Mathematical modelling in the problems of dynamics of brush-commutator unit: taking into account the conditions of the contact conservation possibilities

A I Orlenko\textsuperscript{1a}, A V Eliseev\textsuperscript{2b} and S V Eliseev \textsuperscript{3c}

\textsuperscript{1}Krasnoyarsk Institute of Railway Transport, the branch of ISTU, 89 Lado Ketskhoveli, Krasnoyarsk, Russia
\textsuperscript{2,3}Irkutsk State Transport University, 15 Chernyshevsky, Irkutsk, Russia
\textsuperscript{a}orlenko_ai@krsk.irgups.ru, \textsuperscript{b}eavsh@ya.ru, \textsuperscript{c}eliseev_s@inbox.ru

Abstract. The article focuses on the features of the dynamics of unilateral interactions of the elements of the brush-commutator unit of the traction electric motor of an electric locomotive within the scope of the problems of determining the conditions of motion without a gap interpreted as the quality factor of a current pick-up in dynamic loading processes. A mathematical model for the interaction of a brush, a collector surface and a pressing mechanism is proposed, which takes into account the possibility of contact failure depending on the parameters of the elements of the brush-commutator unit and the force factors including the effects of the vibrations of the traction motor body. The positivity of the complete reaction was used as the criterion for maintaining the contact. An analytical form of the conditions for the continuity of the contact between the elements of the brush-commutator unit is obtained on the basis of a comparison of the static and dynamic components of the complete reaction. The work shows the dependence of the analytical conditions of the continuity of the contact on the choice of the system parameters. It proposes a graphic-analytical method for determining the parameters ensuring continuity of the brush holder for the limiting stiffness of the pressing mechanism.

Keywords: electric locomotive traction motor, brush-commutator unit, unilateral constraints, contact reaction, gap, condition for maintaining contact

1. Introduction

Modern requirements to the peculiarities of rolling stock dynamics draw attention to the problems of reliability of electric locomotive traction electric motors operation, which is shown in a number of works reflecting the features of interaction between the elements of the brush-commutator unit (BCU) in the process of the current pick-up [1]. The efficiency of the operation of the BCU is determined by the vibration modes of the force impacts caused by the traction motor in combination with the features of the interaction of the BCU elements characterized by a number of parameters.

The brush is pressed to the collector by means of a pressing device that is a lever mechanism, which determines the peculiarity of the operation of the brush-commutator unit. The nature of the constructive interaction of the brush, pressing device and collector surface is formed taking into account unilateral constraints. In their turn, unilateral constraints create prerequisites for a variety of dynamic regimes that result in the possibility of contact failure and, as a consequence, cause a negative process of destruction of contacting surfaces with increasing dynamic loads.
Unilateral constraints demonstrate their peculiarities in the formation of various dynamic effects, which is shown in [2]. A number of questions reflecting the peculiarities of the dynamics of the interactions of the elements of a BCU are presented in [3–6]. At the same time, approaches to the development of mathematical models taking into account unilateral constraints, considered within the framework of the problem of determining the contact continuity conditions, have not yet been properly detailed.

The article proposes a method for developing mathematical models with unilateral constraints for the problems of determining the interaction parameters of the elements of the brush-commutator unit ensuring the contact continuity conditions.

2. Basic provisions. Statement of the research task.

A brush with a mass $m_1$, a pressing mechanism with a reduced mass $m_2$ and a collector can be assigned to the elements of the brush-commutator unit. The computational scheme of the interacting elements of the brush-commutator unit is shown in Figure 1. It is assumed that the constraint between the brush and the collector surface is elastic and is represented by a stiffness coefficient. This relationship within the scope of the problem under study is of a restraining (bilateral) nature, so that only one unilateral contact between the brush and the pressing mechanism is evaluated at this stage of the research. It is assumed that, while in a state of non-restraining (unilateral) contact, the brush and the pressing mechanism, having only one degree of freedom, move along the vertical guides.

It is believed that the unevenness of the collector surface, taking into account the possible deflections of the rotation axis in the process of vibrational loading, including from the side of the traction motor body, can be interpolated by a harmonic function:

$$Z_1 = A_1 \sin \omega_1 t,$$

(1)

where $A_1$ and $\omega_1$ are the amplitude and frequency of the conditional unevenness of the collector surface.

