Inflationary attractors from a non-canonical kinetic term

Zhu Yi, and Zong-Hong Zhu

Department of Astronomy, Beijing Normal University, Beijing 100875, China
E-mail: yz@bnu.edu.cn, zhuzh@bnu.edu.cn

Abstract. We show explicitly how the T-model, E-model, and Hilltop inflations are obtained from the inflation models with a non-canonical kinetic term and an arbitrary potential. By this method, any attractor of observables $n_s$ and $r$ is possible. The presence of attractors poses a challenge to differentiate inflation models.
1 Introduction

Inflation is a graceful solution to the monopole, horizon, and flatness problems in the standard big bang cosmology; besides, the quantum fluctuations of the inflaton can provide the initial condition for the formation of the large-scale structure of our Universe [1–5]. The constraints on the inflation from the cosmic microwave background (CMB) anisotropies are $n_s = 0.9649 \pm 0.0042$ (68% C.L.) and $r_{0.002} < 0.064$ (95% C.L.) [6]. For easy comparison with observational data, the predictions of the inflation models are generally expressed in terms of the number of e-folds $N$ at a pivot scale before the end of inflation. According to the observational data, the simplest form of the spectral index may be $n_s = 1 - 2/N$ with the e-folds $N = 60$. If the form of the spectral index $n_s$ or other slow-roll parameters is given, we can reconstruct the inflationary potentials in the slow-roll regime [7–19]. In this way, a wide range of inflation models that consistent with the observational data can be found.

There is an attractor phenomenon in which a lot of inflationary models make the same prediction. For example, the $R^2$ inflation [4] and the Higgs inflation with the nonminimal coupling $\xi \phi^2 R$ [20, 21] make the same prediction of $n_s = 1 - 2/N$ and $r = 12/N^2$. This prediction, then known as the universal attractor, was also derived from a more general inflation model with the nonminimal coupling $\Omega(\phi) = [1 + \xi f(\phi)] R$ and potential $\lambda^2 f^2(\phi)$ for an arbitrary function $f(\phi)$ in the strong coupling limit [22]. This general inflation model is reduced to the Higgs inflation by choosing $f(\phi) = \phi^2$. If the nonminimal coupling is $\Omega(\phi) = \xi f(\phi) R$, then we obtain the induced inflation, which also predicts the universal attractor in the strong coupling limit [23]. For the two fields conformal invariant inflation model with the potential $F(\phi/\chi)(\phi^2 - \chi^2)^2/36$, under the gauge $\chi^2 - \phi^2 = 6$ and in terms of the canonical scalar $\varphi$, the potential becomes $F(\tanh \varphi)$ [24]. For the monomial function of $F$, one gets the T-model $V(\varphi) = V_0 \tanh^{2n} (\varphi/\sqrt{6})$, that also gives the universal attractor [24]. If the gauge is chosen as $\chi = \sqrt{6}$ instead, the kinetic term for the scalar field $\phi$ in the Einstein frame becomes $(\partial \phi)^2/(1 - \phi^2/6)^2$ [25] and has a pole of order 2. In terms of the canonical scalar
\( \varphi = \sqrt{6} \tanh^{-1}(\phi/\sqrt{6}) \), the potential is also the T-model [26, 27] and the universal attractor is obtained. The universal attractor can be generalized to the \( \alpha \) attractors with the same \( n \) and \( r = 12\alpha/N^2 \) by varying the Kähler curvature [28]. For the \( \alpha \) attractors, the kinetic term is generalized to \( \partial \varphi^2/(1 - \varphi^2/6\alpha)^2 \) in the Einstein frame [25], and the potential is generalized to \( V(\varphi) = V_0 \tanh^{2n}(\varphi/\sqrt{6\alpha}) \) in terms of the canonical scalar field [29]. For more about the \( \alpha \)-attractors, see Refs. [30–39]. The common denominator of these attractor inflation models was then elucidated in Ref. [40] that their robust predictions stem from a joint pole of order 2 in the kinetic term of the inflaton field in the Einstein frame formulation prior to switching to the canonical variables. The inflation model with a pole in the kinetic term was discussed in detail as the generalized pole inflation [41, 42]. It was then found that any attractor is possible for the nonminimal coupling \( \Omega(\phi) = [1 + \xi f(\phi)] R \) with potential \( \Omega^2(\phi)U(\sqrt{3/2 \ln \Omega(\phi)}) \) in the strong coupling limit [43].

