NEW SUPERSYMMETRY ALGEBRAS FROM PARTIAL SUPERSYMMETRY BREAKING

JONATHAN A. BAGGER
RICHARD ALTENDORFER
Department of Physics and Astronomy
Johns Hopkins University
Baltimore, MD 21218, USA

In this talk we will study the partial breaking of supersymmetry in flat and anti de Sitter space. We will see that partial breaking in flat space can be accomplished using either of two representations for the massive $N = 1$ spin-$3/2$ multiplet. We will “unHiggs” each representation and find a new $N = 2$ supergravity and a new $N = 2$ supersymmetry algebra. We will also see that partial supersymmetry breaking in AdS space can give rise to a new $N = 2$ supersymmetry algebra, one that is necessarily nonlinearly realized.

1 Introduction

It is a sad fact of life that modern particle physicists can be classified by an integer, $N$, which counts the number of supersymmetries they assume to be active in the physical world. String theorists, for example, live and work in a rarefied region where $N = 8$ supersymmetry appears to hold sway. Most experimentalists, on the other hand, toil in great laboratories where $N = 0$ supersymmetry rules the day. In between are the phenomenologists, who are busy preparing for the time when experimentalists will study the first supersymmetry, that of $N = 1$.

This hierarchy of supersymmetry is not only a sociological fact but a physical necessity as well. From string theory we know that the real world has $N = 8$ supersymmetry. These supersymmetries must be spontaneously broken, either all at once, to $N = 0$, or partially, first to $N = 1$ (or higher), and then to $N = 0$. The spontaneous breaking of extended supersymmetry, from $N = 8$ to $N = 1$ to $N = 0$, is what ties together the different regions of the physical world.

The hierarchy of supersymmetries can be described phenomenologically using the language of effective field theory. In particle physics, this language was first developed during the 1960’s to describe the chiral symmetry breaking associated with pions, protons, and neutrons. By now the formalism has been sufficiently well developed that it can be used as a framework for understanding all of field theory, especially the physics associated with spontaneous symmetry breaking.
From this point of view, there is an ultimate theory, perhaps M theory, that exists at high energies. At each lower energy, one integrates out the degrees of freedom associated with the high energies, and constructs a nonrenormalizable, effective field theory. The effective field theory contains only those degrees of freedom that are relevant for physics at the scale under study. In the effective field theory, unbroken symmetries are realized linearly, while spontaneously broken symmetries are realized nonlinearly. The nonlinear symmetries are the remnants of symmetries spontaneously broken at higher scales.

In the context of supersymmetry, we are interested in the effective field theory that comes from breaking $\mathcal{N} = 8$ down to $\mathcal{N} = 1$. In this talk, for simplicity, we will focus our attention on the easiest case, of $\mathcal{N} = 2$ broken to $\mathcal{N} = 1$. We will construct effective theories that contain an unbroken, linearly realized $\mathcal{N} = 1$ supersymmetry, together with a spontaneously broken, nonlinearly realized, $\mathcal{N} = 2$.

At first glance, it might seem impossible to partially break $\mathcal{N} = 2$ to $\mathcal{N} = 1$. The argument runs as follows. Start with the $\mathcal{N} = 2$ supersymmetry algebra,

$$\{ Q_\alpha, \bar{Q}^{\dot{\alpha}} \} = 2 \sigma^m \alpha_{\dot{\alpha}} P_m ,$$

$$\{ S_\alpha, \bar{S}^{\dot{\alpha}} \} = 2 \sigma^m \alpha_{\dot{\alpha}} P_m ,$$

where $Q_\alpha$ and its conjugate $\bar{Q}^{\dot{\alpha}}$ denote the first, unbroken supersymmetry, and $S_\alpha, \bar{S}^{\dot{\alpha}}$ the second. Suppose that one supersymmetry is not broken, so

$$Q |0\rangle = \bar{Q} |0\rangle = 0 .$$

Because of the supersymmetry algebra, this implies that the Hamiltonian also annihilates the vacuum,

$$H |0\rangle = 0 .$$

Then, according to the supersymmetry algebra,

$$(\bar{S}S + SS) |0\rangle = 0 .$$

The final step is to peel apart this relation and conclude that

$$S |0\rangle = \bar{S} |0\rangle = 0 .$$

From this line of reasoning, one might think that partial breaking is impossible.

Fortunately, this argument has two significant loopholes. The first is that, technically-speaking, spontaneously-broken charges do not exist. Indeed, in a spontaneously broken theory, one only has the right to consider the algebra
of the currents. For the case at hand, the current algebra can be modified as follows,
\[
\{ \bar{Q}_\alpha, J^1_{\alpha m} \} = 2 \sigma^n_{\alpha \dot{\alpha}} T_{mn} \\
\{ \bar{S}_\dot{\alpha}, J^2_{\alpha m} \} = 2 \sigma^n_{\alpha \dot{\alpha}} (\nu^4 \eta_{mn} + T_{mn}),
\]
where the \( J^i_{\alpha m} (i = 1, 2) \) are the supercurrents and \( T_{mn} \) is the stress-energy tensor. Note that Lorentz invariance does not force the right-hand sides of the commutators to be the same. If there were no first supersymmetry, the \( \nu^4 \) term in the second commutator could be absorbed in \( T_{mn} \); it would play the role of the vacuum energy. However, the first supersymmetry can be said to define the stress-energy tensor, in which case there is an extra term in the second commutator. This discrepancy prevents the current algebra from being integrated into a charge algebra, so the no-go theorem is avoided.

The second loophole involves the last step of the theorem. Even if the supercharges were to exist, it is only possible to extract (5) from (4) if the Hilbert space is positive definite. In covariantly-quantized supergravity theories, this is not the case: the gravitino \( \psi_{ma} \) is a gauge field with negative-norm components.

There are, by now, many examples of partial supersymmetry breaking which exploit the first loophole. The first was given by Hughes, Liu and Polchinski, who showed that supersymmetry is partially broken on the world volume of an \( N = 1 \) supersymmetric 3-brane traveling in six-dimensional superspace. Later, Bagger and Galperin used the techniques of Coleman, Wess, Zumino, and Volkov to construct an effective field theory of partial supersymmetry breaking, with the broken supersymmetry realized nonlinearly. They found that the Goldstone fermion could belong to an \( N = 1 \) chiral or an \( N = 1 \) vector multiplet. At about the same time, Antoniadis, Partouche and Taylor discovered another realization in which the Goldstone fermion is contained in an \( N = 2 \) vector multiplet.

