Study on Dynamic Modeling and Anti-slip Control for Four-wheel Independent Driving/Steering Electric Vehicle

Su ZHOU\textsuperscript{1,2}, Xue-lei ZHI\textsuperscript{1,*} and Lu-wei JIANG\textsuperscript{2}

\textsuperscript{1}School of Automotive Studies, Tongji University, 4800 Caoan Road, Shanghai, China
\textsuperscript{2}CDHK, Tongji University, 1239 Siping Road, Shanghai, China

*Corresponding author

Keywords: Four-wheel independent driving/steering electric vehicle, Dynamic model, Driving anti-slip control.

Abstract. This paper presents a vehicle dynamic model of 7-freedom degrees with different road adhesion coefficients based on MATLAB/Simulink platform. The correctness was verified by the dynamic simulation of vehicle model with the same parameters established in CarMaker software. In order to reduce the torque of the slip wheel to make full use of the road adhesion as well as to regulate the torque of non-slip wheel to ensure the stability of the vehicle, the control strategy of driving anti-slip control system with stability torque adjustment function working in full speed range was designed. In terms of torque distribution, according to different four-wheel slip state, it was proposed to different stability torque adjustment mode. The controller model was set up, and the closed-loop simulation of docking and split-road surface were carried out. The effectiveness of the anti-slip control and the torque control strategy are verified based on the simulation platform.

Introduction

The current anti-slip control methods of four-wheel independent drive electric vehicle are mainly logic threshold control, PID control, model tracking control, sliding mode variable structure control and fuzzy logic control. In the literature \cite{1,2,3}, the slip rate is used as the control variable, and its control is direct responsive. However, there are two problems related to the method: firstly, the slip rate is not measurable, usually obtained by estimation, which will lead to poor performance by estimation and measurement of actual wheel speed; secondly, the peak of each road adhesion coefficient is different, the corresponding optimal slip rate is also different, and setting the target slip rate as a fixed value can’t follow the change of road adhesion capacity well \cite{4}.

Independent torque regulation of the sliding wheels will cause undesirable yaw movement of vehicle, and hence affect the stability performance. Therefore, when adjusting torque of sliding wheels, it is necessary to take measures to ensure vehicle stability. The direct yaw moment control method is widely used and its control effect has been verified. As shown in literature \cite{5}, based on yaw moment control method a sliding mode controller is designed for two control targets of sideslip angle and yaw rate. The anti-slip control module is added to realize the anti-slip function. In the literature \cite{6}, a fuzzy slip controller is added to the direct yaw moment control method. The problem of this solution is that the weakened torque is not compensated by other wheels, so the addition of the anti-slip control module increases the execution error of direct yaw torque control.

Therefore, for the four-wheel independent driving/steering (4WID-4WIS) electric vehicles, it is necessary to add torque regulation control for anti-slip control strategy. It can compensate for the loss of power, and reduce the stability problem caused by wheel slip and torque adjustment as well.

Establishment and Verification of Dynamic Model

Establishment of Dynamic Model. The 4WID-4WIS electric vehicle model includes two parts: wheel model and vehicle dynamic model of 7-freedom degrees.
1) Vehicle dynamic model. The motion of a vehicle on the flat road is mainly considered, and the acceleration of the micro electric vehicle considered is assumed small. Therefore, when the model is established, the slope resistance and the acceleration resistance are neglected. The vehicle speed along the body coordinate system is divided into longitudinal and lateral ones, as Eq.1 shows.

\[
\begin{align*}
V_x &= V \cos \beta, \\
V_y &= V \sin \beta 
\end{align*}
\]  

where, \(V_x\) is the longitudinal velocity, \(V_y\) is the lateral velocity and \(\beta\) is the side-slip angle.

Newton’s Second Law is used to list the differential equations of vehicle translation and rotational dynamics as Eq.2.

