Electron Pair Resonance in the Coulomb Blockade

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We study many-body corrections to the cotunneling current via a localized state with energy $\epsilon_d$ at large bias voltages $V$. We show that the transfer of electron pairs, enabled by the Coulomb repulsion in the localized level, results in ionization resonance peaks in the third derivative of the current with respect to $V$, centered at $eV = \pm 2\epsilon_d/3$. Our results predict the existence of previously unnoticed structure within Coulomb-blockade diamonds.

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Introduction.—Current flow through a single localized state (LS) coupled to metallic leads is a paradigm of quantum transport through nanostructures, with applications to many systems such as impurities embedded in tunnel barriers, quantum dots, single-molecule junctions, or carbon nanotubes, see, e.g., Refs. 1, 2, 3, 4, 5. Despite its simplicity, this system exhibits a wide range of transport behaviors, including resonant and sequential tunneling, cotunneling, and the Kondo effect.

All of these regimes are captured remarkably well by a simple extension of the Anderson impurity model

$$
H = \sum_\sigma \epsilon_dd_\sigma^\dagger d_\sigma + U n_1 \uparrow n_1 \downarrow + \sum_{k\sigma\alpha} \epsilon_k c_{k\sigma\alpha}^{\dagger} c_{k\sigma\alpha} + \sum_{k\sigma\alpha} [t_{\alpha} d_\sigma^\dagger c_{k\sigma\alpha} + t_{\alpha}^* c_{k\sigma\alpha}^\dagger d_\sigma], \tag{1}
$$

which describes tunneling of amplitude $t_\alpha$ between the spin-degenerate LS of energy $\epsilon_d$ (with creation operator $d_\sigma^\dagger$ and number operator $n_\sigma = d_\sigma^\dagger d_\sigma$) and two leads $\alpha = L, R$ (with dispersion $\epsilon_k$ and creation operator $c_{k\sigma\alpha}^{\dagger}$). For large on-site Coulomb repulsion $U$, double occupation of the LS is suppressed and the nature of transport depends on both $\epsilon_d$ (tunable by a gate voltage $V_g$) and the bias voltage $V$. Within the shaded areas of the stability diagram in Fig. 1 the average occupation $n_d = n_\uparrow + n_\downarrow$ of the LS is close to integer and current flow is suppressed by the Coulomb blockade. In contrast, current can flow by sequential tunneling processes outside the shaded areas, where the average occupation of the dot is no longer integer. This picture of the Coulomb blockade has been confirmed in numerous experiments performed on various systems 1, 2, 3, 4, 5.

It is the main point of this paper that even the minimal model of Eq. (1) predicts additional structure within the Coulomb-blockaded region, emerging from two-electron ionization of the LS at large biases. This ionization process is an effect of many-body correlations, enabled by the on-site Coulomb repulsion, which is much more robust than the Kondo correlations emerging in the Kondo valley $n_d = 1$ at low temperatures and small voltages. Indeed, the fine structure due to the two-electron ionization process exists in both Kondo and non-Kondo valleys, as illustrated in Fig. 1.

The two-electron ionization requires biases beyond a threshold voltage $V_c$, indicated by the thick black lines in Fig. 1. Below the threshold voltage, correlated two-electron transfers between the two leads constitute a precursor effect to two-electron ionization. While the limit of the Coulomb blockaded region is characterized by a resonance peak in $d^3I/dV^3$ due to opening of two-electron ionization, for $eV$ below the threshold voltage $eV_c = 2\epsilon_d/3$, tunneling of electron pairs is a precursor effect to two-electron ionization.

FIG. 1: (Color online) Schematic stability diagram of a single-level quantum dot. The thick lines within the shaded Coulomb blockaded region are characterized by a resonance peak in $d^3I/dV^3$ due to opening of two-electron ionization. For $eV$ below the threshold voltage $eV_c = 2\epsilon_d/3$, tunneling of electron pairs is a precursor effect to two-electron ionization.

Most of our conclusions carry over to many-level quantum dots (“metallic dots”) where the stability diagram...
from the onset of two-electron ionization occur for
energy amplitudes of the steps (i) and (iii) are proportional to
and (ii) tunnels into the state
energy
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exhibits a sequence of Coulomb diamonds, reflecting the
step-wise population of the dot with increasing gate volt-
age. This is depicted in Fig. 2, where we include the effects
of asymmetric capacitances between dot and electrodes.

