Comment on $V_{ub}$ from Exclusive Semileptonic $B$ and $D$ Decays

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Abstract

The prospects for determining $|V_{ub}|$ from exclusive $B$ semileptonic decay are discussed. The double ratio of form factors $\left(\frac{f(B\to\rho)}{f(B\to K^*)}\right)/\left(\frac{f(D\to\rho)}{f(D\to K^*)}\right)$ is calculated using chiral perturbation theory. Its deviation from unity due to contributions that are non-analytic in the symmetry breaking parameters is very small. Combining experimental data obtainable from $B\to\rho\ell\bar{\nu}_\ell$, $B\to K^*\ell\bar{\nu}_\ell$ and $D\to\rho\bar{\nu}_\ell\nu_\ell$ can lead to a model independent determination of $|V_{ub}|$ with an uncertainty from theory of about 10%.
The next generation of $B$ decay experiments will test the flavor sector of the standard model at high precision. The basic approach is to determine the elements of the CKM matrix using different methods and then check for the consistency of these results. At the present time $CP$ non-conservation has only been observed in kaon decay arising from $K^0 - \bar{K}^0$ mixing. Many extensions of the minimal standard model (e.g., models with several Higgs doublets or low energy supersymmetry) have new particles with weak scale masses that contribute to flavor changing neutral current processes like $K^0 - \bar{K}^0$ mixing, $B^0 - \bar{B}^0$ mixing, $B \to K^{\ast}\gamma$, etc., at a level comparable to the standard model.

At the present time, the magnitude of the $b \to u$ CKM matrix element is determined by comparing experimental results on the inclusive electron spectrum in the endpoint region with phenomenological models \[1\], or by comparing experimental results on $B \to \rho \ell \bar{\nu}_\ell$ and $B \to \pi \ell \bar{\nu}_\ell$ with phenomenological models and lattice QCD results \[2\]. These two approaches yield remarkably consistent determinations of $|V_{ub}|$, but have large uncertainties.

This paper is a comment on the proposal to determine $|V_{ub}|$ \[3,4\] using a combination of heavy quark symmetry \[5\] and $SU(3)$ flavor symmetry. The basic idea is to compare $D \to K^{\ast}\ell\bar{\nu}_\ell$ with the Cabibbo suppressed decay $D \to \rho \ell \bar{\nu}_\ell$. Using heavy quark symmetry the $SU(3)$ violations in the form factors that occur in these decays are related to those that occur in a comparison of $B \to K^{\ast}\ell\bar{\nu}_\ell$ (or $B \to K^{\ast}\nu_\ell \bar{\nu}_\ell$) with $B \to \rho \ell \bar{\nu}_\ell$. Therefore, experimental data on $B \to K^{\ast}\ell\bar{\nu}_\ell$ in conjunction with data on $D \to \rho \ell \nu_\ell$ and $D \to K^{\ast}\ell \nu_\ell$ can be used to determine $|V_{ub}|$. This proposal is complementary to other approaches for determining $|V_{ub}|$, since it relies on the standard model correctly describing the rare flavor changing neutral current process $B \to K^{\ast}\ell\bar{\nu}_\ell$.

In this letter we compute corrections to these form factor relations which violate both chiral and heavy quark symmetry, and are non-analytic in the symmetry breaking parameters. We also reconsider the influence of long distance effects on the extraction of the $B \to K^{\ast}$ form factors from $B \to K^{\ast}\ell\bar{\nu}_\ell$.

We denote the form factors relevant for semileptonic transitions between a pseudoscalar meson $P^{(Q)}$, containing a heavy quark $Q$, and a member of the lowest lying multiplet of
vector mesons, \( V \), by \( g^{(H \rightarrow V)} \), \( f^{(H \rightarrow V)} \) and \( a^{(H \rightarrow V)}_{\pm} \), where

\[
\langle V(p', \epsilon) | \bar{q} \gamma_{\mu} Q | H(p) \rangle = i g^{(H \rightarrow V)} \varepsilon_{\mu\nu\lambda\sigma} \epsilon^{*\nu} (p + p')^\lambda (p - p')^\sigma,  
\]

and \( \epsilon^{0123} = -\varepsilon_{0123} = 1 \). We view the form factors \( g \), \( f \) and \( a \) as functions of the dimensionless variable \( y = v \cdot v' \), where \( p = m_H v \), \( p' = m_V v' \), and \( q^2 = (p - p')^2 = m_H^2 + m_V^2 - 2m_H m_V y \).

