Modeling Simulation and Maintenance Strategy Optimization of Multi-component System

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Abstract. As the system tends to be complicated and refined, the probability and cost of failure also increase. In the actual operation and production, the system will be affected by various factors such as temperature, humidity and other factors to change their original working environment, which will have greater impact on the reliability of the system. Based on the reliability theory, this paper mainly studies the $k$-out-of-$n$: $F$ system, constructs the reliability model of the system in dynamic environment, gives the simulation algorithm of the system reliability. Finally, this paper puts forward two maintenance strategies which are optimized to achieve the purpose of minimizing the long-term average cost during system operation.

Keywords: Multi-component system, maintenance cost, preventive maintenance, dynamic environment, $k$-out-of-$n$ system.

1. Introduction

The improvement of science and technology promotes the change of social production and manufacturing structure, and the product system structure produced by socialized mass production tends to be complicated, randomized and varied. The purpose of the system is to make the product have better performance and longer life. However, the more complex the system, the higher the probability of failure. Therefore, there is a great contradiction between complexity and reliability, so we need to find a balance between them. In the process of modernization in China, the proposal of "Made in China 2025" has aroused widespread concern in the industry about product reliability. The reliability theory was born in the 1930s, and the first field studied was machine maintenance[1]. The optimization of maintenance strategy is to maximize the reliability of the system under the condition of minimizing the cost and to ensure its high availability and security[2-5]; For reducing the probability of system or product failure, the method of update theory by Lotka[6] and Campbell[7] related to component replacement strategy has been discussed in the later part of the paper. Franceschini and Galetto[8] consider the potential efficiency of the system, propose a regular component replacement strategy, and finally establish and optimize the maintenance strategy model targeting cost minimization.

The innovation of this paper is that on the basis of regular replacement, this paper puts forward the strategy of preventive maintenance, and changes the research environment from simple environment to complex dynamic environment. The purpose of this paper is to find out a strategy to minimize the average cost of the system during operation.
2. Reliability Modeling and Simulation of Multi-component System

2.1. Basic Assumptions and Reliability Modelling

The life of a system and the performance of its components are affected by different environments. The system operates in normal (Environment 0) and harsh environment (Environment 1), and the change of environment is independent of the state of the system. Figure 1 shows an example of a k-out-of-n system with \( n=6 \) and \( k=4 \). The basic assumptions of the system are as follows:

- \( n \): The number of components
- \( k \): The minimum number of failed components that makes the system stop working
- \( i \): The environment where the system operates
- \( Y_i \): The duration of Environment \( i \), \( i=0,1 \)
- \( \lambda_i \): The parameter of the exponential distribution which \( Y_i \) submits to, \( i=0,1 \)
- \( H_i(t) \): Cumulative distribution function of \( Y_i \), \( i=0,1 \)
- \( h_i(t) \): Probability density function of \( Y_i \), \( i=0,1 \)
- \( X_i \): The life of the component working in the Environment \( i \), \( i=0,1 \)
- \( \mu_i \): The parameter of the exponential distribution which \( X_i \) submits to, \( i=0,1 \)
- \( F_i(t) \): Cumulative distribution function of \( X_i \), \( i=0,1 \)
- \( f_i(t) \): Probability density function of \( X_i \), \( i=0,1 \)

![Figure 1. The k/n[F] system (k=4, n=6).](image-url)

Based on the above discussion, we establish the system reliability model in two steps. The first step is to model the environment. The system will be assumed to work alternately in two different environments, so we can study the reliability of the system in dynamic situations. The duration of each system in one environment is not fixed, but the duration of the Environment \( i \) is subject to the specified exponential distribution. Therefore, it can be concluded that there is a need to introduce a two-state Markov Mechanism Transformation Model. If \( i \) represents two different states of the environment, the change of the environment can be represented by a two-state Markov process \( \{Y(t), t \geq 0\} \). The transfer rate matrix of the system between different environments is

\[
Q = \begin{bmatrix}
-\lambda_0 & \lambda_0 \\
\lambda_1 & -\lambda_1
\end{bmatrix}
\]

The second step is to establish a reliability model for the k/n[F] system. If the life of a single component is assumed \( X \), the reliability of the component in a stable environment is the probability that the system continues to work longer than the time \( t \), that is

\[
R_0(t) = P\{X > t\} = 1 - F(t) = \bar{F}(t)
\]

