INTERSTELLAR GAS AND A DARK DISK

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ABSTRACT

We introduce a potentially powerful method for constraining or discovering a thin dark matter disk in the Milky Way. The method relies on the relationship between the midplane densities and scale heights of interstellar gas being determined by the gravitational potential, which is sensitive to the presence of a dark disk. We show how to use the interstellar gas parameters to set a bound on a dark disk and discuss the constraints suggested by the current data. However, current measurements for these parameters are discordant, with the uncertainty in the constraint being dominated by the molecular hydrogen midplane density measurement, as well as by the atomic hydrogen velocity dispersion measurement. Magnetic fields and cosmic ray pressure, which are expected to play a role, are uncertain as well. The current models and data are inadequate to determine the disk’s existence, but taken at face value, may favor its existence depending on the gas parameters used.

Key words: dark matter – Galaxy: disk – ISM: abundances – ISM: magnetic fields – ISM: structure – local interstellar matter

1. INTRODUCTION

Fan et al. (2013) proposed the existence of thin disks of dark matter in spiral galaxies, including the Milky Way, in a model termed Double Disk Dark Matter (DDDM). In this model, a small fraction of the dark matter is interacting and dissipative, so that this sector of dark matter would cool and form a thin disk. More recently, Randall & Reece (2014) showed that a dark matter disk of surface density $\sim 10^2 M_\odot pc^{-2}$ and scale height $\sim 10$ pc could possibly explain the periodicity of comet impacts on Earth. It is of interest to know what values of dark disk surface density and scale height are allowed by the current data and whether these particular values are allowed.

Since the original studies by Oort (1932, 1960), the question of disk dark matter has been a subject of controversy. Over the years, several authors have suggested a dark disk to explain various phenomena. Kalberla et al. (2007) proposed a thick dark disk as a way to explain the flaring of the interstellar gas layer. It has also been argued that a thick dark disk is formed naturally in a $\Lambda$CDM cosmology as a consequence of satellite mergers (Read et al. 2008). Besides these, there are also models arguing for a thin dark disk. Fan et al. (2013) put forward a model for dark matter where a small fraction of the total dark matter could be self-interacting and dissipative, necessarily forming a thin dark disk. In Kramer & Randall (2016), we investigated the constraints on such a disk from stellar kinematics. In this paper, we investigate the constraint imposed by demanding consistency between measurements of midplane densities and surface densities interstellar gas.

We assume a Bahcall-type model for the vertical distributions of stars and gas (Bahcall 1984a, 1984b, 1984c) as in Kramer & Randall (2016), with various visible mass components as well as a dark disk. We investigate the visible components in detail, given more recent measurements of both the surface and midplane densities. A dark disk affects the relationship between the two as argued in Kramer & Randall and as we review below. Although current measurements are insufficiently reliable to place strong constraints on or identify a disk, we expect this method will be useful in the future when better measurements are achieved.

2. POISSON–JEANS THEORY

As explained in detail in Kramer & Randall (2016), for an axisymmetric self-gravitating system, the vertical Jeans equation near the $z = 0$ plane reads

$$\frac{\partial}{\partial z}(\rho_i \sigma^2) + \rho_i \frac{\partial \Phi}{\partial z} = 0.$$  (1)

For an isothermal population ($\sigma_z(z) = \text{constant}$), the solution reduces to

$$\rho_i(z) = \rho_i(0) e^{-\Phi(R,z)/\sigma_z^2}.$$  (2)

Combining this with the Poisson equation gives the Poisson–Jeans equation for the potential $\Phi$

$$\frac{\partial^2 \Phi}{\partial z^2} = 4\pi G \sum_i \rho_i(0) e^{-\Phi/Ri}/\sigma_i^2,$$  (3)

which can also be cast in integral form (assuming $z$-reflection symmetry)

$$\frac{\rho_i(z)}{\rho_i(0)} = \exp \left( -\frac{4\pi G}{\sigma_i^2} \sum_k \int_0^z dz' \int_0^z dz'' \rho(z'') \right).$$  (4)

This is the form used in our Poisson–Jeans solver.

2.1. A Toy Model

In Kramer & Randall (2016), we showed that the exact solution to the Poisson–Jeans equation for a thick component (in this case, the interstellar gas which is thick compared with the dark disk) with midplane density $\rho_0$ and vertical dispersion $\sigma$ interacting with an infinitely thin (delta-function profile) dark disk was

$$\rho(z) = \rho_0(1 + Q^2) \sech^2 \left( \frac{\sqrt{1 + Q^2}}{2h}(|z| + z_0) \right),$$  (5)

where $Q \equiv \sigma_0/4\rho_0 h$.

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Thus, the effect of the dark disk is to “pinch” the density distribution of the other components, as we can see in Figure 1. Therefore, although the scale height of the gas disk is proportional to its velocity dispersion according to Equation (7), a dark disk will reduce the gas disk’s thickness relative to this value, and for a fixed midplane density \( \rho(0) \), it implies that their surface densities, \( \Sigma_i \), are less than what it would be without the dark disk or any other mass component. In this approximation, the gas distribution will have a cusp at the origin, but in general the dark disk will have a finite thickness and the solution will be smooth near \( z = 0 \).

Integrating (5) gives the surface density of the visible component as

\[
\Sigma_{\text{vis}}(\Sigma_D) = \sqrt{\Sigma_{\text{vis}}(0)^2 + \Sigma_D^2} - \Sigma_D,
\]

where \( \Sigma_{\text{vis}}(0) = 4\rho_0 h \) is what the surface density would have been without the dark disk. This expression is monotonically decreasing with \( \Sigma_D \).

Another way of explaining this is that the dark disk “pinches” the visible matter disk, reducing its thickness, \( H_{\text{vis}} \). Because the surface density of the visible disk scales roughly as \( \Sigma_{\text{vis}} \sim \rho_{\text{vis}} H_{\text{vis}} \), the effect of the dark disk is to reduce the total surface density for a given midplane density \( \rho_{\text{vis}} \).

