EXTRACTING CHARGINO/NEUTRALINO MASS PARAMETERS FROM PHYSICAL OBSERVABLES

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Abstract

I report on two papers, hep-ph/9806279 and hep-ph/9807336, where complementary strategies are proposed for the determination of the chargino/neutralino sector parameters, $M_1, M_2, \mu$ and $\tan \beta$, from the knowledge of some physical observables. This determination and the occurrence of possible ambiguities are studied as far as possible analytically within the context of the unconstrained MSSM, assuming however no CP-violation.

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1 Introduction

The gauge bosons and Higgs bosons superpartners have every chance to play, in the minimal version of the supersymmetric standard model (MSSM), an important part in the first direct experimental evidence for supersymmetry, if the latter happens to be linearly realized in nature around the electroweak scale. This would go through the study of the direct production of the light states and their subsequent decays, eventually cascading down to leptons (or jets) and missing energy [3], [4], [5].

The chargino/neutralino sector is an over-constrained system in the sense that only a few basic parameters in the Lagrangian are needed to determine all the six physical masses and the mixing angles of the various states. The latter determine the couplings to gauge bosons, Higgs bosons and matter fermions, so that various phenomenological tests could be in principle envisaged in the process of experimental identification. Alternatively, one might hope that a partial experimental knowledge of this sector would be sufficient to allow a reasonably unequivocal reconstruction of the full set of parameters; at stakes, on one hand the determination of the magnitude of the fermion soft susy breaking parameters, on the other, the existence of a heavy neutral stable particle, of prime importance to the cold dark matter issue [6]. Furthermore, the sensitivity to tan $\beta$, the ratio of the two vacuum expectation values of the Higgs fields, and to the supersymmetric parameter $\mu$, brings in a further correlation with the other sectors of the MSSM.

Hereafter we describe two strategies: the first deals with the extraction of $M_2$, $\mu$ and tan $\beta$ form the study of the lightest chargino pair production and decay in $e^+e^-$ collisions [1], the second with the extraction of $M_1$, $M_2$ and $\mu$ form the knowledge of any three ino masses and tan $\beta$ [2]. We start by stating the common features to these complementary approaches as well as their specific assumptions. We then highlight the main ingredients of each of them and illustrate some of their results. Finally we show in what sense they eventually complement one another. [Obviously, the reader is referred to [1] and [2] for more details and references. Still, we add some comments at various places of the ongoing presentation, which differ slightly from, and hopefully complete, the latter references.]

The reconstruction of the basic parameters of the theory involves generically two steps which can be sketched as follows:

\[
\begin{align*}
\text{Experimental Observables} & \quad \xrightarrow{(I)} \quad \text{Physical Parameters} \\
\text{Physical Parameters} & \quad \xrightarrow{(II)} \quad \text{Lagrangian parameters}
\end{align*}
\]

Each of these steps can suffer from equivocal reconstructions due to partial experimental knowledge or to theoretical ambiguities. In the present report we concentrate on the theoretical aspects of both steps.
2 CDDKZ and KM common features

The ino sector is considered in both [1] (referred to as CDDKZ) and [2] (KM) with the following assumptions:

- No reference to model-dependent assumptions about physics at energies much higher than the electroweak scale, like the GUT scale, and their possible implication on the parameters of this sector. [Thus the study is mainly carried out in the unconstrained MSSM, but any model-assumptions can be easily overlaid.]

- R-parity conservation;

- CP-conservation in the ino sector; This assumption is here only for practical reasons and should be eventually removed in future studies in order to cope with the possibility to deal with (complex) phases [7];

- CDDKZ and KM choose \( M_2 > 0 \). This is of course a mere convention due to the partial phase freedom through redefinition of fields, the only physical signs being the relative ones among \( M_1, M_2 \) and \( \mu \) as one can easily see from the relevant terms in the Lagrangian. (Also \( \tan \beta \) is taken positive and the \( \mu \) term convention is that of ref.\([8]\).)

