Separating eccentricity deviation in transfer torque wrench calibrations

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Abstract. An approximation method is proposed to achieve the contribution of the square drive eccentricity in transfer torque wrench calibrations. This systematic contribution can thus be quantified and eliminated. By doing this, the influence of the contact surfaces in the square drive can be estimated more precisely. The approximation succeeds not only at complete 4-position calibrations but also – with the help of a numerical fit – with calibrations over only 3 positions, a procedure which is required in the current German guideline for torque wrench calibrations. The proposed method has been validated through a survey of transfer torque wrench calibrations and through Monte Carlo simulations.

1. Introduction
Transfer torque wrench calibrations include multiple series in various mounting positions of the square drive, consisting of the square adapter plug of the wrench and the corresponding socket of the calibration machine. The results of these series are usually averaged to eliminate specific effects associated with the mounting position. The two most important of these are the eccentricity and the contact effects. Since usually – according to the relevant German guideline – only 3 of the 4 possible positions are used, the sinusoidal contribution of the eccentricity cannot be eliminated completely and the average shows a systematic offset. But even the average over a complete 4-position calibration cannot separate the two effects, which prevent, firstly, the improvement of the process and, secondly, the proper calculation of the standard uncertainty of the position effect.

2. Square drive effects

2.1. Surface effects
Imperfect matching of the combined surfaces in the square drive due to their convexity, obliquity and roughness causes deviations $b_{s,i}$ in the force transmission of the square drive. It acts like a random effect and is specific to each combination of surfaces in the rotational position $i$ of the square drive.

2.2. Eccentricity
The eccentricity $\vec{E}$ is the distance vector between the calibration machine fulcrum and the torque wrench fulcrum in the plane perpendicular to the torque vector. The deviation $b_E$ caused by it, is based on the change in the effective lever length and depends on the torque load and the nominal lever length $l$ [1]. $\vec{E}$ is a superposition of the eccentricity $\vec{E}_p$ of the plug relative to the torque wrench sensor, the
eccentricity $E_p$ of the square drive fit between the plug and the socket and the eccentricity $E_R$ of the socket relative to the calibration device fulcrum.

Since the position of the plug at the wrench is usually defined to be fixed during transfer calibrations, the contribution of $E_p$ is then eliminated. For clamped square drives, $E_p$ is reduced to zero; for loose square drives, the amount depends on the friction and the geometrical conditions in the square drive. Sources of $E_R$ are misalignments of the calibration device and plug size variations.

Further deviations are due to the direction and amount of the cross force $F_c$ and of the weight $F_G$ which create the deviations $b_c$ [2] and $b_G$. These effects are only accessible by position alterations which involve the entire wrench [3] and are not the subject of this paper.

![Figure 1](image)

**Figure 1.** Positions and eccentricity $E$ in a square drive adaption. If the position of the socket is altered from $\vartheta$ to $\vartheta + 1$, the coordinate system of the socket rotates by the same angle and carries $E$ with it to $\vartheta + \vartheta + 1$. The lever always stays upright.

3. Implementation

3.1. Rotation

To achieve useful calibration results despite the square drive effects, as a rule 3 series were analysed under different rotational relations between the plug and the socket [4]. The deviations $b_s$ and $b_E$ can be reduced by square drive rotations if $b_s$ is based on a random effect and if the eccentricity, which is known to be a systematic effect, is carried on by the rotation of the positions. This is certainly provided with clamped connections. But even then, calculating the average of the positions cannot separate the two contributions. Furthermore, the sinusoidal contribution of the eccentricity cannot be eliminated completely in 3 positions proceeding by 90° and the average shows a systematic offset, which can reach $1 \cdot 10^{-4}$ relative to the average value under typical conditions.

3.2. Separation of the eccentricity

Even if measured in 4 positions of 90°, the influences of eccentricity and of matching surfaces cannot be distinguished by averaging in the signal $S$, which includes the influences of both. This is possible with an approach describing the signal $S_i$ at the position $i$ by (1).

$$S_i = S' \left( 1 + \frac{E}{l} \cos(\vartheta_i + \vartheta_E) + b_{s,i} \right),$$

where

$$i = [1,2,3,4], \quad \vartheta_i = \vartheta_{[1,2,3,4]} = [0°, 90°, 180°, 270°].$$

Considering the geometrical circumstances (figure 1), the approximated amount $E^*$ of the eccentricity $E$ becomes

$$E \approx E^* = l \left( \frac{(S_{i,3} - S_{2})^2 + (S_{2,4} - S_{1})^2}{S^2} \right)^{1/2},$$

if $S'$ is similar to the 4-position average $\bar{S}$ and if the averaged contribution $b_{s,i}$ of the surfaces is small compared to that due to the eccentricity. The approximated angle $\vartheta_E^*$ of the eccentricity is then given by

$\bar{S} \approx S'$
In this manner, a 4-position calibration can be exempted from the influence of the eccentricity:

\[
\vartheta_E = \vartheta_E^* = \arcsin\left(\frac{l}{E^*} \left(\frac{S_i}{S} - S_z\right)\right)
\]

(3)

and a signal \(S_{i,i}^*\) including only the influence of surfaces is available. The parameter \(b\), which is the repeatability defined as a standard deviation in [4], could be reduced to the surface effect:

\[
b \approx \sigma(S_{i,i}^*)
\]

(5)

With this method, not only the uncertainty contribution of \(b\) is reduced, but also the information about the eccentricity becomes available for improving the calibration machine and the calibration process.

