A stochastic user-operator assignment game for empirical evaluation of a microtransit service in Luxembourg

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Abstract

We tackle the problem of evaluating and designing a microtransit service. Microtransit operators can allocate resources to improve upon many aspects of operation: vehicle capacities; fleet size; algorithms to improve routing, pricing, repositioning, matching; and more. We conduct the first empirical application of a model from Rasulkhani and Chow (2019) that evaluates such systems using stable matching between travelers and operator-routes. The study is conducted using a real data set of Kussbus shared by industry collaborator UFT (Utopian Future Technologies S.A.) covering 3010 trips made between April to October 2018 in Luxembourg and its French-side and Belgium-side border areas. Several modifications are made to the model to convert it into a stochastic reliability-based model to better allow it to fit to the data. Calibration results led to a VOT of 24.67 euros/hour, a base utility of trips made with Kussbus as 45 Euros, and a significance level for stability of $\alpha = 0.20$ (54% prediction rate, 70% matches with withheld 20% test set). Analysis of the system using the model shows that the current Kussbus operation is not a stable outcome; an increase of ticket prices to a buyer-optimal policy would only reduce ridership from 465 trips to 426 trips while increasing net profit from -4135 euros to 187 euros for 615 ride requests. If the government were to intervene, we recommend they subsidize Kussbus to improve their route operating costs by requiring a buyer-optimal pricing policy as a cost reduction of 50% would increase ridership by 10%. On the other hand, if Kussbus can reduce in-vehicle travel time on their own by 20%, they can significantly increase profit several fold from the baseline.

Keywords: Microtransit ; On-demand mobility ; Stable matching; Assignment game ; Service network design; Luxembourg

1. Introduction

On-demand mobility (MOD) service has been promoted as an effective alternative to reduce traffic congestion and CO2 emissions in many countries (Murphy and Feigon, 2016). Generally, this kind of service is operated by private transport network companies (TNC) to provide travelers with door-to-door rides based on travelers’ needs with flexible pickup and drop-off locations, comfortable and convenient service. A range of such MOD services include microtransit, ridesharing, paratransit, taxi, and ride-hailing, etc. have been successfully deployed in many cities with different service requirements and operation policies (Kwoka-Coleman, 2017; Metro magazine, 2019). As rural areas have low accessibility to public transport service, a microtransit system presents a good potential to compensate for this gap and reduce personal car use. Although microtransit service can overcome the shortage of fixed-route public transit service, one of
the main issues remains its high operating cost, generally much exceeding its revenue from ticketing and require government’s subsidy as many public transport services. A mix of successful ventures like Via and MaaS Global along with failed microtransit services like Kutsuplus (Haglund et al., 2019), Bridj (Bliss, 2017), and Chariot (Hawkins, 2019) show the importance of operating cost allocation decisions for a sustainable service operation. When planning microtransit service, an operator needs to consider a bundle of decision parameters including vehicle dispatching, routing, access distance to bus stops, maximum waiting time and detour time, etc. These operating policies influence operating cost and can have a significant impact on ticket price and ridership.

Most studies address the MOD/microtransit services planning from the supply-side perspective, focusing on vehicle routing and dispatching (Cordeau and Laporte, 2007; Agatz et al., 2012; Zhang and Pavone, 2016; Ma 2017; Ma et al., 2018, 2019), pricing (Sayarshad and Chow, 2015), vehicle rebalancing (Sayarshad and Chow, 2017), and integrated service planning (Horn 2002; Raghunathan et al. 2018). However, the problem in system evaluation is to have descriptive models instead of normative models. Continuous approximation models are used as descriptive models to evaluate on-demand services (e.g. Daganzo, 1978; Chang and Schonfeld, 1991) but tend to assume simple operating policies and fixed demand.

Planning these services needs to jointly consider travellers’ choice preferences as well as operators’ cost allocation policy to predict and evaluate ridership on the service network. Bilevel models have been proposed for fixed route transit markets (Zhou et al., 2005), which are computationally intractable for realistic systems and rely on user equilibrium in route choice on a road network. However, in systems involving mobility-on-demand systems the choice of travellers is not on which links in a road network to traverse but on which modal services to take. The interaction of travellers and operators can also be achieved using dynamic systems simulation via day-to-day adjustment (Djavadian and Chow, 2017a, b). The drawback is that this method requires using agent simulation which limits the generality of findings.

Rasulkhani and Chow (2019) proposed a new user-route assignment approach based on the assignment games (Shapley and Shubik, 1971; Sotomayor, 1992). This approach considers user-route assignment as a matching problem between a set of buyers (travellers) and sellers (operators) under service capacity constraints. Under this modelling approach, travellers receive a net utility from using the service and transfer the cost (ticket price) as a benefit to the operator. The problem is formulated as an assignment game in which travellers and operators need to have sufficient incentives (non-zero profit on each side) to participate. The study argues that Mobility-as-a-Service (MaaS) systems evaluation can be conducted using stable matching criterion instead of the commonly held paradigm of user equilibrium for route assignment. Moreover, travellers’ preference, generalized travel cost, and operators’ routing cost and service design can be explicitly considered. This approach allows one or multiple operators to evaluate the impact of different operating policies on ridership. While subsequent studies have extended the work to include generalized multimodal trips (Pantelidis et al., 2019), no empirical study has been conducted with this methodology yet.

The contribution of this study is threefold. We first propose a stochastic variant of the user-operator assignment model to match users and a set of service lines with capacity constraints. This model takes into account the stochastic nature of users’ travel utility perception, resulting in a probabilistic stable operation cost allocation outcome to design ticket price and ridership forecasting. Second, we develop the methodology to estimate the model parameters and calibrate them to evaluate an operator’s service policy. Third and primarily, we apply the proposed approach to an empirical study of a microtransit service, namely Kussbus¹, in Luxembourg and its French- and Belgium-side border area using real data shared by the company UFT (Utopian Future Technologies S.A.). We conduct a sensitivity analysis to investigate the impact of route cost, in-vehicle travel time and access distance to bus stops on ticket prices, ridership and operator’s profit. The results support the new approach and tool to evaluate different operating and cost allocation policies for operators.

¹ https://kussbus.lu/
### 2. Methodology

The following notation is used.

| Symbol | Description |
|--------|-------------|
| $s \in S$ | a user or a set of homogeneous users |
| $r \in R$ | Index of routes, defined as a sequence of stops visited by a shuttle (bus) |
| $u_{sr}$ | Utility/payoff gained for user $s$ for matching route $r$. |
| $t_{sr}$ | Generalized travel cost for user $s$ taking route $r$ to connect origin to destination. It could be measured as the weighted sum of walking, waiting and in-vehicle riding time and ticket price. |
| $a_{sr}$ | Payoff gained resulting from $(s, r)$ match, defined as $a_{sr} = \max(0, U_{sr} - t_{sr})$ |
| $u_s$ | Payoff gained for user $s$ |
| $\nu_r$ | Profit gained for route $r$ |
| $v_{sr}$ | Profit gained for the operator from the user-route matching $(s, r)$ |
| $p_{sr}$ | Ticket price for user $s$ to use route $r$ |

Note: utility, payoff and profit are measured in monetary units.

