Higgs sector extension of the neutrino minimal standard model with thermal freeze-in production mechanism

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Abstract

The neutrino minimal Standard Model ($\nu$MSM) is the minimum extension of the standard model. In this model, the Dodelson-Widrow mechanism (DW) produces the keV sterile neutrino dark matter, the degenerate GeV heavy Majorana neutrinos leads to the leptogenesis. However, the DW has been excluded from Lyman-\(\alpha\) bounds and X-ray constraints. A scenario, where the sterile neutrino DM is generated by thermal freeze-in mechanism via a singlet scalar has been proposed and it is possible to evade these bounds. In this paper, we consider the Higgs sector extension of the $\nu$MSM to improve dark matter sectors and leptogenesis scenarios with focusing on the thermal freeze-in production mechanism. We discuss various thermal freeze-in production scenarios of the keV-MeV sterile neutrino DM with a singlet scalar, and reinvestigate the Lyman-\(\alpha\) bounds and the X-rays constraints on the parameter regions. Furthermore, we propose thermal freeze-in leptogenesis scenarios in the extended $\nu$MSM. The singlet scalar needs to be TeV scale in order to generate the observed DM relic density or baryon number density with the thermal freeze-in production mechanism.
I. INTRODUCTION

The standard model of particle physics (SM) has demonstrated the great success in high energy physics. However, it is not the complete theory because the Standard Model cannot treat gravity consistently nor explain several observed phenomena, such as neutrino oscillations, cosmological dark matter, baryon asymmetry of the universe, horizon and flatness problems, etc. In the dark matter sector, TeV scale supersymmetry provides weakly interacting massive particles (WIMP) as the leading dark matter candidates [1]. However, the first run of the LHC experiment excludes significant parameter space of the weak super-partners, the recent results from LUX [2, 3] and XENON100 [4–6] have severely restricted the WIMP cross section. This situation is the same in other beyond standard models. Feebly Interacting Massive Particle (FIMP) [7–10] is not constrained by such direct detection experiments due to the much small couplings. Therefore, the FIMP is an interesting candidate of dark matter.

The sterile neutrino can be a FIMP. In particular, the keV sterile neutrino can be candidates of a warm dark matter. The neutrino minimal Standard Model ($\nu$MSM) [11–13] has the three right-handed neutrinos below electroweak-scale and explains the keV sterile neutrino, other experimental facts. The lightest right-handed neutrino $N_1$ becomes the keV sterile neutrino dark matter which is produced from active-sterile neutrino oscillation called Dodelson-Widrow mechanism [14], while the other heavy right-handed neutrinos $N_2$ and $N_3$ lead to leptogenesis via the CP-violating oscillations [15, 16] where the heavy Majorana neutrinos should satisfy $1 \text{ GeV} \leq m_{N_{2,3}} \leq 20 \text{ GeV}$. Furthermore, it is possible to generate Higgs inflation [17, 18] from non-minimal coupling of Higgs and gravity. Thus, the $\nu$MSM can explain the large number of phenomena with the minimum number of parameters.

However, it is restricted severely by the recent observations, especially, there are severe constraints on the sterile neutrino DM. The Dodelson-Widrow mechanism is known to be excluded by the Lyman-$\alpha$ bounds and the X-ray constraints [19]. To evade these bounds, the sterile neutrino DM production by thermal freeze-in mechanism has been considered in Ref.[20–27] 1. Ref.[20] consider the scenario that the inflaton decays into the sterile neutrino DM. Ref.[21, 22] show that a GeV scale scalar singlet produces the sterile neutrino DM.

1 Ref.[27] gives a comprehensive study of the keV sterile neutrino DM via the singlet scalars, but our purpose is the estimate of scale in the extended $\nu$MSM to improve the dark matter sectors and the leptogenesis scenarios rather than such a generic study of the sterile neutrino DM.
Ref.\cite{23, 24} consider the non-thermal decay production via a singlet scalar. Furthermore, the singlet scalar can improve the electroweak vacuum stability or the Higgs inflation as well as the dark matter sectors.

In this paper, we do not mention the theoretical merits of the singlet scalar in the $\nu$MSM such as the electroweak vacuum stability \cite{28, 29}, the Higgs inflation \cite{30} and the scale invariance \cite{31}, although we are motivated by these theoretical aspects, we concentrate on estimating the scale of the singlet scalar to improve the dark matter sectors and the leptogenesis scenarios. We discuss various thermal freeze-in production scenarios of the keV-MeV sterile neutrino in the extended $\nu$MSM with a singlet scalar. Especially, we revisit the Lyman-$\alpha$ bounds and the X-rays constraints and show that the singlet scalar cannot be heavier than TeV scale. We also show thermal freeze-in leptogenesis scenarios, which is able to produce the lepton asymmetry more than the thermal leptogenesis due to the contribution of the singlet scalar. In the leptogenesis scenarios, the singlet scalar need to be lighter than 1 TeV in order to generate the observed baryon asymmetry.

This paper is organized as follows. In section II, we review the scalar singlet extension of the $\nu$MSM. In section III, we consider the two thermal freeze-in scenarios utilizing a thermal or a non-thermal singlet scalar. In section IV, we review the X-rays constraints and the lifetime bounds of the sterile neutrino dark matter. In section V, we investigate the free streaming horizon and the Lyman-$\alpha$ constraints in our scenarios. In section VI, we discuss the thermal freeze-in leptogenesis with the singlet scalar. Section VII is devoted for discussion and conclusions.

II. THE SCALAR SINGLET EXTENSIONS OF THE $\nu$MSM

In this section, we review the extended $\nu$MSM which contains three right-handed sterile neutrinos $N_a$ ($a = 1, 2, 3$) and one real singlet scalar $S$ \footnote{If you think a complex singlet scalar instead of a real singlet scalar, breaking a global lepton number $U(1)_L$ which is unavoidable to construct Majorana mass $M_a$ cause the light Nambu-Goldstone bosons. The presence of such light bosons would make the sterile neutrinos unstable, and they could decay into other particles. It is not desirable to construct sterile neutrino dark matter model.}. In this model, the vacuum expectation value (VEV) of $S$ generates Majorana mass $M_a$ of the right-handed neutrino $N_a$. The Lagrangian is given as follows,

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2} (\partial_\mu S)(\partial^\mu S) + i \overline{N}_a \gamma^\mu N_a - y_{a\alpha} H^\dagger T_{a\alpha} N_{Ra} - \frac{\kappa_a}{2} S N^\dagger_a N_a - V(H, S) + h.c., \quad (1)$$
where \( \ell_\alpha \) are the lepton doublets, \( H \) is the Higgs doublet, \( y_{\alpha a} \) and \( \kappa_a \) are the Yukawa couplings, \( V(H,S) \) is the Higgs potential. After the spontaneous symmetry breaking, the Higgs doublet and the scalar singlet fields develop the VEVs, \( \langle H \rangle = \frac{1}{\sqrt{2}} v \), where \( v = 247 \text{ GeV} \), \( \langle S \rangle = f \), \( S = s + \langle S \rangle \), the right-handed neutrinos acquire Majorana masses \( M_a = \kappa_a \langle S \rangle \).

Without loss of generality, we can choose the mass basis where the Majorana mass term is diagonal. The Lagrangian is written as the following,

\[
\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2} (\partial_\mu s) (\partial^\mu s) + i N_R \not{D} N_{Ri} - y_{\alpha i} H^\dagger \ell_\alpha N_{Ri} - \frac{M_i}{2} \bar{N}_{Ri} N_{Ri} - V(H,S) + h.c.. \tag{2}
\]

If the Dirac masses are much smaller than the Majorana masses, the left-handed neutrino masses is expressed as follows due to the type I seesaw mechanism,

\[
m_{\nu} \simeq m_D M^{-1} m_D \simeq \frac{(y \langle H \rangle)^2}{\kappa \langle S \rangle}, \tag{3}
\]

\[
m_N \simeq M \simeq \kappa \langle S \rangle, \quad \theta \simeq \frac{m_D}{M}. \tag{4}
\]

For the scalar potential \( V(H,S) \), we impose the softly broken discrete symmetry \( \mathbb{Z}_2 \) where the scalar singlet is \( \mathbb{Z}_2 \)-odd \( (S \rightarrow -S) \) and all the other fields are \( \mathbb{Z}_2 \)-even to construct the scalar potential with even powers,

\[
V(H,S) = -\mu_H^2 H^\dagger H - \frac{1}{2} \mu_S^2 S^2 + \lambda_H (H^\dagger H)^2 + \frac{1}{4} \lambda_S S^4 + 2\lambda (H^\dagger H) S^2 + \omega S, \tag{5}
\]

where \( \omega S \) is soft \( \mathbb{Z}_2 \) breaking term. The spontaneous breaking of the discrete symmetries \( \mathbb{Z}_N \) could produce domain walls [32]. The soft \( \mathbb{Z}_2 \) breaking term \( \omega S \) make the vacua of singlet scalar degenerate so that the domain wall problem is evaded [32–34]. The minimum of the scalar potential is given by following equations,

\[
\begin{align*}
\mu_H^2 &= \lambda_H v^2 + 2\lambda f^2, \\
\mu_S^2 &= \lambda_S f^2 + 2\lambda v^2 + \frac{\omega f}{f}.
\end{align*} \tag{6}
\]

