Gapless Color-Flavor-Locked Quark Matter

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In neutral cold quark matter that is sufficiently dense that the strange quark mass $M_3$ is unimportant, all nine quarks (three colors; three flavors) pair in a color-flavor locked (CFL) pattern, and all fermionic quasiparticles have a gap. We argue that as a function of decreasing quark chemical potential $\mu$ or increasing $M_3$, there is a quantum phase transition from the CFL phase to a new “gapless CFL phase” in which only seven quasiparticles have a gap. The transition occurs where $M_3^2/\mu \approx 2\Delta$, with $\Delta$ the gap parameter. Gapless CFL, like CFL, leaves unbroken a linear combination $\bar{Q}$ of electric and color charges, but it is a $\bar{Q}$-conductor with a nonzero electron density. These electrons and the gapless quark quasiparticles make the low energy effective theory of the gapless CFL phase and, consequently, its astrophysical properties qualitatively different from that of the CFL phase, even though its $U(1)$ symmetries are the same. Both gapless quasiparticles have quadratic dispersion relations at the quantum critical point. For values of $M_3^2/\mu$ above the quantum critical point, one branch has conventional linear dispersion relations while the other branch remains quadratic, up to tiny corrections.

To impose color neutrality, it is sufficient to consider the $U(1)_3 \times U(1)_8$ subgroup of the color gauge group generated by the Cartan subalgebra $T_3 = \text{diag}(\frac{1}{2}, -\frac{1}{2}, 0)$ and $T_8 = \text{diag}(\frac{1}{3}, \frac{1}{3}, -\frac{2}{3})$ in color space. We introduce chemical (color-electrostatic) potentials $\mu_3$ and $\mu_8$ coupled to the color charges $T_3$ and $T_8$, and an electrostatic potential $\mu_e$ coupled to $Q_8$ which is the negative of the electric charge $Q = \text{diag}(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$ ($\mu_e > 0$ corresponds to a density of electrons; $\mu_e < 0$ to positrons.) The neutrality condition on $\mu_3, \mu_8, \mu_e$ is

$$\frac{\partial \Omega}{\partial \mu_3} = \frac{\partial \Omega}{\partial \mu_8} = \frac{\partial \Omega}{\partial \mu_e} = 0.$$  \hspace{1cm} (5)

Any condensate of the form (11) is neutral with respect to a “rotated electromagnetism” generated by $\bar{Q} = Q - T_3 - \frac{1}{2}T_8$, so $U(1)_Q$ is never broken, but, depending on the values of the $\Delta_i$, the rest of the gauge group may be spontaneously broken. We emphasize that this does not affect the neutrality condition (5): a macroscopic sample must be neutral under all gauge symmetries (11).

Previous model-independent calculations \cite{dressel_grove, alford_goldman_kusenko, alford_golfman_kusenko, alford_golfman_kusenko_2} have compared the free energy of the CFL phase with that of the 2SC and unpaired phases. In the CFL phase, $\mu_e = \mu_3 = 0$ and $\mu_8 = -M_3^2/(2\mu)$ to leading order in $M_3/\mu$. To this order, the CFL phase has a lower free energy than either 2SC or neutral unpaired quark matter for

$$\frac{M_3^2}{\mu} < 4\Delta_{CFL}. \hspace{1cm} (6)$$

In this paper we show that the CFL phase becomes unstable already at a lower value of $M_3^2/\mu$.

We shall present solutions to the gap equations for $\Delta_1$, $\Delta_2$ and $\Delta_3$ below. First, however, we give a

\begin{align*}
\Delta_3 \approx & \Delta_2 = \Delta_1 = \Delta_{CFL} \quad \text{CFL} \hspace{1cm} (2) \\
\Delta_3 > & 0, \quad \Delta_1 = \Delta_2 = 0 \quad \text{2SC} \hspace{1cm} (3) \\
\Delta_3 > & \Delta_2 > \Delta_1 > 0 \quad \text{gapless CFL} \hspace{1cm} (4)
\end{align*}

We know a lot about the properties of cold quark matter at sufficiently high baryon density from first principles. Quarks near their Fermi surfaces pair, forming a color superconductor \cite{shovkovy}. In this letter we study how the favored pairing pattern at zero temperature depends on the strange quark mass $M_3$, or equivalently on the quark chemical potential $\mu$, using the pairing ansatz \cite{alford_golfman_kusenko_2}

$$\langle \psi^a C\gamma_5 \psi_b \rangle \sim \Delta_1 e^{\alpha_1} e_{a1} + \Delta_2 e^{\alpha_2} e_{a2} + \Delta_3 e^{\alpha_3} e_{a3} \hspace{1cm} (1)$$

Here $\psi^a$ is a quark of color $\alpha = (r, g, b)$ and flavor $a = (u, d, s)$; the condensate is a Lorentz scalar, antisymmetric in Dirac indices, antisymmetric in color (the channel with the strongest attraction between quarks), and consequently antisymmetric in flavor. The gap parameters $\Delta_1, \Delta_2$ and $\Delta_3$ describe down-strange, up-strange and up-down Cooper pairs, respectively.

