πNN and Pseudoscalar Form Factors from Lattice QCD

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Abstract

The πNN form factor $g_{πNN}(q^2)$ is obtained from a quenched lattice QCD calculation of the pseudoscalar form factor $g_P(q^2)$ of the proton with pion pole dominance. We find that $g_{πNN}(q^2)$ fitted with the monopole form agrees well with the Goldberger-Treiman relation and is much preferred over the dipole form. The monopole mass is determined to be $0.75 \pm 0.14 \text{GeV}$ which shows that $g_{πNN}(q^2)$ is rather soft. The extrapolated πN coupling constant $g_{πNN} = 12.7 \pm 2.4$ is quite consistent with the phenomenological values. We also compare $g_{πNN}(q^2)$ with the axial form factor $g_A(q^2)$ to check the pion dominance in the induced pseudoscalar form factor $h_A(q^2) \text{ vis à vis chiral Ward identity.}$

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The $\pi NN$ form factor $g_{\pi NN}(q^2)$ is a fundamental quantity in low-energy pion-nucleon and nucleon-nucleon dynamics. Many dynamical issues like $\pi N$ elastic and inelastic scattering, $NN$ potential, three-body force (triton and $^3$He binding energies), pion photoproduction and electroproduction all depend on it. Similarly, the pseudoscalar form factor is important in testing low-energy theorems, the chiral Ward identity and the understanding of the explicit breaking of the chiral symmetry. Yet, compared with the electromagnetic form factors and the isovector axial form factor of the nucleon, the pseudoscalar form factor $g_P(q^2)$ and the $\pi NN$ form factor $g_{\pi NN}(q^2)$ are poorly known either experimentally or theoretically.

Notwithstanding decades of interest and numerous work, the shape and slope of $g_{\pi NN}(q^2)$ remain illusive and unsettled. Upon parametrizing $g_{\pi NN}(q^2)$ in the monopole form

$$g_{\pi NN}(q^2) = g_{\pi NN} \frac{\Lambda_{\pi NN}^2 - m_\pi^2}{\Lambda_{\pi NN}^2 - q^2}$$

with $g_{\pi NN} \equiv g_{\pi NN}(m_\pi^2)$, the uncertainty in the parametrized monopole mass $\Lambda_{\pi NN}$ can be as large as a factor 2 or 3. For the sake of having a sufficiently strong tensor force to reproduce the asymptotic D- to S- wave ratio and the quadrupole moment in the deuteron, $\Lambda_{\pi NN}$ is shown to be greater than 1 GeV \[1\]. Consequently, $\Lambda_{\pi NN}$ in the realistic NN potentials are typically fitted with large $\Lambda_{\pi NN}$ (e.g. $\Lambda_{\pi NN}$ ranges from 1.3 GeV \[4\] to 2.3 – 2.5 GeV \[4\]). On the other hand, arguments based on resolving the discrepancy of the Goldberger-Treiman relation \[4\] and the discrepancy between the $pp\pi^0$ and $pn\pi^+$ couplings \[3\] suggest a much softer $g_{\pi NN}(q^2)$ with $\Lambda_{\pi NN}$ around 0.8 GeV. Furthermore, hadronic models of baryons with meson clouds like the skyrmion typically have a rather soft form factor (i.e. $\Lambda_{\pi NN} \sim 0.6$ GeV) due to its large pion cloud and such a small $\Lambda_{\pi NN}$ is needed for the high energy elastic $pp$ scattering \[7\].

In view of the large uncertainty in $g_{\pi NN}(q^2)$, it is high time to study it with a lattice QCD calculation. Since our recent calculations of the nucleon axial and electromagnetic form factors are within 10% of the experimental results \[8,9\], a prediction of $g_{\pi NN}(q^2)$ with a similar accuracy should be enough to adjudicate on the controversy over the $\pi NN$ form factor. In this letter, we extend our lattice calculation to the proton pseudoscalar form factor for a range of light quark masses. $g_{\pi NN}(q^2)$ is obtained by considering the pion pole dominance in $g_P(q^2)$ when the latter is extrapolated to the quark mass which corresponds to the physical pion mass.

