Time dependence of accretion flow with a toroidal magnetic field

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ABSTRACT
In this paper, we investigate the time evolution of quasi-spherical polytropic accretion flow with a toroidal magnetic field. We focus in particular on the astrophysically important case in which the adiabatic exponent $γ = 5/3$. In this scenario, we have assumed that the angular momentum transport is a result of viscous turbulence and we have used the $α$-prescription for the kinematic coefficient of viscosity. The equations of accretion flow are solved in a simplified one-dimensional model that neglects the latitudinal dependence of the flow. In order to solve the integrated equations that govern the dynamical behaviour of the accretion flow, we have used a self-similar solution. The solution provides some insight into the dynamics of quasi-spherical accretion flow and avoids many of the strictures of the steady self-similar solution. The effect of the toroidal magnetic field is considered with an additional variable $β = \frac{p_{\text{mag}}}{p_{\text{gas}}}$, where $p_{\text{mag}}$ and $p_{\text{gas}}$ are the magnetic pressure and gas pressure, respectively. The solution indicates a transonic point in the accretion flow, that this point approaches the central object by adding the strength of the magnetic field. Also, by adding the strength of the magnetic field, the radial thickness of the disc decreases and the disc compresses. We indicate analytically that the radial velocity is only a function of Alfvén velocity. The model implies that the flow has differential rotation and is sub-Keplerian at all radii.

Key words: accretion, accretion discs – MHD.

1 INTRODUCTION
Accretion is the main source of energy in many astrophysical objects, including different types of binary stars, binary X-ray sources, quasars and active galactic nuclei (AGN). Although the development of accretion theory first started a long time ago (Bondi & Hoyle 1944; Bondi 1952), the intensive development of the theory only began after the discovery of the first X-ray sources (Giacconi et al. 1962) and quasars (Schmidt 1963). Because the removal of the angular momentum operates on slower timescales compared to the free-fall time, infalling gas with sufficiently high angular momentum can form a disc-like structure around a central gravitating body, which can be thin or thick depending upon the geometrical shape. The models of thin accretion discs are perhaps better developed and seem to have good observational basis (Shakura & Sunyaev 1973). However, for thick accretion discs, no fully developed model exists, and there remain many theoretical uncertainties about their structure and stability (Banerjee et al. 1995; Ghanbari & Abbassi 2004; Ghanbari, Salehi & Abbassi 2007).

It is thought that accretion discs, whether in star-forming regions, in X-ray binaries, in cataclysmic variables or in the centres of AGN, are likely to be threaded by magnetic fields. Consequently, the role of magnetic fields has been analysed in detail by a number of investigators (Blandford & Znajek 1977; Lubow, Papaloizou & Pringle 1994; Banerjee et al. 1995; Shadmehri 2004). A mechanism for angular momentum transport is another key ingredient in the theory of accretion processes and many theoretical uncertainties still remain about its nature. As originally pointed out in Lynden-Bell (1969) and Shakura & Sunyaev (1973), a magnetic field can also contribute to the angular momentum transport. A robust mechanism of the excitation of magnetohydrodynamical (MHD) turbulence was shown to operate in accretion discs as a result of the magnetorotational instability (MRI; Balbus & Hawley 1998; Machida, Hayashi & Matsumoto 1999; Begelman & Pringle 2007).

