Nonlinear-supersymmetric General Relativity Theory I

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Abstract

The geometrical argument of the general relativity principle of Einstein is formulated in unstable Riemann space-time just inspired by the nonlinear representation of supersymmetry, which produces new Einstein-Hilbert type action. They show a new paradigm for the supersymmetric unification of space-time and matter, which gives new insight into the unsolved problems of particle physics and cosmology.

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1 Introduction

The symmetry and its spontaneous breaking are key notions for describing the rationale of being of nature. Supersymmetry (SUSY)\cite{1,2} related naturally to space-time symmetry is promising for the unification of general relativity and the low energy SM in one irreducible representation of the symmetry group. Therefore, the evidences of SUSY and its spontaneous breakdown \cite{3,4,5} should be studied not only in (low energy) particle physics but also in cosmology, i.e., in the framework necessarily including graviton. SUGRA is the most promising framework and the particle assignment of SO(8) SUGRA is studied within the existing field theoretical models\cite{6}. And we have found by group theoretical arguments that among all SO\((N)\) super-Poincaré (sP) groups the SO\((10)\) sP group decomposed as \(N = 10 = 5 + 5^*\) under SO\((10) \supset SU(5)\) may be a unique and minimal group which accommodates all observed particles including graviton in a single irreducible representation of \(N = 10\) linear(L) SUSY \cite{7}. In this case 10 supercharges \(Q_I, (I = 1, 2, \cdots, 10)\) are decomposed as follows: 

\[
\begin{align*}
5_{SU(5)} &= 5_{SU(5)} + 5^*_{SU(5)}, \\
5^*_{SU(5)} &= \{Q_a (a = 1, 2, 3) : \frac{2}{3} \alpha^e, \frac{1}{3} \alpha^e; \left(\frac{e}{3}, \frac{e}{3}, \frac{e}{3}\right)\} + \{Q_m (m = 4, 5) : \frac{1}{3} \alpha^e, \frac{2}{3} \alpha^e (-e, 0)\}.
\end{align*}
\]

(1)

The quintet of the supercharge \(5_{SU(5)}\) have the same quantum numbers as \(5\) of SU\((5)\) GUT, i.e., \([\bar{d}\text{-type supercharge} \; Q_a, \; (e., \nu_e)\text{-type supercharges} \; Q_m]\), Applying the representation theory of sP algebra the massless helicity state \(|h\rangle\) of the gravity supermultiplet of SO\((10)\) sP is specified by \(|h\rangle = Q^n Q^{n-1} \cdots Q^2 Q^1|2\rangle, \ Q^n (n = 0, 1, 2, \cdots, 10)\), for the helicity \(h = (2 - \frac{n}{2})\). Interestingly, \(Q^n Q^{n-1} \cdots Q^2 Q^1, n = 12, \cdots, n\) is the all possible non-trivial combination(product) of the supercharge with the dimension \(d_{[n]} = \frac{10!}{n!(10-n)!}\) (with CPT conjugation) and produces the following massless gravity supermultiplet.

| \(|h|\) | \(\frac{3}{2}\) | \(\frac{1}{2}\) | \(1\) | \(\frac{1}{2}\) | \(0\) |
|-----|-----|-----|-----|-----|-----|
| \(d_{[n]}\) | 1\([10]\) | 10\([9]\) | 45\([8]\) | 120\([7]\) | 210\([6]\) |
| 2 | 1\([0]\) | 10\([1]\) | 45\([2]\) | 210\([5]\) | 252\([4]\) |
| 3 | 2 | 120\([3]\) | 210\([4]\) |  |

In order to extract any possible low energy physical contents from the tower of the helicity states we assume a maximal \(SU(3) \times SU(2) \times U(1)\) invariant superHiggs-like mass generation mechanism among helicity states, i.e., all high helicity states redundant for SM become massive by absorbing lower helicity states as the longitudinal components in SM invariant way. Many lower helicity states disappear
from the physical degrees of freedom. The results are interesting.

In the fermionic sector, just three generations of quark and lepton states and some exotic states survive as shown in the following table.

| SU(3) | Q_e | SU(2) ⊗ U(1) |
|-------|-----|--------------|
| 1     | 0   | (ν_e) (ν_μ) (ν_τ) (E) |
|       | -1  | (e) (μ) (τ) |
|       | -2  | (ν_e) (ν_μ) (ν_τ) |
| 3     | 5/3 | (u) (c) (t) (a) (g) (r) |
|       | 2/3 | (d) (s) (b) (f) |
|       | -1/3| (h) (o) |
|       | -4/3| |
| 6     | 4/3 | (P) (X) |
|       | 1/3 | (Q) (Y) |
|       | -2/3| (R) (Z) |
| 8     | 0   | (N_1) (N_2) |
|       | -1  | (E_1) (E_2) |

The list of spin $\frac{1}{2}$ survivors after superHiggs-like mechanism are shown tentatively as Dirac particles in the table.

In the bosonic sector, gauge fields of SM in vector states and one Higgs scalar field of SM survive. Besides those observed states, one color-singlet neutral massive vector state $S$ and one color-singlet double-charge massive spin $\frac{1}{2}$ state $E^{±2}$ are predicted, which can be tested in the high energy (cosmic ray) experiment. The simple extension of the model [7] to larger $N > 10$ produces massless charged high spin states, ugly generation structures, etc. and is excluded.

We will show in the next section that no-go theorem for constructing non-trivial $SO(N > 8)$susy theory including gravity can be circumvented by adopting the nonliner (NL) representation of SUSY [12], i.e. by introducing the degeneracy of space-time through NLSUSY degrees of freedom.
2 Nonlinear-Supersymmetric General Relativity (NLSUSYGR)

For simplicity we discuss $N = 1$ without the loss of the generality. The extension to $N > 1$ is straightforward. The fundamental action nonlinear supersymmetric general relativity theory (NLSUSYGR) has been constructed by extending the geometric arguments of Einstein general relativity (EGR) on Riemann space-time to new space-time inspired by NLSUSY [8, 15]. The tangent space of new space-time is specified not only by the Minkowski coordinate $x_a$ for $SO(1,3)$ but also by the Grassmann coordinate $\psi_\alpha$ for $SL(2,C)$ related to NLSUSY [8, 9]. They are coordinates of the coset space $\frac{superGL(4,R)}{GL(4,R)}$ and can be interpreted as NG fermions associated with the spontaneous breaking of $superGL(4,R)$ down to $GL(4,R)$. (The noncompact isomorphic groups $SO(1,3)$ and $SL(2,C)$ for tangent space-time symmetry on curved space-time can be regarded as the generalization of the compact isomorphic groups $SU(2)$ and $SO(3)$ for the gauge symmetry of ’t Hooft-Polyakov monopole on flat space-time.) The NLSUSYGR action [9, 8] is given by

