The space charge limited current and huge linear magnetoresistance in silicon

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Huge magnetoresistance in space charge regime attracts broad interest on non-equilibrium carrier transport under high electric field. However, the accurate fitting for the current-voltage curves from Ohmic to space charge regime under magnetic fields has not been achieved quantitatively. We conjecture that the localized intensive charge dynamic should be taken into consideration. Here, by introducing a field-dependent dielectric constant, for the first time, we successfully simulate the current-voltage curves of covalent crystal silicon wafers under different magnetic fields (0–1 Tesla). The simulation reveals that the optical phonon, instead of the acoustic phonon, plays a major role for the carriers transport under magnetic fields in space charge regime.

The space charge effect covers vast applications, such as solar cells\(^1,2\), light-emitting diodes\(^3\), electro-resistance\(^4,5\) and high-power semiconductor devices\(^6\). The theoretical approach on the effect can be traced back to 1950s. Earlier models postulated the density state distribution of band tail, either Gaussian or exponential, for matching experiment data\(^7–9\). Recently, it has been revealed that the interplay between dopants and traps\(^10,11\), or molecular sites and the free-carrier density of states\(^11,12\), controls current-voltage (I-V) characteristics in semiconductors.

Remarkably, in 2009, Delmo et al. achieved a large positive magnetoresistance of more than 1,000 percent in silicon at room temperature that could be explained by the quasi-neutrality breaking of the space-charge effect\(^13\). The phenomena may originate from a spatial inhomogeneous electric field or an inhomogeneous charge distribution\(^13,14\). Experimentally, the magnetoresistance can be controlled by the current, the magnetic field\(^15\), the dopant concentration\(^16\), the electrode configuration\(^14,17\) and the difference of the surface/bulk electron combination rate\(^18\). One could predict a non-saturating huge linear magnetoresistance through a macroscopic random resistor network model\(^19,20\) or evaluate the net-charge distributions via finite element calculations\(^14,17\). However, the relationship between the spatial charge dynamic and the large linear magnetoresistance effect remains obscure.

Here, we consider that under a high electric field, the localized charges are forced to be itinerary; meanwhile, the itinerant charges can be captured by the ion cores. The dynamic of ionization and filling results in a high-energy electron-phonon interaction, which emits (or annihilates) virtual phonons and enhances (or reduces) the local carrier density\(^10,11,21\). By introducing a field-dependent dielectric constant, we successfully simulate the I-V curves of silicon wafers from Ohmic to space-charge regimes under different magnetic fields based on the theorem proposed by Zhang and Pantelides (ZP model). A positive linear magnetoresistance over 2,600% is obtained for an intrinsic N-Si in the space-charge regime under 1.2 T at room temperature. The capacitance of the intrinsic N-Si quasi-linearly increases under applied voltage, but shows a quadratic decrease under magnetic field. Although a heavily-doped N-Si or a P-Si possesses a quite similar I-V curve with an intrinsic N-Si, their magnetoresistances are negligible at room temperature.

Experimental details

The double-sides polished silicon wafers (thickness, 0.5 mm; Ke Jing Materials Technology, HeFei) with different doping type and resistance were used. Naturally oxidized layer on silicon wafer was removed by hydrofluoric acid. Then good ohmic indium/silicon contacts were fabricated under 400 °C for ten minutes. The two-terminal magnetotransport was measured using a source meter (Keithley 4200) under a EMP-7 electromagnet (East Changing Technology, Beijing) at room temperature. The capacitances were measured by an Andeen-Hagerling 2700 A ultra precision capacitance bridge with 10 V alternating excitation signal under different the bias voltage and magnetic field.
Result and Discussion