We first recall the stable matching model in the context of cost allocation policy design for on-demand shuttle (microtransit) service for the passenger transportation problem. The problem considers a set of users to be assigned to a set of routes provided by one or multiple operators. The problem is a many-to-one assignment game in which one route can be matched to multiple users and one user can match to only one route (see an illustrative example in Figure 1). Each route is a sequence of stops visited by one or more vehicles on which line (for multiple vehicles) or vehicle (for single vehicle) capacity constraints need to be satisfied. We consider each user a buyer, and each route a seller with a selling price for using that portion of the service route. When users are assigned to routes, users pay a respective ticket price and gain a payoff upon trip completion, while the seller gains a profit as the revenue received from a user reduced by the cost allocated to the supply of that portion of routes. The objective of the assignment game is to find a seller-buyer matching/assignment such that a total generalized payoff is maximized. The outcome of the model is route flows as well as stable cost/profit allocation outcome, i.e. user payoff and operator profit profiles. The cost allocation outcome can be used to design ticket prices and other travel disutilities (e.g. wait or access times due to matching algorithms which impact the total payoff available for cost allocation) and evaluate their impact on ridership and operator revenue. The reader is referred to Rasulkhani and Chow (2019) for a more detailed description of the model properties.

![Figure 1](image-url)  
**Figure 1.** Example of one user and two routes (r1 and r2). A user’s generalized travel cost includes a door-to-door travel cost as the weighted sum of walking time, waiting time, in-vehicle travel time, and ticket price paid to the operator.

The stable matching model can be formulated as follows. First, an optimal user-route assignment problem is formulated as an assignment game (P1, below) to find user-route matches that maximize total generated payoff. The output of P1 is a set of matched user-route flows on the operator’s service network. Second, given the assignment result of P1, a stable cost allocation problem (P2, described later) is formulated under a desired objective in which operators and users have no incentive to switch (for users this might involve switching to other service routes or a dummy route for no travel or an option external to the market; for
operators this involves matching to other users). The output of P2 is the profile of net payoffs for users and routes of operators.

**P1: User-route assignment model**

\[ \max \sum_{s \in S} \sum_{r \in R} a_{sr} x_{sr} \]  
\[ \text{s.t.} \]  
\[ \sum_{r \in R} x_{sr} \leq q_s, \forall s \in S \setminus \{1\} \]  
\[ \sum_{s \in S \setminus \{1\}} \delta_{asr} x_{sr} \leq u_a, \forall a \in A_r, r \in R \]  
\[ \sum_{s \in S \setminus \{1\}} x_{sr} \leq M(1 - x_{kr}), \forall r \in R \]  
\[ x_{sr} \in \{0, 1\}, \forall s \in S, \forall r \in R \]  

The objective function (1) maximizes total payoff gains form the assignment. The payoff gained by a user or a set of homogeneous (e.g. same origin-destination (OD) pair) users \( s \) for matching with route \( r \) is \( a_{sr} = \max(0, U_{sr} - t_{sr}) \), where \( U_{sr} \) is the utility gain, \( t_{sr} \) is the generalized travel cost for user-route pair \( (s,r) \). The latter parameter \( t_{sr} \) can be tuned to account for many different policy or algorithm designs as well as scenario settings. For example, one can specify \( t_{sr} = t_{sr,IV} + b_1 t_{sr,wait} + b_2 t_{sr,access} \) as three terms for in-vehicle time (IV), waiting, and access with corresponding coefficients \( b_1, b_2 \). In that case, an operator interested in evaluating a new matching algorithm that would on average increase access time for users but reduce wait time and in-vehicle time as well as operating cost \( C_r \) of route \( r \) can use this model to compare the effect of the operating designs. A city agency wanting to measure the effect of increased travel times due to added congestion on the roads can increase the in-vehicle time to see how that impacts the assignment game outcomes.

Equation (2) states for any user \( s \) the summation of flows over routes cannot exceed its demand \( q_s \). Equation (3) states assigned user flow on any route needs to satisfy corresponding route capacity constraint \( u_r \) (passengers per hour). \( \delta_{asr} \) is an indicator being 1 if arc \( a \) is used by user \( s \) for route \( r \) and 0 otherwise. The dummy user \( k \) of not matching with any route is set as a reference alternative, generally with a utility of 0. This assumes that the market has no other travel options outside the system that provides travel utility for matching (i.e. a closed market as opposed to an open market or submarket controlled with a platform). Equation (4) ensures that a route is only matched when its total payoff exceeds a threshold cost; for private operators with no subsidy this would be setting \( a_{kr} = C_r \). \( M \) is a big positive number. Equation (5) ensures that the decision variable \( x_{sr} \) is a non-negative integer.

Departing from Rasulkhani and Chow (2019), we make the following modification to the model to allow us to forecast utility from route-level or user-level attributes. We assume the utility \( U_{sr} \) is an independent random variable composed of a deterministic part \( V_{sr} \) and an unobserved part \( \epsilon_{sr} \) as Equation (6).

\[ U_{sr} = V_{sr} + \epsilon_{sr} \]  

where \( V_{sr} \) is the mean utility gain and \( \epsilon_{sr} \) is a random utility term that follows a Normal distribution with mean 0 and standard deviation \( \sigma \). Given \( U_{sr} \) is probabilistic, so is \( a_{sr} \). The resulting optimal assignment and stable outcome becomes a probabilistic space.
**P2: User-operator stable cost sharing model (stable sharing model)**

Per Rasulkhani and Chow (2019), the stable outcome model is specified as follows in Equations (7) – (12).

\[
\begin{align*}
\text{max } Z \quad & \text{(7)} \\
\text{s.t.} \quad & \sum_{s \in G(r,x)} u_s + v_r \geq \sum_{s \in G(r,x)} a_{sr} - C_r, \forall r \in R \quad & \text{(8)} \\
\sum_{s \in S(r,x)} u_s + v_r &= \sum_{s \in S(r,x)} a_{sr} - C_r, \forall r \in R^* \quad & \text{(9)} \\
v_r &= 0, \forall r \in R \setminus R^* \quad & \text{(10)} \\
u_s &= 0, \text{ if } s \in S = \{s \mid \sum_{r \in R} x_{sr} = 0\} \quad & \text{(11)} \\
u_s &\geq 0, v_r \geq 0, \forall r \in R^* \quad & \text{(12)}
\end{align*}
\]

Equation (7) is the objective function to be maximized. If no cost allocation mechanism is being evaluated, one can set the buyer-optimal and seller-optimal objectives to obtain the vertices for the full range of stable outcomes. Since the problem is convex, the prices based on convex combinations of the two vertices would all be stable as well. For example, we can set \( Z = \sum_{s \in S} u_s \) to maximize total utility gain of users, which would be a vertex of interest to public agencies. Its solution, if any, is a buyer-optimal cost allocation outcome. Alternative, if we aim to maximize total profit gain of operators, the objective function becomes \( Z = \sum_{r \in R} v_r \). The optimal solution is a seller-optimal outcome.

