The New Extended Left-Right Symmetric Grand Unified Model with $SO(3)$ Family Symmetry

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Abstract: We suggest a new left-right symmetric grand unified model by extending Pati-Salam group to contain an isospin $SU(2)$ and a flavor $SO(3)$ subgroup, where the superheavy fermions are introduced as a mirror to the low-energy standard model fermions. The model undergoes three steps to break to the SM by means of the specified Higgs multiplets. The model few parameters can elegantly accommodate whole mass spectra for all the particles at the electroweak scale, especially, two different flavor mixing for the quark and lepton sectors are reproduced in agreement with the current experimental data very well. The strong $CP$ violation is excellently explained. The matter-antimatter asymmetry in the universe is successfully implemented through the $B-L$ violating decays of the superheavy gauge bosons. The model also predicts that the lightest right-handed Majorana neutrino, whose mass is about several hundred GeVs and energy is about $10^{16}$ GeV, is possibly a candidate for the dark matter.

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I. Introduction

The open problems that the origin of the elementary particle masses and flavor mixing, the genesis of the matter-antimatter asymmetry in the universe, and which kind of particle is veritably ingredient of the dark matter have been the focus of attention in particle physics [1]. The precision tests for the electroweak scale physics have established plenty of information about the elementary particles [2]. The impressive puzzles involve mainly as follow some facts. The charged fermion mass spectra emerge large hierarchy ranging from one MeV to a hundred GeVs or so [2]. The neutral fermions have also been verified to have non-zero but Sub-eV masses [3], but that their nature are Majorana or Dirac particle has yet to be identified by experiments such as $0\nu\beta\beta$. On the other hand, two kinds of the flavor mixing in the quark and lepton sectors are very distinctly different. The flavor mixing in the quark sector are small mixing angles but bi-large mixing angles in the lepton sector [4]. The Higgs sector is known the least up to now. The gauge symmetries can not generally set the unique type of Higgs multiplets, accordingly the Higgs particle spectra are ambiguous. Whereas Higgs particles play crucial roles for spontaneous symmetry breaking, Searching for Higgs particles has been one of the most important goals in high energy physics experiments such as LHC [5]. The observations of the universe have confirmed the two important facts. The universe appears to be matter dominated, and the ratio of baryons to photons is very well determined as $\eta \sim 6.1 \times 10^{-10}$ [6]. The various contributions for the universe critical density are the visible matter $\Omega_{\text{em}} \sim 0.04$, the dark matter $\Omega_{\text{dm}} \sim 0.26$ and the dark energy $\Omega_{\text{de}} \sim 0.7$ [7]. The baryogenesis mechanism and the dark matter particle nature have been extensively discussed but they are yet suspense [8].

Any new theory beyond the Standard Model (SM) has to be confronted with the above diverse intractable issues. Some supersymmetric or non-supersymmetric grand unified theory (GUT) [9], in particular, those models based on $SO(10)$ or Pati-Salam symmetric group with flavor symmetry have been proposed to explain the issues to some extent [10]. However, these models seem to be very difficult to solve all the forenamed problems together and satisfactorily with the small number of parameters. It is verily a large challenge for theoretical particle physicists to uncover these mysteries of the nature.

In this works, we attempt to incorporate all the above problems into an unification framework. For this purpose, we propose a new GUT model and consider some new approaches. The left-right symmetric GUT models based on Pati-Salam symmetric group are theoretically well-motivated extension of the SM [11]. It is surely an appreciated idea that the left-handed matter and right-handed matter are perfectly symmetry at high-energy scale but the left-right symmetry is broken at low-energy scale. On the basis of it, we now extend the symmetry to the full gauge symmetric group as $SU(2)_L \otimes SU(2)_R \otimes SU(4)_C \otimes SU(2)_G \otimes SO(3)_F \otimes D_P$. It appends a high-energy isospin subgroup $SU(2)_G$, a flavor subgroup $SO(3)_F$ and a discrete subgroup $Z_2$ which is named as $D_P$ parity. However, these new appended symmetries are
retained only at the high-energy scale but are broken at the low-energy scale. In
addition, we introduce new matter fermions which are considered as a mirror to the
low-energy SM fermions including the right-handed neutrinos. These superheavy
fermions appear only at high-energy scale. The low-energy effective theories such as
the SM are achieved by integrating them out. We also arrange special Higgs field
structures to implement spontaneous symmetry breaking of the model. By means
of chain breaking, in which Higgs potential are broken step by step at the different
energy scales, the model gauge symmetry undergoes three steps to descend to the
SM symmetric group \( SU(2)_L \otimes U(1)_Y \otimes SU(3)_C \). Moreover, the theoretical structure
of the model can automatically eliminate the strong \( CP \) violation [12]. After
electroweak symmetry breaking, all the particle masses and flavor mixing angles are
correctly reproduced and are very well in agreement with the current experimental
data.

Baryogenesis has a few mechanisms. The usual mechanism is baryogenesis
through leptogenesis [13], in which the right-handed Majorana neutrino decays play
key roles. In our model, the lightest right-handed Majorana neutrino mass is only
about one TeV, and the effective Yukawa couplings involving with the right-handed
neutrinos are actually less several order of magnitude than the other Yukawa cou-
plings. The mechanism is herein an infeasible scenario to generate correctly the
matter-antimatter asymmetry in the universe. We therefore consider a new baryoge-
nesis mechanism to replace the old one. In our model, there are gauge bosons acting
as intermedia between quarks and leptons. After the above-mentioned breaking are
fulfilled, they can achieve masses near the GUT energy scale \( \sim 10^{16} \) GeV. Furthermore,
These gauge bosons can decay into some pairs of quark and lepton. These
decays all conserve the quantum number \( B-L \) except the exclusive decays into the
right-handed up-type quarks and the right-handed neutrinos. The such processes,
however, violate the quantum number \( B-L \) in virtue of Majorana property of the
effective right-handed neutrinos. Moveover, the \( CP \) asymmetry of the decays are
also induced through the loop correction owing to the effective complex Yukawa cou-
plings. The out-of-equilibrium decays of the superheavy gauge bosons thus cause an
asymmetry of the quantum number \( B-L \). The \( B-L \) asymmetry is eventually trans-
lated into an asymmetry of the baryon number through sphaleron processes over
the electroweak scale [14]. The new baryogenesis mechanism is implemented suc-
cessfully in our model. The calculating results are also in accord with the universe
observations very well.

In our model, since the right-handed neutrinos have only very weak interaction
with the other leptons and Higgs bosons, they probably become some of weakly
interacting massive particles (WIMPs) [15]. If the lightest right-handed Majorana
neutrino mass is far smaller than the lightest Higgs boson mass, thus it is relatively
a stable particle. The left-handed neutrinos with tiny mass are known as significant
component of hot dark matter [16]. In view of neutrinos possing a special status
among all kinds of the universe particles, we analogously guess that the right-handed
neutrinos, which have not yet been detected, are probably primary ingredient of the
dark matter.

The remainder of this paper is organized as follows. In Section II we outline the model and characterize the symmetry breaking procedure. The particle masses and flavor mixing through the renormalization group running are discussed in Sec. III. We suggest a possible solution for baryogenesis and dark matter in Sec. IV. In Sec. V, a detailed numerical results are given in a specific parameter set satisfying the experimental constraints. Sec. VI is devoted to conclusions.

II. Model and Symmetry Breaking

We now outline the our model based on the symmetry group $SU(2)_L \otimes SU(2)_R \otimes SU(4)_C \otimes SU(2)_G \otimes SO(3)_F \otimes D_P$. The particle contents and they falling into representations are listed as follows. The low-energy matter fields are

$$
\psi_L = \left( \begin{array}{c} Q_L \\ L_L \end{array} \right)_\alpha \sim (2, 1, 4, 1, 3), \quad \psi_R = \left( \begin{array}{c} Q_R \\ L_R \end{array} \right)_\alpha \sim (1, 2, 4, 1, 3),
$$

where

$$Q = \left( \begin{array}{c} u \\ d \end{array} \right), \quad L = \left( \begin{array}{c} \nu \\ e \end{array} \right).$$

The letters $\alpha$ and $i$ are respectively family and color indices. The left-handed and right-handed fields are in the different representations under the left-right symmetric group $SU(2)_L \otimes SU(2)_R$. The quarks and leptons are in 4 representation of the color group $SU(4)_C$, while the three generation fermions are in 3 representation of the flavor group $SO(3)_F$. However, they are all singlets under the high-energy isospin group $SU(2)_G$. The superheavy matter fields as a mirror to the low-energy matter fields are

$$
\lambda_L = \left( \begin{array}{c} \lambda^q_L \\ \lambda^l_L \end{array} \right)_\alpha \sim (1, 1, 4, 2, 3), \quad \lambda_R = \left( \begin{array}{c} \lambda^q_R \\ \lambda^l_R \end{array} \right)_\alpha \sim (1, 1, 4, 2, 3),
$$

where

$$\lambda^q = \left( \begin{array}{c} \lambda^u \\ \lambda^d \end{array} \right), \quad \lambda^l = \left( \begin{array}{c} \lambda^\nu \\ \lambda^e \end{array} \right).$$

