The $\Upsilon$ electroproduction at HERA, EIC, and LHeC within the nonrelativistic QCD framework

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Abstract

Based on the nonrelativistic QCD framework, we carry out the study of the $\Upsilon$ production in the semi-inclusive deeply inelastic electron-proton scattering (SIDIS) at HERA, EIC, and LHeC, respectively, with the main aim of assessing the viability of observing the $\Upsilon$ electroproductions at the three colliders. The color-octet (CO) contributions are found to exert crucial effect on both the integrated and differential cross sections, serving to further establish the significance of the CO mechanism. By setting the kinematic cuts to $p_t^2 > 1$ GeV$^2$, $W > 50$ GeV, $2 < Q^2 < 100$ GeV$^2$, and $0.3 < z < 0.9$, only quite few electroproduced $\Upsilon$ events can be generated by HERA, partially accounting for its lack of measurement on the $\Upsilon$ electroproduction. However, under the same cut conditions, the EIC and the LHeC can accumulate about $8.7 \times 10^2$ and $3.7 \times 10^4$ reconstructed $\Upsilon$ events in one operation year, respectively, which manifestly indicates the prospect of detecting the $\Upsilon$ related SIDIS processes at the two forthcoming $ep(eA)$ colliders.

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I. INTRODUCTION

The heavy quarkonium related production processes, which involve both perturbative (production of a heavy-quark pair) and non-perturbative (evolution of the heavy-quark pair into the physical quarkonium) aspects of QCD, have always been an area of great interest in the high-energy physics. In addition to the inclusive hadro- and photoproductions where the validity of the nonrelativistic QCD (NRQCD) factorization \[^{1}\] has been proved by the success of the results it predicts, the electroproduction in the deeply inelastic lepton-hadron scattering (DIS), commonly viewed as the cleanest probe of partonic behavior in protons and nuclei, is another process of particular importance for studying the properties of the heavy quarkonium. In contrast to the inclusive photoproduction where the virtuality \(Q^2\) of the photon emitted from the initial electron is constrained to a very small value, e.g., \(Q^2 < 1 \text{ GeV}^2\), the DIS process allows a much larger \(Q^2\), which may make the perturbative calculations work better. An evident example is that the next-to-leading order QCD correction to the \(J/\psi\) production in semi-inclusive DIS (SIDIS) exhibits a decent convergence \[^{2}\], significantly better than the cases of hadro- and photoproductions \[^{3}-8\]. Meanwhile, the large \(Q^2\) will make the resolved photon effects less important than the photoproduction case \[^{9}\] due to the rapid decrease in the probability of a photon to appear resolved towards higher \(Q^2\). Furthermore, the background via the diffractive production is also expected to decrease faster with \(Q^2\) than the case of photoproduction \[^{9}\]. On the experimental aspects, the data given by Tevatron, LHC, etc., are mainly correlated to the quarkonium transverse momentum \((p_t)\); however, as for the SIDIS process, much more varieties of physical observable can be measured, e.g., \(p_t, p_t', Y\) (rapidity), \(Y^*, W, z,\) and \(Q^2\), which can help to progress in the understandings of the heavy-quarkonium production mechanism. Throughout the paper we employ the superscript “*” to denote the measured quantities in the center-of-mass (CM) frame of \(\gamma^* p\). Moreover, the distinct signature of the scattered electron in the final state makes the progress easier to be identified.

The Hadron-Electron Ring Accelerator (HERA), as the unique available electron-proton \((ep)\) collider, has released copious data relating to the \(J/\psi\) production in SIDIS \[^{9,12}\], which apparently exceed the predictions given by the color-singlet mechanism (CSM) but coincide with the NRQCD ones \[^{2,13-26}\]. Despite the fruitful \(J/\psi\) concerned measurements, as a result of the low luminosity \((\sim 10^{31} \text{cm}^{-2}\text{s}^{-1})\), HERA has not yet observed significant signal


