Azimuthal asymmetries in high-energy collisions of protons with holographic shockwaves

Jorge Noronha$^1$, and Adrian Dumitru$^2$

$^1$Instituto de Física, Universidade de São Paulo, C.P. 66318, 05315-970 São Paulo, SP, Brazil

$^2$Department of Natural Sciences, Baruch College, CUNY, 17 Lexington Avenue, New York, NY 10010, USA

Abstract

Large azimuthal quadrupole and octupole asymmetries have recently been found in p+Pb collisions at the LHC. We argue that these might arise from a projectile dipole scattering off randomly shaped intrinsic fluctuations in the target with a size on the order of the dipole. Generic energy-momentum fluctuations generate comparable asymmetries for all multipole moments $v_n$. The holographic description of this process involves the calculation of a light-like Wilson loop in the background of a non-uniform holographic shockwave in the presence of a Neveu-Schwarz 2-form.
Recent measurements of the azimuthal momentum distributions of particles produced in high-multiplicity $p + Pb$ collisions at the LHC have revealed large asymmetries $v_n = \langle \cos n\phi \rangle$ [1], c.f. fig. 1. Remarkably, the octupole asymmetry is of the same order of magnitude as the quadrupole. In this paper, we argue that these asymmetries in the final state could reflect the multipoles of a snapshot of the fluctuations in the Pb target taken in the instant of the collision. Rotational symmetry is spontaneously broken (locally) by a fluctuation in the target of arbitrary shape. Such random fluctuations generically lead to large local azimuthal anisotropies in coordinate-space [2, 3]. We provide an alternative to (almost) dissipationless hydrodynamic expansion in pA collisions [4, 5] for the conversion of coordinate-space fluctuations into asymmetries in momentum space. Here, non-zero $v_n$'s emerge from the coupling of the orientation of a projectile “dipole” to the multipoles of a local fluctuation in the target. Other calculations of $v_2$ have been carried out within the “Color Glass Condensate” approach, see [6] for a recent review, though at present it is not clear if a large $v_3$ emerges as well.

FIG. 1. Amplitude of azimuthal asymmetries (top: quadrupole $v_2$; bottom: octupole $v_3$) in high-multiplicity $p + Pb$ collisions (right panels in each figure) at 5 TeV center of mass energy; CMS data.
In the so-called hybrid formalism the proton projectile is treated as a beam of collinear partons with a large light-cone momentum $p^-$ which probe the field of the target. At large Feynman-$x_F$ the contribution from quarks dominates while particles with $x_F \ll 1$ are mainly gluons. We describe here mainly the case of quark scattering but it is straightforward to obtain the contribution from gluons by considering the scattering amplitude in the adjoint representation.

This treatment is distinct from that of Refs. [7] where a pA collision is modeled as an asymmetric collision of holographic shockwaves [8], or from $\mathcal{R}$-current deep inelastic scattering [9]. In our approach the gauge-gravity correspondence is used only to evaluate the “target field averages” such as the light-like Wilson loop, analogous to the computation of the jet quenching parameter $\hat{q}$ in ref. [10]. We do not use it to describe the actual collision; thus, no black hole is formed and the projectile quark is not stopped. The form of the scattering amplitude of a projectile quark with large light-cone momentum is taken from QCD.

Our approach is based on the intuitive picture that a high-energy projectile parton couples weakly to the target field. However, over some intermediate range of semi-hard transverse momenta the target is modelled as a holographic shockwave before it turns into a beam of non-interacting asymptotically free partons at very high transverse momentum $p_\perp$ (far exceeding its saturation scale $Q_s$ computed below). Whether or not the proposed picture has a viable theoretical justification remains to be seen.

To leading order in $p_\perp/p^-$ projectile partons propagate on eikonal trajectories so that the amplitude corresponding to elastic scattering from momentum $p$ to $q$ is [11]

$$\langle \text{out}, q|\text{in}, p \rangle \equiv \bar{u}(q)\tau(q, p)u(p)$$

$$\tau(q, p) = 2\pi\delta(p^- - q^-) \gamma^- \int d^2\vec{x} \left[ V(\vec{x}) - 1 \right] e^{i(p^- - q^-)\cdot\vec{x}}.$$  