$$L_{NLSUSYGR}(w) = -\frac{c^4}{16\pi G} |w| \{ \Omega(w) + \Lambda \},$$

where $G$ is the Newton gravitational constant, $\Lambda$ is a (small) cosmological term and $\kappa$ is an arbitrary constant of NLSUSY with the dimension (mass)$^{-2}$. $w^a_\mu(x)$ = $e^a_\mu + t^a_\mu(\psi)$ and $w^\mu_a = e^\mu_a - t^\mu_a + t^\rho_\sigma t^\rho_\mu a + t^\rho_\sigma t^\rho_\mu a + t^\sigma_\rho t^\sigma_\mu a + t^\sigma_\rho t^\sigma_\mu a$ which terminate at $O(t^4)$ for $N = 1$ are the invertible unified vierbeins of new space-time. $e^a_\mu$ is the ordinary vierbein of EGR for the $SO(1,3)$ and $t^a_\mu(\psi)$ is the mimic vierbein analogue (actually the stress-energy-momentum tensor) of NG fermion $\psi(x)$ for the $SL(2,C)$. (We call $\psi(x)$ superon as the hypothetical fundamental spin $1/2$ particle quantized canonically in compatible with the sP algebra[15].) $\Omega(w)$ is the the unified Ricci scalar curvature of new space-time computed in terms of the unified vierbein $w^a_\mu(x)$. Interestingly Grassmann degrees of freedom induce the imaginary part of the unified vierbein $w^a_\mu(x)$, which represents straightforwardly the fermionic matter contribution. Note that $e^a_\mu$ and $t^a_\mu(\psi)$ contribute equally to the curvature of space-time, which may be regarded as the Mach’s principle in ultimate space-time. (The second index of mimic vierbein $t$, e.g. $\mu$ of $t^a_\mu$, means the derivative $\partial_\mu$.) $s^a_\mu \equiv w^a_\mu \eta_{ab} w^b_\nu$ and $s^{\mu\nu}(x) \equiv w^a_\mu(x) w^a_\nu(x)$ are unified metric tensors of new spacetime.
NLSUSY GR action (2) possesses promising large symmetries isomorphic to \( SO(N) \) (\( SO(10) \)) SP group \([13, 16]\), namely, \( L_{\text{NLSUSYGR}(w)} \) is invariant under

\[
\text{[new NLSUSY]} \otimes \text{[local GL(4,R)]} \otimes \text{[local Lorentz]} \tag{4}
\]

for space-time symmetries and

\[
\text{[globalSO(N)]} \otimes \text{[localU(1)^N]} \tag{5}
\]

for internal symmetries in case of \( N \) superons \( \psi^i, i = 1, 2, \cdots, N \).

For example, \( L_{\text{NLSUSYGR}(w)} \) (3) is invariant under the following NLSUSY transformations:

\[
\delta^N_{\text{L}} \psi = \frac{1}{\kappa^2} \zeta + i\kappa^2 (\bar{\zeta} \gamma^\rho \psi) \partial_\rho \psi, \quad \delta^N_{\text{L}} e_a^\mu = i\kappa^2 (\bar{\zeta} \gamma^\rho \psi) \partial_\rho e_a^\mu, \tag{6}
\]

where \( \zeta \) is a constant spinor parameter and \( \partial_\rho e_a^\mu = \partial_\mu e_a^\rho - \partial_\mu e_a^\rho \), which induce the following GL(4,R) transformations on the unified vierbein \( w_a^\mu \)

\[
\delta_\zeta w_a^\mu = \xi^\nu \partial_\nu w_a^\mu + \partial_\mu \xi^\nu w_a^\nu, \quad \delta_\zeta s_{\mu\nu} = \xi^\kappa \partial_\kappa s_{\mu\nu} + \partial_\mu \xi^\kappa s_{\kappa\nu} + \partial_\nu \xi^\kappa s_{\mu\kappa}, \tag{7}
\]

where \( \xi^\rho = i\kappa^2 (\bar{\zeta} \gamma^\rho \psi) \).

The commutators of two new NLSUSY transformations (6) on \( \psi \) and \( e_a^\mu \) are GL(4,R);

\[
[\delta_{\zeta_1}, \delta_{\zeta_2}] \psi = \Xi^\rho \partial_\rho \psi, \quad [\delta_{\zeta_1}, \delta_{\zeta_2}] e_a^\mu = \Xi^\rho \partial_\rho e_a^\mu + e_a^\rho \partial_\mu \Xi^\rho, \tag{8}
\]

where \( \Xi^\mu = 2i\kappa (\bar{\zeta}_2 \gamma^\mu \zeta_1) - \xi_1^a \xi_2^a (\partial_\rho e_a^\sigma) \). The algebra closes. The ordinary local GL(4,R) invariance is trivial by the construction.

Also NLSUSYGR is invariant under the following local Lorentz transformation: on \( w_a^\mu \)

\[
\delta_L w_a^\mu = e_b^a w_b^\mu \tag{9}
\]

or equivalently on \( \psi \) and \( e_a^\mu \)

\[
\delta_L \psi = -\frac{i}{2} \epsilon_{ab} \sigma^{ab} \psi, \quad \delta_L e_a^\mu = e_a^b e_b^\mu + \frac{\kappa^4}{4} \epsilon^{abcd} \bar{\psi} \gamma_5 \gamma_d \psi (\partial_\mu \epsilon_{bc}), \tag{10}
\]

with the local parameter \( \epsilon_{ab} = (1/2) \epsilon_{[ab]}(x) \). The local Lorentz transformation forms a closed algebra, for example, on \( e_a^\mu \)

\[
[\delta_{L_1}, \delta_{L_2}] e_a^\mu = \beta_{ab} e_b^\mu + \frac{\kappa^4}{4} \epsilon^{abcd} \bar{\psi} \gamma_5 \gamma_d \psi (\partial_\mu \beta_{bc}), \tag{11}
\]

where \( \beta_{ab} = -\beta_{ba} \) is defined by \( \beta_{ab} = \epsilon_{2ac} \epsilon_1^b - \epsilon_{2ac} \epsilon_1^c \).

Note that the no-go theorem is overcome (circumvented) in a sense that the nontrivial \( N(N > 8) \)-extended SUSY theory with the gravitational interaction has been constructed in the global NLSUSY invariant way.
3 Big Collapse of space-time

New (empty) space-time described by NLSUSYGR action $L_{NLSUSYGR}(w)[9, 8]$ of the EH-type equipping cosmological constant is unstable due to NLSUSY structure of tangent space-time and would collapse (called Big Collapse [16]) spontaneously to ordinary Riemann space-time with the cosmological constant and fermionic matter superon (called superon-graviton model (SGM)). Note that Big Collapse induces instanstantaneously the rapid expansion of three dimensional space-time due to the Pauli exclusion principle of NG fermion superon. SGM action is the following;

$$L_{SGM}(e, \psi) = -\frac{c^4}{16\pi G}e|w| - \frac{c^4}{16\pi G}e|w|R(e) + \frac{c^4}{16\pi G}e|w||2t^{(\mu\nu)}R_{\mu\nu}(e)$$

$$+ \frac{1}{2}\left\{g^{\mu\nu}\partial^\rho \partial^\sigma t_{(\mu\nu)} - t_{(\mu\nu)}\partial^\rho g^{\mu\nu} + g^{\mu\nu}\partial^\rho t_{(\mu\sigma)} \partial^\sigma g_{\rho\nu} - 2g^{\mu\nu}\partial^\rho t_{(\mu\nu)} \partial^\sigma g_{\rho\sigma} + \ldots\right\}$$

$$+ \{t^{(\mu\rho)}t^{(\nu\sigma)}R_{\mu\nu\rho\sigma}(e) - \{2t^{(\mu\rho)}t^{(\nu\rho)}R_{\mu\nu\rho\rho} + t^{(\mu\rho)}t^{(\nu\sigma)}R_{\mu\nu\rho\sigma}(e) + \frac{1}{2}t^{(\mu\rho)}(g^{\sigma\rho}\partial_\sigma \partial_\rho t_{(\mu\sigma)} = g^{\sigma\rho}\partial_\sigma \partial_\rho t_{(\mu\sigma)} + \ldots)\} + \{O(t^5)\} + \ldots\}, \quad (12)$$

