We discuss a five dimensional inflationary scenario based on a supersymmetric $SO(10)$ model compactified on $S^1/(Z_2 \times Z_2^\prime)$. Inflation is implemented through scalar potentials on four dimensional branes, and a brane-localized Einstein-Hilbert term is essential to make both brane vacuum energies positive during inflation. The orbifold boundary conditions break the $SO(10)$ gauge symmetry to $SU(4)_c \times SU(2)_L \times SU(2)_R$ ($\equiv H$). The inflationary scenario yields $\delta T/T \propto (M/M_{\text{Planck}})^2$, which fixes $M$, the symmetry breaking scale of $H$ to be close to the SUSY GUT scale of $10^{16}$ GeV. The scalar spectral index $n$ is $0.98 - 0.99$, while the gravitational wave contribution to the quadrupole anisotropy is negligible ($\lesssim 1\%$). The inflaton decay into the lightest right handed neutrinos yields the observed baryon asymmetry via leptogenesis.
There exists a class of supersymmetric models in which a close link exists between inflation and the grand unification scale \[1, 2\]. In particular, the quadrupole microwave anisotropy is proportional to \( (M/M_{\text{Planck}})^2 \), where \( M \) denotes the scale of the gauge symmetry breaking associated with inflation, and \( M_{\text{Planck}} = 1.2 \times 10^{19} \) GeV. Thus, \( M \) is expected to be of order \( 10^{16} \) GeV, to within a factor of 2 or so, depending on the details of the supersymmetric model. This is tantalizingly close to the supersymmetric grand unification scale inferred from the evolution of the minimal supersymmetric standard model (MSSM) gauge couplings, and it is therefore natural to try to realize this inflationary scenario within a grand unified framework \[2\].

The \( SO(10) \) model is particularly attractive in view of the growing confidence in the existence of neutrino oscillations \[3\], which require that at least two of the three known neutrinos have a non-zero mass. Because of the presence of right-handed neutrinos (MSSM singlets), non-zero masses for the known neutrinos is an automatic consequence of the see-saw mechanism \[4\]. Furthermore, the right-handed neutrinos play an essential role in generating the observed baryon asymmetry via leptogenesis \[5\], which becomes especially compelling within an inflationary framework \[6\]. Indeed, an inflationary scenario would be incomplete without explaining the origin of the observed baryon asymmetry, and the kind of models we are interested in here automatically achieve this via leptogenesis.

A realistic supersymmetric inflationary model along the lines we are after was presented in \[7\], based on the \( SO(10) \) subgroup \( SU(4)_c \times SU(2)_L \times SU(2)_R (\equiv H) \) \[8\]. The scalar spectral index \( n \) has a value very close to unity (typically \( n \approx 0.98 - 0.99 \)), while the symmetry breaking scale of \( H \) lies, as previously indicated, around \( 10^{16} \) GeV. The vacuum energy density during inflation is of order \( 10^{14} \) GeV, so that the gravitational contribution to the quadrupole anisotropy is essentially negligible. It is important to note here that the inflaton field in this scenario eventually decays into right handed neutrinos, whose out of equilibrium decays lead to leptogenesis. An extension to the full \( SO(10) \) model is complicated by the notorious doublet-triplet
splitting problem, which prevents a straightforward implementation of the inflationary scenario. Of course, the subgroup $H$ neatly evades this problem and even allows for a rather straightforward resolution of the ‘$\mu$ problem.’

Our objective here is to take advantage of recent orbifold constructions of five dimensional (5D) supersymmetric GUTs, in which a grand unified symmetry such as $SO(10)$ can be readily broken to its maximal subgroup $H$ [9] (an alternative possibility is $SU(5) \times U(1)$ which we will not pursue here), with the doublet-triplet splitting problem circumvented without fine tuning of parameters. Our main challenge then is to develop a 5D framework which can be merged with the four dimensional (4D) supersymmetric inflationary scenario based on $H$. Because of $N = 2$ SUSY (in 4D sense) in 5D bulk, the F-term inflaton potential is allowed only on the 4D orbifold fixed points (branes), where only $N = 1$ SUSY is preserved. We shall see how 4D inflation comes about through scalar potentials localized on the two branes by analyzing the 5D Einstein equation. A brane-localized Einstein-Hilbert term is essential to make both brane vacuum energies positive definite during inflation, which is a condition required by 4D $N = 1$ SUSY.

The four dimensional inflationary model is best illustrated by considering the following superpotential which allows the breaking of some gauge symmetry $G$ down to $SU(3)_c \times SU(2)_{L} \times U(1)_Y$, keeping supersymmetry (SUSY) intact [1, 10]:

$$W_{\text{infl}} = \kappa S(\phi \bar{\phi} - M^2).$$

(1)

Here $\phi$ and $\bar{\phi}$ represent superfields whose scalar components acquire non-zero vacuum expectation values (VEVs). For the particular example of $G = H$ above, they belong to the $(\mathbf{4}, \mathbf{1}, \mathbf{2})$ and $(\mathbf{4}, \mathbf{1}, \mathbf{2})$ representations of $H$. The $\phi$, $\bar{\phi}$ VEVs break $H$ to the MSSM gauge group. The singlet superfield $S$ provides the scalar field that drives inflation. Note that by invoking a suitable $R$ symmetry $U(1)_R$, the form of $W$ is unique at the renormalizable level, and it is gratifying to realize that $R$ symmetries naturally occur in (higher dimensional) supersymmetric theories and can be appropriately exploited. From $W$, it is straightforward to show that the supersymmetric
minimum corresponds to non-zero (and equal in magnitude) VEVs for \( \phi \) and \( \bar{\phi} \), while \( \langle S \rangle = 0 \) \[1\]. (After SUSY breaking à la \( N = 1 \) supergravity (SUGRA), \( \langle S \rangle \) acquires a VEV of order \( m_{3/2} \) (gravitino mass)).

