A Radiative Model for the Weak Scale and Neutrino Mass via Dark Matter

Amine Ahriche\textsuperscript{a,b} Kristian L. McDonald\textsuperscript{c} Salah Nasri\textsuperscript{d}

\textsuperscript{a}Department of Physics, University of Jijel, PB 98 Ouled Aissa, DZ-18000 Jijel, Algeria.
\textsuperscript{b}The Abdus Salam International Centre for Theoretical Physics, Strada Costiera 11, I-34014, Trieste, Italy.
\textsuperscript{c}ARC Centre of Excellence for Particle Physics at the Terascale, School of Physics, The University of Sydney, NSW 2006, Australia.
\textsuperscript{d}Department of Physics, UAE University, P.O. Box 17551, Al-Ain, United Arab Emirates.

E-mail: aahriche@ictp.it, kristian.mcdonald@sydney.edu.au, snasri@uae.ae.ac

Abstract: We present a three-loop model of neutrino mass in which both the weak scale and neutrino mass arise as radiative effects. In this approach, the scales for electroweak symmetry breaking, dark matter, and the exotics responsible for neutrino mass, are related due to an underlying scale-invariance. This motivates the otherwise-independent $\mathcal{O}$(TeV) exotic masses usually found in three-loop models of neutrino mass. We demonstrate the existence of viable parameter space and show that the model can be probed at colliders, precision experiments, and dark matter direct-detection experiments.
1 Introduction

Radiative symmetry breaking [1] offers an interesting alternative to the conventional Higgs mechanism. In this approach, calculable weakly-coupled radiative effects induce symmetry breaking in classically scale-invariant theories, thereby giving birth to mass — a process known as dimensional transmutation. When applied to the Standard Model (SM), it is well known that radiative symmetry breaking is not viable, due to the destabilizing influence of the heavy top quark. However, the SM is known to be incomplete, due to e.g. an absence of massive neutrinos and the need to incorporate dark matter (DM). It is therefore interesting to consider the viability of radiative symmetry breaking within SM extensions.
The addition of massive neutrinos and DM to the SM likely requires new degrees of freedom. When considering radiative symmetry breaking, there are a number of relevant considerations that can guide the choice of beyond-SM fields. The destabilizing radiative corrections from the top quark can be overcome by bosonic degrees of freedom with mass \( \gtrsim 200 \text{ GeV} \). In principle these states could be much heavier than the TeV scale. However, radiative symmetry breaking typically introduces a single scale into a theory, with other mass and symmetry breaking scales related to this scale.\(^1\) Consequently both the electroweak scale and the mass scale for exotics may be related via dimensionless parameters. Thus, absent hierarchically small parameters \(^2\), one anticipates exotics with \( \mathcal{O}(\text{TeV}) \) masses.

In the LHC era, TeV scale exotics are of particular interest. However, efforts to generate tiny neutrino masses via weak-scale exotics can struggle to achieve the necessary mass-suppression, relative to the weak scale, without invoking tiny couplings. Perhaps the most obvious exception are models with radiative neutrino mass, as the inherent loop-suppression in such models can motivate lighter new physics. From this perspective, three-loop models of neutrino mass are particularly compelling, as the new physics is expected to be \( \mathcal{O}(\text{TeV}) \).

These considerations focus our attention on scale-invariant models with three-loop neutrino mass. If we also seek to address the DM problem, a minimal approach would see the DM play a role in either generating neutrino mass or triggering electroweak symmetry breaking. Thus, we arrive at a picture in which both the weak scale and neutrino mass arise as radiative effects, with the weak scale, the DM mass, and the mass scale for the exotics that induce neutrino mass, all finding a common birth, via dimensional transmutation. This picture can address short-comings of the SM, while also explaining why the exotics required in three-loop neutrino mass models have (otherwise independent) masses of \( \mathcal{O}(\text{TeV}) \) — a common ancestry requires that they be related to the weak scale.

In this work we present a scale-invariant model for three-loop neutrino mass that contains a fermionic DM candidate. We explore the model in detail and present feasible parameter space that achieves the correct DM relic abundance, while generating viable symmetry breaking and neutrino masses — all compatible with low-energy constraints. As per usual for scale-invariant frameworks, the model predicts a dilaton. However, here the dilaton has the dual role of allowing electroweak symmetry breaking and simultaneously sourcing the lepton number violation that allows radiative neutrino masses. We note that a number of earlier works studied relationships between the origin of neutrino mass and DM, see e.g. Refs. [3–8]. There has also been much interest in scale-invariant models in recent years, see e.g. Refs. [9–14].

The structure of this paper is as follows. In Section 2 we introduce the model and detail the symmetry breaking sector. We turn our attention to the origin of neutrino mass in Section 3 and discuss various constraints in Section 4. Dark matter is discussed in Section 5 and our main analysis and results appear in Section 6. Conclusions are drawn in Section 7.

\(^{1}\)The exceptions being when a theory also contains a confining gauge sector, as with QCD in the SM, or a completely decoupled hidden sector possessing its own symmetry breaking and/or confining pattern.
2 A Scale-Invariant Three-Loop Model

We consider a classically scale-invariant (SI) extension of the SM in which neutrino mass appears at the three-loop level. The SM is extended by the addition of two charged scalars, \( S_1^+ \sim (1, 1, 2) \), three singlet fermions, \( N_{iR} \sim (1, 1, 0) \), with \( i \in \{1, 2, 3\} \) labeling generations, and a singlet scalar, \( \phi \sim (1, 1, 0) \).2 A \( Z_2 \) symmetry with action \( \{ S_2, N_R \} \rightarrow \{ -S_2, -N_R \} \) is imposed, with all other fields being \( Z_2 \)-even. This symmetry remains exact in the full theory, making the lightest \( Z_2 \)-odd field a stable DM candidate, which should be taken as the lightest fermion, \( N_1 \equiv N_{DM} \), to avoid a cosmologically-excluded stable charged particle. The scalar \( \phi \) plays a key role in triggering electroweak symmetry breaking, as explained below, and also ensures that lepton number symmetry is explicitly broken, thereby allowing radiative neutrino mass.

Consistent with the SI and \( Z_2 \) symmetries, the Lagrangian contains the following terms:

\[
L \supset - \left\{ f_{\alpha\beta} \overline{L^c_\alpha} L^c_\beta S_1^+ + g_{ia} \overline{N^c_i} S_2^+ e_{\alpha R} + H.c \right\} - \frac{1}{2} \bar{y}_i \phi \overline{N^c_i} N_i - V(H, S_{1,2}, \phi),
\]

(2.1)

where Greek letters label SM flavors, \( \alpha, \beta \in \{e, \mu, \tau\} \), and \( f_{\alpha\beta}, g_{ia} \) and \( \bar{y}_i \) are Yukawa couplings. The \( Z_2 \) symmetry forbids the term \( \overline{L} H N_R \), which would otherwise generate tree-level neutrino masses after the SM scalar \( H \sim (1, 2, 1) \) develops a VEV. The potential \( V(H, S_{1,2}, \phi) \) is the most-general potential consistent with the SI and \( Z_2 \) symmetries.

2.1 Symmetry Breaking

We are interested in parameter space where both \( \phi \) and \( H \) acquire nonzero vacuum expectation values (VEVs), \( \langle H \rangle \neq 0 \) and \( \langle \phi \rangle \neq 0 \). This breaks both the SI and electroweak symmetries while preserving the \( Z_2 \) symmetry. The most-general scalar potential includes the terms

\[
V_0(H, S_{1,2}, \phi) \supset \lambda_H |H|^4 + \frac{\lambda_{\phi m}}{2} |H|^2 \phi^2 + \frac{\lambda_\phi}{4} \phi^4 + \frac{\lambda_S}{4} (S_1^-)^2 (S_1^+)^2 + \sum_{a=1,2} \frac{1}{2} (\lambda_{\phi a} |H|^2 + \lambda_{\phi a} \phi^2) |S_a|^2.
\]

(2.2)

A complete analysis of the potential requires the inclusion of the leading-order radiative corrections. In general the full one-loop corrected potential is not analytically tractable. However, a useful approach for approximating the ground state in SI models was presented in Ref. [15]. Taking guidance from Ref. [15], we adopt an approximation for the ground state that allows one to obtain simple analytic expressions. The physical spectrum contains two charged scalars \( S_{1,2}^+ \), and two neutral scalars, denoted as \( h_{1,2} \). As discussed in Appendix A, for the present model, the minimum of the loop-corrected potential can be approximated by neglecting loop corrections involving only the scalars \( h_{1,2} \). The viability of this simplification follows from the dominance of the beyond-SM scalars \( S_{1,2}^+ \) (see Appendix A). Adopting this

---

2 Quantities in parentheses refer to quantum numbers under the SM gauge symmetry \( SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \).
approximation, the one-loop corrected potential for the CP-even neutral scalars is

\[ V_{1-l}(h, \phi) = \frac{\lambda_H}{4} h^4 + \frac{\lambda_{\phi H}}{4} \phi^2 h^2 + \frac{\lambda_\phi}{4} \phi^4 + \sum_{i=\text{all fields}} n_i G \left( m_i^2(h, \phi) \right), \]  

(2.3)

\[ G(\eta) = \frac{\eta^2}{64 \pi^2} \left[ \log \frac{\eta}{\Lambda^2} - \frac{3}{2} \right], \]  

(2.4)

where \( \Lambda \) is the renormalization scale, \( n_i \) are the field multiplicities, and we employ the unitary gauge, with \( H = (0, h/\sqrt{2})^T \). The sum is over all fields, neglecting the light SM fermions (all but the top quark) and the (to be determined) neutral scalar mass-eigenstates \( h_{1,2} \). Due to the SI symmetry, the field-dependent masses can be written as

\[ m_i^2(h, \phi) = \frac{\alpha_i}{2} h^2 + \frac{\beta_i}{2} \phi^2, \]  

(2.5)

the constants \( \alpha_i \) and \( \beta_i \) are given by

\[
\begin{align*}
\alpha_W &= \frac{g^2}{2}, \\
\alpha_Z &= \frac{g^2 + g'^2}{2}, \\
\alpha_t &= y_t^2, \\
\alpha_{S_u} &= \lambda_{H u}, \\
\alpha_{N_i} &= 0, \\
\beta_W &= \beta_Z = \beta_t = 0, \\
\beta_{S_u} &= \lambda_{\phi u}, \\
\beta_{N_i} &= 2 y_t^2,
\end{align*}
\]  

(2.6)

with \( g \) (\( g' \)) and \( y_t \) are the \( SU(2)_L \) (\( U(1)_Y \)) gauge and top Yukawa couplings, respectively.