The experimental values for the \( D \rightarrow K^* \bar{\nu}_\ell \nu_\ell \) form factors assuming nearest pole dominance for the \( q^2 \) dependences are [6]

\[
\begin{align*}
    f^{(D \rightarrow K^*)}(y) &= (1.9 \pm 0.1) \text{ GeV} \\
    a^{(D \rightarrow K^*)}_+(y) &= -\frac{(0.18 \pm 0.03) \text{ GeV}^{-1}}{1 + 0.63 (y - 1)}, \\
    g^{(D \rightarrow K^*)}(y) &= -\frac{(0.49 \pm 0.04) \text{ GeV}^{-1}}{1 + 0.96 (y - 1)}. 
\end{align*} 
\]

The shapes of these form factors are beginning to be probed experimentally [3]. The form factor \( a_- \) is not measured because its contribution to the \( D \rightarrow K^* \bar{\nu}_\ell \nu_\ell \) decay amplitude is suppressed by the lepton mass. The minimal value of \( y \) is unity (corresponding to the zero recoil point) and the maximum value of \( y \) is \( (m_D^2 + m_{K^*}^2)/(2m_D m_{K^*}) \approx 1.3 \) (corresponding to \( q^2 = 0 \)). Note that \( f(y) \) changes by less than 20% over the whole kinematic range \( 1 < y < 1.3 \). In the following analysis we will extrapolate the measured form factors to the larger region \( 1 < y < 1.5 \). The full kinematic region for \( B \rightarrow \rho \bar{\nu}_\ell \nu_\ell \) is \( 1 < y < 3.5 \).

The differential decay rate for semileptonic \( B \) decay (neglecting the lepton mass, and not summing over the lepton type \( \ell \)) is

\[
\frac{d\Gamma(B \rightarrow \rho \bar{\nu}_\ell \nu_\ell)}{dy} = \frac{G_F^2 |V_{ub}|^2}{48 \pi^3} m_B m_\rho^2 S^{(B \rightarrow \rho)}(y). 
\]

Here \( S^{(H \rightarrow V)}(y) \) is the function
\[ S^{(H \to V)}(y) = \sqrt{y^2 - 1} \left[ |f^{(H \to V)}(y)|^2 (2 + y^2 - 6yr + 3r^2) + 4\text{Re} \left[ a_+^{(H \to V)}(y) f^{(H \to V)}(y) \right] m_H^2 r (y - r)(y^2 - 1) + 4|a_+^{(H \to V)}(y)|^2 m_H^4 r^2 (y^2 - 1)^2 + 8|g^{(H \to V)}(y)|^2 m_H^4 r^2 (1 + r^2 - 2yr)(y^2 - 1) \right] \]
\[ = \sqrt{y^2 - 1} \left[ |f^{(H \to V)}(y)|^2 (2 + y^2 - 6yr + 3r^2) [1 + \delta^{(H \to V)}(y)] \right], \quad (4) \]

with \( r = m_V/m_H \). The function \( \delta^{(H \to V)} \) depends on the ratios of form factors \( a_+^{(H \to V)}/f^{(H \to V)} \) and \( g^{(H \to V)}/f^{(H \to V)} \). \( S^{(B \to \rho)}(y) \) can be estimated using combinations of \( SU(3) \) flavor symmetry and heavy quark symmetry. \( SU(3) \) symmetry implies that the \( \bar{B}^0 \to \rho^+ \) form factors are equal to the \( B \to K^* \) form factors and the \( B^- \to \rho^0 \) form factors are equal to \( 1/\sqrt{2} \) times the \( B \to K^* \) form factors. Heavy quark symmetry implies the relations \([3]\)

\[ f^{(B \to K^*)}(y) = \left( \frac{m_B}{m_D} \right)^{1/2} \left( \frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right)^{-6/25} f^{(D \to K^*)}(y), \]
\[ a_+^{(B \to K^*)}(y) = \left( \frac{m_D}{m_B} \right)^{1/2} \left( \frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right)^{-6/25} a_+^{(D \to K^*)}(y), \]
\[ g^{(B \to K^*)}(y) = \left( \frac{m_D}{m_B} \right)^{1/2} \left( \frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right)^{-6/25} g^{(D \to K^*)}(y). \quad (5) \]

The second relation is obtained using \( a_-^{(D \to K^*)} = -a_+^{(D \to K^*)} \), valid in the large \( m_c \) limit.

