The experimental system of automatic wheel drift and vehicle rollover prevention

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Abstract. The research topic relevance is determined by the global significance level of traffic safety improvement. The purpose of the work is to create mathematical model and software for the active vehicle safety experimental intelligent system. To solve this problem, methods of indirect vehicle state variables measurements were used. As a mathematical model, we used a system of equations for the wheels rotation speed on a bend. For estimating the parameters of the mass center and the wheels motion, standard wheel speed sensors and algorithms for processing the measurement data are used. The task of angular drift velocity determining based on wheels rotation frequency measurements is ill-posed and additional conditions are introduced to solve it. To predict the wheel drift and car rollover, extrapolate algorithms are used. These phenomena are prevented by the certain brake formation that limit the object speed at the predicted boundaries level. The research novelty is to create algorithms for an experimental system that provides the solution to the wheel drift and rollover prevention problem in a minimal configuration of technical means.

1. Introduction

The road safety improvement problem is one of the most important problems in modern society and its importance is accepted globally. A systematic analysis of this problem [1] shows that one of the most promising way for its solution is the creation of various vehicle active safety systems.

Full collision avoidance systems claim not only developments in the field of technical facilities but also mathematical models and software. Intellectual properties of modern systems are acquired through the virtual information sensors usage, which allow to increase system information functions in a minimal technical means configuration. Virtual information sensors are algorithmic constructions that replace object state parameters physical sensors. The development of such constructions requires new mathematical models for solving ill-posed problems and algorithms for data processing.

It should be noted that noticeable improvements in the traffic safety should be expected in case of equipping the entire operating fleet, including low price models, with effective active safety systems.

A necessary condition for the unmanned vehicles safe operation is installation of full-featured active safety systems. These circumstances set the trend for the advanced intellectual active safety systems development in the near future.

Among the electronic systems for driving stability improvement historically, an antilock braking system (ABS) stands out, which allows to maintain car controllability and stability during emergency

\[ \text{Equation} \]

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braking [2]. On the basis of ABS an electronic stability control system (ESC) was created and is now included in the configuration of the new cars. This system is mainly aimed at oversteer and understeer suppression of a car [3]. In addition, to increase the driving stability, some manufacturers implement software extension of the basic ABS system, among which are the electronic brake force distribution system and the braking stabilization function in the turn [4]. The traction control system, the extended function of the electronic differential lock, the trailer stabilization function are also referred to the driving stability improvement systems [5].

Despite significant advances in the development of systems for driving stability improvement, each system has shortcomings, caused by both the laid-down operation principles and the physical limitations of the wheel-to-road grip. For example, the ESC system is effective in preventing road accidents of single cars [6], caused by road departure with subsequent overturning [7] and characterized by severe consequences. At the same time, in conditions of dense traffic, ESC systems are often unable to prevent vehicle collisions caused by oversteer or understeer. Among the significant drawbacks of ABS is a significant increase of the braking distance at braking on ice, snow, on roads that do not have a hard coating, on uneven surfaces [8].

This scientific research aims to set and solve the wheel drift and vehicle rollover prevention problem. The developed mathematical model and software is intended for use as part of the experimental intellectual active safety system.

2. Task statement
At the conceptual level, the task of wheel drift and vehicle rollover prevention is reduced to prediction of the dangerous speed limits excess and necessary control actions on the traction, brakes and steering wheel ensuring.

At the content level, the task under consideration is reduced to the dynamic velocity stabilization of the mass center at limited control actions, formed in a hardware/software environment that meets the cost and operating cost constraints.

To assess the solution quality, we use a modified quadratic control quality functional:

\[
Q(t_2) = \int_{t_1}^{t_2} C_0 \left[ V_m(\tau) - V_{\text{bound}}(\tau) \right]^2 \, d\tau + \int_{t_1}^{t_2} \Delta C(R, \tau) \, d\tau + Q(R, t_1)
\]  

at \( U \in U_{\text{add}} \) and \( R \in R_{\text{add}} \), where

\[
V_m(\tau) - \text{longitudinal velocity of the vehicle mass center;}
\]

\[
V_{\text{bound}}(\tau) - \text{high limit of the safe speed, defined as } \min \left[ V_{\text{bound}0}, V_{\text{bound}1}, V_{\text{bound}2} \right];
\]

\[
V_{\text{bound}0} = \sqrt{0.5abh_m^{-1} |\Psi_c|^{-1}};
\]

\[
V_{\text{bound}1} = \text{Re} \sqrt{2[m_{12}gb - Ra_d a_{\text{dr}}]k_{sq} |\Psi_c|^{-1}};
\]

\[
V_{\text{bound}2} = \text{Re} \sqrt{2[m_{34}gb + Ra_d a_{\text{dr}}]k_{sq} |\Psi_c|^{-1}};
\]