Let us now recall briefly the basic ingredients of the ino mass matrices. The physical charginos (resp. neutralinos) are mixtures of charged (resp. neutral) higgsino and gaugino components. The chargino mass matrix reads:

\[
M_C = \begin{pmatrix}
M_2 & \sqrt{2}m_W \sin \beta \\
\sqrt{2}m_W \cos \beta & \mu
\end{pmatrix}
\]  

(2.2)

It has a supersymmetric contribution coming from the \( \mu \) term in the superpotential, the higgsino component, a contribution from the soft susy breaking wino mass term, and off-diagonal terms due to the electroweak symmetry breaking. Since \( M_C \) is not symmetric one needs two independent unitary matrices for the diagonalization. This is but the reflection of the fact that there are two independent mixings involving separately the two higgsino \( SU(2)_L \) doublets. The eigenvalues are most easily obtained from the diagonalization of \( M_C^\dagger M_C \) giving the squares of the chargino masses:

\[
M^2_{\chi_{1,2}^\pm} = \frac{1}{2} [M_2^2 + \mu^2 + 2m_W^2 \pm \sqrt{(M_2^2 + \mu^2 + 2m_W^2)^2 - 4(M_2\mu - m_W^2 \sin 2\beta)^2}] 
\]  

(2.3)

On the other hand, the angles \( \phi_L, \phi_R \) defining the two independent left- and right-chiral mixings among the winos and higgsinos in the four component Dirac representation are given by
\[
\cos 2\phi_L = \frac{M_2^2 - \mu^2 - 2m_W^2 \cos 2\beta}{2(M_1^2 - m_W^2) - M_2^2 - \mu^2}
\]
\[
\sin 2\phi_L = \frac{2\sqrt{2}m_W (M_2 \cos \beta + \mu \sin \beta)}{2(M_1^2 - m_W^2) - M_2^2 - \mu^2}
\]
\[
\cos 2\phi_R = \frac{M_2^2 - \mu^2 + 2m_W^2 \cos 2\beta}{2(M_1^2 - m_W^2) - M_2^2 - \mu^2}
\]
\[
\sin 2\phi_R = \frac{2\sqrt{2}m_W (M_2 \sin \beta + \mu \cos \beta)}{2(M_1^2 - m_W^2) - M_2^2 - \mu^2}
\]

(2.4)

where \( M_{\chi_1} \) is the lightest chargino mass given by eq.(2.3). This form of the mixing angles is such that the eigenvalues of \( M_C \) are always positive definite.

The neutralino mass matrix corresponds to bilinear terms in the photino, zino and neutral higgsino two-component fields. It receives contributions from the \( \mu \) term, the soft mass terms of the gaugino SU(2) \( L \) triplet \( (M_2) \) and singlet \( (M_1) \), while the mixing among states is triggered by the electroweak symmetry breaking:

\[
M_N = \begin{pmatrix}
M_1 & 0 & -m_Z s_w \cos \beta & m_Z s_w \sin \beta \\
0 & M_2 & m_Z c_w \cos \beta & -m_Z c_w \sin \beta \\
-m_Z s_w \cos \beta & m_Z c_w \cos \beta & 0 & -\mu \\
m_Z s_w \sin \beta & -m_Z c_w \sin \beta & -\mu & 0
\end{pmatrix}
\]

(2.5)

In contrast with \( M_C \), \( M_N \) is symmetric so that it can be diagonalized with one unitary matrix. On the other hand the eigenmasses are not positive definite\(^1\). Finally we note that in general the diagonalization of \( M_N \) cannot be achieved through a similarity transformation, unless all three parameters \( M_1, M_2 \) and \( \mu \) are real. This will be a key point in the algorithm we present for the reconstruction of the parameters in the neutralino sector.

