The further estimations of the Q-balls with one-loop motivated effective potential

Yue Zhong    Hongbo Cheng*
Department of Physics, East China University of Science and Technology, Shanghai 200237, China

Abstract
The analytical estimations on the Friedberg-Lee-Sirlin typed Q-balls is performed. The two-field Q-balls are also discussed under the one-loop motivated effective potential subject to the temperature. We argue under the analytical consideration that the parameters from the potential can be regulated to lead the energy per unit charge of Q-balls to be lower to keep the model stable. If the energy density is low enough, the Q-balls can become candidates of dark matter. It is also shown rigorously that the two-field Q-balls can generate in the first-order phase transition and survive while they are affected by the expansion of the universe. The analytical evaluations show that the Q-balls with one-loop motivated effective potential can exist with the adjustment of coefficients of terms. We cancel the infinity in the energy to obtain the necessary conditions consist with those imposed in the previous work. According to the approximate expressions instead of curves versus the model parameters with a series of fixed values, the lower temperature will reduce the energy density, so there probably have been more and more stable Friedberg-Lee-Sirlin typed Q-balls to become the dark matter in the expansion of the universe.

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*E-mail address: hbcheng@ecust.edu.cn
I. Introduction

In contrast to the topological charge resulting from spontaneously symmetry breaking in the course of topological defects, the nontopological solitons involve a conserved Noether charge because of a symmetry of this kind of Lagrangian system [1-4]. The Q-balls as nontopological solitons appear in extended localized solutions of models with a certain self-interacting complex scalar field [5]. The charge of Q-balls and that their mass is smaller than the mass of a collection of scalar fields keep them stable instead of dispersing. The Q-balls are important models which have been studied in many areas of physics. The Q-balls can become boson stars as flat spacetime limits [6-8]. We studied the nontopological solitons in de Sitter and anti de Sitter spacetimes respectively to show the constrains from background on the models [9, 10]. The compact Q-balls in the complex Signum-Gordon model were also explored [11-14]. The Affleck-Dine field fragments into Q-balls which generated in the early universe and change the scenario of Affleck-Dine baryogenesis significantly [15-18]. It should be pointed out that the Q-balls may be studied to explain the common origin of the baryon asymmetry and the dark matter. The Affleck-Dine mechanism produces a scalar field condensating with baryon and the baryon asymmetry while these kinds of nontopological solitons may be considered as candidates of dark matter [16-20]. Further the scalar field configuration of the Q-balls with a step function was discussed to calculate the ratio of the Q-ball decay into the candidates for dark matter [21-23]. There are more descriptions on the detections of Q-balls [24-27]. In the cosmological context, the existence of the Q-balls was formulated and further estimate the net baryon number of the universe, its dark matter and the ratio of the baryon to cold dark matter [28]. The evolution of universe is associated with change of temperature. A new kind of first-order phase transition was induced by that the Q-balls build up quickly with absorbing charged particles from the outside in the process of expanding universe with the sufficiently low temperature [29]. There may exist the phase transitions induced the solitosynthesis that will lead the formation of large Q-balls in the process of graduate charge accretion if some primordial charge asymmetry and initial seed-like Q-balls exist [29-31]. The Q-balls subject to the thermal logarithmic potential were investigated [32-35].

It is fundamental to have a better understanding of the relations of the cosmological phase transition, dark matter Q-balls and the baryon asymmetry. It is interesting that the baryon asymmetry and dark matter in the universe may have common origin [36-45]. In this case the production of the baryon asymmetry and the dark matter may happen. In order to explain the formation of dark matter, a mechanism that some kinds of Q-balls packing the particles were put forward [46]. It is further shown that these kinds of Q-balls generated in the course of the first-order cosmological phase transition [46, 47]. The Friedberg-Lee-Sirlin type Q-balls, one of the simplest models [1, 48-50], confine the particles to become dark matters and the formulae for the dark matter properties such as charge, mass and concentration of dark matter Q-balls are obtained [46]. The effective potential of a particular one-loop motivated form was also introduced to describe the generation of the dark matter Q-balls during the cosmological phase transition that is strongly first order to
relate the dark matter Q-ball parameters and their present mass density to the properties of the first-order phase transition [46]. The phase transition temperature has also been estimated reasonably [46]. As the temperature decreased, these Q-balls as candidates of dark matter shrink, so the Q-ball size is very small [46].