where $|w| = deta^a_b = det(\delta^a_b + t^a_b), e = dete_\mu, t^{(\mu\nu)} = t^{\mu\nu} + t^{\nu\mu}, t_{(\mu\nu)} = t_{\mu\nu} + t_{\nu\mu}$ and $(\psi)^5 \equiv 0$ (for $N = 1$). $|w|$ is the flat space NLSUSY action of $VA[2]$ containing up to $O(t^4)$ and $R(e), R_{\mu\nu}(e)$, and $R_{\mu\nu\rho\sigma}(e)$ are the familiar curvature tensors of Riemann space of GR. Remarkably the first term should reduces to NLSUSY action [2] in Riemann-flat $e^a(x) \rightarrow \delta^a_\mu$ space-time, i.e. the arbitrary constant $\kappa$ of NLSUSY is fixed to

$$\kappa^{-2} = \frac{c^4}{8\pi G} \Lambda. \quad (13)$$

$L_{SGM}(e, \psi) (12)$ can be recasted formally as the following familiar form

$$L_{SGM}(e, \psi) = -\frac{c^4}{16\pi G}|e|\{R(e) + \Lambda + \bar{T}(e, \psi)\}, \quad (14)$$

where $R(e)$ is the Ricci scalar curvature of ordinary EH action and $\bar{T}(e, \psi)$ represents the kinetic term and the gravitational interaction of superons.

We have shown qualitatively that NLSUSYGR/SGM may describe a new paradigm for the SUSY unification of space-time and matter. Considering that the graviton is the universal attractive force constituting all possible nontrivial composites(combinations) of superons, which is equivalent to the all possible products of supercharges used in constructing the massless helicity representation of sP group, The vacuum configuration of $L_{NLSUSYGR}(w) = L_{SGM}(e, \psi)$ may be achieved by producing gravitational composites of superons as the eigen states($LSUSYSupermultiplet$).
of sP space-time symmetry. That is, there may be a possibility that all (observed) low energy particles may be gravitational eigenstates of $SO(N)$ sP expressed uniquely as the SUSY composites of $N$ superons. We study explicitly these possibilities in the next section.

4 Linearizing NLSUSY and vacuum of SGM

4.1 Linearization of NLSUSY

The relation between the global LSUSY representation and the global NLSUSY one in flat space-time is studied in detail [18, 19, 20]. They have shown for $N = 1$ SUSY in flat space-time that NSUINSY can be linearized, i.e., the LSUSY transformation of a specific supermultiplet can be reproduced in terms of NLSUSY transformation of the NG field and the (equivalent) relation of the two actions is shown. We anticipate that $L_{SGM}(e, \psi)$ is linearized as well. Unfortunately due to the high nonlinearity of SGM action, the linearization of SGM action and extracting the (low energy) physical meaning of SGM directly on curved Riemann space-time is yet to be done.

However, considering that the SGM action in Riemann-flat ($e^a_{\mu} \rightarrow \delta^a_{\mu}$) space-time reduces essentially to the $N$-extended NLSUSY action with $\kappa^2 = (\frac{G}{8\pi})^{-1}$, it is interesting from the viewpoint of the low energy physics on the tangent flat space-time to linearize the $N$-extended NLSUSY model and find the equivalent(related) $N$-extended LSUSY theory. We show explicitly in two dimensional space-time ($d = 2$) [24, 25] for simplicity that $N = 2$ LSUSYQED is equivalent(related) to $N = 2$ NLSUSY model.

Firstly we perform the heuristic approach of the linearization based on the commutator of SUSY algebra. The heuristic approach is suggested by the following observations and gives the intuitive understandings of the linearization which is formulated on the various Lorentz tensors and $|w| [20, 21]$

The product of Lorentz tensors composed of $\psi^i$ multiplied by $|w|[21]:

$$b^i_{\ A} \ i_{\ B} \ ... \ j_{\ C} \ n (\bar{\psi}^i)^{2(n-1)} |w| = \kappa^{2n-3} \bar{\psi}^i_{\gamma A} \psi^j_{\gamma B} \ ... \ \bar{\psi}^m_{\gamma C} \psi^n |w|, \quad (15)$$

$$f^{ij}_{\ A} \ kl_{\ B} \ ... \ m_{\ C} \ p (\bar{\psi}^i)^{2(n-1)} |w| = \kappa^{2(n-1)} \bar{\psi}^i_{\gamma A} \psi^j_{\gamma B} \ ... \ \bar{\psi}^m_{\gamma C} \psi^n |w|, \quad (16)$$

play basic roles.
The variations under the NLSUSY transformations become
\[
\delta \phi^{i A}_{jk B} \phi^{l \ldots n}_{C} = \kappa^{2(n-1)} \left[ \left\{ \left( \tilde{\zeta}^{i} A \gamma^{j} B \psi^{l} + \bar{\zeta}^{i} A \gamma^{j} B \psi^{l} \right) \bar{\psi}^{n} \gamma^{m} C \psi^{n} + \cdots \right\} |w| 
+ \kappa \partial_{a} \left( \xi^{a} \bar{\psi}^{i} A \gamma^{j} B \psi^{l} \bar{\psi}^{n} \gamma^{m} C \psi^{n} |w| \right) \right], \quad (17)
\]
\[
\delta \phi^{i j A}_{k l B} \phi^{m n \ldots C} = \kappa^{2n-1} \left[ \left\{ \left( \tilde{\zeta}^{i j} A \gamma^{k l} B \psi^{m} \bar{\psi}^{n} \gamma^{m} C \psi^{n} + \cdots \right\} |w| 
+ \psi^{i} \left( \tilde{\zeta}^{i j} A \gamma^{k l} B \psi^{m} \bar{\psi}^{n} \gamma^{m} C \psi^{n} + \cdots \right) \right| w \right] 
+ \kappa \partial_{a} \left( \xi^{a} \psi^{i j} A \gamma^{k l} B \psi^{m} \bar{\psi}^{n} \gamma^{m} C \psi^{n} |w| \right) \right], \quad (18)
\]
where \( \xi^{a} = \kappa \bar{\zeta}^{i} A \gamma^{a} \psi^{i} \).

These show that Lorentz tensors of \( \psi^{i} \) multiplied by \( |w| \)
give finite representation of NLSUSY algebra, because of \( (\psi^{i})^{n} / n \equiv 0, n > 4N \).

They satisfy the commutator
\[
[\delta Q(\zeta_{1}), \delta Q(\zeta_{2})] = \delta P(v), \quad (19)
\]
where \( \delta P(v) \) is a translation with a parameter \( v^{a} = 2i(\bar{\zeta}^{i}_{1 L} \gamma^{a} \zeta^{i}_{2 L} - \bar{\zeta}^{i}_{1 R} \gamma^{a} \zeta^{i}_{2 R}) \).

These results show that the commutator-based linearization closes on the all possible Lorentz tensors composed of \( \psi^{i} \) and gives a finite dimensional representation of sP algebra.