Equation (8) ensures the stable condition for which no user would have a better payoff other than the current assignment. \( G(r,x) \) is the group of users which can be feasibly assigned on route \( r \) given the solutions \( x \) of P1. In the case of an operator owning multiple routes, constraint (8) is only applied to routes not owned by that operator. In other words, in the case of a centralized operator where costs can freely transfer between routes, constraint (8) would be relaxed. For the Kussbus case study we assume routes do not freely transfer costs between each other.

The feasibility constraints are verified when \( G(r,x) \) satisfies Equation (3). For example, consider an assignment outcome \( x \) assigns users \( \{s_1, s_2, s_3\} \) to a route \( r \). The set of group users \( G(r,x) \) is the union of subsets of users from \( \{s_1, s_2, s_3\} \), i.e. \( \{s_1\}, \{s_2\}, \{s_3\}, \{s_1, s_2\}, \{s_1, s_3\}, \{s_2, s_3\}, \{s_1, s_2, s_3\} \). Equation (9-11) are the feasibility conditions where \( R^* \) is the subset of routes with at least one matched user. \( S(r,x) \) is the set of users matching route \( r \), given an optimal assignment solution \( x \) obtained by P1. Equation (12) ensures the decision variable \( u_s \) and \( v_r \) are non-negative continuous variables. We call \( ([u, v]; x) \) a cost allocation outcome given an optimal assignment \( x \). The cost allocation outcome is the list of payoffs and profits for users and routes.

New in this study and different from the deterministic stable matching model (Rasulkhani and Chow, 2019), Equation (8) and (9) are now stochastic constraints because of the presence of a stochastic \( a_{sr} \). By introducing the Equation (6) in (8), Equation (8) becomes Equation (13).

\[
\begin{align*}
\sum_{s \in G(r,x)} u_s + v_r + C_r &= \sum_{s \in G(r,x)} v_{sr} \geq \sum_{s \in G(r,x)} \varepsilon_{sr}, \quad \forall r \in R \quad & \text{(13)}
\end{align*}
\]

The sum of \( k \) Normal distributions of \( \varepsilon_{sr} \) is also a Normal distribution. Consider the concept of \( \alpha \)-stability, a reliability measure for which matches are perceived to be stable (e.g. \( \alpha = 0.05 \) implies being 95% sure of
matching satisfying the constraint). Then Equation (13) can be expressed deterministically as a chance constraint (14).

$$\Phi \left[ \frac{\sum_{S \in G(r,x)} u_s + v_r + C_r + \sum_{S \in G(r,x)} v_{sr}}{|G(r,x)|} \right] \geq 1 - \alpha, \quad \forall G(r,x), r \in R$$

(14)

where \( \Phi(z) = Pr(Z \leq z) \) is the cumulative density function of \( Z \).

Equation (14) is a nonlinear constraint which can be transformed to a linear inequality in Equation (15) per Shapiro et al. (2009).

$$\sum_{s \in G(r,x)} u_s + v_r + C_r - \sum_{s \in G(r,x)} v_{sr} \geq Z_{1-\alpha}$$

(15)

with new deviation \( \sigma' = \sqrt{|G(r,x)|} \sigma^2 \). The extension is a generalization; when \( \alpha = 0.50 \) the problem simplifies back to the original deterministic model from Rasulkhani and Chow (2019).

The P2 problem then becomes a linear programming problem and can be solved efficiently by the simplex algorithm or interior-point algorithm using existing commercial solvers.

Given the \( \alpha \)-stability outcome \( \{(u, v); x\} \), we can determinate ticket prices as shown in Equation (16).

$$p_{sr} = v_{sr} + c_{sr}, \forall r \in R, \forall s \in S \setminus \{k\}$$

(16)

with \( \sum_{S \in G(r,x)} c_{sr} = C_r \) and \( \sum_{S \in G(r,x)} v_{sr} = v_r, \forall r \in R \)

Operators may determine the ticket prices based on user-defined policy (e.g. equal-share or cost-based share policy) given the stable cost allocation outcome. Given a user-route assignment outcome in P1, one can solve the P2 problem based on a buyer-optimal objective or seller-optimal objective to obtain upper and lower bounds of a stable outcome space to delimit ticket prices for each user. Note that the price is a cost transfer decision between user and operator, which is determined in the stable outcome (P2). Basically \( a_{sr} \) in P1 determines the best flows in which ticket price is left out. The pricing can only be set where that flow is optimal. Operators can then evaluate the ridership and profit as user-route matched flows by integrating ticket prices as a part of user’s generalized travel cost in P1. Consequently, the resulting ridership and operator profit depends on the integrated price schemes.

3. Kussbus trip data and exploratory data analysis

In this section, we apply the stochastic stable matching model on evaluating the service design of a microtransit service in Luxembourg. We first present an exploratory data analysis. Then we detail how to calibrate the model parameters and evaluate the impact of different service designs on ridership and operator’s revenue. For this case study we assume each route operates in a decentralized manner, i.e. there is no cost transfer between routes (they each need to be self-sustaining).

3.1. Kussbus on-demand bus service and exploratory data analysis

Kussbus Smart shuttle service (https://kussbus.lu/en/how-it-works.html) is a first microtransit service operating in Luxembourg and its border area. The service was provided by the Utopian Future Technologies S.A.(UFT) from April 2018 to March 2019. Like most microtransit systems, users use dedicated Smartphone applications to book a ride in advance with desired origin, destination and pickup time as input. Service routes are flexible to meet maximum access distance constraint. Routes are generated in a way that we assume users need to walk from/to the origin/destination to/from shuttle stops given a pre-defined threshold (i.e. around one kilometer). The service started operating between the Arlon region in Belgium and the
Kirchberg district of Luxembourg City on 04/25/2018 and a second line started on 09/24/2018 between Thionville region (France) and Kirchberg district. Both service areas are highly congested on road networks due to high car use during morning and afternoon peak hours (Rifkin et al., 2016).

