Mathematical modelling of fluid flow processes in the fracture-porous reservoir

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Abstract. Authors consider processes the filtration process in the reservoir. The model of the dual porosity of Warren and Root is used to study the processes. The pores (matrix) are represented by rectangular parallelepipeds in this model, which have high porosity and low permeability. The low permeable pores are separated by a system of natural fracture that has high permeability and low porosity. Filtration of liquid in the formation is carried out through a system of fracture, and the matrix is a reservoir that continuously feeds the network of fracture. The redistribution of fluid between the matrix and fracture depends on the shape and size of the matrix blocks, the ratio of pore permeability and fracture system. The pressure distribution in the “system of fracture-matrix” system is described by the piezoconductivity equations. An analytical solution for the Warren and Root model cannot be obtained in a general form, approximate solutions exist for special cases. The results of the computer simulation and the numerical solution of this problem is presented in this paper. The problem was approximated using an implicit difference scheme. The matrix sweep method was used for the calculation. Model pressure recovery curves were obtained. Analysis showed that the specific conductivity coefficient depends on the size of the matrix blocks and possible to evaluate the process of manifestation of the effect of dual porosity.

1. Introduction
The discovery and development of a number of new large deposits in recent years indicate that the role of carbonate reservoirs in the development of the oil industry in Russia is actively growing. As a rule, productive reservoir with dual porosity are not sufficiently studied in comparison with ordinary sandstone in terrigenous reservoirs [1, 2, 3, 4]. The development of oil deposits with carbonate reservoirs is characterized by a number of specific features that are associated with the flow of fluid in environment with a dual porosity [5]. The development of methods for mathematical modeling of fluid flow in a given medium is an urgent problem. Due to their physicochemical properties, susceptibility to cracking, leaching, and recrystallization, carbonate reservoirs form a complex microstructure of the void space. The main characteristics of such rocks are fracture and cavernousness [6]. The main cause of the appearance of fractures in the rock is deformation phenomena when the stresses resulting from the action of mechanical loads of various nature, as well as tectonic movements and sedimentation processes, change. Fractures are violations of the continuity of the rock. Geometrically, they are characterized by a significant difference in dimensions in the fracture plane (width and length of fractures) and in the perpendicular direction (fracture opening or height). Fractures observed in carbonate rocks can be completely or partially filled with various mineral substances, for example
carbonate or sulfates. Along with them, fractures that remain hollow or open can be distinguished. Also, fractures can be filled with oil or bitumen. Disclosure of mineral fractures varies in very wide limits: from fractions of a millimeter to 1 cm or more. As a rule, openness of open fractures does not exceed 20-25 microns [1].

The appearance of a system of interconnected fractures in the rock can change the filtration properties of productive deposits [7, 8].

The technology for reservoir development with dual porosity can be effectively implemented only on the basis of a comprehensive study of the mechanisms of filtration in heterogeneous fracture-porous reservoirs. Hydrodynamic methods for studying the parameters of fractured reservoirs due to strong heterogeneity differ significantly from traditional methods [9, 10]. Such reservoirs are characterized by an intensive exchange fluid flow between the fractures and porous blocks (matrix), which introduces certain corrections to known methods for determining reservoir parameters [11].

Fractured layer is characterized by the discreteness of the properties and parameters of the channels due to the presence of two types of voidness [12]. The matrix has more fine pores (or voids) and has a significant capacity, but low filtration properties. Different authors proposed different methods of development based on simplified reservoir models. The purpose of these studies was to calculate flow characteristics under special conditions of a sharp inhomogeneity in the reservoir. But, despite the wide variety of approaches of such authors as Odeh, Kazemi, DeSwaan and Pollard, they all boil down either to special cases, or to the exclusions of the Warren and Root model[13]. Indeed, the Warren and Root model represents the general case and the best method for describing the process of fluid filtration in a fractured formation under unsteady filtration conditions.

An analytical solution for the Warren and Root model can not be obtained in general form. There is an analytic expression that represents an approximate solution for some particular cases. In this paper, we consider the numerical solution of the filtration process in a fractured-pore-type collector based on the Warren and Root dual porosity model [14].

