Hints of large $\tan \beta$ in flavour physics

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Abstract

Motivated by the first evidence of the $B_u \to \tau \nu$ transition reported by Belle [1] and by the precise $\Delta M_{B_s}$ measurement by CDF [2], we analyse these and other low-energy observables in the framework of the MSSM at large $\tan \beta$. We show that for heavy squarks and $A$ terms ($M_{\tilde{q}}, A_U > \sim 1$ TeV) such scenario has several interesting virtues. It naturally describes: i) a suppression of $\mathcal{B}(B_u \to \tau \nu)$ of (10-40)%, ii) a sizable enhancement of $(g - 2)_\mu$, iii) a heavy SM-like Higgs ($m_{h^0} \sim 120$ GeV), iv) small non-standard effects in $\Delta M_{B_s}$ and $\mathcal{B}(B \to X_s \gamma)$ (in agreement with present observations). The possibilities to find more convincing evidences of such scenario, with improved data on $\mathcal{B}(B_u \to \tau \nu)$, $\mathcal{B}(B_{s,d} \to \ell^+ \ell^-)$ and other low-energy observables, are briefly discussed.

1 Introduction

In many extensions of the SM, including the so-called Minimal Supersymmetric extension of the SM (MSSM), the Higgs sector consists of two $SU(2)_L$ scalar doublets, coupled separately to up- and down-type quarks. A key parameter of all these models is $\tan \beta = v_u/v_d$, the ratio of the two Higgs vacuum expectation values. This parameter controls the overall normalization of the Yukawa couplings. The regime of large $\tan \beta$ [$\tan \beta = \mathcal{O}(m_t/m_b)$] has an intrinsic theoretical interest since it allows the unification of top and bottom Yukawa couplings, as predicted in well-motivated grand-unified models [3].

The large $\tan \beta$ regime of both supersymmetric and non-supersymmetric models has a few interesting signatures in $B$ physics. One of the most clear ones is the suppression of $\mathcal{B}(B_u \to \tau \nu)$ with respect to its SM expectation [4]. Potentially sizable effects are expected also in $\mathcal{B}(B \to X_s \gamma)$, $\Delta M_{B_s}$ and $\mathcal{B}(B_{s,d} \to \ell^+ \ell^-)$. Motivated by the recent
experimental results on both $\mathcal{B}(B_u \to \tau \nu)$ [1] and $\Delta M_{B_s}$ [2], we present here a correlated analysis of all these observables within the large $\tan \beta$ limit of the MSSM.

Because of the effective non-holomorphic terms which break the Peccei-Quinn symmetry of the tree-level Yukawa interaction [5, 6], the phenomenology of the MSSM in the large $\tan \beta$ regime is richer than in non-supersymmetric models. We pay particular attention to re-summation effects beyond the one-loop level, both in charged- and in neutral-current interactions, which play a key role in the correlations among these $B$-physics observables [7–12].

The generic MSSM contains in principle several free parameters in addition to $\tan \beta$. Given the absence of significant non-standard effects both in the electroweak and in the flavour sector, we limit ourselves to a Minimal Flavour Violating (MFV) scenario [13, 14] with squark masses in the TeV range. In addition, we take into account the important information on the model derived by two flavour-conserving observables: the anomalous magnetic moment of the muon and the lower limit on the lightest Higgs boson mass.

The present central values of the measurements of $\mathcal{B}(B_u \to \tau \nu)$ and $(g - 2)_\mu$ are substantially different from the corresponding SM expectations. Although both these effects are not statistically significant yet, we find that these central values can naturally be accommodated within this scenario (for a wide range of $\mu$, $\tan \beta$ and the charged Higgs mass). More interestingly, if the trilinear term $A_U$ is sufficiently large, this scenario can also explain why the lightest Higgs boson has not been observed yet. Finally, the parameter space which leads to these interesting effects can also naturally explain why $\mathcal{B}(B \to X_s \gamma)$ and $\Delta M_{B_s}$ are in good agreement with the SM expectations. We are therefore led to the conclusion that, within the supersymmetric extensions of the SM, the scenario with large $\tan \beta$ and heavy soft-breaking terms in the squark sector is one of the most interesting and likely possibilities.

The plan of the paper is the following: in Section 2 we recall the basic formulae to analyse large-$\tan \beta$ effects in $\mathcal{B}(B_u \to \tau \nu)$, $\mathcal{B}(B_{s,d} \to \ell^+ \ell^-)$, $\Delta M_{B_s}$, and $\mathcal{B}(B \to X_s \gamma)$. We pay particular attention to the $\mathcal{B}(B_u \to \tau \nu)$ case, analysing the resummation of large $\tan \beta$ effects beyond the lowest order and the strategy to decrease the theoretical uncertainty with the help of $\Delta M_{B_d}$. In Section 3 we discuss the implications on the MSSM parameter space derived by $m_{h^0}$ and $(g - 2)_\mu$. The correlated analysis of all the observables is presented in Section 4 together with a discussion about future tests of the model by means of other $P \to \ell \nu$ decays. The results are summarized in the Conclusions.

