ELECTROMAGNETIC PROCESSES IN \(\chi\)EFT

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Nuclear electromagnetic currents derived in a chiral-effective-field-theory framework including explicit nucleons, \(\Delta\) isobars, and pions up to N\(^2\)LO, i.e., ignoring loop corrections, are used in a study of neutron radiative captures on protons and deuterons at thermal energies, and of \(A=2\) and 3 nuclei magnetic moments. With the strengths of the \(\Delta\)-excitation currents determined to reproduce the \(n-p\) cross section and isovector combination of the trinucleon magnetic moments, we find that the cross section and photon circular polarization parameter, measured respectively in \(n-d\) and \(n-d\) processes, are significantly underpredicted by theory.

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1. Introduction

Nuclear electromagnetic currents have recently been derived in a chiral-effective-field-theory (\(\chi\)EFT) framework including nucleons, \(\Delta\) isobars, and pions\textsuperscript{1}. Formal expressions up to one loop have been obtained in time-ordered perturbation theory with non-relativistic Hamiltonians constructed from the chiral Lagrangian formulation of Refs.\textsuperscript{2,3,4}. Thus, the study in Ref.\textsuperscript{1} is similar to the work of Park \textit{et al.}\textsuperscript{5}, albeit it uses a different formalism.

The present talk is a much abridged summary of Ref.\textsuperscript{1}. The currents up to next-to-next-to-leading order (N\(^2\)LO), that is ignoring loop corrections which enter at N\(^3\)LO, are used to calculate the magnetic moments of \(A=2\) and 3 nuclei, and the thermal neutron radiative captures on protons and deuterons. Realistic two- and three-nucleon (for \(A=3\)) potentials are used to generate the bound and continuum wave functions. To have an estimate of the model dependence arising from short-range phenomena, the variation of the predictions is studied as function of the cutoff parameter, which is used to regularize the two-body operators, as well as function of the input potentials—either the Argonne \(v\textsubscript{18}\) (AV18)\textsuperscript{6} or CD-Bonn (CDB)\textsuperscript{7} in combination with respectively the Urbana IX\textsuperscript{8} or Urbana IX\textsuperscript{*}\textsuperscript{9}—used to generate the wave functions (the AV18 and CDB have rather different short-range behaviors).

We find that the N\(^2\)LO calculations do not provide a satisfactory description of the experimental data, particularly for the \(^2\text{H}(n,\gamma)^3\text{H}\) process. It remains an interesting question whether N\(^3\)LO corrections will resolve the present discrepancies
Nucleons, ∆ isobars, pions, and photons are denoted by solid, thick-solid, dashed, and wavy lines, respectively.

between theory and experiment.

2. Currents up to $N^2$LO

The currents up to $N^2$LO are illustrated by the diagrams in Fig. 1, where we show only one of the possible time orderings. The LO and NLO currents, panels a) and b)-c), are well known and will not be given here. At $N^2$LO there is a contribution originating from $(Q/M)^2$ corrections to the LO (one-body) current ($Q$ is the low momentum scale and $M \simeq 1$ GeV is the typical hadronic mass scale); it reads

$$j_{N^2LO}^{RC} = - \frac{e}{8 m_N} \epsilon_{N,i} \left[ 2 \left( K_1^2 + q^2/4 \right) (2 K_1 + i \sigma_1 \times q) + K_1 \cdot q (q + 2 i \sigma_1 \times K_1) \right]$$

$$- \frac{i e}{8 m_N} \kappa_{N,i} \left[ K_1 \cdot q \left( 4 \sigma_1 \times K_1 - i q \right) - (2 i K_1 - \sigma_1 \times q) q^2/2 \right. \right.$$  

$$\left. + 2 (K_1 \cdot q) \sigma_1 \cdot K_1 \right] + 1 \rightarrow 2 , \quad (1)$$

where the momenta $k_i$ and $K_i$ are defined as $k_i = p'_i - p_i$ and $K_i = (p'_i + p_i)/2$, $q$ is the photon momentum, and

$$\epsilon_{N,i} = (1 + \tau_{i,z})/2 , \quad \kappa_{N,i} = (\kappa_S + \kappa_V \tau_{i,z})/2 , \quad \mu_{N,i} = \epsilon_{N,i} + \mu_{N,i} , \quad (2)$$

with $\kappa_S = -0.12$ n.m and $\kappa_V = 3.706$ n.m.

The configuration-space versions of the NLO and $N^2$LO operators have $1/r^2$ and $1/r^3$ singularities ($r$ is the interparticle separation), which need to be regularized in...
order to avoid divergencies in the matrix elements of these operators between nuclear wave functions. We adopt a simple regularization procedure, i.e. a momentum-space cutoff. While its precise functional form is arbitrary, the choice made here of a Gaussian cutoff function, $C_\Lambda(p) = e^{-\left(p/\Lambda\right)^2}$, with the parameter $\Lambda \leq M$, is merely dictated by convenience, since it leads to analytical expressions for the Fourier transforms. It is expected that this arbitrariness be of little relevance, since the dependence of theoretical predictions on variations in the cutoff is (or should be, see next section) largely removed by a renormalization of the theory free parameters, which are fixed by reproducing a given set of observables.

