Strategic Interactions among different Entities in Internet of Things

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Abstract—We investigate the economics of internet of things (IoT). An economic model of IoT consists of end users, advertisers and three different kinds of providers. We model different kinds of interaction among the providers as a combination of sequential and parallel non-cooperative games. We characterize the equilibrium pricing strategy and payoff of providers and corresponding demands of end users in each such setting. We quantify the impact of advertising revenue on the equilibrium pricing and demands, and compare the payoffs and demands for different interaction models.

I. INTRODUCTION

Internet of Things (IoT) is a world-wide network where various kinds of objects such as sensors, RFID tags, robots and smart phones will interact and operate with minimal human intervention. IoT has several applications— such as in smart grid, e-Health service, smart transportation system, building, and home automation. Research has been initiated to combat the technical challenges associated with IoT [1]–[3]. Also compressed sensing techniques have been developed to acquire information in an efficient manner in IoT [4]. However, the economic aspects of IoT remain largely ignored. Large scale proliferation of IoT will critically depend on rendering it profitable to all the entities in the IoT paradigm. We seek to contribute in this space.

We first characterize the entities involved in IoT architecture: end-users (e.g. individuals, organization, government agencies), advertisers, and providers. There are three different kinds of providers in IoT– IoT Service Provider (IoTSP), Wireless Service Provider (WSP), and Cloud Service Provider (CSP). IoT service will be provided by IoTSP (e.g. Apple Hue) similar to Internet service provided by ISP. CSP will help to store and process the enormous amount of data generated in IoT. WSP will enable the communication amongst devices and between the devices and the providers. Online advertisement will generate revenue for IoTSP as well as additional traffic for WSP and CSP, and additional reimbursement for them.

We seek to develop an economic model where different providers choose their actions in order to maximize their individual payoffs. We consider a monopolistic setting, i.e. there is one IoTSP, one WSP and one CSP. We consider two different interaction models. In the push or centralized model, the IoTSP procures the bandwidth and computing resources from the WSP and the CSP respectively and offers a package $p$ to end-users (Fig. 1). On the other extreme in a decentralized setting or pull model, each provider separately quotes a price to end-users, an end-user procures the service by paying all the providers separately (Fig. 1). In both the models, the demand of end-users decrease (increase, respectively) with the increase (decrease, resp.) in price that an end-user has to pay. In the pull model, the price charged by all providers directly influence the demand. In the push model, the price charged by IoTSP (WSP, CSP, respectively) directly (indirectly, resp.) influence the demand. The indirect influence in the push model is the following – if either WSP (or, CSP) increases its price charged to IoTSP, then the IoTSP also has to increase its price quoted to end-users which in turn will decrease the demand. Thus, the providers’ payoffs are not maximized either at very high price or at very low price. We characterize the pricing strategy and payoff of each provider in both models in a game theoretic setting.

In presence of advertisement revenue, it is expected that each provider’s payoff will increase. If advertisement volume increases in the push (pull, respectively) model IoTSP (end-user, resp.) has to pay WSP and CSP more for the additional traffic which increases WSP and CSP’s payoffs but also decreases IoTSP’s revenue (demand of end-users, resp.). Additionally, advertisement revenue may enable IoTSP to decrease its price leading to an increase in demand which in turn also increases the payoff of WSP and CSP. We characterize the impact of advertisement revenue on each provider’s pricing strategy and payoff as well as on the overall demand.

The overall strategic interactions (or, game) consists of three (two, resp.) stages (or, sub-games) in push (pull, resp.) model. First, IoTSP and advertisers jointly decide the advertisement price and the advertisement traffic to WSP and CSP (Sub-game 1) in both the models. IoTSP and advertisers inform the same to all the providers. Then, providers interact either in a push framework or pull framework. In the sub-game
2 of the push model, WSP and CSP quote their prices in parallel, in sub-game 3 of the push model, IoTSP selects its price with the knowledge of the prices quoted by WSP and CSP (Section III-A). We characterize the Sub-game perfect Nash equilibrium (SPNE) pricing strategy of the above game (Section III-B). Our analysis reveals:

- There is a unique SPNE when the advertisement revenue per user is below a certain threshold (say, \( T \)), but there are infinitely many SPNEs when the advertisement revenue per user exceeds \( T \) (Theorem 1).
- Price charged by IoTSP (payoff of IoTSP, respectively) decreases (increases, resp.) as the advertisement revenue per user increases when it is below \( T \) (Theorem 2, Corollary 1, resp.). At and above \( T \), the IoTSP can sustain a service at a free of cost i.e. it sets its price at 0 (Theorem 2). IoTSP’s payoff becomes constant when the advertisement revenue per user exceeds \( T \) (Corollary 2); thus, IoTSP only retains a fixed share from the advertisement revenue and the rest is shared between WSP and CSP in this regime.
- Price (payoff, resp.) of WSP (or CSP) increases with the advertisement revenue per user when it is below \( T \) (Theorem 1, Corollary 1, resp.). At and above \( T \), the price (payoff, resp.) will depend on the selection of equilibrium since there are multiple SPNEs, but the sum of the WSP and CSP is always unique and increases linearly with the advertisement revenue (Corollary 2).
- Since the demand increases monotonically with the decrease in price quoted by IoTSP, thus, the demand increases when advertisement revenue per user is below \( T \) (Corollary 1). At and above \( T \), the demand remains constant at the maximum value (Corollary 2).

In the pull model, since an end-user independently pays IoTSP, WSP, and CSP, thus, in the sub-game 2 of providers jointly select prices for end-users in a non cooperative game setting (Section IV-A). We fully characterize the Nash equilibrium (NE) pricing strategy and the payoffs of providers in this regime (Section IV-B).

- Unlike the push model, there is a unique NE in the pull model for all sets of parameters (Theorem 3).
- Payoff of IoTSP increases with advertisement for all sets of parameters (Corollary 3 and Corollary 4). Payoffs of WSP and CSP increase with advertisement revenues per user when it is below a certain threshold \( T_1 \), say (Corollary 3) and remain constant when the advertisement revenue per user exceeds \( T_1 \) (Corollary 4). Thus, unlike the push model, IoTSP grabs all the advertisement revenue when advertisement revenue per user becomes very high.
- Total payment that an end-user has to make decreases (remains constant, resp.) when the advertisement revenue per user is below \( T_1 \) (at and above \( T_1 \), resp.) (Theorem 3).
- The behavior of demand is similar to the push model (Corollaries 3 & 4). Only difference is that when the advertisement revenue per user exceeds \( T_1 \) then it remains constant at a value lower than the maximum value (Corollary 4).

