New ultra high-speed all-optical coherent D-trigger

S N Bagayev\textsuperscript{1}, V S Egorov\textsuperscript{2}, V G Nikolaev\textsuperscript{3}, I B Mekhov\textsuperscript{2,4,5}, I A Chekhonin\textsuperscript{2}, M A Chekhonin\textsuperscript{2}

\textsuperscript{1}Institute of Laser Physics, Novosibirsk, Russia
\textsuperscript{2}St.Petersburg State University, St. Petersburg, Russia
\textsuperscript{3}ITMO University, St. Petersburg, Russia
\textsuperscript{4}University of Oxford, UK
\textsuperscript{5}University Paris-Saclay, France

E-mail: chekhonin@mail.ru

Abstract. We study the interaction of two counterpropagating unipolar video pulses of electromagnetic radiation in a dense resonant two-level medium. The pulse durations are less than one oscillation period of an atomic transition. We show that a polariton cluster (i.e. the compact long-living strongly coupled state of electromagnetic field and matter polarisation) is created, when two unipolar video pulses collide in a resonant medium of the frequency $\omega_0$ (the pulses correspond to self-induced transparency solitons of the same amplitudes and opposite polarities).

We studied for the first time multiple recording and erasing of a polariton cluster in a thin layer of a resonant medium (quantum dots) placed on the mirror surface. We showed that dynamics of the medium population difference $N(x,t)$ is analogous to the operation of a D-trigger of the pulse rate 60 000 GHz and higher. We found such a method of the polariton cluster recording and erasing that excludes the accumulation of erasing errors. Therefore, the total duration of the optical D-trigger operation time can strongly exceed the phase relaxation time $T_2$.

1. Introduction

Recently, the generation, coherent propagation, and interaction in dense resonant media between ultrashort (few-cycle, single-cycle, sub-cycle) pulses of the electromagnetic field attract strong interest. Modern methods of generating unipolar video pulses are reviewed in [1,2]. Effects of coherent interaction between counterpropagating pulses are considered in a significantly smaller number of works, e.g. [3-5].

Here we theoretically study the processes of formation and dynamics of polariton clusters, which are created during collisions between unipolar pulses of the amplitudes of opposite signs (solitons of the self-induced transparency).

The correct description of propagation and interaction of ultrashort (single-cycle, sub-cycle) solitons is possible only by eliminating the approximations imposed by the slowly varying envelope (SVEA) and rotating-wave approximations (RWA) both in time and spatial coordinate. The theory of propagation of such solitons in dense resonant media is well established, cf. for example [6-12].

Dynamics of a quantum two-level particle with the dipole moment $d$ and transition frequency $\omega_0$ is described by the Bloch equations [6,7] for the pseudospin vector projections $s = (s_1, s_2, s_3)$:
\[ \dot{s}_1 = -\omega_0 s_2 - \frac{1}{T_2} s_1 \]  
\[ \dot{s}_2 = \omega_0 s_1 + 2 \frac{i}{\hbar} E(t,z) s_2 - \frac{1}{T_2} s_2 \]  
\[ \dot{s}_3 = -2 \frac{i}{\hbar} E(t,z) s_3 - \frac{1}{T_1} (s_3 + 1) \]  
\[ P(t,z) = N_0 d \cdot s_1 \]  
(1)  
(2)  
(3)  
(4)

Here \( E(t,z) \) is the electric field amplitude of a pulse propagating along the coordinate \( z \), \( T_1 \) and \( T_2 \) are the longitudinal and transversal relaxation times, \( P(t,z) \) is the medium polarization, \( N_0 \) is the resonant particle density. The physical meaning of the pseudospin projection \( s_3 \) is the normalized population difference of a two-level medium: \( s_3(z,t) = N(z,t)/N_0 \).

Dynamics of the electromagnetic field propagation is described by Maxwell’s equations:

\[ \frac{1}{c} \frac{\partial H_y}{\partial t} = \frac{\partial E_x}{\partial z} \]  
\[ \frac{1}{c} \frac{\partial E_x}{\partial t} = -\frac{\partial H_y}{\partial z} - \frac{4\pi}{c} \dot{P} \]  
(5)  
(6)

or by the wave equation

\[ \frac{\partial^2 E}{\partial t^2} - c^2 \frac{\partial^2 E}{\partial z^2} = -4\pi \dot{P} \]  
(7)

Here \( E_x \) and \( H_y \) are the projections of electric and magnetic fields of the pulse on the transverse axes \( x \) and \( y \), \( c \) is the speed of light in vacuum.

In the absence of relaxation (\( T_1 = T_2 = \infty \)), the Maxwell-Bloch equations have analytical single-soliton solutions of the self-induced transparency theory [7-12]:

\[ E = \frac{h}{\alpha t_p} \text{sech} \left( \frac{t - zV^{-1} - t_p}{t_p} \right) \]  
\[ V = \frac{c}{\sqrt{1 + \frac{2\alpha t_p^2}{1 + \tau_p^2}}} \]  
\[ \alpha = \frac{4\pi}{\hbar} \frac{N_0 d^2}{\omega_o} = \frac{1}{t_p \omega_o} \]  
(8)  
(9)  
(10)

Here \( V \) is the soliton propagation velocity along the coordinate \( z \), \( t_p \) is the soliton duration, \( \tau_p = \omega_o t_p \).

