Research Article
Surface Casing Buckling Effect on the Wellhead Movement of a Subsea Well

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In this work, the wellhead axial movement of a subsea well was predicted with a previously presented multi-string model modified to account for a possible buckling of the well’s surface casing. This model represented the structural casing with a nonlinear spring that considers the casing-soil interaction using $t-z$ curves with hysteretic behavior. In addition, the surface casing was modeled with a nonlinear spring that incorporates buckling deformation when the effective force is compressive and may be assumed not fully cemented due to, e.g., its long length, formation characteristics, or limitations of the rig. Finally, the other casing strings were represented with linear springs. Two case studies were conducted to verify the proposed model. In the first, the wellhead movements during each step of a simplified well’s construction and production were compared with those obtained in an equivalent beam-based FE model. Both models yielded the same results, encouraging the analysis of a second case, which investigated an actual well. In this case, the surface casing buckling led to considerable wellhead movement variations, consequently affecting the surface casing stress, especially in scenarios in which the structural casing’s bearing capacity was surpassed, highlighting the need to account for this phenomenon in the design of subsea wells.

1. Introduction

According to API RP 96 [1], the structural and the fully cemented surface casings comprise the foundation of a subsea well. However, de Souza et al. [2] highlight that these wells’ surface casings may be long enough in some applications to prevent a fully cemented condition, such as in Brazil’s pre-salt offshore hydrocarbon basins. Moreover, several reasons can also promote a partial cement job, such as an equivalent circulating density above the fracture gradient, a long slurry pumping time, and/or a low cement stock at the rig.

To understand the behavior of a partially cemented foundation, de Souza et al. [2] proposed a multi-string model to predict the wellhead axial movements in a subsea well, studying the influence of the top of cement (TOC, Figure 1) on its structural response. The model idealized the well as a structure composed of hysteretic nonlinear springs representing the casing-soil interaction and linear springs representing the other casings, as shown in Figure 1(b). The analyses showed the meaningful impact of the uncemented length on the wellhead movements and the stresses in the casings’ strings. Nevertheless, some of these analyses indicated negative effective forces in the well’s surface casing, possibly inducing its global buckling. The model proposed by de Souza et al. [2] included this effect only in the computation of the stresses in the surface casing. However, its impact on the wellhead’s force equilibrium and displacement compatibility equations has not been assessed. Hence, this article complements the multi-string model from de Souza et al. [2] by including the possibility of surface casing buckling in each step of the well’s construction and operation. The schematic of the modified model is presented in Figure 1(c).
I—_he surface casing buckling modeling is based on the analytical theory proposed by Lubinski and Althouse [3], which is traditionally used in the oil and gas industry in many kinds of applications comprehending drilling, completion, and production phases [4]. I—_he model assumes that the helical buckling of a vertical tube confined in a cylindrical cavity arises when the effective force becomes negative. Buckling may be partial or total depending on the magnitude of the effective force \( F_{ef} \), as can be depicted in Figure 2. This work solves the problem by applying the minimum potential energy theorem regarding the axial force and bending moments. However, the twisting moment and lateral friction resulting from the contact of the pipe with the adjacent elements (borehole or outer string) are neglected.

Despite several structural aspects of the casings’ resistance under different loadings have been dealt with in the public literature, as discussed in Tan et al. [5] and Li et al. [6], very few works addressed the impact of the surface casing top of cement (TOC) on the wellhead movements and axial capacity in either platform or subsea wells during their construction and/or operation.
In the case of platform wells, McCabe [7] proposed the multi-string model composed of elastic bars presented in Figure 1(a) to study the wellhead movements in a platform well during its construction (drilling, completion, and production). Aasen and Aadnoy [8, 9] improved McCabe’s model by including possible uncemented lengths between the TOC and the wellhead. Moreover, Lewis and Miller [10] added the effects of liftoff, the casing pretension during the well’s construction, and the shear stresses at the casing-soil interface. In contrast, Liang [11] constructed a multi-string model where the structural casing is a fixed bar at the midline, and the other strings are fixed at the TOC. The model accounts for the tubing contraction due to the ballooning effect. Liu et al. [12] developed an analytical model by including possible unloading and reloading responses, as proposed by Masing [16]. The structural casing-soil interaction is represented with t-z curves calibrated with field data from Chin and Poulus [17], API RP 2GEO [18], Kim et al. [19], and observations from de Souza et al. [20, 21], based on several FE analyses.

The t-z curves developed in de Souza et al. [2] express z analytically as a function of t. Since the inverse values are required in the calculation, they must be obtained numerically, increasing the numerical solution’s computational effort. Thus, alternative equations for t-z curves were fitted with inverse functions that optimize the approximation error, as shown in Figure 3.

In Figure 3, \( t_{\text{max}} \) is the maximum unit shaft friction and \( z_{\text{peak}} \) is the corresponding displacement. According to API RP 2GEO [18], these quantities can be expressed by the following equations:

\[
 t_{\text{max}} = \alpha \cdot S_u, \quad (1)
\]

\[
 z_{\text{peak}} = 0.02 \cdot r_e, \quad (2)
\]

where \( \alpha \) is the adhesion factor; \( S_u \) is the undrained shear strength; and \( r_e \) is the external radius of the structural casing. The inverse functions are expressed as follows:

\[
 t_f(z) = \frac{1.875(z/z_{\text{peak}})}{1 + 0.8741 \left(\frac{z}{z_{\text{peak}}}\right)^{1/4}}, \quad (3)
\]

\[
 t_u(z) = \frac{t_A}{t_{\text{max}}} = \frac{1.875(z/z_{\text{peak}} - z_A/z_{\text{peak}})}{1 + 0.3482 \left(\frac{z}{z_{\text{peak}}} + z_A/z_{\text{peak}}\right)^{1/3}}, \quad (4)
\]

\[
 t_r(z) = \frac{t_B}{t_{\text{max}}} = \frac{1.875(z/z_{\text{peak}} - z_B/z_{\text{peak}})}{1 + 0.3482 \left(\frac{z}{z_{\text{peak}}} + z_B/z_{\text{peak}}\right)^{1/3}}, \quad (5)
\]

where \( t_f, t_u, \) and \( t_r \) are the unit shaft friction related to the loading, unloading, and reloading paths, respectively, and the subscripts \( A \) and \( B \) refer to the initial points of unloading and reloading, in this order.

2. Method

2.1. Soil Model. The same approach proposed in de Souza et al. [2] is considered to model the soil. This approach includes backbone curves combined with rules that establish the unloading and reloading responses, as proposed by Masing [16]. The structural casing-soil interaction is represented with t-z curves calibrated with field data from Chin and Poulus [17], API RP 2GEO [18], Kim et al. [19], and observations from de Souza et al. [20, 21], based on several FE analyses.

Regarding the wellhead movements in subsea wells, the work of Halal et al. [15] and the previously discussed model from de Souza et al. [2] may be considered references. Halal et al. [15] proposed a multi-string approach that addresses the interactions between the strings’ radial, hoop, and axial stresses. Additionally, equilibrium and compatibility conditions were imposed on the wellhead for each casing string installation, but equipment such as BOPs or Christmas trees was not considered. Finally, the buckling of the surface casing is only considered during the production phase.

Table 1 summarizes the models’ main features, evidencing that few analyze the wellhead movements in subsea wells, but the surface casing buckling is considered with simplifications. Hence, in what follows, the multi-string model proposed by de Souza et al. [2] is revisited, and the implementation of the surface casing buckling effect is presented. Then, the model verification is conducted by comparing its results with those from an equivalent finite element (FE) model developed in commercial software. After that, a realistic case study illustrates the effect of buckling on the wellhead movement during the construction sequence and production life of a subsea well. Finally, the main conclusions of this study are stated.
It should be remarked that the inverse functions given by equations (3) to (5) were adjusted to the t-z curves proposed by de Souza et al. [2] by carefully choosing the functions’ coefficients that minimized the residual of the sum of the squared errors. As a result, the obtained R-squared were 0.98 (loading), 0.96 (unloading), and 0.96 (reloading); i.e., excellent agreements between both functions were achieved.