As shown in Fig. 1a, a typical \(I-V\) curve of an intrinsic N-Si exhibits a slow rise (\(V < 10\,V\)) followed by a sharp, power-law rise at a critical voltage \(V_0\) (black open diamonds). In order to avoid the competition of the surface and bulk parallel paths and the edge effects of a limited-size sample, a symmetric out-of-plane electrode set-up is selected in experiments, as shown in the insert of Fig. 1a. The slow rise is the Ohmic regime, and the sharp rise belongs to the space-charge regime according to space-charge-limited-currents theory. Recently, Zhang and Pantelides evidenced in theory and experiment that the interplay between dopants and traps controls the power-law rise of \(I-V\) curves. The prediction of ZP model is drawn in Fig. 1a (blue solid line) with the calculation parameters: the dielectric constant \(\varepsilon_r = 12\), dopant density \(N_D = 1.2 \times 10^{17}\,\text{cm}^{-3}\), and trap density \(N_T = 2 \times 10^{17}\,\text{cm}^{-3}\). It is clear that the predicted current by ZP model significantly lags behind the experiment data at high electric field. In principle, for a high purity semiconductor, the acoustic and optical phonon scatterings dominate the transport of carriers, while the ionized impurity contribution can be neglected at room temperature. In respect to the matters, as presented in Fig. 1b, the ratios of the relaxation time and the effective mass of carrier \(m^*\) with the external electric fields are evaluated by ZP model for both the optical and the acoustic phonons at room temperature (blue lines in Fig. 1b). Obviously, \(\tau/m^*\) of the optical and acoustic phonons remain a constant with the increase of voltage in ZP model. The acoustic phonon \(\tau_C/m^*\) obeys a power law rise of temperature in a broad temperature range rather than a function of electric field. The higher the temperature is, the lower the carrier mobility is, which is homogenous in sample and offers a minor correction for carrier transport if Joule heat is limited.

ZP model may underestimate the growth of carrier density due to intense electron-phonon excitation. We conjecture that the overall dielectric constant \(\varepsilon_{\text{sum}}\) can be expressed as \(1 + \chi + \chi_L + \chi_0\), where \(\chi\), \(\chi_L\), and \(\chi_0\) are the intrinsic, the non-equilibrium and the orientation terms, respectively. \(\chi_L\) is assumed proportional to the density of non-equilibrium carriers. \(\chi_0 = \eta_0 \Delta n/n_0\), here \(\Delta n\) defines as the density of non-equilibrium carriers, \(n_0\) is the density of thermal-exciters carriers and \(\eta_0\) is an arbitrary parameter for simulation. \(\chi_0\) can be expressed as \(\gamma \ln(1 + \Delta n/n_0)\), where \(\ln\) is a fitting coefficient. Under a high electric field, the uncompensated charge and the concomitant push-back electrostatic field yield the spatial distribution of polarity-conserved charges. Under a high electric field, the uncompensated charge and the term of polarization convergence ought to be considered even for a non-polar one. For our experiments, a symmetric out-of-plane electrode set-up with \(W \ll \tau\) is used in experiments. As shown in Fig. 1d, the ratios of the relaxation time and the effective mass of carrier \(m^*\) with the external electric fields are evaluated by ZP model for both the optical and the acoustic phonons at room temperature (blue lines in Fig. 1b). Obviously, \(\tau/m^*\) of the optical and acoustic phonons remain a constant with the increase of voltage in ZP model. For a high purity semiconductor, the acoustic and optical phonons scatterings dominate the transport of carriers, while the ionized impurity contribution can be neglected at room temperature. In respect to the matters, as presented in Fig. 1b, the ratios of the relaxation time and the effective mass of carrier \(m^*\) with the external electric fields are evaluated by ZP model for both the optical and the acoustic phonons at room temperature (blue lines in Fig. 1b). Obviously, \(\tau/m^*\) of the optical and acoustic phonons remain a constant with the increase of voltage in ZP model. The acoustic phonon \(\tau_C/m^*\) obeys a power law rise of temperature in a broad temperature range rather than a function of electric field. The higher the temperature is, the lower the carrier mobility is, which is homogenous in sample and offers a minor correction for carrier transport if Joule heat is limited.

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classical equilibrium magneto-electric transport theory. The turn-on voltage, $V_0$, grows quasi linearly, from 14 to 104 V, with the increase of magnetic field from 0 to 1 T. Under a magnetic field, the current suppression constrains the field-induced ionization along the transport direction, resulting in the postponed of $V_0$. In fact, although such phenomena are reported in those pioneer works for silicon wafers, to our best knowledge, qualified explanations have not been achieved. Here, we present that I-V curves under different magnetic fields are successfully simulated by LO model with a set of unified fundamental parameters, such as $N_D$, $N_t$, and energy levels, as presented in Fig. 2a by the solid lines (Supplemental Material S3).

