Measuring the energy spectrum of neutral pions in ultra-high-energy proton–air interactions

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Abstract

Fluctuations in the muon content of extensive air showers are anti-correlated to the fluctuations of the energy taken by the neutral pions which emerge from the first interaction of the cosmic ray in the atmosphere. We demonstrate that the high-energy tail of the neutral pion spectrum produced in the first proton-air interaction can be measured, within the uncertainties of present cosmic ray experiments, through the analysis of the probability distribution of showers over the shower muon contents.

I. INTRODUCTION

Ultra-high-energy cosmic rays (UHECRs) have long been seen as a unique opportunity to probe hadronic interaction physics at center-of-mass energies that surpass the 100 TeV center of mass scale, which is well beyond the reach of any human-made collider. Indispensable to this quest is the knowledge of the UHECRs composition, whose average mass number has been shown to be heavier than proton at the highest energies [1]. This unexpected result could be explained by phenomena in the physics of UHECR propagation and astrophysical sources that have not been accounted for [2]. Alternatively, it could be due to an incomplete description of hadronic interactions, which might hamper the interpretation of air shower data in terms of the nature of the primaries.

The description of hadronic interactions in air showers is mostly based on phenomenological models. These models are tuned to accelerator data and then extrapolated to energies and kinematic regions essential to the description of the development of the extensive air shower (EAS). In fact, there are several pieces of evidence showing that the hadronic component of the shower is poorly described. One example is the measurements of the average muon number in showers.

The combination of several experiments has shown that the muon deficit in simulations starts around $\sim 10^{16}$ eV and steadily increases up to the highest energies available $\sim 10^{20}$ eV [3]. The origin and nature of this discrepancy are still unknown. In particular, it is unclear if it is related to a poor description of low energy interactions or due to unexpected new phenomena at the highest energies.

Recently, the relative fluctuations and the event-by-event distribution of the number of muons in showers at the highest energies were measured for the first time [4].

It was also shown that the fluctuations can be traced back to fluctuations of the quantity $\alpha_1$, which is related to the first interaction of the UHECR [5]. The quantity $\alpha_1$ is defined as

$$\alpha_1 \equiv \sum_i \left( \frac{E_i^{\text{had}}}{E_0} \right)^\beta,$$

where we sum over hadronically-interacting particles (basically all baryons, kaons and pions, excluding neutral pions) and where $E_i^{\text{had}}/E_0$ is the fraction of the energy of the primary carried by hadron $i$. The parameter $\beta$ is set to 0.93, motivated by the dependence of the average number of muons with the primary energy in the models. In many practical cases, $\alpha_1$ can be approximated as the energy fraction carried by hadronically-interacting particles, and its complement $f_{\text{e.m.}} \simeq 1 - \alpha_1$ is the fraction of energy taken by the $\pi^0$. The detailed analysis of the probability distribution of showers over the shower muon content opens a new window of observation, which brings new information on the hadronic interactions at the start of the air shower.
Figure 1. Shower-to-shower distribution of $\alpha_1$ (large figure) and $\ln N_\mu$ (inset figure) for proton-induced showers of $E_0 = 10^{19} \text{ eV}$ and $\theta = 67^\circ$. The different lines are the predictions for distinct hadronic interaction models: EPOS-LHC (green); QGSJet-II.04 (red), and Sibyll 2.3c (yellow).

II. EXPLORING SHOWERS WITH LOW MUON CONTENT

The investigations performed in this work were conducted using CONEX simulations [6], which combine Monte Carlo simulations with Cascade Equations for low energy particles. This hybrid technique allows a significant increase in the size of the shower samples while preserving the features of the muon distributions [5,7].

Around one million showers were simulated with the primary energy $E_0 = 10^{19} \text{ eV}$, at a zenith angle of $\theta = 67^\circ$ and the ground level at 1400 m above sea level (the altitude of the Pierre Auger Observatory [8]). High energy interactions were simulated with the post-LHC hadronic interaction models: EPOS-LHC [6], QGSJet-II.04 [10], and Sibyll 2.3c [11].

