CP–Violation in $K$ and $B$ decays

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In this article we give an overview of CP–violation for both $K^0$ ($\bar{K}^0$), $B^0$ ($\bar{B}^0$), and $B_s$ ($\bar{B}_s$) systems. Direct CP–violation and mixing induced CP–violation are discussed for $K^0$ ($\bar{K}^0$), and $B^0$ ($\bar{B}^0$) decays.

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I. INTRODUCTION

Symmetries have played an important role in particle physics. In quantum mechanics a symmetry is associated with a group of transformations under which a Lagrangian remains invariant. Symmetries limit the possible terms in a Lagrangian and are associated with conservation laws. Here we will be concerned with the role of discrete symmetries associated with space reflection ($P$): $\vec{x} \rightarrow -\vec{x}$, Time reversal ($T$): $t \rightarrow -t$ and charge conjugation ($C$): transformation of particle to its antiparticle.

Quantum Electrodynamics (QED) and Quantum Chromodynamics (QCD) respect all these symmetries. Local quantum field theories in which Lorentz invariance is built in are $CPT$ invariant. However in weak interactions $C$ and $P$ are maximally violated. $CP$ violation is a small effect observed in $B$ and $K$ decays. However the weak interaction Lagrangian in the standard model violates $C$ and $P$ but is $CP$- invariant. Thus in the standard model the $CP$ non-conservation is a consequence of Higgs sector i.e. a mismatch between the weak eigenstates and mass eigenstates in quark sector of the standard model.

First we note that under $C$, $P$ and $T$ operations the Dirac spinor $\Psi$ transforms as follow

$$P\Psi (t, \vec{x}) P^\dagger = \gamma^0 \Psi (t, -\vec{x})$$
$$C\Psi (t, \vec{x}) C^\dagger = -i\gamma^2 \gamma^0 \Psi^T (t, \vec{x})$$
$$T\Psi (t, \vec{x}) T^\dagger = \gamma^1 \gamma^3 \Psi (-t, \vec{x})$$

(1)

The effect of transformations $C$, $P$ and $CP$ on various quantities that appear in a gauge theory Lagrangian are given below

| Term/Transformation | Scalar | Pseudoscalar | Vector | Axial vector |
|---------------------|--------|--------------|--------|--------------|
| $P$                 | $\Psi^j \Psi_j$ | $-i\Psi^j \gamma_5 \Psi_j$ | $\eta (\mu) \Psi^j \gamma^\mu \Psi_j$ | $-\eta (\mu) \Psi^j \gamma^\mu \gamma^5 \Psi_j$ |
| $C$                 | $\Psi_j \Psi_i$ | $i\Psi_j \gamma_5 \Psi_i$ | $-\Psi_j \gamma_5 \Psi_i$ | $\Psi_j \gamma^\mu \gamma_5 \Psi_i$ |
| $CP$               | $\Psi^j \Psi_i$ | $-i\Psi^j \gamma_5 \Psi_i$ | $-\eta (\mu) \Psi^j \gamma^\mu \Psi_i$ | $-\eta (\mu) \Psi^j \gamma^\mu \gamma^5 \Psi_i$ |

The vector bosons associated with the electroweak unification group $SU_L (2) \times U (1)$ transform under $CP$ as

$$W^\pm_\mu (\vec{x}, t) \overset{CP}{\rightarrow} -\eta (\mu) W^\mp_\mu (-\vec{x}, t)$$
$$Z_\mu (\vec{x}, t) \overset{CP}{\rightarrow} -\eta (\mu) Z_\mu (-\vec{x}, t)$$
$$A_\mu (\vec{x}, t) \overset{CP}{\rightarrow} -\eta (\mu) A_\mu (-\vec{x}, t)$$

(2)

$$\eta (\mu) = 1 \quad \mu = 0$$
$$\eta (\mu) = -1 \quad \mu = 1, 2, 3$$

*This is an extended version of the talk given on 4th February 2003 at Sharif University, Tehran, Iran, on the occasion of 16th Khwarizmi International Award. I wish to thank Iranian Research Organization for Science and Technology (IROST) for hospitality.
Then it is clear that electro-weak interaction Lagrangian is $CP$-invariant. It is instructive to discuss the restrictions imposed by $CPT$ invariance. $CPT$ invariance implies
\[
\text{out} \langle f | \mathcal{L} | X \rangle = \text{out} \left\langle \hat{f} \left| (CPT)^{-1} \mathcal{L} CPT \right| X \right\rangle = \eta_f^* \eta_T^f \text{in} \left\langle \hat{f} \left| (CP)^\dagger \mathcal{L} (CP)^{-1} \right| X \right\rangle^* = \eta_f^* \eta_T^f \left\langle X \left| (CPT)^{-1} \mathcal{L} (CP) \right| \hat{f} \right\rangle_{\text{in}}
\]
\[
= -\eta_f^* \eta_T^f \eta_{CP} \left\langle \tilde{X} \mathcal{L} S_f | \tilde{f} \right\rangle_{\text{out}} = \eta_f \text{out} \left\langle \hat{f} | S^\dagger \mathcal{L}^{\dagger} | \tilde{X} \right\rangle^* = \eta_f \exp(2i\delta_f)_{\text{out}} \langle f | \mathcal{L} | \tilde{X} \rangle^*, \quad \mathcal{L}^\dagger = \mathcal{L}
\]
(3)

Hence we get
\[
\text{out} \langle f | \mathcal{L} | X \rangle = \eta_f \exp(2i\delta_f)_{\text{out}} \langle f | \mathcal{L} | X \rangle^* = \eta_f \exp(i\delta_f)A^* (X \to f)
\]
(4)

In deriving the above result, we have put $\tilde{f} = f$ where $\tilde{}$ means that momenta and spin are reversed. Since we are in the rest frame of $X,T$ will reverse only magnetic quantum number and we can drop $\tilde{}$. Further we have used
\[
CP |X\rangle = -|X\rangle
\]
(5)
\[
CP |f\rangle = \eta_f^{CP} |\tilde{f}\rangle
\]
(6)
\[
|f\rangle_{\text{out}} = S_f |f\rangle_{\text{in}} = \exp(2i\delta_f) |f\rangle_{\text{in}}
\]
(7)

$\delta_f$ is the strong interaction phase shift. If $CP$-invariance holds then
\[
\text{out} \langle f | \mathcal{L} | X \rangle = \text{out} \left\langle \hat{f} | \mathcal{L} | \tilde{X} \right\rangle
\]
i.e.
\[
A^* = A
\]
(8)

Thus necessary condition for $CP$-violation is that the decay amplitude $A$ should be complex. In view of our discussion above, schematically under $CP$ an operator $O(\vec{x},t)$ is replaced by
\[
O(\vec{x},t) \to O^{\dagger}(\vec{x},t)
\]
(9)

The effective Lagrangian has the structure ( $\mathcal{L}^\dagger = \mathcal{L}$)
\[
\mathcal{L} = aO + a^*O^{\dagger}
\]
(10)

Hence $CP$-violation requires $a^* \neq a$.

Now $CP$-violation can also arise when the $CP$-eigenstates
\[
|X_{1,2}^0\rangle = \frac{1}{\sqrt{2}} \left[ |X^0\rangle \mp |\tilde{X}^0\rangle \right]
\]
(11)
are not the mass eigenstates i.e. $CP$-violation in the mass matrix. $CP$-violation due to mass mixing and in the decay amplitude has been experimentally observed in $K^0$ and $B^0_d$. For $B_s$ decays, the $CP$-violation in the mass matrix is not expected in the Standard Model. In fact time dependent $CP$-violation asymmetry gives a clear way to observe direct $CP$-violation in $B_s$ decays.
II. $K^0 - \bar{K}^0$ COMPLEX

$K^0 - \bar{K}^0$ complex in particle physics, is a simple example of two states system in the sense that states $|K^0\rangle$ and $|\bar{K}^0\rangle$ form a complete set so that an arbitrary state can expanded in terms of them. It has some interesting consequences for CP-violation.

