Weak Scale Threshold Corrections in Supersymmetric Models

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Abstract

I discuss the weak scale threshold corrections in supersymmetric models. I describe the “match and run” approximation to the threshold corrections and compare with the exact one-loop results. With explicit examples I show that in cases without large hierarchies in the mass spectra the “match and run” approximation can lead to order $O(1)$ errors in the determination of the threshold corrections. I demonstrate how to obtain the threshold-corrected Yukawa coupling from the fermion pole mass. I present corrections to the top quark and squark/slepton masses as a function of the GUT scale parameters $m_0$ and $m_{1/2}$ and show that the gauge/Higgs sector corrections to the top quark mass are small while the gluino correction can be larger than the well known gluon correction.

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1 Introduction

I will contrast two methods of accounting for weak scale threshold corrections. Although the discussion will be quite general I will concentrate on obtaining the physical spectrum of particle masses from a set of DR running parameters in the context of supersymmetric GUT models.

In a supersymmetric GUT model with supergravity boundary conditions, we have as inputs at the GUT scale the universal scalar mass $m_0$, the gaugino mass $m_{1/2}$, the A-term $A_0$, the gauge coupling $\alpha_{\text{GUT}}$ and the Yukawa couplings $\lambda_i$. We then run these parameters down to low energies using the two-loop $\overline{\text{DR}}$ renormalization group equations[1]. The weak scale threshold corrections to the masses take us from the running $\overline{\text{DR}}$ masses, evaluated at an arbitrary $\overline{\text{DR}}$ scale $\mu$, to the physical mass. Hence we have the correspondence

$$ \text{Weak scale threshold corrections : } m_{\overline{\text{DR}}} (\mu) \longleftrightarrow m_{\text{pole}}. $$

In the next section I compare the threshold corrections obtained using the “match and run” method with the exact one-loop results. In Sec.3 I discuss how to obtain the threshold corrected Yukawa couplings from the physical quark masses. In Sec.4 I present some results, and I briefly summarize.

2 Comparison

In this section I first describe the “match and run” approximation and then explicitly compare with the exact one-loop results. The “match and run” procedure is often used in the literature for approximating the weak scale threshold corrections. The advantage of this method, and perhaps the main reason for its ubiquity, is that one may approximate the threshold corrections with only a knowledge of the RGE’s. The procedure is based on effective field theory and the decoupling theorem. Solving the $\overline{\text{DR}}$ renormalization group equation for a given mass parameter

$$ \frac{dm}{dt} = \frac{\beta}{16\pi^2} m, \quad t = \ln \frac{\mu^2}{M_{\text{GUT}}^2}, $$

with the boundary condition $m(M_{\text{GUT}}) = m_0$, we obtain the $\overline{\text{DR}}$ running mass $m(\mu)$. In solving this equation with the one-loop $\beta$-function we sum the leading logarithms of the type $\sum_n (\beta/16\pi^2)^n \ln^n (M_{\text{GUT}}^2/\mu^2)$. The one-loop $\beta$-function, with the form $\beta = \sum_i c_{ij} g_i^2$, is (for example) a sum of (gauge or Yukawa) coupling constants squared multiplied by quadratic Casimir coefficients $c_{ij}$, and the contributions in the sum correspond to physical particles circulating in a one-loop diagram.

Consider a particle of mass $m$ which receives contributions to its $\beta$-function from all the various particles in the MSSM. At scales larger than the heaviest particle in the spectrum (e.g. the squarks) the particle’s $\overline{\text{DR}}$ mass evolves according to the full MSSM RGE. As we decrease $\mu$ we eventually encounter the scale of the squark masses. At this point we stop the RG evolution and construct a new effective theory in which the squarks are integrated out. At scales below the squark mass the squarks are not active degrees of freedom; they do not circulate in the loops. Hence, in the effective theory at scales below the squark mass we do not include the squark contributions to the $\beta$-functions. Also we match the two theories at the scale $\mu = m_{\tilde{q}}$. Hence we set $m(m_{sq}^-) = m(m_{sq}^+)$, and continue the RG evolution of the parameter $m$ with the new
\( \beta \)-function until we reach the next heaviest particle that contributes to the RGE (e.g. the \( \chi \)'s). At this point we subtract the chargino and neutralino contributions to the \( \beta \)-functions, require continuity of the parameters, and continue. Likewise we decouple the top quark, the Z-boson, and so on. The RG evolution is then terminated when we reach the scale \( \mu = m(\mu) \). This quantity \( m(\mu) \) is then the approximation to the physical mass in the “match and run” program. This procedure is illustrated in Fig.1.