2.2. Structural Casing Modeling. In the work of de Souza et al. [2], the structural casing is modeled with a one-dimensional finite element that addresses the casing’s structural response and the soil interaction. The casing behavior is linear and elastic, while the hysteretic t-z curves defined by equations (3) to (5) are used to model the soil. The response of the casing is obtained by solving the matrix equation (2):

\[(K_e + K_s) \cdot z = Q, \quad (6)\]

where \(K_e\) is the stiffness matrix of the casing; \(K_s\) is the mobilized soil-pile adhesion matrix; \(z\) is the vector of nodal displacements; and \(Q\) is the vector of nodal forces (body and axial forces). The matrices \(K_e\) and \(K_s\) and the vector of nodal

### Table 1: Main features of the available methodologies for well mechanical analysis.

| Reference | Type of well | Structural casing-soil interaction? | Load capacity of structural casing? | Load sequence to calculate the wellhead movement? | Multi-string analysis due to APB? | Buckling in a multi-string analysis? |
|-----------|--------------|------------------------------------|-------------------------------------|-----------------------------------------------|---------------------------------|-----------------------------------|
| [7]       | Platform     | No                                 | No                                  | D, C, P                                       | No (6)                          | No                                |
| [7]       | Subsea       | No                                 | No                                  | D, C, P (3)                                    | Yes (7)                         | No                                |
| [8, 9]    | Platform     | No                                 | No                                  | D, C, P                                       | No (6)                          | No                                |
| [10]      | Platform     | No                                 | No                                  | D, C, P (4)                                    | No (6)                          | No                                |
| [11]      | Platform     | No                                 | Yes (2)                             | D, C, P                                       | Yes                             | No                                |
| [12]      | Platform     | No                                 | Yes                                 | D, C, P                                       | Yes                             | No                                |
| [13]      | Platform     | No                                 | Yes                                 | D, C, P                                       | Yes                             | No                                |
| [14]      | Platform     | No                                 | Yes                                 | D, C, P                                       | Yes                             | No                                |
| [2]       | Subsea       | Yes                                | Yes                                 | D, C, P                                       | Yes                             | Yes                               |
| This work | Subsea       | Yes                                | Yes                                 | D, C, P                                       | Yes                             | Yes                               |

(1) Notes: D = drilling; C = completion; P = production.
(2) This effect is only considered in the production analysis.
(3) The model does not consider loads related to the BOP or Christmas tree.
(4) The model considers the liftoff analysis and pretension.
(5) The model was employed to investigate an accident in an injection well.
(6) The annulus is vented.
(7) Buckling is considered only in the production analysis.
forces $Q$, in a FE mesh with $n$ nodes and $n-1$ elements, can be obtained based on the connectivity between the nodes and the following matrix expressions:

$$K_E = \sum_{i=1}^{n-1} \left( P_i^T \cdot k_{Ei} \cdot P_i \right),$$

$$K_S = \sum_{i=1}^{n-1} \left( P_i^T \cdot k_{Si} \cdot P_i \right),$$

$$Q = \sum_{i=1}^{n-1} \left( P_i^T \cdot q_i \right),$$

where $k_E$, $k_S$, and $q$ are local element matrices; i.e.,

$$k_{Ei} = \begin{bmatrix} \frac{E_i \cdot A_i}{h_i} & \frac{E_i \cdot A_i}{h_i} \\ \frac{E_i \cdot A_i}{h_i} & \frac{E_i \cdot A_i}{h_i} \end{bmatrix} i = 1 \cdots n - 1,$$

$$k_{Si} = \begin{bmatrix} \frac{\pi}{3} \cdot D_i \cdot h_i \cdot k_i & \frac{\pi}{6} \cdot D_i \cdot h_i \cdot k_i \\ \frac{\pi}{6} \cdot D_i \cdot h_i \cdot k_i & \frac{\pi}{3} \cdot D_i \cdot h_i \cdot k_i \end{bmatrix} i = 1 \cdots n - 1,$$

$$q_i = \begin{bmatrix} \frac{w_f \cdot h_i}{2} + F_i \\ \frac{w_f \cdot h_i}{2} \end{bmatrix} i = 1,$$

$$q_i = \begin{bmatrix} \frac{w_f \cdot h_i}{2} \\ \frac{w_f \cdot h_i}{2} \end{bmatrix} i = 2 \cdots n - 1,$$

where $E, D,$ and $A$ are the Young modulus, the diameter, and the cross-sectional area of the structural casing; $k$ is the stiffness of the soil; $w_f$ is the submerged self-weight per unit length of the structural casing; $F_i$ is the axial force at the top of the casing; and $h$ is the length of the element. The connectivity matrices $P$ are written as follows:

$$P_1 = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 \end{bmatrix},$$

$$P_n = \begin{bmatrix} 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix}.$$

Equation (9) indicates that $k_S$ depends on the soil stiffness $k$, which is related to the axial displacement of the element. Hence, (6) is nonlinear but can be solved numerically, where, in each interaction, the soil stiffness is estimated with

$$k_i = \frac{t_i}{z_i} i = 1 \cdots n - 1,$$

where $t_i$ and $z_i$ are initially related by equation (3). Furthermore, the unloading and reloading matrix equations are given as follows [2]:

$$(K_E + K_S) \cdot z + K_A \cdot t_A - K^T \cdot z_A = q, \quad (13)$$

$$(K_E + K_S) \cdot z \cdot K \cdot t_B - K^T \cdot z_B = q, \quad (14)$$

where subscripts $A$ and $B$ are associated with the vector quantities at points $A$ and $B$ presented in Figure 4; matrix $K_I$ is as follows:

$$K_I = \sum_{i=1}^{n-1} \left( P_i^T \cdot k_{z_i} \cdot P_i \right). \quad (15)$$

Then,

$$k_{z_i} = \begin{bmatrix} \frac{\pi}{3} \cdot D_i \cdot h_i & \frac{\pi}{6} \cdot D_i \cdot h_i \\ \frac{\pi}{6} \cdot D_i \cdot h_i & \frac{\pi}{3} \cdot D_i \cdot h_i \end{bmatrix}, i = 1 \cdots n - 1. \quad (16)$$

Equations (13) and (14) are also nonlinear, and the numerical strategy to solve equation (6) is used again. In solving these equations, the soil stiffness is estimated with

$$k_i = \begin{bmatrix} t_{Ai} - t_i \\ z_{Ai} - z_i \end{bmatrix} i = 1 \cdots n \quad \text{for the unloading path,}$$

$$k_i = \begin{bmatrix} t_{Bi} - t_i \\ z_{Bi} - z_i \end{bmatrix} i = 1 \cdots n \quad \text{for the reloading path,} \quad (17)$$

where $t$ and $z$ are related by equations (4) and (5). Furthermore, the solution of equations (6), (13), and (14) converges, in each load step, if the residual force norm is less than a prescribed value. The value adopted here is 0.1% of the imposed load increment.

Finally, a nonlinear spring with stiffness $k_s$ represents the structural casing by relying on the relation between the imposed axial force and the wellhead displacement, as shown in Figure 4. The stiffness $k_s$ may be obtained with

$$k_s = \frac{\Delta Q}{\Delta Q} = \frac{Q(\delta_0 + \delta_Q) - Q_0}{\delta_Q}, \quad (18)$$

where $\Delta Q = Q - Q_0$ is the increment in axial force regarding a previous load step; $\delta_Q$ is the wellhead displacement variation due to $\Delta Q$. The strategy to determine $k_s$ allows the evaluation of the forces imposed on the structural casing. This strategy is discussed next.