Weak Interactions:

$$K^0 \rightarrow \pi^+ \pi^-$$

$$P(K^0) = -1, \quad P(\pi^+ \pi^-) = (-1)^2 (-1)^{l=0} = 1$$

Parity is not conserved.

Charge conjugation is also not conserved.

$$\pi^+ \overset{C}{\rightarrow} \pi^-$$

$$\mu^+ \overset{C}{\rightarrow} \mu^-$$

$$\nu \overset{C}{\rightarrow} \bar{\nu}$$

$\nu$ is left handed and $\bar{\nu}$ is right handed.

Helicity under $C$ and $P$ transforms as:

$$\mathcal{H} = \frac{\vec{s} \cdot \vec{p}}{|p|} \overset{C}{\rightarrow} \mathcal{H}$$

$$P \overset{P}{\rightarrow} -\mathcal{H}$$

Invariance under $C$ gives

$$\Gamma_{\pi^+ \rightarrow \mu^+(-)\nu} = \Gamma_{\pi^- \rightarrow \mu^-(+)\bar{\nu}}.$$ 

Experimentally

$$\Gamma_{\pi^+ \rightarrow \mu^+(-)\nu} \gg \Gamma_{\pi^- \rightarrow \mu^-(+)\bar{\nu}}$$

showing that $C$ is violated in weak interactions. Under $CP$

$$\Gamma_{\pi^+ \rightarrow \mu^+(-)\nu} \overset{CP}{\rightarrow} \Gamma_{\pi^- \rightarrow \mu^-(+)\bar{\nu}}$$

which is seen experimentally.

Now

$$K^0 \rightarrow \pi^+ \pi^- \rightarrow \bar{K}^0, \quad |\Delta Y| = 2$$

Thus weak interaction can mix $K^0$ and $\bar{K}^0$

$$\langle K^0 | H | \bar{K}^0 \rangle \neq 0.$$ 

Off diagonal matrix elements are not zero. Thus $K^0$ and $\bar{K}^0$ can not be mass eigenstates.

Select the phase:

$$CP|K^0\rangle = -|\bar{K}^0\rangle.$$ 

Define

$$|K_1^0\rangle = \frac{1}{\sqrt{2}} [ |K^0\rangle - |\bar{K}^0\rangle ]$$

$$|K_2^0\rangle = \frac{1}{\sqrt{2}} [ |K^0\rangle + |\bar{K}^0\rangle ]$$

$$CP|K_1^0\rangle = |K_1^0\rangle$$

$$CP|K_2^0\rangle = -|K_2^0\rangle$$
$K_1^0$ and $K_2^0$ are eigenstates of $CP$ with eigenvalues $+1$ and $-1$.

If $CP$ is conserved

$$\langle K_2^0 | H | K_1^0 \rangle = \langle K_2^0 | (CP)^{-1} H (CP) | K_1^0 \rangle = - \langle K_2^0 | H | K_1^0 \rangle$$

Therefore

$$\langle K_2^0 | H | K_1^0 \rangle = 0.$$ 

Thus $|K_2^0\rangle$ and $|K_1^0\rangle$ can be mass eigenstates.

$K_1^0 \rightarrow \pi^+ \pi^-$: large phase space; decay probability large; short lived.

$K_2^0 \rightarrow \pi^+ \pi^- \pi^0$ small phase space, Long lived.

$K_1^0$ and $K_2^0$ are mass eigenstates, they form a complete set.

$$|\psi(t)\rangle = a(t) |K_1\rangle + b(t) |K_2\rangle$$

$$\hbar = c = 1$$

$$i \frac{d |\psi(t)\rangle}{dt} = \begin{pmatrix} m_1 - \frac{i}{2} \Gamma_1 & 0 \\ 0 & m_2 - \frac{i}{2} \Gamma_2 \end{pmatrix} |\psi(t)\rangle.$$ \hspace{1cm} (12)

The Solution is

$$a(t) = a(0) \exp \left( -im_1 t - \frac{1}{2} \Gamma_1 t \right)$$

$$b(t) = b(0) \exp \left( -im_2 t - \frac{1}{2} \Gamma_2 t \right)$$

Suppose we start with $|K^0\rangle$ initially

$$|\psi(0)\rangle = |K^0\rangle,$$

Then we get

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}} \left[ \exp \left( -im_1 t - \frac{1}{2} \Gamma_1 t \right) |K_1\rangle + \exp \left( -im_2 t - \frac{1}{2} \Gamma_2 t \right) |K_2\rangle \right]$$

or

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}} \left\{ \left[ \exp \left( -im_1 t - \frac{1}{2} \Gamma_1 t \right) 
\right.
+ \exp \left( -im_2 t - \frac{1}{2} \Gamma_2 t \right) \left| K^0 \right| 
\left. 
\right] 
+ \exp \left( -im_1 t - \frac{1}{2} \Gamma_1 t \right) \left| K^0 \right| 
\right. 
- \exp \left( -im_2 t - \frac{1}{2} \Gamma_2 t \right) \left| K^0 \right| \right\}.$$ \hspace{1cm} (13)

However in $K^0 - \bar{K}^0$ basis the mass matrix is given by

$$M = m - \frac{i}{2} \Gamma$$

$$= \begin{pmatrix} m_{11} - \frac{i}{2} \Gamma_{11} & m_{12} - \frac{i}{2} \Gamma_{12} \\ m_{21} - \frac{i}{2} \Gamma_{21} & m_{22} - \frac{i}{2} \Gamma_{22} \end{pmatrix}.$$ \hspace{1cm} (14)
Hermiticity of matrices $m_{\alpha\alpha'}$ and $\Gamma_{\alpha\alpha'}$ gives

\begin{equation}
(m)_{\alpha\alpha'} = (m^\dagger)_{\alpha'\alpha}, \quad \Gamma_{\alpha\alpha'} = \Gamma_{\alpha'\alpha}^\ast
\end{equation}

\begin{align*}
\alpha = \alpha' &= 1, 2 \\
m_{21} &= m_{12}, \quad \Gamma_{21} = \Gamma_{12}^\ast
\end{align*}

CTP invariance gives

\begin{align*}
\langle K^0 | M | K^0 \rangle &= \langle \bar{K}^0 | M | \bar{K}^0 \rangle \\
m_{11} &= m_{22}, \quad \Gamma_{11} = \Gamma_{22}
\end{align*}

\begin{equation}
\langle \bar{K}^0 | M | K^0 \rangle = \langle K^0 | M | \bar{K}^0 \rangle \quad \text{Identity.}
\end{equation}

Diagonalization of mass matrix $M$ in eq. (14) gives

\begin{align*}
m_{11} - \frac{i}{2} \Gamma_{11} - pq &= m_1 - \frac{i}{2} \Gamma_1 \\
m_{11} - \frac{i}{2} \Gamma_{11} + pq &= m_2 - \frac{i}{2} \Gamma_2
\end{align*}

\begin{equation}
\text{where}
\end{equation}

\begin{align*}
p^2 &= m_{12} - \frac{i}{2} \Gamma_{12} \\
q^2 &= m_{12}^\ast - \frac{i}{2} \Gamma_{12}^\ast
\end{align*}

Assuming $CP$ conservation

\begin{align*}
\langle \bar{K}^0 | M | K^0 \rangle &= \langle K^0 | M | \bar{K}^0 \rangle \\
m_{21} &= m_{12}, \quad \Gamma_{21} = \Gamma_{12}
\end{align*}

\begin{equation}
\text{Identity.}
\end{equation}

$m_{12}$ and $\Gamma_{12}$ are real. Thus

\begin{align*}
pq &= m_{12} - \frac{i}{2} \Gamma_{12} \\
m_1 &= m_{11} - m_{12}, \quad \Gamma_1 = \Gamma_{11} - \Gamma_{12} \\
m_2 &= m_{11} + m_{12}, \quad \Gamma_2 = \Gamma_{11} + \Gamma_{12} \\
\Delta m &= m_2 - m_1 = 2m_{12}, \\
\Delta \Gamma &= \Gamma_2 - \Gamma_1 = 2\Gamma_{12}
\end{align*}

However it was found experimentally that $CP$ is not conserved in $K^0$ decay. We note

\begin{align*}
CP \left( K_1^0 \right) &= 1 \\
CP \left( \pi^+ \pi^- \right) &= (-1)^l \left( -1 \right)^l = 1
\end{align*}

Thus

\begin{equation}
K_1^0 \longrightarrow \pi^+ \pi^-
\end{equation}

is allowed by $CP$ conservation.

