Study on Design modifications and feasibility in Conversion of scroll compressor into Scroll expander for using as battery charger.

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ABSTRACT

Scroll expanders are well suitable for low grade power production with an ORC. In this paper, a scroll compressor design aspect in conversion to a compact scroll expander is taken. The major application of scroll expander is to generate power. The investigation of its application to charge batteries in automobiles is done to extend its application range. This analysis is done in two stages. In the first stage, a refrigeration scroll type compressor is taken to make it as a scroll type expander, as the availability of scroll type expanders are not wide in range. Its design modification has taken to make it compact and suitable for use as a battery charger with compressed air as working fluid. A mathematical simulation is done to find the design and thermodynamic properties like redesigning of the shaft to maintain eccentricity of rotating scroll and pressure ratio that influence the mechanical power developed in the scroll expander. A proto type of scroll expander is developed to conduct experiments for further analysis. In the second stage, the numerical feasibility analysis is done to verify the factors like speed and torque that influence the electrical power generated through mechanical work. In the trials carried, a voltage of 12.23 has been developed at inlet pressure of 15 bar and temperature of 303k.

Keywords: scroll expander; Low grade energy; Power generation; ORC applications; Design.

1. Introduction

In the present energy scenario, the use of power as well as energy consumption is increasing tremendously. So, the need to search for energy production improvement has attained major importance. Even a small amount of heat available has to be utilized to improve power production to meet the demand. In this context, Scroll expanders are well suitable for low grade power production [1-2] with an ORC. Heat engines of low capacity require proper thermodynamic cycle selection and a correct prime mover. Ammonia- water or ORC are well suitable for the low capacity heat engine cycles[3-10]. ORC is considered in this paper. By coupling this scroll expander with sources from low capacity heat energy like
Geothermal, Solar and in conventional power plants as secondary turbines, we can generate electricity for minimum usage purposes like charging batteries in bikes, cars and in other automobiles. If this scroll expander is made compact, its portability can be increased. This analysis is carried in two stages. In the first stage, a refrigerant scroll compressor is modified to a scroll expander by modifying its design to reduce its robustness. In the second stage, study is conducted to verify the feasibility of its conversion to battery charger. CFD simulation is carried to verify the model obtained with experiment in later stages. Zhiewei Ma [11] modified the geometry of scroll for enhancing the scroll type expander efficiency by decreasing the losses. J.W. Bush and W.P. Beagle [12] worked on profiles of scroll expander and defined a conjugacy method of approach to design scroll profiles. Shen-Jenn Hwang [13] carried out numerical study on elliptical involute and cross section of scroll compressor and stated that an elliptical scroll with normal cross section is efficient. In his research, he concluded that for the compression process, parabolic, trapezoidal cross section produces less efficiency when compared with equal wall thickness cross section of the scroll type expander.

2. Design Modifications:

In general, scroll machines have low-capacity and are used in the refrigeration industry as compressors. Using scroll machines as expanders for power generation is not a new concept, but it was not used widely too. In order to increase capability of low capacity power generation and extend their applications to various industries, vast research has to be done. In the current research scroll compressor ZR28KM-PFZ-582 is taken for converting in to scroll expander and as per manufacturer catalogue, its rated speed is 2900 RPM and shaft power is 2.3 HP with net weight of 4kg working with R22 as working fluid. The basic criteria in this regard is to know the scroll design which was defined by Halm [14], for this purpose, the simplest way is to reverse a refrigeration scroll compressors and make it to work as expanders, as they can work in a reversible manner. In reversing scroll compressor to expander operation [16], similar pressure, temperature and volumetric ratio are to be maintained.

