Coherent virtual absorption for discretized light

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Compiled March 28, 2018

Coherent virtual absorption (CVA) is a recently-introduced phenomenon for which exponentially growing waves incident onto a conservative optical medium are neither reflected nor transmitted, at least transiently. CVA has been associated to complex zeros of the scattering matrix and can be regarded as the time reversal of the decay process of a quasi-mode sustained by the optical medium. Here we consider CVA for discretized light transport in coupled resonator optical waveguides or waveguide arrays and show that a distinct kind of CVA, which is not related to complex zero excitation of quasi-modes, can be observed. This result suggests that scattering matrix analysis can not fully capture CVA phenomena. © 2018 Optical Society of America

OCIS codes: 130.3120, 290.539, 000.1600

The ability of tailoring light absorption in a medium exploiting interference effects has sparked much interest in the past recent years with a wealth of potential applications ranging from optical to sensing to optical modulation, optical switching, polarization control, etc. [1–17]; for a recent review see [18]. Coherent perfect absorption (CPA) refers to the possibility for a dissipative medium to fully absorb suitable patterns of incoming waves; it can be regarded as the time reversal of lasing [2]. In a multiport linear system, incoming and outgoing fields are related by the scattering matrix $S$. CPA corresponds to complex zeros of the scattering matrix, i.e. existence of quasi-bound optical modes. Unlike conventional CPA, in CVA the excitation fields have an amplitude that increases exponentially in time and space, similar to Gamow’s states introduced in earlier theories of quantum mechanical decay [19]. CVA can be viewed as the time reversal process [20, 21] of the light decay process in a high-quality passive resonator [21].

In this Letter we reveal a distinct form of coherent virtual absorption, which is neither related to complex zero eigenvalues nor does it strictly require existence of quasi-bound optical eigenmodes. Specifically, we consider transport of discretized light in coupled resonator optical waveguides (CROW) [22, 23] or in evanescently-coupled optical waveguide arrays [24–26], and show that a kind of virtual absorption can be observed even in structures that do not sustain any resonance mode, i.e. beyond the conditions considered in previous work [17]. Such a kind of CVA stems from the property that specially-designed waveforms, synthesized using time reversal symmetry, can show half-space discrete self-focusing on the lattice.

We consider light transport in a linear chain of CROW [22, 23] [Fig.1(a)] or, similarly, spatial propagation of light in arrays of evanescently coupled optical waveguides [24–26] [Fig.1(b)]. For the sake of definiteness, we will refer mostly to the former physical system. The CROW is composed by a chain resonators with the same resonance frequency $\omega_0$, which are evanescently-coupled with a coupling constant $\kappa$. A defective resonator, with detuned resonance frequency $\omega_0 + \delta\omega_0$, is placed at site $n = 0$, and coupled to its neighboring resonators with a modified coupling constant $x\kappa$, with $x \leq 1$; see Fig.1(a). The defective resonator could be lossy, with a loss rate $\gamma$ ($\gamma = 0$ in the conservative limit). Light transport in the CROW is described by coupled-mode equations [22, 23]

\[
\begin{align*}
    i\frac{dc_n}{dt} &= \omega_0 c_n - \kappa (c_{n+1} + c_{n-1}) \quad |n| \geq 2 \\
    i\frac{dc_{n+1}}{dt} &= \omega_0 c_{n+1} - \kappa c_{n+2} - x\kappa c_0 \\
    i\frac{dc_0}{dt} &= (\omega_0 + \delta\omega_0 - i\gamma)c_0 - x\kappa(c_1 + c_{-1})
\end{align*}
\]
where \(c_n(t)\) is the field amplitude of the resonator mode at site \(n\). The scattering properties of the defective resonator are embodied in the \(2 \times 2\) scattering matrix

\[
S = \begin{pmatrix} r & t \\ t & r \end{pmatrix}
\]

(2)

where \(r = r(q)\) and \(t = t(q)\) are the spectral reflection and transmission amplitudes, respectively. These are determined from the solutions to Eq.(1) of the form

\[
c_n(t) = \begin{cases} 
[A \exp(iqn) + B \exp(-iqn)]\exp(-i\Omega t) & n \leq -1 \\
E \exp(-i\Omega t) & n = 0 \\
[C \exp(-iqn) + D \exp(iqn)]\exp(-i\Omega t) & n \geq 1
\end{cases}
\]

where \((A, C)\) and \((B, D)\) are the amplitudes of outgoing and incoming waves according to [Fig.1(a)], \(q\) is the Bloch wave number with \(0 \leq \text{Re}(q) \leq \pi\), and

\[
\Omega(q) = \omega_0 - 2\kappa \cos q
\]

