The nuclear equation of state probed by $K^+$ production in heavy ion collisions

C Fuchs†, Amand Faessler†, S El-Basaouny†, K Shekhter†, E E Zabrodin‡§, Y M Zheng‡
† Institut für Theoretische Physik, Universität Tübingen, Tübingen, Germany
‡ China Institute of Atomic Energy, Beijing 102413, China
§ Institute for Nuclear Physics, Moscow State University, Moscow, Russia

Abstract. The dependence of $K^+$ production on the nuclear equation of state is investigated in heavy ion collisions. An increase of the excitation function of $K^+$ multiplicities obtained in heavy ($Au + Au$) over light ($C + C$) systems when going far below threshold which has been observed by the KaoS Collaboration strongly favours a soft equation of state. This observation holds despite of the influence of an in-medium kaon potential predicted by effective chiral models which is necessary to reproduce the experimental $K^+$ yields. Phase space effects are discussed with respect to the $K^+$ excitation function.

1. Introduction and model

The original motivation to study the kaon production in heavy ion reactions at intermediate energies, namely to extract information on the nuclear equation of state (EOS) at high densities is a matter of current debate. Already in the first theoretical investigations by transport models it was noticed that the $K^+$ yield reacts sensitive on the nuclear equation of state [1–4]. The yields were found to be about a factor 2–3 larger when a soft EOS was applied compared to a hard EOS. At that time the available data [5] already favoured a soft equation of state. However, calculations as well as the experimental data were still burdened with large uncertainties.

In [6] we studied the question if in the meantime decisive information on the nuclear EOS can be extracted from subthreshold kaon production in heavy ion collisions. There are several reasons why it appears worthwhile to do this: Firstly, there has been significant progress in the recent years towards a more precise determination of the elementary kaon production cross sections [7,8], based also on new data points form the COSY-11 for the reactions $pp \rightarrow pK^+X$ very close to threshold [9]. Secondly, the KaoS Collaboration has performed systematic measurements of the $K^+$ production far below threshold in heavy ($Au + Au$) and light ($C + C$) systems [10]. Looking at the ratios built from heavy and light systems possible uncertainties which might still exist in the theoretical calculations should cancel out to a large extent which allows to draw reliable conclusions. Furthermore, far below threshold the kaon production is a highly collective process and a particular sensitivity to the compression of the participant matter is expected.

The present investigations are based on the Quantum Molecular Dynamics (QMD) transport model [11]. For the nuclear EOS we adopt soft and hard Skyrme forces corresponding to a compression modulus of $K=200$ MeV and $380$ MeV, respectively, and with a momentum dependence adjusted to the empirical optical
nucleon-nucleus potential \([11]\). The saturation point of nuclear matter is thereby fixed at \(E_B = -16\) MeV and \(\rho_{\text{sat}} = 0.17\) fm\(^{-3}\) \([11]\). The calculations include \(\Delta(1232)\) and \(N^*(1440)\) resonances. The QMD approach with Skyrme interactions is well tested, contains a controlled momentum dependence and provides a reliable description of the reaction dynamics in the SIS energy range, expressed e.g. by collective nucleon flow observables as well as particle production. In contrast to AGS energies where the creation of resonance matter may lead to an effective softening of the EOS, baryonic resonances with masses above the \(N^*(1440)\) can safely be neglected for the reaction dynamics at SIS energies \([12]\).

We further consider the influence of an in-medium kaon potential based on effective chiral models \([13\, 14\, 15\, 16]\). The \(K^+\) mean field consists of a repulsive vector part \(V_\mu = 3/8f_\pi^2j_\mu\) and an attractive scalar part \(\Sigma_S = m_K - m_K^* = m_K - \sqrt{m_K^2 - \Sigma_{\text{KN}}/f_\pi^2}\rho_S + V_\mu V_\mu\). Here \(j_\mu\) is the baryon vector current and \(\rho_S\) the scalar baryon density and \(\Sigma_{\text{KN}} = 450\) MeV. Following \([16]\) in the vector field the pion decay constant in the medium \(f_\pi^* = 0\). However, the enhancement of the scalar part using \(f_\pi^*\) is compensated by higher order contributions in the chiral expansion \([16]\), and therefore here the bare value is used, i.e. \(\Sigma_{\text{KN}}\rho_S/f_\pi^2\).