Experimentally it was found that long lived $K^0$ also decay to $\pi^+ \pi^-$ but with very small probability. Small $CP$ non conservation can be taken into account by defining

\begin{align*}
| K_S \rangle &= | K_1^0 \rangle + \varepsilon | K_2^0 \rangle \\
| K_L \rangle &= | K_2^0 \rangle + \varepsilon | K_1^0 \rangle
\end{align*}

\begin{equation}
\text{where} \quad \varepsilon \quad \text{is a small number. Thus $CP$ non conservation manifest itself by the ratio:}
\end{equation}
$$\eta_{+-} = \frac{A(K_L \rightarrow \pi^+ \pi^-)}{A(K_S \rightarrow \pi^+ \pi^-)} = \varepsilon$$  \hspace{2cm} (23)

$$|\eta_{+-}| \simeq (2.286 \pm 0.017) \times 10^{-3}$$

Now CP non conservation implies

$$m_{12} \neq m_{12}^*, \ \Gamma_{12} \neq \Gamma_{12}^*.$$  \hspace{2cm} (24)

Since CP violation is a small effect

$$\text{Im} m_{12} \ll \text{Re} m_{12}$$

$$\text{Im} \Gamma_{12} \ll \text{Re} \Gamma_{12}$$  \hspace{2cm} (25)

Further if CP violation arises from mass matrix then

$$\Gamma_{12} = \Gamma_{12}^*.$$  \hspace{2cm} (26)

Thus CP violation can result by a small term $i \text{Im} m_{12}$ in the mass matrix given in Eq. (12).

$$M = \begin{pmatrix} m_1 - \frac{1}{2} \Gamma_1 & i \text{Im} m_{12} \\ -i \text{Im} m_{12} & m_2 - \frac{1}{2} \Gamma_2 \end{pmatrix}.$$  \hspace{2cm} (27)

Diagonalization gives

$$\varepsilon = \frac{i \text{Im} m_{12}}{(m_2 - m_1) - i (\Gamma_2 - \Gamma_1)/2}.$$  \hspace{2cm} (28)

Then from Eq. (21) up to first order, we get

$$\Delta m = m_2 - m_1 \rightarrow m_{K_L} - m_{K_S}$$

$$= 2 \text{Re} m_{12}$$

$$\Delta \Gamma = \Gamma_2 - \Gamma_1 = \Gamma_L - \Gamma_S = 2\Gamma_{12}$$  \hspace{2cm} (29)

Then Eq. (13) is unchanged; replace

$$m_1 \rightarrow m_S, \ m_2 \rightarrow m_L$$

$$\Gamma_1 \rightarrow \Gamma_S, \ \Gamma_2 \rightarrow \Gamma_L$$

Now

$$\Delta m = m_L - m_S$$

$$\Delta \Gamma = \Gamma_L - \Gamma_S$$

$$\Gamma_S = \frac{\hbar}{\tau_S} = 7.367 \times 10^{-12} \text{ MeV},$$

$$\tau_S = (0.8935 \pm 0.0008) \times 10^{-10} \text{ S}$$

$$\Gamma_L = \frac{\hbar}{\tau_L} = 1.273 \times 10^{-14} \text{ MeV},$$

$$\tau_L = (5.17 \pm 0.04) \times 10^{-8} \text{ S}$$

$$\Delta \Gamma \simeq -\Gamma_S$$

$$m_L = m + \frac{1}{2} \Delta m$$

$$m_S = m - \frac{1}{2} \Delta m$$  \hspace{2cm} (30)

Hence from Eq. (2)

$$|\psi(t)\rangle \approx e^{-\frac{i}{2} \Delta mt} \left\{ \begin{bmatrix} e^{-\frac{i}{2} \Gamma_S t} e^{i \Delta mt} + e^{-i \Delta mt} \\ -e^{-\frac{i}{2} \Gamma_S t} e^{i \Delta mt} - e^{-i \Delta mt} \end{bmatrix} |K^0\rangle \right\}$$  \hspace{2cm} (31)
Therefore probability of finding $\bar{K}^0$ at time $t$ [we started with $K^0$]

$$P (K^0 \rightarrow \bar{K}^0, t) = |\langle \bar{K}^0 | \psi (t) \rangle|^2$$

$$= \frac{1}{4} \left( 1 + e^{-\Gamma_S t} - 2e^{-\frac{1}{2}\Gamma_S t} \cos (\Delta m) t \right)$$

$$= \frac{1}{4} \left( 1 + e^{-t/\tau_S} - 2e^{-\frac{1}{2}t/\tau_S} \cos (\Delta m) t \right)$$

(32)

If kaons were stable ($\tau_S \rightarrow \infty$), then

$$P (K^0 \rightarrow \bar{K}^0, t) = \frac{1}{2} [1 - \cos (\Delta m) t]$$

(33)

which shows that a state produced as pure $Y = 1$ state at $t = 0$ continuously oscillates between $Y = 1$ and $Y = -1$ state with frequency $\omega = \frac{\Delta m}{\hbar}$ and period of oscillation

$$\tau = \frac{2\pi}{\Delta m/\hbar}$$

(34)

Kaons, however decay and oscillations are damped.

By measuring the period of oscillation, $\Delta m$ can be determined.

$$\Delta m = m_L - m_S = (3.489 \pm 0.008) \times 10^{-12} \text{ MeV.}$$

(35)

Such a small number is measured, consequence of superposition principle in quantum mechanics

$$\pi^- p \rightarrow K^0 \Lambda^0 \quad \bar{K}^0 p \rightarrow \pi^+ \Lambda^0$$

$\pi^+$ can only be produced by $\bar{K}^0$ in the final state. This would give the clear indication of oscillation.

Coming back to $CP$-violation

$$\varepsilon = \frac{i \text{Im } m_{12}}{\Delta m - i \Delta \Gamma / 2}, \quad \varepsilon = |\varepsilon| e^{i\phi_\varepsilon}$$

(36)

$$\tan \phi_\varepsilon = -2 \frac{\Delta m}{\Delta \Gamma} = \frac{\Delta m}{\Gamma_S} - \Gamma_L$$

$$\approx \frac{2 \times 0.474 \Gamma_S}{0.998 \Gamma_S}$$

$$\rightarrow \phi_\varepsilon = 43.49 \pm 0.08^0 C.$$
\[ \eta_{++} = |\eta_{-+}| e^{i\phi_{++}} \simeq \varepsilon + \varepsilon' \]
\[ \eta_{00} = |\eta_{00}| e^{i\phi_{00}} \simeq \varepsilon - 2\varepsilon' \]  

(40)

where

\[ \varepsilon' = \frac{i}{\sqrt{2}} e^{i(\delta_2 - \delta_0)} \text{Im} \frac{A_2}{A_0} \]  

(41)

Clearly \( \varepsilon' \) measures the \( CP \)-violation in the decay amplitude, since \( CP \)-invariance implies \( A_2 \) to be real.

After 35 years of experiments at Fermilab and CERN, results have converged on a definitive non-zero result for \( \varepsilon' \)

\[ R = \frac{|\eta_{00}|}{|\eta_{++}|} = \frac{\varepsilon - 2\varepsilon'}{\varepsilon + \varepsilon'}, \quad \varepsilon' \ll \varepsilon \]
\[ \simeq \left| 1 - \frac{3\varepsilon'}{\varepsilon} \right|^2 \simeq 1 - 6 \text{Re} \left( \varepsilon' / \varepsilon \right) \]
\[ \text{Re} \left( \varepsilon' / \varepsilon \right) = \frac{1 - R}{6} \]
\[ = (1.8 \pm 0.4) \times 10^{-3}. \]  

(42)

This is an evidence that although \( \varepsilon' \) is a very small, but \( CP \)-violation does occur in the decay amplitude. Further we note from Eq. (41)

\[ \phi_{\varepsilon'} = \delta_2 - \delta_0 + \frac{\pi}{2} \approx 48 \pm 4^0 \]

where numerical value is based on an analysis of \( \pi \pi \) scattering.