The mass eigenstates of Higgs and singlet scalar are \( h \) and \( s \) where \( h \) is nearly the SM higgs boson. The physical masses of \( h \) and \( s \) are given by

\[
m_h^2 = \lambda_H v^2 - \frac{(2\lambda f v)^2}{\lambda_S f^2 - \lambda_H v^2}, \tag{7}
\]

\[
m_s^2 = \lambda_S f^2 + \frac{(2\lambda f v)^2}{\lambda_S f^2 - \lambda_H v^2}. \tag{8}
\]
The Higgs portal coupling $\lambda$ induces the doublet-singlet mixing. In this paper, we consider the TeV scale singlet scalar which decays into the sterile neutrino DM and the heavy Majorana neutrinos. Therefore, there is essentially no constraint on coupling $\lambda$. The size of coupling $\lambda$ still affects the thermal history of $s$. The references [22, 35] show that $s$ is out of the thermal equilibrium if the Higgs portal coupling satisfies $\lambda \ll 10^{-6}$. In this paper, we assume that the singlet scalar is out of thermal equilibrium for $\lambda \ll 10^{-6}$ and concentrate on the thermal freeze-in production mechanism $^3$.

III. THE STERILE NEUTRINO DARK MATTER BY THE THERMAL FREEZE-IN PRODUCTION MECHANISM

In this section, we consider various production scenarios of the sterile neutrino DM in the extended $\nu$MSM with the singlet scalar. The sterile neutrino could be produced by thermal freeze-out, thermal freeze-in and non-thermal decay. Those scenarios also depend on whether the singlet scalar is generated by freeze-out or freeze-in. In addition, the Dodelson-Widrow mechanism via active-sterile oscillations can produce the sterile neutrino.

It is possible to restrict these scenarios from the mass relation of the seesaw mechanism. For simplicity, we assume the lightest right-handed neutrino $N_1$ is the sterile neutrino DM, and the mass is about 10 keV. We will see later the sterile neutrino with mass above 1 MeV is not favored by X-rays constraints and the lifetime bounds. The Yukawa coupling of the singlet scalar and the right-handed neutrino are $\kappa_1 \approx 10^{-8}$ and the vacuum expectation value of the singlet scalar is $\langle S \rangle \approx 1$ TeV, then the following relations are derived from the seesaw mechanism.

$$m_\nu \simeq m_D M^{-1} m_D \simeq \left( \frac{y \langle H \rangle}{\kappa_1 \langle S \rangle} \right)^2 \simeq y^2 \left( 10^{18} \text{ eV} \right),$$

$$m_{N_1} \simeq M_1 \simeq \kappa_1 \langle S \rangle \simeq 10 \text{ keV}.$$  \hspace{1cm} (9)

The Yukawa couplings $y$, $\kappa_1$ are very small $y \approx 10^{-10}$ and $\kappa_1 \approx 10^{-8}$. If the reheating temperature $T_{RE}$ satisfies $T_{RE} \lesssim 10^{16} \text{ GeV}$, the sterile neutrino DM does not come into the thermal equilibrium for $\kappa_1 \ll 10^{-6}$ [36, 37]. Therefore, we may regard the sterile neutrino DM as non-thermal particles in the early universe. In such case, we find only two realistic dark matter scenarios to realize the keV-MeV sterile neutrino described in the following.

$^3$ If singlet scalar is not directly produced from inflatons and the reheating temperature is low enough, the singlet scalar can be out of thermal equilibrium even if $\lambda > 10^{-6}$.
FIG. 1. We describe the evolution of the relic density $Y_{N_1}$ and $Y_s$ as the temperature $T$ decreases.

The three freeze-in production process are shown on FIG.1(a), 1(b) and 1(c). The sterile neutrino is generated by the thermal freeze-in mechanism from the Higgs boson (FIG.1(a)), by the thermal freeze-in production from singlet scalar (FIG.1(b)), by thermal freeze-in production and the non-thermal singlet scalar decays into the sterile neutrino (FIG.1(c)).

**A. The singlet scalar enter into thermal equilibrium**

If the Higgs portal coupling is relatively large $\lambda > 10^{-6}$, the singlet scalar $s$ enter into thermal equilibrium and the sterile neutrino DM can be produced by the thermal freeze-in mechanism of $s$. In addition, $h$ couples to $N_1\nu$ with suppressed coupling, and after the EW symmetry breaking, there are small mixing between $\nu_L$ and $s$. To check the effect we consider $h \rightarrow \nu_e N_1$ contribution as well. The production from the singlet scalar has been considered in Ref.[21, 22]. The thermal freeze-in production is caused by the Yukawa interaction of $s$ and $N_1$ or $h$ and $N_1$. In our assumption $m_s \gg m_h$, as the universe is expanding, the temperature becomes low, $s$ disappears first. The Higgs boson $h$ however, is still in thermal equilibrium and the freeze-in production by $h$ is effective until $T \sim m_h$. The dark matter relic density can be calculated by solving the Boltzmann equations. In this scenario, the relevant Boltzmann equations for $Y_{N_1} = n_{N_1}/s$ are given as follows,

$$\frac{dY_{N_1}}{dT} = \frac{dY_{Ds_{N1}}}{dT} + \frac{dY_{Dh_{N1}}}{dT}, \quad (11)$$

where $Y_{Ds_{N1}}(Y_{Dh_{N1}})$ is $Y_{N_1}$ from $s$ ($h$) decays respectively.

In FIG.1(a), FIG.1(b) and FIG.1(c), we show numerically the evolution of the sterile
neutrino relic density $Y_{N_1}$ and the singlet scalar $Y_s$ for various thermal freeze-in mechanisms, the sterile neutrino DM are generated by the thermal freeze-in production from $h$ in FIG.1(a), the thermal freeze-in production from $s$ in FIG.1(b), the non-thermal decay production from $s$ in FIG.1(c).

Now, $Y_{N_1}$ is obtained from the following calculation. The Boltzmann equation for the sterile neutrino number density $n_{N_1}$ involving $s$ is written as the following,

$$\frac{d}{dt}n_{N_1} + 3Hn_{N_1} = \sum_{\text{spin}} \int d\pi_s d\pi_{N_1} d\pi_{N_1'} (2\pi)^4 \delta(4) (p_s - p_{N_1} - p_{N_1'}) \left\{ |M|_{s \rightarrow N_1 N_1'}^2 f_s (1 - f_{N_1}) (1 - f_{N_1'}) - |M|_{N_1 N_1' \rightarrow s}^2 f_{N_1} f_{N_1'} (1 - f_s) \right\} . \quad (12)$$

We assume that the initial abundance of the sterile neutrino can be neglected and the singlet scalar enters into thermal equilibrium,

$$\frac{d}{dt}n_{N_1} + 3Hn_{N_1} = \sum_{\text{spin}} \int d\pi_s d\pi_{N_1} d\pi_{N_1'} (2\pi)^4 \delta(4) (p_s - p_{N_1} - p_{N_1'}) |M|_{s \rightarrow N_1 N_1'}^2 f_s \quad (13)$$

The sterile neutrino abundance $Y_{N_1} = \frac{n_{N_1}}{s}$ can be obtained from the entropy density $s = \frac{2\pi^2}{45} h_{\text{eff}} T^3$ and the abundance of $N_1$ is written as the following,

$$\frac{dY_{N_1}^{Ds}}{dT} = -\sqrt{\frac{45}{\pi^3 G_N}} \frac{1}{\sqrt{g_{\text{eff}}}} \frac{K_1(m_s/T)}{T^3 K_2(m_s/T)} \Gamma (s \rightarrow N_1 N_1) Y_s^{eq}$$

$$= -\frac{135\sqrt{5}}{4\pi^{11/2}} \frac{m_{\text{pl}}}{h_{\text{eff}} \sqrt{g_{\text{eff}}}} \frac{m_s^2 K_1(m_s/T)}{T^5} \Gamma (s \rightarrow N_1 N_1). \quad (14)$$