Their the color and flavor quantum numbers are the same as ones of the low-energy fermions, but they are singlets under the low-energy left-right symmetric group $SU(2)_L \otimes SU(2)_R$ and are doublets under the high-energy isospin group $SU(2)_G$. Although the left-handed and right-handed superheavy fermions are uniform, namely have the same gauge quantum numbers, they have actually different properties under $D_P$ transformation which is defined later. The light Higgs fields are

$$
H_L = \left( \begin{array}{cc} H^0_{L2} & H^\pm_{L1} \\ H^-_{L2} & H^0_{L1} \end{array} \right) \sim (2, 1, 1, \overline{2}, 1), \quad H_R = \left( \begin{array}{cc} H^0_{R2} & H^\pm_{R1} \\ H^-_{R2} & H^0_{R1} \end{array} \right) \sim (1, 2, 1, \overline{2}, 1).
$$
In addition, we also introduce $\tilde{H}_L = \tau_2(H_L)^*\tau_2$ and $\tilde{H}_R = \tau_2(H_R)^*\tau_2$. Here and thereafter $\tau_1, \tau_2, \tau_3$ are Pauli matrices. The two Higgs fields respectively play a role in breaking of the left-handed isospin group $SU(2)_L$ and the right-handed isospin group $SU(2)_R$. The superheavy Higgs fields are

$$H_1 \sim (1, 1, 1, 1), \ H_2 \sim (1, 1, 1, 3, 1), \ H_3 \sim (1, 1, 15, 1, 1), \ H_4 \sim (1, 1, 15, 3, 1), \ H_5 \sim (1, 1, 1, 1, 5), \ H_6 \sim (1, 1, 1, 3, 5), \ H_7 \sim (1, 1, 15, 1, 5), \ H_8 \sim (1, 1, 15, 3, 5), \ H_9 \sim (1, 1, 1, 1, 3), \ \Omega \sim (1, 1, 10, 3, 5). \quad (6)$$

Where $H_1, \cdots, H_8$ are all hermitian representations, $\Omega$ is an antisymmetric hermitian representation, $\Omega$ is a symmetric complex representation. These Higgs fields are responsible for breaking of the color, flavor and $D_P$ symmetries. The model $D_P$ transformation, namely the left-right symmetry, is defined as follows

$$\begin{align*}
\left(\begin{array}{c}
\psi_L \\
\psi_R
\end{array}\right) &\rightarrow \tau_2 \left(\begin{array}{c}
\psi_L \\
\psi_R
\end{array}\right) = \left(\begin{array}{c}
-\imath \psi_R \\
\imath \psi_L
\end{array}\right), \quad \left(\begin{array}{c}
\lambda_L \\
\lambda_R
\end{array}\right) &\rightarrow \tau_2 \left(\begin{array}{c}
\lambda_L \\
\lambda_R
\end{array}\right) = \left(\begin{array}{c}
-\imath \lambda_R \\
\imath \lambda_L
\end{array}\right), \\
\left(\begin{array}{c}
H_L \\
H_R
\end{array}\right) &\rightarrow -\tau_1 \left(\begin{array}{c}
H_L \\
H_R
\end{array}\right) = \left(\begin{array}{c}
-H_R \\
-H_L
\end{array}\right), \quad \left(\begin{array}{c}
W^\mu_L \\
W^\mu_R
\end{array}\right) &\rightarrow \tau_1 \left(\begin{array}{c}
W^\mu_L \\
W^\mu_R
\end{array}\right) = \left(\begin{array}{c}
W^\mu_R \\
W^\mu_L
\end{array}\right), \\
H_k &\rightarrow -H_k \quad (k = 1, 2, \cdots, 9), \quad \Omega \rightarrow \Omega, \quad g_L = g_R,
\end{align*}$$

(7)

where $W^\mu_L, W^\mu_R$ and $g_L, g_R$ are respectively gauge fields and gauge coupling coefficients in relation to the left-right symmetric group $SU(2)_L \otimes SU(2)_R$.

Under the above symmetry group, the model gauge invariant Yukawa couplings are such as

$$-\mathcal{L}_{\text{Yukawa}} = y_0 \left(\overline{\psi}_L H_L \lambda_R + \overline{\psi}_R H_R \lambda_L\right) + \sum_{k=1}^9 y_k H_k \lambda_R
$$

$$+ y_{10} \left(\overline{\lambda}_L^c \Omega \lambda_L - \overline{\lambda}_R^c \Omega \lambda_R\right) + h.c., \quad (8)$$

where all the Yukawa coupling coefficients are chosen to be real, thus the model also holds independent $C, P, T$ discrete symmetries. The model Higgs potential is written as

$$-\mathcal{L}_{\text{Higgs}} = V_A + V_B + V_C \quad (9)$$

with

$$\begin{align*}
V_A &= \text{Tr} \left[ -\mu_A^2 \Omega^* \Omega + A_0 (\Omega^* \Omega)^2 \right], \\
V_B &= \text{Tr} \left[ (\mu_{2k}^2 - B_{1k} \Omega^* \Omega) H^2_k + B_{2k} H^4_k \right], \\
V_C &= \text{Tr} \left[ \left(\mu_{C1}^2 - C_1 \Omega^* \Omega - C_2 H^2_k \right) \left( H^H_L H_L + H^H_R H_R \right) - \mu_{C2} H^H_I H_L - H^H_R \right] \\
&+ \frac{1}{2} \left(\mu_{C3}^2 - C_3 \Omega^* \Omega - C_4 H^2_k \right) \left( H^H_L \widetilde{H}_L + H^H_R \widetilde{H}_R + h.c. \right) + \frac{1}{2} \mu_{C1} C_1 H^H_I \left( H^H_L \widetilde{H}_L - H^H_R \widetilde{H}_R + h.c. \right) \\
&+ \frac{C_5}{2} \left( H^H_L H_L + H^H_R H_R \right)^2 - \frac{C_5}{8} \left( H^H_L \widetilde{H}_L + H^H_R \widetilde{H}_R + h.c. \right)^2.
\end{align*} \quad (10)
SU(2)_{L} \otimes SU(2)_{R} \otimes SU(4)_{C} \otimes SU(2)_{G} \otimes SO(3)_{F} \otimes D_{P}

$\langle \Omega \rangle \sim 10^{16}$ GeV.

In this set of equations, the iterative index $k$ sums from 1 to 9. We have divided the Higgs potential into three parts according to sequence of the symmetry breaking, namely the $V_{A}$ term is firstly broken, secondly $V_{B}$, lastly $V_{C}$. Moreover, we assume the following three factors in order to ensure the breaking sequence. The mass dimension $\mu$ parameters in different parts of the potential have very large hierarchy. The couplings among Higgs fields at the different breaking levels are much weaker than that among Higgs fields at the same breaking level. All the coefficients in the formula (10) are positive. Thus the chain breaking can be carried out desirably. Finally, we point out that the model is renormalizable, and also free of anomaly.

By means of chain breaking of the Higgs potential, the GUT model symmetry can descend to the SM symmetry $SU(2)_{L} \otimes U(1)_{Y} \otimes SU(3)_{C}$ through three breaking steps. The breaking procedure is sketched in figure 1. The first step of the model breaking chain is that the subgroup $SU(4)_{C} \otimes SU(2)_{G}$ breaks to $SU(3)_{C} \otimes U(1)_{X}$, and $SO(3)_{F}$ breaks to $S_{2}$ simultaneously. This is accomplished by Higgs field $\Omega$ in $V_{A}$ developing a vacuum expectation value (VEV) at the GUT energy scale about $10^{16}$ GeV. The new subgroup $U(1)_{X}$ is from a linear combination of the original

Figure. 1. The sketch map of the model symmetry breaking. a) The first step of the breaking is accomplished by $\langle \Omega \rangle \sim 10^{16}$ GeV. b) In the second stage, the two discrete symmetries are broken by every $\langle H_{k} \rangle \sim 10^{12} - 10^{14}$ GeV. c) $\langle H_{R} \rangle \sim 10^{10}$ GeV is responsible for the third step of the breaking. d) $\langle H_{L} \rangle \sim 10^{2}$ GeV completes the last breaking.
subgroup $U(1)_{B-L}$ and $U(1)_{I_R}$, while $S_2$ is a permutation group between the second and third generation fermions. The charge quantum numbers and gauge coupling coefficients of the three $U(1)$ subgroups have respectively relations

$$X = I_3^G + \frac{B - L}{2},$$

$$\frac{1}{g_X^2} = \frac{1}{g_c^2} + \frac{1}{g_R^2}. \quad (11)$$

Under the subgroup $SU(3)_C \otimes U(1)_X$, the various field representations in $(1), (3), (5), (6)$ have following decomposition

$$\psi_{(4,1)} = Q_{(3,0)} \oplus L_{(1,-\frac{1}{3})}, \quad \lambda_{(4,2)} = \lambda_{(3,\frac{1}{3})} u \oplus \lambda_{(3,-\frac{1}{3})} d \oplus \lambda_{(1,0)}^e \oplus \lambda_{(1,-1)}^e;$$

$$H_{L(1,2)} = H_{L2(1,-\frac{1}{2})} \oplus H_{L1(1,\frac{1}{2})}; \quad H_{R(1,2)} = H_{R2(1,-\frac{1}{2})} \oplus H_{R1(1,\frac{1}{2})};$$

