A nonlinear time-varying copula using kink approach

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Abstract. The structural change in copula parameter is motivates us to propose a flexible non-linear time varying copula that allows for capturing the structural change in the time varying dependence between variables. In this study, two families of Elliptical copula, namely Gaussian and Student-t copulas, are considered. We conduct a simulation study to examine the performance and accuracy of the proposed model and we obtain the reliable and acceptable results. In addition, the new model is applied to explain the dependence between S&P 500 and FTSE 100 stock markets. The proposed model fits well with these datasets and shows evidence of structural change in the dependence structure overtime. Moreover, the nonlinear time varying copula outperforms the conventional linear time varying copula suggesting that our model can detect better the dependence structure of these two stock markets.

1. Introduction
Modelling of the dependence structure of between variables has been increasingly interested in various fields of research, such as economic, financial, engineer, health, environment, etc. It is well accepted that the normality assumption does not always hold in many types of variables and the correlation between these random variables may not have a linear. Thus, the conventional correlation methods, such as Pearson correlation and Kendall tau, might fail to show perfect dependence if the relationship between two variables is non-linear. Thus, the flexible non-standard multivariate distribution known as copula is proposed to deal with the complexity in dependence structure. Copula is firstly introduced in the Sklar’s theorem and has been further developed by [1]. Then, it has become increasingly popular in various applications such as finance, agriculture, health, and economics. (see [2],[3],[4],[5] and [6]). The model enables us to describe the complicated dependency between random variables as well as measure the joint distribution as it eliminates the multivariate symmetric assumption on the joint distribution. Nevertheless, the stability of the models and the invariability of the parameters over time have been questioned in many studies as the constant copula parameter might not be useful for explaining the presence of change in correlation over time. In recent work of [7], the time varying copula-based models were proposed for dealing with non-normality and dependence dynamics over time. The dependence time evolution allows the dependence parameter to evolve over time according to ARMA process.

Despite its simplicity and usefulness, the dynamic copula approach is no longer considered as a dependence measure in various studies. A number of studies suggested that there exists a structural change in the dependence structure. [8] tested the presence of change in the dependence structure between oil price and stock and provided the evidence of change point in the copula parameter. They
proposed a Markov Switching dynamic copula to deal with the non-linear relationship in the dynamic copula and mentioned that the copula parameters and tail dependence coefficients are greater during financial crisis period, while there are tend low in the boom period. However, their model could not give information about the existence of change point in the time varying dependence. This paper attempts to fill the gap by applying the kink approach of [9] to ARMA (1, 10) process of [7] and thereby allowing a dependence parameter to evolve according to the kink ARMA (1, 10) process. In other words, the dependence parameter is subject to regime-change at unknown kink point or threshold variable and thereby separating this dependence parameter into two or more regimes. In the model, the AR coefficient can be separated into two or more regimes according to $\rho_{t-1} < \gamma$ and $\rho_{t-1} \geq \gamma$.

Our framework can be generalized further by allowing the MA to split into two (or more) regimes based on a threshold indicator too. However, we ignore these extensions here, in order to avoid complex models that would entail more challenging numerical maximizations of the likelihood function. The main goal of our paper is to extend the existing ARMA (1, 10) process in time varying copula models of [7] to accommodate structural break in time varying dependence series. Thus, the nonlinear time varying copula is proposed where the dependence parameter is allowed to vary according to the restricted kink ARMA (1, 10) process. Consequently, our proposed model has gained more ability to capture the structural change of dependence over time and allowing a variety of dependence structures to be captured with more flexibility and parsimony than in conventional model. In particular, it acknowledges shifts in the relationships between variables which can be beneficial for the design of more adequate regulatory frameworks and for the reduction of bias in the dependence measure under the time varying aspect. To the best of our knowledge, our proposed model has not been explored in the literature yet and this is the main contribution of this study.