2.3. Equilibrium Condition at the Wellhead. A subsea well is subjected to several interventions during its construction and operation. From a structural perspective, these interventions essentially involve the direct imposition of axial
forces, variations of the conveying fluid density, and the occurrence of loads typical of the production phase. The loads result in forces that act in the casing strings and need to be evaluated. In this context, the proposed multi-string model assumes a rigid wellhead; therefore, each string’s top has axial displacement constrained to the wellhead movement. Moreover, the translation of the string at the TOC is disregarded. Therefore, all casing strings are fixed at the base, as shown in Figure 5, except for the structural casing modeled with a nonlinear hysteretic spring.

Complementarily, the present work accounts for the curvature of the surface casing due to buckling, which can occur for all loads, including self-weight, since its curvature is considered different from the other strings. However, this analysis is restricted to the surface casing because this work focuses on the well’s foundation. Therefore, it is sufficient to evaluate whether the effective force on the surface casing is negative during the load steps, as depicted in Figure 5. The evaluation of forces’ distribution is detailed in the following subsections, which goes further from the previous work [2] by adding the adjustment procedure to account for buckling effects.

2.3.1. Direct Application of Forces. Following the approach previously proposed in de Souza et al. [2], the behavior of the surface casing is studied. Initially, the effect of its buckling on the well’s response is disregarded. Hence, the equilibrium of forces and displacement compatibility at the wellhead lead to the equation that governs the displacement $\delta_Q$ for a force perturbation $\Delta Q$:

$$\left( \sum_{j=1}^{n_c} k_{eqj} + k_{surf} + k_s \right) \delta_Q = \Delta Q,$$  

where $n_c$ is the number of casing strings, $k_{surf}$ is the stiffness of the surface casing, $k_s$ is the stiffness of the structural casing (a function of $\delta_Q$, i.e., $k_s = f(\delta_Q)$); see equation (18), and $k_{eqj}$ is the stiffness of the other strings. The equivalent axial stiffness $k_{eqj}$ of each string $j$ and $k_{surf}$ is stated as follows:

$$k_{eqj} = \left( \sum_{\ell=1}^{m_cj} \frac{1}{k_{\ell}} \right)^{-1}, \quad j = 1 \ldots n_c - 2,$$

$$k_{surf} = \left( \sum_{\ell=1}^{m_{surf}} \frac{1}{k_{surf,\ell}} \right)^{-1},$$

where $m_cj$ (with $x = j$ or $x = surf$) is the number of different cross sections along each string $j$ or the surface casing $surf$; $k_{\ell}$ is the axial stiffness of each section $\ell$, which is written as follows:

$$k_{\ell} = \frac{E_{\ell,\ell} A_{\ell,\ell}}{L_{\ell}}, \quad j = 1 \ldots n_c - 2, \ell = 1 \ldots m_cj,$$

$$k_{surf,\ell} = \frac{E_{surf,\ell} A_{surf,\ell}}{L_{surf,\ell}}, \quad \ell = 1 \ldots m_{surf},$$

where $E_{xx,\ell}$, $A_{xx,\ell}$, and $L_{xx,\ell}$ are, respectively, the Young modulus, the cross-sectional area, and the length of the string section $\ell$ of each string $j$ or surface casing $surf$.

Since the stiffness $k_s$ is related to $\delta_Q$, equation (18) must be solved iteratively. Once $\delta_Q$ is found by equation (19), the internal load variation at the surface casing is as follows:

$$\Delta N_{surf} = k_{surf} \delta_Q.$$  

From $\Delta N_{surf}$, the effective force is updated, as already presented in Figure 5. If it is negative, the displacement $\delta_Q$ must be adjusted due to surface casing buckling, as presented next.

2.3.2. Adjusted Displacement for Direct Application of Forces. The wellhead displacement increases if the surface casing buckles due to an effective negative force, as depicted in Figure 6. The strategy to adjust the wellhead displacement is to apply small-displacement steps at the wellhead and numerically check the equilibrium of the well’s structure by

Figure 4: FE analysis of a structural casing subjected to axial loading: (a) schematic of the structural casing under axial load and its interaction with the surrounding soil (t-z curve); (b) resulting axial load vs. axial displacement at the wellhead. Initially, a total load $Q_0$ acts at the wellhead and induces an axial displacement $\delta_0$, and a further axial load increment $\Delta Q$ results in an additional wellhead displacement $\delta_Q$. The response is governed by the structural casing-soil interaction, which is represented by the proposed t-z curve, leading to an overall axial stiffness $K_s$. 

Figure 6 Mathematical Problems in Engineering
comparing it with the predefined residue. In this context, the wellhead displacement $\delta_{Q,\text{adjusted}}$ after each step $j$ can be expressed as follows:

$$
\delta_{Q,\text{adjusted}} = \delta_Q + \sum_{j=1}^{n_I} \delta_j, j = 1, \ldots, n_I,
$$

where $\delta_Q$ is determined with equation (19), $\delta_j$ is the displacement increment for step $j$, and $n_I$ is the number of iterations to achieve a force residue smaller than the allowable predefined residue.

Let $\Delta L_H$ be the change in length of the surface casing due to Hooke’s law and $\Delta L_L$, conversely, be the change due to buckling formulas [3]. Thus, the displacement compatibility for an iteration $j$ can be expressed as follows:

$$
\delta_{Q,\text{adjusted}} = \Delta L_H + \Delta L_L,
$$

where the term $\Delta L_H$ is given by equation (25) and $\Delta L_L$ by equation (26). It is important to remark that the computation of $\Delta L_L$ requires the subtraction of the accumulated buckling displacement $\delta_a$ due to the nonlinear response represented in Figure 7 schematically.

$$
\Delta L_H = \frac{\Delta N_{\text{surf}}}{k_{\text{surf}}},
$$

$$
\Delta L_L = \delta_L - \delta_a = \int_0^L \frac{r^2}{4EI} (T_{\text{ele}})^3 dl - \delta_a.
$$

Figure 5: Response of an FE analysis of structural casing subjected to an axial load.

Figure 6: Adjusted displacement at wellhead due to the buckling of the surface casing.
where \( r \) is the clearance (the gap between the outside diameter of the surface casing and the borehole), \( E \) is the Young modulus, \( I \) is the moment of inertia, \( L \) is the buckling length, \( \delta_e \) is the total buckling displacement, and \( \delta_a \) is the accumulated buckling displacement before the force perturbation \( \Delta Q \) is applied. The notation \( F_{\text{efc}} \) means

\[
F_{\text{efc}} = \begin{cases} 0, & \land F_{\text{efc}} > 0 \text{(tension)} \\ F_{\text{efc}}, & \land F_{\text{efc}} \leq 0 \text{(compression)}. \end{cases} \tag{27}
\]

The effective force \( F_{\text{efc}} \) must account for the initial condition \( F_{\text{efc}} \), the load history, and the unknown load variation \( \Delta N_{\text{surt}} \), i.e.,

\[
\delta_Q + \sum \delta_i^j = \frac{\Delta N_{\text{surt}}^j}{k_{\text{surt}}} + \frac{\int_0^L r^2}{4EI} \left( F_{\text{efc}} + \int \Delta N_H + (\Delta p_i)_{HAl} - (\Delta p_e)_{HAc} - \Delta N_{\text{surt}} \right)^2 \text{d}l - \delta_a.
\tag{29}
\]

Equation (29) must be solved numerically to find the load variation \( \Delta N_{\text{surt}}^j \) for each iteration \( j \), until the equilibrium condition is satisfied for a given predefined residue \( R \), as follows:

\[
k_c(\delta_Q + \sum \delta_i^j) + \Delta N_{\text{surt}} + \sum_{i=1}^{n-2} k_{\text{ij}}(\delta_Q + \sum \delta_i^j) - \Delta Q \leq R. \tag{30}
\]

For a surface casing with a constant cross section, the integral of equation (27) has an analytical solution, which depends on the neutral point. The distance \( Z \) from the TOC of the surface casing to the neutral point is as follows:

\[
Z = \frac{F_{\text{efc}}}{Y_{cf}},
\tag{31}
\]

where \( F_{\text{efc}} \) is the effective force at the TOC and \( Y_{cf} \) is the buoyancy weight per length as presented in the following equation:

\[
Y_{cf} = \gamma + \rho_{cAl} - \rho_{eAb},
\tag{32}
\]

where \( \gamma \) is the weight per length, \( \rho_i \) is the internal fluid density, and \( \rho_e \) is the external fluid density along the surface casing string.

