Waves on a Free Surface of Ferrofluid Layer, Laying on a Liquid Substrate

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Abstract. Waves on the free surface of a magnetic fluid located on a liquid substrate were studied experimentally. The wave motion of the surface was induced by a homogeneous oscillating magnetic field orthogonal to the layer. In this case two types of waves can be formed on the surface of a magnetic fluid: a standing wave of the same frequency as the alternating magnetic field, and a standing wave, independent of the field frequency. The paper reviews the main theoretical and experimental studies of wave instability of such systems. The stability of a two-layered liquid system in an alternating vertical magnetic field is investigated. For efficient processing of the experimental results, the optical part of the experimental setup was modified. An algorithm has been developed for processing the profiles of the magnetic fluid surface obtained during the experiment, which helps to determine the length of the generated waves.

1. Introduction
Any arbitrary local disturbance of the horizontal liquid surface induces the appearance of waves that propagate over the surface and quickly decay with its depth. Waves are generated by the combined action of inertia and gravity (gravitational waves) or capillary forces due to the fluid surface tension (capillary waves). The contribution of the gravitational and capillary mechanisms, striving to return the disturbed surface to the state of mechanical equilibrium, is different. The influence of gravity increases with increasing wavelength, while capillary forces are dominant in the case of short-wave disturbances. In the intermediate case, the waves are of a mixed nature and are called gravitational-capillary.

Taking into account the influence of both gravity \(g\) and surface tension \(\sigma\) on the shape of the interface subjected to periodic perturbations, it is possible to obtain a dispersion relation connecting the temporal \(\omega\) and spatial \(k\) frequencies of the emerging traveling wave with the density \(\rho\) and surface tension \(\sigma\) [1]:

\[
\omega^2 = gk + \left(\frac{\sigma}{\rho}\right)k^3.
\]

The system loses its stability at \(\omega^2 = 0\). It is expressed in an unlimited increase in the amplitude of the surface perturbation [2]. As a result, various types of instabilities and resonance phenomena are observed.

Magnetic fluid is commonly used as a seal which comes into contact with other non-magnetic media being exposed to an external magnetic field [3]. Another application of magnetic fluids is an adaptive optics, that pay much attention to the stability of the fluid interface and ability to change its configuration in a predictable manner [4]. The character and stability of an interface "magnetic - nonmagnetic liquid" remain a matter of topical interest of experimental and theoretical research [5, 6]. A magnetic field \(H\) is an efficient tool to control the magnetic fluid free surface due to the volumetric magnetic force \(F_M = \mu_0 MVH\) and magnetic pressure jump \(p\) on the interface [7]:

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\[ p = \mu_0 \int_0^H M(H) dH, \]

where \( \mu_0 = 4\pi \times 10^{-7} \text{ H/m} \) is the magnetic permeability of vacuum, \( M \) is the magnetic fluid magnetization undergoing a linear law \( M(H) = \chi H \) in weak magnetic field, \( \chi \) – magnetic susceptibility.

There are random gravitational-capillary perturbations in a magnetic fluid layer with an initially smooth surface. These perturbations cause a redistribution of a magnetic field and, consequently, the magnetic pressure on the surface. Thus, the so-called Rosenzweig instability arises in a stationary orthogonal magnetic field [8]. The dispersion relation \( \omega(k) \) for gravitational-capillary waves in a magnetic fluid takes into account the influence of various factors on the evolution of surface instability (such as orientation of the magnetic field relative to the surface, thickness of the fluid layer, fluid properties, etc.) and helps to determine the critical parameters of the system \( (H, k) \) [7].

\[ \omega^2 = g k + \left( \frac{\sigma}{\rho} \right) k^3 - \frac{k^2 \mu_0 M^2}{\rho \left( 1 + \frac{\mu_0 M^2}{\mu} \right)} \]

It is possible to decrease the values of the critical parameters of the surface instability if the magnetic fluid layer has two free surfaces [9]. In this case, both surfaces are perturbed on their own, and one of them dominates and subordinates the perturbation to the second. These perturbations sum up when the maximum amplitude is reached at one of the surfaces [10]. Thus, at a certain critical magnetic field strength, the magnetic fluid layer experiences an instability, accompanied with its disintegration into a system of drops [11].

