A Constraint on EHNS Parameters from Solar Neutrino Problem

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Abstract

We suppose that the solar neutrino problem is only due to the mechanism introduced by Ellis, Hagelin, Nanopoulos and Srednicki (EHNS), then a lower limit constraint on EHNS parameters is obtained. We find that \(\gamma \geq 7.4 \times 10^{-22} \text{GeV}\) if \(\alpha < 2\gamma\) and \(\alpha \geq 1.5 \times 10^{-21} \text{GeV}\) if \(\alpha > 2\gamma\), this limit is consistent with the upper limits extract from the \(K^0 - \bar{K}^0\) system and can be detected in the future experiments.

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I. Introduction.

According to Hawking’s postulate on the evaporation of black holes[1-2], a pure quantum mechanical state may evolve into mixed states. Although this violation of quantum mechanics could cause a lot of problem in physics, such as a serious conflict between energy-momentum conservation and locality[3] and CPT violation[4-5] etc., the theory in this field has been set up.

To include the quantum mechanics violation owing to Hawking’s quantum gravity effect, in 1983, J.Ellis, J.S.Hagelin, D.V.Nanopoulos and M.Srednick put forward a modified hamiltonian equation of motion for density matrix and used it to interpret upper bounds on the violation of quantum mechanics in different phenomenological situation[6]. Soon after, T.Banks, L.Susskind and M.E.Peskin (BPS)[3] proposed a generic form of the modified evolution equations for density matrix which preserves the linearity, locality in time, and conservation of probability. Recently, motivated by EHNS and BPS and the other researchers, B.Reznik[7] proposed another modified Liouville equation which constitutes a linear, local, and unitary extension and thus allows a quantum mechanical system for unitary evolution between pure and mixed states. But different from the exponentially decaying (or exponentially increasing) solution of EHNS mechanism, this model yields a oscillatory modification. However, both mechanisms can lead to CP violation. By analysing the experimental datum of CP violation in $K^0 - \overline{K^0}$ system, Ellis et al.[8-9] and Peskin et al.[10] determined the values of two of the three EHNS parameters independently. All the analysis gives the upper limits of EHNS parameters approach the range $0(m_2^2/M_{pl}) \approx 2 \times 10^{-20}$ GeV. In the meantime, Ellis et al. determin these parameters by theoretical consideration furthermore. Based on string theory[10-11] and display a logarithmic divergence in the density matrix of a scalar field in the presence of an Einstein-Yang-Mills black hole in four dimensions[12], they get the values of the size of the estimate above. The latest estimate of these parameters is[20]

$$\alpha \leq 4 \times 10^{-17} GeV, \quad |\beta| \leq 3 \times 10^{-19} GeV, \quad \gamma \leq 7 \times 10^{-21} GeV.$$  

But, if EHNS mechanism is a correct physical principle, the lower limits of these parameters should be known when we do not know their exact values yet.

On the other hand, the solar neutrino problem which consists of the deficit of observed neutrino emitted from the Sun with respect to the theoretically expected amount has been
with the physicists for more than 30 years. To resolve this puzzle, a class of solutions such as neutrino oscillation and the matter effect[13-15], neutrino magnetic moment[16], and a combination of both etc. have been developed. In Ref.[19], we have found that the EHNS mechanism can affect the neutrino oscillation behaviors greatly and hence may be taken as a new solution of the solar neutrino problem[19].

In this work, we suppose that the solar neutrino deficit is due to EHNS mechanism, so we can get a constraint on the lower limits of the EHNS parameters. To ensure this paper is selfcontained, we first introduce EHNS mechanism following Huet and Peskin[10] in section two and list the relative formulas and datum about neutrino oscillation in section three, then, in section four, we show how to estimate the lower limit constraint on the EHNS parameters by using neutrino oscillation datum. The conclusion and discussion are given in the final section.

II. EHNS Mechanism

In conventional quantum mechanics, the evolution of density matrix obeys Liouville equation:

\[ \frac{d}{dt} \rho = [H, \rho]. \]  

(1)

This equation guarantees that the probability is conserved and the purity of the state is not changed in the evolution of a system.

