High Energy Field Theory in Truncated AdS Backgrounds

Walter D. Goldberger\textsuperscript{1,2} and Ira Z. Rothstein\textsuperscript{3}

\textsuperscript{1}Department of Physics, University of California, Berkeley, CA 94720
\textsuperscript{2}Theoretical Physics Group, Lawrence Berkeley National Laboratory, Berkeley, CA 94720
\textsuperscript{3}Department of Physics, Carnegie Mellon University, Pittsburgh, PA 15213

In this letter we show that in five-dimensional anti-deSitter space (AdS) truncated by boundary branes, effective field theory techniques are reliable at high energy (much higher than the scale suggested by the Kaluza-Klein mass gap), provided one computes suitable observables. We argue that in the model of Randall and Sundrum for generating the weak scale from the AdS warp factor, the high energy behavior of gauge fields can be calculated in a cutoff independent manner, provided one restricts Green’s functions to external points on the Planck brane. Using the AdS/CFT correspondence, we calculate the one-loop correction to the Planck brane gauge propagator due to charged bulk fields. These effects give rise to non-universal logarithmic energy dependence for a range of scales above the Kaluza-Klein gap.

I. INTRODUCTION

It is widely believed that there exists a large energy desert separating the weak scale from the scale of grand unification, or perhaps the Planck scale. A primary reason for this belief is the observation that high-scale perturbative unification seems plausible in supersymmetric extensions of the Standard Model. On the other hand, it appears that in models in which the weak scale is fundamental, such as scenarios based on extra dimensions, one has to abandon the idea of gauge unification at high energies.

Several authors have argued that in scenarios where the weak/Planck hierarchy is generated by a single warped extra dimension\textsuperscript{3} (the RS model), it is possible for bulk gauge couplings to evolve logarithmically over a large range of scales\textsuperscript{4,5,6}. This is in sharp contrast to the behavior of a gauge theory propagating on compactified 5D flat space (for instance flat $R^4 \times S^1$), where vacuum polarization effects give rise to power law corrections that become dominant at mass scales comparable to the compactification scale. At the scales where these power corrections dominate, effective field theory techniques break down. In order to make predictions beyond such energies, local field theory must be embedded into a suitable UV completion that resolves its short distance singularities.

While we agree that high energy logarithmic behavior is possible in gauge theories propagating in compactified AdS, we find that this is physically realized in a manner that is somewhat different from the proposal of\textsuperscript{3}. We will show that as in flat space, the one-loop correction to the gauge field zero mode propagator in compactified AdS contains a power law term that is saturated at the Kaluza-Klein mass scale (of order TeV). This power correction is a finite, non-analytic function of the 4D momentum. Because of this, it cannot be removed by any procedure for regulating the ultraviolet divergences that arise in loop calculations. However, while in a flat space theory the emergence of power law behavior at the KK mass scale signifies a breakdown of the higher dimensional field theory description, the same is not true for theories propagating in background AdS spaces. Rather, in AdS, large loop corrections at the TeV scale imply a breakdown of the field theoretic description of the zero mode observable, but not necessarily of other correlators. Thus, high energy gauge theory effects may be accessible via local field theory, provided one calculates the appropriate observable.

In the RS proposal for obtaining the weak scale from the AdS warp factor\textsuperscript{3}, a set of such calculable quantities is the 5D gauge theory correlators with external points restricted to the Planck brane. On the Planck brane the local cutoff for correlators is of order the AdS curvature scale (which is taken to be of order the Planck scale and is much larger than the KK scale). This implies that it is this scale, and not the TeV scale, which suppresses power corrections to these observables. Thus the logarithms due to KK zero modes in loops give the dominant non-analytic corrections for a large range of scales.

To illustrate these claims, in this letter we will work in the context of 5D massless scalar electrodynamics as a model of gauge dynamics. In principle, calculations in the model can be done in 5D, in a manner that is independent of any specific regulator. However, in order to compute the correlators on the Planck brane, it is easier to employ the AdS/CFT correspondence\textsuperscript{7,8,9} as it applies to models with boundary branes\textsuperscript{3,7}. The Planck brane vacuum polarization of our toy model can be computed in a dual 4D field theory that consists of a
massless scalar and a gauge field which are both weakly coupled to a (broken) conformal field theory (CFT). The one-loop logarithm of the 5D theory can be calculated in this framework without any detailed knowledge of the CFT.