of $\Upsilon$ production in SIDIS by now; theoretically, far too little attention has been paid to this topic. In comparison with $J/\psi$, $\Upsilon$ has its own conspicuous advantages. For one thing, the large mass of the $b$ quark makes both the typical coupling constant ($\alpha_s$) and relative velocity ($v$) smaller than the $J/\psi$ case, in general resulting in a better convergent perturbative series over the expansion of $\alpha_s$ and $v^2$. For another thing, $\Upsilon$ can also be straightforwardly tagged by hunting it decaying into lepton pair; moreover, the large $b$-quark mass will further make these decay products more energetic and thereby more easily detectable. Additionally, there is no $b$-hadron feedown contributing to the $\Upsilon$ production. From these points of view, $\Upsilon$, as the most studied $b\bar{b}$ meson in the bottomonium family, may be an even better place to apply the NRQCD framework than the $J/\psi$ case. In combination with the aforementioned DIS-process benefits, the $\Upsilon$ electroproduction is expected to provide an ideal laboratory for the study of the heavy quarkonium, meriting a separate investigation.

As previously stated, the low luminosity of HERA highly suppressed its capability of hunting the $\Upsilon$ productions in SIDIS, subsequently raising obstacle to the corresponding theoretical studies. Fortunately, the forthcoming Large Hadron-Electron Collider (LHeC) \cite{27, 28} and the Electron-Ion Collider (EIC) \cite{29}, which exceed the HERA luminosity by 2-3 orders, bring great opportunities to fulfill the observation of the $\Upsilon$ electroproduction. The LHeC, designed as a second (first) generation DIS $ep$ ($eA$) collider by CERN, takes advantage of a newly built electron beam of 60 GeV, to possibly 140 GeV, to collide with the intense, high-energy hadron beams of the LHC. It has a unique physics programme of DIS which can be pursued with unprecedented precision over a hugely extended kinematic range. The EIC that is proposed by the Brookhaven National Laboratory is designed to utilize a new electron beam facility based on an Energy Recovery LINAC to be built inside the RHIC tunnel to collide with RHICs existing high-energy proton and nuclear beams, with the CM energy varying in $20 - 140$ GeV. The high luminosities assign to the LHeC and EIC excellent abilities to perform a multitude of crucial DIS measurements. In light of these outstanding merits, in this paper we will investigate the $\Upsilon$ production in $ep$ SIDIS at the LHeC, the EIC, and the HERA for comparison, respectively, paving the preliminary way for the future comparisons between the measurements and the corresponding theoretical predictions.

The rest of the paper is organized as follows: In Sec. II, we give a description on the calculation formalism. In Sec. III, the phenomenological results and discussions are
FIG. 1: The illustrative diagram for the $\Upsilon$ production via $e + p \to e + \Upsilon + X$. The subscript “$q$” represents the light quarks ($u, d, s$).

II. CALCULATION FORMALISM

For a start, we schematically depict the $\Upsilon$ production via the SIDIS process $e(p_{e_i}) + p(p_p) \to e(p_{e_f}) + \Upsilon(p_\Upsilon) + X$ in Fig. 1. By setting the mass of electron and proton equal to zero, we introduce some generally employed invariants to characterize the DIS process:

\begin{align}
Q^2 &= -p_{\gamma^*}^2 = -(p_{e_i} - p_{e_f})^2, \\
W^2 &= (p_{\gamma^*} + p_p)^2, \\
S &= (p_p + p_{e_i})^2 = 2 \cdot p_p \cdot p_{e_i}, \\
\hat{s} &= (p_a + p_{\gamma^*})^2, \\
s &= \hat{s} + Q^2 = 2 \cdot p_a \cdot p_{\gamma^*}, \\
z &= \frac{p_p \cdot p_x}{p_p \cdot p_{\gamma^*}}, \quad y = \frac{p_p \cdot p_{\gamma^*}}{p_p \cdot p_{e_i}}. 
\end{align}

In the proton rest frame, $z$, known as the inelasticity variable, can measure the fraction of the virtual-photon energy transferred to the $\Upsilon$ meson; $y$ reflects the relative lepton-energy loss.

