Interaction-induced Landau-Zener transitions

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received 2 June 2014; accepted in final form 18 July 2014
published online 7 August 2014

PACS 03.65.Xp - Tunneling, traversal time, quantum Zeno dynamics
PACS 03.75.-b - Matter waves
PACS 37.10.Gh - Atom traps and guides

Abstract – By considering a quantum-critical Lipkin-Meshkov-Glick model we analyze a new type of Landau-Zener transitions where the population transfer is mediated by interaction rather than from a direct diabatic coupling. For this scenario, at a mean-field level the dynamics is greatly influenced by quantum interferences. In particular, regardless of how slow the Landau-Zener sweep is, for certain parameters almost no population transfer occurs, which is in stark contrast to the regular Landau-Zener model. For moderate system sizes, this counterintuitive mean-field behaviour is not duplicated in the quantum case. This can be attributed to quantum fluctuations and to the fact that multi-level Landau-Zener-Stückelberg interferences have a “dephasing” effect on the above-mentioned phenomenon. We also find a discrepancy between the quantum and mean-field models in terms of how the transfer probabilities scale with the sweep velocity.

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Introduction. – The Landau-Zener (LZ) formula gives the transition probability when a system is swept through an avoided crossing [1,2]. Explicitly, by introducing the diabatic states [1] and [2] and writing a general state as $|\psi(t)\rangle = c_1(t)|1\rangle + c_2(t)|2\rangle$, the LZ problem solves the coupled equations ($\hbar = 1$)

$$\frac{\partial}{\partial t} \begin{bmatrix} c_1(t) \\ c_2(t) \end{bmatrix} = \begin{bmatrix} \lambda & U \\ U & -\lambda \end{bmatrix} \begin{bmatrix} c_1(t) \\ c_2(t) \end{bmatrix},$$

(1)

where $\lambda$ is the sweep velocity and $U$ the coupling strength of the two diabatic states. For an initial state $|\psi(-\infty)\rangle = |1\rangle$, the probability for population transfer from the state $|1\rangle$ to the state $|2\rangle$ at $t = +\infty$ is $1 - P_{LZ}$ with $P_{LZ} = \exp \left(-\frac{\pi U^2}{\lambda} \right)$ and the adiabaticity parameter $\Lambda = \frac{\pi U^2}{\lambda}$. In the adiabatic regime, $\Lambda \gg 1$, we obtain an almost complete transfer of population between the two diabatic states. This LZ formula holds only for initial conditions as the one above (or equivalently $|\psi(-\infty)\rangle = |2\rangle$) and for infinite integration times $t \in [-T, T]$; $T \to \infty$. For finite times or other initial conditions, quantum interference alters the exponential transition formula. As will be discussed in the present work, this phenomenon is especially evident in certain non-linear extensions of the above paradigm LZ model.

Various generalizations of the LZ problem have been considered in the past, especially multi-level problems [3–6], many-body situations [7–11], and non-linear LZ transitions [12–15]. It has been particularly demonstrated that for non-linear models both the exponential dependence and the smoothness of $P_{LZ}$ can be lost due to hysteresis phenomena [12,13]. Furthermore, in the adiabatic regime when $P_{LZ}$ is smooth, the transition probability typically obeys a power-law dependence, i.e. $P_{LZ} \sim \lambda^\nu$ for some exponent $\nu$ [13]. Using classical adiabatic arguments, power-law dependences have also been predicted in many-body LZ problems beyond the mean-field regime [7]. All these works assume infinite integration times, or more precisely choosing an initial state $|\psi(-\infty)\rangle = |1\rangle$ (or the ground-state in the many-body/level setting). At these infinite initial times the diabatic and adiabatic states coincide and as a result, effects deriving from the interference phenomenon mentioned above will be greatly suppressed. It is not clear, however, how other more general initial states will evolve for non-linear models.

