Dirac actions for D-branes on backgrounds with fluxes

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Abstract

The understanding of the fermionic sector of the worldvolume D-brane dynamics on a general background with fluxes is crucial in several branches of string theory, like for example the study of nonperturbative effects or the construction of realistic models living on D-branes. In this paper we derive a new simple Dirac-like form for the bilinear fermionic action for any Dp-brane in any supergravity background, which generalizes the usual Dirac action valid in absence of fluxes. A nonzero world-volume field strength deforms the usual Dirac operator in the action to a generalized non-canonical one. We show how the canonical form can be re-established by a redefinition of the world-volume geometry.

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1 Introduction

The study of string backgrounds with fluxes turned on has recently received considerable attention due to its relevance in the construction of more realistic models within the framework of string theory. Since D-branes play a crucial role in the construction of these models, the understanding of the effect of the background fluxes on the world-volume geometry becomes of obvious interest.

While the bosonic sector of the D-brane action on a general background seems to be completely under control (at least for the Abelian case of a single D-brane), the introduction of fermions on a general background required the use of superspace techniques, which allowed to write the complete superaction in an elegant and compact form [1, 2]. Nevertheless, the use of the superspace as target space hides how the background fields enter the fermionic terms of the action and then any explicit calculation or consideration involving the world-volume fermions cannot be done only using the implicit superspace formalism. An important step for the understanding of the fermionic terms in the Dp-brane actions in backgrounds with fluxes was obtained in [3, 4]. In these papers the complete quadratic fermionic action was given for any Dp-brane on any (bosonic) supergravity background. The explicit form of these terms is indeed necessary in several interesting situations where the contribution of the world-volume fermionic dynamics becomes relevant. For example, they are necessary to write the effect of background fluxes in the effective action governing some phenomenologically interesting brane configurations [5, 6, 7]. Also, Euclidean brane configurations are sources of nonperturbative corrections in lower dimensional effective theories obtained by compactification [8, 9, 10]. Finally, the knowledge of the explicit form of the fermionic terms is indeed necessary for any kind of quantum world-volume computation (see for example [11]).

In a seminal paper [9], Witten found some restrictive conditions on the possibility of having nonperturbative corrections to the effective superpotential, which are generally valid for compactifications on backgrounds without fluxes. Similar kinds of nonperturbative corrections play a crucial role in addressing the problem of moduli-stabilization in the search for realistic flux-compactification models along the lines of [12]. The understanding of nonperturbative effects when fluxes are turned on becomes of obvious importance in this context and this understanding can pass only through a better understanding of the world-volume fermionic physics. Indeed some recent work has gone in this direction. For example, the fermionic quadratic M5-brane action was derived in [13], also discussing the effect of the background flux in the internal symmetries of the fermionic action. This action was then used in [14, 15] to discuss new conditions for having nonperturbative corrections in presence of fluxes. Also, the paper [16] studies the quadratic fermionic D3-brane action in order to gain insight into Euclidean D3-brane instantons generating nonperturbative corrections.
The aim of this paper is to clarify the geometrical structure of the results derived in [3, 4]. In the first of these papers [3] the fermionic bilinear terms in the action of any Dp-brane were presented in a Dirac-like form, making the assumption that the world-volume field-strength $F_{\alpha \beta} = P[B]_{\alpha \beta} + f_{\alpha \beta}$ was vanishing ($P[.]$ indicates the pull-back on the world-volume while $f(2) = dA(1)$ denotes the purely world-volume field strength). This condition was dropped in the second paper [4], where a general $F(2) = B(2) + f(2)$ was included. The net effect of this world-volume field is two-fold. First, it gives a correct redefinition of the $\kappa$-symmetry operator that naturally enters the ($\kappa$-symmetric) action. Secondly, it adds new kinetic terms that apparently destroy the Dirac-like form of the actions [see eq.(3) in the next section]. In this paper we will show how these terms can indeed be reorganized into a more geometrical term that naturally generalizes the Dirac-like operator. It contains a kinetic term of the schematic form

$$\left(M^{-1}\right)_{\alpha \beta} \Gamma_{\beta} \nabla_{\alpha},$$

(1)

where $M_{\alpha \beta} = P[G]_{\alpha \beta} + F_{\alpha \beta}$ (see eq.(17) for the precise form). This is somehow analogous to what happens on the M5 brane with a non-zero $h$ field [13], and these two forms should indeed be related by double dimensional reduction.

We also address the problem of writing this kinetic operator in canonical form. Indeed the presence of a nonzero $F_{\alpha \beta}$ enters not only the kinetic term in the generalized Dirac operator [11] but also the generalized integration measure $\sqrt{-\det M}$ that also appears in the bosonic Dirac-Born-Infeld action. Then it seems natural to see the effect of the field $F_{\alpha \beta}$ as a deformation of the world-volume geometry. This involves a redefinition of the world-volume metric and coupling constant analogous to that found in [17] for describing the noncommutative theories living on a D-brane in presence of a constant background $F_{\alpha \beta}$ field. Once such a redefinition is extended to the vielbein, we show how it is possible to write the bilinear fermionic action in a canonical form.

The paper is structured as follows. In section 2 we recall the results of the papers [3, 4], namely the explicit form for the fermionic bilinear action for any Dp-brane on any (bosonic) supergravity background, and show how it is possible to rewrite this result in a new, more geometrical form [see eq.(17)] that nicely includes the effect of a nonzero $F_{\alpha \beta}$. In section 3 we show how in the new form the action is easily proved to be invariant in form under T-duality. In section 4 we consider the $\kappa$-fixing of the action and discuss explicitly the possible resulting world-volume supersymmetries. In section 5 we make some general observations about the natural world-volume geometry which is deformed by the presence of a non-zero $F_{\alpha \beta}$ and in section 6 we use these observations to show how the fermionic action can be rewritten in a form containing a canonical Dirac operator plus mass terms coming from the background fields and the world-volume configuration itself. Finally, in section 7 we present our conclusions. Appendix A contains the notation and comments on the explicit form of the $\kappa$-fixed action are given in appendix B.
2 The quadratic fermionic action on a general background

We are interested in studying the effective actions of D-branes on a general background at the quadratic order in the fermions. The D-brane actions can be formulated on a general background using the superspace formalism \[1, 2\]. Unfortunately, even if elegant and in principle complete, this formalism hides the explicit couplings between the physical fields of the theory and makes the worldvolume fermionic sector of the theory quite obscure. An expansion of the superactions in background components is required in order to have the possibility to make any calculations involving world-volume fermions. If one starts from the superactions of \[1, 2\] such a calculation can be really cumbersome and requires a case by case study (see for example \[18\] and the recent \[16\]). The fermionic quadratic action for any D-brane on any background was obtained in \[3, 4\] by following a somehow different route. The starting point was the normal coordinate expansion of the M2-brane superaction presented in \[19\]. Then, by dimensional reduction and T-duality all the D\(_p\)-brane actions quadratic in the fermions were derived in a unified and compact form dictated by the consistency with T-duality.

First of all, let us recall the final result of \[3, 4\]. The bosonic part of the D\(_p\)-brane action is given by the standard DBI+CS form

\[
S_{Dp}^{(B)} = -\tau_{Dp} \int d^{p+1}\xi e^{-\phi} \sqrt{-\det(g + F)} + \tau_{Dp} \int \sum_n P[C(n)] e^F ,
\]

where \(\tau_{Dp} = (2\pi\alpha')^2 g_s\) is the brane tension. We use coordinates \(\xi^\alpha, \alpha = 0, \ldots, p\) to parametrize the worldvolume of the brane, \(g_{\alpha\beta} = P[G]_{\alpha\beta}\) is the pull-back of the background metric \(G_{mn}\) on the world-volume, and \(F_{\alpha\beta} = P[B]_{\alpha\beta} + f_{\alpha\beta}\), where \(f(2) = dA(1)\) is the field-strength of the gauge field living on the brane.

The quadratic fermionic term is given by \[3, 4\]:

\[
S_{Dp}^{(F)} = \frac{\tau_{Dp}}{2} \int d^{p+1}\xi e^{-\phi} \sqrt{-\det(g + F)} \bar{\theta}(1 - \Gamma_{Dp})(\Gamma^\alpha D_\alpha - \Delta + L_{Dp})\theta ,
\]

where \(\Gamma_\alpha\) is the pullback of the gamma matrices \(\Gamma_m\) (see for the notation appendix \[A\] where also some differences with the conventions used in \[3, 4\] are spelled out), the fermionic field \(\theta\) is a 10d Majorana spinor in type IIA and a positive chirality doublet Majorana-Weyl spinor in type IIB. The other objects entering the actions are the following. For type IIA D-branes we have

\[
\Gamma_{D(2n)} = \sum_{q+r=n} \frac{(-)^{r+1}(\Gamma_{(10)})^{r+1} e^{\alpha_1 \ldots \alpha_{2q} \beta_1 \ldots \beta_{2r+1}}}{q!(2r + 1)! 2^r \sqrt{-\det(g + F)}} \mathcal{F}_{\alpha_1 \alpha_2 \ldots \alpha_{2q} \gamma} \Gamma_{\beta_1 \ldots \beta_{2r+1} \gamma} D_\gamma ,
\]

