Strong Decays of Scalar Glueball in a Scale-Invariant Chiral Quark Model

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An effective meson Lagrangian including a scalar glueball is constructed on the base of $U(3) \times U(3)$ chiral symmetry. The glueball is introduced into the meson Lagrangian by using the principle of scale invariance of an effective Lagrangian and the dilaton model. The singlet-octet mixing of scalar meson states is described by means of 't Hooft interaction. The contribution of the scalar and pseudoscalar anomalies into the breaking of scale invariance is taken into account. The mixing of quarkonia with the glueball is described. The mass spectrum of scalar mesons together with the glueball and also their strong decay widths are calculated. From comparing the obtained results with experimental data, it follows that $f_0(1500)$ is rather a glueball, whereas $f_0(1710)$ is a quarkonium. This accords with the results obtained in our previous works where radially-excited scalar meson states were described. It is shown that $\rho$-mesons play an important role in the description of glueball decays.
1. INTRODUCTION

The self-interaction of gluons, a peculiarity of QCD, gave an idea that gluons can form bound states that can propagate as particles in the space. Unfortunately, because of theoretical problems, there is no yet the exact answer to whether these states really exist or not. However, from recent lattice simulations [1–3] one can conclude that it is most probably that glueballs are real objects of our world. Having assumed that glueballs exist, one can try to construct a model to describe their interaction with other mesons, their properties, such as, e. g., mass and decay width, and to identify them with observed resonances.

An exact microscopic description of bound gluon states cannot be done systematically in the framework of QCD. In this situation, QCD-motivated phenomenological models are the tool that can help to deal with glueballs as well as with quarkonia which form the most of observed meson states. However, using these models to describe glueballs, we encounter many difficulties concerning, e. g., the ambiguity of the ways of including glueballs into models and identification of experimentally observed meson states. This explains the variety of points of view on this problem.

First of all, we do not know the exact mass of a glueball. From the quenched QCD lattice simulations, Weingarten (see, e. g., [1–3]) concluded that the lightest scalar glueball is expected around 1.7 GeV. Amsler [4] considered the state $f_0(1500)$ as a candidate for the scalar glueball. QCD sum rules [5] and $K$-matrix method [6] showed that both $f_0(1500)$ and $f_0(1710)$ are mixed states with large admixture of the glueball component.

All bound isoscalar $qar{q}$ states are subject to mixing with glueballs, and their spectrum has many interpretations made by different authors. For instance, Palano [7] suggested a scenario, in which the states $a_0(980)$, $K_0^*(1430)$, $f_0(980)$, and $f_0(1400)$ form a nonet. The state $f_0(1500)$ is considered as the scalar glueball. Törnqvist et al. [8] looked upon the states $f_0(980)$ and $f_0(1370)$ as manifestations of the ground and excited $sar{s}$ states, and upon
the state $f_0(400 - 1200)$ as the ground $u\bar{u}$ state. Eef van Beveren et al. [9] considered the states $f_0(400 - 1200)$ and $f_0(1370)$ as ground $u\bar{u}$ states, and the states $f_0(980)$ and $f_0(1500)$ as ground $s\bar{s}$ states. Two states for each $q\bar{q}$ system occur due to pole doubling, which takes place for scalar mesons in their model. Shakin et al. [10,11] obtained from a nonlocal confinement model that the $f_0(980)$ resonance is the ground $u\bar{u}$ state, and $f_0(1370)$ is the ground $s\bar{s}$ state. The state $f_0(1500)$ is considered as a radial excitation of $f_0(980)$. They believe the mass of scalar glueball to be 1770 MeV.

In our recent papers [12], following methods given in [13–16], we showed that all experimentally observed scalar meson states with masses in the interval from 0.4 to 1.71 GeV can be interpreted as members of two scalar meson nonets — the ground state of the meson nonet (lighter than 1 GeV) and its first radial excitation (heavier than 1 GeV). We considered all scalar mesons as $q\bar{q}$ bound states and took into account the singlet-octet mixing caused by ’t Hooft interaction. In [12], we obtained a scalar isoscalar state with mass 1600 MeV and had to choose to which of the experimentally observed states, $f_0(1500)$ or $f_0(1710)$, we should ascribe it. From our analysis of the strong decay rates calculated in our model we found that $f_0(1710)$ fits to the nonet of quarkonia better than $f_0(1500)$. Therefore, we supposed that the state $f_0(1500)$ contains greater admixture of the scalar glueball (see [3,6]). However, the final decision should be made after including the scalar glueball into the model and taking account of its mixing with quarkonia. In the present work, that is devoted to solving this problem, from the analysis of strong decay widths of the glueball we again come to an analogous conclusion.\textsuperscript{1)

To describe the properties of the glueball and its interaction with quarkonia, one should introduce a scalar isoscalar dilaton field $\chi$ into our model, in addition to the quarkonia that have already been described [12]. For this purpose, one can make use of the idea of approximate scale invariance of

\textsuperscript{1) However, radially-excited states are not yet considered.}
effective Lagrangians based on the dilaton model. Such models were studied by many authors (see, e. g., [17–21]). Unfortunately, there is no unique way to introduce the dilaton field into a chiral Lagrangian. This explains the large number of models dealing with glueballs.

The guideline one should follow when introducing the dilaton field into an effective meson Lagrangian is to reproduce the Ward identity connected with the scale anomaly. The latter leads to the following equation for the vacuum expectation value of the divergence of the dilatation current:

\[
\langle \partial_\mu S^\mu \rangle = C_g - \sum_{q=u,d,s} m_0^q \langle \bar{q}q \rangle,
\]

where \( N_c \) is the number of colours; \( N_f \) is the number of flavours; \( \langle \frac{\alpha}{\pi} G^2_{\mu\nu} \rangle \) and \( \langle \bar{q}q \rangle \) are the gluon and quark condensates; \( m_0^q \) is the current quark mass.

In this paper we are going to use the most natural method of introducing the dilaton field into the effective Lagrangian by requiring that, in the chiral limit, our Lagrangian should be scale-invariant except for the dilaton potential and terms induced by gluon anomalies. To realize this program, one should multiply all dimensional parameters of the original Lagrangian (without dilaton) by a corresponding power of the dilaton field divided by its vacuum expectation value \( \chi_c \). Thus, instead of the four-quark coupling constant \( G \), the ’t Hooft coupling constant \( K \), ultraviolet cutoff \( \Lambda \) (necessary for regularizing the divergent integrals coming from quark loops), and the constituent quark masses \( m_q \) \((q = u, s)\), one should use \( G(\chi_c/\chi)^2 \), \( K(\chi_c/\chi)^5 \), \( \Lambda(\chi/\chi_c) \), and \( m_q(\chi/\chi_c) \).

Current quark masses \( m_0^q \) are not multiplied by the dilaton field and violate scale invariance explicitly, as it takes place in QCD. Their contribution to the divergence of dilatation current is determined by quark condensates and disappears in the chiral limit (see (1)).

The scale invariance is also broken by those terms in the effective Lagrangian that are induced by the pseudoscalar and scalar gluon anomalies.
and look as follows [22,23]

\[ L_{an} = -h_{\phi}\phi_0^2 + h_{\sigma}\sigma_0^2, \]  

where \( h_{\phi}, h_{\sigma} \) are constants. \( \phi_0 = \sqrt{2/3}\phi_u - \sqrt{1/3}\phi_s, \phi_0 \) and \( \sigma_0 \) \((\langle \sigma_0 \rangle \neq 0)\)
\( \sigma_0 = \sqrt{2/3}\sigma_u - \sqrt{1/3}\sigma_s, \) where \( \sigma_u \) \((\langle \sigma_u \rangle \neq 0)\) consists of \( u(d)\)-quarks and \( \sigma_s \) \((\langle \sigma_s \rangle \neq 0)\) of \( s\)-quarks.

These terms appear due to the ’t Hooft interaction. When restoring scale invariance of the effective Lagrangian by inserting dilaton fields (the procedure of the restoration of scale invariance is given in Sect. 3), these terms must be treated separately. Moreover, it turns out that these terms determine the most of quarkonia-glueball mixing.

Omitting, for a moment, the ’t Hooft interaction in our approach, we require the Lagrangian to be scale-invariant in the chiral limit both before and after the spontaneous breaking of chiral symmetry (SBCS), except for the dilaton potential. This property can be obtained by considering (after bosonization when the effective Lagrangian is expressed in terms of bosonic scalar and pseudoscalar fields \( \sigma \) and \( \phi \)) the shift of the scalar meson field \( \sigma \)

\[ \sigma = \sigma' - m\frac{X}{\chi_c}, \quad (m^0 = 0), \]

where \( \langle \sigma' \rangle_0 = 0, \quad \langle \sigma \rangle_0 = -m, \) guaranteeing that the relation (1) is satisfied. The nonzero vacuum expectation value of \( \sigma \) appears as a result of SBCS, and thus, the constituent quark mass \( m \) is produced. In the case of nonvanishing current quark masses, (4) changes by including an additional (nonscaled) mass term \( m^0 \) into the r.h.s.

\[ \sigma = \sigma' - m\frac{X}{\chi_c} + m^0. \]

The structure of the paper is as follows. In Section 2, we derive the usual \( U(3) \times U(3)\)-flavour symmetric effective Lagrangian with the ’t Hooft interaction and without dilaton fields. In Section 3, the dilaton field is introduced into the effective Lagrangian obtained in Section 2. In Section 4, the gap equations are investigated, the quadratic (in fields) terms are deduced and
the mixing matrix for scalar isoscalar states is introduced. In Section 5, the numerical estimates for the model parameters are given. The main strong decays of scalar isoscalar mesons are calculated in Section 6. It is shown there that \( \rho \)-mesons play an important role in the decay of a glueball into four pions. Finally, in the Conclusion, we discuss the obtained results.

