Design and implementation of a personal mobility of single spherical drive

Tasuku Hoshino, Miki Yazawa, Ryota Naganuma and Kotaro Takada

Department of Mechanical Engineering, Tokyo University of Science, Suwa,
5000-1 Toyohira, Chino, Nagano 391-0292, Japan
E-mail: hoshino@rs.suwa.tus.ac.jp

Abstract. This paper deals with a personal electric vehicle driven by a single spherical wheel. Using an appropriate feedback control, this driving strategy realizes dynamic stability in all directions and the vehicle can always be kept upright on the road surface of variety of slopes. It also enables immediate mobility to all directions, unlike personal vehicles of two-wheel type. The spherical wheel is driven by omnidirectional wheels as usual; however, since the number and location of wheels have huge effect on the driving performance, the authors firstly analyze kinematics of omnidirectional wheels and sphere and derive new configuration to achieve maximum power. Based on the kinematic analysis, the equation of motion of the vehicle is derived via Lagrangian formulation. The full dynamic model including kinematic constraints is then derived. Using the full model, a stabilizing controller for driving is designed based on partial feedback linearization technique. The vehicle is constructed and tested with a human driver. The proposed configuration of omnidirectional wheels, the controller design model and the control scheme are examined in practice. Results of the experiments, including going over uphill road and uneven ground, show much better driving performance than authors’ previous prototype of the similar.

1. Introduction

Single passenger electric vehicles, especially which are called personal mobility, are getting popular and increasing their importance in our society [2]. They provide means of energy-efficient assistance for short-distance transportation of person. One of currently available such vehicles is 4-wheel type for the elderly. Though the vehicle weighs more than the driver and the center of mass are positioned low in order to preserve static stability while in motion, its stability margin is still small and it sometimes tips over on a sloping surface. The same applies also to i-REAL, Toyota’s 3-wheel vehicle [3].

On the other hand, dynamically-stable mobility devices have an advantage on this point; well-known 2-wheel upright vehicles, Segway and Winglet, e.g., achieve higher stability though the mass center are located upper than 4-wheel vehicles. However, the property and even the immediate moving direction are limited back and forth. Dynamically-stable mobile robots on a sphere fully exploit the high stability in all directions. Ballbot [5, 7] is one of such robots and is designed with a small base and similar height to humans in order that it can easily make its way in a human environment and interact with people. Other similar robots has been developed [6, 8].

In order to comply with the small-size and light-weight requirements for personal mobility,
the authors has developed a small electric vehicle on a single spherical wheel driven by three omnidirectional wheels, utilizing the dynamic stability [1]. It was named OmniRide and is shown in Figure 1(b). Through many mechanical, electrical and controller revisions over a year after completing construction, it is finally stabilized with a human driver and serves as a personal mobility. OmniRide can move freely in any direction the driver wishes, while standing still. However, in order to use it on public road, further improvements turned out to be necessary especially on the driving performance. From the static mechanics point of view, the maximum driving power is determined by the number and location of omnidirectional wheels.

Based on the observation, the authors newly developed the second prototype of similar vehicle OmniRide 2, whose spherical wheel is driven by four omnidirectional wheels. This paper describes OmniRide 2 from mechanical, electrical and mathematical point of view and derives the stabilizing controller. Similar to the previous, since OmniRide 2 has asymmetric shape, development of the complete dynamic model is essential for design of practical controllers. Section 3 describes the modeling process in detail.

2. Vehicle description

OmniRide 2 consists of a single spherical wheel and a rigid vehicle body is simply mounted on it. Figure 1(a) shows the appearance. Whole size is similar to a bicycle, but the body weighs about 35 kg. The spherical wheel is a 1.5 mm-thick steel spherical shell of 300 mm diameter, covered by 6 mm-thick urethane. It can rotate about any of three orthogonal axes using four omnidirectional wheels located under the chassis. Each omnidirectional wheel is driven by an AC servomotor through gear train. A bicycle saddle and handlebars are fixed on the chassis. Each end of the bars is equipped with a multifunctional lever.

Only tasks needed for driving are to shift driver’s weight back and forth or sideways, to turn the handlebars and to pull the multifunction levers. For example, OmniRide 2 turns into operating mode by pulling either lever twice quickly while holding the other. It starts moving by releasing the lever. Shifting driver’s weight determines moving direction and speed. Turning handlebars changes vehicle direction. Pulling either lever twice leaves the operating mode and turns off the control.