An inflationary scenario is realized in the early universe with both \( \phi \), \( \bar{\phi} \) and \( S \) displaced from their present day minima. Thus, for \( S \) values in excess of the symmetry breaking scale \( M \), the fields \( \phi \), \( \bar{\phi} \) both vanish, the gauge symmetry is restored, and a potential energy density proportional to \( M^4 \) dominates the universe. With SUSY thus broken, there are radiative corrections from the \( \phi \)-\( \bar{\phi} \) supermultiplets that provide logarithmic corrections to the potential which drives inflation. In one loop approximation \[1\] \[2\],

\[
V \approx V_{\text{tree}} + \kappa^2 M^4 \frac{\kappa^2 N}{32\pi^2} \left[ 2 \ln \frac{\kappa^2 |S|^2}{\Lambda^2} + (z + 1)^2 \ln(1 + z^{-1}) + (z - 1)^2 \ln(1 - z^{-1}) \right],
\]

where \( z = x^2 = |S|^2/M^2 \), \( \Lambda \) denotes a renormalization mass scale and \( N \) denotes the dimensionality of the \( \phi \), \( \bar{\phi} \) representations. From Eq. (2) the quadrupole anisotropy is found to be \[1\] \[2\]

\[
\left( \frac{\delta T}{T} \right)_Q \approx \frac{8\pi}{\sqrt{N}} \left( \frac{N_Q}{45} \right)^{1/2} \left( \frac{M}{M_{\text{Planck}}} \right)^2 x_Q^{-1} y_Q^{-1} f(x_Q^2)^{-1}.
\]

The subscript \( Q \) is there to emphasize the epoch of horizon crossing, \( y_Q \approx x_Q(1 - 7/12 x_Q^2 + \cdots) \), \( f(x_Q^2)^{-1} \approx 1/x_Q^2 \), for \( S_Q \) sufficiently larger than \( M \), and \( N_Q \approx 45 - 60 \) denotes the e-foldings needed to resolve the horizon and flatness problems. From the expression for \( \delta T/T \) in Eq. (3) and comparison with the COBE result \( (\delta T/T)_Q \approx 6.6 \times 10^{-6} \) \[13\], it follows that the gauge symmetry breaking scale \( M \) is close to \( 10^{16} \) GeV. Note that \( M \) is associated in our \( SO(10) \) example with the breaking scale of \( H \) (in particular the \( B - L \) breaking scale), which need not exactly coincide with the SUSY GUT scale. We will be more specific about \( M \) later in the text.

The relative flatness of the potential ensures that the primordial density fluctuations are essentially scale invariant. Thus, the scalar spectral index \( n \) is 0.98 for the simplest example based on \( W \) in Eq. (1). In some models \( n \) is unity to within a percent.
Several comments are in order:

- The 50-60 e-foldings required to solve the horizon and flatness problems occur when the inflaton field $S$ is relatively close (to within a factor of order 1-10) to the GUT scale. Thus, Planck scale corrections can be safely ignored.

- For the case of minimal Kähler potential, the SUGRA corrections do not affect the scenario at all, which is a non-trivial result [2]. More often than not, supersymmetric inflationary scenarios fail to work in the presence of SUGRA corrections which tend to spoil the flatness of the potential needed to realize inflation.

- Turning to the subgroup $H$ of $SO(10)$, one needs to take into account the fact that the spontaneous breaking of $H$ produces magnetic monopoles that carry two quanta of Dirac magnetic charge [13]. An overproduction of these monopoles at or near the end of inflation is easily avoided, say by introducing an additional (non-renormalizable) term $S(\phi \bar{\phi})^2$ in $W$, which is permitted by the $U(1)_R$ symmetry. The presence of this term ensures the absence of monopoles as explained in Ref. [7]. Note that the monopole problem is also avoided by choosing a different subgroup of $SO(10)$. In a separate publication, we will consider a scenario based on the $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_X$ subgroup of $SO(10)$ whose breaking does not lead to monopoles. Another interesting candidate is $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. The salient features of the model are not affected by the monopole problem [7].

- At the end of inflation the scalar fields $\phi$, $\bar{\phi}$, and $S$ oscillate about their respective minima. Since the $\phi$, $\bar{\phi}$ belong respectively to the $(\bar{4},1,2)$ and $(4,1,2)$ of $SU(4)_c \times SU(2)_L \times SU(2)_R$, they decay exclusively into right handed neutrinos via the superpotential couplings,

$$W = \frac{\gamma_i}{M_P} \bar{\phi} \bar{F}^c_i F^c_i ,$$

where the matter superfields $F^c_i$ belong to the $(\bar{4},1,2)$ representation of $H$, and $M_P \equiv M_{\text{Planck}}/\sqrt{8\pi} = 2.44 \times 10^{18}$ GeV denotes the reduced Planck mass, and $\gamma_i$ are dimensionless coefficients. We will have more to say about inflaton decay, the reheat temperature, as well as leptogenesis taking account of the recent neutrino oscillation.
data. However, we first wish to provide a five dimensional setting for this inflationary scenario.

