Harmonic Analysis of Ionospheric Total Electron Content (TEC) Using Kalman Filter

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Abstract. The ionospheric diurnal variation from TEC Global Ionospheric Maps (GIM) data at the epicenter coordinates of the Lombok earthquake on July 29 2018 (-8.4°, 116.5°) has been analyzed using harmonic function which is the sum of the sine and cosine functions with estimated harmonic coefficients obtained using Kalman Filter (KF) method. The initial value of the estimated KF method was obtained from Least Square Estimation (LSE) on the first day data. The estimated TEC value by the KF method shows that there are seasonal variations of the peak values of diurnal variations which is a result of variation in the zenith angle of the sun that change over a year with peaks in March and November in the equator. The results of the first-order harmonic amplitude analysis, namely the diurnal TEC variation (A24) which is the root value of the sum of the squared values of the A1 and B1 coefficients were further analyzed by estimating polynomials with order 17 to eliminate the seasonal effect on A24. The difference in value of A24 from its estimation with order 17 polynomial (delta A24) shows the influence of geomagnetic and seismic activity. A24 analysis based on solar radiation flux at wavelength 10.7 cm (F10.7) does not indicate a strong influence or relationship. Equatorial geomagnetic disturbance index (etc.) shows the influence of geomagnetic storms on diurnal amplitude and earthquake data around Lombok indicating symptoms of pre-earthquake that affects the ionospheric TEC through a decrease in diurnal amplitude a few days before the earthquake.

1. Introduction
Ionosphere is the region of the Earth’s upper atmosphere extending from altitudes of approximately 60 km from the surface of Earth to the altitudes beyond 1000 km, where the sufficient amount of charged particles exists. In other words, this is the region of a shell of electrons and electrically charged particles (atoms and molecules), also known as plasma that surrounds the Earth. In plasma, the electrostatic force attracts the negative free electrons and the positive ions to each other, but they are too energetic to stay fixed together in an electrically neutral molecule. The plasma concentration may amount to only about 1% of the neutral concentration and the total ionosphere represents only less than 0.1% of the total mass of the Earth's atmosphere [1].
A theory of the ionization of the upper atmosphere of the earth by the ultra-violet light of the sun is developed based on known laws of pressures and constitution of the high atmosphere, ionic recombination, attachment of free electrons to neutral molecules, and diffusion of ions. It is concluded that the solar ultra-violet light is a necessary and sufficient cause of the Kennelly-Heaviside layer, and that hypotheses of other agencies of ionization, such as charged particles from the sun, penetrating radiation, etc., are uncalled for except perhaps in unusual cases [2].

Because the Sun shines over the equator, the ionospheric density was expected to vary from a maximum at the equator to a minimum at high latitudes. But when the density variation was measured [3]. In [4] it was found to exhibit an unexpected large structure with a trough around the equator, crests near ±15° magnetic latitudes and crest-to-trough ratio about 1.6 in daytime peak electron density (Nmax). This large structure known as the equatorial ionization anomaly (EIA) develops in the morning at around 10 LT, continues to exist well beyond sunset and covers about half the global area in 24 hours; and the position of the crests and crest-to-trough ratio vary with various geophysical conditions.

Solar radiation absorbed by neutral molecules and atomic in the upper atmosphere depends on the angle of arrival of solar radiation as measured by the sun's zenith angle. The greater the zenith angle of the sun, the greater the energy of the absorbed solar radiation. Apart from the intensity of solar radiation, the ionosphere is also affected by the transport process because as charged particles that are in the earth's magnetic field also move as a combined effect between the magnetic field and the electric field in ionospheric charged particles. The ionospheric electric field arises as a result of differences in the warming of the upper atmosphere by solar radiation which gives rise to neutral winds in the upper atmosphere that are westward towards the side of the day and eastward at night. It happens because the response of particles with different electrons and ions to this neutral wind arises the variation of charged particles so that an electric field appears east in the afternoon and westward at night. Therefore, during the day the ionosphere moves upwards and at night the ionosphere moves down.

