Squarks Below the Z

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ABSTRACT

We investigate the possibility that the difference between the measurements of $\alpha_3(M_Z)$ from the hadronic branching ratio of the $Z^0$ and the world average of other measurements is due to the decay of the $Z^0$ into quark, anti-squark, and gluino. Consequences for supersymmetry breaking models are discussed.
Recently, some of us have pointed out several features of quarkonium decay suggesting the existence of gluinos in the sub-GeV mass region. Gluinos below 1 GeV are natural in the $m_{1/2} = 0$ model of soft SUSY breaking, which corresponds to the low-energy manifestation of spontaneously broken $N = 1$ supergravity with minimal gauge kinetic term. In this model, the superpartners of the massless gauge bosons are themselves massless at tree level receiving small calculable masses in perturbation theory, and SUSY breaking is dominantly driven by a universal scalar squared mass $m_0^2$ representing the difference between the average squared masses of the fermions and their superpartners. Assuming degenerate super-heavy particles at the GUT scale, minimal SUSY unification with light gluinos and the world average values of the standard model parameters require at the one standard deviation level that

$$75 \text{ GeV} < m_0 < 270 \text{ GeV}. \quad (1)$$

Non-degenerate super-heavies could loosen this prediction. Assuming negligible left-right mixing for the light quark partners, the lower limit in Eq. (1) insures that none of the sfermions except perhaps the lightest stop quark is below half the $Z^0$ mass. The $M_{Z}/2$ experimental lower limit on the stop quark constrains the other parameters of the $m_{1/2} = 0$ model. This entire range of $m_0$ leads naturally in the $m_{1/2} = 0$ model to gluino masses in the sub-GeV region. Values of $m_0$ near the lower end of this allowed range are characterized by squark masses between $M_{Z}/2$ and $M_{Z}$ and it is the purpose of this paper to explore the consequences of this possibility. Contrary results from hadron colliders suggesting 100 GeV level lower bounds for the masses of squarks and gluinos are dependent on the assumption that these particles, once produced, will lead to significant amounts of missing transverse energy due to their ultimate decay into photinos or other weakly interacting stable neutralinos. The hadronization Monte Carlos leading to the quoted bounds are reliable at some level for gluino masses in the multi-GeV or tens of GeV range but are presumably not intended to be used for light gluinos. The following plausible scenario assuming squark masses above half the $Z^0$ mass and a gluino mass below one GeV would invalidate the hadron collider limits. A squark produced in hadronic collisions will immediately decay into the corresponding quark plus a gluino. The decay gluino or a gluino independently produced in the hadronic collision would hadronize into a heavy jet of hadrons one of which would ultimately be the lightest hadron with a gluino constituent. Such a hadron would presumably be in the 1.5 to 2.5 GeV mass region as expected for gluon
bound states. The gluino-containing hadron would decay predominantly into a multi-pion final state plus a (predominantly soft) photino. Such a photino would very rarely carry off a significant amount of transverse energy. Thus if the gluinos are in the sub-GeV region the major effect in hadronic collisions will be only a somewhat higher apparent value of the strong coupling constant with little difference in the jet topology. In this scenario the tightest limits on SUSY particles will come from $e^+e^-$ colliders.

It is well known that the non-observation of the decay of the $Z^0$ into squark-antisquark pairs sets a lower limit of approximately 42 GeV for the lightest squark mass [5]. However, if the gluino is much lighter than $Z^0$, a stronger bound can be put on the squark mass due to the effect on the $Z^0$ hadronic branching ratio of the decay

$$Z^0 \rightarrow \tilde{q}\tilde{q} \tilde{g} + \text{Charge Conjugate}$$

where the tildes indicate supersymmetric partners of the quark $q$ and gluon $g$.

