Renormalization of B-meson distribution amplitudes

N. Offen
Laboratoire de Physique Théorique
CNRS/Univ. Paris-Sud 11 (UMR 8627),
F-91405 Orsay, France
E-mail: nils.offen@th.u-psud.fr

S. Descotes-Genon
Laboratoire de Physique Théorique
CNRS/Univ. Paris-Sud 11 (UMR 8627),
F-91405 Orsay, France
E-mail: sebastien.descotes-genon@th.u-psud.fr

We summarize a recent calculation of the evolution kernels of the two-particle $B$-meson distribution amplitudes $\phi_+$ and $\phi_-$ taking into account three-particle contributions. In addition to a few phenomenological comments, we give as a new result the evolution kernel of the combination of three-particle distribution amplitudes $\Psi_A - \Psi_V$ and confirm constraints on $\phi_+$ and $\phi_-$ derived from the light-quark equation of motion.

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1. Introduction

Exclusive decays of $B$-mesons provide important tools to test the Standard Model and to search for physics beyond it. Hadronic inputs encoding soft physics are not only form factors but also light-cone distribution amplitudes (LCDAs). In particular the $B$-meson LCDAs enter the parametrization of the hard-scattering part of hadronic matrix elements of bilocal current operators where large momentum is transferred to the soft spectator quark [1-10]. Impressive progress has been made in the calculation of the hard scattering amplitudes entering factorization theorems, see e.g. [11-15] for the $B \to PP$ case, but one limiting factor for the extraction of fundamental parameters is the uncertainty coming from the hadronic input. Recent years have seen several analyses concerning the renormalization properties [16, 17, 18] and the shape of the $B$-meson LCDAs [4, 18, 19, 20, 21, 22, 23]. Up to now these analyses were restricted to the two-particle case or to leading order with the exception of [23]. Here we present the results of [24] for the renormalization of the two-particle $B$-meson LCDAs taking into account mixing with three-parton LCDAs and in section (2.3) the results of a new calculation for the combination of three-particle LCDAs $\Psi_A - \Psi_V$ entering the equations of motion.

2. One-loop calculation with three-parton external state

The relevant two- and three-parton distribution amplitudes are defined as $B$ to vacuum matrix-elements of a non-local heavy-to-light operator, which reads in the two-particle case [9]

$$\langle 0|\bar{q}\beta(z)[z,0|\phi_{\alpha}(0)|B(p)| = -i\frac{\hat{J}_{B}(\mu)}{4} \left[(1 + \psi)\left(\phi_+ (t) + \frac{t}{2\mu} [\phi_- (t) - \phi_+ (t)]\right)^{\gamma_5}\right]_{\alpha\beta}$$

(2.1)

and in the three-particle case [21] (the most general decomposition without contraction with a light-like vector is given in [23]):

$$\langle 0|\bar{q}\beta(z)[z,\mu]H_{\nu}(uc)z^V[uc,0|\phi_{\alpha}(0)|B(p)| = \frac{\hat{J}_{B}(\mu)M}{4} \left[(1 + \psi)\left(v_{\mu}\hat{t} - t\gamma_{\mu}\right)\left(\tilde{\psi}_{A}(t,u) - \tilde{\psi}_V(t,u)\right) - i\sigma_{\mu\nu}\tilde{z}\tilde{y}_V(t,u)\right]_{\alpha\beta}.\right]$$

(2.2)

We use light-like vectors $n_{\pm}$ so that every vector can be decomposed as

$$q_{\mu} = (n_{\perp} \cdot q)n_{\perp\mu} + (n_{\perp} \cdot q)n_{\perp\mu} = q_{\perp} n_{\perp\mu} + q_{\perp} n_{\perp\mu} + q_{\perp\mu}.\right]$$

(2.3)

The computation of the renormalisation properties of the distribution amplitudes requires us to consider matrix elements of the relevant operators

$$O^H_+(\alpha) = \frac{1}{2\pi} \int dte^{iut} \langle 0|\bar{q}(z)[z,0|\hat{u}_+ \Gamma \phi_{\alpha}(0)|H\rangle$$

(2.4)

$$O^H_-(\alpha) = \frac{1}{2\pi} \int dte^{iut} \langle 0|\bar{q}(z)[z,0|\hat{u}_- \Gamma \phi_{\alpha}(0)|H\rangle$$

(2.5)

$$O^H_3(\alpha, \xi) = \frac{1}{(2\pi)^2} \int dt d\xi \int dte^{iut} \langle 0|\bar{q}(z)[z,uc]g_H(uc)z^V[uc,0|\Gamma \phi_{\alpha}(0)|H\rangle$$

