Azimuthal MHD stirring of metal in vessels with cross-sections of different configuration

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Abstract. Continuous casting of cylindrical ingots from aluminum and preparation of aluminum-based alloys and composites require intensive mixing of liquid metal phase in the crystallization area of the melt. It is evident that the topology of the flow in the liquid phase of an ingot should influence the processes occurring during crystallization. Contemporary continuous casting machines use MHD-stirrers that generate an azimuthal motion in a crystallizer with a warm top of circular cross-section in the presence of rotating magnetic field. The flow of metal in the liquid phase of an ingot is similar to its rotation in a solid state, and transport processes are most intensively carried out in the near near-wall region and near the ingot solidification front, where shear flows are essential. In this work, we consider the possibility of amplifying transport processes in the entire volume of a stirred metal by making the cross-section shape of the warm top of the crystallizer different from a circle. It has been found numerically that the total energy of the flow in a crucible of square cross-section is twice as lower as that in a crucible with circular cross-section at the same inductor current. Turbulent pulsations in the square crucible, as well as in the circular one, are concentrated mainly in the near-wall region. The energy of pulsations in the square crucible also reduces, but the time of stirring of the passive impurity introduced into the volume of the metal is less than in the circular crucible. The effect of MHD stirring on the vertical temperature distribution on the square crucible is higher than in the “round crucible”.

1. Introduction

MHD stirring of liquid metal has found wide application in metallurgy to produce alloys and in foundry to cast ingots during the crystallization process and is the world-class technology. MHD stirrers that generate a rotating magnetic field (most often used in metallurgical production) create azimuthal stirring of the melt during the continuous casting of cylindrical aluminum alloy-based ingots. This makes it possible to provide a homogeneous distribution of introduced impurities throughout the melt and to crush off dendrites at the crystallization front of the melt.

The study of stirring processes taking place in liquid aluminum and its alloys under the action of a rotating magnetic field is motivated primarily by the fact that this technique ensures the production of aluminum alloy-based composites and cast ingots having the structure suitable for thixoforming.
Furthermore, the application of MHD stirring allows one to improve operational conditions for casting high quality aluminum alloy-based parts that require no further processing [1-4].

The azimuthal flow of the melt generated by the rotating magnetic field in the upper cylindrical warm part of a continuous casting machine crystallizer induces turbulent pulsations in the near-wall area of the crystallizer top part, which increases markedly the diffusion processes in this region. Additionally, the main azimuthal flow induces weak meridional flows in this zone. All these processes cause a uniform distribution of impurities throughout the metal and have an impact on the heat/mass transfer processes in the metal. One can expect that the azimuthal flow and the resulting processes depend on the cross-section geometry of the upper warm part of a crystallizer.

There are a number of works, in which the processes induced in the cylindrical vessels with liquid metal under the action of a rotating magnetic field are explored [5-9]. However, the attention has been mainly focused on the hydrodynamic processes in cylindrical vessels with circular transverse cross-section, whereas the crystallizers with warm top commonly used in contemporary continuous casting machine have different design and geometry. This is a reason for studying here heat transfer processes induced in the vessels of different shape (e.g. those with square cross-section) under the action of a rotating magnetic field.

2. Problem formulation

Consider the flow of liquid metal in cylindrical crucibles of different shape generated by the rotating field of an MHD-stirrer (Fig.1). The stirrer consists of a steel core of complex shape and 12 coils through which an alternating electric current of force I and circular frequency \( \omega \) is supplied. The phases of electric current oscillations in the neighboring pairs of coils differ by \( \pi/3 \) so that the induction of the magnetic field generated by the electric current in the conductors is on average directed horizontally and rotates about the z-axis.