The effect of squarks and gluinos on $e^+e^-$ annihilation into hadrons through a virtual photon was treated a decade ago [6] and these results can be carried over simply to $Z^0$ decay. The hadronic to electronic branching ratio of the $Z^0$ is of the form [7]

$$\frac{\Gamma(Z^0 \rightarrow \text{hadrons})}{\Gamma(Z^0 \rightarrow e^+e^-)} = 19.97 \left[ 1 + 1.05 \frac{\alpha_3(M_Z)}{\pi} + 0.9 \left( \frac{\alpha_3(M_Z)}{\pi} \right)^2 - 13 \left( \frac{\alpha_3(M_Z)}{\pi} \right)^3 + \cdots \right].$$

Including the effect of virtual squark-antisquark pairs and quark-squark-gluino decay channels, the lowest order effect of the SUSY particles is to replace $\alpha_3$ by $\alpha_{\text{eff}}$ in the linear term of Eq. (3).

$$\alpha_3(M_Z) \rightarrow \alpha_{\text{eff}}(M_Z) = \alpha_3(M_Z)(1 + \delta)$$

The $\delta$ parameter, which is a function of the squark and gluino masses, gives the lowest order effect of the SUSY particles assuming the squark mass is above half the $Z^0$ mass. Since the effects of SUSY particle on the other terms are not readily available we make the substitution of Eq. (4) in the quadratic and cubic terms of Eq. (3) also, accepting a theoretical error of order $(\alpha_3(M_Z)/\pi)^2\delta$. $\delta$ can be calculated from Eqs. (3) and (8) of the first paper of Ref. [6] as a function of the squark mass and the gluino mass. The typographical error in the last term of Eq. (8) has been corrected in our calculations. In these equations the quark mass is neglected by comparison to $M_Z$ and $m_{\tilde{q}}$. The results for $\delta$ are presented in Fig. 1 including all quarks except the top and setting all squark masses equal as in Ref. [6]. Results for more realistic calculations in which the squark
masses differ are discussed below. The effective strong coupling constant as deduced by
the experimental value of the left hand side of Eq. (3) is

\[ \alpha_{\text{eff}}(M_Z) = 0.141 \pm 0.017 \]  

which can be compared with the world average value

\[ \alpha_3(M_Z) = 0.113 \pm 0.003 \]  

At the one standard deviation level, Eq. (4) would imply that

\[ \delta = 0.25 \pm 0.15 \]  

One could, conservatively, take Eq. (7) as putting an upper limit of 0.4 to the contribution
of squarks and gluinos to \( \delta \). We would not expect an appreciable fraction of this to be
attributable to the higher order process \( Z^0 \rightarrow q\bar{q}g\bar{g} \). From Fig. 1, assuming negligible
gluino mass, this translates into an experimental bound

\[ m_{\tilde{q}} > 50 \text{ GeV} \]  

Since, however, in the \( m_{1/2} = 0 \) scenario for soft SUSY breaking the gluinos are
expected to be below 1 GeV and the squarks could be between \( M_Z/2 \) and \( M_Z \), it is
interesting to consider the consequences of the assumption that squarks and gluinos are
in fact causing the deviation of \( \delta \) from zero. Arguing against this interpretation is the
fact that the \( R \) parameter of \( e^+e^- \) measured at 35 GeV where there is certainly no squark
contribution also yields an anomalously large value of \( \alpha_3 \) (with large errors) [4]. We might
suggest, however, that at 35 GeV there is room for higher twist, power law, contributions
to the \( R \) parameter at the level of a few percent. This would be sufficient to bring the value
at 35 GeV into line with other measurements. At the \( Z^0 \) mass these should be negligible
and therefore one might expect the hadronic decay rate of the \( Z^0 \) to accurately reflect the
perturbative QCD final states. Eq. (9), at the one standard deviation level, together with
Fig. 1 would then imply

\[ 50 \text{ GeV} < m_{\tilde{q}} < 82 \text{ GeV} \]  

As discussed above, the uncertainties in the hadronization models built into the jet Monte
Carlos are in the case of light gluinos such that Eq. (9) can probably not be reliably ruled
out by the present hadronic collider data. The sfermions, except for the stop quark, are
expected in a good approximation to be unmixed partners of the left and right handed fermions with masses

\[ m_{f,LL}^2 = m_0^2 + m_f^2 + M_Z^2 \cos 2\beta \left[ T_{3,f} - e_f \sin^2 \theta_W \right], \quad (10a) \]

\[ m_{f,RR}^2 = m_0^2 + m_f^2 + M_Z^2 \cos 2\beta e_f \sin^2 \theta_W. \quad (10b) \]