The remainder of the article is structured as follows. Section 2 describes the time varying Copula families and Markov Switching approach. Section 3 presents the data used and reports and discusses empirical results. Section 5 provides some concluding remarks.

2. Modelling framework

2.1. Bivariate copula function and dynamic dependence structure

Formally, copulas are functions for use to join different univariate distributions to form a valid multivariate distribution. According to the theorem of [10], the joint distribution function of continuous random variable $x_{1t}$ and $x_{2t}$ can be decomposed into marginal distributions $F_1(x_{1t})$ and $F_2(x_{2t})$. Let $H(x_{1t}, x_{2t})$ be a joint distribution function with marginal $F_1(x_{1t}), F_2(x_{2t})$, there exists a copula $C: [0,1]^2 \rightarrow [0,1]$ such that

$$H(x_{1t}, x_{2t}) = C(F_1(x_{1t}), F_2(x_{2t}))\theta,$$

where $\theta$ is a copula dependence parameter. For time varying specification, the bivariate joint distribution can be written as

$$H(x_{1t}, x_{2t}) |\Psi = C_1(F_1(x_{1t}), F_2(x_{2t})|\Psi),$$

where $\Psi$ denotes vector of parameters in time varying process and $C_t(\cdot)$ is the time varying copula function. We assume that the marginals can be modelled parametrically, thus the probability transform is given by $U_1 = F_1(x_{1t})$ and $U_2 = F_2(x_{2t})$. Then, the time varying copula joint probability density holds

$$h(U_1, U_2) = c_t(U_1, U_2 |\psi_{t-1}) \prod_{i=1}^{2} f_i(x_{it}) \phi_{t-1},$$

where $\phi_{t-1}$ denotes the marginal probability density functions of $x_{it}$.
where \( f_i(F_i(x_i|\varphi_i)) \) is the density function corresponding to the best fit marginal distribution which is constructed by generalized autoregressive conditional heteroscedasticity (GARCH) process. Then, the full likelihood function can be expressed as:

\[
L(\psi, \varphi) = c(F_1(x_{it}), F_2(x_{2t})|\psi_{t-1}) f_i(x_{it}|\varphi_1) f_j(x_{2t}|\varphi_2),
\]

where \( \psi = \{\omega_0, \omega_1, \omega_2\} \) is the parameter estimates in time varying copula function, \( \varphi_1 \) and \( \varphi_2 \) are the parameter estimates in ARMA(p)-GARCH(1,1). In the optimization of this model, the full likelihood function \( L(\psi, \varphi) \) can be split into two parts namely copula likelihood function and GARCH likelihood function which are estimated separately [1]. Therefore, the two-stage estimator is used.

\[
\tilde{\varphi}_i = \arg \max (\log L(\varphi_i)), \quad i = 1, 2
\]

\[
\tilde{\psi} = \arg \max (\log L(\psi))
\]

2.2. Time varying kink process

[7] proposed time varying copula models by proving Sklar’s theorem for conditional distributions and suggested that the dependence copula parameter (\( \rho \)) can be computed by simple ARMA (1,10) process.

\[
\rho_t = \Lambda(\omega_0 + \omega_1 \rho_{t-1} + \omega_2 \Gamma_{t-10}),
\]

where \( \Lambda(\cdot) \) is the logistic transformation for each copula function, \( \omega_0 \) is intercept term and \( \omega_1 \) is AR coefficient and \( \Gamma_t \) is the forcing variable to ensure that copula parameter is varying in its range.

