Arbitrary Beam Synthesis of Different Hybrid Beamforming Systems

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Abstract—For future mmWave mobile communication systems the use of analog/hybrid beamforming is envisioned be a key aspect. The synthesis of beams is a key technology of enable the best possible operation during beamsearch, data transmission and MU MIMO operation. The developed method for synthesizing beams is based on previous work in radar technology considering only phase array antennas. With this technique it is possible to generate a desired beam of any shape with the constraints of the desired target transceiver antenna frontend. It is not constraint to a certain antenna array geometry, but can handle 1d, 2d and even 3d antenna array geometries like cylindric arrays. The numerical examples show that the method can synthesize beams by considering a user defined tradeoff between gain, transition width and passband ripples.

Index Terms—millimeter Wave, hybrid beamforming, beam synthesis.

I. INTRODUCTION

To satisfy the ever increasing data rate demand, the use of the available bandwidth in the mmWave frequency range is considered to be an essential part of the next generation mobile broadband standard [1]. To attain a similar link budget, the effective antenna aperture of an mmWave system must be comparable to current systems operating at a lower carrier frequency. Since the antenna gain, and thus the directivity increases with the aperture, an antenna array is the only solution to achieve a high effective aperture, while maintaining a 360° coverage.

The antenna array combined with the large bandwidth is a big challenge for the hardware implementation, because the power consumption limits the design space. Analog or hybrid beamforming are considered to be a possible solutions to reduce the power consumption. These solutions are based on the concepts of phased array antennas. In this type of systems the signal of multiple antennas are phase shifted combined and afterwards converted into the analog baseband followed by an A/D conversion. If the signals are converted to only one digital signal we speak of analog beamforming, otherwise hybrid beamforming is used. The transmission of hybrid beamforming systems follows the inverse procedures to the described reception.

To utilize the full potential of the system it is essential that the beams in analog and hybrid beamforming systems are aligned. Therefore, a trial and error procedure is used to align the beams of Tx and Rx [2]. This beamsearch procedure does either utilize beams of different width with additional feedback or many beams of the same width with only one feedback stage [3]. In both cases the beams with specific width and maximum gain and flatness need to be designed.

Based on requirements on the beam shape this work formulates an optimization problem similar to authors of [4], [5]. Afterwards the optimization problem is solved numerically. In contrast this work includes the specific constraints of hybrid beamforming and low resolution phase shifters. In [3] the authors approximate a digital beamforming vector by a hybrid one. We generate our beam by approximating a desired beam instead.

The superscript e and f are used to distinguish between the two different hybrid beamforming systems. Bold small $a$ and capital letters $A$ are used to represent vectors and matrices. The notation $[a]_n$ is the $n$th element of the vector $a$. The superscript $T$ and $H$ represent the transpose and hermitian operators. The symbols $\otimes$ and $\circ$ are the Kronecker and Hadamard product.

II. OPTIMIZATION PROBLEM BASED BEAM SYNTHESIS

In the following we are developing a strategy to synthesis arbitrary beams based on the formulation as an optimization problem. After this we show how different constraints can be used to model the restrictions of different systems.

A. Objective function

The array factor $A(u, a)$ of a antenna array is defined as

$$ A(u, a) = a^T p(u), \quad [p(u)]_n = e^{j 2\pi x_n(u)}, \quad (1) $$

where $a$ is the beamforming vector, $u$ is the spatial direction combining the azimuth and elevation angle. The scalar $x_n(u)$ is the distance to the plane defined by the normal vector $u$ and a reference point. A common choice for the reference point is the position of the first antenna, in this case $x_1(u) = 0$.

The objective of synthesizing an arbitrary beam pattern can be formulated as a weighted $L_p$ norm between the desired pattern $D(u)$ and the absolute value of the actual array factor $|A(u, a)|$

$$ f(a) = \left( \int W(u) |A(u, a)| - D(u) |A(u, a)|^p du \right)^{\frac{1}{p}}, \quad (2) $$
is convex over its domain, but the constraints on $\alpha$ lead to a non-convex optimization problem.

This problem formulation ignores the phase of array factor, since we only desired the magnitude of the array factor to be of a specific shape. By only optimizing over the array factor we don’t take the pattern of the antennas into account. As described in [4] to take this into account we only need to divide $D(u)$ and $W(u)$ by the pattern of the antenna elements.

