Accuracy analysis of three deterministic sampling nonlinear filtering algorithms

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Abstract. To study the estimation accuracy of nonlinear Kalman filter in transfer alignment system, the Taylor expansion of multivariable function is utilized to analyze the estimation accuracy of the three typical deterministic sampling nonlinear Kalman filters, such as high-degree cubature Kalman filter (HCKF), cubature Kalman filter (CKF) and unscented Kalman filter (UKF). The Taylor expansion analyses demonstrate that CKF and UKF produce truncation error since the fourth degree term, while HCKF can capture fifth degree term of Taylor series expansion. The three nonlinear filtering algorithms are applied to carrier-aircraft transfer alignment system under large misalignment angle. The simulation results show that CKF and UKF have the same accuracy, and HCKF has higher estimation accuracy than the other two, which is consistent with the theoretical analysis results.

1. Introduction
Kalman filter is the first method of carrier-aircraft transfer alignment in order to estimate the initial misalignments [1]. The position of carrier-born aircraft landing on the deck is arbitrary, while the carrier's master inertial navigation system (MINS) is installed in the navigation room below the deck. In other words, the initial misalignment angle of the salve inertial navigation system (SINS) may be notably large, which will increase the nonlinearity of the system. At this time, the linear Kalman filter is no longer applicable, and extended Kalman filter (EKF) is also difficult to guarantee performance of the system. With the rapid development of nonlinear filter based on Bayesian framework in recent years, deterministic sampling Kalman filter with higher accuracy than EKF is used to deal with transfer alignment problems under large misalignment angle [2, 3]. Deterministic sampling Kalman filter is a kind of filtering algorithm that uses finite fixed sampling points to approximate the moments of states in a Gaussian system and performs weighted summation estimation after nonlinear function propagation [4]. The three typical deterministic sampling algorithms are high-degree cubature Kalman filter [5], cubature Kalman filter [6] and unscented Kalman filter [7].

Accuracy is an important indicator to characterize the performance of filter. Previous studies on nonlinear filter accuracy mostly analyze single algorithm [8, 9], and there is a lack of systematic analysis and comparison between deterministic sampling algorithms with similar forms. Therefore, this paper will use Taylor series expansion to theoretically analyze the accuracy of three typical deterministic sampling algorithms and compare them horizontally. Finally, the deterministic sampling algorithms are applied to the transfer alignment system of carrier-aircraft under large misalignment angle, and the correctness of the theoretical analysis is verified by Monte-Carlo simulation.
2. Accuracy analysis of deterministic sampling filtering algorithm

The accuracy of a filter is the premise of ensuring system performance. This section will use the Taylor series expansion of multivariable function to analyze the accuracy of HCKF, CKF and UKF, in an effort to obtain an accurate and quantitative representation.

2.1 Taylor expansion of multivariate function

The multivariate function \( f(x) \) can be expanded into a Taylor series around mean \( \bar{x} = [\bar{x}_1, \bar{x}_2, \cdots, \bar{x}_n]^T \) as

\[
f(x) = f(\bar{x}) + \sum_{i=1}^{n} \frac{1}{i!} \left( \delta x_i \frac{\partial}{\partial x_i} + \cdots + \delta x_n \frac{\partial}{\partial x_n} \right)^{i} f(\bar{x})
\]  

(1)

Where \( \delta x_i = x_i - \bar{x}_i, i = 1, 2, \cdots, n \).

By defining \( \delta x = [\delta x_1, \cdots, \delta x_n]^T \), then the equation (1) can be written as

\[
\bar{f}(x) = f(\bar{x}) + E \left[ D_{\delta x} f + \frac{1}{2!} D_{\delta x}^2 f + \frac{1}{3!} D_{\delta x}^3 f + \cdots \right]
\]  

(2)

Where \( D_{\delta x} f = (\delta x \cdot \nabla)^T f(x) \) and \( \nabla \) represents the differential of \( f(x) \).