These results leave open many important questions. First and foremost, one would like to know how partial breaking works in the presence of gravity. Gravity couples to the true stress-energy tensor, so it distinguishes between the right-hand sides of the commutators (6). Some early work on this question was done by Cecotti, Girardello and Porrati and by Zinov’ev. (A geometrical interpretation was given in Ref. 9.) These groups considered nonminimal cases and found that their gravitational couplings utilize the second loophole. One would like to reconcile these results with those above. For reasons that will soon become clear, one would also like to know how partial breaking works in the presence of a nonvanishing cosmological constant.
In this talk we will address these questions using nonlinear realizations. The theory of nonlinear realizations provides a minimal, model-independent approach to the questions associated with partial supersymmetry breaking. We will focus on two of the multiplets of Bagger and Galperin, and couple each of them to supergravity, to lowest nontrivial order.

During the course of our work, we will find that partial breaking in flat space motivates an alternative representation for the $N = 1$ massive spin-3/2 multiplet. When coupled to gravity, this representation gives rise to a new $N = 2$ supergravity with a different $N = 2$ supersymmetry algebra. We shall also see that partial breaking in anti de Sitter space can give rise to a new $N = 2$ supersymmetry algebra.

In each case, our technique will be as follows: We will start by constructing the Lagrangian and supersymmetry transformations for the massive $N = 1$ spin-3/2 multiplet. We shall then “unHiggs” the representation by adding appropriate Goldstone fields and coupling gravity. We will see that the basic technique works in flat and AdS space.

### 2 The Massive $N = 1$ Spin-3/2 Multiplet in Flat Space

The starting point for our investigation is the massive $N = 1$ spin-3/2 multiplet. This multiplet contains six bosonic and six fermionic degrees of freedom, arranged in states of the following spins,

$$
\begin{pmatrix}
\frac{3}{2} \\
1 \\
1 \\
\frac{1}{2} \\
\end{pmatrix}.
$$

The traditional representation of this multiplet contains the following fields: one spin-3/2 fermion, one spin-1/2 fermion, and two spin-one vectors, each of mass $m$. The alternative representation has the same fermions, but just one vector plus one antisymmetric tensor. As we shall see, each representation has a role to play in the theory of partial supersymmetry breaking.

The traditional representation is described by the following Lagrangian:

$$
\mathcal{L} = \epsilon^{mn\rho\sigma} \bar{\psi}_m \sigma_n \partial_\rho \psi_\sigma - i \bar{\zeta} \sigma^m \partial_m \zeta - \frac{1}{4} A_{mn} \bar{A}^{mn} \\
- \frac{1}{2} m^2 A_m \bar{A}^m + \frac{1}{2} m \zeta \zeta + \frac{1}{2} m \bar{\zeta} \bar{\zeta} \\
- m \psi_m \sigma^{mn} \bar{\psi}_n - m \bar{\psi}_m \sigma^{mn} \bar{\psi}^n.
$$

Here $\psi_m$ is a spin-3/2 Rarita-Schwinger field, $\zeta$ a spin-1/2 fermion, and $A_m = A_m + i B_m$ a complex spin-one vector. This Lagrangian is invariant under the
following $N = 1$ supersymmetry transformations,

$$
\delta_\eta A_m = 2\bar{\psi}_m \eta - i \frac{2}{\sqrt{3}} \bar{\zeta} \sigma_m \eta - \frac{2}{\sqrt{3m}} \partial_m (\zeta \eta)
$$

$$
\delta_\eta \zeta = \frac{1}{\sqrt{3}} \bar{A}_{mn} \sigma^{mn} \eta - i \frac{m}{\sqrt{3}} \bar{\sigma}_m \eta A_m
$$

$$
\delta_\eta \psi = \frac{1}{3m} \partial_m (\bar{A}_{rs} \sigma^{rs} \eta + 2im\sigma^n \bar{\eta} A_n) - \frac{i}{2} (H_{+mn} \sigma^n + \frac{1}{3} H_{-mn} \sigma^n) \bar{\eta}
$$

$$
- \frac{2}{3} m (\sigma_m \bar{A}_n \eta + \bar{A}_m \eta)
$$

where $H_{\pm mn} = A_{mn} \pm \frac{i}{2} \epsilon_{mnrs} A^{rs}$.

The alternative representation has the following Lagrangian,

$$
L = \epsilon^{pqrs} \bar{\psi}_p \bar{\sigma}_q \partial_r \psi_s - i \bar{\zeta} \sigma^m \partial_m \zeta - \frac{1}{4} A_{mn} A^{mn} + \frac{1}{2} v_m v_m
$$

$$
- \frac{1}{2} m^2 A_m A^m - \frac{1}{4} m^2 B_{mn} B^{mn} + \frac{1}{2} m \bar{\zeta} \zeta + \frac{1}{2} m \bar{\zeta} \zeta
$$

$$
- m \bar{\psi}_m \sigma^{mn} \psi_n - m \bar{\psi}_m \bar{\sigma}^{mn} \psi_n
$$

where $A_{mn}$ is the field strength associated with the real vector field $A_m$, and $v_m = \frac{1}{2} \epsilon_{mnrs} \partial^p B^{rs}_p$ is the field strength for the antisymmetric tensor $B_{mn}$. This Lagrangian is invariant under the following $N = 1$ supersymmetry transformations,

$$
\delta_\eta A_m = (\psi_m \eta + \bar{\psi}_m \bar{\eta}) + \frac{i}{\sqrt{3}} (\bar{\eta} \sigma_m \zeta - \bar{\zeta} \sigma_m \eta) - \frac{1}{\sqrt{3m}} \partial_m (\zeta \eta + \bar{\zeta} \bar{\eta})
$$

$$
\delta_\eta B_{mn} = \frac{2}{\sqrt{3}} \left( \eta \sigma_m \zeta + \frac{i}{2m} \partial_m (\bar{\zeta} \bar{\sigma}_n \eta) \right) + i \eta \bar{\sigma}_m \bar{\psi}_n + \frac{1}{m} \eta \psi_m + h.c.
$$