III. \( B^0 - \bar{B}^0 \) COMPLEX

For \( B^0 \) meson, one finds (see below)

\[ m_{12} = |m_{12}| e^{2i\phi_M} \]
\[ \Gamma_{12} = |\Gamma_{12}| e^{2i\phi_M} \]
\[ |\Gamma_{12}| \ll |m_{12}| \]  

(44)

\[ p^2 = e^{2i\phi_M} \left( |m_{12}| - i |\Gamma_{12}| \right) \simeq |m_{12}| e^{2i\phi_M} \]
\[ q^2 = e^{-2i\phi_M} \left( |m_{12}| - i |\Gamma_{12}| \right) \simeq |m_{12}| e^{-2i\phi_M} \]
\[ pq = |m_{12}| \]  

(45)

(46)

Hence the mass eigenstates \( B^0_H \) and \( B^0_L \) can be written as:

\[ |B^0_H \rangle = \frac{1}{\sqrt{2}} \left[ |B^0 \rangle - e^{2i\phi_M} |\bar{B}^0 \rangle \right] \]  

(47)

\[ |B^0_L \rangle = \frac{1}{\sqrt{2}} \left[ |B^0 \rangle + e^{2i\phi_M} |\bar{B}^0 \rangle \right] \]  

(48)

\( CP \) violation occurs due to phase factor \( e^{2i\phi_M} \) in mass matrix. Hence one gets [from Eq. (13)],

\[ |B^0(t) \rangle = \frac{1}{\sqrt{2}} \left\{ \exp \left( -i m_1 t - \frac{1}{2} \Gamma_1 t \right) \right. \]
\[ + \exp \left( -i m_2 t - \frac{1}{2} \Gamma_2 t \right) |B^0 \rangle \]
\[ - e^{+2i\phi_M} \left[ \exp \left( -i m_1 t - \frac{1}{2} \Gamma_1 t \right) \right. \]
\[ - \exp \left( -i m_2 t - \frac{1}{2} \Gamma_2 t \right) |\bar{B}^0 \rangle \left\} \]  

(49)
For $B$-decays

$$\Gamma_1 = \Gamma_2 = \Gamma$$

$$\Delta m_B = m_2 - m_1$$

$$m = \frac{1}{2}(m_1 + m_2).$$

(50)

Then from (49), we get

$$|B^0(t)⟩ = e^{-imt}e^{-\frac{i}{2}\Gamma t}\left\{ \cos\left(\frac{\Delta m t}{2}\right) |B^0⟩ + i\epsilon^{+2i\phi_M} \sin\left(\frac{\Delta m t}{2}\right) |B^0⟩ \right\}$$

(51)

Similarly we get

$$|\bar{B}^0(t)⟩ = -e^{-imt}e^{-\frac{i}{2}\Gamma t}\left\{ \cos\left(\frac{\Delta m t}{2}\right) |\bar{B}^0⟩ + i\epsilon^{-2i\phi_M} \sin\left(\frac{\Delta m t}{2}\right) |\bar{B}^0⟩ \right\}$$

(52)

¿From Eq. (51) and (52), the decay amplitudes for

$$B^0(t) \rightarrow f \quad A_f(t) = \langle f | H_w | B^0(t)⟩$$

$$\bar{B}^0(t) \rightarrow \bar{f} \quad \bar{A_f}(t) = \langle \bar{f} | H_w | \bar{B}^0(t)⟩$$

(53)

are given by

$$A_f(t) = e^{-imt}e^{-\frac{i}{2}\Gamma t}\left\{ \cos\left(\frac{\Delta m t}{2}\right) A_f + i\epsilon^{+2i\phi_M} \sin\left(\frac{\Delta m t}{2}\right) A_f \right\}$$

(54)

$$\bar{A_f}(t) = e^{-imt}e^{-\frac{i}{2}\Gamma t}\left\{ \cos\left(\frac{\Delta m t}{2}\right) \bar{A_f} + i\epsilon^{-2i\phi_M} \sin\left(\frac{\Delta m t}{2}\right) \bar{A_f} \right\}.$$  

(55)

Consider the decay for which

$$CP |f⟩ = \eta_f |f⟩$$

For this case we get, from Eqs. (54) and (55),

$$A_f(t) = \frac{\Gamma_f(t) - \bar{\Gamma}_f(t)}{\Gamma_f(t) + \bar{\Gamma}_f(t)} = \cos(\Delta mt) \left|A_f|^2 - |\bar{A_f}|^2\right|$$

$$+ i \sin(\Delta mt) \left( e^{2i\phi_M} A_f^* \bar{A_f} - e^{-2i\phi_M} A_f \bar{A_f} \right) / \left|A_f|^2 - |\bar{A_f}|^2\right|^2$$

(56)

For the above kind of decays which proceed through a single diagram (for example tree graph), $\bar{A_f}/A_f$ is given by

$$\frac{\bar{A_f}}{A_f} = e^{i(\phi + \delta_f)} = e^{2i\phi}$$

where $\phi$ is the weak phase in the decay amplitude. Hence from Eq. (56), we obtain

$$A_f = -\sin(\Delta mt) \sin(2\phi_M + 2\phi)$$

(57)

In particular for the decay
we obtain

$$A_{\psi K_s}(t) = - \sin(2\phi_M) \sin(\Delta m t)$$

and

$$A_{\psi K_s} = \frac{\int_0^\infty \left[ \Gamma_f(t) - \bar{\Gamma}_f(t) \right] dt}{\int_0^\infty \left[ \Gamma_f(t) + \bar{\Gamma}_f(t) \right] dt} \frac{(\Delta m / \Gamma)}{1 + (\Delta m / \Gamma)^2}$$

Expt. : \( \left( \frac{\Delta m}{\Gamma} \right)_{B_0^0} = 0.775 \pm 0.015 \)

The following comments are in order. There are three generations of elementary fermions

- \(( u, d', \nu_e)\) \( m_e = 0.511\) MeV \( m_u, m_d \sim 4 - 5\) MeV
- \(( c, s', \nu_\mu)\) \( m_\mu = 105\) MeV \( m_c \sim 1.4\) GeV
- \(( t, b', \nu_\tau)\) \( m_\tau = 1.777\) GeV \( m_b \approx 4 - 4.5\) GeV \( m_t = 175\) GeV

The left–handed fermions are put into doublet representation of electroweak unification group \( SU_L(2) \times U(1) \) as follows

\[
\begin{align*}
&\left( \begin{array}{c}
u_e \\
\mu \\
\tau \\
e_R \\
u_R \\
d_R
\end{array} \right)_{L} \quad 
\begin{array}{c}SU_L(2) \\ I_{3L} \\ Y\
\end{array} \\
&\quad \left( \begin{array}{c}
u_e \\
\mu \\
\tau \\
e_R \\
u_R \\
d_R
\end{array} \right)_{L}
\end{align*}
\]

\[
Q = I_{3L} + \frac{Y}{2}
\]

Mediators of the weak interactions are put in the: adjoint representation of \( SU_L(2) \)

\[
W^+, W^-, W^\mu, B^\mu, Z^\mu : SU_L(2) \quad \text{Isospin} \quad U(1) \quad \text{Hypercharge}
\]

Since weak forces are short range, the gauge group \( SU_L(2) \times U(1) \) is spontaneously broken:

\[
SU_L(2) \times U(1) \longrightarrow U_{em}(1)
\]

The mass eigenstates are

\[
W^+, W^-, Z_\mu = -\sin \theta_W B_\mu + \cos \theta_W W^\mu_\mu
\]

\[
A_\mu = \cos \theta_W B_\mu + \sin \theta_W W^\mu_\mu
\]

\( W^+, W^-, Z_\mu \) are mediators of weak interactions. \( A_\mu \): photon mediator of electromagnetic interaction.
\[ m_A = 0, \quad m_W^2 = \frac{\pi \alpha}{\sqrt{2} G_F \sin^2 \theta_W}, \quad \sin^2 \theta_W \approx 0.23 \]

\[ m_Z = \frac{m_W}{\cos \theta_W} \]

After radiative corrections \( m_W = 80.39 \text{ GeV}, \ m_Z = 91.18 \text{ GeV} \) in remarkable agreement with the experimental values.

