Multiplicity and pseudorapidity distributions of charged particles in asymmetric and deformed nuclear collisions in the wounded quark model

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Abstract. The charged particle multiplicity ($n_{ch}$) and pseudorapidity density ($d\frac{n_{ch}}{d\eta}$) are key observables to characterize the properties of matter created in heavy-ion collisions. The dependence of these observables on collision energy and the collision geometry are a key tool to understand the underlying particle production mechanism. Recently much interest has been focused on asymmetric and deformed nuclei collisions since these collisions can provide a deeper understanding about the nature of quantum chromodynamics (QCD). From the phenomenological perspective, a unified model which describes the experimental data coming from various kinds of collision experiments is much needed to provide physical insights on the production mechanism. In this paper, we have calculated the charged hadron multiplicities for nucleon-nucleus, such as proton-lead ($p$-Pb) and asymmetric nuclei collisions like deuteron-gold ($d$-Au), and copper-gold (Cu-Au) within a new version of the wounded quark model (WQM) and we have shown their variation with respect to centrality. Further we have used a suitable density function within our WQM to calculate pseudorapidity density of charged hadrons at midrapidity in the collisions of deformed uranium nuclei. We found that our model with suitable density functions describes the experimental data for symmetric, asymmetric and deformed nuclei collisions simultaneously over a wide range of the collision energy.

1 Introduction

Multiparticle production in ultra-relativistic heavy-ion collisions is an important tool to study the perturbative as well as non-perturbative nature of Quantum Chromodynamics (QCD) [1–3]. Enormous data has been collected by the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) from various types of collision, e.g., gold-gold (Au-Au), lead-lead (Pb-Pb), deuteron-gold ($d$-Au), copper-gold (Cu-Au), uranium-uranium (U-U), proton-lead ($p$-Pb), etc., over a wide range of center-of-mass energies [4–19]. The charged particle multiplicity ($n_{ch}$) and their pseudorapidity density ($d\frac{n_{ch}}{d\eta}$) at central rapidity are fundamental quantities measurable in high-energy experiments, which serve as an important tool to analyze these data and characterize the global properties of the systems created in heavy-ion collisions. Due to variation in shape, size and orientation of the colliding nuclei, collisions can have different kinds of initial geometries. The dependence of $n_{ch}$ and ($d\frac{n_{ch}}{d\eta}$)$_{\eta=0}$ on the collision geometry is sensitive to the underlying particle production mechanism. These distributions can be used to quantify the contribution of soft and hard processes in particle generation, stopping and penetrating powers of the colliding nuclei, contribution ratios of leading nucleons, square of the speed of sound, etc. [20].

Particle production in heavy-ion collisions is quite an involved process, since the colliding nuclei in these collisions do not behave as a mere incoherent superposition of their constituent nucleons. Rather, coherence effects become important and they modify not only the partonic flux into the collision, but also the underlying dynamics of particle production in the scattering processes. Therefore we need some baseline nucleon-nucleon collision experiments, e.g., proton-proton ($p$-$p$), proton-antiproton ($p$-$\bar{p}$), etc., and also the nucleon-nucleus (or some very small nucleus colliding

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with a large nucleus) collision experiments. Data from baseline experiments are useful to understand the particle production mechanism in vacuum and nucleon-nucleus (n-A) collisions to study the pure nuclear effects (such as shadowing, anti-shadowing, absorption, saturation, etc.) on the particle production mechanism. The data from these collisions also help to decipher the initial- and final-state effects for a proper characterization, as these effects may lead to qualitatively similar phenomena in observables.

