Automatised full one-loop renormalisation of the MSSM
II: The chargino-neutralino sector, the sfermion sector and some applications

N. Baro¹, F. Boudjema²

1) Institut für Theoretische Physik E, RWTH Aachen University,
   D-52056 Aachen, Germany
2) LAPTH, Université de Savoie, CNRS,
   BP 110, F-74941 Annecy-le-Vieux Cedex, France

Abstract

An on-shell renormalisation programme for the chargino/neutralino and the sfermion sectors within the Minimal Supersymmetric Standard Model as implemented in a fully automated code, $\texttt{SloopS}$, for the calculation of one-loop processes at the colliders and in astrophysics, is presented. This is a sequel to our study in Ref. [1] where an on-shell renormalisation of the Higgs (and the gauge/fermion) sector is performed. The issue of mixing is treated in a unified and coherent manner in all these sectors, in particular we give some new insight into the renormalisation of the mixing angle in the sfermion sector and like with the Higgs sector and the issue of $\tan\beta$ we discuss different schemes. We also perform numerical comparisons between our code $\texttt{SloopS}$ and different results found in the literature. In particular we consider loop corrections to the neutralino and sfermion masses, chargino pair production and stau pair production in $e^+e^-$ colliders, as well as a few decays of the heavier chargino. For all these observables, we analyse the $\tan\beta$ scheme dependence using different definitions of this parameter and comment on the impact of using different renormalisation of the mixing parameter in the sfermion sector.
1 Introduction

The description of the Higgs within the Standard Model is unsatisfactory as it poses the problem of naturalness. Besides, the Higgs particle is still missing. Moreover there is overwhelming evidence that there is a large amount of Dark Matter that can not be accounted for by any of the particles of the Standard Model, SM. All this points to New Physics. The best motivated model of this New Physics is undoubtedly supersymmetry that rests on solid theoretical grounds and allows for full calculability and therefore predictions. Full calculability is not, by itself, a sacrosanct virtue but it must be admitted that supersymmetry addresses some of the problems of the SM. Indeed, although the primary motivation for supersymmetry as implemented in the MSSM, Minimal Supersymmetric Standard Model, was to solve the hierarchy and naturalness problem it was soon realised that the model contained an excellent candidate for cold dark matter besides incorporating almost naturally the gauge unification. However, predictions of the MSSM based on tree-level calculations predict a Higgs that is lighter than the $Z$ mass. By now this is ruled out. It is only through radiative corrections that the MSSM has survived. Radiative corrections are therefore essential. Moreover the next generation of experiments at the colliders will reach unsurpassed precision which will need computations beyond the tree approximation. Extraction of the cosmological parameters that are used to measure the relic density of cold dark matter have recently reached an accuracy that will also soon compete with the accuracy we have been accustomed to from the LEP era. Precision loop calculations within the MSSM are therefore a must. It must be said that quite a lot of these calculations have been performed, even though the bulk of these have been made for collider observables and indirect precision measurements such as $(g-2)\mu, b \rightarrow s\gamma,...$ Very little has been done concerning the cross sections relevant for dark matter annihilation that enter, for example, a precise prediction of the relic density. It rests that these calculations have been done piecemeal and quite often within different renormalisation schemes.

One of the reasons that these calculations have been done piecemeal is that the MSSM, though minimal, still contains a large number of particles and a very large number of parameters especially through the soft-susy breaking terms for example. This explains why different groups have concentrated on different sectors of the model. Performing loop calculations with so large a number of parameters and huge number of interactions is an almost untractable task especially if one has to be ready to perform precision predictions for any process or at least a large number of processes as it occurs for example with the calculation of the relic density where many processes and sub-processes are at play for a particular choice of parameters. One has to rely on a fully automatised code for such calculations. SloopS is such a code with an automatisation starting already from the implementation of the model file. Instead of coding by hand all the Feynman rules which usually constitute the model file and realising that for one-loop applications one needs to also enter the full set of counterterms, SloopS relies on a much improved version of LanHEP \cite{lhep} to automatically generate the model file. Through LanHEP one writes the Lagrangian in a compact form through multiplets and the use of the superpotential. The improved
version of LanHEP has built-in rules for shifting fields and parameters thus easily generating the set of counterterms. This approach therefore takes care of generating the few thousand Feynman rules for all the vertices needed for the calculations of any one-loop process in the MSSM.

The model file thus generated is interfaced to the bundle of packages FeynArts [3], FormCalc [4] and LoopTools [5], that we will refer to as FFL for short. This code has recently been used very successfully for the first calculation of a number of processes that enter the prediction of the relic density of Dark Matter [6] as well as some one-loop induced processes of relevance for indirect detection [7].

The aim of the present paper is to first give some details on the renormalisation scheme that is implemented in SloopS and in particular how the sfermion sector and the neutralino/chargino sector are treated. This is a follow up to our paper detailing the renormalisation of the Higgs sector where apart from the implementation of the scheme we brought up crucial issues related to the definition of $\tan \beta$, the issue of gauge invariance and the impact of different schemes on observables in the Higgs sector. The present paper will also compare one-loop predictions in the sfermion and chargino/neutralino sector based on different schemes for $\tan \beta$. We will also make some interesting observations and analyses concerning the treatment of mixing in these sectors, especially how one could define a process independent mixing angle in the sfermion sector.

The paper is structured as follows. In Section 2 we give a brief summary of the renormalisation scheme used in the code for the Higgs sector and the SM-like sector that includes the gauge and fermion parts. In the same section we also present a general overview of our approach. Section 3 deals with the sfermion sector, both squarks and sleptons, that we use in SloopS. In Section 4 we detail our on-shell renormalisation scheme in the chargino/neutralino sector and comment on some alternatives for the choice of the input parameters. Section 5 illustrates the use of the code for some applications. We will give results for the one-loop corrections to the masses of the heavier neutralinos and the sfermions that are not used as input in our schemes. We also present results for the one-loop calculation of chargino pair production and sfermion pair production at a linear collider, $e^+ e^- \rightarrow \tilde{\chi}_1^\pm \tilde{\chi}_1^\mp$ and $e^+ e^- \rightarrow \tilde{\tau}_i \tilde{\tau}_j$ comparing. Finally we compare our results with those of Grace-SUSY [8] taking as examples a few decay channels of the heavier chargino for a certain choice of parameters. In all these examples the $\tan \beta$-scheme dependence is also studied thus complementing the scheme dependence that we studied for observables within the Higgs sector and for annihilation processes of interest for the relic density computations. Section 6 gives a brief summary and outlook.
2 Renormalisation: The general approach, the gauge, the fermion and the Higgs sector

Our renormalisation of the MSSM, with CP conservation with all parameters taken real, follows the same strategy and the same procedure that we adopted for the renormalisation of the Standard Model, see [9]. In particular we strive for an on-shell renormalisation of the physical parameters. Counterterms to these parameters are gauge independent. Wave function renormalisation is introduced in order that the residue of the two-point function, the propagator, is unity for the physical state on its mass shell, as well as to eliminate any mixing between the physical fields when these are on-shell so that the qualification as a physical field is maintained order by order. Naturally, these field renormalisation constants are not needed if one only requires that the observables of the \( S \)-matrix are finite but one does not insist that all the Green’s function to be finite, see [9]. On the technical side this field renormalisation avoids that one includes in the calculation of matrix elements loop corrections on the external legs. Moreover there is no need to consider field renormalisation for the unphysical fields like the Goldstones bosons or on the current fields before mixing. Talking about the Goldstone fields a very powerful feature of \textsc{SloopS} is the use and implementation of a non-linear gauge fixing condition [10, 9, 1]. The gauge-fixing condition furnishes eight gauge parameters (\( \tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\omega}, \tilde{\kappa}, \tilde{\rho}, \tilde{\epsilon}, \tilde{\gamma} \)) on which we could perform gauge parameter independence checks, beside the ultraviolet finiteness checks. The gauge-fixing writes

\[
\mathcal{L}^{GF} = -\frac{1}{\xi_W}F^+ F^- - \frac{1}{2\xi_Z} |F^Z|^2 - \frac{1}{2\xi_\gamma} |F^\gamma|^2, \text{ with}
\]

\[
F^+ = (\partial_\mu - ie\gamma_\mu - i e c_W s_W \tilde{\beta} Z_\mu) W^{\mu+} + i \xi_W e s_W (v + \tilde{\delta} h^0 + \tilde{\omega} H^0 + i \tilde{\rho} A^0 + i \tilde{\epsilon} G^0) G^+, \\
F^Z = \partial_\mu Z^\mu + \xi_Z e s_W (v + \epsilon h^0 + \gamma H^0) G^0, \\
F^\gamma = \partial_\mu \gamma^\mu.
\] (2.1)

As extensively stressed in [9] and [1] the gauge fixing term is considered renormalised. \( h^0 \) and \( H^0 \) are, respectively, the lightest and heaviest CP-even Higgses, \( A^0 \) is the CP-even Higgs, \( G^0, \pm \) are the Goldstone bosons and, \( W^\pm, Z, \gamma \) are, with obvious notations, the gauge fields. We have \( c_W \equiv \cos \theta_W = M_W/M_Z \) [4]. We work with \( \xi_{W,Z,\gamma} = 1 \) in order not to have to deal with too high a rank tensors concerning the loop libraries, see [1].

Another crucial feature of our renormalisation program is our treatment of the mixing which occurs in all sectors of the MSSM. In general, fields are expressed in the current basis. They, however, mix. Physical mass eigenstates fields are obtained from these current fields through some rotation matrix at tree-level. We consistently take, in all sectors, this matrix to be renormalised and therefore no extra counterterm is introduced to this matrix. At one-loop, this will still leave some transitions between fields, however field renormalisation

\*To avoid clutter we use some abbreviations for the trigonometric functions. For example for an angle \( \theta \), \( \cos \theta \) will be abbreviated as \( c_\theta \), etc... \( t_\beta \) will then stand for \( \tan \beta \).
is defined to precisely get rid of any residual mixing when the physical particles are on-shell. Therefore inducing counterterms for the rotation matrix is redundant and not helpful. Let us now briefly recap on the renormalisation of the gauge, fermion and Higgs sector.

2.1 The fermion and gauge sector

The fermion sector as well as the gauge sector are renormalised on-shell. It means, for example, that the gauge boson masses $M_{W^\pm}$ and $M_{Z^0}$ are defined from the pole mass, imposing the one-loop on-shell condition on the mass counterterms as

$$
\delta M_{W^\pm}^2 = -Re\Sigma^T_{W^\pm W^\pm}(M_{W^\pm}^2), \quad \delta M_{Z^0}^2 = -Re\Sigma^T_{Z^0 Z^0}(M_{Z^0}^2). \quad (2.2)
$$

The electric charge $e$ is defined in the Thomson limit. Since MSSM processes and parameters are taking place at the weak scale, the effective gauge coupling constant is of order $\alpha(M_{Z^0})$ which includes large logarithms from the very light standard model charged fermion masses. It is useful to reparameterise the one-loop corrections in terms of this effective coupling in order to absorb these large logarithms as we will see later.

2.2 The Higgs sector

The renormalisation scheme and renormalisation procedure at one-loop in the Higgs sector that we adopt in the code is detailed in Ref. [1]. The only ingredient that makes its way from the Higgs sector and the Higgs observables to the chargino/neutralino sector and the sfermion sector is the ubiquitous $t_\beta$ and its renormalisation. We use the same notation as in [9]. At tree-level, $t_\beta$ is defined by the ratio of the two vacuum expectation values $t_\beta = v_2/v_1$. At one-loop, as pointed out in Ref. [11, 1] it is difficult to find a proper definition for $t_\beta$. In [1] we critically discussed the issue of gauge invariance as regards different definition of $t_\beta$ and looked quantitatively at the scheme dependence introduced by $t_\beta$ in some Higgs observables. We will extend this investigation in our applications to observables involving the sfermions and the chargino/neutralinos. We therefore consider 4 definitions which are detailed in [1].

- $A_{\tau\tau}$-scheme.
  $t_\beta$ is extracted from the decay $A^0 \rightarrow \tau^+\tau^-$ to which the QED corrections have been subtracted. This leads to a gauge-independent counterterm. In Ref. [12] the decay of the charged Higgs boson $H^+$ into $\tau^+$ and associated neutrino $\nu_\tau$ has been suggested. This would qualify as a gauge independent definition, the advantage of our $A^0 \rightarrow \tau^+\tau^-$ is that the full QED corrections can be extracted most unambiguously.

- $MH$-scheme.
  Here the heaviest CP-even Higgs mass $M_{H^0}$ is taken as input. This definition is obviously gauge independent and process independent but unfortunately, we remarked that it induces large corrections in many cases.
• **$\overline{DR}$-scheme.**
  Here only the ultra-violet part of an observable such as $A^0 \to \tau^+\tau^-$ (or any other definition but within the linear gauge, see [1]), is extracted. In this scheme, the counterterm depends explicitly on a scale $\mu$. This scale $\mu$ is fixed at $M_{A^0}$.

• **DCPR-scheme** [13].
  $\delta t_\beta$ is extracted from the $A^0-Z^0$ transition at $q^2 = M_{A^0}^2$:
  \[
  \frac{\delta t_\beta^{DCPR}}{t_\beta} = -\frac{1}{M_Z s_{2\beta}} \text{Re} \Sigma_{A^0Z^0}(M_{A^0}^2). \quad (2.3)
  \]
  The self-energy of the $A^0 - Z^0$ transition at large $t_\beta$ is dominated by the bottom/tau loops because of the $A^0bb$ vertex which is proportional to $m_b t_\beta$ and thus enhanced when $t_\beta$ becomes large,
  \[
  \frac{\delta t_\beta^{DCPR}}{t_\beta} \approx -\frac{t_\beta}{s_{2\beta}} \frac{g^2}{c_W^2 M_Z^2} \frac{1}{4\pi^2} \left(3m_b^2B_0(M_{3A^0}^2, m_b^2, m_b^2) + m_\tau^2B_0(M_{3A^0}^2, m_\tau^2, m_\tau^2)\right). \quad (2.4)
  \]
  The loop functions $B_0$ is defined in [14]. At large $t_\beta s_{2\beta} \sim 2/t_\beta$, the finite part of $\delta t_\beta/t_\beta$ in the DCPR scheme is of order $t_\beta^2$. This scheme is not gauge independent and would depend on some parameter of the non-linear gauge for example. When comparing the results of observables within this scheme we will set all non-linear gauge parameters to zero, *i.e.* we will be specialising to the linear gauge.

### 3 The sfermion sector and its renormalisation

The sfermion sector comprises the superpartners of the fermions of the Standard Model where the interaction fields are the chiral left and right states. We do not consider generation mixing. For each generation, the field content is therefore the doublet $\tilde{Q}_L = (\tilde{u}_L, \tilde{d}_L)$ and singlets $\tilde{u}_R$ and $\tilde{d}_R$ for the squarks. For the sleptons we have $\tilde{E}_L = (\tilde{\nu}_L, \tilde{e}_L)$ and $\tilde{\nu}_R$. In case the corresponding Yukawa coupling is zero with vanishing fermion masses, we expect no $\tilde{u}_L - \tilde{u}_R$ and $\tilde{d}_L - \tilde{d}_R$ mixing, so that the physical fields are $\tilde{u}_L, \tilde{u}_R$ and $\tilde{d}_L, \tilde{d}_R$ in the squark sector. Let us briefly recall where the mass parameters of the sfermion sector originate from, and how many can be identified and defined solely within the sfermion sector, once for example the Higgs sector and gauge sector have been identified.

• The soft supersymmetry breaking terms
  \[
  \mathcal{L}_{\text{soft}}^\tilde{f} = -\sum_{\tilde{f}_i} M_{\tilde{f}_i}^2 \tilde{f}_i^* \tilde{f}_i \quad \tilde{f}_i = \tilde{Q}_L, \tilde{L}_L, \tilde{u}_R, \tilde{d}_R, \tilde{e}_R \quad (3.1)
  \]
  \[
  = - \epsilon_{ij} \left( \frac{\sqrt{2}m_u}{v_2} A_u H_2^i \tilde{Q}_L^j \tilde{u}_R + \frac{\sqrt{2}m_d}{v_1} A_d H_1^i \tilde{Q}_L^j \tilde{d}_R 
  + \frac{\sqrt{2}m_e}{v_1} A_e H_1^i \tilde{L}_L^j \tilde{e}_R + \text{h.c.} \right). \quad (3.2)
  \]
Our conventions for the Higgs doublet and the vacuum expectation values of these are defined in [1]. Supersymmetry breaking therefore provides the soft scalar masses $M^2_{\tilde{f}}$, Eq. (3.1) and the tri-linear scalar coupling $A_f$ parameters Eq. (3.2), for $f = e, u, d$ of one generation. The contribution of the latter vanishes in the chiral limit where the mass of the fermion, $m_f$, vanishes. The latter generates not only a contribution to the mass of the different sfermions but also contributes to the coupling of the sfermions to Higgses and Goldstones. As known, because of the $SU(2)$ symmetry, there is only one soft mass parameter for the up and down left component of the scalars.

- Sfermion masses get also a contribution from the usual Yukawa mass terms, these are proportional to the corresponding $m^2_f$.

- We also get contributions from the supersymmetry conserving F terms. The $F(f)$ contribution does not mix left and right explicitly (though it is proportional to the corresponding fermion masses, $m^2_f$). This only generates couplings to Higgses. The $F(H_{1,2})$ involve the $\mu$ parameter and generate supersymmetry conserving tri-linear scalar coupling. They lead to left-right mixing which is proportional to $m_f \mu$.

- There are also $D$ term contributions, chirality conserving, proportional to the gauge boson masses. These give contributions to the sfermion mass terms, $\tilde{f}\tilde{f}$, Higgs couplings $\tilde{f}\tilde{f}H, G$ and quartic scalar couplings: $\tilde{f}\tilde{f}\tilde{f}\tilde{f}$ and $\tilde{f}\tilde{f}HH$. Once the gauge and Higgs sector have been renormalised these contributions are also.

These simple observations show that since the $A_f$ terms and $\mu$ contributions do not act similarly on the mass term and the Higgs couplings of sfermions, renormalisation of the sfermion two-point functions (mass, mixing and wave function renormalisation) is not enough to completely renormalise processes with ordinary standard particles and the sfermions. One needs also to define a renormalisation to the $\mu$ parameter. This is most conveniently done from the chargino/neutralino sector. Note however that the Higgs coupling to sfermions, can provide an alternative definition to $\mu$.

3.1 **Renormalisation of the Squark sector**

We show in detail the different steps specialising to those squarks with mixing. The case with no-mixing is then trivial.

