Unusual field and temperature dependence of Hall effect in graphene

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We calculate the classic Hall conductivity and mobility of the undoped and doped (or in the gate voltage) graphene as a function of temperature, magnetic field, and carrier concentration. Carrier collisions with defects and acoustic phonons are taken into account. The Hall resistivity varies almost linearly with temperature. The magnetic field dependence of resistivity and mobility is anomalous in weak magnetic fields. There is the square root contribution from the field in the resistivity. The Hall mobility diverges logarithmically with the field for low doping.

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The widespread attention is attracted to the recent investigations1,2 of a mono-atomical layer of graphite with a honeycomb lattice (graphene). Among the various reasons for this interest, there are the following. The single layer of graphite can be wrapped into 0d fullerenes, rolled into 1d nanotubes, and stacked into 3d graphite. Hence one has a possibility to study the dimensionality effects for the unique substance5. Graphene has a very simple band structure in the range of several eV around the Fermi level. As has long been shown in study of graphite, the energy bands of its single layer are cones ϵ1,2(p) = ±vp at the corners K in the 2d Brillouin zone with v = 10\(^6\) cm/s and the Fermi energy ϵ_F = 0. So the Fermi surface shrink into points. Such a degeneration is conditioned by symmetry because the small group C\(_{3v}\) of the K points has two-dimensional representation.

According to the symmetry consideration, this spectrum (of the Dirac type but massless and two-dimensional) turns out to be stable with respecto to the Coulomb interaction, what was shown in Ref.2 for the case of the 3d dimensionality. Authors of the recent works3,4 argue, that graphene alters its behavior under doping from a marginal Fermi liquid to the ordinary 2d Fermi liquid. Graphene exhibits Shubnikov–de Haas oscillations with the temperature dependence explained in terms of the standard Fermi-liquid theory. They weak-localization corrections to the conductivity can have the different signs3 depending on the interaction range of impurity potentials or be strongly suppressed4 due to the Dirac singularity of the spectrum.

Much theoretical efforts1,2,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18 have been devoted to evaluate in different approaches the minimal conductivity discovered in Refs.1,2. The finite values of conductivity at low temperature means that 2d graphene turns out to be a metal or a semimetal in contradiction with the recent theoretical analysis11.

The challenging task of the carrier interaction with defects in graphene and in the underlying substrate was studied in Refs.14,20,21,22. In order to simplify the transport problem we have considered13 the quantum conductivity of graphene in the collision-less limit, when the electric field frequency or the spatial dispersion are much important in comparison with the collision rate, (ω, kv) ≫ τ\(^{-1}\). We found that the conductivity consists of two terms — one of the Drude type and another describing the interband carrier transitions. Since there is no a gap between the conduction and valence bands, these two terms can compete and the interband contribution becomes larger at high frequencies ω > T. In the opposite case, the intraband contribution plays the leading role. Then the quantum considerations give the same result as the semiclassical Boltzmann equation.

The Hall effect was studied in Refs.22,23,24,25 in both quantum and classic regimes for zero temperature, including the gap due to interaction-induced phenomena26 or interactions with chemical adsorbents27.

In present Letter, we consider the classic Hall effect in both undoped and doped (or in the gate voltage) graphene for finite temperatures taking into account scattering processes by defects as well as by acoustic phonons.

Calculations of graphene transport properties meet a fundamental difficulty because the modern methods (for instance, the diagrammatic approach) are restricted by the requirement that the mean free path ℓ = vτ of carriers must be much larger than the electron wavelength λ = h/ε_F, i.e., ℓτ ≫ 1. This condition cannot evidently be satisfied in the case of undoped graphene where ℓ > 0. One can avoid this difficulty addressing the problem to doped samples or to finite temperatures when T ≫ τ\(^{-1}\). In the last case, the temperature appears instead of the Fermi energy and electrons obey the Boltzmann statistics.

