COMPRESSIBLE MHD TURBULENCE: IMPLICATIONS FOR MOLECULAR CLOUD AND STAR FORMATION

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We review recent results from numerical simulations and related models of MHD turbulence in the interstellar medium (ISM) and in molecular clouds. We discuss the implications of turbulence for the processes of cloud formation and evolution, and the determination of clouds’ physical properties. Numerical simulations of the turbulent ISM to date have included magnetic fields, self-gravity, parameterized heating and cooling, modeled star formation and other turbulent inputs. The structures which form reproduce well observed velocity-size scaling properties, while predicting the non-existence of a general density-size scaling law. Criteria for the formation of gravitationally-bound structures by turbulent compression are summarized. For flows with equations of state $P \propto \rho^\gamma$, the statistics of the density field depend on the exponent $\gamma$. Numerical simulations of both forced and decaying MHD compressible turbulence have shown that the decay rate is comparable to the non-magnetic case. For virialized clouds, the turbulent decay time is shorter than the gravitational free-fall time, so wholesale cloud collapse is only prevented by ongoing turbulent inputs and/or a strong mean magnetic field. Finally, perspectives for future work in this field are briefly discussed.
I. INTRODUCTION

A. Observational Motivation

Present-day star formation in our Galaxy is observed to take place in cold molecular clouds, which appear to be in a state of highly compressible MHD turbulence. Furthermore, the atomic gas, within which the molecular clouds form, is also turbulent on larger scales, from hundreds to thousands of pc (e.g., Braun 1998). In this chapter we review recent results in the areas of cloud formation, structure and evolution, as well as their implications for observed physical and statistical cloud properties; these results are obtained mainly from numerical simulations of compressible MHD turbulence, and related analytical models. An introductory review including a more detailed discussion of turbulence basics and discussions (current up to 1997) on fractality and phenomenological models has recently been given by Vázquez-Semadeni (1999). The review of Ostriker (1997) focuses on the role of MHD turbulence in the internal dynamical evolution of molecular clouds. The reviews of Heiles et al. (1993) and McKee et al. (1993) provide extensive discussions of the observations and general theory, respectively, of magnetic fields in star-forming regions.

The formation of molecular clouds probably cannot be considered separately from the formation of larger diffuse HI structures, since the former are often observed to have HI “envelopes” (e.g., Peters & Bash 1987; Shaya & Federman 1987; Wannier et al. 1991; Blitz 1993; Williams et al. 1995), suggesting that molecular clouds and clumps can be regarded as the “tips of the icebergs” in the general continuum interstellar density field of galaxies. The process of cloud formation quite possibly involves more than a single mechanism, including the passage of spiral density waves and the effects of combined large-scale instabilities (e.g., Elmegreen 1993a, 1995; Gammie 1996) operating preferentially in the formation of the largest high-density structures, and the production of smaller density condensations by either swept-up shells (Elmegreen & Elmegreen 1978; Vishniac 1983, 1994; Hunter et al. 1986), or by a generally turbulent medium (Hunter 1979; Hunter & Fleck 1982; Tohline et al. 1987; Elmegreen 1993b). In Sec. II of this chapter we discuss recent results on the generation of density fluctuations in a turbulent medium, obtained from numerical simulations and semi-phenomenological models. Other mechanisms of cloud formation have been reviewed extensively by Elmegreen (1993a, 1995).
Note, however, that discrete cloud coagulation mechanisms discussed there may not be directly applicable in the context of a dynamically-evolving continuum, such as that considered here.

Structurally, molecular clouds are very complex, with volume-averaged \( n(H_2) \) ranging from \( \lesssim 50 \text{ cm}^{-3} \) for giant molecular clouds of sizes several tens of pc (e.g., Blitz 1993), to \( n(H_2) \gtrsim 10^5 \text{ cm}^{-3} \) for dense cores of sizes 0.03–0.1 pc (e.g., Wilson & Walmsley 1989; Plume et al. 1997; Pratap et al. 1997; Lada et al. 1997; Myers 1995). Kinetic temperatures are relatively constant, typically \( T \sim 10 \text{ K} \pm \) a few degrees (e.g., Pratap et al. 1997), although temperatures larger by factors of a few are found in the vicinity of star-forming regions (e.g., Torrelles et al. 1983; Solomon & Sanders 1985). Their internal velocity dispersions are generally supersonic, with Mach numbers up to \( \gtrsim 10 \), except for the smallest cores (size \( R \lesssim 0.1 \text{ pc} \)) (e.g., Larson 1981; Blitz 1993 and references therein; Goodman et al. 1998). When available (and significant; see, e.g., Crutcher et al. 1993, 1996; Padoan & Nordlund 1998a), Zeeman measurements give typical values of the magnetic field intensity of a few to a few tens of \( \mu \text{G} \) (e.g., Heiles et al. 1993; Crutcher et al. 1993; Troland et al. 1996; Crutcher 1998), consistent with near equipartition between the kinetic and magnetic energies.

Additionally, interstellar clouds seem to follow power-law scaling laws between their average density, velocity dispersion and size (Larson 1981; see also Blitz 1993 and references therein). Together with the near equipartition between kinetic and magnetic energy, these are normally interpreted as evidence for virialized magnetic support for the clouds (e.g., Shu et al. 1987; Mouschovias 1987; Myers & Goodman 1988a,b; see also Whitworth 1996), although they only provide arguments of self-consistency of the virial equilibrium hypothesis rather than conclusive proofs (Heiles et al. 1993). In addition to the clouds that are self-consistent with virialization, however, examples also exist of objects which appear to be highly disturbed (e.g., Carr 1987; Loren 1989; Plume et al. 1997), or simply regions away from map intensity maxima (Falgarone et al. 1992, 1998), which do not satisfy one or more of the scalings. Another example of such scalings is the mass spectrum of the clouds and clumps, which seems to also be a power-law (e.g., Blitz 1993; Williams et al. 1995), although present Galactic cloud identification surveys remain incomplete.

Molecular clouds also exhibit signatures of self-similarity, or more
generally, multifractality. Scalo (1990) and Falgarone et al. (1991) have shown that cloud boundaries have projected fractal dimensions $\sim 1.4$. Also, clouds exhibit hierarchical structure; i.e., the densest structures are nested within larger, less dense ones in a self-similar fashion (e.g., Scalo 1985), at least at large scales. The self-similarity appears to either break down or at least change similarity exponents at some small scale, reported by Larson (1995) to be close to 0.05 pc in Taurus, corresponding to the local Jeans length, although the latter point is controversial (e.g., Simon 1997; Bate et al. 1998). A similar break has also been reported at scales 0.25–0.5 pc (Blitz & Williams, 1997), showing that this question is still largely open.