$$
\delta_\eta \zeta = \frac{1}{\sqrt{3}} A_{mn} \sigma^{mn} \eta - \frac{im}{\sqrt{3}} \sigma^m \bar{\eta} A_m - \frac{1}{\sqrt{3}} m \sigma_{mn} \eta B^{mn} - \frac{1}{\sqrt{3}} v_m \sigma^m \bar{\eta}
$$

$$
\delta_\eta \psi = \frac{1}{3m} \partial_m (A_{rs} \sigma^{rs} \eta + 2im \sigma^n \bar{\eta} A_n) - \frac{i}{2} (H^A_{+mn} \sigma^n + \frac{1}{3} H^A_{-mn} \sigma^n) \bar{\eta}
$$

$$
- \frac{2}{3} m (\sigma_m \bar{A}_n \eta + \bar{A}_n \eta) + \frac{1}{3m} \partial_m (2v_m \sigma^n \bar{\eta} - m \sigma^n \eta B_{rs})
$$

$$
- \frac{2i}{3} (v_m + \sigma_m v_n) \eta - \frac{im}{3} (B_{mn} \sigma^n \bar{\eta} + i \epsilon_{mnrs} B^{rs} \sigma^s \bar{\eta})
$$

where the square brackets denote antisymmetrization, without a factor of 1/2.

These Lagrangians describe the free dynamics of massive spin-3/2 and 1/2 fermions, together with their supersymmetric partners, massive spin-one vector...
and tensor fields. They can be thought of as “unitary gauge” representations of theories with additional symmetries: a second supersymmetry for the massive spin-3/2 fermion, and additional gauge symmetries associated with the massive gauge fields.

To study partial breaking, we need to “unHiggs” these Lagrangians by including appropriate gauge and Goldstone fields. In each case we will need to add a Goldstone fermion and gauge the full $N = 2$ supersymmetry. The supersymmetric partners of the fermion will turn out to be the Goldstone bosons that restore the gauge symmetries associated with the massive bosonic fields. In this way we will construct two theories with $N = 2$ supersymmetry nonlinearly realized, and $N = 1$ represented linearly on the fields. The resulting effective field theories describe the physics of partial supersymmetry breaking, well below the scale where the second supersymmetry is broken.

The trick to this construction is to add the right fields. Because $N = 1$ supersymmetry is not broken, the Goldstone fermion must belong to an $N = 1$ supersymmetry multiplet. For the two cases of interest, we shall see that the Goldstone fermion must be in a chiral or a vector multiplet.

Let us first consider the chiral case. Under the first supersymmetry, a complex boson $\phi$ transforms into a Weyl fermion $\chi$,

$$\delta_\eta \phi = \sqrt{2} \eta^1 \chi .$$  \hfill (12)

If $\chi$ is the Goldstone fermion, it shifts under the second supersymmetry,

$$\delta_\eta^2 \chi = \sqrt{2} v^2 \eta^2 + \ldots ,$$  \hfill (13)

where $v$ is the scale of the second supersymmetry breaking. Therefore the closure of the two supersymmetries on $\phi$ gives

$$[ \delta_\eta^2, \delta_\eta^1 ] \phi = 2 v^2 \eta^1 \eta^2 + \ldots$$  \hfill (14)

We see that the complex scalar $\phi$ undergoes a constant shift. This implies that $\phi$ itself is a Goldstone boson. It expects to be eaten by a complex vector field, which suggests that the chiral Goldstone multiplet should be associated with the traditional representation for the massive spin-3/2 multiplet.

As shown in Figure 1(a), the degree of freedom counting works out just right. We start with the $N = 1$ chiral Goldstone multiplet and add an $N = 1$ vector multiplet. These fields may be thought of as $N = 1$ matter. We then add the gauge fields of $N = 2$ supergravity. As we will see, the full set of fields can be used to construct a Lagrangian which is invariant under $N = 2$ supersymmetry. The results look complicated, but they are actually very simple: In unitary gauge, the two vectors eat the two scalars, while the
Rarita-Schwinger field eats one linear combination of the spin-1/2 fermions. This leaves the massive $N = 1$ multiplet coupled to $N = 1$ supergravity.

With that said, we now present the Lagrangian:

$$
e^{-1} \mathcal{L} =$$

$-$ \( \frac{1}{2 \kappa^2} R + \epsilon^{mnr} \bar{\psi}_m \sigma_n D_r \psi^i - i \bar{\chi} \sigma^m D_m \lambda - i \bar{\sigma}^m D_0 \phi \overline{D_m \phi} \)

$-$ \( \frac{1}{4} A_{mn} \overline{A}^{mn} - \left( \frac{1}{\sqrt{2}} m \psi_m^2 \sigma^m \chi + i m \psi_m^2 \sigma^m \bar{\chi} + \sqrt{2} m \lambda \chi + \frac{1}{2} m \chi \chi \right) + m \psi_m^2 \sigma^m \psi_n^2 + \frac{\kappa}{4} \epsilon_{ij} \psi_i^1 \psi_j^1 \overline{H}^{mn} + \frac{\kappa}{\sqrt{2}} \chi \sigma^m \sigma^n \psi_1^1 \overline{D_m \phi} \)

$+$ \( \frac{\kappa}{2 \sqrt{2}} \bar{\sigma}_m \psi_n \overline{H}^{mn} + \frac{\kappa}{\sqrt{2}} \epsilon^{mnr} \bar{\psi}_m \sigma_n \psi_r \phi \overline{D_s \phi} + h.c. \),

where $\kappa$ denotes Newton’s constant, $m = \kappa v^2$, and

\[
\begin{align*}
A_m &= A_m + i B_m \\
A_{mn} &= \partial_m A_n - \partial_n A_m \\
H_{+mn} &= A_{mn} \pm i \epsilon_{mnr} A^r_s .
\end{align*}
\]

The supercovariant derivatives are as follows,

\[
\begin{align*}
\hat{D}_m \phi &= \partial_m \phi - \frac{\kappa}{\sqrt{2}} \psi^1_m \chi - \frac{1}{\sqrt{2}} \kappa v^2 A_m \\
\hat{A}_{mn} &= A_{mn} + \kappa \psi^1_m \psi^1_n - \frac{\kappa}{\sqrt{2}} \bar{\sigma}_{[m} \psi^1_{n]} .
\end{align*}
\]

