Revisiting the top quark chromomagnetic dipole moment in the SM

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We revisit the anomalous chromomagnetic dipole moment in the Standard Model and show that its triple gluon vertex contribution, with the on-shell gluon ($q^2 = 0$), generates an infrared divergent pole. Consequently, the chromomagnetic dipole should not be perturbatively evaluated at $q^2 = 0$. Focusing on this top quark anomaly, denoted as $\hat{\mu}_t(q^2)$, we compute it with the off-shell gluon at a large momentum transfer, just as the $\alpha_s(m_Z^2)$ convention scale, for both spacelike $q^2 = -m_Z^2$ and timelike $q^2 = m_Z^2$ cases. We found $\hat{\mu}_t(-m_Z^2) = -0.0224-0.000925i$ and $\hat{\mu}_t(m_Z^2) = -0.0133-0.0267i$. Our $Re\hat{\mu}_t(-m_Z^2)$ matches well with the current experimental value $\hat{\mu}_t^{\exp} = -0.024^{+0.013}_{-0.009}(\text{stat})^{+0.016}_{-0.011}(\text{syst})$ and the Im $\hat{\mu}_t(-m_Z^2)$ part is induced by flavor changing charged currents.

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I. INTRODUCTION

The top quark anomalous chromomagnetic dipole moment (CMDM) has been recently measured by the CMS Collaboration at the Large Hadron Collider (LHC), by using $pp$ collisions at the center-of-mass energy of 13 TeV with an integrated luminosity of 35.9 fb$^{-1}$ [1]. In specific, they reported

$$\hat{\mu}_t^{\text{Exp}} = -0.024^{+0.013}_{-0.009}(\text{stat})^{+0.016}_{-0.011}(\text{syst}),$$

whereas for the chromoelectric dipole moment (CEDM)

$$|\hat{d}_t^{\text{Exp}}| < 0.03,$$

at 95% C. L.

In contrast, in the Standard Model (SM) the CMDM is induced at the one-loop level, it receives contributions from both quantum chromodynamics (QCD) and electroweak (EW) sectors. A peculiar feature of this property concerns the existence of an infrared (IR) divergence generated by the Feynman diagram coming from the QCD non-Abelian triple gluon vertex. This issue occurs when the gluon momentum transfer $q$ of the external gluon is on-shell, $q^2 = 0$, which has been pointed out in the Refs. [2-3]. The authors in Ref. [2] were the first in to show the presence of that IR divergence when the gluon is on-shell; they employed the Feynman parameterization (FP) method and realized that the corresponding calculation reported as finite in Ref. [5], through the same method, is incorrect. Nonetheless, this ill result has been considered as the correct SM prediction by the community [6-10]. Also, in Ref. [3], based on the integration-by-parts technique [11], it was indicated the same divergence issue.

In this work we take the divergence discussion one step further, here, we will show, by using dimensional regularization (DR), the IR nature of that divergence by displaying its $1/\epsilon_{IR}$ infrared pole [12-20], which comes from a two-point Passarino-Veltman scalar function (PaVe), identified as $B_0(q^2,0,0)$, when $q^2 = 0$. Consequently, in the context of perturbative QCD (pQCD) it is not suitable to evaluate the CMDM with the on-shell gluon, hence, in pQCD it is not possible to establish a faithful analogy of the CMDM with the quantum electrodynamics (QED) static anomalous magnetic dipole moment defined with the on-shell photon $q^2 = 0$. Because of this, in the Ref. [2] it was proposed to evaluate the CMDM at a large gluon momentum transfer, $q^2 = -m_Z^2$. This choice is justified since in pQCD the strong running coupling constant is characterized at that conventional scale, $\alpha_s(Q^2 = -q^2 = m_Z^2) = 0.1179$ [21], which depends on the momentum transfer $Q^2 = -q^2$, where $q$ is the four-momentum flow of the process; it is established in the spacelike domain $Q^2 > 0$, implying $q^2 < 0$ [22,23]. Although the perturbative $\alpha_s$ is conceived in the

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spacelike regime $q^2 < 0$, its value is indistinctly used in the strong interaction processes, for example, in $\bar{t}t$ production at the LHC where the top quark chromodipoles are assumed in general as complex quantities in the timelike domain $q^2 > 0$. Motivated by this we will evaluate the $\hat{\mu}_q(q^2)$ at $q^2 = -m_Z^2$ and at $q^2 = m_Z^2$, in advance, we have found that both evaluations give rise to complex quantities, of which the Re $\hat{\mu}_q(\pm m_Z^2)$ parts are within the measured statistical error [1], but in particular, our Re $\hat{\mu}_q(\pm m_Z^2)$ matches quite well with the central value. In the Table III from Ref. [4], some of us have already published our numerical results for the CMDM of the top quark in the SM, but without providing details that now in this work we deepen.

Another manner to deal with the IR divergence issue, that we also address in this work, is to apply the FP method [28], but without omitting the $+\varepsilon$ Feynman prescription of the propagators, this in order to analytically show that the gluon propagator in the Feynman-'t Hooft gauge $\xi = 1$ and in the general renormalizable $R_\varepsilon$ gauge leads to the same logarithmic IR divergence when $\varepsilon \to 0$. Furthermore, as it was done in Ref. [2], we also implement the massive gluon propagator artifice, but we do provide an analytical expression to prove that the logarithmic IR divergence arise when $m_g \to 0$.

The outline of this paper is as follows. In the Sec. II is presented the general effective chromoelectromagnetic dipole moment Lagrangian. In the Sec. III the six one-loop diagram contributions to the CMDM of the top quark are calculated. In the Sec. IV the numerical results for the CMDM of the top quark are discussed. The Sec. V is devoted to our conclusions. In the Appendix A are presented the needed Feynman rules and the used input values. In the Appendix B are listed the resulting PaVes $A_0$, $B_0$ and $C_0$. In the Appendix C is performed the DR of the IR divergent two-point PaVe from the CMDM no-Abelian diagram with the on-shell gluon. In the Appendix D the IR divergence issue is addressed by the FP method and also by the massive gluon artifice.

II. THE CHROMOMAGNETIC DIPOLE MOMENT

\begin{equation}
\mathcal{L}_{q\bar{q}g} = -g_s \bar{q}_A \gamma^\mu q_B g_\mu^a T_{AB}^a + \mathcal{L}_{\text{eff}},
\end{equation}

being

\begin{equation}
\mathcal{L}_{\text{eff}} = -\frac{1}{2} \bar{q}_A \sigma^{\mu\nu} (\mu_q + i d_q) q_B G_{\mu\nu}^a T_{AB}^a,
\end{equation}

where $T_{AB}^a$ is the color generator of $SU(3)_C$ ($A$ and $B$ are color indices), $\sigma^{\mu\nu} \equiv \frac{i}{2} [\gamma^\mu, \gamma^\nu]$, $\mu_q$ is the CP-conserving chromomagnetic form factor, $d_q$ is the CP-violating chromoelectric (CEDM) form factor and $G_{\mu\nu}^a = \partial_\mu g_\nu^a - \partial_\nu g_\mu^a - g_s f_{abc} q_\mu g_\nu^b g_\nu^c$ is the gluon strength field, whose last term is not involved in the CMDM calculated below. In the SM the CMDM is induced perturbatively as a quantum fluctuation or radiative correction at the one-loop level [2, 3, 4], while the CEDM arises at the three-loop level [27, 29]. Because the $\mathcal{L}_{\text{eff}}$ have mass dimension 5, it is more suitable to define the dipoles dimensionless [21, 26, 27, 29] as

\begin{equation}
\hat{\mu}_q \equiv \frac{m_q}{g_s} \mu_q \quad \hat{d}_q \equiv \frac{m_q}{g_s} d_q,
\end{equation}

where $m_q$ is the quark mass, $g_s = \sqrt{4\pi \alpha_s}$ is the QCD group coupling constant, being $\alpha_s$ the perturbative strong coupling constant, characterized at the mass of the $Z$ gauge boson, $\alpha_s(m_Z^2) = 0.1179$ [21]. In general, the CEMDM are complex quantities, they may have absorptive imaginary parts, for example, when the momentum transfer is
The complete evaluation of the CMDM, coming from Eq. (9), will be addressed in the Sec. IV. In our analytical and numerical study we have used the software \textit{Mathematica}, \textit{FeynCalc} \cite{31,32}, \textit{FeynHelpers} \cite{33}, \textit{Package-X} \cite{34},
A. The $\gamma$ diagram

The photon Schwinger-type diagram is illustrated in the Fig. 2(a), where its tensor amplitude is

$$\mathcal{M}_q^\mu(\gamma) = \mu^2 \int \frac{d^Dk}{(2\pi)^D} \bar{u}(p') \left[ \frac{-ieQ_q}{k + p'} \delta_{AC_1} \right] \left[ \frac{i}{(k + p')^2 - m_{q_i}^2 + i\epsilon} \delta_{C_1C_2} \right] \left[ \frac{-ig_{sA_2}T_{\alpha_2}^aC_3}{k^2 + i\epsilon} \right] \left( \frac{-g/2 - 1}{k^2 + i\epsilon} \right) \right] \left[ \frac{\overline{u}(p)}{i} \right],$$

with $\delta_{AC_1}\delta_{C_1C_2}T_{\alpha_1}^{\alpha_2}T_{\alpha_2}^aT_{\alpha_3}^b = T_{\alpha_3}^b$, where a sum over repeated indices is assumed.

