The joint strength formula and its application in disaster evaluation of loess slope

Wang Zhijun¹, Luo Jianwen¹, Pan Junyi¹, Li Rongjian²*
¹ Company limited of Chang-qing Science and Technology, WeiYang Road, Xi’an, 710021, China;  
² Institute of Geotechnical Engineering of Xi'an University of Technology, South JinHua Road, Xi’an, 710048, China.  
E-mail: lirongjian@xaut.edu.cn

Abstract. There are plenty of steep loess slopes in loess plateau and the stress distribution field of a structural loess slope is usually composed of tension-shear stress areas and compression-shear stress areas. Based on the joint strength of structural loess that can consider comprehensively the features of tensile strength and shear strength, the structural parameter of stress ratio was also introduced into the joint strength formula of structural loess. Then the triggering mechanism of a structural loess landslide was analyzed. The results show that the increase of water content in loess slope not only lead to a reduction in tensile strength, but also the safety factor of the structural loess slope gradually decreased to the occurrence of instability. The joint strength provides a more practical computational approach for assessing the stability of structured loess slopes.

1. Introduction
Plenty of engineering slopes are found in mountainous areas, and natural disasters such as landslides are often accompanied by excavation, rainfall infiltration, or earthquake. On one hand, the landslide hazard and risk analysis in a regional scale should be evaluated from a macro perspective [1], and the probabilistic method is assumed to assess the regional susceptibility of landslides [2]. On the other hand, the triggering mechanism, kinematic process, and susceptibility of landslides should be analyzed in slope engineering, such as the Hekou sugar-refinery landslide [3] and the Qianjiangping landslide [4], which were induced by rainfall infiltration.

Soil structure refers to the distributive and combination features of soil particles. The structural property of soil is the mechanical effect produced by the soil structure. Many scholars at home and abroad carried out advanced researches on the structural property of the soil by using micro mechanics, solid mechanics and soil mechanics. Currently, representative researches on the structural property of soil and its mechanical effects include the disturbed state concept [5], geotechnical damage mechanics [6], four-dimensional space theory [7], and comprehensive structural potential theory [8].

The above mentioned studies positively promoted the theoretical research on the structural parameter of structural soil. However, these documents are not involved with the tensile property and its assessment in either tests or theoretical studies. Therefore, the strength of the structure soil can be exaggerated in the analysis method used. Thus, some results may become distorted.

Steep loess slopes are found everywhere in a loess plateau. The stress distribution field of a structural loess slope is usually composed of tension-shear areas and compression-shear stress areas.
The slope soil in the tensile and shear areas is often characterized by tensile-shear complex stresses, and its typical failure is tensile-shear strength failure. The slope soil in compressed and shear areas is often characterized by the compressive-shear complex stresses, and its typical failure is compressive-shear strength failure. If Mohr-coulomb strength is adopted directly in the calculation of slope stability, the tensile strength of soil is definitely overestimated, which increases the calculated value. If the tensile strength is underestimated, the calculated value must be too conservative, which cannot explain the upright or steep characteristic of the structural loess slope. Thus, the tensile strength feature of the structural loess should be studied and evaluated reasonably.

In general, the role of the tensile strength of structural loess cannot be ignored. Moreover, the tensile strength and shear strength of structural loess are equally important, Li et. al [9] proposed a preliminary framework of joint strength theory for structural loess.

This study aims to consider the comprehensive tensile and shear properties of structural loess and to analyze the instability mechanism of a structural loess landslide based on the joint strength.

2. Joint strength formula of structural loess

Based on the tensile and triaxial shear test results, by smoothing the Mohr-coulomb strength line within the tensile shear stress area, a strength plot can be obtained via curve fitting in the tensile shear and compressive-shear stress areas. This plot can reflect the failure caused by tensile-shear and compressive-shear complex stresses. Based on the tensile strength ($\sigma_t$), the joint strength formula (Equation (1)) can be derived [9].

$$\tau^2 = (c + \sigma \tan \phi)^2 - (c - \sigma \tan \phi)^2$$  \hspace{1cm} (1)

In addition, a stress ratio structure parameter $m_\eta$ was proposed which can integrate the spherical and shear stress [10], and the corresponding formula is as follows:

$$m_\eta = \left(\frac{q}{p}\right)^2 \left[\left(\frac{q}{p}\right) - \left(\frac{q}{p}\right)_s\right]$$  \hspace{1cm} (2)

where $m_\eta$ is the stress ratio structure parameter, $q$ is the generalized shear stress, $p$ is the spherical stress, subscripts $i, r, s$ stand for intact, remodeled, and saturated soil, respectively. Equation (2) was used to describe the variation in the structural parameter according to the stress-strain curve in the triaxial compression tests.

$$m_\eta = Ae^{\epsilon} + 1$$  \hspace{1cm} (3)

where $A, B$ are the parameters related to water content and consolidation pressure in the triaxial compression tests.

