Total phase shift of terahertz wave in a rectangular waveguide with zero-refractive index metamaterial

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Abstract

In many applications, appropriate spatial phase shifts in propagation of a THz wave are desired. For this purpose, a rectangular waveguide with two-dimensional photonic crystal zero-refractive index metamaterial is studied. In this structure, additional phase shift is computed in a numerical method in comparison with the same waveguide without zero-refractive index metamaterial. Modelling the characteristics of this waveguide, relations are presented which are shown to be compatible with numerical results. Then getting in inverse direction, a procedure is introduced in which, equivalent photonic crystals can be designed in terms of an arbitrary given phase shift with estimated errors of less than 1 degree. We also have calculated the sensitivity of additional phase difference with respect to the refractive index of rods, which showed relatively high dependence.

1. Introduction

Nowadays new techniques are emerged promising more quality to all currently used photonic devices. Like Photonic Integrated Circuits (PICs) by using which, faster and more efficient solutions are introduced. The benefits to PICs can be mentioned as package density, interconnections, improved functionality and of course cost reduction [1–5]. It is so important to have optical dense components in the applications such as optical communication [2, 4, 6], imaging [2, 3, 7], sensing [2, 4], optical signal processing [1, 2, 6, 7], optical computers [1, 6], etc. It is worth mentioning that making these devices is compatible with current planar fabrication technologies [8]. Of course, there are some issues alongside the benefits, such as fundamental diffraction rules [9].

Electromagnetic spatial phase shifters are sub-devices of PICs. The phase shifts are quite sensitive to the structural accuracy of the PICs. One of the useful techniques to realize the phase shifters is using Photonic Crystal (PC) metamaterials. It is represented that PC metamaterials can tune the photonic dispersion so that one can build new optical devices with new functionalities [3, 4, 8, 10–17]. In fact, PC metamaterials could be built in a way which exhibit Dirac-cone photonic dispersion in the center of Brillouin-Zone in an arbitrary frequency using the concept of accidental degeneracy [15]. In modern photonics they are called Zero-Refractive Index Metamaterials (ZRIMs). For the first time, in 2015 two-dimensional (2D) photonic crystal ZRIMs were built on-chip with efforts of Harvard researchers with current silicon photonic fabrication technologies [4, 17]. Such 2D-PC metamaterials with C$_3$ symmetry behave like an effective medium with $\varepsilon_{\text{eff}} \cong 0$ and $\mu_{\text{eff}} \cong 0$ for TEM polarized Electromagnetic Waves (EMWs) [18, 19]. In theory, it is constantly shown that these photonic systems allow EMWs to propagate phase freely through the system [15, 18, 20–30]. Also these waveguides have small footprints and high capabilities of energy transmission [14] and could be used in same level to overcome diffraction issues as non-diffractive photonic devices [16]. In another work [31], It has been reported that the spatial phase can be controlled using Gradient-Metasurfaces (GMSs), which also converts the incident EMWs to propagating Unidirectional Edge States (UESs). One can tune the refractive-index utilizing yttrium-iron garnet (YIG) rods [32, 33] which determines the phase difference. This can be performed by magnetically-thermally controlling the permeability of YIG in a cryogenic system.
In this article, we look at ZRIMs more closely, studying the EMW phase shifting. At first, we numerically compute this phase shifting more accurately for 2D-PC ZRIMs with square lattices having different numbers of columns of rods and sufficient number of rows. In the next section, we illustrate a procedure for designing 2D-PC ZRIMs with square lattices in order to provide arbitrary spatial phase shifts. Then, analyzing the data, we present a constitutive relation for the phase shift resulted from PC ZRIMs. Finally, we have computed the sensitivity of the additional phase difference with respect to the material refractive index of rods for estimating possible tolerances in the experimental results.