Toroidal magnetic fields have been observed in the outer regions of the discs of young stellar objects (YSOs; Aitken et al. 1993; Wright et al. 1993; Greaves, Holland & Ward-Thompson 1997) and in the Galactic Centre (Chuss et al. 2003; Novak et al. 2003). Accretion discs containing a toroidal magnetic field have been studied by several authors (Fukue & Okada 1990; Geroyannis & Sidiras 1992, 1993, 1995; Banerjee et al. 1995; Terquem & Papaloizou 1996; Liffman & Bardou 1999; Machida et al. 1999; Akizuki & Fukue 2006, hereafter AF; Begelman & Pringle 2007; Rempel 2006). Fukue & Okada (1990) examined the oscillations of a gaseous disc, which were penetrated by toroidal magnetic fields. Geroyannis & Sidiras (1992, 1993) described differentially rotating polytropic models distorted by toroidal magnetic fields. Also, Geroyannis & Sidiras (1995) considered dissipative effects by viscous friction of differentially rotating visco-polytropic models that
were further distorted by a toroidal magnetic field. Banerjee et al. (1995) presented a toroidal magnetic field that was generated by the interaction of rotating plasma and the dipolar magnetic field of a central object. They showed that the toroidal magnetic field has an important effect in the structure of the disc. Terquem & Papaloizou (1996) studied the linear stability of a differentially rotating disc containing a purely toroidal magnetic field. They discovered that discs containing a purely toroidal magnetic field are always found to be unstable. Machida et al. (1999) considered the three-dimensional global MHD simulation of a torus treated by toroidal magnetic fields. AF examined the effect of a toroidal magnetic field on a viscous gaseous disc around a central object under an advection dominated stage. Assuming steady and axisymmetric flow and using the steady self-similar method, they found that the nature of the disc was significantly different from that of the weakly magnetized case.

In this study, we want to explore how the dynamics of a rotating and accreting viscous gas depends on its toroidal magnetic field. We answer this question by solving MHD equations for accreting gases that are self-similar in time. We assume that turbulent viscosity is a result of the angular momentum transport of the fluid and that there is efficient radiation cooling in the flow. This paper is organized as follows. In Section 2, we define the general problem of constructing a model for quasi-spherical magnetized polytropic accretion flow. In Section 3, we use the self-similar method to solve the integrated equations that govern the dynamical behaviour of the accreting gas. We present a summary of the model in Section 4.

2 GENERAL FORMULATION

We use the spherical coordinates \((r, \theta, \varphi)\) centred on the accreting object and make the following standard assumptions.

(i) The accreting gas is a highly ionized gas with infinitive conductivity.

(ii) The magnetic field has only an azimuthal component.

(iii) The gravitational force on a fluid element is characterized by the Newtonian potential of a point mass, \(\Psi = -(GM_\ast/r)\), where \(G\) represents the gravitational constant and \(M_\ast\) is the mass of the central star.

(iv) The equations written in spherical coordinates are considered in the equatorial plane \(\theta = (\pi/2)\) and terms with any \(\theta\) and \(\varphi\) dependence are neglected; thus, all quantities are expressed in terms of spherical radius \(r\) and time \(t\).

(v) For the sake of simplicity, self-gravity and general relativistic effects have been neglected.

(vi) The equation of state for the accreting gas is \(p_{\text{gas}} = K\rho^\gamma\) where \(\gamma\) and \(K\) are constant.

The macroscopic behaviour of such a system can be analysed by perfect MHD approximation. As stated in the introduction, we focus on analysing the role of the toroidal magnetic field and viscosity in an accreting gas. Thus, the basic equations are the continuity equation

\[
\frac{\partial \rho}{\partial t} + r \frac{\partial}{\partial r} \left( r^2 \rho v_r \right) = 0,
\]

the equations of motion

\[
\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{GM_\ast}{r^2} = \frac{B_r}{4\pi \rho} \frac{\partial}{\partial r} (r B_r),
\]

the polypotropic equation

\[
p_{\text{gas}} = K \rho^\gamma,
\]

and the field freezing equation

\[
\frac{\partial B_r}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r v_r B_r) = 0.
\]

Here, \(\Omega = \nu \theta/r\) is the angular speed and \(v\) is the kinematic viscosity coefficient. As mentioned above, our understanding of turbulent viscosity is incomplete, and for this reason we adopt an empirical prescription. So, we use the usual \(\alpha\)-prescription (Shakura & Sunyaev 1973) for the viscosity, which we write in the following form for the kinematic coefficient of viscosity:

\[
v = \alpha \frac{p_{\text{gas}}}{\rho \Omega K}.
\]

(Narayan & Yi 1994). Here, \(\alpha\) is constant (Tout 2000; King, Pringle & Livio 2007) and \(\Omega_k\) is the Keplerian angular velocity defined by

\[
\Omega_k = \sqrt{\frac{GM_\ast}{r^3}}.
\]

Note that \(v\) is a function of position and time, as \(\Omega_k\) depends on \(r\), and \(\rho\) varies with \(r\) and \(t\). To study the effect of viscosity, \(\alpha\) is used as a free parameter.

