Universal length dependence of tensile stress in nanomechanical string resonators

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We investigate the tensile stress in freely suspended nanomechanical string resonators, and observe a material-independent dependence on the resonator length. We compare strongly stressed string resonators fabricated from four different material systems based on amorphous silicon nitride, crystalline silicon carbide as well as crystalline indium gallium phosphide. The tensile stress is found to increase by approximately 50% for shorter resonators. We establish a simple elastic model to describe the observed length dependence of the tensile stress. The model accurately describes our experimental data. This opens a perspective for stress-engineering the mechanical quality factor of nanomechanical string resonators.

I. INTRODUCTION

Over the past decades, nanomechanical systems (NEMS) have received considerable attention as versatile elements in mesoscopic devices. For example, they are well-suited for sensor applications, and have been shown to act as highly sensitive mass [1, 2], force [3–6], or infrared [7] detectors. Even more, they serve as amplifiers or oscillators. On the other hand, nanomechanical systems lend themselves to the exploration of fundamental physical phenomena both in the classical [8–10] and in the quantum regime [11–14], with the prospect of serving as hybrid transducer [15] or as storage element [16] in future quantum technologies.

Many of the underlying devices are based on one-dimensional string or two-dimensional membrane resonators [6, 8–14, 17–27] which are both characterized by a strong intrinsic tensile stress in the device layer. Tensile stress in the device layer enables remarkably large mechanical quality factors as a result of a process commonly referred to as dissipation dilution [28–31]. This process relies on the stress-induced increase of the stored vibrational energy of the resonator, while the dissipated energy remains largely unaffected.

Several approaches have been pursued to suppress the limiting dissipation mechanism and to further enhance the potential of this class of nanomechanical resonators. For example, clamping losses can be reduced by means of mechanical impedance mismatch engineering [32] which is exploited, e.g., in trampoline resonators [23, 24, 33]. In addition, phononic bandgaps have been employed to reduce clamping losses [34–36]. Recently, soft clamping has been presented as an innovative approach of boosting the quality factor [25]. It is also based on phononic bandgap engineering, but additionally prevents mechanical strain at the interface between the resonator and its clamping points and thus enhances the dissipation dilution. Soft-clamped membranes have recently enabled measurement-based quantum control [12] or measurements below the standard quantum limit [13], and may find application in magnetic resonance force imaging [6]. For string resonators, soft clamping can be beneficially combined with stress engineering [26, 37], which boosts the tensile stress close to the mechanical limits and enables quality factors $Q$ of around 800 millions and a $Q \times f$ product of more than $10^{15}$ Hz. These developments pave the way towards an ever increasing sensitivity and the observation of mechanical quantum phenomena at room temperature.

Here we describe a previously unappreciated aspect of nanomechanical string resonators: The one-dimensional tensile stress in the string is not solely determined by elastic material properties, but significantly depends on its length as well as other geometric parameters. This allows to increase the tensile stress by approximately 50% by using shorter strings and thus boost the dissipation dilution. The observed length-dependence of the tensile stress is material independent. We demonstrate length-dependent tensile stress for nanomechanical resonators fabricated from four different wafers featuring the three complementary, tensile-stressed device layers silicon nitride (SiN), silicon carbide (SiC), and indium gallium phosphide (InGaP). A simple elastic model is developed which captures the observed features. Our results entail important insights for strain engineering of nanomechanical devices and may be exploited to boost the mechanical quality factor of string resonators.

II. EXPERIMENTAL RESULTS

We investigate nanomechanical string resonators fabricated from three distinct, strongly stressed material
platforms, namely amorphous silicon nitride (SiN), crystalline silicon carbide (SiC) and crystalline indium gallium phosphide (InGaP). The 100 nm thick amorphous stoichiometric SiN film is deposited by low pressure chemical vapor deposition on top of two different substrates, a fused silica wafer (material denoted SiN-FS) and a sacrificial SiO₂ layer atop a silicon wafer (SiN-Si). The 110 nm thick film of crystalline 3C-SiC is epitaxially grown on a Si(111) wafer (SiC). A III-V heterostructures hosting an 100 nm thick In₀.₄₁₅Ga₀.₅₈₅P film is epitaxially grown atop a sacrificial layer of AlₓGa₁₋ₓAs on a GaAs wafer. See Tab. S.1. in the Supplemental Material (SM) for the full details of all wafers.

