Mass Function of Rich Galaxy Clusters and Its Constraint on $\sigma_8$

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Abstract

The mass function of galaxy clusters is a powerful tool to constrain cosmological parameters, e.g., the mass fluctuation on the scale of 8 $h^{-1}$ Mpc, $\sigma_8$, and the abundance of total matter, $\Omega_m$. We first determine the scaling relations between cluster mass and cluster richness, summed r-band luminosity and the global galaxy number within a cluster radius. These relations are then used to two complete volume-limited rich cluster samples which we obtained from the Sloan Digital Sky Survey (SDSS). We estimate the masses of these clusters and determine the cluster mass function. Fitting the data with a theoretical expression, we get the cosmological parameter constraints in the form of $\sigma_8(\Omega_m/0.3)^{\alpha} = \beta$ and find out the parameters of $\alpha = 0.40 - 0.50$ and $\beta = 0.8 - 0.9$, so that $\sigma_8 = 0.8 - 0.9$ if $\Omega_m = 0.3$. Our $\sigma_8$ value is slightly higher than recent estimates from the mass function of X-ray clusters and the Wilkinson Microwave Anisotropy Probe (WMAP) data, but consistent with the weak lensing statistics.

Key words: galaxies: clusters: general --- cosmological parameters

1 INTRODUCTION

Precise determination of cosmological parameters is an important goal in astrophysics. In the linear theory, the present root-mean-square (rms) mass fluctuation on the scale of 8 $h^{-1}$ Mpc, $\sigma_8$, is one of fundamental parameters (see Spergel et al. 2003) to describe the power spectrum of mass fluctuations in the universe. It is one of key parameters in the large scale structure simulations (e.g., Jenkins et al. 1998). The $\sigma_8$ can be determined by galaxy-galaxy correlations (e.g., Tegmark et al. 2004, Cole et al. 2005), fluctuations in the cosmic microwave background (Spergel et al. 2003, 2007, Komatsu et al. 2008), gravitational lensing statistics (e.g., Hoekstra et al. 2004, Kitching et al. 2007, Benjamin et al. 2007), cluster mass function (e.g., White et al. 1993, Bahcall & Fan 1998, Reiprich & B"ohringer 2002), Ly$\alpha$ forest (Jena et al. 2008, McDonald et al. 2005) and galaxy peculiar velocities (Feldman et al. 2003).

The cluster mass function can be determined by the estimated masses for a sample of clusters (e.g., Dahle 2006), or by the X-ray luminosity and temperature function with a prior scaling relation (Viana & Liddle 1996, Allen et al. 2003). Fitting the cluster mass function with a theoretical expression can provide constraints on $\sigma_8$. Generally, $\sigma_8$ is coupled with $\Omega_m$, the abundance of present total matter, in the form of $\sigma_8(\Omega_m/0.3)^{\alpha} = \beta$. Previous studies have found $\alpha$ in the range 0.3–0.6 and $\beta$ in the range 0.6–1.2 (see Table 2 in Section 4). The determined $\sigma_8$ in recent years (2002–2009) has a mean value of 0.73±0.05 assuming $\Omega_m = 0.3$, which is in agreement with the WMAP data (Komatsu et al. 2009), but lower than those by weak lensing statistics (Hetterscheidt et al. 2007), galaxy-galaxy correlations (Tegmark et al. 2004, Cole et al. 2005) and Ly$\alpha$ forest (Jena et al. 2008, McDonald et al. 2005).

The amplitude of cluster mass function has large uncertainties, mainly caused by the uncertain normalization of the mass scaling relation (e.g., Henry 2004). Other uncertainties come from the scatter of mass scaling relation and the incompleteness of the X-ray flux-limited cluster samples (Reiprich & B"ohringer 2002). The cluster mass function may be underestimated if only X-ray clusters are used. Erben et al. (2000) and Dahle et al. (2003) have noticed the existence of a class of X-ray-underluminous massive clusters. Popesso et al. (2007) found that 40% of Abell clusters have a low level or no detection in X-rays. A large complete volume-limited sample of clusters is crucial for the purpose. Using the photometric redshifts of galaxies, we found 39,668 clusters in the redshift range 0.05 < $z$ < 0.6 (Wen et al. 2009). Clusters are approximate volume-limited complete in the redshift range 0.05 < $z$ < 0.42. The richnesses and the summed luminosities of clusters are estimated from their luminous members, and they are tightly related to cluster mass. In Section 2, we carefully determine the scaling relation for cluster mass. In Section 3, we get the cluster mass function for a local sample of clusters and a sample at mediate redshifts, and then fit the cluster mass function with a theoretical expression for constraints on cosmological parameters, $\Omega_m$ and $\sigma_8$. Discussions and conclusions are given in Section 4.
### Table 1. Cluster masses from literature and the mass tracer values for 53 clusters (richness $R \geq 8$ and $0.03 \lesssim z \lesssim 0.3$, sorted with $z$ in the field of the SDSS DR6).

