Study of light scalar mesons through $D_s^+ \rightarrow \pi^0 \pi^0 e^+ \nu_e$ and $K_S^0 K_S^0 e^+ \nu_e$ decays

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Using 6.32 fb$^{-1}$ of $e^+e^-$ collision data recorded by the BESIII detector at center-of-mass energies between 4.178 to 4.226 GeV, we present the first measurement of the decay $D^+_s \to f_0(980)e^+\nu_e$, $f_0(980)\to \pi^0\pi^0$. The product branching fraction of $D^+_s \to f_0(980)e^+\nu_e$, $f_0(980)\to \pi^0\pi^0$ is measured to be $(7.9 \pm 1.4_{\text{stat}} \pm 0.4_{\text{syst}}) \times 10^{-4}$, with a statistical significance of 7.8σ. Furthermore, the upper limits on the product branching fractions of $D^+_s \to f_0(500)e^+\nu_e$ with $f_0(500)\to \pi^0\pi^0$ and the branching fraction of $D^+_s \to K^0_SK^0_Le^+\nu_e$ are set to be $7.3 \times 10^{-4}$ and $3.8 \times 10^{-4}$ at 90% confidence level, respectively. Our results provide valuable inputs to the understanding of the structures of light scalar mesons.

The constituent quark model has been strikingly successful, but the nontrivial quark structures of scalar mesons below 1 GeV, $f_0(500)$, $f_0(980)$, and $a_0(980)(9\bar{0}\pm\bar{0})$ (briefly denoted with $\sigma$, $f_0$, and $a_0(\pm\pm)$, respectively), are not completely classified [1]. Many theoretical hypotheses, such as the tetraquark states [2–13], and two-meson bound states [14–18], have been proposed for these light scalar mesons but with controversial results. Identifying the correct hypothesis is key to exploring chiral-symmetry-breaking mechanisms of nonperturbative QCD in low-energy region [3]. Therefore, conclusive experimental results are required to interpret these states.

Semileptonic charm meson decays provide a clean environment to study scalar mesons [19–25]. Experimentally, the BESIII collaboration has reported the measurements of $D^{0(\pm\pm)} \to a_0^{-(0)}e^+\nu_e$, and $D^+ \to f_0/\sigma e^+\nu_e$ with $f_0/\sigma \to \pi^+\pi^-$ [26, 27], and the search of $D^+_s \to a_0^0e^+\nu_e$ [28]. The CLEO collaboration has also reported the measurement of $D^+_s \to f_0e^+\nu_e$ with $f_0 \to \pi^+\pi^0$ [29]. On the other hand, theoretical studies of neutral channels ($f_0 \to \pi^0\pi^0$) are rare compared to those of charged channels. Like charged channels, the branching fractions (BFs) of the semileptonic $D^+_s$ decays into light scalar meson in their decay to neutral channels and the $\pi^0\pi^0$ invariant mass spectrum aid in understanding the nontrivial nature of light scalar mesons [4, 11, 20, 25]. However, unlike charged channels, there is no background from $\rho(770)^0 \to \pi^+\pi^0$, thereby providing an ideal environment to study $f_0/\sigma$. Therefore, it is of great interest to study this kind of decays in experiment.

In addition, the BaBar collaboration claimed that a
possible \( f_0 \rightarrow K^+K^- \) contribution is found under the dominant decay \( D^+_s \rightarrow \phi e^+\nu_e \) in the study of \( D^+_s \rightarrow K^+K^-e^+\nu_e \) [30]. On the contrary, no other collaboration reported significant \( f_0 \rightarrow K^+K^- \) signal in the same decay [1]. We report the first search for the neutral channel \( D^+_s \rightarrow K^0_S K^0_S e^+\nu_e \), associated with \( f_0 \rightarrow K^0_S K^0_S \), avoiding heavy contamination from \( \phi \rightarrow K^+K^- \) decays. Throughout this paper, charge conjugate channels are always implied.