We consider 5D space-time \((x^\mu, y)\), \(\mu = 0, 1, 2, 3\), where the fifth dimension is compactified on an \(S^1/Z_2\) orbifold. The action is given by

\[
S = \int d^4x \int^{y_c}_{-y_c} dy \sqrt{-g_5} \left[ \frac{M_5^3}{2} R_5 - \Lambda_B + \frac{\delta(y)}{\sqrt{g_{55}}} \left( \frac{M_4^2}{2} \bar{R}_4 - \Lambda_1 \right) - \frac{\delta(y - y_c)}{\sqrt{g_{55}}} \Lambda_2 \right],
\]

where \(R_5 (\bar{R}_4)\) is the 5 dimensional (4 dimensional) Einstein-Hilbert term, \(\Lambda_B\), and \(\Lambda_1, \Lambda_2\) are the bulk and brane cosmological constants, and \(M_5\) and \(M_4\) are mass parameters. The cosmological constants in the bulk and on the branes could be interpreted the vacuum expectation values of some scalar potentials from the particle physics sector. The brane curvature scalar (Ricci scalar) \(\bar{R}_4(g_{\mu\nu})\) is defined with the induced metric of the bulk metric, \(\bar{g}_{\mu\nu}(x) \equiv g_{\mu\nu}(x, y = 0)\) \((\mu, \nu = 0, 1, 2, 3)\). For an inflationary solution, we take the metric ansatz,

\[
ds^2 = \beta^2(y)(-dt^2 + e^{2H_0 t}dx^2) + dy^2,
\]

where \(H_0\) could be interpreted as the 4 dimensional Hubble constant. The non-vanishing components \((\mu, \mu)\) and \((5, 5)\) of the 5 dimensional Einstein equation derived from (5) gives

\[
3 \left[ \left( \frac{\beta'}{\beta} \right)^2 + \left( \frac{\beta''}{\beta} \right) - \left( \frac{H_0}{\beta} \right)^2 - \delta(y) \frac{M_4^2}{M_5^3} \left( \frac{H_0}{\beta} \right)^2 \right] = -\frac{\Lambda_B}{M_5^3} - \frac{\Lambda_1}{M_5^3} - \delta(y - y_c) \frac{\Lambda_2}{M_5^3},
\]

\[
6 \left[ \left( \frac{\beta'}{\beta} \right)^2 - \left( \frac{H_0}{\beta} \right)^2 \right] = -\frac{\Lambda_B}{M_5^3},
\]

where primes denote derivatives with respect to \(y\). The last term in the left hand side in Eq. (7) arises from the brane scalar curvature term, and vanishes when \(H_0 = 0\).

The solutions to the equations Eq. (7) and (8) are given by

\[
\beta(y) = \left( \frac{H_0}{k} \right) \sinh(\pm k|y| + c) \quad \text{for} \quad \Lambda_B < 0,
\]

\[
\beta(y) = \pm H_0|y| + c \quad \text{for} \quad \Lambda_B = 0,
\]

---

3The importance of the brane-localized 4D Einstein-Hilbert term, especially for generating 4D gravity with a non-compact extra dimension was first noted in Ref. [15].
where $k \equiv \sqrt{-\Lambda_B/6M_5^2}$, and $c$ is an integration constant. Without loss of generality, we can take $c$ positive. To avoid the existence of naked singularities within the interval $-y_c < y < y_c$, $\pm ky_c + c > 0$ and $\pm H_0 y_c + c > 0$ should be required. For simplicity of our discussion, let us take $\,'+$ among $\pm$ in Eqs. (9) and (10).

The introduction of the brane scalar curvature term $\bar{R}_4$ does not affect the bulk solutions, (9) and (10), but it modifies the boundary conditions. For $\Lambda_B < 0$, the solution should satisfy the following boundary conditions at $y = 0$ and $y = y_c$,

$$k \coth c - \frac{1}{2} \frac{M_4^2}{M_5^2} \frac{k^2}{\sinh^2 c} = -\frac{\Lambda_1}{6M_5^2},$$

(11)

$$k \coth(ky_c + c) = \frac{\Lambda_2}{6M_5^2}.$$

(12)

Hence, the integration constant $c$ and the interval length $y_c$ are determined by $\Lambda_1$ and $\Lambda_2$. Similarly, the solution for $\Lambda_B = 0$ should satisfy the boundary conditions,

$$\frac{H_0}{c} - \frac{1}{2} \frac{M_4^2}{M_5^2} \frac{H_0^2}{c^2} = -\frac{\Lambda_1}{6M_5^2},$$

(13)

$$\frac{H_0}{c + H_0 y_c} = \frac{\Lambda_2}{6M_5^2},$$

(14)

so $\Lambda_1$ and $\Lambda_2$ determine $H_0/c$ and $y_c$. Note that $\Lambda_2$ must be fine-tuned to zero when $\Lambda_1 = 0$ \[17\]. Hence it is natural that the scalar field which controls inflation is introduced in the bulk.

