Abstract
We show that in ideal hydrodynamic simulations of heavy-ion collisions, initial state fluctuations result in an increase of the mean transverse momentum of outgoing hadrons, \( \langle p_t \rangle \). Specifically, \( \langle p_t \rangle \) is larger by a few percent if realistic fluctuations are implemented than with smooth initial conditions, the multiplicity and impact parameter being kept fixed. We show that result can be traced back to the fact that for a given total entropy, the initial energy contained in the fluid is larger if the density profile is bumpy. We discuss the implication of these results for the extraction of the equation of state of QCD from experimental data.

Keywords: heavy ions, nucleus-nucleus collisions, LHC, equation of state

1. Introduction

The importance of event-to-event fluctuations of the initial density profile created in relativistic heavy-ion collisions has been demonstrated in studies of anisotropic flow \([1,2,3]\). Here, we study their effect on the mean transverse momentum, \( \langle p_t \rangle \), of outgoing hadrons. We assume for simplicity that the evolution of the system is driven by ideal hydrodynamics \([4,5]\). It has long been recognized that initial-state fluctuations, followed by ideal hydrodynamic expansion, enhance the production of high-\( p_t \) particles \([6,7]\). This is understood as the effect of larger pressure gradients in the initial state, leading to higher fluid velocities. Here, we do not study the detailed structure of the \( p_t \) spectrum, but focus on the mean \( p_t \). One of our goals is to check whether the tight correlation between \( \langle p_t \rangle \) and the energy per particle, which has been established with smooth initial conditions, and which can be used to extract the equation of state from experimental data \([8,9]\), is preserved in the presence of fluctuations.

2. Event-by-event hydrodynamics

We model event-to-event fluctuations using the TRENTo parameterization of the initial entropy density profile \([10]\), where parameters are tuned to LHC data \([11]\). We generate 850 events with the same impact
Fig. 1. (Color online) Results of inviscid fluid-dynamical calculations for 850 events with the same impact parameter and entropy, but where the distribution of entropy in the transverse plane fluctuates event to event. Each filled circle corresponds to a single event. The red stars correspond to the event-averaged (smooth) initial density profile. Left: Scatter plot of the mean transverse momentum \( \langle p_t \rangle \) and the effective temperature \( T_{\text{eff}} \), defined by Eq. (1). Right: Scatter plot of \( \langle p_t \rangle \) and initial energy per unit rapidity \( E_i \) in our event-by-event hydrodynamic calculations. The dotted line in the right panel is a one-parameter fit using \( E_i = \alpha \epsilon(T) / s(T) \), where \( T \equiv \langle p_t \rangle / 3.03 \), and \( \alpha \) is a fit parameter (see text).

3. Interpretation

The mean transverse momentum is closely related to the energy per particle [9]. Since all our events have the same multiplicity by construction, this suggests that \( \langle p_t \rangle \) is determined by the fluid energy at freeze-out, \( E_f \), which fluctuates event to event. To study the correlation between \( \langle p_t \rangle \) and \( E_f \), we follow Ref. [9]. We define the effective temperature, \( T_{\text{eff}} \), and the effective volume, \( V_{\text{eff}} \), as those of a uniform fluid at rest which would have the same energy and entropy as the fluid at freeze-out:

\[
E_f = \int_{t_0}^{t_f} T^0 d\sigma_\rho = \epsilon(T_{\text{eff}})V_{\text{eff}},
\]

\[
S_f = \int_{t_0}^{t_f} s d\sigma_\rho = s(T_{\text{eff}})V_{\text{eff}}.
\]

For fixed entropy \( S_f \), the energy \( E_f \) is to a good approximation proportional to \( T_{\text{eff}} \), so that fluctuations of \( E_f \) correspond to fluctuations of \( T_{\text{eff}} \). Figure 1(left) shows that \( \langle p_t \rangle \) is tightly correlated with \( T_{\text{eff}} \), and that...
the relation $\langle p_t \rangle \approx 3.03 \, T_{\text{eff}}$ is a good approximation for all events, including the smooth event. Therefore, the larger $\langle p_t \rangle$ in the presence of fluctuations can be attributed to a larger $T_{\text{eff}}$ or, equivalently, a larger $E_f$.

Now, the reason why fluctuations increase the final energy $E_f$ is simply that they also increase the initial energy $E_i$, evaluated by integrating the energy density over the fluid volume at time $\tau_0$. Figure 1 (right) displays the scatter plot of $\langle p_t \rangle$ and $E_i$ for our sample of events. Comparison between the two panels shows that the correlation of $\langle p_t \rangle$ with $E_i$ is even stronger than the correlation with $T_{\text{eff}}$. More specifically, if one replaces $E_f = \int_{\tau_0} T^{00}d\tau\mu$ with $E_f = xE_i$ in Eq. (1) (where $x \approx 0.41$ is the average fraction of energy remaining after longitudinal cooling), and if one denotes by $T'_{\text{eff}}$ the new value of $T_{\text{eff}}$ obtained by solving the equations, then $3.03T'_{\text{eff}}$ is a better approximation of $\langle p_t \rangle$, represented by the dotted line in Fig. 1 (right), than $3.03T_{\text{eff}}$ in Fig. 1 (left). It is a paradox that the initial energy is a better predictor of $\langle p_t \rangle$ than the energy at freeze-out, since particle emission takes place at freeze-out. We do not have a simple explanation for this striking observation.

