Geometrical substantiation of the graphical technique of building surface contours in perspective view

Lina Kravcova¹, Irina Kostrubova¹

¹Irkutsk National Research Technical University, 664074, 83 Lermontov street, Irkutsk City, Russia

Abstract. The article presents the original method of geometric construction of axonometric essays of spatial forms. The imposition of the horizontal projections of these forms with the horizontal projections of their axonometric outline is used, which makes it possible to find reference points by orthogonal drawing. The relationship between the axonometric outline of the object's surface and the corresponding contour of the same surface on the projection drawing is established.

1 Introduction

In the process of developing architectural or design projects, as a rule, there is a graphic construction of visual images of spatial forms in a drawing [1-4].

As is known, the theoretical basis for the construction of technical, architectural drawings and drawings is the projection method: the central and its particular case is the parallel one. Both methods allow to obtain full visual graphic models of specific products and objects and are called a perspective or axonometry [5, 6]. As a rule, the perspective and axonometry are drawn graphically according to specified orthogonal drawings. The transition from orthogonal projections of spatial forms of objects to their visual images is associated with the construction of newly formed contours of details [7, 8], which can sometimes be difficult, especially when depicting mutually intersecting curvilinear surfaces. The authors noted that this issue is not sufficiently represented in the well-known educational literature on descriptive geometry [9, 10]. Therefore, we propose in this article to consider some methods that allow one to rather simply and accurately build on the visual image, the above-mentioned newly formed contours of details.

2 Problem statement

The solution to this problem will be considered in relation to axonometry. Recall, the word "AXONOMETRY" in translation from Greek means measurement along the axes. The essence of the method of construction of the axonometric image is that the object is referred to the coordinate system and then it, together with the coordinate system, is projected parallel to the axonometric plane. The related geometric constructions are well known [5-7, 10]. Nevertheless, the construction of axonometric reference points of essay generators when depicting intersecting curvilinear surfaces can be difficult, since in the literature on geometry known to authors, this question is not addressed, we consider the solution of a specific problem.
Figure 1 (left) shows an orthogonal drawing of two intersecting surfaces of rotation of the cone and cylinder. The line of intersection of these surfaces, as seen in the drawing, is constructed by the method of spheres. Simple enough and accurate. And how to solve this problem on axonometry?

![Image]

**Figure 1.** Building a line of intersection of the cone and cylinder.

It is clear that the solution of this problem by the method of spheres will be accompanied by distortions of the images of the circles. Such a graphical solution in axonometry will be very difficult, laborious and practically impossible.

More rationally in the drawing (Pic.1, right), the intersection line is determined by another method, the method that exists in practice. The problem is solved in the following sequence:
- first, on the projection drawing we build a line of intersection of the surfaces, using well-known geometric algorithms. In our task the method of spheres is used;
- then, with the help of coordinates, we transfer this line to an axonometry of any one of the intersecting surfaces. In our case, a cylinder is used. It is clear that this line will belong simultaneously to the surface of the cone.

It is easy to see that the sketched generators of these surfaces on the axonometric drawing and the contours of these surfaces on the projection drawing are images of different lines, not related to the projection. Hence there is an additional task - to determine the anchor points of the outline generators on the axonometric drawing. Let us clarify that the points of intersection of the outline generators of one of some surfaces with the surface of another body belong to the anchor points.

To determine the anchor points on the axonometric drawing, you can transfer the essays that form axonometrics to the projection drawing using all the same coordinates, then find the intersection points of these outline forming the surfaces with the previously constructed intersection line. This is for axonometric anchor points. It now remains to bring them back to axonometry using coordinates.

However, as the experience of solving such problems shows, it is not always so easy to determine the support points on the contours curvilinear surfaces, for example, a sphere, an ellipsoid, or a paraboloid. We set the task - to investigate the possibility of finding right along the orthogonal drawing of the contours of the surfaces, which, when further constructed in axonometry, will be sketched and it would be possible to find anchor points on the orthogonal drawing immediately. That is, we establish the relationship between the axonometric outline of the object's surface and the corresponding contour of the same surface in the projection drawing.

To solve this problem, we will use in our reasonings the concept of “wrapping projection surfaces”, by analogy with the concept of “wrapping ray surfaces” known from the theory of constructing shadows [11]. The term “wrapping projection surfaces” will be understood as the set of projecting lines touching the surface of an object. The line of contact of the wrapping projection surface and the
surface of the object will be the main contour, the axonometric projection of which will be a outline, as the line of intersection of the axonometric plane and the enveloping projection surface.

Consider the solution of the problem on examples of the most frequently encountered surfaces of rotation.

We will carry out all our reasoning with reference to a rectangular isometry.

Thus, in picture 2, on the left, a cylinder of rotation is shown, which is related to the coordinate system and its isometry.

**Figure 2.** Rotation cylinder images made by orthogonal and isometric projection.

Figure 2 on the right shows a projection drawing of this cylinder and defines the direction of isometric projection. With In the construction of axonometry, the wrapping projecting surface will represent two elliptical cylinders and two projecting planes, and two circles and two cylinders forming the tangent lines on the surface of the cylinder. We then take them as the main contour for the construction of an axonometric outline.

Figure 3 on the left shows the cone of rotation related to the coordinate system and its isometry. Picture 3 on the right shows a projection drawing of this cone and defines the direction of isometric projection. When constructing axonometry, the wrapping projecting surface will represent two projecting planes and an elliptical cylinder, and the lines of tangency on the surface of the cone will be the two forming cones and a circle. We then take them as the main contour for the construction of an axonometric feature article.

**Figure 3.** Cone rotation images made by orthogonal and isometric projection.

Figure 4 shows a projection drawing of the surface of a sphere and indicates the direction of isometric projection.
Figure 4. Hemisphere images made by orthogonal and isometric projection

When constructing axonometry, the projecting wrapping projection surface will represent the projected cylinder, and the circumference line will be the line of tangency on the surface of the sphere. Note that this circle, which we take for the main contour to construct an axonometric essay, lies in a plane with an angle of 45° and is projected onto the horizontal plane of the projections by an ellipse. We will not build the frontal projection of this circle due to the lack of necessity.

3 Results

Next, we will consider a method of constructing reference points in axonometry using the relationship between an axonometric sketch of the object's surface and the corresponding contour on the projection drawing on the images.

So, picture 5 shows a projection drawing and axonometric view of two intersecting surfaces - a straight prism and a hemisphere. On the projection drawing, the graphical methods of intersection of three faces of a prism with a hemisphere of three pieces of circles are constructed using known graphic techniques. The anchor points are defined and the horizontal projection of the hemisphere contour is shown with an ellipse. As shown above, this contour in axonometry (in Figure 5 on the right) corresponds to an axonometric essay in the form of a semicircle. The anchor points of this essay are its intersections with the edges of the prism, transferred from the horizontal projection with the help of the coordinates.

Figure 5. Construction of the line of intersection of surfaces.
4 Conclusion

In conclusion of this work, we state that as a result of the geometric study, a relationship has been established between an axonometric outline of the object's surface and the corresponding contour of the same surface on the projection drawing. This allows you to geometrically accurately build anchor points on axonometric essays on curved lines.

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