Abstract: In this paper, we present the method for determination of effective electromagnetic parameters of complex building materials. By application of the proposed algorithm, it is possible to analyze electromagnetic field distribution for large-scale problems with heterogeneous materials. The two-dimensional numerical model of building components (hollow brick) with periodic boundary conditions was solved using the finite-difference time-domain method (FDTD) and discussed. On this basis, the resultant transmission coefficient was found and then the equivalent relative permeability and electric conductivity of heterogeneous dielectric structures, in the developed homogenization algorithm, were identified. The homogenization of material properties was achieved by performing a multi-variant optimization scheme and finally, selecting optimal electric parameters. Despite the analysis of heterogeneous building materials, the presented algorithm is shown as a tool for the homogenization of complex structures when scattering of a high-frequency electromagnetic field is considered.

Keywords: effective electromagnetic parameters; computational electromagnetics; finite-difference time-domain method; wireless networks; heterogeneous materials; homogenization techniques

1. Introduction

Wireless networks are common, modern solutions used in telecommunications and data transfer systems and operate in the high-frequency range, e.g., in the microwave range [1–6]. Nowadays, these technologies have become even more important than wired networks and are used almost everywhere, particularly when technical or environmental restrictions make it difficult to use cables or optical fibers. Wireless data transmission systems allow for relatively easy and quick connection of many devices, preserving flexible, dynamic and automatic configuration of the group of communication elements [1,7,8]. The appropriate quality of communication in the selected area, including a connection stability and transmission speed, is an essential issue when it comes to installing wireless communication systems. The structure of a wireless network requires taking into account the number of access points and their locations [5]. The discussed problems become particularly important when wireless networks are installed inside buildings, where complex geometry [3,4,9–14] and electrical properties of materials [15,16] influence the propagation of electromagnetic (EM) waves. The impact of these viabilities was confirmed by the measurements of mobile phone signal attenuation by different building structures [12] in Global System for Mobile Communications (GDM) networks. Therefore, the age of
buildings and their constructions, along with the techniques of the efficient modeling of buildings, should also be taken into account while EM field analysis is performed [13,17].

Design stage and analysis of the network can be performed using computer modeling [2,3,5,10]. Numerical analysis of the propagation of electromagnetic waves allows for taking into account the building structure as well as to perform multi-variant analysis of the wireless communication system [5,6,11]. The analysis of the electromagnetic phenomena can be performed using numerical methods such as finite element method (FEM) [18,19] or finite-difference time-domain method (FDTD) [3,9,11,20,21]. Despite the approximations in the models and calculation errors, numerical methods allow to obtain solutions for complex models. The propagation of electromagnetic waves in building structures is one of the problems in which numerical algorithms are useful tools for the analysis of field phenomena.

The authors performed a numerical analysis of the electric field intensity distribution in models with a single-layer wall [21] or a multi-layer wall [5,6]. The variability of the electrical parameters of the materials was also taken into account. Models composed of homogeneous materials (concrete, aerated concrete, full brick) [21] as well as complex ones (clinker bricks) [6] were also considered. The authors also analyzed models related to concrete and reinforcement. The influence of electrical parameters of concrete and the configuration of reinforcement (diameter, spacing, number of rows) were considered. Additionally, the influence of electric parameters of concrete (electrical permittivity and conductivity) on the electric field intensity near heterogeneous structures was discussed. The performed works were aimed at determining the distribution of the electromagnetic field and evaluation of the phenomena occurring in model systems. As a result, the impact of the electrical parameters and the internal structure of building materials on the field distribution (at frequencies used in wireless networks) were determined. The aim was also to determine effective parameters of complex materials. Homogenization is necessary for large-scale calculations. Through the optimization algorithm presented in Reference [11], which was then improved, multi-variant-tested and described in this article, it is possible to analyze areas of large dimensions without the need to model the details of the structure (e.g., drilling, reinforcement or other materials’ admixtures).

The most important elements defining the building material are electromagnetic parameters (permittivity, permeability and conductivity), type of material (the wall) and its thickness. Electrical conductivity has the greatest impact on the measured or calculated values of the electric field. For the walls made of concrete [14,21], the permittivity ($\varepsilon_r' \in \{5, 6, 7, 8\}$) has a smaller impact, since changes in the electric field intensity are up to a 15% at $\sigma < 0.01$ S/m. However, for $\sigma > 0.09$ S/m, the values of the electric field are almost identical [21,22]. Aerated concrete is a lighter type of concrete and in the wireless communication analysis, $\varepsilon_r' \in \{2, 2.25, 2.5\}$ is assumed [21]. In this case, the variation of permittivity also results in minor effects (changes are up to 2% at $\sigma < 0.05, 0.2 >$ S/m). The influence of solid and different types of clinker bricks was widely considered [5,6,16]. With the rising number of hollows and their dimensions inside a brick, the higher values of the electric field behind the wall will appear. An identical effect was observed for the walls with a lower relative volume of the clay mass, when the conductivity of ceramic increased (e.g., if relative volume of the clay mass will grow by 16%, then at $\sigma = 0.1$ S/m, the field intensity will be 22% higher).