2.1 Study of Scroll Modal

Scroll type expanders comes under classification of positive-displacement machines which consists of two spiral shaped scroll fitted one over the other such that one is fixed and other will be rotating forming orbital motion. Out of the two scrolls, the rotating scroll is inverted over the fixed one and lower scroll is maintained in a particular angular position to allow radial movement and restricts its rotation in orbital path by a specially designed Oldham coupling. Scroll expander consists of scroll wraps which are involute of a circle and inlet port is arranged in the middle of stator and rotor as eccentric arrangement of inlet port leads in asymmetric pressure distribution [15]. The shaft of scroll compressor ZR28KM-PFZ-5982 is 90mm with diameter of 25mm and it has been redesigned to 19.5 mm and diameter of 10mm trial and error method is adopted to reach eccentricity of rotor . The following figure is the redesigned shaft.
2.2 Evaluation parameters for the Performance of scroll type expanders

The expansion process in scroll type expansion is considered as non isentropic. The leakage losses, heat transfer losses, suction losses and uneven expansion in the scroll chambers that occur during the expansion process in the scroll type expander and scroll complex geometry is considered to be the major factor for deviation from actual isentropic cycle. To evaluate these irreversible losses, the important parameter that has to be determined is isentropic efficiency. When scroll type expander used in power generation, the overall mechanical losses along with transmission loss due to belt and pulley can be determined by using the ratio of output mechanical power to the overall enthalpy drop from inlet to outlet. The notations followed in this paper are presented in the nomenclature section. For a scroll expander, the following relations are used for determining isentropic and overall efficiencies.

\[
\frac{h_{\text{in}}-h_{\text{out}}}{h_{\text{in}}-h_{\text{is}}} = \eta_{\text{is}} \quad \text{and} \quad \frac{W_m}{m(h_{\text{in}}-h_{\text{out}})} = \eta_{\text{m}}
\]

In order to perform numerical simulation of scroll expander, its working process need to be evaluated based on the geometric structure.

2.3 Expander working process

The process of working includes suction, expansion and discharging of working fluid under consideration inside the scroll chambers as in fig.2.
The suction process is considered as isenthalpic process and scroll expander inlet port is maintained with small diameter to attain high pressure supply. The expansion process cannot be isentropic due to losses and in the discharging process, uneven expansion loss occurs.

In conversion of compressor to expander, the discharge of compressor is made as suction port of expander and the compressor suction port is made as discharge of expander. Hence the top of the scroll machine as an expander will be high pressure end and low pressure is discharge port. The sequence of processes in compressor like suction process, compression process, discharge process and volume are expressed as functions of involute and scroll orbiting angle of compressor can also be used for scroll expander operation. Wang et al. [17] gave analytical expressions as orbiting angle functions for chambers of scroll type compressors. The volume equation for expander intake is taken as

\[
V_{intake} = HR_0 \cdot R \cdot \theta (\theta - \phi_i + \phi_o + 3\pi); \quad (i)
\]

Where for intake process, \(0 \leq \theta \leq 2\pi\), of scroll expander, one full orbit is taken for this intake process.

The expansion chamber volume of scroll type of expander can be evaluated from the equation

\[
V_{expander} = 2\pi HR_0 \cdot R_0 \cdot (2\theta - (\phi_i - \phi_o - \pi)); \quad (ii)
\]

Where \(2\pi \leq \theta \leq 2\pi\), similarly at \(\phi_i - 2\pi \leq \theta \leq \phi_o\) corresponds to discharge process and discharge chamber volume can be evaluated by

\[
V_{discharge} = HR_0 \cdot R_0 \cdot (2\theta - 2\phi_i + (\phi_o - \theta) - (\phi_i - \phi_o)) \cdot (\phi_o + \phi_o + \pi) + 2(1 - \cos(\phi_i - \theta) - 2(\phi_i - \pi) \sin(\phi_i - \theta) - \sin(2(\phi_i - \theta)); \quad (iii)
\]

The built in volumetric ratio of scroll type expanders can be deduced by the following relation using the above equations (i-iii).
\[ V_{ee} = \frac{V_e(\phi - 2\pi)}{V_i(2\pi)} = \frac{(2\phi - \phi_i + \phi_o - 3\pi)}{(5\pi - \phi_i + \phi_o)} \]

The analysis of compression and expansion process in scroll type machines, volumetric ratio is the most determining parameter which directly influences either in expander or compressor and to determine coefficients of leakage flow using volumetric efficiency and isentropic of scroll. Detailed design modifications to make a compact expander from a scroll type refrigeration compressor is taken as priority to extend its application range is carried out in this paper.

