Corrigendum: On the reliability of yield functions in deep drawing simulations (2022 IOP Conf. Ser.: Mater. Sci. Eng. 1238 012073)

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Figure 9(c) is automatically updated during the revising process, and now it shows a wrong curve for “BBC2005” and no curves for “Barlat89” and “Vegter”. The scatters for “Experiment” and the curve for “Facet-3D” are correct. This wrong change is done only in the revised version, and the figure is currently as follows:

The correct figure with the right data in sub-figure c (as it also appeared in the submitted version) is as follows:

![Corrected Figure 9(c)](image-url)
Figure 9. The results for the best combination of friction coefficients: (a) cup thickness variations, (b) punch forces, and (c) cup height profiles, and comparison to measured values [1].
On the reliability of yield functions in deep drawing simulations

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Abstract. In sheet metal forming simulations, the yield functions are usually calibrated based on experimental data, and validated by comparing the modelled and measured r-values and (in more advanced models) yield stresses along various in-plane directions. The fitted yield function should ideally reproduce (or interpolate) all the experimental values used for the fitting. However, this requirement does not guarantee accurate results in a forming process simulation, and it can even lead to unexpected results. The performance of a yield function, in addition to the fitting procedure, depends on the active loading modes which the material experiences during the simulation. The active loading modes are in turn determined by the die geometry as well as the process parameters like blank holder force and lubrication. As a consequence, the performance of a yield function, which is fitted to simple experimental data, is not identical for different forming conditions. Therefore, the application of each model is usually limited to a range of materials and processes, and this applicability is often evaluated based on experience. In the present study, we examine three phenomenological yield functions and a new crystal plasticity based material model for cup drawing process simulation with an AA6016-T4 aluminium alloy. These functions are different in their input data types used for the calibration. The results surprisingly shows that the models with more experimental data (in particular, yield stresses in different directions) in their formulation not only predict unexpectedly wrong results, but also show strong sensitivity to some of those additional input data and even to modelling parameters like the friction coefficient.

1. Introduction
The accuracy of sheet metal forming simulations depends on the models used for capturing the anisotropy of the material. Traditionally, the yield functions are fitted to experimental data obtained from simple tests (e.g. tensile test) in different in-plane directions. This way the model can exactly reproduce and reflect the input data. However, the behavior of the model is not always reliable for the other loading conditions. The reliability of the yield function is basically influenced by two factors: (1) its mathematical formulation, and (2) the quantity and quality of the input data.

The mathematical formulation of the yield function should be flexible enough to fit to the unpredicted distribution of experimental data. This is obtained by including more terms in the yield function which
in turn may require a larger number of tests to generate the training data set. In addition, the fitting procedure may also be a challenge in more complex yield functions. Therefore, the developers should look for a trade-off between accuracy and efficiency if the model is designed for a wide range of materials.

The training data should be designed and collected such that it covers as much loading modes as possible. This may require advanced tests which may be sometimes unfeasible or even impossible to be carried out in practice. Also, in combination with the first factor, it may end up with elevated experimental costs. Here the difference between the yield functions is in the loading modes which are thought to be more important for the considered forming process.

In addition to the reliability of the yield functions in representing material behavior under various loading modes, their robustness under experimental uncertainties is another important factor which should be tested for a given material and/or process. In other words, the results obtained from a yield function should not be too sensitive to the measured input data or to modelling parameters like the friction coefficient which are usually entered to the simulations with uncertainties. Otherwise, the risk of introducing errors in the simulations will be high unless the precision of experimental measurements and the accuracy of friction models are guaranteed.

In the present study, the performances of three phenomenological yield functions (with different levels of complexity) as well as a crystal plasticity (CP)-based model are assessed for circular cup drawing simulations. It is demonstrated how a simple phenomenological yield function requiring a minimum number of material parameters and the CP-based model can perform better in predicting the earing profile than more advanced phenomenological expressions. This observation cannot be generalized to other materials or processes, and there can be other cases that show opposite results. Moreover, it is shown that more advanced phenomenological models may lead to unrealistic results even for the current process with its simple geometry.

2. Methods and material

All the experimental data for the present study are taken from [1] and [2]. The cup drawing experiments have been carried out on an AA6016-T4 aluminum alloy, and with the following process parameters: punch diameter: 50 mm, punch fillet radius: 6 mm, die diameter: 52.82 mm, die fillet radius: 3 mm, blank diameter: 90 mm, blank thickness: 1 mm, and blank holder force: 10 kN.