3 Specific features

3.1 CDDKZ

The lightest chargino \( \chi_1^+ \) can be produced in pairs in \( e^+e^- \) collisions, at LEP II \(^4\) or NLC \(^5\) energies, through \( \gamma \) and \( Z \) s-channel exchange as well as sneutrino t-channel exchange. The production cross section will thus depend on the chargino mass \( m_{\chi_1} \), the sneutrino mass \( m_{\tilde{\nu}} \) and the mixing angles, eq.(2.4), which determine the couplings of the chargino states to the \( Z \) and the sneutrino. The unpolarized total cross section is illustrated in fig.1 with three representative cases of higgsino, gaugino or mixed content of the lightest

\(^1\)For more details about the ino sector see for instance \(^8\),\(^9\) and references therein.
Figure 1: Total cross section for the charginos pair production for a representative set of $M_2, \mu$, solid line gaugino case, dashed line higgsino case, dot-dashed line mixed case. In (a) $m_\tilde{\nu} = 200\text{GeV}$. (taken from ref.[1])

Chargino mass. The sharp rise near threshold should allow a precise determination of the chargino mass. Also the sensitivity to the sneutrino mass with the typical destructive interference in the gaugino and mixed cases necessitates the knowledge of this parameter. Subsequently the chargino will decay directly to a pair of matter fermions (leptons or quarks) and the (stable) lightest neutralino, through the exchange of a $W$ boson (charged Higgs exchange is suppressed for light fermions) or scalar partners of leptons or quarks. Of course the decay matrix elements will depend on further parameters like the susy scalar masses and couplings to the neutralino. However, CDDKZ propose that, looking at the total production cross section and some polarization components and spin-spin correlations of the final state charginos, one can define measurable combinations for which the details of the chargino decay products cancel out. This allows to isolate to a large extent the chargino system from the neutralino system and thus extract the mixing angles and chargino mass from these observables (step (I) in eq.(1.1)). In fact, for step (I) to work completely for the chargino system, one needs to know, besides the sneutrino mass, also the lightest neutralino mass, as will become clear later on. Once the chargino mass $m_{\chi^\pm}$ and $\cos 2\phi_L, \cos 2\phi_R$ are known one can determine $M_2, \mu$ and $\tan\beta$ up to possible ambiguities, [step (II) of eq.(1.1).]

Before going further let us first describe in some detail the basic ingredients of step (I). The presence of invisible neutralinos, in the final state of the process $e^+e^- \rightarrow \chi_1^+\chi_1^- \rightarrow \chi_1^0\chi_1^0 (f_1\bar{f}_2)(\bar{f}_3 f_4)$, makes it impossible to measure directly the chargino production angle in the laboratory frame. From now on we will thus concentrate on observables where this angle is integrated out. Integrating also over the invariant masses of the fermionic systems $(f_1\bar{f}_2)$ and $(\bar{f}_3 f_4)$ one can write the differential cross section in the following form:

$$d^4\sigma(e^+e^- \rightarrow \chi_1^+\chi_1^- \rightarrow \chi_1^0\chi_1^0 (f_1\bar{f}_2)(\bar{f}_3 f_4)) = \frac{\alpha^2 \beta}{124 \pi s} B r_{\chi^- \rightarrow \chi^+ f_1 \bar{f}_2} B r_{\chi^+ \rightarrow \chi^0 f_3 f_4} \Sigma(\theta^*, \phi^*, \bar{\theta}^*, \bar{\phi}^*) \quad (3.6)$$

where $\alpha$ is the fine structure constant, $\beta$ the velocity of the chargino in the c.m. frame,
Figure 2: Contours for the “measured values” of the total cross section (solid line), \( \frac{P^2}{Q} \), and \( \frac{P^2}{Y} \) (dot-dashed line) for \( m_{\chi^\pm} = 95 GeV \) \[ m_{\tilde{\nu}} = 250 GeV \]. (taken from ref.[1])

\( \theta^* \) (\( \bar{\theta}^* \)) denotes the polar angle of the \( f_1\bar{f}_2 \) (\( \bar{f}_3f_4 \)) system in the \( \chi_1^- \) (\( \chi_1^+ \)) rest frame with respect to the charginos flight direction in the laboratory frame, and \( \phi^* \) (\( \bar{\phi}^* \)) the corresponding azimuthal angle with respect to a canonical production plane. \( \Sigma(\theta^*,\phi^*,\bar{\theta}^*,\bar{\phi}^*) \) is made out of combinations of helicity amplitudes which lead to an unpolarized term plus fifteen other contributions from polarization components and spin-spin correlations. We reproduce here only those components which are relevant to our discussion.