As we mentioned earlier, our framework can be generalized further by allowing the underlying ARMA process for time varying copula to step into non-linear kink ARMA process. Thus, we rewrite (Equation 2.7) as.

\[
\rho_t = \Lambda(\omega_0 + \omega_1^- (\rho_{t-1} - \gamma) + \omega_1^+ (\rho_{t-1} - \gamma) + \omega_2 \Gamma_{t-10}),
\]

\[
\Lambda(\cdot) = \frac{2}{1 + \exp(-\rho)} - 1,
\]

where \( \omega_0 \) is intercept term and \( \omega_2 \) is MA coefficient. \( \omega_1^- \) and \( \omega_1^+ \) are the AR coefficients with respect to variable \( \rho_{t-1} \) for value of \( \rho_{t-1} \leq \gamma \) and with respect to variable \( \rho_{t-1} \) for value of \( \rho_{t-1} > \gamma \), respectively. Following [9], our time varying dependence is subject to regime-change at unknown kink variable \( \gamma \). These kink variable is compact and strictly in the interior of the support of \( \rho_{t-1} \). \( \Lambda(\cdot) \) is logistic transformation for keeping in \([-1,1]\) and \( \Gamma_t \) is the forcing variable which is written as

\[
\Gamma_t = \frac{1}{10} \sum_{j=1}^{10} F_1(U_{1,t-j})F_2(U_{2,t-j}).
\]

In this Elliptical copula case, it is a copula function which has symmetrical tail dependence where its dependence parameter in range \([-1,1]\). In this study, we consider two families of Elliptical copula, Gaussian and student-t copulas. According to [1] and [2], they are briefly presented below

The bivariate normal copula is defined by
\[ C_N(U_1, U_2; \rho) = \int_{-\infty}^{\Phi^{-1}(U_1)} \int_{-\infty}^{\Phi^{-1}(U_2)} \frac{1}{2\pi \sqrt{1-\rho^2}} \exp \left\{ -\frac{x_1^2 - 2\rho x_1 x_2 + x_2^2}{2(1-\rho^2)} \right\} dx_1 dx_2, \rho \in (-1,1), \quad (2.11) \]

where \( \rho \) is a time varying dependence parameter which is the linear correlation coefficient \( \rho \) restricted to the interval \((1,1)\). \( \Phi^{-1}(U_1) \) is the univariate standard normal cumulative distribution. The Gaussian copula is symmetric and has no tail dependence. Thus, its dynamic equation can be written as

\[ \rho_t = \Lambda(\omega_0 + \omega_1 (\rho_{t-1} - \gamma) + \omega_2 (\rho_{t-1} - \gamma), \omega_2 \cdot \frac{1}{10} \sum_{j=1}^{10} \Phi^{-1}(U_{1j}) \Phi^{-1}(U_{2j}), \quad (2.12) \]

The bivariate Student-t copula is defined by

\[ C_T(U_1, U_2; \rho) = \int_{-\infty}^{t^{-1}(U_1)} \int_{-\infty}^{t^{-1}(U_2)} \frac{1}{2\pi \sqrt{1-\rho}} \exp \left\{ \frac{x_1^2 + x_2^2 - 2\rho x_1 x_2}{\nu(1-\rho_t^2)} \right\}^{(\nu+2)/2} dx_1 dx_2, \rho \in (-1,1), \quad (2.13) \]

The Student-t Copula has a symmetric correlation coefficient \( \rho \) restricted to the interval \((-1,1)\) and symmetric tail dependence. \( t^{-1}(U) \) is the univariate standard student-t cumulative distribution with \( \nu \) degree of freedom. The Student-t copula can capture extreme dependence between variables. In addition, its dynamic equation can be written as

\[ \rho_t = \Lambda(\omega_0 + \omega_1 (\rho_{t-1} - \gamma) + \omega_2 (\rho_{t-1} - \gamma), \omega_2 \cdot \frac{1}{10} \sum_{j=1}^{10} T^{-1}(U_{1j}) T^{-1}(U_{2j}), \quad (2.14) \]

\[ v_t = \bar{A}(\omega^*_1 + \beta^*_1 v_{t-1} + \omega^*_2 \cdot \frac{1}{m} \sum_{j=1}^{m} T^{-1}(U_{1j}) T^{-1}(U_{2j}) \quad (2.15) \]

where \( \bar{A}(\cdot) = (\exp(v_t) / (1 + \exp(v_t))) \cdot 98 + 2 \) is the logistic transformation of \( \nu \). In this study, we ignore non-linear kink ARMA process in degree of freedom \( \nu \) in order to avoid complex models that would entail more challenging numerical maximizations of the likelihood function.

