The Statistical Origin of Constituent-Quark Scaling in QGP Hadronization

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(Received 3 December 2012)

Nonextensive statistics in a blast-wave model is implemented to describe the identified hadron production in relativistic p+p and nucleus-nucleus collisions. Incorporating the core and corona components within the TBW formalism allows us to describe simultaneously some of the major observations in hadronic observables at the Relativistic Heavy-Ion Collider (RHIC): the amount of constituent quark scaling (NCQ), the large radial and elliptic flow, the effect of gluon saturation, and the suppression of hadron production at high transverse momentum (p_T) due to jet quenching. In this formalism, the NCQ scaling at the RHIC appears as a consequence of a non-equilibrium process. Our study also provides concise reference distributions with a least χ^2 fit of the available experimental data for future experiments and models.

PACS: 12.38.Mh, 12.40.Ee, 25.75.Dw, 25.75.Ld

Several intriguing features are discovered in relativistic heavy ion collisions[1] when particles emerging from the quark-gluon plasma are detected by the experiments at the Relativistic Heavy-Ion Collider (RHIC). In Au+Au collisions, identified particle yields integrated over the transverse momentum range around the center-of-mass rapidity window have been shown to be at equilibrium at the chemical freeze-out in a statistical analysis.[1,2] The hydrodynamic model with proper equation of state and initial condition can describe the anisotropic flow with small shear viscosity and provides the notion of “perfect liquid”.[3,4] Furthermore, the transverse momentum distributions of identified particles can be described by a hydrodynamic-inspired model with a compact set of parameters.[1,2,5–7]

However, in the intermediate p_T range, particle production exhibits grouping between baryons and mesons with baryons having relatively higher yield and larger elliptic flow than the mesons.[1,2] This feature of constituent quark scaling is not present in the hydrodynamics. A microscopic quark coalescence at the hadronization seems to be inescapable.[8] At even higher p_T, the hard perturbative QCD processes (jets) are relevant. Absorption of jets in the medium formed in A+A collisions has been used for studying the properties of the QGP.[9] Even though hydrodynamics with space-time evolution from an initial condition[8] is so far the most realistic simulation for bulk matter produced in relativistic heavy ion collisions, its applicability is expected to breakdown for p+p and peripheral A+A collisions at the RHIC. A recent study showed that hydrodynamics can not replace the microscopic hadronic cascade at the late stage regardless of freeze-out and the equation of state one chooses[10] because the particle interactions may be dominated by the non-equilibrium hadronic processes.[11] In A+A collisions, the fluctuations at initial impact due to color-glass condensate (CGC) formation or individual nucleon-nucleon collision may not be completely washed out by subsequent interactions in either the QGP phase or hadronic phase.[12] These effects leave footprints in the spectra at low and intermediate p_T.

With its development and success of nonextensive statistics (also known as Tsallis statistics[13,14]) in dealing with non-equilibrated complex systems in condensed matter, many researchers have utilized Tsallis statistics to understand the particle production in high-energy and nuclear physics.[9] Although the implications and understanding of the consequences of such an application are still under investigation, the usual Boltzmann distribution in an m_T exponential form can be readily rewritten as an m_T power-law (Levy) function:[14]

$$\frac{d^2N}{2\pi m_T dm_T dy} \propto \left(1 + \frac{q - 1}{T} m_T\right)^{-1/(q-1)},$$