| Name        | R.A. (deg) | Decl. (deg) | $z$  | $R$    | $L_r$ ($10^{10} K^{-2} L_\odot$) | $GGN/\gamma_{GGN}$ (Mpc$^{-1}$) | $M_{vir}$ ($10^{14} h^{-1} M_\odot$) | Method | Ref. |
|-------------|------------|-------------|------|-------|---------------------------------|----------------------------------|-------------------------------------|--------|------|
| Abell 2199  | 247.15930  | 39.55121    | 0.030| 17.76 | 28.49                           | 18.10                            | 2.39$^{+0.38}_{-0.38}$              | X-ray 1|      |
| Abell 2052  | 229.18536  | 7.02162     | 0.035| 9.50  | 21.97                           | 11.62                            | 3.42$^{+0.26}_{-0.26}$              | X-ray 2|      |
| Abell 2063  | 230.77209  | 8.60922     | 0.035| 15.30 | 22.96                           | 19.93                            | 1.47$^{+0.28}_{-0.28}$              | X-ray 2|      |
| Abell 2147  | 240.57094  | 15.97465    | 0.035| 12.22 | 22.50                           | 14.67                            | 1.58$^{+0.05}_{-0.06}$              | X-ray 2|      |
| Abell 1413  | 216.48611  | 37.81645    | 0.171| 34.16 | 84.15                           | 38.07                            | 2.31$^{+0.18}_{-0.18}$              | X-ray 2|      |
| Abell 2151w | 241.14914  | 17.72156    | 0.037| 11.08 | 22.16                           | 13.29                            | 2.78$^{+0.42}_{-0.42}$              | X-ray 2|      |
| MKW9        | 233.13339  | 4.68100     | 0.040| 8.88  | 15.77                           | 11.44                            | 1.24$^{+0.09}_{-0.09}$              | X-ray 2|      |
| Abell 1983  | 223.23048  | 16.70286    | 0.044| 9.27  | 11.76                           | 11.45                            | 1.01$^{+0.25}_{-0.25}$              | X-ray 3|      |
| Abell 160   | 18.24822   | 15.49129    | 0.045| 16.61 | 34.96                           | 16.54                            | 1.39$^{+0.53}_{-0.53}$              | X-ray 4|      |
| Abell 85    | 10.46029   | −9.30312    | 0.056| 23.61 | 54.65                           | 21.85                            | 1.41$^{+0.55}_{-0.55}$              | X-ray 4|      |
| Abell 91    | 223.63122  | 18.64232    | 0.059| 19.22 | 39.13                           | 17.48                            | 0.96$^{+0.14}_{-0.14}$              | X-ray 2|      |
| Abell 1975  | 207.21877  | 26.59293    | 0.063| 19.56 | 39.70                           | 19.56                            | 4.23$^{+1.15}_{-1.15}$              | X-ray 2|      |
| Abell 2092  | 233.31403  | 31.14515    | 0.067| 9.27  | 15.93                           | 11.45                            | 5.55$^{+0.53}_{-0.53}$              | X-ray 2|      |
| Abell 2065  | 230.60008  | 27.71436    | 0.072| 40.86 | 76.31                           | 40.86                            | 7.54$^{+0.49}_{-0.49}$              | X-ray 2|      |
| ZwCl 1215  | 184.42134  | 3.65584     | 0.075| 20.69 | 43.81                           | 18.71                            | 4.14$^{+4.31}_{-4.31}$              | X-ray 2|      |
| Abell 1800  | 207.34822  | 28.10732    | 0.075| 19.77 | 44.73                           | 18.83                            | 3.07$^{+0.43}_{-0.43}$              | X-ray 2|      |
| Abell 1775  | 205.45477  | 26.37347    | 0.076| 14.52 | 32.62                           | 17.76                            | 7.06$^{+0.69}_{-0.69}$              | X-ray 2|      |
| Abell 2029  | 227.73376  | 5.74478     | 0.135| 76.47 | 20.87                           | 60.63                            | 7.86$^{+0.70}_{-0.70}$              | X-ray 2|      |
| Abell 2555  | 258.11996  | 64.06072    | 0.080| 40.23 | 83.40                           | 33.51                            | 7.90$^{+0.25}_{-0.25}$              | X-ray 2|      |
| Abell 1650  | 194.67288  | −1.76146    | 0.084| 17.08 | 42.92                           | 20.68                            | 8.13$^{+0.21}_{-0.21}$              | X-ray 2|      |
| Abell 1692  | 198.05661  | −0.97448    | 0.085| 14.40 | 24.62                           | 14.73                            | 1.19$^{+0.38}_{-0.38}$              | X-ray 2|      |
| Abell 1750  | 202.71080  | −1.86197    | 0.088| 24.35 | 58.05                           | 27.58                            | 7.48$^{+0.64}_{-0.64}$              | X-ray 2|      |
| Abell 2142  | 239.58334  | 27.23341    | 0.090| 39.33 | 80.90                           | 39.88                            | 11.00$^{+2.86}_{-2.86}$             | X-ray 2|      |
| Abell 2244  | 255.67705  | 34.05999    | 0.097| 31.22 | 62.35                           | 28.67                            | 11.95$^{+2.24}_{-2.24}$             | X-ray 2|      |
| Abell 2034  | 227.54883  | 33.48646    | 0.113| 32.00 | 68.97                           | 31.08                            | 7.23$^{+0.22}_{-0.22}$              | X-ray 2|      |
| Abell 1068  | 160.18541  | 39.95313    | 0.138| 16.38 | 38.87                           | 16.38                            | 7.17$^{+0.30}_{-0.30}$              | X-ray 2|      |
| Abell 1413  | 178.82501  | 23.40491    | 0.143| 37.17 | 93.39                           | 41.82                            | 4.87$^{+0.42}_{-0.42}$              | X-ray 2|      |
| RXJ2172.1+2637 | 260.04184  | 26.62557    | 0.164| 26.87 | 69.11                           | 27.30                            | 2.36$^{+0.33}_{-0.33}$              | X-ray 2|      |
| Abell 1914  | 216.48611  | 37.81645    | 0.171| 34.16 | 84.15                           | 38.07                            | 5.84$^{+0.78}_{-0.78}$              | X-ray 2|      |
| MS 0906.5+1110 | 137.30312  | 10.97475    | 0.176| 36.00 | 119.08                          | 38.61                            | 5.84$^{+0.40}_{-0.40}$              | X-ray 2|      |

