Implications of a Loop-top Origin for Microwave, Hard X-Ray, and Low-energy Gamma-Ray Emission from Behind-the-limb Flares

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Abstract

Fermi has detected hard X-ray (HXR) and gamma-ray photons from three flares, which according to STEREO occurred in active regions behind the limb of the Sun as delineated by near-Earth instruments. For two of these flares, RHESSI has provided HXR images with sources located just above the limb, presumably from the loop-top (LT) region of a relatively large loop. Fermi-GBM has detected HXRs and gamma-rays, and the Radio Solar Telescope Network has detected microwave emissions with similar light curves. This paper presents a quantitative analysis of these multiwavelength observations assuming that HXRs and microwaves are produced by electrons accelerated at the LT source, with emphasis on the importance of the proper treatment of particle escape from the acceleration-source region and the transrelativistic nature of the analysis. The observed spectra are used to determine the magnetic field and relativistic electron spectra. It is found that a simple power law in momentum (with a cutoff above a few 100 MeV) agrees with all observations, but in energy space, a broken power-law spectrum (steepening at \(\sim mc^2\)) may be required. It is also shown that the production of the \(>100\) MeV photons detected by Fermi-LAT at the LT source would require more energy than photospheric emission. These energies are lower than that required for electrons, so that the possibility that all the emissions originate in the LT cannot be ruled out on energetic grounds. However, the differences in the light curves and emission centroids of HXRs and \(>100\) MeV gamma-rays favor a different source for the latter.

Key words: acceleration of particles – Sun: coronal mass ejections (CMEs) – Sun: flares

1. Introduction

The Fermi Gamma Ray Observatory (Fermi; Atwood et al. 2009) observes the Sun once every other orbit. During the past solar active phase, its Large Area Telescope (LAT) has detected \(>100\) MeV photons from more than 40 solar flares. A few of these are detected only during the impulsive phase coincident with hard X-rays (HXRs) that are produced as nonthermal electron bremsstrahlung (NTB) and nuclear gamma-ray lines excited by high-energy ions (mostly protons; Ackermann et al. 2012). There is considerable evidence that the electrons are accelerated in a reconnection region near the loop top (LT) of the flaring loops (Masuda et al. 1994; Petrosian et al. 2002; Krucker et al. 2010; Nitta et al. 2010; Liu et al. 2013), and it is generally assumed that this is the site of acceleration of the impulsive phase protons (and ions) as well. A majority of the LAT-detected flares show only long-duration emission (extending up to tens of hours), however, usually rising after the impulsive phase (Ajello et al. 2014). Some stronger flares show both impulsive and gradual emission (Ackermann et al. 2014). Almost all LAT flares are associated with relatively fast (\(>1000\) km s\(^{-1}\)) coronal mass ejections (CMEs) and are often accompanied by gradual solar energetic particle (SEP) events. This may indicate that the high-energy particles responsible for the LAT gamma-rays are accelerated in the CME shock environment where the SEPs are produced. However, while SEPs are particles escaping the upstream region of the CME shock, the gamma-ray producing particles, if originating at the CME, most likely come from the downstream region of the shock, with magnetic connection to the higher density solar atmosphere, which is the only place at which such high-energy radiation can be produced. This scenario has received further support from a Fermi-LAT detection of three flares that as observed by STEREO, originated from active regions (ARs) behind the limb (BTL) of the Sun, as delineated by near-Earth instruments. The analysis and some preliminary interpretation of the data from Fermi and other instruments on the BTL flares are presented in Pesce-Rollins et al. (2015) and Ackermann et al. (2017, hereafter Ack17).

Our aim here is a more detailed modeling of the BTL flares with particular focus on the determination of electron spectra and energy contents required to produce the multwavelength radiations seen in two of these flares. It should be noted that flares, such as these with occulted footpoints, provide a clearer view of the coronal LT source, which may be the site of particle acceleration. Thus, the analysis presented below provides the most direct information on the acceleration process. Several observations of partially occulted flares in HXRs (see, e.g., Frost & Dennis 1971; Krucker et al. 2007b) and in gamma-ray emissions have been reported (Vestrand & Forrest 1993; Barat et al. 1994; Vilmer et al. 1999). More recently, Effenberger et al. (2017) have provided a complete list of RHESSI-observed partially occulted flares combining those from cycle 24 with the earlier list by Krucker & Lin (2008) from cycle 23. An analysis similar to that presented here can be carried out for any of these flare with contemporaneous microwave coverage.

The next section presents a summary of the relevant observational characteristics of these flares (all taken from Ack17). Section 3 provides a description of the main focus of this paper, which is to describe the emission processes and to determine the characteristics of the nonthermal electrons required for their production. Section 4 contains a brief discussion of the
possibility of an LT origin of the >100 MeV gamma-rays detected by the LAT. A summary and conclusions are presented in Section 5.

2. Review of Relevant Observations

Multiwavelength observations of the BTL flares and their analysis were presented in Ack17, the main source of the data used here. In Table 1 we reproduce some of these, together with a few new results from further analysis of the radio observations that are relevant for our modeling, in particular for the determination of the broadband spectra and numbers (or energy contents) of the accelerated particles. Only two of the three BTL flares, namely SOL2013-10-11 and SOL2014-09-01, had complete sets of HXR, radio, and gamma-ray data. (For the sake of brevity, we refer to these as the October 13 and September 14 flares, respectively.) For each flare we list the spectral parameters averaged over the duration $\Delta T$ of the flare (25 and 18 minutes, respectively). This covers most of the impulsive phase. For HXRs we list the $f(\nu)$ energy flux, in units of erg cm$^{-2}$ s$^{-1}$ at 30 keV (above which the emission is dominated by NTB), the photon number spectral index, $\gamma_X$, and a high-energy exponential cutoff energy, $\epsilon_{c.X}$. Most of these are obtained from Fermi-GBM data, which agree with RHESSI and Konus-WIND data. The same parameters are also given for the LAT >100 MeV gamma-rays. These are fits to the photon counts and can be used to model these observations by either a relativistic NTB or by a pion decay model. Ack17 fit the photon counts directly to the thick-target pion decay model and listed the time-averaged simple power-law (accelerated) proton indexes of 4.4 and 4.6 for these two flares, respectively.

The radio spectral parameters were obtained using the radio spectra shown in Figure 12 of Ack17 (also shown below; Figure 4). These spectra appear to peak at a frequency $\nu_p$ falling as a power law, $f(\nu) \propto \nu^{-\gamma}$, above the peak, and they decrease relatively steeply below it. These are clearly portions of optically thin and thick gyro-synchrotron emission with optical depth $\tau_\nu \sim 1$ at the peak. In Table 1 we give our best estimates for the peak frequency $\nu_p$, $f(\nu)$ flux at $\nu_p$ and at the optically thin part $\nu = 10$ GHz, and the spectral index $\gamma_\nu$. Absorption of microwave radiation in solar flares has many causes (see Ramaty & Petrosian 1972), but the most common cause is synchrotron self-absorption that gives a spectrum $f(\nu) \propto \nu^{3/2}$ for $\nu \ll \nu_p$. The extant data is not accurate enough to distinguish among the various possibilities. In what follows, we consider free–free and self-absorptions. We note that the microwave spectra used for these estimations are for the one-minute interval around the peak of the light curve where the particle and photon spectra are generally harder. This should be kept in mind when comparing the radio with the HXR and $\gamma$-ray spectra, which are integrated over the longer times used for HXRs.

In addition to the spectral observations given in Table 1, we need a few other properties of the emission site for a detailed modeling of these flares. Table 2 gives some of these properties. For each flare, we give the angular size in sr (based on RHESSI images), the height above the photosphere (based on the position of the AR BTL as determined by STEREO), the distance between the centroids of the RHESSI and LAT sources (in arcseconds), the emission measure (EM; usually obtained from fits to the lower energy HXR thermal component), the density $n = \sqrt{\text{EM}/V}$ (with $V$ the volume of the source; see also footnote 7), the durations described above, and the magnetic field estimates based on the spectral fits to the optically thick radio spectra as described in the next section. For the October 13 flare, the EM value is obtained from the RHESSI thermal component by

### Table 1

| Flare            | Angular Size $\Omega$ | Height | $D^*$ | Emission Measure | Density | Duration | Magnetic Field |
|------------------|-----------------------|--------|-------|-----------------|---------|----------|----------------|
| SOL2013-10-11    | $1.5 \times 10^{-8}$  | 1.0    | 65    | $\sim 1.3 \times 10^{18}$ | $\sim 1.4 \times 10^{10}$ | 25       | ~200–500       |
| SOL2014-09-01    | $4.2 \times 10^{-8}$  | 20     | 275   | $< 10^{17}$     | $< 10^{9}$ | 18       | ~2–10          |

### Table Notes

- Distance between the centroids of the LAT and RHESSI sources.