Using Eq. (3) and \( SU(3) \) to get \( \bar{B}^0 \to \rho^+ \ell \bar{\nu}_\ell \) form factors (in the region \( 1 < y < 1.5 \)) from those for \( D \to K^* \bar{\ell} \nu_\ell \) given in Eq. (2) yields \( S^{(B \to \rho)}(y) \) plotted in Fig. 1 of Ref. [4]. The numerical values in Eq. (2) differ slightly from those used in Ref. [4]. This makes only a small difference in \( S^{(B \to \rho)} \), but changes \( \delta^{(B \to \rho)} \) more significantly. In the region \( 1 < y < 1.5 \), \( |\delta^{(B \to \rho)}(y)| \) defined in Eq. (4) is less than 0.06, indicating that \( a_+^{(B \to \rho)} \) and \( g^{(B \to \rho)} \) make a small contribution to the differential rate in this region.

This prediction for \( S^{(B \to \rho)} \) can be used to determine \( |V_{ub}| \) from a measurement of the \( B \to \rho \ell \bar{\nu}_\ell \) semileptonic decay rate in the region \( 1 < y < 1.5 \). This method is model independent, but cannot be expected to yield a very accurate value of \( |V_{ub}| \). Typical \( SU(3) \) violations are at the 10 – 20% level and one expects similar violations of heavy quark symmetry.

Ref. [4] proposed a method for getting a value of \( S^{(B \to \rho)}(y) \) with small theoretical uncertainty. They noted that the “Grinstein-type” [4] double ratio
$$R(y) = \left[ f^{(B \to \rho)}(y) / f^{(B \to K^*)}(y) \right] / \left[ f^{(D \to \rho)}(y) / f^{(D \to K^*)}(y) \right]$$

is unity in the limit of SU(3) symmetry or in the limit of heavy quark symmetry. Corrections to the prediction $R(y) = 1$ are suppressed by $m_s/m_{c,b}$ ($m_{u,d} \ll m_s$) instead of $m_s/\Lambda_{QCD}$ or $\Lambda_{QCD}/m_{c,b}$. Since $R(y)$ is very close to unity, the relation

$$S^{(B \to \rho)}(y) = S^{(B \to K^*)}(y) \left| \frac{f^{(D \to \rho)}(y)}{f^{(D \to K^*)}(y)} \right|^2 \left( \frac{m_B - m_\rho}{m_B - m_{K^*}} \right)^2,$$

(7)

together with measurements of $|f^{(D \to K^*)}|$, $|f^{(D \to \rho)}|$, and $S^{(B \to K^*)}$ will determine $S^{(B \to \rho)}$ with small theoretical uncertainty. The last term on the right-hand-side makes Eq. (6) equivalent to Eq. (3) in the $y \to 1$ limit. The ratio of the $(2 + y^2 - 6y + 3r^2) [1 + \delta^{(B \to V)}(y)]$ terms makes only a small and almost $y$-independent contribution to $S^{(B \to \rho)}/S^{(B \to K^*)}$ in the range $1 < y < 1.5$. Therefore, corrections to Eq. (6) are at most a few percent larger than those to $R(y) = 1$.

$|f^{(D \to K^*)}|$ has already been determined. $|f^{(D \to \rho)}|$ may be obtainable in the future, for example from experiments at $B$ factories, where improvements in particle identification help reduce the background from the Cabibbo allowed decay. The measurement $B(D \to \rho^0 \ell \nu_\ell)/B(D \to \bar{K}^{*0} \bar{\ell} \nu_\ell) = 0.047 \pm 0.013$ already suggests that $|f^{(D \to \rho)}/f^{(D \to K^*)}|$ is close to unity. Assuming SU(3) symmetry for the form factors, but keeping the explicit $m_V$-dependence in $S^{(D \to V)}(y)$ and in the limits of the $y$ integration, the measured form factors in Eq. (2) imply $B(D \to \rho^0 \ell \nu_\ell)/B(D \to \bar{K}^{*0} \bar{\ell} \nu_\ell) = 0.044$. $S^{(B \to K^*)}$ is obtainable from experimental data on $B \to K^* \nu_\ell \bar{\nu}_\ell$ or $B \to K^* \ell \bar{\ell}$. While the former process is very clean theoretically, it is very difficult experimentally. A more realistic goal is to use $B \to K^* \ell \bar{\ell}$, since CDF expects to observe $400 - 1100$ events in the Tevatron run II (if the branching ratio is in the standard model range) \[3\]. There are some uncertainties associated with long distance nonperturbative strong interaction physics in this extraction of $S^{(B \to K^*)}(y)$. To use

1This prediction would be $|V_{cd}/V_{cs}|^2/2 \simeq 0.026$ with $m_\rho = m_{K^*}$. Phase space enhances $D \to \rho$ compared to $D \to K^*$ to yield the quoted prediction.
the kinematic region $1 < y < 1.5$, the form factor ratio $f^{(D\rightarrow \rho)}/f^{(D\rightarrow K^*)}$ in Eq. (7) must be extrapolated to a greater region than what can be probed experimentally. For this ratio, the uncertainty related to this extrapolation is likely to be small.