The empirical ride data was provided by the operator for the period from 4/25/2018 to 10/10/2018. A total of 3258 trips (rides) were collected. Each ride contains the following information: booking date and time, pickup time and drop-off time, pickup and drop-off locations, pickup and drop-off stops, walking distance between stops and origins/destinations, origin-destination pairs of users, and fare. Any abnormal trips (e.g. trip duration less or equal to 5 minutes) were removed. As a result, a total of 3010 trips were used for this study.

We summarize the characteristics of Kussbus service as follows.

- Service areas: two service areas: a.) Arlon region (Belgium) < −→ Kirchberg district (Luxembourg City), and b.) Thionville region (France) < −→ Kirchberg district.
- Operating hours: From 05:30 to 09:30 and from 16:00 to 19:00 from Monday to Friday.
- Vehicle capacity: vehicle capacity differs from 7-seater, 16-seater, and 19-seater.
- Booking and ticket price: users need to book a ride by the dedicated Smartphone application. First 6 trips are free, and then the unit ticket price is around 5 euros per trip.
- Vehicle routing policy: vehicle routes are scheduled based on pre-booked customer requests on previous days. Late-requests could be accepted under certain operational constraints.

More detail about the operation policy and characteristics of Kussbus service can be found at: https://uft.lu/en/news/references/kussbus.

### 3.2. Exploratory data analysis

We conduct an exploratory data analysis of the empirical ride data. Table 1 shows the spatial distribution of users’ pickup and drop-off counties in the studied area. For the morning peak hour period, 90.1% of users commute from Belgium to Luxembourg City. Only 5.1% commuting trips are from the French-side border area to Luxembourg due to this service line launched in September 2018 (i.e. two-week data in the sample). Regarding request booking time, 24% of trips were pre-booked on the same day of riding, 41% one day in advance and 35% at least two days in advance. Figure 2 shows the spatial distribution of pickup and drop-off locations. On Belgium-side, the pickup-up locations are mainly located in the municipality of Arlon. On French-side, users are mainly along the axis of Thionville-Hettange Grande-Luxembourg City.

| Country     | Pickup Morning | Evening | Drop-off Morning | Evening |
|-------------|----------------|---------|-----------------|---------|
| Belgium     | 90.1           | 2.6     | 87.53           |         |
| France      | 5.1            | 0.1     | 5.93            |         |
| Luxembourg  | 4.8            | 97.3    | 100             | 6.54    |

Table 1. Distribution of pickup/drop-off stop locations by country (%).
Figure 2. Spatial distribution of pickup and drop-off locations in the morning (upper part) and in the afternoon (lower part).

Figure 3 shows the cumulative density function (CFD) of trip durations: 60% of trips have a duration between 40 to 60 minutes. Around 20% have shorter trips falling in the range of 20 to 40 minutes. Trips longer than 60 minutes account for around 18% of trips. Figure 3 (right) shows the CDF of walking distance from stops to users’ origin/destination. Around 95% of trips are less than 1 km.

Figure 3. The cumulative probability distribution (CDF) of in-vehicle riding time (Left), and of walking distance from users’ origins to pick-up stops and from drop-off stops to the destination (Right).
Figure 4 shows the temporal demand within a day and within a week. The peak demand hour appears between 6:30 and 7:30 a.m. and 4:30-5:30 p.m. During weekdays, Monday and Tuesday have a higher demand compared to the other weekdays. The average number of requests per day on Monday-Tuesday is 23% higher compared to that on Wednesday-Thursday-Friday. Figure 5 shows weekly-demand evolution from April to September. We found a sharply increasing demand after the first month launch. The demand reached its first peak of 140 rides per week at the end of the 27th month of the year. Then it slowly fell down to 100 rides per week at the 33rd week and restarted a second increasing wave up to 290 rides on the 41st week. The sharp drop on the 41st week is due to the All Saints holiday.

![Figure 4](image1.png)

**Figure 4.** The distribution of users’ pickup time within a day (Left) and over weekdays (Right).

![Figure 5](image2.png)

**Figure 5.** Number of rides per week during 04/2018-10/2018.

4. **Stable matching application case study**

In this section, we present a methodology to estimate and calibrate the model parameters for the stochastic assignment game model and evaluate different service design such as access time, ride time, and paid fare
on the operator’s revenue and the ridership. More precisely, it aims to respond to the following research questions:

- Based on the estimated utility parameters and the characteristics of the routes, the model predicts a stable outcome range for user ticket prices for a given reliability measure $\alpha$. Having the individual observations, how should one calibrate $\alpha$ if the objective is to maximize matches between predicted vehicle-route flow and the observed data?
- What is the impact of different pricing policies on the ridership and operator’s profit?
- If Kussbus should focus on one area to improve upon (i.e. reduction in operating cost, reduction in access time, or in-vehicle time), which should they focus on to increase ridership and what would be the resulting impact on its net profit?

4.1. Case study setting

The entire study period of Kussbus riding data contains 235 commuting periods in the morning or afternoon from April to October 2018. The operator’s routes are generated beforehand based on the observed routes in the data. We solve P1 and P2 under a multi-period, static setting.

There are 429 possible routes observed from Arlon to Kirchberg (see Figure 6) and 449 in the reverse direction. From Thionville to Kirchberg there are 52 routes (see Figure 7) with 50 routes in the reverse direction. The average operation costs takes into account driver and fuel costs. For the operating cost of route (i.e. vehicle-route), it is estimated as the average operating cost per kilometer travelled multiplied by travel distance. Route travel time is estimated by Google Maps API during corresponding peak hour traffic conditions. Table 2 reports the characteristics of Kussbus service and relevant parameter settings for the case study.

We calibrate users’ utility (Equation (6)) to fit observed user-route matches. For this purpose, we divide the data into a training dataset (first 80% rides) and a test dataset (remaining 20% rides). The calibration consists of two steps. The first step consists of estimating the value of in-vehicle travel time (VOT). The estimated VOT can then be used to estimate users’ generalized travel costs. The second step consists of calibrating the users’ utility values to fit observed user-route matches (i.e. user-used route pair) over the studied period. We use the commercial solver intlinprog of MATLAB to solve the P1 and P2 problems based on a Dell Latitude E5470 laptop with win64 OS, Intel i5-6300U CPU, 2 Cores and 8GB memory.

| Table 2. Kussbus service characteristics and parameters settings. |
|------------------------------------------------------------------|
| **Attribute** | **Value** | **Attribute** | **Value** |
| Number of trips | 3010 | User’s maximum waiting time at stop | 10 minutes |
| Value of in-vehicle time (VOT) (euro)$^1$ | 24.21 | Capacity of vehicles | 7, 16 and 19 passenger seats |
| Walking speed | 5km/hr | Average route cost | 61.0 euros |
| Average route distance | 46.5 km | Average travel distance of users | 43.0 km |

