NON-PERTURBATIVE EFFECTS IN $e^+e^-$ EVENT-SHAPE VARIABLES

A. Banfi$^1$ and G. Zanderighi$^2$

$^1$Dipartimento di Fisica, Università di Milano-Bicocca and INFN, Sezione di Milano, Italy;
$^2$Dipartimento di Fisica Nucleare e Teorica, Università di Pavia and INFN, Sezione di Pavia, Italy.

Abstract

We review theoretical methods employed to study non-perturbative contributions to $e^+e^-$ event-shapes and discuss their phenomenological relevance.

1 Introduction

Event-shape variables in $e^+e^-$ annihilation, such as Thrust $T$, Heavy-jet Mass $M_H$, $C$-parameter, Broadening $B$, have provided various tests of QCD and a way to measure $\alpha_s$. Besides the perturbative (PT) results, agreement with data is achieved only by taking into account additional corrections of non-perturbative (NP) origin recently addressed through analytic approaches [1–3].

As explained in Section 2, the net effect of these corrections is to raise the mean value of an event-shape by an amount proportional to $1/Q$, being $Q$ the center of mass energy. Similar features occur for distributions. In Section 3 universality of $1/Q$ corrections is discussed and it is mentioned how hadron mass effects partially spoil the 'simple' universality picture. We conclude in Section 4 giving some outlooks.

2 Power correction to event shapes

To see how NP corrections to event-shapes emerge, we consider, for instance, the mean value of $\tau \equiv 1 - T$. As shown in fig. 1, a next-to-leading order (NLO) calculation alone does not fully describe data and one has to add a power suppressed contribution of the form $C_\tau/Q$, with $C_\tau \simeq 1\text{GeV}$. 
Figure 1: Mean value of $1 - T$ as a function of $Q$ [4]. The dotted line represents the NLO prediction, the solid line the improved prediction including a $1/Q$ correction.

One cannot get rid of this mismatch by simply taking into account higher orders in the PT expansion, since the PT series itself is divergent (for a recent review see [5]). Any attempt to give a meaning to the series leads to an ambiguity (infra-red renormalon) which, for event shapes, is of the order $\Lambda_{QCD}/Q$. This ambiguity must be canceled by a NP contribution with the same power behavior.

For the distribution the situation is more involved. Actually, the full $\tau$ distribution can be expressed as a convolution of a PT distribution with a NP shape function $f_{NP}$. In the region $\tau \gg \Lambda_{QCD}/Q$ one can make the approximation

$$\frac{d\sigma}{d\tau} = \int d\sigma_{PT}(\tau - \frac{\langle \epsilon \rangle}{Q}) f_{NP}(\epsilon),$$

so that hadronization corrections result in a $1/Q$ power-suppressed shift of the PT distribution. In the region $\tau \sim \Lambda_{QCD}/Q$ higher powers become equally important, so that the full shape function should be kept. This is the basis of the Korchemsky-Sterman approach [3], where a parameterization of the shape function is given and the parameters are fitted to the data.

3 Universality of $1/Q$ power corrections

Although $1/Q$ power corrections are intrinsically NP quantities, one is able to predict their relative size from one observable to the next. First one observes that hadron multiplicity $n_h$ is uniform in rapidity ($\eta$):

$$\frac{dn_h}{d\ln k_t d\eta} = \phi_h(k_t),$$

with $k_t$ the particle transverse momentum with respect to the thrust axis. This is the basis of the local parton hadron duality (LPHD) approach [6], where hadron momentum flow is supposed to follow parton flow. Furthermore, a soft particle contribution to an event shape may be written as the product of $k_t/Q$ times an observable-dependent function of rapidity $f_V(\eta)$ (for the Thrust $f_\tau(\eta) = e^{-|\eta|}$).
As a consequence, the NP correction to the mean value of $V$ can be expressed as (see for example [7], and references therein)

$$\langle V \rangle_{NP} = \frac{\langle k_t \rangle_{NP}}{Q} c_V , \quad \text{with} \quad \frac{\langle k_t \rangle_{NP}}{Q} = \int \frac{dk_t}{k_t} \phi_h(k_t), \quad c_V = \int d\eta f_V(\eta) , \quad (3)$$

so that all the observable dependence is contained in the calculable coefficient $c_V$ and one is left with only one unknown NP parameter $\langle k_t \rangle_{NP}$. This property of $1/Q$ power corrections is commonly referred to as \textit{universality}.

