COMPLEX SYSTEMS WITH IMPULSIVE EFFECTS AND LOGICAL DYNAMICS: A BRIEF OVERVIEW

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ABSTRACT. In the past decades, complex systems with impulsive effects and logical dynamics have received much attention in both the natural and social sciences. This historical survey briefly introduces relevant studies on impulsive differential systems (IDSs) and logical networks (LNs), respectively. To begin with, we investigate five aspects of IDSs containing fundamental theory, Lyapunov stability, input-to-state stability, hybrid impulses and delay-dependent impulses. Next, we compactly summarize the research status of some problems of LNs including controllability, stability and stabilization, observability and current research. Moreover, some significant applications of proposed results are illustrated. Finally, based on this overview, we further discuss some future work on complex systems with impulsive effects and logical dynamics.

1. Introduction. Complex systems research plays an increasingly important role in both the natural and social sciences. Due to the varied complexities, such as system dynamics complexity, statistical complexity and so on, some complex systems involve more than one disciplines [40]. From the perspective of control, the complexity of system dynamics has been widely investigated in last decades. Among system dynamics, both the impulsive effects and logical dynamics have attracted increasing attention recently. Hence, it is of interest and importance to investigate the complex systems with impulsive effects and logical dynamics.

It is known that there exists the phenomenon of transient disturbances or sudden jump in many dynamical systems. Compared with the whole evolution processes, this phenomenon lasts for a short time, hence, one can assume that the states change instantaneously at certain instants. This phenomenon is often viewed as the impulsive effect, and the system with the impulsive effects can be described as an impulsive differential system (IDS) [101, 21, 137, 34, 96, 78]. In practice, the impulsive effects can be found in many fields, such as the mechanical systems [130], economic control systems [28], attitude control systems [2, 45]. Although the impulsive effects are always regarded as the disturbances in
nature, we can also use this characteristic to design a control method, which is called the impulsive control method. In fact, impulsive control theory has been developed and applied widely during the past decades. The method of impulsive control can date back to [127], since the impulsive effects on stability were considered therein. Later, the model of IDS was firstly proposed in [42], and its relevant fundamental theories were proposed as well. Generally speaking, this book possesses substantial theoretical and practical value of studies on IDSs. In 2001, the concept of impulsive control was introduced by Yang Tao in [156]. Then, impulsive control systems received much attention and were investigated by many researchers [120, 141, 20, 29, 90, 91].

Actually, impulsive controller can be viewed as a controller with a simple structure that aims to change state trajectories at certain discrete instants. Note that the impulsive control method can overcome the difficulty that some systems cannot be implemented by continuous inputs [85, 80, 97, 76, 83, 75]. For example, the saving rate of a bank is not suitable to adjust all the time, and continuous input may result in antimicrobial resistance problem. Therefore, impulsive control seems to be a better choice to deal with these situations. For some more special systems, impulsive control method may be the only option to acquire desired performance. Moreover, compared with some continuous control method, the closed-loop system under impulsive control may possess better robustness and efficiency. Thus, there emerges much interesting work on the impulsive control systems in many fields [22, 136, 79, 71, 81, 23, 166].

On the other hand, there exist a lot of logical dynamics in both the life sciences and network communication engineering, and logical networks (LNs) can be well used to simulate and analyze dynamic behaviors. LNs are a kind of discrete-time systems, and all of variables including their states, outputs and inputs, only can take value from finite fields. Moreover, each feedback function is a logical function that composes of logical operators such as “and”, “or” and “negation”. The most important feature of LNs is no parameters over other systems such as continue systems and multi-agent systems, because LNs more emphasize on general principles rather than quantitative details. According to different finite domains, LNs naturally can be divided into different types. Specifically, when the variables take value from different finite domains, then LNs are called mixed-valued LNs [35]. While, the variables take value from the same finite domains, then LNs are called multi-valued LNs [111]. Especially, if the finite domain is binary, i.e., \{0, 1\}, then multi-valued LNs become Boolean networks (BNs) [38]. Due to the conceptual simplicity and the universality of applications, BNs have been studied much more thoroughly. Moreover, many results of BNs are also available on mix-valued LNs and multi-valued LNs. Akutsu et al. in [1] first proposed the concept of Boolean control networks (BCNs). Subsequently, many kinds of control strategies were introduced into BCNs, such as state feedback control [110], pinning control [118], sampled control [105] and so forth. Therefore, BNs and BCNs are the most representative and major research models of LNs. BNs are usually used to model gene regulatory networks (GRNs) since 1969 [38], and promote the development of GRNs. BNs can also be classified by different switching signals. When switching signals are probabilistic, then the corresponding systems are called probabilistic BNs [131]. Especially, if the probability is uniform distribution, then probabilistic BNs become switched BNs [61, 58], that are a special type of probabilistic BNs [33]. When the switching signals are stochastic, BNs are called stochastic BNs [150, 149]. While if there are no switching signal, then the systems are generality BNs.

In this paper, some basic theory and recent progress of the complex systems with impulsive effects and logical dynamics are briefly reviewed. Specifically, this investigation intends to introduce the work on two type of complex systems, i.e., IDSs and LNs. The rest
of this paper is arranged as follows. In Section 2, five topics of IDSs containing fundamental theory, Lyapunov stability, input-to-state stability, hybrid impulses and delay-dependent impulses are systemically introduced. Section 3 discusses five aspects of LNs, i.e., basic theory, controllability, stability and stabilization, observability and current research are proposed. Some applications of complex systems with impulsive effects and logical dynamics are illustrated in Section 4. Finally, Section 5 concludes this survey and discusses the further work.

Notations: Let $\mathbb{R}$ and $\mathbb{R}_+$ denote the set of real numbers and nonnegative real numbers, respectively. We denote $\mathbb{Z}_+$ and $\mathbb{N}$ the set of positive integer and nonnegative integer, respectively. Let $\mathbb{R}^n$ be the $n$-dimensional real spaces equipped with the Euclidean norm $| \cdot |$. Let $\mathbb{R}^{n \times m}$ denote the $n \times m$-dimensional real spaces. The notation $A > (\leq) 0$ implies that the matrix $A$ is positive (negative) definite matrix. The notations $A^T$ and $A^{-1}$ presents the transpose and the inverse of $A$, respectively. The matrix set $M_{m \times n}$ denotes the set of $m \times n$ real matrices. If $A$, $B$ are symmetric matrices, $A > B$ means that $A - B$ is positive definite matrix. For any interval $J \subseteq \mathbb{R}$, let $S \subseteq \mathbb{R}^k (1 \leq k \leq n)$, $C(J, S) = \varphi : J \rightarrow S$ is continuous and $C^1(J, S) = \varphi : J \rightarrow S$ is continuously differentiable. A function $\alpha : [0, \infty) \rightarrow [0, \infty)$ belongs to class $\mathcal{K}$ if $\alpha$ is continuous, strictly increasing, and $\alpha(0) = 0$. In addition, if $\alpha$ is unbounded, it belongs to class $\mathcal{K}_\infty$. A function $\beta : [0, \infty) \times [0, \infty) \rightarrow [0, \infty)$ belongs to class $\mathcal{K}_{\infty}$ if $\beta (\cdot, t)$ belongs to $\mathcal{K}$ for every fixed $t \geq 0$ and $\beta (r, t)$ decreases to 0 as $t \rightarrow \infty$ for every fixed $r \geq 0$. In addition, $\otimes$ denotes the Kronecker-product. Let $[t]$ represent the maximum integer of no more than $t$.

2. Complex systems with impulsive effects. In this section, we focus on the investigation of the complex system with impulsive effects. Moreover, we will model it by an impulsive differential system (IDS) and further discussions will be proposed. Namely, five aspects, that is, fundamental theory, Lyapunov stability, input-to-state stability, hybrid impulses and delay-dependent impulse will be taken into consideration.

2.1. Fundamental theory.

2.1.1. General model description. An impulsive differential system (IDS) can be described as follows:

$$\begin{cases}
\dot{x}(t) = f(t, x(t)), (t, x) \notin \Sigma_t, \\
\Delta x(t) = J(t, x), (t, x) \in \Sigma_t,
\end{cases}$$

(1)

where $x(t)$ and $\dot{x}(t)$ denote the system state and its right-hand derivative, respectively. And the state can be influenced by impulsive effects, namely, $x(t_k^+) = x(t_k) + J(t_k, x(t_k))$, when $(t_k, x(t_k)) \in \Sigma_t$. It is assumed that $x(t_0^+) = x(t)$, i.e., the solutions to system (1) are right continuous. In addition, $J(t_k, x(t_k)) \in \mathbb{R}^n$ denotes the difference value of the state $x(t)$ at $t_k$. In general, in order to investigate the stability of IDS (1), we also assume that the impulsive instant sequence $\{t_k\}$ satisfying that $t_k \in \mathbb{R}_+, t_k < t_{k+1}, k \in \mathbb{Z}_+, \lim_{k \rightarrow +\infty} t_k = +\infty$ and the uniqueness of solutions to system (1) with initial condition can be assured on interval $[0, +\infty)$. From above model description, one can observe that IDS (1) consists of three parts, that is,

i) an ordinary differential equation that characterize the the dynamic behaviors of system (1) between two consecutive impulsive instants;

ii) a difference equation that describe the instantaneous changes of states at impulsive instant;

iii) a regulation that determine when the impulsive effect happen.
Actually, one can conclude that the solution to IDS (1) may have following three situations:

1) a continuous function, if the state does not reach set $\Sigma$ or difference value $J(t_k, x(t_k)) = 0$;
2) a piecewise continuous function with finite number of discontinuities, which belong to the first kind, if the number of times that the state reaches set $\Sigma$ is finite and corresponding $J(t_k, x(t_k)) \neq 0$;
3) a piecewise continuous function possessing a countable number of discontinuities, which belong to the first kind, if the number of times that the state reaches set $\Sigma$ is countable and corresponding $J(t_k, x(t_k)) \neq 0$.

Generally, on the basis of the choice of impulsive instants, IDSs can be categorized into following three types that are of interest.

A type-I IDS with impulsive effects at fixed time:

\[
\begin{aligned}
\dot{x}(t) &= f(t, x(t)), \quad t \neq t_k, \\
\Delta x(t) &= J(t, x(t)), \quad t = t_k.
\end{aligned}
\]

In this model, the impulsive effects occur in a given time sequence $\{t_k\}$. It is clear that the solution to (2) is piecewise continuous with finite or countable number of discontinuities if $J(t_k, x(t_k)) \neq 0$. In fact, this type is the most studied by scholars.

A type-II IDS with impulsive effects at variable time:

\[
\begin{aligned}
\dot{x}(t) &= f(t, x(t)), \quad \mathcal{S}(t, x) \neq 0, \\
\Delta x(t) &= J(t, x(t)), \quad \mathcal{S}(t, x) = 0.
\end{aligned}
\]

In this situation, impulsive effects are determined by both the time and states, that is when the state trajectory of IDS (3) reaches the set $\mathcal{S}(t, x) = 0$, then the impulsive effect occurs. In fact, we can rewrite the set $\Sigma$ of system (1), i.e., $\Sigma = \{(t, x)|\mathcal{S}(t, x) = 0\}$. Compared with type-I IDS, type-II IDS seems to be more complex and more difficult to deal with.