We note that the weak eigenstates \( d', s' \) and \( b' \) are not the same as mass eigenstates \( d, s \) and \( b \).

\[
\begin{pmatrix}
  d' \\
  s' \\
  b'
\end{pmatrix}
= V
\begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix}
\]

(61)

\( V \) is called the CKM matrix.

\[
V = \begin{pmatrix}
  V_{ud} & V_{us} & V_{ub} \\
  V_{cd} & V_{cs} & V_{cb} \\
  V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\]

\[
\simeq \begin{pmatrix}
  1 - \frac{1}{2} \lambda^2 & \lambda & A \lambda^3 (\rho - i \eta) \\
  -\lambda & 1 - \frac{1}{2} \lambda^2 & A \lambda^2 \\
  A \lambda^3 (1 - \rho - i \eta) & -A \lambda^2 & 1
\end{pmatrix}
+ O(\lambda^4), \ \lambda = 0.22
\]

(62)

The unitarity of \( V \)

\[ VV^\dagger = 1 \]

gives

\[
V_{ud}^* V_{ub} + V_{cb}^* V_{cd} + V_{td}^* V_{tb} = 0
\]

(63)

The second line in Eq. (62) expresses \( V \) in terms of Wolfenstein parameterization. The unitarity of \( V \) can be graphically represented as triangle shown in Fig. 1.

\[
V_{cb} = A \lambda^2 \\
V_{ub} = |V_{ub}| e^{-i \gamma} \\
V_{td} = |V_{td}| e^{-i \beta}
\]

\( \eta : \ Source \ of \ CP\text{-violation} \)

(64)

We note that

\[
\phi_M = \beta
\]

(65)

\[
A_{\psi K_s} = -\sin 2\beta \frac{(\Delta m/\Gamma)}{1 + (\Delta m/\Gamma)^2}
\]

(66)

\( A_{\psi K_s} \) has been experimentally measured which gives

\[ \sin 2\beta = 0.79 \pm 0.14 \]

But to see that \( \phi_M = \beta \), we note that Fig. 2 represents the transition \( B^0 \to \bar{B}^0 \) ( \( t \)-quark gives the leading contribution) This transition is proportional to

\[
(V_{ub})^2 (V_{td}^*)^2 = |V_{td}|^2 e^{2i \beta} \\
= A^2 \lambda^6 \left[ (1 + \rho)^2 + \eta^2 \right] e^{2i \beta}
\]

(67)

\[
m_{12} = |m_{12}| e^{2i \beta} \\
\Gamma_{12} = |\Gamma_{12}| e^{2i \beta}
\]

(68)

Note that
\[ \frac{\Gamma_{12}}{m_{12}} \sim \frac{m_b^2}{m_t^2} \]

hence \( \Gamma_{12} \ll m_{12} \).

On the other hand: \( B_s^0 \to \bar{B}_s^0 \) transition is proportional to

\[ (V_{tb})^2 (V_{ts}^*)^2 = |V_{ts}|^2 \approx A^2 \lambda^4 \]
\[ m_{12} = |m_{12}| \]
\[ \Gamma_{12} = |\Gamma_{12}| \]
\[ \phi_M = 0 \] (71)

For \( K^- \)–mesons,

\[ V_{ts}V_{td}^* = -A^2 \lambda^5 (1 - \rho + i \eta) \]
\[ V_{cs}V_{cd}^* = \left( 1 - \frac{1}{2} \lambda^2 \right) [ -\lambda + A^2 \lambda^5 (1 - \rho + i \eta) ] \]

It is clear from Eq. (72)

\[ \frac{(V_{ts}V_{td}^*)^2}{(V_{cs}V_{cd}^*)^2} = \frac{\lambda^{10} m_t^2}{\lambda^2 m_c^2} \approx 5 \times 10^{-6} \frac{m_t^2}{m_c^2} \]

Thus the relative contribution of \( t \)-quark as compared with the \( c \) quark is of the order

\[ 5 \times 10^{-6} \frac{m_t^2}{m_c^2} \]

and hence is negligible for \( m_t \approx 175 \) GeV. Thus since \( m_{12} \) and \( \Gamma_{12} \) are proportional to the \( (V_{cs}V_{cd}^*)^2 \), we conclude from Eqs. (40) that

\[ \text{Im} \, m_{12} \ll \text{Re} \, |m_{12}| \]
\[ \text{Im} \, \Gamma_{12} \ll \text{Re} \, |\Gamma_{12}| \]

These facts, we have used in discussing the \( K^0 - \bar{K}^0, B^0 - \bar{B}^0 \) systems. Still we need to determine \( \gamma \), which directly measures the CP–Violation in \( B \) and \( B_s \) decays.

**IV. DIRECT \( CP \)-VIOLATION AND FINAL STATE INTERACTIONS**

Direct \( CP \)-violation in \( B \) decays involves the weak phase in the decay amplitude. The reason for this being that necessary condition for direct \( CP \)-violation is that decay amplitude should be complex as discussed in section 1. But this is not sufficient because in the limit of no final state interactions, the direct \( CP \)-violation in \( B \to f, \bar{B} \to \bar{f} \) decay vanishes. To illustrate this point, we discuss the decays \( B^0 \to \pi^+ \pi^- \). The main contribution to this decay is from tree graph [Fig. 3]. But this decay can also proceed via penguin diagram Fig. 4.

The contribution of penguin diagram can be written as

\[ P = V_{ub} V_{ud} f(u) + V_{cb} V_{cd}^* f(c) + V_{tb} V_{td}^* f(t) \] (73)

where \( f(u), f(c) \) and \( f(d) \) denote the contributions of \( u, c \) and \( t \) quarks in the loop. Now using the unitarity equation (63), we can rewrite Eq. (73) as

\[ P = V_{ub} V_{ud}^* (f(u) - f(t)) + V_{cb} V_{cd}^* (f(c) - f(t)) \] (74)

Due to loop integration \( P \) is suppressed relative to \( T \). But still its contribution is not negligible. The first part of Eq. (74) has the same CKM matrix elements as for tree graph, so we can absorb it in the tree graph. Hence we can write \( f = \pi^+ \pi^- \)

\[ A_f = A(B^0 \to \pi^+ \pi^-) = T e^{i(\gamma + \delta_T)} + P e^{i(\phi + \delta_P)} \] (75)
where $\delta_T$ and $\delta_P$ are strong interaction phases which have been taken out so that $T$ and $P$ are real. $\phi$ is the weak phase in Penguin graph; in fact it is zero for this particular decay. CPT invariance gives

$$A_f \equiv A(B^0 \to \pi^+\pi^-) = T e^{-i(\gamma - \delta_T)} + P e^{-i(\phi - \delta_P)}.$$  

Hence direct CP-violation asymmetry is given by

$$C_{\pi\pi} \equiv \frac{\Gamma(B^0 \to \pi^+\pi^-) - \Gamma(B^0 \to \pi^+\pi^-)}{\Gamma(B^0 \to \pi^+\pi^-) + \Gamma(B^0 \to \pi^+\pi^-)}$$

$$= \frac{1 - |\lambda|^2}{1 + |\lambda|^2} = \frac{2r \sin (\delta_T - \delta_P) \sin (\phi - \gamma)}{1 + 2r \cos (\delta_T - \delta_P) \cos (\gamma - \phi)}$$

$$= \frac{2r \sin (\delta_T - \delta_P) \sin \gamma}{1 + 2r \cos (\delta_T - \delta_P) \cos \gamma + r^2}; \quad r = \frac{P}{T}, \quad \lambda = \frac{\bar{A}_f}{A_f}.$$  

