Holographic image reconstruction by a Josephson junction

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A general problem of magnetic sensors is a trade-off between spatial resolution and field sensitivity: with decreasing sensor size the resolution is improved but the sensitivity is deteriorated. Here we present a novel method of magnetic image reconstruction by a single Josephson junction, which can resolve the problem. The method resembles holography, with the diffraction-like $I_c(H)$ pattern serving as a hologram. It represents an example of quantum holography of electronic wave functions, corresponding in this case to the macroscopic superconducting condensate. We show theoretically and verify experimentally that the method allows super-resolution imaging with a nm-scale spatial resolution not limited by the junction size. We demonstrate that planar Josephson junctions facilitate both high field sensitivity and high spatial resolution, thus obviating the trade-off problem in magnetic scanning probe imaging.

Magnetic scanning probe microscopy (SPM) has been rapidly developing in recent decades. Magnetic force (MFM) [1–7], Superconducting Quantum Interference Device (SQUID) [8–10], Hall-probe [11–13] and NV-center [14–19] microscopies achieved remarkable advances. However, many magnetic sensors suffer from the trade-off problem between spatial resolution and magnetic field sensitivity. For example, SQUIDs detect a fraction of the flux quantum, $\Phi_0$ [20–22]. Therefore, their field sensitivity is inversely proportional to the pickup loop area, while spatial resolution is determined by the loop size. Thus, miniaturization leads to improvement of resolution at the expense of sensitivity.

Coherent nature of the superconductivity enables observation of quantum-mechanical phenomena in macroscopic objects [23–25]. Josephson effect appears as a result of electronic wave function interference between two superconducting electrodes [26]. It leads to diffraction-like Fraunhofer modulation of the critical current as a function of magnetic field, $I_c(H)$. In Ref. [27] it was proposed to use a single sandwich-type Josephson junction (JJ) as an SPM sensor. This enables ultimate miniaturization and improves spatial resolution [13], but the trade-off problem persists. In Ref. [28] it was argued that planar JJs [29–31] would allow at least partial obviating of the problem. Local magnetic field, $H^*(x)$, leads to distortion of $I_c(H)$ [32] and $H^*(x)$ is encoded in the shape of $I_c(H)$. Restoration of this information would allow super-resolution imaging not limited by the JJ size [33]. This requires solution of an inverse problem - reconstruction of unknown $H^*(x)$ from the known $I_c(H)$. This is imaged by a JJ. We assume that the JJ is “short” and does not screen the field. The direct problem, i.e., calculation of $I_c(H)$ for a given $H^*(x)$, was solved in Ref. [32]. The field induces a gradient of Josephson phase shift, $d\varphi/dx = \alpha H_y$. A small magnetic object creates a step-like phase shift,

$$\varphi^*(x) = \alpha \int_0^x H^*(\xi) d\xi. \quad (1)$$

$I_c$ is obtained by maximization of the Josephson current,

$$I_c = \frac{L/2}{-L/2} \int J_c(x) \sin[\alpha H x + \varphi^*(x) + \varphi_0] \, dx, \quad (2)$$

with respect to $\varphi_0$. Here $J_c(x)$ is the critical current density, which may vary along the JJ [33–35]. Our goal is to solve the inverse problem: reconstruct $H^*(x)$ from a given $I_c(H)$. Below we briefly describe our approach with more details available in the Supplementary [41].

