Multi-Objective Optimal Gearshift Control for Multispeed Transmission Electric Vehicles

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ABSTRACT Electric vehicles are being increasingly adopted worldwide to support the sustainable development of society. However, various technologies that are related to electric vehicles still require further investigation. In this study, we consider a gearshift control architecture that combines offline trajectory planning and online control for improved shifting in multispeed transmission electric vehicles. For gearshift trajectory planning, we establish a dynamic model and apply the multi-objective optimization Radau pseudospectral method. Simulations of this method provide a Pareto solution set under three optimization objectives, namely duration, friction work, and jerk, thereby establishing a new approach for satisfying the requirements for shift quality. Moreover, the relations among these objectives are detailed on the Pareto solution set. We conducted a hardware-in-the-loop simulation to verify the performance of the proposed control architecture and trajectory planning method. The simulation results indicate that the gearshift control delivers a suitable response for different driving intentions based on the obtained Pareto solution set.

INDEX TERMS Electric vehicle, gearshift control, multi-objective optimization, Radau pseudospectral method.

NOMENCLATURE

| PHYSICAL QUANTITY | \(\theta_{ssg}\) | Angular displacement of small sun gear |
|--------------------|----------------|----------------------------------------|
| \(\theta_{lsg}\)    | Angular displacement of large sun gear |
| \(\theta_c\)       | Angular displacement of carrier |
| \(\theta_{op}\)    | Angular displacement of outer pinions |
| \(\theta_{ip}\)    | Angular displacement of inner pinions |
| \(\theta_r\)       | Angular displacement of ring |
| \(T_m\)            | Torque of motor |
| \(T_{lsg}\)        | Torque of large sun gear |
| \(T_c\)            | Torque of carrier |
| \(J_t\)            | Duration |
| \(J_w\)            | Friction work |
| \(J_j\)            | Jerk |

ABBREVIATIONS

| Abbreviation | Description |
|--------------|-------------|
| RPM          | Radau pseudospectral method |
| DTD          | Driver torque demand |
| DPD          | Driver power demand |
| BP           | Balanced pattern |
| MDP          | Minimum duration pattern |
| MJCP         | Minimum continuous jerk pattern |
| MFWP         | Minimum friction work pattern |
| HIL          | Hardware-in-the-loop |

I. INTRODUCTION

Fossil fuel is currently an indispensable resource for transportation; however, its intensive consumption is leading to severe shortage and environmental pollution. Consequently, several countries are enforcing laws to reduce emissions from conventional internal combustion engine vehicles and developing incentives for the automotive industry to foster the adoption of electric vehicles (EVs) and hybrid EVs. Considering the current energy and environmental problems and the corresponding policies, researchers have been focusing on novel EV technologies to achieve the sustainable development of the automotive industry [1]–[3].

Currently, EVs are preferred worldwide owing to zero emissions, low noise, and versatility [4]. Because the electric motor has a wide speed range to match the main reducer, most EVs have no transmission [5]. Hence, the power train...
is simplified to reduce production costs and volume for improved distribution of different components.

However, as EV technology advances, the lack of transmission hinders the improvement of efficiency and performance. In some cases, the electric motor of an EV does not operate at optimal conditions, thereby increasing energy consumption. In addition, both speed and acceleration are difficult to improve, limiting the EV dynamic performance [6]–[8]. Moreover, if a motor without transmission is designed, to meet the efficiency and dynamic performance requirements, its volume may be large and its structure may be complex, thereby increasing costs.

EV transmission requires the support of a control module. Recent studies have predominantly focused on two categories of these modules, which are offline trajectory planning and online controller design. Tian et al. [9] proposed a two-speed EV transmission and trajectory planning based on specific rules. Ye et al. [10] applied minimization for EV gearshift trajectory planning for seamless two-speed transmission and proposed a coordinating controller to optimize the shift trajectories and improve the overlapping shift. Lu et al. [11] designed a data-driven predictive gearshift controller to improve shifting and implemented the control on an FPGA platform to improve the execution efficiency. Zhao et al. [12] proposed a complete control architecture for gearshift in a dual-clutch transmission to realize torque-coordinating robust control for shifting.

Various multi-objective optimization methods have been employed to deal with the optimal control problem. Chai et al. [13] applied particle swarm optimization (PSO) to generate the parking movement for ground vehicles. Their results showed that PSO has the advantages of rapid convergence and effective optimization of system parameters. Chai et al. [14] proposed a genetic algorithm (GA) to plan the multi-objective optimal trajectory for spacecraft in the reentry phase. Their approach generates well-distributed Pareto fronts and has excellent convergence performance. Zhou et al. [15] studied vehicle trajectory planning for automated on-ramp merging using the Pontryagin Maximum Principle (PMP), which has high computational accuracy. However, each abovementioned algorithm has its own unique disadvantage: PSO lacks dynamic variable regulation, which influences the convergence accuracy; the GA’s performance is influenced by the initial random population and parameters; and the PMP is sensitive to the initial value; therefore, it needs to deal with a large number of numerical gradient operations.

Compared with PSO, GA, and PMP, the Radau pseudospectral method (RPM) has a high iteration speed and low sensitivity to initial values in multi-objective optimization [16]. Moreover, this method has been proven feasible [17] for solving the global nonlinear problem transformed from the optimal gearshift control. Thus, in this study, we use the RPM to plan the gearshift trajectory of EVs to improve the shift quality. We generate the Pareto solution set to clearly demonstrate the relations among the selected objectives, namely duration, friction work, and jerk, which can reflect the transmission’s service life and vehicle’s dynamic performance and ride comfort. Then, we select specific solutions from the set as different patterns to demonstrate different optimal results.

