Constraints on the Skyrme Equations of State from Properties of Doubly Magic Nuclei

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I use properties of doubly-magic nuclei to constrain nuclear matter and neutron matter equations of state. I conclude that the data determined the value of the neutron equation of state and the symmetry energy near a density of \( \rho_{\text{on}} = 0.10 \) nucleons/fm\(^3\). The slope at that point is constrained by the value of the neutron skin.

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The neutron equation of state (EOS) is important for understanding properties of neutron stars [1], [2]. Recently an extensive study was made of the constraints on the Skyrme energy-density functionals (EDFs) provided by the properties of nuclear matter [3]. Out of several hundred Skyrme EDFs, the 16 given in Table VI of [3] called the CSkP set best reproduced a selected set of experimental nuclear matter properties. Five of these were eliminated [3] since they gave transitions to spin-ordered matter around \( \rho = 0.25 \) nucleons/fm\(^3\) (in this paper the densities are all in units of nucleons/fm\(^3\)). The remaining 11 are those given in Table I and labeled with their name and order in Table VI of [3]. I start with these 11 interactions and use properties of doubly-magic nuclei to provide further constraints on the deduced EOS. I also consider the SLy4 [4] and SkM* [5] EDFs since they are widely used in the literature.

The EOS for nuclear matter, neutron matter and the symmetry energy \( S \) obtained with the CSkP, SLy4 and SkM* parameters given Table VI of [3] are shown in Fig. 1. The symmetry energy EOS is the difference between the neutron EOS and matter EOS. The spread is relatively small. But I will demonstrate that the spread can be reduced if the Skyrme parameters are constrained by the same set of nuclear data.

The data set consists of the ground-state properties of the doubly-magic nuclei used for the Skx family of Skyrme interactions [6], [7], [8], [9], [10]: \( ^{16}\text{O} \), \( ^{24}\text{O} \), \( ^{34}\text{Si} \), \( ^{40}\text{Ca} \), \( ^{48}\text{Ca} \), \( ^{68}\text{Ni} \), \( ^{88}\text{Sr} \), \( ^{100}\text{Sn} \), \( ^{132}\text{Sn} \) and \( ^{208}\text{Pb} \). The properties are binding energies, rms charge radii and single-particle energies. These data are given in [6]. All of the CSkP functionals were constrained to have a range of \( L \) values (defined below) that correspond to a neutron skin thickness (the differences between the neutron and proton rms radii) for \( ^{208}\text{Pb} \) to be near \( R_{np} = 0.20 \) fm. The The Ska25s20 and Ska35s20 are unpublished func-

FIG. 1: EOS obtained from the CSkP set of Skyrme interactions plus SLy4 and SkM*. The black lines are those with \( m^*/m \approx 1.0 \) and the red lines are those with \( m^*/m = 0.70\) - 0.85. The blue lines are those for SLy4 and SkM*.

FIG. 2: EOS obtained from the CSkP set of Skyrme interactions fitted to properties of doubly-magic nuclei and with a constraint of \( R_{np} = 0.20 \) fm for the neutron skin of \( ^{208}\text{Pb} \). See caption to Fig. 1.
FIG. 3: EOS obtained from the CSkP set of Skyrme interactions fitted to properties of doubly-magic nuclei and with a constraint of 0.20 fm for the neutron skin of $^{208}$Pb and $m^*_n/m=0.90$ at $\rho_{on}=0.10$. See caption to Fig. 1.

FIG. 4: EOS obtained from the interactions fitted to properties of doubly-magic nuclei and with values of 0.16 and 0.24 fm for the neutron skin of $^{208}$Pb together with $m^*_n/m=0.90$. See caption to Fig. 1.

