Mixing of $1/2^-$ Octets under SU(3) Symmetry

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We investigate the $J^p = 1/2^-$ baryons in the octets based on flavor SU(3) symmetry. Since baryons with same quantum numbers can mix with each other, we consider the mixing between two octets before their mixing with the singlet. Most predicted decay widths are consistent with the experimental data, and meanwhile we predict two possible Ξ mass ranges of the two octets.

**Keywords:** Flavor symmetry; Hadron decay; Mixing; Octet.

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1. Introduction

The hypothesis of the approximate SU(3) symmetry of strong interactions proposed by Gell-Mann and Ne’eman\cite{123} has been proved quite successful and fruitful in the classification of elementary particles. In this classification scheme, one can group the experimentally known strongly interacting particles with the same quantum numbers of spin and parity into various irreducible representations of the SU(3) group\cite{1}. The assignment of baryons and mesons to definite SU(3) multiplets seems to be very satisfactory for the $1/2^+, 3/2^+$ baryons and $0^-, 1^-$ mesons. It also seems to be satisfactory for the particles with spin-parity $3/2^-, 5/2^-, 7/2^-, 5/2^+, 7/2^+$ and $2^+$, but poor for $1/2^-$ and $1^+$ hadrons. It has been speculated that the mixing between multiplets might be useful for the interpretation to assign these particles to SU(3) multiplets\cite{5}.

Recently, Guzey and Polyakov reviewed the spectrum of all baryons with mass less than approximately 2000-2200 MeV and catalogued them into twenty-one SU(3) multiplets\cite{6}. That work can be viewed as an attempt to update Ref.\cite{4}. In their paper, the masses for (N, Λ, Σ, Ξ) members of an octet are listed in a parenthesis. They introduced the mixing of the octet (8, 1/2$^-$)=(1535, 1670, 1560, 1620-1725) with the singlet (1, 1/2$^-$)=Λ(1405). It is noticed that the small $\Gamma_{\Lambda(1670) \rightarrow \eta K}$ forces
2. Decay widths and coupling constants

For the decay process of a baryon $B^*$ to a baryon $B$ and a pseudoscalar meson $M$

$$B^* \rightarrow B + M,$$

the calculation of decay widths can be performed in the framework of Rarita-Schwinger formalism. The parity-conserving Lagrangian of $B^*_{1/2^-} \rightarrow B_{1/2^+} + M$ interaction is

$$\mathcal{L} = g_{B^*BM} \bar{\Psi} \Phi \phi,$$

where $\Psi$ is the $J^P = 1/2^-$ field, $\Phi$ is the $J^P = 1/2^+$ field, $\phi$ is the pseudoscalar meson field. Meanwhile, the Lagrangian of $B^*_{3/2^-} \rightarrow B_{3/2^+} + M$ interaction is

$$\mathcal{L} = \frac{g_{B^*BM}}{m_\pi} \bar{\Psi} \gamma_5 \Phi^\mu \partial_\mu \phi,$$

where the factor $1/m_\pi$ is introduced to make the coupling constant $g_{B^*BM}$ dimensionless. Accordingly, the decay widths are written as

$$\Gamma_{B^*_{1/2^-} \rightarrow B_{1/2^+} + M} = \frac{g_{B^*BM}^2}{8\pi m_B^2} P_{cm}^2 [(m_B^* + m_B)^2 - m^2],$$

$$\Gamma_{B^*_{3/2^-} \rightarrow B_{3/2^+} + M} = \frac{g_{B^*BM}^2}{24\pi(m_B^* m_\pi)^2} P_{cm}^2 [(m_B^* - m_B)^2 - m^2],$$
where $g_{B^*BM}$ is the physical coupling constant and $P_{cm}$ is the c.m. momentum of final particles. In terms of the baryons masses $m_B^*$, $m_B$ and the meson mass $m$, we have

$$P_{cm} = \frac{1}{2m_B^*}[(m_B^* - m_B + m)^2 - (m_B - m)^2]^{1/2}, \quad (6)$$

In order to calculate the partial decay widths, we must know the physical coupling constant $g_{B^*BM}$, which needs not only computing the Clebsch-Gordan coefficient among the SU(3) irreducible representations of $B^*$, $B$ and $M$, but also computing mixing of two octets and singlet.