Herein, we propose a control architecture combining offline multi-objective optimization with online control. A two-speed planetary transmission is introduced for the EV, as it is a suitable option considering its cost and performance [18]. Compared with automatic transmission, which is widely used in conventional internal combustion engine vehicles, the two-speed planetary transmission for EVs is not equipped with a torque converter because the motor has no idle speed. In addition, compared with the dual-clutch transmission, the planetary transmission provides layout compactness, shift comfort, and reliability [19]. Moreover, although automatic mechanical transmission is easy to operate, its dynamic performance is lower than that of the planetary transmission [20], [21].

The remainder of this paper is organized as follows. Sections II and III present the proposed gearshift control architecture and powertrain model for EVs, respectively. In Section IV, we derive the dynamic model of the two-speed planetary transmission and objective functions. Multi-objective trajectory planning is implemented on the dynamic model by iterating the RPM. Section V details the Pareto solution set, and we apply the obtained trajectories in the proposed control architecture. Finally, we present conclusions in Section VI.

II. GEARSHIFT CONTROL ARCHITECTURE AND POWERTRAIN MODEL

To successfully implement the gearshift, the corresponding control architecture is necessary. The desired shift quality for transmission depends on the control strategy, which should improve the EV efficiency, dynamic performance, and driving comfort [22]–[24]. The two-speed planetary transmission can reduce the performance requirements for electric motors and improve the EV dynamic performance. Thus, we selected this transmission to verify the proposed trajectory planning method and control architecture.

Fig. 1 shows the proposed gearshift control architecture, which comprises modules for optimal shift trajectory selection, state estimation, feedforward and feedback control, and actuation. We utilize a Pareto solution set as a new approach to satisfy different requirements for shift quality. The proposed shift control logic is mainly concerned with power-on upshifting.

A. OPTIMAL SHIFT TRAJECTORY SELECTION MODULE

We formulate gearshift trajectory planning as a multi-objective optimization problem. Therefore, trajectory planning is integrated into an optimal gearshift trajectory selection module intended to improve shifting. The optimal selection process consists of three steps, namely discretization of shift schedule, fuzzy control, and selection from the Pareto
solution set, as shown in Fig. 2. From the various measures available for evaluating the gearshift quality, we selected duration, friction work, and jerk as optimization objectives. The Pareto solution set obtained from simulation demonstrates the relations among these three objectives.

1) DISCRETIZATION OF SHIFT SCHEDULE
Based on the formulation principle of economic gearshift scheduling, we propose the two-parameter schedule for EVs shown in Fig. 3. The pedal position and vehicle speed are selected as inputs to the shift schedule. In the schedule, the upshift and downshift lines are discretized at various shift points according to the storage limits of the control unit. Each point on the shift lines is regarded as a condition for multi-objective optimization. The corresponding Pareto solution sets and their optimal shift trajectories can be obtained and stored in the control unit. According to the vehicle speed and pedal position, the closest point and its Pareto solution set can be chosen for the next trajectory selection based on the driver requirements.

2) FUZZY LOGIC CONTROL
This step enables the identification of the driving intention for selection of the corresponding optimal shift trajectories from the Pareto solution set. First, during gear shifting, the driving intention is identified according to the pedal action for selecting the optimal trajectory. We identify the intention using fuzzy logic control, wherein the pedal position $\alpha$ and its derivative $\dot{\alpha}$ are the inputs, and two optimization objectives in the Pareto solution set, namely duration $J_d$ and jerk $J_j$, are the outputs.

After fuzzification, we formulate the fuzzy control rules shown in the appendix. According to the driver’s behavior, the expected values of the optimization objectives in the Pareto solution set can be obtained in this step. As the Pareto solution set is a three-dimensional surface, the last objective, friction work, can be calculated according to the duration $J_d$ and jerk $J_j$.

3) SELECTION IN PARETO SOLUTION SET
During early trajectory planning, the Pareto solution set is a three-dimensional surface comprising several points conforming to a set of optimal gearshift trajectories. From fuzzy logic control, the three optimization objectives can be obtained. In addition, using a lookup table, the closest point and its corresponding optimal shift trajectories are selected. Owing to the mismatch between the selected shift point by discretization and real shift point, the optimal trajectories are uniformly scaled according to the operation condition.

B. ADDITIONAL MODULES
1) FEEDFORWARD AND FEEDBACK CONTROL MODULE
In this module, the optimal trajectories obtained from the selection module are converted into electric signals and sent to the actuator module through a controller area network (CAN) bus. For the electric motor, the trajectory is converted into a torque control signal. For the engaging elements, the trajectory is converted into either an oil pressure control signal sent to the hydraulic actuators or a current signal sent to the hysteresis actuators.

For accurate gearshift execution, we apply a feedback controller for motor torque control. The estimated speed difference is selected as the input of the feedback controller. The subtraction of the optimized and estimated speed differences allows the execution of the correction of motor torque through
a proportional–integral–derivative (PID) controller, as shown in Fig. 4.

2) ACTUATOR MODULE
For the real torques of the motor and element 1 to track the optimal trajectories, corresponding actuators are necessary. As actuator modules are widely used in automatic manual transmission, dual-clutch transmission, and automatic transmission, we omit the details of this module for brevity.

3) STATE ESTIMATION MODULE
The coordinated control of motor and engaging element 1 substantially influences the shifting quality and effectiveness. Hence, we should measure the real speeds of the vehicle and motor using sensors installed on various parts. The measured speed signals allow the estimation of the speed difference for feedback control of motor torque.