After algebraic manipulations, the resulting part of the CMDM with the off-shell gluon ($q^2 \neq 0$) is

$$\hat{\mu}_q(\gamma) = \frac{-\alpha Q_{q_i}^2 m_{q_i}^2}{2\pi(q^2 - 4m_{q_i}^2)} (B^\gamma_{01} - B^\gamma_{02})$$

$$= \frac{\alpha Q_{q_i}^2 m_{q_i}^2}{2\pi\sqrt{q^2 - 4m_{q_i}^2}} \sqrt{q^2 - 2m_{q_i}^2 - q^2},$$

(11)

where $B^\gamma_{01} \equiv B_0(m_{q_i}, 0, m_{q_i}^2)$ and $B^\gamma_{02} \equiv B_0(q^2, m_{q_i}^2, m_{q_i}^2)$; the explicit form of the PaVes can be consulted in the Appendix B.

i) On-shell gluon case ($q^2 = 0$): here $\hat{\mu}_q(\gamma)$ is a constant, with $B^\gamma_{02} = B_0(0, m_{q_i}^2, m_{q_i}^2)$, therefore

$$\hat{\mu}(\gamma) = \frac{\alpha}{9\pi}.$$  

(12)

An analogous behaviour will occur for the gluon-Schwinger type diagram $\hat{\mu}_q(g)$ (see Fig. 2(e)), this will be shown in the Sec. III E.

ii) Off-shell gluon case ($q^2 = \pm m_Z^2$): from Eq. (11) it can be noticed that $\hat{\mu}_q(\gamma) \propto m_{q_i}^2$, which provides a large value for the top quark.

The resulting evaluations are listed in the Table I and the general behavior of $\hat{\mu}(\gamma)$ is shown in the Fig. 3(a).

B. The $Z$ gauge boson diagram

The $Z$ gauge boson contribution is shown in the Fig. 2(b), the respective tensor amplitude is

$$\mathcal{M}_q^\mu(Z) = \mu^2 \int \frac{d^Dk}{(2\pi)^D} \bar{u}(p') \left[ \frac{-ig_{V_1}}{2c_{\gamma_5}} \gamma^\alpha (g_{V_q} - g_{A_q} \gamma_5) \delta_{AC_1} \right] \left[ \frac{i}{(k + p')^2 - m_{q_i}^2 + i\epsilon} \delta_{C_1C_2} \right] \left[ \frac{-ig_{sA_2}T_{\alpha_2}^aC_3}{k^2 + i\epsilon} \right] \left( \frac{-g/2 - 1}{k^2 + i\epsilon} \right) \right] \left[ \frac{\overline{u}(p)}{i} \right],

with the same color algebra as in Eq. (11).
The off-shell gluon \((q^2 \neq 0)\) CMDM induced by the \(Z\) neutral gauge boson is

\[
\hat{\mu}_{q_i}(Z) = \frac{\alpha}{8 \pi c_1^2 s_1^2 m_{Z_i}^2 (q^2 - 4 m_{W_i}^2)^2} \left( g_{V_{q_i}}^2 \left( m_{Z_i}^2 \left( q^2 - 4 m_{W_i}^2 \right) \left( A_{01}^Z - A_{02}^Z + m_{W_i}^2 \right) \right) + m_{W_i}^2 \left[ 8 m_{q_i}^4 - 2 m_{q_i}^2 \left( 5 m_{Z_i}^2 + q^2 \right) + 2 m_{q_i} q^2 \right] B_{01}^Z \right) + m_{q_i}^2 \left( 2 m_{q_i}^2 + m_{Z_i}^2 \right) \left( q^2 - 4 m_{W_i}^2 \right) (A_{01}^Z - A_{02}^Z + m_{W_i}^2) B_{02}^Z + 2 m_{q_i}^2 m_{W_i}^2 \left( -8 m_{q_i}^2 + 3 m_{W_i}^2 + 2 q^2 \right) C_{01}^Z + g_{A_{q_i}} \left( 2 m_{q_i}^2 + m_{Z_i}^2 \right) (q^2 - 4 m_{W_i}^2) (A_{01}^Z - A_{02}^Z + m_{W_i}^2) + m_{q_i}^2 \left[ 8 m_{q_i}^4 - 2 m_{q_i}^2 \left( 12 m_{Z_i}^2 + q^2 \right) + 6 m_{Z_i}^2 + 9 m_{W_i}^2 q^2 \right] B_{01}^Z + m_{q_i}^2 \left( 20 m_{q_i}^4 - 2 m_{q_i}^2 \left( 5 m_{Z_i}^2 + 4 q^2 \right) + 2 m_{Z_i}^2 q^2 \right) B_{02}^Z + 2 m_{q_i}^2 m_{Z_i}^2 \left[ 24 m_{q_i}^4 - 2 m_{q_i}^2 \left( 9 m_{Z_i}^2 + 7 q^2 \right) + 3 m_{Z_i}^2 + 2 \left( 3 m_{Z_i}^2 + q^2 \right) q^2 \right] C_{01}^Z \right),
\]

with \(A_{01}^Z \equiv A_0 \left( m_{Z_i}^2 \right), A_{02}^Z \equiv A_0 \left( m_{Z_i}^2 \right), B_{01}^Z \equiv B_0 \left( m_{q_i}^2, m_{Z_i}^2, m_{Z_i}^2 \right), B_{02}^Z \equiv B_0 \left( q^2, m_{q_i}^2, m_{Z_i}^2 \right)\) and \(C_{01}^Z \equiv C_1 \left( m_{q_i}^2, m_{q_i}^2, q^2, m_{Z_i}^2, m_{Z_i}^2, m_{q_i}^2 \right);\) because the analytical formula for \(C_{0i}^Z\) with the off-shell gluon is very long, we present a suitable approximation (see Appendix [B]).

i) On-shell gluon case \((q^2 = 0)\): here \(B_{02}^Z \equiv B_0 \left( 0, m_{q_i}^2, m_{Z_i}^2 \right)\) and \(C_{0i}^Z \equiv C_1 \left( m_{q_i}^2, m_{q_i}^2, q^2, m_{Z_i}^2, m_{Z_i}^2, m_{q_i}^2 \right)\);

ii) Off-shell gluon case \((q^2 = \pm m_{Z_i}^2)\): its \(C_{0i}^Z\) function is given in Eq. \([13,15]\).

The numerical values of this contribution to the CMDM are given in the Table [I] and its behavior as a function of the gluon momentum transfer is showed in the Fig. [I, b].

### C. The W gauge boson diagram

The \(W\) gauge boson contribution (see Fig. [2c]) gives rise to the following tensor amplitude

\[
\mathcal{M}_{q_i}^W(W) = \mu_{e} \int \frac{d^D k}{(2\pi)^D} u(p') \left( \frac{i g}{\sqrt{2}} \gamma^{\alpha_1} P_L V_{q_i,q_i}^{*} \delta_{AC_1} \right) \left( i \frac{\not{k} + \not{p}' + m_{q_i}}{(k + p')^2 - m_{q_i}^2} + \frac{\delta_{C_1 C_2}}{i \varepsilon} \right) \left( -i g_{\alpha_2} T^{\alpha_2}_{\gamma} C_3 \right) \times \left( i \frac{\not{k} + \not{p} + m_{q_i}}{(k + p)^2 - m_{q_i}^2} + \frac{\delta_{C_1 C_2}}{i \varepsilon} \right) \left( -i g_{\alpha_2} P_L V_{q_i,q_i}^{*} \delta_{C_4 B} \right) u(p, m_{q_i}) \times \frac{i}{k^2 - m_{W}^2 + i \varepsilon} \left( -g_{\alpha_1 \alpha_2} + \frac{k_{\alpha_1} k_{\alpha_2}}{m_{W}^2} \right),
\]

being \(q_1, q_2, q_3 = d, s, b\), where the same color algebra as in Eq. \([10]\) applies.

The off-shell gluon \((q^2 \neq 0)\) CMDM induced by the \(W\) gauge boson is composed by

\[
\hat{\mu}_{q_i}(W) = \sum_{j=1}^{3} \hat{\mu}_{q_i}(W, q_j)
\]

\[
= \frac{\alpha}{16 \pi c_1^2 s_1^2 m_{W_i}^2 (q^2 - 4 m_{W_i}^2)^2} \left( (q^2 - 4 m_{W_i}^2) \left( m_{W_i}^2 + m_{q_j}^2 + 2 m_{W_i}^2 \right) \left( A_{01}^W - A_{02}^W + m_{q_j}^2 \right) + \left( -m_{q_i}^2 \left( 2 m_{q_j}^2 + \left( 8 m_{q_j}^2 - 18 m_{Z_i}^2 + q^2 \right) \right) + m_{q_i}^2 \left( 2 m_{q_j}^2 \left( 5 m_{q_j}^2 + (5 m_{W_i}^2 + q^2) \right) - m_{W_i}^2 \left( 20 m_{W_i}^2 + 9 q^2 \right) \right) - q^2 \left( m_{q_i}^2 + m_{W_i}^2 \left( m_{q_i}^2 + 2 m_{W_i}^2 \right) \right) B_{01}^W \right) + m_{q_i}^2 \left( 2 m_{q_j}^2 + \left( 12 m_{q_j}^2 - 22 m_{W_i}^2 + q^2 \right) \right) \right) - m_{q_i}^2 \left( 6 m_{q_j}^2 + 3 \left( 2 m_{W_i}^2 + q^2 \right) + 2 m_{W_i}^2 \left( 6 m_{W_i}^2 + 5 q^2 \right) \right) B_{02}^W \left( m_{q_i}^2 \left( 7 m_{q_j}^2 + 2 \left( q^2 - 6 m_{W_i}^2 \right) + m_{W_i}^2 \left( 17 m_{W_i}^2 + 8 q^2 \right) \right) \right) - m_{q_i}^2 \left( m_{W_i}^2 \left( 3 m_{q_j}^2 + q^2 \right) - m_{W_i}^2 \left( 9 m_{W_i}^2 + 6 q^2 \right) - 2 m_{W_i}^2 \left( m_{W_i}^2 + q^2 \right) \left( 3 m_{W_i}^2 + q^2 \right) \right) C_{0i}^W \right),
\]