Such complexity strongly suggests itself as a manifestation of the turbulent regime which permeates molecular clouds. Thus, an understanding of the basics of compressible MHD turbulence and its incarnation in the interstellar case is necessary for explaining molecular cloud structure and evolution, and for diagnosing the consequences of turbulent dynamics for the process of star formation. The disordered, nonlinear nature of interstellar MHD turbulence makes direct numerical simulation the most useful tool for developing this understanding. In Sec. III of this chapter, we summarize recent results on cloud structure and discuss questions such as whether the physical conditions and scaling laws observed in clouds arise naturally in simulations of the turbulent ISM, and what simulations predict for energy balance in the clouds, density fluctuation statistics, and the general characterization of structure in these flows.

Evolutionary aspects of turbulence and the mechanisms of cloud support under highly nonlinear, strongly self-gravitating conditions have remained open questions since the original identification of turbulent molecular clouds in the ISM. Supersonic turbulent maintenance faces the well-known problem of an expected excessive dissipation in shocks (Goldreich & Kwan 1974). Arons and Max (1975) proposed that the “turbulent” motions in molecular clouds may actually be moderate-amplitude (sub-Alfvénic) MHD waves, arguing that shock formation and the associated dissipation would then be significantly diminished. Additionally, it has been suggested that even strong dissipation may be compensated by sufficient energy injection from embedded stars and other sources (e.g. Norman and Silk 1980; Scalo 1987; McKee 1989; Kornreich & Scalo 1998). Sec. IV in this chapter discusses recent nu-
merical MHD results on calculating dissipation rates for turbulence with parameters appropriate for the cold ISM, and the implications of these results for questions of cloud support against self-gravitating contraction. In Sec. V we conclude with a summary and discussion of outstanding goals and challenges for future research.

B. MHD Compressible Turbulence Basics

Before proceeding to the next sections, a few words on the nature and parameters of molecular cloud turbulence are in order. Its properties are very different from laboratory incompressible turbulence and in fact additional non-dimensional parameters are necessary to characterize the flow besides the viscous and magnetic Reynolds numbers (which respectively measure the magnitude of the nonlinear advection term $v \cdot \nabla v$ in the momentum equation with respect to the viscous and Ohmic dissipation terms). Two of these parameters are the sonic (thermal) and Alfvénic Mach numbers $M_s \equiv u/c$ and $M_A \equiv u/v_A$, where $u$ is a characteristic velocity of the flow, $c$ is the isothermal sound speed and $v_A = B/(4\pi \rho)^{1/2}$ is the Alfvén speed, with $B$ the magnetic field strength and $\rho$ the fluid density. The plasma beta, $\beta \equiv c^2 \rho/(B^2/8\pi) = 2(M_A/M_s)^2$ (i.e., the ratio of the thermal to the magnetic pressure) is also frequently used to characterize the importance of magnetic fields. The last basic parameter, which characterizes the importance of self-gravity, is the Jeans number $n_J \equiv L/L_J = L/\sqrt{\pi c^2/G\rho}$ (i.e. this compares the cloud linear scale $L$ with the minimum Jeans-unstable wavelength $L_J$).

Other peculiarities of ISM turbulence are related to the driving mechanisms. In standard (“Kolmogorov”) incompressible turbulence theory, energy is assumed to be injected at large scales; from there it cascades down to the small scales where it is dissipated. In the ISM, forcing occurs at all scales (e.g., Scalo 1987; Norman & Ferrara 1996): some mechanisms operate at large scales ($\gtrsim 1$ kpc), such as the Galactic shear (Fleck 1981, although this is believed to be a very inefficient source; see, e.g., Shu et al. 1987) and supershells; others at intermediate scales ($\gtrsim 100$ pc), such as expanding HII regions and supernova explosions. Yet others act at small scales (a few tenths to a few pc), such as stellar winds or bipolar outflows. All these mechanisms are important sources of kinetic energy.

Small-scale dissipation mechanisms in the cold ISM are of several types. First, there is the usual viscous dissipation operating mostly in
shocks and which remains finite even in the limit of vanishing viscosity. A significant amount of dissipation also results from ambipolar diffusion due to ion-neutral friction at high densities in the presence of a strong enough magnetic field (Kulsrud and Pearce 1969; see McKee et al. 1993; Myers and Khersonsky 1995). Cooling processes are also very efficient and may radiate much of the dissipated energy, but are balanced by heating processes and may result in near-isothermal or quasi-polytropic behavior (i.e. $P \propto \rho^\gamma$) (Sec. II).

Even aside from all these astrophysical properties, compressible turbulence has many distinctive features related to the transfer of energy from large scales to the dissipation (thermalization) scale. The new ingredient with respect to incompressible turbulence is the existence of potential (or compressible) modes, in addition to the solenoidal (vortical) ones. Because compressions and rarefactions can exchange energy between bulk kinetic and microscopic thermal parts, compressible flows do not in general conserve bulk kinetic energy. Strongly radiative shocks, which yield an irreversible energy loss, are the most important aspect of this feature. The presence of additional compressive and thermal degrees of freedom therefore excludes the use of dimensional analysis to determine the slope of the velocity spectrum, as is done (by assumption of an energy-conservative cascade through scales) in the so-called K41 theory (Kolmogorov 1941; Obukhov 1941; see e.g. Landau and Lifshitz 1987), which produces the well-known “universal” spectrum of the form $d u^2 / d k \equiv E(k) \propto k^{-5/3}$ for incompressible, non-magnetic turbulence, where $k = 2\pi/\lambda$ is the wavenumber associated to wavelength $\lambda$. For magnetized, incompressible, strong turbulence, recent theoretical work suggests that the mean magnetic field leads to strong anisotropy in the cascade but the same averaged energy spectrum (Goldreich and Sridhar 1995).

