Assessment of Wind Power Integrated to the Power Systems Using Probabilistic Load Flow Based on PFV with LS

E. Mohammed¹, J. Yu¹, S. Wang¹ and L. Peng¹

¹School of Electrical Engineering and Automation, Harbin Institute of Technology, Harbin 150001, China
moheissa183@gmail.com

Abstract. Power flow analysis (PFA) is one of the most important issues of power system analysis and design. This paper aims at developing and applying a high-speed sampling method to produce large-capacity sample set based on pattern feature vector with layer structure (PFV with LS) of IEEE 14-bus system data analytical methods. In this study, sampling based pattern vectors are constructed in three layers. The first and second layer vectors are very low dimensional, and the sampling of the third layer is in parallel for each local area. Samples generated by this sampling method can be applied to the calculation of probabilistic load flow (PLF) and probabilistic static security assessment. Simulation results show that the proposed method can improve efficiency of PLF analysis and probabilistic static security assessment. The advantages of using PLF based on PFV with LS as the power flow limit can minimize the complicated mathematical equations. Furthermore, the algorithm is very simple and accurate, especially when the system connected to the wind power.

1. Introduction
In recent years, power system operation faces a new challenge due to deregulation and restructuring of electricity markets. The PFA is necessary for planning, operation, economic scheduling and exchange of power between utilities. This paper extends the application of the security assessment index. It aims to show the detailed mathematical model of the uncertainties for the grid connected wind farms. Power grid contains other power generation and wind power uncertainties. However, the effects of the uncertainty can be reduced by accurate calculation.

Power flow (PF) is the very important tool for the analysis power systems, and it is used in operational and planning. The objective of PF is calculating unspecified bus voltage angles and magnitudes, active and reactive powers, as well as line loadings and their associated real and reactive losses for certain generation and load conditions. Different mathematical techniques have been used to examine PF analysis. These techniques are Newton-Raphson (NR), Fast-Decoupled (FD), and Gauss-Seidel (GS) methods.

One of the best PF methods is NR method. The NR process usually converges faster than other methods, but it takes longer computational time per iteration. However, the proposed method in this paper is a concern in accounting for any kind of limit generation, not only line and curve but also the combination of line and curve with discontinuities.

The load flow (LF) studies are performed for power system planning, operation, and control. LF studies data is also used for contingency analysis, outage security assessment, as well as for optimal dispatching and stability. The LF problem has received more attention than all the other power system
problems combined [1]. The LF studies are used to ensure that electric power transfer from generators to consumers through the grid system is stable, reliable and economic.

LF calculation is a standard tool used to analyze the operation and planning of power systems. Normally, a LF calculation solves system states and power flows for a given fixed value of power generations and load demands. This type of calculation is referred to as deterministic LF (DLF). A DLF calculation does not consider uncertain phenomena in power systems, such as generator outages and load variations. Furthermore, modern power systems are commonly integrated with renewable energy based DG units. These renewable generations, such as wind power and photovoltaic generation, are intermittent and not fully controllable due to the stochastic nature of their prime sources.

The PLF analysis technique based on the static security assessment is considered as important to the operation and planning of power systems. PLF technique is an important probabilistic analysis tool for power system, historically suggested by the Borkowska in 1974 [2]. PLF can be used to quantitatively evaluate the impacts of various uncertain factors on PF when the system connected to the wind power is applied. Considerable work related to PLF has been carried out.

Probabilistic reliability techniques are required to model the impacts of wind power on system’s reliability and adequacy [3]. In fact, the correlation between wind power and load directly influences the reliability of system, and for the installation of a new wind farm, the minutely correlation between wind power and load plays a key role in supplying peak load [4]. The static method is widely analyzed for voltage stability assessment when large demand and limited network expansion occur. The relationship between transmitted power and receiving end voltage (PV) curve is one of the powerful tools for stability analysis [5-6]. The probability density functions of loads at different buses show a number of variations and cannot be represented by any specific distribution [7].

Voltage stability is one of the most important and interesting topics for transmission networks. It is defined as the ability of a power system to maintain load voltage magnitudes after disturbances or uncertainty [8]. The increased uncertainty caused by wind power and intermittent energy behavior, it is hard to predict the system operational status. The PV stability analysis should consider how to accurately express the system’s risks. Several researchers have studied the risk-assess based voltage stability problems using probabilistic methods [9].