where \(A_{01}^W \equiv A_0 \left( m_{q_i}^2 \right), A_{02}^W \equiv A_0 \left( m_{W_i}^2 \right), B_{01}^W \equiv B_0 \left( m_{q_i}^2, m_{q_j}^2, m_{W_i}^2 \right), B_{02}^W \equiv B_0 \left( q^2, m_{q_j}^2, m_{W_i}^2 \right), C_{0i}^W \equiv C_0 \left( m_{q_i}^2, m_{q_j}^2, m_{q_j}^2, m_{W_i}^2, m_{W_i}^2 \right).\)
i) On-shell gluon case \((q^2 = 0)\): here \(B_0^W \equiv (0, m_{q_1}^2, m_{q_2}^2)\) and \(C_0^W \equiv C_0(m_{q_1}^2, m_{q_2}^2, 0, m_{q_1}^2, m_{W}^2, m_{q_2}^2)\).

ii) Off-shell gluon case \((q^2 = \pm m_Z^2)\): since the \(C_0^W\) solution is extremely long, instead, we provide the FP formula for its numerical evaluation (see Eq. (B0)).

In the Table I are listed the resulting values for the CMDM, where it should be noted that it has real and imaginary parts. The Fig. 3(c) shows the real part and the Fig. 3(d) the imaginary one.

### D. The Higgs boson diagram

The Higgs boson contribution (see Fig. 2(d)) generates the tensor amplitude

\[
\mathcal{M}_q^\mu = \mu^{2e} \int \frac{d^2k}{(2\pi)^2} \frac{\delta ABC}{2m_W^2} \right[ \frac{i}{(k + p')^2 - m_{q_i}^2 + i\varepsilon} \delta_{C_iC_2} \left( -ig_s\gamma^\mu T_{C_2}^{aC} \delta_{C_1C_3} \right) \right] \right] (17)
\]

Again, the color algebra is the same as in Eq. (10).

The off-shell gluon \((q^2 \neq 0)\) CMDM generated by the Higgs boson is

\[
\hat{\mu}_q(H) = \frac{\alpha m_{\gamma}^2}{16\pi m_{\gamma}^2 s_W^2 (q^2 - 4m_{q_i}^2)} \left\{ (q^2 - 4m_{q_i}^2) (A^H_{01} - A^H_{02} + m_{q_i}^2) \right. \right.
\]

\[
+ [16m_{q_i}^4 - 2m_{q_i}^2 (5m_{\gamma}^2 + 2q^2) + m_{H}\tilde{q}^2] B^H_{01} + 3m_{q_i}^2 (-4m_{q_i}^2 + 2m_H^2 + q^2) B^H_{02}
\]

\[
+ 6m_{q_i}^2 m_H^2 (-4m_{q_i}^2 + m_H^2 + q^2) C^H_0 \right\},
\]

with \(A^H_{01} \equiv A_0(m_{q_i}^2), A^H_{02} \equiv A_0(m_H^2), B^H_{01} \equiv B_0(m_{q_i}^2, m_{q_i}^2, m_{q_i}^2), B^H_{02} \equiv B_0(q^2, m_{q_i}^2, m_{q_i}^2),\) and \(C^H_0 \equiv C_0(m_{q_i}^2, m_{q_i}^2, m_{q_i}^2, m_{q_i}^2, m_{q_i}^2, m_{q_i}^2).\)

i) On-shell gluon case \((q^2 = 0)\): for it \(B^H_{02} \equiv B_0(0, m_{q_i}^2, m_{q_i}^2)\) and \(C_0^H \equiv C_0(m_{q_i}^2, m_{q_i}^2, 0, m_{q_i}^2, m_{q_i}^2, m_{q_i}^2).\)

ii) Off-shell gluon case \((q^2 = \pm m_Z^2)\): the \(C^H_0\) analytical approximation for this situation is given in Eq. (15).

All the resulting values are listed in the Table I. The plot of this contribution can be seen in Fig. 3(c).

### E. The \(g\) diagram

The gluon Schwinger-type diagram (see Fig. 2(e)) offers this tensor amplitude

\[
\mathcal{M}^\mu_{\gamma} = \mu^{2e} \int \frac{d^2k}{(2\pi)^2} \frac{\delta ABC}{2m_W^2} \left[ \frac{i}{(k + p')^2 - m_{q_i}^2 + i\varepsilon} \delta_{C_iC_2} \left( -ig_s\gamma^\mu T_{C_2}^{aC} \delta_{C_1C_3} \right) \right] \right] (19)
\]

where \(T_{AC_1}^{aC} \delta_{C_2C_3} T_{C_4}^{aC} \delta_{C_5C_6} = T_{A_1C_1}^{aC} T_{C_2}^{aC} T_{C_4}^{aC} T_{C_5}^{aC} = (T_{A_1}^{aC} T_{aC}^{aC})_{AB} = (C_F - \frac{1}{2} C_A) \tau^a_{AB} = -\frac{1}{6} \tau^a_{AB}, C_A = N = 3, C_F = (N^2 - 1)/2N = 4/3.\)

The resulting off-shell gluon CMDM is

\[
\hat{\mu}_{\gamma}(g) = \frac{\alpha_s m_{\gamma}^2}{12\pi (q^2 - 4m_{q_i}^2)} \left( B_{01}^g - B_{02}^g \right)
\]

\[
= -\frac{\alpha_s m_{\gamma}^2}{12\pi \sqrt{q^2 (q^2 - 4m_{q_i}^2) / 2m_{q_i}^2}} \ln \frac{\sqrt{q^2 (q^2 - 4m_{q_i}^2) + 2m_{q_i}^2 - q^2}}{2m_{q_i}^2},
\]

being \(B_{01}^g \equiv B_0(0, m_{q_i}^2, 0, m_{q_i}^2), B_{02}^g \equiv B_0(q^2, m_{q_i}^2, m_{q_i}^2)\). As already commented, this virtual gluon case is entirely analogous to the \(\gamma\) Schwinger-type case from Sec. IIIA.

i) On-shell gluon case \((q^2 = 0)\): here results the constant

\[
\hat{\mu}_{\gamma}(g) = -\frac{\alpha_s}{24\pi}.
\]
ii) Off-shell gluon case \((q^2 = \pm m_{\top}^2)\): from Eq. (20) is clear that \(\hat{\mu}_q(g) \propto m_{\top}^2\), which yields a large contribution for the top quark CMDM.

The respective numerical evaluations are listed in the Table I and the corresponding CMDM behavior is presented in the Fig. (3f).

F. The 3g diagram

The triple gluon vertex diagram, characterized by being the only non-Abelian contribution to the CMDM, contains an IR divergence when the gluon is on-shell that the previous literature has not properly addressed, this is why we treated it in details. The associated Feynman diagram is depicted in Fig. (2f), whose tensor amplitude is written as

\[
\mathcal{M}_{\tilde{q},i}^a(3g) = \mu^2 \int \frac{d^D k}{(2\pi)^D} \bar{u}(p') (-ig_s \gamma^\alpha T_A^{a_1}) \left( i \frac{k + m_{\tilde{q}_i}}{k^2 - m_{\tilde{q}_i}^2 + i\varepsilon} \delta_{C_1 C_2} \right) (-ig_s \gamma^\alpha T_{C_2 B}^{a_4}) u(p)
\]

\[
\times \left[ \frac{i}{(k - p')^2 + i\varepsilon} \right] \left[ -g_{\alpha a_2 a_4} T_{999}^{\alpha a_2 a_4}(p' - p, -k + p, k - p') \right] \times \left[ \frac{i}{(k - p)^2 + i\varepsilon} \right] \left[ -g_{\alpha a_3 a_4} \right],
\]

\[
\mathcal{M}_{\tilde{q},i}^a(3g) = -\frac{3g^3}{2T_{AB}} \mu^2 \int \frac{d^D k}{(2\pi)^D} \bar{u}(p') \gamma^\alpha \gamma^\beta (k + m_{\tilde{q}_i})_\gamma^\alpha u(p)
\]

\[
\times \left[ \frac{i}{(k - p')^2 + i\varepsilon} \right] \left[ -g_{\alpha a_2 a_4} T_{999}^{\alpha a_2 a_4}(p' - p, -k + p, k - p') \right] \times \left[ \frac{i}{(k - p)^2 + i\varepsilon} \right] \left[ -g_{\alpha a_3 a_4} \right],
\]

(22)

where 1. \(T_{AC_1}^{a_1} \delta_{C_1 C_2} T_{C_2 B}^{a_4} \delta_{a_1 a_2} f_{a_2 a_4} T_{999}^{\alpha a_2 a_4} (p' - p, -k + p, k - p') = (k - 2p + p')_{a_2} g_{\alpha a_2 a_3} + (2k + p + p')_{a_2} g_{\alpha a_2 a_3} + (k + p + p')_{a_2} g_{\alpha a_2 a_3}.

