KNOTS AND PREONS

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Abstract. It is shown that the four quantum trefoil solitons that are described by the irreducible representations $D_{\frac{3}{2}\lambda\mu\nu}$ of the quantum algebra $SL_q(2)$ (and that may be identified with the four families of elementary fermions $(e, \mu, \tau; \nu e, \nu \mu, \nu \tau; d, s, b; u, c, t)$) may be built out of three preons, chosen from two charged preons with charges $(1/3, -1/3)$ and two neutral preons. These preons are fermions and are described by the $D_{\frac{1}{2}}$, representation of $SL_q(2)$. There are also four bosonic preons described by the $D_{1}$ and $D_{0}$ representations of $SL_q(2)$. The knotted standard theory may be replicated at the preon level and the conjectured particles are in principle indirectly observable.
1 Introduction

One proposal for going beyond the standard theory of elementary particles depends on the use of the quantum groups. A particular form of this proposal, which exploits the fact that the quantum group $SL_q(2)$ is the symmetry group of the knot, has revived interest in the old speculation that the elementary particles are knots.\textsuperscript{1,2,3} This possibility may be empirically tested by comparing the four simplest quantum knots (quantum trefoils) with the four classes of simplest particles (neutrinos, leptons, up quarks, down quarks), and is in fact supported by simple relations that exist between the topology of the trefoil and the charge, hypercharge, and the isotopic structure of the simplest particles. These relations depend on the hypothesis that the kinematic eigenstates of the quantum knot are matrix elements of irreducible representations of $SL_q(2)$, denoted by $D^N_w_r^{N/2}$, where $(N, w, r)$ are (the number of crossings, writhe, and rotation of the knot), respectively. For trefoils one has $(N = 3, w = \pm 3, r = \pm 2)$. The four quantum trefoils may in this representation be uniquely correlated with the four classes of fermions. The three fermions in each class (e.g. $e, \mu, \tau$ in the lepton class) are assumed to be three quantum states of a soliton with the topology of a trefoil.

In a classification where $D^N_w_r^{N/2}$ represents a bosonic knot when $N$ is even, and a fermionic knot when $N$ is odd, the $N = 3$ trefoils represent the simplest classes of fermions. If, however, the new idea that is being proposed to supplement the standard theory is not primarily the idea of knots but rather the introduction of a new symmetry, namely $SL_q(2)$, then the lower representations $D^j_{mm'}$ when $j = 1/2$ and $j = 1$, become of interest. We are here interested in relating these “preon” representations to the trefoil and other representations.
2 The Knot Representation

We replace the point particles of standard theory with quantum knots by attaching to each normal mode a knot state just as one introduces spin by attaching a spin state. The knot states and the corresponding fields are defined only up to a gauge transformation and the action is required to be invariant under these gauge transformations.

We assume that the kinematical quantum states of the knot are derived from the irreducible representations of the symmetry algebra of the knot, namely

\[ D^j_{mm'} (a, b, c, d) = \sum_{s \leq n_+, t \leq n_-} A^j_{mm'} (q, s, t) \delta(s + t, n'_+) a^s b^{n_+ - s} c^t d^{n_- - t} \quad (2.1) \]

where

\[ n_+ = j \pm m \quad (2.2) \]
\[ n'_+ = j \pm m' \quad (2.3) \]

and the arguments of \( D^j_{mm'} \) obey the algebra of \( SL_q(2) \) as follows:

\[ \begin{align*}
ab &= qba & bd &= qdb & bc &= cb & ad - qbc &= 1 & q_1 &= q^{-1} \\
ac &= qca & cd &= qdc & da - q_1 cb &= 1
\end{align*} \quad (2.4) \]

We shall refer to (2.4) as the “knot algebra”. This algebra and \( D^j_{mm'} (a, b, c, d) \) are defined only up to the following gauge transformation

\[ \begin{align*}
a' &= e^{i\varphi_a} a & b' &= e^{i\varphi_b} b \\
d' &= e^{-i\varphi_c} d & c' &= e^{-i\varphi_c} c
\end{align*} \quad (2.5) \]