To derive the physical coupling constants, we first denote the physical states as $|N_8\rangle$, $|N_8'\rangle$, $|A_8\rangle$, $|A_8'\rangle$, $|\Sigma_8\rangle$, $|\Sigma_8'\rangle$, $|\Xi_8\rangle$, $|\Xi_8'\rangle$, and the bare states as $|N_8^0\rangle$, $|N_8'^0\rangle$, $|A_8^0\rangle$, $|A_8'^0\rangle$, $|\Sigma_8^0\rangle$, $|\Sigma_8'^0\rangle$, $|\Xi_8^0\rangle$, $|\Xi_8'^0\rangle$. The denotations without prime are for the octet (1535, 1670, 1560, 1620-1725) and with prime are for the octet (1650, 1800, 1860, 1860-1915). Then, we introduce the mixing between the two octets. We assume that the mixing angles of different baryons in the two octets are different, i.e., the angles are $\delta$, $\theta$, $\beta$, $\gamma$ for $N$, $\Lambda$, $\Sigma$, $\Xi$, respectively. Since the physical states can be written as linear superpositions of the bare states, the physical states $|N_8\rangle$, $|N_8'\rangle$ are

$$\begin{pmatrix} |N_8\rangle \\ |N_8'\rangle \end{pmatrix} = \begin{pmatrix} \cos \delta & \sin \delta \\ -\sin \delta & \cos \delta \end{pmatrix} \begin{pmatrix} |N_8^0\rangle \\ |N_8'^0\rangle \end{pmatrix}. \quad (7)$$

Therefore, the coupling constants of $N_8$ after two octet mixing are

$$\begin{pmatrix} g_{N_8BM} \\ g_{N_8'^0BM} \end{pmatrix} = \begin{pmatrix} \cos \delta & \sin \delta \\ -\sin \delta & \cos \delta \end{pmatrix} \begin{pmatrix} g_{N_8^0BM} \\ g_{N_8'^0BM} \end{pmatrix}, \quad (8)$$

where $g_{B^*BM}$ is the SU(3) universal coupling constant, $g_{B^*BM}$ is the physical coupling constant of mixing. The physical states of $\Sigma$, $\Xi$ have the same form as $N$. However, $A_8$ are special, the couplings after mixing with octet are

$$g_{A(1670)BM} = \cos \theta \ g_{A(1670)BM} + \sin \theta \ g_{A(1600)BM},$$
$$g_{A(1600)BM} = -\sin \theta \ g_{A(1670)BM} + \cos \theta \ g_{A(1600)BM},$$

where $\tilde{A}(1670)$ represents a middle state. And then, we introduce the angle $\xi$ to mix the mixed state $\tilde{A}(1670)$ with $\Lambda(1405)$ and can get the coupling constants of physical state $A(1670)$ and $\Lambda(1405)$

$$g_{A(1405)BM} = \cos \xi \ g_{A(1405)BM} + \sin \xi \ g_{\tilde{A}(1670)BM},$$
$$g_{A(1670)BM} = -\sin \xi \ g_{A(1405)BM} + \cos \xi \ g_{\tilde{A}(1670)BM}. \quad (10)$$

In these physical coupling constants, there are 12 parameters, which are five mixing angle parameters ($\delta$, $\theta$, $\beta$, $\gamma$, $\xi$) and 7 coupling parameters ($\alpha$, $A_8$, $A_{10}$, $A_1$, $A_{10}'$, $A_8'$, $A_{10}'$) of the universal coupling constants $^{[4]}$, where the parameters $\alpha$, $A_8$, $A_{10}'$, $A_8'$ are for the $8 \rightarrow 8 + 8$ decays, $A_{10}$, $A_{10}'$ for the $8 \rightarrow 10 + 8$ decays and $A_1$ for the $1 \rightarrow 8 + 8$ decays. If these parameters are known, we can get all the coupling
constitute and calculate the decay widths. On the other hand, we can also constrain the parameters if we know information on the coupling constants.

Therefore, we use some experimental decay widths of baryons to adjust the parameters of coupling constants, then use these parameters to calculate all decay widths. If the calculated decay widths are consistent with or closer to the experimental data, it supports the feasibility of considering mixing between octets.

