Unitarity and the color confinement

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Abstract

We discuss how confinement property of QCD results in the rational unitarization scheme and how unitarity saturation leads to appearance of a hadron liquid phase at very high temperatures.
Introduction

One of the fundamental problems of QCD is related to a confinement of the color. This phenomenon is associated with collective, coherent interactions of quarks and gluons, and results in formation of the asymptotic states, which are the colourless, experimentally observable particles.

On the other, theoretical side, there is a hypothesis on the completeness of a set of the asymptotic hadronic states. This assumption plays an important role (cf. [1]) in strong interaction theory and leads, e.g. due to unitarity of the scattering matrix, to an optical theorem relating the total cross-section with the value of the elastic scattering amplitude in the forward direction, i.e. at $t = 0$. The unitarity condition is formulated for the asymptotic colorless on-mass shell states and does not constrain the fundamental fields of QCD – colored fields of quarks and gluons.

The Hilbert space, in the axiomatic formulation of the quantum field theory, corresponds to the colorless hadron states. It is constructed using vectors obtained by action of the relevant creation operators on the physical vacuum. The state of physical vacuum is the state without particles, i.e. annihilation operator acting on it produces zero. It is the state of a lowest energy of the system. This state is also invariant under Lorentz transformations.

The point of completeness of a set of the asymptotic states, which includes hadrons only, can be considered as a questionable one in the QCD, and the unitarity could be violated in the above indicated sense (cf. [2]). It was stated that the Hilbert space, which corresponds to colorless hadron states and is constructed using vectors spanned on the physical vacuum state, should be extended. Indeed, nowadays it is often regarded that the vacuum state is not a unique one: colored current quarks and gluons are the degrees of freedom related to another, perturbative vacuum. The vacuum state is not considered anymore as a state of the lowest energy of a system.

According to the confinement property of QCD, isolated colored objects cannot exist in the physical vacuum and there is no room for the objects like quark-proton scattering amplitude, since isolated color object has an infinite energy in the physical vacuum. Transition from physical vacuum to the perturbative one supposed to occur in the process of deconfinement resulting in quark-gluon plasma formation, i.e. a gaseous state of the free colored quarks and gluons. Thus, it is evident that the hadrons and free quarks (and gluons) cannot coexist together since those belong to the different vacua [3].

In this paper we address the issues related to unitarity and its relation to the confinement property of QCD. In the next section we demonstrate how the color confinement could result in the rational unitarization scheme for the scattering matrix. Section 2 is devoted to discussion of the consequences of the rational unitarization scheme, namely appearance of the reflective scattering mode. It will
be shown that this mode could result in the emerging new phase of the hadron matter — hadron liquid. In the section 3 a possible microscopic mechanism for the above transition and existence of the third vacuum state is discussed in the context of the unitarity saturation at very high energies.

1 Unitarity and confinement

In this section we demonstrate that inclusion into a set of the asymptotic states of the fields corresponding to the confined objects would lead to a rational form of S-matrix unitarization provided those fields satisfy a certain simple constraint. We address above issues using a paper by N.N. Bogolyubov [5] as a guide and consider a state vector $|\Phi\rangle$ being a sum of the two vectors

$$|\Phi\rangle = |\varphi\rangle + |\omega\rangle,$$

where $|\varphi\rangle$ corresponds to the physical states and belongs to the Hilbert space $\mathcal{H}_\varphi$ and $|\omega\rangle$ — to the confined states and belongs to the Hilbert space $\mathcal{H}_\omega$. So, we have that $|\varphi\rangle = P|\Phi\rangle$ and $|\omega\rangle = (1 - P)|\Phi\rangle$, where $P$ is a relevant projection operator. Difference with consideration performed in [5] is in the replacement of the states with indefinite metrics by the states of confined objects such as quarks and gluons. A common Hilbert space $\mathcal{H}$ is a sum:

$$\mathcal{H} = \mathcal{H}_\varphi + \mathcal{H}_\omega$$

and the scattering operator $\tilde{S}$ acting in $\mathcal{H}$,

$$|\Phi, out\rangle = \tilde{S}|\Phi, in\rangle,$$

should not, in principle, conserve probability and obey unitarity condition since $\mathcal{H}$ includes $\mathcal{H}_\omega$ — Hilbert space where confined objects reside. The norm of confined objects is not defined.

