Charmonium states in strong magnetic fields

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Abstract

The medium modifications of the masses of the charmonium states (J/ψ, ψ(3686) and ψ(3770)) in asymmetric nuclear matter in the presence of strong magnetic fields are studied using an effective chiral model. The mass modifications arise due to medium modifications of the scalar dilaton field, which simulates the gluon condensates of QCD within the effective hadronic model. The effects due to the magnetic field as well as isospin asymmetry are observed to be appreciable at high densities for the charmonium states, which can have consequences, e.g., in the production of the open charm mesons and the charmonium states, in the asymmetric heavy ion collisions at the compressed baryonic matter (CBM) experiments at the future facility at GSI. The presence of magnetic field leads to Landau quantization of the energy levels of the proton in the asymmetric nuclear matter. The effects of the anomalous magnetic moments of the proton and neutron on the masses of the charmonium states are studied and are observed to lead to larger masses of the charmonium states, as compared to when these effects are not taken into account.
I. INTRODUCTION

The study of hadrons under extreme conditions, e.g., at high densities and/or temperatures is a topic of intense research in strong interaction physics, due to its relevance in the context of ultra relativistic heavy ion collision experiments. In the recent past, the study of medium modifications of the heavy flavour hadrons have also gained considerable interest [1] because of the possibility of formation of the heavy hadrons bound in nuclei due to their attractive interaction in nuclear matter. The medium modifications of the heavy flavour mesons are also of relevance to the experimental observables, e.g., the production and propagation of these mesons in the strongly interacting matter resulting from high energy nuclear collisions. Large magnetic fields are believed to be created in these relativistic heavy ion collision experiments [2] and the time evolution of the magnetic field [3, 4] is still an open problem, which needs the proper estimate of the electrical conductivity of the medium as well as solutions of the magnetohydrodynamic equations. The estimation of the magnetic fields being created in non-central ultra-relativistic heavy-ion collision experiments as huge, has initiated the study of the heavy flavour mesons, e.g. the open charm and open bottom mesons [5–9] as well as the charmonium states [10, 11] in the presence of strong magnetic fields. The isospin asymmetry effects also need to be accounted for the study of the in-medium properties of the hadrons due to the fact that the colliding nuclei in the heavy ion collision experiments have large isospin asymmetry. In the present work, we study the medium modifications of the charmonium states (ground state $J/\psi$ and excited states $\psi(3686)$ and $\psi(3770)$) in isospin asymmetric nuclear matter in the presence of strong magnetic fields using a chiral effective model.

The heavy quarkonium states, which are bound states of heavy quark, $Q = (c, b)$ and heavy antiquark, $\bar{Q} = (\bar{c}, \bar{b})$, have been studied extensively in the literature using the potential models [12–14]. In these models, the mass of the quarkonium state is obtained by solving the Schrodinger equation using an effective potential. The Cornell potential, which has been widely used in the literature to describe the charmonium and bottomonium spectroscopy, has a short distance Coulomb type of interaction along with a long distance confining potential. At finite temperatures, there are color Debye screening effects, which can lead to dissociation of the quarkonium state at a critical temperature, $T_c$. In Refs. [15–17], the heavy quarkonium state has been studied as a non-relativistic bound state of the heavy quark ($Q$) and heavy antiquark
(Q) interacting by the color Coulomb potential and the effects of the gluonic fluctuations on the quarkonium state have been considered in the vacuum/nuclear medium. With the assumption that the separation of Q and \(\bar{Q}\) in the quarkonium bound state is small compared to the characteristic scale of the gluonic fluctuations, the interaction of the quarkonium state with the gluonic field is expanded in multipole expansion. The leading contribution to the mass of the quarkonium state is proportional to the gluon condensates in vacuum/nuclear medium. In the QCD sum rule calculations, the mass modifications of the heavy quarkonium states, e.g., the charmonium states arise due to the medium modifications of the gluon condensate in the nuclear medium [18, 20]. This is contrary to the light vector mesons (\(\omega\), \(\rho\) and \(\phi\)) which are modified in the hadronic medium, due to the medium changes in the light quark condensates in the hadronic medium [21, 22], within the QCD sum rule approach. The open heavy flavour mesons, which consist of a light quark (antiquark) along with a heavy (charm or bottom) antiquark (quark) are also modified due to the interaction with the light quark condensates in the nuclear medium within the framework of QCD sum rule approach [23–25]. The open heavy flavour mesons have also been studied in the literature, using quark meson coupling model [26], where the quarks interact with exchange of scalar and vector mesons [27], using heavy quark symmetry and interaction of these mesons with nucleons via pion exchange [28], heavy meson effective theory [29], heavy flavour meson as an impurity in nuclear matter [30] as well as using the coupled channel approach [31–35]. In Ref. 36, the mass modifications of the charmonium states have been studied using leading order QCD formula [15] and the linear density approximation for the gluon condensate in the nuclear medium. The QCD sum rule calculations for the charmonium states have been generalized to finite temperatures [37], where the medium modifications of these states have been studied arising due to the temperature effects on the gluon condensates, extracted from lattice calculations. Within the chiral effective model [38–40] used in the present investigation, the gluon condensate is mimicked by incorporating a scalar dilaton field. The medium modifications of the charmonium masses are obtained from the modifications of the dilaton field [41–43] within the chiral effective model. Due to the attractive interaction of the J/\(\psi\) in nuclear matter [36–42, 44], the possibility of the J/\(\psi\) forming bound states with nuclei have also been predicted [45].