FIG. 5: The neutron EOS at low density for $R_{np} = 0.16, 0.20$ and 0.24 fm. They are compared to the theoretical results from [11] (dashed lines). See caption to Fig. 1.

and its nuclear matter properties are given in [3]. These parameters are all well determined by the data set. The values for $\sigma$, $x_1$ and $x_2$ were fixed at their original values. A reasonable range of $\sigma$ gives relatively good fits to the data. The $x_1$ and $x_2$ are not determined from the data. The parameters that are fixed are given in Table I together with the value of the $\chi^2$ of the fit. The 4th one in Table VI of [3] (LNS) was not used since it did not give converged results for nuclei. The new results for the EOS are shown in Fig. 2. The EOS are now more tightly constrained. The single-particle energies included in the fit are best reproduced with a nuclear-matter effective mass of near unity. The fit quickly converges for the values of the binding energies and radii. The convergence for single-particle energies is an order of magnitude slower. If all of the fits were fully converged, they would end up with the same low $\chi^2$ values and with nuclear matter effective masses near unity. I stop with the quick convergence part that leaves some range of values for the nuclear matter effective mass.

Some of the CSkP set were eliminated in [3] because the neutron matter effective mass was not less than unity. However, I find that by repeating the fit and allowed the $x_1$ parameter to vary and keeping $x_2$ fixed, the neutron effective mass at $\rho_{on}=0.10$ can be fixed at a value of 0.90 without any significant change to the $\chi^2$ for the fit to nuclear ground-state data. (The neutron effective mass
depends upon both \( x_1 \) and \( x_2 \). In some cases \( x_2 \) and be varied and \( x_1 \) can be fixed. In the case of Sly4 it was only possible to do the latter.) The EOS are shown in Fig. 3. They are similar to those in Fig. 2, but are closer together at large density. The resulting \( x_1 \) values are given on the right hand side of Table I. A good fit would be obtained for a reasonably wide range of values for the neutron effective mass at \( \rho_{on}=0.10 \) (0.8 to 1.0).

The CSKp sets provide a reasonable range of values for nuclear matter incompressibility \( K_m \) from 219 to 240 MeV as given in Table I. The fit to nuclear ground-state data is good for all of this range. The range of values obtained from a QRPA analysis of the giant monopole data is good for all of this range. The range of values varied and for the neutron effective mass at \( \chi_R \) from \( \rho_0 \) take the average value of neutron EOS is most well determined in this range. I from the delta-function part that depends on \( \rho_{n}\) NN forces from [11] (the dashed lines). Other predictions with smaller error bands. Present results will be important for comparing to future calculations based on the Entem and Machleidt N\( _{3\text{LO}} \) NN potential with a cutoff at 500 MeV that include N\( _{3\text{LO}} \) NNN forces from [11] (the dashed lines). Other predictions are shown in Fig. 7 of [14]. The present results for \( R_{np} = 0.20 \) fm are all within the theoretical error band and have the same \( \rho \) dependence as the calculations. The present results will be important for comparing to future calculations with smaller error bands.

All of the curves obtained by Skyrme EDF in these figures are given by the analytical expression

\[
F(\rho) = a\rho + b\gamma + c\rho^{2/3} + d\rho^{5/3},
\]

where \( \gamma = 1 + \sigma \), and \( a, b, c \) and \( d \) are constants that depend on the Skyrme parameters. The first term is from the delta-function part that depends on \( t_0 \) and \( x_0 \), the second term is from the density dependent part that depends on \( t_3 \) and \( x_3 \), the third term is the Fermi-gas kinetic energy, and the fourth term depends on \( t_1, t_2, x_1 \) and \( x_2 \). The effective mass is given by

\[
\frac{m^*(\rho)}{m} = \frac{c}{c + d\rho}.
\]

The first and second derivatives are given by

\[
F'(\rho) = a + b\gamma \rho^{\gamma - 1} + (2/3) c\rho^{-1/3} + (5/3) d\rho^{2/3},
\]

and

\[
F''(\rho) = \gamma (\gamma - 1) b\rho^{\gamma - 2} - (2/9) c\rho^{-4/3} + (10/9) d\rho^{-1/3}.
\]