It is important to explore the Friedberg-Lee-Sirlin Q-balls with an effective potential of one-loop motivated form with the help of virvial theorem in order to describe the models further. This kind of Friedberg-Lee-Sirlin Q-balls as the candidate of dark matter forming in the first-order cosmological phase transition at some temperature $T_c$ and evolving in the expanding universe has been estimated numerically [46]. It is significant to find the analytical expressions for charge, radius and energy of some special Q-balls in order to exhibit the formation and evolution of this kind of dark matter Q-balls in detail and more accurately. It is difficult to reveal the reliable and explicit relations among the model parameters and temperature by performing the burden numerical calculation repeatedly because the field equations for the two kinds of fields consisting of Q-balls are nonlinear and certainly complicated. It was found that a generalized virvial relation for Q-balls with general potential in the spacetime with arbitrary dimensionality was used to derive the analytical description for Q-balls instead of a series of curves [51]. It should be pointed out that only the analytical expressions can show clearly how the potential involving the model variables influences on the existence and the stability of the Q-balls. Here we will follow the procedure of Ref. [51] to discuss the Friedberg-Lee-Sirlin type Q-balls with particular field configuration governed by an effective potential of one-loop motivated form. We hope to understand the necessary conditions for the generation of dark matter Q-balls during the cosmological phase transition and how the model parameters and temperature influence on the Q-balls and the fate of these kinds of dark matter.

We describe the special types of dark matter Q-balls with one-loop motivated effective potential at finite temperature analytically by means of virvial theorem in this paper. We look for the virvial relation for the Friedberg-Lee-Sirlin type Q-balls in the case of zero temperature or nonzero ones respectively. With the help of corresponding ansatz, we hope to find the radius and energy of these Q-balls as functions of model parameters and temperature to show the relation between their existence and the evolution of the universe. Finally we list our results.

II. The virial relation for the Friedberg-Lee-Sirlin type Q-balls

The Lagrangian of the Friedberg-Lee-Sirlin Q-balls as one of the simplest compact objects with a global symmetry and associated conserved charge reads [46, 48-50],

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + (\partial_\mu X)(\partial^\mu X) - h^2 \phi^2 X^+ X - U(\phi)$$

(1)

with the index $\mu = 0, 1, 2, 3$ and the signature $(+, - , - , -)$. The potential is assumed to be,

$$U(\phi) = \lambda (\phi^2 - v^2)^2$$

(2)
and here the parameters $h$, $\lambda$ and $v$ belong to the model. The complex scalar field $X = X(x)$ is used to constitute the Q-ball. The real scalar field $\phi = \phi(r)$ makes up the external influence. There exist the conditions to keep the Q-ball stable, so the Lagrangian of the Q-ball has a conserved $U(1)$ symmetry under the global transformation $X(x) \rightarrow e^{i\alpha}X(x)$ with constant $\alpha$ \cite{2}. The associated conserved current density is defined as $j^\mu \equiv -i(X^+ \partial^\mu X - X \partial^\mu X^+)$ and the corresponding conserved charge can be given by $Q = \int d^3 x j^0$ \cite{2}. The total potential of the system has a global minimum at $X = 0$ and $\phi = v$ outside the ball. The ansatz for field configuration with lowest energy is chosen as \cite{51},

$$X(x) = \frac{1}{\sqrt{2}} F(r) e^{i\omega t} \quad (3)$$

Here the fields $F(r)$ and $\phi(r)$ can be taken to be spherically symmetry meaning $F(r) = \mathbf{F}(r)$ and $\phi(r) = \phi(r)$. \{r\} represent the spatial components and $r = |r|$ is the radial part. The field equations for this Q-ball read,

$$(\nabla^2 + \omega^2) F - h^2 \phi^2 = 0 \quad (4)$$

and

$$\nabla^2 \phi - h^2 F^2 \phi - 4\lambda(\phi^2 - v^2)\phi = 0 \quad (5)$$

The Lagrangian (1) can lead the total energy of the system,

$$E[F, \phi] = \int d^3 x \left[ \frac{1}{2}(\nabla F)^2 + \frac{1}{2}(\nabla \phi)^2 + \frac{1}{2}\omega^2 F^2 + V(F^2, \phi^2) \right] \quad (6)$$

where the reduced potential is,

$$V(F^2, \phi^2) = \frac{1}{2}h^2 F^2 \phi^2 + \lambda(\phi^2 - v^2)^2 \quad (7)$$