In the present work, we investigate the medium modifications of the masses of the charmo-
nium states in the isospin asymmetric nuclear medium in the presence of strong magnetic fields using a chiral effective model. These are studied from the medium modifications of the scalar dilaton field introduced in the model to incorporate the broken scale invariance of QCD. The dilaton field is thus related to the gluon condensates of QCD and its medium modification gives the measure for the medium modification of the gluon condensate, which is used to calculate the in-medium masses of the charmonium states. The chiral effective model has been used extensively in the literature, for the study of finite nuclei [39], strange hadronic matter [40], light vector mesons [46], strange pseudoscalar mesons, e.g. the kaons and antikaons [47–50] in isospin asymmetric hadronic matter, as well as for the study of bulk matter of neutron stars [51]. The model has also been generalized to include the charm and bottom sectors, to study the open (strange) charm mesons [41, 43, 52, 53], open (strange) bottom mesons [54, 55], the charmonium states [42, 43], the upsilon states [56], the partial decay widths of the charmonium states to $D\bar{D}$ in the hadronic medium [43] using a light quark creation model [57], namely the $^3P_0$ model [58]. The in-medium decay widths of the charmonium (bottomonium) to $D\bar{D}$ ($BB$) have also been investigated using a field theoretic model for composite hadrons [59, 60]. The model has recently been used to study the in-medium masses of the open charm ($D$ and $\bar{D}$ mesons) [8] as well as the open bottom ($B$ and $\bar{B}$ mesons) [9] in strongly magnetized nuclear matter.

The outline of the paper is as follows: In section II, we describe briefly the chiral effective model used to study the in-medium masses of the charmonium states in isospin asymmetric nuclear matter in the presence of an external magnetic field. The medium modifications of the charmonium masses arise from the medium modification of a scalar dilaton field introduced in the hadronic model to incorporate broken scale invariance of QCD leading to QCD trace anomaly. The effects of the anomalous magnetic moments of the nucleons are also taken into account in the present work. In section III, we discuss the results obtained in the present investigation of the in-medium masses of the charmonium states in strong magnetic fields and section IV summarizes the findings of the present study.
II. IN-MEDIUM MASSES OF THE CHARMONIUM STATES

We use an effective chiral $SU(3)$ model \cite{39} to study the in-medium masses of the charmonium states in asymmetric nuclear matter in the presence of an external magnetic field. The model is based on the nonlinear realization of chiral symmetry \cite{61, 62, 63} and broken scale invariance \cite{39, 40, 46}. The effective hadronic chiral Lagrangian density contains the following terms

\[ \mathcal{L} = \mathcal{L}_{\text{kin}} + \sum_{W=X,Y,V,A,u} \mathcal{L}_{BW} + \mathcal{L}_{\text{vec}} + \mathcal{L}_{0} + \mathcal{L}_{\text{scalebreak}} + \mathcal{L}_{\text{SB}} + \mathcal{L}_{\text{mag}}, \]  