The magnetoresistance is defined as $\rho(H)/\rho(0) - 1 \times 100\%$, with $\rho(0)$ and $\rho(H)$ the resistance at zero and applied magnetic field, respectively. In Fig. 2b, the out-of-plane magnetoresistances measured at a constant current mode $I = 10$ mA under different $\theta$ are shown together. The measurements are performed below the breakdown voltages to ensure the data stability. The inhomogeneous spatial dynamic of ionization and filling is inevitably influenced by a perpendicular magnetic field under a certain voltage, viz., prompting the traverse filling and suppressing the ionization along carrier transport direction. In our experiment, the magnetoresistance has an excellent linear relationship with magnetic fields under different $\theta$, as shown in Fig. 2b, exhibiting a pronounced anisotropic behavior. As $\theta = 90^\circ$, the magnetoresistance reaches ~2600% at 1.2 T, which is much larger than those reported in the former works. In the case of $\theta = 15^\circ$, the magnetoresistance is ~400% under 1.2 T, which is an order of magnitude smaller than the one at $\theta = 90^\circ$.

Numerous theories implicate spatial variation of the carrier mobility as being responsible for such an anomalously huge magnetoresistance. The spatial variation can be aroused by several factors, such as macroscopic inclusions, geometric configurations, defects, and electric field fluctuations in nonmagnetic materials. The magneto-induced phonon resonance relays on the inelastic inter-Landau-level scatterings. It is known that the value of the inhomogeneous magnetoresistance follows $\Delta\rho/\rho \propto \omega_c \tau^{27,28}$, where $\omega_c$ is

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**Figure 1.** Fits for I-V curves of the intrinsic and heavily doped N-Si without applying magnetic field and the capacitance measurements. (a) The I-V curve for an intrinsic N-Si with nominal resistivity $>10 \, K\Omega\cdot cm$ is measured at room temperature plotted on a double logarithmic scale. The blue solid line is the prediction by ZP model and the red solid line is the simulation by the LO model. The critical voltage $V_0$ is the crossover value by tangent lines of Ohmic and space-charge regimes. (Inset) This device is fabricated on the 16 mm $\times$ 16 mm substrate of thickness $L = 0.5$ mm, and the indium contacts are about 1 mm $\times$ 1 mm. The indium electrodes are pressed on the center of the upper and lower surfaces of the silicon wafer. $\theta$ is defined as the angle between the transport and the applied magnetic field direction; (b) $\tau/m^*$ of the acoustic phonon and LO phonon branches are evaluated by ZP model and LO model, respectively. (c) The I-V curve for a heavily doped N-Si with nominal resistivity 10 $\Omega\cdot$ cm measured at room temperature is plotted on a double logarithmic scale. The blue solid line is the prediction by ZP model and the red solid line is the simulation by the LO model. (d) $V_{IC}$ versus bias voltage, where the open diamonds and the solid squares represent the intrinsic and the heavily doped N-Si, respectively.
the cycling frequency and τ is the relaxation time of carriers. As shown in the inserted plot of Fig. 2b, ω_{co} and \( \tau \) of optical and acoustic phonons are shown by the open diamonds and circles, respectively. The red and the black lines are fitted by eyes. (c) The relationship between magnetoresistance and \( \theta \) follows Asin\( \theta \) under different magnetic fields. (d) MIC versus the square of the magnetic field under \( \theta \) and 20 V are shown as black and red solid diamonds, respectively. The linear fitting is guided by eyes.

Figure 2. Fits for the I-V curves under magnetic field of the intrinsic N-Si and the magnetoresistance response. (a) Experiment data (scatters) are fitted using the LO model (solid lines) with the magnetic field ranging from 0 to 1 T. \( V_0 \) shows a linear growth from 14 to 104 V. (b) The magnetoresistance is measured at a constant current mode \( I = 10 \) mA under different \( \theta \). Insert, \( \omega \tau \) of optical and acoustic phonons are shown by the open diamonds and circles, respectively. The red and the black lines are fitted by eyes. (c) The relationship between magnetoresistance and \( \theta \) follows Asin\( \theta \) under different magnetic fields. (d) MIC versus the square of the magnetic field under \( \theta \) and 20 V are shown as black and red solid diamonds, respectively. The linear fitting is guided by eyes.
modulation of the electron-to-hole density ratio under magnetic fields. By the symmetric out-of-plane electrode set-up, as illustrated in the inserted plot of Fig. 1a, we revisit the magnetoresistance for P-Si. Figure 3c shows the $I$-$V$ curves under different magnetic fields as $\theta = 90^\circ$ for an intrinsic P-Si. It is noted that the breakdown voltage only increases from 21 to 23 V between 0 and 1.2 T. Such phenomenon contracts to the intense magnetic-field variation of the breakdown voltages for an intrinsic N-Si, as demonstrated in the same plot by the gray dots. The relaxation of high-energy carriers depends on the intervalley scattering processes due to the different mobility of band valleys. For silicon, the density states of heavy-hole band are much larger than that of light-hole and spin-orbit splitting bands, resulting in weak intervalley scattering and a small variation of hole mobility under electric fields. The spatial inhomogeneity of hole mobility in P-Si is trivial compared to that of electrons in N-Si. The magnetoresistance of P-Si is inconspicuous accordingly. Capacitances under different magnetic fields are presented in Fig. 3d. The black and red boxes represent the capacitances measured at 0 and 10 V, respectively.

