Firefly Algorithm for Multi-type Vehicle Routing Problem

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Abstract. The problem of distribution is a problem that cannot be separated from the industrial world. These problems are because these problems have a considerable influence on the cost and level of service to consumers. Therefore we need a way so that the distribution process can run smoothly and on time. One way that can be done in the distribution process is to optimize vehicle routes so that the time spent serving two consumers is more efficient and the goods can get to consumers on time. The problem of optimizing vehicle routes is known as vehicle routing problem (VRP). VRP which aims to form an optimal route to serve consumers with capacity constraints and Type of Vehicle is called Multi-type vehicle routing problem (MTVRP). In solving the MTVRP problem, two methods will be combined namely the exact method and the heuristic method. The firefly algorithm has been widely used in solving VRP problems. The results are obtained total distance of 2488 km.

Keywords: Firefly Algorithm (FA), Goods Distribution, Multi-Type Vehicle Routing Problem

1. Introduction

Distribution process is a process of delivering goods or services from producers to consumers, when and where the goods or services are needed. The distribution process basically creates a utility of time and place [1]. One of the factors that influence the distribution process is the distribution route selection [2]. Distribution route is a path that must be passed by the flow of goods from producers to agents or intermediaries or wholesalers to consumers [3]. The problem is how to determine the vehicle route, type of vehicle and scheduling based on the location of the depot along with the distance to the consumer. This problem is known as Vehicle Routing Problem (VRP).

VRP is a matter of how to determine a route consisting of several destination locations. The location is geographically dispersed and has different distances. We will arrange a vehicle visit route that starts at the depot and ends at the depot again. The aim is to minimize the total distance of all routes [4]. There are many types of VRP that have been used by researchers, including Two-Echelon Vehicle Routing Problem (2E-VRP), Asymmetric capacitated VRP (ACVRP), Arc Routing Problem (ARP), CVRP (Capacitated Vehicle Routing Problem), Dial-a-ride Problem (DARP), Distance-Constrained Capacitated Vehicle Routing Problem (DCVRP), The Emissions Vehicle Routing problem (EvRP), Generalized VRP (GVRP), Location Routing Problem (LRP), Multi-Depot Vehicle Routing Problem (MDVRP), OVRP (Open VRP), Periodic Vehicle Routing Problem (PVRP), Split Delivery VRP (SDVRP), Stochastic VRP (SVRP), Time Dependent VRP (TDVRP), CVRP on trees (TCVRP), Truck and Trailer routing Problem (TTRP), Vehicle routing problems with backhauls (VRPB), VRP with heterogeneous fleet (VRPHE), Vehicle Routing problem with multiple routes (VRPM), VRP with Pick Up and Delivery (VRPPD), VRP with time windows (VRPTW), VRP with Private Fleet Common Carrier (VRPPC) and Multi-type Vehicle Routing
Problem (MTVRP) [5]. In this research, we use VRP with MTVRP type because it is influenced by more than one of vehicles with their capacity.

There are many methods used to solve VRP, including the exact method, the heuristic method, the metaheuristics method and the matheuristics method. For the exact method, there are several types of algorithms used including branch and bound algorithm by Laporte and Nobert [6], branch and cut algorithm by Cordeau [7], branch and price algorithm by Pessoa [8], and branch and cut and price algorithm by Baldacci [9].

In this study, we use one of the algorithms from the metaheuristic method, the firefly algorithm. This algorithm has been widely used in solving VRP Problem. For example, research about using firefly algorithm to solve Vehicle Routing Problem with Time Windows by Fengshan Pan [10] and research about using firefly algorithm to solve Rich Vehicle Routing Problem by Eneko Osaba [11]. This algorithm has the advantage of being simpler in its use. In some cases solutions can also be found optimal quickly with a fairly high success rate [12].

Furthermore, this paper is organized as follows. In Section 2, the MTVRP is formulated. In Section 3 discussed the basic concepts of the FA. Section 4 explains the result and discussion. Finally, the conclusions of the study are given in Section 5.