The main purpose of this comment is to examine the deviation of $R$ from unity using chiral perturbation theory. We find that it is at the few percent level. The uncertainty from long distance physics in the extraction of $S^{(B\rightarrow K^*)}$ is also reviewed. On average, in the region $1 < y < 1.5$, this is probably less than a 10% effect on the $B \rightarrow K^*\ell\bar{\ell}$ decay rate. Consequently, a determination of $|V_{ub}|$ from experimental data on $D \rightarrow K^*\ell\bar{\nu}_\ell$, $D \rightarrow \rho\ell\bar{\nu}_\ell$, $B \rightarrow K^*\ell\bar{\ell}$ and $B \rightarrow \rho\ell\bar{\nu}_\ell$ with an uncertainty from theory of about 10% is feasible.

(i) Chiral perturbation theory for $R$

The leading deviation of $R$ from unity can be calculated using a combination of heavy hadron chiral perturbation theory for the mesons containing a heavy quark and for the lowest lying vector mesons. We adopt the notations and conventions of Refs. [10,11]. The weak current transforms as $(\bar{3}_L, 1_R)$, and at the zero recoil kinematic point there are two operators that are relevant for $P^{(Q)} \rightarrow V$ transition matrix elements (where $P^{(b)} = B$, $P^{(c)} = D$, and $V$ is one of the lowest lying vector mesons $\rho, \omega, K^*, \phi$). Demanding that the Zweig suppressed $D_s \rightarrow \omega\ell\bar{\nu}_\ell$ process vanishes relates the two operators, yielding [12]

$$\bar{q}_a \gamma_\mu (1 - \gamma_5) Q = \beta \text{Tr}[N^t_{cb} \gamma_\mu (1 - \gamma_5) H_e^{(Q)} \xi^t_{ba}].$$

(8)

Here repeated $SU(3)$ indices are summed and the trace is over Lorentz indices. $H^{(Q)}$ contains the ground state heavy meson doublet, $N$ is the nonet vector meson matrix [11], and $\beta$ is a constant. The leading contribution to $R(1) - 1$ arises from the Feynman diagrams in Fig. 1. Diagrams with a virtual kaon cancel in the double ratio $R$. Neglecting the vector meson widths, these diagrams yield

\[2\]The only significant width is that of the $\rho$ meson. Since it occurs in the loop graph involving an $\eta$, neglecting the $\rho$ width amounts to treating $\Gamma_\rho/2m_\eta \ll 1$, which is a reasonable approximation.
FIG. 1. Feynman diagram that gives the leading contribution to $R(1) - 1$. The dashed line is a $\pi$ or an $\eta$. The black square indicates insertion of the weak current.

$$R(1) - 1 = -\frac{g g_2}{12 \pi^2} f^2 \left[G(m_{\pi}, \Delta^{(b)}) - G(m_{\eta}, \Delta^{(b)}) - G(m_{\pi}, \Delta^{(c)}) + G(m_{\eta}, \Delta^{(c)})\right],$$  \hspace{1cm} (9)

where $\Delta^{(b)} = m_{B^*} - m_B$, $\Delta^{(c)} = m_{D^*} - m_D$, and for $m \geq \Delta$,

$$G(m, \Delta) = \frac{\pi m^3}{2 \Delta} - \frac{(m^2 - \Delta^2)^{3/2}}{\Delta} \arctan \left(\frac{\sqrt{m^2 - \Delta^2}}{\Delta}\right) - \Delta^2 \ln m. \hspace{1cm} (10)$$

Here $g_2$ is the $\rho \omega \pi$ coupling, $g$ is the $DD^*\pi$ coupling, and $f \simeq 131$ MeV is the pion decay constant. In the nonrelativistic constituent quark model $g = g_2 = 1 \ [10]$, while in the chiral quark model $[13] g = g_2 = 0.75$. Experimental data on $\tau \rightarrow \omega \pi \nu_{\tau}$ in the region of low $\omega \pi$ invariant mass gives $g_2 \simeq 0.6 \ [14]$.

For small $\Delta$, Eq. (9) for $R(1) - 1$ has a non-analytic $\sqrt{m_{\eta}}$ dependence on the light quark masses. This cannot arise from corrections to the current in Eq. (8) or to the chiral Lagrangian, and must come from 1-loop diagrams involving the pseudo-Goldstone bosons $\pi$, $K$, $\eta$. Using the measured values of the pion and eta masses, Eqs. (9) and (10) imply $R(1) = 1 - 0.035 g g_2$. There may be significant corrections from higher orders in chiral perturbation theory. However, the smallness of our result lends support to the expectation that $R(1) - 1$ is very close to zero. There is no reason to expect any different conclusion over the kinematic range $1 < y < 1.5$.