Considering the importance of the pole in the kinetic term in the Einstein frame on predicting the attractors, in this paper, we research the attractor phenomenon in the inflation model with a general non-canonical term. We find that any attractor is possible for this inflation model. We also show that the \( \alpha \) attractors can be obtained from a general non-canonical kinetic term with a pole of order 2 in the Einstein frame for an arbitrary potential. The inflation model with a non-canonical kinetic term can be also used to produce primordial black holes and induce secondary gravitational waves [44–50]. The paper is organized as follows. In Sec. II, we derive the attractor formulas in the inflation model with a non-canonical kinetic term. The examples for the models with different attractors are given in Sec. III. The conclusions are drawn in Sec. IV.

## 2 The attractors

In this paper, we work on the attractor behavior in the inflation model with a non-canonical term in the Einstein frame. The action is

\[
S = \int dx^4 \sqrt{-g} \left[ \frac{1}{2} R - \frac{1}{2} K(\phi)(\partial \phi)^2 - V(\phi) \right],
\]

where \( (\partial \phi)^2 = g^{\mu \nu} \nabla_\mu \phi \nabla_\nu \phi \), \( V(\phi) \) is the potential, and \( K(\phi) > 0 \) is an arbitrary coupling function, and we take the convention \( 8\pi G = 1 \). With the help of this non-canonical coupling function \( K(\phi) \), the power spectrum of the primordial curvature perturbations can be enhanced at small scales to produce primordial black holes and induce secondary gravitational wave, and simultaneously keep small at large scales to satisfy the constraints from the CMB anisotropy [44–50]. In terms of the canonical field \( \varphi \),

\[
d\varphi = \sqrt{K(\phi)} d\phi,
\]

the model (2.1) becomes

\[
S = \int dx^4 \sqrt{-g} \left[ \frac{1}{2} R - \frac{1}{2} \nabla_\mu \varphi \nabla^\mu \varphi - U(\varphi) \right],
\]

where \( \nabla_\mu \varphi \nabla^\mu \varphi = \sqrt{K(\phi)} \nabla_\mu \phi \nabla^\mu \phi \).
where the canonical potential related to potential $V(\phi)$ is

$$U[\varphi(\phi)] = V(\phi).$$  \hfill (2.4)

By combining the relations (2.2) and (2.4), the coupling function $K(\phi)$ expressed by the potentials becomes

$$K(\phi) = \left[ \tilde{U}'(V(\phi))V'(\phi) \right]^2,$$  \hfill (2.5)

where $\tilde{U} = U^{-1}$ is the inverse function of the canonical potential $U(\varphi)$, and a prime denotes the derivative with respect to the argument of the function,

$$\tilde{U}'(V(\phi)) = \frac{d\tilde{U}(x)}{dx} \bigg|_{x=V(\phi)}, \quad V'(\phi) = \frac{dV(\phi)}{d\phi}.$$  \hfill (2.6)

By using the relation (2.5), the action (2.1) becomes

$$S = \int dx^4 \sqrt{-g} \left[ \frac{1}{2} R - \frac{1}{2} \left( \tilde{U}'(V(\phi))V'(\phi) \right)^2 (\partial \phi)^2 - V(\phi) \right].$$  \hfill (2.7)

Action (2.3) and action (2.7) are equivalent and make the same prediction for $n_s$ and $r$. Because the prediction of action (2.3) is determined by the canonical potential $U(\varphi)$, the prediction of action (2.7) will be also determined by the potential $U(\varphi)$ and independent on the potential $V(\phi)$. Therefore, for different potentials $V(\phi)$, the action (2.7) will make the same prediction, which is an attractor and determined by the canonical potential $U(\varphi)$. In the following, we will show explicitly how to obtain the attractors determined by the T-model, the E-model, and the Hilltop potential.