\[
L_{D(2n)} = \sum_{q \geq 1, q+r=n} \frac{(-)^{r+1}(\Gamma_{(10)})^{r+1} e^{\alpha_1 \ldots \alpha_{2q} \beta_1 \ldots \beta_{2r+1}}}{q!(2r + 1)! 2^r \sqrt{-\det(g + F)}} \mathcal{F}_{\alpha_1 \alpha_2 \ldots \alpha_{2q}} \Gamma_{\beta_1 \ldots \beta_{2r+1} \gamma} D_\gamma ,
\]

\[
(2)
\]

\[
(3)
\]

\[
(4)
\]

\[
(5)
\]
whereas for type IIB D-branes

\[ \Gamma_{D(2n+1)} = \sum_{q+r=n+1} \frac{(-)^{r+1}(i\sigma_2)(\sigma_3)^r \epsilon^{a_1 \ldots a_{2q} b_1 \ldots b_{2r}}}{q!(2r)!2^q \sqrt{-\det(g + F)}} \mathcal{F}_{a_1 a_2} \cdots \mathcal{F}_{a_{2q-1} a_{2q}} \Gamma_{b_1 \ldots b_{2r}}, \quad (6) \]

\[ L_{D(2n+1)} = \sum_{q\geq 1, q+r=n+1} \frac{(-)^{r+1}(i\sigma_2)(\sigma_3)^r \epsilon^{a_1 \ldots a_{2q} b_1 \ldots b_{2r}}}{q!(2r)!2^q \sqrt{-\det(g + F)}} \mathcal{F}_{a_1 a_2} \cdots \mathcal{F}_{a_{2q-1} a_{2q}} \Gamma_{b_1 \ldots b_{2r}} \gamma D_\gamma. \quad (7) \]

In the above expressions \( D_m \) (which enters through its pullback \( D_\alpha \)) and \( \Delta \) are the operators appearing in the supersymmetry transformation laws of the background gravitino and dilatino and are defined in appendix A in the equations (81), (82) and (83).

Let us recall that the action \([3]\) was first found in \([3]\) in the restricted case in which \( \mathcal{F}_{\alpha\beta} = 0 \). It is evident from \([3]\) that in this case the action takes an explicit canonical Dirac-like form, where the background fluxes contribute with mass terms through the operators \( D_\alpha \) and \( \Delta \). The effect of a nonzero \( \mathcal{F}_{\alpha\beta} \) is two-fold. First it is included in the \( \Gamma_{D_p} \) operators \([4]\) and \([5]\). Secondly, we have the new terms \( L_{D_p} \), which enter the action and which not only add new non-derivative couplings to the background and worldvolume fields, but also add new kinetic terms. We are now going to show how these new terms can be reorganized such that the fermionic lagrangian \([3]\) is written in a more geometrical form where the effect of the field \( \mathcal{F}_{\alpha\beta} \) can be reabsorbed in a shift of the pulled-back world-volume metric \( g_{\alpha\beta} \). This simplifies the final form of the action considerably and makes its structure more transparent.

First of all, let us introduce the operator

\[ \Gamma_{D_p}^{(0)} = \frac{\epsilon^{a_1 \ldots a_{p+1}}}{(p + 1)!\sqrt{-\det g}} \Gamma_{a_1 \ldots a_{p+1}}. \quad (8) \]

It is useful for the manipulations below to notice that it squares to \((-1)^{(p-1)(p-2)/2}\). We use the formula

\[ \epsilon^{a_1 \ldots a_r b_{r+1} \ldots b_{p+1}} \Gamma_{b_{r+1} \ldots b_{p+1}} = (-)^{r(r-1)/2} (p + 1)! \Gamma_{a_1 \ldots a_r} \Gamma_{D_p}^{(0)} \quad (9) \]

to write the chiral operators \([4]\) and \([6]\) in the form used in the paper \([2]\)

\[ \Gamma_{D(2n)} = \frac{\sqrt{\det g}}{\sqrt{-\det(g + F)}} \Gamma_{D(2n)}^{(0)} (\Gamma_{(10)})^{n+1} \sum_q \frac{(-)^q (\Gamma_{(10)})^q}{q!2^q} \Gamma_{a_1 \ldots a_{2q}} \mathcal{F}_{a_1 a_2} \cdots \mathcal{F}_{a_{2q-1} a_{2q}}. \quad (10) \]

\[ \Gamma_{D(2n+1)} = \frac{\sqrt{-\det g}}{\sqrt{-\det(g + F)}} \Gamma_{D(2n+1)}^{(0)} (\sigma_3)^{n+1} (-i\sigma_2) \sum_q \frac{(\sigma_3)^q}{q!2^q} \Gamma_{a_1 \ldots a_{2q}} \mathcal{F}_{a_1 a_2} \cdots \mathcal{F}_{a_{2q-1} a_{2q}}. \quad (11) \]

From \([5]\) one analogously calculates that

\[ L_{D(2n)} = \frac{-\sqrt{\det g}}{\sqrt{-\det(g + F)}} \Gamma_{D_p}^{(0)} (\Gamma_{(10)})^n \times \]
\[
\sum_{q \geq 1} \left( -\Gamma_{(10)} \right)^{q-1} \frac{\Gamma^{\alpha_1 \ldots \alpha_{2q-1}}}{(q-1)!^{2q-1}} F_{\alpha_1 \alpha_2} \ldots F_{\alpha_{2q-3} \alpha_{2q-2}} F_{\alpha_{2q-1}} \gamma D_{\gamma} .
\] (12)

This expression can in turn be rewritten as

\[
L_{D(2n)} = -\Gamma_{D(2n)} \Gamma_{(10)}^{\alpha \beta} D_{\beta} - \sqrt{-\det g} \Gamma_{Dp}^{(0)} \left( \Gamma_{(10)} \right)^{n+1} \times \\
\sum_{q \geq 2} \left( -\Gamma_{(10)} \right)^{q-2} \frac{\Gamma^{\alpha_1 \ldots \alpha_{2q-3}}}{(q-2)!^{2q-2}} F_{\alpha_1 \alpha_2} \ldots F_{\alpha_{2q-5} \alpha_{2q-4}} F_{\alpha_{2q-3}} \gamma_1 F_{\gamma_1 \gamma_2} \gamma_2 D_{\gamma_2} .
\] (13)

By iterating this last step (and by doing an analogous calculation for the IIB case) the following formulae can be found:

\[
L_{D(2n)} = -\Gamma_{D(2n)} \sum_{q \geq 1} \left( \Gamma_{(10)} \right)^{q} (F^q)^{\alpha \beta} \Gamma_{\alpha \beta} D_{\beta} ,
\]
\[
L_{D(2n+1)} = -\Gamma_{D(2n+1)} \sum_{q \geq 1} (-\sigma_3)^{q} (F^q)^{\alpha \beta} \Gamma_{\alpha \beta} D_{\beta} ,
\]

where \((F^q)^{\alpha \beta} = F_{\gamma_1 \gamma_2} \ldots F_{\gamma_{q-2} \gamma_{q-1}} F_{\gamma_q}^{\gamma-1 \beta}\). We introduce the operator

\[
\tilde{M}_{\alpha \beta} = g_{\alpha \beta} + \tilde{\Gamma}_{(10)} F_{\alpha \beta} ;
\]

where

\[
\text{type IIA} : \tilde{\Gamma}_{(10)} = \Gamma_{(10)} , \quad \text{type IIB} : \tilde{\Gamma}_{(10)} = \Gamma_{(10)} \otimes \sigma_3 .
\] (16)

By putting these results together, it is easy to see that we can write the fermionic action in the compact and elegant form

\[
S_{Dp}^{(F)} = \frac{\tau_{Dp}}{2} \int d^{p+1} \xi e^{-\Phi} \sqrt{-\det (g + \mathcal{F})} \left[ (\tilde{M}^{-1})^{\alpha \beta} \Gamma_{\beta} D_{\alpha} - \Delta \theta \right] .
\] (17)

This action represents one of the main result of this paper. The operators \(L_{Dp}\) that in the action \(\mathcal{F}\) were disturbing, can now be seen as the source of the natural geometrical coupling to the matrix

\[
M_{\alpha \beta} = g_{\alpha \beta} + \mathcal{F}_{\alpha \beta} .
\]

This matrix substitutes the metric already in the bosonic action or in the definition of the natural volume element defined on the brane. In this way we see how the effect of the \(\mathcal{F}_{\alpha \beta}\) field, given by the couplings contained in the operators \(L_{Dp}\), can be schematically reabsorbed in the following redefinition of the kinetic term

\[
g^{\alpha \beta} \Gamma_{\beta} \nabla_{\alpha} \rightarrow (\tilde{M}^{-1})^{\alpha \beta} \Gamma_{\beta} D_{\alpha} .
\] (19)

On the other hand, the effect of the \(\mathcal{F}_{\alpha \beta}\) field included in the \(\Gamma_{Dp}\) operators appears in the most natural form for a \(\kappa\)-symmetric action. This form of the action is related
to the expansion of the super D-brane equations of motion found in [20] for a general background where the structure of (17) and (19) can be recognized.

The effect of a nonzero $\mathcal{F}_{\alpha\beta}$ on the world-volume geometry will be considered more carefully in sections 5 and 6. But let us first of all consider two relevant aspects regarding the action (17), namely the consistency with T-duality and the $\kappa$-fixed form of the above action together with its possible linearly and nonlinearly realized supersymmetries. These will be discussed in the following two sections.