In the above Lagrangian density, the first term $\mathcal{L}_{\text{kin}}$ corresponds to the kinetic energy terms of the baryons and the mesons. $\mathcal{L}_{BW}$ is the baryon-meson interaction term and the vacuum baryon masses are generated by the baryon-scalar meson interaction terms. $\mathcal{L}_{\text{vec}}$ describes the interactions of the vector mesons, $\mathcal{L}_{0}$ contains the meson-meson interaction terms inducing the spontaneous breaking of chiral symmetry. $\mathcal{L}_{\text{scalebreak}}$ is a scale invariance breaking logarithmic potential given in terms of a scalar dilaton field \cite{64}, $\mathcal{L}_{\text{SB}}$ describes the explicit chiral symmetry breaking, and $\mathcal{L}_{\text{mag}}$ is the contribution from the magnetic field, given as \cite{65, 68}

\[ \mathcal{L}_{\text{mag}} = -\bar{\psi}_i q_i \gamma_\mu A^\mu \psi_i - \frac{1}{4} \kappa_{iN} \bar{\psi}_i \sigma^{\mu\nu} F_{\mu\nu} \psi_i - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}, \]  

where, $\psi_i$ corresponds to the $i$-th baryon (proton and neutron for nuclear matter, as considered in the present work). The second term in equation (2) corresponds to the tensorial interaction with the electromagnetic field and is related to the anomalous magnetic moment \cite{65, 71} of the nucleon.

The concept of broken scale invariance of QCD is simulated in the effective Lagrangian at tree level \cite{38} through the introduction of the scale breaking term \cite{39, 40} of the general Lagrangian given by equation (11) as

\[ \mathcal{L}_{\text{scalebreak}} = -\frac{1}{4} \chi^4 \ln \left( \frac{\chi_0^4}{\chi^4} \right) + \frac{d}{3} \chi^4 \ln \left( \frac{I_3}{\det(X)_0} \right), \]

where $I_3 = \det(X)$, with $X$ as the multiplet for the scalar mesons. The effect of these logarithmic terms is to break the scale invariance, which leads to the trace of the energy momentum tensor as \cite{39}

\[ \theta^\mu_\mu = \chi \frac{\partial \mathcal{L}}{\partial \chi} - 4 \mathcal{L} = -\chi^4. \]
The comparison of the trace of the energy momentum tensor of QCD in the massless quarks limit to that of the chiral effective model as used in the present work, leads to the relation of the dilaton field to the scalar gluon condensate as given by

$$\Theta^\mu_\mu = \langle \frac{\beta_{\text{QCD}}}{2g} G^a_{\mu\nu} G^{\mu\nu a} \rangle \equiv -(1 - d)\chi^4$$

The parameter $d$ originates from the second logarithmic term of equation (3). The QCD $\beta$ function at one loop level, for $N_c$ colors and $N_f$ flavors, is given by

$$\beta_{\text{QCD}}(g) = -\frac{11N_c g^3}{48\pi^2} \left(1 - \frac{2N_f}{11N_c}\right),$$

The first term in the above equation is a purely gluonic contribution, whereas the second term arises from the screening contribution of the quark pairs. At the one loop level of the $\beta$ function, the value of $d$ should be $\frac{2N_f}{11N_c}$ as is evident from equations (5) and (6). However, as one cannot rely on the one loop level result, the parameter $d$ is taken as a parameter within the chiral effective model used in the present investigation.