Dimensional transmutation introduces a dimensionful parameter into the theory in exchange for one of the dimensionless couplings. In the present model, an analysis of the potential shows that a minimum with \( \langle h \rangle \equiv v \neq 0 \) and \( \langle \phi \rangle \equiv x \neq 0 \) exists for \( \lambda_{\phi H} < 0 \), and is triggered at the scale where the couplings satisfy the relation

\[ 2 \left\{ \lambda_{H} \lambda_{\phi} + \frac{\lambda_{H}}{x^2} \sum_{i} n_i \left( \beta_i - \alpha_i \frac{v^2}{x^2} \right) G'(m_i^2) \right\}^{1/2} + \lambda_{\phi H} + \frac{2}{x^2} \sum_{i} n_i \alpha_i G'(m_i^2) = 0, \]  

(2.7)

with \( G'(\eta) = \partial G(\eta)/\partial \eta \). The further condition

\[ -\frac{\lambda_{\phi H}}{2 \lambda_H} = \frac{v^2}{x^2} + \left( \sum_{i} n_i \alpha_i \lambda_H x^2 G'(m_i^2) \right), \]  

(2.8)

is also satisfied at the minimum. Thus, for \( \lambda_{\phi H} = \mathcal{O}(1) \) one has \( v \sim x \) and the exotic scale is naively expected around the TeV scale. Note that Eqs. (2.7) and (2.8) ensure that the tadpoles vanish.\(^3\)

Defining the one-loop quartic couplings as

\[ \lambda_{\phi}^{1-l} = \frac{1}{6} \frac{\partial^2 V_{1-l}}{\partial \phi^2}, \quad \lambda_{H}^{1-l} = \frac{1}{6} \frac{\partial^2 V_{1-l}}{\partial h^2}, \quad \lambda_{\phi H}^{1-l} = \frac{\partial^4 V_{1-l}}{\partial h^2 \partial \phi^2}, \]  

(2.9)

vacuum stability at one-loop requires that the following conditions be satisfied:

\[ \lambda_{H}^{1-l}, \lambda_{\phi}^{1-l}, \lambda_{\phi H}^{1-l} + 2 \sqrt{\lambda_{H}^{1-l} \lambda_{\phi}^{1-l}} > 0. \]  

(2.10)\(^3\)

\(^3\)To our level of approximation, Eqs. (2.7) and (2.8) are the loop-corrected generalizations of the standard tree-level results, \( 4 \sqrt{\lambda_H(\Lambda)} \lambda_\phi(\Lambda) + \lambda_{\phi H}(\Lambda) = 0 \) and \( \lambda_{\phi H}/2\lambda_H = v^2/x^2 \) [16].
We must also impose the condition $\lambda_{\phi H}^{1-l} < 0$ to ensure that the vacuum with $v \neq 0$ and $x \neq 0$ is the ground state.\textsuperscript{4} Eq. (2.10) also guarantees that the eigenmasses-squared for the CP-even neutral scalars are strictly positive, and forces one of the beyond-SM scalars $S_{1,2}^+$ to be the heaviest particle in the spectrum.

2.2 The Scalar Spectrum

The mass matrix for the neutral scalars is denoted as

$$ V_{1-l}(h, \phi) \supset \frac{1}{2}(h, \phi) \begin{pmatrix} m_{hh}^2 & m_{h\phi}^2 \\ m_{h\phi}^2 & m_{\phi\phi}^2 \end{pmatrix} \begin{pmatrix} h \\ \phi \end{pmatrix}, \quad (2.11) $$

where the mass parameters $m_{hh}$, $m_{\phi\phi}$ and $m_{h\phi}$ are calculated from the loop-corrected potential $V_{1-l}(h, \phi)$. The mass eigenstates are labeled as

$$ h_1 = \cos \theta_h h - \sin \theta_h \phi, \quad h_2 = \sin \theta_h h + \cos \theta_h \phi, \quad (2.12) $$

with the eigenvalues and mixing angles given by

$$ M_{h_{1,2}}^2 = \frac{1}{2} \left\{ m_{11}^2 + m_{22}^2 \pm \sqrt{(m_{22}^2 - m_{11}^2)^2 + 4m_{12}^4} \right\}, $$

$$ \tan 2\theta_h = \frac{2m_{12}^2}{m_{22}^2 - m_{11}^2}. \quad (2.13) $$

Here $h_1$ is a massive SM-like scalar and $h_2$ is a pseudo-Goldstone boson associated with SI symmetry breaking — the latter is massless at tree-level but acquires mass at the loop-level. One can obtain simple tree-level expressions for the SM-like scalar mass

$$ M_{h_1}^2 = (2\lambda_{H} - \lambda_{\phi H})v^2, \quad (2.14) $$

and the mixing angle,

$$ c_h \equiv \cos \theta_h = \frac{x}{\sqrt{x^2 + v^2}}, \quad s_h \equiv \sin \theta_h = \frac{v}{\sqrt{x^2 + v^2}}, \quad (2.15) $$

though in large regions of parameter space it is important to include loop corrections to these expressions to obtain accurate results. In our numerical analysis we employ the full loop-corrected expressions for the scalar masses and mixing, as is necessary to obtain $M_{h_2} \neq 0$. Due to the SI symmetry, the parameters in the model are somewhat constrained, with $\lambda_{\phi}$ and $\lambda_{\phi H}$ fixed by Eqs. (2.7) and (2.8) while the Higgs mass $M_{h_1} \simeq 125$ GeV fixes $\lambda_{H}$.

The tree-level masses for the charged scalars, $S_{1,2}^+$, are

$$ M_{S_a}^2 = \frac{1}{2} \left\{ \lambda_{\phi a}x^2 + \lambda_{H a}v^2 \right\} \text{ for } a = 1, 2, \quad (2.16) $$

where $S_1^+$ and $S_2^+$ do not mix due to the $Z_2$ symmetry. Note that a useful approximation for $M_{h_2}$ is [15]

$$ M_{h_2}^2 \simeq \frac{1}{8\pi^2 \langle \phi \rangle^2 (\langle h \rangle^2 + \langle h \rangle)} \left\{ M_{h_1}^4 + 6M_W^4 + 3M_Z^2 - 12M_t^4 + 2 \sum_{a=1}^{2} M_{S_a}^4 - 2 \sum_{i=1}^{3} M_{N_i}^4 \right\}, \quad (2.17) $$

\textsuperscript{4}For $\lambda_{\phi H}^{1-l} > 0$ the vacuum with only one nonzero VEV is preferred.
which shows that one of the beyond-SM scalars $S_{1,2}$ must be the heaviest beyond-SM state in order to ensure $M_{h_2} > 0$.

As mentioned already, we expect the VEVs to be of a similar scale, $\langle \phi \rangle \sim \langle h \rangle$, as evidenced by Eq. (2.8). For completeness, however, we note that there is a technically natural limit in which one obtains $\langle \phi \rangle \gg \langle h \rangle$. This arises when all the couplings to $\phi$ are taken to be hierarchically small, namely $\{ \tilde{y}_i, \lambda_{\phi H}, \lambda_{\phi 1,2} \} \ll 1$, with the masses $M_{h_1}, M_N$ and $M_{S_{1,2}}$ held at $\mathcal{O}(\text{TeV})$. This feature reflects the fact that $\phi$ decouples in the limit $\{ \tilde{y}_i, \lambda_{\phi H}, \lambda_{\phi 1,2} \} \to 0$, up to gravitational effects [17]. In this limit we expect the model to be very similar to the KNT model [3], but with a light, very weakly-coupled scalar in the spectrum, $h_2$. Absent a compelling motivation for such hierarchically small parameters, we restrict our attention to values of $\langle \phi \rangle \leq 5 \text{ TeV}$.

### 3 Neutrino Mass

We now turn to the origin of neutrino mass. The $Z_2$-odd fermions, $N_i$, develop masses $M_{N_i} = \tilde{y}_i \langle \phi \rangle$, and do not mix with SM leptons due to the $Z_2$ symmetry. We order their masses as $M_{\text{DM}} \equiv M_{N_1} < M_{N_2} < M_{N_3}$. SM neutrinos, on the other hand, acquire mass radiatively. The combination of the Yukawa interactions in Eq. (2.1) and the term

$$V(H, S_{1,2}, \phi) \supset \frac{\lambda_{\phi}}{4} (S_1^-)^2 (S_2^+)^2,$$

in the scalar potential, explicitly break lepton number symmetry. Consequently neutrino masses appear at the three-loop level as shown in Figure 1.