3 Consistency with T-duality

The original fermionic action (3) was constructed in [4] by using T-duality and assembling the different terms in a rather indirect way, using the partial results obtained previously in [3] and completing them by means of consistency conditions. Let us rederive the proof of the T-duality consistency of the above actions given in [3, 4] starting from their new expression given in (17). This is an important consistency check and it clarifies also the validity of the arguments, given in [3, 4], to obtain the final form of the action (3). In order to do this, let us first of all prove that the term

$$\frac{\tau_{Dp}}{2} \int dp^{p+1} e^{-\Phi} \sqrt{-\det(g + \mathcal{F}) - \bar{\theta}(\bar{\theta}) (\tilde{M}^{-1})^{\alpha\beta} \Gamma_{\beta} D_{\alpha} - \Delta|\theta},$$

in the fermionic action (17) is left invariant in form by T-duality. One can check this property directly by using the usual T-duality rules for the bosonic fields and Hassan’s T-duality rules for the fermions [21]. It is however easier to derive this property in a less direct way. Let us first introduce the following combination of bosonic and fermionic fields

$$\Phi = \Phi - \frac{1}{2} \bar{\theta} \Delta \theta,$$
$$G_{mn} = G_{mn} - \bar{\theta} \Gamma_{(m} D_{n)} \theta,$$
$$B_{mn} = B_{mn} - \bar{\theta} \Gamma_{(10)} \Gamma_{[m} D_{n]} \theta.$$  \hspace{1cm} (21)

These can be seen as superfields expanded up to the second order. One of the basic observations of [22, 4] is that, using Hassan’s T-duality rules for fermions [21], these second order superfields transform (up to second order) in the same way the corresponding bosonic fields do. The next step is to notice that the term (20) can be seen as the second order term arising in the expansion of a DBI action for the superfields (21)

$$S_{Dp} = -\tau_{Dp} \int dp^{p+1} e^{-\Phi} \sqrt{-\det(P[\mathbf{G} + \mathbf{B}] + f)}.$$  \hspace{1cm} (22)

Once we know that the superfields (21) transform as the corresponding bosonic fields under T-duality we can immediately conclude that the action (22) is left invariant in form by T-duality and then also its second order term (20) is invariant by T-duality.
It is now easy to see that also the second contribution in the second order action (17)
\[ -\frac{\tau_{Dp}}{2} \int d^{p+1}\xi e^{-\Phi} \sqrt{-\det(g + \mathcal{F})} \bar{\theta} \Gamma_{Dp} [(\tilde{M}^{-1})^{\alpha \beta} \Gamma_\beta D_\alpha - \Delta] \theta , \]
is left invariant by T-duality. Since we already know that the term \([(\tilde{M}^{-1})^{\alpha \beta} \Gamma_\beta D_\alpha - \Delta]\) is invariant in form under T-duality, we need only that the \( \Gamma_{Dp} \) are transformed into themselves under T-duality. But this is indeed the case by definition, since in [4] these operators were obtained one from the other by using T-duality. Then the whole action (17) is clearly invariant in form under T-duality.

4 \( \kappa \)-fixing and supersymmetry

The action (17) presented in section 2, once completed with the bosonic one (2), has world-volume diffeomorphisms and \( \kappa \)-symmetry as world-volume gauge symmetries. In this section we would like to discuss some aspects regarding their gauge-fixing and the consequent effects on the way the possible background supersymmetries are realized on the brane. Let us start by considering the \( \kappa \)-symmetry. We recall that, up to the fermionic order we are interested in, the \( \kappa \)-symmetry transformations of the bosonic plus fermionic action written in (2) and (17) are given by [4]
\[ \delta_\kappa \bar{\theta} = \dot{\kappa}(1 + \Gamma_{Dp}) \theta, \]
\[ \delta_\kappa x^m = -\frac{1}{2} \delta_\kappa \bar{\theta} \Gamma^m \theta, \]
\[ \delta_\kappa A_\alpha = \frac{1}{2} \delta_\kappa \bar{\theta} \tilde{\Gamma}_{(10)} \Gamma_\alpha \theta - \frac{1}{2} B_{\alpha \sigma} \delta_\kappa \bar{\theta} \Gamma^\sigma \theta. \] (24)

In order to consider the problem of \( \kappa \)-fixing more clearly, it is convenient to write (24) in a double spinor convention for both type IIA and type IIB, as introduced in appendix A. In this notation, the first two transformation rules of (24) can be written in exactly the same form but with a \( \Gamma_{Dp} \) given by\(^1\) (for both type IIA and IIB)
\[ \Gamma_{Dp} = (-)^p \Gamma_{Dp}^{(0)} (\sigma_3)^{\frac{p(p+1)}{2}} (i\sigma_2) \frac{\sqrt{-\det g}}{\sqrt{-\det(g + \mathcal{F})}} \sum_q (\sigma_3)^q \frac{\Gamma^{\alpha_1...\alpha_{2q}}}{q!2^q} \mathcal{F}_{\alpha_1\alpha_2} \cdots \mathcal{F}_{\alpha_{2q-1}\alpha_{2q}}. \] (25)

On the other hand, the last transformation of (24) takes the form
\[ \delta_\kappa A_\alpha = -\frac{1}{2} \delta_\kappa \bar{\theta} \sigma_3 \Gamma_\alpha \theta - \frac{1}{2} B_{\alpha \sigma} \delta_\kappa \bar{\theta} \Gamma^\sigma \theta. \] (26)

\(^1\)Note that, for type IIA, the \( \Gamma_{Dp} \) in double spinor notation is not the same as (10) but is given by \( \Gamma_{Dp}^{\text{double}} = \sigma_1 \Gamma_{Dp} \sigma_1 \) due to the chosen representation of the charge conjugation matrix. On the other hand, the \( \sigma_1 \) factors in (25) have already been extracted from \( \Gamma_{Dp}^{(0)} \), which is thus considered here as the diagonal matrix in the extension index.
As stressed in [23], it is important to note that the $\Gamma_{Dp}$ operators entering the $\kappa$-symmetry transformations (24) are off-diagonal:

$$ \Gamma_{Dp} = \begin{pmatrix} 0 & \hat{\Gamma}_{Dp}^{-1} \\ \hat{\Gamma}_{Dp} & 0 \end{pmatrix}, $$

(27)

where

$$ \hat{\Gamma}_{Dp} = \left( \begin{array}{cc} 0 & \tilde{\Gamma}_{Dp} \\ \tilde{\Gamma}_{Dp} & 0 \end{array} \right), $$

(28)

and $\hat{\Gamma}_{Dp}^{-1}(\mathcal{F}) = (-)^{(p-2)(p-3)} \hat{\Gamma}_{Dp}(\mathcal{F})$ (remember that $\Gamma_{Dp}^{(0)}$ is defined in (8)). Indeed, using this property it is easy to see that the $\kappa$-symmetry transformation rules can be written in terms of an irreducible 16-dimensional spinor $\kappa$. We can use for example a spinor $\kappa$ satisfying the condition $\tilde{\Gamma}_{(10)}\kappa = -\kappa$ and rewrite the transformations (24) in the following way

$$ \delta_{\kappa} \bar{\theta}_1 = \bar{\kappa} \hat{\Gamma}_{Dp} \, , \quad \delta_{\kappa} \bar{\theta}_2 = \bar{\kappa} \cdot $$

(29)

Then, it is clear that, as discussed for example in [24, 23], we can impose a covariant gauge-fixing like $\tilde{\Gamma}_{(10)}\theta = \theta$ (i.e. $\theta_2 = 0$). The resulting $\kappa$-fixed action can be easily seen to be expressible in terms of only $\theta_1$ in the following way

$$ S_{Dp}^{(F)} = \frac{\tau_{Dp}}{2} \int d^{p+1}\xi e^{-\Phi} \sqrt{-\det(g + \mathcal{F})} \left\{ \bar{\theta}_1 \left[ (M^{-1})^{\alpha\beta} \Gamma_{\alpha}D_{\beta}^{(0)} - \Delta^{(1)} \right] \theta_1 + \bar{\theta}_1 \hat{\Gamma}_{Dp}^{-1} \left[ (M^{-1})^{\alpha\beta} \Gamma_{\beta}W_{\alpha} - \Delta^{(2)} \right] \theta_1 \right\}, $$

(30)

where the operators involved are defined in appendix A. The explicit form of this action involves terms that vanish by means of the symmetry properties of the gamma matrices (for example the term containing the gradient of the dilaton present in $\Delta^{(1)}$ is identically zero). See appendix B for more on this point.