We note that in the above extended LZ models the transition is maintained by a constant coupling between the diabatic states. Thus, interaction in these models primarily adds an effective (non-linear) energy shift of the instantaneous (adiabatic) energies. In this work we consider a different scenario where the coupling is solely driven by interaction, such that turning off the interaction implies a trivial decoupled system. In particular, we analyze a
Lipkin-Meshkov-Glick model (LMG) [16], both at a mean-field and at a many-body level. At the mean-field level, by considering initial states as those discussed above [(1) or (2)] they are decoupled and we encounter no population transfer. As a result, to stimulate any population transfer both initial, diabatic or adiabatic, states have to be populated and interferences between the two is unavoidable. In addition, in this “interaction-induced LZ model”, as will be shown, this type of interference has far more drastic influence on the dynamics than in the other LZ models. Beyond mean field, at a full many-body level, quantum fluctuations will, however, act as a sort of “dephasing” and the interference phenomenon is not equally transparent. Like in other extended LZ models, both at the mean-field and the full quantum level we find a power-law dependence on the transition probability, but the exponentials differ in the two cases for the system sizes considered.

Landau-Zener transitions. – Due to the diverging adiabatic energies of the LZ model in the asymptotic time limits, whenever more general initial states of the LZ problem are studied, one encounters a mathematical controversy regarding quantum interferences. The general solution of the LZ problem reads (given in the diabatic representation)

\[
\begin{bmatrix} c_1(+\infty) \\ c_2(+\infty) \end{bmatrix} = \begin{bmatrix} S_2 & S_1 \\ -S_1 & S_2 \end{bmatrix} \begin{bmatrix} c_1(-\infty) \\ c_2(-\infty) \end{bmatrix}. \tag{2}
\]

The scattering matrix elements are [17]

\[
S_1 = \sqrt{1 - P_{LZ}} e^{ix},
S_2 = P_{LZ}, \tag{3}
\]

with the phase

\[
\chi = \frac{3\pi}{4} - \arg \left[ \Gamma \left(\frac{i\Lambda}{2}\right) \right] + 2\Phi, \tag{4}
\]

where the last term is related to the (adiabatic) dynamical phase accumulated throughout the transition,

\[
\Phi = \lim_{t \to -\infty} \frac{\Lambda}{2} t^2 + \frac{\Lambda}{2} \log (\sqrt{2\lambda t}) \tag{5}
\]

and \(\Gamma(x)\) is the gamma function. Obtaining the asymptotic solution above implies studying the limits of functions when their arguments \(|z|\) goes to infinity. These limits may depend on the phase of \(z\), something referred to as the Stokes phenomenon [18]. The lines in the complex plane where the function changes character are called Stokes lines and in particular for the LZ problem the \(t \to -\infty\) and the \(t \to +\infty\) limits belong to different sectors divided by two such Stokes lines [19].

Returning to the expressions (4) and (5) we have that \(\Phi\) diverges in the large time limit, which means that the probability to find the system in, say, state \(|\Phi\rangle\) for an initial state \(|\psi(-\infty)\rangle = \cos \theta |1\rangle + \sin \theta |2\rangle\),

\[
P_1 = \cos^2 \theta (1 - P_{LZ}) + \sin^2 \theta P_{LZ} + \sin 2\theta P_{LZ} \sqrt{1 - P_{LZ}^2} \cos \chi, \tag{6}
\]
is ill-defined. Naturally, this is a result of looking at the asymptotic solution of the LZ problem, while for finite time sweeps the dynamical phase \(\Phi\) is finite. This interference effect is well known from the theory of Landau-Zener-Stückelberg interferometry [20]; the LZ transition depends on the relative phase of the incoming state.

It should be clear that whenever the state is initialized in say \(|\Phi\rangle\) and the integration time is finite the transition probability will always display interference between the corresponding adiabatic states. As will be demonstrated in the next section, for the interaction-induced LZ problem discussed in this work, the influences from this type of interference is greatly enhanced.

