I. INTRODUCTION

Proton radiography\cite{1,2} is a diagnostic tool for time-resolved and spatial-resolved studies of the electromagnetic field structures in inertial confinement fusion (ICF) and high energy density (HED) plasmas. The information about the morphology and strengths of electric and magnetic field is coded in the deflection angle of the proton beams and alters the proton flux after interaction with electromagnetic field. The proton flux is then recorded on a detector. This kind of imaging technique has been used to characterize the electromagnetic fields and carry out measurements in a variety of experiments, including ICF implosion capsules\cite{3-12}, magnetic reconnection\cite{13–18}, self-generated magnetic fields through Biermann battery term\cite{19–25} and plasma instabilities\cite{26–33}, non-ideal MHD effects\cite{34–36}, and laboratory dynamo experiments\cite{37, 38}.

In ICF and HED experiments, two distinct types of proton sources have been developed for high performance diagnostics. First, in a capsule implosion, DD(3MeV) and D\textsuperscript{3}He(14.7MeV) protons from fusion reaction driven by multiple laser beams. The protons leave tracks in CR-39\cite{39} which is etched and scanned to get the absolute location and track characteristics of each proton \cite{39}. Second, broadband proton beams\cite{2,10} up to 60MeV are driven by ultra-intense (\(> 10^{18}\text{W/cm}^2\)) short pulse laser beam through Target Normal Sheath Acceleration (TNSA) mechanism, and the proton flux is recorded on the radiochromic film pack with a sequence of proton energies. In general, the TNSA proton backlighter offers better spatial and temporal resolution, while the D\textsuperscript{3}He fusion-based techniques offers better spatial uniformity and energy resolution.

The understanding of field structure from proton images is limited by the fact that the images are a two-dimensional mapping of the three dimensional field distribution. The general mapping can be nonlinear, degenerate and diffusive. Direct interpretation of the proton images is achievable only under the assumptions of simple field geometries. Some inverse-problem type of general techniques\cite{11,43} have been developed to infer the integral quantities over the line of sight, e.g., magnetic field perpendicular to line of sight (\(\int dz \cdot \mathbf{B}\)) or MHD current along the line of sight (\(\int dz \times \mathbf{B}\)). The comprehensive description of the inverse-problem type of techniques for proton images of stochastic magnetic fields has been developed\cite{42}. However, the caustic and diffusive regimes are still challenging for inferring the fields. The primary focus of this paper to develop a tool to understand the proton image in the diffusive regime, where a ballistically propagating beam from the source is diffused by Coulomb scattering and stopping power.

There are some general-purpose Monte Carlo toolkits, e.g. MCNP\cite{44} and GEANT4\cite{45}, and tools specifically for HED applications\cite{46, 47}, for forward modeling of proton radiography. MCNP and GEANT4 have the features to model the energy lost and collisional scattering of protons in cold matter, but corrections are needed for calculations of plasma stopping power and scattering\cite{48}. In this paper, we take those corrections related to plasma into account and develop a more accurate Monte Carlo and ray-tracing tool called MPRAD\cite{54} for forward modeling of proton radiography. We make some approximations in the models for Coulomb scattering and stopping power used in the code. Those are good approximations under the condition that proton energy \(E_p > 1\text{MeV}\), electron temperature \(\frac{kT_e}{\text{545eV}} < E_p[\text{MeV}]\), density \(\rho/A < 10^4\text{g/cc}\) (\(A\) is the mass number of the matter), and the transition layer between cold matter and fully ionized plasma is thin compared to the rest of the system. This condition covers the conditions of a range of ICF and HED experiments. MPRAD is written in Python with MPI+OpenMPI parallelization among particles or rays, using Cython and

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Keywords: Monte Carlo, ray-tracing, proton radiography
MPI4py package. Cython compilation for the core part of the code is used to improve the performance. The output data from plasma-dynamical modeling such as radiation-magnetohydrodynamics or particle-in-cell (PIC) simulations can be imported into MPRAD. The Python package from Yt-project[49] is used to read the data from FLASH[50] simulations. Each MPI process gets the whole data set of pre-calculated quantities. Each thread solves the Monte Carlo transport for each particle (or ray transport for each ray) independently. And the binned data (or final quantities for the rays) is collected after each process and thread completes the calculation for the targeted number of particles (or rays). The process for making MPRAD an open source code is ongoing. Our tool will be used for designing and analyzing the data for the recent OMEGA experiments of magnetic field generation in shock-shear type of targets[51].

This paper is organized as follows. Sec II describes the features of the MPRAD code, including the model for Coulomb scattering and stopping power. In Sec III we perform benchmark simulations for cold matter with MPRAD and compare the results with MCNP simulations. Some applications and examples for the effect of diffusion process on the proton radiography are discussed in Sec IV. The summary is given in Sec V.

II. FEATURES OF THE CODE

In MPRAD, we solve the relativistic equation of proton motion, i.e. the evolution of position and velocity of the protons in the beam, in electromagnetic field, similar to the features in the existing tools[11, 14, 45, 17]. In addition we implement the stopping power and Coulomb Scattering, both in cold matter approximation and weakly interacting plasmas. Pre-calculated quantities are used to speed up the large scale simulations.