We can now pass to the discussion of how possible background supersymmetries are realized on the world-volume. Let us first of all recall that, if the supergravity background is supersymmetric, i.e. possesses a killing spinor $\varepsilon$, then the gauge-unfixed D-brane action is symmetric under the following (leading order) induced supersymmetry transformations [4] (in standard notation for IIA)

$$ \delta_{\varepsilon} \theta = \varepsilon, \quad \delta_{\varepsilon} x^m = -\frac{1}{2} \bar{\theta} \Gamma^m \varepsilon, \quad \delta_{\varepsilon} A_\alpha = \frac{1}{2} \bar{\theta} \tilde{\Gamma}_{(10)} \Gamma_{\alpha} \varepsilon - \frac{1}{2} B_{am} \bar{\theta} \Gamma^m \varepsilon. $$

(31)

These transformations have the same form in double spinor notation, up to the substitution $\tilde{\Gamma}_{(10)} \rightarrow -\sigma_3$ in the last line. In order to write these transformations in their
gauge-fixed form, we have to compensate the possible breaking of the $\kappa$-fixing condition $\theta = \Gamma_{(10)} \theta$ by some additional $\kappa$-transformation. For example, we have to add a $\kappa$-transformation $\kappa = -\varepsilon_2$. Then, the resulting supersymmetry transformation of the physical fermionic field $\theta$ becomes
\[
\delta_{\varepsilon} \tilde{\theta}_1 = \bar{\varepsilon}_1 - \bar{\varepsilon}_2 \bar{\Gamma}_{Dp} .
\] (32)
It is clear that supersymmetry is preserved by a classical configuration only if
\[
\bar{\varepsilon}_1 = \bar{\varepsilon}_2 \Gamma^{(cl)}_{Dp} ,
\] (33)
which corresponds to the usual condition $\bar{\varepsilon} \Gamma^{(cl)}_{Dp} = \bar{\varepsilon}$ that has to be satisfied in order for the D-brane to preserve the background supersymmetry $\varepsilon$. Then, using the relation (33), it is possible to write the preserved supersymmetry transformations for the $\kappa$-fixed action in terms of only $\varepsilon_1$ as follows
\[
\begin{align*}
\delta_{\varepsilon} \tilde{\theta}_1 & = \bar{\varepsilon}_1 (1 - \bar{\Gamma}^{(cl)}_{Dp}) \Gamma_{Dp} , \\
\delta_{\varepsilon} x^m & = \frac{1}{2} \bar{\varepsilon}_1 (1 + \bar{\Gamma}^{(cl)}_{Dp}) \Gamma^m \theta_1 , \\
\delta_{\varepsilon} A_\alpha & = \frac{1}{2} \bar{\varepsilon}_1 (1 + \bar{\Gamma}^{(cl)}_{Dp}) \Gamma_{\alpha} \theta_1 + \frac{1}{2} B_{am} \bar{\varepsilon}_1 (1 + \bar{\Gamma}^{(cl)}_{Dp}) \Gamma^m \theta_1 .
\end{align*}
\] (34)
These expressions contain only the supersymmetry transformations in lowest order of fermions (without fermion fields for the transformations of the fermions, and linear in fermions for the transformations of the bosons). The supersymmetry of the full action needs higher order terms. However, such transformations are sufficient for determining the variations of the action linear in fermions. Therefore, they are exact supersymmetries for the completely truncated action quadratic in both bosons and fermions around some particular classical configuration. To see what they look like in this linearized approximation, it is convenient to fix the residual gauge invariance under world-volume diffeomorphisms in order to identify the physical worldvolume scalar fields. This could be done by adopting the standard static gauge condition $x^\alpha = \xi^\alpha$, which means that only the fluctuations $\delta x^\hat{m}$ ($\hat{m} = p + 1, \ldots, 9$ labels the transverse directions) are physical, and one has to impose the condition $\delta x^\alpha = 0$. However, since we are working in a general curved space, this kind of gauge fixing is not the most geometrical one due to the arbitrariness of the coordinate choice. It is then natural to break explicitly the local $SO(1,9)$ Lorentz invariance of the theory into $SO(1,p) \times SO(9-p)$ and select a class of adapted co-vielbein $e_{\hat{m}} = (e_\alpha, e_{\hat{m}})$, such that the pull-back on the brane of the $e_{\hat{m}}$ is vanishing and the pulled-back $e_\alpha$ form a world-volume vielbein. Now one can consider the fluctuations of the brane as described by a section $\phi_{\hat{m}}$ of the normal bundle (i.e. $\phi_{\hat{m}} = e_{\hat{m}} \delta x^m$). This means that the natural gauge fixing condition is
\[
e^\alpha_m \delta x^m = 0 .
\] (35)
In order to write the supersymmetry transformations for the completely gauge-fixed linearized action, we now have to compensate the transformation (34) with a world-volume diffeomorphism \( \delta \xi^\alpha(\epsilon) \) defined by the condition
\[
\delta \xi^\alpha(\epsilon) P[e^\alpha_m]_\alpha = -e^\alpha_m \delta \epsilon^m.
\] (36)

Taking into account this compensation and using the fact that \( \phi^\hat{m} = 0 \) when evaluated on the classical configuration, the linearized gauge-fixed supersymmetry transformations (34) become
\[
\delta \epsilon^{\alpha 1} = (M^{-1})^\alpha \beta (\nabla^N_\alpha \phi^\hat{m}(\hat{m}^\beta)) \Gamma^\hat{m} \bar{\epsilon}_1 + (M^{-1})^\alpha \beta \Gamma^\gamma \kappa_\alpha \phi^\hat{m} + \frac{1}{2} (M^{-1})^\alpha \gamma (M^{-1})^\beta \delta (\phi^\hat{m} H^\gamma \kappa_\delta + f_{\gamma \delta}) \Gamma^\alpha \bar{\epsilon}_1,
\]
\[
\delta \phi^\hat{m} = \bar{\epsilon}_1 \Gamma^\hat{m} \theta_1,
\]
\[
\delta \epsilon A_\alpha = \bar{\epsilon}_1 \Gamma^1 \alpha \theta_1 + f^{(cl)}_{\alpha \beta} \bar{\epsilon}_1 \Gamma^\beta \theta_1 + B_{\alpha m} \bar{\epsilon}_1 \Gamma^m \theta_1,
\] (37)

where \( \nabla^N_\alpha = \partial_\alpha + \frac{1}{4} \mathcal{A}_\alpha \hat{m} \hat{n} \bar{\Gamma}^\hat{m} \bar{\Gamma}^\hat{n} \) indicates the normal bundle covariant derivative with connection
\[
\mathcal{A}_\alpha \hat{m} \hat{n} = \Omega_\alpha \hat{m} \hat{n},
\] (38)

where \( \Omega_\alpha \hat{m} \hat{n} \) is the pull-back of the spin connection of the target-space vielbein \( e^\hat{m}_m \) and \( K_{\alpha \beta} \hat{\kappa} \) is the extrinsic curvature of the world-volume of the brane and is defined by
\[
K_{\alpha \beta} \hat{\kappa} = K_{\beta \alpha} \hat{\kappa} = e^\kappa_\beta \Omega_\alpha \hat{m} \hat{n}. 
\] (39)

The derivation of the first of (37) is straightforward but tedious, as it involves several rearrangements using gamma matrix properties along the lines followed to derive (14) from (5) and (7).

5 The world-volume geometry

In section 2 we have seen how it is possible to write the quadratic fermionic action for a D-brane in the form (17) which makes transparent how the background geometry couples to the world-volume theory. In this section we explore further the geometry characterizing the theory that lives on the world-volume of the Dp-brane. In order to do this we consider from now on the dynamics of the brane around some classical configuration and use the condition (35) to fix the world-volume reparametrization invariance. Furthermore, we will always consider the (bosonic) fields as evaluated at their classical value and write the contribution coming from the dynamical fluctuations explicitly.

The first thing that one immediately notices is that the action (17) is not in a canonical form, in the sense that the kinetic term is contained in
\[
(M^{-1})^\alpha \beta \Gamma^\beta D_\alpha,
\] (40)

10
which clearly does not give a canonical kinetic term for the fermions.

This feature is already visible in the bosonic action, as is evident from the fact that the natural integration measure is given by

\[ \sqrt{-\det(g + \mathcal{F})}. \]  

(41)

As we want to study the world-volume physics around a particular background brane configuration with a nonzero \( \mathcal{F}_{\alpha\beta} \), the general fluctuation \( \delta M_{\alpha\beta} \) of (18) has a lagrangian of the schematic form

\[ \sqrt{-\det M(M^{-1} \cdots M^{-1} \delta M \cdots \delta M)} . \]  

(42)

This means that not only the natural volume element is given by \( \sqrt{-\det M} \), but also that the lower indices of the different \( \delta M_{\alpha\beta} \) are raised not with a metric but with \( (M^{-1})^{\alpha\beta} \), analogously to what happens in the term (40) of the fermionic action. The effect is that the kinetic terms arising from the expansion of the bosonic action are not in canonical form, as in the fermionic case.

Such a deviation from the canonical form is of course given by the presence of a nontrivial background \( \mathcal{F} \), which in some sense deforms the world-volume geometry, generating these noncanonical kinetic terms. The strictly related situation one immediately thinks of is the case in which \( \mathcal{F} \) is constant, and the background is flat. It is well known that in this case the effective world-volume theory is described by a noncommutative DBI action with zero background \( \mathcal{F} \) field [17]. In this case the effect of a constant nonzero \( \mathcal{F}_{\alpha\beta} \) can be completely reabsorbed in a (non-isotropic) noncommutative deformation of the world-volume theory. It is also important to remember that the metric and the coupling constant of the noncommutative theory are not the same as the background “commutative” ones but are related to these by the relations

\[
\begin{align*}
g^{(nc)}_{\alpha\beta} &= g_{\alpha\beta} - \mathcal{F}_{\alpha\gamma} g^{\gamma\delta} \mathcal{F}_{\delta\beta}, \\
g^{(nc)}_{s} &= g_s \sqrt{\frac{\det(g + \mathcal{F})}{\det g}}.
\end{align*}
\]  

(43)

We are now going to show how the effect of a nonzero background \( \mathcal{F}_{\alpha\beta} \) can also be reabsorbed in a non-isotropic deformation of the theory that produces a commutative theory with canonical kinetic terms and new coupling terms generated by the presence of the nonzero background \( \mathcal{F}_{\alpha\beta} \). This will involve a redefinition of the world-volume metric and coupling constant (or better, of the dilaton as seen by the brane), analogous to those recalled in [13] for the noncommutative case.