Interaction-induced Landau-Zener transitions. – The LMG model was first introduced in nuclear physics [16], but have since then been shown to be of relevance for numerous other systems; atomic condensates [14,21], ion traps [15], or in cavity/circuit QED [22]. The LMG model can be seen as an infinite range transverse Ising model. The type of LZ model we analyze is given by

\[
\hat{H}_{LMG} = \lambda t \hat{S}_z - \frac{U}{S} \hat{S}_x^2. \tag{7}
\]

Here, \(\hat{S}_x, \hat{S}_y\) and \(\hat{S}_z\) are the \(SU(2)\) angular-momentum operators obeying the commutation relations \([\hat{S}_\alpha, \hat{S}_\beta] = i\varepsilon_{\alpha\beta\gamma} \hat{S}_\gamma\), with \(\varepsilon_{\alpha\beta\gamma}\) the fully antisymmetric tensor. The LZ sweep velocity \(\lambda\) is taken to be positive, and \(U\), the interaction strength, is also positive meaning that we consider the ferromagnetic case. The “classical limit” accounts to take the spin \(S \to \infty\). The diabatic states are the eigenstates of the \(z\)-spin component, \(\hat{S}_z|\vec{S},m\rangle = m|\vec{S},m\rangle\). Importantly, we note that it is the interaction term causing a coupling between these diabatic states. In addition to the continuous \(U(1)\) symmetry arising from conserved spin, \([\hat{S}^2, \hat{H}_{LMG}] = 0\), the model also supports a \(Z_2\) parity symmetry given by \((\hat{S}_z, \hat{S}_y, \hat{S}_x) \to (-\hat{S}_z, -\hat{S}_y, \hat{S}_x)\). If \(S\) is an integer, the ground state at \(t = -\infty\) and at \(t = \infty\) has the same parity, while this parity changes when \(S\) is a half-integer. In the following we will always assume the spin to be an integer such that the instantaneous ground-state parity is preserved through the sweep.

Thinking of \(t\) as a parameter, for large \(|\lambda t|\) the ground state is ferromagnetic; \(|\vec{S}, S\rangle\). For \(\lambda = 0\) instead, the ground state \(|\vec{S}, \pm S\rangle\) is doubly degenerate. In the thermodynamic limit (here equivalent to the classical limit \(S \to \infty\)), the model is critical [23] with critical points at \(\lambda U = \pm 2\). The transitions are of the Ising universality class and for \(|\lambda U| < 2\) the system is in the symmetry broken phase in which the \(Z_2\) parity is broken. The anti-ferromagnetic LMG model (7), i.e. \(U < 0\), is not critical but instead there is a first-order transition at \(\lambda = 0\) separating the two ferromagnetic states \(|\vec{S}, \pm S\rangle\).

As a final remark, we compare the present LMG model to the otherwise frequently analyzed LMG systems, see refs. [14,15,21,22]. In all these cases, a term \(\varepsilon \hat{S}_x\) is included
in the Hamiltonian. Such a term breaks the $\mathbb{Z}_2$ symmetry and thereby split the ground state degeneracy and the model is no longer critical. Equally important, the LZ transition occurs also for zero interaction $U = 0$ in such cases. We call these models for parity-broken LMG systems. We note that the “spin-$S$ LZ problem”, the Hamiltonian is given by eq. (7) by letting $\mathbf{S}\mathbf{S}/S \rightarrow \hat{S}_z$, can be solved exactly for any $S$ [3].

Mean-field analysis. As the spin is preserved, the phase space is the $SU(2)$ Bloch sphere with radius $S$. The classical, or mean-field Hamiltonian, depends thereon on the polar and azimuthal angles $\theta$ and $\phi$. The corresponding classical Hamiltonian

$$\frac{H_{\text{cl}}}{S} = \lambda \cos(\theta) - U \sin^2(\theta) \cos^2(\phi),$$