The trace of the energy-momentum tensor in QCD, using the one loop beta function given by equation (6), for $N_c=3$ and $N_f=3$, is given as,

$$\theta^\mu_\mu = -\frac{9\alpha_s}{8\pi} G^a_{\mu\nu} G^{\mu\nu a}$$

Using equations (5) and (7), we can write

$$\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{\mu\nu a} \rangle = \frac{8}{9}(1 - d)\chi^4$$

We thus see from the equation (8) that the scalar gluon condensate $\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{\mu\nu a} \rangle$ is proportional to the fourth power of the dilaton field, $\chi$, in the chiral SU(3) model. As mentioned earlier, the masses of the charmonium states are modified in the nuclear medium due to the modifications of the gluon condensates, which is calculated within the chiral SU(3) model, from the modification of the dilaton in the medium, by using the equation (8). In the present work of study of the in-medium charmonium masses using the chiral SU(3) model, we use the mean field approximation, where all the meson fields are treated as classical fields. In this approximation, only the scalar and the vector fields contribute to the baryon-meson interaction, $L_{BW}$ since for all the other mesons, the expectation values are zero. In the present investigation,
for given values of the external magnetic field, the scalar-isoscalar fields (non strange, $\sigma$ and strange, $\zeta$ fields), the scalar isovector field $\delta$, and the dilaton field, $\chi$ are solved from the coupled equations of motion, for given values of the baryon density, $\rho_B$, and the isospin asymmetry parameter, $\eta = (\rho_n - \rho_p)/(2\rho_B)$, where $\rho_n$ and $\rho_p$ are the number densities of the neutron and the proton respectively. The mass modifications of the charmonium states ($J/\psi$, $\psi(3686)$ and $\psi(3770)$), as arising from the medium modifications of the dilaton field, $\chi$, in hot asymmetric nuclear (hyperonic) matter \cite{42, 43} have already been studied using the chiral SU(3) model. The mass shift of the charmonium states in the medium due to the gluon condensates \cite{36} in the large charm mass limit has been used to calculate the mass shift of the charmonium states. The medium change of the scalar gluon condensates, which in the nonrelativistic limit, is due to the medium change in the $\langle \alpha_s \vec{E}^2 \rangle$ \cite{36}, similar to the second order Stark effect. In Ref. \cite{36}, the medium modifications of the scalar gluon condensate was obtained in the linear density approximation. In the chiral SU(3) model as used in the present investigation, the mass shift in the charmonium state arises due to the medium modification of the scalar gluon condensate, and hence due to the change in the value of the dilaton field, and is given as \cite{42, 43}

$$\Delta m_\psi = \frac{4}{81} (1 - d) \int dk^2 \frac{k}{k^2/m_c} \left| \frac{\partial \psi(\vec{k})}{\partial \vec{k}} \right|^2 + \epsilon \left( \chi^4 - \chi_0^4 \right), \quad (9)$$

where

$$\langle \left| \frac{\partial \psi(\vec{k})}{\partial \vec{k}} \right|^2 \rangle = \frac{1}{4\pi} \int \left| \frac{\partial \psi(\vec{k})}{\partial \vec{k}} \right|^2 d\Omega, \quad (10)$$

$m_c$ is the mass of the charm quark, taken as 1.95 GeV, $m_\psi$ is the vacuum mass of the charmonium state and $\epsilon = 2m_c - m_\psi$ is the binding energy. The formula for the mass shift of the heavy quarkonium state given by equation (9) is derived assuming Coulomb potential between $Q$ and $\bar{Q}$ in the quarkonium state. The value of $m_c$ as 1.95 GeV is taken so as to reproduce the mass difference of the charmonium states $J/\psi$ and $\psi(3686)$ in vacuum \cite{36}. For $2m_c < m_\psi$, the binding energy becomes negative, when the charmonium state can no longer exist as a bound state. The values of the dilaton field in the nuclear medium and the vacuum are $\chi$ and $\chi_0$ respectively. $\psi(\vec{k})$ is the wave function of the charmonium state in the momentum space, normalized as $\int \frac{dk^3}{(2\pi)^3} |\psi(\vec{k})|^2 = 1$ \cite{37}.

