Linear Coasting in Cosmology and SNe Ia

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Abstract
A strictly linear evolution of the cosmological expansion scale factor is a characteristic feature in several classes of alternative gravity theories as also in the standard (big-bang) model with specially chosen equations of state of matter. Such an evolution has no free parameters as far as the classical cosmological tests are concerned and should therefore be easily falsifiable. In this article we demonstrate how such models present very good fits to the current supernovae 1a data. We discuss the overall viability of such models.

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Large scale homogeneity and isotropy observed in the universe suggests the following [Friedman-Robertson-Walker (FRW)] form for the spacetime metric:

\[ ds^2 = dt^2 - a(t)^2 \left( \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right) \]  

(1)

Here \( k = \pm 1, 0 \) is the curvature constant. In standard “big-bang” cosmology, the scale factor \( a(t) \) is determined by the equation of state of matter and Einstein’s equations. The form for the scale factor determines the response of a chosen model to the three classical cosmological tests, viz: (1) The galaxy number count as a function of redshift; (2) The angular diameter of “standard” objects (galaxies) as a function of redshift; and finally (3) The apparent luminosity of a “standard candle” as a function of redshift. The first two tests are marred by evolutionary effects and for this reason have fallen into disfavour as reliable indicators of a viable model. However, the discovery of Supernovae type Ia [SNe Ia] as reliable standard candles has raised hopes of elevating the status of the third test to that of a precision measurement that could determine the viability of a cosmological model. The main reason for regarding these objects as reliable standard candles are their large luminosity, small dispersion in their peak luminosity and a fairly accurate modeling of their evolutionary features. Recent measurements on 42 high redshift SNe Ia’s reported in the supernovae cosmology project [1] together with the observations of the 16 lower redshift SNe Ia’s of the Callan-Tollolo survey [4, 5] have been used to determine the cosmological parameters \( \Omega_{\Lambda} \) and \( \Omega_M \). The data eliminates the “minimal inflationary” prediction defined by \( \Omega_{\Lambda} = 0 \) and \( \Omega_M = 1 \). The data can however, be used to assess a “non-minimal inflationary cosmology” defined by \( \Omega_{\Lambda} \neq 0, \Omega_{\Lambda} + \Omega_M = 1 \). The maximum likelihood analysis following from such a study has yielded the values \( \Omega_M = 0.28 \pm 0.1 \) and \( \Omega_{\Lambda} = 0.72 \pm 0.1 \) [2, 3, 6, 7].

In this article we explore the concordance of the SNe Ia data with a FRW cosmology in which the scale factor evolves linearly with time: \( a(t) \propto t \). The motivation for such an endeavour comes from several reasons. First of all, such a cosmology does not suffer from the horizon problem. Horizons occur in models with \( a(t) \approx t^\alpha \) for \( \alpha < 1 \). Secondly, linear evolution of the scale factor is supported in alternative gravity theories (eg. non-minimally coupled scalar tensor theories) where it turns out to be independent of the matter equation of state. The scale factor in such theories does not constrain
the matter density parameter thereby curing the flatness problem. Further the age estimate of the \((a(t) \propto t)\) universe, deduced from a measurement of the Hubble parameter, is given by \(t_o = (H_o)^{-1}\). This is \(\approx 50\% \) greater than the age inferred from the same measurement in standard (cold) matter dominated cosmology (without the cosmological constant). This would make the age estimate comfortably concordant with age estimates of old clusters. Finally, a linear coasting cosmology, independent of the equation of state of matter, is a generic feature in a class of models that attempt to dynamically solve the Cosmological constant problem \([8, 9, 10]\). Such models have a scalar field non-minimally coupled to the large scale scalar curvature of the universe. With the evolution of time, the non-minimal coupling diverges, the scale factor quickly approaches linearity and the non-minimally coupled field acquires a stress energy that cancels the vacuum energy in the theory.

There have been other gravity models that also account for a linear evolution of the scale factor. Notable among such models is a recent idea of Allen \([11]\) in which such a scaling results in an \(SU(2)\) cosmological instanton dominated universe. Yet another possibility derives from the Weyl gravity theory of Manheim and Kazanas \([12]\). Here again the FRW scale factor approaches a linear evolution at late times.