A. Stopping power

In MPRAD code, we use the models for stopping power and energy-loss straggling in the literatures[52, 56]. The relativistic effects of protons are taken into account to accurately calculate the motion of non-relativistic to highly relativistic protons with \( \beta = v/c \), where \( v \) is the velocity of the proton, and \( c \) is the speed of light. The velocity for the proton beam is assumed to be much higher than the electron thermal speed \( v_p > v_{e,th} \), which implies \( E_p/m_p/kT_e/m_e > 1 \), i.e.

\[
\frac{kT_e}{546\text{eV}} < \frac{E_p}{1\text{MeV}}
\]  

(1)

Under the \( v_p > v_{e,th} \) assumption, we further assume that the beam–plasma coupling strength\[53]\( \gamma_c \) is small, i.e.

\[
\gamma_c = 6.8 \times 10^{-3} \left( \frac{\rho}{1\text{g/cc}} \right)^{1/2} \left( \frac{E_p}{1\text{MeV}} \right)^{3/2} \frac{1}{A^{1/2}} \ll 1
\]  

(2)

where \( A \) is the mass number of the matter. For \( E_p > 1\text{MeV} \) and \( \rho/A < 10^4 \text{g/cc} \), \( \gamma_c \) is always much less than unity, so that the beam–plasma coupling effect can be neglected[55].

For room temperature, we use the stopping power for cold matter. For proton energy \( E_p > 1\text{MeV} \), the stopping power in cold matter can be written as[52]

\[
\frac{d(E_p)}{dx} = -\frac{4\pi e^4 n_e}{\beta^2 m_e c^2} \left[ f(\beta) + a \right] = -\frac{0.31\text{MeV/cm} \times Z \frac{\rho}{1\text{g/cc}}}{A^{3/2}} \left[ f(\beta) + a \right]
\]  

(3)

where \( m_e \) is the mass of electron, \( n_e \) is the total electron number density (including both free electrons and bound electrons), \( \rho \) is the density of the matter, \( A \) is the mass number of the matter, and \( Z \) is the charge number of the matter. The bracket \( \langle E_p \rangle \) represents the average energy lost, and \( x \) is the path length of the proton. Due to the fact that the collision between the protons and the particles in the matter is random, the energy lost follows a distribution deviating from the average energy lost, which is described as the straggling function as given in Eq(13). The quantity \( a \) is related to the material property and can be found in stopping power database such as PSTAR[57] and SRIM[58]. The function \( f(\beta) \) is

\[
f(\beta) = \ln[\beta^2 / (1 - \beta^2)]
\]  

(4)

For mixture, compound or isotopes, i.e. different \( A \)'s and \( Z \)'s, the stopping power is

\[
\frac{d(E_p)}{dx} = -\frac{0.31\text{MeV/cm} \times Z_1 \frac{\rho}{1\text{g/cc}}}{A_1^{3/2}} \left[ f(\beta) + a_{CM} \right]
\]  

(5)

where \( f_i \) is the atomic number fraction of \( i \)th element, \( A_1 = \sum_i f_i A_i \), \( Z_1 = \sum_i f_i Z_i \), and

\[
a_{CM} = \sum_i Z_i f_i a_i Z_i
\]  

(6)

where the subscript “CM” denotes “cold matter”.

For the calculations of plasma stopping power, only electron contribution is considered, because the contribution of the plasma ions to stopping power is negligible due to the fact that \( m_i/m_e = 1836 A \gg 1 \). The expression of stopping power in plasma is simply the Bethe
formula under our assumptions in Eq(1) and (2)

\[
\frac{d(E_p)}{dx} = \frac{4\pi e^4 n_e}{\beta^2 m_e c^2} \ln \left[ 1.123 \sqrt{\frac{1}{2\pi}} \frac{m_e^3/2 e^2 \beta^2 / (1 - \beta^2)}{\hbar c n_e} \right] - 0.31 \text{MeV/cm} \times Z_1 \frac{\rho}{1 \text{g/cc}} A \beta^2 \times f(\beta) + 0.5 \ln A - 0.5 \ln \left( \frac{\rho}{1 \text{g/cc}} \right)
\]

(7)

where \( h \) is the reduced Planck constant. Eq (7) is consistent with the results from [53–55]. For the plasma composed of multiple ion species

\[
\frac{d(E_p)}{dx} = -0.31 \text{MeV/cm} \times Z_1 \frac{\rho}{1 \text{g/cc}} \frac{A \beta^2}{Z_1} f(\beta) + a_{\text{plasma}}
\]

(8)

where \( A_1 \) and \( Z_1 \) have the same definitions as in the cold matter case but \( f_i \)'s are replaced by the number fractions of ions. And

\[
a_{\text{plasma}} = \sum_i f_i Z_i (10.2 + 0.5 \ln A_i - 0.5 \ln Z_i) - 0.5 \ln \left( \frac{\rho}{1 \text{g/cc}} \right)
\]

(9)

