Near-Field Intelligent Reflecting Surfaces for Millimeter Wave MIMO Full Duplex
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Abstract—Full duplex (FD) systems suffer from very high hardware cost and high power consumption to mitigate the self-interference (SI) in the analog domain. Moreover, in millimeter wave (mmWave) they rely on hybrid beamforming (HYBF) as a signal processing tool to partially deal with the SI, which presents many drawbacks, e.g., high insertion loss, high power consumption, high computational complexity for its configuration. This article proposes the use of near-field (NF-) IRSs for FD systems with the objective to solve the aforementioned issues cost-efficiently. Namely, we propose to truncate the analog stage of the mmWave FD systems and assist them with an NF-IRS, to simultaneously and smartly control the uplink (DL) and downlink (DL) channels, while assisting in shaping the SI channel: this to obtain very strong passive SI cancellation. A novel joint active and passive beamforming design for the weighted sum-rate (WSR) maximization of a NF-IRS-assisted mmWave point-to-point FD system is presented. Results show that the proposed solution fully reaps the benefits of the IRSs only when they operate in the NF, which leads to considerably higher gains compared to the conventional massive MIMO (mMIMO) mmWave FD and half duplex (HD) systems.

Index Terms—Full duplex, Near-Field Intelligent Reflecting Surfaces, Passive Self-Interference Cancellation, Millimeter Wave, Hybrid Beamforming

I. INTRODUCTION

To achieve ubiquitous connectivity for continuously evolving wireless networks, intelligent reflecting surfaces (IRSs) are considered as one of the most prominent hardware technology for beyond fifth generation (B5G) and six generation (6G) networks [1], [2], with the potential to create smart, reconfigurable, and highly energy-efficient wireless systems. An IRS is a metasurface made of passive elements, which can be flexibly programmed to reflect the impinging signals in the desired way [3].

On one hand, both its low hardware cost and the passive nature of its meta elements enable a sustainable evolution of wireless networks [4]. On the other hand, given the limited wireless spectrum, there has been a growing interest in millimeter wave (mmWave) full duplex (FD) technology, which offers simultaneous transmission and reception in the same frequency band, thus doubling the spectral efficiency with respect to half duplex (HD) systems [5]. However, FD systems suffer from self-interference (SI), which can be from 90 to 110 dB higher than the received signal of interest [6]. SI cancellation (SIC) techniques are a viable tool to mitigate this impairment [7], and they can operate both in the analog and digital domains [8]–[10]. Analog SIC techniques entail high hardware cost and power consumption to sufficiently mitigate SI and avoid the analog-to-digital converter (ADC) saturation [11], [12]. Hybrid beamforming (HYBF) is also considered as a promising signal processing tool to reduce SI in mmWave FD systems [13]–[15]. Although enabling the design of mmWave FD systems with fewer radio frequency (RF) chains, the main pitfalls of HYBF systems include high power consumption, high insertion loss [19], [20], and high computational complexity for the phasors’ configuration, which further enhances power consumption [21]. Such inherent characteristics of HYBF motivates the exploration of other alternatives.

A. State-of-the-Art

The mmWave FD systems aided by IRSs can potentially revolutionize the wireless communications and pave the path towards highly spectrally efficient, energy efficient, smart, reconfigurable, and sustainable networks [22]. Recent studies on FD systems assisted with the IRSs, limited only to sub-6 GHz bands, have been presented in [23]–[30]. In [23], a novel beamforming design is proposed for decode-and-forward FD relays assisted by one IRS to maximize the minimum achievable rate. In [24], the authors proposed a joint active and passive beamforming design to cover dead zones in a multi-user multiple-input single-output (MISO) FD network, while suppressing the single-antenna FD user-side self and co-channel interference. In [25], the performance of an IRS-aided FD system in terms of outage and error probabilities and the impact of IRS in mitigating SI on single-antenna FD system is investigated. In [26], the authors presented a novel beamforming design for a MISO FD system with two users (one in uplink (UL) and one in downlink (DL)) to minimize the power consumption of the FD access point while the UL user is subject to the minimum rate constraint. In [27], a novel beamforming design is presented, based on the Arimoto-Blahut algorithm for IRS-aided point-to-point FD systems, to maximize the sum-rate. The importance of IRSs in improving the security of FD systems under imperfect channel state information (CSI) is analyzed in [28], where a novel beamforming design is proposed to maximize the worst-case achievable secrecy rate under a bounded CSI error model. A mixed time-scale joint active and passive beamforming design for an IRS-aided multi-user FD system is presented in [29].
and by resorting to a deep neural network the computational burden of the proposed solution is further reduced. In [30], the capability of IRSs in lowering the SI for a single-antenna FD system is shown on an experimental testbed. In [31], the authors evaluated the potential of an IRS in canceling the SI by placing it close the FD node. However, such work is limited to the sub-6 GHz band and only SI with IRS was investigated, without considering any UL traffic. Moreover, this work is limited to a simple MISO case.

B. Motivation and Main Contributions

Although the aforementioned papers studied the IRS-aided FD systems in many different scenarios, all results are limited to the sub-6 GHz band. Moreover, the advantage of handling SI is analyzed mostly for single-antenna FD users, with IRSs placed in the far field (FF), which can potentially limit the impact of passive SIC, due to a smaller power gain than the SI channel. In [31], the authors considered SI with the IRS in the near-field (NF). However, this work is limited to a MISO case and does not consider UL traffic in their design, which motivates further studies by considering a general case of multiple-input multiple-output (MIMO) FD systems, and a more practical case wherein the IRS is not used only for SIC purposes, but also to simultaneously improve the UL and DL channel quality. Moreover, despite the drawbacks of the mmWave massive MIMO (mMIMO) FD systems such as high hardware cost and power consumption, a solution to overcome them has not yet been explored.

