EXPLORING CLOUDY GAS ACCRETION AS A SOURCE OF INTERSTELLAR TURBULENCE IN THE OUTSKIRTS OF DISKS

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Received 2007 February 20; accepted 2007 April 27; published 2007 May 17

ABSTRACT

High-resolution two-dimensional MHD numerical simulations have been carried out to investigate the effects of the continuing infall of clumpy gas in extended H I galactic disks. Given a certain accretion rate, the response of the disk depends on its surface gas density and temperature. For Galactic conditions at a galactocentric distance of \( \sim 20 \) kpc, and for mass accretion rates consistent with current empirical and theoretical determinations in the Milky Way, the rain of compact high-velocity clouds onto the disk can maintain transonic turbulent motions in the warm phase (\( \sim 2500 \) K) of H I. Hence, the H I line width is expected to be \( \sim 6.5 \) km s\(^{-1}\) for a gas layer at 2500 K, if infall is the only mechanism driving the turbulence. Some statistical properties of the resulting force flow are shown in this Letter. The radial dependence of the gas velocity dispersion is also discussed.

Subject headings: galaxies: ISM — hydrodynamics — intergalactic medium — ISM: kinematics and dynamics — ISM: structure — turbulence

Online material: mpeg animation

1. INTRODUCTION

H I line widths, \( \sigma_w \), are observed to vary from \( \sim 12–15 \) km s\(^{-1}\) in the central parts to 6–8 km s\(^{-1}\) in the outer parts. Beyond the optical disk, the maps of \( \sigma_w \) display a patchy distribution with values populating the interval from 5 to 9 km s\(^{-1}\), independent of galactocentric distance. The median value is fairly constant along the extended outer parts, with about the same universal value of 7 ± 1 km s\(^{-1}\) for all the observed galaxies (Lewis 1984; Dib et al. 2006 and references therein). By fitting two-component Gaussians to the H I profiles in NGC 6822, the second moments for the cold and warm phases of the H I were found to be \( \sigma_w \approx 5 \) km s\(^{-1}\) and \( \sigma_w \approx 8 \) km s\(^{-1}\), respectively (de Blok & Walter 2006).

The observed \( \sigma_w \) are likely to represent turbulent small-scale motions; the existence of a cold phase in the outer disks, the observed level of star formation, and the double exponential radial profile of the star formation rate have also been interpreted as a consequence of the turbulent gas compressions in the outer disks (e.g., Ferguson et al. 1998; Elmegreen & Hunter 2006).

Dib et al. (2006) found that if turbulence is driven by supernovae, the velocity dispersion of the H I gas in the quiescent regime is \( \sim 3 \) km s\(^{-1}\), a factor of \( \sim 2 \) smaller than the observed values. Thus, either there are other physical processes driving the turbulence or the supernova feedback efficiency has been underestimated. There exist several physical mechanisms that could drive the turbulence even in the absence of star formation: hydrodynamic or magnetohydrodynamic instabilities, frequent minor mergers of small satellite clumps, ram pressure, or infalling gas clouds. Among the hydrodynamical instabilities, thermal instability by itself cannot sustain turbulence (Sánchez-Salcedo 2001; Gazol et al. 2001; Brandenburg et al. 2007). Magnetorotational instability, on the other hand, can only account for \( \sim 4.5 \) km s\(^{-1}\) of turbulent motion for the three components, if thermal broadening is not subtracted, and might be completely suppressed by stellar feedback (Dib et al. 2006 and references therein).

Our main goal is to assess how much of the velocity dispersion observed in the interstellar medium (ISM) is due to the impact of high-velocity clouds (HVCs) and intermediate-velocity clouds (IVCs). Ample evidence for the presence of a continuing gaseous infall toward the Galactic disk has been compiled in Beckman et al. (2003). All the studies firmly suggest that the inflow in the Galactic disk has been constant, with an accretion rate of \( \sim 1 \) \( M_\odot \) yr\(^{-1}\), or has even increased during its lifetime. H I observations have revealed the accretion of both diffuse and discrete structures from the extended environment in M31 and M33 (Thilker et al. 2004; Braun & Thilker 2004). Putman (2006) infers that the present total mass in condensed infalling clouds around the Milky Way is \( \sim 6 \times 10^6 \) \( M_\odot \) if they are all at distances <60 kpc. HVCs may be a repository for large amounts of gas if clouds fall into the disk rapidly after they are formed (Maloney & Putman 2003; Maller & Bullock 2004; Putman 2006). In the following sections, we consider the turbulent H I dynamics driven by the infall of clouds. Preliminary results were reported in Sánchez-Salcedo et al. (2007).