Before solving equations (1)–(5), it is convenient to non-dimensionalize the equations. So, the dimensionless variables are introduced according to

\[
r \rightarrow r^\ast, \quad t \rightarrow \tilde{t}, \quad \rho \rightarrow \tilde{\rho}, \quad p_{\text{gas}} \rightarrow \tilde{p} p_{\text{gas}}, \quad v_r \rightarrow \tilde{v}_r, \quad \Omega \rightarrow \tilde{\Omega}, \quad B_r \rightarrow \tilde{B} B_r,
\]

where

\[
\tilde{v} = \sqrt{\frac{GM_\ast}{r^3}} \tilde{t} = \tilde{\Omega},
\]

\[
\tilde{p} = \frac{\tilde{B}^2}{\tilde{B}_o} = \tilde{\rho} \tilde{v}^2, \quad K = \frac{GM_\ast}{\tilde{B}_o \tilde{v}^{\gamma-1}}.
\]

With these transformations and using equations (4), (6) and (7), equations (1) and (5) do not change, but equations (2) and (3) become

\[
\frac{\partial v_r}{\partial \tilde{t}} + v_r \frac{\partial v_r}{\partial \tilde{r}} + \frac{\gamma \rho^{\gamma-2}}{r} \frac{\partial \rho}{\partial \tilde{r}} + \frac{1}{\tilde{r}^2} = \frac{\tilde{v}_r^2}{r} - \frac{2 \tilde{B}_r}{\tilde{B}_o} \frac{\partial}{\partial \tilde{r}} (r \tilde{B}_r),
\]

\[
\rho \left[ \frac{\partial}{\partial \tilde{t}} (r^2 \tilde{\Omega}) + v_r \frac{\partial}{\partial \tilde{r}} (r^2 \tilde{\Omega}) \right] = \frac{\alpha}{\tilde{r}^2} \frac{\partial}{\partial \tilde{r}} \left[ r^{1/2} \rho^{\gamma-1} \frac{\partial \tilde{\Omega}}{\partial \tilde{r}} \right].
\]

3 SELF-SIMILAR SOLUTIONS

3.1 Analysis

The technique of self-similar analysis is useful for an understanding of the physics of accreting viscous gas in a toroidal magnetic field. Of course, this method is familiar from its wide range of applications in the full set of equations of MHD in many research fields of astrophysics. In the self-similar formulation, various physical quantities are expressed as dimensionless functions of a similarity variable. So, this lends itself to a set of partial differential equations, such as those mentioned above, which can be transformed into a set of ordinary differential equations. A similarity solution, although constituting only a limited part of problem, is often useful in understanding the basic behaviour of the system. So, in order to seek
similarity solutions for the above equations, a similarity variable $\eta$ is introduced as

$$\eta = \frac{r}{t}$$

and it is assumed that each physical quantity is given by the following forms:

$$\rho(r, t) = t^{\epsilon_1} R(\eta)$$

$$v_r(r, t) = t^{\epsilon_2} V(\eta)$$

$$\Omega(r, t) = t^{\epsilon_3} \alpha(\eta)$$

$$B_\theta(r, t) = t^{\epsilon_4} B(\eta).$$

The exponents $\epsilon_1, \epsilon_2, \epsilon_3,$ and $\epsilon_4$ are constant and must be determined. By substituting equations (12)–(16) into equations (1), (5), (10) and (11), the following general results are obtained:

$$\epsilon_1 = -\frac{2}{3(\gamma - 1)}, \quad \epsilon_2 = -\frac{1}{3}, \quad \epsilon_3 = -1, \quad \epsilon_4 = -\frac{\gamma}{3(\gamma - 1)},$$

and

$$n = \frac{2}{3}.$$

The above results imply that each physical quantity retains a similar spatial shape as the flow evolves, but the radius of the flow increases proportionally to $t^{3/5}$. Also, the time-dependent density, the pressure and the toroidal magnetic field vary with $\gamma$; however, they decrease by time for $\gamma > 1$.