The wave functions for the charmonium states are taken to be Gaussian and are given as
\psi_{N,l} = N \times Y_l^m(\theta, \phi)(\beta^2 r^2)^{\frac{1}{2}} \exp\left(-\frac{1}{2} \beta^2 r^2\right)L^{l+\frac{1}{2}}_{N-\frac{1}{2}}(\beta^2 r^2) \tag{11}

where \beta^2 = M\omega/h characterizes the strength of the harmonic potential, \( M = m_c/2 \) is the reduced mass of the charm quark and charm anti-quark system, and \( L^k_p(z) \) is the associated Laguerre Polynomial. As in Ref. [36], the oscillator constant \( \beta \) is determined from the mean squared radii \( \langle r^2 \rangle \) as 0.46\(^2\) fm\(^2\), 0.96\(^2\) fm\(^2\) and 1 fm\(^2\) for the charmonium states J/\psi(3097), \psi(3686) and \psi(3770), respectively. This gives the value for the parameter \( \beta \) (in GeV) as 0.51, 0.38 and 0.37 for J/\psi(3097), \psi(3686) and \psi(3770), assuming that these charmonium states are in the 1S, 2S and 1D states respectively. Knowing the wave functions of the charmonium states and calculating the medium modification of the dilaton field in the magnetized nuclear matter, we obtain the mass shifts of the charmonium states, J/\psi, \psi(3686) and \psi(3770) respectively. In the next section we shall present the results obtained for the in-medium charmonium masses in asymmetric nuclear matter in presence of strong magnetic fields.

III. RESULTS AND DISCUSSIONS

In this section, we first investigate the effects of magnetic field, density and isospin asymmetry of the magnetized nuclear medium on the dilaton field \( \chi \), which mimicks the gluon condensates of QCD, within the chiral SU(3) model. The in-medium masses of charmonium states J/\psi, \psi(3686) and \psi(3770) are then calculated from the value of \( \chi \) in the nuclear medium using equation (9).

The dilaton field \( \chi \) as modified in the asymmetric nuclear medium in the presence of strong magnetic fields, is plotted in figure II. The graph shows the behaviour of the dilaton field \( \chi \) with density, for given values of magnetic field and isospin asymmetry parameter, \( \eta \), accounting for the anomalous magnetic moments (AMM) of the nucleons and compared with the situations when AMMs are not taken into consideration. The variations of the dilaton field \( \chi \) with magnetic field, baryon density, and isospin asymmetry, within the chiral SU(3) model, are obtained by solving the coupled equations of motion of the scalar fields, \( \sigma, \zeta, \delta \) and \( \chi \). These equations for the scalar fields contain the number and scalar densities of the nucleons, where the effects of the magnetic field, are in terms of the summation over the Landau energy levels
FIG. 1: (Color online) The dilaton field $\chi$ plotted as a function of the baryon density in units of nuclear matter saturation density, for different values of the magnetic field and isospin asymmetry parameter, $\eta$, including the effects of the anomalous magnetic moments of the nucleons. The results are compared to the case when the effects of anomalous magnetic moments are not taken into consideration (shown as dotted lines).
FIG. 2: (Color online) The mass shift of $J/\psi$ plotted as a function of the baryon density in units of nuclear matter saturation density, for different values of the magnetic field and isospin asymmetry parameter, $\eta$, including the effects of the anomalous magnetic moments of the nucleons. The results are compared to the case when the effects of anomalous magnetic moments are not taken into consideration (shown as dotted lines).
FIG. 3: (Color online) The mass shift of Ψ(3686) plotted as a function of the baryon density in units of nuclear matter saturation density, for different values of the magnetic field and isospin asymmetry parameter, η, including the effects of the anomalous magnetic moments of the nucleons. The results are compared to the case when the effects of anomalous magnetic moments are not taken into consideration (shown as dotted lines).
FIG. 4: (Color online) The mass shift of $\Psi(3770)$ plotted as a function of the baryon density in units of nuclear matter saturation density, for different values of the magnetic field and isospin asymmetry parameter, $\eta$, including the effects of the anomalous magnetic moments of the nucleons. The results are compared to the case when the effects of anomalous magnetic moments are not taken into consideration (shown as dotted lines).
(for proton, the charged nucleon) and the anomalous magnetic moments of the nucleons. It can be observed that with increase in density, the value of $\chi$ decreases and the shift from its vacuum value ($\chi_0 = 409.77$ MeV) increases with density. With increasing magnetic field, for a certain density, $\chi$ is observed to attain a higher value and thus the shift from the vacuum value decreases. When the anomalous magnetic moments of the nucleons are ignored, these deviations from vacuum value, are observed to increase with increasing magnetic field. For the isospin symmetric nuclear matter ($\eta=0$), for baryon density, $\rho_B=\rho_0$, the nuclear matter saturation density, and for values of the external magnetic field with $|eB|$, as $4m^2_\pi$, $8m^2_\pi$, $10m^2_\pi$, $12m^2_\pi$, shown in panels (a), (b), (c) and (d), the observed values of the field $\chi$ (in MeV) with (without) AMM are $406.57$ ($406.49$), $406.574$ ($406.6$), $406.58$ ($406.594$) and $407.12$ ($406.77$) respectively. For density, $\rho_B = 5\rho_0$ in symmetric nuclear matter, for the same magnetic fields, these values of $\chi$ are modified to $396.38$ ($395.6$), $397.2$ ($394.57$), $397.7$ ($394.17$), $398.29$ ($393.88$) respectively. As the isospin asymmetry of the medium increases, the scalar field $\chi$ is also observed to increase. At density of $5\rho_0$, and for magnetic field $|eB| = 4m^2_\pi$, compared to the value of $396.38$ ($395.6$) for symmetric nuclear matter, the scalar field $\chi$ has the values to be $396.51$ ($396$) and $397.64$ ($397.34$) MeV for isospin asymmetry parameter $\eta$ as 0.3 and 0.5 respectively, with (without) accounting for the anomalous magnetic moment effects for the nucleons. For the same density, and for the value of the magnetic field as $|eB| = 12m^2_\pi$, the value of $\chi$ is modified from $398.29$ ($393.88$) for symmetric case, to $398.42$ ($395.61$) and $399.84$ ($397.34$) MeV, for $\eta$ as 0.3 and 0.5 respectively, with (without) AMM effects. For the case of $\eta=0.5$, the medium comprises of only neutrons, and hence the only effect of magnetic field is due to the anomalous magnetic moment of the neutrons. Hence, in the case when the AMM effects are not taken into consideration, the value of the dilaton field remains independent of the magnetic field. This leads to the mass shift of the charmonium states to be independent of the magnetic field, when AMM effects are not taken into account.

