Repercussions of dimension five nonminimal couplings in the electroweak model

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Abstract. We propose two possibilities of CPT-odd, dimension five and Lorentz-violating (LV) nonminimal couplings in the Electroweak model. These gauge-invariant terms couple a fixed LV 4-vector to the electroweak field strengths. After finding the LV contributions to the electroweak currents, we evaluate the decay rate for the vector mediators \( W \) and \( Z \). Using the known experimental uncertainty in these decay rates, upper bounds at the level of \( 10^{-15} \) (eV)\(^{-1} \) and \( 10^{-14} \) (eV)\(^{-1} \) are imposed on the magnitude of the proposed nonminimal interactions.

1. Introduction
The most general effective theory considering the explicit violation of Lorentz and CPT symmetry is the minimal standard model extension (mSME) [1], which is an extension of the \( SU(3) \times SU(2) \times U(1) \) standard model, exhibiting terms violating Lorentz and CPT symmetries in all of its sectors: lepton, quark, Yukawa, Higgs and gauge. Nonminimal LV interactions have also been examined in an extension of the SME encompassing higher-order derivatives in both the gauge [2] and the fermion sectors [3]. Some models possessing higher-dimension operators [4] and higher derivatives [5] have also been proposed and developed. Nonminimal interactions have been a topical issue in the latest years, mainly in the fermion and electromagnetic sectors [7]. A systematic investigation on nonminimal couplings (NMCs) of dimension five and six was recently proposed in Ref. [6].

In the Glashow-Salam-Weinberg electroweak model (GSW), with a \( SU(2)_L \times U(1)_Y \) gauge structure spontaneously broken via the Higgs mechanism, the vector bosons, \( W^\pm, Z^0 \) and \( \gamma \) work as mediators of the interactions, being introduced via minimal coupling to the matter fields. In this theory, left-handed leptons \( (L_l) \) are represented by isodoublets and right-handed leptons \( (R_l) \) are isosinglets,

\[
L_l = \begin{bmatrix} \psi_l \\ \bar{\psi}_l \end{bmatrix}_L = \frac{1 - \gamma_5}{2} \begin{bmatrix} \psi_l \\ \bar{\psi}_l \end{bmatrix}_L, \quad R_l = (\psi_l)_R = \left( \frac{1 + \gamma_5}{2} \right) \psi_l, \tag{1}
\]

and \( l = 1, 2, 3 \) is the lepton flavor label: \( \psi_l = (e, \mu, \tau) \). The part of the electroweak Lagrangian, in which the leptons interact directly with the gauge fields, is \( \mathcal{L}_{EW} = \mathcal{L}_{gauge} + \mathcal{L}_{lepton} \), where

\[
\mathcal{L}_{gauge} = -\frac{1}{4} W_{\mu\nu} W^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}, \quad \mathcal{L}_{lepton} = \bar{L}_{l} \gamma^\mu i D_\mu L_l + \bar{R}_{l} \gamma^\mu i D_\mu R_l, \tag{2}
\]
with $W_\mu = (W_\mu^1, W_\mu^2, W_\mu^3)$ being a four-vector gauge field which is a three-vector in isospin space, and $B_\mu$ a gauge four-vector field. The associated field strengths are

$$ W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu + g W_\mu \times W_\nu, \quad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \tag{3} $$

The covariant derivative involves both gauge fields,

$$ D_\mu = \partial_\mu - ig T : W_\mu - ig' Y B_\mu. \tag{4} $$

where $g, g'$ are the coupling constants, $T = (T_1, T_2, T_3)$ stands for the generators of the group $SU(2)_L$, and $Y$ is the generator of $U(1)_Y$ group. The generators and isovector can be also written as $T = (T_+, T_3, T_-)$. $W_\alpha = (W_\alpha^{(+)} / \sqrt{2}, W_\alpha^3, W_\alpha^{(-)} / \sqrt{2})$, where $T_+ = \sigma_x / 2 + i (\sigma_y / 2)$, $T_3 = \sigma_z / 2$, $W_\mu^{(\pm)} = \frac{1}{\sqrt{2}} (W_\mu^1 \pm i W_\mu^2)$ and $W_\mu^3 = \sin \theta A_\mu + \cos \theta Z_\mu$ with $\sigma_x, \sigma_y, \sigma_z$ being the Pauli matrices, $A_\mu$ and $Z_\mu$ denoting the photon field and the neutral intermediate boson respectively, and $\theta$ standing for the weak mixing angle.

