RELATIVISTIC THEORY OF SPECTRA OF PIONIC AND KAONIC ATOMS: HYPERFINE STRUCTURE, TRANSITION PROBABILITIES FOR NITROGEN

A new theoretical approach to energy and spectral parameters of the hadronic (pionic and kaonic) atoms in the excited states with precise accounting for the relativistic, radiation and nuclear effects is applied to the study of radiation parameters of transitions between hyperfine structure components of the pionic and kaonic nitrogen. The advanced data on the probabilities of radiation transitions between components of the hyperfine structure transitions $5g-4f$, $5f-4d$ in the spectrum of pionic nitrogen and $8k-7i$, $8i-7h$ in the spectrum of kaonic nitrogen are presented and compared with alternative theoretical data.

1. Introduction

Our work is devoted to the further application of earlier developed new theoretical approach [1-8] to the description of spectra and different spectral parameters, in particular, radiative transitions probabilities for hadronic (pionic and kaonic) atoms in the excited states with precise accounting for the relativistic, nuclear and radiative effects. As it was indicated earlier [7-12] nowadays investigation of the pionic, kaonic and at whole the exotic hadronic atomic systems represents a great interest as from the viewpoint of the further development of atomic and nuclear spectral theories as creating new tools for sensing the nuclear structure and fundamental hadron-nucleus strong interactions [6-14]. Spectroscopy of hadronic atoms already in the electromagnetic sector is extremely valuable area of research that provide unique data for different areas of physics, including nuclear, atomic, molecular physics, physics of particles, sensor electronic etc. It should be emphasized that the theory of pion spectra of atoms are highly excited, even in the electromagnetic sector (ie short-range strong pion-N interaction neglects little) is extremely complex and at present, despite the known progress remains very poorly developed. It is about the fundamental theoretical problems describing relativistic atoms considering nuclear, radiation effects, and a completely insufficient spectral data for pion atoms. While determining the properties of pion atoms in theory is very simple as a series of H such models and more sophisticated methods such combination chiral perturbation theory (TC), adequate quantitative description of the spectral properties of atoms in the electromagnetic pion sector (not to mention even the strong interaction sector ) requires the development of High-precision approaches, which allow you to accurately describe the role of relativistic, nuclear, radiation QED (primarily polarization electron-positron vacuum, etc.). pion effects in the spectroscopy of atoms. The most popular theoretical models for pionic and kaonic atoms are naturally based on the using the Klein-Gordon-Fock equation, but there are many important problems connected with accurate accounting for as pion-kaon-nuclear strong interaction effects as QED radiative corrections (firstly, the vacuum polarization effect etc.). This topic has been a subject of intensive theoretical and experimental interest (see [1-16]). The perturbation theory expansion on the physical; parameter $aZ$ is usually used to take into account the radiative QED corrections, first of all, effect of the polarization of electron-positron vacuum etc. This approximation is sufficiently correct and comprehensive in a case of the light pionic atoms, however it becomes incorrect in a case of the heavy atoms with large charge of a nucleus $Z$.

The more correct accounting of the QED, finite nuclear size and electron-screening effects
for pionic atoms is also very serious and actual problem to be solved more consistently in comparison with available theoretical models and schemes. At last, a development of the comprehensive theory of hyperfine structure and computing radiative transitions probabilities between its components is of a great interest and importance in a modern theory of the hadronic atom spectra [1-39].

2. Theory

The basic topics of our theoretical approach have been earlier presented [3-8,27,28], so here we are limited only by the key elements. The relativistic dynamic of a spinless boson (pion) particle is described by the Klein-Gordon-Fock (KGF) equation. As usually, an electromagnetic interaction between a negatively charged pion and the atomic nucleus can be taken into account introducing the nuclear potential $A_\nu$ in the KGF equation via the minimal coupling $p_\nu \rightarrow p_\nu - qA_\nu$. Generally speaking, the Klein-Gordon-Fock equation can be rewritten as the corresponding two-component equation:

$$\left[ (-\sigma_3 + i\sigma_2) \frac{\nabla^2}{2\mu} + \sigma_3 \mu + (\sigma_3 + i\sigma_2) V_{\nu}^{(0)} + V_{C}^{(0)} \right] \Psi = E \Psi,$$

where $\Psi_i$ are the Pauli spin matrices and

$$\Psi_i = \frac{1}{2} \left( \begin{array}{c} 1 + (E - V_{C}^{(0)}/\mu) \phi_i \\ 1 - (E - V_{C}^{(0)}/\mu) \phi_i \end{array} \right).$$