We now show how the “match and run” procedure compares with the exact one-loop result. Consider the threshold correction for a particle of mass \( m \) due to a particle of mass \( M \). First we consider the case \( M > m \). As can be seen from Fig.2, decoupling the particle of mass \( M \) yields the correction

\[
\frac{\Delta m}{m} = \frac{\Delta \beta}{16\pi^2} \ln\left(\frac{M^2}{m^2}\right), \quad (M > m),
\]

where \( \Delta \beta = \beta_h - \beta_l \). \( \beta_h(\beta_l) \) is the \( \beta \)-function including (not including) the contribution from the heavy particle of mass \( M \). For the case \( M \leq m \), the “match and run” procedure gives the correction

\[
\frac{\Delta m}{m} = 0, \quad (M \leq m).
\]

Now we consider the exact one-loop result. Upon evaluating the diagram of Fig.3 we find

\[
\frac{\Delta m}{m} = \frac{\Delta \beta}{16\pi^2} \int_0^1 dx \ln\left(\frac{|(1-x)M_1^2 + xM_2^2 - x(1-x)m^2|}{\mu^2}\right).
\]

In case of \( M_1 = M_2 > m \) (and setting \( \mu = m \)) we find the correction

\[
\frac{\Delta m}{m} = \frac{\Delta \beta}{16\pi^2} \left[ \ln\left(\frac{M^2}{m^2}\right) + \text{finite} \right], \quad (M > m),
\]

In the case \( M_1, M_2 \leq m \) we find

\[
\frac{\Delta m}{m} = \frac{\Delta \beta}{16\pi^2} \left[ 0 + \text{finite} \right], \quad (M \leq m),
\]
where the ‘0’ signifies no logarithmic correction.

The “match and run” procedure leads to good approximations to the pole masses when the mass under consideration \( m \) is much smaller than the masses of the decoupled particles \( M_i \). In the limit \( M_i^2 \gg m^2 \) the large logarithmic corrections proportional to \( \log (M_i^2/m^2) \) are correctly taken into account.

On the other hand, at each threshold there are finite corrections which are entirely missed in the “match and run” framework. These finite (i.e. not logarithmically enhanced) corrections can be as large as the logarithmic corrections when, for example, all of the particle masses are of the same order of magnitude. In fact, in supersymmetric GUT models with universal boundary conditions it is not uncommon that the entire supersymmetric spectrum is of order \( M_Z \). In such a case we can expect the finite corrections to be as large as the logarithms. Hence, the error in evaluating the threshold corrections in the “match and run” procedure can be \( O(1) \).

Since the exact one-loop threshold functions for all of the particles in the MSSM have been calculated\(^2\) both the logarithmic and finite corrections can be consistently incorporated, and this leads to precise results. Furthermore, with the threshold functions in hand, the \( \overline{\text{DR}} \) scale has no significance. In the “match and run” procedure it was important to stop the running of a mass at the scale equal to the mass. However, when using the one-loop threshold functions the \( \overline{\text{DR}} \) parameters can be evaluated at any scale of order the electroweak scale and no decoupling in the RG evolution is necessary. Hence, we can simply run all the \( \overline{\text{DR}} \) parameters down to the scale \( M_Z \) using the original set of RG equations, and then add the threshold corrections to obtain the pole masses.

Of course when using the two-loop RG equations it is important to include the threshold corrections correctly. The corrections due to the two-loop RG running are expected to be numerically of the same order as the one-loop threshold corrections and as we have stated above the “match and run” procedure can lead to \( O(1) \) errors in the determination of the threshold corrections. Hence we emphasize that the following go together:

\[
\left\{ \text{two-loop RGE's, one-loop threshold functions} \right\}.
\]

3 Examples

In this section I list a few examples of finite threshold corrections, then I show some examples comparing the “run and match” approximations with the exact one-loop corrections.