This work proposes to remove the analog stage of the mmWave FD system in both transmission and reception by allowing each FD node to operate with few antennas in a fully digital mode. To tackle the propagation challenges of mmWave, we propose to assist each FD node with an NF-IRS. We remark that the proposed idea herein is also prominent in THz FD systems. However, as the investigation of such systems is still nascent, we restrict our work to the mmWave band. The objective of the NF-IRSs is to simultaneously compensate for the analog beamforming and combining gain of the mmWave FD nodes by providing a smart control of the UL and DL channels. Moreover, since the IRSs operate in the NF, each NF-IRS also jointly assists in shaping the MIMO SI channel, leading to a very strong passive SIC scheme. Note that the MIMO FD nodes also benefit from the active digital beamforming gain for SIC, which, combined with passive SIC, leads to superior SI suppression. We consider the maximization of the weighted sum-rate (WSR) criteria and propose a novel joint active and passive beamforming design for the NF-IRS-aided mmWave FD systems. Compared to mMIMO mmWave FD, our method requires only a few active components, leading to a significant reduction in power consumption, hardware cost, and computational complexity. Moreover, our method also significantly reduces the power and hardware costs of the analog SIC architecture, which strictly depends on the number of antenna elements. Simulation results show that the proposed method outperforms the traditional mmWave mMIMO fully-digital HD system with only a few antenna elements. Moreover, the NF-IRSs not only replace the analog stage but also provide significantly larger gains than the mMIMO FD systems. Namely, a MIMO FD aided with a NF-IRS of size $30 \times 30$ can achieve ~4 times higher gain than the mMIMO HD system while operating with 5 times less antennas; meanwhile, the classical mMIMO FD system could theoretically only achieve 2 times higher gain. Finally, results also show that the MIMO FD systems can fully reap the benefits of the IRSs only when they operate in the NF. IRSs operating in the far field (FF) can provide only a small gain over the conventional mMIMO FD systems.

Paper Organization: The rest of the paper is organized as follows. First, we introduce the system model of the considered system in Section II. The novel joint active and passive beamforming design is then derived in Section III. Finally, Sections IV and V present the numerical results and conclusions, respectively.

Mathematical Notations: Boldface lower and upper case characters denote vectors and matrices, respectively. $\mathbb{E}\{\cdot\}$, $\text{Tr}\{\cdot\}$, $\mathbf{I}$ and $\odot$ denote expectation, trace, identity matrix, and Hadamard product, respectively. The superscripts $(\cdot)^T$ and $(\cdot)^H$ denote transpose and conjugate-transpose (Hermitian) operators, respectively.

II. SYSTEM MODEL

We consider a mmWave point-to-point FD communication system consisting of two mmWave MIMO FD nodes, indicated with indices $l$ and $r$. They are assisted with one NF-IRS each, denoted with the indices $i_l$ and $i_r$, respectively. Both the IRSs are assumed to be placed in such a way that the link between the SI channel and the other node is in line-of-sight (LoS). Each IRS operates in the NF for the assisted FD node and in the FF for the other node, as shown in Fig. 1. The MIMO FD node $l$ is assumed to be equipped with $M_l$ transmit and $N_l$ receive antennas, while node $r$ is assumed to be equipped with $M_r$ transmit and $N_r$ receive antennas. Let $\mathbf{V}_l \in \mathbb{C}^{M_l \times d_l}$ and $\mathbf{V}_r \in \mathbb{C}^{M_r \times d_r}$ denote the digital beamformers of the two nodes, for the unitary-variance data streams $\mathbf{s}_l \in \mathbb{C}^{d_l \times 1}$ and $\mathbf{s}_r \in \mathbb{C}^{d_r \times 1}$, respectively. The IRS $i_l$ has $L_l \times L_c$ elements, and IRS $i_r$ has $R_r \times R_c$, elements. Let $\phi_l$ and $\phi_r$ denote vectors collecting the phase shift response of the IRSs $i_l$ and $i_r$, respectively: the elements of $\phi_l$ and $\phi_r$
at position $i$ and $j$ are phasors of the form $\phi_i(j) = e^{i\theta_i(j)}$ and $\phi_r(j) = e^{i\theta_r(j)}$, respectively. Let $\Phi_l = \text{diag}(\phi_l)$, $\Phi_r = \text{diag}(\phi_r) \in C^{R \times R}$. Let $\Psi_l, \Psi_r \in C^{N \times N}$ denote the diagonal matrices containing the phase responses of the IRSs ij and ir, respectively. Let $H_{g,l,i} \in C^{L \times M}$ and $H_{g,l,i} \in C^{R \times R}$ denote the direct channels from the FD node l to its NF IRS ij and to the other IRS i, operating in FF, respectively. A similar notation holds for node ir. The channel matrices from IRSs ij and ir to the receive antenna array of FD node l are $H_{r,ij} \in C^{N \times M}$ and $H_{r,ir} \in C^{N \times M}$, respectively. The channel matrices from the IRSs ij and ir to the receive antenna array of node r are $H_{r,ij} \in C^{N \times M}$ and $H_{r,ir} \in C^{N \times M}$, respectively. Let $H_{r,ij} \in C^{R \times M}$ and $H_{r,ir} \in C^{R \times M}$ denote the channel from IRS ij to the IRS ir, while the channel from IRS ir to IRS ij is denoted with $H_{r,ij}$. The direct channels from node l to node r and from node r to node l are denoted with $H_{l,ij} \in C^{N \times M}$ and $H_{r,ir} \in C^{N \times M}$, respectively. The SI channel for the MIMO nodes l and r are denoted with $H_{l,ij} \in C^{N \times M}$ and $H_{r,ir} \in C^{N \times M}$.

A. Channel Modeling

Let $a_i(\theta_{ip,mc})$ and $a^T_i(\theta_{ip,mc})$ denote the receive and transmit antenna array response at the MIMO node l and r, respectively, with angle of arrival (AOA) $\theta_{ip,mc}$ and angle of departure (AoD) $\theta_{ip,mc}$. The direct channel $H_{l,ir}$ from node r to node l in mmWave can be modeled as [14]

$$H_{l,ir} = \sqrt{\beta_{l,r}} H_{l,ir} = \sum_{c=1}^{N_c} \sum_{n=1}^{N_p} a_i^T(\theta_{ip,mc}) a_i(\theta_{ip,mc}),$$  

where $N_c$ and $N_p$ are the number of clusters and rays [14].