According to Ref. \cite{51}, the virial relation thought as a generalization of Derrick’s theorem for Q-balls can be expressed as,

$$\langle U(\phi) + h^2 \phi^2 X^+ X \rangle = \frac{1}{2} \frac{Q^2}{\langle F^2 \rangle} - \frac{1}{6} \langle (\nabla F)^2 \rangle - \frac{1}{6} \langle (\nabla \phi)^2 \rangle \quad (8)$$

where $\langle \cdot \cdot \cdot \rangle = \int \cdot \cdot \cdot d^3 x$. The total charge of the Q-ball is,

$$Q = \int j^0 d^3 x = \omega \langle F^2 \rangle \quad (9)$$

Because of $\langle U(\phi) + h^2 \phi^2 X^+ X \rangle \geq 0$, the absolute lower bound for Q-balls to be a preferred energy state becomes,
\[ Q^2 \geq \frac{1}{3}(\langle(\nabla F)^2 \rangle + \langle(\nabla \phi)^2 \rangle)\langle F^2 \rangle \]

The energy per unit charge is shown as,

\[ \frac{E}{Q} = \omega (1 + \frac{\langle(\nabla F)^2 \rangle + \langle(\nabla \phi)^2 \rangle}{6(U(\phi) + h^2 \phi^2 F^2) + \langle(\nabla F)^2 \rangle + \langle(\nabla \phi)^2 \rangle}) \]

The necessary conditions can be imposed in Eq. (11) to keep the Q-balls’ stability.

Now we start to study the large Friedberg-Lee-Sirlin Q-balls. First of all we assume more than one kind of field according to the Coleman issue [5]. We choose both the complex scalar field \( X \) composing the Q-balls and the real scalar field \( \phi \) as external limitation to be step functions. The step function for the complex field is equal to be a constant \( F_c \) within the model and vanishes outside the ball’s volume \( V_c \). In contrast to the matter field of the Q-balls, we construct the step function for the real one as zero inside the Q-ball and the parameter \( v \) beyond the distribution of ball matter. This kind of step functions as the field solutions suggest that the system energy distributes within the ball size. According to the profiles of the fields for Q-balls above, the system energy is,

\[ E = \frac{1}{2} \frac{Q^2}{F_c^2 V_c} + U(0)V_c \]

where \( U(0) = U(\phi)|_{\phi=0} = \lambda v^4 \). By means of extremizing the expression (12) with respect to the volume \( V_c \), the minimum energy per unit charge is,

\[ \frac{E_{\text{min}}}{Q} = \frac{\sqrt{2}}{F_c} \sqrt{U(0)} < m \]

while the expression can prevent the Q-ball from dispersing. The Friedberg-Lee-Sirlin type Q-balls within the frame of Coleman approach can survive.

In order to try to describe the large Friedberg-Lee-Sirlin type Q-balls, we choose the field profile \([51, 52]\),

\[ F(r) = \begin{cases} F_c & r < R \\ F_c e^{-\alpha(r-R)} & r \geq R \end{cases} \]

and

\[ \phi(r) = \begin{cases} 0 & r < R \\ v(1 - e^{-\alpha(r-R)}) & r \geq R \end{cases} \]

where \( \alpha \) is a variational parameter and \( R \) represents the radius of Q-ball. The choice of the same parameter \( \alpha \) is acceptable. It should be pointed out that the so-called ansatz (14) and (15) are just
assumptions keeping the Q-balls boundary conditions. We make use of the field configuration to probe this kind of Q-balls. According to the large-Q-ball ansatz (14) and (15), the energy of model reads,

\[ E[F, \phi] = \frac{1}{2} Q^2 F^2 + \frac{1}{2} \langle (\nabla F)^2 \rangle + \frac{1}{2} \langle (\nabla \phi)^2 \rangle + \langle U(\phi) + h^2 X^+ X \phi^2 \rangle \]

\[ = \frac{1}{2} \left[ \frac{4\pi}{3} F_c^2 R^3 + 8\pi F_c^2 \left( \frac{1}{2\alpha} + \frac{R}{(2\alpha)^2} + \frac{R^2}{4\alpha} \right) \right. \]

\[ + \pi v^2 \left( \frac{1}{2\alpha^2} + \frac{R}{\alpha} + R^2 \right) + \pi F_c^2 \left( \frac{1}{2\alpha^2} + \frac{R}{\alpha} + R^2 \right) \]

\[ + \frac{4\pi}{3} \lambda v^4 R^3 + \pi \lambda v^4 \left( \frac{635}{216\alpha^3} + \frac{89R}{18\alpha^2} + \frac{11R^2}{3\alpha} \right) \]

\[ + \pi h^2 v^2 F_c^2 \left( \frac{115}{432\alpha^3} + \frac{13R}{36\alpha^2} + \frac{R^2}{6\alpha} \right) \]

(16)

where the total conserved charge is,

\[ Q = \frac{4\pi}{3} \omega F_c^2 R^3 + 2\pi \omega F_c^2 \left( \frac{1}{2\alpha^3} + \frac{R}{\alpha^2} + \frac{R^2}{\alpha} \right) \]

(17)

We should further investigate the properties of Friedberg-Lee-Sirlin Q-balls such as their stability. We extremize the energy expression (16) with respect to radius \( R \) and coefficient \( \alpha \) respectively because this kind of Q-ball model does not involve the variables \( R \) and \( \alpha \) which should be confirmed.