Here, let us find the time-dependent self-similar solution of the mass accretion rate

$$\dot{M} = -4\pi r^2 \rho v_r.$$  \hspace{1cm} (19)

We can non-dimensionalize equation (19) under transformation (8) and

$$\dot{M} \rightarrow \dot{\dot{M}} \dot{M},$$

where

$$\dot{\dot{M}} = \dot{r}^2 \dot{\rho} \dot{v}.$$  \hspace{1cm} (21)

Under transformations (8) and (20), equation (19) does not change and its behaviour can be considered under similarity quantities that are implied in equations (12)–(16). The similarity solution shows that the mass accretion rate $\dot{M}$ is proportional to $r^{(\gamma-5)/3(\gamma-1)}$. When $\gamma = 5/3$, the mass accretion rate is independent of time and decreases in $1 < \gamma < 5/3$. The time-dependent behaviour of this quantity is applied in Section 3.2.

Solving equations (1), (10), (11) and (19) under transformations (12)–(15) in a non-magnetic state makes it clear that the behaviour of physical quantities in the non-magnetic and the magnetic disc are the same. The result is one of the strictures of time-dependent self-similar solution.

Subsequently, equations (1), (5), (10) and (11) for the dependence of the physical quantities on the similarity variables are written as

$$-\frac{2}{3(\gamma - 1)} R + \left( V - \frac{2\eta}{3} \right) \frac{dR}{d\eta} + \frac{R}{\eta^2} \frac{d}{d\eta} (\eta^2 V) = 0,$$ \hspace{1cm} (22)

$$-\frac{\gamma}{3} V + \left( V - \frac{2\eta}{3} \right) \frac{dV}{d\eta} + \gamma R \frac{dR}{d\eta} + \frac{1}{\eta^2} = \eta \omega^2 - \frac{2}{\eta} \frac{d(\eta B)}{d\eta},$$ \hspace{1cm} (23)

$$\frac{1}{\eta^2} \frac{d}{d\eta} \left[ \frac{\eta^{11/2} R^{\gamma - 1} \frac{d\omega}{d\eta}}{\eta^2} \right],$$ \hspace{1cm} (24)

$$-\frac{\gamma}{3(\gamma - 1)} B + \left( V - \frac{2\eta}{3} \right) \frac{dB}{d\eta} + \frac{B}{\eta} \frac{d}{d\eta} (\eta V) = 0$$ \hspace{1cm} (25)

This is a system of non-linear ordinary differential equations. Once $\alpha$ and $\gamma$ are selected, the set of equations (22)–(25) can be solved. Before solving the above equations numerically, it was found that equations (22) and (25) imply

$$V = \frac{2\eta}{3} + C \frac{R}{B^2}.$$ \hspace{1cm} (26)

where $C$ is constant of integration; this is calculated in Section 3.2. We can rewrite equation (26) in terms of the Alfvén velocity. The Alfvén velocity in a purely toroidal magnetic field is $v_A B = B^2 / 4\pi \rho$. By using transformations of equations (8), (9), (13), (16), (17) and the Alfvén velocity equation, equation (26) can be rewritten in the following form

$$V = \frac{2\eta}{3} + \frac{2C}{A^2},$$ \hspace{1cm} (27)

where $A = v_A / \dot{v}$. The result implies that the radial velocity of a quasi-spherical accretion flow in the presence of a toroidal magnetic field is a function of the Alfvén velocity.

