Is the Zee model neutrino mass matrix ruled out?

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Abstract

A very economic model of generating small neutrino masses is the Zee model. This model has been studied extensively in the literature with most of the studies concentrated on the simplest version of the model where all diagonal entries in the mass matrix are zero. SNO, and KamLAND data disfavor this simple version, but only when one also combines information from atmospheric and K2K data, can one rule out this model with high confidence level. We show that the simplest version of Zee model is ruled out at $3\sigma$ level. The original Zee model, however, contains more than enough freedom to satisfy constraints from data. We propose a new form of mass matrix by naturalness consideration. This new form of mass matrix predicts that $m_{\nu_3} = 0$, and $\tan^2 \theta_{\text{solar}}$ increases with $|V_{e3}|$. For the best fit value of $\tan^2 \theta_{\text{solar}}$, $|V_{e3}|$ is sizeable but below the upper bound.
There are abundant data [1–6] from solar, atmospheric, laboratory and recent long baseline (K2K and KamLAND) experiments on neutrino mass and mixing. It is certain that some of the neutrinos have non-zero masses and also different neutrino spices mix with each other. In the minimal Standard Model (SM) in which there is just one Higgs doublet in the scalar sector and there are no right-handed neutrinos, neutrinos are massless. In order to have non-zero neutrino masses and mixing, one must go beyond the minimal SM.

There are different possible ways to generate neutrino masses. A very economic way of generating neutrino masses is to introduce a charged scalar and an additional Higgs doublet into the minimal SM as proposed by Zee [7]. The Zee model provides a natural mechanism to generate small neutrino masses because they can only be induced at loop level, and also suggests special forms for the mass matrix. If one imposes a discrete symmetry such that only one of the Higgs doublets couples to the leptons as suggested by Wolfenstein [8], one obtains a simple mass matrix with all diagonal entries zero. We will refer this simple version as the Zee-Wolfenstein model. This model has been studied extensively in the literature [7–12]. In this paper we further study the Zee model using the most recent experimental data. We show that the Zee-Wolfenstein model is ruled out at the 99.73% (3σ) C.L.. However the original Zee model contains more than enough freedom to satisfy experimental constraints. We propose a new form of neutrino mass matrix resulting from naturalness condition. This model predicts that $m_{\nu_3} = 0$, and $\tan^2 \theta_{\text{solar}}$ increases with $|V_{e3}|$. For the best fit value of 0.4 for $\tan^2 \theta_{\text{solar}}$, $|V_{e3}|$ is sizeable but below the 3σ upper bound.

The Zee model contains, in addition to the gauge bosons and the minimal fermion contents, a singlet scalar $h$ and two Higgs doublets $\phi_{1,2}$ transforming under the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ as (1,1,1) and (1,2,-1/2). With these particles it is not possible to have tree level neutrino masses from renormalizeable Lagrangian, but it is possible at one loop level. The relevant terms in the Lagrangian are [7],

$$L = -\bar{l}_{aR}^i \tilde{\phi}_a^{\dagger} d_{b}^j \psi_{bL}^{i} \epsilon_{ij} - \bar{\psi}_{aL}^{T} C \psi_{bL}^{i} \epsilon_{ij} h - M^{\alpha\beta} \phi_{a}^{\dagger \alpha} \phi_{a}^{\beta} \epsilon_{ij} h + H.C.,$$  \hspace{1cm} (1)

where $\psi_{aL}^{i} = (\nu_{aL}, e_{aL})$ and $l_{aR}$ are the left- and right-handed leptons with “a” the generation.
index and “i,j” the SU(2)_L indices. \(\epsilon_{ij}\) is the anti-symmetric symbol. \(C\) is the Dirac charge conjugation matrix. \(\tilde{f}^{\phi,ab}_\gamma\) are the Yukawa couplings responsible for charged lepton masses. \(\tilde{f}^{ab}\) is an anti-symmetric matrix in generation indices \(a\) and \(b\) due to Fermi statistics.

