A Generic Dynamical Model of Gamma-ray Burst Remnants

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ABSTRACT

The conventional generic model is deemed to explain the dynamics of \(\gamma\)-ray burst remnants very well, no matter whether they are adiabatic or highly radiative. However, we find that for adiabatic expansion, the model could not reproduce the Sedov solution in the non-relativistic phase, thus the model needs to be revised. In the present paper, a new differential equation is derived. The generic model based on this equation has been shown to be correct for both radiative and adiabatic fireballs, and in both ultra-relativistic and non-relativistic phase.

Key words: gamma-rays: bursts — hydrodynamics — relativity — shock waves
1 INTRODUCTION

Since the BeppoSAX detection of GRB 970228, X-ray afterglows have been observed from about 15 gamma-ray bursts (GRBs), of which 10 were detected optically and 5 even also in radio wavelengths (Costa et al. 1997; Kulkarni et al. 1998; Bloom et al. 1998; Piran 1998; and references therein). The cosmological origin of at least some GRBs is thus firmly established. The so called fireball model (Goodman 1986; Paczyński 1986; Rees & Mészáros 1992, 1994; Mészáros & Rees 1992; Katz 1994; Sari, Narayan & Piran 1996) is strongly favoured, which is found successful at explaining the major features of the low energy light curves (Mészáros & Rees 1997; Vietri 1997; Tavani 1997; Waxman 1997; Wijers, Rees & Mészáros 1997; Sari 1997; Huang et al. 1998; Dai & Lu 1998a; Dai, Huang & Lu 1998). A variant of this model, where central engines (e.g., strongly magnetized millisecond pulsars) supply energy to postburst fireballs through magnetic dipole radiation, has been proposed to account for the special features of the optical afterglows from GRB 970228 and GRB 970508 (Dai & Lu 1998b, c).

Since the expansion of a fireball may be either adiabatic or highly radiative, extensive attempts have been made to find a common model applicable for both cases (Blandford & McKee 1976; Chiang & Dermer 1998; Piran 1998). As a result, a conventional model was suggested by various authors (see for example, Chiang & Dermer 1998; Piran 1998). A dynamical model should be correct not only in the initial ultra-relativistic phase, which is well described by those simple scaling laws (Mészáros & Rees 1997; Vietri 1997; Waxman 1997), but also in the consequent non-relativistic phase, which is correctly discussed by using the Sedov solution (Sedov 1969; Wijers, Rees & Mészáros 1997). Although the conventional model is correct for the ultra-relativistic phase, we find it could not match the Sedov solution in the non-relativistic limit. So in this paper, we will construct a dynamical model that is really capable of describing generic fireballs, no matter whether they are radiative or adiabatic, and no matter whether they are ultra-relativistic or non-relativistic.
2 CONVENTIONAL DYNAMIC MODEL

A differential equation has been proposed to depict the expansion of GRB remnants (Chiang & Dermer 1998; Piran 1998),

\[ \frac{d\gamma}{dm} = -\frac{(\gamma^2 - 1)}{M}, \]  

(1)

where \( m \) is the rest mass of the swept-up medium, \( \gamma \) is the bulk Lorentz factor and \( M \) is the total mass in the co-moving frame, including internal energy \( U \). Since thermal energy produced during the collisions is \( dE = (\gamma - 1)dmc^2 \), usually we assume: \( dM = (1-\epsilon)dE/c^2 + dm = [(1-\epsilon)\gamma+\epsilon]dm \), where \( \epsilon \) is defined as the fraction of the shock generated thermal energy (in the co-moving frame) that is radiated (Piran 1998). It is putative that equation (1) is correct in both ultra-relativistic and non-relativistic phase, for both radiative and adiabatic fireballs. However, after careful inspection, we find that during the non-relativistic phase of an adiabatic expansion, equation (1) could not give out a solution consistent with the Sedov results (Sedov 1969).