Similarly, $h$ also produces $N_1$ as the following

$$\frac{dY_{N_1}^{Dh}}{dT} = -\sqrt{\frac{45}{4\pi^3 G_N}} \frac{1}{\sqrt{g_{\text{eff}}}} \frac{1}{T^3 K_2(m_h/T)} \Gamma (h \rightarrow N_1 \nu_e) Y_h^{eq}$$

$$= -\frac{135\sqrt{5}}{8\pi^{11/2}} \frac{m_{\text{pl}}}{h_{\text{eff}} \sqrt{g_{\text{eff}}}} \frac{m_h^2 K_1(m_h/T)}{T^5} \Gamma (h \rightarrow N_1 \nu_e), \quad (15)$$

where $m_{\text{pl}} = 1.22 \times 10^{19}$ GeV is the planck mass, $g_{\text{eff}}$ and $h_{\text{eff}}$ are the effective degrees of freedom for energy and entropy, $K_n(x)$ is the modified Bessel functions of the second kind, while the equilibrium abundances $Y_s^{eq}, Y_h^{eq}$ are expressed as

$$Y_s^{eq} = \frac{45 g_s m_s^2 K_2(m_s/T)}{4\pi^4 T^2 h_{\text{eff}}}, \quad (16)$$

$$Y_h^{eq} = \frac{45 g_h m_h^2 K_2(m_h/T)}{4\pi^4 T^2 h_{\text{eff}}}. \quad (17)$$
The partial decay width of $s$ into $N_1 \nu_e$ is obtained by

$$\Gamma (s \to N_1 N_1) = \frac{\kappa^2 m_s}{16 \pi} \left( 1 - \frac{4 M^2_{N_1}}{m^2_s} \right), \quad (18)$$

and the partial decay width of $h$ into $N_1 \nu_e$ is given as

$$\Gamma (h \to N_1 \nu_e) = \frac{y^2 e m_h}{16 \pi} \left( 1 - \frac{M^2_{N_1}}{m^2_h} \right). \quad (19)$$

For the estimates of the relic density, we analytically integrate the relevant Boltzmann equations. The relic density at the today temperature $Y_{N_1} (T_0)$ is given for $s \to N_1 N_1$,

$$Y_{N_1}^{D_s} (T_0) = - \frac{135 \sqrt{5}}{4 \pi^{11/2} h_{\text{eff}}} \left( \frac{m_{N_1}}{m_s} \right) \int_{T_{\text{RE}}}^{T_0} \frac{m^2_h K_1 (m_s/T)}{T^5} \Gamma (s \to N_1 N_1) \, dT \approx - \frac{135 \sqrt{5}}{4 \pi^{11/2} h_{\text{eff}}} \left( \frac{m_{N_1}}{m_s} \right) \int_{0}^{\infty} \frac{m^2_h K_1 (m_s/T)}{T^5} \Gamma (s \to N_1 N_1) \, dT \approx 1.58 \times 10^{14} \left( \frac{m^2_{N_1}}{m_s} \right) \left( \frac{1}{\langle S \rangle} \right)^2, \quad (20)$$

where we assume the effective degrees of freedom as $h_{\text{eff}} \approx g_{\text{eff}} \approx 100$. The sterile neutrino DM relic density in the thermal freeze-in mechanism can be obtained as the following,

$$\Omega_{N_1}^{D_s} h^2 = 2.733 \times 10^8 \cdot Y_0 \cdot \left( \frac{m_{DM}}{\text{GeV}} \right)^3$$

$$= 4.32 \times 10^{-7} \left( \frac{m_{N_1}}{\text{keV}} \right)^3 \left( \frac{\text{TeV}}{m_s} \right) \left( \frac{\text{TeV}}{\langle S \rangle} \right)^2. \quad (21)$$

The observed DM relic density for Planck+WP [38] is estimated as

$$\Omega_{\text{DM}} h^2 = 0.1199 \pm 0.0027. \quad (22)$$

The mass of the sterile neutrino to explain the observed DM relic density is $m_{N_1} \approx 10 \text{ keV}$ for $\langle S \rangle \approx m_s \approx 1 \text{ TeV}$ and $m_{N_1} \approx 1 \text{ MeV}$ for $\langle S \rangle \approx m_s \approx 100 \text{ TeV}$. FIG.2 shows the relic density of the sterile neutrino as the function of $m_s$ for different values of $m_{N_1}$.

Similarly, we integrate the Boltzmann equations of thermal freeze-in production via $h$.

$$Y_{N_1}^{D_h} (T_0) = - \frac{135 \sqrt{5}}{8 \pi^{11/2} h_{\text{eff}}} \left( \frac{m_{N_1}}{m_s} \right) \int_{T_{\text{RE}}}^{T_0} \frac{m^2_h K_1 (m_h/T)}{T^5} \Gamma (h \to N_1 \nu_e) \, dT \approx - \frac{135 \sqrt{5}}{8 \pi^{11/2} h_{\text{eff}}} \left( \frac{m_{N_1}}{m_s} \right) \int_{0}^{\infty} \frac{m^2_h K_1 (m_h/T)}{T^5} \Gamma (h \to N_1 \nu_e) \, dT \approx 1.04 \times 10^7 \cdot \sin^2 \theta \cdot m^2_{N_1}, \quad (23)$$

and the relic density of $N_1$ is given by

$$\Omega_{N_1}^{D_h} h^2 = 2.84 \times 10^{-3} \sin^2 \theta \left( \frac{m_{N_1}}{\text{keV}} \right)^3. \quad (24)$$
Finally, the sterile neutrino DM is produced from the thermal background of active neutrinos via coherent scattering (Dodelson-Widrow mechanism). The dark matter density can be expressed as the following formula [39]

$$\Omega_{N_1}^{DW}h^2 = 5.47 \times 10^7 \sin^2 \theta \left( \frac{m_{N_1}}{\text{keV}} \right)^{1.63}.$$  \hfill (25)

In the keV-MeV sterile neutrino DM, this contribution in Eq.(25) is larger than the thermal freeze-in production of $h$ in Eq.(24). Although, there are additional contributions from $Z$-bosons or $W$-bosons decay due to the neutrino mixing of the same order of Eq.(24), we can safely ignore them in the mass region. Altogether, the total sterile neutrino DM relic density is given by

$$\Omega_{N_1}h^2 = 4.32 \times 10^{-5} \left( \frac{m_{N_1}}{\text{keV}} \right)^3 \left( \frac{\text{TeV}}{m_s} \right) \left( \frac{\text{TeV}}{\langle S \rangle} \right)^2$$

$$+ 5.47 \times 10^7 \sin^2 \theta \left( \frac{m_{N_1}}{\text{keV}} \right)^{1.63}.$$  \hfill (26)

When the mass and the VEV of singlet scalar are near 1 TeV order, the thermal freeze-in mechanism of the singlet scalar dominate the keV-MeV sterile neutrino production.
FIG. 3. The relic density of sterile neutrino as the function of $m_s$ for different values of Higgs portal coupling $\lambda$ or mass $m_{N_1}$ of sterile neutrino.

B. The singlet scalar is out of thermal equilibrium

In this case $\lambda \ll 10^{-6}$, both $s$ and $N_1$ never entered into thermal equilibrium in the early universe. The sterile neutrino DM is generated from thermal freeze-in production of $h$. The singlet scalar $s$ is also be generated by the thermal freeze-in production and decay efficiently into $N_1$. In this section, we assume $m_s \ll m_{N_{2,3}}$, so that $s$ cannot decay into $N_2$ and $N_3$.

To calculate the relic density of the sterile neutrino, we have to solve the Boltzmann equations given by the following two interrelated equations,

$$\frac{dY_s}{dT} = \frac{dY_s^A}{dT} + \frac{dY_s^{Ds}}{dT}, \quad (27)$$

$$\frac{dY_{N_1}}{dT} = \frac{dY_{s}^{Ds}}{dT} + \frac{dY_{N_1}^{Ds}}{dT} + \frac{dY_{s}^{Ds}}{dT} = -\frac{1}{2} \frac{dY_{N_1}^{Ds}}{dT}. \quad (28)$$

The standard model particles in thermal equilibrium can annihilate into singlet scalars. For simplicity, we concentrate on the thermal Higgs annihilation as the dominant production mechanism of singlet scalar and ignore the other standard model effects. The Boltzmann equations of the annihilation process can be expressed as follows,

$$\frac{dY_s^A}{dT} = -\frac{135\sqrt{5}}{64\pi^{17/2}} \frac{m_{\text{pl}}}{h_{\text{eff}} \sqrt{g_{\text{eff}}}} \frac{\lambda^2 m_s}{T^3} \frac{K_1(2m_s/T)}{T}. \quad (29)$$
We integrate Eq.(27) to estimate the abundance of the sterile neutrino \( Y_{N_1} \). There are no initial abundance \( (Y_s(T_{RE}) = 0) \) and no final abundance \( (Y_s(T_0) = 0) \), and therefore, the following equation can be obtained,

\[
\int_{T_{RE}}^{T_0} \frac{dY_s}{dT} dT = -\int_{T_{RE}}^{T_0} \frac{dY^A}{dT} dT.
\]

(30)