The mass matrix \(\tilde{m}\) for the charged leptons is given by,

\[
\tilde{m} = (v_1 \tilde{f}_1^\phi + v_2 \tilde{f}_2^\phi) = v (\sin \beta \tilde{f}_1^\phi + \cos \beta \tilde{f}_2^\phi).
\]

Here \(v_\gamma = \langle \phi_\gamma \rangle\) are the vacuum expectation values (VEV), \(v = \sqrt{v_1^2 + v_2^2} = 174\) GeV and \(\tan \beta = v_1/v_2\). The mass matrix \(\tilde{m}\) can be diagonalized to obtain the eigen-mass matrix \(m = \text{Diag}(m_e, m_\mu, m_\tau)\) by a bi-unitary transformation multiplying two unitary matrices \(V_{L,R}\) from left and right,

\[
m = V_R \tilde{m} V_L^T.
\]

The linear combination \(\phi^-_W = \cos \beta \phi^-_1 + \sin \beta \phi^-_2\) is “eaten” by \(W^-\). The physical combination which mixes with \(h\) is \(\phi^- = \cos \beta \phi^-_1 - \sin \beta \phi^-_2\). We indicate the two mass eigenstates of masses \(M_1\) and \(M_2\) for the charged scalars as \(h_1 = \cos \theta_Z h - \sin \theta_Z \phi^+\) and \(h_2 = \sin \theta_Z h + \cos \theta_Z \phi^+\). Here \(\sin \theta_Z\) is proportional to \(M_{\alpha\beta}\) characterizing the strength of the \(h - \phi^+\) mixing.

The terms responsible for neutrino mass generation in the previous equation, in the mass eigenstates basis for the charged lepton and scalar fields, can be written as

\[
L = -\bar{E}_R m E_L - \bar{E}_R \left( \frac{1}{v \tan \beta} m - \frac{1}{\sin \beta} f^\phi \right) \nu_L (\sin \theta_Z h_1^T - \cos \theta_Z h_2^T)
- 2 \nu_L^T f^{\phi \gamma}_L \nu_L (\cos \theta_Z h_1 + \sin \theta_Z h_2) + \ldots
\]  

(2)

where \(f^\phi = (f^{\phi,ab}_\gamma) = V_R f^\phi V_L^\dagger\), \(f = (f^{ab}) = V_L^* \tilde{f} V_L^\dagger\), \(E_{L,R} = (e, \mu, \tau)_{L,R}\), and \(\nu_L = (\nu_1, \nu_2, \nu_3)_L\).

Exchange of charged scalars \(h_{1,2}\) and charged leptons at one loop level, Majorana neutrino mass term \(L_m = (1/2) \nu_L^T M_\nu C \nu_L\) can be generated with

\[
M_\nu = A[(f m^2 + m^2 f^T) - \frac{v}{\cos \beta} (f m f^\phi + f^\phi f^T m f^T)],
\]

(3)

where \(A = \sin(2\theta_Z) \log(M_2^2/M_1^2)/(16\pi^2 v \tan \beta)\) which is of order \(O(10^{-5})\) if the \(\sin(2\theta_Z)\) and \(\tan \beta\) are both of order one. This is the general mass matrix in the Zee model [12]. The mixing matrix is the unitary matrix \(V\) which diagonalizes the mass matrix and is defined by, \(D = V^T M_\nu V\), with \(D = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})\).
The present experimental data on neutrino masses and mixing angles can be summarized as follows \([13,14]\). The \(3\sigma\) allowed ranges for the mass-squared differences are constrained to be:

\[1.6 \times 10^{-3} \text{ eV}^2 \leq |\Delta m_{atm}^2| \leq 4.8 \times 10^{-3} \text{ eV}^2, \quad \text{and} \quad 4.7 \times 10^{-5} \text{ eV}^2 \leq \Delta m_{solar}^2 \leq 1.7 \times 10^{-4} \text{ eV}^2,\]

with the best fit values given by

\[|\Delta m_{atm}^2| = 2.5 \times 10^{-3} \text{ eV}^2, \quad \text{and} \quad \Delta m_{solar}^2 = 7.0 \times 10^{-5} \text{ eV}^2.\]