At energies of order the KK mass gap, the Planck brane correlates match on to the zero mode Green’s functions. Therefore, there is a calculable relation between high energy couplings and the parameters measured in low energy experiments. In particular, symmetry constraints on the dynamics near the curvature scale (for instance, high energy unification of gauge forces) could have meaningful implications for low energy physics.

Our setup is as follows. We will work in the context of field theory propagating in a background five-dimensional (Euclidean) AdS spacetime with the coordinatization

$$ ds^2 = G_{MN}dX^M dX^N = \frac{1}{(kz)^2} \left( \eta_{\mu\nu} dx^\mu dx^\nu + dz^2 \right), \quad (1) $$

where $z$ parameterizes location in the AdS bulk. The AdS boundary is cut off by a Planck brane at $z = 1/k$, and the AdS horizon removed by the presence of a TeV brane at $z = 1/T$. Here $k$ is the AdS curvature parameter, and $T$ is an energy scale that sets the masses of KK excitations of bulk fields. For applications to the hierarchy problem all parameters are taken to scale as appropriate powers of the Planck scale, except $T \sim$ TeV.

II. FLAT SPACE EFFECTIVE FIELD THEORY AND UNIFICATION

Let us now review the one-loop structure of gauge theory compactified on spaces of the form $R^4 \times S^1$ or $R^4 \times S^1/Z_2$. We will work in the context of massless scalar electrodynamics in five-dimensions. This simple model retains the essential physical features that we wish to discuss while avoiding technicalities that arise in more realistic models of non-Abelian gauge fields and fermions. It is convenient to work with an observable that matches onto measurable 4D quantities at low energies (smaller than the compactification radius). Such an observable is provided by the two-point correlator of the zero mode field which is generated by a 5D effective action evaluated on gauge fields that have no dependence on the compact coordinate. Keeping only terms up to quadratic in the gauge field and working in the $A_5 = 0$ gauge this is given by

$$ S_{eff} = \frac{1}{2} \int \frac{d^4q}{(2\pi)^4} A_\mu(q) \left[ q^\mu q^\nu - q^2 \eta^\mu\nu \right] \Pi(q^2) A_\nu(-q), \quad (2) $$

On $R^4 \times S^1$, summing the tower of scalar KK modes running in the loops, and using dimensional regularization to regulate the short distance divergences yields

$$ \Pi_{S^1}(q^2) = \frac{1}{g_4^2} - \frac{R \sqrt{q^2}}{256} - \frac{1}{8\pi^2} \int_0^1 dx \sqrt{1 - x^2} \ln \left[ 1 - e^{-\pi x R \sqrt{q^2}} \right]. \quad (3) $$

The first term in this expression simply the tree level action, with $g_4$ the coupling of the zero mode gauge field to the tower of KK states. On the circle, this is related to the 5D gauge coupling by $1/g_4^2 = 2\pi R/g_5^2$. The second two terms represent the one-loop contributions. We have written the one-loop corrections as a piece which is identical to the vacuum polarization of an uncompactified 5D gauge theory (with zero external momentum in the fifth direction), and a piece which contains finite radius effects.

Note that on $R^4 \times S^1$, the vacuum polarization is finite. Since $|g_3| = -1/2$, divergent contributions to the zero mode vacuum polarization in a massless theory must scale as $g_3^2 \Lambda$, with $\Lambda$ an ultraviolet cutoff. However, we have used dimensional regularization to regulate divergences, which simply sets these pure counterterm contributions to zero. Indeed, from the 5D point of view the concept of running couplings is not particularly meaningful, since the resummation of power corrections is made moot by the fact that when these contributions are of order one calculability is lost.