C. POWERTRAIN MODEL
Herein, a novel type of two-speed planetary transmission, presented in [9], [25], is adopted as the control object. As shown in Fig. 5, the transmission consists of two coaxial sun gears, inner planet pinions, outer planet pinions, ring gear, and carrier. The small and large sun gears are engaged with the inner and outer planet pinions, respectively.

The configuration of the powertrain model is based on a front transverse engine, front-wheel drive vehicle platform. The powertrain makes the head of the EV lighter than that of an internal combustion engine vehicle with front-wheel drive, thereby theoretically improving the handling performance and reducing the EV front-brake load. Moreover, the structure is compact because it only consists of one planetary gear set, and a torque converter is unnecessary, unlike the automatic transmission in an internal combustion engine vehicle.

The small sun gear is connected to the electric motor, whereas the large sun gear is connected to its brake. In addition, the ring gear is attached to the driving gear of the final drive, and the one-way clutch is fixed on the carrier and located between the carrier and electric motor.

Table 1 lists the gears that can be realized by different control strategies of the carrier brake, large sun gear brake, and one-way clutch (OWC). When the transmission runs at the first gear, the carrier is locked by its brake, and the large sun gear rotates with its brake deactivated.

| Gear    | C Brake | LS Brake | OWC   |
|---------|---------|----------|-------|
| 1st     | engage  | engage   | engage|
| 2nd     | disengage | engage  | disengage |
| Reverse | engage  | disengage | disengage |
| Parking | engage  | engage   | disengage |

C, carrier; LS, large sun gear; OWC, one-way clutch.

Upshifting is divided into torque and inertia phases. In the torque phase, the carrier brake is rapidly deactivated and does not generate friction work, and the OWC is engaged for the carrier to remain stationary. At the end of the torque phase, the OWC does not operate, and the carrier begins to rotate. In the inertia phase, the angular velocity of the large sun gear falls to zero.

As the transmission runs at the second gear, the carrier is released, and the large sun gear is locked by its brake. If the transmission switches to reverse gear, the electric motor rotates reversely. The OWC is deactivated, and the carrier is locked by its brake. At the parking gear, both the carrier and large sun gears lock the whole planetary gear set.

III. MULTI-OBJECTIVE OPTIMIZATION
For multi-objective optimization of gearshift control, we apply the dynamic model, three objective functions, and RPM, as detailed below.

A. SIMPLIFIED DYNAMIC MODEL
The gear acceleration relations are given by (1)–(4).

\[
\ddot{\theta}_{ssg} = \ddot{\theta}_c (r_{ssg} + r_{ip}) - \ddot{\theta}_{ip} r_{ip},
\]

\[
\ddot{\theta}_{lsg} = \ddot{\theta}_c (r_{lsg} + r_{op}) - \ddot{\theta}_{op} r_{op},
\]

\[
\frac{\ddot{\theta}_c (r_{ssg} + r_{op})}{\sin e} = \ddot{\theta}_{op} r_{op} + \ddot{\theta}_{ip} r_{ip},
\]

\[
\ddot{\theta}_r r_r = \ddot{\theta}_c (r_{lsg} + r_{op}) + \ddot{\theta}_{op} r_{op}.
\]

Equations (5)–(8) can be obtained from (1)–(4).

\[
\ddot{\theta}_{ssg} a_1 + \ddot{\theta}_{lsg} a_2 = \ddot{\theta}_c,
\]

\[
\ddot{\theta}_{ssg} b_1 + \ddot{\theta}_{lsg} b_2 = \ddot{\theta}_{ip},
\]
\[
\begin{align*}
\dot{\theta}_{ssg} c_1 + \dot{\theta}_{lsg} c_2 &= \ddot{\theta}_{op}, \\
\dot{\theta}_{ssg} d_1 + \dot{\theta}_{lsg} d_2 &= \ddot{\theta}_r.
\end{align*}
\]

The simplified dynamic model is given by
\[
\begin{align*}
\ddot{\theta}_{ssg} &= g_1 T_m + g_2 T_{lsg} + g_3 T_e + g_4 T_{ef}, \\
\ddot{\theta}_{lsg} &= m_1 T_m + m_2 T_{lsg} + m_3 T_c + m_4 T_{ef},
\end{align*}
\]

where
\[
\begin{align*}
g_0 &= (J_{CA2} + 3J_{pb2} + 3J_{OP2} + J_{LS} + \frac{r_{lsg}}{r_r} + \frac{2r_{op}}{r_r}) J_{ERd1} + (J_{CA1} + J_{M+SS} + 3J_{pb1} + 3J_{OP1} + J_{M+SS} + 3J_{pb1} + 3J_{OPc1} + J_{ERd1}) \\
&\times (J_{M+SS} \frac{r_r}{r_{ssg}} J_{pb1} \frac{r_r}{r_{op}} + J_{OPc1} \frac{r_r}{r_{op}} + J_{ERd1}) \\
&\times (-3J_{pb2} \frac{r_r}{r_{ip}} + 3J_{OP2} \frac{r_r}{r_{op}} J_{LS} \frac{r_r}{r_{lsg}} + J_{ERd2});
\end{align*}
\]

\[
\begin{align*}
g_1 &= \frac{1}{g_0} (J_{CA2} + 3J_{pb2} + 3J_{OP2} + J_{LS} + \frac{r_{lsg}}{r_r} + \frac{2r_{op}}{r_r}) \times J_{ERd2}) \frac{r_r}{r_{ssg}} \frac{r_r}{r_{ip}} + (J_{CA2} + 3J_{pb2} + 3J_{OP2} + J_{LS} \frac{r_r}{r_{lsg}} + J_{ERd2});
\end{align*}
\]