Compared to other chiral approaches \([14,\, 15]\) the resulting kaon dispersion relation shows a relatively strong density dependence. The increase of the in-medium \(K^+\) mass \(\tilde{m}_K\), Eq. (2), with this parameterisation is still consistent with the empirical knowledge of kaon-nucleus scattering and allows to explore in-medium effects on the production mechanism arising from zero temperature kaon potentials. For the kaon production via pion absorption \(\pi B \rightarrow Y K^+\) the elementary cross section of \([17]\) are used. For the \(NN \rightarrow BY K^+\) channels we apply the cross sections of Ref. \([7]\) which give a good fit to the COSY-data close to threshold. For the case of \(N\Delta \rightarrow BY K^+\) and \(\Delta\Delta \rightarrow BY K^+\) reactions experimental data are rare. Thus we rely on the model calculation of ref. \([8]\). In the case that a \(N^*\) resonance is involved in the reaction the same cross section is applied. In the presence of scalar and vector fields the kaon optical potential in nuclear matter has the same structure as the corresponding Schroedinger equivalent optical potential for nucleons
\[
U_{\text{opt}}(\rho, k) = -\Sigma_S + \frac{1}{m_K}k_\mu V_\mu + \frac{\Sigma_S^2 - V_\rho^2}{2m_K}.
\]

and leads to a shift of the thresholds conditions inside the medium. To fulfill energy-momentum conservation the optical potential is absorbed into an newly defined effective mass
\[
\tilde{m}_K(\rho, k) = \sqrt{m_K^2 + 2m_KU_{\text{opt}}(\rho, k)}
\]
which is a Lorentz scalar and sets the canonical momenta on the mass-shell \(0 = k^2 - \tilde{m}_K^2\). Thus, e.g., the threshold condition for \(K^+\) production in baryon induced reactions reads \(\sqrt{s} \geq \tilde{m}_B + \tilde{m}_Y + \tilde{m}_K\) with \(\sqrt{s}\) the centre-of-mass energy of the colliding baryons. For a consistent treatment of the thresholds the scalar and vector baryon mean fields entering into eq. \([4]\) are determined from two versions of the non-linear Walecka model with \(K=200/380\) MeV, respectively \([8]\). The hyperon field is thereby scaled by \(2/3\) which yields also a good description of the \(\Lambda\) flow \([18]\). Since the parameterisations chosen for the non-linear Walecka model yield the same EOS as the Skyrme ones, the overall energy is conserved. The kaon production is treated perturbatively and does generally not affect the reaction dynamics \([13]\).
2. EOS dependence of $K^+$ production

The $K^+$ excitation function for $Au + Au$ and $C + C$ reactions starting from 0.8 A·GeV which is far below threshold ($E_{th} = 1.58$ GeV) has been measured by the KaoS Collaboration [10, 20]. In [6] we calculated this excitation function for a soft/hard EOS including the in-medium kaon potential. For both systems the agreement with the KaoS data [10] is very good when a soft EOS is used. In the large system there was a visible EOS effect which is absent in the light system. The inclusion of the repulsive in-medium $K^+$ potential is thereby essential to reproduce the data [20].

