Neutrino interactions with nuclei

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Abstract. We present a model for neutrino-nucleus scattering in the energy region relevant for present and forthcoming neutrino-oscillation experiments. The model is based on the RPA treatment of the nuclear responses in the quasi-elastic and Delta-resonance region. It includes also in a phenomenological way nucleon knock-out. It aims at the description, within a single framework, of several final state channels i.e. quasi-elastic, incoherent and coherent one-pion production and two- or several-nucleon knock-out.

Keywords: Neutrino-nucleus interactions, Quasi-elastic scattering, Pion production, Coherent scattering

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INTRODUCTION

Various theoretical approaches [1] have been used to interpret the experimental results on neutrino interactions with matter [2, 3, 4, 5, 6, 7, 8, 9] in quasi-elastic processes or coherent and incoherent single pion production. In our work, we explore these interactions in the energy region around 1 GeV using the formalism of the nuclear response functions treated in the random phase approximation (RPA) and incorporating Delta-resonance excitation as in the work of Marteau [10]. This approach has the merit of describing in a unique frame several final state channels i.e. quasi-elastic, incoherent and coherent one-pion production and two- or several-nucleon knock-out. In the following we present the results obtained for each of them.

Several types of nuclear responses enter the total neutrino-nucleus cross-section: the isovector \( R_\tau \), the spin-isospin transverse \( R_{\sigma \tau}^T \) or longitudinal \( R_{\sigma \tau}^L \). In order to illustrate this point, we give below a simplified expression of the double differential cross-section for the reaction \( \nu_l (\bar{\nu}_l) + A \rightarrow l^- (l^+) + X \) which, in particular, ignores the lepton mass contribution and assumes zero \( \Delta \) width:

\[
\frac{d^2\sigma}{d\Omega dQ^2} = \frac{G_F^2 \cos^2 \theta_c(q^2)}{2\pi^2} \cos^2 \frac{\theta_c}{2} \left[ G_E^2 \left( \frac{q^2}{2\pi^2} \right)^2 R_{\tau}^{NN} + G_A^2 \left( \frac{M_\Delta - M}{2q^2} \right)^2 R_{\sigma \tau}^{NN} + G_A^2 \left( \frac{M_\Delta - M}{q^2} \right)^2 R_{\sigma \tau}^{NN} \right]
\]

\[
+ \left( \frac{G_A^2}{q^2} + \frac{G_A^2}{q^2} \right) \left( -\frac{q^2}{q^2} + 2\tan^2 \frac{\theta_c}{2} \right) \left( R_{\sigma \tau}^{NN} + 2R_{\sigma \tau}^{NN} + R_{\sigma \tau}^{NN} \right) \right] \left( R_{\sigma \tau}^{NN} + 2R_{\sigma \tau}^{NN} + R_{\sigma \tau}^{NN} \right) \].

For the variable definitions and for the complete formulas we refer to [10, 11]. We stress that in the actual calculations we make use of the full formulas.

The various responses \( R \) appearing in Eq. (1) are related to the imaginary part of the corresponding full polarization propagators through

\[
R(q, \omega) = -\frac{1}{\pi} \text{Im}\Pi(q, q, \omega).
\]

They are calculated within a RPA ring approximation, starting from “bare” propagators (meaning that the nuclear correlations are switched off) and solving integral equations which have the generic form

\[
\Pi = \Pi^0 + \Pi^0 V\Pi,
\]

where \( V \) denotes the effective interaction between particle-hole excitations.
The bare polarization propagator is density dependent. In a finite system, $\Pi^0(\vec{q}, \vec{q}', \omega)$ is non-diagonal in momentum space. In order to account for the finite size effects we evaluate it in a semi-classical approximation where it can be cast in the form

$$\Pi^0(\vec{q}, \vec{q}', \omega) = \int d\vec{r} e^{-i(\vec{q}-\vec{q}')\cdot\vec{r}} \Pi^0\left(\frac{\vec{q}+\vec{q}'}{2}, \vec{r}, \omega\right).$$

In practice we use a local density approximation,

$$\Pi^0\left(\frac{\vec{q}+\vec{q}'}{2}, \vec{r}, \omega\right) = \Pi^0_{k_F(r)}\left(\frac{\vec{q}+\vec{q}'}{2}, \omega\right),$$

where the local Fermi momentum $k_F(r)$ is related to the experimental nuclear density through:

$$k_F(r) = (3/2 \pi^2 \rho(r))^{1/3}.$$

The bare response is the sum of the following partial components: (1) $NN$ quasi-elastic (as described by the standard Lindhard function); (2) $NN$ $2p-2h$; (3) $N\Delta$ and $(3') \Delta N$ $2p-2h$; (4) $\Delta\Delta$ $2p-2h$; (5) $\Delta\Delta$ $3p-3h$.

The RPA response generically writes

$$\text{Im} \Pi = |\Pi|^2 \text{Im} V + |1 + \Pi|^2 \text{Im} \Pi^0.$$

It splits in two terms. The first implies a cut on the pion exchange potential $V_\pi$. It represents the coherent pion production where the nucleus is left in the ground state. The second, proportional to the bare polarization propagator $\text{Im} \Pi^0$, reflects the type of final state already mentioned in the bare case, modified by collective effects.