Given a fixed \( c \) and \( d \) one can write

\[
[F(\rho)/\rho] = a + b\rho^{\gamma - 1} + A(\rho)
\]

and

\[
F'(\rho) = a + b\gamma \rho^{\gamma - 1} + B(\rho),
\]

where

\[
A(\rho) = c\rho^{-1/3} + d\rho^{2/3},
\]

and

\[
B(\rho) = (2/3)c\rho^{-1/3} + (5/3)d\rho^{2/3}.
\]

These equations provide the results needed to obtain \( a \) and \( b \) in terms of \( F(\rho_0) \) and \( F'(\rho_0) \) at a fixed value \( \rho = \rho_0 \),

\[
b = \frac{F'(\rho_0) - [F(\rho_0)/\rho_0] - A(\rho_0) + B(\rho_0)}{(\gamma - 1) \rho_0^{\gamma - 1}},
\]

and

\[
a = [F(\rho_0)/\rho_0] - b\rho_0^{\gamma - 1} - A(\rho_0).
\]

It is conventional to define the value and derivatives at \( \rho_{on} = 0.16 \) with \( J = F(\rho_{on}) \), \( L = 3\rho_{on} F'(\rho_{on}) \) and \( K = 9\rho_{on}^2 F''(\rho_{on}) \). There are three quantities, the nuclear-matter EOS, \( F_m = (E/N) \), the neutron EOS, \( F_n = (E/N) \), and the symmetry energy \( S = F_{sym} = F_n - F_m \). The results corresponding to one of the best fits to nuclear ground-state data, the Ska25 with \( \sigma = 0.25 \), are given Table II. For a given \( \gamma \) and effective mass \( \rho_0 \), the values of \( F(\rho_0) \) and \( F'(\rho_0) \) determine the entire EOS. For example, for the nuclear matter EOS the values \( F(\rho_{on}) = -16.0, F'(\rho_{on}) = 0.0, c = 75 \) and \( d = 0 \) determine \( a \) and \( b \) and the entire nuclear matter EOS.

Considering the entire CSKp set, the neutron EOS is best determined at \( \rho_{on} = 0.10 \) with a value of \( F_n(\rho_{on}) = (E/N)(\rho_{on}) = 11.3(8) \) MeV. The symmetry energy at this point is \( S(\rho_{on}) = F_n(\rho_{on}) - F_m(\rho_{on}) = 11.3(8) + 14.1(7) = 25.4(8) \) MeV. The derivatives at \( \rho_{on}=0.10 \) are approximately linear with the value of \( R_{np} \) with

\[
S'(\rho_{on}) = p_s R_{np}.
\]

and

\[
F'_n(\rho_{on}) = p_n R_{np}.
\]

with \( p_s = 850 \) and \( p_n = 525 \). For a given value of \( R_{np} \) we have \( S' \) and \( F'_n \) that can be used together with the above values at \( \rho_{on}=0.10 \) in Eqs. (9) and (10) to obtain \( a \) and
TABLE I: Properties of the fitted Skyrme interactions with $R_{np} = 0.20\, \text{fm}$. The results for $x_1$ in the last column were obtained with the additional constraint that $m^+_n(\rho_{on})/m \approx 0.90$.