The magnetic fluid free surface could be perturbed by a mechanical stimulation [12], or an external electromagnetic field [13], or by both mechanisms at the same time [14]. While the mechanical action of any amplitude causes a disturbance of the liquid surface in the form of the wave, its excitement is possible in the applied magnetic field of a specific parameters only [15]. This parametric instability, called the Faraday instability, involves the excitation of parametric waves by ponderomotive forces under the action of an alternating magnetic field [16]. If the magnetic field changes periodically in time, then periodic motion will be excited on the free surface of the magnetic fluid due to the fact that the pressure at any point of the magnetic fluid is proportional to the magnetic strength in it [17].

Two types of waves can be formed on the surface of a magnetic fluid in an alternating magnetic field: a standing wave of the same frequency as the oscillating magnetic field, and a standing wave, independent of the field frequency. Experiments show that the fluid viscosity has a significant effect on waves of the first type, and practically does not affect waves of the second [18].

The action of an oscillating magnetic field was previously considered in vertical [19], horizontal [20], and oblique [21] directions with respect to a thick layer of magnetic fluid located on a solid substrate. The use of a liquid substrate for the magnetic fluid layer, may result in the appearance of some interesting effects, such as the stabilization of the fluid system by an alternating magnetic field with respect to the Rosenweig instability [22]. For example, a stability analysis of the ferrofluid layer free surface subjected to vertical vibrations and a horizontal magnetic field at the same time revealed a nonmonotonic dependence of the stability threshold on the magnetic field at high frequencies of the vibrations [14]. It has also been found that the action of both destabilizing factors (i.e., gravity modulation accompanied with normal magnetic field action) leads to a delay of the Rosensweig instability [15]. Moreover, the addition of disturbances from both deformable surfaces of a two-layer liquid system can lead to interference of two emerging reliefs, which, causes the appearance of a new vibrational mode, and, as a consequence, the emergence of self-oscillations in the system [23]. Fluids in such systems must be immiscible and stratified in density to avoid the occurrence of Rayleigh-Taylor instability [24].

In this paper we report on the experimentally obtained waves on the free surface of the magnetic fluid layer lying on a liquid substrate under the influence of a vertically oscillating uniform magnetic field. The results of the study are based upon the data obtained in the stability investigation of the two-layered liquid system [22].
2. Experimental technique

In the course of the experiments, two-layer liquid system filled a vessel in the form of a short vertical glass cylinder of diameter $D$ (Figure 1). The vessel was placed on a horizontal platform in the center of Helmholtz coils (the non-homogeneity of the magnetic field in the measurement zone did not exceed 2%). In order to organize linearly polarized alternating magnetic field, the Helmholtz coils were connected to a signal generator of special shape GW Instek AFG-72005 through an Emotiva A-300 amplifier. The output signal from a small resistor, connected in series with the coils, was fed to the input of the analogue-to-digital converter ADC LA-I24 USB and processed using the standard software kit. The visualization of the layer surface relief was performed using a circular LED diffuse light source in order to facilitate visual analysis of the wave’s quantitative characteristics. The image of the ferrofluid free surface, being reflected from a mirror (mounted above the coil system at an angle of about 45°) was recorded with a high-speed digital video camera Baumer VCXU-02M located beside the coils (Fig. 1).