Next, let us to impose condition similar to the one used in [5], i.e.

$$|\omega, in\rangle + |\omega, out\rangle = 0.$$

It means that in- and out- vectors corresponding to the states of the confined objects are just the mirror reflections of each other. Those reflections can be associated with the impossibility for confined objects to propagate outside the hadron border. Thus, the rational form of unitary scattering operator $S$

$$S = (1 - U)(1 + U)^{-1},$$

\footnote{The reference to this paper was brought to our attention by V.A. Petrov.}
in the physical Hilbert space $\mathcal{H}_\varphi$,

$$|\varphi, \text{out} \rangle = S|\varphi, \text{in} \rangle$$

can easily be obtained, since

$$|\varphi, \text{out} \rangle = P\tilde{S}(|\varphi, \text{in} \rangle + |\omega, \text{in} \rangle).$$

Operator $U$ has the following form

$$U = (1 - P)\tilde{S}.$$ 

We started with non-unitary scattering operator $\tilde{S}$ and obtained the scattering operator $S$, which automatically satisfy unitarity condition. Crucial assumption there was a constraint for the states of confined objects $|\omega, \text{in} \rangle + |\omega, \text{out} \rangle = 0$, which we assumed to be a cofinement condition since it provides a condition for an existence of a horizon for the colored particles. Colorless particles such as a photon or a pion are not affected by the existence of such a horizon and can travel beyond hadron boundaries. Thus, it is very tempting to claim that unitarity can be related to a confinement.

Rational or $U$-matrix form of unitarization was proposed long time ago [6] in the theory of radiation dumping. Self-damping of inelastic channels was considered in [7] and for the relativistic case such form of unitarization was obtained in [8]. But, an importance of the forgotten paper [5] is, in particular, the following: it provides a clue for the physical interpretation of $U$-matrix. Nowadays it can be used for construction of a bridge between the physical states of hadrons and the states of confined objects – quarks and gluons.

The elastic scattering $S$-matrix, i.e. the $2 \rightarrow 2$ matrix element of the operator $S$, in the impact parameter representation can be written in this unitarisation scheme in the form of linear rational transform (cf. [9] and references therein) and in the case of pure imaginary $U$-matrix

$$S(s, b) = \frac{[1 - U(s, b)]}{1 + U(s, b)},$$

where $U(s, b)$ is the generalized reaction matrix. It is considered to be an input dynamical quantity. And, this is an essential point, an explicit form of the function $U(s, b)$ and the numerical predictions for the observable quantities depend on the particular model used for hadron scattering description.

With account of what was said above we can associate this function with matrix element$^3$ of the operator $(1 - P)\tilde{S}$, i.e. $U(s, b)$ should be related to a scattering dynamics of the confined hadron constituents.

$^3$Imaginary part of $U(s, b)$ gets contributions from inelastic intermediate channels.
Rational representation of the scattering matrix leads to several distinctive features such as a peripherality of inelastic processes \cite{9} and restoration of confined phase of hadronic matter at very high temperatures. We consider the latter issue in the following section.

2 Hadronic liquid at very high temperatures

For the qualitative purposes it is sufficient to know \cite{9} that the function \( U(s, b) \) increases with energy in a power-like way and decreases with impact parameter like a linear exponent or Gaussian\(^3\). It can easily be seen that the new scattering mode, reflective scattering (when \( S(s, b) < 0 \)) starts to appear at the energy \( s_R \), which is determined by a solution of the equation

\[ U(s_R, b = 0) = 1. \]

Indeed, the unitarity relation written for the elastic scattering amplitude \( f(s, b) \) in the high energy limit has the following form

\[ \text{Im} f(s, b) = h_{el}(s, b) + h_{inel}(s, b). \] (2)

The inelastic overlap function \( h_{inel}(s, b) \) is connected with the function \( U(s, b) \) by the relation

\[ h_{inel}(s, b) = U(s, b) / [1 + U(s, b)]^2, \] (3)

and the only condition to obey unitarity is \( U(s, b) \geq 0 \). The elastic overlap function in its turn is related to the function \( U(s, b) \) as follows

\[ h_{el}(s, b) = [U(s, b)]^2 / [1 + U(s, b)]^2. \] (4)

At sufficiently high energies the inelastic overlap function \( h_{inel}(s, b) \) would have a peripheral \( b \)-dependence and will tend to zero for \( b = 0 \) at \( s \to \infty \) (cf. e.g. \cite{2}). Therefore, corresponding behaviour of the elastic scattering \( S \)-matrix (note that \( S(s, b) = 1 + 2if(s, b) \)) can then be interpreted as an appearance of a reflecting ability of scatterer due to increase of its density beyond some critical value. In another words, the scatterer has now not only absorption ability (due to presence of inelastic channels), but it starts to be reflective at very high energies. In central collisions, \( b = 0 \), an elastic scattering approaches to the completely reflecting limit \( S = -1 \) at \( s \to \infty \).

