Exotic Hairy Black Holes

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Abstract

We study black hole solutions in asymptotically $AdS_4$ spacetime with scalar hair. Following AdS/CFT dictionary these black holes can be interpreted as thermal states of 2+1 dimensional conformal gauge theory plasma, deformed by a relevant operator. We discover a rich phase structure of the solutions. Surprisingly, we find thermodynamically stable phases with spontaneously broken global symmetries that exist only at high temperatures. These phases are metastable, and join the stable symmetric phase via a mean-field second-order phase transition.

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1 Introduction

The power of holographic gauge theory/string theory correspondence of Maldacena [1] is that it extends beyond the conformal examples, and thus can be used as a tool to study critical phenomena in strongly coupled systems. Furthermore, while there are few explicit realizations of this duality in a non-conformal setting, the application of the effective Maldacena framework provides a new valuable (phenomenological) tool in addressing the strongly coupled dynamics. Such a framework was applied with intriguing results to formulate the holographic model of QCD [2], non-relativistic AdS/CFT correspondence [3, 4], effective description of viscous hydrodynamics [5, 6], holographic theories of superfluidity [7] and superconductivity [8], to name some. Often a posteriori, these phenomenological models of the Maldacena correspondence were embedded in string theory. The application of the holographic framework itself, rather than its specific string theory realization, is particularly relevant to the physics of the continuous critical phenomena where one expects organization of the physical phenomena in universality classes, thus obliterating the importance of any particular representative in a universality class. While it is certainly interesting to find embedding of the models discussed here in string theory, the latter is not our primary objective in this paper.

In this paper we follow up on a very interesting suggestion by Gubser\textsuperscript{1} [9] and will attempt to engineer finite temperature critical phenomena in non-conformal 2+1 dimensional strongly coupled systems\textsuperscript{2}. We now proceed to discuss the motivation of our model from the field-theoretic perspective. Consider a 2+1 dimensional relativistic conformal field theory. A deformation of this CFT by a relevant operator $O_r$, \begin{equation}
\mathcal{H}_{CFT} \rightarrow \tilde{\mathcal{H}} = \mathcal{H}_{CFT} + \lambda_r O_r
\end{equation}
softly breaks the scale invariance and induces the renormalization group flow. Our experience with similar deformations [12, 13] suggests that in a thermal state $O_r$ will develop a vacuum expectation value. This expectation value tends to zero in a high temperature phase, but it grows as the temperature $T$ becomes comparable to a scale $\Lambda$, set by the coupling $\lambda_r$. We assume that the deformed theory $\tilde{\mathcal{H}}$ has an irrelevant operator $O_i$. Note that we do not (further) deform $\tilde{\mathcal{H}}$ by $O_i$, as this would destroy a

\textsuperscript{1}We would like to thank Omid Saremi for bringing this work to our attention and for valuable discussions concerning his related work [11].

\textsuperscript{2}Our choice of the dimensionality was dictated by the further possibility to study these systems (in a holographic setting) in external magnetic fields [10].
UV fixed point defined by $\mathcal{H}_{CFT}$. We consider the case in which the operator $O_i$ has an associated $\mathbb{Z}_2$ discrete symmetry. As long as the operators $O_r$ and $O_i$ do not mix under the RG flow, the expectation value of $O_i$ is zero. Of course, the generic situation is that the operator mixing is present, in which case $O_i$ will develop a vev in a thermal state. The absence of discrete symmetries would typically produce a nonvanishing expectation value of irrelevant operators even in the high temperature phase [14, 15]. On the other hand, it is natural to expect that a discrete symmetry (preserved by a RG flow) would be protected at least when the operator mixing is small, *i.e.*, in the high temperature phase. As $T \lesssim \Lambda$ one expects that the mixing would be large, producing the condensate of $O_i$ and thus spontaneously breaking $\mathbb{Z}_2$ symmetry. A holographic implementation of this phenomenon was proposed in [9].