2.3. ARMA-GARCH for marginal distributions

To investigate the dependency between variables, the first step consists of modelling the margin of return series. Hence, we combine an ARMA (p, q) process with the generalized autoregressive conditional heteroscedasticity (GARCH) process of model of [11]. In this study, we employ a general ARMA (p, q)-GARCH (1, 1) model which can be written as

\[ y_t = c + \sum_{p=1}^{P} \phi_p y_{t-p} + \sum_{q=1}^{Q} \varphi_q \varepsilon_{t-q} + \sigma_i z_i \quad (2.16) \]

\[ \sigma^2_t = \alpha \varepsilon_{t-1} + \beta \sigma^2_{t-1}, \quad (2.17) \]

where \( y_t \) is the return, and \( \sigma^2_t \) is time varying volatility obtaining from the GARCH process in Equation (2.16), \( c, \phi_p \) and \( \varphi_q \) are a constant term of the conditional mean equation. It is quite obvious the structure of GARCH (1, 1) consisting with parameter \( \alpha \) and \( \beta \) are assumed to be greater than 1 and their summation must less than 1. \( \varepsilon_t = \sigma_i z_i \) is error term where \( z_i \) is a sequence of an i.i.d. random variable with zero mean and unit variance. The marginal distributions that we used to build a
joint bivariate distribution is assumed to be student-t distribution. Then, empirical cumulative distribution function (ECDF) is used to probability transformation.

3. Simulation study
In this experiment, the bivariate Gaussian and Student-t nonlinear time varying copula model of size N=1000, N=2000, and N=3000 is generated. We use to simulate our data set is given by

\[ \rho_t^G = \Lambda(0.5 + 1(|\rho_{t-1}^G - 0.5|)_+ + 2(|\rho_{t-1}^G - 0.5|)_+ + 0.8 \cdot \frac{1}{10} \sum_{j=1}^{10} \Phi^{-1}(U_{1,t-j}) \Phi^{-1}(U_{2,t-j}) , \]  

\[ \rho_t^T = \Lambda(0.5 + 1(|\rho_{t-1}^T - 0.5|)_+ + 2(|\rho_{t-1}^T - 0.5|)_+ + 0.8 \cdot \frac{1}{m} \sum_{j=1}^{m} T_v^{-1}(U_{1,t-j}) T_v^{-1}(U_{2,t-j}) . \]  

\[ \nu_j = \bar{\Lambda}(1-0.1\nu_{t-1}) + 0.8 \cdot \frac{1}{m} \sum_{j=1}^{m} T_v^{-1}(U_{1,t-j}) T_v^{-1}(U_{2,t-j}) . \]  

| Table 1. Simulation results. |
|---------------------------|
| Gaussian Copula Parameter | True | N=1000 | N=2000 | N=3000 |
| \( \omega_0 \) | 0.5 | 0.5301 (0.0703) | 0.4602 (0.0208) | 0.4667 (0.0171) |
| \( \omega_1 \) | 1 | 1.0794 (0.3364) | 0.9156 (0.2291) | 0.9717 (0.1202) |
| \( \omega_2 \) | 2 | 2.1826 (0.1833) | 2.2788 (0.0726) | 2.0416 (0.0716) |
| \( \gamma \) | 0.8 | 0.6592 (0.0524) | 0.6819 (0.0532) | 0.8246 (0.0458) |