At the energy values \( s > s_R \) the equation \( U(s, b) = 1 \) has a solution in the physical region of the impact parameter values, i.e. \( S(s, b) = 0 \) at \( b = R(s) \).

\(^3\)In fact, the analytical properties of the scattering amplitude imply a linear exponential dependence at large values of \( b \).
The probability of reflective scattering at $b < R(s)$ and $s > s_R$ is determined by the magnitude of $|S(s,b)|^2$; this probability is equal to zero at $s \leq s_R$ and $b \geq R(s)$. Those inequalities impose an equation for a horizon for the reflective scattering events. The dependence of $R(s)$ is determined by the logarithmic function, $R(s) \sim \frac{1}{M} \ln s$. This dependence is consistent with analytical properties of the resulting elastics scattering amplitude in the complex $t$-plane and mass $M$ can be related to the pion mass. Thus, at the energies $s > s_R$ the reflective scattering will simulate presence of the repulsive core in the hadron and meson interactions and the reflective elastic scattering will become a dominating process at sufficiently high energies. This kind of the elastic scattering preserving the hadron identities acts against deconfinement. It would lead to a new phase of the hadron matter at very high temperatures.

Presence of the reflective scattering can be accounted for by using the van der Waals method (cf. [17]). This approach originally was used for the description of liquids starting from a gas approximation by introducing a nonzero size of molecules.

The hadron liquid density $n_R(T, \mu)$ can be connected then [18] with the density in the approach without reflective scattering (i.e. poinlike type of interaction) $n(T, \mu)$ by the following relation

$$n_R(T, \mu) = n(T, \mu)/[1 + \kappa(s)n(T, \mu)],$$

where $\kappa(s) = p_R(s)V_R(s)/2$, $p_R(s)$ is the averaged over volume $V_R(s)$ probability of reflective scattering and the volume $V_R(s)$ is determined by the radius of the reflective scattering. At very high energies ($s \rightarrow \infty$)

$$n_R(T, \mu) \sim \frac{1}{\kappa(s)} \sim \frac{M^3}{\ln^3 s}.$$

This limiting dependence for the hadron liquid density appears due to presence of the reflective scattering. In the oversimplified geometrical picture it resembles a scattering of hard spheres in the head-on hadron collisions. It can also be associated with saturation of the Froissart-Martin bound for the total cross-section.

At very high temperatures we could expect that a new confined phase corresponding to hadron liquid reappears. A possible presence of this phase is connected with the unitarity saturation, and a relevant microscopic mechanism will be discussed in the next section.

### 3 Microscopic mechanism

The picture described above implies an existence of the two vacuum states: perturbative and physical ones. It results from assumption on the same scale of transitions responsible for confinement-deconfinement and chiral restoration. This
assumption has a theoretical ground in some of the lattice calculations (cf. e.g. [10]).

However, it is often assumed that the scales relevant to confinement and chiral symmetry breaking are different [11], scale of confinement \( \Lambda_{\text{QCD}} = 100 - 300 \) MeV while chiral symmetry breaking scale — \( \Lambda_{\chi} \simeq 1 \) GeV. Thus, in the range between these two scales the matter is in a deconfined state but chiral symmetry is spontaneously broken there. In line with this picture, which can be treated as a posteriori justification, long time ago, in the pre-QCD era, it was supposed that hadrons have a simple structure and non-relativistic quark model has been commonly adopted. During recent time such a model has evolved and obtained much more solid theoretical grounds [11, 12, 13]. As it will be discussed further, one can assume existence in the hadron’s interior of the third (nonperturbative) vacuum state with colored constituent quarks and pions as the relevant degrees of freedom.

The origin of a nonperturbative vacuum and relevant effective degrees of freedom can be related to the mechanism of spontaneous chiral symmetry breaking (\( \chi_{\text{SB}} \)) in QCD [14]. This mechanism describes transition of the current into the constituent quarks and emerging of the Goldstone bosons. Massive constituent quarks appear as quasi-particles, i.e. current quarks and the surrounding clouds of quark–antiquark pairs. The constituent quarks interact via exchange of the Goldstone bosons; this interaction is mainly due to a pion field. Pions themselves are the bound states of massive quarks. Constituent quark interaction with the Goldstone bosons is strong and could have the following form [13]:

\[
L_I = \bar{Q} [i\partial_\mu - M \exp(i\gamma_5 \epsilon^A \chi_A/F_\pi)] Q, \quad \pi^A = \pi, K, \eta.
\]  

(5)

For simplicity, in what follows we refer to pions only, denoting by this generic word all Goldstone bosons, i.e. pions themselves, kaons and \( \eta \)-mesons.