### Footnotes

1. This is the same as $E^2dN/dE$ used in Ack17 and $e^2J(\epsilon)$ used below for HXR emissivities.

2. X- and gamma-ray fluxes refer to $e^2J(\epsilon)$ in erg cm$^{-2}$ s$^{-1}$, with $J(\epsilon)$ as the number flux averaged over the durations given in Table 2. Radio fluxes $F(\nu)$ are in erg cm$^{-2}$ s$^{-1}$ Hz$^{-1}$ averaged over one minute around the peak.

3. X-ray and gamma-ray indexes refer to photon numbers $J(\epsilon) \propto \epsilon^{-\gamma_X}$; the radio index is the photon energy index $F(\nu) \propto \nu^{-\gamma}$.

4. High-energy cutoffs in MeV.

5. Radio peak flux frequency in GHz.

6. Radio peak flux frequency in GHz.

7. The exact value depends on the index $\gamma_\nu$ and the geometry of the source (see below).
We assume that particles of energy $E$ (in units $m_ec^2$) are either accelerated outside the LT source and injected into it at a rate of $\dot{Q}(E,t)$, or that they are accelerated in this source region with a spectrum $N(E,t)$. As shown below, in either case, because the particle energy loss time $\tau_L \gg \tau_{esc}(E,t)$, the time spent traversing the LT source, they lose a small fraction of their energy and produce thin-target radiation. In the first case, the spectrum of particles integrated over the source region would be $N(E,t) = \dot{Q}(E,t)\tau_{esc}(E,t)$. If the acceleration and emission sites are the same, then $\dot{Q}(E,t) = N(E,t)/\tau_{esc}(E,t)$ will represent the flux of the escaping particles. As evident, the difference between these two scenarios is a matter of semantics, so in what follows we use the first scenario, which gives the number (and energy) flux of particles that escape the LT region (essentially at the injection rate) to the footpoints (FPs) of the AR located BTL, where they lose all their energy and produce the usual thick-target FP radiations. These emissions, the usual focus for disk flares, are obscured by the optically thick solar gas from near-Earth instruments for a BTL flare. STEREO, the only satellite with a direct view of the AR, detected extreme ultraviolet (EUV) radiation from these flares.

Our goal is to use the observations to obtain the spectrum of the injected flux, $\dot{Q}(E)$, and accelerated particle number, $N(E)$. Over the small range of energies commonly provided by observations, one can use a simple power law to describe this spectrum. However, to model the combined HXR and microwave data (and the gamma-rays in case of the September 14 flare), we must consider electron spectra spanning a wide range of energies; from nonrelativistic (for the production of HXRs) to extreme relativistic (for the production of radio and gamma-rays). In this case, a broken power law, or a power law with an exponential cutoff, could provide a better fit. If accelerated protons are responsible for LAT gamma-rays, we need their spectra from 300 MeV to tens of GeV, again straddling the transrelativistic range. This raises two important issues.

1. When dealing with transrelativistic spectra, one must distinguish between spectra in the energy and momentum spaces. A simple power law in energy space [$N(E) \propto E^{-\eta}$] will turn into a broken power law in the momentum space [$N(p) \propto p^{(1-\eta)/2}$] at nonrelativistic and $N(p) \propto p^{-\eta}$ at extreme relativistic momenta), and vice versa, with a break at $p \sim mc$ or $E \sim mc^2$. This should be distinguished from the actual breaks determined by the interplay between the parameters of the acceleration and energy transport and loss mechanisms. In what follows, we consider spectra in both momentum and energy.

2. The energy dependence of $\tau_{esc}(E)$, or the time the particles spend in the LT source. The usual assumption of the thin-target model is that particles cross the length $L$ of the source freely with $\tau_{esc}(E,t) \sim \tau_{cross} \sim L/v$. However, this is the shortest possible escape time. In general, $\tau_{esc}(E,t) \gg \tau_{cross}$ because the source region is highly magnetized and may contain turbulence, in which case magnetic mirroring or scattering by turbulence become important. As a result, the energy dependence of $\tau_{esc}(E)$ is more complex, and here also, it could change across the transrelativistic energy. (Note that even though $\tau_{esc}(E,t) \gg \tau_{cross}$, the thin-target assumption is still valid)

3. Modeling the Loop-top Source

F. Rubio da Costa (2018, private communication) and we determine the volume, $V$, from the source area times an assumed depth comparable to the width of the source. For the September 14 flare, we do not have access to the thermal component, so we assume an upper limit for the EM that gives a lower density, as is appropriate for its height above the photosphere.

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5 Scattering by Coulomb collisions cannot be the agent here because then the energy loss time, which is comparable to scattering time, will also be shorter than the crossing time and we will no longer be in the thin-target regime (Petrosian & Donaghy 1999). This may be the case at electron energies <25 keV (see Figure 1 below).
because as shown below, $T_{\text{esc}}(E, t) < \tau_L$. In the strong diffusion case, i.e., when scattering time $\tau_{sc} \ll \tau_{cross}$, the accelerated particles perform a random walk across the source so that $T_{\text{esc}} \sim \tau_{cross}/\tau_{sc}$ and the magnetic field variations on the scale $L$ have a small effect. On the other hand, in the weak diffusion limit with $\tau_{sc} \gg \tau_{cross}$, and for a converging field geometry, the escape time is determined by how fast particles are scattered into the loss cone, in which case (for injected particles, the pitch-angle distribution is not highly beamed along the field lines) $T_{\text{esc}} \propto \tau_{sc}$, with the proportionality constant increasing with increasing field convergence. In summary, we have

$$T_{\text{esc}} = \begin{cases} 
1 & \text{if } \tau_{sc} \gg \tau_{cross}, \text{ Free stream} \\
\propto \tau_{sc} & \text{if } \tau_{sc} \gg \tau_{cross}, \text{ Converging field} \\
\tau_{cross}/\tau_{sc} & \text{if } \tau_{sc} \ll \tau_{cross}, \text{ Strong diffusion}. 
\end{cases}$$

Combining these three cases, we obtain (see Malyshev & Kulsrud 2001; Figure 2 in Petrosian 2016 (P16))

$$T_{\text{esc}} = \tau_{cross}\left(\eta + \frac{\tau_{cross}}{\tau_{sc}} + \ln \frac{\tau_{sc}}{\tau_{cross}}\right),$$

where $\eta$ is a measure of the convergence rate of the field lines (e.g., the inverse of the ratio of the magnetic field at the top of the loop to where they exit the LT source. In what follows we use $\eta = 3$ (see, e.g., McTiernan & Petrosian 1991).  