Thus, let \( L \) be the free length of the surface casing (portion without cement). According to Lubinski and Althouse [3], for \( Z \leq L \), equation (29) can be rewritten as equation (33) and, for \( Z > L \), as equation (34):

\[
\delta_Q + \sum \delta_i^j = \frac{\Delta N_{\text{surt}}}{k_{\text{surt}}} + \frac{\left( F_{\text{efc}} \right)^2}{8EIY_{cf}} - \delta_0, \quad j = 1, \ldots, n_t, \quad Z \leq L,
\tag{33}
\]

\[
\delta_Q + \sum \delta_i^j = \frac{\Delta N_{\text{surt}}}{k_{\text{surt}}} + \frac{\left( F_{\text{efc}} \right)^2}{8EIY_{cf}} \left[ \frac{Y_{cf}L}{F_{\text{efc}}} \left( 2 - \frac{Y_{cf}L}{F_{\text{efc}}} \right) \right] - \delta_0, \quad j = 1, \ldots, n_t, \quad Z > L,
\tag{34}
\]

where \( F_{\text{efc}} \) is the effective force at TOC (see Figure 4).

Finally, the flowchart in Figure 8 summarizes the methodology to adjust the wellhead displacement regarding the surface casing buckling.

2.3.3. Change in Fluid Density. According to de Souza et al. [2], the impact of a change in fluid density on the casing strings is twofold:

(i) The piston effect at the wellhead is the increment of axial force \( \Delta Q \) resulting from the acting pressure on the exposed area of the casing hanger:

\[
\Delta Q = (\rho_f - \rho_i) \Delta h_{wh} \Delta a_b,
\tag{35}
\]

where \( h_{wh} \) is the wellhead’s depth; \( A_b \) is the exposed area; and \( \rho_i \) and \( \rho_f \) are, in this order, the initial and final densities of the fluid. The displacement of the wellhead, in turn, is obtained by (19), and its adjustment follows the methodology previously presented (adjusted displacement for direct application of forces).

(ii) The ballooning effect is caused by the pressure acting inside the string. The displacement at the wellhead \( \delta_{wh} \) related to the ballooning effect can be obtained with

\[
\sum_{j=1}^{n-2} k_0 \delta_j + k_{\text{surt}} + k_s \delta_0 = k_{\text{eqn}} \delta_{bn},
\tag{36}
\]

where \( \delta_{bn} \) is the length variation in the \( n \)th casing string given as follows:

\[
\delta_{bn} = \sum_{j=1}^{n-2} \frac{v_i k_{0j} f_{\text{efc}}}{F_{\text{efc}}} (\Delta p_{\text{eqn}} - A_{\text{eqn}} - \Delta p_{\text{inj}} - A_{\text{inj}}),
\tag{37}
\]

where subscripts \( n,k \) indicate the cross section \( k \) of string \( n \); \( m_i \) is the number of different cross sections of length \( L_c \) and area \( A_c ; E_c \) is the Young modulus; \( v_0 \)
During production, the effects of thermal loading have to be considered. Initially, the forces at the wellhead are given by the following equation:

\[ \delta_{wh} = \frac{\Delta P_{B}}{k_{surf}} + \frac{\pi r^2 (F_{efe})^2}{8EI\gamma_{ef}} - \delta_{a}, \quad j = 1, \ldots, n_{I} \quad Z \leq L, \]

\[ \delta_{wh} = \frac{\Delta P_{B}}{k_{surf}} + \frac{\pi r^2 (F_{efe})^2}{8EI\gamma_{ef}} \left[ \frac{\gamma_{ef} L}{F_{efe}} \left( 2 - \frac{\gamma_{ef} L}{F_{efe}} \right) \right] - \delta_{a}, \quad j = 1, \ldots, n_{I} \quad Z > L, \]

where \( k_{surf} \) is the average external and internal pressure variation along the string, while \( A_{c} \) and \( A_{t} \) are the areas corresponding to the outer and inner diameters in this cross section.

Since the stiffness \( k_{s} \) is a function of the displacement \( \delta_{wh} \), equation (36) must be solved iteratively. Additionally, as already previously presented (adjusted displacement for direct application of forces), if the effective force at the TOC of the surface casing is compressive, the displacement of the wellhead must be adjusted. The scheme for a change in fluid density in the nth string can be visualized in Figure 9, and the flowchart to adjust the wellhead displacement, considering a constant cross section of surface casing, is depicted in Figure 11.

Figure 7: Nonlinear displacement in buckling analysis.

Figure 8. Flowchart to adjust wellhead displacement, considering a constant cross section of surface casing.

The wellhead displacement due to ballooning and thermal effects depends on the length variation \( \delta_{j} \) in each string \( j \), as discussed in this section.

The force equilibrium is given by the following equation:

\[ \sum_{j=1}^{n-2} k_{c,j} + k_{surf} + k_{s} \delta_{j} = k_{surf} \delta_{surf} + \sum_{j=1}^{n-2} k_{c,j} \delta_{j}, \]

where \( \delta_{j} \) and \( \delta_{surf} \) can be written as follows:

\[ \delta_{j} = \Delta L_{B_{j}} + \Delta L_{T_{j}}, \quad j = 1 \cdots n_{c} - 2, \]

\[ \delta_{surf} = \Delta L_{B_{surf}} + \Delta L_{T_{surf}}, \]

and \( n_{c} \) is the number of different cross sections of length \( L_{c} \) and area \( A_{c} \), \( E_{c} \) is the Young modulus, and \( \nu_{c} \) is the Poisson coefficient of the material; \( \Delta p_{b} \) and \( \Delta p_{i} \) are the average external and internal pressure variation along the string section, while \( A_{b} \) and \( A_{i} \) are the areas corresponding to the outer and inner diameters in this cross section.

The wellhead displacement due to ballooning and thermal effects depends on the length variation \( \delta_{j} \) in each string \( j \), as discussed in this section.

The force equilibrium is given by the following equation:

\[ \sum_{j=1}^{n-2} k_{c,j} + k_{surf} + k_{s} \delta_{j} = k_{surf} \delta_{surf} + \sum_{j=1}^{n-2} k_{c,j} \delta_{j}, \]

where \( \delta_{j} \) and \( \delta_{surf} \) can be written as follows:

\[ \delta_{j} = \Delta L_{B_{j}} + \Delta L_{T_{j}}, \quad j = 1 \cdots n_{c} - 2, \]

\[ \delta_{surf} = \Delta L_{B_{surf}} + \Delta L_{T_{surf}}, \]

and \( A_{c} \) and \( A_{i} \) are the areas corresponding to the outer and inner diameters in this cross section.

Since the stiffness \( k_{s} \) is a function of the displacement \( \delta_{wh} \), equation (36) must be solved iteratively. Additionally, as already previously presented (adjusted displacement for direct application of forces), if the effective force at the TOC of the surface casing is compressive, the displacement of the wellhead must be adjusted. The scheme for a change in fluid density in the nth string can be visualized in Figure 9, and the flowchart to adjust the wellhead displacement, considering a constant cross section of surface casing, is depicted in Figure 11.