We compare the payoffs and the demand associated with these two models (Section V):

- IoTSP’s payoff is the highest in the pull model (Corollary 3).
- Demand and thus, the reach of the IoT technology, is higher in the pull (push, respectively) model for low (high, respectively) advertisement revenue per user regime (Corollary 3).
- The payoff of WSP (or CSP) is the highest (lowest, respectively) in the pull model in the low (high, resp.) advertisement revenue per user regime (Corollary 3).

Related Literature: To the best of our knowledge this is the first investigation of the economic aspect of interactions among providers of different kinds in an IoT setting. However, pricing strategy in a system consisting of only one kind of provider has been extensively studied e.g. ISP [3], WSP [6], CSP [7] or CSP [8]. [9]. But different modes of interactions like push and pull naturally arise when there are multiple kinds of providers as in IoT. Payoff functions of different kinds of providers are also different; for example advertisers directly pay only IoTSP, while WSP and CSP only get indirect share of the advertisement revenue either from end-users or IoTSP.

The closest to our work are some recent works on net-neutrality which consider the interaction between ISP and Content provider (CP) [10], [11], [10] considers a setting similar to the push model where the ISP only charges CP and CP charges end-users. [11] considers that ISP and CP both charge the end-users (similar to the pull model in our setting). But the pricing game in the IoT context requires different problem formulation. For example [11] needs a purely sequential game where first CP selects its price and then ISP selects its price; but the push model which is the closest to [11] needs a combination of a sequential and a parallel game; in the first stage of this interaction, WSP and CSP select their prices in parallel and then IoTSP selects its price. The results are also substantially different. For example, [10] shows that there always exists a unique SPNE. But we find that there may be infinitely many SPNEs in the push model. Moreover, we consider optimal pricing strategy of IoTSP for advertisement volume which is not considered in [10]. [11]. The consideration of optimal pricing strategy for advertisement requires an additional game between IoTSP and advertisers where IoTSP quotes a price for advertisement and advertisers select the advertisement volume before providers select pricing strategies in the push and pull framework. Further, references [10], [11] did not characterize the payoffs in the setting when one of the provider’s price becomes zero, but we have fully characterized the payoffs of all providers in all possible settings. The above characterization enables us to study the impact of high advertisement revenue per user on individual payoff and the results are quite different from the low advertisement revenue setting. We also consider that WSP and CSP charge additionally for advertisement data delivered to end-users. We also analytically provide comparison of the

\[ \text{Recall from the above paragraph that in the sub-game 1 IoTSP and advertisers jointly select advertisement price and advertisement volume.} \]
payoffs of providers and demand in two different models. The consideration of above factors provide us novel insights.

Proofs have been deferred to Appendix.

II. PLANNERS, DECISION VARIABLES AND PARAMETERS

We investigate a monopoly where there is one provider of each kind i.e. there is one IoTSP, WSP and CSP.

IoTSP decides a price \( p_I \geq 0 \) for an IoT application.\(^3\) The price may be a periodic subscription fee \(^4\) or may be the price per application.\(^5\) WSP decides a charge of \( p_w \geq 0 \) per data (in bytes). We consider a usage based pricing scheme, but our result can be easily generalized for flat rate pricing schemes. CSP decides a charge of \( p_c \geq 0 \) for each unit of resource or capacity. Similar kind of pricing is currently employed by Amazon EC2 \(^6\). IoTSP decides \( b \geq 0 \) for per unit of advertisement volume and in response, advertisers decide the advertisement volume \( a \) for each end-user. An IoTSP receives \( ba_1 \) amount of advertisement revenue per user. \( a_1 \) is a function of \( b \), we only assume that \( a_1 \) and \( b \) are selected from a closed intervals, thus, \( ba_1 \) has at least one maximum. Advertisers also decide the additional cloud resources \( a_2 \) required for each end-user for providing customized advertisement. IoTSP informs \( b, a_1, a_2 \) to other providers before they select their prices.

We denote-- i) the average data flow (in bytes) required for each end-user for IoT application as \( a_1 \); ii) cloud capacities required to serve one end-user as \( a_2 \). Note that \( \alpha + a_1 (\beta + a_2) \), respectively, is the total data traffic (cloud capacity required, resp.) for each end-user.

In the subsequent sections we first characterize the equilibrium prices and payoffs, then we study the impact of advertisement revenue on the prices, payoffs of providers and the demand of end-users; subsequently, we analyze how IoTSP should select \( b \) to maximize its own payoff.

III. PUSH MODEL

A. System Model

End-users pay IoTSP and IoTSP shares its revenue with WSP and CSP (fig. 1). The demand of end-users only depends on the price of IoTSP i.e. \( p_I \). We assume that the demand-response of end-users follows a linear relationship--

\[
D_{push} = D_{max} - dp_I
\]

where \( d \) denotes the price sensitivity \(^6\) of end-users, \( D_{max} \) is the maximum demand.

IoTSP attains \( D_{push}ba_1 \) amount from the advertisement, but, it also has to pay \( p_w(a + a_1)D_{push} \) amount to WSP and \( p_c(\beta + a_2)D_{push} \) amount to CSP. Thus, IoTSP’s payoff is

\[
U_{I.push} = p_ID_{push} + bD_{push}a_1 - p_wD_{push}(a + a_1) - p_cD_{push}(\beta + a_2) \tag{2}
\]

WSP’s payoff is

\[
U_{w.push} = p_wD_{push}(a + a_1) \tag{3}
\]

CSP’s payoff is

\[
U_{c.push} = p_cD_{push}(\beta + a_2) \tag{4}
\]

We consider a non co-operative game where each provider only tries to maximize its own payoff. In the first stage IoTSP decides \( b \) and in response, advertisers jointly decide \( a_1 \) and \( a_2 \) (sub-game 1). With the knowledge of \( b, a_1 \), and \( a_2 \) providers then quote prices in the push model where we consider a combination of sequential and parallel pricing games. At the second stage (sub-game 2), WSP and CSP decide their prices they will charge in parallel i.e. the pricing strategy \((p_w^*, p_c^*)\), and in the third stage IoTSP selects its price \( p_I^* \) with the knowledge of prices of WSP and CSP (sub-game 3). We seek to obtain SPNE of the above game.

Definition 1. \([14]\) A Sub-game Perfect Nash equilibrium (SPNE) strategy profile is an NE \([14]\) at every subgame.

In Section III-B we obtain the prices and payoffs of providers in terms of \( b, a_1 \) and \( a_2 \), subsequently in Section III-C we will show how IoTSP should select \( b \).