Variations of the ionosphere can be classified as regular and irregular. Regular variations occur more or less in cycles and therefore can be modeled in advance with reasonable accuracy. Irregular variations, which are mainly due to abnormal solar behavior such as sudden changes in solar radiation, cannot be predicted in advance. Examples of regular variations are diurnal, annual, and solar cycle variations [5]. Possible modulations of the diurnal signal with other frequencies are also identified as regular variations. In this case, the amplitude of the daily signal changes from one season to another. A mathematical foundation is developed to detect and model such occurrences [6].

Since the existence of the ionosphere is due to solar radiation, the relative location of the earth with respect to the sun is responsible for a large part of the ionospheric variations. The daily earth rotation changes the solar zenith angle, which describes a large part of the daily ionospheric variations. The principal diurnal (24 h) component of the daily signal and its higher harmonics (24/n hours, n > 1) are due to the day–night variation of the ionosphere and hence of the total electron contents (TEC) values.

Daily variations of harmonic amplitude can be associated with solar activity, geomagnetic activity and seismic activity. Seismic activity can influence the diurnal amplitude variation with the assumption that before an earthquake, anomaly can occur in the electric field above the epicenter of an earthquake that affects the electric field in the upper atmosphere.

The very early versions of seismo-ionospheric coupling models were based on direct calculations of seismogenic electric field effect from the ground surface to the ionosphere [7]; [8]. All of them are based on the methodology of electric field penetration into the ionosphere from thundercloud [9]. This paper
describes the results of harmonic analysis of ionospheric TEC above the epicenter of the Lombok earthquake in 2018 to investigate the possible ionospheric precursor before Lombok earthquake.

2. Data and Methodology

The ionospheric TEC data above the Lombok earthquake epicenter was taken from TEC GIM data that can be downloaded from ftp://aiub.unibe.ch in the CODE folder and in 2018 with the CODGDOY0.18I file name, with DOY showing the day to year.

The TEC data above the epicenter of the Lombok earthquake from GIM can be extracted or selected for the TEC data closest to the epicenter of the Lombok earthquake on July 28 2018, namely at latitude 8.4° south latitude and longitude 116.5°. After extracting, the TEC data above the epicenter is arranged in a matrix with the number of rows equal to the number of days from doy 1 to 339, and column 1 to 24 contain the value of TEC from 0 to 23. After that, a TEC harmonic analysis was carried out through the KF method with the initial value of the LSE results of the first day of the TEC in 2018. The sine and cosine harmonic coefficients of the equation (1) the results of calculation using the KF method are filtered from the effect of variations in the sun's zenith angle (seasonal variation) with 17th order polynomial. The filtering results of A24 are correlated with F10.7 which is also filtered with order polynomial 27.

Analysis of the effect of earthquake precursors on the A24 diurnal amplitude can be done by making a graph between delta A24 versus DOY and marking the time of an earthquake with a vertical line on the date of the earthquake. Likewise, the effect of geomagnetic storms is seen qualitatively through delta A24 graph.

2.1 Harmonic Analysis

Twelve years of harmonic TEC analysis used modest harmonic analysis and the most dominant diurnal variation is the main diurnal variation (24-hour period) and half-year period (6 months), in addition to 27 days due to solar rotation and annual variation due to the solar cycle [2]. Based on these results, in this paper the ionospheric TEC diurnal variations were only analyzed by two harmonic components, namely periods 24 and 12. Referring to the formulation of harmonic analysis [10] use different notations, the formulation of harmonic analysis for the two main components of diurnal variation can be given as

\[
TEC(t) = A_0 + A_1 \sin \left( \frac{2\pi}{24} t \right) + A_2 \cos \left( \frac{2\pi}{24} t \right) + B_1 \sin \left( \frac{2\pi}{12} t \right) + B_2 \cos \left( \frac{2\pi}{12} t \right) \tag{1}
\]

with the explanation of the harmonic coefficient as follows:

- **Ao**: TEC average in one day
- **A1**: Amplitude of the sine component of period 24
- **B1**: Amplitude of the cos component of period 24
- **A2**: Amplitude of the sine component of period 12
- **B2**: Amplitude of the cosine component of period 1

If the harmonic coefficients were obtained, the amplitude of the wave / diurnal variation is

\[
A_{24} = \sqrt{A_1^2 + B_1^2} \tag{2}
\]

and the wave phase is
The system model used by the KF method can be written as follows [4,11]:

\[ x_{k+1} = Ax_k + Bu_k + \eta_k \]  

and the measurement model

\[ z_k = Hx_k + \epsilon_k \]

The matrix A in the difference equation (6) relates the state at the previous time step (k) to the state at the current step (k+1), in the absence of either a driving function or process noise. Note that in practice A might change with each time step, but here we assume it is constant. The matrix B relates the optional
control input to the state \( x \). The matrix \( H \) in the measurement equation (7) relates the state \( x_k \) to the measurement \( z_k \). In practice \( H \) might change with each time step or measurement, but here we assume it is constant.