(ii) Long distance effects and the extraction of $S^{(B \rightarrow K^*)}$ from $B \rightarrow K^* \ell \bar{\ell}$

The decay rate for $B \rightarrow K^* \nu_\ell \bar{\nu}_\ell$ could determine $S^{(B \rightarrow K^*)}$ free of theoretical uncertainties. However, experimental study of this decay is very challenging. A more practical approach to extracting this quantity is to use $B \rightarrow K^* \ell \bar{\ell}$. The differential decay rate is

$$\frac{d\Gamma(B \rightarrow K^* \ell \bar{\ell})}{dy} = \frac{G_F^2 |V_{ub}^* V_{tb}|^2}{24 \pi^3} \left(\frac{\alpha}{4\pi}\right)^2 m_B m_{K^*}^2 \left[|\tilde{C}_9(y)|^2 + |C_{10}|^2\right] [1 + \Delta(y)]$$

$$\times S^{(B \rightarrow K^*)}(y) [1 + d(y)], \hspace{1cm} (11)$$
This and Eq. (7) allow us to rewrite Eq. (3) as

$$\frac{dΓ(B → ρ ℓνℓ)}{dy} = \frac{|V_{ub}|^2 \cdot 8\pi^2}{|V_{ts}V_{tb}|^2} \frac{1}{\alpha^2} \frac{1}{|C_9(y)|^2 + |C_{10}|^2} \frac{1}{1 + Δ(y)} \frac{1}{1 + d(y)}$$

$$\times \frac{m_o^2}{m_{K^*}^2} \left( \frac{m_B - m_ρ}{m_B - m_{K^*}} \right)^2 \left| \frac{f^{(D→ρ)}(y)}{f^{(D→K^*)}(y)} \right|^2 \frac{dΓ(B → K^*ℓνℓ)}{dy},$$

(12)

which can be directly used to extract $|V_{ub}|$. Unitarity of the CKM matrix implies that $|V_{ts}V_{tb}| ≃ |V_{cs}V_{cb}|$ with less than a 3% uncertainty. The fine structure constant, $\alpha = 1/129$, is evaluated at the $W$-boson mass. $d(y)$ parameterizes long distance effects, and will be discussed below. $Δ(y)$ takes into account the contribution of the magnetic moment operator, $O_7 = (eq/16\pi^2) m_b (s_L σ_{μν} b_R) F^{μν}$ (a factor of $-4G_F V_{ts}V_{tb}/\sqrt{2}$ has been extracted out in the definition of operator coefficients). Ref. [9] (see also Ref. [13]) found using heavy quark symmetry that $Δ(y) ≃ -0.14 - 0.08(y - 1)$ in the region $1 < y < 1.5$. Corrections to this are expected to be small since there are no $1/m_c$ corrections to $Δ(1)$. $C_{10}$ is the Wilson coefficient of the operator $O_{10} = (eq/16\pi^2) (s_L γ_μ b_L)(\bar{ℓ} γ^μ γ_5 ℓ)$. $C_9(y)$ takes into account the contribution of the four-quark operators, $O_1 - O_6$, and the operator $O_9 = (eq/16\pi^2) (s_L γ_μ b_L)(\bar{ℓ} γ^μ ℓ)$. In perturbation theory using the next-to-leading logarithmic approximation [16]

$$C_9(y) = C_9 + h(z, y) (3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) - \frac{1}{2} h(0, y) (C_3 + 3C_4)$$

$$- \frac{1}{2} h(1, y) (4C_3 + 4C_4 + 3C_5 + C_6) + \frac{2}{9} (3C_3 + C_4 + 3C_5 + C_6),$$