2. Mathematical modeling of multi-type vehicle routing problem

The mathematical model used in this paper is derived from a mathematical model created by Brian Kallehauge [13]. We also consider there are many types of vehicles and for simplification of the model we only assume there are two types of vehicles. The two types of vehicles are given the symbols K1 and K2. The number of K1 type vehicles is 2 units and the number of K2 type vehicles is 3 units. Then, we use a mathematical model from Xiaobin Gan because the mathematical model is a development of the mathematical model of Brian Kallehauge and fits this research [14]. The mathematical formulation used by Xiaobin Gan is presented as follows:

Define variables as follow:

\[ x_{ijk} = \begin{cases} 1, & \text{if vehicle } k \text{ traverses route } (i,j) \\ 0, & \text{else} \end{cases} \] (1)

Objective function

Minimize \[ \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{k=1}^{K} c_{ij} x_{ijk} \] (2)

Subject to

\[ \sum_{j=1}^{N} x_{ijk} = \sum_{j=1}^{N} x_{jik} \leq 1 \quad \text{for } i = 0, 1 \leq k \leq K \] (3)

\[ \sum_{i=0}^{N} \sum_{j=0}^{N} x_{ijk} = 1 \quad \text{for } 0 \leq i \leq N \] (4)

\[ \sum_{i=0}^{N} \sum_{j=0}^{N} x_{ijk} = 1 \quad \text{for } 0 \leq j \leq N \] (5)

\[ \sum_{i=0}^{N} \sum_{j=0}^{N} x_{ijk} \leq Q_k \quad \text{for } k = 1, 2, \ldots, K_1 \text{ and } i \neq j \] (6)

\[ \sum_{i=0}^{N} \sum_{j=0}^{N} x_{ijk} \leq Q_k \quad \text{for } k = 1, 2, \ldots, K_2 \text{ and } i \neq j \] (7)

\[ \sum_{i=1}^{N} \sum_{k=1}^{K} x_{ijk} \leq K \quad \text{for } i = 0 \] (8)

\[ 0 \leq \sum_{i=0}^{N} \sum_{j=0}^{N} x_{ijk} \leq N \quad \text{for } 1 \leq k \leq K, \ i \neq j \] (9)

Equation (2) is the goal of this model that minimizes the total cost. Constraint (3) means that each vehicle departs from the depot and comes back the same depot. Constraints (4) and (5) point out...
that each customer must be visited once and only once by exactly one vehicle. Then, constraints (6) and (7) restrict the vehicle load to the specified scope. And constraint (8) explains the number of delivery vehicles within limited scope. Finally, constraint (9) indicates that the customers served by the $k$th vehicle must not exceed total clients $N$.

3. Firefly algorithm

Firefly is a nature-inspired algorithm was developed recently by Yang [15] to imitate a flashing behavior of the fireflies. It was formulated with the following three assumptions:

- All fireflies of the swarm are single sex which means that a firefly attracts all other fireflies regardless of their sex.
- The attraction of a firefly is related to its light. For example, given two fireflies, the brighter one will attract the less bright one; in other words; the less brighter firefly will move toward the brighter one. The attraction decreases as the distance between the fireflies increases. Furthermore, if one firefly is the brightest one of the swarm, it moves randomly.
- The brightness of a firefly is determined by the value of the objective function of problem under consideration.

The main steps of FA are given in Algorithm 1. There are three important factors that should be considered when implementing FA that is the attractiveness, the distance between fireflies and the movement of fireflies. In the basic version of FA, these factors are tackled in the following way. First of all, the attractiveness of a firefly is determined by its light intensity via Equation (10):

$$\beta = \beta_0 e^{-\gamma r^2}$$  \hspace{1cm} (10)

where $\beta_0$ is the attractiveness at distance $r = 0$. Secondly, the distance $r_{ij}$ between any two fireflies $i$ and $j$ is calculated using the Cartesian distance as shown in Equation (11):

$$r_{ij} = \|x_i - x_j\| = \sqrt{\sum_{k=1}^{N} (x_{ik} - x_{jk})^2}$$  \hspace{1cm} (11)

Where $x_{ik}$ and $x_{jk}$ is the $k$th component of the spatial coordinate $x_i$ and $x_j$ of the $i$th and $j$th fireflies. Finally, the movement of a firefly $i$ toward any other brighter firefly $j$ is determined by Equation (12):

$$x_{i}^{t+1} = x_{i}^{t} + \beta_0 e^{-\gamma r_{ij}^2}(x_{j}^{t} - x_{i}^{t}) + \alpha \epsilon_{i}^{t}$$  \hspace{1cm} (12)

where $\alpha$ indicates that the randomization parameter, $\epsilon_{i}^{t}$ is a vector of random numbers drawn from a Gaussian distribution at iteration $t$ and $\gamma$ is a scaling factor that represent the light absorption coefficient in a given medium. $r_{ij}$ is the Cartesian distance between the two fireflies $i$ and $j$ at positions $x_i$ and $x_j$.