2.1 Radiative case

In the highly radiative case, \( \epsilon = 1 \), \( dM = dm \), equation (1) reduces to,

\[ \frac{d\gamma}{dm} = -\frac{(\gamma^2 - 1)}{M_{ej} + m}, \]  

(2)

where \( M_{ej} \) is the mass ejected from the GRB central engine. Then an analytic solution is available (Blandford & McKee 1976; Piran 1998):

\[ \frac{(\gamma - 1)(\gamma_0 + 1)}{\gamma + 1)(\gamma_0 - 1)} = \left(\frac{m_0 + M_{ej}}{m + M_{ej}}\right)^2, \]  

(3)

where \( \gamma_0 \) and \( m_0 \) are initial values of \( \gamma \) and \( m \) respectively. Usually we assume \( \gamma_0 \sim \eta/2 \), \( m_0 \sim M_{ej}/\eta \), where \( \eta \equiv E_0/(M_{ej}c^2) \) and \( E_0 \) is the total energy in the initial fireball (Waxman 1997; Piran 1998).

During the ultra-relativistic phase, \( \gamma \gg 1 \), \( M_{ej} \gg m \), equation (3) gives \( (\gamma + 1)m \approx M_{ej} \), or equivalently the familiar power-law \( \gamma \propto R^{-3} \), where \( R \) is the radius of the blast wave. In the
later non-relativistic phase, $\gamma \sim 1$, $m \gg M_{ej}$, we have: $m^2\beta^2 = 4M_{ej}^2$, or $\beta \propto R^{-3}$, where $\beta = v/c$ and $v$ is the bulk velocity of the material. This is consistent with the late isothermal phase of the expansion of supernova remnants (SNRs) (Spitzer 1968). From these approximations, we believe that equation (2) is really correct for highly radiative fireballs.

2.2 Adiabatic case

In the adiabatic case, $\epsilon = 0$, $dM = \gamma dm$, equation (1) also has an analytic solution (Chiang & Dermer 1998):

$$M = \left[M_{ej}^2 + 2\gamma_0 M_{ej}m + m^2\right]^{1/2}, \quad (4)$$

$$\gamma = \frac{m + \gamma_0 M_{ej}}{M}. \quad (5)$$

During the ultra-relativistic phase, $\gamma_0 M_{ej} \gg m \gg M_{ej}/\gamma_0$, $\gamma \gg 1$, this solution can produce the familiar power-law $\gamma \propto R^{-3/2}$, which is often quoted for an adiabatic blastwave decelerating in a uniform medium. In the non-relativistic limit ($\gamma \sim 1$, $m \gg \gamma_0 M_{ej}$), Chiang & Dermer (1998) have derived $\gamma \approx 1 + \gamma_0 M_{ej}/m$, so that they believe it also agrees with the Sedov solution (Lozinskaya 1992). However we find that their approximation is not accurate enough, because they have omitted some first-order infinitesimals of $\gamma_0 M_{ej}/m$. The correct approximation could be obtained only by retaining all the first and second order infinitesimals, which in fact gives: $\gamma \approx 1 + (\gamma_0 M_{ej}/m)^2/2$, then we have $\beta \propto R^{-3}$. This is not consistent with the Sedov solution! We have also evaluated equation (1) numerically, the result is consistent with equations (4) and (5), all pointing to $\beta \propto R^{-3}$, not the relation of $\beta \propto R^{-3/2}$ as declared popularly in the literature (Chiang & Dermer 1998; Piran 1998).

This question is serious. First, it means that equation (1) is not a dependable model for non-radiative fireballs, although it can reproduce the major features in the ultra-relativistic phase. Second, the expansion of a realistic fireball is widely believed to be highly radiative at first, but after only several days, the expansion will become non-radiative (Sari, Piran & Narayan 1998; Dai, Huang & Lu 1998). So in the non-relativistic phase, the fireball is more likely to be adiabatic rather than highly radiative. However, it is just in this condition that the
conventional model fails. So any calculation made according to equation (1) will lead to serious deviations in the light curves in the non-relativistic phase.

