The role of magnetic moments in the determination of nuclear wave functions of short-lived excited states

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Abstract.
For the last 50 years theory and experiment have made huge strides in the understanding of the structure of low lying levels in nuclei from the perspective of electromagnetic moments. However, while both theoretical and experimental techniques have become considerably more sophisticated and have reached high degrees of precision, a close comparison between calculation and measurements still eludes us, except in a few selected cases. In general, trends can be well described for a given family of nuclei. But even the evolution of structure as a function of \( N \) or \( Z \), or spin within a single nucleus, is still mysterious. A discussion of the latest developments in experimental techniques, measurements with radioactive beams of \(^{76}\text{Kr}\) and \(^{132}\text{Te}\), and future prospects will be presented.

1. Introduction: a historical perspective
Nuclear models have been continuously refined as experimental techniques have been developed. In these supportive activities, magnetic moment determinations have significantly constrained the possible wave function of particular states.

Experiments carried out in the nineteen fifties utilized the hyperfine interaction of nuclei with electronic and solid environments. These methods resulted in the invention and exploitation of a variety of techniques such as time differential and time integral angular correlations, recoil in vacuum (RIV), recoil in gas (RIG). Ground states were measured with high precision. However, in general, states with mean lifetimes of the order of 10\(\mu\)s to nanoseconds yielded magnetic moments with errors of the order of \(\sim 5\%\). For excited states with shorter lifetimes, in the picosecond range, a precision of only 10 - 30\% was usually achievable.

In the nineteen seventies the advent of higher energy beams and corresponding faster recoil ions spawned a new approach through the recognition of the very high “transient” magnetic fields (TF) that acted on the moving ions while they traversed ferromagnetic materials such as polarized iron or gadolinium foils. These techniques allowed measurements to be carried out on nuclear states with lifetimes of the order of fractions of picoseconds and resulted in much higher precision (1\%) when used for states with half lives of 5 -10 picoseconds. Some of the new results presented here, for example those on the Ca isotopes fall in that category.

The availability of yet higher energy ions at Berkeley, Yale, ORNL, MSU, GSI, led to the “re-invention” of the TF in inverse kinematics, an approach which is particularly well suited for work with radioactive beams. A novel approach to the preparation of a radiative beam, the re-cyclotron technique, was developed and used to measure the magnetic moment of the \(2^+\) state
in $^{76}$Kr. More recently the “re-invention” of RIV with large detector arrays was carried out at
ORNL and an example of that approach will be shown for $^{132}$Te. Finally, the “re-utilization” of
fully stripped ions at MSU or GSI was further exploited.

Fifty years ago shell model was in its infancy. The Schmidt limits were the best estimate for
$g$ factors, namely $g_π > 0$ and $g_ν$ either $< 0$ or $> 0$ depending on whether the nucleonic spin
and orbital angular momenta were aligned or anti-aligned. This characteristic of the nucleon $g$
factors provided a distinctive handle on the determination of the microscopic structure of a
state. But, in general, existing estimates of magnetic moments were inadequate. To explain
the observations, it became imperative to consider configuration mixing, core excitation and
mesonic effects.

In this paper, only a very schematic description of the theoretical advances will be presented/

In light nuclei, particularly in the $fp$ space, calculations have been relatively manageable.
Nevertheless, questions arise as to what are the most appropriate configurations that need to be
taken into account, how they mix, what is the exact form of the residual interactions. Several
different interactions have been put forward such as KB3, FPD6, WBT, FPY, VHG, GXPF,
FSZM. Another area of investigation relates to the integrity of the closed shell core. Which of
these nuclei, $^{40}$Ca, $^{48}$Ca, $^{56}$Ni, for example, has the best closed shell?

In heavy nuclei, collective effects become more important. Nevertheless, the remanence
of strong single particle interactions maintains its visibility especially near closed shells, as
expected. How do we probe the strength of these single particle contributions? Magnetic
moment measurements and other techniques such as particle transfer reactions have had an
impact in answering these questions.