In Figure 8, the following equations are employed:

\[ \Delta L_{B_{j}} = \sum_{k=1}^{m_{j}} 2 \nu_{c,jk} L_{c,jk} \left( \Delta p_{b,jk} A_{o,jk} - \Delta p_{i,jk} A_{i,jk} \right), \]

\[ j = 1 \cdots n_{c} - 2, \quad j = surf, \]

\[ \Delta L_{T_{j}} = \sum_{k=1}^{m_{j}} \alpha T_{j,k} \Delta T_{j,k} L_{c,jk}, \quad j = 1 \cdots n_{c} - 2, \quad j = surf, \]

where \( \nu_{c,jk} \) and \( \alpha T_{j,k} \) indicate the cross section of the string \( j \), \( m_{j} \) is the number of different cross sections of length \( L_{c} \) and area \( A_{c} \), \( E_{c} \) is the Young modulus, and \( \nu_{c} \) is the Poisson coefficient of the material; \( \Delta p_{b} \) and \( \Delta p_{i} \) are the average external and internal pressure variation along the string section, while \( A_{o} \) and \( A_{i} \) are the areas corresponding to the outer and inner diameters in this cross section; \( \alpha T \) is the coefficient of linear thermal expansion, and \( \Delta T \) is the average temperature variation. Finally, for conciseness, the subscript \( j \) is used to refer all casing strings except the conductor.

As mentioned previously, if the effective force at the TOC of the surface casing is compressive, the displacement at the wellhead must be adjusted. The scheme for a production load can be visualized in Figure 11, and the flowchart to adjust the wellhead displacement, considering a constant cross section of surface casing, is depicted in
Displacement Compatibility (Mueller method):
Eq. (37), Eq. (42) or Eq. (51)
(Depends on the type of action)

Equilibrium (Predefined Residue $R = 10$ lbf):
Eq. (34), Eq. (44) or Eq. (53)
(Depends on the type of action)

Is the residue $R$ Satisfied?
Yes

New Iteration

$F_{eq} < 0$ (Buckling condition)

$i = 1, \ldots, n_I$ (For each iteration, there is an increment in a displacement)

Displacement: $\delta_{\text{adjusted}} = \delta_0 + \sum \delta_\eta$
(Wellhead movement corrections)

Displacement Compatibility (Mueller method):
Eq. (37), Eq. (42) or Eq. (51)
(Depends on the type of action)

New Iteration

Equilibrium (Predefined Residue $R = 10$ lbf):
Eq. (34), Eq. (44) or Eq. (53)
(Depends on the type of action)

Is the residue $R$ Satisfied?
Yes

Adjusted Displacement

Figure 8: Flowchart of adjusted displacements at the wellhead: direct application of forces (blue); fluid density change (orange); production loads (green).

Figure 9: Adjusted displacement at the wellhead for change in fluid density.

Figure 10: Piston effect at wellhead during production.
2.4 Stresses in the Surface Casing. According to de Souza et al. [2], the structural and surface casings resist the axial loads and the internal pressure variations caused by changes in the drilling fluid or the APB. By hypothesis, in this work, these deformations are directly proportional to the imposed loads and are in the elastic domain. Consequently, the superposition principle is valid. The axial stresses in the surface casing are calculated by summing the contribution of each operation in the load sequence to the stress state immediately after the cement job (initial condition), and the stress components in hoop and radial directions are determined with the Lamé equations. Finally, these stress components are combined, and the yielding of the surface casing is verified using the von Mises yield criterion:

\[
\sigma_{VM} = \sqrt{\left(\frac{F_{efc}}{A_c} + F_b\right)^2 + 3\left(p_l - p_o\right)^2 \left(\frac{A_s}{A_c}\right)^2} \leq \sigma_y, \tag{47}
\]

where \(\sigma_{VM}\) is the von Mises stress, \(F_{efc}\) is the effective force at the TOC of the surface casing (Figure 4), and \(\sigma_y\) is the yield limit of the material. Moreover, the force \(F_b\) is an equivalent axial load caused by the buckling of the surface casing and is given as follows:

\[
F_b = \sigma_b A_c, \tag{48}
\]

where \(\sigma_b\) is the buckling stress, which is stated as in Lubinski and Althouse [3].
where $F_L$ is the maximum force in downward direction imposed during the entire loading sequence; $n_l$ is the number of load increments, which is assumed, here, 20.

(4) Determination of the wellhead movement during loading. The wellhead movement and axial load in the structural casing are obtained in each step of the loading sequence with equation (19) and equation (36) and using the Newton–Raphson method. In each step, the axial force $vs$ axial translation curve (Figure 4(b)) is constructed, and the nonlinear stiffness $k_i$ is calculated with equation (18).

(5) Determination of the wellhead movement during the unloading and reloading paths. The $f vs \delta$ curve of the structural casing (Figure 4(b)) is determined considering each load step, following equations, respectively.

\[
F_{U_i} = F_L - \frac{\Delta F}{n_l} k k = 1 \cdots n_l, \quad (51)
\]

\[
F_{R_i} = F_L - \frac{\Delta F}{n_l} k k = 1 \cdots n_l, \quad (52)
\]

where $\Delta F_U$ is the maximum force variation during the unload sequence in an upward direction and $\Delta F_R$ is the maximum force variation during the reload sequence in a downward direction. Finally, step 4 is repeated, and production loads are considered.

After that, equation (19) is employed to evaluate the displacements associated with the direct application of forces. Moreover, equations (19), (35), and (36) are used to calculate the wellhead movement in the case of a change in fluid density. Finally, equations (19), (39), and (40) are used to calculate the displacements related to the production loads.

(6) For loading, unloading, and reloading sequences, the axial load initial condition in the surface casing and the final loads are determined. If the effective force is negative in any step of a load sequence, the methodology proposed to treat the surface casing buckling is considered. In this case, the flowchart presented in Figure 8 is applied to correct the wellhead movements. After that, the von Mises stresses are checked with equation (47).

In this procedure, all equations are solved in Mathcad [22] using the Mueller method (secant method), assuming a tolerance (relative deviation concerning two consecutive solutions) of $10^{-3}$. Moreover, a load residue of 10 lbf was assumed in the equilibrium condition to correct the wellhead movements when buckling occurs (Figure 8).

3. Results and Discussion

3.1. Comparative Analysis

3.1.1. Description. Initially, the proposed model is verified by analyzing the construction of a subsea well with three strings (structural, surface, and intermediate), schematically presented in Figure 12. Moreover, the considered load sequence (operations) is indicated in Table 2.

The main features of the strings are presented in Table 3, while the (un)cemented lengths, fluid densities, and the strings’ buoyed weights are shown in Table 4. The structural casing is jetted in clay soil, with an adhesion factor with the casing, equation (1), of 0.25, and the undrained shear strength $S_u$ is given as follows:

\[
S_u(h) = 1.3 \frac{kPa}{m} \cdot h, \quad (53)
\]

where $h$ is the depth below the midline in meters.

The results obtained with the proposed model are compared with those from an FE model constructed in the commercial FE package Abaqus [23]. In the FE model, casing strings were modeled using element type PIPE31 [23], a two-node three-dimensional pipe with six degrees of freedom that assumes Euler beam behavior and considers inner and outer pressures. Boreholes were modeled with the same element type. Soil-structure interaction in the structural casing was represented with PSI24 [23], a two-dimensional 4-node pipe-soil interaction element, which requires the definition of the axial force vs displacement relation. The t-z curve proposed in this work (soil model) was employed in the loading path in this comparative analysis. Finally, the default properties of contact are applied. These properties ensure the non-penetration of pipe sections and boreholes, even after buckling. Abaqus’ automatic stabilization was used to process this analysis, thus limiting the dissipated energy fraction to 5% of the strain energy.

In this case study, two different FE analyses were conducted, i.e., buckling and static analyses. The first analysis was conducted to find the surface casing’s buckled shapes (eigenvectors). The second analysis uses the first buckling mode shape as an initial imperfection on the surface casing to induce the instability of the well’s structure.

3.1.2. Buckling Analysis. For buckling analysis purposes, the structure is composed only of the free portion of the surface casing (500 m) inside a cylinder representing the borehole, as depicted in Figure 13(a). A unit load ($\Delta Q = 1N$) is applied at the top of the surface casing, and using the Lanczos method, the first eigenvector (deformed shape) was obtained in Abaqus [23] as illustrated in Figure 13(b).

