The role of rotation on Petersen Diagrams. The $\Pi_{1/0}$ ($\Omega$) period ratios

J.C. Suárez$^{1,2}$ *, R. Garrido$^1$, and M.J. Goupil$^2$

1 Instituto de Astrofísica de Andalucía (CSIC), CP3004, Granada, Spain
2 Observatoire de Paris, LESIA, UMR 8109, Meudon, France

Received ... / Accepted ...

Abstract. The present work explores the theoretical effects of rotation in calculating the period ratios of double-mode radial pulsating stars with special emphasis on high-amplitude δ Scuti stars (HADS). Diagrams showing these period ratios vs. periods of the fundamental radial mode have been employed as a good tracer of non-solar metallicities and are known as Petersen diagrams (PD). In this paper we consider the effect of moderate rotation on both evolutionary models and oscillation frequencies and we show that such effects cannot be completely neglected as it has been done until now. In particular it is found that even for low-to-moderate rotational velocities (15–50 km s$^{-1}$), differences in period ratios of some hundredths can be found. The main consequence is therefore the confusion scenario generated when trying to fit the metallicity of a given star using this diagram without a previous knowledge of its rotational velocity.

Key words. Stars: variables: δ Scut – Stars: rotation – Stars: variables: RR Lyrae – Stars: oscillations – Stars: fundamental parameters – Stars: variables: Cepheids

1. Introduction

Radial pulsators, and more particularly double mode pulsators, have been extensively studied using the well known Petersen diagrams (from now on called PD). Such diagrams show the ratio between the fundamental radial mode and the first overtone as a function of the fundamental mode. Typically, the stars concerned are double-mode Cepheids, RR Lyrae and high-amplitude δ Scuti stars (HADS). Recently, the analysis of data from large-scale projects like OGLE (Optical Gravitational Lensing Experiment Szymanski 2005, Udalski et al. 1997), NSVS (Northern Sky Variability Survey Wozniak et al. 2004), ASAS (All Sky Automated Survey Pojmanski 2002, 2003) or MACHO (Alcock et al. 2000) has permitted deeper studies of the observational properties of such double-mode pulsators.

These period ratios were firstly studied by Petersen (1973, 1978), and they have been used for decades as metallicity indicators, as a function of stellar mass and age, to test mass-luminosity and/or radius-luminosity relations. In addition, they also have been used, for instance, to determine the distance modulus to the SMC Kovács 2000.

While non-radial pulsators like β Cephei or low-amplitude δ Scuti stars (LADS) generally show moderate to fast rotational velocities ($v_{\sin i}$), the double-mode radial pulsators can be considered as slow-to-moderate rotating stars. This fact has lead to neglect systematically the effect of rotation on theoretical period-ratios, in particular F-1O ratios in PD. However, we remind the reader that such objects are generally faint enough as to make difficult the observation of their rotational velocities ($v_{\sin i}$). Figure 1 shows the location of all known double-mode HADS (from Poretti et al. 2005) in the HR diagram, compared with the location of all LADS with measured $v_{\sin i}$. As can be seen, the narrow band (in effective temperature) occupied by double-mode HADS overlaps the region where LADS are located. In luminosity, HADS occupy a broader range. Unfortunately, the number of known HADS constitutes a very poor sample, specially when compared with LADS. This means that we cannot discard, a priori, the possibility of HADS in a wider range of effective temperatures and present larger $v_{\sin i}$ measurements. In addition to this, although most of HADS present $v_{\sin i}$ $\leq$ 20 km s$^{-1}$, this represent a lower limit of their rotational velocities. When varying the angle of inclination of the star $i$, velocities up to 50 km s$^{-1}$ could be reached.

From the theoretical side, only a few works examined partially the effect of rotation on period ratios and mainly focused on very rapidly rotators. In Perez Hernandez et al. (1998), second order effects of rotation on oscillation periods were taken into account in order to discriminate between radial and non-radial modes. The authors expressed the relative period change as $\delta \Pi_{n}^{(rot)}/\Pi_{n} = Z_{n}(\Pi_{n(rot)}/\Pi_{n})^{-2}$, where $\Pi_{n}$ is the unperturbed period of the mode with radial order $n$, and $\Pi_{n(rot)}$ being the rotation period. The coefficient $Z_{n}$ depends on the radial order of the mode and on the structure of the star. Such values were

Send offprint requests to: J.C. Suárez e-mail: jcsuarez@iaa.es

* Associate researcher at institute (2), with financial support from Spanish «Consejería de Innovación, Ciencia y Empresa» from the «Junta de Andalucía» local government.
estimated interpolating from computations derived for a polytropic model with index 3. Recently, Pamyatnykh (2003) studied the behaviour of period ratios of radial modes when near degeneracy effects due to rotation are included for a typical 1.8 $M_\odot$ δ Scuti stellar model. He showed that very large and non-regular perturbations to such ratios are expected to occur. More recently, Suárez et al. (2005a) proposed a limit of validity of the perturbation theory (up to second order) in terms of rotational velocity for rotating models. Such limit is given by the behaviour of period ratios when near degeneracy is considered, which clearly complicates the naive interpretation of the PD.