Fluctuations increase the initial energy $E_i$ for the following reason. At a given point in the transverse plane, the entropy density $s$ fluctuates event to event. The energy density at this point, denoted by $\epsilon$, is a function of $s$. Thermodynamic stability requires that the function $\epsilon(s)$ is convex. This implies that the average value of $\epsilon$ over events is larger than the value of $\epsilon$ corresponding to the average value of $s$, that is, larger than the energy density of the smooth profile. The relative increase depends on the model of fluctuations, in particular the transverse size of the inhomogeneities. A more spiky profile will result in a larger increase. Note that in contrast, anisotropic flow is typically insensitive to the scale of inhomogeneities [16]. The relative increase also depends on how thermalization is achieved in the early stages of the collisions. Assuming that ideal hydrodynamics holds at $\tau_0 = 0.6 \, \text{fm/c}$, as we do in our calculation, is likely to overestimate the energy increase. In order to obtain a realistic estimate of this effect, it is crucial to carefully model the thermalization phase [17].

4. Effective volume

Since all events have the same total entropy $S_f$, inspection of Eq. (1) shows that larger values of the effective temperature $T_{\text{eff}}$ imply smaller values of the effective volume $V_{\text{eff}}$. This is confirmed by the numerical results displayed in Fig. 2. Fluctuating initial conditions result in a smaller effective volume than...
smooth initial conditions. In Ref. [9], it was argued that the effective volume is determined by the initial radius $R_0$, defined by:
\[
(R_0^2) = 2 \int_0^{\tau_0} |r|^2 s(\tau_0, r) d\tau_0 s(\tau_0, r),
\]
where $s(\tau_0, r)$ is the initial entropy density, and the integration runs over the transverse plane. The results in Fig. 2 show that the value of $R_0^2$ for smooth initial conditions, indicated by a star, corresponds to the average of $R_0^2$ for fluctuating initial conditions, as expected. On the other hand, the proportionality factor $V_{eff}/R_0^3$ is consistently smaller if fluctuations are present. This is a corollary of the increase of the initial energy discussed above: Thus, the magnitude of the effect depends on the transverse scale of the fluctuations.

5. Implications for the extraction of the equation of state

Figure 1 (left) shows that the proportionality factor between $T_{eff}$ and $\langle p_t \rangle$ is essentially the same with smooth or fluctuating initial conditions. Thus, the value of $T_{eff} = 222 \pm 9$ MeV obtained in Ref. [9] assuming smooth initial conditions is robust with respect to the inclusion of initial state fluctuations. In order to estimate the corresponding entropy density $s(T_{eff})$ from the second line of Eq. (1), however, one needs the value of $V_{eff}$, which cannot be measured, and must be calculated. The present study suggests that the effect of initial fluctuations on $V_{eff}$ is potentially large. It is likely to be overestimated by the present study, where we use ideal hydrodynamics at early times, while a large pressure anisotropy is expected due to the fast longitudinal expansion. A quantitative study will require to include transport coefficients and a dynamical treatment of the thermalization phase [17].

Acknowledgments

F.G.G. was supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq grant 205369/2018-9 and 312932/2018-9). M.L. acknowledges support from FAPESP projects 2016/24029-6 and 2017/05685-2. F.G.G. and M.L. acknowledge support from project INCT-FNA Proc. No. 464898/2014-5 and G.G., M.L. and J.-Y.O. from USP-COFECUB (grant Uc Ph 160-16, 2015/13).

References

[1] B. Alver, et al., Phys. Rev. Lett. 98 (2007) 242302.
[2] J. Takahashi, B. M. Tavares, W. L. Qian, R. Andrade, F. Grassi, Y. Hama, T. Kodama, N. Xu, Phys. Rev. Lett. 103 (2009) 242301.
[3] B. Alver, G. Roland, Phys. Rev. C81 (2010) 054905, [Erratum: Phys. Rev.C82,039903(2010)].
[4] P. F. Kolb, U. W. Heinz, Hydrodynamic description of ultrarelativistic heavy ion collisions (2003) 634–714, arXiv:nucl-th/0305084.
[5] P. Romatschke, U. Romatschke, Relativistic Fluid Dynamics In and Out of Equilibrium, Cambridge Monographs on Mathematical Physics, Cambridge University Press, 2019. arXiv:1712.05815 doi:10.1017/9781108651998.
[6] R. P. G. Andrade, F. Grassi, Y. Hama, T. Kodama, W. L. Qian, Phys. Rev. Lett. 101 (2008) 112301.
[7] H. Holopainen, H. Niemi, J. K. Eskola, Phys. Rev. C83 (2011) 034901.
[8] L. Van Hove, Phys. Lett. 118B (1982) 138.
[9] F. G. Gardim, G. Giacalone, M. Luzum, J.-Y. Ollitrault, arXiv:1908.09728.
[10] J. S. Moreland, J. E. Bernhard, S. A. Bass, Phys. Rev. C92 (1) (2015) 011901.
[11] G. Giacalone, J. Noronha-Hostler, J.-Y. Ollitrault, Phys. Rev. C95 (5) (2017) 054910.
[12] J.-F. Paquet, C. Shen, G. S. Demiclo, M. Luzum, B. Schenke, S. Jeon, C. Gale, Phys. Rev. C93 (4) (2016) 044906.
[13] J. D. Bjorken, Phys. Rev. D27 (1983) 140–151.
[14] F. Cooper, G. Frye, Phys. Rev. D10 (1974) 186.
[15] A. Bazavov et al., Phys. Lett. B795 (2019) 15–21.
[16] F. G. Gardim, F. Grassi, P. Ishida, M. Luzum, P. S. Magalhães, J. Noronha-Hostler, Phys. Rev. C97 (6) (2018) 064919.
[17] A. Kurkela, A. Mazeliauskas, J.-F. Paquet, S. Schlichting, D. Teaney, Phys. Rev. Lett. 122 (12) (2019) 122302.