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Abstract. Classification of nonabelian T-duals of the flat matric in D=4 with respect to the four-dimensional continuous subgroups of the Poincare group is given. After dualizing the flat background we identify majority of dual models as conformal sigma models in plane-parallel wave backgrounds, most of them having torsion. We give their form in Brinkmann coordinates. Besides the pp-waves we find several diagonalizable curved metrics with nontrivial scalar curvature and torsion.

Due to the nonabelian T-duality, we find general solution of classical field equations of all the sigma models in terms of d’Alembert solutions of wave equation.

1. Introduction
String theory in curved and/or time-dependent background can be formulated as a sigma model satisfying supplementary conditions. Finding solutions of equations of motion in these backgrounds is usually very complicated. That’s why every solvable case attracts considerable attention. An example of such a model is a string theory in the homogeneous plane-parallel wave background, solved in Ref. [1] in terms of Bessel functions. Plane-parallel (pp-)wave backgrounds in string theory were repeatedly investigated in the past (see e.g. references in [2]). They give not only solvable models [3], but also allow to study the behavior of strings near spacetime singularity [4, 5]. Moreover, to extract information about string behavior in general curved background one can take the Penrose limit [6], extended for fields in string theory in [7], and study string behavior in the resulting plane-wave background.

Particular cases of four-dimensional pp-wave background obtained from gauged WZW models were given in [8],[9],[2] as

\[ ds^2 = du dv + \frac{g_1(u')}{g_1(u')g_2(u) + q^2} \, dx_1^2 + \frac{g_2(u)}{g_1(u')g_2(u) + q^2} \, dx_2^2, \]

\[ B_{12} = \frac{q}{g_1(u')g_2(u) + q^2}, \]

where \( u' = au + d \) (\( a, d = \text{const} \)) and the functions \( g_i \) can take any pair of the following values

\[ g(u) = 1, \ u^2, \ \tanh^2 u, \ \tan^2 u, \ \u^{-2}, \ \coth^2 u, \ \cot^2 u. \]

It is mentioned in [8] that this background is dual to the flat space for \( g_1 = 1 \), \( g_2 = u^2 \). We shall show that several other cases of these backgrounds are dual to the flat space as well. Moreover,
we shall use this fact for finding general solution of classical sigma model field equations in these pp-wave backgrounds. Beside pp-waves, we find several curved backgrounds with diagonalizable metrics and solve their field equations as well.

We understand the non-Abelian T-duality \[10\] as a special case of Poisson–Lie T-plurality [11] based on the structure of Drinfeld double. For technical reasons we shall restrict to four spacetime dimensions, but the discussion can be extended to higher dimensions using the spectator fields or subgroups of Poincare group in higher dimensions.

2. Non-Abelian T-duality

The sigma model on a manifold \(M\) is given by the classical action

\[
S_F[X] = - \int d\sigma_+ d\sigma_- ( \partial_- X^\mu F_{\mu\nu}(X) \partial_+ X^\nu ) = \frac{1}{2} \int d\tau d\sigma \left[ - \partial_\tau X^\mu G_{\mu\nu}(X) \partial_\tau X^\nu + \partial_\sigma X^\mu G_{\mu\nu}(X) \partial_\sigma X^\nu - 2 \partial_\tau X^\mu B_{\mu\nu}(X) \partial_\sigma X^\nu \right],
\]

where \(F\) is a second order tensor field on \(M\), with the metric and the NS–NS 2-form (torsion potential) given by the symmetric and antisymmetric part of \(F\)

\[
G_{\mu\nu} = \frac{1}{2}(F_{\mu\nu} + F_{\nu\mu}), \quad B_{\mu\nu} = \frac{1}{2}(F_{\mu\nu} - F_{\nu\mu}).
\]

The worldsheet coordinates are

\[
\sigma_+ = \frac{1}{\sqrt{2}}(\tau + \sigma), \quad \sigma_- = \frac{1}{\sqrt{2}}(\tau - \sigma).
\]

The functions \(X^\mu\) are determined by the composition \(X^\mu(\tau, \sigma) = x^\mu(X(\tau, \sigma))\), where \(X : \mathbb{R}^2 \ni (\tau, \sigma) \mapsto X(\tau, \sigma) \in M\) and \(x^\mu : U_p \rightarrow \mathbb{R}\) are components of a coordinate map on a neighborhood \(U_p\) of an element \(X(\tau, \sigma) = p \in M\).

