Lee-Quigg-Thacker Bounds for Higgs Boson Masses in a Two-Doublet Model

Shinya KANEMURA, Takahiro KUBOTA and Eiichi TAKASUGI

Institute of Physics, College of General Education
Osaka University, Toyonaka, Osaka 560, Japan

Abstract

Upper bounds for neutral as well as charged Higgs boson masses in a two-doublet model are obtained on the basis of tree unitarity conditions à la Lee, Quigg and Thacker. A wide variety of scattering processes are considered so extensively that our bounds are more restrictive than those obtained previously for neutral Higgs bosons and are also of a new kind for charged Higgs boson. It is argued that at least one of the Higgs bosons should be lighter than 580 GeV/$c^2$. 
1. Introduction

Although the success of the standard $SU(2)_W \times U(1)_Y$ gauge theory of the electroweak interactions is overwhelming, the Higgs boson sector in charge of spontaneous symmetry breaking has so far eluded experimental verification and is still a mystery. We all agree that the Higgs boson is one of our central concerns of the present experimental search and will be so even more in the future colliders, JLC, SSC and LHC.

It has been known by now rather well that the mass of the Higgs boson, which is proportional to the Higgs quartic coupling, may be bounded from above, provided that the quartic coupling is not so large as to violate the validity of perturbative calculations [1,2]. In fact in the minimal standard model with a single Higgs doublet, Lee, Quigg and Thacker (LQT) [1] deduced the constraint from the perturbative unitarity which turned out to be $m_h < (8\pi\sqrt{2}/3G_F)^{1/2} \equiv M_{LQT} \sim 1\text{TeV}/c^2$, where $G_F$ is the Fermi constant and $m_h$ is the mass of a neutral Higgs boson.

Extension of their type of analyses was considered by several authors in the presence of more than one Higgs doublet. There are several motivations for increasing the number of Higgs bosons; supersymmetric extension of the standard model, a model of spontaneous CP-violation due to Higgs sector [3], the Peccei-Quinn mechanism [4] and so forth. There is also a surge of phenomenological interest in the two Higgs doublet model in recent literatures [5].

Casalbuoni et al. [6] have raised a question which has close bearings on LQT’s, namely, at what energy strong interaction phenomena would start to show up whenever one or more Higgs masses are sufficiently large. They examined models with two doublets, a doublet plus a singlet and also a supersymmetric model where there exist three Higgs supermultiplets.

Maalampi et al. [7] have recently studied the two-doublet model in the same vein as LQT. They derived an upper bound of the neutral Higgs boson mass by a numerical analysis, which gave them more or less the same bound as of LQT. It should be pointed out herewith that they did not consider a broad class of scattering processes to derive constraints on all of the charged and neutral Higgs boson masses.

The purpose of the present paper is to reexamine the two-doublet model to see whether one can derive an upper bounds for neutral as well as charged Higgs boson masses in the method of LQT. We will answer to this question in the affirmative by taking into our considerations sufficiently large number of scattering processes. Overall allowed regions of three neutral and one charged Higgs boson masses are explored and their maximally possible values are presented (see Eqs. (45)-(48)). Moreover we will argue that at least one of the Higgs bosons ought to be lighter considerably than might have been expected from the LQT’ work (see Eqs. (49) and (50)).

2. Two Higgs Doublet Model

Let us start by specifying the $SU(2)_W \times U(1)_Y$ invariant Higgs potential for two $Y = 1$, $SU(2)_W$ doublets, $\Phi_1$ and $\Phi_2$. To avoid the flavor changing neutral current, we assume the discrete symmetry under $\Phi_2 \rightarrow -\Phi_2$ [8]. The most general potential then
consists of five quartic couplings together with mass terms

\[
V(\Phi_1, \Phi_2) = \sum_{i=1}^{2} \left(-\mu_i^2|\Phi_i|^2 + \lambda_i|\Phi_i|^4\right) + \lambda_3|\Phi_1|^2|\Phi_2|^2 \\
+ \lambda_4(\text{Re}\Phi_1^\dagger \Phi_2)^2 + \lambda_5(\text{Im}\Phi_1^\dagger \Phi_2)^2.
\]  

(1)

The spontaneous symmetry breaking is triggered by two vacuum expectation values, \(v_1\) and \(v_2\) of each doublet field and we write

\[
\Phi_i = \left( \frac{w_i^+}{\sqrt{2}} (v_i + h_i + iz_i) \right).
\]

(2)

In general, we can take \(v_1 > 0\), and \(v_2 > 0\).