Eqs. (2.5) leave the algebra (2.4) invariant and induce on the elements of every representation the following transformation

\[ \begin{align*}
D^j_{mm'} (a', b', c', d') &= e^{i(m+m')} \varphi_a e^{i(m-m')} \varphi_b D^j_{mm'} (a, b, c, d) \\
 &= e^{im(\varphi_a + \varphi_b)} e^{im' (\varphi_a - \varphi_b)} D^j_{mm'} (a, b, c, d) \quad (2.6)
\end{align*} \]

We now associate \( D^j_{mm'} \) with the geometrical knot by setting

\[ \begin{align*}
j &= \frac{N}{2} \\
m &= \frac{w}{2} \\
m' &= \frac{r+1}{2}
\end{align*} \quad (2.7) \]
where \( w \) and \( r \) are the writhe and rotation respectively and where \( N \) is the number of crossings. The writhe and the rotation are topological invariants and we shall also assume that \( N \) is a dynamical invariant. The kinematic states of the quantum knot may be labelled by these three invariants of the motion. Then by (2.6) and (2.7)

\[
D^{N/2}_{w+1} (a', b', c', d') = e^{i \frac{w}{2} \phi_w} e^{i \frac{r}{2} \phi_r} D^{N/2}_{w+1} (a, b, c, d) = e^{-i Q(w) \phi_w} e^{-i Q(r) \phi_r} D^{N/2}_{w+1} (a, b, c, d)
\]

(2.8)

where

\[
Q(w) = -k \frac{w}{2} \\
Q(r) = -k \frac{r + 1}{2}
\]

(2.9) (2.10)

and \( k \) is a constant to be determined. \( Q(w) \) and \( Q(r) \) are two topological integrals of the motion. The gauge transformations (2.8) operate on all the normal modes and therefore on all fields. We require that the action be invariant under these gauge transformations since they are induced by the transformations (2.5) that leave the defining algebra (2.4) invariant. Therefore by Noether’s theorem \( Q(w) \) and \( Q(r) \) behave as conserved charges and will be called the writhe charge and the rotation charge.

We shall now make a direct comparison between \( Q(w) \) and \( Q(r) \) of the quantum knot and the charge and hypercharge of the 4 fermion families, each denoted by \((f_1, f_2, f_3)\).

| Standard Representation | Knot Representation |
|-------------------------|---------------------|
| \((f_1, f_2, f_3)\)     | \((w, r)\)          |
| \([t, t_3, t_0, Q_e]\)  | \(D^{N/2}_{w+1}\) |
| \((e, \mu, \tau)\)     | \((3, 2)\)          |
| \([\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -e]\) | \(D^{3/2}_{\frac{3}{2}}\) | \(-k \left( \frac{3}{2} \right)\) | \(-k \left( \frac{3}{2} \right)\) | \(-3k\) |
| \((\nu_e, \nu_\mu, \nu_\tau)\) | \((-3, 2)\)          |
| \([\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, 0]\) | \(D^{3/2}_{-\frac{1}{2}}\) | \(-k \left( -\frac{3}{2} \right)\) | \(-k \left( \frac{3}{2} \right)\) | 0 |
| \((d, s, b)\)          | \((3, -2)\)          |
| \([\frac{1}{2}, -\frac{1}{2}, \frac{1}{6}, -\frac{3}{2} e]\) | \(D^{3/2}_{\frac{3}{2}}\) | \(-k \left( \frac{3}{2} \right)\) | \(-k \left( -\frac{1}{2} \right)\) | \(-k\) |
| \((u, c, t)\)          | \((-3, -2)\)         |
| \([\frac{1}{2}, \frac{1}{2}, \frac{1}{6}, \frac{2}{3} e]\) | \(D^{3/2}_{\frac{3}{2}}\) | \(-k \left( -\frac{3}{2} \right)\) | \(-k \left( -\frac{1}{2} \right)\) | \(2k\) |

Table 1.
In this table $Q_w$ and $Q_r$ are given by (2.9) and (2.10), and we have adopted a particular relative order of the four trefoils with respect to the four families of fermions. For this order we find

\begin{align*}
Q_w &= et_3 \\
Q_r &= et_0 \\
Q_w + Q_r &= Q_e
\end{align*}

if and only if $k = e/3$. These relations are in agreement with the following independent equation of standard theory:

\[ Q_e = e(t_3 + t_0) \]  

(2.14)

If one aligns the trefoils and the families in any other relative order, one needs more than one value of $k$. In this respect the correspondence between the two sets is unique.