In analogy with the study of the electromagnetic and axial form factors \[8,9\] of the nucleon, we calculate the following two- and three-point functions for the proton

$$G_{pp}^{\alpha\alpha}(t, \vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \langle 0|T(\chi^\alpha(\vec{x})\bar{\chi}^\alpha(0))|0\rangle$$

$$G_{pPp}^{\alpha\beta}(t_f, \vec{p}, t, \vec{q}) = \sum_{\vec{x}_f, \vec{x}} e^{-i\vec{p}\cdot\vec{x}_f + i\vec{q}\cdot\vec{x}} \langle 0|T(\chi^\alpha(\vec{x}_f)P(\vec{x})\bar{\chi}^\beta(0))|0\rangle,$$
where $\chi^\alpha$ is the proton interpolating field and $P(x)$ is the mean-field improved isovector pseudoscalar current for the Wilson fermion

$$P(x) = \frac{2\kappa}{8\kappa_c} e^{m_q a} \bar{\psi}(x) i\gamma_5 \frac{T_3}{2} \psi(x). \quad (4)$$

Here, we have included the $2\kappa/8\kappa_c$ ($\kappa_c = 0.1568$ is the critical $\kappa$ value for the chiral limit for our lattice at $\beta = 6.0$) and the $e^{m_q a}$ ($m_q a = \ln(4\kappa_c/\kappa - 3)$ is the quark mass) factors in the definition of the lattice current operator. These factors take into account the mean-field improvement and finite quark mass correction for the Wilson action [10] and have been shown to be an important improvement in the evaluation of the axial form factor in order to allow the perturbative lattice renormalization to work [8].

Phenomenologically, the pseudoscalar current matrix element is written as

$$\langle \vec{p}s | P(0) | \vec{p}'s' \rangle = g_P(q^2) \bar{u}(\vec{p}, s) i\gamma_5 u(\vec{p}', s'). \quad (5)$$

where $g_P(q^2)$ is the pseudoscalar form factor. It has been shown [8, 9] that when $t_f - t$ and $t >> a$, the lattice spacing, the combined ratios of three-point and two-point functions with different momentum transfers lead to desired form factors related to the probing currents. In the case of the pseudoscalar current in eq. (4), the lattice pseudoscalar form factor $g_L^P(q^2)$ is determined from the following ratio

$$\frac{\Gamma G^{\alpha\beta}_{\rho p p}(t_f, \bar{0}, t, \bar{q}) G^\alpha_{\rho p p}(t, \bar{0})}{G_{\rho p p}(t_f, \bar{0}) G_{\rho p p}^{\omega\alpha}(t, \bar{q})} \rightarrow \frac{q_3}{E_q + m} g_L^P(q^2) \quad (6)$$

where $\Gamma = \left( \begin{array}{cc} \sigma_3 & 0 \\ 0 & 0 \end{array} \right)$, and $m$ and $E_q$ are the proton mass and energy with momentum $\bar{q}$ respectively.

Quark propagators have been generated on 24 quenched gauge configurations on a $16^3 \times 24$ lattice at $\beta = 6.0$ to study the nucleon electromagnetic and axial form factors [8, 9]. We shall use the same propagators for the present calculation. Results are obtained for three light quarks with $\kappa = 0.154, 0.152,$ and $0.148$. They correspond to quark masses $m_\rho$ of about 120, 200, and 370 MeV respectively. (The scale $a^{-1} = 1.74(10)$ GeV is set by fixing the nucleon mass to its physical value.) Results of $g_L^P(q^2)$ for the momentum transfers $\bar{q}^2 a^2 = n(2\pi/L)^2$ ($n = 1$ to 4) are obtained from the plateaus of the ratio in eq. (6) as a function of $t$, the time slice of the current insertion, away from the sink and source of the nucleon interpolation fields [8]. Since the ratio in eq. (6) is proportional to $q_3$, $g_L^P(q^2)$ at $q^2 = 0$ can not be obtained directly. Rather, it will be obtained from extrapolation from the finite $q^2$ data as explained later.