On the basis of the NRQCD and the collinear factorization, we factorize $e + p \to e + \Upsilon + X$
FIG. 2: The typical Feynman diagrams for the subprocess $e + a \rightarrow e + b\bar{b}[n] + a$ with $a = g, q/\bar{q}$. The subscript “$q$” represents the light quarks ($u, d, s$).

as

$$d\sigma(e + p \rightarrow e + \Upsilon + X) = \int dx \sum_{a,n} f_{a/p}(x, \mu_f) d\hat{\sigma} \times \langle O^{\Upsilon}(n) \rangle,$$

where $d\hat{\sigma}$ is the perturbative calculable short distance coefficient (SDC), representing the production of a configuration of the $(b\bar{b})[n]$ intermediate $b$–quark pair with $n(=^{2S+1} L^{(1,8)}_f)$. For the $\Upsilon$ production, $n$ can be $^{3}S^{[1]}_1$, $^1S^0_0$, $^3S^1_1$, and $^3P^8_J$ up to the order of $v^4$. $\langle O^{\Upsilon}(n) \rangle$ is the universal nonperturbative long distance matrix elements (LDMEs) used to describe the evolution probability of $b\bar{b}[n]$ pair into $\Upsilon$. $f_{a/p}(x, \mu_f)$ is the parton distribution function evaluated at the factorization scale $\mu_f$, with $a$ running over all the species of partons, i.e., $g, q/\bar{q}(q = u, d, s)$.

Now we are to discuss the concrete ingredients of $d\hat{\sigma}$. As regards the CS channel, the unique leading-order (LO) partonic process in $\alpha_s$ is $e + g \rightarrow e + (b\bar{b})[^3S^1_1] + g$; in the case of the color-octet (CO) configurations, the LO parton-level processes consist of $e + g \rightarrow e + (b\bar{b})[^1S^0_0]$ and $e + g \rightarrow e + (b\bar{b})[^3P^8_J]$, which only contribute to the upper endpoint of the $\Upsilon$ energy spectrum, $z \approx 1$ and $p^*_t, \Upsilon \approx 0$. Note that the diffractive production which can not be calculated within perturbative QCD might contaminate the region $z \approx 1$ and make it difficult to extract precise information on the CO contributions. To avoid the kinematic overlaps of the diffractive production, it is a common choice to restrict the analysis to the regions of $z \leq 0.9$ and $p^*_t, \Upsilon \geq 1 \text{ GeV}^2$, correspondingly requiring the CO states to be produced in accompany with a hadron jet. Taken together, our considered partonic processes involved
in $d\sigma$ can be written in the general form of ($a = g, q/q$)

$$e(p_{e_i}) + a(p_a) \rightarrow e(p_{e_f}) + (b\bar{b})[n](p_T) + a(p_a),$$

which, to be specific, incorporates

$$e + g \rightarrow e + b\bar{b} \left[ ^3 S_1 \right]^1 \left[ ^3 S_0 \right]^3 \left[ ^1 P \right]^3 P + g,$$

$$e + q/\bar{q} \rightarrow e + b\bar{b} \left[ ^1 S_0 \right]^3 \left[ ^3 S_1 \right]^3 \left[ ^1 P \right]^3 P + q/\bar{q}. \quad (3)$$

For $n = ^3 S_1$, there are 6 diagrams, half of which are presented by Figs. 2a, 2b, and 2c; for $n = ^1 S_0 (^3 P_1)$, 10 diagrams contribute, as illustratively shown by Figs. 2a 2c, 2d, and 2e; for $n = ^3 S_1$, there exist 8 diagrams, which are typically depicted by Figs. 2a 2c, and 2e.