For many NonLinear Programing (NLP) solvers it is of advantage to analytically calculate the gradient of the objective function with respect to its parameters. For the shown objective function the gradient is calculated as

$$\nabla f(\alpha) = \left(1 - p(\alpha) \langle W(u) | A(u, \alpha) \rangle - D(u) \right)^{p-1} \text{sgn} \left(\langle W(u) | A(u, \alpha) \rangle - D(u) \right) \frac{1}{2} | A(u, \alpha) |^{-1} \nabla |A(u, \alpha)|^2 d u.$$

(3)

The gradient of the the absolute value of $A(u, \alpha)$ is calculated as

$$\nabla |A(u, \alpha)|^2 = 2 R \{ A(u, \alpha) \nabla A^*(u, \alpha) \}.$$

(4)

The gradient $\nabla A(u, \alpha)$ depends on the actually parameters of the array and is therefore different for each of the array types.

**B. Constraints**

We consider two different hybrid beamforming designs. These are the systems currently considered in literature [3], [6]. They have different advantages and drawbacks. The special case of analog beamforming and full digital beamforming are included as a special case.

In the first case all antennas $M$ are divided into groups of size $M_C$. Each subgroup consists of a Radio Frequency (RF) chain, a $M_C$ power divider followed by a phase shifter and a Power Amplifier (PA) at each antenna (see Figure 1 (a)). In total there are $M_{RFE}$ RF chains. This restricts the beamforming vector $\alpha$ to have the form

$$\alpha = \mathbf{W}^e \alpha^e = \begin{bmatrix} w_1^e & 0 & \cdots & 0 \\ 0 & w_2^e & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & w_{M_{RFE}}^e \end{bmatrix} \begin{bmatrix} \alpha_1^e \\ \alpha_2^e \\ \vdots \\ \alpha_{M_{RFE}}^e \end{bmatrix},$$

(5)

where the vectors $w_i^e$ models the analog phase shifting of group $i$ and therefore has the form

$$w_i^e = [e^{j \theta_{i,1}^e}, e^{j \theta_{i,2}^e}, \ldots, e^{j \theta_{i,M_C}^e}]^T.$$

(6)

The corresponding gradient of the array factor $A(u, \alpha)$ with respect to the decrees of freedom $\alpha^e$ and $\theta^e$ is calculated as

$$\nabla A(u, \alpha) = \left( \nabla \alpha^e \right) A(u, \alpha) = \left( \frac{(\mathbf{W}^e)^T \mathbf{p}(u)}{j (\alpha^e \otimes 1_{MC}) \circ \mathbf{p}(u) \circ \mathbf{r}^e} \right),$$

(7)

where the vector $\mathbf{r}^e$ contains all phase rotations

$$\mathbf{r}^e = [(w_1^e)^T, (w_2^e)^T, \ldots, (w_{M_{RFE}}^e)^T]^T.$$

(8)

In the second case each of the RF chain is connected to a $M$ power divider followed by a phase shifter for each antenna (see Figure 1 (b)). At each antenna the phase shifted signal from each RF chain is combined and then amplified by a PA followed by the antenna transmission. With this restrictions the beamforming vector $\alpha$ can be decomposed into

$$\alpha = \mathbf{W}^f \alpha^f = \begin{bmatrix} w_1^f & w_2^f & \cdots & w_{M_{RFE}}^f \end{bmatrix} \begin{bmatrix} \alpha_1^f \\ \alpha_2^f \\ \vdots \\ \alpha_{M_{RFE}}^f \end{bmatrix},$$

(9)

The gradient of the array factor with respect to the parameters $\alpha^f$ and $\theta^f$ is then:

$$\left( \begin{bmatrix} \nabla \alpha^f \\ \nabla \theta^f \end{bmatrix} \right) A(u, \alpha) = \left( \frac{(\mathbf{W}^f)^T \mathbf{p}(u)}{j (\alpha^f \otimes 1_{MC}) \circ \mathbf{p}(u) \circ \mathbf{r}^f} \right),$$