Considering the symmetry of \( \delta x \), all the odd degree moments equal to zero. Therefore, equation (2) can be rewritten as

\[
\bar{f}(x) = f(\bar{x}) + E \left[ \frac{1}{2!} D_{\delta x}^2 f + \cdots \right]
\]  

(3)

According to \( \delta x \) with zero-mean and covariance matrix \( P \) obeying a Gaussian distribution \( \delta x \sim N(0, P) \), and the definition of the covariance matrix \( E(\delta x \delta x^T) = P \), the second order term in the Taylor expansion is

\[
E \left[ \frac{D_{\delta x} f}{2!} \right] = E \left[ \frac{(\delta x \cdot \nabla)(\delta x \cdot \nabla) f(\bar{x})}{2!} \right] = \frac{(\nabla^T P \nabla) f(\bar{x})}{2!}
\]  

(4)

Define \( \delta \bar{x} \) obey the Gaussian distribution \( \delta \bar{x} \sim N(0, I) \) ( \( I \) represents the unity matrix) and \( \delta x = \sqrt{P} \delta \bar{x} \). Then with defining \( \sqrt{P} = [p_{ij}]_{n \times n}, i, j = 1, 2, \cdots, n \), \( D_{\delta x}^2 f \) can be expressed as

\[
D_{\delta x}^2 f = \left[ \sum_{i=1}^{n} \sum_{j=1}^{n} p_{ij} \frac{\partial}{\partial x_i} \right]^{\otimes 2k} f(\bar{x})
\]  

(5)

\[
= [\delta \bar{x}_1^{2k} \left(p_{11} \frac{\partial}{\partial x_1} + \cdots + p_{1n} \frac{\partial}{\partial x_n} \right)^{2k} + \cdots + \delta \bar{x}_n^{2k} \left(p_{n1} \frac{\partial}{\partial x_1} + \cdots + p_{nn} \frac{\partial}{\partial x_n} \right)^{2k}]] f(\bar{x}) + \zeta
\]

Where \( \zeta_i \) is a cross term which can be neglected, and \( \zeta_i (i = 1, 2, \cdots) \) are all cross terms in this paper.

The mean of \( D_{\delta x}^2 f, k = 2, 3 \cdots \) can be calculated as
\[
E\left( \mathbf{D}_{x}^{2k} \mathbf{f} \right) = \left[ E\left( \mathbf{\delta} x_{i}^{2k} \right) \left( p_{11} \frac{\partial}{\partial x_{1}} + p_{21} \frac{\partial}{\partial x_{2}} + \cdots + p_{n1} \frac{\partial}{\partial x_{n}} \right)^{2k} \right] \cdots \\
E\left( \mathbf{\delta} x_{i}^{2k} \right) \left( p_{11} \frac{\partial}{\partial x_{1}} + p_{21} \frac{\partial}{\partial x_{2}} + \cdots + p_{n1} \frac{\partial}{\partial x_{n}} \right)^{2k} \right] f(\mathbf{x}) + E(\mathbf{\zeta}_{1})
\]

The even degree moments of the multidimensional Gaussian distribution satisfy
\[
E\left( \mathbf{\delta} x_{i}^{2k} \right) = \cdots = E\left( \mathbf{\delta} x_{i}^{2k} \right) = 1 \times 3 \times \cdots (2k - 1) \tag{7}
\]

Substituting equations (4), (6) and (7) into equation (3), the mean of \( f(\mathbf{x}) \) is expressed as
\[
\tilde{f}(\mathbf{x}) = f(\overline{\mathbf{x}}) + \frac{(\nabla^T \mathbf{P} \nabla)}{2} f(\overline{\mathbf{x}}) + \sum_{k=2}^{2k} \left( \frac{1 \times 3 \times \cdots (2k-1)}{(2k)!} \right) \left( \sum_{i=1}^{n} \left( p_{1i} \frac{\partial}{\partial x_{1}} + \cdots + p_{ni} \frac{\partial}{\partial x_{n}} \right)^{2k} \right) f(\overline{\mathbf{x}}) \right] + \zeta_{2} \tag{8}
\]