\[
\Sigma = \Sigma_{\text{unpol}} + (\kappa - \bar{\kappa}) \cos \theta^* P + \cos \theta^* \cos \bar{\theta}^* \kappa \bar{\kappa} Q \\
+ \sin \theta^* \sin \bar{\theta}^* \cos(\phi^* + \bar{\phi}^*) \kappa \bar{\kappa} Y + \ldots
\]  

(3.7)

Actually, among the sixteen terms which contribute to \( \Sigma \) only ten survive because of CP-invariance (when violation of CP from the Z-boson width or radiative corrections is neglected). Of these ten, three are redundant being CP eigenstates. Of the remaining seven independent components, only those which can be extracted from experimentally measurable angular distributions are explicitly written in eq.(3.7). This means that the others will be integrated out through appropriate projections.

In eq.(3.7) \( \Sigma_{\text{unpol}} \) corresponds to the unpolarized cross section for the chargino pair production and is given in terms of helicity amplitudes by

\[
\Sigma_{\text{unpol}} = \frac{1}{4} \int d\cos \Theta \sum_{\sigma = \pm} |\langle \sigma; ++ \rangle|^2 + |\langle \sigma; + - \rangle|^2 \\
+ |\langle \sigma; - + \rangle|^2 + |\langle \sigma; -- \rangle|^2|
\]  

(3.8)
\( \mathcal{P} \) is a polarization component coming separately from the \( \chi^- \) (or \( \chi^+ \)) system

\[
\mathcal{P} = \frac{1}{4} \int \, d \cos \Theta \sum_{\sigma = \pm} \left[ |\langle \sigma; ++ \rangle|^2 + |\langle \sigma; -+ \rangle|^2 - |\langle \sigma; -+ \rangle|^2 - |\langle \sigma; -- \rangle|^2 \right]
\]  

(3.9)

while \( \mathcal{Q} \) and \( \mathcal{Y} \) describe the spin correlations between the two chargino systems and have the following structure

\[
\mathcal{Q} = \frac{1}{4} \int \, d \cos \Theta \sum_{\sigma = \pm} \left[ |\langle \sigma; ++ \rangle|^2 - |\langle \sigma; -+ \rangle|^2 - |\langle \sigma; -+ \rangle|^2 + |\langle \sigma; -- \rangle|^2 \right]
\]  

(3.10)

\[
\mathcal{Y} = \frac{1}{2} \int \, d \cos \Theta \sum_{\sigma = \pm} \text{Re}\{\langle \sigma; -- \rangle \langle \sigma; ++ \rangle^* \}
\]

where \( \sigma \) is the initial state electron helicity. The strategy of CDDKZ is based on the following two observations:

\begin{itemize}
  \item[i)] The three angular contributions, \( \cos \theta^*, \cos \bar{\theta}^* \) and \( \sin \theta^* \sin \bar{\theta}^* \cos (\phi^* + \bar{\phi}^*) \) are fully determined by the measurable parameters \( E, \langle \vec{P} \rangle \) (the energy and momentum of each of the decay systems \( f_i \bar{f}_j \) in the laboratory frame), and the chargino mass \( m_{\chi^+_1} \);
  \item[ii)] The three quantities \( \Sigma_{\text{unpol}}, \mathcal{P}^2/\mathcal{Q} \) and \( \mathcal{P}^2/\mathcal{Y} \) lead to \( \kappa \) free observables, where \( \kappa \) (and \( \bar{\kappa} = -\kappa \)) measures the asymmetry between left- and right-chirality form factors in the decay products of the chargino;
\end{itemize}