2. THE MODEL

The ideal MHD equations are integrated using the ZEUS code (Stone & Norman 1992a, 1992b). A local Cartesian frame of reference with \( x \) and \( z \) corresponding to the horizontal and vertical directions, respectively, was adopted to simulate a small patch of our Galaxy at a distance \( R_0 \) from the Galactic center. The galactic symmetry plane, \( z = 0 \), is placed in the middle of the computational domain. The simulation domain is a square of size \( L \times L \) with 1024\(^2\) zones. The ambient medium that is interacting with the HVCs is initially in plane-parallel magneto-hydrostatic equilibrium with scale height \( h_0 \) in an external gravitational potential. Here the scale height is defined as the half-width at half-maximum of the vertical volume density of the gas layer. The ambient gas is initially isothermal in space with a \textit{thermal} sound speed \( c_s \), and its evolution is nearly isothermal with a specific heat ratio \( \gamma = 1.01 \). Self-gravity of the gas was disregarded. The magnetic field has only one component along the \( y \)-axis and is initially stratified in the \( z \)-direction, \( B_z(z) \). Thus, there is no magnetic tension, only magnetic compression. Two magnetic configurations are explored. The first

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one is the high-latitude extension of the Galactic thick disk of Boulares & Cox (1990, hereafter the BC model) following Sancchez-Salcedo et al. (2007 for a discussion of this choice). The injection rate of clouds in the whole box domain is 300 Gyr\(^{-1}\), corresponding to an accretion rate defined as \(4\pi R^2 \Sigma \frac{\dot{m}}{H_{1000}}\). While our simulations are two-dimensional, this situation is energetically equivalent to an accretion rate of 750 clouds per gigayear and per square kiloparsec of the galactic disk. Since the kinetic energy of a cloud is \(\sim 10^{50}\) ergs, the energy input rate from impacting clouds is 7.5 \(\times 10^{12}\) ergs Gyr\(^{-1}\) kpc\(^{-2}\), which is equivalent to 75 \(\frac{e}{\text{year}}\) supernova explosions per gigayear and per square kiloparsec of the disk, each with mechanical energy \(e \times 10^{51}\) ergs, where \(e \sim 0.1\) is the efficiency factor. At the end of the simulation (2 Gyr), the total mass in our grid increased by a factor 1.45.

Clouds interact with the ambient medium and are diluted (see Fig. 1). At a height of 500 pc, they lose their identity and mix with the turbulent disk. Hence, the number of “active” clouds is 300 Gyr\(^{-1}\) (1.5 kpc/100 km s\(^{-1}\)) \sim 4–5. The infalling clumpy flow perturbs the disk through MHD waves that interact in a complicated fashion, producing a network of plumes and shells (see Fig. 1). As a consequence, the one-dimensional rms velocity of the gas, \(\sigma_v\), increases in time and saturates after 100 Myr, reaching values that oscillate between 6 and 7.5 km s\(^{-1}\) around the mean value \(\bar{\sigma}\) (see Fig. 2). The difference between \(\bar{\sigma}\) and \(\sigma_v\) is only \(\sim 0.5\) km s\(^{-1}\). The velocity dispersion in each direction was obtained by fitting the mass-weighted velocity profile for cells within \(|z| < h_p\), with \(h_p\) the asymptotic value of the scale height. Examples of mass-weighted velocity profiles are given in Figures 2b and 2c. Recall that \(\sigma_v = (\sigma^2 + c^2_v)^{1/2}\) for isothermal gas; thus, \(\sigma_v \sim 10\) km s\(^{-1}\). For \(v_{cl} = 50\) km s\(^{-1}\), \(\sigma_v\) and \(\sigma\) reach similar values as those for \(v_{cl} = 100\) km s\(^{-1}\). These results suggest that the continuous accretion of a clumpy gas may contribute significantly to the random motions observed.

The vertical density profile averaged on horizontal cuts is shown in Figure 3. Note that even though the injection of clouds occurs only through the upper cap \(z = z_{cl}\), the density profile is quite symmetric, and the net vertical displacement is relatively small. The scale height is \(\sim 640\) pc at \(\sim 2\) Gyr, similar to the observed value at \(R = 20\) kpc.