z in the field of the SDSS DR6.
| Name | R.A. (deg) | Decl. (deg) | z | $R$ | $L_{\nu}$ ($10^{10} h^{-2} L_{\odot}$) | $GGN/r_{GNN}$ (Mpc$^{-1}$) | $M_{\text{vir}}$ ($10^{14} h^{-1} M_{\odot}$) | Method | Ref. |
|------|------------|-------------|---|-----|-----------------------------|--------------------------|------------------------|--------|-----|
| Abell 1689 | 197.87291 | $-1.34108$ | 0.184 | 62.74 | 136.85 | 50.32 | $10.50^{+1.94}_{-1.91}$ | WL | 12 |
| | | | | | | | $11.69^{+1.80}_{-1.78}$ | SL+WL | 13 |
| | | | | | | | $13.24^{+3.99}_{-3.99}$ | X-ray | 14 |
| | | | | | | | $13.43^{+0.96}_{-0.96}$ | X-ray | 2 |
| | | | | | | | $13.51^{+1.40}_{-1.40}$ | WL | 15 |
| | | | | | | | $14.70^{+1.40}_{-1.40}$ | WL | 16 |
| | | | | | | | $17.55^{+1.40}_{-1.40}$ | WL | 17 |
| | | | | | | | $20.53^{+1.74}_{-1.74}$ | WL | 18 |
| | | | | | | | $25.50^{+3.30}_{-3.30}$ | WL | 11 |
| Abell 963 | 154.26515 | 39.04705 | 0.206 | 43.00 | 93.83 | 38.52 | $3.45^{+0.80}_{-0.80}$ | WL | 17 |
| | | | | | | | $6.50^{+2.00}_{-2.00}$ | WL | 11 |
| | | | | | | | $6.53^{+2.00}_{-2.00}$ | X-ray | 14 |
| | | | | | | | $7.00^{+2.45}_{-2.45}$ | X-ray | 7 |
| | | | | | | | $8.01^{+8.14}_{-6.36}$ | WL | 9 |
| | | | | | | | $8.38^{+7.46}_{-7.46}$ | WL | 10 |
| Abell 1423 | 179.32219 | 33.61092 | 0.214 | 20.78 | 64.82 | 21.16 | $14.98^{+7.67}_{-7.67}$ | WL | 10 |
| | | | | | | | $15.10^{+9.40}_{-9.40}$ | X-ray | 7 |
| RX J1504.1−0248 | 226.03130 | $-2.80460$ | 0.215 | 20.08 | 73.32 | 26.82 | $10.63^{+3.27}_{-3.27}$ | X-ray | 14 |
| | | | | | | | $15.33^{+6.65}_{-6.65}$ | WL | 10 |
| | | | | | | | $24.70^{+9.44}_{-9.44}$ | WL | 9 |
| Abell 1682 | 196.70833 | 46.55927 | 0.226 | 40.76 | 112.14 | 39.49 | $3.96^{+3.52}_{-3.52}$ | WL | 9 |
| | | | | | | | $5.49^{+3.72}_{-3.72}$ | WL | 10 |
| | | | | | | | $6.23^{+3.92}_{-3.92}$ | X-ray | 14 |
| | | | | | | | $8.91^{+5.79}_{-5.79}$ | WL | 9 |
| | | | | | | | $10.54^{+3.39}_{-3.39}$ | WL | 10 |
| | | | | | | | $12.28^{+2.17}_{-2.17}$ | WL | 17 |
| | | | | | | | $13.50^{+3.70}_{-3.70}$ | WL | 11 |
| | | | | | | | $8.19^{+3.49}_{-3.49}$ | WL | 10 |
| | | | | | | | $8.57^{+4.46}_{-4.46}$ | WL | 9 |
| | | | | | | | $11.30^{+3.25}_{-3.25}$ | WL | 11 |
| | | | | | | | $18.66^{+3.34}_{-3.34}$ | WL | 17 |
| Abell 1763 | 203.83372 | 41.00115 | 0.228 | 34.10 | 109.20 | 32.77 | $6.12^{+3.64}_{-3.64}$ | WL | 10 |
| | | | | | | | $6.92^{+3.95}_{-3.95}$ | WL | 9 |
| | | | | | | | $2.36^{+3.20}_{-3.20}$ | WL | 17 |
| | | | | | | | $5.36^{+3.30}_{-3.30}$ | X-ray | 14 |
| | | | | | | | $7.50^{+2.83}_{-2.83}$ | WL | 11 |
| | | | | | | | $14.98^{+3.33}_{-3.33}$ | WL | 10 |
| | | | | | | | $16.34^{+4.53}_{-4.53}$ | WL | 9 |
| | | | | | | | $3.15^{+3.25}_{-3.25}$ | WL | 10 |
| | | | | | | | $1.50^{+9.99}_{-9.99}$ | WL | 11 |
| | | | | | | | $5.96^{+3.70}_{-3.70}$ | X-ray | 7 |
| | | | | | | | $7.51^{+2.24}_{-2.24}$ | WL | 10 |
| | | | | | | | $15.49^{+9.24}_{-9.24}$ | WL | 9 |
| | | | | | | | $8.19^{+3.07}_{-3.07}$ | WL | 12 |
| | | | | | | | $8.41^{+2.57}_{-2.57}$ | X-ray | 14 |
| | | | | | | | $10.61^{+5.69}_{-5.69}$ | WL | 10 |
| | | | | | | | $15.62^{+8.56}_{-8.56}$ | WL | 9 |
| | | | | | | | $17.00^{+3.30}_{-3.30}$ | X-ray | 7 |
| | | | | | | | $24.21^{+3.76}_{-3.76}$ | WL | 17 |
| | | | | | | | $5.75^{+3.26}_{-3.26}$ | WL | 9 |
| | | | | | | | $7.30^{+1.90}_{-1.90}$ | WL | 11 |
| | | | | | | | $12.88^{+6.07}_{-6.07}$ | WL | 10 |
| | | | | | | | $6.12^{+2.45}_{-2.45}$ | WL | 10 |
| | | | | | | | $5.26^{+5.79}_{-5.79}$ | WL | 8 |
| | | | | | | | $26.94^{+13.47}_{-13.47}$ | WL | 10 |
| | | | | | | | $39.05^{+13.16}_{-13.16}$ | WL | 9 |
2 MASS SCALING RELATIONS FOR CLUSTERS

