An Efficient Method to Transform SAT problems to Binary Integer Linear Programming Problem

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Abstract—In computational complexity theory, a decision problem is NP-complete when it is both in NP and NP-hard. Although a solution to a NP-complete can be verified quickly, there is no known algorithm to solve it in polynomial time. There exists a method to reduce a SAT (Satisfiability) problem to Subset Sum Problem (SSP) in the literature, however, it can only be applied to small or medium size problems. Our study is to find an efficient method to transform a SAT problem to a mixed integer linear programming problem in larger size. Observing the feature of variable-clauses constraints in SAT, we apply linear inequality model (LIM) to the problem and propose a method called LIMSAT. The new method can work efficiently for very large size problem with thousands of variables and clauses in SAT tested using up-to-date benchmarks.

keywords: SAT(Satisfiability problem); BinaryILP(Integer Linear programing); 3SAT; Reduction

I. INTRODUCTION

P problems are the class of problems that can be solved in polynomial time. This means they are problems which are solvable in time \(O(n^k)\) in Big O notation for any constant \(k\), where \(n\) is the size of the input of the problem. NP problems are the set of problems that can be verified in polynomial time as a function of the given input size using a nondeterministic Turing machine. This means if there is a certificate of a solution, then the certificate can be proved to be correct in time polynomial in the size of the input to the problem. In 1971, Cook defined that problem X polynomial reducible to problem Y if arbitrary instances of problem X can be solved using polynomial number of standard computational steps, plus polynomial number of calls to oracle that solves problem Y. Therefore, the definition of NP complete is a problem Y in NP with the property that for every problem X in NP, \(X \leq_p Y\), that is problem X can be polynomial-time reducible to problem Y. NP-complete problems constitute the class of the most difficult possible NP problems.

NP-complete problems can be divided into six basic genres, i.e., packing problems, covering problems, constraint satisfaction problems, sequencing problems, partitioning problems, numerical problems.

Constraint satisfaction problems include Circuit Satisfiability problems, Satisfiability problem, 3SAT. A specific situation of SAT is 3SAT that each clause of it has exactly three literals, which correspond to distinct variables or the negative form of these variables. The computational complexity of SAT problem in the worst case is \(O(2^n)\), where \(n\) is the number of variables. Because SAT can be transformed to 3SAT, it has similar computational complexity as SAT.

The question whether an arbitrary Boolean formula is satisfiable cannot be solved within polynomial time. A formula with \(n\) variables, possibilities of variables assignment can reach to \(2^n\). If formula length \(\leq n\) is a polynomial length about \(n\), then it will take \((2^n)\) for each assignment. It is a superpolynomial length about to formula length. Due to this fact, this paper aims to propose an efficient method to transform SAT problems to a mixed integer linear programming problem to reduce the handling time of SAT problem.

A. Related Work

Conflict-driven clause learning (CDCL) is an efficient method for solving Boolean satisfiability problems (SAT). Up to now, many heuristics are added in it to improve performance, for example, restart, Variable State Independent Decaying Sum (VSIDS). Hidetomo et al. focus on clause reduction heuristic, which aims to suppress memory consumption and sustain propagation speed. In their study, the reduction consists of two parts: evaluation criteria and reduction strategy. The first step measuring the usefulness of learnt clauses is using LBD (literals blocks distance) and the latter one using a new strategy based on the coverage of used LBDs is to select removing clauses according to the criteria. In experiments, they compare Glucose schema and Coverage schema. The result shows that the new strategy improves performance for both SAT and UNSAT instances. Another strategy used to improve SAT solver is called HSAT (Hint SAT). It is proposed by Jonathan Kalechstain et al. to cut the searching space by using a hint-based partial resolution-graph to get a solution faster. For hint generation, they chiefly use two heuristics. The first one is Avoiding Failing Branches (AFB) which avoids the solver spending too much time on explored branches which are made of decision variables. The second heuristic is Random Hints (RH) which aims to create hints that contradict the instance. This algorithm is based on random assignments and satisfiability checking. Experiments show that AFB can solve 113 instances from SAT 2013 within
half an hour where the total number of satisfiability instances is 150. The experimental consequences of SAT 2014 are almost the same.