In the remainder of the paper, we quantify the behavior
of the current near the two-particle threshold.

Two-electron ionization. — Ionization by means of sin-
gle particle tunneling becomes energetically allowed when
two-particle ionization is enabled by the on-site Coulomb
interaction.

Based on Eq. (3) and energy conservation, the two-
electron ionization rate per spin, at \( T = 0 \), is

\[
\Gamma_{\text{ion}} = \frac{\Gamma_L^2 \Gamma_R}{(2\pi)^2} \int_{-\infty}^{\infty} \frac{dE_1}{d\epsilon_1} \int_{-\infty}^{\infty} \frac{dE_2}{d\epsilon_2} \int_{-eV/2}^{eV/2} d\epsilon \delta(\epsilon_1 + \epsilon_2 - \epsilon_d - E_1),
\]

where \( h = 1 \). Here \( \Gamma_L = 2\pi|t_L|^2\nu \) and \( \Gamma_R = 2\pi|t_R|^2\nu \) are the partial widths of the LS due to escape to source and
and drain, respectively, and \( \nu \) denotes the density of states in
the leads. Performing the integration over \( \epsilon_2 \), we obtain

\[
\Gamma_{\text{ion}} = \frac{\Gamma_L^2 \Gamma_R}{(2\pi)^2} \int_{-\infty}^{\infty} \frac{dE_1}{d\epsilon_1} \int_{-eV/2}^{eV/2} d\epsilon \theta(eV/2 + \epsilon_1 - E_1 - \epsilon_d) \frac{\theta(\epsilon_1 + \epsilon_2 - \epsilon_d)}{(\epsilon_d - \epsilon_1)^2(\epsilon_1 - \epsilon)^2},
\]

The predicted lines in the stability diagram originating
from the onset of two-electron ionization occur for
\( V = \pm V_c \). Thus, they are located within the Coulomb
blockaded region which extends up to \( eV = \pm 2e\epsilon_d \).

Microscopically, the two-electron ionization process proceeds as follows, cf. Fig. 3(a): (i) An electron with
energy \( \epsilon_1 \) from the source electrode (L) enters the LS
and (ii) tunnels into the state \( E_1 \) of the drain (R). In the
same process, (iii) a second electron with opposite spin and
energy \( \epsilon_2 \) tunnels from the source into the LS. The
amplitudes of the steps (i) and (iii) are proportional to
\( t_L \), while the amplitude of step (ii) is proportional to \( t_R^* \).
Thus, the resulting amplitude of two-electron ionization
is given by

\[
A_{\epsilon_1 - \epsilon_d}^{\epsilon_2} = \frac{t_L^2 t_R^*}{(\epsilon_d - \epsilon_1)(E_1 - \epsilon_1)}.
\]

Following standard perturbation theory, the energy
denominators are given by the difference between the inter-
mediate and initial energies. In Eq. 3, we assumed
a large on-site Coulomb repulsion \( U \) so that there is no
contribution from virtual states with double occupation
of the LS. If these states were included, the correspond-
ing terms would exactly cancel the amplitude Eq. 3
in the limit of vanishing \( U \). This makes it manifest that
two-electron ionization is enabled by the on-site Coulomb
interaction.

Based on Eq. 3 and energy conservation, the two-
electron ionization rate per spin, at \( T = 0 \), is

\[
\Gamma_{\text{ion}} = \frac{\Gamma_L^2 \Gamma_R}{(2\pi)^2} \int_{-\infty}^{\infty} \frac{dE_1}{d\epsilon_1} \int_{-\infty}^{\infty} \frac{dE_2}{d\epsilon_2} \int_{-eV/2}^{eV/2} d\epsilon \delta(\epsilon_1 + \epsilon_2 - \epsilon_d - E_1),
\]

where \( h = 1 \). Here \( \Gamma_L = 2\pi|t_L|^2\nu \) and \( \Gamma_R = 2\pi|t_R|^2\nu \) are the partial widths of the LS due to escape to source and
and drain, respectively, and \( \nu \) denotes the density of states in
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\]

of the two-electron ionization rate \( \Gamma_{\text{ion}} \).