Series of nanomechanical string resonators such as the one depicted in Fig. 1 are defined in all four materials. The length of the strings increases from 10 µm to 110 µmconstantly, whereas the thickness of the device layer \( h_1 \) is obtained from the wafer growth protocol, whereas the thickness of the resonator, and \( \sigma \) the tensile stress of the string.

The resonance frequencies shown in Fig. 2 are fitted with Eq. (1). To a very good approximation, the eigenfrequencies of all resonators scale linearly with the mode number as expected for stress-dominated nanostrings for which \( f_n \approx (n/2L) \sqrt{\sigma/\rho} \). The fits of the eigenfrequencies as a function of mode number allow to extract the tensile stress of each nanostring resonator. The obtained values are shown as a function of the resonator length for all four materials in Fig. 3. Clearly, the tensile stress is not constant, but decreases for increasing length of the resonator. The same qualitative behavior is observed in all four material systems.

### III. ELASTIC MODEL

To the best of our knowledge, a length-dependence of the tensile stress in a nanostring has not been reported in the literature. We have developed a simple theoretical model to quantify this previously unappreciated phenomenon. Our model is based on elastic theory. As such, it is material independent and can be applied to all material systems under investigation. The model assumes a prismatic string of length \( L \), width \( w \) and thickness \( h_1 \). Its cross sectional area is \( A_s = w h_1 \). On both ends, the string is attached to a rectangular clamping structure. It consists of a clamping pad in the device layer with lithographic dimensions \( 2a_x \) and \( 2a_y \), as well as thickness \( h_1 \) (Fig. 4(a)), which is supported by a pedestal of height \( h_0 \) in the underlying sacrificial or substrate layer (Fig. 4(b) and (c)). As a result of the isotropic wet etching process required to suspend the nanostrings, the clamping pads exhibit a certain undercut \( a_{uc} \), i.e. the width of the pad \( 2a_x \) in \( x \)-direction, the width \( a_y \) in \( y \)-direction may differ is larger than that of the pedestal \( 2a_y = 2a_x - 2a_{uc} \). The cross sectional area of the clamping pad (in \( y \)-direction) is \( A_p = 2a_y h_1 \). The geometric parameters of the four investigated samples are summarized in Tab. I. The thickness of the device layer \( h_1 \) is obtained from the wafer growth protocol, whereas the
FIG. 3. Experimentally determined tensile stress as a function of the length of the nanostring for all four material systems. Fits of Eq. (9) are included as solid lines. The obtained fit parameters are summarized in Tab. II. The shaded areas indicate the uncertainty resulting from measurement errors of the pedestal height $h_0$ and undercut $a_{uc}$.

TABLE I. Geometric parameters of the investigated samples.

| Material | $h_1$ (nm) | $h_0$ (nm) | $2a_x$ (µm) | $2a_y$ (µm) | $a_{uc}$ (nm) | $w$ (nm) |
|----------|------------|------------|-------------|-------------|---------------|----------|
| SiN-FS   | 100(5)     | 460(20)    | 13.7(2)     | 13.6(2)     | 570(100)      | 420(25)  |
| SiN-Si   | 100(5)     | 365(20)    | 14.1(2)     | 15.0(2)     | 410(150)      | 340(25)  |
| SiC      | 110(15)    | 570(40)    | 14.2(2)     | 15.0(2)     | 860(150)      | 360(30)  |
| InGaP    | 100(1)     | 990(10)    | 12.7(2)     | 13.3(2)     | 640(170)      | 250(15)  |

height of the pedestal $h_0$ was determined by atomic force microscopy. The half-widths of the clamping pad $a_x$ and $a_y$, the undercut $a_{uc}$ and the width of the nanostring $w$ have been extracted using electron beam microscopy. The elastic and material parameters of the samples are listed in Tab. S.2. in the SM.