The BESIII detector [31, 32] records symmetric \( e^+e^- \) collisions provided by the BEPCII storage ring [33]. The cylindrical core of the BESIII detector covers 93% of the full solid angle and consists of a helium-based multilayer drift chamber (MDC), a plastic scintillator time-of-flight system (TOF), and a CsI(Tl) electromagnetic calorimeter (EMC), which are all enclosed in a superconducting solenoidal magnet providing a 1.0 T magnetic field. The charged-particle momentum resolution at 1 GeV/c is 0.5%, and the \( dE/dx \) resolution is 6% for electrons from Bhabha scattering. The EMC measures photon energy with a resolution of 2.5% (5%) at 1 GeV in the barrel (end cap) region. The time resolution in the TOF barrel region is 68 ps, while that in the end cap region is 110 ps. The end cap TOF system was upgraded in 2015 using multi-gap resistive plate chamber technology, providing a time resolution of 60 ps [34].

The analysis is performed based on data samples corresponding to an integrated luminosity of 6.32 fb\(^{-1} \) at \( \sqrt{s} = 4.178, 4.189, 4.199, 4.209, 4.219, \) and 4.226 GeV [35]. The signal events are selected from the process \( e^+e^- \rightarrow D^+_s\bar{D}^+_s \rightarrow \gamma D^+_s D^-_s \). A GEANT4-based [36] Monte Carlo (MC) simulation sample is used to determine detection efficiencies and to estimate background processes. The simulation models the beam energy spread and initial state radiation (ISR) in the \( e^+e^- \) annihilations with the generator KKMC [37]. The inclusive MC sample includes the production of open charm processes, the ISR production of vector charmonium(-like) states, and the continuum processes incorporated in KKMC [37]. The known decay modes are modelled with EVTGEN [38] using BFs taken from the Particle Data Group [1], and the remaining unknown charmonium decays are modelled with LUNDCHARM [39]. Final state radiation (FSR) from charged final state particles is incorporated using PHOTOS [40]. The signal detection efficiencies and signal shapes are obtained from signal MC samples. In the signal MC sample, the \( D^+_s \) decays generically and the signal \( D^+_s \) decays to \( \pi^0\pi^0e^+\nu_e \) or \( K^0_SK^0_S e^+\nu_e \) according to the generators described below. The form factor \( FF \) is parameterized as [41, 42]

\[
FF = p_{had} m_{D^+_s} A \frac{q^2}{1 - \frac{q^2}{m^2_{D^+_s}}},
\]

where \( q^2 \) is the invariant mass squared of the \( e^+\nu_e \) system, \( p_{had} \) is magnitude of the three-momentum of the \( \pi^0\pi^0/K^0_SK^0_S \) system in the \( D^+_s \) rest frame, the pole mass \( m_A \) is expected to be \( m_{D^+_s} \sim 2.5 \text{ GeV}/c^2 \) [1], and \( m_{D^+_s} \) is the nominal \( D^+_s \) mass [1]. The amplitude \( A \) for the \( f_0(980) \) resonance is parameterized by the Flattone formula with parameters fixed to the LHCb measurement [43], that for the \( \sigma \) resonance is described by the Bugg lineshape [44], and that in \( D^+_s \rightarrow K^0_SK^0_S e^+\nu_e \) signal MC sample is set to be one.

The signal process \( e^+e^- \rightarrow D^+_s D^-_s + c.c. \rightarrow \gamma D^+_s D^-_s + c.c \) allows studying semileptonic \( D^+_s \) decays with a tag technique [45, 46] since the neutrino is the only one particle undetected. There are two types of samples used in the tag technique: single tag (ST) and double tag (DT). In the ST sample, a \( D^-_s \) meson is reconstructed through a particular hadronic decay without any requirement on the remaining measured charged tracks and EMC showers. In the DT sample, a \( D^-_s \), designated as “tag”, is reconstructed through a hadronic decay mode first, and then a \( D^+_s \), designated as the “signal”, and the transition photon from the \( D^+_s \rightarrow \gamma D^-_s \) decay are reconstructed with the remaining tracks and EMC showers. The BF of the signal decay is given by [28]

\[
B_{\text{sig}} = \frac{N_{\text{DT}}^{\text{ST}} \epsilon_{\alpha,i}^{\text{DT}} \epsilon_{\alpha,i}^{\text{ST}}}{N_{\text{DT}}^{\text{total}}} \epsilon_{\alpha,i}^{\text{DT}} \epsilon_{\alpha,i}^{\text{ST}},
\]