In the present work, we aim at analysing the consequences of neglecting the effect of rotation on radial period ratios even for low rotational velocities. To do so, up-to-date techniques taking properly into account the rotation in the modelling (in equilibrium models and in oscillation frequencies) are used. Similarly as done in Pamyatnykh (2003) and Suárez et al. (2005a), near degeneracy is also considered.

The paper is structured as follows: A general description of the modelling, focusing on how rotation is taken into account is given in Sect. 2. Fundamental-to-first harmonic ratios in presence of rotation, $\Pi_{1/0}(\Omega)$, are introduced in Sect. 3 and a discussion of their impact on PD analysis is proposed in Sect. 4. Finally, the conclusions are given in Sect. 5.

### 2. The modelling

The evolutionary code CESAM (Morel 1997) is used, which is particularly adapted for our purposes. The numerical precision and the mesh grid (around 2000 mesh points given in the basis of B-splines) of equilibrium models have been adapted according to the oscillation computation requirements.

![Fig. 1. Absolute magnitudes $M_V$ as a function of the effective temperature of all known δ Scuti stars (empty symbols), compared with those of all double-mode HADS known up to date (filled symbols). Different symbol types represent the observed $\sin i$ ranges (taken from Rodríguez et al. 2000). The cross represents typical errors on absolute magnitudes and effective temperature for δ Scuti stars.](image1)

![Fig. 2. Typical PD ($\Pi_0$ in d) containing different tracks of 1.8 $M_\odot$ evolutionary models computed with different initial metal content [Fe/H], from solar composition (bottom, solid line) to -1.00 (top, triple dot-dashed line). Crosses represent non-rotating models. The shaded area corresponds to typical values found for Pop. I stars. The two filled star symbols represent the observed $\Pi_{1/0}$ of the double-mode high-amplitude δ Scuti stars RV Ari and SX Phe, as an illustration of (Pop. I) and (Pop. II) stars respectively. (For clarity, colours are used in the on-line version of the paper).](image2)

Following Kippenhahn & Weigert (1990), a first order effect of rotation is taken into account in equilibrium equations. In particular, the spherical symmetric contribution of the centrifugal acceleration is included by means of an effective gravity $g_{\text{eff}} = g - A_c(r)$, where $g$ represents the local gravity component, $r$ the radial distance and $A_c(r) = \frac{1}{2} r^2 \Omega^2$ the centrifugal acceleration of matter elements at a distance $r$ from the centre of the star. This spherically symmetric contribution of the rotation does not change the shape of the hydrostatic equilibrium equation. Although, the non-spheric components of the centrifugal acceleration are not considered, they are included as a perturbation in the oscillation computation. The total angular momentum of models is assumed to be globally conserved along the evolution of the star. Input physics has been adapted for intermediate mass stars.

Theoretical oscillation spectra are computed from the equilibrium models described in the previous section. For this purpose the oscillation code Filou (Tran Minh & Léon 1995; Suárez 2002) is used. This code, based on a perturbative analysis, provides adiabatic oscillations corrected for the effects of rotation up to second order (centrifugal and Coriolis forces).

Furthermore, for moderate–high rotational velocities, the effects of near degeneracy are expected to be significant (Soufi et al. 1998). Two or more modes, close in frequency, are rendered degenerate by rotation under certain conditions, corresponding to selection rules. In particular these rules select modes with the same azimuthal order $m$ and degrees $\ell$ differing by 2 (Soufi et al. 1998). If we consider two generic modes $\alpha_1 \equiv (n, \ell, m)$ and $\alpha_2 \equiv (n', \ell', m')$ under the aforementioned conditions, near degeneracy occurs for $|\sigma_{\alpha_1} - \sigma_{\alpha_2}| \leq \sigma_{\alpha_2}$, where
σ_1 and σ_2 represent the eigenfrequency associated to modes α_1 and α_2 respectively, and σ_2 represents the stellar rotational frequency. In certain cases, such effect may be dominant in the behaviour of the radial period ratios studied here (Pamyatnykh 2003; Suárez et al. 2005a). However, due to its complexity, we believe such effects should be analysed separately (Suárez et al., in prep.) and, for the sake of clarity, they have not been included in the present work.