### 3.2 Inner limit

When $\gamma = 5/3$, an appropriate asymptotic solution as $\eta \rightarrow 0$ has the form

$$R(\eta) \sim R_0 \eta^{-3/2}$$ \hspace{1cm} (28)

$$V(\eta) \sim V_0 \eta^{-1/2}$$ \hspace{1cm} (29)

$$\omega(\eta) \sim \omega_0 \eta^{-3/2}$$ \hspace{1cm} (30)

$$B(\eta) \sim B_0 \eta^{-1/2}$$ \hspace{1cm} (31)

in which

$$R_0 = \left( \frac{M}{12\pi \alpha} \right)^{3/5}$$ \hspace{1cm} (32)

$$V_0 = -3\alpha \left( \frac{M}{12\pi \alpha} \right)^{2/5}$$ \hspace{1cm} (33)

$$\omega_0 = 1 - \frac{5}{2} \left( \frac{M}{12\pi \alpha} \right)^{2/5} - \frac{9}{2} \alpha^2 \left( \frac{M}{12\pi \alpha} \right)^{4/5}$$ \hspace{1cm} (34)

$$B_0 = \beta_0 \left( \frac{M}{12\pi \alpha} \right).$$ \hspace{1cm} (35)

In order to derive the above relations, the mass accretion rate and the $\beta$ parameter are used (i.e. the ratio of the magnetic pressure to the gas pressure). When $\eta \rightarrow 0$ and $\gamma = 5/3$, the mass accretion rate becomes $\dot{M} \sim -4\pi R_0 V_0$, and the ratio of the magnetic pressure to the gas pressure becomes $\beta(\eta) \sim \beta_0 \eta^{3/2}$, where $\beta_0 = B_0^2 / R_0^3$. These relations were applied to derive equations (32)–(35). In order to consider the effect of the magnetic field in the disc, the $\beta_0$ parameter is used.
The asymptotic solution shows that the $\alpha$ parameter is effective in the inner edge of the disc and physical quantities are sensitive to it; that is, the radial infall velocity increases by adding $\alpha$, the angular velocity is sub-Keplerian for all values of $\alpha$, and the density and the toroidal magnetic field in the inner edge of the disc decrease with increasing $\alpha$. These results, which are achieved for the inner edge of the disc, are qualitatively consistent with the results of AF. Now, it is possible to derive the approximate constant of integration $C$ in equation (26), by using equations (28)–(35):

$$C \sim -3\alpha_0 \left( \frac{M}{12\pi R} \right)^{4/5}.$$

(36)

3.3 Numerical solution

If the value of $\eta_{in}$ is estimated, which is a point very near to the centre, the equations can be integrated from this point outwards by using the above expansions. Examples of such solutions are presented in Figs 1 and 2. The profiles in Figs 1 and 2 are plotted for different $\beta_{in}$, which is the amount of $\beta$ in $\eta_{in}$. From the estimated $\eta_{in}$ and $\beta_{in}$, and equations (28), (31), (32) and (35), we can find $\beta_0 = \beta_{in}/\eta_{in}^{1/2}$. The delineated quantities ($\eta_{1/2} R, \eta_{1/2} V, \ldots$) in Figs 1 and 2 are constant at steady self-similar solutions (Narayan & Yi 1994, 1995; Shadmehri 2004; AF; Ghanbari et al. 2007).

By increasing the $\beta$ parameter, which indicates the role of the magnetic field in the dynamics of accretion discs, the radial thickness of the disc decreases; equations (13) and (17) imply that compression increases with time. Liffman & Bardou (1999) and Campbell & Heptinstall (1998) showed compression of the disc in the height direction by the effect of the toroidal magnetic field, but they did not consider the effect of the toroidal magnetic field in the radial thickness of the disc. Also, by adding the $\beta$ parameter, the radial infall velocity increases; such a property is qualitatively consistent with AF. This is because of the magnetic tension terms that dominate the magnetic pressure term in the radial momentum equation, and which assist the radial infall motion. The flow is differentially rotating, although it is highly sub-Keplerian at large radii.