\[
\begin{align*}
g_2 &= -\frac{1}{g_0} (J_{CA2} + 3J_{pb2} + 3J_{OP2} + J_{LS} + \frac{r_{lsg}}{r_r} + \frac{2r_{op}}{r_r} \times J_{ERd2}) \frac{r_r}{r_{ssg}} \frac{r_r}{r_{ip}} + (J_{CA2} + 3J_{pb2} + 3J_{OP2} + J_{LS} \frac{r_r}{r_{lsg}} + J_{ERd2});
\end{align*}
\]

\[
\begin{align*}
g_3 &= -\frac{1}{g_0} (J_{CA2} + 3J_{pb2} + 3J_{OP2} + J_{LS} \frac{r_r}{r_{lsg}} + J_{ERd2}) \frac{r_r}{r_{ssg}} \frac{r_r}{r_{ip}} + (J_{CA2} + 3J_{pb2} + 3J_{OP2} + J_{LS} \frac{r_r}{r_{lsg}} + J_{ERd2});
\end{align*}
\]

\[
\begin{align*}
g_4 &= \frac{1}{g_0} (J_{CA2} + 3J_{pb2} + 3J_{OP2} + J_{LS} \frac{r_r}{r_{lsg}} + J_{ERd2}) \frac{r_r}{r_{ssg}} \frac{r_r}{r_{ip}} + (J_{CA2} + 3J_{pb2} + 3J_{OP2} + J_{LS} \frac{r_r}{r_{lsg}} + J_{ERd2});
\end{align*}
\]

\[
\begin{align*}
m_0 &= (J_{CA1} + J_{M+SS} + 3J_{pb1} + 3J_{OPc1} + \frac{r_{lsg}}{r_r} + \frac{2r_{op}}{r_r} \times J_{ERd1}) (-3J_{pb2} \frac{r_r}{r_{ip}} + 3J_{OPc2} \frac{r_{ip}}{r_{op}} - J_{LS} \frac{r_r}{r_{lsg}} + J_{ERd2}) \\
&\times J_{ERd1} \frac{r_r}{r_{ssg}} J_{pb1} \frac{r_r}{r_{ip}} + J_{OPc1} \frac{r_r}{r_{ip}} + J_{ERd1};
\end{align*}
\]

\[
\begin{align*}
m_1 &= \frac{1}{m_0} (J_{CA1} + J_{M+SS} + 3J_{pb1} + 3J_{OPc1} + \frac{r_{lsg}}{r_r} + \frac{2r_{op}}{r_r} J_{ERd1} \frac{r_r}{r_{ssg}} - J_{LS} \frac{r_r}{r_{lsg}} - 3J_{pb1} \frac{r_r}{r_{ip}} + 3J_{OPc1} \frac{r_r}{r_{ip}} + J_{ERd1});
\end{align*}
\]

\[
\begin{align*}
m_2 &= -\frac{1}{m_0} (J_{CA1} + J_{M+SS} + 3J_{pb1} + 3J_{OPc1} + \frac{r_{lsg}}{r_r} + \frac{2r_{op}}{r_r} J_{ERd1} \frac{r_r}{r_{ssg}} + J_{ERd1});
\end{align*}
\]

\[
\begin{align*}
m_3 &= -\frac{1}{m_0} (J_{CA1} + J_{M+SS} + 3J_{pb1} + 3J_{OPc1} + \frac{r_{lsg}}{r_r} + \frac{2r_{op}}{r_r} J_{ERd1} \frac{r_r}{r_{ssg}} + J_{ERd1});
\end{align*}
\]

\[
\begin{align*}
m_4 &= \frac{1}{m_0} (J_{CA1} + J_{M+SS} + 3J_{pb1} + 3J_{OPc1} + \frac{r_{lsg}}{r_r} + \frac{2r_{op}}{r_r} J_{ERd1} \frac{r_r}{r_{ssg}} + J_{ERd1});
\end{align*}
\]

\[
T_{ef} = \frac{1}{i_y} (mgsin\alpha + \mu mgcos\alpha + \frac{1}{2} \rho C_d A (\dot{\theta}_w r_w)^2) r_w; J_{M+SS} = J_M + J_{SS}; J_{ER} = J_R + J_{dl1} + \frac{J_R + J_{ld1} + J_{ld1}}{i_y};
\]

\[
\begin{align*}
a_1 &= \frac{r_{lsg}}{r_{lg}} \frac{r_{lsg} + r_{op} (1 - \sin \theta)}{r_{ip} + r_{op} + r_{ssg}}, \\
b_2 &= \frac{r_{lsg} + r_{op} (1 - \sin \theta)}{r_{ip} + r_{op} + r_{ssg}}; \\
c_1 &= \frac{a_1 (r_{lsg} + r_{op})}{r_{op}}; \\
d_1 &= \frac{a_2 (r_{lsg} + r_{op}) + c_2 r_{op}}{r_{op}}.
\end{align*}
\]

In the dynamic model, \(J_{SS}, J_{LS}, J_R, J_{dl1}, J_{ld1}, J_{IP}, J_{OP}, J_C, J_{M}\) represent the inertia of the small and large sun gears, ring gear, driving and driven gear of the final drive, inner and outer pinions, carrier, and motor, respectively. In addition, \(r_r, r_{ssg}, r_{lsg}, r_{ip}, r_{op}\) represent the radius of the ring gear, small and large sun gears and inner and outer pinions, respectively, and \(\theta, \dot{\theta}, \ddot{\theta}\) are the angular displacement, angular velocity, and angular acceleration of the rotating parts, respectively.