Algorithm 1. Pseudo-code of firefly algorithm

1 \hspace{0.5cm} \textbf{begin}
2 \hspace{1.5cm} \textbf{Input: objective function } f(x);
3 \hspace{1.5cm} \textbf{Output: the best solution } x^* ;
4 \hspace{1.5cm} \textbf{Initialize the firefly population } x = (x_1, x_2, ..., x_i);
5 \hspace{1.5cm} \textbf{Evaluate each firefly } x_i \text{ in the initial population by } f(x_i);
6 \hspace{1.5cm} \textbf{Light intensity } I_i \text{ at } x_i \text{ is determined by } f(x_i);
7 \hspace{1.5cm} \textbf{while termination criterion not reached do}
8 \hspace{2cm} \textbf{for } i = 1 \text{ to } n \textbf{ do}

for \( j = 1 \) to \( n \) do

if \( I_i > I_j \) then

Compute the attractiveness based on Equation (10);

Move firefly \( i \) toward \( j \) based on Equation (12);

end

end

end

Evaluate the population;

Rank the fireflies, find the current best;

end

Return global best solution \( x^* \);

end

4. Result and discussion

In this section, we consider there is a depot that distributes goods to 14 stores. The depot has 5 vehicles and has an average speed of 50 km/h. The earliest departure time of each vehicle is 08.00 AM. Whereas is shown in Table 1, respectively.

Table 1. The Demands of The Customers

| Store | Demand |
|-------|--------|
| 1     | 120    |
| 2     | 200    |
| 3     | 120    |
| 4     | 130    |
| 5     | 150    |
| 6     | 140    |
| 7     | 60     |
| 8     | 110    |
| 9     | 100    |
| 10    | 180    |
| 11    | 90     |
| 12    | 160    |
| 13    | 150    |
| 14    | 140    |

Additionally, we also consider that there are two types of vehicles that are notated as type A and type B. Vehicle loads determined from type A and type B are 500 units and 400 units, respectively. The distance between depot 0 and customer \( i \) \( (i = 1, 2, \ldots, 14) \) is shown in Table 2.

Table 2. The distance between the depot and customers

| Distance | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|----------|---|---|---|---|---|---|---|---|---|---|----|----|----|----|----|
| 0        | 0 | 40| 106| 36 | 22 | 41 | 81 | 52 | 41 | 48 | 26 | 95 | 118| 87 | 90 |
| 1        | 0 | 56| 78 | 98 | 87 | 65 | 78 | 45 | 12 | 45 | 35 | 67 | 43 | 76 |
| 2        | 0 | 65| 64 | 54 | 23 | 14 | 56 | 23 | 45 | 78 | 56 | 23 | 34 |
| 3        | 0 | 12| 56 | 23 | 45 | 87 | 15 | 34 | 13 | 76 | 87 | 45 |
| 4        | 0 | 23| 78 | 14 | 24 | 90 | 87 | 68 | 13 | 56 | 24 |
| 5        | 0 | 45| 89 | 08 | 27 | 58 | 16 | 34 | 76 | 67 |
| 6        | 0 | 13| 16 | 78 | 25 | 15 | 26 | 90 | 58 |
| 7        | 0 | 15| 67 | 34 | 89 | 09 | 80 | 98 |
| 8        | 0 | 12| 56 | 74 | 23 | 54 | 35 |
| 9        | 0 | 14| 34 | 76 | 83 | 56 |
| 10       | 0 | 09| 98 | 87 | 77 |
| 11       | 0 | 34| 67 | 78 |
| 12       | 0 | 21| 23 |
| 13       | 0 | 45 |
The optimal vehicle route and distance obtained by firefly algorithm are presented in Table 4. But, this best solution can be accepted in this case. However, there are more than vehicles needed to service a group of customers at the depot to get more optimum distance.

5. Conclusion
Firefly Algorithm can be applied to solve Multi-type Vehicle Routing Problem (MTVRP). From the mathematical model, we consider two factors, namely multiple types of the vehicle and the uncertain number of vehicles of different types. By integrating these two factors, we applied MTVRP to solve routing problem in a depot in Balikpapan. The result is obtained the total distance to distribute goods from one depot to fourteen customers is 2488 km. In the future, we implemented some other development of firefly algorithm and we compare it to get the minimum distance.

Acknowledgement
This article is the result of the research project “Optimalisasi Multi-type Vehicle Routing Problem dalam distribusi bahan pokok di Balikpapan”. Assigned by the Vicechancellorship for Research, innovation and outreach of the Institut Teknologi Kalimantan.

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