In either case, the scattering by turbulence plays a crucial role. Relativistic particles scatter primarily by large-scale fast-mode or Alfvén waves, with $\tau_{sc} \propto E^{q-1}(-q')$, where $q$ is the spectral index of the turbulence, for a Kolmogorov spectrum $\alpha_{tr} = 1/3$. For semi-relativistic and nonrelativistic particles, this relation is more complicated and does not fit a simple power law (see Pryadko & Petrosian 1997 and Petrosian & Liu 2004). Chen & Petrosian (2013, hereafter CP13), applying the inversion method proposed by Piana et al. (2003) to two flares, find energy dependences for the escape, energy loss, and acceleration times, empirically and directly from RHESSI data, in the nonrelativistic regime. These results indicate that we are dealing with the middle case in the above equation with $\alpha_{tr} \sim 0.8$ and 0.2. This is in good agreement with the distribution of $\alpha_{tr}$ determined (also empirically) based on comparison of SEP- and HXR-producing electron spectral indexes (see Figure 4 in P16). In what follows, we treat $\alpha_{tr}$ as a free parameter and use $\alpha_{tr} = 1/3$ (or $q = 5/3$),

$$\tau_{sc}(E) = \tau_{sc} 0 \times (E/E_i)^{\alpha_{tr}} = \frac{1 + (E/E_i)^{\alpha_{tr}}}{1 + (E/E_i)^{\alpha_{tr}}}, \text{ with } E_i \sim 1.\quad (2)$$

These two energy dependencies mold the thin-target spectra. In Figure 1 we show the energy dependences of $\tau_{cross}$, $\tau_{sc}$, $\tau_{esc}$ (in cyan), where we have used a normalization (i.e., $\tau_{sc}$0) that gives them the same relative value as the crossing time and Coulomb energy loss time determined in CP13. Here we also give energy loss times, defined as $\tau_L = E/E_i$, for the energy loss rates $E_L$ due to Coulomb, bremsstrahlung, synchrotron, and inverse Compton (IC). As is evident for the electron energies of $>10$ keV of interest here, the total energy loss time (solid black) is longer than the escape time, which justifies the thin-target assumption. To have a thick-target LT source, we need $T_{\text{esc}} \leq \tau_L$. For the October 13 flare, this would require an escape time that is 10 or 100 times longer for HXR and microwave ranges, respectively. This means a 10–100 times shorter $\tau_{sc}$ or a 10–100 times higher field convergence parameter $\eta$ for strong and weak diffusion cases, respectively.

As also evident from these figures, for most of the relevant energies, we are in the weak diffusion limit. Thus, in order to simplify the analysis, in what follows, we ignore the transition from the weak to the strong diffusion case and set $T_{\text{esc}}(E) \propto \tau_{sc}(E)$.

In what follows, we deal mainly with $\nu(f,v)$ spectra integrated over the LT source region and the specified duration $\Delta T$ around the peak of the impulsive phase emission, so that

$$Q(E) = \int_{\Delta T} \dot{Q}(E, t)dt \quad \text{(or } N(E) = Q/T_{\text{esc}}(E))$$

is the total number of injected (or accelerated) particles, and $T_{\text{esc}}(E)$ is the time-averaged escape time.

### 3.1. Electron Bremsstrahlung and HXRs

The NTB $\nu(f,v)$ spectrum of photons with energy $\epsilon$ (in units of $m_ec^2$) produced by nonthermal electrons (interacting with background ions at nonrelativistic energies, but with both electrons and ions in the relativistic regime) is obtained using the differential cross section (integrated over angles) $\nu(d\nu/dE)$ (see Equation (3BN) of Koch & Motz (1959, hereafter KM59)) as

$$J(\epsilon) = \frac{1}{\epsilon_{\text{brem}}} \int_0^\infty \frac{T_{\text{esc}}(E) f(\epsilon, E) Q(E)}{\beta(E)} dE,$$

where $\beta = v/c$, and

$$\epsilon_{\text{brem}} = \frac{3}{16} \alpha_{\text{eff}}^2 \epsilon^2 \approx 1.1 \times 10^5 (10^{10} \text{ cm}^{-3}/n_d) \, \text{s}^{-1}$$

for $\alpha = 1/137$ and $r_0 = e^2/m_ec^2 = 2.8 \times 10^{-13} \text{ cm}$. The function $f(\epsilon, E)$ is a complicated but slowly varying function of $x = \epsilon/E$; in the nonrelativistic regime $f_{\text{nr}}(x) = \ln(1 + \sqrt{1 + x})$, and for the extreme relativistic regime, $f_{\text{rel}}(x) = (1 - x + 3x^2/4)[\ln(E + 0.19 - \ln(x - 1))].$ (In our numerical calculations we use the exact cross section in KM59). For relativistic energies we add the contribution of electrons as described in footnote 7 (see also Appendix A).

Thus, given the $T_{\text{esc}}(E)$ as described above, we can use Equation (3) to obtain the total flux (in and out of the source), $Q(E)$, of the accelerated electrons during the impulsive phase. In general, for most solar flares, the HXR spectra $J(\epsilon)$ decrease

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6 The IC loss rate is due to interactions with solar optical photons of energy density $u_{\nu} = L_\nu/(4\pi a^2)$, where $L_\nu$ is the solar luminosity and $a \sim R_s$ is the distance from the center of the Sun. It is identical to the synchrotron loss rate, but with an effective magnetic field $B_{\text{eff}} = \sqrt{8\pi u_{\nu}} \sim 7G$ that is usually negligible for prevailing magnetic fields of $B > 100 \text{ G}$. However, for large flaring loops like that of the September 14 flare, $B_{\text{eff}} > B$, and the IC loss becomes important.

7 For fully ionized plasma, the effective density $n_{\text{eff},\text{rel}} = \Sigma(Z_i^2n_i)$ at nonrelativistic energies and $n_{\text{eff},\text{rel}} = n_{\text{eff},\text{nr}} + n_{\text{eff},\text{rel}}$, where $n_{\text{eff},\text{nr}} = (1 + X_1/X) n_p$, and $n_{\text{eff},\text{rel}} = X_1/(XA_1) n_p$. Here $Z_i$, $X_1$, and $X_A$ are the charge, atomic number, and fractional mass of ions, and $n_p$ is the proton density of $X_1 = X$. The background densities are usually obtained from the emission measure of the thermal HXR component, $EM_{\text{th}} = 3V\Sigma(Z_i^2n_i) = V_0 n_p a^2$. For solar abundances ($X_1 = X \sim 0.74$, $X_A = Y + 0.25$ and $Z_i = \Sigma Z_i X_i \sim 0.01$ it is easy to show that $n_{\text{eff},\text{rel}} = (\Sigma Z_i EM_{\text{th}})/1 = (1 + X) \Sigma EM_{\text{th}}/11$. However, for an average $Z_i^2/A_i \sim 4$, we obtain $n_{\text{eff},\text{rel}} = 2.1 EM_{\text{th}}/1$. In what follows we use $n_{\text{eff},\text{rel}} = (1 + 2)\Sigma EM_{\text{th}}$ for nonrelativistic and relativistic regimes, respectively.
rapidly with energy (power-law index $\gamma_X > 3$), with most of the emission in the nonrelativistic regime, so that the lowest energy value of $\epsilon J(\epsilon_0)$ provides a good estimate of the total photon energy integrated over the duration $\Delta T$ of the flare:

$$\mathcal{E}_{\text{NTU}}(>\epsilon_0) = C \int_{\epsilon_0}^{\infty} \epsilon J(\epsilon) d\epsilon = C \frac{\epsilon^2 J(\epsilon_0)}{\gamma_X - 2}$$

(5)

where $C = 4\pi d^2 \Delta T$ and $4\pi d^2 = 2.8 \times 10^{27}$ cm$^{-2}$ for a distance $d$ of 1 AU. This is the case for the October 13 flare with $\gamma_X = 3.2$, but not for the September 14 flare, where the HXR $\text{ph} \nu$ spectrum is flat (i.e., $\gamma_X \sim 2$ or $\epsilon J(\epsilon) \sim \text{const}$) over several decades in energy, $\Delta \ln \epsilon \sim 7$, giving the total photon energy $\mathcal{E}_{\text{brem}}(>\epsilon_0) \propto \epsilon^2 J(\epsilon_0) \Delta \ln \epsilon$. In what follows we relate the photon energies to the total flux and energy of electrons $Q(\epsilon_0) = \int_{\epsilon_0}^{\infty} Q(E) dE$ and $\mathcal{E}(\epsilon_0) = \int_{\epsilon_0}^{\infty} EQ(E) dE$.

