An assumption required to reproduce LEP hadronic cross section data

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It has been shown that the Standard Model can reproduce the inclusive hadronic cross sections measured at LEP for $e^+e^- \rightarrow Z \rightarrow \text{hadrons}$. Embedded in the Standard Model analysis, however, is the assumption that $Z \rightarrow u\bar{u}$ events generally produce different physical states than $Z \rightarrow d\bar{d}$ events. Is that assumption justified? It is argued that for most $Z \rightarrow u\bar{u}$ events, there is a $Z \rightarrow d\bar{d}$ event that produces the same physical state. The matrix elements for these two events should then be added before being squared. If this is done, it may be difficult for the Standard Model to reproduce the LEP inclusive hadronic cross section data.

The experiments at the Large Electron Positron (LEP) collider at CERN made very precise measurements of the inclusive cross section for $e^+e^- \rightarrow Z \rightarrow \text{hadrons}$. The Standard Model prediction for this cross section incorporates the following zeroth order expression for the branching ratio of Z boson decay to hadrons (see eqs 1.43 and 1.37 of [1]):

$$\Gamma_{\text{had}} = \sum_{q=u,d,s,c,b} \Gamma_{q\bar{q}} \propto \left( g_q^V \right)^2 + \left( g_q^A \right)^2,$$

where $g_q^V$ and $g_q^A$ are the vector current and axial vector current couplings of the quark $q$ to the Z boson.

There is an assumption required to use the the above expression. Namely, it is assumed that at zeroth order, it is a good approximation to treat $Z \rightarrow u\bar{u}$ events as if they generate physical states that are different from those of $Z \rightarrow d\bar{d}$ events. That assumption is what allows those matrix elements to be squared independently before being added, as in Fig. 1. Squaring before adding for those hadrons had to have been created in pairs from gluons (or photons).

Most of the $e^+e^-$ collisons at LEP produced two jets [2]. Taking $b$ quarks as an example, the fact that each jet was colorless (and had integral charge and baryon number) implies that close to the $Zb\bar{b}$ vertex, another $\bar{q}'q'$ pair must have been created by a gluon (or photon), with $\bar{q}'$ joining the $b$ jet and $q'$ joining the $\bar{b}$ jet. Additional quark pairs were created between $b$ and $\bar{q}'$, forming a colorless hadronic jet. Instead of $\bar{q}'q'$, a pair of diquarks could have been created, giving each jet a nonzero baryon number, but this less frequent case is ignored for the approximations in this paper.

In other words, a $Z \rightarrow b\bar{b}$ event looked diagramatically like Fig 2, where in the $b$ jet, gluons (or photons) created $n$ quark-antiquark pairs, while in the $\bar{b}$ jet, they created $n'$ of them. Fig 2 does not show the actual gluon interactions that generated the quark pairs, but examples of those interactions are given below.

All of the quarks on the right side of Fig 2 are assumed to be valence quarks for the hadrons generated by the event. Some assumptions are made below about how these valence quarks combine into hadrons (e.g. using quark-meson couplings). In that sense, the valence quark diagrams in this paper represent physical states involving hadrons. It is assumed that these diagrams are better approximations of physical states than the zeroth order single-quark approximations of Fig 1.

Neglecting the very rare cases when $\bar{q}'q' = b\bar{b}$, every diagram depicting a $Z \rightarrow b\bar{b}$ decay like Fig 2 has a different physical state than any $Z \rightarrow q\bar{q}$ diagram where $q \neq b$. This is because each jet of Fig 2 carries a beauty quantum number, whereas none of the jets in non-$b$ quark diagrams do. That being the case, for beauty quarks it

FIG. 1. SM assumption for LEP hadronic cross sections

should be a good approximation for diagrams involving heavy quarks that carry a distinct quantum number like charm or beauty, but is it a good approximation for the up and down quark diagrams?

To address that question, a valence quark approximation of hadronic physical states is utilized. A typical Z boson decay to hadrons at LEP produced a large number of hadrons. Aside from the quark and antiquark that came directly from the Z boson, all of the valence quarks

FIG. 2. $b\bar{b}$ hadronic jets

n...
makes sense to simplify the calculations by approximating physical states at zeroth order by the left side of Fig 2. In other words for beauty quarks, the valence quark approximation of physical states justifies the commonly used single quark zeroth order approximation of physical states. Since \( \bar{q}'q' = \bar{c}c \) is also very rare, the same is true for charm quarks.

But the same justification does not apply to decays of Z bosons to up and down quarks. For this paper, the split pair \( \bar{q}'q' \) is assumed to be either \( \bar{u}u \) or \( \bar{d}d \), created by gluons with equal couplings and probability. This approximation could be relaxed by also considering events with \( \bar{q}'q' = \bar{s}s \), but that would not qualitatively change the conclusions of the paper.

