Formal student thinking in mathematical problem-solving

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Abstract: The purpose of this research is to find out the students' thinking process in formal cognitive development in solving mathematical problems. The problem solving used in this study uses patterns from the pattern that have been adapting to the habits of students in Indonesia, namely the first stage is known, the second stage is asking, and the third stage is answering. This study was included in a qualitative descriptive study using six junior high school students with formal that were used as research subjects. The research instrument used in this study is Test of Logical Operations and mathematical problem-solving tests. Data analysis techniques are used using Milles, Huberman, and Sandana. From the results and discussion of the research carried out, it can be concluded that students with formal cognitive in the first and second stages tend to think about assimilation and abstraction. While in the third stage, students in formal tend to think about assimilation processes even though there are subjects who do the accommodation thinking process.

1. Introduction
Mathematical problem solving is one of the important things to be invested and owned by students [1,2]. Problem-solving is part of the mathematics curriculum which is very important because in the learning process and its completion, students are likely to gain experience [3,4]. Some reasons are first problem solving can train students in making decisions, second problem solving can be used to formulate concepts, develop their ideas, third process standards in mathematics include problem-solving, reasoning and verification, linkage, communication, and representation, fourth the purpose of mathematics learning is that students are expected to foster the ability to think critically, logically, systematically, carefully, effectively, and efficiently in solving problems [5–8].

The low ability of students in solving mathematical problems due to mathematical connection abilities possessed by students is not optimal [9–11], Contextual Teaching and Learning, conventional learning obtained a mean mathematical connection ability in the low so that the ability to solve problems was not optimal [12–14]. Also, students 'unstructured habits in solving problems greatly affect students' ability to solve mathematical problems [15,16]. It is as students are not able to understand the problem at hand so that students are unable to implement plans to solve mathematical problems. Even planning to solve mathematical problems, students also feel unable to do it [17,18].

Many ways can be done by the teacher so that students' problem-solving skills can increase such as using problem-based learning [19,20], using Realistic Mathematics learning [21–24]. Things that can
be done by a teacher in learning include knowing the thinking process of students in solving problems. By knowing the students' thinking process in solving mathematical problems can help the teacher to know mathematics learning can be better [25] because by knowing the thinking process of students a teacher can prepare learning methods, learning models, and learning tools so that learning Mathematics becomes more effective [26].

Students in Junior High school who are at the age of more than 12 years, according to Piaget cognitive are in the last phase of cognitive, namely formal cognitive [7,27,28]. This development can be seen from the ability of students in being able to think logically, thinking with formal theoretical thinking based on propositions and hypotheses, can conclude from what can be observed at the time, and abstract ways of thinking begin to be understood [29,30]. Someone with formal already able to think scientifically, think theoretically, argue and test hypotheses that prioritize thinking skills, children can solve problems logically by involving various related problems [31]. Formal children are generally able to think abstractly in classification reasoning, conservation reasoning, reasoning theoretical, combination reasoning, proportional reasoning, functional reasoning, controlling variables, analogy reasoning, propositional reasoning, correlational reasoning, probability reasoning [32]. As a result of students who have formal or abstract thinking skills, students can be invited to use their minds to solve problems that require high-level thinking such as linking, weighing, testing, deciding, abstract thinking, understanding, analyzing, and solving problems [30].

In this regard, the purpose of this study is to find out the formal thinking processes of students in solving mathematical problems. By knowing the thinking process of these students can make it easier for teachers to develop mathematics learning tools, at least to compile mathematics learning media so that students' thinking processes in solving mathematical problems become more clear and systematic.

2. Method

This research is included in descriptive qualitative research because it intends to uncover students' thinking processes in the phase of cognitive in solving mathematical problems. The mathematical problems used relate to algebra and geometry materials, while problem-solving refers to the problem-solving steps of the Polya model that is understanding the problem, making plans, implementing the plan, and re-examining the answers [33]. Polya's step was reduced and adapted so that it was adjusted to the characteristics of students in Indonesia into three steps, namely being known, asked and answered [34,35].

The selection of research subjects using purposive sampling, namely sampling selected using certain criteria so that the subjects are relevant and meet requirements for use in this study [36–38]. The subjects in this study were four students in seventh grade of Junior High Scholl "X" in Yogyakarta who had formal cognitive. Piaget's cognitive was confirmed using an instrument called the Test of Logical Operations [39,40]. The Test of Logical Operations is adapted from Leongson and Limjap where the indicators are Classification, Seriation, Logical Multiplication, Compensation, Proportional Thinking or Ratio, Probability and Correlational Thinking [41]. In addition to using, the Test of Logical Operations instrument, to find out the thinking process of students in solving mathematical problems, using a mathematical problem-solving test instrument containing questions related to ratio or arithmetic. These two materials were chosen because, in this material, Mathematical questions can be made in the form of stories so students must use all their abilities to make mathematical equations in solving these problems.