This Lagrangian is invariant under two independent abelian gauge symmetries,
as well as the following supersymmetry transformations,

\[ \delta e^a_m = i \kappa (\eta^i \sigma^a \psi^i_m + \bar{\eta}_i \sigma^a \psi^\dagger_{m}) \]
\[ \delta \psi^i_m = \frac{2}{\kappa} D_m \eta^i \]
\[ + \left( -\frac{i}{2} \hat{H}_{mn} \sigma^a \eta^1_m + \sqrt{2} D_m \phi \eta^1 - \kappa \psi^1_m (\bar{\psi} \bar{\eta}_1) + iv^2 \sigma_m \eta_2 \right) \delta_2^i \]
\[ \delta A_m = 2 \epsilon_{ij} \psi^j_m \eta^i + \sqrt{2} \lambda \sigma_m \eta^1 \]
\[ \delta \lambda = \frac{i}{\sqrt{2}} \bar{A}_{mn} \sigma^{mn} \eta^1 - i \sqrt{2} v^2 \eta^2 \]
\[ \delta \chi = i \sqrt{2} \sigma^m \bar{D}_m \phi \eta^1 + 2 v^2 \eta^2 \]
\[ \delta \phi = \sqrt{2} \chi \eta^1, \quad (18) \]

for \( i = 1, 2 \). This result holds to leading order, that is, up to and including terms in the transformations that are linear in the fields. Note that this representation is irreducible in the sense that there are no subsets of fields that transform only into themselves under the supersymmetry transformations.

Let us now consider the vector case. Under the first supersymmetry, the real vector \( B_m \) of a vector multiplet transforms into a Weyl fermion \( \lambda \),

\[ \delta \eta_1 B_m = \sqrt{2} i (\lambda \sigma_m \bar{\eta}^1 - \eta^1 \sigma_m \bar{\lambda}). \quad (19) \]

If \( \lambda \) is the Goldstone fermion, it shifts under the second supersymmetry. Therefore, the closure of the two supersymmetries on \( B_m \) gives

\[ [\delta \eta^2, \delta \eta^1] B_m = 2v^2 (\eta^2 \sigma_m \bar{\eta}^1 - \eta^1 \sigma_m \bar{\eta}^2) + \ldots \quad (20) \]

From this we see that the real vector \( B_m \) is a Goldstone boson. It expects to be eaten by an antisymmetric tensor field. This suggests that the vector Goldstone multiplet should be associated with the alternative representation for the massive spin-3/2 multiplet.

The degree of freedom counting is shown in Figure 1(b). As before, we include the \( N = 2 \) supergravity multiplet. This time, however, the matter fields include the \( N = 1 \) vector Goldstone multiplet, together with one \( N = 1 \) tensor multiplet. In unitary gauge, one vector eats one scalar, while the antisymmetric tensor eats the other vector. These are the minimal set of fields that arise when coupling the alternative spin-3/2 multiplet to \( N = 2 \) supergravity.

The Lagrangian for this system can be worked out following the same procedure described above. We find

\[ e^{-1} \mathcal{L} = \]
- \frac{1}{2\kappa^2} R + \epsilon^{pqrs} \bar{\psi}_p \sigma_q \psi_r \psi_s^i - i\lambda \sigma^m \psi_m \lambda - \frac{1}{2} D^m \phi D_m \phi \\
- \frac{1}{4} F^A_{mn} F^{Amn} - \frac{1}{4} F^B_{mn} F^{Bmn} + \frac{1}{2} \psi^m \psi_m - \left( \frac{1}{\sqrt{2}} m \psi_m^2 \sigma^m \lambda + m i \psi_m^2 \sigma^m \bar{\lambda} \right) \\
+ \sqrt{2} m i \lambda \chi + m \chi \psi_m^2 \psi_m^2 + \frac{\kappa}{2\sqrt{2}} \epsilon_{ij} \psi_i \psi_j F^{Amn} \\
+ \frac{\kappa}{2} \chi \sigma^m \sigma^n \psi_m^1 D_n \phi + \frac{\kappa}{2} \bar{\lambda} \sigma^m \psi_m^1 F^{Bmn} + \kappa \epsilon^{pqrs} \bar{\psi}_p \sigma_q \psi_r \psi_s \phi \\
- i \frac{\kappa}{2} \chi \sigma^m \sigma^n \psi_m^1 \psi_n - \frac{\kappa}{2} \epsilon^{pqrs} \bar{\psi}_p \sigma_q \psi_r \psi_s \psi_s + h.c. \right) \tag{21}

where, as before, \( m = \kappa v^2 \), and

\begin{align*}
D_m \phi &= \partial_m \phi - \frac{m}{\sqrt{2}} (A_m + B_m) \\
F^A_{mn} &= \partial_{[m} A_{n]} + \frac{m}{\sqrt{2}} B_{mn} \\
F^B_{mn} &= \partial_{[m} B_{n]} - \frac{m}{\sqrt{2}} B_{mn}. \tag{22}
\end{align*}

This Lagrangian is invariant under an ordinary abelian gauge symmetry, an antisymmetric tensor gauge symmetry, as well as the following two supersymmetries,

\begin{align*}
\delta_\eta \epsilon^a_m &= i \kappa (\eta^i \sigma^a \bar{\psi}_m^i + \bar{\eta}_i \bar{\sigma}^a \psi_m^i) \\
\delta_\eta \psi_m^1 &= \frac{2}{\kappa} D_m \eta^1 \\
\delta_\eta A_m &= \sqrt{2} \epsilon_{ij} (\psi_m^i \eta^j + \bar{\psi}_m^i \bar{\eta}_j) \\
\delta_\eta B_m &= \bar{\eta}_1 \sigma_m \lambda + \bar{\lambda} \sigma_m \eta^1 \\
\delta_\eta B_{mn} &= 2 \eta^1 \sigma_m \psi_n + i \eta^1 \sigma_m \bar{\psi}_n + i \eta^2 \sigma_m \psi_n + h.c. \\
\delta_\eta \lambda &= i \bar{\psi}_m \sigma^{mn} \eta^1 - i \sqrt{2} \sigma^m \eta^2 \\
\delta_\eta \chi &= i \sigma^m \bar{\eta}_1 D_m \phi - \bar{\eta}_m \sigma^m \eta^1 + 2 \eta^2 \eta^2 \\
\delta_\eta \psi_m^2 &= \frac{2}{\kappa} D_m \eta^2 + i \sigma^m \psi_m^2 - \frac{i}{\sqrt{2}} \bar{\psi}_m \sigma^m \eta^1 \\
&\quad + \bar{\psi}_m \phi \eta^1 + \kappa (\bar{\psi}_m \chi \eta - (\bar{\chi} \eta) \psi_m^1) - i \bar{\psi}_m \eta^1 \\
\delta_\eta \phi &= \chi \eta^1 + \bar{\chi} \bar{\eta}^1 \tag{23}
\end{align*}