2. CHIRAL EFFECTIVE LAGRANGIAN WITH ’T HOOFT INTERACTION

A \( U(3) \times U(3) \) chiral Lagrangian with the ’t Hooft interaction was investigated in paper [24]. It consists of three terms (see below). The first term represents the free quark Lagrangian, the second is composed of four-quark vertices as in the NJL model, and the last one describes the six-quark ’t Hooft interaction [25] that is necessary to solve the \( U_A(1) \) problem.

\[
L = \bar{q} (i\not{\partial} - m^0) q + \frac{G}{2} \sum_{a=0}^{8} ((\bar{q}\lambda_a q)^2 + (\bar{q}\gamma_5\lambda_a q)^2) - K \{\det[\bar{q}(1 + \gamma_5)q] + \det[\bar{q}(1 - \gamma_5)q]\}. \tag{6}
\]

Here \( G \) and \( K \) are coupling constants, \( \lambda_a \) \((a = 1, ..., 8)\) are the Gell-Mann matrices, \( \lambda_0 = \sqrt{2/3} \ 1 \), with 1 being the unit matrix; \( m^0 \) is a current quark mass matrix with diagonal elements \( m^0_u, m^0_d, m^0_s \) \((m^0_u \approx m^0_d)\).

The standard bosonization procedure for local quark models consists in replacing the four-quark vertices by Yukawa coupling of quarks with bosonic fields which enables one to perform the integration over quark fields. The final effective bosonic Lagrangian appears then as a result of the calculation of the quark determinant. To realize this program, it is necessary, using the method described in [24–27], to go from Lagrangian (6) to an intermediate Lagrangian which contains only four-quark vertices

\[
L = \bar{q} (i\not{\partial} - \bar{m}^0) q + \frac{1}{2} \sum_{a,b=1}^{9} \left[G_{ab}(-)(\bar{q}\tau_a q)(\bar{q}\tau_b q) + G_{ab}(+)(\bar{q}\gamma_5\tau_a q)(\bar{q}\gamma_5\tau_b q)\right], \tag{7}
\]

where
\[ \tau_a = \lambda_a \quad (a = 1, \ldots, 7), \quad \tau_8 = \left(\sqrt{2}\lambda_0 + \lambda_8\right)/\sqrt{3}, \]
\[ \tau_9 = (-\lambda_0 + \sqrt{2}\lambda_8)/\sqrt{3}, \]
\[ G_{11}^{(\pm)} = G_{22}^{(\pm)} = G_{33}^{(\pm)} = G \pm 4Km_sI_1^\Lambda(m_s), \]
\[ G_{44}^{(\pm)} = G_{55}^{(\pm)} = G_{66}^{(\pm)} = G_{77}^{(\pm)} = G \pm 4Km_uI_1^\Lambda(m_u), \]
\[ G_{88}^{(\pm)} = G \mp 4Km_sI_1^\Lambda(m_s), \quad G_{99}^{(\pm)} = G, \]
\[ G_{89}^{(\pm)} = G_{98}^{(\pm)} = \pm 4\sqrt{2}Km_uI_1^\Lambda(m_u), \]
\[ G_{ab}^{(\pm)} = 0 \quad (a \neq b; \quad a, b = 1, \ldots, 7), \]
\[ G_{a8}^{(\pm)} = G_{a9}^{(\pm)} = G_{8a}^{(\pm)} = G_{9a}^{(\pm)} = 0 \quad (a = 1, \ldots, 7), \tag{8} \]

and \( \bar{m}^0 \) is a diagonal matrix composed of modified current quark masses:

\[ \bar{m}_u^0 = m_u^0 - 32Km_u m_s I_1^\Lambda(m_u) I_1^\Lambda(m_s), \tag{9} \]
\[ \bar{m}_s^0 = m_s^0 - 32Km_u^2 I_1^\Lambda(m_u)^2. \tag{10} \]

Here \( m_u \) and \( m_s \) are constituent quark masses and the integrals

\[ I_n^\Lambda(m_a) = \frac{N_c}{(2\pi)^4} \int d^4k \frac{\theta(\Lambda^2 - k^2)}{(k^2 + m_a^2)^n}, \quad (n = 1, 2; \quad a = u, s), \tag{11} \]

are calculated in the Euclidean metric and regularized by a simple \( O(4) \)-symmetric ultraviolet cutoff \( \Lambda \). For \( I_1^\Lambda(m_a) \) one gets

\[ I_1^\Lambda(m_a) = \frac{N_c}{16\pi^2} \left( \Lambda^2 - m_a^2 \ln \left( \frac{\Lambda^2}{m_a^2} + 1 \right) \right), \tag{12} \]

where \( m_a \) represents a corresponding constituent quark mass: \( m_u \) or \( m_s \). Note that we have introduced the notation of constituent quark mass already here, although they will be consistently considered only later, when discussing mass gap equations (compare (50) and (51) below) and the related shift of scalar meson fields. However, as we want to use an effective four-fermion interaction instead of the original six-quark one, we have to use full quark propagators with constituent quark masses to calculate quark loop corrections for the constant \( G \) (see (8)). For the definition of the constituent quark masses see (14) and (15) below.
In addition to the one-loop corrections to the constant $G$ at four-quark vertices, we modified the current quark masses $m^0_a$ (see (9) and (10)). This is to avoid the problem of double counting of the ’t Hooft contribution in gap equations which was encountered by the author in [27]. After the redefinition of the constant $G$ and of the current quark masses, we can guarantee that in the large-$N_c$ limit the mass spectrum of mesons and the gap equations, derived from the new Lagrangian with modified four-quark vertices and current quark masses, are the same as those obtained from the original Lagrangian with six-quark vertices.

Now we can bosonize Lagrangian (7). By introducing auxiliary scalar $\sigma$ and pseudoscalar $\phi$ fields, we obtain [13, 14, 24]

$$L(\sigma, \phi) = -\frac{1}{2} \sum_{a,b=1}^{9} \left( \sigma_a (G^{-})^{-1}_{ab} \sigma_b + \phi_a (G^{(+)}-1)_{ab} \phi_b \right) -$$

$$-i \text{Tr} \ln \left\{ i \hat{\partial} - m + 9 \sum_{a=1}^{9} \tau_a (\sigma_a + i \gamma_5 \phi_a) \right\}. \quad (13)$$

As we expect, the chiral symmetry is spontaneously broken due to strong attraction of quarks in the scalar channel and the scalar isoscalar fields acquire nonzero vacuum expectation values $\langle \sigma_a \rangle_0 \neq 0$ ($a = 8, 9$). These values are related to basic model parameters $G$, $m^0$, and $\Lambda$ via gap equations as it will be shown in the next Section. Therefore, we first have to shift the $\sigma$ fields by a proper value so that the new fields have zero vacuum expectation values:

$$\sigma_a = \sigma'_a - \mu_a + \bar{\mu}_a^0, \quad \langle \sigma'_a \rangle_0 = 0,$$

where $\mu_a = 0, \quad (a = 1, \ldots, 7), \quad \mu_8 = m_u, \quad \mu_9 = -m_s/\sqrt{2}$ and $\bar{\mu}_a^0 = 0, \quad (a = 1, \ldots, 7), \quad \bar{\mu}_8^0 = \bar{m}_u^0, \quad \bar{\mu}_9^0 = -\bar{m}_s^0/\sqrt{2}$. After this shift we obtain:

$$L(\sigma', \phi) = L_G(\sigma', \phi) - i \text{Tr} \ln \left\{ i \hat{\partial} - m + 9 \sum_{a=1}^{9} \tau_a (\sigma'_a + i \gamma_5 \phi_a) \right\}, \quad (15)$$

where

$$L_G(\sigma', \phi) = -\frac{1}{2} \sum_{a,b=1}^{9} \left( \sigma'_a - \mu_a + \bar{\mu}_a^0 \right) (G^{-})^{-1}_{ab} \left( \sigma'_b - \mu_a + \bar{\mu}_a^0 \right) -$$

$$-\frac{1}{2} \sum_{a,b=1}^{9} \phi_a (G^{(+)}-1)_{ab} \phi_b. \quad (16)$$
and $m$ is a diagonal matrix of constituent quark masses for different flavors. From Lagrangian (13) we take only those terms (in momentum space) which are linear, squared, cubic, and quadruple in scalar and pseudoscalar fields.

$$\mathcal{L}(\sigma', \phi) = L_G(\sigma', \phi) + \text{tr} \left[ I_2^A(m)((\partial_\mu \sigma')^2 + (\partial_\mu \phi)^2) - 4m I_1^A(m)\sigma' + 
+ 2I_1^A(m)(\sigma'^2 + \phi^2) - 4m^2 I_2^A(m)\sigma'^2 + 
+ 4m I_2^A(m)\sigma'(\sigma'^2 + \phi^2) - I_2^A(m)(\sigma'^2 + \phi^2)^2 + 
+ I_2^A(m)[\sigma' - m, \phi]^2 \right], \quad (17)$$

$$\sigma' = \sum_{a=1}^{9} \sigma^r_a \tau_a, \quad \phi = \sum_{a=1}^{9} \phi^r_a \tau_a, \quad (18)$$

where “tr” means calculating the trace over $\tau$-matrix expressions and $[. . .]_{-}$ stands for a commutator. (The calculation of “tr” is explained in details in [13]) The expression for $I_1^A(m_a)$ in Euclidean metric is given in (12). The integrals $I_2^A(m_a)$ are also calculated in Euclidean space-time

$$I_2^A(m_a) = \frac{N_c}{16\pi^2} \left( \ln \left( \frac{\Lambda^2}{m_a^2} + 1 \right) - \frac{\Lambda^2}{\Lambda^2 + m_a^2} \right). \quad (19)$$