3. Results and Conclusions

At $N^2$LO, the only isoscalar terms are from the (one-body) LO and $N^2$LO-RC operators, which are independent of the cutoff $\Lambda$. In Tables 1 we list their contributions to the deuteron magnetic moment and isoscalar combination of the $^3$He and $^3$H magnetic moments. The $N^2$LO-RC correction is (in magnitude) about 1% of the LO contribution but of opposite sign, so that its inclusion increases the difference between the measured and calculated values. As a result the experimental deuteron and trinucleon isoscalar magnetic moments are underpredicted by theory at the (1.6–2.1)% and (3.0–4.7)% levels, respectively, depending on whether the CDB and CDB/UIX* or AV18 and AV18/UIX combinations are adopted in the $A=2$ and $A=3$ calculations. We note that a recent calculation of these same observables based on variational Monte Carlo (VMC) wave functions corresponding to the AV18/UIX Hamiltonian, finds the magnitude of the $N^2$LO-RC correction somewhat smaller in $A=2$ (–0.0069 n.m.) and significantly larger in $A=3$ (–0.012 n.m.) than obtained here. However, the expression for the magnetic dipole operator is different from that resulting from Eq. 11, and the VMC wave functions are less accurate than the hyperspherical harmonics (HH) wave functions used in this work.

In the isovector sector, the NLO current involves the combination $g_A/F_\pi$, for which we adopt the value $(m_\pi g_A/F_\pi)^2/(4\pi)=0.075$ as inferred from an analysis of nucleon-nucleon elastic scattering data. In the $N^2$LO currents, the parameters $C_{\Delta}$ and $C_{\Delta c}$ are determined by reproducing the $n-p$ radiative capture cross section and $^3$He/$^3$H isovector magnetic moment, respectively. We note that the $\Delta_c$ current
in Eq. (4) gives no contribution in the n-p capture. It contributes in three-body matrix elements only because in the configuration-space version of this operator the \( \delta \)-function is replaced by a finite width Gaussian \(^1\). It is for this reason that one can interpret the contributions resulting from the \( \Delta_c \) current as representing corrections beyond \( \text{N}^2\text{LO} \).

Results for the isovector combination \( \mu_V \) of the trinucleon magnetic moments (without inclusion of the \( \Delta_c \) current contribution) are presented in Table 2. The NLO contribution calculated in Ref. \(^10\) with VMC wave functions and a cutoff of \( 600 \text{ MeV} \) is \(-0.205 \text{ n.m.} \), which is 4\% larger than obtained here. Of course, the parameter \( C_{\Delta_c} \) is adjusted to reproduce, as function of \( \Lambda \), the \( \mu_V \) experimental value.

Predictions for the cross section \( \sigma_T \) and photon circular polarization parameter \( R_c \) measured in the reaction \( ^2\text{H}(n, \gamma)^3\text{H} \) (with unpolarized and polarized neutrons, respectively) are presented in Table 3. At thermal energies this process proceeds through S-wave capture predominantly via magnetic dipole transitions from the initial doublet \( J=1/2 \) and quartet \( J=3/2 \) n-d scattering states. In addition, there is a small contribution due to an electric quadrupole transition from the initial quartet state. At \( \text{N}^2\text{LO} \) the cross section is underpredicted by theory by (11–38)\% as the

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\begin{array}{cccccc}
\text{Table 2. Contributions in units of n.m. to the isovector combination of the trinucleon magnetic moments, obtained with the AV18/UIX and CDB/UIX* Hamiltonian models and cutoff values in the range 500–800 MeV. The LO and N2LO-RC contributions are cutoff independent. The experimental value is } -2.553 \text{ n.m.}.

\begin{array}{cccccc}
\hline
\Lambda (\text{MeV}) & & & & & \\
A & 500 & 600 & 800 & 500 & 600 & 800 \\
\hline
\text{LO} & -2.159 & -2.159 & -2.159 & -2.180 & -2.180 & -2.180 \\
\text{NLO} & -0.156 & -0.197 & -0.238 & -0.113 & -0.156 & -0.200 \\
\text{N2LO-RC} & +0.029 & +0.029 & +0.029 & +0.024 & +0.024 & +0.024 \\
\text{N2LO-}\Delta & -0.258 & -0.253 & -0.250 & -0.205 & -0.292 & -0.200 \\
\text{Sum} & -2.544 & -2.580 & -2.618 & -2.474 & -2.514 & -2.556 \\
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\begin{array}{cccc}
\text{Table 3. Cumulative contributions to the cross section } \sigma_T \text{ (in mb) and photon polarization parameter } R_c \text{ of the reaction } ^2\text{H}(n, \gamma)^3\text{H} \text{ at thermal energies, obtained with the AV18/UIX Hamiltonian model and cutoff values in the range 500-800 MeV. The experimental values for } \sigma_T \text{ and } R_c \text{ are respectively } (0.508\pm0.015) \text{ mb from Ref.}^{14} \text{ and } -0.42\pm0.03, \text{ from Ref.}^{15}.