We identified 39,668 clusters from the SDSS DR6 by discrimination of luminous member galaxies with following steps (Wen et al. 2009). First, we assume that each galaxy at a given photometric redshift $z$ is the central galaxy of a cluster candidate, and we count the number of luminous “member galaxies” of $M_r \leq -21$ within a radius of 0.5 Mpc and a photometric redshift gap of $z \pm 0.04(1+z)$. We set $\Delta z = 0.04(1+z)$ for the gap to allow variable uncertainties of photometric redshifts at different redshifts. Second, we define the center of a cluster candidate to be the position of the galaxy with a maximum number count. The cluster redshift is estimated to be the median value of the photometric redshifts of the recognized “members”. Third, for each cluster candidate at $z$, all galaxies within 1 Mpc from the cluster center and $z \pm 0.04(1+z)$ are assumed to be the member galaxies. Their absolute magnitudes are re-calculated with the cluster redshift. Finally, a cluster at $z$ is identified when the number of member galaxies of $M_r \leq -21$ reaches 8 within a projected radius of 0.5 Mpc and $z \pm \Delta z$. Monte-Carlo simulations show that the detection rate is more than 90% for massive clusters (richness $R \geq 16.7$) if the redshift uncertainty of cluster galaxies is about 0.03(1+z).

We defined the cluster richness, $R$, to be the total number of galaxies ($M_r \leq -21$) within a radius of 1 Mpc and $z \pm 0.04(1+z)$.
Mass Function of Rich Galaxy Clusters

Figure 2. Correlations between cluster mass \( M_{\text{vir}} \) and richness \( R \), summed luminosity \( L_r \) and \( \text{GGN}/r_{\text{GGN}} \) for 17 clusters in the redshift range \( 0.17 < z < 0.26 \). The black dots are the clusters with more than three estimates of their masses. The solid lines are the same shown in Figure 1. The dashed line is the new scaling relation with the same slope but different offsets determined from the data.

Here, we convert the mass of \( M_{200} \) and \( M_{500} \) to the virial mass \( M_{\text{vir}} \) according to Shimizu et al. (2003). We will discuss later the influence on our result from a possible bias conversion. For each cluster with mass estimated, we calculate the cluster richness, the summed \( r \)-band luminosity and \( \text{GGN}/r_{\text{GGN}} \) following the method of Wen et al. (2009). Only clusters of richness \( R \geq 8 \) are listed in Table I since the uncertainties of \( R \) and the summed luminosities become larger for clusters with a smaller \( R \).

We notice that clusters with estimated masses preferentially have low \( z < 0.1 \) and mediate \((0.2 < z < 0.25)\) redshifts (see Table I). To minimize the uncertainty, we determine the scaling relations between the masses and observational tracers for clusters in the two small redshift ranges independently. This is because the discrimination of member galaxies (e.g., completeness or contamination rate) may be different for clusters at different redshifts, and the systematic bias can be ignored in such a small range. In the low redshift range \( z < 0.1 \), the masses of many clusters are available and distributed in a large mass range, which is good for determination of the scaling relations. We get 15 clusters of \( 0.05 < z < 0.1 \). We also include 8 clusters of \( 0.03 < z < 0.05 \) and one cluster of \( z = 0.113 \) to derive the scaling relations at the low redshift range. Several clusters have multiple estimates for mass from literature, we adopt the median value or the average of two middle ones for even measurements.

The mass–richness relation, i.e., the so called halo occupation distribution in some literature (e.g., Popesso et al. 2007b), is described by a power law, \( R \propto M^\nu \). The correlation of cluster mass with the optical luminosity, i.e., the mass-to-light ratio \( M/L \), is also described by a power law, \( M/L \propto L^\nu \), i.e., \( M \propto L^{1+\nu} \). In Figure 1 we show the correlations between cluster mass and cluster richness, summed luminosity and \( \text{GGN}/r_{\text{GGN}} \) for 24 nearby clusters. The uncertainties of richness \( R \), summed luminosity \( L_r \) and \( \text{GGN}/r_{\text{GGN}} \) are about 10%–20% (Wen et al. 2009). We fit the correlations with power-law relations,

\[
\log M_{\text{vir}} = (-1.43 \pm 0.07) + (1.55 \pm 0.06) \log R, \tag{1}
\]
\[
\log M_{\text{vir}} = (-1.77 \pm 0.08) + (1.49 \pm 0.05) \log L_r, \tag{2}
\]

after subtracting the local background, i.e., the average number of luminous galaxies. The summed \( r \)-band luminosity of each cluster, \( L_r \), is calculated as the total luminosity of member galaxies within the region also after subtracting the background. From the radial distribution of member galaxies, we got the cluster radius, \( r_{\text{GGN}} \), where the density of galaxies is as low as background. Here, we defined the Gross Galaxy Number (GGN) of a cluster as the total number of luminous galaxies \( (M_r \leq -21) \) within the radius \( r_{\text{GGN}} \) and the redshift gap of \( z \pm 0.04(1 + z) \) after subtracting the local background. It has been known for a long time that the cluster richness and summed luminosity are related to cluster mass (Schindler 1996). Previous studies usually provided the mass within a large radius \( R_{500} \) or \( R_{200} \) (e.g., Reiprich & Böhringer 2002; Pedersen & Dahl 2007).
and
\[
\log M_{\text{vir}} = (-2.11 \pm 0.10) + (2.03 \pm 0.08) \log(GGN/\sigma_{GGN}).
\tag{3}
\]
Here, \(M_{\text{vir}}\) has a unit of \(10^{14} h^{-1} M_{\odot}\), \(L_r\) has a unit of \(10^{39} h^{-2} L_{\odot}\). The uncertainty of the estimated cluster mass, \(\sigma_{\log M_{\text{vir}}}\), is mainly determined by the uncertainties of the intercept and the slope in the logarithm for three scaling relations in Equations (1)–(3). Yee & Ellingson (2003) defined \(B_{\text{gg}}\) to be the amplitude of galaxy-cluster cross-correlation function and found \(M_{\text{vir}} \propto B_{\text{gg}}^{1.64 \pm 0.28}\). The slope is in agreement with that of our \(M_{\text{vir}}\) to \(GGN/\sigma_{GGN}\) relation. These scaling relations, Equation (1)–(3), will be used to estimate masses of a complete volume-limited sample of clusters in the local universe for cluster mass function.

