Measuring mixed state entanglement through single-photon interference

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Entanglement is a fundamental feature of quantum mechanics, considered a key resource in quantum information processing. Measuring entanglement is an essential step in a wide range of applied and foundational quantum experiments. When a two-particle quantum state is not pure, standard methods to measure the entanglement require detection of both particles. We introduce a method in which detection of only one of the particles is required to characterize the entanglement of a two-particle mixed state. Our method is based on the principle of quantum interference. We use two identical sources of a two-photon mixed state and generate a set of single-photon interference patterns. The entanglement of the two-photon quantum state is characterized by the visibility of the interference patterns. Our experiment thus opens up a distinct avenue for verifying and measuring entanglement, and can allow for mixed state entanglement characterization even when one particle in the pair cannot be detected.

We begin by giving a physical description of our entanglement verification scheme. (The full theoretical treatment is presented in a separate publication [32].) We employ two identical sources, \( Q_1 \) and \( Q_2 \) (Fig. 1), each of which can generate the same two-photon quantum state. They emit in such a way that only one pair of photons is produced at a time, i.e., we generate only one copy of the state [24–29], all rely on the detection of both subsystems.

Whether entanglement of a bipartite mixed state can be verified by performing a measurement on only one subsystem is an open question.

We address this question and demonstrate that it is possible to verify entanglement in a bipartite mixed state by detecting only one subsystem. We choose a polarization entangled mixed state, which can be obtained by generalizing two Bell states. Based on a particular implementation of quantum interference [30, 31], we perform an experiment in which the single-photon interference patterns generated by emissions from two identical twin photon sources contain the complete information about entanglement in a two-photon mixed state. Only one photon pair is produced in each detection run. We find that for certain choices of measurement bases, single-photon interference is possible only when the photon pair is entangled in polarization. It turns out that the interference visibility is linearly proportional to the concurrence, a widely used entanglement measure for qubits. In fact, even though each photon from a completely mixed (separable) two photon polarization state may be described by the same unpolarized state as each photon from a maximally entangled two-photon polarization state, these two scenarios can be distinguished in our experiment without coincidence detection or any post-selection.

We begin by giving a physical description of our entanglement verification scheme. (The full theoretical treatment is presented in a separate publication [32].) We employ two identical sources, \( Q_1 \) and \( Q_2 \) (Fig. 1), each of which can generate the same two-photon quantum state. They emit in such a way that only one pair of photons is produced at a time, i.e., we generate only one copy of the state. We denote the two photons by \( \alpha \). We ensure that \( Q_2 \) can emit photon \( \alpha \) into propagation mode \( \alpha \). We ensure that \( Q_2 \) can emit photon \( \alpha \) only in the same propagation mode (\( \alpha_1 \)). This is done by sending the beam of photon \( \alpha \) generated by \( Q_1 \) through source \( \alpha \) and perfectly aligning the beam with the spatial propagation mode \( \alpha \) generated by \( Q_2 \). Therefore, if one only observes photon \( \alpha \) that emerges from \( Q_2 \), one cannot identify the origin of the photon. Sources \( Q_1 \) and \( Q_2 \)
can emit photon $\beta$ into distinct propagation modes $\beta_1$ and $\beta_2$, respectively. These two modes are superposed by a beamsplitter, $BS$, and one of the outputs of $BS$ is collected by a detector, $PD$, where the single-photon counting rate (intensity) is measured. We also include an additional device, $\Gamma$, which can transform or project the light emerging from the beamsplitter to a particular state of our choice. Note that photon $\alpha$ is never detected. It is known that single-photon interference can be observed (at $PD$) for photon $\beta$ in such a setup [30, 31].

We now introduce a device, $O$, in propagation mode $\alpha_1$ between $Q_1$ and $Q_2$. A striking feature of this kind of interferometer is that the effect of this interaction is observed in the interference pattern recorded at $PD$ although photon $\beta$ never interacts with $O$. Recent imaging, spectroscopy and optical coherence tomography experiments have shown that with the knowledge of the two-photon quantum state, one can retrieve the information about the interaction from the interference pattern [33–40].

Our entanglement verification method is essentially the converse of the imaging method described in Refs. [33, 34]. Here, we retrieve the information about the two-photon quantum state from the interference pattern with the knowledge of the interaction between $O$ and the undetected photon $\alpha$.

