The quantum anomalous Hall effect on a star lattice with spin–orbit coupling and an exchange field

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Abstract

We study a star lattice with Rashba spin–orbit coupling and an exchange field and find that there is a quantum anomalous Hall effect in this system, and that there are five energy gaps at Dirac points and quadratic band crossing points. We calculate the Berry curvature distribution and obtain the Hall conductivity (Chern number \( \nu \)) quantized as integers, and find that \( \nu = -1, 2, 1, 2 \) when the Fermi level lies in these five gaps. Our model can be viewed as a general quantum anomalous Hall system and, in limit cases, can give what the honeycomb lattice and kagome lattice give. We also find that there is a nearly flat band with \( \nu = 1 \) which may provide an opportunity for realizing the fractional quantum anomalous Hall effect. Finally, the chiral edge states on a zigzag star lattice are given numerically, to confirm the topological property of this system.

(Some figures may appear in colour only in the online journal)

1. Introduction

The quantum Hall effect was observed [1] in 1980 in a two-dimensional electron system, in which the Hall conductivity takes quantized values \( \sigma_{xy} = \nu e^2/h \). The integer value \( \nu \) is called the TKNN number (for Thouless, Kohmoto, Nightingale and den Nijs) [2] or Chern number. The essential ingredient of Hall effect is to break the time-reversal symmetry of the system. Thus, introducing an external magnetic field is not the only way to produce this effect. In fact, anomalous Hall conductivity had been observed in ferromagnetic iron and ferromagnetic conductors since 1881 [3]. The internal magnetization plays an essential role in this so called anomalous Hall effect. It is natural to ask whether it is possible to produce the quantum anomalous Hall (QAH) effect without an external magnetic field.

A model constructed by Haldane demonstrated that the integral quantum Hall (IQH) effect can be realized without the Landau level induced by the external magnetic field [4]. In this spinless model on the honeycomb lattice, a staggered magnetic field was introduced to break the time-reversal symmetry, while the magnetic flux per unit cell is zero so no Landau levels are present. For this staggered magnetic field, in other words, the complex next-nearest hopping amplitude opens a gap at the Dirac point. At half-filling, when the Fermi level lies in this gap, the Chern number of this system takes the values \( \nu = \pm 1 \). Besides this toy model, several more realistic models which are based on various systems have been proposed, for example, Anderson insulators [5], the HgMnTe quantum well [6], optical lattices [7, 8], magnetic topological insulators [9], graphene [10, 11], and kagome lattices [12].

In this paper, we consider a star lattice model (see figure 1(a)) which has a close geometrical connection to the honeycomb lattice and the kagome lattice [13] and study its QAH effect. We calculate the Berry curvature in momentum space and find the Hall conductivity quantized as integer values when the Fermi level lies in the topologically nontrivial gap opened by Rashba spin–orbital coupling and the exchange field. Unlike in the proposals based on the honeycomb lattice and the kagome lattice [10–12], five gaps are opened due to the lattice structure complexity of the star lattice. Thus, in our model, there exist five Hall plateaus when the Fermi level lies...
in these gaps and there is a nearly flat band, with nonzero Chern number ($\nu = 1$), which may provide an opportunity to realize the fractional quantum anomalous Hall (FQAH) effect when electron interaction is introduced. We also make calculations using the zigzag ribbon and demonstrate that the excitation states are exactly local on the edge when the Fermi level lies in the bulk gap.

This paper is organized as follows. In section 2, we present the tight-binding Hamiltonian of our model. In section 3 we calculate the Berry curvature in crystal momentum space, and obtain the Hall conductivity. In section 4, we calculate the edge states on the zigzag ribbon. A conclusion is given in section 5.

2. The tight-binding Hamiltonian

The star lattice can be seen as replacing the sites of the honeycomb lattice by triangles, or replacing the sites of the kagome lattice by segments. The number of sites per unit cell in the star lattice is 6, three times the number in the honeycomb lattice and twice the number in the kagome lattice. In this paper, we focus on the nearest hopping term and assume that the hopping amplitude takes the same values $t_1$ in triangles and $t_2$ between triangles (figure 1). The nearest hopping tight-binding Hamiltonian is

$$H_0 = -t_1 \sum_{\langle i,j \rangle, \Delta} c_i^\dagger \sigma x j c_j - t_2 \sum_{\langle i,j \rangle, i \leftrightarrow j, \Delta} c_i^\dagger \sigma x j c_j + \text{h.c.},$$

where $\alpha$ is the spin index, $\Delta$ means ‘within triangle’ (all the gray bonds in figure 1(a)), and $\Delta \leftrightarrow \Delta$ means ‘between triangles’ (all the black bonds).