The mixing angles are in the ranges of

\[0.28 \leq \sin^2 \theta_{atm} \leq 0.7 \quad \text{and} \quad 0.29 \leq \tan^2 \theta_{solar} \leq 0.63.\]

Also the CHOOZ experiment \([4]\) gives an upper bound of \(0.22\) on the \(\nu_e - \nu_x\) (where \(\nu_x\) can be either \(\nu_\mu\) or \(\nu_\tau\) or a linear combination) oscillation parameter for

\[\Delta m_{x1}^2 = |m_x|^2 - |m_{\nu_1}|^2 > 10^{-3} \text{ eV}^2.\]

In the model discussed here the atmospheric neutrino and K2K data can be explained by oscillation between the muon and the tauon neutrinos, and the solar neutrino and KamLAND data explained by oscillation between the electron and muon (or a linear combination of muon and tauon neutrino) neutrinos. In this case the CHOOZ limit applies to the oscillation between the electron and tauon neutrinos \(^*\).

Setting \(f^\phi_2\) in eq. (3) to zero, one obtains the famous Zee-Wolfenstein mass matrix,

\[M_\nu = \begin{pmatrix} 0 & \tilde{a} & \tilde{b} \\ \tilde{a} & 0 & \tilde{c} \\ \tilde{b} & \tilde{c} & 0 \end{pmatrix}, \tag{4}\]

where \(\tilde{a} = Af^{e\mu}(m_\mu^2 - m_e^2)\), \(\tilde{b} = Af^{e\tau}(m_\tau^2 - m_e^2)\) and \(\tilde{c} = Af^{\mu\tau}(m_\tau^2 - m_\mu^2)\). One can redefine the neutrino and charged lepton phases such that all \(\tilde{a}, \tilde{b}\) and \(\tilde{c}\) are real.

Unfortunately the Zee-Wolfenstein model is now ruled out by experimental data. This can be seen from the following.

The above mass matrix satisfies the “zero sum” condition \(m_{\nu_1} + m_{\nu_2} + m_{\nu_3} = 0\), therefore all the neutrino masses are determined in terms of the mass-squared differences \([16]\). We have \([16]\)

\(^*\)There are additional evidences for oscillation between electron and muon neutrinos from LSND experiment \([15]\). If confirmed more neutrinos are needed to explain all the data.
\[ m_{\nu_i}^2 = -\frac{1}{3} \left[ 2\Delta m_{21}^2 + \Delta m_{32}^2 - 2\sqrt{(\Delta m_{32}^2)^2 + \Delta m_{21}^2 \Delta m_{32}^2 + (\Delta m_{21}^2)^2} \right]. \]  

(5)

The other two masses are given by, \( m_{\nu_2}^2 = \Delta m_{21}^2 + m_{\nu_1}^2 \) and \( m_{\nu_3}^2 = \Delta m_{32}^2 + m_{\nu_2}^2 \). The “zero sum” condition admits two types of mass hierarchy if the absolute value of \( r = \Delta m_{21}^2 / \Delta m_{32}^2 \) is much smaller than one (experimentally \(|r| < 0.106 \) at 3\( \sigma \) level), with one of them the normal one: \( m_{\nu_3} > m_{\nu_2} > m_{\nu_1} \) and \( m_{\nu_1} \approx m_{\nu_2} \), and another the inverted one: \( |m_{\nu_2}| > |m_{\nu_1}| > |m_{\nu_3}| \) and \( m_{\nu_2} \approx -m_{\nu_1} \). One finds that \(|x| = |m_{\nu_1}/m_{\nu_2}|\) is determined to be very close to one.