We can also use a much larger cluster sample at redshift \((0.2 < z < 0.25)\) for cluster mass function. Some massive clusters in this redshift range have their masses estimated (see Table I). We obtain masses of 17 clusters in the redshift range of \(0.17 < z < 0.26\), of which 10 clusters have more than three estimates. In Figure 2, we show the correlations between cluster mass and cluster richness, summed luminosity and \(GGN/\sigma_{GGN}\) for the 17 clusters. Most of them are similarly massive of \(10^{15} h^{-1} M_{\odot}\) and few have smaller masses, so that it is difficult to determine a new scaling relations. Here, we calibrate the mass scaling relations by assuming the same slopes of Equations (1)–(3) and finding the offsets. We then get the scaling relations,
\[
\log M_{\text{vir}} = (-1.57 \pm 0.12) + 1.55 \log R,
\tag{4}
\]
\[
\log M_{\text{vir}} = (-2.03 \pm 0.06) + 1.49 \log L_r,
\tag{5}
\]
and
\[
\log M_{\text{vir}} = (-2.33 \pm 0.11) + 2.03 \log(GGN/\sigma_{GGN}).
\tag{6}
\]
The uncertainties in Equation (4)–(6) reflect the scatters of masses to the mean relations (dashed line). We notice that the scatter is the smallest for the \(M_{\text{vir}}-L_r\) relation for the high redshift data, because clusters with more than three estimates (black dots) are very consistent with the fitting relation (dashed line). Therefore, the cluster masses estimated by the \(M_{\text{vir}}-L_r\) relation may be more accurate than other tracers. The offsets between the relations for samples at two redshift ranges may come from the problem of the SDSS galaxy data. The sky background level is overestimated for nearby bright galaxies \((12.5 < r < 15.5)\), so that galaxies have systematically fainter magnitudes by 0.15–0.2 mag than their true magnitude. Adelman-McCarthy et al. (2008). This can result in systematically lower cluster richness and summed luminosity for clusters of \(0.05 < z < 0.1\) than clusters of \(0.2 < z < 0.25\). The two scaling relations are used to samples of clusters at two redshift ranges independently. Hence, the systematic bias does not affect the final \(\sigma_8\) values from each sample.

3 CLUSTER MASS FUNCTIONS

Assuming a Gaussian distribution of mass fluctuation, Press & Schechter (1974) used a linear theory to derive the first theoretical expression of cluster mass function, which is in agreement with mass functions derived from observations and numerical simulations within a large mass range (e.g., White et al. 1993; Reiprich \& Böhringer 2002). Recent simulations show slightly more massive clusters than the Press \& Schechter mass function gives (Sheth \& Tormen 1999; Jenkins et al. 2001; Warren et al. 2006). In this work, we take the form of the cluster mass function as Equation (B4) of Jenkins et al. (2001). The mean differential comoving number density of dark matter halos is
\[
\frac{d\rho}{dM} = \frac{\rho_0}{M^2} \frac{d\ln M'}{d\ln M} \exp\left(\left[-\ln \sigma_1 - 0.67\right]^{3.82}\right).
\tag{7}
\]
Here, \(\rho_0 = 2.78 \times 10^{11} \Omega_m h^2 M_{\odot} \text{ Mpc}^{-3}\) is the comoving density of the universe. \(M\) is the halo mass within a radius with a mean overdensity of 324 times of the mean density of the universe (roughly the virial mass, \(M_{\text{vir}}\), if \(\Omega_m = 0.3\)). \(\sigma^2(M, z)\) is the variance of the linearly evolved density field smoothed by a spherical top-hat filter that enclose mass \(M\). Here, \(\sigma(M, z) = \sigma_8 \times f\), where \(\sigma_8\) is the present linear rms mass fluctuation on the scale of \(8 h^{-1}\) Mpc and \(f\) is a function of \(M, z, \Omega_m\) as well as the Hubble constant \(h\), the abundance of baryons \(\Omega_b\) and the present cosmic microwave background temperature \(T_{\text{CMB}}\). \(d\ln M^{-1}/d\ln M\) can be derived from the expression of \(\sigma(M, z)\) (see details of \(\sigma(M, z)\) in Reiprich \& Böhringer 2002). The values of \(\Omega_m\) and \(\sigma_8\) are the main parameters to define the mass function. The other parameters does not strongly affect the results in our analysis, thus can be fixed.

The \(\sigma_8\) strongly depends on cluster mass function at the high mass end. Since the mass function is steep at high mass end, the data scatter for mass scaling relations induces more low mass to higher mass. Thus, the uncertainty of the mass scaling relation, \(\sigma_{\log M}\), is included in the fitting. We re-write the mass function with the uncertainty on mass estimate to be the Jenkins function convolved by a Gaussian function,
\[
\frac{d\rho}{dM} = \int \frac{d\rho}{dM}g(M - M', \sigma_{\log M})d\log M',
\tag{8}
\]
where \(g(x, \sigma) = \frac{\exp(-x^2/2\sigma^2)}{(\sqrt{2\pi}\sigma)}\).

First, we use a complete volume-limited sample of rich clusters \((R \geq 16.7, 90\%\) complete\) in the local universe \((0.05 < z < 0.1)\) to determine the cluster mass function. Since the photometric redshift was used to identify the cluster member galaxies, the absolute magnitudes of member galaxies could have large uncertainties when the estimated cluster redshift slightly deviates from its true redshift. To reduce the uncertainty at low redshift, we use the spectroscopic redshifts of clusters if its discriminated members are spectroscopically observed. The cluster richness, the summed \(r\)-band luminosity and \(GGN/\sigma_{GGN}\) are re-calculated as Wen et al. (2009). In this sample, 56 clusters have richness \(R \geq 16.7\), which are used to determine the cluster mass function in the local universe.

