TRADER DYNAMICS IN A MODEL MARKET

NEIL F. JOHNSON and MICHAEL HART
Physics Department, Oxford University
Parks Road, Oxford, OX1 3PU, U.K.
n.johnson@physics.ox.ac.uk

PAK MING HUI
Department of Physics, The Chinese University of Hong Kong
Shatin, New Territories, Hong Kong
pmhui@phy.cuhk.edu.hk

DAFANG ZHENG
Department of Applied Physics, South China University of Technology
Guangzhou 510641, People’s Republic of China
phdzheng@scut.edu.cn

We explore various extensions of Challet and Zhang’s Minority Game in an attempt to gain insight into the dynamics underlying financial markets. First we consider a heterogeneous population where individual traders employ differing ‘time horizons’ when making predictions based on historical data. The resulting average winnings per trader is a highly non-linear function of the population’s composition. Second, we introduce a threshold confidence level among traders below which they will not trade. This can give rise to large fluctuations in the ‘volume’ of market participants and the resulting market ‘price’.

1. Introduction

Two obvious practical goals in the study of financial markets are to understand how arbitrage opportunities might arise and consequently be exploited, and to understand quantitatively the origin of price fluctuations. The Minority Game represents a fascinating toy-model of a complex adaptive system in which individual members (e.g., traders) repeatedly compete to be in a minority. The Minority Game offers a simple paradigm for the decision dynamics underlying financial markets: if for example there are more buyers than sellers at a given moment, prices are pushed up and hence it would be better for a trader to be in the minority group of sellers.

Here we explore two extensions of the Minority Game which seem relevant for real markets. First we consider the performance of a heterogeneous population of traders who differ in the ‘time horizon’ employed when making buy/sell decisions based on past market data. We find that the average winnings per trader is a highly non-linear function of the population’s composition. Second, we introduce
a threshold confidence level among traders below which they will not trade. We find that this feature can give rise to large variations in the ‘volume’ of market participants and the resulting market ‘price’.

2. Basic Minority Game

The basic Minority Game consists of a repeated game with an odd number of traders $N$ who must choose independently whether to be in room 0 (e.g. buy) or room 1 (e.g. sell). The winners are those in the room with fewer traders, i.e. the sellers win if there is an excess of buyers. The output is a single binary digit, 0 or 1, representing the winning decision for each time step. This output is made available to all traders, and is the only information they can use to make decisions in subsequent turns. The memory $m$ is the length of the recent history bit-string that a trader uses when making its next decision. In the market context, $m$ can be thought of as a ‘time horizon’ over which a given trader considers the past history to be relevant when making a prediction as to the direction of the next market movement.

The traders randomly pick $s$ strategies at the beginning of the game, with repetitions allowed. After each turn, the trader assigns one (virtual) point to each of his strategies which would have predicted the correct outcome. In addition the trader gets awarded one (real) point if he is successful. At each turn of the game, the trader uses the most successful strategy (i.e., most virtual points) from his bag of $s$ strategies. The strategy-space forms a $2^m$-dimensional hypercube for memory $m$ with strategies at the $2^m$ vertices. If the size of the strategy space is small compared to the total number of traders $N$ (i.e., $2 \cdot 2^m << Ns$) many traders may hold the highest-scoring strategy at any given turn and hence make the same decision. This leads to a large standard deviation in the winning room and hence a relatively low number of total points awarded. Such crowd-effects are a strategy-space phenomenon and have been shown to quantitatively explain the fluctuations for the pure population as a function of $m$ and $s$.

