The $F/D$ Ratios of Spin-Flip Baryon Vertex in $1/N_c$ Expansion

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We calculate the $F/D$ ratios of spin $1/2$ baryon vertex for both the non-relativistic quark model and the chiral soliton model with arbitrary number of color degrees of freedom $N_c$ and examine the results in terms of the consistency condition approach for the baryon vertices recently developed by Dashen, Jenkins and Manohar from the viewpoint of QCD.

We show that the $1/N_c$ corrections have two different origins, i.e., one is from the baryon states or baryon wave functions and the other from the vertex operators. Although in the limit $N_c \to \infty$ the $F/D$ tends to $1/3$ in all models, the $1/N_c$ expansion of $F/D$ ratio does not converge for $N_c=3$ in the chiral soliton model in contrast to the non-relativistic quark model.

§ 1. Introduction

The non-relativistic quark model (NRQM) has successfully described the hadron phenomena in various aspects and played a crucial role in the way of establishing QCD. However, the NRQM description for the hadron states has not been derived convincingly from QCD. This is mainly due to the nonperturbative nature of QCD in the low energy regions. In order to overcome this difficulty, in 1974, 't Hooft introduced a hidden small expansion parameter of QCD, $1/N_c$. In 1979 Witten applied the $1/N_c$ expansion method to baryon and suggested that in the large $N_c$ limit the baryons are realized as solitons of meson fields because the low energy effective Lagrangian does not contain baryons due to their large mass of order $N_c$. From the early 1980's the Skyrme's conjecture that baryons are the soliton of the nonlinear chiral Lagrangian for the chiral fields has been revived. By quantizing the collective modes of rotations of soliton in the spin-isospin space we obtain the "spinning" soliton states which corresponds to baryons and various properties of baryons are well reproduced in terms of the parameters of meson sectors. The most important characteristics of the chiral soliton will be the hedgehog structure which gives $I+J=0$.

On the basis of the $1/N_c$ expansion method we have been able to understand many of the qualitative properties of mesons and baryons, the quantitative properties of hadrons, however, could not be derived up to very recently. The first quantitative relation for hadron phenomena was obtained from the consistency condition for the baryon vertex in the pion-nucleon scattering amplitude which was derived by Gervais and Sakita on the basis of the large $N_c$ expansion and the strong coupling theory. Recently Dashen, Jenkins and Manohar have developed this consistency condition approach from the viewpoint of QCD and derived an induced algebra with respect to the spin-flavor symmetry. On the basis of the consistency conditions they have analysed

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the large $N_c$ behaviors of baryon vertices in terms of model-independent method and examined the vertices in the NRQM and the static $SU(2)$ chiral soliton model (CSM). Furthermore they have obtained $1/3$ for the large $N_c$ limit of $F/D$ ratio from the consistency conditions model-independently.

In this paper we calculate the $F/D$ ratios for arbitrary value of color degrees of freedom $N_c$ for both the NRQMs with flavor $f=2$ and $f=3$ using $SU(4)$ and $SU(6)$ symmetries respectively and for the $SU(2)$ and $SU(3)$ CSMs.

We pay our attention to the fact that the $1/N_c$ corrections have two different origins, i.e., one from the baryon states or baryon wave functions and the other from the vertex operators.

We compare the obtained results of $F/D$ ratios by using magnetic moments of proton and neutron and confirm that in all models the limiting value of $F/D$ ratio of spin-flip vertex is $1/3$. We show that in the case of $SU(3)$, the magnetic moment of proton and neutron depend on how the real baryon states with $N_c=3$ are extrapolated to large $N_c$ baryon states, but the $F/D$ ratio depends only on group structure of the baryon multiplet and not on the way of extrapolation of baryon states. We found also that although in the limit $N_c\to\infty$ the $F/D$ ratio tends to $1/3$ in all models, the $1/N_c$ expansion of $F/D$ ratio does not converge for $N_c=3$ in the $SU(3)$ CSM in contrast to the NRQMs.