As we will show in the following, the tensile stress in the device layer atop an unstressed sacrificial layer or substrate gives rise to a balance of forces which in turn leads to a length- and geometry-dependent change in the one-dimensional tensile stress of the nanostring. To quantify the contributing forces, we follow the process sequence required to fabricate a freely suspended nanosting. First, an anisotropic etch defines the lateral dimensions of the structure. Following this vertical release, the tensile-stressed device layer will contract and induce a certain amount of shear in the pedestal (Fig. 4(b)). As a result, the tensile stress in the pad relaxes to a value $\sigma_p$. Second, an isotropic wet etch releases the nanostring and undercuts the clamping pads. As a result of this lateral release, the two-dimensional stress in the nanostring relaxes in the direction perpendicular to the string. At the same time, the undercut parts of the tensile-stressed clamping pad contract, which applies additional stress on the nanostring (Fig. 4(c)). The combination of the described effects gives rise to the tensile stress experienced by the nanostring $\sigma$ (see Eq. (1) and Fig. 3). The model assumes a clear separation between these two effects, and neglects geometric and elastic reconfigurations of the sheared pedestal and stressed clamping pad arising from the subsequent lateral release, which we can safely assume to be small.

A. Pedestal shear

To evaluate the shear of the pedestal induced by the vertical release of the structure, we will first consider an isolated clamping structure and focus on its cross section.
along the $x$-$z$-direction as indicated in Fig. 4(b). The resonator will be included at a later stage. Following the vertical release of the structure, the strong tensile stress in the device layer leads to a contraction of the clamping pad in order to minimize internal forces. This contraction produces an increasing shear of the pedestal. The reconfiguration of the clamping structure stops once equilibrium between the reduced tensile force and the counteracting shearing force is reached. The shear stress $\tau$ of such a shear-constrained material system can be expressed as [40]

$$\tau = \sigma_{2D} h_1 k \tanh (ka_x), \quad k = \sqrt{\frac{G_0}{h_0 E_1 h_1}}$$  \hspace{1cm} (2)$$

where $h_0$ and $h_1$ is the height of the pedestal and the clamping pad, respectively, $G_0$ is the shear modulus of the pedestal, $E_1$ is Young’s modulus of the clamping pad, and $\sigma_{2D}$ is the initial two-dimensional stress in the device layer. This result in the maximum contraction of the clamping pad $\Delta p$ from its original half-width $a_x$:

$$\Delta p = \frac{h_0}{G_0} \tau = \frac{\sigma_{2D}}{E_1 k} \tanh (ka_x).$$  \hspace{1cm} (3)$$

In consequence, the tensile stress in the clamping pad is reduced to $\sigma_p = \sigma_{2D} - E_1 \frac{\Delta p}{a_x}$ according to Hooke’s law. Note that a similar model which also accounts for additional shear in the device layer is presented in Ref. [41].

For the sake of simplicity we neglect the minute counterforce exerted by the presence of the resonator, which will lead to a slightly reduced contraction of the pad to which it is attached. Similarly, the effect of shear of the pedestal underneath the resonator will be ignored.

An experimental verification of the contraction of the clamping pad following the vertical release is discussed in the SM.