In order to demonstrate our method, we work with two-qubit states determined by three free parameters. One example of such state is expressed by the density operator

$$
\rho = I_H |H_\alpha H_\beta\rangle \langle H_\alpha H_\beta| + I_V |V_\alpha V_\beta\rangle \langle V_\alpha V_\beta| + (\mathcal{J} \sqrt{I_H I_V} e^{-i\phi} |H_\alpha H_\beta\rangle \langle V_\alpha V_\beta| + \text{H.c.},
$$

where $I_H + I_V = 1$ with $0 \leq I_H \leq 1$, $\phi$ is a real number, and $0 \leq \mathcal{J} \leq 1$. This state can be seen as a result of decoherence of the pure state $\sqrt{I_H} |H_\alpha H_\beta\rangle + e^{i\phi} \sqrt{I_V} |V_\alpha V_\beta\rangle$.

Note that state $\rho$ can also be obtained by generalizing the following two Bell States: $|\Phi^+\rangle = (|H_\alpha H_\beta\rangle + |V_\alpha V_\beta\rangle)/\sqrt{2}$ and $|\Phi^-\rangle = (|H_\alpha H_\beta\rangle - |V_\alpha V_\beta\rangle)/\sqrt{2}$.

State $\rho$ is entangled when $0 < I_H \leq 1$ and $\mathcal{J} \neq 0$. It is maximally entangled for $I_H = I_V = 1/2$ and $\mathcal{J} = 1$. When $I_H = 1$ or $I_H = 0$, the state $\rho$ is pure and separable. The state is maximally mixed and separable for $I_H = I_V = 1/2$ and $\mathcal{J} = 0$. A measure of entanglement, commonly used for two-qubit systems, is the concurrence $C$ [41], which for the state, $\rho$ [Eq. (1)], is [32]

$$
C(\rho) = 2 \mathcal{J} \sqrt{I_H I_V}.
$$

For maximally entangled states $C(\rho) = 1$ and for separable states $C(\rho) = 0$.

In the experiment our source of entangled photons is a pair of perpendicularly oriented nonlinear crystals, $C'_H$ and $C'_V$ (Fig. 2a)[5]. However, our scheme also works for any other source producing the state $\rho$ given by Eq. (1), for example, a single type-II non-linear crystal [4]. Horizontally and vertically polarized two-photon states ($|H_\alpha H_\beta\rangle$ and $|V_\alpha V_\beta\rangle$) are produced by spontaneous parametric down-conversion in $C'_H$ and $C'_V$, respectively, where photons $\alpha$ and $\beta$ have distinct wavelengths. Parameters $I_H$ and $I_V$ are proportional to the probability of emissions at $C'_H$ and $C'_V$, respectively. The parameter $\mathcal{J}$ represents the mutual coherence between these emissions and $\phi$ is the relative phase between these emissions. All three parameters are separately tuned in our experiment (Supplemental Information).

As illustrated in Fig. 1, we use two such sources in the experiment (see supplemental information for the detailed experimental setup). As for device $O$, we use a half-wave plate (HWP), which allows us to introduce distinguishability. The device, $\Gamma$, is a combination of wave plates and a polarizer (Supplemental Information) such that we can project photon $\beta$ onto horizontal ($H$), vertical ($V$), diagonal ($D$), anti-diagonal ($A$), right-circular

