Abstract—The era of fully autonomous, electrified taxi fleets is rapidly approaching, and with it the opportunity to innovate myriad on-demand services that extend beyond the realm of human mobility. This project envisions a future where autonomous EV fleets can be dispatched as both a taxi service and a source of on-demand power serving customers during power outages. We develop a PDE-based scheme to manage the optimal dispatch of an autonomous fleet to serve passengers and electric power demand during outages as an additional stream of revenue. We use real world power outage and taxi data from San Francisco for our case study, modeling the optimal dispatch of several fleet sizes over the course of one day; we examine both moderate and extreme outage scenarios. In the moderate scenario, the revenue earned serving power demand is negligible compared with revenue earned serving passenger trips. In the extreme scenario, supplying power accounts for between $1 and $2 million, amounting to between 32% and 40% more revenue than is earned serving mobility only, depending on fleet size. While the overall value of providing on-demand power depends on the frequency and severity of power outages, our results show that serving power demand during large-scale outages can provide a substantial value stream, comparable to the value to be earned providing grid services.

I. INTRODUCTION

A. Motivation and Background

Fully autonomous plug-in electric vehicles (PEVs) have tremendous potential to change the future of mobility. In particular, fleets of autonomous vehicles providing on-demand mobility services will likely play a major role in transportation systems [1]. While the impact of these changes on travel demand is uncertain, it is clear that safety, energy efficiency, and cost of travel will be substantially improved in the future. It is also clear that autonomous on-demand fleets of PEVs will require continued innovation in methods for systems optimization and control.

Autonomous PEV fleets could play an important role in providing flexibility services to the future electric grid. Another potential source of ancillary value provided by these vehicles is supplying electricity to buildings during power outages, when occupants are willing to pay more for energy to avoid damages associated with lack of electric service. The current work examines the additional revenue attained by a fleet of autonomous electric vehicles providing both a mobility-on-demand service and backup power during outages.

B. Relevant Literature

The current personal vehicle ownership paradigm involves gross under-utilization of vehicles, as personal vehicles sit idle for most of the day. This under-utilization makes grid-connected PEV batteries an excellent source of load flexibility by charging or discharging as needed while vehicles are not in use. Numerous studies examine the capabilities [2], [3], [4], [5] and economics [6], [7], [4] of using electric vehicles to provide grid services. However, Sheppard and Bae conclude that privately owned vehicles can earn only about $100 per year (on average) providing ancillary services [7].

Furthermore, technology development and gradual deployment of semi-autonomous safety features suggest that the future of transportation is autonomous. Once autonomous vehicles are deployed at scale, the current paradigm of personal vehicle ownership is likely to change [1]. Although a rightsized, autonomous, commercially operated fleet is likely to be much less flexible than privately owned vehicles, centralized control can increase the magnitude and reliability of aggregate response when price signals for battery charging or discharging are adequate.

C. Focus of this Study

We propose a PDE-based approach, described in [2], to simulate the optimal dispatch of autonomous on-demand PEVs serving time varying, spatially distributed demand for trips and backup power. The fleet is dispatched to maximize profit earned from serving both trips and power. The revenue earned for each trip serviced or kWh provided depends on the origin and destination of the trip, and the location of the power outage. We consider several fleet sizes, examining differences in vehicle dispatch, state of charge, revenue earned, and unserved demand for trips/power. Key contributions of this work include the geospatial modeling of vehicle mobility, charging & discharging, and inclusion of backup power as an ancillary revenue stream.

II. TECHNICAL DESCRIPTION

A. Modeling Aggregations of Autonomous Electric Vehicles

We adopt and extend the scheme developed by [2] for tracking and controlling an aggregation of electric vehicles. The core advantage of the scheme is the recognition that in an autonomous PEV fleet, only the location of vehicles and their state of charge are critical to know at any point in time. Instead of representing individual vehicles explicitly and developing a combinatorial approach to control, we aggregate all vehicles in a node and represent the aggregate distribution of vehicle state
of energy (SOE). Vehicles in any node \( i \) can be in one of three states: charging, idle, or discharging, which we represent by the state variables \( u_i(x,t) \), \( v_i(x,t) \), and \( w_i(x,t) \), respectively. The system is then characterized by the following coupled partial differential equations (see Table I for further nomenclature):

