On the violation of the Fermi-liquid picture in two-dimensional systems in the presence of van Hove singularities

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Abstract

We consider the two-dimensional $t$-$t'$ Hubbard model with the Fermi level being close to the van Hove singularities. The phase diagram of the model is discussed. In a broad energy region the self-energy at the singularity points has a nearly-linear energy dependence. The corresponding correction to the density of states is proportional to $\ln^3 |\varepsilon|$. Both real- and imaginary part of the self-energy increase near the quantum phase transition into magnetically ordered or superconducting phase which implies violation of the Fermi-liquid behavior. The application of the results to cuprates is discussed.
I. INTRODUCTION

Last decade, a possibility of a non-Fermi-liquid (NFL) behavior in two-dimensional (2D) systems has been a subject of many theoretical investigations. This question is especially interesting in connection with the high-temperature superconductors where many unconventional features, including pseudogap phenomena, are observed. Anderson [1] has put forward the idea that a 2D system can demonstrate NFL behavior at arbitrary small interelectron repulsion $U$ owing to a finite phase shift at the Fermi energy. Even in the absence of such effects, a NFL state can occur at small $U$ because of peculiarities of electron or spin and charge fluctuation spectra.

In the usual 2D Fermi liquid the imaginary part of the self-energy (electron damping) has the energy dependence $\text{Im}\Sigma(k_F, \varepsilon) \propto \varepsilon^2 \ln |\varepsilon| \ [2,3]$, and the temperature behavior of resistivity is $\rho \propto T^2 \ln T \ [4]$. However, these dependences do not describe experimental data on high-$T_c$ cooper-oxide compounds. To treat the anomalous behavior of these systems, Varma et al [5] proposed the phenomenological marginal Fermi-liquid (MFL) theory where $\text{Im}\Sigma(k_F, \varepsilon) \propto |\varepsilon|$. Then the electronic specific heat should demonstrate $T \ln T$-dependence, and the resistivity the $T$-linear behavior. A similar behavior in a broad temperature region can be obtained in the presence of strong antiferromagnetic fluctuations for 2D and nested 3D systems [6,7]. The linear energy dependence of the self-energy was also obtained within the spin-fermion theory [8,9] which supposes that the magnetic correlation length of the system is large enough.

Another explanation of anomalous electron properties of 2D lattice systems can be found in the presence of the van Hove (VH) singularities [10]. In this case, the bare electron density of states is logarithmically divergent and to leading (second) order in $U$ we have the behavior $\text{Im}\Sigma(k_{VH}, \varepsilon) \propto |\varepsilon| \ln |1/\varepsilon|$ at $|\varepsilon| \gg |\mu|$ ($k_{VH}$ is assumed to be VH point), which differs slightly from the linear dependence in the MFL theory. However, the important question occurs how this behavior changes with taking into account higher-order terms. RPA calculations [11], numerical calculations within the FLEX approximation [12] and analytical analysis [12] show...
that not too close to van Hove filling the (quasi-) linear energy dependence of the self-energy takes place in a broad range of the energies.

Within the renormalization-group approach, the behavior of the self-energy was discussed in Refs. [13,14]. The conclusions of different approaches turned out to be contradictory: a possibility of a NFL state is discussed in Ref. [13], while a standard Fermi-liquid behavior was found in Ref. [14]. It should be noted that in both the papers only part of all the possible interaction channels was taken into account. However, as it was discussed in Refs. [15,16], all the scattering channels are equally important, which can substantially change the previous results.