The mass terms in (1) may be diagonalized by rotation

\[
\begin{pmatrix}
  h_1 \\
  h_2
\end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\
  \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} h \\
  H \end{pmatrix},
\]

(3)

\[
\begin{pmatrix}
  w_1 \\
  w_2
\end{pmatrix} = \begin{pmatrix} \cos \beta & -\sin \beta \\
  \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} w \\
  G \end{pmatrix},
\]

\[
\begin{pmatrix}
  z_1 \\
  z_2
\end{pmatrix} = \begin{pmatrix} \cos \beta & -\sin \beta \\
  \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} z \\
  \zeta \end{pmatrix}.
\]

(4)

In fact the mixing angles are determined by

\[
\tan \alpha = \frac{-\lambda_1 v_1^2 + \lambda_2 v_2^2 + \sqrt{\left(\lambda_1 v_1^2 - \lambda_2 v_2^2 \right)^2 + \left(\lambda_3 + \lambda_4\right)^2 v_1^2 v_2^2}}{(\lambda_3 + \lambda_4) v_1 v_2}.
\]

(5)

\((-\pi/2 \leq \alpha \leq \pi/2)\) and \(\tan \beta = v_2/v_1\) (\(0 < \beta < \pi/2\)). We note that \(w\) and \(z\) are massless Nambu-Goldstone bosons and are absorbed into the longitudinal components of the gauge bosons. The masses of the other fields, \(h, H, G\), and \(\zeta\) are expressed in terms of \(\lambda_i\) (\(i = 1, \cdots, 5\)) and \(v_i\) (\(i = 1, 2\)). Or conversely, the five quartic couplings are given by [6]

\[
\lambda_1 = \frac{G_F}{\sqrt{2} \cos^2 \beta} \left( m_h^2 \cos^2 \alpha + m_H^2 \sin^2 \alpha \right),
\]

(6)

\[
\lambda_2 = \frac{G_F}{\sqrt{2} \sin^2 \beta} \left( m_h^2 \sin^2 \alpha + m_H^2 \cos^2 \alpha \right),
\]

(7)

\[
\lambda_3 = \frac{\sqrt{2} G_F \sin 2\alpha}{\sin 2\beta} \left( m_h^2 - m_H^2 \right) + 2\sqrt{2} G_F m_G^2,
\]

(8)

\[
\lambda_4 = -2\sqrt{2} G_F m_G^2,
\]

(9)

\[
\lambda_5 = 2\sqrt{2} G_F \left( m_\zeta^2 - m_G^2 \right),
\]

(10)

where we have set \(\sqrt{v_1^2 + v_2^2} = (\sqrt{2} G_F)^{-1/2}\) and have assumed \(\beta \neq 0, \pi/2\) (\(v_1, v_2 > 0\)).
We will take it for granted hereafter that the quartic couplings \( \lambda_i \) \((i = 1, \ldots, 5)\) are lying in the region ensuring the positivity of all the masses squared. The conditions are summarized by

\[
4\lambda_1\lambda_2 > (\lambda_3 + \lambda_4)^2, \quad (11)
\]
\[
\lambda_1v_1^2 + \lambda_2v_2^2 > 0, \quad \lambda_5 > \lambda_4, \quad 0 > \lambda_4. \quad (12)
\]

We will also assume \( m_h > m_H \) without spoiling generality.

3. Eigenvalues of S-Matrix

The formulae (6) - (10) show that, if the quartic couplings \( \lambda_i \) \((i = 1, \ldots, 5)\) are comparable to, say, unity, then the Higgs masses are also on the order of \( G_F^{-1/2} \). The upper bounds of neutral as well as charged Higgs boson masses are derived by assuming that all these quartic couplings are within a perturbative region. As a criterion of the weak quartic couplings we adopt the perturbative unitarity à la LQT, thereby constraining the diagonal elements of the S-matrix.