The relic density of sterile neutrino at the today’s temperature can be obtained by Eq.(29) and Eq.(30),

\[
Y^D_{N_1}(T_0) = -2 \int_{T_{RE}}^{T_0} \frac{dY^D_s}{dT} dT = 2 \int_{T_{RE}}^{T_0} \frac{dY^A_s}{dT} dT = -\frac{135\sqrt{5}}{32\pi^{17/2}} \frac{m_{pl}}{h_{eff}\sqrt{g_{eff}}} \int_{T_{RE}}^{T_0} \lambda^2 m_s T^3 K_1(2m_s/T) dT.
\]

(31)

Therefore, the abundance at the today temperature \( Y_{N_1}(T_0) \) is given as the following,

\[
Y^D_{N_1}(T_0) \approx -\frac{135\sqrt{5}}{32\pi^{17/2}} \frac{m_{pl}}{h_{eff}\sqrt{g_{eff}}} \int_{T_{RE}}^{T_0} \lambda^2 m_s K_1(2m_s/T) dT \\
\approx 1.07 \times 10^{13} \left( \frac{\lambda^2}{m_s} \right).
\]

(32)

The sterile neutrino DM relic density in the non-thermal decay mechanism is obtained as the following,

\[
\Omega^D_{N_1} h^2 = 2.93 \times 10^{-2} \left( \frac{m_{N_1}}{\text{keV}} \right) \left( \frac{\lambda}{10^{-7}} \right)^2 \left( \frac{\text{TeV}}{m_s} \right).
\]

(33)

FIG.3 describes the relic density of sterile neutrino as the function of \( m_s \) for different values of Higgs portal coupling \( \lambda \) or mass \( m_{N_1} \) of sterile neutrino.

In this scenario, the Dodelson-Widrow mechanism can also produce the sterile neutrino DM. Therefore, the total relic density is obtained as the following

\[
\Omega_{N_1} h^2 = 2.93 \times 10^{-2} \left( \frac{m_{N_1}}{\text{keV}} \right) \left( \frac{\lambda}{10^{-7}} \right)^2 \left( \frac{\text{TeV}}{m_s} \right) + 5.47 \times 10^7 \sin^2 \theta \left( \frac{m_{N_1}}{\text{keV}} \right)^{1.63}.
\]

(34)

The relic density formula depends on the Higgs portal coupling \( \lambda \). The coupling \( \lambda \) is bounded as \( \lambda < 10^{-6} \) so that \( s \) does not come into thermal equilibrium. Therefore, \( s \) can not be much larger than TeV scale in order to produce the keV-MeV sterile neutrino dark matter by non-thermal decay production mechanism.
IV. THE X-RAYS CONSTRAINTS AND THE LIFETIME BOUNDS OF STERILE NEUTRINO DARK MATTER

In this section, we will review the X-rays bounds and the lifetime bounds of sterile neutrino DM. At first, the sterile neutrino can decay into standard model particles through active-sterile neutrino mixings. In the keV-MeV mass range, the sterile neutrino decays mainly into three active neutrinos \[40–42\]. For the three-neutrino decay channel, the decay life time is expressed as

\[
\tau_{3\nu} \simeq 2.88 \times 10^{19} \text{ sec} \left( \frac{\text{keV}}{m_{N_1}} \right)^5 \frac{1}{\sin^2 \theta}.
\] (35)

Their lifetime must be longer than age of our universe age \(10^{17} \text{ sec}\) in order to be a dark matter, which constrains the mixing angles and the sterile neutrino mass as follows,

\[
\sin^2 2\theta < 2.88 \times 10^2 \left( \frac{m_{N_1}}{\text{keV}} \right)^{-5}.
\] (36)

When the sterile neutrino constitutes the dark matter, their radiative decay \(N_1 \rightarrow \gamma \nu\) would lead to the cosmic X-ray background. We have not seen such X-rays excess except for the recent observation of the 3.5 keV signal \[43, 44\] in galactic clusters. This puts an upper limit to neutrino mixing angles for a given sterile neutrino mass. From the diffuse X-ray background observations XMM-Newton \[45, 46\] and HEAO-1 \[47\], the authors of Ref.\[48\] obtain the simple empirical formula,

\[
\sin^2 2\theta < 1.15 \times 10^{-4} \left( \frac{m_{N_1}}{\text{keV}} \right)^{-5} \left( \frac{\Omega_{N_1}}{0.26} \right).
\] (37)

The XMM-Newton observations of the Virgo and Coma galaxy clusters present the more stringent constraints \[49\],

\[
\sin^2 2\theta < 8 \times 10^{-5} \left( \frac{m_{N_1}}{\text{keV}} \right)^{-5.43} \left( \frac{\Omega_{N_1}}{0.26} \right).
\] (38)

The more precise X-rays constraints have been reported in Ref.\[19\]. Note that these bounds are given for the sterile neutrino DM which explains the current dark matter density. If the sterile neutrino DM is a partial dark matter, the X-rays bounds become weaker.

V. THE FREE STREAMING HORIZON AND THE LYMAN-\(\alpha\) CONSTRAINTS

Recent observations such as WMAP and Planck mission have proven that the ΛCDM, which contains cold dark matter is an extremely successful cosmological model \[38\]. However
the ΛCDM can not solve the small scale crisis including the missing satellite problem and the cuspy halo problem. The warm dark matter (WDM), which has adequate free streaming horizon and suppresses the structure of dwarf galaxies size may solve the problem. The upper bound of the free streaming scale of WDM is obtained from the observed Lyman-α forest which is the Lyman-α absorption lines by intergalactic neutral hydrogen in the spectra of distant quasars and galaxies.

The free streaming horizon is a travel distance of the DM particle and a good measure to classify CDM, WDM and HDM. The free streaming horizon is given as the following,

$$\lambda_{FS} = \int_{t_{in}}^{t_0} \frac{\langle v(t) \rangle}{a(t)} dt. \quad (39)$$

Where $t_{in}$ is the DM production time, $t_0$ is the current time, $\langle v(t) \rangle$ is the thermal average velocity of the DM particles, and $a(t)$ is the scale factor. In this paper, we assume that the free streaming scale of CDM, WDM and HDM satisfy $\lambda_{FS} < 0.01 \text{ Mpc}$, $0.01 \text{ Mpc} < \lambda_{FS} < 0.1 \text{ Mpc}$, $0.1 \text{ Mpc} < \lambda_{FS}$. It is not an accurate definition, but they give an useful criteria of the thermal property. Note that HDM is excluded by the observations of Lyman-α forest.

We now consider the thermal average velocity $\langle v(t) \rangle$ of the sterile neutrino DM $N_1$ in order to decide the free streaming horizon. We define $t_{nr}$ which is the time when $N_1$ becomes non-relativistic by the equality $\langle p(t_{nr}) \rangle = m_{N_1}$ and the approximate thermal average velocity $\langle v(t) \rangle$ is given as the following,

$$\langle v(t) \rangle \simeq \begin{cases} 1 & t < t_{nr}, \\ \frac{\langle p(t) \rangle}{m_{N_1}} & t \geq t_{nr}. \end{cases} \quad (40)$$

The non-relativistic thermal velocity is expressed by thermal average momentum, which can be extracted from the distribution function $f(p)$. It depends on the DM production mechanism. In this section, we will consider the thermal average momentum and the free streaming horizon in thermal freeze-in mechanism of singlet scalar, the Dodelson-Widrow mechanism and the non-thermal singlet scalar. Finally, we show the Lyman-α constraints and allowed parameter region on the each production mechanisms.
A. The thermal freeze-in production mechanism from the singlet scalar

For thermal freeze-in mechanism via singlet scalar boson, the momentum distribution of sterile neutrino DM \([50, 51]\) is given by

\[
f(p) = \frac{\beta}{(p/T)^{1/2}} \, g_{5/2}(p/T), \tag{41}\]

where

\[
g_\nu(x) = \sum_{n=1}^{\infty} \frac{e^{-nx}}{n^5}. \tag{42}\]

The normalization factor \(\beta\) is determined by the Yukawa coupling \(\kappa\) and the mass of singlet scalar \(m_s\), and then, \(\beta \propto \kappa^2 m_s^{-1}\). The thermal average momentum \(\langle p(t) \rangle\) can be calculated by

\[
\langle p(t) \rangle = \frac{\int_0^\infty dp \sqrt{T} p^5 \sum_{n=1}^{\infty} e^{-n(p/T)/(n^{5/2})}}{\int_0^\infty dp \sqrt{T} p^3 \sum_{n=1}^{\infty} e^{-n(p/T)/(n^{5/2})}} \approx 2.4527 \, T. \tag{43}\]

This thermal average momentum \(\langle p(t) \rangle\) leads to the thermal average velocity \(\langle v(t) \rangle\),

\[
\langle v(t) \rangle \simeq \begin{cases} 
1 & t < t_{nr}, \\
\frac{2.45 T}{m_{N_1}} = \frac{a(t_{nr})}{a(t)} & t \geq t_{nr}.
\end{cases} \tag{44}\]

The time when DM particles become non-relativistic is \(t_{nr}^{1/2} \approx 2.45 \left(\frac{\text{MeV}}{m_{N_1}}\right)\) sec. The free streaming horizon is calculated by

\[
\lambda_{FS} = \int_{t_{in}}^{t_{eq}} \frac{\langle v(t) \rangle}{a(t)} dt \\
= \int_{t_{in}}^{t_{nr}} \frac{dt}{a(t)} + \int_{t_{nr}}^{t_{eq}} \frac{\langle v(t) \rangle}{a(t)} dt + \int_{t_{eq}}^{t_{in}} \frac{\langle v(t) \rangle}{a(t)} dt \\
=\frac{5\sqrt{t_{eq}t_{nr}}}{a(t_{eq})} + \sqrt{t_{eq}t_{nr}} \ln \left(\frac{t_{eq}}{t_{nr}}\right) - \frac{2\sqrt{t_{eq}t_{in}}}{a(t_{eq})} - \frac{3\sqrt{t_{eq}t_{nr}}}{a(t_{eq})^{1/2}} \\
\simeq \frac{\sqrt{t_{eq}t_{nr}}}{a(t_{eq})} \left[5 + \ln \left(\frac{t_{eq}}{t_{nr}}\right)\right]. \tag{45}\]

To obtain the last line, we neglect the third and the last term of the third line.