up to linear order in the fields. The supercovariant derivatives are given by

\begin{align*}
\bar{D}_m \phi &= D_m \phi - \frac{\kappa}{2} (\bar{\psi}_m \chi + \bar{\psi}_m \bar{\chi})
\end{align*}
\( \hat{F}^{A}_{mn} = F^{A}_{mn} + \frac{\kappa}{\sqrt{2}} (\psi^2_m \psi^1_n + \psi^2_m \bar{\psi}^1_n) \)

\( \hat{F}^{B}_{mn} = F^{B}_{mn} - \frac{\kappa}{2} (\bar{\lambda} \bar{\sigma}^n_m \psi^1_m + \psi^1_m \bar{\sigma}^n_m \bar{\lambda}) \)

\( \hat{v}_m = v_m + \left( i\kappa \psi^1_m \sigma^m_n \chi - \frac{i\kappa}{2} \epsilon^{mnrs} \psi^1_n \sigma^r \bar{\psi}^2_s + \text{h.c.} \right) \). \quad (24)

These fields form an irreducible representation of the \( N = 2 \) algebra.

Each of the two Lagrangians has a full \( N = 2 \) supersymmetry (up to the appropriate order). The first supersymmetry is realized linearly, so it is not broken. The second is realized nonlinearly, so it is spontaneously broken. In each case, the transformations imply that

\( \zeta = \frac{1}{\sqrt{3}} (\chi - i\sqrt{2}\lambda) \) \quad (25)

does not shift, while

\( \nu = \frac{1}{\sqrt{3}} (\sqrt{2}\chi + i\lambda) \) \quad (26)

does. Therefore \( \nu \) is the Goldstone fermion for \( N = 2 \) supersymmetry, spontaneously broken to \( N = 1 \).

In the chiral case, we find

\[
[\delta_{\eta_1}, \delta_{\eta_2}] \phi = 2\sqrt{2} v^2 \eta_1 \eta_2 \\
[\delta_{\eta_1}, \delta_{\eta_2}] A_m = \frac{4}{\kappa} \partial_m \eta_1 \eta_2 .
\]

(27)

The complex scalar \( \phi \) is indeed the Goldstone boson for a gauged central charge. Moreover, in unitary gauge, where

\( \phi = \nu = 0 \), \quad (28)

this Lagrangian reduces to the usual representation for a massive \( N = 1 \) spin-\( 3/2 \) multiplet.

In the vector case, we have

\[
[\delta_{\eta^2}, \delta_{\eta^1}] A_m = \frac{2\sqrt{2}}{\kappa} \partial_m (\eta^1 \eta^2 + \bar{\eta}^1 \bar{\eta}^2) - \sqrt{2} i v^2 (\eta^2 \sigma_m \bar{\eta}^1 - \eta^1 \sigma_m \bar{\eta}^2) \\
[\delta_{\eta^2}, \delta_{\eta^1}] B_m = \sqrt{2} i v^2 (\eta^2 \sigma_m \bar{\eta}^1 - \eta^1 \sigma_m \bar{\eta}^2) \\
[\delta_{\eta^2}, \delta_{\eta^1}] B_{mn} = \frac{2i}{\kappa} \epsilon^{mnrs} \eta^1 \sigma_r \bar{\eta}^2 .
\]

(29)

We see that the real vector \(- (A_m - B_m) / \sqrt{2}\) is the Goldstone boson for a gauged vectorial central extension of the \( N = 2 \) algebra. In addition, the real
scalar $\phi$ is the Goldstone boson associated with a single real gauged central charge. In the unitary gauge, with

$$-\frac{1}{\sqrt{2}} (A_m - B_m) = \phi = \nu = 0,$$

(30)

this Lagrangian reduces to the alternative representation for the massive $N = 1$ spin-3/2 multiplet.

Now that we have explicit realizations of partial supersymmetry breaking, we can go back and see how they avoid the no-go argument presented in the introduction. We first compute the second supercurrent. In each case it turns out to be

$$J^2_{m\alpha} = v^2 (\sqrt{6} i \sigma_{\alpha\dot{\alpha}} \bar{\nu}^{\dot{\alpha}} + 4 \sigma_{\alpha\dot{\beta}mn} \psi^{2n\dot{\beta}})$$

(31)

plus higher-order terms. Computing, we find

$$\{ \bar{Q}_{\dot{\alpha}}, J^1_{m\alpha} \} = 2 \sigma^n_{a\dot{\alpha}} T_{mn}$$

$$\{ \bar{S}_{\dot{\alpha}}, J^2_{m\alpha} \} = 2 \sigma^n_{a\dot{\alpha}} T_{mn}.$$

(32)

Now, however, $J^1_{a\alpha}$ and $T_{mn}$ contain contributions from all of the fields, including the second gravitino. When covariantly-quantized, the second gravitino gives rise to states of negative norm. Indeed, it is not hard to check that

$$(\bar{S}S + S\bar{S}) |0\rangle = 0,$$

(33)

even though

$$S |0\rangle \neq 0 \quad \bar{S} |0\rangle \neq 0.$$  

(34)

The supergravity couplings exploit the second loophole to the no-go theorem!

The Lagrangian in the chiral case is a truncation of the supergravity coupling found by Cecotti, Girardello and Porrati and by Zinov'ev. Their results were based on linear $N = 2$ supersymmetry; they involved $N = 2$ vector- and hyper-multiplets. The Lagrangian for the vector case is new. It contains a new realization of $N = 2$ supergravity. In each case, the couplings presented here are minimal and model-independent. They describe the superHiggs effect in the low-energy effective theories that arise from partial supersymmetry breaking.