We further note that (cf. Eqs. (47) and (48)):

$$\frac{\Gamma(B^0_H \to \pi^+\pi^-)}{\Gamma(B^0_L \to \pi^+\pi^-)} \approx \tan^2 (\beta + \gamma) \left[ 1 + 4r \frac{\sin (\phi - \gamma)}{\sin (2\beta + 2\gamma)} \right] + O(r^2)$$

$$= \tan^2 (\beta + \gamma) \left[ 1 - 4r \frac{\sin \gamma}{\sin (2\beta + 2\gamma)} \right].$$  

This may be of academic interest; unless one can experimentally distinguish between $B^0_H$ and $B^0_L$. For the time dependent CP-asymmetry for $B^0 \to \pi^+\pi^-$ decay we obtain from Eqs. (56) and (75)

$$A(t) = C_{\pi\pi}(\cos \Delta mt) - S_{\pi\pi}(\sin \Delta mt),$$

where the direct CP-violation $C_{\pi\pi}$ is given in Eq. (4.5). The mixing induced parameter $S_{\pi\pi}$ is given by

$$S_{\pi\pi} = \frac{\text{Im}[e^{2i\phi e^{60}}]}{1 + |\lambda|^2} = \frac{\sin (2\beta + 2\gamma) + 2r \cos (\delta_T - \delta_P) \sin (2\beta + \gamma)}{1 + 2r \cos (\delta_T - \delta_P) + r^2}.$$  

The recent BELLE and BABAR results are (see ref.)

BELLE: $S_{\pi\pi} = -1.23 \pm 0.41$ (stat) $^{+0.08}_{-0.07}$ (syst.)

BABAR: $C_{\pi\pi} = -0.77 \pm 0.27$ (stat) $^{+0.08}$ (syst.)

These results give clear indication of direct and mixing-induced CP-violation in $B^0 (\bar{B}^0) \to \pi^+\pi^-$ decays. However it is clear from Eqs. (79a) and (79b), that without the knowledge of strong interaction phases it is not possible to extract weak interaction phases $\beta$ and $\gamma$ from the data.

In order to discuss the final state interactions (FSI) it is useful to consider $\Delta C = \pm 1, \Delta S = -1$ decays of $B$. In $B$-decays, the $b$ quark is converted into $c$ or $u$-quark:

$$b \to c + q + \bar{q}, \quad b \to u + q + \bar{q}.$$  

For the $\Delta C = 1, \Delta S = -1$ decays, $b \to c + q + \bar{q}$ is relevant whereas for $\Delta C = -1, \Delta S = -1$ decays the transition $b \to u + q + \bar{q}$ enters.

In the tree graphs (See Fig. 5) the configuration is such that $q$ and $\bar{q}$ essentially go together into the color singlet states, the third quark recolling, there is a significant probability that the system will hadronize as a two body final state. In this picture final state interactions may be neglected for the tree graphs (i.e. strong phase shifts are expected to be small). The following decays proceed through tree graphs. They are dominant decay modes and phase shifts for these decay modes are expected to be small.

$$\bar{B}^0 \to K^- D^+, \quad A_{-+} = a_- e^{i\delta_-} : \quad T e^{i\delta_T}$$

$$B^+ \to K^- D^0, \quad A_{0-} = a_0 e^{i\delta_0} : \quad T e^{i\delta_T} + C e^{i\delta_C}$$

$$\bar{B}^0 \to K^- D^+_s, \quad B_{s+} = b_s e^{i\delta_s} : \quad T e^{i\delta_T}$$

$$\bar{B}^0 \to K^+ D_s^-, \quad \bar{A}_{s-} = \bar{a} e^{-i(\gamma + \delta)} : \quad T e^{i(\delta_T + \gamma)}$$

$$\bar{B}^0 \to K^+ D_s^-, \quad B_{s-} = b_s e^{i\delta_s} : \quad T e^{i\delta_T}$$

where $V_{us}$ and $V_{cb}$ are CKM matrix elements. The complex parameters $a_0, a_-, a_+,$ and $a_-$ are related to the strong parameters $\delta_0, \delta_-, \delta_+$ through Eqs. (81).
corrections may be important. Rescattering is depicted in Fig. 7.

Further we obtain

\[ \frac{V_{ub}V_{cs}^*}{V_{cb}V_{as}^*} = \sqrt{\rho^2 + \eta^2} e^{i\gamma}, \quad \sqrt{\rho^2 + \eta^2} = 0.36 \pm 0.09 \]  \hspace{1cm} (82)

\[ \frac{T}{T} \simeq 0.72 \times \sqrt{\rho^2 + \eta^2} \]  \hspace{1cm} (83)

Color suppressed decays: (Fig. 6) are given below

\[ \bar{B}^0 \to \bar{K}^0 D^0, \quad A_{00} = a_{00} e^{i \delta_{00}} = C e^{i \delta_c} \quad \Rightarrow V_{cb}V_{as}^* \]

\[ \bar{B}^0 \to \bar{K}^0 D^0, \quad \tilde{A}_{00} = a_{00} e^{i \tilde{\delta}_{00} + \gamma} = C e^{i (\delta_c + \gamma)} \quad \Rightarrow \frac{C}{T} = \frac{\tilde{C}}{T} \simeq \left( \frac{a_2}{a_1} \right) \approx 0.22 \]  \hspace{1cm} (84)

We have neglected annihilation diagrams which are suppressed. For color suppressed graphs the rescattering corrections may be important. Rescattering is depicted in Fig. 7.

Rescattering is essentially determined by the scattering amplitudes of the following processes.

\[ P_a + D \to P_b + D \]

\[ P_a + \bar{D} \to P_b + \bar{D} \]

\( P \) stands for \( K \) and \( \pi \).

Using Regge phenomenology, one notes that pomeron, \( \rho - A_2 \) and \( \omega - f \) trajectories in \( t \)-channel contribute. Using \( SU(3) \) and the fact that these trajectories are exchange degenerate, one finds that the rescattering corrections can be expressed in terms of two parameters \( \epsilon \) and \( \theta \). For example, one finds (see Ref. [5])

\[ A (\bar{B}^0 \to K^0 D^0)_{\text{FSI}} = \epsilon e^{i \theta} A (\bar{B}^0 \to K^- D^+) \quad \epsilon \simeq 0.08, \quad \theta = 73^\circ \]  \hspace{1cm} (85)

After taking into account rescattering corrections, we get

\[ A_{00} = a_{00} e^{i \delta_{00}} + \epsilon e^{i \theta} a_{-+} e^{i \delta_{-+}} \]

\[ A_{-+} = a_{-+} e^{i \delta_{-+}} \]

\[ A_{-0} = a_{-0} e^{i \delta_{-0}} + \epsilon e^{i \theta} a_{+-} e^{i \delta_{+-}} \]

\[ B_{-+} = b_{+} e^{i \delta_{+}} \]  \hspace{1cm} (87a)

\[ \tilde{A}_{00} = (\tilde{a}_{00} e^{i \tilde{\delta}_{00}} + \epsilon e^{i \theta} \tilde{a} e^{i \tilde{\delta}}) e^{i \gamma} \]

\[ \tilde{A}_{-0} = (\tilde{a}_{-0} e^{i \tilde{\delta}_{-0}} + \frac{1}{2} \left( 1 - i \frac{1}{3} \right) \tilde{a} e^{i \tilde{\delta}}) e^{i \gamma} \]

\[ \tilde{A}_{0-} = (\tilde{a}_{0-} e^{i \tilde{\delta}_{0-}} + \frac{1}{2} \left( 1 + i \frac{1}{3} \right) \tilde{a} e^{i \tilde{\delta}}) e^{i \gamma} \]

\[ \tilde{B}_{-+} = \tilde{b}_{+} e^{i \tilde{\delta}_{+}} \]  \hspace{1cm} (87b)

