We present a parametrization for the Dark Energy Equation of State “EoS” which has a rich structure, performing a transition at pivotal redshift $z_T$ between the present day value $w_0$ to an early time $w_i = w_a + w_0 \equiv w(z \gg 0)$ with a steepness given in terms of $q$ parameter. The proposed parametrization is $w = w_0 + w_a(z/z_T)^q/[1 + (z/z_T)]^q$, with $w_0$, $w_i$, $q$ and $z_T$ constant parameters. It reduces to the widely used EoS $w = w_0 + w_a(1 - a)$ for $z_T = q = 1$. This transition is motivated by scalar field dynamics such as for example quintessence models. We study if a late time transition is favored by BAO measurements combined with local determination of $H_0$ and information from the CMB. According to our results, an EoS with a present value of $w_0 = -0.92$ and a high redshift value $w_i = -0.99$, featuring a transition at $z_T = 0.28$ with an exponent $q = 9.97$ was favored by data coming from local dynamics of the Universe (BAO combined with $H_0$ determination). We find that a dynamical DE model allows to simultaneously fit $H_0$ from local determinations and Planck CMB measurements, alleviating the tension obtained in a ΛCDM model. Additionally to this analysis we solved numerically the evolution of matter over-densities in the presence of dark energy both at background level and when its perturbations were considered. We show that the presence of a steep transition in the DE EoS gets imprinted into the evolution of matter overdensities and that the addition of an effective sound speed term does not erase such feature.

I. INTRODUCTION

We live in a particular epoch of the cosmic history characterized by the acceleration in the expansion rate of the Universe. Although its cause is unknown this acceleration is described as the consequence of a Cosmological Constant, $\Lambda$, with density $\rho_\Lambda$ constant in space and time. Despite its simplicity, there is no fundamental understanding of its origin and this framework has serious theoretical issues namely the coincidence and fine-tunning problems (1, 2). For this reason alternative models that either modify gravity at large scales as prescribed by General Relativity or introduce a dynamical Dark Energy (DE) component have arisen. Dynamical dark energy models are often characterized by the DE equation of state (EoS), $w \equiv P/\rho$, which is the ratio of the DE pressure to its density. Since DE properties are still unknown, several models to parametrize its EoS as a function of time, $w(z)$, have arisen in the literature (3–13). One of the most popular among them is the CPL parametrization (14, 15), widely used in cosmological observational analysis. The present value of DE EoS is restricted by observations to be close to $-1$ ($w = -1.019^{+0.075}_{-0.086}$ according to the 95% limits imposed by Planck data combined with other astrophysical measurements [16]). Nevertheless, the DE behavior and its properties at different cosmic epochs are much poorly constrained by current cosmological observations. According to astrophysical observations our Universe is flat and dominated at present time by the DE component (16), so data coming from late-time, low-redshift measurements such as Baryon Acoustic Oscillations (BAO) from Large Scale Structure surveys are those best suited for its analysis.

The aim of this work is to determine the late time dynamics of DE through its EoS, and in particular we are interested in studying if a transition in $w(z)$ takes place. To that end the parametrization used is $w(z) = w_0 + w_a(z/z_T)^q/[1 + (z/z_T)]^q$, with $w_0$, $w_a = w_i - w_0$, $q$, and $z_T$ constant parameters. This EoS allows for a steep transition for a large value of $q$ at the pivotal point $z_T$, which is prompted by scalar field dynamics such as quintessence models and motivated in [13], where a new parametrization that captures the dynamics of DE is presented.

The scientific community is devoting a large amount of time and resources in the quest to understand the dynamics and nature of DE, working on current (SDSS-IV [18], DES [19]) and future (DESI [20–22], Euclid [23], LSST [24]) experiments to study with very high precision the expansion history of the Universe and thus be able to test interesting models beyond a Cosmological Constant or Taylor expansions of the EoS of DE.