For a two states system, the density matrix can be expanded by using the Pauli sigma matrix:

\[ \rho = \rho^0 1 + \rho^i \sigma^i \]  

(2)

where \( i = 1, 2, 3 \) and repeat index represents summation. It is evidently that \( \rho^0 = \frac{1}{2} \) and to ensure the density of probability is semi-positive definite, \( \rho^i \) should satisfy the following constraint

\[ (\rho^0)^2 \geq \sum_{i=1}^{3} (\rho^i)^2. \]  

(3)

When expanding the hamiltonian in the same way, Eq.(1) becomes

\[ \frac{d}{dt} \rho = 2\varepsilon^{ijk} H^i \rho^j \sigma^k \]  

(4)

where \( \varepsilon^{ijk} \) is the totally antisymmetric tensor and

\[ \varepsilon^{123} = +1 \]  

(5)

For a non-Hermitian Hamiltonian, the case is little more complicated, we refer the readers to the original paper of Huet and Peskin[10] or Ellis et al[8][20].
Now, to include the modification due to Hawking’s quantum gravity effect which permits quantum mechanical system evolve into mixed states, the most general linear terms

\[ -h^{0j}\rho^j 1 - h^{j0}\sigma^j - h^{ij}\sigma^i\rho^j \]  

should be added to Eq.(4). But there are two natural restrictions on these terms: probability conservation and entropy of the density never decrease. These requirements set \( h^{0j} = 0 \) and \( h^{j0} = 0 \) respectively. Then we get

\[ \frac{d}{dt}\rho = 2\epsilon_{ijk}H^i\rho^j\sigma^k - h^{ij}\sigma^i\rho^j \]  

Because the antisymmetric part of \( h^{ij} \) can be absorbed into \( H^i \), we may assume that \( h^{ij} \) is symmetric. For \( K^0 - \overline{K^0} \) system, EHNS further assume that this new effect does not change strangeness. So, \( h^{1j} = 0 \) and

\[ h = 2\begin{pmatrix} 0 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta & \gamma \end{pmatrix} \]  

where \( \alpha, \beta, \gamma \) are the EHNS parameters and [10]

\[ \alpha, \gamma > 0, \quad \alpha\gamma > \beta^2 \]  

Notice that the neutrino oscillation among different flavors is very similar to the strangeness oscillation in \( K^0 - \overline{K^0} \) system, we think that it is reasonable to generalize the above assumption from \( K^0 - \overline{K^0} \) system to neutrino system.

Here, three EHNS parameters are present. As we have mentioned before, all the consideration from either theory or experiments by Ellis et al and Peskin et al only give their upper limits. We have not yet known their definite values - if this mechanism is a correct physical law. The central purpose of this paper is intend to determine their lower limits constraint by neutrino oscillation.

III. Neutrino Oscillation and the Present Situation of Solar Neutrino Problem

The neutrino weak eigenstates may not coincide with their mass eigenstates. Because of the different time evolution, oscillation can occur[13-15]. For simplicity, we only consider the case of two flavors. Suppose that the neutrinos mix through a vacuum mixing angle \( \theta \), i.e,

\[ \begin{pmatrix} |\nu_e> \\ |\nu_\mu> \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\nu_1> \\ |\nu_2> \end{pmatrix} \]  

\[ \begin{pmatrix} |\nu_e> \\ |\nu_\mu> \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\nu_1> \\ |\nu_2> \end{pmatrix} \]
where $|\nu_e>, |\nu_\mu>$ are the weak eigenstates and $|\nu_1>, |\nu_2>$ the mass eigenstates with masses $m_1$ and $m_2$ respectively. Because the two states evolve differently,

$$
\begin{pmatrix}
|\nu_e(t) > \\
|\nu_\mu(t) > 
\end{pmatrix} =
\begin{pmatrix}
\cos \theta e^{-iE_1 t} & \sin \theta e^{-iE_2 t} \\
-sin \theta e^{-iE_1 t} & \cos \theta e^{-iE_2 t}
\end{pmatrix}
\begin{pmatrix}
|\nu_1 > \\
|\nu_2 > 
\end{pmatrix}
$$

