]. For time response of frequency regulation quality problems, in literature [2], WEI Shou-ping had studied the theory, and carried out experiment and numerical simulation of hydropower plant without surge tank. He also analysed the governor parameter set and its influence on the time response of frequency. However, for hydropower plant with surge tank, there are less research results. Surge tank is a crucial flat pressure measure for...
the hydropower plant, and the unit time response of frequency is affected by the fluctuation of water level in the tank, which generates both main wave and tail wave [3]. These waves are significantly different from the one in a hydropower plant without surge tank. Using a numerical simulation method in literature [4], Zou Jin analysed the influence of different order generator model on the time response of frequency of hydraulic turbine. By using another numerical simulation method in literature [5], WEI Shou-ping analysed the influence of governor power regulation on the regulation quality under secondary frequency modulation. He also compared the differences between the hydropower and thermal power governor systems connected with grid. However, it could be found in literatures [2-5] that the models used were incomplete; some considered the influence of surge tank without a power grid while others had taken the effect of the power grid into account but without a surge tank. In both cases, they failed to consider the influence of the coupled effect of the surge tank and the power grid comprehensively.

To overcome the aforementioned deficiencies, in this study, the influence of coupled action of surge tank of different sectional areas and power grid of different scales on the time response of frequency[6] of hydro-turbine governing system have been examined by adding a grid model. Further, on the basis of a mathematical model of hydro-turbine governing system [7] operating in an isolated grid with a surge tank, whose stability was analysed [8], six groups of different parameters were set to describe six grids of different scales. In this way, a complete mathematical model of the hydro-turbine governing system, which is close to the real situation, has been developed. Then, taking the disturbance of a hydro-turbine as the input signal and the speed of a hydro-turbine as the output signal, a comprehensive transfer function has been developed using Laplace transform of the complete mathematical model. Further, the mathematical model of a diversion power system has been developed using MATLAB-Simulink to simulate numerically the time response of frequency of the system under the coupled action of the grid of different scales and the surge tank of different sectional areas. Horizontally, this study has compared the differences of the time response of frequency of the system with surge tank, connected with power grid and the system without surge tank, connected with isolated grid. The mechanism of the coupled action of surge tank and power grid was analysed. Vertically, the influence of the surge tank section area and the power grid scale on the time response of frequency of the system under coupled action of surge tank and power grid was also analysed. Finally, conclusions are drawn based on the results of the analyses.

2. Mathematical model

Fig. 1 shows a mathematical model of a complete diversion power system [9]. It consists of a hydraulic system model, a model of hydro-turbine control system [10], a governor system model [11], a generator model [12-13] and a power grid model.

![Figure 1. Model of diversion power system](image)

2.1 Basic equations of hydraulic system model [14]
The hydraulic system simulates the relationship between work head and work flow. In this study, a rigid water hammer model is used, whose basic equations are:
Momentum equation of flow in diversion tunnel:
\[ z = T_m \frac{dq_y}{dt} + \frac{2h_o}{H_o} q_y \]  
(1)

Continuity equation of flow in surge tank:
\[ q_y = -T_f \frac{dz}{dt} + q_i \]  
(2)

Momentum equation of flow in pressure pipeline:
\[ h = -T_{st} \frac{dq_i}{dt} - \frac{2h_o}{H_o} q_i - z \]  
(3)

2.2 Basic equations of hydro-turbine control system model [1][6]

Moment equation:
\[ m_i = e_i h + e_x x + e_y y \]  
(4)

Discharge equation:
\[ q_i = e_{qi} h + e_{xi} x + e_{yi} y \]  
(5)

2.3 Basic equation of governor system model [1][6]
Governor usually operates in three regulation modes: (1) frequency regulation, (2) power regulation, and (3) opening regulation. This study examined the frequency regulation mode only, without considering factors such as characteristics of dead zone and saturation in the electro-hydraulic servo system. Assuming that the governor system is a linear element, the basic equation is:
\[ b_i \frac{dy}{dt} = -(x + T_d \frac{dx}{dt}) \]  
(6)

2.4 Basic equations of generator model
In this study, the first order model of a generator is used and the basic equation [6] is:
\[ T_a \frac{dx}{dt} = m_i - (m_x + e_x x) \]  
(7)

In which, \( m_x \) can be expressed as follows [6]:
\[ m_x = K_x \int x dt + D_x x + \Delta P_{em} \]  
(8)

2.5 Power grid model
In this study, the power grid has been substituted by an equivalent generator unit [15][16], as shown in Fig. 2.

