Optimization of the drive mechanism with lead screw using Isight software

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Abstract. The ball screws are characterized with better positioning accuracy and higher efficiency and load ratings than the same size lead screw, but still cannot fully replace their functionality mostly due to the self-locking. Leads screws are recommended especially for such applications as grippers, lifters, presses and vertical positioning drives. By proper selection of the lead screw geometrical features and application of the modern materials, it is possible to obtain much more advantageous parameters of the drive system. In this paper, the methodology of optimization of the drive mechanism with lead screw using Isight software is presented. The performed sensitivity analysis allowed not only to find the influence of various constructional features such as: size of the thread, length of the screw and material properties, on the maximum load capacity of the drive mechanism and its inertia, but also determine its scale impact. The obtained results can improve the design process of lead screw mechanism.

1. Introduction

Nowadays, in the linear motion the ball screw seems to be the most obvious choice, however it still is not able to fully replace the lead screw in the machine design. The ball screw (fig. 1) uses rolling friction between the set of balls and the surface of the grooves in the screw and the nut. Their continuous flow is provided by the return tube in the nut and the wiper maintain the cleanness of the contact surface and helps with the lubrication.

Comparing it to the lead screw (fig. 2) it can be observed at the first sight that it has more complex construction which affects in higher costs as well as greater mass of the ball screw for respective size of both linear drives. Considering the properties of both linear drives, the ball screw has higher efficiency (over 90%) than the lead screw (20-80%). The main issue is that the ball screw is not self-locking, which means that for some applications (e.g. lifters, grippers, presses or vertical positioning devices) it is recommended to use lead screw with efficiency lower than 50%, which guarantee the self-locking. If one consider the load capacity of both linear drives, the ball screw has a slight advantage, but what is more important its load capacity does not strongly depends on the screw velocity as it does for the lead screw. Discussing the precision of motion ball screw provides a better one due to the ease of preloading and smaller backlash. Although based on the above discussion the ball screw seems to be an unquestionable more advantageous solution, there are few aspects in which the lead screw may be better option. If properly lubricated the lead screw provide quieter operation and better corrosion resistance than the ball screw. It also requires lower maintenance and provides less limited design freedom [1, 2].

Based on the above analysis it can be observed that by proper design of the linear drive with lead screw it is possible to obtain properties which can be competitive to the ball screw. For that reason authors decided to analyse to influence of the geometrical features of the screw on the linear drive
properties. In this paper the methodology of optimization with using numerical model and Isight software of such linear drives is presented.

Figure 1. Ball screw [3]  

Figure 2. Lead screw [4]

The lead screw more often has a symmetrical trapezoidal thread which basic profile (fig. 3) and its sizes are defined in the ISO 2904-2020 (previously ISO 2904-1977). Based on the three main parameters: basic diameter \( d \), tip clearance \( a_p \) and pitch \( P \) it is possible to determine the rest dimensions of the screw. These correlations were used to derive the mathematical model of screw geometry.

Figure 3. Basic profile of the symmetrical trapezoidal thread according to norm ISO 2904-1977 [5]

The constructions of the screw lifters which are the main focus in this research can be various. One can distinguish linear drives with the rotary screw or the rotary nut. Regardless of that the movable part of the screw lifter may be either the screw or the nut. For the further analysis the screw mechanism with the rotary and movable screw will be taken in consideration (fig. 4). In the drawing the construction is rotated 90° anticlockwise (force \( Q \) acts along the direction of gravity).
2. Materials and methods

In order to build the numerical model of the linear drive with movable and rotary lead screw, which will enable to perform a sensitivity analysis and optimisation, three stages of mathematical modelling were performed. They are connected with the properties of the designed mechanisms such as the load capacity, the acceleration torque of the lead screw and the cost of the material.