3. The Π_1/0 (Ω) ratios

In general, low order radial period ratios can be considered as dependent of the distribution of mass (or density) throughout the star, and the thermodynamical properties of the stellar matter. In Fig. 2 a classic PD for 1.8 M_⊙ evolutionary tracks is depicted. It illustrates the well-known dependence on the metallicity of Π_1/0 = Π_1/Π_0 ratios similar as those shown for instance in Petersen (1973), Petersen & Christensen-Dalsgaard (1996) and Petersen & Christensen-Dalsgaard (1999). As can be seen, the period ratios increase when decreasing the stellar initial metal content. This property is commonly used to discriminate, in the context of radial pulsators, Pop. I from Pop. II stars. The shaded region indicates the typical Π_1/0 values found for Pop. I stars, in the range of Π_1/0 = [0.772, 0.776]. As a reference, the observed period ratios of two stars: RV Ari (Pop. I) and SX Phe (Pop. II) are also depicted (values obtained from Poretti et al. 2005, and references therein). For a given chemical composition the period ratio Π_1/0 is determined by mass and the radius (or Π_0 which is scaled by the mean density ρ, and then

Π_1/0 = Π_1/0 (M, R, Z).

In addition, period ratios can be easily written in terms of observed quantities through the so-called pulsation constant Q_n,ff = Π_n,ff / Π_0,ff. In the context of radial modes, this constant can be expressed as

Q_n,ff = Π_n,ff (g / g_⊙)^1/2 (L / L_⊙)^-1/4 (T_eff / T_eff_⊙),

allowing us to assign to each observed radial period Π_n,ff a value Q_n,ff, which can be compared with theoretical predictions.

However, it is well known that rotation modifies the structure of stars and thereby the cavity where modes propagate. It is thus plausible to consider the period ratios to be Ω-dependent. Moreover, the angle of inclination of the star is generally unknown (only rotational projected velocities (vsini) are provided by observations), and thereby it follows

Π_1/0 (Ω) = Π_1/0 (M, R, Z, Ω(i)).

From a theoretical point of view, following Soufi et al. (1998); Suárez et al. (2005b), the adiabatic oscillation eigenfrequencies ω_n,ℓ,m can be expressed in terms of a perturbative theory as:

ω_n,0,0 = ω_n,0,0 + ω_2,0,0

for radial modes (ℓ = 0, m = 0), where ω_n,0,0 represents the unperturbed frequency and ω_2,0,0 the second order correction term. Both terms implicitly include the symmetric component of the centrifugal force (mainly the departure from sphericity of the star) given by the equilibrium model. First order correcting terms ω_1,0 (Coriolis force effect) are proportional to the azimuthal order m, and therefore they are zero for radial modes. For shortness, from now, only the subscript n is kept. Figure 3 illustrates the effect of rotation on normalised frequencies corresponding to the fundamental mode ω_1,0 (top panel) and the first overtone ω_1,1 (bottom panel) for a given metallicity. As can be seen, the first overtone frequencies are visibly more affected by rotation than the fundamental ones. Without entering in details (a theoretical work analysing the behaviour of Π_1/0 (Ω) is currently in preparation) this suggests the possibility that ω_1,1 is proportional to the distribution of the density inside the star (affected by rotation) rather than simply the mean density.

Fig. 3. Fundamental radial order mode ω_0,Ω (top panel) and the first overtone ω_1,Ω (bottom panel) normalised frequencies as a function of the logarithm of the fundamental radial order period (in d). (For clarity, colours are used in the on-line version of the paper.)

\[ \text{Fig. 3. Fundamental radial order mode } \omega_{0,\Omega} \text{ (top panel) and the } \text{first overtone } \omega_{1,\Omega} \text{ (bottom panel) normalised frequencies as a function of the logarithm of the fundamental radial order period (in d).} \]
4. The $\Pi_{1/0}$ ($\Omega$) period ratios and the metallicity determinations

In order to analyse the impact of considering $\Pi_{1/0}$ ($\Omega$) rather than $\Pi_{1/0}$ period ratios on metallicity determinations, these quantities, obtained from models taking into account rotation $\Pi_{1/0}$ ($\Omega$), are compared with those $\Pi_{1/0}$. To do so, several evolutionary tracks have been computed as described in Sect. for six different metallicities: [Fe/H] = 0, −0.1, −0.2, −0.35, −0.50 and −1.00 dex; and three different initial rotational velocities $\Omega_i = 25, 50$ and 100 km s$^{-1}$. The case of rotational velocities between 50 and 100 km s$^{-1}$ is purely illustrative, since there are no HADS known in that range. The discussion is thus mainly focused on models with rotational velocities up to 50 km s$^{-1}$.