A type-III autonomous system with impulses:

\[
\begin{aligned}
\dot{x}(t) &= f(x(t)), \quad x \notin \Sigma, \\
\Delta x(t) &= J(x), \quad x \in \Sigma.
\end{aligned}
\]

In this case, one can see that the state $x(t)$ is not explicitly dependent on $t$ and the difference equation satisfying that $J(t, x) = J(x)$. It is clear that the impulsive effects of this type is state-dependent, and due to this fact, there exist some very interesting examples of type-III IDS (see [42]).

2.1.2. Comparison with ordinary differential systems. Actually, the studies on impulsive differential systems (IDSs) are more abundant than that of ordinary differential systems without impulsive effects. For instance, there maybe exist no solution with certain initial value in certain IDS even when the corresponding ordinary differential system possesses a solution for arbitrary initial value; some basic characteristics such as continuous dependence with respect to initial value may no longer hold, and Lyapunov stability of IDSs may need a modified definition. Furthermore, a simple IDS may have some new behaviors that are of interest. For example, one can see rhythmical beating, merging of solutions, and noncontinuability of solutions in certain IDS. Specifically, we propose the following IDSs to illustrate that the continuability of certain solutions can be tremendously impacted by
impulsive effects. Consider such an IDS:
\[
\begin{align*}
\dot{x} &= 0, \ t \neq k, \ k \in \mathbb{Z}_+, \\
\Delta x &= 1 \frac{1}{x-1}, \ t = k.
\end{align*}
\] (5)

We can conclude that its solution \(x(t)\) with initial value \(x(0) = 1\) can exist only for \(0 < t < 1\). Actually, this solution is not continuable for \(t > 1\), due to the fact that the function \(\Delta x = \frac{1}{x-1}\) make no sense when \(x = 1\). Conversely, the solutions \(x(t)\) to the corresponding ordinary differential system \(\dot{x} = 0\) with arbitrary initial value can exist for all \(t > 0\). From this example, one can conclude that the impulses may destroy the continuability of certain solutions to original system.

Consider another IDS:
\[
\begin{align*}
\dot{x} &= 1 + x^2, \ t \neq k\frac{\pi}{4}, \ k \in \mathbb{Z}_+, \\
\Delta x &= -1, \ t = k\frac{\pi}{4}.
\end{align*}
\] (6)

We can obtain that the solution \(x(t)\) with initial value \(x(0) = 0\) can not be continued for all \(t > 0\). Further, the solution can be derived as follows, \(x(t) = \tan(t - k\frac{\pi}{4}), \ t \in (k\frac{\pi}{4}, (k+1)\frac{\pi}{4}]\), for all \(k \in \mathbb{Z}_+\). Nevertheless, the corresponding ordinary differential system possesses the solution \(x(t) = \tan(t)\) that can not be continued for \(t = \frac{\pi}{2}\) since \(\lim_{t \to (\frac{\pi}{2})^-} x(t) = \infty\). Hence, this example illustrates that the impulsive effects may enhance the continuability of certain solutions to original system.

Recall the assumption that the impulsive instant sequence \(\{t_k\}\) satisfies that \(t_k \in \mathbb{R}_+, \ t_k < t_{k+1}, \ k \in \mathbb{Z}_+\) and \(\lim_{k \to \infty} t_k = +\infty\). One can observe that this assumption indicates that in system (1), these impulsive effects can not destroy the continuability of solutions to original system. In fact, in order to study Lyapunov stability more effectively, we assume this condition holds generally.

2.2. Lyapunov stability. For the study on Lyapunov stability of impulsive differential systems (IDFs), the approach of constructing Lyapunov function is still popular with many researchers.

For stability analysis, note that the major difference between impulse-free systems and IDFs is that we have to deal with the uncontinuity of the Lyapunov function at impulsive instants for IDFs. In [132], a Lyapunov inverse theorem was obtained to analyze the stability of IDFs. In detail, this result presents a necessary condition of exponentially stability for IDFs, that is, there exists a discontinuous Lyapunov function \(V(t, x)\) such that it is nonincreasing at the impulsive instants and its derivative is negative between successive impulsive instants. However, under certain situation, it may be not easy to select a single Lyapunov function that meets such a necessary condition. Then, [133] developed some stability and boundedness criteria for IDFs by so-called perturbing Lyapunov functions which can traced back to [41].

Discuss the following IDS:
\[
\begin{align*}
\dot{x} &= f(t, x), \ t \neq t_k, \ k \in \mathbb{Z}_+, \\
x(t_k) &= g(t_k, x(t_k^-)), \ t = t_k,
\end{align*}
\] (7)

where \(t \in \mathbb{R}_+, \ t_k < t_{k+1}\) and \(\lim_{k \to \infty} t_k = \infty\), and we denote such type of impulsive instant sequence by set \(\mathfrak{F}_0\). We suppose that \(f(t, 0) = 0, g(t, 0) = 0\) for all \(t \in \mathbb{R}_+\).
Definition 2.1. [83, 84] Assume that $x(t)$ is the solution to system (7). Then the trivial solution is said to be

- stable, if for any $\varepsilon > 0$, there exists a positive constant $\delta = \delta(\varepsilon, t_0)$ such that for any $|x_0| < \delta(\varepsilon, t_0)$, then $|x(t, t_0, x_0)| < \varepsilon$, $t > t_0$.
- uniformly stable, if the same $\delta(\varepsilon)$ in the above applies for all $t_0$
- uniformly asymptotically stable, if the trivial solution is uniformly stable, and for any $t_0 > 0$ and $\varepsilon > 0$, there exist $\sigma(t_0) > 0$ and $T(\varepsilon, x_0) > 0$, such that $|x_0| < \delta$ implies that $|x(t, t_0, x_0)| < \varepsilon$ for any $t > t_0 + T$, namely, $\lim_{t \to \infty} x(t, t_0, x_0) = 0$.
- exponentially stable, if there exist some $M > 0$, $\lambda > 0$ such that

$$\|x(t)\| \leq M|x_0|e^{-\lambda(t-t_0)},$$

for all $t \geq 0$.

For simplicity, system (7) is said to be exponentially stable if its trivial solution is exponentially stable. Note that this definition is dependent on the selection of the impulsive instant sequence $\{t_k\}$. Further, we are interested in characterizing Lyapunov stability over classes of sequences in $\mathcal{F}_0$. In this subsection, we mainly talk about some results on exponential stability.

Definition 2.2. [84] System (7) is said to be uniformly exponentially stable (UES) over the class $\mathcal{F}_0$ if there exist $M > 0$, $\lambda > 0$ such that the estimate (8) holds for each $\{t_k\} \in \mathcal{F}_0$.

Before presenting some interesting results on UES, some necessary notations are given as follows.

Definition 2.3. [84] $V : \mathbb{R}_+ \times \mathbb{R}^n \rightarrow \mathbb{R}_+$ is said to be in $\mathcal{V}_0$ if

1. $V(t, x)$ is continuous in $(t_{k-1}, t_k) \times \mathbb{R}^n$ and $\lim_{(t,y) \to (t_k^+,x)} V(t,y) = V(t_k^+, x)$.
2. $V$ is locally Lipschitz in $x$ and $V(t, 0) = 0$ for all $t > 0$.
3. there exist positive scalars $a_1$, $a_2$ and $p$ such that $a_1|x|^p \leq V(t, x) \leq a_2|x|^p$ holds for all $x \in \mathbb{R}^n$.

Moreover, in order to characterize the frequency of impulsive effects, [119] introduced the concept of average impulsive interval (AII) which is inspired by that of average dwell time.

Definition 2.4. [119] (Average Impulsive Interval) For the impulsive instant sequence $\mathcal{F}_0 = \{t_1, t_2, \ldots\}$, if there exist positive integer $N_0$ and positive number $T_a$, such that

$$\frac{T-t}{T_a} - N_0 \leq N(T,t) \leq \frac{T-t}{T_a} + N_0, \forall T \geq t \geq t_0,$$

where $N_0 > 0$ is called the chatter bound and $N(T,t)$ denotes the number of impulses occur in interval $(t, T]$, then the average impulsive interval (AII) is equal to $T_a$.

Further, based on this concept, a significant result was derived in [119] as follows.

Lemma 2.5. [119] Suppose that there exist function $V \in \mathcal{V}_0 : \mathbb{R} \times \mathbb{R}^n \rightarrow \mathbb{R}^+$, such that following conditions hold

$$D^+V(t, x) \leq -cV(t, x), \quad \forall (t, x) \in [t_{k-1}, t_k) \times \mathbb{R}^n,$$

$$V(t_k, g(t_k, x(t_k^+))) \leq aV(t_k, x(t_k^+)), \quad k \in \mathbb{Z}_+,$$

for all $x \in \mathbb{R}^n$ and $k \in \mathbb{Z}_+$. If all of impulsive instant sequence $\mathcal{F}_0$ satisfies that

$$-cT_a + \ln \alpha < 0,$$

then, system (7) is globally UES over the class $\mathcal{F}_0$. 

Remark 1. In fact, before concept of AII came along, many investigators tried to obtain the stability criterion of IDSs by restricting the upper and lower bounds of impulsive interval [159, 24]. It is clear that compared with these results, Lemma 2.5 possesses less conservativeness by the concept of AII. Furthermore, note that Lemma 2.5 holds regardless of $c > 0$ or $c < 0$ as long as condition (11) is satisfied.

Recently, the concept of AII was generalized by [142], i.e., the limiting form of average impulsive interval that is of interest. For convenience, we call this limiting form the generalized average impulsive interval (GAII).

Definition 2.6. [142] (Generalized Average Impulsive Interval) The generalized average impulsive interval (GAII) of impulsive sequence $\mathcal{F}_1 = \{t_1, t_2, \cdots \}$ is equal to $T_{ga} > 0$ if

$$\lim_{t \to \infty} \frac{t - t_0}{N(t, t_0)} = T_{ga}, \quad (12)$$

where $N(t, t_0)$ denotes the number of impulses occur in interval $[t_0, t]$. It is clear that condition (12) is weaker than (9), since from condition (9), we can also obtain the GAII $T_{ga} = \lim_{t \to +\infty} \frac{t - t_0}{N(t, t_0)}$. Furthermore, Definition 2.6 allows the case that GAII $T_{ga} = +\infty$, which is not considered in Definition 2.4, and this situation can happen when impulsive effects are infinite but sparse. For example, if $N(t, t_0) = |t - t_0|$, we obtain that GAII $T_{ga} = 1$; but if $N(t, t_0) = [0.1 \sqrt{t - t_0}]$, then it yields that GAII $T_{ga} = +\infty$, which is depicted in Fig. 1.

Although the concept of GAII is more general than that of AII, by Definition 2.6 we can only derive the “attractability”, but not the “stability” of corresponding IDS. Namely, we can only obtain the information of the state as $t \to +\infty$ based on condition (12). In fact, according to the Definition 2.6, we can always select a impulsive instants sequence which satisfies the condition (12), but the corresponding IDS becomes unstable. Specifically, we can better understand it by following example.