3.1.1 **Fields and parameters at tree-level**

The tree level kinetic and mass term for the squarks $\tilde{q} = \tilde{u}, \tilde{d}$ are given by,

$$L^{\tilde{q}} = -\frac{1}{2} \left( \partial^\mu \tilde{q}^*_L \partial^\nu \tilde{q}^*_R \right) \left( \partial^\mu \tilde{q}_L \partial^\nu \tilde{q}_R \right) + \left( \tilde{q}^*_L \tilde{q}^*_R \right) M^2_{\tilde{q}} \left( \tilde{q}_L \tilde{q}_R \right),$$

(3.3)
with the $2 \times 2$ non-diagonal mass matrix

\[ \mathcal{M}_2^\tilde{q} = \begin{bmatrix} M_{\tilde{q}LL} & M_{\tilde{q}LR} \\ M_{\tilde{q}RL} & M_{\tilde{q}RR} \end{bmatrix}. \]  

(3.4)

The different components of this matrix are,

\[ M_{\tilde{q}LL} = M_{\tilde{Q}L}^2 + m_q^2 + c_2\beta(T_3^q - Q_q s_W^2)M_Z^2, \]  

(3.5)

\[ M_{\tilde{q}RR} = M_{\tilde{q}R}^2 + m_q^2 + c_2\beta Q_q s_W^2 M_Z^2, \]  

(3.6)

\[ M_{\tilde{q}LR} = m_q(A_q - \mu t_\beta)^{-2T_3^q}. \]  

(3.7)

$M_{\tilde{Q}L}^2$ is the soft-SUSY-breaking mass parameter of the $SU(2)_L$ doublet, whereas $M_{\tilde{q}R}$ is the soft-SUSY-breaking mass parameter of the singlet. $T_3^q$ and $Q_q$ are the third component of the isospin and the electric charge respectively. $M_{\tilde{q}LR}$ is the mixing parameter that has contributions from both the higgsino supersymmetry conserving mass parameters and the tri-linear supersymmetry breaking term. This induces mixing between the left and right components. This mixing vanishes for sfermions associated to massless quarks but is important especially for the third family squarks. Note that this mixing can also vanish, at tree-level, even for massive quarks for exceptional $A_t = \mu/t_\beta$ for stops and $A_b = \mu t_\beta$ for sbottoms.

If $\mu$ is to be determined from the chargino/neutralino sector, this sector involves 5 new parameters, $M_{\tilde{Q}L}, M_{\tilde{Q}R}, M_{\tilde{d}R}, A_u, A_d$ and thus requires 5 renormalisation conditions. For a physical on-shell renormalisation this requires trading these Lagrangian parameters with 5 physical parameters. Owing to $SU(2)$ invariance, the soft-breaking mass parameters $M_{\tilde{Q}L}^2$ of the left-chiral scalar fermions of each isospin doublet are identical. Thus, one of the physical squark masses, say $\tilde{u}_1$, could be expressed in terms of the other masses which will be used as input. The mass of the $\tilde{u}_1$ would then receive a finite shift at the one-loop level.

In order to find the physical fields $\tilde{q}_{1,2}$, we introduce a rotation matrix $R_\tilde{q}$ such as

\[ \begin{pmatrix} \tilde{q}_1 \\ \tilde{q}_2 \end{pmatrix} = R_\tilde{q} \begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix}, \quad R_\tilde{q} = \begin{pmatrix} c_\theta_q & s_\theta_q \\ -s_\theta_q & c_\theta_q \end{pmatrix}. \]  

(3.8)

This transformation diagonalises the mass matrix $\mathcal{M}_2^\tilde{q}$,

\[ M_2^\tilde{q} = R_\tilde{q} \mathcal{M}_2^\tilde{q} R_\tilde{q}^\dagger = \text{diag}(m_{\tilde{q}_1}^2, m_{\tilde{q}_2}^2), \quad m_{\tilde{q}_1}^2 > m_{\tilde{q}_2}^2. \]  

(3.9)

The physical masses are expressed in terms of the soft-susy mass terms as

\[ m_{\tilde{q}_{1,2}} = \frac{1}{2} \left( M_{\tilde{q}LL}^2 + M_{\tilde{q}RR}^2 \right) \pm \frac{1}{2} \sqrt{(M_{\tilde{q}LL}^2 - M_{\tilde{q}RR}^2)^2 + 4(M_{\tilde{q}LR}^2)^2}. \]  

(3.10)

For further reference it is useful to express $s_2\theta_q$, the parameter that measures the amount of mixing, in terms of the Lagrangian mixing parameter and the physical masses.
\[ s_{2\theta_q} = \frac{2M_{\tilde{q}L}}{m_{\tilde{q}_1}^2 - m_{\tilde{q}_2}^2}. \]  

(3.11)

Note also the trivial fact that \( s_{2\theta_q} \) as expressed through Eq. (3.11) is regular in the limit \( m_{\tilde{q}_1}^2 \to m_{\tilde{q}_2}^2 \) since this necessarily corresponds to no mixing with \( M_{\tilde{q}L}^2 = 0 \). In this limit we can take \( \theta_q = 0 \). At tree-level \( s_{2\theta_q} \) can be accessed directly through the \( \tilde{q}_1 \to \tilde{q}_2 Z^0 \) (or \( Z^0* \to \tilde{q}_1 \tilde{q}_2 \)) which is described by the Lagrangian

\[
\mathcal{L}_{\tilde{q}_1 \tilde{q}_2 Z} = i g_Z T^a_f \frac{s_{2\theta_f}}{2} \left( \frac{\tilde{f}_1^a}{\partial} \frac{\tilde{f}_2^a}{\partial} + \tilde{f}_2^a \frac{\partial}{\partial} \tilde{f}_1^a \right) Z^0. \]  

(3.12)

Provided both parameters \( \theta_{u,d} \) have been determined along side the physical masses of \( \tilde{u}_2, \tilde{d}_2, \tilde{d}_1 \) one determines the tree-level \( \tilde{u}_1 \) mass

\[
m_{\tilde{u}_1}^2 = \frac{1}{c_{\theta_u}^2} \left( c_{\theta_d}^2 m_{\tilde{d}_1}^2 + s_{\theta_d}^2 m_{\tilde{d}_2}^2 - s_{\theta_u}^2 m_{\tilde{u}_2}^2 + m_u^2 - m_d^2 + c_{2\beta} M_W^2 \right), \]  

(3.13)

In principle we could also use all four masses as input and trade this input with one of the mixing parameters, leading to

\[
s_{2\theta_u} = \frac{c_{\theta_d}^2 m_{\tilde{d}_1}^2 + s_{\theta_d}^2 m_{\tilde{d}_2}^2 - m_{\tilde{u}_1}^2 + m_u^2 - m_d^2 + c_{2\beta} M_W^2}{m_{\tilde{u}_2}^2 - m_{\tilde{u}_1}^2}. \]  

(3.14)

Note however that the appearance of \( (m_{\tilde{u}_2}^2 - m_{\tilde{u}_1}^2) \) in the denominator makes this definition subject to large uncertainties especially for nearly degenerate masses of \( \tilde{u}_1 \) and \( \tilde{u}_2 \). The definition from a decay such as \( \tilde{u}_1 \to \tilde{u}_2 Z^0 \), if open, is more direct. This is reminiscent of our discussion about the choice of a good definition of the parameter tan \( \beta \) in [1]. Compared to the case of the neutralino/chargino system, the extraction of the underlying parameters in terms of the physical mass parameters is rather trivial. In fact the most important underlying parameter to extract here is \( A_f \) as this will be needed for the coupling to Higgses.

### 3.1.2 Counterterms

So far all fields and the parameters of the Lagrangian should be considered as bare quantities. The bare parameters for example labeled as \( \mathcal{P}_0 \) will now be split into a renormalised parameter \( \mathcal{P} \) and its counterterm \( \delta \mathcal{P} \).

It is very important to stress that the rotation matrix is defined as renormalised in our approach. This we have pursued consistently throughout all the sectors. Therefore from Eq. (3.8)

\[
\begin{pmatrix} \tilde{q}_1 \\ \tilde{q}_2 \end{pmatrix}_0 = R_{\tilde{q}} \begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix}_0, \quad \text{implies} \quad \begin{pmatrix} \tilde{q}_1 \\ \tilde{q}_2 \end{pmatrix} = R_{\tilde{q}} \begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix}. \]  

(3.15)
This allows to introduce the wave function renormalisation directly on the "physical" fields after rotation to the mass basis. These field renormalisation constants will be chosen so that one gets rid of the mixing introduced by the mass shifts, at least one of these physical particles are on their mass shell. We therefore introduce the following counterterms

\[
\tilde{q}_{i0} = (\delta_{ij} + \frac{1}{2} \delta Z_{ij}^q) \tilde{q}_j, \tag{3.16}
\]

\[
\mathcal{M}_{\tilde{q}0}^2 = \mathcal{M}_{\tilde{q}}^2 + \delta \mathcal{M}_{\tilde{q}}^2. \tag{3.17}
\]

The shifts on the parameters induce,

\[
\delta m_{\tilde{q}_{ij}}^2 = \begin{bmatrix}
\delta M_{\tilde{q}_L}^2 + \delta (m_{\tilde{q}}^2 + c_{2\beta}(T_{3q}^3 - Q_q s_W^2)M_Z^2) & \delta (m_{\tilde{q}} A_q) - \delta \left( m_{\tilde{q}} \mu t_{\beta}^{-2T_{3q}^3} \right) \\
\delta (m_{\tilde{q}} A_q) - \delta \left( m_{\tilde{q}} \mu t_{\beta}^{-2T_{3q}^3} \right) & \delta M_{\tilde{q}_R}^2 + \delta (m_{\tilde{q}}^2 + c_{2\beta} Q_q s_W^2 M_Z^2)
\end{bmatrix}. \tag{3.18}
\]

After shifting the parameters and the fields, the renormalised self-energies for the squarks are given by

\[
\hat{\Sigma}_{\tilde{q},\tilde{q}}(q^2) = \Sigma_{\tilde{q},\tilde{q}}(q^2) + \frac{1}{2}\delta Z_{ij}^q(q^2 - m_{\tilde{q}_i}^2) - \frac{1}{2}\delta Z_{ji}^q(q^2 - m_{\tilde{q}_j}^2). \tag{3.19}
\]

The counterterm \(\delta m_{\tilde{q}_{ij}}^2\) is connected to the counterterm \(\delta \mathcal{M}_{\tilde{q}}^2\) through the relation,

\[
\delta m_{\tilde{q}_{ij}}^2 = \left( R_{\tilde{q}} \delta \mathcal{M}_{\tilde{q}}^2 R_{\tilde{q}}^\dagger \right)_{ij}. \tag{3.20}
\]

### 3.1.3 Constraining the wave function renormalisation constants

The residue condition at the pole for the diagonal self-energy propagator imposes 4 conditions on the diagonal wave function renormalisation constants, for \(\tilde{q} = (\tilde{u}, \tilde{d})\):

\[
\delta Z_{11}^\tilde{q} = Re \Sigma_{\tilde{q}_1\tilde{q}_1}^\prime(m_{\tilde{q}_1}^2), \\
\delta Z_{22}^\tilde{q} = Re \Sigma_{\tilde{q}_2\tilde{q}_2}^\prime(m_{\tilde{q}_2}^2). \tag{3.21}
\]

We impose that no mixing occurs between the two squarks \(\tilde{q}_1\) and \(\tilde{q}_2\) when on-shell, constraining the non-diagonal wave function renormalisation constants accordingly:

\[
\delta Z_{12}^\tilde{q} = \frac{2}{m_{\tilde{q}_2}^2 - m_{\tilde{q}_1}^2} (Re \Sigma_{\tilde{q}_1\tilde{q}_2}(m_{\tilde{q}_2}^2) + \delta m_{\tilde{q}_{12}}^2), \\
\delta Z_{21}^\tilde{q} = \frac{2}{m_{\tilde{q}_1}^2 - m_{\tilde{q}_2}^2} (Re \Sigma_{\tilde{q}_1\tilde{q}_2}(m_{\tilde{q}_1}^2) + \delta m_{\tilde{q}_{12}}^2). \tag{3.22}
\]

In our approach, the non-diagonal wave functions are not completely determined at this stage because the mixing counterterm \(\delta m_{\tilde{q}_{12}}^2\) appears in their definitions. It is also important to point out that unless \(\delta m_{\tilde{q}_{12}}^2\) is chosen judiciously these non diagonal wave functions
are ill-defined in the limit \( m_{\tilde{q}_1}^2 \to m_{\tilde{q}_2}^2 \). For further reference it is interesting to define

\[
\delta Z_{12}^{\tilde{q}S} = \frac{1}{m_{\tilde{q}_2}^2 - m_{\tilde{q}_1}^2} \left( \text{Re} \Sigma_{\tilde{q}_1 \tilde{q}_2}(m_{\tilde{q}_2}^2) - \text{Re} \Sigma_{\tilde{q}_1 \tilde{q}_2}(m_{\tilde{q}_1}^2) \right),
\]

\[
\delta Z_{12}^{\tilde{q}A} = \frac{1}{m_{\tilde{q}_2}^2 - m_{\tilde{q}_1}^2} \left( \text{Re} \Sigma_{\tilde{q}_1 \tilde{q}_2}(m_{\tilde{q}_2}^2) + \text{Re} \Sigma_{\tilde{q}_1 \tilde{q}_2}(m_{\tilde{q}_1}^2) + 2\delta m_{\tilde{q}_{12}}^2 \right),
\]

such that

\[
\delta Z_{12}^{\tilde{q}12} = \delta Z_{12}^{\tilde{q}S} \pm \delta Z_{12}^{\tilde{q}A}.
\]

Only \( \delta Z_{12}^{\tilde{q}A} \) is now potentially singular in the limit \( m_{\tilde{q}_2}^2 \to m_{\tilde{q}_1}^2 \). We will come back to this issue when fixing a renormalisation for \( \delta m_{\tilde{q}_{12}}^2 \).

### 3.1.4 Renormalisation of the mass parameters, physical masses as input

The default scheme in SloopS takes \( m_{\tilde{d}_1} \), \( m_{\tilde{d}_2} \) and \( m_{\tilde{u}_2} \) (the lightest up-type squark) as input parameters considered to be the physical masses of \( \tilde{d}_1 \), \( \tilde{d}_2 \) and \( \tilde{u}_2 \) respectively. This fixes 3 counterterms:

\[
\delta m_{\tilde{d}_1}^2 = -\text{Re} \Sigma_{\tilde{d}_1 \tilde{d}_1}(m_{\tilde{d}_1}^2),
\]

\[
\delta m_{\tilde{d}_2}^2 = -\text{Re} \Sigma_{\tilde{d}_2 \tilde{d}_2}(m_{\tilde{d}_2}^2),
\]

\[
\delta m_{\tilde{u}_2}^2 = -\text{Re} \Sigma_{\tilde{u}_2 \tilde{u}_2}(m_{\tilde{u}_2}^2).
\]

### 3.1.5 Renormalisation of the mass parameters, the issue of the mixing parameter at one-loop

To complete the renormalisation of the squark sector for each generation, as we need 5 renormalisation conditions, we have to impose two additional conditions on what measures the mixing in the up squarks and down squarks and therefore fixes \( \delta m_{\tilde{q}_{12}}^2 \) for \( \tilde{q} = \tilde{u}, \tilde{d} \). Once this is fixed, the remaining heaviest up squark \( \tilde{u}_1 \) mass receives a finite correction at one-loop. One possibility is to define these mixing parameters through physical observables. One can for example choose the two decays \( \tilde{d}_1 \to \tilde{d}_2 Z \) and \( \tilde{d}_1 \to \tilde{u}_2 W^- \) as inputs provided they are open. This is within the spirit we have followed to define a gauge-invariant \( \tan \beta \) from the decay \( A^0 \to \tau^+ \tau^- \) \cite{ref}. This will then define \( A_d \) and \( A_u \) at one-loop respectively.

The one-loop radiative corrections to sfermions into gauge bosons have been studied in previous work \cite{ref1, ref2}. Since the issue of mixing is quite subtle with many definitions based on two-point functions being rather ad-hoc, we look at the problem rather afresh. Moreover the discussion is the same for sleptons with mixing, we therefore generalise this for sfermions in general and consider that the counterterm \( \delta m_{\tilde{f}_{12}}^2 \) absorbs the ultra-violet divergence of the decay \( \tilde{f}_1 \to \tilde{f}_2 Z^0 \). We have just seen for example that at tree-level this coupling is a direct measure of the mixing. Taking a physical observable will unravel how
to possibly extract a gauge invariant universal definition based on the two-point functions. With $\mathcal{M}_0$ representing the tree-level amplitude, $\mathcal{M}_0 = ig_Z T_f^z s_{2\theta_f}/2$, the one-loop correction can be written as

$$\mathcal{M}_1^{f_1 f_2 z^0} = \mathcal{M}_0^{f_1 f_2 z^0} \left( 1 + \delta_{V_1} f_2 z^0 + \frac{\delta e}{e} - \frac{c_{2W}}{c_W^2} \delta s_W + \frac{1}{2} \delta Z_{11} + \frac{1}{2} \delta Z_{22} \right)$$

$$+ i g_Z T_f^3 \delta_{V_2} f_2 z^0$$

$$+ i g_Z T_f^3 \left( 1 - 4 s_W^2 |Q_f| \right) \left( \frac{Re \Sigma_{f_1 f_2} (m_{f_2}^2) - Re \Sigma_{f_1 f_2} (m_{f_1}^2)}{m_{f_1}^2 - m_{f_2}^2} \right)$$

$$+ i g_Z T_f^3 c_{2\theta_f} \left( \frac{2 \delta t_{12} + \delta \delta_{f_1 f_2} (m_{f_1}^2) + Re \Sigma_{f_1 f_2} (m_{f_2}^2)}{m_{f_1}^2 - m_{f_2}^2} \right). \quad (3.26)$$

The first part of the correction proportional to the tree-level contribution is due to diagonal wave function renormalisation and renormalisation of the gauge parameters. Just like the tree-level contribution this part is regular in the limit $(m_{f_1}^2 - m_{f_2}^2) \to 0$, see the trivial remark we made after Eq. (3.11). $\delta_{V_2} f_2 z^0$ represents purely one-loop virtual corrections which do not necessarily vanish in the limit of a vanishing tree-level mixing with $\theta_f = 0$ much like the one-loop induced $f_1 \to f_2 \gamma$. The corrections in the third and fourth line of Eq. (3.26) are due to $f_1 \leftrightarrow f_2$ transitions triggered from the diagonal couplings $f_i f_i Z$.

$\mathcal{M}_1^{f_1 f_2 z^0}$ contains pure QED corrections that can be unambiguously extracted, these contain infra-red singularities that need to be combined with the bremsstrahlung corrections. Subtracting these pure QED virtual corrections and the corresponding gluonic QCD corrections defines a gauge invariant, infrared safe observable that does not depend on any experimental cut-off on the energy of the bremsstrahlung photon or gluon. Let us define this observable as $\overline{\mathcal{M}}_1^{f_1 f_2 z^0}$. $\delta m_{f_1 f_2}^2$, defined from $\overline{\mathcal{M}}_1^{f_1 f_2 z^0}$ by requiring that the one-loop correction, $(\overline{\mathcal{M}}_1^{f_1 f_2 z^0} - \mathcal{M}_0^{f_1 f_2 z^0})$, vanishes constitutes a fully gauge invariant, although process dependent, definition of $\delta m_{f_1 f_2}^2$. In this definition process dependent vertex corrections combine with self-energy contributions leading to a gauge independent definition.