where \( \alpha \) denotes various tag modes, \( i \) denotes different \( \sqrt{s} \), \( \epsilon_{\alpha,i}^{\text{DT(ST)}} \) denotes the DT (ST) reconstruction efficiencies, \( B_{\text{sig}} \) represents the BF of \( D^+_s \rightarrow \gamma D^-_s \), \( N_{\text{DT}}^{\text{ST}} \) is the signal yield for all six data sets, and \( N_{\text{DT}}^{\text{total}} \) is the ST yields for various tag modes. The tag candidates are reconstructed with charged \( K \) and \( \pi, \pi^0, \eta(\prime) \), and \( K^0_S \) mesons in nine tag modes, \( D^-_s \rightarrow K^0_SK^- \), \( K^+K^-\pi^- \), \( K^0_SK^+\pi^- \), \( K^0_SK^+\pi^+ \), \( K^0_S\pi^+\pi^- \), \( K^0_S\pi^0\pi^0 \), \( K^0_SK^+\pi^0 \), \( K^-\pi^+\pi^- \), \( K^-\pi^0\pi^0 \), \( K^-\pi^0\eta \), \( K^-\pi^0\eta' \). Requirements on the recoiling mass are applied to the tag candidates in order to identify the process \( e^+e^- \rightarrow D^+_s D^-_s \). If there are multiple candidates for a tag mode, the one with recoiling mass closest to the nominal \( D^+_s \) mass [1] is chosen. A detailed description of the requirements on the mass and the recoiling mass of tagged \( D^-_s \), and the selection criteria for charged and neutral particle candidates is provided in Ref. [28]. The ST yields of data for tag modes \( N_{\text{ST}}^{\alpha,i} \) are determined from fitting to the tag \( D^-_s \) invariant mass \( (M_{\text{tag}}) \) distributions [47]. The signal shape is modeled with the MC-simulated shape convolved with a Gaussian function, and the background is parameterized as a second-order Chebyshev function. The efficiencies \( \epsilon \) for ST are obtained from the inclusive MC samples [47].

After a tag \( D^-_s \) is identified, the signal decays are selected recoiling against the tag side, requiring that there is no track other than those accounted for in the tagged \( D^+_s \), the positron, and the semileptonic-side hadrons \( (N_{\text{extra}} = 0) \). A joint kinematic fit, in which four-momentum of the missing neutrino needs to be deter-
minded, is performed to select the best transition photon candidate from $D_s^{\pm} \rightarrow \gamma D_s^{\pm}$. The fit includes: The total four-momentum of reconstructed particles and the missing neutrino is constrained to the four-momentum of $e^+ e^-$ system. Invariant masses of the two $\pi^0/K_S^0$ candidates, the $D_s^-$ tag, the $D_s^+$ signal, and the $\gamma D_s^+$ are constrained to the corresponding nominal masses [1]. The transition photon candidate leading to the minimum $\chi^2$ of the joint kinematic fit is chosen. Furthermore, the largest energy of the remaining EMC showers that are not used to in the event reconstruction, $E_{\gamma \max}$, is required to be less than 0.2 GeV to suppress backgrounds with photon(s). The square of the recoil mass against the transition photon and the $D_s^-$ tag ($M_{\text{rec}}^2$) is expected to peak at the nominal $D_s^\pm$ meson mass-squared before the kinematic fit for signal $D_s^{\pm} D_s^{\pm}$ events. Therefore, $M_{\text{rec}}^2$ is required to satisfy $3.75 \pm 0.15 \text{ GeV}^2 / c^4 < M_{\text{rec}}^2 < 4.05 \text{ GeV}^2 / c^4$ to suppress the backgrounds from non-$D_s D_s^{\pm}$ processes.