2. Numerical simulations

Considering the concept of phase freely propagation of the appropriate EMW through ZRIM we are going to compute and analyze the accurate phase shift in this system by comparing the EMW shape with and without ZRIM structure in a rectangular waveguide. A two-port rectangular waveguide system with appropriate dimensions 6000 \( \mu m \times 1800 \mu m \) (according to the wave propagation rules) is considered in which we have a 2D-PC ZRIM square lattice, shown in the figure 1. As presented in [18], we need to create a Dirac-point in the center of Brillouin-zone \((\kappa = \Gamma)'\) in PC. in order to set the PC ZRIM structural parameters we need to obtain the lattice constant and radius of the rods. We have considered a lossless 2D-PC at the operating frequency of 1.082 THz, and thus we have the lattice constant as \( a = 0.54125 \times 150 \mu m \), and the radius of the rods as \( r = 0.2a = 30 \mu m \). The resulted photonic dispersion in 2D-PC ZRIM is shown figure 2.

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It is well-known that the term ‘photonic crystal’ is ideally refered to an infinite periodic structure, but even for a single column structure, like \( 1 \times 12 \), the purpose of phase shifting is also achieved, i.e. ZRIM works. For various lattice structures with 1...16 columns and 12 rows, we numerically computed spatial phase shifts in systems with and without the mentioned phase shifter. A Tehrahertz (THz) plane wave with frequency of 1.082 THz is applied from the left side (input port) and illuminated on ZRIM structure to validate the additional phase shift.
Obtaining the accurate phase shift, we compute the spatial difference between a point with location of several wavelengths (input wavelength) before the ZRIM and its corresponding same phase point with location of several wavelengths after ZRIM (Figure 1). This spatial difference does not necessarily correspond to zero phase difference in a waveguide without ZRIM structure. Hence we can compute the corresponding additional phase shift using the following relation:

$$\Delta \theta = \left[ \text{Floor}\left( \frac{\Delta x}{\lambda} \right) - \frac{(\Delta x)}{\lambda} \right] \times 360^\circ,$$

where $$\Delta x$$ is the obtained spatial difference, and the Floor function is noted for integer truncation operation. Computed values are given in Table 1.

### 3. Design

Here, we want to compensate an arbitrary phase shift using a ZRIM. This phase compensation is based on phase freely propagation in 2D-PC ZRIM, so we represent the following relation to predict this phase:

$$a \times N = \frac{\lambda \times (\Delta \phi + \Delta \phi_2)}{2\pi} + M \times \lambda,$$

where $$N$$, $$\Delta \phi$$, and $$M$$ are, number of columns in PC lattice, arbitrary phase shift in radians, and an integer, respectively. $$\Delta \phi_2$$ is noted as a possible small phase error for the assumed ZRIM and corresponding integer value of $$M$$. Using this relation, for arbitrary given values of $$\Delta \phi$$ and $$\Delta \phi_2$$, Therefore one can obtain corresponding couples of integer values $$N$$ and $$M$$. To do this, we swept $$N$$ from 1 to 100, constraining the maximum value of $$\Delta \phi_2$$ with the desired accepted phase error ($$\pm1^\circ$$ in Table 2), and then we found the values of $$M$$ and $$\Delta \phi_2$$. Results for some of the spatial phase angles close to Table 1 data are shown in Table 2.

### 4. Analysis

#### 4.1. Presenting an analytic relation for phase shift

We can predict the exact phase shift resulting from a 2D-PC ZRIM with an arbitrary number of columns of rods, substituting $$\Delta x$$ in (1) with $$a \times N$$, reaching the following constitutive relation:

$$\Delta \theta = \left[ \text{Floor}\left( \frac{a \times N}{\lambda} \right) - \frac{(a \times N)}{\lambda} \right] \times 360^\circ,$$

As shown in Figure 3, we simultaneously have plotted computed phase shifts by full wave simulations (Table 1) and the phase shifts resulting from (3), versus the number of columns $$N$$. It is apparent that these two sets of data are highly in agreement.