2.1. Modelling of the load capacity

The first stage concerned with the maximum force which can be carried by such linear drive. The direction of the force is presented in fig. 4. Since the compressive force is applied it is necessary to check the buckling of the screw, because it is a slender element. The slenderness of the screw \( \lambda \) is calculated for the weakest section of the screw, which is the core with diameter \( d_3 \), based on the equation 1. In order to determine if it is an elastic-plastic or an elastic buckling it has to be compared with the critical slenderness, which can be determined based on the material properties such as Youngs’ modulus \( E \) and Limit of proportionality \( R_h \) (equation 2).

\[
\lambda = \frac{4 \cdot L_b}{d_3} = \frac{4 \cdot \alpha_b \cdot L}{d_3}
\]

\[
\lambda_{cr} = \pi \left( \frac{E}{R_h} \right)^{1/2}
\]

If the slenderness of the lead screw \( \lambda \) is greater than the critical one \( \lambda_{cr} \) the elastic buckling may occur and can cause the visible deflection of the screw. In order to determine the maximum load which can be applied without causing the buckling one can use equation 3 (Euler formula). If this condition is not fulfilled, the elastic-plastic can be observed, if the applied force exceeds the value calculated based on equation 4 (Johnson-Ostenfeld model). In both cases the safety factor \( x \) were used which is selected by a designer based on the application of the screw mechanism (for screw lifters one can take factors between 4 to 6).

\[
Q_b \leq \frac{\pi^3 \cdot E \cdot d_3^2}{4 \cdot \lambda^2 \cdot x}
\]

\[
Q_b \leq \frac{R_e \cdot \pi \cdot d_3^2}{4x} - \frac{R_e^2 \cdot L_b^2}{\pi \cdot x \cdot E}
\]

Although in most cases the limit force calculated from buckling corresponds with the load capacity of the screw mechanism it is necessary to check the force connected with the screw core strength. In the weakest cross-section of the screw the complex state of stress occurs – the combination of compressive...
stress (normal stress \(\sigma\)) and twisting stress (tangential stress \(\tau\)) [6]. Since the stresses are in perpendicular planes for determining the reduced stress the Huber-Mises hypothesis will be used (equation 5).

\[
\sigma_{\text{red}} = \left( \frac{4Q_{cs}}{\pi d_3^2} \right)^2 + 3 \cdot \left( \frac{16 \cdot T_t}{\pi \cdot d_3^3} \right) \right)^{1/2}
\]  

Assuming that the header of the screw is supported by the rolling bearing (the additional frictional drag can be neglected) the twisting torque equals the friction torque between the screw and the nut (equation 6) [7].

\[
T_t = \frac{1}{2} \cdot Q_{cs} \cdot d_2 \tan(\gamma + \rho'),
\]  

where:

- \(d_2\) – pitch diameter of the thread,
- \(\gamma\) – lead angle of the thread (strictly dependent on the geometry of the thread),
- \(\rho'\) – apparent angle of friction (connected with the contact properties – coefficient of friction and thread profile angle).

The final mathematical model to determine the maximum load which can be applied without destroying the screw is presented below (equation 7).

\[
Q_{cs} \leq \frac{R_e}{x} \left( \frac{16}{\pi^2 \cdot d_3^2} + \frac{192 \cdot d_2^2 (\tan(\gamma + \rho')^2)^2}{\pi^2 \cdot d_3^6} \right)^{-1/2}
\]  

To determine the load capacity of the lead screw the minimum value of both discussed forces should be taken into consideration.

\[
F_0 = \min(Q_b, Q_{cs})
\]  

2.2. Modelling of the additional torque during acceleration

In order to determine the additional torque during the acceleration in the rotary motion of the screw the mathematical model of the moment of inertia of the screw was derived. Each coil of the screw can be divided into four segments: two identical conical ones with moment of inertia \(I_{cc}\), each, and two cylindrical segments with diameters \(d\) and \(d_3\) (moments of inertia respectively \(I_{cd}\) and \(I_{cd3}\)). The moment of inertia of a single coil should equal to the sum of moments for all these segments (equation 9).

\[
I_c = 2 \cdot I_{cc} + I_{cd} + I_{cd3}
\]  