B. Results

1) Equilibrium Price: NE Pricing strategy of IoTSP at Sub-game 3:

Lemma 1. Unique NE pricing strategy \( p_I^* \) in sub-game 3 is

\[
p_I^* = \max\{\frac{D_{max}}{2d} - \frac{ba_1}{2} + \frac{p_w(a + a_1)}{2} + \frac{p_c(\beta + a_2)}{2}, 0\}. \tag{5}
\]

NE pricing strategy of WSP and CSP at sub-game 2:

Theorem 1. When \( dba_1 < 5D_{max} \), then the unique NE \((p_w^*, p_c^*)\) in the sub-game 2 is

\[
p_w^* = \frac{D_{max}}{3d(a + a_1)} + \frac{ba_1}{3(a + a_1)} \tag{6}
\]

\[
p_c^* = \frac{D_{max}}{3d(\beta + a_2)} + \frac{ba_1}{3(\beta + a_2)} \tag{7}
\]

When \( dba_1 \geq 5D_{max} \), then any \((p_w^*, p_c^*)\) which satisfies the following conditions constitutes an NE in this sub-game

\[
p_w^*(a + a_1) \in [2D_{max}/d, ba_1 - 3D_{max}/d] \tag{8}
\]

\[
p_c^*(\beta + a_2) \in [2D_{max}/d, ba_1 - 3D_{max}/d] \tag{9}
\]

such that \( p_c(\beta + a_2) + p_w(a + a_1) = ba_1 - D_{max}/d \). \tag{10}

Note that \( ba_1 - 3D_{max}/d \geq 2D_{max}/d \) when \( dba_1 \geq 5D_{max} \). Thus, the interval from which WSP and CSP select their prices given in \( [8] \) and \( [9] \) respectively is non-empty. Though there is a unique equilibrium when \( dba_1 \leq 5D_{max} \), there are infinitely many equilibria in the sub-game 2 when \( dba_1 > 5D_{max} \). But the sum of payment received by WSP and CSP from IoTSP for each end-user is unique and increases linearly (by \( 10 \)) even when \( dba_1 > 5D_{max} \). Note that both

\[3\]There may be an additional fixed installation cost which we do not consider.

\[4\]AT&T has already announced monthly subscription scheme for its smart home secure appliances \([12]\).

\[5\]One such example is that Amazon charges a price only when a book is downloaded via Kindle. End-users do not need to pay a periodic subscription fee.

\[6\]High price sensitivity indicates that demand will decrease (increase, respectively) at high rate with increase (decrease) in price.
Theorem 2. At SPNE,
\[
p_1^* = \max \left\{ \frac{5D_{\max}}{6d} - \frac{ba_1}{6}, 0 \right\}
\]

(11)

Though \(p_1^*, p_w^*, p_c^*\) are not always unique, \(p_1^*\) is always unique. \(p_1^*\) decreases with \(ba_1\) and ultimately becomes 0 when \(ba_1 \geq 5D_{\max}\). Though IoTSP has to share its revenue with WSP and CSP, still IoTSP sets price at \(p_1^*\) (fig. 2). Thus, when \(ba_1\) is very high, then IoTSP can procure high advertisement revenue by selecting small price which in turn increases the demand \(D_{\text{push}}\).

2) Payoffs of providers and Demand of end-users: -

a) \(ba_1 < 5D_{\max}\):

Corollary 1. When \(ba_1 < 5D_{\max}\), then at equilibrium
\[
D_{\text{push}} = \frac{D_{\max}}{6d} + \frac{ba_1}{6}
\]
\[
U_{1,\text{push}} = \frac{D_{\max}}{6d} + \frac{ba_1}{6}
\]
\[
U_{w,\text{push}} = U_{c,\text{push}} = \frac{D_{\max}}{6d} + \frac{ba_1}{6}
\]

Corollary 1 shows that the demand \(D_{\text{push}}\) increases with increase in \(ba_1\), when \(ba_1 < 5D_{\max}\) since price charged by IoTSP decreases with \(ba_1\). The payoffs of providers increase quadratically with \(ba_1\) (fig. 2). Thus, if IoTSP can procure high \(ba_1\), then it not only increases the payoff of IoTSP, but it also increases the payoffs of WSP and CSP. WSP and CSP attain strictly higher payoffs compared to IoTSP since WSP and CSP act as the leaders and IoTSP acts as the follower in the sequential game.

b) \(ba_1 \geq 5D_{\max}\): Now we evaluate the demand and payoffs at equilibrium when \(ba_1 \geq 5D_{\max}\). Since the SPNE is not unique in this case we consider the worst possible payoffs for WSP \(U_{w,\text{push}}\) and CSP \(U_{c,\text{push}}\).

Corollary 2. When \(ba_1 \geq 5D_{\max}\), then
\[
D_{\text{push}} = \frac{D_{\max}}{6d} + dp_1 = D_{\max}
\]
\[
U_{1,\text{push}} = D_{\max}^2/d, \quad \text{at each SPNE (12)}
\]
\[
U_{w,\text{push}} = U_{c,\text{push}} = \frac{D_{\max}^2}{6d} \quad \text{if } \text{ba}_1 D_{\max} - D_{\max}^2/d
\]
\[
U_{w,\text{push}} + U_{c,\text{push}} = \text{ba}_1 D_{\max} - D_{\max}^2/d
\]

Note that though there are infinite SPNEs in this case, IoTSP’s payoff as well as the sum of the payoffs of WSP and CSP are always unique. This happens because the price selected by IoTSP and the sum of payments received by WSP and CSP are always unique. The worst case payoffs of WSP and CSP are strictly higher compared to the payoff of IoTSP. From (12), the payoff of IoTSP is independent of \(ba_1\) (fig. 2). Thus, IoTSP retains only a constant amount of revenue from the advertisement. The rest of the advertisement revenue is shared between CSP and WSP (by [13]). From Corollary 1 and 2 it is easy to verify that each provider’s payoff is strictly higher compared to the maximum possible payoff attainable at the setting when \(ba_1 < 5D_{\max}\). Corollary 2 shows that the demand becomes equal to the maximum possible value since price charged by IoTSP becomes 0.

C. IoTSP’s optimal price for advertisers in Sub-game 1

Note from Corollaries 1 and 2 that \(U_{1,\text{push}}\) is a non-decreasing function in \(ba_1\). Since IoTSP selects \(b\), and \(a_1\) is a function of \(b\), thus, IoTSP will select \(b\) in order to maximize \(ba_1\). Recall from Section IV that there exists at least one \(b\) where \(ba_1\) is maximized, thus, IoTSP will select \(b\) among one of those global maxima. If the maximum \(ba_1 < 5D_{\max}/d\), then \(U_{1,\text{push}}\) is given by Corollary 1 otherwise it is equal to the expression given in Corollary 2.