We compute the shifts of the masses of the charmonium states, $J/\psi$, $\psi(3686)$ and $\psi(3770)$, from their vacuum values, as arising from the medium change of the dilaton field $\chi$ in the chiral SU(3) model. These are plotted in figures 2, 3 and 4 as functions of baryon density (in units of nuclear matter saturation density). In the massless QCD limit, the mass shift is observed to be proportional to the medium change in the fourth power of the $\chi$ field, as can be seen.
from equation (9), for given values of the magnetic field, and for different values of the isospin asymmetry parameter, \( \eta \). We have shown the results for the values of the magnetic field as \( |eB|, 4m^2, \ 8m^2, \ 10m^2, \ 12m^2 \) in panels (a), (b), (c) and (d) for values of isospin asymmetry parameter, \( \eta \) as 0, 0.3 and 0.5. The effects of anomalous magnetic moments on the in-medium charmonium masses are also investigated and compared to the cases when these effects are not taken into account.

The change in the mass of \( J/\psi \) as a function of nuclear matter density is observed to be relatively small as compared to that of \( \psi(3686) \) and \( \psi(3770) \). There is a significant drop in the masses of \( \psi(3686) \) and \( \psi(3770) \) as a function of density. The drop in these masses are the largest for isospin asymmetry parameter \( \eta = 0 \) in both the cases of accounting for AMM and without AMM effects. The mass shifts of all these charmonium states from the vacuum values, are observed to decrease, when the isospin asymmetry of the medium is increased. There is observed to be comparatively less drop in the masses of charmonium states when we account for the calculations with AMM as compared to the case of without AMM calculations. The difference in the mass shift of charmonium states accounting the AMM and without accounting the AMM increases as we increase the magnetic field.