Then, we renormalize the fields in (17) so that the kinetic terms of the effective Lagrangian are of the conventional form, and diagonalize the isoscalar sector.

$$\mathcal{\bar{L}}(\sigma^r, \phi^r) = \mathcal{\bar{L}}_G(\sigma^r, \phi^r) + 
+ \text{tr} \left[ \frac{1}{4}((\partial_\mu \sigma^r)^2 + (\partial_\mu \phi^r)^2) - 4mg I_1^A(m)\sigma^r + 2g^2 I_1^A(m)(\sigma^{r^2} + Z\phi^{r^2}) + 
+ \frac{1}{4}[m, \phi^r]^2 - m^2 \sigma^{r^2} + mga \sigma^r(\sigma^{r^2} + Z\phi^{r^2}) - \frac{g}{2}[m, \phi^r]_-(\sigma^r, \phi^r) - 
- \frac{g^2}{4}((\sigma^{r^2} + Z\phi^{r^2})^2 - [\sigma^r, \phi^r]^2) \right], \quad (20)$$

$$\sigma^r = \sum_{a=1}^{9} \sigma^r_a \tau_a, \quad \phi^r = \sum_{a=1}^{9} \phi^r_a \tau_a. \quad (21)$$

2) Despite that the scalar fields are of the main interest in this paper, we still need pseudoscalar fields to fix the model parameters.
For $\tilde{L}_G$ we have:

$$\tilde{L}_G(\sigma^r, \phi^r) = -\frac{1}{2} \sum_{a,b=1}^{9} (g_a \sigma_a^r - \mu_a + \bar{\mu}_a^0) (G^{(-)})_{ab}^{-1} (g_b \sigma_b^r - \mu_b + \bar{\mu}_b^0) -$$

$$-\frac{Z}{2} \sum_{a,b=1}^{9} g_a \phi_a^r (G^{(+)}_{ab})^{-1} g_b \phi_b^r.$$  \hspace{1cm} (22)

Here we introduced Yukawa coupling constants $g_a$:

$$\sigma'_a = g_a \sigma_a^r, \quad \phi_a = \sqrt{Z} g_a \phi_a^r,$$  \hspace{1cm} (23)

$$g_1^2 = g_2^2 = g_3^2 = g_8^2 = g_9^2 = [4I_2^A(m_u)]^{-1}, \quad g_4^2 = g_5^2 = g_6^2 = g_7^2 = [4I_2^A(m_u, m_s)]^{-1},$$

$$g_2^2 = g_3^2 = [4I_2^A(m_s)]^{-1},$$  \hspace{1cm} (24)

$$I_2^A(m_u, m_s) = \frac{N_c}{(2\pi)^4} \int d^4k \frac{\theta(\Lambda^2 - k^2)}{(k^2 + m_u^2)(k^2 + m_s^2)} =$$

$$= \frac{3}{(4\pi)^2(m_s^2 - m_u^2)} \left[ m_s^2 \ln \left( \frac{\Lambda^2}{m_s^2} + 1 \right) - m_u^2 \ln \left( \frac{\Lambda^2}{m_u^2} + 1 \right) \right],$$  \hspace{1cm} (25)

$$Z = \left( 1 - \frac{6m_u}{M_{A_1}^2} \right)^{-1} \approx 1.44,$$  \hspace{1cm} (26)

where we have taken into account $\pi$-$A_1$-transitions leading to an additional $Z$ factor, with $M_{A_1}$ being the mass of axial-vector meson (see [13]). The renormalized scalar and pseudoscalar fields in (20)–(23) are marked with the superscript $r$.

The mass formulae for isovectors and isodoublets follow immediately from (20). One just has to look up the coefficients at $\sigma^r 2$ and $\phi^r 2$. There are still nondiagonal terms in (22) in the isoscalar sector. This problem is solved by choosing the proper mixing angles both for the scalars and pseudoscalars (see, e. g., [24]). As we are going to introduce the glueball field, the mixing with scalar isoscalar quarkonia will change the situation. One has to consider the mixing among three states, which cannot be described by a single angle. For simplicity, in our estimations we resort to a numerical diagonalization procedure, not to the algebraic one. Concerning the pseudoscalar sector, one
can avail oneself with the results given in [24]. All what concerns dealing with the glueball is discussed in the next Section.

3. NAMBU–JONA-LASINIO MODEL WITH DILATON

As we have already mentioned above, we introduce the glueball field into our effective Lagrangian obtained in the previous Section, as a dilaton. For this purpose, we use the following principle. Insofar as the QCD Lagrangian is scale-invariant in the chiral limit, we suppose that our effective meson Lagrangian, motivated by QCD, has also to be scale-invariant both before and after SBCS in the case when the current quark masses are equal to zero. As a result, we come to the following prescription: the dimensional model parameters $G$, $\Lambda$, $K$, and $m_a$ are replaced by the following rule: $G \to G(\chi_c/\chi)^2$, $K \to K(\chi_c/\chi)^5$, $\Lambda \to \Lambda(\chi/\chi_c)^2$, $m_a \to m_a(\chi/\chi_c)$, where $\chi$ is the dilaton field with the vacuum expectation value $\chi_c$. But there are terms that break scale invariance. They are the terms containing current quark masses; the scale anomaly of QCD, reproduced by the dilaton potential; and terms of the type $h_\phi \phi_0^2$ and $h_\sigma \sigma_0^2$ (see (3)), induced by gluon anomalies in the meson Lagrangian.

As it was mentioned in the previous paragraph, the current quark masses break scale invariance and, therefore, should not be multiplied by dilaton fields. The modified current quark masses $\bar{m}^0_a$ are also not multiplied by dilaton fields. In particular, this transforms formula (14) to what follows:

$$\sigma_a = \sigma'_a - \mu_a \frac{\chi}{\chi_c} + \bar{m}_a^0.$$  \hspace{1cm} (27)

Finally, we come to the following Lagrangian:

$$\bar{\mathcal{L}}(\sigma^r, \phi^r, \chi) = \mathcal{L}(\chi) + L_{\text{kin}}(\sigma^r, \phi^r) + \bar{\mathcal{L}}_G(\sigma^r, \phi^r, \chi) + L_{1-\text{loop}} + \Delta L_{\text{an}}.$$  \hspace{1cm} (28)

Here $\mathcal{L}(\chi)$ is the pure dilaton Lagrangian

$$\mathcal{L}(\chi) = \frac{1}{2} (\partial_\nu \chi)^2 - V(\chi)$$  \hspace{1cm} (29)
with the potential
\[ V(\chi) = B \left( \frac{\chi}{\chi_0} \right)^4 \left[ \ln \left( \frac{\chi}{\chi_0} \right)^4 - 1 \right] \]  \hspace{1cm} (30)
that has a minimum at \( \chi = \chi_0 \), and the parameter \( B \) representing the vacuum energy when there are no quarks. The curvature of the potential at its minimum determines the bare glueball mass
\[ m_g = \frac{4\sqrt{B}}{\chi_0}. \]  \hspace{1cm} (31)

The part \( L_{\text{kin}}(\sigma^r, \phi^r) \) of Lagrangian (28) contains pure kinetic terms:
\[ L_{\text{kin}}(\sigma^r, \phi^r) = \frac{1}{2} \sum_{a=1}^{9} \left[ (\partial_\nu \sigma^r_a)^2 + (\partial_\nu \phi^r_a)^2 \right]. \]  \hspace{1cm} (32)

The next term reads
\[
\bar{L}_G(\sigma^r, \phi^r, \chi) = -\frac{1}{2} \left( \frac{\chi}{\chi_c} \right)^2 \sum_{a,b=1}^{9} \left( g_a \sigma^r_a - \mu_a \frac{\chi}{\chi_c} + \bar{\mu}_a^0 \right) (G^{(-)})^{-1}_{ab} \times \\
\times \left( g_b \sigma^r_b - \mu_b \frac{\chi}{\chi_c} + \bar{\mu}_b^0 \right) - \\
-\frac{Z}{2} \left( \frac{\chi}{\chi_c} \right)^2 \sum_{a,b=1}^{9} g_a \phi^r_a (G^{(+)})^{-1}_{ab} g_b \phi^r_b. \]  \hspace{1cm} (33)

The sum of one-loop quark diagrams is denoted as \( L_{\text{1-loop}} \):
\[
L_{\text{1-loop}} = \text{tr} \left[ -4mgI_1^A(m)(\sigma^r \left( \frac{\chi}{\chi_c} \right)^3 + 2g^2I_1^A(m)(\sigma^r \phi^r)^2 \left( \frac{\chi}{\chi_c} \right)^2 - \\
-m^2g^2 \sigma^r \phi^r \left( \frac{\chi}{\chi_c} \right)^2 + mg \frac{\chi}{\chi_c} \sigma^r (\sigma^r \phi^r)^2 - \\
- \frac{g^2}{4} (\sigma^r \phi^r)^2 \right]. \]  \hspace{1cm} (34)

As one can see, expanding (33) in a power series of \( \chi \), we can extract a term that is of order \( \chi^4 \). It can be absorbed by the term in the pure dilaton potential which has the same degree of \( \chi \). Obviously, this leads only to a redefinition of constants \( B \) and \( \chi_0 \) which anyway are not known from
the very beginning. Moreover, saying in advance, the terms like \( \chi^4 \) do not contribute to the divergence of the dilatation current \( \Pi \) because of their scale invariance.