Eq. (3.26) is also instructive in that it reveals how to extract a process and gauge independent definition of $\delta m_{f_1 f_2}^2$. Indeed Eq. (3.26) exhibits a specific pole structure in $(m_{f_1}^2 - m_{f_2}^2)$, the residue of the pole must be gauge independent. Therefore considering a Laurent series of the amplitude in the pole $(m_{f_1}^2 - m_{f_2}^2)$, a gauge and process independent definition based on two-point functions can be defined as

$$\delta m_{f_1 f_2}^2 = - \frac{1}{2} \lim_{m_{f_1}^2 - m_{f_2}^2} \left( Re \Sigma_{f_1 f_2} (m_{f_1}^2) + Re \Sigma_{f_1 f_2} (m_{f_2}^2) \right) \equiv - Re \Sigma_{f_1 f_2} (m_{f_1}^2, m_{f_2}^2). \quad (3.27)$$

\[1\] This is in line with the definition of the $Z^0$ mass from $e^+ e^- \to \mu^+ \mu^-$ through a Laurent series based on analyticity properties of the $S$-matrix, see [17].
The value at the pole $Re\Sigma_{f_1f_2}^P(m_{f_1}^2, m_{f_2}^2)$ is gauge-invariant and universal. All the remaining contributions in Eq. (3.20) are then regular in the limit $(m_{f_1}^2 - m_{f_2}^2) \to 0$ and in particular the contribution in the third line of Eq. (3.20). Care should be taken in defining these limits. It is useful to express $m_{f_1,2}^2$ in terms of $m_{f_\pm}^2$:

$$m_{f_\pm}^2 = \frac{m_{f_1}^2 \pm m_{f_2}^2}{2},$$  \hspace{1cm} (3.28)

in order to make the dependence in the pole $m_{f_\pm}^2$ explicit. Then $\Sigma_{f_1f_2}(m_{f_\pm}^2) = \Sigma_{\tilde{f}_1\tilde{f}_2}(m_{f_+}^2, \pm m_{f_-}^2)$, so that $\Sigma_{\tilde{f}_1\tilde{f}_2}(m_{f_\pm}^2)$ is a function of these two variables. These functions should be expanded in $m_{f_\pm}^2$, such that

$$\Sigma_{\tilde{f}_1\tilde{f}_2}(m_{f_\pm}^2, \pm m_{f_-}^2) = \Sigma_{\tilde{f}_1\tilde{f}_2}(m_{f_+}^2, 0) \pm m_{f_-}^2 \frac{\partial \Sigma'_{\tilde{f}_1\tilde{f}_2}(m_{f_+}^2, 0)}{\partial m_{f_-}^2} + \ldots$$  \hspace{1cm} (3.29)

We then have

$$\frac{Re\Sigma_{\tilde{f}_1\tilde{f}_2}(m_{f_1}^2) + Re\Sigma_{\tilde{f}_1\tilde{f}_2}(m_{f_2}^2)}{m_{f_1}^2 - m_{f_2}^2} = \frac{Re\Sigma_{\tilde{f}_1\tilde{f}_2}(m_{f_+}^2, 0)}{m_{f_-}^2} + \frac{m_{f_-}^2}{2} Re\Sigma'_{\tilde{f}_1\tilde{f}_2}(m_{f_+}^2, 0) + O((m_{f_-}^2)^3)$$

$$\frac{Re\Sigma_{\tilde{f}_1\tilde{f}_2}(m_{f_1}^2) - Re\Sigma_{\tilde{f}_1\tilde{f}_2}(m_{f_2}^2)}{m_{f_1}^2 - m_{f_2}^2} = Re\Sigma'_{\tilde{f}_1\tilde{f}_2}(m_{f_+}^2, 0) + O((m_{f_-}^2)^2).$$  \hspace{1cm} (3.30)

We can identify

$$Re\Sigma_{\tilde{f}_1\tilde{f}_2}^P(m_{f_1}^2, m_{f_2}^2) = Re\Sigma_{\tilde{f}_1\tilde{f}_2}(m_{f_+}^2, 0).$$  \hspace{1cm} (3.31)

By looking at the pole structure of the amplitude it is now clear that $Re\Sigma_{\tilde{f}_1\tilde{f}_2}(m_{f_+}^2, 0)$ is gauge independent. However, $Re\Sigma'_{\tilde{f}_1\tilde{f}_2}(m_{f_+}^2, 0)$ in Eq. (3.30), for example, is not guaranteed to be gauge independent. Its gauge dependent part cancels against those contained in the vertex corrections.

One should be aware not to systematically equate

$$Re\Sigma_{\tilde{f}_1\tilde{f}_2}(m_{f_+}^2, 0) = Re\Sigma_{\tilde{f}_1\tilde{f}_2}(p^2 = m_{f_+}^2).$$  \hspace{1cm} (3.32)

Indeed a naive replacement $Re\Sigma_{\tilde{f}_1\tilde{f}_2}(p^2 = m_{f_+}^2)$ may still give extra contributions that are of order $m_{f_-}^2$. This is exactly what happens when we calculate $Re\Sigma_{\tilde{f}_1\tilde{f}_2}(p^2)$ in a gauge which is not the Feynman gauge with $\xi_{W,Z} \neq 1$. One finds that the gauge dependent part of the quantity $Re\Sigma_{\tilde{f}_1\tilde{f}_2}(p^2 = m_{f_+}^2)$ proportional to $(1 - \xi_{W,Z})$ are of order $m_{f_-}^2$, see [18, 19]. Let us mention that the choice based on $Re\Sigma_{\tilde{f}_1\tilde{f}_2}(p^2 = m_{f_+}^2)$ had been advocated to improve the scale independence of the mixing angle [18]. Note that after the renormalisation of the mixing has been set according to Eqs. (3.27, 3.31),
the last term in Eq. (3.26) contributes an ultraviolet finite part. This, on the other
hand, is not the case of the contribution from the third line in Eq. (3.26). Indeed its
$\text{Re} \Sigma'_{f_1 f_2} (p^2 = m^2_{f_+})$ might still be needed to absorb possible infinities from the vertex virtual corrections for example.

In $\text{SloopS}$ we work in the Feynman gauge with $\xi_W = \xi_Z = 1$. At one-loop $\Sigma_{f_i f_j}$ is insensitive to the non-linear gauge parameters in Eq. (2.1). We therefore obtain the same result for $\Sigma_{f_i f_j}$ as in the usual linear gauge within the Feynman gauge. Therefore one can afford using Eq. (3.32). Taking this into account with Eq. (3.27) and Eq. (3.31), the default scheme in $\text{SloopS}$ is

$$ \delta m^2_{f_1 f_2} = - \text{Re} \Sigma_{f_1 f_2} (p^2 = m^2_{f_+}). $$

(3.33)

To compare with results in the literature we have also implemented the prescription,

$$ \delta m^2_{f_1 f_2} = - \frac{1}{2} \left( \text{Re} \Sigma_{f_1 f_2} (m^2_{f_1}) + \text{Re} \Sigma_{f_1 f_2} (m^2_{f_2}) \right). $$

(3.34)

which is equivalent to the condition introduced in Ref. [20]. In the Feynman gauge the difference with the default scheme is ultraviolet safe and numerically small, see the examples in Section 5.1 and 5.5. As we stressed repeatedly, in our approach we do not introduce counterterms to the rotation matrices since non-diagonal wave function renormalisation is necessary in any case. For the sfermions this reveals more easily the correct prescription to take for the renormalisation of the mixing parameter. In practically all other approaches counterterms to mixing matrices are introduced and therefore $\theta_f \rightarrow \theta_f + \delta \theta_f$. We can recover these approaches by, for example, looking at the example of $\tilde{f}_1 \rightarrow \tilde{f}_2 Z^0$ and considering the shift to the angle, rather than introducing the shift $\delta m^2_{f_1 f_2}$ indirectly through the non-diagonal wave function renormalisation constants. From $\delta s_{2\theta_f} = 2 c_{2\theta_f} \delta \theta_f$ we make the identification

$$ \delta \theta_f = \frac{\delta m^2_{f_1 f_2}}{m^2_{f_1} - m^2_{f_2}}. $$

(3.35)

3.1.6 SUSY QCD corrections and the squark mixing angle

There have been many proposals in defining this angle or alternatively the mixing parameter when considering purely supersymmetric QCD corrections. The different proposals relied on constraining the mixing angle, Eq. (3.35), through a combination of two-point functions in order that some specific observable be finite. This rather ad hoc approach would of course guarantee finiteness for that observables but does not necessarily guarantee that this observable or quantity is gauge invariant with this choice of counterterm. What is worse is that if one uses the same prescription when considering one-loop electroweak corrections to the same quantity even finiteness is lost. The prescription based on the residue of the pole would have given the correct procedure. The aim of this subsection is to understand
why finiteness is obtained in the case of supersymmetric QCD corrections. Pure QCD contributions to \( \Sigma_{\tilde{q}_1 \tilde{q}_2} \) are from the gluino \( \tilde{g} \) exchange self energies and the tadpole squark exchange. The results can be written in a very compact form, see for example \[21\]

\[
\Sigma_{\tilde{g} \tilde{q}_2}(p^2) = \frac{4\alpha_s}{3\pi} m_{\tilde{g}} m_q c_{2\theta_q} B_0(p^2, m_{\tilde{g}}, m_q),
\]

\[
\Sigma_{\tilde{q} \tilde{q}_2}(p^2) = \frac{\alpha_s}{3\pi} c_{2\theta_q} s_{2\theta_q} \left( A_0(m_{\tilde{q}_2}^2) - A_0(m_{\tilde{q}_1}^2) \right). \tag{3.36}
\]

The loop functions \( A_0 \) and \( B_0 \) are as defined in \[14\]. It is evident that \( \Sigma_{\tilde{q} \tilde{q}_2}(p^2) \) is of order \( m_{\tilde{q}_2}^2 - m_{\tilde{q}_1}^2 \). It independently vanishes for \( s_{2\theta_q} \rightarrow 0 \). Note that the QCD contribution of the gluino does not depend on the squark masses for a general \( p^2 \). Therefore, \( \Sigma_{\tilde{g} \tilde{q}_2}(m_{\tilde{q}_1}^2) - \Sigma_{\tilde{g} \tilde{q}_2}(m_{\tilde{q}_2}^2) \) is finite. This explains why different schemes work fine, in the sense of leading to finite results, for SUSY QCD corrections to processes involving squarks. One of the most complicated is based on tuning combinations of \( \Sigma_{\tilde{q} \tilde{q}_2} \) such that a finite results for \( e^+e^- \rightarrow \tilde{q}_i \tilde{q}_j \) obtains as far as QCD corrections are concerned \[22\]. With the coupling of the \( Z \) to squarks defined as \( c_{ij} \) for \( Z \tilde{q}_i \tilde{q}_j \), the following combination is used to define the counterterm,

\[
\frac{c_{22} Re \Sigma_{\tilde{q}_1 \tilde{q}_2}(m_{\tilde{q}_1}^2) - c_{11} Re \Sigma_{\tilde{q}_1 \tilde{q}_2}(m_{\tilde{q}_1}^2)}{c_{22} - c_{11}} \tag{3.37}
\]

This can be rewritten as

\[
\frac{c_{22} Re \Sigma_{\tilde{q}_1 \tilde{q}_2}(m_{\tilde{q}_1}^2) - c_{11} Re \Sigma_{\tilde{q}_1 \tilde{q}_2}(m_{\tilde{q}_1}^2)}{c_{22} - c_{11}} = \frac{Re \Sigma_{\tilde{q}_1 \tilde{q}_2}(m_{\tilde{q}_1}^2) + Re \Sigma_{\tilde{q}_1 \tilde{q}_2}(m_{\tilde{q}_2}^2)}{2} \tag{3.38}
\]

\[
+ \frac{c_{22} + c_{11} Re \Sigma_{\tilde{q}_1 \tilde{q}_2}(m_{\tilde{q}_1}^2) - Re \Sigma_{\tilde{q}_1 \tilde{q}_2}(m_{\tilde{q}_2}^2)}{c_{22} - c_{11}} \frac{2}{2}.
\]

The much simpler scheme based on the use of \( Re \Sigma_{\tilde{q}_1 \tilde{q}_2}(m_{\tilde{q}_1}^2) \) \[23\] is in fact a very special case of the scheme in Eq. \[3.38\], we can see that it is obtained as \( c_{11} \rightarrow 0 \) in Eq. \[3.38\]. For the electroweak case the extra terms proportional to \( Re \Sigma_{\tilde{q}_1 \tilde{q}_2}(m_{\tilde{q}_1}^2) - Re \Sigma_{\tilde{q}_1 \tilde{q}_2}(m_{\tilde{q}_2}^2) \) in Eq. \[3.38\] are not finite apart from the gauge invariance issue. However as we have seen the ultraviolet divergent part can be cancelled in \( (Re \Sigma_{\tilde{q}_1 \tilde{q}_2}(m_{\tilde{q}_1}^2) + Re \Sigma_{\tilde{q}_1 \tilde{q}_2}(m_{\tilde{q}_2}^2))/2 \) as suggested in \[24\]. However this suggestion was not based on a very strong theoretical or physical argument apart from it being more symmetric or democratic in the two squarks.

### 3.1.7 Deriving the counterterms

We are now in a position to derive all the needed counterterms. First of all with both prescriptions for \( \delta m_{\tilde{q}_1 \tilde{q}_2} \) either based on Eq. \[3.33\] or the naive Eq. \[3.34\], the non-diagonal wave function renormalisation constants \( \delta Z_{ij}^q \) and \( \delta Z_{ij}^d \) are now regular in the limit \( m_{\tilde{q}_i}^2 \rightarrow \)
\[ m^2_{\tilde{q}_2}, \] where any potential ultraviolet divergence is contained in \( \delta Z^{\tilde{q},S}_{12} \). In fact in the scheme of Eq. (3.34) only this part remains and therefore \( \delta Z^{\tilde{q}}_{12} = \delta Z^{\tilde{q},S}_{12} \).

The remaining counterterm \( \delta m_{\tilde{u}_{11}} \) is completely constrained,

\[
\delta m^2_{\tilde{u}_{11}} = \frac{1}{c_{\theta_u}^2} \left( c_{\theta_d}^2 \delta m^2_{d_{11}} + s_{\theta_d}^2 \delta m^2_{d_{22}} - s_{2\theta_d} \delta m^2_{d_{12}} - s_{\theta_d}^2 \delta m^2_{u_{12}} + s_{2\theta_d} \delta m^2_{u_{12}} + \delta m_u^2 - \delta m_d^2 + M_W^2 \left( c_{2\beta} \frac{\delta M_W^2}{M_W^2} - s_{2\beta} \frac{\delta t_\beta}{t_\beta} \right) \right),
\]

(3.39)

For \( c_{\theta_u}^2 \ll 1 \) this scheme is not appropriate as it will induce large radiative corrections. One should prefer the use of \( m^2_{\tilde{u}_1} \) as input parameter in lieu of \( m^2_{\tilde{u}_2} \). With \( m^2_{\tilde{u}_2} \) as input parameter the physical mass of \( \tilde{u}_1 \) will then receive a finite correction at one-loop,

\[
m^2_{\tilde{u}_1} = m^2_{\tilde{u}_1} + \delta m^2_{\tilde{u}_{11}} + Re \Sigma_{\tilde{u}_1\tilde{u}_1}(m^2_{\tilde{u}_1}).
\]

(3.40)

Alternatively we can use \( m_{\tilde{u}_1} \) as input like we have done with the other squark masses. This will allow to define \( t_\beta \) from the sfermion sector through

\[
\frac{\delta t_\beta}{t_\beta} = \frac{1}{\frac{1}{2} s_{2\beta} M_W^2} \left( c_{\theta_d}^2 \delta m^2_{d_{11}} + s_{\theta_d}^2 \delta m^2_{d_{22}} - s_{2\theta_d} \delta m^2_{d_{12}} - \delta m_d^2 \right.
- \left. c_{\theta_d}^2 \delta m^2_{u_{11}} + s_{\theta_d}^2 \delta m^2_{u_{22}} - s_{2\theta_d} \delta m^2_{u_{12}} + \delta m_u^2 + c_{2\beta} \delta M_W^2 \right).
\]

(3.41)

Using Eq. (3.20), we find the relations between the counterterms \( \delta m^2_{\tilde{q}_{ij}} \) and the counterterms \( \delta M_{Q_L}, \delta M_{\tilde{u}_R}, \delta M_{\tilde{d}_R}, \delta A_u \) and \( \delta A_d \) of the underlying parameters at the Lagrangian level

\[
\delta M_{Q_L} = \frac{1}{2M_{Q_L}^2} \left( c_{\theta_d}^2 \delta m^2_{d_{11}} + s_{\theta_d}^2 \delta m^2_{d_{22}} - s_{2\theta_d} \delta m^2_{d_{12}} - \delta m_d^2 \right.
- \left. M_Z^2 \left( -\frac{1}{2} + \frac{1}{3} s_W^2 \right) \left( c_{2\beta} \frac{\delta M_Z^2}{M_Z^2} + s_{2\beta} \frac{\delta t_\beta}{t_\beta} \right) - c_{2\beta} \frac{1}{3} M_Z^2 \delta s_W^2 \right),
\]

\[
\delta M_{\tilde{u}_R} = \frac{1}{2M_{\tilde{u}_R}^2} \left( s_{\theta_d}^2 \delta m^2_{u_{11}} + c_{\theta_d}^2 \delta m^2_{u_{22}} + s_{2\theta_d} \delta m^2_{u_{12}} - \delta m_u^2 \right.
- \left. \frac{2}{3} s_W^2 M_Z^2 \left( c_{2\beta} \frac{\delta M_Z^2}{M_Z^2} + s_{2\beta} \frac{\delta s_W^2}{s_W^2} \right) - c_{2\beta} \frac{1}{3} M_Z^2 \delta s_W^2 \right),
\]

\[
\delta M_{\tilde{d}_R} = \frac{1}{2M_{\tilde{d}_R}^2} \left( s_{\theta_d}^2 \delta m^2_{d_{11}} + c_{\theta_d}^2 \delta m^2_{d_{22}} + s_{2\theta_d} \delta m^2_{d_{12}} - 2m_d \delta m_d \right.
+ \left. \frac{1}{3} s_W^2 M_Z^2 \left( c_{2\beta} \frac{\delta M_Z^2}{M_Z^2} + s_{2\beta} \frac{\delta s_W^2}{s_W^2} \right) - c_{2\beta} \frac{1}{3} M_Z^2 \delta s_W^2 \right),
\]

\[
\delta (m_u A_u) = \frac{s_{2\theta_u}}{2} \left( \delta m^2_{u_{11}} - \delta m^2_{u_{22}} \right) + c_{2\theta_u} \delta m^2_{u_{12}} + m_u \frac{t_\beta}{t_\beta} \left( \delta \mu + \mu \frac{\delta m_u}{m_u} - \delta t_\beta t_\beta \right),
\]

\[
\delta (m_d A_d) = \frac{s_{2\theta_d}}{2} \left( \delta m^2_{d_{11}} - \delta m^2_{d_{22}} \right) + c_{2\theta_d} \delta m^2_{d_{12}} + m_d t_\beta \left( \delta \mu + \mu \frac{\delta m_d}{m_d} + \frac{\delta t_\beta}{t_\beta} \right).
\]

(3.42)
3.2 Slepton sector

After having shown in detail how the squark sector is renormalised in the case of mixing, it is straightforward to treat the slepton sector. Again for the sleptons, the case with mixing is for all practical purposes only relevant for the $\tilde{\tau}$. In the code we have implemented mixing for all generations, in the first and second generation this is used only in to conduct high precision checks on the results, for applications the unmixed case is used. Here we will show only the case with mixing, the unmixed case is then trivial.