Moreover, the K41 theory is based on the assumption of locality in Fourier space of the nonlinear cascade (i.e., transfer between Fourier modes of similar wavelengths), a hypothesis probably not valid in the compressible case because coupling among very different scales occurs in shocks. In these highly intermittent (i.e., inhomogeneous in space and time) structures, all Fourier modes decay at the same rate (Kadomtsev & Petviashvili 1973; Landau & Lifshitz 1987), thus invalidating the notion of inviscid cascade along the “inertial range”, defined as the range in Fourier space where the energy flux is constant. Note
II. TURBULENT CLOUD FORMATION

An important question is whether structures formed by either turbulent compressions or passages of single shock waves can become gravitationally unstable and collapse (Ögelman & Maran 1976; Elmegreen & Lada 1977; Elmegreen & Elmegreen 1978; Hunter 1979; Hunter & Fleck 1982; Hunter et al. 1986; Tohline et al. 1987; Stevens et al. 1992; Elmegreen 1993b). As pointed out by Hunter et al. (1986), when both heating and cooling are present, the isothermal approximation, often used to describe radiative flows, is just one out of a continuum of possibilities. In cases where the heating and cooling rates are reasonably approximated by power-law functions of the density and temperature, and faster than the dynamical rates, the gas can be described as a barotropic fluid with power-law equation of state $P \propto \rho^\gamma$ (e.g., de Jong et al. 1980; Maloney 1988; Elmegreen 1991; Vázquez-Semadeni, Passot & Pouquet 1996); we use the term “effective polytropic exponent” to refer to $\gamma$.

In general, $\gamma$ is different from 1 for the global ISM (Myers 1978), and possibly even for molecular clouds (de Jong et al. 1980; Scalo et al. 1998). Note, however, that, since cooling laws are often approximated by piecewise power-laws, the effective polytropic exponent is also approximately piecewise constant. Numerical simulations explicitly including heating and cooling appropriate for atomic gas in-

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1 We remark that the present effective polytropic exponent, which describes the thermal behavior of gases with local heating and cooling in equilibrium, is different from the adiabatic exponent $c_p/c_v$ (describing thermal behavior with no heating or cooling) and that its usage does not imply in any form that the system is in hydrostatic equilibrium. Also, note that strictly speaking the polytropic equation is not properly an equation of state, since the ideal-gas equation of state is also satisfied at all times.
Indeed show that the corresponding rates are faster than the dynamical rates by factors of at least 50 (Passot et al. 1995; Vázquez-Semadeni et al. 1996), confirming earlier estimations (Spitzer & Savedoff 1950; Elmegreen 1993b). However, those simulations depart from pure power-law equations of state near star formation sites. For optically thin gas with rapid heating and cooling, an effective polytropic equation of state with parameterized $\gamma$ may therefore be considered as the next level of refinement over isothermal or adiabatic laws; from the above simulations, $\gamma$ is found to take values between 0 and $1/2$ in the warm and cold “phases”. For molecular material which is optically thick and turbulent, it is not yet clear from simulations how well an isothermal or effective polytropic law may be satisfied.

The choice of cooling and heating functions in the above-mentioned simulations implies the absence of an isobaric thermal instability (Field, Goldsmith & Habing 1969) at densities $\sim 0.5-5$ cm$^{-3}$ and temperatures between $10^2$ and $10^4$ K. There exist realistic values of interstellar parameters, such as the background UV field, coolant depletion and grain abundances, for which such an instability is indeed not realized (e.g., Draine 1978; Wolfire et al. 1995). This choice of parameters has allowed the analysis of the role of turbulence separately from that of the thermal instability. Even though it is desirable to study these processes in combination, the thermal instability is unlikely to be the main driver of interstellar motions, since thermal pressure is smaller, by at least a factor of 3, than each of the other principal forms of interstellar pressure (turbulent, magnetic and cosmic ray), and at least $\sim 10$ times smaller than their sum (e.g., Boulares & Cox 1990).

The stability of fluid parcels compressed in $n$ dimensions by shocks or turbulence requires $\gamma > \gamma_{cr} \equiv 2(1 - 1/n)$ (Chandrasekhar 1961; McKee et al. 1993; Vázquez-Semadeni et al. 1996). Shocks are likely to have $n = 1$, but generic turbulent compressions can have any dimensionality $n \leq 3$. The latter authors have also shown numerically the production of small-scale collapsing regions within a globally stable turbulent medium, both in magnetic and non-magnetic cases, although the former still requires the collapsing region to be super-critical (cf. §IV.B). They also noted that turbulence-induced collapse generally involves small fractions of the mass available in the flow, in contrast to the case of globally unstable regions, and showed that the density jump $\rho_2/\rho_1 \equiv X$ across shocks in a polytropic medium satisfies
$X^{1+\gamma}-(1+\gamma M_s^2)X+\gamma M_s^2=0$. In particular, this recovers the well known result that $X=M_s^2$ for $\gamma=1$, but also implies that in the limit of vanishing $\gamma$, $X \to \exp(M_s^2)$. An important consequence of this effective polytropic behavior is that, if $0<\gamma<1$, denser regions are colder. Upon the production of turbulent density fluctuations, the flow develops a temperature distribution similar to that resulting from isobaric thermal instabilities ($\gamma<0$), but without the need for them. Essentially, turbulent ram pressure provides the drive which in the thermally unstable case is provided by thermal pressure. Note, however, that in the turbulent $\gamma>0$ case there are no sharp phase transitions. In this scenario, the near-constancy of the thermal pressure among the various moderate-density phases of the ISM is an accidental consequence of a small value of $\gamma$ rather than a cloud confining agent (Ballesteros-Paredes, Vázquez-Semadeni & Scalo, 1998, hereafter BVS98). The increase in thermal pressure in molecular clouds is a consequence of $\gamma$ being closer to unity at the relevant densities (Scalo et al. 1998), although the thermal pressure is still subdominant with respect to other sources of pressure for scales $\lesssim 0.1$ pc.

For three-dimensional compressions, the minimum Mach number $M_0$ necessary to induce collapse by the velocity field has been computed by several authors as a function of $\gamma$ and the mass $m$ of the cloud in units of the Jeans mass. It is found that $M_0 \propto \ln m$ for the isothermal ($\gamma=1$) case (Hunter 1979), $M_0 \propto m^{\gamma-1/(4-3\gamma)}$ for $4/3 > \gamma > 1$ (Hunter & Fleck 1982) and $M_0 \geq \sqrt{10/3(1-\gamma)}$ for $0 < \gamma < 1$ (Tohline et al. 1987). Note that the latter result is independent of the cloud’s mass, at least for perfect spherical geometries.