\( R_0(t) \) is the probability of that the component \( X \) does not fail in time \([0, t]\). Now we study \( n \) components in the system, assuming that the life of each component is \( X_1, X_2, X_3, \ldots, X_n \), and they
are independent of each other, the reliability of each component is \( R_0(t) \). At the beginning, all parts of the system start working at the same time \( t=0 \), and they are all initially intact parts. So the reliability of the system above is

\[
R(t) = \sum_{j=k}^{n} \binom{n}{j} P\{X_{j+1}, \ldots, X_n \leq t < X_1, \ldots, X_j\} = \sum_{j=k}^{n} \binom{n}{j} R_0^j(t)[1 - R_0(t)]^{n-j}
\]

\[
= \frac{n!}{(n-k)!(k-1)!} \int_0^{R_0(t)} x^{k-1}(1-x)^{n-k} dx
\]

(3)

Then, we take the factors of environmental change into account, and the system will work alternately in two different environments. After the environment changes \( (i=i-1) \), the system starts to work in the new environment, and judge whether the number of invalid components is more than \( k \) in the duration of the new environment.

2.2. The Simulation of System Reliability

In this section, the simulation results of the system reliability function are given by the MATLAB through simulation. The steps of system reliability simulation are as follows:

Step 1: Assign an initial value to the relevant parameters. Set \( n, k \), the state of the environment \( i = 0 \), the number of failed components \( S = 0 \), and the life of the system \( H = 0 \).

Step 2: Randomly generate values for each component life and current environmental duration. The life of components \( X_1, X_2, X_3, \ldots, X_n \) in the same working environment are randomly generated by the MATLAB, which obey the exponential distribution of parameter \( \mu_i \) and arranged in order from small to large. And then generates \( Y_i \).

Step 3: Compare. After the initial \( n+1 \) random numbers are generated, the life of each component is compared with the environmental duration: if \( X_1, X_2, \ldots, X_n > Y \), Then \( H = H + Y \), \( i = 1 - i \) and returns to Step 2.

Step 4: Judge whether system fails. If \( X_1, X_2, \ldots, X_n > Y \), then count the number of failed components \( SUM(X > Y) \) and judge them: if at least \( k - S \) components have failed, the system will fail, Then and turn to Step 5; Otherwise count the number of failed components \( j \) at this stage. Then \( S = S + j \), \( H = H + X_{k-S} \), \( i = 1 - i \) and return to Step 2.

Step 5: End. Obtain the simulation result of system’s life \( H \).

3. Maintenance Modeling and Replacement Strategy Optimization

This section will continue to take the system in dynamic environment as the research object, and put forward two maintenance policies: Strategy I is Breakdown Maintenance, Strategy II is Preventive Maintenance, and the case is used to simulate the long-term average cost of the system under each Strategy.

3.1. Strategy I: Breakdown Maintenance Modeling and Simulation

3.1.1. Basic assumptions and modeling. The Breakdown Replacement Strategy means that when the system shuts down because of failure, we need to replace all the components. Every time the system is repaired, the system will start operating from the Environment 0. Figure 2 shows an example of \( k\)-out-of-\( n \) system with \( n=6 \) and \( k=4 \). The basic parameters of the system are as follows:

\( Y_i \): The duration of Environment \( i \), \( i=0,1 \)

\( \lambda_i \): The parameter of the exponential distribution which \( Y_i \) submits to, \( i=0,1 \)

\( X_j \): The life of the component working in the Environment \( i \), \( i=0,1 \)

\( X\mu_i \): The parameter of the exponential distribution which \( X_i \) submits to, \( i=0,1 \)
$C_r$: The cost of the replacing all components after shutdown each time

$C_s$: The daily loss cost caused by the shutdown

$d$: The maintenance time required after each shutdown of the system

$dmu$: The parameter of the exponential distribution which $d$ submits to

$C_T$: The long-term cost of operation and maintenance

The long-term cost of operation and maintenance will be the sum of the replacement cost of the components and the cost of the system shutdown. So

$$C(T) = \frac{C_p \cdot W(T) + C_s \cdot D(T)}{T}$$

(4)

$W(T)$ is the expected number of system failures in time $(0, T]$ and $D(T)$ is the expected total number of days of system shutdown in time $(0, T]$. When time $T$ tends to infinity, the cost of long-term operating is

$$C(T) = \lim_{{T \to \infty}} \frac{\text{Expected cost in time}(0, T]}{T}$$

(5)

3.1.2. The Simulation of Cost. The simulation steps are as follows:

**Step 1**: Assign initial value to the relevant parameters. Set $n$, $k$, $C_f$, $C_s$; The number of days the system has run $R = 0$; the total number of system failures $W = 0$; the total number of days the system stops $D = 0$; The initial $\text{Environment } i = 0$; the total cost $C = 0$, the average cost of long-term operation and maintenance $AC = 0$; the number of failed parts $S = 0$.