When windows are taken into account, their EM properties and thickness are equally important. Electrical parameters of a glass are similar to those of concrete [22]. However, windows have lesser impact on wireless communication quality, which is due to the fact that the standard window is 4 mm thick, while the thickness of a concrete wall is not less than 0.12 m [12,22]. If reinforcement is present in the wall, local fading of a signal and significant reduction in the field intensity will appear (up to 40%, compared to a wall made of solid bricks). When concrete walls with reinforcement are analyzed, it is not possible to easily predict the EM field distribution, because it strongly depends on the size of reinforcements, spacing between them and the number of rows in the wall [14,22]. Therefore, when analyzing the influence of building structures, it is necessary to take many factors into account and perform individual computations for each variant [12,17,22,23].
In order to determine the effective electrical parameters of the complex material, an example of a model with four types of walls composed of clinker bricks was used. The model presented in this article was considered for single-layer walls (Figure 1) at frequency 2.4 GHz [16]. The analysis of the presented model was extended to the variant with two-layer walls [5]. In order to compare the results at different frequencies, numerical analysis was used at typical frequencies for Wi-Fi with the grid size [24] equal to 0.0016667 m [6]. In this article, the single-layer walls and the double-layer walls at two frequencies (2.4 and 5 GHz) were calculated with the assumption that the Yee cell size is 0.001 m for more accurate results. Also, in this article, the search area for the maximum value of the electric field intensity has been doubled. The receipt of accurate data guaranteed the correctness of the obtained results for the effective parameters of the complex material. It is worth noting that the developed optimization algorithm is universal because it determines the effective parameters for other heterogeneous structures, e.g., concrete with reinforcement.

The crucial part in modeling large-scale systems is a precise reproduction of complex materials, e.g., hollow bricks [2,5] or concrete with reinforcement [11]. For this reason, it is necessary to replace a composite material with a homogeneous structure possessing equivalent electrical parameters [19]. The determination of properties and formulation of the homogeneous models of complex materials is an important and constantly developed topic [2,15,18]. Effective electromagnetic parameters of complex materials (e.g., bricks) can be taken into account when calculating large-scale models, in which, due to the size of the numerical models, the discrete arrangement of air and ceramic mass cannot be fully reproduced.

In this article, a method of homogenization of complex building structures is presented. In Section 2, the developed, two-dimensional model of considered systems is introduced. Using the FDTD method, the numerical models of several exemplary walls made of hollow bricks are solved. The phenomena are analyzed until reaching a steady state and taking into account multiple reflections of EM waves. Then, the calculated transmission coefficient is used for the identification of equivalent permittivity and conductivity of the homogeneous wall, by comparing the obtained value with the analytical solution (transmission coefficient) of a homogeneous slab.

In Section 3, in this article, the whole diagram of the developed optimization algorithm, the adopted limitations and assumptions, the resolution of the solution search and the results for exemplary variants of the model (Section 1) composed of different bricks for single-layer and double-layer walls at 2.4 and 5 GHz are presented. The tables also include some of the best variants of solutions for bricks with 18 and 30 vertical hollows, taking into account the conductivity values most commonly used in the literature. The solution of the homogenization problem and identification of the properties, presented in Section 3, are achieved by finding a pair of optimal parameters: relative permeability and electric conductivity. The results obtained for \( f = 2.4 \) GHz and \( f = 5 \) GHz are presented and discussed.
2. Analyzed Models

2.1. Complex Building Materials—Hollow Clay Brick

One of the commonly used building materials is a hollow brick made of clay or other materials with the addition of sand, then dried and fired at high temperatures [25–28]. This material will be used to present the developed algorithm for determining the effective electrical parameters. The dimensions of the bricks vary greatly, but they are based on the ratio 1:2:4 of height \( h \), width \( w \) and length \( l \), respectively [5,25,28] (Figure 1). In the macroscopic approach, building materials (hollow bricks) are classified as complex, heterogeneous structures. The presented analysis is related with heterogeneous building walls made of two types of commonly used building materials presented in References [6,16]:

1. Hollow clay brick with 18 vertical holes (marked as B18) (Figure 1a),
2. Hollow clay brick with 30 vertical holes (B30) (Figure 1b).

The discussed computational problem is limited to the analysis of porous elements, where the electromagnetic wave propagates through a multi-layered structure composed of an air and a non-ideal dielectric. The dimensions of the brick and holes (at frequency \( f = 2.4 \) or \( 5.0 \) GHz) are comparable to the EM wavelength in air \( \lambda_a = 0.125 \) m at \( f = 2.4 \) GHz), as shown in Figures 1 and 2, hence it is necessary to take into account the distribution and size of the holes.

![Figure 2](image)

**Figure 2.** Relative dimensions of two types of analyzed bricks for \( f = 2.4 \) GHz: (a) model B18 and (b) model B30.

The dimensions given in Figure 1 are average values calculated based on the performed measurements, wherein the method of determining the test parameters is specified in the standard [25–28]. The standard specifies the number of samples required to test each property of a masonry element. In this study, the measurements were performed on 50 randomly selected samples of each type of brick. The main reason was the lack of regulations related to the dimensions of the hollows inside the bricks. The standard defines the permissible deviations of the external dimensions of the bricks, resulting from forming, drying or firing ceramic material [26].