Here,

$$V(\vec{x}) = \mathcal{P} \exp \left[ ig \int dx^- A^+(x^-, \vec{x}) \right]$$

is a Wilson line along the light cone. Upon squaring the amplitude [12] the scattering cross section can be written as [13]

$$\frac{d\sigma_{qA}}{d^2\vec{b}d^2\vec{q}} = \frac{1}{(2\pi)^2} \int d^2\vec{x} e^{-i\vec{q}\cdot\vec{x}} \left\langle \frac{1}{N_c} \text{tr} \left( W(\vec{x}, \vec{b}) - V(\vec{b} - \vec{x}/2) - V^\dagger(\vec{b} + \vec{x}/2) \right) \right\rangle + 1.$$
Here, $\vec{b}$ denotes the impact parameter of the collision and $W(\vec{x}, \vec{b})$ is a light-like Wilson loop of width given by $|\vec{x}|$. In covariant gauge $W(\vec{x}, \vec{b}) = V^\dagger(\vec{b} + \vec{x}/2)V(\vec{b} - \vec{x}/2)$ and so is commonly referred to as the dipole unintegrated gluon distribution [14]. The size of the dipole corresponds to the shift of the transverse coordinate of the eikonal quark line from the amplitude to the complex conjugate amplitude, respectively.

Eq. (4) can be turned into a physical $pA \rightarrow h + X$ single inclusive cross section for production of a hadron of type $h$ via a convolution with a proton-parton distribution and a corresponding $q \rightarrow h$ fragmentation function [15–17]. Here we will only need Eq. (4). Below, we employ the holographic correspondence [18] to compute the light-like Wilson loop in the field of a shockwave in strongly-coupled $\mathcal{N} = 4$ Supersymmetric Yang-Mills (SYM) theory with a large number of colors, $N_c$. The essential point is to consider scattering of a dipole, whose angular orientation couples to the multipoles of local fluctuations in the target.

The nucleus is traveling along the $x^+$ axis with a light-cone momentum $p^+$. We are interested in shockwave solutions of the form

$$\langle \hat{T}_{--}(x^-, \vec{x}) \rangle = \frac{N_c^2}{2\pi^2} p^+ \delta(x^-) \mu^2 f(\vec{x})$$

(5)

where $f(\vec{x})$ describes the energy density distribution of the target in the transverse plane, and $\mu^2 \sim A^{1/3}$ [20, 21] is the transverse density scale of the shockwave. This non-uniform shockwave at the boundary can be obtained from a source $J(\vec{x})\delta(z - 1/\mu)$ in the bulk convoluted with the Green’s function found in [22],

$$f(\vec{x}) = \int d^2\vec{x}' \frac{J(\vec{x}')}{(1 + \mu^2|\vec{x} - \vec{x}'|^2)^3}.$$  

(6)

The metric is a solution of the 5 dimensional Einstein’s equations with a negative cosmological constant $\sim 1/L^2$ with a source [22, 23]. It has the form

$$ds^2 = \frac{L^2}{z^2} \left( p^+ \delta(x^-) \mu^2 \mathcal{F}(z, \vec{x}) z^4 dx^- dx^+ - 2 dx^+ dx^- + d\vec{x}^2 + dz^2 \right),$$

(7)

where $\lim_{z \to 0} \mathcal{F}(z, \vec{x}) = f(\vec{x})$.

One can now compute the light-like Wilson loop in this background [25]. The rectangular loop $C$ is defined on the $x^-$ axis with transverse size $\vec{d}$ and it is the boundary for a minimal surface in the bulk [26, 27]. When the radius of AdS$_5$ is much larger than the string length $\ell_s$, i.e., $L^2/\alpha'^2 \gg 1$, where $\alpha' = \ell_s^2$, this is obtained by minimizing the Nambu-Goto action

$$S_{NG} = \frac{1}{2\pi \alpha'} \int_\Omega d^2\sigma \sqrt{-\det h_{ab}}$$

(8)
where $\Omega$ denotes the worldsheet, $h_{ab} = g_{MN} \partial_a X^M \partial_b X^N$ is the worldsheet metric, and $X^M(\sigma)$ is the embedding function that describes the string worldsheet in the bulk. Our calculation for the light-like Wilson loop closely follows the one performed in [10] to obtain the jet quenching parameter $\hat{q}$ coefficient in the strongly coupled $\mathcal{N} = 4$ SYM plasma. However, for our shockwave the gauge theory is not at finite temperature and, thus, the background geometry does not have a horizon. The light-like Wilson loop is given in terms of the on-shell action as $\langle \text{tr} W(\mathcal{C}) \rangle / N_c = e^{iS_{\text{on-shell}} \text{NG}}$.

The string worldsheet coordinates are $\tau = x^-$ and $\sigma$ and the worldsheet embedding function is $X^M = (x, \vec{b} + \sigma \vec{d}, 0, du(\sigma))$ with $\sigma \in (-1/2, 1/2)$. The endpoints of the string are located at the boundary at $\vec{b} - \vec{d}/2$ and $\vec{b} + \vec{d}/2$. With the metric (7) the Nambu-Goto action becomes

$$iS_{\text{NG}} = -\frac{\sqrt{\lambda}}{2\pi} A^{1/6} \mu d \int_{-1/2}^{1/2} d\sigma \sqrt{f(du(\sigma), \vec{b} + \sigma \vec{d})} \sqrt{1 + u'(\sigma)^2}.$$  \hspace{1cm} (9)

The explicit factor of $A^{1/6}$ arises from the integration over the longitudinal coordinate $x^-$ [20, 21, 28].