The major electrical components are listed in Table 1 and their connection is shown in Figure 2. Four 200 W AC servomotors with servo drivers are used in current control mode; thus each output torque can be used as control input. The vehicle body configuration and its angular velocity are measured by an inertial measurement unit (IMU). A mini-ITX sized board
### Table 1. Major electrical components

| Component                                      | Part number         | #  | Manufacturer                      |
|------------------------------------------------|---------------------|----|-----------------------------------|
| AC servomotor (200 W)                         | TS4607N3302E620     | 4  | Tamagawa Seiki Co., Ltd.          |
| AC servo driver                                | TA8410N7518E297     |    |                                   |
| Communication unit                             | TA8433N211          | 1  |                                   |
| 3 axes inertial measurement unit               | AU7428N2100         | 1  |                                   |
| PC/mini-ITX (Geode LX800)                      | ALIX.1E             | 1  | PC Engines GmbH                   |
| PCI pulse counter & parallel I/O               | LPC-631204          | 1  | Interface Corp.                   |
| LiPo battery (22.2 V, 4.2 Ah)                  | LG335-4200-6S       | 6  | Hyperion HK Ltd.                  |

**Figure 2.** Electric schematic diagram

PC is used as an embedded controller. Two sets of lithium-polymer batteries supply the power for the system and the motors.

Physical parameters of OmniRide 2 are listed in Table 2. All values except two radii are obtained in the mass properties generated by 3D CAD software. Symbols used in the following sections are listed in Table 3 and vectors are shown in Figure 3.

### 3. Mathematical description of the vehicle

#### 3.1. Kinematics of a sphere and omnidirectional wheels

Let us begin with describing kinematics of a sphere driven by omnidirectional wheels. Assume that there is no slipping between the sphere and omnidirectional wheels in tangential direction.
Table 2. Physical parameters of OmniRide 2

| Physical parameter                     | Symbol | Value              |
|----------------------------------------|--------|--------------------|
| Radius of spherical wheel              | $r_s$  | $1.55 \times 10^{-1}$ m |
| Radius of omnidirectional wheel        | $r_o$  | $4.50 \times 10^{-2}$ m |
| Height of COG of vehicle body          | $s_z$  | $1.64 \times 10^{-1}$ m |
| Mass of vehicle body                   | $m_b$  | $3.42 \times 10^1$ kg |
| Mass of spherical wheel                | $m_s$  | $5.40 \times 10^0$ kg |
| Moment of inertia of vehicle body (with respect to center of spherical wheel) | $I_{bxx}$ | $1.94 \times 10^0$ kg m$^2$ |
|                                        | $I_{bxy}$ | $-0.03 \times 10^{-1}$ |
|                                        | $I_{bxz}$ | $-1.40 \times 10^{-1}$ |
|                                        | $I_{byy}$  | $2.72 \times 10^0$ |
|                                        | $I_{byz}$  | $0.00 \times 10^{-1}$ |
|                                        | $I_{bzz}$  | $1.67 \times 10^0$ |
| Moment of inertia of spherical wheel (with respect to its center) | $I_s$  | $8.18 \times 10^{-2}$ kg m$^2$ |
| Moment of inertia of omnidirectional wheel (about axis; including gears, shafts and rotor) | $I_o$  | $9.77 \times 10^{-4}$ kg m$^2$ |

Table 3. Symbols used in mathematical modeling

| Vehicle body (subscript: $b$) | Spherical wheel (subscript: $s$) |
|-------------------------------|----------------------------------|
| RPY angles                    | Downward radius $z_s$            |
| Angular velocity              | Rotational angle $\theta_s$      |
| Translational velocity        | Angular velocity $\omega_s$      |
| Inertia tensor                | Translational velocity $v_s$      |
| COG position                  | Inertia tensor $I_s$              |
|                               | Driving Torque $\tau_s$          |
| Omnidirectional wheel ($k$: 1, $\cdots$, 4) | Constants                      |
| Rotational angle              | Gravity acceleration $g$         |
| Angular velocity              | $n$-by-$n$ identity matrix $I_n$ |
| Driving torque                | $n$-by-$n$ zero matrix $O_n$     |
| Contact point position        | $m$-by-$n$ zero matrix $O_{m \times n}$ |
| Unit tangent at contact point |                                  |
| Radius at contact point       |                                  |