In this production mechanism, the DM is produced at high temperature \(T \gtrsim 1\) TeV and the entropy dilution affects the free streaming horizon. The effect of the entropy dilution can be estimated by the factor \(\xi^{-1/3}\) which is given by

\[
\xi = \frac{g_{\text{eff}}(\text{high } T)}{g_{\text{eff}}(\text{current } T_0)} \approx 109.5 \frac{3.36}{3.6}, \tag{46}\]

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FIG. 4. The X-rays bounds, the free streaming horizon, HDM, WDM, CDM regions and other constraints for thermal freeze-in mechanism of the singlet scalar for (a) \( \langle S \rangle = 1 \) TeV and for (b) \( \langle S \rangle = 100 \) TeV.

Now we assume that both \( s \) and \( N_1 \) contribute to the effective number of freedom and ignore the other heavy right-handed neutrinos \( N_2 \) and \( N_3 \). It’s not accurate expression of the entropy dilution factor, but the numerical difference is very tiny [23, 24]. Taking the entropy dilution into account and using conversion factor \( c = 10^{-14} \text{(Mpc/sec)} \), the final expression is given by

\[
\lambda_{FS} = \frac{c}{a} \left[ 5 + \ln \left( \frac{t_{eq}}{t_{nr}} \right) \right] \frac{1}{\xi^{1/3}}. \tag{47}
\]

The Lyman-\( \alpha \) bounds and HDM regions are given by

\[
m_{N_1} < 1.57 \text{ keV}. \tag{48}
\]

The region of WDM sterile neutrino can be obtained

\[
1.57 \text{ keV} < m_{N_1} < 20.5 \text{ keV}. \tag{49}
\]

In FIG. 4, we show the X-rays bounds, HDM, WDM and CDM regions for the thermal freeze-in production mechanism by the singlet scalar. In this figure, we assume that \( m_s \) is larger than \( m_h = 125 \text{ GeV} \) and smaller than \( \langle S \rangle = 1,100 \text{ TeV} \) to evade non-perturbative effects. The DW dominant region shows that the sterile neutrino by the DW mechanism
is larger than 1% of the total DM. When the scale of singlet scalar becomes higher, the produced sterile neutrino becomes colder but the scenario suffers from the X-rays constraints. The scale related with the singlet scalar is about 1 TeV to regard the sterile neutrino as a warm dark matter candidate. This production mechanism produces the cool sterile neutrino and does not come into conflict with the Lyman-α bounds and the X-rays constraints.

B. The non-thermal decay production mechanism from the singlet scalar

If the Higgs portal coupling is small and the single scalar is out of thermal equilibrium, it decays into the sterile neutrino. The free streaming horizon has been considered in [23, 24]. The momentum distribution of sterile neutrino DM is given as the following [52–55].

\[
f(p) = \frac{\beta}{p/T_{DM}} \exp \left( -\frac{p^2}{T_{DM}^2} \right),
\]

\(\beta\) is the normalization factor and the DM temperature is \(T_{DM} = \frac{m_s a(t)}{2a(t)}\). The thermal average momentum is given as the following.

\[
\langle p(t) \rangle = \frac{\int d^3 p f(p)}{\int d^3 p f(p)} = \frac{\int_0^\infty dp p^2 e^{-p^2/T_{DM}^2}}{\int_0^\infty dp pe^{-p^2/T_{DM}^2}} = \frac{\sqrt{\pi}}{2} T_{DM}.
\]

From the above result, thermal average velocity is expressed as,

\[
\langle v(t) \rangle \simeq \begin{cases} 
1 & t < t_{nr}, \\
\frac{\sqrt{\pi m_s a(t)}}{4m_{N_1} a(t)} & t \geq t_{nr}.
\end{cases}
\]

Now, we assume that the production time is \(t_{in} = t_{fe} + \tau\), the freeze-in time of \(s\) is \(t_{fe} \simeq \left( \frac{\text{MeV}}{T_{fe}} \right)^2 \text{sec}\), the freeze-in temperature is \(T_{fe} \simeq m_s \) GeV and the lifetime of \(s\) is \(\tau = \hbar/\Gamma (s \rightarrow N_1 N_1)\). The non-relativistic time is given by \(t_{nr} = \frac{\pi m_s^2}{16 m_{N_1}^2} t_{in} \) sec and the matter-radiation equal time is \(t_{eq} = 1.9 \times 10^{11} \) sec. We estimate the free streaming horizon of the DM sterile neutrino by using this formula

\[
\lambda_{FS} = \frac{c \sqrt{t_{eq} t_{nr}}}{a(t_{eq})} \left[ 5 + \ln \left( \frac{t_{eq}}{t_{nr}} \right) \right] \frac{1}{\zeta^{1/3}}.
\]

For \(m_s = 1 \) TeV and \(\kappa_1 = 10^{-8}\), the bound of Lyman-α and HDM region are obtained as

\[
m_{N_1} < 4.36 \text{ keV}.
\]
FIG. 5. The bounds of X-rays, HDM, WDM, CDM regions and other constraints for the non-thermal decay mechanism by the singlet scalar for (a) $m_s = 1$ TeV and for (b) $m_s = 100$ TeV.

The WDM sterile neutrino mass can be constrained as,

$$4.36 \text{ keV} < m_{N_1} < 64.3 \text{ keV}. \quad (55)$$

This constraints are larger than the DW mechanism because the DM sterile neutrino is produced by the decay of non-thermal heavy particles. When the lifetime of singlet scalar is not small, the Lyman-α constrains becomes stronger. For $m_s = 100$ TeV and $\kappa_1 = 10^{-8}$, the Lyman-α bounds and HDM regions are given as

$$m_{N_1} < 64.2 \text{ keV}. \quad (56)$$

The WDM sterile neutrino mass can be obtained as the following

$$64.2 \text{ keV} < m_{N_1} < 840 \text{ keV}. \quad (57)$$

Therefore, in this scenario, the singlet scalar can not be heavier than TeV scale \footnote{Ref. [27] describes the more detailed calculations in this scenario by solving numerically the system of the Boltzmann equations and the mass of the singlet scalar could be more restricted.}.

In FIG. 5, we show the X-rays bounds, HDM, WDM and CDM regions for the non-thermal decay production from the singlet scalar. FIG.5(a) describes the non-thermal decay
The thermal freeze-in mechanism by Higgs boson

CDM region

WDM region

The Dodelson-Widrow mechanism

Lifetime constraints

Cosmic X rays

Background

Ω_{DM} h^2 = 0.1199 line

Ω_{DM} h^2 = 0.01199 dash line

(a) $h \to \nu_e N_1$

(b) $\nu_e \nu_\mu \nu_\tau \to N_1$

FIG. 6. The bounds of X-rays, HDM, WDM, CDM regions and other constraints for (a) the thermal freeze-in mechanism of Higgs boson and for (b) the Dodelson-Widrow mechanism. The red lines describe that the sterile neutrino DM produces all DM density $\Omega_{DM} h^2 = 0.1199$ and the red dash lines give that the sterile neutrino DM produces the partial DM density $\Omega_{DM} h^2 = 0.01199$.

mechanism in Higgs portal coupling $\lambda = 10^{-7.4}, 10^{-7.7}, 10^{-8}$ and singlet scalar mass $m_s = 1$ TeV, the sterile neutrino DM does not suffer from the the Lyman-$\alpha$ bounds. FIG.5(b) shows in portal coupling $\lambda = 10^{-6.4}, 10^{-6.7}, 10^{-7}$ and singlet scalar mass $m_s = 100$ TeV. When the mass of the singlet scalar becomes larger, the produced sterile neutrino becomes warmer and this scenario suffers from the Lyman-$\alpha$ bounds.