3. ANALYSIS

Here, we explain how we compare the surface densities of the various gas components estimated in the next section with those predicted by their midplane densities and velocity dispersions to place self-consistency constraints on the mass model. Section 2.1 explains how the presence of the dark disk decreases the surface density of each component if the midplane density is held fixed (as it is in a Poisson–Jeans solver). Thus, given fixed midplane densities, we can assign a probability to a model with any dark disk surface density, \( \Sigma_D \), and scale height, \( h_D \), based on how well the predicted surface densities, \( \Sigma_i \), determined from the Poisson–Jeans solver match the observed values.

Starting with the midplane densities and dispersions of Section 4, we solved the Poisson–Jeans equation for \( \Sigma_D \) values between 0 and 24 \( M_\odot \text{pc}^{-2} \). Each time the Poisson–Jeans equation was solved, the density distributions were integrated to give the total surface densities of \( H_2 \) and \( H_1 \). Each model was then assigned a probability according to the chi-squared distribution with 2 degrees of freedom, based on the deviation of this surface density from the measured values. We did this using different central values and uncertainties for the midplane densities. Thus, for example, using \( n_H = 0.19 \text{ cm}^{-3} \), we would take \( \rho_{H_2 + H_1}(0) = 1.42 \times n_H \times 0.19 \text{ cm}^{-3} = 0.013 M_\odot \text{pc}^{-2} \). If, for a model with this value of \( \rho(0) \) and with a certain dark disk surface density value of \( \Sigma_D \), we find an \( H_2 \) surface density, \( \Sigma_{H_2} = 1.0 M_\odot \text{pc}^{-2} \), then according to Section 4.1, we should assign this model a chi-squared value, \( \chi^2_{H_2} = (1.0 - 1.3)^2/\Delta^2_{\Sigma_{\text{H}_2}} \). We would then assign a probability to this model according to the Gaussian cumulative distribution, \( p_{H_2} = \int d\chi \exp(-\chi^2/2)/\sqrt{2\pi} \), where the limits of integration are from \(-\infty\) to \(-\chi_{H_2}\) and \(\chi_{H_2}\) to \(\infty\). We would similarly compute probabilities \( p_{H_1} \), \( p_{H_1+H_2} \), and a combined probability, \( p = p_{H_2} \times p_{H_1} \times p_{H_1+H_2} \). We note here that this is not the absolute probability of the model given the data; rather, it is the probability of the data given the model. We define a model for which the data is less probable than 5% to be excluded.

An important question is what to use for \( \Delta^2_\Sigma \). There are two uncertainties here. Namely, (1) the uncertainty in the surface density measurements, \( \Delta \Sigma_i \), and (2) the uncertainty in our output values of \( \Sigma(\rho, \Sigma_D) \), resulting from the uncertainty in the input midplane density measurements \( \rho_i \). Formally, this is \( |\partial \Sigma_i / \partial \rho| \Delta \rho_i \). Assuming Gaussian distributions for the measurements \( \Sigma_i \) and \( \rho_i \), and a uniform prior for \( \Sigma_D \), one can show...
We computed
\[ p(\hat{\Sigma}_i, \hat{\rho}_i|\Sigma_\text{D}) \sim \int d\rho_i \exp \left( -\frac{(\hat{\Sigma}_i - \Sigma(\rho_i, \Sigma_\text{D}))^2}{2\Delta_{\Sigma_i}^2} \right) \times \exp \left( -\frac{(\rho_i - \hat{\rho}_i)^2}{2\Delta_{\rho_i}^2} \right), \]
where \( \rho_i \) are the true midplane densities and that expanding \( \Sigma(\rho_i, \Sigma_\text{D}) \) to first order in \( \rho_i \), this integrates to give an approximately Gaussian distribution for \( \hat{\Sigma}_i - \Sigma(\hat{\rho}_i, \Sigma_\text{D}) \), with width
\[ \Delta_{\hat{\Sigma}_i} \approx \sqrt{\Delta_{\Sigma_i}^2 + \left( \frac{\partial \Sigma_i}{\partial \rho_i} \right)^2 \Delta_{\hat{\rho}_i}^2}, \]
The purpose of this section is to determine accurate values for \( \rho_i(0), \sigma_i \), and \( \Sigma_i \) (midplane density, velocity dispersion, and surface density) for the different components of interstellar gas based on existing measurements. Using Equation (2), these can then be compared for a given dark disk model in order to check for self-consistency.

We now discuss in detail the various measurements of the gas parameters and the uncertainties in each. Our starting point is the Bahcall model used by Flynn et al. (2006, Table 2). These values are updated from the ones used in Holmberg & Flynn (2000). Values for the stellar components were updated using the values of McKee et al. (2015), and are shown in rows 5–15 in Table 2 of Kramer & Randall (2016).

In these models, the gas and stars are both separated into approximately isothermal components as in Bahcall (1984b), so that each component \( i \) is characterized by a midplane density \( \rho_{i0} \) and a vertical dispersion \( \sigma_i \). Using only these values for all of the components, we can solve the Poisson–Jeans Equation (4) for the system. A major difference between our model and that of Flynn et al. (2006) is that their gas midplane densities were fixed by the values needed to give the correct surface densities in accordance with the Poisson–Jeans equation. We, on the other hand, use measured values of the midplane densities as we explain in this section.

We explain the various literature values that were included in the determination of the gas parameters. We also compare these to the recent values of McKee et al. (2015). In Section 5, the analysis is conducted separately for the values we determine by combining the results in the literature and the values obtained solely from the recent paper by McKee et al. (2015).