Application of the inverse Fourier transform to Eq. (2) yields a system of two equations for $\varphi^*(x)$ [41]:

$$J_c(x) \sin[\varphi^*(x)] = \frac{\alpha}{2\pi} \int_{-\infty}^{\infty} \cos[\alpha x H + \varphi_0(H)] I_c(H) dH, \quad (3)$$

$$J_c(x) \cos[\varphi^*(x)] = \frac{\alpha}{2\pi} \int_{-\infty}^{\infty} \sin[\alpha x H + \varphi_0(H)] I_c(H) dH. \quad (4)$$
The unknown \( \varphi_0(H) \) should be obtained from the maximization criterion, \( \partial I_c/\partial \varphi_0 = 0 \), which gives:

\[
\varphi_0(H) = \frac{\pi}{2} - \arctan \left( \frac{A(H)}{B(H)} \right). \tag{5}
\]

Here \( A(H) = \int_{-L/2}^{L/2} J_c(x) \sin \left( \alpha H x + \varphi^*(x) \right) dx \) and \( B(H) = \int_{-L/2}^{L/2} J_c(x) \cos \left( \alpha H x + \varphi^*(x) \right) dx \). In the absence of the object, \( \varphi^* = 0 \), for a uniform JJ, \( J_c(x) = I_c/L \), the term \( A \) vanishes because the integrand is odd in \( x \). In this case, Eq. (5) yields \( \varphi_0 = \pi/2 \) and \( I_c(f) \) exhibits Fraunhofer modulation, \( I_{c0} \sin(\pi f)/\pi f \), where \( f = \Phi/\Phi_0 = H/H_0 \) is the normalized flux and \( H_0 \) is the flux quantization field. Substitution of \( \varphi_0 = \pi/2 \) and the Fraunhofer \( I_c(H) \) in Eqs. (3,4) leads to \( \sin(\varphi^*) = 0 \), \( \cos(\varphi^*) = 1 \), verifying reconstruction of the trivial case.

For \( H^* \neq 0 \), \( \varphi_0 \) may depend both on \( H \) and \( H^* \), preventing a straightforward solution. As usual, the inverse problem requires additional knowledge about the object. In SPM we are primarily interested in imaging of small magnetic objects, such as vortices or domain walls, with spatially symmetric \( H^*(x) \). When a symmetric object is placed in the middle of a JJ with a symmetric \( J_c(x) \), the term \( A(H) \) in Eq. (5) vanishes again, so that \( \varphi_0 = \pi/2 \) and the inverse solution, Eqs. (3,4), remains unambiguous. The most accurate reconstruction is achieved using \( \tan[\varphi^*] \) obtained by solving both Eqs. (3) and (4). Mutual division of Eqs. (3) and (4) eliminates the \( J_c(x) \) term. This is important for practical application when \( J_c(x) \) is not confidently known. All solutions presented below are obtained this way.

To verify the method, first we consider the well calibrated case of AV. Vortex stray fields induce the Josephson phase shift \( \varphi^* \),

\[
\varphi^*(x) = -V \arctan \left( \frac{x-x_v}{|z_v|} \right). \tag{6}
\]

Here \( V \) is the vorticity, and \( x_v, z_v \) are AV coordinates. Figure 2(a) shows the calculated total flux induced by AV at different locations \((x_v, z_v)\) \cite{29}. It corresponds to SPM scan with zero height, \( y_v \) = 0. When the JJ approaches the vortex along the middle line, \( x = 0 \), the growing induced flux and phase shift \( \varphi^* \), Eq. (6), lead to a progressive shift and distortion of \( I_c(H) \) patterns \cite{32, 37, 38, 42}. This is shown in Fig. 2(b) for three positions of an antivortex, indicated by arrows. Dashed black lines in Fig. 2(c) represent inverse problem solutions, \( H^*(x) \), reconstructed from these \( I_c(H) \) patterns. They coincide with the actual profiles, shown by blue, olive and red lines in Fig. 2(c), confirming the successful image reconstruction. Interestingly, unlike a conventional pixel-by-pixel SPM scanning, here the complete one-dimensional distribution \( H^*(x) \) within the JJ is obtained at once. Therefore, it would be sufficient to scan just in the \( z \)-direction to obtain the full two-dimensional \( H^*(x, z) \) map, speeding up the imaging process.