2.2 Estimation accuracy of HCKF

For the Gaussian distribution \( \mathbf{x} \sim N(\overline{\mathbf{x}}, \mathbf{P}) \), the high-degree (5th degree) spherical-radial cubature rule can be expressed as
\[
\int_{x_{i}} f(\mathbf{x}) N(\mathbf{x}, \mathbf{P}) d\mathbf{x} = \frac{2}{n+2} f(\overline{\mathbf{x}}) + \frac{4-n}{2(n+2)} \sum_{i=1}^{n} \left[ f(\beta \sqrt{\mathbf{s}_{i}^{+} + \mathbf{x}}) + f(-\beta \sqrt{\mathbf{s}_{i}^{+} + \mathbf{x}}) \right] + \frac{1}{(n+2)^{2}} \sum_{i=1}^{n(n+1)/2} \left[ f(\beta \sqrt{\mathbf{s}_{i}^{+} + \mathbf{x}}) + f(-\beta \sqrt{\mathbf{s}_{i}^{+} + \mathbf{x}}) \right] \tag{9}
\]

\[
\left\{ s_{i}^{+} \right\} \triangleq \left\{ \sqrt{1/2} (\mathbf{e}_{k} + \mathbf{e}_{l}) : k < l, k, l = 1, 2, \cdots n \right\}
\]
\[
\left\{ s_{i}^{-} \right\} \triangleq \left\{ \sqrt{1/2} (\mathbf{e}_{k} - \mathbf{e}_{l}) : k < l, k, l = 1, 2, \cdots n \right\} \tag{10}
\]

Where \( \mathbf{e}_{i} \) represents a unit vector in \( \mathbb{R}^{n} \) with the i-th element being 1, and the other elements are zero, the parameter \( \beta = \sqrt{n+2} \), \( \sqrt{\mathbf{P}} \) represents the lower triangle of \( \mathbf{P} \) through Cholesky decomposition, viz. \( \mathbf{P} = \sqrt{\mathbf{P}} \left( \sqrt{\mathbf{P}} \right)^{T} \).

The Taylor expansion around \( \overline{\mathbf{x}} \) of each cubature point through nonlinear transformation is given as follows:
\[
f_{HCKF}(\mathbf{x}_{i}) = f(\overline{\mathbf{x}}) + D_{\overline{\mathbf{x}}_{i}} \mathbf{f} + \frac{D_{\overline{\mathbf{x}}_{i}}^{2} \mathbf{f}}{2!} + \frac{D_{\overline{\mathbf{x}}_{i}}^{3} \mathbf{f}}{3!} + \cdots \quad i = 0, 2, \cdots, 2n^{2} \tag{11}
\]

\[
\delta_{\overline{\mathbf{x}}_{i}} = \begin{cases} 
[0,0,\cdots,0]^T & i = 0 \\
\beta(\sqrt{\mathbf{P}})_{i} & i = 1, 2, \cdots, n \\
-\beta(\sqrt{\mathbf{P}})_{n+i} & i = n+1, \cdots, 2n \\
\beta\sqrt{\mathbf{P}} s_{i-2n}^{+} & i = 2n+1, \cdots, n(n+3)/2 \\
-\beta\sqrt{\mathbf{P}} s_{i-2n(n+1)/2}^{+} & i = n(n+3)/2 + 1, \cdots, n(n+1) \\
\beta\sqrt{\mathbf{P}} s_{i-2n(n+1)/2}^{-} & i = n(n+1) + 1, \cdots, 3n(n+1)/2 \\
-\beta\sqrt{\mathbf{P}} s_{i-3n(n+1)/2}^{-} & i = n(3n+1)/2 + 1, \cdots, 2n^{2} 
\end{cases} \tag{12}
\]

Where \( n \) is the dimension of state vector, and \( \left( \sqrt{\mathbf{P}} \right)_{i} \) is the i-th column of \( \sqrt{\mathbf{P}} \).
According to equation (9), and considering that \(2n^2 + 1\) cubature points of the HCKF algorithm are symmetric, then all the odd degree moments sum up to zero. Therefore, the mean of \(f(x)\) is calculated as

\[
\overline{f}_{HCKF}(x) = \frac{2}{n+2} f(\overline{x}) + \sum_{k=1}^{2n^2} w_i \left[ f(\overline{x}) + \frac{D^2_{\delta x_k} f}{2!} + \cdots + \frac{D^{2k}_{\delta x_k} f}{(2k)!} \right]
\]

\[
= f(\overline{x}) + \frac{4-n}{2(n+2)^2} \sum_{i=1}^{2n} \left[ D_{\delta x_i} f \right] + \frac{1}{(n+2)^2} \sum_{i=2n+1}^{2n^2} \left[ D_{\delta x_i} f \right]
\]

