A New Model for Filamentary Molecular Clouds

Jason D. Fiege and Ralph E. Pudritz

Dept. of Physics and Astronomy, McMaster University, Hamilton, ON

Abstract. We develop a theory for filamentary molecular clouds including the effects of ordered magnetic fields, and external pressure. We first derive a new virial equation appropriate for filamentary clouds. By comparing with observational results collected from the literature, we find that the fields are likely helical. Secondly, we construct numerical, MHD models of filamentary clouds that agree with the observational constraints. We find that our models produce more realistic density profiles $r \sim r^{-1.8}$ to $r^{-2}$ than previous models, where the density falls off as $r^{-4}$.

1. Introduction

Most molecular clouds are filamentary structures that are supported by non-thermal, MHD turbulence, as well as large scale ordered magnetic fields (eg. review, McKee et. al 1993). Nevertheless, virtually all theoretical models assume spheroidal geometry. While spheroidal models obviously apply to molecular cloud cores, they cannot adequately describe molecular clouds on larger scales.

Observations suggest that some filamentary clouds may be wrapped by helical fields (Heiles 1987; Bally 1987). A few authors have previously modeled filamentary clouds with helical fields (cf. Nakamura, Hanawa, & Nakano 1993; Hanawa et al., 1993), but the fields in these models simply rescale the Ostriker (1964) solution for unmagnetized filaments. They are also unconstrained by observational data.

The role of the external pressure has been almost completely ignored by previous models of filamentary clouds. As we discuss in Section 2, real molecular clouds are truncated at finite radius by the pressure of the external medium. By ignoring the external pressure, most existing models cannot adequately describe real molecular clouds.

2. Surface Pressures on Molecular Clouds

Molecular clouds and the surrounding atomic gas are dominated by non-thermal motions, which result in total pressures that greatly exceed the thermal pressure. The total pressure of the atomic gas has been evaluated at the Galactic midplane by Boulares and Cox (1990), who find pressures on the order of $10^4 K cm^{-3}$. However, it is likely that the molecular clouds are surrounded by atomic gas at significantly higher pressures. Some are associated with HI clouds at a pressure of $\sim 10^5 K cm^{-3}$ (Chromey, Elmegreen, and Elmegreen 1989). Therefore, we
conservatively adopt surface pressures in the range of $10^{4–5} \text{ K cm}^{-3}$ for our analysis.

3. A New Form of the Virial Equation for Filamentary Molecular Clouds

We consider the virial equilibrium of a long filamentary cloud with an external pressure $P_S$ that truncates the cloud at some cylindrical radius $R_S$, and a general magnetic field of helical geometry. By considering only radial equilibrium, we derive a new virial equation from the tensor virial theorem:

$$\frac{P_S}{\langle P \rangle} = 1 - \frac{m}{m_{\text{vir}}} \left(1 - \frac{M}{|W|}\right),$$  \hspace{1cm} (1)

where

$$\langle P \rangle = \frac{\int P dV}{V}$$

$$m_{\text{vir}} = \frac{2\langle \sigma^2 \rangle}{G}$$

$$W = = -m^2G$$

$$M = \frac{1}{4\pi} \int B_z^2 dV - \left(\frac{B_{zS}^2 + B_{\phi S}^2}{4\pi}\right) V.$$  \hspace{1cm} (2)

In this equation, $P_S$ and $\langle P \rangle$ are the surface and average internal pressures, $m$ is the mass per unit length, $m_{\text{vir}}$ is the “virial” mass per unit length. $W$ and $M$ are respectively the gravitational and total magnetic energies per unit length. We note that $W$ is independent of the filament radius.

The total magnetic energy $M$ may be either positive or negative depending on whether the poloidal field or the toroidal field dominates the overall energetics of the cloud. Thus, the poloidal field component helps to support the cloud radially against self-gravity, while the toroidal component works with gravity and the external pressure to squeeze the cloud.