![Figure 1. Interaction of the elements of the brush-commutator unit. In Fig. 1, the following designations are accepted: $Z_1$ is the collector surface, $P_i$ is viscous frictional forces of the BCU elements, $Q_i$ is gravity forces of the elements, $F_i$ is additional constant forces, $N_{12}$, $N_{21}$ are complete reactions in contact, $a_2$, $b_1$ are contact surfaces.](image-url)
scheme in Figure 1, the values of $X_1$ and $X_2$ are the coordinates of the brush with the mass $m_1$ and the pressing device with the reduced mass $m_2$. It is believed that in the process of motion from the stationary medium, the forces of viscous friction $P_1$ and $P_2$ act on the brush and the pressing device, respectively. $Q_1$ and $Q_2$ are the weights acting on the brush and pressing device. In addition to the forces of weight and friction, additional constant forces $F_1$, $F_2$ are applied to the brush and pressing device, in accordance with Fig. 1.

Taking into account the unilateral nature of the contact between the brush and the pressing device in the state of contact failure is reflected in the form of an inequality:

$$X_1 + d_1 \leq X_2,$$

where $d_1$ is the conventional "dimension" of the brush.

The analytical record of a contact has the form:

$$X_1 + d_1 = X_2.$$  

In the phase of contact between the brush and the pressing device, a reaction occurs that characterizes the interaction of the elements. The criterion for the occurrence of a critical state, in a number of cases preceding the occurrence of a gap, is the complete contact reaction being equal to zero. The criterion for the contact continuity is the positivity of the complete reaction. The combination of the parameters of the elements of the system, including the features of the force impact, determines the possibility of a gap between the brush and the pressing mechanism.

The task of the study is to develop a method for constructing a mathematical model with allowance for non-restraining (unilateral) relationships and determining the parametric dependencies characterizing the conditions for the continuity of the contact between the brush-commutator unit elements.

3. Mathematical model

The total displacement of the brush $X_1^{\Pi}$, the total displacement of the pressing device $X_2^{\Pi}$ and the complete contact reaction $N_{21}^{\Pi}$ in the contact can be represented as the sum of the corresponding static and dynamic components. To determine the conditions for a zero gap motion in the contact between the brush and the pressing element, a system of equations is made, which makes it possible to find the components of a complete contact reaction. The constant and dynamic components act as components of the complete contact reaction. The static component is determined by the static components of the forces applied to the system at the initial instant of time. The dynamic component of the reaction is represented as the difference between the total reaction and the static response:

$$X_1^{\Pi} = X_1^\Sigma + X_1^\Delta,$$
$$X_2^{\Pi} = X_2^\Sigma + X_2^\Delta,$$
$$N_{21}^{\Pi} = N_{21}^\Sigma + N_{21}^\Delta,$$

where three upper indices are used to designate the corresponding component: $\Sigma$ is the static component, $\Pi$ is full one and $\Delta$ is the dynamic one.

The system of differential equations for determining the dynamic displacement components $X_1^\Delta$, $X_2^\Delta$ on the contact interval with initial zero conditions, depending on the dynamic component of the contact reaction $N_{12}^{\Delta} = N_{12}^\Delta = N_{21}^{\Delta}$, takes the form:
\[
\begin{align*}
&\begin{cases}
m_1\dddot{X}_1^A + p_1\dddot{X}_1^A + k_1\dot{X}_1^A = f_{z_1} - N^A,
\end{cases}
\\
&\begin{cases}
m_2\dddot{X}_2^A + p_2\dddot{X}_2^A = N^A,
\end{cases}
\\
&\begin{cases}
\dot{X}_1^A(0) = 0, \dot{X}_2^A(0) = 0,
\end{cases}
\\
&\begin{cases}
X_1^A(0) = 0, X_2^A(0) = 0,
\end{cases}
\end{align*}
\]  
(5)

where \(p_1\) and \(p_2\) are the coefficients of viscous friction with the medium, respectively, during the motion of the pressing device and the brush.

