Quantification of stochastic fragmentation of self-gravitating discs

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ABSTRACT

Using 2D smoothed particle hydrodynamics, we investigate the distribution of wait times between strong shocks in a turbulent, self-gravitating accretion disc. We show the resulting distributions do not depend strongly on the cooling time or resolution of the disc and that they are consistent with the predictions of earlier work (Young & Clarke 2015; Cossins et al. 2009, 2010). We use the distribution of wait times between shocks to estimate the likelihood of stochastic fragmentation by gradual contraction of shear-resistant clumps on the cooling time scale. We conclude that the stochastic fragmentation mechanism (Paardekooper 2012) cannot change the radius at which fragmentation is possible by more than ~ 20%, restricting direct gravitational collapse as a mechanism for giant planet formation to the outer regions of protoplanetary discs.

1 INTRODUCTION

It is widely believed that planets can form via fragmentation of disc gas only in the outermost regions of protoplanetary discs (i.e. > 50 A.U., Rafikov 2005; Clarke 2009; Rice & Armitage 2009). This mechanism is thus favoured for explaining planets in wide orbits as detected via direct imaging (Marois et al. 2008, 2010) for which the time to form by the alternative core accretion model is uncomfortably long. Core accretion (i.e. the assembly of a rock core and subsequent accretion of disc gas) is therefore favoured for the formation of the vast majority of gas giant planets detected to date.

The argument against forming planets by fragmentation in the inner disc hinges on the fact that self-gravitating discs are very optically thick at smaller radii which renders $\beta$ (the ratio of cooling time to dynamical time) very large. A large number of numerical experiments have shown that whether a self-gravitating disc fragments or whether it is maintained in a self-regulated (non-fragmenting) state depends on the value of $\beta$. We will not here repeat the considerable debate about the exact value of the critical $\beta$ value for fragmentation, nor its dependence on numerical technique and resolution (Rice et al. 2003, 2005; Gammie 2001; Meru & Bate 2011, 2012; Lodato & Clarke 2011; Young & Clarke 2015; Rice et al. 2012, 2014). Suffice it to say that the range of values of $\beta_{\text{crit}}$, claimed in the literature ($3 \sim 30$), corresponds to a rather narrow range of minimum radii for fragmentation because the radial dependence of $\beta$ in the outer regions of a steady state self-gravitating disc is steep ($\propto r^{-4.5}$) (Rafikov 2005, 2009; Clarke 2009). All of the above numerical experiments refer to prompt fragmentation (i.e. fragmentation that occurs within a cooling time of the disc being set up in a self-gravitating state). Paardekooper (2012) however, reported a distinct fragmentation mode in long timescale integrations using the grid code FARGO 2D. In this experiment the disc apparently settled into the self-regulated state and then eventually formed a fragment after a large number of cooling times. He dubbed this alternative mode of fragmentation “stochastic fragmentation”.

The prospect of eventual fragmentation even at high $\beta$ values could potentially overshadow the conclusions drawn above about the impossibility of forming planets by gravitational fragmentation at small radii. Given that the self-gravitating lifetime of a disc is $> 10^5$ dynamical times at the radius of Jupiter, it follows that an extremely small (but non-zero) probability of fragmentation per dynamical time could lead to planet formation by this route.

The possibility of stochastic fragmentation in self-gravitating media has also been discussed by Hopkins & Christiansen (2013). In their picture, stochastic fragmentation may result from the collapse of extremely rare nonlinear density fluctuations on a scale much smaller than the Jeans length; their study quantifies the probability of this outcome using the statistics of isothermal turbulence. Although those studies that have attempted to quantify the power-spectrum of the turbulence have found little power at sub-Jeans scales (Boley et al. 2007; Cossins et al. 2009), we cannot rule out the possibility that such a mechanism is operative in self-gravitating discs (indeed no simulations performed to date have the extremely sub-Jeans length resolution required for this analysis). However, this is definitely not the mechanism that is operating in the simulations of Paardekooper (2012). Here collapse is initiated on the Jeans scale (as expected) and the proto-clump contracts on the (long) cooling time of the simulation. With such a very slow contraction, the expected outcome is that proto-clumps are disrupted by collisions with the spiral arms. The “stochas-
tic” (and unexpected) outcome of the Paardekooper simulation was that clumps can occasionally avoid such collisions over a sufficiently long time window that they manage to collapse.