In accordance with the relations $\langle O^T (^3 P_2^0) \rangle = 5\langle O^T (^3 P_0^0) \rangle$ and $\langle O^T (^3 P_1^0) \rangle = 3\langle O^T (^3 P_0^0) \rangle$, we synthesise the three SDCs for $^3 P_0^0$, $^3 P_1^0$, and $^3 P_2^0$ as

$$d\sigma \left( e + a \rightarrow b\bar{b} \left[ ^3 P_0^0 \right] + e + a \right)$$

$$\equiv d\sigma \left( e + a \rightarrow b\bar{b} \left[ ^3 P_0^0 \right] + e + a \right)$$

$$+ 3d\sigma \left( e + a \rightarrow b\bar{b} \left[ ^3 P_1^0 \right] + e + a \right)$$

$$+ 5d\sigma \left( e + a \rightarrow b\bar{b} \left[ ^3 P_2^0 \right] + e + a \right). \quad (4)$$

The SDC in Eq. (2) can be further expressed as

$$d\sigma = \frac{1}{2xS N_c N_s} |M|^2 d\Phi, \quad (5)$$

with

$$|M|^2 = L^{\mu\nu} H_{\mu\nu}, \quad (6)$$

where $1/(N_c N_s)$ is the color and spin average factor; $|M|^2$ and $d\Phi$ are the squared matrix element and the 3-body phase space, respectively; $L^{\mu\nu}$ and $H_{\mu\nu}$ mean the leptonic and hadronic tensor, respectively.

The leptonic tensor $L^{\mu\nu}$ follows as \[2, 20, 21\]

$$L^{\mu\nu} = \frac{8\pi\alpha}{Q^2} \left[ C_1 (-g^{\mu\nu}) + C_2 p_\mu^a p_\nu^\alpha + C_3 \frac{p_\mu^a p_\nu^\alpha}{2} + C_4 p_\mu^a p_\nu^\alpha \right], \quad (7)$$

6
where

\[ \epsilon_L = \frac{1}{Q} \left( p_{\gamma^*} + \frac{2Q^2}{s} p_a \right), \]
\[ \epsilon_T = \frac{1}{p_t^2} (p_T - \rho p_a - zp_{\gamma^*}), \]
\[ C_1 = A_g, \]
\[ C_2 = \frac{4Q^2}{s^2} (A_L - 2\beta A_{LT} + \beta^2 A_T), \]
\[ C_3 = \frac{4Q}{p_t^2 s} (A_{LT} - \beta A_T), \]
\[ C_4 = \frac{1}{p_t^2 s} A_T, \]  \hspace{1cm} (8)

with

\[ A_g = 1 + \frac{2(1 - y)}{y^2} - \frac{2(1 - y)}{y^2} \cos(2\psi^*), \]
\[ A_L = 1 + \frac{6(1 - y)}{y^2} - \frac{2(1 - y)}{y^2} \cos(2\psi^*), \]
\[ A_{LT} = \frac{2(2 - y)}{y^2} \sqrt{1 - y \cos(\psi^*)}, \]
\[ A_T = \frac{4(1 - y)}{y^2} \cos(2\psi^*), \]
\[ \rho = \frac{(p_t^2 + M_T^2)}{z + zQ^2}, \]
\[ \beta = \frac{(p_t^2 + M_T^2)}{2p_t^2 Q}. \]  \hspace{1cm} (9)

Here \( p_t^2 \) is the square of the \( \Upsilon \) transverse momentum; \( \psi^* \) refers to the azimuthal angle of the \( \Upsilon \) production plane around the \( z \) axis relative to the lepton plane expanded by the incoming and the outgoing electrons.

In order to achieve \( H_{\mu\nu} \), one can directly calculate the hadronic part in Fig. 2 i.e., \( \gamma^* + a \rightarrow b\bar{b}[^{n}] + a \). In deriving \( H_{\mu\nu} \) as well as its contraction with \( L_{\mu\nu} \), the Mathematica-Fortran package \texttt{MALT@FDC} that has been employed to deal with several heavy quarkonium related SIDIS processes \[2, 20, 21, 30\] is utilized.

