Low-scale Quantum Gravity and Double Nucleon Decay

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Abstract

In models with a low quantum gravity scale, one might expect sizable effects from nonrenormalizable interactions that violate the global symmetries of the standard model. While some mechanism must be invoked in such theories to suppress higher-dimension operators that contribute to proton decay, operators that change baryon number by two units are less dangerous and may be present at phenomenologically interesting levels. Here we focus on $\Delta B = 2$ operators that also change strangeness. We demonstrate how to compute explicitly a typical nucleon-nucleon decay amplitude, assuming a nonvanishing six-quark cluster probability and MIT bag model wave functions. We then use our results to estimate the rate for other possible modes. We find that such baryon-number-violating decays may be experimentally accessible if the operators in question are present and the Planck scale is less than $\sim 400$ TeV.
I. INTRODUCTION

One of the exciting implications of extra spacetime dimensions with large radii of compactification is the possibility that the scale of quantum gravity may be brought down to TeV energies [1]. While much effort has focused on understanding the experimental signals of the extra dimensions themselves (for example, through the effects of Kaluza-Klein excitations in the effective four-dimensional theory), much less has been said on the physics that originates at the cut off [2]. This is understandable for two reasons. First, any detailed knowledge of Planck-suppressed operators in a theory with a low quantum gravity scale requires a complete theory of quantum gravity, which is not yet at hand. Alternatively, a general effective field theory approach, in which one includes all operators consistent with the gauge symmetries of the standard model and suppressed by powers of the cut off, leads to baryon- and lepton-number-violating effects far in excess of the experimental bounds [3,4]. The most common approach to this dilemma is to assume that some mechanism forbids the undesirable operators, and then to ignore the issue altogether. Here we will explore the possibility that the mechanism responsible for maintaining proton stability does not forbid the complete set of operators that violate the global symmetries of the standard model. Assuming \( \Delta B = 1 \) interactions are absent, higher-dimension operators that violate baryon number \( B \) by two units are far less problematic, and may be present in low Planck scale scenarios at a phenomenologically interesting level. Moreover, such operators are not suppressed by separating quarks and leptons in an extra dimension, as has been suggested as a remedy to the proton decay problem [5]. In this letter we study the two-body double nucleon decays that are induced by \( \Delta B = 2 \) operators and determine the sensitivity of existing experiments to the scale of the new physics.

The idea that the most dangerous baryon-number-violating operators may be absent, while others are present is not at all a radical one. Consider any model in which baryon-number is promoted to a gauge symmetry and then spontaneously broken: higher-dimension operators that violate baryon number by \( \Delta B \) units are induced in the low-energy theory, but the smallest possible value of \( \Delta B \) is controlled by the charge of the Higgs field that is responsible for the spontaneous symmetry breaking [4]. From a low-energy perspective, the original continuous gauge symmetry is irrelevant, and one concludes that a discrete remnant is sufficient to eliminate the unwanted interactions. Such “discrete gauge symmetries” are known to be preserved by quantum gravitational effects [6], are well defined as fundamental symmetries [7], and arise in string theory [8]. It is not hard to imagine scenarios in which operators that contribute to nucleon decay are forbidden by some residual discrete symmetry below the string scale, while operators of higher-dimension that violate baryon number remain in the low-energy theory [9].

Past interest in effective \( \Delta B = 2 \) interactions has appeared in the context of grand unified theories [10], R-parity-violating supersymmetry [11], and Planck-scale physics [12]. The relevant dimension-nine operators have been cataloged in the literature [13,14], but have not been studied in their entirety. This is due in part to the difficulty in evaluating hadronic matrix elements, and the relatively large number of operators involved. The fact that each operator has an undetermined coefficient of \( O(1) \) leads to an unavoidable theoretical uncertainty, and makes the value added in undertaking a complete analysis somewhat small. We will proceed by selecting a typical operator and decay process that is convenient for an
explicit matrix element evaluation (in fact, one that has never appeared in the literature); we then use this result to estimate the size of other accessible modes.

