A Four-point three-dimensional spatial localization algorithm based on RSSI

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Abstract. The three-dimensional positioning of nodes is a major and basic problem in wireless sensor networks. It has an important application value in areas such as search and rescue, target tracking, disaster reduction and intelligent environment. This paper presents a three-dimensional four-point centroid location algorithm based on RSSI. Based on signal strength between unknown nodes and known nodes, a received signal strength indicator (RSSI) propagation model in shadow mode is established. The Gaussian model is employed to RSSI. The signal strength is rectified to obtain a more accurate ranging model. In a three-dimensional space, a reasonable detection point is selected, and the centroid iteration algorithm is used twice. The weighted centroid algorithm is utilized for the first time and the average centroid algorithm is used for the second time to find the target. The coordinates of the nodes realizes the three-dimensional spatial positioning of the unknown nodes. Through simulation experiments, the positioning algorithm has improved accuracy and stability compared with the least square method and weighted centroid algorithm.

1. Introduction

In many applications of wireless sensor network technology, the target localization takes a significant position. Therefore, wireless localization technology plays a critical role in military, smart home, public safety and other fields [1]. According to the different localization methods, the wireless localization technology is divided into two methods based on ranging and no distance measurement [1, 2]. The method based on ranging mainly uses the positioning algorithm, such as trilateration method, triangulation method and the maximum likelihood method, to estimate the position of unknown nodes by measuring the distance or the angle information between nodes [3]. The method mainly uses spatial geometric relations or network multi-hop routing to complete positioning, such as DV-hop using network multi-hop routing, centroid algorithm using spatial geometric relationship, convex programming and MDS-MAP.

In the ranging-based localization algorithm [4], the ranging techniques include time of arrival (TOA) technology, time difference of arrival (TDOA) technology, and angle of arrival (AOA) technology, RSSI technology, etc. [5] The hardware without the need for ranging algorithm has low hardware requirements with are commonly used simple calculation, resulting in the low positioning accuracy. The most widely used algorithm is built on the signal strength indication RSSI. The RSSI ranging is a
distance that can be obtained by the signals transmitting between a wireless signal transmitting node and an accepting node, which are simple and easy to operate, and has low cost. And it is appropriate for indoor short-range wireless positioning. Therefore, our design mainly realizes the positioning method of the wireless sensor network using RSSI.

In the research of RSSI-based four-point localization algorithm, Feng Dongdong et al. [5] and Wang Fei et al. [6] use the four-point localization algorithm, but the algorithm is applied to the two-dimensional plane. In the literature [7], the six-node COLA three-dimensional positioning algorithm is mentioned. Six-node information is associated with the operation to improve the positioning accuracy. However, the mutual influence of the signals between the six nodes makes the positioning accuracy deteriorate, and the calculation amount increases sharply. Based on the above situation, in order to reduce the complexity of the algorithm and satisfy the requirement of three-dimensional spatial positioning, this paper proposes a three-node spatial positioning algorithm based on four-node RSSI which arranges four nodes with relative positions in the three-dimensional space and then uses the four known node signal strength to obtain the distance \( d \) from the unknown node. The main point of views of the proposed algorithm is as follows: Assuming that \( P_1, P_2, P_3 \) and \( P_4 \) are four spherical centers, the distance \( d \) from the unknown node to a known node calculated by formula (6) is taken as the radius of a new sphere, and then the four spherical centers are combined into four triangles, each of which has a center of mass, and the coordinates of each center of mass are obtained respectively. Then, the four centers of mass are related to form a vertex. Finally, the weighting algorithm is utilized to calculate the weighting factors to get the coordinates of the target nodes, thus, the location of unknown nodes can be obtained.

The rest of the paper is organized as follows. In Section 1, the RSSI ranging model is introduced. In Section 2, three-dimensional weighted centroid localization algorithm (WCLA) are introduced. Section 3 consists of two parts: Influence of noise on positioning error and Influence of path attenuation index. Finally, concluding remarks are made in Section 4.

2. related work

2.1. RSSI ranging model

In practical application environment, signal transmission in the wireless channel is often affected by various factors such as noise, path loss and so on. Doppler effect and instability will also occur when the transmitter and receiver move. According to the law of wireless transmission, the attenuation of the wireless signal will occur in the process of transmission. The attenuation degree is related to distance. RSSI ranging is based on this principle [1–4].