The 3-body phase space \( d\Phi \) in association with \( dx \) reads

\[ dx d\Phi = dx (2\pi)^4 \delta (p_{\gamma^*} + p_a - p_T - p_{\Upsilon^*}) \frac{d^3 p_{\gamma}}{(2\pi)^3 2p_{\gamma}} \frac{d^3 p_{e_f}}{(2\pi)^3 2p_{e_f}} \frac{d^3 p_{\Upsilon}}{(2\pi)^3 2p_{\Upsilon}}. \]  \hspace{1cm} (10)

By integrating over the azimuthal angle of the outgoing electron, the three momentum of \( p_{a^*} \), and \( x \), respectively, we finally have \[2, 20, 21\]

\[ dx d\Phi = \frac{1}{(4\pi)^4 S(W^2 + Q^2)z(1 - z)} dQ^2 dW^2 dp_t^2 dz d\psi^*. \]  \hspace{1cm} (11)
Note that in Eq. (11) the value of $x$ has been fixed by the conservation relation
\[ \delta (p_{1,a}^0 + p_{1,b}^0 - p_{1,c}^0) \] at
\[ x = \frac{b + Q^2}{W^2 + Q^2}, \text{ with } b = \frac{p_1^2 + M^2}{z} + \frac{p_1^2}{1 - z}. \]  

\[ (12) \]

### III. PHENOMENOLOGICAL RESULTS

Prior to demonstrating the numerical results, we at this point talk about the choice of parameters used in the calculations:

1) The respective collision energy of HERA, EIC, and LHeC are summarized in Tab. I, where $E_e$ and $E_p$ refer to the electron and proton beam energies, respectively, and $\sqrt{S}$ is the CM energy.

|       | $E_e$ | $E_p$ | $\sqrt{S}$ |
|-------|------|------|----------|
| HERA  | 27.5 | 920  | 318      |
| EIC   | 21   | 100  | 91.65    |
| LHeC  | 60   | 7000 | 1296     |

2) Seeing no evident signal of $\Upsilon$ production via SIDIS has been captured by far, we will take the $J/\psi$ electroproduction at HERA for reference to assume the kinematic cuts for the $\Upsilon$ case. We can learn from [9–12] that the $J/\psi$ related SIDIS processes at HERA mainly cover the scope of $p_{t, J/\psi}^2 > 1 \text{ GeV}^2$, $0.3 < z < 0.9$, $2 < Q^2 < 100 \text{ GeV}^2$, and $W > 50 \text{ GeV}$. The cuts $p_{t, J/\psi}^2 > 1 \text{ GeV}^2$ together with $z < 0.9$ are applied to suppress the diffractive effects; $z > 0.3$ is taken to exclude the contributions of the $b$-hadron decaying and the resolved photon. With respect to the $\Upsilon$ electroproduction, the situation is analogous to the $J/\psi$ case, hence in our calculation we take the following cut conditions

\[ p_{t, \Upsilon}^2 > 1 \text{ GeV}^2, W > 50 \text{ GeV}, \]
\[ 2 < Q^2 < 100 \text{ GeV}^2, \]
\[ 0.3 < z < 0.9. \]  

\[ (13) \]
3) The $b$–quark mass is taken as $m_b = M_\Upsilon / 2 = 4.75$ GeV \[31\]; the fine structure constant
is $\alpha = 1/137$.

4) The factorization and renormalization scales are chosen to be $\mu_f = \mu_r = \xi \sqrt{Q^2 + M_\Upsilon^2}$
with varying $\xi$ between 1/2 and 2 around the default value 1.

5) To determine the NRQCD LDMEs, we take advantage of the two linear combinations
given by \[31\]^1,

\begin{align*}
M_{0,r_0}^\Upsilon &= \langle \mathcal{O}^\Upsilon (1S_0^{[8]}) \rangle + \frac{r_0}{m_b^2} \langle \mathcal{O}^\Upsilon (3P_0^{[8]}) \rangle, \\
M_{1,r_1}^\Upsilon &= \langle \mathcal{O}^\Upsilon (3S_1^{[8]}) \rangle + \frac{r_1}{m_b^2} \langle \mathcal{O}^\Upsilon (3P_0^{[8]}) \rangle,
\end{align*}

where $r_0 = 3.8$, $r_1 = -0.52$, $M_{0,r_0}^\Upsilon = 13.7 \times 10^{-2}$ GeV$^3$, and $M_{1,r_1}^\Upsilon = 1.17 \times 10^{-2}$ GeV$^3$.