C. The sterile neutrino DM production mechanism in the $\nu$MSM

In the $\nu$MSM, the sterile neutrino dark matter can be generated by the thermal freeze-in production mechanism via Higgs boson or the Dodelson-Widrow mechanism. The thermal freeze-in mechanism via Higgs boson has the free streaming horizon as small as that of the sterile neutrino DM via the freeze-in production form. However, this production mechanism comes into conflict with life time bounds and X-rays of bounds (see FIG.6(a)).

In the Dodelson-Widrow mechanism, the sterile neutrino is produced from the thermal background of active neutrino via coherent scattering. Therefore, the momentum obeys
thermal distribution of the Fermi-Dirac type [14].

\[ f(p) = \frac{\beta}{e^{p/T} + 1}, \]  

(58)

where \( p \) denotes the comoving momentum of \( N_1 \) and \( \beta \propto \theta^2 M_1 \). For the thermalized sterile neutrino, the thermal average momentum \( \langle p(t) \rangle \) can be given by,

\[ \langle p(t) \rangle = \frac{\int_0^\infty dp \frac{p^3}{e^{p/T} + 1}}{\int_0^\infty dp \frac{p^2}{e^{p/T} + 1}} = \frac{7\pi^4 T}{180\zeta(3)} \approx 3.1513 \ T. \]

(59)

The thermally produced sterile neutrino has the relation \( \langle p \rangle / 3.15 T \approx 1 \), but DW mechanism produce the sterile neutrino distribution cooler than thermal background \( \langle p \rangle / 3.15 T \approx 0.9 \). The free streaming horizon of the sterile neutrino DM is given by the thermal average momentum [41]

\[ \lambda_{FS} \approx 0.84 \text{ Mpc} \left( \frac{\text{keV}}{m_{N_1}} \right) \left( \frac{\langle p \rangle}{3.15 T} \right). \]

(60)

The observations of Lyman-\( \alpha \) forest lead to the severe restriction on the DW mechanism \( m_{N_1} > 10 \text{ keV} \) [56, 57]. This limit is in conflict with the X-ray bounds (see FIG.6(b)) and the DW mechanism scenario is excluded.

If the lepton asymmetry is relatively large in the early universe, the thermal production of sterile neutrinos can be enhanced by MSW effect (Shi-Fuller mechanism). The Shi-Fuller mechanism lead to the colder thermal distribution \( \langle p \rangle / 3.15 T \approx 0.6 \). The Shi-Fuller production mechanism can evade this bounds although it needs to the relative large lepton asymmetry. In conclusion, thermal background production such as DW is severely constrained due to the large free streaming scale and X-rays bounds. It should be emphasized that the \( \nu \)MSM fails perfectly in the dark matter sector because the sterile neutrino dark matter can’t be generated by DW nor thermal freeze-in production by Higgs boson. Therefore, we have to extend \( \nu \)MSM and consider another DM production scenarios.

VI. THE THERMAL FREEZE-IN LEPTOGENESIS FROM THE SINGLET SCALAR

In this section, we discuss leptogenesis scenarios with the thermal freeze-in production mechanism. In this scenario, we assume that heavy Majorana neutrinos are generated from thermal freeze-in production or non-thermal decay production of the singlet scalar. The produced Majorana neutrinos generate the lepton asymmetry. The produced lepton
asymmetry is transferred into the baryon asymmetry by non-perturbative electroweak effects (Sphaleron). We assume that $N_1$ becomes the sterile neutrino DM and does not affect the leptogenesis scenarios, while $N_2$ and $N_3$ satisfy $T_{EW} < M_2 < M_3$.

In our model, the Majorana masses are around TeV scale, but TeV scale leptogenesis comes into conflict with the Davidson-Ibarra bound [58] which constraints Majorana masses $M_2 > 10^9$ GeV. It is possible to escape this lower bound when the mass difference between $N_2$ and $N_3$ is order of their decay width, In that case, resonant leptogenesis can occur [59, 60]. The resonant CP asymmetry is obtained as the following,

$$\epsilon_i = \frac{\Gamma (N_i \rightarrow \ell_{\alpha} H) - \Gamma (N_i \rightarrow \ell_{\alpha} H^*)}{\Gamma (N_i \rightarrow \ell_{\alpha} H) + \Gamma (N_i \rightarrow \ell_{\alpha} H^*)} \simeq \epsilon'_i + \epsilon_i.$$  \hspace{1cm} (61)

The $\epsilon'$-type CP violation is obtained from the vertex contribution,

$$\epsilon'_i = \frac{\text{Im}(y^i y)^2}{(y^i y)^2} \cdot \frac{(\Gamma_{N_j} m_{N_j})}{(m_{N_i} m_{N_j})^2} f \left( \frac{m_{N_i}^2}{m_{N_j}^2} \right),$$  \hspace{1cm} (62)

where $\Gamma_{N_i}$ is the tree-level decay width and $f(x)$ is the loop function as the following,

$$\Gamma_{N_i} = \frac{(y^i y)^2}{8\pi} m_{N_j}, \quad f(x) = \sqrt{x} \left[ 1 - (1 + x) \ln \left( \frac{1 + x}{x} \right) \right].$$  \hspace{1cm} (63)

In the degenerate heavy Majorana neutrino mass limit ($m_{N_2} \approx m_{N_3}$), the CP violation $\epsilon'_2$ cancels against CP violation $\epsilon'_3$ and no net CP violation can be obtained from the vertex contribution.

The $\epsilon$-type CP violation arises from the self-energy contribution.

$$\epsilon_i = \frac{\text{Im}(y^i y)^2}{(y^i y)^2} \cdot \frac{(\Gamma_{N_j} m_{N_j})}{(m_{N_i} m_{N_j})^2} \cdot \frac{(m_{N_i}^2 - m_{N_j}^2)}{(m_{N_i}^2 - m_{N_j}^2)^2 + m_{N_i}^2 \Gamma_{N_j}^2}. $$  \hspace{1cm} (64)

In the limit ($m_{N_2} \approx m_{N_3}$), $\epsilon$-type CP violation $\epsilon_i$ dominates over $\epsilon'_i$. Furthermore, $\epsilon_2$ and $\epsilon_3$ have the same sign. In the limit $m_{N_2} \approx m_{N_3}$, the total CP asymmetry involving $N_2$ contribution is given by,

$$\epsilon_2 \simeq \frac{\text{Im}(y^i y)^2}{(y^i y)^2} \cdot \frac{(\Gamma_{N_3} m_{N_3})}{(m_{N_2} m_{N_3})^2} \cdot \frac{(m_{N_2}^2 - m_{N_3}^2)}{(m_{N_2}^2 - m_{N_3}^2)^2 + m_{N_2}^2 \Gamma_{N_3}^2}. $$  \hspace{1cm} (65)

In order to generate the $O(1)$ lepton asymmetry, it is necessary to satisfy following two conditions [59],

$$m_{N_3} - m_{N_2} \approx \frac{1}{2} \Gamma_{N_{3,2}}, \quad \frac{\text{Im}(y^i y)^2}{(y^i y)^2} \cdot \frac{(\Gamma_{N_3} m_{N_3})}{(m_{N_2} m_{N_3})^2} \approx 1.$$  \hspace{1cm} (66)
In general, mass difference is larger than tree-level decay width ($\Delta m_{N_{32}} > \Gamma_{N_{3,2}}$).

$$
\epsilon_2 \simeq \frac{\text{Im}(y\,y)_{23}^2}{(y\,y)_{22}(y\,y)_{33}} \frac{m^2_{N_2} - m^2_{N_3}}{m_{N_2} m_{N_3}} \left( \frac{\Gamma_{N_{3,2}}}{m_{N_3}} \right)
$$

$$
\simeq - \frac{\text{Im}(y\,y)_{23}^2}{(y\,y)_{22}(y\,y)_{33}} \frac{\Delta m_{N_{32}} m^2_{N_2} m_{N_3}}{\Delta m_{N_{32}} m^2_{N_2} + m^2_{N_2} \Gamma_{N_{3,2}}^2} \left( \frac{\Gamma_{N_{3,2}}}{m_{N_3}} \right)
$$

(67)

when we assume that $\frac{\text{Im}(y\,y)_{23}^2}{(y\,y)_{22}(y\,y)_{33}} \approx 10^{-3}$. The masses of heavy Majorana neutrinos are 1 TeV and the mass difference is $\Delta m_{N_{32}} \approx 1$ MeV order, relation of the mass difference and the tree-level decay width is $\Delta m_{N_{32}} = 10^6 \Gamma_{N_{3,2}}$, and therefore, CP asymmetry factor becomes $\epsilon_2 \approx 10^{-9}$. In this section, we assume that heavy Majorana neutrinos are 1 TeV, mass difference is $\Delta m_{N_{32}} \approx 1$ MeV and CP asymmetry factor is $\epsilon_2 \approx 10^{-9}$ order.