This article is organized as follows: we introduce our theoretical framework and the data sets used in Section I. Section II details the analysis performed at Background level and the results obtained, Section IV discusses the analysis at perturbative level while Section V summarizes our Conclusions.
II. METHOD AND DATA

Within the General Relativity framework for a flat Universe and a FLRW metric we have

\[ H(z) = H_0 \sqrt{\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_{DE} F(z)} \]

where \( H \equiv (da/dt)/(1+z) \) is the Hubble parameter, \( a = (1+z)^{-1} \) the scale factor of the Universe and \( H_0 = 100 \cdot h \) is the Hubble parameter at redshift zero in units of \( \text{km} \cdot \text{s}^{-1} \text{Mpc}^{-1} \). The present value of matter, radiation, and DE fractional densities are given by \( \Omega_m, \Omega_r, \Omega_{DE} \), respectively. The function \( F(z) = \rho_{DE}(z)/\rho_{DE}(0) \) in equation (1) encodes the evolution of DE component in terms of its EoS, \( w(z) \), according to

\[ F(z) = \exp \left( -3 \int_0^z dz' \frac{1+w(z')}{1+z'} \right) \]

where \( w(z) \) specifies the evolution of the DE fluid and accordingly, the Universe expansion rate at late times, following the dynamics set by equation (1).

\[ w(z) = w_0 + w_a \left( \frac{z}{z_T} \right)^q \]

with \( w_a = w_i - w_0 \) where \( w_i \) and \( w_0 \) represent the value for \( w(z) \) at large redshifts and at present day, respectively, whereas the term \( f(z) = \left( \frac{z}{z_T} \right)^q \) modulates the dynamics of this parametrization in between both values, and takes the values \( f(z) = 0 \), \( f(z \to \infty) = 1 \) and \( f(z = z_T) = 1/2 \), i.e. \( 0 \leq f(z) \leq 1 \). This EoS makes a transition between the two regimes: \( w(z = 0) \to w_0 \), \( w(z \gg 0) \to w_i \), at redshift \( z = z_T \), taking a value of \( w(z_T) = (w_0 + w_i)/2 \). The parameter \( q \) modulates the steepness of the transition featured: a larger value for \( q \) has a steeper transition, as figure 1 shows.

For \( q = z_T = 1 \), equation (3) includes the well known CPL parametrization ([14, 15]) as a particular case but it allows for a richer physical behavior. CPL EoS written in terms of scale factor reads:

\[ w(a) = w_0 + w_a (1-a) \]

from where we see that its slope is constant and with a value \( dw(a)/da = -w_a = -(w_i - w_0) \), meaning that the late time dynamics of DE is fixed from the present and initial values of the EoS.

Clearly, taking \( w_0 = w_i = -1 \) in (3), the Cosmological Constant solution, \( w_A = -1 \), is recovered.

Ever since its first detection (Cole et al. 2005 [25], Eisenstein et al 2005, [26] the Baryon Acoustic Oscillation feature has been widely used as a powerful tool for cosmology becoming the standard ruler of choice. It has become the best way to probe late time dynamics of the Universe and in consequence that of DE. For that reason it is the cosmological tool used by several experiments like 6dF [27], WiggleZ [28], SDSS-III [29], (and most recently [30]), SDSS-IV [18] and Dark Energy Survey (DES) [19] and the main probe to be implemented in future experiments like the Dark Energy Spectroscopic Instrument (DESI) [20][22] and Euclid [23].

The corresponding size, \( r_{BAO}(z) \), is obtained by performing a spherical average of the galaxy distribution both along and across the line of sight (Bassett and Hlozek 2010 [31]):

\[ r_{BAO}(z) \equiv \frac{r_s(z_d)}{D_V(z)} \]

The comoving sound horizon at the baryon drag epoch is represented by \( r_s(z_d) \) and the dilation scale, \( D_V(z) \), contains information about the cosmology used in \( H(z) \):

\[ r_s(z_d) \equiv \int_{z_d}^\infty \frac{dz}{H(z) \sqrt{3(H(z) + 1)}} \]

\[ D_V(z) \equiv \left[ \frac{(1+z)^2}{H(z)} A(z)^2 \right]^{1/3} \]

FIG. 1: Evolution of \( w(z) \) from equation (3) with \( q = 1 \) (solid red), \( q = 4 \) (black dot-dashed), \( q = 6 \) (blue dotted), and \( q = 10 \) (green dashed). The other parameters were fixed to \( w_0 = -0.9 \), \( w_i = -0.5 \) and \( z_T = 1 \). The solid red curve takes the special case \( q = z_T = 1 \), representing the CPL parametrization [4].
where
\[ D_A(z) = \frac{1}{1+z} \int_0^z \frac{dz'}{H(z')} \]  
(8)

In this way, the BAO standard ruler which is set by a particular size in the spatial distribution of matter, can be used to constrain the parameters in equation [9].