From Eqs. (11)–(14), we note that the brane cosmological constants $\Lambda_1$ and $\Lambda_2$ should have opposite signs in the absence of the brane curvature scalar contribution at $y = 0$. However, a suitably large value of $M_4/M_5$ can even make the sign of $\Lambda_1$ positive. Since the introduction of the brane curvature term does not conflict with any symmetry that may be present, there is no reason why such a term with a parameter $M_4$ that is large compared to $M_5$ is not allowed \[15\]. Thus, $\Lambda_1$ and $\Lambda_2$ could both be positive and this fact will be exploited for implementing the inflationary scenario. We will later suggest a model for explaining how a large $M_4/M_5$ ratio may be realized.

From (13) and (14), we also note that $\Lambda_1$ and $\Lambda_2$ in the $\Lambda_B = 0$ case are related to the 4 dimensional Hubble constant $H_0$, unlike the $\Lambda_B < 0$ case in Eqs. (11) and
While their non-zero values are responsible for the 3-space inflation, vanishing brane cosmological constants guarantee an effective 4 dimensional flat space-time. On the other hand, for $\Lambda_B < 0$, the relations between the bulk and brane cosmological constants are responsible for inflation. To obtain a static solution, we should take $H_0 \to 0$ and $c \to \infty$ (or $\Lambda_{1(2)}/6M^3_5 \to -k$ (+k)) while letting the ratio $H_0 e^c/2k \to 1$.

Our main task is to embed the 4D supersymmetric inflationary scenario in 5D space-time, employing the framework and solutions discussed above. In order to extend the setup to 5D SUGRA, a gravitino $\psi_M$ and a vector field $B_M$ should be appended to the graviton (fünfbein) $e^M_M$. Through orbifolding, only $N = 1$ SUSY is preserved on the branes. The brane-localized Einstein-Hilbert term in Eq. (5) is still allowed, but should be accompanied by a brane gravitino kinetic term as well as other terms, which is clear in off-shell SUGRA formalism. In a higher dimensional supersymmetric theory, a F-term scalar potential is allowed only on the 4 dimensional fixed points which preserve $N = 1$ SUSY. We require a formalism in which inflation and the Hubble constant $H_0$ are controlled only by the brane cosmological constants, such that during inflation the positive vacuum energy slowly decreases, and the minimum of the scalar potential corresponds to a flat 4D space-time. The solution for $\Lambda_B = 0$ meets these requirements in the presence of the additional brane scalar curvature term at $y = 0$, and so we will focus only on this case.

We have tacitly assumed that the interval separating the two branes (orbifold fixed points) remains fixed during inflation. The scenario is quite different from what is often called ‘D-brane inflation’. The dynamics of the orbifold fixed points, unlike the D-brane case, is governed only by the $g_{55}(x, y)$ component of the metric tensor. The real fields $e^5_5, B_5$, and the chiral fermion $\psi_{5R}$ in the 5D gravity multiplet are assigned even parity under $Z_2$, and they compose an $N = 1$ chiral multiplet on the branes. The associated superfield can acquire a superheavy mass and its scalar component can develop a VEV on the brane. With superheavy brane-localized mass
terms, their low-lying Kaluza-Klein (KK) mass spectrum is shifted so that even the lightest mode obtains a compactification scale mass \[22\]. Since this is much greater than \(H_0\), the interval distance is stable even during inflation. The stabilization of the interval distance leads to the stabilization also of the warp factor \(\beta(y)\), because the fluctuation \(\delta\beta(y)\) of the warp factor near the solution in Eq. \[10\] (also Eq. \[9\]) turns out to be proportional to the interval length variation \(\delta g_{55}\) by the linearized 5D Einstein equation \[23\].

With \(\Lambda_B = 0\), the effective 4 dimensional reduced Planck mass squared \(M_P^2 \equiv 1/(8\pi G_N)\) is given by

\[
M_P^2 \equiv M_5^3 \int_{-y_c}^{y_c} dy \beta^2 + M_4^2 \beta^2|_{y=0},
\]

\[
= M_5^3 y_c \left[ \frac{2}{3} H_0^2 y_c^2 + 2c H_0 y_c + 2c^2 \right] + M_4^2 c^2. \tag{15}
\]

For \(M_3^3 y_c << M_4^2 \sim M_P^2\), gravity couples universally at low energy to fields localized at \(y = y_c\) and \(y = 0\) and in the bulk, with the strength controlled by \(1/M_4^2\) \[24\]. The 4 dimensional effective cosmological constant turns out to be

\[
\Lambda_{\text{eff}} = \int_{-y_c}^{y_c} dy \beta^4 \left[ M_5^3 \left( 4 \left( \frac{\beta''}{\beta} \right) + 6 \left( \frac{\beta'}{\beta} \right)^2 \right) + \delta(y) \Lambda_1 + \delta(y - y_c) \Lambda_2 \right]
\]

\[
= 3H_0^2 \left[ M_5^3 y_c \left( \frac{2}{3} H_0^2 y_c^2 + 2c H_0 y_c + 2c^2 \right) + M_4^2 c^2 \right] = 3H_0^2 M_P^2, \tag{16}
\]

where the first two terms in the first line are the warp factor contributions. Hence, from Eqs. \[13\] and \[14\] \(\Lambda_{\text{eff}}\) vanishes when \(\Lambda_1 = \Lambda_2 = 0\). Note that for \(H_0 y_c << 1\),

\[
\Lambda_{\text{eff}} \approx \frac{c^2}{12} \frac{\Lambda_5^2 M_P^2}{M_5^6}. \tag{17}
\]

We can directly adapt these results for the \(S^1/(Z_2 \times Z_2')\) case.

