Are Cluster Magnetic Fields Primordial?

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We present results of a detailed and fully non-linear numerical and analytical investigation of magnetic field evolution from the very earliest cosmic epochs to the present. We find that, under reasonable assumptions concerning the efficiency of a putative magnetogenesis era during cosmic phase transitions, surprisingly strong magnetic fields $10^{-13} - 10^{-11}$ G, on comparatively small scales $100$ pc $- 10$ kpc may survive to the present. Building on prior work on the evolution of magnetic fields during the course of gravitational collapse of a cluster, which indicates that pre-collapse fields of $\sim 4 \times 10^{-12}$ G extant on small scales may suffice to produce clusters with acceptable Faraday rotation measures, we question the widely hold view that cluster magnetic fields may not be entirely of primordial origin.

Magnetic fields exist throughout the observable Universe. They exist in the interstellar medium, in galaxies, and clusters of galaxies (for reviews cf. [1]). The origin of galactic- and cluster- magnetic fields is still unknown. A plausible, though by far not convincingly established possibility is the generation of magnetic seed fields and their subsequent amplification via a galactic dynamo mechanism. Seed fields may be due to a variety of processes (and with a variety of strengths), such as the Biermann battery within intergalactic shocks [2], stellar magnetic fields expelled in planetary nebulae, or during supernovae explosions, either into the intergalactic, or in the presence of galactic outflows into the intergalactic medium [2], as well as due to quasar outflows of magnetized plasma [4]. Seed fields may also be of primordial origin with a multitude of proposed scenarios. These include generation during first-order phase transitions (e.g. QCD or electroweak), around cosmic defects, or during an inflationary epoch (with, nevertheless, extremely small amplitudes). For a review of proposed scenarios we refer the reader to [2].

The philosophy in prior studies of primordial magnetogenesis is often (but not always) as follows. After establishing a battery mechanism (e.g. separation of charges and production of currents) and a “prescription” or estimate for the final, non-linearly evolved magnetic field strength (e.g. equipartition of magnetic energy with turbulent flows), subsequent evolution is approximated by simply assuming frozen-in magnetic field lines into the plasma. Though such an approximation may be appropriate on the very largest scales, it should be clear, that this may not be the case on the “integral” or coherence scale of the field. Here, coupling of the magnetic fields to the gas induces non-linear cascades of energy in Fourier space. The characteristics of initially created magnetic field are thus vastly modified during cosmic evolution between the epoch of magnetogenesis and the present.

The final step in such studies is then often to determine field strengths on some prescribed large scale (e.g. $10$ Mpc) typically falling in the range $10^{-30}$ G $\lesssim B \lesssim 10^{-20}$ G, inferring that this may act as seed for a sufficiently efficient dynamo to produce galactic- and cluster- magnetic fields of order $10^{-6}$ G. This is observed in negligence of the fact that much stronger fields on smaller scales, result not only from a variety of astrophysical seeds, but from these very same primordial scenarios. Considering the likelihood of a magnetized plasma in the early Universe, it seems important to be able to make predictions on such final remnant fields surviving a magnetogenesis scenario to the present epoch, particularly also since such fields may fill voids of galaxies and may potentially be observable by upcoming technology [6].

The purpose of this letter is twofold. We have attempted to develop a coherent picture of gross features of non-linear, cosmic MHD evolution of primordial fields, including all relevant dissipative processes, such as viscosity due to diffusing- or free-streaming- neutrinos and photons, as well as ambipolar damping. A subset of the results of our numerical and analytical analysis is presented here, whereas details are presented elsewhere [7]. Our study allows us, for the first time, to make predictions for magnetic field energy and coherence length at the present epoch, for broad ranges of initial magnetic configurations, parametrized by spectral index, initial helicity, initial energy, as well as era of magnetogenesis. Second, drawing on a prior numerical MHD study of the gravitational collapse of a cluster of galaxies [3], we challenge the often-cited conclusion that cluster magnetic fields may not be entirely of primordial origin. Rather, we stress that it seems not clear at the moment if such fields on comparatively small scales may not, after all, produce cluster magnetic fields as widely observed.