In the case of known signal strength transmitted by the transmitting node, the distance of signal propagation is calculated according to the signal strength received by the receiving node. In the propagation process, the physical model and the empirical model are used to convert the loss in the signal propagation process into a distance, and then the positioning algorithm is used to calculate the unknown node position. The function of signal propagation loss and distance can be expressed by the following equation, expressed as follows

\[
RSSI(d) = -10\gamma \log 10d + A
\]  

Where: \( \gamma \) represents the signal propagation index, \( A \) is the signal intensity collected at the distance of 1 m between the node and the transmitting node, and \( d \) is the distance between the node and the transmitting node. \( A \) has a great relationship with the RF circuit and the wireless signal transmission environment, and \( \gamma \) depends on the location of the positioning node. Therefore, it is difficult to obtain the exact values of \( A \) and \( \gamma \) in real time. This algorithm combines the values of \( \gamma \) and environmental impact factors in general engineering. Studies have shown that the path loss factors in different environments are different, as shown in Table 1. The value of \( A \) is generally the intensity of the received signal at a distance of 1 m, and the optimum range value of the experimental A value is from 45 to 49. According to Table 1, the value of \( \gamma \) is usually from 1.4 to 2.0 indoors.
Table 1  Average path loss index of different buildings \[^{[3]}\]

| Building                  | average path loss index $\gamma$ | building             | average path loss index $\gamma$ |
|---------------------------|----------------------------------|----------------------|----------------------------------|
| Office, hard segmentation | 1.9                              | stone wall           | 3.5                              |
| Apartment corridor        | 2.0                              | fence surrounded by  | 4.9                              |
|                           |                                   | plants               |                                  |
| Stairs, balcony           | 1.9                              | grass                | 3.6                              |
| Big yard                  | 3.2                              | park                 | 3.0                              |
| Alley                     | 2.7                              | seaside beach        | 4.2                              |

2.2. RSSI Measurement Value Correction

Considering that in practical application, the signal strength can be calculated only by formula. Due to the interference of other electromagnetic signals in the air and the hardware itself, the measured signal strength will have a large error. Therefore, this paper uses the Gaussian distribution model to correct the RSSI measurement value $Z \sim N (0, \sigma^2)$, and $Z$ obeys the Gaussian distribution with mean and variance. In fact, the actual position is related to the density of the RSSI values received in $m$ times, where the higher the probability density is the closer it will meet the actual position. Therefore, the $m$ signal strength values received by the unknown node at the same location are placed in the corresponding array $R$, and then the array data are processed by the Gaussian distribution function, and the probability density function of the RSSI value is shown in formula (2).

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left(-\frac{(x - \overline{RSSI})^2}{2\sigma^2}\right)$$  \(2\)

where: $\overline{RSSI}$ is the average of $n$ RSSI measurements, it is shown in formula (3).

$$\overline{RSSI} = \frac{1}{n} \sum_{i=1}^{m} RSSI_i$$  \(3\)

$x_i$ is the $i$-th received measurement.

$$\sigma^2 = \frac{1}{n-1} \sum_{i=1}^{m} (RSSI_i - \overline{RSSI})^2$$  \(4\)

And, $\alpha$ is set as the threshold, then the RSSI measurement distribution is valid in the region where the probability density is greater than or equal to $\alpha$, and then all the effective values are averaged, then formula (2) needs to be satisfied formula (5).

$$\alpha \leq \frac{1}{\sigma \sqrt{2\pi}} \exp \left(-\frac{(x - \overline{RSSI})^2}{2\sigma^2}\right) \leq f(x) \leq 1$$  \(5\)

After the RSSI and the variance $\sigma^2$ are obtained by (3) and (4), the range of the RSSI value can be determined by (2) and (5), and then all the eligible values are averaged to obtain the RSSI signal optimization Value $\overline{RSSI}$, where $\alpha$ is 0.6 to 0.7. After the above optimization, using (1), we can get the distance calculation formula as

$$d_i = 10 \alpha - \overline{RSSI} d_i = 10^{-\frac{\Delta - \overline{RSSI}}{10\gamma}}$$  \(6\)

3. Improved localization algorithm

In this paper, the four-point-three-dimensional spatial localization algorithm (FPTDSLA) proposed is based on the two-dimensional weighted centroid localization algorithm.
3.1. Two-dimensional weighted centroid localization algorithm

The weighted centroid localization algorithm is based on the centroid localization method [9]. The weighting factor is used to reflect the weight of the reference node's decision on the centroid coordinate. It is used to reflect the influence of each reference node on the centroid position, which implies the inner relationship of the difference between them.