The mass matrix element \( M_{11} = 0 \) leads to, \( V_{e1}^2 m_{\nu_1} + V_{e2}^2 m_{\nu_2} + V_{e3}^2 m_{\nu_3} = 0 \), which can be rewritten as

\[ V_{e2}^2 = \frac{-x + (1 + 2x)V_{e3}^2}{1 - x}. \]  

(6)

Since \(|x|\) is smaller but close to one, the above equation only allows negative \( x \) for small \( V_{e3}^2 \) implying that only the inverted mass hierarchy is possible. One thus obtains a minimal \( V_{e2,\text{min}}^2 \) of \( V_{e2}^2 \) close to \((1 - V_{e3,\text{max}}^2)/2 \approx 0.47\), while data from SNO and KamLAND prefers a smaller \( V_{e2}^2 \). Therefore SNO and KamLAND data disfavor the Zee-Wolfenstein model.

This has been noticed in Ref. [11]. However, we would like to point out that although the Zee-Wolfenstein model can not produce the central values for the mixing and mass difference from solar and KamLAND data, the present data can not rule out the model at more than even 2\( \sigma \) level.

To have a more quantitative statement, we have carried out a detailed study and the results are shown in Figure 1. The dashed lines in Figure 1 are for \( \tan^2 \theta_{\text{solar}} \) \((\sin^2 2\theta_{\text{solar}} = 4|V_{e1}|^2|V_{e2}|^2)\) with two values (0.22 and 0.15) of \( V_{e3} \) as a function of \( r \). When \(|V_{e3}|\) decreases, \( \tan^2 \theta_{\text{solar}} \) increases. \( \tan^2 \theta_{\text{solar}} \) is about 0.53 for the 3\( \sigma \) upper bound of \(|V_{e3}|\), and becomes larger than the 3\( \sigma \) allowed value of 0.63 when \(|V_{e3}|\) decreases to be lower than 0.15. One therefore can take \(|V_{e3}|\) to be larger than 0.15 at 3\( \sigma \) level. It is clear that the model is not possible to produce the best fit value of 0.4 for \( \tan^2 \theta_{\text{solar}} \). However at 2\( \sigma \), \( \tan^2 \theta_{\text{solar}} \) can be as large as 0.54 [14]. Therefore it is not possible to rule out the model at more than 2\( \sigma \) level from data on solar and KamLAND.
FIG. 1. The dashed lines $S_1$ and $S_2$ are for $\tan^2 \theta_{\text{solar}}$ as functions of $r$. The two solutions for $\sin^2 \theta_{\text{atm}}$ are indicated by solid lines $A_1a$ and $A_2a$, and $A_1b$ and $A_2b$, respectively. Here the indices “1” and “2” indicate the cases with $|V_{e3}|$ equals to 0.22 and 0.15, respectively.

Data on $\sin^2 \theta_{\text{atm}}$ can provide further constraints on the model. The condition $M_{22} = m_1 V^2_{\mu_1} + m_2 V^2_{\mu_2} + m_3 V^2_{\mu_3} = 0$ in the model can be used to determine $\sin^2 \theta_{\text{atm}} = V^2_{\mu_3}$. The mixing matrix $V$ can be parameterized using three rotation angles, for example $V_{e2} = s_{12} c_{13}$, $V_{e3} = s_{13}$ and $V_{\mu_3} = s_{23} c_{13}$. Here $s_{ij} = \sin \theta_{ij}$ and $c_{ij} = \cos \theta_{ij}$. Two of the angles, $\theta_{12,13}$ can be determined in terms of $V_{e3}$ and $r$ from previous discussions. The condition

\[ M_{22} = m_{\nu_1} (s_{12} + c_{12} s_{13} t_{23})^2 + m_{\nu_2} (c_{12} - s_{12} s_{13} t_{23})^2 + m_{\nu_3} c^2_{13} t^2_{23} = 0, \] (7)

then determines $\theta_{23}$ in terms of $V_{e3}$ and $r$. Here $t_{23} = s_{23}/c_{23} = \tan \theta_{23}$. There are two solutions for $\tan \theta_{23}$ for given $V_{e3}$ and $y$ which we indicate by “a” and “b”.