By writing this Hamiltonian in momentum space and diagonalizing the Hamiltonian matrix $H(k)$, the band structures are obtained, as shown in figures 2(a)–(c). These

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band structures demonstrate the close connections among the star lattice, honeycomb lattice and kagome lattice. When \( t_2 < \frac{1}{3} t_1 \), the probability for electrons hopping out of the triangle is smaller than that for hopping between triangles. This means that the three points are bound together strongly and can be seen as shrinking to one point. As a result, the low energy bands in this case look like graphene (see figure 2(a)). In contrast, when \( t_2 > \frac{1}{3} t_1 \), the low energy bands look like those of a kagome lattice (see figure 2(c)). We also notice that at \( t_2 = \frac{1}{3} t_1 \), the gap closes.

As shown in figure 2(a), a band gap already exists when only considering the nearest hopping. However, this is not a topological gap. In order to open topological gaps, we introduce Rashba spin–orbit coupling and an exchange field as follows:

\[
H_{\text{RSO}} = i RSO \sum_{\langle ij \rangle, \alpha \beta} (\tilde{\sigma}_{\alpha \beta} \times \hat{d}_j) \gamma_3 c_{i \alpha} + \text{h.c.}, \tag{2a}
\]

\[
H_{\lambda} = \lambda \sum_{i, \alpha} c_{i \alpha}^\dagger c_{i \alpha} \sigma^z_{\alpha}, \tag{2b}
\]

where \( t_{\text{RSO}} \) is the strength of the spin–orbit coupling, \( \tilde{\sigma} \) are the vector Pauli matrices in spin space, \( \hat{d}_j \) is the unit vector pointing from site \( j \) to \( i \), and \( \lambda \) is the strength of the exchange field.

So, the total Hamiltonian is

\[
H = H_0 + H_{\text{RSO}} + H_{\lambda}. \tag{3}
\]

The band evolution of this Hamiltonian is shown in figure 2. Without Rashba spin–orbit coupling and an exchange field, the six-site unit cell forms six bands which are doubly degenerate. With only Rashba spin–orbit coupling \( H_{\text{RSO}} \) added, the spin degeneracy is lifted, except at a few \( k \) points. The exchange field alone has a similar effect, but leaves degeneracy at different \( k \) points. And band gaps can be opened when both interactions are added into the hopping Hamiltonian. In the following, we will confirm that these band gaps are topologically nontrivial and that they can realize the quantum anomalous Hall effect.

3. The Berry curvature and the Chern number

The Berry curvature in crystal momentum space is an essential concept and should be included when considering a solid state system [14]. Here, the intrinsic Hall conductivity can be written as the summation of the Berry curvatures of all bands under the Fermi level [15] as follows:

\[
\sigma_{xy} = \frac{e^2}{h} \frac{2\pi}{N V} \sum_{k, E_n \leq E_F} \Omega_z(E_n, k), \tag{4}
\]

where \( N \) is the number of primitive unit cells, and \( V \) is the volume of the unit cell. The \( n \)th band’s Berry curvature \( \Omega_z(E_n, k) \) can be obtained from

\[
\Omega_z(E_n, k) = \sum_{E_m(\neq E_n)} \left| \langle \psi_{nk} | \partial H(k) / \partial k_\lambda | \psi_{mk} \rangle \langle \psi_{mk} | \partial H(k) / \partial k_\lambda | \psi_{nk} \rangle \right| ^2 / (E_n - E_m)^2. \tag{5}
\]

The Chern number is given by

\[
\nu = \frac{2\pi}{N V} \sum_{k, E_n(\leq E_F)} \Omega_z(E_n, k), \tag{6}
\]

or

\[
\nu = \frac{1}{2\pi} \int_{E_n(\leq E_F)} \int_{BZ} 2 d^2 k \Omega_z(E_n, k), \tag{7}
\]

when \( N \to \infty \).

Writing the Hamiltonian equation (3) in momentum space and calculating the Hall conductivity by using equation (4), we get the Hall conductivity \( \sigma_{xy} \) as a function of the Fermi level \( E_F \), as shown in figure 3(a). Five Hall plateaus are obtained when the Fermi level lies in the gaps (as suggested by the density of states shown in figure 3(b)). The Chern number takes quantized values \( \nu = -1, 2, 1, 2, 0 \).