In the heuristic commutator based linearization we consider two steps:
(i) SUSY compositeness based on LSUSY algebra: Find composite LSUSY supermultiplet, i.e. every component field including the auxiliary field of LSUSY supermultiplet should be expressed as the Lorentz tensors composed of the products of the NLSUSY NG fermion \( \psi \) and simultaneously the familiar LSUSY transformation on the supermultiplet should be reproduced(satisfied) in terms of the composite supermultiplet under the NLSUSY transformations of the constituent NG fermion \( \psi \).
(ii) NL/L SUSY relation(equivalence): Show LSUSY action \( L_{LSUSY} \) reduces to NL-SUSY action \( L_{NLSUSY} \) when SUSY compositeness is substituted into LSUSY field of \( L_{LSUSY} \).

Now we consider explicitly the vacuum structure of \( N = 2 \) LSUSY QED in the SGM scenario in \( d = 2 \) [25], which enables to study the vacuum structure of SGM scenario. (Note that the minimal realistic SUSY QED in SGM composite scenario is given by \( N = 2 \) SUSY [23].) \( N = 2 \) NLSUSY action for two superons (NG fermions) \( \psi^{i} (i = 1, 2) \) in \( d = 2 \) is written as follows,
\[
L_{N=2NLSUSY}
\]
\[= -\frac{1}{2\kappa^2} |w| \]
\[= -\frac{1}{2\kappa^2} \left\{ 1 + t^a_a + \frac{1}{2!} (t^a_a t^b_b - t^a_b t^b_a) \right\} \]
\[= -\frac{1}{2\kappa^2} \left\{ 1 - \frac{i}{2} \frac{\kappa}{\bar{\psi}} \psi^i \frac{\partial}{\partial \psi^i} - \frac{1}{2} \kappa^4 \left( \bar{\psi}^i \frac{\partial}{\partial \psi^i} \psi^j - \bar{\psi}^j \frac{\partial}{\partial \psi^j} \psi^i \right) \right\} \]
(20)

where \(\kappa\) is a constant whose dimension is \((\text{mass})^{-1}\) and \(|w| = \det(w^a_b) = \det(\delta^a_b + t^a_b)\), \(t^a_b = -i \kappa^2 \bar{\psi}^i \gamma^a \partial_b \psi^i\). While, the helicity states of \(N = 2\) LSUSY QED are the vector supermultiplet containing \(U(1)\) gauge field

\[
\begin{pmatrix}
+1 \\
\frac{1}{2}, \frac{1}{2} \\
0 \\
0
\end{pmatrix} + \text{[CPT conjugate]},
\]

and the scalar supermultiplet for matter fields

\[
\begin{pmatrix}
+\frac{1}{2} \\
0, 0 \\
-\frac{1}{2}
\end{pmatrix} + \text{[CPT conjugate]}.
\]

The most general \(N = 2\) LSUSY QED action in the Wess-Zumino gauge for the massless case in \(d = 2\), is written as follows[25],

\[
L_{N=2\text{SUSYQED}} = -\frac{1}{4} (F_{ab})^2 + \frac{i}{2} \bar{\lambda}^i \frac{\partial}{\partial \lambda^i} + \frac{1}{2} (\partial_a A)^2 + \frac{1}{2} (\partial_a \phi)^2 + \frac{1}{2} D^2 \\
-\frac{1}{\kappa} \chi \frac{\partial}{\partial \chi} + \frac{i}{2} \chi \frac{\partial}{\partial \phi} D^2 + \frac{i}{2} \bar{\nu} \frac{\partial}{\partial \nu} + \frac{1}{2} (F^i)^2 \\
+ f(A \bar{\lambda}^i \lambda^i + \epsilon^{ij} \phi \bar{\lambda}^i \gamma_5 \lambda^j - A^2 D + \phi^2 D + \epsilon^{ab} A \phi F_{ab}) \\
+ e \left\{ i \nu_a \bar{\chi} \gamma^a \nu - \epsilon^{ij} \nu^a B^i \partial_a B^j + \bar{\lambda}^i \chi B^j + \epsilon^{ij} \bar{\lambda}^i \nu B^j \\
- \frac{1}{2} D (B^i)^2 + \frac{1}{2} (\bar{\chi} \chi + \bar{\nu} \nu) A - \bar{\chi} \gamma_5 \nu \phi \right\} \]
\[+ \frac{1}{2} e^2 (v^2_a - A^2 - \phi^2) (B^i)^2, \]
(21)

where \((v^a, \lambda^i, A, \phi, D) \ (F_{ab} = \partial_a v_b - \partial_b v_a)\) is the \textit{minimal} off-shell vector supermultiplet containing \(v^a\) for a \(U(1)\) vector field, \(\lambda^i\) for doublet (Majorana) fermions, \(A\) for a scalar field in addition to \(\phi\) for another scalar field and \(D\) for an auxiliary scalar field, while \((\chi, B^i, \nu, F^i)\) is the \textit{minimal} off-shell scalar supermultiplet containing \((\chi, \nu)\) for two (Majorana) fermions, \(B^i\) for doublet scalar fields and \(F^i\) for auxiliary
scalar fields. The linear term of $F$ is forbidden by the gauge invariance\cite{25}. Also $\xi$ is an arbitrary dimensionless parameter giving a magnitude of SUSY breaking mass, and $f$ and $e$ are Yukawa and gauge coupling constants with the dimension (mass)$^1$, respectively.

$N = 2$ LSUSY QED action (21) is invariant under the following LSUSY transformations parametrized by $\zeta^i$,

$$
\delta \zeta v^a = -ie^{ij} \bar{\zeta}^i \gamma^a \lambda^j, \quad \delta \zeta \lambda^i = (D - i\phi A) \zeta^i + \frac{1}{2}e^{ab}e^{ij} F_{ab} \gamma_5 \zeta^j - ie^{ij} \gamma_5 \phi \zeta^j, \quad (22)
$$

$$
\delta \zeta A = \bar{\zeta}^i \lambda^i, \quad \delta \zeta \phi = -e^{ij} \bar{\zeta}^i \gamma_5 \lambda^j, \quad \delta \zeta D = -i\bar{\zeta}^i \phi \lambda^j.
$$

for the vector multiplet and

$$
\delta \zeta \chi = (F^i - i\bar{\zeta} B^i) \zeta^i - e^{ij} V^i B^j, \quad \delta \zeta \nu = e^{ij} (F^i + i\phi B^i) \zeta^j + e V^i B^i, \quad (23)
$$

$$
\delta \zeta B^i = \bar{\zeta}^i \chi - e^{ij} \bar{\zeta}^j \nu,
$$

$$
\delta \zeta F^i = -i\bar{\zeta}^i \phi \chi - e^{ij} \bar{\zeta}^j \phi \nu - e \{e^{ij} V^j \chi - \bar{V}^i \nu + (\bar{\zeta}^i \lambda^j + \bar{\zeta}^j \lambda^i) B^j - \bar{\zeta}^i \lambda^j B^i\}
$$

with $V^i = iv_a \gamma^a \zeta^i - e^{ij} A \zeta^j - \phi \gamma_5 \zeta^i$ for the scalar multiplet.

For extracting the low energy physical contents of $N = 2$ SGM (NLSUSY GR) we consider SGM in asymptotic Riemann-flat space-time, where $N = 2$ SGM reduces to essentially $N = 2$ NLSUSY action. We will show the relation(equivalence) of $N = 2$ NLSUSY action and $N = 2$ SUSYQED action(called $NL/L$ SUSY relation.) :

$$
L_{N=2SGM} \longrightarrow (e^a_\mu \rightarrow \delta^a_\mu) \longrightarrow L_{N=2NLSUSY} + [surface \ terms] = f_\xi L_{N=2SUSYQED}, \quad (24)
$$

where $f_\xi$ is the function of vacuum values of auxiliary fields. NL/L SUSY relation indicates the equivalence(relation) of two theories irrespective of the renormalizability. NL/L SUSY relation is shown explicitly by substituting the following SUSY compositeness condition\cite{25} into the LSUSY QED theory.