In order to determine the influence of the drillings on the EM field distribution, calculations were performed for the width of drillings, which was changed along the axis of the longest dimension, \( s \), of the brick (Figure 1), \( s \in [0, 0.005, 0.007, 0.011, 0.013, 0.015, 0.017, 0.019] \) m. The size of the hole affects the percentage of the lossy dielectric (clay mass) in the brick (Table 1). This percentage was estimated as:

\[
V_{\%mc} = \frac{V_c - V_d}{V_c} \cdot 100\%
\]

where \( V_d \) represents volume of all holes in a given brick, and \( V_c = h \times w \times l \) is the volume of a brick.
Table 1. Clay percentage inside analyzed bricks.

| Geometric Size of Holes (s) Inside the Brick (m), Changed along the Ox Axis (Figure 1) | Relative Volume of Clay in the Brick (V_{\text{mc}}) (%) | Model of the Brick B18 | Model of the Brick B30 |
|---|---|---|---|
| 0.005 | 90.40 | 87.50 |
| 0.007 | 86.56 | 82.50 * |
| 0.009 | 82.72 * | 77.50 |
| 0.011 | 78.88 * | 72.50 |
| 0.013 | 75.04 | 67.50 * |
| 0.015 | 71.20 | 62.50 * |
| 0.017 | 67.36 * | 57.50 |
| 0.019 | 63.52 | 52.50 |

The value of V_{\text{mc}} for the considered brick is shown in Table 1. The symbol (*) signifies values of the typical size of hole inside the brick, while symbol (**) was used to signify the two closest V_{\text{mc}} values in various sizes of holes. The electrical parameters of bricks, based on available literature, are presented in Table 2.

Table 2. Different electrical properties used for characterization of the bricks.

| No. | f (GHz) | $\varepsilon_r'$ | $\varepsilon_r^2$ | $\sigma$ (S/m) | Literature | Comments |
|---|---|---|---|---|---|---|
| 1. | 0.9 | 4.6 | - | 0.0175 | [29] | - |
| 2. | 1.7 ÷ 2.6 | 4.44 | - | 0.01 | [30] | - |
| 3. | 1.7 ÷ 18.0 | 4.62 ÷ 4.11 | - | 0.0174 ÷ 0.0364 | [31] | - |
| 4. | 2.0 | 4.44 | - | - | [31] | - |
| 5. | 3.0; 9.0; 24.0 | 3.7 ÷ 4.0 | 0.12 | - | [31,32] | - |
| 6. | 3.0; 9.0; 24.0 | 3.7 ÷ 19 | 0.12 ÷ 3.7 | - | [32,33] | - |
| 7. | 5.0 | 4.12 | 0.16 | 0.0445 | [34] | brick with holes |
| 8. | 5.0 | 3.3 | 0.01 | 0.00278 | [34] | full brick |
| 9. | 5.0 | 3.56 | 0.34 | 0.0946 | [34] | - |
| 10. | 5.3 | 4.1 | 0.15 | - | [35] | - |
| 11. | 5.8 | 3.58 | - | 0.11 | [36] | measurements with 1.8% water volume |
| 12. | 5.8 | 3 | - | 0.12 | [36] | measurements with 1.8% water volume |
| 13. | 24.0 | 3.7 ÷ 4.0 | 0.6 | - | [32] | - |
| 14. | 60.0 | 3.95 | 0.073 | 0.244 | [34] | brick with holes |
| 15. | 60.0 | 2.82 | 0.44 | 0 | [29] | full brick |

Mapping of the relative dimensions of analyzed bricks, assuming that relative permittivity of the material is $\varepsilon_r' = 4.44$ and conductivity $\sigma = 0$ S/m, is shown in Figure 2. The dimensions of the ceramic elements are related to the wavelength inside the brick ($\lambda_b = 0.0593$ m at $f = 2.4$ GHz).

In real cases, the dimensions of the building materials are not so strict, and their variability may also affect the propagation of the EM wave. Still, they can be taken into account using the discussed homogenization procedure. In this article, we have assumed averaged values based on 50 test samples. While considering the influence of building materials on the field intensity in wireless communication, a model with exact dimensions, even up to millimeters, is widely used [2,28,35–39]. This is due to the fact that only external dimensions are subject to standardization of bricks and changes in the structure inside the bricks are not standardized. In the literature, the polynomial chaos expansion has been described to estimate the influence of uncertainties in geometrical and physical properties of a building facade on the scattered electric field [13].

Four types of wall in an area, $\Omega_S$, in the base model were analyzed (Figure 3). Discussed variants were taking into account the number of brick layers and the type of brick. For example, 1w_B18 refers to a model of a single-layer wall made of B18 bricks. Based on the literature, for the analysis of considered variants, the constant value of permittivity of the ceramic mass was assumed ($\varepsilon_r' = 4.44$) and the conductivity, $\sigma$, was changing from 0 to 0.2 S/m.
The real component $\varepsilon'$ determines the ability of the dielectric to accumulate energy in the electric field, while the imaginary component $\varepsilon''$ is responsible for energy loss related to displacement currents. Due to changes of polarization depending on the frequency, the values of the permeability...
components in the medium are not constant. When describing the properties of the dielectric medium, the effective parameters are defined as:

$$\varepsilon_{ef}(\omega) = \varepsilon' + \frac{\sigma}{\omega\varepsilon_0} = \varepsilon' - j \left(\varepsilon'' + \frac{\sigma}{\omega\varepsilon_0}\right)$$  \hspace{1cm} (4)