(8)
gives the classical equations of motion

$$\dot{\phi} = \lambda \sin(\theta) + U \sin(2\theta) \cos^2(\phi),$$

$$\dot{\theta} = U \sin^2(\theta) \sin(2\phi),$$

(9)

with the dot representing the time derivative. Note that the above mean-field equations are “exact” when the quantum state is enforced to populate a spin-coherent state $|\theta, \phi\rangle$, and in particular one would expect the accuracy of this approach to be good for spins $S \gg 1$. Initially, $t_i = -T$, we assume the magnetization $z \equiv \cos(\theta) \approx 1$. Thus, the precession arounds the north pole. This marks an important difference between the present model and previously studied ones; if we let $z = 1$ we see that at a mean-field level the dynamics is frozen, i.e., no population transfer takes place. This derives from the fact that the transitions are emerging from interaction and when the “target mode” is empty there are no (quantum) fluctuations stimulating a transition. Thereby, we automatically have to initialize $z \neq \pm 1$ and as a result the population transfer will depend on the above-discussed quantum interference occurring between the adiabatic states. Note that this is so regardless of integration time —also in the limit $T \rightarrow \infty$. As a result, we expect the LZ interference effect to be particularly pronounced in the present model.

In the following we will integrate the classical equations of motion from $t_i = -200$ to $t_f = +200$. The initial magnetization $z_{t_i} = 0.98$, i.e., the initial spin is very close to pointing to the north pole. The magnetization after the LZ sweep, $z_{t_f}$, as a function of the LZ sweep velocity is displayed in fig. 1. In the adiabatic regime, typically $\lambda/U < 1$, the lower bound of $z_{t_f}$ follows a power-law behaviour $\sim \lambda^\nu$ with $\nu = 1/2$. The smallest $\lambda$ in the figure is $\lambda = 0.02$ meaning that for $t_i = -200$ the lowest adiabatic state is predominantly populated. The LZ-type interferences are evident throughout the parameter regime in terms of rapid oscillations where the transition is greatly suppressed. It is important to appreciate that the amplitude of these oscillations is much larger than what could be expected from the expression (6); with $z = \cos \theta = 0.98$ the amplitude of the LZ oscillations

$$\sin(2\phi)P_{\text{LZ}} \sqrt{1 - P_{\text{LZ}}} < 0.15.$$ We have numerically integrated the regular LZ problem with the same initial state and over the same time interval and found that the oscillations are often an order of magnitude smaller in amplitude in the regular LZ model. Furthermore, when the integration time is increased in the present model, the amplitude of oscillations grows and in the limit $t_i \rightarrow -\infty$ our numerical results suggest that the (adiabatic) transfer can be largely suppressed also for infinitely small $\lambda$’s. This is in stark contrast to the analytical result (6) for the regular LZ problem.

Let us look closer at the behaviour of fig. 1 and especially at how the LZ interference can be understood in this classical picture. For large negative $\lambda t$, the azimuthal angle $\phi$ oscillates rapidly while the polar angle $\theta$ evolves on a much longer time scale (adiabatic regime). Put in other words, whenever $\lambda|t| \gg U$, which warrants adiabatic evolution, the classical action $I = \int_{t_0}^{t_f} z d\phi$, with the integration curve along the classical phase space trajectory, stays constant (equivalently, during one classical orbit the Hamiltonian change is minimal) [24]. In the vicinity of the crossing, $\lambda t \sim 0$, there is, however, no clear separation of time scales between the two variables and it is here that adiabaticity breaks down (also called sudden or critical regime). In the limit of adiabatic evolution, the state follows the instantaneous constant energy curves, $H_{\text{cl}}[\theta, \phi, t] = \text{const}$. The extrema of the Hamiltonian functional give the fix points of eq. (9). The north and south pole on the Bloch sphere are two hyperbolic fix points of the classical equations of motion. There are two additional (elliptic) fix points: $(\theta_{\text{ep}}, \phi_{\text{ep}}) = (\arccos(-\lambda t/2U))$, 0 and $(\theta_{\text{ep}}, \phi_{\text{ep}}) = (\arccos(-\lambda t/2U), \pi)$. For large times $|t|$ these coincide with the other two fix points. For $|\lambda t|/2U \leq 1$ they traverse the Bloch sphere along the meridians $\phi = 0$, $\pi$. The four fix points define the (nonlinear) adiabatic energy curves via $E_{\text{ad}}(t) = H_{\text{cl}}[\theta_{\text{fp}}, \phi_{\text{fp}}]$.