![Figure 1: Entanglement verification scheme. Two identical sources, $Q_1$ and $Q_2$, individually generate the same two-photon state ($\hat{\rho}$). Source $Q_1$ can emit a photon pair ($\alpha$, $\beta$) into propagation modes $\alpha_1$ and $\beta_1$. Source $Q_2$ is restricted to emit photon $\alpha$ also in the mode $\alpha_1$. Photon $\alpha$, which is never detected, interacts with a device, $O$, between $Q_1$ and $Q_2$. Source $Q_2$ can emit photon $\beta$ in propagation mode $\beta_2$. Modes $\beta_1$ and $\beta_2$ are combined by a beamsplitter ($BS$) and an output of $BS$ is collected by a photo-detector ($PD$). Another device ($\Gamma$), placed before $PD$, allows us to choose the measurement basis. Sources $Q_1$ and $Q_2$ never emit simultaneously. When it is impossible to know the source of a detected photon, single-photon interference is observed at $PD$. For certain choices of basis, the entanglement of the two-photon state determines the visibility of the interference pattern. Information about the entanglement is retrieved from the single-photon interference patterns.](image-url)
Likewise, photon-pair emissions at \(C^v\) can be used to determine \(t\), and vice versa. In such a situation photon-pair emissions at \(\beta\) become maximum. (This result is fully consistent with Refs. [30, 31].) Note that in this case the polarization-entangled photon pair: Each source \(\alpha\) or \(\beta\) is illuminated by mutually coherent laser propagation modes (not shown) such that the horizontal \((H)\) components of the possible emissions at the separate sources are coherent. Highest interference visibility is observed at PD if \(H\) polarized photons are detected. (c) For \(\theta = \pi/4\), we probe the indistinguishability between emissions at \(C^v_H\) and \(C^v_V\) and also between emissions at \(C^r_H\) and \(C^r_V\) (not shown) by detecting diagonally \((D)\) linearly polarized \(\beta\)-photons. The visibility of the resulting interference pattern depends on the entanglement in the two-photon state.

\(\) \(R\), and left-circular \((L)\) polarization states. Therefore, we choose the measurement basis by the use of \(\Gamma\). The phase in the interferometer is changed by moving the position of the beamsplitter \((BS)\).

\(\) The two sources \((Q_1\) and \(Q_2\) are illuminated by mutually coherent laser beams. In such a situation photon-pair emissions at \(C^v_H\) and \(C^v_V\) are fully coherent. If the HWP is set at angle \(\theta = 0\) and the device \(\Gamma\) is set such that only \(H\)-polarized photons \((|H\rangle)\) are detected at PD (Fig. 2b), visibility of the recorded interference pattern becomes maximum. (This result is fully consistent with the results presented in Refs. [30, 31].) Note that in this case, no photon emitted by \(C^v_H\) or \(C^v_V\) arrives at the detector. Likewise, photon-pair emissions at \(C^r_H\) and \(C^r_V\) are also fully coherent when \(Q_1\) and \(Q_2\) are illuminated coherently. However, as mentioned before, pair emissions at \(C^v_H\) and \(C^v_V\) (and also at \(C^r_H\) and \(C^r_V\)) may not be fully coherent and the mutual coherence between them is given by \(\mathcal{F}(Q_1, Q_2)\). If emission at \(C^v_H\) is fully coherent to \(C^r_V\) and the mutual coherence between emissions at \(C^v_H\) and \(C^r_V\) is \(\mathcal{F}\), then the mutual coherence between pair emissions at \(C^v_H\) and \(C^v_V\) is also given by \(\mathcal{F}\) (Ref. [32], Eq. (11)). The same is true for the mutual coherence between emissions at \(C^r_H\) and \(C^r_V\).

When the HWP is set at \(\theta = \pi/4\), the polarization components of \(\alpha_1\) are rotated as \(|H\rangle \rightarrow |V\rangle\) and \(|V\rangle \rightarrow -|H\rangle\). The quantum state produced at Q2 is not affected by the rotation of the HWP. If we now detect photon \(\beta\) after projecting onto the \(|H\rangle, |V\rangle\) basis, no interference is observed for all values of \(\mathcal{F}\) and \(I_H\), i.e., the corresponding values of visibility are \(V_H|\theta = \pi/4 = V_V|\theta = \pi/4 = 0\). This is because if we were to jointly measure the polarization state of photon \(\alpha\) after \(Q_2\) we would know from which crystal photon \(\beta\) had arrived. It is important to note that measurement in \(|H\rangle, |V\rangle\) basis does not yield any information about entanglement.

We now detect photon \(\beta\) after projecting onto \(|D_H\rangle = (|H\rangle + |V\rangle)/\sqrt{2}\) while the HWP is set at \(\theta = \pi/4\). Photon \(\beta\) can now arrive at the detector in four alternative ways: 1) from \(C^v_H\), 2) from \(C^r_H\), 3) from \(C^v_V\), and 4) from \(C^r_V\). We first note that alternative 1 is fully distinguishable from alternative 2 for the reason discussed in the previous paragraph. Likewise, alternative 3 is fully distinguishable from alternative 4. For very similar reasons, alternatives 1 & 3 are also distinguishable from each other, as are alternatives 2 & 4. According to the laws of quantum mechanics, the distinguishable alternatives do not result in interference. Let us now consider the remaining two options: alternatives 1 & 4, and alternatives 2 & 3. These two sets of alternatives are fully equivalent to each other. For the sake of brevity, we only present arguments for alternatives 1 & 4 (Fig. 2c).