To see how inflation is realized in this 5D setting, let us consider the 4D \(SU(4)_c \times SU(2)_L \times SU(2)_R(\equiv H)\) supersymmetric inflationary model \[7\]. An effective 4D theory with the gauge group \(H\) is readily obtained from a 5D \(SO(10)\) gauge theory if the fifth dimension is compactified on the orbifold \(S^1/(Z_2 \times Z_2')\) \[9\], where \(Z_2\) reflects \(y \to -y\), and \(Z_2'\) reflects \(y' \to -y'\) with \(y' = y + y_c/2\). There are two
independent orbifold fixed points (branes) at \( y = 0 \) and \( y = y_c/2 \), with \( N = 1 \) SUSYs and gauge symmetries \( SO(10) \) and \( H \) respectively \cite{9}. The \( SO(10) \) gauge multiplet \((A_M, \lambda^1, \lambda^2, \Phi)\) decomposes under \( H \) as

\[
V_{45} \rightarrow V(15,1,1) + V(1,3,1) + V(1,1,3) + V(6,2,2)
\]

\[ + \Sigma(15,1,1) + \Sigma(1,3,1) + \Sigma(1,1,3) + \Sigma(6,2,2), \]

where \( V \) and \( \Sigma \) denote the vector multiplet \((A_\mu, \lambda^1)\) and the chiral multiplet \(((\Phi + iA_5)/\sqrt{2}, \lambda^2)\) respectively, and their \((Z_2, Z'_2)\) parity assignments and KK masses are shown in Table I.

| (\( Z_2, Z'_2 \)) Masses | \((n=0)\pi/y_c\) & \((n=1)\pi/y_c\) & \((n=2)\pi/y_c\) & \((n=2)\pi/y_c\) |
|--------------------------|----------------|----------------|----------------|----------------|
| Chiral \((\Sigma)\)     | \(\Sigma(15,1,1)\) & \(\Sigma(1,3,1)\) & \(\Sigma(1,1,3)\) & \(\Sigma(6,2,2)\) |

**Table I.** \((Z_2, Z'_2)\) parity assignments and Kaluza-Klein masses \((n = 0, 1, 2, \cdots)\) for the vector multiplet in \( SO(10) \).

The parities of the chiral multiplets \( \Sigma \)'s are opposite to those of the vector multiplets \( V \)'s in Table I and hence, \( N = 2 \) SUSY explicitly breaks to \( N = 1 \) below the compactification scale \( \pi/y_c \). As shown in Table I, only the vector multiplets, \( V(15,1,1), V(1,3,1), \) and \( V(1,1,3) \) contain massless modes, which means that the low energy effective 4D theory reduces to \( N = 1 \) supersymmetric \( SU(4)_c \times SU(2)_L \times SU(2)_R \). The parity assignments in Table I also show that the wave function of the vector multiplet \( V(6,2,2) \) vanishes at the brane located at \( y = y_c/2 \) (B2) because it is assigned an odd parity under \( Z'_2 \), while the wave functions of all the vector multiplets should be the same at the \( y = 0 \) brane (B1). Therefore, while the gauge symmetry at B1 is \( SO(10) \), only \( SU(4)_c \times SU(2)_L \times SU(2)_R \) is preserved at B2 \cite{25}.

The 5D inflationary solution requires positive vacuum energies on both branes B1 and B2. While the scalar potential in Eq. (2) would be suitable for B2, an appropriate
scalar potential on B1 is also required. Since the boundary conditions in Eq. (13) and (14) require \( \Lambda_1 \) and \( \Lambda_2 \) to simultaneously vanish, it is natural to require \( S \) to be a bulk field. Then, the VEVs of \( S \) on the two branes can be adjusted such that the boundary conditions are satisfied. As an example, consider the following superpotential on B1,

\[
W_{B1} = \kappa_1 S (Z \bar{Z} - M_1^2) ,
\]

where \( Z \) and \( \bar{Z} \) are \( SO(10) \) singlet superfields on the B1 brane with opposite \( U(1)_R \) charges. The condition for a positive brane cosmological constant on B1 is found from (13) to be \( (H_0/c)(M_4^2/M_5^3) > 2 \). For \( \kappa \sim 10^{-3} \), say, and \( c \sim 1 \), we have \( H_0 \sim 10^{10} \) GeV and \( M_5 \sim 10^{15} \) GeV (so that \( M_4 \sim M_P \)). Thus, there exists a hierarchy of order \( 10^3 \) between the 5D bulk scale \( M_5 \) and the four dimensional brane mass scale \( M_4 \). To see how this hierarchy could arise, consider the case where the brane-localized gravity kinetic term has the canonical form but not the bulk term. Thus,

\[
\mathcal{L}/e = \frac{M_5^3}{2} e^{-f(|\phi|)} R_5 + \frac{\delta(y)}{e_5^2} \left( \frac{M_5^2}{2} \partial_4 - V(|\phi|) \right) - \frac{1}{2y_e} \partial_\phi \bar{\phi} M_\phi \phi^* + \cdots ,
\]

where \( \phi \) is some scalar field, \( V(|\phi|) \) its associated potential, we take \( M_4 = M_P \). Let us assume that like \( e_5 \), \( \phi \) acquires a Planck scale mass and VEV on the brane. Then, at the minimum of \( V(|\phi|) \), the 5D Einstein equation determining the background geometry is effectively given by Eqs. (7) and (8), with \( M_5 = e^{-\langle f(|\phi|)/3 \rangle} M_\phi \). Taking \( f(|\phi|) = 2|\phi|/M_P \), for instance, and \( \langle \phi \rangle \approx 10 M_P \) would lead to \( M_5 \sim 10^{15} \) GeV as required.