Using the Laplace transform, the resulting system (5) of differential equations is reduced to the system:

\[
\begin{align*}
&\begin{cases}
m_1s^2\dddot{X}_1 + p_1s\dddot{X}_1 + k_1\dot{X}_1 = \ddot{f}_{z_1} - N^A,
\end{cases}
\\
&\begin{cases}
m_2s^2\dddot{X}_2 + p_2s\dddot{X}_2 = N^A.
\end{cases}
\end{align*}
\]  
(6)

Transforms \(\ddot{X}_1, \ddot{X}_2, \ddot{f}_{z_1}, \dddot{N}^A\), considered as functions of a complex variable \(s\), are placed in accordance with the originals \(\dddot{X}_1^A, \dddot{X}_2^A, f_{z_1}, N^A\), considered as functions of time, on the basis of the Laplace transform:

\[
\dddot{X}(s) = \frac{\infty}{0} X(t)e^{-st} dt
\]  
(7)

It is assumed that all the necessary conditions for the existence of the corresponding integrals are fulfilled [6].

The transform of the dynamic component of the contact reaction is expressed from the system (6) in the form:

\[
\dddot{N}^A = \frac{L_1(s)L_2(s)}{L_1(s)+L_2(s)} \frac{\dddot{f}_{z_1}}{L_1(s)}
\]  
(8)

Where \(L_1(s) = m_1s^2 + p_1s + k_1\), \(L_2(s) = m_2s^2 + p_2s\).

Taking into account the representation of \(\dddot{f}_{z_1} = k_1\dddot{Z}_1\) (\(Z_1\) means the transform of \(Z_1\)), the transfer function, for which the input signal is a kinematic perturbation \(\dddot{Z}_1\), and the output signal is the dynamic component of the contact reaction \(\dddot{N}^A\), has the form:

\[
W_{\frac{\dddot{N}^A}{\dddot{Z}_1}}(s) = \frac{k_1L_2(s)}{L_1(s)+L_2(s)}
\]  
(9)

Thus, it is shown that the dynamic component of a complete contact reaction can be represented by a transfer function.

The developed method can be used to estimate the vibration range of the dynamic component of the constraint reaction in the contact between the brush and the pressing mechanism.

Sufficient conditions for the contact continuity, when the complete contact reaction \(N_{21}^{\Pi}\) takes only positive values, can be written in the form:

\[
A_1 \cdot A(\omega) < N^\Sigma,
\]  
(10)
where, \( A_1 \cdot \omega_1 \), respectively, are the amplitude and frequency of oscillations in the unevenness of the collector surface, \( A(\omega_i) \) is the amplitude-frequency characteristic of the transfer function, \( W_{\pi_i}^\pm (s) \), \( N^\Sigma \) is the static component of the reaction.

4. Estimation of the parameters of zero gap motion for the limiting variant of stiffness.

The analytical form of sufficient conditions for the contact continuity (10) makes it possible to estimate the regions of the system parameters that ensure the maintaining of contact, as well as to consider limiting cases.

The mathematical model of the system of the pressing device and the brush connected by an elastic element with a vibration surface, shown in Fig. 1, with the stiffness coefficient \( k_1 \to \infty \) results in a mathematical model in which it can be assumed that the pressing device directly has a unilateral contact with the unevenness of the collector surface. The peculiarity of the mathematical model of the direct interaction of the pressing device with the surface of the collectors is the decreasing of the critical amplitude that causes the opening, as the frequency increases.

In the case of finite stiffness \( k_1 \), the set of parameters ensuring motion in the contact is determined from the inequality:

\[
A_1 \sqrt{\frac{(m_1 k_1 \omega_1)^2 + (p_1 k_1 \omega_1)^2}{(k_1 - (m_1 + m_2) \omega_1)^2 + ((p_1 + p_2) \omega_1)^2}} < m_2 g + f_c,
\]

where \( f_c \) is the magnitude of the additional constant force.

The conditions (11) for implementing the contact interaction between the pressing device and the brush can be represented in the form:

\[
\omega_1 A_{12} \sqrt{m_2^2 \omega_1^2 + p_2^2} < m_2 g + f_c,
\]

where the value \( A_{12} \) is an amplitude that depends on various characteristics of the system, including the magnitude of the oscillation of the unevenness of the collector surface \( A_1 \):

\[
A_{12} = \frac{A_1 k_1}{\sqrt{(k_1 - (m_1 + m_2) \omega_1)^2 + ((p_1 + p_2) \omega_1)^2}}.
\]

It should be noted that in the case of finite stiffness \( k_1 \), examination of the conditions (11) of the continuity of the contact of the pressing device with the brush surface is determined by the existence of a dependence of the conditions of the contact continuity on the two masses: the mass of the pressing device and the mass of the brush.