§ 2. The $F/D$ ratio from $SU(4)$ and $SU(6)$ symmetric quark model

In the NRQMs with the $SU(4)$ and $SU(6)$ symmetries the spin $1/2$ baryons are given by the completely symmetric representation with respect to spin and flavor which are represented by the Young diagrams with the first row of length $N_c=2k+1$ ($k=0, 1, 2, \cdots$) only and have dimensions $sH_{N_c}$ and $tH_{N_c}$, where $sH_r$ is a repeated combination $sH_r=n+r-1C_r=(n+r-1)!/r!(n-1)!$.

In the $SU(4)$ symmetric model the magnetic moment of baryon $B$ is given by

$$
(\mu_B)^{SU(4)}_{SU(4)} = \left( \begin{array}{cc} \frac{15}{s_Q} & sH_{N_c} \\ 0 & sH_{N_c}^* \end{array} \right) \mu,
$$

where $s_Q$ represents spin up or down and $Q=(\tau_3+1)/2$ the charge of the nucleon and $\mu$ is an unknown constant to the leading order in $1/N_c$. It is noted here that in the $SU(4)$ symmetry, both of the isoscalar part and isovector part of the spin-flip vertex are generators which belong to the same $SU(4)$ supermultiplet $15$.

Similarly in the $SU(6)$ symmetric model the magnetic moment of spin $1/2$ baryon $B$ is given by

$$
(\mu_B)^{SU(6)}_{SU(6)} = \left( \begin{array}{cc} \frac{35}{s_Q} & sH_{N_c} \\ 0 & sH_{N_c}^* \end{array} \right) \mu.
$$

Here $Q=\lambda_3/2+\lambda_8/2\sqrt{3}$ is the charge operator, and the isoscalar part and isovector part of the spin-flip vertex belong to the same supermultiplet $35$ of $SU(6)$.

Both of the $SU(4)$ and $SU(6)$ symmetric models give the same magnetic moment
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for proton and neutron for arbitrary $N_c$:

$$\begin{align*}
(\mu_p)_{SU(4),SU(6)} &= (k + 2)\mu, \\
(\mu_n)_{SU(4),SU(6)} &= -(k + 1)\mu.
\end{align*}$$

(1)

(2)

These results are already derived by Karl and Paton.\(^6\)

The most general flavor octet vertex constructed from spin 1/2 baryon $B$ is given by\(^5\)

$$M \text{Tr}[\overline{B}T^aB] + N \text{Tr}[\overline{B}B^a],$$

where $T^a$ a flavor octet matrix. The $F/D$ ratio of flavor octet vertex is given by

$$\frac{F}{D} = \frac{M-N}{M+N}.$$ 

By the use of this $F/D$ ratio the magnetic moments of nucleons are given by

$$\begin{align*}
\mu_p &= \mu_F + \frac{1}{3}\mu_D, \\
\mu_n &= -\frac{2}{3}\mu_D,
\end{align*}$$

(3)

(4)

where $\mu_D$ and $\mu_F$ are the $D$ and $F$ type contributions to the magnetic moment of baryons. For arbitrary $N_c$ we calculate the $N/\bar{N}$ ratio and $F/D$ ratio for the "baryon" with spin 1/2 and the obtained results are

$$\begin{align*}
\left(\frac{N}{\bar{N}}\right)_{SU(4),SU(6)} &= \frac{N_c - 1}{2(N_c+2)}, \\
\left(\frac{F}{D}\right)_{SU(4),SU(6)} &= \frac{N_c + 5}{3(N_c+1)}.
\end{align*}$$

(5)

(6)

The same result can be derived from the magnetic moments of nucleons given by (1), (2), (3) and (4). It is noted that in the case of NRQMs with $SU(4)$ and $SU(6)$ symmetries the coefficient $\mu$ of magnetic moment is different depending on how the large $N_c$ baryon states are defined. However the $F/D$ ratio does not depend on the way of extrapolation to large $N_c$ baryons.