Substituting the effective permittivity of lossy dielectric (Equation (4)), its wave impedance will be expressed by the formula:

$$Z_S = \sqrt{\frac{\mu}{\varepsilon}} = \frac{\sqrt{\frac{\mu_0 \mu_r}{\varepsilon_\omega}}}{\varepsilon'} \left[1 + j \left(\frac{\sigma + 2\pi f \cdot (\varepsilon_\omega \varepsilon_r''')}{2 \cdot (2\pi f \cdot (\varepsilon_\omega \varepsilon_r'))}\right)\right]$$  \hspace{1cm} (5)

With the perpendicular incidence of the wave on boundary, the transmission coefficient in the $\Omega_S$ area is given by the following equation [40]:

$$T_e = \left|\frac{T_{e2+}}{T_{e1-}}\right| = \left|\frac{T_{e1} \cdot T_{e2} \cdot e^{-j2\varepsilon_\omega}}{1 + \Gamma_1 \cdot \Gamma_2 \cdot e^{-2j2\varepsilon_\omega}}\right|$$  \hspace{1cm} (6)

where the respective field coefficients are

$$T_{e1} = 1 + \Gamma_1 = 1 + \frac{Z_0 - Z_4}{Z_0 + Z_4}$$  \hspace{1cm} (7)

$$T_{e2} = 1 + \Gamma_2 = 1 + \frac{Z_0 - Z_5}{Z_0 + Z_5}$$  \hspace{1cm} (8)

and the wavenumber is defined as:

$$k = \left(2\pi f \sqrt{(\mu_0 \mu_r) \cdot (\varepsilon_\omega \varepsilon_r')}\right) \cdot \left[1 - j \left(\frac{\sigma + 2\pi f \cdot (\varepsilon_\omega \varepsilon_r''')}{2 \cdot (2\pi f \cdot (\varepsilon_\omega \varepsilon_r'))}\right)\right]$$  \hspace{1cm} (9)

Equation (6) makes it possible to verify the results obtained in numerical calculations using the FDTD, FDFD and FEM methods. Its applicability is limited to cases with isotropic, homogeneous materials such as concrete or brick. Maximum values of the electric field in the area behind the wall are used for determining the transmission coefficient based on Equation (6). When time domain methods are considered, the analysis in $\Omega_2$ area must be performed in a steady state, where the multiple reflections of the wave in the wall area are present.

2.3. Mathematical Model

The dimensions of modeled structures are comparable or greater than wavelength in the microwave range (Figure 2). The considered task comes to a solution of the boundary value problem described by partial differential equations [40,41]. The constituent materials present in the systems are continuous, isotropic media characterized by parameters $\varepsilon_r$, $\mu_r$ and $\sigma$. The finite-difference time-domain method (FDTD) was used to determine the distribution of the electromagnetic field. The FDTD is based on Maxwell’s equations in the time-domain [20,24,42]:

$$\nabla \times \mathbf{E} = -\mu_r \frac{\partial \mathbf{H}}{\partial t},$$  \hspace{1cm} (10)

$$\nabla \times \mathbf{H} = \varepsilon_r \frac{\partial \mathbf{E}}{\partial t}$$  \hspace{1cm} (11)

where: $\mathbf{E}$—electric field in (V/m), and $\mathbf{H}$—magnetic field in (A/m). The propagation of the EM wave in building structures can be simplified to 2D space. Equations (10) and (11) are discretized on the
rectangular differential grid [24]. In the time-domain, the calculations of instantaneous values of magnetic field components $H_x$ and $H_y$ are interlaced by calculations of electric field component $E_z$.

2.4. Numerical Model

The system consisting of the wall made of building materials (hollow bricks) is analyzed. An open space with the air properties is attached to both sides of the wall, as shown in Figure 5. It was also assumed that the dimensions of the wall perpendicular to the direction of wave propagation (wall width and height) are much greater than the wavelength ($\lambda_0 = 0.125$ m at $f = 2.4$ GHz and $\lambda_0 = 0.06$ m for $f = 5$ GHz). Therefore, apart from the phenomena occurring at the ends of the wall, near edges or at the contact with another wall, it was possible to reduce the size of the model by application of periodic boundary conditions. These assumptions allowed for determining the influence of the considered materials on the distribution of the electromagnetic field in an isolated system.

![Figure 5](image-url)

**Figure 5.** Numerical model used for calculation of systems with the heterogeneous material.

The analysis of the electric field distribution in individual variants was carried out on the basis of observation of the maximum value of the $E_z$ component in a specific area behind the wall, marked in Figure 5 by the green rectangle. The source field in the model was a harmonic, linearly polarized wave propagating in the direction of the $Oy$ axis ($k = 1_y$) [20,24,42]

$$E(x, y, t) = E_z \mathbf{1}_z = \sin(\omega t) \cdot 1(t) \cdot \mathbf{1}_z. \quad (12)$$

The phenomena of wave propagation in open space were imitated by adopting PML (perfectly matched layer) conditions at the edges perpendicular to the direction of plane wave propagation [42,43]. Periodic conditions were applied on the edges parallel to the wave vector. The area of the considered models was discretized using uniform Yee cells. To determine the effective electromagnetic properties of complex materials, it is important to obtain correct results from the numerical model, since the transmission coefficient calculated numerically ($T_{e,FDTD}$) will be compared with the value found analytically ($T_e$). Numerical analysis of the influence of the variability of electrical conductivity and the
size of holes in the brick on the electric field was performed earlier in Reference [6], where the size of mesh in the models was $\Delta = 0.0016667$ m. In this article, calculations of $T_{e,\text{FDTD}}$ were performed for the grid with smaller size, $\Delta = 0.001$ m. The area of the considered model was discretized using uniform Yee cells in order to fulfill the required conditions [24,42] and as a result, due to the frequency of the EM field (and the wavelength in the air and brick), the maximum linear size of the Yee cell was $\Delta_x = \Delta_y = 1$ mm.