2. Formation of a polariton cluster during the collision of two solitons

In this work we have studied the process of inelastic collision between two self-induced transparency solitons (8) in a resonant medium. We theoretically considered a problem of two identical counterpropagating unipolar solitons of either the same or different polarities. We solved numerically the Maxwell-Bloch equations (1)-(3) and (5)-(6) without using the slow amplitude approximations (SVEA, RWA) both in time and spatial coordinate. We took into account the finite relaxation times of a medium \( T_1 \) and \( T_2 \). The details of the simulation methods are described in [4].

In Figure 1, we show dynamics of the normalized population difference \( s_3 = N(z,t)/N_0 \) during the collision of two unipolar solitons. One can see that the long-living polariton cluster is formed during the inelastic collision between the solitons of the opposite polarity (right panel) near the collision point \( z_o \) (\( Kz_o = 15 \)). The cluster has a double spatial structure. By varying the soliton duration \( t_p \), we have
shown that the quantity $<N(z)>$ averaged over the cluster existence area $K_z = [14...16]$ has a well-defined maximum at the soliton duration $\omega_{otp} = 0.75$.

Figure 1. Dynamics of the normalized population difference $s_3 = N(z,t)/N_0$ during the collision of two unipolar solitons. Arrows show the directions of pulse propagation. Left panel: collision between the pulses of the same polarity; right panel: collision of the pulses of the opposite polarity.

$K_z = 2\pi/\lambda$ is the wave vector, $\omega_{otp} = 1$, $T_1 = 10$ psec, $T_2 = 10$ psec, $d = 5D, N_0 = 1.77 \cdot 10^{20}$ cm$^{-3}$, $\omega_o = 1 \cdot 10^{15}$ rad/sec, $2dE_o/\hbar \omega_o = 2$.

Such features have a simple physical explanation. Analysing the temporal behaviour of the sum fields of two solitons $E(t)$ in various spatial points, one sees that there exist two special points with the coordinates $z'$ and $z''$ (cf. Figures 2). In these points, the field $E(t)$ has a form of bipolar single-cycle pulse, which can resonantly excite the medium at the atomic transition frequency $\omega_o$. The condition of resonant excitation corresponds to the optimal soliton duration $\omega_{otp} = 0.75$.

Figure 2. Temporal behaviour of the field $e(t)$ in two points of the polariton cluster $z'$ and $z''$ near the point of the pulse collision $z_o (K_{z_o} = 15)$. Left panel: $K_{z'} = K_{z_o} - 1.8$, right panel: $K_{z''} = K_{z_o} + 1.8$.

$e(t)$ is the dimensionless field amplitude: $e(t) = 2dE(t)/\hbar \omega_o$.

Figure 3 shows the spatial distribution of the medium polarization $s_1$ and population difference $s_3$ immediately after the collision between the solitons of opposite polarities. One sees that $s_3$ has a double structure (cf. also Figure 1), while $s_1$ oscillates with the amplitude close to the maximal one ($s_3 \approx 1$). Nevertheless, the radiation of cluster in the far field is negligibly small. The reason is that the spatial Fourier spectrum of the medium polarization $P(t,z) = N_c d s_1$ is concentrated at the wave vectors $Q >> 2\pi/\lambda$, which do not satisfy the phase matching conditions. Therefore, the cluster field $E(t)$ exists in the near field only. An analogous mechanism of the forbidden radiation for long-living
polariton clusters, which are formed during the collision between the few-cycle self-induced transparency solitons, has been discussed in [4].

![Figure 3](image1.png)

**Figure 3.** Spatial distribution of the normalized medium polarization $s_1$ (left panel) and normalized population difference $s_3$ (right panel) in a polariton cluster at the time moment $\omega_o t = 28.85$ immediately after the pulse collision. The collision happens at $\omega_o t = 25.38$, $K_{z0} = 15$. The arrows show the propagation directions.

3. Recording and erasing polariton clusters. D-trigger.

In this part of the work, we studied the propagation of a periodic sequence of counterpropagating unipolar pulses through a dense resonant medium. The repeated action of two pulses of opposite polarities on the polariton cluster leads to its complete destruction (cf. Figure 4). The next pair of pulses reproduces the polariton cluster again. Such a process can be repeated many times, and dynamics of the population difference $N(t)$ is analogous to the action of an electronic D-trigger.

Such recording and erasing processes are coherent, because they happen at time intervals $\Delta t \ll T_2$. Therefore, the pulse repetition period $T$ should satisfy the condition $T = (2n + 1)T_o/2$, where $T_o = 2\pi/\omega_o$ is the oscillation period of the two-level atomic system, $n = 0, 1, 2 ...$ is an integer.