The non-Abelian T-duality \[10\] of sigma models is a special case of Poisson–Lie T-duality [11],[12] that can be formulated by virtue of the Drinfeld double – a connected Lie group \(\tilde{G}\) can be decomposed into a pair of equally dimensional subalgebras \(\mathfrak{g}, \tilde{\mathfrak{g}}\) that are maximally isotropic with respect to a symmetric ad-invariant non-degenerate bilinear form \(<...,>\) on \(\mathfrak{g}\).

The Drinfeld double suitable for non-Abelian T-duality is the semidirect product \(G \ltimes \tilde{G}\), where the group \(G\) can be taken as a subgroup of the isometry group of the background, which, in our case, will be flat. The group \(\tilde{G}\) has to be chosen Abelian in order to satisfy the conditions of dualizability of the sigma model [11]. We shall focus on the case, when the isometry subgroup acts freely and transitively on the manifold, so that we can identify \(G \approx M\). This is usually referred to as atomic duality. Let us summarize the main points of the construction of dual models.

Given the four-dimensional subgroup \(G\) of the isometry group generated by Killing vectors of the flat metric, the tensor \(F\) is symmetric and can be written as

\[
F_{\mu\nu}(x) = G_{\mu\nu}(x) = e^a_\mu(g(x))(E_0)_{ab} e^b_\nu(g(x)),
\]

where \(E_0\) is a constant non-singular symmetric matrix, and \(e^a_\mu(g(x))\) are components of right-invariant forms \((dg)g^{-1}\) expressed in coordinates \(\{x^\mu\}\) on the group \(G\) and basis of its Lie algebra \(\{T_a\}\).

Denoting the mutually dual bases of \(\mathfrak{g}\) and Abelian \(\tilde{\mathfrak{g}}\) as \(\{T_i\}, \{\tilde{T}^j\}\), we construct subspaces \(\varepsilon^+ = \text{Span}(T_i + E_{0,ij} \tilde{T}^j), \varepsilon^- = \text{Span}(T_i - E_{0,ij} \tilde{T}^j)\) that are orthogonal w.r.t. \(<.,.>\) and span the
The whole Lie algebra $\mathfrak{g}$. The field equations for the sigma model on the group $G$ can be rewritten as equation

$$< (\partial_\pm l) l^{-1}, \varepsilon^\pm >= 0$$

(5)

for mapping $l$ from the worldsheet in $\mathbb{R}^2$ into the Drinfeld double $D$.

Due to Drinfeld, there exists a unique decomposition (at least in the vicinity of the unit element of $D$) of an arbitrary element $l$ of $D$ as a product of elements from $G$ and $\tilde{G}$. Solutions of equation (5) and solution of the equations of motion for the sigma model $X^\mu(\tau, \sigma) = x^\mu(\tau, \sigma)$ are related by

$$l(\tau, \sigma) = g(\tau, \sigma)\tilde{h}(\tau, \sigma) \in D,$$

where $\tilde{h}(\tau, \sigma) \in \tilde{G}$ fulfills the equations

$$\partial_\tau \tilde{h}_j = -v_j^\lambda G_{\lambda\nu} \partial_\sigma X^\nu,$$  

(6)

$$\partial_\sigma \tilde{h}_j = -v_j^\lambda G_{\lambda\nu} \partial_\tau X^\nu,$$  

(7)

with $v_j^\lambda$ representing components of the left-invariant fields $v_j$ on $G$ in the group coordinates $x^\mu$.