\[
\begin{align*}
\frac{\partial u_i}{\partial t}(x,t) &= -\frac{\partial}{\partial x}[q_C(x)u_i(x,t)] + \sigma_{i\rightarrow C_i}(x,t) \\
\frac{\partial v_i}{\partial t}(x,t) &= \sum_{j \in \mathbb{Z}} \left[ \sigma_{i\rightarrow C_j}(x,t) + \sigma_{j\rightarrow i}(x,t) \right] \\
&\quad - \sigma_{i\rightarrow I_i}(x,t) - \sigma_{I_i\rightarrow D_i}(x,t) \\
\frac{\partial w_i}{\partial t}(x,t) &= -\frac{\partial}{\partial x}[q_D(x)w_i(x,t)] + \sigma_{i\rightarrow D_i}(x,t)
\end{align*}
\]

Where:

\[
q_C(x) = \frac{7}{E_{max} 60}, \quad q_D(x) = \frac{-7}{E_{max} 60}
\]

The equations make use of an advection term (when the time derivative is linearly related to the spatial derivative) to represent how SOE changes over time in the charging or discharging states, with SOE advecting toward 1 or 0, respectively. The model is spatially disaggregated, so the three PDEs are repeated for every node in the system and indexed by \( i \).

Flow terms \( \sigma_{i\rightarrow C_i}(x,t) \) and \( \sigma_{i\rightarrow D_i}(x,t) \) capture the transport of vehicles between the three distributions within each node. Additional flow terms capture transport between the Idle curves of distinct nodes. For a given node \( i \) and any other node \( j \), four separate terms are used to represent trips with and without passengers (\( \sigma' \) and \( \sigma'' \) respectively) and departing trips versus arriving trips (\( \sigma_{j\rightarrow i} \) and \( \sigma_{i\rightarrow j} \), respectively).

The inter-nodal flow terms are then constrained through the optimization scheme such that departures from a node \( i \) to node \( j \) are equivalent to the arrivals of vehicles from \( i \) to \( j \) at a future time and with a lower SOE, corresponding to the travel time and energy requirements of that trip. The distinction between trips with and without passengers becomes critical in the context of the economic optimization that places monetary value on transporting people over moving empty vehicles.

### B. Optimization Formulation

1) **Objective:** The objective of the optimization is to maximize the operational profit of dispatching the fleet of autonomous on-demand PEVs:

\[
\max_{\sigma_{i\rightarrow D_i}, \sigma_{i\rightarrow C_i}} K = \sum_{i \in \mathbb{Z}} \int_0^T \left[ \frac{\rho_{dis}(i) Q_{dis,i}(t)}{60} + \sigma_{i\rightarrow D_i} Q_{ch,i}(t) \right] dt
\]

\[
Q_{dis,i}(t) = \int_0^1 \rho_{mob,i,j} Q_{mob,i,j}(t) dx
\]

\[
Q_{ch,i}(t) = \int_0^1 \left[ \frac{\rho_{mob}(i,j) Q_{mob,i,j}(t)}{60} - \frac{C}{60} Q_{ch,i}(t) \right] dx
\]

Where \( \rho_{mob}(i,j) \), \( \rho_{dis}(i) \), and \( C \) are the fares charged to passengers, the price charged to serve load during outages, and the cost to purchase electricity from the grid, respectively. The constant 60 converts kWh to kW-minutes and the constant 7 is the charging and discharging rate of each vehicle.

2) **Constraints:** The equations of state are discretized using a first-order upwind scheme for numerically solving hyperbolic PDEs. They appear in the formulation as a set of equality constraints. In addition to the equations of state there are other constraints on the flows which we use to enforce realistic transport between nodes and the overall conservation of vehicles in the system.