After applying DR to the above amplitude, the CMDM with the off-shell gluon \((q^2 \neq 0)\) can be extracted, being

\[
\hat{\mu}_q(3g) = \frac{3\alpha_s}{4\pi} \frac{m_{\tilde{q}_i}^4}{q^2 - 4m_{\tilde{q}_i}^2} \left[ 8 - 2\frac{q^2}{m_{\tilde{q}_i}^2} + \left( 8 + \frac{q^2}{m_{\tilde{q}_i}^2} \right) (B_{01}^{3g} - B_{02}^{3g}) - 6q^2 C_0^{3g} \right],
\]

(23)

with \(B_{01}^{3g} = B_0(m_{\tilde{q}_i}^2, 0, m_{\tilde{q}_i}^2), B_{02}^{3g} = B_0(q^2, 0, 0), C_0^{3g} = C_0(m_{\tilde{q}_i}^2, m_{\tilde{q}_i}^2, q^2, 0, m_{\tilde{q}_i}^2, 0)\) (see Appendix B). We emphasize that this contribution of the CMDM is strictly valid only when \(q^2 \neq 0\).

Nonetheless, an IR divergence arises when \(q^2 \to 0\), specifically, from the part (in Eq. (23))

\[
B_{01}^{3g} - B_{02}^{3g} = -\ln \frac{m_{\tilde{q}_i}^2}{-q^2}.
\]

(24)

This behavior comes from \(B_{02}^{3g}\); for more details see Eq. (23) in the Appendix B. Then, by considering \(q^2\) small enough we get

\[
\hat{\mu}_q(3g) \approx \frac{3\alpha_s}{8\pi} \left( 1 - \ln \frac{m_{\tilde{q}_i}^2}{-q^2} \right)
\]

(25)

which diverges if \(q^2 \to 0\).

This problematic logarithm in Eq. (23) was also pointed out in the Eq. (37) from Ref. [3], but it was not indicated the source that induces the IR divergence. On the other hand, in the Eq. (11) from Ref. [2] the IR divergence was presented through the FP method without considering the \(+i\varepsilon\) prescription.

In order to go into details of the IR singularity and provide a wide panorama of the different approaches for dealing with it, we present four different schemes that lead to the same divergent issue. It can be appreciated in the Appendices C and D. Firstly, in the Appendix C we treat the IR problem by using DR, which represent the most formal procedure in quantum field theory. Secondly, in the Appendix D we focus on the problem by applying the FP

---

1 It is important to stand out that in Eq. (9) from Ref. [2], their \(T_{ji}^c T_{ji}^b f_{abc} = -i T_{ji}^b / 4\) is incorrect.
method considering the $+i\epsilon$ Feynman prescription in all the propagators, which results crucial to keep track of the IR divergence in the triple gluon vertex contribution. We begin employing the gluon propagator in the Feynman-$t$ Hooft gauge $\xi = 1$, afterwards, the general renormalizable $R_{g}$ gauge is taken into account. Finally, we also apply the fictitious mass regularization scheme for virtual gluons or massive gluons artifice. In summary, all these different procedures reveal the same IR divergence issue when the gluon is on-shell.

i) On-shell gluon case ($q^{2} \rightarrow 0$): the resulting dimensional regularized two-point scalar function $B_{02}^{3g} = B_{0}(q^{2}, 0, 0)$, for $q^{2} \rightarrow 0$, is given in Eq. (C12), which it is now expressed in terms of the UV and IR poles as

\[
B_{02}^{3g} = B_{0}(0, 0, 0) = \frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}} = \Delta_{UV} - \Delta_{IR},
\]

with $\Delta_{UV}$ and $\Delta_{IR}$ defined in Eqs. (12) and (C13), respectively. Besides, the last term from Eq. (23) vanishes

\[
q^{2}c_{0}^{3g} = 0,
\]

when $q^{2} \rightarrow 0$. Therefore, the CMDM from the triple gluon vertex diagram given in Eq. (24), with the on-shell gluon, takes the final form

\[
\lim_{q^{2} \rightarrow 0} \hat{\mu}_{q_{i}}(3g) = \frac{3\alpha_{s}}{8\pi} \left( \Delta_{IR} + \ln \frac{\mu^{2}}{m_{q_{i}}^{2}} + 3 \right),
\]

where $\Delta_{IR}$ contains the pole $1/\epsilon_{IR}$ of IR nature. Numerically, this divergent behavior can also be appreciated, for the top quark, in the Fig. 3(g) when $q^{2} = \pm M \rightarrow 0$.

ii) Off-shell gluon case ($q^{2} = \pm m_{Z}^{2}$): from Eq. (23), the spacelike value, $q^{2} = -m_{Z}^{2}$, only yields a real part, while the timelike value, $q^{2} = m_{Z}^{2}$, provides real and imaginary parts; these values are listed in the Table I. The behavior of $\hat{\mu}_{\ell}(3g)$ as a function of $q^{2} = \pm M^{2}$ is shown in the Figs. 3(g) and 3(h).

IV. RESULTS

As it was already commented in the Introduction and in the Subsection III F because of the IR divergence presence in $\hat{\mu}_{q_{i}}(3g)$ when $q^{2} \rightarrow 0$, the complete $\hat{\mu}_{q_{i}}$ in Eq. 9 cannot be evaluated with the on-shell gluon, nevertheless, as it was proposed in the Ref. 2, it makes sense to evaluate it at a conventional large gluon momentum transfer scale $q^{2} = \pm m_{Z}^{2}$, as it happens for the perturbative strong running coupling constant $a_{s}(-q^{2} = m_{Z}^{2}) = 0.1179$, which is conceived in the spacelike domain $q^{2} < 0$. The $\hat{\mu}_{\ell}$ evaluations for $q^{2} = -m_{Z}^{2}, 0, m_{Z}^{2}$ for each contribution are listed in the Table I. In the Fig. 3 the six different contributions to the top quark CMDM are shown, where it is displayed the behavior of each individual CMDM as a function of the gluon momentum transfer $q^{2} = \pm M^{2}$, running the interval $M = [0, 200]$ GeV, the value $M = m_{Z}$ is highlighted with the vertical blue line. It is included the on-shell gluon value which is finite for all the contributions except for the triple gluon vertex one. In the Table I it can be appreciated that the on-shell gluon, $q^{2} = 0$, evaluation for $\Re \hat{\mu}_{\ell}(X)$ (for $X = \gamma, Z, W, H, g$) essentially corresponds to the central value of the two other values at $q^{2} = \pm m_{Z}^{2}$; it is remarkable that these resulting values are very close to each other. The Fig. 3(a) presents the $\hat{\mu}_{\ell}(\gamma)$ photon Schwinger-type contribution; the values are entirely real and positive. The Fig. 3(b) displays the behavior of the $Z$ neutral gauge boson contribution $\hat{\mu}_{\ell}(Z)$, this provides only negative real values that are one order of magnitude larger and even closer to each other than in the photon case. The Figs. 3(c) and (d) correspond to the $W$ charged gauge boson contribution $\hat{\mu}_{\ell}(W)$, it results in real and imaginary parts, the $\Re \hat{\mu}_{\ell}(W)$ part is plotted in (c), and its $\Im \hat{\mu}_{\ell}(W)$ part in (d), notice that at $q^{2} = \pm m_{Z}^{2}$ the magnitude of the imaginary part is one order of magnitude larger than the real one, and these values are mainly due to the virtual bottom quark. The Fig. 3(e) exhibits the Higgs boson contribution $\hat{\mu}_{\ell}(H)$, where it should be noted that its values are quite similar to those of the $Z$ case but with opposite sign. The Fig. 3(f) presents the gluon Schwinger-type contribution $\hat{\mu}_{\ell}(g)$, it only has real part that is of the same order of magnitude $\sim 10^{-3}$ than the $Z$ and $H$ cases.

The triple gluon vertex contribution, $\hat{\mu}_{\ell}(3g)$, is shown in the Figs. 3(g) and 3(h), where its real and imaginary parts are displayed, respectively. Because $\hat{\mu}_{\ell}(3g)$ is the responsible for the largest value to the complete CMDM, $\hat{\mu}_{\ell}$, at $q^{2} = \pm m_{Z}^{2}$ (see Table I), we compare it with the central experimental measure given in Eq. (I), which we indicate in the plots with an horizontal red line. In Fig. 3(g) the $\Re \hat{\mu}_{\ell}(3g)$ part with $q^{2} = \pm M^{2}$ is plotted, both curves manifest the IR divergence feature when the external gluon is on-shell; notice that the spacelike evaluation produces only real values, its imaginary part is exactly zero. On the other hand, the timelike evaluation yields a complex quantity, this can be appreciated in Fig. 3(h).
The total CMDM of the top quark $\hat{\mu}_t(q^2)$ is shown in Figs. 4(a) and (b), for the spacelike and the timelike domains, respectively. From this, our main result is the spacelike evaluation (see Table I), since the $\text{Re} \hat{\mu}_t(-m_Z^2)$ part matches with the experimental central value $\hat{\mu}_t^{\text{Exp}} = -0.024^{+0.013}_{-0.009} \text{(stat)}^{+0.016}_{-0.011} \text{(syst)}$ [1]. In contrast, our result for the $\text{Im} \hat{\mu}_t(-m_Z^2)$ part is purely an EW effect induced by the $W$ gauge boson loop. Moreover, the timelike domain produces also complex values (see Fig. 4(b)) and its real part is entirely within the experimental statistical error; here, the $\hat{\mu}_t(3g)$ generates by itself an imaginary part.