| Student-t Copula Parameter | True | N=1000 | N=2000 | N=3000 |
|---------------------------|
| \( \omega_0 \) | 0.5 | 0.6994 (0.1311) | 0.2921 (0.0999) | 0.6886 (0.0634) |
| \( \omega_1 \) | 1 | 1.7106 (0.6839) | 0.7733 (0.3692) | 1.1759 (0.1236) |
| \( \omega_2 \) | 2 | 3.8798 (0.6723) | 1.7468 (0.4368) | 2.0951 (0.1125) |
| \( \gamma \) | 0.8 | 0.6423 (0.1649) | 0.9681 (0.1213) | 0.7688 (0.0506) |
| \( \omega_0 \) | 0.5 | 0.6881 (0.1854) | 0.4382 (0.0503) | 0.6269 (0.0096) |
| \( \omega_1 \) | 1 | 0.4126 (0.1158) | 1.4698 (0.1027) | 0.7010 (0.0891) |
| \( \omega_2 \) | -0.1 | -0.1004(0.0025) | -0.0912(0.0001) | -0.0908(0.0001) |
| \( \omega_0 \) | 0.8 | 1.0333 (0.0688) | 1.3331 (0.0660) | 1.0369 (0.0239) |

Note: standard error is ()

In this simulation study, we try to evaluate the performance of our proposed model and compare the results to the true values of each case. Table 1 shows the result of the Monte Carlo simulation our proposed model. We find that the estimated parameters are very close to their true values with satisfactory standard error, meaning that our proposed model could perform very well. Besides, that standard error decreases as the sample size increases. For both cases, the results are expected and the model has a better performance when the sample size increases. Furthermore, figure 1 presents the estimated time varying dependence parameters (red line) comparing with the true and true parameters (black line). We can find that our model is quite accurate in this simulation study. The black lines and red dot lines are close to each other in all sample sizes (N=1000, 2000, and 3000). Therefore, overall, the Monte Carlo simulation suggests that our proposed model is quite accurate.
Figure 1. Simulated time varying dependence parameters (black line) and estimated parameters (red line).

### 4. Empirical application

#### 4.1. Marginal distribution

| Table 2. Estimate result. |
|---------------------------|
|                         |
| **S&P500**               | **FTSE100**            |
| \( \phi_0 \)             | 0.0004 (0.0001)***     | 0.0003 (0.0001)***     |
| \( \text{ar1} \)         | 1.6137 (0.0085)***     | 1.3623 (0.0024)***     |
| \( \text{ar2} \)         | -0.6316 (0.0084)***    | -1.4027 (0.0027)***    |
| \( \text{ar3} \)         |                         | 0.8788 (0.0067)***     |
| \( \text{ar4} \)         |                         | 0.0161 (0.0031)***     |
| \( \text{ar5} \)         |                         | -0.0148 (0.0064)**     |
| \( \text{ma1} \)         | -1.6933 (0.000011)***  | -1.3975 (0.0001)***    |
| \( \text{ma2} \)         | 0.7042 (0.0001)***     | 1.4323 (0.0001)***     |
| \( \text{ma3} \)         |                         | -0.935 (0.000033)***   |
| \( \alpha_0 \)          | 0.000001 (0.000002)*** | 0.000001 (0.000001)**  |
| \( \beta_1 \)           | 0.0875 (0.0034)***     | 0.0969 (0.0072)***     |
| \( \beta_2 \)           | 0.9009 (0.0024)***     | 0.8906 (0.0071)***     |
| Log Likelihood          | 15406.93               | 15331.48               |

Note: *, **, *** denote significant at 90%, 95% and 99% respectively and () is standard error.

We apply model in analysis of top five stock exchange by market capitalization consist US Stock exchange (S&P 500) and London Stock Exchange (FTSE). The study used the daily data for period July 20, 2001 to February 2, 2018 and then transform the data to log-return. The data were collected from Thomson Reuters. Then, Stock exchange markets returns are modelled by AR- GJR-GARCH with skewed normal distribution. Table 2 reports ML estimates for the conditional marginal distributions of the daily log returns of all series. The degree of skewness of parameter \( \gamma \) are significant and high, these indicates that all returns have heavy tails and skew to the left. In addition,
the sum of ARCH and GARCH coefficients, $\beta_1$ and $\beta_2$, are quite close to unity and significant in all returns. These results indicate that S&P500 and FTSE to the s has long-run persistence and there are volatility shock effects in our returns.