The basic task of PLF analysis is to obtain the statistic or probability-distributions of the PF responses from the probability-distributions of stochastic input variables such as loads, PV, and wind power generations. The PLF methods can be classified into two types, first one Monte Carlo simulation (MC), and the second analytical methods. On the other hand, the MC technique is regarded as a benchmark to assessing the accuracy and effectiveness of other approximate PLF analytical methods [10-11].

Connecting wind farm to the power network increases the variability and uncertainty of power grid. Therefore, applying accurate calculation techniques, such as probabilistic static security assessment, static security analysis method of a power system with large scale of wind power, and stochastic response surface method, is essential to assure system stability [12-14]. To deal with the correlations between the stochastic input variables in the PLF based on PFV with LS analysis. Therefore, the main contributions of this paper, which are not considered in previously published works, can be listed as follows.

- A new PLF method based on PFV with LS is proposed. It can model stochastic input variables following both normal distributions as loads and non-normal distributions as wind power.
- The correlations between all the stochastic input variables in the PLF based on PFV with LS analysis are considered.
- The correctness and effectiveness of the proposed method are demonstrated by comparing the probabilistic static security assessment power flow results for the IEEE 14-bus systems obtained using the proposed method, the amplitude of the bus voltage combined with PLF by traditional load flow method.
2. Steady state analysis

2.1. Overview of Newton–Raphson load flow equations
In this paper, we used base LF equations for the process of calculation, and the obtained results are compared with those of the proposed method, the exponential model for representation the dependence of active power, and reactive power, on the bus voltage magnitude at a load bus in an electric power network. In conventional LF studies the general equations normally solved are:

\[
P_i = f_i(\delta, V) = V_i \sum_{j=1}^{n} V_j (G_{ij} \cos(\delta_j - \delta_j) + B_{ij} \sin(\delta_j - \delta_j)) \tag{1}
\]

\[
Q_i = g_i(\delta, V) = V_i \sum_{j=1}^{n} V_j (G_{ij} \sin(\delta_j - \delta_j) - B_{ij} \cos(\delta_j - \delta_j)) \tag{2}
\]

- General power balance equations as:

\[
P_i = \sum_{j=1}^{n} |V_i||V_j|(G_{ij} \cos \delta_j + B_{ij} \sin \delta_j) = P_{Gi} - P_{Di} \tag{3}
\]

\[
Q_i = \sum_{j=1}^{n} |V_i||V_j|(G_{ij} \sin \delta_j - B_{ij} \cos \delta_j) = Q_{Gi} - Q_{Di} \tag{4}
\]

- Numerical iteration as:
The set of LF equation (1) & (2) are non-linear and solved by the NR iterative method, which requires finding a Jacobin matrix to update the current estimates of improved solutions. The general forms of the iteration for the load flow analysis are as follows:

Update values = old value + iteration matrix * error

\[
p^{(k+1)} = p^{(k)} + A^{-1}[y - f(p^{(k)})] \tag{5}
\]

\[
\Delta p_i^{(k)} = \Delta c_i^{k} \left( \frac{df_i}{dp} \right)^{k} \tag{6}
\]

The variables that need to optimize from equation (1) & (2) are \( \delta \) and \(|V|\) so the general forms of the iterations equation can be written as follows:

\[
\delta_i^{(k+1)} = \delta_i^{(k)} + \Delta \delta_i^{(k)} \tag{7}
\]

\[
\|V_i^{(k+1)}\| = \|V_i^{(k)}\| + \Delta \|V_i^{(k)}\| \tag{8}
\]

Then the above equations can be written as:

\[
\begin{bmatrix}
\Delta \delta \\
\Delta |V|
\end{bmatrix} =
\begin{bmatrix}
J_1 & J_2 \\
J_3 & J_4
\end{bmatrix}^{-1}
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} \tag{9}
\]

\[
\Delta P_i^{(k)} = P_i^{sch} - P_i^{(k)} \tag{10}
\]

\[
\Delta Q_i^{(k)} = Q_i^{sch} - Q_i^{(k)} \tag{11}
\]
The $J_1, J_2, J_3$, and $J_4$ actually are related to kind of buses, for PV-buses the variable need to optimize are $\delta$ and $Q$. Figure 1 shows the flow chart of the NR based PLF. The LF calculations by NR method are embedded within a cycle that simulates many load scenarios for different realizations.