It is crucial that energy exchange between electrons in
the leads does not require direct interaction between
them. Instead, this process is enabled by the finite
coulomb repulsion in the LS alone. In this regard, the
underlying physics of two-particle ionization is similar
to that of energy exchange between electrons in a bulk
metal, facilitated by a magnetic impurity [6]. Indeed, it
is the non-zero on-site coulomb repulsion \( U \) that ulti-
marily generates the magnetic impurity [7]. Curiously,
similar many-body processes can also be enabled by the
pairing interaction in devices consisting of two Josephson
junctions in series, where they lead to subgap structure
in the current [8].
After entering the empty LS with rate $\Gamma_\text{ion}$ by two-electron ionization, the electron rapidly escapes into source or drain electrode by single-electron tunneling. These depopulation processes occur with rates $\Gamma_L$ and $\Gamma_R$, respectively. Thus, the average occupation of the LS is governed by the rate equation

$$2\Gamma_\text{ion}(1-n_1)(1-n_1) = (\Gamma_L + \Gamma_R)n_d. \quad (7)$$

Here, the factor 2 accounts for spin. Eq. (7) yields $n_d \approx 2\Gamma_\text{ion}/(\Gamma_L + \Gamma_R)$. Since the net charge transfer is $2e$, when the electron tunnels out to the drain (source) electrode, the “two-electron ionization” current $I(V)$ between the leads becomes

$$I(V) = e(2\Gamma_R + \Gamma_L)n_d \approx 2e\Gamma_\text{ion} \frac{2\Gamma_R + \Gamma_L}{\Gamma_L + \Gamma_R}. \quad (8)$$

Due to $\Gamma_\text{ion}$, the ionization current $I(V)$ also exhibits the threshold behavior $I(V) \propto (V - V_c)^2\theta(V - V_c)$.

Clearly, the ionization current, Eqs. (7) and (8), constitutes but a small fraction $\sim \Gamma_L(V - V_c)^2/V_c^2$ of the cotunneling current $\sim \frac{2\Gamma_L}{V_c}V$. Thus, it is an important question how the threshold anomaly Eq. (7) can be distinguished from the background cotunneling current. Eq. (8) predicts that two-electron ionization induces a jump in $dI/dV$ located at $V = V_c$. We now turn to a more careful analysis of this jump, focusing first on the two-electron current below threshold, before deriving a general interpolation formula.

Two-electron current below threshold.—For voltages below the threshold, $V < V_c$, ionization of the LS is no longer possible by two-electron processes. But two-electron processes can still excite electrons in the leads to just below the energy of the LS. We will now show that this constitutes a precursor effect to two-electron ionization which contributes a logarithmically singular threshold dependence to the differential conductance.

For large on-site Coulomb repulsion $U$, the two-electron process below threshold proceeds microscopically as follows, cf. Fig. 3(b): i) A spin-up electron from lead $\alpha_1$ with energy $\epsilon_1$ enters the LS; (ii) the electron tunnels out to state $E_1$ in lead $\alpha'_1$; (iii) a spin-down electron from lead $\alpha_2$ with energy $\epsilon_2$ enters the LS and (iv) leaves into state $E_2$ in lead $\alpha'_2$. The corresponding amplitude is

$$A_{\alpha'_2-E_2}^{\alpha_2-E_1} = \frac{t_{\alpha_1}\epsilon_2 t_{\alpha'_1}^{\epsilon_1}}{(\epsilon_d - \epsilon_1)(E_1 - \epsilon_1)(\epsilon_d - E_2)} + (1 \leftrightarrow 2), \quad (9)$$

where the second term accounts for the four-step process described above with the interchanged order (iii) $\leftrightarrow$ (i). This results in a scattering rate

$$\Gamma_{\alpha_2-\alpha'_2}^{\alpha_1-E_1} = 2\pi\hbar \int \sum_{i=1}^{2} |d\epsilon_i E_i\{1 - f(E_i - \epsilon_i')\}|$$

$$\times \ |A_{\alpha'_2-E_2}^{\alpha_2-E_1}|^2 \delta(\epsilon_1 + \epsilon_2 - E_1 - E_2). \quad (10)$$

Here $f(\epsilon) = [e^{\epsilon/T} + 1]^{-1}$ and $\mu_{L/R} = \pm eV/2$. The resulting two-electron tunneling current contains two contributions: $I = I^{(1e)} + I^{(2e)}$, where $I^{(2e)} = 2e\Gamma_{L-R}^{\alpha_1-\alpha'_2}$ corresponds to two-electron transfer between the leads, while $I^{(1e)} = e\sum_{\alpha}(\Gamma_{L,R}^{\alpha-\alpha'} + \Gamma_{L,R}^{\alpha'-\alpha})$ accounts for one-particle transfer between the leads, accompanied by the creation of a particle-hole excitation in one lead.