The second moment calculation process is as follows:

\[
\sum_{i=1}^{2n} D^2_{\delta x_i} f = \frac{4-n}{2(n+2)^2} \left\{ \sum_{i=1}^{n} \left[ \nabla^T (n+2)(\sqrt{P})_{(i)} (\sqrt{P})_{(i)}^T \nabla \right] f(\overline{x}) \right\}
\]

\[
+ \sum_{i=n+1}^{2n} \frac{1}{2} \left[ \nabla^T (n+2)(\sqrt{P})_{(i)} (\sqrt{P})_{(i)}^T \nabla \right] f(\overline{x})
\]

\[
+ \frac{1}{(n+2)^2} \left\{ \sum_{i=1}^{n} \frac{1}{2} \left[ \nabla^T (n+2) \left[ (\sqrt{P})_{(i)} + (\sqrt{P})_{(i)}^T \right] V \right] f(\overline{x}) \right\}
\]

\[
+ \sum_{i=n+1}^{2n} \frac{1}{2} \left[ \nabla^T (n+2) \left[ (\sqrt{P})_{(i)} - (\sqrt{P})_{(i)}^T \right] V \right] f(\overline{x})
\]

\[
= \frac{1}{2} [\nabla^T P \nabla] f(\overline{x})
\]

Consider the other even degree Taylor expansion terms below, when \(i = 1, 2, \cdots, 2n\)

\[
\sum_{i=1}^{2n} w_i \left[ \frac{1}{(2k)!} D^{2k}_{\delta x_i} f \right] = \frac{2}{(2k)!} \frac{4-n}{2(n+2)^2} \sum_{i=1}^{n} D^{2k}_{\delta x_i} f
\]

\[
= \frac{1}{(2k)!} (4-n)(n+2)^{k-2} \times \sum_{i=1}^{n} \left[ p_{ii} \frac{\partial}{\partial x_1} \cdots + p_{ii} \frac{\partial}{\partial x_n} \right] f(\overline{x}) + \sum_{i=2n+1}^{2n^2} w_i \zeta
\]

And for \(i = 2n + 1, 2n+2, \cdots, 2n^2\) have

\[
\sum_{i=2n+1}^{2n^2} w_i \left[ \frac{1}{(2k)!} D^{2k}_{\delta x_i} f \right] = \frac{1}{(2k)!} (n+2)^{k-2}(n-1) \times \sum_{i=1}^{n} \left[ p_{ii} \frac{\partial}{\partial x_1} \cdots + p_{ii} \frac{\partial}{\partial x_n} \right] f(\overline{x}) + \sum_{i=2n+1}^{2n^2} w_i \zeta
\]

Substituting equations (14), (15) and (16) into equation (13), \(\overline{f}_{HCKF}(x)\) can be rewritten as

\[
\overline{f}_{HCKF}(x) = f(\overline{x}) + \frac{3}{2} [\nabla^T P \nabla] f(\overline{x}) + \sum_{k=2}^{n} \left[ \frac{3}{(2k)!} \sum_{i=1}^{n} \left[ p_{ii} \frac{\partial}{\partial x_1} \cdots + p_{ii} \frac{\partial}{\partial x_n} \right] f(\overline{x}) \right] + \zeta,
\]

Where \(\zeta = \sum_{i=1}^{2n^2} w_i \zeta\).

### 2.3 Estimation accuracy of CKF and UKF

Since the derivation process is similar to the accuracy analysis of HCKF, the Taylor expansion mean of the CKF around \(\overline{x}\) is directly given by [9]
\[ f_{\text{CKF}}(x) = f(x) + \frac{1}{2} \|V^TPV\| f(x) + \sum_{k=2}^{n} \left( \frac{(n+k-1)!}{(2k)!} \right) \sum_{i=1}^{n} \left( p_{i1} \frac{\partial}{\partial x_1} + \cdots + p_{in} \frac{\partial}{\partial x_n} \right)^{2k} f(x) \right] + \zeta_s \quad (18) \]

The Taylor expansion mean of the UKF around \( \bar{x} \) is given by [8]

\[ f_{\text{UKF}}(x) = f(x) + \frac{1}{2} \|V^TPV\| f(x) + \sum_{k=2}^{n} \left( \frac{(n+k-1)!}{(2k)!} \right) \sum_{i=1}^{n} \left( p_{i1} \frac{\partial}{\partial x_1} + \cdots + p_{in} \frac{\partial}{\partial x_n} \right)^{2k} f(x) \right] + \zeta_s \quad (19) \]