The average squark squared mass as well as the average slepton squared mass therefore satisfies

\[ \langle m_f^2 \rangle = m_0^2 + \langle m_f^2 \rangle. \quad (11) \]

In the \( m_{1/2} = 0 \) scenario the angle \( \beta \), the arc-tangent of the ratio of Higgs vevs is constrained by current data to lie in one of two ranges \([1]\)

\[-0.674 < \cos^2 \beta < -0.385 \quad \text{or} \quad 0.406 < \cos^2 \beta < 0.536 \quad (12)\]

Some squark masses in the range of Eq. \( (11) \) would follow in this scenario if \( m_0 \) were near the lower end of the range in Eq. \( (1) \). Because of the non-linearity of \( \delta \) as a function of the squark masses, the effective mass in Eq. \( (1) \) for determining \( \delta \) lies between the lightest and the average squark mass.

We ask the question: If a \( \delta \) in the range of Eq. \( (11) \) is to be produced by squarks and gluinos what must be the values of \( m_0 \) and \( \beta \) and what would be the consequences for slepton and top quark masses? If these consequences can be ruled out experimentally alternate interpretations must be sought for the \( \delta \) anomaly. Of course the anomaly is only a two standard deviation effect which could easily disappear with better statistics. In either case the parameters of the \( m_{1/2} = 0 \) model would be further constrained with important implications for the gluino and squark masses.

From Eq. \( (11) \) we can see that, if \( \cos^2 \beta \) is positive, the “down-type” squarks \( (T_3 = -1/2) \) would be below \( m_0 \) and the “up-type” squarks would be above \( m_0 \), with the reverse situation holding if \( \cos^2 \beta \) were negative. The squared coupling to the \( Z^0 \) is proportional to \( T_{3,f}^2 + (T_{3,f} - 2 e_f \sin^2 \theta_W)^2 \) which is greater for “down-type” squarks than for “up-type”. Thus, for fixed \( m_0 \), a greater contribution to \( \delta \) would result if \( \cos^2 \beta \) were positive. However, current experimental constraints are such that lower \( m_0 \) is allowed if \( \cos^2 \beta \) is negative so that the largest effect consistent with current data is obtained if \( \cos^2 \beta \) is negative and is dominated by the effect of “up-type” squarks. In Figs. 2 and 3 we show the \( \delta \) contours as functions of \( m_0 \) and the gluino mass in the cases of positive and negative \( \cos^2 \beta \) respectively. However, apart from the window at very low gluino mass, most of the parameter space of
Figs. 2 and 3 is ruled out by collider and beam dump searches for SUSY particles. In Fig. 4 we show the $\delta$ contours as a function of $m_0$ and $\cos2\beta$ assuming negligible gluino mass as suggested in the second article of Ref. [1]. The currently allowed regions of parameter space in the $m_{1/2} = 0$ model with $m_0 < 95$ GeV lie in the trapezoid in the upper right of Fig. 4 and in the pentagonal region in the lower right. The region to the left of the slanted lines of postive slope is ruled out by the current lower limit of 65 GeV on the mass of the selectron [9,5] while the region to the left of the slanted line of negative slope is ruled out by the current limit on sneutrino masses. As can be seen from Fig. 4, squarks below the $Z^0$ can account for the anomalously large apparent value of $\alpha_3(M_Z)$ seen in $Z^0$ decay if $m_0$ is in the range

$$60 \text{ GeV} < m_0 < 85 \text{ GeV}$$

(13)

with values above $\delta = 0.16$ allowed only for negative values of $\cos2\beta$.

Such a low value of $m_0$ would also have strong consequences for the top quark mass since minimal SUSY unification with gluinos below the $Z^0$ leads to the effective SUSY mass

$$M_S = 150 \text{ GeV} \times e^{-518.5(\sin^2\theta_W - 0.2333)} e^{1.85(\alpha_3^{-1}(M_Z) - 0.113^{-1})}. \tag{14}$$