SOL2013-10-11: As described above, the October 13 flare has a well-defined nonthermal spectrum, a simple power law with $\gamma_X = 3.2$, between 30 and 100 keV (based on RHESSI and Fermi-GBM data). Thus, we can use the nonrelativistic approximations ($\beta^2 \sim 2E$, $T_{\text{esc}} = T_{\text{esc,0}} e^{\epsilon/\Gamma}$, $f_{\text{nr}}$), and for a power-law electron spectrum $Q(E) = Q_0 E^{-\delta}$, we obtain (see, e.g., Lin & Hudson 1971; Brown 1972; Petrosian 1973)

$$J(\epsilon) = J_0 \epsilon^{-\gamma_X} \text{ with } J_0 = \frac{T_{\text{esc,0}} Q_0}{\sqrt{2} \mathcal{E}_{\text{brem}}} I(\gamma_X - 1)$$

(6)

and $\gamma_X = \delta + 0.5 - \alpha_{\text{nr}}$, where

$$I(n) = \int_0^n x f_{\text{nr}}(x) dx = \frac{\Gamma(n + 1) \Gamma(1/2)}{(n + 1) \Gamma(n + 3/2)},$$

(7)

where $\Gamma$ stands for the gamma function. From these and the observed photon flux $J_0$ and $\gamma_X = 3.2$, we can obtain the injected electron flux at $E = m c^2$

$$Q_0 = J_0 \frac{\sqrt{2} \mathcal{E}_{\text{brem}}}{T_{\text{esc,0}} d(2.2)} = 10^5 J_0 \eta_0,$$

(8)

where we have defined $\eta_0 \equiv \frac{10^{10} \text{ cm}^{-3} \text{ 5s}}{n_{\text{eff}} T_{\text{esc,0}}}$, or the average accelerated electron spectrum

$$N(E) = Q(E) T_{\text{esc}} (E) = \frac{\sqrt{2} \mathcal{E}_{\text{brem}}}{I(\gamma_X - 1)} J_0 E^{-\gamma_X + 0.5}.$$

(9)

Note that the energy dependence of the escape time does not enter in the determination of the spectrum of the accelerated electrons, $\delta_\eta = -d \ln N(E) / d \ln E = \Gamma(\gamma_X - 0.5 - 2.7$, while the index of the total flux $Q$ is $\delta = 2.7 + \alpha_{\text{nr}}$, so that for the range $0 < \alpha_{\text{nr}} < 1.0$, we have $2.7 < \delta < 3.7$. Figure 2 shows the calculated photon spectra for simple power-law electron spectra (in both momentum and energy) for the flux $Q$ (the very high energy exponential cutoff is not relevant here) and for the escape time index $\alpha_{\text{nr}}$ as a free parameter. (We use the exact bremsstrahlung cross section; formula 3BN, KM59.) As expected, in the nonrelativistic range, the two spectra agree with each other (but, of course, with different indexes) and with the observations based on Fermi-GBM data (which agree with RHESSI data for this flare). However, the two model spectra begin to diverge in the relativistic regime (with the power law in energy predicting higher emission). These deviations are beyond the observed HXR range, where we have only upper limits (open circles; except for two possible detection with large error bars).

As shown below, radio observations shed light on the spectra at these energies. Note also that the values of the index obtained from numerical fits, $\delta_\eta = \alpha_{\text{nr}} + 3.0 \pm 0.05$ (or $\delta_\eta = 2 \alpha_{\text{nr}} + 5.0 \pm 0.05$), are slightly different than the above values ($\delta = 2.7 + \alpha_{\text{nr}}$), which assume the nonrelativistic approximations (e.g., $\delta / \beta / d \ln E = 1/(E + 1)(E + 2) \sim 0.5$).

Using these fit parameters, the observed $\epsilon^2 J(\epsilon_0) = 9 \times 10^{-8}$ erg cm$^{-2}$ s$^{-1}$ at $\epsilon_0 = 0.06$, we can obtain the total number and the energy of the injected (or escaping) electrons for the duration $\Delta T = 25$ minutes of the flare as follows. Using the normalization value (for energy fit) of 0.25 shown in Figure 2, we obtain

$$Q_0 = 0.25 \frac{\mathcal{E}_{\text{brem}} C \epsilon^2 J(\epsilon_0)}{T_{\text{esc,0}} m c^2} = 2.5 \times 10^{34} \eta_0,$$

(10)

Given $Q_0$, we then obtain the total electron number and energy (above $E_0 = 30$ keV) as

$$Q(>E_0) = Q_0 \delta - \alpha_{\text{nr}} - 1 = 3.6 \times 10^{36} \eta_0,$$

(11)

If we use the approximate nonrelativistic relations in Equations (6) and (8) and the fact that $I_0 = C \epsilon^2 J(\epsilon_0) \epsilon^{1/2}/(mc^2)$, we obtain the normalization $\frac{1}{\epsilon^2 / (1/2)} \epsilon^{12} = 0.15$ instead of 0.25.
and
\[ E_c(>E_0) = Q_0 \frac{E_0^{\delta - 3}}{\delta - \alpha_{nr} - 2} = 3.4 \times 10^{30} \eta_0 \text{ erg}, \quad (12) \]

where we have used the fitted index \( \delta = 3 - \alpha_{nr} \), with \( \alpha_{nr} = 0.0 \). For \( \alpha_{nr} = 0.5 \), the number and energy values will be larger by factors of 5.5 and 8.2, respectively.

**SOL2014-09-01:** We can carry out a similar analysis for the September 14 flare as well. However, because here we have a nearly flat \( \nu f(\nu) \) flux extending over three decades in energy from the nonrelativistic to the extreme relativistic regime (30 keV to 30 MeV), we need to rely on numerical solutions. In fact, as shown in Appendix A, it is difficult to obtain such a spectrum via bremsstrahlung emission because of the changes in the energy–momentum–velocity relation and the bremsstrahlung cross section across the transrelativistic region. For a simple power-law spectrum of the accelerated particles, \( N(E) = Q(E)T_{esc}(E) \propto E^{-\beta_{nr}} \), and using the nonrelativistic and extreme relativistic forms of the function \( f(\epsilon, E) \) in Equation (3), it is easy to show that one obtains photon spectra \( J_{nr}(\epsilon) \propto \epsilon^{-(\beta_{nr}+0.5)} \) and \( J_{er} \propto \epsilon^{-(\beta_{er}+0.5)}(\ln \epsilon + c_1) \) (with \( c_1 \) a constant of order unity), respectively, which indicates spectral hardening of \( \sqrt{\epsilon} \ln \epsilon \) or a photon index change of \( \gamma_X - \gamma_X^1 = 0.5 + 1/(\ln \epsilon + c_1) \). Thus, to obtain a power-law photon spectrum, we need a BPL spectrum of accelerated electrons, \( N(E) \), that steepens for \( E > 1 \). However, this spectral hardening can be compensated for by a break in \( T_{esc}(E) \), which, as can be seen from Equation (2), is the case for \( \alpha_{er} > \alpha_{nr} = 1/3 \), so that a simple power law of injected electrons \( Q(E) \) can reproduce the observations. As shown in the top panel of Figure 3, this is the case for \( \alpha_{nr} = 1.0 \), \( \alpha_{er} = 1/3 \) and \( \delta_{er} = 2.5 \pm 0.1 \).

Similarly, for a simple power law in momentum, \( N(p) \propto p^{-\beta_{nr}} \), we have \( J_{nr}(\epsilon) \propto \epsilon^{-(1+\beta_{nr}/2)} \) and \( J_{er} \propto \epsilon^{-(1+\beta_{er}/2)}(\ln \epsilon + c_1) \); again with spectral index \( \gamma_X \) changing from \( 1 + \beta_{nr}/2 \) to \( 1 + \beta_{er}/2 \). In this case we have a spectral softening (or steepening) for \( \delta_{er} > 3 \) (which is usually the case). Thus, in momentum space we need an electron spectrum that becomes harder (flattens) in the relativistic range. However, as shown in Appendix A, for \( 2.3 < \delta_{er} < 2.5 \), the logarithmic part can compensate for this steepening and give a nearly flat \( \epsilon^{2J(\epsilon)} \) spectrum across the transrelativistic range. Again, for the injected (or escaping) spectrum, \( Q(p) \), we need to include the energy dependence of \( T_{esc} \). This discussion implies that we need a weaker (or no) energy dependence for \( T_{esc} \). As shown in the bottom panel of Figure 3, we obtain acceptable fits for \( \alpha_{nr} = 0.3 \) and \( \delta_{er} = 2.8 \pm 0.1 \).

In summary, power-law injected spectra (with an exponential cutoff at above few MeV) both in momentum and energy space can explain the observations with different values of index \( \alpha_{nr} \) but well within the range obtained empirically by CP13 and P16. Note, however, that if we include the transition from weak to strong diffusion, the photon spectra will be steeper than shown in the above figures at (low) energies below the observed range.