We recall that the mutual coherence between emissions at \(C^v_H\) and \(C^v_V\) is given by \(\mathcal{F}\). Therefore, if \(\mathcal{F} = 0\), alternatives 1 & 4 become fully distinguishable [42] and no interference occurs. If \(I_H = 0\) or \(I_V = 0\), no emission occurs at \(C^v_H\) or \(C^v_V\). In this case alternatives 1 & 4 are also fully distinguishable. When \(\mathcal{F} = 1\) and \(I_H = I_V = 1/\sqrt{2}\), alternatives 1 & 4 are fully indistinguishable and interference occurs with maximum visibility. In any intermediate case interference occurs with reduced visibility. Following this argument, we find that the visibility is given by (c.f. [32])

\[
V_D|\theta = \pi/4 \propto \mathcal{F}\sqrt{I_H I_V}.
\]

It follows from Eqs. (2) and (3) that the single-photon interference visibility \(V_D|\theta = \pi/4\) is linearly proportional to the concurrence, i.e., the visibility contains information about the entanglement. Figure 3 shows experimentally obtained interference patterns for four states. The data clearly show that when HWP angle \(\theta = \pi/4\), the visibility measured for \(|D_H\rangle\) increases with the amount of entanglement.

Likewise, single-photon interference patterns obtained in circular polarization basis \(|R_\beta\rangle, |L_\beta\rangle\) also contain information about the entanglement [32]. Obtaining non-zero visibility after projecting photon \(\beta\) onto \(|D_\beta\rangle, |A_\beta\rangle\), \(|R_\beta\rangle\) or \(|L_\beta\rangle\) confirms that the two-photon state is entangled.

Equations (2) and (3) suggest that the concurrence can be determined from the visibility of the interference patterns. However, for an accurate measurement of the
concerning one needs to consider the experimental loss of photons in propagation mode \( \alpha \) between \( Q_1 \) and \( Q_2 \) because such losses lead to loss of visibility. In fact, visibilities measured for \( \ket{D^\beta}, \ket{A^\beta}, \) and \( \ket{R^\beta}, \) and \( \ket{L^\beta} \) (while \( \theta = \pi/4 \)) will always be smaller or equal to \( \mathcal{C}(\hat{\rho})/\sqrt{2} \) (c.f. [32] Eqs. (26) and (28)). Since losses for \( H \) and \( V \) polarization components are different in our experiment, we need to calibrate the system by measuring single-photon visibility in \( \{\ket{H^\beta}, \ket{V^\beta}\} \) basis for \( \theta = 0 \). We find that the concurrence is given by [32]

\[
\mathcal{C}(\hat{\rho}) = \sqrt{2 \left( \frac{\mathcal{V}_D|_{\theta = \pi/4}}{\mathcal{V}_H|_{\theta = 0}} \right)^2 + \left( \frac{\mathcal{V}_R|_{\theta = \pi/4}}{\mathcal{V}_V|_{\theta = 0}} \right)^2}.
\]

The experimentally measured values of concurrence for five mixed states, \( \hat{\rho}_1, \ldots, \hat{\rho}_5 \), are shown in Fig. 4. For comparison, we also make tomographic reconstruction of these states (Supplemental Information) and determine the concurrences independently. As can be clearly seen in Fig. 4, the values of concurrence obtained by our method (without coincidence detection) are in excellent agreement with those values obtained from quantum state tomography (with coincidence detection).

Note that by measuring the relative horizontally polarized and vertically polarized photon count rates produced by one single source \( (Q_1 \) or \( Q_2 \)), one can obtain the parameter \( \mathcal{I}_H \). One can then use the value of \( \mathcal{C}(\hat{\rho}) \), obtained from the single photon interference visibilities, to determine the parameter \( \mathcal{J} \). The corresponding results are in very good agreement with those obtained from full quantum state tomography (Supplemental Information).