**Figure 1.** Three-loop diagram for neutrino mass in a scale-invariant model.

Calculating the loop diagram, the mass matrix has the form

$$\left( M_\nu \right)_{\alpha\beta} = \frac{\lambda_\phi}{(4\pi)^2} \frac{m_\sigma m_\rho}{M_{S_2}} g_{\rho i}^* g_{\sigma i}^* f_{\alpha\sigma} f_{\beta\rho} \times F_{\text{loop}} \left( \frac{M_{S_1}^2}{M_{S_2}^2}, \frac{M_{S_1}^2}{M_{S_2}^2} \right),$$

where $m_{\sigma, \rho}$ denote charged lepton masses and the function $F_{\text{loop}}(x, y)$ encodes the loop integrals [5]

$$F_{\text{loop}}(\alpha, \beta) = \frac{\sqrt{\alpha}}{8\beta^2} \int_0^\infty dr \frac{r}{r + \alpha} \left( \int_0^1 dx \ln \frac{x(1-x)r + (1-x)\beta + x}{x(1-x)r + x} \right)^2.$$
One can relate the neutrino mass matrix to the elements of the Pontecorvo-Maki-Nakawaga-Sakata (PMNS) mixing matrix \[18\] elements, we parameterize the latter as

\[
U_\nu = \begin{pmatrix}
c_{12}c_{13} & c_{13}s_{12} & s_{13}e^{-i\delta_d} \\
-c_{23}s_{12} - c_{12}s_{13}s_{23}e^{i\delta_d} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_d} & c_{13}s_{23} \\
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_d} & -c_{12}s_{23} - c_{23}s_{12}s_{13}e^{i\delta_d} & c_{13}c_{23}
\end{pmatrix} \times U_m, \tag{3.4}
\]

with \(\delta_d\) the Dirac phase and \(U_m = \text{diag}(1, e^{i\theta_{12}/2}, e^{i\theta_{13}/2})\) encoding the Majorana phase dependence. The shorthand \(s_{ij} = \sin \theta_{ij}\) and \(c_{ij} = \cos \theta_{ij}\) refers to the mixing angles. For our numerical scans (discussed below) we fit to the best-fit experimental values for the mixing angles and mass-squared differences: 

\[
s_{12}^2 = 0.320^{+0.016}_{-0.017}, \quad s_{23}^2 = 0.43^{+0.03}_{-0.03}, \quad s_{13}^2 = 0.025^{+0.003}_{-0.003},
\]

\[
|\Delta m_{13}^2| = 2.55^{+0.06}_{-0.06} \times 10^{-3} \text{eV}^2 \quad \text{and} \quad |\Delta m_{21}^2| = 7.62^{+0.19}_{-0.18} \times 10^{-5} \text{eV}^2 \tag{19}.\]

Furthermore, we require that the contribution to neutrino-less double beta decay in this model satisfies the current bound. Within these ranges, one determines the parameter space where viable neutrino masses and mixing occur in the model.

4 Experimental Constraints

In this section we discuss the constraints on the model from the lepton flavor violating process \(\mu \to e\gamma\), the electroweak precision tests, the invisible Higgs decay, and the effect on \(h\gamma\gamma\) process.

4.1 Lepton flavor

Flavor changing processes like \(\mu \to e + \gamma\) arise via loop diagrams containing virtual charged scalars and give important constraints on the model. At one-loop the branching ratio for \(\mu \to e + \gamma\) is

\[
B(\mu \to e\gamma) = \frac{\Gamma(\mu \to e + \gamma)}{\Gamma(\mu \to e + \nu + \bar{\nu})} \approx \frac{\alpha \pi^4}{384\pi} \times \left\{ \frac{|f_{\mu
u}f_{\nu\gamma}|^2}{M_{S_1}^2} + \frac{36}{M_{S_2}^2} \sum_i g_i e g_{i\mu} F_2(M_i^2/M_{S_2}^2) \right\}^2, \tag{4.1}
\]

where \(F_2(R) = |1 - 6R + 3R^2 + 2R^3 - 6R^2 \log R|/[6(1 - R)^4].\) The corresponding expression for \(B(\tau \to \mu + \gamma)\) follows from a simple change of flavor labels in Eq. (4.1). Similarly, the one-loop contributions to the anomalous magnetic moment of the muon are

\[
\delta a_\mu = -\frac{m_\mu^2}{16\pi^2} \left\{ \sum_{\alpha \neq \mu} \frac{|f_{\mu\alpha}|^2}{6M_{S_1}^2} + \sum_i \frac{|g_{i\mu}|^2}{M_{S_2}^2} F_2(M_i^2/M_{S_2}^2) \right\}. \tag{4.2}
\]

Null-results from searches for neutrino-less double-beta decay give an additional constraint of \((M_\nu)_{ee} \lesssim 0.35 \text{ eV}\) \[20\], though we find this is easily satisfied.
4.2 Electroweak precision tests

In principle, precision electroweak measurements can provide additional constraints. The oblique parameters characterizing new physics effects are given by [21]

\[
\frac{\alpha}{4s_W^2c_W^2} S = \frac{A_{ZZ} (M_Z^2) - A_{ZZ} (0)}{M_Z^2} - \frac{\partial A_{\gamma\gamma} (q^2)}{\partial q^2} \bigg|_{q^2=0} + \frac{c_W^2 - s_W^2}{c_W s_W} \frac{\partial A_{\gamma Z} (q^2)}{\partial q^2} \bigg|_{q^2=0},
\]

(4.3)

\[
\alpha T = \frac{A_{WW} (0) - A_{ZZ} (0)}{M_Z^2}.
\]

(4.4)

Here, \( \alpha = e^2 / (4\pi) = g^2 s_w^2 / (4\pi) \) is the fine-structure constant, \( s_w = \sin \theta_w \) and \( c_w = \cos \theta_w \) are the sine and cosine, respectively, of the Weinberg angle \( \theta_w \), and the functions \( A_{VV'} (q^2) \) are the coefficients of \( g^{\mu\nu} \) in the vacuum-polarization tensors \( \Pi^{\mu\nu}_{VV'} (q) = g^{\mu\nu} A_{VV'} (q^2) \) + \( q^\mu q^\nu B_{VV'} (q^2) \), where \( VV' \) could be either \( \gamma\gamma, \gamma Z, ZZ, \) or \( WW \). In our model, the oblique parameters are given by [22]

\[
\Delta T = \frac{3}{16\pi s_w^2 c_w^2 M_W^2} \{ \chi_w^2 [ F (M_{h_1}^2, M_{h_1}^2) - F (M_W^2, M_W^2)] + s_w^2 [ F (M_{h_2}^2, M_{h_2}^2) - F (M_W^2, M_W^2)] \}
\]

\[
- [ F (M_{h_2}^2, M_{h_2}^2) - F (M_W^2, M_W^2)] \} ,
\]

(4.5)

\[
\Delta S = \frac{1}{24\pi} \left\{ 4s_w^4 G (M_{S_1}^2, M_{S_1}^2, M_Z^2) + 4s_w^4 G (M_{S_2}^2, M_{S_2}^2, M_Z^2) + c_w^2 \ln \frac{M_{h_2}^2}{M_{h_1}^2} + s_w^2 \ln \frac{M_{h_2}^2}{M_{h_1}^2} \right. \\
\left. + c_w^2 \hat{G} (M_{h_1}^2, M_{h_1}^2) + s_w^2 \hat{G} (M_{h_2}^2, M_{h_2}^2) - \hat{G} (M_{h_1}^2, M_{h_2}^2) \} \right.,
\]

(4.6)

where the functions \( F, G \) and \( \hat{G} \) are given in the appendix and \( M_h = 125.09 \) GeV denotes the reference value.

4.3 Higgs invisible decay

The model can also face constraints from the invisible Higgs decay, \( B (h \to inv) < 17\% \) [23]. In our case we have \( inv \equiv \{ h_2 h_2 \}, \{ N_{DM} N_{DM} \} \), when kinematically available. The corresponding decay widths are given by

\[
\Gamma (h_1 \to h_2 h_2) = \frac{1}{32\pi} \frac{(\lambda_{122})^2}{M_{h_1}} \left( 1 - \frac{4M_{h_2}^2}{M_{h_1}^2} \right)^{\frac{3}{2}} \Theta (M_{h_1} - 2M_{h_2}),
\]

\[
\Gamma (h_1 \to N_{DM} N_{DM}) = \frac{g_{DM}^2 s_h^2}{16\pi} M_{h_1} \left( 1 - \frac{4M_{DM}^2}{M_{h_1}^2} \right)^{\frac{3}{2}} \Theta (M_{h_1} - 2M_{DM}).
\]

(4.7)

The effective cubic coupling \( \lambda_{122} \) is defined below in Eq. (5.8). Due to the SI symmetry, we find that \( \lambda_{122} \) vanishes at tree-level, with the small (loop-level) coupling sufficient to ensure the decay to \( h_2 h_2 \) pairs is highly suppressed.\(^5\)

\(^5\)Note that \( h_2 \) decays to SM states, much like a light SM Higgs boson but with suppression from the mixing angle, \( s_h^2 \). However, currently there are no dedicated ATLAS or CMS searches for light scalars in the channels \( 2h, 2\tau \) or \( 2\gamma \), so we classify the decay \( h_1 \to h_2 h_2 \) as invisible. In practice the suppression of \( \Gamma(h_1 \to h_2 h_2) \) due to SI symmetry renders this point moot.
4.4 The Higgs decay channel $h \to \gamma \gamma$

The existence of extra charged scalars modifies the two Higgs branching ratios $B(h \to \gamma \gamma, \gamma Z)$, and this deviation can be parameterized by the ratios:

$$R_{\gamma \gamma} = \frac{B(h \to \gamma \gamma)}{B_{SM}(h \to \gamma \gamma)} = 1 + \frac{v^2}{2 c_h} \left( \frac{d_1}{m_{S_1}^2} A_0^{\gamma \gamma} (\tau_{S_1}) + \frac{d_2}{m_{S_2}^2} A_0^{\gamma \gamma} (\tau_{S_2}) \right)^2, \quad (4.8)$$

$$R_{\gamma Z} = \frac{B(h \to \gamma Z)}{B_{SM}(h \to \gamma Z)} = 1 + \frac{s_w v}{c_h} \left( \frac{d_1}{m_{S_1}^2} A_0^{\gamma Z} (\tau_{S_1}, \lambda_{S_1}) + \frac{d_2}{m_{S_2}^2} A_0^{\gamma Z} (\tau_{S_2}, \lambda_{S_2}) \right)^2 + \frac{2(1 - s_w^2 / 3)}{c_w} A_1^{\gamma Z} (\tau_{t}, \lambda_{t}), \quad (4.9)$$

where $\tau_X = M_X^2 / 4M_X^2$ and $\lambda_X = M_X^2 / 4M_X^2$, with $M_X$ is the mass of the charged particle $X$ running in the loop, $N_c = 3$ is the color number, $Q_t$ is the electric charge of the top quark in unit of $|e|$, and the loop amplitudes $A_i$ for spin 0, spin 1/2 and spin 1 particle contribution [24], which are given in the appendix. Here $\theta_a$ are the SM-like Higgs couplings to the pairs of charged scalars $S_{1,2}^\pm$, which are given by

$$\theta_a = c_h \lambda_{h_a} + s_h \lambda_{d_a} x. \quad (4.10)$$

The effect of the charged scalars on (4.8) and (4.9) depends on the masses for $S_{a}^\pm$, the sign and the strength of their couplings to the SM Higgs doublet and the neutral singlet and on the mixing angle $\theta_h$. One can use the reported results from LHC to constraints these parameters.

