The source of THz radiation based on dielectric waveguide excited by sequence of electron bunches

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Abstract. We present a new method for excitation of THz Cherenkov radiation in a dielectric waveguide by relativistic electron bunches. A sequence of bunches generates monochromatic radiation. The frequency of radiation is defined by the distance between the bunches. The studies were carried by using the newly updated BBU-3000 code which permits taking into account a number of additional options: an external quadrupole focusing system, group velocity of the wakefield, and the dielectric material loss factor. In this paper, we present our algorithm for optimizing the number and sequential positions of bunches for generation of narrow band high power THz radiation.

1. Introduction
Many applications benefit from the imaging capabilities of high power THz radiation. Laser driven THz emitters, solid state oscillators (high frequency diodes), and QCL (quantum cascade lasers) are currently considered as viable THz sources [1,2]. At the same time, a high power narrow bandwidth THz source can also be implemented using coherent Cherenkov radiation (CCR) generated by an electron bunch train passing through an all-metal or dielectric slow wave structure. In [3] observation of coherent Cherenkov radiation in the terahertz regime emitted by a relativistic electron pulse train passing through a dielectric lined cylindrical waveguide was presented. With the proposed technique, selective excitation of modes beyond the fundamental were demonstrated by the use of the appropriate bunch spacing [3]. Use of bunch train with adjustable spacing to selectively excite high order modes in a multimode structure was demonstrated in [4]. Narrowband THz generation by a bunch train in a multimode dielectric loaded wavefield structure was studied experimentally. The structure parameters (geometry and permittivity) provided a wide spacing between the modes in the frequency domain and a number of different frequencies were individually excited by a tunable bunch train [4].

In this paper we consider THz radiation based on a dielectric lined waveguide (DLW). THz radiation is produced by sequence of relativistic electron bunches passing along the DLW, figure 1. This work was initiated by the experimental studies presented in [4]. The bunch train was produced following three major steps: energy modulation of long bunch by self-wakefield generation, energy modulation conversion into spatial modulation by using a chicane technique, and finally THz radiation by the electron bunch train passing through the DLW. This scheme allows tuning the bunch separation in the train providing coherent THz generation and mode selectivity as well. The spectrum of THz
radiation is defined by both the DLW parameters and bunch separation of the train. In this paper, we present our results of numerical modelling of the THz radiation generation by the train consisting of 100 µm long electron bunches taking in account the group velocity effects and loss factor at the propagation distance of the wave envelope in the DLW. We also considered the selectivity of the first three TM modes generated by the bunch train of up to 10 bunches. Special attention has been paid to the generation of the $TM_{03}$ mode in the 0.8 THz frequency range. Monochromaticity and field amplitude of the first three THz modes have been studied as well.

2. Problem formulation
We studied the wakefield created by Gaussian bunch train by using the Green function for DLW, figure 1. All bunches in train are ultrarelativistic so the phase velocities of each mode are close to speed of light $V_c = \beta c$, $\beta \geq 1$. If the radial offsets of the bunches are equal to zero, we can take into account only the TM-modes with a high value of the longitudinal component of electric field and no transverse deflecting force for relativistic electrons. The charge distribution in bunch train is described by the equation:

$$p(\zeta) = \frac{Q}{\sqrt{2\pi} \sigma_\zeta} \sum_{n=1}^{N} \exp \left(-\frac{(\zeta - l_n)^2}{2\sigma_\zeta^2} \right)$$

where $Q$ - charge of single bunch, $\sigma_\zeta$ - is the rms length of a single bunch, $\zeta = z - V_c t$ is the axial position behind the first bunch, $l_n$ - is the position of the $n$th bunch in the train.

Figure 1. The bunch train passing along the axis of cylindrical dielectric waveguide.

We take into account the attenuation coefficient and group velocity for each TM mode. The Green function is defined by equation:

$$G_E(\zeta) = \sum_{m=0}^{\infty} \left\{ E_m \left\{ \begin{array}{ll}
\cos(\kappa_m \zeta) \exp(-\alpha_m \zeta) & \text{if } \zeta < L \left(1 - \nu_{gm}/c\right) \\
0 & \text{if } 0 \leq \zeta \leq L \left(1 - \nu_{gm}/c\right)
\end{array} \right\} \right\},$$

where $E_m$, $\kappa_m = \omega_m / V_c$, $\alpha_m$, $\nu_{gm}$ -amplitude, wave number, attenuation coefficient and group velocity of $TM_{0m}$-mode, $L$- way passed by bunch train in waveguide, $\beta = V_c / c$. The attenuation coefficient depends on losses in waveguide according to:

$$\alpha_m = \frac{\omega_m}{2 Q_{gm} \nu_{gm}}, \quad \frac{1}{Q_{gm}} = \frac{1}{Q_{gm}^{\text{dil}}} + \frac{1}{Q_{gm}^{\text{wall}}}.$$ 