Now we find the SUSY compositeness of LSUSY gauge multiplet as follows.

Starting tentatively from natural and simple SUSY compositeness for $v^a, A, \lambda$ as $v^a = \xi ke^{ij} \bar{\psi}^i \gamma^a \psi^j |w\rangle, \quad A = \frac{1}{2} \xi k \bar{\psi}^i \psi^j |w\rangle, \quad \lambda^i = \xi \bar{\psi}^i |w\rangle$ and then operating NL-SUSY transformation on the constituents $\psi$, the results contains various Lorentz tensors composed of $\psi$. Comparing these results with the familiar LSUSY transformation of the gauge supermultiplet including the auxiliary field, we get the SUSY compositeness.
The SUSY compositeness condition for the minimal vector supermultiplet \((v^a, \lambda^i, A, \phi, D)\) in the Wess-Zumino gauge\([26]\) are

\[
v^a = -\frac{i}{2} \xi \kappa \epsilon^{ij} \bar{\psi}^i \gamma^a \psi^j |w|,
\]

\[
\lambda^i = \xi \left[ \psi^i |w| - \frac{i}{2} \kappa^2 \partial_a \{ \gamma^a \psi^i \bar{\psi}^j \psi^j (1 - i \kappa^2 \bar{\psi}^k \bar{\phi} \psi^k) |w| \} \right],
\]

\[
A = \frac{1}{2} \xi \kappa \bar{\psi}^i \psi^i |w|,
\]

\[
\phi = -\frac{1}{2} \xi \kappa \epsilon^{ij} \bar{\psi}^i \gamma^5 \psi^j |w|,
\]

\[
D = \frac{\xi}{\kappa} |w| - \frac{1}{8} \xi \kappa^3 \partial^a (\bar{\psi}^i \gamma^a \psi^j \bar{\psi}^j |w|).
\]

(25)

While for the minimal scalar supermultiplet \((\chi, B^i, \nu, F^i)\) the SUSY compositeness condition is

\[
\chi = \xi^i \left[ \psi^i |w| + \frac{i}{2} \kappa^2 \partial_a \{ \gamma^a \psi^i \bar{\psi}^j \psi^j (1 - i \kappa^2 \bar{\psi}^k \bar{\phi} \psi^k) |w| \} \right],
\]

\[
B^i = -\kappa \left( \frac{1}{2} \xi^i \bar{\psi}^j \psi^j - \xi^j \bar{\psi}^j \psi^j \right) |w|,
\]

\[
\nu = \xi^i \epsilon^{ij} \left[ \psi^j |w| + \frac{i}{2} \kappa^2 \partial_a \{ \gamma^a \psi^j \bar{\psi}^k \psi^k (1 - i \kappa^2 \bar{\psi}^l \bar{\phi} \psi^l) |w| \} \right],
\]

\[
F^i = \frac{1}{\kappa} \xi^i \left\{ |w| + \frac{1}{8} \kappa^3 \partial^a (\bar{\psi}^j \gamma^a \psi^j \bar{\psi}^k \psi^k |w|) \right\}
- \frac{3}{4} \epsilon \kappa^2 \xi^i \bar{\psi}^j \psi^j \bar{\psi}^k \psi^k |w|,
\]

(26)

where \(\xi\) and \(\xi^i\) are factors of the vacuum expectation values of auxiliary fields \(D\) and \(F^i\). SUSY compositeness (25) and (26) satisfy (i) and (ii). Furthermore substituting these relations into the \(N = 2\) SUSYQED action (21) we can show directly NL/L SUSY relation (19), i.e., \(N = 2\) SUSYQED action (21) is equivalent (related) to \(N = 2\) NLSUSY action provided

\[
f_\xi(\xi, \xi^i) = \xi^2 - (\xi^i)^2 = 1.
\]

(27)

Note that for the SUSY compositeness (26) of the scalar supermultiplet it is interesting that the four-fermion self-interaction term appearing only in the auxiliary fields \(F^i\) is the origin of the familiar local \(U(1)\) gauge symmetry of LSUSY theory. The introduction of new auxiliary fields in the supermultiplet improves (clarifies) these situations. Especially the total derivative term in SUSY compositeness (25) and (26) disappears by introducing a new auxiliary fields composed of fermion self-interactions. Note that in the straightforward linearization the commutator algebra
does not contain U(1) gauge transformation even for the vector U(1) gauge field[23]. NL/L SUSY relation of SGM scenario for the larger supermultiplet predicts the magnitude of the bare gauge coupling constant as shown below. These situations are easily seen by using the superfield formulation of the linearization of NLSUSY[27].

Now we discuss the linearization by the superfield formalism[25]. We adopt for the $N=2$ general vector supermultiplet of the general superfield

$$V(x, \theta^i) = C(x) + \bar{\theta}^i \lambda^i(x) + \frac{1}{2} \bar{\theta}^i \theta^j M^{ij}(x) - \frac{1}{2} \bar{\theta}^i \bar{\theta}^j M^{jj}(x) + \frac{1}{4} \epsilon^{ij} \bar{\theta}^i \gamma_5 \theta^j \phi(x) - \frac{i}{4} \epsilon^{ij} \bar{\theta}^i \gamma_a \theta^j \nu^a(x) - \frac{1}{2} \bar{\theta}^i \bar{\theta}^j \lambda^j(x) - \frac{1}{8} \bar{\theta}^i \bar{\theta}^j \bar{\theta}^j D(x),$$

and for the $N=2$ scalar supermultiplet

$$\Phi^i(x, \theta^i) = B^i(x) + \bar{\theta}^i \chi^i(x) - \epsilon^{ij} \bar{\theta}^i \nu^j(x) - \frac{1}{2} \bar{\theta}^i \theta^j F^j(x) + \bar{\theta}^i \bar{\theta}^j F^j(x) - i \bar{\theta}^i \bar{\theta} \partial B^i(x) \theta^i + \frac{i}{2} \bar{\theta}^i \theta^j (\bar{\theta}^j \phi(x) - \epsilon^{jk} \theta^k \phi^i(x)) + \frac{1}{8} \bar{\theta}^i \bar{\theta}^j \bar{\theta}^k \theta^j \phi^i \partial^a B^i(x),$$

The N=2 SUSYQED action and its NL/L SUSY relation by the superfield is expressed as:

$$S_{N=2NLSUSY} = S_{\text{free}} + S_{\text{Y}} + S_{\text{gauge}} = f_\xi (\xi, \xi^i, \xi^c) S_{N=2NLSUSY}. \quad (30)$$

$S_{\text{free}}$ is the kinetic (free) action with the Fayet-Iliopoulos (FI) $D$ term for the $N=2$ vector supermultiplet $V$

$$S_{\text{free}} = \int d^2x \left\{ \int d^2 \theta^i \left[ \frac{1}{32} (\bar{D}^i \bar{W}^{jk}_5 D^i W^{jk}_5 + \bar{D}^i \bar{W}^{jk}_5 D^j W^{jk}_5) + \int d^4 \theta^i \frac{\xi}{2\kappa} V \right] \right\}_{\theta^i = 0} \quad (31)$$

where

$$W^{ij} = \bar{D}^i D^j V, \quad W^{ij}_5 = \bar{D}^i \gamma_5 D^j V. \quad (32)$$

The FI $D$ term gives the correct sign of the NLSUSY action.