On the other hand, asymmetric and deformed nuclei collisions, e.g., Cu-Au collisions, U-U collisions, etc., are interesting to study due to the different initial geometrical configurations of the colliding systems [17]. For example a substantial initial geometrical asymmetry, which exists in Cu-Au collisions, could lead to naturally arising odd harmonics, from both the core and/or the corona [19]. In peripheral Cu-Au collisions, a sizable directed flow from the Au to Cu nucleus is generated due to initial asymmetry of electric charges on two nuclei and can thus be useful to study the electromagnetic and chiral magnetic properties of quark gluon plasma [21]. Since the Cu nucleus is completely swallowed by the Au one in central collisions, it could also provide an opportunity to measure an extensive range of initial energy densities for this system. Recently an analysis made by the PHENIX Collaboration shows that the hadron multiplicities in Cu-Au collisions show the same centrality trend as observed in Au-Au collisions at 200 GeV center-of-mass energy. However the transverse energy calculated for Cu-Au is higher than for the Au-Au system, which is contrary to earlier expectations [22]. Similarly, the study of deformed uranium-uranium (U-U) collisions at RHIC has gained much of interest in the past few years. The unique geometry and shape of the uranium nucleus provides opportunities to understand the particle production mechanism and the elliptic flow [23,24]. The uranium nucleus is not spherical and has a prolate shape, which leads to different collision geometry shapes from body-on-body to tip-on-tip configurations [25,26], even in fully overlap collisions. In the case of tip-on-tip collisions, the general expectation was that the produced particle multiplicity should be higher due to the larger number of binary collisions, while the elliptic flow is smaller since the overlapping region is symmetric while, in the case of body-on-body collisions, there is smaller multiplicity associated with larger elliptic flow. However recent data on U-U collisions for most central 1% event from the PHENIX experiment show almost no enhancement of multiplicity in comparison to symmetric Au-Au collisions [17]. These results can only be understood with the help of theoretical models. These models can only provide physical insights onto the underlying particle production mechanism. Much effort has been put forward in this direction [24–48]. However, a phenomenological model which describes these multiplicity distribution data simultaneously for various types of collision is still needed. Recently we have proposed a parametrization which is based on a phenomenological model involving wounded quarks interactions [49–51]. Our model with a minimal number of parameters successfully explains the charged hadron multiplicity distributions for symmetric nucleus-nucleus collisions, such as Cu-Cu, Au-Au, Pb-Pb, etc. [52,53]. In this paper our main aim is to extend our model suitably, so that we can accommodate asymmetric and deformed nuclei collisions along with symmetric collisions. Here we first improve our parametrization for nucleon-nucleon collisions by including recent experimental data and then we extend our WQM by changing its density profile function to describe the various types of collisions having different initial geometrical configurations.

The rest of the paper is organised as follows: In sect. 2, we firstly provide a parametrization for the total multiplicity and pseudorapidity density at midrapidity of charged hadrons which are produced in p-p collisions. Section 3 will provide a brief description of WQM. Here we also present the different nuclear density functions for symmetric, asymmetric and deformed nuclei. In sect. 4 we show the results obtained in our model and their comparison with the corresponding experimental data, wherever available. Finally we summarize our present study.

2 Parametrization for p-p collisions

The production of charged hadrons in nucleus-nucleus collisions are deeply linked with p-p collisions at various energies. Initially, Feynman pointed out that the charged hadron multiplicity distribution in p-p collisions have no dependence on the available energy as centre-of-mass energy (√s) → ∞. This implies that total charged hadron multiplicity, after summing over rapidity, has ln √s dependence, since y_{max} = ln (√s/m_N), where m_N is the nucleon mass. Further additional gluons arising from gluon-bremsstrahlung processes give QCD radiative corrections to the total charged hadron production; hence total charged hadron multiplicity has ln^2 √s dependence. Recent data regarding pseudorapidity distributions of charged hadrons coming from the PHOBOS experiment for p-p and p-$\bar{p}$ in the central plateau region, i.e., (dN_{ch}/dη)_{η=0} shows ln^2 √s dependence, which will finally give ln^3 √s type dependence to the total charged hadron multiplicity distribution [54–56]. Based on these experimental findings and exploiting the QCD hypothesis of universal particle production in various hadronic collisions, we have proposed a parametrization involving a cubic logarithmic term to calculate the charged hadron mean multiplicity in p-p collisions at any collision energy as follows:

$$\langle n_{ch} \rangle_{pp} = (a' + b' \ln \sqrt{s_A} + c' \ln^2 \sqrt{s_A} + d' \ln^3 \sqrt{s_A}) - \alpha.$$  \hspace{1cm} (1)