### 3 Examples

#### 3.1 $\alpha$-attractors

We first discuss how to obtain the $\alpha$-attractors from the model (2.7). The $\alpha$-attractors are [28]

$$n_s = 1 - \frac{2}{N}, \quad r = \frac{12\alpha}{N^2},$$  \hfill (3.1)

and consistent well with the Planck 2018 constraints [6],

$$n_s = 0.9649 \pm 0.0042, \quad r_{0.002} < 0.064.$$  \hfill (3.2)

The canonical potentials that make the predictions of the $\alpha$-attractors are T-model and E-model.
3.1.1 T-model

The T-model is

\[
U(\varphi) = U_0 \tanh^{2n} \left( \frac{\varphi}{\sqrt{6} \alpha} \right),
\]

with \( \alpha > 0 \) and \( n > 0 \). Substituting T-model (3.3) into Eq. (2.5), we obtain

\[
K(\phi) = \frac{3\alpha}{2n^2} \left( \frac{1}{U_0} \right)^{1/n} \frac{V^{\frac{1}{n}-2}}{1 - (V/U_0)^{1/n}} \left[ \frac{dV(\phi)}{d\phi} \right]^2.
\]

For different potentials \( V(\phi) \), the model (2.1) combined with Eq. (3.4) will give the same prediction (3.1). The coupling function \( K(\phi) \) has a pole of order 2, which is consistent with the statement in Ref. [40], that all the cosmological attractors can be brought to the inflation model (2.1) with the coupling function \( K(\phi) \) having a pole of order 2. Therefore, we obtain the \( \alpha \) attractors from the general non-canonical term (3.4) with a pole of order 2 for an arbitrary potential. To show the T-model (3.4) more specifically, in the following, we choose the potential \( V(\phi) \) as the chaotic inflation potential or the natural inflation potential.

For the chaotic inflation, the potential is [51]

\[
V(\phi) = V_0 \phi^p,
\]

the corresponding coupling function Eq. (3.4) becomes

\[
K(\phi) = \frac{3p^2 \alpha}{2\beta n^2} \frac{\phi^{\frac{p}{n}-2}}{(1 - \beta^{-1}\phi^{p/n})^2},
\]

with \( \beta = (U_0/V_0)^{1/n} \); and the action (2.7) becomes

\[
S = \int dx^4 \sqrt{-g} \left[ \frac{1}{2} R - \frac{3p^2 \alpha}{2\beta n^2} \frac{\phi^{\frac{p}{n}-2}}{(1 - \beta^{-1}\phi^{p/n})^2} \frac{(\partial \phi)^2}{2} - V_0 \phi^p \right],
\]

which is the general form of the chaotic inflation that gives the T-model.

To make the non-canonical term reduces to the canonical case after inflation at the low energy regimes \( \phi \ll 1 \), we choose

\[
p = 2n, \quad \beta = 6\alpha.
\]

And then the coupling function (3.6) becomes

\[
K(\phi) = \frac{1}{\left[ 1 - \phi^2/(6\alpha) \right]^2},
\]

the attractor model (3.7) becomes [25, 28]

\[
S = \int dx^4 \sqrt{-g} \left[ \frac{1}{2} R - \frac{1}{\left[ 1 - \phi^2/(6\alpha) \right]^2} \frac{(\partial \phi)^2}{2} - V_0 \phi^{2n} \right].
\]
Therefore, we obtain the $\alpha$-attractors action derived in Ref. [25, 28]. And from the above discussion, we know that this action, reducing to the canonical model at the lower energy regimes $\phi \ll 1$, is a special case of action (3.7).