Following the same procedure as above, we can also derive the total number and energy flux of the electrons. We use the fit parameters in the energy space, which is simpler. The fitted index \( \delta_e = 2.5 \) and normalization \( Q_0 T_{esc,0}/\tau_{brem} = 2.8 \) used in obtaining the fit (top panel Figure 3) implies
\[ Q_0 = 2.8 \frac{\tau_{brem}}{T_{esc,0}} \frac{C \epsilon^2 J(\epsilon_0)}{m e^2} = 4.5 \times 10^{35} \eta_b, \quad (13) \]

where we set \( \epsilon_0^2 J(\epsilon_0) = 4 \times 10^{-7} \text{ erg cm}^{-2} \text{ s}^{-1} \), \( C = 4 \pi m_e^2 \Delta T = 3.0 \times 10^{30} \text{ cm}^2 \text{ s} \) (for a duration \( \Delta T = 18 \) minutes), and we have defined \( \eta_b = \frac{n_{eff}}{\tau_{esc,0}} \). From this we can obtain the total number and energy flux of injected particles.
Radio Spectra $F(\nu) = F_0 (\nu/\nu_p)^{2.5} \times (1 - e^{-r})$

Figure 4. Observed (points; from Ack17) and self-absorbed fitted (curves) spectra for the October 13 (red) and September 14 (blue) BTL flares. The dashed green curve includes free–free absorption, which provides a better fit for the October 13 flare (see Appendix B).

Electrons above an energy $E_0 = 0.059$ (30 keV) as

$$Q(E > E_0) = 2.1 \times 10^{33} \text{Jy}$$

Here we have ignored the exponential cutoff, which will reduce these numbers by a factor $< 1 - \sqrt{E_c/E_0} < 0.99$.

3.2. Electron Synchrotron and Microwaves

3.2.1. General Synchrotron Spectra

Figure 4 shows the observed microwave spectra (points) of the two flares. Synchrotron emission by relativistic electrons is the most likely mechanism of these emissions. The high-frequency optically thin portion is observed over only one decade ($\sim 1 < \nu < \sim 10$ GHz, with the spectral index $\gamma_0$) so that only a fit to a simple power-law electron density spectrum $n(E) = n_0 E^{-\delta}$ is possible. In addition, because of the unusually great height (above the photosphere) of these sources, we are most likely dealing with lower than usual magnetic fields, lower gyrofrequencies, $\nu_B = 2.8 \times 10^6 (B/G)$, high harmonic $\nu / \nu_B > (300 G)/B$, and a Lorentz factor ($\gamma \sim 14 \sqrt{G}/B$). Thus, we are most likely in the relativistic regime, with no difference between the spectra in energy and momentum space, and we can use the usual relativistic formulation of the synchrotron emission and absorption coefficients $J(\nu)$ and $\kappa(\nu)$ (see, e.g., Rybicki & Lightman 1979):

$$J(\nu) = \alpha h \nu_B n_0 a(\delta) x^{(1-\delta)/2},$$

and

$$\kappa(\nu) = \alpha n_0 \left( \frac{h \nu_B}{4 \pi m c^2} \right) \left( \frac{c}{\nu} \right)^2 b(\delta) x^{-(\delta-2)/2},$$

where $a(\delta)$ and $b(\delta)$ are slowly varying functions on the order of unity (see Table 4 in Appendix B). From these we obtain the source term

$$S(\nu) = \frac{J(\nu)}{4\pi \kappa(\nu)} = \frac{m v_B^2 c(\nu)}{2} x^{(1-\delta)/2},$$

and the spatially integrated radio flux $F(\nu) = S(\nu) \Omega f (\tau_0)$, where $\Omega$ is the angular size (in sr) and

$$\tau_0 = \int \kappa d l \approx \tau_0 x^{-2/\delta}$$

is the optical depth (integrated over the source depth along the line of sight, $L = V/A$). The function $f(\tau)$ depends on the source shape and size and on the magnetic field geometry. However, as shown in Appendix B, the spatially integrated results depend weakly on the exact form of this function. In what follows, we use primarily the plane-parallel radiative transfer relation $f(\tau) = 1 - e^{-\tau}$. In general, however, in the optically thin ($\tau_0 \ll 1$) regime $f(\tau) = \tau$ and

$$F(\nu) = \alpha n_0 \frac{L}{4 \pi} \nu_B c(\nu) x^{(1-\delta)/2},$$

and in the optically thick ($\tau_0 \gg 1$) regime $f(\tau) = 1$, and we obtain

$$F(\nu) = \left( \frac{L}{4 \pi} \nu_B c(\nu) \right) x^{(1-\delta)/2},$$

with a peak flux $F_p = S(\nu_p) \Omega f (\tau_p \sim 1)$ at frequency $\nu_p$.

3.2.2. Electron Characteristics

From the observed spectral index $\gamma_0$ of microwave flux in the optically thin regime, we determine the electron index $\delta = 2 \gamma_0 + 1$ (and hence $a(\delta), b(\delta)$). As is well known, flux measurements in this regime are not sufficient to determine the number (or energy) of the electrons because of the degeneracy between $n_0$ and the magnetic field $B$ (or $\nu_B$). Observations in the optically thick regime (Equation (20)) provide the second datum that allows us to break this degeneracy and determine both $n_0$ and $B$. Using the expression for the source in Equation (17), it is easy to show that we can write (see Appendix B for more details)

$$\nu_B \approx \left[ c(\delta) f(\tau_0) \right]^{1/2} \frac{m \Omega}{T_p^2} \nu_p^5,$$

which then can be used in Equation (19) along with flux measurements in the optically thin regime to determine $n_0$, or the spatially and temporally integrated number

$$N_0 = \int dt \int n_0(i, t) dV$$

where we have used $V/(\nu \Delta \Omega) = d^2$ and $F$ is the average flux for the duration of the microwave flare. The spectra shown in Figure 4 are for a duration of about one minute around the peak of the radio light curve. For the purpose of comparison with

9 This and all of the above relativistic relations are valid for low magnetic fields ($\nu_B \ll \nu$) and high Lorentz factors ($\gamma \sim \sqrt{\nu / \nu_B}$). As shown in Petrovian (1981), see also Petrovian & McTiernan (1983), these relations are more complicated in the semi-relativistic regime. In general, for a power-law electron index, the synchrotron spectra steepen at lower frequencies (see, e.g., Ramaty 1969), so that the relation between $\delta$ and $\gamma_0$ varies slowly with frequency (see Ramaty & Petrovian 1972). Using numerical results, Dulk (1985) gives the semi-relativistic relation $\delta \sim 1.1 \gamma_0 + 1.36$, which is an approximate average value.
electron numbers and spectra obtained from the analysis of the NTB emission, we need the value of flux that is averaged over the same durations used above (ΔT = 25 and 18 minutes for the October 13 and September 14 flares, respectively). Since the radio light curves are almost triangular (see Figures 2 and 5 in Ack17), we estimate average fluxes of 1/2 and 3/4 of the peak-time fluxes shown in Figure 4 and given in Table 1 for the October 13 and September 14 flares, respectively.

In Figure 4 we show self-absorbed spectra based on the above equations superimposed on observations of the Radio Solar Telescope Network of the two flares, from which we can determine νB and FB. These are not very accurate fits, especially for the October 13 flare, but they allow us to obtain rough estimates of the required quantities. In particular, the value of B thus obtained is very uncertain for several reasons. First, as evident from Equation (21), νB is very sensitive to the measured parameters; it depends on the fifth power of νt and square of FB. Second, inhomogeneities in the source can bias the result. Third, there may be other absorption processes. In particular, as shown by Ramaty & Petrosian (1972), free–free absorption may be important in a situation with high elevation and a low magnetic field. In fact, the spectrum of the October 13 flare in Figure 4 shows some flattening around 5 GHz, perhaps due to free–free absorption, with possible emergence of self-absorption around 1 GHz. As described in Appendix B and shown by the dashed green curve, including free–free absorption improves the fit considerably. As also indicated in Appendix B, this model also implies optically thin free–free emission from 5 GHz to soft X-rays of ∼10 keV well below the observed microwave fluxes, which is in rough agreement with the thermal bremsstrahlung flux observed below 10 keV (see Pesce-Rollins et al. 2015).