\[
\begin{align*}
ma_x &= m\left(V_x - V_y \gamma\right) = F_x - F_{xw} - F_f \\
ma_y &= m\left(V_y + V_x \gamma\right) = F_y \\
J_{z}\ddot{\gamma} &= M_z
\end{align*}
\]  
in which, \(F_x = \sum F_{x_i} \cos \delta_i, F_y = \sum F_{y_i} \sin \delta_i, F_{xw} = C_p A \rho \frac{V_x^2}{2}, F_f = G \cdot f, M_z = \sum M_i, i = fl, fr, rl, rr\), and \(M_{fl} = -0.5F_{xfl} \cos \delta_{fl} \cdot B_f + F_{xfl} \sin \delta_{fl} \cdot L_f\), \(M_{fr} = 0.5F_{xfr} \cos \delta_{fr} \cdot B_f + F_{xfr} \sin \delta_{fr} \cdot L_f\), \(M_{rl} = -0.5F_{xrl} \cos \delta_{rl} \cdot B_r - F_{xrl} \sin \delta_{rl} \cdot L_r\), \(M_{rr} = 0.5F_{xrr} \cos \delta_{rr} \cdot B_r - F_{xrr} \sin \delta_{rr} \cdot L_r\). \(m\) is the weight of full-load-vehicle (kg), \(a_i/a_y\) the body longitudinal/lateral acceleration (m/s²), \(\gamma\) the yaw rate (rad/s); \(F_{x, y}\) is the tire longitudinal force from the ground (N), \(F_{f, l}\) the resultant force of \(F_{fl, fr}\) in the longitudinal/lateral direction of the vehicle body, \(F_{xw}\) the wind resistance of the body, \(F_f\) the rolling resistance force from the ground; \(M_i\) is the torque acting on the center of mass by longitudinal force of the wheel (Nm), \(M_{fl}\) the combined torque of \(M_i\); \(L_f/L_r\) represents the distance from the center of mass to the front/rear axles (m), \(J_{z}\) the rotational inertia of the Z-axis of the vehicle body coordinate system passing through the center of mass (kg.m²); \(C_{D}\) the wind resistance coefficient, \(A\) the windward area (m²), \(\rho\) the air density (kg/m³), \(f\) the rolling resistance coefficient, \(G\) the vehicle gravity (N).

For 4WID-4WIS electric vehicles, the wheels are independently controlled. In order to improve the accuracy of the driving anti-slip control system, the translational velocity of each wheel along the longitudinal direction of the body coordinate system are calculated.

\[
\begin{align*}
u_{sfl} &= (V_x - \gamma \cdot B_f / 2) \cos \delta_{fl} + (V_y + \gamma \cdot L_f) \sin \delta_{fl} \\
u_{sfr} &= (V_x + \gamma \cdot B_f / 2) \cos \delta_{fr} + (V_y + \gamma \cdot L_f) \sin \delta_{fr} \\
u_{srl} &= (V_x - \gamma \cdot B_r / 2) \cos \delta_{rl} + (V_y - \gamma \cdot L_r) \sin \delta_{rl} \\
u_{srr} &= (V_x + \gamma \cdot B_r / 2) \cos \delta_{rr} + (V_y - \gamma \cdot L_r) \sin \delta_{rr}
\end{align*}
\]  

where \(u_{sfl}\) is the translational velocity of each wheel along the longitudinal direction of the body coordinate system (m/s).

The longitudinal slip rate of each wheel is calculated as Eq.4.

\[
S_{si} = \frac{\omega_{fl} \cdot -u_{sfl}}{\omega_{fl}} \times 100\%.
\]  

2) Wheel model. The relationship between the road adhesion coefficient \(\mu_i\) and the wheel slip rate \(S_{si}\) can be described more accurately by the expression proposed by Burckhardt[7,8], as Eq.5 shows.

\[
\begin{align*}
\mu_i(S_{si}) &= C_1 (1 - e^{-C_3 S_{si}}) - C_2 S_{si} \\
S_{sopt} &= \frac{\ln(C_1C_2 / C_3)}{C_3} \\
\mu_{max}(S_{sopt}) &= C_1 - (C_3 / C_2)(1 + \ln(C_1C_2 / C_3))
\end{align*}
\]
where $S_{\text{sopt}}$ is the optimal slip rate corresponding to the peak road adhesion coefficient, $\mu_{\text{max}}(S_{\text{sopt}})$ is the peak adhesion coefficient, and $C_1, C_2, C_3$ are the fitting parameters.

Six common standard roads are selected according to the fitting road condition of Burckhardt by the peak adhesion coefficient from high to low. The roads are classified into dry asphalt, dry cement, wet asphalt, wet pebble, snow and ice. The specific fitting parameters, the peak adhesion coefficient and the optimal slip ratio are shown in Table 1 [9], and the relationship between the adhesion coefficient and slip rate of different roads are shown in the left picture of Figure 1. The longitudinal force of the wheel can be calculated from the adhesion coefficient and the vertical load of the wheel. Taking the dry asphalt road as an example, the relationship between the longitudinal force and the slip rate of the wheel under different vertical loads are shown in the right picture of Figure 1.