For the natural inflation, the potential is [52]

$$V(\phi) = V_0 [1 + \cos (\phi/f)] , \quad (3.11)$$

the corresponding coupling function Eq. (3.4) becomes

$$K(\phi) = \frac{3\alpha}{2n^2f^2\beta_2} \frac{\cos^{2-2}(\phi/2f) \sin^2(\phi/2f)}{[1 - \beta_2^{-1} \cos^{2/n}(\phi/2f)]^2} , \quad (3.12)$$

with $\beta_2 = \beta/2^{1/n}$. Substituting relations (3.12) and potential (3.11) into action (2.1), we obtain the general natural inflation model that gives the T-model. If we choose

$$\beta_2 = 1, \quad n = 1, \quad f = \sqrt{3\alpha} , \quad (3.13)$$

the equation (3.12) becomes

$$K(\phi) = \frac{1}{[1 - \cos(\phi/\sqrt{3\alpha})]} , \quad (3.14)$$

and the attractor model (3.7) becomes

$$S = \int dx^4 \sqrt{-g} \left[ \frac{1}{2} R - \frac{1}{[1 - \cos(\phi/\sqrt{3\alpha})]^2} \frac{1}{2} \frac{\partial \phi}{\partial \phi} \right] - V_0 \left[ 1 + \cos \left( \phi/\sqrt{3\alpha} \right) \right] . \quad (3.15)$$

Therefore, we obtain the T-model from the natural inflation.

3.1.2 E-model

The E-model is

$$U(\varphi) = U_0 \left[ 1 - \exp \left( -\sqrt{\frac{2}{3\alpha}} \varphi \right) \right]^{2n} . \quad (3.16)$$

Substituting it into Eq. (2.5), the corresponding coupling function becomes

$$K(\phi) = \frac{3\alpha}{8n^2 \left( U_0 \right)^{1/n}} \frac{V^{\frac{1}{2} - 2}}{[1 - (V/U_0)^{1/2n}]^2} \left[ \frac{dV(\phi)}{d\phi} \right]^2 . \quad (3.17)$$

The E-model and the T-model both predict the $\alpha$-attractors (3.1). Therefore, the non-canonical coupling function for the E-model and that for the T-model are almost the same, except that the point of the pole in E-model is $(V/U_0)^{1/2n}$ while that in T-model is $(V/U_0)^{1/n}$. To show the E-model (3.17) more specifically, we also choose the potential $V(\phi)$ as the chaotic inflation potential or the natural inflation potential.
For the chaotic inflation with the potential (3.5), the non-canonical kinetic term becomes

$$K(\phi) = \frac{3p^2\alpha}{8\beta n^2} \frac{\phi^{n-2}}{(1 - \phi^{p/(2n)} / \sqrt{\beta})^2},$$

(3.18)

and the action (2.7) becomes

$$S = \int dx^4 \sqrt{-g} \left[ \frac{1}{2} R - \frac{3p^2\alpha}{8\beta n^2} \frac{\phi^{n-2}}{(1 - \phi^{p/(2n)} / \sqrt{\beta})^2} \frac{(\partial \phi)^2}{2} - V_0 \phi^p \right].$$

(3.19)

This is the general form of the chaotic inflation that gives the E-model. To get a canonical kinetic term after inflation at the low energy regime $\phi \ll 1$, we choose

$$p = 2n, \quad \beta = \frac{3\alpha}{2}.$$  

(3.20)

And then the coupling function becomes

$$K(\phi) = \frac{1}{\left(1 - \sqrt{\frac{2}{3\alpha}} \phi\right)^2},$$

(3.21)

the attractor action (3.19) becomes

$$S = \int dx^4 \sqrt{-g} \left[ \frac{1}{2} R - \frac{1}{\left(1 - \sqrt{\frac{2}{3\alpha}} \phi\right)^2} \frac{(\partial \phi)^2}{2} - V_0 \phi^{2n} \right].$$

(3.22)

In Ref. [25], the E-model is obtained from the non-canonical kinetic coupling function (3.9) with the potential $[\phi/(\sqrt{6\alpha} + \phi)]^{2n}$. In this paper, we show that the E-model can be also obtained from the chaotic inflation with a non-canonical term, that reduces to the canonical case at the lower energy regimes $\phi \ll 1$.