(10)
with $r^f$ being the concatenation of all phase shifting vectors $w^f_i$

$$r^f = \left( w^f_1 \right)^T \left( w^f_2 \right)^T \cdots \left( w^f_{M_{RFE}} \right)^T \right)^T. \quad (11)$$

The next constraints models the limited maximum output power of each PA. This we look at a system normalized by the maximum transmit power the elements of $a$ have to be less or equal to 1

$$[a]_m \leq 1 \ \forall m = \{1, 2, \cdots, M\}. \quad (12)$$

It is important to keep in mind that this restriction is after the solution of the phase shifters. This means that the values of $\theta^c_{i,j}$ are from a finite set of possibilities

$$\theta^c_{i,j} = -\pi + \frac{2\pi}{K} \ \forall i, j \text{ and } k \in \{0, 1, \cdots, K-1\}, \quad (13)$$

where $K$ is the number of possible phases. In this systems a possible phase shift in the digital domain needs to be taken into account. In the case without quantization this phase shift is redundant with the analog phase shift. Therefore, in addition to the scaling $\alpha^f$ or $\alpha^c$, we need to take a phase shift $\psi^f$ or $\psi^c$ into account. For the case of exclusive antennas with limited resolution RF phase shifters the beamforming vector $a$ takes the form

$$a = W^c (\alpha^c \circ \psi^c), \quad (14)$$

where $\psi^c$ are the digital phase shifts. The formulation for the fully connected case follows is same procedure.

$$\psi^c = \left[ e^{j \psi_1}, e^{j \psi_2}, \cdots, e^{j \psi_{M_{RFE}}} \right]^T. \quad (15)$$

It is important to keep in mind that with the resolution constraints on $\theta^c_{i,j}$ we now have a Mixed Integer NonLinear Programing (MINLP) problem.

C. Problem formulation

Combining the objective function with the constraints associated with the hardware capabilities lead to the following optimization problem

$$\min f(a) \quad \text{s.t. } g(a) \leq 0, \quad h(a) = 0, \quad (16)$$

where $g(a)$ and $h(a)$ are the constraints modeling the desired hardware capabilities. The weighting $W(u)$, the desired pattern $D(u)$ and the choice of $p$ in $f(a)$, determine which point in the trade-off gain, passband ripple and transition width is going to be hit (Figure 2).

III. NUMERICAL RESULTS

In the following section beams synthesized by the described method are shown. All systems utilized 64 Antennas ($M = 64$) and 4 RF-chains ($M_{RFE} = 4$) and have a Uniform Linear Array (ULA) with $\lambda/2$ spacing. Since the antenna array is one dimensional it is sufficient to look at only one spatial direction. All plots are in terms of $\psi = \frac{\lambda}{2} \sin(\phi)$, where $\phi$ is the geometric angle between a line connecting all antennas and the line having the same phase of a planar wavefront.

For each system three beams with the width $b = \pi, \pi/2, \pi/4$ are synthesized. For a ULA the weighting $W(\psi)$, the desired pattern $D(\psi)$ and the array factor $A(\psi, a)$ the spatial direction $u$ is fully represented by $\psi$. Since the absolute value of each element of $a$ is less or equal to one, if a perfect flat beam without sidelobes could be constructed, it would have the array-factor $D_{\text{max}} = \sqrt{N}2\pi/b$. As also described in [4] such a beam cannot be realized, therefore $D(\psi) = \beta D_{\text{max}}$ at the desired directions and 0 elsewhere. The parameters $\beta$ is relaxes the constraint and is given logarithmic scale $20 \log_{10}(\beta)$. 

![Fig. 2. Illustration of the trade-off associated with the beam pattern synthesis.](image)

![Fig. 3. Beams of different width with gain (a) 18 dB, (b) 22 dB and (c) 25 dB of exclusive antenna hybrid beamforming array.](image)

![Fig. 4. Beams of different width with gain (a) 18 dB, (b) 22 dB and (c) 25 dB of fully connected hybrid beamforming array.](image)
The weighting of different parts of the beam pattern $W(\psi)$ is uniformly set to 1, except for a small transition region around the edge of the desired directions. For all systems $p = 4$ was used in the objective function. This ensures equal gain and side lobe ripples. The integral over all spatial direction in the objective function is approximated by a finite sum. To ensure a sufficient approximation the interval is split into 512 elements. As described in [4] the computational complexity can significantly reduced by using a FFT/IFFT to calculate $A(\psi, \alpha)$ and it the derivatives of the objective function.