3. Mass

We denote the bare masses of baryons as $N_8^0$, $N_{8'}^0$, $\Sigma_8^0$, $\Sigma_{8'}^0$, $\Xi_8^0$, $\Xi_{8'}^0$, $\Lambda_1^0$, $\Lambda_8^0$, $\Lambda_{8'}^0$ and the physical masses as $N_8$, $N_{8'}$, $\Sigma_8$, $\Sigma_{8'}$, $\Xi_8$, $\Xi_{8'}$, $\Lambda_1$, $\Lambda_8$, $\Lambda_{8'}$. The bare masses of the two octets are\footnote{1}:

\[
\begin{align*}
N_8^0 &= M_1 - x_1 + y_1, & N_{8'}^0 &= M_2 - x_2 + y_2, \\
\Lambda_8^0 &= M_1 - 2x_1, & \Lambda_{8'}^0 &= M_2 - 2x_2, \\
\Sigma_8^0 &= M_1 + 2x_1, & \Sigma_{8'}^0 &= M_2 + 2x_2, \\
\Xi_8^0 &= M_1 - x_1 - y_1, & \Xi_{8'}^0 &= M_2 - x_2 - y_2.
\end{align*}
\]

A consequence of Eq. (11) is the Gell-Mann–Okubo (GMO) relation for octet masses

\[
\frac{N_8 + \Xi_8}{2} = \frac{3\Lambda_8 + \Sigma_8}{4}.
\]

The physical masses can be obtained from the diagonalization of the matrices

\[
\begin{pmatrix}
N_8^0 & V_N \\
V_N & N_{8'}^0
\end{pmatrix}, \quad \begin{pmatrix}
\Sigma_8^0 & V_\Sigma \\
V_\Sigma & \Sigma_{8'}^0
\end{pmatrix}, \quad \begin{pmatrix}
\Xi_8^0 & V_\Xi \\
V_\Xi & \Xi_{8'}^0
\end{pmatrix}, \quad \begin{pmatrix}
\Lambda_1^0 & V_1 & V_2 \\
V_1 & \Lambda_8^0 & V_3 \\
V_2 & V_3 & \Lambda_{8'}^0
\end{pmatrix}.
\]

Take the special $\Lambda$ for example, which mixes twice. The first is the $\Lambda_8$ and $\Lambda_{8'}$ mixing, the second is the $\Lambda_1^0$ and $\tilde{\Lambda}_8$ mixing

\[
\left(\begin{array}{c}
|\tilde{\Lambda}_8\rangle \\
|\Lambda_{8'}\rangle
\end{array}\right) = \begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix} \left(\begin{array}{c}
|\Lambda_8^0\rangle \\
|\Lambda_{8'}^0\rangle
\end{array}\right),
\]

\[
\left(\begin{array}{c}
|\Lambda_1\rangle \\
|\Lambda_8\rangle
\end{array}\right) = \begin{pmatrix}
\cos \xi & \sin \xi \\
-\sin \xi & \cos \xi
\end{pmatrix} \left(\begin{array}{c}
|\Lambda_1^0\rangle \\
|\Lambda_8^0\rangle
\end{array}\right). \quad (14)
\]

For simplicity, we use $\Lambda$ instead of $|\Lambda\rangle$ to denote the state. We write the bare states from the physical states using the mixing of $3 \times 3$ matrices as follows:

\[
\begin{pmatrix}
\Lambda_1^0 \\
\Lambda_8^0 \\
\Lambda_{8'}^0
\end{pmatrix} = \begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \theta & -\sin \theta \\
0 & \sin \theta & \cos \theta
\end{pmatrix} \begin{pmatrix}
\Lambda_1^0 \\
\Lambda_8^0 \\
\Lambda_{8'}^0
\end{pmatrix},
\]

\[
= \begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \theta & -\sin \theta \\
0 & \sin \theta & \cos \theta
\end{pmatrix} \begin{pmatrix}
\cos \xi & -\sin \xi & 0 \\
\sin \xi & \cos \xi & 0 \\
0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
\Lambda_1 \\
\Lambda_8 \\
\Lambda_{8'}
\end{pmatrix}. \quad (15)
\]
Using (13) and (15), we can get the result

\[
\left( \begin{array}{ccc}
\Lambda_1 & 0 & 0 \\
0 & \Lambda_8 & 0 \\
0 & 0 & \Lambda_{8'}
\end{array} \right) = \left( \begin{array}{ccc}
\Lambda_1 & \Lambda_S & \Lambda_{8'}
\end{array} \right) \left( \begin{array}{ccc}
\Lambda_1 & 0 & 0 \\
0 & \Lambda_8 & 0 \\
0 & 0 & \Lambda_{8'}
\end{array} \right),
\]