The electroweak lepton sector of the mSME [1] is composed of CPT-even and CPT-odd terms,

$$ \mathcal{L}_{lep}^{\text{even}} = (c_L)_{\mu\nu AB} \tilde{L}_A \gamma^\mu D^\nu L_B + (c_R)_{\mu\nu AB} \tilde{R}_A \gamma^\mu D^\nu R_B, \tag{5} $$

$$ \mathcal{L}_{lep}^{\text{odd}} = -(a_L)_{\mu AB} \tilde{L}_A \gamma^\mu L_B - (a_R)_{\mu AB} \tilde{R}_A \gamma^\mu R_B, \tag{6} $$

where $A, B = 1, 2, 3$ are the lepton flavor labels and the constant coefficients $(c_L)_{\mu\nu AB}$ and $(a_L)_{\mu AB}$ with $I = L, R$, produce the breakdown of Lorentz symmetry. LV studies in the electroweak sector were initially developed in connection with meson decays $(\pi^\pm \to \mu^\pm + \nu_\mu)$, where the LV effects were considered at the level of the Feynman propagator of the $W$ boson [8], $\langle W^{\mu\nu} W^{\nu\alpha} \rangle = -i (g^{\mu\nu} + \chi^{\mu\nu}) / M_W^2$, implying upper bounds of 1 part in $10^4$. Detailed contributions to the $W$ propagator were more explicitly considered in Ref. [9], with several implications on the nuclear $\beta$ decays [10], [11]. It was also examined the possibility of LV electroweak terms making feasible forbidden processes ($Z_0 \to \gamma + \gamma$) [12] or modifying reactions such as $\gamma + e \to W + \nu_e$, $\gamma + \gamma \to W + W$ [13]. Lepton flavor violating decays triggered by renormalizable and nonrenormalizable (dimension five) terms belonging to the Higgs sector were recently considered as well [14].

In the present work, we discuss a different way of considering Lorentz violation in the electroweak model: a NMC in the lepton sector [16]. In this way, we introduce two possibilities of CPT-odd LV nonminimal interactions in the Electroweak sector, the first one being proposed in the $U(1)_Y$ sector of the GSW (Glashow-Salam-Weinberg) model, while the second is considered in its $SU(2)_L$ sector, both as extensions of the covariant derivative. After determining the terms engendered in the interaction Lagrangian, we evaluate the Lorentz-violating corrections to the decay rates of the following mediators: $Z_0 \to l + l$ and $W^\pm \to l + \nu_l$. With these results and the experimental uncertainty in the measurements, we impose upper limits on the magnitude of the LV parameters, achieving upper bounds as tight as $10^{-6}$ (GeV)$^{-1}$.

2. A nonminimal coupling in the $U(1)_Y$ sector of the GSW model

Gauge invariant NM interactions in the electroweak sector can be proposed in the context of the covariant derivative (4). A first possibility, in the $U(1)_Y$ sector of the GSW model, is to include a dimension five term in it,

$$ D_\mu = \partial_\mu - ig T : W_\mu - ig' Y B_\mu + ig_2 Y B_{\mu\nu} C^\nu, \tag{7} $$

involving $C^\nu$ as a fixed 4-vector that establishes a preferred direction in spacetime and violates Lorentz symmetry. Replacing such a derivative in Lagrangian (2), the nonminimal coupling
yields additional electromagnetic and neutral LV interactions, $\mathcal{L}_{LV}^{(1)} = J_{EM(LV)}^{(1)} A_\nu + J_{0(LV)}^{(1)} Z_\nu$, given explicitly as

$$J_{EM(LV)}^{(1)} = \frac{g_2}{\sqrt{2}} \cos \theta \left[ \tilde{\psi}_\nu \gamma^\mu (1 - \gamma_5) \psi_\nu \right] C^\nu \partial_\mu - \frac{g_2}{\sqrt{2}} \cos \theta \left[ \tilde{\psi}_\nu \gamma^\nu (1 - \gamma_5) \psi_\nu \right] C^\mu \partial_\nu + g_2 \sin \theta \left[ j_1^\mu C^\nu \partial_\mu \right] - g_2 \cos \theta \left[ j_1^\nu C^\mu \partial_\mu \right],$$