Finally, the question of the modification of the shell structure in nuclei far from stability
and the known shells and the effect of these modifications on effective $g$ factors is one that
could be answered by experiment, particularly when beams of radioactive nuclei are produced
in abundance at the new facilities, such as the Rare Isotope Accelerator (RIA).

In this presentation, a brief overview of the new techniques and present examples of cases
where, already in the first few excited states, the competition between collectivity and single
particle excitation is very important, will be discussed.

2. The techniques of transient fields in inverse kinematics/Coulomb excitation

The technique of transient field in inverse kinematics that is mostly used for the experiments
described here has been described in detail in various publications [1, 2, 3, 4]. The technique
is applicable to nuclear states with meanlife times of the order of picoseconds and has been
applied across the periodic table. The technique has the advantage of providing high statistics,
therefore higher precision, and of allowing measurements that were not feasible in previous years.
Furthermore, Coulomb excitation leads to spectral simplicity, as only few states are excited,
and backgrounds are minimized. Such techniques are particularly appropriate for work with
radioactive beams.

In general, the beam of interest interacts with lighter nuclei in the first layer of the target.
The excited nucleus traverses a subsequent layer of a polarized, ferromagnetic, material where
it experiences the transient magnetic field and either stops in a final, hyperfine interaction-free
layer such as copper, or exits through a hole in a particle detector located at $0^\circ$. The light
target ions are scattered forward into this detector. The $\gamma$ rays are detected in four detectors
placed at angles where the particle-$\gamma$ angular correlation has an optimum slope. The analysis is
carried out in a manner similar to that used in the traditional forward-scattering method. While
multi-detector arrays such as Gammasphere have been used for $g$ factor measurements in the
past in reactions involving fusion evaporation, these detectors have not been exploited for low
multiplicity events like Coulomb excitation. Recently, both particle and $\gamma$ ray multi-detector
arrays have been used, as described below.
3. Heavy nuclei: the competition between collective effects and single particle configurations: the Nd isotopes

The $g$ factors of higher excited states in $^{144,146,148,150}$Nd isotopes have been measured. The data clearly indicate that, while $^{148,150}$Nd are well described by collective excitations for which the $g$ factor is independent of the spin $I$, the structure of the low lying levels in the lighter isotopes is actually dominated by neutrons occupying the $2f_{7/2}$ shell [5, 6]. The results are plotted in Fig 1.

![Figure 1. Measured $g(I)$ factors in the $^{144,146,148,150}$Nd isotopes.](image)

4. Nuclei in the $fp$ shell

4.1. The Ti and Cr isotopes

The magnetic moments of the $2^+_1$ and $4^+_1$ states in $^{46,48}$Ti and $^{50,54}$Cr have been measured with sufficient accuracy to carry out critical tests of different shell model calculations [7]. The results are displayed in Fig.2. The calculations were carried out in a variety of modes using different interactions such as KB3, FDP6, FPY and VHG interactions including the whole $fp$ shell.

The $g$ factors and B(E2) values in particular for $^{50,52}$Cr compare well with the theoretical calculations. The Ti isotopes, on the other hand, seem to exhibit more collectivity than the model allows, suggesting that the $^{40}$Ca core is not as good a closed shell nucleus as $^{48}$Ca. These results have been exploited in the work on the Ca isotopes described below.

4.2. The Ca isotopes

The $^{40,48}$Ca are doubly magic nuclei. It was expected that shell model calculations would reproduce the experimental spectroscopic information. However, already in the sixties, it became obvious that more complex configurations involving $0p-0h$, $2p-2h$, and $4p-4h$ with particles and holes restricted to the $f_{7/2}$ and $d_{3/2}$ respectively needed to be taken into consideration [8, 9, 10, 11]. A strong argument in favor of this model is the fact that the $0^+_5$ state supports a rotational band. Recent Monte Carlo calculations [12] have implied that the core of $^{48}$Ca is more inert core than the core of $^{40}$Ca. In order to investigate further the nature of the core in
the Ca isotopes, $g$ factor measurements of the $2^+_1$ states of the even Ca isotopes were undertaken. The discussion below summarizes the work described in a series of papers \[13, 14, 15, 16\]. Fig. 4 displays the measured $g$ factors in the Ca isotopes and Table I summarizes the data.