The cup drawing simulations were performed with AutoForm Forming R10 with the mesh settings as follows: element type: thick shell with 11 through thickness integration points (TS-11), initial maximum element size: 1 mm, minimum element size: 0.5 mm, and number of initial elements: 18707. Four friction conditions were tested in two different ways: (1) traditional Coulomb friction with three coefficients 0.05, 0.1, and 0.15, and (2) physical modelling with TriboForm which is available in AutoForm. With TriboForm, the default settings for a contact between Al 6000 series and cast iron with drawing oil were used.

For all the simulations, an isotropic hardening behavior was assumed according to Hockett-Sherby model given by

$$\sigma = \sigma_{\text{sat}} - (\sigma_{\text{sat}} - \sigma_i) e^{-ae_p^p}$$

where $\sigma_{\text{sat}} = 438.56$ MPa, $\sigma_i = 115.13$ MPa, $a = 2.74$, and $p = 0.59$ were found by fitting Eq. (1) to the stress-strain data obtained from the tensile test in the rolling direction.

The phenomenological yield functions were selected among those available in AutoForm R10. Hill48 [3] and Hill90 [4] were not used as they are not recommended for Al alloys due to their low r-values. The other yield functions in AutoForm R10, i.e., Barlat89 [5], BBC2005 [6], and Vegter [7] were tested. In addition, a new and reduced version of Facet yield function [8], named Facet-3D, was also implemented as an R&D Plugin material model in AutoForm R10 and tested for the simulations. This crystal plasticity based yield function is dedicated to the generalized plane stress conditions in sheet metal forming simulations. Facet-3D has the same formulation as the general 5D Facet (see[8]), except
that the out-of-plane shear terms are neglected in its formulation, and the virtual experiments are executed on a grid in 3D stress space created by the spiral method [9].

For calibrating the phenomenological yield functions, the material properties were measured through tensile tests in various in-plane directions. Facet-3D however is calibrated to virtual experimental data generated by the ALAMEL crystal plasticity model [10] which requires only one texture measurement as the input. The required experimental input data for all the studied yield functions are summarized in Table 1. In this table, $r$, $YS$, $PS$, and $PSH$, stand for $r$-value, yield stress, plane strain point, and pure shear point, respectively.

Table 1. The required experimental input data for all the studied yield functions.

| model     | Rolling Direction | Diagonal Direction | Transverse Direction | Biaxial | Texture |
|-----------|-------------------|--------------------|----------------------|---------|---------|
|           | $r$   | $YS$  | $PS$  | $PSH$ | $r$   | $YS$  | $PS$  | $PSH$ | $r$   | $YS$  |
| Facet-3D  | *     | *     | *     | *     | *     | *     | *     | *     | *     | *     |
| Barlat89  | *     | *     | *     | *     | *     | *     | *     | *     | *     | *     |
| BBC2005   | *     | *     | *     | *     | *     | *     | *     | *     | *     | *     |
| Vegter    | *     | *     | *     | *     | *     | *     | *     | *     | *     | *     |

Uniaxial and equi-biaxial yield stresses and $r$-values were experimentally measured (figure 1 and Table 2). The $r$-values in RD, DD, and TD were obtained at 20% plastic strain, and the equi-biaxial $r$-value in the point with identical plastic work to the test in RD [11]. The plane strain and pure shear points, which are required only for Vegter, were obtained by virtual experiments using Virtual Experimentation Framework (VEF) software [2] (Table 3).

Table 2. Material parameters obtained by physical experiments [2].

| Angle to RD (°) | $r$-value | Yield Stress ($\sigma_x/\sigma_0$) | Equi-biaxial point ($\sigma_y/\sigma_0$) | $r$-value stress ($\sigma_y/\sigma_0$) |
|----------------|-----------|----------------------------------|-----------------------------------|----------------------------------|
| 0              | 0.552     | 1.000                            |                                   |                                  |
| 45             | 0.409     | 0.963                            | 1.012                             | 1.037                            |
| 90             | 0.549     | 0.979                            |                                   |                                  |

The texture of the material was measured by XRD method on plane samples taken at three different positions through the sheet thickness: sub-surface, 25% thickness, and 50% thickness. The ODF plots of the measured textures are shown in figure 2. This material exhibits a strong through-thickness texture gradient, where the cube component becomes weaker and more distributed from surface to the mid-section. The numerically merged texture (using weight factors 0.25, 0.5, and 0.25 for textures (a), (b), and (c), respectively) that was used for the virtual experiments with ALAMEL is shown in figure 2(d).
Figure 2. The ODF sections at $\phi_2 = 45^\circ$ of the AA6016-T4 alloy, measured by XRD at (a) 50% thickness, (b) 25% thickness, (c) sub-surface, and (d) the numerically merged texture.