In the present paper we consider the energy-dependence of the self-energy with account of all the scattering channels. We demonstrate that a violation of the Fermi-liquid picture takes place provided that the system is close to quantum phase transition into a magnetically ordered or superconducting state. Our approach can be considered as a generalization of that of Ref. [17] to the case of a non-nested Fermi surface. Note that our RG analysis takes into account the contribution of the vicinity of VH points only. For fillings close to VH one, this contribution is most singular and vertices with the corresponding momenta are most divergent. This can be checked by analysis of the results of complete RG approach which takes into account the contribution of entire Fermi surface [18]. Unlike this approach, however, present approach gives the possibility to take into account all channels of electron scattering, as discussed in details in Ref. [15]. Recent analysis [16] which permits to consider the interplay of different channels gives the results close to that obtained earliar within two-patch and parquet equations approach [15]. Thus we can expect that the approach based on consideration of the vicinities of VH singularities is reliable and gives qualitatively correct results for fillings close to VH one. The behavior of some physical properties and application of the results obtained to cuprates is discussed in the Conclusion.
II. THE MODEL AND QUANTUM PHASE TRANSITIONS

We consider the $t$-$t'$ Hubbard model on the square lattice:

$$H = \sum_k \varepsilon_k c_k^\dagger c_k + U \sum_i n_i^\uparrow n_i^\downarrow \quad (1)$$

with the electron spectrum

$$\varepsilon_k = -2t(\cos k_x + \cos k_y) - 4t'(\cos k_x \cos k_y + 1) - \mu \quad (2)$$

Hereafter we assume $t > 0, t' < 0$ (which is the case for hole-doped systems), $0 \leq |t'/t| < 1/2$. For $t' = 0$ the Fermi surface is nested, which results in peculiarities of physical properties [17,7]. However, nesting is removed for $t'/t \neq 0$. For arbitrary $t'/t$, the spectrum (2) contains VH singularities connected with the points $A = (\pi, 0), B = (0, \pi)$. The chemical potential $\mu$ is determined by the electron concentration $n$ and can be obtained from the condition

$$n = \sum_k f_k \quad (3)$$

with $f_k = f(\varepsilon_k)$ being the Fermi function, so that $\mu = 0$ corresponds to Van Hove filling. Being expanded near the VH singularity points, the spectrum takes the form

$$\varepsilon^A_k = -2t(\sin^2 \varphi \overline{k}_x^2 - \cos^2 \varphi \overline{k}_y^2) - \mu \quad (4a)$$

$$\varepsilon^B_k = 2t(\cos^2 \varphi \overline{k}_x^2 - \sin^2 \varphi \overline{k}_y^2) - \mu \quad (4b)$$

where $\overline{k}_x = \pi - k_x, \overline{k}_y = \pi - k_y, \varphi$ is the half of the angle between asymptotes at VH singularity, $2\varphi = \cos^{-1}(-2t'/t)$.

We have a set of quantum phase transitions (QPT’s) for the model (1), see Ref. [13]. We restrict ourselves to the case of small enough $U \lesssim 6t$, so that we may neglect the correlation effects connected with Hubbard’s subband formation (e.g., the Mott-Hubbard metal-insulator transition). Provided that the electron concentration is not too close to its van Hove value, i.e. the chemical potential satisfies $|\mu| > \mu_c, \mu_c$ being the critical value, we have the normal (paramagnetic and non-superconducting) phase. When approaching the
VH singularity, a QPT to antiferromagnetic, superconducting or ferromagnetic state occurs. The type of the ground state depends on $t'/t$ and $U$. In particular, for $U = 4t$ we have the antiferromagnetic ground state at $|t'/t| < 0.30$ and ferromagnetic one for larger $|t'/t|$ [15]. The critical electron concentrations for stability of these phases differ by several percents from van Hove filling.