2.4 **Comparison of estimation accuracy of three deterministic sampling filtering algorithms**

Comparing equations (17), (18) and (19) with the Taylor expansion of multivariate function equation (8) we can see that the high-degree (5th-degree) cubature Kalman filter can accurately propagate to the 5th-degree term, and the truncation error starts from the 6th-degree term. That is, there is a 5th-degree Taylor expansion accuracy. However, CKF and UKF can only accurately propagate to the 3rd-degree term, and the truncation error starts from the 4th-degree term, that is, there is a 3rd-degree Taylor expansion accuracy.

3. The model of rapid transfer alignment

In this section, the nonlinear state transition and measurement model of the rapid transfer alignment (RTA) will be described.

3.1 **The nonlinear state transition model of RTA**

The state vector of a RTA Kalman filter is

\[ X = \begin{bmatrix} \psi_{mz} & \psi_{my} & \psi_{mx} & \delta V_n & \delta V_y & \delta V_x & \epsilon_x & \epsilon_y & \epsilon_z & \psi_{ay} & \psi_{az} & \psi_{ax} \end{bmatrix}^T \quad (20) \]

Where \( \psi_{mz}, \psi_{my}, \psi_{mx} \) are the attitude errors expressed as \( \psi_{m} \), \( \delta V_n, \delta V_y, \delta V_x \) are velocity errors expressed as \( \delta V \) in NUE navigation frame, \( \epsilon_x, \epsilon_y, \epsilon_z \) are gyroscope biases in body frame expressed as \( \epsilon \), and \( \psi_{ax}, \psi_{ay}, \psi_{az} \) are the installation deviation angles expressed as \( \psi_a \).

The transition functions of RTA are given by [10, 11]

\[
\begin{align*}
\delta \dot{V} &= C_b^n \left( I - C_a^n C_b^n \right) \dot{f}_b^n - \left( 2 \Omega_b^n + \Omega_a^n \right) \times \delta V \\
\psi_m &= \left( I - C_a^n C_b^n \right) \delta \omega_{bn} + \epsilon \\
\psi_a &= 0 \quad \epsilon = 0
\end{align*}
\]

Where \( C_b^n \) is the direction cosine matrix from the body frame to the navigation frame, \( \dot{f}_b^n \) represents the specific forces in body frame, and \( \Omega_a^n \) is the turn rate of the Earth with respect to inertial frame expressed in navigation frame. \( \Omega_{bn} \) represents the turn rate expressed in navigation frame with respect to Earth frame expressed in navigation frame, and \( \delta \omega_{bn} \) represents the turn rate of body frame with respect to navigation frame expressed in body frame. \( C_b^a \) and \( C_a^b \) are related to \( \psi_a \) and \( \psi_m \) respectively given as follows

\[
C_a^b = \begin{bmatrix} 
\cos \psi_{az} \cos \psi_{ay} & -\sin \psi_{az} \cos \psi_{ay} & \cos \psi_{az} \sin \psi_{ay} + \sin \psi_{az} \cos \psi_{ay} & \sin \psi_{az} \cos \psi_{az} \sin \psi_{ay} + \sin \psi_{az} \cos \psi_{az} \\
\sin \psi_{az} & -\cos \psi_{az} & \sin \psi_{az} \cos \psi_{az} & \sin \psi_{az} \sin \psi_{az} + \cos \psi_{az} \cos \psi_{az} \\
-\cos \psi_{az} \sin \psi_{ay} & \sin \psi_{az} \sin \psi_{ay} \cos \psi_{az} + \cos \psi_{az} \sin \psi_{ay} & -\sin \psi_{az} \sin \psi_{az} \sin \psi_{ay} + \cos \psi_{az} \cos \psi_{az} \\
-\sin \psi_{az} \sin \psi_{az} \sin \psi_{ay} & \cos \psi_{az} \sin \psi_{az} \sin \psi_{ay} & -\sin \psi_{az} \sin \psi_{az} \cos \psi_{az} + \cos \psi_{az} \cos \psi_{az} 
\end{bmatrix} \quad (22)
\]
\[ C^s = \begin{bmatrix} \cos \psi_{ax} \cos \psi_{ay} & \sin \psi_{ax} & -\cos \psi_{ax} \sin \psi_{ay} \\ -\sin \psi_{ax} \cos \psi_{ay} \cos \phi_{ax} + \sin \psi_{ax} \sin \phi_{ax} & \cos \psi_{ax} \cos \phi_{ax} - \sin \psi_{ax} \sin \phi_{ax} & \sin \psi_{ax} \sin \phi_{ax} \sin \psi_{ay} + \cos \psi_{ax} \cos \phi_{ax} \cos \psi_{ay} \\ \sin \psi_{ax} \sin \psi_{ay} \sin \phi_{ax} + \sin \psi_{ax} \cos \phi_{ax} & \cos \psi_{ax} \sin \phi_{ax} & -\sin \psi_{ax} \sin \phi_{ax} \sin \psi_{ay} - \cos \psi_{ax} \cos \phi_{ax} \cos \psi_{ay} \end{bmatrix} \] (23)