IV. PULL MODEL

A. System Model

IoTSP, WSP and CSP all directly charge prices to end-users (fig. 1). An end-user needs to pay \(p_w(\alpha + a_1)\) and \(p_c(\beta + a_2)\) amount to WSP and CSP respectively. Thus, the demand is
\[
D_{\text{pull}} = D_{\max} - (p_1 + p_w(\alpha + a_1) + p_c(\beta + a_2))
\]

(14)

We consider the same price sensitivity parameter for the prices of different providers.

Since IoTSP does not pay either WSP or CSP, thus
\[
U_{1,\text{pull}} = \frac{D_{\max}}{4d} - \frac{3ba_1}{4} \quad \text{if } ba_1 \leq D_{\max}/3
\]
\[
0 \quad \text{if } ba_1 > D_{\max}/3
\]

(15)

WSP’s payoff is
\[
U_{w,\text{pull}} = U_{c,\text{pull}} = \frac{D_{\max}}{4d} - \frac{3ba_1}{4} \quad \text{if } ba_1 \leq D_{\max}/3
\]
\[
0 \quad \text{if } ba_1 > D_{\max}/3
\]

(16)

CSP’s payoff is
\[
U_{c,\text{pull}} = p_c(\beta + a_2)D_{\max}/3
\]

(17)

In the first stage (Sub-game 1) IoTSP selects \(b\) and in response advertisers select \(a_1\) and \(a_2\). With the knowledge of \(b\), \(a_1\) and \(a_2\), providers quote their prices to end-users. Since each provider independently quotes its price, thus, in the second stage (Sub-game 2) we consider a non co-operative game where each provider simultaneously selects its price. In Section IV-B we find NE pricing strategy profile \((p_1^*, p_w^*, p_c^*)\) and payoffs of providers in terms of \(b, a_1\) and \(a_2\); subsequently, in Section IV-C we discuss how IoTSP should select \(b\).

B. Results

1) Equilibrium prices \((p_1^*, p_w^*, p_c^*)\) at Sub-game 2:

Theorem 3. The NE strategy profile at sub-game 2 is unique–
\[
p_1^* = \begin{cases} 
\frac{D_{\max}}{4d} - \frac{3ba_1}{4} & \text{if } ba_1 \leq D_{\max}/3 \\
0 & \text{if } ba_1 > D_{\max}/3 
\end{cases}
\]
\[
p_w^* = \begin{cases} 
\frac{D_{\max}}{3d(\alpha + a_1)} + \frac{ba_1}{4(\alpha + a_1)} & \text{if } ba_1 \leq D_{\max}/3 \\
\frac{D_{\max}}{3d(\alpha + a_1)} & \text{if } ba_1 > D_{\max}/3
\end{cases}
\]
\[
p_c^* = \begin{cases} 
\frac{D_{\max}}{3d(\beta + a_2)} + \frac{ba_1}{4(\beta + a_2)} & \text{if } ba_1 \leq D_{\max}/3 \\
\frac{D_{\max}}{3d(\beta + a_2)} & \text{if } ba_1 > D_{\max}/3
\end{cases}
\]

(18)

The generalization of our model to account for different price sensitivity parameters for different providers is straightforward.
When increases quadratically with for each provider (fig. 3) unlike in the push model. Each payoff the providers select prices such that the payoffs are the same linearly increases when such that the price of IoTSP decreases with becomes static when , but , even for high , the payoff of IoTSP becomes independent of in the push model, but in the pull model, the payoff of IoTSP strictly increases with even for high .

2) Payoffs of providers and Demand of end-users:
   a) \( \text{dba} \leq \text{D}_{\text{max}}/3 \):

Corollary 3. When \( \text{dba} \leq \text{D}_{\text{max}}/3 \), then at equilibrium

\[
\begin{align*}
D_{\text{pull}} & = \frac{\text{D}_{\text{max}}}{3} + \frac{\text{d} \text{dba}}{4} \\
U_{1,\text{pull}} & = U_{w,\text{pull}} = U_{\text{c,pull}} = d \left( \frac{\text{D}_{\text{max}}}{4d} + \frac{\text{dba}}{4} \right)^2
\end{align*}
\]

Note that the demand \( D_{\text{pull}} \) increases with \( \text{dba} \) despite that end-users pay WSP for the additional advertisement traffic \( a_1 \); the increase in \( \text{dba} \) enables the IoTSP to decrease its price which enhances the demand. Also note that \( a_2 \) does not play any role in the demand though end-users have to pay CSP for additional computational resource \( a_2 \). Corollary 3 entails that the providers select prices such that the payoffs are the same for each provider (fig. 5) unlike in the push model. Each payoff increases quadratically with \( \text{dba} \).

b) \( \text{dba} > \text{D}_{\text{max}}/3 \):

Corollary 4. When \( \text{dba} > \text{D}_{\text{max}}/3 \), then at equilibrium

\[
\begin{align*}
D_{\text{pull}} & = \frac{\text{D}_{\text{max}}}{3} \\
U_{1,\text{pull}} & = \text{dba} \frac{\text{D}_{\text{max}}}{3} \\
U_{w,\text{pull}} & = U_{\text{c,pull}} = d \text{D}_{\text{max}}/(9d)
\end{align*}
\]

Corollary 3 entails that the demand becomes independent of \( \text{dba} \) when \( \text{dba} > \text{D}_{\text{max}}/3 \). This is because the total payment that an end-user incurs becomes independent of \( \text{dba} \) when \( \text{dba} > \text{D}_{\text{max}}/3 \) by Theorem 3. Each end-user has to pay WSP and CSP under all cases unlike the push model, thus, the demand never reaches the maximum possible value, \( \text{D}_{\text{max}} \) even when \( p_1 = 0 \). (18) shows that IoTSP’s payoff linearly increases with \( \text{dba} \) and retains all the revenues from advertisement for high values of \( \text{dba} \) (fig. 3). Note that IoTSP’s payoff is strictly higher compared to the payoffs of WSP and CSP when \( \text{dba} > \text{D}_{\text{max}}/3 \) (fig. 5).

C. IoTSP’s optimal price for advertisers in sub-game 1

Note from Corollaries 3 and 4 that \( U_{1,\text{pull}} \) is a monotonically increasing function in \( \text{dba} \). Thus, IoTSP’s payoff is maximized at the maximum possible value of \( \text{dba} \). As discussed in Section III-C IoTSP will select \( b \) such that maximum \( \text{dba} \) is attained. If the maximum \( \text{dba} \leq \text{D}_{\text{max}}/(3d) \), then \( U_{1,\text{pull}} \) is given by Corollary 3, otherwise it is equal to the expression given in Corollary 3.