Since omnidirectional wheels are free in binormal direction, the translational velocities of the first omnidirectional wheel in the tangential direction and the sphere at the contact point coincide. It is written using scalar triple products as follows:

$$b_\omega_1 \times b_r_o \cdot b_t_1 = b_\omega_s \times b_p_1 \cdot b_t_1.$$  \hfill (1)

The upper left $b$ indicates that the vector is represented in the body frame. Rewriting (1) gives

$$\dot{b}_\theta_1 = b_p_1 \times b_t_1 \cdot b_\omega_s / b_r_o.$$  \hfill (2)
where \( r_o = |r_o| \). Combining similar equations for the other three omnidirectional wheels,

\[
\dot{\theta}_o = T^b\omega_s, \quad T := \frac{1}{r_o} \begin{bmatrix}
(b_1 \times b_2)^T \\
(b_2 \times b_3)^T \\
(b_3 \times b_4)^T \\
(b_4 \times b_1)^T 
\end{bmatrix}
\] (3)

is obtained; \( \dot{\theta}_o := [\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3, \dot{\theta}_4]^T \) denotes rotational velocities of the omnidirectional wheels.

The principle of virtual work states virtual work done by omnidirectional wheels and that done by sphere satisfy

\[
\tau_o^T \delta \theta_o = \tau_s^T \delta \theta_s, \quad (4)
\]

where \( \tau_s := [\tau_x, \tau_y, \tau_z]^T \) is a vector of the sphere torques and \( \tau_o := [\tau_1, \tau_2, \tau_3, \tau_4]^T \) is a vector of the torques of omnidirectional wheels. Combining (3) and (4), \( \tau_s \) determines \( \tau_o \) of minimum norm using the right generalized inverse of \( T \) as

\[
\tau_o = T(T^TT)^{-1}\tau_s. \quad (5)
\]

Since there exists maximum rotational speed for each omnidirectional wheel, maximum angular speed \( |\omega_s| \) for specific direction can be computed by evaluating (3). Similarly, (5) determines maximum driving torque \( |\tau_s| \) based on the maximum torque of each omnidirectional wheel. Figure 4 is these driving performance obtained in two cases: (i) using three omnidirectional wheels and (ii) using four wheels.

When driven by three wheels as of the first OmniRide, both driving speed and driving force in back-and-forth direction is less that those of sideways. Values of their product are 1.5 and 2 in each directions, which means only 1.5 and 2 of 3 motor power are utilized. On the other hand, when driven by four wheels, both directions have the same performance and their product becomes 4. It means 4 of 4 motor power are utilized in these directions.
According to the analysis above, the authors mounted four omnidirectional wheels on OmniRide 2 symmetrically about these two directions. Since each wheel is driven by 200 W AC servomotor, driving performance of 800 W power is expected, which is more than twice of that of the first one.

3.2. Vehicle kinematics

Let $\alpha$, $\beta$, and $\gamma$ be yaw, pitch, and roll angles of the vehicle body and take these angles as configuration variables. The rotation matrix $R_b$ that transforms a vector represented in the body coordinate into that in the base coordinate is

$$R_b = \text{Rot}(Z, \alpha)\text{Rot}(Y, \beta)\text{Rot}(X, \gamma),$$

where Rot($A$, $\theta$) denotes a rotation matrix of angle $\theta$ about axis $A$. Since the center of gravity (COG) of the body is a fixed point in the body frame, $b\mathbf{s}_b = [0, 0, s_z]$ is constant. The body COG in the base frame is then given by

$$s_b = R_b b\mathbf{s}_b.$$  

(7)

The body angular velocity results from three rotations about $X$, $Y$, and $Z$ axes. Taking (6) into account, it is represented in the base frame as

$$\omega_b = \begin{bmatrix} 0 \\ 0 \\ \dot{\alpha} \end{bmatrix} + \text{Rot}(Z, \alpha) \begin{bmatrix} 0 \\ \dot{\beta} \\ 0 \end{bmatrix} + \text{Rot}(Z, \alpha)\text{Rot}(Y, \beta) \begin{bmatrix} \dot{\gamma} \\ 0 \\ 0 \end{bmatrix}.$$  

(8)

Since $b\omega_s$ is relative to the vehicle body, angular velocity of the spherical wheel in the base frame is given by

$$\omega_s = b\omega_s + \omega_b.$$  

(9)

Assuming no slipping between the spherical wheel and the floor, the translational velocities of the spherical wheel and the vehicle body in the base coordinate are computed by

$$v_s = \omega_s \times z_s, \quad v_b = \frac{ds_b}{dt} + v_s.$$  

(10)
3.3. Vehicle dynamics

Once the vehicle kinematics is developed, a set of equations of motion can be systematically derived through the Lagrange formulation.