In Figure 1 the solid lines show $\sin^2 \theta_{\text{atm}}$ as a function of $r$ for $|V_{e3}|$ equals to its 3σ allowed upper value of 0.22 and the allowed lower value of 0.15. From the figure we see that $\sin^2 \theta_{\text{atm}}$ decreases for solution “a”, and $\sin^2 \theta_{\text{atm}}$ increases for solution “b” as $r$ increases from the 3σ lower bound of -0.106 to the allowed upper bound of 0. All solutions for $\sin^2 \theta_{\text{atm}}$ are outside the 3σ allowed range of 0.3 $\sim$ 0.7. For $|V_{e3}|$ smaller than 0.15, it is possible for $\sin^2 \theta_{\text{atm}}$ of solution “b” to become smaller than the 3σ allowed upper bound. However $|V_{e3}|$ smaller than 0.15 will drive $\tan^2 \theta_{\text{solar}}$ to move out the 3σ allowed range. Therefore the combined neutrino data on $\tan^2 \theta_{\text{solar}}$ and $\sin^2 \theta_{\text{atm}}$ rule out the Zee-Wolfenstein model at
more 3σ level.

The above discussions show clearly that the Zee-Wolfenstein neutrino mass matrix is in trouble. That does not, however, mean that the Zee model itself is in trouble. The mass matrix given in eq. (3) contains more than enough freedom to fit data. Here we encounter a common problem for physics beyond the SM that there are too many new parameters. Additional theoretical considerations have to be applied to narrow down the parameters.

We find that an interesting neutrino mass matrix emerges if one requires that there should be no large hierarchies among the new couplings, that is all $f_{ij}$ and $f_{\phi,ab}^2$ are of the same order of magnitude, respectively. This can be considered as a naturalness requirement. From eq. (3) one sees that all terms in the mass matrix are either proportional to $m_l$ or $m_l^2$. Since $m_\tau >> m_{\mu,e}$, the leading contributions to the neutrino mass matrix are proportional to $f^{ij}m_\tau^2$ and $f_{\phi,ab}^2 m_\tau$. To this order we have

$$M_{11} = -2A \frac{v}{\cos \beta} f^{e\tau} f_{\phi,\tau e}^2 m_\tau, \quad M_{22} = -2 \frac{v}{\cos \beta} f^{\mu\tau} f_{\phi,\tau \mu}^2 m_\tau, \quad M_{33} = 0,$$

$$M_{12} = -\frac{v}{\cos \beta} A(f^{e\tau} f_{\phi,\tau \mu}^2 + f^{\mu\tau} f_{\phi,\tau e}^2) m_\tau,$$

$$M_{13} = A f^{e\tau} m_\tau (m_\tau - \frac{v}{\cos \beta} f_{\phi,\tau \tau}^2), \quad M_{23} = A f^{\mu\tau} m_\tau (m_\tau - \frac{v}{\cos \beta} f_{\phi,\tau \tau}^2).$$

(8)

Without loss of generality, by appropriate choices of neutrino filed phases, the 11, 13, 23 entries can be made real with just one physical phase $\delta$ in the mass matrix. One can rewrite the above mass matrix as

$$M_\nu = a \begin{pmatrix} 1 & (ye^{i\delta} + x)/2 & z \\ (ye^{i\delta} + x)/2 & xye^{i\delta} & xz \\ z & xz & 0 \end{pmatrix},$$

(9)

with $a = |M_{11}|$, $x = |f^{\mu\tau}|/|f^{e\tau}|$, $y = |M_{22}|/xa$, $z = |M_{13}|/a$.