Further we obtain

\[ \tilde{\delta}_+ = \delta_+, \quad \tilde{\delta} = \tilde{\delta}_{-+}, \quad \tilde{\delta}_{-0} = \tilde{\delta}_{00}, \quad \tilde{\delta}_{0-} = \tilde{\delta}_{00} \]  \hspace{1cm} (87c)

In order to give some feeling, how the above results are obtained, we note that using time reversal invariance, we get

\[ A_f = \text{out} \langle f | H | B \rangle = \text{out} \langle f | T^{-1} T H T^{-1} T | B \rangle = \text{out} \langle f | T^{-1} H T | B \rangle = \text{in} \langle f^* | H^* | B \rangle^* = \text{out} \langle f | S^1 H | B \rangle^*. \]  \hspace{1cm} (89)
Hence

\[
A_f^* = \text{out} \langle f \mid S^\dagger H \mid B \rangle = \sum_n \text{out} \langle f \mid S^\dagger \mid n \rangle \text{out} \langle n \mid H \mid B \rangle \\
= \sum_n S_{nf}^* A_n \\
= \sum_n (\delta_{nf} - 2iM_{nf}^*) A_n
\]  

(90)

\[
A_f^* - A_f = -2i \sum_n M_{nf}^* A_n \\
\text{Im} A_f = \sum_n M_{nf}^* A_n
\]  

(91)

where \(M_{nf}\) is the scattering amplitude for \(f \rightarrow n\). The dominant contribution to the decay amplitude in Eq. (91) is from those two body decays of \(B\) which proceed through the channel \(n \rightarrow f'\), where \(A_f'\) is given by tree graph (See Fig. 5). In Eq. (91), the two particle unitarity gives \(\text{Im} A_f\) in terms of the scattering amplitude \(M_{f'f}\) and the tree amplitude \(A_f'\); where the scattering amplitude \(M_{f'f}\) is obtained by using Regge phenomenology. Then using unsubtracted dispersion relation for the decay amplitude gives the rescattering corrections to \(A_f'\) in the form \(\epsilon \bar{e} e^{i\theta} A_f'\). The relationship between the phase shifts given in Eq. (88) follow from the argument given below. First the equality \(\delta_s = \delta_s\) is the consequence of the \(C\)–invariance of strong interactions viz.

\[
\langle K^- D_s^- \mid S \mid K^- D_s^+ \rangle = \langle K^- D_s^+ \mid C^{-1}CSC^{-1}C \mid K^- D_s^+ \rangle \\
= \langle K^- D_s^+ \mid C^{-1}SC \mid K^- D_s^+ \rangle \\
= \langle K^- D_s^- \mid S \mid K^- D_s^- \rangle
\]  

(92)

For the rest of the relationships between phase shifts given in Eq. (89), we note that since \(\gamma_{\rho D^+D^-} = -\gamma_{\omega D^+D^-}\); the \(\rho\) trajectory does not contribute to the scattering \(K^-D^+ \rightarrow K^-D^+\); \(\rho\) trajectory also does not contribute to the channel \(\pi^+D_s^- \rightarrow \pi^+D_s^-, \rho = \omega\) are not coupled to \(D_s^+D_s^-\). Thus only pomeron contribute to these channels and since pomeron is \(SU(3)\) singlet; hence it follows that \(\delta = \delta_{++}\). Similarly since \(-\gamma_{\rho K^0K^0} = \gamma_{\omega K^0K^0}\), only pomeron contributes to the scattering channels \(K^0 D^0 \rightarrow K^0 D^0, K^0 \bar{D}^0 \rightarrow K^0 \bar{D}^0\) so that again \(\delta_{00} = \delta_{00}\). Similar argument holds for the equality of \(\delta_{-0} = \delta_{0-}\).

We now discuss the effect of FSI on \(CP\)-asymmetry. Define

\[
D_{\mp}^0 = \frac{1}{2} (D^0 \mp \bar{D}^0)
\]

\(D_{\mp}^0\) are eigenstates of \(CP\), with eigenvalue \(\pm 1\). We define \(CP\)-asymmetry

\[
A_\mp = \frac{\Gamma (B^- \rightarrow K^- D_{\mp}^0) - \Gamma (B^+ \rightarrow K^+ D_{\mp}^0)}{\Gamma (B^0 \rightarrow K^- D^+)}
\]

(93)

Then one finds after taking out the weak interaction phase \(e^{i\gamma}\):

\[
A_\mp = \pm 2 \sin \gamma \left[ \frac{\text{Re} \, A_{-0} \text{Im} \, \bar{A}_{-0} - \text{Im} \, A_{-0} \text{Re} \, \bar{A}_{-0}}{|A_{-+}|^2} \right]
\]

(94)

Now using Eqs. (87a,87c ), one gets

\[
A_\mp = \pm 2 \sin \gamma \left[ -f \bar{f} \sin (\delta_{-0} - \bar{\delta}_{-0}) \\
-\bar{c} f \sin (\theta + \delta_{-+} - \bar{\delta}_{-0}) \\
+ \frac{\sqrt{10}}{6} c f \sin (\theta - \phi + \bar{\delta} - \delta_{-0}) \right], \text{where}
\]

\[
f = \frac{a_{-0}}{a_{-+}} = \left( 1 + \frac{C}{T} \right) \approx 1.22, \quad \left( 1 + \frac{i}{\sqrt{3}} \right) = \frac{\sqrt{10}}{3} e^{\pm \phi}
\]

15
Using $\delta$ $f$

For $\bar{A}$, using Eqs. (54 and 55) and Eqs. (87a,87c,88 ), we get

$$\bar{A} = \frac{\bar{a}}{a} + \frac{\bar{T}}{T} \quad \phi = \tan^{-1} \frac{1}{3} = 18^0$$

(96)

In the limit that the phase shifts $\delta$'s $\to$ 0, the rescattering corrections give

$$A_\pi \sim 10^{-2} \sin \gamma$$

This may regarded as an upper limit.

$$A_\pi \leq 10^{-2} \sin \gamma$$

(97)

Thus $A_\pi$ will be zero in the absence of FSI. A reliable estimate is not possible, because it is not easy to estimate the strong interaction phase shifts $\delta$'s. Our estimate for $\delta$'s gives

$$A_\pi \sim 10^{-3} \sin \gamma.$$  

(98)

For $B_s$ decays, defining

$$B_\pi = \frac{1}{\sqrt{2}} (B_s^0 - \bar{B}_s^0),$$

(99)

we get

$$2 |A(B_\pi \to K^+ D_s^-)|^2 - b_s^2 - \bar{b}_s^2$$

$$= \mp \left[ \cos (\gamma - \delta_s + \bar{\delta}_s) \right]$$

$$= \mp \left[ \cos (\gamma) \cos (\delta_s - \bar{\delta}_s) + \sin (\gamma) \sin (\delta_s - \bar{\delta}_s) \right]$$

(100)

$$2 |A(B_\pi \to K^- D_s^+)|^2 - b_s^2 - \bar{b}_s^2$$

$$= \mp \left[ \cos (\gamma + \delta_s + \bar{\delta}_s) \right]$$

$$= \mp \left[ \cos (\gamma) \cos (\delta_s + \bar{\delta}_s) - \sin (\gamma) \sin (\delta_s + \bar{\delta}_s) \right]$$

(101)

Therefore

$$\Gamma (B_\pi \to K^+ D_s^-) - \Gamma (B_\pi \to K^- D_s^+)$$

$$= 2b_s \bar{b}_s \sin (\gamma) \sin (\delta_s - \bar{\delta}_s)$$

(102)

Using $\delta_s = \bar{\delta}_s$, we get

$$\Gamma (B_\pi \to K^+ D_s^-) = \Gamma (B_\pi \to K^- D_s^+)$$

(103)