![Figure 2. Block diagram of power grid model](image)

In Fig. 2, \( T_i \) includes the rotation inertia effect of all the units in the grid, whose value has accounted for the grid scale. \( D_i \) account for the damping characteristics about the grid load and frequency and determine the corresponding power change for a certain degree of frequency deviation. Usually, it is difficult to measure \( D_i \) accurately. However, it has a positive correlation with the scale of the grid [17]. Furthermore, \( \frac{1}{R_g (T_g s + 1)} \) denotes the equivalent speed regulating system of the grid, accounting for the dynamic characteristics of the speed regulating system of all the units in the grid. \( R_g \) is determined
by $e_{pi}$ of all the units in the grid jointly, satisfying the relation $R_e = \sum_i e_{pi}$. Due to the difference between the rated power base value of the generator and that of the grid system, it is necessary to use B to link them.

### 2.6 Overall model of system

Fig. 3 shows a block diagram of the system model.

![Figure 3. Block diagram of system model](image)

### 3. The time response of frequency of system under the coupled action of surge tank and power grid

In this study, a relatively complete model of the diversion power system has been developed by using MATLAB-Simulink. Combining this model with the models of hydraulic system, generator and power grid, in which the hydraulic system uses the rigid water hammer model; the hydro-turbine uses the ideal model; and the generator uses the first order model. In this study, by setting the basic parameter as follows: $e_h=1.5$, $e_x=-1$, $e_y=1$, $e_qh=0.5$, $e_qx=0$, $e_qy=1$, $b_T=0.5$, $T_a=10s$, $T_s=10s$, $e_g=1$, $e_a=0.2$, $T_{wt}=2.33s$, $T_{wy}=2.33s$, $T_{x}=23.84s$, $T_s=10s$, $h_0=5.53$, $h_0=7.57$, $H_0=89m$, the time response of frequency of the system under single action of surge tank and coupled action of the surge tank and the power grid have been analysed.

#### 3.1 Time response of frequency of system under single action of surge tank

Under step load disturbance $m_{0g}=-0.1$, $n_f$ are 0.73, 1, 1.33, and 1.67 for the time response of frequency, showed in Fig.4, of both systems, in isolated grid, without surge tank and with surge tank separately.
Based on Fig. 4, under the step load disturbance, the transient characteristic of the system without surge tank rise rapidly and then decays rapidly, forming a head wave. On the other hand, for the system with surge tank, it also forms a head wave but there is also a tail wave of low frequency, which is due to the presence of the surge tank. Further, \( n_f \) has no effect on the head wave, but has enormous impact on the tail wave. The bigger the \( n_f \), the more gentle the tail wave fluctuates, and easier for the system to become steady. The converse is true for smaller \( n_f \).

3.2 The time response of frequency of system under coupling action of surge tank and power grid

3.2.1 Establishment and analysis of space state equation of power grid model

In the power grid model, the disturbance of unit frequency \( x \) and the load \( M_{g0} \) are taken as the input signal. \( \Delta P_{em} \) and \( \Delta X_s \) are taken as the output signal. Further, Fig. 5 shows the spatial state parameters \( X_1, X_2, X_3 \).

![Figure 5. Schematic diagram of spatial state parameters of power grid model](image)

The space state equations of the power grid model are:
The system matrix is denoted as $A$ and it accounts for the characteristics of the system [18]:

$$
A = egin{bmatrix}
0 & 0 & -\frac{1}{T_g} \\
0 & -\frac{1}{T_g} & \frac{1}{T_g} \\
Bk_a & -\frac{1}{R_g T_g} & -\frac{D_g B + D_s}{T_s}
\end{bmatrix}
$$