During the evolution, the rotational velocity of models decrease up to 0.75$\Omega_i$ at TAMS, due to the global conservation of the total angular momentum (see Sect. for more details). The mass of models is fixed to 1.8, $M_\odot$ which typically corresponds to a $\delta$ Scuti star.

Adiabatic oscillations are then computed from these rotating models, obtaining thus the corresponding $\Pi_{1/0}$ ($\Omega$) period ratios. In Fig. 4, such rotational PD (hereafter RPD) are displayed, from top to bottom, for tracks computed from $\Omega_i = 25$ to 100 km s$^{-1}$ respectively. It can be noticed that $\Pi_{1/0}$ ($\Omega$) period ratios increase for increasing rotational velocities (left panels). Such effect is similar to decreasing the metallicity in classic PD. As explained in the previous section, this can be understood in terms of variations of the density distribution in stellar model interiors due to rotation effects (Suárez et al., work in preparation). In addition, the shift to larger period ratios is dependent on the metallicity. In particular, the higher is the rotational velocity the closer are the models of different metallicity in RPD (left panels). This means that the effect of rotation on
period ratios is systematically larger for increasing metallicity values (up to the solar value, in the present study).

In Fig. 5, $\Pi/\Omega$ (right panels) are displayed together with classic ones $\Pi/\Omega$. A first quantitative comparison is performed, in terms differences of period ratios

$$\delta \Pi/\Omega = \left[ \frac{\Pi}{\Omega} - \frac{\Pi}{\Omega} \right]$$

for a given metallicity. For the lowest initial rotational velocity considered, $\Omega_i = 25 \text{ km s}^{-1}$, $\delta \Pi/\Omega$ reach up to $2 - 3 \times 10^{-3}$, and for $\Omega_i = 50 \text{ km s}^{-1}$, differences increase up to $6 - 8 \times 10^{-3}$, and finally, for $\Omega_i = 100 \text{ km s}^{-1}$, they can be of the order of $10^{-2}$. The main effect on PD (right panels) is to shift and compress tracks of same metallicity (and mass) toward higher period ratios with respect to classic tracks.

The previous differences in period ratios can also be analysed in terms of metallicity. For shortness, tracks will be specified, from now on, with the subscript corresponding to the rotational velocity considered. For instance, the track computed with $\Omega_i = 25 \text{ km s}^{-1}$ and $[\text{Fe/H}] = -0.1$ will be called as $[-0.1]_{25}$. As it will be shown, such analysis leads to a confusing scenario: For $\Omega_i = 25 \text{ km s}^{-1}$ (top, right) solar metallicity models tracks are similar to classic ones. However, when decreasing $[\text{Fe/H}]$, rotating and non-rotating tracks are located quite close. In particular, rotating $[-0.1]_{25}$ tracks may be confused with $[-0.2]_{0}$ ones, and $[-0.2]_{25}$ may be confused with non-rotating $[-0.35]_{0}$ ones. The analysis of the middle right panel reveals that $[0.0]_{50}$ tracks are located closely to $[-0.2]_{0}$ ones. Similarly, $[-0.1]_{50}$ tracks may be confused with $[-0.2], [-0.35], and finally, $[-0.2]_{50}$ tracks are close to $[0.50]_{0}$ ones. As can be seen, the mix-up is critical for Pop. II stars when considering 1.8 $M_\odot$ rotating models evolved with $\Omega_i = 25, 50 \text{ km s}^{-1}$.

Up to this point, all the discussion has been based on the results obtained only for 1.8 $M_\odot$ models. When other masses and metallicities are taken into account, the confusion in metallicities and rotational velocities significantly increases. In order to illustrate this, two solar metallicity tracks for higher mass (1.9 and 2.00 $M_\odot$) non-rotating models are displayed in the RPD of Fig. 5 where they are compared with other rotating non-solar tracks. As can be seen, the effect of increasing the mass of the models goes in the same direction as increasing the rotational velocity, that is, it increases the period ratios. It is worth noting that such shift toward larger period ratios systematically occurs for any rotating track. Therefore the previous discussion based on 1.8 $M_\odot$ models is equivalent whatever the mass and/or rotational velocity considered. Such behaviour may extend the confusing scenario to Pop. II stars. Considering 1.8 $M_\odot$ models, it would be necessary to consider initial rotational velocities up to $100 \text{ km s}^{-1}$. However, this lower limit rapidly decrease when increasing the mass of the models. For instance, 2 $M_\odot$ and $\Omega_i = 50 \text{ km s}^{-1}$ tracks may be misinterpreted with $[-1.00]_{0}$ ones.