Since in the following section we will work with fermions, it is convenient to define from the beginning the deformed theory in terms of the vielbein instead of in terms of the metric. In particular, since we will work around some fixed classical configuration,
we can restrict to the class of adapted co-vielbeins $e^m = (e_\alpha, e_\hat{m})$ such that $P[e_\hat{m}] = 0$, introduced in the previous section. The presence of a world-volume field $\mathcal{F}$ generically breaks the world-volume $SO(1, p)$ symmetry into $SO(1, 1) \times [SO(2)]^{[(p-1)/2]}$, naturally selecting a subclass of world-volume vielbeins $e_\alpha$ such that

$$\mathcal{F} = \tanh \phi_0 \ e_\alpha \wedge e_\beta + \sum_{r=1}^{[(p-1)/2]} \tan \phi_r \ e_\alpha^r \wedge e_\beta^{2r+1}. \quad (44)$$

This form is clearly invariant under the residual $SO(1, 1) \times [SO(2)]^{[(p-1)/2]}$ symmetry. This decomposition has been used in [24] to write the world-volume chiral operators (10) and (11) (up to a sign) in a nice form. Using the same approach, we will show that the effect of the field $\mathcal{F}$ can be reabsorbed in a non-isotropic deformation of the world-volume metric.

Let us first make a preliminary observation. If we define

$$X_\alpha^\beta = \mathcal{F}_\alpha^\beta, \quad (45)$$

then the action of the matrix $(1 + X)$ can be seen as the product of a rotation $\Lambda \in SO(1, 1) \times [SO(2)]^{[(p-1)/2]}$ and an operator $T$ defined as

$$T = \sqrt{1 - X^2}, \quad \Lambda = (1 + X)T^{-1}. \quad (46)$$

Note that $[T, \Lambda] = 0$ and $T$ does not break $SO(1, 1) \times [SO(2)]^{[(p-1)/2]}$. These properties can be immediately understood by writing $\Lambda$ and $T$ in our preferred vielbein satisfying (44):

$$T_\alpha^\beta = \left( \begin{array}{cccccccc} \frac{1}{\cosh \phi_0} & 0 & 0 & 0 & \cdots \\ 0 & \frac{1}{\cosh \phi_0} & 0 & 0 & \cdots \\ 0 & 0 & \frac{1}{\cosh \phi_1} & 0 & \cdots \\ 0 & 0 & 0 & \frac{1}{\cosh \phi_1} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{array} \right) \quad (47)$$

and

$$\Lambda_\alpha^\beta = \left( \begin{array}{cccccccc} \cosh \phi_0 & \sinh \phi_0 & 0 & 0 & \cdots \\ \sinh \phi_0 & \cosh \phi_0 & 0 & 0 & \cdots \\ 0 & 0 & \cos \phi_1 & \sin \phi_1 & \cdots \\ 0 & 0 & -\sin \phi_1 & \cos \phi_1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{array} \right). \quad (48)$$

Here and in the following we often do not write explicitly the pull-back symbol $P[.]$ and our notation does not distinguish between the world-volume vielbein and the $e_\hat{m}$ belonging to the target space vielbein. The resolution of these ambiguities should be clear from the context.
Using the matrix $T$ we can define a new non-isotropically deformed vielbein

$$\hat{e}^\alpha = e^\beta T_{\beta}{}^\alpha,$$  

and consequently a deformed metric

$$\hat{g}_{\alpha\beta} = \eta_{\alpha\beta} \hat{e}^\alpha \hat{e}^\beta.$$  

(50)

It is immediate to see that the definition of the deformed metric (50) is completely identical to the “noncommutative” one defined in (43), i.e.

$$\hat{g}_{\alpha\beta} = g_{\alpha\beta} - F_{\alpha\gamma} g^{\gamma\delta} F_{\delta\beta}.$$  

(51)

Note that this world-volume metric coincides, up to an overall constant factor, also with the on-shell metric found in [25] from a Polyakov-like action for D-branes.

We also expect a redefinition of the effective world-volume coupling constant analogous to the noncommutative one presented in (43). Since in this case we allow a non-constant $F_{\alpha\beta}$, we expect here a rescaling of the coupling to the background dilaton. Indeed, this effect can be easily seen writing the natural volume element entering the action in the following form

$$\sqrt{-\det(g + F)} = \sqrt{-\det g \det(1 + X)} = \frac{\sqrt{-\det g}}{\sqrt{\det(1 + X)}}.$$  

(52)

The most straightforward interpretation of this result, guided by the noncommutative case [43], is that we have a world-volume theory defined in a deformed string frame with a standard volume element $\sqrt{-\det g}$, and a coupling to a world-volume rescaled dilaton $\hat{\Phi}$ defined by

$$e^{\hat{\Phi}} = e^\Phi \sqrt{\det(1 + X)}.$$  

(53)

Let us finally turn to the original motivation of this deformation, that is the need to obtain a canonical kinetic term. This effect will become evident in the following section where we consider the fermionic action, but already in the bosonic action it is possible to see such an effect by noting that the $(M^{-1})^{\alpha\beta}$ entering the general expansion (42) takes the form

$$(M^{-1})^{\alpha\beta} = \hat{g}^{\alpha\beta} - \hat{F}^{\alpha\beta},$$  

(54)

where $\hat{g}^{\alpha\beta}$ is the inverse of $\hat{g}_{\alpha\beta}$ and $\hat{F}^{\alpha\beta} = \hat{e}^\alpha \hat{e}^\beta X^{\alpha\beta}$. Then we see how in the deformed theory the inverse metric $\hat{g}^{\alpha\beta}$ separates completely from the contribution given by $\hat{F}^{\alpha\beta}$ which can be directly identified with the “deformed” version of the background world-volume field-strength. Then, when formulated in terms of the new deformed geometry, the kinetic terms come in a canonical form. Obviously, as the parent noncommutative
formulation suggests, we cannot expect that the effect of a non-zero background $\mathcal{F}_{\alpha\beta}$ can be completely reabsorbed in a deformation of the metric. Indeed, the $\hat{\mathcal{F}}_{\alpha\beta}$ appearing in (51) adds other couplings in the expansion (42), involving also derivatives of the bosonic world-volume fields, but since $\hat{\mathcal{F}}_{\alpha\beta}$ is antisymmetric, these are different in nature from kinetic terms and can be interpreted as generalized electromagnetic couplings. In the following section we will see how the deformation of the world-volume geometry introduced here will allow us to isolate a kinetic term, writing the fermionic action as a standard Dirac action plus mass terms coming from the embedding in a curved background with fluxes and from the world-volume background field strength $\mathcal{F}_{\alpha\beta}$.

6 A canonical fermionic action

In this section we reconsider the fermionic action (17) discussed in section 2, and rewrite it in terms of the deformed vielbein (49) defined in the previous section. Let us start by first noticing that, following [24], it is possible to write our chiral operators (4) and (10) that enter the fermionic action in the form

$$\Gamma_{Dp} = e^{R\hat{\Gamma}_{(10)}(\Gamma(0))} e^{-R\hat{\Gamma}_{(10)}}.$$  

We have defined the operators

$$\Gamma_{Dp}^{(0)} = \begin{cases} \Gamma_{Dp}^{(0)}(\Gamma(10))^{\frac{p+2}{2}} & \text{for type IIA}, \\ \Gamma_{Dp}^{(0)}(i\sigma_2)(\sigma_3)^{\frac{p-1}{2}} & \text{for type IIB}, \end{cases}$$

and

$$R = \frac{1}{4} Y_{\alpha\beta} \Gamma_{\alpha\beta}$$

is a Lorentz generator expressed easily in our preferred vielbein in the following way:

$$Y_{(2)} = \phi_0 e^{\partial_0} \wedge e^{\frac{1}{4}} + \sum_{r=1}^{[(p-1)/2]} \phi_r e^{2r} \wedge e^{2r+1}.$$  

Let us observe that $R\hat{\Gamma}_{(10)}$ generates on each irreducible component of a type IIA or IIB spinor a Lorentz transformation belonging to the unbroken $SO(1,1) \times [SO(2)]^{[(p-1)/2]}$, but also that it rotates the two irreducible components of a type IIA/IIB spinor in opposite directions. Then, we can now define, for both type IIA and IIB, a new “rotated” fermionic field (recalling that the two irreducible components are rotated in opposite directions)\(^3\)

$$\Theta = e^{R\hat{\Gamma}_{(10)}} \theta.$$  

\(^3\)A similar rotation of the fermions accompanied by a vielbein redefinition of the kind given in (49) was discussed in [20, 22].
This sort of generalized chiral rotation is naturally accompanied by the following redefinition of the operators entering the fermionic action

\[
\hat{D}^{(0)}_\alpha = e^{R\tilde{\Gamma}^{(10)}_\alpha} D^{(0)}_\alpha e^{-R\tilde{\Gamma}^{(10)}_\alpha}, \\
\hat{W}_\alpha = e^{-R\tilde{\Gamma}^{(10)}_\alpha} W_\alpha e^{-R\tilde{\Gamma}^{(10)}_\alpha}, \\
\hat{\Delta}^{(1)} = e^{-R\tilde{\Gamma}^{(10)}_\alpha} \Delta^{(1)} e^{-R\tilde{\Gamma}^{(10)}_\alpha}, \\
\hat{\Delta}^{(2)} = e^{R\tilde{\Gamma}^{(10)}_\alpha} \Delta^{(2)} e^{-R\tilde{\Gamma}^{(10)}_\alpha},
\]

(60)

where the operators involved are defined in appendix A.

