Non-leptonic decays in an extended chiral quark model

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We consider the color suppressed (nonfactorizable) amplitude for the decay mode $B_0 \rightarrow \pi^0 \pi^0$. We treat the $b$-quark in the heavy quark limit and the energetic light $(u,d,s)$ quarks within a variant of Large Energy Effective Theory combined with an extension of chiral quark models.

Our calculated amplitude for $B_0 \rightarrow \pi^0 \pi^0$ is suppressed by a factor of order $\Lambda_{QCD}/m_b$ with respect to the factorized amplitude, as it should according to QCD-factorization. Further, for reasonable values of the (model dependent) gluon condensate and the constituent quark mass, the calculated nonfactorizable amplitude for $B_0 \rightarrow \pi^0 \pi^0$ can easily accommodate the experimental value. Unfortunately, the color suppressed amplitude is very sensitive to the values of these model dependent parameters. Therefore fine-tuning is necessary in order to obtain an amplitude compatible with the experimental result for $B_0 \rightarrow \pi^0 \pi^0$.

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I. INTRODUCTION

The decay modes of the type $B \rightarrow \pi \pi$ are dynamically different. For the case $B_0 \rightarrow \pi^+ \pi^-$ there is a substantial factorized contribution which dominates. In contrast, the decay mode $B_0 \rightarrow \pi^0 \pi^0$ has a small factorized contribution, being proportional to a small Wilson coefficient combination. However, for the decay mode $B_0 \rightarrow \pi^0 \pi^0$ there is a sizeable nonfactoriz-

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able (color suppressed) contribution due to soft (long distance) interactions, which dominate the amplitude.

In spite of tremendous effort within QCD factorization \[1\], soft collinear effective theory (SCET) \[2, 3\], the so-called pQCD model \[4\], and QCD sum rules, the obtained theoretical amplitude for $\overline{B}_d \rightarrow \pi^0 \pi^0$ is still a factor $\sim 2$ off \[5\].

Chiral quark models bridge between the quark and meson picture and has shown to be suitable for calculating color suppressed decays \[6–19\]. In this talk I report on a calculation \[20\] of $\overline{B}_d \rightarrow \pi^0 \pi^0$ within a extension of chiral quark model \[21\].

II. EFFECTIVE THEORIES

The effective non-leptonic quark level Lagrangian at a scale $\mu$ has the form:

$$L_W = \sum_i C_i(\mu) \hat{Q}_i(\mu),$$

where $C_i$ are Wilson coefficients containing loop effects from scales above $\mu$. Typically, the operators $\hat{Q}_i$'s are products of two left-handed quark currents. Genericly, for non-leptonic processes with two numerically relevant operators $\hat{Q}_{X,Y}$ one obtains

$$\langle M_1 M_2 | L_W | M \rangle = \left( C_X + \frac{C_Y}{N_c} \right) \langle M_1 | j^L_1(1) | 0 \rangle \langle M_2 | j^L_2(2) | M \rangle + C_Y \langle M_1 M_2 | \hat{Q}_Y^{color} | M \rangle.$$ (1)

Here, for “flavor mismatch”, as shown in the right-hand diagram of fig.2, one has used a Fierz transformation. The operator $\hat{Q}_Y^{color}$ is a product of two colored currents of the type $j^a_\alpha = \bar{q}_1 \gamma_\alpha L t^a q_2$, where $q_{1,2}$ are quark fields, $t^a$ a color matrix and $L$ the left-handed projector in Dirac space.

In some cases the coefficient combination $\left( C_X + \frac{C_Y}{N_c} \right)$ is close to zero. Then matrix elements of $\hat{Q}_Y^{color}$ might dominate. It is important to note that the matrix elements of colored operators might be calculated within chiral quark models.