Firstly, we constrain the size of the flows between states \( u, v, \) and \( w \) to be no greater than the number of vehicles in those states:

\[
\begin{align*}
-\sigma_{i\rightarrow C_i}(x,t) &\leq u_i(x,t) / \Delta t \\
\left\{ \sigma_{i\rightarrow C_i}(x,t) + \sigma_{i\rightarrow D_i}(x,t) + \sigma_{i\rightarrow I_i}(x,t) \right\} &\leq v_i(x,t) / \Delta t \\
\left\{ \sigma_{i\rightarrow I_i}(x,t) - \sigma_{i\rightarrow I_j}(x,t) \right\} &\leq w_i(x,t) / \Delta t
\end{align*}
\]

We also require that as charging vehicles reach an SOE of 1 or...
as discharging vehicles reach an SOE of 0, they immediately flow to the Idle state.

\[-\sigma_{I_i \rightarrow O_i}(1, t) = \frac{u_i(1, t)}{\Delta t} \]

\[-\sigma_{I_i \rightarrow D_i}(0, t) = \frac{w_i(0, t)}{\Delta t} \]

Next, we require that trips be conserved between origin-destination pairs, where arrivals are shifted to a later time step and a lower SOE, based on the time (\(\Delta t\)) and energy (\(\Delta x\)) requirements of the trip.

\[\sigma_{I_i \rightarrow I_j}(x, t) = \sigma_{I_j \leftarrow I_i}(x) - \Delta x_{i,j} + \Delta t_{i,j} \]

\[\sigma_{I_i \rightarrow D_j}(x, t) = \sigma_{D_j \leftarrow I_i}(x) + \Delta x_{i,j} + \Delta t_{i,j} \]

\[\{(i, j) \in \mathbb{Z} \times \mathbb{Z}\} \]

The values of \(\Delta x\) and \(\Delta t\) for each node (I, II and IV) are derived empirically based on real San Francisco taxi fare data collected over the course of a month in June 2012. We assume a decline in personal vehicle ownership accompanies deployment of autonomous vehicles. We account for increasing reliance on mobility-on-demand services by scaling travel demand by a factor of 10 relative to 2012. We averaged the measured trip durations and trip distances for trips from each node \(i\) to each node \(j\), scaling the average distance by 5.05 km/kWh to derive \(\Delta x_{i,j}\) and taking the average time as \(\Delta t_{i,j}\). The derived values are shown in Table II.

| TABLE II | FLOW CONSTRAINTS |
|----------|-----------------|
| Node Flows \((i \rightarrow j)\) | Derived \(\Delta x\) (kWh) | Derived \(\Delta t\) (s) |
| I→I     | 0.42            | 476              |
| I→II    | 0.82            | 792              |
| I→IV    | 0.93            | 1000             |
| II→I    | 0.84            | 760              |
| II→II   | 0.38            | 489              |
| II→IV   | 0.77            | 698              |
| IV→I    | 0.93            | 956              |
| IV→II   | 0.77            | 725              |
| IV→IV   | 0.37            | 403              |

Vehicle dispatch is constrained such that the number of vehicles servicing passenger trips or power demand cannot exceed mobility and power demand at that time step.

\[Q_{dis,i}(t) \leq D_{dis,i}(t) \]

\[Q_{mob,i,j}(t) \leq D_{mob,i,j}(t) \]

The demands \(D_{dis,i}\) and \(D_{mob,i,j}\) are exogenously defined; derivation of \(D_{dis,i}\) is described below. The choice of inequality constraints when constraining \(Q_{dis,i}\) and \(Q_{mob,i,j}\) serves three purposes: 1) it allows the solution of the optimization to prioritize between serving the two types of demand; 2) it enables simulations where the fleet of vehicles is not sized to meet the peak demand in the system; and 3) it allows the system to be used in an application where power outages occur spontaneously and without foresight.

Finally, we require that the vehicles have sufficient state of energy to make trips:

\[\sigma_{I_i \rightarrow I_j}(x, t) = 0, \quad x < \Delta x_{i,j} \]

\[\sigma_{I_i \rightarrow D_j}(x, t) = 0, \quad x < \Delta x_{i,j} \]

C. Application

1) Spatial Discretization: We have divided the City of San Francisco, CA into a highly simplified 4-zone, equal-area network (Figure 1). As described above, we analyzed taxi data to characterize the constraints realted to mobility and the prices used in the objective. Below we describe how power outages are characterized from real world data. We observe very little demand for mobility and few outages in Node III; due to additional computational complexity of modeling a four node system, we exclude Node III from the current analysis.

Fig. 1. We divide San Francisco into 4 equal-area nodes. Origins and destinations of taxi trips over one month (June 2012) are plotted as red dots.