In the $SU(4)$ and $SU(6)$ NRQMs taking the limit $N_c \to \infty$ the $F/D$ ratio tends to 1/3. For $N_c=1$ the $F/D$ ratio becomes 1 reflecting the fact that the baryons are quarks themselves for $N_c=1$.

§ 3. The $F/D$ ratio in $SU(2)$ and $SU(3)$ chiral soliton model

In the $SU(2)$ CSM the spin 1/2 baryon state is represented by the elements of $SU(2)$ matrix in the fundamental representation of $SU(2)$ which is independent of color degrees of freedom $N_c$.\(^7\) In this model the isoscalar part and the isovector part of magnetic moment of baryon have distinct origins. That is, the isovector part is space integral of conserved isovector current which is the Noether current reflecting the symmetry of the chiral Lagrangian and is of order $O(N_c)$. On the other hand the
isoscalar part comes from the baryon number current which is a topologically conserved current and of order $O(1/N_c)$ in the $1/N_c$ expansion. Therefore the magnetic moment of the spin 1/2 baryon is given by

$$\langle \mu_B \rangle_{SU(2)\text{CSM}} = \left( \begin{array}{ccc} 3 & 2 & 2 \\ \sigma_3 & B & \bar{B} \end{array} \right) \mu^{I=1} + \cdots,$$

where $r^3/2 = Q^{I=1}$ is the isovector part of charge. The ellipsis in (7) denotes contributions from time derivative of dynamical variables of the spin-isospin rotation of chiral soliton in which the isoscalar part of magnetic moment given by the topological or baryon number current is contained. The magnetic moments of proton and neutron are

$$\langle \mu_p \rangle_{SU(2)\text{CSM}} = \frac{1}{2} \mu^{I=1} + \cdots,$$

$$\langle \mu_n \rangle_{SU(2)\text{CSM}} = -\frac{1}{2} \mu^{I=1} + \cdots.$$  

Then the $F/D$ ratio in the $SU(2)$ CSM is

$$\left( \frac{F}{D} \right)_{SU(2)\text{CSM}} = \frac{1}{3} + \cdots.$$  

In the $SU(2)$ CSM the isovector part is given by the vector current of soliton which is a Noether current, but the isoscalar part denoted by the ellipsis is given by the baryon number current which is of topological origin. That is to say, there is no direct relation between the isovector and the isoscalar part.

On the other hand, in the $SU(3)$ CSM the spin 1/2 baryon states are represented by the $SU(3)$ matrix of the $(1, k) = (1+k)(3+k)$ dimensional representation were $k = (N_c - 1)/2$. $(1, k)$ denotes the representation of $SU(3)$ with the Young diagram which has the first row of length $k+1$ and the second row of length $k$. In the case $N_c = 3$ this is the octet or the regular representation of $SU(3)$.

In contrast to the $SU(2)$ CSM, in the $SU(3)$ chiral soliton model the magnetic moment of spin 1/2 baryon $B$ is given by

$$\langle \mu_B \rangle_{SU(3)\text{CSM}} = \sum_n \left( \begin{array}{ccc} 8 & (1, k) & (1, k)^*_n \\ Q & B & \bar{B} \end{array} \right) \left( \begin{array}{ccc} 8 & (1, k) & (1, k)^*_n \\ \sigma_3 & B & \bar{B} \end{array} \right) \mu^{I=1} + \cdots,$$

where the summation over $n$ denotes two orthogonal states of baryons of $(1, k)$ representation and the ellipsis denotes corrections from the time derivative of the $SU(3)$ matrix valued dynamical variable $A(t)$ describing the "rotations" in spin-flavor space, and contains the higher order term of $1/N_c$ expansion. The results for the magnetic moments are

$$\langle \mu_p \rangle_{SU(3)\text{CSM}} = \frac{k+3}{3(k+4)} \mu + \cdots,$$

$$\langle \mu_n \rangle_{SU(3)\text{CSM}} = \frac{k^2+5k+3}{3(k+2)(k+4)} + \cdots.$$
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Fig. 1. $F/D$ ratios in the $SU(6)$ symmetric NRQM (lower curve) and $SU(3)$ CSM (upper curve). The large $N_c$ limiting value 1/3 is shown by an arrow and the experimental value by a bullet at $N_c=3$.