3. Mixed population of traders

Consider a population containing $N_{m_1}$ traders with memory $m_1$, and $N_{m_2} = N - N_{m_1}$ traders with memory $m_2$. We define the average points per trader per turn, $W$, to be the total number of points awarded in that turn divided by the total number of traders. For small $m$, $W$ is substantially less than 0.5 due to the crowd-effects mentioned above. The maximum possible $W$ would correspond to the number of winners remaining at $(N - 1)/2$. Therefore $W$ is always less than or equal to $(N - 1)/2N$, hence $W < 0.5$. Note that an external (i.e. non-participating) gambler using a coin-toss to predict the winning decision, would have a 50% success rate since he would not suffer from this intrinsic crowding in strategy-space. The history-space forms an $m$-dimensional hypercube whose $2^m$ vertices correspond to all possible recent history bit-strings of length $m$. For a pure population of traders with the
same memory $m$, where $2 \cdot 2^m << Ns$, there is information left in the history time-
series however this information is hidden in bit-strings of length greater than $m$
and hence is not accessible to these traders. For large $m$, there is information left in
bit-strings of any length, however the traders have insufficient strategies to further
exploit this information.

---

**Fig. 1.** Numerical results for average winnings per trader per turn $W$: traders with memory $m = 3$
possess $s_1 = 2, 3, \ldots, 7$ strategies (each data set corresponds to a different $s_1$ value) whereas traders
with memory $m = 6$ have $s_2 = 7$ strategies for each data set.

Figure 1 shows the numerical results for the average points per trader per turn
$W$ in a mixed population of $N = 101$ traders, with memory $m = 3$ or 6. The
number of $m = 3$ traders $N_3$ is shown on the $x$-axis, hence the number of $m = 6$
traders is given by $N_6 = 101 - N_3$. The $m = 3$ traders possess $s_1 = 2, 3, \ldots, 7$
strategies (each data set corresponds to a different $s_1$) while the $m = 6$ traders
possess $s_2 = 7$ strategies. The data were collected in the limit of long times, and
averaged over many runs. We observe from Fig. 1 that the average winnings $W$
can show a maximum at finite mixing.

Figure 2 shows results for the opposite case where the $m = 3$ traders each possess
$s_1 = 7$ strategies while the $m = 6$ traders possess $s_2 = 2, 3, \ldots, 7$ strategies. Note
that simulations in which traders are fed a random (as opposed to real) history do
not reproduce the numerical results of Figs. 1 and 2. This is essentially because the
$m = 6$ traders in the real-history game have the opportunity to exploit correlations
in the real-history time-series left by the $m = 3$ traders (see also Refs. [1],[2] and
[8]). In other words, the long-memory (i.e., $m = 6$) traders can identify and exploit
arbitrage opportunities that are inaccessible to the short-memory (i.e., $m = 3$)
traders.

Any fluctuation in the number of winners away from $(N - 1)/2 = 50$ implies
wastage of total points. Hence $W \sim 0.5 - \frac{\sigma}{\sqrt{N}}$, where $\sigma$ is the standard deviation
of the number of traders making a given decision, say ‘buy’. In order to develop
an analytic theory, we assume that the corresponding $\sigma$ can be obtained by adding
separately the contributions to the variance from the $m = 3$ traders and the $m = 6$
traders. This amounts to assuming that the system has managed to remove any internal frustration between traders with different memories, and the two groups of traders behave independently. Hence $\sigma^2 \sim \sigma_3^2 + \sigma_6^2$, where $\sigma_3$ ($\sigma_6$) is the variance due to the $m = 3$ ($m = 6$) traders. Following Ref. [4], and defining the concentration of $m = 3$ traders as $x = N_3/N$, we obtain $\sigma_3 \sim C_3 x N$ and $\sigma_6 \sim C_6 (1-x) N$ where $C_3$ and $C_6$ are given by the general expression

$$C_m = \frac{1}{2} \left[ \sum_r \left( \left[ 1 - \frac{r - 1}{2 \cdot 2^m} \right]^s - \left[ 1 - \frac{r}{2 \cdot 2^m} \right]^s \right)^2 \right]^{1/2}. \quad (3.1)$$