The two-dimensional weighted centroid localization algorithm based on the distance from the three known reference nodes to the unknown node, and three circles are utilized to obtain the position of the unknown node. But in reality, the three circles are often not exactly intersected at one point, and the fact is that they mostly generate an area. Therefore, we select three approximated points \( m_1, m_2, \) and \( m_3 \) in this region, and make them form a triangle, and then obtain the position of the unknown node in the weighted centroid approximation of the triangle.

Assume that the nodes \( A, B, \) and \( C \) are known to have coordinates \( A(x_1, y_1), B(x_2, y_2), C(x_3, y_3), \) and their radii are \( r_1, r_2, r_3, \) respectively. Firstly, the coordinates of the three vertices in the common area of the three circles are calculated, \( M_1, M_2 \) and \( M_3 \) are shown in figure 1. The calculation method of the coordinates of the point \( M_1(x_{m1}, y_{m1}) \) is as shown in the formula (7).

\[
\begin{align*}
(x - x_2)^2 + (y - y_2)^2 &= r_2^2 \\
(x - x_3)^2 + (y - y_3)^2 &= r_3^2 \\
(x - x_1)^2 + (y - y_1)^2 &\leq r_1^2
\end{align*}
\]  

(7)

Similarly, the coordinates of \( M_2(x_{m2}, y_{m2}) \) and \( M_3(x_{m3}, y_{m3}) \) can be obtained, and then the approximate position of unknown node \( m(x_m, y_m) \) can be calculated according to the algorithm formula of weighted centroid (8).

![Figure 1 Diagram of three-point localization algorithm](image)

In the weighted centroid algorithm, the weighting factors \( \frac{1}{d_1 + d_2}, \frac{1}{d_2 + d_3}, \frac{1}{d_3 + d_1} \), are used to represent the size of the reference node's decision on the centroid coordinate. Weighting factors are used to describe the influence of known nodes on the location of unknown node, and the intrinsic relationship between them. The two-dimensional weighted centroid coordinates are expressed as shown in equations (8) and (9)

\[
x_m = \frac{x_{m1}}{d_1 + d_2} + \frac{x_{m2}}{d_2 + d_3} + \frac{x_{m3}}{d_3 + d_1}
\]  

(8)
3.2. Generalized to three-dimensional weighted centroid localization algorithm

In this paper, the two-dimensional weighted centroid localization algorithm is improved and extended to three-dimensional. A three-node spatial algorithm based on four-node RSSI is proposed step-by-step.

Suppose there are n known nodes and one unknown node are placed in the space. The position distribution is shown by the black dot in Figure 2. Suppose that the coordinates of the unknown node are \( M(x, y, z) \). The fixed coordinates of the relative positions of n nodes are defined as \( P_i = (x_i, y_i, z_i) \), and one of the nodes is selected as the origin, such as \( P_1 = (x_1, y_1, z_1) \) is the origin of coordinates, the coordinates of \( P_2, P_3, ... P_n \) are: \( P_2 = (x_2, y_2, z_2), P_3 = (x_3, y_3, z_3), ... P_n = (x_n, y_n, z_n) \).

The first step, according to the equation (6), the distance from the unknown node \( M \) to n known nodes are calculated, it is \( d_i \) \( \{d_1, d_2, ... d_n\} \).

The second step, the four nodes with the shortest distance between the known node and the unknown nodes are selected from \( d_i \) \( \{d_1, d_2, ... d_n\} \). It is determined that the four nodes are not on a straight line, and the four nodes are denoted as \( P_1, P_2, P_3, \) and \( P_4 \).

The third step, suppose that \( P_1, P_2, P_3, \) and \( P_4 \) are four spheres respectively, and the distance \( d_i \) from the unknown node to the known node is used as the radius. There are five relationships between the four spheres: intersection, external cutting, incision, inclusion, and separation(3). Then, the four spheres are combined into three triangles, and the four triangles have four centroids, and then the four centroids are connected as vertices to form a tetrahedron or a quadrilateral, as shown in Figure 2.

\[
y_m = \frac{y_{m1}}{d_1} + \frac{y_{m2}}{d_2} + \frac{y_{m3}}{d_3} + \frac{y_{m4}}{d_4}
\]

The four centroids are connected as vertices to form a tetrahedron or a quadrilateral, as shown in Figure 2.

\[
x_{m1} = \frac{x_1 + x_2 + x_3}{3} \tag{10}
\]
\[
y_{m1} = \frac{y_1 + y_2 + y_3}{3} \tag{11}
\]
\[ z_{ml} = \frac{z_1 + z_2 + z_3}{3} \]  