Recently, BVS98 have investigated some implications of the scenario that clouds are turbulent density fluctuations by exploring the properties of clouds formed in 2D numerical simulations of the ISM (Passot et al. 1995) on scales between 1.25 pc and 1 kpc, including standard atomic-gas cooling rates, diffuse background heating, modeled stellar ionization heating, self-gravity, the Coriolis force, galactic shear and magnetic fields. It should be stressed that the clouds form during the evolution of the simulations, rather than being started with some pre-defined conditions, and have estimated lifetimes (BVS98) comparable to observational estimates (e.g., Bash et al. 1977; Blitz & Shu 1980; Blitz 1994). BVS98 find that the velocity field is in general continuous across the boundaries, with kinetic surface terms in the Eulerian
form of the Virial Theorem (McKee & Zweibel 1992) of comparable magnitude to the total kinetic energy contained within the clouds.

BVS98 also suggest that the formation of hydrostatic structures within a turbulent medium is not possible unless the effective polytropic exponent $\gamma$ increases during the process of collapse initiated by a turbulent compression. Such a change in $\gamma$ will not occur until protostellar densities are reached if thermal pressure alone is considered, so in general the production of hydrostatic structures appears unlikely.

III. CLOUD STRUCTURE

A. Scaling Relations and Energy Spectra

One of the most conspicuous properties of molecular clouds (shared with diffuse HI clouds), are the so-called Larson (1981) relations, which in their most common form read $\Delta v \propto R^{1/2}$ and $\rho \propto R^{-1}$, where $R$ is a characteristic spatial scale (e.g. cloud size), $\Delta v$ is a characteristic velocity difference (e.g. linewidth or line centroid difference), and $\rho$ the cloud’s mean density.\(^1\) The most generally accepted explanation of the origin of these relations, as stated in the Introduction, is that the clouds are in virial equilibrium between gravity and turbulent support (which causes one of the two relations to become a consequence of the other), together with some other assumption, such as that the clouds are magnetically critical and that the magnetic field does not vary much from one region to another (e.g., Shu et al. 1987; Mouschovias 1987), which fixes the other relation. However, molecular clouds may not always be gravitationally bound (Blitz 1994); indeed, except at the largest scales, observed internal clumps within clouds are not gravitationally confined (Bertoldi and McKee 1992). Being highly dynamic entities, even bound clouds may generally not be in static virial equilibrium (Ballesteros-Paredes & Vázquez-Semadeni 1997).

Alternative explanations under turbulent conditions have been proposed as well. Kolmogorov-like arguments based on, for example, cascades of angular momentum (Henriksen & Turner 1984) or kinetic energy density (Ferrini et al. 1983; Fleck 1996), are reviewed in Vázquez-Semadeni (1998). Here we briefly discuss results from numerical simulations.

\(^1\) Note that Larson’s original exponents were slightly different, being 0.38 for the linewidth-size relation and $-1.1$ for the density-size relation.
As mentioned in § I.B, for highly compressible regimes, the turbulent energy spectrum is expected to approach the form \( E(k) \propto k^{-2} \) of shock-dominated Burgers turbulence. If we assume that the observed linewidths measure the mean square turbulent velocity \( u_l \) over scales smaller than \( l \), we can write 

\[
\frac{u_l^2}{l} = 2 \int_{2\pi/l}^{\infty} E(k) dk \propto \int_{2\pi/l}^{\infty} k^{-2} dk \propto l,
\]

so that the observed velocity dispersion-size relation appears to emerge naturally. However, the identification of \( \Delta v(l) \) with \( u_l \) is not trivial. While \( \Delta v(l) \) is the linewidth within a beam of width \( l \) integrated over the whole line of sight, \( u_l \) is an average over an idealized ensemble (and, in practice, over all space) for volumes of size \( l^3 \). Thus, the identification of \( u_l \) with \( \Delta v(l) \) may depend on each beam being dominated by a single component of scale \( \sim l^3 \); hierarchical density fluctuations could potentially produce the required structure. The data of Issa et al. (1990) and Falgarone et al. (1992), which include positions in the sky away from brightness maxima and still exhibit a similar relation (slope \( \sim 0.4 \)), seem to support the turbulent origin of the \( \Delta v-R \) relationship (Issa et al. also considered random positions in their maps). Peng et al. (1998) have recently studied a sample of CCN clumps, most of which are reported to be gravitationally unbound, yet seem to follow Larson’s linewidth-size relation as well. However, Bertoldi & McKee (1992) found that the smaller, non-self-gravitating clumps in their large study sample did not follow Larson’s linewidth-size relation.

Vázquez-Semadeni, Ballesteros-Paredes & Rodríguez (1997, hereafter VBR97) have surveyed the clouds appearing in the 2D numerical simulations of Passot et al. (1995), finding a velocity dispersion-to-size relation with a logarithmic slope \( \sim 0.4 \) as well (albeit with a large scatter). The energy spectrum of those simulations is indeed of the form \( k^{-2} \). To study the development of the energy spectrum, simulations of MHD turbulence in which driving is localized in wavenumber space at scales smaller than the box have been performed in slab geometry (“1 2/3 D”) (Gammie & Ostriker 1996), and fully 3D geometry (Stone 1998; Stone, Ostriker, & Gammie 1998). For both cases, extended spectra develop in \( k \)-space above and below the range of driving frequencies, indicating the development of both “direct” and “inverse” cascades. The 1 2/3 D models, which were evolved over very long times, show spectral slopes \(-2\) or slightly steeper both above and below the forcing scale. The 3D models, although more limited in dynamic range, show combined \( u_k^2 + B_k^2 \) slopes between \(-2\) and \(-5/3\) for \( k \) larger than the
forcing scale, with the former (latter) occurring in weaker (stronger) magnetic field models. The slope of $u_k^2$ alone is $-2$ (stronger fields) or slightly steeper (weaker fields).

Overall, turbulent MHD simulations tend to evolve toward global energy equipartition. For example, Ostriker et al. (1998) and Stone et al. (1998) find perturbed magnetic energies between 30-60% of kinetic energies over a wide range of $\beta$. Kinetic and magnetic spectra are within factors of a few from each other (Passot et al. 1995), implying equipartition at all scales.

VBR97 also find cloud mass spectra of the form $dN(M)/dM \propto M^{-1.44 \pm 0.1}$, consistent with the low end of observational estimates (e.g., Blitz 1993), suggesting that the simulations reproduce well a number of observational cloud properties, even though they are two-dimensional. Additionally, VBR97 find that Larson’s density-size relation is not satisfied in general, but rather seems to be the upper envelope in a log $\rho$-log $R$ diagram of the clouds’ locus, i.e., it is satisfied only by the densest clouds at a given size (implying largest column densities). Low-column density clouds would naturally escape observational surveys utilizing limited amounts of integration time (Larson 1981; Kegel 1989; Scalo 1990), suggesting that this relation may be an observational artifact. The off-peak data of Falgarone et al. (1992) are consistent with this suggestion. In summary, these results can be interpreted as implying that a $Dv-R$ relation comparable to the observed scalings may be established globally as a consequence of the development of compressible MHD turbulence, while the density-size relation may occur only for gravitationally bound clouds within this turbulent field. Further work is necessary to confirm this possibility.