2) Demand for Backup Power: We estimate the magnitude and location of power outages using real outage data collected from the Pacific Gas & Electric Company website. These data report the number and spatial distribution of outages in the region; we aggregate outages spatially by node. We estimate the magnitude of unserved load based on the number of customers affected, expected distribution by customer type (i.e., residential, commercial, industrial), and average power demand by customer type (as reported in EIA form 861). We use local population and economic census data to estimate the distribution of customer types affected by outages in each node.

We examine two days of outage data, including one extreme outage scenario (December 31, 2014) and one moderate outage scenario (September 29, 2014). Figure 2 shows the estimated power demand at each node for both scenarios. We highlight
that demand in the Extreme outage scenario exceeds demand in the Moderate outage scenario by two orders of magnitude.

Finally, we estimate the value of providing backup power on demand. To do so, we compute the cost of damages incurred due to outages in each node for both outage scenarios using the ICE Calculator [9], a tool commonly used by electric utilities to estimate the economic benefits of measures to improve reliability. Inputs for the damage calculations include: time of day/year, the type and size of the affected customers, and the duration of the outage. Table III gives the estimated value of backup power in each node for the two outage scenarios in $ per unit energy delivered (kWh) and $ per time step (10 minutes). Although power demand is much higher in the Extreme outages scenario, the cost per kWh is greater in the Moderate outages scenario.

For comparison, Table IV lists the fares associated with passenger trips to and from each node in terms of dollars per unit energy consumed (or $ per time step). These fares are empirically derived from the San Francisco taxi data. The value earned per kWh serving passenger trips is remarkably similar to the value earned per kWh of power demand served.

### III. Results

We present simulation results for the two outage scenarios with various fleet sizes, including 7,500, 10,000 and 15,000 vehicles for the Moderate outage scenario, and 7,500, 15,000 and 40,000 vehicles for the Extreme outage scenario. The following sections detail the results. We highlight the revenue earned in different scenarios, and differences in dispatch among different fleet sizes.

#### A. Revenue

Figure 3 presents the revenue earned in each scenario by the entire fleet and per vehicle. Contributors to overall revenue include: the cost to charge (G2V), revenue earned serving trips (Trips), and revenue earned serving power demand (V2B). The total revenue earned (Total) in each scenario and maximum possible revenue (Max) are also shown. The maximum possible revenue includes servicing all passenger trips and all power demand, with no charging costs.

Charging costs are almost negligible compared with the revenue earned because the cost of charging (0.25 $/kWh) is small compared with the revenue earned serving power and trip demand (see Tables III and IV).

Next we consider the revenue earned at each node serving power and mobility demand in the Extreme outages scenario, shown in Figure 4. Very little revenue is earned at Node I; this is attributable to limited demand for trips and low power demand. Nodes II and IV have higher demand for passenger trips, and experience power outages in the afternoon and morning, respectively.

The revenue peaks at Nodes II and IV, coincide with the power outages at those nodes (see Figure 2). At Node II, the revenue earned per unit time serving power demand ($11 per 10 minute interval) is marginally higher than the revenue earned per unit time serving passenger trips. Trips are primarily within node II, and provide $10 per 10 minute interval. Thus there is only a marginal increase in revenue for the 7,500 vehicle fleet during the outage at Node II. We do see a significant increase in revenue during the same outage in the over-sized fleets (15,000 and 40,000 vehicles). At Node IV, the revenue earned serving power demand is $18 per 10 minute interval, which is considerably more than the revenue to be earned serving passenger trips.
Next we consider the benefits and drawbacks of different fleet sizes. Nearly all demand for mobility and power can be served with a 40,000 vehicle fleet in the Extreme scenario, and a 15,000 vehicle fleet in the Moderate scenario. Figures 5 and 6 show the number of vehicles in each state in the Extreme outages scenario with 40,000 and 7500 vehicles. States include: in transit with and without passengers, charging, discharging, and idle.

Figure 5 reveals that a 40,000 vehicle fleet spends most of the simulation in the idle state; the fleet is only fully utilized between 800 and 900 seconds when power demand peaks. Low revenue per vehicle in Figure 3 provides further evidence that the 40,000 vehicle fleet is under-utilized. On the other hand, the 7,500 vehicle fleet in Figure 6 earns less revenue overall, but spends very little time in the idle state. In fact, the vehicles spend more time charging than in any other state; faster charging infrastructure would increase fleet utilization, and should be evaluated as an alternative to increasing the fleet size.