After inflation is over, the oscillating system consists of the complex scalar fields \( \Phi = (\delta \bar{\phi} + \delta \phi) \), where \( \delta \bar{\phi} = \bar{\phi} - M \) (\( \delta \phi = \phi - M \)), and \( S \), both with masses equal to \( m_{\text{infl}} = \sqrt{2} \kappa M \). Through the superpotential couplings in Eq. (14), these fields decay into a pair of right handed neutrinos and sneutrinos respectively, with an approximate decay width [7]

\[
\Gamma \sim \frac{m_{\text{infl}}}{8\pi} \left( \frac{M_i}{M} \right)^2 ,
\]

where \( M_i \) denotes the mass of the heaviest right handed neutrino with \( 2M_i < m_{\text{infl}} \), so that the inflaton decay is possible. Assuming an MSSM spectrum below the GUT
scale, the reheat temperature is given by \[26\]

\[ T_r \approx \frac{1}{3} \sqrt{\Gamma M_P} \approx \frac{1}{12} \left( \frac{55}{N_Q} \right)^{1/4} \sqrt{y_Q M_i} . \]  

(22)

For \( y_Q \sim \text{unity} \) (see below), and \( T_r \lesssim 10^{9.5} \text{ GeV} \) from the gravitino constraint \[27\], we require \( M_i \lesssim 10^{10} - 10^{10.5} \text{ GeV} \).

In order to decide on which \( M_i \) is involved in the decay \[28\], let us start with atmospheric neutrino (\( \nu_\mu - \nu_\tau \)) oscillations and assume that the light neutrinos exhibit an hierarchical mass pattern with \( m_3 >> m_2 >> m_1 \). Then \( \sqrt{\Delta m_{\text{atm}}^2} \approx m_3 \approx m_{D3}^2/M_3 \), where \( m_{D3} (= m_4(M)) \) denotes the third family Dirac mass which equals the asymptotic top quark mass due to SU(4).c. We also assume a mass hierarchy in the right handed sector, \( M_3 >> M_2 >> M_1 \). The mass \( M_3 \) arises from the superpotential coupling Eq. (4) and is given by \( M_3 = 2\gamma_3 M^2 / M_P \sim 10^{14} \text{ GeV} \), for \( M \sim 10^{16} \text{ GeV} \) and \( \gamma_3 \sim \text{unity} \). This value of \( M_3 \) is in the right ball park to generate an \( m_3 \sim \frac{1}{20} \text{ eV} \) (\( \sim \sqrt{\Delta m_{\text{atm}}^2} \)), with \( m_t(M) \sim 110 \text{ GeV} \) \[26\]. It follows from (22) that \( M_i \) in (21) cannot be identified with the third family right handed neutrino mass \( M_3 \). It should also not correspond to the second family neutrino mass \( M_2 \) if we make the plausible assumption that the second generation Dirac mass should lie in the few GeV scale.

The large mixing angle MSW solution of the solar neutrino problem requires that \( \sqrt{\Delta m_{\text{solar}}^2} \sim m_2 \sim \text{GeV}^2 / M_2 \sim \frac{1}{160} \text{ eV} \), so that \( M_2 \gtrsim 10^{11} - 10^{12} \text{ GeV} \). Thus, we are led to conclude \[28\] that the inflaton decays into the lightest (first family) right handed neutrino with mass

\[ M_1 \sim 10^{10} - 10^{10.5} \text{ GeV} , \]  

(23)

such that \( 2M_1 < m_{\text{infl}} \).

The constraint \( 2M_2 > m_{\text{infl}} \) yields \( y_Q \lesssim 3.34\gamma_2 \), where \( M_2 = 2\gamma_2 M^2 / M_P \). We will not provide here a comprehensive analysis of the allowed parameter space but will be content to present a specific example, namely

\[ M \approx 8 \times 10^{15} \text{ GeV} , \ \kappa \approx 10^{-3} , \ m_{\text{infl}} \sim 10^{13} \text{ GeV} \ (\sim M_2) , \]  

(24)
with \( y_Q \approx 0.4 \) (corresponding to \( x_Q \) near unity, so that the inflaton \( S \) is quite close to \( M \) during the last 50–60 e-foldings).

Note that typically \( \kappa \) is of order \( 10^{-2} \)– few \( \times 10^{-4} \), so that the vacuum energy density during inflation is \( \sim 10^{-4} - 10^{-8} \, M_{\text{GUT}}^4 \). Thus, in this class of models the gravitational wave contribution to the quadrupole anisotropy \( (\delta T/T)_Q \) is essentially negligible \( (\lesssim 10^{-8}) \). With \( \kappa \sim \text{few} \times 10^{-4} \left(10^{-3}\right) \), the scalar spectral index \( n \approx 0.99 \left(0.98\right) \).