Rather surprisingly, we find that in our specific holographic realization of Gubser’s proposal [9], spontaneous breaking of a discrete symmetry occurs at high temperature. The thermal phases with the broken symmetry exist only above certain critical temperatures — different for each distinct broken phase. In a holographic setting, each broken phase is identified by the number of nodes in the wavefunction of the supergravity field dual to an irrelevant operator $O_i$. Although thermodynamically stable, these phases have higher free energy than the symmetric phase. Some of them connect to a symmetric phase via a second-order mean-field transition, with $\langle O_i \rangle$ as an order parameter.

In the next section we discuss our holographic model in detail. The results of the thermodynamic analysis are presented in section 3. We conclude in section 4 with future directions and further conjectures.

## 2 A holographic model of an exotic critical phenomena

In this section we discuss an explicit realization of Gubser’s holographic model of critical phenomena [9].

The effective four-dimensional gravitational bulk action, dual to a field-theoretic setup discussed in the introduction, takes form\(^3\)

$$S_4 = S_{CFT} + S_r + S_i = \frac{1}{2\kappa^2} \int dx^4 \sqrt{-\gamma} [\mathcal{L}_{CFT} + \mathcal{L}_r + \mathcal{L}_i] , \quad (2.1)$$

$$\mathcal{L}_{CFT} = R + 6 , \quad \mathcal{L}_r = -\frac{1}{2} (\nabla \phi)^2 + \phi^2 , \quad \mathcal{L}_i = -\frac{1}{2} (\nabla \chi)^2 - 2\chi^2 - g\phi^2\chi^2 , \quad (2.2)$$

\(^3\)We set the radius of an asymptotic $AdS_4$ geometry to unity.
where we split the action into (a holographic dual to) a CFT part $S_{\text{CFT}}$; its deformation by a relevant operator $\mathcal{O}_r$; and a sector $S_i$ involving an irrelevant operator $\mathcal{O}_i$ along with its mixing with $\mathcal{O}_r$ under the RG dynamics. The four dimensional gravitational constant $\kappa$ is related to a central charge $c$ of the UV fixed point as

$$c = \frac{192}{\kappa^2}. \quad (2.3)$$

In our case the scaling dimension of $\mathcal{O}_r$ is either 1 or 2, depending on which of the two normalizable boundary modes of a gravitational scalar $\phi$ [16] we treat as a field-theoretic parameter (and thus keep it constant as we vary the temperature), and the scaling dimension of $\mathcal{O}_i$ is 4. In order to have asymptotically $\text{AdS}_4$ solutions, we assume that only the normalizable mode of $\mathcal{O}_i$ is nonzero near the boundary. Finally, we assume that $g < 0$ in order to holographically induce the critical behavior [9].

Effective action (2.1) has a $\mathbb{Z}_2 \times \mathbb{Z}_2$ discrete symmetry that acts as a parity transformation on the scalar fields $\phi$ and $\chi$. The discrete symmetry $\phi \rightarrow -\phi$ is softly broken by a relevant deformation of the $\text{AdS}_4$ CFT; while as we will see, the $\chi \rightarrow -\chi$ symmetry is broken spontaneously.

We will study Schwarzschild black hole solutions in (2.1) with translation invariant horizons and nontrivial scalar hair. Thus, we assume:

$$ds_4^2 = -c_1(r)^2 \, dt^2 + c_2(r)^2 \left[ dx_1^2 + dx_2^2 \right] + c_3^2 \, dr^2, \quad \phi = \phi(r), \quad \chi = \chi(r). \quad (2.4)$$