III. ELECTRON SELF-ENERGY IN THE SECOND ORDER

Consider first zero-temperature perturbative results. To second order the expression for the electron self-energy has the form

$$\Sigma^{(2)}(k, \varepsilon) = U^2 \sum_{pq} \frac{f_{p+q-k}(1 - f_p - f_q) + f_pf_q}{\varepsilon + \varepsilon_{p+q-k} - \varepsilon_p - \varepsilon_q}$$

(5)

This contribution can be represented as a sum of three diagrams (Fig.2 a-c):

$$\Sigma^{(2)}(k, \varepsilon) = \Sigma_1(k, \varepsilon) + \Sigma_2(k, \varepsilon) + \Sigma_3(k, \varepsilon)$$

(6)

When picking out the singularities we can put $\varepsilon_p = \varepsilon_p^A$ or $\varepsilon_p = \varepsilon_p^B$ for $p$ being close to $A(B)$ van Hove points. Note that the term $\Sigma_1$ was investigated earlier [13, 14, 22]. We restrict our consideration to the VH points at the Fermi surface, $k_F = (0, \pi)$ or $(\pi, 0)$. The calculations at $|\varepsilon| \gg |\mu|$ yield:

$$\text{Re}\Sigma_1(k_F, \varepsilon) = -(\ln 2)(g_0/\sin 2\varphi)^2\varepsilon \ln^2(\Lambda^2|t/\varepsilon|)$$

$$\text{Re}\Sigma_{2,3}(k_F, \varepsilon) = -(g_0/\sin 2\varphi)^2\varepsilon \begin{cases} A_{2,3} \ln(\Lambda^2|2t/\varepsilon|) & |\varepsilon| \ll 2t \cos 2\varphi \\ k_{2,3} \ln 2 \ln^2(\Lambda^2|2t/\varepsilon|) & |\varepsilon| \gg 2t \cos 2\varphi \end{cases}$$

(7)

where $g_0 = U/(4\pi^2t)$ is the dimensionless coupling constant, $\Lambda \sim 1$ is the ultraviolet momentum cutoff, $k_2 = 1$, $k_3 = 2/3$,

$$A_2 = \int_0^{\cos 2\varphi} \frac{dx}{x} \ln \frac{\varepsilon_x - x}{\varepsilon_x} + \frac{1}{2} \int^{1/\cos 2\varphi}_{\cos 2\varphi} \frac{dx}{x} \ln \frac{\varepsilon_x + x}{\varepsilon_x}$$

$$+ \frac{1}{2} \int_{-\infty}^0 \frac{dx}{x} \ln \frac{\varepsilon_x}{\varepsilon_x + x}$$

(8a)
\[
A_3 = \int_0^{\cos 2\varphi} \frac{dx}{2\varepsilon_x + x} \ln \left( -\frac{x}{2\varepsilon_x} \right) + \frac{1}{2} \int \frac{dx}{2\varepsilon_x + x} \ln \frac{2(\varepsilon_x + x)}{x} + \frac{1}{2} \int_{-\infty}^0 \frac{dx}{2\varepsilon_x + x} \ln \frac{x}{2(\varepsilon_x + x)}
\]

(8b)

with \(\varepsilon_x = (x - \cos 2\varphi)(1 - x \cos 2\varphi)/\sin^2 2\varphi\). For small energies \(|\varepsilon| \ll |\mu|\) the real part remains linear in energy with the difference that logarithmic divergences are cut at \(|\mu|\) rather than at \(|\varepsilon|\). The corresponding imaginary parts at \(|\varepsilon| \gg |\mu|\) read

\[
\text{Im}\Sigma_1(k_F, \varepsilon) = -(\pi \ln 2)(g_0/\sin 2\varphi)^2 |\varepsilon| \ln(\Lambda^2 t/|\varepsilon|) \\
\text{Im}\Sigma_{2,3}(k_F, \varepsilon) = -\pi (g_0/\sin 2\varphi)^2 |\varepsilon| \begin{cases} 
B_{2,3} & |\varepsilon| \ll 2t \cos 2\varphi \\
k_{2,3} \ln 2 \ln(\Lambda^2 t/|\varepsilon|) & |\varepsilon| \gg 2t \cos 2\varphi 
\end{cases}
\]