Although there are no ultraviolet logarithms, finite logarithms can arise from the infrared region of loop integrals. For $\sqrt{q^2} R \ll 1$, Eq. (3) becomes

$$ \Pi_{S^1}(q^2) \simeq \frac{1}{g_4^2} - \frac{1}{24\pi^2} \left[ \ln \left( 2\pi R \sqrt{q^2} \right) - \frac{4}{3} \right]. \quad (4) $$

Taking the limit $R \rightarrow 0$ with $g_4$ fixed, we see that this finite logarithm becomes singular. But this is precisely the 4D limit, in which the massive KK states decouple. Thus the singular $R \rightarrow 0$ limit manifests itself as the usual one-loop UV logarithm of a 4D theory. The large

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1 This one-loop logarithm should not be confused with the classical high energy logarithmic behavior of the gauge field propagator on the Planck brane. That tree-level logarithm (attributed to CFT effects in the dual 4D theory) is universal, and would not be observed in the relative evolution of different gauge couplings at high energy.
logarithms that arise in this regime can be resummed by using the renormalization group in 4D. Besides the logarithm that appears in this equation, the expansion for \( \sqrt{q^2}R \ll 1 \) of the integral in Eq. (3) also yields a non-analytic \( \sqrt{q^2}R \) piece as well as a series of integer powers of \( q^2 \). The non-analytic power-law term generated by this expansion cancels exactly the explicit power law that appears in Eq. (3), leaving behind the logarithm plus an analytic function of \( q^2 \). This is exactly what we expect for the behavior of the zero mode correlator if the low energy limit of the 5D compactified theory is to be reproduced by an effective 4D theory of massless modes plus a tower of non-renormalizable local operators.

For later comparison with the compactified AdS theory, we also consider the zero mode two-point correlator for flat field theory on a finite interval. Taking the space to be \( R^4 \times S^1/Z_2 \) and making the bulk scalar even under the orbifold action, one finds in this case:

\[
\Pi_{S^1/Z_2}(q^2) = \frac{1}{48\pi^2} \left[ \frac{1}{\epsilon} + \ln(4\pi) - \frac{\gamma}{2} + \frac{4}{3} \right] - \frac{1}{96\pi^2} \ln \left( \frac{q^2}{\mu^2} \right) + \frac{1}{2} \Pi_S(q^2),
\]

where \( \mu \) is a subtraction scale. In contrast to the result for \( S^1 \), the orbifold calculation encounters logarithmic divergences, represented here by a \( 1/\epsilon \) pole. This is because the presence of the orbifold fixed planes has modified the divergence structure of the field theory. Indeed, it is now possible to write down brane localized gauge kinetic operators

\[
\mathcal{L} = \frac{1}{4} \left[ \lambda_0 \delta(z) + \lambda_1 \delta(z - \pi R) \right] F_{\mu\nu} F^{\mu\nu}
\]

with \( \lambda_{0,1} \) dimensionless. By dimensional analysis, a divergent one-loop correction to these boundary couplings depends on an ultraviolet cutoff as \( \ln \Lambda \), which corresponds to the \( 1/\epsilon \) pole in Eq. (3). Thus loop effects induce non-trivial RG flows for boundary couplings.

How do we reconcile the fact that 5D bulk couplings do not run with the statement of “power-law running” and unification that is often made in the literature? In general, gauge unification implies that above some energy scale \( M_G \) associated with symmetry breaking of a unified gauge group, the forces mediated by the exchange of unbroken gauge bosons are equal in magnitude. In our 5D gauge theories, this equality should hold for the full gauge two-point correlators evaluated at arbitrary external positions in the fifth direction. In particular, averaging the 5D correlators over the compact direction generates a condition on the zero mode vacuum polarization of the unbroken gauge bosons at the unification scale (assuming, for instance, a symmetry breaking pattern \( G \to G_1 \times G_2 \))

\[
\Pi^{G_1}(M_G^2) = \Pi^{G_2}(M_G^2).
\]

This equation can be used to obtain a relation between the gauge couplings in the 5D Lagrangian. From this, one can extract predictions for low energy observables, leading to the usual prediction for the couplings at the weak scale with power law running. However, one might expect that unknown UV effects could be large and put into question the reliability of predictions when the power law starts to play an important role.