5 Dark Matter

5.1 Relic Density

The lightest $Z_2$-odd field is a stable DM candidate. As mentioned already, the lightest exotic fermion $N_{DM} \equiv N_1$ is the only viable DM candidate in the model. The relic density is given by [25]

$$\Omega_{DM} h^2 = \frac{1.04 \times 10^9 \text{GeV}^{-1}}{M_P} \frac{1}{\sqrt{g_*(T_f) < \sigma v_r (x_f) >}}, \quad (5.1)$$

where $M_P = 1.22 \times 10^{19}$ GeV is the Planck scale, $g_*(T)$ is the total effective number of relativistic particle at temperature $T$, and

$$\langle \sigma(N_{DM} N_{DM}) v_r \rangle = \sum_X \langle \sigma(N_{DM} N_{DM} \to X)v_r \rangle = \frac{1}{8T M_{DM}^4 K_2^2 \left( \frac{M_{DM}}{4 \pi} \right)^2} \times \int_{4M_{DM}^2}^\infty ds \sigma_{N_{DM} N_{DM} \to all}(s) (s - 4M_{DM}^2) \sqrt{s} K_1 \left( \frac{\sqrt{s}}{T} \right), \quad (5.2)$$

is the thermally averaged DM annihilation cross-section, $v_r$ is the relative velocity, $s$ is the Mandelstam variable, $K_{1,2}$ are the modified Bessel functions and $\sigma_{N_{DM} N_{DM} \to all}(s)$ is the
annihilation cross into all kinematically accessible final state particles at the CM energy $\sqrt{s}$.

The parameter $x_f = M_{\text{DM}} / T_f$ represents the freeze-out temperature, and can be computed from

$$x_f = \ln \frac{0.03 M_{\text{Pl}} M_{\text{DM}} < \sigma v_r(x_f) >}{\sqrt{T_f x_f}}. \quad (5.3)$$

As will be discussed in the next section, we require that $\Omega_{\text{N}\text{DM}} h^2$ to be in agreement with the observed value of the dark matter relic density [26].

The thermally averaged annihilation cross-section can be approximated in the non-relativistic limit as $< \sigma v_r > = a + b v_r^2$, where $v_r$ is the relative DM velocity and $a$ and $b$ are the $s$-wave and $p$-wave factors, which receives contributions from different annihilation channels. In this limit, the velocity squared is approximated by $v_r^2 \approx 6 / x_f$. Here, we evaluate the thermally averaged cross section exactly following (5.2).

### 5.2 Annihilation cross section

In our model, there are many contributions, where the channels can be classified into three types according to their Feynman diagrams types: (1) annihilation into charged leptons $N_{\text{DM}} N_{\text{DM}} \rightarrow \ell^+ \ell^-$ (Fig. 2-a and -b), which are $t$-channel diagrams mediated by charged scalars$^6$ (2) annihilation into SM fermions and gauge bosons pairs $N_{\text{DM}} N_{\text{DM}} \rightarrow f \bar{f}, \ W^- W^+, \ ZZ$ (Fig. 2-c), which occur through $s$-channel $h_{1,2}$-mediated diagrams, and (3) the annihilations into scalars, $N_{\text{DM}} N_{\text{DM}} \rightarrow h_{1,2} h_{1,2}$ (Fig. 2-d, -e and -f), which occur through both $s$- and $t$-channel diagrams.

![Figure 2](image.png)

**Figure 2.** Different diagrams for DM annihilation.

**Charged leptons annihilation channel**

The DM $N_1$ couples to SM leptons through the Yukawa couplings $g_{1\alpha}$, and can annihilate into charged lepton pairs as shown in Fig. 2-a and -b. The cross section for annihilation

---

$^6$Actually, for the same flavor case there two $s$-channel diagrams mediated by $h_{1,2}$, however we neglect them due to the suppressed Higgs charged leptons couplings.
into charged leptons \(^7\) is given by [27]

\[
\sigma(N_{\text{DM}}N_{\text{DM}} \rightarrow \ell_\alpha^+ \ell_\beta^-) = \frac{1}{8\pi s} \frac{|g_{1\alpha}g_{1\beta}|^2}{s(M_{S}^{2} + M_{\text{DM}}^{2} + \frac{s}{2})^2} \left[ \frac{m_{\ell_\alpha}^2 + m_{\ell_\beta}^2}{2} \left( \frac{s}{2} - M_{\text{DM}}^2 \right) \right] + \frac{8}{3} \left( \frac{M_{S}^{2} - M_{\text{DM}}^{2}}{M_{S}^{2} - M_{\text{DM}}^{2} + \frac{s}{2}} \right)^2 \left( \frac{s}{4} - M_{\text{DM}}^2 \right), \quad (5.4)
\]

**SM fermions and gauge boson channels**

The processes \(N_{\text{DM}}N_{\text{DM}} \rightarrow b\bar{b}, t\bar{t}, W^{+}W^{-} \) and \(ZZ\) can occur as shown in Fig. 2-c. The corresponding amplitude can be written as

\[
\mathcal{M} = i c_{h} s_{h} y_{1} u(k_{2}) u(k_{1}) \left( \frac{i}{s - M_{h_{1}}^2} - \frac{i}{s - M_{h_{2}}^2} \right) \mathcal{M}_{h \rightarrow \text{SM}}(m_{h} \rightarrow \sqrt{s}), \quad (5.5)
\]

where \(\mathcal{M}_{h \rightarrow \text{SM}}(m_{h} \rightarrow \sqrt{s})\) is the amplitude of the Higgs decay \(h \rightarrow X_{\text{SM}} \bar{X}_{\text{SM}}\), with the Higgs mass replaced as \(m_{h} \rightarrow \sqrt{s}\). This leads to the cross section

\[
\sigma(N_{\text{DM}}N_{\text{DM}} \rightarrow X_{\text{SM}} \bar{X}_{\text{SM}}) v_{r} = 8 \sqrt{s} m_{h}^{2} \left[ \frac{1}{s - M_{h_{1}}^2} - \frac{1}{s - M_{h_{2}}^2} \right]^{2} \Gamma_{h \rightarrow X_{\text{SM}} \bar{X}_{\text{SM}}}(m_{h} \rightarrow \sqrt{s}), \quad (5.6)
\]

where \(\Gamma_{h \rightarrow X_{\text{SM}} \bar{X}_{\text{SM}}}(m_{h} \rightarrow \sqrt{s})\) is the total decay width, with \(m_{h} \rightarrow \sqrt{s}\).

**Higgs channel**

The DM can self-annihilate to \(h_{1,2}h_{1,2}\), as seen in Fig. 2-d, -e and -f. The amplitude squared is given by

\[
|\mathcal{M}|^2 = 2 y_{\text{DM}}^{2} c_{h} s_{h} \left( \frac{c_{h} \lambda_{1ik}}{s - M_{h_{1}}^2} + \frac{s_{h} \lambda_{2ik}}{s - M_{h_{2}}^2} \right)^{2} + 4 c_{1} c_{2} c_{h} \bar{y}_{\text{DM}} M_{\text{DM}} \left( \frac{c_{h} \lambda_{1ik}}{s - M_{h_{1}}^2} + \frac{s_{h} \lambda_{2ik}}{s - M_{h_{2}}^2} \right) \left( \frac{t - M_{\text{DM}}^2}{t - M_{h_{1}}^2} + a \left( \frac{s - M_{h_{1}}^2 + M_{h_{2}}^2}{u - M_{h_{2}}^2} \right) \right) + \frac{2 c_{2} c_{2} \bar{y}_{\text{DM}}^{2}}{(t - M_{\text{DM}}^2)} \left( 4 M_{\text{DM}}^{2} M_{h_{k}}^{2} + t - M_{h_{k}}^{2} \right) \left( t - M_{h_{k}}^{2} + t - M_{h_{k}}^{2} \right) + a^{2} \frac{2 c_{2} c_{2} \bar{y}_{\text{DM}}^{2}}{(u - M_{\text{DM}}^2)} \left( 4 M_{\text{DM}}^{2} M_{h_{k}}^{2} + t - M_{h_{k}}^{2} \right) \left( t - M_{h_{k}}^{2} + t - M_{h_{k}}^{2} \right) + a^{2} \frac{2 c_{2} c_{2} \bar{y}_{\text{DM}}^{2}}{(t - M_{\text{DM}}^2)} \left( t - M_{h_{k}}^{2} \right) \left( t - M_{h_{k}}^{2} \right) + \left( M_{\text{DM}}^{2} + M_{h_{k}}^{2} \right) \left( M_{\text{DM}}^{2} + M_{h_{k}}^{2} \right) \left( s - 4 M_{\text{DM}}^2 \right) \left( s - M_{h_{k}}^{2} - M_{h_{k}}^{2} \right), \quad (5.7)
\]

with \(s, t, u\) being the Mandelstam variables, the Yukawa couplings defined as \(y_{\text{DM}} \equiv y_{1}, c_{1} \equiv c_{h} \) and \(c_{2} \equiv s_{h}\). Here, we integrate numerically on the phase space in order to get the

\(^7\)Indeed for same flavor charged leptons \((\alpha = \beta)\), there are \(h_{1,2}\) mediated \(s\)-channel processes that are proportional to their Yukawa couplings; we ignore these due to the Yukawa suppression.
cross section for a given $s$ value. At tree-level the effective cubic scalar couplings ($\lambda_{1ik}$ and $\lambda_{2ik}$) are given by

$$
\begin{align*}
\lambda_{111} &= 6\lambda_H c_h^2 v - 3\lambda_{\phi H} c_h^2 s_h v + 3\lambda_{\phi H} c_h s_h^2 x - 6\lambda_{\phi} s_h^2 x, \\
\lambda_{112} &= \lambda_{\phi H} c_h^2 x + 2c_h s_h (3\lambda_H - \lambda_{\phi H}) v + 2c_h s_h^2 (3\lambda_{\phi} - \lambda_{\phi H}) x + \lambda_{\phi H} s_h^3 v, \\
\lambda_{222} &= \lambda_{122} = 0,
\end{align*}
$$

(5.8)

though for completeness we use the full one-loop results that can be derived from the loop-corrected potential following [28]. The absence of cubic interactions $h_1 h_2^2$ and $h_3^2$, at leading order, is a general feature of SI models.