The missing neutrino is inferred by the missing mass squared ($MM^2$), defined as

$$MM^2 = \frac{1}{c^2} \left( p_{\text{cm}} - p_{\text{tag}} - p_{\text{had}} - p_e - p_\gamma \right)^2,$$

where $p_{\text{cm}}$ is the four-momentum of the $e^+ e^-$ center-of-mass system, $p_{\text{tag}}$ for the tag $D_s^-$, $p_{\text{had}}(e)$ for the semileptonic-side hadrons (positron), and $p_\gamma$ for the transition photon from the $D_s^{\pm}$ decay. To partially recover the energy lost due to FSR and bremsstrahlung, the four-momenta of photon(s) within 5$^\circ$ of the initial positron direction are added to the positron four-momentum measured by the MDC. The invariant mass distributions of semileptonic-side hadrons of the selected candidates for $D_s^+ \rightarrow \pi^0 \pi^0 e^+ \nu_e$ and $D_s^+ \rightarrow K_S^0 K_S^0 e^+ \nu_e$ are shown in Fig. 1. Notable $f_0$ signals are found in the $\pi^0 \pi^0$ mass distribution while no significant signals of $\sigma \rightarrow \pi^0 \pi^0$ and $f_0 \rightarrow K_S^0 K_S^0$ are observed. The background is mostly caused by miscellaneous backgrounds with multiple photons.

A two-dimensional unbinned maximum likelihood fit to the $MM^2$ versus $M_{\pi^0\pi^0}$ distribution is performed to extract the DT yield of $D_s^+ \rightarrow f_0 e^+ \nu_e, f_0 \rightarrow \pi^0 \pi^0$. The signal and background components are described by the simulated shape from the signal and inclusive MC samples, respectively, using a kernel estimation method [48] implemented in RooFit [49]. The fit result is shown in Fig. 2. The obtained signal yields is $N_{\text{signal}}^{\text{DT}} = 54.8 \pm 10.1$ with a statistical significance of 7.8$\sigma$. Using the DT efficiencies from the signal MC samples (see Ref. [47]), and $B_{\gamma i}$, the resulting $B(D_s^+ \rightarrow f_0 e^+ \nu_e, f_0 \rightarrow \pi^0 \pi^0)$ is $(7.9 \pm 1.4_{\text{stat}} \pm 0.4_{\text{sys}}) \times 10^{-4}$. The second uncertainty is systematic, which are described in the following.

Since no significant signals are observed for the decays $D_s^+ \rightarrow \sigma e^+ \nu_e$ with $\sigma \rightarrow \pi^0 \pi^0$ and $D_s^+ \rightarrow K_S^0 K_S^0 e^+ \nu_e$, the upper limits of the BF's for these decays are determined. The candidate events for the former decay are required to satisfy $M_{\pi^0\pi^0} < 0.66 \text{ GeV} / c^2$. A veto $0.458 < M_{\pi^0\pi^0} < 0.520 \text{ GeV} / c^2$ is applied to suppress the background from $D_s^+ \rightarrow K_S^0 (\rightarrow \pi^0 \pi^0) e^+ \nu_e$. Unbinned maximum-likelihood fits are performed to the corresponding $MM^2$ distributions. The signal and background are modeled by the simulated shapes obtained from the signal and inclusive MC samples, respectively. The $MM^2$ distributions and the likelihoods of fit results as functions of assumed BFs are presented in Fig. 3. The upper limits, set at 90$\%$ C.L., of the BFs of $D_s^+ \rightarrow \sigma e^+ \nu_e, \sigma \rightarrow \pi^0 \pi^0$ and $D_s^+ \rightarrow K_S^0 K_S^0 e^+ \nu_e$ are $7.3 \times 10^{-4}$ and $3.8 \times 10^{-4}$, respectively. The method to incorporate systematic uncertainty is discussed in the following.

The sources of the systematic uncertainties for the BF measurement of $D_s^+ \rightarrow f_0 e^+ \nu_e$, as summarized in Table I, are described below. Note that most systematic uncertainties on the tag side cancel due to the DT technique. Any residual effects are negligible.