In the compression process, the volume will be in the closing phase and hence will be less when compared with the volume at end of expansion. The expander designed from compressor should meet this criteria. In this conversion at initial stage, it is assumed to have no leakage and later it can be evaluated by applying the energy balance equation.

Certain design modifications are made to make it compact and to accustom dynamometer to calculate power and to convert it into a portable battery charger. The following figure 3 is the converted compressor to expander.

![Fig 3: Scroll compressor ZR28KM-PFZ-582 converted to expander with dynamometer on the top of the shaft.](image)

In Scroll expanders, the unsteady expansion process and thermal parameters change with orbiting angle. The gas forces that act on scroll while rotating will be transferred to the shaft which is rotating. The final output torque in scroll expander is majorly caused by the tangential forces. The current numerical feasibility analysis is done using force analysis of the design as expansion chamber thermal properties can be calculated only through cfd simulation. The force distribution for the eccentric shaft is given in fig 4.
2.4 Simulation of scroll type expander

Simulation is done using ANSYS v15 and a working model is built using CREO 5.0 software as shown in fig 5. As per design criteria adopted by manufacturing companies for verification of numerical values with the trail calculations of the expander. The numerical values obtained from the mathematical equations and results from simulation are in well agreement with trail results of the experiment. For simulation, dynamic mesh modal has been adopted.

Scroll expander consists of scroll type wraps which are involute of a circle with uniform thickness and inlet port is arranged in the middle of stator and rotor as eccentric arrangement of inlet port leads in asymmetric pressure distribution. As per this model volume in each chamber is determined by considering a base line at which the moving scroll is engaged with fixed scroll forming crescent shaped chambers also termed as conjugate points. For geometric models the parameters taken are listed in Table1.
Table 1: Geometric parameters of constant thickness.

| Parameter                        | Dimension  |
|----------------------------------|------------|
| Height of scroll                 | 50 mm      |
| Base radius                      | 11 mm      |
| Distance between outer and inner scroll | 8.055mm   |
| Width                            | 4.2mm      |
| Entire curve length              | 483.749mm  |

2.4.1 Flow Model

To evaluate the scroll profile, Ansys v15 software modeled on FVM (finite volume model) is used for numerical simulation. For flow analysis solving in Ansys, RANS turbulence model, renormalization and k-epsilon model transport equations are used, such that the accuracy improves and swirl flows can also be identified.

2.4.2 Dynamic Mesh

The domain under consideration consists of stator, rotor, outlet, case and working chambers as in fig 6. PRESTO pressure segregation method is adopted for calculation of pressure-velocity coupled equations. Scroll geometry is complex for evaluation because of rotor and stator parts. It is quite difficult to evaluate the interaction between moving parts and the fixed part. Simulation is carried to determine leakage losses, pressure variations and thermodynamic analysis and the parameters which have impact on the power output.

Fig 6: Meshing of components in bottom plate of scroll type expander.
Dynamic mesh approach is adopted to simulate the scroll expander. The mesh motion is controlled using CG_Motion macro in udf of fluent. Local remeshing and smoothing method are used for maintaining quality and skewness of mesh.

3. Feasibility for converting into battery charger

In order to convert a scroll compressor to expander for using it as a battery requires design modifications. In general based on the scroll compressor design, the scrolls i.e., both fixed and rotated are arranged at an eccentric angle mounted on a shaft of 19.5cm. For making it a portable expander, the first step is to redesign the shaft to accommodate the scroll weight and to maintain eccentric rotation of the rotating scroll. Shaft is also modified to hold a direct contact dynamometer [18] to convert rotational energy to electrical energy. From the dynamometer, DC electricity is generated which can be converted into AC by using an adaptor, the calculations for mechanical loads are given by Mirzaeva [19-21]. Initially experiments have to be conducted to verify the amount of power that is produced with this compact mode. Later, it can be remodeled for automobile applications. Based on the application, the shaft size has to be designed to accommodate corresponding weight and size of the scroll under usage.