As in [14], for the precision numerical analysis we find it convenient to introduce a new radial coordinate $x$ as follows

$$1 - x \equiv \frac{c_1(r)}{c_2(r)}, \quad (2.5)$$

so that $x \rightarrow 0$ corresponds to the boundary asymptotic, and $y \equiv 1 - x \rightarrow 0$ corresponds to a regular Schwarzschild horizon asymptotic. Further introducing

$$c_2(x) = \frac{a(x)}{(2x - x^2)^{1/3}}, \quad (2.6)$$

the equations of motion obtained from (2.1), (2.2) with the background ansatz (2.4), and the boundary conditions discussed above, imply

$$a = \alpha \left( 1 - \frac{1}{40} \, p_1^2 \, x^{2/3} - \frac{1}{18} \, p_1 p_2 \, x + \mathcal{O}(x^{4/3}) \right),$$

$$\phi = p_1 \, x^{1/3} + p_2 \, x^{2/3} + \frac{3}{20} \, p_1^2 x + \mathcal{O}(x^{4/3}), \quad (2.7)$$

$$\chi = \chi_4 \left( x^{4/3} + \left( \frac{1}{7} \, g - \frac{3}{70} \right) \, p_1^2 \, x^2 + \mathcal{O}(x^{7/3}) \right).$$
near the boundary $x \to 0_+$, and

$$a = \alpha \left( a_0^h + a_1^h y^2 + \mathcal{O}(y^4) \right), \quad \phi = p_0^h + \mathcal{O}(y^2), \quad \phi = c_0^h + \mathcal{O}(y^2),$$

(2.8)

near the horizon $y = 1 - x \to 0_+$. Apart from the overall scaling factor $\alpha$ (which is related to the temperature), the background is uniquely specified with 3 UV coefficients \{p_1, p_2, \chi_4\} and 4 IR coefficients \{a_0^h, a_1^h, p_0^h, c_0^h\}. Recall that both modes of $\phi$ near the boundary are normalizable. Thus we have a freedom to interpret the relevant deformation (1.1) as due to $\mathcal{O}_r$ with dim[$\mathcal{O}_r$] = 2, in which case we have to keep the combination $p_1 \alpha \equiv \Lambda$ fixed, or due to $\mathcal{O}_r$ with dim[$\mathcal{O}_r$] = 1, in which case we have to keep $p_2 \alpha \equiv \Lambda^2$ fixed. We can determine these scalings by carefully matching the boundary asymptotic finite temperature background (2.4) to the boundary asymptotic of the zero temperature background — in the latter case it is clear what is meant to keep $\lambda_r$ fixed\(^4\). As we point out below, the highly nontrivial check on our identification is the consistency of the hairy black hole thermodynamics with the first law of thermodynamics. The scale $\Lambda$ is the scale set by the relevant coupling $\lambda_r$ in (1.1). Notice that the two solutions

$$(p_1, p_2, \chi_4, a_0^h, a_1^h, p_0^h, c_0^h) \quad \& \quad (-p_1, -p_2, -\chi_4, a_0^h, a_1^h, -p_0^h, -c_0^h)$$

are equivalent. Thus we can assume $p_1 > 0$ for dim[$\mathcal{O}_r$] = 2, and $p_2 > 0$ for dim[$\mathcal{O}_r$] = 1.

It is straightforward to compute the temperature $T$ and the entropy density $s$ of the black hole solution (2.4)

$$\left( \frac{8\pi T}{\alpha} \right)^2 = \frac{6(a_0^h)^3(6 - 2(\chi_0^h)^2 + (p_0^h)^2 - g(p_0^h)^2(c_0^h)^2))}{3a_1^h + a_0^h},$$

(2.9)

$$s = \frac{c}{384} 4\pi \alpha^2 (a_0^h)^2.$$  (2.10)

The energy density $\mathcal{E}$ and the free energy density $\mathcal{F}$ can be computed using the techniques of the holographic renormalization [17]. There is a slight subtlety in holographic renormalization in our case associated with the fact that both modes of $\mathcal{O}_r$ are normalizable near the boundary [18]:

\(^4\)Identical procedure has been used in [14].
when \(\text{dim}[\mathcal{O}_r] = 2\) we have
\[
\hat{F} \equiv \frac{384}{c} \mathcal{F} = \alpha^3 \left( 2 - \frac{1}{6} p_1 p_2 - \frac{(a_0^h)^3}{2} \sqrt{\frac{6a_0^h(6 - 2(c_0^h)^2 + (p_0^h)^2 - g(p_0^h)^2(c_0^h)^2)}{3a_1^h + a_0^h}} \right),
\]
\[
\hat{E} \equiv \frac{384}{c} \mathcal{E} = \alpha^3 \left( 2 - \frac{1}{6} p_1 p_2 \right),
\]
(2.11)

when \(\text{dim}[\mathcal{O}_r] = 1\) we have
\[
\hat{F} \equiv \frac{384}{c} \mathcal{F} = \alpha^3 \left( 2 + \frac{1}{3} p_1 p_2 - \frac{(a_0^h)^3}{2} \sqrt{\frac{6a_0^h(6 - 2(c_0^h)^2 + (p_0^h)^2 - g(p_0^h)^2(c_0^h)^2)}{3a_1^h + a_0^h}} \right),
\]
\[
\hat{E} \equiv \frac{384}{c} \mathcal{E} = \alpha^3 \left( 2 + \frac{1}{3} p_1 p_2 \right).
\]
(2.12)

In both cases we independently evaluated \(\{\mathcal{F}, \mathcal{E}, s\}\). A nontrivial check on the holographic renormalization is the automatic fulfillment of the basic thermodynamic relation
\[
\mathcal{F} = \mathcal{E} - Ts.
\]

The UV fixed point is described by \(AdS_4\) Schwarzschild-black hole solution
\[
a(x) = \frac{\alpha}{(2x - x^2)^{1/3}}, \quad \phi(x) = \chi(x) = 0,
\]
(2.13)
in which case we have
\[
T = \frac{3\alpha}{4\pi}, \quad \frac{\hat{F}}{(\pi T)^3} = -\frac{64}{27}, \quad \frac{\hat{E}}{(\pi T)^3} = \frac{128}{27}.
\]
(2.14)

We conclude this section with comments on the numerical procedure. We use numerical technique identical to the one developed in [14]. For \(\text{dim}[\mathcal{O}_r] = 2\) we vary \(p_1\); for each given value of \(p_1\) we compute the remaining UV and the IR coefficients \((p_2, \chi_4, a_0^h, a_1^h, p_0^h, c_0^h)\). Notice that there are just the right number of these coefficients to uniquely determine the solution of a system of 3 second order differential equations (the equations of motion) for the background functions \(\{a(x), \phi(x), \chi(x)\}\). Given the numerical data, we can use (2.9) and the condition \(p_1\alpha = \Lambda\) to present the data, say for the free energy, as \(\hat{F} / \pi T\) versus \(\frac{\Lambda}{T}\). The procedure is then repeated for \(\text{dim}[\mathcal{O}_r] = 2\) with appropriate replacements of \(p_1\) with \(p_2\).
Figure 1: The free energy density of hairy black holes in asymptotic $AdS_4$ geometry for different deformations of the UV fixed point. The red curves correspond to symmetric phases with $\langle \mathcal{O}_i \rangle = 0$. The other curves correspond to phases with spontaneously broken $\mathbb{Z}_2$ symmetry, $\langle \mathcal{O}_i \rangle \neq 0$. The purple curves represent symmetry broken phases with the holographic condensate wavefunction $\chi(x)$ not having any nodes for $x \in [0, 1]$. The green curves represent symmetry broken phases with the holographic condensate wavefunction $\chi(x)$ having exactly one node for $x \in [0, 1]$.

3 Thermodynamics of hairy black holes in asymptotic $AdS_4$ geometry

In this section we present results of the thermodynamic analysis of the hairy black holes. We choose $g = -100$ in (2.2).

Figures 1 and 2 represent the free energy density and the energy density (correspondingly) of the hairy black holes in asymptotic $AdS_4$ geometry. Each plot is labeled by the dimension of the relevant operator $\mathcal{O}_r$ used to generate the holographic RG flow, see (1.1). The red curves correspond to $\mathbb{Z}_2$ symmetric phases with a vanishing expectation value of the irrelevant operator $\mathcal{O}_i$ (recall in our model $\text{dim}[\mathcal{O}_i] = 4$). The remaining curves represent phases with the broken $\mathbb{Z}_2$ symmetry, and the non-vanishing expectation value for $\mathcal{O}_i$. The purple curves denote symmetry broken phases in which the wavefunction of the gravitational holographic dual to $\mathcal{O}_i - \chi(x)$ — has no nodes for $x \in [0, 1]$. The green curves denote symmetry broken phases in which the wavefunction $\chi(x)$ has exactly one node for $x \in [0, 1]$.