To find which phases occur in realistic quark matter, one must take into account the strange quark mass and equilibrium under the weak interaction, and impose neutrality under the color and electromagnetic gauge symmetries. The arguments that favor \cite{alford_golfman_kusenko_2} are unaffected by these considerations, but there is no reason for the gap parameters to be equal once $M_3 \neq 0$. Previous work \cite{dressel_grove, alford_golfman_kusenko, alford_golfman_kusenko_2, alford_golfman_kusenko_3} compared the color-flavor-locked (CFL) phase (favored in the limit $M_3 \rightarrow 0$ or $\mu \rightarrow \infty$), and the two-flavor (2SC) phase (favored in the limit $M_3 \rightarrow \infty$). In this paper we show that in fact a transition between these phases does not occur. Above a critical $M_3^2/\mu$, the CFL phase gives way to a new “gapless CFL phase”, not to the 2SC phase. The relevant phases are

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model-independent argument for the instability of the CFL phase above some critical $M_s^2/\mu$. In the condensate $|\langle \bar{s}s \rangle|$, the $(gs, bd)$, $(ru, rs)$, and $(rd, gu)$ quarks pair with gap parameters $\Delta_1$, $\Delta_2$, and $\Delta_3$ respectively, while the $(ru, gd, bs)$ quarks pair among each other involving all the $\Delta$’s. The gap equations for the three $\Delta$’s are coupled, but we can, for example, analyze the effect of a specified $\Delta_1$ on the $gs$ and $bd$ quarks without reference to the other quarks. The leading effect of $M_s$ is like a shift in the chemical potential of the strange quarks, so the $bd$ and $gs$ quarks feel “effective chemical potentials” $\mu_{bd}^{\text{eff}} = \mu - 3\mu_b$ and $\mu_{gs}^{\text{eff}} = \mu + \frac{\mu_s}{\Lambda} = \frac{M_s^2}{2\mu}$. In the CFL phase $\mu_{gs} = -M_s^2/(2\mu)$, so $\mu_{bd}^{\text{eff}} - \mu_{gs}^{\text{eff}} = M_s^2/\mu$. The CFL phase will be stable as long as the pairing makes it energetically favorable to maintain equality of the $bd$ and $gs$ Fermi momenta, despite their differing chemical potentials. It becomes unstable when the energy gained from turning a $gs$ quark near the common Fermi momentum into a $bd$ quark (namely $M_s^2/\mu$) exceeds the cost in lost pairing energy $2\Delta_1$. So the CFL phase is stable when

$$\frac{M_s^2}{\mu} < 2\Delta_{\text{CFL}}.$$

For larger $M_s^2/\mu$, the CFL phase is replaced by some new phase with unpaired $bd$ quarks, which from (6) cannot be neutral unpaired or 2SC quark matter because the new phase and the CFL phase must have the same free energy at the critical $M_s^2/\mu = 2\Delta_{\text{CFL}}$.

For a more detailed analysis, we use a NJL model with a pointlike four-quark interaction with the quantum numbers of single-gluon exchange, as in the first paper in Ref. 1 but with chemical potentials $\mu_e$, $\mu_3$, and $\mu_8$ introduced as in Ref. 4. Whereas in nature, the conditions are enforced by the dynamics of the gauge fields whose zeroth components are $\mu_e$, $\mu_3$, and $\mu_8$. The NJL model Eqs. in an NJL model Eqs. must be imposed. The model has two parameters, the four-fermion coupling $G$ and a three-momentum cutoff $\Lambda$, but we quote results in terms of the physical quantity $\Delta_0$ (the CFL gap at $M_s = 0$), since varying $\Lambda$ by 20% while tuning $G$ to keep $\Delta_0$ fixed changes all of our results by at most a few percent. We use $\Lambda = 800$ MeV in all results that we quote.