(13)

where $z = m_c/m_b$. Here

$$h(u, y) = -\frac{8}{9} \ln u + \frac{8}{27} + \frac{4}{9} x - \frac{2}{9} (2 + x) \sqrt{1 - x} \left\{ \ln \frac{1 + \sqrt{1 - x}}{1 - \sqrt{1 - x} - iπ}; \quad x < 1 \right\}$$

$$\times 2\text{arctan}(1/\sqrt{x - 1}); \quad x > 1,$$

(14)

where $x ≡ 4u^2m_o^2/(m_B^2 + m_{K^*}^2 - 2m_B m_{K^*} y)$. Using $m_t = 175 \text{GeV}$, $m_b = 4.8 \text{GeV}$, $m_c = 1.4 \text{GeV}$, $α_s(m_W) = 0.12$, and $α_s(m_b) = 0.22$, the numerical values of the Wilson coefficients are $C_1 = -0.26$, $C_2 = 1.11$, $C_3 = 0.01$, $C_4 = -0.03$, $C_5 = 0.008$, $C_6 = -0.03$, $C_7 = -0.32$, $C_9 = 4.26$, and $C_{10} = -4.62$. Of these, $C_9$ and $C_{10}$ are sensitive to $m_t$ (quadratically for $m_t \gg m_W$).
In Eq. (13) the second term on the right-hand-side, proportional to \( h(z, y) \) comes from charm quark loops. Since the kinematic region we are interested in is close to \( q^2 = 4m_c^2 \), a perturbative calculation of the \( c \bar{c} \) loop cannot be trusted. Threshold effects which spoil local duality are important. It is these long distance effects that give rise to the major theoretical uncertainty in the extraction of \(|V_{ub}|\) from the \( B \to K^* \ell \bar{\ell} \) differential decay rate using Eq. (12). The influence of this long distance physics on the differential decay rate is parameterized by \( d(y) \) in Eq. (11), where setting \( d(y) = 0 \) gives the perturbative result.

For the part of the \( c \bar{c} \) loop where the charm quarks are not far off-shell, a model for \( h(z, y) \) which sums over \( 1^{--} \) resonances is more appropriate than the perturbative calculation. Consequently, we model the part of \( h(z, y) \) with explicit \( q^2 \)-dependence in Eq. (14) with a sum over resonances \[17\] calculated using factorization

\[
h(z, y) \to -\frac{8}{9} \ln z + \frac{8}{27} - \frac{3\pi \kappa}{\alpha^2} \sum_n \frac{\Gamma_{\psi(n)} B(\psi(n) \to \ell \bar{\ell})}{(q^2 - M_{\psi(n)}^2)/M_{\psi(n)} + i\Gamma_{\psi(n)}}.
\]

The resonances \( \psi^{(n)} \) have masses 3.097 GeV, 3.686 GeV, 3.770 GeV, 4.040 GeV, 4.160 GeV, and 4.415 GeV, respectively, and their widths \( \Gamma_{\psi(n)} \) and leptonic branching ratios \( B(\psi^{(n)} \to \ell \bar{\ell}) \) are known \[18\]. The factor \( \kappa = 2.3 \) takes into account the deviation of the factorization model \[19\] parameter \( a_2 \) from its perturbative value. Denoting the value of \( \tilde{C}_9(y) \) in this model by \( \tilde{C}_9'(y) \), its influence on the differential decay rate is given by \( d(y) \) defined as

\[
|\tilde{C}_9'(y)|^2 + |C_{10}|^2 = (|\tilde{C}_9(y)|^2 + |C_{10}|^2) [1 + d(y)].
\]

\( d(y) \) is plotted in Fig. 2 (solid curve). The physical interpretation of the \( 1^{--} \) resonances above 4 GeV is not completely clear. It might be more appropriate to treat them as \( D \bar{D} \) resonances than as \( c \bar{c} \) states. It is possible that for these resonances factorization as modeled by Eq. (15) with \( \kappa = 2.3 \) is not a good approximation. Including only the first three \( 1^{--} \) resonances in Eq. (14), yields \( d(y) \) plotted with the dashed curve in Fig. 2.

\[3\]The four-quark operators involving light \( u, d, \) and \( s \) quarks also have uncertainty from long distance physics. However, this is expected to have a very small effect on the \( B \to K^* \ell \bar{\ell} \) rate.
FIG. 2. $d(y)$ defined in Eq. (16). The solid curve takes into account all six $1^- c\bar{c}$ resonances according to Eq. (15), whereas the dashed curve is obtained including only the three lightest ones.

This estimate of $d(y)$ based on factorization and resonance saturation differs from that in Ref. [4] in two respects. Firstly, the phase of $\kappa$ is viewed as fixed because recent data has determined the sign of the ratio of factorization model parameters, $a_2/a_1$, and the phase of $a_1$ is expected to be near its perturbative value [20]. Secondly, since the resonance saturation model only represents the $c\bar{c}$ loop for charm quarks that are not far off-shell, we have only used it for the part of $h(z,y)$ in Eq. (14) with explicit $q^2$ dependence, retaining the perturbative expression for the first two terms, $-(8/9)\ln z + 8/27$. The $\ln z$ term has dependence on $m_b$, which is clearly short distance in origin. This reduces somewhat the magnitude of $d(y)$ and makes it more symmetric about zero (compare Fig. 2 with Fig. 6 of Ref. [4]). It would be interesting to have a more physical separation between the long and short distance parts of the amplitude.