We perform the derivation \( \frac{\partial E}{\partial R} \big|_{R=R_{cl}} = 0 \) to find the approximate critical radius of Q-balls,

\[ R_{cl} \approx \left( \frac{9Q^2}{32\pi^2 \lambda v^4 F_c^2} \right)^{\frac{1}{3}} - \left( \frac{1}{12\pi \lambda v^4} \right) (\alpha v^2 + \alpha F_c^2 + \frac{11\lambda v^4}{3\alpha} + \frac{h^2 v^2 F_c^2}{6\alpha}) \]

(18)

where we keep several dominant terms in the expression of the energy for simplicity and this approximation is acceptable for large Q-balls. The enormous amount of Q-ball charge can keep the radius positive while huge. According to the condition \( \frac{\partial E}{\partial \alpha} \big|_{\alpha=\alpha_c} = 0 \) imposing to the energy, we obtain,

\[ \alpha_c^2 = \frac{22\lambda v^4 + h^2 v^2 F_c^2}{6(v^2 + F_c^2)} \]

(19)

It is obvious that \( \alpha_c^2 \) denoted in Eq. (19) is positive, which keeps \( \alpha_c \) real, or the model consisting of fields \( X(x) \) and \( \phi(x) \) can not constitute the Q-balls according to the ansatz (14) and (15).

Combining Eq. (18) and Eq. (19), we find the minimum energy of large Friedberg-Lee-Sirlin Q-ball per unit charge,

\[ E_{\text{min}} = E \big|_{R=R_{cl}, \alpha=\alpha_c} \]

\[ \approx \frac{\sqrt{2\lambda v^4}}{F_c} Q \left( 1 + \frac{\xi v^2}{\frac{4}{3}} \right) \]

(20)

where
The $\xi_c$-term in Eq. (20) is due to the dominant terms. This term is tiny compared to the first one. In the case of huge Q-ball, its minimum energy per unit charge is,

$$\lim_{Q \to \infty} \frac{E[F_c, v]|_{R= R_{cl}, \alpha= \alpha_c}}{Q} = \frac{\sqrt{2} \lambda \xi_c v^2}{F_c}$$

(22)

The explicit expression above shows that the minimum energy over total charge for this kind of large Q-balls is finite even the number of particles is extremely large. We can choose the values of $\lambda$, $v$ and $F_c$ to keep the particles of the system to distribute within a definite region which is lower than the necessary kinetic energy of a free particle in the system, and this kind of Q-balls keep stable. The Eq. (14) and Eq. (15) show that the field $F(r)$ decreases while the field $\phi(r)$ increases as $r > R$. When $r$ as Q-ball size is large enough, the fields $F(r)$ and $\phi(r)$ approach to the zero and $v$ respectively, leading the total energy (6) with the reduced potential (7) to vanish and satisfying the boundary conditions. The numerical solutions to field equations (4) and (5) satisfy the requirements of $F(r)$ and $\phi(r)$ for large Friedberg-Lee-Sirlin Q-ball in the Figure 1.

The minimum energy density certainly depends on the model parameters such as $h$, $\lambda$ and $v$. The influence from the variable $\lambda$ on the relation between the minimum energy per unit charge of the Friedberg-Lee-Sirlin Q-ball and the charge amount $Q$ is depicted in the Figure 2. The variable $\lambda$ with larger magnitude will increase the energy. The corrections from the other parameters $h$ and $v$ are similar to the results above. The solutions corresponding to the ansatz (14) and (15) can compose the Q-balls.