**Conclusion**

In this letter, we reveal that under high electric field, the dynamic of ionization and filling arouses the strong electro-LO phonon interaction in N-type silicon, accompanied with the virtual phonon processes and the large lattice distortion. The dielectric response of silicon is thus a function of fields especially in non-equilibrium regimes, as evidenced by capacitance measurements. Further experiment and theory on carrier transport under the intense spatial charge dynamic are encouraged, which can shed light on applications in novel devices, ranging from energy-harvesting cells to novel magneto-electric devices.

**References**

1. Li, G. *et al.* High-efficiency solution processable polymer photovoltaic cells by self-organization of polymer blends. *Nature Mater.* 4, 864–868 (2005).
2. Mihailetchi, V. D., Wildeman, J. & Blom, P. W. M. Space-Charge Limited Photocurrent. *Phys. Rev. Lett.* 94, 126602 (2005).
3. Cho, K.-S. *et al.* High-performance crosslinked colloidal quantum-dot light-emitting diodes. *Nature photon.* 3, 341–345 (2009).
4. Bray, M. G. & Werner, D. H. Passive switching of electromagnetic devices with memristors. *Appl. Phys. Lett.* 96, 073504 (2010).

Figure 3. The $I$-$V$ curves and the capacitance measurements of the heavily doped N-Si and the intrinsic P-Si. (a) The $I$-$V$ curves for the heavily doped N-Si with nominal resistivity $10 \Omega\cdot cm$ are measured at room temperature under magnetic field ranging from 0 to 1.2 T. (b) MIC of a heavily-doped silicon has a 0.4% change from 0 to 1.2 T under 50 V bias voltage (open triangles) which is much smaller than the change of MIC of the intrinsic N-Si (grey solid squares). (c) The $I$-$V$ curves for the intrinsic P-Si with nominal resistivity $>1000 \Omega\cdot cm$ are measured at room temperature under magnetic field ranging from 0 to 1.2 T. In contrast to the significant variation of $V_0$ in N-Si, there is a slight $V_0$ shift in P-Si. The data of N-Si are re-plotted here for comparison. (d) Capacitances under different magnetic field measured at 0 and 10 V bias voltage are shown in black and red boxes, respectively.
5. Zhang, J. J. et al. AgInSbTe memristor with gradual resistance tuning. Appl. Phys. Lett. 102, 183513 (2013).
6. Morčko, H. et al. Large-band-gap SiC, III-V nitride, and II-VI ZnSe-based semiconductor device technologies. J. Appl. Phys. 76, 1363 (1994).
7. Smith, R. W. & Rose, A. Space-Charge-Limited Currents in Single Crystals of Cadmium Sulfide. Phys. Rev. 97, 1531–1537 (1955).
8. Rose, A. Space-charge-limited currents in solids. Phys. Rev. 97, 1538–1544 (1955).
9. Lampert, M. A. Simplified theory of space-charge-limited currents in an insulator with traps. Phys. Rev. 103, 1648–1656 (1956).
10. Zhang, X. G. & Pantelides, S. T. Theory of Space Charge Limited Currents. Phys. Rev. Lett. 108, 266602 (2012).
11. Basile, A. F. & Fraboni, B. Numerical modeling of current-voltage characteristics to extract transport properties of organic semiconductors. J. Appl. Phys. 116, 194507 (2014).
12. Pasveer, W. F. et al. Unified description of charge-carrier mobilities in disordered semiconducting polymers. Phys. Rev. Lett. 94, 206603 (2005).
13. Delmo, M. P., Yamamoto, S. Y., Kasai, S., Ono, T. & Kobayashi, K. Large positive magnetoresistive effect in silicon induced by the space-charge effect. Nature 457, 1112–1116 (2009).
14. Wan, C. H. et al. Nonlocal magnetoresistance due to Lorentz force in linear transport regime in bulk silicon. Appl. Phys. Lett. 103, 262406 (2013).
15. Delmo, M. P., Kasai, S., Kobayashi, K. & Ono, T. Current-controlled magnetoresistance in silicon in non-Ohmic transport regimes. Appl. Phys. Lett. 95, 132106 (2009).
16. Porter, N. A. & Marrows, C. H. Dependence of magnetoresistance on dopant density in phosphorous doped silicon. J. Appl. Phys. 109, 07C703 (2011).