The difference between cold matter stopping power and plasma stopping power is only in the expressions for \( a_{\text{CM}} \) and \( a_{\text{plasma}} \), i.e. Eq(1) and Eq(9), while other parts of the two equations are identical. The typical values of \( a_{\text{CM}} \) or \( a_{\text{plasma}} \) are around 10. In a typical HED target system, there are both cold matter and plasma. We use the ratio of Debye length \( \lambda_D \) to Fermi radius \( a_Z \) for quantifying the partition between cold matter and fully ionized plasma.

\[
\frac{\lambda_D}{a_Z} = \sqrt{\frac{kT}{4\pi e^2 n_e}} = 0.885 a_0 Z^{-1/3} = 0.5 Z^{1/3} \sqrt{\frac{T}{1 \text{eV}}} \sqrt{\frac{n_e}{10^{23} \text{cc}}} \text{ cm}
\]

(10)

where \( a_0 \) is the Bohr radius and \( n_e \) is free electron density, i.e. not including the bond electrons, which is different from total electron density \( n_e \), i.e. including both bond electrons and free electrons. For matter composed of multiple elements, we use logarithm averaged charge number \( Z_{1g} = \exp(\sum_i f_i \log Z_i) \) in Eq(10). The total stopping power with combined cold matter and fully ionized plasma is

\[
\frac{d(E_p)}{dx}_{\text{total}} = -0.31 \text{MeV/cm} \times Z_1 \frac{\rho}{1 \text{g/cc}} \frac{A \beta^2}{Z_1} f(\beta) + a_{\text{total}}
\]

(11)

where

\[
a_{\text{total}} = \frac{\lambda_D^2}{\lambda_D + a_Z^2} a_{\text{CM}} + \frac{a_Z^2}{\lambda_D + a_Z^2} a_{\text{plasma}}
\]

(12)

The combination using Eq(12) is a good approximation if the transition layer between cold matter and plasma is thin compared to the fully cold matter or fully plasma regions, i.e. most regions in the modeling has \( a_Z \ll \lambda_D \) or \( a_Z \gg \lambda_D \). If \( a_Z \approx \lambda_D \) dominates, then one need more precise combination model in the transition region.

In Figure 1, we plot the ratio between the total stopping power with combined cold matter and plasma given by Eq(11) and cold matter approximation given by Eq(6) for plastic(CH with C:H=1) and copper, and for proton energy 14.7MeV and 3MeV. We use the mean ionization state from PROPACEOS [59] equation of state table to calculate the free electron density. The cold matter approximation is good for the matter near or above solid density, i.e. 1g/cc for CH and 8.9g/cc for Cu. For low densities, the correction from Eq(7) has significant contribution to total stopping power, especially for Cu. For high temperatures, i.e. \( T > 50 \text{eV} \), the correction from Eq(7) has larger contribution for CH than for Cu.

The straggling function of the proton energy, i.e. the variation of stopping power along the path of motion follows a Gaussian distribution [52]

\[
F(\Delta, s) = \frac{1}{\sqrt{2\pi} \Omega} \exp\left[\frac{-(\Delta - \Delta_{av})^2}{2\Omega^2}\right]
\]

(13)

with a variance \( \Omega^2 \), and a mean value \( \Delta_{av} \) equal to the product of path length \( s \) and the stopping power. The expression for the variance is [52] [56]

\[
\Omega = \sqrt{4\pi e^4 n_e \frac{1 - \beta^2/2}{1 - \beta^2} s} = 0.16 \text{MeV} \sqrt{\frac{n_e}{10^{23} \text{cc}}} \frac{s}{1 \text{cm}} \frac{1 - \beta^2/2}{1 - \beta^2}
\]

(14)

which only depends on the electron density \( n_e \), the path length \( s \) and the normalized velocity \( \beta \) of the proton.

In both Monte Carlo and ray-tracing calculations, three quantities are pre-calculated, (1) the coefficient for stopping power \( \frac{0.31 \text{MeV/cm} \times Z_1 \frac{\rho}{1 \text{g/cc}}}{A_1 \beta^2} \), (2) \( a_{\text{total}} \), (3) \( 4\pi e^4 n_e \). In each time step of the particle motion, in both Monte Carlo and ray-tracing, the energy lost can be calculated from the pre-calculated quantities and the current value of \( \beta \) of the particle or ray, as given by Eq(11). In the Monte Carlo calculation, the straggling of proton energy is sampled at each time step using Eq(13). In the ray-tracing calculation, the total variable of proton energy is calculated by the numerical integration

\[
\Omega_{total}^2 = \sum_{\Delta s} \frac{4\pi e^4 n_e 1 - \beta^2/2}{1 - \beta^2} \Delta s
\]

(15)

where the value of \( \beta \) is the current value for the particle and \( n_e \) is at the particle location.
Figure 1. Subfigures (a) to (d) are the contours of the ratio between the total stopping power with the combined cold matter and plasma given by Eq(11) and cold matter approximation given by Eq(5), for four different cases: (a) plastic (C:H=1), proton energy $E = 14.7\text{MeV}$, (b) plastic (C:H=1), $E = 3.0\text{MeV}$, (c) copper, $E = 14.7\text{MeV}$, (d) copper, $E = 3\text{MeV}$. Subfigure (e) and (f) are the contours of the ratio between the scattering angle using the full characteristic small scattering angle given by Eq(20) and cold matter approximation, for four different case (e) CH with proton energy $E = 14.7\text{MeV}$ and $E = 3\text{MeV}$, (f) Cu with proton energy $E = 14.7\text{MeV}$ and $E = 3\text{MeV}$. There is slight difference between $E = 14.7\text{MeV}$ and $E = 3\text{MeV}$. The column density is $\rho_L = 0.005\text{g/cm}^2$ for (e) and (f). The horizontal axes of all the subfigures are densities, and the vertical axes of all the subfigures are electron temperatures.