Now we pay attention to the small Friedberg-Lee-Sirlin type Q-balls. According to Ref. [51, 52], we also bring about a Gaussian ansatz like,

$$F(r) = F_c e^{-\frac{r^2}{R^2}}$$

(23)

and

$$\phi(r) = v(1 - e^{-\frac{r^2}{R^2}})$$

(24)

to give an example. We explore the special Q-balls whose field profiles obey the ansatz (23) and (24). We substitute the ansatz (23) and (24) into the expression (6) to find the total energy belonging to the small Q-balls as follows,

$$E = \iiint d^3 x \left[ \frac{1}{2} \omega^2 F^2 + \frac{1}{2} (\nabla F)^2 + \frac{1}{2} (\nabla \phi)^2 + U(\phi) + \frac{1}{2} h^2 \phi^2 F^2 \right]$$

$$= aR^{-3} + bR + cR^3$$

(25)

where the charge is,
replacing the frequency $\omega$. In the process similar to that of Ref. [51], we extremize the expression of the energy with respect to $R$ like $\frac{\partial E}{\partial R}|_{R=R_{cs}} = 0$ to set up the equation for the critical radius $R_{cs}$ as follows,

$$3cR_{cs}^6 + bR_{cs}^4 - 3a = 0$$

(27)

where

$$a = \frac{2\frac{3}{2}Q^2}{\pi^\frac{3}{2}F_c^2}$$

(28)

$$b = \frac{3}{2}(\frac{\pi}{2})^\frac{3}{2}(F_c^2 + v^2)$$

(29)

$$c = (\frac{1}{8} - 4 \times 3^{-\frac{3}{2}} + 2^{\frac{3}{2}})\pi^\frac{3}{2}\lambda v^4 + (2^{-\frac{3}{2}} - 3^{-\frac{3}{2}} + \frac{1}{16})\pi^\frac{5}{2}h^2 v^2 F_c^2$$

(30)

The acceptable approximate solution for special critical radius becomes,

$$R_{cs} = [1 - \frac{b}{2(9cR_0^6 + 2b)}]R_0$$

(31)

with

$$R_0 = \frac{2^{\frac{7}{2}}}{\pi^\frac{3}{2}F_c^2}Q^\frac{1}{2}\left[\left(\frac{1}{8} - \frac{4}{3^\frac{3}{2}}\right)\pi^\frac{3}{2}\lambda v^4 + \left(\frac{1}{2^\frac{3}{2}} - \frac{1}{3^\frac{3}{2}} + \frac{1}{16}\right)\pi^\frac{5}{2}h^2 v^2 F_c^2\right]^{\frac{1}{7}}$$

(32)

The minimum energy in the case of small Q-balls can be shown in terms of the critical radius as,

$$E_{min} = E[F_c]|_{R=R_{cs}}$$

$$\approx (\frac{2^\frac{7}{2}}{\pi^\frac{3}{2}F_c^2})Q^{\frac{1}{2}}[2c + b(\frac{c}{\hat{a}})^\frac{1}{2}Q^{-\frac{2}{3}} - \frac{b^2}{18}(\hat{a}^2 c)^{-\frac{1}{3}}Q^{-\frac{4}{3}}]Q$$

(33)

where $\hat{a} = \frac{\sqrt{2}}{\pi^\frac{3}{2}F_c^2}$. The amount of charge for smaller balls can also extremely large. In the limit of huge quantity of charge, the minimum energy per unit charge for small Q-balls becomes,

$$\lim_{Q \to \infty} \frac{E[F_c]|_{R=R_{cs}}}{Q} = 2(\frac{2^\frac{7}{2}c}{\pi^\frac{3}{2}F_c^2})Q^{\frac{1}{2}}$$

(34)

It is clear that the energy density will not be divergent. For the Gauss ansatz (23) and (24) [51, 52], the small Friedberg-Lee-Sirlin typed Q-ball constitutions $F(r)$ and $\phi(r)$ are decreasing and increasing functions respectively at first and approach to the constants such as zero and Q-ball variable $v$. In the case of small balls, the profiles of the solutions to field equations (4) and (5) shown numerically in the Figure 3 meet the Q-balls requirements including the necessary boundary
conditions. According to the Figure 4, the shapes of the curves for small balls energy density resemble those of large Friedberg-Lee-Sirlin typed Q-balls. The model variable \( \lambda \) with larger value can also make the total energy greater. This kind of small Q-balls can be composed of the solutions subject to the ansatz (23) and (24). The energy per unit charge can be small enough if the amount of model variables is selected reasonably. As an example, these special Q-balls with sufficiently small energy density can gather the particles instead of decentralizing and become the dark matter.

