The Shears Mechanism in $^{142}$Gd in the Skyrme-Hartree-Fock Method with the Tilted-Axis Cranking

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We report on the first Skyrme-Hartree-Fock calculations with the tilted-axis cranking in the context of magnetic rotation. The mean field symmetries, differences between phenomenological and self-consistent methods and the generation of shears-like structures in the mean field are discussed. Significant role of the time-odd spin-spin effective interaction is pointed out. We reproduce the shears mechanism, but quantitative agreement with experiment is rather poor. It may have to do with too large core polarization, lack of pairing correlations or properties of the Skyrme force.

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1. Introduction

It is known, that if a quantum system exhibits rotational excitations there must be a factor breaking its spherical symmetry. In the nuclear case, deformation of the charge distribution is the most familiar one, and was the only known till the early 1990’s. In that period a new situation was found in some rotational bands of light lead isotopes. These bands are characterized by very weak E2 intraband transitions, implying almost spherical shape, and strong M1 transitions, suggesting large magnetic dipole moment as the main factor breaking sphericity. This is why the new phenomenon was given the name of magnetic rotation, see Ref. [1] for a review.

The following mechanism is believed to underly the so-called magnetic dipole bands. At the bandhead, the spin of the valence protons and that of

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the valence neutrons form the angle of $90^\circ$. When the system rotates, both spins align towards the axis of rotation due to the gyroscopic effect, resembling the closing of shears blades. Hence the name of the shears mechanism.

The cranked mean-field is a tool suitable for describing rotational excitations. There is a well established correspondence between the spin-parity sequences in rotational bands, and the discrete symmetries of the mean field [2]. Properties of the standard E2 bands imply conservation of parity and signature. With these symmetries imposed, the cranking frequency can be applied only along the axis to which the signature refers, and all average-spin vectors obtained from the solution point along this unique direction. Such a case is called the one-dimensional or principal-axis cranking. For the magnetic dipole bands, the only conserved spatial symmetry is the parity. In the context of the mean field, this allows for any direction of the cranking-frequency and average-spin vectors which complies with the perpendicular orientation in the shears picture. Such an approach is referred to as the tilted-axis cranking.

There are two basic versions of the mean field approach - phenomenological, employing a pre-defined potential, and self-consistent, in which the potential is obtained from averaging a two-body effective interaction. Up to now, only the former one has been used for tilted-cranking calculations, with the notable exception of a self-consistent calculation performed within the relativistic mean field applied to $^{84}$Rb [3]. In the phenomenological method, one can either fix the orientation of the potential and vary the direction of the cranking-frequency vector when minimizing energy or vice-versa. The first possibility is used in existing codes. Self-consistently, the orientation of the potential is not available as a parameter, and the only feasible way is to fix the cranking vector and let the self-consistent potential reorient and conform to it in course of the Hartree-Fock iterations.

Although the phenomenological description of the shears bands has led to a considerable success, the self-consistent methods have to be applied in order to study such structures in more detail. This includes the stability of the proposed configurations with respect to the core excitation, the full minimization of the underlying energies with respect to all deformation variables, and the inclusion of the spin-current interactions.

2. The code HFODD

In the present study, the self-consistent solutions for rotational bands in $^{142}$Gd were obtained with the new version (v1.91) of the code HFODD [4]. Previous versions are described in Ref. [5]. The code solves the nuclear Hartree-Fock equations for the Skyrme effective interaction. All time-even and time-odd terms of the mean field can be included. The wavefunctions
are expanded onto the deformed Cartesian harmonic-oscillator basis. Four symmetry modes are available, conserving either both parity and signature, only simplex, only parity, or no point symmetries. The cranking-frequency vector can take any direction. If the solution becomes tilted with respect to the laboratory frame, the program finds the principal axes of the mass quadrupole moment and recalculates all quantities in the corresponding intrinsic frame. Pairing correlations are not taken into account.

3. Results obtained for $^{142}$Gd

It turns out, that the Skyrme-Hartree-Fock calculations with broken signature are very vulnerable to divergences and often parallel coupling of angular momenta is obtained where a perpendicular one is expected. We attribute these effects to overestimated strengths of the time-odd components of the Skyrme mean field. Terms originating from the interaction between intrinsic spins have a particularly strong influence here. This observation complies with what is known about the nuclear spin-spin interaction, that it prefers parallel coupling. The terms in question are $\vec{s}^2$ and $\vec{s} \cdot \Delta \vec{s}$ - see Refs. [6, 5] for details. We tested the SKM* [7] and SLy4 [8] forces and observed a similar behavior in both cases. In the present calculations the SLy4 parametrization was used and the time-odd coupling constants corresponding to $\vec{s}^2$ (both density dependent and independent), $\vec{s} \cdot \Delta \vec{s}$ and $\vec{s} \cdot \vec{T}$ were set to zero. Only the ones multiplying the terms $\vec{j}^2$ and $\vec{s} \cdot (\nabla \times \vec{j})$ were kept at the original values of the SLy4 parametrization, in order to conserve the local gauge invariance of the force [6].