3 OUR GENERIC MODEL

Equation (1) is not consistent with the Sedov solution, we need to revise it. In the fixed frame, since the total kinetic energy of the fireball is $E_K = (\gamma - 1)(M_{ej} + m)c^2 + (1 - \epsilon)\gamma U$ (Panaitescu, Mészáros & Rees 1998), and the radiated thermal energy is $\epsilon\gamma(\gamma - 1)dm c^2$ (Blandford & McKee 1976), we have:

$$d[(\gamma - 1)(M_{ej} + m)c^2 + (1 - \epsilon)\gamma U] = -\epsilon\gamma(\gamma - 1)dm c^2.$$  \hspace{1cm} (6)

For the item $U$, it is usually assumed: $dU = (\gamma - 1)dm c^2$ (Panaitescu, Mészáros & Rees 1998). Equation (1) has been derived just in this way. However, the jump conditions (Blandford & McKee 1976) at the forward shock imply that $U = (\gamma - 1)mc^2$, so we suggest that the correct expression for $dU$ should be: $dU = d[(\gamma - 1)mc^2] = (\gamma - 1)dm c^2 + mc^2d\gamma$. Here we simply use $U = (\gamma - 1)mc^2$ and substitute it into equation (6), then it is easy to get:

$$\frac{d\gamma}{dm} = -\frac{\gamma^2 - 1}{M_{ej} + \epsilon m + 2(1 - \epsilon)\gamma m}.$$  \hspace{1cm} (7)

We expect this equation should describe a generic fireball correctly.

Indeed, in the highly radiative case ($\epsilon = 1$), equation (7) reduces to equation (2) exactly. While in the adiabatic case ($\epsilon = 0$), equation (7) reduces to:

$$\frac{d\gamma}{dm} = -\frac{\gamma^2 - 1}{M_{ej} + 2\gamma m}.$$  \hspace{1cm} (8)

This equation has an analytic solution:

$$(\gamma - 1)M_{ej}c^2 + (\gamma^2 - 1)mc^2 \equiv E_{K0},$$  \hspace{1cm} (9)

where $E_{K0}$ is the initial value of $E_K$. In the ultra-relativistic phase ($\gamma_0 M_{ej} \gg m \gg M_{ej}/\gamma$), we get the familiar relation of $\gamma \propto R^{-3/2}$. And in the non-relativistic phase ($m \gg M_{ej}$), we get $\beta \propto R^{-3/2}$ as required by the Sedov solution.
For any other $\epsilon$ value between 0 and 1, equation (7) describes the evolution of a partially radiative fireball. Unfortunately, we now could not find an exact analytic solution for equation (7). But in the non-relativistic phase, by assuming $m \gg M_{\text{ej}}$, we still can get $m(\gamma - 1)(2-\epsilon)/2 \equiv \text{const}$, that is:

$$\beta \propto R^{-3/(2-\epsilon)}.$$  \hspace{1cm} (10)

### 4 NUMERICAL RESULTS

We have evaluated equation (7) numerically, bearing in mind that (Huang et al. 1998):

$$dm = 4\pi R^2 nm_p dR,$$  \hspace{1cm} (11)

$$dR = \beta c\gamma (\gamma + \sqrt{\gamma^2 - 1}) dt,$$  \hspace{1cm} (12)

where $n$ is the number density of interstellar medium, $m_p$ is the mass of proton, and $t$ is the time measured by an observer. We take $E_0 = 10^{52}$ ergs, $n = 1 \text{ cm}^{-3}$, $M_{\text{ej}} = 2 \times 10^{-5} \text{ M}_\odot$. Figures (1) – (4) illustrate the evolution of $\gamma$, $v$, $R$, and $E_K$ respectively. In these figures, we have set $\epsilon = 0$ (full lines), 0.5 (dotted lines), and 1 (dashed lines). It is clearly shown that our generic model overcomes the shortcoming of equation (1).