Fig. 1, respectively, and $a_i(\theta_{ip,mc}) \sim CN(0,1)$ is a complex Gaussian random variable. Thus, the channels have amplitude and phase distributed according to the Rayleigh and uniform distribution, respectively. Note that, compared to [13], 1 is scaled by $\beta_{l,r} = D_{l,r}/D_{l,r}$, which captures its dependency on the distance, with $D_{l,r}$ being a distance between the centers of the MIMO FD nodes l and r: this is equal to one for the direct link [1] and will be larger than 1 for all the channels closer than $D_{l,r}$, emphasizing the stronger channel gain. Namely, for any channel matrix $H_{m,n}$, its distance-dependent scale factor is $\beta_{m,n} = D_{m,n}/D_{m,n}$, with $D_{m,n} \leq D_{m,n}, \forall m,n$. The remaining FF channels $H_{l,ir}, H_{l,ir}, H_{r,ir}, H_{r,ir}, H_{r,ir}, H_{r,ir}$, and $H_{r,ir}$ can be modeled similarly to [1]. The SI channel for node l can be modeled as [14]

$$H_{l,ir} = \sqrt{\beta_{l,r}} H_{l,ir} + \sum_{m=1}^{M} \frac{1}{\kappa_l + 1} H_{l,ir}^{\text{LoS}} + \frac{1}{\kappa_l + 1} H_{l,ir}^{\text{Re}}$$.

$$H_{l,ir} = \frac{\rho}{r_{m,n}} e^{-i2\pi d_{m,n}/\lambda},$$

where $\lambda$ denotes the wavelength and the power normalization constant $\rho$ assures that $\mathbb{E}(|H_{l,ir}^{\text{LoS}}|^2) = M_{l,r}$. The scalar $d_{m,n}$ denotes the distance between m-th receive and n-th transmit antenna of node l $\in \mathcal{F}$, which depends on the transmit and receive array geometry (9) [14]. A similar modelling holds also for the LoS component $H_{l,ir}^{\text{LoS}}$ of the SI channel $H_{l,ir}$.

Matrix $H_{l,ir}^{\text{LoS}}$ considers the LoS path between the transmit and receive antennas, which are in the NF and its modeling will be discussed after a brief introduction to the NF concept. Due to the NF-IRSs, channels $H_{l,ir}, H_{l,ir}, H_{r,ir}, H_{r,ir}$ cannot be modeled as in [1]. To shed light on the NF concept, let $D$ denote the size of the radiating element, which could be either the transmit array of MIMO FD node i $\in \mathcal{F} = \{l, r\}$, or the IRS operating in NF. The space surrounding any transmitting device can be divided into three regions: 1) Reactive NF (RE-NF), 2) Radiating NF (RA-NF), and 3) FF, as shown in Fig. 2. The radius $R_1$ and $R_2$ are the boundaries between the regions and depend on both the size $D$ and the wavelength $\lambda$. Regions RE-NF, RA-NF, and FF are also known as inductive NF, Fresnel zone, and Fraunhofer zone, respectively. In the RE-NF zone, the reactive fields dominate the radiation fields, and therefore its not of interest for wireless communications. In the RA-NF zone, the radiation fields dominate the reactive fields, however, the received wave has a spherical wave-front. In the FF zone, the received wave-front is plane and the receiver sees the transmitter as a point-wise source. In mmWave FD systems, to compensate for the analog stage and assist with SIC, NF-IRSs should be placed in the RA-NF zone, i.e., at a distance $R_1 \leq z \leq R_2$. Therefore, in contrast to the FF case, we consider a spherical wave-front for the channels $H_{l,ir}, H_{l,ir}, H_{r,ir}, H_{r,ir}, H_{r,ir}$, and $H_{r,ir}$, which is also true for the LoS components of the SI channels for FD nodes l and r.

Since the LoS path between the transmit and receive antennas of the FD node l is in NF, the elements of channel matrix $H_{l,ir}^{\text{LoS}}$ can be modelled as [14]

$$H_{l,ir}^{\text{LoS}}(m,n) = \frac{\rho}{r_{m,n}} e^{-i2\pi d_{m,n}/\lambda},$$

where $\lambda$ denotes the wavelength and the power normalization constant $\rho$.
on the NF-IRS $i_l$ transmitted from node $l \in \mathcal{F}$ is spherical; therefore, similar to \cite{3}, we consider a variation in the phase-shift due to distance from each antenna element of FD node $l$ to each element of IRS $i_l$ as

$$H_{i_l}(m, n) = \sqrt{\beta_{i_l}} \left[ \frac{p_{i_l}}{d_{m,n}} e^{-jkd_{m,n}} \right],$$

(4)

where $k = 2\pi/\lambda$ is the wave number and $d_{n,m}$ denotes the distance between the $n$-th transmit antenna and $m$-th element of the IRS, with $1 \leq n \leq M_l$, $1 \leq m \leq L_r$, $p_{i_l}$ is a normalization constant to assure $E[|H_{i_l}(m, n)|^2] = L_r L_c M_l$, and $\beta_{i_l} = D_{l,r}/D_{i_l}$ is the distance-dependent average channel gain.

Similarly, for the NF channel from the IRS $i_l$ to the receive antenna array of FD node $l$, we have

$$H_{i_l,i}(i, j) = \sqrt{\beta_{i_l}^{ij}} \left[ \frac{p_{i_l}}{d_{i,j}} e^{-jkd_{i,j}} \right],$$

(5)

where $d_{i,j}$ denotes the distance from the $j$-th element of IRS $i_l$ to the transmit antenna $i$ at the FD node $l$, with $1 \leq j \leq L_r L_c$ and $1 \leq i \leq N_l$, $p_{i_l}^{ij}$ is the power normalization constant which assures $E[|H_{i_l,i}(i, j)|^2] = L_r L_c N_l$, and $\beta_{i_l}^{ij} = D_{i,l}/D_{i,l}$ is the distance dependent average channel gain. The elements of the channel matrices $H_{i_l,i}$ and $H_{i_l,i}$ can also be modeled similarly as \cite{4} and \cite{5}, respectively. For a concise summary of the aforementioned notations, see Table I.

| $V_I$ | Digital beamformer for FD node $l$ |
|-------|----------------------------------|
| $V_R$ | Digital beamformer for IRS $i_I$ |
| $\Phi_I$ | Diagonalized phase response of IRS $i_I$ |
| $\Phi_R$ | Diagonalized phase response of IRS $i_R$ |
| $H_{i_l,i}$ | Channel matrix from FD node $l$ to IRS $i_l$ |
| $H_{i_l,r}$ | Channel matrix from FD node $l$ to IRS $i_r$ |
| $H_{i_r,i}$ | Channel matrix from FD node $r$ to IRS $i_l$ |
| $H_{i_r,r}$ | Channel matrix from FD node $r$ to IRS $i_r$ |
| $H_{l,i}$ | Channel matrix from IRS $i_l$ to FD node $l$ |
| $H_{l,r}$ | Channel matrix from IRS $i_r$ to FD node $r$ |
| $H_{l,l}$ | Channel matrix from IRS $i_l$ to IRS $i_l$ |
| $H_{l,r}$ | Channel matrix from IRS $i_l$ to IRS $i_r$ |
| $H_{r,l}$ | Channel matrix from IRS $i_r$ to IRS $i_l$ |
| $H_{r,r}$ | Channel matrix from IRS $i_r$ to IRS $i_r$ |
| $H_{l,r}$ | Channel matrix from FD node $l$ to IRS $i_r$ |
| $H_{r,l}$ | Channel matrix from IRS $i_l$ to FD node $r$ |
| $H_{l,l}$ | Channel matrix from IRS $i_l$ to IRS $i_l$ |
| $H_{l,r}$ | Channel matrix from IRS $i_l$ to IRS $i_r$ |
| $H_{r,l}$ | Channel matrix from IRS $i_r$ to IRS $i_l$ |
| $H_{r,r}$ | Channel matrix from IRS $i_r$ to IRS $i_r$ |