In this approximation, the leading diagrams for \( Z \to u\bar{u} \) events are those shown in Fig 3, where the first (second) diagram produces charged (neutral) jets. For every

FIG. 3. The leading valence quark diagrams from \( u\bar{u} \) events

\[ Z \to u\bar{u} \] charged jet diagram, it is possible to create an analogous \( Z \to \bar{d}d \) diagram with the same valence quark content – and the same physical states. As a result, the matrix elements of those two diagrams should be added before squaring, as in Fig 4.

FIG. 4. Charged jet diagrams from \( u\bar{u} \) and \( \bar{d}d \) events

The fact that the diagrams of Fig 4 should be added before squaring could lead one to ask whether the standard approach of Fig 1 is justified. It is worth looking at the simplest specific cases in more detail. To be able to analyze results in terms of hadronic physical states, it is assumed that valence quarks can produce a meson via the interaction depicted in Fig 5. The vertex for this

FIG. 5. Quark-meson vertex

quark-meson interaction is assumed to have a form such as that in [3]. In particular, the vertex involves a unit matrix in color space, a Pauli matrix (or unit matrix) in isospin space, one or more gamma matrices in spin space (depending on the spin of the meson), and momentum dependencies that are symmetric between the incoming quark and antiquark and are also consistent with overall 4-momentum conservation. As an example, for \( \pi^0 \) the vertex is proportional to \( \gamma^5 \tau^3 \), where \( \tau^3 \) is a Pauli matrix in isospin space.

Using this vertex, it is possible to draw a diagram for a Z boson decaying to two charged mesons as in Fig 6. Due to the fermion loops, each matrix element (before squaring) involves a trace. Due to the momentum symmetry of the meson vertices, the two diagrams of Fig 6 are the same except for their coupling with the Z boson. In other words, Fig 6 is proportional to

\[
\text{Tr} \left( \ldots \left( g_\nu^V + g_\nu^A \right) + \gamma_5 \left( g_\mu^A + g_\mu^A \right) \ldots \right). \tag{2} \]

The logic for Fig 6 can be extended to any diagram with more complicated gluon interactions depicting the decay of a Z boson to two charged mesons. In particular, for any complicated gluon interaction for \( Z \to u\bar{u} \) (the left diagram of Fig 6), an analogous \( Z \to \bar{d}d \) diagram with the same complicated gluon interaction can be drawn. The sum of all of these diagrams is still proportional to eq (2).

The situation is more complicated for the decay of a Z boson to two neutral mesons such as shown in Fig 7. For spin 0 mesons, the \( u\bar{u} \) and \( \bar{d}d \) valence quarks could

FIG. 6. Z boson decaying to two charged mesons

FIG. 7. Z boson decaying to two neutral mesons

generate \( \pi^0, \eta, \) or \( \eta' \). These have different quark meson couplings in isospin space (\( \tau^3 \) vs. \( \tau^0 \)), so it is helpful to make some approximations to simplify the problem.

It has been noted that the 2-jet events produced by \( e^+e^- \) at LEP were “pencil-like” with very small \( p_T \) dif-
ferences for hadrons within each jet relative to the thrust axis [2]. For small pr differences, one would expect π\(^0\) production to dominate over η (or η') production (see also [4]). For π\(^0\) mesons, the meson vertex involves τ\(^3\) in isospin space, so that \(d\bar{d}\) production of a π\(^0\) has the opposite sign to \(u\bar{u}\) production of a π\(^0\). Extending this spin 0 approximation to mesons with higher spin, the following simplifying assumption is made:

\[
\text{Simplifying Assumption 1}
\]

\[\text{At a meson vertex: } dd \simeq -u\bar{u}. \tag{3}\]

With that Simplifying Assumption, the sum of all diagrams depicting the decay of a Z boson to two neutral mesons (e.g. Fig 7) is proportional to eq (2).

Most jets have many hadrons, not just one, so the above simple cases must be generalized. A convenient next level of generalization is to consider jets that only have u and d valence quarks (and antiquarks) and that only combine to form mesons. For any number of mesons in a jet (or diagram), the jet (or diagram) can be characterized as "odd" or "even" if it has an odd or even number of mesons generated by \(dd\).

As the number of mesons in a jet gets large, the number of odd jets becomes very similar to the number of even jets, for both \(Z \rightarrow u\bar{u}\) and \(Z \rightarrow d\bar{d}\) events. This can be seen iteratively. Consider a neutral u jet with one meson (the upper jet in the left diagram of Fig 7). There is one configuration, and it is even: \(u\bar{u}\). For a neutral u jet with three mesons, there are 3 even configurations (\(u\bar{u}, u\bar{u}, u\bar{u}; u\bar{u}, ud, \bar{d}\bar{u};\) and \(ud, \bar{d}u, u\bar{u}\)) and 1 odd configuration (\(ud, \bar{d}d, \bar{d}u\)). For a neutral u jet with five mesons, there are 10 even and 6 odd configurations. Neutral d jets follow an analogous pattern with odd=even. As the number of mesons in the jet grows, the number of odd and even configurations grow closer in number in each type of jet.