4. GAS PARAMETERS

We now explain the various measurements of the molecular hydrogen volume density and surface densities and how they are corrected. As molecular hydrogen cannot be observed directly, it must be inferred from the amount of CO present, derived from the intensity of the \( J = 1 \rightarrow 0 \) transition photons. These are related by the so-called X-factor, defined by
\[ N_{\text{H}_2} \equiv X_{\text{CO}} \]
where \( N_{\text{H}_2} = N_{\text{H}_2, \text{l.o.s.}} \) is the line of sight column density of \( \text{H}_2 \) molecules and \( X_{\text{CO}} \) is the total, velocity-integrated CO intensity along the line of sight (Draine 2011). Column densities perpendicular to the galactic plane can then be obtained by simple trigonometry:
\[ N_{\perp} = N_{\text{H}_2, \text{l.o.s.}} \sin b \]
and volume densities can be obtained by dividing the intensity density in velocity space \( dW_{\text{CO}}/dv \) by the rotation curve gradient \( dv/dr \), or by estimating the distance along the line of sight using other means. The volume and surface densities can also both be found by fitting an assumed distribution to measurements of the gas' vertical scale height \( \Delta z \). Surface densities can then be given, for example, by
\[ \Sigma_{\text{H}_2} = m_{\text{H}_2} N_{\perp, \text{H}_2} = m_{\text{H}_2} X_{\text{CO}} \sin b. \]

4.1. Molecular Hydrogen

The purpose of this section is to determine accurate values for \( \rho_i(0), \sigma_i \), and \( \Sigma_i \) (midplane density, velocity dispersion, and surface density) for the different components of interstellar gas based on existing measurements. Using Equation (2), these can then be compared for a given dark disk model in order to check for self-consistency.

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helium respectively. Thus,
\[ \Sigma_{\text{H}_2 + \text{He}} = 1.42 \Sigma_{\text{H}_2}. \]  

Note that Binney & Merrifield (1998, p. 662) did not include helium in the total ISM mass. Also Read (2014) did not distinguish between H I results including and not including helium.

We now explain how we obtain midplane volume densities \( n_{\text{H}_2}(z = 0) \) and surface densities \( \Sigma_{\text{H}_2 + \text{He}} \) from the various references in the literature. Bronfman et al. (1988) measured the molecular hydrogen over different radii within the solar circle. Their data are shown as one of the data sets in Figure 2. Averaging the values from the northern and southern Galactic plane in Table 4 of the latter, we find, for the measurements closest to the Sun, \( \Sigma_{\text{H}_2} = 2.2 \ M_\odot \, \text{pc}^{-2} \) and \( n_{\text{He}} = 0.2 \text{ cm}^{-3} \). Since surface densities depend only on the total integrated intensity along the line of sight, they are independent of the value of \( R_\odot \), the Sun’s radial position from the center of the Galaxy. On the other hand, it follows from this that old values for volume densities (which scale as \( R_\odot^{-1} \)) must be rescaled by \( R_\odot \text{old}/R_\odot \text{new} \) (Scoville & Sanders 1987). Since Bronfman et al. used the old value \( R_\odot = 10 \, \text{kpc} \), this value needs to be rescaled...
by $0.833^{-1}$ to take into account the new value of $R_\odot = 8.33 \pm 0.35 \text{kpc}$ (Gillessen et al. 2009). They also used an X-factor of $X = 2.8 \times 10^{20} \text{cm}^{-2}$ $(K^{-1} \text{km s}^{-1})^{-1}$. We correct this using a more recent value of $X = 1.8 \pm 0.3 \times 10^{20} \text{cm}^{-2}$ $(K^{-1} \text{km s}^{-1})^{-1}$, obtained by Dame et al. (2001). The most recent value of $R$, obtained by Okumura & Kamae (2009), is $X = 1.76 \pm 0.04 \times 10^{20} \text{cm}^{-2}$ $(K^{-1} \text{km s}^{-1})^{-1}$, although the value of Dame et al. that we use is still cited by Draine (2011) as the most reliable. These corrections give $n_{\text{HI}} = 1.50 M_\odot \text{pc}^{-2}$ and $\Sigma_{\text{HI}} = 1.4 M_\odot \text{pc}^{-2}$.

On the other hand, Clemens et al. (1988), found the local CO emissivity $J = dW_{\text{CO}}/dr$ in the first galactic quadrant for radii through $R_\odot$. For $R < R_\odot$ and $R > R_\odot$, respectively, they found these to be $J = 3.1$ and $2.3 \text{K km s}^{-1} \text{pc}^{-1}$, which, using $X = 1.8 \times 10^{20} \text{cm}^{-2}$ $(K^{-1} \text{km s}^{-1})^{-1}$, and rescaling for $R_\odot$ by $(0.833)^{-1}$, gives interpolated density $n_{\text{HI}}(R_\odot) = 0.19 \text{cm}^{-3}$. Using their FWHM measurements for $R_\odot$, we can convert their measurements to surface density values according to Equation (19). As before, the surface density values are independent of $R_\odot$. We have, interpolating to $R_\odot$, $\Sigma_{\text{HI} + \text{He}} = 2.0 M_\odot \text{pc}^{-2}$. The rescaled data are shown in Figure 2.

Another measurement is provided by Burton & Gordon (1978), who had already measured Galactic CO emissivity $J = dW_{\text{CO}}/dr$ between $R \approx 2$–16 kpc, assuming $R_\odot = 10 \text{kpc}$, from which we obtain $n_{\text{HI}}(R)$ after correcting for $R_\odot$, shown in Figure 2. Interpolating linearly, this gives $n(R_\odot) = 0.31 \text{cm}^{-3}$ (Sanders et al. 1984) also measured CO in the first and second Galactic quadrants within and outside the solar circle. They used the values $R_\odot = 10 \text{kpc}$ and $X = 3.6 \times 10^{20} \text{cm}^{-2}$ $(K^{-1} \text{km s}^{-1})^{-1}$. Their results for both volume and surface density, corrected to $R_\odot = 8.33 \text{kpc}$ and $X = 1.8 \times 10^{20} \text{cm}^{-2}$ $(K^{-1} \text{km s}^{-1})^{-1}$, are also shown in Figure 2. In particular, after rescaling and interpolating their volume densities, we have $n(0.95R_\odot) = 0.39 \text{cm}^{-3}$. For surface density, we obtain $\Sigma_{\text{HI} + \text{He}} = 2.7 M_\odot \text{pc}^{-2}$. This is the highest value in the literature. Grabelsky et al. (1987) also measured CO in the outer Galaxy, which, with a $1.8/2.8$ correction factor for $X$, as well as correcting $R_\odot$ from 10 to 8.33 kpc, their results near the Sun read $n(1.05R_\odot) = 0.14 \text{cm}^{-3}$ and $\Sigma_{\text{HI} + \text{He}}(1.05R_\odot) = 1.4 M_\odot \text{pc}^{-2}$ (Digel 1991) also measured H$_2$ in the outer Galaxy. Using his results, we find $n(1.06R_\odot) = 0.13 \text{cm}^{-3}$ and $\Sigma_{\text{HI} + \text{He}}(1.06R_\odot) = 2.1 M_\odot \text{pc}^{-2}$.