(2.6)
Renormalization of B-Meson distribution amplitudes

N. Offen

\[ A : -g_s e_+ \varepsilon \left[ \delta (\omega - k_+ - q_+) - \delta (\omega - k_+) \right] \bar{v}_l \Gamma T^a u \]

\[ B : -g_s v \cdot \varepsilon \delta (\omega - k_+) \bar{v}_l \Gamma T^a u \]

\[ C : g_s \frac{1}{(k + q)^2} \delta (\omega - k_+ - q_+) \bar{v}_d (k + q) u \Gamma T^a u \]

Figure 1: The three leading-order contributions to the matrix element of \( O_\pm \) with a three-parton external state.

with \( z \) parallel to \( n_+ \), i.e. \( z_\mu = tn_{+\mu}, t = v \cdot z = z_-/2 \) and the path-ordered exponential in the \( n_+ \) direction:

\[ [z, 0] = P \exp \left[ ig_s \int_0^\infty dy_\mu A^\mu (y) \right] \]

\[ = 1 + ig_s \int_0^1 d\alpha z_\mu A^\mu (\alpha z) - g_s^2 \int_0^1 d\alpha \int_0^\alpha d\beta z_\mu A^\mu (\alpha z) A^\nu (\beta z) + \ldots \] (2.7)

The Fourier-transforms of the different distribution amplitudes are then defined via

\[ \phi_\pm (\omega) = \frac{1}{2\pi} \int dt e^{i\omega t} \tilde{\phi}_\pm (t) \quad F(\omega, \xi) = \frac{1}{(2\pi)^2} \int dt \int du e^{i(\omega + u\xi)t} \tilde{F}(t, u) \] (2.8)

where \( F = \Psi_Y, \Psi_A, X_A, Y_A \). Since the renormalization of the operators is independent of the infrared properties of the matrix-elements, we can choose an on-shell partonic external state consisting of a light quark, a heavy quark and a gluon in equation (2.6). The resulting leading-order diagrams are shown in figures 1 and 2. Next-to-leading order (NLO) diagrams are obtained by adding a gluon or a quark loop (a ghost loop) in all possible places (for a complete list of diagrams, see [24]). Since the operators give rise to \( \delta \)-distributions in the \( + \)-component of the momenta, we chose to proceed via the theorem of residues. To be more explicit, we decomposed the loop momentum \( l \) in light-cone components, picked up the poles in the \( l^- \)-integral and performed the \( l^- \)-integration in dimensional regularization with \( D = 2 - 2\varepsilon \) dimensions. Additional \( \frac{1}{\varepsilon} \)-poles arise through the \( l^- \)-integration for diagrams where a gluon is exchanged between the Wilson-line from the operator and the heavy-quark field. These are related to the cusp anomalous dimension, see e.g. [26, 27], stemming from the intersection of one light-like Wilson line from the path ordered exponential in the operator and one time-like Wilson line from the interaction of soft gluons with the heavy quark. The additional poles give rise to \( \frac{1}{\varepsilon^2} \)-terms as well as Sudakov logarithms.
where we defined the three-particle term. The renormalization group equation to order $\alpha_s$ can be written as

$$\frac{\partial \phi_+(\omega; \mu)}{\partial \log \mu} = -\frac{\alpha_s(\mu)}{4\pi} \left( \int d\omega' \gamma_+^{(1)}(\omega, \omega'; \mu) \phi_+(\omega'; \mu) \right) + \int d\omega'd\xi' \gamma_+^{(1)}(\omega, \omega', \xi'; \mu) [\Psi_A - \Psi_V](\omega, \xi'; \mu)$$

(2.11)

where $\gamma_+^{(1)}$ is the result from [17]

$$\gamma_+^{(1)}(\omega, \omega'; \mu) = \gamma_+^{(1)} - \Gamma^{(1)}_{\text{cusp}} \frac{\theta(\omega' - \omega)}{\omega'}$$

(2.12)

and $\gamma_{-3}^{(1)}$ from [24]

$$\gamma_{-3}^{(1)}(\omega, \omega', \xi') = 4 \left[ \frac{\Theta(\omega)}{\omega'} \left( (C_A - 2C_F) \left[ \frac{1}{\xi' \omega' + \xi' - \omega} \Theta(\xi' - \omega) + \frac{\Theta(\omega' + \xi' - \omega)}{(\omega' + \xi')^2} \right] - C_A \left[ \frac{\Theta(\omega' + \xi' - \omega)}{(\omega' + \xi')^2} - \frac{1}{\xi' \omega' + \xi' - \omega} \Theta(\omega - \omega') - \Theta(\omega - \omega' - \xi') \right] \right] \right]_+$$