For experimental verification we use planar Nb-CuNi-Nb JJs, as shown in Figure 3(d). Several devices were studies, each containing one or two JJs with the lengths \( L \approx 5.4 \mu m \) and a vortex trap in the middle of the electrode, \( x_v = 0 \), at different distances, \( z_v \), from the JJs. Details about device fabrication, characterization and the experimental setup can be found in Refs. \cite{29, 31, 37, 42} and the Supplementary \cite{41}. Black symbols in Fig. 3(a-c) show measured \( I_c(H) \) patterns at \( T \approx 6.6 K \), (a) in the absence of a vortex, and with a trapped antivortex at (b) \( z_v = 0.94 \mu m \) and (c) \( z_v = 0.36 \mu m \). A progressive distortion, similar to that in Fig. 2(b) can be seen. Red lines represent fits using \( \varphi^* \) from Eq. (6) with the actual \( L, x_v \) and \( z_v \), and the prefactor \( V \) as the only fitting parameter \cite{37}. Black line in Fig. 3(e) represents the reconstructed AV field, obtained from the experimental \( I_c(H) \) from (c). The red line shows the expected \( H^*(x) \) obtained from the fit using Eq. (6) in Fig. 3(c). The quantitative agreement is apparent. The width at half-maximum of the reconstructed \( H^*(x) \) is \( \sigma \approx 400 \mu m \leq 1 L \), confirming the super-resolution ability of the method. However, the accuracy of reconstruction is much better than \( \sigma \). The inset in Fig. 3(e) shows a close-up on the half-maximum region. It is seen that the discrepancy of fitted and reconstructed profiles is \( \Delta \sigma \approx 20 \mu m \). It represents the actual spatial resolution of the method.

As shown in the Supplementary \cite{41}, the resolution is limited only by the maximum flux range \( \Phi/\Phi_0 \), i.e., the number of lobes in \( I_c(H) \). In Fig. 3(f) we show the relative accuracy of reconstruction of the width, \( \Delta \sigma/\sigma \), and the height, \( \Delta H^*/H^*(0) \), as a function of the inverse flux range. It can be seen that the accuracy of both quantities rapidly improves for \( \Phi/\Phi_0 > 5 \) and practically vanishes.

![FIG. 1. A sketch of the considered SPM experiment. A planar Josephson junction is employed as a sensor for imaging of a small magnetic objects with a local field \( H^* \).](image-url)
for $\Phi/\Phi_0 > 10$. The experimental reconstruction in Fig. 3 (e) is made for $\Phi/\Phi_0 = \pm 8$, see Fig. 3 (c), which is enough to achieve the remarkable $\sim 20$ nm accuracy.

Our method resembles the Fourier-transform holography [33–35], with diffraction-like $I_c(H)$ patterns serving as holograms. In holography the image quality increases with increasing the size of the hologram, i.e., with increasing the number of stored interference fringes. In our case the number of fringes corresponds to the number of lobes, i.e., to the flux range $\Phi/\Phi_0$. However, the specifics of our case is that the hologram is created by interference of the object with electronic wave functions of the condensate. In this respect it has a connection with electronic quantum holography [36], which, however occurs at a macroscopic scale in superconducts.

To demonstrate holographic imaging of an external object, we place the JJ in a low-temperature MFM and measure its response to the local field induced by the MFM tip (for details, see the Supplementary [41]). The tip creates a monopole-like field with a large $\sim 190$ Oe field at the tip [43]. To reduce its invasiveness, we placed the tip at a significant height $h \simeq 1.7 \mu m$ above the center of the JJ. The black line in Fig. 3 shows corresponding experimental $I_c(H)$ modulation. Even at this height the tip strongly shifts and distorts the $I_c(H)$ pattern. The black line in Fig. 3 (h) shows reconstructed tip-induced phase shift $\varphi^*(x)$. The blue line is a smooth fit, the derivative of which yields the tip-induced field, shown by the blue line in Fig. 3 (i). The red line in (i) represents the expected tip field at the tip-JJ height of $\simeq 1.7 \mu m$ [41]. The agreement is very good. To cross check the correctness, we also calculated expected $I_c(H)$ modulation for the reconstructed $\varphi^*$. It is shown by the red line in Fig. 3 (g). The agreement with experimental $I_c(H)$ is good, confirming the validity of reconstruction.