2.5. Initial Results of Numerical Analysis

The main results of the FDTD method are electric field distributions for all the time stamps. For this reason, it was necessary to develop an additional algorithm, where the aim was to determine the map describing the maximum values of the field by processing the sequence of instantaneous field distributions calculated at equal intervals of time. Figure 6 shows maximum values of the $E_z$ component (relative to source $E_z$ value) depending on the conductivity of the brick material and the size of the holes ($s$).

![Figure 6](image)

Figure 6. The relative maximum values of the $E_z$ component behind the wall made of: (a) B18 bricks (one layer), (b) B30 bricks (one layer), (c) B18 bricks (two layer), (d) B30 bricks (two layer).

While passing through different areas of the brick, a local change of the speed of the electromagnetic wave leads to the temporary images of the field and proves the occurrence of interference. This effect is especially visible behind the wall made of B30 bricks (the field behind the wall has higher minimum and maximum values). Additionally, small changes in the size of holes resulted in a change of the electric field. Both high percentage of clay and sizes of holes may cause a change in transmission speed. Some waves passing through the area with $\varepsilon > 1$ (clay) may be delayed with relation to waves passing through mixed areas (air holes–clay). Due to these delays, especially, double-layer walls will decrease the original transmission speed. In order to evaluate possible data transfer problems, a precise visualization of the EM field distribution will be required. For this reason, it is necessary to accurately
reproduce brick’s structure. On the other hand, when large-scale models are analyzed, it is the biggest problem to reproduce all details of the structure, and for this reason, the homogenization is necessary. Since the discussed systems operate at typical Wi-Fi frequencies, it is worth to consider an impact of the wall structure on the stability of communication. Firstly, the electrical conductivity has an influence on the wave absorption. If a receiver is located behind the wall, for less conductive material \((\sigma < 0.08 \text{ S/m})\), the maximum value of \(E_z\) is greater than 0.1 \(E_{\text{source}}\), while for higher \(\sigma\), it tends to be 0.01 \(E_{\text{source}}\). Conductivity of the brick may vary throughout the year, thus walls with higher content of the absorbed water may cause significant loss of the signal and decrease communication stability. A similar conclusion may come from the comparison of one- and two-layer walls. For all single \(\sigma\) permittivity, \(E_{\text{wall}}(1w)\) cases (Figure 6a,b) \(E_z\) behind the wall is higher than 0.05 \(E_{\text{source}}\), however for all cases of double-layer \(2w)\) wall, electric field intensity is lower (for the corresponding conductivity). Secondly, a higher percentage of clay material in the wall causes smaller distortion of the wave front in the area behind the wall. When discussing the wall in which the damping is insignificant \((\sigma < 0.01 \text{ S/m})\) or equal to zero, interference effects play a much greater role. This may explain the non-monotonous course of characteristics and visible minimal values for some sizes of holes. A local destructive interference of electric field also increases the probability that the receiver will be present in an area where \(E_z\) is very low.

### 3. Identification of Effective Electromagnetic Parameters

In Section 2, the model and results related to heterogeneous walls made of B18 and B30 hollow bricks were presented [5,6]. The results of this type of analysis will be used to develop an algorithm for determination of effective electromagnetic properties of heterogeneous material. Here, the transmission coefficient calculated numerically is the crucial parameter used for identification of effective permittivity and conductivity of the homogenized wall. The solution of the homogenization problem can be achieved by choosing optimal values of \(\varepsilon_{r,\text{opt}}\) and \(\sigma_{\text{opt}}\) based on optimization task.

#### 3.1. General Algorithm for Determining the Effective Parameters

The identification of the effective electric parameters by optimization algorithm (Figure 7) is based on minimization of the error between transmission coefficient, calculated numerically \((T_{e,FDTD})\) and defined analytically \((T_e)\). As an example, the wall made of clinker bricks was used.

![Figure 7. Homogenization scheme and connection of input and output data in the implementation of the developed algorithm \((A_{\text{opt}})\).](image)

The effective parameters \(\varepsilon_{r,\text{opt}}\) and \(\sigma_{\text{opt}}\) are determined assuming identical wall thickness in the model under consideration and the equivalent medium \((w)\). The optimization algorithm \((A_{\text{opt}})\) aims to determine the values of isotropic effective parameters of homogenized material \((\varepsilon_{r,\text{opt}}\) and \(\sigma_{\text{opt}})\) based on a non-homogeneous material (e.g., hollow bricks):

\[
A_{\text{opt}} = \{\varepsilon_{r,\text{min}}, \varepsilon_{r,\text{max}}, \sigma_{\text{min}}, \sigma_{\text{max}}, \Delta_e, \Delta_s, \delta_A, f_g\}
\]