**B. Coulomb Scattering**

The cross-section of single and multiple Coulomb scattering in thick foil was studied in [60, 61], which has been used in GEANT4[45] and MCNP[44]. The cross-section for large angle scattering remains unchanged from cold matter to plasma

$$N \sigma(\chi) d\chi = 2 \chi^2 \chi d\chi q(\chi)/\chi^4$$

(16)

where $\sigma(\chi) d\chi$ is the differential scattering cross section into the angular interval $d\chi$ by each atom(or ion), $s$ is thickness of the material, $N$ is the number of scattering atoms(or ions) per volume, $q$ is the ratio of actual to Rutherford scattering and approaches unity for large angle scattering, and

$$\chi_c^2 = 4\pi N se^4 Z(Z+1)/(pv)^2$$

(17)

where $p$ is the proton momentum and $v$ the velocity of the proton beam. The physical meaning of $\chi_c$, is that the total probability of single scattering through an angle greater than $\chi_c$, is exactly one. For mixture, compound or isotopes

$$\chi_c^2 = 4\pi N se^4(Z_2^2 + Z_1)/(pv)^2$$

(18)

where $Z_2 = \sqrt{\sum Z_i^2 f_i}$ and $Z_1$ is the same as that in Eq(6). The expression for the numerical value of $\chi_c^2$ in terms of density $\rho$ is

$$\chi_c^2 = 1.8 \times 10^{-7} \times \frac{\rho s}{\text{g/cm}^2} \frac{(Z_2^2 + Z_1) 1 - \beta^2}{A \beta^4}$$

(19)

For Coulomb scattering in plasmas, we replace the Fermi radius $a_Z$ of the atom with the Debye length $\lambda_D$ of the plasma in the calculation of characteristic small scattering angle where $q(\chi)$ approaches zero. In general, for the regions with both cold matter and plasma, the characteristic small scattering angle is

$$\chi_0 = \frac{1}{\lambda_D} \sqrt{\frac{1}{a_Z^2} + \frac{1}{\lambda_D^2}}$$

(20)

where $\lambda$ is the De Broglie wavelength of the proton. For cold matter approximation, $\lambda_D/a_Z \to \infty$, Eq(20) recovers the characteristic small scattering angle in cold matter, which is identical to Eq(8) in Ref. [60].

For thick target where many scattering events occur, $B_c > 5$ for the variable $B_c$ given by the following equations (we use $B_c$ instead of $B$ as in Ref. [60] to avoid
confusion with magnetic fields)

\[ B_c - \ln B_c = b = \ln \frac{\chi_c^2}{1.167 \chi_a^2} \]  

\[ \chi_a^2 = \chi_0^2 (1.13 + 3.76(Z_2 e^2)/((hv)^2)) \]  

where \(3.76(Z_2 e^2)/((hv)^2)\) is the second order term in the Born approximation. In \(\lambda_D/\alpha_Z \to \infty\) limit, the expression for the numerical value of \(e^b\) without second order term in Born approximation is

\[ e^b \approx \frac{6680 \rho_{\text{cm}}^* (Z_2^2 + Z_1)}{\beta^2 A Z_{\text{ig}}^{2/3}} \]  

which is consistent with Eq(22) in Ref. [60]. The distribution of the scattering angle \(\theta\) is expanded in a series of \(B_c\). The tabulated numerical values of the distributions are in Ref. [60]. We keep the first three terms, i.e. the Gaussian distribution (zeroth order term) with

\[ \sigma_{\text{Gauss}} = \chi_c (B_c/2)^{1/2} \]  

and the terms in \(\frac{1}{B_c}\) and \(\frac{1}{B_c^2}\). The distribution is closer to Gaussian distribution when \(B_c\) becomes larger. At each time step of proton motion, \(B_c\) is calculated, using the length step \(\Delta s\) as the target thickness \(s\) in Eq(15). In Monte Carlo calculation, if \(B_c\) is small, i.e. \(B_c < 5\), we fallback to use the cross section for single scattering. The method we implement for Coulomb scattering as a random process has been used in other Monte Carlo codes such as MCNP [62] and GEANT4 [15]. Two quantities in each cell are pre-calculated before the Monte Carlo or ray-tracing calculations. (1) the coefficient for large angle scattering cross section, i.e. \(4\pi N e^4 (Z_2^2 + Z_1)\), (2) the characteristic small scattering angle \(\chi_0\).