The spherical wheel is symmetric and its inertial tensor is given by a diagonal matrix

\[ \hat{I}_s = \hat{I}_s = I_3, \]

where hatted (\( \hat{\} \)) symbols denote inertia tensors or moments with respect to their center of mass. The kinetic energy is a sum of those of rotational and translational motion and is given in quadratic form:

\[ K_s = \frac{1}{2} \hat{I}_s \omega_s^T \omega_s + \frac{1}{2} m_s v_s^T v_s. \]  

(11)

The kinetic energy of the vehicle body is similarly given by

\[ K_b = \frac{1}{2} (R_b^{-1} \omega_b)^T b \hat{I}_b (R_b^{-1} \omega_b) + \frac{1}{2} m_b v_b^T v_b. \]  

(12)

Since the inertia tensor remains constant only in the body frame and the entries of only \( b \hat{I}_b \) (or \( bI_b \)) are known, the body angular velocity \( \omega_b \) is transformed into the body frame in (12) and the quadratic form is computed there. The potential energy of the vehicle body is negative of the work done by gravity:

\[ U_b = -m_b g^T s_b. \]  

(13)

Using the Lagrangian of the whole system

\[ L := K_s + K_b - U_b, \]  

(14)

the equations of motion are given by

\[ \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} = \tau, \]  

(15)

where \( \theta := [\gamma, \beta, \alpha, b \theta_x, b \theta_y, b \theta_z]^T \) is a vector of the configuration variables and \( \tau := [0, 0, 0, \tau_x, \tau_y, \tau_z]^T \) is a vector of the generalized forces. (15) can be written in a matrix form of

\[ M(\theta) \ddot{\theta} + h(\theta, \dot{\theta}) = \tau, \]  

(16)

where \( M(\theta) \in \mathbb{R}^{6 \times 6} \) is the inertia matrix and \( h(\theta, \dot{\theta}) \in \mathbb{R}^6 \) is the vector of Coriolis, centrifugal and gravity forces. \( M(\theta) \) is non-singular.

In the derivation of (16), slipping between the spherical wheel and the floor only in horizontal directions is considered in (10). The other condition on slipping around vertical axis should also be taken into account. Since the spherical wheel is assumed not to rotate about the vertical axis, \( \omega_{sz} = 0 \) holds; where \( \omega_{sz} \) denotes rotational speed of the sphere around vertical axis. In matrix form,

\[ J_s(\theta) \dot{\theta} = 0, \quad J_s(\theta) := \frac{\partial \omega_{sz}}{\partial \dot{\theta}}. \]  

(17)

Equation (17) serves as a nonholonomic, algebraic constraint with respect to \( \theta \) and \( \dot{\theta} \) upon the motion governed by (16). Hereafter, dependence of matrices and vectors to \( \theta \) or \( \dot{\theta} \) are omitted for notational simplicity.

The constrained dynamics is described by a set of the algebraic equation (17) and the differential equations (16) with additional constraint force as follows:

\[ \begin{cases} M\ddot{\theta} + h = \tau + J_s^T \lambda, \\ J_s \dot{\theta} = 0. \end{cases} \]  

(18)
The constraint force $J^T \lambda$ is determined by Lagrange’s method of undetermined multipliers. By differentiating the second equation of (18), a block matrix equation

$$
\begin{bmatrix}
M & -J^T \\
J_s & 0 \\
\end{bmatrix}
\begin{bmatrix}
\dot{\theta} \\
\lambda \\
\end{bmatrix}
= 
\begin{bmatrix}
\tau - h \\
-J_s \dot{\theta} \\
\end{bmatrix}
$$

is obtained. Since $M$ is non-singular, the block matrix inversion exists and the multiplier $\lambda$ is determined as:

$$
\lambda = (J_s M^{-1} J_s^T)^{-1} \left\{ J_s M^{-1} (h - \tau) - J_s \dot{\theta} \right\}.
$$

By substituting (20) into the first equation of (18), the complete nonlinear equations of motion for the OmniRide 2 is finally given by

$$
M \ddot{\theta} + (I_6 - J^T \Phi J_s M^{-1}) \dot{h} + J_s^T \Phi J_s \dot{\theta} = (I_6 - J^T \Phi J_s M^{-1}) \tau, \quad \Phi := (J_s M^{-1} J_s^T)^{-1}.
$$

The concrete, analytical description of (21) is also available by using symbolic computation tools for equations, e.g., Mathematica, Maxima, or others.