**Step 2**: Randomly generate $d$, and generate the system life $H$ according to Section 2 algorithm.

**Step 3**: Define the total number of running days $Long T$ and compare with $R$. If $R < Long T$, then $R = R + H + d$, $D = D + d$ and the running times of MATLAB progress $W = W + 1$, then turn **Step 4**; if $R \geq Long T$, then $C = C_f \cdot W + C_s \cdot D$, turn to **Step 2**.

**Step 4**: End. Obtain the long-term average cost of operation and maintenance $AC$.

3.2. Strategy II: Preventive Maintenance Modeling and Simulation

3.2.1. Basic assumptions and modeling. Preventive replacement Strategy means that the system will be checked in a fixed time during the process of system operation. If it is found that the number of failure components reaches the pre-set threshold $G$, these invalid components would be replaced. Of course, if the system fails due to the excessive number of failure parts before checks, it would be replaced afterwards. Our aim is to find the optimal preventive maintenance threshold $G$ to minimize the average cost of the system during a certain period of operation. Figure 3 shows a possible sample path of the system under Strategy II. When system failed, maintenance personnel will replace the all components. The basic parameters of the system are as follows:

$Y_i$: The duration of $\text{Environment } i, i=0, 1$

$\lambda_i$: The parameter of the exponential distribution which $Y_i$ submits to, $i=0, 1$
$
X_i$: The life of the component working in the Environment $i$, $i=0,1$  

$\mu_{X_i}$: The parameter of the exponential distribution which $X_i$ submits to, $i=0,1$  

$C_f$: The cost of the replacing all components after shutdown each time  

$C_s$: The daily loss cost caused by the shutdown  

d$: The maintenance time required after each shutdown of the system  

$d_{mu}$: The parameter of the exponential distribution which $d$ submits to  

$C_r$: The long-term cost of operation and maintenance  

t$: The interval for checking point  

$G$: The preventive maintenance threshold  

$m$: The number of invalid components  

$C_d$: The cost of invalid component for replacement during the check  

\begin{figure}[h]
  \centering
  \includegraphics[width=\textwidth]{figure3.png}
  \caption{Strategy II: A sample path of $k/n[F]$ system ($n=6, k=4, G=3$).}
\end{figure}

If $m < G$, the system will continue to work properly without any cost;  
If in time $(0, T]$, $G \leq m < k$, the system does not fail, and only the failed components need to be pre-replaced, the total cost is

$$C(T) = C_d \cdot m(T)$$  \hspace{1cm} (6)  

$m(T)$ is the number of pre-replaced components in time $(0, T]$. The average cost is

$$AC(T) = \frac{C_d \cdot m(T)}{T}$$  \hspace{1cm} (7)  

If in time $(0, T]$, the system has not only been pre-replaced processing, but also experienced Breakdown Replacement, then the total cost is

$$C(T) = C_d \cdot m(T) + C_p \cdot W(T) + C_s \cdot D(T)$$  \hspace{1cm} (8)  

$W(T)$ is the number of system downtime in time $(0, T]$ and $D(T)$ is the total number of downtime in time $(0, T]$. The average cost is

$$AC(T) = \frac{C_d \cdot m(T) + C_p \cdot W(T) + C_s \cdot D(T)}{T}$$  \hspace{1cm} (9)  

3.2.2. The Simulation of Cost. The simulation steps are as follows:  

**Step 1**: Assign initial value to the relevant parameters. Set $n$, $k$, $C_f$, $C_s$, $C_d$, $t$, $G$; The number of days the system has run $R = 0$; The total number of system failures $W = 0$; The total days of the system stopped $D = 0$; The environmental status $i = 0$; The total cost $C = 0$; The average cost of long-term operation and maintenance $AC = 0$; The number of failed components $S = 0$, the number of failed parts at the time of check $m = 0$ and the cumulative detection time $T = 0$.  

**Step 2**: Randomly generate values for each component life $X_1, X_2, ..., X_n$ and current environmental duration $Y_i$.  