3.1.3. Static Analysis. Aiming to find a better response to the FE simulation in Abaqus [23], firstly, a mesh convergence analysis was carried out considering the same structure presented in Figure 13(a), but with a realistic load $\Delta Q = 1000$ kbf (4448 kN) was adopted. The initial shape of the surface casing was assumed to be the first eigenvector of the buckling simulation (see Figure 13(b)) but with a scale factor of 20% of the clearance between the surface casing and the borehole. The post-buckled result after load application is shown in Figure 14, where Figure 14(a) shows that a helical buckling with a reversible helix occurs. The maximum lateral displacement was 0.127 m (5 in), corresponding to the clearance...
between the 20" surface casing and borehole, as depicted in Figure 14(b).

Figures 15 and 16 summarize the mesh convergence analyses' results by showing the wellhead's relative displacement for an increasing number of elements and the relation between the ALLSD (stabilization energy) and the ALLIE (internal energy), respectively. Figure 15 indicates that convergence was achieved for a mesh composed of 250 elements and the approximation error is 2%, compared with the analytical solution [3]. Therefore, this mesh was used subsequently. Figure 16 shows that ALLSD is a small fraction of ALLIE, which means that the physical response of the problem was not affected by the stabilization technique. Finally, Figure 17 shows the final structural model used to represent the well-studied in this work (Figure 12), and Table 5 presents the respective FE mesh characteristics.

3.1.4. Comparisons. The results of the comparative analysis are outlined next. Figures 18 and 19 present the wellhead axial displacement and the axial load in the structural casing after each operation step. Figure 20 illustrates the von Mises stress and the post-buckling equilibrium geometry of the surface and intermediate casings after the last operation, simulated in Abaqus. Moreover, Figure 21 indicates the relation between the ALLSD (stabilization energy) and ALLIE (internal energy) throughout the time simulation in Abaqus.

Figures 18 and 19 evidence a very good agreement between the FE and the proposed models. Significant wellhead axial displacements are observed, as depicted in Figure 18, with a maximum value of 41 cm in operation 5 (external force perturbation). In operations 1 and 2, the structural casing stiffness governed the wellhead movement, and small wellhead movements were obtained. However, from operations 3 to 5, the axial load resisted by the structural casing is constant, as indicated in Figure 19, and the structural casing no longer resists the imposed axial load. Hence, the excess load is redistributed to the other casing strings, mainly the surface casing. In this context, the axial translations are much larger than those observed at the beginning of the load sequence.

In the FE analysis, the von Mises equivalent stress at the TOC of the surface casing equals 29,630 psi (204 MPa), while the proposed methodology calculated 28,310 psi (195 MPa). These values are quite close and lower than the yield limit of the material that constitutes the surface casing (56,000 psi, Table 3). Moreover, Figure 20 shows the von Mises equivalent stress and the deformed shape of both surface and intermediate casings after operation 5. It can be noticed that both casing strings buckled helically [3], which means that the surface casing buckling induced, in turn, the buckling of the intermediate casing since it remains in tension (effective force is positive). Although the buckling of the intermediate casing may affect the equilibrium and compatibility conditions, the results show that this influence was negligible, and the methodology presented, considering only surface casing buckling in a multi-string context, agrees well with the FE analysis. It is also observed that the lateral displacement obtained along the string was identical to that illustrated in Figure 14, i.e., the clearance of 0.127 m. This behavior is similar to the theory of helical buckling proposed by Lubinski and Althouse [3] and helps explain the excellent agreement of the models compared in this section. Finally, Figure 21 shows a small ratio between ALLSD and ALLIE, which means that the physical response of the problem is not affected by the stabilization technique.

3.2. Production Well

3.2.1. Description. This case study aims to verify the influence of the surface casing buckling in the wellhead movement of an exploratory well during its construction and testing. A similar analysis has been conducted by de Souza et al. [2] but without considering the possibility of the surface casing buckling. The well's structure is schematically presented in Figure 22, and the main characteristics of the strings are shown in Tables 6 and 7. Furthermore, the load history considered in the analyses is indicated in Table 8.
In the well testing, a constant oil production rate is 10,000 bbl/day (1589.9 m$^3$/day) during 100 hours, and the only subsea equipment installed was the BOP, with a buoyed weight of 400 klbf (1779.3 kN). Moreover, the thermal expansions and the APB were estimated using WELLCAT [24], and the obtained values are presented in Tables 7 and 9, respectively. It is also important to emphasize that de Souza et al. [2] considered the tubing string unrestricted by the packer, allowing displacements without transmitting forces. Finally, the surface casing is routinely considered fully cemented in the design of subsea wells. Nevertheless, a poor cement job may occur, as discussed previously, and the impact of the TOC position should be investigated by varying the TOC distance to the midline ($x$ in Figure 22) from 100 m to 500 m with increments of 100 m. Moreover, two values of adhesion factor $\alpha$ were considered, i.e., 0.50 and 0.75, and a borehole of 30 in (0.76 m) was assumed in the buckling analysis.

### Table 3: Main characteristics of the casing strings: verification well.

| String (material)          | Weight (lb/ft) (kN/m) | Outer diameter (in) (m) | Inner diameter (in) (m) | Yield strength (ksi) (MPa) |
|---------------------------|-----------------------|-------------------------|-------------------------|---------------------------|
| Structural casing (X60 steel) | 457 (6.667)         | 30.000 (0.762)          | 27 (0.686)              | 60 (414)                  |
| Surface casing (X56 steel)  | 133 (1.941)          | 20.000 (0.508)          | 18.750 (0.476)          | 56 (386)                  |
| Intermediate casing (P110 steel) | 72 (1.051)       | 13.375 (0.340)          | 12.347 (0.314)          | 110 (758)                 |

### Table 4: (Un)cemented lengths, buoyed weights, and fluid densities: verification well.

| String (casing) | Uncemented length (m) | Cemented length (m) | Buoyed weight (klbf) (kN) | Internal fluid (lb/gal) (kN/m$^3$) | External fluid (lb/gal) (kN/m$^3$) | Cement slurry (lb/gal) (kN/m$^3$) |
|----------------|-----------------------|---------------------|---------------------------|-------------------------------------|------------------------------------|-----------------------------------|
| Structural     | 0                     | 0                   | 109.6 (487.5)             | 8.5 (9.988)                         | 8.5 (9.988)                        | —                                 |
| Surface        | 500                   | 356                 | 176.6 (785.6)             | 8.5 (9.988)                         | 8.5 (9.988)                        | 12.2 (14.336)                     |
| Intermediate   | 1156                  | 500                 | 260.8 (1160.1)            | 10.4 (12.221)                       | 10.4 (12.221)                      | 16.2 (19.037)                     |

#### 3.2.2. Results. Initially, Figure 23 presents the axial translation of the wellhead during the construction load sequence of the well (Table 8), assuming an adhesion factor $\alpha$ of 0.50, while Figures 24 and 25 show the related variations of the axial load applied to the structural casing and the von Mises stress in the surface casing.

Figure 23 indicates that the maximum values of the wellhead axial translations in the proposed model were about 10% higher than the reference (non-buckling) model [2]. Furthermore, these translations may be noteworthy, with the maximum values occurring after operation 9 (tubing installation). The proposed model indicates translations varying between 4 cm and 24 cm, while the corresponding translations are 5 cm to 22 cm in the reference model, depending on the uncemented length. In operations 1 to 6 (initial load steps), the models’ responses in Figure 23 are almost the same. In these operations, the axial movements at the wellhead are associated with the response of the structural casing, whose axial stiffness is high compared with the axial stiffness of the other strings that comprise the well’s structure. Therefore, small axial translations were observed at the wellhead, and buckling effects were absent. However, Figure 24 shows constant axial forces acting on the structural casing from operations 7 to 9, which indicates that the bearing resistance of the structural casing was surpassed. In this scenario, the excess load was mainly transferred to the surface casing, which started to buckle. Therefore, the well’s structure response is more affected by buckling in axial translations, as illustrated in Figure 23, and less in axial load (Figure 24) and stress (Figure 25). It is essential to highlight that the reference model [2] adds stresses due to the surface casing buckling in a post-processing step following Lubinski and Althouse’s [3] approach. Therefore, there are no significant differences in the stresses shown in Figure 25.