Example 1. Discuss a simple IDS as follows:

$$\begin{align*}
\dot{x} &= -x, \ t \neq t_k, \ k \in \mathbb{Z}_+,
\Delta x &= x(t_k^-),
\end{align*} \quad (13)$$

then, we can select a function $V(t, x) = |x(t)|$, such that following conditions hold

$$(H_1) \quad D^\gamma V(t, x) \leq -V(t, x), \quad \forall (t, x) \in [t_{k-1}, t_k] \times \mathbb{R}^n,$n

$$(H_2) \quad V(t_k, g(t_k, x(t_k^-))) \leq 2V(t_k, x(t_k^-)), \quad k \in \mathbb{Z}_+, \quad (14)$$

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{An example of GAII $T_{ga} = +\infty$.}
\end{figure}
for all \( x \in \mathbb{R}^n \) and \( k \in \mathbb{Z}_+ \). By applying Lemma 2.5, one can conclude that IDS (13) is globally UES over the class \( \mathcal{F}_0 \) if \( A^{II} \) satisfies that \( T_a > \ln 2 \). Now, we give an impulsive instant sequence \( \{t_k\} \) such that \( t_k = k, \ k \in \mathbb{Z}_+ \), which implies that \( A^{II}T_a = 1 \). We can check that all conditions in Lemma 2.5 holds, hence IDS (13) is globally UES over the sequence \( \{t_k\} \) (see thin solid line in Fig. 2). However, we can not claim that IDS (13) is globally UES over the class \( \mathcal{F}_1 \) when \( G^{II} \) satisfies that \( T_{ga} > \ln 2 \). In fact, there exist \( \epsilon = 1, T = 1 \) and sequence \( \{t^0_k\} \) such that for arbitrary small number \( \delta > 0, |x_0| < \delta \), it yields that \( x(T) > 1 \). Specifically, we can select a finite number of impulsive instants in interval \([0, 1]\) such that the impulsive effects can make the state \( x(1) > 1 \) (see thick solid line in Fig. 2). Then, after \( t > 1 \), let the impulsive instants are the same with \( \{t_k\} \). It can be calculated that \( G^{II} \) of sequence \( \{t^0_k\} \) satisfies condition (12) in Definition 2.6, while system (13) that is attractive, can not be stable over the class \( \mathcal{F}_1 \).

**Figure 2.** Behaviors of IDS (13) with initial value \( x_0 = 0.1 \).

### 2.3. Input-to-state stability

While studying the dynamics of a control system, the influence of external inputs should be characterized and investigated. The concepts of input-to-state stability (ISS) and integral input-to-state stability (iISS), proposed by Sontag, have been extensively applied in demonstrating the impact of the inputs [134, 135]. Considering both the impulsive effects and external input, [30] proposed the appropriate concept of ISS and iISS for IDS. First, we present the IDS with external input in both the continuous dynamics and impulses:

\[
\begin{align*}
\dot{x}(t) &= f(x(t), u(t)), \quad t \neq t_k, \\
x(t_k) &= g\left(x(t_k^-), u(t_k^-)\right),
\end{align*}
\]  

(15)

where \( k \in \mathbb{Z}_+, t_k < t_{k+1} \) and \( \lim_{k \to \infty} t_k = \infty \); the state \( x(t) \in \mathbb{R}^n \) is absolutely continuous between two successive impulsive instants; \( u(t) \in \mathbb{R}^m \) is a locally bounded, Lebesgue-measurable input; and \( f \) and \( g \) are functions: \( \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n \) and \( f \) is locally Lipschitz. Further, the uniqueness of solutions to system (15) with initial condition is assumed on interval \([t_0, +\infty)\) [30].

**Definition 2.7.** [30] Given a sequence \( \{t_k\} \), system (15) is called input-to-state stable (ISS) if there exist functions \( \beta \in \mathcal{K} \mathcal{L} \) and \( \gamma \in \mathcal{K} \mathcal{L}_\infty \), such that for each initial condition and each input \( u \), the solution to (15) satisfies

\[
|x(t)| \leq \beta\left(|x(t_0)|, T - t_0\right) + \gamma\left(\|u\|_{t_0, t}\right), \quad \forall t \geq t_0,
\]  

(16)

where \( \| \cdot \|_J \) presents the supremum norm on interval \( J \).
We can observe that ISS property describes robustness to inputs in the $L_\infty$ sense. Another situation is to discuss the robustness to inputs in the $L_1$ sense, that is iISS.

**Definition 2.8.** [30] Given a sequence $\{t_k\}$, system (15) is called integral input-to-state stable (iISS) if there exist functions $\beta \in \mathcal{K}L$ and $\gamma, \alpha \in \mathcal{K}_\infty$, such that for each initial condition and each input $u$, it yields

$$\alpha(|x(t)|) \leq \beta(|x(t_0)|, t - t_0) + \int_{t_0}^{t} \gamma(|u(s)|) ds + \sum_{t_k \in [t_0, t]} \gamma(|u(t_k^-)|), \quad \forall t \geq t_0.$$  

(17)

Note that Definitions 2.7 and 2.8 are dependent on the selection of the impulsive instant sequence $\{t_k\}$. Further, the researchers are often interested in characterizing ISS and iISS over classes of sequences $\{t_k\}$ in $\mathfrak{G}_0$. Hence, based on All condition, there also are many interesting results for ISS and iISS of IDSs [30, 15, 165, 16, 36, 86]. For instance, [15] investigated ISS for a class of IDS which jump map depends on time. Additionally, [165] considered ISS of IDSs with distributed-delayed impulses. Moreover, [86, 103] studied the effect of delay-dependent impulses on ISS of nonlinear systems. It is shown that the delay in impulses may bring a destabilizing effect to the ISS property of nonlinear systems with delay-dependent impulses. In addition, some interesting results on generalized concepts of ISS, e.g., input/output-to-state stability (IOSS) can be found in [79, 36].

### 2.4. Hybrid impulses.

In general, the research on dynamics of impulsive systems can be categorised into two groups, that is, impulsive perturbation problem (IPP) and impulsive control problem (ICP). If the impulses destroy original performance of the system, e.g., destabilize the stable system, then we call such phenomenon an IPP. Many interesting results on IPP have been derived, such as [82, 31, 152, 74, 73, 158] and references therein. On the contrary, if the impulses bring a certain positive performance to the system, e.g., stabilize the unstable system, then it is regarded as an ICP. Note that an impulsive controller contains two elements, that is, a set of control instants $T = \{t_k\}$, $t_k \in \mathbb{R}$, $t_k < t_{k+1}$, $k = 1, 2, \cdots$ and control laws $U(k, x) \in \mathbb{R}^n$, $k = 1, 2, \cdots$. At each $t_k$, state $x$ is changed impulsively by $x(t_k^+) = x(t_k^-) + U(k, x)$ such that some desired performance (e.g., stability) can be achieved under this impulsive control. We can also find some significant work on ICP (see [85, 80, 97, 76, 83, 75]).

For instance, discuss a simple IDS as follows:

$$\begin{cases} \dot{x} = 0, & t \neq n, \, n \in \mathbb{Z}_+, \\ \Delta x = 2^{(-1)^n} x(t^-), & t = n. \end{cases}$$  

(18)

We can verify that the equilibrium $x = 0$ is stable under impulsive perturbation, (see thin solid line in Fig. 3) hence it belongs to the group of IPP.

Similarly, we consider another IDS

$$\begin{cases} \dot{x} = 0, & t \neq n, \, n \in \mathbb{Z}_+, \\ \Delta x = 2^{-1} x(t^-), & t = n. \end{cases}$$  

(19)

One can check that the equilibrium $x = 0$ turns to be globally asymptotically stable under impulsive control (see thick solid line in Fig. 4). Due to the fact that the corresponding ordinary differential system does not possess such stability property, this belongs to the group of ICP.
Similarly, for stability problem, corresponding impulses can also be divided into three types: stabilizing impulses, destabilizing impulses and neutral impulses. We call the impulses belong to the type of stabilizing impulses, if the impulses can enhance the stability of original systems. On the contrary, we call the impulses belong to destabilizing impulses, if the impulses may destroy the stability of original systems or result in undesired performance. While, another one, i.e., the neutral impulses implies that the impulses do not have impact on the stability property [77]. In fact, in order to study these three types of impulses, three stability criteria for impulsive positive systems were proposed in [32]. Specifically, we take linear impulses $x(t_k) = \mu x(t^-_k)$ as an example, and there emerge three situations:

- If $|\mu| > 1$, the impulses may disorganize the stability property and result in certain undesired behaviors. Thus, this impulses are called destabilizing impulses. It is clear that destabilizing impulses are often regarded as disturbances, and corresponding phenomenon is of IPP.
- If $|\mu| = 1$, the impulses neither suppress nor enhance the stability of IDS since the norm of each state is not changed. Thus, the impulses with impulsive gain $|\mu| = 1$ belong to the type of neutral impulses.
- If $|\mu| < 1$, the impulses can increase the stability of corresponding ordinary differential systems. Thus, we claim that this impulses belong to stabilizing impulses. Further, stabilizing impulses can be considered as a impulsive controller, under which the closed-loop system can achieve the stability property. Hence, corresponding situation is of ICP.

Recall IDS (18), there exist both destabilizing impulses and stabilizing impulses according to the above analysis. Hence, it is natural to ask how to deal with more than one type of impulses simultaneously? In fact, the investigators systematically discussed this question in [142]. Moreover, they call the impulses containing more than one type of impulse the hybrid impulses. Let us see the following example concluding destabilizing impulses, stabilizing impulses and hybrid impulses.

**Example 2.** Consider simple impulsive systems as follows:

$$
\begin{cases}
\dot{x} = 0, \ t \neq n, \ n \in \mathbb{Z}_+,
\Delta x = \alpha x(t^-), \ t = n,
\end{cases}
$$

(20)

with initial condition $x_0 = 3$. Different state trajectories of system (20) for different values of $\alpha$ are depicted in the Fig. 3.

Specifically, we can observe that when $\alpha = 2$, system (20) becomes unstable due to the impulsive perturbation (see thin dashed line in Fig. 3); when $\alpha = 0.5$, system (20) becomes asymptotic stable under the impulsive control (see thick solid line in Fig. 3); when $\alpha = 2^{(-1)^p}$, system (20) possesses stability with hybrid impulses (see thin solid line in Fig. 3).

In order to analyze the hybrid impulses from a holistic perspective, the concept of average impulsive gain (AIG) was introduced in [142].

**Definition 2.9.** [142] (Average Impulsive Gain) The average impulsive gain (AIG) of impulsive instant sequence $\mathfrak{I}_0 = \{t_1, t_2, \cdots\}$ is defined as follows:

$$
\mu = \lim_{t \to +\infty} \frac{|\mu_1| + |\mu_2| + \cdots + |\mu_{N(t,t_0)}|}{N(t,t_0)} > 0,
$$

(21)

where $\mu_i$ is the impulsive gain of the $i$-th impulse, and $N(t,t_0)$ presents the number of impulsive instants during the interval $(t_0, t)$. 

In fact, due to the lack of this concept, the majority of work only investigated either
the destabilizing impulses (|μ_k| < 1) or destabilizing impulses (|μ_k| > 1), and there exist
few results that considered both of them simultaneously. It is permitted that more than one
type of impulses exist in the IDS based on concept of AIG. Further, by this concept, the
synchronization criteria of Lur’e networks with hybrid impulses were obtained in [144].
Hybrid impulses means that the impulses whose impulsive gains are not changeless and
may be different between each node. Namely, the strengths of impulsive effects may be
dependent on both the time and state [145]. Thus, compared with the previous existing
results, hybrid impulses seems to be more general. In addition, some more results on
hybrid impulses can be found in [92, 146].

2.5. Delay-dependent impulses. It is proved in [5] that, in the networked systems under
impulsive control, there inevitably exist communication delays and sampling delays. Gen-
erally speaking, these delays may result from the networked factors, such as controller-to-
actuator delays and sensor-to-controller delays. Further, this phenomenon can be modeled
by the IDS with delay-dependent impulses [138].