2.1 Electromagnetic forces

The rotating magnetic field induces an electric current in liquid aluminum. The interaction of this current with the magnetic field generates a volumetric force that entrains liquid metal. Under real production conditions the rotational velocity of liquid metal is markedly less than that of the rotating magnetic field. For this reason, we ignore the inverse effect of liquid metal on the magnetic field, which allows us to separate the electromagnetic and hydrodynamic problems and to solve them.
Electromagnetic phenomena are described by the system of Maxwell equations and the Ohm law:

\[ \nabla \times \vec{H} = \vec{j}, \quad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad \nabla \cdot \vec{B} = 0, \tag{1} \]

\[ \vec{j} = \sigma \vec{E}, \tag{2} \]

In equations (1) and (2), we use generally recognized notation for magnetic induction, electric field strength, vacuum magnetic permeability, electric current density and medium conductivity. For the media interface, the standard conditions for equality of the normal components of magnetic induction and the jump of tangential components are assumed:

\[ B_{n1} = B_{n2}, \quad \frac{1}{\mu_1} B_{r1} = \frac{1}{\mu_2} B_{r2} \tag{3} \]

Let us introduce the vector potential \( \vec{A} = B \times \vec{B} \). According to the harmonic law for changes in the electric current and assuming that the medium is homogeneous and non-dispersed, the electromagnetic problem can be essentially simplified. The time dependence of magnetic field and electric current can be represented as \( \vec{A} = \vec{A}_0 e^{i\omega t} \) and \( \vec{j} = \vec{j}_0 e^{i\omega t} \), which allows us to reduce the initial problem to the stationary problem for complex amplitudes:

\[ \nabla^2 \vec{A}_0 - i\omega \mu_0 \sigma \vec{A}_0 = -\mu_0 \vec{j}_0 \]

Equation (4) is solved by the finite-element method. The computational domain includes the MHD-stirrer, the crucible with molten metal and the surrounding extended area filled with air. Almost all the material, except the steel core, are non-ferromagnetic (\( \mu = 1 \)), and the magnetic permeability of the core is assumed to be constant: \( \mu = 2500 \). For the outside boundary \( \Gamma \) of the computational domain we assume that the normal component of the vector potential is equal to zero:

\[ \vec{A}_0 \big|_\Gamma = 0 \tag{5} \]

Solving the boundary-value problem (4)-(5) makes it possible to determine the field of electromagnetic forces in the molten metal:

\[ \vec{f}_{em} = \text{Re} \vec{j} \times \vec{B} \tag{6} \]

2.2. Hydrodynamic part

In this section we investigate the flow of liquid aluminum induced by the rotating magnetic field in crucibles with circular and square transverse cross-section. The height of liquid metal in these crucibles is 300 mm. The diameter of the round crucible is 127 mm, and the size of the side wall of the crucible is 112 mm.

The hydrodynamic processes taking place in liquid aluminum are described by the Navier-Stokes and continuity equations as

\[ \frac{\partial \vec{v}}{\partial t} + (\vec{v} \nabla) \vec{v} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \vec{v} + \frac{\vec{j}_{em}}{\rho}, \quad \nabla \cdot \vec{v} = 0 \tag{7} \]

No-slip boundary conditions are specified for the lower and lateral edges of cylinders \( \Lambda \), and slip conditions for the upper edge \( M \):

\[ \vec{v} \big|_\Lambda = 0, \quad \vec{v} \big|_M = 0 \tag{8} \]
Thus, the upper edge is considered to be the free surface of liquid, but the change in its shape caused by the fluid motion is not taken into account. The 3D boundary-value problem (7)-(8) has been solved using the finite-element method as well.

3. Results

3.1. Flow

Analysis of the results obtained during the solution of electromagnetic problem (4)-(5) indicates that the rotating magnetic field creates electromagnetic forces with predominantly azimuthal and radial directions in the crucibles of a shape symmetric relative to the z-rotational axis. Under these forces, the liquid metal performs the azimuthal rotation about the z-axis. Figure 2 show the trajectories of motion of particles for time \( \Delta t = 5 \, \text{c} \). The trajectories are spiral, because apart from the azimuthal component the velocity has the axial component oriented along the z-axis.

![Figure 2](image)

**Figure 2.** Liquid metal flow structure in cylindrical crucibles with circular (a) and square (b) profiles at \( I=8 \, \text{A} \) (9.2mTl): particle motion trajectories over the period of 5 s (left), and velocity field in the cross-section \( y=0 \). Velocity is measured in m/s.