**B. OBJECTIVE FUNCTIONS**

1) TORQUE PHASE

In this phase, the carrier is locked by the OWC, which is a path constraint:
\[
\theta_c = 0.
\]

Thus, (5) can be expressed as follows:
\[
\hat{\theta}_{ssg} a_1 + \hat{\theta}_{lsg} a_2 = 0.
\]

The path constraints on motor torque change and small sun gear speed are expressed as follows:
\[
T_{m, \min} \leq T_m \leq T_{m, \max},
\]
\[
\dot{\theta}_{ssg, \min} \leq \dot{\theta}_{ssg} \leq \dot{\theta}_{ssg, \max}.
\]
The dynamic model can be derived from (1) and (2) as follows:

\[
\ddot{\theta}_{ssg} = g_1 T_m + g_2 T_{lsg} + g_3 T_c + g_4 T_{ef}, \quad (15)
\]
\[
\ddot{\theta}_{lsg} = m_1 T_m + m_2 T_{lsg} + m_3 T_c + m_4 T_{ef}. \quad (16)
\]

The duration is given by

\[
J_{t}^{(1)} = t_f^{(1)} - t_0^{(1)}. \quad (17)
\]

The friction work is given by

\[
J_{w}^{(1)} = \int_{t_0^{(1)}}^{t_f^{(1)}} T_{lsg} \dot{\theta}_{lsg}^{(1)} dt. \quad (18)
\]

The jerk is given by (19), as shown at the bottom of the page.

The point constraints exist at the end of the torque phase

\[
\dot{\theta}_c = 0, \quad (20)
\]
\[
T_c = 0. \quad (21)
\]

2) INERTIA PHASE

In this phase, the carrier releases, and the angular velocity of the large sun gear gradually decreases until it reaches zero at the end of the inertia phase.

The path constraints are given by

\[
T_c = 0, \quad (22)
\]
\[
T_{m,\min} \leq T_m \leq T_{m,\max}, \quad (23)
\]
\[
\dot{\theta}_{ssg,\min} \leq \dot{\theta}_{ssg} \leq \dot{\theta}_{ssg,\max}. \quad (24)
\]

The dynamic model is given by

\[
\ddot{\theta}_{ssg} = g_1 T_m + g_2 T_{lsg} + g_4 T_{ef}, \quad (25)
\]
\[
\ddot{\theta}_{lsg} = m_1 T_m + m_2 T_{lsg} + m_4 T_{ef}. \quad (26)
\]

The duration is given by

\[
J_{t}^{(2)} = t_f^{(2)} - t_0^{(2)}. \quad (27)
\]

The friction work is given by

\[
J_{w}^{(2)} = \int_{t_0^{(2)}}^{t_f^{(2)}} T_{lsg} \dot{\theta}_{lsg}^{(2)} dt. \quad (28)
\]

The jerk is given by (29), as shown at the bottom of the page.

C. SOLUTION OF RPM

We set the angular velocities of the small and large sun gears and the torques of the motor, large sun gear, and carrier as state variables. In addition, we set the torque variations of the motor, large sun gear, and carrier as control variables.

Before obtaining the angular velocities of the small and large sun gears, the corresponding accelerations are required, as given by (9) and (10).

First, a time-domain transformation for the gearshift optimization problem is applied. Then, the state and control variables for optimization are discretized according to the RPM. Next, Lagrange polynomial interpolation is used to approximate the state variables and control variables for the differential operations of state variables and integral operations of control variables to be transformed into algebraic operations. Finally, the gearshift optimization is transformed into a nonlinear programming problem [16], [26], as detailed below.

1) TIME-DOMAIN TRANSFORMATION

To obtain orthogonality for the Legendre polynomial, we transform duration \([t_0^{(p)}, t_f^{(p)}]\) to each gearshift phase respectively into the orthogonal polynomial interval \([-1, 1]\).

\[
\tau = [2t - (t_0 + t_f)]/(t_f - t_0), \quad \tau \in [-1, 1]. \quad (30)
\]

2) COLLOCATION AND DISCRETIZATION

The collocation points of the RPM are Legendre–Gauss–Radau points with roots to \(P_N(\tau) = P_{N-1}(\tau)\), wherein \(P_N(\tau)\) is the \(N\)-th-degree Legendre polynomial. We set the collocation interval to \(\tau \in (-1, 1)\).

The number of Legendre–Gauss–Lobatto points is \(N + 1\), comprising \(N\) Legendre–Gauss–Radau points and initial point \(t_0 = -1\). We use the angular velocities of the small and large sun gears as state variables for simulation and discretize them at the collocation points. For Lagrange interpolation, the interpolation basis functions are given by

\[
L_i(\tau) = \prod_{j=0, j \neq i}^{N} (\tau - \tau_j)/(\tau_i - \tau_j), \quad (31)
\]

and each state variable can be expressed as a corresponding interpolation polynomial:

\[
\dot{\theta}_{ssg}(\tau) \approx \sum_{i=0}^{N} L_{ssg,i}(\tau) \dot{\theta}_{ssg,i}, \quad (32)
\]
\[
\dot{\theta}_{lsg}(\tau) \approx \sum_{i=0}^{N} L_{lsg,i}(\tau) \dot{\theta}_{lsg,i}. \quad (33)
\]
where \(k_i\) and \(T_{q,i}\) (\(i = 0, 1, \ldots, N + 1\)) represent the state variable and control variable in phase \(i\), respectively, and \(L_{q,i}\) represents the corresponding interpolation basis function.