The uncertainty in the total number of the ST $D_s^-$ mesons is assigned to be 0.4$\%$ by examining the changes of the fit yields when varying the signal shape, back-
ground shape, and taking into account the background fluctuation in the fit. The uncertainty from the quoted BF of $D_s^+ \to \gamma D_s$ is 0.7% [1]. The systematic uncertainties from tracking and PID efficiencies of $e^+$ are assigned as 1.0% for each by using radiative Bhabha events. The systematic uncertainties from reconstruction efficiencies of $\gamma$ and $\pi^0$ are studied by using control samples of the decay $J/\psi \to \pi^+\pi^-\pi^0$ [50, 51] and the process $e^+e^- \to K^+K^-\pi^+\pi^-\pi^0$, respectively. A conservative 2% systematic uncertainty is assigned for each $\pi^0$(the transition photon) in the analysis of $D_s^+ \to \sigma e^+\nu_e$, since no significant signal is available to check the data-MC consistency. As for the analysis of $D_s^+ \to f_0 e^+\nu_e$, a momentum-weighted correction factor for each $\pi^0$ is calculated to be 99.4% and the residual uncertainty of 0.8% is assigned as the corresponding systematic uncertainty along with a 1% systematic uncertainty for the transition photon. The uncertainties of the $E_{\text{extr}}^{\gamma}$, $E_{\text{extr}}^{\pi^0}$, $N_{\text{char}}^{\gamma}$, and $N_{\text{char}}^{\pi^0}$ requirements are assigned as 0.7% and 0.8%, respectively, by analyzing DT hadronic events of $\pi^+\pi^0\eta$. The uncertainty due to the limited MC statistics is obtained by $\sqrt{\sum_{\alpha}(f_{\alpha}^2/c_{\alpha}^2)}$, where $f_{\alpha}$ is the tag yield fraction in data, and $c_{\alpha}$ and $\delta_{\alpha}$ are the signal efficiency and the corresponding uncertainty of tag mode $\alpha$, respectively. The systematic uncertainty associated with signal models is studied by replacing the parameters of $f_0$ from LHCb [43] by those from BES [52] in generating the signal MC sample. The difference of the measured BFs, where the effects of the signal efficiencies and the two-dimensional signal shape have been taken into account, is assigned as the associated systematic uncertainty. The background shape is altered by varying the relative fractions of major backgrounds from $e^+e^- \to q\bar{q}$ and non-$D_s^+D_s^+$ open-charm processes within 30% according to the uncertainties of their input crossing section in the inclusive MC sample. The effects caused by the smoothing parameter of the kernel estimation method [48, 49] is negligible. The largest change is taken as the corresponding systematic uncertainty.

The sources of systematic uncertainties on the upper limit measurements are classified into two types: additive ($\sigma_n$) and multiplicative ($\sigma_r$).

Additive uncertainty is dominated by the background shape description. The systematic uncertainty is studied by altering the nominal MC background shape with two methods. First, alternative simulated shapes are used, where the relative fractions of the dominant backgrounds from $e^+e^- \to q\bar{q}$ and non-$D_s^+D_s^+$ open-charm processes are varied within 30% according to the uncertainties of their input crossing section in the inclusive MC sample. Second, the alternative background shapes are obtained from the inclusive MC sample using the kernel estimation method [48, 49] with the smoothing parameter varied to be 0, 1, and 2.

Multiplicative uncertainties, as summarized in Table I, are related to the efficiency determination and the quoted BFs. All systematic uncertainties are the same as those for $D_s^+ \to f_0 e^+\nu_e$ except for the following. The uncertainty for the $K_0^0$ reconstruction efficiency is assigned as 1.5% per $K_0^0$ using control samples of $J/\psi \to K_0^0K^+\pi^-\pi^0$ and $\phi K_0^0 K^+\pi^0$ decays. The uncertainties of the $E_{\gamma}$, $E_{\pi^0}$, $N_{\text{char}}^{\gamma}$, and $N_{\text{char}}^{\pi^0}$ requirements in the $D_s^+ \to K_0^0 K_0^0 e^+\nu_e$ study are assigned as 0.5% and 0.9%, respectively, by analyzing DT hadronic events of $D_s^+ \to K^+K^-\pi^\pm$ and $K_0^0 K^\pm$. The systematic uncertainty of the $\sigma_n$ modeling is considered by replacing the lineshape of $\sigma$ in the signal MC sample with a conventional relativistic Breit-Wigner function with the mass and width fixed to the BES measurements [53]. The systematic uncertainty related to the $K_0^0 K_0^0 e^+\nu_e$ model is estimated by replacing the nominal model in the signal MC sample by a uniform distribution in phase space.

The additive uncertainty is taken into account by extracting likelihood distributions using different alternative background shapes and the one resulting the most conservative upper limit is chosen. Then, the multiplicative systematic uncertainty is incorporated in the calculation of the upper limit via [54, 55]

$$L(B) \propto \int_0^1 L \left( \frac{B - c_0}{c_0} \right) \frac{-(c_0^2 - 1)^2}{2(c_0)^2} \, de , \tag{4}$$
where $L(B)$ is the likelihood distribution as a function of BF; $e_F$ is the expected efficiency and $e_0$ is the averaged MC-estimated efficiency.