This is a highly constrained form of mass matrix. This matrix is rank two implying that one of the neutrinos has zero mass. The non-zero eigenvalues are given by

$$m_\pm^2 = \frac{a^2}{4}(\sqrt{1 + 2xy \cos \delta + x^2 + y^2} \pm \sqrt{(1 + x^2)(1 + y^2 + 4z^2)})^2.$$

(10)
Since experimentally $\Delta m^2_{21} > 0$, there are two types of eigen-mass hierarchies, a) $m_{\nu_1} = 0$, $|m_{\nu_2}| = \sqrt{\Delta m^2_{21}} = m_-$, $|m_{\nu_3}| = \sqrt{\Delta m^2_{32} - \Delta m^2_{21}} = m_+$; and b) $|m_{\nu_1}| = \sqrt{|\Delta m^2_{32}| - \Delta m^2_{21}} = m_-$, $|m_{\nu_2}| = \sqrt{|\Delta m^2_{32}|} = m_+$, $m_{\nu_3} = 0$. The five parameters in the mass matrix are severely constrained from data on $\Delta m^2_{21} , \Delta m^2_{32} , V_{e2} , V_{e3}$ and $V_{\mu 3}$.

To have some idea about what parameter space may satisfy experimental constraints, let us discuss the situation with the phase $\delta$ set to be zero for simplicity. For type a) of mass hierarchy since $|r| = |\Delta m^2_{21}/\Delta m^2_{32}|$ is much smaller than 1, one would have $(1 + xy)^2$ to be almost equal to $(1 + x^2)(1 + y^2 + 4z^2)$. To satisfy this, $x$ should be close to $y$ and $z$ to be much smaller than 1. Expanding the mixing matrix elements around $x = y$ and small $z$, we find that $(V_{e2} , V_{\mu 2} , V_{r 2})$ to be proportional to $(z, xz, -(1 + x^2))$. Since $z$ is much smaller than 1, one would obtain too small a $V_{e2}$ in contradiction with solar and KamLAND data. This qualitative feature is not changed even if a non-zero $\delta$ is introduced. There is no solution for the normal mass hierarchy of type a).

For type b) of mass hierarchy, one has $(V_{e3} \ V_{\mu 3} \ V_{r 3})$ is proportional to $(-2xz , 2z , x - y)$. A small $|r|$ requires $xy$ to be close to -1. Then small $V_{e3}$, and large $|V_{\mu 3}|$ and $|V_{r 3}|$ require $x$ to be small and $2xz$ to be of order one. We indeed find solutions for the mixing matrix satisfying experimental constraints. We also find that the size of $V_{e3}$ anti-correlates with $\tan^2 \theta_{solar}$ strongly, that is, when $|V_{e3}|$ decreases, $\tan^2 \theta_{solar}$ increases. If $\tan^2 \theta_{solar}$ is close to its best fit value of 0.4, $|V_{e3}|$ is close to, but below, the $3\sigma$ upper bound of 0.22. In the following we present a sample solution with $\Delta m^2_{21,32}$ have their best fit values,

$$m_{\nu_1} = 4.93 \times 10^{-2} \text{eV}, \ m_{\nu_2} = -5.00 \times 10^{-2} \text{eV}, \ m_{\nu_3} = 0.$$  

$$V = \begin{pmatrix} 0.8244 & -0.5312 & -0.1953 \\ 0.2961 & 0.6989 & -0.6511 \\ 0.4823 & 0.4789 & 0.7335 \end{pmatrix}.$$  \hspace{1cm} (11)

The $\tan^2 \theta_{solar}$ is 0.415 close to the best fit value. The value $-0.1953$ for $V_{e3}$ is below, but close to the $3\sigma$ allowed upper bound.