(16)

where \( \Lambda \) physical masses are

\[
\Lambda_1 = \cos^2 \xi \Lambda_1^0 + \cos^2 \theta \sin^2 \xi \Lambda_1^0 + \sin^2 \theta \sin^2 \xi \Lambda_{8'},
\]

+ \cos \theta \sin 2\xi V_1 + \sin \theta \sin 2\xi V_2 + \sin 2\theta \sin^2 \xi V_3,
\]

\[
\Lambda_8 = \sin^2 \xi \Lambda_1^0 + \cos^2 \theta \cos^2 \xi \Lambda_1^0 + \sin^2 \theta \cos^2 \xi \Lambda_{8'},
\]

− \cos \theta \sin 2\xi V_1 - \sin \theta \sin 2\xi V_2 + \sin 2\theta \cos^2 \xi V_3,
\]

\[
\Lambda_{8'} = \sin^2 \theta \Lambda_1^0 + \cos^2 \theta \Lambda_{8'}^0 - \sin 2\theta V_3.
\]

(17)

Meanwhile the physical masses of other baryons are \( \Xi \)

\[
N_{s,s'} = \frac{1}{2} \left( N_8^0 + N_{8'}^0 \pm \sqrt{(N_8^0 - N_{8'}^0)^2 + 4V_N^2} \right),
\]

\[
\Sigma_{s,s'} = \frac{1}{2} \left( \Sigma_8^0 + \Sigma_{8'}^0 \pm \sqrt{\Sigma_8^0 - \Sigma_{8'}^0)^2 + 4V_\Sigma^2} \right),
\]

\[
\Xi_{s,s'} = \frac{1}{2} \left( \Xi_8^0 + \Xi_{8'}^0 \pm \sqrt{\Xi_8^0 - \Xi_{8'}^0)^2 + 4V_\Xi^2} \right),
\]

(18)

where

\[
V_N = \frac{1}{2} (N_8 - N_{8'}) \sin 2\delta,
\]

\[
V_\Sigma = \frac{1}{2} (\Sigma_8 - \Sigma_{8'}) \sin 2\beta,
\]

\[
V_\Xi = \frac{1}{2} (\Xi_8 - \Xi_{8'}) \sin 2\gamma,
\]

\[
V_1 = \frac{1}{2} \cos \theta \sin 2\xi (\Lambda_1 - \Lambda_8),
\]

\[
V_2 = \frac{1}{2} \sin \theta \sin 2\xi (\Lambda_1 - \Lambda_8),
\]

\[
V_3 = \frac{1}{2} \sin 2\xi (\sin^2 \xi (\Lambda_1 + \cos^2 \xi \Lambda_8 - \sin 2\theta \Lambda_{8'}). \]

(19)

From (17), (18), we get a new relation among the baryon masses

\[
3(\sin^2 \xi \Lambda_1 + \cos^2 \xi \Lambda_8 + \Lambda_8') + (\Sigma_8 + \Sigma_{8'}) = 2(N_8 + N_{8'} + \Xi_8 + \Xi_{8'}). \]

(20)

For getting more information of \( \Xi \), we need to know the parameters. However, there are 12 parameters \( (M_1, x_1, y_1, M_2, x_2, y_2, M_{\Lambda_1}, \delta, \theta, \xi, \beta, \gamma) \), while we only know 7 physical masses \( (N_8, N_{8'}, \Sigma_8, \Sigma_{8'}, \Lambda_1, \Lambda_8, \Lambda_{8'}) \) from PDG. Therefore we use the angles obtained from decay widths and physical masses to calculate the parameters \( M_1, x_1, y_1, M_2, x_2, y_2, M_{\Lambda_1} \). Then, we can get all bare masses.

Meanwhile, there is a relation

\[
2V_\Xi = (\Xi_8^0 - \Xi_{8'}^0) \tan 2\gamma.
\]

(21)
Using (18), (20), (21) and according to the experimental $\Gamma_{\Xi_{\text{total}}}$, we predict the mixing angle ranges and the physical mass ranges of the two $\Xi$’s.