## 2 $B$-physics observables

### 2.1 $B_u \to \tau \nu$

The SM expectation for the $B_u \to \tau \nu$ branching fraction is

$$\mathcal{B}(B_u \to \tau \nu)^{\text{SM}} = \frac{G_F^2 m_B m_\tau^2}{8 \pi} \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 f_B^2 |V_{ub}|^2 \tau_B .$$  \hspace{1cm} (1)
Using $|V_{ub}| = (4.39 \pm 0.33) \times 10^{-3}$ from inclusive $b \to u$ semileptonic decays [16], $\tau_B = 1.643 \pm 0.010$ ps [17], and the recent unquenched lattice result $f_B = 0.216 \pm 0.022$ GeV [18], this implies $\mathcal{B}(B_u \to \tau \nu)_{\text{SM}} = (1.59 \pm 0.40) \times 10^{-4}$. This prediction should be compared with Belle’s recent result [1]:

$$
\mathcal{B}(B^- \to \tau^- \bar{\nu})_{\text{exp}} = (1.06_{-0.28}^{+0.34}\text{(stat)})_{-0.16}^{+0.18}\text{(syst)}) \times 10^{-4}.
$$

(2)

Within two-Higgs doublet models, the charged-Higgs exchange amplitude induces an additional tree-level contribution to semileptonic decays. Being proportional to the Yukawa couplings of quarks and leptons, this additional contribution is usually negligible. However, in $B \to \ell \nu$ decays the $H^\pm$ exchange can compete with the $W^\pm$ exchange thanks to the helicity suppression of the latter. Interestingly, in models where the two Higgs doublets are coupled separately to up- and down-type quarks, the interference between $W^\pm$ and $H^\pm$ amplitudes is necessarily destructive [4].

Taking into account the resummation of the leading tan $\beta$ corrections to all orders, the charged-Higgs contributions to the $B_u \to \tau \nu$ amplitude within a MFV supersymmetric framework lead to the following ratio:

$$
R_{B\tau\nu} = \frac{\mathcal{B}^{\text{SUSY}}(B_u \to \tau \nu)}{\mathcal{B}^{\text{SM}}(B_u \to \tau \nu)} = \left[1 - \left(\frac{m_B^2}{m_{H^\pm}^2}\right) \frac{\tan^2 \beta}{(1 + \epsilon_0 \tan \beta)}\right]^2,
$$

(3)

where $\epsilon_0$ denotes the effective coupling which parametrizes the non-holomorphic correction to the down-type Yukawa coupling induced by gluino exchange (see Section 2.2). We stress that the result in Eq. (3) takes into account all the leading tan $\beta$ corrections both in the redefinition of the bottom-quark Yukawa coupling and in the redefinition of the CKM matrix.\footnote{The result in Eq. (3) can easily be obtained by means of the charged-Higgs effective Lagrangian in Eq. (52) of Ref. [14], which systematically takes into account the redefinition of Yukawa couplings and CKM matrix elements. The explicit application to $B_u \to \tau \nu$ has been presented first in Ref. [15].}

For a natural choice of the parameters ($30 \lesssim \tan \beta \lesssim 50, 0.5 \lesssim M_{H^\pm}/\text{TeV} \lesssim 1, \epsilon_0 \sim 10^{-2}$) Eq. (3) implies a (5-30)\% suppression with respect to the SM. This would perfectly fit with the experimental result in (2), which implies

$$
R_{B\tau\nu}^{\text{exp}} = \frac{\mathcal{B}^{\text{exp}}(B_u \to \tau \nu)}{\mathcal{B}^{\text{SM}}(B_u \to \tau \nu)} = 0.67_{-0.27}^{+0.30} = 0.67_{-0.21}^{+0.24}\text{exp} \pm 0.14|f_B| \pm 0.10|V_{ub}|.
$$

(4)

Apart from the experimental error, one of the difficulties in obtaining a clear evidence of a possible deviation of $R_{B\tau\nu}$ from unity is the large parametric uncertainty induced by $|f_B|$ and $|V_{ub}|$. As suggested by Ikado [19], an interesting way to partially circumvent this problem is obtained by normalizing $\mathcal{B}(B_u \to \tau \nu)$ to the $B_d \bar{B}_d$ mass difference ($\Delta M_{B_d}$). Neglecting isospin-breaking differences in masses and decay constants between $B_d$ and $B_u$ mesons, we can write

$$
\mathcal{B}(B_u \to \tau \nu)_{\tau\nu,\Delta M_{B_d}} = \left.\frac{3\pi}{4m_B(m_b)\lambda (m_t^2/M_W^2)B_{B_d}(m_b)M_W^2}\right\frac{m_{\tau}^2}{m_B^2} \left(1 - \frac{m_{\tau}^2}{m_B^2}\right)^2 \left|\frac{V_{ub}}{V_{td}}\right|^2.
$$

(5)

$$
= 1.24 \times 10^{-4} \left(\frac{|V_{ub}/V_{td}|}{0.473}\right)^2 \left(\frac{0.836}{B_{B_d}(m_b)}\right).
$$