![Flow chart of the NR based PLF](image)

**Figure 1.** Flowchart of the Newton-Raphson based load flow calculation.

### 2.2. Probabilistic load flow analysis

This section contains two parts about the PLF analysis. The first one focuses on describes the techniques for PLF calculations. First, a short review of PLF methods is provided. Then, the MC based PLF method is discussed, followed by the convolution based and cumulates based PLF methods. The IEEE 14-bus system is used to demonstrate the different methods. The second part of the section investigates the influence of temporal correlation in stochastic generation and load on the accuracy of PLF based on PFV with LS results.

The solution to a PLF calculation can be obtained in three ways: a numerical approach, an analytical approach and a combined approach. The main advantage of the numerical approach is that nonlinear LF equations can be directly used in the PLF analysis, whereas the disadvantage is that a MC simulation is usually very-time consuming. On the other hand, the numerical approach is much faster in computation speed. However, the main issue is that it involves complex mathematical computation, which usually requires linearization of the LF equations.

### 2.3. PLF based on the PFV with LS method

The PLF analysis method based on the PFV with LS is proposed in this paper. First, stochastic input variables including loads $P_L$, PV power outputs $P_{PV}$, and the outputs of wind power $P_W$ are expressed by standard random variables. Second, traditional load flow method is used to represent the PF responses using the standard random variables. Third, the unknown coefficients in the traditional LF are determined by several deterministic PF. Finally, stochastic samples, statistics and probability distributions of the PF responses are established.

The input variables such as: PV generations, wind power, and loads follow different probability distributions. Gaussian, and normal distributions are usually used to model the randomness of $P_{PV}, P_W$, and $P_L$, respectively. The inputs in the PLF based on PFV with LS must be standard random variables.
3. Methodology

In this section, we will describe the detailed procedure of the new approach for pattern feature vector with layer structure (PFV with LS), probabilistic load flow (PLF) distribution of power that considers load and wind power generation uncertainties. The simulation data used are the IEEE data test 14 buses. To show the advantages of using PLF based on PFV with LS, the load data each bus is increased.

3.1. Procedure of the proposed method

Based on the above subsections, the overall procedure of the proposed method for PLF based on the PFV with LS can be summarized as follows:

- Firstly, the PFV is composed of total load power of the system, so its dimension is very low.
- Secondly, the PFV dimension equals to the number of targeted sub-systems, and its value is relatively low; the PFVs for the second layer and the third layer are respectively composed of the power ratio coefficients of the second layer to the first layer and of the third layer to the second layer.
- In addition, the PFV for the final can be processed in parallel by decomposing the sub-systems of the second layer into multiple sub-vectors. Thus, the low-dimension of vector and parallel processing of sub-vectors enable high sampling efficiency.

The proposed method is tested by simulation on IEEE 14-bus corrected system, and problem of probabilistic static security assessment of the power system with wind power integration was analyzed in terms of bus voltage, branch current values, CDF, etc. The framework of the proposed assessment based PLF method is shown in figure 2, which shows the process of classification and steps of security assessment from PLF. The 1st and 2nd steps are data processing; the 3rd and 4th steps are using approaches; the 5th to 8th steps assess static security, and the 9th step displays the results.

![Figure 2. Framework of the PLF method process of security assessment.](image-url)
4. Case study

The verification of the proposed method in this paper includes three parts. The first proposed method, then the PPF by traditional load flow, and finally the base cases are applied to the PLF based on PFV with LS of the IEEE 14-bus test system. The simulation results of the three methods are compared to demonstrate the accuracy and efficiency of the proposed method.

4.1. Comparisons between proposed method and PPF by traditional load flow method

Accuracy of the proposed method in this section: The modified IEEE 14-bus test system given in figure 3 is composed of 14 buses and 19 branches. The performance is evaluated on the IEEE 14-bus test system using three indexes: power generation, inter-area flow and total load for the system.