There is a difference in the magnetic moments of nucleons in the chiral soliton models between flavor 2 and 3 contrary to the NRQMs. This comes from the fact that the nucleon states belong to the fundamental representation 2 in the $SU(2)$ soliton irrespective of $N_c$ while in the $SU(3)$ case the states belong to the regular representation $(1, k)$.

From these results we obtain the $F/D$ ratio of spin-flip baryon vertex in the $SU(3)$ CSM

$$\left( \frac{F}{D} \right)_{SU(3)CSM} = \frac{N_c^2 + 8N_c + 27}{3(N_c^2 + 8N_c + 3)} + \cdots. \quad (13)$$

In Fig. 1 we compare the $F/D$ ratio in the NRQM with $SU(2f)$ symmetries and the ratio in the $SU(3)$ CSM. Two ratios coincide at both $N_c=1$ and $N_c \to \infty$. The experimental value of the $F/D$ ratio of spin-flip baryon vertex $\mu_{\text{exp}} = 0.65 \pm 0.02$ lies between the two lines nearer to that of $SU(3)$ chiral soliton.

§ 4. The $1/N_c$ expansion of $F/D$ ratio in the nonrelativistic quark model and chiral soliton model

If the value of $N_c$ is large enough we can expand $F/D$ ratios as follows,

$$\left( \frac{F}{D} \right)_{SU(4), SU(6)} = \frac{1}{3} + \frac{4}{3N_c} - \frac{4}{3N_c^2} + \cdots, \quad (14)$$

$$\left( \frac{F}{D} \right)_{SU(3)CSM} = \frac{1}{3} + \frac{0}{N_c} + \frac{8}{N_c^2} + \cdots, \quad (15)$$

$$\left( \frac{F}{D} \right)_{SU(2)CSM} = \frac{1}{3} + \cdots. \quad (16)$$
First we note here that the limiting value $1/3$ of the $F/D$ ratio is the consequence of the large $N_c$ counting rules of the spin-flip baryon vertex. Generally the isoscalar part of spin-flip vertex is $1/N_c$ of the isovector part of spin-flip vertex. Therefore we can neglect the isoscalar part in the large $N_c$ limit and we obtain the limiting value $F/D=1/3$. This is natural from the viewpoint of the $SU(4)$ and $SU(6)$ symmetric NRQMs since $u$ and $d$ quarks are in the totally symmetric states with respect to the spin and isospin and for the ground state baryons the $I=0$ ($I=1$) states have $J=0$ ($J=1$) and the spin-flip transitions are allowed only for the isospin-flip transitions. Similar situations also occur in the CSM. Due to the hedgehog Ansatz $I=J$ structure is realized in the static chiral soliton.

The second point to be noted here is that the $1/N_c$ correction to $F/D$ ratio has two different origins. One is the $1/N_c$ correction from baryon states and the other is from the $1/N_c$ correction of the dynamical quantities. In the $SU(4)$ symmetric model the $1/N_c$ correction comes only from baryon states and not from the vertex operators. Contrary to the $SU(4)$ symmetric model, in the $SU(2)$ CSM the $1/N_c$ corrections come from the dynamical variables (time derivative of spin-isospin rotation matrix $A(t)$) not from the baryon states.

The third point is that in the $SU(4)$ and $SU(6)$ symmetric models the $1/N_c$ expansion of $F/D$ ratio (14) is a convergent expansion for $N_c>1$, while in the $SU(3)$ CSM the $1/N_c$ expansion (15) is a divergent expansion at $N_c=3$, because the convergence radius is $1/N_c=(4-\sqrt{13})/3=1/7.6$ and $N_c$ must be larger than 8 for the convergence of expansion. Therefore it is not clear whether the quantitative feature of the chiral soliton model with $N_c=3$ is the same as that with large enough $N_c$ or not.