(2.13)

where we defined the $+$-distribution with three variables as

$$\left[ f(\omega, \omega', \xi') \right]_+ = f(\omega, \omega', \xi') - \delta(\omega - \omega' - \xi') \int d\omega f(\omega, \omega', \xi')$$

(2.14)
2.3 $\Psi_A - \Psi_V$-renormalization and equation of motion constraints

Here we report on an up to now unpublished calculation of the renormalization of the three-particle LCDAs $\Psi_A - \Psi_V$. We project on the relevant distribution amplitudes in equation (2.3) using $\Gamma = \gamma_+^{\mu} d_+ d_- \gamma_5$ (although a $\gamma_5^{\mu}$ instead of $\gamma_5^{\mu}$ yields the same result). The calculations go along the same lines as in the previous two cases, even though there is only one leading-order structure (shown in figure 3) and NLO diagrams must have one gluon attached to the vertex in order not to vanish trivially. For convenience the result is splitted into $C_F$- and $C_A$-colour structures.

$$\gamma_{3,3,C_4}^{(1)}(\omega, \xi, \omega', \xi') = 2 \left[ \delta(\omega - \omega') \left\{ \frac{\xi}{\xi'} \Theta(\xi' - \xi) - \frac{\Theta(\xi' - \xi)}{\xi' - \xi} \right\} + \delta(\omega + \xi - \omega' - \xi') \times \left\{ \frac{1}{\xi'} \Theta(\omega - \omega') - \frac{\Theta(\omega - \omega')}{\omega - \omega'} - \frac{\Theta(\omega') - \omega}{\omega - \omega} \right\} + \delta(\omega + \xi - \omega' - \xi') \times \left\{ \frac{\theta}{\xi'} \Theta(\omega - \omega') - \frac{\Theta(\omega - \omega')}{\omega - \omega'} - \frac{\Theta(\omega') - \omega}{\omega - \omega} \right\} + \frac{\omega - \xi}{\omega'} (\omega + 2 \xi') - \Theta(\omega' - \omega) \Theta(\omega - \omega') + \frac{\omega - \xi}{\xi'} \frac{\omega'}{\omega - \omega} \Theta(\omega - \xi') \Theta(\xi') \right\} \right]$$

(2.15)

$$\gamma_{3,3,C_4}^{(1)}(\omega, \xi, \omega', \xi') = \gamma_+^{(1)}(\omega, \omega'; \mu) \delta(\xi - \xi') + \gamma_+^{(1)}(\omega, \xi, \omega', \xi')$$

$$\gamma_{R,3,3}^{(1)}(\omega, \xi, \omega', \xi') = 4 \delta(\omega + \xi - \omega' - \xi') \times \left\{ \frac{\xi^2}{\omega^2} \frac{\Theta(\omega') - \omega}{\omega + \xi} \Theta(\xi) + \frac{\omega}{\xi'} \frac{\Theta(\omega) - \omega}{\omega + \xi} \Theta(\omega) \left( \frac{\xi}{\omega + \xi} - \frac{\omega - \xi}{\xi'} \right) \right\}$$

(2.16)

with $\gamma_+^{(1)}$ the same as in equation (2.10) and $\gamma_{3,3}^{(1)}$ defined as in (2.11) with obvious changes. Part of this calculation, namely the light-quark-gluon part, has been calculated in a different context and a different scheme, e.g. in [29, 28].

In [21] two equations from the light- and heavy-quark equations of motion were derived

$$\omega \phi_+^{(1)}(\omega; \omega; \mu) + \phi_+^{(1)}(\omega; \omega; \mu) = I(\omega; \mu), \quad (\omega - 2\lambda) \phi_+^{(1)}(\omega; \omega; \mu) + \omega \phi_-^{(1)}(\omega; \mu) = J(\omega; \mu),$$

(2.17)

where $I(J)(\omega; \mu)$ are integro-differential expressions involving the three-particle LCDAs $\Psi_A - \Psi_V$ ($\Psi_A + X_A$ and $\Psi_V$) respectively. While the second equation was shown not to hold beyond leading order in [17, 23] we have checked that the first one is valid once renormalization is taken into account. Taking the derivative of the first equation with respect to $\log \mu$, inserting

$$I(\omega; \mu) = \frac{2 d}{d \omega} \int_0^\omega d \rho \int_{\omega - \rho}^\omega \frac{d \xi}{\xi} \frac{\partial}{\partial \xi} [\Psi_A(\rho, \xi; \mu) - \Psi_V(\rho, \xi; \mu)]$$