The injected energy is mostly dissipated in radiative shell compressions and behind shock fronts, with the dissipation timescale for turbulent kinetic energy being \(\sim 8\) Myr. The energy spectrum contains information about how the kinetic en-

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**Figure 1**—Density (color logarithmic scale) and velocity field (arrows) at \(t = 1.7\) Gyr for our run BCa. A general view of the disk in the range \(-2\) kpc \(< z < 2\) kpc (note that the z-axis has been cut off) is shown (top). A magnification zoom right into the marked region is displayed (bottom). Notice the presence of vortical motions and the dilution of the infalling clouds. This figure is also available as an mpeg animation in the electronic edition of the *Astrophysical Journal*.

\[
\lambda_{\text{acc}} = \left( \frac{h_p}{c_s} \right) \left( \frac{\dot{m}}{\Sigma} \right), \text{ with } \Sigma \text{ being the initial column density of the disk. Note that once } c_s \text{ is fixed, the scale height } h_p \text{ is determined by the depth of the external gravitational potential. The collisions of infalling clouds onto the gaseous disk will take the gas out of hydrostatic equilibrium. Consequently, the scale height } h \text{ of the H I layer may evolve in time.}
\]

**3. RESULTS**

The disk gas is subject to a stochastic driving force caused by the MHD pull of randomly injected clouds, plus the restoring gravitational force that tends to push back the gas to the midplane. The level of substructure in the resulting fluid depends on the structural parameters of the clouds and the injection velocity. We focus first on describing the evolution of our fiducial BC model, labeled as run BCa, that is intended to represent conditions at a galactocentric distance \(R_g = 20\) kpc. In this model, the parameters of the disk are \(c_s = 8\) km s\(^{-1}\), \(\Sigma = 1.2 \times 10^2\) pc\(^{-2}\), and \(h_p = 220\) pc. For the clouds, we adopted \(v_{cl} = 100\) km s\(^{-1}\), \(z_{cl} = 2\) kpc, \(n_{cl} = 0.1\) cm\(^{-3}\), and \(R_{cl} = 50\) pc (see Sánchez-Salcedo et al. 2007 for a discussion of this choice). The injection rate of clouds in the whole box domain is 300 Gyr\(^{-1}\), corresponding to an accretion rate defined as \(4\pi R^2 \Sigma \frac{\dot{m}}{H_{1000}}\). While our simulations are two-dimensional, this situation is energetically equivalent to an accretion rate of 750 clouds per gigayear and per square kiloparsec of the galactic disk. Since the kinetic energy of a cloud is \(\sim 10^{50}\) ergs, the energy input rate from impacting clouds is 7.5 \(\times 10^{12}\) ergs Gyr\(^{-1}\) kpc\(^{-2}\), which is equivalent to 75 \(\frac{e}{\text{year}}\) supernova explosions per gigayear and per square kiloparsec of the disk, each with mechanical energy \(e \times 10^{51}\) ergs, where \(e \sim 0.1\) is the efficiency factor. At the end of the simulation (2 Gyr), the total mass in our grid increased by a factor 1.45.
energy is distributed at different scales and about the nature of the driving mechanisms of the random motions in the compressible ISM. Due to the stratification and the anisotropy of the random motions, we have calculated the power spectrum along horizontal cuts for each component of the velocity separately. The power spectrum follows a power law with a different exponent for each component $E(k) \sim k^{-\alpha_i}$, with $\alpha_i = 3$ and $\alpha_j = 4$.

Vázquez-Semadeni (1994) and Padoan et al. (1997) reported lognormal density probability distribution functions (PDFs) for two- and three-dimensional isothermal turbulent flows. In fact, our PDF is approximately lognormal, with excesses at large and low densities, probably because, in our case, the turbulence is not isotropic; in addition, there is some probability of finding density substructure that does not have a turbulent origin, being the remnants of infalling clumps.

When the clouds are randomly injected along both caps, the mass enhancement in the central disk ($|z| < 600$ pc) is slightly larger (about 10%) than when the injection only proceeds along one side, because the symmetry in the injection leads to a more efficient balance of momentum. However, $\sigma(t)$ is quantitatively similar, and thus there is no need to show it again.

The elasticity of the disk, in relation to collisions, depends on the speed of sound and on the magnetic field strength. In order to see the sensitivity of $\alpha$ on the adopted magnetic configuration, we started with an $\alpha = 1$ Spitzer model and with the same $c_s$ and total pressure at the midplane as in run BCa; $\sigma_i$ and $\sigma_j$ were found to be 5 and 6.2 km s$^{-1}$, respectively.