For ray tracing calculation, we use the numerical integration of the R.H.S. of Eq(18) and the ion+atom density weighted value of \(\ln \chi_0\)

\[ \ln \chi_0 = \frac{\sum \Delta s \left( N \ln \chi_0 \right)_{\text{local}} \Delta s}{\sum \Delta s N_{\text{local}} \Delta s} \]  

which is an analog of Eq(16) in Ref. [60]. In Figure 1(e) and (f), we show the ratio between the scattering angle using the full characteristic small scattering angle given by Eq(20) and using cold matter approximation. The scattering angle is calculated using Eq(24), and \(B_c, \chi_c\) are calculated by ray-tracing of proton beam through a material with column density \(\rho L = 0.005 g/cm^2\). For low densities or high temperatures, the correction from finite \(\lambda_D\) has significant contribution to total scattering angle as shown in the top left corner of Figure 1(e) and (f). The difference between 3MeV and 14.7MeV protons is more prominent in CH than in Cu, which can be explained by the sensitivity of \(b\) to the proton energy or proton velocity given Eq(21) and Eq(22). For CH, \(3.76(Z_2 e^2)/((hv)^2)\) is 0.6 for \(E_p = 3\)MeV and 0.1 for \(E_p = 14.7\)MeV, both less than 1.13, thus \(e^b \sim e^{1.13+3.76(Z_2 e^2)/((hv)^2)}\) is sensitive to proton energy. For Cu, \(3.76(Z_2 e^2)/((hv)^2)\) is 26 for \(E_p = 3\)MeV and 5 for \(E_p = 14.7\)MeV, both much larger than 1.13, thus \(e^b \sim e^{1.13+3.76(Z_2 e^2)/((hv)^2)} \sim 1/1.13 \sim \text{constant}\).

III. BENCHMARK AGAINST MCNP CODE FOR COLD MATTER

Under cold matter approximation, we test the Monte Carlo calculation in MPRAD code by the setup as shown in Fig 2. The mono-energetic(\(E_p = 0\)) and collimated proton source with \(E_p = 15\)MeV is placed 1cm from the slab of matter with given material and thickness. We use \(10^6\) particles in the simulations and the proton velocity is perpendicular to the detector plane. The detector plane is 20cm from the slab of matter, and the particles reaching the detector plane are binned by spatial grid with \(\Delta x = \Delta y = 0.01\)cm and energy grid with \(\Delta E = 0.05\)MeV. The simulation with the same setup is also carried out using MCNP code [44].

For all the test cases, both the spatially binned proton image and the proton spectrum are consistent between MPRAD and MCNP. An example is shown in Fig 3. The protons in the narrow beam are isotropically scattered by colliding with the matter in the slab, so a circular spot on the detector plane is produced as shown in Fig 3(b) and (c). The protons lose energy and have a finite width in the spectrum at the detector as shown in Fig 3(a), because different protons have different path length in the matter due to scattering. For a given composition of the slab material, different density \(\rho\) but same column density \(pt\) produces similar image and spectrum. Quantitative comparison between the results from MPRAD
and MCNP is shown in Table I and Table II. The slight difference between results from MPRAD and MCNP is tolerant for typical proton radiography setup in HED experiments, where the spectrum width of the source is a few keV to MeV[2][30][38].

IV. EXAMPLE APPLICATIONS

For small angle deflection, the deflection angle of protons by magnetic field is [43]

$$\alpha = 1.80 \times 10^{-2} \text{rad} \times \left( \frac{E_p}{14.7 \text{MeV}} \right)^{-1/2} \times \left( \frac{B}{10^6 \text{G}} \right) \times \left( \frac{l_i}{0.1 \text{cm}} \right)$$  \hspace{1cm} (26)

where $E_p$ is the energy of proton, $B$ is the strength of magnetic field, and $l_i$ is the longitudinal size of the interaction region. From Eq (24) and Eq (26) we can calculate the ratio between the deflection angle by magnetic field and the Coulomb scattering angle

$$\frac{\alpha}{\sigma_{\text{Gauss}}} = \frac{2\sqrt{A}}{\sqrt{B_c(Z_2^2 + Z_1)}} \sqrt{\frac{E_p}{m_p c^2}} \frac{v_A}{c} \sqrt{\frac{l_i m_p c^2}{e^2}}$$

$$= \frac{6}{\sqrt{2\pi^3}} \left( \frac{E_p}{14.7 \text{MeV}} \right)^{1/2} \times \frac{v_A}{2.8 \times 10^4 \text{cm/s}} \left( \frac{l_i}{0.1 \text{cm}} \right)$$ \hspace{1cm} (27)

where $v_A$ is the Alfvén speed, i.e. $v_A = \frac{B}{\sqrt{4\pi m_p}}$.