If the procedure of the scale invariance restoration of this Lagrangian is implemented, the part induced by gluon anomalies also becomes scale-invariant. To avoid this, one should subtract this part in the scale-invariant form and add it in a scale-breaking (SB) form. This is achieved by including the term \( \Delta L_{\text{an}} \):

\[
\Delta L_{\text{an}} = -L_{\text{an}} \left( \frac{\chi}{\chi_c} \right)^2 + L_{\text{an}}^{\text{SB}}. \tag{35}
\]

The term \( L_{\text{an}} \) was introduced in (3). In \( L_{\text{an}} \), we will use renormalized \( \sigma_0^r \) and \( \phi_0^r \) fields in place of \( \sigma_0 \) and \( \phi_0 \), however, taking into account the effects of nonzero vacuum expectation value of \( \sigma_0 \). Let us define the scale-breaking term \( L_{\text{an}}^{\text{SB}} \). The coefficients \( h_{\sigma} \) and \( h_{\phi} \) in (3) can be determined by comparing them with the terms in (33) that describe the singlet-octet mixing. We obtain

\[
h_{\phi} = -\frac{3}{2\sqrt{2}} g_u g_s Z (G^{(+)\!}_8)_{89}^{-1}, \quad h_{\sigma} = \frac{3}{2\sqrt{2}} g_u g_s (G^{(-)\!}_8)_{89}^{-1}. \tag{36}
\]

If these terms were to be made scale invariant, one should insert \( (\chi/\chi_c)^2 \) into them. However, as the gluon anomalies break scale invariance, we introduce the dilaton field into these terms in a more complicated way. The inverse matrix elements \( (G^{(+)\!})_{ab}^{-1} \) and \( (G^{(-)\!})_{ab}^{-1} \),

\[
(G^{(+)\!})_{89}^{-1} = \frac{-4\sqrt{2} m_u K I_{1}^A (m_u)}{G^{(+)\!}_{88} G^{(+)\!}_{99} - (G^{(+)\!}_{89})^2}, \tag{37}
\]

\[
(G^{(-)\!})_{89}^{-1} = \frac{4\sqrt{2} m_u K I_{1}^A (m_u)}{G^{(-)\!}_{88} G^{(-)\!}_{99} - (G^{(-)\!}_{89})^2}, \tag{38}
\]

are determined by two different interactions. The numerators are fully defined by the ’t Hooft interaction that leads to anomalous terms (3) breaking scale invariance, therefore we do not introduce here dilaton fields. The denominators are determined by constant \( G \) describing the main four-quark
interaction, and the dilaton field is inserted into it, according to the pre-
scription given in the beginning of this Section. Finally, we come to the
following form of $L_{SB}^{\text{an}}$:

$$L_{SB}^{\text{an}} = \left( -h_\phi \phi_0^2 + h_\sigma \left( \sigma_0^r - F_0 \frac{X}{X_c} + F_0^0 \right)^2 \right) \left( \frac{X}{X_c} \right)^4,$$

(39)

$$F_0 = \frac{\sqrt{2} m_u}{3 g_u} + \frac{m_s}{\sqrt{6} g_s}, \quad F_0^0 = \frac{\sqrt{2} \bar{m}_u^0}{3 g_u} + \frac{\bar{m}_s^0}{\sqrt{6} g_s}. \quad (40)$$

From it, we immediately obtain the term $\Delta L_{\text{an}}$:

$$\Delta L_{\text{an}} = \left( h_\phi \phi_0^2 - h_\sigma \left( \sigma_0^r - F_0 \frac{X}{X_c} + F_0^0 \right)^2 \right) \left( \frac{X}{X_c} \right)^2 \left( 1 - \left( \frac{X}{X_c} \right)^2 \right).$$

(41)

Let us now consider the vacuum expectation value of the divergence of the
dilatation current calculated from the potential of the effective meson-dilaton
Lagrangian:

$$\langle \partial_\mu S^\mu \rangle = \left( \sum_{a=8}^9 \sigma_a^r \frac{\partial V}{\partial \sigma_a^r} + \chi \frac{\partial V}{\partial \chi} - 4V \right) \bigg|_{\chi=\chi_c, \sigma_a^r=0} =$$

$$= 4B \left( \frac{X}{X_0} \right)^4 - 2h_\sigma \left( F_0 - F_0^0 \right)^2 - \sum_{q=u,d,s} \bar{m}_q^0 \langle \bar{q}q \rangle. \quad (42)$$

Here $V = V(\chi) + \bar{V}(\sigma^r, \phi^r, \chi)$, and $\bar{V}(\sigma^r, \phi^r, \chi)$ is the potential part of La-
grangian $\bar{L}(\sigma^r, \phi^r, \chi)$ that does not contain the pure dilaton potential. The
expression given in (42) is simplified by using the following relation of the
quark condensates to integrals $I_1^A(m_u)$ and $I_1^A(m_s)$:

$$4m_q I_1^A(m_q) = -\langle \bar{q}q \rangle_0, \quad (q = u, d, s), \quad (43)$$

and that these integrals are connected with constants $G_{ab}^{(-)}$ through gap equa-
tions, as it will be shown in the next Section (see (47) and (48) below). Com-
paring the QCD expression (1) with (42), one can see that the term $\sum m_q^0 \langle \bar{q}q \rangle$
on the left-hand side is canceled by the corresponding contribution on the
right-hand side. Equating the right hand sides of (1) and (42),
\[ C_g - \sum_{q=u,d,s} m_q^0 \langle \bar{q}q \rangle = \]
\[ = 4B \left( \frac{X_c}{X_0} \right)^4 - 2h_\sigma \left( F_0 - F_0^0 \right)^2 - \sum_{q=u,d,s} \bar{m}_q^0 \langle \bar{q}q \rangle, \]  
(44)

we obtain the correspondence

\[ C_g = 4B \left( \frac{X_c}{X_0} \right)^4 + \sum_{a,b=8} \left( \mu_a^0 - \mu_a \right) (G^{(-)})_{ab}^{-1} (\mu_b - \bar{\mu}_b) - 2h_\sigma \left( F_0 - F_0^0 \right)^2, \]  
(45)

where \( \mu_a^0 = 0 \) \( (a = 1, \ldots 7), \mu_8^0 = m_u^0, \) and \( \mu_9^0 = -m_s/\sqrt{2}. \) This equation relates the gluon condensate, whose value we take from other models (see, e. g., [28]), to the model parameter \( B. \) The next step is to investigate the gap equations.

4. EQUATIONS

As usual, gap equations follow from the requirement that the terms linear in \( \sigma^r \) and \( \chi' \) should be absent in our Lagrangian:

\[
\frac{\delta \bar{L}}{\delta \sigma_8^r} \bigg|_{(\phi^r, \sigma^r, \chi')} = 0, \quad \frac{\delta \bar{L}}{\delta \sigma_8^r} \bigg|_{(\phi^r, \sigma^r, \chi')} = 0, \quad \frac{\delta \bar{L}}{\delta \chi} \bigg|_{(\phi^r, \sigma^r, \chi')} = 0. \]  
(46)

Here, the field \( \chi' = \chi - \chi_c \) with a zero vacuum expectation value \( \langle \chi' \rangle_0 = 0, \) is associated with the glueball field. In further calculations, the Lagrangian is expanded in power series of \( \chi' \). As a result the following equations are obtained:

\[
\left( m_u - \bar{m}_u^0 \right) (G^{(-)})_{88}^{-1} - \frac{m_s - \bar{m}_s^0}{\sqrt{2}} (G^{(-)})_{89}^{-1} - 8m_u I_1^A (m_u) = 0, \]  
(47)

\[
\left( m_s - \bar{m}_s^0 \right) (G^{(-)})_{99}^{-1} - \sqrt{2}(m_u - \bar{m}_u^0) (G^{(-)})_{98}^{-1} - 8m_s I_1^A (m_s) = 0, \]  
(48)

\[
4B \left( \frac{X_c}{X_0} \right)^4 \frac{1}{X_0} \ln \left( \frac{X_c}{X_0} \right)^4 + \frac{1}{X_c} \left( \sum_{a,b=8} \bar{\mu}_a^0 (G^{(-)})_{ab}^{-1} (\bar{\mu}_b - 3\mu_b) \right) - \frac{2h_\sigma}{X_c} \left( F_0 - F_0^0 \right)^2 = 0. \]  
(49)

Using (9) and (10), one can rewrite equations (47) and (48) in the well-known form [27]:
\begin{align}
m_u^0 &= m_u - 8G m_u I_1^A (m_u) - 32 K m_u m_s I_1^A (m_u) I_1^A (m_s), \quad (50) \\
m_s^0 &= m_s - 8G m_s I_1^A (m_s) - 32 K (m_u I_1^A (m_u))^2. \quad (51)
\end{align}

To define the masses of quarkonia and the glueball, let us consider the part of Lagrangian \((\text{23})\) which is quadratic in fields \(\sigma^r\) and \(\chi'\) and which we denote as \(L^{(2)}\)

\[
L^{(2)}(\sigma, \phi, \chi') = -\frac{1}{2} g_8^2 \left\{ \left[ (G^{(-)})^{-1}_{ss} - 8I_1^A (m_u) \right] + 4m_u^2 \right\} \sigma^r g - \\
-\frac{1}{2} g_9^2 \left\{ \left[ (G^{(-)})^{-1}_{ss} - 8I_1^A (m_s) \right] + 4m_s^2 \right\} \sigma^r g - \\
g_8 g_9 (G^{(-)})^{-1}_{ss} \sigma^r g - \frac{M_g^2}{2} \chi'^2 + \\
+ \sum_{a,b=8} \bar{\mu}_a^0 (G^{(-)})^{-1}_{ab} g_b \sigma^r_b \chi' + \frac{4h_\sigma (F_0^2 - F_0^0)}{\chi_c \sqrt{3}} \left( \sigma^r_9 - \sigma^r_8 \sqrt{2} \right) \chi', \quad (52)
\]

where

\[
M_g^2 = \chi_c^{-2} (4C_g + \sum_{a,b=8} \bar{\mu}_a^0 (G^{(-)})^{-1}_{ab} (2\bar{\mu}_b^0 - \mu_b) + \\
+ \sum_{a,b=8} 4\mu_a^0 (G^{(-)})^{-1}_{ab} (\mu_b - \bar{\mu}_b^0) - h_\sigma 4F_0^2 + 4h_\sigma (F_0^0)^2) \quad (53)
\]

is the glueball mass before taking account of mixing effects.