Assume that the case study system contains three local areas, the first area containing three load buses L1, L2 and L3, then the second area containing two load buses L4 and L5, and the last area containing three load buses L6, L7 and L8. For simplifying the analysis, power factors of all loads and wind farms are assumed to be constant. Suppose also the bus loads in the system are assumed to follow normal distributions with the mean values being original bus loads and the standard deviation values being 5% of the mean values, the original system is modified which includes two generators of wind farms and eight load buses. 24.65 & 11.70 MW PV generators are added at buses 6 and 7, respectively.

In PFV with LS, taking the calculate of load as example, assume that pattern feature vector has 3-layer structure.

The 1st layer is system total load, denoted as $P_1$

$$X_1 = (P_1)^T$$  \hspace{1cm} (12)

The 2nd layer includes loads of three local areas, denoted as $P_2(1), P_2(2), P_2(3)$

$$X_2 = (P_2(1)/P_1, P_2(2)/P_1, P_2(3)/P_1)^T$$  \hspace{1cm} (13)

The 3rd layer includes loads of eight buses, denoted as $P_3(1-1), P_3(1-2), P_3(1-3), P_3(2-1), P_3(2-2), P_3(3-1), P_3(3-2), P_3(3-3)$

$$X_3 = (P_3(1-1)/P_2(1), P_3(1-2)/P_2(1), P_3(1-3)/P_2(1), P_3(2-1)/P_2(2), P_3(2-2)/P_2(2), P_3(3-1)/P_2(3), P_3(3-2)/P_2(3), P_3(3-3)/P_2(3))^T$$  \hspace{1cm} (14)

As shown in figure 4, the P-V curve is obtained based on the continuation PF, plotted by the change of (p.u) power injection on buses on the horizontal axis and the voltage of node on the vertical.

![Figure 3. Single-line diagram of the modified IEEE 14-bus system.](image-url)
The simulation results of proposed method are usually over optimistic because it takes the uncertainties of load and wind power into consideration. Get the change of PV curves before (base case), and after traditional and proposed methods) connecting wind farm to power grid, at the bus 6 are shown in figure 4. The results show the voltage increment due to injection of wind power. The voltage steady-state stability of the system is more than the case without wind power.

![Figure 4](image-url)

**Figure 4.** The P-V curve of load bus without and with wind power, with different methods.

LF problem has been solved for the case study considering the wind farms and the bus voltages have been obtained. Figure 5 shows the voltage profile of the network buses without wind farm (base case), with wind farm connected to the system by traditional method, and proposed method. From figure 5 shows that the proposed method gives a better solution for voltage improvement. In this figure, the blue line corresponds to the PLF by traditional LF, and the green line corresponds to the base case.

![Figure 5](image-url)

**Figure 5.** Voltage profile of 14-bus system at the different methods.

In figure 6, the voltage stability margin is obtained by proposed, traditional and base case sequence with the different cases, and cumulative distribution function (CDF) curve of voltage stability margin are plotted. Figure 6 shows the CDF of the voltage at bus 6 in a modified IEEE 14-bus system obtained by using three different methods: base case simulation, traditional method and proposed method. For comparison, the CDF of the voltage obtained from the proposed method match the voltage from traditional method simulation very well. The traditional method is slightly worse than the proposed method, but the result also resembles the proposed results adequately.
Figure 6. The voltage CDF of bus 6 before and after wind power integration.

4.2. Comparisons of numerical values with different methods
The proposed method (denoted as the PLF based on PFV with LS in the tables and figures in this section), the PFV with LS, and the traditional method are applied to solve the PLF of the IEEE 14-bus test system. The wind farms are located at the buses 6 and 7; simulations are carried out for the three cases. For easy comparison the results are summarized and given in table 1. The means of the bus voltage magnitudes were calculated and are given in table I.

