An improved weighted fusion algorithm of multi-sensor

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Abstract. Multi-sensor data fusion is to take full advantage of the complementary nature of multivariate data to improve the feasibility of the statistics. The weighted fusion algorithm is commonly used due to its easiness to achieve. Among the relative factors, the weight directly impacts the results of the data fusion, therefore, the selection of weight is particularly important, as choosing an inappropriate weight will lead to the instability of algorithm performance. To find the best weight, we develop an improved weighted fusion algorithm, introducing the concept of the optimal proportion weight and using secondary weighted approach ---- the single sensor will be weighted individually before the whole sensor system is weighted in order to achieve the optimal algorithm performance.

1. Introduction
Multi-sensor data fusion is a rapidly developing integrated information processing technology in recent years, it will process the information and data from multi-sensor or multi-source so as to make the conclusion more accurate and credible. This technology will not only enhance the accuracy and reliability of the system, but will also improve the measuring scope of the system, increase the credibility of the system and shorten the response time. The weighted fusion algorithm is a relatively more mature kind of fusion algorithm. Its optimality, unbiasedness, minimum mean square error have been proved by many research results. The core issue of the weighted fusion algorithm is how to determine the weights, because the selecting of the weights directly impacts the results of fusion[1].

The weighted average fusion algorithm is one of the most simple and popular ways, it takes the weighted mean of the redundant information provided by multi-sensors as the value of fusion. This method can deal with real-time dynamic raw data, but it does not achieve the optimal result practically due to the subjectivity of weight selection. To make some improvement, the secondary weighted approach and the concept of the optimal proportion weight are introduced. Firstly, a single sensor will be weighted, and then the whole sensor system will be weighted to derive the improved weighted fusion formula. Through simulation and the comparison with the equally weighted fusion method applied in the weighted average method, its effectiveness will be verified.

2. Multi-sensor data fusion weighted
The weighted data fusion refers to the measurement of the data of the same characteristic parameters with the help of multi-sensor in a certain environment. Combined with the local estimation of the individual sensor, it designates the weight for each sensor under a certain principle and gets the optimal estimated value through synthesizing all the local estimated values[2].
2.1. Description of the state observation equation of Multi-sensor

While observing moving targets, the sensors distributed in different locations will create a collection of observations of different time and space. If the volume of sensors is $S$, and the volume of time is $N$, then the number of observations is $S \times N$, which can be expressed with the set $Z$:

\[ Z = \{Z_j\} \quad (j=1, 2, \ldots, s) \]  
\[ Z_j = \{Z_j(k)\} \quad (k=1, 2, \ldots, n) \]

Set up the multi-sensor state observation equation:

\[ Z =HX + V \]  

In this formula, $Z$ is the $s$-dimensional observation vector; $H$ is the known $s \times s$ matrix; $X$ is the $s$-dimensional state vector to be tested; $V$ is the $s$-dimensional noise vector, and

\[ V = (v_1, v_2, \ldots, v_s)^T \]

Assuming the measurement noise of all of the sensors is the white noise independent of each other and subject to normal distribution, then

\[ E(v_j) = 0 \quad (j=1, 2, \ldots, s) \]  
\[ E(v_j^2) = E[(x_j - z_j^2)] = R_j \quad (j=1, 2, \ldots, s) \]

2.2. The weighted average fusion algorithm

Assuming sensors $S_1, S_2, \ldots, S_n$ estimate the state of the same target in a $n$-sensor fusion system, and the local state estimated value of $S_i$ in the $k$ moment is $\hat{x}_k^i$ $(i=1, 2, \ldots, n)$. Here we assume that $\hat{x}_k^i$ is an unbiased estimated value, and the estimated errors are unrelated between any two of the local sensors.