Here the kinematic configuration is similar to that of a \( \tau \) lepton pair production with successive decays in light leptons or quarks plus missing energy. However in the present context the invisible particle has a non negligible mass whose knowledge is necessary to relate the energy of the \( f_i \bar{f}_j \) system in the chargino rest frame to that in the laboratory frame. Thus the neutralino mass is actually necessary in the reconstruction of the angular contributions in i). The crucial point in ii) is that the dependence on the final state decay fermions through the asymmetry in the left- and right-chiral structure cancels out. Thus \( \Sigma_{\text{unpol}}, \mathcal{P}^2/\mathcal{Q} \) and \( \mathcal{P}^2/\mathcal{Y} \) allow to study the chargino sector independently of the details of the decay products. In the same time, their extraction from the experimental data, via convolution with appropriate moments, requires the measurement of the energies and momenta of the two \( f_i \bar{f}_j \) systems, the chargino mass (ex. via threshold effects, see fig.1), as well as the neutralino mass (ex. from the energy distribution of the final particles).

Once extracted, one can combine \( \Sigma_{\text{unpol}}, \mathcal{P}^2/\mathcal{Q} \) and \( \mathcal{P}^2/\mathcal{Y} \) which depend on the c.m. energy \( \sqrt{s} \), the sneutrino mass \( m_{\tilde{\nu}} \), and \( \cos 2\phi_L, \cos 2\phi_R \) to determine the latter cosines. An illustration is given in fig. 2 of a unique consistent solution corresponding to the intersection point of the contour plots at \( (\cos 2\phi_L = -0.8, \cos 2\phi_R = -0.5) \). The requirement that the three curves should meet in one point offers clearly a very stringent consistency check of the model. On the other hand, while \( \Sigma_{\text{unpol}} \) is a quadratic polynomial in \( \cos 2\phi_L, \cos 2\phi_R \), the two other observables are quartic in these variables. A potential ambiguity in the determination of \( (\cos 2\phi_L, \cos 2\phi_R) \) will be, however, very unlikely, especially if \( m_{\tilde{\nu}} \) is fixed independently and the c.m. energy varied. We do not dwell here on further aspects
of step (I) which can be found in [1].

We now go to step (II) of eq.(1) and describe briefly how to determine $M_2$, $\mu$ and $\tan \beta$. Starting from eq.(2.4) and $m_{\chi^+_1}$ in eq.(2.3), CDDKZ give closed expressions for $M_2$, $\mu$, $\tan \beta$ in terms of the quantities $p = \cot(\phi_R - \phi_L)$, $q = \cot(\phi_R + \phi_L)$. They considered all possible cases and concluded to the existence of at most a two-fold ambiguity in the determination of the Lagrangian parameters, traceable to a sign ambiguity in $\sin 2\phi_{L,R}$, (see [1] for details). Here we only sketch an equivalent discussion which shows that, when it occurs, this two-fold ambiguity is always associated with opposite $\mu$ sign solutions. This can be most easily seen as follows: from $\cos 2\phi_L$, $\cos 2\phi_R$ in eq.(2.4) one easily determines $M_2$ uniquely (remember that $M_2$ is positive in our convention) and $\mu$ with a global sign ambiguity, as functions of $m_{\chi^+_1}$, $\tan \beta$, $\cos 2\phi_L$ and $\cos 2\phi_R$. Plugging those functions in the $m_{\chi^+_1}$ part of eq.(2.3) one gets, thanks to some cancellations, a simple quadratic equation in $\tan \beta$. The two solutions encompass automatically the sign of $\sin 2\phi_{L,R}$. Furthermore, each of them is consistent only with (at most) one $\mu$ sign reproducing the correct $m_{\chi^+_1}$, since eq.(2.3) is not invariant under $\mu \rightarrow -\mu$ for a given $M_2$, $\tan \beta$. As a numerical illustration, taking the input of fig.2, $\sigma_{tot} = 0.37$pb, $P^2/Q = -0.24$, $P^2/Y = -3.66$, CDDKZ find the following two-fold solution

$$[\tan \beta; M_2, \mu] = \begin{cases} 
(A) & [1.06; 83\text{GeV}, -59\text{GeV}] \\
(B) & [3.33; 248\text{GeV}, 123\text{GeV}] 
\end{cases}$$  