HYBF to achieve performance comparable to the fully digital beamforming, \cite{14}, \cite{32}. Assuming uniform linear arrays (ULAs) with antenna elements separated by half-wavelength, for a typical antenna array with 32 antenna elements, we have $D = 16\lambda$. Typically, the IRS are made of hundreds of meta elements, and therefore, the NF region of each MIMO FD node is strictly dictated by the size of the NF-IRS. Assuming an NF-IRS $i_l$ of size $50\lambda \times 50\lambda$, the boundary regions can be identified as $R_1 = 0.62 \sqrt{L_{max}^3/\lambda}$ and $R_2 = 2L_{max}^2/\lambda$, where $L_{max} = \max\{L_r, L_c\}$, and in mmWave band, typical values are reported in Table II.

| $\lambda$ | $R_1$ | $R_2$ |
|----------|-------|-------|
| 1 mm     | 0.22 m| 5 m   |
| 10 mm    | 2.19 m| 50 m  |

Note that, given the shorter distance between the FD nodes and IRSs, it is possible to control to the IRS with the IRS controller connected through a wired connection instead of a wireless connection, which can lead to enhanced security and reliability of the control signaling.

### B. Problem Formulation

Let $\hat{H}_{l,r}, \hat{H}_{l,i}, \hat{H}_{r,l}$ and $\hat{H}_{r,r}$ denote the effective channels $H_{l,r}, H_{l,i}, H_{r,l}$ and $H_{r,r}$, respectively, including the smartly controlled paths by the IRS response, given as in \cite{6}. Let $y_l$ and $y_r$ denote the total received signal at the FD nodes $l$ and $r$, respectively, which can be written as

$$y_l = \hat{H}_{l,i} V_i s_l + \hat{H}_{l,r} V_r s_l + n_l,$$

(7a)

$$y_r = \hat{H}_{r,r} V_r s_r + \hat{H}_{r,r} V_r s_r + n_r,$$

(7b)

with $n_l \sim CN(0, I)$ and $n_r \sim CN(0, I)$ denoting the additive white Gaussian noise (AWGN) vectors. Let $\bar{k}$ denote the indices in the set $\mathcal{F}$ except the element $k$. Let $(R_l, R_f)$ and $(R_l, R_f)$ denote the (signal and) interference plus noise covariance matrices, which, with the effective channel matrices, can be written as

$$R_l = \hat{H}_{l,r} V_r V_r^H \hat{H}_{l,r}^H + \hat{H}_{l,i} V_i V_i^H \hat{H}_{l,i}^H + I,$$

(8a)

$$R_f = \hat{H}_{r,r} V_r V_r^H \hat{H}_{r,r}^H + \hat{H}_{r,r} V_r V_r^H \hat{H}_{r,r}^H + I,$$

(8b)

$$R_f = R_l - S_l, \quad R_f = R_r - S_r.$$

(8c)

The WSR maximization problem for the NF-IRSs assisted mmWave MIMO FD system, under the sum-power constraint

$$\hat{H}_{l,r} = H_{l,r} + \hat{H}_{i,r} \Phi_l \hat{H}_{i,r}, \quad \hat{H}_{l,i} = H_{l,i} + \hat{H}_{i,i} \Phi_l \hat{H}_{i,i}, \quad \hat{H}_{r,r} = H_{r,r} + \hat{H}_{i,r} \Phi_r \hat{H}_{i,r}, \quad \hat{H}_{r,i} = H_{r,i} + \hat{H}_{i,i} \Phi_r \hat{H}_{i,i}.$$
for the FD nodes and the unit-modulus constraint for the NF-IRSs can be formally stated as

\[
\begin{align*}
\max_{V_f, V_r, \Phi_f, \Phi_r} & \quad w_l \ln \det (R_l^{-1} R_l) + w_r \ln \det (R_r^{-1} R_r) \\
\text{s.t.} & \quad \text{Tr}(V_k V_k^H) \leq p_k, \quad \forall k \in \mathcal{F}, \\
& \quad |\phi_f(i)| = 1 \& |\phi_f(j)| = 1, \forall i, j,
\end{align*}
\]  

(9a)

(9b)

(9c)

where \(w_l\) and \(w_r\) denote the rate weights for node \(l\) and \(r\), respectively, \(p_l\) and \(p_r\) denote their total sum power constraint, respectively.

III. JOINT ACTIVE AND PASSIVE BEAMFORMING

The WSR maximization problem stated above is not concave, and finding its global optimum is challenging. To obtain a simpler solution, we adopt the weighted minimum mean squared error (WMMSE) method, which solves \(\mathcal{P}\) based on alternating optimization, leading to a sub-optimal solution \(\mathcal{P}_3^\beta\).

To proceed, we assume that the MIMO FD nodes \(l\) and \(r\) deploy the digital combiners \(F_l\) and \(F_r\) to estimate the received data streams, i.e.,

\[
\hat{s}_l = F_l r_l, \quad \hat{s}_r = F_r r_r.
\]  

(10)

Let \(E_l\) and \(E_r\) denote the mean squared error (MSE) matrices for the MIMO FD nodes \(l\) and \(r\), respectively. Given \(\mathcal{P}_3\), they can be written as

\[
\begin{align*}
E_l &= \mathbb{E}[(F_l r_l - s_l)(F_l r_l - s_l)^H] = F_l \hat{H}_{l,r} V_r V_r^H \hat{H}_{l,r}^H F_l^H \\
& \quad - F_l \hat{H}_{l,r} V_r + F_l \hat{H}_{l,r} V_l V_l^H \hat{H}_{l,r}^H F_l^H + F_l F_l^H \\
& \quad - V_r^H \hat{H}_{l,r}^H F_l^H + I, \\
E_r &= \mathbb{E}[(F_r r_r - s_l)(F_r r_r - s_l)^H] = F_r \hat{H}_{r,l} V_l V_l^H \hat{H}_{r,l}^H F_r^H \\
& \quad - F_r \hat{H}_{r,l} V_r + F_r \hat{H}_{r,l} V_r V_r^H \hat{H}_{r,l}^H F_r^H + F_r F_r^H \\
& \quad - V_r^H \hat{H}_{r,l}^H F_r^H + I.
\end{align*}
\]  