In the above redefinition, it can be useful to write the operator \( W_m \) entering the action (see appendix A for its explicit definition), in terms of a new operator \( \hat{W} \) defined by the relation

\[
W_m = \hat{W} \Gamma_m
\]

and then

\[
\hat{W} = e^{-R\tilde{\Gamma}^{(10)}_\alpha} W e^{R\tilde{\Gamma}^{(10)}_\alpha}.
\]

(61)

Then it is possible to write the fermionic action (17) in a canonical form. Indeed, using the fact that

\[
e^{-R\Gamma_\alpha} e^R = \Lambda_\alpha^\beta \Gamma_\beta
\]

(62)

and that for example

\[
\Lambda_\alpha^\beta \varepsilon^m_\beta = (1 + X)_\alpha^\beta \varepsilon^m_\beta,
\]

(63)

it is possible to show that the action (17) can be written in the form

\[
S^{(F)}_{Dp} = \frac{\tau_{Dp}}{2} \int d^{p+1} \xi e^{-\tilde{\Phi}} \sqrt{-\det \hat{g}} \left\{ \tilde{\Theta}(1 - \Gamma^{(0)}_{Dp})(\hat{\Gamma}^\alpha \hat{D}^{(0)}_\alpha - \hat{\Delta}^{(1)})\Theta + \Theta(1 - \Gamma^{(0)}_{Dp}) e^{-2R\tilde{\Gamma}^{(10)}_\alpha}(\tilde{g}^{\alpha\gamma}[(1 + \hat{\Gamma}^{(10)}_\alpha)\tilde{X}^{-1}]_\gamma^\beta \hat{\Gamma}_\beta \hat{W} \hat{\Gamma}_\alpha - \hat{\Delta}^{(2)})\Theta \right\},
\]

(64)

where

\[
\hat{\Gamma}_\alpha = e^{\beta} \hat{\Gamma}_\beta, \quad \hat{\Gamma}^\alpha = \hat{g}^{\alpha\beta} \hat{\Gamma}_\beta.
\]

(65)

Using the equations (62) and (63), it is possible to write the explicit form of the new operators defined in (60) and (61). To do this, let us first of all extend the operator \( X_{\underline{\alpha} \beta} \) to an operator acting on all the indices, by simply putting all the remaining components equal to zero. This redefinition can then be extended in an obvious way to all the other operators constructed from \( X_{\underline{\alpha} \beta} \). Then, for example, we can define the complete deformed vielbein \( \hat{e}^m_\alpha \), by simply generalizing the definition (49) into the definition \( \hat{e}^m_\alpha T^m_{\alpha m} \). Of course, these kind of extended redefinitions do only make sense when restricted to the world-volume of the brane. The operators \( \Delta^{(1)} \) and \( \Delta^{(2)} \) are defined in terms of operators \( T \) of the form (see appendix A for the explicit expression)

\[
T \sim T_{m_1 m_2} \cdots \Gamma^{m_1 m_2} \cdots.
\]

(66)
Then it is possible to see that the corresponding “hatted” operators have the form

\[ \hat{T} \sim T_{k_1 k_2 \ldots} \hat{\Gamma}_{m_1 m_2 \ldots}(1 + \hat{\Gamma} (10) X)_{m_1}^{k_1} (1 + \hat{\Gamma} (10) X)_{m_2}^{k_2} \ldots. \]  

(67)

One can do a completely analogous computation for \( \hat{W} \), where however in this case \( \hat{\Gamma} (10) \) in (67) is replaced by \( -\hat{\Gamma} (10) \). Note further that

\[ e^{-2R\hat{\Gamma} (10)} = \frac{1}{\sqrt{1 + X}} \sum_q (-)^q (\hat{\Gamma} (10))^q \cdot X_{\hat{\alpha}_1 \alpha_2} \ldots X_{\hat{\alpha}_{2q-1} \alpha_{2q}} \hat{\alpha}_{2q+1}^{\alpha_{2q}} \ldots \hat{\alpha}_1^{\alpha_1} \ldots \hat{\alpha}_q^{\alpha_1} \hat{\alpha}_q^{\alpha_2} \ldots \hat{\alpha}_1^{\alpha_2}. \]  

(68)

and that the operator \( \Gamma^{(0)}_{D\hat{p}} \) entering the definition of \( \Gamma^{(0)’}_{D\hat{p}} \) in (66) can be written in terms of the deformed quantities by simply adding “hats” everywhere in the definition (8).

It remains to rewrite \( \hat{\Gamma}^\alpha \hat{D}^{(0)}_\alpha \) in terms of the deformed variables. The contribution by the \( B \)-field deforms analogous to (67). The pull-back of the target space covariant derivative can be rewritten as:

\[
\hat{\Gamma}^\alpha \hat{\nabla}_\alpha = \hat{\Gamma}^\alpha \hat{\nabla}_\alpha + \frac{1}{4} \hat{\Gamma}^\alpha \hat{B}_{\alpha \beta} \hat{\Gamma}^\alpha \hat{\Gamma}^\beta - \frac{1}{2} X^\beta K^\alpha_{\beta} \hat{\Gamma} (10) \hat{\alpha}_{\beta} + \frac{1}{4} X^\alpha K^\alpha_{\beta} \hat{\Gamma} (10) \hat{\alpha}_{\beta} + \frac{1}{2} \hat{\Gamma}^\alpha \hat{A}_{\alpha \beta} \hat{\Gamma}^\beta + \frac{1}{4} \hat{\Gamma}^\alpha \hat{A}_{\alpha \beta} \hat{\Gamma}^\beta + \frac{1}{4} \hat{\Gamma}^\alpha A_{\alpha \beta} \hat{\Gamma}^\beta + \frac{1}{2} \hat{\Gamma}^\alpha \hat{A}_{\alpha \beta} \hat{\Gamma}^\beta + \frac{1}{4} \hat{\Gamma}^\alpha A_{\alpha \beta} \hat{\Gamma}^\beta + \frac{1}{4} \hat{\Gamma}^\alpha A_{\alpha \beta} \hat{\Gamma}^\beta. \]

(69)

In this expression we have made use of the splitting of the pull-backed target space connection into a world-volume connection plus a part related to the extrinsic curvature and the normal bundle connection, introduced in (38) and (39). The normal bundle connection and extrinsic curvature are given by the embedding in the target space, and as such they do not depend on the world-volume geometry. As we want to find the kinetic term in a canonical form, we needed to introduce in (69) a covariant derivative \( D \) with respect to the deformed frame, since this is the frame in which the world-volume geometry is naturally described. This new derivative is defined using the connection \( \hat{\omega} \) of the deformed vielbein, which is related to the original world-volume connection \( (\omega^{\alpha \beta} = \Omega^{\alpha \beta}) \) by:

\[ \omega^{\alpha \beta} = \hat{\omega}^{\alpha \beta} + \hat{B}_{\alpha \beta}, \]

(70)

with,

\[ \hat{B}_{\alpha \beta} = (1 + \hat{\Gamma} (10) X)^{\hat{\omega} \alpha \beta} [ (1 + \hat{\Gamma} (10) X)^{-1} \hat{D}_{\alpha} X (1 + \hat{\Gamma} (10) X)^{-1} - (1 - X^2) \hat{D}_{\alpha} X ] \frac{\hat{\alpha} \hat{\beta}}{2} \hat{\Gamma} (10) - [ \hat{D}_{\alpha} X (1 + \hat{\Gamma} (10) X)^{-1} ]^{\hat{\alpha} \hat{\beta}} \hat{\Gamma} (10) \].

(71)

where we have raised and lowered the flat indices by using \( \eta_{\alpha \beta} \) and its inverse as usual.

This discussion makes explicit that the operator \( \hat{\Gamma}^\alpha \hat{D}^{(0)}_\alpha \) contains the covariant derivative

\footnote{Since \( \hat{X}_{\alpha \beta} = \hat{e}_\alpha \hat{e}_\beta \hat{X}_{\alpha \beta} = e_\alpha e_\beta X_{\alpha \beta} = X_{\alpha \beta} \), the objects in (64) and (67) are unambiguously defined.}
with respect to the deformed metric together with some covariant couplings of the world-volume fields to the background. If one takes the world-volume geometry as given by the deformed metric (50), it can be seen that (64) consists of a canonical Dirac operator together with some additional interactions, given by the embedding and the fluxes.