In Appendix B, using a self-absorbed model for the September 14 flare, we obtain magnetic field values ranging from 2 to 20 G. The fact that for this flare with a height of 1010 cm we obtain magnetic fields lower than the usual B = 100 G associated with low-lying (∼1015 cm) LT sources is encouraging. A self-absorbed fit to the October 13 flare gives B values in the range 300–3000 G. This is most likely not correct because of the poor fit. Using the fit parameters including free–free absorption yields a more reasonable value of ∼200 G. We use these values of B (or νB) and the fluxes at ν = 10 GHz (in the optically thin range) in Equation (22) to calculate the number of electrons required for the production of the microwaves. For the October 13 flare with fit parameters δ = 5.2, a(δ) = 2.6 and B = 200 G (obtained from the fit including free–free absorption) and the observed flux F(ν = 10 GHz) = 10 SFU, we obtain N0 or Q0 = N0/Teesc.0 to be

\[ Q_0 = 5.5 \times 10^{34} \left( \frac{5 s}{T_{esc,0}} \right) \left( \frac{200 G}{B} \right)^{3.1}. \] (23)

This should be compared with 2.5 × 1034 obtained in Equation (10). There are two uncertain parameters, however: neff and B. For example, the two estimates would agree for neff = 5 × 1010 cm−3 and B = 70 G. Note that for this magnetic field, νB = 0.2 GHz, and the lowest reliable observed microwave point of ν = 0.6 GHz is produced roughly by electrons with a Lorentz factor γ ∼ (ν/νB)1/2 = 1.7 so that the relativistic expressions used here begin to break down and one should use the semi-relativistic expressions. However, at such low frequencies, we are in the optically thick regime, while the values of B and neff are determined by data points with higher frequency. For the September 14 flare, using self-absorbed fit parameters δ = 2.7, B = 2.5 G, a(δ) = 0.1 and F(ν = 10 GHz) = 10 SFU, we obtain

\[ Q_0 = 4.4 \times 10^{34} \left( \frac{10 s}{T_{esc,0}} \right) \left( \frac{2.5 G}{B} \right)^{1.85}, \] (24)

which is somewhat fortuitously exactly what was obtained from the X- and gamma-ray observations given in Equation (13).

### 3.3. Combined Electron Spectra

We now combine the results obtained for the electron characteristics from HXR and microwave data. In Table 3 we summarize our results on electron spectral indexes assuming Kolmogorov turbulence with αnr = 1/3. For the October 13 flare, the index of 5.1–5.5 obtained from the microwave data agrees with momentum index based on HXRs with αnr = 0.05 – 0.25, but agreement with the energy index requires either an unusually large αnr > 2 or a spectral steepening (by 1–2 units) above E ∼ mc2. For the September 14 flare, the radio index of 2.8 for N(E) and 3.1 ± 0.1 for Q(E) is closer to the HXR momentum index of 2.8 ± 0.1 than the energy index of 2.5 ± 0.1. For this flare the values of Q0 (or number of electrons at E = mc2) obtained from radio and x-gamma-ray data are in excellent agreement. As described above, however, some adjustments of uncertain parameter values (neff, Teesc, etc.) are needed for an acceptable agreement for the October 13 flare. Figure 5 summarizes these findings.

In addition to the preliminary analysis in Ack17 we referred to at the beginning, there have been similar determinations of electron spectra based on HXRs (Plotnikov et al. 2017; Share et al. 2017) assuming both thin and thick target, based only on electron energy spectra, and without considering the energy dependence of the escape time. As expected, the electron indexes derived in these papers are different than those presented here, which not only are for a thin-target model, but also include the energy dependence of the escape time. In the case of the September 14 flare, the analysis here moreover includes the exact relativistic bremsstrahlung cross section. These factors can account for such differences.

### 3.4. Emissions by Escaping Electrons

Some of the particles escape along closed field lines to the FPs to the AR located BTL and visible only to STEREO. They
lose all their energy at the FPs and produce thick-target HXRs and microwaves. Some escape out of the corona along open field lines and eventually reach Earth, where they are detected as SEPs by near-Earth instruments. As shown in P16, the escape times up, $T^{a}_{\text{esc}}(E)$, and down, $T^{d}_{\text{esc}}(E)$, will most likely have different values and energy dependences, so that the flux of SEPs will be different than those traveling to the FPs and produce HXRs. As shown in Krucker et al. (2007a), observations indicate that most of the particles are directed downward and produce thick-target HXR and microwave emission more efficiently than in the LT region. For example, the NTB spectrum would be

$$J_{FP}(\varepsilon) = \frac{1}{c_{\text{brem}}} \int_{\varepsilon}^{\infty} \frac{\tau_{L}(E)f(\varepsilon,E)\beta(E)}{\varepsilon} d\varepsilon \left( \frac{1}{E} \int_{E}^{\infty} Q(E')dE' \right), \tag{25}$$

which is similar to the thin-target expression given in Equation (3), but with two differences. The first is that instead of $Q(E)$ we now have the effective electron spectrum given by the integral in the parenthesis, which for a power-law injected spectrum is equal to $Q(E)/(\delta - 1)$. The second is that instead of escape time, the integrand contains the energy loss time $\tau_{L}(E) = E/E_{L}$ shown by the solid black lines in Figure 1. In the nonrelativistic limit (e.g., for the October 13 flare) with $T^{d}_{\text{esc}}(E) \propto E^{2.5}$ and $\tau_{L} \propto E^{3.5}$, this will lead to an FP photon spectrum with index $\gamma_{FP}^{L} = \delta - 1$ instead of $\gamma_{X}^{L} = \delta + 1/2 - \alpha_{fr}$, implying that $\gamma_{FP}^{L} = \gamma_{X}^{L} - 1.5 + \alpha_{fr} = 1.7 + \alpha_{fr}$. As shown in Figure 1 for the energy range of 10 to a few 100 keV, $T_{\text{esc}} \sim$ constant ($\alpha_{fr} = 0$) so that the FP HXR emission will be much harder. Furthermore, since the loss time is about 10 times longer than the escape time in this energy range, the FP flux will be correspondingly higher (modulo the factor $\delta - 1 \sim 2$). These relations are more complicated for the September 14 flare with HXRs extending into the relativistic range, but in general, we would expect an even harder and higher flux of FT emission. The same is true for synchrotron emission by relativistic electrons, where one must also consider the synchrotron emission, absorption, and loss process in higher magnetic fields at the FPs, which affect both the emission and energy loss rates. This implies that 10–100 times higher fluxes of HXRs and microwaves are emitted from the FPs (in the AR BTL) than those emitted from the LT.

The above equation is also applicable if the LT source was a thick- rather than a thin-target source. As stated in Section 3, this will require an unusually short scattering mean free path (i.e., short $\tau_{sc}$) or a highly converging magnetic field structure. If this were the case, however, it would require more steeply accelerated electron spectra. For example, in the nonrelativistic HXR emission case, instead of $\delta_{\text{thin}} = \gamma_{X} + \alpha_{fr} - 0.5$ (see discussion related to Equations (6) and (7)), one needs $\delta_{\text{thick}} = \gamma_{X} + 1$, which is larger (e.g., 1.5, for $\alpha_{fr} \sim 0$). Similarly, the required energy fluxes of electrons will be lower by a factor equal to the average value of $T_{\text{esc}}(E)/\tau_{L}(E)$ in the relevant energy range. Thus, all the curves in Figure 5 would be lower and steeper, and the transitions from nonrelativistic to relativistic range would be somewhat different.

4. LAT Gamma-Rays and Accelerated Protons

The Fermi-LAT emission of $>100$ MeV photons is different from the impulsive emissions considered in this paper in two important ways. The first difference is that centroids of the LAT sources are located $\sim 65^\circ$ and $275^\circ$ away from the centroids of the RHESSI LT sources for the October 13 and September 14 flares, respectively. The second is that like most flares detected by the Fermi-LAT, the LAT light curves of the flares under consideration are very different than the light curves of impulsive emissions. They rise somewhat later and decay much more slowly, with a duration more similar to gradual SEPs that are believed to be accelerated in the CME environment. Since Fermi-LAT flares are almost always associated with fast CMEs, the possibility that the LAT emission is produced by particles that are accelerated in the CME environment that escape from the shock downstream toward the Sun has gained some momentum. For the BTL flares under consideration here, this scenario requires a magnetic connection between the downstream region and areas in the photosphere in the visible disk far away from the AR where these flares originated. Recent simulations (Plotnikov et al. 2017; Jin et al. 2018) indicate that this is a likely scenario.