Gilles Audemard et al. [20] study how to measure SAT instances. They give 5 indicators: the number of decision levels, the number of decisions between two conflicts, the number of successive conflicts, the number of non-binary glue clauses and the number of unit propagation. They also mention the restart polarity policy which is added to Glucose. The new version of Glucose solves 20% more problems than the original one and increases the speed for UNSAT instances. Further, Mathan Mull et al. [18] analysis the structure of industrial benchmarks. Previous studies hold that the reason why CDCL solver is efficient for industrial benchmarks is due to its good community structure (high modularity). However, Mathan Mull et al. get the different result. They use random unsatisfiable instances produced by pseudo-industrial community attachment model to do experiment. The result shows that community structure is not adequate to explain the good performance of CDCL on industrial benchmarks.

Symmetry is another characteristic of SAT problem. Jo Devriendt et al [15] present symmetric explanation learning (SEL), in which symmetric clauses are learned only when they are unit or conflicting. 1300 benchmark instances indicate that among GLUCOSE, BREAKID, SEL, SP, SLS, SEL outperforms other four solver configurations and as a dynamic symmetry handling technique, SEL is the first one competitive with static symmetry breaking which is known to the most effective way to handle symmetry. In terms of symmetry, C.K Cuong et al. [19] as well propose a method transforming unavoidable sub-graphs to SAT, which can make up for the shortcomings of SAT solvers.

With the development of Machine Learning, combing SAT problem with Machine Learning is a good idea. Quanrun Fan et al. [23] use clustering method basing on divide and conquer to deal with Boolean satisfiability problems. In this way, the original problem will be divided into many small ones. There- to deal with Boolean satisfiability problems. In this way, the original problem will be divided into many small ones. There-

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2) Reduction from a SAT problem to SSP by the method introduced in [2]; obtaining a matrix $A$ which has $2(n + k)$ by $(n + k)$ dimension, we denote the matrix $A_1$ which is $2n$ by $n$ from $A$, and $A_2$ which is $2n$ by $k(n + 1 : n + k)$ from $A$.

3) To meet the constraints, we need $x A_1 \leq b_1, b_2 \leq x A_2 \leq b_3$, where $b_1$ and $b_2$ are all ones, and $b_3$ are all threes. We only need $b_1$ and $b_2$ for our problem. Set target value as an array in $b$, where $b = [\text{ones}(1, n), -\text{ones}(1, k)]$, and solve integer linear equation $x A_1 - A_2 \leq b$, where $b = [b_1, -b_2]$, if there exits solution to $x$, then the original SAT problem is satisfiable, otherwise, it is not. Our model is as following:

$$\min \ c^T x \quad (2)$$

s.t. $x A \leq b \ x \in (0, 1)$

where $c$ is coefficient(default as all ones)

III. EXPERIMENTAL RESULTS

The test cases come from SAT 2016 competition [14]. There are 5 categories: Application Benchmarks from Main/Parallel Tracks, Crafted Benchmarks from Main/Parallel Tracks, Agile Track Benchmarks, Random Track Benchmarks and Incremental Track Benchmarks. In our research, we focus on main-crafted Track in which a majority of instances come from Crafted Benchmarks from Main Tracks. In fact, these instances are limited in 5000 seconds. We carry out our algorithm in Gurobi 7.5.1. In order to avoid being out of memory, we use sparse matrix as input. All the experiments were performed on Intel Xeon CPU(2.4GHz) with 20G memory which is similar to the configuration in SAT 2016 [14]. The time limit was set to 5000s.

In order to verify the correctness of our algorithm, firstly we test one hundred instances named uf250-1065 coming from [13]. These instances with 250 variables and 1065 clauses are all SAT, and our experimental result is the same as the given result. Table [II] containing part of the testing result shows the algorithm proposed in this paper can solve problems correctly. Next, we implement our algorithm for SAT competition problems. For main-crafted instances, we test 104, 68 of them is SAT, the rest is unknown. Table [III] lists successfully solved instances.

### TABLE I: Reduction from 3SAT to SubSet-Sum

| $X_1$ | $X_2$ | $X_3$ | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|-------|-------|-------|-------|-------|-------|-------|
| $v_1$ | 1     | 0     | 0     | 1     | 0     | 1     |
| $v_2$ | 1     | 1     | 0     | 0     | 0     | 1     |
| $v_3$ | 0     | 1     | 1     | 1     | 1     | 1     |
| $v_4$ | 0     | 0     | 0     | 0     | 0     | 0     |
| $v_5$ | 0     | 0     | 0     | 1     | 0     | 1     |
| $v_6$ | 0     | 0     | 0     | 1     | 1     | 1     |
| $v_7$ | 0     | 0     | 0     | 2     | 0     | 0     |
| $v_8$ | 0     | 0     | 0     | 1     | 0     | 1     |
| $v_9$ | 0     | 0     | 0     | 0     | 2     | 0     |
| $v_{10}$ | 1     | 1     | 1     | 1     | 1     |
| $v_{11}$ | 1     | 1     | 1     | 1     | 1     |