The nucleus $^{142}$Gd ($Z=64$ and $N=78$) belongs to the so-called transitional region of nuclei, i.e., it has a small but non-negligible deformation $\beta$ and a substantial triaxiality angle $\gamma$. It may, therefore, exhibit an interplay between the shears mechanism and the standard collective rotation. Fig. 1 shows a partial level scheme of $^{142}$Gd taken from Ref. [9]. Bands denoted by $\pi h_{11/2}^2$ and $\nu h_{11/2}^{-2}$, as well as the ground-state band, have the electric quadrupole character. Band denoted by $\pi h_{11/2}^2 \nu h_{11/2}^{-2}$ is a magnetic dipole one.

In our calculations all the four bands correspond to the same slightly triaxial deformation of about $\beta = 0.19$ and $\gamma = 40^\circ$, that does not significantly change with rotational frequency. It is known, that spherical mean-field orbitals split with deformation. For oblate shapes the suborbitals with high Nilsson number $\Omega$ (angular momentum projection onto the symmetry axis) become lower in energy than those with low $\Omega$. Our solution is close to the oblate shape and we observe a similar behavior. In particular, the lowest suborbitals of $h_{11/2}$ have large alignments along the shortest axis of the nucleus and small alignments along the longest one. The opposite holds for
the highest suborbitals.

In the ground state obtained from our calculations there are two protons in \( h_{11/2} \) occupying its two lowest suborbitals. They carry alignments of about \( \pm 11/2 \) along the shortest axis, and the total equals zero. When either of them is excited to the state of alignment \( \mp 9/2 \), spin of 10 units along the shortest axis is obtained. This corresponds to the bandhead of the band denoted as \( \pi h_{11/2}^2 \) in Fig. 1. By applying the cranking field along this direction one obtains the subsequent states of the band. Analogous excitation of neutron holes yields spin 10, but along the longest axis, because the holes are located in the highest suborbitals of \( h_{11/2} \). This leads to the band marked as \( \nu h_{11/2}^{-2} \).
Fig. 2. Single-particle routhians in $^{142}$Gd calculated for the $\pi h_{11/2}^2 \nu h_{11/2}^{-2}$ configuration. The two proton and two neutron routhians responsible for the shears blades are marked by arrows.

When the neutron and proton excitations are combined together, perpendicular coupling is obtained, corresponding to the magnetic dipole band $\pi h_{11/2}^2 \nu h_{11/2}^{-2}$. The single-particle routhians calculated for this band are shown in Fig. 2. Note, that they behave quite differently than in the one-dimensional cranking, namely, all components of $h_{11/2}$ split strongly with rotational frequency. The proton particles and neutron holes responsible for the two shears blades are marked by arrows.

Fig. 3 shows the angular momentum vectors for several cranking frequencies. Spins of the $h_{11/2}$ protons and neutrons, that is, the shears blades (dashed lines), and contributions from the remaining proton and neutron cores (solid lines) are shown separately. Closing of the shears with rotational frequency is clearly visible. At the same time, polarization of the core increases rapidly. As expected, both cores polarize in the direction of the longest axis, which plays the role of the collective one, because the nucleus is approximately oblate. The angular momentum of the proton core is much larger than that of the neutron one. The two core subsystems differ by the six $d_{5/2}$ neutrons, so the properties of this orbital may be responsible
for the obtained asymmetry.

Experimentally, core polarization in $^{142}$Gd certainly affects the properties of the $\pi h_{11/2}^2 \nu h_{11/2}^{-2}$ band - it is observed up to spin 22, which is by 2 units more than the two blades of spin 10 can account for. Similar core-rotation effects have already been considered in the phenomenological analysis of the $^{197}$Pb shears band [10]. It seems, however, that the core polarization is too strong in our calculations, because we can build the band far beyond the spin 22 without even achieving zero angle between the shears. For example, solution for $\omega=0.6$ MeV/$h$ illustrated in Fig. 3, corresponds to $I\approx26$. Another point is that the calculated moment of inertia is too large, see Fig. 4.
Fig. 4. Energy as a function of spin for the magnetic dipole band $\pi h_{11/2}^2 \nu h_{11/2}^{-2}$ in $^{142}$Gd. Vertical scale shows the total energy obtained from the present calculations (line). Experimental data (dots) are arbitrarily shifted in energy in order to make the point at spin 16 coincide with the line.

4. Conclusions

In the present paper we report on the first Skyrme-Hartree-Fock calculations performed within the tilted-axis cranking method. It seems, that the strength of the time-odd components of the Skyrme mean field is crucial for the existence of the shears-like solutions, and that these strengths are incorrect when taken directly from typical Skyrme-force parametrizations. Our results for the $\pi h_{11/2}^2 \nu h_{11/2}^{-2}$ band in $^{142}$Gd indicate, that both the shears mechanism and collective rotation of the core are responsible for the generation of angular momentum, although the latter effect is probably too strong in the calculations. Whether the reason for the obtained discrepancies is the lack of pairing correlations or inadequate time-odd terms is not clear at the moment, and will be the subject of further investigations.

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