In fact, the delay-dependent impulses mean that the impulsive effects are determined by
both their current states and historical states. It is clear that such model is more general
and practical. The effects of delay-dependent impulses on stability of corresponding IDS
have been investigated widely. It should be noted that global existence and uniqueness
are fully studied in [102], which also should be assumed to hold in this subsection. In
[148], the researchers proposed a generalized Halanay differential inequality with respect
to delay-dependent impulses. Based on the work in [148], some researchers investigated
the synchronization problem of coupled memristor-based recurrent neural networks with
delay-dependent impulses in [164]. However, in most existing results, the delay in impulses
is regarded as a negative factor for the stability of the IDS [165]. Then, it is illustrated
that the IDS possesses robustness to small delay in impulses [6]. More interestingly, it
is proved that certain IDS can hold exponentially stability no matter how large the delay
in impulses. Later, [83] investigated the delay-dependent impulsive control of delayed
systems, in which it is proved that the delay in impulses may be beneficial for stability.
Recently, [84] proposed the stability criteria for nonlinear systems with delay-dependent
impulses. Moreover, an interesting example was given to illustrate the effects of the delay
in impulses.
Discuss the following IDS

\[
\begin{aligned}
\dot{x}(t) &= f(t, x(t)), \ t \neq t_n, \ t \geq t_0, \\
x(t) &= g(x(r^-)), \ t = t_n, \ r = r_n \in \mathcal{J}_\{t_n\},
\end{aligned}
\]

(22)

where \( n \in \mathbb{Z}_+, x(t) \in \mathbb{R}^n \) is the system state, \( \dot{x}(t) \) denotes the right-hand derivative of \( x(t) \), and \( x(r^-) \) denotes the left limit at instant \( t \). Note that functions \( f \) and \( g \) are assumed to satisfy suitable conditions so that global existence and uniqueness of solutions to system (22) with given initial condition are guaranteed [102]. For any given impulse sequence \( \{t_n\} \in \mathcal{S}_0 \) and constant \( \delta \in (0, 1] \), a set \( \mathcal{J}_\{t_n\}^\delta \) is introduced below:

\[
\mathcal{J}_\{t_n\}^\delta = \{r_n : r_n = (1 - \delta)t_{n-1} + \delta t_n, n \in \mathbb{Z}_+\}. \tag{23}
\]

System (22) is said to be globally UES over the class \( \mathcal{H}[\mathcal{S}_0, \mathcal{J}_\{t_k\}] \) which consists of the sequences \( \{t_k, r_k\} \), if there exist \( M > 0 \) and \( \lambda > 0 \) such that estimate (8) holds for each \( \{t_k, r_k\} \in \mathcal{H}[\mathcal{S}_0, \mathcal{J}_\{t_k\}] \).

Lemma 2.10. [84] Suppose that there exists a function \( V \in \mathcal{V}_0 : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^+ \), such that following conditions hold

\[
\begin{aligned}
(H_1) \quad &D^+ V(t, x) \leq -eV(t, x), \ \forall (t, x) \in [t_{k-1}, t_k) \times \mathbb{R}^n, \\
(H_2) \quad &V(t_k, g(x)) \leq e^{-\frac{d}{c} t_k} V(r, x), \ r = r_n \in \mathcal{J}_\{t_k\}^\delta,
\end{aligned}
\]

(24)

for all \( x \in \mathbb{R}^n \) and \( k \in \mathbb{Z}_+ \). Then system (22) is globally UES over the \( \mathcal{H}[\mathcal{S}_0, \mathcal{J}_\{t_k\}] \), if \( \{t_k\} \in \mathcal{S}_0 \) satisfies condition (9), and \( \lambda \) constant satisfies one of following conditions:

1) \( T_a > \frac{d}{c0} \), and \( c > 0, \ d < 0 \),

2) \( T_a < \frac{d}{c0} \), and \( c < 0, \ d > 0 \).

Remark 2. Note that in Lemma 2.10, it is required that the delay \( \tau_n = (1 - \delta)(t_n - t_{n-1}) \) and design parameter \( \delta \in (0, 1] \), which implies that the delay will be certainly determined by the impulsive instant sequence \( \mathcal{S}_0 = \{t_1, t_2, \ldots\} \) and the given parameter \( \delta \). Thus, condition (23) seems to be a little conservative. Inspired by this condition, a new concept of average impulsive delay, which overcomes the difficulty that the delays in impulses can be flexible and even beyond the impulsive interval, is proposed in [37]. Moreover, Lemma 2.10 only consider the delay in impulses, but not in continuous dynamics. Hence, an interesting topic is to obtain stability criteria for delayed systems with delay-dependent impulses.

We can see the bilateral effects of time delay in impulses from the following example:

\[
\begin{aligned}
\dot{x}(t) &= -\nu x(t), \ t \neq n, \\
x(t) &= 2^\nu x(t-h), \ t = n,
\end{aligned}
\]

(25)

with initial condition \( x_0 = 3 \), where \( n \in \mathbb{Z}_+, h \geq 0 \) is the time delay in impulses. If \( \nu = 1 \) and \( h = 0 \), one can conclude that system (25) is stable under the delay-free impulsive control (see thin solid line in Fig. 4), however, it turns to be unstable if there exists delay \( h = 0.5 \) in impulses (see thin dashed line in Fig. 4). It illustrates that the time delay in impulses may have negative impact on the stability. Conversely, if \( \nu = -1 \) and \( h = 0 \), system (25) with delay-free impulses is unstable (see thick dashed line in Fig. 4), however, it turns to be stable if there exist delay \( h = 0.5 \) in impulses (see thick solid line in Fig. 4). It is shown that the time delay in impulses may be beneficial for the stabilization.
3. Complex systems with logical dynamics. In this section, we will mainly consider the complex system with logical dynamics. It is known that if value domain is equal to set \( \{0, 1\} \), then logical networks (LNs) become Boolean networks (BNs). Note that BNs have attracted much attention in last decades, and many complex systems with logical dynamics can be modeled by BNs. Hence, five aspects containing basic theory, controllability, stability and stabilization, observability and current research will be introduced.

3.1. Model description and preliminaries. A BN can be described as follows:

\[
\begin{align*}
    x_1(t+1) &= f_1(x_1, x_2, \cdots, x_n), \\
    & \vdots \\
    x_{n-1}(t+1) &= f_{n-1}(x_1, x_2, \cdots, x_n), \\
    x_n(t+1) &= f_n(x_1, x_2, \cdots, x_n).
\end{align*}
\]   

(26)

Here, \( x_i \in \{0, 1\}, i \in \{1, 2, \cdots, n\} \) and \( f_i : \{0, 1\}^n \to \{0, 1\} \) is a logical function. In 2001, Prof. Cheng published a paper on the semi-tensor product (STP) of matrices in [7], announcing the birth of the STP. It is a generalization of convention matrices product, and breaks through the dimension matching condition of matrices product. Specifically, for any given two matrices \( A \in M_m \times n \) and \( B \in M_p \times q \), the STP of these two matrices is defined as

\[
A \bowtie B = (A \otimes I_l)(B \otimes I_l).
\]

Here, \( l = lcm(n, p) \) represents the least common multiple of \( n \) and \( p \). Then, in 2009, Prof. Cheng utilized the method of STP to realize the matrices expressions of logical functions [11], based on which, BNs can further be converted into multi-linear discrete systems, as:

\[
x(t+1) = L \bowtie x(t),
\]

(27)

where \( x(t) = \bigoplus_{i=1}^n x_i \), and \( L \) is called the transition matrix of system (26) and it is a logical matrix. Thus, based on the algebraic expression, a lot of approaches of analysis and control on the modern control theory are applicable to BNs. Early research methods on BNs can not establish an algebraic expression of BNs, then many problems were difficult to be investigated. For example, in [14], the authors needed to compute all the possible matrix-vector multiplications to find the optimal control strategy, but failed to provide an efficient algorithm to the obtained strategy. While under the framework of STP, [176] proposed new algorithms to find an optimal control strategy, which reduces both space complexity.
and computational complexity. Over the past decades, BNs have developed into a relatively mature stage under the help of STP. Among the existing publications, we want to emphasize attention on the following works.

3.2. Controllability. Controllability is one of the non-trivial and fundamental problems of control systems, especially for biology systems. As [1] pointed out that “One of the major goals of systems biology is to develop a control theory for complex biological systems”. In the early period, the controllability of BCNs have been investigated by several investigators [14, 18]. While, due to in the absence of a standard algebraic expression, it is difficult to achieve a well controllability criteria. Until 2009, Cheng et al. utilized the method of STP to obtain the algebraic expressions of BNs, then some necessary and sufficient conditions (NSCs) of controllability with two kinds of control inputs can be obtained [11]. However, the computational complexity of the criteria will be exponentially increased with steps. To solve the problem, a controllability matrix was constructed in [43] where there exist some states constraints, then several controllability criteria, which are independent of steps, were proposed under the help of Perron-Frobenius theory. Based on [43], Li et al. considered the controllability of switched BCNs with states and inputs constraints, then two algorithms were given to find switching sequence and control inputs that minimize the cost function [61]. Similarly, the controllability of probabilistic BCNs also was investigated in [106, 117]. In [117], both state controllability and trajectory controllability of BCNs with time-delay were investigated by the Perron-Frobenius theory. It pointed out that if one system with time-delay is trajectory controllable, then it must be state controllable, but the reverse is not true. Additionally, the controllability of some more complex BCNs were analyzed. In [171], Zhong et al. investigated the controllability of a family of BCNs, and considered two kinds of forms of control: the free control sequence and input Boolean network. Moreover, the authors also considered the controllability of identical-hierarchy mixed-valued LCNs in [172]. Then, some NSCs for group-controllability and simultaneously-controllability were derived. Note that, in these results [43, 61, 106, 171, 172, 117], the dimension of controllability matrices will increases exponentially with the number of nodes. In [93], Liang et al. obtained some improved controllability criteria by the Warshall algorithm, which reduced the computational complexity to $O(2^{3n})$ with $n$ being the number of state variables in a BCN. Subsequently, Zhu et al. also presented an improved method via combining the well known Tarjan’s algorithm and depthfirst search technique to investigate the controllability in [177], where the computational complexity is reduced from $O(2^{3n})$ to $O(2^{2n})$ compared with [93]. Actually, [1] has pointed out that it is a NP-hard for the control problems of BCNs. Numerous fundamental and important results about controllability of BCNs are based on the case that every node is under controlled. Lu et al. in [118] first proposed the definition of pinning control in BCNs. Under the pinning controllers, the authors obtained some NSCs for controllability, but pinning nodes were given in advance. Fortunately, in [59], Li et al. showed the selection algorithm of pinning nodes by solving some certain logical matrix equations.