3) STATE-SPACE EQUATIONS

After collocation and discretization, global interpolation polynomials approximate state variables and control variables, whose differential expressions are given by

\[
\dot{\theta}_{ssg,i}(\tau,k) = \sum_{i=0}^{N} L_{1,i}(\tau,k) \ddot{\theta}_{ssg,i} = \sum_{i=0}^{N} D_{ki} \ddot{\theta}_{ssg,i}, \quad (37)
\]

\[
\ddot{\theta}_{bsg,i}(\tau,k) = \sum_{i=0}^{N} L_{2,i}(\tau,k) \dddot{\theta}_{bsg,i} = \sum_{i=0}^{N} D_{ki} \dddot{\theta}_{bsg,i}, \quad (38)
\]

\[
\dot{T}_m(\tau,q) \approx \sum_{i=0}^{N} \dot{L}_{m,i}(\tau,q) \dot{T}_{m,i} = \sum_{i=0}^{N} D_{ki} \dot{T}_{m,i}, \quad (39)
\]

\[
\dot{T}_{lsg}(\tau,q) \approx \sum_{i=0}^{N} \dot{L}_{lsg,i}(\tau,q) \dot{T}_{lsg,i} = \sum_{i=0}^{N} D_{ki} \dot{T}_{lsg,i}, \quad (40)
\]

\[
\dot{T}_c(\tau,q) \approx \sum_{i=0}^{N} \dot{L}_{c,i}(\tau,q) \dot{T}_{c,i} = \sum_{i=0}^{N} D_{ki} \dot{T}_{c,i}, \quad (41)
\]

where \(k = 0, 1, 2, \ldots, N\), \(D_{ki}\) is a differential matrix of dimension representing the differential value of the Lagrange interpolation basis function at each discretized point. The dynamic constraints can be expressed as the algebraic constraints in (34) and (35) at collocation point \(\tau_{q,k}\).

\[
\sum_{i=0}^{N} D_{ki} \ddot{\theta}_{ssg,i} = \frac{T_f - T_0}{2} (g_1 T_m + g_2 T_{lsg} + g_3 T_c + g_4 T_{ef}), \quad (42)
\]

\[
\sum_{i=0}^{N} D_{ki} \dddot{\theta}_{bsg,i} = \frac{T_f - T_0}{2} (m_1 T_m + m_2 T_{lsg} + m_3 T_c + m_4 T_{ef}), \quad (43)
\]

4) TRANSFORMATION FOR OBJECTIVE FUNCTIONS

The objective functions contain Lagrange integral terms only, which can be approximated by Gauss–Radau integration (44), (45), and (46), as shown at the bottom of the next page, where \(J_1\) is the duration, \(J_\omega\) is the friction work, and \(J_2\) is the squared continuous jerk.

Thus, optimal gearshift control can be transformed into a nonlinear problem to be solved by the corresponding program.

\[T_m(\tau) \approx \sum_{i=0}^{N} L_{m,i}(\tau)T_{m,i}, \quad (34)\]

\[T_{lsg}(\tau) \approx \sum_{i=0}^{N} L_{lsg,i}(\tau)T_{lsg,i}, \quad (35)\]

\[T_c(\tau) \approx \sum_{i=0}^{N} L_{c,i}(\tau)T_{c,i}, \quad (36)\]

where \(\dot{\theta}_{ssg}\) and \(T_{q,i}\) (\(i = 0, 1, \ldots, N + 1\)) represent the state variable and control variable in phase \(i\), respectively, and \(L_{q,i}\) represents the corresponding interpolation basis function.

IV. RESULTS AND DISCUSSION

We performed the optimization and conducted simulations to verify the performance of the proposed gearshift trajectory planning method and control architecture through an implementation on MathWorks MATLAB/Simulink.

A. PARETO SOLUTION SET

We generated the Pareto solution set for the objective functions to unveil the relations among duration, friction work, and squared continuous jerk during the gearshift process. Usually, the vehicle’s gearshift is set to DTD mode, which means the torque of the engine or motor is determined by the throttle’s position. In this paper, we propose DPD mode in which the vehicle’s acceleration after gear shifting does not change. Because we set the throttle at the 50% position, we regard them as 50% DTD and 50% DPD in the following part 1) and 2), which represents two of the three optimal scenes.

1) PARETO SOLUTION SET IN 50% DTD MODE

Fig. 6 shows a pareto solution set in which all points of the three objective functions, i.e., duration, friction work, and squared continuous jerk, consist of enveloping surfaces.

To highlight the relations between two objectives, we plotted planar projections of Fig. 6. Figs. 7 to 9 show the relation between squared continuous jerk and friction work.
squared continuous jerk and duration, and friction work and duration, respectively. Each variable is inversely proportional to the other two variables. For example, Fig. 7 shows that the squared continuous jerk decreases as the friction work increases at a fixed duration. Similar trends are shown in Figs. 8 and 9.

2) PARETO SOLUTION SET IN 50% DPD MODE
The pareto solution set in Fig. 10 shows that the relation among the three objective functions follows the same trend as 50% DTD modes. However, the range of each objective value is different, especially regarding friction work.