The measured r-values and yield stress ratios are compared with those calculated with the various yield functions in figure 3. All the three phenomenological yield functions exactly reproduce the measured r-values at 0° (RD), 45° (DD) and 90° (TD) as these are used as input parameters for their calibration. Facet-3D overestimates the planar anisotropy. Whereas its r-values in RD and TD are comparable to the experimental values, its r-value in DD is underestimated. This can be a consequence of texture gradient in the material, and a through thickness wide field EBSD measurement could be a better method than XRD for obtaining a representative texture. When the yield stress ratios are taken into account in the yield function formulation, the yield stress anisotropy is also reproduced (see BBC2005 and Vegter curves in figure 3(b)). Otherwise, the yield stress anisotropy may be different from the experiment. This is seen with both Facet-3D and Barlat89 in figure 3(b). The main reason for the inaccurate yield stress anisotropy with Facet-3D is that the ALAMEL model accounts for the crystallographic texture but neglects other microstructural elements that may be responsible for the observed yield stress anisotropy, such as particles and their distribution. The measured equi-biaxial r-value (1.037) and yield stress ratio (1.012) are exactly reproduced by BBC2005 and Vegter (as they include these parameters in their formulation); however, these values are different than the measured ones for Facet-3D ($r_b = 1.049$, $Y_b = 0.98$) and Barlat89 ($r_b = 1.060$, $Y_b = 0.97$).

Figure 3. Measured material parameters [1] and those exploited from the yield functions: (a) r-values, and (b) yield stress ratios.

3. Results and discussion

3.1. Punch forces

The variation of punch force as a function of punch displacement for all the yield functions and friction conditions are demonstrated in figure 4. For each friction condition, the curves predicted by all the yield functions are similar to each other. With TriboForm, the peak is underestimated, and the decreasing slope after the peak is more pronounced than the measured value. These aspects both improve when Coulomb friction with a coefficient 0.05 is used. With increasing the friction coefficient to 0.1 and 0.15, the punch force peaks increase beyond the experimentally measured value, and the
peaks predicted by Facet-3D and Vegter also become larger than those of Barlat89 and BBC2005. The curves here suggest that a friction coefficient around 0.1 can be a good choice for the simulations.

![Figure 4](image)

**Figure 4.** The measured punch force variation [1] and those predicted using: (a) TriboForm, and friction coefficients (b) 0.05, (c) 0.1, and (d) 0.15.

### 3.2. Cup height profiles

The measured and predicted cup heights were compared by calculating the relative height values using the following formula: \( \text{relative height} = \text{absolute height} - \text{average height} \). Figure 5 illustrates the cup heights predicted by all the yield functions and friction conditions with the measured values. The relative heights show how the earing profile is predicted by each model-friction combination. With TriboForm, BBC2005 predicts the ear at 45º, but all the other yield functions predict acceptable earing profiles. Vegter shows the best match with the experiment, and Facet-3D and Barlat89 predict stronger earing behaviors. When Coulomb friction with a coefficient 0.05 is used, Facet-3D and Barlat89 are improved, but BBC2005 and Vegter results become worse. With increasing the friction coefficient, the gradual improvement of Facet-3D and Barlat89 continues, whereas BBC2005 and Vegter results continuously worsen and lead to erroneous positions of the ears. Vegter shows the most severe sensitivity to the friction coefficient.

As shown in figure 5(e), the average cup heights are lower than the measured value for all the studied friction conditions, but they improve with an increase in the friction coefficient. If the best friction coefficient is selected solely based on the absolute cup height values, then this figure suggests to use a higher friction coefficient even beyond 0.15. However, it was observed in the previous section that a coefficient 0.15 is too large concerning the punch force variations. If only the earing profile (i.e. the relative cup height) is considered as the reference, then a lower friction coefficient or TriboForm would be more logical as, in addition to Facet-3D and Barlat89, Vegter also predicts a better cup height profile. However, with TriboForm or a low friction coefficient like 0.05, the average cup heights become smaller and their differences with the measured value become larger.