Fig. 1: (Colour on-line) The imbalance $z$ for large times $t_f = 200$ as a function of $\lambda/U$ (we actually average $z(t)$ over some periods around $t_f$ in order to avoid additional fluctuations). The initial condition is taken as $z(t_i) = \cos \theta = 0.98$ and $\phi(t_i) = 0$. The insets display zooms of the imbalance. The green solid line is the result from a TWA simulation with 5% fluctuations in the initial imbalance $z(t_i)$ and a fully random initial phase $\phi(t_i)$. 
which with the above expressions becomes $E_{\text{ad}}(t) = \pm S\lambda t$ and $E_{\text{ad}}(t) = -S(U + \frac{\lambda t^2}{4})$.

Historically, the rapid changes in the transition probability for non-linear LZ problems has been traced back to a hysteresis effect (the adiabatic energies build up so-called swallow-tail loops) which is present above some critical strength of non-linearity [13]. Dynamically, this is explained from two fix points “colliding” in phase space and the solution is not able to precess around a single fix point any longer. The interferences of fig. 1 can also be understood by returning to the phase space evolution. Initially, the system adiabatically encircles the north pole. At some instant, in the terminology of a transcritical bifurcation, two elliptic fix points begin to depart from the north pole. This is the critical regime where there exist no clear separation of time scales. The solution can here “choose” between encircling the hyperbolic or elliptic fix point as they separate in phase space. In the latter case, the system ends up with a large fraction of population centered around the south pole. Thus, the rapid variations in the population transfer again stems from a “collision” of fix points, but this time it coincides with a critical point in the original quantum model. This is indeed the crucial difference between this model and the earlier studies; the fate of the system which is determined from which fix point it will “follow” occurs in the critical regime while in other models the evolution can be smooth up to the “hysteresis jump”. Another way to see the difference is to note that for the parity-broken LMG system, the bifurcation is of the imperfect type. To make the picture more clear, the adiabatic energy curves are depicted in fig. 2 as blue lines (solid lines are the stable and dashed lines the unstable classical solutions). Breaking of the parity symmetry implies opening up a gap between the two elliptic solutions. In fact, it has been shown that for the ferromagnetic ($U > 0$) parity-broken LMG model a non-zero interaction $U$ increases the population transfer [25] contrary to the LMG model analyzed in this work. We see that the present model also displays swallow-tail loops, but contrary to earlier studies these additional solutions are always present for non-zero $U$.

A most relevant question is whether the interferences survive in the quantum case where quantum fluctuations could destabilize the classical solutions. To explore the influence of quantum fluctuations of the initial states we apply the truncated Wigner approximation (TWA) [26] which solves the classical equations of motion for a set of initial states $(\phi_i(t_f), \theta_i(t_f))$ which are taken randomly according to the initial quantum distribution $|\Psi(\theta, \phi, t_f)|^2$. The resulting semi-classical results are obtained by averaging over the set of classical solutions, i.e. the trajectories are added incoherently meaning that any dynamical quantum interference effects are neglected. The results of a TWA simulation is presented as the green line in fig. 1. Expectedly, the initial fluctuations smear out the rapid variations in the fully classical results. Here, since the initial state is close to the north pole the initial phase $\phi(t_f)$ is taken fully random. Note, however, that deep in the classical regime (large $\mathcal{S}$) and by starting slightly off the north pole, these fluctuations could, in principle, be made arbitrary small and the interferences would reappear. We can use that the fluctuations $\sim 1/\sqrt{S}$ away from the poles and numerically estimate how large the spin $S$ must be in order to restore the oscillations. For the given integration interval we find that $S > 20000$.

Full quantum analysis. We now go beyond the classical and semi-classical approaches and analyze the evolution of the full quantum system defined by the Hamiltonian (7). One of the main objections is to explore whether the interference structure found in the transition probabilities in the classical model survives also in the quantum problem. Before presenting the results we may note that there are some earlier studies of related problems. More precisely, driving the ferromagnetic LMG model through its critical point was analyzed in refs. [9], and it was found that the non-adiabatic corrections obey a power-law dependence of the sweep velocity $\lambda$. A similar behaviour was also demonstrated in the Tavis-Cummings model describing $N$ spin-(1/2) particles collectively interacting with a single-boson mode [7]. Also the LZ problem of the parity-broken LMG model has been considered [14].