What makes the above ideas particularly appealing is a recent demonstration \([13]\) of primordial nucleosynthesis not to be an impediment for a linear coasting cosmology. For the currently favoured value of 65 km /sec /Mpc for the Hubble parameter, it follows that for baryon entropy ratio \(\eta = 5 \times 10^{-9}\) we can get just the right amount of Helium 23.8\%. Besides, one also gets a primordial metallicity quite close to the lowest observed metallicities. The only problem that one has to contend with is the significantly low yields of deuterium in such a cosmology. However, as pointed out in \([13]\), the amount of Helium produced is quite sensitive to \(\eta\) in such models. In an inhomogeneous universe, therefore, one can have the heliump to hydrogen ratio to have a large variation. Deuterium can be produced by a spallation process much later in the history of the universe \([14]\). If one considers spallation of a helium deficient cloud onto a helium rich cloud, it is easy to produce deuterium as demonstrated by Epstein - but without overproduction of Lithium. This result can be used to soften nucleosynthesis constraints on a general power law cosmology.
Finally we must mention that within the framework of standard cosmology there are rather strong constraints on the allowable equations of state. Perlmutter et al. [15] have demonstrated that a general equation of state: $P = wρ$ in standard cosmology, is constrained to $w \leq -2/3$. Linear coasting, possible in standard cosmology for $w = -1/3$ as also for empty models, would therefore appear to be disfavoured. However, as pointed out in [1], such a linear coasting, though a bit improbable, is definitely not ruled out. Any mechanism for dimming the high redshift SNe Ia by .15 magnitude would make linear coasting concordant. This is not at all discouraging - considering the fact that the intrinsic uncertainty in the luminosity for most high redshift objects is in excess of .15. Further, as reported in this article, statistical measures of fit such as the $\chi^2$ per degree of freedom for a linear coasting cosmology turn out to be comparable to the best fits reported in [1] ≈ 1.17. In any case the results presented in this article would be of interest for alternative gravity models mentioned earlier.

**A coasting cosmology and Type Ia Supernovae**

In this section we explore the concordance of a linear coasting cosmology with the recent extra-galactic Type Ia supernovae apparent magnitude - redshift data [1, 2].

The apparent magnitude of an object is related to the luminosity distance $d_L$ by the following relation,

$$m = 25 + M + 5logd_L$$  \hspace{1cm} (2)

with $d_L$ defined as

$$d_L = a_o r (1 + z)$$  \hspace{1cm} (3)

Here $r$ is the comoving radial coordinate. In the standard model,

$$D_L(z; \Omega_M, \Omega_\Lambda) \equiv d_L H_o = \frac{c(1 + z)}{\sqrt{|\kappa|}}$$  \hspace{1cm} (4)

$$S(\sqrt{|\kappa|}) \int_0^z \left( \frac{dz'}{\sqrt{(1 + z')^2(1 + \Omega_M z') - z'(2 + z') \Omega_\Lambda}} \right)$$  \hspace{1cm} (5)
where for $\Omega_M + \Omega_\Lambda > 1$, $S(x) = \sin(x)$ and $\kappa = 1 - \Omega_M - \Omega_\Lambda$; for $\Omega_M + \Omega_\Lambda < 1$, $S(x) = \sinh(x)$ and $\kappa$ as above; and for $\Omega_M + \Omega_\Lambda = 1$, $S(x) = x$ and $\kappa = 1$ [3]. In the low redshift limit, this equation reduces to the usual linear Hubble relation between $m$ and $\log(cz)$

$$m(z) = M + 5\log(cz), \quad (6)$$

where $M = M - 5\log(H_o) + 25$ This quantity can be measured from the apparent magnitude and redshift of low-redshift samples of the standard candle independent of $H_o$. Using this value on recent measurements on 42 high redshift SNe Ia’s reported in the supernovae cosmology project [4] together with the observations of the 16 lower redshift SNe Ia’s of the Callan-Tololo survey [4], one can determine the cosmological parameters $\Omega_\Lambda$ and $\Omega_M$. The data eliminates the “minimal inflationary” prediction $\Omega_\Lambda = 0, \Omega_M = 1$ [2, 3, 6, 7].