(4)

is the lattice dispersion relation of the homogeneous lattice [26]. The scattering matrix relates the amplitudes of outgoing and incoming waves according to \((B, D)^T = S(A, C)^T\). The explicit form of \(r(q)\) and \(t(q)\) read

\[
r(q) = \frac{\rho/\kappa - 2(x^2 - 1)\cos q}{2x^2 \exp(iq) - 2 \cos q - \rho/\kappa}
\]

(5)

\[
t(q) = \frac{2ix^2 \sin q}{2x^2 \exp(iq) - 2 \cos q - \rho/\kappa}
\]

(6)

where we have set \(\rho = \delta\omega_0 - i\gamma\). Note that, for a real Bloch wave number \(q\), the frequency \(\Omega(q)\) is real and the scattered states (3) are stationary and propagative waves.

Like for light scattering in continuous media, CPA and CVA based on real and complex zeros of the scattering matrix can be found for discretized light on a lattice. The condition \(\det(S) = 0\), corresponding to a zero eigenvalue of \(S\), implies \(r = -t\) or \(r = t\). Only the former condition \(r = -t\), corresponding to symmetric mode excitation \(A = C\), is relevant and yields the following equation for the Bloch wave number \(q\)

\[
(x^2 - 1) \cos q - ix^2 \sin q = (\delta\omega_0 - i\gamma)/(2\kappa)
\]

(7)

For CPA, we require \(q\) to be real. Like in other CPA systems [2], this implies some dissipation \(\gamma\) in the medium and a constraint between frequency detuning \(\delta\omega_0\) and dissipation \(\gamma\). In a dissipationless structure, i.e. for \(\gamma = 0\), the condition \(\det(S) = 0\) can be satisfied for a complex Bloch wave number \(q = q_0\), corresponding to incoming (Gamow’s-like) waves with an exponentially increasing amplitude in time. For the sake of simplicity, let us assume \(\delta\omega_0 = 0\) and \(x \leq 1\), so that the defect does not sustain any bound state. A resonance (quasi-bound) state, with lifetime determined by the imaginary part of \(\Omega(q_0)\), is found as the complex root of Eq.(7) with \(\gamma = \delta\omega_0 = 0\)

\[
q_0 = \pi/2 - (i/2)\ln(1 - 2x^2).
\]

(8)

Note that for \(x \ll 1\) one has \(q_0 \approx \pi/2 + i\pi^2\), and the complex energy of the resonance mode is given by \(\Omega \approx \omega_0 + 2\kappa x^2\). In this limit, two incoming waves, exponential growing in time \(t\) and space \(n\) according to

\[
c_n(t) \simeq \exp[-(i/(2n))|n| + x^2|n| + 2\kappa x^2 t - i\omega_0 t]
\]

(9)

\((n \neq 0)\), realize CVA, in which the energy of incoming waves is fully stored in the defective resonator. Figure 2 shows an example of CVA based on complex zero excitation, in which the exponential waves at initial time are truncated at \(|n| = 200\) [Fig.2(b)]. This is precisely the scenario considered in Ref. [17], and therefore it will not be further analyzed.

Here we wish to suggest a different route toward virtual absorption, which is not associated to the existence of resonance modes. To this aim, let us consider the behavior of the Bloch wave number \(q_0\) [Eq.(8)] in complex plane, which is shown in Fig.1(c) together with the corresponding behavior of the frequency \(\Omega = \omega_0 - 2\kappa \cos q_0\).

Clearly, a phase transition is found at \(x = 1/\sqrt{2}\). For \(x < 1/\sqrt{2}\), \(\text{Re}(q_0) = \pi/2\) is independent of \(x\) while \(\text{Im}(q_0)\) increases as \(x\) increases, corresponding to lowering of the \(Q\)-factor (lifetime) of the quasi-mode. At \(x = 1/\sqrt{2}\), \(q_0\) goes to infinity, i.e. \(\det(S)\) does not vanish nowhere in complex \(q\) plane and the quasi-mode disappears. For \(1/\sqrt{2} < x \leq 1\), \(q_0\) is purely imaginary and the frequency \(\Omega\) becomes real, indicating that the solution (3) with \(B = D = 0\) is stationary in time but exponentially growing in space (anti-evanescent wave), and thus unphysical.