(8)

$$J_{0(LV)}^{(1)} = -\frac{g_2}{\sqrt{2}} \sin \theta \left[ \tilde{\psi}_\nu \gamma^\mu (1 - \gamma_5) \psi_\nu \right] C^\nu \partial_\mu + \frac{g_2}{\sqrt{2}} \sin \theta \left[ \tilde{\psi}_\nu \gamma^\nu (1 - \gamma_5) \psi_\nu \right] C^\mu \partial_\nu - g_2 \sin \theta \left[ j_1^\mu C^\nu \partial_\mu \right] + g_2 \sin \theta \left[ j_1^\nu C^\mu \partial_\mu \right],$$

(9)

with $j_0^\mu(x) = \tilde{\psi}_l(x) \gamma^\mu (3 + \gamma_5) \psi_l(x)/2$. These expressions are useful to show the processes that are directly affected, at tree-level, by the nonminimal derivative (7). We now examine the effect of this nonminimal coupling on the $Z_0$ decay in a pair lepton and antilepton, $Z_0 \rightarrow l + \bar{l}$, evaluating the contributions implied to the decay rate. The total neutral current that contributes for this process is

$$\left( J_{0(LV)}^{(1)} + J_{0(LV)}^{(0)} \right) Z_\mu = \frac{-g}{4 \cos \theta} \tilde{\psi}_l(x) \gamma^\mu (g'_\nu - \gamma_5) \psi_l(x) Z_\mu(x) - g_2 \sin \theta \left[ j_1^\mu C^\nu \partial_\mu \right],$$

(10)

where $g'_\nu = 1 - 4 \sin^2 \theta$. The first term in the above equation is the usual Lorentz invariant contribution, while the second and third terms stemming from Eq. (9) encoded the Lorentz-violating effects.

The scattering matrix for such a process is

$$S = -i \int d^4 x \left( J_{0(LV)}^{(1)} + J_{0(LV)}^{(0)} \right) Z_\mu = S_0 + S_{LV(1)} + S_{LV(2)},$$

(11)

where the zero order and first order contributions in the LV parameters are

$$S_0 = i \frac{-g}{4 \cos \theta} \int d^4 x \tilde{\psi}_l(x) \gamma^\mu (g'_\nu - \gamma_5) \psi_l(x) Z_\mu(x),$$

(12)

$$S_{LV(1)} = i g_2 \sin \theta \int d^4 x \left[ j_1^\mu C^\nu \partial_\mu \right],$$

(13)

$$S_{LV(2)} = -i g_2 \sin \theta \int d^4 x \left[ j_1^\mu C^\nu \partial_\mu \right].$$

(14)

In order to evaluate these elements, we propose plane wave expansions, $Z_\mu^0(x) = N_k e_\mu(k, \lambda) \exp(-ik \cdot x)$, $\psi_l(x) = N_q u_l(q, s) \exp(-iq \cdot x)$, $\tilde{\psi}_l(x) = N_q^* v_l(q', s') \exp(iq' \cdot x)$, where $k, q, q'$ stand for the 4-momentum of the $Z_0^0$ boson and the emerging leptons, respectively, and $N_q = (2V_{q0})^{-1/2}$.

