Modal and structural defect analysis for a new docking mechanism

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Abstract: In medium or small-sized spacecrafts' docking process, it's feasible to adopt a flexible probe for the probe-cone docking mechanism instead a buffer to reduce the docking impact force. The governing equations of kinematics and dynamics are derived from Lagrange analytical method. The influence parameters of the impact force are separated by simplifying the impact mathematical modeling. The effects of the influence parameters in the impact force, such as the length of docking beam, and the material of the docking mechanism, are discussed by finite element method. The variation amplitude and variation frequency are obtained by docking experiment, and reliability and correctness of the analysis results are verified.

1. INTRODUCTION

Studies on dynamics of flexible multibody system have been done in many previous fields. An in-orbit satellite flexible capture technology, which reduces the elastic vibration during oblique docking process, is a simply structure[1, 2]. However, the impacting dynamics analysis of spacecrafts was complicated due to the nonlinear chaos in the process of dock and capture[3]. Despite this, researchers obtained excellent insights into this field and their achievements mainly focuses on these aspects:

(1) the analysis of dynamics model. As a research core issue of docking process, the dynamics analysis of the docking system was mature and its theoretical studies gradually rised from general kinematic analysis to classical dynamics method[4, 5].

(2) the study of the control strategy. The impact force was controlled within desired value by using a combination of active force-limited control approach and passive damping mechanism[6, 7].

(3) the optimization of the model. A Hill Clohessy Wiltshire model with additional disturbance and line of sight constraint, was also used to optimize control the spacecraft rendezvous and docking based on robust model predictive controller (MPC) and genetic algorithms[8, 9].

The probe-cone docking mechanism has the advantages of simple structure, light weight and mature technology compared with other types of docking mechanisms. The vibration analysis of probe-cone docking mechanism differs from that of other types of docking mechanisms and two main research methods are concerned: the dynamics of flexible docking probe is simplified to cantilever beam and the impact dynamics can be analyzed by numerical coupling and simulations. To consider the effects of the simplified flexible docking cantilever beam in the process of docking impact and further analyze the vibration of the docking mechanism, the numerical coupling simulation method was mainly used in previous vibration analyses of the docking mechanisms of traditional structure[10, 11]. However, for the numerical coupling simulation, the modified modal equations of docking cantilever beam were huge and the complexity of the derivation and solving process were height[12]. Since finite element method combines the characteristics of analytical dynamics and nodal dynamics,
it was especially suitable for non-linear systems[13]. Therefore, in this paper, a numerical coupling simulation is proposed that is based on finite element method that verify the correctness of the modified modal equation.

Most of the above papers focus on the dynamics of docking mechanism and the analysis method of docking impact for the classical probe-cone docking mechanism. The vibration problems are not studied in detail in the process of docking, and the modal characteristics of the mechanism are the main factors affecting the docking vibration. Therefore, the vibration effect and modal characteristics are studied based on a new docking mechanism in this paper.

2. MATHEMATICAL MODELING

The displacements vector model of impacting and docking in base coordinate system O-XYZ can be written as

\[
\dot{\mathbf{q}} = \frac{\partial}{\partial t} \mathbf{q} + \mathbf{R} \dot{\mathbf{q}} = \begin{bmatrix} e_1(t) \\
0 \\
0 \\
0 \\
0 \\
0 \end{bmatrix}
\]

\[
\mathbf{R} = \begin{bmatrix}
1 & 0 & 0 & e_1(t) \\
0 & 1 & 0 & e_1(t) \\
0 & 0 & 1 & e_1(t) \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

where \(\mathbf{R}\) is the coordinate transformation matrix from coordinate system O-XYZ to coordinate system \(\mathbf{O}_2-\mathbf{u}_2-\mathbf{v}_2-\mathbf{w}_2\). \(\mathbf{q}'\) and \(\mathbf{q}\) are the displacements vector of point \(q'\) and \(q\), respectively. \(e_1\) is the distance between the principal axis of docking mechanism and mass center of the floating satellite in X and Y direction, respectively. \(t\) is the contact time of the joint with the cone.

The kinetic energy \(T\) of docking mechanism is

\[
T = \frac{1}{2} M \dot{\mathbf{q}}^T \dot{\mathbf{q}} + \frac{1}{2} \mathbf{I}_m \dot{\mathbf{q}}^T \dot{\mathbf{q}} + \frac{1}{2} \rho A \int_0^t \dot{\mathbf{r}}^T \dot{\mathbf{r}} dl
\]

(2)

where \(M\) is the mass of floating satellite. \(m\) is the mass of docking joint. \(\mathbf{I}_m\) is inertia matrix of floating satellite. \(\rho\) is the density of docking beam. \(A\) is the cross-sectional area of docking beam. \(\mathbf{I}\) is the displacements vector of docking beam. \(\Theta\) is the angle vector of floating satellite. \(\mathbf{q}'\) is the displacements vector of docking joint \(q\).

The potential energy \(V\) of docking mechanism is

\[
V = \frac{1}{2} E A \int_0^t \left( \frac{\partial \mathbf{A}(l,t)}{\partial l} \right)^2 dl + \frac{1}{2} E I \int_0^t \left( \frac{\partial^2 \mathbf{N}(l,t)}{\partial l^2} \right)^2 dl + \frac{1}{2} E I \int_0^t \left( \frac{\partial^2 \mathbf{W}(l,t)}{\partial l^2} \right)^2 dl
\]

(3)

where \(E\) and \(G\) are the modulus of elasticity and modulus of shear of docking beam, respectively. \(I\) is the sectional inertia moment of docking beam.