The major surprise in the data presented in Fig. 4 is the fact that the $g$ factors of the $2^+_1$ in $^{42,44,46}$Ca are positive and not negative. This result led to the latest experiments on $^{46}$Ca where, indeed, the expected negative $g$ factor makes its appearance again.

Calculations were carried out considering pure $(f_{7/2})_\nu$ configurations, the whole $1fp$ shell \[13, 14\], as well as a large scale shell model (LSSM) \[16\]. Finally, following the work of Gerace and Green \[17\] and Towsley, Cline and Horoshko \[18\] and Bjerrgard and Hansen \[19\], an approach that does not rely on a specific knowledge of the particle hole excitations that make up the wave functions was used. The wave function of a particular state was represented by two-components with amplitudes $C$ and $D$ where $C$ represents the amplitude of the single particle configurations, while $D$ represents the amplitude of deformed components of the wave function, a term which includes all core excitations \[14\]:

$$\langle \Psi(2^+_1) \rangle = C[(fp)^6_\nu] J=2 + D[1\Psi_{def} J=2]$$

The $g$ factor becomes:

$$g(2^+_1)_{meas} = C^2 [g(fp)^6_\nu] + D^2 [g(\Psi_{def})]$$

The new experimental $g(2^+_1)$ values were used to estimate the intensity of the core-excited component in the $2^+_1$ states in $^{42,44,46}$Ca. In Eq.2, $g(\Psi_{def}) = Z/A$ while $g((fp)^6_\nu) = -0.3085$ obtained from the full $(fp)_\nu$ shell model calculation with the KB3 and FPD6 interactions.

Table 2 shows the resulting intensities $C^2$ and $D^2$ for the even Ca isotopes. For $^{42,44}$Ca the intensity of the deformed component in the wave function is large. Deformation is nearly negligible in $^{46}$Ca.
Figure 3. Measured $g$ factors of the ground states of the odd Ca isotopes (open circles), of $2^+_1$ states of even Ca isotopes (solid symbols) as well as of the $6^+_1$ state in $^{42}$Ca. The solid lines describe the results of $(fp)_ν$ shell-model calculations for the ground state $g$ factors of the odd-$A$ Ca isotopes using the KB3 and FPD6 interactions.

Table 1. Measurements of $g$ factors of $2^+_1$ states in the $^{42,44,46}$Ca isotopes. For $^{44}$Ca and $^{46}$Ca the average $g$ factors from Refs.[13-16] are also shown.

| Nucleus | $g(2^+_1)$ | $^{42}$Ca | $^{44}$Ca | $^{46}$Ca |
|---------|------------|----------|----------|----------|
| $^{42}$Ca | +0.04(6)Ref.[16] | +0.17(3)Ref.[16] | -0.19(12)Ref.[15] | +0.12(5)Ref.[13] | -0.26(5)Ref.[14] |
| $^{44}$Ca | $<g>$+0.04(6) | +0.16(3) | $-0.26(5)$ |

Table 2. Comparison of the relative intensity components, single particle ($C^2$) and deformed ($D^2$), in the $2^+_1$ wave functions of the even mass Ca isotopes. For $^{44}$Ca and $^{46}$Ca the average $g$ factors from Refs.[13,14,15,16] are shown.

| Nucleus | $<g(2^+_1)>$ | $C^2$ | $D^2$ |
|---------|--------------|------|------|
| $^{42}$Ca | Gerace and Green Ref.[17] | 0.50 | 0.50 |
| $^{44}$Ca | Stripping reactions Ref.[19] | +0.04(6) | 0.45(6) | 0.55(6) |
| | $Q_{meas}$ Ref.[18] | 0.47 | 0.53 |
| $^{46}$Ca | -0.25(5) | 0.93(7) | 0.07(7) |
5. Radioactive beams

5.1. $^{76}$Kr

A beam of $^{76}$Kr was prepared at the Berkeley LBNL 88-Inch cyclotron by the "recirculation" method. The activity was prepared in batch mode by bombarding a target of isotopic $^{74}$Se with about 6 $\mu$A of alpha particles. Then the Kr was separated from the target material and re-injected into the AECR-U ion source. Beam intensities as high as $3 \times 10^8$ particles per second were observed with an integrated beam current of $6 \times 10^{11}$ particles per 24-hour batch. A total of three production and acceleration cycles yielded six hours of beam on target with peak rates of $10^8$ particles/sec.