2. Formulation of the problem. The Warren-Ruth dual porosity model

In the process of filtration in reservoirs of the fractured-pore type, it is necessary to take into account the exchange of fluids between the low permeable pores and the system of natural fractures. The model considers a porous collector, schematized by the same rectangular parallelepipeds as shown in Figure 1 and which have high porosity and low permeability. The low-permeability matrix is divided by a system of natural fractures that have high permeability and low porosity. It is believed that the movement of fluid to the well is carried out through a system of fractures, and the matrix is a capacitance that continuously feeds the entire system of natural fractures. The redistribution of the fluid between the matrix and the fractures depends on the shape and size of the matrix blocks, the smaller the blocks, the easier the fluid flow between them [13, 15, 16]. The matrix and crack have individual properties and are characterized by their own permeability, compressibility and porosity in the dual porosity model.

![Figure 1. The dual porosity model.](image-url)
To describe the filtering mechanism in the "system of fractures - matrix" system, the following equations of mathematical physics are used:

\[ \varphi_f c_{tf} \frac{\partial P_f}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{k_f}{\mu} r \frac{\partial P_f}{\partial r} \right) + S \frac{k_m}{\mu} (P_m - P_f), \]

The following initial and boundary conditions are considered:

\[ P_f \bigg|_{r=0} = P_0 - \Delta P, \quad P_f \bigg|_{r=r_w} = P_0, \]

\[ 2\pi h \frac{k_f}{\mu} \left( r \frac{\partial P_f}{\partial r} \right)_{r=r_w} = Q(t); \]

\[ S = \frac{4 \cdot n \cdot (n + 2)}{L^2}, \]

\[ L = \frac{3 \cdot a \cdot b \cdot c}{a \cdot b + b \cdot c + c \cdot a}, \]

where, \( \varphi_f \) – is the porosity of the natural fractures system, \( \varphi_m \) – is the porosity of the matrix, \( c_{tf} \) – is the compressibility of the fractures system (1/Pa), \( c_{tm} \) – is the compressibility of the matrix (1/Pa), \( k_f \) – is the permeability of the fractures system (m²), \( k_m \) – is the permeability of the matrix (m²), \( \mu \) – is the oil viscosity (Pa·s), \( P_f \) – is the formation pressure in the fractures system (MPa), \( P_m \) – is the reservoir pressure in the matrix (MPa), \( h \) - is the effective thickness of the formation (m), \( q \) - flow rate of liquid (m³/ day), \( \pi \approx 3.14159 \), \( S \) – is the coefficient of fractured rock (1/ m²), \( n \) – is the number of mutually perpendicular fracture groups, \( L \)– is the block size(m²), \( a \) – is the block side length (m), \( b \) – block side width m atrium (m), \( c \) – height of the matrix block side (m).

Unlike the model of a homogeneous formation, the dual porosity model is characterized by two additional parameters: storativity ratio \( (\omega) \) and transmissivity ratio\( (\lambda) \). \( \omega \) is the fraction of fractures in the total formation system, the higher the coefficient, the greater the fracture-cavernous capacity in the reservoir.

\[ \omega = \frac{\varphi_f c_{f}}{\varphi_f c_{f} + \varphi_m c_{m}}, \]

\[ \lambda = \frac{k_m}{k_f} r_w^2, \]

where, \( S \) – is the characteristic coefficient of the fractured rock (1/m³), \( r_w \) – is the radius of the well (m).

Based on various calculations, the following order of magnitude of these parameters was established:

\[ 10^{-3} < \lambda < 10^{-9} \]  
- corresponds to small values of \( S \) - blocks of large sizes, small values of \( k_m \) - impermeable matrix, and high values of \( k_f \) - significant crack opening.

\[ 10^{-2} < \lambda < 10^{-4} \]  
- corresponds to \( \varphi_f c_{f} \gg \varphi_m c_{m} \), and often \( \varphi_f \gg \varphi_m \).
The limits of applicability of the parameters \( \omega \rightarrow 0, \lambda \rightarrow 0 \) and \( \omega \rightarrow 1, \lambda \rightarrow \infty \) are due to the basic physical parameters, such as the voidness (porosity), permeability, crack density and block size. In some limiting cases, a system with a double porosity can be reduced to a system with one type of voidness.