| name         | $\sigma$ | $x_1$ | $x_2$ | $\chi^2$ | $K_m$ (MeV) at $\rho_{on}=0.10$ | $m^+_n/m$ at $\rho_{on}=0.16$ | $m^+^2/m$ | $x_1$ |
|--------------|----------|-------|-------|-----------|-------------------------------|----------------------------|------------|-------|
| KDE0v1       | s3       | 1/6   | -0.35 | -0.93     | 1.88                          | 219                         | 0.79       | 0.79  | 0.18  |
| LNS          | s4       | 1/6   | 0.06  | 0.66      |                               |                            |            |       |       |
| NRAPR        | s6       | 0.14  | -0.05 | 0.03      | 2.77                          | 227                         | 1.00       | 0.85  | -0.07 |
| Ska25s20     | s7       | 0.25  | -0.80 | 0.00      | 0.88                          | 219                         | 0.99       | 0.99  | -1.35 |
| Ska35s20     | s8       | 0.35  | -0.80 | 0.00      | 0.74                          | 240                         | 1.00       | 1.00  | -1.46 |
| SKRA         | s9       | 0.14  | 0.00  | 0.20      | 1.70                          | 215                         | 0.99       | 0.79  | -0.45 |
| SkT1         | s10      | 1/3   | -0.50 | -0.50     | 0.79                          | 236                         | 0.98       | 0.97  | -1.02 |
| SkT2         | s11      | 1/3   | -0.50 | -0.50     | 0.82                          | 237                         | 0.95       | 0.97  | -0.99 |
| SkT3         | s12      | 1/3   | -1.00 | 1.00      | 0.76                          | 236                         | 0.98       | 0.98  | -1.52 |
| SQMC750      | s15      | 1/6   | 0.00  | 0.00      | 2.50                          | 227                         | 0.92       | 0.71  | -0.08 |
| SV-sym32     | s16      | 0.30  | -0.59 | -2.17     | 0.83                          | 233                         | 1.12       | 0.91  | -1.77 |
| SLy4         | s17      | 1/6   | -0.34 | -1.00     | 3.42                          | 230                         | 0.73       | 0.70  |       |
| SkM*         | s18      | 1/6   | 0.00  | 0.00      | 1.71                          | 221                         | 0.98       | 0.78  | -0.36 |

TABLE II: Nuclear matter properties of the Ska25 fits. $J$, $L$ and $K$ are defined at $\rho_{on}=0.16$. $F(\rho_{on})$ and $F'(\rho_{on})$ are defined at $\rho_{on}=0.10$.

| $R_{np}$ | $F_m = E/N$ | $F_n = E/N$ | $F_{sym} = S$ | $F_{sym} = S$ | $F_{sym} = S$ | $F_{sym} = S$ |
|----------|-------------|-------------|---------------|---------------|---------------|---------------|
| 0.16     | $-307$      | $-444$      | $374$         | $-568$        | $250$         |               |
| 0.20     | $-307$      | $-444$      | $374$         | $-568$        | $250$         |               |
| 0.24     | $-307$      | $-444$      | $374$         | $-568$        | $250$         |               |

The $\sigma$, $\gamma$, $J$, $L$, $K$, $R_{np}$, $F_n(\rho_{on})$, $F'(\rho_{on})$ and $R_{np}$. Although the $\gamma=1+\sigma$ value and the $d$ term are not the same for all of the Skyrme EDFs considered, all of the lines in the Fig. 3, 4 and 5 are within a narrow band that includes Ska25 lines. Thus the parameters given in Table II are close to a universal parametrisation of the Skyrme EOS that is constrained by nuclear data and its dependence on the neutron skin. An assumption for all of the Skyrme EDFs considered is that $\gamma = 1 + \sigma$ is the same for both the matter and neutron EOS. The $\gamma$ for nuclear matter is constrained by the $K_m$ value obtained from the energy of the giant monopole resonance. The value of $\gamma$ may be different for neutron matter and this would effect the $K_{sym}$ value. The $^{208}$Pb neutron skin thickness obtained from the PREX parity-violating electron scattering experiment is $R_{np} = 0.302(0.175)_{\exp}^{0.026} \pm 0.005_{\text{model}} \pm 0.005_{\text{strange}}\, \text{fm}$ \[^{15},^{16}\]. A PREX-II experiment has been approved that will reduce the error bar to about 0.06 fm. This will put an important constraint on the neutron EOS. Other data that will constrain the neutron EOS come from properties of the dipole resonance \[^{17},^{18}\] and from nuclear reactions \[^{19}\].

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