It turns out to be very difficult to infer from basic principles how $\sigma$ scales with $c_s$, or with any other parameter (Sánchez-Salcedo et al. 2007). In order to gain some insight, Spitzer configurations with $\alpha = 1$ but different $c_s$, between 5 and 12 km s$^{-1}$ (spaced by 1 km s$^{-1}$), were considered. If the total pressure at the midplane $z = 0$ is identical in all the models and is equal to the value in the standard model,$^1$ we find that $\sigma$ scales linearly with $c_s$; a good fit to the data is $\sigma = 0.55c_s + 1.16$ (thermal broadening has been subtracted) when $v_c = 50$ km s$^{-1}$. If, instead of the total pressure, the surface gas density is taken to be the same as in case BCa in all the experiments, then $\sigma \propto \sqrt{c_s}$. For instance, by using $v_c = 50$ km s$^{-1}$ again, $\sigma$ varies from 7.4 to 5.0 km s$^{-1}$ when the sound speed drops by a factor of 2 (from 12 to 6 km s$^{-1}$).

In order to simulate various Galactic conditions at different galactocentric distances, runs have been carried out that vary $\lambda_{acc}$ and $R_{acc}/\rho_{crit}$. Assuming that $R_{acc}$ does not depend strongly on radius, $R_{acc}/\rho_{crit}$ should decrease with $R$ because the observed $h$ increases with $R$, but it is uncertain since $R_{acc}$ could also increase with $R$ as well. The radial variation of $\lambda_{acc}$ is also uncertain because the accreting mass flux $\Sigma(R)$ and the temperature structure $c_s(R)$ are still unknown. As a departure assumption, consider that the accretion of an intergalactic plane-parallel flow at infinity occurs ballistically, $\Sigma \propto R^{-1/2}$ (López-Corredoira et al. 2002). For our Galaxy, $\Sigma$ decreases by a factor of $\sim 10$ between 15 and 25 kpc, whereas $h$ increases by a factor of 4. Thus, $\lambda_{acc}$ varies by a factor of $\sim 30$ assuming that $c_s$ is constant with $R$. In order to bracket the conditions at these galactocentric distances, we have varied $\lambda_{acc}$ by a factor of 25 for run BCb as compared to run BCc, whereas we have varied $R_{acc}/\rho_{crit}$ by a factor of 4 (see Table 1). In runs BCb and Sp-b, $\sigma$ reaches a maximum and decreases afterward partly because the accreted mass in the disk is no longer negligible in the length of the run. Since, in these experiments, $v_c$ deviates significantly from a Gaussian distribution, whereas $v_x$ is still well described by a Gaussian, only the values of $\sigma/\sqrt{c_s}$ are reported in Table 1. According to run BCc, which represents Galactic conditions at $R \sim 15$ kpc, we find that for $c_s = 4.5$ km s$^{-1}$, $\sigma \approx 3.4$ km s$^{-1}$ and $\sigma_w = 5.6$ km s$^{-1}$ if thermal broadening is taken into ac-

$^1$ The fact that the total pressure at $z = 0$ is the same in these models implies that the surface gas density increases for low values of $c_s$. 

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**Fig. 3.**—The vertical density profile averaged on horizontal cuts is shown at three selected times, $t = 0$, 1.27, and 1.9 Gyr, for run BCa. The corresponding scale heights are 220, 660, and 640 pc, respectively.
count. At $c_\| = 8 \text{ km s}^{-1}$, $\bar{\sigma} \approx 4.5 \text{ km s}^{-1}$ (and thus $\sigma_v = 9 \text{ km s}^{-1}$). At the far outer disk ($\sim 25 \text{ kpc}$), $\bar{\sigma}$ is found to be 7.5 and 10 km s$^{-1}$ for $c_\|$ = 4.5 and 8 km s$^{-1}$, respectively ($\sigma_v \approx 9-13 \text{ km s}^{-1}$). To derive these values, we recaled the results of run BCb appropriately using our previous result that $\bar{\sigma} \propto \sqrt{c_\|}$. Slightly smaller $\sigma_\|$ are derived for Spitzer models (see Table 1). Line widths of $\sim 10 \text{ km s}^{-1}$ as predicted at the edge of the disk seem rather high, but note that empirical determinations are scarce and very uncertain at low surface gas densities ($\Sigma < 10^{-3} M_\odot \text{ pc}^{-2}$; e.g., Shostak & van der Kruit 1984).

If accretion occurs through rare mergers of very massive clouds, it would be hard to see how such a localized event could produce the observed uniform level of turbulence. Nevertheless, experiments with input rates of new clouds 25 times smaller than in our fiducial simulation (i.e., 12 Gyr$^{-1}$ in our box domain) but with cloud radii 5 times larger (250 pc in physical units), so that the mass accretion rate is unchanged, still maintain a rather uniform level of turbulence everywhere.