After operation 9, Figure 24 evidences a load relief caused by the well’s testing with load reversal, i.e., compressive loads to tensile loads, for uncemented lengths higher than 200 m. Moreover, the influence of the surface casing buckling is more pronounced in terms of wellhead displacement.

Figure 25 shows that the von Mises stresses induced in the surface casing increase significantly after operation 4 (increased fluid density) due to the consequent increase in
Figure 14: Post-buckling geometry (lateral displacement in meter) in the verification well: (a) lateral view; (b) top view.

Figure 15: Variation of the relative displacement at the wellhead with the number of elements in the FE mesh: verification well.

Figure 16: Variation of the energy ratio with the imposed load: verification well. The solution evolution corresponds to the percentage of the solution procedure that has already been completed (0: analysis starts; 1: analysis is concluded).

Figure 17: Structural model using finite elements: verification well.
Table 5: FE mesh characteristics: verification well.

| Material                  | Element | Number of elements | Number of nodes |
|---------------------------|---------|--------------------|-----------------|
| Soil                      | PSI24   | 84                 | 85              |
| Structural casing         | PIPE31  | 84                 | 85              |
| Surface casing            | PIPE31  | 250                | 251             |
| Intermediate casing       | PIPE31  | 578                | 579             |
| Borehole of 30 inches     | PIPE31  | 250                | 251             |
| Borehole of 17.5 inches   | PIPE31  | 578                | 579             |

Operations:
1) Structural casing installation.
2) Surface casing installation.
3) BOP installation.
4) Intermediate casing installation.
5) Force of 500 klbf [2224 kN].

Figure 18: Wellhead axial translation at each operation: verification well.

Figure 19: Axial load in the structural casing at each operation: verification well.
internal pressure. However, these stresses are almost constant in operations 5 and 6 but increase when the axial resistance of the structural casing is surpassed (operation 7 onwards). Moreover, this figure indicates that the maximum stress occurs for an uncemented length of 500 m and is about 42,600 psi (294 MPa). This stress corresponds to 76% of the material’s yield limit.

Figure 26 indicates that the maximum axial translations occur in operation 9 and vary between 2.1 cm ($x = 100$ m) and 2.9 cm ($x = 500$ m), which leads to a variation of 0.8 cm regarding the uncemented length. This value is significantly lower than the 19 cm obtained in the analysis with an adhesion factor of 0.50, highlighting the need to evaluate this
parameter adequately. Moreover, as in Figure 23, Figure 27 indicates a load relief in the structural casing after operation 10. Nonetheless, in this figure, there is no load reversal.

The von Mises equivalent stress in the surface casing, as illustrated in Figure 28, increases after operation 4 (fluid density change) due to the increase in internal pressure, as discussed before. However, the stresses are nearly constant in the following operations until operation 10 (well’s testing). The noteworthy increase after this operation is mainly related to the thermal expansion and the APB. The maximum von Mises stress is verified with an uncemented length of 500 m. It equals 190 MPa, which corresponds to 49% of the material’s yield limit and is significantly lower than the stress induced in the analysis with an adhesion factor of 0.50.

| String (material)                      | Weight (lbf/ft) (kN/m) | Outer diameter (in) (m) | Inner diameter (in) (m) | Yield strength (ksi) (MPa) |
|---------------------------------------|------------------------|-------------------------|-------------------------|---------------------------|
| Structural casing (X60 steel)         | 457 (6.667)            | 30.000 (0.762)          | 27 (0.686)              | 60 (414)                  |
| Surface casing (X56 steel)            | 133 (1.941)            | 20.000 (0.508)          | 18.750 (0.476)          | 56 (386)                  |
| Intermediate casing (P110 steel)      | 72 (1.051)             | 13.375 (0.340)          | 12.347 (0.314)          | 110 (758)                 |
| Production casing (C110 steel)        | 53.5 (0.781)           | 9.625 (0.244)           | 8.535 (0.217)           | 110 (758)                 |
| Tubing string (L80 steel)             | 29.8 (0.435)           | 5.500 (0.140)           | 4.778 (0.121)           | 80 (552)                  |

| Table 7: (Un)cemented lengths, buoyed weights, and fluid densities: subsea well. |
|---------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| String (casing)                 | Uncemented length (m) | Cemented length (m) | Buoyed weight (kN) | Thermal expansion (m) | Internal fluid (kN/m³) | External fluid (kN/m³) | Cement slurry (kN/m³) |
| Structural                       | —               | 756             | 487.5           | —                | 8.5             | 8.5             | —                        |
| Surface                          | 100             | 656             | 785.6           | 0.048            | 8.5             | 8.5             | 12.2                     |
| Surface                          | 200             | 556             | 873.6           | 0.096            | 8.5             | 8.5             | 16.2                     |
| Surface                          | 300             | 456             | 961.7           | 0.142            | 8.5             | 8.5             | 16.2                     |
| Surface                          | 400             | 356             | 1000.0          | 0.186            |                  |                  |                          |
| Surface                          | 500             | 500             | 1137.8          | 0.227            |                  |                  |                          |
| Intermediate                     | 1156            | 700             | 1160.1          | 0.791            | 10.4            | 10.4            | 16.2                     |
| Production                       | 1756            | —               | 1406.9          | 1.254            | 11.9            | 11.9            | 16.2                     |
| Tubing string                    | 2356            | 756             | 758.0           | —                | 12.0            | 12.0            | —                        |

| Table 8: Load sequence (operations) considered in constructing the exploratory subsea well. |
|---------------------------------|-----------------|
| Operation | Action |
| 1         | Installation of the structural casing |
| 2         | Installation of the surface casing |
| 3         | Installation of the BOP |
| 4         | Change in the drilling mud density within the surface casing |
| 5         | Installation of the intermediate casing |
| 6         | Change in the drilling mud density within the intermediate casing |
| 7         | Installation of the production casing |
| 8         | Change in the drilling mud density within the production casing |
| 9         | Installation of the tubes’ string |
| 10        | Test of the well |

| Table 9: APB values during testing. |
|---------------------------------|-----------------|-----------------
| Annulus (see Figure 10)         | APB (psi) (MPa) | Boundary conditions |
| A                                | 6965 (48.0)     | Non-vented annulus |
| B                                | 432 (3.0)       | Fracture at the 13.375″ shoe |
| C                                | 199 (1.4)       | Fracture at the 20.000″ shoe |
Figure 23: Axial displacements at the wellhead during the construction of the well: effect of the surface casing uncemented length and $\alpha = 0.50$.

Figure 24: Axial load in the structural casing during the construction of the well: effect of the surface casing uncemented length and $\alpha = 0.50$. 

Operations:
1) Structural casing installation.
2) Surface casing installation.
3) BOP installation.
4) Change of mud density.
5) Intermediate casing installation.
6) Change of mud density.
7) Production casing installation.
8) Change of mud density.
9) Drillstring installation.
10) Well testing.
1) Structural casing installation.
2) Surface casing installation.
3) BOP installation.
4) Change of mud density.
5) Intermediate casing installation.
6) Change of mud density.
7) Production casing installation.
8) Change of mud density.
9) Drillstring installation.
10) Well testing.

Figure 25: von Mises stress in the surface casing during the construction of the well: effect of the surface casing uncemented length and $\alpha = 0.50$.

Figure 26: Axial displacements at the wellhead during the well’s construction: effect of the surface casing uncemented length and $\alpha = 0.75$. 
Operations:
1) Structural casing installation.
2) Surface casing installation.
3) BOP installation.
4) Change of mud density.
5) Intermediate casing installation.
6) Change of mud density.
7) Production casing installation.
8) Change of mud density.
9) Drillstring installation.
10) Well testing.

Figure 27: Axial load in the structural casing during the well’s construction: effect of the surface casing uncemented length and $\alpha = 0.75$.