III. The virial relation for the Friedberg-Lee-Sirlin typed Q-balls with one-loop motivated effective potential

We start to investigate the Friedberg-Lee-Sirlin Q-balls subject to one-loop motivated effective potential involving temperature from Ref. [46] analytically. Certainly we should choose the potential to be [46],

\[
U(\phi, T) = \alpha'(T^2 - T_c^2)\phi^2 - \gamma T \phi^3 + \lambda \phi^4
\]  

(35)

where \( T_c^2, \alpha' \) and \( \gamma \) are model parameters. With the relation \( \alpha' T_c^2 = 2 \lambda v^2 \), the potential (35) can be thought as the generalization of potential (2) for the Friedberg-Lee-Sirlin type Q-balls without thermal corrections like [46],

\[
U(\phi, T) = U(\phi) + (-\gamma T \phi^3 + \alpha' T_c^2 \phi^2 - \lambda v^4)
\]  

(36)

We continue our research on this kind of Q-balls in virtue of technique in Ref. [51]. The total energy from the Lagrangian of the system is chosen as,

\[
E[F, \phi, T] = \frac{1}{2} \omega Q + \frac{1}{2} \langle (\nabla \phi(r))^2 \rangle + \frac{1}{2} \langle (\nabla F(r))^2 \rangle + \langle U(\phi, T) \rangle
\]  

(37)

We also research on the special Friedberg-Lee-Sirlin models with ansatz (14), (15), (23) and (24) in the cases of large size and small ones respectively. At first we substitute the large-Q-ball ansatz (14) and (15) [51, 52] in Eq. (37) to find that the first three terms are the same as the parts of Eq. (16). We discuss the potential terms at finite temperature,

\[
\langle U(\phi, T) \rangle = \int U(\phi, T) d^3 x
\]

\[
= 4\pi v^2 [\alpha'(T^2 - T_c^2) - \gamma T v + \lambda v^2] \int_R^\infty r^2 dr
\]

\[
+ 8\pi v^2 \alpha'(T^2 - T_c^2) \sum_{k=0}^{2} \frac{1}{k!} \left( \frac{1}{2^{3-k}} - 2 \right) \frac{R^k}{\alpha^{3-k}}
\]

\[
+ 8\pi v^3 \gamma T \sum_{k=0}^{2} \frac{1}{k!} (3 - 3 \cdot 2^{3-k} + \frac{1}{3^{3-k}}) \frac{R^k}{\alpha^{3-k}}
\]

\[
+ 8\pi v^4 \lambda \sum_{k=0}^{2} \frac{1}{k!} (-4 + \frac{6}{2^{3-k}} - \frac{4}{3^{3-k}} + \frac{1}{4^{3-k}}) \frac{R^k}{\alpha^{3-k}}
\]  

(38)
It is manifest that the integral form $\int_0^\infty r^2 dr$ is equal to the infinity, which leads the total energy to be divergent. In order to eliminate the infinity, we choose the coefficient of the integral term to be zero,

$$\lambda v^2 - \gamma T v + \alpha'(T^2 - T^2_c) = 0$$

(39)

like that imposed in Ref. [46]. We have to keep the following condition,

$$T < T_c$$

(40)

where the critical temperature is defined as [46],

$$T^2_c = \frac{4\alpha' \lambda}{4\alpha'^2 - \gamma^2 T^2_c}$$

(41)

to keep the parameter $v$ to be real. The condition (40) appeared in Ref. [46] when the authors discussed the effective potential. Here we should pointed out that the necessary condition prevents the total energy of the system from infinity and leads the variable $v$ to be real. Having imposed the constraint (39) on the temperature-corrected potential (38), we obtain the energy of large Friedberg-Lee-Sirlin type Q-balls containing the temperature as follows,

$$E[\phi, F] = \frac{1}{2} \omega Q + \frac{1}{2}((\nabla \phi(r))^2) + \frac{1}{2}((\nabla F(r))^2) + (U(\phi) + \lambda h^2 \phi^2 X^+ X)$$

$$= \frac{3Q^2}{8\pi F_c^2} \frac{1}{1 + \frac{1}{2} \frac{1}{\alpha R}} R^{-3}$$

$$+ 4\pi (\alpha^2 v^2 + \alpha^2 F_c^2 + h^2 v^2 F_c^2) (\frac{1}{8\alpha^2} + \frac{R}{4\alpha^2} + \frac{R^2}{4\alpha})$$

$$+ 8\pi v^2 \alpha (T^2 - T^2_c) \sum_{k=0}^2 \frac{1}{k!} \left( \frac{1}{2^2 - k} - 2 \right) \frac{R^k}{\alpha^{3-k}}$$

$$+ 8\pi v^2 \gamma T \sum_{k=0}^2 \frac{1}{k!} (1 - \frac{1}{2^2 - k} + \frac{1}{3^2 - k}) \frac{R^k}{\alpha^{3-k}}$$

$$+ 8\pi v^4 \lambda \sum_{k=0}^2 \frac{1}{k!} (-2 + \frac{5}{2^3 - k} - \frac{4}{3^3 - k} + \frac{1}{4^3 - k}) \frac{R^k}{\alpha^{3-k}}$$

$$+ 8\pi h^2 \alpha F_c^2 \left( \frac{15}{64 \alpha^3} + \frac{7}{16 \alpha^2} + \frac{1}{6} \frac{R^2}{\alpha} \right)$$