In thermal leptogenesis, the right-handed neutrino abundance $Y_{N_2}$ and the lepton asymmetry $Y_{\Delta L}$ are written by the following two Boltzmann equations,

$$
\frac{dY_{N_2}}{dT} = (D_2 + S) \left( Y_{N_2} - Y^\text{eq}_{N_2} \right),
$$

(68)

$$
\frac{dY_{\Delta L}}{dT} = -\epsilon_2 D_2 \left( Y_{N_2} - Y^\text{eq}_{N_2} \right) + W_{ID} Y_{\Delta L}.
$$

(69)

The scattering term $S$ describes the $\Delta L = 1$ scattering effects, but we neglect this contribution for simplicity. The decay and washout terms are expressed as the following,

$$
D_2(T) = \frac{\Gamma_{N_3}}{H(T)T} \frac{K_1 \left( m_{N_2}/T \right)}{K_2 \left( m_{N_2}/T \right)}, \quad W_{ID}(T) = \frac{1}{2} D_2(T) \frac{Y^\text{eq}_{N_2} \left( m_{N_2}/T \right)}{Y^\text{eq}_{\ell}}.
$$

(70)

The equilibrium abundances of Majorana neutrinos and leptons are given by,

$$
Y^\text{eq}_{N_2}(T) = \frac{45m^2_{N_2} K_2 \left( m_{N_2}/T \right)}{2\pi^4 T^2 h_{\text{eff}}}, \quad Y^\text{eq}_{\ell} = \frac{15}{4\pi^2 h_{\text{eff}}}.
$$

(71)

The analytical solution for the lepton asymmetry $Y_{\Delta L}$ is given by the following formula [61–63],

$$
Y_{\Delta L}(T) = Y_{\Delta L}(T_{RE}) e^{\int_{T_{RE}}^T dT' W_{ID}(T')} - \int_{T_{RE}}^T dT' \epsilon_2 e^{\int_{T'}^T dT'' W_{ID}(T'')}
$$

(72)

$$
\simeq -\epsilon_2 \int_{T_{RE}}^T dT' Y_{N_2} \left( m_{N_2}/T' \right) e^{\int_{T'}^T dT'' W_{ID}(T'')}
$$

(73)

We assume that there is no preexisting lepton asymmetry $Y_{\Delta L}(T_{RE}) = 0$ and neglect the washout term,

$$
Y_{\Delta L}(T_0) \simeq -\epsilon_2 \int_{T_{RE}}^{T_0} dT' \frac{dY_{N_2}}{dT'} = \epsilon_2 Y_{N_2}(T_{RE}).
$$

(74)
In general, heavy Majorana neutrinos come into thermal equilibrium. The initial abundance is given by the thermal equilibrium abundance $Y_{N_2}(T_{RE}) = Y_{N_2}^{eq} \simeq 0.004$ and the lepton asymmetry can be approximately given by $Y_{\Delta L}(T_0) \simeq 0.004 \epsilon_2$. If we take care of washout effects, the exact final lepton asymmetry can be obtained as the followings \cite{63},

$$Y_{\Delta L}(T_0) \simeq -\frac{27}{16} \epsilon_2 \left( \frac{\Gamma_{N_2}}{H(m_{N_2})} \right)^2 Y_{N_2}^{eq}. \quad (75)$$

In this section, we consider two thermal freeze-in leptogenesis scenarios from the singlet scalar.

A. From the singlet scalar enter into thermal equilibrium

In this subsection, we will discuss new leptogenesis scenario where the singlet scalar comes into thermal equilibrium and the Majorana neutrinos are generated by the thermal freeze-in mechanism of the scalar singlet. In this scenarios, the Higgs portal coupling has to be relatively large $\lambda > 10^{-6}$ and the Yukawa coupling needs to be small $\kappa_2 < 10^{-6}$. The relevant Boltzmann equations are given by the following formula,

$$\frac{dY_{N_2}}{dT} = D_2 \left( Y_{N_2} - Y_{N_2}^{eq} \right) - 2D_s Y_s^{eq}, \quad (76)$$

$$\frac{dY_{\Delta L}}{dT} = -\epsilon_2 D_2 \left( Y_{N_2} - Y_{N_2}^{eq} \right) + W_{1D} Y_{\Delta L}. \quad (77)$$

The relevant terms are given by

$$D_2 Y_{N_2} = -\sqrt{\frac{45}{4\pi^3 G_N}} \frac{1}{\sqrt{g_{eff}}} \frac{1}{T^3} K_2 \left( \frac{m_{N_2}}{T} \right) \Gamma_{N_2} Y_{N_2}$$

$$= -\frac{3\sqrt{5}}{2\pi^{3/2}} \frac{m_{pl}}{h_{eff} T^3} \frac{1}{K_2 \left( \frac{m_{N_2}}{T} \right)} \Gamma_{N_2} Y_{N_2}, \quad (78)$$

$$D_2 Y_{N_2}^{eq} = -\sqrt{\frac{45}{4\pi^3 G_N}} \frac{1}{\sqrt{g_{eff}}} \frac{1}{T^3} K_2 \left( \frac{m_{N_2}}{T} \right) \Gamma_{N_2} Y_{N_2}^{eq}$$

$$= -\frac{135\sqrt{5}}{4\pi^{11/2}} \frac{m_{pl}}{h_{eff} \sqrt{g_{eff}}} \frac{m_{N_2}^2 K_1 \left( \frac{m_{N_2}}{T} \right)}{T^5} \Gamma_{N_2}, \quad (79)$$

$$D_s Y_s^{eq} = -\sqrt{\frac{45}{4\pi^3 G_N}} \frac{1}{\sqrt{g_{eff}}} \frac{1}{T^3} K_2 \left( \frac{m_{s}}{T} \right) \Gamma \left( s \rightarrow N_2 N_2 \right) Y_s^{eq}$$

$$= -\frac{135\sqrt{5}}{8\pi^{11/2}} \frac{m_{pl}}{h_{eff} \sqrt{g_{eff}}} \frac{m_{s}^2 K_1 \left( \frac{m_{s}}{T} \right)}{T^5} \Gamma \left( s \rightarrow N_2 N_2 \right). \quad (80)$$

We can write down the analytical solution of the lepton asymmetry $Y_{\Delta L}$,

$$Y_{\Delta L}(T) = -\epsilon_2 \int_{T_{RE}}^{T} \left( \frac{dY_{N_2}}{dT} + 2D_s Y_s^{eq} \right) e^{\int_{T'}^{T} \frac{dT''}{H(T'')}} dT'. \quad (81)$$
We assume that the Majorana neutrinos and the initial lepton asymmetry are zero $Y_{N_{1,2,3}} = Y_{\Delta L} (T_{RE}) = 0$ and neglect the washout term in Eq.(81),

$$Y_{\Delta L} (T_0) \simeq \epsilon_2 Y_{N_2} (T_{RE}) - \epsilon_2 \int_{T_{RE}}^{T_0} 2D_s Y_s^{eq} \, dT$$

(82)

$$\simeq -\epsilon_2 \int_{T_{RE}}^{T_0} 2D_s Y_s^{eq} \, dT.$$  

(83)

Finally, we will analytically integrate Eq.(83) from $T_0 = 0$ to $T_{RE} = \infty$ as the estimate of the lepton asymmetry,

$$Y_{\Delta L} (T_0) \simeq 135\sqrt{5} \frac{m_{pl}}{4\pi^{11/2}} \frac{m_s}{h_{\text{eff}}} \epsilon_2 \int_{T_{RE}}^{T_0} \frac{m_s^2 K_1 (m_s/T)}{T^5} \frac{\Gamma (s \rightarrow N_2 N_2)}{T} \, dT$$

$$\simeq 135\sqrt{5} \frac{m_{pl}}{4\pi^{11/2}} \frac{m_s}{h_{\text{eff}}} \epsilon_2 \int_{0}^{\infty} \frac{m_s^2 K_1 (m_s/T)}{T^5} \frac{\Gamma (s \rightarrow N_2 N_2)}{T} \, dT$$

$$\simeq -1.59 \times 10^{-12} \left( \frac{\epsilon_2}{10^{-9}} \right) \left( \frac{\kappa_2}{10^{-7}} \right)^2 \left( \frac{m_s}{\text{TeV}} \right)^2.$$  

(84)

The $(B + L)$ violating interactions of sphalerons come into thermal equilibrium at temperatures $T > T_c \approx 200$ GeV above the electroweak phase transition. The lepton asymmetry can be converted into the $B - L$ asymmetry and the baryon asymmetry as the following [64–66]

$$Y_{\Delta B} (T) = \frac{28}{79} Y_{\Delta B - L} (T) = -\frac{28}{51} Y_{\Delta L} (T).$$  

(85)