At low temperatures, collisions with defect are important. Using the Fermi golden rule, one can find the collision rate for scattering by defects,

\[ \tau_{imp}^{-1}(ε) = \frac{n_{imp}}{2π} \int d^2p |u(p - p')|^2 |1 - \cos \theta| δ[ε - ε(p')] , \]

(1)

where n_imp is the defect concentration per the unit surface and u(p - p') is the Fourier component of the defect potential. As noticed previously, we are interested in the carrier energy of the order of temperature ε ≃ T, whereas the defect potential varies on interatomic distances. Therefore, the potential in integrand (1) for the intravalley scattering can be considered as a constant...
The magnetic field of Ref. obtains the residual resistance independent of temperature. It is important that the collision rate be where phonon scattering are independent, the total scattering rate can be written in the form

\[ \tau^{-1} = \tau_{\text{imp}}^{-1}(\varepsilon) + \tau_{\text{el-ph}}^{-1}(\varepsilon) = |\varepsilon| n_d^s, \]

where \( n_d = n_{\text{imp}}(u_0/v)^2 \) is the defect concentration per lattice unit. It is important that the collision rate becomes proportional to the energy. Since the electron density of states is also proportional to the energy, one obtains the residual resistance independent of temperature.

The temperature dependence arises due to scattering by phonons. At low temperatures, the electron collisions with acoustic phonons are essential. Extending the result of Ref. to the 2d electron system, we obtain

\[ \tau_{\text{el-ph}}^{-1}(\varepsilon) = \alpha |\varepsilon| T/T_D, \]

where \( \alpha \) is a constant of the order of the unity and \( T_D \) is the Dedye temperature (around 2000 K for graphene). For the important values of energy \( \varepsilon \approx T \), Eq. (2) gives \( \tau^{-1} \approx T^2/T_D \) in comparison with \( \tau^{-1} \approx T^3/T_D^2 \) for 3d systems. Notice, that all scattering angles are essential now, since the Fermi surface (or the chemical potential) is assumed to be small.

As the processes of electron-defect and electron-phonon scattering are independent, the total scattering rate can be written in the form

\[ \tau^{-1} = \tau_{\text{imp}}^{-1}(\varepsilon) + \tau_{\text{el-ph}}^{-1}(\varepsilon) = |\varepsilon| n_d^s, \]

where the notation

\[ n_d^s = n_d + \alpha T/T_D \]

is introduced.

The solution to the Boltzmann equation in the \( \tau \)-approximation results in the electrical conductivity

\[ \sigma_{\alpha\beta} = -\frac{e^2v^2}{\pi} \sum_{\text{bands}} \int d\varepsilon \frac{df_{\alpha}}{d\varepsilon} \frac{m\tau}{1 + (\Omega\tau)^2} \left( \begin{array}{c} 1 \\ -\Omega \tau \\ 1 \end{array} \right), \]

where the factor 4 is acquired due to summation over spin and two \( K \) points per the Brillouin zone. The integration is performed over \( \varepsilon \) from 0 to \( \infty \) in the conduction band and from \( -\infty \) to 0 in the valence band, the cyclotron mass \( m = \varepsilon/v^2 \), the cyclotron frequency \( \Omega = eH/mc \), the magnetic field \( H \) is assumed to be normal to the graphene layer, and \( f_0(\varepsilon - \mu) \) is the Fermi function.

As the result, the Hall conductivity tensor takes the form

\[ \sigma_{xx} = \frac{e^2}{4\pi\hbar n_d^s} \int_{-\infty}^{\infty} \frac{\varepsilon^2 d\varepsilon}{\varepsilon^4 + \eta^2} \text{sech}^2 \frac{\varepsilon}{2}, \]

\[ \sigma_{xy} = \frac{e^2}{4\pi\hbar n_d^s} \int_{-\infty}^{\infty} \frac{\varepsilon^2 \text{sign}(\varepsilon) d\varepsilon}{\varepsilon^4 + \eta^2} \text{sech}^2 \frac{\varepsilon - \varphi}{2}, \]

where we restore the Planck constant and introduce the dimensionless \( \eta \) magnetic field and the dimensionless chemical potential \( \varphi \):

\[ \eta = |e|\hbar v^2/cn_0^s k_B^2 T^2, \quad \varphi = \mu/k_B T. \]