Without pole zero cancellation in the transfer function of the grid system model, the state stability is consistent with the output stability. So, the characteristics equation of matrix $A$ is the characteristics equation of the system and the characteristics root of the system matrix $A$ is the pole of the system transfer function. According to equation $|A - \lambda I| = 0$, the characteristics equation of the transfer function of grid system model is:

$$
T_s R_s T_s \lambda^3 + (R_s T_s D_g + R_s T_s D_s + R_s T_s) \lambda^2 + (R_s D_s + BK_a R_s T_s + R_s D_s B + 1) \lambda + BK_a R_g = 0
$$

Further, as $T_s R_s T_s > 0$, $R_s T_s D_g + R_s T_s D_s + R_s T_s > 0$, $R_s D_s + BK_a R_s T_s + R_s D_s B + 1 > 0$, $BK_a R_g > 0$,

and $-\frac{1}{R_s T_s D_g + R_s T_s D_s + R_s T_s} \frac{T_s R_s T_s}{R_s T_s D_g + R_s T_s D_s + R_s T_s} > 0$, the grid system model is steady according to the Routh criterion.

In this study, six power grids of different scales have been examined: (1) small grid, (2) medium grid I, (3) medium grid II, (4) large grid I, (5) large grid II, (6) infinite grid. Table 1 shows their basic parameters:

| B     | Ts    | Ds    | Rg    | Tg    |
|-------|-------|-------|-------|-------|
| small grid | 1     | 0.1   | 1     | 8     |
| medium grid I | 0.2   | 0.2   | 0.3   | 20    |
| medium grid II | 0.1   | 0.4   | 0.2   | 403   |
| large grid I | 0.01  | 400   | 4     | 1     | 400   |
| large grid II | 0.02  | 4000  | 0.2   | 4000  |
| infinite grid | $\varepsilon \to 0^+$ | $\infty$ | 1000  | $\varepsilon \to 0^+$ | $\infty$ |

In order to simplify the model, equating $D_s=0.073$. Further, Table 2 shows the poles of transfer function of the grid model at six different scales.
Table 2. Poles of transfer function of grid model at six different scales

| Scale       | $\lambda_1$                  | $\lambda_2$                  | $\lambda_3$                  |
|-------------|------------------------------|------------------------------|------------------------------|
| small grid  | -0.014338+0.51453i           | -0.014338-0.51453i           | -0.11795                     |
| medium grid | -0.012493+0.1668i           | -0.012493-0.1668i           | -0.035744                    |
| medium grid II | -0.0098509 +0.089317i     | -0.0098509 -0.089317i       | -0.015481                    |
| large grid I | -0.0057553+0.0096421i   | -0.0057553-0.0096421i       | -0.00099132                  |
| large grid II | -0.0000212                  | -0.00061456                  | -0.0096143                   |
| infinite grid | $-\varepsilon \to 0^-$     | $-\varepsilon \to 0^-$     | $-\varepsilon \to 0^-$     |

It is apparent all the poles of transfer function of the grid model system at different scales have negative real part, which indicates that the grid model system developed in this study is stable. With larger grid scale, both the absolute value of imaginary part and real part become smaller. In fact, the absolute value of the imaginary part becomes smaller faster than the real part. This means that for power grids of larger scale, the oscillation period of the time response of frequency becomes longer. In fact, for grids larger than Grid II, the oscillations under the external disturbance were extremely slow. It could even be considered to have achieved a new stable state after a certain period of time.

Figs. 6 show the fluctuation of $\Delta P_{em}$ of the grid model system at different scales under the load disturbance $M_{e0}=-0.1$.

![Figure 6](image)

**Figure 6.** Fluctuation of $\Delta P_{em}$ of grid model system under load disturbance

In view of the preceding results, the grid model can simulate the performance of real-life power grid system. For power grids of larger scale, the greater the grid system inertia, the more robust is the system. Moreover, the electromagnetic effect of the load disturbance on the generator set is amplified by the power grid. For larger grids, the amplification effect is more significant.

3.2.2 The time response of frequency of system under the coupled action of power grid and surge tank

The influences of the coupled action of the surge tank and the power grid on the time response of frequency of system have been analysed. It has been carried out under the disturbance of $\Delta P_{em}=-0.1$ with different $n_f$ for four groups of time response of frequency when the system was connected to the power grid of different scales and one isolated grid.