This study assumes that the cohesion and internal friction angle in the strength parameter are functions of the stress ratio structural parameter, so that the stress ratio structural parameter can be introduced into the joint strength formula of structured loess. Therefore, this study modified Equation (1) as

$$\tau^2 = (c(m_\eta) + \sigma \tan \phi(m_\eta))^2 - (c(m_\eta) - \sigma \tan \phi(m_\eta))^2$$  \hspace{1cm} (4)

The joint strength theory of the structured loess describes the cohesion and internal friction angle as functions of the structural parameter. Therefore, the Equation (4) provides a theoretical basis for assessing the stability of structural loess slope.

In the strength reduction finite element calculation of the structured loess slope stability, after Equation (3) and Equation(4) are introduced into numeric computation, the corresponding correction of failure stress is performed.

In the computation of strength reduction finite element method, at first, initial reduction factor is chosen, according to which, reduction of strength of every element in slope is in progress. That is to say, soil parameter value $c$, and $\phi$ are divided by a reduction factor $F$, a new group of value $c_F$ and $\phi_F$ gotten, as new parameter. Besides, the corresponding yield criterion of strength is updated, and computation of elasto-plastic finite element method is in progress by the new yield criterion of strength.

To every computational step of reduction strength:

2
\[ c_F = \frac{c}{F} \]  
\[ \tan \phi_F = \frac{\tan \phi}{F} \]  

According to the initial effective stress field, the strength parameter of every element in the soil slope should be reduced based on the initial reduction factor and should be carried out via finite element calculation. If the program calculation obtains convergence, the soil slope is still in a stable state. As such, the reduction factor continues to increase until it does not converge when the slope reaches the critical sliding state and the reduction factor of the previous calculated step becomes the safety factor of the slope. Finally, the location of potential sliding surface is confirmed based on the distribution fields of the corresponding displacement increment near the critical state. Moreover, the changed distribution fields of the structural parameters can also be confirmed by calculating the corresponding strain and by assessing the stability of the structured loess slope.

### 3. Mechanism analyses of structural loess landslide

A landslide of an upright loess slope occurred in the county of Jingbian, and this accident is typical landslide in loess area. Thus, conducting an in-depth analysis on the instability mechanism of such structural loess slope is necessary.

To analyze the instability mechanism of a loess slope, the profile of slope is shown in figure 1. The height of the slope is 10 m, the width is 19.5 m at the top of the slope, the width is 10.5 m at the bottom of the slope, and the thickness of foundation is 10.0 m. The corresponding finite element model (figure 2) is established using four-node tetrahedral elements. The number of elements are 1000, and the number of nodes are 1073. A horizontal restraint is adopted in both left and right boundaries of the model, and a fixed constraint is adopted in the bottom boundary. The relevant parameters of the soil are shown in table 1.

This study carries out analysis and comparisons of slope stability at an initial water content of 15% and 25% after rainfall to study the structural loess slope stability and variations in structural parameters.

| Parameters             | Unit  | Value |
|------------------------|-------|-------|
| Water content (w)      | %     | 15    | 25    |
| Modulus (E)            | MPa   | 90    | 40    |
| Density (\(\rho\))    | g/cm³ | 1.63  | 1.80  |
| Poisson’s ratio (\(\nu\)) | -   | 0.29  | 0.33  |
| Cohesion (c)           | kPa   | 90    | 26    |
| friction angle (\(\phi\)) | °   | 28    | 28    |
| Tensile strength (\(\sigma_t\)) | kPa | 151.3 | 13.3  |
| A                      |       | 4.15  | 2.38  |
| B                      |       | 0.4   | -0.43 |
The joint strength of the structural loess can be introduced into the finite element analyses. Hence, in the calculation, the slope with an initial water content of 15% was named Case 1, where the initial state of the structured loess slope was analyzed. The slopes with water contents of 25% were named as Cases 2, respectively, where effects of water content change caused by rainfall on the instability of the structured loess slope were investigated.

Cases 1 to Case 2 had safety factors of 2.82 and 0.97, respectively. The results indicate that the influence of rainfall on the stability of the structural loess slope. When the water content caused by rainfall is gradually increased, the safety factor of structural loess slope decreases and the slope finally loses stability.

Figure 3 shows the contour of structural parameter in the loess slope with strength reduction in each case. Figure 4 shows the contour of increment displacement of the loess slope, which is calculated using the slope stability in each case condition. The contour of increment displacement shown in the figure 4 is the difference of the two slope displacements. Based on the maximum grades of the contour of increment displacement, the location of the potential surface can be determined.