Thus, we will assume that the vacuum state \( |0\rangle_{\text{ph}} \) has a perturbative nature at short distances with current quarks and gluons as degrees of freedom, at large distances the physical vacuum state \( |0\rangle_{\text{ph}} \) has relevant colorless hadrons as degrees of freedom, and inside a hadron the vacuum \( |0\rangle_{np} \) has a nonperturbative origin with constituent quarks and Goldstone pions being the relevant degrees of freedom. We suppose the picture of a hadron consisting of constituent quarks interacting with pions. The latter have a dual role: Goldstone and physical particles.

There are different approaches to the deconfinement dynamics. It was recently proposed [15, 16] to use percolation theory as a candidate for the mechanism of deconfinement in the form of the analytical crossover (i.e. without first and second order phase transitions). This form of deconfinement was found in the experimental studies at RHIC. Evidently, such purely geometrical perolation approach should be amended by a dynamical mechanism and color dynamics of deconfinement due to formation of molecular-like aggregations was proposed in [3]. The
vacuum inside the hadron was taken to be a perturbative one and quark interactions have origin in the colour dynamics. It seems, however, that for crossover nature of deconfinement dynamics it is more natural to expect transition of physical to nonperturbative vacuum $|0\rangle_{ph} \rightarrow |0\rangle_{np}$ instead of transition $|0\rangle_{ph} \rightarrow |0\rangle_{pt}$. Indeed, using effective quark-pion interaction inside hadron and hadron-pion interaction outside hadron, we have a pion field as an universal interaction agent for both confined and deconfined states and this could serve as a natural explanation of deconfinement as a crossover transition.

Experimentally, deconfined state of matter has been discovered at RHIC where the highest values of energy and density have been reached. This deconfined state appears to be a strongly interacting collective state with properties of a perfect liquid. The matter is strongly correlated and reveals the high degree of coherence when it is well beyond the critical values of density and temperature. In the framework of the approach under consideration this state can be interpreted as a Quark (constituent)-pion liquid in the nonperturbative vacuum $|0\rangle_{np}$.

A natural question arises then: what one should expect at higher temperatures, e.g. at the LHC energies, i.e. would one observe transition $|0\rangle_{np} \rightarrow |0\rangle_{pt}$, or additional possibilities exist? Indeed, due to a large kinetic energy of the constituent quarks in the nonperturbative vacuum there should be a finite probability to form the colorless clusters again, i.e. confinement mechanism could take place, transition $|0\rangle_{np} \rightarrow |0\rangle_{ph}$ would happen instead of the transition $|0\rangle_{np} \rightarrow |0\rangle_{pt}$ and hadrons would reappear again. This possibility obtain support from unitarity saturation at very high energies discussed in the previous section.

Thus, the following chain of phase transitions of hadron matter could exist as the temperature increases at the constant value of chemical potential $\mu$:

$$|0\rangle_{ph} (\text{Hadron gas}) \rightarrow |0\rangle_{np} (\text{Quark-pion liquid}) \rightarrow |0\rangle_{ph} (\text{Hadron liquid}).$$

In the above chain a physical vacuum suffers a loop transition to a nonperturbative one and back to a physical vacuum. It is also useful to keep in mind that due to Lorentz invariance of the vacuum states energy difference between those states should also be Lorentz invariant [1], i.e. several vacua would be either degenerate or have infinite energy difference. On that basis it can be supposed that crossover form of deconfinement implies degeneracy of the states: $|0\rangle_{ph}$ and $|0\rangle_{np}$.

The picture described above is certainly an oversimplified one. The reflective scattering is always accompanied by the absorptive scattering at moderate and large impact parameters in hadronic collisions. Such highly energetic peripheral hadron collisions might imply the transitions $|0\rangle_{np} \rightarrow |0\rangle_{pt}$ and, in fact, at very high temperatures we might have not a homogeneous phase of a hadronic liquid.
Conclusion

It was shown how unitary form of the scattering matrix could be inherited from the confinement property of QCD. It was conjectured also that saturation of unitarity would restore confinement, and percolation mechanism alone is not sufficient for the deconfinement as it was assumed in [15, 16, 17, 18]. In general, we would like to note that at very high temperatures there is a certain probability that the matter would return to a confined state if the unitarity saturation would occur there. Hopefully, the experimental studies with heavy ions at the LHC will be able to reveal the new phases of hadronic matter.

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