The analysis will be carried out to find out the formal thinking process of students using steps (1) Examining all collected data from various sources, (2) making classification of students' cognitive types, namely concrete, transitional, and formal, (3) determining formal students which will be used as research subjects, (4) reviewing the results of formal student work in solving mathematical problem solving questions, (5) Verifying data and data sources that have been classified and transcribed in data presentation or exposure [42].
3. Result and discussion

As revealed in the method section that the test instrument for problem-solving given to students contains questions about the comparative and artificial material. The problem of comparison given to two formal subjects F1 and F2 is (1) The distance between Yogyakarta and Semarang is 135km. If in a plan the distance of Jogjakarta with Semarang is 15 cm, what is the scale stated in the plan, (2) Dadang invites his friends to an event watching the ball together? Dadang served coffee as a drink. One glass of coffee needs two teaspoons of coffee and three tablespoons of sugar. If available 300 sugar how much coffee is needed, and (3) Hanifah height compared to Arifa is 8: 6, the difference in height is both 40cm. What is the height of Hanifah And Arifah?

While the arithmetic problem given to the two formal subjects F3 and F4 is (1) Fajar buys a bicycle at a price of IDR 700,000, after being used for several months the bicycle has suffered minor damage. Fajar repaired the bicycle for IDR 150,000. The bike is sold again at a price of IDR 950,000. What are the advantages or disadvantages experienced by dawn!, (2) A fruit trader buys 1000 oranges with a net weight of 200 kg. The price per kilogram is IDR 12,000; it turns out 40 pieces were damaged. If the transportation costs are 200,000, and the trader sells the fruit at a price of 15,000, determine the percentage of profit or loss !, and (3) Adi buys ten bu manga boxes weighing 100 kg each. in the written box there is 5% if purchased at 5,000 each kilogram determines the loss or profit earned if the manga is sold at a price of 6,000.

3.1. Ratio

F1 and F2 subjects can write down what is known and what is asked. The subjects were able to integrate the old scheme with the new scheme. Between the old scheme and the new knowledge, the subjects did not experience cognitive conflict so that the subjects did not experience any obstacles in identifying the problem at hand. If the old scheme process and the new scheme do not occur conflict, the cognitive structure is said to be in a state of equilibrium with its environment [43], so that the process of pooling new schemes into existing schemes uses the assimilation thinking process [44]. The assimilation process can occur if there is a match between new information and existing cognitive structures so that there is equilibration [45,46]. Related to this, the schema structure that has been owned by students is used to deal with the problems it faces. The combination of new data with existing cognitive structures causes the thought processes that formal students do at a known stage and are asked to think of assimilation.

F1 subjects use the JS symbol as a substitute for actual distance and JP as the distance on the map to solve the first problem. In the third problem, subjects used TB as a change in height. In connection with these results, F1 subjects were able to construct a symbol as a substitute for distance so that it was not too long to solve the comparison problem. Likewise in subject F2, on the first problem the subject uses the JS symbol for the actual distance, JD for the distance on the map or floor plan, and S for the scale. On the issue of the three, F2 used the symbol of the front letter as a substitute for names such as H for Hanifa and A for Arifa. The use of symbols like this is one indicator of the process of estimating abstraction in solving problems [47]. In this regard, F1 and F2 in addition to the assimilation process of thinking also carry out the process of thinking abstraction.

In the third or step stage answered, F1 and F2 subjects can solve the problem in the comparison questions correctly and smoothly. The subject did not find difficulties in solving the problem; this meant the subject was able to integrate old knowledge and new knowledge to solve this problem. The process of integration between old knowledge and new knowledge has experienced equality so that the subject performs the assimilation process of thinking. As stated earlier that assimilation can occur if there is a match between new information and existing cognitive structures so that equilibration occurs [45], so that the schema structure that has been owned by students is used to deal with the problems it faces. Related to the symbols used by subjects in the previous stages, in this third stage, the symbol is still used by the subject to solve the problem at hand. In connection with this matter, the subject also performs the process of thinking abstraction in addition to carrying out the assimilation process of thinking in solving assimilation problems.
In connection with the thinking process carried out by F1 and F2 in solving the problem of comparison can be categorized that the subject performs the thinking process assimilation and abstraction in stages are known, asked and answered. The process of thinking assimilation can be seen from the compatibility between new information with existing cognitive structures so that there is equilibration [45], the old scheme process and new schemes do not occur so that the balance of the scheme occurs with the environment [43].