We see that the two-fold ambiguity comes with a sign change for $\mu$ in accord with the general pattern just described. To eliminate this discrete ambiguity one would clearly need an independent information about any of the three parameters. Finally, the reconstruction obviously depends on the quality of the experimental accuracy with which the needed observables can be determined. This requires among other things:

- running at different c.m. energies: at threshold for a good determination of $m_{\chi^+_1}$, away from threshold to increase the sensitivity to chargino polarization;

- a good reconstruction of the final state fermion systems for a good determination of the neutralino mass;

- identification of the chargino electric charge, necessary for the extraction of $P$;

- an independent knowledge of the sneutrino mass, to avoid a three parameter fit to the observables;

### 3.2 KM

In this section we describe another strategy for extracting the Lagrangian parameters [2]. It consists in assuming that only ino masses are known. Among other things, this strategy will be complementary to the one described in the previous section in the sense
that it provides (within the CDDKZ strategy) an algorithm for the determination of $M_1$, the only parameter which was not reconstructed in ref. [1]. KM concerns mainly step (II) of eq.(1). The emphasis is put on the extent to which the reconstruction can be made through a controllable analytical procedure including all possible ambiguities, if three ino masses and $\tan \beta$ were known$^2$. This is particularly relevant for the neutralino sector where the analytical reconstruction is far from trivial.

The next aim in [2] is to provide a numerical code which uses as much of the analytical solutions as possible and allows a direct reconstruction of $M_1$, $M_2$ and $\mu$ from the physical ino masses. We do not address here the more realistic issues when only mass differences are measured$^2$, however it is clear that the study provides a useful building block even in this case, and practically allows to avoid parameter scanning numerical procedures as well as model-dependent assumptions. KM distinguish two cases:

$S_1$: The two charginos and one neutralino masses are input;  

$S_2$: One chargino and two neutralino masses are input;

Although $S_1$ is phenomenologically less compelling than $S_2$ as far as the generic pattern of low lying states is concerned, it turns out that it leads to a full analytical reconstruction. In contrast, $S_2$ needs partly a purely numerical algorithm which is, however, minimized through the use of the $S_1$ solutions. The bottom line is that the resulting algorithms are very fast, the first being fully analytical and the second needing seldom more than a few iterations to reach numerical convergence (see [2] for more details).

Let us now describe briefly the solutions for $S_1$.

**Chargino sector:**
Starting from eq. (2.3) one can determine analytically $\mu^2$ and $M_2$ in terms of $M_{\chi_1^+, \chi_2^+}$, $\tan \beta$ and $m_W$. Without further information in the chargino sector, $\mu$ and $M_2$ will be determined, but up to a $|\mu| \leftrightarrow M_2$ ambiguity (that is, one cannot determine uniquely at this level the Higgsino and Gaugino content of the charginos). On the other hand the global sign ambiguity in $\mu$, due to the fact that only $\mu^2$ is known, is actually lifted by the relation

$$M_2 \mu = m_W^2 \sin 2\beta \pm M_{\chi_1^+} M_{\chi_2^+}$$

(3.12)

since $M_2$ is positive by definition. Nonetheless there remains a two-fold ambiguity coming from the relative $\pm$ sign in eq. (3.12). On the other hand, some constraints will come from the requirement of real-valuedness of $M_2$ and $\mu$. All these aspects are analytically delineated in [2] in terms of domains of $\tan \beta$ and the sum and difference of the input chargino masses.

**Neutralino sector:**
Let us now assume that $M_2$, $\mu$, $\tan \beta$ and one neutralino mass have been determined, and address the question of reconstructing $M_1$ and thus the three remaining neutralino masses.