(9)

where

\[
B_2 = (A_{21} + A_{23})\theta(\varepsilon) + A_{22}\theta(-\varepsilon) \\
B_3 = (A_{31} + A_{33})\theta(-\varepsilon) + A_{32}\theta(\varepsilon)
\]

and \(A_{2n}\) and \(A_{3n}\) are \(n\)-th summands in the definition of \(A_2\) and \(A_3\), Eq. (8), respectively, \(\theta(\varepsilon)\) is the step function. At \(t'/t \neq 0\) we have \(\text{Im}\Sigma(k_F, \varepsilon) \neq \text{Im}\Sigma(k_F, -\varepsilon)\) because of the absence of the particle-hole symmetry for the less singular terms \(\Sigma_{2,3}(k_F, \varepsilon)\). This fact results in an asymmetry of the electron density of states near the Fermi level and can be important for some physical properties, e.g., thermoelectric power. Unlike the real part of the self-energy, the imaginary part changes its dependence to quadratic one at \(|\varepsilon| \ll |\mu|\) demonstrating a conventional Fermi-liquid behavior in this region.

IV. RENORMALIZATION

Both real and imaginary parts of the self-energy contain large logarithms at \(|\mu| \ll |\varepsilon| \ll t\). Therefore we can introduce the logarithmic variable \(\lambda = \ln(\Lambda|t/|\varepsilon|^{1/2})\). Then the leading terms of the expansion in the powers of interaction strength can be written down as
\[ \Sigma(\lambda) = A\lambda g_0^2(1 + C_1 g_0 + D_1 g_0^2 \lambda + ...) \]

To perform the summation of leading logarithms in the self-energy, we introduce the vertices \( \gamma_i(\lambda) \), \( i = 1, \ldots, 4 \) (Fig. 2), and consider the renormalization of \( \Sigma \). As discussed in Refs. [23,24,15], \( \gamma_i(\lambda) \) can be determined from the renormalization-group (RG) equations

\begin{align*}
\gamma_1' &= 2d_1(\lambda)\gamma_1(\gamma_2 - \gamma_1) + 2d_2\gamma_1\gamma_4 - 2d_3\gamma_1\gamma_2 \\
\gamma_2' &= d_1(\lambda)(\gamma_2^2 + \gamma_3^2) + 2d_2(\gamma_1 - \gamma_2)\gamma_4 - d_3(\gamma_1^2 + \gamma_2^2) \\
\gamma_3' &= -2d_0(\lambda)\gamma_3\gamma_4 + 2d_1(\lambda)\gamma_3(2\gamma_2 - \gamma_1) \\
\gamma_4' &= -d_0(\lambda)(\gamma_3^2 + \gamma_4^2) + d_2(\gamma_1^2 + 2\gamma_1\gamma_2 - 2\gamma_2^2 + \gamma_4^2) \tag{10}
\end{align*}

where \( \gamma_i' \equiv \frac{d\gamma_i}{d\lambda} \),

\begin{align*}
d_0(\lambda) &= 2c_0\lambda; \\
d_1(\lambda) &= 2 \begin{cases} \\
&\lambda, \quad \lambda < 2z_Q \\
&z_Q, \quad \lambda > 2z_Q \\
\end{cases} \\
d_2 &= 2z_0; \quad d_3 = 2c_Q \tag{11}
\end{align*}

The quantities

\[ z_0 = c_0 = \frac{1}{\sin(2\varphi)} = \frac{1}{\sqrt{1 - R^2}} \]

are the prelogarithmic factors in small-momentum particle-hole and particle-particle bubble, while

\[ z_Q = \ln[(1 + \sqrt{1 - R^2})/R] \]
\[ c_Q = \tan^{-1}(R/\sqrt{1 - R^2})/R \tag{12} \]

are the prelogarithmic factors in particle-hole and particle-particle bubble with momenta close to \( Q = (\pi,\pi) \), \( R = -2t'/t \). Equations (10) should be solved with the initial condition \( \gamma_i(0) = g_0 \). Because of the presence of double-logarithmic terms, the corresponding RG equations are only approximate. However, the comparison of the results of their solution
[15] with the parquet approach [15] and the RG approach which takes into account the contribution of the whole Fermi-surface [16] shows that they reproduce well the renormalization of the couplings.