where: \(\varepsilon_{r,\text{min}}\) and \(\varepsilon_{r,\text{max}}\)—lower and upper limits on the domain of optimization for relative electric permittivity, \(\sigma_{\text{min}}\) and \(\sigma_{\text{max}}\)—lower and upper limits on the domain of optimization for conductivity, \(\Delta_e\) and \(\Delta_s\)—resolution of the domain optimization, \(\delta_A\)—objective function and \(f_g\)—function of
generating sequential solution variants. The objective function $\delta_A$ which classifies calculated variants, depends on assumed values of parameters \(\{\varepsilon', \sigma\}:
\[
\delta_A = \frac{|T_e - T_{e,FDTD}|}{T_{e,FDTD}},
\]
where: \(T_e\)—transmission coefficient module determined by the analytical method (Equation (6)) for a given, iterative variable value of material parameters \(\{\varepsilon', \sigma\}\) and wall thickness \(w\), and \(T_{e,FDTD}\)—transmission coefficient calculated based on the numerical solution (FDTD) for given properties of the ceramic material \(\varepsilon',\sigma_{FDTD}\) and \(\sigma_{FDTD}\), wall thickness and brick’s structure.

The search for the optimal solution was carried out taking into account (Figure 8):

1. Boundaries of the optimization domain for effective permittivity \(\varepsilon'\in<2, 6>\) and conductivity \(\sigma\in<0, 0.1>\) S/m (the limits were determined based on the literature [29–36]).
2. Limited resolutions, \(\Delta_{\varepsilon}, \Delta_{\sigma}\), constituting the maximum differences in all values of \(\varepsilon'\) and \(\sigma\), determining the number of iterations performed for \(\varepsilon'_{opt}(N_{\varepsilon})\) and \(\sigma_{opt}(N_{\sigma})\):
\[
N_{\varepsilon} = \left(\varepsilon'_{max} - \varepsilon'_{min}\right) / \Delta_{\varepsilon},
\]
\[
N_{\sigma} = (\sigma_{max} - \sigma_{min}) / \Delta_{\sigma}.
\]
where: \(\Delta_{\varepsilon} = 0.01\) and \(\Delta_{\sigma} = 0.001\).

\[ \text{Figure 8. Boundaries of the search area when determining the equivalent electric parameters for heterogeneous material.} \]

3.2. Algorithm Implementation

To identify effective electric parameters, an optimization algorithm was developed (Figure 9) based on the analytical [40,41] and numerical methods [20,24]. The overall structure was divided into two main stages, where stage I is related to the elements defined before the start of calculations, while stage II presents the process of optimization calculations.

Stage I:
- Reading and processing input data related to the considered wall variant.
- Reading the value of the maximum electric field determined in Section 3 and calculating \(T_{e,FDTD}\).
- Reading the data defining the range of the search domain of effective parameters and determining the accuracy of their selection.
- Creating vectors in order to find the best solutions’ effective parameters \((\varepsilon'_{opt} \text{ and } \sigma_{opt})\), for which the value of relative error \(\delta_A\) is lowest.

Stage II relies on iteratively performing calculations for assumed, subsequent values of \(\varepsilon'\) and \(\sigma\):
• Calculating temporary value of transmission coefficient for the successively adopted values of effective parameters ($\varepsilon_r'$ and $\sigma$), assuming that the wall is made of a solid brick (Equation (6)).
• Finding the value of relative error ($\delta_A$) using Equation (14).
• Sorting solutions using the sort algorithm to create a list of calculated variants with the smallest approximation error ($\delta_A$).

Figure 9. Diagram of the algorithm for the selection of effective electrical parameters.

3.3. Effective Parameters of Hollow Bricks at $f = 2.4$ GHz

At specific frequency $f = 2.4$ GHz and conductivities $\sigma_{\text{FDTD}} \in <0.01, 0.04>$ S/m, usually used for the description of ceramic materials (Table 2), the effective electric parameters were found. Figures 10–13 show the distribution of the relative error, $\delta_A$, depending on the values of $\varepsilon_r'$ and $\sigma$. The position of the optimal variant, in the global sense described by the minimum value of the objective function, is presented as a point ($\varepsilon_r', \sigma_{\text{opt}}, \delta_{\text{opt}}$) marked with a white dot. The figures present a whole set of local minima that give solutions with greater error. Four models of walls were considered: 1w_B18, 1w_B30, 2w_B18 and 2w_B30 (as described in Section 2).

Regardless of the number of brick layers in the wall and the conductivity of the material, variants with the smallest error, $\delta_A$, were found within the area $\sigma_{\text{opt}} \in <0.01, 0.03>$ S/m. When the increase of conductivity, $\sigma_{\text{FDTD}}$, in the numerical model occurred, the point of optimal solution was also moved towards higher values of $\sigma_{\text{opt}}$. Still, the selected equivalent conductivity in exemplary models was lower by 0.01 S/m than the input data. Parameters describing single-layer walls were characterized by similar dependencies within the same conductivity (Figures 10 and 11) and a similar tendency was also noticed for double-layer walls (Figures 12 and 13). It was found that in the analyzed range $\varepsilon_r' \in <2, 6>$, optimal effective conductivity ($\sigma_{\text{opt}}$) was higher than for single-layer walls.