### TABLE II: Part of result of uf250-1065 instances (others are solved within a few seconds)

| Filename | Variable | Clause | Result | Time(s) |
|----------|----------|--------|--------|---------|
| $u f_{250} - 02$. cnf | 250 | 1065 | sat | 63 |
| $u f_{250} - 04$. cnf | 250 | 1065 | sat | 64 |
| $u f_{250} - 01$. cnf | 250 | 1065 | sat | 65 |
| $u f_{250} - 00$. cnf | 250 | 1065 | sat | 66 |
| $u f_{250} - 03$. cnf | 250 | 1065 | sat | 67 |
| $u f_{250} - 04$. cnf | 250 | 1065 | sat | 68 |
| $u f_{250} - 05$. cnf | 250 | 1065 | sat | 69 |
| $u f_{250} - 06$. cnf | 250 | 1065 | sat | 70 |
| $u f_{250} - 01$. cnf | 250 | 1065 | sat | 71 |
| $u f_{250} - 00$. cnf | 250 | 1065 | sat | 72 |

### TABLE III: Successfully solved main-Craft benchmark instances

| Filename | Result | CPUTime(s) |
|----------|--------|------------|
| craft_fixedbandwidth-eq-31 | unsat | 0.99 s |
| craft_fixedbandwidth-eq-32 | unsat | 0.74 s |
| craft_fixedbandwidth-eq-33 | unsat | 0.71 s |
| craft_fixedbandwidth-eq-34 | unsat | 0.74 s |
| craft_fixedbandwidth-eq-35 | unsat | 0.71 s |
| craft_fixedbandwidth-eq-36 | unsat | 1.58 s |
| craft_fixedbandwidth-eq-37 | unsat | 1.70 s |
| craft_fixedbandwidth-eq-39 | unsat | 0.87 s |
| craft_fixedbandwidth-eq-40 | unsat | 0.81 s |
| craft_fixedbandwidth-eq-42 | unsat | 0.79 s |
| craft_rphp4_065 | unsat | 107.51 s |
| craft_rphp4_070 | unsat | 233.01 s |
| craft_rphp4_075 | unsat | 209.19 s |
| craft_rphp4_080 | unsat | 136.75 s |
| craft_rphp4_085 | unsat | 177.09 s |
| craft_rphp4_090 | unsat | 118.25 s |
| craft_rphp4_095 | unsat | 197.94 s |
| craft_rphp4_100 | unsat | 217.43 s |
| craft_rphp4_105 | unsat | 401.78 s |
| craft_rphp4_110 | unsat | 698.25 s |
| craft_rphp4_115 | unsat | 865.15 s |
| craft_rphp4_120 | unsat | 1296.42 s |
| craft_rphp4_125 | unsat | 1280.32 s |
| craft_rphp4_130 | unsat | 1690.78 s |
| craft_rphp4_135 | unsat | 1530.62 s |
| craft_rphp4_140 | unsat | 2562.48 s |
| craft_rphp4_145 | unsat | 1266.03 s |
| craft_rphp4_150 | unsat | 2100.18 s |
| craft_rphp4_155 | unsat | 3982.60 s |
| craft_rphp4_160 | unsat | 4671.99 s |
| craft_rphp5_035 | unsat | 69.89 s |
| craft_rphp5_040 | unsat | 66.08 s |

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From Table III it is clear that Gurobi solver based on 0-1 Linear Inequalities Model performances well. In SAT competition 2016, the best solver tc_glucose for main-craft totally solves 58 instances used servers with good configuration [14].

IV. DISCUSSION AND CONCLUSION

In this paper, we presented 0-1 ILP. A key idea is to reduce the size of SSP matrix from \(2(n+k)(n+k)\) to \(n(n+k)\) using the property of this matrix, i.e., lines \(v_i\) and \(v'_i\) located in can not be chosen in the same time and slack variables are all in the lower right corner.

Being different from reduction in [2], LIMSAT works for general SAT problems including 3SAT but dos not need transforming SAT to 3SAT. We can construct a new SSP matrix according the CNF file. Comparing to the original SSP matrix introduced in [2], the new one only contains variables and clauses without slack variables.

The experimental results show that our algorithm is effective. For future work, we will improve the efficiency of our implementation will be improved. Specifically, we consider using parallel algorithm to deal with SAT problems.

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