(11a)

(11b)

We assume that combiners \(F_l\) and \(F_r\) are optimized based on the minimum MSE (MMSE) criteria, and therefore, they can be obtained by solving the following optimization problem

\[
\min_{F_r, F_r} \quad \text{Tr}(E_l) + \text{Tr}(E_r),
\]  

which can be written as the following expressions for the MMSE combiners

\[
\begin{align*}
F_l &= V_l^H \hat{H}_{l,r}^H (\hat{H}_{l,r} V_r V_r^H \hat{H}_{l,r}^H + \hat{H}_{l,r} V_r V_r^H \hat{H}_{l,r}^H + I)^{-1}, \\
F_r &= V_r^H \hat{H}_{r,l}^H (\hat{H}_{r,l} V_l V_l^H \hat{H}_{r,l}^H + \hat{H}_{r,l} V_l V_l^H \hat{H}_{r,l}^H + I)^{-1}.
\end{align*}
\]  

(13a)

(13b)

Assuming the combiners to be fixed according to \(\hat{F}_l\) and \(\hat{F}_r\), by plugging these expressions in the error covariance matrices, it is immediate to show that the error covariance matrices can be written as

\[
\begin{align*}
E_l &= (I + V_l^H \hat{H}_{l,r}^H R_l \hat{H}_{l,r} V_l)^{-1}, \\
E_r &= (I + V_r^H \hat{H}_{r,l}^H R_r \hat{H}_{r,l} V_r)^{-1}.
\end{align*}
\]  

(14)

(15)

Therefore, maximizing the WSR is equivalent to minimizing the MSE error and problem \(\mathcal{P}\) can be restated with respect to the digital beamformers and IRSs’ phase response with the minimization of the MSE as

\[
\begin{align*}
\min_{V_f, V_r, \Phi_f, \Phi_r} & \quad \text{Tr}(W_l E_l) + \text{Tr}(W_r E_r) \\
\text{s.t.} & \quad \text{Tr}(V_k V_k^H) \leq p_k, \quad \forall k \in \mathcal{F}, \\
& \quad |\phi_f(i)| = 1 \& |\phi_f(j)| = 1, \forall i, j,
\end{align*}
\]  

(16a)

(16b)

(16c)

where \(W_l\) and \(W_r\) are the constant weight matrices associated with the MIMO FD nodes \(l\) and \(r\), respectively. The gradient of the WSR and the WMMSE optimization problems turns out to be the same when the weight matrices are chosen as \(\mathcal{P}_3^\beta\), i.e.,

\[
W_l = \frac{w_l}{\ln 2} E_l^{-1}, \quad W_r = \frac{w_r}{\ln 2} E_r^{-1}.
\]  

(17)

A. Active Digital Beamforming

We assume the combiners and the IRSs’ response to be fixed and consider the optimization of the digital beamformers \(V_l\) and \(V_r\) for the MIMO FD nodes \(l\) and \(r\), respectively. The active digital beamforming optimization problem can be formally stated as follows

\[
\begin{align*}
\min_{V_l, V_r} & \quad \text{Tr}(W_l E_l) + \text{Tr}(W_r E_r), \\
\text{s.t.} & \quad \text{Tr}(V_k V_k^H) \leq p_k, \quad \forall k \in \mathcal{F},
\end{align*}
\]  

(18a)

(18b)

By taking the partial derivative of the Lagrangian function of \(\mathcal{P}_3^\beta\) with respect to the digital beamformers \(V_l\) and \(V_r\), we obtain the following optimal beamformers

\[
\begin{align*}
V_l &= (X_l + \mu_l I)^{-1} \hat{H}_{l,r}^H F_l^H W_l, \\
V_r &= (X_r + \mu_r I)^{-1} \hat{H}_{r,l}^H F_r^H W_r,
\end{align*}
\]  

(19a)

(19b)

where the matrices \(X_l\) and \(X_r\) are defined as

\[
\begin{align*}
X_l &= \hat{H}_{l,r}^H F_l^H W_r F_r H_{l,l}, \\
X_r &= \hat{H}_{r,l}^H F_r^H W_l F_l H_{r,r},
\end{align*}
\]  

(20a)

(20b)

and scalars \(\mu_l\) and \(\mu_r\) denote the Lagrange multipliers for the sum-power constraint of the MIMO FD nodes \(l\) and \(r\), respectively. The Lagrange multipliers can be calculated while meeting the sum-power constraints. Namely, to find the Lagrange multiplier, we can consider the singular value decomposition of \(X_l = U_l \Sigma U_l^H\) and \(X_r = U_r \Sigma U_r^H\) and write the power constraints in \(\mathcal{P}\), after simple steps, as

\[
\begin{align*}
\text{Tr}(V_k V_k^H) &= \sum_{i=1}^{N_l} \frac{G_l(i,i)}{(\mu_l + \Sigma(i,i))^2} = p_l, \\
\text{Tr}(V_k V_k^H) &= \sum_{i=1}^{N_r} \frac{G_r(i,i)}{(\mu_r + \Sigma(i,i))^2} = p_r,
\end{align*}
\]  

(21a)

(21b)

where the matrices \(G_l\) and \(G_r\) are defined as follows

\[
\begin{align*}
G_l &= U_l \hat{H}_{l,r}^H F_l^H W_r W_l F_l H_{l,l}, \\
G_r &= U_r \hat{H}_{r,l}^H F_r^H W_l W_r F_l H_{r,r}.
\end{align*}
\]  

(22a)

(22b)

To find the optimal values of \(\mu_l\) and \(\mu_r\), a linear search method can be adopted and we consider finding the multipliers with the Bisection method. If the values of the Lagrange multipliers result to be negative, then they are set to zero.
B. Passive Beamforming With IRSs

We assume the digital combiners and beamformers to be fixed and consider the optimization of $\Phi_r$ and $\Phi_t$, which are the passive beamforming response of the IRSs $i_r$ and $i_t$, respectively. As shown in the Appendix, the optimization problem with respect to $\Phi_r$ and $\Phi_t$ after some algebraic manipulations, can be formally stated as

$$\min_{\phi_r} \phi_r^H \Sigma_r \phi_r + s_r^H \phi_r^* + s_r^T \phi_r,$$

s.t. $|\phi_r(i)| = 1, \ \forall i,$

$$\min_{\phi_t} \phi_t^H \Sigma_t \phi_t + s_t^H \phi_t^* + s_t^T \phi_t,$$

s.t. $|\phi_t(i)| = 1, \ \forall i.$

(23a)