Set the weight of each sensor as $w_1, w_2, \ldots, w_n$, then $\hat{x}_k$ and $w_i$ should meet the following conditions:

\[ \hat{x}_k = \sum_{i=1}^{n} w_i \hat{x}_k^i \]
\[ \sum_{i=1}^{n} w_i = 1 \]

The mathematics-average formula is often used in the weighted average fusion algorithm, which means the weight of each sensor is approximately equal. Set weight as $w$, according to formula (6),

\[ w \sum_{i=1}^{n} l_i = 1 \]
\[ w = \frac{1}{n} \]

and so

\[ \hat{x}_k = \frac{1}{n} \sum_{i=1}^{n} w_i \hat{x}_k^i \]

3. The Improved weighted fusion algorithm

This section mainly discusses the concept of the weight, and proposes the improved weighted fusion algorithm. Firstly, a single sensor will be weighted, and then the whole sensor system will be weighted to achieve the optimal algorithm performance[3-5].

3.1. The first weighting process to a single sensor

The general method of obtaining observation data is to adopt a single sensor. Because the variance of the sensor system is fixed, the only way of reducing estimated mean square error is to increase the observational data, which, on the other hand, will augments the calculated load and slows down the convergence rate. Multi-sensor data fusion can solve this problem. However, in the multi-sensor system, the fuse data of one or more sensors will also lead to the instability of system performance and serious estimated bias under the circumstances of a large observation noise or the estimated value divergence. Therefore, the estimated value of each single sensor should be weighted before
multi-sensor data fusion, which makes the estimated value rapidly converge in order to input the stable data to the fusion system and make the state estimated value optimal after data fusion[6].

The single sensor is weighted according to the following process: in a specific moment when the variance is the minimum, the optimal proportion weight is the ratio of the estimated state value to the sum of the estimated state value and the observational value. Using the optimal proportion weight is to correct the estimated value that is divergent or largely biased and make it convergent. In this way, the stable data source will be put into the multi-sensor data fusion system.

\[ v_k = z_k + \hat{x}_k \]  

(8).

When \( \min P_k \), the optimal proportion weight:

\[ W_k = \hat{x}/(z_k + \hat{x}_k) = \hat{x}/v_k \]  

(9).

Therefore, the single sensor weighted fusion formula:

\[ \hat{x}_j = W_j v_j \quad (j = 1, 2, \ldots, t) \]  

(10).

\( W_k \) is the optimal proportion weight under the circumstances of minimum variance at the K moment; \( V_j \) is the sum of the observational values and the state estimated values in t moments. \( \hat{x}_j \) is the weighted state estimated value in t moments[7-8].

3.2. Secondary weighted fusion of multi-sensor

Multi-sensor data fusion can improve the accuracy of the estimated value of the target. Because the variance of sensor is fixed, the impacts brought by the variance of sensor on the weight of fusion should be taken into consideration.

Set the weight of multi-sensor fusion as \( \alpha_i \) \( (i = 1, 2, \ldots, n) \)  

(11)

To derive the conditions which make the value of \( f(\alpha_i) = \sum_{i=1}^{n} \alpha^2 \sigma_i^2 \) minimum, we can give a multivariable objective function with constrained conditions:

\[ F(\alpha_i) = f(\alpha_i) + \lambda \theta(\alpha_i) \]  

(12)

Constrained conditions:

\[ \theta(\alpha_i) = \sum_{i=1}^{n} \alpha_i - 1 \]  

(13)

For the partial derivatives \( \alpha_i \) of the objective function:

\[ \frac{\partial F}{\partial \alpha_i} = \frac{df}{d\alpha_i} + \lambda = 2\alpha_i \sigma_i^2 + \lambda \quad (i = 1, 2, \ldots, n) \]  

(14)

Set \( \frac{\partial F}{\partial \alpha_i} = 0 \), the function \( F(\alpha_i) \) obtains the minimum value. So we get:

\[ \alpha_i = \frac{\lambda}{2\sigma_i^2} \quad (i = 1, 2, \ldots, n) \]  

(15)

From the formula (13) and (15) we can deduce that:

\[ \lambda = \frac{2}{\sum_{i=1}^{n} 1} \quad (i = 1, 2, \ldots, n) \]  

(16)

Put (16) into (15) it comes to the conclusion that the multi-sensor fusion weight is:

\[ \alpha_i = \frac{1}{\sigma_i^2 + \sum_{i=1}^{n} \frac{1}{\sigma_i^2}} \]  

(17)

Therefore, from the formula (11) and (10) we can derive the improved secondary weighted fusion algorithm formula:
\[ \hat{x}_j = \sum_{i=1}^{n} \alpha_i W_i v_j \] (18)

Constrained conditions:
\[ \sum_{i=1}^{n} \alpha_i = 1 \quad (i=1, 2, \ldots, n) \quad (j=1, 2, \ldots, t) \] (19)

\( \hat{x}_j \) is the estimated fusion value at moment \( j \); \( \alpha_i \) is the fusion weight; \( W_i \) is the optimal proportion weight of sensor \( I \) at moment \( K \); \( v_j \) is the sum of the observational values and the state estimated values of sensor \( I \) at moment \( j \).