Dame et al. (1987), by directly observing clouds within 1 kpc of the Sun only, found local volume density $n_{\text{HI}} = 0.10 \text{cm}^{-3}$ and surface density $\Sigma_{\text{HI} + \text{He}} = 1.3 M_\odot \text{pc}^{-2}$, which, correcting for $X = 2.7$, to $1.8 \times 10^{20} \text{cm}^{-2}$ $(K^{-1} \text{km s}^{-1})^{-1}$, gives $0.08 \text{cm}^{-3}$ and $0.87 M_\odot \text{pc}^{-2}$. This volume density is lower than many other measurements, and may represent a local fluctuation in the solar region on a larger scale than the Local Bubble. On the other hand, their surface density value is not the lowest. Luna et al. (2006), using $X = 1.56 \times 10^{20} \text{cm}^{-2}$ $(K^{-1} \text{km s}^{-1})^{-1}$, found $\Sigma_{\text{HI} + \text{He}}(0.975R_\odot) = 0.24 M_\odot \text{pc}^{-2}$. Correcting for $X$ gives $0.29 M_\odot \text{pc}^{-2}$, which is the lowest value in the literature. However, they admit that their values beyond $0.875R_\odot$ are uncertain. Another determination from 2006 (Nakanishi & Sofue 2006) gives, after interpolation, $n_{\text{HI}}(R_\odot) = 0.17 \text{cm}^{-3}$ and $\Sigma_{\text{HI}}(R_\odot) = 1.4 M_\odot \text{pc}^{-2}$, or $\Sigma_{\text{HI}}(R_\odot) = 2.0 M_\odot \text{pc}^{-2}$.

Figure 2 shows the various measurements described here, as well as the overall average and standard error. Although not all measurements are equally certain, in computing average values for $n_{\text{HI}}$ and $\Sigma_{\text{HI}}$, we treated all measurements with equal weight. We estimated the resulting uncertainty as the standard deviation divided by the the square root of the number of measurements available at each $R$. We found the mean values and standard errors of volume and surface densities near the Sun to be

$$n_{\text{HI}}(R_\odot) = 0.19 \pm 0.03 \text{cm}^{-3},$$

$$\Sigma_{\text{HI} + \text{He}}(R_\odot) = 1.55 \pm 0.32 M_\odot \text{pc}^{-2}.$$
or $\Sigma_{\text{H}^1+\text{He}} = 7.1 \, M_\odot \, \text{pc}^{-2}$. Another measurement is provided by Burton & Gordon (1978), who measured volume densities for $R \sim 2 - 16 \, \text{kpc}$. We interpolate their data (and correct for $R_0 = 10 \, \text{kpc} \rightarrow 8.33 \, \text{kpc}$) to obtain $n_{\text{H}^1} = 0.49 \, \text{cm}^{-3}$. Although they did not determine surface densities, we can estimate them by assuming a single Gaussian component with FWHM given Dickey & Lockman (220–230 pc). A better estimate is perhaps obtained by assuming, rather than a Gaussian distribution, a distribution with the same shape as Dickey & Lockman. This amounts to assuming an effective Gaussian FWHM of $\sim 330 \, \text{pc}$. This gives a surface density near the Sun of $\Sigma_{\text{H}^1+\text{He}} = 5.9 \, M_\odot \, \text{pc}^{-2}$. Liszt (1992), however, argues that the midplane density of Dickey & Lockman was artificially enhanced to give the correct surface density. He measures midplane density $n_{\text{H}^1} = 0.41 \, \text{cm}^{-3}$, which, assuming as for Burton & Gordon a Gaussian distribution with effective FWHM 330 pc, gives a surface density of only $\Sigma_{\text{H}^1+\text{He}} = 5.1 \, M_\odot \, \text{pc}^{-2}$. Nakaniishi & Sofue (2003) also measured the Galactic H I, from the Galactic center out to $\sim 25 \, \text{kpc}$. Their results are shown in Figure 3. Interpolating to $R_0$, we have $n_{\text{H}^1}(R_0) = 0.28 \, \text{cm}^{-3}$ and $\Sigma_{\text{H}^1+\text{He}} = 5.9 \, M_\odot \, \text{pc}^{-2}$, in agreement with the value of Burton & Gordon.

On the other hand, there are several authors who report much larger mass parameters for Galactic H I. They are Wouterloot et al. (1990) and Kalberla & Dedes (2008). Wouterloot et al. used 21 cm observations from outside the solar circle. Their data are shown in Figure 3. Closest to the Sun, their data show $\Sigma_{\text{H}^1+\text{He}}(1.06 R_0) = 8.6 \, M_\odot \, \text{pc}^{-2}$ with a FWHM of 300 pc. This corresponds to a midplane density of roughly $n_{\text{H}^1} = 0.73 \, \text{cm}^{-3}$. The Kalberla & Dedes data (also shown in Figure 3) show $\Sigma_{\text{H}^1+\text{He}} \approx 10 \, M_\odot \, \text{pc}^{-2}$. A more refined estimate gives $\Sigma_{\text{H}^1+\text{He}} \approx 9 \, M_\odot \, \text{pc}^{-2}$ (McKee et al. 2015). This is consistent with a midplane density of roughly 0.8 cm$^{-3}$. This is much higher than the value of Kalberla & Kerp (1998), who obtained $n_{\text{CNM}} = 0.3 \, \text{cm}^{-3}$ and $n_{\text{WNM}} = 0.1 \, \text{cm}^{-3}$. However, there is reason to expect a relatively high H I midplane density. Based on extinction studies, Bohlin et al. (1978) find a total hydrogen nucleus density $2n_{\text{HI}} + n_{\text{H}^1} = 1.15 \, \text{cm}^{-3}$. Updating this for the newer value of the Galactocentric radius of the Sun $R_0$ as in Section 4.1, we have $2n_{\text{HI}} + n_{\text{H}^1} = 1.38 \, \text{cm}^{-3}$. According to the average midplane density determined for molecular hydrogen in Section 4.1, $n_{\text{HI}} = 0.19 \pm 0.03 \, \text{cm}^{-3}$, and including an additional $0.15 \pm 0.07 \, \text{cm}^{-3}$ for the dark molecular hydrogen, we therefore expect an atomic hydrogen density $n_{\text{H}^1} = 0.70 \pm 0.18 \, \text{cm}^{-3}$. Optical thickness corrections, which we explain below, increase this number to 0.84 cm$^{-3}$. The results are shown in Figure 3. As in the case of molecular hydrogen, all measurements were treated with equal weight and the uncertainty was estimated as the standard error at each $R$.