(23b)

(23c)

(23d)

where matrices $\Sigma_r$, $\Sigma_t$, and vectors $s_t, s_r$ are defined in the Appendix. Problem (23) is non-convex due to the unit-modulus constraint. To render a feasible solution, we adopt the majorization-maximization optimization method. Its objective is to solve a difficult problem through a series of more tractable sub-problems. We carry out its explanation only for $\phi_r$ and a similar reasoning follows also for $\phi_t$. Let $f(\phi_r(n))$ denote the value of the objective function as a function of $\phi_r$ at the $n$-th iteration. We proceed by constructing its upper bound with $g(\phi_r|\phi_r(n))$. In particular, we construct an approximate problem by using $g(\phi_r|\phi_r(n))$ at the $n+1$-th iteration. If $g(\phi_r|\phi_r(n))$ satisfies the following conditions

1) $g(\phi_r|\phi_r(n)) = f(\phi_r(n))$,

2) $\nabla_{\phi_r} g(\phi_r|\phi_r(n)) = \nabla_{\phi_r} f(\phi_r(n))|_{\phi_r = \phi_r(n)}$,

3) $g(\phi_r|\phi_r(n)) \geq f(\phi_r(n))$,

then the solutions of the obtained at each iteration will result in a monotonically decreasing objective function and converge to the actual solution of $f(\cdot)$. The first two conditions dictate that $g(\phi_r|\phi_r(n))$ and its first order gradient should be the same, while the third condition states that it should construct an upper bound for the original function. It has been shown in [36] that a simple upper bound $g(\phi_r|\phi_r(n))$ or $g(\phi_r|\phi_r(n))$ for $\Phi_t$ in our problem (see (36c-36d) in the Appendix) is given by

$$g(\phi_r|\phi_r(n)) = 2Re\{s_r^H q_r(n)\} + o_r,$$

(24a)

$$g(\phi_t|\phi_t(n)) = 2Re\{s_t^H q_t(n)\} + o_t,$$

(24b)

where $o_r$ and $o_t$ denote constant terms, and $q_r(n)$ and $q_t(n)$ are given by

$$q_r(n) = (\lambda_{r}^{max} - \Sigma_r)\phi_r(n) - s_r^*,$$

(25a)

$$q_t(n) = (\lambda_{t}^{max} - \Sigma_t)\phi_t(n) - s_t^*,$$

(25b)

and $\lambda_{r}^{max}$ and $\lambda_{t}^{max}$ denote the maximum eigenvalues of $\Sigma_r$ and $\Sigma_t$, respectively. Then our optimization problem (see (36c-36d) in the Appendix), by ignoring all the terms which are constant, can be restated by using the upper bounds (24a) and (24b), respectively, as

$$\min_{\phi_r} 2Re\{s_r^H q_r(n)\},$$

s.t. $|\phi_r(i)| = 1, \ \forall i,$

(26a)

(26b)
By fixing the digital beamformers and the IRSs response, the weight matrices and the combiners are chosen to be the MMSE combiners as (13). By replacing the optimal MMSE receivers given in (28) in (11a) and (11b), respectively, we obtain a new cost function

$$
\min_{\mathbf{w}_i, \mathbf{w}_f, \mathbf{V}_i, \mathbf{V}_f} \sum_{f \in \mathcal{F}} \text{Tr}(\mathbf{W}_f \mathbf{E}_f) - w_f \log \det(\frac{\ln 2}{w_f} \mathbf{W}_f) + d_f \frac{w_f}{\ln 2}.
$$

The optimization of (29) with respect to the weight matrix \( \mathbf{W}_f \) yields the solution \( \mathbf{W}_f = \frac{w_f}{\ln 2} \mathbf{E}_f^{-1} \). By plugging the optimized weight matrices in (29), we obtain the new cost function

$$
\min_{\mathbf{V}_i, \mathbf{V}_f, \Phi_i, \Phi_f} \sum_{f \in \mathcal{F}} -w_f \ln \det((\mathbf{E}_f)^{-1})
$$

s.t. \( \text{Tr} (\mathbf{V}_k \mathbf{V}_k^H) \leq p_k, \forall k \in \mathcal{F} \)

$$
|\phi_r(i)| = 1 \& |\phi_r(j)| = 1, \forall i, j,
$$

which is the original WSR cost function (2). Every update of the digital beamformers \( \mathbf{V}_l \) and \( \mathbf{V}_r \) and the IRSs’ phase response by minimizing the MSE leads to a monotonic increase in the WSR, which assures convergence of the proposed joint active and passive beamforming design.

A typical average convergence behavior of the proposed beamforming design for the NF-IRS assisted FD systems is shown in Fig. 3. We recall that the WMMSE method is a sub-optimal solution for WSR maximization [33], and therefore Algorithm 2 converges to a local optimum.

IV. NUMERICAL RESULTS

In this section, we present simulation results to evaluate the performance of the proposed joint active and passive beamforming design to analyze its potential to replace HYBF and combining simultaneously for the FD systems, while shaping the SI with NF-IRSs.

For comparison, we define the following benchmark schemes:

- A mMIMO HD system with fully digital beamforming capability, 100 transmit and 50 receive antennas for each MIMO HD node, which is denoted as HD-100 × 50.
- A mMIMO FD system with fully digital beamforming capability, 100 transmit and 50 receive antennas for each MIMO FD node, denoted as FD-100 × 50.

Note that digital beamforming designs represent upper bounds for the HYBF designs. Therefore, we compare our solution against the mMIMO fully digital systems, which provide the maximum achievable gain for the HYBF designs.

We define the signal-to-noise ratio (SNR) for the NF-IRSs assisted FD system as \( \text{SNR} = \frac{p_r}{\sigma_i^2} = \frac{p_t}{\sigma_i^2}, \) where \( p_i \) and \( \sigma_i^2 \) denote the total transmit power and noise variance for node \( i \), respectively, with \( i = l \) or \( i = r \). We assume that the considered systems operate at the frequency of 30 GHz, i.e., \( \lambda = 10 \) mm. The noise variance is set to be \( \sigma_i^2 = 1 \) and the total transmit power is chosen to meet the desired SNR.