At a fixed duration, the friction work and squared continuous jerk are larger than those in the 50% DTD mode. For example, the friction work and squared continuous jerk are 7200–8700 J and 0.87–2.25 (m/s$^3$)$^2$ for the duration of 0.74 s in the 50% DPD mode, respectively, whereas the values are 4550–4800 J and 0.80–0.95 (m/s$^3$)$^2$ for the same duration in the 50% DTD mode. As in the 50% DTD mode, any two variables are inversely proportional in the 50% DPD mode, as shown in Figs. 11 to 13. Owing to space constraints, we have not included the details here.

B. CONTROL TRAJECTORIES SELECTED PATTERNS
The trajectories were adopted in the hardware-in-the-loop (HIL) experiment shown in Fig. 14. The HIL simulation is based on the Simulink real-time platform of MATLAB. It comprises two computers, one running the proposed gearshift logic control architecture, whereas the other executing the dynamic model of the EV with the two-speed planetary transmission. The two computers communicate over a pair of CAN buses. The real-time vehicle data are collected and transmitted to the host computer, where the control architecture determines whether the gearshift is carried out according to the data. If the gearshift condition is satisfied, the shift is performed, otherwise shift is not performed. According to the actual shift point and driver’s

\[ J_f = t_f - t_0, \]
\[ J_w = \frac{t_f - t_0}{2} \sum_{i=0}^{N} (T_{lsg} \dot{\theta}_{lsg}), \]
\[ J_j = \frac{t_f - t_0}{2} \sum_{i=0}^{N} \left( d_1 (g_1 \dot{T}_m^{(1)} + g_2 \dot{T}_{lsg}^{(1)} + g_3 \dot{T}_c^{(1)}) + d_2 (m_1 \dot{T}_m^{(1)} + m_2 \dot{T}_{lsg}^{(1)} + m_3 \dot{T}_c^{(1)}) \right)^2. \]

as shown in Figs. 11 to 13. Owing to space constraints, we have not included the details here.
intention, the appropriate trajectory is selected. To evaluate the effectiveness of the obtained Pareto solution set and proposed control architecture, we selected four trajectories from the solution set, namely balanced pattern (BP), minimum duration pattern (MDP), minimum friction work pattern (MFWP), and minimum continuous jerk pattern (MCJP).

1) EXPERIMENTAL RESULTS IN 50% DTD MODE
Herein, we analyze the four trajectories from the Pareto solution set in the 50% DTD mode. The angular velocity, torque, and jerk results are shown in Figs. 15–17, respectively.

Figs. 15 and 16 show the angular velocities of the motor and large sun gear, and the torques of the motor, large sun gear, and carrier. During the torque phase of gearshift, the angular velocity of the motor increases and the carrier is released, thereby not transferring torque. Before the end of the inertia phase, the slope of the motor angular velocity decreases mainly because of the falling and rising motor torque.

Tables 2–4 list the optimization and simulation results. Considering the duration, the results from minimum to maximum are obtained from MDP, BP, MFWP, and MCJP. For
friction work, these results are obtained from MFWP, BP, MDP, and MCJP. Regarding squared continuous jerk, these results are obtained from MCJP, BP, MCJP, and MFWP. In each trajectory, the friction work is larger in the inertia phase than that in the torque phase, and the squared continuous jerk follows the opposite trend.

By comparing the optimization and HIL simulation results in each trajectory, it can be seen that the two-speed planetary transmission in the HIL simulation completes gearshift in a comparable time to that of optimization, suggesting that the proposed control architecture can perform gearshift. Likewise, the other measures of shift quality in the HIL simulation are similar within an order of magnitude to those obtained from optimization. The error of duration between the HIL simulation and optimization ranges from 1.4% to 3.2%, whereas the error of friction work ranges from 0.4%
2) EXPERIMENTAL RESULTS IN 50% DPD MODE

Herein, we analyze the four trajectories from the Pareto solution set in the 50% DPD mode. The angular velocity, torque, and jerk results are shown in Figs. 18–20, respectively.

In Figs. 18–20, the general trends of the angular velocities and torques are similar to those from the 50% DTD mode. Table 5 shows that the duration in MDP is the shortest in the 50% DPD mode, similar to that in the 50% DTD mode. The duration in MFWP is much longer than that in the other three trajectories. Although the total duration in each trajectory differs, the total duration in each torque phase is 0.35 s, because it differs to the duration from the 50% DTD mode. Because the motor torque at the end of gearshift reaches 150 Nm in 50% DPD mode, the large sun gear brake will generate much friction work to lock the large sun gear considering the duration and jerk. From a different perspective, the jerk is larger than that in 50% DTD mode if the friction work or the duration is being controlled. Table 6 shows that the friction work in both the torque and inertia phases are larger in the 50% DPD than in the 50% DTD mode for each trajectory. A similar trend is obtained for the overall duration of the

![Figure 18. Angular velocity from four trajectories in 50% DPD mode.](image)

![Figure 19. Torque in 50% DPD mode for (a) BP, (b) MDP, (c) MCIP, and (d) MFWP.](image)

| Patterns                  | Duration (s) | Friction work (J) | Squared continuous jerk (m/s^2)^2) |
|---------------------------|--------------|-------------------|-----------------------------------|
| Balance pattern (BP) Opt  | 0.80         | 8837              | 0.31                              |
| Sim                       | 0.79         | 8850              | 0.40                              |
| Minimum duration pattern (MDP) Opt | 0.69 | 8378              | 2.46                              |
| Sim                       | 0.68         | 8417              | 1.93                              |
| Minimum continuous jerk pattern (MCIP) Opt | 0.82 | 9435              | 0.21                              |
| Sim                       | 0.82         | 9454              | 0.26                              |
| Minimum friction work pattern (MFWP) Opt | 1.01 | 4460              | 6.16                              |
| Sim                       | 0.84         | 4465              | 4.24                              |
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FIGURE 20. Jerk in 50% DPD mode for (a) BP, (b) MDP, (c) MCJP, and (d) MFWP.