In eq. (1), α is the leading particle effect which arises due to the energy carried away by the spectator quarks and its value is experimentally determined as 0.85; $\sqrt{s_A}$ is the available center-of-mass energy, i.e., $\sqrt{s_A} = \sqrt{s} - m_B - m_T$, where m_B and m_T are the masses of the projectile and target, respectively.
where \( m_B \) is the mass of projectile and \( m_T \) the mass of the target nucleon, respectively; \( a', b', c' \) and \( d' \) are constants having values \( a' = 1.8, b' = 0.37, c' = 0.43 \) and \( d' = 0.04 \) [41,42]. To understand the particle production mechanism from high-energy \( h-h \) to \( A-A \) collisions, pseudorapidity distribution is a very useful quantity. It was pointed out that \( d_{\eta}/d\eta \) is suitably used to find the information about the temperature and energy density of QGP. For pseudorapidity density at midrapidity, we have proposed a parameterization in our earlier publications [50,51]. Here we use the same parameterization but do a reasonable fit again (see fig. 1) to accommodate the new data coming from \( p-p \) collision data at 0.9, 1.8, 7 TeV energies [4]:

\[
\langle (d_{\eta}/d\eta)_{\eta=0} \rangle = \left( a_1' + b_1' \ln \sqrt{s_A} + c_1' \ln^2 \sqrt{s_A} + d_1' \ln^3 \sqrt{s_A} \right) - \alpha_1'.
\] (2)

We have obtained slightly changed values of the parameters as compared to the values obtained in our earlier publications [52,53]. The values are \( a'_1 = 1.15, b'_1 = 0.16, \) and \( c'_1 = 0.05. \)

3 Model formalism

Extrapolating our parametrization from \( p-p \) collisions to \( h-A \) collisions we have assumed that the quark-quark picture is more suitable than nucleon-nucleon picture for determining charged hadron multiplicity distribution. We consider here that a wounded quark which participates in the reaction suffered multiple collisions before it hadronizes. Based on this, we have used the expression for average charged hadron multiplicity in \( h-A \) collisions as follows [52,53]:

\[
\langle n_{ch} \rangle_{hA} = N_q \left[ a' + b' \ln \left( \frac{\sqrt{s_A}}{N_q} \right) + c' \ln^2 \left( \frac{\sqrt{s_A}}{N_q} \right) + d' \ln^3 \left( \frac{\sqrt{s_A}}{N_q} \right) \right] - \alpha.
\] (3)

In eq. (3), \( \sqrt{s_A} \) is related to \( s_A \), as \( \sqrt{s_A} = \nu_{qA} s_A \), where \( \nu_{qA} \) represents the mean number of collisions of the wounded quark inside the nucleus \( A \) and is defined by

\[
\nu_{qA} = \frac{A \sigma_{inN}^q}{\sigma_{qA}^in}.
\] (4)

Here \( \sigma_{inN}^q \) and \( \sigma_{qA}^in \) are the inelastic cross-sections for quark-nucleon \( (q-N) \) and quark-nucleus \( (q-A) \) interactions, respectively, and \( A \) is the atomic mass of the target nucleus. In the above equation the mean number of constituent particles
quarks which become wounded in $h$-$A$ collisions shares the total available centre-of-mass energy $\sqrt{s_A}$, according to the law of equipartition of energy. Due to this, the energy available in each interacting quarks becomes $\sqrt{s_A}/N_q$. The parameters $a, b, c$ and $d$ do not change their value in $h$-$A$ collisions as compared to $p$-$p$ collisions and thus give us hint that they are directly related to quark-quark and quark-gluon interaction processes in QCD. In earlier publications by Singh et al. [49–51], the importance of the parameter $c$ is shown where it comes automatically from the gluon-bremsstrahlung process in QCD. Further, in eq. (3), $N_q$ is the mean number of inelastically interacting quarks with the nuclear targets and is defined as

$$N_{qA}^{hA} = \frac{N_q \sigma^{in}_{qA}}{\sigma^{in}_{hA}}. \quad (5)$$

Here, $\sigma^{in}_{qA}$ and $\sigma^{in}_{hA}$ are the scattering cross-sections for quark-nucleus and hadron-nucleus interactions, respectively, obtained from Glauber’s theory. The quark-nucleus inelastic interaction cross-section $\sigma^{in}_{qA}$ is determined from $\sigma^{in}_{qN}(=1/3\sigma^{in}_{NN})$ by using Glauber’s approximation as follows:

$$\sigma^{in}_{qA} = \int d^2b \left[ 1 - (1 - \sigma^{in}_{qN}D_A(b))^A \right], \quad (6)$$

where the profile function $D_A(b)$ is related to nuclear density, $\rho(b, z)$ by the relation

$$D_A(b) = \int_{-\infty}^{\infty} \rho(b, z)dz. \quad (7)$$

The fireball size formed in the hadronic and nuclear collisions depends on the overlap cross-section of colliding systems, which indirectly results in the change in the mean number of participating quarks as we move towards peripheral collisions. $N_q$ becomes maximum in central collisions and it decreases when we move to peripheral collisions. This fact shows that $N_q$ has a centrality as well as colliding-system dependence.