Remark: based on the estimation in this study.
4.2. VOT estimation

To estimate the VOT for commuting trips in the study area, we use a mobility survey conducted in October-November 2012 for the EU officials and temporary employees working in the European institutions at the Kirchberg district of Luxembourg. The survey contains samples living in Luxembourg and its French, Belgium and Germany border areas, which perfectly matches Kussbus’s service area. A total of 370 valid samples (individuals) were collected in which there are 131 individuals from the European Investment Bank (~6.2% of total staff in 2012) and 239 individuals from the Court of Justice of the European Union (~11.2% of total staff). After a data cleaning process, a total of 309 individuals’ commuting trip data were used for
the analysis. The spatial distribution of respondents’ residential locations appeared as Luxembourg (78.3%), France (9.4%), Belgium (7.8%) and Germany (4.5%). Note that Belgium employees live mainly in Arlon (45.8% of Belgium employees). French employees live mainly in Thionville, Hettange-Grande, and Yutz (44.8% of French employees). As only 5% of the sample use ‘walk’ and ‘bicycle’ as commuting mode, these samples are excluded from the analysis. We focuses on bimodal (car and public transport) mode choice case, which is consistent with the current mode share in the study area (“Luxmobil” survey, 2017).

Based on previous studies (Gerber et al., 2017; Ma, 2015, Ma et al, 2017), explanatory variables for mode choice include individual-specific socio-demographic variables (gender, age, presence of children etc.), and alternative-specific variables (i.e. travel time and travel cost etc.). Two discrete choice models are specified: a multinomial probit model (MP) and a mixed logit model (ML). The mixed logit model allows random preference coefficient specification to capture travelers’ preference heterogeneity (Train, 2003). As no convergent estimation results were obtained for the mixed logit model, we only report the estimation results of the MP model in Table 3. The first model (MP-1) considers relevant socio-demographic variables and mode-specific variables. The second model (MP-2) further incorporates spatial-specific variables related to the municipality of respondents’ residential locations. The likelihood ratio test shows the MP-2 outperforms the MP-1 at a statistical significance level of 0.05 (Prob. > chi-square=0.0148). We retain the MP-2 model as the final selected model.

Regarding the estimated coefficients in the final model, the results are consistent. Travel time and travel cost have negative effects on individuals’ choices on car use. Free parking at the workplace encourages individuals to use car. Similarly, season ticket subscriptions might be related to frequently public transport users who prefer public transport. Number of children and number of cars in the household positively influence individuals’ preferences to use car as a commuting mode. This result might be explained by the convenience of using cars for pickup/drop-off needs when children are present in the household. Luxembourg residents have significant preference for using car as a commuting mode due to lower accessibility to public transport in rural area, and other reasons related to habits, social and cultural norm. The estimated VOT for the MP-2 is 24.67 euro/hour which is consistent with existing VOT studies related to Luxembourg’s situation2 (Wardman et al., 2016).

Table 3. Estimation results of the multinomial probit models with different model specifications.

| Variable       | MP-1 Coef. | Std.  | MP-2 Coef. | Std.  |
|----------------|------------|-------|------------|-------|
| Travel_time    | -0.013     | 0.009 | -0.023*    | 0.012 |
| Cost           | -0.155***  | 0.060 | -0.057     | 0.072 |
| Free_parking   | 0.589*     | 0.349 | 0.608*     | 0.354 |
| Season_ticket  | -1.050***  | 0.249 | -1.025***  | 0.254 |
| Gender         | -0.183     | 0.236 | -0.176     | 0.238 |
| Couple         | -0.669**   | 0.329 | -0.720**   | 0.333 |
| Age34          | -0.377     | 0.411 | -0.297     | 0.417 |
| Age35_44       | -0.169     | 0.385 | -0.138     | 0.389 |
| Age45_54       | -0.711*    | 0.398 | -0.722*    | 0.405 |
| N_children     | 0.329***   | 0.124 | 0.350***   | 0.127 |
| N_car          | 1.193***   | 0.224 | 1.225***   | 0.230 |
| Flex_time      | 0.014      | 0.320 | -0.091     | 0.331 |
| Res_lux        | 1.711**    | 0.767 |            |       |

2 Wardman et al. (2016) estimated the values of time (€ per hour based on 2010 incomes and prices) for car commute is 18.06 (urban free flow) and 25.68 (urban congestion) in Luxembourg. For car business travel, it is 37.94 euros in urban free flow situation and 53.95 euros in urban congestion situation.
4.3. Utility calibration

We calibrate the utility $U_{sr}$ using the first 80% training dataset to maximize the user-route matches between observation and model predicted results. As no available survey data is available to directly estimate user commuting trip utility, we approximate it as an equivalent door-to-door car-use generalized travel cost ($U_s^{car}$) plus a constant utility ($U_s^0$) to be calibrated (i.e. $U_{sr} = U_s^0 + t_{sr} + e_{sr}$). Note that $U_s^{car}$ represents the perceived cost of the reference mode, and $U_s^0$ is the differentiation value between car and Kussbus service (Breidert, 2005). We estimate users’ car-use generalized travel cost as $VOT \times t_s + \bar{c}_{car} \times d_s$, where $t_s$ is a user’s trip travel time and $d_s$ is the trip travel distance. $\bar{c}_{car}$ is the average cost per kilometre travelled by car estimated as 0.2534 euros/km by considering fuel cost, vehicle purchase cost and annual assurance cost, which is consistent with an existing study (Victoria Transport Policy Institute, 2009). The user generalized travel cost $t_{sr}$ in Equation (1) is estimated by considering walking time $T_{walk}$, waiting time $T_{wait}$, and riding time $T_{ride}$ of trip, estimated as Equation (17).

$$t_{sr} = \tau_1 T_{o_{v_1}} + \tau_2 T_{v_1} + \tau_3 T_{v_1 v_2} + \tau_4 T_{v_2 d}$$  \hspace{2cm} (17)

where $o$ and $d$ are user origin and destination, respectively. $v_1$ and $v_2$ are pickup and drop-off stops for user $s$ and route $r$, respectively. $\tau_1$, $\tau_2$ and $\tau_3$ are the value of walking time, value of waiting time and VOT, respectively. We set $\tau_1 = 1.5 \tau_3$ and $\tau_2 = 2 \tau_3$ (Wardman et al., 2016).

The calibration result is shown in Figure 6. We vary $U_s^0$ from 0 to 100 and solve the P1 problem to match users and routes. We found $U_s^0 \geq 45$ euros fits observed user-route matches with 79.03% corrected prediction rate on the training data. We retain $U_s^0 = 45$ as the calibrated constant user’s trip utility value. For the remaining 20% test data, its corrected prediction rate of user-route matches is 65.45%. The mean and standard deviation of $U_{sr}$ is 73.39 and 3.57 for Belgium-side rides, and these numbers become 72.96 and 8.75, respectively, on the French-side.