where the left-hand side is the invariant differential particle yield and q is a parameter characterizing the degree of non-equilibrium, m_T = \sqrt{m^2 + p_T^2} is the transverse mass of the given particle with mass of m and T interpreted as the average temperature of the system with large temperature fluctuation. The distribution can be derived from the usual procedure in statistical mechanics, starting from a non-equilibrium q-entropy.[13] The successful application of Levy functions (TBW_{pp}) to the spectra in p+p collisions at...
the RHIC resulted in q values significantly larger than unity and are different between the groups of baryons and mesons. However, in the central Au+Au collisions, the spectra at low p_T show a characteristic Boltzmann distribution with the q value being close to unity (q → 1, Eq. (1) becomes a Boltzmann distribution) even though there are still significant power-law tails with considerable particle yields at high p_T. In addition to the escaping jets at high p_T, coalescence with non-equilibrated quarks has also been proposed to study the power-law behavior. Difficulty in accounting for these processes so far seems to be a major limitation of the TBW statistical description of the experimental data over a wide range of p_T. To bridge the hydrodynamic nature of the spectra at low p_T and power-law tails at high p_T with smooth transition at intermediate p_T, models which include a hot and dense core with a corona of a jet-like process have been proposed.

In order to take into account the collective flow and the azimuthal anisotropy in the transverse direction in relativistic heavy ion collisions, the Levy distribution needs to be embedded in the framework of hydrodynamic expansion. We follow the recipe of the blast-wave model provided in literature and change sources of particle emission from a Boltzmann distribution to a Levy distribution:

\[
\frac{dN}{m_T dm_T d\phi} \propto m_T \int_0^{2\pi} d\phi_0 \int_{y_0}^{y_0 + \rho_0} dy \sqrt{\frac{y^2}{y^2 + \rho_0^2}} \cosh(y) \cdot \int_0^R rdr \left(1 + \frac{q - 1}{T} E_T \right)^{-1/(q-1)},
\]

where \( y_0 = \ln \left( \sqrt{\frac{y}{m_T}} \right) \) is the beam rapidity and the rapidity distribution can be approximated as a Gaussian with a width of \( \sigma_y = 2.27 \pm 0.02 \) at the center-of-mass energy of \( \sqrt{s_{NN}} = 200 \) GeV. The transverse energy is \( E_T = m_T y \cos(y) \cosh(\rho) - p_T \sinh(\rho) \cos(\phi) - \phi \), \( \rho = \sqrt{\left( r \cos \left( \phi_0 / R_X \right) \right)^2 + \left( r \sin \left( \phi_0 / R_Y \right) \right)^2} \) is the flow profile in transverse rapidity, and \( \tan(\phi_0) = (R_X / R_Y)^2 \tan(\phi_L) \) relates the azimuthal angle of the coordinate space (\( \phi_L \)) to the angle of the flow direction (\( \phi_0 \)) of the emitting source. Equation (2) extends the nonextensive statistics in a blast-wave model to incorporate particle emission from an elliptic source (\( R_X \) and \( R_Y \) are the axes in the coordinate space, respectively) with an elliptic expansion. In addition to this core component, it is important to include the corona with jet-like particle emission at high momentum resembling an ensemble of individual p+p collisions. The combined core and corona formula reads

\[
\frac{dN}{m_T dm_T d\phi} \bigg|_{\Lambda \Lambda} = \text{TBW}_{\text{core}} + f_{pp} N_{\text{bin}} (1 + v_T^{\text{jet}} \cos(2\phi)) \text{TBW}_{pp},
\]

where \( v_T^{\text{jet}} \) is the jet-like flow at mid-rapidity at \( \sqrt{s_{NN}} = 200 \) GeV. The symbols represent experimental data. The curves represent the TBW fit. Only fits to the particle yields and power-law tails at high p_T are shown since the model has the same spectra in panel (a), the spectra of a meson (baryon) are scaled to match with that of \( \pi^+ \) (p) at \( m_T = 1.5 \) GeV/c^2 for \( \phi (\Xi^-) \), at 4 GeV/c^2 for J/ψ and at 1 GeV/c^2 for the rest.