(42)

According to the similar procedure above [51] and the reduced energy, the approximate critical radius of Q-ball associated with the temperature is denoted as,

$$R_{clT} = \left( \frac{9Q^2}{16\pi v^2} \right)^\frac{1}{3} \left[ \pi \alpha' (v^2 + F_c^2) + \frac{4\pi v^3}{3\alpha} \gamma T - \frac{7\pi v^4}{3\alpha} \lambda + \frac{\pi h^2 v^2 F_c^2}{6\alpha} \right]^{-\frac{1}{3}}$$

(43)

The stable Q-balls expand with decreasing temperature. With the imposing condition $\frac{\delta E}{\partial \alpha}|_{\alpha=\alpha_{cl}} = 0$, the result is listed as,
\[ \alpha_c T = \frac{v^2}{6(v^2 + F_c^2)}(8v\gamma T - 14v^2\lambda + h^2 F_c^2) \] (44)

and the minimum energy density of large Friedberg-Lee-Sirlin typed Q-ball at finite temperature is,

\[ \frac{E_{min}}{Q} = \frac{E|_{R = R_{cT}, \alpha = \alpha_{cT}}}{Q} \]
\[ = \left( \frac{2}{3} \right)^{\frac{3}{8}} + \left( \frac{3}{2} \right)^{\frac{3}{8}} \right) \left[ \frac{9\pi(v^2 + F_c^2)^3}{8F_c^4} \right]^{\frac{1}{2}} \frac{\alpha_{cT}^4}{\alpha} Q^{-\frac{1}{2}} \] (45)

It should be pointed out that the energy density has something to do with the charge \( Q \) and temperature. For the huge balls with large amount of charge \( Q \), their energy densities will not be infinite, but to be smaller. It is even more possible for the huge Q-balls to become the dark matter. According to the Eq. (44) and Eq. (45), the smaller parameter \( \alpha_c T \) also leads the minimum energy density to be smaller. The Figure 5 indicates that the Q-ball energy will become smaller when the temperature drops. The expansion of the universe impels more of these kinds of models to stand for the dark matter because the decreasing temperature diminishes the parameter \( \alpha_c T \) to make the energy densities to be sufficiently low.

Following the same procedure of Ref. [51] with the help of ansatzs (23) and (24) [51, 52], we have the critical radius of small Friedberg-Lee-Sirlin typed Q-ball under the one-loop motivated effective potential as follows,

\[ R_{cT} = \left[ 1 - \frac{b}{2(9c_T R_{0T}^2 + 2b)} \right] R_{0T} \] (46)

where

\[ R_{0T} = \frac{2^{\frac{1}{3}} Q^{\frac{1}{6}}}{\pi^{\frac{1}{2}} v^2 F_c^\frac{3}{2}} \left[ \left( \frac{2 - \sqrt{2}}{2} + \frac{\sqrt{3}}{9} \right) \gamma v T - \left( \frac{15 - 10\sqrt{2}}{8} + \frac{4\sqrt{3}}{9} \right) \lambda v^2 + \left( \frac{2\sqrt{2} + 1}{16} - \frac{\sqrt{3}}{9} \right) h^2 F_c^2 \right]^{-\frac{1}{2}} \] (47)

Further the minimum energy per unit charge of this kind of Q-balls with small size is,

\[ E_{T min} = E[F_c]|_{R = R_{cT}} \]
\[ \approx \left( \frac{2^{2\gamma}}{\pi^{\frac{1}{2}} c_T F_c^\frac{3}{2}} \right)^{\frac{2}{3}} 2 c_T + b(c_T a)^{\frac{1}{3}} Q^{-\frac{2}{3}} - \frac{b^2}{18} (a^2 c_T)^{-\frac{1}{2}} Q^{-\frac{4}{3}} |Q \] (48)

where

\[ c_T = \left[ \left( \frac{2 - \sqrt{2}}{2} + \frac{\sqrt{3}}{9} \right) \gamma v T - \left( \frac{15 - 10\sqrt{2}}{8} + \frac{4\sqrt{3}}{9} \right) \lambda v^2 + \left( \frac{2\sqrt{2} + 1}{16} - \frac{\sqrt{3}}{9} \right) h^2 F_c^2 \right]^{\frac{1}{2}} v^2 \] (49)