For jets with a large number of mesons, it is possible to match almost every \(Z \rightarrow u\bar{u}\) diagram with a \(Z \rightarrow d\bar{d}\) diagram that has the same even or odd designation and has the same collection of final-state mesons with the same momenta. In some cases, two matching diagrams have exactly the same functional form. An example is shown in Fig 8. When diagrams like this are added, the result is proportional to eq (2).

Other matching diagrams will have different functional dependencies, since the matching mesons will not be in exactly the same places in each diagram. An example of this type (with gluons suppressed) is shown in Fig 9. To address these types of diagrams, it is helpful to make the following additional simplifying assumption:

\[
\text{Simplifying Assumption 2}
\]

Net differences in functional dependencies are small in matching \(Z \rightarrow u\bar{u}\) and \(Z \rightarrow d\bar{d}\) diagrams.

With Simplifying Assumption 2, the sum of all matching \(Z \rightarrow u\bar{u}\) and \(Z \rightarrow d\bar{d}\) diagrams with large numbers of mesons in the jets is also proportional to eq (2).

In the context of these assumptions, for \(Z\) decays to two jets that do not carry quantum numbers of beauty, charm, strangeness or baryon number, a better zeroth order approximation than Fig 1 is obtained in the following manner: First, define the following effective couplings of the up and down quarks with the \(Z\) boson:

\[
g^V_{u,\text{eff}} = g^V_{d,\text{eff}} = \frac{1}{\sqrt{2}} (g^V_u + g^V_d) \]

\[
g^A_{u,\text{eff}} = g^A_{d,\text{eff}} = \frac{1}{\sqrt{2}} (g^A_u + g^A_d). \tag{5}\]

Then using these couplings, implement the standard method of eq (1), treating all the quarks as if they were physical states.

The factor of \(1/\sqrt{2}\) corrects for the fact that even though the couplings have already incorporated the effects of both up and down quarks, eq (1) still sums over up and down.

If eq (5) was the correct zeroth order approximation, the Standard Model would not be able to reproduce the LEP inclusive hadronic cross section. The reason is this: In the usual analysis, Fig 1 accounts for \(\sim 39\%\) of the total inclusive hadronic cross section. In comparison, Fig 1 with the effective couplings of eq (5) generates a value that is \(\sim 1\%\), due to significant cancellations in the sum of the up and down quark couplings with the \(Z\) boson. So if the above simplifying assumptions were approximately valid, the Standard Model would underpredict the measured cross section by about \(38\%\).

As an aside, if the same arguments and Simplifying Assumptions were applied to the “R Value” \(\sigma(e^+e^- \rightarrow \)
hadrons)/σ(e^+e^- → μ^+μ^-), the Standard Model would also underpredict the data. For example, above the \( b \bar{b} \) threshold, it would underpredict the data by \( \sim 36\% \) (\( \frac{7}{3} \) vs. \( \frac{14}{7} \) \[5\]).

The approximation could be improved by also taking into account gluon-generated \( ss \) pairs or diquark-antidiquark pairs (to form baryons), but those next levels of approximation would not be able to overcome the significant under-prediction by the Standard Model.

It could be argued that the Simplifying Assumptions are not adequately justified. But even if the Simplifying Assumptions are relaxed and the discrepancy is < 38% at the Z pole, it is difficult to see how the standard approximation of Fig 1 with the standard couplings could be justified. The fact that matrix elements like Fig 4 must be added before squaring should have some impact on zeroth order approximations. Further research should look into this question.

Standard analyses using fragmentation functions are not well suited to addressing this question. This is because fragmentation functions do not first calculate a matrix element and then square it in order to obtain a cross section. Instead, the functions directly determine probabilities (squares of matrix elements) of various hadrons, given a particular quark as an input. Since they are run independently for each quark flavor, fragmentation function methods implicitly assume Fig 1 from the beginning.

In summary, Standard Model comparisons to LEP inclusive hadronic cross section data require the assumption that it is a good zeroth order approximation to treat the individual quarks \( q \) coming from \( Z \rightarrow q \bar{q} \) decays as if they were unique physical states. This paper employs a valence quark approximation to physical states to analyze the validity of that assumption. The analysis suggests that the assumption is justified for charm and beauty quarks, but it is difficult to justify for up and down quarks.

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