A natural question then arises: while for \(x < 1/\sqrt{2}\) the structure sustains a quasi-bound mode and CVA can be observed by complex zero eigenvalue excitation, what happens when crossing the phase transition point at \(x = 1/\sqrt{2}\)? Can virtual absorption be found for \(x > 1/\sqrt{2}\)? The main result of this Letter is that virtual absorption does not strictly require the existence of quasi bound modes. Exploiting the very general concept
of time-reversal symmetry, one can synthesize half-space self-focusing wave packet distributions such that, for a finite (yet arbitrarily long) time interval, an initially broad wave, localized at $n < 0$ sites, shrinks and stores energy in few lattice sites around $n = 0$, with negligible transmission in the $|n| > 1$ half space.

To gain some insights into the synthesis method, let us indicate by $U_{n,m}(t)$ the propagator of Eq.(1) with $\delta \omega_0 = \gamma = 0$, so that

$$c_n(t) = \sum_m U_{n,m}(t)c_m(0) \exp(-i\omega_0 t).$$

The propagator $U_{n,m}(t)$ can be calculated in a closed form for special values of $x$, namely for $x = 1$ (homogeneous lattice) and for $x = 1/\sqrt{2}$, i.e. at the phase transition point. For $x = 1$, one has $U_{n,m}(t) = i^n J_n(t)$, while for $x = 1/\sqrt{2}$ the expression of $U_{n,m}(t)$ is more involved and not given here for the sake of brevity (see [27]). For other values of $x$, $U_{n,m}(t)$ can be computed numerically.

To realize CVA with one-side excitation, let us assume the simplest case $x = 1$ and let us assume $c_n(0) = \delta_{n,0} + i\Delta (\delta_{n,1} + \delta_{n,-1})$; one then readily obtains

$$c_n(T) = i^n J_n(2\kappa T) \left(1 + \frac{n\Delta}{\kappa T}\right).$$

Note that the waveform broadens as $\kappa T$ increases and the symmetry $|c_{-n}|^2 = |c_n|^2$ is broken for a non-vanishing value of $\Delta$. Since the Bessel $J_n$ function has its maxima at $n \simeq \pm 2\kappa T$ for $\kappa T \gg 1$, by choosing $\Delta = -1/2$ the distribution $|c_n(T)|^2$ turns out to be strongly asymmetric around $n = 0$, with the major excitation in the $n < 0$ half lattice. Time reversal thus corresponds to a self-focusing wave with dominant excitation in the left half-space of the lattice. If more than three waveguides with properly-tailored amplitudes and phases are excited at $t = 0$, the pattern $|c_n(T)|^2$ can be entirely localized in the half space $n \leq 0$, so that its time reversal would correspond to CVA with unidirectional excitation of the lattice. This is a distinctive feature as compared to CVA based on complex zeros of the scattering matrix eigenvalues, where bidirectional excitation is required [see Eq.(9)].

To realize CVA with one-side excitation, let us assume the initial condition

$$c_n(0) = U_{n,0}^*(T)H_n$$

with envelope $H_n$ defined by

$$H_n = \begin{cases} \exp(\beta n) & n \leq -1, \\ 0 & n \geq 0. \end{cases}$$

and $0 < \beta \ll 1$. The waveform (12) is localized in the $n < 0$ half-space and is obtained from the deformation and truncation of the time reversal of the impulse response $U_{n,0}$ of the lattice. Extended numerical simulations indicate that the waveform (12) undergoes half-space transient self focusing in the time interval $(0,T)$, with $c_n(T)$ being non vanishing for few sites around $n = 0$ (focusing condition). The instantaneous transmit-
In this work the phenomenon of coherent virtual absorption, recently introduced in [17], has been investigated for discretized light transport. The main conclusion is that virtual absorption does not necessarily require complex zero excitation of quasi-modes. This result suggests that scattering matrix analysis does not fully capture the entire physics of the phenomenon and motivates further theoretical and experimental research, opening up some major questions. For example, can virtual absorption without complex zero eigenvalues be observed for light waves in continuous media? Can transient virtual absorption be controlled by exploiting material nonlinearity?