Since we are discussing the collisions happening among small-large nuclei, deformed-deformed nuclei along with large-large nuclei, so we have to choose an appropriate density function to properly describe the charged density of different nuclei. We have used the following functions.

### 3.1 For large nuclei

We have used the Woods-Saxon charge distribution for large nuclei which can be expressed as follows:

$$\rho(b, z) = \frac{\rho_0}{1 - \exp(\frac{\sqrt{b^2 + z^2 - R}}{a})}, \quad (8)$$

where all the notations have their usual meaning [52,53].

### 3.2 For small deuteron nuclei

We have used the following Hulthén function for expressing charge density of deuterium nuclei [71,72]

$$\rho(r) = \rho_0 \left( \frac{e^{-ar} + e^{-br}}{r} \right)^2, \quad (9)$$

where $a = 0.457$ fm$^{-1}$ and $b = 2.35$ fm$^{-1}$.

### 3.3 For deformed nuclei

For deformed nuclei we have used the following modified form of the Woods-Saxon charge distribution [71,72]:

$$\rho(x, y, z) = \rho_0 \frac{1}{1 + \exp \left( \frac{r - R(1 + \beta_2 Y_{20} + \beta_4 Y_{40})}{a} \right)}, \quad (10)$$

where $Y_{20} = \sqrt{\frac{5}{192\pi}}(\cos^2(\theta) - 1), Y_{40} = \frac{a}{16\sqrt{\pi}}(35\cos^4(\theta) - 30\cos^2(\theta) + 3)$ are the spherical harmonics with the deformation parameters $\beta_2$ and $\beta_4$. 
The different parameters value for uranium nuclei is taken from refs. [71,72].

The generalization of the \( b-A \) to nucleus-nucleus collisions goes along the same line in WQM and can be given as follows [52,53]:

\[
(n_{ch})_{AB} = N_q^{AB} \left[ a' + b' \ln \left( \frac{\sqrt{s_{AB}}}{N_q^{AB}} \right) + c' \ln^2 \left( \frac{\sqrt{s_{AB}}}{N_q^{AB}} \right) + d' \ln^3 \left( \frac{\sqrt{s_{AB}}}{N_q^{AB}} \right) - \alpha \right],
\]

(11)

where \( \sqrt{s_{AB}} = A(\nu_q^{AB}s_A)^{1/2} \), where \( A \) is the mass number of the colliding nucleus and the mean number of inelastic quark collision, \( \nu_q^{AB} \), can be given as

\[
\nu_q^{AB} = \nu_q A = \frac{A \sigma_{qN}^{in}}{\sigma_{qA}^A} \cdot \frac{B \sigma_{qN}^{in}}{\sigma_{qB}^B}.
\]

(12)

Furthermore, the mean number of participating quarks \( N_q^{AB} \) can be calculated by generalizing eq. (9) in the following manner:

\[
N_q^{AB} = \frac{1}{2} \left[ \frac{N_B \sigma_{qA}^{in}}{\sigma_{AB}^{in}} + \frac{N_A \sigma_{qB}^{in}}{\sigma_{AB}^{in}} \right],
\]

(13)

where \( \sigma_{AB}^{in} \) is the inelastic cross-section for \( A-B \) collision and can be expressed as

\[
\sigma_{AB}^{in} = \pi r^2 \left[ A^{1/3} + B^{1/3} - \frac{c}{A^{1/3} + B^{1/3}} \right]^2,
\]

(14)

where \( c \) is a constant and has a value of 4.45 for nucleus-nucleus collisions.