The metric and the torsion potential of the non-Abelian T-dual model can be obtained from the tensor $\tilde{F}$

$$\tilde{F}_{\mu\nu}(\tilde{x}) = [E_0 + \tilde{\Pi}(\tilde{g}(\tilde{x}))]^{-1},$$  

(8)

where the matrix $\tilde{\Pi}$ is given by the adjoint representation of the Abelian subgroup $\tilde{G}$ on the Lie algebra of the Drinfeld double in the mutually dual bases

$$Ad(\tilde{g})^T = \left( \begin{array}{cc} 1 & 0 \\ \tilde{\Pi}(\tilde{g}) & 1 \end{array} \right).$$

The relation between solution $X^\mu(\tau, \sigma)$ of the equations of motion of the sigma model given by $F$ and solution $\tilde{X}^\mu(\tau, \sigma) := \tilde{x}^\mu(\tilde{g}(\tau, \sigma))$ of the sigma model given by $\tilde{F}$ follows from two possible decompositions of elements $l$ of the Drinfeld double

$$g(\tau, \sigma)\tilde{h}(\tau, \sigma) = \tilde{g}(\tau, \sigma)h(\tau, \sigma),$$  

(9)

where $g, h \in G$, $\tilde{g}, \tilde{h} \in \tilde{G}$. The map $\tilde{h} : \mathbb{R}^2 \rightarrow \tilde{G}$ that we need for this transformation is obtained from the equations (6),(7).

The equation (9) defines the Poisson–Lie transformation between solutions of the equations of motion of the original sigma model and its dual. Its application may be rather complicated. To use it for finding solution of the dual model, the following three steps must be done:

- **Step 1:** One has to know the solution $X^\mu(\tau, \sigma)$ of the sigma model given by $F_{\mu\nu}(x)$.
- **Step 2:** Given $X^\mu(\tau, \sigma)$, one has to find $\tilde{h}(\tau, \sigma)$, i.e. solve the system of PDEs (6),(7).
- **Step 3:** Given $l(\tau, \sigma) = g(\tau, \sigma)\tilde{h}(\tau, \sigma) \in D$, one has to find the dual decomposition $l(\tau, \sigma) = \tilde{g}(\tau, \sigma)h(\tau, \sigma)$, where $\tilde{g}(\tau, \sigma) \in \tilde{G}, h(\tau, \sigma) \in G$. Functions $\tilde{X}^\mu(\tau, \sigma) := \tilde{x}^\mu(\tilde{g}(\tau, \sigma))$ then solve the field equations of the dual sigma model.

### 3. Strings in pp-wave background

We will be interested in special subclass of metrics called pp-waves. Their metric in the so called Brinkmann coordinates $(u, v, z_3, z_4, \ldots, z_D)$ can be written as

$$ds^2 = 2du dv - K(u, \tilde{v})du^2 + d\tilde{v}^2,$$  

(10)
where $d\tilde{z}^2$ is the Euclidean metric in the transversal space with coordinates $\tilde{z} = (z_3, z_4, \ldots, z_D)$. We denote the number of transversal coordinates by $d$, such that $D = 2 + d$. The NS–NS 2-form of particular interest to us has the form

$$B = B_j(u, \tilde{z}) du \wedge dz_j.$$  \hfill (11)

The metric (10) has covariantly constant null Killing vector $\partial_v$ and particularly simple curvature properties, because the Ricci tensor has only one nonzero component

$$R_{uu} = \frac{1}{2} (\partial^2_3 K + \partial^2_4 K + \ldots + \partial^2_D K),$$

and the scalar curvature vanishes. Special case is background resulting from the Penrose–Güven limit [6, 7], with

$$K(u, \tilde{z}) = K_{ij}(u) z_i z_j,$$  \hfill (12)

and the torsion

$$H = H_{ij}(u) du \wedge dz_i \wedge dz_j$$  \hfill (13)

that follows from the NS–NS 2-form (11) if $B_j(u, \tilde{z})$ is linear in $z$. The one-loop conformal invariance conditions then simplify to solvable differential equation for the dilaton $\Phi = \Phi(u)$

$$\Phi''(u) - K_{jj}(u) + \frac{1}{4} H_{ij}(u) H_{ij}(u) = 0.$$  \hfill (14)