The two \( \nu = 2 \) plateaus come from the gaps opened at Dirac points, which are similar to those in a honeycomb lattice [10] and a kagome lattice [12]. The two \( \nu = 1 \) plateaus come from the gaps opened at quadratic band crossing points (QBCP), which are similar to those in a kagome lattice [12]. However, there is only one \( \nu = 1 \) plateau in the kagome lattice. What is more, one of these nonzero Chern numbers belongs to the nearly flat band. This novel property provides great potential for realizing the fractional quantum anomalous Hall effect in the star lattice. The \( \nu = -1 \) plateau also comes from the gap opened at the Dirac point. But this gap will
Figure 4. The total Berry curvature distribution in momentum space $\Omega_z(k) = \sum_{E_n \leq E_F} \Omega_z(E_n, k)$ for a star lattice with $t_2 = t_1$, $t_{\text{RSO}} = 0.2t_1$, and $\lambda = 0.2t_1$, when the Fermi level lies at (a) $E_F = -2.77$, (b) $E_F = -2.32$, (c) $E_F = -0.21$, (d) $E_F = 0.36$, (e) $E_F = 1.27$ and (f) $E_F = 1.0$.

disappear when $\frac{t_2}{t_1}$ increases. The reason that $\nu = -1$ can appear in our model is that the bands of the star lattice (figure 2) are narrower than those of the honeycomb lattice and kagome lattice. This means that a direct gap can be opened above the lowest band. So, when $\frac{t_2}{t_1}$ increases, the bands become wider, the direct gap becomes an indirect gap, and the Hall conductivity becomes not quantized.

To further confirm the above interpretation, we plot the total Berry curvature of the bands below the Fermi level $\Omega_z(k) = \sum_{E_n \leq E_F} \Omega_z(E_n, k)$. When the Fermi level lies in the gaps, peaks at the minimal energy difference points are clearly seen (figures 4(a)–(e)). Also, in figure 4(f), we present the Berry curvature distribution for when the Hall conductivity does not take quantized values.

4. The chiral edge states

The nonzero Chern number is also manifested in the presence of chiral edge states which localize at edges and propagate in one direction. Now, we study the edge state property on the zigzag star lattice ribbon (figure 5). A periodic boundary condition is taken along the $y$ direction (parallel to the edge). An open boundary condition is taken along the $x$ direction. The band structure of a ribbon with 324-site width is shown in figure 6.

To further demonstrate the topological property of these gaps, we calculate the edge state functions corresponding to the points marked on the edge bands (figure 6). The probability density quickly decreases on going into the bulk. The group velocity $\frac{\partial E(k)}{\partial k}$ suggests that these edge states propagate in different directions at different edges (figure 7). So chiral edge states appear when the Fermi level lies in the topologically nontrivial bulk gaps. The numbers of these edge
Figure 5. Illustration of a zigzag star lattice ribbon which is infinite in the y direction and has width $W$ in the x direction. The unit cell is indicated by the dashed lines.

Figure 6. The band structure of a $W = 324$ width zigzag star lattice ribbon with $t_2 = t_1$, $t_{\text{RESO}} = 0.2t_1$, and $\lambda = 0.2t_1$. Edge bands appear in the topologically nontrivial gaps. The points A–D indicate the edge states whose probability along the x direction will be presented in figure 7. The red/blue color indicates positive/negative group velocity along the y direction.

5. Conclusion

In summary, we have studied the quantum anomalous Hall effect on the star lattice in the presence of both a Rashba spin–orbit effect and an exchange field. The Chern number calculated from the Berry curvature in momentum space and the edge states calculated on the zigzag ribbon confirm the emergence of a QAH effect when the Fermi level lies in the topological gaps. Due to the complex lattice structure of the star lattice, our results include those obtained for the honeycomb lattice which has Chern number $\nu = 2$ and the kagome lattice which has Chern number $\nu = 1$ or 2. What is more, a nearly flat band with nonzero Chern number ($\nu = 1$) also appears in our model, which provides great potential for realizing the fractional quantum Hall effect without tuning the ratio between nearest and next-nearest hopping amplitudes [16–18].

Finally, the physical realization of the star lattice would not be very difficult, considering that its two cousins, the honeycomb lattice and the kagome lattice, have been realized in optical lattices [19, 20]. We also notice that spin–orbit coupling has also been realized in optical lattices [21]. Both of these features offer the possibility of realizing a QAH effect in a star optical lattice. In the solid state system, a polymeric
iron(III) acetate with an underlying star lattice has also been reported [22].

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