Let us next consider the effect of gauge fixing kappa-symmetry. We impose the gauge fixing condition
\[ \Theta = \tilde{\Gamma}_{(10)} \Theta, \] (72)
which simply means that the second component of \( \Theta \) is equal to zero. It can then easily be seen that the action (64) written in terms of the first component of \( \Theta \) (that we indicate again with \( \Theta \)) reduces to:

\[ S^{\prime(F)}_{d^{p + 1}} = \frac{T_{d^{p + 1}}}{2} \int d^{p + 1} \xi e^{-\Phi} \sqrt{-\det \tilde{g}} \left\{ \Theta (\tilde{\Gamma}^{\alpha} \tilde{D}_{\alpha}^{(0)} - \tilde{\Delta}^{(1)}) \Theta - \Theta \tilde{\Gamma}_{d^{p + 1}}^{-1}(\tilde{g}^{\alpha \gamma}[(1 + X)^{-1}]_{\gamma}^{\beta} \tilde{W} \tilde{\Gamma}_{\alpha} - \tilde{\Delta}^{(2)}) \Theta \right\}. \] (73)

The explicit couplings defined by this action are obtained in appendix B.

We conclude this section by writing the linearized supersymmetry transformation rules (37) in the new deformed variables:

\[ \delta_{\varepsilon} \Theta = \tilde{\Gamma}^{\alpha} \nabla_{\alpha} N \tilde{\phi}_{\tilde{m}} \tilde{m} \chi - X_{\alpha}^{\beta} K_{\beta \gamma} \tilde{m} \tilde{\phi}_{\tilde{m}} \tilde{\Gamma}^{\alpha \gamma} \chi - \frac{1}{2}(\tilde{\phi}_{\tilde{m}} \tilde{H}_{\tilde{m} \alpha \beta} + f_{\alpha \beta}) \tilde{\Gamma}^{\alpha \beta} \chi, \]
\[ \delta_{\chi} \phi_{\tilde{m}} = \tilde{\chi} \tilde{\Gamma}^{\tilde{m}} \Theta, \]
\[ \delta_{\chi} A_{\alpha} = \tilde{\chi} \tilde{\Gamma}_{\alpha} \Theta + B_{\alpha m} e_{m}^{\tilde{m}} \tilde{\chi} \tilde{\Gamma}^{\tilde{m}} \Theta, \] (74)

where \( \chi = e^{R} \varepsilon_{1} \) and the scalar fields \( \tilde{\phi}_{\tilde{m}} \) describe the brane fluctuations in the normal directions and \( f_{\alpha \beta} \) is the dynamical world-volume field-strength. Note that the world-volume part of these transformations takes the usual form valid when the world-volume field-strength \( F_{\alpha \beta} \) is vanishing while the terms in (74) involving explicitly \( B_{(2)} \) and its field-strength \( H_{(3)} \) are non-zero only if some of the off-diagonal components \( B_{\alpha m} e_{m}^{\tilde{m}} \) of \( B_{(2)} \) are nonvanishing.

### 7 Conclusions

The main results of this paper are the transparent formulae for the quadratic fermionic part of the Dp-brane actions on any supergravity background including possible fluxes. In particular, (17) gives this parametrization and \( \kappa \)-symmetric action, and (73) gives a convenient \( \kappa \)-fixed form.

We started by re-expressing the results of \( [3, 4] \) by re-organizing terms in a more compact notation. We clarified the underlying geometric structure, using the tensor \( \bar{M}_{\alpha \beta} \), see (15), in both type IIA and IIB. The formulation makes the invariance under
T-duality easy to be verified. Using a similar doublet notation for IIA and IIB, the $\kappa$-gauge fixing can be discussed uniformly. The preserved supersymmetry transformations after gauge-fixing the $\kappa$-symmetry and worldvolume reparametrizations are obtained in linearized form. In order to clarify the world-volume geometry we have identified a new natural world-volume vielbein such that the measure of integration is its determinant and all kinetic terms for the fermions are recollected in the standard Dirac operator. Also the supersymmetry transformations in this new metric are obtained, and the explicit form of the terms entering the $\kappa$-fixed action is discussed.

These new results can be useful for any kind of quantum calculation on the brane, in particular for the understanding of non-perturbative effects in string theory. The results can also be useful for constructing effective actions for string configurations where D-branes are involved.

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A Conventions

In this paper we use Latin indices $m, n, \ldots = 0, \ldots, 9$ for 10-dimensional curved coordinates, whereas for D$p$-brane world-volume coordinates we use Greek indices $\alpha, \beta, \ldots = 0, \ldots, p$. The corresponding flat indices are underlined, e.g. the vielbein is given by $e^m = e^m_n dx^n$. The ten dimensional (flat) gamma matrices are $\Gamma^m$ and the 10-dimensional chiral operator is $\Gamma_{(10)} = \Gamma^01 \ldots 9$. Pulled back gamma matrices are then $\Gamma_\alpha = \Gamma^m e^m_\alpha \partial_\alpha x^m$.

The Levi-Civita symbol $\epsilon_{\alpha_1 \ldots \alpha_{p+1}}$ is a density, i.e. it takes values $\pm 1$.

Differently from [3, 4], in this paper we use the standard convention for the expansion of the forms in components, i.e. a $p$-form $\chi^{(p)}$ is expanded as

$$\chi^{(p)} = \frac{1}{p!} \chi_{m_1 \ldots m_p} dx^{m_1} \wedge \ldots \wedge dx^{m_p}, \quad (75)$$

whereas the authors of [3, 4] used the superspace convention where the $dx^m$ in the above expression are multiplied in the opposite order. Furthermore, since we consider branes with a positive CS term, the RR-gauge fields $C^{(n)}$ are related to those used in [3, 4] by the substitution

$$C_{m_1 \ldots m_n} \rightarrow (-)^{\frac{n(n+1)}{2}} C_{m_1 \ldots m_n}, \quad (76)$$

in such a way that the associated differential forms in the two conventions are the same. Another difference with these papers is that we have changed the definition of the charge conjugation matrix such that we avoid a factor i for any barred spinor. I.e. we take $\bar{\theta} = i \theta^T \Gamma^0$.

To write the supergravity actions, let us start by introducing the generalized RR field-strengths$^5$

$$F_{(1)} = dC_{(0)} \quad , \quad F_{(2)} = dC_{(1)} \quad , \quad F_{(3)} = dC_{(2)} + C_{(0)} H_{(3)} \quad , \quad F_{(4)} = dC_{(3)} + H_{(3)} \wedge C_{(1)} \quad , \quad F_{(5)} = dC_{(4)} + H_{(3)} \wedge C_{(2)}, \quad (77)$$

where $H_{(3)} = dB_{(2)}$.

The type IIA supergravity action is the following

$$S_{IIA} = \frac{1}{2 \kappa^2_{10}} \int d^{10} x \sqrt{-G} \left\{ e^{-2 \Phi} [R + 4(\partial \Phi)^2 - \frac{1}{2 \cdot 3!} (H_{(3)})^2] + \frac{1}{2\cdot 2!} (F_2)^2 - \frac{1}{2\cdot 4!} (F_4)^2 \right\} - \frac{1}{4 \kappa^2_{10}} \int B_{(2)} \wedge dC_{(3)} \wedge dC_{(3)}, \quad (78)$$

whereas the type IIB supergravity action is

$$S_{IIB} = \frac{1}{2 \kappa^2_{10}} \int d^{10} x \sqrt{-G} \left\{ e^{-2 \Phi} [R + 4(\partial \Phi)^2 - \frac{1}{2 \cdot 3!} (H_{(3)})^2] + \frac{1}{2\cdot 2!} (F_2)^2 - \frac{1}{2\cdot 4!} (F_4)^2 \right\} - \frac{1}{4 \kappa^2_{10}} \int B_{(2)} \wedge dC_{(3)} \wedge dC_{(3)}, \quad (78)$$

$^5$We are using essentially the same conventions as in [27].
\[-\frac{1}{2 \cdot 3!} (F_3)^2 - \frac{1}{4 \cdot 5!} (F_4)^2 \} +
+ \frac{1}{4 \kappa^2_{10}} \int dC_{(2)} \wedge H_{(3)} \wedge (C_{(4)} + \frac{1}{2} B_{(2)} \wedge C_{(2)}) \].

(79)

In both actions $2 \kappa^2_{10} = (2\pi)^7 \alpha'^4 g_s^2$ and in type IIB one has to add the selfduality condition $F_{(5)} = \ast F_{(5)}$ by hand at the level of the equations of motion.