![Figure 1. Time-dependent self-similar solution for $\gamma = 5/3$, $\alpha = 0.5$ and $M = 1.0$. The solid lines represent $\beta_{in} = 0.5$, the dotted lines represent $\beta_{in} = 1.0$ and the dashed lines represent $\beta_{in} = 1.5$, where $\beta_{in}$ is the value of $\beta$ in $\eta_{in}$.](image1)

![Figure 2. Time-dependent self-similar solution for $\gamma = 5/3$, $\alpha = 0.5$ and $M = 1.0$. The solid lines represent $\beta_{in} = 0.5$, the dotted lines represent $\beta_{in} = 1.0$, and the dashed lines represent $\beta_{in} = 1.5$, where $\beta_{in}$ is the value of $\beta$ in $\eta_{in}$.](image2)
To investigate the existence of the transonic point, the square of the sound velocity is introduced, which subsequently can be expressed as

\[ v_s^2 = \frac{\gamma P_{\text{gas}}}{\rho} = \frac{GM}{r} r^{-3/2} \gamma R^{p-1} \] (37)

Here, \( S = (\gamma R^{p-1})^{1/2} \) is the adiabatic sound velocity in the self-similar flow, which is rescaled in the course of time. The Mach number referred to the reference frame is defined as (Gaffet & Fukue 1983; Fukue 1984)

\[ \mu = \frac{v_t - v_F}{v_s} = \frac{V - n \eta}{S} \] (38)

where

\[ v_F = \frac{dr}{dt} = \frac{r}{t} \] (39)

is the velocity of the reference frame which is moving outward here as time goes by. The Mach number introduced so far represents the instantaneous and local Mach number of the unsteady self-similar flow. As seen in Fig. 2, there is a transonic point, which denotes that the square of the Mach number is equal to unity (\( \mu^2 = 1 \)).

By adding the strength of the magnetic field, the transonic point approaches the central object. The solution shows that the parameter \( \beta \) varies by radii and is important at larger radii, while in the steady self-similar solution, this parameter is constant. The \( \beta \) parameter shows that the dominant pressure in the outer region of the disc is the magnetic pressure; this result is consistent with observed YSO discs (Aitken et al. 1993; Wright et al. 1993; Greaves et al. 1997).

4 SUMMARY

In this paper, we have solved the equations of time-dependent quasi-spherical accretion flow with a toroidal magnetic field using semi-analytical similarity methods. The flow is able to radiate efficiently, so we have substituted the polytropic equation instead of the energy equation. A solution was found for the important case \( \gamma = 5/3 \), which has differential rotation and viscous dissipation. The flow avoids many of the strictures of steady self-similar solutions (Narayan & Yi 1994, AF). Thus, the radial dependence of the calculated physical quantities in this sense are different from the steady self-similar solution.

The flow has differential rotation in small radii and has Keplerian behaviour at large radii, which at large radii is similar to steady self-similar solutions. Also, the flow is sub-Keplerian at all radii, which is consistent with the findings of AF, when they considered the disc under a moderate strength of the magnetic field. The solution shows that in time-dependent quasi-spherical accretion flow, there is a transonic point, where the point approaches the central object by increasing the strength of the toroidal magnetic field. By increasing the strength of the toroidal magnetic field, the radial thickness of the disc decreases and the disc becomes compressed.

Here, the latitudinal dependence of the physical quantities is ignored, although some authors have shown that latitudinal dependence is important in the structure of a disc (Narayan & Yi 1995; Ghanbari et al. 2007). It is possible to investigate the latitudinal behaviour of such discs. Also, it is assumed that there is efficient radiation cooling in the flow and we have used the polytropic equation for the energy equation. During recent years, one type of accretion disc has been studied, in which the energy released through viscous processes in the disc may be trapped within the accreting gas. This type of flow is known as advection-dominated accretion flow (ADAF). The solution of AF shows that the physical quantities of the disc vary with the advection parameter. In future studies, we will improve our model with a realistic energy equation.

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