In the Figs. 4(c) and (d) the EW contributions are displayed, which are given by $\hat{\mu}_t(q^2)^{\text{EW}} = \hat{\mu}_t(\gamma) + \hat{\mu}_t(Z) + \hat{\mu}_t(W) + \hat{\mu}_t(H)$ (see Table I). In the Figs. 4(e) and (f) the QCD contributions are displayed, these are composed by $\hat{\mu}_t(q^2)^{\text{QCD}} = \hat{\mu}_t(g) + \hat{\mu}_t(3g)$ (see Table I). It is obvious that both Figs. 4(e) and (f) essentially resemble the respective (a) and (b) plots, this is because the EW contribution is up to two orders of magnitude smaller than the QCD one.

Finally, it is worth to compare the absolute values of our results with the experimental one, this is plotted in the Fig. 4(g), where $|\hat{\mu}_t(-m_Z^2)| = 0.0224$, $|\hat{\mu}_t(m_Z^2)| = 0.0298$ and $|\hat{\mu}_t^{\text{Exp}}| = 0.024$.

### V. CONCLUSIONS

We have revisited the anomalous CMDM in the SM and demonstrated by DR the existence of an IR pole $1/\epsilon_{\text{IR}}$ when the gluon is on-shell. The IR divergence is induced by the contribution of the non-Abelian triple gluon vertex diagram. In consequence, the perturbative CMDM must be evaluated with the off-shell gluon momentum transfer at the reference scales $q^2 = \pm m_Z^2$ [2], this choice is based on the strong coupling constant, that is perturbatively evaluated at the conventional spacelike value, $\alpha_s(-q^2 = m_Z^2) = 0.1179$ [21, 22]. The most important prediction of our work is the evaluation of the CMDM of the top quark at the spacelike scenario $\hat{\mu}_t(-m_Z^2) = -0.0224 - 0.000925i$, whose real part coincides quite well with the recent experimental report $\hat{\mu}_t^{\text{Exp}} = -0.024^{+0.013}_{-0.009} \text{(stat)}^{+0.016}_{-0.011} \text{(syst)}$ [1], while our predicted imaginary quantity is an EW effect induced by the $W$ gauge boson. Comparing the absolute values we have $|\hat{\mu}_t(-m_Z^2)| = 0.0224$, $|\hat{\mu}_t(m_Z^2)| = 0.0298$, and $|\hat{\mu}_t^{\text{Exp}}| = 0.024$. However, according to our results for the timelike scenario, our predictions for $\hat{\mu}_t(m_Z^2)$ should not be discarded, since it falls within the experimental range of measurement.

From our obtained results for the top quark CMDM we appreciate that both perturbative parameters $\alpha_s$ and $\hat{\mu}_t$ have similar behaviors: they are undetermined when $q^2 \to 0$ and describe very well the strong interaction processes at the spacelike conventional scale $q^2 = -m_Z^2$.

| $\hat{\mu}_t$ | $-m_Z^2$ | $q^2$ | $m_Z^2$ |
|----------------|----------|-------|-------|
| $\gamma$      | $2.62 \times 10^{-4}$ | $2.74 \times 10^{-4}$ | $2.88 \times 10^{-4}$ |
| $Z$           | $-1.78 \times 10^{-3}$ | $-1.84 \times 10^{-3}$ | $-1.90 \times 10^{-3}$ |
| $W$           | $-2.91 \times 10^{-3} - 9.25 \times 10^{-4}i$ | $6.29 \times 10^{-7} - 1.21 \times 10^{-3}i$ | $1.44 \times 10^{-4} - 1.16 \times 10^{-3}i$ |
| $H$           | $1.86 \times 10^{-3}$ | $1.92 \times 10^{-3}$ | $1.99 \times 10^{-3}$ |
| $g$           | $-1.50 \times 10^{-3}$ | $-1.56 \times 10^{-3}$ | $-1.64 \times 10^{-3}$ |
| $3g$          | $-2.12 \times 10^{-2}$ | IR div. | $-1.22 \times 10^{-2} - 2.55 \times 10^{-2}i$ |
| Total         | $-2.24 \times 10^{-2} - 9.25 \times 10^{-4}i$ | IR div. | $-1.33 \times 10^{-2} - 2.67 \times 10^{-2}i$ |

**TABLE I:** The different contributions to the top quark CMDM: the experimental value is $\hat{\mu}_t^{\text{Exp}} = -0.024^{+0.013}_{-0.009} \text{(stat)}^{+0.016}_{-0.011} \text{(syst)}$ [1].

| $\hat{\mu}_t$ | $-m_Z^2$ | $q^2$ | $m_Z^2$ |
|----------------|----------|-------|-------|
| EW             | $0.000315 - 0.000925i$ | $0.000357 - 0.00121i$ | $0.000514 - 0.00116i$ |
| QCD            | $-0.0227$ | IR div. | $-0.0138 - 0.0255i$ |
| Total          | $-0.0224 - 0.000925i$ | IR div. | $-0.0133 - 0.0267i$ |

**TABLE II:** The top quark CMDM separated into EW and QCD contributions; the experimental value is $\hat{\mu}_t^{\text{Exp}} = -0.024^{+0.013}_{-0.009} \text{(stat)}^{+0.016}_{-0.011} \text{(syst)}$ [1].
FIG. 3: Contributions to the CMDM of the top quark as function of the gluon momentum transfer $q^2 = \pm M^2$, where $M = [0, 200]$ GeV; the blue line indicates $M = m_Z$. The largest contribution to the CMDM of the top quark, $\text{Re} \hat{\mu}_t(3g)$, is shown in (g), where it is compared with the experimental measure $\hat{\mu}_t^{\text{Exp}}$.

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FIG. 4: Top quark CMDM as a function of the gluon momentum transfer \( q^2 = \pm M^2 \), where \( M = [0, 200] \) GeV; the blue line indicates \( M = m_Z \) and the experimental value \( \hat{\mu}_t^{\text{Exp}} \) is displayed. In (a) and (b) the total contributions are shown, in (c) and (d) the EW parts are depicted, and in (e) and (f) the QCD ones are presented. In (g) the absolute values of the total contributions for the spacelike domain, the timelike one and the experimental value are compared.
Appendix A: Feynman rules

\begin{align*}
&\gamma_\mu^a \quad \rightarrow \quad q_B \\
&\mu \quad \rightarrow \quad \gamma_\mu \quad \rightarrow \quad -ieQ_q \gamma_\mu \delta_{AB} \\
&W_\mu^a \quad \rightarrow \quad d_j^B \quad u_i^A \\
&\mu \quad \rightarrow \quad W^a_\mu \quad \rightarrow \quad -ie\frac{g}{\sqrt{2}} P_L V_{\mu j} \delta_{AB} \\
&H \quad \rightarrow \quad q_B \quad q_A \\
&\mu \quad \rightarrow \quad H \quad \rightarrow \quad -ie \frac{m_m}{2m_W} \delta_{AB} \\
&g_\mu (p_1) \quad \rightarrow \quad g_\mu (p_2) \quad g_\mu (p_3) \\
&\mu \quad \rightarrow \quad g_\mu (p) \quad \rightarrow \quad \frac{i}{p^2 + i\varepsilon} (-g_{\mu\nu}) \\
&H(p) \quad \rightarrow \quad H(p) \quad \rightarrow \quad \frac{i}{p^2 - m_H^2 + i\varepsilon} \\
&\mu, a \quad \rightarrow \quad \nu, b \\
&\mu \quad \rightarrow \quad \nu \quad \rightarrow \quad \frac{i}{p^2 - m_Z^2 + i\varepsilon} (-g_{\mu\nu}) \delta_{ab}
\end{align*}

TABLE III: Feynman rules.