Yukawa interaction terms for $V$ is

$$S_{\text{Y}} = \frac{1}{8} \int d^2 x f \left[ \int d^2 \theta^i W^{jk} (W^{ji} W^{kl}_5 + W^{kl}_5 W^{ij}_5) + \int d^2 \theta^i d \theta^j 2 \{ W^{ij} (W^{kl} W^{kl}_5 + W^{kl}_5 W^{kl}_5) + W^{jk} (W^{jl} W^{kl}_5 + W^{kl}_5 W^{jl}_5) \} \right]_{\theta^i = 0} \quad (33)$$

By means of cancelations among four NG-fermion self-interaction terms General mass terms for $\tilde{V}$ vanishes and $\tilde{\Phi}$ as well.
The most general $U(1)$ gauge invariant action for $\Phi^i$ coupled with $V$

$$S_{\text{gauge}} = -\frac{1}{16} \int d^2x \int d^4\theta^i e^{-4eV(\Phi^i)}.$$  \hspace{1cm} (34)

Now we consider the following $\psi^i$-dependent superspace supertranslations with $-\kappa \psi(x)$,

$$x'^a = x^a + i\kappa \bar{\theta}^i \gamma^a \psi^i, \quad \theta'^i = \theta^i - \kappa \psi^i,$$

and denote the resulting superfields on $(x'^a, \theta')$ and their $\theta$-expansions as

$$\mathcal{V}(x'^a, \theta') = \tilde{\mathcal{V}}(x^a, \theta^i(x)), \quad \Phi(x'^a, \theta') = \tilde{\Phi}(x^a, \theta^i; \psi^i(x)).$$  \hspace{1cm} (35)

and consider the supertranslation of $x', \theta'$, we obtain

$$\delta \tilde{\mathcal{V}}(x^a, \theta^i; \psi^i(x)) = \xi_{\mu} \partial^\mu \tilde{\mathcal{V}}(x^a, \theta^i; \psi^i(x)), \quad \delta \tilde{\Phi}(x^a, \theta^i; \psi^i(x)) = \xi_{\mu} \partial^\mu \tilde{\Phi}(x^a, \theta^i; \psi^i(x)),$$

where $\xi_{\mu} = -i\bar{\psi}^i \gamma_{\mu} \psi^i$. Therefore, we obtain the following generalized global SUSY invariant constraints for (the component fields in) $\tilde{\mathcal{V}}$ and $\tilde{\Phi}$,

$$\tilde{\phi}^I_V(x) = \xi_{V}^I(\text{constant}) \quad \tilde{\phi}^I_{\Phi}(x) = \xi_{\Phi}^I(\text{constant}).$$  \hspace{1cm} (38)

Solving these equations with respect to the initial components of supermultiplet we obtain the SUSY compositeness conditions in terms of $\tilde{\phi}^I_V(x)$, $\tilde{\phi}^I_{\Phi}(x)$ and $\psi^i(x)$.

Now we impose the constraints from the physical viewpoint, i.e. the Lorentz invariance of the vacuum expectation value of auxiliary scalar field, the following simple SUSY invariant constraint is the minimal one:

$$C = \tilde{M}^i = \tilde{\phi} = \tilde{\nu}^a = \tilde{\lambda}^i = 0, \quad D = \frac{\xi}{\kappa}, \quad B^i = \tilde{\lambda} = \tilde{\nu} = 0, \quad F^i = \frac{\xi^i}{\kappa},$$

which produces the previous simple SUSY compositeness condition (25) and (26) and NL/L SUSY relation (30) with (27). By changing the integration variables $(x^a, \theta^i) \rightarrow (x'^a, \theta'^i)$, NL/L SUSY relation can be confirmed straightforwardly.

More general and physical SUSY invariant constraint corresponding to vev of general scalar auxiliary fields:

$$C = \xi_c, \quad \tilde{M}^i = \tilde{\phi} = \tilde{\nu}^a = \tilde{\lambda}^i = 0, \quad D = \frac{\xi}{\kappa}, \quad B^i = \tilde{\lambda} = \tilde{\nu} = 0, \quad F^i = \frac{\xi^i}{\kappa}.$$  \hspace{1cm} (39)

gives SUSY compositeness condition for larger supermultiplet and produces for NL/L SUSY relation[27]

$$f_{\xi}(\xi, \xi^i, \xi_c) = \xi^2 - (\xi^i)^2 e^{-4\xi_c} = 1, \quad i.e., \quad e = \frac{\ln(\frac{\xi^i}{\xi})}{4\xi_c}.$$  \hspace{1cm} (41)

Remarkably the bare gauge coupling constant $e$ is determined in terms of the the vev of the scalar fields. This mechanism seems natural and favorable from the viewpoint of theory of everything.
4.2 Vacuum structure of NLSUSY

NL/L SUSY relation of N=2 SUSY means that the physical configuration of the vacuum of NLSUSY theory is given by the the vacuum structure of $N = 2$ SUSY QED action (21) [29]. The vacuum of $N = 2$ SUSY QED action is given by the minimum of the potential $V(A, \phi, B^i, D)$ of $L_{N=2SUSYQED}$.

$$V(A, \phi, B^i, D) = -\frac{1}{2}D^2 + \left\{ \frac{\xi}{\kappa} - \frac{1}{2}e|A|^2 + \frac{1}{2}|B^i|^2 \right\} D + \frac{e^2}{2}(A^2 + \phi^2)|B^i|^2. \quad (42)$$

Substituting the solution of the equation of motion for the auxiliary field $D$ we obtain

$$V(A, \phi, B^i) = \frac{1}{2}f^2 \left\{ A^2 - \phi^2 - \frac{e}{2f}|B^i|^2 - \frac{\xi}{f\kappa} \right\}^2 + \frac{1}{2}e^2(A^2 + \phi^2)|B^i|^2 \geq 0. \quad (43)$$

The configuration of the fields corresponding to the vacua in $(A, \phi, B^i)$-space which are $SO(1,3)$ symmetric classified according to the signatures of the parameters $e, f, \xi, \kappa$.

We obtain two different types of vacua $V = 0$: (I) $A = \phi = 0$, $|\hat{B}^i|^2 = -k^2$ and (II) $|\hat{B}^i|^2 = 0$, $A^2 - \phi^2 = k^2$, $(\hat{B}^i = \sqrt{\frac{f}{k}}B^i, k^2 = \frac{\xi}{f\kappa})$. Parametrizing the potential configuration as for case (I), $A = -(k + \rho)\cos \theta \cos \varphi \cosh \omega$, $\phi = (k + \rho)\sinh \omega$, $\hat{B}^1 = (k + \rho)\sin \theta \cosh \omega$, $\hat{B}^2 = (k + \rho)\cos \theta \cos \varphi \cosh \omega$. and for case (II), $A = (k + \rho)\sin \theta \cosh \omega$, $\phi = (k + \rho)\sin \omega$, $\hat{B}^1 = (k + \rho)\cos \theta \cos \varphi \cosh \omega$, $\hat{B}^2 = (k + \rho)\cos \theta \sin \varphi \cosh \omega$. Expanding fields around the vacuum values of the parameter space and substituting these expressions into $L_{N=2SUSYQED}(A, \phi, B^i)$ we obtain the physical particle configuration around the true vacuum. For example, we have for the type (II) vacuum

