Diffusive and arrested-like dynamics in currency exchange markets

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This work studies the symmetry between colloidal dynamics and the dynamics of the Euro–US Dollar currency exchange market (EURUSD). We consider the EURUSD price in the time range between 2001 and 2015, where we find significant qualitative symmetry between fluctuation distributions from this market and the ones belonging to colloidal particles in supercooled or arrested states. In particular, we find that models used for arrested physical systems are suitable for describing the EURUSD fluctuation distributions. Whereas the corresponding mean squared price displacement (MSPD) to the EURUSD is diffusive for all years, when focusing in selected time frames within a day, we find a two-step MSPD when the New York Stock Exchange market closes, comparable to the dynamics in supercooled systems. This is corroborated by looking at the price correlation functions and non-Gaussian parameters, and can be described by the theoretical model. We discuss the origin and implications of this analogy.

Slow dynamics is common to multiple physical systems such as atoms, granular, and soft-matter systems, as well as from which exhibit universal features when approaching the transition towards glass or jammed states [11–17]. A particular hallmark of such framework is the transition towards fluctuation distributions far from Gaussian, usually characterized with long tails, that depict slow structural relaxations within such arrested systems [8–13]. Different models have been proposed to reproduce these observations [11–15]. Within this regard, soft matter systems such as colloids, polymers or surfactants have been established as canonic in non equilibrium systems by their own right [16, 17], also exhibiting strong symmetries with many other fields, for example, with atomic or molecular systems [18–20]. Such symmetry is originated because these are many-body systems of interacting particles, described by equilibrium and non-equilibrium statistical mechanics [21].

Another field that is strongly amenable to be described by statistical mechanics is financial markets [22], where statistical mechanics has proven to be a useful tool. Louis Bachelier’s PhD Thesis, Theory of Speculation triggered, a century ago, an increasing interest for finance from a physical and mathematical point of view [23–26]. The introduction of computational techniques allowed the development of models such as the fractal one from Mandelbrot, which in fact resembles fractal descriptions also appearing in colloidal structures [27, 28]. However, it has been after the works by Stanley et al. [29] when the amount of research papers published by physicists, in economics in general and in finance in particular, has become relevant. Other achievements such as the GARCH model or the Black-Scholes equation helped in the rise of the field known as econophysics, which aims to employ physical theories and models in finance [30–34]. Still, when considering a physical scope, market dynamics is not fully understood; usually stochastic processes, statistical mechanics or non-linear physics are considered when describing market dynamics, but an unified body that describes the equivalence between mass or lengths with financial magnitudes is lacking.

In this paper we study financial markets, namely foreign exchange markets focused on the Euro-US Dollar exchange rate (EURUSD), from a physical approach, typical of undercooled systems. We study the distribution of the variation of the EURUSD price, and analyze it with a theoretical model of supercooled colloids, where the particles are transiently trapped, but can escape on a large time scale. EURUSD dynamics is considered by computing the mean squared price displacement (MSPD), the analogous to the particle mean squared displacement (MSD). While yearly EURUSD fluctuation distributions can be described by the colloidal glass model, MSPDs are diffusive at all times, without a clear hallmark of glassy arrest. Arrested states are however found in particular daily time frames, hallmarked by two steps in the evolution of the MSPD, the corresponding correlation function and also, by the equivalent to the non-Gaussian parameter. The description of the dynamics of colloids and the EURUSD exchange rate, poses the question of a symmetry or possible unification between both fields. Therefore, not only a descriptive approach but a physical origin to foreign exchange markets is proposed, where the EURUSD market, is analog to a supercooled colloidal system.

We have selected the EURUSD instantaneous price at time intervals of 1 minute. The price trajectory strikingly resembles the trajectory of a colloidal particle in suspension, shown in Fig. 1. Intrigued by this resemblance, we wonder if the dynamics from currency exchange markets and the one from colloids exhibit a more significant symmetry and if ultimately can be described by equivalent
Figure 1. Time evolution of the EURUSD price (a) and cartesian trajectories from a colloid embedded in a quasi 2-dimensional colloidal glass (b).

physical laws. Note that colloidal dynamics is driven by parameters such as interparticle interactions, density or applied shear \[35, 38\], whereas market dynamics is determined by parameters of very different nature, such as trading volume, number of investors at a given period or balance between supply and demand \[39, 41\].