From this Lagrangian, after diagonalization, we obtain the masses of three scalar meson states: \(\sigma_I, \sigma_{II},\) and \(\sigma_{III},\) and a matrix of mixing coefficients \(b\) that connects the nondiagonalized fields \(\sigma_8 \equiv \sigma_u, \sigma_9 \equiv \sigma_s, \chi'\) with the physical ones \(\sigma_I, \sigma_{II}, \sigma_{III}\)

\[
\begin{pmatrix}
\sigma_u \\
\sigma_s \\
\chi'
\end{pmatrix} =
\begin{pmatrix}
b_{\sigma_u \sigma_I} & b_{\sigma_u \sigma_{II}} & b_{\sigma_u \sigma_{III}} \\
\sigma_{II} \\
\sigma_{III}
\end{pmatrix}

\begin{pmatrix}
\sigma_I \\
\sigma_{II} \\
\sigma_{III}
\end{pmatrix} \quad (54)
\]

5. MODEL PARAMETERS AND NUMERICAL ESTIMATES

The basic parameters of our model are \(G, K, \Lambda, m_u,\) and \(m_s.\) After the dilaton fields are introduced, they keep their values \([13][24]:\)
\[ m_u = 280 \text{ MeV}, \ m_s = 420 \text{ MeV}, \ \Lambda = 1.26 \text{ GeV}, \]
\[ G = 4.38 \text{ GeV}^{-2}, \ K = 11.2 \text{ GeV}^{-5}. \]  \tag{55}

Moreover, new three parameters \( \chi_0, \chi_c, \) and \( B \) appear. To fix the new parameters, one should use equations (45), (49), and the physical glueball mass. As a result we obtain for \( \chi_0 \) and \( B \):

\[ \chi_0 = \chi_c \exp \left( -\frac{\sum_{a,b=8} G(-1)_{ab}(3\mu_b - \bar{\mu}_b)}{4C_g - (\bar{\mu}_a - \mu_a)(G(-1)_{ab}(\bar{\mu}_b - \mu_b) + 2h_\sigma (F_0 - F_0^0)^2)} \right), \]  \tag{56}

\[ B = \frac{C_g - (\bar{\mu}_a - \mu_a)(G(-1)_{ab}(\bar{\mu}_b - \mu_b) + 2h_\sigma (F_0 - F_0^0)^2}{4} \left( \frac{\chi_0}{\chi_c} \right)^4. \]  \tag{57}

We adjust the parameter \( \chi_c \) so that the mass of the heaviest scalar meson, \( \sigma_{\text{III}} \), would be either 1500 MeV or 1710 MeV. The result of our fit for both cases is given in Table 1. One also will find the mixing coefficients in Table 2.

### 6. DECAY WIDTHS

Once all parameters are fixed, we can estimate the decay widths for the main strong decay modes of scalar mesons: \( \sigma_l \to \pi \pi, KK, \eta \eta, \eta \eta' \), and \( 4\pi \) where \( l = I, II, III \).

Note that, in the energy region under consideration (\( \sim 1500 \text{ MeV} \)), we work on the brim of the validity of exploiting the chiral symmetry that was used to construct our effective Lagrangian. Thus, we can consider our results as rather qualitative.

Let us start with the lightest scalar isoscalar meson state \( \sigma_1 \), associated with \( f_0(400 - 1200) \). This state decays into pions. This is the only strong decay mode, because \( \sigma_1 \) is too light for other channels to be open. The amplitude describing its decay into pions has the form:

\[ A_{\sigma_1 \to \pi^+ \pi^-} = 2A^g_{\pi^+ \pi^-} b_{\chi\sigma_1} + 2A^u b_{\sigma_1 \sigma_1}, \]  \tag{58}

\[ A^g_{\pi^+ \pi^-} = -\frac{M_\pi^2}{\chi_c}, \quad A^u = 2g_u m_u Z, \]  \tag{59}
where \( A^g_{\pi^+\pi^-} \) is the contribution from the glueball component; and \( A^u \), from the \((\bar{u}u)\) quarkonium one. The coefficients \( b_{\chi'\sigma_1} \) and \( b_{\sigma_u\sigma_1} \) represent the corresponding elements of the \(3 \times 3\) mixing matrix for scalar isoscalar states (see Table 2). Both contributions have equal signs and add to the the width of \( \sigma_1 \).

To calculate the decay width of a meson into two mesons, one can use the following formula:

\[
\Gamma = \frac{|A|^2}{16\pi M^3} \frac{\lambda^{1/2}(M^2, M_1^2, M_2^2)}{r},
\]

where \( A \) is the amplitude of the process; \( M \) is the mass of a decaying particle; \( M_1 \) and \( M_2 \) are masses of secondary particles; \( r \) is the dimension of the permutation symmetry group in the phase space of final states. The function \( \lambda(x, y, z) \) is defined as follows [30]:

\[
\lambda(x, y, z) = (x - y - z)^2 - 4yz.
\]

For the decay of \( \sigma_1 \) into pions, formula (60) can be rewritten in a simpler form

\[
\Gamma_{\sigma_1 \rightarrow \pi^+\pi^-} = \frac{|A_{\sigma_1 \rightarrow \pi^+\pi^-}|^2}{16\pi M_{\sigma_1}} \sqrt{1 - \frac{4M_{\pi}^2}{M_{\sigma_1}^2}}.
\]

Using isotopic symmetry, we obtain the total width

\[
\Gamma_{\sigma_1 \rightarrow \pi\pi} = \frac{3}{2} \Gamma_{\sigma_1 \rightarrow \pi^+\pi^-} \approx 820 \text{ MeV}
\]

for the case when the model parameters are fixed for the state \( \sigma_{III} \) identified with \( f_0(1500) \), and

\[
\Gamma_{\sigma_1 \rightarrow \pi\pi} \approx 830 \text{ MeV}.
\]

for the case \( \sigma_{III} \equiv f_0(1710) \). The experimental value is known with a large uncertainty and is reported to lie in the interval from 600 to 1000 MeV [29].

The amplitude describing the decay of the state \( \sigma_{II} \) which we identify with \( f_0(980) \) into pions also consists of two parts.
Here the glueball contribution is small again and the quarkonium determines the decay width, however, in this case both contributions are opposite in sign and slightly compensate each other. The width of the state $\sigma_{\Pi}$ is close to that obtained in the model without glueballs \[24\]. We obtain

$$
\Gamma_{\sigma_{\Pi} \to \pi\pi} \approx 28 \text{ MeV},
$$

if $\sigma_{\Pi} \equiv f_0(1500)$ and

$$
\Gamma_{\sigma_{\Pi} \to \pi\pi} \approx 26 \text{ MeV},
$$

if $\sigma_{\Pi} \equiv f_0(1710)$. The experiment gives for the decay of $\sigma_{\Pi}$ into pions a value lying within the range 30 – 70 MeV \[29\].

Now let us proceed with decays of $\sigma_{\Pi}$. The process $\sigma_{\Pi} \to \pi^+\pi^-$ is given by the amplitude

$$
A_{\sigma_{\Pi} \to \pi^+\pi^-} = 2A_{\pi^+\pi^-}^g b_{\chi'} \sigma_{\Pi} + 2A^u b_{\sigma_u} \sigma_{\Pi}
$$

that consists of two parts. The first part represents the contribution from the pure glueball. This contribution is small (since it is proportional to the pion mass squared), and the process is determined by the second part that describes the decay of the quarkonium component. As a result, the width of the decay $\sigma_{\Pi} \to \pi\pi$ if $\sigma_{\Pi} \equiv f_0(1500)$ is

$$
\Gamma_{\sigma_{\Pi} \to \pi\pi} = \frac{3}{2} \Gamma_{\sigma_{\Pi} \to \pi^+\pi^-} \approx 14 \text{ MeV},
$$

and, if $\sigma_{\Pi} \equiv f_0(1710)$,

$$
\Gamma_{\sigma_{\Pi} \to \pi\pi} \approx 8 \text{ MeV}.
$$

In the case of $K\bar{K}$ channels, the contribution of the pure glueball is also proportional to the kaon mass squared, and is rather large as compared to the pions case. The amplitude of the decay $\sigma_{\Pi} \to K^+K^-$ consists of three parts.
\[ A_{\sigma_{\text{III}} \rightarrow K^+ K^-} = A_{KK}^g b_{\sigma_{\text{III}}} + A_{KK}^u b_{\sigma_{\text{III}}} + A_{KK}^s b_{\sigma_{\text{III}}}, \tag{71} \]

where the pure glueball decay into \( K^+ K^- \) is given by the amplitude

\[ A_{KK}^g = -\frac{2M_K^2}{\chi_c}. \tag{72} \]