In Figure 1, the distribution of the natural logarithm of the muon content of showers, $\ln N_\mu$, and the distribution of $\alpha_1$ from the first interaction are shown for an ensemble of proton-induced showers. The peak at $\alpha_1 = 1$ corresponds to quasi-elastic scattering in the first interaction, which corresponds to a diffractive interaction between the cosmic ray and the atmospheric nuclei. The region of low values of both distributions was fitted to the functions $A \exp(\alpha_1/\Lambda_\alpha)$ and $B \exp(\ln N_\mu/\Lambda_\mu)$, with $A$ and $\Lambda_\alpha$, and $B$ and $\Lambda_\mu$ as free parameters. The fitting ranges were chosen so that the deviation from the pure exponential function would not exceed 5% [1]. The values of $\Lambda_\alpha$ and $\Lambda_\mu$ for the three available models are represented in Figure 2 by the solid dots. The relation between $\Lambda_\alpha$ (related to the physics of the first interaction) and $\Lambda_\mu$ (related to the muon content of the showers) was further studied. Small perturbations introduced in the $\alpha_1$ distribution of each model would result in a new value for $\ln N_\mu$. These perturbations were emulated by re-sampling the original data set and keeping pairs $(\Lambda_\mu, \Lambda_\alpha)$ with a probability $\propto \exp(\alpha_1/\delta \Lambda_\alpha)$, where $\delta \Lambda_\alpha$ is the size of the perturbation. Note that in this way, the slope of the tail of the $\alpha_1$ was changed while preserving any non-exponential features. The new $\alpha_1$ and $\ln N_\mu$ distributions were fitted again to exponentials, obtaining new values of $\Lambda_\alpha$ and $\Lambda_\mu$.

The result of this procedure is presented in Figure 2. The points are the nominal values $(\Lambda_\mu, \Lambda_\alpha)$ for each hadronic interaction model, and the lines are the results when the $\alpha_1$-distribution is varied with the re-sampling method. Notice that the re-sampling method assumes that the properties of the hadronic interaction remain unchanged. Hence, the conversion curves for each model cover only a limited region around the nominal model value. Nevertheless, there is a monotonic relation between $\Lambda_\mu$ and $\Lambda_\alpha$, where the maximum deviation between models in $\alpha_1$ is $\sim 12\%$. Through this conversion curve, one can transform a measurement of $\Lambda_\mu$ into a quantity characteristic of the production cross-section of the first interaction of the UHECR, $\Lambda_\alpha$. It should be noted that the exponential tail in the distribution of $\ln N_\mu$ is most prominent for proton-induced showers (largest $\Lambda_\mu$). For heavier primaries, the tail becomes steeper, (smaller $\Lambda_\mu$), such that their $\ln N_\mu$-distribution is hidden by the proton $\ln N_\mu$-distribution. While this means that the calibration between $\Lambda_\mu$ and $\Lambda_\alpha$ is only valid for proton primaries, it also means that the tail in the distribution of $\ln N_\mu$, for protons can be measured even in the case of the scenario of a mixed mass composition. For that it is only required that an enough number of proton showers is present. The effects of different mass composition scenarios will be discussed in Section III.

Each $\alpha_1$ value contains information about the energy distribution of the products arising from the first interaction, while $\Lambda_\alpha$ gives information about the shower-to-shower distribution of $\alpha_1$ itself. In this sense, the latter is a more fundamental variable being more directly relatable to the hadronic interaction and the measurements performed in accelerators.

To see this, let us, for simplicity, consider pions as the only products of the hadronic interactions [2]. Due to the isospin symmetry, the number and energy carried by the bulk of neutral and charged pions emerging from the interaction should be equal. As such, the energy imbalance towards $\pi^0$, characteristic of showers with a reduced muon number, cannot be justified by low $\pi^\pm$.

1 This technique has the advantage of being more independent of the model and closer to what could be done in an experimental analysis. It was also checked that the qualitative behavior was insensitive to small variations of the fit range.