Plotted in fig. 1 are the lattice isovector pseudoscalar form factors $g_L^P(q^2)$ of the proton as a function of the quark mass in dimensionless unit $m_q a$ which takes into account the tadpole-improved definition for the quark mass. They include different
momentum transfers with $\vec{q}^2$ from 1 to 4 times $(2\pi/La)^2$ (N.B. $q^2 = (E - m_N)^2 - \vec{q}^2$ for the four-momentum transfer squared). The errors are obtained through the jackknife in this case. The extrapolation of $g^L_P$ to the quark mass $m_qa$ which corresponds to the physical pion mass is carried out with the correlated fit to a linear dependence on the quark mass $m_qa$ for $\kappa = 0.154, 0.152$ and 0.148. The data covariance matrix is calculated with the single elimination jackknife error for $g^L_P$ which takes into account the correlation among the gauge configurations [8, 11]. This fitting gives $\chi^2/N_{DF} = 0.005, 0.008, 0.65, \text{and } 1.7$ for $\vec{q}^2a^2$ form 1 to 4 $(2\pi/L)^2$.

To extract the $\pi NN$ form factor $g_{\pi NN}(q^2)$, we take the pion pole dominance in the dispersion relation for $g^L_P(q^2)$ so that

$$g^L_P(q^2) = \frac{G_\pi g_{\pi NN}(q^2)}{m^2_\pi - q^2}$$

where $G_\pi = \langle 0|P(0)|\pi \rangle$ can be obtained from the two-point function

$$\langle 0|\sum_\vec{x} P(\vec{x}, t) P(0, 0)|0 \rangle \xrightarrow{t \gg a} \frac{G^2_\pi}{2m_\pi} e^{-m_\pi t}$$

Plotted in Fig. 2 is $g_{\pi NN}(q^2)$ defined via eqs. (7) and (8). There is a caveat to extracting $g_{\pi NN}(q^2)$ this way which we wish to point out. Strictly speaking eq. (7) is equivalent to PCAC where the physical pion field dominates and is thus valid for small $q^2$. For $q^2$ as large as $m^2_\pi$, with $\pi'$ being the radially excited pion at 1.3 GeV, higher mass contribution to $g^L_P(q^2)$ may not be negligible. However, $g_{\pi' NN}$ is expected to be an order of magnitude smaller than $g_{\pi NN}$ due to the fact that a node in the internal $q\bar{q}$ wavefunction of $\pi'$ will lead to cancellation in the vertex function. Therefore, we estimate that the pion pole dominance (eq. (7)) may have an error as large as 5 to 10% at the highest $q^2$ we calculated. This is much smaller than the statistical error we have at the highest $q^2$. This is also consistent with the estimate that PCAC and chiral perturbation is good to a scale of $4\pi f_\pi$. Keeping this in mind, we discuss the behavior of $g_{\pi NN}(q^2)$. We fitted it with both a monopole form (i.e. eq. (7)) and a dipole form. We found that the monopole form with $\Lambda_{\pi NN} = 0.75 \pm 0.14$ GeV and a $\chi^2/N_{DF} = 0.13/2$ is only slightly better than the dipole form with a dipole mass of 1.32 $\pm$ 0.17 GeV and a $\chi^2/N_{DF} = 0.57/2$, in so far as the $\chi^2$ is concerned. However, we can inject our knowledge at $q^2 = 0$. The Goldberger-Treiman (GT) relation which relates coupling constants at $q^2 = 0$