B. Turbulent Pressure

Vázquez-Semadeni, Cantó & Lizano (1998) have performed 2D and 3D numerical simulations of isothermal gravitational collapse in initially turbulent clouds, following the evolution of the velocity dispersion as the mean density increases during the collapse. They found power-law behavior of the form $P_t \sim \rho^{\gamma_t}$ for the “turbulent pressure”. In particular, for slowly-collapsing magnetic simulations, $\gamma_t \sim 3/2$, consistent with the result of McKee & Zweibel (1995) for the adiabatic exponent of Alfvén waves upon slow compression. However, non-magnetic and rapidly-collapsing (shorter Jeans length) magnetic simulations have $\gamma_t \sim 2$. Gammie & Ostriker (1996) also verified the
McKee & Zweibel scalings $P_{\text{wave}} \propto \rho^{3/2}$ and $P_{\text{wave}} \propto \rho^{1/2}$ for Alfvén wave pressure of, respectively, an adiabatically contracting medium, and propagation along a density gradient. For freely-evolving decay simulations, however, they found only a weak, variable correlation between perturbed magnetic pressure and density. These results are incompatible with the so-called “logatropic” equation $P \sim \ln \rho$, warning against its usage in dynamical situations.

C. The Density Field

The turbulent density field has a number of relevant statistical and physical properties. Besides the density-size scaling relation (Sec. III.A above), which may or may not be a true property of clouds, these exhibit hierarchical nesting (e.g., Scalo 1985; Houllahan & Scalo 1990, 1992; Vázquez-Semadeni 1994) and evidence for fractal (e.g., Falgarone et al. 1991) or even multi-fractal structure (Chappell & Scalo 1998). The spatial and statistical distribution of the density field is crucial in the study of star formation and the understanding of the stellar Initial Mass Function (IMF).

In order to construct a real theory of the IMF (or at least of the cloud mass spectrum, if the actual masses of stars turn out to be rather independent of their parent clumps) it is necessary to have a complete knowledge of the density statistics. A simple theory based exclusively on the probability density function (pdf, also commonly referred to as the density distribution function) of the fluid density (Padoan 1995; Padoan, Nordlund & Jones 1997) has been criticized by Scalo et al. (1998) who point out that, in addition to the probability of occurrence of high density sites, it is also necessary to know how much mass is contained in these fluctuations; this requires information on multi-point statistics.

As a first step towards the understanding of density fluctuations in the ISM, the pdf of the density in polytropic gas dynamics ($P \propto \rho^\gamma$) has been investigated as a function of the rms Mach number, $\gamma$, and the mean magnetic field strength (Vázquez-Semadeni 1994; Scalo et al. 1998; Passot & Vázquez-Semadeni 1998; Nordlund & Padoan 1998; Ostriker, Gammie, and Stone 1998; Stone et al. 1998). In the isothermal case ($\gamma = 1$) the density pdf is close to a lognormal distribution for every value of the Mach number. For polytropic cases with $\gamma < 1$ or $\gamma > 1$, a power law develops at densities larger than the mean or smaller than the mean, respectively, the effect being enhanced as the
Mach number increases. This behavior is a consequence of the dependence of the local Mach number with the density in each case (Passot & Vázquez-Semadeni 1998; Nordlund & Padoan 1998). It has been verified in one-dimensional numerical simulations of a forced polytropic gas, but appears to be supported also in several dimensions and in the presence of thermal heating and cooling which yield “effective” polytropic behavior (Scalo et al. 1998). In the limit of high Mach numbers and/or vanishing effective polytropic exponent, the behavior does not coincide with that of the Burgers equation (Passot & Vázquez-Semadeni 1998). One implication of these results is that it should be possible in principle to determine the actual effective $\gamma$ of the medium from observational determinations of its pdf, provided the problem of deconvolving the projected pdf is solved and $\gamma$ does not vary much in the observed region. Also, for example, the observation by Scalo et al. (1998) that full simulations of the ISM including the turbulent magnetic field have pdfs consistent with $\gamma < 1$ suggests that the MHD waves do not give appreciably large values of $\gamma$ (as is also suggested by the $P \propto \rho^{1/2}$ scaling of propagating linear-amplitude waves; cf. McKee & Zweibel 1995).

The mean mass-averaged value of $\log(\rho/\rho_0)$ increases with the Mach number $M_s$; for isothermal models in 2.5D and 3D respectively, Ostriker et al. (1998) and Padoan, Jones & Nordlund (1997) both find a logarithmic dependence on $M_s^2$. In large-Mach number, high-resolution 1D isothermal models by Passot & Vázquez-Semadeni (1998), a linear dependence is found between the variance of the density logarithm and $M_s^2$, leading to a mass-averaged value of $\log(\rho/\rho_0)$ that varies like $M_s^2$; this difference with higher-dimensional simulations (linear vs. logarithmic scaling with $M_s^2$) likely arises because the 1D simulations have a purely compressive velocity field and the higher-dimension simulations do not. Ostriker et al. (1998) and Stone et al. (1998) show that the largest mean contrasts in the density logarithm occur in models with the strongest mean magnetic fields (see also Pouquet, Passot and Léorat 1990).

The spectrum of the density field has recently started to be investigated. Padoan, Jones & Nordlund (1997) have reported a steep logarithmic slope $\sim -2.6 \pm 0.5$ for low-resolution 3D simulations of isothermal turbulence. Scalo et al. (1998) have reported two regimes, one with a slope $\sim -0.9$ at low $k$ and another with slope $\sim -2.4$ at
large $k$ for high-resolution 2D simulations of the ISM with heating and cooling. The steeper slopes may be due to inadequate resolution. On the other hand, Lazarian (1995) has produced an algorithm for deconvolving projected HI interferometric spectra, favoring a slope $\sim -1$. Equivalent work for molecular clouds and high-resolution simulations in the corresponding regimes are necessary to resolve this issue.