While the sound horizon, \( r_s(z_d) \), depends upon the physics prior to the recombination era, given by \( z_d \approx 1059 \) [16] and the baryon to photon ratio, \( R(z) \equiv \frac{3\Omega_c(z)}{4\Omega_b(z)} \), the dilation scale, \( D_V(z) \), is sensitive to the physics of much lower redshifts, particularly to those probed by Large Scale Structure experiments.

In this work we make use of the observational points from the six-degree-field galaxy survey (6dFGS [27]), Sloan Digital Sky Survey Data Release 7 (SDSS DR7 [32]) and the reconstructed value (SDSS(R) [34]), as well as the latest result from the complete BOSS sample SDSS DR12 ([33]), and the Lyman-α forest (Lyα-F) measurements from the Baryon Oscillation Spectroscopic Data Release 11 (BOSS DR11 [34], [35]). Table I summarizes them all. Since the volume surveyed by BOSS and WiggleZ partially overlap we do not use data from the latter in this work (see details in [50]).

**B. Local value of the Hubble Constant**

The present value of Hubble constant has been determined observationally from direct measurement of the local dynamics, as in the latest work of A. Riess et al in [37], but also from BAO measurements either from galaxy surveys or from the Lyman-α forest and it can be derived as well from CMB experiments such as Planck.

Regarding the work of A. Riess et al (AR16), their best estimate in units of \( km \cdot s^{-1} \cdot Mpc^{-1} \) reports a value of
\[ H_0 = 73.21 \pm 1.74, \]  
(9)
the accuracy of which was achieved in great deal due to the utilization of maser system in NGC1258 both to calibrate and as an independent anchor for the cosmic distance ladder.

**C. Cosmic Microwave Background**

The Cosmic Microwave Background is the most precise cosmological data set. The angle subtended by the first peak is determined with exquisite precision ([16]):
\[ \theta_\star = 1.04077 \pm 0.00032 \times 10^{-2} \]  
(10)

Following the latest report of the Planck collaboration (P15) [38] and [39] we use Planck TT+TE+EE+lowP which denotes the combination of likelihood at \( l \leq 30 \) using TT, TE, and EE spectra with the low-\( l \) temperature+polarization likelihood.

However, it has been shown ([30], [31], [38]) that the information of CMB power spectra can be compressed within few observables such as the angular scale of sound horizon at last scattering, \( l_A = \pi/\theta_\star \), and the scaled distance to last scattering surface,
\[ R = \sqrt{\Omega_M H_0^2} d_A(z_\star). \]

We keep the flat geometry and the baryon density fixed and thus we can add the CMB information to BAO and \( H_0 \) measurements by means of the observables \( \{\theta_\star, \omega_c \equiv \Omega_c h^2\} \). The corresponding covariance matrix is
\[ C_{CMB} = \begin{pmatrix} \omega_c & \theta_\star \\ \theta_\star^T & 1 \end{pmatrix} \]
\[ \times 10^{-7} \]  
(11)

The angle of horizon at last scattering is defined to be
\[ \theta_\star = \frac{r_s(z_\star)}{d_A(z_\star)} \]  
(12)

where \( r_s(z_\star) \) is the horizon size at the decoupling epoch \( (z_\star \approx 1090.06 \) according to Planck [16], defined by the integral in equation [9] evaluated from \( z_s \) to \( \infty \), and \( d_A(z_\star) \) is the comoving distance to last scattering surface:
\[ d_A(z_\star) = \int_0^{z_\star} \frac{dz'}{H(z')} \]  
(13)

The reported value for the Hubble constant by P15 is \( H_0 = 67.8 \pm 0.9 \) [10], which assumes a ΛCDM universe and is known to be in tension with AR16 at the 3.4σ level.