We set $\langle \mathcal{O}^\Upsilon (1S_0^{[8]}) \rangle$ to $\zeta M_{0,r_0}^\Upsilon$, correspondingly $\langle \mathcal{O}^\Upsilon (3P_0^{[8]}) \rangle = (1 - \zeta) (m_b^2/r_0) M_{0,r_0}^\Upsilon$, and
vary $\zeta$ between 0 and 1 around the default value 1/2. The CS LDME $\langle \mathcal{O}^\Upsilon (3S_1^{[1]}) \rangle$ is
related to the $S$–wave function at the origin by the following formula:

\[ \frac{\langle \mathcal{O}^\Upsilon (3S_1^{[1]}) \rangle}{6N_c} = \frac{1}{4\pi} |R_\Upsilon(0)|^2, \]

with $|R_\Upsilon(0)|^2 = 6.477$ GeV$^3$ \[32\].

| TABLE II: The integrated cross section (unit: pb) of the $\Upsilon$ electroproduction in correspondence to
$1S_0^{[8]}$, $3S_1^{[8]}$, $3P_0^{[8]}$, and $3S_1^{[1]}$, respectively. $p_t^2 > 1$ GeV$^2$, $W > 50$ GeV, $2 < Q^2 < 100$ GeV$^2$, and
$0.3 < z < 0.9$. |
| --- | --- | --- | --- |
| $\sigma[n]$ | $n = 1S_0^{[8]}$ | $n = 3S_1^{[8]}$ | $n = 3P_0^{[8]}$ | $n = 3S_1^{[1]}$ |
| HERA | 0.342 | 0.009 | 0.448 | 1.065 |
| EIC | 0.033 | 0.001 | 0.043 | 0.097 |
| LHeC | 1.691 | 0.035 | 2.200 | 5.362 |

The integrated cross section of the $\Upsilon$ production in SIDIS corresponding to different Fock
states are listed in Tab. \[11\]. As can be seen from this table, including the CO ($1S_0^{[8]}$, $3S_1^{[8]}$, and
$3P_0^{[8]}$) contributions can to a large extent enhance the CS ($3S_1^{[1]}$) predictions, by about

\[1\] In our calculation, we just concentrate on the direct $\Upsilon(1S)$ production, completely excluding the feedown
contributions via the higher excited states.
75% − 80%; among the CO channels, the contributions of $^1S_0^{[8]}$ and $^3P_J^{[8]}$ play the leading role, absolutely dominating over that of the $^3S_1^{[8]}$ configuration.

By summing up the CS and CO contributions, we finally get for the NRQCD predictions:

$$\sigma_{\text{HERA}} = 1.864^{+0.695+0.109}_{-0.489-0.109} \text{ pb},$$
$$\sigma_{\text{EIC}} = 0.174^{+0.099+0.010}_{-0.058-0.010} \text{ pb},$$
$$\sigma_{\text{LHeC}} = 9.288^{+1.743+0.525}_{-1.620-0.525} \text{ pb},$$

(16)

where the two columns of uncertainties are caused by varying $\xi$ from 1/2 to 2 around 1 and varying $\zeta$ from 0 to 1 around 1/2, respectively. For HERA and EIC, which are equipped with low CM energies, the ambiguity of the choice of the renormalization and factorization scales serves as the most important source of the theoretical uncertainties. For example, halving (doubling) the default value of $\mu_r$ and $\mu_f$ simultaneously will enlarge (diminish) the cross sections by about 57%(34%) for EIC, and 37%(26%) for HERA. Regarding the LHeC, which runs at a much higher collision energy, varying $\xi$ from 1 to 1/2(2) just brings about a 19%(17%) fluctuation of the integrated cross section. The variation in $\zeta$ from 0 to 1 around 1/2 only changes the results by about ±5%.