The magnetic or superconducting instabilities manifest in the divergence of the vertices $\gamma_i(\lambda)$ at some critical scale $\lambda_c$. This is connected with the critical energy scales discussed in Sect. II as $\mu_c \sim T_c \sim \Lambda \exp(-2\lambda_c)$, for a detailed discussion see Ref. [15]. (In the absence of interlayer coupling the quantity $T_c$ has the meaning of a temperature of crossover into the state with pronounced short-range order (or pseudogap state) rather than of a phase transition temperature.) For $\lambda$ close to $\lambda_c$ the solution of the Eqs. [10] can be represented in the form

$$\gamma_i = \frac{\gamma_i^c}{\lambda_c - \lambda}$$

(13)

The corresponding correlation length is given by

$$\xi^{-2} = C_\xi |\mu|(\lambda_c - \lambda)/t$$

(14)

where $C_\xi$ is the universal number. For $T \ll |\mu|$ we should stop the scaling at

$$\lambda^* = \ln[\Lambda t^{1/2}/\max(|\mu|, |\epsilon|)^{1/2}]$$

(15)

while $T \gg |\mu|$ at

$$\lambda^* = \ln[\Lambda t^{1/2}/\max(T, |\epsilon|)^{1/2}]$$

(16)

The condition $|\mu| > \mu_c$ or $T > T_c$ guarantees that $\lambda^* < \lambda_c$, i.e. the system is not ordered and RG approach is applicable.

Now we consider the renormalization of the self-energy. We follow the method of Refs. [25–27]. Defining the scale-dependent quasiparticle residue

$$Z(\lambda) = Z_1(\lambda)Z_2(\lambda)Z_3(\lambda)$$

where $Z_i$ ($i = 1, 2, 3$) is the contribution of $i$-th diagram, we have the RG equations
\[
\frac{d \ln Z_1(\lambda)}{d \lambda} = -(8 \ln 2)\lambda \gamma_4^2 / \sin^2 2\varphi \\
\frac{d \ln Z_2(\lambda)}{d \lambda} = -D_2(\lambda)(\gamma_2^2 + \gamma_1^2 - \gamma_1\gamma_2) / \sin^2 2\varphi \\
\frac{d \ln Z_3(\lambda)}{d \lambda} = -D_3(\lambda)\gamma_3^2 / \sin^2 2\varphi 
\] (17)

Here

\[D_{2,3}(\lambda) = \begin{cases} 
4k_{2,3}\lambda \ln 2 & \lambda < (1/2) \ln(1/\cos 2\varphi) \\
A_{2,3} & \lambda > (1/2) \ln(1/\cos 2\varphi)
\end{cases}\]

and the summation over spin indices in vertices is performed. Then the real part of the self-energy can be found as

\[\text{Re} \Sigma(k_F, \varepsilon) = \varepsilon \ln Z(\lambda^*) \] (18)

(see, e.g., Ref. [27]). After calculating Re\Sigma, the imaginary part of the self-energy can be obtained from the Kramers-Kronig relations.

**V. RESULTS OF CALCULATIONS**

First, we demonstrate our approach in a simple case with the only non-zero vertex, \(\gamma_4 \neq 0\). As discussed in Ref. [13], this case corresponds to \(t' \to -t/2\), the ground state at van Hove filling being ferromagnetic (flat-band ferromagnetism). Then we have

\[\gamma_4 = \frac{g_0}{1 + g_0(c_0\lambda^2 - 2z_0\lambda)} \] (19)