In this Letter, we present the procedure of implementing nonextensive statistics in the blast-wave model with azimuthally anisotropic particle emission, and use it to fit the identified particle spectra and for the first time to elliptic flow at mid-rapidity at the RHIC. The model uses the TBW function obtained from p+p data as the corona and an additional TBW function as the core to fit Au+Au data. The formalism thus provides a systematic comparison between p+p and the central Au+Au collisions in one macroscopic statistical model framework and gives an accurate numeric description of the experimental data over a wide range of \( p_T \). Examples of such successful applications in the related subjects are the chemical fit to the particle yields and the global fit of the parton distribution function (PDF) of a proton. Good TBW fits can also offer a simple formula for developing ideas and building models in a reasonably realistic environment and provide a practical experimental tool to extract particle yields by extrapolating to unmeasured kinematic ranges.
where TBW\textsubscript{core} is obtained from Eq. (2), \( f_{pp} \) and \( v_2^{jet} \) represent the fraction and the anisotropy of the escaping jet compared to the expected number of binary \( p + p \) collisions in Au+Au collisions (\( N_{bin} \), \( \epsilon = p_T^2/(p_T^2 + Q_S^2) \) takes into account the gluon saturation effect from \( p + p \) to Au+Au collisions with saturation scale \( Q_S = 1.5 \) GeV/c[22,23] and TBW\textsubscript{pp} are the adopted fit results from the spectra in \( p + p \) collisions without any additional free parameter for either baryons or mesons. Since radial flow velocity \( \rho \) was found to be zero in \( p + p \) collisions,[7] TBW\textsubscript{pp} can be simplified to

\[
TBW_{pp} \propto m_\pi \int_{y_{min}}^{y_{max}} dy e^\sqrt{m_\pi^2 - v_2^2} \cosh(y) \cdot \left( 1 + \frac{q - 1}{T} m_\pi \cosh(y) \right)^{-1/(q-1)},
\]

(4)

The STAR and PHENIX collaborations have published the most complete series of particle spectra and \( v_2 \) at mid-rapidity for \( p + p \) and Au+Au collisions at \( \sqrt{s_{NN}} = 200 \) GeV. The identified particle spectra and \( v_2 \) include \( \pi^\pm, K^\pm, K^0, \phi, \Lambda, \Sigma, \bar{\rho}, \bar{\Lambda}, \) and \( \Sigma^+ \) in STAR publications[1,2,24–35]. The \( \eta, \eta' \) and \( \omega \) spectra in \( p + p \) collisions, \( K^\pm \) spectra, \( \pi^0 \) and \( v_2 \) in Au+Au collisions are from PHENIX publications.[36–43] Figure 1(a) shows the invariant differential yields in \( p + p \) collisions at \( \sqrt{s} = 200 \) GeV. All of the mesons or baryons have the common \( m_\pi \) spectral shape. The solid lines represent the fit to Eq. (4) for mesons and baryons separately. The need for separation between baryons and mesons indicates that the assembly from \( p + p \) collisions cannot be treated as a thermodynamic system even with a non-equilibrium assumption. The common fit parameters and the best \( \chi^2 \) per fitting degree of freedom (nDoF) are listed in Table 1. In addition to the common fit parameters, a global normalization factor is applied for each particle spectrum. Figure 1(b) shows the invariant differential yields together with our fit results in 10–40% centrality Au+Au collisions. The results of \( v_2 \) from the simultaneous fit with spectra are displayed in Fig. 2. The fit parameters and the best \( \chi^2 \) per nDoF are tabulated in Table 2. In addition to these parameters common to all particles, a fit parameter is required as a normalization factor for TBW\textsubscript{core} of each particle spectrum. The TBW\textsubscript{pp} in the fit function is directly adopted from the fit results shown in Fig. 1(a) for each particle species without any additional free parameter. With only 7 common parameters, one global normalization factor for each \( m_\pi \) spectrum \( m \) and the parameterized \( m_\pi \) spectra in \( p + p \) collisions as shown in Fig. 1(a), the TBW model simultaneously describes 13 \( m_\pi \) spectra and 7 \( v_2 \) versus \( p_T \) distributions (more than 300 data points in total) in 10–40% Au+Au collisions with reasonably good \( \chi^2 \). The \( R_{AA} \) (ratio of the \( N_{Au} \) normalized \( p_T \) spectra in \( A + A \) collisions to the underlying \( p + p \) spectrum) from model reproduces the data very well, as shown in Fig. 3.
and high $v_2$ at the intermediate $p_t$. (4) The baryon and meson yields (TBW$_{pp}$) are grouped in $p+p$ collisions with baryon yields systematically lower than meson yields as observed in the experimental data.\cite{Adams:2005dq,Adams:2004ep} (5) The combination of the non-equilibrium tail from the core and the baryon-meson separation from the corona brings down the bulk $v_2$, produces the baryon enhancement, and causes the NCQ scaling at the intermediate $p_t$. (6) The medium quenches the jet and reduces it to a fraction ($f_{pp} = 0.36$) of its underlying binary nucleon-nucleon collisions, resulting in a finite azimuthally anisotropic emission ($\langle v_2^\parallel \rangle = 8.7\%$).\cite{He:2010iu} (7) The high-precision (statistical uncertainty of $\pm 10^{-4}$) experimental $v_2$ data points concentrate at low $p_t$, dominating the fit $\chi^2$. Additional high-quality data in a higher $p_t$ range (e.g., $v_2$ of baryons) will balance the contributions to the $\chi^2$ from components with different physical origins. The not-quite-ideal $\chi^2$/nDoF value indicates significant tensions among the different datasets and models, and warrants further detailed assessment and categorization similar to the PDF fit\cite{Adams:2004ep} in the future. For example, it is necessary to assess the uncertainty between the experimental $v_2$ value and the actual hydrodynamic flow which represents of all the particle species at low $p_t$.