In Figure 4, the 7,500 vehicle fleet earns less revenue at Node II than the larger fleets for almost the entire simulation. This result suggests that the 7,500 vehicle fleet is under-sized. Figure 7 shows the dispatch of a 15,000 vehicle fleet serving...
mobility only in the Moderate outages scenario. The results indicate that with charging constraints, upwards of 15,000 vehicles are needed to meet all of the demand for mobility.

**IV. DISCUSSION**

The fundamental question underlying the current work is whether on-demand backup power provides a substantial value stream for the fleet. To answer that question, we must consider the relative frequency of Extreme and Moderate outage days, and the marginal increase in revenue associated with serving power demand in addition to passenger trips.

We consider several scenarios for the number of Extreme versus Moderate outage days in a year. We then compute the marginal annual revenue earned serving both power and mobility demand, compared with serving mobility only. We treat the Moderate outages scenario as a mobility-only case, as the revenue earned serving power demand in that scenario is negligible.

We calculate the marginal revenue earned serving power demand by taking the difference between a year with Extreme outages and a year with only Moderate outages for equivalent fleet sizes. The results, summarized in Table V, suggest that fleet operators can earn $1,400-$3,400 (or ~1-3%) more revenue per vehicle per year serving power demand during outages, depending on fleet size and the number of major power outages.

| Extreme Days | New Revenue ($/year/vehicle) | Percent Increase (%) |
|--------------|------------------------------|----------------------|
| 10           | 1400                         | 7,500                     | 2000                          | 0.9                           | 1.6                           |
| 12           | 1700                         | 2300                     | 1.0                           | 1.8                           |
| 14           | 2000                         | 2600                     | 1.2                           | 2.0                           |
| 16*          | 2200                         | 2800                     | 1.4                           | 2.2                           |
| 18           | 2500                         | 3100                     | 1.5                           | 2.4                           |
| 20           | 2800                         | 3400                     | 1.7                           | 2.6                           |

* Actual number of days with major power outages in the Pacific Gas and Electric Company service territory in 2014 [10].

These results are sensitive to numerous assumptions in our analysis, including but not limited to: outage cost, outage frequency/duration, vehicle battery size, battery discharge rate, optimization window, and foresight into demand for power and passenger trips.

**A. State of Energy**

Figure 8 shows the aggregate SOE of the fleet with respect to time for the various fleet sizes and outage scenarios. We initialize the fleet with an aggregate SOE of 0.5. For all fleet sizes, the aggregate SOE then drops to below 5% before any charging occurs. Figures 5 and 6 show that charging begins at about 250 and 50 minutes for the 40,000 and 7,500 vehicle fleets, respectively. The entire fleet operates at a very low SOE, cycling out of charging before vehicles reach full SOE.

The fleet operates at a low SOE because the current model dispatches the fleet based on a planning horizon of only 50 minutes. We assume no knowledge of demand for trips or power more than 50 minutes ahead of time, and assign no penalty for entering the next planning horizon with low SOE. Thus the fleet is dispatched to maximize profit within each 50 minute window, and vehicles spend only as much time charging as is needed to serve near-term demand for trips and power. Furthermore, when vehicles are not needed or have insufficient charge to meet demand within the planning horizon, charging is less cost effective than remaining idle (at zero cost) with low SOE.

Future work will examine more realistic assumptions around vehicle charging. Examples could include a penalty for failing to achieve some minimum SOE at the end of each planning horizon, or a fee charged upon entry into the charging state, incentivizing vehicles to charge until reaching full SOE.

**V. SUMMARY**

We demonstrate a method for simulating a fleet of autonomous PEVs in San Francisco dispatched to serve mobility...
and electricity demand during power outages throughout the city. We use a PDE-based approach to model the aggregate state of energy of the fleet as vehicles charge, discharge, and travel throughout the system. We optimize vehicle dispatch over a 50 minute planning horizon, assuming perfect foresight into both mobility and power demand within that time frame. We consider two outage scenarios, including both Moderate and Extreme outages based on real outage data for San Francisco. Finally, we compute the revenue earned in each scenario with various fleet sizes, ranging from 7,500 to 40,000 vehicles. We find that serving power demand increases fleet revenue by $1,400-$3,400 per vehicle, or 30-40%, in the Extreme outages scenario. Given that power outages are rare, these results translate to ∼1-3% more revenue per year, depending on the number of major power outages in a year.

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