The supersymmetry transformations for both IIA and type IIB can be written in the form

$$\delta \psi_m = D_m \varepsilon \ , \ \delta \lambda = \Delta \varepsilon \ , \quad (80)$$

where $\varepsilon$ is a Majorana spinor for type IIA and a doublet of Majorana-Weyl spinors of positive chirality for type IIB. It is useful to split the operators $D_m$ and $\Delta$ in two

$$D_m = D^{(0)}_m + W_m \ , \ \Delta = \Delta^{(1)} + \Delta^{(2)} \ . \quad (81)$$

In type IIA we have

$$D^{(0)} = \nabla_m + \frac{1}{4 \cdot 2!} H_{mnp} \Gamma^{np} \Gamma^{(10)} \ ,$$

$$W_m = -\frac{1}{8} e^\Phi \left( \frac{1}{2} F_{np} \Gamma^{np} \Gamma^{(10)} + \frac{1}{4!} F_{npqr} \Gamma^{npqr} \right) \Gamma_m \ ,$$

$$\Delta^{(1)} = \frac{1}{2} \left( \Gamma^m \partial_m \Phi + \frac{1}{2 \cdot 3!} H_{mnp} \Gamma^{mnp} \Gamma^{(10)} \right) \ ,$$

$$\Delta^{(2)} = \frac{1}{8} e^\Phi \left( \frac{3}{2!} F_{mn} \Gamma^{mn} \Gamma^{(10)} - \frac{1}{4!} F_{mnpq} \Gamma^{mnpq} \right) \ , \quad (82)$$

while in type IIB

$$D^{(0)} = \nabla_m + \frac{1}{4 \cdot 2!} H_{mnp} \Gamma^{np} \sigma_3 \ ,$$

$$W_m = \frac{1}{8} e^\Phi \left[ F_n \Gamma^m (i \sigma_2) + \frac{1}{3!} F_{npq} \Gamma^{npq} \sigma_1 + \frac{1}{2 \cdot 3!} F_{npqrt} \Gamma^{npqrt} (i \sigma_2) \right] \Gamma_m \ ,$$

$$\Delta^{(1)} = \frac{1}{2} \left( \Gamma^m \partial_m \Phi + \frac{1}{2 \cdot 3!} H_{mnp} \Gamma^{mnp} \sigma_3 \right) \ ,$$

$$\Delta^{(2)} = -\frac{1}{2} e^\Phi \left[ F_m \Gamma^m (i \sigma_2) + \frac{1}{2 \cdot 3!} F_{mnp} \Gamma^{mnp} \sigma_1 \right] \ , \quad (83)$$

where $\nabla_m = \partial_m + \frac{1}{4} \Omega_{nm} \Gamma_m \Gamma_{np} \sigma_3$ is the covariant derivative.

Finally we end this appendix with some comments on the double spinor notation used from section 4 onwards. From the start, the spinors in type IIB are doublets. This means that $\theta$ stands for the 64-component spinor

$$\begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} \ , \quad (84)$$
whose 32-component parts are both left-handed, i.e. \( \theta_i = \Gamma_{(10)} \theta_i \) with \( i = 1, 2 \). The \( \Gamma \) matrices do not mix with the extension index, i.e. \( \Gamma_m \theta \) stands for

\[
\begin{pmatrix}
\Gamma_m \theta_1 \\
\Gamma_m \theta_2
\end{pmatrix},
\]

(85)
of which both components are now right-handed. In other words, the Clifford matrices in the large space act as \( \Gamma_m \otimes 1_2 \). The conjugate spinor \( \bar{\theta} \) is represented by

\[
( \bar{\theta}_1 \; \bar{\theta}_2 ),
\]

(86)
and is from the right projected onto itself by \( \frac{1}{2} (1 - \Gamma_{(10)}) \otimes 1_2 \).

For type IIA, in sections 2 and 3 and in the heavily-used references [3, 4] as in many other papers, the two spinors are combined in a 32-component Majorana spinor \( \theta = \theta_1 + \theta_2 \), where \( \theta_1 = \Gamma_{(10)} \theta_1 \) (left-handed) and \( \theta_2 = -\Gamma_{(10)} \theta_2 \) (right-handed). We now define here also the doublet spinor (84), where now both 32-component parts have opposite chiralities. To obtain formulae that are similar to the IIB formulae, we use in the 64-component representation different Clifford representations. The Clifford matrices in the large space are represented by

\[
(0 \quad \Gamma_m) = \Gamma_m \otimes \sigma_1.
\]

(87)
The charge conjugation matrix in the large space is taken to be \( C \otimes \sigma_1 \), where \( C \) is the 32 \( \times \) 32 charge conjugation matrix. This implies that the conjugate spinor is

\[
( \bar{\theta}_2 \; \bar{\theta}_1 ).
\]

(88)
These two choices imply e.g. that the expression \( \delta_\kappa \bar{\theta} \Gamma_m \theta \) maintains its form when we go from the 32-component notation to the 64-component notation. The matrix \( \Gamma_{(10)} \) is still represented by \( \Gamma_{(10)} \otimes 1_2 \), but on the doublet (84) it acts as \( 1_{32} \otimes \sigma_3 \). Therefore \( \Gamma_{(10)} \), see (10), is represented on \( \theta \) in both IIA and IIB as \( 1_{32} \otimes \sigma_3 \). In any case it anticommutes with the representations of the \( \Gamma \)-matrices.

**B  Explicit form of \( \kappa \)-fixed action**

As we observed in section 4, the \( \kappa \)-fixed action (30) contains terms that simplify thanks to the symmetry properties of the gamma matrices, and for example the term containing the gradient of the dilaton disappears. Such simplifications become, however, more visible in the formulation with the deformed variables given in (73) of section 6, and so we discuss explicitly this case. The results obtained can be directly applied to the action (30) when \( F_{(2)} = 0 \), since in this limit the two actions coincide.
One can extract the explicit couplings included in (73) in the following way. The first two terms in the $\kappa$-fixed action can be simplified by making a standard analysis based on the symmetry properties of $\Gamma$-matrices. For example the coupling to the trace of the extrinsic curvature and that to the gradient of the dilaton in $\Delta^{(1)}$ drop. The argument for the couplings contained in $\Delta^{(2)}$ is a bit more subtle; one will encounter terms of the form

$$\hat{\Theta} \Gamma^{(0)}_{Dp} e^{-2R} F_{m_1 \cdots m_n} \hat{\Gamma}^{k_1 \cdots k_n} (1 + X)_{k_1}^{m_1} \cdots (1 + X)_{k_n}^{m_n} \Theta .$$

Using symmetry properties of the $\Gamma$-matrices, it can be shown that those are equal to

$$(-1)^{\frac{p-n}{2} + a + 1} \hat{\Theta} \Gamma^{(0)}_{Dp} e^{-2R} F_{m_1 \cdots m_n} \hat{\Gamma}^{k_1 \cdots k_n} (1 - X)_{k_1}^{m_1} \cdots (1 - X)_{k_n}^{m_n} \Theta .$$

The sign in front depends on $p$, on $n$ (which is the number of $\hat{\Gamma}$-matrices that are contracted with the fluxes) and on the number $a$ of world-volume indices of the fluxes. One notices that (90) can be obtained from (89) by replacing $X$ with $-X$, up to a possible overall sign. Suppose first that the values of $p$, $n$ and $a$ are such that the sign in front of (90) is equal to $+1$. After expanding $e^{\pm 2R}$ along the lines of (68), one can compare (89) and (90) at equal order in this expansion of $e^{\pm 2R}$. If one looks at an odd order in this expansion, it is easy to see that only the terms with an odd power of $X$ that are contracted with the $\hat{\Gamma}^{k_1 \cdots k_n}$ are non-vanishing, while for an even order in the expansion of $e^{\pm 2R}$, the terms with an even power of $X$ that are contracted with $\hat{\Gamma}^{k_1 \cdots k_n}$ survive. So, in the end, one sees that only terms with an even power of $X$ (now also counting the ones in $e^{\pm 2R}$) survive. If the sign in (90) is equal to $-1$ a similar reasoning shows that only the terms with an odd power of $X$ will survive. For the couplings contained in $W$, one first splits the matrix $\hat{g}^{\alpha \gamma} [(1 + X)^{-1}]_{\gamma \beta}$ into its symmetric and its antisymmetric part which, once expanded, contain an even and odd power of $X$’s respectively. One can then do a reasoning similar as before on both parts. In this, one has to take into account that for the antisymmetric part there is an extra minus sign in front of the analogue of (90).

Indeed, the analogues of (89) and (90) are now:

$$\hat{\Theta} \Gamma^{(0)}_{Dp} e^{-2R} F_{m_1 \cdots m_n} \hat{\Gamma}^{k_1 \cdots k_n} \hat{\Gamma}_\alpha (1 + X)_{k_1}^{m_1} \cdots (1 + X)_{k_n}^{m_n} \Theta ,$$

$$(-1)^{\frac{p-n}{2} + a + 1} \hat{\Theta} \Gamma^{(0)}_{Dp} e^{-2R} F_{m_1 \cdots m_n} \hat{\Gamma}_\alpha \hat{\Gamma}^{k_1 \cdots k_n} \hat{\Gamma}_\beta (1 - X)_{k_1}^{m_1} \cdots (1 - X)_{k_n}^{m_n} \Theta ,$$

where the $\alpha$- and $\beta$-indices are contracted with $\hat{g}^{\alpha \gamma} [(1 + X)^{-1}]_{\gamma \beta}$.

In order to compare these two expressions along the lines just described, one needs to exchange the order of $\hat{\Gamma}_\alpha$ and $\hat{\Gamma}_\beta$. For the piece with the antisymmetric part of $\hat{g}^{\alpha \gamma} [(1 + X)^{-1}]_{\gamma \beta}$ this will give an extra minus sign. In summary, terms with an overall even (odd) power of $X$ survive only if $\frac{p-n}{2} + a + 1$ is even (odd).
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