(6)
Following standard notations, we have denoted by $S_0$, $\eta_B$ and $B_{B_d}$ the Wilson coefficient, the QCD correction factor and the bag parameter of the $\Delta B = 2$ operator within the SM (see e.g. Ref. [11]). Using the unquenched lattice result $B_{B_d}(m_b) = 0.836 \pm 0.068$ [20] and $|V_{ub}/V_{td}| = 0.473 \pm 0.024$ from the UTfit Collaboration [21], we then obtain

$$R_{B\tau\nu}^{\exp} = \frac{B_{B_u \to \tau\nu}^{\exp}}{B_{B_u \to \tau\nu}^{SM}} \frac{\Delta M_{B_d}^{\exp}}{\Delta M_{B_d}^{SM}} = 1.03^{+0.39}_{-0.33} = 1.03^{+0.37}_{-0.31} \exp \pm 0.08|B_{B_d}| \pm 0.10|V_{ub}/V_{td}|.$$ 

(7)

The following comments follow from the comparison of Eqs. (4) and (8):

- The two results are compatible and with similar overall relative errors. However, the parametric/theoretical component is smaller in Eq. (8). The latter could therefore become a more stringent test of the SM in the near future, with higher statistics on the $B_u \to \tau\nu$ channel.

- In generic extensions of the SM, $R_{B\tau\nu}$ and $R_{B\tau\nu}^{\exp}$ are not necessarily the same. However, they should coincide if the non-SM contribution to $\Delta M_{B_d}$ is negligible, which is an excellent approximation in the class of models we are considering here.

- For consistency, the $|V_{ub}/V_{td}|$ combination entering in Eq. (8) should be determined without using the information on $\Delta M_{B_d}$ and $B_u \to \tau\nu$ (condition which is already almost fulfilled). In the near future one could determine this ratio with negligible hadronic uncertainties using the relation $|V_{ub}/V_{td}| = |\sin \beta_{\text{CKM}}/\sin \gamma_{\text{CKM}}|$.

2.2 $B(B_{s,d} \to \ell^+\ell^-)$, $B(B \to X_s\gamma)$, and $B_s-\overline{B}_s$ mixing

The important role of these observables in the MSSM with MFV and large $\tan \beta$ has been widely discussed in the literature [8–11] (see also [22–25]). We recall here only a few ingredients which are necessary to analyse their correlations with $B(B_u \to \tau\nu)$.

We are interested in a scenario with heavy squarks, where the $SU(2)_L$-breaking corrections of $O(M_W/M_{\tilde{g}})$ can be treated as a small perturbation. In this limit, the $\tan \beta$-enhanced corrections to the down-type Yukawa couplings are parameterized by the following effective couplings [5]

$$\epsilon_0 = -\frac{2\alpha_s\mu}{3\pi M_{\tilde{g}}} H_2 \left( \frac{M_{\tilde{q}L}^2}{M_{\tilde{g}}^2}, \frac{M_{\tilde{d}_R}^2}{M_{\tilde{g}}^2} \right), \quad \epsilon_Y = -\frac{A_U}{16\pi^2\mu} H_2 \left( \frac{M_{\tilde{q}L}^2}{\mu^2}, \frac{M_{\tilde{u}_R}^2}{\mu^2} \right),$$

(9)

where

$$H_2(x,y) = \frac{x \ln x}{(1-x)(1-y)} + \frac{y \ln y}{(1-y)(1-x)}$$

(10)

and, as usual, $\mu$ denotes the supersymmetric Higgs mass term and $A_U$ the three-linear soft-breaking term.
In $\mathcal{B}(B_{s,d} \to \ell^+\ell^-)$ and $B_s-\bar{B}_s$ the only relevant contributions in the limit of heavy squarks are the effective tree-level Higgs-mediated neutral currents. In the $\mathcal{B}(B_{s,d} \to \ell^+\ell^-)$ case this leads to [9,10,14]:

$$R_{B\ell\ell} = \frac{\mathcal{B}^{\text{SUSY}}(B_q \to \ell^+\ell^-)}{\mathcal{B}^{\text{SM}}(B_q \to \ell^+\ell^-)} = (1 + \delta_S)^2 + \left(1 - \frac{4m_t^2}{M_{B_q}^2}\right)\delta_S^2,$$

$$\delta_S = \frac{\pi \sin^2 \theta_w M_{B_q}^2}{\alpha_{\text{em}}M_A^2 C_{10A}(m_t^2/M_W^2)} \frac{\epsilon_Y \lambda_t^2 \tan^3 \beta}{1 + (\epsilon_0 + \epsilon_Y \lambda_t^2 \tan \beta)[1 + \epsilon_0 \tan \beta]},$$

where $\lambda_t$ is the top-quark Yukawa coupling, $C_{10A}(m_t^2/M_W^2)$ is the SM Wilson coefficient ($\lambda_t \approx 1$, $C_{10A} \approx 1$) and $M_A$ is the mass of the physical pseudoscalar Higgs (at the tree level $M_A^2 = M_{H^+}^2 - M_W^2$). As discussed in [9–11,22], for $\tan \beta \approx 50$ and $M_A \sim 0.5 \text{ TeV}$ the neutral-Higgs contribution to $\mathcal{B}(B_{s,d} \to \ell^+\ell^-)$ can easily lead to an $\mathcal{O}(100)$ enhancement over the SM expectation. This possibility is already excluded by experiments: the CDF bound $\mathcal{B}(B_s \to \mu^+\mu^-) < 8.0 \times 10^{-8}$ [26] implies

$$R_{B\ell\ell} < 23 \quad [90\% \text{ C.L.}].$$

As we will discuss in Section 4, this limit poses severe constraints on the MSSM parameter space; however, it does not prevent a sizable charged-Higgs contribution to $\mathcal{B}(B_u \to \tau\nu)$. This can easily be understood by noting that the effect in Eq. (11) vanishes for $A_U \to 0$ and has a stronger dependence on $\tan \beta$ than $R_{B\tau\nu}$.