$$L_{N=2SUSYQED}$$

$$= \frac{1}{2}\{ (\partial_\mu \hat{A})^2 - 4f^2k^2 \hat{A}^2 \} + \frac{1}{2}\{ |\partial_\mu \hat{B}^i|^2 + |\partial_\mu \hat{B}^2|^2 - e^2k^2(|\hat{B}^1|^2 + |\hat{B}^2|^2) \} + \frac{1}{2}(\partial_\mu \phi)^2$$

$$- \frac{1}{4}(F_{ab})^2 + \frac{1}{2}(i\lambda^i\partial^i - 2f k \lambda^i) + \frac{1}{2}\{ i(\bar{\chi}\partial \chi + \nu \partial \nu) - ek(\bar{\chi}\chi + \bar{\nu}\nu) \} + \cdots. \quad (44)$$

and the following mass spectra of particles(tilded fields below) in the true vacuum:

$$m_{\hat{A}}^2 = m_{\hat{\chi}}^2 = 4f^2k^2 = \frac{4\xi f}{\kappa},$$

$$m_{\hat{B}^1}^2 = m_{\hat{B}^2}^2 = m_{\hat{\chi}}^2 = m_{\hat{\nu}}^2 = e^2k^2 = \frac{\xi e^2}{\kappa f},$$

$$m_{\nu} = m_\phi = 0. \quad (45)$$
with the mass hierarchy by the factor $\frac{e}{f}$. The non-zero vacuum value of the scalar field breaks SUSY and gives mass to the fermion. The resulting model describes: one massive charged Dirac fermion ($\psi^c D_+ \sim \chi + i\nu$), one massive neutral Dirac fermion ($\lambda^0 D_0 \sim \lambda_1 - i\lambda_2$), one massless vector (a photon) ($v_a$), one charged scalar ($B^1 + iB^2$), one neutral complex scalar ($\phi + i\hat{\phi}$), $\phi$ is Goldstone mode in SO(1.3) plane, which are composites of superons.

Apparently the identification of the supersymmetric partners is not manifest in the resulting spectra. (without manifest superpartners). From these arguments we conclude that $N = 2$ SUSY QED is equivalent (related) to $N = 2$ NLSUSY action, i.e., to the matter sector of $N = 2$ SGM produced by Big Collapse of new space-time $N = 2$ NLSUSYGR. And the true vacuum of $N = 2$ NLSUSY model is achieved by the LSUSY model where all particles of the LSUSY supermultiplet are SUSY composites of NG fermions.

By the similar computations for (I) we obtain the vacuum which breaks both SUSY and the local $U(1)$ spontaneously.

### 4.3 SU(5) Superon-quintet model (SQM) of particles

The graviton is the universal attractive force and dictates the evolution of $L_{SGM}$, which creates composite objects of all possible combinations of superons. We have found that the massless representation of sP group is generated by the all possible parallel helicity products of supercharges. And the supercharge reduces to superon in the low energy (current-field identity in the low energy), i.e., the leading term of the supercurrent is the superon field in the low energy.

The canonical quantization of superon is carried out in compatible with the SUSY algebra.

NL/L SUSY equivalence (relation) is shown for $N = 2$ SUSY QED and $N = 3$ SUSY YM, in $d = 2[30]$.

From these considerations we speculate that all (observed) particles are such composite objects of superons as eigenstates of sP group constructed group theoretically by the non-trivial multiplication products of the supercharge. SM, GUT may be regarded as the low energy effective theory of $N = 10$ NL-SUSYGR/SGM, where $N = 10 = 5 + 5^*$ under $SO(10) \supset SU(5)$, which we call...
**superon-quintet model (SQM)** of particles. In order to survey the potential of SQM we adopt tentatively the following naive L-R symmetric assignment for the SM particles:

for \( (e, \nu_e) \): \( \delta^{ab} Q_a Q^*_b Q_m \), \( (\mu, \nu_\mu) \): \( \delta^{ab} Q_a Q^*_b Q_i Q^*_m Q_n \), \( (\tau, \nu_\tau) \): \( \epsilon^{abc} Q_b Q_c Q^*_d Q^*_e Q_m \)

for \( (u, d) \): \( \epsilon^{abc} Q_b Q_c Q^*_d Q^*_e Q_m \), \( (c, s) \): \( \epsilon^{lm} Q_l Q_m Q^*_n \), \( (t, b) \): \( \epsilon^{abc} Q_a Q_b Q_c Q^*_d Q_m \),

for the neutral Higgs particle we choose \( Q_a Q^*_a Q_i Q^*_l \),

for gauge bosons we have

\( Q_a Q^*_a, Q_i Q^*_m, Q_i Q^*_n, Q_a Q^*_l, Q_a Q_m, \cdots \),

\( (a, b, \cdots = 1, 2, 3; l, m, \cdots = 4, 5) \)

By specifying the superon content of SM particles explicitly we interpret the Feynman diagram of the SM/GUT in terms of the composite superon pictures, i.e. the single line of the propagator of the SM particle in the Feynman diagram is replaced by the multiple lines of the constituent superons. We find that most Feynman diagrams of the observed physical process of SM/GUT are reproduced in terms of the composite picture of particles. However the diagrams of the dangerous (no evidence) process in SM/GUT, e.g. FCNC and the proton decay, etc. are not reproduced due to the selection rule for the superon number conservation at the vertex, i.e., the decay mode \( p \to \bar{e} + \nu + \pi^0 \) is forbidden in SQM. The diagram of the dangerous process looks forbidden automatically in the tentative superon-quintet composite model for GUT. As an example of the superon diagram, \( \beta \)-decay diagram of SM is recasted as follows in SQM.

![Feynman Diagram](image-url)
Revisiting SM and GUT from the diagramatic SQM composite viewpoints may give new insights into unsolved problems. As for the assignment of particles, the different assignment \( e^{abc}Q_aQ_bQ_cQ_m \) of the Higgs particle disfavors the left-right symmetric one and needs further study. Finally we just mention that there is no (non-gravitational) excited states of quarks, leptons, gauge bosons, \( \cdots \) in the SGM/SQM picture.

5 Cosmology of NLSUSYGR

5.1 Big Collapse of ultimate space-time(NLSUSYGR)

Now we discuss the cosmological implications of NLSUSYGR. NLSUSYGR space-time described by \( L_{\text{NLSUSYGR}} \) is unstable and spontaneously collapses (Big Collapse) to \( L_{\text{SGM}} \) of familiar Riemann space-time (graviton) and Nambu-Goldstone fermion matter (superon). The Big Collapse may induce the rapid expansion of three dimensional space-time by the Pauli exclusion principle[14]. NLSUSYGR scenario predicts the four dimensional space-time. Because we assume space-time supersymmetry at the local coordinate frame as the origin of ordinary particle supersymmetry, i.e. we ask the isomorphism of \( SO(1,D-1) \) and \( SL(d,C) \):

\[
\frac{D(D-1)}{2} = 2(d^2 - 1),
\]

which holds for only

\[
D = 4, \quad d = 2
\]

and predicts the four dimensional space-time.