The summation is over weakly correlated groups: each group $r$ comprises a crowd of like-minded (i.e., correlated) traders who use the same strategy, and a corresponding anticrowd of opposite-minded traders who use the anticorrelated strategy. The short-memory (i.e., $m = 3$) sub-population of traders tends to lie in the crowded regime, hence we only need to include $r = 1$ in the summation to obtain $C_3$. The long-memory (i.e., $m = 6$) sub-population of traders will tend to lie in the crowded regime if the number $s_2$ of strategies they hold is large, but will form crowd-anticrowd pairs if $s_2$ is small. To obtain $C_6$ we sum up the terms from $r = 1$ to $r = 101(1-x)$ if $\frac{2^{m+1}}{2} \geq 101(1-x) N$, or to $r = \frac{2^{m+1}}{2}$ if $\frac{2^{m+1}}{2} < 101(1-x)$. The analytic expression for the average winnings is given by

$$W \sim 0.5 - \left[ C_3^2 x^2 + C_6^2 (1-x)^2 \right]^{1/2}. \quad (3.2)$$

Figures 3 and 4 show the analytic results corresponding to the numerical simulations of Figs. 1 and 2 respectively. As can be seen the agreement is fairly good. Intriguingly, the theory has a tendency to underestimate the actual winnings suggesting that the actual population is somehow exhibiting an additional degree of co-operation (correlation). A fuller theory of this mixed system thus requires the inclusion of higher-order inter-trader correlations.
Fig. 3. Analytic results corresponding to Figure 1.

Fig. 4. Analytic results corresponding to Figure 2.
4. Threshold confidence level among traders

In the Minority Game with either a homogeneous or heterogeneous population, traders must either buy or sell at every time-step. In a real market, however, traders are likely to wait on the sidelines until they are reasonably confident of winning at a given time-step. They will observe the market passively, mentally updating their various strategies, until their confidence overcomes some threshold value - then they will jump in and trade.

We now attempt to incorporate this general behavior as follows. Taking the simplest generalization, we assign a threshold confidence level \( r_{\text{min}} \) below which a trader will not trade. A trader’s confidence level at a given time-step is determined by the success rate \( r \) of his best-performing strategy over the last \( T \) time-steps. Hence the number of traders who actively trade \( N_{\text{active}} \) will fluctuate in time - this feature is reminiscent of the grand canonical ensemble in statistical mechanics.\(^1\)\(^2\) We will call \( N_{\text{active}}(t) \) the ‘volume’ of active traders at time \( t \). Given that \( N_{\text{active}}(t) = N_{\text{buy}}(t) + N_{\text{sell}}(t) \), we can form a simple-minded ‘price’ time-series \( P(t) \) by setting

\[
P(t+1) = P(t) + [N_{\text{buy}}(t) - N_{\text{sell}}(t)]/D
\]

(4.3)

where \( D \) is a parameter characteristic of a particular market. For simplicity we take \( D \) to be time-independent. Similar linear expressions for the price \( P(t) \) have been discussed recently by other authors.\(^9\)

A real market is not a zero-sum game - the commission charged by the market maker and/or general transaction costs imply that the success rate will be less than 50%, similar to the Minority Game. Hence it is reasonable to expect that typically \( r_{\text{min}} \geq 0.5 \). Interestingly, the fluctuations in \( N_{\text{active}} \) are much larger at \( r_{\text{min}} = 0.5 \) than for \( r_{\text{min}} \neq 0.5 \): in fact the resulting plot of fluctuations as a function of \( r_{\text{min}} \) resembles a \( \lambda \)-transition in statistical mechanics.\(^10\) Hence \( r_{\text{min}} = 0.5 \) seems to represent a ‘critical’ value, thereby adding weight to the conjecture that real markets may be positioned at some kind of critical point.