Some of the comments below follow from the data presented; some other comments rely on the results of the analysis we omit here.
Figure 2: The energy density of hairy black holes in asymptotic $AdS_4$ geometry for different deformations of the UV fixed point. The color coding is as in figure 1.

Figure 3: The speed of sound of hairy black holes in asymptotic $AdS_4$ geometry for different deformations of the UV fixed point. The color coding is as in figure 1.

- We verified the consistency of our analysis and the identification of the mass scale in deformation (1.1) by checking the first law of thermodynamics:

$$dF = -sdT.$$ 

In our numerical results

$$\left| 1 + \frac{dF}{sdT} \right| \lesssim 10^{-7}. \quad (3.1)$$

- All the symmetry broken phases for the deformation with $\dim[\mathcal{O}_r] = 2$ (and some of the phases generated by the $\dim[\mathcal{O}_r] = 2$ RG flow) exist only above certain critical
\[ \Delta = \ln \left( \frac{\tilde{F}}{\pi T^4} + \frac{64}{27} \right) \quad \dim[\mathcal{O}_r] = 2 \]

Figure 4: The difference between the free energy density of the "purple" symmetry broken phase and the symmetric phase (see figure 1) for \( \frac{A}{T} \to 0 \) as a function of \( \ln(-g) \). The straight line fit to the data produces the following functional dependence: \( \Delta \approx 2.32861 - 1.01493 \ln(-g) \).

Temperature, \( T_c \). Notice that this \( T_c \) is different for phases in which the condensate \( \chi(x) \) has different number of nodes — \( T_{c,\text{purple}} \neq T_{c,\text{green}} \) (recall that in purple phases \( \chi(x) \) does not have any nodes, and in green phases \( \chi(x) \) has one node ).

- All the symmetry broken phases have a higher free energy density compare to a symmetric phase. Thus, at best, they are metastable. Further work is needed to determine if phases with spontaneous symmetry breaking are stable \[19\]. Note that if these phases are shown to be perturbatively unstable (having a tachyon in the spectrum of fluctuations), it will be rather unusual given that these phases survive to arbitrary high temperatures \( \frac{A}{T} \to 0 \).

- All the broken phases for the \( \dim[\mathcal{O}_r] = 2 \) deformation, and the phases for the \( \dim[\mathcal{O}_r] = 1 \) deformation which "end" on the symmetric phase (the red curves), undergo a mean-field second-order phase transition. In all cases as \( T \to T_c^+ \)

\[
\langle \mathcal{O}_i \rangle \propto \chi_4 \propto (T - T_c)^{1/2}, \quad \quad (3.2)
\]

\[
\hat{\mathcal{F}}^{\text{broken}} - \hat{\mathcal{F}}^{\text{symmetric}} \propto \begin{cases} 
+ (T - T_c)^2, & \text{as } T > T_c, \\
0, & \text{as } T \leq T_c.
\end{cases} \quad (3.3)
\]

- All the phases we discuss are thermodynamically stable, at least in the temperature range under consideration. Figure 3 represents the dependence of the speed of sound...
squared $c_s^2$ on $\frac{\Lambda^2}{T}$.

- We found symmetry broken phases with $\chi(x)$ having more than one mode for $x \in [0, 1]$, thus we believe there are infinitely many high temperature phases with $\langle O_i \rangle \neq 0$. We expect that all these phases share the same general features as the phases we discussed in details above.