Heavy ($b$ eventually also $c$-) quarks might be treated within Heavy Quark Effective Theory (HQET) \[22, 23\]. In this theory one projects out the movement of a heavy quark: $p_Q = m_Q v + k$, where the heavy quark velocity $v$ satisfies $v^2 = 1$, and $m_Q$ is the heavy quark mass. One obtains an effective Lagrangian for the reduced heavy quark field $Q_v$ and the corresponding propagator $S(p_Q)$:

$$L_{HQET} = \overline{Q}_v i v \cdot D Q_v + \mathcal{O}(m_Q^{-1}) ; \quad S(p_Q) = \frac{(1 + \gamma \cdot v)}{2 k \cdot v}.$$ (2)
For energetic light \((u, d, s)\) quarks in the final state one might use \textit{Large Energy Effective Theory} \([24, 25]\) which was later developed into SCET \([2, 3]\). In this case one projects out movement of light energetic quark with momentum \(p^\mu = E n^\mu + k^\mu\), where \([21]\]

\[
n(\text{or } \tilde{n}) = (1, 0, 0, \pm \eta) \quad ; \quad \eta = \sqrt{1 - \delta^2} , \quad n^2 = \tilde{n}^2 = \delta^2 , \quad v \cdot n = v \cdot \tilde{n} = 1. \tag{3}
\]

Here \(\delta \sim \Lambda_{QCD}/m_Q \ll 1\). In the original version \([25]\), \(\delta = 0\), but it was found later \([21]\) that it was necessary to have \(\delta \neq 0\) in order to combine LEET with chiral quark models. One obtains an effective Lagrangian for reduced light energetic quark field \(q_n\):

\[
\mathcal{L}_{\text{LEET}\delta} = \bar{q}_n \left( \frac{1}{2} (\gamma \cdot \tilde{n} + \delta) \right) (i n \cdot D) q_n + \mathcal{O}(E^{-1}) \quad ; \quad S(p_q) = \frac{\gamma \cdot n}{2n \cdot \tilde{n}} . \tag{4}
\]

In the formal limits \(M_H \to \infty\) and \(E \to \infty\), \(\langle P \left| V^\mu \right| H \rangle\) of the form \([25]\):

\[
\langle P \left| V^\mu \right| H \rangle = 2E \left[ \zeta^{(v)}(M_H, E)n^\mu + \zeta_1^{(v)}(M_H, E)v^\mu \right] ,
\]

where the form factors scale as:

\[
\zeta^{(v)} = C \frac{\sqrt{M_H}}{E^2} \quad , \quad C \sim (\Lambda_{QCD})^{3/2} \quad , \quad \frac{\zeta^{(v)}}{\zeta^{(v)}} \sim \delta \sim \frac{1}{E} . \tag{5}
\]

The behavior of the form factors are consistent with the energetic quark having \(x\) close to one, where \(x\) is the quark momentum fraction of the outgoing pion.

Within the mesonic picture one has low energy effective theories, mainly chiral perturbation theories, both for the pure light sector and the heavy-light sector, with effective cut-offs \(\Lambda_\chi \sim 4\pi f_\pi \sim 1\ \text{GeV}\). Such effective theories contain the light meson fields \((\pi, K, \eta_8)\) in a 3 by 3 matrix \(\Pi\). The chiral Lagrangians will contain the fields \([26]\)

\[
\xi \equiv \exp \left( \frac{i}{2f_\pi} \Pi \right) \quad ; \quad H^{(\pm)} = P^{(\pm)}(v) (P^{(\pm)}_\mu \gamma^\mu - i P^{(\pm)}_5 \gamma^5) , \tag{6}
\]
where $H_v$ is the combined field consisting of one light and one heavy ($b$- eventually also $c$-quark) with spin-parity $0^-$ and $1^-$. 

**III. CHIRAL QUARK MODELS**

Chiral quark models are the bridge between the quark and the meson pictures. Loop momenta in $\chi$QM’s should not exceed the chiral symmetry breaking scale $\Lambda_\chi$. In the pure light ($q = u, d, s$) sector the Lagrangian might be written [27–31]:

$$\mathcal{L}_{\chi QM} = \overline{\chi} \left[ \gamma^\mu (i D_\mu + V_\mu + \gamma_5 A_\mu) - m \right] \chi + \mathcal{O}(m_q)$$

where $m$ is the *constituent* light quark mass, due to chiral symmetry breaking. This implies meson-quark couplings modelling confinement! The quark fields $\chi$ are flavor rotated versions of the ordinary left and righthanded quark fields $q_{L,R}$, namely: $\chi_L = \xi q_L$ , $\chi_R = \xi^\dagger q_R$. A color suppressed suppressed decay $M \rightarrow M_1 M_2$ will within chiral quark models generically look like in Fig 2.