This equation is equivalent to the stationary Klein-Gordon-Fock equation. The corresponding non-stationary Klein-Gordon-Fock equation can be written as follows:

$$\mu^2 c^5 \Psi(x) = \left\{ \frac{1}{c^2} \left[ i\hbar \hat{\nabla} + eV_\nu(r) \right]^2 + \hbar^2 \nabla^2 \right\} \Psi(x)$$

where $c$ is the speed of light, $\hbar$ is the Planck constant, $m$ is the reduced mass of the pion-nuclear system, and $\Psi_0(x)$ is the scalar wave function of the space-temporal coordinates. Usually one considers the central potential $[V_\nu(r), 0]$ approximation with the stationary solution:

$$\Psi(x) = \exp(-iEt/\hbar) \phi(x),$$

where $\phi(x)$ is the solution of the equation:

$$\left\{ \frac{1}{c^2} [E + eV_\nu(r)]^2 + \hbar^2 \nabla^2 - \mu^2 c^2 \right\} \phi(x) = 0$$

Here $E$ is the total energy of the system (sum of the mass energy $mc^2$ and binding energy $e_0$). In principle, the central potential $V_\nu$ is the sum of the following potentials: the electric potential of a nucleus, vacuum-polarization potential. The strong interaction potential can be added below. Generally speaking, an energy of the pionic atomic system can be represented as the following sum:

$$E \approx E_{KG} + E_{FS} + E_{QED} + E_N,$$

where $E_{KG}$ is the energy of a pion in a nucleus $(Z, A)$ with the point-like charge, $E_{FS}$ is the contribution due to the nucleus finite size effect, $E_{VP}$ is the radiation QED correction, $E_N$ is the energy shift due to the strong (pion- or kaon- nuclear) interaction $V_N$. In principle, the central potential $V_\nu$ should include the central Coulomb potential, the radiative (in particular, vacuum-polarization) potential as well as the electron-screening potential in the atomic-optical (electromagnetic) sector. Surely, the full solution of the pionic atom energy especially for the low-excited state requires an inclusion the hadron-nuclear strong potential.

The next step is accounting the nuclear finite size effect or the Breit-Rosenthal-Crawford-Schawlow one. In order to do it we use the widespread Gaussian model for nuclear charge distribution. The advantages of this model in comparison with usually used models such as for example an uniformly charged sphere model and others had been analysed in Ref. [3]. Usually the Gauss model is determined as follows:

$$\rho(r) = \left( \frac{4\pi \gamma^3}{R^2} \right) \exp(-\gamma r^2),$$

where $\gamma = 4\pi R^2$, $R$ is an effective radius of a nucleus.

In order to take into account very important radiation QED effects we use the radiative potential from the Flambaum-Ginges theory [15]. In includes the standard Uelsing-Serber...
potential and electric and magnetic form-factors plus potentials for accounting of the high order QED corrections such as:

\[ \Phi_{\text{rad}}(r) = \Phi_U(r) + \Phi_g(r) + \Phi_f(r) + \ldots \]

\[ + \Phi_f(r) + \frac{2}{3} \Phi^{\text{high-order}}_U(r) \]  

(8)

where

\[ \Phi^{\text{high-order}}_U(r) = -\frac{2\alpha}{3\pi} \Phi(r) \frac{0.092Z^2\alpha^2}{1 + (1.62r/r_c)^4} \]

\[ \Phi_f(r) = -\frac{B(Z)}{e} Z^4 \alpha^5 mc^2 e^{-Zr/\alpha} \]  

(9)

Here \( e \) – a proton charge and universal function \( B(Z) \) is defined by expression: \( B(Z) = 0.074 + 0.35Z_a \).

At last to take into account the electron screening effect we use the standard procedure, based on addition of the total interaction potential SCF potential of the electrons, which can be determined within the Dirac-Fock method by solution of the standard relativistic Dirac equations. It should be noted however, that contribution of these corrections is practically zeroth for the pionic nitrogen, however it can be very important in transition to many-electron as a rule heavy hadronic atoms.

Further in order to calculate probabilities of the radiative transitions between energy level of the pionic atoms we have used the well-known relativistic energy approach (c. g. [16-28]). Other details are in Refs. [4,7,8].