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$^2$The fact that $\tan \beta$ needs to be an input is actually a marginal point here, as one can assume that this parameter has been determined from elsewhere, like for instance in [1] or from the study of yet another sector of the MSSM.
It should be clear that the answer to this question is not straightforward independently of whether it can be phrased analytically or not. Indeed, with all parameters but $M_1$ fixed in eq. (2.3), and the knowledge of the mass of just one neutralino state (say the lightest), it could well be that multiple branch solutions exist which would be lifted only through extra information about the couplings in this sector. It turns out, however, not to be the case (at least when phases are ignored): there is basically a unique solution, apart from the fact that one should allow for negative and positive values for the input neutralino mass since $\mathcal{M}_N$ can have negative eigenvalues. (This sign liberty is actually the only ambiguity which can be eventually fixed through the study of the couplings and will not be discussed further here.)

The trick is to write down the four independent combinations of the entries of $\mathcal{M}_N$ which are invariant under similarity transformations, and thus relate them simply to the four eigenvalues of $\mathcal{M}_N$. One can then express the correlations between the eigenvalues and the basic parameters in the following form:

$$M_1 = \frac{P_{ij}^2 + P_{ij}(\mu^2 + m_2^2 + M_2 S_{ij} - S_{ij}^2) + \mu m_2^2 M_2 S_{ij} \sin 2\beta}{P_{ij}(S_{ij} - M_2) + \mu (c_w^2 m_2^2 \sin 2\beta - \mu M_2)}$$  \hspace{1cm} (3.13)$$

$$M_2 = \frac{S_{ij} P_{ij}^2 + P_{ij} m_2^2 \sin 2\beta - (P_{ij}^2 + (\mu^2 + m_2^2) P_{ij} + S_{ij} m_2^2 \mu \sin 2\beta) M_1}{P_{ij}^2 + P_{ij}(\mu^2 + s_w^2 m_2^2) + \mu S_{ij} (s_w^2 m_2^2 \sin 2\beta - \mu M_1)}$$  \hspace{1cm} (3.14)$$

where

$$S_{ij} \equiv \tilde{M}_{N_i} + \tilde{M}_{N_j}$$

$$P_{ij} \equiv \tilde{M}_{N_i} \tilde{M}_{N_j}$$

and $i \neq j$ index any neutralino mass parameter. (The tilde denotes the fact that the mass parameters can be negative valued) The nice thing about the above equations is that if any of the neutralino masses is taken as input (say $\tilde{M}_{N_i}$), then the other three are determined analytically through a simple cubic equation. A unique value for $M_1$ is then determined from eq. (3.13) after plugging any of these solutions. Eqs. (3.13, 3.14) express in a specially convenient way the various correlations among the four eigenvalues and the basic parameters. It is also noteworthy that the input set $(M_2, \mu, \tan \beta)$ plus one neutralino mass is optimal for a fully analytical determination. In particular this is precisely the input set required in ref. [1]. We illustrate here the complementarity of the two approaches by taking the two sets of numbers (A) and (B) in eq. (3.11) and a lightest neutralino $M_{N_1} = 30 GeV$, to reconstruct $M_1$ and the remaining neutralino masses from eqs. (3.13, 3.14):
\[
\begin{bmatrix}
M_1, & \tilde{M}_{N_2}, & \tilde{M}_{N_3}, & \tilde{M}_{N_4}
\end{bmatrix}
\]

\[(A) \begin{cases}
(+); [30\text{GeV}, 59\text{GeV}, -107\text{GeV}, 122\text{GeV}] \\
(-); [-52\text{GeV}, 58\text{GeV}, -119\text{GeV}, 120\text{GeV}]
\end{cases} \quad (3.15)
\]

\[(B) \begin{cases}
(+); [46\text{GeV}, 110\text{GeV}, -130\text{GeV}, 284\text{GeV}] \\
(-); [-25\text{GeV}, 101\text{GeV}, -132\text{GeV}, 284\text{GeV}]
\end{cases}
\]

Here (±) refer to the two possible signs of the 30GeV lightest eigenmass input. The effect of this sign tends to be more important for \(M_1\) than for the neutralino masses. Of course a minus sign should be accompanied with the appropriate sign change in the Feynman rules involving neutralinos (see [8]). A further study of the left and right form factors in the chargino decay system could then lift partially the four-fold ambiguity in eq.(3.15). Lifting the remaining two-fold ambiguity will still necessitate further measurements from the ino sector.