$$H_{1/5} = H^{A}_{(1,0,1/5)};$$

$$H_{2/6} = H^{B}_{(1,1,1/5)} \oplus H^{B}_{(1,0,1/5)} \oplus H^{B}_{(1,1,1/5)};$$

$$H_{3/7} = H^{C}_{(1,0,1/5)} \oplus H^{C}_{(3,\frac{1}{3},1/5)} \oplus H^{C}_{(3,-\frac{1}{3},1/5)} \oplus H^{C}_{(8,0,1/5)};$$

$$H_{4/8} = H^{D}_{(1,1,1/5)} \oplus H^{D}_{(1,0,1/5)} \oplus H^{D}_{(1,1,1/5)} \oplus H^{D}_{(3,\frac{1}{5},1/5)} \oplus H^{D}_{(3,\frac{1}{3},1/5)} \oplus H^{D}_{(3,-\frac{1}{3},1/5)} \oplus H^{D}_{(8,0,1/5)} \oplus H^{D}_{(8,1,1/5)};$$

$$H_9 = H^{A}_{(1,0,3)};$$

$$\Omega = \Omega_{(1,0)} \oplus \Omega_{(1,1)} \oplus \Omega_{(1,2)}$$

$$\oplus \Omega_{(\bar{3},-\frac{1}{3})} \oplus \Omega_{(\bar{3},\frac{1}{3})} \oplus \Omega_{(\bar{3},\frac{1}{3})}$$

$$\oplus \Omega_{(\bar{3},-\frac{1}{3})} \oplus \Omega_{(\bar{3},-\frac{1}{3})} \oplus \Omega_{(\bar{3},\frac{1}{3})}. \quad (12)$$

On the other hand, under the discrete subgroup $S_2$, the relevant representations of the flavor $SO(3)_F$ have following decomposition

$$3 = 1 \oplus 2, \quad 5_S = 1 \oplus 1' \oplus 1'' \oplus (-1') \oplus (-1''), \quad 3_A = 1 \oplus (-1') \oplus (-1''). \quad (13)$$

It can be seen from the above results that the breaking really arises from the singlet component $\Omega_{(1,0)}$ under the subgroup $SU(3)_C \otimes U(1)_X$, while the breaking in flavor space occurs only along directions of the three $S_2$ singlets $1, 1', 1''$. A detailed calculation shows that the actual vacuum state is only $1$ and $1'$ developing non-zero complex VEVs $\omega_1, \omega_2 e^{i\delta_0}$, where the $\delta_0$ phase is non-removable by the $\lambda_L$ and $\lambda_R$ field phases redefining. Put these together, the VEV of $\Omega$ is given by

$$\langle \Omega \rangle = \frac{v_G}{\sqrt{2}} \left( \begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) \otimes \left( \begin{array}{c} 1 \\ 0 \\ 0 \\ 0 \end{array} \right) \otimes \left( \begin{array}{cccc} -2 \cos \beta_G & 0 & \sin \beta_G e^{i\delta_0} & 0 \\ 0 & \sqrt{2} & 0 & 0 \\ \sin \beta_G e^{i\delta_0} & 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 & \sqrt{3} \end{array} \right), \quad (14)$$
where \( v_G = \sqrt{\omega_1^2 + \omega_2^2} \approx 10^{16} \text{ GeV} \), \( \tan \beta_G = \frac{\omega_1}{\omega_2} \), and \( \text{Tr}|\langle \Omega \rangle|^2 = v_G^2 \). The \( v_G \) value signifies the GUT energy scale of the model.

After the first stage of breaking is over, twelve gauge bosons achieve superheavy masses near the GUT energy scale by Higgs mechanism. For example, the gauge fields in the 15 adjoint representation of \( SU(4)_C \) decompose as \( 8 \oplus 3 \oplus 3 \oplus 1 \) under the subgroup \( SU(3)_C \). The 3 and \( \overline{3} \) representation gauge fields, which are denoted by \( X^\pm \mu_\nu \), have fractional charge and color charge, and act as intermediaries between quarks and leptons. This three pairs of gauge fields achieve masses such as

\[
M_{X^\mu} = \frac{g_C v_G}{\sqrt{2}}.
\]

In like manner, a pair of charged and a neutral superheavy gauge bosons are as intermediaries of the high-energy isospin interaction, three superheavy gauge bosons with flavor charge are as intermediaries among the three generation fermions. Because these gauge bosons are very heavy, the low-energy processes such as proton decay and flavor violation are drastically suppressed. In addition, the \( V_A \) term breaking can also cause large numbers of superheavy Higgs bosons or even massless Goldstone particles. A whole discussion about their mass spectrum are very difficult. However, these Higgs or Goldstone particles have not any couplings with the low-energy matter fields, in other words, they have directly no effect on the low-energy phenomenology. Here we do not deeply discuss about them.

It is inferred from the \( \mathcal{L}_{\text{Yukawa}} \) term that the first step of breaking directly brings about the three results as follow. i) The neutral superheavy fermions \( \lambda^c \) attain Majorana mass terms. ii) The model \( C \) and \( CP \) symmetries are deprived. iii) The two sets of the singlet Higgs fields \( H_A^{(1,0,1/5)}, H_B^{(1,0,1/5)}, H_C^{(1,0,1/5)}, H_D^{(1,0,1/5)} \); each group has the same quantum number under the subgroup \( SU(3)_C \otimes U(1)_X \), therefore they can respectively mix to generate two new sets of orthogonal states such as

\[
\begin{pmatrix}
H^u(1,0,1/5) \\
H^d(1,0,1/5) \\
H^c(1,0,1/5)
\end{pmatrix} = \frac{1}{\sqrt{8}} \begin{pmatrix}
\sqrt{3} & \sqrt{3} & 1 & 1 \\
\sqrt{3} & -\sqrt{3} & 1 & -1 \\
1 & 1 & -\sqrt{3} & -\sqrt{3} \\
1 & -1 & -\sqrt{3} & \sqrt{3}
\end{pmatrix} \begin{pmatrix}
H_A^{(1,0,1/5)} \\
H_B^{(1,0,1/5)} \\
H_C^{(1,0,1/5)} \\
H_D^{(1,0,1/5)}
\end{pmatrix}.
\]

The elements of the above unitary transform matrix which is denoted by \( U_{CX} \) are justly Clebsch-Gordon coefficients under the subgroup decomposition. The Yukawa coupling term can now be rewritten as

\[
-\mathcal{L}_{\text{Yukawa}} = y_0 \left[ \overline{Q_L} H_{L2}^f \lambda_R^f + \overline{Q_L} H_{L1}^f \lambda_R^d + \overline{Q_R} H_{R2}^f \lambda_L^d + \overline{Q_R} H_{R1}^f \lambda_L^d \\
+ \overline{L_L} H_{L2}^c \lambda_R^c + \overline{L_L} H_{L1}^f \lambda_R^c + \overline{L_R} H_{R2}^c \lambda_L^c + \overline{L_R} H_{R1}^c \lambda_L^c \right] + \lambda_L \left[ \sum_f y_{L1}^f H_{f(1,0,1)}^f + \sum_f y_{L1}^f H_{f(1,0,5)}^f + y_9 H_{f(1,0,3)}^f + \text{non-singlet terms} \right] \lambda_R
\]

\[
+ \frac{1}{2} \left[ (\lambda_L)^T M_{\lambda^c}^M \lambda_L^c - (\lambda_R)^T M_{\lambda^c}^M \lambda_R^c \right] + h.c.,
\]

(17)
where $f = (u, d, v, e)$, $y^f_I = (y_1, y_2, y_3, y_4) U_C X$, $y^f_{II} = (y_5, y_6, y_7, y_8) U_C X$, and Majorana mass matrix of the neutral superheavy fermions is

$$M^M_{\lambda^\nu} = 2y_{10}(\Omega) = \sqrt{2} y_{10} v_G \left( \begin{array}{ccc} -2 \cos \beta \delta_0 & \sin \beta \delta_0 e^{i\delta_0} & \sin \beta \delta_0 e^{i\delta_0} \\ \sin \beta \delta_0 e^{i\delta_0} & \sqrt{2} \cos \beta \delta_0 & \sqrt{2} \\ \sin \beta \delta_0 e^{i\delta_0} & \sqrt{2} \cos \beta \delta_0 & 0 \end{array} \right). \quad (18)$$

$M^M_{\lambda^\nu}$ is of the order of $10^{15}$ GeV after taking account of the factor $y_{10}$. By reason of the $\delta_0$ phase in $M^M_{\lambda^\nu}$ arising, the $C$ and $CP$ symmetries are deprived by this time, but the left-right symmetry $D_P$ is still non-breaking.