4. Results

The detailed calculating processes are as follows. At first, we use the $N$ and $\Lambda$ experimental decay width data and the least square method to get the parameters $(\delta, \theta, \xi, \alpha, A_8, A_{10}, A_1, \alpha', A'_8, A'_{10})$. Secondly, we use $\Sigma$ width data to get $\beta$. Thirdly, we use the angles obtained from decay widths and physical masses to calculate the parameters $M_1, x_1, M_2, x_2, y_2, M^0_{A_1}$. Finally, we use the obtained angles and the mass parameters to get $\gamma$ range and the $\Xi$ mass ranges of the two octets, and further, to predict the decay width ranges of the two $\Xi$’s. The parameters are

$$
\delta = -4.5^\circ, \quad \theta = -26.4^\circ, \quad \xi = -34.9^\circ, \quad \beta = -31.4^\circ,
$$

$$
\alpha = -0.697, \quad A_8 = 0.889, \quad A_{10} = 2.04, \quad A_1 = 1.65,
$$

(22)

and

$$
M_1 = 1601.2, \quad x_1 = -12.5, \quad y_1 = -77.9, \quad M^0_{A_1} = 1491.7,
$$

$$
M_2 = 1680.4, \quad x_2 = -38.4, \quad y_2 = -69.5.
$$

(23)

According to the experimental $\Gamma_{\Xi_{\text{total}}}$, we predict that the $\Xi$ mixing angle range $\gamma$ is between $-36.0^\circ$ and $-29.0^\circ$, meanwhile, we predict that two possible $\Xi$ mass ranges of the two octets are $\Xi(1583-1649)$ and $\Xi'(1831-1896)$. We list all results in Table 1. Most predicted decay widths are consistent with the experimental data.

5. Discussions

In our results, most predicted widths are consistent with the experimental data. The predicted $\Gamma_{N(1535)\rightarrow N\eta}$ and $\Gamma_{N(1650)\rightarrow N\pi}$ become broader, which fit the experimental data. The theoretical $\Gamma_{\Sigma(1560)_{\text{total}}}$ agrees with the experimental data. The mixing angle between the octet and the singlet is $-34.9^\circ$, which is smaller and in the range $15^\circ < |\xi| < 35^\circ$. It also predicts two possible $\Xi$ mass ranges. However, the $\sqrt{\frac{1}{N_K} \cdot \Sigma(1385)_{\pi}}$ is lower. There are three reasons for this. One is that the coupling and the phase space of $\Gamma_{N(1535)\rightarrow \Delta\pi}$ are both bigger than that of $\Gamma_{N(1670)\rightarrow \Sigma(1385)_{\pi}}$; the second is that the small $\Gamma_{N(1535)\rightarrow \Delta\pi}$ (most 1.75 MeV) makes $A_{10}$ not large; and the third is that the mixing of two octets suppresses $\Gamma_{N(1670)\rightarrow \Sigma(1385)_{\pi}}$ by a factor 2.03.

We also have tried three possible ways to include the mixing. The first is just what we have done above, but to introduce same octet’s mixing angles before their mixing with the singlet, the second is to consider only two octet mixing without their mixing with the singlet, the third is to consider the mixing of the octet with the singlet firstly and then consider the mixing between the two octets. However, we cannot find appropriate results which are consistent with all experimental data. So as an example, we only show the results from the first way, i.e., to consider
the two octet mixing first, and then mix the mixed state $\tilde{\Lambda}(1670)$ with $\Lambda(1405)$. The results suggest that the octet mixing might be a feasible effect to improve the agreement between theory and experiments. However, it also suggests us that other mechanism and/or dynamical effects need to be introduced for a better description of all available experimental data.

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References

1. M. Gell-Mann, California Institute of Technology, Report CTSL-20, March, 1961 (unpublished); *Phys. Rev.* **125**, 1067 (1962).
2. Y. Ne’eman, *Nucl. Phys.* **26**, 222 (1961).
3. M. Gell-Mann, Y. Ne’eman, *The Eightfold Way*, W.A. Benjamin, Inc., Amsterdam and New York, 1964.
4. N.P. Samios, M. Goldberg and B.T. Meadows, *Rev. Mod. Phys.* **46**, 49 (1974).
5. L. Gomberoff, V. Tolmachev, *Phys. Rev.* **187**, 2185 (1969).
6. V. Guzey, M.V. Polyakov, arXiv:hep-ph/0512355.
7. Particle Data Group, W.-M.Yao, et al., *J.Phys.G* **33**, 1 (2006).
8. J.G. Rushbrooke, *Phys. Rev.* **143**, 1345 (1966).
9. D. Diakonov, V. Petrov, *Phys. Rev. D* **69**, 094011 (2004).