We have argued above that it is essential to quantify the probability of stochastic fragmentation before ruling out fragmentation of the inner disc. Such quantification is however extremely difficult through direct simulation. By definition, stochastic fragmentation simulations must employ large $\beta$ values in order to avoid the well studied prompt fragmentation regime. Spiral structure is weak at large $\beta$ (Cossins et al. 2009) and so high resolution is required in order that weak spiral heating is not swamped by artificial viscosity (Lodato & Clarke 2011). The nature of stochastic fragmentation also implies that long integration times (many cooling times) are required. The combination of high resolution and long integration times implies that direct simulation is not likely to be a feasible strategy in the foreseeable future.

Here we adopt a different approach. We argued in Young & Clarke (2015) that the $\beta$ threshold for prompt fragmentation can be simply understood in terms of the mean spiral structure in self-gravitating discs. (Cossins et al. 2009) presented a detailed characterisation (wavenumbers, pattern speed, pitch angles) of the spiral modes present in such discs which allows the calculation of the mean time between spiral arm passages. Young & Clarke (2015) interpreted the fragmentation boundary as corresponding to the criterion that a proto-fragment can cool on a timescale less than this mean inter-spiral time; if this condition is not fulfilled then proto-fragments are disrupted by spiral arm passage before they collapse to the small scales where they are immune to such disruption.

If this were the full story then there would be no possibility of stochastic fragmentation at larger $\beta$. However a casual inspection of simulations of self-gravitating discs is enough to show that individual spiral features can dissolve and re-form on a timescale that is close to dynamical. Although the Cossins et al. (2009) description is a reasonable representation of the time-averaged behaviour, it does not address the possible variability in the inter-spiral time. In the fluctuating density field encountered in the simulations it may be possible for individual proto-fragments to avoid spiral shocks over significantly longer periods.

In this paper we therefore analyse the intermittent nature of spiral structure in self-gravitating discs, recording the distribution of inter-spiral times for fluid elements in the disc. We expect from Cossins et al. (2009) that the geometry (though not the amplitude) of spiral features is not a strong function of either $\beta$ or resolution but test this hypothesis with a range of simulations. We emphasise that we are not seeking to model fragmentation itself (and choose simulation parameters with sufficiently high $\beta$ that prompt fragmentation is avoided) but instead use the Lagrangian nature of SPH to analyse particle histories and infer the distribution of wait times between spiral arm encounters. In going on to draw conclusions about the survival prospects of any proto-fragments that might eventually form in the disc, we will be assuming that such fragments would be sampling the same fluctuating density field that we analyse here.

2 DISC MODEL

We construct a disc with a power law surface density profile, assume a locally isothermal Gaussian in the vertical direction with scale height $H$ and set the initial temperature such that $Q = Q_0$ at all radii. To ensure the simulation is scale free and the resolution $h/H$ and aspect ratio $H/R$ are independent of radius, we choose a surface density power law index of $-2$ (Young & Clarke 2015). This choice gives,

$$\Sigma = \frac{M_D}{2\pi \log(\xi) R^2}$$

(1)

c_s = \sqrt{\frac{GM}{R} \frac{qQ_0}{2\log(\xi)}}$$

(2)

where $\Sigma$ is the surface density, $M_D$ the disc mass, $q = M_D/M$, $c_s$ is the sound speed of the gas, and $Q_0$ is the initial value of $Q$, $\xi = R_0/R_1$ where $R_0$ and $R_1$ are the outer and inner radii of the disc respectively. The disc’s aspect ratio is,

$$\frac{H}{R} = \frac{qQ_0}{2\log(\xi)}$$

(3)

and the ratio of SPH smoothing length to disc scale height, $h/H$ is,

$$\frac{h}{H} = \left(\frac{8\pi\log(\xi)^3}{Nq^2Q_0^2}\right)^{1/2}$$

(4)

Our simulations were performed using the same modified version of GADGET2 (Springel 2005) used by Young & Clarke (2015). The code modifications include artificial conductivity (Monaghan 1997), $\beta$ cooling (i.e. loss of internal energy on a timescale that is a fixed multiple ($\beta$) of the local dynamical time), particle accretion and the correct treatment of softening with variable smoothing lengths (Price & Monaghan 2007) and the artificial viscosity method of Cullen & Dehnen (2010). We set the artificial viscosity (and conductivity) to the values that produce the best results in test problems where the correct result is known (e.g., shock tube test, Sedov blast wave, Kelvin-Helmholtz instability).