Table 1. Means of the bus voltage magnitudes (p.u) in the IEEE 14-bus modified test system.

| Bus No | Calculate mean voltage values at: | Bus No | Wind injected at bus 6, 7 by |
|--------|----------------------------------|--------|-----------------------------|
|        | Base case | Traditional | Proposed | Base case | Traditional | Proposed |
| 1      | 1.04000   | 1.06000     | 1.06700   | 8        | 1.07000   | 1.09000   | 1.09700   |
| 2      | 1.02500   | 1.04500     | 1.01200   | 9        | 0.97994   | 1.00070   | 1.00613   |
| 3      | 0.99900   | 1.01000     | 1.01700   | 10       | 0.98120   | 1.00190   | 1.00750   |
| 4      | 0.97587   | 0.99601     | 1.00273   | 11       | 1.00970   | 1.03010   | 1.03640   |
| 5      | 0.98110   | 1.00120     | 1.00800   | 12       | 1.02200   | 1.04300   | 1.04870   |
| 6      | 1.05000   | 1.07000     | 1.07700   | 13       | 1.00840   | 1.02890   | 1.03480   |
| 7      | 1.00900   | 1.02940     | 1.03560   | 14       | 0.94146   | 0.96393   | 0.96597   |

When the power generated from the RES is supplied at the load bus locally, the overall demand on the system is significantly reduced. This improves the voltage profile and minimizes the power loss of power grid. Table 1 describe the numerical results of the bus voltages for the IEEE 14-bus system at two cases: without and with wind power. From the table it can be inferred that the voltage profile at all the load buses is improved.

Table 2. Means of the branch voltage magnitudes (p.u) in the IEEE 14-bus modified test system.

| Branch No | Branch voltage values at: | Branch No | Wind injected at bus 6, 7 by |
|-----------|--------------------------|-----------|-----------------------------|
|           | Base case | Traditional | Proposed | Base case | Traditional | Proposed |
| 1-2       | 0.330680 | 0.332681     | 0.337681 | 7-8       | 0.220465   | 0.223468 | 0.224893 |
| 2-4       | 0.438033 | 0.440056     | 0.445011 | 1-14      | 0.391069   | 0.393069 | 0.398069 |
| 3-4       | 0.355585 | 0.357585     | 0.362585 | 3-13      | 0.408001   | 0.410011 | 0.415022 |
| 4-5       | 0.364624 | 0.366624     | 0.371624 | 12-13     | 0.413213   | 0.415203 | 0.420218 |
| 5-6       | 0.075854 | 0.076851     | 0.079894 | 12-14     | 0.418905   | 0.420000 | 0.425011 |
| 5-9       | 0.372630 | 0.374631     | 0.379632 | 11-12     | 0.378136   | 0.380136 | 0.385136 |
| 6-9       | 0.423022 | 0.425011     | 0.430210 | 10-11     | 0.428001   | 0.430956 | 0.435051 |
| 6-7       | 0.166247 | 0.166330     | 0.167049 | 8-12      | 0.433011   | 0.435012 | 0.440055 |
| 6-8       | 0.197949 | 0.200705     | 0.206138 | 8-11      | 0.345760   | 0.347762 | 0.352762 |
| 5-8       | 0.388358 | 0.390358     | 0.395358 |

As can be seen from table 1 and table 2, the calculated value of the PLF result by using the traditional method has been barely affected, but the deviation is much bigger than that using the
proposed method. This shows that the relevance of node power injection increases the fluctuation of the node voltage and the branch current values. Therefore, when the system is close to the limit running state, the result of node voltage amplitude and branch, current flows are more likely to go beyond the range by using the proposed method. This is because when the relevance of node power injection is high, the probability of the output of each wind farm or the load of each node increasing or decreasing at the same time is higher. The traditional probability method does not take this into account, so it is easy to underestimate the risk of system operation.

5. Conclusion
This paper presents a method for assessing power flow for wind power integrated into the power systems with the consideration of wind and load-power generation correlations. Using PLF based on PFV with LS for calculations, the possibility load flow can be computed and distribution functions of the voltage magnitude. The effectiveness of the proposed method is verified via the modified IEEE 14-bus system. Simulation numerical results show that the proposed method is appropriate for security assessing load flow of wind-integrated power systems. It is observed that the types, and the parameters of uncertain wind power distribution would affect the security assessment indices of load flow. Thus, appropriate shape and scale parameters of Gaussian distribution for wind power should be carefully tuned. In addition, wind resource and load power generation correlations would also impact the security assessment indices of load flow, and needed to be considered when assessing the load flow security. Security assessment indices with the consideration of random variable correlation would more accurately reflect the system security and provide more effective information for the decision-maker.

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