Our canonical decay calculation will be for the process \( D \to K^* K \), where \( D \) represents a deuteron. We choose this mode for a number of reasons: (i) The underlying operator changes strangeness and is unconstrained by neutron-antineutron oscillation bounds. (We will have more to say about the relationship between the operator we consider and others that do not change strangeness in the final section.) (ii) the operator we consider contributes to the matrix element of this process via precisely one Feynman diagram, and (iii) the spin-flavor-color-spatial wave function of the initial state is easily cross-checked with studies of the deuteron structure that appear in the literature \([15]\). We compute the effective lifetime for this decay, and then extrapolate to other NN modes of interest. We will show that such two-body decays may be accessible if the Planck scale is less than \( \sim 400 \) TeV, and we discuss the dependence of our result on the flavor structure of the theory.

How then can a significant number of quarks within a deuteron be annihilated by a contact interaction, if the quarks are spatially separated within the neutron and proton?

### II. DEUTEROMONY

A favorable answer to this question is that the deuteron is not entirely made of a proton and neutron, but also includes a significant admixture of a six-quark (6q) cluster state \([15]\). In the case at hand, the 6q cluster is a six quark state that is totally antisymmetric in color, spin, and flavor, with isospin 0 and spin 1. Such a state is not factorable into a simple product of two color-singlet, three-quark states.

When the constituents of the nucleus are far apart, a description in terms of neutrons and protons is accurate. The question is what happens to the material inside the nucleus when the pieces come close to each other. The quark cluster viewpoint is that if two nucleons come sufficiently close together, the quarks within them reorganize into a new state where each quark is in the lowest energy spatial state, and the color-spin-flavor part of the quark wave function is uniquely fixed \([15]\) by the requirement that it be totally antisymmetric, colorless, and of the desired spin and isospin.

Quark clustering gained impetus \([16]\) in explaining \(^3\)He data in kinematic regions inaccessible to scattering off a stationary nucleon and where contributions due to Fermi motions calculated in standard models were insufficient. Further, differences between quark distributions in 6q clusters and in nucleons provide one straightforward explanation \([17]\) of the nuclear EMC effect.

The deuteron state may be expressed as a linear combination

\[
|D\rangle = (1 - f)^{1/2} |D, NN\rangle + f^{1/2} |D, 6q\rangle,
\]

where \( f \) is the 6q cluster probability. The crucial point is that there is no reason to expect the overlap of the quark spatial wave functions to be negligibly small in a 6q cluster. Providing \( f \) is also non-negligible, we avoid the possibility of obtaining an uninteresting result due to wave function suppression.

Values for \( f \) have been estimated from 6q cluster models of the nuclear EMC effect \([17]\); from calculations of the probability for nucleons to overlap using realistic deuteron wave
functions [18]; from descriptions of high energy SLAC electron-deuteron data at \( x > 1.0 \) [19]; from studies of the deuteron electromagnetic structure functions [20]; and from calculations of fast backward tagged nucleons coming from deuteron breakup [21]. The estimates range between 0.01 and 0.07. We shall quote results assuming \( f = 0.01 \) for free deuterons, and scaling appropriately for 6q clusters within the nuclear medium.

It will be convenient for us to represent the spin-flavor-color structure of the deuteron 6q state in a field-theoretic form. Letting \( q^\dagger \) represent an (anticommuting) quark creation operator, the highest projection spin state may be written

\[
|D, 6q, S_z = 1\rangle = N \epsilon^{\alpha\beta\gamma} \epsilon^{\delta\epsilon\zeta} \epsilon^{ijkl} \times q^\dagger_{\alpha ai} q^\dagger_{\beta bj} q^\dagger_{\gamma c k} q^\dagger_{\delta dk} q^\dagger_{\epsilon el} q^\dagger_{\zeta f l} |0\rangle ,
\]

where the Greek indices represent SU(3) color, \( \ldots e \) SU(2) flavor, and \( \ldots l \) SU(2) spin, and \( q^\dagger \) is a creation operator for a quark state satisfying

\[
\{q_{\alpha ai}, q^\dagger_{\beta bj}\} = \delta_{\alpha\beta} \delta_{ab} \delta_{ij} .
\]

One may see by inspection that the state has the quantum numbers of the deuteron. Note that the SU(2) spin space corresponds exactly to the MIT bag model wave functions described below (so the reader should not think that our calculation is nonrelativistic). In this representation, it is straightforward (though tedious) to compute the normalization factor \( N \); we find \( N = 1/(48\sqrt{10}) \).