The virial quantities $m/m_{\text{vir}}$ and $P_S/\langle P \rangle$, in equation [1], may be determined from observations; thus, we may easily infer the magnetic parameter $M/|W|$. We have calculated $m/m_{\text{vir}}$ and $P_S/\langle P \rangle$ for several filaments, based on observational results gathered from the literature. The reader is referred to Fiege & Pudritz (1999a) for the full data tables and references. We refer to Figure 1 of Pudritz and Fiege (1999), in this volume, where we plot contours of $M/|W|$ over these virial parameters. We find that most of the filaments in our sample fall in the range

$$0.11 \leq m/m_{\text{vir}} \leq 0.43$$

$$0.012 \leq P_S/\langle P \rangle \leq 0.75,$$  \hspace{1cm} (3)

and that $M/|W| < 0$, which is consistent with a helical magnetic field.
4. Numerical Magnetostatic Models

Our theoretical models involve three parameters, two to describe the mass loading of the poloidal and toroidal field lines, and a third that specifies the radial concentration of the filament. We fully sample our parameter space by a Monte Carlo method, in which models are randomly generated and tested for agreement with the observational constraints given in equation 3. In addition to equation 3, we demand that the magnetic and kinetic energies are nearly in equipartition; $0.2 \leq \frac{M}{K} \leq 5$, where $M$ and $K$ are the average magnetic and kinetic energy densities in the cloud. This has been observationally determined for many clouds (Myers & Goodman 1988a,b).

Figure 1 shows the structure of a few models that are consistent with our constraints. We find that the density falls off as $r^{-1.8}$ to $r^{-2}$ in the outer regions, which is in excellent agreement with recent results by Alves et al. (1998) and Lada et al. (1998) for the L977 and IC5146 filamentary molecular clouds. We find that the peak poloidal field in our models is always much stronger than the peak toroidal field; only the outer regions are dominated by the toroidal field component. In this sense, our helical fields are actually quite weakly wrapped. These results are robust; a Monte Carlo sampling of our parameter space shows that nearly all allowed models have these characteristics.

5. Stability of Filamentary Molecular Clouds

We consider the stability of filamentary molecular clouds against axisymmetric fragmentation into cores. By solving the linearized equations of MHD and self-gravity, we determine how pressure truncation and the individual magnetic field components affect the stability of our models. We find that pressure truncation has a stabilizing effect, which results from the decreased mass per unit length of the filament. Both field components also stabilize clouds against gravitational fragmentation. However, we find that sufficiently strong toroidal fields may trigger MHD-driven instabilities. We refer the reader to Fiege & Pudritz (1999b), where we fully explore the stability of our models.
6. Discussion

We have found indirect evidence, based on a very general virial analysis, suggesting that most filamentary molecular clouds may be wrapped by helical magnetic fields. We have also constructed numerical MHD models that agree with the observations. These models differ from previous models in that they have much shallower density profiles, which are in good agreement with the available observational results; we find that the density falls off as $r^{-1.8}$ to $r^{-2}$, compared with the $r^{-4}$ behaviour of previous models. This behaviour is entirely due to the toroidal character of the magnetic field in the outer regions. We also find that models with purely poloidal fields have steep density gradients that are not allowed by the observations.

References

Alves J., Lada C.J., Lada E.A., Kenyon S.J., Phelps R., 1998, Ap.J., 506, 292
Bally J., 1989, in Proceedings of the ESO Workshop on Low Mass Star Formation and Pre-main Sequence Objects, ed. Bo Reipurth (Garching:European Southern Observatory), p.1
Boulares A., Cox D.P., 1990, ApJ, 365, 544
Chromey F.R., Elmegreen, B.G., Elmegreen, D.M., 1989, ApJ, 98, 2203
Fiege, J.D., & Pudritz, R.E., 1999a, MNRAS, astro-ph/9901096
Fiege, J.D., & Pudritz, R.E., 1999b, MNRAS, astro-ph/9902385
Hanawa T., et al., 1993, ApJ, 404, L83
Heiles C., 1987, Ap.J., 315, 555
Lada C.J., Alves J., Lada E.A., 1998, Ap.J. in Press
McKee C.F., Zweibel E.G., Goodman A.A., Heiles C., 1993, in Protostars and Planets III, ed. Levy E.H. & Lunine J.I. (Tucson:University of Arizona Press), 327
Myers P.C., Goodman A.A., 1988a, ApJ, 326, L27
Myers P.C., Goodman A.A., 1988b, ApJ, 329, 392
Nakamura F., Hanawa T., Nakano T., 1993, PASJ, 45, 551
Ostriker J., 1964, ApJ, 140, 1056
Pudritz R.E., Fiege J.D., 1999, in Proceedings of the Naramata Workshop on the Interstellar Medium