Now, the central average charge hadron multiplicity can be found in following way:

\[
(n_{ch})_{AB}^{central} = A \left[ a' + b' \ln (\nu_q^{AB} s_A)^{1/2} + c' \ln^2 (\nu_q^{AB} s_A)^{1/2} + d' \ln^3 (\nu_q^{AB} s_A)^{1/2} - \alpha \right].
\]

(15)

Now, we want to provide an expression to calculate pseudorapidity distribution of charged hadrons with respect to pseudorapidity. For this we enlist the help of the wounded nucleon two-component model. The physical interpretation of the usual two-component model is based on hadron production by longitudinal projectile nucleon dissociation (soft component) and transverse large-angle scattered parton fragmentation (hard component). However, here, in our wounded quark picture the hard component scales with the number of quark-quark collisions and the soft component scales with the participant quarks number. Thus in \( A-B \) collisions, central mean pseudorapidity density can be parametrized in terms of \( p-p \) rapidity density as follows [52,53]:

\[
\left( \frac{dn_{ch}}{d\eta} \right)^{AA}_{\eta=0} = \left( \frac{dn_{ch}}{d\eta} \right)^{pp}_{\eta=0} \left[ (1 - x) N_q^{AB} + x N_q^{AB} \nu_q^{AB} \right],
\]

(16)

where \( \left( \frac{dn_{ch}}{d\eta} \right)^{pp}_{\eta=0} \) is calculated from eq. (2) using the new parameter values, and where \( x \) signifies the relative contributions of hard and soft processes in two component model. The value of \( x \) varies from 0.1 to 0.125 with centre-of-mass energy.

4 Results and discussions

4.1 Total multiplicity and pseudorapidity density

In a previous publication we have shown that WQM provides a suitable description to the various features of charged hadron production in high-energy collisions, such as multiplicity distributions with respect to pseudorapidity and collision energy for symmetric collisions of nuclei with varying sizes [52]. Recently the multiplicity data for symmetric Pb-Pb collisions at 2.76 TeV and 5.02 TeV has been presented by the LHC experiment. Thus we first want to show our results corresponding to \( n_{ch} \) and \( dn_{ch}/d\eta \) for this symmetric collision and then move towards asymmetric and deformed nuclei collisions. In fig. 2 we have shown the variation of \( \sigma_{qPb}^{in} \) with respect to the transverse size of the collision zone for the Pb-Pb collision at 2.76 TeV. Here we have shown the selection criteria in the present model for different centrality bins using a cut on \( \sigma_{qPb}^{in} \). We have used the same centrality criteria to calculate the charged hadron multiplicity with respect to the centrality for Pb-Pb collisions at 2.76 TeV in fig. 3. Further we have compared our results with the corresponding experimental data. We find that the model suitably describes the data. We have also shown WQM results and compared them with experimental data in tabular form in table 1 for four centrality classes.
Fig. 2. Variation of the quark-nucleus inelastic cross-section ($\sigma_{qA}^{\text{in}}$) in our model as a function of centrality for Pb-Pb collisions $\sqrt{s_{NN}} = 2.76$ TeV at LHC energy.

Fig. 3. Variation of mean charged hadron multiplicity with respect to centrality for Pb-Pb collisions at 2.76 TeV. Open triangles are the WQM results and solid circles are the experimental data taken from ref. [5].

Rapidity density of charged particles is quite useful in providing a qualitative measure of the final entropy per unit rapidity produced in a collision. It depends essentially on system size, centrality of the collision event and collision energy. Additionally, it also gives an indication towards the viscous effect during expansion of the hot and dense QCD matter created in a collision as viscous effects contribute to the entropy generation and, in turn, enhance the final
Table 1. The total charge hadron multiplicities as a function of centrality in Pb-Pb collisions LHC energy. The data shown here are taken from the ALICE experiment [5].

| Centrality bin | √sNN = 2.76 TeV |
|----------------|------------------|
| Model          | Experimental     |
| 0–5%           | 14764            | 14963 ± 666 |
| 5–10%          | 12667            | 12272 ± 561 |
| 10–20%         | 9484             | 9205 ± 457  |
| 20–30%         | 6384             | 6324 ± 330  |

Fig. 4. Variation of pseudorapidity density with respect to centrality for Pb-Pb collisions at 2.76 TeV [6].

charged particle multiplicity. In fig. 4 we have presented the variation of charged hadron pseudorapidity density at midrapidity with respect to centrality for Pb-Pb collisions at 2.76 TeV. Further, we have shown our model results with the experimental data along with other model results for a comparative study. We have shown here the results obtained from HIJING [73], AMPT [73] with different sets of parameters along with the results obtained from the recent version of DPMJET [35]. Here we find that most of these models are able to satisfy the data obtained in peripheral collisions. However, for semi-peripheral and central collisions, a varying level of agreement is observed as some of these models overpredict and some of these underpredict the data. In particular, the difference between the results obtained from HIJING with jet quenching and the experimental data is quite large in central collisions. On the other hand, WQM results satisfy the experimental data quite well in a consistent manner from central to peripheral collisions.