Note that, the observed underestimation of the cup heights, as well as the inversion of the height profile in DD, is also reported in several other studies for the same material and cup drawing process [1, 2, 12] with different finite element solvers, more complex yield functions, and even higher friction coefficients. Therefore, it seems that considering the punch force variations as the reference for selecting
the friction condition is safer, and this suggests that in this study a Coulomb friction with a coefficient 0.1 is the best choice among the other friction conditions.

Figure 5. Measured cup heights [1] and those predicted with the yield functions for different friction conditions: (a) TriboForm, and friction coefficients (b) 0.05, (c) 0.1, and (d) 0.15, and (e) comparison of the average cup heights.

3.3. Cup thickness variations
The sheet thickness variation from center to the rim of the cup is another important factor which should be looked at. This was done in the present study in the rolling, diagonal, and transverse directions for all the yield functions and friction conditions. The sections on which the thickness variations were measured as well as the typical points on the observed curves are shown in figure 6.

Figure 6. Thickness measurement sections in rolling (RD), diagonal (DD), and transverse (TD) directions.

The points A, B, C, D, E, and F were present in all the curves for all the yield functions and friction conditions. The curves for all the yield functions with a friction coefficient 0.1 are compared in figure 7. The studied yield functions predict roughly similar thickness variation from center of bottom to the rim of the cup. However, there are a few differences between the measured and predicted values. For instance, the thinning on the cup bottom (AB section) is somewhat overestimated by all the yield functions. Since the loading condition on the cup bottom is close to equi-biaxial stress mode, the yield
functions which take this point into account (i.e. BBC2005 and Vegter) are slightly better than the others. The predicted thickness on this section is similar for all the three directions, but the measured values are a bit larger in the diagonal direction.

In addition to the equi-biaxial stress point, the friction condition between punch and sheet can also affect the thinning on the cup bottom. Figure 8 demonstrates this only for Facet-3D, but the other yield functions also behave in the same manner. As can be seen, with a fixed friction coefficient 0.1 for the die and binder, a larger friction coefficient for the punch improves the predicted thickness on the cup bottom and corner. This variation does not influence the thickness change on the cup wall, the punch force variation, and the cup height profile. However, it affects the average cup heights as the thickness change takes place in a wide area on the cup bottom and corner, and less thinning there results in less extension and therefore a smaller contribution to the total cup height. This figure suggests that a larger friction coefficient can be assigned to the punch in the simulations to have a better prediction of thickness on the cup bottom and corner. This can be reasonable because the extremely low relative sliding speed between the punch and blank leads to more severe tribological condition in the punch-blank contact area than die-blank and binder-blank contacts [13].

![Figure 7](image1.png)

**Figure 7.** Comparison of the measured cup thickness variations in RD, DD, and TD [1] with those predicted by the studied yield functions.

![Figure 8](image2.png)

**Figure 8.** The effect of friction coefficient of punch on (a) cup thickness variations, (b) punch forces, (c) relative cup heights, and (d) average cup heights, with a fixed coefficient 0.1 for die and binder, and comparison to the measured values [1].
For the best friction condition, i.e., Coulomb friction with coefficients 0.1 for die-blank and binder-blank contacts, and 0.15 for the punch-blank contact, the thickness variation curves in RD, DD, and TD, punch forces, and cup height profiles are illustrated in figure 9. Except the cup profiles by BBC2005 and Vegter, the other quantities are predicted with acceptable accuracy by all the yield functions. The inverted cup profile prediction by BBC2005 and Vegter is unexpected because these models are designed to accurately reproduce the anisotropic properties of the material, especially the yield stress anisotropy (see figure 3). Note that the same phenomenon was also observed in an earlier study conducted with BBC2008 coupled to Abaqus/Explicit solver, for the same material and cup drawing process [2].