Combining all these results, we have, in the absence of optical thickness corrections,

$$n_{\text{H}^1}(R_0) = 0.53 \pm 0.10 \, \text{cm}^{-3}, \quad (27)$$

$$\Sigma_{\text{H}^1+\text{He}}(R_0) = 7.2 \pm 0.7 \, M_\odot \, \text{pc}^{-2}. \quad (28)$$

In the Dickey & Lockman (1990) model, 70% of this H I midplane density is in CNM and the remaining 30% is WNM. In Kalberla & Kerp (1998), the numbers are 75% and 25%. We will take the average of these two results, 72.5% and 27.5%, which give $n_{\text{CNM}} = 0.38 \, \text{cm}^{-3}$ and $n_{\text{WNM}} = 0.15 \, \text{cm}^{-3}$.

McKee et al. (2015) pointed out that these numbers must be corrected for the optical depth of the CNM. Assuming the CNM to be optically thin leads to an underestimate of the CNM column density by a factor $R_{\text{CNM}}$. McKee et al. (2015) estimate this factor to be $R_{\text{CNM}} = 1.46$, which they translate, for the total H I column density, to $R_{\text{HI}} = 1.20$. Correcting for this gives

$$n_{\text{CNM}} \approx 0.56 \, \text{cm}^{-3}, \quad (29)$$

$$n_{\text{WNM}} \approx 0.15 \, \text{cm}^{-3}. \quad (30)$$

with totals

$$n_{\text{HI}}(R_0) = 0.71 \pm 0.13 \, \text{cm}^{-3}, \quad (31)$$

$$\Sigma_{\text{H}^1+\text{He}}(R_0) = 8.6 \pm 0.8 \, M_\odot \, \text{pc}^{-2}. \quad (32)$$

which we use for this analysis.

On the other hand, McKee et al. (2015) argues that the model of Heiles et al. (1981) is more accurate, and recommends increasing the amount of H I in the ISM by a factor of 7.45/6.2. McKee et al. (2015)'s values are therefore $n_{\text{CNM}} = 0.69 \, \text{cm}^{-3}$, $n_{\text{WNM}} = 0.21 \, \text{cm}^{-3}$, $n_{\text{HI}} = 0.90 \, \text{cm}^{-3}$, and $\Sigma_{\text{HI}} = 10.0 \pm 1.5 \, M_\odot \, \text{pc}^{-2}$. Although these numbers are different from our average of conventional measurements (Equations (29)–(32)), it agrees with the extinction result of Bohlin et al. (1978) mentioned above once the latter is corrected for the optical depth of the CNM. To account for any discrepancy, we perform our analysis separately using the values of Equations (29)–(32) and the results of McKee et al. (2015). We present both results in Section 5.

For the atomic hydrogen’s velocity dispersion, Heiles & Troland (2003), found $\sigma_{\text{CNM}} = 7.1 \, \text{km} \, \text{s}^{-1}$ and $\sigma_{\text{WNM}} = 11.4 \, \text{km} \, \text{s}^{-1}$, while Kalberla & Dedes (2008) found $\sigma_{\text{CNM}} = 6.1 \, \text{km} \, \text{s}^{-1}$ and $\sigma_{\text{WNM}} = 14.8 \, \text{km} \, \text{s}^{-1}$. Earlier, Belfort & Crouviér (1984) measured $\sigma_{\text{HI}} = 6.9 \pm 0.4 \, \text{km} \, \text{s}^{-1}$, and Dickey & Lockman (1990) found $\sigma_{\text{HI}} = 7.0 \, \text{km} \, \text{s}^{-1}$ but did not specify if the gas was CNM or WNM. Since these are comparable to more recent measurements of the CNM component of H I, we assume both of these to correspond to $\sigma_{\text{CNM}}$. The averages of these values are

$$\sigma_{\text{CNM}} = 6.8 \pm 0.5 \, \text{km} \, \text{s}^{-1}, \quad (33)$$

$$\sigma_{\text{WNM}} = 13.1 \pm 2.4 \, \text{km} \, \text{s}^{-1}. \quad (34)$$

### 4.3. Ionized Hydrogen

Besides the H$_2$ and the two types of H I (CNM and WNM), there is a fourth, warm, ionized component of interstellar hydrogen, denoted H II. Holmberg & Flynn (2000) and Flynn et al. (2006) included this component. Binney & Merrifield (1998) did not include the ionized component in the value for $\Sigma_{\text{ISM}}$, possibly because of its very large scale height. Its density is typically obtained by measuring the dispersion of pulsar signals that have passed through the H II clouds. The time delay for a pulse of a given frequency is proportional to the dispersion measure