In summary, we have verified and measured entanglement in bipartite mixed states without detecting one subsystem. Our method is particularly useful when, for any reason, detectors are not available for one of the subsystems.

It is important to note that the method is independent of the structure of each source, these need not be composed of two crystals. In addition, there is no need for two identical sources, as a double pass of the laser in the same source would work (Supplementary Information). We demonstrated the method by working with a mixed state that is obtained by generalizing two Bell states. Our method also applies to the mixed state which can be obtained by generalizing the other two Bell states [55]. Furthermore, the method could also be extended to transverse spatial entanglement [43, 44] or orbital angular momentum entanglement, if devices \( O \) and \( \Gamma \) (Fig. 1) are appropriately chosen.

Our experiment shows that the concept of induced coherence, which has applications to entanglement production [45–49], fundamental tests of quantum mechanics [50–52] and superconducting cavities [53], leads to a distinct entanglement measurement scheme for bipartite states. It also inspires to ask a more general question:
what other information can be learned about a quantum state by detecting only one of its parts.

Acknowledgments. — The experiment was performed at the Institute for Quantum Optics and Quantum Information, Austrian Academy of Sciences, Boltzmanngasse 3, Vienna A-1090, Austria. We acknowledge support from the Austrian Academy of Sciences (OAW–462 IQOQI, Vienna) and the Austrian Science Fund (FWF) with SFB F40 (FOQUIS) and W1210-2 (CoQuS). M.L. also acknowledges support from College of Arts and Sciences and the Office of the Vice President for Research, Oklahoma State University. R.L. was supported by National Science Centre (Poland) grants 2015/17/D/ST2/03471, 2015/16/S/ST2/00424, the Polish Ministry of Science and Higher Education, and the Foundation for Polish Science (FNP) under the FIRST TEAM project “Spatiotemporal photon correlation measurements for quantum metrology and super-resolution microscopy” cofinanced by the European Union under the European Regional Development Fund.

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[55] This mixed state has the form \( \hat{\rho} = I_1 \ket{H_\alpha, V_\beta} \bra{H_\alpha, V_\beta} + I_2 \ket{V_\alpha, H_\beta} \bra{V_\alpha, H_\beta} + (e^{-i\phi}) \sqrt{I_1 I_2} \ket{H_\alpha, V_\beta} \bra{V_\alpha, H_\beta} + \text{H.c.} \). It is obtained by generalizing the two Bell states \( \ket{\Psi^+} = \frac{\ket{H_\alpha, V_\beta} + \ket{V_\alpha, H_\beta}}{\sqrt{2}} \) and \( \ket{\Psi^-} = \frac{\ket{H_\alpha, V_\beta} - \ket{V_\alpha, H_\beta}}{\sqrt{2}} \).
Supplementary material

A. Details of Experimental Setup

The experimental setup is shown in Fig. 5. The sources $Q_1$ and $Q_2$ are each composed of two periodically poled Potassium titanyl phosphate (PPKTP) crystals of dimensions 2mm×2mm×1mm in a single home-made Peltier heater and a continuous wave diode laser (Toptica DL Pro HP 405) tuned to the wavelength $\lambda = 405.8$nm. In each two-crystal parametric down-conversion source [4], one crystal had its extraordinary axis aligned with the horizontal laboratory reference and the other crystal was aligned the vertical laboratory reference. The crystals were placed in temperature controlled ovens heated to $\sim 70^\circ C$, in order to obtain the desired collinear non-degenerate type-0 phase matching.

In pump paths $p_1$ and $p_2$ quarter wave plates and half wave plates are used to define the polarization before illumination of the crystals. To produce the state $|\rho_1\rangle$ the pump had vertical linear polarization. For $|\rho_1\rangle$ illumination of the crystals. To produce the state $|\rho_1\rangle$ wave plates are used to define the polarization before illumination of the crystals. To produce the state $|\rho_1\rangle$ the pump had vertical linear polarization. For $|\rho_1\rangle$ illumination of the crystals. To produce the state $|\rho_1\rangle$ wave plates are used to define the polarization before illumination of the crystals. To produce the state $|\rho_1\rangle$ wave plates are used to define the polarization before illumination of the crystals.