An interesting application has been given by Padoan, Jones & Nordlund (1997), who have shown that a turbulent density field with a power-law spectrum (though steep) and a lognormal pdf produces simulated plots of extinction dispersion vs. mean extinction that compare well with the analogous observational diagram for an extinction map of a dark cloud. Padoan & Nordlund (1998a) also suggest that models with weaker mean magnetic fields (such that $M_A \sim 10$) have extinction dispersion vs. mean extinction plots that show better agreement with observations than do models with stronger mean magnetic fields (such that $M_A \sim 1$); however, these models do not include self-gravity, which would affect the distribution of column densities (i.e. extinctions).

D. Correlations Among Variables

Numerical simulations are especially useful in the investigation of the spatial correlation among physical variables, a crucial ingredient in the formation of stars. Of particular interest is the correlation of magnetic field strength with density, as well as the correlation between field direction and the topology of density features.

The topology of the clouds is extremely clumpy and filamentary – for an example, see Figure 1. In general, the magnetic field exhibits a morphology indicative of significant distortion by the turbulent motions, with greater magnetic field tangling in cases with weak mean magnetic fields (Ostriker et al. 1998, Stone et al. 1998). Magnetic fields may be either aligned with or perpendicular to density features (cf. Figure 1; Passot et al. 1995; Vázquez-Semadeni & Passot 1998; Ostriker et al. 1998; Stone et al. 1998), with the former trend most visible at the boundaries of supershells (Gazol & Passot 1998a), and the latter most prominent for strong-field simulations in which field kinks and density maxima coincide (Gammie & Ostriker 1996; Ostriker et al. 1998).

Globally, no clear trend between density and magnetic field intensity is found in the simulations, but very weak correlations are observed (Passot et al. 1995, Gammie & Ostriker 1996). For example, in the
large-scale 2D ISM models of Passot et al. (1995), the field strength $B$ varies from $\sim 10^{-2} \, \mu G$ in the low density intercloud medium to $\sim 25 \, \mu G$ in the densest clouds ($n \sim 50 \, \text{cm}^{-3}$, size $\sim$ several tens of pc), although vanishing field strengths are also found in those regions. In 3D simulations with weak mean magnetic fields ($B_0 = 1 \, \mu G$), Padoan & Nordlund (1998a) found a large dispersion in the values of $B$ as a function of density $n$, but a power-law correlation $B \propto n^{0.4}$ in the upper envelope of this distribution. In stronger-mean-field simulations ($B_0 = 30 \, \mu G$), they found little variation of $B$ with $n$. Padoan & Nordlund argue that the $B - n$ upper-envelope correlation found in the former case supports the notion that the mean fields in molecular clouds are weak, because a similar $B - n$ scaling has been observed for measured Zeeman field strengths (e.g. Crutcher 1998). However, because the densities over which the envelope correlation is found in simulations is much smaller than the density regime in which a power-law $B - n$ relation is observed in real clouds, and because the simulations do not include gravity, the conclusion remains controversial.

To explore how line spectra may vary spatially in simulated clouds, Padoan et al. (1998) have produced synthetic non-LTE spectra of various molecular transitions from models with weak and strong mean magnetic fields. They find that their super-Alfvénic model reproduces the observational trend of line-width vs. integrated temperature found by Heyer, Carpenter & Ladd (1996), while the equipartition model gives a weaker trend. However, these models do not include self-gravity, which would affect the density distributions and density-velocity correlations. Thus, it remains uncertain whether the weak-mean-field model of molecular clouds advanced by Padoan & Nordlund (1998a) truly provides a better fit to observations.

IV. CLOUD EVOLUTION

A. Dissipation Rates and Turbulence Maintenance

As described above, the first observations of supersonic internal cloud velocities immediately led to the questions – which have persisted up to the present – of what creates these large-amplitude motions, and how they are maintained. Since the large-scale ISM is itself turbulent, turbulent motions may be incorporated into cold clouds from their formation stages, being part of the same continuum. In addition,
there may be ongoing inputs which tap energy from larger scales in the
Galaxy, or from smaller scales within the cloud, particularly associated
with various aspects of star formation (see e.g. Scalo 1987; Miesch and
Bally 1994; Kornreich & Scalo 1998). One of the first steps in under-
standing cloud evolution, and relating this evolution to the initiation of
star formation and dynamical feedback, is to assess the rate of turbu-

ten decay under the range of conditions representative of dark clouds
and GMCs.

From dimensional analysis, the decay rate of kinetic energy per
unit mass must scale as \( \dot{E} \sim v^3/R \), where \( v \) is some characteristic
speed (or weighted product of two or more speeds), and \( R \) is some
characteristic scale. In incompressible turbulence (cf. §1.B), the only
characteristic scales are that of the box \((L)\), that of the flow velocity
difference \( u_L \) on the largest scale, and the value of the small-scale vis-
cosity \( \nu \). The velocity dispersion over the whole box \( \sigma_v \sim u_L \) provided
the spectral slope is \(-1\) or steeper. The first two quantities deter-
mine the decay rate, \( \sigma_v^3/L \), while the last sets the spatial dissipation
scale to \( l_{\text{min}}/L \sim (\nu/L\sigma_v)^{3/4} \equiv Re^{-3/4} \). For a nearly pressureless
(i.e. highly compressible) fluid, again possessing the same three char-
acteristic scales, the same dissipation rate would apply, except that
energy would be transferred by a shock directly from the largest scale
to the dissipation scale within a flow crossing time. For a magnetized
flow of finite compressibility, on the other hand, other velocity scales
in addition to \( \sigma_v \) – namely those associated with thermal pressure and
magnetic stress, \( c \) and \( v_A \) – enter the problem, and may potentially
influence the scaling of the dissipation rate.

In molecular clouds, the thermal pressure is very low, and there-
fore strong dissipation in shocks is expected. However, from the time
of early observations until quite recently, it has widely been consid-
ered likely that the “cushioning” effect of magnetic fields would signif-
ically reduce kinetic energy dissipation for motions transverse to the
mean field, provided that turbulent velocities remain sub-Alfvénic. Low
dimension (1 2/3D) simulations (Gammie & Ostriker 1996) provided
some support for this idea, in that they found a scaling of dissipation
rate with \( \beta \) in quasi-steady-state as \( \dot{E} \propto \beta^{1/4} \), such that magnetic fields
of a few tens of \( \mu G \) could potentially provide a factor \(~3\) reduction
in dissipation compared to that in weak-field (\( \beta = 1 \)) cases. However,
very recent higher-dimension numerical simulations of both forced and
decaying turbulence have shown that although some differences remain between weak-field and strong-field cases, dissipation rates are never substantially lower than the predictions of unmagnetized turbulence. Below we describe specific results.