An important ingredient in our treatment of color suppressed amplitudes is the emission of soft gluons making (model dependent) gluon condensates [32]:

$$g_8^2 G_{\mu\nu}^a G_{\alpha\beta}^a \rightarrow 4\pi^2 \left( \frac{\alpha_s}{\pi} G^2 \right) \frac{1}{12} (g_{\mu\alpha} g_{\nu\beta} - g_{\mu\beta} g_{\nu\alpha}) \ . \ (7)$$

Within chiral quark models the logarithmic- and quadratic divergent integrals are identified with the physical $f_\pi$ and the quark condensate, respectively [27–31, 33].
FIG. 3: Bosonization of the heavy meson \((H_v)\) to light energetic meson \((M_n)\) current (vertex in red) involving the the heavy reduced quark field \((Q_v\) in blue), the light energetic quark field \((q_n\) in green) and the soft light (flavor rotated) quark field \((\chi\) in black)

For the heavy light case \((HL\chiQM)\), including heavy quarks our description includes \(L_{HQET}, L_{\chiQM}\) and an additional the meson-quark interaction \(^{33-37}\):

\[
L_{Int} = -G_H Q_v H_v \chi + h.c.
\]  

(8)

Integrating out quarks (by loop diagrams) should give the known HL\(\chi\)PT terms with definite constants in front. The physical and model dependent parameters \(f_\pi, \langle q\bar{q}\rangle, f_H, g_A\) are linked to (divergent) loop integrals as in the pure light \(\chiQM\). A fit in strong sector gives \(^{33}\): \(m \sim 220\) MeV, \((\frac{m}{\pi^2} G^2)^{1/4} \sim 315\) MeV, \(G_H^2 = \frac{2m}{f}\rho\) where \(\rho \sim 1\) and \(\rho\) depend on \(f_\pi, \langle q\bar{q}\rangle, m, g_A\). An ideal case for our method is \(B - \overline{B}\)-mixing, where a result very close to the lattice result was obtained \(^{38}\).

Recently, chiral quark models are extended \(^{21}\) to incorporate also light energetic quarks described by LEET. The meson-quark interaction is then assumed to have the form (- the light meson field \(M\) is a 3 by 3 matrix):

\[
L_{intq} = G_A \bar{q} \gamma_\mu \gamma_5 (\partial^\mu M) q_n + h.c.
\]  

(9)

For mesons containing a reduced field \(q_n\) we use a corresponding meson field \(M_n\).

The coupling \(G_A\) is determined by a loop diagram for \(\zeta^{(v)}\) in Fig. 3. We find that the obtained ratio of formfactors obtained in LE\(\chiQM\) is fulfilling the requirements of \(^{25}\) in eq. (7). The bosonized current for \(H_v \rightarrow M_n\) transition is \((H_v\) and \(M_n\) represent the heavy B- and the energetic \(\pi\)-meson):

\[
J^\mu_{tot}(H_v \rightarrow M_n) = \left(-i\frac{G_H G_A}{2} m^2 F\right) \text{Tr} \left\{ \gamma^\mu LH_v [\gamma \cdot n] \xi^1 M_n\right\},
\]  

(10)
where \( F = N_c/(16\pi) + ... \sim 10^{-1} \). Using \( \delta = m/E \), which is the chiral quark model version of \( \Lambda_{QCD}/m_b \), we obtain a result of the type (recalling \( M_B \approx 2E \) in eq. (7)):

\[
G_A \sim \frac{1}{N_c} \frac{1}{E^2}
\]

The coupling \( G_A \) is then fixed from light cone sum rules: \( \zeta^{(e)} \approx 0.3 \) \[39, 40\]. Using \( G_A \) and \( \zeta^{(e)} \) it was found \[21\] that the nonfactorizable amplitude accounted for 2/3 of the experimental amplitude for \( B_0 \rightarrow D^0 \pi^0 \). There are also additional meson contributions.

Our prescription for calculating non-leptonic amplitudes is then the following: Integrate out \( W \) and heavy quarks to obtain eq. (1). Then, bosonize the quark operators by integrating out the quark fields, and obtain an effective (chiral) Lagrangian at meson level. This is however often an idealized situation. In reality some meson loops cannot be calculated as chiral loops in the ordinary sense. They might be suppressed by say \( 1/M_B \) or must be treated by other methods. In any case, final state interactions are in general present \[41, 42\].