To determine the moment of inertia of each segment one can use either the well-known correlations (for a cylindrical parts) or solve a triple integral (for a conical part – equation 10) [8]. The parameters used in the model (in the polar coordinate system) for a single integration point (IP) are presented in fig. 5. In order to establish the limits of the integration for variables \(r\), \(\phi\) and \(z\), it is required to find the correlation which defines the \(r\) coordinate for each integration point (equation 11).

\[
I_c = 2 \cdot I_{cc} + I_{cd} + I_{cd3}
\]  

Figure 5. Profile of the trapezoidal symmetrical thread along with the parameters used for integration
\[ I_{cc} = \int_{0}^{h} \int_{0}^{2\pi} \int_{0}^{r_3} \rho \, dq \, dr \, dz \]

where: \( h = H_3 \cdot \tan \frac{\alpha}{2} \) 

By solving the integral from equation 10 the final solution is as presented in equation 12. For the cylindrical parts the correlations are presented in equations 13 and 14.

\[ I_{cc} = \frac{\pi \rho h}{160} \cdot \frac{d_3^5 - d^5}{d_3 - d} \]

\[ I_{cd} = \frac{\pi \rho}{32} \cdot d^4 \cdot \left( \frac{P - 2h}{2} \right) \]

\[ I_{cd3} = \frac{\pi \rho}{32} \cdot d_3^4 \cdot \left( \frac{P - 2h}{2} \right) \]

Applying the equations 12-14 results in the final mathematical model of the moment of inertia of a single coil of the lead screw in function on the basic geometrical parameters of the thread and the density \( \rho \) of the selected material (equation 15). To determine the total moment of inertia for the whole lead screw it has to be multiplied by the number of coils, which is connected with the length \( L \) and the pitch \( P \) (equation 16).

\[ I_c = \frac{\pi \rho}{16} \cdot \left( \frac{1}{5} H_3 \tan \frac{\alpha}{2} \cdot \frac{d_3^5 - d^5}{d_3 - d} + \frac{1}{4} \left( P - 2H_3 \tan \frac{\alpha}{2} \right) \left( d^4 + d_3^4 \right) \right) \]

\[ I_{LS} = I_c \cdot n_c = I_c \cdot \frac{L}{P} \]

In order to validate obtained model numerical calculations were compared with the value of the moment of inertia taken from the 3D models of the selected types of screw according to norm ISO 2904-2020. The dispersion of the relative error \( \delta \) is between -0.6% to 1.23%, while the mean relative error equals 0.05%. The small difference can be caused by neglecting the effect of the helix of the thread and not taking the filleting of the edges into consideration.

To determine the acceleration torque \( T_a \) the moment of inertia should be multiplied by the angular acceleration \( \varepsilon \). During the numerical tests it was assumed that the time of acceleration and linear velocity are constant, which results in various angular velocity and acceleration for different pitches \( P \).

2.3. Modelling of the raw material cost

The last output which was taken into consideration is the economical aspect in the form of the cost of the material for the screw. It can be assumed that the cost of the material per kg is in the linear correlation with the yield point \( R_y \) of the material. The approximation of such correlation was made based on the actual data from the stock about the price of two most commonly used steels: low carbon steel S235 and carbon steel for hardening C40 (fig. 6). In order to determine the total cost the volume of the screw and as a result its mass has to be determined similar to the described earlier methodology of derivation of the moment of inertia (equations 17-22).

\[ V_c = 2 \cdot V_{cc} + V_{cd} + V_{cd3} \]

\[ V_{cc} = \frac{1}{3} \pi \cdot H \cdot \left( d^2 + d \cdot d_3 + d_3^2 \right) \]
\[ V_{cd} = \frac{\pi d^2}{4} \left( \frac{P - 2h}{2} \right) \] (19)

\[ V_{cd3} = \frac{\pi d_3^2}{4} \cdot \frac{P - 2h}{2} \] (20)

\[ V_{LS} = V_c \cdot n_c \] (21)

\[ m_{LS} = V_{LS} \cdot \rho \] (22)