Whether it is reasonable to use factorization for the resonances above 4 GeV can be tested experimentally, since these states cause a very distinctive pattern in $d\Gamma/dy$. In Fig. 3 the shape of $d\Gamma/dy$ is plotted in the region $1 < y < 1.5$ using the resonance saturation model for $d(y)$ (solid curve). Experimental support for this shape would provide evidence that this model correctly describes the long distance physics parameterized by $d(y)$. Although
FIG. 3. $d\Gamma(B \rightarrow K^*\ell\bar{\ell})/dy$ in units of $\left[|C_9(1)|^2 + |C_{10}|^2\right]m_Bm_{K^*}G_F^2|V_{ts}V_{tb}|^2\alpha^2/(384\pi^5)$ as given in Eq. (11). The solid curve takes into account all six $1^{--}$ $c\bar{c}$ resonances, the dashed curve includes only the three lightest ones, and the dotted curve is the perturbative result (i.e., $d(y) = 0$).

$d(y)$ gets as large as $\pm 0.2$, since it oscillates, its influence on the $B \rightarrow K^*\ell\bar{\ell}$ decay rate in the region $1 < y < 1.5$ is about $-8\%$ compared to the perturbative result (which is plotted with the dotted curve in Fig. 3). Even if our estimates of this long distance physics based on factorization and resonance saturation has a $100\%$ uncertainty (a prospect that we do not consider particularly unlikely), it will only cause about a $4\%$ uncertainty in this determination of $|V_{ub}|$. Including only the first three $1^{--}$ resonances in the sum in Eq. (15) yields the dashed curve in Fig. 3. In this case $d(y)$ causes a $-13\%$ change in the $B \rightarrow K^*\ell\bar{\ell}$ decay rate in the region $1 < y < 1.5$.

(iii) nearer term prospects

Without information on the $y$ spectrum for the $B$ decay rates in Eq. (12), it is still possible to determine $|V_{ub}|$ by comparing the branching ratios for $B \rightarrow \rho \ell \bar{\nu}_\ell$ and $B \rightarrow K^*\ell\bar{\ell}$ in the region $1 < y < 1.5$. Integrating Eq. (12) over $1 < y < 1.5$ we can write
\[
\Gamma(B^0 \to \rho^+ \ell \bar{\nu}_\ell) \bigg|_{y<1.5} = \left| \frac{|V_{ub}|^2}{|V_{ts}^* V_{tb}|^2} \right|^2 \frac{8\pi^2}{\alpha^2} \frac{1}{C_9^2 + C_{10}^2} \left[ \frac{1}{(1 + \Delta)} - \frac{1}{(1 + d)} \right] \times \frac{m_{\rho}^2}{m_{K^*}^2} \left( \frac{m_B - m_{\rho}}{m_B - m_{K^*}} \right)^2 \left| \frac{f(D \to \rho)}{f(D \to K^*)} \right|^2 \left| \frac{f(D \to \rho)}{f(D \to K^*)} \right| \right|_{y<1.5}.
\]

Here the barred quantities, \( \bar{C}_9 \), \( \Delta \), and \( \bar{d} \) denote the averages of \( |\bar{C}_9(y)|^2 \), \( \Delta(y) \), and \( d(y) \) weighted with \( S(y) \). Using the shape for \( S(y) \) predicted from heavy quark symmetry, we find \( \bar{C}_9 = 4.58 \), \( \Delta = -0.16 \), and \( \bar{d} = -0.08 \). Note that the \( y \)-dependence of \( \bar{C}_9 \) is small and \( \bar{C}_9 \) is close to \( C_9 \). In Eq. (17) the \( y \)-dependence of the ratio \( f(D \to \rho)/f(D \to K^*) \) has been neglected. If the shape of these form factors can be approximated with a pole form, then the pole masses of 2.56 GeV for \( f(D \to K^*) \) and 2.45 GeV for \( f(D \to \rho) \) (corresponding to \( D_s^* \) and to \( D^* \)) imply that \( |f(D \to \rho)/f(D \to K^*)|^2 \) varies by less than 1% over the range \( 1 < y < 1.5 \). \( SU(3) \) symmetry and the measured \( D \to K^* \) form factors imply that \( \delta(D \to \rho) \) contributes only about 23% of the \( D \to \rho \ell \bar{\nu}_\ell \) decay rate. Using this prediction for \( \delta(D \to \rho) \), and assuming that \( f(D \to \rho) \) and \( f(D \to K^*) \) have the same \( y \)-dependence, yields

\[
\mathcal{B}(D \to \rho^0 \ell \bar{\nu}_\ell)/\mathcal{B}(D \to \rho \ell \bar{\nu}_\ell) = 0.044 \left| \frac{f(D \to \rho)}{f(D \to K^*)} \right|^2.
\]