For the examples we show in this Section, we use the setup as shown in Figure 2, with a magnetic field in the slab. We carry out Monte Carlo runs with a flux rope of toroidal fields, a flux rope of poloidal fields, and a turbulent field that satisfies the power law energy spectrum. The $z$ axis is along the line of sight, and the detector plane is $x - y$ plane. The interaction region is filled with plastic(C:H=1). The thickness of the interaction region is $l_i = 1000\mu m$ with uniform tunable density $\rho$ and fixed temperature $T_e = 1000$K. And the field is centered at $(0,0,0)$. The source is at $(0,0,-1cm)$, mono-energetic and collimated with $E_p = 15$MeV.

We use the same notation from Ref. [33] to define the contrast field

$$\Lambda(x_{\perp}) = \frac{\Psi(x_{\perp}) - \psi_0}{\psi_0}$$  \hspace{1cm} (28)

where $\psi_0$ is the unperturbed proton flux, which is uniform by assumption, $\Psi(x_{\perp})$ is the perturbed proton flux by both deflection and diffusion, and $x_{\perp}$ is the position vector on the image plane. Eq(19) in Ref. [43] gives the expression for the contrast field as a map of MHD current

$$\Lambda(x_{\perp}) = \frac{e r_s(r_s - r_i)}{r_s \sqrt{2 m_p c^2 E_p}} \sigma \cdot \int dz \nabla \times B$$ \hspace{1cm} (29)

where $r_s$ is the distance between the interaction region and the screen, $r_i$ is the distance between the source and the image plate. For the parameters we use, we have

$$\Lambda(x_{\perp}) = 1.7 \times 10^{-6} \text{G}^{-1} \times \left( \frac{E_p}{14.7 \text{MeV}} \right)^{-1/2} \times \sigma \cdot \int dz \nabla \times B$$ \hspace{1cm} (30)

We use the field strength that makes $|\Lambda(x_{\perp})| < 1$ to avoid caustics.

A. Localized magnetic fields

Toroidal magnetic fields have been observed and measured in some HED experiments[1][2][18][20][22][34][35], especially for the reconnection geometry. The typical geometry of self-generated magnetic field in the plasma plume produced by single laser spot is toroidal. We follow the expression for toroidal magnetic field in literatures[11][43][47], which is the characteristic distribution of a localized toroidal field

$$B = \frac{B_0}{a} \exp\left(-\frac{x^2 + y^2 + z^2}{a^2}\right)(-y,x,0)$$  \hspace{1cm} (31)

where we use $B_0 = 1 \times 10^5 \text{G}$, $a = 200\mu m$ for our numerical tests.

As shown in Figure 4(a), the quasi-monoenergetic proton beam has become a beam with a broad energy distribution as it goes through the target region, and the mean energy becomes lower than the source energy. In the analysis for real data from the experiments, one also has to take the spectrum width of the source into account. The diffusion can affect the interpretation of proton image. As the density increases, the theoretical peak value of the contrast drops as $E_p^{-1/2}$ given by Eq (30) if no diffusion is considered. However, the peak value of the contrast in the simulation drops faster than the theoretical value given by Eq (30), as shown in Figure 4(c). One deduces smaller field or MHD current from the image for large density case. For large densities, the variation level of proton number in each pixel can potentially become comparable or even smaller than the poisson noise for the CR-39 image, and the variation level of the proton flux can become smaller than the sensitivity of radiochromic film.

A few HED experiments have generated and characterized poloidal magnetic field, such as supersonic jets with mega-gauss self-generated magnetic fields localized in the interaction region[24][25]. We follow the expression for poloidal magnetic field in literatures[11][47], which is the characteristic distribution of a localized poloidal field

$$B_y = \frac{B_0}{a} \exp\left(-\frac{x^2 + z^2}{a^2}\right)$$ \hspace{1cm} (32)

where we use $B_0 = 4 \times 10^5 \text{G}$, $a = 200\mu m$ for our numerical tests. The results are shown in Figure 4(d) and (e).
Table I. Comparison of average energy and energy variation from MPRAD and MCNP for different material, density and thickness

| Material, density, thickness | $\bar{E}$ (MeV), MPRAD | $\bar{E}$ (MeV), MCNP | $\sqrt{E^2 - \bar{E}^2}$ (MeV), MPRAD | $\sqrt{E^2 - \bar{E}^2}$ (MeV), MCNP |
|-----------------------------|------------------------|------------------------|---------------------------------------|---------------------------------------|
| Be, 1.85g/cc, 200µm        | 13.96                  | 13.96                  | 0.0546                                | 0.0565                                |
| Be, 7.40g/cc, 50µm         | 13.96                  | 13.96                  | 0.0546                                | 0.0565                                |
| Mg, 1.74g/cc, 16µm         | 14.93                  | 14.92                  | 0.0157                                | 0.0138                                |
| Mg, 6.96g/cc, 4µm          | 14.93                  | 14.92                  | 0.0157                                | 0.0138                                |
| Cu, 8.96g/cc, 16µm         | 14.70                  | 14.71                  | 0.0355                                | 0.0350                                |
| Cu, 17.92g/cc, 8µm         | 14.70                  | 14.71                  | 0.0355                                | 0.0350                                |