The problem was approximated using an implicit difference scheme. This scheme was chosen for calculation, because when using an explicit scheme, the time step size \( \tau \) is further limited by the Courant condition:

\[
\tau < \frac{h^2}{2\omega_I}
\]

Thus, the amount of calculations for the explicit scheme is significantly increased (approximately by 3 orders of magnitude) [17]. Using an implicit difference scheme [18] allows you to select an arbitrary grid, including an uneven grid.

\[
\varphi_m \left( \frac{p_{mi}^{j+1} - p_{mi}^j}{\tau} \right) = \frac{S_k m}{\mu} \left( p_{mi}^{j+1} - p_{fi}^{j+1} \right) + \frac{S_k m}{\mu} \left( p_{mi}^{j+1} - p_{fi}^{j+1} \right)
\]

To calculate the implicit scheme, the matrix sweep method was used [19]. Matrix sweep refers to direct methods for solving difference equations. In comparison with other direct methods for solving difference problems, matrix sweeping is more universal, since it allows solving equations with variable coefficients and does not impose strong restrictions on the form of the boundary conditions.

3. The algorithm of matrix sweep

It is necessary to solve a system of linear algebraic equations with a block tridiagonal matrix, which looks like this:

\[
AP_{i-1} - CP_i + BP_{i+1} = -F_i
\]

The solution of the system is found recursively by the formulas:

\[
\alpha_1 = C_0^{-1} B_0;
\]

\[
\beta_1 = C_0^{-1} F_0;
\]

\[
\alpha_{i+1} = (C_i - A_i \alpha_i)^{-1} B_i, \quad i = 1, 2, \ldots N - 1
\]

\[
\beta_{i+1} = (C_i - A_i \beta_i)^{-1} (A_i \beta_i + F_i), \quad i = 1, 2, \ldots N
\]

\[
P_i = \alpha_{i+1} P_{i+1} + \beta_{i+1}, \quad i = N - 1, N - 2, \ldots 1, 0
\]

\[
P_N = \beta_{N+1},
\]

where \( \alpha \) and \( \beta \) are coefficients. The elements of the tridiagonal matrix are matrices (the dimension of the matrix in question is 2 * 2):

\[
A_i = \begin{bmatrix}
0 & 1 & k_f \\
0 & 1 & \frac{1}{r \mu \varphi f_c f}
\end{bmatrix},
\]

\[
C_i = \begin{bmatrix}
1 + \frac{S_k m \tau}{\mu \varphi f c f} & 0 & 0 \\
\frac{S_k m \tau}{\mu \varphi f c f} & - \frac{S_k m \tau}{\mu \varphi f c f}
\end{bmatrix} + \begin{bmatrix}
\frac{1}{r \mu \varphi f c f} & \frac{1}{h^2 (r_{i+2}^1 - r_{i+1}^1)} + \frac{S_k m \tau}{\mu \varphi f c f} \\
\frac{1}{r \mu \varphi f c f} & \frac{1}{h^2 (r_{i+2}^1 - r_{i+1}^1)} + \frac{S_k m \tau}{\mu \varphi f c f}
\end{bmatrix},
\]

where \( \varphi f \) is the matrix in question is 2 * 2.)
\[ B_i = \begin{bmatrix} 0 & k_f & 1 \\ 0 & \frac{1}{r \mu A F c_{tf}} & \frac{1}{r^{1/2}} \end{bmatrix}, \]  
\[ F_i = \begin{bmatrix} P_m \\ P_f \end{bmatrix}. \] (18) 

The matrix sweep for the system (10) - (11) under consideration is stable, since it satisfies Theorem 1 [20]:

**Theorem 1:** Let \( A_i, B_i \) be non-zero matrices, \( i = 1, 2, \ldots N-1 \), and let there exist matrices \( C_i^{-1}, i = 1, 2, \ldots N \). If the inequalities

\[ \| C_i^{-1} A_i \| + \| C_i^{-1} B_i \| \leq 1, \quad i = 1, 2, \ldots N-1 \]  
\[ \| C_0^{-1} B_0 \| \leq 1, \| C_N^{-1} A_N \| < 1, \]  

then the matrix sweep is stable.