We apply the scaling relations of Equations (1)–(3) to these 56 rich clusters in the local universe and calculate the number of clusters as a function of mass. Figure 3 shows the cluster mass functions and the best fit with Equation (8). From the probability contours in the \(\sigma_8-\Omega_m\) plane for three mass tracers (Figure 4), we find that the \(\sigma_8\) and \(\Omega_m\) are coupled in the form of \(\sigma_8(\Omega_m/0.3)^\alpha = \beta\). From the cluster mass distribution using the mass–richness scaling relation, we find
\[
\sigma_8\left(\frac{\Omega_m}{0.3}\right)^{0.42 \pm 0.03} = 0.82 \pm 0.04.
\tag{9}
\]
From the cluster mass distribution using the mass–luminosity scaling relation, we find
\[
\sigma_8\left(\frac{\Omega_m}{0.3}\right)^{0.46 \pm 0.03} = 0.90 \pm 0.04.
\tag{10}
\]
Figure 3. Mass function for a sample of 56 rich clusters \((R \geq 16.7, 0.05 < z < 0.1)\). The error bars on the horizontal axis are calculated from the uncertainties of Equation (1)–(3), and the error bars on the vertical axis are calculated by Poisson statistics. The solid line is the best fit with the cluster mass function of Equation (8). The dashed line is the cluster mass function of Equation (8) with \(\Omega_m = 0.273\) and \(\sigma_8 = 0.813\) from the WMAP5 data (Komatsu et al. 2009). Data for the mass functions from Reiprich & Böhringer (2002) and Rines et al. (2007) are plotted for comparison.

Figure 4. The probability contour in the \(\sigma_8-\Omega_m\) plane for three corresponding mass tracers in Figure 3, 68% confidence level for the inner curve and 99% for the outer curve.

\[
\sigma_8 \left( \frac{\Omega_m}{0.3} \right) = 0.40 \pm 0.03 = 0.83 \pm 0.04.
\] (11)

During the fitting, we have taken into account only statistical uncertainties. Assuming \(\Omega_m = 0.3\), the value of \(\sigma_8\) is \(0.82 \pm 0.04, 0.90 \pm 0.04\) and \(0.83 \pm 0.04\) for masses scaled from cluster richness, summed luminosity and \(GGN/r_{GGN}\), respectively.

We also apply the scaling relations of Equation (4)–(6) to the a complete volume-limited sample of 810 rich clusters \((R \geq 16.7)\) of \(0.2 < z < 0.25\) to calculate their masses, and get the cluster mass function. Again, spectroscopic redshifts of 466 clusters are used since they are available from the SDSS, otherwise photometric redshifts are used. Figure 5 shows the cluster mass functions and

Figure 5 shows the contours in the \(\sigma_8-\Omega_m\) plane based on three mass tracers. Since there are much more clusters in this sample, the mass functions have small errors than those of \(0.05 < z < 0.1\). We fit the data to Equation (8), and find

\[
\sigma_8 \left( \frac{\Omega_m}{0.3} \right) = 0.42 \pm 0.01 = 0.85 \pm 0.02,
\] (12)

\[
\sigma_8 \left( \frac{\Omega_m}{0.3} \right) = 0.46 \pm 0.01 = 0.94 \pm 0.02,
\] (13)

\[
\sigma_8 \left( \frac{\Omega_m}{0.3} \right) = 0.39 \pm 0.01 = 0.82 \pm 0.02.
\] (14)
Figure 5. The same as Figure 3 but for a sample of 810 rich clusters ($R \geq 16.7$, $0.2 < z < 0.25$). The curve from the WMAP5 result by Komatsu et al. (2009) is plotted for comparison.

Figure 6. The same as Figure 4 but corresponding to the three mass tracers in Figure 5 for the cluster sample of $0.2 < z < 0.25$.

for the cases using the mass tracer of richness, summed luminosity and the $GGN/r_{GGN}$, respectively. Assuming $\Omega_m = 0.3$, the value of $\sigma_8$ is $0.85 \pm 0.02$, $0.94 \pm 0.02$ and $0.82 \pm 0.02$, respectively. They are consistent with those from the cluster sample of $0.05 < z < 0.1$ for each mass tracer.

### 4 DISCUSSIONS AND CONCLUSIONS

Cluster mass function can be accurately determined from a complete volume-limited sample. The scaling relations of cluster mass have been determined for three optical observations, cluster richness, summed luminosity and $GGN/r_{GGN}$. The scaling relations are then used to estimate cluster mass for two samples of rich clusters. We get cluster mass functions and fit them with a theoretical expression. Cosmological parameters are constrained in the form of $\sigma_8(\Omega_m/0.3)^{\alpha} = \beta$, with $\alpha = 0.40$–0.50 and $\beta = 0.8$–0.9. For $\Omega_m = 0.3$, we get $\sigma_8 = 0.8$–0.9 using different mass tracers or using the rich cluster samples at different redshift ranges.

The $\sigma_8$ values from the mass tracers of richness $R$ and $GGN/r_{GGN}$ obtained using both cluster samples are consistent, while $\sigma_8$ values derived from $L_r$ are higher. This discrepancy may come from some potential systematic bias on the mass scaling relations. If the $M_{vir}-L_r$ relations for both samples are really unbiased, then the cluster masses tracer by richness $R$ and $GGN/r_{GGN}$ are systematically underestimated. However, it is hard to assess which one is a better mass tracer. Given the scarce of mass estimates from different methods for the same clusters in Table 1 for the scaling relations, it is also hard to estimate the systematic bias on these mass estimates due to different methods (X-ray or weak lensing).
Mass Function of Rich Galaxy Clusters

Table 2. Comparison of results on $\sigma_8$-$\Omega_m$ derived from cluster mass function (upper part) and cosmic microwave background (CMB) measurement (middle part). See Table 5 of [Hetterscheidt et al. 2007] for the results derived from weak lensing statistics.