In passing we note that there exists a number of analytical [3, 10, 11, 12, 13, 14, 15, 16] and numerical [14, 17], studies on the evolution of non-helical and helical primordial magnetic fields, which, nevertheless, for one or the other reason either remain inutile in predicting final field properties, or do so only for a specific scenario. The inutility of results is related to facts such as, the adaption of an evolutionary model not supported by numerical simulations, an inadequate treatment of viscosity due to photons, or simply, the analysis being linear in nature or being performed in Minkowski space and not properly transferred to the expanding Universe.
The generation of primordial magnetic fields in magnetogenesis scenarios is generally believed to occur during well-defined periods (e.g. QCD-transition). Subsequent evolution of these magnetic fields should therefore be well approximated by a “free decay” without any further input of kinetic or magnetic energy, i.e. as freely decaying MHD. The exceedingly large Prandtl numbers in the early Universe allow for the interpretation of the MHD equations allow for the interpretation of results obtained with Minkowski metric to results applicable for a Friedman-Robertson-Walker metric. Nevertheless, due to dissipation effects arising from the “imperfectness” of the fluid due to neutrino- and photon- diffusion and free-streaming play an important role in early MHD evolution. One may further show that for field strength as considered in this letter the assumption of fluid incompressibility is appropriate.

To verify theoretical expectations we have performed numerical simulations of incompressible, freely decaying, ideal, but viscous MHD. These simulations are performed with the help of a modified version of the code ZEUS-3D in a non-expanding (Minkowski) background and on 128$^3$ to 512$^3$ grids. Modifications lie in the inclusion of fluid viscosities. It is shown elsewhere that conformal- or near-conformal-invariance of the MHD equations allow for the interpretation of results obtained with Minkowski metric to results applicable for a Friedman-Robertson-Walker metric. Results of such simulations, in particular the decay of magnetic energy $E_{\text{mag}}$ with time, for a variety of physical regimes are shown in Fig. 4.

We have found that results of our simulations may be understood in a comparatively simple manner. In particular, non-linear MHD processing of the initial spectrum at epoch with Hubble constant $H(T)$ occurs for all scales $l$, which obey

$$v(l)/l \gtrsim H(T) ,$$

irrespective of the Reynolds number $R_e$ of the flow. Here $v(l)$ may be written as the Alfvén velocity $v_A(l) = B(l)/\sqrt{4\pi(\rho + p)}$ when turbulence holds, $R_e \gtrsim 1$, and as $v(l) = v_A(l) R_e \langle v = v_A, l \rangle$ for viscous MHD ($R_e \lesssim 1$). This holds equally during the photon diffusion ($l_p \ll l$) regime $R_e = v l/\eta$ and the photon free-streaming ($l_p \gg l$) regime $R_e = v/\alpha l$, where $\eta$, $\alpha$, and $l_p$ are photon shear viscosity, drag coefficient, and mean free path, respectively. Defining $L(T)$, the integral- or coherence- scale, as the scale where equality applies in Eq. (1), one finds that $L(T)$ is in fact, the scale containing most of the energy of the flow. This is due to a non-linear and rapid cascade developing on all scales $l \lesssim L$, with energy in fluid eddies transported down to the dissipation scale $l_{\text{diss}}$ and transferred to heat. Since the resultant small-scale spectrum is red and we assume the initial as yet unprocessed large-scale ($l \gtrsim L$) spectrum to be blue, $L(T)$, as the smallest unprocessed scale, remains as the magnetic coherence scale of the field at epoch with temperature $T$. Magnetic energy is then approximately given as the initial energy on scales $L(T)$.