That is, we set $\alpha_{\text{cond}} = 1.0$, $\alpha_{\text{max}} = 5.0$, $\alpha_{\text{min}} = 0.0$ and $l = 0.05$ (Cullen & Dehnen 2010). Note that the high value of $\alpha_{\text{max}}$ translates in practice to an average per-particle value of $\alpha_{\text{SPH}} \sim 0.1$ away from shocks. In order to include the effects of the vertical distribution of mass in our calculation of gravity, we softened all gravitational interactions on the scale $H$ using the same method as (Young & Clarke 2015).

All our simulations were run in 2D, with $\xi = 5$ and $q = 0.2$ to minimise computational expense (Young & Clarke 2015). Each simulation was run for at least 10 cooling times at the outer edge, ensuring that the disc reached a “settled” state. We limited our analysis to the final $500t_{\text{dyn}}$ of each simulation ($200t_{\text{dyn}}$ for the highest resolution run), where the disc was in the $Q \sim 1$ gravo-turbulent state.

3 RESULTS

Paardekooper (2012) and Young & Clarke (2015) interpreted stochastic fragmentation as arising from unstable over-densities that survive for long enough to become bound and resist disruption. That is, the gravo-turbulent, $Q \sim 1$
state is constantly producing gravitationally unstable overdensities. The majority of these overdensities are disrupted before they can become bound enough to survive disruption by the environment. If an over-density is “lucky” and can survive for long enough without being disrupted, it will form a fragment.

Paardekooper (2012) found that over-densities consistent with this explanation could be found at all points in high resolution simulations which attain $Q \sim 1$ but do not promptly fragment and that the number of these “potential fragments” declined as $1/\beta$. This heuristic picture was further developed by Young & Clarke (2015), who proposed that the survival of clumps depended on their ability to survive encounters with spiral shocks. That is, for fragmentation to occur, an over-density must be able to collapse sufficiently before encountering a spiral shock. On average, the time between successive spiral shocks can be shown to be, 

$$t_{spi} = \frac{2\pi}{m\xi} t_{dyn} \quad (5)$$

where $t_{dyn} = \Omega^{-1}$, $m$ is the azimuthal wavenumber of the spiral pattern, $\Omega$ is its pattern speed and $\xi = |\Omega_p - \Omega|/\Omega$ (Young & Clarke 2015). Realistic disc assumptions yield $t_{spi} = 9 \times 10^2$, although Equation 5 is only likely to be accurate to about a factor 2.

To quantify the likelihood of stochastic fragmentation, we measured the distribution of times between successive spiral shock wave encounters for many particles within our quasi-stable simulations. Because reliably detecting shock fronts is numerically challenging (Cullen & Dehnen 2010; Morris & Monaghan 1997), we only consider the relatively strong shocks in the disc. We tracked individual particles over roughly 45, 90, 180 or 360 dynamical times and marked all periods of increase in the particle’s entropy and surface density (some of which spanned multiple snapshots) as potential encounters with shocks. We deemed a shock to have been encountered whenever a particle’s entropy and surface density increased by more than the median increase across all potential shocks. We limited our analysis to particles within $R = 2 - 3R_h$ to avoid corruption by edge effects.

Figure 1 shows the resulting distributions of wait time between shocks for simulations at a range of resolutions, values of $\beta$ and particle tracking times. There is no significant difference in the average wait times between simulations at different resolutions or with different cooling rates. Furthermore, the distributions are converged with respect to the length of time a particle is tracked for, indicating that neither increased resolution nor longer simulation runs would change the result.