B. Power law energy spectrum in magnetic turbulence

Magnetic turbulence and dynamo have been studied in HED experiments[37, 38]. In turbulent magnetic fields, magnetic energy cascades to small scales, and the magnetic energy spectrum follows a power law distribution. The power law spectrum can be inferred using inverse-problem type of technique[37, 38, 43]. As an example for using MPRAD to study how diffusion affect the inferred spectrum, we use a power law in a recently designed turbulent dynamo experiment on the OMEGA-EP[62], where the magnetic energy spectrum follows $E(k) \propto k^{-2.3}$. The method for generating the magnetic field by random numbers for numerical tests is discussed in Ref. [42]. The vector potential is multiplied by $\exp(-\frac{x^2+y^2+z^2}{a^2})$ where $a = 200\mu m$ to get the localized field. We assume the RMS value of the magnetic field in the $i^3$ box is $B_{\text{rms}} = 1 \times 10^5 G$, the maximum field strength is $B_{\text{max}} = 1 \times 10^6 G$, the same as the test problem for localized toroidal and poloidal magnetic fields. We use the algorithm in Ref. [43] to reconstruct the divergence free turbulence spectrum. For reconstruction, Eq(52) in Ref. [43] gives the expression for the inferred magnetic energy density

$$E_B(\frac{r_s}{r_i}, 2\pi L | n |) = \frac{2\pi}{c^2(r_s - r_i)^2l_i^2L^2} (\hat{\Lambda}(n)\hat{\Lambda}(n)^*)$$ (33)

where $l_i$ is the longitudinal size of the interaction region, $L$ is the length and width of the image plate, $\hat{\Lambda}(n)$ is the discretized Fourier transform of $\Lambda(x \perp)$, and the average is over cells in $n$-space for the discretized Fourier transform.

As shown in Figure 4(f), the diffusion affects the cutoff length scale $\pi/k_c$ of the spectrum given by the inversion algorithm. The results for $\rho = 10^{-7} g/cc$ shows little scattering and the spectrum around $k \sim 10^3 cm^{-1}$ agrees being a the power law, and $k > 3 \times 10^3 cm^{-1}$ is beyond the resolution limit. For densities from $\rho = 3.0 \times 10^{-4}$ and above, there is a critical wavevector $k_c$ that the diffusion affect damps the small scale feature for $k > k_c$ but retains

Similar to the case for the toroidal magnetic fields, the diffusion of the beam affect the final spectrum of the protons and the peak value of proton flux contrast, and thus some care are needed for interpreting the proton images.
Table II. Comparison of weighted (by proton flux) average of $x^2 + y^2$ from MPRAD and MCNP for different material, density and thickness

| Material, density, thickness | $\sqrt{x^2 + y^2}$ (cm), MPRAD | $\sqrt{x^2 + y^2}$ (cm), MCNP |
|-----------------------------|-------------------------------|-------------------------------|
| Be, 1.85g/cc, 200µm         | 0.381                         | 0.369                         |
| Be, 7.40g/cc, 50µm          | 0.381                         | 0.369                         |
| Mg, 1.74g/cc, 16µm          | 0.161                         | 0.161                         |
| Mg, 6.96g/cc, 4µm           | 0.161                         | 0.161                         |
| Cu, 8.96g/cc, 16µm          | 0.535                         | 0.536                         |
| Cu, 17.92g/cc, 8µm          | 0.535                         | 0.536                         |

Figure 4. The results for the example applications for 1000µm thickness. The colors of the curves are consistent among all panels, red for $\rho = 10^{-7}$ g/cc, blue for $\rho = 0.385$ g/cc, green for $\rho = 0.852$ g/cc, and black otherwise. Subfigure (a) shows the spectrum of the protons in the detector plane for different densities. Subfigure (b) is the contrast field of the proton image for the test case for localized toroidal magnetic field, the feature is coaxial as we can see from the symmetry of the field. A line-out cross the center is shown in (c), for different densities. The dashed lines are the theoretical value of contrast given by Eq. (30), and the solid lines are from the MPRAD simulations. The results for the localized poloidal magnetic field is in (d) and (e). Subfigure (f) is the inferred turbulence magnetic energy spectrum using Eq. (33), for different densities.

the large scale feature for $k < k_c$. For $\rho = 7.3 \times 10^{-3}$ g/cc and $\rho = 3.6 \times 10^{-2}$ g/cc, the energy density at low k, i.e. $k < 50$ cm$^{-1}$ becomes higher than other densities. The inverse of the cutoff scale $k_c$ is roughly the scattering angle multiplied by $r_i$, thus

$$k_c \approx \frac{\pi}{10 r_i \sigma_{\text{Gauss}}}$$

$$\approx 20 \text{cm}^{-1} \left( \frac{\rho}{1 \text{g/cc}} \right)^{-1/2}$$  \hspace{1cm} (34)

where the factor 10 in the denominator is an estimate of the scattering angle in $\frac{1}{B_c}$ term. $B_c$ is roughly 6 for $\rho = 1.5 \times 10^{-3}$ g/cc so that $\frac{1}{B_c}$ term is not neglectable.
The estimate for $k_c$ is in good agreement with the results in Figure[3](f). In the analysis for real data from the experiments, one also has to take the angular distribution of the source into account. The composition and temperature can also affect $k_c$.