3.3. Stability and stabilization. Stability and stabilization of BNs also are interesting topics. In [12], the authors first utilized the information of incidence matrices obtaining sufficient conditions for stability of BNs, then based on the transition matrices, some NSCs for stability and stabilization were derived. Similarly, based on the algebraic expressions, [64] further investigated the stability of switched BNs. The authors not only presented one necessary and sufficient condition (NSC) for point-wise stability, but also gave an verifiable NSC for switched BNs with arbitrary switching signal. Moreover, [64] also pointed out that one switched BN with $n$ nodes is globally stable at a given fixed point with time $2^{n}$
under any switching signal. As reported in [180], the asymptotical stability of a probability BN was successfully characterized by the solution uniqueness to its induced equations. Furthermore, bounded state delays were proved to be inoperative without consideration of the convergence rate. Considering impulsive effects, [56] presented some NSCs for stability and stabilization of the BNs with impulsive effects. Note that control gain matrices are known in [12, 56], but there is no algorithm for how to design stabilization controllers. Subsequently, in [69] and [70], Li et al. first proposed general state feedback controllers design approaches of BCNs and probabilistic BCNs to achieve global stabilization by constructing some reachable sets, respectively. While, [69] and [70] can not obtain all the possible state feedback controllers. To obtain all the controllers, in [62], Li et al. further analyzed reachable sets and obtained all the complete family of reachable sets, then not only all the possible state feedback controllers were designed, but also all output feedback controllers were obtained. In [98], the authors showed how to design state feedback stabilization controllers of BCNs with states and inputs delays. If the delays are stochastic, then [125] used an augmented method such that BNs with stochastic delays were converted into two coupled Markovian switching systems without delays. Finally, controller can be obtained by solving a convex programming problem.

The global stability implies that the BNs are finally convergent to a fixed point, while set stability determines whether the systems are convergent to a given set. Set stability and set stabilization were first formulated in [27], where the largest invariant subset and largest control invariant subset were found. Some NSCs for set stability and set stabilization were derived by analyzing invariant subsets. Then, [87] and [72] generalized the set stability and set stabilization to switched BNs. Moreover, output feedback controllers were designed to achieve stabilization in [99]. Additionally, In [157], the stabilization of dynamic-algebraic BCNs was investigated.

To reduce control cost, in [51], Li first considered the pinning control design for the stabilization of BNs. The selection algorithm of pinning nodes were given, then by solving some matrix equations, pinning controllers were designed. Moreover, Li further applied the pinning scheme into the investigations of the stability of BNs with delays and set stabilization of multi-valued LNs [53] and [54], respectively. In [121], the authors investigated the minimum number of pinning controllers such that BCNs achieve stabilization. When considering BNs with disturbances, the pinning control strategy also can be applicable to analyze the global robust stability and stabilization of BNs [167]. Actually, most of the selection strategies are dependent of transition matrices such as [51] and [167], then the design of pinning controllers depends on all of the nodes. However, in [50] and [168], the selection of pinning nodes depend on incident matrices rather than transition matrix, which is more applicable to large-scale BNs. Moreover, the pinning controllers were designed depending on the neighbors of controlled nodes. In both state feedback controllers and pinning controllers, the controllers need to be updated at every moment. In [105], Liu et al. formulated the sampled-data state feedback control, where controllers only were updated in each sampling period, to achieve stabilization. Then, the controllers were applied to solve the set stabilization of BCNs in [178, 153]. Note that the sampling period is invariant. In order to further reduce the number of controllers updated, Lu et al. designed the aperiodic sampled-data control to realize the stabilization [116, 139]. Moreover, event-triggered controllers also were designed to achieve the stabilization of probabilistic BCNs in [179].

In continuous systems, Lyapunov functions are one of the main tools to investigate the stability. The concept of Lyapunov function in BNs was first proposed in [143], and several Lyapunov-based stability results were obtained. Then, in [63], the authors proposed two approaches to construct Lyapunov functions to achieve the stability of BNs, and applied the
results to analyze the stability of switched BNs. Additionally, the Lyapunov function for stochastic BNs was defined in [126], based on which some NSCs for global stability were provided. Subsequently, in [124], the authors investigated the stability and guaranteed cost of time-triggered BNs. It was proved that the global stability can be guaranteed under a designed switching signal by constructing Lyapunov functions, based on which the bound of the infinite time cost function was presented. Additionally, it should be pointed out that the problem of output tracking control can be converted into that of stabilization such as [65, 88].

3.4. **Observability.** The observability also plays an important role in control systems. In [11], Cheng *et al.* first investigated the observability of BNs, and constructed the observability matrix. It was proved that if all of columns of the matrix are different, then the BNs are observable, vice versa. Subsequently, the observability of BCNs with impulsive effects and time delays were investigated in [55] and [57], respectively, where control inputs networks were known in [55] and [57]. Then, Zhang *et al.* investigated the observability of BCNs with time-variant delays in states in [163]. One algorithm was presented to design control sequences. In [19], the definition of distinguishable was proposed, then free control sequences were designed to achieve observability. Moreover, the reconstructibility also was analyzed, then the relation between observability and reconstructibility was revealed. In [161], four different definitions of observability were given, and implication relations were revealed. Some algorithms were provided to verify whether a given BCN is observable based on the theories of finite automata and formal languages. Then, the idea was generalized to investigate the observability of switched BCNs in [162]. More effective sufficient or necessary conditions for observability were obtained by constructing weighted pair graphs. Later on, the observability problem was converted into a set controllability problem in [10] and [175]. Moreover, in [175, 95], an observability graph was constructed to verify the observability of BCNs. In [26], Guo *et al.* proposed a new definition of observability called the output-feedback observability, then proved that the output-feedback observability can be regarded as an equivalent stabilizability problem of the corresponding LNs. As shown in [44] that determining a BN whether is observable is NP-hard, then some results about observability were obtained by combining the method of STP with the graph-theoretic approach, which also is exponential complexity. In [147], the authors considered how to add a minimal number of nodes such that conjunctive BNs with \( n \) nodes become observable. Then, an efficient algorithm with complexity \( O(n^2) \) was given to solve the minimal observability problem.

3.5. **Current research problems of BNs.** Except for the three basic problems described above, there are many interesting problems for BNs. In the following, we will list some of them.

The key point of disturbance decoupling problem (DDP) is to design controllers such that disturbances have no effect on outputs. Cheng *et al.* in [8] firstly gave the definition of DDP for BCNs, then free control sequences were found by constructing constant mapping matrices. Subsequently, the method of redundant variables separation was proposed in [155] to design state feedback controllers such that the DDP is solvable. Based on the method, the DDP of switched BCNs and singular BCNs were investigated in [66] and [108], respectively. Additionally, [46] and [109] designed event-triggered controllers and pinning controllers achieving DDP solvability, respectively. However, Li *et al.* in [47] pointed out that the definition of DDP proposed by [8] is somewhat strict if the aim of DDP is to make outputs undisturbed. Then, the definition of outputs robustness for BCNs was proposed,
and state feedback controllers as well as pinning controllers were designed to achieving DDP solvability by constructing permutation systems in [47] and [48], respectively.

The synchronization problem also attracted considerable attention, and is to see whether the trajectories of master-slave systems can keep consistent after some period of time. Zhong et al. have done a series of works about the analysis of synchronization [173, 172]. In [110], Liu et al. first proposed the designing algorithm of state feedback controllers by converting the synchronization problem into set stabilization. Later on, pinning controllers and event-triggered controllers were designed to achieve synchronization [52, 154, 122]. Additionally, local synchronization also was investigated in [3].

There are many other interesting topics, like normalization of singular BNs [104], function perturbations [107], local convergence [49], block decoupling [160], output regulation [68], robust control invariance [140, 94] and so forth.

4. Applications. Due to the fact that impulsive controller has simple structure and strong robustness, it has been widely applied to more and more control systems, such as neural networks, population modeling, ecosystems management, sample-data systems and synchronization of chaotic secure communication systems [128, 39, 129, 4].

\[
x_1(t)(m(t)+x_3(t))
\]

\[
X(t_i)
\]

\[
X u
delay
\]

\[
F
\]

IGURE 5. Cryptosystem based on impulsive synchronization.

To be specific, [128] studied exponential stability criteria for IDS and convert the sampled-data system into the corresponding IDS. In [39], the authors obtained some applications to secure communication by impulsive synchronizing chaotic systems. In fact, the encryption process can be achieved by chaotic masking and modulation as follows. If \( m(t) \) denotes the message, then by chaotic masking we can convert it into encrypted form, i.e., \( f(t) = e(m(t)) = x_1(t)(m(t)+x_3(t)) \). Further, a cryptosystem with the transmission of impulses was proposed in [39] (see Fig. 5). In addition, [129] studied the stability of delay impulsive systems and applied it into the networked control systems. In [4], we can also find some applications to image encryption by impulsive synchronization.

Moreover, some applications of complex with logical dynamics have also been achieved successfully. Specifically, feedback shift registers (FSRs) are the main building block of stream ciphers. FSRs are a kinds of logical systems. The main tool for investigating FSRs is algebraic normal form (ANF), based on which, a lot of interesting problems on FSRs were investigated, such as irreducibility, equivalent transformation, decomposition. The Fig. 6 shows two kinds of configurations for FSRs, that are: Fibonacci and Galois, which can be modeled by BNs. While, there exist some barriers on the approach of ANF. For example, when consider the transformation between Fibonacci FSRs and Galois FSRs by
ANF, the uniform conditions need to be guaranteed [17]. Recently, BNs were proposed to model FSRs, and some interesting results were obtained [114, 170, 115]. Zhong et al. utilized the method of STP obtaining characters of transition matrices for Fibonacci FSRs [169]. Based on which, in [113], Lu et al. relaxed the conditions of the transformation compared with [17].

$$f(x_1, x_2, \ldots, x_n)$$

$$x_{n-1} \quad x_n \quad \ldots \quad x_1$$

(1)

$$f' \quad x' \quad f' \quad x' \quad \ldots \quad f'$$

(2)

**Figure 6.** Two kinds of configurations for FSRs based on BNs.

In [9], Cheng et al. viewed finite evolutionary game dynamics as LNs, then under the framework of STP, the existence of stationary stable profiles was studied. Zhao et al. [25] established a matrix-based framework for networked evolutionary games with finite memories. Interesting results also include [13, 67, 123, 100, 89]. The STP also can be applied to wireless communication [151], smart grid [174] and fault detection of circuits [60] and so forth. Some comprehensive introductions about the applications of STP method can refer to [112].

5. **Conclusion and further work.** Due to the fact that more and more dynamics can be modeled by complex systems with impulsive effects and logical dynamics, many investigators pay increasing attention to this type of complex system. Therefore, the corresponding relevant work has been systematically reviewed in this paper. First, we have compactly summarized five topics of IDS, that is, some basic theory of IDS, Lyapunov stability, input-to-state stability, hybrid impulses and delay-dependent impulses. Second, for LNs, we have emphasized attention on some problems including controllability, stability and stabilization, observability and current research. Finally, some significant applications of existing results on the complex system with impulsive effects and logical dynamics have been proposed.

Although the studies on complex systems have been developed rapidly, there also exist some challenging and interesting topics that should be further considered in the future. For example, in most existing results, it is required that the delay in impulses can not be larger than the length of impulsive interval; although [83] have proved that the delay in impulses may contribute to the stability, it only has considered the “pure” delay-dependent impulsive control, i.e., $$x(t_k) = Jx((t_k - \tau_k)^-)$$ . Namely, [83] has not considered the impulsive control “mixed” form: $$x(t_k) = J_1x((t_k)^-) + J_2x((t_k - \tau_k)^-)$$ . Therefore, one future work is to study the impact of large delay in impulses, i.e., $$\tau_k > t_k - t_{k-1}$$ . Another further work is to investigate the “mixed” impulsive control of delayed systems, in which the delay may have positive effects on stability. As for complex systems with logical dynamics, many results about BNs and BCNs have been obtained under the framework of STP, which leads to the computation complexity increases exponentially with networks nodes. Therefore, we have to seek some methods to reduce the computation complexity. On the other hand, how to
apply the existing and mature BNs method to other aspects especially on cyber-security is of great importance.