4. The computer simulation of the improved weighted fusion algorithm

4.1. Computer Simulation

Firstly, the single sensor will be simulated by comparing the two types of fusion algorithm mentioned above. Consider the observation equation of the sensor system as:
\[ z(t) = [1,0]x(t) + v(t) \] (20)

Target model as:
\[ x(t+1) = \Phi x(t) + \Gamma w(t) \]
\[ \Gamma = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad \Phi = \begin{bmatrix} 0.9 & 0 \\ 0.8 & 0.5 \end{bmatrix} \] (21)

Through the 50-step recursive calculation with Matlab, the state estimated value can be obtained, as is shown in Figure1 and Figure2:

Figure1. The state filtering curve of the single sensor
Figure 2. The state filtering curve of the single sensor after weighting.

Through the comparison between Figure 1 and 2, we can see that the estimated accuracy of the single sensor weighted by the optimal proportion weight is much higher than that of the single sensor which is not weighted by the optimal proportion weight. The improved weighted fusion algorithm can improve the accuracy of the estimated value.

Compare the two algorithm simulations and consider the two-dimensional tracking system composed of three sensors:

\[ x(t + 1) = \Phi x(t) + \Gamma w(t) \]  \hspace{1cm} (22)
\[ z_i(t) = H x(t) + v_i(t) \hspace{1cm} i = 1, 2, 3 \]  \hspace{1cm} (23)

\( T \) is the sampling period.

\[ x(t) = [x_1(t), x_2(t)]^T, \quad x_1(t), \quad x_2(t) \quad \text{and} \quad w(t) \] are the location, speed and acceleration of the moving target at the moment \( tT \), and \( z(t) \) is the observation signal for \( x(t) \); \( v(t) \) is the observation noise.

Set \( w(t) \) and \( v(t) \) as the zero mean, and the variance \( \sigma^2_w \) and \( \sigma^2_v \) as independent Gaussian white noise:

\[ \Phi = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} 0.5T^2 \\ T \end{bmatrix}, \quad H = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad T = 2, \quad \sigma^2_w = 4, \quad \sigma^2_v = \sigma^2_v, \quad \sigma^2_v, \quad \sigma^2_v. \]

Through the Matlab simulation, we can calculate the state estimated value of the target tracked by three sensors in 200 periods and the fusion data of the two types of fusion algorithms.
Figure 3. The comparison between the fusion value and the true value of the two types of fusion algorithms.

Figure 4. Comparison between the state filtering error curves of the two types of fusion algorithms.

Figure 3 is the comparison between the fusion value and the true value of the two types of fusion algorithms, and Figure 4 is the comparison between the state filtering error curves of the two types of...
fusion algorithms. From the two figures we can see that no matter which fusion algorithm is applied, the state estimated value will be much more reliable after the multi-sensor fusion.

It can be concluded from this paper that the effect of the improved weighted fusion algorithm in multi-sensor fusion system is obviously superior to that of the single sensor. Through comparing the fusion estimated value and the variance of the two types of fusion algorithms in the Figure 3 and 4, we come to the conclusion that the improved weighted fusion algorithm is superior to the weighted average fusion algorithm.

5. Conclusion
In the process of the multi-sensor data fusion, the large variance of the sensor system has a negative effect on the weighted fusion algorithm. To find the best weight, we develop an improved weighted fusion algorithm, introducing the concept of the optimal proportion weight and using secondary weighted approach ---- the single sensor will be weighted individually before the whole sensor system is weighted and calculate the weighted fusion formula. Through analysis of computer simulation and comparison of the weighted average fusion algorithm, the improved weighted fusion algorithm can be proved validity.

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