$$m_N g_A(0) = f_\pi g_{\pi NN}(0)$$

predicts $g_{\pi NN}(0) = 12.66 \pm 0.04$ from the known $g_A(0) = 1.2573 \pm 0.0028$ and $f_\pi = 93.15 \pm 0.11$ MeV [12]. Extrapolation of the monopole fit of $g_{\pi NN}(q^2)$ gives $g_{\pi NN}(0) = 12.2 \pm 2.3$ which agrees with the GT relation. Yet, the dipole fit giving $g_{\pi NN}(0) = 10.8 \pm 1.3$ falls outside the prediction of the GT relation. Thus, we conclude that
the monopole form is much preferred over the dipole form. The monopole mass $\Lambda_{\pi NN} = 0.75 \pm 0.14$ GeV thus obtained is much smaller than those typically used in the NN potential, but agrees well with those based on the consideration of the GT relation [4]. The apparent discrepancy between $g_{\pi^0\eta}$ and $g_{\pi^0\eta}$ [5], and the nucleon models like the skyrmion [3]. To salvage the nice fit of the NN scattering data and the deuteron properties based on a hard $\pi NN$ form factor, attempts have been made to incorporate a soft $g_{\pi NN}(q^2)$ either by appending a heavy pion at $\approx 1.2$ GeV [13] or by including multi-meson exchanges [14] (e.g. $\pi\rho$ and $\pi\sigma$). Extrapolating $g_{\pi NN}(q^2)$ to $q^2 = m_{\pi}^2$, we obtain $g_{\pi NN}$, the $\pi N$ coupling constant, to be $12.7 \pm 2.4$. This compares favorably with the empirical value of $13.40 \pm 0.17$ [13] and $13.13 \pm 0.07$ [16]. The 4% change in $g_{\pi NN}(q^2)$ from $q^2 = 0$ to $m_{\pi}^2$ indeed can account for the 4% discrepancy in the GT relation when the physical $g_{\pi NN}$ is used in eq. (9) instead of the $g_{\pi NN}(0)$ [4].

Putting the chiral Ward identity $\partial_{\mu}A_\mu = 2m\bar{\psi}\gamma_5\tau_\alpha/\psi$ with pion pole dominance or equivalently PCAC ($\partial_{\mu}A_\mu = f_\pi m_{\pi}^2\phi^\alpha$) between nucleon states, we find

$$2m_{\pi}g_A(q^2) + q^2h_A(q^2) = \frac{2m_{\pi}^2 f_\pi g_{\pi NN}(q^2)}{m_{\pi}^2 - q^2}$$

(10)

In addition to PCAC, if one further assumes that the induced pseudoscalar form factor $h_A(q^2)$ is dominated by the pion pole, i.e. $h_A(q^2) = 2f_\pi g_{\pi NN}(q^2)/(m_{\pi}^2 - q^2)$, then $g_{\pi NN}(q^2) = (m_{\pi}/f_\pi)g_A(q^2)$. In other words, $g_{\pi NN}(q^2)$ has the same $q^2$ dependence as $g_A(q^2)$ which has been frequently used in the literature [17, 18]. As there is no a priori reason why $g_{\pi NN}(q^2)$ should have the same falloff as $g_A(q^2)$ at all $q^2$ and, furthermore, chiral perturbation calculation [19] at one loop suggests that they acquire different contributions, we compare $g_{\pi NN}(q^2)$ from eq. (9) and $g_A(q^2)$ obtained on the same set of gauge configurations [3] for the light quark cases. Both $g_{\pi NN}(q^2)$ and $g_A(q^2)$, normalized at $q^2 = 0$, are plotted in Fig. 3 for $\kappa = 0.148, 0.152, 0.154,$ and 0.1567. The last $\kappa$ corresponds to the physical pion mass. We find that in all these light quark cases, there is a tendency for the normalized $g_{\pi NN}(q^2)$ to lie lower/higher than the normalized $g_A(q^2)$ at lower/higher $-q^2$. This presumably reflects the preferred monopole vs dipole fit for the $g_{\pi NN}(q^2)$ and $g_A(q^2)$. Our data do not discern this well though. If this behavior is verified, it would imply that the induced pseudoscalar form factor $h_A(q^2)$ (not the pseudoscalar form factor $g_P(q^2)$) is not entirely dominated by the pion for higher $-q^2$ as it is at very low $-q^2$ (< 0.1 GeV$^2$ say).

Lastly, from the chiral Ward identity (eq. (11)), we can obtain the induced pseudoscalar form factor $h_A(q^2)$ from $g_A(q^2)$ [3] and $g_{\pi NN}(q^2)$. The pion decay constant $f_\pi$ needed in eq. (11) is calculated from the two-point functions $\langle A_\tau A_\tau(t, \vec{x})P(0, 0) \rangle$ and eq. (8). It is found to be $89.8 \pm 4.5$ MeV when the finite lattice renormalization is taken into account. We plot $h_A(q^2)$ in Fig. 4. Also plotted in the insert are experimental data obtained from pion electroproduction [18]. It turns out that the momentum transfer ranges of our lattice calculation and the available experiment do not overlap. We can not compare them directly. However, if we use the monopole
fit of \( g_{\pi NN}(q^2) \) and the dipole fit of \( g_A(q^2) \) \[8\], we find the extrapolation of \( h_A(q^2) \) (solid line in Fig. 4) does agree with the experimental data at small \(-q^2\). We note the errors of the fit start to diverge as \(-q^2 \to 0\) due to the \( q^2 \) singularity in eq. (11). As a result we are not able to extrapolate to \(-q^2 = 0\).