5.3 Direct Detection

Concerning direct-detection experiments, the effective low-energy Lagrangian responsible for interactions between the DM and quarks is given by

$$
\mathcal{L}_{N_{1-}q}^{(eff)} = a_q \bar{q} q N_{DM}^c N_{DM},
$$

(5.9)

with

$$
a_q = \frac{s_h c_h M_q M_{DM}}{2 v x} \left[ \frac{1}{M_{h_1}^2} - \frac{1}{M_{h_2}^2} \right].
$$

(5.10)

Consequently, the nucleon-DM effective interaction can be written as

$$
\mathcal{L}_{DM-N}^{(eff)} = a_N \bar{N} N N_{DM}^c N_{DM},
$$

(5.11)

with

$$
a_N = \frac{s_h c_h (M_N - \frac{7}{9} M_B) M_{DM}}{v x} \left[ \frac{1}{M_{h_1}^2} - \frac{1}{M_{h_2}^2} \right].
$$

(5.12)

In this relation, $M_N$ is the nucleon mass and $M_B$ the baryon mass in the chiral limit [29]. Thus, the approximate expression of the spin-independent nucleon-DM elastic cross section at low momentum transfer reads

$$
\sigma_{det} = \frac{s_h^2 c_h^2 M_N^2 (M_N - \frac{7}{9} M_B)^2 M_{DM}^4}{\pi v^2 x^2 (M_{DM} + M_B)^2} \left[ \frac{1}{M_{h_1}^2} - \frac{1}{M_{h_2}^2} \right]^2.
$$

(5.13)

As will be discussed below, the most stringent constraint on $\sigma_{det}$ comes from the present as well the recent upper bound reported by LUX experiment [30, 31].

6 Numerical Analysis and Results

In our numerical scan we enforce the minimization conditions, Eqs. (2.7) and (2.8), vacuum stability, the Higgs mass $M_{h_2} = 125.09 \pm 0.21$ GeV, as well as the constraints from LEP (OPAL) on a light Higgs [32]. The constraint from the Higgs invisible decay $B(h \rightarrow inv) < 17\%$ [23] is also enforced. All dimensionless couplings are restricted to perturbative values and we consider the range $200 \text{ GeV} < \langle \phi \rangle < 5 \text{ TeV}$ for the beyond-SM VEV. We find a range
of viable values for $M_{h^2}$, consistent with the OPAL bounds, as shown in Figure 3. For the parameter space in our scan we tend to find $M_{h^2}$ in the range $\mathcal{O}(1) \mathrm{GeV} \lesssim M_{h^2} \lesssim 90 \mathrm{GeV}$. Lighter values of $M_{h^2}$ appear to require a degree of engineered cancelation among the radiative mass-corrections from fermions and bosons; see Eq. (2.17). We noticed that regions with $\langle \phi \rangle \gtrsim 700 \mathrm{GeV}$ tend to be preferred in our scans.

![Figure 3](image_url)

**Figure 3.** Scalar mixing versus the light scalar mass. The palette gives the branching ratio for invisible Higgs decays, with an overwhelming majority of the points shown satisfying the constraint $B(h_1 \rightarrow inv) < 17\%$.

We also scan for viable neutrino masses and mixing, subject to the LFV and muon anomalous magnetic moment constraints, while also demanding a viable DM relic density. In Figure 4 we plot viable benchmark points for the Yukawa couplings $g_{i\alpha}$ and $f_{\alpha\beta}$, along with the corresponding LFV branching ratios and $\delta a_\mu$ contributions. It is clear that the couplings $f_{\alpha\beta}$ are generally smaller than the couplings $g_{i\alpha}$, and that the bound on $\tau \rightarrow \mu \gamma$ is readily satisfied, while the constraint from $\mu \rightarrow e\gamma$ is more severe. We observe in Figure 4 that the model requires the largest coupling in the set $g_{i\alpha}$ to take $\mathcal{O}(1)$ values. This feature is a generic expectation for three-loop models of neutrino mass, as one cannot make the new physics arbitrarily heavy, while reducing the Yukawa couplings, and retain viable SM neutrino masses. Thus, the testability of such models, which predict new physics at the TeV scale, is generally coupled with a need for $\mathcal{O}(1)$ couplings. Consequently one expects such couplings to encounter a Landau pole in the UV, requiring a new description. We note that, when considering only one or two generations of singlet fermions, no solutions that simultaneously accommodate the neutrino mass and mixing data, low-energy flavor constraints, and the DM relic density, were found. Therefore at least three generations of exotic fermions are required. Also, we verified that the constraints from neutrino-less double-beta decay searches are easily satisfied for all benchmark points.

Recall that, with regards to the DM relic density, there are many classes of annihilation channels, namely $N_{DM}N_{DM} \rightarrow X$ ($X = \ell^+\ell^-, b\bar{b}, t\bar{t}, WW + ZZ, h_{1,2}h_{1,2}$). According to the DM mass, each channel could be significant or suppressed. In order to probe the role of
Figure 4. Left: Viable benchmark points for the Yukawa couplings $g_{i\alpha}$ and $f_{\alpha\beta}$, in absolute values, where the dashed line represents the fully degenerate case, i.e., $\min|f| = \max|f|$. Right: The LFV branching ratios, scaled by the experimental bounds, versus the muon anomalous magnetic moment. The vertical line represents the muon anomalous magnetic moment experimental constraint.

each channel, we plot the relative contribution of each channel to the total cross section, i.e. the ratio $\sigma_{X}\sigma_{\text{tot}}$ at the freeze-out versus the DM mass, in Figure 5-left. We see that the channel $N_{\text{DM}} N_{\text{DM}} \rightarrow \ell^+ \ell^-$ is always fully dominant except for a few benchmark points. For DM masses smaller than 80 GeV the contribution of $X = b\bar{b}$ can be significant, while in the range between 80 GeV < $M_{\text{DM}}$ < 100 GeV, both gauge bosons $X = WW + ZZ$ and $X = t\bar{t}$ contributions can be important. In the range 200 GeV < $M_{\text{DM}}$ < 400 GeV, their contribution can reach 20%. For large DM masses $M_{\text{DM}}$ > 200 GeV, the $X = hh$ contribution can reach at most 8%. The fact that the $X = t\bar{t}$ contribution could be important around 100 GeV, i.e., for $M_{\text{DM}} < M_t$, can be understood due to thermal fluctuations. Figure 5-right shows the corresponding charged scalar masses. For lighter DM masses of $M_{\text{DM}}$ < 300 GeV, the charged scalar masses $M_{S_{1,2}}$ should not exceed 450 GeV, while for larger values of $M_{\text{DM}}$, the scalar masses $M_{S_{1,2}}$ can be at the TeV scale. Such light charged scalars can be within reach of collider experiments [33].

Next we discuss the constraints from direct-detection experiments. We plot the direct-detection cross section versus the DM mass for our benchmark points in Figure 6. One observes immediately that the direct-detection limits impose serious constraints on the model, with a large number of the benchmarks excluded by LUX [30] as well the improved LUX bounds [31]. We find that only few benchmarks with $M_{\text{DM}} \lesssim 10$ GeV or $M_{\text{DM}} \gtrsim 400$ GeV survive the LUX bounds. As is clear from the figure, the surviving benchmarks will be subject to future tests in forthcoming direct-detection experiments. The palette in Figure 6 shows the corresponding values for $M_{h_2}$, in units of GeV. In the region of parameter space for which $N_{\text{DM}}$ gives viable dark matter, we find that the $M_{h_2}$ must be greater than 20 GeV.

We emphasize that we only found a few benchmarks for which the DM relic density was primarily determined by annihilations into scalars. On the surface, this claim may appear contrary to the results of Refs. [34, 35], which consider Majorana DM coupled to a
Figure 5. Left: The relative contributions of each channel to the annihilation cross section at the freeze-out temperature versus the DM mass. Right: The corresponding charged scalar masses versus the DM mass.

Figure 6. The direct detection cross section versus the DM mass compared to the recent results from LUX. The palette shows the mass for the neutral beyond-SM scalar, $M_{h_2}$, in units of GeV.

A singlet scalar that communicates with the SM via the Higgs portal (called the Indirect Higgs Portal [34]). Naively one may expect our model to admit parameter space where the DM relic-density is determined primarily by the annihilations $N_{DM} N_{DM} \rightarrow hh$, in analogy with the results of Refs. [34, 35]. However, due to the SI symmetry, our model contains no bare mass terms, which reduces the number of free parameters in the Lagrangian. Consequently the DM mass $M_{DM}$ is related to both the coupling between $N_{DM}$ and $\phi$, and the mixing angle $\theta_h$. This reduction in parameters means we cannot evade the LUX constraints whilst generating a viable relic density by annihilations into scalars, explaining the difference between our results and Refs. [34, 35]. It also explains some features of the benchmark distributions in Figure 6. The benchmarks with larger contributions from the channel $N_{DM} N_{DM} \rightarrow hh$ have a stronger coupling between $N_{DM}$ and $\phi$. This increases the direct-
detection cross section due to $h_{1,2}$ exchange, creating conflict with the bounds from LUX, so the corresponding benchmarks are strongly ruled out. Indeed, with the smaller number of parameters in the SI model, it is a non-trivial result that viable regions of parameter space were found in Figure 6.