### B. Undercut of clamping pads

The lateral release of the nanostrings results in an undercut of the clamping pads. More specifically, the width of the pedestal is reduced by $a_{uc}$ from either side such that the rim of the clamping pad gets freely suspended as shown in Fig. 4(c). This enables a relaxation of the tensile force in the undercut parts of the pads which gives rise to a contraction by an amount $\Delta c$. The resulting contracting force acting on the interface between the clamping pad and the nanostring can be expressed as

$$F_c = \sigma_p A_p - E_1 \frac{\Delta c}{a_{uc}} A_p,$$  \hspace{1cm} (4)$$

where $\sigma_p$ is the remaining tensile stress in the clamping pad following the vertical release, and $E_1 \Delta c / a_{uc}$ is its reduction in the undercut part of the clamping pad, again according to Hooke’s law. Note that in absence of the nanostring, the suspended part of the clamping pad fully relaxes such that $F_c = 0$. In the presence of the nanostring, however, the contracting force of the clamping pad is countered by a second force acting on the interface between the clamping pad and the nanostring which is associated with the elongation $\Delta L$ of the nanostring

$$F_s = \sigma_\infty A_s + E_1 \frac{\Delta L}{L} A_s,$$  \hspace{1cm} (5)$$

where $\sigma_\infty$ is the one-dimensional stress of an infinitely long nanostring after the lateral release, and $E_1 \Delta L / L$ is its modification according to Hooke’s law.

The equilibrium condition for the clamping pad - nanostring interface

$$F_c = F_s$$  \hspace{1cm} (6)$$

determines the final geometric reconfiguration of the clamping pad and the string, under the boundary condition that the total length of the compound between the centers of the clamping pads has to be conserved,

$$2\Delta p + 2\Delta c = \Delta L.$$  \hspace{1cm} (7)$$

Equations 6 and 7 form a second order system of linear equations with the unknown parameters $\Delta L$ and $\Delta c$. The third unknown $\Delta p$ is determined using Eq. (3). Solving for the elongation of the resonator yields

$$\Delta L = 2L \left( \frac{A_p a_{uc} \sigma_p + A_p E_1 \Delta p - A_s a_{uc} \sigma_\infty}{E_1 (2A_s a_{uc} + A_p L)} \right).$$  \hspace{1cm} (8)$$

This length change of the resonator directly translates into an additional strain $\varepsilon = \Delta L/L$, giving rise to a length-dependent stress $\sigma(L)$ of the doubly clamped string resonator via Hooke’s law

$$\sigma(L) = \sigma_\infty + \frac{E_1 \Delta L}{L}.$$  \hspace{1cm} (9)$$

### IV. DISCUSSION

To validate the theoretical model, we fit Eq. (9) to the experimental data measured on all four material systems, using the geometric and material parameters specified in Tabs. I and S.2. The initial two-dimensional stress $\sigma_{2D}$ can be calculated from the epitaxial lattice mismatch of the crystalline InGaP sample. The mismatch of the lattice constants of the In$_{1-y}$Ga$_y$As sacrificial layer induces an in-plane strain $\varepsilon_{\parallel}(x) = (a_x^\infty - a_x^\parallel(x))/a_x^\infty(x)$, where $a_x^\infty(x)$ is the lattice constant of In$_{1-y}$Ga$_y$As and $a_x^\parallel(x)$ is the in-plane lattice constant of the strained In$_{1-y}$Ga$_y$As$_z$P film. This allows to compute the two-dimensional stress of the thin In$_{1-y}$Ga$_y$As$_z$P layer $\sigma_{2D} = \varepsilon_{\parallel}(x) / (1 - \nu_1)$ [42]. The ratio $E_1/(1 - \nu_1)$ including the Poisson ratio $\nu_1$ (see Tab. S.2 in the SM) represents the biaxial modulus of the stressed thin film, which is required as no stress occurs in the $z$-direction normal to the substrate. The obtained $\sigma_{2D}$
value of 0.95 GPa has also been used as input parameter for the model. In principle, the same argument can be made for SiC which is also an epitaxially grown crystalline thin film material. However, the crystallization of 3C-SiC atop a Si wafer is more complex than that of the III-V heterostructures. SiC and Si feature the same crystal structure, but exhibit a lattice mismatch of approx. 20%. This implies a nontrivial, commensurate growth of the SiC film which strongly depends on growth conditions, such that the two-dimensional stress cannot be predicted from the crystal structure [43–48]. Hence, we set $\sigma_{2D}$ as an additional fit parameter for SiC. The same applies for the amorphous thin film materials SiN-FS and SiN-Si. The one-dimensional stress $\sigma_\infty$ is employed as fit parameter for all material systems.