TABLE 6. Optimized friction work in 50% DPD mode.

| Patterns                          | Friction work |
|----------------------------------|---------------|
|                                  | Torque phase | Inertia phase |
| Balance pattern (BP)             | 3636         | 5174          |
| Minimum duration pattern (MDP)   | 3773         | 4576          |
| Minimum continuous jerk pattern (MCJP) | 3897         | 5509          |
| Minimum friction work pattern (MFWP) | 1406         | 3037          |

TABLE 7. Optimized squared continuous jerk in 50% DPD mode.

| Patterns                          | Square of continuous jerk |
|----------------------------------|----------------------------|
|                                  | Torque phase | Inertia phase |
| Balance pattern (BP)             | 0.03         | 0.28          |
| Minimum duration pattern (MDP)   | 0.13         | 2.31          |
| Minimum continuous jerk pattern (MCJP) | 0.08         | 0.13          |
| Minimum friction work pattern (MFWP) | 2.68         | 3.46          |

TABLE 8. Optimization results from proposed control.

| Patterns                          | Duration (s) | Friction work (J) | Squared continuous jerk (m/s²²) |
|----------------------------------|--------------|------------------|---------------------------------|
| Balance pattern (BP)             | 0.63         | 5336             | 0.89                            |
| Minimum duration pattern (MDP)   | 0.63         | 5438             | 0.90                            |
| Minimum continuous jerk pattern (MCJP) | 0.73         | 5492             | 0.64                            |
| Minimum friction work pattern (MFWP) | 0.72         | 4152             | 1.40                            |

TABLE 9. Optimized friction work in DTD mode.

| Patterns                          | Friction work |
|----------------------------------|---------------|
|                                  | Torque phase | Inertia phase |
| Balance pattern (BP)             | 2084         | 3237          |
| Minimum duration pattern (MDP)   | 2138         | 3285          |
| Minimum continuous jerk pattern (MCJP) | 2055         | 3372          |
| Minimum friction work pattern (MFWP) | 1425         | 2674          |

The optimization and HIL simulation results in the 50% DPD mode suggest that the proposed control architecture can suitably perform gearshift. In MFWP, at approximately 0.85 s, the speed of the driving and that of the driven parts of the large sun gear brake are mostly equal. Gearshift is almost completed in the control architecture. However, in MFWP, the requirement of the same 50% DPD extends the duration of the shift process. The duration error ranges from 0.0% to 1.5%, whereas the friction work error ranges from 0.1% to 0.5% and the squared continuous jerk error ranges from 21.5% to 31.2%.

C. ALGORITHM PERFORMANCE

To show the algorithm’s performance with model uncertainty and disturbances, we implemented another 50% DTD mode with a 400 kg load added to the vehicle as another optimal scene.
To briefly describe the trajectory trend, we will not present the figures for all four trajectories. Tables 8–10 present the optimization results. Compared with the results of the previous 50% DTD mode, there is no obvious change in the duration values, as shown in Table 8; the friction work values in BP, MDP, and MFWP are larger, but that in MCJP is smaller, as shown in Tables 8 and 9. In terms of squared continuous jerk, only the value in BP is obviously larger than that in the previous 50% DTD mode, and other patterns’ values are not noteworthy.

In terms of the solution process for each mode mentioned above, the generated values in the RPM have less effect on the solution results. Overall, the optimization results have not changed significantly compared with the previous 50% DTD optimization. This demonstrates that the RPM’s robustness can satisfy more gearshift conditions. However, an inappropriate preset time will result in the algorithm failing to generate solutions. For example, the maximum preset time in the main function of the RPM should not be overly small, otherwise solution generation will fail.

V. CONCLUSION

In this study, we proposed a gearshift control architecture for EV transmissions and offline multi-objective trajectory planning method that generates the Pareto set. The control architecture selects the appropriate trajectory from the Pareto set to satisfy the driver’s intention. We considered a two-speed planetary transmission for EVs to evaluate the proposed control architecture. Using the corresponding dynamic model, we formulated gearshift trajectory planning as a multi-objective optimization problem. Multi-objective optimization was performed for each mode by iterating the RPM. Then, we conducted HIL experiments on four specific trajectories. Our findings are as follows. (1) The proposed combination of offline multi-objective trajectory planning with online control establishes a new approach that can suitably reflect the driver’s behavior and satisfy different driving requirements. (2) The proposed approach can improve the shift quality and increase EV operation efficiency. (3) This approach may be applied to various multi-objective control problems.

APPENDIX

The fuzzification of the control rules are presented in Tables 11 and 12. Pedal position $\alpha$, ranging from 0 to 1, is fuzzified as {very low, low, middle, high, very high}, denoted as {VL, L, M, H, VH}. Derivative $\dot{\alpha}$, ranging from $-1.2$ to $1.2$, is fuzzified as {negative large, negative small, zero, positive small, positive large}, denoted as {NL, NS, Z, PS, PL}. Duration $J_d$, ranging from maximum to minimum in the Pareto solution set, is fuzzified as {very small, small, medium, large, very large}, denoted as {A, B, C, D, E}. Jerk $J_j$, ranging from maximum to minimum in the Pareto solution set, is fuzzified as {very small, small, medium, large, very large}, denoted as {A, B, C, D, E}.

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