From the consistency conditions Dashen, Jenkins and Manohar derived $1/N_c$ expansion of $\mathcal{H}/\mathcal{M}$ ratio,\(^5\)

\[
\frac{\mathcal{H}}{\mathcal{M}} = \frac{1}{2} + \frac{a}{N_c} + \ldots, \tag{17}
\]

which gives the $1/N_c$ expansion of $F/D$ ratio

\[
\frac{F}{D} = \frac{1}{3} - \frac{8a}{9N_c} + \ldots. \tag{18}
\]

In this expansion, $a=-3/2$ in the $SU(4)$ or $SU(6)$ symmetric model. On the other hand, in the $SU(2)$ and $SU(3)$ CSM $a=0$ in the static limit.

§ 5. Consistency condition, $F/D$ ratios of $SU(6)$ symmetry and $SU(3)$ chiral soliton model

We study whether the above results for the NRQMs and the CSMs satisfy the consistency conditions derived by Dashen, Jenkins and Manohar.\(^5\) For simplicity, we consider here explicitly only the $SU(2)$ part of the matrix elements of the axial vector baryon-pion vertex expanding it in $1/N_c$ as

\[
(A^{ia})_{\nu \pi} = N_c(X_6^{ia})_{\nu \pi} + (X_1^{ia})_{\nu \pi} + \frac{1}{N_c} (X_2^{ia})_{\nu \pi} + \ldots. \tag{19}
\]
The Gervais-Sakita condition is

(I) \( ([X^a_0, X^b_0])_{B'B} = 0 \). \hfill (20)

Dashen, Jenkins and Manohar obtain the consistency condition

(II) \( ([X^a_0, X^j_0, X^i_0])_{B'B} + ([X^a_0, X^b_0, X^i_0])_{B'B} = 0 \) \hfill (21)

and

(III) \( ([X^a_0, X^j_0, X^i_0])_{B'B} + ([X^a_0, X^b_0, X^i_0])_{B'B} + ([X^a_0, X^j_0, X^i_0])_{B'B} = 0 \). \hfill (22)

The consistency condition (III) that we use here for the sake of simplicity is equivalent to but has slightly different expression from the one in Ref. 5).

(i) the NRQMs with the \( SU(4) \) and \( SU(6) \) symmetries

The isospin, spin and spin-isospin operators in the NRQMs are given by

\[
G^a = q^* \tau^a q ,
I^a = q^* \tau^a q ,
J^i = q^* \sigma^i q ,
\]

where \( q^* \) and \( q \) are respectively creation and annihilation operators.

These operators satisfy \( SU(2f) \) algebra

\[
[I^a, G^b] = i \varepsilon_{abc} G^c ,
\]

\[
[J^i, G^b] = i \varepsilon_{iaj} G^b ,
\]

\[
[G^a, G^b] = \frac{i}{2f} \varepsilon_{ijk} \delta_{ab} J^k + \frac{i}{4} f_{abc} \delta_{ij} I^c + \frac{i}{2} \varepsilon_{ijk} \delta_{abc} G^k .
\]

These operators themselves have no \( N_c \) dependence in the \( SU(2f) \) symmetric NRQM.

The axial current \( A^{ia} \) in QCD can be expanded by using the NRQM operators as\(^{10}\)

\[
A^{ia} = G^{ia} + q^* \frac{a}{N_c} J^i I^a + \ldots .
\]

From large \( N_c \) counting rules, \( (G^{ia})_{B'B} \sim O(N_c) \), \( (I^a)_{B'B} \sim (J^i)_{B'B} \sim O(N_c^0) \). So the operator \( J^i I^a \) is the higher order term in the \( 1/N_c \) expansion and the breaking term for the \( SU(4) \) \( (SU(6)) \) symmetry. By taking the matrix element of (25) between the initial and the final state baryons \( B \) and \( B' \) we obtain the following large \( N_c \) relations for the \( SU(4) \) symmetric NRQM:

\[
([X^a_0, X^b_0])_{B'B} = 0 ,
\]

\[
([X^a_0, X^j_0])_{B'B} + ([X^a_0, X^b_0])_{B'B} = 0 ,
\]

\[
([X^a_0, X^j_0])_{B'B} + ([X^a_0, X^j_0])_{B'B} + ([X^a_0, X^j_0])_{B'B} = 0 .
\]
Here the contribution from the $d$-term in the case of $SU(6)$ symmetry is absorbed into the isoscalar part.