The results of the fits are included in Fig. 3 as solid lines. The shaded area represents the model’s uncertainty arising from the error of the input parameters. As long as $A_\perp \ll A_p$ and $a_{uc} \ll L$, the length dependence of Eq. (9) can be approximated as $\sigma(L) \propto 1/L$. This holds true for all nanostrings under investigation, such that an $1/L$ dependence of the stress can be assumed. We find a remarkable agreement between the model and the experimental data. This is particularly noteworthy for the case of the InGaP samples for which only one fit parameter, $\sigma_\infty$, is employed, which, in the above approximation of small $A_\perp a_{uc}$ corresponds to a vertical offset and thus the limit $\sigma(L \to \infty)$. Also the results for SiN and SiC, which involve two fitting parameters, show good agreement between the model and the experimental data. Again, $\sigma_\infty$ can be interpreted as the tensile stress of an infinitely long string, whereas the two-dimensional stress in the as-grown device layer $\sigma_{2D}$ can at least to some extent be compared to literature values.

Table II summarizes the parameters obtained from the elastic model as well as the fit parameters for the case of the longest strings ($L = 110 \mu m$). The as-grown two-dimensional stress in LPCVD-grown stoichiometric SiN on silicon is found to depend on growth conditions, but has been reported to amount to 1.1 GPa [26, 49] and 1.4 GPa [50], which is close to the value found here. The same applies for high stress 3C-SiC(111), for which an as-grown two-dimensional stress of 1.3 GPa has been reported [45], which is somewhat lower than our result. The growth of high stress SiN on a fused silica substrate is poorly characterized, and no comparison with literature could be obtained. Certainly, all observed two-dimensional stress values are well within the yield strength of the respective material, which amounts to approx. 6 – 7 GPa (or even 12 GPa according to Ref. [51]) for high stress LPCVD-deposited SiN [26, 49, 52], and 21 GPa for SiC [53].

The geometric changes also summarized in Table II are in the range of only a few nm. This is below the resolution of our scanning electron microscopy and can thus not be experimentally confirmed. However, the finite element simulations discussed in the SM qualitatively confirm the reported geometric reconstructions.

| Material     | $\Delta p$ (nm) | $\Delta c$ (nm) | $\Delta L$ (nm) | $\sigma_{2D}$ (GPa) | $\sigma_\infty$ (GPa) |
|--------------|-----------------|-----------------|-----------------|---------------------|----------------------|
| SiN-FS       | 8               | 6               | 27              | 3.1$^a$             | 1560$^b$             |
| SiN-Si       | 3               | 2               | 8               | 1.2$^a$             | 900$^b$              |
| SiC          | 3               | 16              | 16              | 2.5$^a$             | 1110$^a$             |
| InGaP        | 3               | 5               | 15              | 0.95$^b$            | 540$^b$              |

*From fit $^b$Calculated with $\sigma_{2D} = E_p/(1 - \nu_1)$ [42]}

Finally, we discuss the relation between $\sigma_\infty$ and $\sigma_p$. For a one-dimensional nanostring processed from a thin film under biaxial and isotropic stress, the one-dimensional stress follows from the initial two-dimensional stress according to

$$\sigma_{1D} = \sigma_{2D}(1 - \nu_1).$$

For the nanostrings under investigation, this simple picture does not hold, as the stress relaxation along the $y$-direction upon releasing the string assumes a more complicated stress configuration following the contraction of the device layer described in Sec. III A. Not only does the contraction of the clamps by an amount $\Delta p$ reduced the two-dimensional stress in the clamping pads from $\sigma_{2D}$ to $\sigma_p$. A similar contraction also occurs along the $y$-direction of the string, such that the tensile stress in the string before the lateral release can not be considered isotropic.