1. If \( k_1 \to \infty \), then \( A_{12} \to A_1 \). Approximation of \( A_{12} \) to the amplitude \( A_1 \) can mean that the problem under consideration is reduced to the "limiting" problem of determining the conditions for the continuity of the contact between the pressing device and the surface of the vibration itself, in the presence of viscous friction and an additional constant force. The contacting of the pressing device with the collector surface instead of the brush surface is understood in the sense that the limiting shape of the brush motion repeats the form of the unevenness of the collector surface interpolated by the harmonic oscillation with amplitude \( A_1 \) and frequency \( \omega_1 \).

When considering the limiting case, conditionally understood as the case of "large" stiffness \( k_1 \), the limiting condition of contact (13) when \( k_1 \to \infty \) assumes the form in which only the mass of the pressing device is significant:
Thus, in the considered limiting case, it is possible to interpret the interaction of the pressing device with the brush surface as direct interaction of the pressing device with the collector surface.

2. The squaring of both sides of inequality (14) leads to the expression:

$$
\left( (A_1 \omega_1)^2 \omega_1^2 - g^2 \right) m_2^2 - 2gf_m m_2 + (A_1 \omega_1)^2 p_2^2 - f_c^2 < 0 .
$$

(15)

Inequality (15) makes it possible to determine those masses $m_2$ of the pressing device that will ensure a zero gap oscillations of the elements, taking into account the pre-contraction forces and viscous friction. The left expression in inequality (15) is a polynomial, in particular, a second-order polynomial depending on the mass of the pressing device $m_2$. This means that, if the solution exists, the set of mass values satisfying inequality (15) is an interval or a half-interval. By varying the parameters of the system, it is possible to obtain the fulfillment of the contact conditions for different intervals of the values of the masses of the pressing device.

To determine the intervals of the values of the masses that ensure the contact continuity conditions, let us consider the quantities:

$$
\mu = (A_1 \omega_1)^2 \omega_1^2 - g^2 , \quad \lambda = 1 - \left( \frac{A_1 \omega_1}{\gamma} \right) , \quad \gamma = \left| \frac{f_c}{p_2} \right| ,
$$

$$
D_4 = (A_1 \omega_1)^2 p_2^2 \left( \frac{\omega_1 f_c^2}{p_2^2} + g^2 - (A_1 \omega_1)^2 \right).
$$

(16)

Quantities (16) will characterize the location of the roots of the quadratic equation corresponding to (15):

$$
\left( (A_1 \omega_1)^2 \omega_1^2 - g^2 \right) m_2^2 - 2gf_m m_2 + (A_1 \omega_1)^2 p_2^2 - f_c^2 = 0 .
$$

(17)

Analysis of the dependence of the characteristics $\mu$, $\lambda$, $D_4$ on the parameters $\omega_1$, $A_1$, $f_c$, $p_2$ allows us to determine the range of values of masses at which the conditions of contact are implemented. The variables $\mu$, $\lambda$, $D_4$ are functions of the system parameters and can be considered as generalized characteristics of the system in determining the frequency and amplitude regions providing the contact. For each region, the range of detachment masses has its own peculiarities.

3. In Figure 2, in the parameter plane, which is the Cartesian product of the variable frequency $\omega_1$ and the amplitude $A_1$ of the surface oscillation, three curves 1, 2, and 3 are depicted.

For a fixed parameter $\gamma$, the curves 1, 2, 3 (Fig. 2) represent the set of zeros of the equations with respect to amplitudes and frequencies:

$$
D_4 = 0 , \quad \lambda = 0 , \quad \mu = 0 .
$$

(18)

Curve 1 (Figure 2) displays on the plane a set of amplitudes and frequencies for which the following equality holds:

$$
D_4 = (A_1 \omega_1)^2 p_2^2 \left( \frac{\omega_1 f_c^2}{p_2^2} + g^2 - (A_1 \omega_1)^2 \right) = 0 .
$$

(19)
At the points of this curve, the quadratic equation with respect to the mass (17) has a coincidence of the roots. Coincidence of the roots can reflect two options. The first option is that the contact condition is fulfilled only for a pressing device with a fixed mass; the second variant is the inverting of the first: that is, the contact condition is implemented for all masses, except for some fixed mass. The "transition" of the parameters through the curve under consideration means the occurrence or disappearance of a certain interval of values of the masses for which the contact continuity condition is fulfilled.