The neutral-Higgs contribution to $\Delta M_{B_s}$ leads to [10,11,14]:

$$R_{\Delta M_s} = \frac{(\Delta M_{B_s})^{\text{SUSY}}}{(\Delta M_{B_s})^{\text{SM}}} = 1 - m_b(\mu_b^2)m_s(\mu_b^2) \frac{64\pi \sin^2 \theta_w}{\alpha_{\text{em}}M_A^2 S_0(m_t^2/M_W^2)} \times \frac{\epsilon_Y \lambda_i^2 \tan^2 \beta}{1 + (\epsilon_0 + \epsilon_Y \lambda_t^2 \tan \beta)[1 + \epsilon_0 \tan \beta]^2}.$$  

where $m_b(\mu_b^2)$ denotes the bottom- and strange-quark masses renormalized at a scale $\mu_b \approx m_b$.\footnote{For simplicity we have set to one the ratio of bag parameters between SM and scalar-current operators.} The parametric dependence on $A_U$ and $\tan \beta$ in (13) and (11) is quite similar, but the $m_b(\mu_b^2)$ factor implies a much smaller non-standard effect in $\Delta M_{B_s}$ (typically of a few %). Note that, similarly to the $\mathcal{B}(B_u \to \tau\nu)$ case, also in $\Delta M_{B_s}$ one expects a negative correction with respect to the SM. As we will show, thanks to the high experimental precision achieved by CDF [2], at present $\Delta M_{B_s}$ is comparable with $\mathcal{B}(B_s \to \mu^+\mu^-)$ in setting bounds in the MSSM parameter space. According to the SM expectation $(\Delta M_{B_s})^{\text{SM}} = 21.5 \pm 2.6 \text{ ps}^{-1}$ of the UTfit Collaboration [21], the CDF result $(\Delta M_{B_s})^{\text{exp}} = 17.35 \pm 0.25 \text{ ps}^{-1}$ implies

$$R_{\Delta M_s}^{\text{exp}} = \frac{(\Delta M_{B_s})^{\text{exp}}}{(\Delta M_{B_s})^{\text{SM}}} = 0.80 \pm 0.12.$$
The last $B$-physics observable we will consider is $\mathcal{B}(B \to X_s \gamma)$. This observable is particularly sensitive to possible non-standard contributions. However, contrary to $B_u \to \tau \nu$, $B_{s,d} \to \ell^+ \ell^-$, and $\Delta M_{B_s}$, there is no effective tree-level contribution by charged- or neutral-Higgs exchange in $B \to X_s \gamma$. Non-standard contributions from the Higgs sector appear only at the one-loop level and are not necessarily dominant with respect to the chargino-squark contributions, even for squark masses of $\mathcal{O}(1 \text{ TeV})$. In the numerical analysis presented in Section 4 we have implemented the improved chargino-squark amplitude computed in Ref. [8] and the charged- and neutral-Higgs exchange amplitudes of Ref. [14]. A key point to note is that for $A_U < 0$ charged-Higgs and chargino amplitudes tend to cancel themselves. This leads to a particularly favorable situation, given the small room for new physics in this observable. According to the SM estimate $\mathcal{B}(B \to X_s \gamma)^{\text{SM}} = (3.70 \pm 0.30) \times 10^{-4}$ by Gambino and Misiak [27], and the world average $\mathcal{B}(B \to X_s \gamma)^{\text{exp}} = (3.52 \pm 0.30) \times 10^{-4}$ [16], we set

$$0.76 < R_{BXs\gamma} = \frac{\mathcal{B}(B \to X_s \gamma)^{\text{SUSY}}}{\mathcal{B}(B \to X_s \gamma)^{\text{SM}}} < 1.15 \quad [90\% \text{ C.L.}] . \quad (16)$$

### 3 Flavour-conserving observables: $m_{h^0}$ and $(g - 2)_\mu$

In the previous section we have analysed the impact of a MFV supersymmetric scenario with large $\tan \beta$ on various $B$-physics observables. As we have seen, large $\tan \beta$ values can lead to huge enhancements of $\mathcal{B}(B_{s,d} \to \ell^+ \ell^-)$, or a visible depression of $\Delta M_{B_s}$, which are already strongly constrained from data. However, the correlation between the large $\tan \beta$ effects in these observables crucially depends on magnitude (and sign) of the trilinear term $A_U$. The question we address in this Section is which kind of additional information we can extract on $A_U$ and $\tan \beta$ from two key flavour-conserving observables, the lightest Higgs boson mass and the anomalous magnetic moment of the muon.

#### 3.1 The lightest Higgs boson mass

One of the most suggestive prediction of the MSSM is the existence of a relatively light neutral Higgs boson ($h^0$) with $m_{h^0} \lesssim 135 \text{ GeV}$ [28]. One of the most serious consistency problem of the model is why this particle has not been observed yet [29].