3.2. Arithmetic
The F3 subject on arithmetic problems has been able to write down what is known and what is asked of the problem faced correctly so that F3 can directly integrate new perceptions or experiences into the scheme that is in his mind. Similar to the thinking process carried out with F1 and F2 in solving the problem of comparison, that the old scheme and the new scheme are in a state of equilibrium with the environment [43] so that the process of pooling new schemes into existing schemes uses the assimilation thinking process [46]. This assimilation process can occur if the response to an object or event is by the scheme that has been owned [48]. In this regard, F3 can be categorized as carrying out the assimilation process of thinking at the stage of being known and asked.

The F3 subject correctly answers the first and second artificial problems without experiencing problems in implementing the plan. In connection with this, the subject can directly integrate the knowledge that has been possessed with new knowledge. The old knowledge integration process does not conflict with new knowledge, so the subject is in a balanced condition related to the scheme of profit and loss. Because it is in a balanced condition between old and new knowledge, the subject can be categorized as conducting assimilation thinking processes [43,46].

In the third arithmetic problem, F3 at the answering stage has difficulty in this case as seen from the answers of the subjects crossed out in the previous answer. In this regard, subject conducts disequilibration or imbalance between old knowledge and new knowledge. Disqualified is a situation where the old scheme with the new scheme is incompatible so that this situation can lead to accommodation [49]. Even though they have carried out the accommodation process for old and new knowledge, subjects have not been able to solve arithmetic problems correctly. It is because the subject does not yet know the concept of tara in arithmetic so that the accommodation process is carried out by the subject so that the imbalance of the scheme becomes balanced cannot occur so that the subject continues to be miscalibrated even though he has made the process of thinking accommodation.

The process of integrating the old knowledge that F3 has had in solving all three arithmetic problems, in general, does not experience obstacles so that the F3 subject performs the assimilation process of thinking. In contrast to the third arithmetic problem, the subject performs the equilibrium thinking process.

Subject F4 on arithmetic problems does not write down what is known and what is asked of the problem at hand. To find out the subject thinking process needs to be interviewed.

Researcher : Why on your answer sheet, there is no writing what is known and what is asked?
Subject F4 : I am not used to writing that.
Researcher : what do you mean?
Subject F4 : In answering Mathematics questions I am used to directly answering problems by writing answers without writing known and answered.
Researcher : why?
Subject F4 : It's efficient because the time I use to write down what is known and what I ask can I use to write an answer.
Researcher : Do you know what was asked of the first problem?
Subject F4 : know sir, the proof is that I can solve the problem number one. Likewise in the second and third questions.
The results of the interview concluded that subject F4 had a habit of solving problems without writing down what was known and what was asked of the problem at hand. Errors like this are students' habits in solving mathematical problems. This habitual error is like what students do in solving the problem of type divergence proves [50]. The results of such subjects cannot be justified that the subject is unable to understand but must be seen in the process of solving the problem in the next step [17]. Although the subject did not write down what was known and answered, the students could do the process of solving the problem in the next stage (the answering stage). Based on this, the subject of F4 can be categorized as doing the assimilation process of thinking. It is because of the response to an object or event by the scheme that has been owned [48].

Subject F4 in answering the first and second artificial problems correctly without experiencing problems in implementing the plan. In this regard, the subject can directly integrate the knowledge that is already possessed with new knowledge in other words the old scheme and the new scheme are in a balanced condition [43,45,46], so the subjects do assimilation thinking processes. On the third problem, subject F4 has not been able to integrate the initial scheme it has with new knowledge. With this condition, subject F4 performs a disequilibration thinking process at the third stage or answers. The subject does not yet know the concept of tara in arithmetic so that the accommodation process is carried out by the subject so that the imbalance becomes balanced cannot occur so that the subject continues to be miscalibrated.

The process of integrating the old knowledge that F3 and F4 already have in solving the three arithmetic problems, in general, does not experience obstacles so that the subject performs the assimilation process of thinking. Whereas in the third arithmetic problem, subjects F3 and F4 carry out the process of thought disequilibration.

4. Conclusion
From the results of the research and discussion that has been described, it can be concluded that most formal students carry out the assimilation thinking process because the subject can integrate old knowledge with new knowledge. Although there are subjects who make the process of thinking abstraction in solving ratio problems, and there are subjects who do disequilibration thinking processes in solving arithmetic problems. In this regard, learning devices such as learning media and student work are needed so that formal students who are disequilibrarion can solve mathematical problems.

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