Extracting the phase shift values from (3) along extrapolating the data for higher numbers of columns, we have illustrated the results shown in Figure 4. As illustrated in the figure, there seems to be a quasi-periodic

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### Table 1. Numerically computed spatial phase shifts for lattices with 1 to 16 numbers of columns and 12 rows.

| ZRIM structure | Spatial phase shift |
|---------------|---------------------|
| 1 × 12        | 192.3262°           |
| 2 × 12        | 27.6367°            |
| 3 × 12        | 222.9465°           |
| 4 × 12        | 58.2399°            |
| 5 × 12        | 253.5676°           |
| 6 × 12        | 88.9146°            |
| 7 × 12        | 284.2086°           |
| 8 × 12        | 119.5654°           |
| 9 × 12        | 314.8519°           |
| 10 × 12       | 150.3949°           |
| 11 × 12       | 345.4660°           |
| 12 × 12       | 180.8104°           |
| 13 × 12       | 16.07653°           |
| 14 × 12       | 211.4195°           |
| 15 × 12       | 46.72901°           |
| 16 × 12       | 242.0372°           |
Table 2. For shown phase angles (column 1), corresponding number of PC columns are given in column 2. Number of equally confident waves in without ZRIM system (compared to the ZRIM area in ZRIM system), and spatial phase shift errors computed in column 3 and 4, respectively. In column 5, the differences between corresponding spatial phase shifts from table 1 and arbitrary spatial phase shifts (column 1) in this table, are shown.

| Arbitrary spatial phase shift (degree) | M (number of equally confident waves in without ZRIM system) | Δφ-F-Phase error (degree) | Δθ–Δφ (degree) |
|--------------------------------------|-----------------------------------------------------------|--------------------------|-----------------|
| 11°                                  | 13                                                         | 7                        | 0.88°           |
| 29°                                  | 2                                                          | 1                        | 0.52°           |
| 41°                                  | 15                                                         | 8                        | 0.40°           |
| 59°                                  | 4                                                          | 2                        | 0.04°           |
| 88°                                  | 6                                                          | 3                        | 0.56°           |
| 118°                                 | 8                                                          | 4                        | 0.08°           |
| 147°                                 | 10                                                         | 5                        | 0.60°           |
| 177°                                 | 12                                                         | 6                        | 0.12°           |
| 194°                                 | 1                                                          | 0                        | 0.76°           |
| 206°                                 | 14                                                         | 7                        | 0.64°           |
| 224°                                 | 3                                                          | 1                        | 0.28°           |
| 236°                                 | 16                                                         | 8                        | 0.16°           |
| 253°                                 | 5                                                          | 2                        | 0.80°           |
| 283°                                 | 7                                                          | 3                        | 0.32°           |
| 312°                                 | 9                                                          | 4                        | 0.84°           |
| 342°                                 | 11                                                         | 5                        | 0.36°           |

Figure 3. Comparison of Δθ versus the number of columns of rods N for a 2D-PC ZRIM structure resulted from full wave simulations data (blue) and analytic calculations of phase shift according to (3) (red).

Figure 4. Quasi periodic behavior of phase shifts resulting from analytic calculations according to (3), Δθ, versus number of columns of rods N for 1 to 200 columns. Three upper data tips show quasi-period of 61. This comportment is repeated for three lower data tips.
function having a period of 61 for number of columns. A similar behaviour is reported in a metamaterial based
2D-PC Negative-Index Metamaterials (NIMs) and Positive-Index Metamaterials (PIMs) in [20] specification is
due to $61 \times a \cong p\lambda$, where $p$ is an integer. Reaching to better precisions, one may have to go to higher numbers
of columns in the following periods.

In this work, we assumed that the imaginary part of the refractive index ($\textit{i}k$, the absorption loss)
is negligible, otherwise this loss may affect the spatial phase especially in the case of big number of columns and hence there
will be no periodic behaviour in the transmission function. The study of loss and its effects will be the future
work of the authors.