We are going to show that the sigma models in pp-wave backgrounds with special forms of functions $K_{ij}$ (12) and $H_{ij}$ (13) can be obtained as non-Abelian T-duals of sigma models in flat background. Killing vectors of the flat metric $\eta = \text{diag}(-1, 1, 1, 1)$ in coordinates $(t, x, y, z)$ are

$$P_0 = \partial_t, \quad P_j = \partial_j, \quad L_j = -\epsilon_{ijk} x^k \partial_k, \quad M_j = -x^j \partial_t - t \partial_j,$$  \hfill (15)

and form ten-dimensional Poincare Lie algebra. To apply the (atomic) non-Abelian T-duality on sigma models in the flat background, we shall need four-dimensional subalgebras of Poincare Lie algebra, classified in [13].

4. Example

As an example we shall present dual model obtained from subalgebra

$$\text{Span}\{K_1 = L_3 + \epsilon (P_0 + P_3), K_2 = P_1, K_3 = P_2, K_4 = P_0 - P_3\}, \quad \epsilon = \pm 1$$

that produces dual model with torsion. The commutation relations of this subalgebra are

$$[K_1, K_2] = K_3, \quad [K_1, K_3] = -K_2.$$  \hfill (16)

Transformation of coordinates in the flat background

$$t = x^1 \epsilon + x^4, \quad x = x^2, \quad y = x^3, \quad z = x^4 \epsilon - x^4; \quad \epsilon = \pm 1$$

yields components of the flat metric in the group coordinates as

$$F_{\mu\nu}(x) = \begin{pmatrix}
0 & 0 & 0 & -2\epsilon \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
-2\epsilon & 0 & 0 & 0
\end{pmatrix}. $$
Dual background in this case is
\[
\tilde{F}_{\mu\nu}(\tilde{x}) = \begin{pmatrix}
0 & 0 & 0 & -\frac{1}{2\epsilon} \\
0 & 1 & 0 & \frac{x_3}{2\epsilon} \\
0 & 0 & 1 & \frac{x_2}{2\epsilon} \\
-\frac{1}{2\epsilon} - \frac{x_3}{2\epsilon} & \frac{x_2}{2\epsilon} & -\frac{x_2^2 + x_3^2}{4\epsilon^2}
\end{pmatrix},
\]
and transformation to Brinkmann coordinates
\[
\tilde{x}_1 = -v, \quad \tilde{x}_2 = z_3, \quad \tilde{x}_3 = z_4, \quad \tilde{x}_4 = 2\epsilon u,
\] (17)
brings the dual metric to the homogeneous and isotropic form
\[
ds^2 = 2dudv - \left(z_3^2 + z_4^2\right) du^2 + dz_3^2 + dz_4^2.
\] (18)

Torsion in Brinkmann coordinates is constant
\[
H = -2du \wedge dz_3 \wedge dz_4,
\] (19)
and the dilaton is
\[
\Phi(u) = c_1 + c_2 u.
\]