In fig. 5, we have plotted the variation of pseudorapidity density of charged hadrons at midrapidity divided by the number of participating quarks with respect to centrality for Pb-Pb and Au-Au collisions at √sNN = 2.76 TeV and 200 GeV, respectively. We have shown the corresponding p-p data in the most peripheral centrality class of Pb-Pb and Au-Au collisions since the mean number of participating quarks for p-p collisions is 2, which is almost equal to the number of participating quarks in Pb-Pb and Au-Au collisions in their most peripheral collisions. One can observe from the graph that (dnch/dη)η=0, divided by the number of participating quarks, is almost independent of centrality, which is actually the basic theme of the wounded quark picture. Some deviation is due to different transverse momentum or different entropy production at different centralities during the evolution of the system [37,38]. Further, the mean number of charged hadrons per unit pseudorapidity coming from a participating wounded quark pair is the same within error bars for both Pb-Pb and p-p collisions. We have also found this scaling feature in Au-Au and p-p collisions at RHIC energy. These observations support the idea of independent particle production from a wounded quark source.
Fig. 5. Variation of $(dn_{ch}/d\eta)/(2N_{q})$ as a function of centrality for Pb-Pb and Au-Au collisions at $\sqrt{s_{NN}} = 2.76$ TeV and 200 GeV, respectively. We have also shown $(dn_{ch}/d\eta)/(2N_{q})$ for $p-p$ collisions at corresponding energies. Here $N_{q}$ is calculated using WQM and experimental data of $(dn_{ch}/d\eta)$ is taken from refs. [4,6,12,65].

Fig. 6. Variation of pseudorapidity density at midrapidity with respect to centrality for Pb-Pb collisions at 2.76 TeV [6] and 5.02 TeV [14].

In fig. 6, we have shown the model results for charged hadron pseudorapidity density at midrapidity at 2.76 TeV and 5.02 TeV and compare them with the experimental data at the two LHC energies. Model and experimental results at the two LHC energies show a good level of agreement. Further, the observed increase in the multiplicity at 5.02 TeV with respect to the multiplicity at 2.76 TeV is seen, which is quite obvious due to the increase in the collision energy, resulting into the higher production at 5.02 TeV.
Fig. 7. Variation of $\sigma_{\text{q-d}} \cdot \sigma_{\text{q-Au}}$ in our model as a function of the transverse coordinate $b$ (fm) of the collision zone formed in $d$-Au collisions at 200 GeV.

Fig. 8. Variation of total charge hadron multiplicity with respect to centrality for $d$-Au collisions at 200 GeV [9].

In fig. 7, we have presented the variation of $\sigma_{\text{q-d}} \cdot \sigma_{\text{q-Au}}$ with respect to transverse coordinate of the fireball formed in $d$-Au collision to determine the different centrality class. Figure 8 presents the variation of charged hadron multiplicity with respect to centrality for $d$-Au collisions. We have used the Hulthén density function to describe the charged distribution in the deuteron nucleus and Woods-Saxon density function for the gold nucleus. We compared our model result with the corresponding experimental data from the PHENIX experiment and found a good agreement between them.
Fig. 9. Variation of $\sigma_{h Pb}^{\text{in}} \cdot \sigma_{q Pb}^{\text{in}}$ in our model as a function of the transverse coordinate $b$ (fm) of collision zone formed in $p$-Pb collisions at 5.02 TeV.

Fig. 10. Variation of total mean multiplicity with respect to centrality for $p$-Pb collision at 5.02 TeV [10].

Figure 9 represents the product of cross-sections of proton-lead collisions and quark-lead collisions with respect to transverse coordinate $b$ to define the various centrality class in $p$-Pb collisions. In fig. 10, the model results for the variation of mean multiplicity with respect to centrality is shown for $p$-Pb collisions at 5.02 TeV, along with its comparison to experimental data at LHC energy. Further, we have shown the results from the wounded nucleon model and the color glass condensate for the sake of comparison. Our model results agree with the experimental data quite...
Table 2. The total charged hadron multiplicities as a function of centrality in \( d-Au \) collisions at RHIC energy. The experimental data were taken from the PHOBOS experiment [9].