For each system the optimization process was started with a number of random initializations. Since the used NLP and MINLP solvers only guarantee to find a local minimum for a non-convex problem the results were compared and the one with the minimum objective function selected.

In Figure 3 and 4 the synthesized beams for exclusive antenna and fully connected hybrid beamforming are shown. For (a), (b) and (c) the gain penalty $\beta$ was selected to be 3dB, 2dB and 2dB. In all cases the transition width from desired to undesired direction was kept the same. Compared to the fully connected case, exclusive antenna hybrid beamforming has more gain ripples and higher sidelobe energy, while having the same transition width. The reason for this is, that in the fully connected case there are more degrees of freedom, that are in the case of same requirements used to improve the ripples and reduce the sidelobe energy.

In Figure 5 and 6 fully connected hybrid beamforming with quantized phase shifters was used. As described in [3] the beam is optimized to simultaneously transmit into 2 sectors. The power constraint for this case is also different, here only the sum power is constraint to be less or equal to 1. For our evaluation we used the same constraints.

In 6 especially in (a) there are multiple points where both beams almost overlap. In these directions a clear distinction which beam offers the better channel is very likely to fail. This can possible lead to a wrong decision and leads to large errors in a multi-stage beam training procedure. In contrast the solution in 5 offers a more sharp transmission from pass to stop directions. The stop directions attenuation is also close to uniform to enable a uniform performance. The only disadvantage is the larger ripples inside the center main beam.

The shortcomings for 6 are due to the fact that as described in the algorithm to generate $\alpha$. In [3] this method is approximating a version of $a_d$ generated with the assumption of full digital beamforming. Since for a low number of RF-chains this vector cannot be well approximated the resulting beam pattern does not correspond well to the digital one. It is also important to mention that there is no one to one mapping between in the error in approximating $a_d$ and the errors of the corresponding beam. As shown in [3], the method works well if $a_d$ can be well approximated by a larger number of RF chains.

IV. CONCLUSION

The developed approach can synthesized any beam-pattern with the restriction of hybrid-beamforming systems. The numerical example showed that a sufficient solution to the underlying optimization problem can be found with reasonable computational complexity. The numeric examples also demonstrated that it is possible to adapt the approach to any type of constraint arising in the context of hybrid beamforming in communication.

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REFERENCES

[1] F. Boccardi, R. Heath, A. Lozano, T. Marzetta, and P. Popovski, “Five disruptive technology directions for 5g,” IEEE Commun. Mag., vol. 52, no. 2, pp. 74–80, Feb. 2014.

[2] “IEEE Standard for Information technology–Telecommunications and information exchange between systems–Local and metropolitan area networks–Specific requirements-Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications Amendment 3: Enhancements for Very High Throughput in the 60 GHz Band,” IEEE Std 802.11ad, Dec. 2012.

[3] J. Palacios, D. D. Donno, D. Giustiniano, and J. Widmer, “Speeding up mmwave beam training through low-complexity hybrid transceivers,” in 2016 IEEE 27th Annu. Int. Symp. on Personal, Indoor, and Mobile Radio Commun. (PIMRC), 2016.

[4] D. P. Scholnik, “A parameterized pattern-error objective for large-scale phase-only array pattern design,” IEEE Trans. Antennas Propag., vol. 64, no. 1, pp. 89–98, Jan. 2016.

[5] A. F. Morabito, A. Massa, P. Rocca, and T. Isernia, “An effective approach to the synthesis of phase-only reconfigurable linear arrays,” IEEE Trans. Antennas Propag., vol. 60, no. 8, pp. 3622–3631, Aug. 2012.

[6] W. Roh, J. Y. Seol, J. Park, B. Lee, J. Lee, Y. Kim, J. Cho, K. Cheun, and F. Aryanfar, “Millimeter-wave beamforming as an enabling technology for 5g cellular communications: theoretical feasibility and prototype results,” IEEE Commun. Mag., vol. 52, no. 2, pp. 106–113, Feb. 2014.