For the NF-IRS assisted FD system, the number of antennas is chosen to be 5 times less, i.e., only \( M_l = M_r = 20 \) transmit and \( N_l = N_r = 10 \) receive antennas, respectively, and the number of data streams is \( d_l = d_r = 2 \). The FD nodes are assumed to be \( D_{l,r} = 200 \) m far. We consider the NF-IRSs of size \( 10 \times 10, 20 \times 20, \) and \( 30 \times 30 \) for both the MIMO FD nodes, with the center between two IRS elements located at \( \lambda/2 \). They are placed at the minimum distance of 3 m from their FD nodes, which is varied up to 90 m. We assume that both the FD nodes are equipped with ULAs, and their transmit and receive arrays are assumed to be separated by a distance \( D_{l,r} = 20 \) cm, with a relative angle \( \Theta_{l,r} = 180^\circ \) and \( r_{m,n} \) in (3) is set given \( D_{l,r} \) and \( \Theta_{l,r} \) as in [14] (9). The Rician factor is set as \( k_p = 1 \) and the rate weights are chosen as \( w_l = w_r = 1 \). The number of paths and number of clusters are set as \( N_{c,l} = N_{c,r} = 20 \) and the AoA \( \theta_{l,r}^{p,n,c} \) and AoD \( \phi_{l,r}^{p,n,c} \) are assumed to be uniformly distributed in the interval \( \mathcal{U} \sim [-30^\circ, 30^\circ] \). The digital beamformers are initialized as the dominant eigenvectors of the effective channel covariance matrices, and the response of the NF-IRSs is initialized with random phases. The results reported herein are averaged over 200 channel realizations.

Let \( \hat{x}, \hat{y}, \) and \( \hat{z} \) denote the 3 versors on the three dimensional space and any point on the 3 dimensional vector space can be written as \( (x\hat{x}, y\hat{y}, z\hat{z}) \). We assume the FD nodes with ULAs to be aligned with \( x \) direction and the first transmit antenna of the FD nodes \( l \) and \( r \) are assumed to be placed in the positions \( (0,0,0) \) and \( (0,D_{l,r}\hat{y},0) \), respectively, with other antennas placed in positions \( x \). Both the NF-IRSs are assumed to be at the same distance from their FD nodes, and placed on the x-y plane with first element of the NF-IRSs \( i_l \) and \( i_r \) placed in positions \( (0,D_{IRS}\hat{y},0) \) and \( (0,D_{l,r} - D_{IRS}\hat{y},0) \), respectively. Note that when the IRS operate in the NF of the FD node, joint passive SIC and intelligent control of the UL and DL channels also depend on the orientation of the NF-IRSs. Namely, the NF-IRSs can provide better SIC by jeopardizing the UL and DL channel control if they are more oriented towards the receive antennas of the NF FD node, or vice versa. The chosen
Fig. 4. Average WSR as a function of SNR with NF-IRSs of size $30 \times 30$, $20 \times 20$, and $10 \times 10$, placed at a distance of 3 m.

Fig. 5. Average WSR as a function of SNR with NF-IRSs of size $30 \times 30$, $20 \times 20$, and $10 \times 10$, placed at a distance of 30 m.

Fig. 6. Average WSR as a function of the distance between the FD nodes and the NF-IRSs at SNR= 0 dB.

Fig. 7. Average WSR as a function of the distance between the FD nodes and the NF-IRSs at SNR= 30 dB.

Orientation of the NF-IRSs in our setup provides an optimal trade-off between the passive SIC capabilities of NF-IRSs and control over the UL and DL channels.

Fig. 4 shows the achieved average WSR as a function of the SNR for the FD systems assisted with NF-IRSs placed at 3 m from the transmit array. We can see that despite using 5 times fewer antennas than the mMIMO FD systems, the proposed approaches significantly outperform the benchmark schemes. Namely, with an NF-IRSs of size $30 \times 30$, they can achieve $\sim$ 4 times higher gain than the traditional mMIMO HD system. Fig. 4 shows that the achievable gains of the NF-IRSs assisted FD systems are limited only by the size of the IRS, and having larger NF-IRSs will lead to higher gains. Fig. 5 shows the performance of the proposed design as a function of SNR with the NF-IRSs placed at 30 m far from the transmit array. We can see that when the NF-IRSs are placed far from the FD node, they achieve significantly less gain. The reason lies in the fact that the IRSs placed far from the FD node cannot aid in shaping the SI channel, which is handled only with the digital beamformers, which leads to less achievable gain.

Fig. 6 shows the performance of the proposed joint active and passive beamforming designs, as a function of the distance between the FD node and their NF-IRSs, which is varied in the interval $3 \sim 90$ m, with the two FD nodes operating at the distance of 200 m at SNR= 0 dB. From Fig. 6 we note that the FD systems benefit the most with NF-IRSs instead of the FF-IRSs by obtaining a significantly higher achievable WSR compared to the traditional mMIMO FD systems. Placing the IRSs far from the FD systems leads to small performance gains.
as the FD systems improve only the quality of the direct links and cannot assist in shaping the SI channel. Fig. 7 shows the performance as a function of the distance between the NF-IRSs from their FD node at SNR = 30 dB. It is clearly visible that, regardless of the SNR at which the NF-IRS-assisted FD systems operate, the distance between the FD nodes and their NF-IRSs dictates the maximum achievable gain.

Fig.s 8-9 show the cumulative distribution function (CDF) of the average WSR rate for SNR of 0 dB and 30 dB with the NF-IRSs at a distance of 3 m and 30 m, respectively. We can see that the gains are stable over the CDF and also consistent with the results reported in the previous figures. Therefore, we can conclude that with very high probability the FD systems benefit from the IRSs the most, only when they are placed in the NF. Placing the IRSs in the FF is not desirable for the FD system, as it leads significant less gain compared to the NF case.

Finally, Fig. 10 shows the achievable performance of MIMO FD system assisted with FF-IRS of variable size, in comparison with the NF-IRS-assisted system and the benchmark schemes. We can see that the FF-IRSs even with 80 × 80 elements achieves a significantly lower gain than the NF-IRSs of size 30 × 30. This results showcase the importance of the NF-IRS in paving the path towards cost-efficient and energy-efficient mmWave FD systems with very high performance gain.

V. CONCLUSIONS

We proposed to truncate the analog stage of the mmWave FD nodes by shifting the mMIMO FD paradigm back to MIMO FD. Each FD node is then assisted with a NF-IRS which aims to smartly control the UL and DL channel, while performing very strong passive SIC. Therefore, the proposed design leads to significant reduction in the hardware cost and power consumption. A novel joint active and passive beamforming design for the NF-IRSs assisted mmWave FD system has been proposed to maximize the WSR. Simulation results show that the proposed method significantly outperform the conventional HD system. Moreover, it is also shown that the NF-IRSs not only are capable of replacing the analog stage of the mmWave FD systems, but they yield 2 times more gain than the mMIMO FD systems with a NF-IRS of size 30 × 30. Larger gains can be obtained if the size of the NF-IRSs become large.