where $Q_{gm}^{\text{dil}}$, $Q_{gm}^{\text{wall}}$ quality factors caused by losses for $TM_{0m}$-mode in dielectric volume and metallic wall respectively. The group velocities significantly limit the length of wave pocket:
$V_{gm} = \frac{d\omega}{dk} = \frac{P_{m}}{U_{m}}$

where $P_{m}$, $U_{m}$ - power flow through cross section, energy stored per length of waveguide for $TM_{0m}$ mode. The wakefield (resulted THz radiation) is calculated by a convolution of the Green function with the charge density of the bunch train:

$$E_{\zeta} = \int G_{E_{\zeta}}(s - \zeta) p(s) ds.$$  

3. Calculation of THz radiation

We performed a calculation for a DLW with parameters from table 1 in the updated BBU-3000 code [5,6] with a multibunch module. BBU-3000 is used for calculation of wakefield and beam dynamics for different geometries of DLW based on the analytical form of the Green function, figure 2.

![Figure 2. Capabilities of BBU-3000 code.](image)

| Table 1. Parameters of waveguide and bunch |
| Parameter                  | Value  |
|---------------------------|--------|
| Inner dielectric radius (um) | 600    |
| Outer dielectric radius (um) | 850    |
| Length (cm)               | 4      |
| Epsilon                   | 3.8    |
| Loss tangent of dielectric| 0.001  |
| Conductivity of wall (S/m) | $5.7 \cdot 10^7$ |
| Charge (nC)               | 1      |
| $\sigma_2$ (um)           | 100    |

For the THz band we chose a dielectric waveguide with a small aperture and thin dielectric layer (250 um), because the frequency intervals in the discrete spectrum of Green functions grow as the thickness of the dielectric layer is reduced, so that it is possible to adjust frequencies over a large band. The parameters of the TM modes are presented in table 2. The main purpose of using a bunch train is the selection of only one mode with the desired frequency without exciting any of the other modes. The most interesting case is the possibility to generate radiation consisting only of the $TM_{03}$ mode.
Table 2. Properties of TM modes

| Mode   | frequency, GHz | Vg/c | Attenuation coefficient (1/cm) |
|--------|----------------|------|--------------------------------|
| TM$_{01}$ | 149.5          | 0.351 | 0.0466                         |
| TM$_{02}$ | 439.3          | 0.553 | 0.059                          |
| TM$_{03}$ | 768.0          | 0.616 | 0.0813                         |

The radiation is characterized by four main parameters: frequency, amplitude, monochromaticity and number of periods. Monochromaticity is defined as the ratio of the amplitude of the selected mode to the sum of the amplitudes of all radiated modes. In figure 3 the calculated results are shown for radiation with the frequency of TM$_{03}$ mode. From figure 3 (a) we can see that amplitude spectrum distributions of the bunch train and the Green function are covered in frequency by this mode. Amplitude spectrum distribution of result radiation contain principally TM$_{03}$ mode, figure 3 (b). It is possible only for a unique distance between bunches: 0.0765 cm. As we can see from figure 3 (c) the number of wavelengths of result radiation is limited by influence of the group velocities. The high
value of group velocity means to the decrease of wave pocket for each mode. Therefore, the length of the result radiation is defined by the maximal value of the group velocity (0.616c for $TM_{03}$ mode).

The figure 4 show dependence of radiation parameters of $TM_{01}, TM_{02}, TM_{03}$ modes as a function of number of bunches in train. It is shown that there is possibility to generate monochromatic radiations based on $TM_{01}, TM_{02}$ and $TM_{03}$ modes by adjusting distances between the bunches. The amplitude of wakefield depend on number of bunches only for $TM_{01}$ mode, figure 4 (a). Monochromaticity, in particular, does not depend on bunch number in train for $TM_{01}$ and $TM_{03}$ modes, figure 4 (b). It is possible to excite $TM_{01}, TM_{02}$ modes with different distances between the bunches, but the excitation of $TM_{03}$ mode is possible only with train consisting 5, 7 and 8 bunches with single value of distance between bunches 0.0765 cm.

4. Conclusion
In conclusion, we have numerically modeled narrowband THz generation by a bunch train in a multimode dielectric loaded structure. We showed that for a bunch train for 8 bunches passing through the selected dielectric lined waveguide the length of the wave envelope at 768 GHz does not exceed 1 cm for the bunch separation of 765 µm. This limitation is defined by the group velocity value for that particular mode. It was also demonstrated that the THz radiation monochromaticity and field amplitude are reduced for higher frequencies. The radiation monochromaticity is in the range of 0.55 caused by the weak lower $TM_{01}$ and $TM_{02}$ mode generation, figure 4 (b).

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