It has been known that a potential threat to unitarity comes from longitudinal gauge bosons and Higgs particles. Scatterings involving the longitudinal gauge bosons are replaced in the high energy limit by those of corresponding Nambu-Goldstone bosons thanks to the equivalence theorem [1, 9, 10]. Our problem therefore boils down just to focusing the Higgs-Goldstone system described by (1). Furthermore, the dominant contribution in the high energy comes only from the quartic couplings, which we will consider henceforth exclusively. The S-matrix then becomes independent of energy, i.e., just a constant.

We are interested in various two-body scatterings between fourteen neutral states, \( w^+w^-, w^+G^-, G^+w^-, G^+G^-, zz, z\zeta, \zeta\zeta, hh, hH, HH, hz, Hz, \zeta h, \) and \( \zeta H \). The interactions in (1) are, however, extremely involved in terms of \( w, G, z, \zeta, h, \) and \( H \), and so will be the S-matrix, too, not to mention the diagonalization thereof. It is now important to notice that the S-matrix evaluated in the mass eigenstate bases can always be transferred into the one in the original fields \( w, z, \zeta, h \) by making a unitary transformation. The S-matrix in the original field bases is to be calculated by using the quartic part of the interactions (1) expressed in terms of the original fields which looks much simpler of course. Since all we need to know are the eigenvalues of the S-matrix, it suffices to deal with the S-matrix in the original field bases. We are thus able to streamline our calculations just by using the original fields, \( w, z, \zeta, h \) from the outset.

The use of the original fields are justified by the following considerations. In the high energy scattering, dominant contributions to the amplitudes come merely from the quartic couplings as was mentioned before, i.e., Feynman diagrams containing triple Higgs couplings are suppressed in energy on the dimensional account. The absence of Higgs propagators in our calculations amounts to ignoring admittedly the mass differences of various Higgs states originating from propagators. As in the usual particle mixing among degenerate states (such as massless neutrinos), we are allowed to choose freely the most suitable field variables.

We evaluate scattering matrix on the basis of the states, \( |w^+_iw^-_j>, |z_iz_j>, |h_ih_j>, |h_iz_j> \quad (i, j = 1, 2) \) and/or combinations thereof. We will find following orthonormal
bases the most convenient:

\[ |A_i> = \frac{1}{2\sqrt{2}} |2w_i^+ w_i^- + z_i z_i + h_i h_i>, \] (13)

\[ |B_i> = \frac{1}{2\sqrt{2}} |2w_i^+ w_i^- - z_i z_i - h_i h_i>, \] (14)

\[ |C_i> = \frac{1}{2} |z_i z_i - h_i h_i>, \] (15)

\[ |D_i> = |h_i z_i>, \] (16)

\[ |E_1> = \frac{1}{\sqrt{2}i} |w_1^+ w_2^- - w_2^+ w_1^->, \quad |E_2> = \frac{1}{\sqrt{2}} |h_1 z_2 - h_2 z_1>, \] (17)

\[ |F_±> = \frac{1}{2} |w_1^+ w_2^- + w_2^+ w_1^- ± z_1 z_2 ± h_1 h_2>, \] (18)

\[ |F_1> = \frac{1}{\sqrt{2}} |z_1 z_2 - h_1 h_2>, \quad |F_2> = \frac{1}{\sqrt{2}} |h_1 z_2 + h_2 z_1>. \] (19)

The tree level high energy scatterings consist dominantly of S-wave, since they are described by the contact interactions with quartic couplings in the above bases. The transition matrix of S-wave amplitudes turns out to be of a block-diagonal form diag \((A, B, C, D, E, f_+, f_-, f_1, f_2)\) with the following submatrices

\[ A = \frac{1}{16\pi} \begin{pmatrix} 2\lambda_3 + \frac{1}{2}(\lambda_4 + \lambda_5) & 2\lambda_3 + \frac{1}{2}(\lambda_4 + \lambda_5) \\ 2\lambda_3 + \frac{1}{2}(\lambda_4 + \lambda_5) & 6\lambda_1 \end{pmatrix}, \] (20)

\[ B = \frac{1}{16\pi} \begin{pmatrix} \frac{1}{2}(\lambda_4 + \lambda_5) & \frac{1}{2}(\lambda_4 + \lambda_5) \\ \frac{1}{2}(\lambda_4 + \lambda_5) & \lambda_3 \end{pmatrix}, \] (21)