Back to the \(S_1\) strategy, we give in fig.3 an illustration of the sensitivity to a chargino mass, fixing the other two masses of chargino and neutralino. The behavior of the restructured \((M_1, M_2, \mu)\) turns out to be fairly simple, up to the two-fold ambiguity induced by \(\mu\) in the chargino sector. A simple behavior shows as well for the remaining three neutralino masses (see [2]), when the input neutralino mass is varied. This behavior which is fully controlled analytically can be used to discuss the generic gross features of the spectrum even when one deviates from the present input strategy. For instance the sensitivity to \(\tan \beta\) turns out to be rather mild, and the effect of the sign change in the input neutralino mass tends to be negligible apart from well localized regions (see ref.[2] for further illustrations, including a reconstruction of the parameters at the GUT scale). In fig.4 we illustrate the \(S_2\) strategy. The input set in this case requires a partial numerical algorithm since the output becomes controlled by high degree polynomials. However using eqs.(3.13, 3.14) in conjunction with the chargino sector relations allows an optimized iterative algorithm. Fig.4 shows a rather intricate behavior of \((M_1, M_2, \mu)\) when one chargino and two neutralino masses are taken as input, a reflection of the above mentioned high degree polynomials, which nevertheless boils down (at least in our numerical trials) to at most a four-fold ambiguity. The regions of many-fold ambiguities or no ambiguity at all are separated by domains where the output parameters become complex valued (the shaded areas). Furthermore the singular behavior in some small regions is generically traced back to zeroing some parameters (see ref.[2] for more details).
Figure 3: $\mu, M_1$ and $M_2$ (with the “higgsino-like” convention $|\mu| \leq M_2$) as functions of $M_{\chi^2}$ for fixed $M_{\chi^1} (= 400 \text{ GeV})$, $M_{N_2} (= 50 \text{ GeV})$, and $\tan \beta (= 2)$. (taken from ref.[2])

Figure 4: $\mu$, $M_2$ and $M_1$ as function of $M_{\chi^2}^0$ for fixed $M_{N_3} (= -100\text{GeV})$, $M_{\chi^1} (= 80 \text{ GeV})$ and $\tan \beta (= 2)$. (taken from ref.[2])
4 Final comments

In this talk we presented two possible theoretical strategies for the extraction of the ino sector parameters from physical observables. The first relied on the study of the production and decay of the lightest chargino in $e^+e^-$ collisions, the second on the knowledge of some ino masses. We also illustrated a full reconstruction of the ino sector when the two complementary approaches are brought together. Generally speaking, these approaches provide with efficient tools for the study of the ino sector. In the same time, they suggest the need in some cases for further experimental information due to the occurrence of possible discrete ambiguities in the reconstruction.

Furthermore, although we only considered real-valued parameters, some of the material presented here goes through unaltered if phases are allowed. This is the case in CDDKZ for the chargino sector, even though extra information will still be needed to determine those phases. The inclusion of phases is less obvious in KM, especially in the neutralino sector, and deserves a separate study by itself. One should, however, keep in mind that the above strategies can give indirect information about the need for phases, whenever the experimental data place the parameters in the forbidden regions delineated in KM. In any case, a by-product of the analytical study would have been the construction of fast and flexible algorithms which can be used in various ways when reconstructing the ino parameters.

Finally, it should be stressed that the strategies we presented here are just at the theoretical level. Obviously a more realistic examination of the experimental extraction of observables and related errors is still needed to assess their degree of efficiency. In addition, these strategies should eventually be placed in a wider context involving the other sectors of the MSSM, taking into account plausible discovery scenarios of the susy partners. The inter-correlations between these sectors, endemic to supersymmetry, will then hopefully allow a unique determination of all the parameters of the model.

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