Already in the light system the $K^+$ yield is reduced by about 50%. To extract more clear information on the nuclear EOS, in Fig. 1 we considered the ratio $R$ of the kaon multiplicities obtained in $Au + Au$ over $C + C$ reactions, normalised to the corresponding mass numbers. The kaon potential is included since without the in-medium potential one is not able to reproduce the experimental $K^+$ yields [6, 21]. The calculations are performed under minimal bias conditions with $b_{max} = 11$ fm for $Au + Au$ and $b_{max} = 5$ fm for $C + C$ and normalised to the experimental reaction cross sections [11, 20]. Both calculations show an increase of $R$ with decreasing incident energy down to 1.0 A·GeV. However, this increase is much less pronounced when the stiff EOS is employed. In the latter case $R$ even decreases at 0.8 A·GeV whereas the soft EOS leads to an unrelieved increase of $R$. At 1.5 A·GeV which is already very close to threshold the differences between the two models become small. The strong increase of $R$ can be directly related to higher compressible nuclear matter. The comparison to the experimental data from KaoS [10] where the increase of $R$ is even more pronounced strongly favours a soft equation of state. We would like to mention that similar results
were also obtained by independent IQMD calculations [21, 24]. These also include an in-medium kaon potential derived in relativistic mean field theory (RMF) [23] which is somewhat less repulsive than that one used in our calculations. For the soft EOS the IQMD calculations almost coincide with the present results [5]. For the hard EOS there exists still deviations concerning the slope of $R$ going far below threshold. This could be due to the different in-medium potentials and is an open question which has to be resolved by future investigations. However, the compared to earlier works [21] improved statistics has now led to a relatively good overall agreement of the two sets of transport calculations.

The dependence of $R$ on the various $K^+$ production channels is shown in Fig.2. There the ratios $R_i$ are built separately for the production channels with initial states $i = NN, \pi N, N\Delta, \pi \Delta, \Delta \Delta$. Here one has to keep in mind that the $NN$ and $\pi N$ cross sections are quite well under control whereas the $N\Delta, \pi \Delta$ channels are experimentally unknown. Thus one has to rely on model predictions [8]. However, the shape of $R$ is not strongly influenced by these two channels which are the most insecure ones. The excitation function for the $N\Delta$ contribution varies only little as a function of energy and is similar using the different EOSs. The contribution of the $\pi \Delta$ channel is decreasing for both, a hard and a soft EOS. The shape of $R$ is to most extent determined by the $NN$ and $\pi N$ contributions. In our calculations the latter channel is responsible for the decrease of $R$ very far below threshold when the hard EOS is applied. Since we consider ratios theoretical uncertainties in the knowledge of elementary cross sections cancel out in first order anyway, which makes the conclusions more reliable. Also the Nantes group [24] reported that varying e.g. the $N\Delta$ cross section by a factor of two does hardly affect the final shape of $R$.

Figure 2. Dependence of the excitation function of $R$ on the various $K^+$ production channels. Central (b=0 fm) $Au + Au$ and $C + C$ reactions are considered. The calculations are performed with in-medium kaon potential.
3. Phase space for $K^+$ production

To obtain a quantitative picture of the explored density effects in Fig. 3 the baryon densities are shown at which the kaons are created. The energy is chosen most below threshold, i.e. at 0.8 $A \text{ GeV}$ and only central collisions are considered where the effects are maximal. $dM_{K^+}/d\rho$ is defined as

$$dM_{K^+}/d\rho = \sum_i dP_i \frac{d\rho_B(x_i, t_i)}{d\rho}$$

(3)

where $\rho_B$ is the baryon density at which the kaon $i$ was created and $P_i$ is the corresponding production probability. For the comparison of the two systems the curves are normalised to the corresponding mass numbers. Fig. 3 illustrates several features: Only in the case of a soft EOS the mean densities at which kaons are created differ significantly for the two different reaction systems, i.e. $<\rho/\rho_{sat}> = 1.46/1.40$ for $C + C$ and $1.47/1.57$ for $Au + Au$ using the hard/soft EOS. Generally, in $C + C$ reactions densities above $2\rho_{sat}$ are rarely reached whereas in $Au + Au$ the kaons are created at densities up to three times saturation density. Furthermore, for $C + C$ the density distributions are weakly dependent on the nuclear EOS. The situation changes completely in $Au + Au$. Here the density profile shows a pronounced EOS dependence [3]. Moreover, the excess of kaons obtained with the soft EOS originates almost exclusively from high density matter which demonstrates that compression effects are probed. Similar as the density shown before a quantitative measure for the collectivity probed by the $K^+$ production and for phase space effects is shown.
The nuclear equation of state probed by $K^+$ production in heavy ion collisions