\[ C = D = \frac{1}{16\pi} \begin{pmatrix} \frac{1}{2}(\lambda_4 - \lambda_5) & \frac{1}{2}(\lambda_4 - \lambda_5) \\ \frac{1}{2}(\lambda_4 - \lambda_5) & \lambda_3 + \frac{1}{2}(3\lambda_5 - \lambda_4) \end{pmatrix}, \] (22)

\[ E = \frac{1}{16\pi} \begin{pmatrix} \lambda_3 + \frac{1}{2}(3\lambda_5 - \lambda_4) & -\lambda_5 \\ \lambda_3 + \frac{1}{2}(3\lambda_5 - \lambda_4) & \lambda_3 + \frac{1}{2}(\lambda_4 + \lambda_5) \end{pmatrix}. \] (23)

The rows and columns of these \(2 \times 2\) matrices are spanned by the states, (13)-(17), respectively. The diagonal elements of the amplitudes \(f_+, f_1, f_2\) are those of (18) and (19) and are given by

\[ f_+ = \frac{1}{16\pi} (\lambda_3 + \frac{5}{2}\lambda_4 - \frac{1}{2}\lambda_5), \quad f_- = \frac{1}{16\pi} (\lambda_3 + \frac{1}{2}\lambda_4 - \frac{1}{2}\lambda_5), \] (24)

\[ f_1 = f_2 = \frac{1}{16\pi} (\lambda_3 + \frac{1}{2}\lambda_4 + \frac{1}{2}\lambda_5). \] (25)

We are lucky enough to obtain the fourteen eigenvalues of the transition matrix analytically. The eigenvalues of (20)-(23) are given respectively as follows;

\[ a_± = \frac{1}{16\pi} \left\{ \frac{3}{2}(\lambda_1 + \lambda_2) \pm \sqrt{9(\lambda_1 - \lambda_2)^2 + \left\{2\lambda_3 + \frac{1}{2}(\lambda_4 + \lambda_5)\right\}^2} \right\}, \] (26)
\[ b_\pm = \frac{1}{16\pi} \left\{ (\lambda_1 + \lambda_2) \pm \sqrt{(\lambda_1 - \lambda_2)^2 + \frac{1}{4}(\lambda_4 + \lambda_5)^2} \right\}, \]  

\[ c_\pm = d_\pm = \frac{1}{16\pi} \left\{ (\lambda_1 + \lambda_2) \pm \sqrt{(\lambda_1 - \lambda_2)^2 + \frac{1}{4}(\lambda_4 - \lambda_5)^2} \right\}, \]  

\[ e_1 = \frac{1}{16\pi} (\lambda_3 - \frac{1}{2}\lambda_4 + \frac{5}{2}\lambda_5), \]  

\[ e_2 = \frac{1}{16\pi} (\lambda_3 - \frac{1}{2}\lambda_4 + \frac{1}{2}\lambda_5). \]  

The reason for the block diagonal form of our transition matrix is explained in the following way. First of all let us recall the discrete symmetry under \( \Phi_2 \rightarrow -\Phi_2 \). This indicates that the number of suffix “2” is conserved modulo two throughout scattering processes. In other words, the states \( |A_i>, |B_i>, |C_i>, \) and \( |D_i> \) are not able to communicate with the other states \( |E_i>, |F_\pm>, \) and \( |F_1> \).

Besides this discrete symmetry the quartic part of Higgs potential (1) is endowed with the following discrete symmetry

\[ C : \quad \Phi_i \rightarrow \Phi_i^\dagger, \]  

\[ Y_\pi : \quad \Phi_i \rightarrow \exp(i\frac{\pi}{2})\Phi_i, \]  

\[ G : \quad \Phi_i \rightarrow \exp(i\frac{\pi\sigma_2}{2})\Phi_i^\dagger. \]  

The various states (13)-(19) are classified according to the properties under these discrete transformations. It is straightforward to see that \( |A_i>, |B_i>, |C_i>, |F_\pm> \) and \( |F_1> \) are \( C \)-even states, while the others are \( C \)-odd. We also note that \( |A_i>, |B_i>, |E_i> \) and \( |F_\pm> \) are \( Y_\pi \)-even and the others are \( Y_\pi \)-odd. On the other hand, not all of the states are the eigenstates of the G-transformation: there are only two \( G \)-even states (\( |A_i>, |F_\pm> \)) and two \( G \)-odd states (\( |B_i> \) and \( |F_\pm> \)). These discrete symmetries are efficient enough to set up selection rules and to understand vanishing elements of the transition matrix between those states with different quantum numbers.