In (a)-(f) and (h) the A, B are quark color indices running from 1 to 3. In (f)-(g) and (l) a, b, c are gluon color indices running from 1 to 8. For the gluon trivertex (g) we abbreviate

\[ T_{\mu_1\mu_2\mu_3}^{ggg} (p_1, p_2, p_3) \equiv (p_1 - p_2)_{\mu_3} g_{\mu_1\mu_2} + (p_2 - p_3)_{\mu_1} g_{\mu_2\mu_3} + (p_3 - p_1)_{\mu_2} g_{\mu_3\mu_1}. \]  

(A1)

In our calculations we use the electron unit charge \( e = \sqrt{4\pi\alpha} \) and the QCD group strong coupling constant \( g_s = \sqrt{4\pi\alpha_s} \). We took input values from PDG 2020 [21]: the strong coupling constant \( \alpha_s (m_Z) = 0.1179 \), the weak-mixing angle \( s_W = \sin \theta_W (m_Z) = 0.23121 \), the quark masses \( m_d = 0.00467 \), \( m_s = 0.093 \), \( m_b = 4.18 \), \( m_t = 172.76 \) GeV, the boson masses \( m_W = 80.379 \), \( m_Z = 91.1876 \), \( m_H = 125.1 \) GeV and the quark-mixing matrix of Cabibbo–Kobayashi–Maskawa (CKM) is

\[ V_{\text{CKM}} = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix} = \begin{pmatrix}
0.9737 & 0.2245 & 0.00382 \\
0.221 & 0.987 & 0.041 \\
0.008 & 0.0388 & 1.013
\end{pmatrix}. \]  

(A2)

The fine-structure constant \( \alpha (m_Z) = 1/129 \) is taken from [40]. Besides, the electric charges of the quarks \( Q_t = 2/3 \) and the weak couplings \( g_{Vt} = (3 - 8s_W^2) / 6 \), \( g_{At} = 1/2 \).
Appendix B: The Passarino-Veltman scalar functions

We follow the FeynCalc definitions for the scalar functions arguments. The Feynman parameterization formulas are:

i) the one-point scalar function

\[ A_0(m_0^2) = m_0^2 \left( \Delta_{UV} + \ln \frac{\mu^2}{m_0^2 - i\varepsilon} + 1 \right), \quad (B1) \]

\[ \Delta_{UV} \equiv (4\pi)^{\epsilon_{UV}} \Gamma(\epsilon_{UV}) \approx \frac{1}{\epsilon_{UV}} - \frac{\gamma_E + \ln 4\pi}{\epsilon_{UV}}, \quad (B2) \]

\[ \epsilon_{UV} = \epsilon = \frac{4 - D}{2} \gg 0; \quad (B3) \]

ii) the two-point scalar function

\[ B_0(q_1^2, m_0^2, m_1^2) = \Delta_{UV} + \int_0^1 dx_1 \ln \frac{\mu^2}{\Delta B_0}, \quad (B4) \]

\[ \Delta B_0 \equiv q_1^2 x_1^2 + (m_0^2 - m_1^2 - q_1^2) x_1 + m_1^2 - i\varepsilon; \quad (B5) \]

iii) the three-point scalar function

\[ C_0(q_1^2, (q_1 - q_2)^2, q_2^2, m_0^2, m_1^2, m_2^2) = \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{-1}{\Delta C_0}, \quad (B6) \]

\[ \Delta C_0 = q_2^2 x_1^2 + (q_1 - q_2)^2 x_2^2 + (m_0^2 - m_1^2 - q_2^2) x_1 + [m_1^2 - m_2^2 - (q_1 - q_2)^2] x_2 \\
+ [-q_1^2 + q_2^2 + (q_1 - q_2)^2] x_1 x_2 + m_2^2 - i\varepsilon. \quad (B7) \]

Explicit solutions:

\[ B_0(m_q^2, 0, m_q^2) = \Delta_{UV} + \ln \frac{\mu^2}{m_q^2} + 2. \quad (B8) \]

\[ B_0(q^2, m_q^2, m_q^2) = \Delta_{UV} + \ln \frac{\mu^2}{m_q^2} + 2 + \frac{R_q}{q^2} \ln \frac{R_q + 2m_q^2 - q^2}{2m_q^2}, \quad (B9) \]

with \( R_q \equiv \sqrt{q^2 - 4m_q^2}. \)

\[ B_0(0, m_q^2, m_q^2) = \Delta_{UV} + \ln \frac{\mu^2}{m_q^2}. \quad (B10) \]

\[ B_0(m_q^2, m_q^2, m_X^2) = \Delta_{UV} + \ln \frac{\mu^2}{m_q^2} + 2 + \frac{m_X^2}{2m_q^2} \ln \frac{m_X^2}{m_q^2} + \frac{m_X}{m_q^2} R_X \ln \frac{R_X + m_X}{2m_q}, \quad (B11) \]

with \( R_X \equiv \sqrt{m_X^2 - 4m_q^2}. \)

\[ B_0 \left( m_q^2, m_q^2, m_W^2 \right) = \Delta_{UV} + \ln \frac{\mu^2}{m_W^2} + 2 - \frac{m_q^2 + m_q^2 - m_W^2}{2m_q^2} \ln \frac{m_q^2}{m_W^2} + \frac{\sqrt{a}}{m_q^2} \ln \frac{\sqrt{a} - m_q^2 + m_q^2 + m_W^2}{2m_q m_W}, \quad (B12) \]
with \( a \equiv m_{q_i}^4 - 2 m_{q_i}^2 (m_{q_i}^2 + m_W^2) + (m_{q_i}^2 - m_W^2)^2 \).

\[
B_0(q^2, 0, 0) = \Delta_{UV} + 2 + \ln \frac{\mu^2}{q^2}.
\]

\[
C_0 \left( m_{q_i}^2, m_{q_i}^2, 0, m_{q_i}^2, m_X^2, m_{q_i}^2 \right) = - \frac{1}{2m_{q_i}^2} \left( \ln \frac{m_{q_i}^2}{m_X^2} + \frac{2m_X}{R_X} \ln \frac{R_X + m_X}{2m_{q_i}} \right).
\]

\[
C_0 \left( m_{q_i}^2, m_{q_i}^2, q^2, m_{q_i}^2, m_X^2, m_{q_i}^2 \right) \approx \frac{1}{60m_{q_i}^2} \left( - [30m_{q_i}^4 + (5m_{q_i}^2 + q^2) q^2] \ln \frac{m_{q_i}^2}{m_X^2} + m_{q_i}^2 q^2 \left\{ 4m_{q_i}^2 [20m_{q_i}^2 - (5m_X^2 - 6q^2)] - 3m_X^2 q^2 \right\} \right)
\]\
\[
- \frac{2m_X}{(m_X^2 - 4m_{q_i}^2)^2} \left\{ 120m_{q_i}^6 [4m_{q_i}^2 - (2m_X^2 - q^2)] + 10m_{q_i}^4 [3m_X^4 + (3q^2 - 5m_X^2)q^2] \right\}
\]\
\[
+ m_X^2 q^2 \left\{ 5m_{q_i}^2 (m_X^2 - 2q^2) + m_X^2 q^2 \right\} \ln \frac{R_X + m_X}{2m_{q_i}},
\]

where \( m_{q_i}^2 > m_X^2 \geq q^2 \).

\[
C_0(m_{q_i}^2, m_{q_i}^2, 0, m_{q_i}^2, m_X^2, m_{q_i}^2) = - \frac{1}{m_{q_i}^2} \left[ \frac{1}{2} \ln \frac{m_{q_i}^2}{m_X^2} + \frac{m_{q_i}^2 - m_X^2 + m_W^2}{\sqrt{a}} \ln \frac{a - m_{q_i}^2 + m_{q_i}^2 + m_W^2}{2m_{q_i}m_W} \right],
\]

with \( a \equiv m_{q_i}^4 - 2m_{q_i}^2 (m_{q_i}^2 + m_W^2) + (m_{q_i}^2 - m_W^2)^2 \).

\[
C_0(m_{q_i}^2, m_{q_i}^2, q^2, 0, m_{q_i}^2, 0) = \frac{1}{6q^2 \sqrt{1 - \frac{4m_{q_i}^2}{q^2}}} \left\{ 4\pi^2 + 3 \ln \left[ 1 + \frac{q^2}{2m_{q_i}^2} \left( \sqrt{1 - \frac{4m_{q_i}^2}{q^2}} - 1 \right) \right] + 12 \ln \left[ 1 + \frac{q^2}{2m_{q_i}^2} \left( \sqrt{1 - \frac{4m_{q_i}^2}{q^2}} - 1 \right) \right] \right\}.
\]

**Appendix C: Dimensional regularization and the IR divergence**

Starting from D dimensions for the two-point scalar function \( B^{3,2}_{D} \equiv B_0(q^2, 0, 0) \), see Eq. (B13), it is found that this function is responsible for the IR divergence when the gluon is on-shell, \( q^2 = 0 \) (it appears in Eqs. (23) and (24)). This procedure will help to reveal their ultraviolet \( 1/\epsilon_{UV} \) and infrared \( 1/\epsilon_{IR} \) poles, for which we follow the Refs. [19, 20]. The integral representation that give rise to that PaVe is

\[
B_0(q^2, 0, 0) = -i16\pi^2 \mu^{2\epsilon} \int \frac{d^Dk}{(2\pi)^D} \frac{1}{(k - p')^2(k - p)^2}
\]

\[
= -i16\pi^2 \mu^{2\epsilon} \int \frac{d^Dk}{(2\pi)^D} \frac{1}{k^2(k + q)^2}.
\]

To dimensional regularize \( B_0(q^2, 0, 0) \) when \( q^2 \to 0 \) we use\(^2\) the space-time dimension \( D = 4 - 2\epsilon \) as in the Refs. [19, 30], being \( \epsilon_{UV} \equiv \epsilon \lesssim 0 \) for the UV divergence and \( \epsilon_{IR} \equiv \epsilon \gtrsim 0 \) for the IR one. The FP for the integrand in

---

\(^2\) In the Ref. [20] it is used \( D = 4 + 2\epsilon \), where \( \epsilon \lesssim 0 \) stands for the UV divergence and \( \epsilon \gtrsim 0 \) for the IR one.
Eq. (C1) is

\[
\frac{1}{k^2(k+q)^2} = \int_0^1 dx_1 \frac{\Gamma(2)}{[x_1 k^2 + (1-x_1)(k+q)^2]^2} \\
= \int_0^1 dx_1 \frac{\Gamma(2)}{(\ell^2 - \Delta B_0)^2},
\]  