Fig. 7 shows the time response of frequency of the system under the coupled action of the surge tank and the power grid.
It is apparent that the surge tank affects the tail wave only, while the power grid has a significant effect on both the head wave and the tail wave. The head wave is affected by the grid generating high frequency harmonics, and disperses into several wavelets, whose peak gradually decreases with increasing grid scale. While the tail wave is suppressed by the grid: under action of the small grid, the tail wave is similar to the one under the action of the isolated grid but it is easier for the former to become steady, which is consistent with the practical case. The time response of frequency of the system under the action of the small grid is quite close to those in the isolated grid when $n_f$ exceeds a certain value. Under this condition, the isolated grid can be simulated by the small grid: tail wave almost disappears when the grid scale increases to medium or large scale. This shows that the influence of surge tank on the tail wave almost disappears when the system is connected to a large grid. Further, for the grid is close to the infinite grid, after a quick oscillation, the time responses of frequency almost remain constant. This shows that it is acceptable to assume the speed of the generator connected to the infinite power grid is almost constant.

4. Conclusions

1) The time response of frequency of hydro-turbine regulation system consists of the head wave and the tail wave, and the surge tank affects the tail wave only. The bigger the $n_f$, there are less fluctuations in the tail wave. As such, it is easier for the system to become steady.

2) The grid model developed in this study can simulate a real-life power grid system. For power grids of larger scale, the grid system inertia is greater, the system becomes more robust. Moreover, the electromagnetic effect of the load disturbance on the generator set is amplified by the power grid. For larger grids, the amplification effect is more significant.
3) The power grid has a significant effect on both the head wave and the tail wave. The head wave is affected by the grid dispersing into several wavelets, whose peak gradually decreases with increasing grid scale. While the tail wave is suppressed by the grid, when \( n_f \) exceeds a certain value, the isolated grid can be simulated by a small grid. The influence of the surge tank on the tail wave almost disappears when system is connected to a large grid.

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**Nomenclature**
- \( b_t \): Transient transfer coefficient
- \( B \): Power conversion factor
- \( D_s \): Equivalent damping coefficient
- \( D_i \): Self-regulating coefficient of the equivalent load of a power grid
- \( e_s \): Load self-regulation coefficient
- \( e_y, e_x, e_h \): Moment transfer coefficients of turbine
- \( e_y, e_q, e_b \): Discharge transfer coefficients of turbine
- \( e_p \): Permanent difference coefficient of all the units in the grid
- \( E \): Generator potential
- \( F \): Actual sectional area of surge tank
- \( F_{th} \): The critical stable sectional area of surge tank
- \( f_s, f_f \): Sectional area of pressure pipe, diversion tunnel
- \( H \): Work head of unit
- \( K_s \): Equivalent synchronization coefficient
- \( L_s, L_y \): Length of pressure pipe, diversion tunnel
- \( m \): Electromagnetic torque
- \( M_t \): Torque of hydro-turbine
- \( n \): Speed of hydro-turbine
- \( n_f \): Section amplification coefficients of surge tank, and \( n_f = F/F_{th} \)
- \( \triangle P_{em} \): Electromagnetic power variation
- \( \triangle P_i \): Power disturbance of the grid
- \( Q_y, Q_t \): Diversion tunnel flow, pressure pipe flow
- \( R_y \): Equivalent permanent difference coefficient of the grid
- \( T_F \): Time constant of surge tank, and \( T_F = FH_y/Q_0 \)
- \( T_{w_1}, h_{01} \): Flow inertia time constant of pressure pipe and its head loss, and \( T_{w_1} = L_yQ_{01}/g_fH_0 \)
- \( T_{w_2} \): Flow inertia time constant of diversion tunnel and its head loss, and \( T_{w_2} = L_yQ_{02}/g_fH_0 \)
- \( T_a \): Unit inertia time constant
- \( T_s \): Inertia time constant of the grid equivalent unit
- \( T_g \): Inertia time constant of the servomotor of grid equivalent unit
- \( \triangle X_s \): Frequency fluctuation of unit
- \( Y \): Gate opening
- \( z, h, q_1, q_r \): Deviation relative value of \( Z, H, Q, Q, M, n, Y \)
- \( Z \): Change of surge tank water level (positive direction is downward)

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