To explore the concordance of a linear coasting cosmology, we consider a power law cosmology with the scale factor $a(t) = \bar{k}t^\alpha$, with $\bar{k}$, $\alpha$ arbitrary constants. The expression for the luminosity distance is easily seen to be

$$d_L = \left(\frac{\alpha}{H_o}\right)^\alpha (1 + z) S\left[\frac{1}{1 - \alpha} (\frac{\alpha}{H_o})^{1-\alpha} (1 - (1 + z)^{1-\frac{1}{\alpha}})\right] \quad (7)$$

It is straightforward to discover the following relation between the apparent magnitude $m(z)$, the absolute magnitude $M$ and the redshift $z$ of an object for such a cosmology:

$$m(z) = M + 5\log H_o + 5\log\left(\frac{\alpha}{H_o}\right)^\alpha (1 + z) S \left[\frac{1}{1 - \alpha} (\frac{\alpha}{H_o})^{1-\alpha} (1 - (1 + z)^{1-\frac{1}{\alpha}})\right] \quad (8)$$

The above equations have simple forms in the the $\alpha \to 1$ limit:

$$m(z) = M + 5\log H_o + 5\log\left(\frac{1 + z}{H_o} S (\log(1 + z))\right) \quad (9)$$

For example for an open $k = -1$, model the above exact expression reduces to:

$$m(z) = 5\log\left(\frac{z^2}{2} + z\right) + M \quad (10)$$

Figure ‘1’ [4] sums up the Supernova Cosmology project data for supernovae with redshifts between 0.18 and 0.83 together with the set from the Calan / Tololo supernovae, at redshifts below 0.1.
To get an estimate of the goodness of concordance, we fitted the value of $M$ using the low redshift data set [4]. The value comes out to be $M = -3.325$ and confirms the results of Perlmutter et al [1].

Next we used the 54 SN 1a data described by Perlmutter et al. [1, 2, 3] to obtain the value of the parameter $\alpha$ in eqn(8) by minimizing $\chi^2$:

$$\chi^2 = \sum \left[ \frac{m_B(z_i) - m_{Bi}}{\sigma_{mBi}} \right]^2$$  \hspace{1cm} (11)

The summation is over all the data points. Here $m_B(z_i)$ and $m_{Bi}$ are respectively the theoretical and observed values for the apparent magnitude of a supernova and $\sigma_{mBi}$ the corresponding errors are displayed in tables I & II of reference [1]. For $\alpha = 1$ exactly, the limiting form for eqn(8), i.e. eqn(9) is used. The best fit turns out to be $\alpha = 1.001$, $k = -1$. We used standard Monte-Carlo method to generate $10^4$ data sets with a normal distribution around each data point consistent with the error bars for each of the supernovae. The best fit $\alpha$ for each of the data sets was determined and the results are displayed in the histograms (Figures '3(a, b, c)'). Table I sums up the results for all $k = 0, \pm 1$ values. Figure 2 shows the hubble diagram corresponding to the best fit values of $\alpha$ for $k = \pm 1, 0$ in a power law cosmology. We find that linear coasting can not be eliminated for any of the models though the best fit is for the open model with $\alpha = 1.001 \pm 0.043$, and $\chi^2_{\nu}$ (the minimum $\chi^2$ per degree of freedom) 1.18. This is comparable to the corresponding values (1.17) reported by Perlmutter et al for non-minimal inflationary cosmology parameter estimations. Thus a linear coasting is accommodated even in the 68% confidence region. This finds a passing mention in the analysis of Perlmutter [4] who noted that the curve for ($\Omega_\Lambda = \Omega_M = 0$ (for which the scale factor would have a linear evolution) is “practically identical to bestfit plot for an unconstrained cosmology”. 
Table 1: Summary of the Fit Results

| $k$ | $\alpha$       | $\chi^2$ |
|-----|----------------|----------|
| -1  | 1.004 ± 0.043  | 1.179    |
| 0   | 1.182 ± 0.123  | 1.183    |
| 1   | 0.990 ± 0.099  | 1.397    |

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Figure 1:  Hubble diagram, the magnitude residual and the uncertainty - normalized residual plots taken from the supernova cosmology project. “The curve for \((\Omega_\Lambda, \Omega_M) = (0, 0)\) is practically identical to the best fit unconstrained cosmology” [1].
Figure 2: The Hubble diagram for the best fit values of $\alpha$ for $k = \pm 1, 0$ for a power law cosmology projected against the data points of [1, 2].
$\alpha_{\text{mean}} = 1.81(1 \pm 0.04)$

$\alpha_{\text{mean}} = 1.182 \pm 0.122$
Figure 3: Histogram for $\alpha$ from $10^4$ data sets for $k = -1, 0, +1$ respectively.