Compared to the squark sector, as seen from Eq. (3.2), one has, for each generation, only 3 parameters: $M_{\tilde{L}_L}, M_{\tilde{\nu}_R}, A_e$ and one field is missing, $\tilde{\nu}_R$. $\tilde{e}_L$ and $\tilde{e}_R$ will mix leading to the physical fields $\tilde{e}_1$ and $\tilde{e}_2$. In the unmixed case we associate $\tilde{e}_1$ with $\tilde{e}_L$. The mixing matrix is described in exactly the same way as in Eq. (3.4) with $\tilde{q} \rightarrow \tilde{e}$ and the different components given by Eqs. (3.5) - (3.7) with $Q \rightarrow \tilde{L}$ with the corresponding quantum charges. Shifting the fields and parameters we can write the self-energies (in the case of $\Sigma$ diagonal and non-diagonal) as

$$
\hat{\Sigma}_{\tilde{e}_{i} \tilde{e}_{j}}(q^2) = \Sigma_{\tilde{e}_{i} \tilde{e}_{j}}(q^2) + \delta m_{\tilde{e}_{ij}}^2 + \frac{1}{2} \delta Z_{ij}^\tilde{e}(q^2 - m_{\tilde{e}_{i}}^2) + \frac{1}{2} \delta Z_{ij}^\tilde{e}(q^2 - m_{\tilde{e}_{j}}^2),
$$

$$
\hat{\Sigma}_\tilde{\nu}(q^2) = \Sigma_\tilde{\nu}(q^2) + \delta m_\tilde{\nu}^2 - \delta Z_\tilde{\nu}(q^2 - m_\tilde{\nu}^2).
$$

(3.43)

We take the physical selectron masses as input parameters through the usual on-shell condition. We require the residue of the propagators of $\tilde{e}_i$ and $\tilde{\nu}$ to be equal to unity and no mixing between $\tilde{e}_1$ and $\tilde{e}_2$ when these are on-shell. These conditions imply

$$
\delta m_{\tilde{e}_{ii}}^2 = -Re \Sigma_{\tilde{e}_{i} \tilde{e}_{i}}(m_{\tilde{e}_{i}}^2),
$$

$$
\delta Z_{ii}^\tilde{e} = Re \Sigma'_{\tilde{e}_{i} \tilde{e}_{i}}(m_{\tilde{e}_{i}}^2),
$$

$$
\delta Z_\tilde{\nu} = Re \Sigma'_{\tilde{\nu}}(m_\tilde{\nu}^2),
$$

$$
\delta Z_{12}^\tilde{e} = \frac{2}{m_{\tilde{e}_{2}}^2 - m_{\tilde{e}_{1}}^2} \left(Re \Sigma_{\tilde{e}_{1} \tilde{e}_{2}}(m_{\tilde{e}_{2}}^2) + \delta m_{\tilde{e}_{12}}^2\right),
$$

$$
\delta Z_{21}^\tilde{e} = \frac{2}{m_{\tilde{e}_{1}}^2 - m_{\tilde{e}_{2}}^2} \left(Re \Sigma_{\tilde{e}_{1} \tilde{e}_{2}}(m_{\tilde{e}_{1}}^2) + \delta m_{\tilde{e}_{12}}^2\right).
$$

(3.44)

The remaining parameter $\delta m_{\tilde{e}_{12}}^2$ describing mixing is fixed analogously as in the squark sector. The default scheme in SloopS is

$$
\delta m_{\tilde{e}_{12}}^2 = -Re \Sigma_{\tilde{e}_{1} \tilde{e}_{2}}((m_{\tilde{e}_{1}}^2 + m_{\tilde{e}_{2}}^2)/2).
$$

(3.45)

As in the squark sector, a better definition would be to relate this counterterm to a physical observable like the slepton decay $\tilde{e}_1 \rightarrow \tilde{e}_2 Z^0$ for example, see (3.26). The naive scheme

$$
\delta m_{\tilde{e}_{12}}^2 = -\frac{1}{2} \left(Re \Sigma_{\tilde{e}_{1} \tilde{e}_{2}}(m_{\tilde{e}_{1}}^2) + Re \Sigma_{\tilde{e}_{1} \tilde{e}_{2}}(m_{\tilde{e}_{2}}^2)\right),
$$

(3.46)

is also implemented. Another possible scheme uses the mass of the sneutrino as an input parameter such that

$$
\delta m_\tilde{\nu}^2 = -Re \Sigma_\tilde{\nu}(m_\tilde{\nu}^2),
$$

(3.47)
and the counterterm $\delta m_{e_{12}}^2$ is given by,

$$
\delta m_{e_{12}}^2 = \frac{1}{s_{2\theta_e}} \left( c_{\theta_e}^2 \delta m_{e_{11}}^2 + s_{\theta_e}^2 \delta m_{e_{22}}^2 - \delta m_{\nu}^2 - \delta m_e^2 + M_W^2 \left( c_{2\beta} \frac{\delta M_Z^2}{M_Z^2} - s_{2\beta} \frac{\delta t_\beta}{t_\beta} \right) \right). \tag{3.48}
$$

However this definition is to be avoided since the mixing in the slepton sector is usually very small, $s_{2\theta_e} \sim 0$, even for $\tau$’s which would lead to large corrections.

The extraction of the counterterms of the parameters at the Lagrangian follows

$$
\delta M_{LL} = \frac{1}{2 M_{LL}} \left( c_{\theta_e}^2 \delta m_{e_{11}}^2 + s_{\theta_e}^2 \delta m_{e_{22}}^2 - s_{2\theta_e} \delta m_{e_{12}}^2 - \delta m_e^2 - M_Z^2 \left( -\frac{1}{2} + s_W^2 \right) \left( c_{2\beta} \frac{\delta M_Z^2}{M_Z^2} - s_{2\beta} \frac{\delta t_\beta}{t_\beta} \right) - c_{2\beta} M_Z^2 \delta s_W^2 \right),
$$

$$
\delta M_{e_R} = \frac{1}{2 M_{e_R}} \left( s_{\theta_e}^2 \delta m_{e_{11}}^2 + c_{\theta_e}^2 \delta m_{e_{22}}^2 + s_{2\theta_e} \delta m_{e_{12}}^2 - \delta m_e^2 + s_W^2 M_Z^2 \left( c_{2\beta} \left( \frac{\delta M_Z^2}{M_Z^2} + \frac{\delta s_W^2}{s_W^2} \right) - s_{2\beta} \frac{\delta t_\beta}{t_\beta} \right) \right),
$$

$$
\delta (m_e A_e) = \frac{s_{2\theta_e}}{2} \left( \delta m_{e_{11}}^2 - \delta m_{e_{22}}^2 \right) + c_{2\theta_e} \delta m_{e_{12}}^2 + m_e \mu t_\beta \left( \frac{\delta \mu}{\mu} + \frac{\delta m_e}{m_e} + \frac{\delta t_\beta}{t_\beta} \right). \tag{3.49}
$$

If the sneutrino mass is not used as input then it is predicted with a finite correction from its value at tree-level.

$$
m_{\tilde{\nu}}^{1\text{-loop}} = m_{\tilde{\nu}}^{\text{tree}} + \left( \text{Re} \Sigma_{\tilde{\nu}}(m_{\tilde{\nu}}^{\text{tree}}) - \text{Re} \Sigma_{\tilde{e}_{1\tilde{e}_1}}(m_{e_{11}}^2) \right) + M_W^2 \left( c_{2\beta} \frac{\delta M_Z^2}{M_Z^2} - s_{2\beta} \frac{\delta t_\beta}{t_\beta} \right) + s_{\theta_e} \left( \text{Re} \Sigma_{\tilde{e}_{1\tilde{e}_1}}(m_{e_{11}}^2) - \text{Re} \Sigma_{\tilde{e}_{2\tilde{e}_2}}(m_{e_{22}}^2) \right) - s_{2\theta_e} \delta m_{e_{12}}^2 - \delta m_e^2. \tag{3.50}
$$

In the limit of massless fermions, the term in the second line vanishes and we identify, as said earlier, $\tilde{e}_1$ with $\tilde{e}_L$. This is a very good limit for the selectron and smuon sector but we have to consistently take the electron and muon Yukawa couplings to zero.

## 4 The chargino/neutralino sector and its renormalisation

### 4.1 Fields and parameters

The charginos and neutralinos are mixtures of the spin-1/2 fermions which are part, on the one hand, of the two Higgses chiral multiplets, $\tilde{H}_{1,2}$ which constitute the higgsinos, and on the other hand, the electroweak gauginos within the gauge supermultiplet for the
\(U(1)\) and \(SU(2)\) gauge groups of the Standard Model. In terms of the two-component (left-handed) Weyl spinors the two higgsino doublets in accordance with our definition in the Higgs sector \([1]\) are

\[
\tilde{H}_1 = (\tilde{H}_0^1, \tilde{H}_-^1), \quad \tilde{H}_2 = (\tilde{H}_+^2, \tilde{H}_0^2).
\]

We denote the \(U(1)\) gaugino (bino) as \(\tilde{B}_0\) and the \(SU(2)\) one as \(\tilde{W}_i, i = 0, 1, 2\) with \(\tilde{W}_i^\pm = \frac{1}{2}(\tilde{W}_i^1 \mp i\tilde{W}_i^2)\). Due to electroweak symmetry breaking the electrically charged components will mix and lead to the charginos that will be collected as Dirac spinors \(\tilde{\chi}_1^\pm\), \(\tilde{\chi}_2^\pm\), while the electrically neutral ones will mix leading to the neutralinos that will be described as Majorana fermions, \(\tilde{\chi}_1^0, \tilde{\chi}_2^0\), \(\tilde{\chi}_3^0, \tilde{\chi}_4^0\). In this sector soft masses enter only through the soft masses of the gauginos

\[
L_{\text{soft}}^\tilde{V} = -\frac{1}{2} \left( M_1 \tilde{B}_0 \tilde{B}_0^0 + M_2 \sum_i \tilde{W}_i^i \tilde{W}_i^i \right), \quad (4.1)
\]

which is the only source of mass for the gauginos before electroweak symmetry breaking. The higgsinos get a mass from the supersymmetry preserving \(\mu\) term in the superpotential

\[
L_\mu^\tilde{H} = \mu \epsilon_{ij} \tilde{H}_i^1 \tilde{H}_j^1 + h.c. \quad (4.2)
\]

Supersymmetric gauge matter interactions lead to mass mixing terms between these states after symmetry breaking through

\[
L_{\text{mix}}^{\tilde{H}, \tilde{V}} = -\frac{1}{\sqrt{2}} \left( \tilde{H}_1^i (g W^i \tau^i - g' \tilde{B}_0) \tilde{H}_1^1 + \tilde{H}_2^i (g \tilde{W}_i^i \tau^i + g' \tilde{B}_0) \tilde{H}_2^1 + h.c. \right), \quad (4.3)
\]

with \(\tau^i\) the Pauli matrices. At this point let us give our convention on the sign of the parameters \(\mu, M_1, M_2\). We can always take \(M_2 > 0\) since any other phase can be transformed away by a field redefinition in Eq. (4.1), however because of the mixing term in Eq. (4.3) we loose the freedom to redefine the phases of the Higgsino and bino fields and hence the sign of \(\mu\) and \(M_1\).

The kinetic term in terms of the current fields writes as

\[
L_{\text{kin}} = i \overline{\tilde{W}}^i \sigma^\mu (\partial_\mu \tilde{W}^i) + i \overline{\tilde{B}}^0 \sigma^\mu (\partial_\mu \tilde{B}^0) + i \overline{\tilde{H}}^i_1 \sigma^\mu (\partial_\mu \tilde{H}^1_1) + i \overline{\tilde{H}}^i_2 \sigma^\mu (\partial_\mu \tilde{H}^1_2). \quad (4.4)
\]

Collecting all terms in the chargino mass matrix and defining

\[
\psi_R^c = \begin{pmatrix} \tilde{W}^-_1 \\ \tilde{H}^-_1 \end{pmatrix}, \quad \psi_L^c = \begin{pmatrix} \tilde{W}^+_1 \\ \tilde{H}^+_2 \end{pmatrix}, \quad (4.5)
\]

leads to

\[
L^c = i \left[ \psi_R^c t^\mu \partial_\mu \overline{\psi}_R^c + \overline{\psi}_L^c t^\mu \partial_\mu \psi_L^c \right] - \left[ \psi_R^c t^X \psi_L^c + \overline{\psi}_L^c t^X \overline{\psi}_R^c \right], \quad (4.6)
\]

\(t^\) stands for the transpose operation and the mass mixing matrix is given by

\[
X = \begin{pmatrix} M_2 & \sqrt{2} M_W s_\beta \\ \sqrt{2} M_W c_\beta & \mu \end{pmatrix}. \quad (4.7)
\]
The system can be diagonalised by two unitary matrices $U$ and $V$ that define the physical (Weyl) fields as

$$\chi_R^c = U \psi_R^c, \quad \chi_L^c = V \psi_L^c.$$  \hfill (4.8)

In the case of $CP$ conservation that we will cover here, we can take both $U$ and $V$ real. We write the diagonalised mass matrix $\tilde{X}$

$$\tilde{X} = U X V^t = \tilde{X}^t = V X^t U = \begin{pmatrix} m_{\tilde{\chi}_1^±} & 0 \\ 0 & m_{\tilde{\chi}_2^±} \end{pmatrix}. \hfill (4.9)$$

$m_{\tilde{\chi}_{1,2}^±}$ are the (positive) eigenvalues of the hermitian matrix $XX^\dagger$ with $m_{\tilde{\chi}_1^±} < m_{\tilde{\chi}_2^±}$. In our implementation in order to have positive eigenvalues we take

$$\det U = +1 \text{ and } \det V = \text{sign}(\det X) = \epsilon_\mu \quad \text{with} \quad \det X = M_2 \mu - M_W^2 s_{2\beta}. \hfill (4.10)$$

The physical masses are also defined from the invariant basis independent quantities that are the trace and the determinant of the square matrix $XX^t$, which give

$$m_{\tilde{\chi}_1^±}^2 + m_{\tilde{\chi}_2^±}^2 = M_2^2 + \mu^2 + 2M_W^2,$$

$$m_{\tilde{\chi}_1^±} m_{\tilde{\chi}_2^±} = (\det X)^2, \hfill (4.11)$$

and

$$m_{\tilde{\chi}_{1,2}^±}^2 = \frac{1}{2} \left( M_2^2 + \mu^2 + 2M_W^2 \right.\$$

$$\left. + \left[ (M_2^2 - \mu^2)^2 + 4M_W^4 c_{2\beta}^2 + 4M_W^2 (M_2^2 + \mu^2 + 2\mu M_2 s_{2\beta}) \right]^{\frac{1}{2}} \right). \hfill (4.12)$$

The corresponding chargino Dirac spinor $\tilde{\chi}_i^c (i = 1, 2)$ is constructed as

$$\tilde{\chi}_i^+ = \begin{pmatrix} \chi_{L_i}^c \\ \chi_{R_i}^c \end{pmatrix} \quad \Rightarrow \quad \overline{\chi}_i^+ = \begin{pmatrix} \chi_{R_i}^t \\ \chi_{L_i}^t \end{pmatrix} = \tilde{\chi}^{-t}_i \quad i = 1, 2. \hfill (4.13)$$

Similarly the Lagrangian for neutralinos writes

$$\mathcal{L}^n = \frac{i}{2} [\psi^n t \sigma^\mu \partial_\mu \overline{\psi}^n + \overline{\psi}^a t \sigma^\mu \partial_\mu \psi^n] - \frac{1}{2} [\psi^n t Y \psi^n + \overline{\psi}^a t Y^\dagger \overline{\psi}^a], \hfill (4.14)$$

where

$$\psi^n = \begin{pmatrix} \tilde{B}^0 \\ \tilde{W}^0 \\ \tilde{H}_1^0 \\ \tilde{H}_2^0 \end{pmatrix}. \hfill (4.15)$$
The mass matrix \( Y \)
\[
Y = \begin{pmatrix}
M_1 & 0 & -M_Z s_W c_\beta & M_Z s_W s_\beta \\
0 & M_2 & M_Z c_W c_\beta & -M_Z c_W s_\beta \\
-M_Z s_W c_\beta & M_Z c_W c_\beta & 0 & -\mu \\
M_Z s_W s_\beta & -M_Z c_W s_\beta & -\mu & 0
\end{pmatrix},
\]
(4.16)
can be diagonalized by an unitary complex matrix with the physical states being
\[
\chi^n = N\psi^n.
\]
(4.17)

We will refer to the diagonal matrix as
\[
\tilde{Y} = N^* Y N^\dagger = \text{diag}(m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_4^0}), \quad 0 < m_{\tilde{\chi}_1^0} < m_{\tilde{\chi}_2^0} < m_{\tilde{\chi}_3^0} < m_{\tilde{\chi}_4^0}.
\]
(4.18)

Note that \( N \) can be written as \( J\hat{N} \) where \( \hat{N} \) is real and \( J = \text{diag}(j_1, j_2, j_3, j_4) \). \( \hat{N} \) diagonalises \( Y \) but leads to masses that are not necessarily positive. A positive mass obtained with \( \hat{N} \) corresponds to \( j_i = 1 \), a negative mass corresponds to \( j_i = i \). The corresponding neutralino (4-component) Majorana spinor \( \tilde{\chi}_i^0 (i = 1, 2, 3, 4) \) is given by
\[
\tilde{\chi}_i^0 = \begin{pmatrix}
\chi_i^0 \\
\chi_i^0
\end{pmatrix}.
\]
(4.19)