The pump polarization used to produce $|\rho_1\rangle$ was diagonal, $|\rho_1\rangle = \cos \theta |V\rangle + \sin \theta |H\rangle$. Another DM joins the path of photon $\beta$ with that of the pump before the second source. Lenses guarantee a good spatial overlap of signal and idler photons from either source. In order to induce coherence in the setup there are two optical path length requirements: (i) the sum of the lengths of path $p_1$ and $p_2$ to within the coherence length of the laser; (ii) the sum of the lengths of $p_1$ and $p_2$ to within the coherence length of the down-converted photons, which is determined by the $3\mu m$ bandpass filter at detection. Extra non-linear crystals (CC) were used control the degree of coherence of the state $|\rho_1\rangle$.

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The thicker the non-linear crystal (CC) and the more birefringent it is, the lower $|\rho_1\rangle$ in the produced polarization state. To produce the state $|\rho_2\rangle$ a 3 mm thick calcite (CaCO$_3$) crystal was introduced in each signal path $d$ and $e$. To produce the states a $|\beta_3\rangle$ and $|\beta_4\rangle$ a 763$\mu m$ thick Yttrium orthovanadate (YVO4) crystal was placed in each signal path $d$ and $e$.

In a counterintuitive scenario where the idler photons (subsystem $\beta$) were unpolarized, which corresponds to $I_H \approx 0.5$. In front of the single-mode fiber coupled homemade detectors at the signal outputs were placed a quarter wave plate, a half wave plate, a polarizer and a 3$\mu m$ band-

Figure 6: An alternative setup. Our method would also work in an experimental configuration using only one source of entangled photons, which can produce photon pairs in the forward and backward directions. The signal propagation modes originating in either case are combined at a beamsplitter (BS) and detected. The idler photons are separated from the signal propagation mode with a dichroic mirror (DM) and remain undetected.

Figure 7: Subsystem $\alpha$ of the state $|\rho\rangle$ (Eq. (1)) produced in source $Q_1$ passes through the HWP before reaching $Q_2$. As the HWP is rotated, which-source information is introduced. This graph shows the consequent reduction in the visibility of interference in the polarization component $|H\rangle_\beta$. In red is the cosine curve fit $V_H^\alpha \propto \cos \theta$. Although this result is expected for any state $|\rho\rangle$, this data was collected in the particularly counterintuitive scenario where the idler photons (subsystem $\alpha$) were unpolarized, which corresponds to $I_H \approx 0.5$.

In front of the single-mode fiber coupled homemade detectors at the signal outputs were placed a quarter wave plate, a half wave plate, a polarizer and a 3$\mu m$ band-
pass spectral filter centered at 849nm. For the red bars shown in Fig.5 of the main text the idler photons were detected with single-mode fiber coupled homemade detectors preceded by a quarter wave plate, a half wave plate, a polarizer, and a 3nm (full width half maximum) bandpass filter centered at 777nm.

B. Alternative Experimental Setup

In Fig.6 we present an alternative setup that does not require two identical sources because the laser passes twice the same pair of crystals. Similar “double pass” arrangements have been suggested for imaging and other applications [37, 54]. After the first pass of the pump through the crystals, the signal propagation mode is reflected by a dichroic mirror (DM1), and a mirror reflects the pump and idler photons back into the source. The signal and idler photons are reflected at another dichroic mirror (DM2). The source can produce photon pairs in the forward and backward directions, the signal propagation modes originating in either case are combined at a beamsplitter and detected. The idler photons are discarded. The development of the argument and the main theoretical results found in the present work can be directly applied to this alternative experiment.

C. Obtaining $I_H$ and $\mathcal{I}$ by measuring only $\beta$ photons

We can obtain $I_H$ by measuring the ratio of horizontally polarized in the total $\beta$ photon counting rate produced by one source $Q_i$. After obtaining $C$ using the interference visibilities (Eq.4), we use Eq.2 to extract the value of $\mathcal{I}$. In Fig.8 we show that $I_H$ and $\mathcal{I}$ parameter values obtained without coincidence detection match well the results extracted from the full two-qubit tomography.