Tables 3 and 4 show the best solutions (min $\delta_A$) obtained using the optimization algorithm. The calculated values of $\varepsilon_r', \sigma_{\text{opt}}$ and $\delta_{\text{opt}}$ are given as well as the relative error. The obtained results
indicate that for a single-layer walls, the equivalent value of $\varepsilon_{\varepsilon',\text{opt}}$ has an average value of 3.3 and the conductivity is $\sigma_{\text{opt}} \in <0.004, 0.02>$ S/m. On the other hand, the double-layer structure of the walls reduced the permittivity value and increased conductivity for numerically modeled structures, assuming that significant values are $\sigma \in <0.03, 0.04>$ S/m (Table 4).

Figure 10. Relative error between numerical and analytical values of transmission coefficient (single-layer wall made of bricks B18) for: (a) $\sigma_{\text{FDTD}} = 0.01$ S/m, (b) $\sigma_{\text{FDTD}} = 0.02$ S/m, (c) $\sigma_{\text{FDTD}} = 0.03$ S/m, (d) $\sigma_{\text{FDTD}} = 0.04$ S/m.

Figure 11. Relative error between numerical and analytical values of transmission coefficient (single-layer wall made of bricks B30) for: (a) $\sigma_{\text{FDTD}} = 0.01$ S/m, (b) $\sigma_{\text{FDTD}} = 0.02$ S/m, (c) $\sigma_{\text{FDTD}} = 0.03$ S/m, (d) $\sigma_{\text{FDTD}} = 0.04$ S/m.
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Figure 12. Map of relative error depending on the values $\varepsilon'$ and $\sigma$ for the double-layer wall: (a) made of B18 and $\sigma_{\text{FDTD}} = 0.01$ S/m, (b) made of B18 and $\sigma_{\text{FDTD}} = 0.02$ S/m.

Figure 13. Relative error value depending on the values $\varepsilon'$ and $\sigma$ for the double-layer wall: (a) made of B30 and $\sigma_{\text{FDTD}} = 0.01$ S/m, (b) made of B30 and $\sigma_{\text{FDTD}} = 0.02$ S/m.

Table 3. Calculated effective electric parameters, one-layer wall (B18 and B30).

| Model of the Wall | Electric Parameters of Brick Adopted in Numerical Analysis (FDTD) | Calculated Effective Parameters of the Material | Relative Error $\delta_{\text{A}}$ (%) |
|-------------------|---------------------------------------------------|---------------------------------------------|---------------------------------|
|                   | $\varepsilon', \sigma_{\text{FDTD}}$           | $\varepsilon'_\text{opt}$ | $\sigma_{\text{opt}}$ S/m |                             |
| B18 (1w)          | 4.44                                              | 3.63                           | 0.005                          | 0.13                          |
|                   | 0.01                                              | 3.27                           | 0.009                          | 0.80                          |
|                   | 0.02                                              | 3.52                           | 0.015                          | 0.45                          |
|                   | 0.03                                              | 3.15                           | 0.022                          | 0.42                          |
| B30 (1w)          | 4.44                                              | 2.91                           | 0.002                          | 0.22                          |
|                   | 0.01                                              | 2.99                           | 0.005                          | 0.19                          |
|                   | 0.02                                              | 3.41                           | 0.014                          | 0.03                          |
|                   | 0.03                                              | 3.22                           | 0.013                          | 0.46                          |

Table 4. Calculated effective electric parameters, double-layer wall (B18 and B30).

| Model of the Brick | Electric Parameters of Brick Adopted in Numerical Analysis (FDTD) | Calculated Effective Parameters of the Material | Relative Error $\delta_{\text{A}}$ (%) |
|-------------------|---------------------------------------------------|---------------------------------------------|---------------------------------|
|                   | $\varepsilon', \sigma_{\text{FDTD}}$           | $\varepsilon'_\text{opt}$ | $\sigma_{\text{opt}}$ S/m |                             |
| B18 (2w)          | 4.44                                              | 4.82                           | 0.016                          | 0.29                          |
|                   | 0.01                                              | 3.95                           | 0.024                          | 0.27                          |
|                   | 0.02                                              | 2.24                           | 0.026                          | 1.26                          |
|                   | 0.03                                              | 2.45                           | 0.033                          | 2.33                          |
| B30 (2w)          | 4.44                                              | 2.74                           | 0.016                          | 0.42                          |
|                   | 0.01                                              | 3.48                           | 0.014                          | 0.19                          |
|                   | 0.02                                              | 2.75                           | 0.019                          | 0.16                          |
|                   | 0.03                                              | 4.31                           | 0.031                          | 0.04                          |
3.4. Effective Parameters of Hollow Bricks at $f = 5 \text{ GHz}$

Figures 14–16 show the distribution of the relative error ($\delta_A$) depending on the assumed equivalent values of the relative permittivity and conductivity. Based on Table 3, it was decided to distinguish the cases with $\sigma_{\text{FDTD}} \in <0.04, 0.1>$ S/m, including the values adopted for modeling ceramic materials. More than a two-fold increase in the conductivity taken in the analysis by the FDTD method results in a more than four-fold increase in the selected range for $\sigma_{\text{opt}}$ (Figure 14b).

![Figure 14](image1.png)

**Figure 14.** An example map of the relative error of approximated $\varepsilon'_r$ and $\sigma$ for the 1w_B18 model ($f = 5 \text{ GHz}$) with the initial assumption: (a) $\sigma_{\text{FDTD}} = 0.04$ S/m, (b) $\sigma_{\text{FDTD}} = 0.1$ S/m.