### III. MATRIX ELEMENT

With the state defined, we now turn to one possible operator of interest

\[
O = [u^T_R C u^\beta_R] [d^T_R C d^\beta_R] [s^T_R C s^\tau_R] \epsilon_{\alpha\gamma\rho} \epsilon_{\beta\delta\tau},
\]

which contributes to the decay \( D \to K^+K \). Here, \( C \) represents the charge conjugation matrix, and all the fermion fields are right-handed, so that \( O \) is manifestly SU(3)×SU(2)×U(1) invariant. Since strange quarks are present only in the final state, the four remaining fields annihilate quarks in the deuteron, leaving two spectators. We represent the quark cluster component of the deuteron as well as the outgoing kaons as MIT bag model states (described in more detail below), and define the spatial origin as the point at which our \( \Delta B = 2 \) operator acts. When quark fields are replaced by ground state bag wave functions multiplied by appropriate creation or annihilation operators, the spin-flavor-color (SFC) matrix element may be factored from the spatial one. It may be determined by allowing

\[
\hat{O} = \epsilon^{\alpha\gamma\rho} \epsilon^{\beta\delta\tau} \epsilon^{ijkl} \epsilon^{mnn} u_{\alpha i} u_{\beta j} d_{\gamma k} d_{\delta l} s^\dagger_{\rho mn} s^\dagger_{\tau ln}
\]

to act on the state Eq. (2.2), where the symbols now represent creation or annihilation operators rather than fields; one then takes the overlap with a similarly constructed two kaon state. We have computed the SFC matrix element by hand and by symbolic mathematics code, and obtain

\[
\eta_{SFC} = \langle K^0, K^{*+}, S_z = 1 | \hat{O} | D, 6q, S_z = 1 \rangle_{SFC} = -4\sqrt{5} .
\]
To evaluate the spatial part of the matrix element, we must take into account that the desired external states are eigenstates of momentum. To relate these to bag states, we suppose that the momentum eigenstates form a complete set, so that

\[ |D(\vec{R})\rangle_B = \int \frac{d^3P}{(2\pi)^3 2E} \phi(\vec{P}) e^{i\vec{P} \cdot \vec{R}} |D(\vec{P})\rangle , \]  

(3.4)

and similarly for the other states. The state with the subscript “B” is a bag state centered at \( \vec{R} \), spin variables are tacit, and the momentum eigenstate is normalized by

\[ \langle D(\vec{P}')|D(\vec{P})\rangle = 2E(2\pi)^3 \delta^3(\vec{P} - \vec{P}') . \]  

(3.5)

By inversion,

\[ |D(\vec{P})\rangle = \frac{2E}{\phi(\vec{P})} \int d^3R e^{-i\vec{P} \cdot \vec{R}} |D(\vec{R})\rangle_B . \]  

(3.6)

The normalization condition determines \( \phi(P) \equiv \phi(\vec{P}) \),

\[ \phi^2(P) = 2E \int d^3r e^{-i\vec{P} \cdot \vec{r}} I_n(r) = 2EI_n(P) , \]  

(3.7)

where \( I_n \) is the wave function overlap of two \( n \)-quark states centered at different points,

\[ I_n(r) = \langle D(\vec{R} - 1/2\vec{r})|D(1/2\vec{r})\rangle_B . \]  

(3.8)

For applications where the wave function is written as the product of \( n \) independent quark wave functions, one has

\[ I_n(r) = (I_1(r))^n , \]  