$$\text{DM} = \int n_e \, ds, \quad (35)$$

where the integral is performed along the line of sight to the pulsar, and where $n_e$ is the electron number density, equal to the number density of ionized gas. The dispersion measure perpendicular to the plane of the Galaxy, $\text{DM} = \text{DM}/\sin b$, therefore corresponds to the half surface density 1/2 $\Sigma_{\text{HII}}$. Fitting a spatial distribution (e.g., exponential profile),
provides midplane density information. For its midplane density, Kulkarni & Heiles (1987) found $n_{\text{H}_2} = 0.030$ cm$^{-3}$; Cordes et al. (1991) found $n_{\text{H}_2} = 0.024$ cm$^{-3}$; Reynolds (1991) found $n_{\text{H}_2} = 0.040$ cm$^{-3}$. The average of these values is
\begin{equation}
    n_{\text{H}_2} = 0.031 \pm 0.008 \text{ cm}^{-3}.
\end{equation}
This agrees with the traditional model of Taylor & Cordes (1993), refined by Cordes & Lazio (2002), who found a midplane density of
\begin{equation}
    n_{\text{H}_2} = 0.034 \text{ cm}^{-3}.
\end{equation}
For the H$^+$ surface density, Reynolds (1992) reports $\Sigma_{\text{H}_2+\text{He}} = 1.57 M_\odot$ pc$^{-2}$. This is slightly higher than what was found by Taylor & Cordes (1993), who found a one-sided column density $1/2 N_{\text{H}_2} = 16.5$ cm$^{-3}$ pc, or $\Sigma_{\text{H}_2+\text{He}} = 1.1 M_\odot$ pc$^{-2}$, but it is slightly lower than the more recent value of Cordes & Lazio (2002), who found $1/2 N_{\text{H}_2} = 33$ cm$^{-3}$ pc.
or $\sum_{\text{H II,He}} \approx 2.3 M_\odot$ pc$^{-2}$. Assuming an exponential profile, with the scale height of 0.9 kpc of Taylor & Cordes (1993), the Reynolds (1992) result agrees with the midplane densities of Equations (36) and (37). However, Gaensler et al. (2008) argued for a scale height of 1.8 kpc that a distribution with midplane density of

$$n_{\text{H II}} = 0.014 \text{ cm}^{-3}.$$  

(38)

Similarly, Schnitzeler (2012) also argues for large scale heights of $\sim 1.4$ kpc. For DM values between 20 and 30 cm$^{-3}$ pc, this gives a midplane density of $\sim 0.015$ cm$^{-3}$ pc, as preferred by McKee et al. (2015). As we explain in Section 5, we do not find our model to be consistent with these large scale heights, even without a dark disk. We therefore do not use H II parameters in this paper as a constraint.

For its velocity dispersion, Holmberg & Flynn (2000) used the value $\sigma_{\text{H II}} = 40$ km s$^{-1}$. This value seems to have been inferred from scale height measurements of the electrons associated with this ionized gas from Kulkarni & Heiles (1987). From the data in Reynolds (1985), however, we find a turbulent component to the dispersion of only $\sigma_{\text{H II}} = 21 \pm 5$ km s$^{-1}$. On the other hand, temperatures between 8000 and 20,000 K give a thermal contribution of $\sigma_{\text{H II,thermal}} = \sqrt{2.1 k_B T/m_p} \approx 12 - 19$ km s$^{-1}$ (Ferrière 2001, p.14). Summing these in quadrature gives $\sigma_{\text{H II}} = 25 - 29$ km s$^{-1}$. As we will explain in Section 4.4, including magnetic and cosmic ray pressure contributions pushes this up to 42 km s$^{-1}$. Similarly, Kalberla (2003) also finds $\sigma_{\text{H II}} = 27$ km s$^{-1}$ while assuming $P_{\text{mag}} = P_{\text{vis}} = 1/3 P_{\text{turb}}$, for a total effective dispersion of 35 km s$^{-1}$ but did not include a thermal contribution. This gives an average total effective dispersion of $39 \pm 4$ km s$^{-1}$, which, removing magnetic, cosmic ray, and thermal contributions, gives a turbulent dispersion of $\sigma_{\text{turb}} = 22 \pm 3$ km s$^{-1}$.

The new gas parameter estimates, obtained in this work by incorporating a broad range of literature values, are summarized in Table 1 alongside the old (Flynn et al. 2006) values. The values of McKee et al. (2015) are also included for comparison.

### 4.4. Other Forces

Boulares & Cox (1990) considered the effect of magnetic forces and cosmic ray pressure on the interstellar gas. The effect of the magnetic field is a contribution to the force per unit volume on the $i$th component of the gas:

$$f_i = J_i \times B$$  

(39)

where $J_i$ is the current density associated with gas component $i$, $B$ is the magnetic induction field due to component $i$, and where

$$B = \sum_i B_i$$  

(40)

is the total magnetic field from all the gas components. Using Ampère’s law, we can rewrite the $z$-component of the force as

$$f_z = \frac{1}{\mu_0} ((\nabla \times B_i) \times B)_z$$  

(41)

$$= \frac{1}{\mu_0} (B \cdot \text{div}) B_z - \frac{1}{\mu_0} B \cdot \frac{\partial B_i}{\partial z}$$  

(42)

Since according to Parker (1966), the magnetic field is, on average, parallel to the plane of the Galaxy, $(B_z = 0)$ we will make the approximation that the first term vanishes in equilibrium. The second term couples each gas component to the remaining components, since $B$ represents the total magnetic field. However, summing all components, we have

$$f_z = \sum_i f_z = -\frac{1}{\mu_0} B \cdot \frac{\partial B}{\partial z}$$  

(43)

$$= -\frac{\partial}{\partial z} \left( \frac{B^2}{2\mu_0} \right).$$  

(44)

We recognize the form of this expression as the gradient of the magnetic pressure $P_B = B^2/2\mu_0$. To include this effect in the Poisson–Jeans Equation, we note that the first term on the left-hand side of Equation (1) has the interpretation (up to an overall mass factor) as the gradient of a “vertical pressure.” This pressure term is a correct description of a population of stars or of gas clouds. In a warm gas, this term has the interpretation as the turbulent pressure of the gas. However, in this case, one also needs to take into account the thermal pressure of the gas

$$P_{\text{thermal}} = c_i n_i k_B T_i$$  

(45)

where $c_i$ is a factor that takes into account the degree of ionization of the gas, and $n_i = \rho_i/m_p$ is the number density of the gas atoms. The correct Poisson–Jeans equation in this case therefore reads

$$\frac{\partial}{\partial z} (\rho_i \sigma_i^2 + \rho_i c_i k_B T_i) + \rho_i \frac{\partial \Phi}{\partial z} = 0.$$  

(46)

If we define a “thermal dispersion” as

$$\sigma_{T,i}^2 = c_i k_B T_i,$$  

(47)

then we can rewrite this as

$$\frac{\partial}{\partial z} (\rho_i (\sigma_i^2 + \sigma_{T,i}^2)) + \rho_i \frac{\partial \Phi}{\partial z} = 0.$$  

(48)

Clearly, to account for the magnetic pressure, we would include the average of the magnetic pressure term in precisely the same way.
where $\rho$ is the total mass density of the gas. In the following subsections, we describe how we model this magnetic pressure term.