| Centrality bin | \( \sqrt{s_{NN}} = 200 \text{ GeV} \) |
|---------------|--------------------------|
|               | Model          | Experimental |
| 0–20%         | 167            | 167\(^{+14}_{-11}\) |
| 20–40%        | 122            | 109\(^{+10}_{-8}\)  |
| 40–60%        | 85             | 77\(^{+7}_{-5}\)    |
| 60–80%        | 52             | 48\(^{+3}_{-3}\)    |
| 80–100%       | 30             | 29\(^{+3}_{-3}\)    |

![Graph](image)

Fig. 11. Variation of pseudorapidity density at midrapidity with respect to centrality for \( p-Pb \) collision at 5.02 TeV [15].

well, whereas other model results show a varying level of agreement. The charged hadron multiplicity is \( \approx 43 \) for \( p-Pb \) collisions in the most central bins, which is almost one-third of the multiplicity in the most central bins of \( d-Au \) collisions, as tabulated in table 2, where charged hadron multiplicity is 167. In spite of the big size of the Pb nucleus in comparison to that of Au, the multiplicity is low for the \( p-Pb \) case. Thus it suggests that a relatively larger fireball is expected to be formed in \( d-A \) collisions due to the large overlap area than in \( p-Pb \) collisions at the higher energy of 5.02 TeV. Also, the number of collisions suffered by each wounded quark traveling through the medium is less in \( p-Pb \) collisions as compared to \( d-Au \) collisions even at the cost of increased collisional energy. It clearly shows the importance of the role played by collision geometry as smaller collision systems (such as \( p-A \)) will create less matter compared to \( d-A \) collisions and will exist for a shorter span of time for particle production in the final state. Similarly, in fig. 11 we have shown the model results for pseudorapidity density at midrapidity with centrality for \( p-Pb \) collisions at 5.02 TeV and a comparison is made with the available experimental data for the same energy. Again, it is noticeable that model results explain the experimental data quite well for all centrality bins.

To define the centrality bin in asymmetric collisions like Cu-Au is a bit complex process. In our model we have first plotted \( \sigma_{qCu}^m \) and \( \sigma_{qAu}^m \) with respect to the transverse size of the collision zone. After that we have plotted the product of both with respect to the transverse size (as shown in fig. 12) and then we divided the whole transverse fireball area into different centrality regions. Based on this analysis, we have calculated the model results for the variation of \( \frac{dN_{ch}}{d\eta} \) with respect to centrality for asymmetric Cu-Au collisions along with the experimental data for a comparative study (as shown in fig. 13). Furthermore, we have shown comparison of WQM results with the corresponding results obtained in the IP-Glasma model. We found that both models, i.e., WQM and IP-Glasma, suitably describe the experimental data.
Fig. 12. Variation of $\sigma_{qCu}^{in} \cdot \sigma_{qAu}^{in}$ in our model as a function of the transverse coordinate $b$ (fm) of the collision zone formed in Cu-Au collisions at 200 GeV.

Fig. 13. Variation of pseudorapidity density at midrapidity with respect to centrality for Cu-Au collisions at 200 GeV [12].

Now we move towards the deformed U-U nuclei collisions. To describe the charge density function in U nuclei, we have taken the deformed Woods-Saxon density function. For a particular centrality class there are different types of orientation configuration possible for U-U collisions. Measurement of charged particle multiplicity density in deformed nuclei is very sensitive to the orientation of the two colliding nuclei, as the number of collisions suffered by each participating quark inside the other colliding nucleus will be affected, due to the available travel path according to
Table 3. The pseudorapidity distribution of U-U in minimum bias and tip-tip at $\sqrt{s_{NN}} = 193$ GeV by our model compared with IP-Glasma. Experimental data taken from ref. [12].

| Colliding nuclei | $(dn_{ch}/d\eta)_{\text{central}}^{(0-5\%)}$ | $(dn_{ch}/d\eta)_{\text{tip-tip}}^{(0-5\%)}$ |
|------------------|---------------------------------------------|---------------------------------------------|
| Our model        | U-U 797                                     | 739                                         |
| IP-Glasma model  | U-U 824                                     | 815                                         |
| Experimental data| U-U $830.4 \pm 67.8$ (0-5%)                  |                                              |

their orientation. In a heavy-ion collision experiment, it is quite difficult to control the orientation of the two colliding nuclei. In our model, we have calculated the pseudorapidity density for central U-U collisions in two ways: first we have taken the average over all types of configuration for central collision and, second, we have only taken the tip-tip configuration by fixing both angles, $\theta_1$ and $\theta_2$, to zero and then calculated $dn_{ch}/d\eta$ for central collisions. We tabulated our model results in table 3 along with IP-Glasma model results. Further we compared both model results with the experimental data. We found that the IP-Glasma model as well as WQM provide a reasonable agreement with the data.