The quarkonium contributions are

\[ A_{KK}^u = 2g_u Z \left( \frac{m_u + m_s}{2} \left( \frac{F_{\pi}}{F_K} \right)^2 + \frac{m_s(m_u - m_s)}{m_u + m_s} \right), \tag{73} \]

\[ A_{KK}^s = -4\sqrt{2}g_s Z \left( \frac{m_u + m_s}{2} \left( \frac{F_s}{F_K} \right)^2 + \frac{m_u(m_s - m_u)}{m_u + m_s} \right), \tag{74} \]

where \( F_{\pi} \) and \( F_K \) are the pion and kaon weak decay constants, respectively, and \( F_s = m_s/(g_s\sqrt{Z}) \). In the case when \( \sigma_{\text{III}} \) is \( f_0(1500) \), we have

\[ \Gamma_{\sigma_{\text{III}} \rightarrow K K} = \Gamma_{\sigma_{\text{III}} \rightarrow K^+ K^-} + \Gamma_{\sigma_{\text{III}} \rightarrow K^0 \bar{K}^0} = 2\Gamma_{\sigma_{\text{III}} \rightarrow K^+ K^-} \approx 29 \text{ MeV}, \tag{75} \]

and in the other case (\( \sigma_{\text{III}} \equiv f_0(1710) \))

\[ \Gamma_{\sigma_{\text{III}} \rightarrow K K} \approx 60 \text{ MeV}. \tag{76} \]

The amplitude of the decay of \( \sigma_{\text{III}} \) into \( \eta \eta \) and \( \eta \eta' \) can also be considered in the same manner. The only complication is the singlet-octet mixing in the pseudoscalar sector and additional vertices coming from \( \Delta L_{an} \). The corresponding amplitude is

\[ A_{\sigma_{\text{III}} \rightarrow \eta \eta} = 2A_{\eta \eta}^g b_{\chi'_{\sigma_{\text{III}}} + 2A^u \sin^2 \bar{\theta} b_{\sigma_u \sigma_{\text{III}}} + 2A^s \cos^2 \bar{\theta} b_{\sigma_s \sigma_{\text{III}}} + 2A_{\phi}^\text{an} \sin^2 \theta b_{\chi'_{\sigma_{\text{III}}}}, \tag{77} \]

\[ A_{\eta \eta}^g = -\frac{M_\eta^2}{\chi_c}, \tag{78} \]

\[ A_{\phi}^\text{an} = -\frac{2h_{\phi}}{\chi_c}, \tag{79} \]

where \( \bar{\theta} = \theta - \theta_0 \), with \( \theta \) being the singlet-octet mixing angle in the pseudoscalar channel, \( \theta \approx -19^\circ \), and \( \theta_0 \) the ideal mixing angle, \( \tan \theta_0 = 1/\sqrt{2} \).

The decay widths thereby are: if \( \sigma_{\text{III}} \equiv f_0(1500) \),
\[ \Gamma_{\sigma_{\text{III}} \rightarrow \eta} \approx 25 \text{ MeV}, \]  

(80)

and if \( \sigma_{\text{III}} \equiv f_0(1710) \),

\[ \Gamma_{\sigma_{\text{III}} \rightarrow \eta} \approx 43 \text{ MeV}. \]  

(81)

For the decay of \( \sigma_{\text{III}} \) into \( \eta \eta' \), we have the following amplitude

\[ A_{\sigma_{\text{III}} \rightarrow \eta \eta'} = -A^u \sin 2\theta b_{\sigma_\sigma} \sigma_{\text{III}} + A^s \sin 2\theta b_{\sigma_\sigma} \sigma_{\text{III}} - A^a_\sigma \sin 2\theta b_{\chi'} \sigma_{\text{III}}. \]  

(82)

The direct decay of a bare glueball into \( \eta \eta' \) is absent here. This process occurs only due to the mixing of the glueball and scalar isoscalar quarkonia and the anomaly contribution. The decay widths are as follows:

\[ \Gamma_{\sigma_{\text{III}} \rightarrow \eta \eta'} \sim 10 \text{ MeV}, \]  

(83)

for \( \sigma_{\text{III}} \equiv f_0(1500) \), and

\[ \Gamma_{\sigma_{\text{III}} \rightarrow \eta \eta'} \approx 30 \text{ MeV}. \]  

(84)

for \( \sigma_{\text{III}} \equiv f_0(1710) \). The estimate for the decay \( f_0(1500) \) into \( \eta \eta' \) is very rough, because the decay is allowed only due to a finite width of the resonance as its mass lies a little bit below the \( \eta \eta' \) threshold. The calculation is made for the mass of \( f_0(1500) \) plus its half-width. For \( f_0(1710) \), we have a more reliable estimate, since its mass is large enough for the decay to be possible.

Up to this moment we considered only decays into a pair of mesons. For the state \( \sigma_{\text{III}} \), there is a possibility to decay into 4 pions. This decay can occur through intermediate \( \sigma \) \( (f_0(400 - 1200)) \) resonance.

The decay through the \( \sigma \)-resonance can be represented as two processes: with two resonances \( \sigma_{\text{III}} \rightarrow \sigma \sigma \rightarrow 4\pi \) and one resonance \( \sigma_{\text{III}} \rightarrow \sigma 2\pi \rightarrow 4\pi \). The vertices determining these decays follow from Lagrangian (28). The decay of a glueball into two \( \sigma \) is given by the amplitude:

\[ A_{\sigma_{\text{III}} \rightarrow \sigma \sigma} \approx 2A^g_{\sigma \sigma} b_{\chi' \sigma_{\text{III}}} + 3Z^{-1} A^u_{b_{\sigma_\sigma} \sigma_{\text{III}}} b_{\sigma_\sigma} \sigma_1 b_{\sigma_\sigma} \sigma_1 + +2A^a_{\sigma} b_{\chi' \sigma_{\text{III}}} b_{\sigma_\sigma} \sigma_1. \]  

(85)
where $A^g_{\sigma\sigma}$ is the pure glueball amplitude\(^3\):

$$A^g_{\sigma\sigma} \approx -\frac{M^2_{\sigma_u}}{\chi_c},$$

(86)

and the anomaly amplitude $A^{an}_{\sigma\sigma}$ coming from $\Delta L_{an}$ is

$$A^{an}_{\sigma\sigma} = \frac{2h_\sigma}{3\chi_c}.$$

(87)

The total amplitude describing the decay into 4 pions through two $\sigma$-resonances is

$$A_{\sigma_{III} \rightarrow \sigma\sigma \rightarrow 2\pi^+2\pi^-} = 2A_{\sigma_{III} \rightarrow \sigma\sigma}A^2_{\sigma \rightarrow \pi^+\pi^-}(\Delta_\sigma(s_{12})\Delta_\sigma(s_{34}) +$$

$$+\Delta(s_{14})\Delta(s_{23})),$$

(88)

where the function $\Delta_\sigma(s)$ appears due to the resonant structure of the processes

$$\Delta_\sigma(s) = (s - M^2_{\sigma_I} + iM_{\sigma_I}\Gamma_{\sigma_I})^{-1},$$

(89)

where $\Gamma_\sigma$ is the decay width of the $\sigma_I$ resonance (see (53)). This function depends on an invariant mass squared $s_{ij}$ defined as follows

$$s_{ij} = (k_i + k_j)^2, \quad (i, j = 1, \ldots, 4).$$

(90)

Here $i$ and $j$ enumerate the momenta $k_i$ of pions $\pi^+(k_1), \pi^-(k_2), \pi^+(k_3)$, and $\pi^-(k_4)$.

Now let us consider the decay into $4\pi$ through one $\sigma$-resonance. The amplitude describing this process is as follows:

$$A_{\sigma_{III} \rightarrow \sigma2\pi} = A^g_{\sigma2\pi}(b_{\sigma_u\sigma_I}b_{\chi^I\sigma_{III}} + b_{\sigma_u\sigma_{III}}b_{\chi^I\sigma_I}) + A^{u}_{\sigma2\pi}b_{\sigma_u\sigma_{III}}b_{\sigma_u\sigma_I},$$

(91)

The glueball amplitude is

\(^3\) To obtain an approximate estimate for the glueball contribution, we used the mass of $\sigma_u$ state before diagonalization (see the term with $\sigma^r_u$ in (53))
\[ A_{\sigma_{2\pi}}^g = \frac{4m_u g_u Z}{\chi_c} \] (92)

and the quarkonium:

\[ A_{\sigma_{\text{III}} \rightarrow \sigma_{2\pi}}^u = -4g_u^2 Z. \] (93)

The glueball contribution prevails over the quarkonium one in magnitude and is opposite in sign.