- The data in figures 1-3 correspond to RG flows in which the mixing between $O_r$ and $O_i$ (the coupling constant $g$ in (2.2)) is fixed, and negative. We studied the dependence of the thermodynamics on $g$. As $|g|$ decreases, the broken phase free energy density evaluated at a given temperature increases. This increase is particularly dramatic as $g \sim -4 \cdots -3$ (for a purple phase) suggesting that there is a critical mixing $g_c$ (presumably different for different broken phases) at which this symmetry breaking thermodynamic phase disappears. We did not find any symmetry broken phases for positive $g$, thus it is likely that $g_c < 0$. Perhaps even more interesting is the behavior of the symmetry broken phases for large mixing between $O_r$ and $O_i$, i.e., when $g \to -\infty$.

In figure 4 we show that as the mixing between the operators along the RG flow increases, the broken phases approach the symmetric phase at high temperatures. The latter fact is evident both in the behavior of the free energy density and in the vanishing of $\langle O_i \rangle$ for the high temperature broken phases as $g \to -\infty$.

4 Conclusion

In this paper we used the ideas of Gubser [9] to holographically engineer RG flows in strongly coupled 2 + 1 dimensional gauge theory models with exotic finite temperature phases. Specifically, we considered a relevant deformation of a relativistic $CFT_3$ with an operator $O_r$, $\text{dim}[O_r] = \{1, 2\}$. We allowed a mixing of an irrelevant operator $O_i$, $\text{dim}[O_i] = 4$, with $O_r$ along the RG flow. We discovered a plethora of exotic phases in the system at finite temperature. We found the phases that exist only at high temperature, spontaneously break $\mathbb{Z}_2$ symmetry of the model and join the symmetric phase with a mean-field second-order phase transition. All the phases we discuss are thermodynamically stable. The phases with the broken symmetry have a higher free energy density than the symmetric phase; thus, at best, they are metastable.

Still a lot of work (both technical and conceptual) is needed to understand these exotic phases.

- It would be interesting to study perturbative stability of the phases with the bro-
ken symmetry. Using gauge theory/string theory correspondence it is easy to argue that thermodynamic instabilities of the gravitational backgrounds reveal themselves as (perturbative) dynamical instabilities [20] as well. The reverse statement is not necessarily true. For such analysis one needs to compute the spectrum of the quasinormal modes associated with the fluctuations of $\chi(x)$.

- We made a lot of conjectures about the phase structure of the theory based on extensive numerical work. It would be interesting to analytically prove the disappearance of the symmetry breaking phases for small mixing, $g > g_c$. Likewise, it is important to show analytically that there is no phase crossing in the system as $g \to -\infty$. If such a phase crossing does occur, the high temperature symmetry broken phases would be thermodynamically preferable; in addition, there could be interesting critical behavior associated with such crossing.

- We did not study the question of embedding the RG flows discussed here in string theory — it might very well be that there is an (unknown to us at this stage) obstruction for such an embedding. It would be interesting to see if similar phenomena occur in string theory realizations of holographic dualities of seemingly related systems [21].

- We want to emphasize once again the unusual features of the phases with the broken symmetry. It is well known that a two-dimensional Ising model allows for a dual reformulation in terms of a "disorder" operator$^5$. The Hamiltonian of the Ising model is local when expressed in terms of the disorder operators. While the order parameter in the spin formulation vanishes above certain temperature, in the disorder formulation, the disorder parameter (probing the condensation of the kinks) vanishes in the low temperature phase, akin to what we see in our model here. The relation between the original spin and the dual disorder operator in the Ising model is nonlocal. In our model both the operators $O_r$ and $O_i$ are local. If $O_i$ is interpreted as an analog of the disorder operator in the Ising model, how can it be mutually local with $O_r$ — an analog of a local operator in the spin formulation of the Ising model?

- There are known condensed matter systems which have a disordered high temperature phase [23]. It would be interesting to study if a close analogy between those systems and the holographic models discussed here could be established.

- Finally, it would be interesting to develop in more details the idea of ‘microengineering’ the renormalization group flow by specifying not only the operators that deform the (UV) fixed point of the theory, or simply are present in the theory, but also impos-

$^5$See [22] for a review.
ing — by hand — the operator mixing under the renormalization group flow. While this is quite unusual in a standard field-theoretic setting, it is a real possibility in the holographic constructions.

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