Prediction is to predict the state and covariance process at time \( k + 1 \) depending on information at time \( k \).

**Prediction stage:**

- **Forecast real data**
  \[ \bar{x}_{k+1} = A x_k \]  
  \[ (8) \]

- **Error covariance forecast**
  \[ \bar{P}_{k+1} = A P_k A^T + Q \]  
  \[ (9) \]

The next phase of correction as follows:

- **Kalman Gain**
  \[ K_{k+1} = P_{k+1} H_{k+1}^T (H_{k+1} P_{k+1} H_{k+1}^T + R)^{-1} \]  
  \[ (10) \]

- **Error covvariance estimation**
  \[ P_{k+1} = (I - KH) \bar{P}_{k+1} \]  
  \[ (11) \]

- **Real data estimation**
  \[ \hat{x}_{k+1} = \bar{x}_{k+1} + K_{k+1}(z_{k+1} - H \bar{x}_{k+1}) \]  
  \[ (12) \]

### 2.3 Establishment of a System Model

Based on the TEC equation a system model can be formed as follows:

\[ x_{k+1} = A x_k \]  
\[ (13) \]

with \( x = [A_0 A_1 B_1 A_2 B_2 TEC(1) \ldots TEC(24)]^T \), index 1,2,3...24 represents time (hours) and matrix \( A \) is

\[
A = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & \cdots & 0 \\
1 & a1(0) & a2(0) & a3(0) & a4(0) & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
1 & a1(23) & a2(23) & a3(23) & a4(23) & 0 & \cdots & 0 \\
\end{bmatrix}_{29 \times 29}
\]

Where:

- \( a1(t) = \sin((2\pi/24)t) \), \( t = 0, 1, 2, \ldots, 23 \)
- \( a2(t) = \cos((2\pi/24)t) \), \( t = 0, 1, 2, \ldots, 23 \)
- \( a3(t) = \sin((2\pi/12)t) \), \( t = 0, 1, 2, \ldots, 23 \)
- \( a4(t) = \cos((2\pi/12)t) \), \( t = 0, 1, 2, \ldots, 23 \)

The measurement model is

\[ z_k = H x_k \]  
\[ (14) \]
Where

\[ H = \begin{bmatrix} 0_{24 \times 5} & I_{24 \times 24} \end{bmatrix} \]

The system model and measurement of the KF algorithm were obtained from equations (13) and (14). With the initial conditions \( A_0, A_1, B_1, A_2, B_2 \) obtained from the LS method while the initial TEC (1), TEC (2), ..., TEC (24) were obtained from the TEC of first day or the initial TEC. The harmonic coefficients of TEC model is estimated using Kalman filter algorithm as seen in equation (8) to (12).

3. Results and Discussion

By using MATLAB software, software estimation of second order harmonic coefficients has been made as in equation (1) with the KF algorithm as in equation (8) - (12) can be obtained estimation of harmonic coefficients as shown in Figure 1. In Figure 1 can be seen that the coefficient harmonic order 0 (A0) and A1 (sine coefficient period 24) is the most dominant with a value of more than 5 TECU throughout 2018 (blue and red). The peak values of A0 and A1 during 2018 occur at two times, namely at doy between 61 to 91 (March) and between 301 and 331 (November). This shows the peak asymmetry of TEC values in 2018, namely in March and November and not in March and September which is the time at which the sun's zenith is highest. This indicates a delay in the time the TEC reaches its peak in September to November (delayed by 2 months). The delay in the peak value of the TEC in September can be caused by several reasons. First, the epicenter position of the Lombok earthquake in southern latitude could cause changes in the peak value of the TEC due to the presence of meridional neutral winds in the direction of the south which caused an accumulation of ionospheric electron density to peak in November. The second can be caused by the high value of solar radiation in November 2018, but the alleged cause of this asymmetry needs to be supported by the results of previous studies regarding both the occurrence and the explanation of the mechanism.