For both the 5D circle and orbifold theories, the non-analytic power corrections arising from the ultraviolet parts of 5D loop integrals dominate at a scale which is, up to a phase space factor, of order \( 1/R \). At such energies, the 5D flat space gauge theory becomes strongly coupled and ceases to be a useful effective description of the physics. While in flat space all Green’s functions associated with the effective gauge theory exhibit this type of behavior, the same is not necessarily true in curved backgrounds, such as AdS. Then, the relevant question for compactified AdS is, for a given observable, at what scale does power law behavior become the dominant effect?

### III. EFFECTIVE FIELD THEORY AND OBSERVABLES IN ADS

We would like to understand which AdS observables can be studied in the framework of effective field theory (EFT) at a given energy scale. Consider a simple model consisting of a bulk scalar with higher derivative “interaction” terms

\[
S = \frac{1}{2} \int d^5X \sqrt{G} (\partial \Phi)^2 + \frac{\lambda_2}{2} \int d^5X \sqrt{G} \left( \Box \Phi \right)^2 + \sum_{n=3}^{\infty} \frac{\lambda_n}{2} \int d^5X \sqrt{G} \Phi^{\circ n} \Phi,
\]

where \( \Box \) is the scalar Laplacian on AdS. First we examine an \( \mathcal{O}(\lambda_2) \) correction to the two-point function of \( \Phi \).

\[
\langle \Phi(X)\Phi(X') \rangle \sim D(X, X') + \lambda_2 \frac{\delta^n(X - X')}{\sqrt{G}}.
\]
In this equation, \( D(X,X') \) is the scalar propagator derived from the free scalar action. Performing a Fourier transform along the 4D coordinates \( x'^\mu \), this becomes (with \( p^\mu \) the coordinate momentum, \( p \cdot x = \eta_{\mu\nu} p^\mu x^\nu \), and \( p = \sqrt{\eta_{\mu\nu} p^\mu p^\nu} \))

\[
G_p(z,z') \equiv \int d^4xe^{ip \cdot x} \psi(x,z)\psi(0,z') \sim D_p(z,z') + \lambda_2(kz)^5 \delta(z-z'), \quad (10)
\]

Projecting onto the \( m \)-th KK wavefunction, \( \psi_m(z) \), we find

\[
\int \frac{dz'}{(kz')^3} \psi_m(z')G_p(z,z') \sim \psi_m(z) \left[ \frac{1}{p^2 + m_n^2} + \lambda_2(kz)^2 \right], \quad (11)
\]

where we have used the representation of the propagator in terms of modes, as well as the orthogonality condition for scalar KK wavefunctions \( (m_n \text{ is the mass of the } n\text{-th KK state}) \). It is natural to take \( \lambda_2 \) to scale as \( \lambda_2 \sim M_5^{-2} \), where \( M_5 \) is the fundamental scale in 5D. For fixed \( n \) and \( p \gg m_n \), the \( \lambda_2 \) correction to the propagator becomes leading when

\[
p \sim M_5/kz. \quad (12)
\]

This signals the breakdown of the EFT description for the given mode. Eq. (12) is simply the well known result \([1]\) that the local cutoff scales with \( z \). Here we have made clear the relevance of the sliding cutoff for particular modes, and the importance of the localization of the mode to the breakdown of the EFT becomes apparent. From this estimate, it is clear that the loss of predictivity has nothing to do with loop effects, or with the nature of a specific choice of regulator.

An estimate based on a single insertion of the \( \lambda_2 \) coupling is insufficient for the correlator involving only external KK zero modes. In this case insertions higher derivatives operators are necessary. Consider the 1PI zero mode two-point correlator in the presence of all the higher dimension terms in Eq. (8)

\[
G^{-1}(p) = p^2 + \lambda_2 p^4 + \sum_{n=3}^{\lambda_n} \frac{1}{M^2} \left( \frac{k}{MT} \right)^{2n-2} \frac{T^4}{k^2} \left( \frac{p}{T} \right)^{2n}, \quad (13)
\]

with \( \lambda_n \sim M^{2n-2} \lambda_n \). In order to have a sensible low energy theory, the \( p^2 \) term must dominate the corrections from operators with more derivatives. This means that we must have

\[
\frac{p}{T} < \frac{M}{k} \left( \frac{k}{T} \right)^{1/(n-1)}, \quad (14)
\]

where constants of order unity have been dropped. Interestingly, operators with more derivatives dominate at lower energy scales, and an operator with \( n \gg 1 \) becomes as important as the zeroth order propagator at a scale \( p \sim T \).