| Reference            | Sample or method | No. of clusters or observation | $\sigma_8$-$\Omega_m$ relation | $\sigma_8$ ($\Omega_m = 0.30$) |
|----------------------|------------------|---------------------------------|--------------------------------|--------------------------------|
| Viana & Liddle (1996)| X-ray            | 25                              | $\sigma_8 = 0.60\Omega_m^{0.59} + 0.16\Omega_m^{0.66}$ | 1.16                           |
| Eke et al. (1996)    | X-ray            | 25                              | $\sigma_8 = 0.52 \pm 0.04\Omega_m^{0.52} + 0.13\Omega_m$ | 0.93$\pm$0.07                   |
| Markevitch (1998)   | X-ray            | 30                              | $\sigma_8 = 0.78 \pm 0.04$ with $\Omega_m = 0.30$ fixed | 0.78$\pm$0.04                   |
| Pen (1998)           | X-ray            | 70                              | $\sigma_8 = 0.53\Omega_m^{0.53}$ | 1.00                           |
| Borgani et al. (1999)| X-ray           | 70                              | $\sigma_8 = 0.58 \pm 0.06\Omega_m^{0.47} + 0.16\Omega_m$ | 0.96                           |
| Viana & Liddle (1999)| X-ray            | 10                              | $\sigma_8 = 0.56\Omega_m^{0.47}$ | 0.99                           |
| Blanchard et al. (2000)| X-ray       | 25                              | $\sigma_8 = 0.96$ with $\Omega_m = 0.30$ fixed | 0.96                           |
| Oukbir & Arnaud (2001)| X-ray         | 69                              | $\sigma_8 = 0.59\Omega_m^{0.57} + 1.45\Omega_m^{0.60} - 3.48\Omega_m^{0.71} + 3.77\Omega_m^{0.67} - 1.49\Omega_m^{0.63}$ | 0.91                           |
| Wu (2001)            | X-ray            | 25                              | $\sigma_8 = 0.47\Omega_m^{0.06}$ | 0.87                           |
| Borgani et al. (2001) | X-ray           | 103                             | $\sigma_8 = 0.66 \pm 0.05\Omega_m^{0.35} \pm 0.13$ | 0.48$\pm$0.07                   |
| Pierpaoli et al. (2001)| X-ray         | 30                              | $\sigma_8 = 0.49\Omega_m^{0.37}$ | 1.02$\pm$0.07                   |
| Viana et al. (2002)  | X-ray            | 452                             | $\sigma_8 = 0.38\Omega_m^{0.48} + 0.77\Omega_m^{0.67}$ | 0.61                           |
| Reiprich & Böhringer (2002) | X-ray       | 106                             | $\sigma_8 = 0.43\Omega_m^{0.38}$ | 0.68                           |
| Selić (2002)         | X-ray            | 30                              | $\sigma_8 = 0.77 \pm 0.07$ | 0.77$\pm$0.07                   |
| Viana et al. (2003)  | X-ray            | 40                              | $\sigma_8 = 0.78 \pm 0.06$ with $\Omega_m = 0.35$ fixed | 0.77$\pm$0.05                   |
| Dahle (2006)         | X-ray            | 35                              | $\sigma_8 = 0.71 \pm 0.05\Omega_m^{0.31} \pm 0.03\Omega_m$ | 0.77$\pm$0.05                   |
| Rines et al. (2007)  | X-ray            | 66                              | $\sigma_8 = 0.77 \pm 0.04$ with $\Omega_m = 0.30$ fixed | 0.77$\pm$0.04                   |
| Henry et al. (2009)  | X-ray            | 48                              | $\sigma_8 = 0.86 \pm 0.04$ for $\Omega_m < 0.32$ | 0.88$\pm$0.04                   |
| Vikhlinin et al. (2009)| X-ray          | 49                              | $\sigma_8 = 0.78 \pm 0.04$ for $\Omega_m > 0.32$ | 0.75$\pm$0.02                   |
| White et al. (1993)  | Optical          | 111                             | $\sigma_8 = 0.50\Omega_m^{0.019}\Omega_m^{0.25} \pm 0.024$ | 0.69$\pm$0.05                   |
| Bahcall & Fan (1998) | Optical          | 3                               | $\sigma_8 = 0.97 \pm 0.24\Omega_m^{0.047}$ | 0.84$\pm$0.03                   |
| Girardi et al. (1998)| Optical          | 152                             | $\sigma_8 = 0.86 \pm 0.04$ for $\Omega_m < 0.32$ | 0.88$\pm$0.04                   |
| Bahcall et al. (2003)| Optical          | 300                             | $\sigma_8 = 0.86 \pm 0.04$ for $\Omega_m > 0.32$ | 0.75$\pm$0.02                   |
| Eke et al. (2006)    | Optical          | 13832                           | $\sigma_8 = 0.92 \pm 0.05\Omega_m^{0.25} \pm 0.41$ | 0.83$\pm$0.03                   |
| Rozo et al. (2010)   | Optical          | 13832                           | $\sigma_8 = 0.92 \pm 0.05\Omega_m^{0.25} \pm 0.41$ | 0.83$\pm$0.03                   |
| Komatsu et al. (2009)| CMB              | WMAP5                           | $\sigma_8 = 0.92 \pm 0.04\Omega_m^{0.32} \pm 0.03$ | 0.82$\pm$0.04                   |
| Liu & Li (2009)      | CMB              | WMAP5                           | $\sigma_8 = 0.92 \pm 0.04\Omega_m^{0.32} \pm 0.03$ | 0.82$\pm$0.04                   |
| Larson et al. (2010) | CMB              | WMAP7                           | $\sigma_8 = 0.92 \pm 0.04\Omega_m^{0.32} \pm 0.03$ | 0.82$\pm$0.04                   |
| Readhead et al. (2004)| CMB         | CBI                             | $\sigma_8 = 0.96 \pm 0.06$ | 0.95$\pm$0.04                   |
| Dawson et al. (2006) | CMB              | BIMA                            | $\sigma_8 = 0.93 \pm 0.04$ | 0.95$\pm$0.04                   |
| Reichardt et al. (2009)| CMB        | ACBAR                           | $\sigma_8 = 0.93 \pm 0.04$ | 0.95$\pm$0.04                   |
| Sievers et al. (2009)| CMB             | CBI                             | $\sigma_8 = 0.92 \pm 0.05$ | 0.95$\pm$0.04                   |
| Savers et al. (2009) | CMB              | Bolocam                         | $\sigma_8 < 1.57$ (90% confidence level) | 0.96$\pm$0.04                   |
| Veneziani et al. (2009)| CMB         | BOOMERANG                       | $\sigma_8 < 0.92$ (95% confidence level) | 0.96$\pm$0.04                   |