Finally, we discuss advantages of the planar geometry. Although the holographic method is applicable to any type of JJs, good resolution requires 5-10 lobes of $I_c(H)$ and the field range $\pm 5 - 10H_0$. This field should be small enough to be noninvasive for both object and sensor. Therefore, JJs with a high field sensitivity (small $H_0$) are preferred. In this respect, planar JJs with inherently small $H_0$ [29,31] have a major advantage compared to conventional overlap JJs. For our JJs $H = 6 - 8$ Oe, see Figs. 3 (a-c), is sufficient for achieving the spatial accuracy of $\sim 20$ nm. Furthermore, as demonstrated in Ref. [29], the planar geometry allows simple implementation of a control line for producing homogeneous magnetic field locally in the JJ. This facilitates acquisition of many $I_c(H)$ lobes without disturbance of the object.
FIG. 3. Experimental verification of holographic reconstruction of a field from Abrikosov vortex in the junction electrode (a-c,e) and external MFM tip (g-i). (a-c) Measured $I_c(H)$ (black symbols) (a) without a vortex and with a trapped antivortex at (b) $z_v = 0.94 \mu m$ and (c) $z_v = 0.36 \mu m$ from a junction. Red lines show fits, using Eq. (6). (d) SEM image of a planar Nb-CuNi-Nb junction with a vortex trap (false color). (e) Field profiles from AV. The back line represents holographic reconstruction of the experimental $I_c(H)$ pattern from (c). The red line shows the expected profile from the corresponding fit by Eq. (6). The inset represents a close-up on the half-maximum region and demonstrates the spatial accuracy of reconstruction of $\sim 20$ nm. (f) The relative accuracy of reconstruction versus the inverse flux range $\Phi_0/\Phi$ for the case in (c). Blue and olive symbols show the relative accuracy of the width at half-maximum, $\Delta \sigma/\sigma$, and height of the maximum $\Delta H^* / H^*(0)$, respectively. (g) Black symbols represent measured $I_c(H)$ pattern with MFM tip at $h \simeq 1.7 \mu m$ above the JJ, as sketched in the inset. Red line represents $I_c(H)$ calculated using the reconstructed $\varphi^*(x)$. (i) Reconstructed tip-induced phase shift in the JJ (black) and its spline (blue). (h) Field of the tip obtained from the reconstructed $\varphi^*(x)$ (blue) and the anticipated monopole-like tip field at the tip height of $\simeq 1.7 \mu m$ (red).

The ultimate field resolution of such sensor is determined by the flux noise. For our JJs it is $\sim 10^{-7} \Phi_0/\sqrt{Hz}$ at $T = 4.2$ K [29]. Taking into account the flux quantization field $H_0 \simeq 1$ Oe, it translates to the ultimate field sensitivity of $10^{-11}$ Oe/$\sqrt{Hz}$. It is remarkable that, contrary to conventional imaging techniques, which suffer from the trade-off problem between sensitivity and resolution, in the discussed holographic method the high field sensitivity is accompanied by the high spatial resolution.

To conclude we derived theoretically and verified experimentally a novel method of magnetic image reconstruction by a single Josephson junction. The method resembles the holography, with the diffraction-like $I_c(H)$ pattern serving as a hologram. It allows super-resolution
image reconstruction with nano-scale spatial resolution not limited by the junction size. The method allows ob-
viation of the trade-off problem between sensitivity and resolution, typical for conventional imaging techniques, which directly probe the total flux or field in a sensor. We demonstrated that application of a planar Joseph-
son junction for such holographic imaging facilitates both high field sensitivity and high spatial resolution, which is beneficial for scanning probe microscopy.

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