**Figure 6.** Estimation of the material cost per kilogram based on the available data from the stock

2.4. Construction of the numerical model in Isight and MS Excel

Based on the mathematical models derived in sections 2.1-2.3 the numerical model in MS Excel (fig. 7) was prepared. It contains of 7 segments: Inputs, Outputs, Material properties, Kinematic parameters, Thread size, Interim calculations and XY coordinates with the visualisation. In the input section one should put parameters which influence are important for the results of the outputs. In this case these are: pitch \( P \), diameter \( d \), tip clearance \( a_p \), profile angle \( \alpha \), length \( L \), safety factor \( x \), friction coefficient \( \mu \) and buckling factor \( \alpha_b \). Material properties section contains such parameters as Youngs’ modulus, yield point and density. If one want to check the influence of the material selection in the design process the yield point can be also set as input value. Outputs represents the parameters which will be minimized, maximized or targeted during optimization, such as load capacity, acceleration torque and material cost, but also the ratio diameter to pitch which helps to determine if the thread has fine pitch (\( d/P > 8 \)), normal (\( d/P = 4-8 \)) pitch or large pitch (\( d/P < 4 \)). Kinematic parameters describes the motion of the machine as stated in section 2.2. In the interim calculations the numerical model determine the parameters which are necessary to calculate the outputs, but are strictly dependant of input parameters. Thread size section contains the geometrical parameters of the thread based on the ISO 2904-2020 norm and input parameters. These parameters are also connected with XY coordinates section and its visualisation, which helps to observe the course of the numerical analysis performed in Isight software. What is very important, when the connection between Isight and Excel is made, is to tag the cells in which inputs and outputs are (highlighted in red in fig. 7).

In the Isight software two models were built with using two different modules: Design of Experiment (DOE) and Optimization. DOE model (fig. 8) allows to perform the sensitivity analysis in order to find the correlation between the input and output parameters [9]. The most important part is to select the DOE technique, which will define how will the design point be distributed in the design area. In the presented analysis the Optimal Latin Hypercube with 51 points was used, which graphical representation can be observed on the fig. 8. After specifying the technique the factors and their range have to be defined. The algorithm will then generate the design matrix combining the values of factors selected within the specified range (in this case these factors are pitch \( P \), diameter \( d \), tip clearance \( a_p \), length \( L \) and yield point \( R_e \)). In the postprocessing section the target value or the trend (maximize or minimize) of the outputs can be specified (for example that the load capacity should be 10 kN).
The second type of model is the optimization one (fig. 9). In this case the most crucial is to select proper optimization technique (in the presented model the Hooke-Jeeves model was used) and then properly setup the variables, constraints and objectives. For the variables it is possible to define the initial value, lower and upper bounds or even specify specific values like normalized pitch. The constraints are connected with the outputs of the model and they can be scaled or weighted with some factors. The scale factor helps to match the magnitude of outputs to prevent one value to overcome others (for example acceleration torque is in range 0.1-0.3 Nm while the load capacity is in thousands of Newtons). The weight factor helps to increase or decrease the importance of one output over another. In the presented model all outputs had the same weight factor. By applying the constraints one can define either lower and/or upper bound as well as specific target value near which the optimal design point should be. In the objective section it is possible to use both the variables and outputs, with proper weight.
and scale factor. In the presented analysis the acceleration torque and material cost were minimized, while the friction coefficient were maximized.

![Figure 9. Construction of the optimization numerical model in Isight](image)

3. Results and discussion

In the first stage of research the numerical calculations were performed in the MS Excel model for all the unified sizes of the lead screw with trapezoidal symmetrical thread according to norm ISO 2904-2020 in the range of diameters 8-44 mm. The obtained results are presented in figs. 10-12.

In the fig. 10 one can observe the positive slightly non-linear (especially for the smaller diameters) correlation between the lead screw diameter and the load capacity. By comparing the obtained results it is visible that the ranges of unified threads lie within the almost linear range, since at these point the compromise between various properties of the screw mechanism should be found. Analysing the load capacity for the same diameter but various pitches it can be observed that the finer pitch provides better strength because less material is taken away during the thread machining.