Acknowledgments. This work was supported by the National Natural Science Foundation of China under Grant No. 61973078, the Natural Science Foundation of Jiangsu Province of China under Grant No. BK20170019, “333 Engineering” Foundation of Jiangsu Province of China under Grant BRA2019260, and “the Fundamental Research Funds for the Central Universities under Grant No. 2242019k1G013”.

REFERENCES

[1] T. Akutsu, M. Hayashida, W. Ching and M. Ng, Control of Boolean networks: Hardness results and algorithms for tree structured networks, Journal of Theoretical Biology, 244 (2007), 670–679.
[2] J. Benford and J. Swegle, Applications of high power microwaves, 1992 9th International Conference on High-Power Particle Beams, (1992).
[3] H. Chen and J. Liang, Local synchronization of interconnected Boolean networks with stochastic disturbances, IEEE Transactions on Neural Networks and Learning Systems, 31 (2019), 452–463.
[4] W.-H. Chen, S. Luo and W. X. Zheng, Impulsive synchronization of reaction–diffusion neural networks with mixed delays and its application to image encryption, IEEE Transactions on Neural Networks and Learning Systems, 27 (2016), 2696–2710.
[5] W.-H. Chen, D. Wei and W. Zheng, Delayed impulsive control of Takagi-Sugeno fuzzy delay systems, IEEE Transactions on Fuzzy Systems, 21 (2013), 516–526.
[6] W.-H. Chen and W.-X. Zheng, Exponential stability of nonlinear time-delay systems with delayed impulse effects, Automatica, 47 (2011), 1075–1083.
[7] D. Cheng, Semi-tensor product of matrices and its application to Morgens problem, Science in China Series: Information Sciences, 44 (2001), 195–212.
[8] D. Cheng, Disturbance decoupling of Boolean control networks, IEEE Transactions on Automatic Control, 56 (2011), 2–10.
[9] D. Cheng, F. He, H. Qi and T. Xu, Modeling, analysis and control of networked evolutionary games, IEEE Transactions on Automatic Control, 60 (2015), 2402–2415.
[10] D. Cheng, C. Li and F. He, Observability of Boolean networks via set controllability approach, Systems & Control Letters, 115 (2018), 22–25.
[11] D. Cheng and H. Qi, Controllability and observability of Boolean control networks, Automatica, 45 (2009), 1659–1667.
[12] D. Cheng, H. Qi, Z. Li and J. Liu, Stability and stabilization of Boolean networks, International Journal of Robust and Nonlinear Control, 21 (2011), 134–156.
[13] D. Cheng, H. Qi and Z. Liu, From STP to game-based control, Science China Information Sciences, 61 (2018), 010201.
[14] W. Ching, S. Zhang, Y. Jiao, T. Akutsu, N. Tsing and A. Wong, Optimal control policy for probabilistic Boolean networks with hard constraints, IET Systems Biology, 3 (2009), 90–99.
[15] S. Dashkovskiy and P. Feketa, Input-to-state stability of impulsive systems and their networks, Nonlinear Analysis: Hybrid Systems, 26 (2017), 190–200.
[16] S. Dashkovskiy and A. Mironchenko, Input-to-state stability of nonlinear impulsive systems, SIAM Journal on Control and Optimization, 51 (2013), 1962–1987.
[17] E. Dubrova, Finding matching initial states for equivalent NLFSRs in the Fibonacci and the Galois configurations, IEEE Transactions on Information Theory, 56 (2010), 2961–2966.
[18] B. Faryabi, A. Datta and E. Dougherty, On approximate stochastic control in genetic regulatory networks, IET Systems Biology, 1 (2007), 361–368.
[19] E. Fornasini and M. Valcher, Observability, reconstrucitbility and state observers of Boolean control networks, IEEE Transactions on Automatic Control, 58 (2013), 1390–1401.
[20] L. Gao, D. Wang and G. Wang, Further results on exponential stability for impulsive switched nonlinear time-delay systems with delayed impulse effects, Applied Mathematics and Computation, 268 (2015), 186–200.
[21] K. Gopalsamy and B. Zhang, On delay differential equations with impulses, Journal of Mathematical Analysis and Applications, 139 (1989), 110–122.
[22] Z.-H. Guan, G. Chen and T. Ueta, On impulsive control of a periodically forced chaotic pendulum system, IEEE Transactions on Automatic Control, 45 (2000), 1724–1727.
[23] Z.-H. Guan and N. Liu, Generating chaos for discrete time-delayed systems via impulsive control, Chaos, \textbf{20} (2010), 013135.

[24] Z.-H. Guan, Z.-W. Liu, G. Feng and W. Yan-Wu, Synchronization of complex dynamical networks with time-varying delays via impulsive distributed control, IEEE Transactions on Circuits and Systems I: Regular Papers, \textbf{57} (2010), 2182–2195.

[25] G. Zhao, Y. Wang and H. Li, A matrix approach to the modeling and analysis of networked evolutionary games with finite memories, IEEE/CAA Journal of Automatica Sinica, \textbf{5} (2018), 818–826.

[26] Y. Guo, Observability of Boolean control networks using parallel extension and set reachability, IEEE Transactions on Neural Networks and Learning Systems, \textbf{29} (2018), 6402–6408.

[27] Y. Guo, P. Wang, W. Gui and C. Yang, Set stability and set stabilization of Boolean control networks based on invariant subsets, Automatica, \textbf{61} (2015), 106–112.

[28] D. Haltara and G. Ankhbayar, Using the maximum principle of impulse control for ecology-economical models, Ecological Modelling, \textbf{216} (2008), 150–156.

[29] W. He, G. Chen, Q. L. Han and F. Qian, Network-based leader-following consensus of nonlinear multi-agent systems via distributed impulsive control, Information Sciences, \textbf{380} (2017), 145–158.

[30] J. P. Hespanha, D. Liberzon and A. R. Teel, Lyapunov conditions for input-to-state stability of impulsive systems, Automatica, \textbf{44} (2008), 2735–2744.

[31] J. Hu, G. Sui, X. Lu and X. Li, Fixed-time control of delayed neural networks with impulsive perturbations, Nonlinear Analysis: Modelling and Control, \textbf{23} (2018), 904–920.

[32] M.-J. Hu, J.-W. Xiao, R.-B. Xiao and W.-H. Chen, Impulsive effects on the stability and stabilization of positive systems with delays, Journal of the Franklin Institute, \textbf{354} (2017), 4034–4054.

[33] C. Huang, J. Lu, D. W. Ho, G. Zhai and J. Cao, Stabilization of probabilistic Boolean networks via pinning control strategy, Information Sciences, \textbf{510} (2020), 205–217.

[34] A. Ignatyev and A. Soliman, Asymptotic stability and instability of the solutions of systems with impulse action, Mathematical Notes, \textbf{80} (2006), 491–499.

[35] G. Jia, M. Meng and J. Feng, Function perturbation of mix- valued logical networks with impacts on limit sets, Neurocomputing, \textbf{207} (2016), 428–436.

[36] B. Jiang, J. Lu, X. Li and J. Qiu, Input/output-to-state stability of nonlinear impulsive delay systems based on a new impulsive inequality, International Journal of Robust and Nonlinear Control, \textbf{29} (2019), 6164–6178.

[37] B. Jiang, J. Lu, J. Lou and J. Qiu, Synchronization in an array of coupled neural networks with delayed impulses: Average impulsive delay method, Neural Networks, \textbf{121} (2020), 452–460.

[38] S. Kauffman, Metabolic stability and epigenesis in randomly constructed genetic nets, Journal of Theoretical Biology, \textbf{22} (1969), 437–467.

[39] A. Khadra, X. Liu and X. Shen, Impulsively synchronizing chaotic systems with delay and applications to secure communication, Automatica, \textbf{41} (2005), 1491–1502.

[40] J. Ladyman, J. Lambert and K. Wiesner, What is a complex system?, European Journal for Philosophy of Science, \textbf{3} (2013), 33–67.

[41] V. Lakshmikantham and S. Leela, On perturbing Lyapunov functions, Mathematical Systems Theory, \textbf{10} (1976), 85–90.

[42] V. Lakshmikantham, D. D. Bainov and P. S. Simeonov, Theory of Impulsive Differential Equations, vol. 6, World Scientific, 1989.

[43] D. Laschov and M. Margaliot, Controllability of Boolean control networks via perron-frobenius theory, Automatica J. IFAC, \textbf{48} (2012), 1218–1223.

[44] D. Laschov, M. Margaliot and G. Even, Observability of Boolean networks: A graph-theoretic approach, Automatica, \textbf{49} (2013), 2351–2362.

[45] Y.-J. Lee, J. H. Olof and N. Karin, Spatio-temporal dynamics of impulse responses to figure motion in optic flow neurons, Plos One, \textbf{10} (2015), e0126265.

[46] B. Li, Y. Liu, K. Kou and L. Yu, Event-triggered control for the disturbance decoupling problem of Boolean control networks, IEEE Transactions on Cybernetics, \textbf{48} (2018), 2764–2769.

[47] B. Li, Y. Liu, J. Lou, J. Lu and J. Cao, The robustness of outputs with respect to disturbances for Boolean control networks, IEEE Transactions on Neural Networks and Learning Systems, \textbf{31} (2020), 1046–1051.

[48] B. Li, J. Lu, Y. Liu and Z. Wu, The outputs robustness of Boolean control networks via pinning control, IEEE Transactions on Control of Network Systems, \textbf{7} (2020), 201–209.

[49] B. Li, J. Lu, Y. Liu and W. Zheng, The local convergence of Boolean networks with disturbances, IEEE Transactions on Circuits and Systems II: Express Briefs, \textbf{66} (2019), 667–671.

[50] B. Li, J. Lu, J. Zhong and Y. Liu, Fast-time stability of temporal Boolean networks, IEEE Transactions on Neural Networks and Learning Systems, \textbf{30} (2019), 2285–2294.
[51] F. Li, Pinning control design for the stabilization of Boolean networks, IEEE Transactions on Neural Networks and Learning Systems, 27 (2016), 1585–1590.

[52] F. Li, Pinning control design for the synchronization of two coupled Boolean networks, IEEE Transactions on Circuits and Systems II: Express Briefs, 63 (2016), 309–313.

[53] F. Li, Stability of Boolean networks with delays using pinning control, IEEE Transactions on Control of Network Systems, 5 (2018), 179–185.

[54] F. Li, H. Li, L. Xie and Q. Zhou, On stabilization and set stabilization of multivalued logical systems, Automatica, 80 (2017), 41–47.

[55] F. Li and J. Sun, Observability analysis of Boolean control networks with impulsive effects, IET Control Theory & Applications, 5 (2011), 1609–1616.

[56] F. Li and J. Sun, Stability and stabilization of Boolean networks with impulsive effects, Systems & Control Letters, 61 (2012), 1–5.

[57] F. Li, J. Sun and Q. Wu, Observability of Boolean control networks with state time delays, IEEE Transactions on Neural Networks, 22 (2011), 948–954.

[58] F. Li and Y. Tang, Set stabilization for switched Boolean control networks, Automatica, 78 (2017), 223–230.

[59] F. Li, H. Yan and H. Karimi, Single-input pinning controller design for reachability of Boolean networks, IEEE Transactions on Neural Networks and Learning Systems, 29 (2018), 3264–3269.