For the natural inflation with potential (3.11), the coupling function reduces to

$$K(\phi) = \frac{3\alpha}{8n^2\beta^2 f^2} \frac{\cos^{n-2}(\phi/2f) \sin^2(\phi/2f)}{\left[1 - \beta_2^{-1/2} \cos^{1/n}(\phi/2f)\right]^2}.$$

(3.23)

If we choose

$$n = \frac{1}{2}, \quad \beta_2 = 1, \quad f = \sqrt{\frac{3\alpha}{2}},$$

(3.24)

the coupling function becomes

$$K(\phi) = \frac{1 + \cos \left(\sqrt{\frac{2}{3\alpha}} \phi\right)}{1 - \cos \left(\sqrt{\frac{2}{3\alpha}} \phi\right)},$$

(3.25)
and the action becomes
\[
S = \int dx^4 \sqrt{-g} \left[ \frac{1}{2} R - \frac{1 + \cos \left( \sqrt{\frac{2}{3\alpha}} \phi \right)}{1 - \cos \left( \sqrt{\frac{2}{3\alpha}} \phi \right)} \frac{(\partial \phi)^2}{2} - V_0 \left[ 1 + \cos \left( \sqrt{\frac{2}{3\alpha}} \phi \right) \right] \right]. \tag{3.26}
\]

Therefore, the E-model can be also from the natural inflation.

### 3.2 Hilltop inflation

The other observational data favored model is the Hilltop inflation. The potential of the Hilltop inflation is \[53\]
\[
U(\varphi) = U_0 \left[ 1 - \left( \frac{\varphi}{\mu} \right)^n + \cdots \right]. \tag{3.27}
\]
where the dots indicates higher-order terms. Because the Hilltop inflation is supposed to take place near a maximum of the potential, the inflation field satisfies $\phi/\mu \ll 1$ and the higher-order terms can be neglected. To achieve the small field inflation, we take $\mu < 1$, and then the predictions for the spectral index and the tensor-to-scalar ratio are \[43\]
\[
\frac{n_s - 1}{2} = -\frac{2(n - 1)}{(n - 2)N}, \tag{3.28}
\]
\[
r = \frac{8n^2}{\mu^2} \left[ \frac{\mu^2}{n(n - 2)N} \right]^{(2n-2)/(n-2)}, \tag{3.29}
\]
where we suppose $n > 2$. Taking the $e$-folds $N = 60$, we obtain the observational constraints (3.2) on the power index of the Hilltop inflation,
\[
n > 7.59. \tag{3.30}
\]
Combining Eq. (2.5) and the Hilltop inflation potential (3.27), we obtain the non-canonical term with the Hilltop inflation attractors,
\[
K(\phi) = \frac{\mu^2}{n^2 U_0^2} \frac{1}{\left[ 1 - \left( \frac{\mu}{\nu_0} \right)^{2-\frac{n}{2}} \right]^2} \left[ \frac{dV(\phi)}{d\phi} \right]^2. \tag{3.31}
\]
From condition (3.30), we have
\[
1.74 < 2 - \frac{2}{n} < 2. \tag{3.32}
\]
Therefore, the non-canonical coupling for the Hilltop attractor has a pole of order from 1.74 to 2.

For the chaotic inflation (3.5), the non-canonical coupling function (3.31) becomes
\[
K(\phi) = \frac{\mu^2 p^2}{\gamma n^2} \frac{\phi^{2p-2}}{(1 - \gamma^{-1/2} \phi^p)^{2-2p}}. \tag{3.33}
\]
where $\gamma = (U_0/V_0)^2$. If we want the non-canonical term to reduce to canonical term after inflation at the low energy regime $\phi \ll 1$, we choose

$$p = 1, \quad \gamma = \frac{\mu^2}{n^2}, \quad (3.34)$$

and the corresponding coupling function is

$$K(\phi) = \frac{1}{(1 - n\phi/\mu)^{2-\frac{2}{n}}}, \quad (3.35)$$

and the action becomes

$$S = \int dx^4 \sqrt{-g} \left[ \frac{1}{2} R - \frac{1}{(1 - n\phi/\mu)^{2-\frac{2}{n}}} \frac{(\partial \phi)^2}{2} - V_0 \right]. \quad (3.36)$$

Therefore, we obtain the Hilltop inflation from the chaotic inflation model. This action is similar to the action of the E-model (3.22), except that the poles in each kinetic term are different.