17. Wan, C. H., Zhang, X. Z., Gao, X. L., Wang, J. M. & Tan, X. Y. Geometrical enhancement of low-field magnetoresistance in silicon. Nature 477, 304–307 (2011).
18. Chen, J. J. et al. Enhanced linear magnetoresistance of germanium at room temperature due to surface imperfection. Appl. Phys. Lett. 106, 175303 (2015).
19. Parish, M. M. & Littlewood, P. B. Non-saturating magnetoresistance in heavily disordered semiconductors. Nature 426, 162–165 (2003).
20. Parish, M. M. & Littlewood, P. B. Classical magnetotransport of inhomogeneous conductors. Phys. Rev. B 72, 094417 (2005).
21. Patterson, D. J. & Bailey, B. C. Solid-State Physics (Springer Berlin Heidelberg, New York, 2005).
22. Grundmann, M. The Physics of Semiconductors. (Springer-Verlag Berlin Heidelberg, Germany, 2010).
23. Feynman, R. P., Leighton, P. B. & Sands, M. The Feynman's Lectures on Physics. (Scientific & Technical Publishers, Shanghai, 2012).
24. Yuan, K., Chen, L. & Chen, Y. W. Direct anisotropic growth of CdS nanocrystals in thermotropic liquid crystal templates for heterojunction optoelectronics. Chem. Eur. J. 20, 11488–11495 (2014).
25. Lee, S. J. et al. An electrical switching device controlled by a magnetic field-dependent impact ionization process. Appl. Phys. Lett. 97, 253505 (2010).
26. Wu, L. H. et al. Room-temperature non-saturating magnetoresistance of intrinsic bulk silicon in high pulsed magnetic fields. Appl. Phys. Lett. 98, 112115 (2011).
27. Xu, R. et al. Improved magnetoresistance in non-magnetic silver chalcogenides. Nature 390, 57–60 (1997).
28. Herring, C. Effect of random inhomogeneities on electrical and galvanomagnetic measurements. J. Appl. Phys. 31, 1939–1953 (1960).
29. Stroud, D. Generalized effective-medium approach to the conductivity of an inhomogeneous materials. Phys. Rev. B 12, 3368–3373 (1975).
30. Magier, R. & Bergman, D. J. Strong-field magnetotransport of two-phase disordered media in two and three dimensions: Exact and approximate results. Phys. Rev. B 74, 094423 (2006).
31. Solin, S. A., Thio, T., Hines, D. R. & Heremans, J. J. Enhanced room-temperature geometric magnetoresistance in inhomogeneous narrow-gap semiconductors. Nature 289, 1530–1532 (2000).
32. Porter, N. A. & Marrows, C. H. Linear magnetoresistance in n-type silicon due to doping density fluctuations. Sci. Rep. 2, 565 (2012).
33. Usher, A. & Elliott, M. Magnetometry of low-dimensional electron and hole systems. J. Phys: Condens Matter 21, 103202 (2009).
34. Schousson, J. J. H. M., Bloom, F. L., Wagemans, W., Swagten, H. J. M. & Koopmans, B. Extremely Large Magnetoresistance in Boron-Doped Silicon. Phys. Rev. Lett. 100, 127202 (2008).
35. Delmo, M. P., Shikoh, E., Shinjo, T. & Shiraishi, M. Bipolar-driven large linear magnetoresistance in silicon at low magnetic fields. Phys. Rev. B 87, 245301 (2013).
36. Bulutay, C., Ridley, B. K. & Zakhleniuk, N. A. Electron momentum and energy relaxation rates in GaN and AlN in the high-field transport regime. Phys. Rev. B 68, 115205 (2003).
37. Hada, Y. & Eto, M. Electronic states in silicon quantum dots: Multivalley artificial atoms. Phys. Rev. B 68, 155322 (2003).
38. Ottaviani, G., Reggiani, L., Canali, C., Nava, F. & Alberigi-Quaranta, A. Hole drift velocity in silicon. Phys. Rev. B 12, 3318 (1975).

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Author Contributions
Y. Liu fabricated and measured the samples, analyzed the data, and performed the numerical calculations. H. Wang supervised the project. Y. Liu and H. Wang wrote the manuscript. X. Jin prepared the prose work including the equipment debugging and parameter simulation. M. Zhang helped to check the stability and reliability of the experiment independently.

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