### 4.2 Renormalisation: Counterterms and Self-energies

We could have treated the chargino and neutralino system that we have just exposed within a common notation, deriving in a compact form the neutralino sector on the basis of its Majorana nature. This could have been done through a mass matrix \( M \) that stands for either \( X \) (charginos) or \( Y \) of the neutralinos and the two fields \( \psi^c_{R,L} \) that represent either \( \psi^c_{R,L} \) or the single Majorana field \( \psi^a \). To make our renormalisation procedure of this sector as transparent as possible we will take this common approach to show that the approach in renormalising the chargino and neutralino sector is exactly the same and that it corresponds to the approach that we have taken in the Higgs sector and the sfermion sector as concerns the issue of mixing. In particular we stress that we do not renormalise the rotation matrices that express the mass eigenstates from the current eigenstates. Summarising what we have just seen in the sfermion sector and splitting as usual the bare Lagrangian (denoted by \( \mathcal{L}_0 \)) into a renormalised Lagrangian and counterterms, the kinetic term and the mass term of a fermion field \( \psi \) with an arbitrary number of components can be written as
\[
\mathcal{L}_0^f = i[\psi_R^t\sigma^\mu\partial_\mu\bar{\psi}_R^0 + \bar{\psi}_L^0\sigma^\mu\partial_\mu\psi_L^0] - [\psi_R^tM_0\psi_R^0 + \bar{\psi}_L^0M_0^t\bar{\psi}_R^0],
\]
(4.20)
where \( \psi_{R/L}^0 \) represents the the fermion field and \( M_0 \) the non-diagonal mass matrix at bare level. At tree-level this mass matrix is diagonalised by rotating the fields with two unitary
matrices $D_R$ and $D_L$ which define the current fields so that at bare level we write these fields as
\[\chi_{R,0} = D_R\psi_{R,0}, \quad \chi_{L,0} = D_L\psi_{L,0}.\]  
(4.21)
The corresponding diagonal mass matrix $\tilde{M}$ is then given by,
\[\tilde{M} = D_R^* M D_L^\dagger = \tilde{M}^\dagger = D_L M^t D_R^t = \text{diag}(m_{\tilde{x}_1}, m_{\tilde{x}_2}, \ldots),\]  
(4.22)
and gives the physical masses $m_{\tilde{x}_i}$. The ensuing Dirac/Majorana spinors $\tilde{\chi}_{i,0}$ are constructed with these Weyl spinors
\[\tilde{\chi}_{i,0} = \left(\begin{array}{c} \chi_{L,i} \\ \chi_{R,i} \end{array} \right)_0.\]  
(4.23)
After the diagonalisation is performed, the counterterms for the different parameters involved in the mass matrix are introduced,
\[\tilde{M}_0 = M + \delta M,\]  
(4.24)
and also the wave function renormalisation constants $\delta Z^{R,L}_{ij}$ for each chiral “physical” field $\chi_{R/L}$,
\[\chi_{R,L,ij,0} = \left(\delta_{ij} + \frac{1}{2}\delta Z_{ij}^{R,L}\right)\chi_{R,L,j}.\]  
(4.25)
These transformations for the chiral fields are equivalent to the following transformation for the four-component spinor $\tilde{\chi}_i$,
\[\tilde{\chi}_{i,0} = \tilde{\chi}_i + \frac{1}{2}\left[\delta Z_{ij}^L P_L + \delta Z_{ij}^R P_R\right] \tilde{\chi}_j.\]  
(4.26)
We stress again that in our renormalisation scheme, we do not use the extra shifts on the diagonalisation matrices $D_{L,R}, D_{L,R} \rightarrow D_{L,R} + \delta D_{L,R}$, in other words $\delta D_{L,R} = 0$ as done in Ref. [25]. This is in the same spirit as within the Higgs sector and the sfermion sector. So, we consider that the diagonalisation matrices $D_{L,R}$ at tree-level and at the one-loop level are the same, $D_{L,R}$ are renormalised. With the renormalisation counterterms (4.24), (4.25) and
\[\delta \tilde{M} = D_R^* \delta M D_L^\dagger,\]  
(4.27)
the renormalised self energies $\hat{\Sigma}_{\tilde{\chi}_i\tilde{\chi}_j}$ can be cast into
\[\hat{\Sigma}_{\tilde{\chi}_i\tilde{\chi}_j}(q) = \Sigma_{\tilde{\chi}_i\tilde{\chi}_j}(q) - P_L \delta \tilde{M}_{ij} - P_R \delta \tilde{M}_{ij}^* - \frac{1}{2}(\delta - m_{\tilde{x}_j})[\delta Z_{ij}^L P_L + \delta Z_{ij}^R P_R] + \frac{1}{2}[\delta Z_{ji}^R P_R + \delta Z_{ji}^L P_L](\delta - m_{\tilde{x}_j}).\]  
(4.28)
Eq. 4.28 shows clearly that the wave function renormalisation constants are not involved in the renormalisation of the Lagrangian parameters contained in the mass matrices $\tilde{M}$ which in our case involve $M_1, M_2, \mu$.

It is useful to decompose the self-energy into the independent Lorentz structures through the projectors $P_{L,R} = \frac{1+\gamma^5}{2}$,

$$\Sigma_{\tilde{\chi}_i\tilde{\chi}_j}(q) = P_L \Sigma_{\tilde{\chi}_i\tilde{\chi}_j}^{LS}(q^2) + P_R \Sigma_{\tilde{\chi}_i\tilde{\chi}_j}^{RS}(q^2) + \frac{g}{\hat{\sigma}_i} P_L \Sigma_{\tilde{\chi}_i\tilde{\chi}_j}^{LV}(q^2) + \frac{g}{\hat{\sigma}_i} P_R \Sigma_{\tilde{\chi}_i\tilde{\chi}_j}^{RV}(q^2) . \quad (4.29)$$

Hermiticity imposes the following constraints on the elements of the Lorentz decomposition

$$\Sigma_{\tilde{\chi}_i\tilde{\chi}_j}^{RS}(q^2) = \Sigma_{\tilde{\chi}_j\tilde{\chi}_i}^{RS}(q^2), \quad \Sigma_{\tilde{\chi}_i\tilde{\chi}_j}^{LV}(q^2) = \Sigma_{\tilde{\chi}_j\tilde{\chi}_i}^{LV}(q^2) , \quad \Sigma_{\tilde{\chi}_i\tilde{\chi}_j}^{RV}(q^2) = \Sigma_{\tilde{\chi}_j\tilde{\chi}_i}^{RV}(q^2) \quad (4.30)$$

These are also satisfied by the corresponding covariants of the renormalised self-energies in Eq. 4.28. For a Majorana fermion (like a neutralino in the following), the additional Majorana symmetry imposes

$$\Sigma_{\tilde{\chi}_i\tilde{\chi}_j}^{RS}(q^2) = \Sigma_{\tilde{\chi}_j\tilde{\chi}_i}^{RS}(q^2), \quad \Sigma_{\tilde{\chi}_i\tilde{\chi}_j}^{LV}(q^2) = \Sigma_{\tilde{\chi}_j\tilde{\chi}_i}^{LV}(q^2) , \quad \Sigma_{\tilde{\chi}_i\tilde{\chi}_j}^{RV}(q^2) = \Sigma_{\tilde{\chi}_j\tilde{\chi}_i}^{RV}(q^2) \quad (4.31)$$

Some of these properties are used in our code as an extra test.

To fix the wave function renormalisation constants $\delta Z_{ij}^{RL}$, we require that

- the propagators of all the charginos and neutralinos are properly normalised with residue of 1 at the pole mass. This pole mass may get one-loop correction. For our treatment at one-loop it is sufficient to impose the residue condition by taking the tree-level mass. Taking the one-loop mass is a higher order effect, see Section 4.7 of [1] of our treatment in the Higgs sector. This condition implies

$$\lim_{q^2 \to m_{\tilde{\chi}_i}^2} \frac{\frac{g}{\hat{\sigma}_i} + m_{\tilde{\chi}_i}}{q^2 - m_{\tilde{\chi}_i}^2} \tilde{\Re} \Sigma_{\tilde{\chi}_i\tilde{\chi}_i}(q) u_{\chi_i}(q) = u_{\chi_i}(q)$$

and

$$\lim_{q^2 \to m_{\tilde{\chi}_i}^2} \overline{u}_{\chi_i}(q) \tilde{\Re} \Sigma_{\tilde{\chi}_i\tilde{\chi}_i}(q) \frac{\frac{g}{\hat{\sigma}_i} + m_{\tilde{\chi}_i}}{q^2 - m_{\tilde{\chi}_i}^2} = \overline{u}_{\chi_i}(q) \quad (4.32)$$

- No mixing between the physical fields when these are on mass-shell

$$\tilde{\Re} \Sigma_{\tilde{\chi}_i\tilde{\chi}_j}(q) u_{\chi_i}(q) = 0 \text{ for } q^2 = m_{\chi_i}^2 , \ (i \neq j) . \quad (4.33)$$

With these conditions we do not have to consider any loop correction on the external legs. Note that as usual $\tilde{\Re}$ signifies that the imaginary dispersive part of the loop function is discarded so as to maintain hermiticity at one-loop. Eq. 4.33 gives the diagonal element of the wave function renormalisation constants

$$\delta Z_{\tilde{\chi}_i\tilde{\chi}_i}^{L} = - \tilde{\Re} \Sigma_{\tilde{\chi}_i\tilde{\chi}_i}^{LV}(m_{\chi_i}^2) - m_{\tilde{\chi}_i}^2 \left( \tilde{\Re} \Sigma_{\tilde{\chi}_i\tilde{\chi}_i}^{LV}(m_{\chi_i}^2) + \tilde{\Re} \Sigma_{\tilde{\chi}_i\tilde{\chi}_i}^{RV}(m_{\chi_i}^2) \right) - 2 m_{\tilde{\chi}_i} \tilde{\Re} \Sigma_{\tilde{\chi}_i\tilde{\chi}_i}^{LS'}(m_{\chi_i}^2) , \quad (4.34)$$

$$\delta Z_{\tilde{\chi}_i\tilde{\chi}_i}^{R} = - \tilde{\Re} \Sigma_{\tilde{\chi}_i\tilde{\chi}_i}^{RV}(m_{\chi_i}^2) - m_{\tilde{\chi}_i}^2 \left( \tilde{\Re} \Sigma_{\tilde{\chi}_i\tilde{\chi}_i}^{LV}(m_{\chi_i}^2) + \tilde{\Re} \Sigma_{\tilde{\chi}_i\tilde{\chi}_i}^{RV}(m_{\chi_i}^2) \right) - 2 m_{\tilde{\chi}_i} \tilde{\Re} \Sigma_{\tilde{\chi}_i\tilde{\chi}_i}^{RS'}(m_{\chi_i}^2) , \quad (4.34)$$

$$\delta Z_{\tilde{\chi}_i\tilde{\chi}_j}^{L} = - \tilde{\Re} \Sigma_{\tilde{\chi}_i\tilde{\chi}_j}^{RV}(m_{\chi_i}^2) - m_{\tilde{\chi}_i}^2 \left( \tilde{\Re} \Sigma_{\tilde{\chi}_i\tilde{\chi}_j}^{LV}(m_{\chi_i}^2) + \tilde{\Re} \Sigma_{\tilde{\chi}_i\tilde{\chi}_j}^{RV}(m_{\chi_i}^2) \right) - 2 m_{\tilde{\chi}_i} \tilde{\Re} \Sigma_{\tilde{\chi}_i\tilde{\chi}_j}^{RS'}(m_{\chi_i}^2) , \quad (4.34)$$

$$\delta Z_{\tilde{\chi}_i\tilde{\chi}_j}^{R} = - \tilde{\Re} \Sigma_{\tilde{\chi}_i\tilde{\chi}_j}^{RV}(m_{\chi_i}^2) - m_{\tilde{\chi}_i}^2 \left( \tilde{\Re} \Sigma_{\tilde{\chi}_i\tilde{\chi}_j}^{LV}(m_{\chi_i}^2) + \tilde{\Re} \Sigma_{\tilde{\chi}_i\tilde{\chi}_j}^{RV}(m_{\chi_i}^2) \right) - 2 m_{\tilde{\chi}_i} \tilde{\Re} \Sigma_{\tilde{\chi}_i\tilde{\chi}_j}^{RS'}(m_{\chi_i}^2) , \quad (4.34)$$
where we have used the fact that in the case of CP conservation \( \Sigma^{LS}_{\chi_0^i \chi_0^j}(m^2_{\chi_0^i}) = \Sigma^{RS}_{\chi_0^i \chi_0^j}(m^2_{\chi_0^i}) \).

The prime on a function such as \( \Sigma^{RV'}_{\chi_0^i \chi_0^j}(m^2_{\chi_0^i}) \) stands for the derivative \( \partial^2 \Sigma^{RV}_{\chi_0^i \chi_0^j}(q^2)/\partial q^2|_{q^2=m^2_{\tilde{\chi}_i}} \).

The non diagonal elements \((i \neq j)\) of \( \delta Z^{L,R} \) are derived from the constraints of Eq. 4.35

\[
\begin{align*}
\delta Z^L_{ij} &= \frac{2}{m^2_{\chi_i} - m^2_{\chi_j}} \left( m_{\chi_i} \Re \Sigma^{LS}_{\chi_0^i \chi_0^j}(m^2_{\chi_0^i}) + m_{\chi_j} \Re \Sigma^{RS}_{\chi_0^i \chi_0^j}(m^2_{\chi_0^i}) \right. \\
&\quad + \left. m^2_{\chi_j} \Re \Sigma^{LV}_{\chi_0^i \chi_0^j}(m^2_{\chi_0^i}) - m_{\chi_i} \delta \tilde{M}_{ji} - m_{\chi_j} \delta \tilde{M}^*_{ij} \right), \\
\delta Z^R*_{ij} &= \frac{2}{m^2_{\chi_i} - m^2_{\chi_j}} \left( m_{\chi_j} \Re \Sigma^{LS}_{\chi_0^i \chi_0^j}(m^2_{\chi_0^i}) + m_{\chi_i} \Re \Sigma^{RS}_{\chi_0^i \chi_0^j}(m^2_{\chi_0^i}) \right. \\
&\quad + \left. m^2_{\chi_i} \Re \Sigma^{LV}_{\chi_0^i \chi_0^j}(m^2_{\chi_0^i}) - m_{\chi_j} \delta \tilde{M}_{ji} - m_{\chi_i} \delta \tilde{M}^*_{ij} \right). 
\end{align*}
\]

Specialising to the case of the charginos we will have to take \( D_R = U \) and \( D_L = V \), see Eq. (4.7) and \( M = X \), see Eq. (4.8) where both \( U \) and \( V \) are real matrices as is the mass matrix \( X \) in our case with \( CP \) conservation. In this case \( \delta Z^{L,R} \) can be taken real. For the neutralinos \( D_L = D_R = N \), see Eq. (4.17) and \( M = Y \) is a symmetric real matrix, see Eq. (4.16). In this case, as expected, we have \( \delta Z^L = \delta Z^R = \delta Z^0 \) which is a result of the symmetry of \( Y \) and the Majorana constraints of Eq. 4.31. In fact Eq. 4.35 can be recast into

\[
\begin{align*}
\delta Z^0_{ij} &= \frac{1}{m^2_{\chi_i} - m^2_{\chi_j}} \left( m_{\chi_j} \left( \tilde{\Re} \Sigma^{LV}_{\chi_0^i \chi_0^j}(m^2_{\chi_0^i}) - \tilde{\Re} \Sigma^{LV*}_{\chi_0^i \chi_0^j}(m^2_{\chi_0^i}) \right) \\
&\quad + \left( \tilde{\Re} \Sigma^{LS}_{\chi_0^i \chi_0^j}(m^2_{\chi_0^i}) - \tilde{\Re} \Sigma^{LS*}_{\chi_0^i \chi_0^j}(m^2_{\chi_0^i}) \right) \left( \delta \tilde{Y}_{ij} + \delta \tilde{Y}^*_{ij} \right) \right) \\
&\quad + \frac{1}{m^2_{\chi_i} + m^2_{\chi_j}} \left( -m_{\chi_j} \left( \tilde{\Re} \Sigma^{LV}_{\chi_0^i \chi_0^j}(m^2_{\chi_0^i}) - \tilde{\Re} \Sigma^{LV*}_{\chi_0^i \chi_0^j}(m^2_{\chi_0^i}) \right) \\
&\quad + \left( \tilde{\Re} \Sigma^{LS}_{\chi_0^i \chi_0^j}(m^2_{\chi_0^i}) - \tilde{\Re} \Sigma^{LS*}_{\chi_0^i \chi_0^j}(m^2_{\chi_0^i}) \right) \left( \delta \tilde{Y}_{ij} - \delta \tilde{Y}^*_{ij} \right) \right). 
\end{align*}
\]

Note that while \( Z^0_{ii} \) is real, \( Z^0_{ij} (i \neq j) \) is either purely real (given by the first two lines in Eq. 4.36) or purely imaginary (given by the last two lines in Eq. 4.36) when using \( N = JN \) in order to have positive masses. Using \( N = \tilde{N} \) we can have \( Z^0_{ij} \) real but we have to keep track of the sign of the masses in Eq. 4.36. For example, with \( N = \tilde{N} \) (real), when both \( m_{\tilde{\chi}_{i,j}} > 0 \) are obtained, we get

\[
\delta Z^0_{ij} = \frac{2}{m^2_{\chi_i} - m^2_{\chi_j}} \left( \tilde{\Re} \Sigma^{LS}_{\chi_0^i \chi_0^j}(m^2_{\chi_0^i}) + m_{\chi_j} \tilde{\Re} \Sigma^{LV}_{\chi_0^i \chi_0^j}(m^2_{\chi_0^i}) - \delta \tilde{Y}_{ij} \right). 
\]

It is important to note a common feature of our approach that we already encountered in the case of the mixing in the Higgs sector and the sfermion sector. The non diagonal wave function renormalisation constants in Eqs. 4.35, 4.36, 4.37 are fully determined only once
the mass counterterms \( \delta \hat{M} \) are fixed. In our case this requires fixing \( \delta M_1, \delta M_2 \) and \( \delta \mu \) to which we turn in the next section.