For example, for highly radiative expansion, the dashed lines in these figures approximately satisfy $\gamma \propto t^{-3/7}$, $R \propto t^{1/7}$, $\gamma \propto R^{-3}$, $E_K \propto t^{-3/7}$ when $\gamma \gg 1$, and $v \propto t^{-3/4}$, $R \propto t^{1/4}$, $v \propto R^{-3}$, $E_K \propto t^{-3/4}$ when $\gamma \sim 1$. While for adiabatic expansion, the full lines satisfy $\gamma \propto t^{-3/8}$, $R \propto t^{1/4}$, $\gamma \propto R^{-3/2}$ when $\gamma \gg 1$, and satisfy $v \propto t^{-3/5}$, $R \propto t^{2/5}$, $v \propto R^{-3/2}$ when $\gamma \sim 1$.

### 5 CONCLUSION AND DISCUSSION

The conventional dynamic model is successful at describing highly radiative GRB remnants, however it has difficulty in reproducing the Sedov solution for adiabatic fireballs. This is completely unnoticed in the literature. We have constructed a new generic model to overcome this shortcoming. Numerical evaluation has proved that our model is highly credible. We hope
this work would remind researchers of the importance of the transition from ultra-relativistic to non-relativistic phase, which might occur as early as $10^6 - 10^7$ s since the initial burst (Huang, Dai & Lu 1998).

In the above analysis, for simplicity, we have assumed that $\epsilon$ is a constant. But in realistic fireballs, $\epsilon$ is expected to evolve from 1 to 0 due to the changes in the relative importance of synchrotron-radiation-induced and expansion-induced loss of energy (Dai, Huang & Lu 1998). Assuming electrons in the co-moving frame carry a fraction $\xi_e = 1$ of the total thermal energy and that the magnetic energy density is a fraction $\xi_B^2 = 0.01$ of it, we re-evaluate equation (6) numerically. The results are plotted in Figs (1) – (4) with dash-dotted lines. We see from Fig. (4) that the evolution of $\epsilon$ changes $E_K(t)$ dramatically.

It is worth mentioning that SNRs evolve from non-radiative to radiative stage, but GRB remnants are just on the contrary. This is not surprising because GRB remnants radiate mainly through synchrotron radiation while SNRs lose energy due to excited ions. It is reasonable to deduce that at very late stages, when the cooling due to ions becomes important, GRB remnants may become highly radiative again, just in the same way that SNRs do. The transition may occur when the temperature drops to below $\sim 10^6$ K and the velocity is just several tens kilometer per second. This needs to be addressed in more detail.

Another interesting problem is the possibility that HI supershells might be highly evolved GRB remnants (Loeb & Perna 1998; Efremov, Elmegreen & Hodge 1998). Our Figs (3) and (4) have shown that typical adiabatic GRB fireballs can evolve to $R \sim 1$ kpc at $t \sim 10^6 - 10^7$ yr, with $v \sim 10$ km/s, but highly radiative fireballs are obviously not powerful enough. To discuss this possibility in detail, we should pay attention to the possible adiabatic-to-radiative transition mentioned just above.

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Figure Captions

Figure 1. Evolution of the bulk Lorentz factor $\gamma$. We take $E_0 = 10^{52}$ ergs, $n = 1$ cm$^{-3}$, $M_{ej} = 2 \times 10^{-5}$ M$_\odot$. The full, dotted, and dashed lines correspond to $\epsilon = 0$ (adiabatic), 0.5 (partially radiative), and 1 (highly radiative) respectively. The dash-dotted line is plotted by allowing $\epsilon$ to evolve with time (see Sect. (5) in the text).

Figure 2. Evolution of the bulk velocity $v$. Parameters and line styles are the same as in Fig. 1.

Figure 3. Evolution of the shock radius $R$. Parameters and line styles are the same as in Fig. 1.

Figure 4. Evolution of the total kinetic energy $E_K$. Parameters and line styles are the same as in Fig. 1.