Basic requirement for charging an automobile battery is the power input. Depending on the amount of power generated, the time of charging varies. The basic equations for determining voltage from a DC dynamometer are as follows

From ohm's law

\[ I = \frac{V}{R} \]

Where

\[ I \] – current;
\[ V \] – voltage;
\[ R \] – resistance.

The above formula can be applicable in many conditions like measuring the current that has been consumed and voltage applied and to calculate the generator resistance. The power output is evaluated by using

\[ P_{out} = V \cdot I_L \]

Where

\[ P_{out} \] – output power, measured in watts (W);
\[ V \] – Terminal voltage (volts);
\[ I_L \] – current.

Rotational speed obtained by measuring rpm using a tachometer is used for evaluating the angular speed can be calculated from the relation

\[ \omega = \frac{N \cdot 2\pi}{60} \]

Where

\[ \omega \] – Angular speed (rad/s);
N – RPM;
\( \pi \approx 3.14. \)

Torque measurement is a crucial task which requires special instruments. Current voltages are measured by using a multimeter. It can be numerically evaluated by

\[
\tau = \frac{P_{\text{out}} \times 60}{2\pi N}
\]

\( N \) – Speed in rpm;
\( \tau \) – Torque;
\( P_{\text{out}} \) – Power output (Watts);

4. Results and Discussions

From the above analysis, it is well understood that the major determining factors for converting scroll compressor to a compact scroll type expander in generating electricity include shaft design of scroll to maintain exact eccentricity, inlet port and outlet port arrangement. In the trail run of the prototype, it was observed that the electrical voltage generated is 12.23 volts. For a pressure ratio of 3.3, 3.5 and 3.8 with inlet pressure of 15 bar and 17 bar and outlet pressure of 5.15, 4.8 and 4.47 bar at various speeds, various power output and torque can be observed and optimum of all are considered and are shown in the following table 2,3 and 4 for 15 bar and table 5, 6 and 7 for maximum pressure of 17 bar. It can be observed from numerical calculations, with the rise in pressure ratio, power output decreases. Depending on load conditions and rotation of the scroll, the power generated varies and hence in conversion to a battery charger devices like DC to AC converters and resistors are used.

4.1 Simulation Results

From the simulation results, mechanical power output decreases with the increase in speed and loss increases which in turn decreases voltage and current. So an optimum speed is to be determined for maximum voltage generation with simulation. In later stages, the resulting numerical values are compared with results obtained in experiment along with simulation for final result. From the following figures 7 and 8, it can be clearly understood that the mechanical power is maximum at 1600 rpm and a pressure ratio of 3.3 bar.

![Fig 7: Mechanical Power and Efficiency Vs Volumetric ratio at 30°C.](image-url)
4.2 Numerical validation

From the mathematical evaluation, at different speeds and at 15 bar pressure, the results obtained from simulation well agreed with the numerical values within specified volumetric ratio. The mechanical power output increases with increase in speed, within the specified pressure ratio.

Table 2 : Relation between speed, power and torque at 3.3 pressure ratio at 15 bar inlet pressure.

| Rotational Speed rpm | Mechanical power at suction (watts) | Mechanical Power output (watts) | Mechanical Torque at inlet NM | Mechanical torque at output NM |
|----------------------|-------------------------------------|---------------------------------|-------------------------------|------------------------------|
| 1600                 | 645.07                              | 201.062                         | 3.85                          | 1.2                          |
| 2000                 | 806.34                              | 251.32                          | 3.85                          | 1.2                          |
| 2500                 | 1007.92                             | 314.15                          | 3.85                          | 1.2                          |

Table 3 : Relation between speed, power and torque at 3.5 pressure ratio at 15 bar inlet pressure.

| Rotational Speed rpm | Mechanical power at suction (watts) | Mechanical Power output (watts) | Mechanical Torque at inlet NM | Mechanical torque at output NM |
|----------------------|-------------------------------------|---------------------------------|-------------------------------|------------------------------|
| 1600                 | 645.07                              | 184.3                           | 3.85                          | 1.1                          |
| 2000                 | 806.34                              | 230.38                          | 3.85                          | 1.1                          |
| 2500                 | 1007.92                             | 287.97                          | 3.85                          | 1.1                          |