\begin{array}{cccccc}
\hline
\Lambda (\text{MeV}) & \sigma_T & & & & \\
& 500 & 600 & 800 & 500 & 600 & 800 \\
\hline
\text{LO} & 0.229 & 0.229 & 0.229 & -0.060 & -0.060 & -0.060 \\
\text{LO+NLO} & 0.272 & 0.260 & 0.243 & -0.218 & -0.182 & -0.123 \\
\text{LO+····+N2LO-RC} & 0.252 & 0.241 & 0.226 & -0.152 & -0.109 & -0.041 \\
\text{LO+····+N2LO-}\Delta & 0.438 & 0.416 & 0.389 & -0.432 & -0.418 & -0.397 \\
\text{LO+····+N2LO-}\Delta_c & 0.450 & 0.382 & 0.315 & -0.437 & -0.398 & -0.331 \\
\hline
\end{array}
\end{array}
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cutoff is increased from 500 MeV to 800 MeV. This rather drastic cutoff dependence is mostly due to the contribution of the N^2LO-Δ current. Indeed removing it leads to a much weaker variation of the cross section—roughly ±5% about the value obtained with Λ = 600 MeV (next to last row of Table 3). It will be interesting to see to what extent, if any, loop corrections at N^3LO will improve the present predictions, and in particular reduce the cutoff dependence.

The photon polarization parameter is very sensitive to contributions of NLO and N^2LO currents, which produce more than a sixfold increase, in absolute value, of the LO result, and bring it into much closer agreement with the measured value. All results listed in Table 3 for R_c (and σ_T) include the small e_{44} RME, although it only has a significant effect for the LO prediction (R_c=−0.060 versus −0.072 depending on whether e_{44} is retained or not).

We conclude by summarizing our results. Up to N^2LO, the only isoscalar terms are those generated in a non-relativistic expansion of the one-body current, and provide a (cutoff-independent) 1% correction—relative to LO—to the deuteron and isoscalar combination of the trinucleon magnetic moments. This correction is of opposite sign to the LO contribution, and therefore increases the underprediction of the corresponding experimental values from (0.9 ± 0.3)% for the deuteron and (2.7 ± 0.9)% for the trinucleons at LO to, respectively, (1.9 ± 0.3)% and (3.8 ± 0.8)% at N^2LO. The spread reflects differences in the short-range behavior of the AV18 and CDB potentials, in particular the weaker tensor components of the latter relative to the former in this range.

At NLO, isovector terms arise from the pion seagull and in-flight contributions, while at N^2LO, in addition to the relativistic corrections mentioned above, isovector terms due to Δ-isobar excitation are also obtained. The parameters C_Δ and C_{Δc} of the N^2LO two-body Δ-excitation currents have been determined, as functions of the cutoff Λ and for the Hamiltonian model of interest, by reproducing the cross section for the n-p radiative capture at thermal neutron energies and the isovector combination of the trinucleon magnetic moments. This current has then been used to make predictions—with the AV18/UIX model only, since HH continuum wave functions are not yet available for the CDB/UIX* model—for the cross section σ_T and photon circular polarization parameter R_c measured in the capture of, respectively, unpolarized and polarized neutrons by deuterons. The experimental σ_T (|R_c|) is found to be underestimated by 11% (overestimated by 4%) for Λ=500 MeV and 38% (underestimated by 21%) for Λ=800 MeV.

The results display a significant cutoff dependence, particularly so for the N^2LO contributions associated with Δ isobar degrees of freedom. Indeed these contributions are much larger than those at NLO. This is partly due to the fact that the two NLO (pion seagull and in-flight) terms interfere destructively. For example, the seagull (in-flight) contributions to doublet m_{22} and quartet m_{44} M1 matrix elements, in units of fm^3/2 and for Λ=500 MeV, are respectively −9.1 (+6.5) and −0.8 (+0.6). As a result σ_T = 0.425 mb and R_c = −0.425 at LO+NLO (seagull only), which should be compared to σ_T = 0.272 mb and R_c = −0.218 at LO+NLO
(seagull+in-flight) from the second row of Table 3. The relatively large $\Delta$-excitation contributions also point to the need for including loop corrections at $N^3$LO, which these $N^2$LO currents, because of the procedure adopted here to determine their strength, are implicitly making up for.

The next stage in the research program we have undertaken is to incorporate the $N^3$LO operators derived in Ref. 1 into the calculations of the captures and magnetic moments involving light nuclei (with mass number $A \leq 8$), and indeed to extend these calculations to also include $p$-$d$ capture at energies up to a few MeV’s, and possibly four-nucleon processes, in particular $^3\text{He}(n, \gamma)^4\text{He}$ at thermal energies. Of course, at $N^3$LO three-body currents also occur, and will need to be derived. Work along these lines is being pursued vigorously.

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