In their analysis of the S-matrix, LQT argued that there was an \( O(4) \) symmetry in the single-doublet Higgs potential in the high energy limit. They made a full use of this in close analogy with isospin symmetry, classifying states into \( O(4) \) representations. Although our states (13)-(16) are reminiscent of their \( O(4) \)-analysis, our Higgs potential (1) does not have such continuous symmetries.

In passing our S-matrix was once studied partly by Casalbuoni et al. [6]. They restricted themselves, however, to a special case that \( m_h \) was much greater than \( G_F^{-1/2} \) and the mixing angle \( \alpha \) was negligibly small. Note also that Maalampi et al. [7] have studied scattering processes with seven elastic channels \( w^+w^-, G^+G^-, zz, \zeta\zeta, hh, hH, \) and \( HH \) (diagonal channel analysis).

4. Analysis of the Tree Unitarity Constraints
Now that we have the eigenvalues of the transition matrix, we are in a position to impose the tree unitarity conditions. The unitarity is respected if the eigenvalues are lying within the so-called Argand circle on the complex plane. The tree level amplitudes, however, are necessarily real and therefore outside the circle. If higher order radiative corrections are included, the amplitudes will eventually settle down within the circle and unitarity will be restored. For such perturbative calculations to be meaningful, the tree level amplitudes should not be very far away from the circle. As a criterion for the perturbative recovery of unitarity, LQT argued that the eigenvalues should not exceed unity,

$$|a_\pm|, \ |b_\pm|, \ |c_\pm|, \ |d_\pm|, \ |e_i|, \ |f_\pm|, \ |f_i| \leq 1.$$  (34)

In some literatures, the RHS of (34) is replaced by $1/2$, the radius of the Argand circle. We, however, use more conservative conditions (34), LQT’s, since it will be instructive to see how the inclusion of extra Higgs bosons will modify the LQT’s result.

For illustration, we take up the first one $|a_+| \leq 1$. If we express $\lambda_i$’s in $a_+$ in terms of $m_h$, $m_H$, $m_G$, $m_\zeta$, $\alpha$, $\beta$, and $G_F$, with the help of Eqs. (6)-(10), this inequality provides us with a relation that must be satisfied by various Higgs bosons masses together with the mixing angles. After a little manipulation we find $|a_+| \leq 1$ equivalent to the following two conditions

$$\frac{9}{9 - 5 \sin^2 2\alpha} \left( X - \frac{8\pi}{3} \right)^2 - \left( Y - Y_0 \right)^2 \geq R^2, \quad (35)$$

$$\frac{8\pi}{3} \sin^2 2\beta \geq X - Y \cos 2\alpha \cos 2\beta, \quad (36)$$

where our notations are

$$X = \frac{1}{\sqrt{2}} G_F(m_h^2 + m_H^2), \quad (37)$$

$$Y = \frac{1}{\sqrt{2}} G_F(m_h^2 - m_H^2), \quad (38)$$

$$Y_0 = \frac{1}{9 - 5 \sin^2 2\alpha} \left\{ 24\pi \cos 2\alpha \cos 2\beta - \sqrt{2} G_F(m_\zeta^2 + 2m_G^2) \sin 2\alpha \sin 2\beta \right\}, \quad (39)$$

$$R = \frac{1}{9 - 5 \sin^2 2\alpha} \left\{ 16\pi \sin 2\alpha \cos 2\beta + \frac{3}{\sqrt{2}} G_F(m_\zeta^2 + 2m_G^2) \sin 2\beta \cos 2\alpha \right\}. \quad (40)$$

The second inequality (36) excludes the right half of the hyperbola (35) on the $(X, Y)$ plane and the allowed region is the shaded one in Fig. 1, where $Y \geq 0 \ (m_h > m_H)$ and $X - Y = \sqrt{2} G_F m_H^2 \geq 0$ are taken into account.