For completeness let us give the corresponding counterterm matrices \( \delta X \) and \( \delta Y \). We have

\[
\delta X = \begin{bmatrix} \delta M_2 & \delta X_{12} \\ \delta X_{21} & \delta \mu \end{bmatrix}, \quad \delta Y = \begin{bmatrix} \delta M_1 & 0 & \delta Y_{13} & \delta Y_{14} \\ 0 & \delta M_2 & \delta Y_{23} & \delta Y_{24} \\ \delta Y_{13} & \delta Y_{23} & 0 & -\delta \mu \\ \delta Y_{14} & \delta Y_{24} & -\delta \mu & 0 \end{bmatrix}. \tag{4.38}
\]

with

\[
\begin{align*}
\delta X_{12} &= +\sqrt{2} M_W s_\beta \left( \frac{1}{2} \delta M^2_\beta + c_\beta \frac{2 \delta t_\beta}{t_\beta} \right), \\
\delta X_{21} &= +\sqrt{2} M_W c_\beta \left( \frac{1}{2} \delta M^2_\beta - s_\beta \frac{2 \delta t_\beta}{t_\beta} \right), \\
\delta Y_{13} &= -s_W M_Z c_\beta \left( \frac{1}{2} \delta M^2_\beta + \frac{1}{2} \delta s_\beta \frac{2 \delta t_\beta}{t_\beta} \right), \\
\delta Y_{14} &= +s_W M_Z s_\beta \left( \frac{1}{2} \delta M^2_\beta + \frac{1}{2} \delta s_\beta \frac{2 \delta t_\beta}{t_\beta} \right), \\
\delta Y_{23} &= +c_W M_Z c_\beta \left( \frac{1}{2} \delta M^2_\beta + \frac{1}{2} \delta c_\beta \frac{2 \delta t_\beta}{t_\beta} \right), \\
\delta Y_{24} &= -c_W M_Z s_\beta \left( \frac{1}{2} \delta M^2_\beta + \frac{1}{2} \delta c_\beta \frac{2 \delta t_\beta}{t_\beta} \right).
\end{align*} \tag{4.39}
\]

### 4.3 Fixing \( \delta M_1, \delta M_2, \delta \mu \)

\( \delta M_1, \delta M_2, \delta \mu \) can be fixed through the diagonal self-energies of the chargino-neutralino system which we have not fully exploited yet and which constrain the physical masses of the charginos and neutralinos. The most straightforward and simple choice is based on the fact that the chargino system is a \( 2 \times 2 \) system which is easier to handle that the \( 4 \times 4 \) system of the neutralinos. In S1oops the default scheme is to choose the two chargino masses \( m_{\tilde{\chi}_1^\pm} \) and \( m_{\tilde{\chi}_2^\pm} \) as inputs to define the two parameters \( M_2 \) and \( \mu \) and one neutralino mass to define \( M_1 \). The lightest neutralino mass \( m_{\tilde{\chi}_1^0} \) is used by default to fix \( M_1 \). The three other neutralino masses \( m_{\tilde{\chi}_{2,3,4}^0} \) are derived and receive one-loop quantum corrections. At one-loop these three input parameters translate into the usual definition of the pole masses in the on-shell scheme through the renormalised self-energies of the charginos and the lightest neutralino

\[
\tilde{\text{Re}} \Sigma_{\tilde{\chi}_i \tilde{\chi}_i}(q) u_{\chi_i}(q) = 0 \quad \text{for} \quad q^2 = m_{\chi_i}^2, \quad \text{for} \quad \chi_i \to \chi_i^\pm, \chi_2^\pm, \chi_1^0. \tag{4.40}
\]

This translates into

\[
\begin{align*}
\delta \tilde{X}_{11} &= \delta m_{\tilde{\chi}_1^\pm} = \tilde{\text{Re}} \Sigma_{\tilde{\chi}_1 \tilde{\chi}_1}^{LS}(m_{\chi_1}^2) + \frac{1}{2} m_{\tilde{\chi}_1^\pm} \left( \tilde{\text{Re}} \Sigma_{\tilde{\chi}_1 \tilde{\chi}_1}^{LV}(m_{\chi_1}^2) + \tilde{\text{Re}} \Sigma_{\tilde{\chi}_1 \tilde{\chi}_1}^{RV}(m_{\chi_1}^2) \right), \\
\delta \tilde{X}_{22} &= \delta m_{\tilde{\chi}_2^\pm} = \tilde{\text{Re}} \Sigma_{\tilde{\chi}_2 \tilde{\chi}_2}^{LS}(m_{\chi_2}^2) + \frac{1}{2} m_{\tilde{\chi}_2^\pm} \left( \tilde{\text{Re}} \Sigma_{\tilde{\chi}_2 \tilde{\chi}_2}^{LV}(m_{\chi_2}^2) + \tilde{\text{Re}} \Sigma_{\tilde{\chi}_2 \tilde{\chi}_2}^{RV}(m_{\chi_2}^2) \right), \\
\delta \tilde{Y}_{11} &= \delta m_{\tilde{\chi}_1^0} = \tilde{\text{Re}} \Sigma_{\tilde{\chi}_1 \tilde{\chi}_1}^{LS}(m_{\chi_1}^2) + m_{\tilde{\chi}_1^0} \tilde{\text{Re}} \Sigma_{\tilde{\chi}_1 \tilde{\chi}_1}^{LV}(m_{\chi_1}^2). \tag{4.41}
\end{align*}
\]
These three counterterms can be inverted to derive the counterterms parameters $\delta M_1, \delta M_2, \delta \mu$ through $\delta \tilde{Y} = N^* \delta Y N^\dagger$ and $\delta \tilde{X} = U^* \delta XV^\dagger$, see Eq. \[1.27\]. In fact $\delta M_2, \delta \mu$ can be derived more directly without going through the mixing matrices from Eq. \[4.12\]. We get

\begin{align*}
\delta M_2 &= \frac{1}{M_2^2 - \mu^2} \left( (M_2 m_{\tilde{\chi}_1^0}^2 - \mu \text{det} X) \frac{\delta m_{\tilde{\chi}_1^+}}{m_{\tilde{\chi}_1^+}} + (M_2 m_{\tilde{\chi}_2^0}^2 - \mu \text{det} X) \frac{\delta m_{\tilde{\chi}_2^+}}{m_{\tilde{\chi}_2^+}} \\
& \quad - M_W^2 (M_2 + \mu s_{2\beta}) \frac{\delta M_W^2}{M_W^2} - \mu M_W^2 s_{2\beta} c_{2\beta} \frac{\delta t_\beta}{t_\beta} \right), \\
\delta \mu &= \frac{1}{\mu^2 - M_2^2} \left( (\mu m_{\tilde{\chi}_1^+}^2 - M_2 \text{det} X) \frac{\delta m_{\tilde{\chi}_1^+}}{m_{\tilde{\chi}_1^+}} + (\mu m_{\tilde{\chi}_2^+}^2 - M_2 \text{det} X) \frac{\delta m_{\tilde{\chi}_2^+}}{m_{\tilde{\chi}_2^+}} \\
& \quad - M_W^2 (\mu + M_2 s_{2\beta}) \frac{\delta M_W^2}{M_W^2} - M_2 M_W^2 s_{2\beta} c_{2\beta} \frac{\delta t_\beta}{t_\beta} \right), \tag{4.42}
\end{align*}

\begin{align*}
\delta M_1 &= \frac{1}{N_{11}^2} (\delta m_{\chi_1^0} - N_{12}^* \delta M_2 + 2 N_{13}^* N_{14}^* \delta \mu \\
& \quad - 2 N_{11}^* N_{13}^* \delta Y_{13} - 2 N_{12}^* N_{13}^* \delta Y_{23} - 2 N_{11}^* N_{14}^* \delta Y_{14} - 2 N_{12}^* N_{14}^* \delta Y_{24}). \tag{4.43}
\end{align*}

The physical masses of the other three neutralinos ($i = 2, 3, 4$) receive a correction at one-loop given by

\begin{align*}
m_{\tilde{\chi}_i^0}^{\text{phys}} &= m_{\tilde{\chi}_i^0} + \delta \tilde{Y}_{ii} - \text{Re} \Sigma_{\tilde{\chi}_i^0 \chi_i^0}^L (m_{\chi_i^0}^2) - m_{\tilde{\chi}_i^0} \text{Re} \Sigma_{\tilde{\chi}_i^0 \chi_i^0}^Y (m_{\chi_i^0}^2) \\
& \quad + 2 N_{11}^* N_{13}^* \delta Y_{13} + 2 N_{11}^* N_{14}^* \delta Y_{14} + 2 N_{12}^* N_{13}^* \delta Y_{23} + 2 N_{13}^* N_{14}^* \delta Y_{24}. \tag{4.44}
\end{align*}

Checking the cancellation of the ultraviolet divergences in Eq. \[1.41\] is an important non trivial test on the validity and correctness of the procedure and its implementation. Other schemes in the neutralino/chargino sector can be implemented in \texttt{SloopS} as will be shown in a forthcoming publication. A deviation from the commonly used scheme adopted here was taken in Ref. \[26\] where the input parameters are the masses of $\tilde{\chi}_1^0, \tilde{\chi}_2^0$ and $\tilde{\chi}_2^\pm$. There are a few important remarks to make about Eq. \[4.42\] and Eq. \[4.43\]. The choice of $m_{\chi_1^0}$ as an input parameter is appropriate only if the lightest neutralino is mostly bino or if the bino-like neutralino is not too heavy compared to the other neutralinos. Otherwise the extraction of $M_1$ would be subject to uncertainties. This shows in Eq. \[4.43\] since $N_{11}$ would be too small which would in turn induce large radiative corrections. Another difficulty arises with the special configuration $M_2 \sim \pm \mu$. Eq. \[4.42\] shows that an apparent singularity might be present. We had already pointed out in \[\Gamma\] that this configuration can induce a large $t_\beta$-scheme dependence in the counterterms $\delta M_{1,2}$ and $\delta \mu$ and therefore to the annihilation of the LSP into $W$’s for a mixed LSP, see also \[27\]. Let us look at this
cause trouble. The problem is confined to the finite part (in the ultraviolet sense) of the scheme in order not to induce too large corrections or ill-defined constants.

In practice, in the on-shell scheme that is generally used for the chargino/neutralino sector and that we adopt here we need to reconstruct from experiments the value of $\mu$, $M_2$ and $M_1$ from three physical masses. If we invert the mass relations of the chargino sector, we would in general get four solutions $(M_2, \mu)$ for one set of chargino masses $(m_{\tilde{\chi}_1^+}, m_{\tilde{\chi}_2^+})$

\[
\mu^2 = \frac{m^2_{\tilde{\chi}_1^+} + m^2_{\tilde{\chi}_2^+} - 2M^2_W}{2} - \frac{\epsilon_\chi}{2} \left[ (m^2_{\tilde{\chi}_1^+} + m^2_{\tilde{\chi}_2^+} - 2M^2_W)^2 - 4(M^2_W s_{2\beta} + \epsilon_\mu m_{\tilde{\chi}_1^+} m_{\tilde{\chi}_2^+})^2 \right]^{1/2},
\]

\[
M_2 = [m^2_{\tilde{\chi}_1^+} + m^2_{\tilde{\chi}_2^+} - 2M^2_W - \mu^2]^{1/2},
\]

where $\epsilon_{\mu,\chi}$ can take the value $\pm 1$ and summarize the ambiguities in the reconstruction [28]. $\epsilon_\mu$ represents the sign of $\mu$ so that $\mu = \epsilon_\mu \sqrt{\mu^2}$. $\epsilon_\chi$ represents the $M_2 \leftrightarrow \mu$ symmetry in the reconstruction so that $\text{Sgn}\epsilon_\chi = \text{Sgn}(\mu^2 - M^2_2)$. In the numerical computations of the one-loop correction to the neutralino masses in Section 5.2, we have taken the set corresponding to $\epsilon_\chi = \epsilon_\mu = 1$. Once $M_2$ and $\mu$ are known, the remaining parameter $M_1$ can be extracted from the knowledge of one of the masses of the neutralinos. For example, in the case where the neutralino is mostly bino-like and corresponds to the lightest neutralino with mass $m_{\tilde{\chi}_1^0}$

configuration again. We can rewrite Eq. (4.42) as

\[
\delta M_2 = \frac{1}{M_2^2 - \mu^2} \left( \epsilon_\mu \mu \delta E_\chi + (M_2 - \epsilon_\mu \mu) \delta F_\chi \right) = \frac{1}{M_2^2 - \mu^2} \left( |\mu| \delta E_\chi + (M_2 - |\mu|) \delta F_\chi \right),
\]

\[
\delta \mu = \frac{1}{\mu^2 - M_2^2} \left( \epsilon_\mu M_2 \delta E_\chi + (\mu - \epsilon_\mu M_2) \delta F_\chi \right) = \frac{\epsilon_\mu}{\mu^2 - M_2^2} \left( M_2 \delta E_\chi + (|\mu| - M_2) \delta F_\chi \right),
\]

\[
\delta E_\chi = \frac{1}{2} \delta (m_{\tilde{\chi}_1^+} - m_{\tilde{\chi}_2^+})^2 - M^2_W \left( \frac{\delta M^2_W}{M^2_W} (1 + \epsilon_\mu s_{2\beta}) + \epsilon_\mu s_{2\beta} c_{2\beta} \right),
\]

\[
\delta F_\chi = \frac{1}{2} \left( \delta m^2_{\tilde{\chi}_1^+} + \delta m^2_{\tilde{\chi}_2^+} \right) - \delta M^2_W.
\]

(4.45)

It is important to note that the contributions proportional to $\delta F_\chi$ are regular in the limit $M_2 \to |\mu|$, moreover $\delta F_\chi$ does not introduce any $t_\beta$ dependence. Only terms in $\delta E_\chi$ may cause trouble. The problem is confined to the finite part (in the ultraviolet sense) of $\delta E_\chi$. Indeed, we have checked explicitly that in the limit $M_2 \to |\mu|$, $\delta E_\chi$ is finite. This is a strong check on the validity of the code. Therefore any non regular term comes from the finite part (in the ultraviolet sense) of $\delta E_\chi$ and calls for a good choice of the renormalisation scheme in order not to induce too large corrections or ill-defined constants.

4.4 Input parameters and parameter reconstruction

In practice, in the on-shell scheme that is generally used for the chargino/neutralino sector and that we adopt here we need to reconstruct from experiments the value of $\mu$, $M_2$ and $M_1$ from three physical masses. If we invert the mass relations of the chargino sector, we would in general get four solutions $(M_2, \mu)$ for one set of chargino masses $(m_{\tilde{\chi}_1^+}, m_{\tilde{\chi}_2^+})$
as what occurs with the models with gaugino mass unification at the GUT scale, we have

\[
M_1 = \frac{m_{\tilde{\chi}_1^0}^4 - M_2 m_{\tilde{\chi}_1^0}^2 - (\mu^2 + M_Z^2) m_{\tilde{\chi}_1^0}^2 - (s_{2\beta} M_Z^2 \mu - (\mu^2 + s_W^2 M_Z^2) M_2) m_{\tilde{\chi}_1^0} + s_{2\beta} s_W^2 M_Z^2 \mu M_2}{m_{\tilde{\chi}_1^0}^3 - M_2 m_{\tilde{\chi}_1^0}^2 - (\mu^2 + s_W^2 M_Z^2) m_{\tilde{\chi}_1^0}^2 - s_{2\beta} s_W^2 M_Z^2 \mu + \mu^2 M_2}.
\]

(4.47)

Having \( M_1, M_2, \mu \), a consistency check can be made to make sure that \( M_1 \) is indeed given through Eq. (4.47) with \( m_{\tilde{\chi}_1^0} \) as input and not some other neutralino. This shows somehow the ambiguity, already encountered in extracting \( M_1 \) through Eq. (4.47) with \( m_{\tilde{\chi}_1^0} \) as input and not some other neutralino. This shows somehow the effect of the radiative corrections on the masses, discovery of both charginos almost certainly guarantees the discovery of the two Higgsinos and the wino-like neutralinos with masses of the same order as the corresponding parameters, therefore allowing to select the correct \((M_2, \mu)\) from the chargino reconstruction. If the bino is not too heavy it will then be easy to single out and hence measure \( M_1 \).

Another exploration about the correct extraction of \( M_1, M_2, \mu \) can also be done through the measurements of some couplings of the charginos (see for example [29] for a tree-level analysis) and the neutralinos (see for example [30]). We will see below how one can extract these parameters in decays involving the neutralinos combined with the measurements of the chargino masses. Although the situation here is quite different from the mixing in the sfermions, exploiting decays as inputs, to fix the underlying parameters less unambiguously when mixing takes place is promising. We will get back to this issue in a forthcoming publication.

Meanwhile let us give an example about the reconstruction. As an example we take the measured masses that we take as input are the two chargino masses with \( m_{\tilde{\chi}_1^\pm} = 232 \) GeV and \( m_{\tilde{\chi}_1^0} = 426 \) GeV and the lightest neutralino mass \( m_{\tilde{\chi}_1^0} = 98 \) GeV. From the chargino masses we obtain four solutions for \((M_2, \mu)\) according to Eq. (4.48). For each one of these solutions we first reconstruct the corresponding \( M_1 \) by imposing the mass of the \( \tilde{\chi}_1^0 \) taken as input. In the example we have taken, the four solutions for \((M_2, \mu, M_1)\) are (all given in GeV)

\[
\begin{align*}
(250.39, 399.78, 100.38) & \quad \text{for } \epsilon_\chi = 1, \epsilon_\mu = 1, \\
(240.39, -405.86, 98.22) & \quad \text{for } \epsilon_\chi = 1, \epsilon_\mu = -1, \\
(399.78, 250.39, 103.68) & \quad \text{for } \epsilon_\chi = -1, \epsilon_\mu = 1, \\
(405.86, -240.39, 100.05) & \quad \text{for } \epsilon_\chi = -1, \epsilon_\mu = -1.
\end{align*}
\]

(4.48)

Each solution will lead to different predictions on the observables in the chargino and neutralino sector as well as the sfermion/Higgs sector. Comparing the theoretical predictions to the measurements of a minimal set of these observables lifts the four-fold ambiguity. Theoretically with each set of solutions in Eq. (4.48) we can give one-loop predictions.

\footnote{Equation (4.47) was derived in [25] however there is a typo. \( s_W^2 \) in the last term in the numerator of Eq. (4.47) is missing in [25].}
given some other model parameters that indirectly enter in the one-loop calculation. For simplicity and to avoid having to deal with QED corrections we consider the prediction on the 3 other neutralino masses and the decays $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 (\gamma, Z^0)$. The former is a pure one-loop effect. We take the pseudo-scalar mass $M_A^0 = 300 \text{ GeV}$, a common soft-susy sfermion mass $M_{\tilde{f}} = 500 \text{ GeV}$, a common $A_f = 0$, the $SU(3)$ gaugino mass is set at $M_3 = 1000 \text{ GeV}$ and $t_\beta = 10$. For $t_\beta$ the results we present below are within the $MH$ scheme. $\delta \Gamma(\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 Z^0)$ is the one-loop correction to the rate $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 Z^0$.

The results in Table 4.4 show that disentangling between the possible solutions is in principle possible even if not all neutralino masses are measured. For example the rate $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 Z^0$ is a clear cut indicator for the sign of $\epsilon_\chi$, since this rate is an order of magnitude larger if the higgsino-like neutralino is lighter than the wino-like neutralino. If a precision measurement below the 10% level can be achieved on this observable it can, by itself, also disentangle between all four solutions. Considering the smallness of the rate $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 Z^0$ this observable is perhaps of academic interest. Note however that it can in principle be used to lift the degeneracy between all four solutions. Combining measurements like this with measurements of some of the other neutralino masses or measuring all the neutralino masses is certainly a good way to lift the ambiguity.