6

12345
$<N_C>$

Au+Au

C+C

12345
$<N_C>_{Au+Au}/<N_C>_{C+C}$

0.5 1.0 1.5 2.0
$E_{lab}$ [GeV]

Figure 4. As a measure for the available phase space for $K^+$ production the mean number of collisions $<N_C>$ per particle which the hadrons ($N, \Delta, \pi$) did undergo before they produce a $K^+$ meson is considered. The upper panel shows $<N_C>$ in central $Au+Au$ and $C+C$ collisions. The lower panel shows the ratio of this quantity in $Au+Au$ over the same in $C+C$ reactions. The calculations are performed with/without in-medium kaon potential and using a hard/soft nuclear EOS.

in Fig.4. There the average number of collisions for those hadrons ($N, \Delta, \pi$) which were involved in the $K^+$ production are displayed. Again only central collisions are considered where the effects are maximal. $<N_C>$ is defined as

$$<N_C> = \sum_{i} \frac{1}{2} (N_{C_i}^1 + N_{C_i}^2) P_i / \sum P_i$$

(4)

with $N_{C_i}^j$ being the number of collisions which particles $(1, 2)$ experienced before they produced kaon $i$, and $P_i$ is the corresponding production probability. It is seen that in average the particles undergo about twice as much relevant collisions in the heavy compared to the light system. Furthermore, the collectivity, i.e. the accumulation of energy by multiple scattering, increases with decreasing incident energy. Thus one can conclude that the increase of $R$ is not due to a trivial phase space effect, namely the fact that far below threshold the $C+C$ system is simply too small to provide enough collectivity for the kaon production. If such a scenario - which could model independently as well explain the rise of $R$ seen in the KaoS data - would be true, $<N_C>$ would have to saturate for $C+C$ collisions at low energies. One can expect such a saturation from the number of binary collisions at even lower incident energies but here this is obviously not yet the case. Moreover, building also here the ratio (lower
panel of Fig.4) it seems that the relative enhancement of available phase space for $K^+$ production in the large system is decreasing at low energies. This demonstrates that $K^+$ production far below threshold always requires a certain amount of collectivity which can be provided also in a very small colliding system, though such processes are rare. There is, however, no sharp limit were such collision histories become impossible. Thus trivial phase space effects can be excluded for an explanation of the increase of $R$. In [10] a similar argument was based on the measurement of high energy pions which can test the phase space available for particle production.

4. $K^+$ flow

![Diagram](image)

**Figure 5.** Average in-plane transverse $K^+$ flow in 1.93 A.GeV $^{58}$Ni + $^{58}$Ni reactions. The full squares (circles) represent old (new) experimental data from FOPI [29] ([30]). The full down (up) triangles denote the calculated results with (without) kaon potential in the nuclear medium.

Finally in Fig.5 the transverse $K^+$ flow in $Ni + Ni$ reactions at 1.93 A.GeV is considered in order to obtain a conclusive picture of the consistency of in-medium effects with data [24]. As proposed in [20] the $K^+$ in-medium potential is treated in its full covariant form, i.e. including the Lorentz force contribution which arises from the existence of the vector field. As discussed in [26] the Lorentz force leads to a cancellation of the anti-flow [15, 27, 28] due to the repulsive time-like part of the vector potential. Thus there is no difference of the $K^+$ flow for the calculations with and without in-medium effects around mid-rapidity. However, at target and spectator rapidities the reduced error bars of more recent FOPI data [30] allow a distinction between the two scenarios. Both calculations show flow there but the flow signal is much weaker using the in-medium potential. Only the latter case is consistent with the data. The necessity to include the in-medium effects, on the other hand side, is consistent with knowledge from other dynamical observables [27, 31].
5. Summary