The amplitude describing the decay \( \sigma_{\text{III}} \rightarrow 2\pi^+ 2\pi^- \) through one \( \sigma \)-resonance is:

\[ A_{\sigma_{\text{III}} \rightarrow \sigma_{2\pi} \rightarrow 2\pi^+ 2\pi^-} = -A_{\sigma_{\text{III}} \rightarrow \sigma_{2\pi}} A_{\sigma \rightarrow \pi^+ \pi^-} (\Delta_{\sigma}(s_{12}) + \Delta_{\sigma}(s_{34}) + \Delta_{\sigma}(s_{14}) + \Delta_{\sigma}(s_{23})), \] (94)

The total amplitude of the decay into \( 2\pi^+ 2\pi^- \) via \( \sigma \)-resonances is obtained as a cumulative contribution from both one and two intermediate \( \sigma \) mesons:

\[ A_{\sigma_{\text{III}} \rightarrow 2\pi^+ 2\pi^-} = A_{\sigma_{\text{III}} \rightarrow \sigma \rightarrow 2\pi^+ 2\pi^-} + A_{\sigma_{\text{III}} \rightarrow \sigma_{2\pi} \rightarrow 2\pi^+ 2\pi^-}. \] (95)

The amplitude describing the decay into \( 2\pi^0 \pi^+ \pi^- \) has the following form:

\[ A_{\sigma_{\text{III}} \rightarrow 2\pi^0 \pi^+ \pi^-} = A_{\sigma_{\text{III}} \rightarrow \sigma \rightarrow 2\pi^0 \pi^+ \pi^-} + A_{\sigma_{\text{III}} \rightarrow \sigma_{2\pi} \rightarrow 2\pi^0 \pi^+ \pi^-}, \] (96)

where

\[ A_{\sigma_{\text{III}} \rightarrow \sigma \rightarrow 2\pi^0 \pi^+ \pi^-} = 4A_{\sigma_{\text{III}} \rightarrow \sigma \rightarrow 2\pi^0} A_{\sigma \rightarrow \pi^+ \pi^-} \Delta_{\sigma}(s_{12}) \Delta_{\sigma}(s_{34}), \] (97)

\[ A_{\sigma_{\text{III}} \rightarrow \sigma_{2\pi} \rightarrow 2\pi^0 \pi^+ \pi^-} = -2A_{\sigma_{\text{III}} \rightarrow \sigma_{2\pi}} A_{\sigma \rightarrow 2\pi^0} (\Delta_{\sigma}(s_{12}) + \Delta_{\sigma}(s_{34})). \] (98)

In this case, \( k_1 \) and \( k_2 \) are momenta of the two \( \pi^0 \), and \( s_{12} \) is their invariant mass squared. Indices 3 and 4 stand for \( \pi^+ \) and \( \pi^- \), respectively. The amplitude \( A_{\sigma \rightarrow 2\pi^0} \) is equal to \( 0.5A_{\sigma \rightarrow \pi^+ \pi^-} \).

In the case of the decay into \( 4\pi^0 \), we have

\[ A_{\sigma_{\text{III}} \rightarrow 4\pi^0} = A_{\sigma_{\text{III}} \rightarrow \sigma \rightarrow 4\pi^0} + A_{\sigma_{\text{III}} \rightarrow \sigma_{2\pi} \rightarrow 4\pi^0}, \] (99)

where
\[ A_{\sigma_{\text{III}} \to \sigma \to 4\pi^0} = 4A_{\sigma_{\text{III}} \to 2\sigma} A_{\sigma \to 2\pi^0} (\Delta_\sigma (s_{12}) \Delta_\sigma (s_{34}) + \Delta_\sigma (s_{13}) \Delta_\sigma (s_{24}) + \\
+ \Delta_\sigma (s_{14}) \Delta_\sigma (s_{23})), \]  
(100)

\[ A_{\sigma_{\text{III}} \to 2\pi \to 2\pi^0 \pi^+ \pi^-} = -2A_{\sigma_{\text{III}} \to 2\pi} A_{\sigma \to 2\pi^0} (\Delta_\sigma (s_{12}) + \Delta_\sigma (s_{13}) + \Delta_\sigma (s_{14}) + \\
+ \Delta_\sigma (s_{23}) + \Delta_\sigma (s_{24}) + \Delta_\sigma (s_{34})). \]  
(101)

Let us give numerical estimates for these decay modes. The decay width of the glueball into four particles is calculated using the prescript given in [30]

\[ \Gamma_{4\pi} = \frac{1}{64(2\pi)^6 r M_{\sigma_{\text{III}}}^2} \times \]
\[ \times \int_{s_{123}^-}^{s_{123}^+} ds_{123} \int_{s_{12}^-}^{s_{12}^+} ds_{12} \int_{s_{34}^-}^{s_{34}^+} ds_{34} \int_{s_{23}^-}^{s_{23}^+} ds_{23} \int_{-1}^{1} \frac{|A_{\sigma_{\text{III}} \to 4\pi}|^2 d\zeta}{\sqrt{\lambda(s_{123}, s_{12}, M_{\pi}^2)(1 - \zeta^2)}}, \]  
(102)

where \( A_{\sigma_{\text{III}} \to 4\pi} \) is the amplitude describing one of the processes discussed above, \( M_{\sigma_{\text{III}}} \) is the mass of \( \sigma_{\text{III}} \). The corresponding two-particle invariant masses are defined in (90) except for \( s_{123} \), the invariant mass of three pions

\[ s_{123} = (k_1 + k_2 + k_3)^2. \]  
(103)

The cosine between the plane formed by 3-momenta \( k_1, k_2 \) and the plane formed by \( k_3, k_4 \) in the rest frame of three mesons \( (k_1 + k_2 + k_3 = 0) \) is denoted by \( \zeta \). The limits of integration are as follows

\[ s_{123}^- = 9M_{\pi}^2, \quad s_{123}^+ = (M_{\sigma_{\text{III}}} - M_{\pi})^2, \]
\[ s_{12}^- = 4M_{\pi}^2, \quad s_{12}^+ = (\sqrt{s_{123} - M_{\pi}})^2, \]
\[ s_{34}^\pm = 2M_{\pi}^2 + \frac{1}{2s_{123}} [(s_{123} + M_{\pi}^2 - s_{12})(M_{\sigma_{\text{III}}}^2 - M_{\pi}^2 - s_{123}) \pm \\
+ \sqrt{\lambda(s_{123}, s_{12}, M_{\pi}^2)\lambda(M_{\sigma_{\text{III}}}^2, s_{123}, M_{\pi}^2)}], \]
\[ s_{23}^\pm = 2M_{\pi}^2 + \frac{1}{2s_{12}} [s_{12}(s_{123} - M_{\pi}^2 - s_{12}) \pm \\
+ \sqrt{\lambda(s_{12}, M_{\pi}^2, M_{\pi}^2)\lambda(s_{123}, s_{12}, M_{\pi}^2)}]. \]  
(104)

Formula (102) is similar to that given in [31], however, we used here different kinematic variables. As a result, we obtain for the decay into \( 4\pi \):
\[ \Gamma_{\sigma_{III} \to 2\pi^+2\pi^-} \approx 2.2\text{MeV}, \quad \Gamma_{\sigma_{III} \to 2\pi^0\pi^+\pi^-} \approx 1.2\text{MeV}, \quad \Gamma_{\sigma_{III} \to 4\pi^0} \approx 0.1\text{MeV}. \tag{105} \]

The total width is
\[ \Gamma_{\sigma_{III} \to 4\pi}^{\text{tot}} \approx 3.5\text{MeV} \tag{106} \]

and, in the other case \((\sigma_{III} \equiv f_0(1710))\),
\[ \Gamma_{\sigma_{III} \to 2\pi^+2\pi^-} \approx 6\text{MeV}, \quad \Gamma_{\sigma_{III} \to 2\pi^0\pi^+\pi^-} \approx 3.3\text{MeV} \quad \Gamma_{\sigma_{III} \to 4\pi^0} \approx 0.3\text{MeV}. \tag{107} \]

The total width is
\[ \Gamma_{\sigma_{III} \to 4\pi}^{\text{tot}} \approx 10\text{MeV}. \tag{108} \]

As one can see, these values are very small.

The other possibility of the state \(\sigma_{III}\) to decay into 4 pions is to produce two intermediate \(\rho\)-resonances \((\sigma_{III} \to 2\rho \to 4\pi)\). Contrary to the decay through scalar resonances, where strong compensations take place, in the process with \(\rho\)-resonances, no compensation occurs, and it turns out that the decay through \(\rho\) determines the most part of the decay width of \(\sigma_{III}\).

To calculate the amplitude describing the process \(\sigma_{III} \to 2\rho\), we need a piece of the Lagrangian with \(\rho\)-meson fields. Although we did not consider vector mesons in the source Lagrangian, an extended version of NJL model \[13\] contains the vector and axial-vector fields. Taking the mass term for \(\rho\) mesons from \[13\] and including dilaton fields into it according to the principle of scale invariance, we obtain:
\[ \frac{M_\rho^2}{2} \left( \frac{\chi}{\chi_c} \right)^2 (2\rho_\mu^+\rho_\mu^- + \rho_\mu^0\rho_\mu^0), \tag{109} \]

where \(M_\rho = 770\) MeV is the \(\rho\)-meson mass. From this, we derive the vertex describing the decay \(\sigma_{III} \to \rho\rho\):
\[ \frac{M_\rho^2}{\chi_c} b_{\chi'\sigma_{III}} \chi' (2\rho_\mu^+\rho_\mu^- + \rho_\mu^0\rho_\mu^0). \tag{110} \]
The decay of a $\rho$-meson into pions is described by the following amplitude:

$$g_\rho(p_1 - p_2)^\mu.$$  \hfill (111)

where $g_\rho = 6.14$ is the $\rho$-meson decay constant, $p_1$ and $p_2$ are the momenta of $\pi^+$ and $\pi^-$. Finally, we come to the following formula for the amplitude of the process $\sigma_{\text{III}} \to \rho^0 \rho^0 \to 2\pi^+ 2\pi^-:

$$A_{\sigma_{\text{III}} \to \rho^0 \rho^0 \to 2\pi^+ 2\pi^-} = \frac{M_\rho^2 g_\rho^2 b_X \sigma_{\text{III}}}{\chi_c} \left((s_{13} + s_{24} - s_{14} - s_{23})\Delta_\rho(s_{12})\Delta_\rho(s_{34}) + \right.
\left. + (s_{13} + s_{24} - s_{12} - s_{34})\Delta_\rho(s_{14})\Delta_\rho(s_{23})\right),$$  \hfill (112)

The function $\Delta_\rho(s)$ is the following:

$$\Delta_\rho(s) = (s - M_\rho^2 + iM_\rho\Gamma_\rho)^{-1}. \hfill (113)$$

Here $\Gamma_\rho = 150$ MeV is the decay width of the $\rho$ resonance. The decay into $2\pi^0\pi^+\pi^-$ occurs through a pair of charged $\rho$-resonances: $\rho^+$ and $\rho^-$. The amplitude of this process is the same as for the decay with intermediate $\rho^0$. The decay into $4\pi^0$ cannot go via $\rho$-resonances.