(11)

As a result, a state start with $|\nu_e>$ may oscillate into $|\nu_\mu>$ with the probability

$$
p(\nu_e \rightarrow \nu_\mu(t)) = \sin^2(2\theta)\sin^2\left(\frac{1}{2}(E_2 - E_1)t\right)
$$

(12)

and the probability for it to remain as itself is:

$$
p(\nu_e \rightarrow \nu_e(t)) = 1 - \sin^2(2\theta)\sin^2\left(\frac{1}{2}(E_2 - E_1)t\right)
$$

(13)

Notice that the neutrino mass is very small, so their energy and momentum are very close. Hence we can rewrite the survival probability as following[21]:

$$
p(\nu_e \rightarrow \nu_e(t)) = 1 - \sin^2(2\theta)\sin^2\left(\frac{\delta m^2 L}{4E}\right)
$$

(14)

where $\delta m^2 = m_\nu^2 - m_{\nu_e}^2$. Numerically, $\delta m^2 L/(4E) = (1.266932 \ldots)\delta m^2 L/E$, here $\delta m^2$ is measured in $eV^2$, $L$ in $m$ while $E$ in $MeV$ (or $L$ in $Km$ while $E$ in $GeV$). Because in all cases of this paper, $p(\nu_e \rightarrow \nu_e(t)) + p(\nu_e \rightarrow \nu_\mu(t)) = 1$, i.e., the probability is conserved, so we only need to write down one of them.

Now, let’s turn to the solar neutrino problem. This problem was first put forward by the Homestake chlorine experiment around 1970. It was recognized that the observed capture rate was significantly smaller than the standard solar model(SSM) prediction. After the middle of 1980s, some other experiment groups began to collect datum. The recent results from the four solar neutrino experiments[21-25] and the SSM predictions calculated by Bahcall and Pinsonneault(B-P)[26] and by Turck-Chieze and Lopes(T-C-L)[27] are listed in the following table. Where, for Homestake, GALLEX and SAGE, the

| Experiment   | Data          | B-P          | T-C-L       |
|--------------|---------------|--------------|-------------|
| Homestake    | 2.55 ± 0.17 ± 0.18 | 8.0 ± 3.0   | 6.4         |
| GALLEX       | 79 ± 10 ± 6   | 131.5 ± 21 - 17 | 122.5      |
| SAGE         | 73 ± 18 ± 5   | 131.5 ± 21 - 17 | 122.5      |
| Kamiokande   | 2.89 ± 0.22 ± 0.35 | 5.7 ± 2.4 | 4.4         |

Table 1: Experimental and Theoretical Datum of the Solar Neutrino

data are capture rates given in solar neutrino units ($SNU.1SNU = 10^{-36}$ capture per
atom per second). For Kamiokande, the datum is \(8 \times 10^6 \text{ cm}^{-2}\text{s}^{-1}\). The first errors are statistical and the second errors are systematic. The errors associated with the B-P calculation are “theoretical” 3 standard deviations according to the authors.

IV. A Constraint on EHNS Parameters from Solar Neutrino Problem

Let’s first review the effect of EHNS mechanism on neutrino oscillation. For neutrino, its Hamiltonian is diagonalized in the basis of \(|\nu_1>, |\nu_2>\),

\[
H = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}
\]

so,

\[
H^1 = H^2 = 0 \\
H^3 = (E_1 - E_2)/2 \approx -\delta m^2/(4E)
\]

and Eq.(7) becomes

\[
\frac{d}{dt} \begin{pmatrix} \rho^0 \\ \rho^1 \\ \rho^2 \\ \rho^3 \end{pmatrix} = 2 \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\delta m^2/(4E) & 0 \\ 0 & -\delta m^2/(4E) & -\alpha & -\beta \\ 0 & 0 & -\beta & -\gamma \end{pmatrix} \begin{pmatrix} \rho^0 \\ \rho^1 \\ \rho^2 \\ \rho^3 \end{pmatrix}
\]

we only need to consider the case which the neutrino is originally in the state \(|\nu_e>\), from Eq.(10)

\[
\rho_{\nu_e} = \begin{pmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{pmatrix}
\]