**IV. THE AMPLITUDE FOR \( \overline{B}^0 \rightarrow \pi^0 \pi^0 \) IN LE\(\chi\)QM**

Now we use the results of \[21\] to calculate the color suppressed amplitude for \( \overline{B}^0 \rightarrow \pi^0 \pi^0 \) \[20\].

The colored \( H_b \rightarrow M_n \) current (representing \( B \rightarrow \pi \) (hard)) is found to be:

\[
J_{1G}^\mu(H_b \rightarrow M)^a = g_s G_{\alpha\beta}^a \frac{G_H G_A}{128\pi} \epsilon^{\sigma\alpha\beta\lambda} \tilde{n}_\sigma \text{Tr} \left( \gamma^\mu L H_v \gamma_\lambda \xi^1 M_n \right), \quad (11)
\]

Similarly the colored current for outgoing hard \( M_\tilde{n} \) (representing the hard pion \( \pi_\tilde{n} \)) is:

\[
J_{1G}^\mu(M_\tilde{n})^a = g_s G_{\alpha\beta}^a 2 \left( -\frac{G_A E}{4} \right) Y \tilde{n}_\sigma \epsilon^{\sigma\alpha\beta\mu} \text{Tr} \left[ \lambda^X M_\tilde{n} \right], \quad (12)
\]

where \( \lambda^X \) = appropriate SU(3) flavor matrix, and \( Y \) is a loop factor:

\[
Y = \frac{f_\pi^2}{4m^2N_c} \left( 1 - \frac{1}{24m^2f_\pi^2} \frac{\alpha_s}{\pi} G^2 \right). \quad (13)
\]

The ratio of non-factorizable to factorizable amplitudes is found to be \[20\] (refering to eq. (2) for the Wilson coefficient):

\[
r \equiv \frac{M(B_0^0 \rightarrow \pi^0 \pi^0)_{\text{Non-Fact}}}{M(B_0^0 \rightarrow \pi^+ \pi^-)_{\text{Fact}}} = \left( \frac{C_Y \sigma}{(C_X + C_Y/N_c)} \right) \frac{1}{N_c} \frac{m}{E}, \quad (14)
\]
FIG. 4: Non-factorizable contribution to $B \to \pi^0 \pi^0$ containing large energy light fermions and mesons. Also corresponding diagram where the outgoing anti-quark $\bar{u}$ is hard.

where $\sigma$ is a model-dependent hadronic factor, dimension-less and $\sim (N_c)^0$. Our calculations show that the ratio $r \sim 1/N_c$ and $r \sim m/2E \simeq \Lambda_{QCD}/m_b$ as it should according to QCD factorization [1]. The ratio is plotted in Fig 5.

The experimental value of the $\overline{B}_d^0 \to \pi^0 \pi^0$ amplitude can be accomodated for $m \sim 220$ Mev and $\langle \frac{\alpha_s}{\pi} G^2 \rangle^{1/4} \sim 315$ MeV. - as in previous work [6–19]. But the result is very sensitive to variations of $m$ and $\langle \frac{\alpha_s}{\pi} G^2 \rangle$, as seen by loop factor $Y$. In addition there are meson loops not yet calculated.

There is an extension of the models under construction in order to incorporate meson loops including (energetic) vector mesons ($V_n$) in loops and we will need couplings for $V_n \to M_n + (\text{soft } \pi)$. One might also include light non-energetic vector mesons.

V. CONCLUSIONS

We have considered the amplitude $B^0 \to \pi^0 \overline{\pi}^0$ within an extended chiral quark model LE$\chi$QM which is in accordance with the requirements of [25].

For the color suppressed $\sim 1/N_c$ part of the $\overline{B}_d^0 \to \pi^0 \pi^0$ amplitude , we have found a
FIG. 5: Plot for the ratio $r$ in terms of $m$ and $\langle \alpha_s \pi G^2 \rangle^{1/4}$. For reasonable values of these parameters the ratio $r$ can take a wide range of values such that fine-tuning is required to reproduce the experimental value.

\[ r \sim m/2E \approx \Lambda_{QCD}/m_b \] suppression as we should according to [1]. Unfortunately the obtained amplitude is very sensitive to $m$ and $\langle \alpha_s \pi G^2 \rangle$.

Extension of the model including energetic light vectors, in order to be able to consider meson loops, are under construction.
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