![Figure 15](image2.png)

**Figure 15.** An example of the variability of the relative error of approximated $\varepsilon'_r$ and $\sigma$ for the 1w_B30 model ($f = 5 \text{ GHz}$) with the initial assumption: (a) $\sigma_{\text{FDTD}} = 0.01$ S/m, (b) $\sigma_{\text{FDTD}} = 0.02$ S/m.

![Figure 16](image3.png)

**Figure 16.** An example of the variability of the relative error of approximated $\varepsilon'_r$ and $\sigma$ for the 2w_B30 model ($f = 5 \text{ GHz}$) with the initial assumption of $\sigma_{\text{FDTD}} = 0.04$ S/m.
Table 5 shows the values of effective parameters ($\varepsilon'_r,_{\text{opt}}$ and $\sigma_{\text{opt}}$) determined with the use of the optimization algorithm ($A_{\text{opt}}$) and characterized by the smallest relative error, $\delta_A$. Assuming the initial conductivity value $\sigma_{\text{FDTD}} = 0.04$ S/m, it was found that for the description of B30 bricks used in a single-layer wall, a value lower by approximately 30% can be taken as effective conductivity, with a simultaneous reduction of the relative permittivity to $\varepsilon'_r,_{\text{opt}} = 3.66$ S/m.

### Table 5. Identified effective electric parameters for 1 w and 2 w walls (B18 and B30) at $f = 5$ GHz.

| Model of the Wall | Electric Parameters of Brick Adopted in Numerical Analysis (FDTD) | Calculated the Equivalent Electric Parameters for the Material | Relative Error $\delta_A$ (%) |
|------------------|---------------------------------------------------------------|---------------------------------------------------------------|-------------------------------|
|                  | $\varepsilon_r^{\prime,_{\text{FDTD}}} \quad \sigma_{\text{FDTD}}$ (S/m) | $\varepsilon_r'^{\prime,_{\text{opt}}} \quad \sigma_{\text{opt}}$ (S/m) |                                |
| 1w_B18           | 4.44 0.04                                                     | 3.02 0.002                                                   | 0.005                         |
| 1w_B30           | 4.44 0.04                                                     | 3.66 0.027                                                   | 0.09                          |
| 2w_B18           | 4.44 0.04                                                     | 2.98 0.017                                                   | 0.85                          |
| 2w_B30           | 4.44 0.04                                                     | 2.16 0.018                                                   | 0.29                          |

Comparing the effective electrical parameters for a single-layer wall made of bricks with a smaller number of hollows (B18), we noticed that values of $\varepsilon_r'^{\prime,_{\text{opt}}}$ were close to 3.0. However, for B30 bricks, regardless of the type of wall, $\varepsilon_r'^{\prime,_{\text{opt}}} \approx 4$. As the initial conductivity, $\sigma_{\text{FDTD}}$, of the ceramic increases, the value of equivalent conductivity, $\sigma_{\text{opt}}$, was also higher. Additionally, the analysis of the double-layer wall showed that regardless of the initial $\sigma_{\text{FDTD}}$ value in the model, the effective conductivity of hollow bricks ($\sigma_{\text{opt}}$) did not exceed 0.1 S/m.

### 4. Conclusions

The effective electromagnetic parameters of heterogeneous materials, identified by the proposed algorithm, can be applied in the modeling of large-scale systems, when reproduction of the complex structure of walls made of hollow bricks is not possible. The analysis of the obtained results has shown that at frequency $f = 2.4$ GHz and $\sigma_{\text{FDTD}} = 0.01$ S/m, the effective conductivity was less than 0.008 S/m. At higher frequency and for $\sigma_{\text{FDTD}} = 0.04$ S/m, determined values were smaller ($\sigma_{\text{opt}} \in <0.001, 0.033>$ S/m). Furthermore, at $f = 2.4$ GHz and for single-layer walls (and all types of bricks), the average effective relative permittivity was $\varepsilon_r'^{\prime,_{\text{opt}}} = 3.3$, while the effective conductivity was in the range $\sigma_{\text{opt}} \in <0.004, 0.02>$ S/m. For the double-layer walls, a decrease of $\varepsilon_r'^{\prime,_{\text{opt}}}$ and an increase in the effective conductivity, $\sigma_{\text{opt}}$, were observed.

The developed algorithm can be used to determine the effective parameters of complex structures (e.g., hollow brick) and multi-layer structures. The variability and possible range of the permittivity and conductivity as well as size of holes or inclusions, frequency and wall thickness were simultaneously taken into account. By the application of the presented methodology, it is possible to numerically analyze models in two- or three-dimensional space, with much larger sizes as well as number of elements. The algorithm is a general scheme, that can be applied to similar homogenization problems of various complex structures (e.g., clinker bricks, concretes with admixtures or multi-layer materials, etc.), with different electrical parameters, size of drillings or admixtures or wall thickness.

### Author Contributions:
The paper was written by A.C. The methodology and analysis presented in this paper have been developed by A.C. and B.B. The results have been performed by A.C. The resources were provided by A.S. and J.M.S. The review, editing and improvements to the content have been made by A.C., B.B., A.S. and J.M.S. All authors have read and agreed to the published version of the manuscript.

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### Conflicts of Interest:
The authors declare no conflict of interest.
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