**Step 3**: Determine whether $T$ is 0. If $T > 0$, turn to Step 4; Otherwise, $T = T + t$ and turn to Step...
Step 4: Determine the relationship between the check point and the environmental change point.
If $T < Y$, count the number of failed components in check $m = SUM(X < Y)$, then turn to Step 5;
If $T \geq Y$, count the number of failed components in check $m = SUM(X < Y)$, and turn to Step 6.

Step 5: Determine whether to carry out preventive replacement Strategy. Set the value of $dmu$.
(1) If $m \geq k$, randomly generate $d$, then $H = H + X_{k-S}$, $C = C + C_f + C_s * d$, $R = R + H + d$
and turn to Step 7;
(2) If $m < k$, return to Step 3;
(3) If $G \leq m < k$, then $C = C + C_d * m$, $H = H + t$, $R = R + H$ and return to Step 7.

Step 6: Determine whether the system is invalid.
(1) If $m < k$, then $H = H + Y$, $i = 1 - i$, $S = m$. Then $T = t - \text{mod}(H, t)$ and return to Step 2;
(2) If $m \geq k$, randomly generate $d$, then $H = H + X_{k-S}$, $C = C + C_f + C_s * d$, $R = R + H + d$
and turn to Step 7.

Step 7: Define the total running days $LongT$ and compare it with $R$. If $R < LongT$, return to Step 1, $i = 0$, $T = 0$, $S = 0$, $H = 0$; If $R \geq LongT$, turn to Step 8.

Step 8: $AC = C / R$. Obtain the average cost of long-term operation and maintenance of system under Strategy II.

3.3. Case Study: 7-out-of-16 Bearing System
This section selects bearings to study the replacement Strategy for the system. Bearing is the core system of many mechanical equipment, and it is also the most vulnerable to environmental damage. The bearing will suffer from excessive temperature and fatigue damage under repeated loads, the temperature distribution of the bearing will also affect the life of bearing in varying degrees\[9\]. So we assume that a bearing contains 16 balls, and it will be considered to be invalid if the number of deformed balls is greater than or equal to 7. Therefore, the system studied in this section is a $k/n[F]$ system with $n = 16$, $k = 7$, and the life of each component is exponentially distributed. The bearing will work alternately in normal Environment 0 with moderate load and low temperature, or in Environment 1 with heavy load and high temperature. Basic assumptions of this case are shown in Table 1.

Table 1. Basic assumption parameter of the bearing system.

| Parameter | Value |
|-----------|-------|
| $\lambda_0$ | 1/5 |
| $X_{\mu_0}$ | 1/20 |
| $\lambda_1$ | 1/3 |
| $X_{\mu_1}$ | 1/10 |
| $C_f$ | 24 yuan/time |
| $C_s$ | 60 yuan/time |
| $C_d$ | 1.5 yuan/time |
| $dmu$ | 1/5 |
| $LongT$ | 10,000 |

The probability distribution diagram of bearing’s life can be obtained by running program in MATLAB for 100,000 times. Figure 4 shows the result.
The specific optimization of the replacement policies are as follows:

Strategy I: The case is simulated according to the parameters, the average cost $AC$ after simulation is 22.83 yuan / day.

Strategy II: Optimize the preventive replacement Strategy for the case. By enumerating different $G$, we can optimize the average cost of long-term operation and maintenance of the case. The enumeration interval of $G$ is [1,6], and the cost distribution map is obtained as shown in Figure 5.

It can be seen that when the pre-maintenance threshold is set to 4, the average cost of long-term operation and maintenance of the system will reach the minimum cost, which is about 13.0 yuan / day. Compared with the Strategy I, the average cost of Strategy II is much lower. Therefore, when adopting Strategy II and set the pre-maintenance threshold $G = 4$, the purpose of maintenance strategy and cost optimization is achieved.

4. Conclusion
In this paper, the reliability of the k-out-of-n: F system in dynamic environment and the corresponding replacement Strategy optimization are studied. According to the state change caused by the number of parts failure, considering the change of the system in different working environment and the various costs during operation, the simulation and analysis of the bearing case are carried out with both replacement policies. The results show that the long-term cost of preventive maintenance Strategy is much lower than that of Breakdown replacement Strategy.

There are still many shortcomings in our study of multi-component systems. The research will consider the other possible aspects in reality, such as extension of the system state, extension of system structure and dependence between components.

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