From this analysis, we conclude that we should not expect to be able to calculate zero mode observables (and in particular RG flows) for momenta much larger than a TeV, contrary to what was proposed in [2]. Instead, if we are interested in calculating at such large momenta we must restrict ourselves to observables localized to the Planck brane.

### IV. GAUGE FIELDS IN ADS

We will study scalar QED propagating in the curved background of Eq. (3)

\[
S = \int d^5X \sqrt{G} \left[ \frac{1}{4g_5^2} F_{MN}F^{MN} + |D_M \Phi|^2 \right]. \quad (15)
\]

First, consider the gauge field zero mode propagator. Performing the computation explicitly in 5D, we find using dimensional regularization and working in \( A_5 = 0 \) gauge (irrelevant constants have been dropped)

\[
\Pi_{\text{ads}}(q^2) = \frac{1}{g_4^2} + \frac{1}{48\pi^2} \left[ \frac{1}{\epsilon} - \ln \left( \frac{q^2}{kT} \right) + \frac{1}{2} \ln \left( \frac{\mu^2}{kT} \right) \right] - \frac{1}{16\pi^2} \int_0^1 dx x \ln N \left( \frac{\sqrt{q^2x}}{2} \right) \quad (16)
\]

where \( N(p) = I_1(p/T)K_1(p/k) - I_1(p/k)K_1(p/T) \), with \( I_1(x), K_1(x) \) modified Bessel functions of order one, and as in flat space \( 1/g_4^2 = R/g_4^2 \), with \( R \) the proper distance between the boundaries. As on the flat orbifold, it can be shown that the \( 1/\epsilon \) pole represents logarithmic divergences that renormalize gauge kinetic terms localized on the boundary branes at \( z = 1/k \) and \( z = 1/T \) [3], leading to the same RG equations for the boundary terms. Physically, this is to be expected, since the UV divergences of field theory in curved space arise from distances shorter than the curvature scale and are therefore identical to those in flat space. In particular, the RG flows for couplings to operators that are present in both flat and

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3 As we will show in a subsequent paper [13], the finite nonanalytic parts of this quantity can be computed in a regulator independent manner.
curved space should be the same. It follows that as in flat space, the 5D bulk gauge coupling does not run.

For $\sqrt{q^2} \ll T$, the zero mode of the scalar dominates Eq. (10) giving rise to low energy logarithmic energy dependence. For $\sqrt{q^2} \gg T$ the behavior is power law

$$\Pi_{\text{AdS}}(q^2) \sim \frac{\sqrt{q^2}}{T}$$

and the zero mode observable becomes strongly coupled, in accord with the analysis of the previous section. In fact, one does not have to consider loop effects to see power-law behavior. The non-renormalizable operator

$$S = \cdots + \frac{\lambda}{2} \int d^5X \sqrt{G} F_{MN} \Box F^{MN}$$

will give rise to an analytic power-law contribution to Eq. (10) of the form $\Pi_{\text{AdS}}(q^2) \sim \cdots + k\lambda q^2/T^2$. In [3], it was argued that the zero mode gauge propagator can be studied at energies much higher than the KK scale, provided a suitable momentum cutoff is employed. Because the contribution from Eq. (14) is a tree-level effect, it is clear the breakdown of the zero mode observable cannot be avoided by a mere choice of regulator.