We make the ansatz for the diquark condensate in the quark propagator, using the Nambu-Gorkov formalism, and then evaluate the free energy $\Omega$. We shall present the details of our calculation elsewhere, but the formalism is as in Ref. 11, with the additional constraints of electrical and color neutrality. To simplify the analysis, we neglect the color and flavor symmetric contributions $\mu_b$, $\mu_3$, and $\mu_8$, set the light quark masses to zero, and treat the constituent strange quark mass $M_s$ as a parameter, as in Ref. 4, leaving a treatment like that of Refs. 3, 4 in which one solves for the $(\bar{s}s)$ condensate for the future. We incorporate $M_s$ only via its leading effect, a shift $-M_s^2/2\mu$ in the effective chemical potential for the strange quarks. This requires that $M_s^2/\mu$ be small, and neglects the dependence of the gap parameters on the Fermi velocity of the strange quark $\mu_s$ meaning that we find $\Delta_3 = \Delta_2 = \Delta_1$ in the CFL phase instead of finding $\Delta_3$ larger than the other two by a few percent. We work to leading nontrivial order in $M_s$, $\Delta_3$, $\mu_e$, $\mu_3$, and $\mu_8$, since these are all small compared to $\mu$. Finally, we neglect the effects of antiparticles. None of these approximations precludes a qualitative understanding of the new phase we shall describe.

We calculate the free energy $\Omega$, and solve six coupled integral equations, the neutrality conditions and the gap equations $\partial \Omega/\partial \Delta_1 = \partial \Omega/\partial \Delta_2 = \partial \Omega/\partial \Delta_3 = 0$. Our solutions depend on three parameters: $\mu$, $M_s$, and $\Delta_0$. We always take $\mu = 500$ MeV, which is reasonable for the center of a neutron star. We quote results only for $\Delta_0 = 25$ MeV, which is within the plausible range and ensures that the transition occurs where $M_s^2/\mu^2$ corrections are under control. Although we have obtained our results by varying $M_s$ at fixed $\mu$, we typically quote results in terms of the important combination $M_s^2/\mu^2$.

Note that in nature $M_s$ increases with decreasing $\mu$.

In Fig. 1, we show the gaps as a function of $M_s^2/\mu$, for $\Delta_0 = 25$ MeV. We see a phase transition occurring at a critical $M_s^2/\mu$ that, in our model calculation with $\mu = 500$ MeV, lies between $M_s = 153$ MeV and $M_s = 154$ MeV. Below $(M_s^2/\mu)_c$, i.e. at high enough density, we have the CFL phase. At $M_s = 153$ MeV, $\Delta_1 = \Delta_2 = 23.5$ MeV and $M_s^2/\mu = 46.8$ MeV: the model-independent prediction is in good agreement with our model calculation. For $M_s^2/\mu > (M_s^2/\mu)_c$, i.e. at densities below those where the CFL phase is stable, we find the gapless CFL $(g\text{CFL})$ phase with $\Delta_3 > \Delta_1 > \Delta_3 > 0$, and all the gaps changing much more rapidly with $M_s^2/\mu$. We have checked that upon varying $\Delta_0$, the critical $M_s^2/\mu$ changes quantitatively as predicted by (7) and our results are otherwise qualitatively unchanged.

By evaluating the free energy, we have confirmed that the CFL $\rightarrow$ gCFL transition at $(M_s^2/\mu)_c$ is not

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Gap parameters $\Delta_1$, $\Delta_2$, and $\Delta_3$ as a function of $M_s^2/\mu$ for $\mu = 500$ MeV, in a model where $\Delta_0 = 25$ MeV (see text). There is a continuous transition between the CFL phase and the gapless CFL phase.}
\end{figure}
first order, and found a first-order gCFL → unpaired quark matter transition at $M_2^2/\mu \approx 129$ MeV. The (now metastable) gCFL phase continues to exist up to $M_2^2/\mu \approx 144$ MeV where, as we show below, it ceases to be a solution.

We see from the $bd$-$gs$ quasiquark dispersion relations (dashed lines in Fig. 2) that there are gapless excitations at momenta $p_1^{bd}$ and $p_2^{bd}$. The analysis of Ref. [14] demonstrates that, as expected from the model-independent argument above, these bound a “blocking region” $[1\ldots14]$ or “breached pairing” $[8]$ region. One $bu$-$rs$ mode is gapless with an almost exactly quadratic dispersion relation. The five quark quasiparticles not plotted all have gaps, throughout the CFL and gCFL phases.