Therefore, the final baryon abundance is given by

$$Y_{\Delta B} (T_0) \approx 0.87 \times 10^{-12} \left( \frac{\epsilon_2}{10^{-9}} \right) \left( \frac{\kappa_2}{10^{-7}} \right)^2 \left( \frac{m_{N_2}}{\text{TeV}} \right)$$

$$\approx 0.87 \times 10^2 \left( \frac{\epsilon_2}{10^{-9}} \right) \left( \frac{m_{N_2}}{\text{TeV}} \right) \left( \frac{\text{TeV}}{m_s} \right) \left( \frac{\text{TeV}}{\langle S \rangle} \right)^2.$$  

(86)

From the BBN results, the observed baryon abundance is given by

$$Y_{\Delta B}^{BBN} = (8.10 \pm 0.85) \times 10^{-11}.$$  

(87)

From the CMB measurements, the observed baryon abundance is given by

$$Y_{\Delta B}^{CMB} = (8.79 \pm 0.44) \times 10^{-11}.$$  

(88)

FIG.7 shows the dependence of baryon asymmetry on CP asymmetry factor $\epsilon_2$ and Yukawa coupling $\kappa_2$. The limits of Yukawa coupling $\kappa_2 < 10^{-6}$ and Eq.(86) lead to the following constraints on $m_s$,

$$\left( \frac{m_s}{\text{TeV}} \right) < 1.09 \left( \frac{\epsilon_2}{10^{-9}} \right).$$  

(89)
Therefore the mass of the singlet scalar cannot be larger than 1 TeV in order to produce the observed amount of baryon asymmetry. In section III A, we discuss the thermal freeze-in production of the keV-MeV sterile neutrino DM and conclude that the singlet scalar should not be heavier than TeV scale. However, the mass of the scalar singlet that achieve the leptogenesis is more severely restricted than dark matter scenarios.

B. The singlet scalar is out of the thermal equilibrium

When the Higgs portal coupling is small $\lambda \ll 10^{-6}$ and both $s$ and $N_{2,3}$ do not exist in the early Universe, they do not come into thermal equilibrium. The singlet scalar is produced from thermal freeze-in production by the Yukawa interaction and decay efficiently into Majorana neutrinos $N_{2,3}$ which generate net lepton asymmetry. In this scenario, we

5 To satisfy both observed DM density and baryon asymmetry by the singlet scalar, the mass of the sterile neutrino must be around 100 MeV from Eq.(21) and Eq.(86).
have to solve the following Boltzmann equations in order to get these abundance.

\[
\frac{dY_s}{dT} = \frac{dY_A}{dT} + D_s Y_s, \tag{90}
\]

\[
\frac{dY_{N_2}}{dT} = D_2 \left(Y_{N_2} - Y_{N_2}^{eq}\right) - 2D_s Y_s, \tag{91}
\]

\[
\frac{dY_{\Delta L}}{dT} = -\epsilon_2 D_2 \left(Y_{N_2} - Y_{N_2}^{eq}\right) + W_{1D} Y_{\Delta L}. \tag{92}
\]

The relevant term in the annihilation process can be expressed as,

\[
\frac{dY_A}{dT} = -\frac{135\sqrt{5}}{64\pi^{17/2}} \frac{m_{pl}}{\hbar_{\text{eff}} \sqrt{g_{\text{eff}}}} \frac{m_s}{T^3} K_1 \left(2 m_s / T \right). \tag{93}
\]

The singlet scalar decays into the right-handed neutrino, the decay term is given by,

\[
D_s Y_s = \sqrt{\frac{45}{4\pi^3 G_N}} \frac{1}{\sqrt{g_{\text{eff}}}} \frac{1}{T^3} K_2 \left(m_s / T \right) \Gamma \left(s \rightarrow N_2 N_2\right) Y_s
\]

\[
= \frac{3\sqrt{5}}{2\pi^{3/2}} \frac{m_{pl}}{\sqrt{g_{\text{eff}}}} \frac{1}{T^3} K_2 \left(m_s / T \right) \Gamma \left(s \rightarrow N_2 N_2\right) Y_s. \tag{94}
\]

We integrate Eq.(90) to estimate the abundance of right handed neutrinos \(Y_{N_2}\),

\[
\int_{T_{RE}}^{T_0} \frac{dY_s}{dT} dT = \int_{T_{RE}}^{T_0} \frac{dY_A}{dT} dT + \int_{T_{RE}}^{T_0} D_s Y_s dT. \tag{95}
\]

The initial abundance \(Y_s(T_{RE})\) and the final abundance \(Y_s(T_0)\) are zero, and then the following equation is obtained,

\[
\int_{T_{RE}}^{T_0} D_s Y_s dT = -\int_{T_{RE}}^{T_0} \frac{dY_A}{dT} dT. \tag{96}
\]

The lepton asymmetry \(Y_{\Delta L}\) at today temperature is obtained as the following,

\[
Y_{\Delta L}(T_0) \approx -\epsilon_2 \int_{T_{RE}}^{T_0} 2D_s Y_s dT
\]

\[
\approx -\frac{135\sqrt{5}}{32\pi^{17/2}} \frac{m_{pl}}{\hbar_{\text{eff}} \sqrt{g_{\text{eff}}}} \epsilon_2 \int_{T_{RE}}^{T_0} \frac{\lambda^2 m_s}{T^3} K_1 \left(2 m_s / T \right) dT
\]

\[
\approx -\frac{135\sqrt{5}}{32\pi^{17/2}} \frac{m_{pl}}{\hbar_{\text{eff}} \sqrt{g_{\text{eff}}}} \epsilon_2 \int_{0}^{\infty} \frac{\lambda^2 m_s}{T^3} K_1 \left(2 m_s / T \right) dT
\]

\[
\approx 1.07 \times 10^{-13} \left(\frac{\epsilon_2}{10^{-9}}\right) \left(\frac{\lambda}{10^{-7}}\right)^2 \left(\frac{\text{TeV}}{m_s}\right). \tag{97}
\]

The final baryon asymmetry is expressed as,

\[
Y_{\Delta B} \approx 0.59 \times 10^{-13} \left(\frac{\epsilon_2}{10^{-9}}\right) \left(\frac{\lambda}{10^{-7}}\right)^2 \left(\frac{\text{TeV}}{m_s}\right). \tag{98}
\]
FIG. 8. These figure show the relations with the baryon asymmetry, CP asymmetry factors $\epsilon_2$ and Higgs portal coupling $\lambda$. Higgs portal coupling has to be smaller $\lambda < 10^{-6}$ if not singlet scalar come into thermal equilibrium.

In this scenario, TeV scale singlet scalar can generate the observed amount of baryon asymmetry. FIG.8 describes the dependence of baryon asymmetry on CP asymmetry factor $\epsilon_2$ and Higgs portal coupling $\lambda$. The limit of Higgs portal coupling $\lambda < 10^{-6}$ and Eq.(98) lead to the following constraints on $m_s$,

$$\left(\frac{m_s}{\text{TeV}}\right) < 0.13 \left(\frac{\epsilon_2}{10^{-9}}\right).$$ (99)

In this leptogenesis scenario, the singlet scalar cannot be also larger than 1 TeV in order to produce the observed abundance of baryon asymmetry. In section III B, we consider the non-thermal decay production of the keV-MeV sterile neutrino DM and show that the singlet scalar needs to be lower than TeV scale. However, this leptogenesis scenario constrains more severely singlet scalar than the scenario of sterile neutrino DM.

Note that one singlet scalar cannot explain both dark mater sectors and leptogenesis scenarios with the thermal freeze-in production because the heavy Majorana neutrinos dominate the non-thermal decay production via the singlet scalar.
VII. CONCLUSION

We consider an extended $\nu$MSM with one additional singlet scalar. The existence of the singlet scalar improves the dark matter and leptogenesis tensions in the $\nu$MSM through the thermal freeze-in production mechanism. In this model, there are two scenarios for thermal properties of the singlet scalar. If the Higgs portal coupling is relatively large so that the singlet scalar enters into thermal equilibrium, the sterile neutrino and the heavy Majorana neutrinos are produced by the thermal freeze-in production. If the Higgs portal coupling is much small and the singlet scalar is out of thermal equilibrium, the single scalar is produced by thermal freeze-in production. Then, the sterile neutrino and the heavy Majorana neutrinos are generated by the non-thermal decay production of the singlet scalar. In these scenarios, the sterile neutrino can evade the Lyman-$\alpha$ bounds and the X-ray constraints. We found the latter scenario more strongly restricted from the Lyman-$\alpha$ bounds. In this model, the singlet scalar needs to be lower than 100 TeV to produce the keV-MeV sterile neutrino DM. The thermal freeze-in leptogenesis scenarios restrict severely the singlet scalar mass less than 1 TeV to generate the observed baryon asymmetry. Namely, in the extended $\nu$MSM with a singlet scalar, the singlet scalar needs to be TeV scale in order to generate the observed abundance of dark matter or baryon asymmetry with the thermal freeze-in production mechanism.

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