From the above expressions, it is obvious that $X^i_\delta i$, $X^i_\tau$ and $X^i_i$ satisfy the consistency conditions (I), (II) and (III).

(ii) The SU(2) and SU(3) CSMs

The isospin, spin and spin-isospin operators in the CSM are given by

$$\bar{I}^a = i\, \text{Tr}(A^i \tau^a A),$$

$$\bar{J}^i = -i\, \text{Tr}(\sigma^i A^a \dot{A}),$$

$$\bar{G}^{ia} = g \, N_c \, \text{Tr}(\sigma^i A^a \tau^a A),$$

respectively where elements of the $SU(2)$ or $SU(3)$ matrix $A(t)$ are the collective coordinates describing spin-isospin rotations, dot means time derivative. In (30) and (31) $\lambda$ is the moment of inertia of spin-isospin rotation of soliton. This $\lambda$ is order $N_c$ and $\dot{A}$ is order $O(1/N_c)$ thus $\bar{I}^a$ and $\bar{J}^i$ are of order $O(1)$. These operators satisfy an algebra

$$[\bar{I}^a, \bar{G}^{jb}] = i\epsilon_{abc} \bar{G}^{jc},$$

$$[\bar{J}^i, \bar{G}^{jb}] = i\epsilon_{abc} \bar{G}^{kb},$$

$$[\bar{G}^{ia}, \bar{G}^{jb}] = 0.$$  

The axial vertex operator of baryon can be expanded by using these bases

$$A^{ia} = \bar{G}^{ia} + \frac{\alpha}{N_c} \bar{J}^i \bar{I}^a + \cdots.$$  

From large $N_c$ counting rules, $(\bar{G}^{ia})_{B'B} \sim O(N_c)$, $(\bar{I}^B)_{B'B} \sim (\bar{J})_{B'B} \sim O(N_c^0)$. So the operator $\bar{J}^i \bar{I}^a$ is the higher order term in the $1/N_c$ expansion and the breaking term for the contracted $SU(4)$ ($SU(6)$) symmetry.

In the SU(2) CSM case, there are no $1/N_c$ corrections from baryon states. Thus we obtain

$$(X^i_\delta i)_{B'B} = (\bar{G}^{ia})_{B'B}, \quad (X^i_\tau)_{B'B} = 0, \quad (X^i_i)_{B'B} = a'(J^i)_{B'B}(I^a)_{B'B} + \cdots.$$  

It is obvious that the SU(2) CSM satisfies the consistency condition derived by Dashen, Jenkins and Manohar.\textsuperscript{5)}

On the other hand, in the SU(3) CSM there are $1/N_c$ corrections originating from baryon states.

In this case we obtain the following relations:

$$([X^i_\delta, X^i_\tau])_{B'B} = 0,$$
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\[(X_i^a, X_j^b)_{B^B} + ([X_i^a, X_j^b])_{B^B} = 0, \]  
(39)

\[(X_i^a, X_j^b)_{B^B} + ([X_i^a, X_j^b])_{B^B} + ([X_i^a, X_j^b])_{B^B} \]

\[= i\varepsilon^{ijk} \left( a' (I^a)^{B'B'} (X_i^{ka})_{B'B'} + a' (I^a)^{B'B'} (X_j^{kb})_{B'B'} \right) \]

\[+ i\varepsilon^{abc} \left( a' (X_i^{ka})_{B'B'} (j')_{B'B'} + a' (X_j^{kb})_{B'B'} (j')_{B'B'} \right) + \ldots. \]  
(40)

It is obvious from these expressions that $X_i^a$ satisfy consistency conditions (I), (II) and (III) in the $SU(3)$ CSM.