**III. BACKGROUND EVOLUTION**

Additionally to parameters in equation [3] we also investigate the constraints on the physical density of cold dark matter \( \omega_c \equiv \Omega_c h^2 \) and \( H_0 \) (or equivalently \( h \)), resulting in the set \( \alpha = \{w_0, w_i, z_T, q, \omega_c, h\} \) and consider uniform priors on these: \( h \in [0.5, 1], \omega_c \in [0.001, 0.99], w_0 \in [-1, 0], w_i \in [-1, 0], q \in [1, 10] \) and \( z_T \in [0, 3] \). To determine the best-fitting values (BFV), we minimize the \( \chi^2 \) goodness-of-fit estimator,
\[ \chi^2 = (m - d)^T C^{-1} (m - d) \]  
(14)
where \( m \) are theoretical values for each observable (namely \( r_{BAO}(z), H_0, \omega_c, \theta_\star \)) and \( d \) the data. The
The joint analysis of the different data sets is done by adding their respective \(\chi^2\) functions. Further details can be found in the Appendix A.

The reported value of \(\Omega_m, \rho_{DE} \equiv \Omega_{DE} h^2\) and \(\rho_b \equiv \Omega_b h^2\) in tables summarizing our results was obtained by taking the BFV for the DE and local value of \(H_0\) for each model, as well as the present value \(\Omega_b h^2 = 0.02225\) from P15 \([15]\).

Results from this section are discussed below and summarized in Table I and in figures 2 and 3.

Local measurements (labeled model A in table I) point to a dynamical DE presenting a very late and abrupt transition \((z_T = 0.28, q = 9.97)\) from an initial value \(w_i = -0.99\) to a present value \(w_0 = -0.91\). This behavior is portrayed in figure 2 and corresponds to the black dot-dashed curve. The value for \(H_0\) holds in agreement with the reported measurement from AR16 used as prior for this calculation.

The dynamics for DE resulting from the use of BAO data and CMB reduced likelihood (outcome B, Table I) indicates the preference for a steep transition \((q = 9.8)\) from the initial value \(w_i = -0.77\) to the present value \(w_0 = -0.92\) at a pivotal redshift \(z_T = 0.63\). This corresponds to the dotted line in figure 2. The value for \(\omega_c\) lies within the range imposed by CMB priors and the BFV for \(H_0\) is lower, in agreement with P15 \([15]\).

Model C in table I shows that a late time and smooth transition \((z_T=1.31, q = 1.5)\) was preferred by data, with an initial value \(w_i = 0\) to a present value \(w_0 = -0.96\). The blue dashed line in 2 displays this particular dynamics. The amount of matter is very similar in DE and \(\Lambda\)CDM models (cases C and \(C_\Lambda\)), however we obtain a larger amount of DE \(\rho_{DE} > \rho_\Lambda\) at present time and therefore a larger \(H_0\). We see that the dynamics of DE allows to consistently fit the variables from CMB along with the local value of \(H_0\), since the inclusion of \(H_0\) in model C only increased \(\chi^2\) by 0.2% compared to model B. However, from Table II we see that the addition of \(H_0\) to \(\Lambda\)CDM model \((B_\Lambda\) and \(C_\Lambda\)) severely penalizes the fit by increasing \(\chi^2\) by 19%, showing a tension in the value for \(H_0\) from CMB and local measurements.

In this case, the DE density at early times is not negligible since it has \(w_i = 0\). Figure 3 shows that its contribution at decoupling is of order \(\Omega_{DE} = 10\%\), adding an extra component that behaves like dust \((\propto a^{-3})\) at large redshifts. The ratio of DE density to ordinary matter \((\omega_c + \omega_b)\) is nearly constant from \(z \gtrsim 5\) and has a value \(\rho_{DE}(z_9)/\rho_m(z_9) = 0.16\) (Figure 4b). This changes several cosmological parameters, for instance the equivalence epoch, \(a_{eq} \equiv \rho_r(a_{eq})/\rho_m(a_{eq})\), is smaller modifying the distance to the last scattering surface and the sound horizon at recombination. This is an interesting toy model worthwhile of further studies, and it will also impact CMB power spectrum and Large Scale Structure formation.

Having a non-negligible DE at earlier times, allows to put better constraints on its parameters: \(\{w_i, q, z_T\}\).

A DE component which is non-negligible at early times as been studied in the literature and is known as Early Dark Energy (see for example [12]).