The second stage of the breaking chain is that the two discrete symmetries $S_2$ and $D_P$ are broken together. It is achieved by every Higgs fields $H_k$ in the $V_B$ term developing VEVs in the approximate range of $10^{12} - 10^{14}$ GeV. The previous breaking can induce that the $\mu^2$ effective coefficients of the $H_k^2$ terms now become negative, namely $\mu^2_{2k} - B_{2k}(\Omega)^2 < 0$, consequently, this second step of breaking is triggered. This breaking takes place along directions of the singlets $H^f_{(1,0,1)}, H^f_{(1,0,5)}, H_{(1,0,3)}$ of $SU(3)_C \otimes U(1)_X$. In the flavor space, the breaking is exactly along diagonal elements of the symmetric $H^f_{(1,0,1)}, H^f_{(1,0,5)}$ as well as off-diagonal elements of the antisymmetric $H_{(1,0,3)}$. It can be seen from the second term in the formula (17) that all the superheavy fermions now acquire Dirac mass terms, namely

$$M^D_{\lambda^\nu} = y^f_I \langle H^f_{(1,0,1)} \rangle + y^f_{II} \langle H^f_{(1,0,5)} \rangle + y_9 \langle H_{(1,0,3)} \rangle = \left( \begin{array}{ccc} \rho_1^f & i\rho_4 & i\rho_5 \\ -i\rho_4 & \rho_2^f & -i\rho_6 \\ -i\rho_5 & i\rho_6 & \rho_3^f \end{array} \right). \quad (19)$$

The above mass matrix elements can virtually be about $10^{10} - 10^{15}$ GeV after taking account of the factors $y^f_I, y^f_{II}, y_9$. All the pure imaginary off-diagonal elements originate from the antisymmetric hermitian representation $H_{(1,0,3)}$. They are another source of the $C$ and $CP$ violation. In this way all of the discrete symmetries are broken, but the $SU(3)_C \otimes U(1)_X$ symmetry is retained.

The third step of the breaking chain is that $SU(2)_R \otimes U(1)_X$ breaks to $U(1)_Y$, namely the right-handed isospin symmetry breaking. It occurs at the energy scale $10^{10}$ GeV or so. The right-handed Higgs field $H_R$ in the $V_C$ term is responsible for this breaking. After the second step of breaking is over, the left-right symmetry $D_P$ has been invalid. The VEVs $\langle \Omega \rangle$ and $\langle H_k \rangle$ can induce that provided the coefficients $C_3, C_{4k}$ are not enough small, the $\mu^2$ effective coefficients of the $(H_R^+ H_R + h.c.)$ term become negative but ones of the $(H_L^+ H_L + h.c.)$ term are still positive. Accordingly, the $SU(2)_R$ breaking takes place prior to the $SU(2)_L$ breaking. The charge quantum number and gauge coupling coefficient of $U(1)_Y$ are respectively given by

$$\frac{Y}{2} = I^R_3 + X,$$
$$\frac{1}{g_Y^2} = \frac{1}{g_R^2} + \frac{1}{g_X^2}.$$  \quad (20)
Under the subgroup $U(1)_Y$, the relevant fields in the formula (12) have following decomposition

$$
Q_L = Q_L(\frac{1}{2}), \quad Q_R = u_R(\frac{1}{2}) \oplus d_R(-\frac{1}{2}),
$$

$$
L_L = L_L(-1), \quad L_R = \nu_R(0) \oplus e_R(-2),
$$

$$
H_{L1} = H_{L1}(+1), \quad H_{R1} = H_{R1}^+ \oplus H_{R1}^0,
$$

$$
H_{L2} = H_{L2}(-1), \quad H_{R2} = H_{R2}^0 \oplus H_{R2}^-(2).
$$

(21)

It can be seen from this that the breaking is implemented by the neutral singlets $H_{R1}^0, H_{R2}^0$ of $H_R$ developing VEVs, namely

$$
\langle H_R \rangle = \begin{pmatrix}
  v_{R2} & 0 \\
  0 & v_{R1}
\end{pmatrix} = v_R \begin{pmatrix}
  \sin \beta_R & 0 \\
  0 & \cos \beta_R
\end{pmatrix},
$$

(22)

where

$$
v_R^2 = v_{R1}^2 + v_{R2}^2 = -\mu_{R1}^2 + \mu_{R2}^2 \tan^2 \beta_R/C_5 (1 - \tan^2 \beta_R),
$$

$$
\sin 2\beta_R = \frac{-2\mu_{R1}^2}{\mu_{R1}^2 + \mu_{R2}^2},
$$

$$
\mu_{R1}^2 = \mu_{C1}^2 - \frac{1}{2} C_2 k \text{Tr}(H_k)^2 + \mu_{C2}(H_1) > 0,
$$

$$
\mu_{R2}^2 = \mu_{C1}^2 - C_1 v_G^2 - \frac{1}{2} C_2 k \text{Tr}(H_k)^2 + \mu_{C2}(H_1) > 0,
$$

$$
\tilde{\mu}_R^2 = \mu_{C3}^2 - \frac{1}{2} C_3 v_G^2 - \frac{1}{2} C_4 k \text{Tr}(H_k)^2 - \mu_{C4}(H_1) < 0.
$$

(23)

The value of $v_R \sim 10^{10}$ GeV signifies the energy scale of the right-handed isospin symmetry breaking. Because $\mu_{R1}^2 > \mu_{R2}^2$, tan $\beta_R$ is more than one. In fact $v_{R1}$ is approximately equal to $v_{R2}$, namely the difference of the both is very small, therefore tan $\beta_R$ is very close to one. A detailed discussion about a part of potential only involving $H_R$ shows that this breaking leads to three massive right-handed gauge bosons $W_{\mu R}^\pm, Z_{\mu R}^0$ and five massive right-handed Higgs bosons, namely two CP-even neutral $h_R^0, H_R^0$, one CP-odd neutral $A_R^0$, and a pair of charged $H_R^\pm$. Their masses are given by relations as

$$
M_{W_R^\pm}^2 = \frac{g_R^2 v_R^2}{2}, \quad M_{Z_R}^2 = \frac{(g_R^2 + g_X^2) v_R^2}{2};
$$

$$
M_{H_R^+}^2 = \frac{\mu_{R1}^2 - \mu_{R2}^2}{\cos 2\beta_R} = -\frac{C_1 v_G^2}{\cos 2\beta_R}, \quad M_{H_R^-}^2 = \frac{M_{A_R^0}^2}{C_5 v_R^2};
$$

$$
M_{H_R^0}^2 = \frac{M_{A_R}^2}{2} + \frac{1}{2} \sqrt{M_{H_R^+}^4 - 8 \left( M_{H_R^+}^2 - M_{A_R}^2 \right) \left( 2M_{A_R}^2 - M_{H_R^-}^2 \right) \cos^2 2\beta_R},
$$

$$
M_{H_R^-}^2 = \frac{M_{A_R}^2}{2} - \frac{1}{2} \sqrt{M_{H_R^+}^4 - 8 \left( M_{H_R^-}^2 - M_{A_R}^2 \right) \left( 2M_{A_R}^2 - M_{H_R^+}^2 \right) \cos^2 2\beta_R}.\quad (24)
$$
From the above equations we can see that these Higgs particle masses have following relations

\[ M_{h_R^0}^2 + M_{H_R^0}^2 = M_{H_R^\pm}^2, \]
\[ M_{h_R^0}^2 < \frac{1}{2} M_{H_R^\pm}^2 < \left( M_{H_R^0}^2, M_{X_R^0}^2 \right) < M_{H_R^\pm}^2. \]  

(25)

Moreover, the lightest right-handed Higgs boson meets the mass limit \( M_{h_R^0}^2 \leq \sqrt{2} C_5 v_R \). Because their masses are relatively heavy, the right-handed gauge and Higgs bosons are impossibly detected at low energy scale.

Below the \( v_R \) scale, the model symmetry now descends to the SM symmetry \( SU(2)_L \otimes U(1)_Y \otimes SU(3)_C \). At this point all the superheavy fermion are actually decoupling on account of their superheavy masses, therefore, they can be integrated out from the model Lagrangian. The low-energy effective Yukawa Lagrangian is then derived from the formula (17)-(19) such as

\[
\mathcal{L}_{\text{Yukawa}}^{\text{eff}} = \bar{Q}_L H_{L2} Y_u u_R + \bar{Q}_L H_{L1} Y_d d_R + \bar{\ell}_L H_{L1} Y_e e_R \\
+ \frac{1}{2} \left( \overline{\nu}_R, \nu_R \right) \left( \begin{array}{cc} \nu_{L} & \nu_{LR} \\ Y_{LL}^T & -M_{RR} \end{array} \right) \left( \begin{array}{c} H_{L2}^T L_L^c \\ \nu_R \end{array} \right) + \text{h.c.},
\]  

(26)

where

\[
Y_u = \frac{y_0^2 v_R \sin \beta_R}{M_{\lambda_u}^D}, \quad Y_d = \frac{y_0^2 v_R \cos \beta_R}{M_{\lambda_d}^D}, \quad Y_e = \frac{y_0^2 v_R \cos \beta_R}{M_{\lambda_e}^D},
\]
\[
Y_{LR} = \frac{-y_0^2 v_R \sin \beta_R}{M_{\lambda_e}^M + M_{\lambda_e}^M (M_{\lambda_e}^D)^{-1} M_{\lambda_e}^M},
\]
\[
Y_{LL} = \frac{-y_0^2 v_R^2 \sin^2 \beta_R}{M_{\lambda_e}^M + (M_{\lambda_e}^D)^2 (M_{\lambda_e}^M)^{-1} (M_{\lambda_e}^D)^T}.
\]

(27)