\( a \) and \( b \) – wheel track width and wheelbase, respectively;

\( h_m \) – height of the mass center;

\( \Psi_c \) – resulting angle of the steerable wheels, taking into account the angle of lateral drift;

\( m_{12} \) and \( m_{34} \) – the masses of the front and rear axle wheels respectively;

\( g \) – gravity acceleration;

\( Ra_d \) – dynamic radius of the driving wheels;

\( a_{\text{dr}} \) – traction-braking acceleration;

\( k_{sq} \) – sliding friction coefficient of the wheels in the transverse direction;

\( U = \left( U_1, U_2, U_3, U_4 \right)^T \) – control actions vector on the actuator, including gearbox \( U_1 \), accelerator \( U_2 \), brakes \( U_3 \) and steering wheel \( U_4 \);
The best solution to the problem (1) in the above statement is the control actions $U(t)$ sequence realized in the hardware and software environment $R$, which ensures the stabilization of $\mathbf{v}(t)$ at the level $V_{\text{bound}}(t)$ in a system with minimal operating costs and initial cost.

3. Research results

To solve this problem indirect measurements are used based on the mathematical models of the object and algorithms for solving ill-posed problems. The resulting virtual information sensors, replacing physical ones can significantly reduce the cost of operation and the resulting system cost.

As an indirect measurements mathematical model of the vehicle wheel drift angular velocity a system of equations for the linear speeds of wheels rotation $V_i, 1 \leq i \leq 4$ on the bend is used:

$$\begin{align*}
V_1 &= V_m + 0.5 ab^{-1} V_m \Psi_c + \Delta V_{S1} + 0.5 a \Delta \omega_m; \\
V_2 &= V_m - 0.5 ab^{-1} V_m \Psi_c + \Delta V_{S2} - 0.5 a \Delta \omega_m; \\
V_3 &= V_m + 0.5 ab^{-1} V_m \Psi_c + \Delta V_{S3} + 0.5 a \Delta \omega_m; \\
V_4 &= V_m - 0.5 ab^{-1} V_m \Psi_c + \Delta V_{S4} - 0.5 a \Delta \omega_m,
\end{align*}$$

where $\Delta \omega_m$ – drift angular velocity of the car wheels; $\Delta V_{S1}$ – longitudinal slip speed of the i-th wheel; $V_i$ – linear rotation speed of the i-th wheel ($1 \leq i \leq 4$); $V_m$ – linear velocity of the mass center longitudinal motion.

The solution (3), taking into account Euler’s formula $\omega_m = V_m \cdot R_m^{-1}$ and $R_m = b \cdot \Psi_c^{-1}$, in discrete time is reduced to the form:

$$\Psi_m(k) = \Psi_m(k - 1) + b^{-1} \int_{t_{k-1}}^{t_k} V_m(\tau) \Psi_c(\tau) d\tau + \int_{t_{k-1}}^{t_k} \Delta \omega_m(\tau) d\tau$$

The increment of the course angle $\Delta \Psi_m(k)$ at the k-th step is:

$$\Delta \Psi_m = \Psi_m(k) - \Psi_m(k - 1) = b^{-1} \int_{t_{k-1}}^{t_k} V_m(\tau) \Psi_c(\tau) d\tau + \int_{t_{k-1}}^{t_k} \Delta \omega_m(\tau) d\tau$$

In case of rear wheels oversteer, $\Delta \omega_m$ coincides in sign with $\Psi_c$ at $V_m > 0$, which leads to an increase in the module of the increment of the course angle. In the case of front wheels understeer, $\Delta \omega_m$ has the opposite sign to $\Psi_c$, which reduces the increment module of the course angle. Additional rotation (drift) with the frequency $\Delta \omega_m$ occurs around the front wheels axis center when they are oversteer and around the rear wheels in case of understeering.
The reason for the car wheels drift is the excess of the centrifugal force acting on the front and rear axle wheels, the sliding friction forces of the corresponding wheel pairs in the tire contact patch with the coating in the transverse direction.

The estimates $\hat{V}_i (k) = \hat{R}_{ct}(k) \cdot \hat{\omega}_i (k), 1 \leq i \leq 4$ are formed from the rotation frequencies of the wheels $\omega_i (k)$ and the tuning data of the wheels free radii $R_{ct}(k)$.

The equations system of wheels rotation speeds for the car on a steep turn, contains 4 equations with 7 variables, and the task of their definition concerns to ill-posed ones.

To convert the original ill-posed problem to the correct one, we introduce the following additional conditions characterizing the physical properties of the object.

The additional condition number 1 is the longitudinal acceleration and wheel slip longitudinal velocities signs consistency.