The crucial observation is that $I$ is singular as $V$ approaches $V_c$ from below. The singularity arises from the domain $E_1 \simeq -eV/2$, $E_2 \simeq 3eV/2$, $\epsilon_1 \simeq \epsilon_2 \simeq eV/2$. To see this, we first note that in this domain, the amplitude $A_{\alpha'_2-E_2}^{\alpha_2-E_1}$ simplifies, $A_{\alpha'_2-E_2}^{\alpha_2-E_1} \simeq -t_{\alpha_1}^{\epsilon_2} t_{\alpha'_1}^{\epsilon_1} / (eV)^2$. Using the Golden Rule Eq. (10), and performing the integrals over $\epsilon_1$, $\epsilon_2$, and $E_1$, we obtain for $eV_c > T$

$$I = \frac{2e}{\hbar} \frac{\Gamma_{L}^2 \Gamma_{R} (\Gamma_{L} + \frac{1}{2} \Gamma_{R})}{(2\pi)^2 (eV_c)^4} \int dE_2 1 - f(E_2 + eV/2)$$

$$\times \ f(E_2 - 3eV/2) \left[(\pi T)^2 + (E_2 - 3eV/2)^2\right]. \quad (11)$$

Since both $E_2$ and $\epsilon_d$ in the denominator of Eq. (11) are close to $3eV/2$, the remaining integration yields the singular contribution

$$\frac{dI}{dV} = \frac{2e^2}{\hbar} \frac{3\Gamma_L^2 \Gamma_R (\Gamma_L + \frac{1}{2} \Gamma_R)}{(2\pi)^2 (eV_c)^4} \ln \left(\frac{eV_c}{\max\{eV_c - eV_cT\}}\right) \quad (12)$$

to the differential conductance. The logarithmic singularity in the two-electron tunneling current at $V_c$ signals the opening of the two-particle ionization channel in Eq. (8) which is lower order in the tunneling amplitudes and involves real occupation of the LS.

The appearance of $T$-dependence in Eq. (12) at $eV_c > T$ resembles the behavior of the conductance near the onset of sequential tunneling at $eV = \pm 2e\epsilon_d$, cf. Fig. 1. In fact, we find that the analogy between the onset of sequential tunneling at $eV = \pm 2e\epsilon_d$ and the onset of two-electron ionization at $V = \pm V_c$ goes much further. The lines $eV = \pm 2e\epsilon_d$ in the stability diagram separate transport regimes with real occupation (sequential tunneling) and virtual occupation (cotunneling) of the LS. Similarly, the lines $V = \pm V_c$ separate regimes with real occupation.
(two-electron ionization) and virtual occupation (pair-tunneling) of the LS. We now explore this analogy on a quantitative level.

Correspondence of one-electron and two-electron ionization.---We start by noting that Eqs. (8) and (12) yield $d^2I/dV^2 \propto (V-V_c)$ and $d^2I/dV^2 \propto 1/(V_c-V)$, which are the familiar voltage dependencies of the sequential-tunneling and cotunneling currents, respectively, provided we make the replacement $\epsilon V_c \mapsto 2\epsilon_d$. This suggests that the currents near the onsets of sequential tunneling and two-electron ionization are related to one another more generally by two voltage derivatives. To establish this relation, although approximately, we incorporate the lifetime broadening $\Gamma = \Gamma_L + \Gamma_R$ of the LS into Eq. (11), and cast it into the form

$$I \simeq \frac{2e}{h} \Gamma_L \Gamma_R (\Gamma_L + \frac{4}{3} \Gamma_L) \times \int \frac{d\epsilon}{2\pi} \frac{\left\{ f(\epsilon - eV) - f(\epsilon + eV) \right\}}{\left( \epsilon - (\epsilon_d - eV/2) \right)^2 + (\Gamma/2)^2},$$

where $\epsilon \equiv E_2 - eV/2$. The finite lifetime provides a physical cutoff of the singularity in Eq. (12). Most importantly, Eq. (13) captures processes involving both virtual and real occupations of the LS, i.e., it describes the two-electron resonance. Indeed, it can be easily verified that the above and below threshold limits, Eqs. (8) and (11), of the pair resonance are reproduced by Eq. (13).