4.2 Speed of sound

Transport properties are a useful tool to quantify the behaviour of the matter created in heavy-ion collisions [74]. Recently the data on the collective velocity obtained from RHIC and LHC experiments indicate that a perfect-fluid–like system has been created in these collisions, which is in contradiction to earlier prediction of the creation of an ideal QGP gas. The speed of sound is an important transport coefficient which can possibly hint on the nature of matter created in these collisions, since in an ideal gas, the square of the speed of sound can only go up to 0.33 in magnitude. However, if the system is in liquid form, then the speed of sound can cross this limit (since the speed of sound is large in a liquid in comparison to a gas). To calculate the square speed of sound, $c_s^2$, from the pseudorapidity distribution, we have used the prescription, as given in ref. [20],

$$c_s^2 = \frac{1}{3\sigma^2} \left[ \sqrt{16 \ln^2 \left( \frac{\sqrt{s_{NN}}}{2m_N} \right) + 9\sigma^2} - 4 \ln \left( \frac{\sqrt{s_{NN}}}{2m_N} \right) \right]. \quad (17)$$

where $\sigma$ is the rapidity distribution width and $m_N$ denotes the mass of a proton. The value of $\sigma$ is taken from refs. [52, 53] for different colliding nuclei, Cu-Cu, Au-Au and Pb-Pb at different RHIC and LHC energies, respectively. Our fitting function for pseudorapidity distribution with respect to pseudorapidity is a little bit different from the function used in ref. [20]. However, the role of $\sigma$ is similar to that in ref. [20], which is to provide a width to the pseudorapidity distribution and thus one can use the prescription to calculate $c_s^2$.

In fig. 14, we show the variation of the square of the speed of sound ($c_s^2$) in Au-Au and in Pb-Pb systems at different RHIC and LHC energies. Here, we find that the value of $c_s^2$ goes from 0.33 to 0.37 in Au-Au collisions and its value in Pb-Pb collisions at LHC energy is found to be 0.435. From this we infer that the systems created in Au-Au and Pb-Pb collisions is not behaving like an ideal hadronic gas, for which the maximum value of $c_s^2$ can go up to 0.33. In these collisions, higher values of $c_s^2$ indicate the formation of a medium behaving as a viscous fluid in contradiction to an ideal gas. Further the larger value at LHC energy shows that the medium formed at this energy is more viscous in comparison to the medium formed at RHIC energy. In fig. 15, the variation of the square of the speed of sound in the Au-Au and Cu-Cu systems is shown. We see that in the Cu-Cu system, $c_s^2$ is higher than in the Au-Au system at some energies and, at some energies, $c_s^2$ has almost the same value as in the case of Au-Au collisions. The medium created in smaller collision systems, like Cu-Cu, the finite-size effects play an important role, which causes fluctuation in the mean value of $c_s^2$ [75], which makes the speed of sound a bit random.

In summary, we have analyzed the pseudorapidity density and multiplicity data of charged hadrons with respect to different control parameters in view of a new wounded quark model. We proposed a new parametrization to calculate particle production in $p-p$ collisions and then extend them for $A-A$ collisions using the wounded quark picture, in which a quark suffers multiple collisions before hadronization. A different type of nuclear density function has been implemented in this model to calculate the particle production in symmetric, asymmetric and deformed nuclei collisions. We first demonstrated the variation of the total multiplicity and pseudorapidity density of charged hadrons in symmetric Pb-Pb collisions with respect to centrality and compared them with various theoretical models. Further, particle mean multiplicities in $d$-Au and $p$-Pb collisions have been calculated within WQM and their comparison with
wounded nucleon as well as colour glass condensate model were shown. Then, we have calculated $dn_{ch}/d\eta$ of charged hadrons at midrapidity in the asymmetric collisions of Cu-Au as well as deformed U-U nuclei collisions and compared them with the corresponding results of other models. The agreement between data and WQM results suggests that the quark picture suits more as a particle production mechanism in ultra-relativistic heavy-ion collisions. We have also used WQM to calculate the speed of sound in heavy-ion collisions.
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