Table 1. Fitting parameters, peak adhesion coefficient and optimal slip rate of 6 standard roads.

| Roads      | $C_1$  | $C_2$  | $C_3$  | $S_{\text{sopt}}$ | $\mu_{\text{max}}$ |
|------------|--------|--------|--------|-------------------|--------------------|
| Dry asphalt| 1.2801 | 23.99  | 0.52   | 0.17              | 1.17               |
| Dry cement | 1.1973 | 25.168 | 0.5373 | 0.16              | 1.09               |
| Wet asphalt| 0.857  | 33.822 | 0.347  | 0.13              | 0.8013             |
| Wet pebble | 0.4004 | 33.708 | 0.1204 | 0.14              | 0.34               |
| Snow       | 0.1946 | 94.129 | 0.0646 | 0.06              | 0.1907             |
| Ice        | 0.05   | 306.39 | 0      | 0.03              | 0.05               |

Figure 1. Road adhesion curves and longitudinal force under different vertical loads.

**Verification of Dynamic Model Based on Carmaker.** The vehicle dynamics model is established based on MATLAB/Simulink software, and the nonlinear vehicle dynamic model of 7-freedom degrees is simulated and analyzed. The basic parameters of the vehicle are shown in Table 2.

Table 2. Basic technical parameters for simulation.

| No. | Parameters                              | Symbol | Value |
|-----|-----------------------------------------|--------|-------|
| 1   | Full-load weight                        | $M$    | 500(kg) |
| 2   | Distance from barycenter to front axle  | $L_f$  | 1050(mm) |
| 3   | Distance from barycenter to rear axle   | $L_r$  | 1150(mm) |
| 4   | Front axle track                        | $B_f$  | 1400(mm) |
| 5   | Rear axle track                         | $B_r$  | 1400(mm) |
| 6   | Centroid height                         | $h_{\text{cg}}$ | 445(mm) |
| 7   | Wheel radius                            | $R$    | 252.1(mm) |
| 8   | Moment of inertia of the vehicle through the vertical axis of barycenter | $J_z$ | 588(kg·m²) |
| 9   | Moment of inertia of the wheel around the axis of rotation | $J_w$ | 1.5(kg·m²) |
| 10  | Drag coefficient                        | $C_D$  | 0.284 |
| 11  | Rolling resistance coefficient          | $F$    | 0.015 |
| 12  | Frontal Area                            | $A$    | 2m²  |
In order to verify the dynamic performance and accuracy of the model, a model of in-wheel motor driving electric vehicle was established in CarMaker software. The parameters are the same as the MATLAB / Simulink model. The structure of the model is shown in Figure 2.

1) Simulation of the maximum speed. The left picture of Figure 3 is the maximum speed comparison between MATLAB/Simulink model and CarMaker model. It can be seen from the figure that the maximum speed of the two models are 66 km/h and 66.5 km/h with a small difference, and it meets the requirement of the maximum design speed stipulated in China's "Technical Specifications for Micro Low-speed Electric Vehicles"[10], i.e. more than or equal to 40 km/h and less than 70 km/h.

2) Simulation of the acceleration time. According to the description of the acceleration time in "Technical Specifications for Micro Low-speed Electric Vehicles", the acceleration capability is characterized by the time taken by the vehicle to accelerate from zero to 70% of the maximum speed [10]. The acceleration capability of the two models is shown in the right picture of Figure 3. It can be seen from the figure, the acceleration time of the two models were 12.68s and 12.5s, and the trend of the speed is consistent.
Design of Anti-slip Control System

The flow chart of designed driving anti-slip control strategy is shown in Figure 4. When the vehicle works under certain conditions, the longitudinal speed and the slip rate can be estimated by the vehicle dynamics model and the tire model. And according to the judgment of the wheel slip state, the controller determines whether to start the anti-slip control module.

**Figure 4. Flow chart of anti-slip control system.**

**Determination of Wheel Sliding State.** Because the driving torque of wheel motors in this case are relatively small. Hence, the wheel slip rates will be low under good road condition. If the slip rate is used as the target value for anti-slip control, it is impossible to improve the vehicle's power by using the road adhesion capability. Therefore, the wet pebbles pavement with less attaching ability was chosen to carry out the uniform adhesion coefficient pavement simulation. The slip rate of four wheels on the same pavement are basically the same, so the slip rate of left front wheel was taken as an example. The slip rate curve is shown in the left picture of Figure 5.