5 Applications and examples at one-loop

Our code has been checked extensively. We have written a script that automatically calculates cross sections for all $2 \to 2$ process in the MSSM at one-loop. We check ultraviolet finiteness as well as the independence in each of the non-linear gauge parameters. Results of these extensive checks can be found in [31]. Moreover we have compared the results of the code and the renormalisation procedure with quite a few observables that have appeared in the literature. Apart from these comparisons which we will report here the flexibility of the code allows us to study the scheme dependence of the result. We show here a few examples, taken from studies by different groups, of comparisons ranging from mass corrections, two-body decays as well as $2 \to 2$ processes paying a particular attention to the important $t_\beta$ scheme dependence. For the latter we consider the schemes introduced in [1] and summarised in Section 2. The examples we will review here cover the sectors we studied in this paper, leaving aside the Higgs sector that we studied at length in [1].

Before embarking on showing our results for some observables at one-loop, let us briefly describe how we treat infrared divergences. The one-loop corrections can still contain infrared divergences due to photon virtual exchanges. These are regulated by a small photon mass. The photon mass regulator contribution contained in the virtual correction should cancel exactly against the one present in the photon final state radiation. The photonic contribution is in fact split into a soft part, where the photon energy is less than some small cut-off $k_c$, $M_{\gamma}^{soft}(E_\gamma < k_c)$ and a hard part with $M_{\gamma}^{hard}(E_\gamma > k_c)$. The former requires a photon mass regulator. We use the usual universal factorised form with a
| Solution | $\epsilon_\chi = 1, \ \epsilon_\mu = 1$ | $\epsilon_\chi = 1, \ \epsilon_\mu = -1$ | $\epsilon_\chi = -1, \ \epsilon_\mu = 1$ | $\epsilon_\chi = -1, \ \epsilon_\mu = -1$ |
|----------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $m_{\chi_2}^{\text{tree-level}}$ | $232.34, m_{\chi_2}^{\text{phys}} = 232.19, m_{\chi_2}^{\text{phys}} = 232.19$ | $231.83, m_{\chi_2}^{\text{phys}} = 231.74, m_{\chi_2}^{\text{phys}} = 231.74$ | $236.17, m_{\chi_2}^{\text{phys}} = 236.17, m_{\chi_2}^{\text{phys}} = 236.17$ | $231.76, m_{\chi_2}^{\text{phys}} = 232.59, m_{\chi_2}^{\text{phys}} = 232.59$ |
| $m_{\chi_3}^{\text{tree-level}}$ | $405.26, m_{\chi_3}^{\text{phys}} = 407.41, m_{\chi_3}^{\text{phys}} = 407.41$ | $414.02, m_{\chi_3}^{\text{phys}} = 414.19, m_{\chi_3}^{\text{phys}} = 414.19$ | $256.54, m_{\chi_3}^{\text{phys}} = 254.71, m_{\chi_3}^{\text{phys}} = 254.71$ | $249.64, m_{\chi_3}^{\text{phys}} = 249.53, m_{\chi_3}^{\text{phys}} = 249.53$ |
| $m_{\chi_4}^{\text{tree-level}}$ | $425.69, m_{\chi_4}^{\text{phys}} = 425.77, m_{\chi_4}^{\text{phys}} = 425.77$ | $422.79, m_{\chi_4}^{\text{phys}} = 423.46, m_{\chi_4}^{\text{phys}} = 423.46$ | $425.00, m_{\chi_4}^{\text{phys}} = 425.81, m_{\chi_4}^{\text{phys}} = 425.81$ | $425.80, m_{\chi_4}^{\text{phys}} = 425.65, m_{\chi_4}^{\text{phys}} = 425.65$ |
| $\Gamma(\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 \gamma)$ | $0.308 \times 10^{-8}$ | $0.182 \times 10^{-7}$ | $0.142 \times 10^{-7}$ | $0.368 \times 10^{-10}$ |
| $\Gamma(\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 Z^0)_{\text{tree-level}}$ | $0.223 \times 10^{-2}, \delta \Gamma(\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 Z^0) = 0.533 \times 10^{-4}$ | $0.202 \times 10^{-2}, \delta \Gamma(\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 Z^0) = 0.780 \times 10^{-4}$ | $0.197 \times 10^{-1}, \delta \Gamma(\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 Z^0) = 0.271 \times 10^{-2}$ | $0.277 \times 10^{-1}, \delta \Gamma(\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 Z^0) = 0.157 \times 10^{-2}$ |

Table 1: Disentangling between the four solutions for $M_2$, $\mu$ and $M_1$ from the input with $m_{\chi_+} = 232$ GeV, $m_{\chi_0} = 426$ GeV and $m_{\chi_1} = 98$ GeV. All masses and decay widths are in GeV units. $\delta \Gamma$ is the one-loop correction in the MH-scheme.
simple rescaling for the case of the gluon correction in all processes we have studied where the non-abelian coupling of the gluon is not at play. The test on the infrared finiteness is performed by including both the loop and the soft bremsstrahlung contributions and checking that there is no dependence on the fictitious photon mass $\lambda_\gamma$ or gluon mass $\lambda_g$.

For the bremsstrahlung part we use VEGAS adaptive Monte Carlo integration package provided in the FFL bundle and verify the result of the cross section against Comphep [32]. We choose $k_c$ small enough and check the stability and independence of the result with respect to $k_c$.

5.1 Corrections to the sbottom and stau masses

We compare our results with those of Ref. [33] where an approach similar to ours in this sector is taken. For $t_\beta$, the authors of [33] take a DCPR scheme and compare with $\overline{DR}$. The mixing parameter in [33] is however defined through the naive scheme of Eq. (3.34). In order to conduct this comparison we first need to implement the same set of input parameters as in [33]. We therefore slightly change our scheme to predict the heaviest sbottom mass $m_{\tilde{b}_1}$ at one-loop instead of the heaviest stop mass $m_{\tilde{t}_1}$ which is therefore taken as input. Our code being quite flexible this change can be made very easily. The set of parameters corresponds to the (tree-level) choice $\mu = 100$ GeV, $M_1 = 95$ GeV, $M_2 = 200$ GeV, $M_3 = 719$ GeV, $M_{A^0} = 150$ GeV and $M_{f_{LR}} = M_{f_{RL}} = A_f = 300$ GeV. This assumes implicitly that these Lagrangian parameters have been reconstructed from the physical inputs. Let us discuss our results first, taking the same scheme for the sfermion mixing parameter as in Ref. [33] before commenting on the impact of taking the SloopS default scheme for this parameter. As Fig. 1 shows, the corrections are almost insensitive to the $t_\beta$-scheme in the case of the correction to the sbottom mass, which is very welcome. Indeed, the $A_{\tau\tau}$, $\overline{DR}$ and DCPR are within 0.03% and thus indistinguishable, they are shown as one prediction in Fig. 1. The $MH$-scheme deviates very slightly from the other schemes especially for small $t_\beta$, this difference is at most of order 0.3%. However in this case the uncertainty introduced by the $MH$-scheme is an order of magnitude smaller compared to the total correction which is of order 3 – 4%. For the sbottom, the corrections are due essentially to the QCD/SQCD corrections increasing with $t_\beta$ from 3% to about 4%. This correction is by itself small. The correction in the case of the stau mass is even smaller by an order of magnitude at least. However, here the $MH$ uncertainty at small $t_\beta$ is noticeable at small $t_\beta$ of order 0.1 – 0.2% from the other three $t_\beta$ schemes which agree with each other to better than 0.01%. The reason for the (almost) scheme independence is that the $t_\beta$-scheme dependence of the sbottom mass as well as of the stau mass is proportional to $s_{2\beta}^2 \simeq 4/t_\beta^2$ which is strongly suppressed for large $t_\beta$. Our results for the DCPR and $\overline{DR}$ schemes are in excellent agreement with those of Ref. [33]. Concerning the choice of the mixing parameter $\delta m_{\tilde{f}_{12}}^2$, we observe a small difference between the default choice in SloopS given by Eq. (3.33) and the one given by Eq. (3.34). To give an idea, the difference is about 0.2% in the sbottom mass correction for both $t_\beta = 10$ and $t_\beta = 50$. 

30
5.2 Corrections to the masses of the heaviest neutralinos, $m_{\chi_{2,3,4}^0}$

We calculated the quantum corrections to the masses of the three neutralinos for the different schemes of $t_\beta$ implemented in our code and compared our results with Ref. [25] which works within the $DCPR$-scheme but otherwise takes the same input parameters, namely the chargino masses and the LSP mass. The input chargino/neutralino parameters are $m_{\chi_1^+} = 180$ GeV, $m_{\chi_2^+} = 350$ GeV, $m_{\chi_3^0} = 160$ GeV, we choose as in [25] the model corresponding to $\epsilon_\mu = \epsilon_\chi = 1$ in Eqs. (4.46), (4.47) in order to reconstruct the three fundamental parameters $M_1$, $M_2$ and $\mu$. The other input parameters given in Ref. [25] which enter indirectly in the loop calculation are $M_{A^0} = 150$ GeV, $M_3 = 600$ GeV and for the sfermion sector, $M_{\tilde{e}_L} = M_{\tilde{\mu}_L} = M_{\tilde{\tau}_L} = 300$ GeV, $A_e = A_d = 900$ GeV and $A_u = 100$ GeV.

Fig. 2 shows our findings. The results obtained by using the $DCPR$-scheme is in complete
Figure 2: Neutralino masses at tree-level (solid/black) and at one-loop by using the $A_{\tau\tau}$-scheme (dashed/blue), the DR scheme (dotted/light green), the DCPR-scheme (dash-dotted/purple) and the MH-scheme (dash-dot-dotted/red) as a function of $t_\beta$. 
agreement with Fig. 2 of the Ref. [25]. The corrections within the $A_{\tau \tau}$-scheme, $\overline{D}\!R$-scheme and $DCPR$-scheme are very modest. They are largest for $m_{\chi_0^3}$ where they reach a maximum of 5 GeV, which corresponds to a mere 2.5% relative correction. The results between the $A_{\tau \tau}$-scheme and the $\overline{D}\!R$-scheme are almost indistinguishable, for all value of $t_\beta$ and all three masses. $DCPR$-scheme is also very close to the latter schemes, a slight deviation occurs for values of $t_\beta$ in excess of 30. The largest corrections are found with the $MH$-scheme which deviates considerably from all other schemes when $t_\beta$ is in excess of 30. Therefore once again this scheme does not look very suitable.

5.3 Some decays of the two charginos

We compute the full electroweak corrections to a few decays of the charginos that were considered in Ref. [34] with the help of the code Grace-SUSY at one-loop. One of the main differences between our approach and the one adopted in [34] is the definition of $t_\beta$. In [34] $t_\beta$ is closely related to our $MH$ definition. [34] works with renormalised mixing matrices apart from the case of sfermions where a shift in the angle defining the diagonalising matrix is performed. To conduct the comparison we take set(A) of [34] given in Table 2, moreover we have $m_{\tilde{\chi}_1^\pm} = 184.2$ GeV, $m_{\chi_2^\pm} = 421.2$ GeV, $m_{\chi_1^0} = 97.75$ GeV. We study also the $t_\beta$ scheme dependence of the result. As we find an excellent agreement at tree-level, Table 3

| $t_\beta$ | $M_{A^0}$ | $\mu$ | $\tilde{e}$ | $\tilde{\mu}$ | $\tilde{\tau}$ |
|----------|-----------|-------|----------|-------------|-------------|
| 10.00    | 424.90    | 399.31| 184.12   | 184.11      | 182.19      |
| $M_1$    | $M_2$     | $M_3$ | $M_{\tilde{L}}^0$ | 118.01      | 117.99      | 111.29      |
| 100.12   | 197.52    | 610.00| $A_\tilde{t}$ | -398.93     | -452.58     | -444.84     |

| $\tilde{u}$ | $\tilde{d}$ | $\tilde{c}$ | $\tilde{s}$ | $\tilde{t}$ | $\tilde{b}$ |
|-------------|-------------|-------------|-------------|-------------|-------------|
| $M_{\tilde{Q}_L}$ | 565.97     | 565.91     | 453.05     |
| $M_{\tilde{t}_R}$ | 546.78     | 544.95     | 546.84     | 544.97     | 460.52     | 538.13     |
| $A_q$       | -775.58    | -979.08    | -784.72    | -1025.74   | -535.40    | -938.50    |

Table 2: Set of supersymmetric parameters defined as set (A) in [34]. All mass parameters are in [GeV].

shows the tree-level result for both codes in one column. For one loop results the agreement is generally good when we switch to the $MH$ scheme apart from the corrections to the $\chi_3^0, \chi_4^0$ masses where a difference is noticeable\footnote{Note that we have found perfect agreement with Ref. [25] as concerns corrections to all $\chi_i^0$ ($i=2,3,4$) masses in the $\overline{D}\!R$ and DCPR scheme, see Section 5.2.}. The correction to the $\chi_3^0$ mass is quite good. The corrections to the masses are negligible especially in the $\overline{D}\!R$ scheme and $A_{\tau \tau}$ scheme. In the one-loop corrections to the decays this additional negligible mass correction is not
taken into account in a decay such as $\tilde{\chi}_1^+ \to W^+ \chi_2^0$ for example especially because of the large mass difference between $\tilde{\chi}_2^+$ and $\chi_2^0$. For the decays, the largest discrepancy is for $\tilde{\chi}_1^+ \to W^+ \chi_1^0$ and $\tilde{\chi}_2^+ \to Z \chi_1^+$. However we note that when this discrepancy is largest, the correction within our $MH$ scheme deviates drastically from the prediction within the $A_{\tau\tau}$ and $\overline{DR}$ schemes. The $MH$ scheme leads, in some decays, to too large corrections. For example for $\tilde{\chi}_1^+ \to W^+ \chi_1^0$ the $MH$ scheme gives 23% correction whereas the correction in $\overline{DR}$ is only 5%. A similar observation can be made for $\chi_2^0 \to \tilde{\tau}_2^+ \nu_\tau$ where in the $A_{\tau\tau}$ scheme the correction is $\sim 0\%$ whereas it reaches 24% within our $MH$ scheme. These examples also show that for all decays considered in Table 3 the predictions of the $A_{\tau\tau}$ and $\overline{DR}$ are within 2% and very often even much better. Once more these examples show that the $MH$ scheme is not to be recommended, we suspect strongly that the differences we find between Grace-SUSY and SloopS are essentially due to the peculiar choice of the scheme based on the heavy neutral CP-even Higgs that greatly amplifies the corrections and the differences.

| Decays [GeV] | Tree Level | Grace | SloopS $MH$ | SloopS $\overline{DR}$ | SloopS $A_{\tau\tau}$ |
|--------------|------------|-------|-------------|-----------------------|-----------------------|
| $\chi_1^+ \to \nu_\tau \tilde{\tau}_1^+$ | 3.91 x 10^{-2} | 3.78 x 10^{-2}(-3\%) | 3.79 x 10^{-2}(-3\%) | 4.18 x 10^{-2}(+7\%) | 4.15 x 10^{-2}(+6\%) |
| $\chi_1^+ \to e^+ e^-$ | 1.47 x 10^{-2} | 1.48 x 10^{-2}(0\%) | 1.47 x 10^{-2}(0\%) | 1.44 x 10^{-2}(-2\%) | 1.49 x 10^{-2}(+1\%) |
| $\chi_2^0 \to W^+ \chi_1^0$ | 9.65 x 10^{-4} | 1.28 x 10^{-3}(+33\%) | 1.19 x 10^{-3}(+23\%) | 1.01 x 10^{-3}(+52\%) | 1.03 x 10^{-3}(+70\%) |
| $\chi_2^0 \to e^+ e^-$ | 1.54 x 10^{-4} | 1.48 x 10^{-3}(-4\%) | 1.40 x 10^{-3}(-9\%) | 1.52 x 10^{-3}(-1\%) | 1.51 x 10^{-3}(-2\%) |
| $\chi_2^0 \to e^+ e^-$ | 6.89 x 10^{-2} | 5.70 x 10^{-2}(-17\%) | 5.27 x 10^{-2}(-24\%) | 6.75 x 10^{-2}(-2\%) | 6.88 x 10^{-2}(0\%) |
| $\chi_2^0 \to W^+ \chi_1^0$ | 1.93 x 10^{-1} | 2.07 x 10^{-1}(+7\%) | 2.02 x 10^{-1}(+5\%) | 2.08 x 10^{-1}(+7\%) | 2.08 x 10^{-1}(+7\%) |
| $\chi_2^0 \to W^+ \chi_2^0$ | 8.67 x 10^{-1} | 9.93 x 10^{-1}(+15\%) | 9.75 x 10^{-1}(+12\%) | 8.75 x 10^{-1}(+1\%) | 8.80 x 10^{-1}(+1\%) |
| $\chi_2^0 \to Z \chi_1^+$ | 7.53 x 10^{-1} | 8.56 x 10^{-1}(+14\%) | 8.06 x 10^{-1}(+7\%) | 7.64 x 10^{-1}(+1\%) | 7.68 x 10^{-1}(+2\%) |

Neutralino masses [GeV]

| Neutralino | $\chi_1^+$ | $\chi_2^+$ | $\chi_3^+$ | $\chi_4^+$ |
|------------|------------|------------|------------|------------|
| $\chi_1^+$ | 184.55 | 184.62 | 184.60 | 184.44 |
| $\chi_2^+$ | 405.14 | 398.30 | 405.93 | 407.51 |
| $\chi_3^+$ | 420.49 | 413.39 | 420.23 | 419.54 |
| $\chi_4^+$ | 419.60 | 419.60 | 419.60 | 419.60 |

Table 3: Some $\tilde{\chi}_1^\pm, 2$ decays at tree level and at one-loop with three different $t_\beta$-schemes in SloopS compared to Grace-SUSY for set (A) defined in Table 2. Corrections to the masses of $\chi_{2,3,4}^0$ are also given.

### 5.4 $e^+ e^- \to \tilde{\chi}_1^+ \tilde{\chi}_1^-$

We now turn to the full $O(\alpha)$ correction to chargino production at a linear collider. We consider the same process as the one computed in [34] within Grace-SUSY, namely $e^+ e^- \to \chi_1^+ \chi_1^-(\gamma)$. We use the same set of parameters Set(A) defined in Table 2 and study the energy dependence of the total cross section. The same cross section has been studied in [35, 36]. The QED radiation in view of an event generator has been studied in [37]. Fig. 8 shows the cross section of this process computed at tree-level and also at one-loop for different $t_\beta$-schemes. We find excellent agreement with the results of Ref. [34] when specialising to the $MH$-scheme. The $A_{\tau\tau}, \overline{DR}$ and $DCPR$ give corrections within the per-nil level and one can hardly distinguish between the three schemes. For this process and with Set(A), the $MH$ scheme gives systematically about $-1\%$ to $-1.5\%$ difference from the other schemes which is very small compared to the discrepancies we have noted for some decays of the
charginos with the same set of parameters. This suggests that the \( t_\beta \) scheme dependence is quite small and explains why our results for this process agree very well with those of Grace-SUSY. In any case over the whole range of energies the full \( \mathcal{O}(\alpha) \) corrections in the \( \overline{\text{DR}} \) scheme amounts to about \(-9\%\) for a centre of mass energy \( \sqrt{s} = 500 \text{ GeV} \) reaching a maximum of about \(-7\%\) at \( \sqrt{s} = 700 \text{ GeV} \) and dropping to about \(-11\%\) at \( \sqrt{s} = 1300 \text{ GeV} \).