We further test the normality assumption of $U_{sr}$. The skewness and kurtosis test for Normality shows the distribution of $U_{sr}$ for Belgium-side follows the normal distributions with p-value $(p > \chi^2) > 0.05$. For French-side, the Normality test is unable to be conducted due to its small sample size.
4.4. Result

4.4.1. Calibration of $\alpha$

We first calibrate the reliability parameter $\alpha$ (Equation (14)) to fit observed user-route matches based on the stable cost allocation outcome obtained from the stable matching model. The P2 problem is solved based on the buyer-optimal policy (Equation (7)) to maximize the ridership. Given the outcome obtained by P2, we set up ticket prices based on the equal-share policy. For example, consider a route $r$ with an operating cost of 40 euros and shared by 5 users. The portion of the payoff allocated to route $r$ from the solution of P2 is 20 euros. Under the equal-share policy, ticket prices for route $r$ are calculated as $40/5 + 20/5 = 12$ euros.

A set of values for the reliability parameter are tested using the training dataset, i.e. $\alpha \in (0.05, 0.1, 0.2, 0.3, 0.4, 0.5)$. The calibration result is shown in Figure 9. We found $\alpha = 0.2$ has the best-fit of user-route matches with the corrected prediction rate of 63.45%. Figure 10 reports the CDFs of ticket prices given different values of $\alpha$, showing a small perturbation on ticket prices given different values of $\alpha$. 

Figure 6. The calibration of constant part $U_0$ of trip utility.
Figure 9. Corrected prediction rates of observed user-route matches for the training dataset over different \( \alpha \).

Figure 10. The cumulative probability density of ticket prices given different values of \( \alpha \).

Table 4 reports the result of the stable matching model for the training and test dataset. For the training dataset, 76.38% of ride requests match Kussbus’s operating routes given \( \alpha = 0.2 \). For the test dataset, its user-route matches are 70.11% with a 54.43% corrected prediction rate. Having the individual observations, we might expect to identify users who continue to use the service (stable) and those who do not after some initial tries (unstable), and use this information to calibrate \( \alpha \). However, as we only have observed rides in the data and not on other modes the users may have taken, we calibrate the reliability parameter \( \alpha \) based only on observed rides to fit model prediction and observations.

Table 4. User-route assignment result of Kussbus rides for 235 periods.

| Data                        | Number of ride requests (users) | Number of user-route assignment | User-route assignment rate | Number of rides matched with observations | Matched rate (observation v.s. prediction) | Computational time (second) |
|-----------------------------|---------------------------------|---------------------------------|---------------------------|------------------------------------------|-------------------------------------------|-----------------------------|
| Training dataset (80% obs.)| 2395                            | 1829                            | 0.7638                    | 1520                                     | 0.6345                                    | 84.7                        |
| Test dataset (20% obs.)     | 615                             | 431                             | 0.7011                    | 335                                      | 0.5443                                    | 29.0                        |

Remark: \( \alpha = 0.2 \). The reported result is the average based on 5 runs.

4.4.2. Detailed breakdown of two commuting periods using the stable matching model

We now illustrate the detailed result of the stable matching model by considering two commuting periods on 06/27/2018 (Luxembourg-> Arlon in the evening) and 06/28/2018 (Arlon->Luxembourg in the morning) (see Table 5). There are 9 and 14 rides observed during these two periods. For the first period, 5 routes are matched with 9 users of which four routes are observed in the data. Only one route is different. The average ridership is 1.8 users/vehicle under the buyer-optimal policy. The ticket price ranges from 18.8 euros to 46.1 euros to ensure route operating cost could be covered from its revenue. As a comparison, when setting ticket prices under the seller-optimal policy, it would result in higher ticket prices for shared-ride users compared to that based on the buyer-optimal policy. For the second period, 4 routes are matched with 14 users which are observed to be identical. The average ridership is 3.5 users/vehicle with ticket price ranging from 9.2
euros for 6-users share and 28.8 euros for 2-users share. Figures 11 and 12 illustrate the detail of the spatial distribution of users’ origins, destination and the operated routes based on observation and the model prediction.

Table 5. Example of detailed result of the user-route assignment model.

| Period          | Number of users | Route attributes | Assigned routes |
|-----------------|-----------------|------------------|-----------------|
| 06/27/2018      | 9               | ID               | 235 238 178 236 237 |
| Afternoon       |                 | Operating cost   | 56.4 46.1 41.4 51.3 52.7  |
| (Luxembourg-   | Number of users | 3 1 1 2 2        |                 |
| > Arlon)        |                 | Ticket price ($p_{sr}$): | 18.8 46.1 41.4 25.7 26.4 |
|                 |                 | →Buyer-opt.      |                 |
|                 |                 | 47.1 50.1 48.8 55.6 42 |
|                 |                 | →Seller-opt.     |                 |
| 06/28/2018      | 14              | ID               | 242 240 239 241 |
| Morning         |                 | Operating cost ($C_r$) | 57.6 55.2 52.1 54.1 |
| (Arlon- >      | Number of users | 2 6 3 3          |                 |
| Luxembourg)     |                 | Ticket price ($p_{sr}$): | 28.8 9.2 17.4 18.0 |
|                 |                 | →Buyer-opt.      |                 |
|                 |                 | 49.7 50.0 46.4 50.4 |
|                 |                 | →Seller-opt.     |                 |

Remark: Ticket price and profit are measured in euros.

![Figure 11. User-route match results of the stable matching model (Luxembourg to Arlon, 06/27/2018)](image_url)
4.4.3. Comparison of the Kussbus pricing policy to the buyer-optimal and seller-optimal policies

We compare the result of the stable matching model based on buyer-optimal and seller-optimal cost allocation policies using calibrated $\alpha$ and the test dataset. The CDFs under different pricing policies are shown in Figure 13. For the buyer-optimal policy, the 50- percentile of the ticket price is 10.98 euros, and the 75- percentile is 13.18 euros. However, for the seller-optimal policy, a user’s ticket price becomes 49.91 euros and 52.02 euros for the 50- and 75- percentiles, respectively. Compared to taxi fare\(^3\) in Luxembourg (i.e. 2.5 euros for the initial charge and 2.6 euros per kilometer traveled), a single-ride Kussbus price is much cheaper compared to the current taxi fare. Note that Kussbus operated pricing policy gave 6 free rides to users and then charge around 5 euros per ride. Given no subsidy, the total revenue from its service operation is unable to compensate its total operating cost.