(C2)

with \( k \equiv \ell + q(x_1 - 1) \), \( dk = d\ell \) and \( \Delta B_0 \equiv -q^2 x_1 (1-x_1) \). Using the \( D \)-dimensional Minkowski integral given in Eq. (A.44) from [28]

\[
\int \frac{d^D\ell}{(2\pi)^D} \frac{1}{(\ell^2 - \Delta)^n} = \frac{(-1)^n i}{\Gamma(n-D/2)} \left( \frac{(2\pi)^D}{D} \right) \frac{1}{\Delta^{n-D/2}},
\]  

(C3)

and the Euler beta function

\[
B(x, y) = \int_0^1 dz \ z^{x-1}(1-z)^{y-1} \\
= \frac{\Gamma(x) \Gamma(y)}{\Gamma(x+y)},
\]  

(C4)

we obtain

\[
B_0(q^2, 0, 0) = -i 16\pi^2 \mu^2 \left( \frac{q^2}{-q^2} \right)^\epsilon \int_0^1 dx_1 \frac{\Gamma(2)}{(\ell^2 - \Delta B_0)^2} \\
= \Gamma(\epsilon) \int_0^1 dx_1 \left( \frac{4\pi \mu^2}{-q^2} \right)^\epsilon \\
= \Gamma(\epsilon) \left( \frac{4\pi \mu^2}{-q^2} \right)^\epsilon \int_0^1 dx_1 \frac{1}{x_1^\epsilon (1-x_1)^\epsilon} \\
= \Gamma(\epsilon) \left( \frac{4\pi \mu^2}{-q^2} \right)^\epsilon B(1-\epsilon, 1-\epsilon) \\
= \Gamma(\epsilon) \left( \frac{4\pi \mu^2}{-q^2} \right)^\epsilon,
\]  

(C5)

where \( B(1-\epsilon, 1-\epsilon) = 1 \), for \( \epsilon \to 0 \). In addition, the term \( 1/(-q^2)^\epsilon \) with \( \epsilon \to 0 \) and \( q^2 \to 0 \) is an indetermination of the type \( 1/0^0 \), where the procedure of taking the limit must be carefully done; this problem can be faced following the procedure from the Ref. [20]. First, according to the spirit of the Eq. (8.22) from Ref. [20], we have that

\[
\frac{1}{(-y)^\epsilon} = \epsilon \int_{-\infty}^{\infty} dx \frac{1}{x^\epsilon}, \quad \text{Re} \ \epsilon > 0,
\]  

(C6)

which then splits into two regions \( \int_{-\infty}^{\infty} = \int_{-\infty}^{a} + \int_{-a}^{0} \), hence \( \left( \frac{4\pi \mu^2}{-q^2} \right)^\epsilon \) from Eq. (C5) results in

\[
\left( \frac{4\pi \mu^2}{-q^2} \right)^\epsilon = \epsilon \int_{-\infty}^{\infty} dx \frac{1}{r^\epsilon} \left( \frac{4\pi \mu^2}{r^2} \right)^\epsilon \\
= \epsilon \int_{-\infty}^{\infty} dx \frac{1}{r^\epsilon} \left( \frac{4\pi \mu^2}{r^2} \right)^\epsilon + \epsilon \int_{-a}^{0} dx \frac{1}{r^\epsilon} \left( \frac{4\pi \mu^2}{r^2} \right)^\epsilon.
\]  

(C7)

In the same context of Eqs. (8.24) from Ref. [20], it can be written that

\[
\text{UV region : } \int_{a}^{\infty} dx \left( \frac{a}{x} \right)^\epsilon = \frac{1}{\epsilon} \equiv \frac{1}{\epsilon_{\text{UV}}}, \quad \text{Re} \ \epsilon \gtrsim 0, \ a > 0,
\]  

(C8)

\[
\text{IR region : } \int_{0}^{a} dx \left( \frac{a}{x} \right)^\epsilon = -\frac{1}{\epsilon} \equiv -\frac{1}{\epsilon_{\text{IR}}}, \quad \text{Re} \ \epsilon \lesssim 0, \ a > 0.
\]  

(C9)
Then, by replacing the Eq. (C7) in Eq. (C5) and performing $q^2 \to 0$ by means of Eqs. (C8) and (C9), we get

$$B_0(q^2, 0, 0) = \Gamma(\epsilon) \int_{4\pi\mu^2}^{\infty} \epsilon \frac{d\epsilon}{\epsilon^2} \left( \frac{4\pi\mu^2}{r^2} \right) + \Gamma(\epsilon) \int_{-q^2}^{4\pi\mu^2} \epsilon \frac{d\epsilon}{\epsilon^2} \left( \frac{4\pi\mu^2}{r^2} \right). \quad (C10)$$

By taking the limit

$$\lim_{q^2 \to 0} B_0(q^2, 0, 0) = \Gamma(\epsilon) \Gamma(\epsilon) - \Gamma(\epsilon) \approx \frac{1}{\epsilon} - \frac{1}{\epsilon}, \quad (C11)$$

where $\Gamma(\epsilon) \approx 1/\epsilon - \gamma_E$ and $\Gamma(\epsilon) \approx 1/\epsilon - \gamma_E$, it can be expressed as

$$B_0(0, 0, 0) = \frac{1}{\epsilon} - \frac{1}{\epsilon}, \quad (C12)$$

Now, the UV and IR poles are explicit, being suitable to express the poles through $\Delta_{UV}$ from Eq. (B2) and an analogous definition for the IR divergence

$$\Delta_{IR} \equiv (4\pi)^{\epsilon_{IR}} \Gamma(\epsilon_{IR}) \approx \frac{1}{\epsilon_{IR}} + \gamma_E + \ln 4\pi, \quad (C13)$$

$$\epsilon_{IR} \equiv \epsilon = \frac{4 - D}{2} \lesssim 0. \quad (C14)$$

Finally, with $B_{01}^{3g} = B_0(m_{q_i}^2, 0, m_{q_i}^2)$ from Eq. (B8) and the new $B_{02}^{3g} = B_0(0, 0, 0)$ from Eq. (C12) for $q^2 = 0$, the on-shell case of the Eq. (24) takes the form

$$B_{01}^{3g} - B_{02}^{3g} = B_0(m_{q_i}^2, 0, m_{q_i}^2) - B_0(0, 0, 0) = \Delta_{IR} + \ln \frac{\mu^2}{m_{q_i}^2} + 2, \quad (C15)$$

which exhibits the IR nature of the divergence contained in the CMDM of the triple gluon vertex diagram.

**Appendix D: Feynman parameterization of the triple gluon vertex diagram with the $+i\epsilon$ prescription**

The triple gluon vertex contribution to the CMDM (see Fig. 2(f)), with the on-shell gluon, was calculated in the SM in Refs. 2, 3, by means of the FP method. Nonetheless, the authors of the Ref. 3 did not consider the $+i\epsilon$ Feynman prescription for the propagator 28, 41–43, whilst in Ref. 2 even if the IR divergence was indicated and they also implemented the artifice of massive gluons, they did not provide analytical solutions for the parameterized integrals.

In what follows, we demonstrate analytically through the FP method, with the strict use of the $+i\epsilon$ prescription of the propagators, that $\hat{\mu}_{q_i}(3g)$ with the on-shell gluon generates a logarithmic IR divergence. By keeping $+i\epsilon$ throughout the calculation and once the parameterized integrals have been completely solved, we can now apply the limit $\epsilon \to 0$; this in order to find out whether the solution is finite or not. In consequence, only after knowing that the solution is truly finite it is correct to set $\epsilon = 0$ since the beginning. In the case of $\hat{\mu}_{q_i}(3g)$, a logarithmic divergence arise when $\epsilon \to 0$. We will approach this problem by two different ways: in the Subsection D1, the gluon propagator in the Feynman-’t Hooft gauge ($\xi = 1$) is considered and in the Subsection D2 the gluon propagator in the general renormalizable $R_\xi$ gauge is implemented. Furthermore, in the Subsection D3, the artifice of the massive gluon propagator is carried out.
1. The gluon propagator in the Feynman’ Hooft gauge

The gluon propagator in the general $R_\xi$ gauge is

$$\frac{i}{p^2 + i\varepsilon} \left[ -g_{\mu\nu} + (1 - \xi) \frac{p_\mu p_\nu}{p^2 + i\varepsilon} \right] \delta_{ab}. \quad (D1)$$

This propagator (see Table III) in the Feynman-’t Hooft gauge was used in the loop integral given in the Eq. (22), in order to get $\hat{\mu}_{q_i}(3g)$ from Eq. (23) with the off-shell gluon.