The fourth step, according to formula (1), the RSSI value received by the unknown node can be converted into a distance value, and the distance can be used as the weighting factor \( w_i = 1/d_i \), so that the coordinates of the unknown node can be obtained as formula (13), (14), (15).

\[
x = \frac{x_{m1}}{d_1 + d_2 + d_3} + \frac{x_{m2}}{d_1 + d_2 + d_4} + \frac{x_{m3}}{d_1 + d_3 + d_4} + \frac{x_{m4}}{d_2 + d_3 + d_4}
\]

\[
y = \frac{y_{m1}}{d_1 + d_2 + d_3} + \frac{y_{m2}}{d_1 + d_2 + d_4} + \frac{y_{m3}}{d_1 + d_3 + d_4} + \frac{y_{m4}}{d_2 + d_3 + d_4}
\]

\[
z = \frac{z_{m1}}{d_1 + d_2 + d_3} + \frac{z_{m2}}{d_1 + d_2 + d_4} + \frac{z_{m3}}{d_1 + d_3 + d_4} + \frac{z_{m4}}{d_2 + d_3 + d_4}
\]

4. Simulation Analysis

Based on the method of measuring the distance between each known node and unknown nodes based on RSS, this paper proposes a four-point three-dimensional spatial localization algorithm. According to the calculation method of the above design, the simulation of the algorithm was carried out by MATLAB software, and the performance was analyzed. Preset four known nodes in the three-dimensional space at \((0,0,0)\), \((0,0,3)\), \((30,0,0)\), \((30,0,3)\), the unknown node position coordinates are set at \((15, 20, 2)\) point. At the same time, \(A=45dB\) is set in advance, assuming that the RSS noise variance between the unknown node and the four known nodes is \(\delta^2\). The location error of the unknown node is analyzed by root mean square error (RMSE), as shown in equation (16). The RMSE positioning error simulation of this algorithm was run 3,000 times, and the average value of 3000 running results was used as formula (16)

\[ \text{Error} = \sqrt{(x - x_n)^2 + (y - y_n)^2 + (z - z_n)^2} \]  

Where: \(x, y, z\) are calculated values, and \(x_n, y_n, z_n\) are true values.

4.1. Influence of noise on RMSE

The RMSE of the FPTDSL, LSM and WCLA is compared and tested by simulation. The RSS measurement noise \(\delta^2\) is set to vary from 0.2dB to 16dB, Figure3 plots the RMSE of least squares method( LSM) with the RSS measurement noise, the RMSE of LSM increases from 36.10 to 59.53; The RMSE of WCLA increases from 1.36 to 22.39, and the RMSE of FPTDSL increases from 0.57 to 5.74, as shown in Figure 4. It can be observed that as the RSS measurement noise increases, the RMSE also increases. The three methods of WCLA, LSM and FPTDSL are compared, the RMSE of the LSM is larger than the WCLA, and the RMSE of the FPTDSL is smaller than the WCLA.
4.2. Influence of path attenuation index

The measurement noise $\delta^2$ is set equal to 4, and the path attenuation index $\gamma$ value varies from 0.1 to 5.0. Figure 5 plots the influence of the path attenuation index on the RMSE of the LSM, and Figure 6 shows the influence of path attenuation index on the RMSE of WCLA and FPTDSLA. As shown in Figure 5 and Figure 6, the RMSE declines with the increase of path attenuation index gamma. As the path attenuation index $\gamma$ increases, the RMSE gradually increases. When the path attenuation index $\gamma$ is equal to 2, the RMSE of LSM, the WCLA and the above FPTDSLA are 38.74m, 6.29m, and 5.29m, respectively. The three methods of WCLA, LSM and FPTDSLA are compared, the change of path attenuation index $\gamma$ has the greatest impact on the RMSE of the LSM, while the impact on the RMSE of the FPTDSLA is smaller.
5. Conclusion

In this paper, it is presented that a four-node three-dimensional localization algorithm based on RSSI. The RSSI ranging model is modified through Gauss filtering. By combining the position coordinates of four known reference nodes, the average centroid algorithm is used first, and then the weighted centroid algorithm is utilized to realize the three-dimensional coordinates of unknown node positioning. The influence of noise and path loss factor on RMSE is analyzed, and compared with the least square method and the weighted centroid algorithm. The simulation results are to be submitted. It is obvious that the four-node three-dimensional space location algorithm based on RSSI is feasible, with high accuracy and universal applicability.
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