![Figure 4](image2.png)

**Figure 4.** Dynamics of the normalized population difference $s_3 = N(z,t)/N_o$ during the recording and erasing polariton cluster by unipolar pulses of the opposite polarities. The repetition period $T$ of recording and erasing pulses is 8.5 of the atomic oscillation period ($\omega_o T = 53.5$).

$\omega_o, \omega_o T = 0.8, T_1 = 10$ psec, $T_2 = 10$ psec, $d = 5D, N_o = 1.77 \cdot 10^{19}$ cm$^{-3}$, $\omega_o = 1 \cdot 10^{15}$ rad/sec, $2dE_o/\hbar\omega_o = 2.5$.

From a practical point of view, a convenient method to obtain two counterpropagating unipolar pulses of opposite polarities is based on the pulse reflection from a mirror (cf. Figure 5).
Below we consider a case, where a thin layer (≈ λ/2, 300 nm) of a resonant medium is placed on the mirror surface, while the counterpropagating pulse of the duration \( t_p \) equal to 1/8 of the atomic transition oscillation period (\( t_p = 0.24 \) fsec) is created as a result of the reflection from the mirror (cf. Figure 5). The light-induced stationary polariton cluster is situated at the distance of ≈ λ/10 (60 nm) from the mirror. Within the cluster, the medium polarisation oscillates with the maximal amplitude, nevertheless its radiation in the far field is forbidden.

**Figure 5.** Spatial distribution of the normalized population difference \( N(z)/N_0 \) in polariton cluster.

- a – region occupied by resonant medium,
- b – mirror,
- c – incident pulse,
- d – reflected pulse.

\[ K = \frac{2\pi}{\lambda} \] is the wave vector, \( \lambda = 600 \) nm.

In Figure 6, we show the result of multiple recording and erasing the polariton cluster by a periodic sequence of unipolar pulses incident on the mirror. It turns out that, if one uses the pulse sequence with polarity inversion (+1, +1, -1, -1, +1, +1, -1, -1, ...), then the cluster erasing error approaches zero. Moreover, the time \( \Delta t \) of the stable operation of the D-trigger increases multiple times: \( \Delta t \gg \gg \gg T_2 \).

**Figure 6.** Dependence \( s_3 = N(t)/N_0 \) for the cluster point \( Kz = 3.5 \). The arrows show the moments of pulse actions. Pulse rate is 60 000 GHz (60 THz). The function \( f(t) = \exp[-(t/T_2)] \).

Pulse duration is \( t_p = 0.24 \) fsec (\( \omega_o \tau_p = 0.75 \)), \( T_1 = 10 \) psec, \( T_2 = 100 \) fsec,
\( d = 2D, N_0 = 2.22 \cdot 10^{18} \) cm\(^{-3} \), \( \omega_o = 3.142 \cdot 10^{15} \) rad/sec, \( 2dE_o/\hbar \omega_o = 2.67 \).

To confirm this, we performed calculations for very small relaxation times: \( T_1 = 10 \) psec, \( T_2 = 100 \) fsec.
Figure 6 shows the influence of the finite relaxation time $T_2$ on the decrease of the value of $|N(t)/N_0|$ in the regime ON/OFF of the D-trigger. The degradation of the cluster coherence happens at the moments of the cluster switch, i.e. during the time interval $\approx \tau_p$ and it is absent in the interval $T$ between the pulses. At the chosen pulse repetition frequency, the decrease rate of $|N(t)/N_0|$ is 18 times smaller than the decrease rate of the function $f(t) = \exp[-(t/T_2)]$.

Varying the amplitude reflection coefficient $R_a$ has shown that the D-trigger operates stably at the values $R_a = 0.8 \ldots 1.0$.

4. Conclusion

In conclusion, we have systematically studied an effect of creation of polariton cluster (i.e. a compact long-living strongly coupled state of the electromagnetic field and medium polarization). We performed numerical solution of the problem of inelastic collision between two counterpropagating solitons of the self-induced transparency effect.

We physically explained the fact, that for creating a polariton cluster, the best choice is the use of a pair of solitons with the field amplitudes of opposite signs. We have shown that the reason of the long life-time of the cluster is due to the forbiddance of its radiation in the far field.

We studied the processes of multiple recording and erasing of the polariton cluster, and showed that dynamics of the population difference of the medium is analogous to the operation of an electronic D-trigger of high pulse rate 60 000 GHz and higher.

We studied the processes of recording and erasing of a polariton cluster in a thin layer of a resonant medium (quantum dots) near a mirror, and showed that the cluster size is of the order of $\approx \lambda/10$. This makes it a very perspective basic element for all-optical signal processing.

We found a method of cluster recording and erasure, which excludes the erasure error accumulation. Therefore the total time of the optical D-trigger operation can multiply exceed the phase relaxation time $T_2$.

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