APPENDIX

To highlight the dependence of the IRSs’ response on the WSR, we first rewrite the effective channel responses as

\[
\hat{H}_{i,r} = A_{1,r} + A_{2,r} \Phi_i, H_{i,r} = B_{1,r} + H_{r,i} \Phi_i B_{2,r},
\]

(31a)
$$\tilde{H}_{l,r} = C_{1,r} + H_{l,i} + \Phi_{C_{2,r}}, \quad \tilde{H}_{r,r} = D_{1,r} + D_{2,r} + \Phi_{H_{i,r}},$$  
(31b)

to optimize $\Phi_r$. Alternatively, we write them as

$$\tilde{H}_{l,r} = A_{1,r} + H_{l,i} + \Phi_{A_{2,r}}, \quad \tilde{H}_{r,r} = B_{1,r} + B_{2,r} + \Phi_{H_{i,r}},$$  
(32a)

$$\tilde{H}_{l,l} = C_{1,l} + C_{2,l} + \Phi_{H_{i,l}}, \quad \tilde{H}_{r,r} = D_{1,l} + D_{2,l} + \Phi_{H_{i,r}},$$  
(32b)

to highlight the effective channel responses as a function of $\Phi_i$, where the auxiliary matrices appearing in (31), (32) are defined as

$$A_{1,r} = H_{i,r} + H_{i,j} + \Phi_{H_{i,r}},$$  
(33a)

$$B_{1,r} = H_{i,j} + \Phi_{H_{i,j}},$$  
(33b)

$$C_{1,r} = H_{i,l} + H_{i,j} + \Phi_{H_{i,l}},$$  
(33c)

$$D_{1,r} = H_{i,r} + \Phi_{H_{i,r}},$$  
(33d)

$$A_{1,l} = H_{r,l} + \Phi_{H_{r,l}},$$  
(33e)

$$B_{1,l} = H_{r,l} + \Phi_{H_{r,l}},$$  
(33f)

$$C_{1,l} = H_{r,l} + \Phi_{H_{r,l}},$$  
(33g)

$$D_{1,l} = H_{r,l} + \Phi_{H_{r,l}},$$  
(33h)

Let $S_r$, $T_r$, $S_l$, $T_l$ denote additional auxiliary matrices defined as

$$S_r = H_{r,i} + V_{r} H_{r,i} F_{r} H_{r,i},$$  
(34a)

$$Z_r = H_{r,i} H_{r,i} F_{r} H_{r,i} + D_{r,i} H_{r,i} F_{r} H_{r,i},$$  
(34b)

$$T_r = B_{r,i} H_{r,i} F_{r} H_{r,i} + H_{r,i} V_{r} H_{r,i} H_{r,i} + H_{r,i} V_{r} H_{r,i} H_{r,i},$$  
(34c)

$$S_l = A_{l,r} + V_{r} A_{l,r} H_{r,i} F_{r} H_{r,i} + H_{r,i} V_{r} H_{r,i} C_{l} A_{l,r} W_{r} A_{l,r},$$  
(35a)

$$Z_l = H_{i,j} H_{i,j} + C_{l} A_{l,r} W_{r} A_{l,r},$$  
(35b)

$$T_l = A_{l,r} + V_{r} A_{l,r} H_{i,j} + H_{i,j} V_{r} H_{i,j} H_{i,j} + H_{i,j} V_{r} H_{i,j} H_{i,j},$$  
(35c)

By substituting $34^a$ and $35^a$ in the expressions of the error covariance matrices, the minimization of the MSE optimization problem with respect to $\Phi_i$ and $\Phi_r$ can be formally stated as

$$\min_{\Phi_i} \text{Tr}(\Phi_{r} H_{r,i} Z_{r} F_{r} T_{r}) + \text{Tr}(\Phi_{r} H_{r,i} S_{r}) + \text{Tr}(\Phi_{r} S_{r} F_{r} T_{r}) + c_r$$  
(36a)

$$\text{s.t.} \quad |\phi_r(i)| = 1, \quad \forall i,$$  
(36b)

$$\min_{\Phi_r} \text{Tr}(\Phi_{r}^H Z_{r} F_{r} T_{r}) + \text{Tr}(\Phi_{r}^H S_{r} F_{r} T_{r}) + \text{Tr}(\Phi_{r} S_{r} F_{r} T_{r}) + c_r$$  
(36c)

$$\text{s.t.} \quad |\phi_r(j)| = 1, \quad \forall j,$$  
(36d)

where $c_l$ and $c_r$ denote constant terms which are independent of $\Phi_1$ and $\Phi_r$, respectively. We remark that, when solving (36a) and (36b), $\Phi_i$ is assumed to fixed, and similarly, when solving (36c) and (36d), $\Phi_r$ will be assumed to fixed.

Recall that $\phi_i$ and $\phi_r$ are vectors containing the diagonal elements of $\Phi_i$ and $\Phi_r$, respectively. As the off-diagonal elements of the $\Phi_r$ are zero, we wish to maximize the WSR or minimize the MSE with respect to $\phi_i$ and $\phi_r$. For such purpose, from identity 1.10.6 we restate the first trace term appearing in (36a) and (36c) as

$$\text{Tr}(\Phi_{r}^H Z_{r} F_{r} T_{r}) = \phi_r^H \Sigma_r \phi_r,$$  
(37a)

$$\text{Tr}(\Phi_{r}^H Z_{r} F_{r} T_{r}) = \phi_r^H \Sigma_r \phi_r, \quad \text{where} \quad \Sigma_r = Z_{r} \odot T_{r}^T.$$  
(37b)

Let $s_r$ and $s_l$ be the vectors containing the diagonal elements of the matrices $S_r$ and $S_l$, respectively, given as $s_r = [S_{r}(1,1), \ldots, S_{r}(r,r)]^T$ and $s_l = [S_{l}(1,1), \ldots, S_{l}(r,r)]^T$. The second and third terms appearing in (36a) and (36c) can be restated as

$$\text{Tr}(\Phi_{r}^H S_{r} F_{r} T_{r}) = s_r^H \phi_r,$$  
(38a)

$$\text{Tr}(\Phi_{r}^H S_{l} F_{r} T_{r}) = s_l^H \phi_r.$$  
(38b)

By using the aforementioned properties, the optimization problems (36a) and (36c) to optimize $\phi_i$ and $\phi_r$, respectively, can be written as (25).

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