After a long evaluation, the total decay rate for the decay, $Z_0 \rightarrow l + \bar{l}$, is found to be:

$$\Gamma_{ll} = \frac{g^2 (8M_Z)}{1536 \pi \cos^2 \theta} \left\{ \left[ (g'_\nu + 1) - 6g'_\nu \frac{m_l^2}{M_Z^2} \right] - \frac{g_2}{g} \frac{2\theta}{2} (C \cdot k) \left[ (3g'_\nu - 2) - 27g'_\nu \frac{m_l^2}{M_Z^2} \right] \right\} \cdot \Theta (M_Z - 2m_l),$$

(15)

We now use $k^2 = M_Z^2$ and $C \cdot k = C_0 M_Z$. As the $Z_0$ mass ($M_Z = 9.1 \times 10^{10}$ eV) is much larger than lepton masses, we can neglect the mass ratios for the electron, muon and tau.
(m_\nu^2/M_Z^2 \simeq 2 \times 10^{-11}, m_\mu^2/M_Z^2 \simeq 10^{-6}, m_\tau^2/M_Z^2 \simeq 4 \times 10^{-4}), which are smaller than the experimental uncertainty in decay rate measurements. Thus, the result is written as

$$\Gamma_{ll} = \frac{g^2_\nu (g^2_\nu + 1) M_Z}{192 \pi \cos^2 \theta} \left[ 1 - 8 \times |g^{2}_\nu C_0| M_Z \right] \Theta (M_Z - 2m_l),$$

(16)

with the LV contribution appearing as a direct correction to the usual decay rate. We have used $g = e/\sin \theta$, $g_V = 1 - 4 \sin^2 \theta$, $\sin^2 \theta = 0.23$. In accordance with Ref. [15], the Z_0 decay rate (considering lepton universality) is $\Gamma_{ll} = (83.985 \pm 0.086)$ MeV, or $\Gamma_{ll} = 83.985 (1 \pm 0.001)$ MeV, so that the experimental uncertainty is of 1 part in 10^5. We thus impose $8 |g^{2}_\nu C_0| M_Z < 1.0 \times 10^{-3}$, which leads to the upper bound $|g^{2}_\nu C_0| < 1.3 \times 10^{-6}$ (GeV)^{-1}, that is,

$$|g^{2}_\nu C_0| < 1.3 \times 10^{-6} \text{ (GeV)}^{-1}.$$ (17)

3. **A nonminimal coupling in the SU(2)_L sector of the GSW model**

A gauge invariant dimension five nonminimal coupling in the SU(2)_L sector of the GSW model can be proposed as

$$D_\mu = \partial_\mu - ig T \cdot W_\mu - ig^\prime_3 (T \cdot W_{\mu \nu}) V^\nu,$$ (18)

where $V^\nu$ is a fixed 4-vector that establishes a preferred direction in spacetime and violates Lorentz symmetry. The coupling term, $L_1 i \gamma^\mu (i g^\prime_3 T \cdot W_{\mu \nu} V^\nu) L_1$ embraces the following interactions at tree-level, $\mathcal{L}_{LV(2)} = J^{(0)\nu}_{(LV)} W_L^{(-)} + J^{(1)\nu}_{(LV)} W_L^{(+)} + J^{(2)\nu}_{(LV)} Z_\nu$, involving the vector bosons, in which the related currents are

$$J^{(0)\nu}_{(LV)} = \frac{g^\prime_3}{2 \sqrt{2}} \bar{\psi}_\nu \gamma^\mu (1 - \gamma_5) \psi_l V^\nu \partial_\mu + \frac{g^\prime_3}{2 \sqrt{2}} \bar{\psi}_\nu \gamma^\mu (1 - \gamma_5) \psi_l V^\mu \partial_\nu,$$ (19)

$$J^{(1)\nu}_{(LV)} = \frac{g^\prime_3}{2 \sqrt{2}} \bar{\psi}_l \gamma^\mu (1 - \gamma_5) \psi_\nu V^\nu \partial_\mu + \frac{g^\prime_3}{2 \sqrt{2}} \bar{\psi}_l \gamma^\nu (1 - \gamma_5) \psi_\nu V^\nu \partial_\mu,$$ (20)

$$J^{(2)\nu}_{(LV)} = \frac{g^\prime_3 \cos \theta}{4} \{ \bar{\psi}_\nu \gamma^\mu (1 - \gamma_5) \psi_l V^\nu \partial_\mu - \bar{\psi}_l \gamma^\mu (1 - \gamma_5) \psi_\nu V^\mu \partial_\nu - \bar{\psi}_l \gamma^\nu (1 - \gamma_5) \psi_\nu V^\mu \partial_\nu + \bar{\psi}_l \gamma^\nu (1 - \gamma_5) \psi_\nu V^\nu \partial_\mu \}.$$ (21)