Due to the properties of the function $f$, the integrand of the action is finite at the boundary, $u \to 0$, as opposed to the case of a time-like rectangular Wilson loop in vacuum [20] where it diverges as $\sim 1/u^2$. Therefore, any configuration where $u'(\sigma) \neq 0$ will necessarily increase the worldsheet area. Hence, the minimal surface must be the one in which the string remains at the boundary for all $\sigma$, i.e., the string does not fall into the bulk. In fact, $u(\sigma) = 0$ is clearly a solution of the equations of motion that satisfies the boundary conditions, which is consistent with the fact that a light-like string configuration costs zero energy in AdS$_5$ [29].

Therefore, the on-shell action is obtained by setting $u'(\sigma) = 0$ and $u(\sigma) = 0$ which leads to

$$iS_{\text{on-shell \text{NG}}} = -\frac{\sqrt{\lambda}}{2\pi} A^{1/6} \mu d \int_0^{1/2} d\sigma \left( \sqrt{f(\vec{b} + \sigma \vec{d})} + \sqrt{f(\vec{b} - \sigma \vec{d})} \right).$$  \hspace{1cm} (10)

Note the explicit $\vec{d} \to -\vec{d}$ symmetry to which we shall return below. In what follows we assume that the nucleus is much larger than the dipole and that its density over large distance scales is homogeneous. Thus, we can set $\vec{b} = 0$. Furthermore, in the absence of fluctuations we have $f = 1$ and so the forward dipole scattering amplitude becomes

$$W(d) = e^{-\frac{\sqrt{\lambda}}{2\pi} A^{1/6} \mu d}.$$  \hspace{1cm} (11)
The “saturation scale” where $W \sim e^{-1}$ therefore is
\[ Q_s = \frac{\sqrt{\lambda}}{2\pi} A^{1/6} \mu \, . \quad (12) \]

$Q_s$ obtained by averaging the Wilson loop in a shockwave background increases very rapidly with the thickness of the nucleus, $Q_s \sim A^{1/3}$ [20]. The calculations above provide a new and simple starting point for the discussion of deep inelastic scattering on a large nucleus at strong coupling [20].

The transverse momentum distribution of scattered quarks is
\[ \frac{d\sigma_{qA}}{d^2b \, d^2q_{\perp}} = \frac{Q_s}{(Q_s^2 + p_{\perp}^2)^{3/2}} \, . \quad (13) \]

We should stress that this result is not supposed to apply at very large $p_{\perp}$ where from perturbative QCD $d\sigma_{qA}/d^2p_{\perp} \sim \alpha_s^2/p_{\perp}^4$. However, in the non-linear regime at intermediate $p_T$ one does indeed expect a “flatter” transverse momentum distribution similar to (13).

We introduce fluctuations of the density of the shockwave in terms of their Fourier spectrum,
\[ f \left( \vec{b} + \sigma \vec{d} \right) = 1 + \delta f(\vec{b} + \sigma \vec{d}) \]
\[ \delta f(\vec{x}) = \int \frac{d^2k}{(2\pi)^2} \delta f(\vec{k}) e^{i\vec{k} \cdot \vec{x}} \, . \quad (15) \]

For simplicity we assume that
\[ \delta f(\vec{k}) = \frac{1}{2}(2\pi)^2 A(1/|\vec{k}_0|) \left[ \delta(\vec{k} - \vec{k}_0) + \delta(\vec{k} + \vec{k}_0) + i(\delta(\vec{k} - \vec{k}_0) - \delta(\vec{k} + \vec{k}_0)) \right] , \quad (16) \]
i.e., that the fluctuation is dominated by a single wave number and direction though one could also average over some suitable distribution. We shall denote the typical length scale $1/|\vec{k}_0|$ of fluctuations as $\ell$, and the azimuthal orientation of the dipole as $\phi$ so that $\vec{d} \cdot \vec{k}_0 = d/\ell \cos \phi$. Expanding the square root in (10) for small amplitude fluctuations we find
\[ iS_{\text{on-shell}} = -Q_s \, d \left[ 1 + A(\ell) \frac{\sin \left( \frac{d}{\ell} \cos \phi \right)}{\frac{d}{\ell} \cos \phi} \right] \, . \quad (17) \]