Although we cannot use the zero mode observables to study high energy gauge theory, the Planck brane correlator is still perturbative at energies much larger than the KK scale. To see that loop corrections to this quantity are logarithmic for a large range of scales, consider the gauge propagator (in $A_5 = 0$ gauge) with one point on the Planck brane for external momenta larger than $T$:

$$D_{\mu\nu}(z, 1/k) \simeq \frac{kz}{p} \frac{K_1(pz)}{K_0(p/k)} \eta_{\mu\nu} + \text{pure gauge.}$$

For $z$ near $1/T$, $pz \gg 1$, and $K_1(pz) \sim \sqrt{\pi/(2pz)} \exp(-pz)$. We then see that this quantity has almost no overlap with the excited KK states of the bulk scalar, which are localized towards the TeV brane. It follows that the contribution from these states to the one-loop Planck correlator, which could potentially give rise to a power law of the form $\sqrt{q^2}/T$ is practically zero (it is suppressed by powers of $T/k \ll 1$). However, because the scalar zero mode profile is flat, it does not suffer from the exponential suppression, and gives the dominant contribution to the correlator. We thus expect the zero mode logarithm to be the leading energy dependence of the one-loop Green’s function. As the external momentum reaches the curvature scale $k$, the heavy KK modes which have support near the Planck brane start to contribute. These modes give rise to power law behavior that becomes dominant near the scale $k$.

Instead of performing the 5D calculation just described, we will show how to use AdS/CFT duality as it applies to the RS model to obtain quantitative information about the one-loop Planck correlator. Our toy gauge theory in AdS corresponds to the 4D theory of a $U(1)$ gauge field, a charged scalar, and a CFT explicitly broken by a UV cutoff (the dual of the Planck brane) and spontaneously broken in the IR (the TeV brane in the 4D context) [5]. The gauge field couple minimally to the scalar, and couples weakly to an anomaly free $U(1)$ subgroup of the global symmetries of the CFT. By similar arguments as those of [4] for the graviton, the presence of the Planck brane in the 5D theory renders the KK zero mode of the bulk scalar normalizable, and therefore implies the existence of a 4D massless scalar which couples to the CFT through a dimension four operator. (This can be verified by checking that the KK corrections to scalar exchange on the Planck brane in the 5D theory match the contributions to the force law from the CFT in the 4D dual). Thus our 5D theory has a 4D dual description (ignoring couplings to 4D gravity) given by

$$\mathcal{L}_{4D} = \mathcal{L}_{\text{CFT}} + \frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} + A_\mu J^\mu_{\text{CFT}} + |D_\mu \phi|^2 + c(\phi \mathcal{O}_4 + \text{h.c.}),$$

where $c$ is a coupling of order $1/k$. While we do not know precisely what CFT is dual to the RS model, it is still possible to use this Lagrangian to compute Planck brane correlators of bulk fields. For instance, the Planck brane vacuum polarization is given by

$$\Pi^{\mu\nu}(q^2) = (q^\mu q^\nu - q^2 \eta^{\mu\nu}) \left[ \frac{1}{g^2} - \frac{1}{48\pi^2} \ln\left(\frac{q^2}{\mu^2}\right) \right] + \int d^4xe^{iqz}(J_{\text{CFT}}^\mu(x)J_{\text{CFT}}^\nu(0))_{\text{CFT}} + \mathcal{O}(|c|^2).$$

The first line is simply the contribution of a 4D scalar to the vacuum polarization. The second line is the correction to the correlator due to pure CFT effects. Writing

$$\int d^4xe^{iqz}(J_{\text{CFT}}^\mu(x)J_{\text{CFT}}^\nu(0))_{\text{CFT}} = (q^\mu q^\nu - q^2 \eta^{\mu\nu}) \Pi(q^2),$$

it can be shown [6] that for $\sqrt{q^2} \gg T$ (note that the effects of being in a vacuum state with broken conformal symmetry are suppressed exponentially at energies much larger than $T$)

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4However, it is possible for new operators composed from powers of the background curvature to play a role in the RG flows.
\[ \Pi(q^2) \simeq \frac{1}{2g_k^2 k} \ln \left( \frac{q^2}{k^2} \right). \quad (23) \]

In the AdS description, this term corresponds to the contribution of the KK modes of the gauge field to the tree level Planck correlator. Because in the bulk theory Eq. (22) represents a tree level effect, it gives the leading behavior of the correlator. In settings where the AdS/CFT correspondence has been well tested, the CFT is a large $N$ $SU(N)$ gauge theory, so Eq. (22) is expected to be larger than the scalar correction by powers of $N$. However, the coefficient of this classical logarithm is universal, so pure CFT effects will cancel in predictions for the difference of low-energy couplings in terms of high energy parameters.