These two differences point to a different origin for the LAT observations than the LT source considered above. However, based on the localization data alone, the possibility that the LAT gamma-rays may also be thin-target emission coming from the LT RHESSI location cannot be ruled out with high confidence. So it is important to explore this possibility as well. Just as in the case of HXRs described above, a thin-target LT emission would require energy contents for the accelerated protons that are higher by a factor equal to $\tau_{L}/T_{\text{esc}}$ at the LT. The Coulomb loss time for 500 MeV protons is $\tau_{L} \sim 10^3$ s (for $n = 10^{10}$ cm$^{-3}$), but unlike for electrons, we have no fact-based information on the escape time. Assuming the same
we estimate escape times of \(10^{-100}\) s. This means that the production of the LAT gamma-rays at the LT would require \(10^{-7} - 10^{-8}\) times more energy for protons than is required for the thick-target photospheric emission. Ack17, assuming thick-target photospheric emission, estimate proton energies of \(E_p(E > 500 \text{ MeV}) \sim 1\) and \(7 \times 10^{25} \text{ erg}\) for the October 13 and September 14 flares, respectively. This means that the LT thin-target model would require proton energies in the range of \(10^{28-29}\) erg. These, though higher, are still about 10 times lower than the energies of the electrons shown in Figure 5. The proton energies would become comparable and could exceed the electron energies if their spectra are extrapolated to 10s of MeV. However, the absence of a strong signature of nuclear deexcitation lines rules this possibility out. We therefore conclude that the possibility of a thin-target LT source for gamma-rays cannot be ruled out with high confidence on energetic grounds alone. However, the differences in the light curves and centroids of HXR and \(\gamma\)-rays of the thick-target photospheric emission, estimate proton energies of \(\sim 10^{-100}\) MeV. This requires careful consideration of two important aspects. The first is the question of the time that the accelerated particles spend in the source region, which we call the escape time, and the second is that because the observations span the transrelativistic region, we should distinguish between spectra in momentum and energy space. Using empirically determined values and the energy dependence of the escape time in the 10–100 keV range reported by CP13 and P16 and their extension to relativistic energies based on theoretical considerations, we show that these are thin-target processes. This then allows us to obtain the electron characteristics. Our results can be summarized as follows:

1. From modeling the NTB emission of the October 13 flare, we find that simple power-law electron spectra in both momentum and energy space can reproduce the observed HXRs. For the September 14 flare, a simple power law in momentum can describe the broad range of the observed HXRs more readily and with more reasonable values for the escape time index than a simple power law in energy. Based on these fits, we determine the spectral index, numbers, and energy content of the accelerated electrons.

2. The radio spectra for both flares show distinct optically thin emissions that peak around 1 GHz and a well-defined turnover at lower frequencies, indicating the emergence of an optically thick spectrum. A self-absorbed synchrotron spectrum provides an adequate fit for the September 14 flare, but for the October 13 flare, a self-absorbed synchrotron spectrum does not fit the observations in the range \(0.5 < \nu < 5\) GHz. We show that a model in which free–free absorption starts at about 7 GHz with self-absorption becoming dominant below 2 GHz provides an acceptable fit. These modelings allow us to determine both the spectrum and numbers of relativistic electrons and the magnetic field (that turn out to be lower than usually appropriate for the great height of the source).

3. We then compare the two electron spectra obtained by these two methods. We show that for both flares, an extrapolation of the spectra based on HXRs to the relativistic regime agrees with those based on radio data assuming a simple power law (with an exponential cutoff at several 100 MeV) in momentum but not in energy space. The latter requires a broken power law with a break at \(E < mc^2\). The numbers and energy content of these flares are in the correct range and allow us to predict the FP emissions from AR located BTL.

4. We also consider the possibility of thin-target LT emission of the LAT gamma-rays and find that this requires 100–1000 times more energy of the accelerated protons compared to thick-target photospheric emission. However, even these energies are lower than those of the electrons, so that this scenario of high-energy gamma-ray
LT emission cannot be ruled out on energetic grounds. This is also true for the production of these higher energy gamma-rays by GeV electrons at the photosphere. Nevertheless, because an acceleration of electrons to several GeV is difficult, the pion decay scenario is favored, and the differences in the light curves and centroids of HXR and $>100$ MeV emissions indicate a different acceleration site and mechanisms for (pion-producing) protons than (HXR-microwave producing) electrons.

5. The radiative signatures of occulted flares, such as those considered here, provide the most direct information on spectra and energy content of accelerated particles, and hence on the acceleration mechanism, uncontaminated by the stronger FP emission. For example, the differences between the required spectra in energy and momentum spaces can shed light on the details of the acceleration process. This important aspect of the problem will be studied in subsequent papers.

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Appendix A
Some Aspects of Bremsstrahlung Emission

1. Approximate cross section: The nonrelativistic and extreme relativistic approximations given after Equation (3) (the same as expressions 3BNa and 3BNb of KM59,

\[
\frac{d\sigma}{dx} \approx \begin{cases} 
-3.0, -2.5, \ldots, -2.5, +3.0; \text{left to right} \\
\text{Combined (3BNa+3BNb)/(1+E)} 
\end{cases}
\]

Figure 6 compares this cross section (dashed green) with the exact angularly averaged bremsstrahlung cross section (solid black) using Equation (3BN) of KM59 with the approximate one in Equation (26) (dashed green), showing excellent agreement except for deviations of less than a few percent around energies $\sim m_e c^2$.

\[
\epsilon^2 \frac{d\sigma}{d\epsilon} = \frac{16\alpha m_0^2 \epsilon f_0(x) x^{-2} + E \times f_0(x)}{1 + E}; \quad x = \frac{\epsilon}{E}. \quad (26)
\]

Figure 7. Bremsstrahlung energy spectra for power laws in energy (top) and momentum (bottom) with exponential cutoffs. For power laws in $E$, flat spectra can be obtained for $\delta_e \approx 1.6$ only in the relativistic regime and for $\delta_e \approx 2.1$ only in the nonrelativistic regime. This is due to the logarithmic dependence of $J(\epsilon) \propto (\ln \epsilon + a)$ in the relativistic regime. However, for power-law spectra in momentum, this term is compensated for by the steepening of the spectra in the relativistic regime, and fairly flat spectra can be obtained in both regimes for $\delta_p \approx 2.3$. respectively) can be combined as

Electron Number Spectrum $Q_\epsilon(E) = Q_\epsilon(E/m_0 c^2)^{\delta_e} \exp(-E/100)$

Electron Number Spectrum $Q(p) = Q_\epsilon(p/m_0 c)^{\delta_p} \exp(-p/100)$
energies $\epsilon = mc^2$ and $E = mc^2$. This expression can be used to derive the photon spectra analytically. To include the contribution of relativistic electron–electron bremsstrahlung, $E \to 2E$ in the numerator needs to be changed.

2. Flat $\nu f(\nu)$ bremsstrahlung spectra: Figure 7 shows $\epsilon^2 J(\epsilon) \propto \int_0^\infty \epsilon^2 (d\sigma/d\epsilon) \beta(E) Q(E) dE$ NTB photon spectra obtained for power-law (with exponential cutoff) electron spectra in energy and momentum space. As evident, flat photon spectra extending over several decades in photon energy are not possible for these electron spectra in the energy space, but can be achieved for a power law in momentum space for $\delta \sim 2.3$.