The correspondence between the quantum knots and the isospin and charge of the elementary fermions may be summarized by the following relations, which may also be read directly from Table 1:

\begin{align*}
t &= \frac{N}{6} \\
t_3 &= -\frac{w}{6} \\
t_0 &= -\frac{r+1}{6} \\
Q_e &= -\frac{e}{6}(w + r + 1)
\end{align*}

or

\begin{align*}
t &= \frac{i}{3} \\
t_3 &= -\frac{m}{3} \\
t_0 &= -\frac{m'}{3} \\
Q_e &= -\frac{e}{3}(m + m')
\end{align*}

(2.15)

Note also that

\begin{align*}
Q_e &= -\frac{e}{N}\left(\frac{w + r + 1}{2}\right) \\
Q_e &= -e\left(\frac{m + m'}{2j}\right)
\end{align*}

(2.16)

holds for all the elementary fermions.

While $t_3$ and $t_0$ are initially defined in the standard theory with respect to $SU(2) \times U(1)$, here they label the gauge transformations on the knot algebra, and are also described by $(w, r)$ and $(m, m')$ as shown in Table 1 and Eq. (2.15). In the limit of the standard theory, where $q = 1$, $t_3$ and $t_0$ assume their usual meaning in $SU(2)$ and $U(1)$ respectively.

The states labelled by the quantum numbers $(j, m, m')$ or $(t, t_3, t_0)$ are the multinomials lying in the algebra (2.4) and given by (2.1). These multinomials are associated with the knot $(N, w, r)$ and, like the Jones (Laurent) polynomial, label the knot.
These conclusions may be summarized by assigning $D^{3/2}_{-3t_3-3t_0}$ to the fermion families with $(t, t_3, t_0)$. The members of each family are described by the quantum states $D^{3/2}_{-3t_3-3t_0}|n\rangle$ where $|n\rangle$ are the states defined by the knot algebra.\textsuperscript{3}

Remark: Although the $D^{3/2}_{\frac{N}{2}+1}$ representation permits a unified description of the electroweak properties of the four families, it does not also imply that they transform under a 4-dimensional representation of $SL_q(2)$. Rather Eq. (2.8) is the only formal property of $D^{N/2}_{\frac{N}{2}+1}(a, b, c, d)$ that has been invoked. The only new invariance beyond the standard theory that has been introduced is invariance under the global gauge transformations of the knot algebra that are expressed by (2.5) and (2.6).
3 The Preon Representation

Ignoring numerical normalization one finds by (2.1) that the four fermionic knots are represented by four monomials in the knot algebra according to Table 2:

| Solitons   | $D^{N/2}_{r+1} \frac{y}{2}$ | $D^{3/2}_{r+1} \frac{y}{2}$ | $Q \epsilon$ | $et_0$ |
|------------|-----------------------------|-----------------------------|--------------|--------|
| $(e^-, \mu^-, \tau^-)$ | $D^{3/2}_{3\frac{r}{2}}$ | $a^3$ | $-e$ | $-\frac{e}{2}$ |
| $(\nu_e, \nu_\mu, \nu_\tau)$ | $D^{3/2}_{-\frac{3}{2}+\frac{r}{2}}$ | $c^3$ | $0$ | $-\frac{e}{2}$ |
| $(d, s, b)$ | $D^{3/2}_{\frac{3}{2}-\frac{r}{2}}$ | $\sim ab^2$ | $-\frac{1}{3}e$ | $\frac{e}{6}$ |
| $(u, c, t)$ | $D^{3/2}_{\frac{3}{2}-\frac{r}{2}}$ | $\sim cd^2$ | $\frac{2}{3}e$ | $\frac{e}{6}$ |

Table 2.

This table may be re-interpreted by regarding the element $a$ as a creation operator for a preon of charge $-e/3$, and hypercharge $-e/6$ and by regarding $d$ as a creation operator for a preon of charge $+e/3$ and hypercharge $+e/6$ while $b$ and $c$ are regarded as creation operators for neutral preons with hypercharge $e/6$ and $-e/6$ respectively. This interpretation is consistent with our conclusion from earlier work\textsuperscript{2,3} that adjoint operators $(a, d)$ correspond to opposite charges and that the $(b, c)$ sector describes neutral states. According to the same picture the fermion knots, like the nucleons, are composed of three fermions, which are now preons.