This equation with the world average values of $\alpha_3$ and $\sin^2\theta_W$ leads to Eq. (1). However, since we are exploring the consequences of light SUSY particles, for $\alpha_3(M_Z)$ we use the tightly constrained value coming from the best fit to the quarkonia data allowing for light gluinos [1], $\alpha_3(M_Z) = 0.113 \pm 0.001$. Then requiring an $M_S$ consistent with Eq. 13 leads to a tight prediction for $\sin^2\theta_W$. The experimental result for $\sin^2\theta_W$ is strongly dependent on the top quark mass [10]

$$\sin^2\theta_W(M_Z) = 0.2327 \pm 0.0004 + 0.0043 \left[ 1 - \frac{m_t}{125 \text{ GeV}} \right]. \tag{15}$$

The quoted error here comes in equal parts from the experimental error and the uncertainty due to the Higgs mass which is assumed to lie between 50 GeV and 1 TeV. It is clear from Eqs. (14) and (15) that lower values of $m_t$ are correlated with lower values of $M_S$.

$$m_t = \left[ 125 + 56.1 \ln \left( \frac{M_S}{205} \right) \right] \pm 14 \text{ GeV}. \tag{16}$$

The observation of Ref. [3] would increase the error here unless the GUT-scale particles are very nearly degenerate. Equating $M_S$ with the average squark mass squared leads via Eq. (11) to

$$M_S^2 = m_0^2 + 1608 \text{ GeV}^2 \tag{17}$$
where, in calculating the average quark mass, we have used for \( m_t \) a value near the current experimental lower limit since in order to bring \( m_0 \) into the range of Eq. (13), one needs small values of the top quark mass. As \( m_0 \) increases through the range of Eq. (13), the top quark mass runs from \( 67 \pm 14 \text{ GeV} \) to \( 81 \pm 14 \text{ GeV} \). These values are very close to the best fit values of \( m_t \) arising from the total unconstrained LEP data [8]. However only the highest values here are consistent with the recent CDF limit \([11] m_t > 91 \text{ GeV} \). Although the possibility of SUSY decays might loosen the CDF limit, their result taken at face value would disfavor a value of \( \delta \) greater than 0.1 coming from the mechanism discussed here which however is sufficient to bring \( \alpha_{\text{eff}}(M_Z) \) into agreement with the world average value \( \alpha_3(M_Z) \) at the one standard deviation level. This would require a value of \( m_0 \) between 75 and 85 GeV in agreement with the lower limit of Eq. (1).

The following conclusions can be drawn from our analysis. The current experimental constraints on the \( m_{1/2} = 0 \) model of SUSY breaking are such that the discrepancy between the \( \alpha_3(M_Z) \) value from the \( Z^0 \) width and the world average is consistent (at the one standard deviation level) with an explanation in terms of \( Z^0 \) decay into quark-antisquark-gluino. This explanation is however not a prediction of the \( m_{1/2} = 0 \) model since it requires that \( m_0 \) be near the bottom of the currently allowed range. If this explanation is in fact chosen by nature, many of the SUSY particles are below the \( Z^0 \) and will be clearly seen at LEP II. The non-collinear lepton pairs \( Z^0 \to \mu^+ \mu^- \tilde{\gamma} \tilde{\gamma} \) coming from the decay \( Z^0 \to \mu^+ \mu^- \tilde{\gamma} \tilde{\gamma} + \text{C.C.} \) should be seen even at LEP I when increased statistics are available. In addition the top quark should then be at the very low end of the currently allowed range and should be seen in future Fermilab experiments.

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Figure Captions

Fig. 1. Values of $\delta$ as a function of squark and gluino mass. Contours of constant $\delta$ are plotted with the value of $\delta$ shown to the right of the contour. It is assumed that all squark masses are equal.

Fig. 2. Values of $\delta$ as a function of $m_0$ and the gluino mass for $\cos 2\beta = 0.5$. Contours of constant $\delta$ are plotted with the value of $\delta$ shown to the right of the contour.

Fig. 3. Values of $\delta$ as a function of $m_0$ and the gluino mass for $\cos 2\beta = -0.5$. Contours of constant $\delta$ are plotted with the value of $\delta$ shown to the right of the contour.

Fig. 4. Values of $\delta$ as a function of $m_0$ and $\cos 2\beta$ for zero gluino mass. That part of the graph to the right of the dashed lines is the region allowed by the restriction that the squark masses are above $M_Z/2$. The two enclosed areas are the regions allowed by Eq. (12) and the restrictions that the slepton mass must be above 65 GeV and the sneutrino mass must be greater than 41 GeV.