This TBW model describes the number of constituent quark (NCQ) dependencies observed in the data which have also been described in recombination models.\cite{Liao:2009mr,Tang:2009fi,Adams:2004ep,Lacey:2006mr,Tang:2009fi} The NCQ dependence indicates that baryon production at intermediate $p_t$ increases faster going from $p+p$ to Au+Au collisions than meson production does. The division into a core and corona component in Eq. (3) represents a division into two samples, a core and corona where the core has a larger fraction of baryons than the corona. We describe both the core and corona with a TBW statistical model. However, whereas the recombination picture attempts to provide a microscopic description for why the bulk in Au+Au collisions has a larger fraction of baryons, the macroscopic TBW statistical model only describes the existing states as they are, and is not sensitive to the underlying mechanism responsible for producing those states. Regardless of whether the TBW$_{pp}$ component is indeed from individual $p+p$ collision or part of the hadrons from coalescence of the escaping partons, Eq. (3) does provide a unified tool to describe simultaneously a variety of measurements over a wide range in $p_t$ for future studies. These results also provide a macroscopic foundation for discussing the entropy issues associated with the system's underlying microscopic subprocesses.\cite{Adams:2005dq,Liao:2009mr,Tang:2009fi}

In summary, we have implemented the nonextensive statistics in a blast-wave model and incorporated a core-corona model to simultaneously describe (with limited number of parameters) a complete data set on identified particle spectra and elliptic flow versus transverse momenta at mid-rapidity measured at the RHIC. Such a formalism simultaneously describes several novel observations reported at the RHIC over a broad $p_t$ range. Specifically it provides an alternative physical picture, through the role of the core-corona and non-equilibrium effects in understanding the baryon-meson differences in the nuclear modification factor and the elliptic flow at intermediate $p_t$.

The authors thank Y. Bai for clarification of STAR $v_2$\cite{Adams:2004ep} and $v_2\{\text{EP}\}$ data, and A. Tang, P. Sorensen, B. Christie, J. Dunlop, J. Liao, V. Koch, Profs. F. Q. Wang, Q. Wang and G. Wilk for discussions.

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