To find general solution of field equations of the dual sigma model with torsion we have to express the coordinates \(\tilde{x}_\mu\) in terms of \(x^\nu\) and \(\tilde{h}_k\). Solution of the equation (9) is
\[
\tilde{x}_1 = \tilde{h}_1 + x^2\tilde{h}_3 - x^3\tilde{h}_2, \quad \tilde{x}_2 = \tilde{h}_2 \cos x^1 - \tilde{h}_3 \sin x^1, \quad \tilde{x}_3 = \tilde{h}_2 \sin x^1 + \tilde{h}_3 \cos x^1,
\]
\[
\tilde{x}_4 = \tilde{h}_4.
\]
Combining this with (17) and (16), we find general solution of field equations of the sigma model with metric (18) and torsion (19) as
\[
U(\tau, \sigma) = \frac{\tilde{h}_4(\tau, \sigma) }{2\epsilon},
\]
\[
V(\tau, \sigma) = -\tilde{h}_1(\tau, \sigma) - \tilde{h}_3(\tau, \sigma)W^x(\tau, \sigma) + \tilde{h}_2(\tau, \sigma)W^y(\tau, \sigma),
\]
\[
Z_3(\tau, \sigma) = \cos(\Omega(\tau, \sigma))\tilde{h}_2(\tau, \sigma) - \sin(\Omega(\tau, \sigma))\tilde{h}_3(\tau, \sigma),
\]
\[
Z_4(\tau, \sigma) = \cos(\Omega(\tau, \sigma))\tilde{h}_3(\tau, \sigma) + \sin(\Omega(\tau, \sigma))\tilde{h}_2(\tau, \sigma),
\]
where \(W^I(\tau, \sigma)\) are solutions of the wave equations,
\[
\Omega(\tau, \sigma) = \frac{W^t + W^z}{2\epsilon},
\]
and \(\tilde{h}_\mu\) are solutions of the PDEs (6), (7),
\[
\tilde{h}_1 = \gamma_1 + \int \left[ \epsilon \left( \partial_r W^t - \partial_r W^z \right) + W^x \partial_r W^y - W^y \partial_r W^x \right] d\sigma,
\]
\[
\tilde{h}_2 = \gamma_2 - \int \partial_r W^x d\sigma, \quad \tilde{h}_3 = \gamma_3 - \int \partial_r W^y d\sigma,
\]
\[
\tilde{h}_4 = \gamma_4 + \int (\partial_r W^t + \partial_r W^z) d\sigma.
\]
String-type solutions in the light-cone gauge are obtained if we choose

\[ W^x(\tau, \sigma) + W^\bar{x}(\tau, \sigma) = 2 \epsilon \kappa \sigma, \]

\[ W^z(\tau, \sigma) = \sum_{n=\infty}^{\infty} e^{2i n \sigma} \int Z_3^n(\tau)(\kappa \sin(\kappa \sigma) - 2i n \cos(\kappa \sigma)) \]
\[ - Z_4^n(\tau)(\kappa \cos(\kappa \sigma) + 2i n \sin(\kappa \sigma)) d\tau, \]

\[ W^y(\tau, \sigma) = \sum_{n=\infty}^{\infty} e^{2i n \sigma} \int Z_3^n(\tau)(\kappa \cos(\kappa \sigma) + 2i n \sin(\kappa \sigma)) \]
\[ + Z_4^n(\tau)(\kappa \sin(\kappa \sigma) - 2i n \cos(\kappa \sigma)) d\tau. \]

where \( Z_3^n(\tau) \) and \( Z_4^n(\tau) \) solve the system of differential equations

\[ Z_3^{n''}(\tau) + (4 n^2 + \kappa^2) Z_3^n(\tau) - 4i n \kappa Z_4^n(\tau) = 0, \]

\[ Z_4^{n''}(\tau) + (4 n^2 + \kappa^2) Z_4^n(\tau) + 4i n \kappa Z_3^n(\tau) = 0. \]

5. Results for other subalgebras

The classification of subalgebras of the Poincare algebra in [13] was carried out up to group of inner automorphisms of the connected component of the Poincare group (proper orthochronous Poincare transformations). Up to this equivalence, there are 35 four–dimensional subalgebras of the Poincare algebra spanned by Killing vectors (15). Only 17 of them act transitively and freely on the flat manifold and can be used for nonabelian T–duality.

Eleven of the duals are plane-parallel wave backgrounds whose metrics can be brought to the Brinkmann form

\[ ds^2 = 2dudv - [K_3(u)z_3^2 + K_4(u)z_4^2]du^2 + dz_3^2 + dz_4^2, \]

and the torsion is

\[ H = dB = H(u)du \wedge dz_3 \wedge dz_4. \]