V. COMPARISON AMONG DIFFERENT MODELS

Corollary 5. • When \( \text{dba} > \text{D}_{\text{max}} \), \( D_{\text{pull}} > D_{\text{push}} \).
   • When \( \text{dba} = \text{D}_{\text{max}} \), \( D_{\text{pull}} = D_{\text{push}} \).
   • When \( \text{dba} < \text{D}_{\text{max}} \), \( D_{\text{pull}} < D_{\text{push}} \).

At initial stages of deployment of IoT technology, it is expected that \( \text{dba} \) will be small. Thus, if a social planner wants to increases the reach of IoT technology it may recommend the pull model at the initial stage. When \( \text{dba} \) becomes sufficiently high, social planner then may recommend the push model. When \( \text{dba} \) is small (large, respectively) the total payment that an end-user incurs is larger (smaller, resp.) in the push model compared to the pull model. This explains the above variation of demand with \( \text{dba} \).

A. IoTSP’s payoff

Corollary 6. For all values of \( \text{dba} \), \( U_{1,\text{pull}} > U_{1,\text{push}} \).

Even though the demand is high in the push model compared to that of pull model for high \( \text{dba} \), the IoTSP’s payoff is strictly less compared to the pull model (fig. 3). This is because, for high \( \text{dba} \), the payoff of IoTSP becomes independent of \( \text{dba} \) in the push model, but in the pull model, the payoff of IoTSP strictly increases with \( \text{dba} \) even for high \( \text{dba} \).

B. Payoffs of WSP and CSP

Corollary 7. • When \( 0.414 \text{D}_{\text{max}} > \text{dba} \), \( U_{w,\text{pull}} > U_{w,\text{push}} \) and \( U_{c,\text{pull}} > U_{c,\text{push}} \).
   • At \( \text{dba} = 0.414 \text{D}_{\text{max}} \), \( U_{w,\text{pull}} = U_{w,\text{push}} \) and \( U_{c,\text{pull}} = U_{c,\text{push}} \).
   • When \( \text{dba} > 0.414 \text{D}_{\text{max}} \), \( U_{w,\text{push}} > U_{w,\text{pull}} \) and \( U_{c,\text{push}} > U_{c,\text{pull}} \).

Note that for \( \text{dba} \geq 5 \text{D}_{\text{max}} \), even the worst case payoffs of WSP and CSP in the push model are strictly higher compared to that in the pull model. This happens because when \( \text{dba} \) is small (high, respectively) the demand in the push model is smaller (higher, resp.) compared to the pull model.

VI. FUTURE WORK

Generalization of our framework to account for an oligopolistic setting is a work for the future.
Thus, CSP’s payoff is

\[ p_c(\beta + a_2)D_{\text{max}} \]

which is a strictly increasing function in \( p_c \). Since \( p_c(\beta + a_2) < ba_1 - D_{\text{max}}/d - p_c(\alpha + a_1) \), thus, we can always find a small enough \( \epsilon > 0 \) such that \( (p_c + \epsilon)(\beta + a_2) < ba_1 - D_{\text{max}}/d - p_c(\alpha + a_1) \), but the payoff at \( p_c + \epsilon \) is \((p_c + \epsilon)(\beta + a_2)D_{\text{max}}\) which is strictly higher than the payoff at price \( p_c \) which contradicts the fact that \( p_c \) is an NE. Thus, NE can not arise in this case.

Case ii:

\[ p_w(\alpha + a_1) + p_c(\beta + a_2) > \frac{ba_1}{2} - \frac{D_{\text{max}}}{2d} \]

Replacing the value of \( p_1^* \) from (5) in (1) we get

\[ D_{\text{push}} = \frac{D_{\text{max}}}{2} - d(\frac{ba_1}{2} + p_w(\alpha + a_1) + p_c(\beta + a_2)) \]

Using (23) and (3), we obtain

\[ U_{w,\text{push}} = p_w(\alpha + a_1)\left(\frac{D_{\text{max}}}{2} - d(\frac{ba_1}{2} + p_w(\alpha + a_1) + p_c(\beta + a_2))\right) \]

We also obtain from (23) and (4)

\[ U_{c,\text{push}} = p_c(\beta + a_2)\left(\frac{D_{\text{max}}}{2} - d(\frac{ba_1}{2} + p_w(\alpha + a_1) + p_c(\beta + a_2))\right) \]

From first order condition the optimal prices are

\[ p_w^* = \frac{2d(\alpha + a_1)}{2d(\beta + a_2)} - \frac{p_c(\beta + a_2) + ba_1}{2(\alpha + a_1)} \]

\[ p_c^* = \frac{D_{\text{max}}}{2d(\beta + a_2)} - \frac{p_c^*(\beta + a_2) + ba_1}{2(\beta + a_2)} \]

Since the payoff functions of WSP and CSP are strictly concave in \( p_w \) and \( p_c \), respectively in this case, thus, the first order condition is also the sufficient one if they satisfy the condition in (22). Thus, at NE pricing strategy \((p_w^*, p_c^*)\), we must have from (26) and (27)

\[ p_w^* = \frac{D_{\text{max}}}{2d(\beta + a_2)} - \frac{p_c(\beta + a_2) + ba_1}{2(\beta + a_2)} \]

\[ p_c^* = \frac{p_c^*(\beta + a_2) + ba_1}{2(\beta + a_2)} \]

Note that condition (22) is satisfied when

\[ p_c^*(\beta + a_2) + p_w^*(\alpha + a_1) > ba_1 - D_{\text{max}}/d \]

\[ \frac{2D_{\text{max}}}{3d} + \frac{2ba_1}{3} > ba_1 - D_{\text{max}}/d \]

\[ ba_1 < 5D_{\text{max}}/d \]
Thus, NE strategy profile exists in this case only when \( dba_1 < 5D_{max} \).

**Case iii:**

\[
\frac{p_w(\alpha + a_1)}{2} + \frac{p_c(\beta + a_2)}{2} = \frac{ba_1}{2} - \frac{D_{max}}{2d} \quad (31)
\]

Here, \( p_1^* = 0 \) by (5). Thus, the demand becomes

\[
D = D_{max} - dp_1^* = D_{max} \quad (32)
\]

So, WSP’s payoff becomes

\[
p_w(\alpha + a_1)D_{max} \quad (33)
\]

and CSP’s payoff is

\[
p_c(\beta + a_2)D_{max} \quad (34)
\]

Now, we will find out the conditions for which there exists a NE \((p_w, p_c)\) in this case. To this end, we have to rule out any profitable unilateral deviation by either WSP or CSP.

We first show that when a WSP selects a price less than \( p_w \), then WSP’s payoff will be strictly less than the payoff at \( p_w \).