![Figure 1. Experimental setup](image)

Magnetite in kerosene stabilized by oleic acid was used as the magnetic fluid (called also ferrofluid) of density $\rho_1 = 1.45 \text{ g/cm}^3$, surface tension $\sigma_1 = 27.0 \text{ mN/m}$, dynamic viscosity $\eta_1 = 6.1 \text{ mPa\cdot s}$, initial magnetic susceptibility $\chi_0 = 72$, particle concentration $n = 1.58 \cdot 10^{25} \text{ /m}^3$, diameter of magnetic particles $d = 9.8 \text{ nm}$, average magnetic moment $<m> = 3.25 \cdot 10^{-19} \text{ A\cdot m}^2$ [25]. Perfluorocarbonate C$_8$F$_{18}$ (density $\rho_2 = 1.76 \text{ g/cm}^3$, surface tension $\sigma_2 = 15.8 \text{ mN/m}$, dynamic viscosity $\eta_2 = 1.4 \text{ mPa\cdot s}$) was chosen as a transparent immiscible liquid substrate, since it has higher density and lower surface tension compared to magnetic fluid [26]. The thickness of the ferrofluid layer was determined as $h = m/S$, where $m$ was the mass of the liquid, and $S = (\pi D^2)/4$ was the area of the continuous magnetic fluid layer, which was assumed to be flat, $D$ was the inner diameter of the cuvette. The measurement of the liquid’s mass and, accordingly, the layer’s thickness was carried out by weighing the syringe with the ferrofluid before and after pouring it into the cuvette using an electronic scales Vesta VM 2202 with a measurement accuracy of 0.01 g. In the experiments, the thickness of the ferrofluid layer varied from 1 to 5 mm with accuracy of 0.1 mm. The thickness of the liquid substrate was several times greater than the MF layer, ranging from 15 to 20 mm. The ambient temperature for the experiment was kept at $(26 \pm 1) ^\circ\text{C}$.

3. Experimental data processing

The surface data was extracted from the images using an algorithm written with the help of GNU Octave – free software system for mathematical calculations using a high-level language compatible with MATLAB. The images with an explicit wave pattern were selected for processing the results. Then the most suitable sector of the surface in terms of illumination quality was determined. The software searched the image for the characteristic intensity profile of the image, by estimating the brightness throughout the radius of the selected sector from the center to the periphery. These computer-processed profiles were further processed to get information on the surface average intensity, corresponding to its deviation from equilibrium at the moment. The use of Fourier analysis helped to determine the period of the most pronounced periodic disturbance. The illustration of the surface wave profile recovered by this technique is shown in figure 2.
4. Results and discussion

In fig. 3a the upper view of horizontal ferrofluid layer is depicted under the action of direct (ν = 0 Hz) magnetic field of the amplitude $H_m = 0.9H_c$. The free surface of ferrofluid has the form of relief preceding the disintegration of the layer into an ordered system of drops (fig. 3b, $H_m = H_c$) [11]. The origin of the relief is the gravitational-capillary waves arising on the free surface of the layer in the absence of setup damping and enforced by the magnetic field.

The action of oscillating magnetic field leads to competition of gravitational capillary and magnetocapillary perturbations due to the action of magnetic force $F_M$. These perturbations become predominant at the frequency of oscillating field higher than 1 Hz so the magnetic fluid layer undergoes periodical oscillations of the surface. Since the layer of ferrofluid in the near-wall zone has a lower demagnetization factor than in the center of the cell (in view of the approximation of the layer by the plane), the meniscus was involved in the vibrational motion earlier than the rest of the layer. In addition, tangential stresses arising from the inhomogeneity of the field act near the boundary of the cell [23]. With this configuration, the oscillating meniscus turns to be the source of waves traveling along the surface of the ferrofluid layer. The increase of external magnetic field oscillation frequency promotes the regime of traveling waves turns into a regime of standing waves (see Fig. 3, c, d).

It was found [22] that the two-layered liquid system passes from the standing wave regime to the state of instability at some values of the amplitude $H_m$ and the frequency $\nu$ of the magnetic field strength. This transition is accompanied by the decomposition of the ferrofluid layer continuity into droplet structures (Fig. 4), similar to the observed droplets in the case of uniform stationary magnetic field (Fig. 3, b). The region, highlighted in Fig. 4, correspond to the amplitude $H_m$ of the alternating magnetic field equals to the intensity $H_c$ of the stationary magnetic field (Fig. 3, b). The stability map shows that in various cuvettes, the magnetic fluid layers of the same initial thickness are described by similar $H_m(\nu)$ dependencies. The critical magnetic field amplitude $H_m$ exhibits an asymptotic behavior with the growth
of the alternating field frequency. Thus, the imposition of periodic perturbations on the deformed surface of the magnetic fluid layer can shift the system instability threshold.