The eigenenergies $\varepsilon_n$ of $\hat{H}_{\text{LMG}}$ are displayed in fig. 2. In the thermodynamic limit, the critical points are at $\lambda_t/U = \pm 2$ for which the two lowest parity states become degenerate. Since the spectrum is symmetric with respect to $\lambda_t/U = 0$ it follows that the spectrum of the antiferromagnetic LMG is simply $-\varepsilon_n$. This demonstrates the fact that the antiferromagnetic LMG is not critical but hosts a first-order quantum phase transition.

When the initial ground state evolves, it passes through a seam of level crossings starting at $t = -2U/\lambda$ and continuous until $t = 2U/\lambda$. Thus, the system realizes a multi-channel Landau-Zener-Stillclberg interferometer. We note that this multi-level LZ crossings cannot
be described by the LZ bow-tie model [27]. Interferences between the different paths (adiabatic states) could lead to final populations divided among the different diabatic states. In order to compare the amount of excitations in the present system to the LZ formula, we introduce the projectors \( \hat{P}_n(t) = |\psi_n(t)\rangle\langle \psi_n| \), where \( |\psi_n(t)\rangle \) is the \( n \)-th instantaneous eigenstate of \( \hat{H}_{\text{LMG}} \), and define the excitation fraction as

\[
P_{\text{ex}} = \lim_{t \to -\infty} \frac{1}{2S+1} \sum_{n=0}^{2S} n \langle \psi(t) | \hat{P}_{n+1}(t) | \psi(t) \rangle.
\] (10)

Here, \( |\psi(t)\rangle \) is the solution of the full time-dependent problem. \( P_{\text{ex}} \) measures the amount of non-adiabatic excitations; \( P_{\text{ex}} = 0 \) corresponds to the case when only the ground state is populated while \( P_{\text{ex}} = 1 \) is the opposite limit of a maximally excited system. Note that \( P_{\text{ex}} \) is the mean of the final (scaled) distribution \( P(n) \) of population of the various states \( |\psi_n(t)\rangle \). In the asymptotic limit \( t \to +\infty \), when the diabatic and adiabatic states coincide, \( \langle \hat{S}_z \rangle = \langle \hat{S}\rangle = 2 \lambda - \frac{1}{2} \lambda_{\text{ex}} \). To fully characterize the final distribution one would need all moments \( \Delta^{(k)} \equiv \sum_n n^k P(n) \). Of particular interest is the Mandel Q-parameter [28],

\[
Q = \frac{\Delta^{(2)} - \langle \Delta^{(1)} \rangle^2}{\Delta^{(1)}} - 1,
\] (11)

which says whether the distribution \( P(n) \) is sub- \((Q < 0)\) or super-Poissonian \((Q > 0)\).

The full time-dependent problem has been integrated from \( t_i = -200 \) to \( t_f = +200 \). We consider various spins \( S \) and sweep velocities \( \lambda \). The results for \( P_{\text{ex}} \) are shown in fig. 3. We see in the figure that for growing spin \( S \) the system becomes more excited which can be understood from the increased density of states. Indeed, this is a result deriving from the critical slowing-down mechanism in the vicinities of critical points. In the adiabatic and in the intermediate regimes we in particular find (numerically) that \( P_{\text{ex}} \approx \sqrt{S} \).