### 3.3. Demonstration of separation reliability

To demonstrate the ability of the method, 4-position measurements were repeated under 3 carefully prepared and geometrically measured eccentricities at a setup with a clamped square drive. The separation according to (1 - 5) delivers \(E\) and \(\vartheta_E\) with sufficient compliance to the geometrically determined sets of parameters (table 1).

Despite the high eccentricity in the cases II and III, the rotational symmetry of the effect keeps the 4-position averages of these measurements stable within a relative margin of \(5 \cdot 10^{-5}\) at loads higher than 10 %, which is less than the expanded contribution of the repeatability in this situation. The relative spread of the 3-position measurement, \(b\) according to [4], improves significantly by about the amount of \(b_E\) when the separation is executed for correction in this range. Because eccentricity also acts on the relative contribution of lever arm length variation, \(b_l\) [4], the correction here causes a similar reduction. Altogether, the separation avoids the drawbacks of eccentricity almost completely in the given examples.

### 3.4. Complementation of the 4th position

At measurements consisting of only 3 positions, the absent 4th position signal can be substituted by a fit according to the approach described above (4). Since the nodes of the fit are rather numerous (48 nodes in 3 series of 8 steps at clockwise and anticlockwise torque each), the stability of the fit is adequate. As could be demonstrated in a survey of 4-position calibrations, the deviation of the averaged calibration result from the virtual 4-position average could be reduced by the complementation in nearly every case (figure 2), even if the contribution of eccentricity due to the incomplete compensation of the position \(b_{E,2}\) is small. Where no improvement is possible, the worsening remains at low amounts, which are non-critical for the uncertainty budget. The method turns out to be appropriate both for

### Table 1. Example of reproducing intentionally prepared eccentricities by separation.

| No. | \(E / \text{mm geom.}\) | \(\vartheta_E / \text{deg geom.}\) | \(E / \text{mm separ.}\) | \(\vartheta_E / \text{deg. separ.}\) |
|-----|-----------------|-----------------|-----------------|-----------------|
| I   | 0.07            | 45              | 0.06            | 57              |
| II  | 0.43            | 134             | 0.42            | 137             |
| III | 0.70            | 135             | 0.68            | 138             |

### Figure 2. Yield of the complementation method against the method required in [4] in a survey of 4-position calibrations expressed as a relative uncertainty improvement, plotted against the absolute value of the eccentricity contribution in the 2nd position (90°). Blue dots are of clamped square drives, red dots of loose ones.
clamped and for loose square drives and for a wide range of eccentricities.

3.5. Monte Carlo simulations

To confirm the method of complementation, Monte Carlo simulations were executed altering the eccentricity amount $E$, the eccentricity angle $\vartheta_E$ and the influence of the surfaces $b_s$. The results were rated on the basis of the relative improvement $Y$ in the calibration average against the average calculated according to [4] (figure 3). The dependence of $Y$ on the eccentricity was found to be linear with an inclination of about $0.3/l$ at maximum ($\vartheta_E = [90^\circ, 270^\circ]$). The improvement varies with $\vartheta_E$ according to a $\sin^2$ function with zero points at $0^\circ$ and $180^\circ$, where the influence of the eccentricity is eliminated perfectly in the positions 1 and 3, while position 2 contributes no effect. The influence of $b_s$ on $Y$ is also found to be linear with a proportionality factor of about 0.1. The simulations demonstrate the method of complementation to be sufficiently robust and definite in the practice-relevant range of parameters.

4. Conclusions

Calibrations of torque wrenches according to the appropriate German guideline require the average calculation of 3 positions out of a possible 4 x 90° positions. Measuring all 4 positions can prevent relative deviations of about $1 \times 10^{-4}$ and – by approximated separation – the contribution of the square drive eccentricity can be determined and eliminated. At measurements consisting of only 3 positions, a complementation of the 4th position by fitting the sinusoidal eccentricity effect is possible and offers the advantage of the proposed separation method to these calibrations, too. With the help of a survey on numerous wrench calibrations and Monte Carlo simulations, the validation of this method has been proved. The separation method can reduce the uncertainty contributions of the eccentricity, of the contact effects and of the effect due to the lever arm length significantly compared to the usual 3-position calibration.

5. References

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