4.2. Model comparison
This section, we discuss of the abilities of competing specifications of the dynamic copula process to capture the dependence structure of between US Stock market (S&P500) and London Stock market (FTSE 100). As we mentioned earlier, this study considers a Gaussian and Student-t copulas to capture the dependency of the returns. Table 3 shows the result of AIC and BIC for the comparison of various specifications. we can observe that nonlinear Gaussian is the appropriate copula for data since the lowest AIC and BIC are obtained.

|                | Linear ARMA of [7] | Kink ARMA | MS ARMA of [8] |
|----------------|--------------------|-----------|----------------|
|                | Gaussian           | Student-t | Gaussian       |
| AIC            | -129.27            | -1625.59  | -1618.796      |
| BIC            | -109.72            | -1593.3   | -1576.486      |

4.3. Estimation result
The estimated parameters of nonlinear time varying Gaussian copula, this model can capture a nonlinear effect of variables. We found that the model can present between the data set in which most of the parameters are the strictly sign and statistic significant. The result of nonlinear time varying gaussian copula shows in table 4. We can observe a different sign of effect of US. Stock exchange (S&P 500) and London Stock exchange (FTSE 100) in these two regimes. We can observe a different sign of effect between US. Stock exchange (S&P 500) and London Stock exchange (FTSE) in these two regimes.  In this study, we explain regime 1 and regime 2 as low and high in stock, respectively. The result of US. Stock exchange (S&P 500) and London Stock exchange (FTSE) shows negative coefficient is $-2.6511$ in regime 1 and positive coefficient is $2.9076$ in regime 2. Consider the kink ($\gamma$) point is 0.5523, we can observe that these exists a statistically significant kink effect.

| Parameter | Coefficient | S.E.   |
|-----------|-------------|--------|
| $\omega_0$| 1.0539***   | 0.0145 |
| $\omega^-$ (regime 1) | -2.6511*** | 0.1103 |
| $\omega^+$ (regime 2) | 2.9076*** | 0.0299 |
| $\omega_2$ | 0.1615***   | 0.0155 |
| $\gamma$ | 0.5523***   | 0.0023 |

Note: *, **, *** denote significant at 90%, 95% and 99% respectively
These findings are consistent with the plots of the nonlinear time varying Gaussian copula between S&P500 and FTSE 100 figure 2, which change over time around 0.55. While in figure 3 are plot of the linear, the dependence estimated parameters change around 0.18. These results indicate that the assumption of nonlinear time varying copula is more appropriate than linear time varying copula.

5. Conclusions
This study proposes a kink time varying copula to capture the structure change in the time varying dependence between variables. Before applying the model to the data, we proceed a simulation study to confirm the infallibility of our model. In the empirical study, we find that a nonlinear time varying Gaussian copula is chosen to be linked between S&P500 and FTSE100, indicating a nonlinear dynamic dependence in the real data. Future research could improve our methodological approach by considering to applied a KARMA process in the Archimedean copulas such as Clayton, Gumbel, Frank, and Joe copulas, which are typically used to capture asymmetry between lower and upper tail dependences. Thus, the model will gain more ability to capture the asymmetric dependence of the variables. Moreover, another possible extension that can be considered for future research is the nonlinear time varying multivariate copula. Finally, the further a self-tuning longitudinal slip ratio controller (LSC) based on the non-singular and fast terminal sliding mode (NFTSM) control method [12] can apply to our model in order to estimate and predict real-time dependence coefficient under the nonlinear context.

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