For the natural inflation (3.11), the non-canonical term (3.31) becomes

$$K(\phi) = \frac{\mu^2}{f^2 n^2 \gamma_2} \frac{\cos^2(\phi/2f) \sin^2(\phi/2f)}{\left[1 - \cos^2(\phi/2f)/\sqrt{\gamma_2}\right]^{2-\frac{2}{n}}}, \quad (3.37)$$

where $\gamma_2 = \gamma/4$. Choosing

$$\gamma_2 = 1, \quad f = \frac{\mu}{2^{1/n}n}, \quad (3.38)$$

the non-canonical coupling term becomes

$$K(\phi) = \frac{1 + \cos(2^{1/n}n\phi/\mu)}{\left[1 - \cos(2^{1/n}n\phi/\mu)\right]^{1-\frac{2}{n}}}, \quad (3.39)$$

and the action becomes

$$S = \int dx^4 \sqrt{-g} \left[ \frac{1}{2} R - \frac{1 + \cos(2^{1/n}n\phi/\mu)}{\left[1 - \cos(2^{1/n}n\phi/\mu)\right]^{1-\frac{2}{n}}} \frac{(\partial \phi)^2}{2} - V_0 \left[1 + \cos(2^{1/n}n\phi/\mu)\right] \right]. \quad (3.40)$$

Therefore, we obtain the Hilltop inflation from the natural inflation model.

### 4 Conclusion

There exists a phenomenon of attractor that many inflationary models make the same prediction for $n_s$ and $r$. The inflation model with the non-canonical kinetic $K(\phi) = [\dot{U}'(V(\phi))V''(\phi)]^2$ and an arbitrary potential $V(\phi)$ is equal to the canonical inflation model with the canonical potential $U(\varphi)$. Therefore, for different inflation
potentials $V(\phi)$, the models with that $K(\phi)$ make the same prediction which is determined by the canonical potential $U(\varphi)$. Thus, by reconstructing the potential from the observable $n_s$ or $r$, we can get any attractor in the inflation models with the non-canonical kinetic. In particular, we show explicitly the models that give the attractors determined by the T model, the E model, and the Hilltop potential, respectively. For an arbitrary potential, the kinetic terms $K(\phi)$ in these models all have a pole. The order of the pole determined by the T-model and the E-model is 2 and that determined by the Hilltop potential is $2 - 2/n$.

In conclusion, any attractor is possible for the inflation model with a non-canonical term, and the presence of attractors poses a challenge to differentiate inflation models.

Acknowledgments

This work was supported by the National Natural Science Foundation of China under Grants Nos. 11633001, 11920101003 and 12021003, the Strategic Priority Research Program of the Chinese Academy of Sciences, Grant No. XDB23000000 and the Inter-discipline Research Funds of Beijing Normal University. Z. Y. was supported by China Postdoctoral Science Foundation Funded Project under Grant No. 2019M660514.

References

[1] A.H. Guth, *The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems*, Phys. Rev. D 23 (1981) 347.

[2] A.D. Linde, *A New Inflationary Universe Scenario: A Possible Solution of the Horizon, Flatness, Homogeneity, Isotropy and Primordial Monopole Problems*, Phys. Lett. B 108 (1982) 389.

[3] A. Albrecht and P.J. Steinhardt, *Cosmology for Grand Unified Theories with Radiatively Induced Symmetry Breaking*, Phys. Rev. Lett. 48 (1982) 1220.

[4] A.A. Starobinsky, *A New Type of Isotropic Cosmological Models Without Singularity*, Phys. Lett. 91B (1980) 99.

[5] K. Sato, *First Order Phase Transition of a Vacuum and Expansion of the Universe*, Mon. Not. Roy. Astron. Soc. 195 (1981) 467.

[6] PLANCK collaboration, *Planck 2018 results. X. Constraints on inflation*, Astron. Astrophys. 641 (2020) A10 [1807.06211].