Table 4 : Relation between speed, power and torque at 3.8 pressure ratio at 15 bar inlet pressure.

| Rotational Speed rpm | Mechanical power at suction (watts) | Mechanical Power output (watts) | Mechanical Torque at inlet NM | Mechanical torque at output NM |
|----------------------|-------------------------------------|---------------------------------|-------------------------------|------------------------------|
| 1600                 | 645.07                              | 167.88                          | 3.85                          | 1.1                          |
| 2000                 | 806.34                              | 209.85                          | 3.85                          | 1.1                          |
| 2500                 | 1007.92                             | 262.32                          | 3.85                          | 1.1                          |
At 17 bar pressure

Table 5 : Relation between speed, power and torque at 3.3 pressure ratio at 17 bar inlet pressure.

| Rotational Speed rpm | Mechanical power at suction (watts) | Mechanical Power output (watts) | Mechanical Torque at inlet NM | Mechanical torque at output NM |
|----------------------|------------------------------------|--------------------------------|--------------------------------|--------------------------------|
| 1600                 | 723.82                             | 221.67                         | 4.32                           | 0.323                          |
| 2000                 | 904.77                             | 276.01                         | 4.32                           | 0.323                          |
| 2500                 | 1130.97                            | 345.01                         | 4.32                           | 0.323                          |

Table 6 : Relation between speed, power and torque at 3.5 pressure ratio at 17 bar inlet pressure.

| Rotational Speed rpm | Mechanical power at suction (watts) | Mechanical Power output (watts) | Mechanical Torque at inlet NM | Mechanical torque at output NM |
|----------------------|------------------------------------|--------------------------------|--------------------------------|--------------------------------|
| 1600                 | 723.82                             | 207.76                         | 4.32                           | 1.24                           |
| 2000                 | 904.77                             | 258.01                         | 4.32                           | 1.24                           |
| 2500                 | 1130.97                            | 322.5                          | 4.32                           | 1.24                           |

Table 7 : Relation between speed, power and torque at 3.8 pressure ratio at 17 bar inlet pressure.

| Rotational Speed rpm | Mechanical power at suction (watts) | Mechanical Power output (watts) | Mechanical Torque at inlet NM | Mechanical torque at output NM |
|----------------------|------------------------------------|--------------------------------|--------------------------------|--------------------------------|
| 1600                 | 723.82                             | 192.68                         | 4.32                           | 1.15                           |
| 2000                 | 904.77                             | 240.8                          | 4.32                           | 1.15                           |
| 2500                 | 1130.97                            | 301.01                         | 4.32                           | 1.15                           |

4.3 Experimental Results

In the experimental trial conducted, at 15 bar pressure and at 3.3 pressure ratio a voltage of 12.23 has been produced. In later stages, an experiment is conducted to verify the temperature effect and to find most optimum pressure ratio and built in volume and its working procedure as a charger. Different working fluids are also evaluated for maximum voltage and mechanical power. With more advanced fluids, design changes can be made through simulation after the experiment [22-24].

5. Conclusions

From the experimental trails, with increase in pressure ratio to a limit, the power output can be increased also with increase in speed, the voltage decreases. The volumetric ratio has to be maintained constant in determining pressure ratio and speed. The efficiency of scroll depends on speed and volumetric ratio. With increase in speed, efficiency decreases due to increase in losses.
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### Nomenclature

| Description of the parameter                              | Symbol |
|------------------------------------------------------------|--------|
| Radius of the basic circle of the scroll                   | $R_b$  |
| Height of scroll vanes                                     | $H$    |
| Initial angle of the outer involute                        | $\varphi_{o,i}$ |
| Initial angle of the inner involute                        | $\varphi_{i,o}$ |
| Starting angle of the outer involute                       | $\varphi_{o,s}$ |
| Starting angle of the inner involute                       | $\varphi_{i,s}$ |
| Rolling angle (involute ending angle)                      | $\varphi$ |
| Orbiting radius of the rotating scroll                     | $R_o$  |