In comparison with the SM, this low-energy effective theory, which is valid in scope of \( 10^2 - 10^9 \) GeV, has two Higgs doublets and three generation of the right-handed neutrino singlets. It contains a Majorana mass term of the right-handed neutrinos, and a non-renormalizable Majorana-type coupling of the left-handed lepton doublet with the second Higgs doublet, which can generate the left-handed neutrino Majorana masses when \( SU(2)_L \) is broken later. On the basis of the foregoing discussion, we can roughly estimate these quantities in the formula (27). The charged fermion Yukawa couplings \( Y_u, Y_d, Y_e \) are about the magnitude of \( 10^{-5} - 1 \), while the neutrino Dirac-type coupling \( Y_{LR} \) is of the order of \( 10^{-10} - 10^{-5} \) or so. On the average, the later is significantly much less than the former. The non-renormalizable coupling \( Y_{LL} \), which has one minus mass dimension, is about \( 10^{-15} \) GeV\(^{-1}\). It is thus clear
that these effective Yukawa couplings emerge large hierarchy and flavor mixing. In addition, the effective Majorana masses of the right-handed neutrinos are about several ten TeVs. Since $Y_{LR}$, which is actually the only coupling of the right-handed neutrinos with the other particles, is so small, the right-handed neutrinos have essentially decoupled from the low-energy interactions. They probably become some of WIMPs. Furthermore, if the lightest right-handed neutrino is far lighter than the lightest Higgs boson, it is relatively a stable particle, and is able to be a candidate for the dark matter. However, if Higgs bosons are found in future, the decay mode $H_{L2} \rightarrow L_L + \nu_R$ provides an approach to detect the lightest right-handed neutrino as the dark matter.

The last step of the breaking chain is that $SU(2)_L \otimes U(1)_Y$ breaks to $U(1)_{em}$, namely electroweak symmetry breaking. It is completed by the residual $H_L$ Higgs field in the $V_C$ term. The discussion about this breaking is parallel to the last $SU(2)_R$ breaking. The electric charge quantum number and gauge coupling coefficient of $U(1)_{em}$ are respectively given by

$$Q = I_3^L + \frac{Y}{2},$$

$$\frac{1}{e^2} = \frac{1}{g_L^2} + \frac{1}{g_Y^2}. \quad (28)$$

Under the subgroup $U(1)_{em}$, the relevant fields in the formula (21) have following decomposition

$$Q_{L(\frac{1}{3})} = u_L^\frac{2}{3} \oplus d_L^{-\frac{1}{3}}, \quad u_{R(\frac{1}{3})} = u_R^\frac{1}{3}, \quad d_{R(-\frac{1}{3})} = d_R^{-\frac{1}{3}},$$

$$L_{L(-1)} = \nu_L^0 \oplus e_L^{-1}, \quad \nu_{R(0)} = \nu_R^0, \quad e_{R(-2)} = e_R^{-1},$$

$$H_{L1(+1)} = H_{L1}^+ \oplus H_{L1}^0, \quad H_{L2(-1)} = H_{L2}^0 \oplus H_{L2}^- . \quad (29)$$

These things are exactly well-known particle contents of the SM but adding the right-handed neutrino singlet and the second Higgs doublet. The breaking method is the same as the previous ones. The former $\langle H_R \rangle$ can now trigger that the $\mu^2$ effective coefficients of the $(H_L^2 \tilde{H}_L + h.c.)$ terms become negative, accordingly $SU(2)_L$ is broken by the $H_L$ neutral singlets $H_{L1}^0, H_{L2}^0$ developing VEVs, namely

$$\langle H_L \rangle = \begin{pmatrix} v_{L2} & 0 \\ 0 & v_{L1} \end{pmatrix} = v_L \begin{pmatrix} \sin \beta_L & 0 \\ 0 & \cos \beta_L \end{pmatrix}, \quad (30)$$
and (32) that there is a relation between the left-handed Higgs mass $M_{H_L^+}$ and (33) that there is a relation between the left-handed Higgs mass $M_{H_L^+}$ and the right-handed Higgs mass $M_{A_R^0}$ such as
\[ M_{H_L^+}^2 \cos 2\beta_L = M_{A_R^0}^2 \cos 2\beta_R. \]
After the electroweak breaking, the whole symmetry only remains $U(1)_{em} \otimes SU(3)_C$. It is now inferred from the effective Yukawa Lagrangian (26) that all the fermions obtain Dirac masses and the left-handed neutrinos acquire Majorana masses. All the fermion mass terms are written as follows

$$-\mathcal{L}_{mass} = \overline{u}_L M_u u_R + \overline{d}_L M_d d_R + \overline{e}_L M_e e_R + \frac{1}{2} \left( \overline{\nu}_L, \overline{\nu}_R \right) \begin{pmatrix} M_{LL} & M_{LR} \\ M_{LR}^T & M_{RR} \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} + \text{h.c.},$$

(35)

where

$$M_u = -Y_u v_L \sin \beta_L, \quad M_d = -Y_d v_L \cos \beta_L, \quad M_e = -Y_e v_L \cos \beta_L,$$

$$M_{LR} = -Y_{LR} v_L \sin \beta_L, \quad M_{LL} = -Y_{LL} v_L^2 \sin^2 \beta_L.$$  

(36)

Finally, the effective Majorana masses of the left-handed and right-handed neutrinos are achieved by diagonalizing the neutrino mass matrix in the formula (35). Because $M_{LL} \ll M_{LR} \ll M_{RR}$, they are easily given by seesaw mechanism such as [17]

$$M_{\nu_L} \approx M_{LL} - M_{LR} M_{RR}^{-1} M_{LR}^T = \frac{y_0^2 v_L^2 \sin^2 \beta_L}{M_{\nu_M}^2}, \quad M_{\nu_R} \approx M_{RR}.$$  

(37)

It can be seen from (36) and (37) that the quark, lepton and effective left-handed Majorana neutrino masses are of the right order of magnitude comparing with the experiment data [2], while the effective right-handed Majorana neutrino masses are about several ten TeVs.

To summarize all the above discussions, all the masses of the low-energy fermions, gauge bosons and Higgs bosons are naturally solved from our model by the model symmetry breaking step by step. The fermion mass hierarchy and flavor mixing are successfully accomplished. Moreover, the $CP$ violation availably originates from the model spontaneous symmetry breaking. Finally, we especially point out that the strong $CP$ problem can also be resolved very well. Above the GUT scale, the model holds strictly the $C,T,P$ discrete symmetries. The $P$ parity conservation assures that $\theta_{QCD}$ is zero in the model. On the other hand, since all the quark mass matrices from the symmetry breaking are hermitian in virtue of the model itself characteristics, the tree-level $\theta_{QFD} = \text{Arg}[\text{Det}(M_u M_d)]$ is also nought. In a word, the theoretical structure of the model eliminates the strong $CP$ violation automatically.

III. Particle Masses and Flavor Mixing

From the last section discussion, we have seen that below the $v_R$ breaking scale, the model particle spectrum is actually identical to one of the SM with two Higgs doublets but adding the singlet right-handed neutrinos. After taking account of loop correction effects, the fermions masses running from the $v_R$ scale to the electroweak
scale are determined by the renormalization group equations (RGEs). However, the effective right-handed Majorana neutrinos masses running can be neglected since they are actually decoupled below the \( v_R \) scale. We introduce the Yukawa coupling squared matrices for the charged fermions as follows

\[
S_u = Y_u Y_u^\dagger, \quad S_d = Y_d Y_d^\dagger, \quad S_e = Y_e Y_e^\dagger.
\]

The one-loop closed RGEs consisting of these Yukawa coupling squared matrices, the effective left-handed Majorana neutrino mass matrix and the three gauge coupling coefficients are then given by \[18\]

\[
\frac{d\alpha_i(\chi)}{d\chi} = \frac{b_i}{2\pi} \alpha_i^2, \quad (i = 1, 2, 3) \tag{39}
\]

\[
\frac{dS_f(\chi)}{d\chi} = \frac{1}{16\pi^2} (S_f K_f + K_f S_f), \quad (f = u, d, e) \tag{40}
\]

\[
\frac{dM_{\nu_L}(\chi)}{d\chi} = \frac{1}{16\pi^2} [M_{\nu_L} K_{\nu} + (K_{\nu})^T M_{\nu_L}], \tag{41}
\]

with

\[
K_u = \frac{3}{2} S_u + \frac{1}{2} S_d + \left[ \text{Tr}(3S_u) - 4\pi \left( \frac{17}{20} \alpha_1 + \frac{9}{4} \alpha_2 + 8\alpha_3 \right) \right] I,
\]

\[
K_d = \frac{1}{2} S_u + \frac{3}{2} S_d + \left[ \text{Tr}(3S_d + S_e) - 4\pi \left( \frac{1}{4} \alpha_1 + \frac{9}{4} \alpha_2 + 8\alpha_3 \right) \right] I,
\]

\[
K_e = \frac{3}{2} S_e + \left[ \text{Tr}(3S_d + S_e) - 4\pi \left( \frac{9}{20} \alpha_1 + \frac{9}{4} \alpha_2 \right) \right] I,
\]

\[
K_{\nu} = \frac{1}{2} S_e + \left[ \text{Tr}(3S_u) - 4\pi \left( \frac{9}{20} \alpha_1 + \frac{9}{4} \alpha_2 \right) \right] I,
\]