D. Data Analysis

The interference visibility uncertainties are those obtained in the sine function fits to the photon counts obtained as the final beamsplitter is displaced. The blue error bars in Fig.5 of the main text were obtained by propagating the visibility uncertainties for Eq.4. The red error bars in that figure were obtained using a standard procedure, where Monte Carlo simulation considering the photon statistics and waveplate position uncertainties in the data acquired for the two-qubit tomography.

E. Full two-qubit tomography

In the main text (Figure 5) we compare the concurrence obtained with our method (without coincidence detection) with that obtained through full two-qubit tomography, using coincidence detection[17]. The density operators obtained by tomography are listed below and the real part of these matrices are shown in Fig. 9.

In each case, the values of $I_H$ and $\mathcal{I}$ for the two separate sources, $Q_1$ and $Q_2$, obtained from the respective tomographies agree to within the error margin, thus justifying the assumption that the states produced at either source are equal.
Figure 9: Tomographies. (a-e) show the reconstruction of $\text{Re}[\hat{\rho}_1], \ldots, \text{Re}[\hat{\rho}_5]$, respectively. (a) $\hat{\rho}_1$ is approximately pure, separable. (b) $\hat{\rho}_2$ is (almost) maximally mixed. (c, d) $\hat{\rho}_3$ and $\hat{\rho}_4$ are (non-maximally) mixed and (non-maximally) entangled. (e) $\hat{\rho}_5$ is (almost) maximally entangled.

\[
\rho_1 = \begin{pmatrix}
0.95 + 0.1i & -0.07 + 0.01i & 0.04 + 0.03i & -0.01 - 0.01i \\
-0.07 - 0.01i & 0.02 + 0.1i & 0.00 - 0.00i & 0.00 + 0.00i \\
0.04 - 0.03i & 0.00 + 0.00i & 0.03 + 0.1i & 0.00 - 0.00i \\
-0.01 + 0.01i & 0.00 - 0.00i & 0.00 + 0.00i & 0.00 + 0.1i
\end{pmatrix}; \quad (5)
\]

\[
\rho_2 = \begin{pmatrix}
0.48 + 0.4i & 0.00 + 0.00i & 0.04 - 0.01i & 0.01 - 0.02i \\
0.00 - 0.00i & 0.02 + 0.1i & 0.00 + 0.00i & -0.01 + 0.01i \\
0.04 + 0.01i & 0.00 - 0.00i & 0.03 + 0.1i & 0.04 - 0.04i \\
0.01 + 0.02i & -0.01 - 0.01i & 0.04 + 0.04i & 0.47 + 0.1i
\end{pmatrix};
\]

\[
\rho_3 = \begin{pmatrix}
0.50 + 0.1i & -0.03 + 0.02i & 0.00 - 0.01i & 0.14 + 0.08i \\
-0.03 - 0.02i & 0.02 + 0.1i & 0.00 - 0.00i & 0.00 + 0.00i \\
0.00 + 0.01i & 0.00 + 0.00i & 0.02 + 0.1i & -0.00 + 0.03i \\
0.14 - 0.08i & 0.00 - 0.00i & 0.00 - 0.03i & 0.46 + 0.1i
\end{pmatrix};
\]

\[
\rho_4 = \begin{pmatrix}
0.62 + 0.1i & 0.02 - 0.04i & 0.06 + 0.03i & -0.17 - 0.01i \\
0.02 + 0.04i & 0.03 + 0.1i & -0.01 + 0.01i & -0.01 + 0.01i \\
0.06 - 0.03i & -0.01 - 0.01i & 0.03 + 0.1i & -0.01 + 0.03i \\
-0.17 + 0.01i & -0.01 + 0.01i & -0.01 - 0.03i & 0.32 + 0.1i
\end{pmatrix};
\]

\[
\rho_5 = \begin{pmatrix}
0.46 + 0.1i & -0.04 - 0.00i & 0.02 - 0.01i & 0.40 + 0.04i \\
-0.04 + 0.00i & 0.02 + 0.1i & -0.00 - 0.00i & 0.00 - 0.00i \\
0.02 + 0.01i & -0.00 + 0.00i & 0.01 + 0.1i & 0.01 + 0.01i \\
0.40 - 0.04i & 0.00 + 0.00i & 0.01 - 0.01i & 0.51 + 0.1i
\end{pmatrix};
\]