4.4.1. Magnetic Pressure: Thermal Scaling Model

An important phenomenon noted by Parker (1966) is that the magnetic field $B$ is confined by the weight of the gas through which it penetrates. We therefore would like to solve this equation by following Parker in assuming that the magnetic pressure is proportional to the thermal pressure term, $p_i = \rho_i c_B T_i$. Since each gas component contributes to the total thermal pressure with a different temperature $T_i$, we write:

$$\left(\frac{B^2(z)}{2\mu_0}\right) = \alpha \sum_i \rho_i(z) \sigma^2_{i,T}$$

where $\alpha$ is a proportionality constant fixed by $<B^2(0)>$ and $\sum_i \sigma^2_{i,T}$, and where we have defined the “magnetic dispersion” $\sigma^2_{i,B} = \alpha \sigma^2_{i,T}$ i.e., the effective dispersion arising from the magnetic pressure. The Poisson–Jeans equation then reads

$$\sum_i \frac{\partial}{\partial z} \left(\rho_i(z) \sigma^2_{i,T}+\sigma^2_{i,B}\right) + \rho \frac{\partial \Phi}{\partial z} = 0.$$  

The above equation admits many solutions. However, we will assume that the unsmeared equation

$$\frac{\partial}{\partial z} \left(\rho_i(z) \sigma^2_{i,T}+\sigma^2_{i,B}\right) + \rho \frac{\partial \Phi}{\partial z} = 0.$$  

holds for each component individually. This amounts to assuming that all gas components confine the magnetic field equally. Other solutions can be found by substituting $\sigma^2_{i,B} \rightarrow \sigma^2_{i,B} + \Delta S_i(z)$, such that $\sum_i \rho_i(z) \Delta S_i(z) = 0$. However, if we restrict our analysis to “isothermal” solutions (constant $\sigma^2_{i,B}$) the solution $S_i = 0$ will be unique. We can also include the effects of cosmic ray pressure in a similar way, by assuming that the partial cosmic ray pressure is also proportional to the density

$$p_{i,cr}(z) = \rho_i(z) \sigma^2_{i,cr}$$

and where $\sigma^2_{i,cr} = \beta \sigma^2_{i,T}$ for some other constant $\beta$. The Poisson–Jeans Equation then reads

$$\frac{\partial}{\partial z} \left(\rho_i \sigma^2_{i,cr}\right)$$

where we have defined

$$\sigma^2_{i,cr} = \sigma^2_{i} + \sigma^2_{i,T} + \sigma^2_{i,B} + \sigma^2_{i,cr}.$$  

The solution to the Poisson–Jeans Equation for each component will then be

$$\rho_i(z) = \rho_i(0) \exp\left(-\frac{\Phi(z)}{\sigma^2_{i,cr}}\right).$$  

Note that since the pressure is additive, the dispersions add in quadrature. Boulaires & Cox (1990) estimate for the magnetic pressure $p_B \approx (0.4 - 1.4) \times 10^{-12} \text{dyn cm}^{-2}$. For the cosmic ray pressure, they estimate $p_{cr} \approx (0.8 - 1.6) \times 10^{-12} \text{dyn cm}^{-2}$. The dispersions for each component are shown in Table 2 for comparison, as well as the effective dispersions in this model.

4.4.2. Magnetic Pressure: Warm Equipartition Model

Here we describe a second possible model to describe the magnetic and cosmic ray pressures in the interstellar medium. Namely, it has been observed that within the CNM, energy densities in magnetic fields and in turbulence are often roughly equal (Heiles & Troland 2003; Heiles & Crutcher 2005). Although the ratio between these energies is observed to vary greatly over different molecular clouds, this so-called “energy equipartition” seems to be obeyed on average. Physically, this happens because the turbulence amplifies the magnetic field until it becomes strong enough to dissipate through van Alfvén waves. Similarly, we expect magnetic fields to trap cosmic rays within the gas until they become too dense and begin to escape. We might, therefore, expect the cosmic ray and magnetic field energy densities to be similar. For these reasons, an alternative to the first model (Equation (50)) would be to assume equipartition of pressure between turbulence, magnetic fields, and cosmic rays:

$$\sigma^2_i = \sigma^2_{i,B} = \sigma^2_{i,cr}.$$  

for each component $i$. The effective dispersion, which is the sum of turbulent, magnetic, cosmic ray, and thermal contributions, would then be

$$\sigma^2_{i,eff} = \sigma^2_i + \sigma^2_{i,B} + \sigma^2_{i,cr} + \sigma^2_{i,T} \approx 3 \sigma^2_i + \sigma^2_{i,T}$$

for each component. One important factor that we should not overlook here, however, is that the molecular hydrogen and CNM condense to form clouds. Thus, although turbulence, magnetic fields, and cosmic rays may affect the size of the individual clouds, we expect the overall scale height of the cold components to be determined only by the cloud–cloud dispersion and not by these forces. We therefore assume Equations (58)–(60) only for the warm components WNM and H II. For the cold components (H$_2$ and CNM), we assume $\sigma_{eff} = \sigma_i$. We perform calculations separately for the two different magnetic field and cosmic ray models. The effective dispersions in both models are shown in Table 2. As we will
5. RESULTS AND DISCUSSION

We now present the results of the analysis described above. Using the midplane densities of Section 4, we calculate according to the Poisson–Jeans equation the corresponding \( \text{H}_2 \) and HI surface densities, and from these we compute the chi-squared value, \( \chi^2 \), from the disagreement between these values and the measured values. This is done over a range of values for \( S_D \) and \( h_D \). We thereby determine the regions of parameter space where the disagreement exceeds the 68% and 95% bounds, as will be displayed in the plots below. The scale height \( h_D \) is defined such that

\[
\rho(Z = h_D) = \rho(Z = 0) \sech^2(1/2).
\]

We begin by determining the bounds without including the contribution from magnetic fields and cosmic rays. The result is shown in Figure 4. The uncertainty, \( \text{H}_2 \), is dominated by that of the dark molecular gas, while the uncertainty of HI is dominated by that of the WNM velocity dispersion (18%). We can see that although the \( \text{H}_2 \) parameters are consistent with dark disk surface densities of (for low-scale height) up to \( 10-12 \, M_\odot \, \text{pc}^{-2} \), the HI parameters point toward lower surface densities, and that the combined probabilities are lower than 9% for all models. These results make it apparent that the model without magnetic fields is inconsistent.