(2.18)

and using the relation from [17]

$$-\omega \frac{d}{d \omega} \int_0^\eta \frac{d \omega'}{\eta} \gamma_+^{(1)}(\omega, \omega'; \mu) = \gamma_+^{(1)}(\omega, \eta; \mu)$$

(2.19)
one arrives at the following equation
\[ \omega \frac{d}{d\omega} \int d\omega' d\tilde{\xi}' \gamma_{-3}^{(1)}(\omega, \omega'; \tilde{\xi}') \left( \Psi_A(\omega', \tilde{\xi}'; \mu) - \Psi_V(\omega', \tilde{\xi}'; \mu) \right) \]
\[ + 2 \int d\omega' \gamma_{+}^{(1)}(\omega, \omega'; \mu) \frac{d}{d\omega'} \int_0^\omega dp \int_{\omega' - p}^\infty \frac{d\tilde{\xi}}{\tilde{\xi}} \frac{\partial}{\partial \tilde{\xi}} \left( \Psi_A(\rho, \tilde{\xi}; \mu) - \Psi_V(\rho, \tilde{\xi}; \mu) \right) \]
\[ = 2 \int d\omega' d\tilde{\xi}' \frac{d}{d\omega} \int_0^\omega dp \int_{\omega' - p}^\infty \frac{d\tilde{\xi}}{\tilde{\xi}} \frac{\partial}{\partial \tilde{\xi}} \gamma_{3,3}^{(1)}(\rho, \tilde{\xi}, \omega' \tilde{\xi}'; \mu) \left( \Psi_A(\omega', \tilde{\xi}'; \mu) - \Psi_V(\omega', \tilde{\xi}'; \mu) \right), \]
\[ (2.20) \]
which can be proven to hold at order \( \alpha_s \) by simple insertion of the respective evolution kernels \( (2.10), (2.13), (2.15), (2.16) \). This non-trivial outcome gives us further confidence concerning the renormalization group properties of the LCDAs.

3. Conclusions

The presence of \( \delta(\omega - \omega') \log(\mu/\omega) \) in the renormalization matrices gives rise to a radiative tail falling off like \( \log(\omega)/\omega \) for large \( \omega \). Therefore non-negative moments of the LCDAs are not well defined and have to be considered with an ultraviolet cut-off.\[16, 17, 22, 23\]

\[ \langle \omega^N \rangle_{\pm}(\mu) = \int_0^{\Lambda_{UV}} d\omega \omega^N \phi_{\pm}(\omega; \mu) \]  
(3.1)

For \( \phi_- \) it is interesting to examine the limit
\[ \lim_{\Lambda_{UV} \to \infty} \int_0^{\Lambda_{UV}} d\omega \omega^N \phi_{-3}(\omega, \omega', \tilde{\xi}') = 0 \quad N = 0, 1, \quad \phi_{-3} = \frac{1}{2\epsilon} \gamma_{-3}^{(1)}, \]
(3.2)

which is relevant for the calculation of the three-particle contributions to the moments:
\[ \int_0^{\Lambda_{UV}} d\omega \omega^N \phi_{-}(\omega; \mu) = 1 + \frac{\alpha_s}{4\pi} \left( \int d\omega' \phi_{-}(\omega') \int_0^{\Lambda_{UV}} d\omega \omega^N \phi_{-3}(\omega, \omega'; \mu) \right) - \int d\omega' d\tilde{\xi}' (2 - D) [\Psi_A - \Psi_V](\omega', \tilde{\xi}') \int_0^{\Lambda_{UV}} d\omega \omega^N \phi_{-3}(\omega, \omega', \tilde{\xi}'), \]
(3.3)

Therefore as stated in \[17\] three-particle distribution amplitudes give only subleading contribution to the first two moments \( (N = 0, 1) \) and we have explicitly checked that this statement cannot be extended to higher moments \( (N \geq 2) \).

The next step consists in using the renormalization properties as a guide to go beyond the existing models derived from a leading-order sum-rule calculation resulting in \( \Psi_A = \Psi_V \[21\] \) and to analyze their influence on \( \phi_- \). Finally, for practical calculations involving three-particle contributions, one would need the evolution kernels for the all relevant LCDAs, which will be the subject of a future work.

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