While the $D^{3/2}$ representation describes the standard elementary fermions, the $D^{1/2}$ representations describes preons as follows:

$$
\begin{array}{c|cc}
\text{m} \setminus \text{m'} & \frac{1}{2} & -\frac{1}{2} \\
\hline
\frac{1}{2} & a & b \\
-\frac{1}{2} & c & d \\
\end{array}
$$

If one assigns $(t_3, t_0, Q)$ to the preons by the same rules that are given in (2.14) and have been validated for the standard fermions, one has

$$
t_3 = -\frac{m}{3}, \quad t_0 = -\frac{m'}{3} \quad \text{and} \quad Q = e(t_3 + t_0)
$$
Then we find Table 3

|   | $t_3$ | $t_0$ | $Q$  |
|---|-------|-------|------|
| $a$ | $-\frac{1}{6}$ | $-\frac{1}{6}$ | $-\frac{2}{3}$ |
| $c$ | $\frac{1}{6}$ | $-\frac{1}{6}$ | $0$  |
| $d$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{2}{3}$ |
| $b$ | $-\frac{1}{6}$ | $\frac{1}{6}$ | $0$  |

Table 3.

Note that the charge assignments of Table 3 are consistent with the preon interpretation of Table 2.

On the other hand $N = 1$ if we maintain $j = \frac{N}{2}$. Therefore the preon representation as a representation of the quantum algebra $D_{mm'}^{1/2}$, does not correspond to a knot. It may be pictured as a twisted loop. If we maintain $D_{-3t_3,-3t_0}^{3t}$, then $t = 1/6$.

Let us refer to the $D_{mm'}^{1/2}$ particles as the fermionic preons and to the $D_{mm'}^{1}$ particles as the bosonic preons. We will extend the definition of $(t, t_3, t_0)$ to all representations by maintaining the rules for labelling by isotopic labels as follows:

$$m = -3t_3 \quad j = 3t \quad Q = e(t_3 + t_0)$$

and

$$m' = -3t_0 \quad w = -6t_3$$

$$r + 1 = -6t_0$$

Now $j = 1$ implies $N = 2$ and $t = \frac{1}{3}$. Then $D_{mm'}^{1}$ represents a “dipreon” with 2 crossings.

For $D_{mm'}^{1}$ we find Table 4 (again ignoring normalizing numerical factors)

|   | $t_3$ | $t_0$ | $Q/e$ | $D_{mm'}^{1}$ | $t_3$ | $t_0$ | $Q/e$ | $D_{mm'}^{1}$ | $t_3$ | $t_0$ | $Q/e$ | $D_{mm'}^{1}$ |
|---|-------|-------|------|--------------|-------|-------|------|--------------|-------|-------|------|--------------|
| $D_{11}^{1}$ | $-\frac{1}{3}$ | $-\frac{1}{3}$ | $-\frac{2}{3}$ | $b^2$ | $D_{00}^{1}$ | $0$ | $0$ | $0$ | $ab$ | $D_{01}^{1}$ | $0$ | $-\frac{1}{3}$ | $-\frac{1}{3}$ | $ac$ | $D_{-11}^{1}$ | $\frac{1}{3}$ | $-\frac{1}{3}$ | $0$ | $c^2$ |
| $D_{10}^{1}$ | $-\frac{1}{3}$ | $0$ | $-\frac{1}{3}$ | $ab$ | $D_{00}^{1}$ | $0$ | $0$ | $0$ | $ab$ | $D_{01}^{1}$ | $0$ | $0$ | $0$ | $ad + bc$ | $D_{-10}^{1}$ | $\frac{1}{3}$ | $0$ | $\frac{1}{3}$ | $cd$ |
| $D_{1-1}^{1}$ | $-\frac{1}{3}$ | $\frac{1}{3}$ | $0$ | $b^2$ | $D_{0-1}^{1}$ | $0$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $bd$ | $D_{-11}^{1}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $d^2$ | $D_{-1-1}^{1}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $d^2$ |

Table 4.
Again in Table 4 \(a \) and \(d \) can be interpreted as creation operators for particles of charge \(-e/3\) and \(e/3\) respectively, while \(b \) and \(c \) are creation operators for neutral particles.