Even if the $h^0$ mass depends on the whole set of the MSSM parameters (after the inclusions of loop corrections), it is well known that $m_{h^0}$ mainly depends on the left-right mixing term in the stop mass matrix $m_t \tilde{M}_t^R = m_t (A_U - \mu / \tan \beta) \simeq m_t A_U$ (for large $\tan \beta$ values), on the average stop mass (which we identify with the average squark mass, $\overline{M}_t$), and on $\tan \beta$.

In Fig. 1 (left) we show $m_{h^0}$ as a function of $A_U / \overline{M}_t$ for $\overline{M}_t = 200, 500, 1000 \text{ GeV}$ and $M_A = 500 \text{ GeV}$. A maximum for $m_{h^0}$ is reached for about $A_U / \overline{M}_t \approx \pm 2$, which is usually denoted as the “maximal mixing” case. A minimum is reached around $A_U / \overline{M}_t \approx 0$, which we refer to as the “no mixing” case. As can be seen, even with the most favorable choice of $\overline{M}_t$ and $\tan \beta$, relatively large values for $m_{h^0} \geq 120 \text{ GeV}$ are possible only if $A_U \gtrsim \overline{M}_t$. 

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The maximal mixing case certainly favours large effects in $\mathcal{B}(B_{s,d} \to \ell^+ \ell^-)$ (we recall that $\epsilon_Y \sim \mu A_U/M_{\tilde{q}}^2$). However, it is not possible to establish a well defined correlation between $\mathcal{B}(B_{s,d} \to \ell^+ \ell^-)$ and $m_{h^0}$ since $\mathcal{B}(B_{s,d} \to \ell^+ \ell^-)_{\text{SUSY}} \sim \tan^6 \beta$ while $m_{h^0}$ is rather insensitive to $\tan \beta$ for $\tan \beta \geq 10$ (as clearly shown in the right plot of Fig. 1). Moreover, supersymmetric effects in $\mathcal{B}(B_{s,d} \to \ell^+ \ell^-)$ decouple as $M_4^2$ and are proportional to $\mu$, while the $m_{h^0}$ dependence on $\mu$ and $M_A$ is quite mild. In Fig. 1 (right) we show $m_{h^0}$ as a function of $M_{\tilde{q}}$ in the maximal mixing case for different $\tan \beta$ values.

The two plots in Fig. 1 have been obtained including the full one-loop corrections and the leading two-loop ones, cross-checking the results with those of FeynHiggs [30]. In summary, we find that in the absence of fine-tuned solutions, the present experimental lower bounds on $m_{h^0}$ [29] provide a strong support in favour of heavy squarks, $A_U \gtrsim M_{\tilde{q}}$, and $\tan \beta$ well above unity. A scenario which enhances the correlations between $\mathcal{B}(B_u \to \tau \nu)$, $\mathcal{B}(B_{s,d} \to \ell^+ \ell^-)$, $\Delta M_{B_s}$, and $\mathcal{B}(B \to X_s \gamma)$ at large $\tan \beta$. We stress that this scenario is quite different from a non-supersymmetric two-Higgs doublet model, even in the limit where $M_{\tilde{q}}^2 \gg M_{H^\pm}^2$.

### 3.2 $(g - 2)_\mu$

The possibility that the anomalous magnetic moment of the muon [$a_\mu = (g - 2)_\mu/2$], which has been measured very precisely in the last few years [31], provides a first hint of physics beyond the SM has been widely discussed in the recent literature. Despite substantial progress both on the experimental and on the theoretical sides, the situation
is not completely clear yet (see Ref. [32] for an updated discussion). Most recent analyses converge towards a $2\sigma$ discrepancy in the $10^{-9}$ range [32]:

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} \approx (2 \pm 1) \times 10^{-9}.$$  \hfill (17)

If confirmed and interpreted within the MSSM, this result would unambiguously signal a large value of $\tan \beta$ (see Ref. [33, 34] and references therein).

The main SUSY contribution to $a_\mu^{\text{MSSM}}$ is usually provided by the loop exchange of charginos and sneutrinos \([a_\mu^{\text{MSSM}}]_\chi \sim \alpha_2 M_2 \tan \beta/\mu M_\chi^2]\). But if $\mu$ is very large –the scenario where $B(B_s \to \mu^+\mu^-)$ gets its maximum value– then $a_\mu^{\text{MSSM}}$ turns out to be dominated by the neutralino (Bino type) amplitude \([a_\mu^{\text{MSSM}}]_B \sim \alpha_1 M_1 \mu \tan \beta/M_\tilde{\ell}^2\). A useful tool to illustrate basic features of the supersymmetric contribution to $a_\mu$ is the expression

$$\frac{a_\mu^{\text{MSSM}}}{1 \times 10^{-9}} \approx 2.5 \left( \frac{\tan \beta}{50} \right) \left( \frac{500 \text{ GeV}}{M_\tilde{\chi}} \right)^2 \left( \frac{M_\tilde{\ell}}{M_\chi} \right),$$  \hfill (18)

which provides a good approximation to the full one-loop result [34] in the limit of almost degenerate higgsinos and electroweak gauginos ($M_1 \sim M_2 \sim \mu \gg M_W$), and allowing a moderate splitting between slepton and chargino masses. From this expression it is clear that values of $a_\mu^{\text{MSSM}}$ in the $10^{-9}$ range, as suggested by Eq. (17), require large $\tan \beta$ values.