RESULTS AND COMPARISON WITH DATA

Coherent pion production

The response naturally associated to the coherent process is the spin-isospin longitudinal one, since it has the same coupling as the pion. We have tested our description of the coherent responses on the elastic $\pi^+{^{12}}C$ scattering which is sensitive to collective effects in the longitudinal channel. We have also checked the compatibility of our evaluation of neutrino differential cross-section in the forward direction with the experimental $\pi^-{^{12}}C$ elastic scattering cross-section, according to Adler’s theorem.

Figure II displays our evaluations of the coherent pion production off $^{12}$C as a function of the pion kinetic energy, both for charged and neutral current, for several neutrino incident energies. The resulting total coherent cross-sections are also shown as a function of the neutrino energy.
FIGURE 2. Left panel: differential $\nu_\mu^{+}{}^{12}\text{C}$ charged current cross-section as a function of transferred energy in the various channels for $E_\nu=1$ GeV. Right panel: total $\nu_\mu^{+}{}^{12}\text{C}$ charged current cross-section in the various channels.

Quasi-elastic and multi-nucleon channels

The quasi-elastic channel corresponds to a single-nucleon knock-out. In contrast to the coherent channel, the quasi-elastic process is dominated by the transverse response. The quasi-elastic cross-section is displayed in Fig. 2 as function of the energy transfer $\omega$, for a neutrino energy of 1 GeV, both in the bare and in the RPA cases. The RPA effects tend to reduce the cross-section, as expected from the repulsive character of the particle-hole interaction which dominates in the transverse channel. In the same figure we display the sum of the two- and three-nucleon knock-out cross-section, which represents a sizable fraction of the quasi-elastic one. Singling out the genuine quasi-elastic process requires the insurance that no more than one proton is ejected. This issue will appear in connection with the comparison to data. Part of multi-nucleon channels arises from the modification of the Delta width in the medium where other decay channels are possible \cite{12}. The remaining contribution is taken from \cite{13,14}. In neutrino interactions this last part of the cross-section is important but not very well constrained by phenomenology.

Incoherent pion emission

Turning now to incoherent pion emission, the pion arises from the pionic decay of the Delta leaving the nucleus in a particle-hole excited state. For the nucleus that we consider, the incoherent pion cross-section is much larger than the coherent one. As compared to a free nucleon, the emission probability is already reduced in the bare case by the change in the Delta width. Moreover the RPA effects, which are moderate, also contribute to this reduction. The reduction due to the modification of the Delta width has a counterpart in the presence of the multi-nucleon knock-out component discussed before.

All the previous results are summarized in Fig. 2 which displays the muon-neutrino differential cross-section in the various channels as a function of the energy transfer for the case of $^{12}\text{C}$ and a neutrino energy of 1 GeV. The total neutrino cross-section is also displayed. The incoherent $\pi^+$ channel includes all possible charge states. In our evaluation the incoherent $\pi^+$ channel results to be 5/6 of the total.

Comparison with data

Experimental data concern ratios between different cross-sections. For charged current, the K2K collaboration has established a 90% confidence-level upper bound on the ratio of coherent pion production to the total cross-section, giving a limit of $0.60 \times 10^{-2}$ averaged over the neutrino flux with a mean energy of 1.3 GeV \cite{4}. More recently, the SciBooNE collaboration found for the same quantity $0.67 \times 10^{-2}$ at neutrino energy of 1.1 GeV \cite{8}. We report in the left panel of Fig. 3 our prediction for this quantity. Our curve is just compatible with the experimental limit.
Another measured quantity is the ratio of $\pi^+$ production to quasi-elastic cross-section for charged current. The MiniBooNE collaboration has used a CH$_2$ target. In order to compare with ANL [2] and K2K [7] data, they presented the results applying an isoscalar rescaling correction [9]. The issue of pion loss by final state interaction, which is not incorporated in our description, has also been taken into account by MiniBooNE who corrects data for this effect. We can thus compare our $\pi^+$ over quasi-elastic ratio (solid line in the central panel of Fig.3) to the final-state-interaction-corrected MiniBooNE results. Our curve is fully compatible with experimental data.

As an additional information, MiniBooNE also gives a ratio more directly related to the measurements, namely the ratio of pion-like events (defined as events with exactly one $\mu^-$ and one $\pi^+$ escaping the struck nucleus) and quasi-elastic signal (defined as those with one $\mu^-$ and no pions). In our language the last quantity represents the $np-nh$ (including the quasi-elastic for $n=1$) exclusive channel. We have compared this second experimental information with the ratio between our calculated pion production (which however ignores final state interactions) and our total $np-nh$ contribution to the total charged current neutrino cross-section (right panel of Fig.3). The comparison shows an agreement up to $E_\nu \simeq 1.2$ GeV. This may be an indication that final state interactions for the pion is not essential here. Our theoretical approach predicts a strong contribution of $2p-2h$ and $3p-3h$ channels which seems to be supported by the last comparison.

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