Figure 2. Areas of characteristic modes of interaction of the pressing device with the brush, which are specified by curves 1, 2, 3.

Curve 2 (Figure 2) determines the frequencies and amplitudes that satisfy the equation $\lambda = 0$. At the points of the given curve one of the roots of the quadratic equation (17) is zero. When "crossing" the curve, one of the roots changes its sign. This means that there may be a certain mass interval of pressing devices for which a contact is implemented or there is no gap.

Curve 3 (Figure 2) is a set of pairs of amplitudes and frequencies that satisfy the equation $\mu = 0$. For each point of the given curve there is a unique root, and the quadratic equation degenerates into a linear one. For the process of interaction of a pressing device with a brush, "crossing" the curve is the implementations of the contact "inversion". In the areas of parameters in which the contact conditions are fulfilled, the conditions cease to be fulfilled, that is, a gap occurrence is possible. And vice versa, if the contact conditions are not fulfilled for a pressing device with a certain mass, then such a pressing device will "stick" to the brush, that is, the contact continuity condition starts fulfilling.

4. To determine the details of the contacting of the pressing device for each area or its boundary, indicated in accordance with Fig. 2, the corresponding range of mass values is determined.

Table 1 shows the ranges of mass values of the pressing device for the areas (c), (e) and the boundaries $\Gamma_0$, $\Gamma_1$, $\Gamma_2$, $\Gamma_3$ shown in Figure 2.

Table 1 contains summary information on 6 sets of 11 regions, boundaries and point $\Gamma_0$ represented in accordance with Figure 2. The ranges of mass values providing contact for different options for directing an additional constant force are presented in columns 5.6 and 7. Column 7 reflects the option $f_c > 0$, when the constant force is directed towards the gravity force. Column 6 presents the option $f_c = 0$, of no additional strength. Column 7 presents an option $f_c < 0$ and $|f_c| < m_2 g$, when the force is directed against the gravity force.
Table 1. Intervals of the values of the masses of the pressing device, for which the conditions of the contact continuity are fulfilled.

| Region | \( D_4 \) | \( \mu \) | \( \lambda \) | \( f_c > 0 \) | \( f_c = 0 \) | \( f_c < 0 \) |
|--------|----------|----------|----------|----------|----------|----------|
| 1      | >0       | >0       | >0       | \((\sigma) (m'_1, m'_2)\) | No contact | No contact |
| 2      | >0       | =0       | =0       | \(\Gamma_0 (0, \infty)\) | No contact | No contact |
| 3      | =0       | >0       | <0       | \(\Gamma_1 (m'_0, m'_1)\) | No contact | No contact |
| 4      | >0       | =0       | <0       | \(\Gamma_2 (m'_1, \infty)\) | No contact | \((0, m''_1)\) |
| 5      | >0       | =0       | =0       | \(\Gamma_3 (0, \infty)\) | No contact | No contact |
| 6      | >0       | <0       | >0       | \((\rho) (0, \infty)\) | \((0, m''_1)\) \(\cup (m''_2, \infty)\) |

In accordance with Table 1, in the line for the parameters of the region \((c)\), the contact condition is fulfilled when the constant additional force is co-directed to the gravity force \( f_c > 0 \), on the range of mass \((m'_1, m'_2)\) values:

\[
m'_1 = \frac{gf_c - \sqrt{D_4}}{\mu}, \quad m'_2 = \frac{gf_c + \sqrt{D_4}}{\mu}.\]  

(20)

5. To determine the mass interval, a certain parameter is registered, for example, \(\gamma \approx 0.03\) m/s, which characterizes the relationship between the constant force and the viscous friction force. It is assumed that the constant additional force is directed toward the gravity and \( f_c = 0.2\) H. The amplitude of the surface oscillation is \(A_1 = 0.0002\) m. The range of values of the masses, providing the contact, is implemented only for the set \((c)\) in accordance with Fig. 2. The oscillation frequency is registered so that the point \((\omega, A_1)\) is inside the region \((c)\). It is assumed that this will be the frequency \(\omega = 240\) rad/s. To determine the range of values of the masses of the pressing device, providing contact, a graph of the contact characteristic is constructed:

\[
\left((A_1\omega_1)^2\omega_1^2 - g^2\right)m_2^2 - 2gf_m_2 + \left(A_1\omega_1\right)^2 p^2 - f_c^2 < 0.
\]  

(21)

In Figure 3.5, line 1 depicts a graph of the characteristic of the contact (21).
Figure 3. Characteristic of the contact in the form of a polynomial of the second order:  
1 is the characteristic curve, \( m_1', m_2' \) are critical masses of the pressing device.