Figure 28: von Mises stress in the surface casing during the well’s construction: effect of the surface casing uncemented length and $\alpha = 0.75$. 
4. Conclusions

In this work, a previously proposed multi-string model was modified to account for the possibility of surface casing buckling and used to predict the wellhead axial movements in a subsea well. The structural casing of the well was represented by a nonlinear spring that addresses the casing-soil interaction using hysteretic $t$-$z$ curves. The surface casing may be partially cemented and was modeled with a nonlinear spring considering its buckled configuration when the effective force is compressive. All other casings were incorporated as linear springs, whose displacements were compatibilized at the wellhead, ensuring the structure equilibrium throughout the load history. Moreover, the load history accounts for the casing strings and subsea equipment installation, the changes in the drilling fluid, and, finally, the production loads and well testing.

The proposed modified model predictions were first compared with those obtained with a FE model built in the commercial FE package Abaqus. This comparative study simulated five steps of constructing a subsea well with three strings. Both the FE and the proposed models indicated the buckling of the surface casing and predicted almost the same wellhead axial displacements and forces in the structural casing during the load sequence. After that, the impact of the surface casing buckling on the wellhead axial displacements of an exploratory subsea well was analyzed. A study of this well has been previously presented but without considering the possible buckling of the surface casing. From the direct comparison of results, the effect of the surface casing buckling becomes more evident in cases where the load capacity of the structural casing was overcome. The wellhead axial displacements were considerably higher than those observed in the analyses without considering the surface casing buckling. Still, the load resisted by the structural casing and the related stresses are not much affected. On the other hand, when the structural casing capacity is not exceeded, the effect of the surface casing buckling is limited.

Hence, the impact observed in the performed analyses indicates that the surface casing buckling is essential in predicting the wellhead movements in partially cemented subsea wells. However, the buckling modeling is complex, as shown in the presented FE analyses, which required reasonable computational effort and had some convergence difficulties due to the contact and nonlinearities inherent to the phenomenon. In this context, the relatively simple approach proposed in the article makes the representation of this effect accessible to the design of subsea wells. Moreover, the stability analysis of multiple long confined pipes may be found in other engineering problems (e.g., the instability of tubes within steel tube umbilicals [251]). Therefore, the proposed approach may also serve as a start point for developing models to deal with these similar problems. Finally, the authors believe that the presented model may help in the safe design of these wells despite the recognized need for in-field verification and validation.

Data Availability

All data used to support the findings of this study are included within the article and can be obtained from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

[1] AM Institute, API RP 96, “Deepwater Well Design and Construction,” American Petroleum Institute, Washington, D.C., USA, 2013.

[2] C. O. de Souza, J. R. M. de Sousa, and G. B. Ellwanger, “Wellhead axial movements in subsea wells with partially cemented surface casings,” Journal of Petroleum Science and Engineering, vol. 194, Article ID 107537, 2020.

[3] A. Lubinski and W. S. Althouse, “Helical buckling of tubing sealed in packers,” Journal of Petroleum Technology, vol. 14, no. 6, pp. 655–670, 1962.

[4] R. F. Mitchell, “Buckling analysis in deviated wells: a practical method,” SPE Drilling and Completion, vol. 14, no. 1, pp. 11–20, 1999.

[5] C. Tan, W. Yan, Q. Tang et al., “Casing Damage Prediction Model Based on the Data-Driven Method,” Mathematical Problems in Engineering, vol. 2020, Article ID 8315908, 9 pages, 2020.

[6] D. Li, F. Yu, H. Fan, R. Wang, S. Yang, and X. Yan, “Casing collapse strength analysis under nonuniform loading using experimental and numerical approach,” Mathematical Problems in Engineering, vol. 202112 pages, Article ID 5577250, 2021.

[7] A. C. McCabe, “Well vertical movement on platform wells,” in Proceedings of the SPE Offshore Europe, Europe Aberdeen UK, September 1989.

[8] J. A. Aasen and B. S. Aadnøy, “Multistring analysis of well growth,” in Proceedings of the IADC/SPE Asia Pacific Drilling Technology Conference And Exhibition, Kuala Lumpur, Malaysia, September 2004.

[9] J. A. Aasen and B. S. Aadnøy, “Multistring analysis of wellhead movement,” Journal of Petroleum Science and Engineering, vol. 66, no. 3–4, 116 pages, 2009.

[10] D. Lewis and R. A. Miller, “Casing Design,” in Advanced Drilling And Well Technology, B. S. Aadnøy, I. Cooper, S. Z. Miska, R. F. Mitchell, and M. L. Payne, Eds., pp. 153–166, Society of Petroleum Engineers, Pennsylvania, PA USA, 2009.

[11] Q. Liang, “Casing thermal stress and wellhead growth behaviors analysis,” in Proceedings of the SPE Asia Pacific Oil
and Gas Conference and Exhibition, Perth, Australia, October 1992.

[12] X. Liu, G. Chen, Y. Chang, L. Zhang, W. Zhang, and H. Xie, “Multistring analysis of wellhead movement and uncemented casing strength in offshore oil and gas wells,” Petroleum Science, vol. 11, no. 1, 138 pages, 2014.

[13] U. B. Sathuvalli and P. V. Suryanarayana, “Structural-casing/soil interaction effects on wellhead motion,” SPE Drilling and Completion, vol. 31, no. 4, 285 pages, 2016.

[14] Z. Zhang, Z. Zhou, Y. He, and J. Zhou, “Study of a model of wellhead growth in offshore oil and gas wells,” Journal of Petroleum Science and Engineering, vol. 158, no. 144, 151 pages, 2017.

[15] A. S. Halal, R. F. Mitchell, and R. R. Wagner, “Multi-string casing design with wellhead movement,” in Proceedings of the SPE Production Operations Symposium, Oklahoma City, OK USA, March 1997.

[16] G. Masing, “Eigenspannugen und verfestigung beim messing (internal stress and hardening in brass),” in Proceedings of the Second International Congress on Applied Mechanics, pp. 332–335, Corfu, Greece, June 1926.

[17] J. T. Chin and H. G. Poulus, “A "T-Z" approach for cyclic axial loading analysis of single piles,” Computers and Geotechnics, vol. 12, no. 4, 320 pages, 1991.

[18] API RP 2GEO, Geotechnical and Foundation Design Considerations, American Petroleum Institute, Washington, DC., USA, 2014.

[19] Y. H. Kim, M. S. Hossain, D. Wang, and M. F. Randolph, “Numerical investigation of dynamic installation of Torpedo anchors in clay,” Ocean Engineering, vol. 108, pp. 820–832, 2015.

[20] J. R. M. de Sousa, C. S. de Aguiar, G. B. Ellwanger, E. C. Porto, D. Foppa, and C. J. de Medeiros, “Undrained load capacity of Torpedo anchors embedded in cohesive soils,” Journal of Offshore Mechanics and Arctic Engineering, vol. 133, no. 2, 2011.

[21] J. R. M. de Sousa, R. G. B. C. Genzani, E. C. Porto, A. T. Borges, E. F. Nogueira, and G. B. Ellwanger, “On the axial holding capacity of Torpedo bases in clay,” in Proceedings Of the ASME 2017 36th International Conference On Ocean, Offshore And Arctic Engineering, Trondheim, Norway, June 2017.

[22] P. TC, Mathcad 15.0 User’s Guide, Parametric Technology Corporation, Boston, MA, USA, 2010.

[23] Simulia, ABAQUS 6.14 Documentation, Dassault Systèmes, Vélizy-Villacoublay, France, 2014.

[24] Halliburton, WELLCAT 5000.16 Documentation, Halliburton Landmark, Rio de Janeiro, Brazil, 2019.

[25] Q. Lu, Z. Yang, J. Yan, and Q. Yue, “Design of cross-sectional layout of Steel tube umbilical,” Journal of Offshore Mechanics and Arctic Engineering, vol. 136, no. 4, Article ID 41401, 2014.