The vertex (19) diverges in the critical point

\[\lambda_c = \frac{1}{2} \ln \frac{\Lambda^2 t}{\max(\mu, T)} = 1 - \sqrt{1 - 1/(z_0g_0)} \] (20)

where we have put \(z_0 = c_0\) in \(\lambda_c\) according to (12). From (17) we obtain \(Z_2 = Z_3 = 1\) and

\[Z_1 = \exp \left[-\frac{4 \ln 2}{\sin^2 2\varphi (1 - g_0z_0)} \frac{g_0^2 \lambda(\lambda - 1)}{(1 + g_0z_0\lambda^2 - 2g_0z_0)} \right] \] (21)

For \(\lambda\) being close to \(\lambda_c\) we have
\[ \gamma_4 \sim \xi^2(T = 0) \sim (\lambda_c - \lambda)^{-1} \] (22)

As well as in the nesting case (see, e.g., Ref. [17]), the quasiparticle weight \( Z \) vanishes exponentially in the QPT point. Note that this vanishing is much faster than the inverse-logarithmic dependence

\[ Z \sim \frac{1}{\ln q(\lambda_c - \lambda)}, \quad q \simeq 0.35 \] (23)

obtained in Ref. [13] where only one scattering channel was taken into account. Besides nearly-linear dependences in \( \varepsilon \) (with logarithmic corrections), both real and imaginary part of the self-energy contain large \( \varepsilon \)-dependent factors of the order of \( 1/(\lambda_c - \lambda)^2 \propto \xi^4(0) \), which occur because of the divergence of the vertex (19) at \( \lambda \to \lambda_c \).

As discussed in Ref. [15], for \( |t'/t| \) being not very close to \( 1/2 \) we have an interplay of all the scattering channels, so that we have to solve Eqs. (10), (17) numerically. We also calculate the quasiparticle spectral weight in VH points of the Fermi surface,

\[ A(k_{VH}, \varepsilon) = -\frac{1}{\pi |\varepsilon + \mu - \text{Re} \Sigma(k_{VH}, \varepsilon)|^2 + |\text{Im} \Sigma(k_{VH}, \varepsilon)|^2} \] (24)

The results of the calculations for \( t' = -0.45t, \mu/t = 0.3, T = 0 \) (nearly ferromagnetic ground state, \( \mu_c/t = 0.04 \)) are shown in Fig. 3. As well as in the second order of the perturbation theory, at \( |\varepsilon| \ll |\mu| \) we have the 2D behavior of self-energy

\[ \text{Re} \Sigma(k_F, \varepsilon) \propto \varepsilon, \quad \text{Im} \Sigma(k_F, \varepsilon) \propto \varepsilon^2, \] (25)

while \( |\mu| \ll |\varepsilon| \ll t \) it changes to van Hove behavior,

\[ \text{Re} \Sigma(k_F, \varepsilon) \propto \varepsilon \ln(\Lambda t/\varepsilon), \quad \text{Im} \Sigma(k_F, \varepsilon) \propto |\varepsilon| \ln(\Lambda t/\varepsilon) \] (26)

Finally, at \( \varepsilon = \varepsilon_0 \sim t \) the real part of self-energy has a maximum and then decreases with farther increasing of \( \varepsilon \), while imaginary part is almost a constant at \( \varepsilon \sim t \). The slope of RG results in the region \( |\mu| \ll |\varepsilon| \ll t \) is substantially higher that the results of the second-order perturbation theory, the imaginary part of the self-energy at \( |\varepsilon| \geq |\mu| \) also becomes large enough. The peak in the quasiparticle weight obtained within RG approach is more broad.
than in the second-order perturbation theory, which is the consequence of larger electron
damping in the letter case. With the increasing the chemical potential (so that the system is
moved away from QPT), the higher-order renormalizations become less important and the
behavior of the self-energy reproduces the result of the second-order perturbation theory.

The imaginary part of self-energy at finite temperature and \( \varepsilon = 0 \) (the inverse quasipar-
ticle lifetime at the Fermi surface) can be obtained from the scaling arguments,

\[
\gamma(T) = -[\text{Im} \Sigma(k_F, \varepsilon)|_{T=0}]_{\varepsilon \to T}
\]

Thus it is given by the same Figs. 3b with the replacement \( \varepsilon \to T \) and also demonstrates
the linear behavior in some temperature region.