**Case iii.a:** If WSP selects a lower price \( x \), then by condition in (5), \( x(\alpha + a_1)/2 + p_c(\beta + a_2)/2 < ba_1/2 - D_{max}/(2d) \) i.e. it satisfies the condition in case i. Thus, \( p_1^* = 0 \). Thus, WSP’s payoff becomes \( x(\alpha + a_1)D_{max} \) which is strictly less than \( p_w(\alpha + a_1) \) since \( x < p_w \). Thus, WSP has no incentive to lower its price.

Similarly, we can show that CSP will also have no incentive to lower its price compared to \( p_c \).

Now, we obtain the condition under which the WSP will not have any incentive to select a price larger than \( p_w \).

**Case iii.b:** Suppose WSP increases its price and selects \( x > p_w \). thus, \( x(\alpha + a_1) + p_c(\beta + a_2) > ba_1 - D_{max} \); i.e. it satisfies the condition in case ii. Thus, from (24) WSP’s payoff at the price \( x \) is

\[
x(\alpha + a_1)(D_{max}/2 - d(-ba_1/2 + x(\alpha + a_1)/2 + p_c(\beta + a_2)/2))
\]

Now, using (31) in place of \( p_c(\beta + a_2) \), the above expression can be written as

\[
x(\alpha + a_1)(D_{max}/2 - d(-ba_1/2 + x(\alpha + a_1)/2)) \quad (35)
\]

Hence, there will be no profitable unilateral deviation if payoff at \( p_w \) is greater than or equal to the payoff at \( x \), thus from (33) and (35), we must have

\[
p_w(\alpha + a_1)D_{max} - x(\alpha + a_1)(D_{max}/2 - d(-ba_1/2 - p_w(\alpha + a_1)/2 + x(\alpha + a_1)/2)) \geq 0
\]

\[
(p_w(\alpha + a_1) - x(\alpha + a_1))(D_{max} - dx(\alpha + a_1)/2) \geq 0
\]

(36)

Since \( x > p_w \), thus the above condition will be satisfied only when \( 2D_{max}/d \leq x(\alpha + a_1) \). If \( 2D_{max}/d > p_w(\alpha + a_1) \), then, we can find \( x > p_w \) such that \( x(\alpha + a_1) < 2D_{max}/d \). Hence, for no profitable unilateral deviation for WSP, we must have

\[
2D_{max}/d \leq p_w(\alpha + a_1) \quad (37)
\]

Similarly, we can also show that for no profitable unilateral deviation for CSP, we must have

\[
2D_{max}/d \leq p_c(\beta + a_2) \quad (38)
\]

Now if \( p_w(\alpha + a_1) > ba_1 - 3D_{max}/d \), then by (31)

\[
p_c(\beta + a_2) < 2D_{max}/d
\]

Thus, we must have \( p_w(\alpha + a_1) \leq ba_1 - 3D_{max}/d \).

Again, since \( p_w(\alpha + a_1) \geq 2D_{max}/d \), thus, for feasible \( p_w \)

we must have \( 2D_{max}/d \geq ba_1 - 3D_{max}/d \) i.e. \( dba_1 \geq 5D_{max} \).

Similarly, in order to satisfy the condition that \( p_c(\beta + a_2) \geq 2D_{max}/d \) and (31), we must have

\[
p_w(\alpha + a_1) \geq ba_1 - 3D_{max}/d
\]

Since \( p_c(\beta + a_2) \geq 2D_{max}/d \), thus, similarly, \( p_c \) is feasible only when \( dba_1 \geq 5D_{max} \).

Thus, from case iii.b an NE can exist in this case only when \( dba_1 \geq 5D_{max} \) and the NE strategy \((p_w^*, p_c^*)\) must be of the following from

\[
p_w(\alpha + a_1) \in [2D_{max}/d, ba_1 - 3D_{max}/d]
\]

\[
p_c(\beta + a_2) \in [2D_{max}/d, ba_1 - 3D_{max}/d]
\]

such that \( p_w(\alpha + a_1) = p_c(\beta + a_2) = 2D_{max}/d \).

Note that when \( dba_1 \geq 5D_{max} \), there may exist multiple NE pricing strategy in subgame 1. Note that when \( dba_1 = 5D_{max} \) then, the NE strategy becomes unique and \( p_w^*(\alpha + a_1) = p_c^*(\beta + a_1) = 2D_{max}/d \).

Hence, from case ii, when \( dba_1 < 5D_{max} \), the NE strategy must be of the form given in (9) and from case iii, when \( dba_1 \geq 5D_{max} \), then NE strategy profile must be of the form given in (8).

**Proof of Corollary 7** From (5) and (1) we obtain

\[
D_{push} = D_{max} - dp_1 = \frac{D_{max}}{6} + d\frac{ba_1}{6} \quad (37)
\]

From (2), (5), (6), (7) and (37) we obtain

\[
U_{i, push} = \left(\frac{5D_{max}}{6} - \frac{ba_1}{6}\right)\left(\frac{D_{max}}{6} + d\frac{ba_1}{6}\right)
\]

\[
+ ba_1\left(\frac{D_{max}}{6} + d\frac{ba_1}{6}\right)
\]

\[
- \frac{D_{max}}{6} \left(\frac{ba_1}{3} + \left(\frac{D_{max}}{6} + d\frac{ba_1}{6}\right)\right)
\]

\[
= \left(\frac{3d(\alpha + a_1)}{6} + \frac{ba_1}{3}\right)(\alpha + a_1)\left(\frac{D_{max}}{6} + d\frac{ba_1}{6}\right)
\]

\[
- \left(\frac{3d(\beta + a_2)}{6} + \frac{ba_1}{3}\right)(\beta + a_2)\left(\frac{D_{max}}{6} + d\frac{ba_1}{6}\right)
\]

\[
U_{i, push} = \left(\frac{5D_{max}}{6} + \frac{ba_1}{6}\right)\left(\frac{D_{max}}{6} + d\frac{ba_1}{6}\right)
\]

\[
U_{i, push} = d\left(\frac{D_{max}}{6} + \frac{ba_1}{6}\right)^2
\]

We obtain from (36) and (37)

\[
U_{w, push} = p_w^*(\alpha + a_1)\left(\frac{D_{max}}{6} + d\frac{ba_1}{6}\right)
\]

\[
= \left(\frac{D_{max}}{3d} + \frac{ba_1}{3}\right)\left(\frac{D_{max}}{6} + d\frac{ba_1}{6}\right)
\]

\[
= 2d\left(\frac{D_{max}}{6} + d\frac{ba_1}{6}\right)^2
\]
Similarly from [4], [7] and [37] we obtain
\[
U_{c,push} = 2d(V_{\max} - \frac{b_a1}{d})^{2}
\] (40)

Corollary 1 readily follows from (37)–(40).