### 3.4. Linear approximate model

Let a vector of the RPY angles of the vehicle body be $\theta_b := [\gamma, \beta, \alpha]^T$ and a vector of the rotational angles of the sphere be $\theta_s := [\theta_x, \theta_y, \theta_z]^T$. Taking $[\theta_b^T, \theta_s^T]^T := x \in \mathbb{R}^9$ as the state variables, the linear approximation of (21) around an equilibrium point $x^* := O_{9 \times 1}$ is computed and can be rewritten in a block matrix form:

$$
\begin{bmatrix}
M_{11} & M_{12} \\
M_{12}^T & M_{22} \\
\end{bmatrix}
\begin{bmatrix}
\dot{\theta}_b \\
\dot{\theta}_s \\
\end{bmatrix}
+ 
\begin{bmatrix}
H_1 \\
O_3 \\
\end{bmatrix}
\theta_b = 
\begin{bmatrix}
B_1 \\
B_2 \\
\end{bmatrix}
\tau_s,
$$

where the constant component matrices are analytically given by

$$
M_{11} := \begin{bmatrix}
I_s + I_{bxx} + 2m_b s_z r_s & I_{bxy} & I_{bzz} \\
I_{bxy} & I_{byy} + 2m_b s_z r_s & I_{byz} \\
I_{bzz} & I_{byz} & I_{bzz} \\
\end{bmatrix},
$$

$$
M_{12} := \begin{bmatrix}
I_s + m_b s_z r_s & 0 & 0 \\
0 & I_s + m_b s_z r_s & 0 \\
0 & 0 & \hat{I}_s \\
\end{bmatrix},
$$

$$
M_{22} := \begin{bmatrix}
I_s & 0 & 0 \\
0 & I_s & 0 \\
0 & 0 & \hat{I}_s \\
\end{bmatrix},
$$

$$
H_1 := -m_b s_z g \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0 \\
\end{bmatrix},
$$

$$
B_1 := \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & -1 \\
0 & 0 & 0 \\
\end{bmatrix},
$$

$$
B_2 := \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0 \\
\end{bmatrix}.
$$

The entries of the inertia tensor and the moment of inertia are defined using those with respect to their COG:

$$
\begin{bmatrix}
I_{bxx} & I_{bxy} & I_{bzz} \\
I_{bxy} & I_{byy} & I_{byz} \\
I_{bzz} & I_{byz} & I_{bzz} \\
\end{bmatrix} = \hat{b} I_b + m_b s_z \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0 \\
\end{bmatrix},
$$

$$
I_s := \hat{I}_s + (m_s + m_b) r_s^2.
$$

Repartitioning state variables and matrices and defining $\theta_1 := [\gamma, \beta]^T \in \mathbb{R}^2$ and $\theta_2 := [\alpha, \theta_x, \theta_y]^T \in \mathbb{R}^3$, the linearly approximated model of OmniRide 2, whose state-space realization is controllable, is described as follows:

$$
\begin{bmatrix}
M_{11} & M_{12} \\
M_{12}^T & M_{22} \\
\end{bmatrix}
\begin{bmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2 \\
\end{bmatrix}
+ 
\begin{bmatrix}
H_1 \\
O_{3 \times 2} \\
\end{bmatrix}
\theta_1 = 
\begin{bmatrix}
O_{2 \times 3} \\
B_2 \\
\end{bmatrix}
\tau_s,
$$

(23)
where
\[
\mathbf{M}_{11} := \begin{bmatrix}
I_s + I_{bxx} + 2m_b s_z r_s & I_{bxy} \\
I_{bxy} & I_s + I_{byy} + 2m_b s_z r_s
\end{bmatrix},
\mathbf{M}_{12} := \begin{bmatrix}
I_{bxy} & I_{bzz} - m_b r_z s_z \\
I_{bzz} - m_b r_z s_z & 0
\end{bmatrix},
\mathbf{M}_{22} := \begin{bmatrix}
I_{bxx} & 0 & 0 \\
0 & I_s & 0 \\
0 & 0 & I_s
\end{bmatrix},
\mathbf{H}_1 := -m_b s_z g I_2, \quad \mathbf{B}_2 := \begin{bmatrix}
0 & 0 & -1 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}.
\]

The repartition of state variables is intending convenience for controller design purposes which is described in the next section. Basically, for the variables in \( \theta_1 \), position control is required in stabilizing OmniRide 2. For the variables in \( \theta_2 \), on the other hand, their derivatives are subject to control. For convenience, the upper half equation in (23) is hereafter denoted (23a), and the other half is denoted (23b).