From both, table I and figure 3 we can draw the following general results. The value for \(w_0\) is tightly constrained by observations. The scenario \(w_0 = -1\) is included within 1\(\sigma\) error for all the cases. Generally speaking, for the outcomes where DE density becomes negligible at earlier times, we obtained weak or no constraints for the initial value of the EoS, \(w_i\), the transition time, \(z_T\), and the exponent \(q\). In all outcomes, the values \(q = 1\) and \(z_T = 1\) are contained within 1\(\sigma\) of significance Figure 3 shows that \(w_i\) is highly degenerated with \(w_0\). The results for \(\Lambda\)CDM are summarized in Table II.

| Data set       | Redshift | \(r_{BAO}(z)\)  |
|----------------|----------|-----------------|
| 6dF            | 0.106±0.027 | 0.336±0.015   |
| SDSS DR7       | 0.15±0.032   | 0.2239±0.0084  |
| SDSS(R) DR7    | 0.35±0.013   | 0.1137±0.0021  |
| SDSS-III DR12  | 0.38±0.010   | 0.100±0.0011   |
| SDSS-III DR11  | 2.34±0.033   | 0.0320±0.0013  |

**Table I:** \(r_{BAO}(z)\) measurements used in this work. The ones corresponding to SDSS data were inverted from the published values of \(D_V(z)/s_d\) and those corresponding to Lyα-F data were obtained from the reported quantities \(D_A(z)/s_d\) and \(D_H(z)/s_d\).

**IV. GROWTH OF PERTURBATIONS**

We are interested in studying the effect that the transition featured by the EoS \(\sigma\) has in the evolution of matter overdensities well inside the horizon in the matter-DE domination era. We do so by means of the following system of linearized equations:
TABLE II: BFV and 1σ errors for the free parameters as result from the combined analysis of BAO data (table I) along with the local value of $H_0$ (9) and CMB priors (11). The value for $\Omega_m$ and $\rho_{DE} \equiv \Omega_{DE}h^2$ were derived as explained in the text.

| Alias | Data sets used | $\chi^2$ | $w_0$ | $w_1$ | $q$ | $z_T$ | $\omega_c$ | $H_0$ (km/s/Mpc) | $\Omega_m$ | $\rho_{DE}$ (10^{-5}h^2/M_{pc}^3) |
|-------|----------------|--------|--------|--------|------|-------|-----------|-----------------|----------|-----------------|
| A     | BAO + $H_0$   | 9.59   | -0.92  | -0.99  | -0.67| 9.97  | 0.28      | 0.1568 ± 0.0244 | 73.22 ± 1.2 | 0.334 ± 0.052   |
| B     | BAO+CMB       | 9.77   | -0.92  | -0.77  | -0.27| 9.8   | 0.63 (0.10)| 0.1195 ± 0.0031 | 73.80 ± 0.9 | 0.308 ± 0.008   |
| C     | BAO+CMB+$H_0$ | 7.97   | -0.96  | -0.17  | 1.5  | 1.31  | 1.31 ± 0.44| 0.1195 ± 0.0034 | 73.26 ± 1.0 | 0.264 ± 0.008   |

TABLE III: Similar to Table II but assuming $w = -1$ as equation of state. The reported parameters are $\omega_c$ and $H_0$. The value for $\Omega_m$ and $\rho_A \equiv \Omega_A h^2$ were derived as explained in the text.

| Alias | Data sets used | $\chi^2$ | $\omega_c$ | $H_0$ (km/s/Mpc) | $\Omega_m$ | $\rho_A$ (10^{-5}h^2/M_{pc}^3) |
|-------|----------------|--------|------------|-----------------|----------|-----------------|
| A     | BAO + $H_0$   | 10.05  | 0.1476    | 73.56 (2.0)     | 0.3139 ± 0.026 | 0.3712   |
| B     | BAO + CMB     | 11.74  | 0.1201 (1.0)| 70.20 ± 0.5    | 0.2889 ± 0.004 | 0.3504   |
| C     | BAO + CMB + $H_0$ | 13.98 | 0.1203 (0.001) | 70.99 ± 0.5 | 0.2829 ± 0.004 | 0.3614   |

FIG. 2: Evolution of the EoS $w(z)$ in equation (15) according to the best fit values reported in Table II.

For this part of the analysis we take the values for $m$, $\delta$ and $\Omega$ from Table II and $w$ from equation (15) for an DE EoS modeled by result A (Table II) and $c_{ad}^2 = 0.31$.
of $c^2_{f1f}$ is to reduce the magnitude in the growth of $\delta_m$, keeping the shape the same. It is also notorious that the evolution of $\delta_m$ becomes very non-linear when solved coupled to $\delta_{DE}$, due to the term $\left(\frac{c^2_{f1f}}{\sigma_{DE}(a)} - \frac{3}{2} \Omega_{DE}(a)\right) \delta_{DE}(a)$ in equation (15b).