3.4. Sensitivity of cup height profiles to friction condition
One of the most important simulation parameters including uncertainty is the friction coefficient in a sliding contact, and its value is often decided by comparing experimental and predicted forces, thickness variations, geometrical parameters, etc. The friction coefficient is not constant everywhere in the model, and using a fixed value is in fact a simplifying assumption. Therefore, the material models should be robust under small variation of tuning parameters like the friction coefficient. In other words, sensitivity of important results (e.g. the earing profile) to the friction coefficient in a simple forming process with a regular geometry like a circular cup drawing should be limited (unless such a sensitivity to friction conditions would be evidenced by experiments). It is more usual that average cup heights can significantly change by the friction coefficient, but not the position of the ears. As can be seen in figure 10, this robustness is observed with Facet-3D, Barlat89, and to some extent BBC2005 (although with inaccurate ear position). However, for Vegter, the position of ear changes from 0° and 90° (with low friction coefficient) to around 30°-45° (with higher friction coefficients) w.r.t the rolling direction.

3.5. Sensitivity of cup height profiles to experimental data
The experimental data can include errors, and this may influence the material models. However, the materials showing high sensitivity to the experimental input parameters should not be used in forming process simulations. In the present study, BBC2005 and Vegter take into account the yield stresses in RD, DD, and TD in their formulation, and at the same time they wrongly predict the position of ear. This stimulates investigating the sensitivity of these models to those additional input data. Figure 11 shows the variation of relative cup heights to only 2% error in the yield stress value in the diagonal
direction (Y45). Both models show high sensitivity to Y45, and with Vegter, the position of the ear even changes for this small variation. This amount of uncertainty in the yield stress is not so rare, and may easily happen in 6XXX Al alloys (e.g. due to inhomogeneous distribution of precipitates and particles). Such sensitivity in these yield functions may be reduced by e.g. least-squares fitting procedure in conjunction with more material data points generated by physical or virtual experiments [2, 14, 15].

![Figure 10](image1.png)

Figure 10. Sensitivity of the predicted cup height profiles by (a) Facet-3D, (b) Barlat89, (c) BBC2005, and (d) Vegter to the friction condition, and comparison to the measured values [1].

![Figure 11](image2.png)

Figure 11. Sensitivity of the predicted cup height profiles by (a) BBC2005 and (b) Vegter to the yield stress ratio in the diagonal direction (Y45), and comparison to the measured values [1].

4. Conclusions
Circular cup drawing of an AA6016-T4 aluminium alloy was simulated with AutoForm R10 using three phenomenological yield functions (Barlat89, BBC2005, and Vegter) and a new crystal plasticity based yield function (Facet-3D). The performance of the yield functions was assessed by comparing the experimental and predicted punch forces, cup height profiles, and cup thickness variations under various friction conditions (TriboForm and Coulomb friction with three different coefficients). The results revealed that

- the punch force variations obtained by all the models are similar for a fixed friction condition, but their peak value varies with the friction;
- a friction coefficient 0.1 leads to the closet punch force profile to the experiment; TriboForm and lower friction coefficients underestimate the punch force peak, and a higher friction coefficient overestimates it;
- for all the studied yield functions and friction conditions, the average cup heights are underestimated; this result is in line with previous studies [1, 2, 12];
for all the friction conditions, Facet-3D and Barlat89 predict accurate earing profiles with the correct ear position at 0° and 90°, but BBC2005 wrongly predicts the ear at 45°;

- Vegter with combination of TriboForm predicts the most accurate earing profile, but its ear position rapidly changes with an increase in the friction coefficient, and moves to around 30°;

- Vegter shows the highest sensitivity of earing profile to the friction condition;

- all the yield functions predict acceptably accurate thickness on the cup bottom, corner, and wall with close variations to the observed one;

- the punch force and cup thickness variations suggest that the best friction coefficients for the present study are 0.1 for die-blank and binder-blank contacts, and 0.15 for punch-blank contact;

- the earing profiles by BBC2005 and Vegter are very sensitive to small variations (2%) in the yield stress at 45°, where even the ear position for Vegter changes from 0° and 90° to around 30° with this variation.

The cup drawing results demonstrate the accuracy obtained with the proposed crystal plasticity based yield criterion. Thanks to its specific formulation that guarantees convexity, offers flexibility, and proves to be robust, the Facet yield locus is expected to also capture the plastic anisotropy under arbitrary loading modes. This is especially important for more complex forming geometries, and for materials with different textures and corresponding plastic anisotropy.

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