The total revenue, route cost and profit of the operator over the test dataset is shown in Table 6. The result is obtained from solving the stable matching model based on the four pricing schemes: Kussbus-operated ticket price, buyer-optimal ticket price, seller-optimal ticket price, and taxi fare. We find that Kussbus’ operated policy would accumulate a financial loss up to -4135 euros for 465 matched users due to its lower ticket price compared to its route operating cost. By setting ticket prices based on the buyer-optimal policy, 426 users should match with the routes with a positive profit of 187 euros. By contrast, setting ticket prices based on the seller-optimal policy results in a relatively high ticket price (see Figure 9) due to the high operating cost (i.e. 61.0 euros/route on average, see Table 2). Consequently, only 6 users are matched with routes with a positive profit of 128 euros. Note that the counter intuitive result of why the seller-optimal case ends up with lower net profit is due to integrating the higher seller-optimal ticket price in the disutility function as explained in Section 2. Again, applying a taxi tariff results in no rides, given the high taxi fare for the long commuting distances of users in the studied area (i.e. average travel distance of users is 43 km, see Table 2). We conclude that the buyer-optimal cost allocation policy is preferred to maximizing ridership and keeping the service at a minimum profitable level over the long term.

\(^3\) https://www.bettertaxi.com/taxi-fare-calculator/luxembourg/
Figure 13. The cumulative probability distribution of user’s ticket prices under different pricing policy for the test dataset.

Table 6. Revenue, operating cost and profit of different pricing policies for the test dataset.

| Policy                | Ridership | Revenue | Operating cost | Net profit |
|-----------------------|-----------|---------|----------------|------------|
| Kussbus’s tariff       | 465 (75.54%) | 1266    | 5401           | -4135      |
| Buyer-optimal ticket price | 426 (69.24%)  | 4831    | 4644           | 187        |
| Seller-optimal ticket price | 6 (0.98%)    | 231     | 103            | 128        |
| Taxi                  | 0 (0.00%) | 0       | 0              | 0          |

Remark: Measured in euros. The reported result is the average of 5 runs.

4.4.4. Sensitivity analysis of policy
We further evaluate different system parameters to provide useful information for the operator to improve its operating policy design in the future. The considered decision parameters and the test scenarios are as follows.

- Scenarios 1: Route operating cost: -10%, -30%, -30%, -40%, -50%. Examples of route operating cost changes include improvements in routing, repositioning, and matching algorithms that save idle time of vehicles, setting of common meeting points to streamline routes serving passengers, or reduction in congestion that leads to improvements in travel speed.
- Scenarios 2: In-vehicle travel time: -10%, -30%, -30%, -40%, -50%. Examples include reduction in congestion leading to improvements in travel times for passengers.
- Scenarios 3: Access distance to bus stops: -10%, -30%, -30%, -40%, -50%. Examples include algorithms that bring vehicles closer to travellers and reduce their access time.

Two ticket-pricing policies based on the buyer-optimal and seller-optimal setting are evaluated. The aim is to demonstrate the sensitivity of the model to the impact of different decision parameters on the ridership and profit of the operator.
We run the stable matching model based on the test dataset for different scenarios. The ticket price changes under different scenarios as shown in Table 7. For scenario 1, we find reducing route cost is most beneficial for users with lower ticket prices under the buyer-optimal policy. When reducing from -10% to -50% of the route cost, the ticket price would reduce from -6.9% to -37.9%. However, under the seller-optimal policy, the ticket price would keep stable with less than 1% variation.

Table 7. Ticket price variation based on different scenarios.

| Reduction | Route cost | In-vehicle travel time | Access distance to bus stops |
|-----------|------------|------------------------|-----------------------------|
|           | BO         | SO                     | BO                          | SO                         | BO     | SO       |
| 0%        | 11.6       | 49.1                   | 11.7                        | 49.3                       | 11.6   | 49.6     |
| -10%      | 10.8       | -6.9%                  | 49.4                        | 0.6%                       | 11.7   | 0.9%     | 49.4   | -0.4%    |
| -20%      | 10         | -13.8%                 | 49.4                        | 0.6%                       | 11.8   | 1.7%     | 49.9   | 0.6%     |
| -30%      | 9          | -22.4%                 | 49.3                        | 0.4%                       | 11.8   | 3.7%     | 49.9   | 0.6%     |
| -40%      | 8.2        | -29.3%                 | 49.1                        | 0.0%                       | 12.2   | 4.3%     | 50.0   | 0.8%     |
| -50%      | 7.2        | -37.9%                 | 48.9                        | -0.4%                      | 11.9   | 13.0%    | 50.3   | 1.4%     |

Remark: BO: Buyer-optimal; SO: Seller –optimal. The result is based on the average of 5 runs.

For scenario 2, we find there is little change (less than 5%) observed for ticket prices based on the buyer-optimal policy. This is because the operator’s route cost estimation depends on vehicle travel distance only. More elaborate route cost estimation that considers vehicle travel time can be integrated in the future. However, under the seller-optimal policy, reducing in-vehicle travel time between -10% to -50% would increase ticket prices between 3.7% to 13%. This is because the savings in travel time are absorbed by the operator in a seller-optimal policy.

For scenario 3, only a marginal variation (less than 3%) of the ticket price is observed for both pricing policies. As more than 95% of access distance to Kussbus bus stops is less than 1 km, it is expected that reducing the access distance further would have an insignificant impact on ticket prices. Figure 14 shows the cumulative probability distributions of ticket prices for different scenarios based on the buyer-optimal and seller-optimal policies.

Figure 14. Influence of route cost reduction on ticket prices based on the buyer-optimal (on the left) and the seller-optimal policies (on the right).
4.4.5. Policy recommendations under buyer- and seller-optimal policies

The impact of different scenarios on ridership and the operator's profit under the buyer-optimal ticket price is shown in Table 8 and Figure 15. We found that reducing route cost is more effective to increase the ridership (up to +10% when 50% reduction of route cost) compared to reducing in-vehicle travel time and access distance to bus stops. However, it is not beneficial for the operator. For scenario 2, reducing in-vehicle travel time would slightly increase user-route matches (less than 2%) given fixed ride requests. However, it significantly increases the profit of the operator (i.e. +151.5% profit for 50% in-vehicle travel time reduction). Our study provides a benchmark under fixed demand. For the future extension, it would be interesting to consider flexible travel demand under a multimodal transport market setting. For scenario 3, a marginal impact on ridership and operator’s profit is observed due to the short access distance to bus stops. As a conclusion of this comparison, we recommend government subsidy in support of scenario 1 while requiring Kussbus to operate under a buyer-optimal policy, as funding improvements in routing algorithms can significantly improve the consumer surplus of travellers.