To conclude, we have calculated the isovector pseudoscalar form factor of the nucleon in a lattice QCD calculation for quark masses from about one to about two times that of the strange quark. From these we extracted \( g_{\pi NN}(q^2) \) with the help of the pion pole dominance. The main results we gleaned are the following:

1) Incorporating the Goldberger-Treiman relation at \( q^2 = 0 \), we find that \( g_{\pi NN}(q^2) \) is much better described by a monopole than a dipole form. The monopole mass \( \Lambda_{\pi NN} = 0.75 \pm 0.14 \text{ GeV} \) is much smaller than commonly used in the NN potential.

2) \( g_{\pi NN} = 12.7 \pm 2.4 \) agrees well with the phenomenological values of \( 13.40 \pm 0.17 \) \[15\] and \( 13.13 \pm 0.07 \) \[16\]. It is also consistent with the lattice calculation of \( 14.8 \pm 0.6 \) with staggered fermion \[20\].

3) The falloff of \( g_{\pi NN}(q^2) \) is about the same as \( g_A(q^2) \) at very small \(-q^2 (< 0.3 \text{ GeV}^2)\), but is likely to fall slower at higher \(-q^2\). This suggests that the induced pseudoscalar form factor \( h_A(q^2) \) is not entirely dominated by the pion pole at higher \(-q^2\). This point needs to be verified further with higher statistics study.

4) From the chiral Ward identity and PCAC, we obtain \( h_A(q^2) \) which can be checked experimentally in the future.

The soft \( g_{\pi NN}(q^2) \) form factor agrees with the predictions based on the discrepancy of the Goldberger-Treiman relation \[4\] and between the \( pp\pi^0 \) and \( pn\pi^+ \) couplings \[5\]. This will have a large impact on the study of NN potential, the three-body force, and other processes which involve the \( \pi N \) coupling. For future studies, it is essential to improve the calculation by expanding the volume in order to access smaller \(-q^2\) and incorporating dynamical fermions effects.

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Figure Captions

Fig. 1  The lattice isovector pseudoscalar form factors at various $-q^2$ as obtained from eqs. (3) are plotted as a function of $m_q a$, the quark mass in lattice unit, for the three light quark cases (Wilson $\kappa = 0.148, 0.152, \text{ and } 0.154$). The top curve is for $\vec{q}^2 = (2\pi/La)^2$, the rest are for $\vec{q}^2$ from 2 to 4 times of $(2\pi/La)^2$ in descending order.

Fig. 2  $g_{\pi NN}(q^2)$ at the quark mass which corresponds to the physical pion mass. The solid and the dashed curves represent the monopole and dipole fits with the respective monopole and dipole mass $\Lambda$. They give quite different extrapolations at $q^2 = 0$.

Fig. 3  Comparison of $g_A(q^2)$ and $g_{\pi NN}(q^2)$ (both normalized to 1 at $q^2 = 0$) as a function of $-q^2$ for the light quark cases ($\kappa = 0.148, 0.152, 0.154, \text{ and } 0.1567$).

Fig. 4  The induced pseudoscalar form factor $h_A(q^2)$ from eq. (10). The solid line is from the fits to $g_A(q^2)$ and $g_{\pi NN}(q^2)$. Also plotted in the insert are data from the electroproduction of pion [18]. The typical size of the error bars for the solid line is indicated in the insert.
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$g_{\pi NN}(q^2)$

- **Monopole ($\Lambda=0.747\text{GeV}$)**
- **Dipole ($\Lambda=1.32\text{GeV}$)**
Induced pseudoscalar form factor

\( \kappa_\pi = 0.1567 \)