**Proof of Corollary 2** When \( dba1 \geq 5D_{\max} \), then \( p^*_1 = 0 \) by [5], the equilibrium demand is thus-
\[
D_{push} = D_{\max} - dp_1 = D_{\max}
\] (41)

Note that \( p^*_w(\alpha + a1) + p^*_c(\beta + a2) \) is equal to \( ba1 - D_{\max}/d \) by (10) when \( dba1 \geq 5D_{\max} \), thus from (2) and (5) we obtain
\[
U_{1,push} = 0 + D_{push} + ba1D_{push} = \left(p^*_w(\alpha + a1) + p^*_c(\beta + a2)\right)D_{push} = \frac{D_{\max}^2}{d}
\] (42)

Note from (38) and (39) that the lowest possible values for \( p_w(\alpha + a1) \) and \( p_c(\beta + a2) \) are \( 2D_{\max}/d \). Thus, from (41) the worst possible payoffs for WSP and CSP are respectively-
\[
\bar{U}_{w,push} = \frac{2D_{\max}^2}{d}
\] (43)
\[
\bar{U}_{c,push} = \frac{2D_{\max}^2}{d}
\] (44)

From (10) and (41) we obtain
\[
(p^*_w(\alpha + a1) + p_c(\beta + a2))D_{push} = ba1D_{\max} - \frac{D_{\max}^2}{d}
\] (45)

Corollary 2 readily follows from (41)–(45).

**B. Proofs of Results in Section 7**

We first prove Theorem 3 and then Corollaries 3 and 4.

**Proof of Theorem 3** Replacing (14) in (15) we obtain
\[
U_{1,pull} = p_1(D_{\max} - d(p_1 + p_w(\alpha + a1) + p_c(\beta + a2))) + ba1(D_{\max} - d(p_1 + p_w(\alpha + a1) + p_c(\beta + a2)))
\]

Replacing (14) in (16) and (17) we also obtain
\[
U_{w,pull} = p_w(\alpha + a1)(D_{\max} - d(p_1 + p_w(\alpha + a1) + p_c(\beta + a2)))
\]
\[
U_{c,pull} = p_c(\beta + a2)(D_{\max} - d(p_1 + p_w(\alpha + a1) + p_c(\beta + a2)))
\]

From the first order condition
\[
p^*_1 = \max\left\{ \frac{D_{\max}}{2d} - \frac{p^*_w(\alpha + a1)}{2} - \frac{p^*_c(\beta + a2)}{2} - \frac{ba1}{2}, 0 \right\}
\] (46)
\[
p^*_w = \max\left\{ \frac{D_{\max}}{2d(\alpha + a1)} - \frac{p^*_1}{2(\alpha + a1)} - \frac{p^*_c(\beta + a2)}{2(\alpha + a1)}, 0 \right\}
\]
\[
p^*_c = \max\left\{ \frac{D_{\max}}{2d(\beta + a2)} - \frac{p^*_1}{2(\beta + a2)} - \frac{p^*_w}{2(\beta + a2)}, 0 \right\}
\]

Note that \( p^*_1 \leq D_{\max}/2d \) and \( p^*_c(\beta + a2) \leq D_{\max}/2d \). Thus, \( \frac{2d(\alpha + a1)}{\alpha + a1} - \frac{\frac{k}{2(\alpha + a1)}}{1} \geq 0 \). Thus, we can write \( p^*_w \) as
\[
p^*_w = \frac{D_{\max}}{2d(\alpha + a1)} - \frac{p^*_1}{2(\alpha + a1)} - \frac{p^*_c(\beta + a2)}{2(\alpha + a1)}
\]

Also note that, \( p^*_w(\alpha + a1) \leq D_{\max}/2d \), thus \( \frac{D_{\max}}{2d(\beta + a2)} - \frac{p^*_1}{2(\beta + a2)} - \frac{p^*_w(\alpha + a1)}{2(\beta + a2)} \geq 0 \). Hence, we can write \( p^*_w \) as
\[
p^*_w = \frac{D_{\max}}{2d(\beta + a2)} - \frac{p^*_1}{2(\beta + a2)} - \frac{p^*_w(\alpha + a1)}{2(\beta + a2)}
\] (48)

Since \( U_{1,pull}, U_{w,pull} \) and \( U_{c,pull} \) are strictly concave in \( p_1, p_w \) and \( p_c \) respectively, thus, the first order condition is also sufficient. Thus, (46)–(48) are optimal.

We consider the following two cases
\[
\text{case i: } \frac{D_{\max}}{2d} - \frac{p^*_w(\alpha + a1) - p^*_c(\beta + a2)}{2} - \frac{ba1}{2} \geq 0
\]

In this case solving for NE strategy profile \((p^*_1, p^*_c, p^*_w)\) from (49)–(50), we obtain
\[
p^*_1 = \frac{D_{\max}}{4d} - \frac{3ba1}{4}
\]
\[
p^*_w = \frac{D_{\max}}{4d(\alpha + a1)} + \frac{ba1}{4(\alpha + a1)}
\]
\[
p^*_c = \frac{D_{\max}}{4d(\beta + a2)} + \frac{ba1}{4(\beta + a2)}
\] (51)

From (50) and (51) note that this case only arises when \( dba1 \leq D_{\max}/3 \).

\[
\text{case ii: } \frac{D_{\max}}{2d} - \frac{p^*_w(\alpha + a1) - p^*_c(\beta + a2)}{2} - \frac{ba1}{2} < 0
\]

Thus, from (47) and (48) we obtain
\[
p^*_w = \frac{D_{\max}}{3d(\alpha + a1)}
\]
\[
p^*_c = \frac{D_{\max}}{3d(\beta + a2)}
\] (54)

Note that
\[
\frac{D_{\max}}{6d} - \frac{p^*_w(\alpha + a1) - p^*_c(\beta + a2)}{2} - \frac{ba1}{2} < 0 \quad \text{when } dba1 > D_{\max}/3d
\]

Thus, this case arises only when \( dba1 > D_{\max}/3d \).

Thus from case i and ii, we obtain when \( dba1 \leq D_{\max}/3 \), then the NE price strategy must be of the form given in (49)–(51) and when \( dba1 > D_{\max}/3 \), then the NE price strategy is of the form given in (52)–(54).