[7] V. Mukhanov, *Quantum Cosmological Perturbations: Predictions and Observations*, Eur. Phys. J. C 73 (2013) 2486 [1303.3925].

[8] D. Roest, *Universality classes of inflation*, JCAP 1401 (2014) 007 [1309.1285].

[9] J. Garcia-Bellido and D. Roest, *Large-N running of the spectral index of inflation*, Phys. Rev. D 89 (2014) 103527 [1402.2059].

[10] L. Barranco, L. Boubekeur and O. Mena, *A model-independent fit to Planck and BICEP2 data*, Phys. Rev. D 90 (2014) 063007 [1405.7188].
[11] L. Boubekeur, E. Giusarma, O. Mena and H. Ramírez, Phenomenological approaches of inflation and their equivalence, Phys. Rev. D 91 (2015) 083006 [1411.7237].

[12] T. Chiba, Reconstructing the inflaton potential from the spectral index, PTEP 2015 (2015) 073E02 [1504.07692].

[13] P. Creminelli, S. Dubovsky, D. López Nacir, M. Simonović, G. Trevisan, G. Villadoro et al., Implications of the scalar tilt for the tensor-to-scalar ratio, Phys. Rev. D 2 (2015) 123528 [1412.0678].

[14] R. Gobbetti, E. Pajer and D. Roest, On the Three Primordial Numbers, JCAP 1509 (2015) 058 [1505.00968].

[15] J. Lin, Q. Gao and Y. Gong, The reconstruction of inflationary potentials, Mon. Not. Roy. Astron. Soc. 459 (2016) 4029 [1508.07145].

[16] Q. Fei, Y. Gong, J. Lin and Z. Yi, The reconstruction of tachyon inflationary potentials, JCAP 1708 (2017) 018 [1705.02545].

[17] Q. Gao and Y. Gong, Reconstruction of extended inflationary potentials for attractors, Eur. Phys. J. Plus 133 (2018) 491 [1703.02220].

[18] Q. Gao, Reconstruction of constant slow-roll inflation, Sci. China Phys. Mech. Astron. 60 (2017) 090411 [1704.08559].

[19] Q. Fei, Z. Yi and Y. Yang, The reconstruction of non-minimal derivative coupling inflationary potentials, Universe 6 (2020) 213 [2009.14819].

[20] D.I. Kaiser, Primordial spectral indices from generalized Einstein theories, Phys. Rev. D 52 (1995) 4295 [astro-ph/9408044].

[21] F.L. Bezrukov and M. Shaposhnikov, The Standard Model Higgs boson as the inflaton, Phys. Lett. B 659 (2008) 703 [0710.3755].

[22] R. Kallosh, A. Linde and D. Roest, Universal Attractor for Inflation at Strong Coupling, Phys. Rev. Lett. 112 (2014) 011303 [1310.3950].

[23] G.F. Giudice and H.M. Lee, Starobinsky-like inflation from induced gravity, Phys. Lett. B 733 (2014) 58 [1402.2129].

[24] R. Kallosh and A. Linde, Universality Class in Conformal Inflation, JCAP 1307 (2013) 002 [1306.5220].

[25] R. Kallosh, A. Linde and D. Roest, Large field inflation and double α-attractors, JHEP 08 (2014) 052 [1405.3646].

[26] R. Kallosh and A. Linde, Non-minimal Inflationary Attractors, JCAP 1310 (2013) 033 [1307.7938].

[27] R. Kallosh and A. Linde, Multi-field conformal cosmological attractors, JCAP 12 (2013) 006 [1309.2015].

[28] R. Kallosh, A. Linde and D. Roest, Superconformal Inflationary α-Attractors, JHEP 11 (2013) 198 [1311.0472].

[29] R. Kallosh and A. Linde, Cosmological attractors and asymptotic freedom of the inflaton field, JCAP 06 (2016) 047 [1604.00444].

[30] A. Linde, Single-field α-attractors, JCAP 05 (2015) 003 [1504.00663].
[31] R. Kallosh and A. Linde, Planck, lhc, and α-attractors, Phys. Rev. D 91 (2015) 083528 [1502.07733].