According to Eqs. (1)-(3), the dynamics model of the docking mechanism can be obtained based on Lagrange analysis mechanics, and the vibration equations of the docking beam can be established based on Euler Bernoulli beam.

\[
\begin{align*}
E A \dddot{x}(z,t) + \rho A \ddot{x}(z,t) &= 0 \\
E I \dddot{y}(z,t) + \rho A \ddot{y}(z,t) &= 0
\end{align*}
\]

(4)

where \(\rho A\) is the mass per unit length. Galerkin’s method is used to discretize the deflections \(x(z, t)\) and \(y(z, t)\) as
where $i$ is the mode order. Substituting Eq. (5) to Eq. (4)
\[
\begin{align*}
\frac{EI}{\rho A} \phi_i(z) &= \bar{u}(t) = \omega_k x_i \\
\frac{EI}{\rho A} \phi_j(z) &= \bar{y}(t) = \omega_k y_i
\end{align*}
\]
(6)

According to Eq. (6), we obtain
\[
\begin{align*}
\phi_i(z) + \beta_i^2 \phi_i(z) &= 0 \\
\phi_j(z) + \beta_j^2 \phi_j(z) &= 0
\end{align*}
\]
(7)

where $\beta_i = \sqrt{\frac{\rho A \omega^2}{EI}}$. The general solution in Eq. (7) can be expressed as
\[
\begin{align*}
\phi_i(z) &= C_i \cos(\beta_i z) + C_2 \sin(\beta_i z) + C_3 \cosh(\beta_i z) + C_4 \sinh(\beta_i z) \\
\phi_j(z) &= D_i \cos(\beta_j z) + D_2 \sin(\beta_j z) + D_3 \cosh(\beta_j z) + D_4 \sinh(\beta_j z)
\end{align*}
\]
(8)

where $C_i$ and $D_i$ are the integral constant. When the docking beam vibrates, the inertial force is produced at the free end of the docking beam due to the mass of the docking joint. Therefore, the free end shear force of the docking beam is equal to the inertial force of the mass during vibration. The bending model of the docking beam is
\[
\begin{align*}
x(0,t) &= x'(0,t) = 0, \quad Ex''(0,t) = 0, \quad Ex''(L_{soc},t) = mx''(L_{soc},t) \\
y(0,t) &= y'(0,t) = 0, \quad Ey''(0,t) = 0, \quad Ey''(L_{soc},t) = my''(L_{soc},t)
\end{align*}
\]
(9)

3. ANALYSIS AND VALIDATION

3.1. Finite element modeling

In order to simplify the process of the simulation and calculation, the eight-nodded hexahedral element is adopted in the process of grid meshing. Total number of cell is 35972 and total number of node is 43687 of the finite element model for the space docking mechanism. The finite element model of this docking system is shown in Fig. 1.

Fig. 1 Finite element model of the probe-cone docking system

3.2. Numerical analysis results

The values of parameters for the docking impact model are listed in Table 1.
Table 1. Docking impact model parameters

| Floating satellite | Docking beam | Docking joint | Target satellite(cone) |
|--------------------|--------------|---------------|------------------------|
| Mass (kg)          | 0.508        | 0.0275        | 0.019                  | 0.693                  |
| Density (kg/m³)    | -            | 2830          | 2830                   | 7850                   |
| Inertia moment (kg·m²) | Iₓ=0.0013 | Iᵧ=0.0020 | Iₗ=0.0021 | -                     |
| length (m)         | -            | 0.0705        |                       |                        |
| radius (m)         | -            | -             | 0.0247                 | -                      |
| cross-sectional area (m²) | - | 0.0005542 | -                      |                        |
| Initial velocity (m/s) | 0.5        | 0.5           | 0.5                    | 0                      |

As commonly used physical parameters, the modal frequencies reflect the inherent vibration characteristics of the docking mechanism. To explore the modal frequencies of the docking mechanism, the analysis model of space docking mechanism can be simplified as two-dimensional analysis models.

Along with the modified modal equations of the docking mechanism and based on the finite element analysis software, the modal frequency values corresponding to each mode of the flexible beam can be obtained, as shown in Table 2.

Table 2. Modal frequencies of the docking mechanism

| Modal order n | 1 | 2 | 3 | 4 | 5 | 6 |
|----------------|---|---|---|---|---|---|
| Modal frequencies /Hz | 675.38 | 677.05 | 1158.5 | 4118.5 | 4175.1 | 4582.0 |

Table 2 shows the modal frequencies of the docking mechanism before the vibration modal defect is modified. The corresponding vibration of each order modal frequencies are shown in Fig. 2.

Fig. 2 Vibration of each order modal of docking mechanism
3.3. Experimental verification

In order to verify the correctness and effectiveness of the modal analysis in Section 3.2, the vibration amplitude of the docking beam can be measured by building an experimental platform, as shown in Fig. 3.

The experimental results are shown in Fig. 4. According to the measured time history of the vibration displacement curve, the vibration frequency of each curve can be calculated.

According to Fig. 4, the vibration amplitude and frequency of the docking beam can be obtained. The experimental results show that the vibration frequency is less than the natural frequency of the docking beam. The experimental results also indicate that the error between the vibration amplitude of the simulation result and the docking experiment is less than 10%.

4. CONCLUSIONS

The docking beam of BSCDM is simplified as a cantilever beam in this paper. Based on the basic equations of Euler Bernoulli beams, the vibration equations and modal models of the docking beam are derived. The vibration amplitude and modal characteristics of the docking beam can be obtained by the finite element method based on the theoretical model, and the correctness of the numerical analysis results is verified by the docking experiment. This method can provide guidance for solving the dynamic contact mode function of the cantilever beam.

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