where \( \alpha_i = \left( \frac{5}{3}, \frac{7}{4}, \frac{3}{4} \right), b_i = (21/5, -3, -7), \chi = \ln(\mu/v_R) \) and \( I \) is a \( 3 \times 3 \) unit matrix. If the values of \( \alpha_i(\chi), S_f(\chi), M_{\nu_L}(\chi) \) at the \( v_R \) scale are taken as input values, we can solve the above RGEs numerically and figure out their corresponding values at the electroweak scale. All of fermion mass eigenvalues at the electroweak scale are subsequently obtained by diagonalizing the Yukawa coupling squared matrices and the effective neutrino mass matrices such as

\[
U_u S_u(\chi_w) U_u^\dagger = \frac{1}{v_w^2 \sin^2 \beta_L} \text{diag} \left( m_u^2(\chi_w), m_c^2(\chi_w), m_t^2(\chi_w) \right),
\]

\[
U_d S_d(\chi_w) U_d^\dagger = \frac{1}{v_w^2 \cos^2 \beta_L} \text{diag} \left( m_u^2(\chi_w), m_c^2(\chi_w), m_t^2(\chi_w) \right),
\]

\[
U_e S_e(\chi_w) U_e^\dagger = \frac{1}{v_w^2 \cos^2 \beta_L} \text{diag} \left( m_u^2(\chi_w), m_c^2(\chi_w), m_t^2(\chi_w) \right),
\]

\[
U_{\nu_L} M_{\nu_L}(\chi_w) U_{\nu_L}^T = \text{diag} \left( m_1(\chi_w), m_2(\chi_w), m_3(\chi_w) \right),
\]

\[
U_{\nu_R}^* M_{\nu_R} U_{\nu_R}^T = \text{diag} \left( M_1, M_2, M_3 \right), \tag{43}
\]
Figure. 2. Tree level and one-loop diagrams contributing to superheavy gauge boson decays into the right-handed up-type quarks and the effective right-handed Majorana neutrinos.

where \( \chi_w = \ln(M_Z/v_R) \) denotes that the fermion masses and mixing are evaluated at the \( M_Z \) scale. Accordingly, the quark and lepton mixing matrices are given by

\[
U_u U_d^\dagger = U_{CKM}^q(\chi_w), \quad U_e U_{\nu_L}^\dagger = U_{CKM}^l(\chi_w) \, \text{diag}(e^{i\beta_1}, e^{i\beta_2}, 1),
\]

where \( \beta_1, \beta_2 \) are two Majorana phases in the lepton mixing matrix. Finally, the mixing angles and \( CP \)-violating phases in the unitary matrices \( U_{CKM}^q(\chi_w) \) are worked out by the standard parameterization in ref. [2].

IV. Baryogenesis and Dark Matter

The usual baryogenesis mechanism is through leptogenesis. In view of the effective right-handed Majorana neutrino property \( N_R = \nu_R + \overline{\nu}_R \), their non-equilibrium decays \( N_R \rightarrow L_L + H_{L2} \) can lead to the lepton number asymmetry. It afterward is converted into the baryon number asymmetry by the sphaleron processes over the electroweak scale. In our model, because the neutrino Dirac-type coupling \( Y_{LR} \) is much smaller than ones of the charged fermions, the \( CP \) asymmetry generated by the above decay processes is actually less than \( 10^{-12} \). This mechanism is therefore out of work for our model. We here suggest the following new mechanism to implement baryogenesis successfully. It can be seen from the section II discussions that after \( SU(2)_R \) breaking but before \( SU(2)_L \) breaking, the superheavy gauge bosons \( X_{\mu}^{\pm \frac{2}{3}} \) have gauge couplings with the low-energy SM fermions as follows

\[
- \mathcal{L}_{\text{gauge}} = \frac{g_C}{\sqrt{2}} \left( \overline{Q}_L \gamma^\mu X_{\mu} L_L + \overline{u}_R \gamma^\mu X_{\mu} \nu_R + \overline{d}_R \gamma^\mu X_{\mu} e_R \right) + h.c.
\]

It tell us that the superheavy gauge bosons have the only \( B-L \) violating decay modes \( X_{\mu}^{\pm \frac{2}{3}} \rightarrow u_R^{\pm \frac{2}{3}} + N_R \). If we take the loop correction through Yukawa couplings (26) into account, see figure 2, The interference between the tree level graph and the one-loop graph can lead to the \( CP \) asymmetry of their decay widths. This is owing
to Yukawa couplings $Y_u, Y_{LR}$ containing non-removable complex phases. The $CP$ asymmetry $\varepsilon$ is calculated to be

$$
\varepsilon = \frac{\sum \alpha \left[ \Gamma \left( X_{\mu}^+ \leftrightarrow u_{R\alpha}^+ + N_{R\alpha} \right) - \Gamma \left( X_{\mu}^- \leftrightarrow u_{R\alpha}^- + N_{R\alpha} \right) \right]}{\sum \alpha \left[ \Gamma \left( X_{\mu}^+ \leftrightarrow u_{R\alpha}^+ + N_{R\alpha} \right) + \Gamma \left( X_{\mu}^- \leftrightarrow u_{R\alpha}^- + N_{R\alpha} \right) \right]} \approx -\frac{\text{Im} \left[ \text{Tr} \left( Y_u^\dagger Y_{LR} \right) \right]}{24\pi}.
$$

(46)

In accordance with the previous discussions, the $\varepsilon$ value is estimated to be of the order of $10^{-8}$ or so. The above decay processes of the superheavy gauge bosons have three characteristics, viz. $B$-$L$ violating one minus unit, generating the $CP$ asymmetry and being out of thermal equilibrium. The third item can be seen from that $\Gamma/H(M_{X_{\mu}}) = g_C^2 M_{pl}/(16\pi)1.66\sqrt{g_s}M_{X_{\mu}} < 1$ so long as $g_C^2 \sim 0.5, \sqrt{g_s} \sim 10, M_{X_{\mu}} \sim 10^{16}$ GeV. Consequently, an asymmetry of the $B$-$L$ quantum number naturally comes into being after the decay processes are over. It is related to the $CP$ asymmetry by the relation

$$
Y_{B-L} = \frac{n_{B-L} - \bar{n}_{B-L}}{s} = \kappa \frac{(-1)\varepsilon}{g_*},
$$

(47)

where $\kappa$ is the so called dilution factor which accounts for the wash out effects, and $g_*$ is the effective number of relativistic degrees of freedom. Counting the effective right-handed Majorana neutrinos in, $g_* = 116$ in our model. Subsequently, the sphaleron processes over the electroweak scale can violate $B+L$ and eras rapidly whatever $B+L$ asymmetry, at the same time, whereas they conserve $B-L$ and cause that the $B-L$ asymmetry is eventually translated as the baryon asymmetry. The asymmetry $Y_B$ and $Y_{B-L}$ are related by

$$
Y_B = \frac{n_B - \bar{n}_B}{s} = cY_{B-L},
$$

(48)

where $c = (8N_F + 4N_H)/(22N_F + 13N_H) = \frac{8}{23}$. Finally, after the electroweak breaking the matter-antimatter asymmetry in the universe is just as observed nowadays.

We simply discuss a possibility that the lightest right-handed Majorana neutrino is a candidate for the dark matter. Since the right-handed Majorana neutrinos come from the superheavy gauge bosons decay products, one right-handed Majorana neutrino energy is almost an half of one superheavy gauge boson mass, namely $E_{N_R} \approx \frac{1}{2}M_{X_{\mu}}$ for $M_{X_{\mu}} \gg (M_{N_R}, m_u)$. Because $E_{N_R} \gg M_{N_R}$, the right-handed Majorana neutrinos are actually relativistic radiation gas in our universe. In the case of the lightest right-handed Majorana neutrino, it can only decay into the SM fermions through the left-handed Higgs or the right-handed gauge boson intermedia because of $M_{N_{R1}} \ll (M_{W_R}, M_{H_L})$, see figure 3. The decay widths of it’s two decay
modes are approximately

\[ \Gamma_a \lesssim \frac{\text{Tr} \left( Y_u^\dagger Y_u \right) \left( Y_{LR}^\dagger Y_{LR} \right) M_1^5}{4(8\pi)^3 M_{h_L}^3}, \quad \Gamma_b \lesssim \frac{3 g_R^4 M_1^5}{4(8\pi)^3 M_{W_R}^4}. \] (49)

If the lightest Higgs particle mass is too heavy, the dominated contribution for the total width is \( \Gamma_b \). On the contrary, the \( \Gamma_a \) contribution is dominant. If the lightest right-handed Majorana neutrino is a dark matter particle, its decay width should be larger than the current Hubble parameter about \( 10^{-42} \) GeV, in other words, its lifetime should exceed the age of the universe. This universe constraint can give a lower bound for the lightest Higgs particle mass. The dominated processes for the lightest right-handed Majorana neutrino pair annihilation are shown in figure 4. The annihilation into electron pair involves the right-handed charged current as well as the right-handed neutral current, while the annihilation into other fermion pair involves only the right-handed neutral current. The total annihilation cross-section is approximately

\[ \sigma \approx \frac{(21 - 40 \sin^2 \theta_{WR} + 64 \sin^4 \theta_{WR}) g_R^4}{768 \pi M_{W_R}^4} s, \] (50)

where \( s \) is the squared center-of-mass energy, \( \sin^2 \theta_{WR} = \frac{g_R^2}{g_R^2 + g_X^2} \) is the right-handed weak gauge mixing angle. The actual numerical results show \( \sin^2 \theta_{WR} \approx \frac{1}{2} \). If the right-handed gauge boson mass is \( M_{W_R} \approx 10^9 \) GeV, and the center-of-mass energy for the lightest right-handed Majorana neutrino pair annihilation is \( \sqrt{s} \approx M_{X_{\mu}} \approx 10^{16} \) GeV, the annihilation cross-section is able to approach \( \sigma \approx 10^{-10} \) GeV\(^{-2}\). The value is a typical annihilation cross-section for the dark matter particles. It can give a right relic density of the dark matter.