The intensive motion of the fluid the azimuthal direction leads to turbulence evolution in the near-wall area. Turbulent pulsation energy distribution over the crucible cross-section is shown in Fig.3. Turbulence manifests itself only near the walls; in most cases it affects the flow only slightly. This is a characteristic feature of a cylinder with square shape, in which turbulence is less developed. The reason is that at equal values of currents in inductors the intensity of flows in a square crucible is significantly lower. For comparison, Table 1 presents for different excitation currents the flow characteristics, which are integral in the volume of the crucible: full kinetic energy \( E \), turbulent pulsation energy \( Et \), and kinetic energy of motion in the axial direction \( Ez \) (along the z-axis). Index 1 corresponds to the region with circular cross-section, and index 2 to the region with square cross-section. Based on the results given in this table, we can conclude that the intensity of azimuthal rotation in the circular crucible exceeds that in the square crucible by 30-50%. The same is true for turbulent pulsations (mechanisms governing turbulent diffusion), because their strength reduces with decreasing flow rate.

The kinetic energy of fluid motion in the axial direction \( Ez \) (Table 1) indicates that the flow intensity along the z-axis in the square vessel is essentially higher. In comparison with the main azimuthal rotation, this motion is rather weak; its rate is twice less. It is the secondary flow, which is
induced by the main azimuthal motion in the crucible. As a result, one vortex that moves vertically is formed. However, in the case when the vertical/horizontal size ratio is small, there occur relatively weak vortices induced additionally. This vertical flow has an essential effect on the heat transfer process and the stirring of impurities. The structure of vertical convection in crucibles is shown in Fig. 2, where one can see the velocity components $v_x$ and $v_z$ in the cross-section $y = 0$; the component $v_y$ that is perpendicular to the section plane is not shown. The vertical flow experiences high pulsations and therefore only the time-averaged flow pattern is given in the figure. The most intensive motion (main vortex) is observed in the lower part, where the fluid rises in the center and goes downward near the walls; The induced motion whose intensity is weaker and the size is smaller is observed in the upper part, where the fluid goes downward in the center and rises near the walls.

![Figure 2](image2.png)

**Figure 3.** The energy of turbulent pulsations at $I = 8$ A (9.2mTl) for crucibles with round (a) and square (b, c) profiles in the sections $y = 0$ (a, b) and $y = x$ (c). The isolines pass through $10^3$ m$^2$/s$^2$. In the center – zero value.

**Table 1.** Integral characteristics of the flow

| $I$, A | $E_1$, J | $E_2$, J | $E_{t_1}$, J | $E_{t_2}$, J | $E_{z_1}$, mJ | $E_{z_2}$, mJ |
|-------|---------|---------|--------------|--------------|--------------|--------------|
| 2     | 0.452   | 0.182   | 0.0036       | 0.0018       | 0.051        | 0.085        |
| 4     | 1.718   | 0.653   | 0.0138       | 0.0063       | 0.160        | 0.270        |
| 6     | 4.980   | 1.810   | 0.0299       | 0.0132       | 0.280        | 0.495        |
| 8     | 7.675   | 2.748   | 0.0517       | 0.0224       | 0.486        | 0.809        |

3.2. Influence of stirring on temperature vertical distribution

Let us consider the effect of MHD mixing of a liquid metal on the temperature distribution in a crucible, which is described by equation:

$$\frac{\partial T}{\partial t} + (\bar{v} \nabla) T = \chi \nabla^2 T$$ (9)

We assume that the lower end boundary exhibits temperature being close to aluminum crystallization temperature ($933$ K), the side boundaries are thermally insulated, and on the upper end boundary the heat flow $W = 10^4$ W/m$^2$ is imposed. Under such conditions, a vertical temperature gradient is set in the region. In the absence of stirring, this gradient is constant and can be expressed as $\nabla T = W/\kappa$, where $\kappa$ is the heat conduction coefficient. The vertical stirring changes the linear temperature profile, the upper boundary temperature and simultaneously the average vertical gradient.
decrease, and the temperature profile deviates from linear distribution. Since there is no turbulent stirring near the upper boundary (see Fig.3), the temperature profile varies due to the vertical convective flow, which is stronger in the crucible with square cross-section. Figure 4 presents vertical temperature profiles in the absence and presence of stirring. It is seen that stirring in the crucible with square cross-section has a stronger effect on vertical temperature distribution.