The decay of the (lightest) right handed neutrinos generates a lepton asymmetry which is given by

\[
\frac{n_L}{s} \approx \frac{10}{16\pi} \left(\frac{T_r}{m_{\text{infl}}}\right) \left(\frac{M_1}{M_2}\right) c_\theta^2 s_\theta^2 \sin 2\delta \left(\frac{m_{D2}^2 - m_{D1}^2}{\langle h \rangle^2 (m_{D2}^2 s_\theta^2 + m_{D1}^2 c_\theta^2)}\right),
\]

where the VEV \( |\langle h \rangle| \approx 174 \text{ GeV} \) (for large \( \tan \beta \)), \( m_{D1,2} \) are the neutrino Dirac masses (in a basis in which they are diagonal and positive), and \( c_\theta \equiv \cos \theta, s_\theta \equiv \sin \theta \), with \( \theta \) and \( \delta \) being the rotation angle and phase which diagonalize the Majorana mass matrix of the right handed neutrinos. Assuming \( c_\theta \) and \( s_\theta \) of comparable magnitude, taking \( m_{D2} >> m_{D1} \), and using (23) and (24), Eq. (25) reduces to

\[
\frac{n_L}{s} \approx 10^{-8.5} c_\theta^2 \sin 2\delta \left(\frac{T_r}{10^{9.5} \text{ GeV}}\right) \left(\frac{M_1}{2 \cdot 10^{10.5} \text{ GeV}}\right) \left(\frac{10^{13} \text{ GeV}}{M_2}\right) \left(\frac{m_{D2}}{10 \text{ GeV}}\right)^2,
\]

which can be in the correct ball park to account for the observed baryon asymmetry \( n_B/s \approx -28/79 \, n_L/s \).

In conclusion, our goal in this paper has been to demonstrate the existence of realistic models which nicely blend together four particularly attractive ideas, namely supersymmetric grand unification, extra dimension(s), inflation and leptogenesis. The doublet-triplet problem is circumvented by utilizing orbifold breaking of \( SO(10) \), which may also help in suppressing dimension five proton decay. There are two predictions concerning inflation that are particularly significant. Namely, the scalar spectral index \( n \) lies very close to unity \( (\approx 0.98 - 0.99) \), and the gravitational wave contribution to \( (\delta T/T)_Q \) is highly suppressed \( (\sim 10^{-8} - 10^{-9}) \). Finally, the inflaton decay produces heavy right handed Majorana neutrinos (in our case the lightest one),
whose subsequent out of equilibrium decay leads to the baryon asymmetry via leptogenesis. We expect to generalize this approach to other symmetry breaking patterns of $SO(10)$ in a future publication.

**Acknowledgments**

Q.S. thanks Gia Dvali and George Lazarides for useful discussions. We also acknowledge useful discussion with Jim Liu. The work is partially supported by DOE under contract number DE-FG02-91ER40626.

**References**

[1] G. Dvali, Q. Shafi, and R. Schaefer, Phys. Rev. Lett. **73**, 1886 (1994) [hep-ph/9406319].

[2] For a comprehensive review and additional references, see G. Lazarides, [hep-ph/0111328](http://arxiv.org/abs/hep-ph/0111328).

[3] S. Fukuda et. al. [Superkamiokande Collaboration], Phys. Rev. Lett. **85**, 3999 (2000); S. Fukuda et. al., Phys. Lett. **B539**, 179 (2002).

[4] M. Gell-Mann, P. Ramond, and R. Slansky, in Supergravity: proceedings, eds. P. van Nieuwenhuizen, and D. Z. Freedman (North Holland Publ. Co., 1979), p. 315 [Print-80-0576 (CERN)]; T. Yanagida, in Workshop on the Unified Theory and Baryon Number in the Universe, eds. O. Sawada, and A. Sugamoto, (KEK, Tsukuba), 95 (1979).

[5] M. Fukugita, and T. Yanagida, Phys. Lett. **B174**, 45 (1986).

[6] G. Lazarides, and Q. Shafi, Phys. Lett. **B258**, 305 (1991).

[7] R. Jeannerot, S. Khalil, G. Lazarides, and Q. Shafi, JHEP **0010**, 012 (2000) [hep-ph/0002151].

[8] J. C. Pati, and A. Salam, Phys. Rev. **D10**, 275 (1974).
[9] R. Dermisek, and A. Mafi, Phys. Rev. D65, 055002 (2002) [hep-ph/0108139]. See also T. Asaka, W. Buchmüller, and L. Covi, Phys. Lett. B523, 199 (2001) [hep-ph/0108021]; L. Hall, Y. Nomura, T. Okui, and D. Smith, Phys. Rev. D65, 035008 (2002) [hep-ph/0108071].

[10] E. J. Copeland, A. R. Liddle, D. H. Lyth, E. D. Stewart, and D. Wands, Phys. Rev. D49, 6410 (1994).

[11] S. F. King, and Q. Shafi, Phys. Lett. B422, 135 (1998) [hep-ph/9711288].

[12] S. Coleman and E. Weinberg, Phys. Rev. D7, 1888 (1973).

[13] D. H. Lyth, and A. Riotto, Phys. Rep. 314, 1 (1999) [hep-ph/9807278]; E. F. Bunn, A. R. Liddle, and M. White, Phys. Rev. D54, 5917 (1996).