[60] H. Li and Y. Wang, Boolean derivative calculation with application to fault detection of combinational circuits via the semi-tensor product method, Automatica, 48 (2012), 688–693.

[61] H. Li and Y. Wang, Controllability analysis and control design for switched Boolean networks with state and input constraints, SIAM Journal on Control and Optimization, 53 (2015), 2955–2979.

[62] H. Li and Y. Wang, Further results on feedback stabilization control design of Boolean control networks, Automatica, 83 (2017), 303–308.

[63] H. Li and Y. Wang, Lyapunov-based stability and construction of Lyapunov functions for Boolean networks, SIAM Journal on Control and Optimization, 55 (2017), 3437–3457.

[64] H. Li, Y. Wang and Z. Liu, Stability analysis for switched Boolean networks under arbitrary switching signals, IEEE Transactions on Automatic Control, 59 (2014), 1978–1982.

[65] H. Li, Y. Wang and L. Xie, Output tracking control of Boolean control networks via state feedback: Constant reference signal case, Automatica, 59 (2015), 54–59.

[66] H. Li, Y. Wang, L. Xie and D. Cheng, Disturbance decoupling control design for switched Boolean control networks, Systems & Control Letters, 72 (2014), 1–6.

[67] P. Guo, Y. Wang and H. Li, Stable degree analysis for strategy profiles of evolutionary networked games, Science China Information Sciences, 59 (2016), 052204.

[68] H. Li, Y. Wang and P. Guo, Output reachability analysis and output regulation control design of Boolean control networks, Science China Information Sciences, 60 (2017), 022202.

[69] R. Li, M. Yang and T. Chu, State feedback stabilization for Boolean control networks, IEEE Transactions on Automatic Control, 58 (2013), 1853–1857.

[70] R. Li, M. Yang and T. Chu, State feedback stabilization for probabilistic Boolean networks, Automatica, 50 (2014), 1272–1278.

[71] S. Li, X. Song and A. Li, Strict practical stability of nonlinear impulsive systems by employing two Lyapunov-like functions, Nonlinear Analysis Series B: Real World Applications, 9 (2008), 2262–2269.

[72] X. Li, J. Lu, J. Qiu, X. Chen, X. Li and F. Alsaadi, Set stability for switched Boolean networks with open-loop and closed-loop switching signals, Science China Information Sciences, 61 (2018), 092207.

[73] X. Li, Further analysis on uniform stability of impulsive delay differential equations, Applied Mathematics Letters, 25 (2012), 133–137.

[74] X. Li and M. Bohner, An impulsive delay differential inequality and applications, Computers & Mathematics with Applications, 64 (2012), 1875–1881.

[75] X. Li, M. Bohner and C. K. Wang, Impulsive differential equations: Periodic solutions and applications, Automatica, 52 (2015), 173–178.

[76] X. Li, J. Cao and D. W. C. Ho, Impulsive control of nonlinear systems with time-varying delay and applications, IEEE Transactions on Cybernetics, 50 (2020), 2661–2673.

[77] X. Li and F. Deng, Razumikhin method for impulsive functional differential equations of neutral type, Chaos Solitons and Fractals, 101 (2017), 41–49.

[78] X. Li, D. W. C. Ho and J. Cao, Finite-time stability and settling-time estimation of nonlinear impulsive systems, Automatica, 99 (2019), 361–368.

[79] X. Li, P. Li and Q. G. Wang, Input/output-to-state stability of impulsive switched systems, Systems & Control Letters, 116 (2018), 1–7.
[80] X. Li, D. O’Regan and H. Akca, Global exponential stabilization of impulsive neural networks with unbounded continuously distributed delays, *IMA Journal of Applied Mathematics*, 80 (2015), 85–99.

[81] X. Li, R. Rakkiyappan and C. Pradeep, Robust μ–stability analysis of Markovian switching uncertain stochastic genetic regulatory networks with unbounded time-varying delays, *Communications in Nonlinear Science and Numerical Simulation*, 17 (2012), 3894–3905.

[82] X. Li and K. Zhang, Input-to-state stability of time-delay systems with delay-dependent impulses, *IEEE Automatica*, 57 (2015), 7–16.

[83] X. Li and J. Shen, Exponential synchronization of delayed neural networks with impulsive effects, *Neural Networks*, 32 (2012), 1–10.

[84] X. Li and G. Ballinger, Existence and continuability of solutions for differential equations with delays and impulses, *Journal of Mathematical Analysis and Applications*, 252 (2000), 578–594.

[85] X. Li, X. Zhang and S. Song, Effect of delayed impulses on input-to-state stability of nonlinear systems, *Applied Mathematics and Computation*, 219 (2013), 549–560.

[86] Y. Li, J. Lou, Z. Wang and F. Alsaadi, Synchronization of dynamical networks with nonlinearly coupling state-dependent impulses, *IEEE Transactions on Neural Networks and Learning Systems*, 29 (2018), 3301–3306.

[87] Y. Li, H. Li, X. Xu and Y. Li, Semi-tensor product approach to minimal-agent consensus control of networked evolutionary games, *IET Control Theory & Applications*, 12 (2018), 2269–2275.

[88] Y. Li, Impulsive stabilization of stochastic neural networks via controlling partial states, *Neural Processing Letters*, 46 (2017), 59–69.

[89] Y. Li, J. Lou, Z. Wang and F. E. Alsaadi, Synchronization of dynamical networks with nonlinearly coupling function under hybrid pinning impulsive controllers, *Journal of the Franklin Institute*, 355 (2018), 6520–6530.

[90] Z. Li, W. Zhang, G. He and J.-A. Fang, Synchronisation of discrete-time complex networks with delayed heterogeneous impulses, *IET Control Theory & Applications*, 9 (2015), 2648–2656.

[91] J. Liang, H. Chen and J. Lam, An improved criterion for controllability of boolean control networks, *IEEE Transactions on Automatic Control*, 62 (2017), 6012–6018.

[92] Y. Liu, J. Lu, Z. Wang and Y. Liu, Output tracking of Boolean control networks driven by constant reference signal, *IEEE Access*, 5 (2017), 112572–112577.

[93] R. Liu, J. Lu, Y. Liu, H. Li, X. Xu and Y. Li, Semi-tensor product approach to minimal-agent consensus control of networked evolutionary games, *IET Control Theory & Applications*, 12 (2018), 2269–2275.

[94] J. Liu and Z. Zhao, Multiple solutions for impulsive problems with non-autonomous perturbations, *Applied Mathematics Letters*, 64 (2017), 143–149.

[95] J. Liu and X. Li, Impulsive stabilization of high-order nonlinear retarded differential equations, *Applications of Mathematics*, 58 (2013), 347–367.

[96] R. Liu, J. Lu, Y. Liu, J. Cao and Z. Wu, Delayed feedback control for stabilization of Boolean control networks with state delay, *IEEE Transactions on Neural Networks and Learning Systems*, 29 (2018), 3283–3288.

[97] R. Liu, J. Lu, W. Zheng and J. Kurths, Output feedback control for set stabilization of Boolean control networks, *IEEE Transactions on Neural Networks and Learning Systems*, 29 (2018), 3283–3288.

[98] X. Liu and J. Zhu, On potential equations of finite games, *Automatica*, 68 (2016), 245–253.

[99] X. Liu, Stability results for impulsive differential systems with applications to population growth models, *Dynamics and Stability of Systems*, 9 (1994), 163–174.

[100] X. Liu and G. Ballinger, Existence and continuability of solutions for differential equations with delays and state-dependent impulses, *Nonlinear Analysis*, 51 (2002), 633–647.

[101] X. Liu and K. Zhang, Input-to-state stability of time-delay systems with delay-dependent impulsive, *IEEE Transactions on Automatic Control*, 65 (2012), 1676–1682.

[102] Y. Liu, J. Cao, B. Li and J. Lu, Normalization and solvability of dynamic-algebraic Boolean networks, *IEEE Transactions on Neural Networks and Learning Systems*, 29 (2018), 3301–3306.

[103] Y. Liu, J. Cao, L. Sun and J. Lu, Sampled-data state feedback stabilization of Boolean control networks, *Neural Computation*, 28 (2016), 778–799.

[104] Y. Liu, H. Chen, J. Lu and B. Wu, Controllability of probabilistic Boolean control networks based on transition probability matrices, *Automatica*, 52 (2015), 340–345.

[105] Y. Liu, B. Li, H. Chen and J. Cao, Function perturbations on singular Boolean networks, *Automatica*, 84 (2017), 36–42.
[108] Y. Liu, B. Li and J. Lou, Disturbance decoupling of singular Boolean control networks, IEEE/ACM Transactions on Computational Biology and Bioinformatics, 13 (2016), 1194–1200.

[109] Y. Liu, B. Li, J. Lu and J. Cao, Pinning control for the disturbance decoupling problem of Boolean networks, IEEE Transactions on Automatic Control, 62 (2017), 6595–6601.

[110] Y. Liu, L. Sun, J. Lu and J. Liang, Feedback controller design for the synchronization of Boolean control networks, IEEE Transactions on Neural Networks and Learning Systems, 27 (2016), 1991–1996.

[111] Z. Liu, Y. Wang and H. Li, New approach to derivative calculation of multi-valued logical functions with application to fault detection of digital circuits, IET Control Theory & Applications, 8 (2014), 554–560.

[112] J. Lu, H. Li, Y. Liu and F. Li, Survey on semi-tensor product method with its applications in logical networks and other finite-valued systems, IET Control Theory & Applications, 11 (2017), 2040–2047.

[113] J. Lu, M. Li, T. Huang, Y. Liu and J. Cao, The transformation between the Galois NLFSRs and the Fibonacci NLFSRs via semi-tensor product of matrices, Automatica, 96 (2018), 393–397.

[114] J. Lu, M. Li, Y. Liu, D. Ho and J. Kurths, Nonsingularity of grain-like cascade FSRs via semi-tensor product, Science China Information Sciences, 61 (2018), 010204, 12 pp.

[115] B. Li and J. Lu, Boolean-network-based approach for the construction of filter generators, Science China Information Sciences, in press.

[116] J. Lu, L. Sun, Y. Liu, D. Ho and J. Cao, Stabilization of Boolean control networks under aperiodic sampled-data control, SIAM Journal on Control and Optimization, 56 (2018), 4385–4404.

[117] J. Lu, J. Zhong, D. W. C. Ho, Y. Tang and J. Cao, On controllability of delayed Boolean control networks, SIAM Journal on Control and Optimization, 54 (2016), 475–494.

[118] J. Lu, J. Zhong, C. Huang and J. Cao, On pinning controllability of Boolean control networks, IEEE Transactions on Automatic Control, 61 (2016), 1658–1663.

[119] J. Lu, D. W. C. Ho and J. Cao, A unified synchronization criterion for impulsive dynamical networks, Automatica, 46 (2010), 1215–1221.

[120] J. Lu, J. Kurths, J. Cao, N. Mahdavi and C. Huang, Synchronization control for nonlinear stochastic dynamical networks: Pinning impulsive strategy, IEEE Transactions on Neural Networks and Learning Systems, 23 (2012), 285–292.

[121] J. Lu, R. Liu, J. Lou and Y. Liu, Pinning stabilization of Boolean control networks via a minimum number of controllers, IEEE Transactions on Cybernetics, (2019), 1–9.

[122] J. Lu, J. Yang, J. Lou and J. Qiu, Event-triggered sampled feedback synchronization in an array of output-coupled Boolean control networks, IEEE Transactions on Cybernetics, (2019), 1–6.