TABLE I. The systematic uncertainties (%) in the BF measurements. Uncertainties associated with background shapes for $\sigma^+\nu_e$ and $K_S^0K_S^0e^+\nu_e$ are additive in the upper limit measurements and not listed in this table.

| Source | $f_0e^+\nu_e$ | $\sigma^+\nu_e$ | $K_S^0K_S^0e^+\nu_e$ |
|--------|---------------|-----------------|---------------------|
| $D_s^+$ yield | 0.4 | 0.4 | 0.4 |
| $B(D_s^{\pm}\rightarrow \gamma D_s^{\pm})$ | 0.7 | 0.7 | 0.7 |
| $e^+$ tracking efficiency | 1.0 | 1.0 | 1.0 |
| $e^+$ PID efficiency | 1.0 | 1.0 | 1.0 |
| $\gamma$ and $\pi^0$ reconstruction | 2.6 | 5.0 | 1.0 |
| $K_S^0$ reconstruction | - | - | 3.0 |
| $E_{\text{extra}}^{\text{max}} < 0.2$ GeV | 0.7 | 0.7 | 0.5 |
| $N_{\text{char}} = 0$ | 0.8 | 0.8 | 0.9 |
| MC statistics | 0.5 | 0.5 | 0.5 |
| Signal model | 1.3 | 3.3 | 8.8 |
| Background shape | 3.0 | see text | see text |
| Total | 4.7 | 6.3 | 9.5 |

In summary, the first BF measurement of $D_s^+\rightarrow f_0e^+\nu_e, f_0\rightarrow \pi^0\pi^0$ and searches for $D_s^+\rightarrow \sigma^+\nu_e, \sigma\rightarrow \pi^0\pi^0$ and $D_s^+\rightarrow K_S^0K_S^0e^+\nu_e$ are performed using 6.32 fb$^{-1}$ of data taken at $\sqrt{s} = 4.178 - 4.226$ GeV with the BESIII detector.

The BF of $D_s^+\rightarrow f_0e^+\nu_e, f_0\rightarrow \pi^0\pi^0$ is determined to be $(7.9 \pm 1.4_{\text{stat}} \pm 0.4_{\text{syst}}) \times 10^{-4}$. According to isospin symmetry expectation $\frac{B(f_0\rightarrow \pi^0\pi^0)}{B(f_0\rightarrow \pi^+\pi^-)} = 0.5$, our result is consistent with the measurement of $D_s^+\rightarrow f_0e^+\nu_e$ with $f_0\rightarrow \pi^+\pi^-$ by the CLEO collaboration [29]. An upper limit on the BF of $D_s^+\rightarrow \sigma^+\nu_e, \sigma\rightarrow \pi^0\pi^0$ is set to be $7.3 \times 10^{-4}$ at 90% C.L. This upper limit is an overestimation due to omitting the non-$\sigma$ contribution in the region of $M_{\pi^0\pi^0} < 0.66$ GeV/$c^2$. Our results agree with the statement that the $s\bar{s}\rightarrow \tau$ transition is negligibly small in comparison with that of $s\bar{s} \rightarrow f_0$ given by Refs. [20, 23], which follow the four-quark structure or meson–meson interaction hypothesis for $f_0$ and $\sigma$ mesons. Furthermore, the upper limit on $B(D_s^+\rightarrow K_S^0K_S^0e^+\nu_e)$ is set to be $3.8 \times 10^{-4}$ at 90% C.L., indicating that contribution from $B(f_0 \rightarrow K\bar{K})$ is not comparable to $B(f_0 \rightarrow \pi\pi)$ in semileptonic $D_s^+$ decays. Assuming $B(f_0 \rightarrow \pi^0\pi^0)$ contributes one third of the $f_0$ decays, our results leads to $B(D_s^+\rightarrow f_0e^+\nu_e) = (2.4 \pm 0.4) \times 10^{-3}$, which is consistent with the prediction given by Refs. [23, 24] when assuming $f_0$ to be the admixture of $s\bar{s}$ and other light quark-antiquark pairs.

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