In the region \( q^2 = (p_\ell + p_\ell)^2 < m_{J/\psi}^2 \) (corresponding roughly to \( y > 2 \)), one cannot use the double ratio and Eq. (12). Moreover, the \( O_7 \) contribution to the \( B \to K^* \ell \bar{\nu}_\ell \) rate is large and proportional to \( 1/q^2 \), so the (leading order) heavy quark symmetry relations between the tensor and (axial-)vector form factors introduce a significant uncertainty. For \( q^2 < m_{J/\psi}^2 \), one can do better using \( SU(3) \) flavor symmetry alone to predict \( d\Gamma(B \to \rho \ell \bar{\nu}_\ell)/dq^2 \) from a measurement of \( d\Gamma(B \to K \ell \bar{\nu}_\ell)/dq^2 \). Since this region is far from \( q_{\text{max}}^2 \), the \( B^* \) pole contribution is unlikely to upset the \( SU(3) \) relations. The \( O_7 \) contribution to \( d\Gamma(B \to K \ell \bar{\nu}_\ell)/dq^2 \) is at the 10 – 15% level, fairly independent of \( q^2 \). In the region \( (1 - 2)\text{GeV}^2 < q^2 < m_{J/\psi}^2 \), neglecting \( m_{K^*\ell}/m_B \),

\[
\frac{d\Gamma(B^0 \to \pi^+ \ell \bar{\nu}_\ell)}{dq^2} = \frac{|V_{ub}|^2}{|V_{ts}^* V_{tb}|^2} \frac{8\pi^2}{\alpha^2} \frac{1}{|C_9(q^2)| + 2C_7|q^2| + |C_{10}|^2} \frac{d\Gamma(B \to K \ell \bar{\nu}_\ell)}{dq^2}.
\]

A similar relation also holds for integrated rates.

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\[4\] It was argued in Ref. [21] that heavy quark symmetry can be used even at small \( q^2 \).
A measurement of the $B \to K^* \ell \bar{\ell}$ decay rate is unlikely before the Tevatron run II. Without this measurement, one has to rely on predicting the $B \to \rho$ form factors from $D \to \rho$ using heavy quark symmetry, or from $D \to K^*$ using both chiral and heavy quark symmetries. As discussed following Eq. (4), recent experimental data [8] suggests that the $SU(3)$ relation between $f^{(D \to K^*)}$ and $f^{(D \to \rho)}$ is not violated by more than 15%. Heavy quark symmetry and the measured $D \to K^*$ form factors in Eq. (2) imply that the $\bar{B}_0 \to \rho^+ \ell \bar{\nu}_\ell$ branching ratio in the region $1 < y < 1.5$ is $5.9 |V_{ub}|^2$. The measured decay rate $\mathcal{B}(\bar{B}_0 \to \rho^+ \ell \bar{\nu}_\ell) = (2.5 \pm 0.4^{+0.5}_{-0.7} \pm 0.5) \times 10^{-4}$ [2] together with $|V_{ub}| \sim 0.003$ imply that about 20% of $\bar{B}_0 \to \rho^+ \ell \bar{\nu}_\ell$ decays are in the range $1 < y < 1.5$.

Despite the presence of long distance effects associated with the $c \bar{c}$ resonance region, the $B \to K^* \ell \bar{\ell}$ rate can be used in Eq. (12) to determine $|V_{ub}|$ with a theoretical uncertainty that is about 10%. Experimental verification of the distinctive $y$-dependence of the differential rate associated with the $1_{-}^{−}$ resonances above 4 GeV (see Fig. 3) would reduce the theoretical uncertainty from long distance effects. A precise value of $|V_{ub}|$ may be available from other processes, e.g., the hadronic invariant mass spectrum in inclusive $\bar{B} \to X_u \ell \bar{\nu}_\ell$ decay [23] or from lattice QCD results on exclusive form factors [24] before the $B \to K^* \ell \bar{\ell}$ decay rate is measured. In that case, Eq. (12) gives an accurate standard model prediction for the $B \to K^* \ell \bar{\ell}$ decay rate in the region $1 < y < 1.5$. Comparison with data may signal new physics or provide stringent constraints on extensions of the standard model.

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