(3.9)

with

\[ I_1(r) \equiv \langle B|q(-1/2\vec{r})|q(1/2\vec{r})\rangle_B . \]  

(3.10)

In the bag model, this may be written explicitly as

\[ I_1(r) = (4\pi) \int_0^{R_B - r/2} dz \int_0^{R_B^2 - (z + r/2)^2} \rho d\rho (u_u + l_u \hat{R}_+ \cdot \hat{R}_-) \]  

(3.11)

where,

\[ R_{\pm} = [(r/2 \mp z)^2 + \rho^2]^{1/2} , \quad \hat{R}_+ \cdot \hat{R}_- = (R_+^2 + R_-^2 - r^2)/(2R_+R_-) \]  

(3.12)

and where the factors \( u_i = u(R_i) \) and \( l_i = l(R_i) \) are the upper and lower components of bag wave functions

\[ \psi(\vec{r}) = \begin{pmatrix} u(r) \chi \\ il(r) \vec{\sigma} \cdot \vec{r} \chi \end{pmatrix} = \frac{N}{\sqrt{4\pi}} \begin{pmatrix} j_0(\omega r) \chi \\ i j_1(\omega r) \vec{\sigma} \cdot \vec{r} \chi \end{pmatrix} . \]  

(3.13)
Here $\chi$ is a Pauli spinor, $\omega = x/R_B = 2.043/R_B$, and $N^2R_B^3 = x/[2(x-1)j_0^2(x)] \approx 5.15$.

To calculate the matrix element for the decay $D(\bar{P} = 0) \rightarrow K^*(\bar{p})K(-\bar{p})$, we apply the foregoing formalism to the quark cluster part of the deuteron as well as to the kaons. For simplicity, we assume the same bag radius $R_B \approx 1$ fm for all states. We obtain

$$\tilde{M} = \langle K(-\bar{p}); K^*(\bar{p}), S_z = 1|O|D(\bar{P} = 0), 6q, S_z = 1 \rangle = \left[ \frac{2m_D2E_12E_2}{I_0(0)I_2^2(p)} \right]^{1/2} \times \tilde{N} \quad (3.14)$$

and

$$\tilde{N} = \int d^3R d^3R_1 d^3R_2 \ e^{ip(z_1-z_2)}$$

$$\times \left[ B\langle K(\bar{R}_1); K^*(\bar{R}_2), S_z = 1|O|D(\bar{R}), 6q, S_z = 1 \rangle \right]_B , \quad (3.15)$$

where $E_1$ and $E_2$ are the energies of the $K$ and $K^*$, respectively, and the $z$-direction is taken parallel to $\bar{p}$. This may be re-expressed as

$$\tilde{N} = \frac{1}{8^{\eta_{SF C}}} \int d^3R d^3R_1 d^3R_2 \ e^{ip(z_1-z_2)}$$

$$\times I_1(r_{i0})I_1(r_{j0})(u^2 + l^2)^2(u_1u_2 + l_1l_2) \hat{R}_1 \cdot \hat{R}_2 , \quad (3.16)$$

where $u = u(R), l = l(R)$, and $r_{j0} = |\bar{R}_j - \bar{R}|$. The factors of $I_1$ come from the overlap of spectator quark wave functions. All the integrals in Eq. (3.14) may be evaluated numerically. For $pR_B = 3.12$, we find $\tilde{N} = \eta_{SF C}(N^2R_B^3/8\pi)^3 \times 0.31$, $I_2(p) = R_B^3 \times 0.41$, and $I_0(0) = R_B^3 \times 0.13$.