As is seen from the above analysis the difference between the NRQM with $SU(6)$ symmetry and the $SU(3)$ chiral soliton model arises from the difference of $X_i^a$ in these two models. This is consistent with the analysis of Dashen, Jenkins and Manohar. The solution of consistency condition $X_i^a \sim X_i^a$ gives the limiting value $1/3$ for $F/D$ ratio irrespective of $X_i^a$ which gives the next order correction.

Here we note that in the $SU(2)$ CSM there are no $1/N_c$ corrections from baryon states, but in the $SU(3)$ CSM baryon states have $1/N_c$ corrections and give the $1/N_c$ to the matrix elements of the baryon vertices. This is the consequence of the fact that the wave functions of proton and neutron given in the $SU(2)$ and $SU(3)$ CSMs are different.

The situation is quite different in the NRQM with the $SU(4)$ symmetry and the $SU(2)$ CSM. Both in the $SU(4)$ symmetric model and the $SU(2)$ CSM the nucleons belong to the same fundamental representation, i.e., the isospin doublet and gives the same contributions at the leading order in $1/N_c$ expansion as discussed by Manohar

\[11\] and Bardakci.\[12\]

§ 6. Conclusion and discussion

We have calculated the $F/D$ ratios in the NRQMs with the $SU(4)$ and $SU(6)$ symmetries and the $SU(2)$ and $SU(3)$ CSMs to all orders of $1/N_c$ expansion. We have confirmed that the limiting value of $F/D$ ratio tends to $1/3$ and this result is model independent.

In general the $1/N_c$ corrections to the axial vector coupling of baryons have two origins in the model independent QCD. One is the $1/N_c$ corrections from baryon states and the other is the $1/N_c$ corrections of operators. In the $SU(2f)$ symmetric NRQM the $N_c$ dependence comes only from the wave function of baryon states and

| Table 1. $N_c$ dependence of operators and baryon states. |
|-------------|-------------|-------------|
| **Model**   | **Operators** | **Baryon States** |
| QCD         | yes         | yes         |
| $SU(4)$ symmetric NRQM | no         | yes         |
| $SU(6)$ symmetric NRQM | no         | yes         |
| $SU(2)$ CSM | yes         | no          |
| $SU(3)$ CSM | yes         | yes         |
not from the operators. On the other hand in the CSMs the operators have $N_c$ dependence and the wave functions of the $SU(2)$ model have no $1/N_c$ corrections while those of the $SU(3)$ model have the $1/N_c$ corrections. These model dependences of $1/N_c$ correction are summarized in Table I.

In contrast to the $1/N_c$ expansion in the NRQM, we found that the $1/N_c$ expansion of the $SU(3)$ CSM does not converge at $1/N_c=1/3$. For the $1/N_c$ expansion of the $SU(3)$ CSM to converge the value of $N_c$ must be larger than 8.

In connection with convergence problem of the $1/N_c$ expansion we note here that though the limiting value $1/3$ of $F/D$ ratio can also be derived from the analyses of vertices of the other baryons such as $\Sigma$ and $\Lambda$, some of the wave functions of large $N_c$ baryons contain $N_c$ dependence which cannot be expanded at $1/N_c=1/3$. For example, if we use transitions containing the $\Sigma$ and $\Lambda$ vertices

$$\frac{\Sigma^+ \to \Sigma^0 \pi^+}{\Sigma^+ \to \Lambda \pi^+} = 1 + O(1/N_c^2) = \sqrt{1 + \frac{2}{k} \left( \frac{kM - M}{kM + M} \right)}, \quad (41)$$

we can derive the correct limiting value $1/3$ of the $F/D$ ratio. But the wave functions of $\Lambda$ and $\Sigma$ cannot be expanded for $N_c \leq 5$.

References

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Note added: After this work was completed, the second paper of Dashen, Jenkins and Manohar has appeared which concerns similar problems discussed here but from slightly different viewpoint.