So, in particular, object is characterized by positive sliding of the wheels on the horizontal surface during acceleration and negative during decelerations, which corresponds to the consistency of the $a_m$ and $\Delta V_{sl}$ ($1 \leq i \leq 4$) signs:

$$
\begin{align*}
&\text{if } a_m \geq 0, \text{then } \Delta V_{sl} \geq 0; \\
&\text{if } a_m < 0, \text{then } \Delta V_{sl} \leq 0.
\end{align*}
$$

(6)

The additional condition number 2 is the limited longitudinal acceleration.

The longitudinal mass center acceleration $a_m$ is limited by the maximum sliding frictional forces of the wheels, which corresponds to the constraint system:

$$
a_{bound 1}^l \leq a_m \leq a_{bound 1}^u,
$$

(7)

$a_{bound 1}^l$ – low mass center acceleration limit at braking, and $a_{bound 1}^u$ – high mass center acceleration limit during acceleration.

The additional condition number 3 is the determined magnitude and sign of the angular drift velocity.

The magnitude and sign of the wheels drift angular frequency $\Delta \omega_m$ depends on the ratio of the centrifugal force and the wheels sliding frictional force in the transverse direction for the wheels of the front and rear axle and reduces to the following condition:

$$
\begin{align*}
&\text{if } V_m \leq \min [V_{bound 1}, V_{bound 2}], \text{then } \Delta \omega_m = 0; \\
&\text{if } V_{bound 1} > V_{bound 2} \text{ and } V_m > V_{bound 2}, \text{then } \text{sgn}(\Delta \omega_m) = \text{sgn}(\Psi_c); \\
&\text{if } V_{bound 2} > V_{bound 1} \text{ and } V_m > V_{bound 1}, \text{then } \text{sgn}(\Delta \omega_m) = -\text{sgn}(\Psi_c),
\end{align*}
$$

(8)

where the boundary understeer and oversteer wheels velocities $V_{bound 1}$ and $V_{bound 2}$ are determined from the equality conditions of sliding frictional forces to the corresponding centrifugal forces of the front and rear wheels.

The solution of the original problem with the additional conditions in discrete time is reduced to the form:

$$
\hat{V}_m (k) = \begin{cases} 
V_m (k) + 0.5 |\Delta V_{sl} + \Delta V_{sj}|, & \text{if } a_{bound 1}^l \leq a_m (k) \leq a_{bound 1}^u; \\
\hat{V}_m (k - 1) + \Delta T a_{bound 1}^l, & \text{if } a_m (k) > a_{bound 1}^u; \\
\hat{V}_m (k - 1) + \Delta T a_{bound 1}^l, & \text{if } a_m (k) < a_{bound 1}^l,
\end{cases}
$$

(9)

where $|\Delta V_{sl} + \Delta V_{sj}| = \min \{ |\Delta V_{s1} + \Delta V_{s2}|, |\Delta V_{s3} + \Delta V_{s4}|, |\Delta V_{s1} + \Delta V_{s4}|, |\Delta V_{s2} + \Delta V_{s3}| \}$;

$$
\Psi_c (k) = \Psi_c (k) + a^{-1} b V_m^{-1} (\Delta V_{sl} - \Delta V_{sj}) + b V_m^{-1} \Delta \omega_m;
$$

(10)
\[ \Delta \tilde{\omega}_m(k) = \begin{cases} 
0, & \text{if } \frac{\tilde{V}_m(k)}{V_{\text{bound} 1}} \leq V_{\text{bound} 1} \text{ and } \frac{\tilde{V}_m(k)}{V_{\text{bound} 2}} \leq V_{\text{bound} 2}; \\
- b^{-1} \tilde{V}_m(k) \left[ \Psi_c(k) - |\Psi_{\text{bound} 2}| \text{sgn}(\Psi_c) \right], & \text{if } V_{\text{bound} 1} > V_{\text{bound} 2} \text{ and } \frac{\tilde{V}_m(k)}{V_{\text{bound} 1}} > \frac{\tilde{V}_m(k)}{V_{\text{bound} 2}}; \\
- b^{-1} \tilde{V}_m(k) \left[ \Psi_c(k) - |\Psi_{\text{bound} 1}| \text{sgn}(\Psi_c) \right], & \text{if } V_{\text{bound} 2} > V_{\text{bound} 1} \text{ and } \frac{\tilde{V}_m(k)}{V_{\text{bound} 1}} > \frac{\tilde{V}_m(k)}{V_{\text{bound} 2}}. 
\end{cases} \quad (11) \]

where \( |\Psi_{\text{bound} 1}| = 2 [m_{12} g b - R_d a_{dr}] k_{sq} V_m^{-2}; |\Psi_{\text{bound} 2}| = 2 [m_{34} g b + R_d a_{dr}] k_{sq} V_m^{-2}. \)

The traction-braking acceleration \( a_{dr} \) is determined from the equation of the longitudinal motion of the vehicle mass center with the identified parameters:

\[ a_{dr} = a_m + k_x m_0^{-1} V_m^2 + k_{fr} g + \alpha_p g, \quad (12) \]

\( k_x \) – wind drag coefficient;  
\( m_0 \) – vehicle weight;  
\( k_{fr} \) – tire rolling friction coefficient;  
\( \alpha_p \) – pitch angle.