To show that the interpretation of \((a, b, c, d)\) as creation operators for preons holds in all representations let us introduce the operators for the writhe, \(W\), the rotation \(R\), and the charge \(Q\), as follows:

\[
\frac{1}{2} W \, D_{mm'}^j = m \, D_{mm'}^j \tag{3.3}
\]

\[
\frac{1}{2} R \, D_{mm'}^j = m' \, D_{mm'}^j \tag{3.4}
\]

\[
Q \, D_{mm'}^j = -\frac{e}{3} \left( W + \frac{R}{2} \right) D_{mm'}^j \tag{3.5}
\]

where

\[
W = \omega_a - \omega_d + \omega_b - \omega_c \tag{3.6}
\]

\[
R = \omega_a - \omega_d - \omega_b + \omega_c \tag{3.7}
\]

\[
Q = -\frac{e}{3} (\omega_a - \omega_d) \tag{3.8}
\]

Here the \(\omega_x\) are defined by their action on every term of \(D_{mm'}\) according to

\[
\omega_x(x^n x^m \ldots) = n_x(x^n x^m \ldots) \tag{3.9}
\]

Then by (3.6), (3.9), and (2.1)

\[
W \, D_{mm'}^j = (n_a - n_d + n_b - n_c) D_{mm'}^j \tag{3.10}
\]

and by (3.3)

\[
n_a - n_d + n_b - n_c = 2m \tag{3.11}
\]

By (3.7) and (3.9) and (2.1)

\[
R \, D_{mm'}^j = (n_a - n_d - n_b + n_c) D_{mm'}^j \tag{3.12}
\]

and by (3.4)

\[
n_a - n_d - n_b + n_c = 2m' \tag{3.13}
\]

Eqs. (3.10) and (3.12) depend on the fact that \(n_a - n_d = m + m'\) and \(n_b - n_c = m - m'\) are the same for every term of (2.1). \(^3\) By (3.1) and (3.11)

\[
t_3 = -\frac{1}{6} (n_a - n_d + n_b - n_c) \tag{3.14}
\]
By (3.1) and (3.13)

\[ t_0 = -\frac{1}{6} (n_a - n_d - n_b + n_c) \]  

(3.15)

Then

\[ Q = e(t_3 + t_0) = \frac{e}{3} (n_d - n_a) \]  

(3.16)

Eqs. (3.14), (3.15), and (3.16) hold in every representation and agree with the interpretation of \( a \) and \( d \) as creation operators for preons of charge \(-e/3\) and \(+e/3\) respectively. According to (3.15) and (3.14) all four of the preons contribute \( \pm \frac{1}{6} \) to the hypercharge as well as to \( t_3 \) in agreement with the Tables 2, 3, 4.

The operator for \( N (= 2j) \) is

\[ \mathcal{N} = \omega_a + \omega_b + \omega_c + \omega_d \]

and the eigenvalues of \( \mathcal{N} \) are

\[ N = n_a + n_b + n_c + n_d \]

Therefore the total number of preons in the knot described by \( D^{N/2}_{\frac{N}{2} + 1} \) is equal to the number of crossings \( (N) \). If we assume that the preons are fermions, then a knot with \( N \) crossings is a boson or fermion depending on the parity of \( N \), as we have previously assumed.

In this dual description of the knotted soliton the number of crossings, the writhe and the rotation are dual to the number of preons, \( t_3 \) and \( t_0 \) respectively. The particle-knot duality resembles the particle-wave duality, since on the particle-knot side one has by (3.2)

\[ t_3 = -\frac{w}{6} \]
\[ t_0 = -\frac{r+1}{6} \]

while on the particle-wave side one has

\[ p_\mu = \hbar k_\mu \]

and neither the knot nor the wave can be localized.

Let us also apply the knot relations to the \( j = 1/2 \) and \( j = 1 \) representations. By (2.7) we have \( N = 1 \) for \( j = 1/2 \), and \( N = 2 \) for \( j = 1 \). Although these are not knots,
but twisted loops, $w$ and $r$ still have meaning and may be computed according to (2.7). Although the writhe and rotation are no longer topologically conserved, for these cases $w$ and $r$ will remain constants of the motion if the Hamiltonian, $\mathcal{H}$, is so chosen that

$$
[\mathcal{H}, \mathcal{W}] = 0 \\
[\mathcal{H}, \mathcal{R}] = 0
$$

(3.17)

In particular $w$ and $r$ are still constants of the motion if $\mathcal{H}$ is a function of $bc$. 