The correction to the top quark mass due to the gluon loop is well known\(^2\)

\[
m_t^{\text{pole}} = m_t(m_t) \left( 1 + \frac{5\alpha_s}{3\pi} \right),
\]

where \( m_t(m_t) \) signifies the running \( \overline{\text{DR}} \) mass evaluated at the scale \( m_t \). The left hand side of the above equation is not really the pole mass, as the top quark mass receives many other corrections. The next most important is the gluino/squark correction

\[
\frac{\Delta m_t^{\tilde{g} \tilde{q}}}{m_t} = -\frac{\alpha_s}{3\pi} \text{Re} \left[ B_1(m_t, m_{\tilde{g}}, m_{\tilde{t}_1}) + B_1(m_t, m_{\tilde{g}}, m_{\tilde{t}_2}) \right]
\]

\(^*\)Actually this correction is more commonly seen as \( 4\alpha_s/(3\pi) \) which is the result in the \( \overline{\text{MS}} \) scheme.

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Figure 4: The gluino mass vs. the DR scale $\mu$. The dashed lines show the running mass with and without decoupling the squarks. The solid line indicates the pole mass, and the horizontal dashed line shows the points $\mu = m_{\tilde{g}}$. In Fig.(a) The dash-dotted line indicates the exact one-loop squark correction. The asterisks indicate $m_{\tilde{g}}(m_{\tilde{g}})$ before and after adding the gluon correction.

\[-2m_{\tilde{g}}(A_t + \mu \cot \beta) m_{\tilde{t}}^2 - m_{\tilde{t}}^2 \mu \]

where $B_0$ and $B_1$ are the two point functions

\[B_n(p; m_1, m_2) = -\int_0^1 dx x^n \ln \left( \frac{(1-x)m_1^2 + x m_2^2 - x(1-x)p^2}{\mu^2} \right)\]

This correction for $m_{\tilde{g}} > m_t$ and/or $m_{\tilde{t}} > m_t$ is actually a logarithmic correction plus a finite correction. For the top quark the gluon correction is about 6%. As we show in the last section, the gluino/squark correction is typically of the same order as the gluon correction.

For the gluino mass we have the finite correction due to the gluon loop

\[m_{\tilde{g}}^{\text{pole}} = M_3(M_3) \left( 1 + \frac{15\alpha_s}{4\pi} \right)\]

where $M_3(M_3)$ is the DR gluino mass parameter $M_3$ evaluated at $M_3$. Here again the left hand side is not really the pole mass, as there are finite corrections (and potentially large logarithmic corrections) from the quark/squark loops as well [4, 5].

Another example of an important finite correction arises for the bottom quark mass, where the second line in the gluino/squark correction to the top quark mass shown above yields in the case of the bottom quark

\[\frac{\Delta m_b}{m_b} \sim -\frac{2\alpha_s \mu m_{\tilde{g}}}{3\pi m_b^2} \tan \beta \quad \text{(large } \tan \beta)\]

and this correction, which is entirely missed in the “match and run” procedure, can be as large as 50% (see S. Pokorski’s talk in the SUSY-94 conference proceedings).

Next I compare the exact and “run and match” results quantitatively. In the following figures I show the gluino mass, first in the case where the squark masses are much larger than the gluino mass, then in the case where the squarks and gluino are approximately degenerate. As we expect, the “run and match” approximation is a better approximation in the first case than in the second.

In Fig. 4(a) I plot the gluino mass versus the DR scale $\mu$ setting $m_0=700$ GeV and $m_{1/2} =80$ GeV. The dashed lines show the DR running gluino masses. The lower dashed line shows the undecoupled DR mass, and the upper dashed line shows the running gluino mass in which the squarks are decoupled. The squarks are decoupled at around 750 GeV in this case. Parallel to this line and just below it is a dash-dotted line which indicates the exact one-loop result obtained when taking into account the squark corrections. The difference between these two lines indicates the finite correction. The fact that these two lines are straight and parallel indicates that the
exact and “match and run” corrections both account for the same leading logarithm. If we add to the dot-dashed line the exact one-loop gluon correction we obtain the physical mass shown as a solid horizontal line, at 255 GeV. The fact that it is independent of $\mu$ shows that we can evaluate the physical mass at any $\overline{\text{DR}}$ scale in the vicinity of the electroweak scale. This physical mass is not exactly independent of $\mu$ because solving the RG equations includes all orders in perturbation theory; the first order part of the RG result is exactly cancelled by the one-loop threshold function, and the higher order parts of the running are small when considering scales in the range $M_Z$ to 1 TeV.