In the above solution, the input parameters are: $x = -0.3$, $y = 3.455$, $z = 1.667,$
\( a = 1.94 \times 10^{-2} \) eV. One can choose different signs for the parameters \( x, y \) and \( z \). As long as the signs for \( x \) and \( y \) are simultaneously changed, the magnitudes of the eigen-masses and the mixing matrix elements are not changed. We will stick to the signs with \( x \) to be negative, \( y \) and \( z \) to be positive in our later discussions.

One can also find solutions with smaller \( |V_{e3}| \). For example, with \( x = -0.165 \), \( y = 6.531 \) and \( z = 3.030 \), we obtain \( V_{e3} = -0.11 \), but \( \tan^2 \theta_{\text{solar}} = 0.624 \) which is close to the 3\( \sigma \) upper bound.

We searched for other solutions. We find that it is also possible to have solutions with non-zero CP violating phase \( \delta \). For example with \( a = 1.92 \times 10^{-2} \) eV, \( x = -0.276 \), \( y e^{i\delta} = 3.467 - i0.0573 \), and \( z = 1.571 \), we have

\[

m_{\nu_1} = 4.93 \times 10^{-2} e^{-i16.9^\circ} \text{eV}, \ m_{\nu_2} = -5.00 \times 10^{-2} e^{i11^\circ} \text{eV}, \ m_{\nu_3} = 0.
\]

\[

V = \begin{pmatrix}
0.8147 & -0.5048 - i0.2311 & -0.1676 - i0.0024 \\
0.3110 - i0.1995 & 0.7035 & -0.6071 - i0.0087 \\
0.4166 - i0.1619 & 0.4402 - i0.0561 & 0.7767
\end{pmatrix}.
\] (12)

The value for \( \tan^2 \theta_{\text{solar}} \) is about 0.464 which is within the 1\( \sigma \) region. The value \( |V_{e3}| = 0.168 \) is below the 3\( \sigma \) upper bound, but not far below. The CP violating Jarlskog parameter \( J = Im(V_{11}V_{22}^*V_{12}V_{21}^*) \) is predicted to be \( -0.0165 \) which may be studied in future neutrino factories. We have kept masses in the form with phases to illustrate the existence of Majorana phases which can be rotated away by multiplying a phase matrix from the right on \( V \) obtained above.

The neutrino masses obtained in the model are in the interesting ranges. The sum of the absolute neutrino masses, \( m_{\text{sum}} = |m_{\nu_1}| + |m_{\nu_2}| + |m_{\nu_3}| \), in this model is around 0.1 eV which is several times smaller than the recent bound of 0.69 eV from WMAP [17], but can be probed in the near future by the PLANK experiment where the sensitivity on \( m_{\text{sum}} \) can be as low as 0.03 eV. Laboratory neutrino mass experiments can also test the model. A non-zero value \( a = |m_{\nu e}| \) can induce neutrinoless double beta decays. \( |m_{\nu e}| \) obtained here is about 0.02 eV which is safely below the present bound [1,18] of 0.4 eV. However it can be probed
by future experiments, such as GENIUS, MOON and CUORE, where sensitivity of about 0.01 eV may be reached. The effective mass \( m_e = \sqrt{|m_{\nu_1} V_{e1}|^2 + |m_{\nu_2} V_{e2}|^2 + |m_{\nu_3} V_{e3}|^2} \) measured by the end point spectrum of beta decay in our case is around \( \sim 0.05 \) eV which is unfortunately a factor of 2 smaller than the sensitivity of future KATRIN experiment.