Use equation (2) and the phase according to equation (3) which is calculated for all days from doy 1 to 339 as shown in Figure 2.

![Figure 1. Estimated value of 2nd order TEC harmonic coefficients (A0, A1, B1, A2, B2) with Kalman filters](image-url)
Figure 2. Diurnal (blue) and semidiurnal (brown) amplitude

It can be seen from Figure 2 that the peak values of the second monthly variation in a year (the first in March) of diurnal and semi-diurnal amplitude were slightly different. In October, semi-diurnal amplitude has reached the highest while the amplitude of the diurnal variation has only reached a maximum in November. In addition, in August (doy 240) there was a sharp increase in diurnal and semidiurnal amplitude. To see the diurnal amplitude amplitude daily filtering or separation of diurnal amplitude from the effect of long-term variation with 17th order polynomial as shown in Figure 3. There are semi-annual and 3-month long-term variations in the diurnal amplitude. After reducing the diurnal amplitude value with the estimated value of 17th order polynomial, the diurnal amplitude amplitude is obtained as shown in Figure 4.

Figure 3. Seasonal variation in amplitude of ionospheric TEC diurnal variation (blue line) and its estimation with 17th order polynomial function (brown line)
Daily diurnal amplitude variations can be caused by daily variations in solar radiation that can be measured by F10.7. Then the daily variation of F10.7 after filtering with long-term seasonal variations can be correlated with delta A24 as shown in Figure 5 which shows very little effect of daily variation F10.7 on diurnal amplitude daily with a very small correlation indication (0.02) and unpolished relationship between these two parameters.

**Figure 5.** The diurnal delta amplitude relationship with F10.7

Daily diurnal amplitude variations can also be affected by electric fields that appear before earthquakes and geomagnetic disturbances. Then the next analysis is the relationship between prominent diurnal variations in diurnal amplitude and earthquake precursors and geomagnetic storms as shown in Figure 6.
In Figure 6 an earthquake is indicated by a vertical straight line. The first vertical line shows the time of the earthquake on July 29, 2019 to be exact at 6:47 a.m., with a strength of 6.4 M or on July 28 at 22.47 UT according to global time or DOY = 209. In the case of this earthquake, ionospheric TEC anomaly or earthquake precursor in the ionosphere was detected 3 days earlier, namely on July 25 (doy 206) in the form of a decrease in the A24 amplitude until it reaches a minimum with a value of -2 TECU. In the Lombok earthquake on August 5 (doy 217) the ionospheric earthquake precursor was detected 4 days before that on August 1 (doy 213) in the form of a decrease in amplitude reaching -2 TECU and in the case of the August 19 earthquake (doy 231) a two-day TEC anomaly was detected previously, on August 17 (doy 229) in the form of a decrease in amplitude A24 to -3.3 TECU. The greatest anomaly caused by the geomagnetic storm August 26 2018 (doy 238) in the form of an increase in diurnal amplitude reached 6 TECU on August 27 (doy 239) one day after the geomagnetic storm.

4. Conclusion
The geomagnetic storm affects the amplitude of the diurnal variation, one day after geomagnetism. It can be seen that the amplitude of the variation in the ionospheric TEC is influenced by the strong zenith angle of the sun that changes in a year where in March and October it reaches the highest zenith in Indonesia. The daily variation of solar radiation flux at 10.7 cm does not affect the daily diurnal amplitude variation. Daily variations in diurnal amplitude are more influenced by the portion of transport that varies on a daily basis as indicated by changes in geomagnetic disturbances during geomagnetic storms and electric field anomalies that are suspected as precursors of earthquakes a few days before the earthquake occurred. In this case, to further explore the effects of transport on diurnal variations of TEC, it is necessary to analyze...
the diurnal TEC variation extracted from variations in the sun's zenith angle by harmonic analysis on a fixed phase ie at 12 LT as the peak of solar radiation.

5. References

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