4 The Interpretation of $SU_q(2)$ Preons

The preceding discussion is based on $SL_q(2)$. One may, however, describe the knots by $SU_q(2)$ instead of by $SL_q(2)$ if one sets

\[
\begin{align*}
\bar{a} &= d \\
\bar{b} &= -qc
\end{align*}
\]

Then the $SU_q(2)$ algebra is

\[
\begin{align*}
ab &= qba & a\bar{a} + b\bar{b} &= 1 & \bar{b}b &= \bar{b}b \\
\bar{a}b &= q\bar{b}a & \bar{a}a + q^2\bar{b}\bar{b} &= 1
\end{align*}
\]

The gauge invariance of the $SU_q(2)$ algebra is expressed by

\[
\begin{align*}
{a}' &= e^{i\varphi_a}a & {b}' &= e^{i\varphi_b}b \\
\bar{a}' &= e^{-i\varphi_a}\bar{a} & \bar{b}' &= e^{-i\varphi_b}\bar{b}
\end{align*}
\]

The important relation (2.6) holds exactly if $D^l_{mm'}$ refers to $SU_q(2)$ as well as to $SL_q(2)$. One passes from the representations of $SL_q(2)$ to those of $SU_q(2)$ by (4.1). In the $SU_q(2)$ language $a$ and $d$ are not creation operators for distinct preons but are creation operators for preons and antipreons. Similarly $b$ and $c$ are related as antiparticles. Then all the solitons in Table 2 are constructed out of $a$ and $b$ particles and their antiparticles.

Ref. (3) is written in the $SU_q(2)$ language, but most of this paper is written in $SL_q(2)$ language.
5 Preons as Physical Particles

We have so far viewed the preons only as a simple way to describe the algebraic structure of the knot polynomials. If these preons are in fact physical particles, the following decay modes of the quarks are possible

\[
\text{down quarks: } D^{3/2}_{\frac{1}{2}-\frac{1}{2}} \rightarrow D^{1/2}_{\frac{1}{2}+\frac{1}{2}} + D^1_{1-1} \quad \text{or} \quad ab^2 \rightarrow a + b^2
\]

\[
\text{up quarks: } D^{3/2}_{-\frac{1}{2}-\frac{1}{2}} \rightarrow D^{1/2}_{-\frac{1}{2}+\frac{1}{2}} + D^1_{1-1} \quad cd^2 \rightarrow c + d^2
\]

and the preons could play an intermediary role as virtual particles in quark processes.

The justification for considering the preons seriously as physical particles would then no longer depend exclusively on the knot conjecture but rather on a more general role of $SL_q(2)$ gauge invariance. Then the preons would appear as matrix elements of the fundamental and adjoint representations of $SL_q(2)$ just as the fermionic and bosonic knots appear in the $j = 3/2$ and $j = 3$ representations of $SL_q(2)$.

In this scenario knots would be just one of the manifestations of a $SL_q(2)$ related symmetry. There would also be no need to introduce a new Lagrangian for the preons since all particles described by representations of $SL_q(2)$ would be subject to the same modified standard action as follows:

In the knotted standard theory at the level of the quarks and leptons we attach elements of the $D^{3/2}_{m n'}$ representation to the standard quark and lepton normal modes, and elements of the $D^{3}_{m n'}$ representation to the normal modes of the standard vector boson fields. At the preon level one attaches elements of the fundamental $D^{1/2}_{m n'}$ representation to the spinor fields and elements of the adjoint $D^1_{m n'}$ representation to the vector fields.

The simple knot model predicts an unlimited number of excited states but it appears that there are only three generations, e.g. $(d, s, b)$. According to the preon scenario, however, it may be possible to avoid this problem by showing that the quarks will dissociate into preons if given a critical “dissociation energy” less than that needed to reach the level of the fourth predicted flavor. In that case one would also expect the formation of a preon-quark plasma at sufficiently high temperatures. It may be possible to study the thermodynamics of the plasma composed of quarks and these hypothetical particles.
Since the $a$ and $\bar{a}$ particles are charged ($\pm e/3$) one should expect their electroproduction according to

$$e^+ + e^- \rightarrow a + \bar{a} + \ldots$$

at sufficiently high energies of a colliding $(e^+, e^-)$ pair.

References.

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