As pointed out in Sec. I, HERA has not yet liberated any measurement on the $\Upsilon$ production in SIDIS. To clarify this issue quantitatively, we here use the numerical result in Eq. (16) to estimate the possible event number of the $\Upsilon$ electroproduction at HERA. According to the integrated luminosity (63 pb$^{-1}$) corresponding to $E_p = 920$ GeV during 1997-2000 [9], only about 5 electroproduced $\Upsilon$ events$^2$, which are established by hunting $\Upsilon \rightarrow l^+l^-$ ($\simeq 5\%$), can be accumulated, thereby making the detection extremely hard.

By assuming $\mathcal{L}_{\text{EIC}} = 10^{34}\text{cm}^{-2}\text{s}^{-1}$ [29] and $\mathcal{L}_{\text{LHeC}} = 0.8 \times 10^{34}\text{cm}^{-2}\text{s}^{-1}$ [27, 28, 33]$^3$, and the detection efficiency to be 100%, up to about $3.7 \times 10^4$ and $8.7 \times 10^2$ reconstructed $\Upsilon$ events are estimated to be generated in one operation year ($10^7$ seconds running time$^4$) at the LHeC and EIC, respectively, which suggests the two forthcoming $ep(eA)$ collider are capable enough of observing the $\Upsilon$ production in SIDIS.

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$^2$ By taking into account the detection efficiency, the event number should be further reduced.

$^3$ As reported in Ref. [33], by the parasitic operation in parallel to the HL-LHC $pp$ collision, the up-to-date luminosity of LHeC could be improved to be $\mathcal{L}_{\text{LHeC}} = 0.8 \times 10^{34}\text{cm}^{-2}\text{s}^{-1}$.

$^4$ Approximately, 1 year $\approx \pi \times 10^7$ seconds, but it is common that a collider only operates about $1/\pi$ year, i.e., $10^{34}\text{cm}^{-2}\text{s}^{-1} \approx 10^5 \text{ pb}^{-1}/\text{year}$. 
FIG. 3: The distributions of $p_t^2$ and $p_t^*^2$ under the kinematic cuts in Eq. (13). The shaded bands are induced by the variations of $\xi$ and $\zeta$.

The distributions of $p_t^2$, $p_t^*^2$, $Q^2$, $z$, $Y$, and $Y^*$ are shown in Figs. 3, 4, and 5 where the NRQCD predictions are compared with the CS ones. Following the conventions of HERA, the forward direction of $Y^*$ is defined as that of the incident virtual photon; $Y$ is taken to be positive in the direction of the incoming proton. In each diagram, the shaded band denotes the theoretical uncertainties stemming from the variations of $\xi$ and $\zeta$. From the figures we can see:

1) As $p_t^2(p_t^*^2)$ rises, the corresponding differential cross sections continuously decrease,
FIG. 5: The distributions of $Q^2$ and $z$ under the kinematic cuts in Eq. (13). The shaded bands are induced by the variations of $\xi$ and $\zeta$.

with the CO contributions playing an increasingly crucial role. When $p_t^2(p_t^2) < 10$ GeV$^2$, the NRQCD predictions are larger than the CS ones by less than two times; however, when $p_t^2(p_t^2)$ is scattered around 100 GeV$^2$, the former ones notably go beyond the latter ones.

2) The differential cross sections as a function of $Y^*$ show up serious asymmetry. Taking LHeC for example, the value of $d\sigma/dY^*$ at $Y^* = 4$ is about four orders of magnitude in the excess of that at $Y^* = 0$, which signifies, in the $\gamma^*p$ CM frame, $\Upsilon$ is much more likely to be produced in the direction of the virtual photon rather than that of the incoming proton. From the three $Y^*$-distribution figures in Fig. 4 one can find the available $Y^*$ range will extend with the increment of the CM energy. Referring to $d\sigma/dY$, it is also asymmetric, as is manifestly sketched by the triple $Y^*$-distribution figures in Fig. 4. This asymmetry tells us, in the laboratory frame, more $\Upsilon$ events will be generated along the direction of the incident electron.