In the adiabatic regime different power-law dependences \( P_{\text{ex}} \sim \lambda^\nu \) have been reported in various types of LZ models; \( \nu = 3/4 \) for parity-broken LMG model [13] and \( \nu = 1 \) for the Tavis-Cummings model [7]. For a sweep through one of the critical points of the parity LMG it was found that \( \nu = 2 \) deep in the adiabatic regime and \( \nu = 3/2 \) in the intermediate regime [9]. Such a dynamical situation is different from a full LZ sweep taken in this work where Landau-Zener-Stückelberg interferences can alter the excitations. Nevertheless, one may expect similar power-law dependences and this is indeed also the case as has been verified numerically. Thus, for small sweep velocities \( \lambda \), \( P_{\text{ex}} \sim \lambda^2 \) (i.e. \( \nu = 2 \)) and for the regime where breakdown of adiabaticity considerably sets in \( P_{\text{ex}} \sim \lambda^{3/2} \) (i.e. \( \nu = 3/2 \)) and finally in the diabatic regime we recover an exponential dependence \( P_{\text{ex}} \sim [1 - \exp(-\kappa/\lambda)]^3 \) for some \( \lambda \)-independent constant \( \kappa \) (which is however \( N \)-dependent). Note that the corresponding quantum and semi-classical models display different scaling in the adiabatic regime. The inset of fig. 3 displays the excitations in the adiabatic regime, and we can hint that for large spins the exponential \( \nu \) is actually smaller than 2 in the limit \( \lambda \to 0 \) which could explain the discrepancy between the quantum and classical results — we can only expect agreement in the classical limit \( S \to \infty \).

We now return to the question raised in the beginning of this section; will the interference phenomenon discussed in the classical model survive also in the quantum case? Clearly, from fig. 3 we see that the classical oscillations are absent in the quantum simulations. One may say that this is no surprise since the oscillations were already gone in the TWA result. However, as already pointed out, the limit \( S \to \infty \) should reproduce the classical results. In the quantum case, for spins as large as \( S = 500 \) we have not been able to see any signatures of the classical oscillations. This is still, however, a small number compared to the estimated one of \( S > 20000 \) of the previous section (which is beyond what is computationally accessible). Furthermore, apart from the different scalings in the classical and quantum cases, we see that the semi-classical TWA evolution is in general more adiabatic compared to the quantum one for spins \( S > 20 \). For example, at \( \lambda/U = 1 \) the quantum results gives \( z(t_f) \approx 0.65 \) for \( N = 50 \) particles, while from the TWA results one has \( z(t_f) \approx 0.15 \). The difference between the quantum and semi-classical TWA results should be ascribed to the multi-level Landau-Zener-Stückelberg interferences. We expect that such interferences should generate large fluctuations in the distribution \( P(n) \). If this is the case one should find large Mandel Q-parameters as \( S \) grows. In the quasi-adiabatic and intermediate regimes we have numerically found that \( Q \sim S \), while from fig. 3 we can extract that in the corresponding regime \( P_{\text{ex}} \sim \sqrt{S} \). So the fluctuations relative to the non-adiabatic excitations in the system \( Q/P_{\text{ex}} \sim \sqrt{S} \), which agrees with the findings of [7]. Thus, as \( S \) is increased, a larger number of final diabatic/adiabatic states will become populated.

Conclusions. — The LZ problem of a LMG model where the transition is driven by particle interaction was studied. At the mean-field level it was demonstrated that

![Fig. 3: (Colour on-line) The average (scaled) number of excitations created during the LZ sweep for different spins S: following the arrow, 6 (blue), 12 (red), 24 (green), 50 (black), and 74 (magenta). In the adiabatic regime (inset) \( P_{\text{ex}} \sim \lambda^2 \).](image-url)
ergy states and by tuning the ratio $\omega / \omega_y$ the trap prevents the atoms from decaying into other energy states. The quasi-degenerate and with a mean-field and quantum results predicted different scaling of the transition probabilities are suppressed. The quantum level, on the other hand, quantum fluctuations and interference can drastically affect the transition probabilities are suppressed. The anharmonicity of the atom-atom interaction which converts two $p_x$-orbital atoms or vice versa [30]. The anharmonicity of the trap prevents the atoms from decaying into other energy states and by tuning the ratio $\omega_x/\omega_y$, the LZ sweep is realizable.

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The author thanks Fernanda Pinheiro for discussions, and acknowledges VR (Vetenskapsrådet) for financial help.

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