Most importantly, all our simulations show an exponential decay in the likelihood of a patch of disc remaining unshocked, with long periods of time (100s of dynamical times or more) having effectively zero probability in all cases. This is shown more clearly in Figure 2, which shows the log$_{10}$ cumulative probability of surviving for longer than a certain number of dynamical times without encountering a shock. The slope change at the extreme tail of each distribution is caused by low number statistics resulting from our finite sample of particles and finite tracking time. However, it is clear that the probability of a clump surviving for longer than $\sim 10t_{dyn}$ without encountering a strong shock decreases exponentially.

We now relate this wait time distribution to the fragmentation probability per unit time at radius $R$ ($P_{frag}(R)$). This involves assumptions about the numbers of “potential fragments” produced per unit time and also a criterion for the collapse of such potential fragments. In regard to the former, Paardekooper (2012) found that the frequency with which such clumps form scales as $1/\beta$. Regarding the collapse criterion, we first assume that a fragment can collapse only if it does not encounter a shock within a cooling timescale of formation. Since such clumps collapse quasistatically on the cooling time, this criterion is equivalent to requiring that the fragment is able to change its mean density by order unity before encountering a spiral shock. That is, it is well within its Hill sphere at the point that it receives...
It is clear from Figure 3 that the transition between fragmenting and non-fragmenting takes place over a very narrow range of radii. This is true whether we require a clump to meet a density cut-off or to survive for a cooling time before we consider it to have fragmented: indeed the two criteria are visually nearly indistinguishable in Figure 3 because by the time one enters the high $\beta$ regime at small radius where the two criteria diverge, the probabilities are vanishingly small. We thus conclude that stochastic fragmentation cannot decrease the maximum radius at which fragmentation is possible by more than a few 10$s$ of percent.

4 DISCUSSION

In this study, we have chosen to only perform 2D simulations where higher resolution in $h/h$ can be obtained more easily. It has recently been shown that 2D simulations are problematic for studying fragmentation, due to the effect gravitational softening has on preventing pressure-supported collapse (Young & Clarke 2015). However, it is important to note that the suppression of fragmentation in 2D results entirely from the modification to the gravitational force on small scales. Our conclusions rely entirely on results derived from the large scale spiral structure of the disc, which is unaffected by any small scale differences in the gravitational force law between 2D and 3D. As such, we expect the distribution of times between spiral wave encounters to be unchanged in 3D.

Figures [1] and [2] show that the spiral morphology of the disc, and hence the time between successive shock fronts, does not vary significantly with $\beta$ or resolution. This is consistent with the findings of Cossins et al. (2009), who investigated the spiral geometry of 3D gravito-turbulent accretion discs and found no $\beta$ or resolution dependence. To the extent to which there is any trend with $\beta$ and resolution, it is towards times between shock fronts becoming shorter at higher resolution and/or $\beta$, which would only strengthen our conclusions (i.e. make stochastic fragmentation at small radii more difficult).

Naturally, the cooling rates in a realistic disc are not likely to be well described by cooling at constant $\beta$ once the fragments enter the regime of non-linear collapse. This consideration is unlikely to have a major bearing on our analysis since what is relevant is the cooling regime while the fragments’ over-density with respect to the surroundings is still relatively modest and where the cooling rates are therefore not expected to deviate greatly from those appropriate to the background disc. Following Cossins et al. (2010), who examined the effect of temperature dependent cooling on prompt fragmentation, we would not expect this to facilitate fragmentation apart from close to strongly temperature dependent features in the opacity law, e.g. due to ice sublimation (see also Johnson & Gammie 2003). Although this enhanced cooling of over-dense regions may bring the fragmentation boundary inwards by a modest factor, it is unlikely to change our conclusions regarding the sharpness of this transition.