The recent 3D MHD decay simulations (Mac Low et al. 1998; Stone et al. 1998; Padoan & Nordlund 1998a) have followed the evolution of Mach-5 turbulence with a variety of initial velocity spectra and $\beta$ ranging from 0.02 to 2, and compared to unmagnetized models. These models have uniform initial $B$. Stone et al. (1998) have also included simulations of decay from fully-saturated turbulence, and from saturated turbulence with initial density fluctuations suppressed. These experiments all show kinetic energy decay times (defined as the time for kinetic energy reduction by 50%) in the range $\sim 0.4 - 1t_f$, where $t_f$ is the flow crossing time $l/u_l$ on the main energy-containing scale. In Stone et al. (1998), the difference in decay time between the strongest-field case ($\beta = 0.02$, corresponding to $44\mu G$ for $n_{H_2} = 10^3 \text{cm}^{-3}$ and $T = 10\text{K}$) and the unmagnetized run is less than a factor of two. In decay models, Mac Low et al. (1998) and Stone et al. (1998) find late-time power law dependence of the turbulent energy as $E \propto t^{-\eta}$ with $\eta = 0.8 - 1$.

To assess turbulent decay rates for a quasi-steady state, Stone et al. (1998) performed simulations in which a fixed mechanical power $\dot{E}_K$ is input to the flow in the form of random, uncorrelated velocity perturbations. A saturated state with energy level $E_K$ is reached after $\sim t_f$, with only relatively small differences in the saturation energy between magnetized and unmagnetized models; the turbulence dissipation times $E_K/\dot{E}_K$ range between $0.5 - 0.7t_f$. Mac Low (1998) has found similar results, and suggests that the power-law time dependence of decay models arises from a secular increase in the smallest scale of turbulence. The implications of the short turbulent dissipation time for potential cloud support are discussed below in §IV. B.

Kornreich & Scalo (1998) have recently considered the problem of turbulence maintenance in molecular clouds, comparing the average time between external shock wave passages through a cloud with the energy decay time, finding that they are comparable and implying that this “shock pump” is capable of sustaining the cloud turbulence. An additional result is the proposal of a cascade-like mechanism for the compressible case, in which vorticity is generated behind shocks, which
in turn rapidly produces new smaller-scale shocks, and so on. This is due to an interesting asymmetry of the evolution equations for the vorticity and divergence of the velocity field (for recent discussions, see Vázquez-Semadeni et al. 1996; Kornreich & Scalo 1998): the nonlinear transfer will produce compressible modes out of purely rotational ones, but the converse is not true. This can also be understood in terms of the well-known Kelvin theorem of conservation of circulation.

The mechanisms of vorticity production have also been investigated numerically. Simulations of compressible turbulence with purely potential forcing indicate that a negligible amount of kinetic energy is transferred from compressible to solenoidal modes (Kida & Orszag 1990; Vázquez-Semadeni et al. 1996). Vorticity generation behind curved shocks or shock intersections, as well as the vortex stretching term do not seem to be efficient processes to maintain a non-negligible level of vorticity in the flow. However, in the presence of the Coriolis force, thermal heating (through the baroclinic term) or magnetic field, near equipartition between solenoidal and compressible modes is easily obtained (Vázquez-Semadeni et al. 1996).

B. Turbulence and Cloud Support Against Gravity

An issue that is often discussed in tandem with turbulent dissipation is the question of cloud support against self-gravity. Cold, dark clouds and GMCs have typical Jeans lengths $\sim 2$ pc, such that the whole cloud entities exceed the Jeans mass $M_J = \rho L_J^3$ by factors of a thousand or much more. This is just another way of stating that thermal pressure gradients would be powerless to prevent self-gravitating runaway. If not for the intervention of other dynamical processes, wholesale cloud collapse would be the rule.

“Turbulent pressure” is often invoked as a means to counter gravity. For a weakly compressible medium, the $\rho v^2$ Reynolds stresses are naturally associated with this turbulent pressure. The effect of turbulence on the gravitational instability in absence of magnetic field was first investigated by Chandrasekhar (1951) who suggested an increase of the Jeans length and later by Bonazzola et al. (1987, 1992) and Vázquez-Semadeni & Gazol (1995) who predicted a reversal of the Jeans criterion when certain conditions on the energy spectrum are met.

In a strongly compressible and radiative medium, the collisions of supersonically converging streams of gas are nearly inelastic, so the Reynolds stress does not itself act as an effective pressure. Turbu-
lent motions can, however, generate turbulent magnetic fields (and vice versa); these fluctuating magnetic fields exert pressure and tension forces on the medium. Several authors (Shu, Adams, & Lizano 1987; Fatuzzo and Adams 1993; McKee and Zweibel 1995) have suggested that, in particular, the time-dependent magnetic field perturbations associated with Alfvén waves could potentially be important in providing “wave pressure” support against gravity along the mean magnetic field direction in a cloud. This mean-field axis is the most susceptible to collapse; in the orthogonal directions, magnetic pressure suppresses gravitational instability in a homogeneous cloud as long as \( n_J \equiv L/\delta L_J < (\beta/2)^{-1/2} \) (Chandrasekhar and Fermi 1953), and suppresses instability in a cloud pancake of surface density \( \Sigma \) provided \( \Sigma/B < 1/(2\pi\sqrt{G}) \) (cf. Mouschovias and Spitzer 1976; Tomisaka, Ikeuchi, and Nakamura 1988).

An exact derivation of the Jeans criterion is hardly possible in the MHD turbulent case. An attempt in this direction has been made by considering the linear stability of a self-gravitating medium, permeated by a uniform magnetic field \( B_0 \) along which a finite amplitude circularly polarized Alfvén wave propagates. For perturbations along \( B_0 \) it is found that the Alfvén wave increases the critical Jeans length (Lou 1997). In the case of perturbations perpendicular to the mean field Gazol & Passot (1998b) have shown that the medium is less stable in presence of a moderate amplitude Alfvén wave. For large amplitude waves however, McKee & Zweibel (1995) show that the waves have an isotropic stabilizing effect.