Additionally, we wish to note that high resolution X-ray diffraction measurements performed on In$_{1-x}$Ga$_x$P wafers have shown a compositional variation in the direction normal to the substrate [42]. This can furthermore lead to strain gradients inside the device layer. A similar observation has been made for 3C-SiC in Ref. [48]. This suggests that a more thorough analysis of the length-dependent stress should assume a three-dimensional strain tensor accounting for a vertical strain gradient rather than a biaxial isotropic thin film stress $\sigma_{2D}$.

V. CONCLUSION

In conclusion, we report on the observation of a length-dependence of the tensile stress in nanomechanical string resonators. This previously unappreciated effect is material independent, and experimentally observed on samples fabricated from four different wafers, featuring the three complementary material platforms amorphous silicon nitride, crystalline silicon carbide and crystalline indium gallium phosphide. We develop a simple elastic model which describes the observed $1/L$ dependence of the tensile stress, and which allows to explain the observed length-dependence by a combination of the elastic reconfiguration of the device under the vertical as well
as the subsequent lateral release of the one-dimensional nanostring. The one-dimensional tensile stress relaxes to a value considerably smaller than the initial two-dimensional stress value for long strings. For shorter strings, this value increases by approximately 50\%. Besides the length, the height of the supporting pedestal and the size of the undercut of the clamping pads influence the resulting stress. Thus, the geometric parameters of the nanostring allow to engineer the tensile stress, and thus to control the quality factor of the device without the need for complex phononic metamaterial processing.

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Supplemental Material: Universal length dependence of tensile stress in nanomechanical string resonators

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This Supplemental Material provides further information about wafer and material parameters in Appendix A, the finite element simulations performed to validate the geometric reconfiguration of a fully released nanostring in Appendix B, and the experimental confirmation of the shearing contraction of the clamping pedestals in Appendix C.

Appendix A: Wafers and material parameters

In Tab. S.1 the growth parameters of the four wafers employed in this work are summarized, stating the thickness of the device layer, sacrificial layer (if the system has one), substrate, and the corresponding supplier. The two SiN wafers were grown by Low Pressure Chemical Vapor Deposition (LPCVD), the SiC in a two-stage Chemical Vapor Deposition process, and the InGaP using Metal-Organic Chemical Vapor Deposition (MOCVD).

| TABLE S.1. Basic parameters of the wafers on which the string resonators were fabricated. |
|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|
| resonator/device layer | sacrificial layer | substrate | source |
| SiN-FS | 100 nm SiN | — | SiO₂ | HSG-IMIT |
| SiN-Si | 100 nm SiN | 400 nm SiO₂ | Si(100) | HSG-IMIT |
| SiC | 110 nm 3C-SiC | — | Si(111) | NOVASiC |
| InGaP | 100 nm In₀.₄₁₅Ga₀.₅₈₅P | 1000 nm Al₀.₈₅Ga₀.₁₅As | GaAs | CNRS |
All material parameters employed in the theoretical calculations, i.e. the Young’s modulus $E$, the shear modulus $G$, the Poisson ratio $\nu$, and the mass density $\rho$ of the respective materials, are listed in Tab. S.2.

**TABLE S.2.** Young’s modulus, shear modulus and density of the materials used within the main text. All shear mouduli where calculated via $G = \frac{E}{2(1+\nu)}$.

| Material | Young’s modulus $E$ (GPa) | Shear modulus $G$ (GPa) | Poisson’s ratio $\nu$ | Density $\rho$ (g/cm$^3$) |
|----------|--------------------------|-------------------------|----------------------|--------------------------|
| SiN      | 260 [1]                  | 104                     | 0.25 [2]             | 3.1 [2]                  |
| SiO$_2$  | 73 [2]                   | 31                      | 0.17 [2]             | 2.2 [2]                  |
| Si       | 160 [2]                  | 66                      | 0.22 [2]             | 2.4 [2]                  |
| SiC      | 419 [3]$^a$             | 184                     | 0.14 [2]             | 3.166 [4]                |
| InGaP    | 124 [5]$^a$             | 47                      | 0.32 [5]$^b$         | 4.418 [5]                |
| GaAs     | 75 [2]                   | 29                      | 0.31 [5]             | 5.3 [2]                  |

$^a$Determined by utilizing the elastic constants and the stiffness tensor as described in [6].