Finally we discuss the time dependent analysis of $B$-decays to get information about weak phases, Define time–dependent $CP$-asymmetry parameter:

$$\mathcal{A}(t) \equiv \frac{[\Gamma_f(t) + \tilde{\Gamma}_f(t)] - [\tilde{\Gamma}_f(t) + \Gamma_f(t)]}{\Gamma_f(t) + \tilde{\Gamma}_f(t)}$$

(104)

For $f \equiv K_s D^0$ and $\tilde{f} \equiv K_s \bar{D}^0$, using Eqs. (54 and 55) and Eqs. (87a,87c,88 ), we get

$$\mathcal{A}(t) = -4 \frac{\Gamma (\bar{B}_s^0 \to K^- D_s^+) (a_2/a_1) (C/T) \sqrt{\rho^2 + \eta^2}}{\Gamma (B_s^0 \to K^0 D^0) + \Gamma (\bar{B}_s^0 \to K_s D^0)}$$

$$\times \left[ \sin (\Delta m_B t) \sin (2\beta + \gamma) \right]$$

$$\times \left[ 1 + 2 \varepsilon (a_2/a_1) \cos \theta + \varepsilon^2 (a_1/a_2) \right]$$

$$= -4 \sqrt{\rho^2 + \eta^2} \frac{\sin (\Delta m_B t) \sin (2\beta + \gamma)}{1 + \rho^2 + \eta^2} \times 1.34$$

$$= -1.94 \sin (\Delta m_B t) \sin (2\beta + \gamma)$$

(105)
Rescattering corrections are of the order of 34%.

For $B_0^s$, again using Eqs. (54 and 55), (87b,87d), we get

$$\mathcal{A}_s(t) = \frac{\Gamma_{fs}(t) - \bar{\Gamma}_{fs}(t)}{\Gamma_{fs}(t) + \bar{\Gamma}_{fs}(t)} = \frac{b_s \bar{b}_s}{b_s^2 + b_s^2} \sin(\Delta m_B t) \left[ S + \bar{S} \right]$$

$$f_s = K^+ D_s^-, \quad \bar{f}_s = K^- D_s^+$$

$$S = \sin (2\phi_{Ms} + \gamma + \delta_s - \bar{\delta}_s)$$

$$\bar{S} = \sin (2\phi_{Ms} + \gamma - \delta_s + \bar{\delta}_s)$$

Since for $B_0^s$, $\phi_{Ms} = 0$ and $\delta_s = \bar{\delta}_s$, we get

$$\mathcal{A}_s(t) \approx 2 \sqrt{\rho^2 + \eta^2} \frac{T}{1 + (\rho^2 + \eta^2) T/T} \sin(\Delta m_B t) \sin \gamma \approx 0.49 \sin (\Delta m_B t) \sin \gamma$$

We note that for time integrated CP-asymmetry

$$\mathcal{A}_s \equiv \frac{\int_0^\infty \left[ \Gamma_{fs}(t) - \bar{\Gamma}_{fs}(t) \right] dt}{\int_0^\infty \left[ \Gamma_{fs}(t) + \bar{\Gamma}_{fs}(t) \right] dt}$$

$$= \frac{b_s \bar{b}_s}{b_s^2 + b_s^2} \frac{\Delta m_B / \Gamma}{1 + (\Delta m_B / \Gamma)^2} \left[ S + \bar{S} \right]$$

Thus for $\mathcal{A}_s$, we get

$$\mathcal{A}_s \approx 0.49 \sin \gamma \frac{\Delta m_B / \Gamma_s}{1 + (\Delta m_B / \Gamma_s)^2}$$

The CP–asymmetry $\mathcal{A}_s(t)$ or $\mathcal{A}_s$ involves two experimentally unknown parameters $\sin \gamma$ and $\Delta m_{B_s}$. Both these parameters are of importance in order to test the unitarity of CKM matrix viz whether CKM matrix is a sole source of CP–violation in the processes in which CP–violation has been observed. We note that

$$\sin 2\beta = \frac{2\eta (1 - \rho)}{\eta^2 + (1 - \rho)^2}$$

$$\sin \gamma = \frac{\eta}{\sqrt{\eta^2 + \rho^2}}$$

$$\frac{\Delta m_{B_s}}{\Delta m_{B_d}} = \frac{|V_{td}^2|}{|V_{ts}^2|} \xi = \frac{1}{\lambda^2 \left[ \eta^2 + (1 - \rho)^2 \right] \xi}$$

where $\xi$ is a measure of SU(3) violation; lattice calculation gives its value: 1.15±0.4.

Experimental value for $\sin 2\beta$ is 0.79±0.14. Thus measurement of $\sin \gamma$ and $\Delta m_{B_s}$ will check the consistency of CP-violation in B-decays.

V. CONCLUSION

Effective weak interaction Lagrangian in the standard model can accommodate $CP$-violation due to a mismatch between the weak eigenstates and mass eigenstates. These eigenstates are related by a unitary transformation given by CKM matrix $V$. More than two generations of quarks are necessary to have weak phases responsible for $CP$-violation. For three generations of quarks, only two of phases $\alpha$, $\beta$ and $\gamma$ are independent because unitarity of $V$ gives $\alpha + \beta + \gamma = \pi$. Both the direct and mixing-induced CP–violation has been experimentally observed and has been measured in $K$–decays. The mixing-induced CP–violation involves the mass difference $\Delta m_K$, $\Delta m_B$ or $\Delta m_{B_s}$ and arises because the mass eigenstates are not the CP–eigenstates. The mixing-induced CP–violation in the decays
$B^0 (\Bar{B}^0) \rightarrow K_s J/\psi$ has been observed and sin $\beta$ involving CKM phase $\beta$ has been measured. BELLE and BaBar groups have observed both direct and mixing–induced CP–violation in $B^0 (\Bar{B}^0) \rightarrow \pi^+ \pi^-$ decays and the parameters characterising these decays will be experimentally determined more accurately shorty. However it must be pointed out that direct CP–violation involves strong interaction phases due to final state interactions. Thus it will not be easy to extract the weak phase $\gamma$ in the decays $B^0 (\Bar{B}^0) \rightarrow \pi^+ \pi^-$. In this respect the observation of mixing induced CP–violation in the decay channels $\Bar{B}_s^0 (B_s^0) \rightarrow K^- D^+_s (K^+ D^-_s)$ and $B^0_s (\Bar{B}_s^0) \rightarrow K^+ D^-_s (K^- D^+_s)$ which involve the weak phase $\gamma$ and mixing parameter $\Delta m_B$ will be of much interest. There is every indication that CP-violation in $K^0 - \Bar{K}^0$ and $B^0 - \Bar{B}^0$, $B_s^0 - \Bar{B}_s^0$ systems will be understood in terms of CKM matrix. There is now strong evidence from neutrino oscillations that in the lepton sector, weak eigenstates are also related to mass eigenstates by a unitary transformation. Since for neutrinos, it is experimentally established that there are three generations of neutrinos, the question of CP-violation in lepton sector is open one. Finally for baryogenesis, both $C$ and CP-violation are required. How the CP-violation in meson sector is related to CP-violation of baryogenesis? There is no answer to this question as yet.

It is a very selective list of references which have been consulted in the preparation of this article.

VI. FIGURE CAPTIONS

1. The CKM–Unitarity triangle
2. Box diagrams for $\Bar{B}^0 \rightarrow B^0$ transition
3. Tree graph for the decay $\Bar{B}^0 \rightarrow \pi^+ \pi^-$
4. Penguin graph for the decay $\Bar{B}^0 \rightarrow \pi^+ \pi^-$
5. Tree graph for $\Delta C = \pm 1$, $\Delta S = -1$ decays
6. Color suppressed graphs for $\Bar{B}^0 \rightarrow \Bar{K}^0 D^0$, $\Bar{B}^0 \rightarrow K^0 \Bar{D}^0$ decays
7. Rescattering graphs for the decays $\Bar{B}^0 \rightarrow K^- D^+ \rightarrow \Bar{K}^0 D^0$, $\Bar{B}^0 \rightarrow \pi^+ D^- \rightarrow K^0 \Bar{D}^0$ decays
Fig. 2
Fig. 5
Fig. 7