**Figure 7.** Left: The oblique parameters $\Delta S$ versus $\Delta T$ for the benchmarks used previously. The palette shows the mixing $\sin^2 \theta_h$ and all the points are inside the ellipsoid of 68% CL. Right: Ratio of the widths for $h \rightarrow \gamma\gamma$ and $h \rightarrow \gamma Z$ relative to the SM values. The constraints from ATLAS and CMS are shown, along with projected sensitivities after Run II at the LHC.

Finally, we mention that the exotics in the model allow for new contributions to the Higgs decays $h \rightarrow \gamma\gamma$ and $h \rightarrow \gamma Z$. We plot the ratio of the corresponding widths relative to the SM values in Figure 7-right. We observe that a significant portion of the benchmarks are consistent with existing constraints from ATLAS and CMS. Importantly, the model can be probed through more precise measurements by ATLAS and CMS after Run II. We note that all benchmark points are consistent with the oblique parameter constraints, as shown in Figure 7-left.

### 7 Conclusion

We presented a scale-invariant extension of the SM in which both the weak scale and neutrino mass were generated radiatively. The model contains a DM candidate, in the form of a sterile neutrino $N_{DM}$. A new light neutral scalar is also predicted, namely the pseudo Goldstone-boson associated with the broken scale-invariance, $h_2$, along with two charged scalars $S_{1,2}$. The masses for the latter are generically expected to be near the TeV scale, due to the related birth of the exotic scale and the weak scale via dimensional transmutation. The constraints on the model are rather strong, particularly the direct-detections constraints from LUX. However, we demonstrated the existence of viable parameter space with $M_{DM} \lesssim 10$ GeV or $M_{DM} \gtrsim 400$ GeV. The model can be tested in a number of ways, including future direct-detection experiments, collider searches for the charged scalars, improved LFV searches, and precision measurements of the Higgs decay width to neutral gauge bosons. We note that the model does not possess an obvious mechanism for baryogenesis - it would
be interesting to study this matter further. In a partner paper we shall study the scale-invariant implementation of the Ma model in Ref. [4].

Acknowledgments

AA wants to thank the ICTP for the hospitality during the last stage of this work. AA is supported by the Algerian Ministry of Higher Education and Scientific Research under the CNEPRU Project No D01720130042. KM is supported by the Australian Research Council.

A Multi-Scalar Scale-invariant Theories

In a general multi-scalar theory one cannot minimize the full one-loop corrected potential analytically. However, with recourse to the underlying SI symmetry, there exists a simple analytic approximation that captures the leading features [15]. A general tree-level SI potential for a set of scalars \( \{\phi_A\} \) can be written as

\[
V_0(\{\phi_A\}) = g_{ABCD} \phi_A \phi_B \phi_C \phi_D,
\]

(A.1)

where the dimensionless couplings \( g_{ABCD} \) are symmetric. In general, these couplings are running parameters that depend on the energy scale, \( g_{ABCD} = g_{ABCD}(\mu) \), and one can freely select a value of \( \mu \) that simplifies the analysis. A convenient choice is the value \( \mu = \Lambda \), at which the tree-level potential vanishes along the direction of an assumed non-trivial minimum in field space, namely

\[
g_{ABCD}(\Lambda) \hat{\phi}_A \hat{\phi}_B \hat{\phi}_C \hat{\phi}_D = 0.
\]

(A.2)

Here, the minimum is defined by \( \langle \phi_A \rangle = R \hat{\phi}_A \), with \( \hat{\phi}_A \) a unit vector in field space and \( R \) a (yet to be determined) radius. Combining Eq. (A.2) with the minimization conditions, \( \partial V_0/\partial \phi_A = 0 \), determines the angular VEVs \( \hat{\phi}_A \) in terms of the couplings \( g_{ABCD} \). Subsequently expanding around the ground state in the tree-level potential reveals a spectrum containing a massless scalar, corresponding to the flat direction.

Eq. (A.2) implies that the tree-level potential vanishes, at the scale \( \mu = \Lambda \), to an accuracy on the order of the loop corrections:

\[
V_0(\{R\hat{\phi}_A\}; \mu = \Lambda) \lesssim \mathcal{O}(V_{\text{1-loop}}),
\]

(A.3)

where we display the renormalization scale dependence and write the full loop-corrected potential as \( V = V_0 + V_{\text{1-loop}} + \ldots \). Thus, one-loop corrections can be comparable to \( V_0 \) along the direction \( \hat{\phi}_A \), so the interplay of the two terms allows a non-trivial minimum that lifts the flat direction to fix the radial VEV \( \langle R \rangle \). Adding the one-loop corrections along the direction \( \hat{\phi}_A \) gives

\[
V(\{R\hat{\phi}_A\}; \mu = \Lambda) = V_0(\{R\hat{\phi}_A\}; \mu = \Lambda) + V_{\text{1-loop}}(\{R\hat{\phi}_A\}; \mu = \Lambda) + \ldots,
\]

(A.4)
which can be written as [15]

\[ V(\{ \hat{R} \phi_A \}; \mu = \Lambda) = A R^4 + B R^4 \log \frac{R^2}{\Lambda^2} + \ldots, \tag{A.5} \]

with

\[ A = \frac{1}{64\pi^2 \langle R \rangle^4} \left\{ \text{Tr} \left[ \mathcal{M}_S^2 \log \frac{\mathcal{M}_S^2}{\langle R \rangle^2} \right] - \text{Tr} \left[ \mathcal{M}_F^2 \log \frac{\mathcal{M}_F^2}{\langle R \rangle^2} \right] + \text{Tr} \left[ \mathcal{M}_V^2 \log \frac{\mathcal{M}_V^2}{\langle R \rangle^2} \right] \right\}, \tag{A.6} \]

and

\[ B = \frac{1}{64\pi^2 \langle R \rangle^4} \left\{ \text{Tr} \mathcal{M}_S^4 - \text{Tr} \mathcal{M}_F^4 + \text{Tr} \mathcal{M}_V^4 \right\}. \tag{A.7} \]

Here \( \mathcal{M}_{S,F,V} \) are the mass matrices for scalars, fermions and vectors, respectively, and the trace runs over both particle species and internal degrees of freedom. Minimizing the one-loop corrected potential lifts the flat-direction to give

\[ \langle R \rangle = e^{-\frac{A}{2B} + \frac{1}{4}} \Lambda. \tag{A.8} \]

The dilaton acquires a loop-level mass, given by \( M_{\text{dilaton}}^2 = 8B\langle R \rangle^2 \). Thus, radiative corrections successfully induce a non-trivial VEV for one or more of the scalars \( \phi_A \), by introducing a dimensionful parameter, \( \langle R \rangle \propto \Lambda \), in exchange for one of the dimensionless couplings in Eq. (A.2). This manifests dimensional transmutation.

In the present model, demanding that \( M_{\text{dilaton}}^2 = 8B\langle R \rangle^2 > 0 \), requires that \( B \) be dominated by the term \( \text{Tr} \mathcal{M}_S^4 \), meaning that one (or both) of the scalars \( S_{1,2}^+ \) must be the heaviest state in the spectrum. In practise, this implies that \( A \) is also dominated by the contribution of \( S_{1,2}^+ \) to the \( \mathcal{M}_S^4 \) term in Eq. (A.6). Thus, loop corrections from the scalars \( h_{1,2} \) along the flat direction are sub-dominant to the corrections from \( S_{1,2}^+ \). Therefore, simply dropping the corrections from \( h_{1,2} \) will not introduce a significant error in the analysis (the error is expected to be \( \mathcal{O}(M_{h_1}^4/M_{S_{1,2}}^4) \)). As discussed in the text, this simplification has the advantage of allowing one to obtain analytic expressions for the ground state by minimizing the one-loop corrected potential directly. As a point of comparison, for the present model, the minimization in Eq. (A.2) gives \( 4\sqrt{\lambda_R(\Lambda)}\lambda_\phi(\Lambda) + \lambda_{\phi h}(\Lambda) = 0 \), and we see from Eq. (2.7) that our approach incorporates loop corrections to this expression, up to \( \mathcal{O}(M_{h_1}^4/M_{S_{1,2}}^4) \) effects. Taking the heaviest scalar as \( M_S \gtrsim 300 \text{ GeV} \) (which we can always do - see Figure 5), the error in the loop terms is typically \( \lesssim 3\% \). Once we have found the ground state, we reintroduce loop corrections from \( h_{1,2} \) to determine the mass eigenvalues, reducing the error in the expressions for the scalar masses and mixings.

\[ \text{For parameter space of interest in this work, corrections from } h_{1,2} \text{ are also smaller than those from the top quark and, in large regions of parameter space, one or more of the fermions } N. \]
B  Oblique Parameter Functions

The functions employed in the calculation of the oblique parameters in Section 4 are defined as follows:

\[
F(I, J) = \begin{cases} 
\frac{I^2 - J^2}{2} \ln \frac{I}{J} & \iff I \neq J, \\
0 & \iff I = J, 
\end{cases} 
\]

(B.1)

\[
G(I, J, Q) = \frac{16}{3} + \frac{5(I + J)}{Q} - \frac{2(I - J)^2}{Q^2} \\
+ \frac{3}{Q} \left[ \frac{I^2 + J^2}{I - J} - \frac{I^2 - J^2}{Q} + \frac{(I - J)^3}{3Q^2} \right] \ln \frac{I}{J} + \frac{r}{Q^3} f(t, r), 
\]

(B.2)

\[
\dot{G}(I, Q) = -\frac{79}{3} + \frac{9Q}{I} - \frac{2Q^2}{I} + \left(-10 + 18 \frac{I}{Q} - 6 \frac{I^2}{Q^2} + \frac{I^3}{Q^3} - 9 \frac{I + Q}{I - Q} \right) \ln \frac{I}{Q} \\
+ \left(12 - 4 \frac{I}{Q} + \frac{r^2}{Q^2} \right) f\left(I, I^2 - 4IQ\right), 
\]