### 3 The Massive $N = 1$ Spin-3/2 Multiplet in Anti de Sitter Space

In the last part of this talk, we will examine the question of partial supersymmetry breaking in anti de Sitter space. Before we do this, let us first recall
the AdS $N = 2$ supersymmetry algebra, $OSP(2, 4)$. The relevant parts of the algebra are specified by the following commutators:

\[
\begin{align*}
\{Q^i_\alpha, \bar{Q}^j_\dot{\beta}\} &= 2\sigma^a_{\alpha\dot{\beta}} R^i_a \\
\{Q^i_\alpha, Q^j_\beta\} &= 2i\Lambda\sigma^{ab}_{\alpha\beta} M_{a b} \delta^{i j} + 2i\delta^{\beta}_\alpha \varepsilon^{i j} T \\
[T^{ij}, Q^k] &= \imath\Lambda(\delta^k_i Q^j - \delta^k_j Q^i).
\end{align*}
\]  

(35)

When the cosmological constant $\Lambda \to 0$, this contracts the usual $N = 2$ supersymmetry algebra. The generator $R_a$ contracts to the momentum generator $P_a$, while $T$ contracts to a single real central charge. Since the flat-space constructions relied on either a complex central charge or a vector central charge, one is led to wonder how partial breaking works in AdS space.

In what follows we consider the analog of the chiral case discussed above. (The vector case is presently under investigation.) We find it useful to follow the same procedure as before. We start with the massive $N = 1$ spin-3/2 multiplet in AdS space. This multiplet contains the following AdS representations:

\[ D(E + 1, 1) \oplus D(E + 1, \frac{1}{2}) \oplus D(E + 1, -\frac{1}{2}) \oplus D(E, -1) \oplus D(E, -\frac{1}{2}) \oplus D(E, -\frac{1}{2}) \oplus D(E, 0) \oplus D(E, -1) \oplus D(E, -\frac{1}{2}) \]

(36)

where $D(E, s)$ denotes the eigenvalues under $U(1) \times SU(2) \subset SO(3, 2)$ and unitarity requires $E > 2$. ($E$ is the AdS generalization of the mass. For this representation, $E \to 2$ corresponds to the massless limit.)

The Lagrangian for this multiplet is given by

\[
e^{-1} \mathcal{L} = e^{-1} \epsilon^{mnr} \bar{\psi}_m \nabla_r \psi_s - \bar{\psi} \nabla^m \psi_m - \frac{1}{4} A_m A^m - \frac{1}{4} B_m B^m \\
- \frac{1}{2} (m^2 - m\Lambda) A_m A^m - \frac{1}{2} (m^2 + m\Lambda) B_m B^m \\
+ \frac{1}{2} m \zeta \bar{\zeta} + \frac{1}{2} m \bar{\zeta} \zeta - m \psi_m \sigma_m \psi_n - m \bar{\psi}_m \bar{\sigma}_m \bar{\psi}_n + \frac{1}{\sqrt{3} m} \partial_m (\zeta \psi_n - \bar{\zeta} \bar{\psi}_n) \]

(37)

where $\Lambda \neq 0$ and $\nabla$ denotes the AdS covariant derivative. This Lagrangian is invariant under the following supersymmetry transformations,

\[
\begin{align*}
\delta_{\eta} A_m &= \sqrt{1 + \epsilon}(\psi_m \eta + \bar{\psi}_m \bar{\eta}) \\
&+ \frac{1}{\sqrt{1 - \epsilon}} \left( \frac{1}{\sqrt{3}} (1 - \epsilon) (\bar{\eta} \bar{\sigma}_m \zeta - \bar{\zeta} \bar{\sigma}_m \eta) - \frac{1}{\sqrt{3} m} \partial_m (\zeta \eta + \bar{\zeta} \bar{\eta}) \right) \\
\delta_{\eta} B_m &= \sqrt{1 + \epsilon} (1 + \epsilon)(\bar{\eta} \bar{\sigma}_m \zeta + \bar{\zeta} \bar{\sigma}_m \eta) + \frac{1}{\sqrt{3} m} \partial_m (\zeta \eta - \bar{\zeta} \bar{\eta}) \\
&+ \frac{1}{\sqrt{1 - \epsilon}} \left( \frac{1}{\sqrt{3}} (1 + \epsilon) (\bar{\eta} \bar{\sigma}_m \zeta - \bar{\zeta} \bar{\sigma}_m \eta) - \frac{1}{\sqrt{3} m} \partial_m (\zeta \eta + \bar{\zeta} \bar{\eta}) \right)
\end{align*}
\]
\[
\delta_\eta \zeta = \sqrt{1 - \epsilon} \left( \frac{1}{\sqrt{3}} A_{mn} \sigma^{mn} \eta - \frac{m}{\sqrt{3}} \sigma^{m} \bar{\eta} A_{m} \right) \\
+ \sqrt{1 + \epsilon} \left( - \frac{1}{\sqrt{3}} B_{mn} \sigma^{mn} \eta + \frac{m}{\sqrt{3}} \sigma^{m} \bar{\eta} B_{m} \right)
\]

\[
\delta_\eta \psi_m = \frac{1}{\sqrt{1 + \epsilon}} \left( \frac{1}{3m} \nabla_m (A_{rs} \sigma^{rs} \eta + 2im \sigma^{m} \bar{\eta} A_n) - \frac{i}{2} (H^A_{+mn} \sigma^n + \frac{1}{3} H_{-mn} \sigma^n) \bar{\eta} \\
- \frac{2}{3} m (\sigma^n A_n \eta + A_m \eta) - \frac{i}{2} \epsilon H^A_{+mn} \sigma^n \eta - \epsilon m A_m \eta \right) \\
+ \frac{1}{\sqrt{1 - \epsilon}} \left( - \frac{i}{3m} \nabla_m (B_{rs} \sigma^{rs} \eta - 2im \sigma^{m} \bar{\eta} B_n) + \frac{1}{2} (H^B_{+mn} \sigma^n) \bar{\eta} + \frac{1}{2} \epsilon H^B_{+mn} \sigma^n \eta - \epsilon m B_m \eta \right),
\]

where \( \epsilon = \Lambda/m \). Note that the “mass” \( m \) is defined to be \( m = (E - 1)\Lambda \), with \( E > 2 \). This definition gives masses consistent with the AdS representations in eq. (36). The fact that \( E > 2 \) implies that \( 0 \leq \epsilon \leq 1 \).

