Location of Sinabung volcano magma chamber on 2013 using levenberg-marquardt inversion scheme

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Abstract. Sinabung Volcano has been monitoring using GPS after his eruption on August 2010. We Applied Levenberg-Marquardt Inversion Scheme to GPS data on 2013 because deformation of Sinabung Volcano in this year show an inflation and deflation, first we applied Levenberg-Marquardt to velocity data on 23 January 2013 then we applied Levenberg-Marquardt Inversion Scheme to data on 31 December 2013. From our analysis we got the depth of the pressure source modeling results that indicate some possibilities that Sinabung has a deep magma chamber about 15km and also shallow magma chamber about 1km from the surface.

1. Introduction
Before the eruption in 2010, the eruption of Sinabung volcano was never recorded in history. The first period of eruptive activity of Mount Sinabung began on August 27, 2010, which then turned into a classification Sinabung Sinabung volcanoes type A is a solitary volcanic which has a single peak, where the activity before the eruption of Sinabung 2010 just a solfatar gas emissions and fumarole [1]. Based on data and information from PVMBG, the period of the next eruption started on 15 September 2013, which is still a phreatic eruption, then turned into a magmatic eruption since 23 November 2013 and was followed by the emergence of a lava dome since December 16, 2013. From this case we want to know the deep and radius of Sinabung magma source.
2. Methods

2.1. Point Pressure source

Surface deformation of a volcano is controlled by the shape and size of the source, the increment of pressure, and the elastic properties of the medium. The deformation is proportional to the ratio of the cavity pressure change to the half-space elastic modulus \( \Delta P/G \), and Poisson’s ratio [2]. Mogi model assumes that the crust is half-elastic medium and surface deformation caused by pressure source in the form of a spherical of magma which is locate data certain depth. If changes in hydrostatic pressure take place on chamber, the deformation will occur symmetrically. The model input required in the model Mogi point pressure source are the depth \( d \) and changes in volume \( \Delta V \).

\[
\begin{pmatrix}
u \\ v \\ w \\
\end{pmatrix} = \alpha^3 \Delta P \frac{(1-v)}{G} \begin{pmatrix}
\frac{x}{R^2} \\ \\ \frac{y}{R} \\ \\ \frac{z}{d} \\
\end{pmatrix}
\]

(1)

Where \( \alpha \) = radius of magma chamber, \( v \) = poisons ratio, \( G \) = shear modulus, and \( \Delta P \) = Pressure change. \( u, v, w \) are the changes of positional the point \( x, y, 0 \) respectively and \( R = (x^2 + y^2 + d^2)^{1/2} \) is the radial distance from a point to the surface. It is illustrated graphically in Fig. 2. An example of surface deformation based on Mogi model is shown in Fig. 3 and Fig. 4 in term of horizontal and vertical displacements. The input are \( \alpha = 1000 \text{ m}, d = 1000 \text{ m}, x-min= -1000, x-max = -1000, \Delta P / G = 0.001, \) and \( v = 0.25 \) respectively. The displacement vector which has radial direction to the source has a magnitude

\[
U_r = (u^2 + v^2 + w^2)^{1/2} = \alpha^3 \Delta P \frac{(1-v)}{G} \frac{1}{R}
\]

(2)

Its magnitude varies inversely with squared distance from the center of cavity. The location of the source depth can be estimated by calculating \( U_r \) where \( U_r = (u^2 + v^2)^{1/2} \). Pressure changes
can be estimated using the relationship between volume of surface deformation and speed of lava discharge. Relationship between of surface deformation \( \Delta V_{\text{uplift}} \) with the change of volume on the center of pressure \( \Delta V_{\text{injection}} \) is expressed as the equation \( V_{\text{uplift}} = 2(1 - \nu)\Delta V_{\text{injection}} \) with \( \nu \) is poisons’ ratio. Mogi model has been used for examples by Murray et al. [3], Fialko [4], Larson [5], Materlark [6], and Srigutomo [7].

![Figure 2. Geometry of source in elastic half space (redrawn based on Lisowski[2])](image)

**Figure 3.** (a) Horizontal and vertical displacements using Mogi model, (b) 3D surface displacement
2.2. Levenberg Marquardt Inversion Scheme

Menke (1984) defines the inverse theory as a whole technical of mathematical and statistical methods to obtain useful information regarding a physical system based on the observation of the systems [8]. In the non-linear inversion scheme, we can use relation between the forward operator $G$, the model parameters $m$, and the data $d$ as this equation

$$ G(m) = d $$

On Mogi equation the models parameter is $(d$ and $\alpha$) that best fit with the data. By assuming the measurement errors are normally distributed, then we minimize the sum squared error that normalized by the maximum likelihood principle, respective to their standard deviations $\sigma$,

$$ F(m) = \sum_{i=1}^{m} \left( \frac{G(m)_i - d_i}{\sigma_i} \right)^2 $$

If we let

$$ F(m) = \begin{bmatrix} f_1(m) \\ \vdots \\ f_m(m) \end{bmatrix} $$

and $J(m)$ is the Jacobian matrix,

$$ j(m) = \begin{bmatrix} \frac{\partial f_1(m)}{\partial m_i} & \ldots & \frac{\partial f_1(m)}{\partial m_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m(m)}{\partial m_1} & \ldots & \frac{\partial f_m(m)}{\partial m_n} \end{bmatrix} $$

Figure 4. (a) Surface deformation, (b) Displacement vectors using Mogi model.
In the Levenberg-Marquardt (LM) scheme, the update of model parameters at $n$ iteration is found by solving the equation

$$
\left( J(m^k)^T J(m^k) + \lambda I \right) \Delta m = - J(m^k)^T F(m^k)
$$

where $\lambda$ is a dumping parameter which should be adjusted during the inversion until convergence.

3. Results and Discussion
PVMBG has placed several GPS stations at Sinabung volcano since 2010. The locations of these stations are shown in Fig below. We use data of deformation recorded at these stations in 2013 published by PVMBG because in this period we have condition that sinabung being inflation and deflation and we can observed shallow magma chamber and deep magma chamber on this period by appplying Levenberg Marquard (LM) inversion scheme.

![Coordinate of GPS stations placed at Sinabung volcano in 2013](image_url)

Figure 5. Coordinate of GPS stations placed at Sinabung volcano in 2013
Figure 6. Source inversion on phase 1 (23 Januari 2013), and we get deep magma chamber about 12 km.

Figure 7. On (31 December 2013) we get shallow magma chamber about 1 km from the surface.

The Levenberg Marquard (LM) Inversion Scheme was applied to the 2013 data of deformations recorded at 7 GPS stations, first we tried to apply Levenberg Marquard on gps velocity data on 23 Januari 2013, we choose that time because at 23 januari 2013 kondition of sinabung is tend to be stable, from our inversion we get deep magma chamber about 1km with volume of magma chamber about $3 \times 10^9$ m$^3$. Then we applied Levenberg Marquard inversion on velocity data on 31 Desember 2013, on this period we get shallow magma chamber about 1 km and volume of magma chamber about $0.8 \times 10^9$ m$^3$. From tha calculation we predict that the magma sources in 2013 migrated from deeper part to shallower beneath summit of Sinabung before the lava dome appearance in December 2013.
4. Conclusion
From our analysis using Levenberg Marquard (LM) Inversion Scheme, we get prediction that the magma sources in 2013 migrated from deeper part to shallower beneath summit of Sinabung before the lava dome appearance in December 2013, from our inversion we got deep magma chamber about 14 km and also shallow magma chamber about 1 km under the surface.

5. References
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