2 Pions constitute about 70% of the particles arising from a high-energy interaction.
multiplicity, but instead, by fast π^03. These particles are, in essence, connected with valence quarks and forward production. Provided that the largest share of energy is taken by a single π^0, the remaining energy budget (shared among mesons and baryons) feeds the hadronic cascade, and it is partially used to produce muons. Hence, it is only natural to investigate the connection between Λ_µ and the high end of the π^0 energy spectrum. To do so, returning to the same shower simulations, the fraction of energy carried by the leading pion of the first interaction, x_L, was calculated for each event, giving the distribution shown in Figure 3. The tail of this distribution was also fitted to an exponential function of the form C \exp(x_L/\Lambda_π), leaving C and Λ_π as free parameters. Once again, these simulations were re-sampled so that the non-exponential features of the distribution in the tail at large x_L are preserved. As seen in the inset figure, this leads to a change in the tail of the muon number distribution with little effect to the rest of the distribution. Moreover, it can be seen that the harder the pion energy spectrum is, the smaller is the slope of the lnN_µ distribution, as there is less energy available to create muons.

The conversion between Λ_µ and the high energy tail of the π^0, Λ_π, is shown in Figure 4 for each model. From this figure, it is possible to see that this conversion is well defined under the interpretation of a given model. There is a remnant model dependence reaching up to ~ 35% difference. Despite this, provided that Λ_π can be experimentally accessed, this conversion could already be used to investigate possible exotic scenarios in the hadronic interactions at energy scales that surpass 100 TeV center of mass, which corresponds to 10^{18.7} eV in the laboratory system.

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3 the most energetic particle arising from an hadronic interaction is usually called the leading-particle

III. MEASUREMENT OF Λ_µ

The previous results demonstrate the quantitative relationship between observables at the air shower level (Λ_µ) and the production cross-section of the highest energy π^0 in proton-air collisions (Λ_π). In this section, we will discuss under what conditions it is feasible to measure Λ_µ in proton-air interactions.

A realistic distribution of the muon content in showers with energy of E_0 = 10^{18.7} eV and zenith angle of
\( \theta = 67^\circ \) was built. Inclined showers are one of the few nearly-direct measurements of muons for showers initiated by UHECRs \[15\]. Different primaries were used to emulate scenarios of a possible mixed primary composition \[16\], namely: proton, helium, nitrogen, iron, and photons. The experimental uncertainties were conservatively considered by smearing each entry in the histogram of muon numbers by 20\% \[17\].

The result of the above procedure for one of the considered mass composition scenarios can be seen in Figure 5. In this scenario, which is inspired by the mass composition from measurements of \( X_{\text{max}} \) at these energies \[1\], the ratio between p:He:N:Fe is taken to be 2:1:1:0. There has been no observation of ultra-high energy photons; so far, only upper limits have been determined by experiments \[18\]. However, pure electromagnetic showers produce a small number of muons at the ground, which can fall in the region where the low-ln \( N_\mu \) tail is expected to be measured. As such, it was decided to include also the possible impact of photon contamination in the measurement of \( \Lambda_\mu \). At the studied energies, the fraction of photons could be as high as 0.5\% \[18\]. A close inspection of Figure 5 allows us to immediately see that the main sources of background in the measurement of the proton-induced showers \( \Lambda_\mu \) are photons and helium nuclei. Nevertheless, as shown in this same figure, these contributions can be minimized by selecting an appropriate region, as marked by the shaded band. The lower limit of the fit region removes bias on the \( \Lambda_\mu \) caused by photon induced showers while the upper limit cancels possible contributions from higher masses.

The overlap in Figure 5 between the total distribution and the distribution for protons in the shaded area is a good indication of the feasibility of the measurement within the current experimental conditions. Nevertheless, to make a more quantitative statement, the parameter \( \Lambda_\mu \) was extracted by fitting an exponential function to the tail of the total distribution. The obtained value was compared to the \( \Lambda_\mu \) value accessed from the proton distribution in the same fit region. In this way, the bias and the relative accuracy of the measurement could be determined. The later quantity is defined as \( \delta \Lambda_\mu \equiv 1 - (\Lambda_\mu^{\text{fit}} / \Lambda_\mu^{\text{p}}) \), where \( \Lambda_\mu^{\text{fit}} \) and \( \Lambda_\mu^{\text{p}} \) are the exponential slopes of the tails of the total distribution and the distribution for protons, respectively. The bias and \( \delta \Lambda_\mu \) were analyzed for different hadronic interaction models, different primary mass composition scenarios, and the number of shower events necessary to accurately measure \( \Lambda_\mu \).