To unHiggs the representation, note that as \( E \to 2 \), the representation \( (36) \) splits into

\[
D(\frac{5}{2}, \frac{3}{2}) \oplus D(2, 1) \oplus D(3, 1) \oplus D(\frac{5}{2}, \frac{1}{2}) \\
\oplus D(\frac{5}{2}, \frac{5}{2}) \oplus D(3, 0).
\]

These are precisely the degrees of freedom of a massless spin-3/2 multiplet plus a massive spin-one vector multiplet. We see that the resulting symmetry group is actually \( OSP(1, 4) \times U(1) \), where the \( U(1) \) must be spontaneously broken because \( E > 2 \). (Usually, \( E \to 3/2 \) is necessary to unHiggs a vector multiplet in AdS space. For the case at hand, this would spoil the unitarity of the spin-3/2 field.)

The steps to derive the unHiggsed Lagrangian are very similar to those for the chiral multiplet in flat space. We find

\[
e^{-1} \mathcal{L} = \\
- \frac{1}{2\kappa^2} R + \epsilon^{mnr} \bar{\psi}_m \sigma_n D_r \psi_s - i \bar{\lambda} \sigma^m D_m \lambda - i \bar{\chi} \sigma^m D_m \chi \\
- \frac{1}{4} A_{mn} A^{mn} - \frac{1}{4} B_{mn} B^{mn} - \frac{1}{2} D_m \phi_A D^m \phi_A - \frac{1}{2} D_m \phi_B D^m \phi_B \\
- \left( \frac{1}{\sqrt{2}} m \sqrt{1 - \epsilon^2} \psi^2_m \sigma^m \bar{\chi} + m \sqrt{1 - \epsilon^2} \psi^2_m \sigma^m \bar{\chi} \right).
\]
This Lagrangian is invariant (to lowest order in the fields) under the following supersymmetry transformations,

\begin{align}
\delta e^a_m &= i \kappa \eta^a \psi_{mi} + i \kappa \bar{\eta}^a \bar{\psi}^i_m \\
\delta \psi^i_m &= \frac{2}{\kappa} D_m \eta^i + i \frac{e m}{\kappa} \sigma^a \bar{\eta}^a \\
\delta \eta^i &= \sqrt{1 + \epsilon} \frac{1}{\sqrt{2}} \bar{\psi}_{mn} \bar{\sigma}^m \eta^i + \sqrt{1 - \epsilon} \frac{1}{\sqrt{2}} \bar{\psi}_{mn} \sigma^m \eta^i \\
\delta \eta_A &= \sqrt{1 + \epsilon} \frac{1}{\sqrt{2}} \bar{A}_{mn} \sigma^m \eta^i + \sqrt{1 - \epsilon} \frac{1}{\sqrt{2}} \bar{A}_{mn} \sigma^m \eta^i \\
\delta \eta_B &= \sqrt{1 - \epsilon} \frac{1}{\sqrt{2}} \bar{A}_{mn} \sigma^m \eta^i + \sqrt{1 + \epsilon} \frac{1}{\sqrt{2}} \bar{A}_{mn} \sigma^m \eta^i \\
\delta \lambda &= \sqrt{2} i e m \phi_A \eta^i - i \sqrt{2} \frac{m}{\kappa} \sqrt{1 - \epsilon^2} \eta^i \\
\delta \chi &= i \sigma^m \eta^i \bar{D}_m \phi_A - \sigma^m \eta^i \bar{D}_m \phi_B - 2 e m \phi_A \eta^i + 2 \frac{m}{\kappa} \sqrt{1 - \epsilon^2} \eta^i \\
\delta \psi^2_m &= \frac{2}{\kappa} D_m \eta^2 + i \frac{m}{\kappa} \sigma^a \bar{\eta}^a - \frac{1}{2} \bar{\psi}_{mn} \sigma^m \eta^2 - \frac{1}{2} \sqrt{1 + \epsilon} \bar{H}^{A}_{mn} \sigma^m \eta^1 - m \sqrt{1 + \epsilon} A_m \eta^1 \\
&+ \frac{1}{2} \sqrt{1 - \epsilon} \bar{H}^{B}_{mn} \sigma^m \eta^1 + \sqrt{1 - \epsilon} \frac{1}{1 + \epsilon} (\partial_m \phi_A - i D_m \phi_B) \eta^1 \\
&- \kappa \frac{1}{2} \sqrt{1 - \epsilon} \psi^2_m (\delta_{\eta^2} \phi_A - i \delta_{\bar{\eta}^2} \phi_B) - i e m \sqrt{1 - \epsilon} \frac{1}{1 + \epsilon} \phi_A \sigma^a \bar{\eta}^a \\
\delta \phi_A &= \chi \eta^1 + \bar{\chi} \bar{\eta}^1
\end{align}

This expression is valid under the conditions

\begin{align}
\bar{\psi}_{mn} \sigma^m \eta^i &= 0 \\
\bar{\psi}_{mn} \sigma^m \bar{\eta}^i &= 0
\end{align}
\[ \delta_\eta \phi_B = -i \chi \eta^1 + i \bar{\chi} \bar{\eta} \]  

\[(41)\]

where

\[ D_m \phi_A = \partial_m \phi_A - m \sqrt{1 - \epsilon A_m} \]
\[ D_m \phi_B = \partial_m \phi_B - m \sqrt{1 + \epsilon B_m} \]  

and

\[ \hat{D}_m \phi_A = \partial_m \phi_A - m \sqrt{1 - \epsilon A_m} - \frac{\kappa}{2} (\psi^1_{[m} \chi + \bar{\psi}^1_{m]} \bar{\chi}) \]
\[ \hat{D}_m \phi_B = \partial_m \phi_B - m \sqrt{1 + \epsilon B_m} + \frac{\kappa}{2} (\psi^1_{[m} \chi - \bar{\psi}^1_{m]} \bar{\chi}) \]
\[ \hat{A}_{mn} = A_{mn} + \frac{\kappa}{2} \sqrt{1 + \epsilon} (\chi \bar{\sigma}_{[m} \psi^1_{n]} + \bar{\chi} \bar{\sigma}_{[m} \bar{\psi}^1_{n]}) \]
\[ - \sqrt{1 - \epsilon} \frac{\kappa}{2 \sqrt{2}} (\lambda \bar{\sigma}_{[m} \psi^1_{n]} + \bar{\lambda} \bar{\sigma}_{[m} \bar{\psi}^1_{n]}) \lambda) \]
\[ \hat{B}_{mn} = B_{mn} - i \frac{\kappa}{2} \sqrt{1 - \epsilon} (\psi^2_{[m} \psi^1_{n]} - \bar{\psi}^2_{[m} \bar{\psi}^1_{n]}) \]
\[ + i \sqrt{1 + \epsilon} \frac{\kappa}{2 \sqrt{2}} (\lambda \bar{\sigma}_{[m} \psi^1_{n]} - \bar{\lambda} \bar{\sigma}_{[m} \bar{\psi}^1_{n]}) \lambda). \]  

\[(42)\]

Note that this Lagrangian depends on \( \kappa, v \) and \( \Lambda \), with \( m = \sqrt{\Lambda^2 + 4 \kappa^2 v^2} \) and \( \epsilon = \Lambda/m \). As before, \( 0 \leq \epsilon \leq 1 \).