For symmetric nuclear matter (\( \eta = 0 \)), at \( \rho_B = \rho_0(5\rho_0) \), including the effects of AMM, the mass shift (in MeV) of \( J/\psi \) is obtained as \(-8.16 \) (\(-32.93\)), \(-8.16 \) (\(-31\)), \(-8.13 \) (\(-29.8\)) and \(-8.22 \) (\(-28.44\)) for \( |eB| \) as \( 4m^2, \ 8m^2, \ 10m^2, \ 12m^2 \) respectively. When the AMM effects are not accounted for, these values are observed to be \(-8.37 \) (\(-34.745\)), \(-8.42 \) (\(-37.14\)), \(-8.45 \) (\(-38.05\)) and \(-8.5 \) (\(-38.73\)) MeV respectively. The value of mass shift of \( J/\psi \) at nuclear matter density, \( \rho_0 \), in the presence of magnetic field, may be compared to the value of around \(-8.6 \) MeV in the absence of magnetic field \[42\] in the chiral effective model as used in the present investigation. As has already been mentioned the in-medium masses of the charmonium states are computed from the medium changes of the gluon condensates calculated from the modifications of the dilaton field within the chiral effective model and there is no assumption of linear density approximation. The results of the present work are thus not restricted to low densities, but are also valid for high densities as well. The value of mass shift of \( J/\psi \) of around \(-8.6 \) MeV, at \( \rho_B = \rho_0 \) for zero magnetic field, is similar to the value (\( \sim -8\)MeV) obtained using the QCD Stark effect, where the change in the scalar gluon condensate in the nuclear medium
is due to the change in the expectation value of \( \langle \frac{2}{m} \vec{E}^2 \rangle \), computed using the linear density approximation [36]. In the absence of the magnetic field, at the nuclear matter saturation density, the values of the mass shifts of the excited charmonium states \( \psi(3686) \) and \( \psi(3770) \), calculated within the chiral effective model [42] are \(-117\) and \(-155\) MeV respectively in nuclear matter. These values may be compared with the values of the mass shifts of \(-100\) and \(-140\) MeV, for these charmonium states, as calculated using the linear density approximation [36].

The medium modifications of the masses of the charmonium states given by equation (9) are due to the medium modification of the gluon condensate, calculated within the effective chiral model by the medium change of the dilaton field. The wave function of the charmonium state, for the harmonic oscillator potential is given by equation (11). Applying the formula to the case of \( \eta_c \), which is also a 1S state, we observe a similar trend of the mass shift of \( \eta_c \) as for \( J/\psi \). We calculate the harmonic oscillator potential strength, \( \beta \) for \( \eta_c \) meson, to be 535.2 MeV, by linear extrapolation from the vacuum mass versus \( \beta \) graph, for \( J/\psi \) and \( \psi(3686) \). In symmetric nuclear matter, for density, \( \rho_B = \rho_0(5\rho_0) \), the mass shift of \( \eta_c \) is observed to be 6.87 (26.98), 6.77 (25.36), 6.77 (24.30) and 6.77 (23.33) MeV for \( |eB| \) as \( 4m_\pi^2 \), \( 8m_\pi^2 \), \( 10m_\pi^2 \), and, \( 12m_\pi^2 \) respectively, when AMM effects are taken into account. These values are modified to 6.89 (28.39), 6.93 (30.39), 6.95 (31.15) and 6.99 (31.61) MeV respectively, when the AMMs of the nucleons are not taken into consideration. In the absence of magnetic field, the mass shift of \( \eta_c \) meson is observed to be \(-7.1 \) (27.65) MeV for density of \( \rho_0(5\rho_0) \). The value at nuclear matter density may be compared to the value of \(-5.69\) MeV calculated using the QCD sum rule approach [20], as well as to the value of \(-3\) MeV calculated from the \( \eta_c \)–nucleon scattering length in Ref. [73] similar to the calculations in Ref. [16] for \( J/\psi \) nucleon scattering amplitude.