V. SUMMARY

A simulation tool MPRAD is developed in this work, which extends the capability of Monte Carlo calculations for proton radiography, especially for the conditions where Coulomb scattering and stopping power are not neglectable. The model for Coulomb scattering and stopping power in fully ionized plasma and in cold matter are combined to improve the accuracy of modeling, especially in the plasma region. Ray tracing can be used as a quick way to study the effects of Coulomb scattering and stopping power. Synthetic Monte Carlo radiograph by using the imported data of fields, density, mass fraction and power. Synthetic Monte Carlo radiograph by using way to study the effects of Coulomb scattering and stop- etching process.

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[1] C. K. Li, F. H. Séguin, J. A. Frenje, J. R. Rygg, R. D. Petrasso, R. P. J. Town, S. P. Hatchett, O. L. Landen, A. J. Mackinnon, P. K. Patel, V. A. Smalyuk, T. C. Sangster, and J. P. Knauer, Physical Review Letters 97 (2006), 10.1103/physrevlett.97.135003
[2] A. B. Zylstra, C. K. Li, H. G. Rinderknecht, F. H. Séguin, R. D. Petrasso, C. Stoekl, D. D. Meyerhofer, P. Nilson, T. C. Sangster, S. L. Pape, A. Mackinnon, and P. Patel, Review of Scientific Instruments 83, 013511 (2012)
[3] J. R. Rygg, F. H. Séguin, C. K. Li, J. A. Frenje, M. J.-E. Manuel, R. D. Petrasso, R. Betti, J. A. Delettrez, O. V. Gotchev, J. P. Knauer, D. D. Meyerhofer, F. J. Marshall, C. Stoekl, and W. Theobald, Science 319, 1223 (2008)
[4] C. K. Li, F. H. Séguin, J. R. Rygg, J. A. Frenje, M. Manuel, R. D. Petrasso, R. Betti, J. Delettrez, J. P. Knauer, D. D. Meyerhofer, F. J. Marshall, C. Stoekl, O. L. Landen, R. P. J. Town, C. A. Back, and J. D. Kilkenny, Physical Review Letters 100 (2008), 10.1103/physrevlett.100.225001
[5] C. K. Li, F. H. Séguin, J. A. Frenje, R. D. Petrasso, P. A. Amendt, R. P. J. Town, O. L. Landen, J. R. Rygg, R. Betti, J. P. Knauer, D. D. Meyerhofer, J. M. Soures, C. A. Back, J. D. Kilkenny, and A. Nikroo, Physical Review Letters 102 (2009), 10.1103/physrevlett.102.205001
[6] C. K. Li, F. H. Séguin, J. A. Frenje, M. Rosenberg, R. D. Petrasso, P. A. Amendt, J. A. Koch, O. L. Landen, H. S. Park, H. F. Robey, R. P. J. Town, A. Casner, F. Philippe, R. Betti, J. P. Knauer, D. D. Meyerhofer, C. A. Back, J. D. Kilkenny, and A. Nikroo, [physicsofplasmas](2006), 10.1103/physrevlett.97.045001
[7] C. K. Li, F. H. Séguin, J. A. Frenje, M. J. Rosenberg, H. G. Rinderknecht, A. B. Zylstra, R. D. Petrasso, P. A. Amendt, O. L. Landen, A. J. Mackinnon, R. P. J. Town, S. C. Wilks, R. Betti, D. D. Meyerhofer, J. M. Soures, J. Hund, J. D. Kilkenny, and A. Nikroo, Physical Review Letters 108 (2012), 10.1103/physrevlett.108.025001
[8] F. H. Séguin, C. K. Li, M. J.-E. Manuel, H. G. Rinderknecht, N. Sinenian, J. A. Frenje, J. R. Rygg, D. G. Hicks, R. D. Petrasso, J. Delettrez, R. Betti, F. J. Marshall, and V. A. Smalyuk, Physics of Plasmas 19, 012701 (2012)
[9] I. V. Igumenshchev, A. B. Zylstra, C. K. Li, P. M. Nilson, V. N. Goncharov, and R. D. Petrasso, Physics of Plasmas 21, 062707 (2014)
[10] A. J. Mackinnon, P. K. Patel, M. Borgesi, R. C. Clarke, R. R. Freeman, H. Habara, S. P. Hatchett, D. Hey, D. G. Hicks, S. Kur, M. H. Key, J. A. King, K. Lancaster, D. Neely, A. Nikkoo, P. A. Norreys, M. M. Notley, T. W. Phillips, L. Romagnani, R. A. Snavely, R. B. Stephens, and R. P. J. Town, Physical Review Letters 97 (2006), 10.1103/physrevlett.97.045001
[11] L. Volpe, D. Batani, B. Vauzour, P. Nicolai, J. J. Santos, C. Regan, A. Morace, F. Dorcheis, C. Fourment, S. Hulin, F. Perez, S. Baton, K. Lancaster, M. Galimberti, R. Heathcote, M. Tolley, C. Spindloe, P. Koester, L. Labate, L. A. Gizzi, C. Benedetti, A. Sgattoni,