[32] S.D. Odintsov and V.K. Oikonomou, Inflationary α-attractors from F(R) gravity, Phys. Rev. D 94 (2016) 124026 [1612.01126].

[33] K.S. Kumar, J. Marto, P. Vargas Moniz and S. Das, Non-slow-roll dynamics in α−attractors, JCAP 04 (2016) 005 [1506.05366].

[34] T. Krajewski, K. Turzyński and M. Wieczorek, On preheating in α-attractor models of inflation, Eur. Phys. J. C 79 (2019) 654 [1801.01786].

[35] Z. Yi, Y. Gong and M. Sabir, Inflation with Gauss-Bonnet coupling, Phys. Rev. D 98 (2018) 083521 [1804.09116v2].

[36] M. Sabir, W. Ahmed, Y. Gong and Y. Lu, α-attractor from superconformal E-models in brane inflation, Eur. Phys. J. C 80 (2020) 15 [1903.07892].

[37] L. Aresté Saló, D. Benisty, E.I. Guendelman and J. de Haro, α attractors in Quintessential Inflation motivated by Supergravity, Phys. Rev. D 103 (2021) 123535 [2103.07892].

[38] R. Shojaee, K. Nozari and F. Darabi, α-attractors and reheating in a class of galileon inflation, Int. J. Mod. Phys. D 30 (2021) 2150036.

[39] Y. Aldabergenov, A. Chatrabhuti and H. Isono, α-attractors from supersymmetry breaking, Eur. Phys. J. C 81 (2021) 166 [2009.02203].

[40] M. Galante, R. Kallosh, A. Linde and D. Roest, Unity of Cosmological Inflation Attractors, Phys. Rev. Lett. 114 (2015) 141302 [1412.3797].

[41] B.J. Broy, M. Galante, D. Roest and A. Westphal, Pole inflation — shift symmetry and universal corrections, JHEP 12 (2015) 149 [1507.02277].

[42] T. Terada, Generalized pole inflation: Hilltop, natural, and chaotic inflationary attractors, Phys. Lett. B 760 (2016) 674 [1602.07867].

[43] Z. Yi and Y. Gong, Nonminimal coupling and inflationary attractors, Phys. Rev. D 94 (2016) 103527 [1608.05922].

[44] J. Lin, Q. Gao, Y. Gong, Y. Lu, C. Zhang and F. Zhang, Primordial black holes and secondary gravitational waves from k and G inflation, Phys. Rev. D 101 (2020) 103515 [2001.05909].

[45] Z. Yi, Y. Gong, B. Wang and Z.-H. Zhu, Primordial black holes and secondary gravitational waves from the higgs field, Phys. Rev. D 103 (2021) 063535 [2007.09957].

[46] Z. Yi, Q. Gao, Y. Gong and Z.-H. Zhu, Primordial black holes and scalar-induced secondary gravitational waves from inflationary models with a noncanonical kinetic term, Phys. Rev. D 103 (2021) 063534 [2011.10606].

[47] Q. Gao, Y. Gong and Z. Yi, Primordial black holes and secondary gravitational waves from natural inflation, 2012.03856.

[48] F. Zhang, Y. Gong, J. Lin, Y. Lu and Z. Yi, Primordial non-gaussianity from g-inflation, JCAP 04 (2021) 045 [2012.06960].

[49] Z. Yi and Z.-H. Zhu, Nanograv signal and ligo-virgo primordial black holes from higgs inflation, 2105.01943.
[50] Q. Gao, *Primordial black holes and secondary gravitational waves from chaotic inflation*, 2102.07369.

[51] A.D. Linde, *Chaotic Inflation*, *Phys. Lett.* **129B** (1983) 177.

[52] K. Freese, J.A. Frieman and A.V. Olinto, *Natural inflation with pseudo - Nambu-Goldstone bosons*, *Phys. Rev. Lett.* **65** (1990) 3233.

[53] L. Boubekeur and D.H. Lyth, *Hilltop inflation*, *JCAP* **0507** (2005) 010 [hep-ph/0502047].