References

1. M. Cai, O. Painter, and K. J. Vahala, Phys. Rev. Lett. 85, 74 (2000).
2. Y.-D. Chong, L. Ge, H. Cao, and A.D. Stone, Phys. Rev. Lett. 105, 053901 (2010).
3. W. Wan, Y. Chong, L. Ge, H. Noh, A.D. Stone and H. Cao, Science 331, 889 (2011).
4. S. Longhi, Phys. Rev. A 82, 031801(R) (2010).
5. Y.-D. Chong, L. Ge, and A.D. Stone, Phys. Rev. Lett. 106, 093902 (2011).
6. S. Longhi, Phys. Rev. A 83, 055804 (2011).
7. S. Dutta-Gupta, O.F.J. Martin, S. Dutta Gupta, and G.S. Agarwal, Opt. Express 20, 1330 (2012).
8. H. Noh, Y.-D. Chong, A.D. Stone, and H. Cao, Phys. Rev. Lett. 108, 186805 (2012).
9. K.N. Reddy and S. Dutta Gupta, Opt. Lett. 38, 5252 (2013).
10. M. Kang, F. Liu, T.-F. Li, Q.-H. Guo, J. Li, and J. Chen, Opt. Lett. 38, 3086 (2013).
11. Y. Sun, W. Tan, H.-Q. Li, J.-S. Li, and H. Chen, Phys. Rev. Lett. 112, 143903 (2014).
12. S. Longhi, Opt. Lett. 40, 1278 (2015).
13. T. Roger, S. Vezzoli, E. Bolduc, J. Valente, J. J. F. Heitz, J. Jeffers, C. Soci, J. Leach, C. Couteau, N. I. Zheludev, and D. Faccio, Nat. Commun. 6, 7031 (2015).
14. Z. J. Wong, Y. Xu, J. Kim, K. O’Brien, Y. Wang, L. Feng, and X. Zhang, Nature Photon. 10, 796 (2016).
15. L. Jin, P. Wang, and Z. Song, Sci. Rep. 6, 32919 (2016).
16. H. Ramezani, Y. Wang, E. Yablonovitch, and X. Zhang, IEEE J. Select. Top. Quantum Electron. 22, 115 (2016).
17. D.G. Baranov, A. Krasnok, and A. Alù, Optica 4, 1457 (2017).
18. D.G. Baranov, A. Krasnok, T. Shegai, A. Alù, and Y. Chong, Nat. Rev. Mater. 2, 17064 (2017).
19. A. Bohm, M. Gadella, and G. Bruce Mainland, Am. J. Phys. 57, 1103 (1989).
20. G. Lerosey, J. de Rosny, A. Tourin, A. Derode, G. Montaldo, and M. Fink, Phys. Rev. Lett. 92, 193904 (2004).
21. G. Leuchs and M. Sondermann, Physica Scr. 85, 058101 (2012).
22. A. Yariv, Y. Xu, R. K. Lee, and A. Scherer, Opt. Lett. 24, 711 (1999).
23. S. Mookherjea and A. Yariv, IEEE J. Sel. Top. Quantum Electron. 8, 448 (2002).
24. D. N. Christodoulides, F. Lederer, and Y. Silberberg, Nature 424, 817 (2003).
25. A. Szameit and S. Nolte, J. Phys. B 43, 163001 (2010).
26. I.L. Garanovich, S. Longhi, A.A. Sukhorukov, and Y.S. Kivshar, Phys. Rep. 518, 1 (2012).
27. S. Longhi, Phys. Rev. E 74, 026602 (2006).
28. A. Szameit, F. Dreisow, M. Heinrich, T. Pertsch, S. Nolte, A. Tünnemann, E. Suran, F. Louradour, A. Barthelmy, and S. Longhi, Appl. Phys. Lett. 93, 181109 (2008).
29. R. Keil, Y. Lahini, Y. Shechtman, M. Heinrich, R. Pugatch, F. Dreisow, A. Tünnemann, S. Nolte, and A. Szameit, Opt. Lett. 37, 809 (2012).
30. S. Longhi, Opt. Lett. 42, 2551 (2017).
31. F. Dreisow, A. Szameit, M. Heinrich, T. Pertsch, S. Nolte, and A. Tünnemann, Opt. Lett. 33, 2689 (2008).