![Graph](image)

**Figure 3:** Total cross section of \( e^+e^- \to \chi^+_1\chi^-_1(\gamma) \) as a function of \( \sqrt{s} \) at tree-level (solid/black) and at one-loop (full order \( \mathcal{O}(\alpha) \)) in the \( A_{\tau\tau} \)-scheme (dashed/blue), the \( \overline{\text{DR}} \) scheme (dotted/light green), the DCPR-scheme (dash-dotted/purple) and the MH-scheme (dash-dot-dotted/red). The right panel gives the percentage correction. In the left panel considering that the \( A_{\tau\tau} \)-scheme, the \( \overline{\text{DR}} \) and the DCPR-scheme are not distinguishable we therefore only show the result of the \( A_{\tau\tau} \)-scheme beside the tree-level and the MH-scheme.

5.5 \( e^+e^- \to \tilde{\tau}_i\tilde{\tau}_j \)

\( e^+e^- \to \tilde{\tau}_1\tilde{\tau}_1, \tilde{\tau}_2\tilde{\tau}_2, \tilde{\tau}_1\tilde{\tau}_2 \) have been calculated in Ref. [38, 39, 40]. In Ref. [38, 39] only the electroweak non QED corrections are computed, the QED corrections are dismissed on a diagrammatic level by leaving out one-loop Feynman diagrams with virtual photon exchange. In Ref. [40] the full \( \mathcal{O}(\alpha) \) is performed with a resummation of the leading log QED corrections within a structure function approach for the universal initial state radiation. We perform here a complete \( \mathcal{O}(\alpha) \) calculation of these processes and compare our results to those of [39] as concerns the electroweak non QED corrections. We therefore take scenario 1 of [39] with the following set of parameters: \( t_\beta = 20, \mu = 1000 \text{ GeV}, \ M_1 = 94.92 \text{ GeV}, \ M_2 = 200 \text{ GeV}, \ M_3 = 669.18 \text{ GeV}, \ M_{A^0} = 300 \text{ GeV}, \ M_{\tilde{\ell}_R} = M_{\tilde{\ell}_L} = M_{\tilde{\nu}_R} = M_{\tilde{\nu}_L} = M_{\tilde{d}_R} = 400 \text{ GeV}, \ A_f = -500 \text{ GeV}, \ M_{\tilde{t}_R} = 360 \text{ GeV} \) and \( M_{\tilde{b}_R} = 440 \text{ GeV}. \) In [39] the electromagnetic coupling is not taken in the Thomson limit but is fixed from \( \alpha_{\text{MS}}(M_Z^2) \) with \( \alpha_{\text{MS}}(M_Z^2) = 1/127.934. \) This absorbs large logarithms compared to our on-
shell scheme based on $\alpha(0) = 1/137.036$. In \cite{39} the mixing parameter in the stau sector is parameterised through the mixing angle which is renormalised according to Eq. (3.34). For the sake of comparison we will here also switch to this scheme for the sfermion mixing.

Figure 4: Total cross section of $e^+e^- \rightarrow \tilde{\tau}_i \tilde{\tau}_j(\gamma)$ as a function of $\sqrt{s}$ at tree-level (solid/black) and at full one-loop in the DCPR scheme (dashed/blue). We also show the tree-level improved cross section with $\alpha^{\text{MS}}(M_Z^2)$ (dash-dotted/red) and the pure weak correction in the on-shell scheme as defined in the text (dotted/purple). The full $\mathcal{O}(\alpha)$ relative corrections for the three channels with respect to the tree-level cross sections with $\alpha(0)$ is shown in the panel on the right (dashed/blue). We also show the weak non QED relative correction (dotted/purple) where the improved tree-level cross sections with $\alpha^{\text{MS}}(M_Z^2)$ is used to absorb large logs from the running of $\alpha$. This correction should be contrasted to the one obtained in Ref. \cite{39}. In order not to crowd the figure the channels are not labeled. They can be easily identified as they have different thresholds.

In addition to the tree level cross section calculated with $\alpha(0) = 1/137.036$ and the complete $\mathcal{O}(\alpha)$ one-loop correction, we compute the improved tree-level cross section with $\alpha^{\text{MS}}(M_Z^2) = 1/127.934$. Our evaluation of the weak non QED correction is different from the one in \cite{39}. In our case the weak correction is obtained by subtracting the leading QED corrections. The initial state radiation factor including the virtual photon correction and the soft bremsstrahlung photon below the cut-off energy $k_c$ is universal and known, see for example \cite{41},

$$\delta^{\text{QED}}_{V+S} = \frac{2\alpha}{\pi} \left( (L_e - 1) \ln \frac{k_c}{E_b} + \frac{3}{4} L_e + \frac{\pi^2}{6} - 1 \right), \quad L_e = \ln(s/m_e^2).$$

(5.1)

where $m_e$ is the electron mass and $E_b$ the beam energy $s = 4E_b^2$. To subtract not only the initial but also the final state radiation and the final-initial interference QED effect, we take the result of the virtual one-loop correction and the soft radiation factor obtained by the code and subtract the following

$$\sigma^{\text{weak}}(\sqrt{s}) = \sigma^{\text{virtual+soft}}(\sqrt{s}, k_c) - \frac{\alpha}{\pi} A(\sqrt{s}) \ln \left( \frac{2k_c}{\sqrt{s}} \right) - \frac{3\alpha}{2\pi} \sigma^{\text{tree}}(\sqrt{s}) \ln \left( \frac{s}{m_e^2} \right).$$

(5.2)
The last term in Eq. (5.2) stems from the collinear singularity due to initial state radiation and we neglect non log terms, the latter that arise from initial radiation are negligible of order 0.3% relative correction. The term \( A(\sqrt{s}) \) is extracted numerically based on the fact that the weak non QED correction is independent of the cut-off \( k_c \). We take two small enough cut-off \( k_{c1}, k_{c2} \) to extract \( A(\sqrt{s}) \),

\[
\frac{\alpha}{\pi} A(\sqrt{s}) = \frac{\sigma^{\text{virtual+soft}}(\sqrt{s}, k_{c1}) - \sigma^{\text{virtual+soft}}(\sqrt{s}, k_{c2})}{\ln \left( \frac{k_{c2}}{k_{c1}} \right)}.
\]

We have checked that \( \sigma^{\text{weak}}(\sqrt{s}) \) defined this way is independent of the cut-off \( k_c \) by taking other values of \( k_c \). Such a definition of the weak correction has been introduced in [42].

Our tree-level results for the improved tree-level with \( \alpha = \alpha_{\overline{MS}}(M_Z^2) \) reproduces the corresponding cross section in Ref. [39] perfectly. To help compare our results with those Ref. [39], the right panel of Fig. 4 shows also the relative weak non QED correction with \( \alpha_{\overline{MS}}(M_Z^2) \) as input rather than \( \alpha(0) \), hence subtracting large logs from the running of \( \alpha \). Our predictions for the weak correction defined this way are within 1% of those in [39] within the DCPR scheme used in [39]. We have traced this small difference to the different ways the weak correction is defined from the subtraction of the QED corrections. The energy dependence of the weak corrections matches perfectly.

We can now comment on the \( t_\beta \) scheme dependence and the sfermion mixing renormalisation scheme. The corrections induced by the different \( t_\beta \) schemes are very small. Even the MH scheme departs by not more than 0.3% from the \( \overline{DR} \). The other schemes, DCPR and \( A_{\tau \tau} \), agree within better than 0.01% with \( \overline{DR} \). The difference in the choice of the sfermion mixing parameter \( \delta m_{\tilde{g}_{12}}^2 \) is even more negligible here. For example, for a centre of mass energy \( \sqrt{s} = 1000 \) GeV, the one-loop correction to the process \( e^+e^- \rightarrow \tilde{\tau}_1 \tilde{\tau}_2 \) differs only about 0.003% when we switch from the default definition in \texttt{SloopS} Eq. (3.33) to the one that has been usually used Eq. (3.34).

These calculations show that not only it is important to take into account the QED corrections but also that the pure electroweak corrections are certainly not negligible, for example even after absorbing the effect due to the running of \( \alpha \), the weak corrections for \( \tilde{\tau}_1 \tilde{\tau}_2 \) production is about \(-15\%\).

6 Conclusions

We have presented in detail a complete renormalisation of the sfermion sector as well as of the chargino/neutralino sector of the MSSM in the case of CP conservation. We critically analysed the renormalisation of the mixing parameter in the sfermion sector and discussed different ways to define it in a consistent manner. This paper is a sequel to our study in Ref. [1] and completes the presentation of all the ingredients that are built into our automated code for one-loop calculations in the MSSM, \texttt{SloopS}. Although other approaches
to renormalising the MSSM have been worked out, we believe that our approach treats all the sectors consistently within the same general on-shell framework in particular about the treatment of mixing and how one deals with the rotation and diagonalising matrices. Moreover our code permits powerful gauge checks with the help of the non-linear gauge fixing condition and allows to easily switch between different renormalisation schemes. Some very powerful and extensive tests have been conducted on the code as concerns ultraviolet finiteness and gauge parameter independence on an almost exhaustive list of $2 \rightarrow 2$ processes, see \[31\]. In the present paper we choose to concentrate on a few key observables in the sfermion and chargino/neutralino sector and compared our results with some that are found in the literature while at the same time studying the impact of different renormalisation schemes. We have calculated one-loop corrections to sfermion masses and also neutralino masses. We have also derived some chargino decay widths and presented a calculation of the production of charginos and sleptons at $e^+e^-$ colliders. We find the genuine electroweak corrections in these cross sections to be rather important and should therefore be taken into account. Having at our disposal a code that allows the one-loop calculation for any process in the MSSM, it is now possible to envisage revisiting analyses for the extraction of the fundamental supersymmetric parameters from precision measurements at the colliders and use them in turn for a precision calculation of the relic density for example. Finally, let us mention that other renormalisation schemes, with different choices of the input parameters from the one described in this paper, for the chargino/neutralino sector are already implemented in the code and would be part of a forthcoming study. Although in the many examples we have shown here the QCD corrections are calculated, a complete treatment of the gluon/gluino sector within an automated code such as S1oopS and in particular how to easily implement within the code a regulator for the infrared singularity is work in progress.

**Acknowledgments**

We would first like to thank Andrei Semenov whose help was invaluable in the first stages of the project. We also owe much to our friends of the Minami-Tateya group and the developers of the Grace-SUSY code, in particular we learned much from Masaaki Kuroda. We benefited a great deal from discussions with Guillaume Chalons, Sun Hao, Karol Kovarik and Peter Zerwas. This work is supported in part by GDRI-ACPP of the CNRS (France). This work is part of the French ANR project, ToolsDMColl. This work is also supported in part by the European Community’s Marie-Curie Research Training Network under contract MRTN-CT-2006-035505 “Tools and Precision Calculations for Physics Discoveries at Colliders”, the DFG SFB/TR9 “Computational Particle Physics”, and the Helmholtz Alliance “Physics at the Terascale”.

38
References

[1] N. Baro, F. Boudjema, A. Semenov, Phys. Rev. D78 (2008) 115003, arXiv:0807.4668 [hep-ph].

[2] A. Semenov, hep-ph/9608488
   A. Semenov, Nucl. Inst. Meth. and Inst. A393 (1997) 293;
   A. Semenov, Comp. Phys. Commun. 115 (1998) 124;
   A. Semenov, hep-ph/0208011;
   A. Semenov, arXiv:0805.0555 [hep-ph].

[3] J. Küblbeck, M. Böhme, A. Denner, Comp. Phys. Commun. 60 (1990) 165;
   H. Eck, J. Küblbeck, Guide to FeynArts 1.0, Würzburg, 1991;
   H. Eck, Guide to FeynArts 2.0, Würzburg, 1995;
   T. Hahn, Comp. Phys. Commun. 140 (2001) 418, hep-ph/0012260.

[4] T. Hahn, M. Perez-Victoria, Comp. Phys. Commun. 118 (1999) 153, hep-ph/9807565;
   T. Hahn, hep-ph/0406288; hep-ph/0506201.

[5] T. Hahn, LoopTools, http://www.feynarts.de/looptools/.

[6] N. Baro, F. Boudjema, A. Semenov, Phys. Lett. B660 (2008) 550, arXiv:0710.1821 [hep-ph].

[7] F. Boudjema, A. Semenov, D. Temes, Phys. Rev. D72 (2005) 055024, hep-ph/0507127.

[8] J. Fujimoto et al., Comput. Phys. Commun. 153 (2003) 106, hep-ph/0208036.

[9] G. Bélanger, F. Boudjema, J. Fujimoto, T. Ishikawa, T. Kaneko, K. Kato, Y. Shimizu, Phys. Rep. 430 (2006) 117, hep-ph/0308080.

[10] F. Boudjema, E. Chopin, Z. Phys. C73 (1996) 85, hep-ph/9507396.

[11] A. Freitas, D. Stöckinger, Phys. Rev. D66 (2002) 095014, hep-ph/0205281.

[12] J.A. Coarasa, D. Garcia, J. Guasch, R.A. Jimenez, J. Sola, Eur. Phys. J. C2 (1998) 373, hep-ph/9607485;
    J.A. Coarasa, D. Garcia, J. Guasch, R.A. Jimenez, J. Sola, Phys. Lett. B425 (1998) 329, hep-ph/9711472;
    J.A. Coarasa, D. Garcia, J. Guasch, J. Sola, W. Hollik, Phys. Lett. B442 (1998) 326, hep-ph/9808278.

[13] P.H. Chankowski, S. Pokorski, J. Rosiek, Nucl. Phys. B423 (1994) 437, hep-ph/9303309.

[14] A. Denner, Fortsch. Phys. 41 (1993) 307, arXiv:0709.1075 [hep-ph].

[15] A. Arhrib, R. Benbrik, Phys. Rev. D71 (2005) 095001, hep-ph/0412349.

[16] A. Bartl, H. Eberl, K. Hidaka, S. Kraml, W. Majerotto, W. Porod, Y. Yamada, Phys. Lett. B419, 243 (1998), hep-ph/9710286.

[17] R.G. Stuart, Phys. Lett. B262 (1991) 113.
[18] J.R. Espinosa, I. Navarro, Phys. Rev. D66 (2002) 016004, hep-ph/0109126.
[19] J.R. Espinosa, Y. Yamada, Phys. Rev. D67 (2003) 036003, hep-ph/0207351.
[20] J. Guasch, J. Sola, W. Hollik, Phys. Lett. B437 (1998) 88, hep-ph/9802329.
    H. Eberl, S. Kraml, W. Majerotto, JHEP 9905 (1999) 016, hep-ph/9903413.
[21] S. Kraml, PhD dissertation, hep-ph/9903257.
[22] H. Eberl, A. Bartl, W. Majerotto, Nucl. Phys. B472 (1996) 481, hep-ph/9603206.
    S. Kraml, H. Eberl, A. Bartl, W. Majerotto, W. Porod, Phys. Lett. B386, 175 (1996),
    hep-ph/9605412.
    A. Bartl, H. Eberl, K. Hidaka, S. Kraml, W. Majerotto, W. Porod, Y. Yamada, Phys.
    Lett. B419, 243 (1998), hep-ph/9710286.
    A. Bartl, H. Eberl, K. Hidaka, S. Kraml, W. Majerotto, W. Porod, Y. Yamada, Phys.
    Rev. D59, 115007 (1999), hep-ph/9806299.
[23] A. Djouadi, W. Hollik, C. Junger, Phys. Rev. D55, 6975 (1997), hep-ph/9609419.
[24] J. Guasch, J. Sola, W. Hollik, Phys. Lett. B437 (1998) 88.
[25] T. Fritzschke, W. Hollik, Eur. Phys. J. C24 (2002) 619, hep-ph/0203159.
[26] M. Drees, W. Hollik, Q. Xu, JHEP 02 (2007) 032. hep-ph/0610267.
[27] J. Guasch, W. Hollik, J. Sola, JHEP 0210 (2002) 040, hep-ph/0207364.
[28] J.L. Kneur, G. Moultaka, Phys. Rev. D59 (1999) 015005, hep-ph/9807336.
[29] S. Y. Choi, A. Djouadi, M. Guchait, J. Kalinowski, H. S. Song, P. M. Zerwas, Eur.
    Phys. J. C14 (2000) 535, hep-ph/0002033.
[30] S.Y. Choi, J. Kalinowski, G.A. Moortgat-Pick, P.M. Zerwas, Eur. Phys. J. C22 (2001)
    563, Addendum-ibid. C23 (2002) 769, hep-ph/0108117.
[31] N. Baro, PhD thesis, Renormalisation and predictions at one-loop in supersymmetry,
    applications to dark matter and collider physics,
    http://tel.archives-ouvertes.fr/tel-00329722/fr/.
[32] [CompHEP Collaboration], E. Boos et al., Nucl. Instrum. Meth. A534 (2004) 250,
    hep-ph/0403113.
    A. Pukhlov et al., ”CompHEP user’s manual, v3.3”, Preprint INP MSU 98-41/542 (1998)
    hep-ph/9908288.
    http://theory.sinp.msu.ru/comphep/.
[33] W. Hollik, H. Rzehak, Eur. Phys. J. C32 (2003) 127, hep-ph/0305328.
[34] J. Fujimoto, T. Ishikawa, Y. Kurihara, M. Jimbo, T. Kon, M. Kuroda, Phys. Rev.
    D75 (2007) 113002, hep-ph/0701200.
[35] T. Fritzschke, W. Hollik, Nucl. Phys. Proc. Suppl. 135 (2004) 102, hep-ph/0407095.
[36] W. Öller, H. Eberl, W. Majerotto, Phys. Rev. D71 (2005) 115002, hep-ph/0504109.
[37] W. Kilian, J. Reuter, T. Robens, Eur. Phys. J. C48 (2006) 389, hep-ph/0607127.
[38] A. Arhrib, W. Hollik, *JHEP* **0404** (2004) 073, hep-ph/0311149.

[39] K. Kovarik, C. Weber, H. Eberl, W. Majerotto, *Phys. Lett.* **B591** (2004) 242, hep-ph/0401092.

[40] K. Kovarik, C. Weber, H. Eberl, W. Majerotto, *Phys. Rev.* **D72** (2005) 053010, hep-ph/0506021.

[41] G. Bélanger, F. Boudjema, J. Fujimoto, T. Ishikawa, T. Kaneko, K. Kato, Y. Shimizu, *Phys. Lett.* **B559** (2003) 252, hep-ph/0212261.

[42] N. Bouayed and F. Boudjema, *Phys. Rev.* **D77** (2008) 013004, arXiv:0709.4388 [hep-ph].