With this beam, the $g$ factor of the first $2^+$ state of radioactive $^{76}$Kr ($\tau = 14.8$ hrs) was measured [4] extending the systematics of the previously measured stable $^{78,80,82,84,86}$Kr isotopes [20]. Here again the technique of the transient field in inverse kinematics coupled with Coulomb excitation by a C target deposited on a gadolinium foil was used. While the detector arrangement was similar to that used in other such measurements, the beam itself was stopped in a moving tape and taken away from the counting region. The $g$ factor of $g(^{76}$Kr;$2^+_1$) = + 0.37(11) was obtained by direct comparison to the known $g(^{78}$Kr;$2^+_1$) value re measured after the $^{76}$Kr runs with the same setup and under almost identical conditions. The data are not statistically sufficient to allow a clear choice between the prediction of the IBA model with either shell closure at $N = 2$ or $N = 38$. But this experiment provided the first measurement of a $g$ factor by the Coulomb excitation/transient field technique on a radioactive beam. The experiment was not hindered by the fact that the beam was radioactive and therefore signals the feasibility of future measurements with this technique.

5.2. $^{132}$Te

The radioactive beam $^{132}$Te was accelerated and at ORNL and Coulomb excited on a target of natural C. The whole Clarion + HyBall arrays were used to detect the decay $\gamma$ rays and C recoils. The excited $^{132}$Te ions recoiled into vacuum where the hyperfine interactions between the nuclear state spin aligned by the Coulomb excitation reaction and the electronic environment aligned acted in such a way as to attenuate the angular distribution of the $\gamma$ rays. The measured attenuation is proportional to $g^2$ of the $2^+_1$ state. The analysis of the data depends on the choice of model describing the electronic distribution. In this particular case the “static” model in which the electronic lifetime is long compared to the mean nuclear lifetime was chosen for the analysis and the method was further calibrated by measuring the attenuation of the angular correlation of the decay $\gamma$ rays of the $2^+_1$ states of the neighboring $^{122,126,130}$Te nuclei. The absolute value of the $g$ factor was obtained, $| g(2^+_1;^{132}$Te) | = 0.35(5) [21]. Table 3 displays recent calculated values for the $g$ factors of heavy Te isotopes close to the closed shell at $N = 82$ and compares them to theoretical calculations [22, 23, 24].

6. Future measurements

In spite of the weakness of the radioactive beams available today for measurements of spectroscopic information on nuclei far-from-stability, techniques are being developed that show promise for the the fast beams of MSU [25] and for beams from the next generation of accelerators such as RIA.

These techniques, being more demanding, will also allow higher precision to be obtained on stable nuclei at higher spins and energies allowing much better constraints on theoretical predictions.

Overall, as the measurements of magnetic moments of nuclear states far-from-stability, at higher spins and energies continue on a steady path of improvements, the theoretical approaches
Table 3. Calculated and experimental $g$ factors for $2^+_1$ states in Te isotopes close to $N = 82$.  

|         | $^{130}$Te | $^{132}$Te | $^{134}$Te | $^{136}$Te |
|---------|------------|------------|------------|------------|
| Experiment | +0.295(35) (+)0.35(5) | | | |
| QRPA       | +0.491     | +0.695     | -0.174     | Ref.[22]   |
| Shell model| +0.347     | +0.488     | +0.862     | +0.360     | Ref.[23]   |
| $(g_{\text{eff}})$ | +0.275     | | | |
| Shell model| +0.275     | | | Ref.[24] |
| $(g_{\text{free}})$ | | | | |

involving the interplay of collective and single particle excitations will have to be fine tuned. The general trends are well understood, but individual cases need specific evaluations. The next fifty year will likely uncover more surprises in this field.

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