As pointed out in [23], there is a very stringent correlation between $a_\mu^{\text{MSSM}}$ and $B(B_s \to \mu^+\mu^-)^{\text{MSSM}}$ in specific frameworks, such as the constrained minimal supergravity scenario. However, this correlation is much weaker in a more general context, such as the one we are considering here. Given the mild $\tan \beta$ dependence of $a_\mu^{\text{MSSM}}$ compared to $B(B_s \to \mu^+\mu^-)^{\text{MSSM}}$, it is easy to generate a sizable contribution to $a_\mu$ while keeping $B(B_s \to \mu^+\mu^-)$ well below its actual experimental resolution. As we will illustrate in the next section, there is a stronger model-independent correlation between $a_\mu^{\text{MSSM}}$ and $B(B_u \to \tau \nu)^{\text{MSSM}}$.

4 Discussion

The correlations among the various observables discussed in the previous sections are illustrated in Figures 2 and 3. Here we show the regions in the $M_{H^\pm}\tan \beta$ plane which could give rise to a detectable deviation from the SM in $B(B_u \to \tau \nu)$ and $(g - 2)_\mu$, while satisfying the present experimental constraints from $B(B \to X_s \gamma)$, $\Delta M_{B_s}$ and $B(B_s \to \mu^+\mu^-)$.

All plots have been obtained setting $M_\tilde{q} = 1$ TeV, $M_\tilde{\tau} = 0.5$ TeV, $M_2 = 0.3$ TeV, $M_1 = 0.2$ TeV, and changing the values of the two key parameters, $\mu$ (0.5 or 1 TeV) and $A$ ($0, \pm 1, \pm 2$ TeV), as indicated in the captions. We have explicitly checked that the structure of the plots remains essentially unchanged for variations of $M_2$ and $M_1$ in the range 0.2–0.5 TeV. Thus these plots can be considered as representatives of a wide area of the parameter space (for squark masses in the TeV range). Note that, with the exception
Figure 2: $B$-physics observables and $(g - 2)_{\mu}$ in the $M_{H^\pm} - \tan \beta$ plane. The four plots correspond to: $[\mu, A_U] = [0.5, 0]$ TeV (upper left); $[\mu, A_U] = [1, 0]$ TeV (upper right); $[\mu, A_U] = [0.5, -1]$ TeV (lower left); $[\mu, A_U] = [0.5, -2]$ TeV (lower right). The exclusion regions for $\mathcal{B}(B_s \to \mu^+ \mu^-)$ and $\mathcal{B}(B_s \to X_s \gamma)$ correspond to the limits in Eqs. (13) and (16), respectively (see main text for more details).
Figure 3: $B$-physics observables and $(g-2)_{\mu}$ in the $M_{H^\pm}$-$\tan\beta$ plane. The four plots correspond to: $[\mu, A_U] = [1.0, -1.0]$ TeV (upper left); $[\mu, A_U] = [1.0, -2.0]$ TeV (upper right); $[\mu, A_U] = [1.0, 1.0]$ TeV (lower left); $[\mu, A_U] = [1.0, 2.0]$ TeV (lower right). The exclusion regions for $B(B_s \to \mu^+\mu^-)$ and $B(B_s \to X_{s\gamma})$ correspond to the limits in Eqs. (13) and (16), respectively (see main text for more details).
of \((g-2)\mu\) (which depends on \(M_\tilde{e}\)) and \(\mathcal{B}(B \rightarrow X_s\gamma)\) (which depends on \(M_\tilde{q}\)), the other observables are completely independent from the absolute sfermion mass scale.

The dashed areas denote regions which yield a suppression of \(\mathcal{B}(B_u \rightarrow \tau\nu)\) of (10–40)% or (20–30)% (inner region), with a contribution to \(\Delta a_\mu\) in the 1σ range defined by Eq. (17). The exclusion regions from \(\mathcal{B}(B_s \rightarrow \mu^+\mu^-)\) and \(\mathcal{B}(B \rightarrow X_s\gamma)\) correspond to the bounds in Eqs. (13) and (16), respectively. In all plots we have also indicated which is the SM), and the possible future impact of a more stringent bound on \(\Delta M_{B_s}\) (i.e. no more than 10% suppression with respect to the SM), and the possible future impact of a more stringent bound on \(\mathcal{B}(B_s \rightarrow \mu^+\mu^-)\).

A list of comments follows:

- In all cases there is a wide allowed region of the parameter space with sizable (measurable) effects in \(\mathcal{B}(B_u \rightarrow \tau\nu)\), which would also provide a natural explanation of the \((g-2)\mu\) problem. The only scenario where this does not happen is for \(A_U \gtrsim 0.5\) TeV and \(\mu \lesssim 0.5\) TeV (plot not explicitly shown), where the \(\mathcal{B}(B \rightarrow X_s\gamma)\) constraint becomes particularly stringent.