**Figure 5. The slip rate of left front wheel and the change rate of the slip rate.**

As can be seen from the left picture of Figure 5, when the vehicle runs on the wet pebbles road, the steady state slip rate is 5.5%, and it can be used as a judgment condition for the wheel slip. When the
driving anti-slip control module is activated, the torque adjustment module also begins to work. In this situation, the torque adjustment module may improve torque of non-slip wheels to increase the power, resulting in the slip rate of these wheels increasing to over 5.5%. If the increase of slip rate is slow or not more than a certain value, it can be considered that the slip rate is within the acceptable range, the anti-slip control module does not needed to reduce the torque.

In terms of the growth of the slip rate, when it works on ice pavement, the slip rate of the sliding wheel increases with high growth rate. In order to determine the change rate of wheel slip rate when the adhesion coefficient changes rapidly, the following simulation is carried out: the vehicle is running on the uniform wet pebbles for 1s, after that the left front wheel runs on the ice road, and the other wheels keep on the original pavement. The right picture of Figure 5 is the change rate curve of the left front wheel slip rate. When the left front wheel traveled on the ice at 1s, the slip rate increased rapidly. Its growth rate is more than 100% in 0.01s. In term of setting the threshold value of the maximum slip ratio, taking reference of Figure 1 and general anti-slip control threshold is 10% to 30%[11] and considering the driving torque of this electric vehicle is small, 10% is selected as the slip determination threshold in this study. The two judgment conditions of slip are as follows: a) When the wheel slip rate is more than 5.5% and the change rate is greater than 100%, the wheel is determined to slip; b) When the wheel slip rate is more than 10%, the wheel is determined to slip.

**Design of Anti-slip Controller.** Torque of four wheels for the objected vehicle can be controlled separately. Therefore, four anti-slip controllers are designed to control the four wheels respectively. The PID controller was selected, as shown in Figure 6. The inputs are the difference between estimated wheel slip rate in real time $S_{si}$ and the steady-state target slip rate $S_{siref}$, and the output is driving torque of wheels $T_{di}$. When the wheel slip rate satisfies the judgment condition a), the steady-state slip rate of the control target is set as 5.5%. And when the wheel slip rate satisfies judgment condition b), the steady state slip rate of control target is set as 10%.

![Figure 6. Block diagram of driving anti-slip PID controller.](image)

**Simulation Investigation of Driving Anti-slip Control System**

In MATLAB/Simulink platform, the driving anti-slip PID controller and the upper status tracking torque regulation PID controller are built up. And the simulation investigation are carried out under the typical running conditions, i.e. docking and split-road surface. In the simulation, the input of front axle driving torque is set to 100 Nm and the rear axle driving torque is set as 90 Nm. The initial vehicle speed is set to 1.8 km/h. The initial values of the angular velocity of the four wheels are set to 1.98 rad/s to ensure that the initial slip ratio of four wheels are equal.

**Simulation of Anti-slip Control System for Docking Road.** The wet pebble pavement is selected for the high adhesion coefficient road, and the ice pavement for the low adhesion coefficient road. The situation road conditions are shown in the left picture of Figure 7. Two groups of simulation are set under the same conditions, one group without anti-slip control (open-loop simulation), the other with anti-slip control (close-loop simulation). The results are shown in Figure 7 and Figure 8.

In the open-loop simulation, when front and rear wheels are running on the road with low adhesion coefficient, the wheels are sliding obviously, resulting in significant reduction of longitudinal acceleration and power weakened. In the close-loop simulation, when the two front wheels run on the road with low adhesion coefficient, the change of the sliding rate meets the condition of anti-slip
module. So the anti-slip module is started and the slip rate of the sliding wheel is maintained at around 5.5%. At the same time, the torque regulation module also starts, the yaw rate is 0 and slip rate of coaxial wheel are the same, so the torque regulation control module starts the two-wheel regulation mode. Under this mode, the torque of two rear wheels increase, and their slip rates increase. When the slip rates reach 10%, the anti-slip module begins to work to make the slip rates remained at about 10%. Even when the two rear wheels running on low adhesion coefficient Road, the slip rate controller works well.

![Figure 7. Road adhesion coefficient and output slip rate for 4 wheels.](image)

![Figure 8. Trajectory of the centroid, longitudinal/lateral velocity, side slip angle and yaw rate.](image)

It can be seen from Figure 8 that the vehicle's driving trajectories under both open-loop and close-loop control are straight. And after the same simulation time, the vehicle travels farther in close-loop simulation. The output result of vertical speed proves that the vehicle dynamic performance has been improved with the anti-slip system. It can be summed up that the anti-slip system can reduce the slip rate when the wheel slips, and improve the vehicle's power without producing the undesired yaw motion.