![FIG. 2: Dispersion relations at $M_2^2/\mu = 80$ MeV for $gs$ and $bd$ quarks (dashed lines) and for $bu$ and $rs$ quarks (solid lines). There are gapless $gs$-$bd$ modes at $p_1^{bd} = 469.8$ MeV and $p_2^{bd} = 509.5$ MeV, which are the boundaries of the $bd$-filled “blocking” $[1\ldots14]$ or “breached pairing” $[8]$ region. One $bu$-$rs$ mode is gapless with an almost exactly quadratic dispersion relation. The five quark quasiparticles not plotted all have gaps, throughout the CFL and gCFL phases.](image1)

![FIG. 3: The upper and lower curves bound the region of $\mu_e$ where CFL or gCFL solutions are found, if electrons are ignored. Between the curves the quark matter is a $\tilde{Q}$-insulator. Taking electrons into account, the correct solution has $\mu_e = 0$ for $M_2^2/\mu < (M_2^2/\mu)_c$ in the CFL phase (dashed line), and has $\mu_e$ below but very close to the lower curve for $M_2^2/\mu > (M_2^2/\mu)_c$ in the gCFL phase (see text).](image2)

than a $\tilde{Q}$-insulator. Electrons play a crucial role in this, but let us first understand the quark matter on its own, setting the electron mass to infinity and in so doing keeping the gCFL phase a $\tilde{Q}$-insulator.

In the absence of electrons, both the CFL and gCFL phases have a degenerate set of neutral free energy minima over a range of $\mu_{\tilde{Q}} = -\frac{1}{2}(\mu_e + \mu_0 + \frac{1}{2}\mu_8)$, with the two orthogonal chemical potentials and the three gap parameters fixed. The limits of this range (giving $\mu_e$ rather than $\mu_{\tilde{Q}}$) are shown in Fig. 3. Because $\Omega$ is independent of $\mu_{\tilde{Q}}$ in this range, the material is a $\tilde{Q}$-insulator. At the upper limit in $\mu_{\tilde{Q}}$ (lower limit in $\mu_e$), the $bu$ and $rs$ quarks, which have $\tilde{Q} = +1$ and $\tilde{Q} = -1$ respectively, start to unpair: they develop a blocking region of unpaired $bu$ quarks bounded by gapless modes, meaning that the $\tilde{Q}$-neutrality condition cannot be satisfied. At the lower limit in $\mu_{\tilde{Q}}$ (upper limit in $\mu_e$) an analogous dielectric breakdown occurs with the $rd$-$gu$ pairs breaking and a blocking region of unpaired $gu$ quarks with $\tilde{Q} = -1$ developing. The solid curves in Fig. 3 thus define the “band gap” for $\tilde{Q}$-charged fermionic excitations. In the region between the curves, $\tilde{Q}$-insulating CFL or gCFL quark matter exists, but outside that region no neutral solution exists. At $M_2^2/\mu = 144$ MeV, which is so large that the gCFL phase is anyway already metastable with respect to unpaired quark matter, the two boundaries cross, meaning that no gCFL solution can be found.

In the real world there are electrons, which we take to be massless. Consequently, in the CFL phase $[M_2^2/\mu < (M_2^2/\mu)_c]$, neutrality requires $\mu_e = 0$, so no electrons are present and the material remains $\tilde{Q}$-neutral and a $\tilde{Q}$-insulator, as before $[14]$. However, in the gCFL phase, $[M_2^2/\mu > (M_2^2/\mu)_c]$, $\mu_e = 0$ is below the allowed range. In this case, the true solution lies “just below” the lower curve in Fig. 3 where the $bu$ and $rs$ quarks have become gapless, allowing a small density of unpaired $bu$ ($\tilde{Q} = +1$)
quarks to cancel the charge of the electrons. The density of electrons is \( \mu_e^2/(3\pi^2) \), and the density of unpaired \( bu \) quarks is \( \left( \mu_{bu}^2/3 - \left( \mu_{bu}^2/3 \right)^3 \right) \), so they cancel when \( \left( \mu_{bu}^2 - \mu_{pu}^2 \right) = \mu_e^3/3\pi^2 \), where \( \bar{\mu} \) is the average of the momenta \( \mu_{pu} \) and \( \mu_{bu} \) that bound the \( bu \) blocking region. At \( M_s^2/\mu = 80 \) MeV, where \( \mu_e = 14.6 \) MeV at the lower curve in Fig. 3, this implies \( \left( \mu_{bu}^2 - \mu_{pu}^2 \right) = 0.0046 \) MeV! (To resolve \( \mu_{pu}^2 - \mu_{bu}^2 \), we solved the equations assuming 200 and 500 “flavors” of massless electrons.) Because \( \left( \mu_{bu}^2 - \mu_{pu}^2 \right) \) is so small, at the true \( \tilde{Q} \)-neutral solution \( \mu_e \) is very close to the lower curve in Fig. 3 and the gaps are almost unaffected by the inclusion of electrons. However, the effect of including electrons is profound: because \( \mu_e \) cannot be zero, \( gCFL \) quark matter is deformed slightly away from being a \( \tilde{Q} \)-insulator, so that it can carry a positive \( \tilde{Q} \)-charge to compensate the negatively charged electrons. The \( gCFL \) phase is therefore a \( U(1)_{\tilde{Q}} \) conductor. The quantum phase transition at \( M_s^2/\mu = (M_s^2/\mu)_c \) is a “metal-insulator transition”, with electron density \( n_e \sim \left( \mu_e^2 - m_e^2 \right)^{3/2} \) as the most physically relevant order parameter. The phase transition is continuous but higher-than-second order (\( d n_e/d (M_s^2/\mu) \) is continuous).