Besides the terms explicitly shown in Eq. (21), there are also corrections which involve insertions of the operator $O_A$. These terms are suppressed by powers of $k$, and correspond to the exponentially damped scalar KK corrections to the 5D correlator. They can be calculated in terms of CFT correlators (in the vacuum with broken conformal symmetry) of $O_A$ and $J_{CFT}^O$. Such correlators can be obtained using the rules of [6] for relating CFT correlators to solutions of classical AdS field equations.

So far we have only considered the evolution of the Planck brane vacuum polarization due to one-loop effects of bulk matter. It is easy to see that brane localized fields will also contribute logarithms with the usual 4D beta function coefficients. The effects of Planck localized fields persist for all energies up to the curvature scale. However, for $\sqrt{q^2} \gg T$, the contribution due to TeV brane fields on the Planck brane correlator is suppressed by an exponential of $\sqrt{q^2}/T$, since for high energies this is a highly non-local effect (explicitly, the suppression is by two powers of Eq. (19) evaluated at $z = 1/T$). This can be understood also in the 4D dual description [15]. There, Planck brane fields are spectators to the CFT dynamics, so it is clear that they give rise to the usual 4D vacuum polarization effects. On the other hand, TeV brane localized fields arise in the 4D dual as condensates of CFT states in the IR. These bound states do not contribute to the running of the couplings above TeV scale energies.

Although we only explicitly considered a toy scalar model, we expect that loops of bulk non-abelian gauge and fermion fields will also generate logarithmic corrections to the Planck correlator with the usual 4D coefficient. Because at energies below the KK scale the Planck correlators match on to the Green’s functions of zero mode fields, gauge coupling evolution in the RS model with Standard Model gauge fields in the bulk is generically similar to the situation in the minimal (4D) Standard Model with an energy desert. It is therefore possible to make perturbative predictions for the low energy couplings of the model in terms of the parameters in the underlying 5D Lagrangian.

V. CONCLUSIONS

It is clear that effective field theory in curved backgrounds is more subtle than in flat space. We have seen that in truncated AdS geometries, we can determine the scale at which effective field theory breaks down for classes of correlators. For instance, it is not possible to calculate above the TeV scale for zero mode correlators, but correlators whose end points are restricted to the boundary (“Planck”) brane do succumb to field theoretic techniques reliably, all the way up to the curvature scale. Furthermore, there are certain observables, e.g., differences of gauge couplings, which can be calculated without detailed knowledge of the dual CFT. As was previously postulated [2], [3], these couplings run logarithmically.

This leads to the intriguing possibility, first noted by [2], that a version the RS proposal for addressing the hierarchy problem with bulk Standard Model gauge fields could also predict coupling constant unification within a grand unified context [15]. In order to maintain the hierarchy in the presence of bulk gauge fields, the Higgs scalar responsible for electroweak symmetry breaking must be confined to the TeV brane. It only contributes to gauge coupling evolution up to the TeV scale. Standard Model fermions may propagate either in the bulk or on the TeV brane. However, since the fermions come in complete $SU(5)$ multiplets, the RS prediction for low energy coupling constant relations is model independent, and qualitatively similar to that of the Standard Model. Achieving unification, though, could necessitate additional, perhaps ad hoc, physics since the running above the TeV scale excludes the Higgs. As in the Standard Model, there are potential threshold corrections to logarithmic evolution near the GUT scale. There are also TeV threshold corrections that arise from the matching of the Planck to the zero mode Green’s functions. Although these corrections are not accessible to our 4D CFT calculation, they can be unambiguously calculated in 5D. The details for

\footnote{In the proposal of [3], the Standard Model is localized to the Planck brane, so supersymmetry must be employed to stabilize the weak scale. In that case, the AdS warp factor is used to generate an exponentially low supersymmetry breaking scale.}
plausible models have yet to be worked out.

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