**IV. RESULTS**

Using the results from the previous section, it is straightforward to derive the decay width. In keeping with the proton decay literature, we will instead express our result in terms of a ‘partial lifetime’ (i.e. the lifetime if the branching fraction to the given mode were 100%). We find

$$\tau(D \rightarrow K^*K) = 2.18 \times 10^7 \text{ yrs} \cdot \left( \frac{M}{1 \text{ TeV}} \right)^{10} f^{-1} \quad (4.1)$$

where $M^5$ is the dimensionful factor that suppresses the dimension-nine operator of interest. (While we identify $M$ with the Planck scale in the present discussion, it is worth mentioning that our calculational framework is applicable to any model, e.g. R-parity violating supersymmetry, in which such effective operators are induced.) For a free deuteron (for example, in $D_2O$), we set $f = 0.01$ and find $\tau \sim 10^{34}$ years for $M \sim 293$ TeV. For a deuteron within the $O^{16}$ nucleus, the quark cluster probability will scale as the nuclear density; for a reasonable estimate, $f = 0.2$, one finds $\tau \sim 10^{34}$ years for $M \sim 395$ TeV. Varying $M$ by a factor of 2 changes the lifetime by $2^{10} \sim 10^3$, which allows the result to vary from well below to significantly above the lifetimes usually associated with the maximum proton decay reach of Super-Kamiokande.
A more realistic estimate of the decay rate would also include flavor suppression factors that provide a small dimensionless number multiplying our operator. Without a model of flavor, however, we cannot exclude the extreme possibility in which Eq. (3.1) has no further suppression and all other $\Delta B = 2$ operators are simply absent. On the other hand, we may consider how our results change if we impose a simple flavor ansatz: we could assume that fields of the first (second) generation are suppressed by a factor of $\lambda^3 (\lambda^2)$, where $\lambda \sim 0.2$ is of comparable size to the Cabibbo angle. (This could arise if a $\Delta B = 2$ operator involving only third generation fields is allowed, and all others are generated from it via CKM-like rotations on the fields [9].) In this case, our previous estimate for $\tau$ is extended by $\lambda^{-32} \sim 10^{22}$; for decays in $O^{16}$ we now find $\tau \sim 10^{34}$ years for $M = 2.3$ TeV. With this ansatz, we can now also say something about the strangeness conserving operators that contribute to $n-\pi$ oscillation. Estimates of the relevant matrix elements already exist [14], from which we find

$$\tau \sim 10^{-12} \text{ yrs} \cdot \left( \frac{M}{1 \text{ TeV}} \right)^5 \lambda^{-18} = 7.6 \text{ yrs} \cdot \left( \frac{M}{1 \text{ TeV}} \right)^5$$

which exceeds the best experimental bound, 3.8 yrs., 90% CL [23] for any value of $M$ that could be plausibly identified with the Planck scale.

Table 1 shows other possible nucleon-nucleon decay modes, and the multiplicative correction that must be applied to our $D \to KK^*$ result to obtain an estimate for the lifetime. This factor takes into account the differing phase space, spatial wave function overlaps and flavor suppression following from our previous ansatz. We simply assume the SFC matrix elements are of comparable size. While we don’t compute these matrix elements explicitly (given the far larger uncertainty from the proliferation of operator coefficients) the contribution of any given operator of interest may be evaluated explicitly using the approach we have presented. Observation of any of these modes in the absence of conventionally expected nucleon decays would be a remarkable and unexpected sign of exotic physics not too far beyond directly accessible energies.

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|             | ΔS = 0 (×λ⁻⁴) | ΔS = 1 (×λ⁻²) | ΔS = 2 (×λ⁰) |
|-------------|----------------|----------------|---------------|
| \(\pi\pi\) | 2.54           | 1.91           | 1.00          |
| \(\rho\pi\) | 1.40           | 1.25           | 1.49          |
| \(\rho\rho\) | 0.90           | 1.11           | 1.10          |
| \(\pi\eta\) | 1.85           | 0.90           |               |
| \(\rho\eta\) | 1.06           | 1.40           |               |
| \(\pi\eta'\) | 1.18           | 0.96           |               |
| \(\rho\eta'\) | 0.96           | 0.96           |               |
| \(\eta\eta'\) | 0.94           | 1.87           |               |
| \(\eta\eta'\) | 1.32           |               |               |

**TABLE I.** Lifetime correction factors for typical \(NN\) decay modes. The numbers do not include the flavor parameter \(\lambda \approx 0.2\), which should be multiplied in as indicated.