[14] G. Lazarides, M. Magg, and Q. Shafi, Phys. Lett. B97, 87 (1980).

[15] G. Dvali, G. Gabadadze, and M. Porrati, Phys. Lett. B485, 208 (2000) [hep-th/0005016]; G. Dvali, G. Gabadadze, M. Kolanovic, and F. Nitti, Phys. Rev. D64, 084004 (2001) [hep-ph/0102216].

[16] T. Nihei, Phys. Lett. B465, 81 (1999) [hep-ph/9905487]; N. Kaloper, Phys. Rev. D60, 123506 (1999) [hep-th/9905210]; H. B. Kim, and H. D. Kim, Phys. Rev. D61, 064003 (2000) [hep-ph/9909035]. See also J. E. Kim, B. Kyae, and H. M. Lee, Nucl. Phys. B582, 296 (2000) [hep-th/0004005]; Erratum-ibid. B591, 587 (2000); J. E. Kim, and B. Kyae, Phys. Lett. B486, 165 (2000) [hep-th/0005139]; P. Binetruy, C. Deffayet, and D. Langlois, Nucl. Phys. B565, 269 (2000) [hep-th/9905012]; S. Nojiri, and S. D. Odintsov, Phys. Lett. B484, 119 (2000) [hep-th/0004097]; C. Csaki, M. Graesser, C. Kolda, and J. Terning, Phys. Lett. B462, 34 (1999) [hep-ph/9905133]; J. M. Cline, C. Grojean, and G. Servant, Phys. Rev. Lett. 83, 4245 (1999) [hep-ph/9906523]; H. B. Kim, Phys. Lett. B478, 285 (2000) [hep-th/0001209].
[17] See also J. E. Kim, hep-th/0210117. In this paper inflation is discussed based on a self-tuning mechanism for the cosmological constant.

[18] L. Randall, and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999) hep-ph/9905221.

[19] Higher dimensional cosmology has a long history. See, for example, Q. Shafi and C. Wetterich, Phys. Lett. B129, 387 (1983); ibid., B152, 51 (1985); Nucl. Phys. B289, 787 (1987). For a recent discussion and additional references, see M. Bastero-Gil, V. Di Clemente, and S. F. King, hep-ph/0211012.

[20] M. Zucker, Phys. Rev. D64, 024024 (2001) hep-th/0009083; B. Kyae, and Q. Shafi, Phys. Rev. D66, 095009 (2002) hep-ph/0204041.

[21] G. Dvali, and S.-H. H. Tye, Phys. Lett. B450, 72 (1999) hep-ph/9812483; G. Dvali, Q. Shafi, and S. Solganik, hep-th/0105203; C. P. Burgess, M. Majumdar, D. Nolte, F. Quevedo, G. Rajesh, and R.-J. Zhang, JHEP 0107, 047 (2001) hep-th/0105204; G. Shiu, and S.-H. H. Tye, Phys. Lett. B516, 421 (2001) hep-th/0106274; C. Herdeiro, S. Hirano, and R. Kallosh JHEP 0112, 027 (2001) hep-th/0110271; B. Kyae, and Q. Shafi, Phys. Lett. B526, 379 (2002) hep-ph/0111101; J. Garcia-Bellido, R. Rabadan, and F. Zamora, JHEP 0201, 036 (2002) hep-th/0112147; R. Blumenhagen, B. Körs, D. Lüst, and T. Ott, Nucl. Phys. B641, 235 (2002) hep-th/0202124.

[22] Y. Nomura, D. Smith, and N. Weiner Nucl. Phys. B613, 147 (2001) hep-ph/0104041; N. Arkani-Hamed, L. Hall, Y. Nomura, D. Smith, and N. Weiner, Nucl. Phys. B605, 81 (2001) hep-ph/0102090; Z. Chacko, M. A. Luty, and E. Ponton, JHEP 0007, 036 (2000) hep-ph/9909248.

[23] Z. Chacko, and P. J. Fox, Phys. Rev. D64, 024015 (2001) hep-th/0102023; C. Csaki, M. L. Graesser, and G. D. Kribs; Phys. Rev. D63, 065002 (2001) hep-th/0008151; J. E. Kim, B. Kyae, and H. M. Lee, Phys. Rev. D66, 106004 (2002) hep-th/0110103.
[24] B. Kyae, hep-th/0207272.

[25] A. Hebecker, and J. March-Russell, Nucl. Phys. B625 (2002) 128 hep-ph/0107039.

[26] G. Lazarides, R. K. Schaefer, and Q. Shafi, Phys. Rev. D56, 1324 (1997) hep-ph/9608256.

[27] J. Ellis, J. E. Kim, and D. Nanopoulos, Phys. Lett. B145, 181 (1984); M. Yu. Khlopov, and A. D. Linde, Phys. Lett. B138, 265 (1984). For a review and additional references, see W. Buchmüller, Nato Science Series II, Vol. 34, 2001, eds. G. C. Branco, Q. Shafi, and J. I. Silva-Marcos.

[28] See also J. C. Pati, hep-ph/0209160. In this paper it is also discussed how large neutrino mixings compatible with observations can arise from SO(10) and \( SU(4)_c \times SU(2)_L \times SU(2)_R \).

[29] G. Lazarides, Q. Shafi, and N. D. Vlachos, Phys. Lett. B427, 53 (1998), and references therein.