- If we require that charged Higgs effects account for a suppression of \(\mathcal{B}(B_u \rightarrow \tau\nu)\) of at least 10%, then we naturally have large SUSY effects in \((g-2)\mu\) (except for unnaturally heavy sleptons). The viceversa is also true, but only if \(M_{H^\pm}\) is sufficiently light \((M_{H^\pm} \lesssim 600\) GeV), or if we lower the “detectability threshold” of non-standard effects in \(\mathcal{B}(B_u \rightarrow \tau\nu)\) to a few %.

- The \(A_U = 0\) case (Figure 2 upper plots) is shown only for illustrative purposes, to demonstrate that if \(A_U\) is sufficiently small there is no connection between \(\mathcal{B}(B_u \rightarrow \tau\nu)\), \(\mathcal{B}(B_s \rightarrow \mu^+\mu^-)\) and \(\Delta M_{B_s}\). We do not consider this scenario very appealing because of the too light \(m_{h^0}\) (see Section 3.1).

- In the interesting cases with large \(A_U\), the present data on \(\mathcal{B}(B_s \rightarrow \mu^+\mu^-)\) and \(\Delta M_{B_s}\) imply only marginal limitations of the selected regions for \(\mathcal{B}(B_u \rightarrow \tau\nu)\) and \((g-2)\mu\). However, in all cases but for \([\mu, A_U] = [0.5, 1]\) TeV (Figure 2 lower left plot), a future limit on \(\mathcal{B}(B_s \rightarrow \mu^+\mu^-)\) at the \(10^{-8}\) level would cut a large fraction of the interesting region. In particular, we can conclude that if charged Higgs effects account for a suppression of \(\mathcal{B}(B_u \rightarrow \tau\nu)\) of at least 20%, then it is very likely that \(\mathcal{B}(B_s \rightarrow \mu^+\mu^-)\) exceeds \(10^{-8}\) (to be compared with the SM level of \(\approx 3.5 \times 10^{-9}\)).

- In the scenario of maximal mixing \((A_U/M_\tilde{q} \approx 2)\) and negative \(A_U\), the \(\Delta M_{B_s}\) bound is always at the border of the dashed areas. This implies that in this scenario, which is quite interesting given the heavy \(m_{h^0}\) and the effective cancellation of non-standard effects in \(B \rightarrow X_s\gamma\), \(\Delta M_{B_s}\) can receive a small but non negligible suppression \((\lesssim 10\%)\) compared to its SM expectation. A clear evidence of this effect, together with a clear evidence of a larger suppression \((20–40\%)\) in \(\mathcal{B}(B_u \rightarrow \tau\nu)\), would represent the unambiguous signature of this scenario.

\footnote{Note that in all cases we have set a severe limitation on possible non-standard contributions to \(\mathcal{B}(B \rightarrow X_s\gamma)\): the allowed regions are determined by the upper bound in Eq. (19), i.e. by an increase of \(\mathcal{B}(B \rightarrow X_s\gamma)\) not exceeding 15%.
4.1 Other signatures in $P \rightarrow \ell \nu$ decays

Besides $\mathcal{B}(B_u \rightarrow \tau \nu)$, $\mathcal{B}(B_{s,d} \rightarrow \ell^+\ell^-)$ and $(g - 2)_\mu$, which represent the most promising probes of the MSSM scenario with heavy squarks and large $\tan \beta$, specific signatures of this framework can be obtained by means of a systematic analysis of all $P \rightarrow \ell \nu$ modes:

$R_{P\ell\nu}$ In absence of sizable sources of flavour violation in the lepton sector, the expression in Eq. (3) holds for all the $B$ purely leptonic decays. The corresponding expressions for the $D \rightarrow \ell \nu$ case $m_B^2 \rightarrow (m_s/m_c)m_D^2$. It is then easy to check that a 30% suppression of $\mathcal{B}(B \rightarrow \tau \nu)$ should be accompanied by a 0.3% suppression (relative to the SM) in $\mathcal{B}(D \rightarrow \ell \nu)$ (see Ref. [35]) and $\mathcal{B}(K \rightarrow \ell \nu)$. At present, the theoretical uncertainty on the corresponding decay constants does not allow to observe such effects. However, given the excellent experimental resolution on $K \rightarrow \mu \nu$ [36] and the recent progress from the lattice on kaon semileptonic form factors (see e.g. Ref. [38]), the identification of tiny deviations from the SM in this channel (compared to the SM prediction inferred from $K_{e3}$ modes [3]) is not hopeless in a future perspective.

$R_{P\ell/\tau}$ As pointed out in Ref. [39], if the model contains sizable sources of flavour violation in the lepton sector (possibility which is well motivated by the large mixing angles in the neutrino sector), we can expect observable deviations from the SM also in the ratios

$$R_{P\ell_1/\ell_2} = \frac{\mathcal{B}(P \rightarrow \ell_1 \nu)}{\mathcal{B}(P \rightarrow \ell_2 \nu)}. \quad (19)$$