By analyzing the structural parameter distribution of intact soil slope at different water contents as shown in figure 3, the calculation results show that the larger water content of the slope indicates a smaller structural parameter of the soil. Thus, slope stability decreases. A comparatively wide band region in the contour of the upper slope can be observed, and the variation gradient in this region is comparatively smaller. Based on Equation (3), the strain near the potential sliding surface is larger. Therefore, the corresponding changes of structural parameter near the potential sliding surface vary more obviously.
Potential sliding surface

(a). Increment displacement at w=15%.
(b). Increment displacement at w=25%.

Figure 4. Contour of increment displacement of loess slope.

After analyzing the contour of the increment displacement at different water contents as shown in figure 4, an obvious dense band is observed. Based on the contour gradient of increment displacement, the location of the potential sliding surface can be confirmed, that is, the symmetry center of this dense band is the location of the potential sliding surface.

Rainfall is known to increase the water content of the soil slope. After soaking, the original curing cohesion of intact soil decreases, which also decreases the structural properties of the structural loess. For instance, the tensile strength of the structural loess decreases; thus, the safety factor of the structural loess slope gradually decreases until the slope becomes unstable. Moreover, the location of the slope’s potential sliding surface has an obvious upward trend.

In addition, by analyzing the potential sliding surface location in figure 3 and figure 4, a specific relationship is observed between the band location of the structure parameter contour in the structural loess slope and the location of the potential sliding surface, which can be confirmed by the contour distribution of increment displacement.

4. Conclusions
Based on the joint strength formula of structural loess that can consider the tensile and shear features comprehensively, this analysis method effectively overcame the exaggerated assessment of tensile strengths of soils in Mohr-coulomb strength and provided a more practical computational approach for assessing the stability of structural loess slopes. The conclusions are as follows:

1) Stress ratio structural parameter was introduced to the joint strength of structural loess, which considered the cohesion and internal friction angles as functions of structural parameter. Therefore, the joint strength of structural loess can accurately reflect the tensile and shear strength characteristics of the structural loess.

2) By the joint strength formula of structural loess, the following instability mechanism of landslide was analyzed: owing to rainfall, the water content of the structured loess slope increases. Thus, the structural properties of soil decrease, which means that the tensile strength and the safety factor of structural loess slope decrease gradually. As such, the slope instability may occur.

5. Conflict of interest
The authors declared that they do not have any commercial or associative interest that represents a conflict of interest in connection with this work.

Acknowledgement
This work was supported by the project (CTEC(2014)Z-KY-013) supported by Company limited of Chang-qing Science Technology, and the National Natural Science Foundation of China (11072193).
References

[1] A. M. Das, N. S. Kumar, and M. S. Kanti, "Landslide Hazard and Risk Analysis in India at a Regional Scale", Disaster Advances, vol. 4(2), pp. 26-39, April 2010.

[2] J. J. Sun, L. M. Wang, and G. Q. Chen, "A Probabilistic Method to assess the Regional Susceptibility of Landslides induced by Earthquake in Kitakyushu City Japan", Disaster Advances, vol. 4(1), pp. 7-18, January 2011.

[3] W. Xu, R. L. Hu and T. F. Li, "Formation mechanism and stability analysis on the Hekou sugar-refinery landslide in Xinping County Yunnan Province", Disaster Advances, vol. 6(7), pp. 20-25, July 2013.

[4] Q. H. Jiang, Z. H. Zhang, W. Wei, N. Xie, and C. B. Zhou, "Research on triggering mechanism and kinematic process of Qianjiangping Landslide", Disaster Advances, vol. 5(4), pp. 631-636, October 2012.

[5] C. S. DESAI, "A consistent finite element technique for work-softening behavior", the Proceedings of the International Conference on Computation Methods in Nonlinear Mechanics, Austin, America, 1974.

[6] Shen, Z.J. and Chen, T.L, "The rock damage mechanics: basic concept, purpose and task", The seventh conference of the Chinese Society for rock mechanics and Engineering, Beijing, China, 2002.

[7] M. D. Liu, and J. P. Carter, "Volumetric Deformation of Natural Clays", International Journal of Geomechanics, vol. 2(3), pp. 236-252, November 2003.

[8] D. Y. Xie, and J. L. Qi, "Soil structure characteristics and new approach in research on its quantitative parameter", Chinese Journal of Geotechnical Engineering, vol. 21(6), pp. 651-656, December 1999.

[9] R. J. Li, J. D. Liu, R. Yan, W. Zheng, S. J. Shao, "Characteristic of Structural Loess Strength and Its Preliminary Framework of Joint Strength Formula", Water Science and Engineering, vol. 7(3), pp. 319-330, July 2014.

[10] G. H. Deng, S. J. Shao, C. L. Chen and F. T. She, "A structural parameter reflecting coupling action between shear stress and spherical stress", Rock and Soil Mechanics, vol. 33(8), pp. 2310-2314, August 2012.