3) With regard to the $Q^2$ distributions, the differential cross sections gradually fall off with the $Q^2$ rising; the ratio of the NRQCD prediction to the CS one appears to be insensitive to $Q^2$. As for the $z$ distributions, when $z$ approaches 0.3 the CS contribution almost saturates the NRQCD prediction alone; however, towards higher $z$ values, the
CO contribution rises sharply\(^5\), which can be primarily attributed to the \(1S^0\) and \(3P^J\) channels listed in Eq. (3).

### TABLE III: The integrated cross section (unit: pb) of the \(\Upsilon\) electroproduction from the LDMEs of Refs. [34–36]. \(p^2_\gamma > 1\text{ GeV}^2, W > 50\text{ GeV}, 2 < Q^2 < 100\text{ GeV}^2, \text{ and } 0.3 < z < 0.9.\)

|             | Sharman ([34]) | Gong ([35]) | Feng ([36], Tab. I) |
|-------------|----------------|-------------|---------------------|
| HERA CSM    | 1.263          | 1.065       | 1.065               |
| NRQCD       | 2.833          | 1.454       | 1.516               |
| EIC CSM     | 0.115          | 0.097       | 0.097               |
| EIC NRQCD   | 0.266          | 0.134       | 0.140               |
| LHeC CSM    | 6.356          | 5.362       | 5.362               |
| NRQCD       | 14.06          | 7.289       | 7.594               |

One of the crucial issue of the NRQCD research is the determination of the LDMEs, of which there are several independent extractions, with different strategy, having obtained different results. Therefore, to serve as a sound reference, we at last employ three other typical sets of these parameters [34–36] to present our numerical results. From the data in Tab. III, one can find the predicted integrated cross sections given by the LDMEs in Ref. [35] is approximately identical with that from the LDMEs of [36], similar to the results in Eq. (16). However, the large values of the CO LDMEs in [34] significantly increase the CO contributions, subsequently making its NRQCD predictions about two times bigger in magnitude than the ones from [31, 35, 36].

To summarize, the CO configurations, especially \(1S^0\) and \(3P^J\), have a pivotal role in the \(\Upsilon\) production via SIDIS. The predicted remarkable discrepancies between the NRQCD predictions and the CS ones awaiting for the future identification by LHeC and EIC can serve as a useful evidence in favor or disfavor of the CO processes.

\(^{5}\) Near the endpoint region, \(z \approx 1\), the perturbative and velocity expansions maybe break down, similar to the inclusive \(J/\psi\) photo- and electroproductions. Resuming the series in \(\alpha_s\) and \(v\) perhaps to certain degree smear the CO predicted steep ascent of \(d\sigma/dz\) at large \(z\) values.
IV. SUMMARY

In order to provide a deeper insight into the heavy-quarkonium production, we in this work investigate the Υ production via the $ep$ SIDIS at HERA, EIC, and LHeC, respectively, within the NRQCD framework. We find the CO states, especially $^1S_0^{[8]}$ and $^3P_J^{[8]}$, supply substantial contributions to the Υ electroproductions, leading to distinct dissimilarities between the CS predictions and the NRQCD ones, which is beneficial to distinguish between the CS and CO mechanism. Under the assumed constraints $p_{T^2}^{υ} > 1 \text{GeV}^2$, $W > 50 \text{GeV}$, $2 < Q^2 < 100 \text{GeV}^2$, and $0.3 < z < 0.9$, just a rather small number of Υ electroproduction events can be gathered by HERA, which is responsible, in part, for the fact that it has not yet released any data of the Υ production in SIDIS up to now. However, as high as $3.7 \times 10^4$ and $8.7 \times 10^2$ electroproduced Υ events can be collected in one operation year at LHeC and EIC, respectively, clearly revealing the experimental prospect of observing the Υ production via SIDIS at the two future $ep(eA)$ colliders.

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