Table 8. Influence of different scenarios on the ridership and profit of the operator based on the buyer-optimal ticket price.

| Reduction | S1 # | S2 # | S3 # | S1 Euro | S2 Euro | S3 Euro |
|-----------|------|------|------|--------|--------|--------|
|           | ±%   | ±%   | ±%   | ±%     | ±%     | ±%     |
| 0%        | 431  | 430  | 430  | 202    | 196    | 214    |
| -10%      | 428  | -0.7 | 419  | -2.6   | 431    | 0.2    |
| -20%      | 436  | 1.2  | 431  | 0.2    | 447    | 4.0    |
| -30%      | 460  | 6.7  | 458  | 6.5    | 436    | 1.4    |
| -40%      | 467  | 8.4  | 433  | 0.7    | 434    | 0.9    |
| -50%      | 474  | 10.0 | 438  | 1.9    | 436    | 1.4    |

Remark: S1: Route operating cost, S2: In-vehicle travel time, S3: Access distance to bus stops. The result is based on the average of 5 runs.

Figure 15. Influence of different scenarios on (a) ridership and (b) profit of the operator based on the buyer-optimal ticket price.
For the seller-optimal policy, we find reducing route cost and in-vehicle travel time could significantly increase both the ridership and profit of the operator compared to its benchmark as shown in Table 9 and Figure 16. Low ridership for the benchmark results from higher ticket prices. Under scenario 1 and 2, the number of rides would increase from initial 2 rides (over 615 requests) to 61 (scenario 1) and 72 (scenario 2). For scenario 3, its effect on the ridership and profit of the operator is less significant compared to the other two scenarios. In conclusion, if Kussbus were to operate on its own without government intervention, it can seek a seller-optimal policy and invest in algorithms that improve operating cost and/or in-vehicle travel time for passengers.

Table 9. Influence of different scenarios on the ridership and profit of the operator based on the seller-optimal ticket price.

| Reduction | Ridership | Profit |
|-----------|-----------|--------|
|           | S1 | ±% | S2 | ±% | S3 | ±% | S1 | ±% | S2 | ±% | S3 | ±% |
| 0%        | 2  | 2  | 3  |     |     |     | 81 | 71 | 85 |     |     |     |
| -10%      | 2  | 0  | 4  | 100 | 6   | 100 | 84 | 3.7| 145| 104.2| 160| 88.2|
| -20%      | 12 | 500| 15 | 650 | 6   | 100 | 441| 444.4| 590| 731.0| 190| 123.5|
| -30%      | 24 | 1100| 33 | 1550| 7   | 133.3| 966| 1092.6| 1284| 1708.5| 229| 169.4|
| -40%      | 38 | 1800| 39 | 1850| 5   | 66.7| 1549| 1812.3| 1610| 2167.6| 151| 77.6|
| -50%      | 61 | 2950| 72 | 3500| 10  | 233.3| 2594| 3102.5| 3063| 4214.1| 394| 363.5|

Remark: S1: Route operating cost, S2: In-vehicle travel time, S3: Access distance to bus stops. The result is based on the average of 5 runs.

![Graph 1](image1)

![Graph 2](image2)

Figure 16. Influence of different scenarios on the ridership and profit of the operator based on the seller-optimal ticket price.

Our sensitivity analysis shows how the proposed stable matching model can be applied to evaluate different service designs. The operator can apply this methodology to set up ticket prices by considering the price ranges from buyer-optimal and seller-optimal policies.
5. Conclusions
We tackle the problem of evaluating and designing a microtransit service. Microtransit operators can allocate resources to improve upon many aspects of operation: vehicle capacities; fleet size; algorithms to improve routing, pricing, repositioning, matching; and more. We conduct the first empirical application of a model from Rasulkhani and Chow (2019) that evaluates such systems using stable matching between travellers and operator-routes. The study is conducted using a real data set shared by industry collaborator Kussbus covering 3010 trips made between April to October 2018 in Luxembourg and its French-side and Belgium-side border areas.

In order to make it work empirically, we made several modifications to the model, primarily a conversion of the utility $U_{sr}$ into a function of different components including stochastic variables and making the stable matching model into a stochastic model. Doing so allows us to better fit the model to the data using the concept of $\alpha$-reliability.

We calibrated the model to the data. A separate data set from a mobility survey conducted in October to November 2012 covering a similar study area was used to estimate the value of time of travellers as 24.67 euros/hour, which we found consistent with existing VOT studies in Luxembourg. A base utility constant was then estimated for travellers in the Kussbus data and found to be 45 Euros to obtain 79.03% prediction rate with the training data. Validation using the 20% test data showed a user-route match rate of 65.45%. The value of $\alpha$ was calibrated to a value of 0.20 as the best fit to the observations with a prediction rate of 63.45% resulting in 76.38% ride matches. Validation with the 20% test set resulted in 54.43% prediction rate with 70.11% matches.

Our stable matching model, as illustrated with two commuting periods, shows the existence of a stable outcome space between buyer-optimal and seller-optimal policies. We show that Kussbus current pricing policy falls below the buyer-optimal policy, which is not sustainable. By increasing the ticket price to the buyer-optimal policy it would reduce ridership from the current 465 trips to 426 trips and changing the net profit from -4135 euros to 187 euros for 615 ride requests. Increasing the pricing allocation further to the seller-optimal policy significantly reduces the ridership and reduces net profit, while following taxi pricing policy would lead to zero trips.

A sensitivity analysis is then conducted to compare the effects that equal, unilateral reductions in route operating cost, in-vehicle travel time, and access distance to bus stops, can have on the microtransit service. We find that government can intervene by offering to subsidize Kussbus to improve their routing algorithms and reduce operating cost while requiring operation under a buyer-optimal policy. Such an intervention can increase ridership by 10% with an operating cost reduction of 50%. Alternatively, an independent Kussbus can operate in a seller-optimal policy and invest in algorithms to improve in-vehicle travel time which can improve profit by 731% (bearing in mind the low ridership if operating a seller-optimal policy in the current baseline setting) with a 20% reduction in in-vehicle travel time.

These analyses can be further conducted with other operational variables like fleet size, fleet mix in vehicle size, service coverage area, and more.

New insights have been made as a first empirical study of microtransit operation using the stable matching modelling framework. However, more research can be done to improve this work further. A study that includes travellers as part of a whole market system would capture their utility preferences better, allowing us to specify choice models and using the utility functions for the stable matching model. Alternatively, methodological extensions can be made to allow us to evaluate platforms (see Chapter 3.5 in Chow, 2018) controlling submarkets in the presence of external operators/platforms. Evaluation of the Kussbus service as a potential component of a multimodal MaaS trip (see Pantelidis et al., 2019) would be a much more powerful study that can relate its operational policies to impacts to the MaaS market. In this study, a static multi-period model is used to fit to the data; a more realistic model would be a dynamic model that considers dynamic cost allocations (e.g. Furuhata et al., 2014).
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