For the value of isospin asymmetry parameter \( \eta \) as 0.3, at \( \rho_B = \rho_0(5\rho_0) \), when the AMM effects are not accounted for, the mass shift (in MeV) of \( \psi(3686) \) is \(-109.8 \) (458.72), \(-109.88 \) (468.42), \(-109.88 \) (470.13) and \(-109.88 \) (471.14) for value of the magnetic field \( |eB| \) as \( 4m_\pi^2 \), \( 8m_\pi^2 \), \( 10m_\pi^2 \), and, \( 12m_\pi^2 \) respectively. These values are modified to \(-106.38 \) (442.47), \(-106.38 \) (421.2), \(-102.72 \) (401.63) and \(-102.95 \) (381.41), when the AMM effects are taken into account. In the same conditions (\( \eta=0.3 \) and density of \( \rho_0(5\rho_0) \)), the mass shift in \( \psi(3770) \) (in MeV) is \(-145.13 \) (606.3), \(-145.23 \) (619.14), \(-145.23 \) (621.4) and \(-145.23 \) (622.73) respectively for \( |eB| \) as \( 4m_\pi^2 \), \( 8m_\pi^2 \), \( 10m_\pi^2 \), and, \( 12m_\pi^2 \) when the AMM effects are not accounted
for, and, \(-140.6\) \((-584.84)\), \(-140.6\) \((-556.72)\), \(-135.77\) \((-530.86)\) and \(-136.07\) \((-504.13)\), when the AMM effects are taken into account.

For the maximum isospin asymmetry in the system \((\eta = 0.5)\), which corresponds to neutron matter, at \(\rho_B = \rho_0(5\rho_0)\), without accounting for the effects of AMM, the mass shifts (in MeV) in \(J/\psi\), \(\psi(3686)\) and \(\psi(3770)\) are observed to be \(-7.86\) \((-30.68)\), \(-106.61\) \((-416.13)\) and \(-140.92\) \((-550.03)\) MeV respectively, and are independent of the magnetic field. As has already been mentioned, this is because there are no charged fermions (protons) in the system, and, the neutrons interact with magnetic field only due to their AMM. When the AMM effects are taken into account, for \(\eta=0.5\), for density \(\rho_B = \rho_0(5\rho_0)\), the mass shifts for the \(J/\psi\) are \(-7.36\) \((-29.96)\), \(-7.24\) \((-27.83)\), \(-7.17\) \((-26.31)\) and \(-7.09\) \((-24.73)\) for the value of the magnetic field \(|eB|\) as \(4m_{\pi}^2\), \(8m_{\pi}^2\), \(10m_{\pi}^2\) and, \(12m_{\pi}^2\) respectively. For \(\psi(3686)\), for the same conditions, these are modified to \(-99.8\) \((-406.48)\), \(-98.24\) \((-377.56)\), \(-97.31\) \((-356.89)\) and \(-96.25\) \((-335.45)\) and, for \(\psi(3770)\), to \(-131.91\) \((-537.26)\), \(-129.85\) \((-499)\), \(-128.62\) \((-471.72)\) and \(-127.21\) \((-443.4)\) respectively.

IV. SUMMARY

To summarize, we have investigated the effects of density, isospin asymmetry, magnetic field and AMM of nucleons on the mass modifications of the charmonium states \(J/\psi(3097)\), \(\psi(3686)\) and \(\psi(3770)\) from the modification of the scalar dilaton field in isospin asymmetric nuclear medium in presence of strong magnetic fields using a SU(3) chiral model. The excited charmonium states are observed to show significant modifications in their masses in the medium, as compared to the mass of \(J/\psi\) in the nuclear medium. The value of the mass shift of the \(J/\psi\) as well as other charmonium states are observed to increase with density. For a given value of density and magnetic field, the effect of the isospin asymmetry of the medium is to increase the masses of the charmonium states. The effect of the magnetic field on the charmonium states is found to decrease the values of mass shift when the AMM effects are taken into account, as compared to when these are not taken into consideration. The mass shifts of the charmonium states in the nuclear medium in presence of magnetic fields, seem to be appreciable at high densities and these should show in observables like the production of these charmonium states, as well as of the open charmed mesons in the compressed baryonic
matter (CBM) experiment at the future facility at GSI, where baryonic matter at high densities and moderate temperatures will be produced.

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