### 3.2 The measure model of RTA

The RTA adopts the ‘attitude plus velocity’ as the matching scheme. Thus, the measurable misalignment angles and velocity errors are chosen as the measurement data, and the resulting measurement vector is given by:

\[ Z = \begin{bmatrix} \delta V^s \\ \varphi' \end{bmatrix} = \begin{bmatrix} V^s - V^m \\ g(C_z) \end{bmatrix} \] (24)

Where \( V^s \) and \( V^m \) represent the velocity of SINS and MINS respectively, \( C_z \) and \( g(C_z) \) are defined as follows

\[ C_z = C^s \begin{bmatrix} C^s \\ \end{bmatrix} \quad g(C_z) = \begin{bmatrix} \arctan(-C_z(3,2)/C_z(2,2)) \\ \arctan(-C_z(1,3)/C_z(1,1)) \\ \arcsin(C_z(1,2)) \end{bmatrix} \] (25)

### 4. Mathematical Simulation

#### 4.1 Simulation conditions

In order to verify the correctness of the accuracy analysis of HCKF, CKF and UKF algorithms, they are applied to the transfer alignment of the carrier-aircraft under large misalignment angle respectively. The transfer alignment is carried out in calm sea condition \( \gamma = 1' \), \( \varphi = 1' \), \( \theta = 1' \). The gyroscope biases and the accelerometer biases of MINS are 0.0012(°)/h (1σ) and 0.02mg (1σ) respectively. The gyroscope biases and the accelerometer biases of SINS are 0.01(°)/h (1σ) and 0.1mg (1σ) respectively. The horizontal installation deviation angle ranges from -3° to 3°, the heading installation deviation angle value ranges from -30° to 30°, and 200 simulations run with Monte-Carlo method.

#### 4.2 Simulation result

The estimation error curves of the three installation deviation angles based on the three kinds of nonlinear filtering algorithms are shown in Figure 1, 2 and 3, respectively. In addition, Table 1 gives the statistical results of 200 Monte-Carlo runs and shows the relationship between the accuracy of HCKF, CKF and UKF. From the simulation results, we can see that the CKF and UKF estimation
accuracy is equivalent, and the HCKF estimation accuracy is the highest, which is consistent with the accuracy analysis results based on the Taylor series expansion of multivariate function.

![Figure 3](image)

**Figure 3.** Estimation error of installation deviation angle of Z axis

| The error of transfer alignment | HCKF | CKF | UKF |
|-------------------------------|------|-----|-----|
| The error of $\psi_x$ estimation, (°) | 0.79 | 0.87 | 0.88 |
| The error of $\psi_y$ estimation, (°) | 2.95 | 3.55 | 3.53 |
| The error of $\psi_z$ estimation, (°) | 0.78 | 0.86 | 0.87 |

**5. Summary**

In this paper, aiming at the estimation accuracy of three deterministic sampling nonlinear filter, the Taylor series expansion of multivariable function is used to compare and analyze the estimation accuracy of HCKF, CKF and UKF. Theoretical analysis shows that CKF and UKF have a third-degree Taylor expansion accuracy, while HCKF has a fifth-degree Taylor expansion accuracy. The three algorithms are applied to the transfer alignment system of the carrier-aircraft under large misalignment angle. Monte-Carlo simulation results verify the correctness of the accuracy analysis results based on the Taylor series expansion of multivariate function.

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