The Lagrangian (40) describes the spontaneous breaking of \( N = 2 \) supersymmetry in AdS space. It has \( N = 2 \) supersymmetry and a local \( U(1) \) gauge symmetry. In unitary gauge, it reduces to the massive \( N = 1 \) Lagrangian of eq. (37).

In it instructive to consider the Lagrangian (40) in two limits. The first, with \( \Lambda \to 0 \), but fixed \( \kappa \) and \( v \), corresponds to the case \( \epsilon \to 0 \). The Lagrangian (40) reduces to the previous case, of partially broken supersymmetry, coupled to supergravity, in a flat Minkowski background. The second limit, with \( \kappa \to 0 \), but fixed \( v \) and \( \Lambda \), corresponds to \( \epsilon \to 1 \). In this limit, the Lagrangian (40) describes partially broken \( N = 2 \) supersymmetry in a fixed AdS background. The full manifold of the \( N = 2 \) supergravities is presented in Figure 2, where \( \epsilon \) is plotted as a function of \( \kappa \) and \( \Lambda \). The two limits correspond to edges of the plot. The center region contains the new AdS supergravity presented here.

To find the algebra associated with the partial supersymmetry breaking, let us consider the second limit, with \( \kappa \to 0 \) and fixed \( v, \Lambda \). A simple computation shows that

\[ \left[ \delta_\eta^1, \delta_\eta^1 \right] \phi_A = 2v^2 (\eta^1 \eta^2 + \bar{\eta} \bar{\eta}^2) \]
\[ \left[ \delta_\eta^2, \delta_\eta^1 \right] A_m = 0 \]  

\[(44)\]
while

\[
\begin{align*}
\left[ \delta_{\eta^2}, \delta_{\eta^1} \right] \phi_B &= -2i v^2 (\eta^1 \eta^2 - \bar{\eta}_1 \bar{\eta}_2) \\
\left[ \delta_{\eta^2}, \delta_{\eta^1} \right] B_m &= -\sqrt{2}i v^2 \frac{\partial_m}{\eta^2} (\eta^1 \eta^2 - \bar{\eta}_1 \bar{\eta}_2).
\end{align*}
\]

Equation (44) tells us that the real scalar $\phi_A$ is the Goldstone boson associated with the $U(1)$ generator of the AdS algebra. (It is this generator which contracts to a real central charge in flat space.) Equation (45) indicates that the scalar $\phi_B$ is also a Goldstone boson associated with a spontaneously-broken $U(1)$ symmetry, but one which is gauged by the vector field $B_m$.

These results imply that when $v, \Lambda \neq 0$, the full current algebra is actually $OSP(2,4) \rtimes_s U(1)$, nonlinearly realized. The $\rtimes_s$ denotes a semi-direct product, because (45) shows that the supersymmetry generators close into the local $U(1)$ symmetry. Note that this construction evades the constraints of the Coleman-Mandula/Haag-Lopuszański-Sohnius theorem because the broken supercharges do not exist. The $OSP(2,4) \rtimes_s U(1)$ symmetry only exists at the level of the current algebra. The $U(1)$ symmetry is always spontaneously broken.

4 Summary

In this talk we have examined the partial breaking of supersymmetry flat and anti de Sitter space. We have seen that partial breaking in flat space can be accomplished using either of two representations of the massive $N = 1$ spin-$3/2$ multiplet. We unHiggsed each representation, and found a new $N = 2$
supergravity and a new $N = 2$ supersymmetry algebra. We also saw that the partial supersymmetry breaking in AdS space can give rise to a new $N = 2$ supersymmetry algebra, albeit one that is necessarily nonlinearly realized.

Acknowledgements

We would like to thank A. Galperin and S. Osofsky for enjoyable collaboration. This work was supported by the National Science Foundation, grant NSF-PHY-9404057.

References

1. J. Hughes, J. Liu, and J. Polchinski, Phys. Lett. 180B (1986) 370; J. Hughes and J. Polchinski, Nucl. Phys. B278 (1986) 147.
2. J. Bagger and A. Galperin, Phys. Lett. B336 (1994) 25; Phys. Rev. D55 (1997) 1091.
3. J. Bagger and A. Galperin, Phys. Lett. B412 (1997) 296.
4. S. Coleman, J. Wess, and B. Zumino, Phys. Rev. 177 (1969) 2239.
5. D. V. Volkov, Sov. J. Particles and Nuclei 4 (1973) 3; V. I. Ogievetsky, Proceedings of the X-th Winter School of Theoretical Physics in Karpacz, Vol. 1, p. 227 (Wroclaw, 1974).
6. I. Antoniadis, H. Partouche, and T. R. Taylor, Phys. Lett. B372 (1996) 83.
7. S. Cecotti, L. Girardello, and M. Porrati, Phys. Lett. B168 (1986) 83.
8. Yu. M. Zinov’ev, Sov. J. Nucl. Phys. 46 (1987) 540.
9. S. Ferrara, L. Girardello, and M. Porrati, Phys. Lett. B366 (1996) 155.
10. S. Ferrara and P. van Nieuwenhuizen, Phys. Lett. B127 (1983) 70.
11. V. I. Ogievetsky and E. Sokatchev, J. Phys. A Math. Gen. 10 (1977) 2021.
12. H. Nicolai, “Representations of supersymmetry in Anti-de Sitter space,” in Supersymmetry and Supergravity ’84: Proceedings of the Trieste Spring School on Supersymmetry and Supergravity, (World Scientific, 1984).