Figures 5 and 6 show the resulting “price” and “volume” series for trader populations with long and short memories respectively. In the simulations we take \( T = 500 \), however the results are similar for larger \( T \). For a population of the long-memory traders (Fig. 5), the volume is non-zero and large jumps in the price do not occur at a single time-step. Although similar general patterns can be found in the time-series for a population of short-memory traders (Fig. 6) the volume is often zero (the market becomes illiquid) and exhibits large spikes - corresponding large jumps in the price also arise (Fig. 6). Although this study is at a preliminary stage, we note in passing that large jumps in real stock prices can indeed be accompanied by large jumps in trading volume: see, for example, the behavior of Vodafone stock\(^11\) in November 1995, and Hanson stock\(^11\) at the end of January 1996.
Fig. 5. A section of the “price” and “volume” time-series generated by minority game with confidence threshold $r_{\text{min}} = 0.51$ and long trader memory ($m = 6$). $N = 1001$ and $s = 2$.

Fig. 6. A section of the “price” and “volume” time-series generated by minority game with confidence threshold $r_{\text{min}} = 0.51$ and short trader memory ($m = 2$). $N = 1001$ and $s = 2$. 
5. Conclusion

In summary, we have discussed two generalizations of the basic Minority Game. When the trader population is characterized by two different memories ('time horizons') the average winnings per trader can exceed that of a pure population. When traders can opt not to trade based on their confidence level, we find that large fluctuations can arise in the ‘volume’ of market participants and the resulting market ‘price’.

Acknowledgment

We thank D. Challet for discussions.

References

1. D. Challet and Y.C. Zhang, Emergence of cooperation and organization in an evolutionary game, Physica A 246 (1997) 407-418; Y.C. Zhang, Modeling market mechanism with evolutionary games, Europhys. News 29 (1998) 51-4.
2. D. Challet and Y.C. Zhang, On the minority game: Analytical and numerical studies, Physica A 256 (1998) 514-532.
3. R. Savit, R. Manuca and R. Riolo, Adaptive competition, market efficiency, and phase transition, Phys. Rev. Lett. 82 (1999) 2203-2206.
4. N.F. Johnson, M. Hart and P.M. Hui, Crowd effects and volatility in markets with competing agents, Physica A 269 (1999) 1-8.
5. R. D’Hulst and G.J. Rodgers, The Hamming distance in the minority game, Physica A 270 (1999) 514-525; A. Cavagna, J.P. Garrahan, I. Giardina and D. Sherrington, A thermal model for adaptive competition in a market, LANL preprint cond-mat/9903413.
6. M.A.R. de Cara, O. Pla and F. Guinea, Competition, efficiency and collective behavior in the “El Farol” bar model, Euro. Phys. J. B 10 (1999) 187-191.
7. A. Cavagna, Irrelevance of memory in the minority game, Phys. Rev. E 59 (1999) R3783-R3786.
8. N.F. Johnson, P.M. Hui, D. Zheng and M. Hart, Enhanced winnings in a mixed-ability population playing a minority game, J. Phys. A: Math. Gen. 32 (1999) L427-L431.
9. J.P. Bouchaud and R. Cont, A Langevin approach to stock market fluctuations and crashes, Euro. Phys. J. B 6 (1998) 543-550; Y.C. Zhang, Towards a theory of marginally efficient markets, Physica A 269 (1999) 30-44.
10. N.F. Johnson, P.M. Hui, S.W. Lim and T.S. Lo, unpublished.
11. A. Blair, Guide to Charting: An analysis for the intelligent investor, (Pitman Publishing, London, 1996).
number of m = 3

average points per trader

s1=2, s2=7
s1=3, s2=7
s1=4, s2=7
s1=5, s2=7
s1=6, s2=7
s1=7, s2=7

number of m=3 traders
number of $m=3$ traders

average points per trader

- $s_1=2, s_2=7$
- $s_1=3, s_2=7$
- $s_1=4, s_2=7$
- $s_1=5, s_2=7$
- $s_1=6, s_2=7$
- $s_1=7, s_2=7$
number of $m=3$ traders

average points per trader

- $s_1=7, s_2=2$
- $s_1=7, s_2=3$
- $s_1=7, s_2=4$
- $s_1=7, s_2=5$
- $s_1=7, s_2=6$
- $s_1=7, s_2=7$
"Price"

N=1001
m=6
s=2

"Volume"
"Price"

N=1001
m=2
s=2

"Volume"