In the following, we compute $\hat{\mu}_{q_i}(3g)$ when $\xi = 1$, first for the off-shell gluon and after with the on-shell gluon.

i) The off-shell gluon case ($q^2 \neq 0$). From (22), with its denominator parameterized via

$$\frac{1}{D_1D_2D_3} = \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{\Gamma(3)}{[x_1 D_1 + x_2 D_2 + (1 - x_2 - x_2) D_3]^3}, \quad (D2)$$

where $D_1 \equiv k^2 - m_{q_i}^2 + i\varepsilon$, $D_2 \equiv (k - p')^2 + i\varepsilon$, $D_3 \equiv (k - p)^2 + i\varepsilon$ and the shift $k = \ell - p x_1 + (p' - p) x_2 + p$, the resulting CMDM is

$$\hat{\mu}_{q_i}(3g)_{\xi = 1} = \frac{3\alpha_s}{4\pi} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{m_{q_i}^2 x_1 (x_1 - 1)}{m_{q_i}^2 x_1^2 + q_i^2 (x_1 + x_2 - 1) x_2 - i\varepsilon}. \quad (D3)$$

This reproduces the same numerical results as those obtained with Eq. (23), where the Passarino-Veltman tensor decomposition method was employed. We have implemented high numerical precision to successfully evaluate (D3) via Mathematica.

ii) The on-shell gluon case ($q^2 = 0$). Note that if $i\varepsilon$ is ignored in Eq. (D3), it results that

$$\hat{\mu}_{q_i}(3g)_{\xi = 1} = -\frac{3\alpha_s}{4\pi} \int_0^1 dx_1 \frac{(1 - x_1)^2}{x_1} \left. \right|_0^1$$

which diverges when $x_1 \to 0$; this behavior was pointed out in the Eq. (11) from Ref. [2]. However, we prefer to work the Eq. (D3) keeping the $+i\varepsilon$ prescription, since this allows to entirely solve the integral and, at the end, to analyze when $\varepsilon \to 0$. Thereby,

$$\hat{\mu}_{q_i}(3g)_{\xi = 1} = \frac{3\alpha_s}{16\pi} \left\{ 6 - \left( 1 + \frac{i\varepsilon}{m_{q_i}^2} \right) \ln \left( 1 - \frac{m_{q_i}^2}{i\varepsilon} \right)^2 - \left( \frac{i\varepsilon}{m_{q_i}^2} \right)^2 \ln \left( \frac{i\varepsilon/m_{q_i}^2 + 1}{i\varepsilon/m_{q_i}^2 - 1} \right)^4 \right\} \approx \frac{3\alpha_s}{8\pi} \left( 3 - \ln \frac{m_{q_i}^2}{-i\varepsilon} \right), \quad 0 < \varepsilon \ll 1 \quad (D5)$$

which diverges when $\varepsilon \to 0$. 

2. The gluon propagator in the general renormalizable $R_\xi$ gauge

By considering the gluon propagator in the $R_\xi$ gauge (see Eq. (D1)), the integral in Eq. (22) takes a more complicated form

$$
\mathcal{M}_\xi^\mu(3g) = \int \frac{d^Dk}{(2\pi)^D} \tilde{u}(p') \left( -i g_s \gamma^{\alpha_1} T^{(1)}_{A(1)} \left( i \frac{\hat{k} + m_q}{k^2 - m_q^2 + i\varepsilon} c_1 c_2 \right) ( -i g_s \gamma^{\alpha_2} T^{(2)}_{C(2)B} ) u(p) \right.
\times \left\{ \frac{i}{(k - p')^2 + i\varepsilon} \left[ -g_{\alpha_1\alpha_2} + (1 - \xi) \frac{(k - p'_1)\alpha_1 (k - p')_{\alpha_2}}{(k - p')^2 + i\varepsilon} \right] \delta_{\alpha_1\alpha_2} \right\}
\times \left\{ \frac{i}{(k - p)^2 + i\varepsilon} \left[ -g_{\alpha_3\alpha_4} + (1 - \xi) \frac{(k - p')\alpha_3 (k - p')_{\alpha_4}}{(k - p')^2 + i\varepsilon} \right] \delta_{\alpha_3\alpha_4} \right\}
\times \left\{ \frac{3g_s^3 T^{(2)}_{AB}}{2} \int \frac{d^Dk}{(2\pi)^D} \tilde{u}(p') \gamma^{\alpha_1} (\hat{k} + m_q) \gamma^{\alpha_2} u(p) \right\}
\times \left\{ \frac{1}{x_1 D_1 + x_2 D_2 + (1 - x_2 - x_2) D_3} \right\},
$$

(D6)

whose denominator is parameterized as

$$
\frac{1}{D_1 D_2 D_3} = \frac{\Gamma(5)}{\Gamma(1)\Gamma(2)\Gamma(2)} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{x_2(1 - x_1 - x_2)}{[x_1 D_1 + x_2 D_2 + (1 - x_2 - x_2) D_3]^5},
$$

(D7)

where $D_{1,2,3}$ and the shift of $k$ are the same as in Eq. (D2). Thus, the resulting CMDM is

$$
\hat{\mu}_{\xi}(3g)_{R_\xi} = \frac{3\alpha_s}{64\pi} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \left\{ \frac{4m_q^2 (5x_1 - 3)x_1 + 2(1 - \xi)m_q^2 (5x_1 - 2)x_1}{m_q^2 x_1^2 + q^2 (x_1 + x_2 - 1)x_2 - i\varepsilon} \right.
\left. + 2m_q^2 \frac{m_q^2 x_1^2 (4 - 5x_1) + i\varepsilon x_1 (3 - 4x_1) + (1 - \xi)m_q^2 m_q^2 x_1^2 (6 - 5x_1) + i\varepsilon x_1 (1 - 2x_1)}{m_q^2 x_1^2 + q^2 (x_1 + x_2 - 1)x_2 - i\varepsilon} \right]
\left. + 2(x_1 - 1)x_1 \frac{m_q^2 x_1^2 + i\varepsilon m_q^2}{m_q^2 x_1^2 + q^2 (x_1 + x_2 - 1)x_2 - i\varepsilon} \right] \right\}.
$$

(D8)

Solving $\hat{\mu}_{\xi}(3g)_{R_\xi}$ for $q^2 = 0$, yields

$$
\hat{\mu}_{\xi}(3g)_{R_\xi} = \hat{\mu}_{\xi}(3g)_{\xi = 1} + \hat{\mu}_{\xi}(3g)_{\xi},
$$

(D9)

which is divergent, since the first term, $\hat{\mu}_{\xi}(3g)_{\xi = 1}$, is the same as the one obtained with the Feynman-'t Hooft gauge in Eq. (D3); because it should be remembered that this diverges when $\varepsilon \rightarrow 0$. On the other hand, the second term, $\hat{\mu}_{\xi}(3g)_{\xi}$, is proportional to the $\xi$ gauge parameter as

$$
\hat{\mu}_{\xi}(3g)_{\xi} \approx \frac{3\alpha_s}{64\pi} (1 - \xi) \left\{ \frac{2i\varepsilon}{m_q^2} - 3 \ln \left( 1 - \frac{m_q^2}{i\varepsilon} \right)^2 \right\} - \left( \frac{3i\varepsilon}{m_q^2} \right) \ln \frac{i\varepsilon/m_q^2 + 1}{i\varepsilon/m_q^2 - 1}
$$

(D10)

but it vanishes when $\varepsilon \rightarrow 0$, so it is proven that $\hat{\mu}_{\xi}(3g)_{R_\xi}$ is independent of the $\xi$ gauge parameter.

3. The massive gluon propagator artifact

Another way to address the IR divergence is providing small fictitious masses $m_g$ in the gluon propagator. This method was used in the Ref. (2) to show numerically that the triple gluon vertex contribution to the CMDM for $q^2 = 0$
diverges when $m_g \to 0$. Applying such artifice to the loop integral in Eq. (22) it takes the form

$$\mathcal{M}_{\eta_r}(3g) = - \frac{3g^3}{2T_{AB}} \int \frac{d^4k}{(2\pi)^D} \frac{\bar{u}(p')\gamma_{\alpha}(k + m_g)\gamma_{\alpha+}u(p)}{(k^2 - m_{q_i}^2 + i\epsilon) [(k - p')^2 - m_g^2 + i\epsilon] [m_g^2 - m_q^2 + i\epsilon]}
$$

Here, $+i\epsilon$ can be omitted, since if the limit $\varepsilon \to 0$ is taken at the end will lead to a finite solution as long as $m_g \neq 0$; we will keep it by formality. The starting point is to parameterize the denominator as in Eq. (D2), with the same shift for $k$ but with $D_1 \equiv k^2 - m_{q_i}^2 + i\epsilon$, $D_2 \equiv (k - p')^2 - m_g^2 + i\epsilon$ and $D_3 \equiv (k - p)^2 - m_g^2 + i\epsilon$. The derived CMDM with the on-shell gluon is

$$\bar{\mu}_{\eta_r}(3g) = \frac{3\alpha_s}{4\pi} \int_0^1 dx_1 \int_{1-x_1}^{1-x_2} dx_2 \frac{m_{q_i}^2 x_1(x_1 - 1)}{m_{q_i}^2 x_2 - m_{q_i}^2(x_1 - 1) - i\epsilon}$$

$$= \frac{3\alpha_s}{8\pi} \left( 3 - 2 \frac{m_{q_i}^2}{m_{q_i}^2} + \left( 1 - 3 \frac{m_{q_i}^4}{m_{q_i}^2} + \frac{m_{q_i}^4}{m_{q_i}^2} + i\epsilon \right) \ln \frac{m_{q_i}^2 - i\epsilon}{m_{q_i}^2 - i\epsilon} \right)$$

which also diverges when $m_g \to 0$. Notice that this final expression is independent of the $\varepsilon$ parameter; this is why $\varepsilon$ can be removed at the outset.

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