### 4.2. Testing the designed 2D-PC ZRIM structure

We have selected one of the 2D-PC ZRIM structures as a sample to illustrate the output predicted by the design
section. We have put another simple waveguide (without ZRIM) alongside the main waveguide (with ZRIM)
and applied both with electromagnetic plane waves with the same frequency (1.082 THz) and different input
phases corresponding to the given phase difference. According to the results of the previous section, we chose a
PC lattice with 12 columns with predicted $177\degree$ phase difference. Two output waves are highly phase matched
upon exiting ZRIM structure, as shown in figure 5.

It is obvious that both outputs have the same amplitude which is resulting from the fact that the waveguide is
not lossy and 2D-PC ZRIM is working in the ideal situation. By analogy to the Voltage Controlled Oscillator
(VCO) systems this waveguide has as a frequency controlling section which yields to phase tuning at the output.

### 5. Phase difference sensitivity

We chose to obtain spatial phase difference sensitivity with respect to the material refractive index of rods
($\Delta \theta / \Delta n$) as a sample to investigate the fabrication tollerance of silicon quality.

We simulated the proposed waveguide one more time with $n_2 = n_1 - 0.005n_1$ in $12 \times 12$ lattice, where $n_1$
and $n_2$ are primitive refractive index of standard silicon crystal, and perturbed refractive index of rods,
respectively.

Shown in figure 6, we have plotted electric field distribution for both cases simultaneously in order to obtain
comparison objectives. In this figure two upper tips are represented as two same-phase points in the case of
perturbed refractive index (black solid curve). This spatial difference corresponds to induced spatial phase difference of $\pm 207.52^\circ$ and finally for the sensitivity we have: $S = \frac{\Delta \theta_1 - \Delta \theta_2}{n_2 - n_1} \approx -1556.8$ degrees per unit value of refractive index, with $\Delta \theta_1 = 207.52^\circ$, $\Delta \theta_2 = 180.81^\circ$ (table 1), and $|n_2 - n_1| = 0.005n_1 \approx 0.0177$.

6. Conclusion

A waveguide system is studied in which its capability of phase tuning is thoroughly investigated. We have included a 2D-PC ZRIM structure which helped us acting as such. Its structural properties are set to make a Dirac-point at 1.082 THz. In the first step we numerically computed THz EMW’s spatial phase shifts caused by 2D-PC ZRIMs in a waveguide in comparison with same value in the situation where there is no ZRIM in the waveguide.

We have presented a creative approach for calculating the phase shift induced by an intermediate structure in the waveguide. The maximum error of the electrical field numerical data extracted from simulations is directly related to ratio of the mesh size in FEM method with respect to the wavelength of input wave, which here is restricted to $0.5 \mu m/277.264 \mu m$ corresponding to 0.65 degrees. Afterwards, we designed these PCs to provide any given arbitrary spatial phase shift. Constitutive relations were presented to analyze such structures. The data brought in table 2 show reasonable phase errors for designing the appropriate waveguides, which are less than 1 degree. It is demonstrated that these predictions are almost identical to numerical simulation results which shows a very well agreement. Based on the naturally quasi-periodic behavior of phase shift in 2D-PC ZRIMs, one may have to consider higher numbers of columns in the following periods reaching to more precise phase shifts. Although, there will be some problems such as scattering, PC size, energy loss due to material absorption, etc We also have calculated the sensitivity of additional phase difference with respect to the material refractive index of rods in the case of 0.5% perturbation, which showed relatively high dependence of $-1556.8$ degrees per unit value of refractive index.

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**Figure 6.** Z components of the electric field distribution in the proposed waveguide in the case of $n = \sqrt{1.25}$, green solid curve-just like green dashed curve in figure 5(a), and with perturbed refractive index by %0.5, black solid curve. Two upper data tips show two same phase points.
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