Gammie & Ostriker (1996) verified, using simulations in 1 2/3 D, that Alfvén waves of sufficient amplitude (such that \( n_J \lesssim \delta v_A/2c_s \)) can indeed prevent collapse of slab-clouds along the mean field direction. These simulations included both cases with decaying and quasi-steady forced turbulence. For 1D decay models, the turbulent dissipation rate is low enough such that clouds with initial turbulent energy above the limit remain uncollapsed for times up to \( t_g \equiv L_J/c \) (\( \sim 10 \) Myr for typical conditions). However, more recent simulations performed in higher dimensions have shown that the greater dissipation rates quench turbulence too rapidly for magnetic fluctuations to prevent mean-field collapse. Because 3D dissipation times for both magnetized and unmagnetized flows are smaller than the flow crossing time \( t_f \) (cf. §IV A), they will also be less than the gravitational collapse times \( \approx 0.3t_g \) for clouds
that are virialized (e.g., observed cloud scalings yield $t_f \sim 0.5 t_g$); thus cloud support cannot be expected for un-regenerated turbulence. Self-gravitating, magnetized 2 1/2D simulations of Ostriker et al. (1998) have already demonstrated this result directly, and concluded that without ongoing energy inputs, only the strength of the mean magnetic field is important in determining whether or not clouds collapse in times $< 10\text{Myr}$. Numerical simulations have also been performed which allow for ongoing turbulent excitation. In unmagnetized 2D models, Léorat et al. (1990) showed that large-scale gravitational collapse can be prevented indefinitely provided a high enough Mach number is maintained by forcing, and the energy is injected at small enough scales. For magnetized models, Gammie & Ostriker (1996) found similar results for 1 2/3 D simulations (as well as for unpublished 2D simulations).

Ballesteros-Paredes & Vázquez-Semadeni (1997) have measured the overall virial balance of clouds in a “survey” of the 2D high resolution simulations of the ISM of Passot et al. (1995). It was found that the gravitational term appearing in the virial theorem is comparable to the sum of the other virial terms for the largest clouds, but progressively loses importance on the average as smaller clouds are considered. Nevertheless, the scatter about this average trend is large, and a small fraction of small clouds have very large gravitational terms which overwhelm the others and induce collapse. In this scenario, the low efficiency of star formation is understood as a consequence of the intermittency of the turbulence.

C. Evolution of the Star Formation Rate

Numerical simulations on the large scales provide information on the star formation (SF) history as well (Vázquez-Semadeni et al. 1995; Gazol & Passot 1998a). In these models, stars form whenever a certain density threshold is exceeded, rather than being treated as a separate fluid (e.g., Chiang & Prendergast 1985; Rosen, Bregman & Norman 1993). A nontrivial result is the development of a self-sustained cycle of SF in which the turbulence, aided by self-gravity, contains enough power to produce star-forming clouds, while the energy injected by the stars is sufficient to regenerate the turbulence. Due to the strong non-linearity of the SF scheme, this cycle is highly chaotic and particularly intermittent in the presence of SN. Self-propagating SF in supershells is very efficient, increasing the SF rate in active periods, but the destructive power of superbubbles is so large that the system requires a longer
time to create new SF sites once the shells have dispersed. A similar result was obtained by Vázquez & Scalo (1989) in a simple model for gas-accreting galaxies. This behavior is consistent with recent observations suggesting that the SF history in galaxies is highly irregular (e.g., Grebel 1998). Another interesting point concerns the influence of the strength of the uniform component of the magnetic field $B_0$ (Gazol & Passot 1998a). The star formation rate is found to increase with $B_0$ (with $B_0 = 5\mu G$ it is larger by a factor $\sim 3$ compared with the case $B_0 = 0$), as long as $B_0$ is not too large. For very large $B_0$, it decreases due to the rigidification of the medium.

V. CONCLUSIONS

In this chapter we have reviewed a vast body of results which address the problem of MHD compressible (MHDC) turbulence and its implications for problems of cloud and star formation. Most of these results are new, appearing after the last Protostars & Planets Conference. Interstellar turbulence is inherently a multi-scale and non-equilibrium phenomenon, and thus appears to play a fundamental role in the formation, evolution and determination of cloud structural properties. Most of the studies reviewed here have relied on direct numerical simulations of MHDC turbulence in a variety of regimes, ranging from isothermal to polytropic to fully thermodynamic, the latter including parameterized heating and cooling and modeled star formation. High-resolution 2D and 3D simulations of the ISM at “box” scales from 1 kpc down to a few pc have shown that the clouds formed in them reproduce well a number of observational cloud properties (such as clumpy structure and linewidth-size scalings), while suggesting that some others (e.g. Larson’s (1981) density-size relation) may either arise from selection effects or require other special conditions. Simulations to date have shown that the old paradigm that MHDC turbulence should dissipate much more slowly than non-magnetic turbulence may be incorrect, bringing back the necessity of strong large-scale fields for magnetic cloud support and/or of continued energy injection in order to sustain the turbulence. The idea for self-regulation of star formation by turbulent feedback remains promising, and it is also possible that physical processes not yet incorporated in simulations may reduce the turbulent dissipation rate.

A large number of questions remain unanswered, however. At the larger scales, it is necessary to understand the interplay between tur-
bulence, large-scale instabilities, and spiral waves in the formation of molecular clouds and complexes, as well as to investigate the processes that lead to cloud destruction. Quantitative assessments must be developed for the efficiency of turbulent excitation from internal and external sources. A theory of the IMF, or at least of the cloud mass spectrum, requires the knowledge and understanding of multi-point density statistics. Better understanding of the parameter dependence of these and other questions in cloud evolution and structure will be required to address the problem of “deriving” the star formation rate and efficiency, in answer to the oft-posed challenge of extragalactic astronomers. Comparison with observations is crucial to discriminate among models, but will always face the degeneracy limitations associated with projection effects.

Future research in these areas is likely to include more physical processes (e.g., ambipolar diffusion, radiative transfer, and global environment effects, such as the galactic spiral potential and variations in external radiation with galactic position), to perform direct statistical comparison with observations of molecular lines, and polarization and IR measurements, and to work at higher resolutions in 3D, if the multiscale nature of the problem is to be captured adequately. With so much still untried, there is considerable ground to cover before reaching the long-range goal of synthesizing the results of disparate numerical simulations into a coherent theory of the turbulent ISM. Nevertheless, the important advances since PPIII show the success of numerical methods in answering many longstanding questions in the theory of interstellar MHD turbulence, and the opportunity for great progress before the next Protostars and Planets.

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FIGURE CAPTIONS

Figure 1. Sample density, velocity, and magnetic field structure in two turbulent cloud simulations. Density contours are logarithmic, with dark blue running from the mean density to ten times the mean, and light blue 10-100 times the mean; magnetic field lines are shown in green, and the velocity field is shown in red. For both models, the Mach number is ten; for the upper (lower) figure, $\beta = 0.01 (0.1)$, corresponding to stronger (weaker) mean magnetic fields. See Ostriker et al. (1998) for more details.