We first study price fluctuations in the EURUSD market, i.e. the difference between prices separated by a lag time \(\tau\), \(\delta p(\tau) = \langle p(t_0 + \tau) - p(t_0) \rangle\), where the brackets indicate averaging over different time origins, \(t_0\) \[42\]. By fixing the magnitude of \(\tau\), we construct the probability distribution function (pdf) of \(\delta p(\tau)\), an analogue to the standard particle displacement distribution. Fig. 2 shows the pdf considering a total time period from 2010 up to 2015, with \(\tau = 5, 25, 125, 625\) and 3125 min., shifted vertically for clarity. All pdf exhibit a symmetric \(\tau\) up to 2015, with shows the pdf considering a total time period from 2010.

\begin{equation}
G(p, t) = \tau_1 f_{\text{vib}}(p) \phi_1(t) + FT^{-1} \left[ \tilde{f}_{\text{vib}}(q) \tilde{f}(q) \tau_2 \times \exp\left\{ \left( \tilde{f}(q) - 1 \right) t / \tau_2 \right\} - \exp\left( -t / \tau_1 \right) \right] / \tau_2 - \tau_1 + \tilde{f}(q) \tau_1
\end{equation}

Here \(\tilde{f}(q) = \tilde{f}_{\text{vib}}(q) \tilde{f}_{\text{jump}}(q)\), \(\tilde{f}(q)\) is the Fourier transform of function \(f(p)\), \(q\) is the conjugate variable of price \(p\) in the Fourier space and \(FT^{-1}\) denotes the Inverse Fourier Transform. In this original model, the particle is assumed to explore its cage on a time scale much shorter than \(\tau_1\) or \(\tau_2\), thus \(f_{\text{vib}}(r)\) is time independent. Because this assumption can not be made a priori in the EURUSD system, the model is modified to introduce short time diffusion, implying thus a finite time to explore the cage. For this purpose, we consider an Ornstein-Uhlenbeck process to calculate \(f_{\text{vib}}(p, t) = \sqrt{\alpha / 2\pi D(1 - e^{-2\alpha t})} \exp\{-\alpha p^2 / 2D(1 - e^{-2\alpha t})\}\), with \(D\) the diffusion coefficient and \(\alpha = D / l^2\) \[47, 49\]. This de-
picts a particle describing Brownian motion with a linear central force pulling it towards its origin.

Using this new model, the experimental pdf are fitted, as shown in Fig. 2 (lower panel), using \( D, l, d, \tau_1 \) and \( \tau_2 \), as fitting parameters, identical for all values of \( \tau \). The values of the parameters are: \( D = 2 \cdot 10^{-8} \text{min}^{-1} \), \( l = 30 \cdot 10^{-4} \), \( d = 15 \cdot 10^{-4} \), \( \tau_1 = 400 \text{ min.} \), \( \tau_2 = 300 \text{ min.} \), resulting in very good agreement with the experimental data, except for the lowest \( \tau \). The comparison of the complementary cumulative distribution functions (ccdf) from the model and experimental data, which is focused on the behaviour at long distances, is also satisfactory, as shown in the supplemental material [50]. Note that parameters indicate the EURUSD price caged within intervals of ca. 0.30 cents, and jumps out of this range occur on a time scale of approximately five hours.

A hallmark of undercooled systems, as mentioned previously, is the separation between microscopic and structural dynamics, which results in correlation functions decaying in two steps, or intermediate plateaus in the tagged particle mean squared displacement (MSD) \[57–59\]. Analogously, we propose the mean squared price displacement (MSPD):

\[
\langle \Delta p^2(\tau) \rangle = \langle (p(t_0 + \tau) - p(t_0))^2 \rangle \tag{2}
\]

Figure 3. EURUSD MSPDs as calculated from 1min. experimental data for the period 2010 – 2015 (open circles), to the year 2001 (filled circles) and to the year 2007 (crosses). The line is the MSPD according to the van Hove function of Eq. 1 with the same parameters as Fig. 2b.

Despite fluctuation pdf being characteristic to arrested systems, the MSPD is linear, as indicated in Fig. 3. Even more, such diffusive behavior is found in all years, shown in Fig. 3 and all MSPDs exhibit a common origin, indicating that the diffusion coefficient, therefore market dynamics, is characteristic of the EURUSD regardless of the year considered, including years of conflict to economy, such as 2001 or 2007. In passing we note that short time dynamics is Brownian, what leads to establish the analogy with undercooled colloids, either glasses or gels, rather than atomic glasses. The MSPD calculated from our model is also presented in Fig. 3, correctly reproducing the data. The precise combination of the parameters \( \{l, d, \tau_1, \tau_2 \} \) and \( D \) produces this linear evolution of \( \Delta p^2(\tau) \) although the pdf differ clearly from Gaussian.