In an extended NJL model [13], there are no vertices describing the decay of a quarkonium into $\rho$ mesons. As a result, only the glueball part determines the decay of $\sigma_{\text{III}}$ into 4 pions through $\rho$ resonances unlike the case with $\sigma$ resonances. This leads to a large decay rate through $\rho$ mesons (contrary to decays through $\sigma$).

Now let us give the numerical estimates for the decay into 4 pions. In the case, where $\sigma_{\text{III}} \equiv f_0(1500)$ we have:

$$\Gamma_{\sigma_{\text{III}} \to \rho\rho \to 2\pi^+ 2\pi^-} \approx 50, \quad \Gamma_{\sigma_{\text{III}} \to \rho\rho \to 2\pi^0\pi^+\pi^-} \approx 90 \text{ MeV}, \hfill (114)$$

with the total width:

$$\Gamma_{\sigma_{\text{III}} \to 4\pi}^{\text{tot}} \approx 140 \text{MeV}. \hfill (115)$$

In the other case ($\sigma_{\text{III}} \equiv f_0(1710)$),

$$\Gamma_{\sigma_{\text{III}} \to \rho\rho \to 2\pi^+ 2\pi^-} \approx 350 \text{MeV}, \quad \Gamma_{\sigma_{\text{III}} \to \rho\rho \to 2\pi^0\pi^+\pi^-} \approx 650 \text{ MeV}, \hfill (116)$$
\[ \Gamma_{\sigma_{\text{III}} \rightarrow 4\pi}^{\text{tot}} \approx 1 \text{GeV}. \] \hfill (117)

Now we can estimate the total width of the state \( \sigma_{\text{III}} \). If \( \sigma_{\text{III}} \) is identified with \( f_0(1500) \) we have

\[ \Gamma_{\sigma_{\text{III}}}^{\text{tot}} \approx 220 \text{ MeV}, \] \hfill (118)

which is in qualitative agreement with the experimental value 112 MeV \[29\], and, in the other case (\( \sigma_{\text{III}} \equiv f_0(1710) \)),

\[ \Gamma_{\sigma_{\text{III}}}^{\text{tot}} \approx 1.2 \text{ GeV}, \] \hfill (119)

which is in great contradiction with experimental data. In the last case (\( f_0(1710) \)) \( \rho \) mesons can show up as on-mass-shell decay products at large probability. The decay width is estimated as \( \sim 1 \) GeV. The absence of this decay mode in experimental observations is a reason that \( f_0(1710) \) is not a glueball.

Our estimates for the decay widths of the scalar meson states \( \sigma_1 \), \( \sigma_{\text{II}} \), and \( \sigma_{\text{III}} \) are collected in Table 3.

### 7. CONCLUSION

In the approach presented here, we assume that (with the exception of the dilaton potential and the ’t Hooft interaction) scale invariance holds for the effective Lagrangian before and after SBCS in the chiral limit. On the other hand, we take into account effects of scale invariance breaking that come from three sources: the terms with current quark masses, the dilaton potential reproducing the scale anomaly of QCD, and term \( L_{\text{an}} \) induced by gluon anomalies (see (3) in the Introduction).

The scale invariance breaking that is connected with the term \( L_{\text{an}} \) was not taken into account in our previous paper \[32\]. This led to a small quarkonia-glueball mixing proportional to current quark masses, disappearing in the

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4) Note that there was wrong sign at the term in formula (43) that describes the quarkonia-glueball mixing, which led to incorrect estimates for the decay widths of the scalar glueball.
chiral limit. If the term $\Delta L_{\text{an}}$ taken into account in (28) the quarkonia-glueball mixing becomes much greater and does not disappear in the chiral limit, being proportional to constituent quark masses (quark condensates). This corresponds to results obtained from QCD in [33]. This contribution to the quarkonia-glueball mixing turns out to have decisive effect on the strong decay widths of scalar mesons.

For the scalar meson states $f_0(400 - 1200)$ and $f_0(980)$, we obtain good agreement with experimental data [29]. Their decay widths are determined by quarkonium parts of decay amplitudes.

Strong decays of the scalar meson state $\sigma_{\text{III}}$ (“glueball”) are considered for two different masses: 1500 MeV and 1710 MeV. In the $\pi\pi$ channel, the contribution from quarkonia prevails over that from the glueball and thereby determines the decay rate. In the case of $KK$, $\eta\eta$, $\eta\eta'$, and $\pi\pi$ channels, there are noticeable compensations among decay amplitudes.

A similar situation with compensations takes place in the decay into $4\pi$ with intermediate $\sigma$-mesons. Here we have a strong compensation among the glueball and quarkonia contributions. But there is a possibility for the state $\sigma_{\text{III}}$ to decay through $\rho$-resonances. In this case, as the quarkonium component is absent, no compensation occurs, and this channel determines the most of the total decay width of $\sigma_{\text{III}}$.

We performed calculations for both candidates for the scalar glueball state: $f_0(1500)$ and $f_0(1710)$, and found that $f_0(1500)$ is rather the glueball. The main decay mode is that into 4 pions. The decay rate into a pair of kaons is next by order of magnitude and is followed by the $\eta\eta$, $\eta\eta'$, and $\pi\pi$ decay modes.

The total width of the third scalar isoscalar state is estimated to be about 220 MeV for $M_{\sigma_{\text{III}}} = 1500$ MeV and 1.2 GeV for $M_{\sigma_{\text{III}}} = 1710$ MeV. The experimental width of $f_0(1500)$ is 112 MeV and that of $f_0(1710)$ is 130 MeV. Unfortunately, the detailed data on the branching ratios of $f_0(1500)$ and $f_0(1710)$ are not reliable and are controversial [29].
Our calculations are rather qualitative. However, they allow us to conclude that $f_0(1500)$ is a scalar glueball state, whereas $f_0(1710)$ is a quarkonium, for the following reasons: 1) The total decay width of the glueball in our model better fits to its experimental value if $f_0(1500)$ is assumed to be the glueball, rather than $f_0(1710)$. 2) As it follows from our calculations, the main decay mode of the scalar glueball is that into four pions. This is true for the state $f_0(1500)$. A decay of $f_0(1710)$ into four pions, however, was not seen in experiment. 3) Moreover, a direct decay of into a pair of $\rho$ mesons on their mass-shell is possible for a scalar glueball with the mass about 1.7 GeV. It has also not been observed. Our conclusion concerning the nature of $f_0(1710)$ as a quarkonium state is in agreement with the conclusion made in our papers [12].

We are going to use this approach in our future work for describing both glueballs and ground and radially excited scalar meson nonets which lie in the energy interval from 0.4 to 1.71 GeV.

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8. TABLE CAPTIONS

Table 1. The masses of physical scalar meson states $\sigma_I$, $\sigma_{II}$, $\sigma_{III}$ and the values of the parameters $\chi_c$, $\chi_0$, bag constant $B$, and (bare) glueball mass $m_g$ (in MeV) for two cases: 1) $M_{\sigma_{III}} = 1500$ MeV and 2) $M_{\sigma_{III}} = 1710$ MeV

Table 2. Elements of the matrix $b$, describing mixing in the scalar isoscalar sector. The upper table refers to the case $\sigma_{III} \equiv f_0(1500)$, the lower one to the case $\sigma_{III} \equiv f_0(1710)$

Table 3. The partial and total decay widths (in MeV) of the scalar meson states $f_0(400-1200)$, $f_0(980)$ and of the glueball for two cases: $\sigma_{III} \equiv f_0(1500)$ and $\sigma_{III} \equiv f_0(1710)$, and experimental values of decay widths of $f_0(1500)$ and $f_0(1710)$ [29].
9. TABLES

### TABLE 1.

|     | $\sigma_1$ | $\sigma_{II}$ | $\sigma_{III}$ | $\chi_c$ | $\chi_0$ | $B$, GeV$^4$ | $M_g$  |
|-----|------------|---------------|---------------|----------|----------|---------------|--------|
| I   | 400        | 1100          | 1500          | 206      | 190      | 0.009         | 1447   |
| II  | 400        | 1100          | 1710          | 180      | 166      | 0.009         | 1665   |

### TABLE 2.

|     | $\sigma_1$ | $\sigma_{II}$ | $\sigma_{III}$ |
|-----|------------|---------------|---------------|
| $\sigma^r_u$ |      0.939 |       0.240 |         0.247 |
| $\sigma^r_s$ |     −0.214 |       0.968 |       −0.128 |
| $\chi'$     |     −0.270 |       0.067 |         0.960 |

### TABLE 3.

|     | $\Gamma_{\pi\pi}$ | $\Gamma_{KK}$ | $\Gamma_{\eta\eta}$ | $\Gamma_{\eta\eta'}$ | $\Gamma_{4\pi}$ | $\Gamma_{\text{tot}}$ | $\Gamma_{\text{exp}}^\text{tot}$ |
|-----|--------------------|---------------|----------------------|----------------------|------------------|---------------------|------------------------|
| $f_0(400 − 1200)$ |        820       |      −        |      −                |      −                |      −            |         −           | 600–1200               |
| $f_0(980)$       |        28        |      −        |      −                |      −                |      −            |         −           | 40–100                 |
| $f_0(1500)$      |        14        |       28      |       25              |       13              |      −            |         −           | 112                    |
| $f_0(1710)$      |         8        |       60      |       43              |       31              |      −            |         −           | 130                    |