4. DISCUSSION AND CONCLUSIONS

We have shown that rms velocities of 3–8 km s$^{-1}$ in the warm phase of H I naturally arise if outer disks are continuously stirred by a clumpy accretion flow at a rate consistent with observations. The exact radial variation of the H I line width depends on the uncertain contribution of the thermal broadening as a function of radius and on the adopted accreting mass flux $\Sigma(R)$, which ultimately depends on the origin of the condensing clouds. An interesting possibility is that the outer disk is being rained on by Galactic fountain material driven by inner starbursts (Benjamin & Shapiro 1993) or is due to matter circulation in the halo. Under the rather extreme assumption that clouds fall ballistically from infinity, $\sigma_v$ is expected to vary only $\sim 60\%$ between 1 and 1.7 times the optical radius. The coupling between gas accretion and stellar feedback could account for the uniformity of the H I line widths.

So far, our quasi-isothermal simulations do not capture all the physics of the multiphase ISM to permit a detailed comparison between predictions and observations. Three-dimensional simulations including cooling and heating in a thermally bistable medium, plus a spectrum of the masses and the radii of the clouds, are being undertaken. These experiments will allow us to derive the velocity dispersion for each component (cool, warm, and hot phases) and compare it with high-resolution H I observations that have been able to identify cool and warm neutral components in the ISM of external galaxies (e.g., de Blok & Walter 2006). The accretion of clumpy gas can induce turbulence-compressed regions and can trigger star formation, which should also be included.

It is important to consider the other phenomena arising from the present accretion scenario. For instance, if turbulence is driven by oblique accretion flows, a large asymmetry in the scale height of the H I disk like that observed between the northern and southern halves of the Milky Way (Levine et al. 2006) is expected because oblique flows have an azimuthal dependence (see Sánchez-Salcedo 2006). In addition, infalling clouds moving supersonically at the upper warm galactic disk develop optically emitting shocked regions. At a density of $5 \times 10^{-1} \text{ cm}^{-3}$ for the preshocked gas, the face-on $H \alpha$ surface brightness ($I_\alpha$) of a certain shocked region may range between 10 and 32.5 mR (where mR is the unit millirayleigh) for shock velocities of 50 and 100 km s$^{-1}$, respectively (Raymond 1979). In an edge-on galaxy, the expected surface brightness, on scales of $\sim 1.5 \text{ kpc}$, is the result of the contribution of all the shocked regions through the sight line: $I_\alpha \approx (3 \times 2 \Sigma N_j R_j^2 R(I_\alpha))$, where $N_j$ is the number of clouds per unit volume, in our models $N_j \approx 2.5-5 \text{ kpc}^{-3}$. At $R = 20 \text{ kpc}$, $I_\alpha \approx 12-40 \text{ mR}$ depending on the shock velocity, implying emission measures $EM \approx \int n_e^2 dl = 0.03-0.1 \text{ pc cm}^{-6}$ and assuming gas at 8000 K. While the detection of such a $H \alpha$ brightness is challenging, extremely deep $H \alpha$ images of edge-on external galaxies might constrain the form in which galaxies accrete mass.

We thank J. Cantó, A. Hidalgo-Gámez, E. Levine, E. Vázquez-Semadeni, and the referee for very useful comments. The numerical calculations were performed in the Computer Center at UNAM. This work has been partially supported by DGAPA-UNAM grant IN104306.

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TABLE 1

| Run  | $v_l/c_\|$, km s$^{-1}$ | $n_i/n_0$, 10$^{12}$ cm$^{-3}$ | $R_j/h_0$, kpc | $\bar{\sigma}_i/c_\|$, km s$^{-1}$ |
|------|------------------------|-------------------------------|-----------------|----------------------------------|
| BCa  | 12.5                   | 1                             | 0.23            | 1.2                              |
| BCb  | 12.5                   | 1                             | 0.11            | 6.0                              |
| BCa  | 12.5                   | 1                             | 0.46            | 0.24                             |
| BCd  | 12.5                   | 0.1                           | 0.23            | 1.2                              |
| BCe  | 1.25                   | 1                             | 0.23            | 1.2                              |
| Sp-a | 8.33                   | 0.9                           | 0.17            | 2.2                              |
| Sp-b | 8.33                   | 0.9                           | 0.17            | 11.55                            |

a BC refers to the extension of the Boulares & Cox (1990) model. Sp indicates that a Spitzer model with $\alpha = 1$ was used.

b In physical units, an accretion rate of 0.6 $M_\odot$ yr$^{-1}$ corresponds to 1.2 in the BC model and to 2.2 in the Spitzer model, for the Galactic surface density at 20 kpc.