The transition performed in the model A occurs at $z_T = 0.28$ with an steepness given by $q = 9.97$. The corresponding time of transition, $a_T = 1/(1 + z_T) = 0.78$, is marked by a blue dashed vertical line in figures [6] and [8].

In figure 7 we analyze in more detail the effect of a steep transition. We fix $c^2_{f1f} = 0$ to focus on the effect of $c_{m, eff}^2$ only (displayed in figure 5). For comparison we take the CPL with the same values for $w_0$ and $w_1$ as in Result A and we take the ratio of both solutions. The result is displayed in the lower panel of figure 7. We note the difference between $q = 1$ and $q = 9.8$ at the transition time, $a_T$, as a sudden increase during the transition.

Figure 8 shows the normalized growth function $D_m(a) = \frac{\delta_m(a)}{\delta_m(a_0)}$ to the present value for the same models as in figure 7. The bottom panel shows the ratio to $\Lambda$CDM instead.

It is customary to take $\delta_{DE} = 0$. In such case, the system of equations (15) reduces to:

$$a^2 \delta_m'' + a \frac{3}{2} (1 - w(a)\Omega_{DE}(a)) \delta_m' - \frac{3}{2} \Omega_m(a)\delta_m = 0$$

In figure 9 we show the solution to equation (18) for the case of a $\Lambda$CDM scenario, the best fit corresponding to Result A in table II and its CPL limit [4]. The bottom panel shows the relative difference from each model to $\Lambda$CDM, this is $\delta_m(z)/\delta_m, \Lambda(z)$.
where \(\delta_m,\Lambda(z)\) corresponds to the growth of matter contrast when we assume a Cosmological Constant solution. From \(\delta_m(z)/\delta_{m,\Lambda}(z)\) we find differences from \(\Lambda\text{CDM}\) of around 1\% for a \(k\)-mode of \(k = 0.01\, Mpc^{-1}\).

A deeper analysis on the impact of DE perturbations will be subject of a next paper [43].

\[FIG. 5:\] Adiabatic sound speed \(c_{ad}^2\) as function of scale factor assuming the EoS in equation (3) with \(w_0 = -0.9, w_i = -0.6, z_T = 1\) and different values for the exponent: \(q = 1\) (dotted black curve), \(q = 2\) (dot-dashed red), \(q = 4\) (dashed blue) and \(q = 10\) (solid orange line).

\[FIG. 6: (Upper panel)\] Solution for \(\delta_m\) from the system of equations (15) taking values \(c_{eff} = 1\) (dashed pink line), \(c_{eff} = 1/3\) (violet dotted line), and \(c_{eff} = 0\) (purple dot-dashed line) added to the adiabatic speed of sound, \(c_{ad}^2\) (solid orange line). The dashed vertical line marks the transition time, \(a_T\). (Lower panel) Ratio of solution with \(c_{eff}^2 = 1\) to \(\Lambda\text{CDM}, \Delta\delta_m = \frac{\delta_m(a)}{\delta_{m,\Lambda CD M}}\).

\[FIG. 7: (Upper panel)\] Growth of matter overdensities from the system of equations (15) taking \(c_{eff}^2 = 0\) for the “BAO + \(H_0\)” model and its corresponding CPL limit, i.e., \(q = 1\). (Lower panel) Ratio of “BAO + \(H_0\)” solution to CPL limit, \(\Delta\delta_m = \frac{\delta_m(a)}{\delta_{m,CPL}}\).

\[FIG. 8: (Upper panel)\] Same as in figure 7 but displaying the growth function normalized to the present day. (Lower panel) Ratio of “BAO + \(H_0\)” and CPL solutions to \(\Lambda\text{CDM}\) scenario, \(\Delta D_m = \frac{D_m}{D_{m,\Lambda CD M}}\).

V. SUMMARY AND CONCLUSIONS

We presented a parametrization for the EoS of DE and found the constraints deduced by using BAO measurements contained in Table I combined with the latest local determination of Hubble constant (37). Additionally we used the compressed CMB likelihood from Planck (16), by means of the sound horizon at decoupling, \(\theta_s\), and \(\omega_c h^2\).