The off-diagonal entries of the couplings \( f^{ab} \) and \( f^{\phi, ab} \) can induce flavor changing interactions. One should make sure that constraints on related parameters will not rule out the regions of the parameters to produce the mass matrix discussed above. It is not possible to completely determine the couplings using just information from neutrino masses and mixing. We therefore take a simple situation with \( f^{\phi, \tau \tau}_2 = 0 \) for illustration. In this case for the example given in eq. (12): \( f^{\phi, \tau e}_2 / \cos \beta = -0.33 \times 10^{-2}, f^{\phi, \tau \mu}_2 / \cos \beta = -(1.14 - i0.02) \times 10^{-2} \), \( A f^{\tau e} = 0.93 \times 10^{-11} \) (GeV\(^{-1}\)), and \( A f^{\mu \tau} = -0.26 \times 10^{-11} \) (GeV\(^{-1}\)). It is interesting to note that the solution obtained here is consistent with the naturalness requirement that \( f^{\phi, \tau e}_2 \) to be the same order of magnitude as \( f^{\phi, \tau \mu}_2 \), and \( f^{\tau e} \) to be the same order of magnitude as \( f^{\mu \tau} \). If one chooses a smaller \( x \) one would obtain bigger hierarchy for the parameters, \( f^{\mu \tau} \) and \( f^{\tau e} \). The qualitative features will not change when other values for the parameters are used.

Exchange of the neutral Higgs boson \( \phi_1 \) (with mass \( M_0 \)) can induce at tree level \( l_i \rightarrow l_j, k \bar{k} \) decays. For the values of \( f^{\phi, \tau \mu}_2 \) and \( f^{\phi, \tau e}_2 \) obtained in the example of eq.(12) we have

\[
B(\tau \rightarrow \mu \mu \bar{\mu}, \mu e \bar{e}) \approx 3.5 \times 10^{-9} B_\tau, \ 0.80 \times 10^{-13} B_\gamma; \ B(\tau \rightarrow \mu \gamma) \approx 0.76 \times 10^{-8} B_\gamma;
\]

\[
B(\tau \rightarrow e \mu \bar{\mu}, e e \bar{e}) \approx 2.9 \times 10^{-10} B_\tau, \ 0.67 \times 10^{-14} B_\gamma; \ B(\tau \rightarrow e \gamma) \approx 0.63 \times 10^{-9} B_\gamma.
\]

In the above \( B_\tau = (100 \) (GeV)/\( M_0 \tan \beta)^4 B^{SM}(\tau \rightarrow \nu_\tau \mu \bar{\nu}_\mu) \) with \( B^{SM}(\tau \rightarrow \nu_\tau \mu \bar{\nu}_\mu) \approx 17\% \).

There are experimental constraints on the above decays with the 90% C.L. bounds given by [1]: \( B(\tau \rightarrow \mu \mu \bar{\mu}, \mu e \bar{e}) = 1.9 \times 10^{-6}, 1.7 \times 10^{-6}, B(\tau \rightarrow e \mu \bar{\mu}, e e \bar{e}) = 1.8 \times 10^{-6}, 2.9 \times 10^{-6}, B(\tau \rightarrow \mu \gamma, e \gamma) = 1.1 \times 10^{-6}, 2.7 \times 10^{-6} \). For \( \tan \beta \) of order one, all the branching ratios predicted above are safely below the experimental values if the mass \( M_0 \) is of order 100 GeV.

Non-zero \( f^{ij} \) can also induce radiative charged lepton decays by exchanging charged scalars. If the parameter \( A \) is not too much smaller than a natural value of \( A = 10^{-5} \) (GeV\(^{-1}\)), their contributions for the rare decays mentioned will be much smaller. The rare
decays of the types discussed in the above will not provide significant constraints.

From the above discussions we see that the new form of mass matrix proposed is consistent with present experimental data. It also predicts $m_{\nu_3} = 0$ and a sizeable $|V_{e3}|$. In particular, if the error on $\tan^2 \theta_{\text{solar}}$ is reduced and the present best fit value holds, $|V_{e3}|$ will be close to the $3\sigma$ allowed upper bound. The model can be tested in the future.

ACKNOWLEDGMENTS

I thank A. Zee for many useful suggestions. This work was supported in part by NSC under grant number NSC 91-2112-M-002-42, and by the MOE Academic Excellence Project 89-N-FA01-1-4-3 of Taiwan,
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