3.1 Dispersive method

A useful parameterization of $\langle k_t \rangle_{NP}$ is provided by the dispersive approach [2], in which the running coupling is defined at any scale through a dispersion relation. The NP parameter $\langle k_t \rangle_{NP}$ is related to $\alpha_0$, the average of the dispersive coupling below a certain low scale $\mu_I$ (conventionally chosen to be $\mu_I = 2$ GeV):

$$\frac{\langle k_t \rangle_{NP}}{Q} = \frac{4C_F}{\pi^2} M^{\mu_I} \alpha_0(\mu_I) + O \left( \alpha_s(Q) \frac{\mu_I}{Q} \right) , \quad \alpha_0(\mu_I) = \frac{1}{\mu_I} \int_0^{\mu_I} dk \alpha_s(k) . \quad (4)$$

Here the Milan factor $M$ accounts for the non-inclusiveness of shape variables [7]. The value of $\alpha_0$ has been measured by performing a simultaneous fit of $\alpha_s$ and $\alpha_0$ to both mean values and distributions, and is found to be consistent with the universality hypothesis [8] as shown in fig. 2.

![Figure 2: 2-σ contours for fits to various two-jet observables [8]. Solid curves indicate fits to distributions, while dashed lines indicate fits to mean values.](image)

These fits have been performed only with two-jet event shapes. Only very recently these studies have been extended to three-jet events, and the power corrections to the thrust-minor and $D$-parameter have been addressed [9].

3.2 Mass effects and universality

Until now hadron masses have been neglected in the definition of the event-shapes. As shown in [10], hadron masses give rise to additional power corrections

\footnote{Actually, the $1/Q$ corrections to the wide-jet broadening $B_W$ distribution are not yet believed to be fully understood. This seems to be a common problem of less inclusive quantities, such as $B_W$ and $M_H$, where one particular hemisphere is chosen.}
\(\delta V_m\) which, in general, are not proportional to \(c_V\), thus spoiling the universality picture. However, with a suitable redefinition of the observables (E-scheme) one is able to eliminate the non-universal contributions leaving just universal mass corrections of the form

\[
\delta V_m = c_V \frac{\mu_\ell}{Q} \ln^A \frac{\Lambda_{QCD}}{Q}, \quad A = 4C_A/\beta_0 \simeq 1.6, \tag{5}
\]

with \(\mu_\ell\) a new unknown parameter which depends on the hadron level considered. Unfortunately, currently available data are not precise enough to extract \(\alpha_0\) and \(\mu_\ell\) simultaneously. However, changing the definition scheme or the hadron level results in systematic uncertainties in \(\alpha_s\) and \(\alpha_0\) fits, thus revealing the presence of mass effects of the form predicted by eq. [5].

4 Conclusions

During this decade much theoretical and experimental effort has been devoted to the study of hadronization effects in \(e^+e^-\) event shapes. Experiments have confirmed the universality of \(1/Q\) power corrections, thus supporting the validity of the LPHD approach. However, more refined analyses have revealed the need to include higher power corrections through a NP shape function and the existence of mass effects. These and related topics need further experimental investigation.

Acknowledgements We are thankful to Matteo Cacciari, Pino Marchesini, Gavin Salam and Graham Smye for helpful comments and suggestions, and to the organizers of LEPTRE conference for the pleasant days we spent in Rome.

References

[1] A.V. Manohar and M.B. Wise, Phys. Lett. B344 (1995) 407 [hep-ph/9406392]; R. Akhoury and V.I. Zakharov, Phys. Lett. B357 (1995) 646 [hep-ph/9504248].

[2] Yu.L. Dokshitzer, G. Marchesini and B.R. Webber, Nucl. Phys. B469 (1996) 93 [hep-ph/9512336].

[3] G.P. Korchemsky and G. Sterman, Nucl. Phys. B555 (1999) 335 [hep-ph/9902341].

[4] P.A. Movilla Fernández, O. Biebel and S. Bethke, hep-ex/9807007.

[5] M. Beneke, Phys. Rep. 317 (1999) 1 [hep-ph/9807443].

[6] Ya.I. Azimov, Yu.L. Dokshitzer, V.A. Khoze and S.I. Troyan, Phys. Lett. B165 (1985) 147; Z. Phys. C27 (1985) 65.

[7] Yu.L. Dokshitzer, A. Lucenti, G. Marchesini and G.P. Salam, JHEP 05 (1998) 003 [hep-ph/9802381].

[8] G.P. Salam, G. Zanderighi, Nucl. Phys. Proc. Suppl. 86 (2000) 430 [hep-ph/9909324].

[9] A. Banfi, Yu.L. Dokshitzer, G. Marchesini and G. Zanderighi, hep-ph/0104162; JHEP 03 (2001) 007 [hep-ph/0101203]; hep-ph/0010267.

[10] G.P. Salam and D. Wicke, hep-ph/0102343.