4. Controller design

4.1. Position PID servo for the sphere

Since the matrices in (23) contain only kinematic and inertial parameters and no friction effects are modeled, there should exist a significant input disturbance to \( \tau_s \). In order to make the (outer-loop) stabilizing controller design simple, the well-known resolved acceleration control is used, localizing the disturbance effect elsewhere (23).

Firstly, the control input \( \tau_s \) in (23) is changed to \( \hat{\theta}_2 \). Replacing (23b) with \( \hat{\theta}_2 = u \) leads to
\[
\begin{bmatrix}
\mathbf{M}_{11} & \mathbf{O}_{2 \times 3} \\
\mathbf{O}_{3 \times 2} & I_3
\end{bmatrix}
\begin{bmatrix}
\hat{\theta}_1 \\
\hat{\theta}_2
\end{bmatrix}
+ \begin{bmatrix}
\mathbf{H}_1 \\
\mathbf{O}_{3 \times 2}
\end{bmatrix} \theta_1 = -\begin{bmatrix}
\mathbf{M}_{12} \\
I_3
\end{bmatrix} u. \tag{24}
\]

The removed equation will be used later to compute \( \tau_s \). The stabilizing controller based on (24) will generate \( u \), that is the acceleration reference for \( \theta_2 \):
\[
\hat{\theta}_{2d} = \begin{bmatrix}
\hat{\theta}_{xd} \\
\hat{\theta}_{yd}
\end{bmatrix} = u = \begin{bmatrix}
u_1 \\
u_2 \\
u_3
\end{bmatrix}. \tag{25}
\]

Here, the second subscript \( d \) indicates that it is the desired value. Integration of the acceleration reference generates velocity and position references for the sphere respectively:
\[
b\omega_{xd} = \int_0^t u_2 dt, \quad b\theta_{xd} = \int_0^t b\omega_{xd} dt, \quad b\omega_{yd} = \int_0^t u_3 dt, \quad b\theta_{yd} = \int_0^t b\omega_{yd} dt.
\]

By constructing a position PID servo for sphere rotations except around vertical axis, the modified acceleration reference \( \tilde{u} \) is given by
\[
\tilde{u} = \begin{bmatrix}
u_1 \\
u_2 + K_I \int_0^t (b\theta_{xd} - b\theta_x) dt + K_P (b\theta_{xd} - b\theta_x) + K_D (b\omega_{xd} - b\omega_x) \\
u_3 + K_I \int_0^t (b\theta_{yd} - b\theta_y) dt + K_P (b\theta_{yd} - b\theta_y) + K_D (b\omega_{yd} - b\omega_y)
\end{bmatrix}, \tag{26}
\]
where
\[
b\theta_x = \int_0^t b\omega_x dt, \quad b\theta_y = \int_0^t b\omega_y dt.
\]
The angular velocity of sphere in (26) is determined based on rotational velocity of four omnidirectional wheels using the left generalized inverse of $T$ as the least square estimate:

$$\begin{bmatrix}
  \dot{b}_x \\
  \dot{b}_y \\
  \dot{b}_z 
\end{bmatrix} = \dot{b}_\omega = (T^T T)^{-1} T^T \dot{\theta}_o. \tag{27}$$

Using $\tilde{u}$, input disturbance due to mainly unmodeled friction effects is expected to be localized. $K_P$, $K_I$, and $K_D$ are feedback gain constants.