**Proof of Corollary 4** When \( dba1 \leq D_{\max}/3 \), the equilibrium demand from (14), and Theorem 3 is
\[
D_{pull} = D_{\max} - d\left(\frac{D_{\max}}{4d} - \frac{3ba1}{4} + \frac{D_{\max}}{4d} + \frac{ba1}{4}\right)
\]
\[
= \frac{D_{\max}}{4d} + d\frac{ba1}{4}
\] (55)
From (55), (15) and Theorem 3, we obtain
\[ U_{I, pull} = \left( \frac{D_{\max}}{4d} - \frac{3ba_1}{4} \right) \left( \frac{D_{\max}}{4} + b_1 \right) 
+ ba_1 \left( \frac{D_{\max}}{4} + \frac{d}{4} \right)^2 
= d \left( \frac{D_{\max}}{4d} + \frac{ba_1}{4} \right)^2 \] (56)

Also from (16), (17) and Theorem 3, we obtain
\[ U_{w, pull} = d \left( \frac{D_{\max}}{4d} + \frac{ba_1}{4} \right)^2 \] (57)
\[ U_{c, pull} = d \left( \frac{D_{\max}}{4d} + \frac{ba_1}{4} \right)^2 \] (58)

Hence, the result directly follows from (55)-(58).

**Proof of Corollary 4** When \( ba_1 > \frac{D_{\max}}{3} \), then by Theorem 3 and (14) we obtain
\[ D_{\max} = \frac{D_{\max}}{3d} - \frac{D_{\max}}{3d} = \frac{D_{\max}}{3} \] (59)

From Theorem 3 and (15), we obtain
\[ U_{I, pull} = \frac{D_{\max}}{9} + \frac{ba_1 D_{\max}}{3} = \frac{ba_1 D_{\max}}{3} \] (60)

From (16), (59), and Theorem 3, we obtain
\[ U_{w, pull} = pu(\alpha + a_1)D_{\max}/3 = D_{\max}/9d \] (61)

From (17), (59), and Theorem 3, we obtain
\[ U_{c, pull} = pc(\beta + a_2)D_{\max}/3 = D_{\max}/9d \] (62)

The result readily follows from (59)-(62).

**C. Proofs of Results in Section V**

We prove Corollaries 5 and 7.

**Proof of Corollary 5** From Corollary 1 and 3, it is easy to verify that \( D_{\max} > D_{\max} / 3 \). Next, from Corollary 4, \( D_{\max} \) remains constant at \( D_{\max} / 3 \) when \( ba_1 > D_{\max} / 3 \). But from Corollary 1, \( D_{\max} = \frac{D_{\max}}{6} + dba_1 / 6 \). When \( 5D_{\max} > dba_1 > D_{\max} / 3 \), thus,
\[ \begin{align*}
    D_{\max} > D_{\max} / 3 & \quad \text{if } 5D_{\max} > dba_1 > D_{\max} \\
    D_{\max} = D_{\max} / 3 & \quad \text{if } dba_1 = D_{\max} \\
    D_{\max} < D_{\max} \quad & \quad \text{if } D_{\max} / 3 < dba_1 < D_{\max} \quad \text{(63)}
\end{align*} \]

Also note from Corollary 2 that \( D_{\max} = D_{\max} \) when \( ba_1 \geq 5D_{\max} \) which is strictly higher compared to \( D_{\max} / 3 \) which is the demand in the pull model when \( ba_1 \geq 5D_{\max} \) (by Corollary 4). Hence, the result follows.

**Proof of Corollary 7** When \( ba_1 \leq D_{\max} / 3 \), then comparing Corollary 1 and 3, we obtain \( U_{I, pull} < U_{I, pull} \).

When \( 5D_{\max} > dba_1 > D_{\max} / 3 \), then from Corollary 1 and 4, the difference between \( U_{I, pull} \) and \( U_{I, pull} \) is
\[ d \left( \frac{D_{\max}}{6d} + \frac{ba_1}{6} \right)^2 - ba_1 D_{\max} / 3 \]

It is easy to verify that the above expression is strictly decreasing in \( ba_1 \) for \( dba_1 < 5D_{\max} \). But when \( dba_1 = D_{\max} / 3 \), the above expression is negative. Thus, \( U_{I, pull} < U_{I, pull} \) for \( 5D_{\max} > dba_1 > D_{\max} / 3 \).

Also note from Corollary 2 that when \( dba_1 > 5D_{\max} \), then \( U_{I, pull} = D_{\max} / d \), but from Corollary 4, \( U_{I, pull} \) is \( ba_1 D_{\max} / 3 \) which is strictly higher compared to \( D_{\max}^2 / d \) for \( dba_1 \geq 5D_{\max} \). Thus, \( U_{I, push} < U_{I, pull} \) when \( ba_1 < 5D_{\max} / d \).

Thus, from the above discussion, we obtain \( U_{I, push} < U_{I, pull} \) for all sets of values of parameters. Also note that \( U_{I, pull} \) is strictly increasing in \( ba_1 \), and \( U_{I, push} \) becomes constant when \( dba_1 \geq 5D_{\max} \) thus, the difference between \( U_{I, pull} \) and \( U_{I, push} \) increases with \( dba_1 \) when \( dba_1 \geq 5D_{\max} \). □

**Proof of Corollary 7** We only show the relationship of the payoff of WSP between the push and pull model. The relationship between the payoff of CSP in the push model and the pull model will be similar to that of WSP. Comparing Corollary 1 and 3 it is easy to discern that \( U_{w, push} < U_{w, pull} \) when \( dba_1 \leq D_{\max} / 3 \).

Note from Corollary 1 and 4 that when \( 5D_{\max} > dba_1 > D_{\max} / 3 \) the difference between WSP’s payoff in the pull model and that of in the pull model is
\[ d \left( \frac{D_{\max}}{6d} + \frac{ba_1}{6} \right)^2 - \frac{D_{\max}^2}{9d} \]

The above expression monotonically decreases with \( ba_1 \). The two payoffs are equal when \( ba_1 = 6\left( \frac{D_{\max}}{18d} \right) = 0.414D_{\max} / d \), after that WSP’s payoff is higher in the push model compared to the pull model.

On the other hand note from Corollary 2 that the worst case WSP’s payoffs in the push model is \( 2D_{\max} / d \) when \( dba_1 \geq 5D_{\max} \) which is strictly higher compared to \( D_{\max}^2 / 9d \) which is the payoff obtained by WSP in the pull model when \( dba_1 \geq 5D_{\max} \) (by Corollary 3).

Thus, WSP’s payoff is higher in the pull model when \( dba_1 < 0.414D_{\max} \) and higher in the push model when \( dba_1 > 0.414D_{\max} \). Payoffs of WSP in the push model and in the pull model are equal when \( dba_1 = 0.414D_{\max} \).

Thus, summarizing the above results we have
\[ \begin{align*}
    U_{w, pull} & > U_{w, push} \quad \text{when } dba_1 < 0.414D_{\max} \\
    U_{w, pull} & = U_{w, push} \quad \text{when } dba_1 = 0.414D_{\max} \\
    U_{w, pull} & < U_{w, push} \quad \text{when } dba_1 > 0.414D_{\max}
\end{align*} \]

Thus, Corollary 7 readily follows.