**Simulation of Anti-slip Control System for Split-road.** Two different adhesion coefficient for simulation are here set as the same as the docking road. The situations of pavement are shown in the left picture of Figure 9. Two groups of simulation are conducted under the same conditions. One group applies open-loop control and the other applies close-loop control. The results of the two groups are shown in Figure 9-Figure 11.

As the results shown in Figure 9 and Figure 11. In the open-loop case, when the left front wheel and the left rear wheel running on low adhesion coefficient road, the wheels are sliding obviously. The slip state causes severe counterclockwise yaw movement and right lateral movement, and the longitudinal speed is significantly reduced. When the anti-slip system is added, once the left front wheel starts to slide, the slip control module starts and reduces the slip rate. At the same time, the torque regulation module starts, and selects the single-wheel torque regulation mode, which reduces the torque of right front wheel to reduce the yaw rate and slip rate. When the left rear wheel has a clear
sliding tendency, the anti-slip module starts to work, reducing the torque of the right rear wheel in order to reduce the slip rate and yaw rate.

Figure 10 shows that anti-slip control significantly reduces the lateral and yaw movement of the vehicle. From the comparison of the yaw rate, the side slip angle and the lateral velocity in Figure 11, after a relatively short time, these variables can be controlled in an expectable value, so the effect of the anti-slip control based on driving stability is better.
Summary
Based on a 4WID-4WIS electric vehicle, a nonlinear dynamic model of 7-freedom degrees is established and a corresponding simulation analysis of the influence of four-wheel slip states on dynamics and stability is carried out. The driving anti-slip control strategy with torque regulation function is designed for full speed range. According to the different four-wheel slip states, it is proposed to different torque regulation modes. The anti-slip PID controller and the torque regulation PID controller are designed. The control strategy models are established in MATLAB/Simulink software, and the simulation studies are carried out under two typical conditions of docking and split-road. The rationality and effectiveness of the proposed control strategy are investigated.

Acknowledgement
This research was financially supported by ‘RoboCar Program (861000)’ which belongs to Beyond Europe – Program for Internationalisation of RTI Projects within the frame of the Austrian Research Promotion Agency (FFG).

References
[1] L. Zhou, L. Xiong, Z.P. Yu, A research on anti-slip regulation for 4WD electric vehicle with In-wheel motors, Applied Mechanics and Materials. Trans Tech Publications, 347(2013): 753-757.
[2] M. Boisvert, P. Micheau, Estimators of wheel slip for electric vehicles using torque and encoder measurements, Mechanical Systems and Signal Processing, 76-77(2016): 665-676.
[3] Mutoh N, Hayano Y, Yahagi H, et al., Electric braking control methods for electric vehicles with independently driven front and rear wheels, IEEE Transactions On Industrial Electronics, 54(2007): 1168-1176.
[4] H. Alipour, M. Sabahi, M. B. Sharifian, Lateral stabilization of a four wheel independent drive electric vehicle on slippery roads, Mechatronics, 30(2015): 275-285.
[5] L.S. Jin, L.L. Gao, Y.Y. Jiang, et al., Research on the control and coordination of four-wheel independent driving-steering electric vehicle, Advances in Mechanical Engineering, 9-4(2017): 1-13.
[6] G.C. Zou, Y.G. Luo, K.Q. Li, Tire longitudinal force optimization distribution for independent 4WD EV, Journal of Tsinghua University (Science and Technology), 49-5(2009): 719-722.
[7] H.Y. Guo, R. Yu, X.Y. Bai, et al., Vehicle traction control based on optimal slip using sliding mode controller, Proceedings of the 33rd Chinese Control Conference, (2014): 251-256.
[8] M. Duan, W.T Guo, G. Li, Q. Yu, Study on acceleration slip regulation control of electric vehicle based on road identification, Mechanical & Electrical Engineering Magazine, 32-9(2015):1257-1262.
[9] W.Q. Bu, Traffic State Estimation and Pavement Identification of Full Drive-by-wire Electric Vehicle, master dissertation, Jilin University, Changchun, 2014.
[10] China Miniature Electric Vehicle Standardization Technical Committee, “T/TBPS 1001-2016. Technical specifications for Micro low-speed electric vehicles” 2016.
[11] X.D. Zhang, D. Goehlich, Integrated traction control strategy for distributed drive electric vehicle with improvement of economy and longitudinal driving stability, Energies, 10-1(2017): 126-143.