This result (48) is just the replacement of \( c \) by \( c_T \) in Eq. (33). Also the minimum energy per unit charge for small one-loop-potential-controlled Q-ball with larger quantity of charge is,
The energy density relating to the temperature subject to $c_T$ is finite. According to Eq. (49), the parameter $c_T$ will become smaller when the temperature is lower. The small Q-ball energy densities as functions of charge $Q$ for the temperature are described in the Figure 6. The lower temperature also leads the energy densities of the small Friedberg-Lee-Sirlin typed Q-balls with one-loop motivated potential to be smaller. As the universe expands with the decreasing temperature, the energy densities of more and more Friedberg-Lee-Sirlin typed Q-balls with one-loop motivated effective potential will be small enough. Certainly these small Q-balls can survive and further act as candidates of dark matter.

IV. Discussion and Conclusion

The Q-balls with two kinds of scalar fields satisfying the special ansatz produced in the first-order phase transition are considered analytically. The system of two kinds of fields composing the Q-balls can become the candidate of dark matter while can also be used to explain the baryon asymmetry [46]. This kind of Q-balls are powerful [46] and their existence, stability and evolution need to be analyzed in detail. It should be emphasized that the nonlinear field equations of the two kinds of scalar fields describing the Q-balls are complicated and there are diverse numerical estimations corresponding to the model constructions [46]. In the cases of large special balls and relatively small ones we prove strictly to confirm that the two-field Q-balls will be stable instead of dispersing if their energy density is smaller than the kinetic energy per unit charge. Further the Q-balls can become the dark matter with the sufficiently low energy density. We study the Friedberg-Lee-Sirlin Q-balls with the appointed ansatz under the one-loop motivated effective potential with the burden of derivation. We cancel the divergence in the energy of the Friedberg-Lee-Sirlin typed Q-ball with one-loop motivated effective potential and keep the scalar field $\phi$ to be real to confirm the critical temperature $T_c$ and show the existence of the Q-balls when the temperature $T < T_c$ instead of imposing the conditions based on the numerical calculation like Ref. [46]. Our explicit expressions for the energy also indicate that the temperature-dependent Q-balls can act as dark matter when the energy per unit charge is low enough. The evolution of the Q-balls in the cosmological background is exhibited without numerical estimation. The expansion of the universe helps more and more this kind of Friedberg-Lee-Sirlin type Q-balls under the one-loop motivated effective potential to become the dark matter because the decreasing temperature leads the energy per unit charge of the models to be lower. Here the large or small Friedberg-Lee-Sirlin typed Q-balls that we study analytically are special because their fields configurations are particular, but we exhibit their properties mentioned above and they are consistent with those of Ref. [46] and of universal significance. The analytical discussion is reliable and shows the model properties clearly.
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Figure 1: The profile function $F(r)$ of large Friedberg-Lee-Sirlin typed Q-balls with $Q = 10^5$, $F_c = 2$, $v = 2$ and $h = 1$ shown as solid, dotted curves for coefficient $\lambda = 0.05, 0.5$ respectively and the profile function $\phi(r)$ of the same Q-balls as dashed, dot-dashed curves for coefficient $\lambda = 0.05, 0.5$ respectively.
Figure 2: The solid, dot, dashed curves of the minimum energy per unit charge of large Friedberg-Lee-Sirlin typed Q-balls as functions of charge $Q$ for coefficient $\lambda = 2, 4, 6$ respectively.
Figure 3: The profile function $F(r)$ of small Friedberg-Lee-Sirlin typed Q-balls with $Q = 10^3$, $F_c = 2$, $v = 2$ and $h = 1$ shown as solid, dotted curves for coefficient $\lambda = 0.05$, 0.5 respectively and the profile function $\phi(r)$ of the same Q-balls as dashed, dot-dashed curves for coefficient $\lambda = 0.05$, 0.5 respectively.
Figure 4: The solid, dot, dashed curves of the minimum energy per unit charge of small Friedberg-Lee-Sirlin typed Q-balls as functions of charge $Q$ for coefficient $\lambda = 2, 4, 6$ respectively.
Figure 5: The solid, dot, dashed curves of the minimum energy per unit charge of large Friedberg-Lee-Sirlin typed Q-balls with one-loop motivated effective potential as functions of charge $Q$ for temperature $T = 2, 4, 6$ respectively.
Figure 6: The solid, dot, dashed curves of the minimum energy per unit charge of small Friedberg-Lee-Sirlin typed Q-balls with one-loop motivated effective potential as functions of charge $Q$ for temperature $T = 2, 4, 6$ respectively.