The constraints for the free parameters, \(\{w_0, w_i, q, z_T, \omega_c, H_0\}\), and their 68\% errors resulting from the combined analysis of the datasets
were obtained.

Our results show that a dynamical DE is favored by data and that a steep transition is preferred by local measurements, i.e. BAO and $H_0$, and by BAO with CMB Planck observables (figure 2).

Whereas for a LCDM model, the tension between the local determination of $H_0$ ([37]) and the value derived from Planck ([16]) remains (table III), we find that it is possible to simultaneously conciliate the observations from BAO, $H_0$ and CMB in a single model (table II) by means of a dynamical Dark Energy.

For the perturbative analysis it was shown that the feature from the shape of $c_{ad}$ due to the particular form of the EoS got imprinted in the evolution of $\delta_m$. We modeled the speed of sound splitting it into an adiabatic contribution and an effective term, $c_s^2 = c_{ad}^2 + c_{eff}^2$, where $c_{eff}$ encapsulates the physics beyond the EoS of the dark fluid. The addition of this effective term did not erase the features from the bump in the adiabatic speed of sound but suppresses the exponential evolution of the over-densities by several orders of magnitude. This should be studied in detail and will be subject to discussion in a future paper [39].

The solution $\delta_{DE} = 0$ is only exact for the case of a cosmological constant. To be consistent we need to take DE perturbations into account. The solution to [15] with $c_s^2$ taken as the adiabatic contribution [17] for the model “BAO+$H_0$” reported in table II was shown in figure 8. From this we saw that if we take $c_s^2 = c_{ad}$, the solutions were highly unstable and became non-linear extremely fast during the evolution of over-densities.

To summarize, the study of dynamics of Dark Energy is a matter of profound implications for our understanding of the Universe and its physical laws. Although the measurements from CMB are the most precise data sets in Cosmology, the best way to analyze the properties of DE comes from the low redshift regime, where the BAO feature is the most robust cosmic ruler. In this work we have contributed towards that direction, and we have presented the constraints for a dynamical DE model coming from the analysis of BAO distance measurements combined with the most recent $H_0$ determination and CMB information.

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A. Appendix

For the BAO measurements we use the $\chi^2$ function defined as

$$\chi^2_{BAO} = y_{BAO}^T C_{BAO}^{-1} y_{BAO},$$

(A1)

where $y_{BAO} \equiv r_{BAO}^{Th}(\alpha|z_i) - r_{BAO}^{obs}(z_i)$ is the difference between theoretical prediction for $r_{BAO}(z)$ according to (5) and the values listed in table II and $C_{BAO}$ is the inverse of the covariance matrix containing the observational errors for the measurements. Since the data points used in this work are not correlated we have a diagonal matrix whose elements are the square-root of the errors reported in table II.

Additionally to BAO data we make use of the local determination of $H_0$ by means of

$$\chi^2_{H_0} = \frac{(H(\alpha|z = 0) - H_{0}^{obs})^2}{\sigma_{H_0}^2},$$

(A2)

where $H(\alpha|z = 0)$ is the Hubble function [1] evaluated in $z = 0$ taking the model described by $\alpha$ and $H_{0}^{obs} = 73.24 km \cdot s^{-1} Mpc^{-1}$ and $\sigma_{H_0} = 1.74 km \cdot s^{-1} Mpc^{-1}$, according to the primary fit obtained by A. Riess et al. in [37].

Finally, for the CMB, we use the determination of $\theta_c$ and $\omega_c$, made by the Planck Collaboration (P. A. R. Ade et al. 2015, [16]). In particular we use $Planck \ TT + \ TE + \ EE + \ low \ P$ values from...
the table 4 of [16] with the covariance matrix displayed in [11]. With this matrix we can build the $\chi^2_{CMB}$ function:

$$\chi^2_{CMB} = y_{CMB}^T C_{CMB}^{-1} y_{CMB}$$

(A3)

where $y_{CMB}$ is the corresponding data vector defined as $y_{CMB} = [\omega_c^{Planck} - \omega_c^{Planck}, \theta_s(\alpha) - \theta_s^{Planck}]^T$, and $C_{CMB}^{-1}$ is the inverse of matrix [11].

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