### Appendix B

#### Some Details of Synchrotron Emission

1. Numerical coefficients: In Section 3.2 we introduced two coefficients that depend only on the spectral index of the electrons. In the relativistic regime, these coefficients are, for example, Rybicki & Lightman 1979

   \[
   a(\delta) = \frac{3^{3/2}}{\delta + 1} \Gamma \left( \frac{3\delta + 19}{12} \right) \Gamma \left( \frac{3\delta - 1}{12} \right) \left( \sin \theta^{(\delta+1)/2} \right) \tag{27}
   \]

   and

   \[
   b(\delta) = \frac{3^{\delta+1/2}}{2} \Gamma \left( \frac{3\delta + 22}{12} \right) \Gamma \left( \frac{3\delta + 2}{12} \right) \left( \sin \theta^{(\delta+2)/2} \right) \tag{28}
   \]

   where $\Gamma$ stands for the Gamma function and $\theta$ is the angle between the line of sight and the B field. In the LTE sources, the magnetic field may be radial or horizontal with respect to the limb, so that we have $\theta = \pi/2$ and the angular terms are equal to one. In the opposite case of chaotic field lines, the last terms in the above equations are equal to $(\sqrt{\pi}/2)^{\delta/4} \Gamma(\delta + 5)/\Gamma(\delta + 7)/4$ and $(\sqrt{\pi}/2)^{\delta/4} \Gamma(\delta + 6)/\Gamma(\delta + 8)/4$, respectively. An accurate determination of these coefficients is important because the magnetic field estimates are sensitive to their values. Table 4 gives the values of these and other parameters for the range $3 \lesssim \delta < 5$ of interest here.

2. Optical depths and magnetic fields: We are interested in the spatially integrated flux

   \[
   F(\nu) = S(\nu) \Omega f(\tau_\nu) \tag{29}
   \]

   where $S(\nu)$ is the average source term and $f(\tau)$ depends on the shape and geometry of the source. For example, for the plane-parallel approximation $f(\tau) = 1 - e^{-\tau}$ and for a spherically symmetric source $f(\tau) = 1 - 2/\tau + (1 - e^{-\tau})/\tau^2$. Setting the derivative of the flux to zero, we obtain $d\ln f(\tau)/d\ln \tau = 5/(\delta + 4)$, and peak optical depths $\tau_\nu, f(\tau_\nu)$ and $c(\delta)/f(\tau_\nu)$ shown in Table 3 for plane-parallel and spherical (in parenthesis) geometries. Inserting these values in Equations (29) and using Equation (17), we calculate the gyrofrequency as

   \[
   \nu_b = \nu_\nu [c(\delta)/f(\tau_\nu)] \Omega m^2_{\nu} / F(\nu_\nu) \tag{30}
   \]

   Inserting the observed values shown in Tables 1 and 2 and the coefficients in Table 3, we find gyro-frequencies and magnetic field values of 1.0(0.6) GHz and 360(220) G for the October 13 flare, and 10(4) MHz and 3.6(1.5) G for the September 14 flare (spherical geometry in parenthesis). (Note that $c(\delta) = a(\delta)/b(\delta)$, and hence the B field, varies more slowly with the spectral index $\delta$ [than $a(\delta)$ and $b(\delta)$] and the angle $\theta$ (it would change by 10% going from a random field to an ordered field with $\theta = \pi/2$).)

We can obtain the B field with an alternative method that is independent of $F(\nu_\nu)$, the most uncertain observationally determined parameter. In this method we first eliminate one of the unknowns, namely $n_b$, using Equations (18) and (19) to obtain $\nu_b$. From the first equation evaluated at $\nu_b$ and the second equation at any frequency in the optically thin regime, we obtain

   \[
   \frac{c(\delta) \Omega m^2_{\nu} / F(\nu)}{\nu_b} = \frac{F(\nu)}{a(\delta)\Omega} \left( \frac{\nu}{\nu_b} \right)^{1+\delta/2} \tag{31}
   \]

   which then gives

   \[
   \nu_b = \nu_\nu [c(\delta)/\tau_\nu \Omega m^2_{\nu} / F(\nu)] \left( \nu_\nu / \nu \right)^{1+\delta/2} \tag{32}
   \]

   Using the fluxes given in Table 1 and the parameters in Table 3, we obtain gyro-frequencies and magnetic field values of 1.6(9.7) GHz and 560(3500) G for the October 13 flare, and 22(58) MHz and 7.9(21) G for the September 14 flare (spherical geometry in parenthesis). These values are sensitive to $\delta$; e.g., for $\delta = 5$ (instead of 4.7) for the October 13 flare and $\delta = 3$ (instead of 2.7) for the September 14 flare, we obtain $B = 370(2300)$ and $B = 3.4(9.0)$, respectively.

3. Free-free absorption: As mentioned above, free–free absorption with the absorption coefficient (see, e.g., Benz 1993)

   \[
   \kappa_{ff} = 0.2 \nu^{-2} T^{-1.5} (EM/V) \left[ 1 + 0.05 \ln \frac{T/10^7 K}{\nu/(GHz)} \right] \tag{33}
   \]

   can be important for high densities and low magnetic fields. For the October 13 flare with large emission measure $EM \sim 1.3 \times 10^{48}$ cm$^{-3}$ at $T = 0.63 \times 10^{5}$ K (obtained from RHESSI data; F. Rubio da Costa 2018, private communication), we obtain an optical depth of $\tau_{ff} = 50(GHz/\nu)^2$, where we have used an area $A = V/L \sim 10^{18}$ cm$^2$, so that free–free absorption can be important below $\sim 7$ GHz. The dashed green curve in Figure 4 shows a model spectrum that includes both free–free and synchrotron self–absorption with a total optical depth

   \[
   \tau(\nu) = (\nu/\nu_{ff})^{-2} + (\nu/\nu_{sy})^{-\delta/2} \tag{34}
   \]

   so that $\tau_{ff} = 1$ at $\nu_{ff} = 7.5$ GHz and self-absorption, with an optical depth of unity at $\nu_{sy} = 2.7$ GHz, becomes dominant for $\nu < 1.2$ GHz for an electron index $\delta = 5.2$. This will require $EM/(T^{3.5}A) \sim 3 \times 10^{20}$, which is within a factor of 3 of the values quoted above. This will change the required magnetic
field to a lower value. Following the steps of the second (alternative) method used above, we can again eliminate \( n_0 \) and obtain \( \nu_B = B \). After some algebra, we obtain

\[
\nu_B = \nu_{\text{fl}} [c(\delta) \Omega_n^2 / F_0] = 0.59 \text{GHz}
\]

or \( B = 210 \text{ G} \), similar to the lower values obtained above.

Since the inclusion of free–free absorptions improves the fit to the data, we use this value of the magnetic field.

The above values of free–free absorption coefficients imply free–free emissivity (in the microwave range) of \( J_{\text{fl}}(\nu) = 4\pi B(\nu, T) n_{\text{ff}}(\nu) \), where \( B(\nu, T) = 2kT(\nu/c)^2 \) is the blackbody brightness in the Rayleigh–Jeans limit \((\hbar \nu \ll kT)\). It is easy to show that the expected free–free flux at optical depth of one (or \( \kappa(\nu_{\text{fl}}) = (1 - 1/e) / L \approx 0.6\alpha / V \))

\[
F(\nu > \nu_{\text{fl}}) = 2kT \Omega(1 - 1/e)(\nu_{\text{fl}}/c)^2 \sim 5 \text{ SFU},
\]

(36)

(for \( T = 0.6 \times 10^7 \text{ K}, n_{\text{ff}} = 7 \text{ GHz}, \) and \( \Omega \sim 10^{-8} \text{ sr} \)), which is about a factor of 6 below the observed synchrotron flux.

In summary, averaging the above result, we obtain magnetic field values of 2–10 G (September 14) and 200–500 G (October 13), which we have entered in the last column of Table 2.

It is interesting to note that this emission, extrapolated to the range of a few keV, should also agree with the thermal HXR flux. This involves extrapolation over a large frequency range (from \( \sim 10^{10} \) to \( 10^{18} \text{ Hz} \)) and differences between the Gaunt factors at microwaves of \( \sim 15 \) and HXRs of unity. Nevertheless, dividing the above flux by this factor and the Boltzmann factor \( e^\nu / kT \sim 10^{-4.5} \) (for \( \epsilon = 10 \text{ keV} \) and \( T = 10^7 \text{ K} \)), we obtain a 10 keV thermal bremsstrahlung flux of \( F(\epsilon = 10 \text{ keV}) \sim 10^{-22} \text{ erg cm}^{-2} \text{ s}^{-1}, \text{ Hz}^{-1} \) or \( \nu f(\nu) \) flux of \( 10^{-3} \) versus the observed value of \( \sim 10^{-4} \text{ erg cm}^{-2} \text{ s}^{-1} \) (see Figure 3 in Pesce-Rollins et al. 2015). Considering the scale of the extrapolation, this is a satisfactory agreement.

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