**Table I.** Table with the number of shower events in the fit region (tail) and in all the distribution (total) to perform the measurement of \( \Lambda_\mu \) with a precision of \( \delta \Lambda_\mu = 0.2 \). The results are shown for the different hadronic interaction models and distinct mass composition scenarios (p:He:N:Fe).

| Scenario | Model  | \( N_{\text{evt tail}} \) | \( N_{\text{evt total}} \) |
|----------|--------|---------------------------|---------------------------|
| 1:1:1:0 | QGSJet-II.04 | 21 | 11564 |
| 1:1:1:0 | EPOS-LHC | 20 | 1926 |
| 1:1:1:0 | Sibyll2.3c | 30 | 1667 |
| 1:2:1:0 | QGSJet-II.04 | 32 | 7086 |
| 1:2:1:0 | EPOS-LHC | 36 | 5505 |
| 1:2:1:0 | Sibyll2.3c | 33 | 4411 |
| 1:6:2:0 | QGSJet-II.04 | 205 | 385776 |
| 1:6:2:0 | EPOS-LHC | 132 | 136212 |
| 1:6:2:0 | Sibyll2.3c | 123 | 78482 |

Table I summarizes the results obtained in this study. It was found that if the fit converges, then the bias on the measurement becomes negligible even in the most extreme scenarios. In this table, the number of events necessary to reach an accuracy of 20\% on the measurement of \( \Lambda_\mu \) is also shown. The 20\% accuracy was chosen since it is the requirement to start distinguishing between hadronic interaction models. From these results, it becomes clear that, even in scenarios with an extreme mass composition, like the 1:6:2:0, it is still possible to measure the slope of the tail, provided that the number of events is large enough. This is because the exponential tail of the ln \( N_\mu \) distribution for helium falls off more quickly \[18\], which facilitates the measurement of \( \Lambda_\mu \). Given that the mean number of muons in a shower scales almost linearly with the shower energy, the energy spread of the data set translates directly into a spread in \( N_\mu \). This effect can

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4 In case of the Pierre Auger Observatory, \( \sigma^2 = \sigma_{\text{res}}^2(N_\mu) + \sigma_{\text{rms}}^2(E) = 0.15^2 + 0.08^2 = 0.17^2 \) \[1\].

5 The \( N_\mu \) distribution becomes more Gaussian with the increase of the primary mass composition.
be mitigated by rescaling the muon number as $N_{\mu}/E_{\text{rec}}$, where $E_{\text{rec}}$ is the reconstructed primary energy. Provided that the experimental resolution on $E_{0}$ has a Gaussian form, $N_{\mu}/E_{\text{rec}}$ does not hide the exponential tail.

The precise determination of $\Lambda_{\mu}$ requires the measurement of $N_{\mu}$ for around 3000 shower events, which is still a difficult number to reach [13, 20]. However, future experiments like GRANDProto300 [21, 22] or upgrades, such as AugerPrime [23], where the muonic component is directly measured, will enable this measurement within a few years. An important measurement of the number of muons is being conducted at the Pierre Auger Observatory using underground muon detectors [24] and in the future the number of muons will also be measured with resistive plate chambers [25]. Both these extensions measure air showers in an energy range of about $E \approx 10^{17}$ eV, corresponding to the equivalent center-of-mass energy reached by the Large Hadron Collider. Therefore, the proposed measurement can be compared to measurements of accelerator experiments such as LHCf [26], which are already measuring the forward energy flow carried by neutral pions in proton–proton and proton–lead collisions.

IV. CONCLUSIONS

In this paper, we show that the distribution of the muon number in air showers is sensitive to the properties of multi-particle production in the first interaction between the primary cosmic ray and the nuclei in the air. In typical scenarios with different UHECR masses, showers with the lowest muon number can be attributed to proton showers, which exhibit fluctuations to extremely low muon numbers. We find that the shape of the distribution at low muon numbers can be described by an exponential function with a characteristic slope $\Lambda_{\mu}$. We demonstrate that there exists a relation between the slope $\Lambda_{\mu}$ and the slope $\Lambda_{\pi}$ of the production cross-section of $\pi^{0}$ in proton–air interactions. This allows direct measurements of a well defined multi-particle production variable at center-of-mass energies ranging from current accelerator possibilities to the 100 TeV scale. Finally, it was demonstrated that this measurement is already possible within the present experimental uncertainties, being only dependent on the event statistics, which shall be significantly improved by future upgrades.

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