To summarise, we find that at incident energies far below the free threshold $K^+$ production is a suitable tool to study the dependence on the nuclear equation of state. Using a light system as reference frame there is a visible sensitivity on the EOS when ratios of heavy ($Au+Au$) over light ($C+C$) systems are considered. Transport calculations indicate that the $K^+$ production gets hardly affected by compressional effects in $C+C$ but is highly sensitive to the high density matter ($1 \leq \rho/\rho_{sat} \leq 3$) created in $Au+Au$ reactions. Results for the $K^+$ excitation function in $Au+Au$ over $C+C$ reactions as measured by the KaoS Collaboration, strongly support the scenario with a soft EOS. This statement is also valid when an enhancement of the in-medium kaon mass as predicted by chiral models is taken into account. Since the explanation of the total $K^+$ yields and the $K^+$ flow requires the presence of in-medium effects a consistent picture for the $K^+$ dynamics is obtained.

The authors would like to acknowledge valuable discussions with J. Aichelin, Ch. Hartnack, H. Oeschler, P. Senger and C. Sturm.

References

[1] Aichelin J and Ko C M 1985 Phys. Rev. Lett. 55 2661
[2] Huang S W et al 1993 Phys. Lett. B 298 41; Hartnack C et al. 1994 Nucl. Phys. A 580 643
[3] Li G Q and Ko C M 1995 Phys. Lett. B 349 405
[4] Li Bao-An 1994 Phys. Rev. C 50 2144
[5] Miskowiec D et al., KaoS Collaboration 1994 Phys. Rev. Lett. 72 3650
[6] Fuchs C, Faessler A, Zabrodin E, Zheng Y M 2001 Phys. Rev. Lett. 86 1974
[7] Sibirtsev A 1995 Phys. Lett. B 359 29
[8] Tsushima K, Sibirtsev A, Thomas A W, Li G Q 1999 Phys. Rev. C 59 369
[9] Balewski J T et al. 1996 Phys. Lett. B 338 859; 1998 Phys. Lett. B 420 211
[10] Sturm C et al., KaoS Collaboration 2001 Phys. Rev. Lett. 86 39
[11] Aichelin J 1991 Phys. Reports 202 233
[12] Hofmann M, Mattiello R, Sorge H, Stöcker H, Greiner W 1995 Phys. Rev. C 51 2095
[13] Kaplan D B, Nelson A E 1986 Phys. Lett. B 175 57
[14] Waas T, Kaiser N, Weise W 1996 Phys. Lett. B 379 34
[15] Li G Q, Ko C M and Li B A 1995 Phys. Rev. Lett. 74 235; Li G Q, Ko C M 1995 Nucl. Phys. A 594 460
[16] Brown G E and Rho M 1996 Nucl. Phys. A 596 503
[17] Tsushima K, Huang S W, Faessler A 1994 Phys. Lett. B 337 245; 1995 J. Phys. G 21 33
[18] Wang Z S, Faessler A, Fuchs C, Waindzoch T 1998 Nucl. Phys. A 645 177
[19] Fang S X, Ko C M, Li G Q and Zheng Y M 1998 Phys. Rev. C 49 R 608; Nucl. Phys. A 575 766
[20] Laue F et al., KaoS Collaboration 1999 Phys. Rev. Lett. 82 1640
[21] Hartnack C, Aichelin J 2001 J. Phys. G 27 571
[22] Sturm C et al. 2001 annual GSI report
[23] Schaffner J et al. 1997 Nucl. Phys. A 625 325
[24] Hartnack C, contribution to this proceedings
[25] Zheng Y M, Chu Z L, Fuchs C, Faessler A, Xiao W, Hua D P Chin. Phys. Lett., in press
[26] Fuchs C, Kosov D, Faessler A, Wang Z S and Waindzoch T 1998 Phys. Lett. B 434 245
[27] Bratkovskaya E L and Cassing W 1999 Phys. Rep. 308 65
[28] Bratkovskaya E L, Cassing W, Mosel U 1998 Phys. Lett. B 424 244
[29] Ritman J et al., FOPI Collaboration 1995 Z. Phys. A 352 355
[30] Herrmann H 1999 Prog. Part. Nucl. Phys. 42 187
[31] Wang Z S et al. 1997 Phys. Rev. Lett. 79 4096; 1999 Eur. Phys. J. A 5 275