The points $\mu = m_{\tilde{g}}$ are also indicated in Fig. 4(a) as a dotted line. In the “match and run” procedure one would follow the decoupled running mass line from large scales until it intersects the line $\mu = m_{\tilde{g}}$ (indicated by large asterisks in the figures) and this mass $m_{\tilde{g}}(m_{\tilde{g}})$ (230 GeV) would be the approximation to the physical mass. Of course the finite gluon correction could be included, which yields 263 GeV (shown as a small asterisk). In this case, because the squarks are much heavier than the gluino the logarithmic squark correction is larger than the finite squark correction. The threshold correction due to the squarks is approximated by the “match and run” leading logarithm. At the scale $\mu = m_{\tilde{g}}$ the “match and run” correction is 22 GeV and the exact result is 14 GeV. Even in this case in which $m_0 \gg m_{1/2}$ the approximation is not so good; this indicates that the “large logarithm” $\log(m_{\tilde{q}}^2/m_{\tilde{g}}^2)$ with $m_{\tilde{q}} = 750$ GeV and $m_{\tilde{g}} = 250$ GeV cannot really be considered large.

Of course if the squark masses are lighter than or equal to the gluino mass, the correction due to the squarks is 0 in the “match and run” scheme. This is illustrated in Fig. 4(b), where I show again the running (squark decoupled and undecoupled) $\overline{\text{DR}}$ gluino masses and the pole mass as well as the line $\mu = m_{\tilde{g}}$ in the case $m_0=100$ GeV, $m_{1/2} = 180$ GeV. The “match and run” mass of 456 GeV underestimates by 8% the pole mass of 492 GeV. If the finite gluon contribution is added to the “match and run” mass, the result overestimates the pole mass by 4%.

In Fig.5 I show similar results for the top quark mass. Here the $\overline{\text{DR}}$ mass evaluated at the scale $m_t$ yields the approximate pole mass $m_t(m_t) = 162$ GeV. In this example $m_0$ and $m_{1/2}$ are chosen small so that the squark and gluino masses are of order $m_t$. The gluino/squark correction in this case is small. The running mass with only the standard gluon correction taken into account gives 171 GeV, a satisfactory approximation to the complete one-loop mass of 173 GeV.

4 Yukawa coupling corrections

In this section I explain how to obtain the threshold corrections to the Yukawa couplings using the mass threshold corrections. Here we consider the top quark mass $m_t$ as an input. We want
to then determine the DR Yukawa coupling which runs according to the full undecoupled MSSM RGE. This will give us the boundary condition on the Yukawa coupling which we can then run up to the GUT scale. This DR Yukawa coupling contains in it all of the weak scale threshold corrections.

We know the relation between the top quark pole mass and the running DR mass

\[ m_t^{\text{pole}} = m_t^{\text{DR}}(\mu) - \Sigma_t(m_t) \]

where \( \Sigma_t \) is the top quark self energy\(^\dagger \) and the running DR quark mass is related to the DR vev and DR Yukawa coupling by

\[ m_t^{\text{DR}}(\mu) = \frac{1}{\sqrt{2}} \lambda_t^{\text{DR}}(\mu) v_2^{\text{DR}}(\mu) \, . \]

Hence we can solve for the threshold-corrected DR Yukawa coupling if we know the DR vev. The DR vev is determined from the gauge couplings and the Z-boson mass,

\[ M_{Z\text{pole}}^2 = \frac{1}{4}(g^2 + g'^2)(v_1^2 + v_2^2) - \Pi_{ZZ}^T(M_Z^2) \]

where \( g \) and \( g' \) are the DR SU(2)_L and U(1)_Y gauge couplings, \( v_1 \) and \( v_2 \) are the DR vev’s, and \( \Pi_{ZZ}^T \) is the transverse part of the Z-boson self energy. Thus we have the threshold corrected Yukawa coupling,

\[ \lambda_t^{\text{DR}}(\mu) = \left[ \frac{g^2 + g'^2}{M_{Z\text{pole}}^2 + \Pi_{ZZ}^T} \right]^{1/2} \frac{m_t^{\text{pole}} + \Sigma_t}{\sqrt{2} \sin \beta} \, . \]

This formula gives the threshold-corrected Yukawa coupling in terms of two point functions, and the two-point function self energy formulae are much simpler than those obtained by using three point diagrams. Thus we demonstrate a simple way to go from the measured top quark mass to the threshold-corrected Yukawa coupling.