The variation of SGM action \( L_{\text{SGM}} \) with respect to \( e^a\mu \) gives the equation of motion for \( e^a\mu \) recasted as follows

\[
R^a_{\mu\nu}(e) - \frac{1}{2} g_{\mu\nu} R(e) = \frac{8\pi G}{c^4} \{ \tilde{T}^a_{\mu\nu}(e,\psi^j) - g_{\mu\nu} \frac{e^4 A}{8\pi G} \},
\]

where \( \tilde{T}^a_{\mu\nu}(e,\psi^i) \) (\( i, j = 1, 2 \) for \( N = 2 \)) abbreviates the stress-energy-momentum of superon(NG fermion) matter including the gravitational interactions. Note that \( -\frac{e^4 A}{8\pi G} \) can be interpreted as the negative energy density of Riemann space-time, i.e. the dark energy density \( \rho_D \). (The negative sign is provided uniquely, which produces simultaneously the correct kinetic term of superons in NLSUSY.) We anticipate
that the graviton is the universal attractive force which dictates the evolution of the superon-graviton world (SGM action) $L_{SGM}$ and constitutes gravitational (massless) composites eigenststes of space-time symmetry $SO(N)$ sP, which ignites Big Bang model of the universe and continues to the SMs. If superon-graviton phase(SGM action(12)) with the cosmological constant of space-time survives(swicthed off) in the evolution after Big Collapsec and oexists after the ignition of Big Bang, such SGM supace-time behaves as the dark side of the universe, i.e. the dark energy inducing the acceleration of the present universe, which is recognized only by the gravitational interaction.

5.2 Cosmology and particle physics

On tangent flat space-time, by the Noether theorem[15] we obtain the conserved supercurrent $S^{k\mu} = i\sqrt{\frac{c^4 \Lambda}{8\pi G}} \gamma^a \psi^k + \cdots, j, k = 1, 2$ associated with the NLSUSY invariance and obtains the following superon(massless NG fermion)-vacuum coupling and $\sqrt{\frac{c^4 \Lambda}{8\pi G}}$ is the coupling constant $g_{sv}$ of superon with the vacuum via the supercurrent.

$$<\psi_\alpha^j|S^{k\mu}|0> = i\sqrt{\frac{c^4 \Lambda}{8\pi G}}(\gamma^a)_{\alpha\beta} \delta^{jk}, \quad (49)$$

Further we have seen in the preceding section that the right hand side of (48) for $N=2$ is essentially $N=2$ NLSUSY VA action and it is equivalent to the broken $N = 2$ LSUSYQED action (21) with the non-zero vacuum expectation value of the auxiliary field(Fayet-Iliopoulos term).

For the vacuum of the case (II) it gives the mass scale of the SUSY breaking

$$M_{SUSY}^2 \sim \sqrt{\frac{c^4 \Lambda}{8\pi G}} f \xi, \quad (50)$$

to the component fields of the (massless) LSUSY supermultiplet. We find NL-SUSYGR(SGM) scenario gives interesting relations among the important quantities of the cosmology and the low energy particle physics, i.e.,

$$\rho_D \sim \frac{c^4 \Lambda}{8\pi G} \sim g_{sv}^2. \quad (51)$$

Suppose that in the (low energy) LSUSY supermultiplet the stable and the lightest massive particle retains the mass of the order of the spontaneous SUSY breaking. And if we identify the neutrino with such a particle and with $\lambda'(x)$, i.e.

$$m^2_\nu \sim \sqrt{\frac{c^4 \Lambda}{8\pi G}}, \quad (52)$$
then SGM predicts the observed value of the (dark) energy density of the universe and naturally explains the mysterious numerical relations between $m_\nu$ and $\rho^{\text{obs}}_D$, provided $f\xi \sim O(1)$:

$$\rho^{\text{obs}}_D \sim (10^{-12}\text{GeV})^4 \sim m_\nu^4(\sim g_{sv}^2).$$

(53)

The tiny neutrino mass is the direct evidence of SUSY (breaking) in the NLSUSYGR scenario, i.e. Big collapse of space-time and the subsequent creation of gravitational (massless) composites in advance of the Big Bang. The extension to the large $N$ NLSUSY and the linearization of $\tilde{T}_{\mu\nu}(c,\psi)$ necessary for building the realistic broken LSUSY model, which contains the mass scale in the higher order with $\psi[28]$. In Riemann flat space-time of SGM, ordinary LSUSY gauge theory with the spontaneous SUSY breaking emerges as (massless) composites of NG fermion originating from the NLSUSY cosmological constant of SGM. SM and GUT may be regarded as the low energy effective composite theory of NLSUSYGR in flat space-time.

6 Summary

We have proposed NLSUSYGR(SGM) scenario for unity of nature. The ultimate shape of nature is unstable four dimensional space-time specified by $[x^a,\psi_\alpha^N; x^\mu]$ and described by NLSUSYGR $L_{\text{NLSUSYGR}}(w^a_{\mu})$ equipping the cosmological term with $\Lambda > 0$. Big Collapse(BC) of space-time would occur (due to false vacuum $\Lambda > 0$) and create ordinary Riemann space-time $[x^a; x^\mu]$ and massless fermionic matter superon $\psi_\alpha^N$, which is described by SGM action $L_{\text{SGM}}(e,\psi)$.

The universal attractive force graviton dictates the evolution of SGM world by forming all possible gravitational (massless) composites of superons which is, due to the field-current identity analogue, equivalent to the (massless) representation of sP algebra of space-time symmetry and ignites simultaneously the scenario of the Big Bang model of the Universe.

The true vacuum configuration is achieved by forming gravitational composite massless LSUSY supermultiplet and the subsequent oscillations around the true vacuum. We have shown in flat space-time that broken $N$-LSUSY theory emerges from the NLSUSY cosmological term of $L_{\text{SGM}}(e,\psi)$ (NL/L SUSY relation). Interestingly SGM and its evolution may regarded as the superfluidity of space-time and matter. The vacua of composite SGM scenario created by the BC of new space-time possesses rich structures promising for the unified description of nature. SGM gives
new insights into the unsolved problems of cosmology and particle physics, for example, the origin of the three-generations structure for quarks and leptons, the tiny neutrino mass, the proton decay, FCN, the dark matter and the dark energy, the space-time dimension four, etc. SGM predicts two color-singlet particles, i.e., a double-charge spin $\frac{1}{2}$ fermion $E^{\pm 2}$: $\epsilon^{abc}Q_aQ_bQ_ce^{lm}Q_l^*Q_m^*$ and a neutral spin 1 boson $S$: $\delta^{ab}Q_aQ_b$, which may give clear signals in the high energy and the cosmic ray experiment. The decay modes of particles strongly depend upon the choice of the assignment of particles to the superon composites eigenstates of sP group.

We have shown explicitly in 2 dimensional flat space-time that $N = 2$ LSUSY QED theory for the realistic $U(1)$ gauge theory appears as the physical field configurations of the vacuum of $N = 2$ NLSUSY theory on Minkowski space-time, which relate the particle physics with cosmology.

In the NLSUSY/SGM scenario, the global SUSY and NG fermion superon play essential roles and apparently the manifest local SUSY invariance may be unnecessary in flat space so far as shown in the type (II) vacuum, for the massless NG fermion disappears from the physical states.

Establishing NL/L SUSY relation for NLSUSYGR/SGM in curved space-time, i.e. linearizing directly SGM action $L_{SGM}(e, \psi)$ to find the broken (global) LSUSY SUGRA-like equivalent theory with the mass generation in the tower of the helicity states of sP and the extension to large $N$, especially to $N = 10 = 5 + 5^*$[8], is essential and under study.

Finally we just mention that NLSUSYGR and the subsequent SGM scenario for the spin $\frac{3}{2}$ NG fermion[13, 33] is in the same scope.

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