In our work, one potential systematic bias may come from the conversion of cluster mass from measured radii to the virial radius. Here, we use $\gamma = M_{\text{vir}}/M_{\text{vir,true}}$ to stand for the systematic bias of masses in Table 1 where $M_{\text{vir,true}}$ stands for the true virial mass of a cluster. Assuming a $\gamma$, we get $M_{\text{vir,true}}$ and then fit the mass function of clusters to obtain $\sigma_8$. Figure 2 shows the variation of $\sigma_8$ (with $\Omega_m = 0.3$ fixed) as a function of $\gamma$ based on the $M_{\text{vir}}-L_r$ relation. We are only concerned about the cases $\gamma \geq 1$. For example $\gamma = 1.3$, i.e., masses systematically overestimated by 30%, the values of $\sigma_8$ are lower by about 10%. In fact, the deviation of $\gamma$ from 1.0 is related to the uncertainty of intercept in the logarithm scaling relations in Equation (1)–(6). The other possible systematic bias on $\sigma_8$ may come from the slope uncertainties of the scaling relations. Here, we illustrate the dependence of $\sigma_8$ on the slope uncertainty, $\Delta \nu$. We only apply to the $M_{\text{vir}}-L_r$ relation, for example. Given a $\Delta \nu$, i.e., $M_{\text{vir}} = A L_r^{1+\nu+\Delta \nu}$, here $\nu = 1.49$ according to Equation (4) and (5), we fit the power law with the data in Figure 4 and 2 to get $A$, and then get the cluster mass function and fit for
The value of $\sigma_8$ (with $\Omega_m = 0.3$ fixed) from cluster masses based on the $M_{\text{vir}}-L_r$ relation varies with a possible systematic bias on mass conversion of $\gamma = M_{\text{vir}}/M_{\text{vir,true}}$.

Figure 7. The value of $\sigma_8$ (with $\Omega_m = 0.3$ fixed) from cluster masses based on the $M_{\text{vir}}-L_r$ relation varies with a possible systematic bias on the slope of the scaling relation by $\Delta \nu$.

Figure 8. The value of $\sigma_8$ (with $\Omega_m = 0.3$ fixed) from cluster masses based on the $M_{\text{vir}}-L_r$ relation varies with a possible systematic bias on the slope of the scaling relation by $\Delta \nu$.

$\sigma_8$. Figure 8 shows the $\sigma_8$ value varies with $\Delta \nu$. We find that the $\sigma_8$ from the cluster sample of $0.05 < z < 0.1$ does not change significantly with $\Delta \nu$, while the $\sigma_8$ decreases from 1.05 to 0.81 for the cluster sample of $0.2 < z < 0.25$ when the slope varies by $\Delta \nu$ from -0.4 to 0.4.

We can compare our results of $\sigma_8$ with previous determinations from cluster mass function, as listed in Table 2. Most of previous results are based on X-ray flux-limited cluster samples. Our results are systematically larger than those from the mass function of X-ray clusters. Rozo et al. (2010) used the largest number of clusters from SDSS maxBCG catalog to determine the amplitude of cluster mass function. They did not estimate the mass for each cluster, but gave the $\Omega_m = 0.26$ derived from WMAP7, then our values of $\sigma_8$ should become larger by a factor of $(0.26/0.3)^{-0.22} = 1.06$, roughly equal to adding 0.05 to the our $\sigma_8$ value in Table 2. Therefore, the $\sigma_8$ values we derived from galaxy clusters are slightly larger than the those from the WMAP data (Komatsu et al. 2009; Larson et al. 2010). While some reanalysis of the WMAPS data independently (Li et al. 2003) gives $\sigma_8 = 0.921 \pm 0.036$ for $\Omega_m = 0.32 \pm 0.03$ (see Liu & Li 2009). Some studies of cosmic microwave background at small scales also give higher values of $\sigma_8$ than that from WMAP (Readhead et al. 2004; Dawson et al. 2006; Reicardt et al. 2009; Sievers et al. 2009).

Our result of $\sigma_8$ is consistent with many recent studies using other methods. For example, the $\sigma_8$ by weak lensing method has a mean value of $0.85 \pm 0.03$ (see previous results in Table 5 of Hetterscheidt et al. 2007), which is higher than previous results from X-ray clusters. Tegmark et al. (2004) studied the power spectrum of galaxies from the SDSS to constrain cosmological parameters. They obtained $\sigma_8 = 0.89 \pm 0.02$ and $\Omega_m = 0.30 \pm 0.03$. Led (2009) studied the normalization of the power spectrum via the ellipticity function of giant galaxy voids from SDSS DR5 and obtained $\sigma_8 = 0.90 \pm 0.04$ (Iena et al. 2005) used the Ly$\alpha$ data and found $\sigma_8 = 0.9$ and $\Omega_m = 0.27$. Feldman et al. (2003) used the galaxy peculiar velocities to probe the growth rate of the structure and found that $\sigma_8 = 1.13^{+0.22}_{-0.23}$ and $\Omega_m = 0.30^{+0.17}_{-0.07}$.

In this work, we get six values of $\sigma_8$ by cluster mass function. Basically, the results are consistent. However, the precise value of $\sigma_8$ is still to be determined since our constraint is not only coupled with $\Omega_m$, but also has large uncertainties on the scaling relations.

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