$^b$Calculated with $\nu = \frac{c_{12}}{c_{11}+c_{12}}$ where $c_{ij}$ are the elastic constants.

**Appendix B: Finite Element Method Simulations**

The geometric reconfiguration of the pedestal, the clamping pad and the string was explored in more detail by finite element method (FEM) simulations to validate our theoretical considerations. To this end, the individual 10 $\mu$m long and 300 nm wide SiN-FS string resonator held in place by two SiO$_2$ pedestals on a SiO$_2$ substrate shown in Fig. S.1 was considered. The thickness of the device layer was set to 100 nm, a 500 nm undercut and a pedestal height of 1 $\mu$m were assumed, as well as an initial two-dimensional tensile stress of 2.9 GPa. A perfectly matched layer was included to mimic an infinite substrate, but did not noticeably influence the result. A close look at Fig. S.1 clearly reveals the shearing of the pedestal as well as the contraction of the clamping pad due to the stressed device layer. Also apparent is the resulting elongation of the string and the enhanced tensile stress in the string, which, for the case of the extremely short length of the simulated string, even exceeds the remaining tensile stress in the clamping pad. These observations qualitatively support all assumptions of the elastic model put forward in the main text.
FIG. S.1. FEM simulations of a single string resonator with an initial stress of 2.9 GPa. Furthermore, we set a thickness of 100 nm for the device layer, a 500 nm undercut and a pedestal height of 1 µm.

**Appendix C: Measuring the pedestal contraction**

To further support our elastic model, we have experimentally quantified the shearing of the pedestal using the test structures discussed in the following. An array of quadratic pedestals is fabricated on SiN-FS (see Fig. S.2(a) and (c)), the material for which the biggest contraction is expected (c.f. Tab. II). As shown in Fig. S.2(a) and (b), the uncontracted width of a pedestal is $2a$ and the pedestal-pedestal distance is $d$. An anistropic ICP-RIE etch step (etching depth of around 350 nm) allows for a contraction of the pedestal by $2\Delta p$ to $2a_{\text{con}} = 2a - 2\Delta p$. Because the contraction is in the nanometer regime and the pedestal in the micrometer regime, we can not simply image the whole pedestal and directly measure $2a$ and $2a_{\text{con}}$ and calculate the contraction $2\Delta p$, as this is beyond the resolution of our scanning electron microscopy. However, as indicated schematically in Fig. S.2(b), the separation of two closely-spaced pedestals of the test structure can be mapped out with a higher resolution. Comparison of their spacing before and after the contraction, $d$ and $\tilde{d}$, respectively, indeed yields in increase of the gap as shown in Fig. S.2(d), which indicates a contraction of the clamping structure. For our sample chip we measure an average value of $d = 793(6)$ nm and $\tilde{d} = 824(7)$ nm (over the entire gap, not just the section close to the center shown in Fig. S.2(d)), corresponding to a contraction of $\Delta p = \frac{\tilde{d}-d}{2} = 15(7)$ nm. The theoretical value of $\Delta p = 8$ nm is just within the uncertainty of the measured value.
FIG. S.2. Array of pedestals (a) and a close up (b) including length annotations. Dashed lines and solid lines correspond to the pedestal before and after contraction, respectively. (c) SEM image of the array structure before it was etched. (d) SEM image of the gap between two pedestals before (left) and after (right) contraction.
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