Given this unexpected result, we seek a particular case or regime where the non-Gaussian components are important enough to produce deviations in the MSPD with respect to Brownian diffusion. Thus, we de-aggregate the data according to the commencing time, and study the dynamics for the next 24 hours. Additionally, we change the database to second-resolved prices, to access the short time dynamics. In particular, the year 2015 is studied and we first set the starting point for the calculation of \( \langle \delta p^2(\tau) \rangle \) at the opening of the New York Stock Exchange (NYSE), 9:30 am Eastern Time (ET). The MSPD for the forthcoming 24 hours after this commencing time, \( t_0 \), is averaged over all possible days. The same procedure is repeated for different values of \( t_0 \), covering the range of 24
The same behaviour in the MSPD suggests that price dynamics considered at daily periods with the starting time between 3:00pm and 6:00pm ET undergoes to a temporary dynamic arrest. The transient trapping has a typical time scale of about $\tau = 30\text{min}$, and is followed by diffusive dynamics. Commencing times later than 6:00pm reverse this trend; MSPDs increase and the intermediate plateau vanishes, approaching diffusion when the starting point of the MSPD calculation is between 2:00am and 4:00am, with the maximum MSPD at 9:30am. This observation is consistent with the correlation of market behaviour with activity, as done by Ito and Hashimoto for the US Dollar - Japanese Yen market [62].

The theoretical model can rationalize this behaviour, as shown in Fig. 4, for the two extreme cases, $t_0 =$9:30am and 6:00pm. To fit the model, $D$ and the time scales $\tau_1$ and $\tau_2$ where fixed for all $t_0$, as these are expected to be intrinsic to the EURUSD system, and only $l$ and $d$ are varied. The fits shown in the figure are obtained for $l = 3 \cdot 10^{-3}$ and $d = 1.5 \cdot 10^{-5}$ for $t_0 =$9:30am, and $l = 0.1 \cdot 10^{-5}$ and $d = 0.15 \cdot 10^{-5}$ for $t_0 =$6:00pm. Note that in the latter, both $l$ and $d$ are much smaller, but also $l < d$, characteristic of arrested systems, and the cage size is of order $\sim l$.

Given this similarity at the level of the MSPD, we study other properties which serve to identify undercooled states or glasses, in particular, the dynamic structure factor, $S(q, \tau) = \langle \exp\{iq(p(t_0 + \tau) - p(t_0))\} \rangle$, and the one-dimensional non-Gaussian parameter, $\alpha_2(\tau) = \langle \Delta p(\tau)^4 \rangle / 3 \langle \Delta p(\tau)^2 \rangle^2 - 1$. $S(q, \tau)$ starts from 1 decaying to zero in fluids and to a finite value in glasses. In undercooled systems, it shows an intermediate plateau, that depends on the wavevector, $q$. For the EURUSD system, we select a value of $q \sim 2\pi/l$ to probe this range of price variations, and the resulting correlation functions are shown in the inset to Fig. 4(b) and further detailed at the supplemental material [50]. The non-Gaussian parameter, $\alpha_2$, on the other hand, quantifies the deviation of the pdf from Gaussian. In undercooled fluids, $\alpha_2$ starts from zero (the pdf is Gaussian at short times), describes a maximum when the particles are caged and start to break free, to become zero again at long times. The non-Gaussian parameter for the EURUSD system, shown in the inset to Fig. 4(b) and detailed at the supplemental material [50], indeed shows a maximum when $t_0 =$6:00pm in the time range where the shoulder of the MSPD appears. Both $S(q, t)$ and $\alpha_2$ confirm the analogy between the dynamics of supercooled fluids and the EURUSD market.

The dynamics of economic markets is often described as diffusive, based on the linear MSPD profile when considering long periods, where the efficient market hypothesis (EMH) or fractal market hypothesis (FMH) can be invoked to explain such behavior [63, 64]. The EMH states that market prices are due to all market information being available, making it impossible to beat the market;
such a statement is in fact compatible with a random walk. We show in this study that this diffusion-like behaviour is found only when averages over long times are taken in the MSPD, $\langle \Delta p^2(\tau) \rangle$, but disappear when local time averages at 24 hours periods are performed and where either diffusive or arrested dynamics are featured. The FMH states, on the other hand, that investment strategies converge when considering short time frames, arresting the market dynamics and making it more inefficient. Our analysis suggests a fluid-to-glass transition upon the choice of the reference MSPD time, implying that markets become either efficient or fractal-like, depending on their activity.

In particle systems, the different dynamical regimes typical of fluid-to-glass transitions are obtained switching a physical parameter that controls the interaction between particles. It must be then further sought, which is the origin of the non-trivial dynamics shown above and if the analogy can be exploited to identify the equivalent set of parameters that control market dynamics, such as traded volume, number of investors in the market or even policy decisions from regulatory institutions. Furthermore, the symmetry between colloidal systems and the EURUSD exchange market is as well found in the EURCHF. A new approach appears, where exchange rate currency pairs can be regarded as colloidal systems and due to their coupled dynamics, the whole foreign exchange market can be considered a single undercooled system. This view and its natural extension to stock and other markets lays promising when modeling financial markets and needs to be further addressed.

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