Then the initial condition is:

\[
\rho(t = 0) = \rho_{\nu_e}
\]

i.e:

\[
\rho^0 = \frac{1}{2} \quad \rho^1 = \frac{1}{2}\sin(2\theta) \quad \rho^2 = 0 \quad \rho^3 = \frac{1}{2}\cos(2\theta)
\]

The survival probability of \(|\nu_e>\) is

\[
p(\nu_e \to \nu_e) = \text{Tr}[\rho(t) \rho_{\nu_e}]
\]
solution, $\delta m^2/(4E)$ is about the order of $10^{-21}$ $GeV$[21] if we take the magnitude order of the solar neutrino energy as $1$ $MeV$, but the upper limit of $Max[\alpha, \beta, \gamma]$ given by J.Ellis et al.[20] for the $K^0 - \overline{K^0}$ system is $4 \times 10^{-17}$ $GeV$, when transforming it into the neutrino system, as discussed in [19], its magnitude is about $10^{-22}$ $GeV$. This is just one magnitude order less than $\delta m^2/(4E)$.

When approximate to the first order, we can write down the neutrino survival probability analytically as

$$p(\nu_e \rightarrow \nu_e) \simeq \frac{1}{2} + \frac{1}{2} \cos^2(2\theta) e^{-2\gamma L} + \frac{1}{2} \sin^2(2\theta) \cos \left( \frac{\delta m^2}{2E} L \right) e^{-\alpha L} \tag{22}$$

From Eq.(22), it is obviously that as $L \rightarrow \infty$, $p(\nu_e \rightarrow \nu_e(t)) \rightarrow \frac{1}{2}$. This is the most important result of EHNS mechanism on neutrino oscillation in the case of two generations.

To generalize the case of two generations to the case of three generations, we can begin with Eq.(1), expand the Hamiltonian $H$ and the density matrix $\rho$ by using the generators of $SU(3)$, then eight EHNS parameters which have no any experiental restriction will be presence, so, such generation is meaningless.

Here, we further suppose that the direct oscillation between $\nu_e$ and $\nu_\tau$ can be neglect, we only consider the case that the oscillation between $\nu_e$ and $\nu_\tau$ takes the following way: first, $\nu_e$ oscillates into $\nu_\mu$, then, due to the oscillation between $\nu_\mu$ and $\nu_\tau$, it oscillates into $\nu_\tau$. If the system starts with $|\nu_e>$, then the oscillation looks like a waterfall. After making such supposition, we can deal with the neutrino oscillation by including EHNS mechanism in the case of two generations. Due to the discussion of the effect of EHNS mechanism on the oscillation between $\nu_\mu$ and $\nu_\tau$ is the same as that between $\nu_e$ and $\nu_\mu$, we will not repeat it here. But it is evidently that because of the effect of EHNS mechanism,

$$L \rightarrow \infty, \ p(\nu_e \rightarrow \nu_e(t)) \rightarrow \frac{1}{3}$$

when we consider the case of three generations.

Now, return to table 1, it is easily to see that the ratios of experimental central values to theoretical values are between

$$0.32 \sim 0.6.$$ 

On the other hand, if the EHNS effect on neutrino oscillation has not yet developed sufficiently, i.e., the probability decay has not yet developed sufficiently, then the survival probability happens to local in this domain. Hence we can get the experimental constraint on the EHNS parameters.
A detail calculation of the dependence of the survival probability \( p(\nu_e \rightarrow \nu_e) \) on the time \( T \) or distance \( L \) is difficult, as a estimation, we have

\[
\frac{1}{2} + \frac{1}{2} \cos^2(2\theta) e^{-2\gamma L} + \frac{1}{2} \sin^2(2\theta) \cos(\frac{\delta m^2}{2E} L) e^{-\alpha L} \leq 0.6
\]

(23)

where \( \theta \) is the vacuum mixing angle of neutrino and

\[
L \simeq 7.58 \times 10^{26} \text{GeV}^{-1}
\]

is the distance from the sun to the earth in natural units.

Eq.(23) is the most important result in this work, it is just the constraint on EHNS parameters that we want to find, from which we can extract some concrete conclusion.