The phenomenology of \( gCFL \) quark matter in compact stars will be dominated by the modes with energy less than or of order the temperature, which is in the range keV to hundreds of keV. For the \( gs-bd \) quasiparticles (Fig. 2), the gapless \( \tilde{Q} = 0 \) quasiparticles at \( p_{bd}^2 \) and \( p_{bu}^2 \) have conventional linear dispersion relations, except for \( M_s^2/\mu \rightarrow (M_s^2/\mu)_c \) where \( p_{bd}^2 - p_{bu}^2 \rightarrow 0 \). For the \( bu-rs \) quarks, the dispersion relation is strictly speaking also quadratic only at the quantum critical point, but the gapless points separate so slowly in the \( gCFL \) phase that this dispersion relation remains very close to quadratic. For example, at \( M_s^2/\mu = 80 \) MeV the maximum in the quasiparticle energy between \( p_{bu}^2 \) and \( p_{bu}^2 \) is \( \left( \mu_{bu}^2 - \mu_{pu}^2 \right)^2/(\Delta s) = \left( \mu_e^2/2\pi^2 \Delta s_0 \right) \sim 0.13 \) eV, which is negligible at compact star temperatures. The requirement of \( \tilde{Q} \)-neutrality naturally forces the gapless \( bu-rs \) dispersion relation to be (very close to) quadratic, without requiring fine tuning to a critical point.

The low energy effective theory of the \( gCFL \) phase must incorporate gapless fermions which have number densities \( \sim \mu_s^2/\sqrt{\Delta s} T \) (the gapless quarks with quadratic dispersion relation), \( \sim \mu_s^2 T \) (the gapless quarks with linear dispersion relations), and \( \sim \mu_s T \) (the electrons). In contrast, the (pseudo-)Goldstone bosons present in both the \( CFL \) and \( gCFL \) phases have number densities at most \( \sim T^3 \). This means the \( gCFL \) phase will have very different phenomenology. It will be particularly interesting to compute the cooling of a compact star with a \( gCFL \) core, because neutrino emission will be associated with a weak \( bd \leftrightarrow \mu \) transition, requiring conversion between quasiparticles with linear and quadratic dispersion relations.

Although we have studied the \( gCFL \) phase in a model, all of the qualitative features that we have focussed on appear robust, and we have also offered a model-independent argument for the instability that causes the transition. It remains a possibility, however, that the \( CFL \) gap is large enough that baryonic matter supplants the \( gCFL \) phase before \( M_s^2/\mu > 2\Delta \). Assuming that the \( gCFL \) phase does replace the \( CFL \) phase, it is also possible that gaps are small enough that a third phase of quark matter could supplant the \( gCFL \) phase at still lower density, before the transition to baryonic matter. In our analysis with \( \Delta s_0 = 25 \) MeV, this third phase would be the unpaired quark matter at \( M_s^2/\mu > 129 \) MeV but, unlike our central results, this is model dependent. Other possibilities include the gapless 2SC phase \( \tilde{Q}_2 \), a three-flavor extension of the crystalline color superconducting phase \( \tilde{Q}_3 \), or weak pairing between quarks with the same flavor \( \tilde{Q}_1 \).

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The band gap for $\tilde{Q}$-charged bosonic excitations in the CFL phase is narrower than that for fermionic excitations, but still includes $\mu_e = 0$. The CFL phase at $\mu_e = 0$ can exhibit a $\tilde{Q}$-neutral kaon condensate. See P. F. Bedaque and T. Schäfer, Nucl. Phys. A 697, 802 (2002); D. B. Kaplan and S. Reddy, Phys. Rev. D 65, 054018 (2002); and Refs. [1]. The gCFL phase has the same Goldstone and pseudo-Goldstone bosons as the CFL phase, since the same symmetries are broken. However, the pseudo-Goldstone bosons receive new contributions to their squared masses, of order differences between $\Delta^2$’s, which stabilize against meson condensation.

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