To determine the wheels sliding friction coefficient in the transverse direction \( k_{sq} \), the circle Kamma property is used:

\[ k_{Sdi}^2 + k_{Sqi}^2 = (k_{Si}^*)^2, \quad 1 \leq i \leq 4 \quad (13) \]

where \( k_{Sdi} \) and \( k_{Sqi} \) are the sliding friction coefficients of the i-th wheel in the longitudinal and transverse directions, respectively;  
\( k_{Si}^* \) is the top (maximum) value of the i-th wheel sliding friction coefficient.

The \( k_{Sqi} \) value, determined from (13) is equal to:

\[ k_{Sqi} = k_{Si}^* \sqrt{1 - k_{Sdi}^2 (k_{Si}^*)^{-2}} \quad (14) \]

The \( k_{Sdi} \) values are determined in accordance with Newton's third law from the equilibrium equation of traction-braking forces and sliding friction forces: the traction-braking force \( F_i \) is balanced by the sliding friction force \( F_{Si} = F_{Ni} \cdot k_{Sdi} \), where \( F_{Ni} \) is the normal component of the dynamic load on the i-th wheel.

Module \( |k_{Sdi}| \) is determined from the forces equilibrium equation: \( |k_{Sdi}| = |F_i| F_{Ni}^{-1} \).

When the traction-braking forces modulus are small, according to (14), the sliding friction coefficient of the i-th wheel in the transverse direction is \( k_{Sqi} = k_{Si}^* \).

Identification of \( k_{Si}^* \) is performed using software in the process of vehicle movement [9]. The input data for solving this problem are the measured wheel slip \( \tau_e \) and the current sliding friction coefficients of the wheels in the longitudinal direction \( k_{Sdi} \).

The exceeding of the boundary velocities are predicted by extrapolating the boundary velocities \( V_{\text{bound} 1}, V_{\text{bound} 2}, \) and \( V_m \) by the time \( \tau_e \) and verifying the inequality fulfillment:

\[ V_{\text{bound} e} < V_{me}, \quad (15) \]

where \( V_{\text{bound} e} = \min[V_{\text{bound} 0}, V_{\text{bound} 1}, V_{\text{bound} 2}]; \)
\[ V_{\text{bound} 0} = \sqrt{0.5 a b h^{-1} |\Psi_{ce}|^{-1}}; \]
\[ V_{\text{bound} 1} = \text{Re} \sqrt{2 [m_{12} g b - R_d a_{dr}] k_{sq} |\Psi_{ce}|^{-1}}; \]
\[ V_{\text{bound} 2} = \text{Re} \sqrt{2 [m_{34} g b + R_d a_{dr}] k_{sq} |\Psi_{ce}|^{-1}}; \]
\[ \Psi_{ce} = \Psi_c(k) + \tau_e \Delta T^{-1} [\Psi_c(k) - \Psi_c(k - 1)]; \]
\[ V_{me} = \tilde{V}_m(k) + \tau_e \Delta T^{-1} [\tilde{V}_m(k) - \tilde{V}_m(k - 1)]. \]
The braking deceleration $a_{dr} = a_d - a_T$ required to prevent drift and rollover is determined from (12) at $a_m(k) = [V_{bound} - V_{me}]\Delta T^{-1}$.

The technical devices of the experimental wheel drift and car rollover preventing system include:
- standard wheel speed sensors;
- actuator of the brake pedal;
- control board;
- autonomous power source.

Currently, the system under development is being tested at the NAMI testing ground in Dmitrov. The analysis of preliminary test results does not disprove the adequacy of the used model and algorithms.

4. Conclusion
The analysis of the conducted studies results allows us to formulate the following conclusions:
- formulated and solved the task of safe speed dynamic stabilization for bends passing, wheel drift and car rollover prevention;
- developed algorithm of traction-braking acceleration control allows to integrate functions of separate regular active safety systems into one, implemented in a minimal hardware configuration;
- experimental system for wheel drift and car rollover prevention can function with incomplete wheel speed sensors configuration, reduced power consumption and higher operational reliability.

Acknowledgments
This scientific article was prepared based on the results of applied scientific research financially supported by the Education and Science Ministry of Russia, agreement is No. 14.625.21.0042, the unique identifier of the project is RFMEFI62517X0042.

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