Functions \( K_3(u), K_4(u), H(u) \) acquire one of the following forms

\[ K_3(u) = K_4(u) = 1, \quad H(u) = -2, \quad (20) \]

\[ K_3(u) = \frac{3}{(u^2 + 1)^2}, \quad K_4(u) = -\frac{(2u^2 - 1)}{(u^2 + 1)^2}, \quad H(u) = \pm \frac{2}{u^2 + 1}, \quad (21) \]

\[ K_3(u) = 2 \text{sech}^2(u), \quad K_4(u) = 2 \delta \text{sech}^2(u), \quad \delta = 0, 1, \quad H(u) = 0, \quad (22) \]

\[ K_3(u) = -2 \text{csch}^2(u), \quad K_4(u) = -2 \delta \text{csch}^2(u), \quad \delta = 0, 1, \quad H(u) = 0, \quad (23) \]

\[ K_3(u) = K_4(u) = \frac{(1 + 2\beta^2 \text{sech}^2(u))}{\beta^2}, \quad H(u) = -\frac{2}{\beta}, \quad (24) \]

\[ K_3(u) = K_4(u) = \frac{(1 - 2\beta^2 \text{csch}^2(u))}{\beta^2}, \quad H(u) = -\frac{2}{\beta}, \quad (25) \]
In most of the transformed backgrounds the functions $g$ background forms (1) by the standard transformation from Brinkmann to Rosen coordinates. and corresponding sigma models are exactly conformal [3].

Even though the B-fields obtained by T-duality are usually not of the form $B = B_i(u) du \wedge dz_i$, they are gauge equivalent to

$$B' = H(u) du \wedge (z_3 dz_4 - z_4 dz_3),$$

and corresponding sigma models are exactly conformal [3].

Except for (24), (25), these pp-wave backgrounds can be transformed to the gauged WZW background forms (1) by the standard transformation from Brinkmann to Rosen coordinates. In most of the transformed backgrounds the functions $g_1, g_2$ acquire forms $g_1(u) = 1$ and $g_2$ any of the functions (2), but some other combinations of functions $(g_1, g_2)$ also arise, namely $(u^{-2}, \tanh^2 u), (u^{-2}, \coth^2 u), (\tanh^2 u, \tanh^2 u)$ and $(\coth^2 u, \coth^2 u)$.

Consequently, the pp-waves of the form (1) are duals of the flat metric not only for $g_1(u) = 1$ and $g_2(u) = u^2$, as mentioned in the Introduction, but also for many other combinations of functions $g_1, g_2$ from the set (2).

Besides the pp-waves, we get dual metrics with non-vanishing scalar curvature

$$ds^2 = -dy_1^2 + dy_2^2 + \frac{y_1^2}{y_1^2 + \alpha^2} dy_3^2 + \frac{1}{y_1^2 + \alpha^2} dy_4^2,$$

$$H = \frac{2y_1 \alpha}{(y_1^2 + \alpha^2)^2} dy_1 \wedge dy_3 \wedge dy_4,$$

$$ds^2 = \frac{1}{y_3^2 - \alpha^2} dy_1^2 + \frac{\alpha^2}{\alpha^2 - y_3^2} dy_2^2 + dy_3^2 + dy_4^2,$$

$$H = \frac{2y_3 \alpha}{(y_3^2 - \alpha^2)^2} dy_1 \wedge dy_2 \wedge dy_3,$$

$$ds^2 = -dy_1^2 + dy_2^2 + \frac{y_2^2}{y_2^2 + \alpha^2} dy_3^2 + \frac{1}{y_2^2 + \alpha^2} dy_4^2$$
\[ H = \frac{2y_2\alpha}{(y_2^2 + \alpha^2)^2} dy_2 \wedge dy_3 \wedge dy_4 \]

They are obtained as non-Abelian duals with respect to \( S_{11}, S_{18}, S_{19} \). Note that isomorphic (but non-equivalent under proper orthochronous Poincare transformation) subalgebras \( S_{17} \) respectively \( S_{18}, S_{19} \) lead to backgrounds with vanishing and non vanishing curvature.

The metrics (28)–(30) remind black hole [15] and cosmological backgrounds [16] rewritten in [2] to diagonal forms depending again on particular functions \( g_1, g_2 \). The difference from (28) – (30) is in these functions.

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