In the above consideration we neglected completely the damping of the particle-particle
and particle-hole excitations as well as other non-singular contributions. These non-singular
terms can be neglected provided that the condition

\[
\xi^{-2} \gg \max(|\mu|, |\varepsilon|)/t
\]

is satisfied, i.e. not too close to quantum phase transition into the corresponding ordered
state and at not too high energies. Close to the quantum phase transition into the ordered
state, the peaks in the self-energy should become asymmetric, as discussed in Ref. [11], and
the pseudogap can be formed. On the other hand, in the limit of large enough \(|\mu|\), i.e. at

\[
\max(|\varepsilon|/t, \xi^{-2}) \ll |\mu|/t
\]

the spin-fermion theory [9] is applicable.

VI. DISCUSSION AND CONCLUSIONS

In the present paper we have investigated within a scaling approach the energy depen-
dence of the real and imaginary part of the electron self-energy in the presence of van Hove
singularities. We have restricted ourselves to the regions of the \( \mu - T \) phase diagram, where
the Fermi level is not too close to VH points (disordered ground state, \( \mu_c < |\mu| \ll t \) with
\( \mu_c \) is the critical chemical potential) or temperature is above the critical value \( (T > T_c \text{ with } T_c \sim \mu_c) \).

Provided the system is not too close to QPT (at \( \mu_c < |\mu| \ll t \)), the imaginary part of the self-energy at VH points demonstrates in a broad energy region \(|\mu| \ll |\varepsilon| \ll t \) a nearly-linear behavior, \( \text{Im}\Sigma(k_F, \varepsilon) \propto |\varepsilon| \ln(t/|\varepsilon|) \), which is close to that in the marginal Fermi-liquid concept \( [5] \) (however, in MFL the linear behavior of \( \text{Im}\Sigma(k_F, \varepsilon) \) takes place for arbitrary \( k_F \)). The real part of the self-energy behaves as \( \varepsilon \ln^2(t/|\varepsilon|) \). The linear energy dependence of the self-energy \( (26) \) and the behaviour of the slope with changing \( \mu \) are in agreement with the RG results of Ref. \( [19] \) which takes into account of the contribution of the whole Fermi-surface. These dependences are also similar to those obtained within the spin-fermion model for a nearly antiferromagnetic state \( [8,9] \), although in our case they have a different nature and are governed by van Hove singularities themselves rather than by closeness to antiferromagnetic quantum phase transition. The role of a characteristic spin-fluctuation frequency \( \omega_{sf} \), which separates the Fermi-liquid and MFL regimes, belongs in our case to the chemical potential \(|\mu| \) (\( \mu = 0 \) corresponds to VH filling). Another difference is that in the presence of VH singularities the linear dependence of self-energy takes place already in the weak-coupling regime.

Near QPT, the renormalizations become important because of the large ground-state correlation length which enters renormalized vertices. Therefore both real and imaginary parts of the self-energy increase considerably as \( \xi^2(T = 0) \). Such an anomalous behavior also implies a strong violation of the Fermi-liquid and even MFL picture. Note that the anomalies under consideration may induce the electron topological transition with the truncation of the Fermi surface (see, e.g. Ref. \( [28] \)). The RG approach used is not able to describe the magnetically ordered or superconducting state. By this reason, the renormalized-classical regime \( T < T_c, |\mu| < \mu_c \) should be considered within other approaches, see, e.g., Ref. \( [29] \).