The fluctuations generate asymmetries for the multipole moments of the $p_{\perp}$ distribution,
\[ \frac{d\sigma_{qA}}{d^2b \, p_{\perp} \, dp_{\perp} \, d\phi_p} = \frac{1}{(2\pi)^2} \int x_{\perp} \, dx_{\perp} \, d\phi \, e^{-ip_{\perp} x_{\perp} \cos(\phi - \phi_p)} e^{iS_{\text{on-shell}} \, NG} \, . \quad (18) \]
where we denoted the transverse size of the dipole as $\vec{x}_\perp$. Parametrically, nonzero moments of this distribution will be of order $\sim Q_s \ell A(\ell)$. However, the action (17) is even under $\phi \to \phi + \pi$ and, thus, it can only generate even moments of the angular distribution.

Odd moments of the angular distribution can be generated in this approach as follows. First, we note that the dipole-nucleus $S$-matrix $\langle V^\dagger(\vec{x})V(\vec{y}) \rangle$ includes $C$-even “pomeron” and odd “odderon” exchanges. Projecting on odd-even in-out states, the latter corresponds to the imaginary part $\Im m S(\vec{x}, \vec{y}) = \langle O(\vec{x}, \vec{y}) \rangle$ where the odderon operator is given by

$$O(\vec{x}, \vec{y}) = \frac{1}{2iN_c} \text{tr} \left( V^\dagger(\vec{x})V(\vec{y}) - V^\dagger(\vec{y})V(\vec{x}) \right). \quad (19)$$

The odderon has been identified with the fluctuations of the anti-symmetric Neveu-Schwarz-Neveu-Schwarz (NS-NS) 2-form $B_{MN}$ in the bulk [30, 31]. In the dual holographic description used here, the contribution to the action that is odd under the $\vec{d} \to -\vec{d}$ operation should arise from the coupling of the NS 2-form field to the string in the Nambu-Goto action

$$S_{\text{NS-NS}} = \frac{1}{4\pi\alpha'} \int_\Omega d^2\sigma B_{MN} \epsilon_{ab} \partial_a X^M \partial_b X^N \quad (20)$$

where $\epsilon_{ab}$ is the Levi-Civita symbol on the worldsheet [32]. For a single shockwave we consider a pure gauge NS-field $B_{MN}$ which does not alter the equations of motion of supergravity [33] so that the solutions for the background metric and for the string remain valid. [After a collision of two shockwaves this is no longer the case, just as the metric is no longer of the form (7).]

Contributions such as (20) should indeed lead to parity odd moments of the angular distribution (18). Choosing a gauge where $B_{-M} = 0$, this term in the action becomes (using the worldsheet embedding defined earlier)

$$S_{\text{NS-NS}} = \frac{1}{2\pi\alpha'} \int_{-\infty}^{\infty} dx^- \int_0^{1/2} d\sigma \, d\vec{d} \cdot \left[ \vec{B}_{-\vec{x}_\perp}(x^-, \vec{b} + \vec{d}\sigma, 0) + \vec{B}_{-\vec{x}_\perp}(x^-, \vec{b} - \vec{d}\sigma, 0) \right], \quad (21)$$

Thus, it contributes a phase to the amplitude $\exp\{iS_{\text{NG}} + iS_{\text{NS-NS}}\}$ which is odd under $\vec{d} \to -\vec{d}$. Therefore, in this more general scenario, odd moments for the angular distribution such as $v_3$ should be nonzero and, again, of order $\ell Q_s$. Specifically, terms such as

$$i \int_{-1/2}^{1/2} d\sigma \, d\vec{d} \cdot \vec{\nabla} f(\sigma d) = -2iA(\ell) \sin \left( \frac{d}{2\ell} \cos \phi \right), \quad (22)$$

can arise. Indeed, this is parity ($\phi \to \phi + \pi$) odd and generates odd $v_n$ up to $n \sim d/\ell$. 7
In summary, we have argued that azimuthal asymmetries \( v_n \) in p+A collisions arise from scattering of a dipole on random fluctuations in the target. The contribution from the metric to the Nambu-Goto action produces parity even distributions while a NS-NS 2-form field can generate odd moments. More detailed numerical calculations of \( v_n(p_L) \) could potentially provide information on the spectrum of fluctuations, such as if there is a dominant length scale and amplitude.

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* noronha@if.usp.br
† adrian.dumitru@baruch.cuny.edu

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Given the matter content of $\mathcal{N} = 4$ SYM theory, the Wilson loop also contains the coupling to the six $SU(N_c)$ adjoint scalars $X^I$. We follow \cite{24} and consider an average over all the angles $\theta^I$ on $S^5$ to eliminate the dependence of the string worldsheet on modes that carry nonzero Kaluza-Klein charge.

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