(B.3)

with \( t \equiv I + J - Q \) and \( r \equiv Q^2 - 2Q(I + J) + (I - J)^2 \), and

\[
f(t, r) = \begin{cases} 
\sqrt{t} \ln \left| \frac{t - \sqrt{t}}{t + \sqrt{t}} \right| & \iff r > 0, \\
0 & \iff r = 0, \\
2 \sqrt{-r} \arctan \frac{\sqrt{-r}}{t} & \iff r < 0. 
\end{cases} 
\]

(B.4)

C  Loop induced Higgs decay functions

The functions used to evaluate the Higgs decay rate of \( h \rightarrow \gamma\gamma \) are given by

\[
A_0^\gamma(x) = -x^{-2} [x - f(x)], \\
A_1^{\gamma_1}(x) = 2x^{-2} [x + (x - 1) f(x)], \\
A_1^{\gamma_2}(x) = -x^{-2} \left[2x^2 + 3x + 3(2x - 1) f(x)\right], 
\]

(C.1)

with

\[
f(x) = \begin{cases} 
\arcsin^2(\sqrt{x}) & x \leq 1 \\
-\frac{1}{4} \left[ \log \frac{1 + \sqrt{1 - x}}{1 - \sqrt{1 - x}} - i\pi \right]^2 & x > 1, 
\end{cases} 
\]

(C.2)

and those used in the decay rate of \( h \rightarrow \gamma Z \) are given by

\[
A_0^{\gamma_2}(x, y) = I_1(x, y), \\
A_1^{\gamma_2}(x, y) = I_1(x, y) - I_2(x, y), \\
A_1^{\gamma_2}(x, y) = \left[(1 + 2x) \tan^2 \theta_w - (5 + 2x)\right] I_1(x, y) + 4 \left(3 - \tan^2 \theta_w\right) I_2(x, y), 
\]

(C.3)

with

\[
I_1(x, y) = -\frac{1}{2(x-y)} + \frac{f(x) - f(y)}{2(x-y)^2} + \frac{y [g(x) - g(y)]}{(x-y)^2}, \\
I_2(x, y) = \frac{f(x) - f(y)}{2(x-y)}, 
\]

(C.4)
and

\[ g(x) = \begin{cases} 
\sqrt{x^{-1} - 1} \arcsin \left( \sqrt{x} \right) & x \leq 1 \\
\sqrt{\frac{1}{1-x} - 1} \left[ \log \frac{1 + \sqrt{1-x}}{1 - \sqrt{1-x}} - i\pi \right] & x > 1.
\end{cases} \] (C.5)

References

[1] S. R. Coleman and E. J. Weinberg, Phys. Rev. D 7, 1888 (1973).

[2] A. Kobakhidze and K. L. McDonald, JHEP 1407, 155 (2014) [arXiv:1404.5823 [hep-ph]].

[3] L. M. Krauss, S. Nasri and M. Trodden, Phys. Rev. D 67, 085002 (2003) [hep-ph/0210389].

[4] E. Ma, Phys. Rev. D 73, 077301 (2006) [hep-ph/0601225]; M. Aoki, S. Kanemura and O. Seto, Phys. Rev. Lett. 102, 051805 (2009) [arXiv:0807.0361 [hep-ph]]; M. Aoki, S. Kanemura and O. Seto, Phys. Rev. D 80, 033007 (2009) [arXiv:0904.3829 [hep-ph]]; M. Aoki, S. Kanemura, T. Shindou and K. Yagyu, JHEP 1007, 084 (2010) [JHEP 1011, 049 (2010)] [arXiv:1005.5159 [hep-ph]]; S. Kanemura, O. Seto and T. Shimomura, Phys. Rev. D 84, 016004 (2011) [arXiv:1101.5713 [hep-ph]]; M. Aoki, S. Kanemura and K. Yagyu, Phys. Lett. B 702, 355 (2011) [Erratum-ibid. B 706, 495 (2012)] [arXiv:1105.2075 [hep-ph]]; M. Lindner, D. Schmidt and T. Schwetz, Phys. Lett. B 705, 324 (2011) [arXiv:1105.4626 [hep-ph]]; S. Kanemura, T. Nabheshima and H. Sugiyama, Phys. Rev. D 85, 033004 (2012) [arXiv:1111.0599 [hep-ph]]; Y. H. Ahn and H. Okada, Phys. Rev. D 85, 073010 (2012) [arXiv:1201.4436 [hep-ph]]; S. C. Law and K. L. McDonald, Phys. Lett. B 713, 490 (2012) [arXiv:1204.2529 [hep-ph]]; G. Guo, X.-G. He and G.-N. Li, JHEP 1210, 044 (2012) [arXiv:1207.6308 [hep-ph]]; P. S. Bhupal Dev and A. Pilaftsis, Phys. Rev. D 87, 053007 (2013) [arXiv:1212.3808 [hep-ph]]; M. Gustafsson, J. M. No and M. A. Rivera, Phys. Rev. Lett. 110, no. 21, 211802 (2013) [arXiv:1212.4806 [hep-ph]].

[5] A. Ahriche and S. Nasri, JCAP 1307, 035 (2013) [arXiv:1304.2055]; A. Ahriche, C. S. Chen, K. L. McDonald and S. Nasri, Phys. Rev. D 90, 015024 (2014) [arXiv:1404.2696 [hep-ph]]; A. Ahriche, K. L. McDonald and S. Nasri, JHEP 1410, 167 (2014) [arXiv:1404.5917 [hep-ph]]; C. S. Chen, K. L. McDonald and S. Nasri, Phys. Lett. B 734, 388 (2014) [arXiv:1404.6033 [hep-ph]]; A. Ahriche, K. L. McDonald, S. Nasri and T. Toma, arXiv:1504.05755 [hep-ph]; A. Ahriche, K. L. McDonald and S. Nasri, Phys. Rev. D 92 (2015) 9, 095020 [arXiv:1508.05881 [hep-ph]].

[6] M. Aoki, J. Kubo and H. Takano, Phys. Rev. D 87, 116001 (2013) [arXiv:1302.3936 [hep-ph]]; Y. Kajiyama, H. Okada and K. Yagyu, Nucl. Phys. B 874, 198 (2013) [arXiv:1303.3463 [hep-ph]]; Y. Kajiyama, H. Okada and T. Toma, Phys. Rev. D 88, no. 1, 015029 (2013) [arXiv:1303.7356]; S. S. C. Law and K. L. McDonald, JHEP 1309, 092 (2013) [arXiv:1305.6467 [hep-ph]]; D. Restrepo, O. Zapata and C. E. Yaguna, JHEP 1311, 011 (2013) [arXiv:1308.3655 [hep-ph]]; E. Ma, I. Picek and B. Radovec Phys. Lett. B 726, 744 (2013) [arXiv:1308.5313 [hep-ph]]; V. Brdar, I. Picek and B. Radovec, Phys. Lett. B 728, 198 (2014) [arXiv:1310.3183 [hep-ph]]; H. Okada and K. Yagyu, Phys. Rev. D 89, no. 5, 053008 (2014) [arXiv:1311.4360 [hep-ph]]; S. Baek, H. Okada and T. Toma, arXiv:1312.3761 [hep-ph]; S. Baek, H. Okada and T. Toma, arXiv:1401.6921 [hep-ph]; H. Okada, arXiv:1404.0280 [hep-ph]; A. Ahriche, C. S. Chen, K. L. McDonald and S. Nasri, Phys. Rev. D 90, 015024 (2014) [arXiv:1404.2696 [hep-ph]].

[7] J. N. Ng and A. de la Puente, Phys. Rev. D 90, no. 9, 095018 (2014) [arXiv:1404.1415 [hep-ph]]; S. Kanemura, T. Matsui and H. Sugiyama, Phys. Rev. D 90, 013001 (2014) [arXiv:1405.1935 [hep-ph]]; H. Okada and K. Yagyu, Phys. Rev. D 90, 035019 (2014)
[arXiv:1405.2368 [hep-ph]]; S. Kanemura, N. Machida and T. Shindou, Phys. Lett. B 738, 178 (2014) [arXiv:1405.5834 [hep-ph]]; M. Aoki and T. Toma, JCAP 1409, 016 (2014) [arXiv:1405.5870 [hep-ph]]; H. Ishida and H. Okada, arXiv:1406.5808 [hep-ph]; H. Okada and Y. Orikasa, Phys. Rev. D 90, 075023 (2014) [arXiv:1407.2543 [hep-ph]]; H. Okada, T. Toma and K. Yagyu, Phys. Rev. D 90, 095005 (2014) [arXiv:1408.0961 [hep-ph]]; H. Hatanaka, K. Nishiwaki, H. Okada and Y. Orikasa, arXiv:1412.8664 [hep-ph]; S. Baek, H. Okada and K. Yagyu, arXiv:1501.01530 [hep-ph]; L. G. Jin, R. Tang and F. Zhang, Phys. Lett. B 741, 163 (2015) [arXiv:1501.02020 [hep-ph]]; H. Okada, arXiv:1503.04557 [hep-ph]; H. Okada, N. Okada and Y. Orikasa, arXiv:1504.01204 [hep-ph].

[8] P. Culjak, K. Kumericki and I. Picek, Phys. Lett. B 744, 237 (2015) [arXiv:1502.07887 [hep-ph]]; D. Restrepo, A. Rivera, M. Sanchez-Pelaez, O. Zapata and W. Tangarife, arXiv:1504.07892 [hep-ph]; S. Kashiwase, H. Okada, Y. Orikasa and T. Toma, arXiv:1506.00261 [hep-ph]; M. Aoki, T. Toma and A. Vicente, arXiv:1507.01591 [hep-ph]; K. Nishiwaki, H. Okada and Y. Orikasa, arXiv:1507.02412 [hep-ph].