From measurements, longitudinal resistivity \( \rho \) and Hall angle \( \theta_H \) (or Hall mobility \( \mu_H \)) can be obtained:

\[ \rho = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2}, \quad \tan \theta_H = \frac{\sigma_{xy}}{\sigma_{xx}}, \quad \mu_H = \frac{\sigma_{xy}}{\rho \sigma_{xx}}. \]

The chemical potential \( \mu = 0 \) for undoped graphene and

\[ n_0 = 2 \times 10^{-3} \text{ a}^{-2} \]

FIG. 1: Chemical potential versus temperature for two doping values \( n_0 \) per the lattice unit.

in the absence of the gate voltage. In the field effect experiment, i.e. under the gate voltage (or for the doped material), \( \mu \) is determined by the fixed carrier concentration \( n_0 \):

\[ n_0 = \frac{2}{\pi v^2} \int_0^{\infty} \varepsilon [f_0(\varepsilon - \mu) - f_0(\varepsilon + \mu)] d\varepsilon. \]

As seen from Fig. 1 while the temperature grows, the chemical potential tends to its value \( \mu = 0 \) in the undoped graphene.

One can see from Eqs. (3) - (4), that as it must the diagonal component of conductivity is an even function of the chemical potential and the off-diagonal one is an odd function.

The expansion of conductivity tensor components in terms of magnetic field (\( \eta \ll 1 \)) for a case of nondegenerate carrier statistic (\( \varphi \ll 1 \)) takes a form

\[ \sigma_{xx} = \sigma_0 [1 - (\pi/4\sqrt{2}) \sqrt{\eta}], \]

\[ \sigma_{xy} = 0.25 \sigma_0 \eta \varphi \ln (1/\eta), \]

\[ 2 \times 10^{-3} \text{ a}^{-2} \]
where \( \sigma_0 \) is the conductivity in zero field,

\[
\sigma_0 = \frac{e^2}{\pi \hbar n_d^2}.
\]

The resistivity in the absence of field grows linearly with temperature (see Fig. 2) and is independent of the chemical potential \( \mu \), i.e., of doping.

![FIG. 2: Resistivity (in units \( \hbar/e^2 \)) versus temperature for two values of the magnetic field \( H \) (in units \( h_0 = c k_B T/e \hbar v^2 = 0.11 \text{ G} \)) and the doping level \( n_0 \) indicated in Fig. The electron collisions with uncharged defects (of concentration 0.02 per the lattice unit here and everywhere in Figs.) and acoustic phonons are taken into account.

Let us underline that the resistivity corresponding to conductivity [5] grows as the square root of the magnetic field (also see Fig. 3, the upper curve) and acoustic phonons are taken into account.

As expected, these equations result, first, in the resistivity 

\[
\rho = 1/\sigma_0 \quad \text{as a function of magnetic field (in units } h_0 = 0.11 \text{ G) at given temperature and doping } n_0 \text{ labeled at the curves.}
\]

For the degenerate statistic (\( \varphi \gg 1 \)), we obtain from Eqs. [3] – [4]

\[
\begin{align*}
\sigma_{xx} &= \sigma_0 \varphi^4 / (\eta^2 + \varphi^4), \\
\sigma_{xy} &= \sigma_0 \eta \varphi^2 / (\eta^2 + \varphi^4). \tag{7}
\end{align*}
\]

As expected, these equations result, first, in the resistivity \( \rho = 1/\sigma_0 \) independent of the field (see the lower curve in Fig. 3) and, second, in the Hall coefficient \( R = 1/ecn_0 \) which gives the carrier concentration \( n_0 \).

The Hall mobility as a function of temperature is shown in Fig. 4. In relatively high magnetic fields, the mobility is governed by resistivity, but it has more complicated behavior in weak fields.
In conclusions, considering electrons and holes in graphene as ordinary Fermi liquids, we found that the longitudinal resistivity varies mainly linearly with temperature. Hall resistivity and mobility vary anomalously in weak magnetic fields for a case of low carrier concentrations while the degenerate statistic holds. There is the square root contribution from the field in the resistivity. The Hall mobility diverges logarithmically with the field for the low doping. This anomalous behavior results from the linear dependence of both electron density of state and collision rate for the Dirac fermions in graphite.

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