The lepton-flavour violating (LFV) effects can be quite large in $e$ or $\mu$ modes, while in first approximation are negligible in the $\tau$ channels. In particular, the leading parametric dependence of the most interesting $\mathcal{B}(B \rightarrow \ell \nu)$ ratios is described by the following universal expression

$$\left( \frac{R_{B}^{\ell/\tau}}{R_{B \ell/\tau}} \right)^{\text{MSSM}} = \left( \frac{R_{B}^{\ell/\tau}}{R_{B \ell/\tau}} \right)^{\text{SM}} \left[ 1 + \frac{1}{R_{B \tau \nu}} \left( \frac{m_B^4}{M_{H^\pm}^4} \right) \left( \frac{m^2}{m^2_{\ell}} \right) |\Delta_{\ell}^R| |\Delta_{\tau}^R| |\Delta_{\ell}^\tau| |\Delta_{\tau}^\ell| (1 + \epsilon_0 \tan \beta)^2 \right], \quad (20)$$

where the one-loop effective couplings $\Delta_{\ell/\tau}^R$ can reach $O(10^{-3})$ [39]. In the most favorable scenarios, taking into account the constraints from LFV $\tau$ decays [40], Eq. (20) implies spectacular order-of-magnitude enhancements for $R_{B}^{\ell/\tau}$ and $O(10\%)$ deviations from the SM in $R_{B}^{\mu/\tau}$ (a detailed discussion about these effects is beyond the scope of the present work and will be presented in a forthcoming publication).

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\(4\) To be more specific, the charged-Higgs exchange implies a $(0.1 - 0.2)\%$ suppression of the value of $|V_{us}|$ extracted from $K_{\mu2}$ with respect to the one determined from $K_{e3}$ decays. This possibility is perfectly compatible (even slightly favoured . . . ) with present data [37].
5 Conclusions

The observation of the $B_u \rightarrow \tau \nu$ transition [1] represents a fundamental step forward towards a deeper understanding of both flavour and electroweak dynamics. The precise measurement of its decay rate could provide a clear evidence of a non-standard Higgs sector with large $\tan \beta$ [4]. In this work we have analysed the interesting correlations existing between this observable and $\mathcal{B}(B \rightarrow X_s \gamma)$, $\Delta M_{B_s}$, $\mathcal{B}(B_{s,d} \rightarrow \ell^+ \ell^-)$, and $(g-2)_\mu$, in the large $\tan \beta$ regime of the MSSM. We have shown that this scenario is particularly interesting, especially in the limit of heavy squarks and trilinear terms. In this framework one could naturally accommodate the present (non-standard) central values of both $\mathcal{B}(B_u \rightarrow \tau \nu)$ and $(g-2)_\mu$, explain why the lightest Higgs boson has not been observed yet, and why no signal of new physics has been observed in $\mathcal{B}(B \rightarrow X_s \gamma)$ and $\Delta M_{B_s}$.

One of the virtues of the large $\tan \beta$ regime of the MSSM, with MFV and heavy squarks, is its naturalness in flavor physics and in precise electroweak tests. As we have shown, no fine tuning is required to accommodate the precise SM-like results in $\mathcal{B}(B \rightarrow X_s \gamma)$ and $\Delta M_{B_s}$. On the other hand, the scenario could clearly be distinguished by the SM with more precise results on $\mathcal{B}(B_u \rightarrow \tau \nu)$, and possibly on $\mathcal{B}(B_{s,d} \rightarrow \ell^+ \ell^-)$. In particular, we have discussed how to decrease the theoretical/parametric uncertainties in the SM prediction of $\mathcal{B}(B_u \rightarrow \tau \nu)$ by normalizing this observable to $\Delta M_{B_d}$. Moreover, we have shown that in the most favorable scenarios for $m_{h^0}$ (i.e. for $A_U > M_{\tilde{q}}$), a (20–30)% suppression of $\mathcal{B}(B_u \rightarrow \tau \nu)$ is accompanied by enhancements of the $B_{s,d} \rightarrow \ell^+ \ell^-$ rates by more than a factor of 3 compared to the corresponding SM expectations.

The observables $\mathcal{B}(B_u \rightarrow \tau \nu)$, $\mathcal{B}(B_{s,d} \rightarrow \ell^+ \ell^-)$ and $(g-2)_\mu$ can be considered as the most promising low-energy probes of the MSSM scenario with heavy squarks and large $\tan \beta$. Nonetheless, as discussed in Section 4, interesting consequences of this scenario could possibly be identified also in $\Delta M_{B_d}$, and in other $P \rightarrow \ell \nu$ modes. In particular, if $A_U$ is large and negative we expect a $\approx 5\%$ suppression of $\Delta M_{B_s}$ with respect to the SM expectation (possibility which is certainly not excluded by present data [2]). A non-standard effect of this magnitude in $\Delta M_{B_s}$ is very difficult to be detected, but it is not hopeless in view of improved lattice data (see e.g. Ref. [38]) and more refined CKM fits (see e.g. Ref. [21, 41]). The model also predicts a few per-mil suppression of the $|V_{us}|$ value extracted from $K_{\mu2}$ (compared to the $K_{e3}$ one). Finally, if the slepton sector contains sizable sources of flavour violation, we could even hope to observe large violations of lepton universality in the ratios $\mathcal{B}(B \rightarrow \mu \nu)/\mathcal{B}(B \rightarrow \tau \nu)$ and $\mathcal{B}(B \rightarrow e \nu)/\mathcal{B}(B \rightarrow \tau \nu)$, as well as few per-mil effects in $\mathcal{B}(K \rightarrow e \nu)/\mathcal{B}(K \rightarrow \mu \nu)$ [39].

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