Finally, solving (23a) for $\ddot{\theta}_1$ and substituting it into (23b), the torque command for the sphere is computed via $\tilde{u}$ as follows:

$$\tau_s = \mathbf{B}^{-1} \left( \mathbf{M}_{22} - \mathbf{M}_{12} \mathbf{M}_{11}^{-1} \mathbf{M}_{12} \right) \tilde{u} - \mathbf{B}^{-1} \mathbf{M}_{12} \mathbf{M}_{11}^{-1} \mathbf{H}_1 \dot{\theta}_1, \tag{28}$$

and that for each omnidirectional wheel is computed as:

$$\tau_o = T(T^T T)^{-1} \tau_s + \dot{\hat{I}}_o \ddot{\theta}_o. \tag{29}$$

4.2. Stabilizing controller

The purpose of control while OmniRide 2 is moving, is to regulate the vehicle body upright and keep the heading direction unchanged; rotation of the spherical wheel does not concern. To achieve this purpose, a 5th-order subspace

$$\mathbf{x}_d := [\theta_1^T, \dot{\theta}_1^T, \dot{\alpha}]^T \in \mathbb{R}^5$$

of $\mathbf{x}$ is selected as the state space and the corresponding subsystem of (24):

$$\dot{x}_d = A_d x_d + B_d u \tag{30}$$

where

$$A_d := \begin{bmatrix}
  O_2 & I_2 & O_{2 \times 1} \\
  -\mathbf{M}_{11} \mathbf{H}_1 & O_{1 \times 2}
\end{bmatrix}, \quad B_d := \begin{bmatrix}
  O_{2 \times 3} \\
  -\mathbf{M}_{11} \mathbf{M}_{12}
\end{bmatrix}.$$ is taken as the controller design model. An optimal static feedback control

$$u = K_d x_d \tag{31}$$

which minimizes a quadratic criterion function

$$J_d = \int_0^\infty (x_d^T Q_d x_d + u^T u) dt \tag{32}$$

is used to stabilize (30); where $Q_d$ is a weighting matrix. The overall controller structure is shown in Figure 5. Under this feedback control, the driver can change the speed and moving direction of OmniRide 2 by shifting his/her weight back and forth or sideways, as stated before.

Breaking control can be implemented in a similar manner to the way described above. Taking another 7th-order subspace

$$\mathbf{x}_b := [\theta_1^T, \dot{\theta}_1^T, \dot{\theta}_2^T]^T \in \mathbb{R}^7$$

of $\mathbf{x}$ and zeroing it result in stopping the vehicle motion while keeping the body upright.
5. Control experiments
Real-time Linux is installed on the mini-ITX PC together with a kernel-2.4 based Linux distribution. The control software is coded in C language and developed using various programming tools. The generated executable codes run on the real-time Linux with control period of 0.6 ms. The feedback controller is designed using GNU/Octave.

Driving experiments with a human driver were carried out; the driver tried to track a three meters square trajectory while keeping the vehicle direction the same. Figures 7 are the experimental results. The vehicle path in Figure 7(a) shows that OmniRide 2 is able to move both back and forth and sideways. As seen in Figure 7(b), since the roll and pitch angles are regulated within 2 degrees, the vehicle body is kept upright during experiment. While yaw angle varies up to 10 degrees, it depends on the driver’s skill. These results validate the mathematical model and the model-based controller and show the availability of OmniRide 2 as a personal mobility.

Figure 5. The controller structure for OmniRide 2; numbers in parentheses indicate the corresponding equations.

Figure 6. Driving experiments with a human driver
Figure 7. A driving experiment of OmniRide 2: the driver tried to track a 3 meters square trajectory while keeping the heading direction the same.

6. Conclusion
The authors proposed a personal mobility of single spherical drive which effectively utilizes the dynamic stability. The second prototype, OmniRide 2, has been constructed and the complete model and a model-based feedback controller were presented in this paper. Their values were examined through experiments, which shows feasibility of a personal mobility of single spherical drive.

References
[1] Hoshino T, Yokota S and Chino T 2013 Int. Conf. Robotics, Automation and Embedded Systems
[2] Kamata M and Shino N 2006 IATSS Research 30 1 52
[3] Toyota Personal Mobility 2016 http://www.toyota-global.com/innovation/personal_mobility/
[4] Honda Robotics 2016 http://world.honda.com/UNI-CUB/
[5] Lauwers T, Kantor G and Hollis R 2006 Proc. Int. Conf. on Robotics and Automation
[6] Kumagai M and Ochiai T 2009 Proc. Int. Conf. on Robotics and Automation
[7] Dynamically-Stable Mobile Robots in Human Environments 2016 http://www.msl.ri.cmu.edu/projects/ballbot/
[8] Endo T and Nakamura Y 2005 Int. Conf. on Advanced Robotics