From Eq.(23), we achieve,

\[
\sin^2(2\theta) \left\{ 1 - 2\sin^2(\frac{\delta m^2}{4E} L) \right\} e^{-\alpha L} - e^{-2\gamma L} + e^{-2\gamma L} \leq 0.2
\]

(24)

and

\[
\cos^2(2\theta) \left\{ e^{-2\gamma L} - e^{-\alpha L} \left[ 1 + 2\tan^2(2\theta) \sin^2(\frac{\delta m^2}{4E} L) \right] \right\} + e^{-\alpha L} \leq 0.2
\]

(25)

Hence, we obtain

\[
e^{-2\gamma L} \leq 0.2 \quad \text{if} \quad \alpha < 2 \gamma + \frac{1}{L} \log \left[ 1 - 2\sin^2(\frac{\delta m^2}{4E} L) \right]
\]

and

\[
e^{-\alpha L} \leq 0.2 \quad \text{if} \quad \alpha > 2 \gamma + \frac{1}{L} \log \left[ 1 + 2\tan^2(2\theta) \sin^2(\frac{\delta m^2}{4E} L) \right].
\]

Notice that \( \log[1 - 2\sin^2(\frac{\delta m^2}{4E} L)] < 0 \) and \( \log[1 + 2\tan^2(2\theta) \sin^2(\frac{\delta m^2}{4E} L)] > 0 \), then we arrive

\[
\gamma \geq 1.06 \times 10^{-27} \text{ GeV} \quad \text{if} \quad \alpha < 2 \gamma
\]

and

\[
\alpha \geq 2.12 \times 10^{-27} \text{ GeV} \quad \text{if} \quad \alpha > 2 \gamma
\]

But according to EHNS and Huet and Peskin, when transfering these parameters produced in neutrino system into \( K^0 - \overline{K^0} \) system, a certain ratio should be considered. As discussed in [19], we will multiply them by a factor of \( \left( \frac{500 \text{MeV}}{E_{\nu}(\text{in MeV})} \right)^2 \). Where as a estimate, we take \( E_{\nu} \) as the average energy of the solar neutrino, then \( E_{\nu} \sim 0.6 \text{MeV} \)[21]. Finally, we get

\[
\gamma \geq 7.4 \times 10^{-22} \text{ GeV} \quad \text{if} \quad \alpha < 2 \gamma
\]

(26)
\[ \alpha \geq 1.5 \times 10^{-21} \text{GeV} \]  
\[ \text{if } \alpha > 2 \gamma. \]  
\( (27) \)

This is consistent with their upper limits\cite{20}, \( \alpha \leq 4 \times 10^{-17} \text{GeV} \) and \( \gamma \leq 7 \times 10^{-21} \text{GeV} \).

\textbf{V. Conclusion and discussions}

In conclusion, we have derived a lower limit constraint on the EHNS parameters from the solar neutrino problem by supposing that the defect of the solar neutrino is only due to the EHNS mechanism. We find \( \gamma \geq 7.4 \times 10^{-22} \text{GeV} \) if \( \alpha < 2 \gamma \) and \( \alpha \geq 1.5 \times 10^{-21} \text{GeV} \) if \( \alpha > 2 \gamma \). This constraint is coincide with the latest result given by Ellis et al.\cite{20} and can be detected in the future experiments.

Here, the uncertainty arise in the amplification factor \( \left( \frac{500 \text{MeV}}{E_{\nu}(\text{in MeV})} \right)^2 \). Because the solar neutrino energy spectrum has a certain width, so when we do not take its average value as the input parameter of the amplification factor, departure will be present, and the lower limits will be risen for the sector of small energy. But, notice that the present experiments have their threshold, even if we take \( E_{\nu} \) as 0.2 \text{MeV}, the lower limits constraint determined above is still consistent with Ellis et al.\cite{20} and Huet and Peskin\cite{10}.

In this work, we have supposed that the solar neutrino problem is only due to the EHNS mechanism. We have not considered the MSW effect yet. The detail discussion on how about the correction due to the MSW effect will be reported in the future.

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