V. Numerical Results
In this section, we present numerical results of our model. As noted earlier, if a set of parameters at the high-energy scale \( \mu_{\text{high}} = v_R \) are chosen as the input values, we can calculate, at the electroweak scale \( \mu_{\text{weak}} = M_Z = 91.2 \) GeV, three gauge couplings and all the fermion masses and mixing by the RGEs evolution. Moreover, the model can also predict all of the values including the gauge boson masses and Higgs particle masses, the baryon asymmetry and the relic density of dark matter. Of course, all the output results should be compared with the current and future experimental data.

The model input parameters involve the following quantities. First of all, the three critical energy scales marking the symmetry breaking steps are fixed as (in GeV unit)

\[
v_L = 174, \quad v_R = 1 \times 10^{10}, \quad v_G = 4.1 \times 10^{16}.
\] (51)

Secondly, the three gauge coupling coefficients in the model are set as

\[
g_L = g_R = 0.570, \quad g_C = 0.657, \quad g_G = 1.08.
\] (52)

In addition, herein Yukawa coupling \( y_0 \) is a non-independent parameter, we fix \( y_0 = 1 \) without loss of generality. The independent Yukawa coupling \( y_{10} \) and Higgs couplings \( C_1, C_5 \) are chosen as

\[
y_{10} = 0.0343, \quad C_1 = 1 \times 10^{-8}, \quad C_5 = 3.5 \times 10^8.
\] (53)

Thirdly, the parameters related to ratios and phase of the VEVs are taken such as

\[
\tan \beta_L = 6.5, \quad \tan \beta_R = 1.00003, \quad \tan \beta_G = 2.25, \quad \delta_0 = 0.056 \pi.
\] (54)

Lastly, the superheavy fermion mass matrix elements in the formula (19) are input by the following values (in \( 10^{12} \) GeV unit)

\[
\rho^u_1 = 948, \quad \rho^u_2 = -2.72, \quad \rho^u_3 = 0.0106; \quad \rho^\nu_1 = 3000, \quad \rho^\nu_2 = 9000, \quad \rho^\nu_3 = -500; \\
\rho^d_1 = 83.5, \quad \rho^d_2 = 1.32, \quad \rho^d_3 = 0.132; \quad \rho^e_1 = -448, \quad \rho^e_2 = 0.99, \quad \rho^e_3 = 0.126; \\
\rho_4 = 22.59, \quad \rho_5 = 0.831, \quad \rho_6 = 0.0536.
\] (55)
To sum up the above analysis, there are in all twenty-eight independent input parameters. Here we only pick a set of the typical input values without fine tuning instead of the detailed numerical analysis for the parameter space. For all kinds of the following quantities, however, the model predicting results are excellently in agreement with the recent experimental data.

According to eq.(39), the three gauge coupling coefficients at the electroweak scale are firstly solved to be

$$\alpha_1(\chi_w) \approx 0.0168, \quad \alpha_2(\chi_w) \approx 0.0335, \quad \alpha_3(\chi_w) \approx 0.1178.$$  \hspace{1cm} (56)

The gauge boson masses are immediately given by (15), (24) and (32) (in GeV unit).

$$M_{W_L^\pm}(\chi_w) \approx 79.8, \quad M_{Z_L^0}(\chi_w) \approx 91.1; \quad M_{W_R^\pm} \approx 4.03 \times 10^9, \quad M_{Z_R^0} \approx 5.66 \times 10^9; \quad M_{X_\mu} \approx 1.90 \times 10^{16}. \hspace{1cm} (57)$$

The above results are completely consistent with the current experimental measures \cite{2}. Although the Higgs sector RGEs are not discussed intensively, the tree level masses of the left-handed and right-handed Higgs bosons are straightly acquired from (24) and (32) (in GeV unit).

$$M_{H_L^0} \approx M_{A_L^0} \approx M_{H_R^0} \approx 4.06 \times 10^{12}, \quad M_{h_L^0} \approx 4.39 \times 10^6; \quad M_{H_R^0} \approx M_{H_R^0} \approx 7.31 \times 10^{14}, \quad M_{A_R^0} \approx 7.07 \times 10^{14}, \quad M_{h_R^0} \approx 7.75 \times 10^9. \hspace{1cm} (58)$$

The lightest Higgs particle mass should be $M_{h_L^0} \gtrsim 3.8 \times 10^6$ GeV if the lifetime of the lightest right-handed Majorana neutrino as dark matter exceeds the age of the universe. It is thus evident that except the discovered $W_L^\pm, Z_L^0$, the other gauge and Higgs bosons are too heavy to be detected in the future experiments such as LHC.

After using eq.(43) and eq.(44), all the fermion mass eigenvalues and mixing angles are together solved out. For the quark sector, they are

$$m_u(\chi_w) \approx 0.00233 \text{GeV}, \quad m_c(\chi_w) \approx 0.678 \text{GeV}, \quad m_t(\chi_w) \approx 181 \text{GeV}; \quad m_d(\chi_w) \approx 0.00469 \text{GeV}, \quad m_s(\chi_w) \approx 0.0933 \text{GeV}, \quad m_b(\chi_w) \approx 2.99 \text{GeV}; \quad s_{12}^q(\chi_w) \approx 0.2257, \quad s_{23}^q(\chi_w) \approx 0.0415, \quad s_{13}^q(\chi_w) \approx 0.00359, \quad \delta_{13}^q(\chi_w) \approx 58.7^\circ. \hspace{1cm} (59)$$

where $s_{\alpha\beta} = \sin \theta_{\alpha\beta}$. For the lepton sector, they are

$$m_e(\chi_w) \approx 0.000487 \text{GeV}, \quad m_\mu(\chi_w) \approx 0.103 \text{GeV}, \quad m_\tau(\chi_w) \approx 1.75 \text{GeV}; \quad m_1(\chi_w) \approx 0.0106 \text{eV}, \quad m_2(\chi_w) \approx 0.0138 \text{eV}, \quad m_3(\chi_w) \approx 0.050 \text{eV}; \quad s_{12}^l(\chi_w) \approx 0.567, \quad s_{23}^l(\chi_w) \approx 0.692, \quad s_{13}^l(\chi_w) \approx 0.0341, \quad \delta_{13}^l(\chi_w) \approx 0.665 \pi, \quad \beta_1(\chi_w) \approx 0.371 \pi, \quad \beta_2(\chi_w) \approx -0.103 \pi. \hspace{1cm} (60)$$
The above results are very well in accordance with the current status of the fermions masses and mixing at the $M_Z$ scale \cite{4,20}. In particular, the mass and mixing angle parameters for the effective left-handed neutrinos are

$$\Delta m^2_{21} \approx 7.88 \times 10^{-5} \text{ eV}^2, \quad \Delta m^2_{32} \approx 2.31 \times 10^{-3} \text{ eV}^2,$$

$$\tan^2 \theta_{12} \approx 0.473, \quad \sin^2 2 \theta_{23} \approx 0.998, \quad \sin \theta_{13} \approx 0.0341,$$

where $\Delta m^2_{\alpha\beta} = m^2_{\alpha} - m^2_{\beta}$. These results are excellently in agreement with the recent neutrino oscillation experimental data \cite{4}. In view of the value of $\sin \theta_{13}$ being close to zero, detecting it is still a challenge for the future neutrino oscillation experiments. However, the leptonic \textit{CP}-violating phases, including Dirac phase $\delta_{13}^l$ and two Majorana phases $\beta_1, \beta_2$, are relatively large. They are all promising to be detected by the leptonic \textit{CP}-violating experiments such as $0\nu\beta\beta$ \cite{21}.