On the other hand, when we include the pressure contribution from the magnetic fields and from cosmic rays, we find that both the HI and \( \text{H}_2 \) parameters allow non-zero surface densities, \( S_D \), with an upper bound of \( \Sigma_D \approx 10 \, M_\odot \, \text{pc}^{-2} \) in both models, for low scale heights. Higher scale heights are consistent with even higher dark disk surface densities. The results are shown in Figure 5.

For comparison, we also include the corresponding results using the values of McKee et al. (2015). These are shown in Figure 6. Using these values and including magnetic fields and cosmic ray contributions, the data favors a non-zero surface density for the dark disk of between 5 and \( 15 \, M_\odot \, \text{pc}^{-2} \). Note that when neglecting magnetic field and cosmic rays pressures, only low dark disk surface densities seem consistent with the data.

### 5.0.3. Ionized Hydrogen Results and Issues

As was mentioned in Section 4.3, various authors have measured DM values for the H II component of the Milky Way in the range 20–30 cm\(^{-3}\) pc. Older models favored low-scale height with midplane densities as high as 0.034 cm\(^{-3}\), while newer models favor large scale height models with midplane densities as low as 0.014 cm\(^{-3}\). However, using our model, the results of our Poisson–Jeans solver are consistent with

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**Table 2**

| Component | \( \sigma \) (km s\(^{-1}\)) | \( \sigma_T \) (km s\(^{-1}\)) | \( \sigma_B \) (km s\(^{-1}\)) | \( \sigma_{cr} \) (km s\(^{-1}\)) | \( \sigma_{ef} \) (Thermal Scaling) (km s\(^{-1}\)) | \( \sigma_{ef} \) (Warm Equipartition) (km s\(^{-1}\)) |
|-----------|----------------|----------------|----------------|----------------|----------------|----------------|
| H\(_2\)    | 3.7            | 0.2            | 0.3            | 0.3            | 3.7            | 6.4            |
| H\(_2\)CNM| 6.8            | 0.8            | 1.2            | 1.3            | 7.1            | 11.8           |
| H\(_2\)WNM| 13.1           | 6.7            | 10.3           | 10.9           | 21             | 23.7           |
| H\(_\text{II}\) | 22             | 11.8           | 18.1           | 19             | 36.2           | 39.9           |

**Figure 4.** Confidence bounds on DDDM parameter space as a function of \( h_D \), the dark disk sech\(^2(c/2h_D) \) scale height, using averages and uncertainties from Sections 4.1 to 4.3. Solid lines represent 95% bounds and dashed lines represent 68% bounds.
only low-scale heights. Following the models described in Section 4.4, we find scale heights for the H II of 0.9 kpc for the thermal scaling model and 1.0 kpc for the warm equipartition model, assuming $\Sigma_D = 0$. Incorporating a more massive dark disk makes these scale heights smaller. Possible reasons for this might be:

1. The magnetic field model must be modified to include a different value of $\alpha$ for H II. This could be in correspondence with the result of Beuermann et al. (1985), who found that Galactic magnetic fields contained two components, one with short scale height and one with larger scale height. These two components would likely be described by different $\alpha$. We could then attribute the low-scale height component to the molecular and atomic gas and the large scale height component to the ionized gas. We know of no such alternative in the warm equipartition model.

2. The isothermal assumption may not be valid for H II. In fact, as explained in Gaensler et al. (2008), the volume filling fraction of H II may also vary a lot with scale height. If this is the case, then it would be incorrect to treat the H II as an isothermal component, as the degrees of freedom that the temperature describes (the gas clouds) vary with distance from the Galactic midplane.

5.0.4. Stability Issues and Kinematic Constraints

In Figure 7, we show the bound that we obtained from the kinematics of A stars in the solar region, accounting for
nonequilibrium features of the population, namely a net displacement and vertical velocity relative to the Galactic midplane. We also note that there will exist disk stability bounds. The true analysis is subtle, but a step toward the analysis is done by Shaviv (2016b), who develops the stability criterion for a heterogeneous Milky Way disk including a thin dark matter disk. We convert his bound to a bound in the $S_{\text{hDD}}$ plane and superimpose this bound on the gas parameter bound of the present work. We see that a disk with significant mass ($S_{D}$) and $h_{D} > 30$ pc is consistent with all current bounds.

In addition to stability issues, Hessman (2015) argued that there exist other issues with using the vertical Jeans equation to constrain the dynamical mass in the MW disk. In particular, spiral structure must be taken into account when performing these analyses. Indeed, Shaviv (2016a) pointed out that the effect of spiral arm crossing is to induce a “ringing” in the dynamics of tracer stars. However, the present analysis assumes that the timescales for this ringing are much shorter in gas components so that the analysis is valid. Spiral arm crossing could also induce nonequilibrium features in the tracer population, such as discussed in Kramer & Randall (2016), but as in Kramer & Randall, including this effect would allow for more dark matter.

6. CONCLUSION

In this paper, we showed how to use measured midplane and surface densities of various galactic plane components to constrain or discover a dark disk. Although literature values of atomic hydrogen midplane densities are discordant, their mean value is consistent with the remaining gas parameters when magnetic and cosmic ray pressures are included. Using the global averages of literature values of gas parameters that we compiled, we find that the data are consistent with dark disk surface densities as high as $10 M_{\odot}$ pc$^{-2}$ for low scale height, and as low as zero. The gas parameters of McKee et al. (2015) seem to favor an even higher non-zero dark disk surface density. Current data are clearly inadequate to decide this definitively. Further measurements of visible and dark H$_2$ density and WNM density and dispersion, as well as further refinements of magnetic field and cosmic ray models for cold gas, could allow placing more robust bounds on a dark disk.

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