Fig. 1 shows clearly that the allowed region of $m_H$ and $m_h$ is bounded for fixed values of mixing angles, $m_\zeta$ and $m_G$. We can read off bounds for $m_H$ and $m_h$ from the shaded region

$$X - Y = \sqrt{2} G_F m_H^2 \leq X_P, \quad (41)$$

$$X + Y = \sqrt{2} G_F m_h^2 \leq 2X_Q, \quad (42)$$
where $X_P$ and $X_Q$ are the $X$-coordinates of the points $P$ and $Q$, respectively in Fig. 1:

$$X_P = 8\pi - \frac{1}{3} \sqrt{(9 - 5 \sin^2 \alpha)(Y_0^2 + R^2)},$$  

$$X_Q = \frac{1}{5 \sin^2 \alpha} \left[ 24\pi - (9 - 5 \sin^2 \alpha)Y_0 - \sqrt{(9 - 5 \sin^2 \alpha) \left( 5R^2 \sin^2 \alpha + (8\pi - 3Y_0)^2 \right)} \right].$$  

We have similarly analyzed other eigenvalues, and found after all that (41) and (42) are the most stringent conditions upon $m_H$ and $m_h$. Since the points $P$ and $Q$ are both on the left half of the hyperbola, $X_P \leq 8\pi/3$ and $X_Q \leq 8\pi/3$ should hold in general. We thus have upper bounds for neutral Higgs boson masses

$$m_H \leq \left( \frac{4\pi \sqrt{2}}{3G_F} \right)^{1/2} = \frac{1}{\sqrt{2}} M_{LQ},$$  

$$m_h \leq \left( \frac{8\pi \sqrt{2}}{3G_F} \right)^{1/2} = M_{LQ},$$

whatsoever the values of $\alpha$, $\beta$, $m_\zeta$ and $m_G$.

The relations (41) and (42) contain more information than (45) and (46). To elucidate this, let us have a closer look at (41). In the $(G_F m_H^2, G_F m_G^2, G_F m_\zeta^2)$-space, The inequality (41) tells us that the inner region surrounded by the solid curves in Fig. 2 are allowed exclusively. This means that the charged Higgs boson masses $m_G$ and $m_\zeta$ are also bounded. In fact Fig. 2 shows apparently

$$m_G \leq \left( \frac{4\pi \sqrt{2}}{G_F} \right)^{1/2} = \frac{\sqrt{3}}{2} M_{LQ},$$  

$$m_\zeta \leq \left( \frac{8\pi \sqrt{2}}{G_F} \right)^{1/2} = \sqrt{3} M_{LQ},$$

The most interesting bound is derived by a geometrical inspection of Fig. 2. We obtain the bound on the lightest Higgs boson mass among $m_H$, $m_G$, and $m_\zeta$. 


\[ M(\text{the lightest Higgs boson mass}) \leq \sqrt{\frac{\sin^2 2\beta}{4 - \sin^2 2\beta}} M_{LQT} \]

\[ = \sqrt{\frac{x^2}{1 + x^2 + x^4}} M_{LQT}, \quad (49) \]

where \( x = v_2/v_1 = \tan \beta \). This is our main result. The upper limit in (49) is realized at point K on the surface in Fig. 2 where the vector \( OK \) is parallel to \((1, 1, 1)\).

5. Summary

In the present paper we have investigated the consequences of applying the tree unitarity conditions to the two Higgs doublet model. We have seen that, by considering a wide class of scattering processes, not only neutral but also charged Higgs boson masses are bounded from above. Our results are listed in Eqs. (45)-(48).

The most important results of our calculations is that at least one of the Higgs bosons should be much lighter than has been anticipated from LQT’s work [1]. The maximum value of the RHS of (49) is reached when \( x = 1 \) (\( \sin^2 2\beta = 1 \)). It is therefore concluded that the lightest Higgs boson mass has to satisfy

\[ M(\text{the lightest Higgs boson mass}) \leq \frac{1}{\sqrt{3}} M_{LQT} = 580 \text{ GeV}/c^2, \quad (50) \]

that is, the bound is \( 1/\sqrt{3} \) of the one derived by LQT for a single doublet Higgs case. It is very likely that inclusion of more Higgs doublets would give us more tight upper bound.