The current, $J^{(0)\nu}_{(LV)}$, given by Eq. (20), affects the processes mediated by the $W^-$ particle, including the decay $W^- \rightarrow l + \bar{\nu}_l$. The total electroweak current that contributes to this process is

$$\left( J^{(-)}_l + J^{(-)}_{(LV)} \right) W^{(-)}_\mu = \frac{1}{2 \sqrt{2}} \left\{ g j_2^{\mu} W^{(-)}_\mu (x) - g j_3^{\nu} V^\nu \partial_\nu W^{(-)}_\mu (x) + g j_3^{\nu} V^\lambda \partial_\lambda W^{(-)}_\mu (x) \right\},$$ (22)

where $j_2^{\mu} (x) = \bar{\psi}_l (x) \gamma^\mu (1 - \gamma_5) \psi_\nu$, and the first term is the usual Lorentz invariant contribution. The scattering matrix for the process $(W^- \rightarrow l + \bar{\nu}_l)$, at leading order, can be written as $S = -i \int d^4 x \left( J^{(-)}_l + J^{(-)}_{(LV)} \right) W^{(-)}_\mu$, that implies $S = S_0 + S_{LV(1)} + S_{LV(2)}$, with

$$S_0 = -i \frac{g}{2 \sqrt{2}} \int d^4 x \left[ j_2^{\mu} \left( W^{(-)}_\mu \right) \right],$$ (23)

$$S_{LV(1)} = i \frac{g^\prime_3}{2 \sqrt{2}} \int d^4 x \left[ j_3^{\nu} V^\nu \partial_\nu \left( W^{(-)}_\mu \right) \right],$$ (24)

$$S_{LV(2)} = -i \frac{g^\prime_3}{2 \sqrt{2}} \int d^4 x \left[ j_3^{\nu} V^\lambda \partial_\lambda \left( W^{(-)}_\mu \right) \right].$$ (25)
The total decay rate for the decay, $W^- \rightarrow l + \bar{\nu}_l$, is read as

$$\Gamma = \frac{g^2}{48\pi} M_W \left[ 1 + (g_3' V_0) \frac{5M_W}{4g} \right] \Theta(M_W - m_t),$$

(26)

where $V \cdot k = V_0 M_W$ for the rest frame of the $W^-$ mediator, and we have neglected the contributions in $m_t^2/M_W^2, m_t^4/M_W^4$. Considering that the experimental uncertainty in the measures of this decay is at the level of $\sim 4.0 \times 10^{-2}$, and using $g = E/\sin \theta$, $\sin^2 \theta = 0.23$, we impose $7(g_3' V_0) M_W < 4.0 \times 10^{-2}$, yielding $|g_3' V_0| < 7 \times 10^{-14}$ (eV)$^{-1}$, or

$$|g_3' V_0| < 7 \times 10^{-5} \text{ (GeV)}^{-1}.$$  

(27)

As the current (19), involving the mediator $W^+$, is analogue to the current (20), we conclude that these latter developments equally hold to the decay $W^+ \rightarrow l + \nu_l$, which becomes constrained by a bound similar to Eq. (27).

4. Final remarks and Conclusion

We have explicitly evaluated the corrections implied by two CPT-odd nonminimal couplings to the decay rates of the electroweak processes, $Z_0 \rightarrow \bar{l} + l$ and $W^- \rightarrow l + \bar{\nu}_l$ ($W^+ \rightarrow l + \nu_l$). Considering that the experimental imprecision in the measurements, upper limits were set on the magnitude of the LV nonminimal coupling at the level of $10^{-6}$ (GeV)$^{-1}$ and $10^{-5}$ (GeV)$^{-1}$. Other impacts of these NMC in electroweak phenomena are to be investigated, including differential decay rates of polarized processes, which could, in principle, yield improved upper bounds. More details about this investigation were recently published [16].

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