The construction of complete RPD for a wide range of metallicities, initial rotational velocities and masses becomes necessary for the exhaustive analysis of double-mode pulsators (work currently in progress), however this exceeds the scope of this paper.

5. Conclusions

The impact of taking into account the effect of rotation on Petersen Diagrams has been examined here, focusing on main sequence double-mode pulsators. Detailed seismic models have been computed considering rotation effects on both equilibrium models and on adiabatic oscillation frequencies. For 1.8 $M_\odot$ stellar models, period ratios have been calculated for different rotational velocities (Rotational Petersen Diagrams) and metallicities, and then compared with classic non-rotating ones (PD).

The analysis of these RPD reveals that the difference in period ratios increases with the rotational velocity for a given metallicity. It remains around $10^{-3}$ for rotational velocities up to $50 \text{ km s}^{-1}$ and it can reach up to $10^{-2}$ for rotational velocities close to $100 \text{ km s}^{-1}$. Such difference have been found enough to produce a significant confusing scenario when analysing RPD in terms of metallicity variations. In particular, for 1.8 $M_\odot$ stellar models, differences in metallicity up to $\delta [\text{Fe/H}] \sim 0.30$ dex can be found when considering models evolved with initial rotational velocities of $50 \text{ km s}^{-1}$. Furthermore, such confusion may still increase when including other stellar masses, rotational velocities and metallicities, as well as other physical parametrisation (work in progress).

The results presented here should thus be taken into account when analyzing double-mode pulsators with Petersen Diagrams, in particular when accurate metallicity and/or mass determinations are required.

A work on the detailed analysis of period ratios in presence of near degeneracy is currently in preparation. Such work would provide new constraints on the modelling of HADS.

Acknowledgements. This study would not have been possible without the financial support from the European Marie Curie action MERGY-CT-2004-513610. As well, this project was also partially financed.
by the Spanish "Consejería de Innovación, Ciencia y Empresa" from the "Junta de Andalucía" local government, and by the Spanish Plan Nacional del Espacio under project ESP2004-03855-C03-01. JCS gratefully thank J. Christensen-Dalsgaard and W. Dziembowski for their interesting and fruitful discussions about this work.

References

Alcock, C., Allsman, R. A., Alves, D. R., et al. 2000, ApJ, 536, 798
Kippenhahn, R. & Weigert, A. 1990, "Stellar structure and evolution", Astronomy and Astrophysics library (Springer-Verlag)
Kovács, G. 2000, A&A, 360, L1
Morel, P. 1997, A&AS, 124, 597
Pamyatnykh, A. A. 2003, Ap&SS, 284, 97
Perez Hernandez, F., Claret, A., & Belmonte, J. A. 1995, A&A, 295, 113
Petersen, J. O. 1973, A&A, 27, 89
—. 1978, A&A, 62, 205
Petersen, J. O. & Christensen-Dalsgaard, J. 1996, A&A, 312, 463
—. 1999, A&A, 352, 547
Pojmanski, G. 2002, Acta Astronomica, 52, 397
—. 2003, Acta Astronomica, 53, 341
Poretti, E., Suárez, J. C., Niarchos, P. G., et al. 2005, ArXiv Astrophysics e-prints, A&A in press
Rodríguez, E., López-González, M. J., & López de Coca, P. 2000, A&AS, 144, 469
Soufi, F., Goupil, M. J., & Dziembowski, W. A. 1998, A&A, 334, 911
Suárez, J. C. 2002, Ph.D. Thesis
Suárez, J. C., Bruntt, H., & Buzasi, D. 2005a, A&A, 438, 633
Suárez, J. C., Goupil, M. J., & Morel, P. 2005b, A&A, submitted
Szymanski, M. K. 2005, Acta Astronomica, 55, 43
Tran Minh, F. & Léon, L. 1995, Physical Process in Astrophysics, 219
Udalski, A., Kubiak, M., & Szymanski, M. 1997, Acta Astronomica, 47, 319
Woźniak, P. R., Vestrand, W. T., Akerlof, C. W., et al. 2004, AJ, 127, 2436