In 2D situation, the physical properties near QPT should demonstrate singularities which are stronger than those in the 3D case. The correction to the electron density of states \( N(\varepsilon) \) reads
\[ \delta N(\varepsilon) = - \sum_{k\sigma} \left[ \text{Re}\Sigma(k,\varepsilon)\delta'(\varepsilon - \varepsilon_k) + \frac{1}{\pi} \text{Im}\Sigma(k,\varepsilon)/(\varepsilon - \varepsilon_k)^2 \right] \] (30)

Main contribution to the integral comes from the vicinity of VH points where the bare density of states is logarithmically divergent. Taking into account that \( \text{Re}\Sigma^{(2)}(k,\varepsilon) \propto \varepsilon \ln \max\{|\varepsilon|,|\varepsilon_k|\} \), we obtain for the first (coherent) term in the square brackets which originates from renormalization of quasiparticle spectrum

\[ \delta N_{\text{coh}}(\varepsilon) \propto \ln^3(t/|\varepsilon|), \varepsilon \gg \mu. \] (31)

The calculation of the second (incoherent, non-quasiparticle) term requires the full form of \( \Sigma^{(2)}(k,\varepsilon) \), Eq.(5), and leads to the result

\[ \delta N_{\text{incoh}}(\varepsilon) \propto \ln^2(t/|\varepsilon + \mu|). \] (32)

Although this divergence is slightly weaker than of the coherent term, it is not cut at \( \varepsilon = -\mu \). Thereby the bare VH singularity becomes considerably enhanced. Note that the divergence of the density of states together with its asymmetry in \( \varepsilon \) may lead to peculiarities of thermoelectric power owing to impurity scattering, cf. Ref. [7].

To leading (second) order the contribution to electronic specific heat owing to VH singularities has the form \( \delta C \propto T \ln^3(t/\max\{|\mu|,T\}) \). The resistivity (inverse transport relaxation time) should demonstrate at \( T > \mu \) the behavior \( \rho \propto T \ln^2(t/T) \). The calculations are similar to those of Ref. [7] for the antiferromagnetic state, extra logarithmic factors coming from VH singularities. The crossover from quadratic to nearly linear temperature dependence of resistivity is confirmed by experimental data for cuprates (see, e.g., the results of Ref. [30] for the LaSrCuO system).

Thus the divergences in the many-electron system with VH singularities are stronger than those in the MFL theory. Near QPT, we can expect that all the physical properties are strongly renormalized, the renormalizations being dependent on the type of the ordered phase. This problem will be considered elsewhere.

Finally we consider the application of the results obtained to cuprate systems. A nearly linear energy dependence of \( \text{Im}\Sigma(k_F,\varepsilon) \) at VH points, which is similar to our results, was
observed for the system Bi2212 in ARPES experiments [31]. For the system La$_{2-x}$Sr$_x$CuO$_4$ with the doping $x_c \simeq 0.2$ (which is slightly larger than the optimal one) the Fermi surface crosses VH points [32]. The density of states [33], specific heat coefficient and Pauli susceptibility [34] substantially grow near this doping. The mass enhancement factor $m^*/m$ demonstrates a similar behavior. The additional experimental investigations of the self-energy near $(\pi, 0)$ point of La$_{2-x}$Sr$_x$CuO$_4$ would be interesting in this respect.

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Fig.1. A qualitative $T - \mu$ phase diagram in the vicinity of quantum phase transitions. The chemical potential $\mu$ is referred to the Van Hove singularity. Bold line denotes the ordered ground state.

Fig.2. The second-order diagrams for the electron self-energy. Solid and dashed lines correspond to electrons with momenta close to $(0, \pi)$ and $(\pi, 0)$ Van Hove singularities respectively.
\[
\text{Re} \Sigma(k_{VH}, \varepsilon)/(2t)
\]

\[
\text{Im} \Sigma(k_{VH}, \varepsilon)/(2t)
\]
Fig. 3. The energy dependences of real (a) and imaginary (b) parts of the self-energy at VH points, and the spectral weight (c) for $t'/t = -0.45$, $U = 4t$, $\mu/(2t) = 0.15$ and $T = 0$. The dashed line corresponds to the second-order perturbation result.