[9] R. Hempfling, Phys. Lett. B 379, 153 (1996) [hep-ph/9604278]; K. A. Meissner and H. Nicolai, Phys. Lett. B 648, 312 (2007) [hep-th/0612165]; W. F. Chang, J. N. Ng and J. M. S. Wu, Phys. Rev. D 75, 115016 (2007) [hep-ph/0701254 [HEP-PH]]; R. Foot, A. Kobakhidze and R. R. Volkas, Phys. Lett. B 655, 156 (2007) [arXiv:0704.1165 [hep-ph]]; R. Foot, A. Kobakhidze, K. L. McDonald and R. R. Volkas, Phys. Rev. D 76, 075014 (2007) [arXiv:0706.1829 [hep-ph]]; T. Hambye and M. H. G. Tytgat, Phys. Lett. B 659, 651 (2008) [arXiv:0707.0633 [hep-ph]]; R. Foot, A. Kobakhidze and R. R. Volkas, Phys. Rev. D 82, 035005 (2010) [arXiv:1006.0131 [hep-ph]].
[12] S. Abel and A. Mariotti, Phys. Rev. D 89, no. 12, 125018 (2014) [arXiv:1312.5335 [hep-ph]]; C. T. Hill, Phys. Rev. D 89, no. 7, 073003 (2014) [arXiv:1401.4185 [hep-ph]]; J. Guo and Z. Kang, arXiv:1401.5609 [hep-ph]; M. Hashimoto, S. Iso and Y. Orikasa, Phys. Rev. D 89, no. 5, 056010 (2014) [arXiv:1401.5944 [hep-ph]]; S. Benic and B. Radovic, Phys. Lett. B 732, 91 (2014) [arXiv:1401.8183 [hep-ph]]; A. Salvio and A. Strumia, JHEP 1406, 080 (2014) [arXiv:1403.4226 [hep-ph]]; J. Kubo, K. S. Lim and M. Lindner, Phys. Rev. Lett. 113, 091604 (2014) [arXiv:1403.4262 [hep-ph]]; V. V. Khoze, C. McCabe and G. Ro, JHEP 1408, 026 (2014) [arXiv:1403.4953 [hep-ph], arXiv:1403.4953]. G. C. Dorsch, S. J. Huber and J. M. No, Phys. Rev. Lett. 113, 121801 (2014) [arXiv:1403.5583 [hep-ph]]; H. Davoudiasl and I. M. Lewis, Phys. Rev. D 90, no. 3, 033003 (2014) [arXiv:1404.6260 [hep-ph]]; J. Kubo, K. S. Lim and M. Lindner, JHEP 1409, 016 (2014) [arXiv:1405.1052 [hep-ph]]; M. Lindner, S. Schmidt and J. Smirnov, JHEP 1410, 177 (2014) [arXiv:1405.6204 [hep-ph]].

[13] K. Kannike, A. Racioppi and M. Raidal, JHEP 1406, 154 (2014) [arXiv:1405.3987 [hep-ph]]; V. V. Khoze and G. Ro, JHEP 1410, 61 (2014) [arXiv:1406.2291 [hep-ph]]; D. F. Litim and F. Sannino, JHEP 1412, 178 (2014) [arXiv:1406.2337 [hep-th]]; G. M. Pelaggi, Nucl. Phys. B 893, 443 (2015) [arXiv:1406.4104 [hep-ph]]; O. Antipin, E. Melgaard and F. Sannino, JHEP 1506, 030 (2015) [arXiv:1406.6166 [hep-th]]; W. Altmannshofer, W. A. Bardeen, M. Bauer, M. Carena and J. D. Lykken, JHEP 1501, 032 (2015) [arXiv:1408.3429 [hep-ph]]; Y. Hamada, H. Kawai, K. y. Oda and S. C. Park, Phys. Rev. D 91, no. 5, 053008 (2015) [arXiv:1408.4864 [hep-ph]]; T. G. Steele, Z. W. Wang and D. G. C. McKeon, Phys. Rev. D 90, no. 10, 105012 (2014) [arXiv:1409.3489 [hep-ph]]; O. Antipin, M. Redi and A. Strumia, JHEP 1501, 157 (2015) [arXiv:1410.1817 [hep-ph]]; K. Allison, C. T. Hill and G. G. Ross, Nucl. Phys. B 891, 613 (2015) [arXiv:1409.4029 [hep-ph]]; M. B. Einhorn and D. R. T. Jones, JHEP 1503, 047 (2015) [arXiv:1410.8513 [hep-th]]; Z. Kang, arXiv:1411.2773 [hep-ph]; G. F. Giudice, G. Isidori, A. Salvio and A. Strumia, JHEP 1502, 137 (2015) [arXiv:1412.2769 [hep-ph]]; H. Okada and Y. Orikasa, arXiv:1412.3616 [hep-ph].

[14] Y. Hamada, H. Kawai and K. y. Oda, arXiv:1501.04455 [hep-ph]; J. Guo, Z. Kang, P. Ko and Y. Orikasa, arXiv:1502.00508 [hep-ph]; K. Kannike, G. HÝtysÃ, A. Salvio and A. Strumia, arXiv:1502.01334 [astro-ph.CO]; N. G. Nielsen, F. Sannino and O. Svendsen, arXiv:1503.00702 [hep-ph]; K. Endo and Y. Sumino, arXiv:1503.02819 [hep-ph]; P. Humbert, M. Lindner and J. Smirnov, arXiv:1503.03066 [hep-ph]; S. Oda, N. Okada and D. s. Takahashi, arXiv:1504.06291 [hep-ph]; Y. Ametani, M. Aoki, H. Goto and J. Kubo, arXiv:1505.00128 [hep-ph]; Y. Hamada, K. Kawana and K. Tsumura, arXiv:1505.01721 [hep-ph]; C. D. Carone and R. Ramos, arXiv:1505.04448 [hep-ph]; Z. Kang, arXiv:1505.06554 [hep-ph]; S. Di Chiara and K. Tuominen, arXiv:1506.03285 [hep-ph]; Y. Hamada and K. Kawana, arXiv:1506.06553 [hep-ph]; J. Kubo and M. Yamada, arXiv:1506.06460 [hep-ph]; K. Endo and K. Ishiwata, arXiv:1507.01739 [hep-ph]; A. Farzinnia, arXiv:1507.06926 [hep-ph]; D. M. Ghilencea, arXiv:1508.00595 [hep-ph].

[15] E. Gildener and S. Weinberg, Phys. Rev. D 13, 3333 (1976).

[16] R. Foot, A. Kobakhidze, K. L. McDonald and R. R. Volkas, Phys. Rev. D 77, 035006 (2008) [arXiv:0709.2750 [hep-ph]].

[17] R. Foot, A. Kobakhidze, K. L. McDonald and R. R. Volkas, Phys. Rev. D 89, 115018 (2014) [arXiv:1310.0223 [hep-ph]].

[18] B. Pontecorvo, Sov. Phys. JETP 26, 984 (1968) [Zh. Eksp. Teor. Fiz. 53, 1717 (1967)]; Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. 28, 870 (1962).
[19] D. V. Forero, M. Tortola and J. W. F. Valle, Phys. Rev. D 86, 073012 (2012) [arXiv:1205.4018 [hep-ph]].

[20] F. Simkovic, A. Faessler, H. Muther, V. Rodin and M. Stauf, Phys. Rev. C 79, 055501 (2009) [arXiv:0902.0331 [nucl-th]].

[21] B.W. Lynn, M.E. Peskin, and R.G. Stuart, ‘Radiative corrections in SU2xU1’ in Physics at LEP, J. Ellis and R.D. Peccei eds. (CERN, Geneva, 1986); D.C. Kennedy and B.W. Lynn, Nucl. Phys. B 322 (1989) 1; M.E. Peskin and T. Takeuchi, Phys. Rev. Lett. 65 (1990) 964; G. Altarelli and R. Barbieri, Phys. Lett. B 253 (1991) 161; M.E. Peskin and T. Takeuchi, Phys. Rev. D 46 (1992) 381. G. Altarelli, R. Barbieri, and S. Jadach, Nucl. Phys. B 369 (1992) 3 [erratum ibid. B 376 (1992) 444].

[22] W. Grimus, L. Lavoura, O. M. Ogreid and P. Osland, Nucl. Phys. B 801, 81 (2008) [arXiv:0802.4353 [hep-ph]].

[23] P. Bechtle, S. Heinemeyer, O. Stal, T. Stefaniak, and G. Weiglein, JHEP 1411 (2014) 039, [arXiv:1403.1582].

[24] A. Djouadi, Phys. Rept. 457 (2008) 1.

[25] G. Jungman, M. Kamionkowski and K. Griest, Phys. Rept. 267 (1996) 195; G. Bertone, D. Hooper and J. Silk, Phys. Rept. 405 (2005) 279; L. Bergstrom, Rept. Prog. Phys. 63 (2000) 793.

[26] P. A. R. Ade et al. [Planck Collaboration], arXiv:1502.01589 [astro-ph.CO].

[27] K. Cheung and O. Seto, Phys. Rev. D 69 (2004) 113009 doi:10.1103/PhysRevD.69.113009 [hep-ph/0403003].

[28] A. Ahriche, A. Arhrib and S. Nasri, JHEP02 (2014) 042.

[29] X.G. He, T. Li, X.Q. Li, J. Tandean and H.C. Tsai, Phys. Rev. D79 (2009) 023521 (arXiv:0811.0658 [hep-ph]).

[30] D. S. Akerib et al. [LUX Collaboration], arXiv:1310.8214 [astro-ph.CO].

[31] D. S. Akerib et al. [LUX Collaboration], arXiv:1512.03506 [astro-ph.CO].

[32] OPAL Collaboration (G. Abbiendi et al.), Eur. Phys. J. C27 (2003) 311-329.

[33] A. Ahriche, S. Nasri and R. Soualah, arXiv:1403.5694 [hep-ph].

[34] L. Lopez-Honorez, T. Schwetz and J. Zupan, Phys. Lett. B 716, 179 (2012) [arXiv:1203.2064 [hep-ph]].

[35] M. Dutra, C. A. d. S. Pires and P. S. R. da Silva, arXiv:1504.07222 [hep-ph].