During the experiment we applied to the ferrofluid layer alternating linearly polarized magnetic field which amplitude was fixed and was lower than the corresponding critical field strength $H_m$ according to the stability map (Fig.4). The regime of standing waves in the cuvettes of different diameter is shown in Fig. 5. The pictures enclosed might be regarded as self-explanatory. In general, they reflect the process of decreasing length of the standing wave with an increase in the frequency of magnetic field oscillations. These pictures were used in data processing algorithm, described in section 3, in order to compare the computer-extracted data with that obtained with the help of the Comef and Virtual Dub software packages [23]. The wavelength was determined as the spatial period of the wave process (as the distance between the hills and troughs, which remained unchanged as the wave propagated from the periphery to the center of the cuvette. The result of comparison is shown in Fig. 6, where the waves on the ferrofluid layer of thickness $h = 3.5 \text{ mm}$ in the cuvettes of diameters $59.4 \text{ mm}$ ($1-4$) and $42.8 \text{ mm}$ ($5-7$) were analyzed. The dependences $\lambda(\nu)$ were defined manually in Comef ($1-3$ and $5-6$) and processed in Octave ($4, 7$) at the magnetic field amplitudes $H_m=0.75H_c$ ($1$), $H_m=0.8H_c$ ($2, 5$), $H_m=0.9H_c$ ($3-4, 6-7$).

![Figure 4](image1.png)

**Figure 4.** The dependence of the alternating magnetic field amplitude $H_m$ on the frequency $\nu$, at which the ferrofluid layer of the thickness $h = 3.5 \text{ mm}$ experiences instability in different cuvettes

![Figure 5](image2.png)

**Figure 5.** Oscillations of the magnetic fluid free surface of the layer thickness $h = 3.5 \text{ mm}$ in the cuvettes of $D = 42.8 \text{ mm}$ ($a-d$) and $D = 59.4 \text{ mm}$ ($f-j$) under the action of alternating magnetic field with an amplitude $H_m < H_c$ and frequency $\nu$, Hz: $7$ ($a, f$), $8$ ($b, g$), $9$ ($c, h$), $10$ ($d, i$), $15$ ($d, j$)
In particular, both data analyses show good agreement with each other. Thus, the use of the algorithm will speed up the data processing and will be used in further investigation of the magnetic fluid surface waves.

![Figure 6](image)

Figure 6. The length of the wave $\lambda$ on the ferrofluid layer ($h = 3.5 \text{ mm}$) free surface versus the oscillation frequency $\nu$ of the magnetic field amplitude $H_{am} = 0.75H_c$ (1), $H_{am} = 0.8H_c$ (2, 5), $H_{am} = 0.9H_c$ (3–4, 6–7) in cuvettes of diameter $D = 59.4 \text{ mm}$ (1–4) and $D = 42.8 \text{ mm}$ (5–7). Here markers (1–3) and (5–6) – are manually measured values in Comef and markers (4, 7) – processed values in Octave.

5. Conclusion
A number of experiments were carried out to study the effect of a vertically oscillating magnetic field on the thick ferrofluid layer on a liquid substrate. The wave process exerted on the free ferrofluid surface was captured with the high-speed video and processed with an algorithm, written specially for defining the wavelength. It was shown that the length of the waves decreases with increasing of the oscillation’s frequency of the magnetic field strength and increases with the growth of diameter of the cell. The results of the work need further elaboration in the form of a systematic study, accompanied by a significant enumeration of the experimental parameters, as well as in a quantitative comparison of the data obtained with the results of currently existing theoretical models.

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