We compare Eq. (13) with a single-particle resonance

$$I^{1PR}[V, \epsilon_d] = \frac{2e}{h} \Gamma_L \Gamma_R \int \frac{d\epsilon}{2\pi} \frac{f(\epsilon - eV) - f(\epsilon + eV)}{\left( \epsilon - (\epsilon_d - eV/2) \right)^2 + (\Gamma/2)^2}.$$

The qualitative difference between the two expressions arises from the appearance of the Fermi-liquid phase space factor $\left( \pi T \right)^2 + (\epsilon - eV)^2$ in the two-particle resonance Eq. (13). This phase space factor can be removed by taking two derivatives with respect to voltage of Eq. (14). In this way, we find the relation

$$\frac{d^3I}{dV^3} \simeq \frac{\Gamma_L (2\Gamma_R + \Gamma_L)}{(2\pi)^3 e^2 V_c^2} \frac{d}{dV} I^{1PR}[2V, \epsilon_d - eV/2],$$

with the explicit replacements $V \rightarrow 2V$ and $\epsilon_d \rightarrow \epsilon_d - eV/2$. In view of the known properties of the single-particle resonance, this result constitutes our principal prediction. For $T \ll \Gamma$, Eq. (15) predicts a Lorenzian peak in $d^3I/dV^3$ inside the Coulomb blockade regime. Importantly, at $V = V_c$ both $d^3I/dV^3$ and $d^3I^{1PR}/dV^3$ have the same order of magnitude $\sim \Gamma^2/(hV_c^4)$. These results are illustrated in Fig. 4. Note that, for $T \gg \Gamma$, Eq. (15) also predicts temperature broadening of the peak in $d^3I/dV^3$.

FIG. 4: (a) The two-particle resonance induces a peak (dashed line) in $d^3I/dV^3$ of width $\max(T, \Gamma)$, centered at $V = V_c$. The full line curve shows that the two-particle resonance can be observed on top of the smoothly varying single-particle background. (b) The single-particle contribution to $d^3I^{1PR}/dV^3$ becomes singular at $V = 2\epsilon_d$.

Metallic dots.---Our predictions for transport via a single LS also extend to metallic islands with essentially zero level spacing. Transport through these islands can be modeled by the electric circuit shown in the inset of Fig. 2. The corresponding stability diagram includes a sequence of Coulomb diamonds , cf. Fig. 2. It is straightforward to see that for metallic dots, the boundaries of two-electron ionization translate into a sequence of inner diamonds, as shown in Fig. 2.

Discussion and conclusion.---Previously it was believed that in the course of cotunneling through a LS, electrons from the source arrive at the drain one by one. Here we demonstrated that there exists a well-pronounced, although more delicate, transport regime where two-electron processes contribute to the current. We emphasize that this regime is captured by the standard Anderson Hamiltonian Eq. (1).

Intriguingly, our reasoning is easily extended to regimes associated with $N$-particle ionization of the LS ($N > 2$). These induce additional boundaries in the stability diagram Fig. 4 at even lower voltages $eV < eV_c(N) = 2\epsilon_d/(2N - 1)$. A naive estimate of the corresponding near-threshold behavior of the current gives $\sim \left( V - V_c(N) \right)^4 e^2 V_c(N)^4 (2N - 1)/\left( eV_c(N)^4 \right)$. However, destructive interference between different sequences of $N$-electron transitions might lead to further reduction of the current.

Throughout this paper, we considered an empty LS at zero bias (non-Kondo valley $\epsilon_d > 0$). The analysis of the ionization process of the occupied LS (Kondo valley $\epsilon_d < 0$) is entirely analogous, and differs only by the order of virtual transitions.

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