A comment to be made herewith is that, if we would use \( |a_+| \leq 1/2 \) as a criterion of perturbative recovery of unitarity [11], all the bounds of Higgs masses that we have obtained would be reduced by a factor \( 1/\sqrt{2} \). In particular, the upper bound of the lightest Higgs boson mass (50) would be replaced by \( M_{LQT}/\sqrt{6} = 410 \text{ GeV}/c^2 \).

Our final remark is that, so far, we have assumed \( v_1, v_2 \neq 0 \). If one of these vacuum expectation values e.g. \( v_2 \) happens to vanish, then the neutral Higgs boson \( h_2 \) becomes massless at the tree level. This is a pseudo- Nambu-Goldstone boson because there does not exist any particular symmetry prohibiting \( h_2 \) from acquiring a mass on the loop level. Then it is expected that \( v_2 \) becomes non-zero by the higher loop corrections as far as \( \Phi_2 \) is coupled with \( \Phi_1 \).

ACKNOWLEDGEMENTS

Our sincere thanks should go to Takeshi Kurimoto for invaluable discussions. One of us (E.T.) is supported in part by Grant in Aid for Scientific Research, from the Ministry of Education, Science and Culture (No. 02640230).
REFERENCES

[1] B.W. Lee, C. Quigg and H.B. Thacker, Phys. Rev. Lett. 38 (1977) 883; Phys. Rev. D16 (1977) 1519.
[2] D.A. Dicus and V.S. Mathur, Phys. Rev. D7 (1973) 3111.
[3] S. Weinberg, Phys. Rev. Lett. 37 (1976) 657.
[4] R.D. Peccei and H.R. Quinn, Phys. Rev. Lett. 38 (1977) 1440.
[5] A.J. Buras, P. Krawczyk, M.E. Lautenbacher and C. Salazar, Nucl. Phys. B337 (1990) 284; V. Barger, J.L. Hewett and R.J.N. Phillips, Phys. Rev. D41 (1990) 3421; J.F. Gunion and B. Gradkowski, Phys. Lett. B245 (1991) 591; J.F. Gunion, H.E. Haber, G. Kane ans S. Dawson, “The Higgs Hunter’s Guide” (Addison-Wesley Pub. Co. 1990)
[6] R. Casalbuoni, D. Dominici, F. Feruglio and R. Gatto, Nucl. Phys. B299 (1988) 117; Phys. Lett. B200 (1988) 495; R. Casalbuoni, D. Dominici, R. Gatto and C. Giunti, Phys. Lett. B178 (1986) 235;
[7] J. Maalampi, J. Sirkka and I. Vilja, Phys. Lett. B265 (1991) 371.
[8] S.L. Glashow and S. Weinberg, Phys. Rev. D15 (1977) 1958.
[9] J.M. Cornwall, D.N. Levin and G. Tiktopoulos, Phys. Rev. Lett. 30 (1973) 1268; Phys. Rev. D10 (1974) 1145.
[10] M.S. Chanowitz and M.K. Gaillard, Nucl. Phys. B261 (1985) 379; Y.P. Yao and C.P. Yuan, Phys. Rev. D38 (1988) 2237; M. Veltman, Phys. Rev. D41 (1990) 2294; J. Bagger and C. Schmit, Phys. Rev. D41 (1990) 264; H.J. He, Y-P. Kuang and X. Li, Phys. Rev. Lett. 69 (1992) 2619; K. Aoki, in Proceedings of the Meeting on Physics at TeV Energy Scale (KEK Report 89-20, Nov. 1987) p. 20.
[11] W. Marciano, G. Valencia, and S. Willenbrock, Phys. Rev. D40 (1989) 1725; M. Lüscher and P. Weisz, Phys. Lett. B212 (1989) 472.
FIGURE CAPTIONS

Fig. 1 The allowed region of Higgs boson masses given by (35) and (36). Our variables are $X = G_F(m_h^2 + m_H^2)/\sqrt{2}$ and $Y = G_F(m_h^2 - m_H^2)/\sqrt{2}$.

Fig. 2 Schematic view of the allowed region of Higgs boson masses $m_H$, $m_\zeta$ and $m_G$ given by Eq. (41) for given values of mixing angles. The inner region surrounded by the solid curves is allowed. The vector $OK$ is in the direction of $(1, 1, 1)$. 