The contact characteristic is a second-order polynomial depending on the mass of the pressing device. The abscissa values, at which the values of the contact characteristic are negative, determine the set of values of the masses implementing the contact for the given characteristics of the forces and kinematic perturbation. The critical values of the masses of the pressing device, at which the characteristic is zero, take the values \( m_1' \approx 0.02 \text{ kg}, \ m_2' \approx 0.09 \text{ kg} \). Accordingly, for pressing devices whose mass values are within the range of \((m_1', m_2')\), the conditions for the contact continuity will be fulfilled.

Figure 4. The values of the mass ranges of the pressing device, depending on the frequency of the surface oscillation: 1 is the graph of the critical mass \( m_1' \), 2 is the graph of the critical mass \( m_2' \).

6. Accordingly, the variation in the parameters of the system will affect the ranges of mass values that ensure the contact conditions. In particular, a change in the frequency of the oscillation of the surface inside the regions \((d), (c)\) can result in the changes in the range of values of the masses that ensure contact (Fig. 4).

With increasing frequency as the region \((e)\) passes, all the pressing devices are in contact. When passing through the area \((d)\), only pressing devices that do not exceed a certain mass will be in contact with the surface. As the frequency increases in contact, there are pressing devices with a mass from the range of \((m_1', m_2')\). If the surface oscillation frequency is 248 \text{ rad/s}, then according to the presented
model data, only pressing devices with a mass close to 0.036 kg will be in contact. With further increase in frequency there is no contact.

Thus, within the framework of the model problem under consideration, the system is characterized with sufficiently large values of stiffness $k_1$. Reduction of the stiffness results in a change in the amplitude of oscillation of the BCU elements in contact. A more detailed calculation of the parameters and peculiarities of the transformations are presented in [3-6].

5. Conclusions
The above material allows us to draw the following preliminary conclusions.

1. An approach is proposed to investigate the modes of contact interaction between the elements of a brush-commutator unit under conditions of vibrational loading. The criterion for the maintaining contact between the BCU elements is the positive value of the complete contact reaction in the steady-state oscillation regime. The characteristic of the critical state of the elements’ interaction, after which the contact is likely to be broken, is the equality of the total contact reaction to zero.

2. A method has been developed for determining the zero gap motion regimes for the BCU elements with allowance for additional compression forces and viscous friction forces. The analytical dependence between the parameters of zero gap motion of the BCU elements and the characteristics of the oscillation of the unevenness of the collector surface is determined. It is established that the mass of the pressing device, the characteristic of viscous friction, and the additional force of compression can act as a contact factor. The possibility of regulation using the conditions for maintaining contact is shown by varying the system parameters and the characteristics of the force impact.

References
[1] Orlenko A I, Petrov M N and Teregulov O A 2016 Complex diagnostics of electric locomotive traction electric motor (Krasnoyarsk: Polikom) p 218
[2] Blekhn I I 2000 Vibrational Mechanics: Nonlinear Dynamic Effects, General Approach, Applications (Singapore, New Jersey, London, Hong Kong: World Scientific Publishing Co. Pte. Ltd) p 509
[3] Eliseev A V, Orlenko A I and Eliseev S V 2017 Not-holding connections as a characteristic feature of dynamic interactions of elements of technical systems 21st Conference of Open Innovations Association (Helsinki: IEEE) 100-107
[4] Eliseev A V, Artyunin A I and Eliseev S V 2016 Generalized gap function in the dynamic interaction problems of elements of vibrational technological machines with «not holding» ties Vibroengineering Procedia (Kaunas: JVE International Ltd.) 8 495-500
[5] Orlenko A I and Eliseev A V 2017 Mathematical model of brush-commutator interactions taking account of not-holding ties The priorities of the world science: experiments and scientific debate: Proceedings of the XVI International scientific conference (North Charleston: CreateSpace) 42-45
[6] Eliseev A V, Selvinsky V V and Eliseev S V 2015 Dynamics of vibrational interactions of the elements of technological systems taking into consideration unilateral constraints (Novosibirsk: Nauka) p 332