3. Results and conclusions

As example of application of the presented approach, in table 1 we present the data on radiative transition probabilities (in \( s^1 \)) for hyperfine transitions 5g-4f in the spectrum of the pionic nitrogen: Th1- data by Trassinelli-Indelicato; Th2- our data.

| F-F' | T.I : P (5g-4f) | T.II : P (5g-4f) |
|------|-----------------|------------------|
| 5-4  | \( 7.13 \times 10^{13} \) | \( 7.04 \times 10^{13} \) |
| 4-3  | \( 5.47 \times 10^{13} \) | \( 5.41 \times 10^{13} \) |
| 4-4  | \( 5.27 \times 10^{13} \) | \( 5.23 \times 10^{13} \) |
| 3-2  | \( 4.17 \times 10^{13} \) | \( 4.12 \times 10^{13} \) |
| 3-3  | \( 0.36 \times 10^{13} \) | \( 0.34 \times 10^{13} \) |
| 3-4  | \( 0.01 \times 10^{13} \) | \( 0.009 \times 10^{13} \) |

In theory by Trassinelli-Indelicato (look, for example, [6]) it has been used the standard atomic spectroscopy amplitude scheme when the transitions energies and probabilities are calculated in the known degree separately. At the same time this computing within the relativistic energy approach is performed more correctly and self-consistently (look details in [4,9] and Refs. therein).

In table 2 we present our data for radiative transition probabilities (in \( s^1 \)) for hyperfine transitions 5f-4d in the spectrum of the pionic nitrogen: our data.

| F-F' | Our data (5f-4d) |
|------|-----------------|
| 4-3  | \( 4.57 \times 10^{13} \) |
| 3-2  | \( 3.16 \times 10^{13} \) |
| 3-3  | \( 2.98 \times 10^{13} \) |
| 2-1  | \( 2.13 \times 10^{13} \) |
| 2-2  | \( 2.25 \times 10^{13} \) |
| 2-3  | \( 0.01 \times 10^{13} \) |

Table 1. Radiative transition probabilities (in \( s^1 \)) for hyperfine transitions 5g-4f in the spectrum of the pionic nitrogen: Th1- data by Trassinelli-Indelicato; Th2- our data.

Table 2. Radiative transition probabilities (s1) for hyperfine transitions 5f-4d in the spectrum of the pionic nitrogen: our data.
The radiative transition probabilities (in s\(^{-1}\)) for the 8k-7i transition in the k-N atom: Th1- Trassinelli-Indelicato; Th2- our data

| F-F' | TI, P | T.II: our data |
|------|-------|----------------|
| 8-7  | 1.54 \times 10^{13} | 1.51 \times 10^{13} |
| 7-6  | 1.33 \times 10^{13} | 1.32 \times 10^{13} |
| 7-7  | 1.31 \times 10^{13} | 1.29 \times 10^{13} |
| 6-5  | 1.15 \times 10^{13} | 1.12 \times 10^{13} |
| 6-6  | 0.03 \times 10^{13} | 0.02 \times 10^{13} |
| 6-7  | 0.00 \times 10^{13} | 0.004 \times 10^{13} |

In Table 4 we present our data for radiative transition probabilities (in s\(^{-1}\)) for hyperfine transitions 8i-7h in the spectrum of the kaonic nitrogen: our data. In whole, the computed radiative transition probabilities values for considered transitions between hyperfine structure components in the spectrum of the pion within theory by Trassinelli-Indelicato and ours demonstrate physically reasonable agreement, however our values are a little lower. This circumstance fact can be reasonably explained by difference in the computing schemes and different level of accounting for nuclear finite size, QED and other effects (c.g. [1-3,20,21]). In any case the data obtained can be considered as sufficiently accurate ones and used in the corresponding applications, indicated in the introduction.

Radiative transition probabilities (in s\(^{-1}\)) for hyperfine transitions 8i-7h in spectrum of the kaonic nitrogen: our data

| F-F' | Our data |
|------|----------|
| 7-6  | 1.16 \times 10^{13} |
| 6-5  | 0.99 \times 10^{13} |
| 6-6  | 0.96 \times 10^{13} |
| 5-4  | 0.81 \times 10^{13} |
| 5-5  | 0.02 \times 10^{13} |
| 5-6  | 0.005 \times 10^{13} |

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Keywords: relativistic theory, hyperfine structure, hadronic atoms