Figure 5 gives temperature distribution (in Kelvin degrees) over the cross-section $y=0$ at $I=8$ A (9.2 mTl) for the crucibles of circular (a) and square (b) cross-section.

### 3.3. Stirring of passive impurity

The transport of passive impurity is determined by diffusive and convective mechanisms and is described by the equation:

$$\frac{\partial C}{\partial t} + (\nabla V)C = D\nabla C$$  \hspace{1cm} (10)

Where $D$ is the diffusion coefficient, which includes both molecular and turbulent parts $D_T = \nu_T/Sc$, $\nu_T$ is turbulent viscosity, and $Sc$ is the Schmidt number.

A high quality alloy can be prepared through its homogenization. At the same time, the stirring period should be reduced, which will make it possible to decrease energy consumption and to its overheating. The degree of alloy homogenization is evaluated on the basis of the inhomogeneity coefficient $q(t)$ [10]:

$$q(t) = \sqrt{\frac{1}{n} \left[ (\bar{C} - C_1)^2 + (\bar{C} - C_2)^2 + ... + (\bar{C} - C_n)^2 \right]}$$  \hspace{1cm} (11)

i.e. the ratio of root-mean-square deviation of the impurity concentration at all the points $n$ of the computational domain (at time instant $t_1$) from the final concentration after perfect stirring to the root-mean-square deviation at initial time $t = 0$. Also, we introduce the parameter $t_{\text{mix}}$ (stirring time) – time interval within which $q$ decreases by a factor of 5.
Figure 6 illustrates the dynamics of the inhomogeneity coefficient $q(t)$ for three cases of initial distribution of impurity: 1 – impurity occupies 1/10 of the lower part of the crucible, 2 – impurity is in the crucible center, 3 – impurity near the upper boundary. The solid lines correspond to the crucible with circular cross-section, and the dashed lines to the crucible with square cross-section.

For the case when at the initial time instant the impurity occupies 1/10 of the crucible lower part, the characteristic stirring time is found for different values of current in the inductor (Fig.7).

**Figure 6.** Time dependence of the degree of impurity homogenization $I=8$ A (9.2mTl). $I, 2, 3$ – variants of the initial distribution of impurity: in the upper, lower and middle parts of the region. Solid lines - crucible with circular cross-section, and dashed lines - crucible with square cross-section.

**Figure 7.** Time interval within which the impurity propagates in the entire volume of aluminum $t_{\text{mix}}$ depending on the current strength for the crucibles of circular ($I$) and square (2) cross-section.

**4. Conclusion**
Our numerical experiments have indicated that the topology of flows in the square and circular crucibles is similar. A distinction between them lies in the fact that the profile of the azimuthal flow in the crucible of square cross-section differs more from the solid-state one than the profile in the crucible with square cross-section (Fig.8).

**Figure 8.** Azimuthal velocity profiles (a) in the plane $z=0.15$ at $I=8$ A (9.2mTl): 1-crucible of circular cross-section along its radius, and 2,3 – crucible of square cross-section (b) along the directions OA and OB.
This somewhat extends the region of shear flows, which improves the distribution of the introduced impurity. The area of largest turbulent pulsations, as in circular crucibles, is located near the walls. However, the flow intensity in the square crucible is much lower than in the circular crucible at the same current in the inductor, which causes the lower coefficient of turbulent diffusion. At the same time the vertical velocities of metal in the square crucible are significantly higher than in the circular crucible, which intensifies heat transfer processes and provides a more rapid distribution of impurity in the volume of the crucible.

It should be noted that heat and mass transfer processes can be strengthened by increasing the current in the inductor. However, in the circular crucible this will increase the azimuthal velocity, which in turn will cause an increase on the size of the funnel on the metal surface and splash the metal out of the crucible.

In the case of a square crucible, the azimuthal velocity is much lower, the surface deformation of metal in the crucible is insignificant and the azimuthal velocity can be increased further, which will intensify heat-and-mass transfer processes in the liquid metal inside the crucible.

5. References
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