In focusing on the time between spiral wave passages, we have assumed that contracting clumps are disrupted by shocks. In apparent conflict with this assumption, it has been shown that when fragmentation is immediate, frag-
mments arise out of the dense post-shock gas that makes up the spiral arms (Rogers & Wadsley 2012). However, in this scenario the post-shock gas is the seed of the instability (which then rapidly contracts to form a fragment), but does not promote the contraction of an existing clump. It is possible that the dense post-shock gas is the location where the gravitational instability is most commonly triggered. Nonetheless, for stochastic fragmentation to occur, this unstable patch of disc must contrast significantly from its “birth density”. Paardekooper (2012) found no evidence that the stochastic fragments formed in his simulations were driven to fragment by spiral wave encounters. Additionally, if it were the case that spiral waves promoted clump contraction, it is difficult to see how any gravitationally unstable patch of disc could fail to progress into a fragment.

Figure 1 shows that the most common wait time between shocks is \( \sim 4 - 5 t_{\text{dyn}} \). This suggests that \( m_\xi \approx 1 \) in Equation 5 in good agreement with the findings of Cossins et al. (2009). Note that the “fragmentation boundary” measured by Meru & Bate (2011, 2012; Rice et al. 2012, 2014) corresponds to the point where the probability of surviving for more than \( \beta t_{\text{dyn}} \) becomes low, not to the most common wait time. Given this, our results suggest the fragmentation boundary should be in the range \( \beta = 7 - 12 \), although the uncertainty in the number of over-dense regions formed per dynamical time (see below) prevent us from placing stronger constraints.

Paardekooper (2012) found that clumps formed in the gravo-turbulent state for all values of \( \beta \) up to \( \beta \approx 50 \), but was only able to find stochastic fragmentation up to \( \beta \approx 20 \). The probability of a simulation fragmenting in its lifetime is approximately the probability shown in Figure 2 multiplied by the number of over-dense clumps that form in the simulations lifetime. Paardekooper (2012) found roughly one clump per \( \sim 50 t_{\text{dyn}} \) in his \( \beta = 9 \) simulations and found two simulations fragmented after \( \sim 600 \) and \( \sim 800 t_{\text{dyn}} \) while two simulations remained stable for \( 1000 t_{\text{dyn}} \). This implies a probability of fragmentation per clump of \( \sim 2/68 \approx 3\% \) \((3400/50 = 68 \text{ clumps in } 3400 t_{\text{dyn}}) \), which is broadly consistent with the \( \sim 10\% \) predicted by Figure 2. By \( \beta = 20 \), Figure 2 predicts that only \( \sim 0.5\% \) of clumps will fragment, suggesting fragmentation is unlikely but not impossible.

We therefore conclude that stochastic fragmentation only modifies the “fragmentation boundary” in radius by \( \sim 20\% \) (see Figure 5) and rules out the formation of giant planets via any type of gravitational collapse (stochastic or otherwise), except in the outermost parts of protoplanetary discs. This agrees with the conclusions of earlier disc fragmentation studies (Rice et al. 2005; Meru & Bate 2012; Rice et al. 2014; Paardekooper et al. 2011). The only remaining avenue for fragmentation at low radii is via extremely large sub Jean’s length density fluctuations (Hopkins & Christiansen 2013), which cannot presently be directly probed computationally.

5 CONCLUSION

In this paper we have used simple, 2D SPH simulations of self-gravitating discs to place hard, quantitative limits on the values of the cooling time parameter, \( \beta \), for which massive discs can stochastically fragment. We find that the average time any patch of disc spends between spiral wave encounters is broadly consistent with the findings of Young & Clarke (2015) and that the stochastic nature of the disc’s spiral structure can only increase this time by at most a factor of a few.

Consistent with earlier studies (Cossins et al. 2009), we find that the distribution of wait times between strong shocks is independent of both resolution and \( \beta \). Furthermore, the distribution of wait times decreases exponentially for wait times beyond \( \sim 15 t_{\text{dyn}} \). Combining this finding with the expected \( \beta \propto R^{-9/2} \) scaling of cooling time with radius, we find that stochastic fragmentation is unable to modify the radius at which fragmentation is possible by more than a few 10s of percent. This finding restricts direct collapse as a mechanism for giant planet formation to the outer parts of protoplanetary discs.

6 MATERIALS & METHODS

In the interests of reproducibility and transparency, all code and data used in performing this work have been made freely available online at https://bitbucket.org/constantAmateur/discfragmentation.

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