[123] Y. Mao, L. Wang, Y. Liu, J. Lu and Z. Wang, Stabilization of evolutionary networked games with length-r information, Applied Mathematics and Computation, 337 (2018), 442–451.

[124] M. Meng, J. Lam, J. Feng and K. Cheung, Stability and guaranteed cost analysis of time-triggered Boolean networks, IEEE Transactions on Neural Networks and Learning Systems, 29 (2018), 3893–3899.

[125] M. Meng, J. Lam, J. Feng and K. Cheung, Stability and stabilization of Boolean networks with stochastic delays, IEEE Transactions on Automatic Control, 64 (2019), 790–796.

[126] M. Meng, L. Liu and G. Feng, Stability and $L_1$ gain analysis of Boolean networks with Markovian jump parameters, IEEE Transactions on Automatic Control, 62 (2017), 4222–4228.

[127] V. Milman and A. Myshkis, On the stability of motion in the presence of impulses, Sib. Math. J., 1 (1960), 253–237.

[128] P. Naghshtabrizi, J. P. Hespanha and A. R. Teel, Exponential stability of impulsive systems with application to uncertain sampled-data systems, Systems & Control Letters, 57 (2008), 378–385.

[129] P. Naghshtabrizi, J. P. Hespanha and A. R. Teel, Stability of delay impulsive systems with application to sampled-data control systems, Transactions of the Institute of Measurement and Control, 32 (2010), 511–528.

[130] F. G. Pfeiffer and M. O. Foerg, On the structure of multiple impact systems, Nonlinear Dynamics, 42 (2005), 101–112.

[131] I. Shmulevich, E. R. Dougherty, S. Kim and W. Zhang, Probabilistic Boolean networks: A rule-based uncertainty model for gene regulatory networks, Bioinformatics, 18 (2002), 261–274.

[132] P. S. Simeonov and D. D. Bainov, Exponential stability of the solutions of singularly perturbed systems with impulse effect, J. Math. Anal. Appl., 151 (1990), 462–487.

[133] X. Song and A. Li, Stability and boundedness criteria of nonlinear impulsive systems employing perturbing Lyapunov functions *, Applied Mathematics and Computation, 217 (2011), 10166–10174.

[134] E. D. Sontag, Smooth stabilization implies coprime factorization, IEEE Transactions on Automatic Control, 34 (1989), 435–443.

[135] E. D. Sontag, Comments on integral variants of ISS, Systems & Control Letters, 34 (1998), 93–100.

[136] I. Stamova and G. Stamov, Stability analysis of impulsive functional systems of fractional order, Communications on Nonlinear Science and Numerical Simulation, 19 (2014), 702–709.
[137] I. M. Stamova and G. T. Stamov, Lyapunov–Razumikhin method for impulsive functional differential equations and applications to the population dynamics, Journal of Computational and Applied Mathematics, 130 (2001), 163–171.

[138] J. Sun, Q. L. Han and X. Jiang, Impulsive control of time-delay systems using delayed impulse and its application to impulsive master-slave synchronization, Physics Letters A, 372 (2008), 6375–6380.

[139] L. Sun, J. Lu and W. Ching, Switching-based stabilization of aperiodic sampled-data Boolean control networks with all modes unstable, Frontiers of Information Technology & Electronic Engineering, 21 (2020), 260–267.

[140] L. Tong, Y. Liu, Y. Li, J. Lu, Z. Wang and F. Alsaadi, Robust control invariance of probabilistic Boolean control networks via event-triggered control, IEEE Access, 6 (2018), 37767–37774.

[141] G. Wang and Y. Shen, Second-order cluster consensus of multi-agent dynamical systems with impulsive effects, Communications in Nonlinear Science and Numerical Simulation, 19 (2014), 3220–3228.

[142] N. Wang, X. Li, J. Lu and F. E. Alsaadi, Unified synchronization criteria in an array of coupled neural networks with hybrid impulses, Neural Networks, 101 (2018), 25–32.

[143] Y. Wang and H. Li, On definition and construction of lyapunov functions for Boolean networks, in Proceedings of the 10th World Congress on Intelligent Control and Automation, IEEE, (2012), 1247–1252.

[144] Y. Wang, J. Lu, J. Liang, J. Cao and M. Perc, Pinning synchronization of nonlinear coupled Lur’e networks under hybrid impulses, IEEE Transactions on Circuits and Systems II: Express Briefs, 66 (2019), 432–436.

[145] Y. Wang, J. Lu, J. Lou, C. Ding, F. E. Alsaadi and T. Hayat, Synchronization of heterogeneous partially coupled networks with heterogeneous impulses, Neural Processing Letters, 48 (2018), 557–575.

[146] Y. Wang, J. Lu and Y. Lou, Halanay-type inequality with delayed impulses and its applications, Science China Information Sciences, 62 (2019), 192206, 10 pp.

[147] E. Weiss and M. Margaliot, A polynomial-time algorithm for solving the minimal observability problem in conjunctive Boolean networks, IEEE Transactions on Automatic Control, 64 (2019), 2727–2736.

[148] Q. Wu, J. Zhou and L. Xiang, Impulses-induced exponential stability in recurrent delayed neural networks, Neurocomputing, 74 (2011), 3204–3211.

[149] Y. Wu and T. Shen, Policy iteration approach to control residual gas fraction in ic engines under the framework of stochastic logical dynamics, IEEE Transactions on Control Systems Technology, 25 (2017), 1100–1107.

[150] Y. Wu and T. Shen, A finite convergence criterion for the discounted optimal control of stochastic logical networks, IEEE Transactions on Automatic Control, 63 (2018), 262–268.

[151] D. Xie, H. Peng, L. Li and Y. Yang, Semi-tensor compressed sensing, Digital Signal Processing, 58 (2016), 85–92.

[152] F. Xu, L. Dong, D. Wang, X. Li and R. Rakkiyappan, Globally exponential stability of nonlinear impulsive switched systems, Mathematical Notes, 97 (2015), 803–810.

[153] M. Xu, Y. Liu, J. Lou, Z.-G. Wu and J. Zhong, Set stabilization of probabilistic boolean control networks: A sampled-data control approach, IEEE Transactions on Cybernetics, (2019), 1–8.

[154] J. Yang, J. Lu, L. Li, Y. Liu, Z. Wang and F. Alsaadi, Event-triggered control for the synchronization of Boolean control networks, Nonlinear Dynamics, 96 (2019), 1335–1344.

[155] M. Yang, R. Li and T. Chu, Controller design for disturbance decoupling of Boolean control networks, Automatica, 49 (2013), 273–277.

[156] T. Yang, Impulsive Control Theory Lecture Notes in Control and Information Sciences, Springer Science and Business Media, 2001.

[157] X. Yang, B. Chen, Y. Li, Y. Liu and F. E. Alsaadi, Stabilization of dynamic-algebraic Boolean control networks via state feedback control, Journal of the Franklin Institute, 355 (2018), 5520–5533.

[158] X. Yang and J. Lu, Finite-time synchronization of coupled networks with markovian topology and impulsive effects, IEEE Transactions on Automatic Control, 61 (2016), 2256–2261.

[159] Z. Yang and D. Xu, Stability analysis and design of impulsive control systems with time delay, IEEE Transactions on Automatic Control, 52 (2007), 1448–1454.

[160] Y. Yu, J. Feng, J. Pan and D. Cheng, Block decoupling of Boolean control networks, IEEE Transactions on Automatic Control, 64 (2019), 3129–3140.

[161] K. Zhang and L. Zhang, Observability of Boolean control networks: A unified approach based on finite automata, IEEE Transactions on Automatic Control, 61 (2016), 2733–2738.

[162] K. Zhang, L. Zhang and L. Xie, Finite automata approach to observability of switched Boolean control networks, Nonlinear Analysis: Hybrid Systems, 19 (2016), 186–197.

[163] L. Zhang and K. Zhang, Controllability and observability of Boolean control networks with time-variant delays in states, IEEE Transactions on Neural Networks and Learning Systems, 24 (2013), 1478–1484.
[164] W. Zhang, C. Li, T. Huang and X. He, Synchronization of memristor-based coupling recurrent neural networks with time-varying delays and impulses, *IEEE Transactions on Neural Networks and Learning Systems*, 26 (2015), 3308–3313.

[165] X. Zhang and X. Li, Input-to-state stability of nonlinear systems with distributed-delayed impulses, *IET Control Theory and Applications*, 11 (2017), 81–89.

[166] Y. Zhang and Y. Liu, Nonlinear second-order multi-agent systems subject to antagonistic interactions without velocity constraints, *Applied Mathematics and Computation*, 364 (2020), 124667, 14 pp.

[167] J. Zhong, D. Ho, J. Lu and W. Xu, Global robust stability and stabilization of Boolean network with disturbances, *Automatica*, 84 (2017), 142–148.

[168] J. Zhong, B. Li, Y. Liu and W. Gui, Output feedback stabilizer design of Boolean networks based on network structure, *Frontiers of Information Technology & Electronic Engineering*, 21 (2020), 247–259.

[169] J. Zhong and D. Lin, Stability of nonlinear feedback shift registers, *Science China Information Sciences*, 59 (2016), 1–12.

[170] J. Zhong and D. Lin, On minimum period of nonlinear feedback shift registers in grain-like structure, *IEEE Transactions on Information Theory*, 64 (2018), 6429–6442.

[171] J. Zhong, Y. Liu, K. Kou, L. Sun and J. Cao, On the ensemble controllability of Boolean control networks using STP method, *Applied Mathematics and Computation*, 358 (2019), 51–62.

[172] J. Zhong, J. Lu, T. Huang and D. Ho, Controllability and synchronization analysis of identical-hierarchy mixed-valued logical control networks, *IEEE Transactions on Cybernetics*, 47 (2017), 3482–3493.

[173] J. Zhong, J. Lu, Y. Liu and J. Cao, Synchronization in an array of output-coupled Boolean networks with time delay, *IEEE Transactions on Neural Networks and Learning Systems*, 25 (2014), 2288–2294.

[174] B. Zhu, X. Xia and Z. Wu, Evolutionary game theoretic demand-side management and control for a class of networked smart grid, *Automatica*, 70 (2016), 94–100.

[175] Q. Zhu, Y. Liu, J. Lu and J. Cao, Observability of Boolean control networks, *Science China Information Sciences*, 61 (2018), 092201, 12 pp.

[176] Q. Zhu, Y. Liu, J. Lu and J. Cao, Optimal control of Boolean control networks, *SIAM Journal on Control and Optimization*, 56 (2018), 1321–1341.

[177] Q. Zhu, Y. Liu, J. Lu and J. Cao, Further results on the controllability of Boolean control networks, *IEEE Transactions on Automatic Control*, 64 (2019), 440–442.

[178] S. Zhu, Y. Liu, J. Lou, J. Lu and F. Alsaadi, Sampled-data state feedback control for the set stabilization of Boolean control networks, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 50 (2020), 1580–1589.

[179] S. Zhu, J. Lou, Y. Liu, Y. Li and Z. Wang, Event-triggered control for the stabilization of probabilistic Boolean control networks, *Complexity*, 2018 (2018), 1–7.

[180] S. Zhu, J. Lu and Y. Liu, Asymptotical stability of probabilistic Boolean networks with state delays, *IEEE Transactions on Automatic Control*, 65 (2020), 1779–1784.

Received September 2019; revised January 2020.

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