5 Results and summary

In Fig.6(a) I show the correction to the top quark mass versus \( m_{1/2} \) for \( m_0 = 200 \text{ GeV} \), \( \tan \beta = 3 \) and \( A_0 = 0 \). I indicate the gauge/Higgs, gluino and gluon contributions separately. The gauge/Higgs contribution is very small, between \(-1\) and \(0\)% on this plot. The gluon contribution is constant at \(6\)% and the gluino contribution grows logarithmically with \( m_{1/2} \), surpassing the gluon correction for \( m_{1/2} > 480 \text{ GeV} \). Fig. 6(b) shows the top quark mass correction vs. \( m_0 \) for \( m_{1/2} = 200 \text{ GeV} \), \( \tan \beta = 2 \) and \( A_0 = 0 \text{ GeV} \). Here the gauge/Higgs contribution is smaller than \(0.4\)% while the gluino contribution is an almost constant \(3.3\)%.

In Figs. 7(a) and (b) I show the corrections to the third generation squark and slepton masses for the same values of parameters as in Figs. 6(a) and (b). For this choice of parameters the squark mass corrections are negative and in the range \(-2\) to \(-7\)%, and the slepton masses receive small \(<1\)% corrections. For each curve in these plots the renormalization scale is set equal to

\(^\dagger \Sigma_t \) signifies \( \Sigma_1 + m_t \Sigma_\gamma \), where the quark self energy is written \( \Sigma_1 + \Sigma_\gamma \not\!p + \Sigma_\gamma \not\!p_\gamma + \Sigma_\gamma \not\!p_\gamma \).
the mass. The quark, squark, lepton and slepton mass corrections are treated more thoroughly in Ref.[3].

In this talk I have emphasized that the leading logarithmic weak scale threshold corrections to the masses in the MSSM do not generally dominate the finite corrections. The logarithmic corrections are taken into account in the “match and run” procedure. Hence, when large mass hierarchies are present the “match and run” procedure may be useful in approximating the corrections. In the low energy mass spectra of the MSSM the particle masses are not generally widely separated and the “match and run” approximation to the threshold corrections can lead to errors of order 1. The finite and logarithmic corrections are taken into account consistently when using the exact one-loop threshold functions and this leads to precise results. By using the two-loop RGE’s and by taking into account the weak scale thresholds correctly we can reliably investigate the implications of GUT scale boundary conditions.

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\[ m(\mu) \]

\[ \mu = m(\mu) \]
\[
\frac{dm}{dt} = \frac{\beta_h}{16\pi^2} m
\]
\[
\frac{dm}{dt} = \frac{\beta_l}{16\pi^2} m
\]

\[
\frac{\Delta m}{m} = \frac{\Delta \beta}{16\pi^2} \ln \left( \frac{M^2}{m^2} \right)
\]
$m_0 = m_{1/2} = 75 \text{ GeV}; \tan \beta = 2$

$\mu = m_t$

$m_t^{\text{pole}} = 173$

$m_t(\mu) \left(1 + \frac{5\alpha_s}{3\pi}\right)$

$m_t(\mu)$
$m_0=200$ GeV, $\tan\beta=3$, $A_0=0$

$\Delta m_t/m_t$

$\Delta m_t$ vs $m_{1/2}$ [GeV]

$\Delta m_t$ vs $m_0$ [GeV]

(a) $m_1/2=200$ GeV, $\tan\beta=3$, $A_0=0$ GeV

(b) $m_1/2=200$ GeV, $\tan\beta=2$, $A_0=0$ GeV

- total
- $g$
- $\tilde{g}$
- gauge/Higgs