The effective right-handed neutrinos masses are obtained straightforward by eq.(43) as follows (in GeV unit)

$$M_1 \approx 508, \quad M_2 \approx 1.56 \times 10^4, \quad M_3 \approx 9.08 \times 10^4.$$  

(62)

It can be seen from this that the lightest right-handed Majorana neutrino mass $M_1$ is less than one TeV, furthermore, it is far smaller than the lightest Higgs mass $M_{h^0_{L}}$. Its lifetime can be estimated by (49) such as

$$\tau_{N_{R1}} \approx \frac{\gamma}{\Gamma_a + \Gamma_b} \gtrsim 2.47 \times 10^{10} \text{ Year},$$

(63)

where $\gamma = \frac{E_{N_{R1}}}{M_1} \approx \frac{M_{X_{\mu}}}{2M_1}$ is Lorentz dilation factor. It is obviously longer than the age of the universe, in other words, it is also one of the significantly stable particles in the universe such as the left-handed Majorana neutrino, electron, proton. In addition, the relic density of the lightest right-handed Majorana neutrinos can be calculated by pair annihilation cross-section eq.(50) such as

$$\Omega_{N_{R1}} \approx \frac{0.1\text{Pb}}{(\sigma v_{rel})} \approx 0.255,$$

(64)

where $v_{rel} \approx 2$ is the relative velocity between the two $N_{R1}$ particles in their center-of-mass system, the thermal averaging of $s$ is $\langle s \rangle \approx \frac{M_{X_{\mu}}^2}{2}$. The result is exactly in accord with the part of energy density contributed by the dark matter in the current universe \cite{7}. For these reasons, we conclude that the lightest right-handed Majorana neutrino is indeed able to be a candidate for the dark matter.

Finally, the actual numerical results show $\Gamma(M_{X_{\mu}})/H(M_{X_{\mu}}) \sim 0.2$, therefore, the decay processes of the superheavy gauge bosons are indeed out of thermal equilibrium. A complete discussion about the wash-out effects is maybe more suitable in another paper. However, a detailed analysis for the inverse decay process, the $\Delta(B - L) = 1$ and $\Delta(B - L) = 2$ scattering processes shows that the processes reaction rates are actually very weak mainly because the superheavy gauge bosons
masses are far larger than the other particles. Therefore, we can safely neglect the wash-out effects and set the dilution factor as \( \kappa \approx 1 \). The baryon asymmetry \( \eta_B \) is then calculated by (46), (47) and (48) such as

\[ \eta_B = \frac{n_B - \bar{n}_B}{n_\gamma} \approx 7.04 Y_B \approx 6.15 \times 10^{-10}. \] (65)

It is in accord with the universe observations very well [6]. In summary, all the current detected values including the particle masses and mixing, the matter-antimatter asymmetry, the energy density portion of the dark matter are correctly reproduced by our model. All the non-detected values are also predicted in experimental limits.

VI. Conclusions

We have suggested the new left-right symmetric grand unified model based on the symmetric group \( SU(2)_L \otimes SU(2)_R \otimes SU(4)_C \otimes SU(2)_G \otimes SO(3)_F \otimes D_P \). The model symmetries undergo the three breaking steps to descend to the SM symmetries. The model can elegantly explain that all the elemental particle masses and flavor mixing at the electroweak scale, the matter-antimatter asymmetry in the universe, the dark matter and the strong \( CP \) violation. All the current experimental data for the above problems are correctly reproduced by our model without any fine tuning. The other gauge and Higgs bosons in the model are predicted to be relative heavy except \( W^\pm_L, Z^0_L \), thus they are not promising to be detected at LHC. In particular, the model predicts that both the matter-antimatter asymmetry and the dark matter in the universe are closely related to the right-handed Majorana neutrinos. The search for the right-handed Majorana neutrinos, whose mass is several hundred GeVs or so and energy is about \( 10^{16} \) GeV, will perhaps provide us some important information about the universe. The propositions are expected to be tested in future experiment on the ground and in the sky. However, the deeper investigation is worth being made an effort for understanding the mystery of the universe.

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References

[1] H. Fritzsch and Z. Z. Xing, Prog. Part. Nucl. Phys. 45, 1 (2000); R. N. Mohapatra and A. Y. Smirnov, Annu. Rev. Nucl. Part. Sci. 56, 569 (2006); M.
Dine and A. Kusenko, Rev. Mod. Phys. 76, 1 (2004); W. Buchmuller, R. D. Peccei and T. Yanagida, Annu. Rev. Nucl. Part. Sci. 55, 311 (2005); K. Olive, astro-ph/0301505.

[2] C. Amsler et al. [Particle Data Group], Phys. Lett. B 667, 1 (2008).

[3] Y. Fukuda et al. [Super-Kamiokande Collaboration], Phys. Rev. Lett. 81, 1562 (1998); Phys. Rev. Lett. 85, 3999 (2000); M. Apollonio et al. [CHOOZ Collaboration], Phys. Lett. B 466, 415 (1999); Eur. Phys. J. C 27, 331 (2003); K. Eguchi et al. [KamLAND Collaboration], Phys. Rev. Lett. 90, 021802 (2003); Q. R. Ahmad et al. [SNO Collaboration], Phys. Rev. Lett. 89, 011301 (2002).

[4] A. Hocker and Z. Ligeti, Annu. Rev. Nucl. Part. Sci. 56, 501 (2006); R. N. Mohapatra, et al., Rep. Prog. Phys. 70, 1757 (2007); G. L. Fogli, E. Lisi, A. Marrone, A. Palazzo, Pro. Part. Nucl. Phy. 57, 742 (2006); T. Schwetz, M. Tortola and J. Valle, New J. Phys. 10, 113011 (2008).

[5] V. Buscher and K. Jakobs, Int. J. Mod. Phys. A 20, 2523 (2005); K. Jakobs and M. Schumacher, Int. J. Mod. Phys. A 23, 5093 (2008).

[6] C. L. Bennett, et al., (WMAP Collaboration), 2003, astro-ph/0302207 G. Steigman, Int. J. Mod. Phys. E 15, 1 (2006).

[7] J. Garcia-Bellido, hep-ph/0407111, M. Trodden and S. M. Carroll, astro-ph/0401547.

[8] S. Weinberg, Cosmology, Oxford University Press, New York (2008); G. Bertone, D. Hooper and J. Silk, Phys. Reps. 405, 279 (2005).

[9] R. N. Mohapatra, Unification and Supersymmetry–The Frontiers of Quark-Lepton Physics, third edition, Springer-Verlag, New York (2003); D. J. H. Chung, L. I. Everett, G. L. Kane, S. F. King, J. Lykken, Lian-Tao Wang, Phys. Reps. 407, 1 (2005).

[10] J. C. Pati, Int. J. Mod. Phys. A 18, 4135 (2003); M. C. Chen and K. T. Mahanthappa, Int. J. Mod. Phys. A 18, 5819 (2003); G. L. Kane, S. F. King, I. N. R. Peddie and L. V. Sevilla, JHEP 08, 083 (2005); S. F. King, JHEP 08, 105 (2005); I. M. Varzielas and G. G. Ross, Nucl. Phys. B733, 31 (2006); C. Hagedorn, M. Lindner and R. N. Mohapatra, JHEP 06, 042 (2006); G. Altarelli and F. Feruglio, Nucl. Phys. B741, 215 (2006); W. M. Yang and Z. G. Wang, Nucl. Phys. B707, 87 (2005).

[11] J. C. Pati and A. Salam, Phys. Rev. D 10, 275 (1974); R. N. Mohapatra and G. Senjanoviç, Phys. Rev. D 23, 165 (1981).

[12] C. Callan, R. Dashen, and D. Gross, Phys. Lett. B 63, 334 (1976); R. Jackiw, C. Rebbi, Phys. Rev. Lett. 37, 172 (1976).
[13] M. Fukugita and T. Yanagida, *Physics of Neutrinos and Application to Astrophysics*, Springer-Verlag, Germany (2003); M. C. Chun, hep-ph/0703087; S. Davidson, E. Nardi and Y. Nir, Phys. Reps. 466, 105 (2008).

[14] V. A. Kuzmin, V. A. Rubakov, M. A. Shaposhnikov, Phys. Lett. B 155, 36 (1985).

[15] E. W. Kolb and M. E. turner, *The Early Universe*, Addison-Wesley (1990).

[16] R. N. Mohapatra and P. B. Pal, *Massive Neutrinos in Physics and Astrophysics*, third edition, World Scientific, Singapore (2004); A. D. Dolgov, Phys. Reps. 370, 333 (2002); S. Hannestad, New J. Phys. 6, 108 (2004).

[17] M. Gell-Mann, P. Ramond, R. Slansky, in *Supergravity*, eds. P. van Nieuwenhuizen and D. Z. Freeman (North-Holland, Amsterdam, 1979); T. Yanagida, in *Proc. of the Workshop on Unified Theory and Baryon Number in the Universe*, eds. O. Sawada and A. Sugamoto, Tsukuba, Japan (1979); R. N. Mohapatra, G. Senjanović, Phys. Rev. Lett. 44, 912 (1980).

[18] V. Barger, M. S. Berger and P. Ohmann, Phys. Rev. D 47, 1093 (1993); D. J. Castano, E. J. Piard and P. Ramond, Phys. Rev. D 49, 4882 (1994); K. S. Babu, C. N. Leung and J. Pantaleone, Phys. Lett. B 319, 191 (1993); S. Antusch, M. Drees, J. Kersten, M. Lindner and M. Ratz, Phys. Lett. B 525, 130 (2002).

[19] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973); B. M. Pontecorvo, Sov. Phys. JETP 6, 429 (1958); Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. 28, 870 (1962).

[20] C. R. Das and M. K. Parida, Eur. Phys. J. C 20, 121 (2001).

[21] H. Nunokawa, S. Parke and J. Valle, Prog. Part. Nucl. Phys. 60, 338 (2008); F. T. Avignone III, S. R. Elliott, J. Engel, Rev. Mod. Phys. 80, 481 (2008).