![Figure 10. The correlation between the lead screw diameter and the load capacity for various pitches](image)
In the fig. 11 one can observe the positive non-linear correlation between the lead screw diameter and its acceleration torque. Although the differences are not that significant, it can be concluded that for the same diameter and finer pitch the acceleration torque will rise.

**Figure 11.** The correlation between the lead screw diameter and its acceleration torque for various pitches

In the fig. 12 one can observe the non-monotous strongly non-linear and correlation between the lead screw length and the load capacity. The presented results were made only for selected threads. By analysing obtained results it is visible that after exceeding certain length the force calculated from buckling become the one which defines the load capacity of the lead screw, while for a shorter screws the core strength determines this value. For a finer pitches a sharp transition between the elastic plastic and elastic buckling is present.

**Figure 12.** The correlation between the lead screw length and the load capacity for various threads

Based on the characteristics presented in figs. 10-12 it is possible to quickly select proper lead screw for the assumed application, load capacity and acceleration torque, but it is not possible to take into consideration the variety of more parameters. For that reason in the second stage of research the Isight software was used.
3.1. Design of experiment – sensitivity analysis of the influence of constructional features of the lead screw

Using the DOE module it is possible to gain the information about the influence of either the single variable or their combination on the output values. In fig. 13 the plot obtained from the performed analysis is presented. As can be observed in case of the load capacity the highest % effect on its value has the combination of diameter and length followed by its parameters itself. The pitch, yield point and friction coefficient has much smaller impact. On the other hand the pitch has much greater influence on the acceleration torque than the length, but it is still the diameter which affects this output the most.

**Figure 13.** The % effect of the lead screw variables on the load capacity and acceleration torque

In order to find out the possible range of outputs for a various set of variable the characteristics presented in fig. 14 can be drawn. As can be observed by changing the variables in the range defined during model building the acceleration torque may vary in range 0.05-0.3 Nm while the load capacity may change even up to 40 kN. This information can be useful for planning the design and specify the restrictions assigned to the next models to find the optimal solution. It can be observed that for some values the output is lower than 0, which is physically impossible. It is connected with the set of parameters which was randomly generated and cannot exist (like diameter \( d = 8 \) mm and pitch \( P = 10 \) mm). Working on the optimization model and adding some additional constraints should eliminate such weird results. On the characteristics in fig 14 one can observe similar correlations as discussed for fig. 13.

In order to make the results more useful for an engineers the design maps can be created. Such maps can be put in the mechanical parts catalogues in order to simplify the selection process. The example of such map for a lead screw is presented in fig. 15. Finding the combination of the desired diameter and length will give information about the maximum load which can be applied to the designed mechanism.
3.2. Optimization of the lead screw geometry

Although the DOE module provided a lot of useful information, selecting the optimal solution based solely on its result is not possible. In order to find such design point the optimization was performed. In fig. 16 the results of such optimization is presented. In Isight it is possible to observe how did the course from the starting design point to optimal one look, which in this case is thread Tr32×7. It is worth mentioning that such thread is not the unified one. The most important result of optimization is the
correlation between the load capacity and acceleration torque on which the Pareto frontiers design point can be spotted [10]. By analyzing neighboring points it is possible to find the best solution for the assumed entry date. Based on that it is visible how complex is building the optimization model and that a properly designed one may lead to great improvement of the machines. During optimization it is also possible to check which of the input variables has the most effect on output and with using this knowledge improve the optimization model.

![Figure 16. The results of the optimization of lead screw with trapezoidal thread](image)

**Figure 16.** The results of the optimization of lead screw with trapezoidal thread

![Figure 17. The correlation map between input and output variables](image)

**Figure 17.** The correlation map between input and output variables
4. Conclusions
Results presented in the paper shown how important is to properly build an optimization model and how it can improve both design process and the properties of the machine. Based on the presented results the Isight software is suitable to perform such process, however it is very important to properly use mathematical modelling in machine design. This show that conventional approach for machine design cannot be fully replaced by the optimization software, but using such software may be very helpful. The methodology presented in this paper can be surely used for any other design issues or to explain this process for the future engineers.

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