4. Results of numerical simulation

At the initial time, the production well was put into operation with a flow rate of 150 m³/day. After working out some period, at the time of 100 days the well is closed on the face. During stopping (\( q = 0 \) m³/day), the pressure in the formation begins to recover. The following initial and boundary parameters are set for the well and the region under consideration:

| Parameters                        | Value             | Unit of measurement |
|-----------------------------------|-------------------|---------------------|
| oil viscosity, \( \mu \)          | 2.216123467E-3    | Pa·s               |
| initial fracture pressure, \( P_f \) | 250.0E5           | MPa                |
| initial pressure in the matrix, \( P_m \) | 250.0E5           | MPa                |
| permeability of fractures, \( k_f \) | 1000.0E-15        | m²                 |
| permeability of the matrix, \( k_m \) | 1.0E-15           | m²                 |
| compressibility of fractures, \( c_{tf} \) | 3.0E-9            | 1/Pa               |
| compressibility of the matrix, \( c_{tm} \) | 3.0E-10           | 1/Pa               |
| compressibility of oil, \( c_o \) | 3.0E-9            | 1/Pa               |
| compressibility of water, \( c_w \) | 3.0E-9            | 1/Pa               |
| porosity of the natural fractures system, \( \varphi_f \) | 0.01              |                     |
| porosity of the matrix, \( \varphi_m \) | 0.10              |                     |
| number of mutually perpendicular fracture groups, \( n \) | 3                 |                     |
| block length, \( a \)            | 100               | m                  |
| width of the block, \( b \)      | 100               | m                  |
| block height, \( c \)            | 0.6               | m                  |
| reservoir thickness, \( h \)     | 10                | m                  |
| radius of the well, \( r_w \)    | 0.1               | m                  |
| well supply circuit, \( R_e \)   | 100.0             | m                  |
Analysis of the modeling of hydrodynamic studies by the method of the pressure recovery curve in the production well was carried out [21]. The figures 2 below show the results of numerical simulation.

![Figure 2. Results of numerical simulation. Pressure dynamics.](image)

The figure 2 shows the calculations for two cases: for a block height of 1 m and for a block height of 40 m. Note that with a block height of 40 m, the pressure is restored more slowly.

For the obtained pressure curves, derivatives were constructed (Figure 3 and Figure 4).

The effect of dual porosity is manifested in the early times in a short period of time (a derivative jump downwards) due to the small size of the matrix blocks.

![Figure 3. Log-log plot, block height 1 m.](image)

For larger block sizes, the effect of dual porosity appears in later times.

The analysis showed that the transmissivity ratio ($\lambda$) depends on the size of the matrix blocks, namely: as the size of the matrix block increases, the transmissivity ratio decreases, and the ability of the matrix to participate in filtering the system decreases accordingly.

It was also noted that of the three sides of the block, the height of the matrix block makes a greater contribution. The effect of dual porosity manifests itself in the earlier time region as it increases, and vice versa, the effect of dual porosity is manifested late when height decreases (redistribution of pressure between the crack and the matrix).

When the parameter $\omega$ is varied, i.e. change in porosity and compressibility of fractures and matrix, conclusions were drawn too. As this coefficient increases, the volume of fractured-cavernous capacity decreases in the reservoir, respectively, the effect of dual porosity occurs later.

Thus, to predict the productivity of wells, the success of various geological and technological measures, permeability and current skin factor, the geometry of the distribution of conductive fractures must be taken into account.
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Acknowledgments
The reported study was funded by RFBR according to the research projects № 18-07-00341.