For fields which are maximally helical, i.e. $H = H_{\text{max}} \approx (B^2(l)/l) \approx B^2(L) L$ Eq. (1) may still be used to obtain the coherence scale of the field. Nevertheless, due to dissipation effects the maximally helical case large-scale fields undergo growth even on scales $l \gtrsim L(T)$. Surprisingly, we find that during this process of large length scale magnetic field amplification the initial spectral index $n$ is conserved. The decay of magnetic energy, is thus described by the requirement of conservation of helicity, in conjunction with an increase of $L(T)$ described via Eq. (1). Due to a vast increase of $L(T)$ in the early Universe, even initially sub-maximally helical fields, i.e. $H_{\theta} \approx H_{\text{max}} \approx (B^2(l)/l) \approx B^2(L) L$, Eq. (1) ultimately reach a maximal helical configuration. Here, and throughout, an index “$g$” denotes properties at the magnetogenesis epoch. Parameterizing the initial (comoving) magnetic energy spectrum by $B_{g} \approx B_{g}(l/L_g)^{-\gamma}$, where $L_g$ is the initial (magnetogenesis) coherence scale obeying Eq. (1) and $B_{g}$ is approximate initial magnetic energy, one finds that fields have reached a maximally helical state when $L(T)$ has grown beyond

$$L_{\text{max}} \simeq L_g h_{g}^{-1/(\gamma - 1)} .$$

This picture may be employed to derive damping exponents, i.e. $E_{\text{mag}} \sim t^{-\gamma}$, and compare to those inferred from numerical simulations (cf. Fig. 4). Whereas the comparison is quite favorable in the viscous regime, turbulent decay is observed somewhat slower than predicted. For example, for non-helical, turbulent MHD with a $n = 3$ spectrum we predict $\gamma = 1.2$ whereas the best fit to the numerical simulation yields $\gamma = 1.05$. Nevertheless, we argue that this trend, seen also by others, must be due to limited numerical resolution. Its explanation, i.e. a putative additional increase
initial magnetic field configurations with FIG. 2: Evolution of comoving coherence length for different of relaxation time beyond $l/v$ with continuing evolution, requires the existence of additional dimensionful quantities (of dimension velocity or length) associated with the flow. As we may find none, other than $l_{\text{dias}}$ (which is mostly widely separated from $L$ in the early Universe, but not in the simulation) or $L_{h_{\text{box}}}$, the size of the simulation box, we believe the effect to be numerical in nature. We have noted, that spectra at late times show a peak region $\Delta L$ quite spread, and are likely only marginally resolved by the simulations. In any case, larger numerical simulations are required to address this effect, with resulting predictions for the surviving magnetic fields, given in this letter, being on the conservative side.

We have undertaken the in practice straightforward but arduous effort to assemble these results and, under the inclusion of all appropriate dissipation terms, and for quite general initial conditions, followed the growth of magnetic coherence length and energy density from the very earliest times to the present \[ \mathcal{A}, \] Fig. 2 shows examples for the growth of $L(T)$ for a number of scenarios of magnetogenesis at the QCD phase transition. The evolution is observed as an alternation between turbulent MHD and viscous MHD. “Viscosity” here is early on due to neutrinos, some time before recombination due to photons, and after recombination due to hydrogen-ion scattering and hydrogen-hydrogen scattering. Particularly notable are phases where the growth of $L(T)$ is halted completely. This occurs either at epochs before recombination in the viscous regime with diffusing photons or neutrinos, as well in part of the regime when those particles are free-streaming or at epochs after recombination, due to the peculiar redshifting of Eq. 14 and/or the effects of hydrogen diffusion and ambipolar drag. Note, however, that the growth of $L(T)$ and concomitant decrease of $B(T)$ during the late phases of viscous MHD with free-streaming photons (neutrinos) may be faster than the growth of $L(T)$ during turbulent MHD. Note also that initial conditions leading to relatively strong magnetic fields at recombination result in a rapid increase of $L(T)$ at $T_{\text{rec}} \approx 0.3$ eV, whereas for weaker fields $B \lesssim 10^{-13}$ G a similar jump occurs at reionization. Note that effects due to structure formation are not taken into account here (see below, however).

We give here the final (pre-structure formation) results on the coherence scale $L(T_0)$ and field amplitude $B(T_0)$, where $T_0$ is present CMB temperature, derived by employing Eq. (4), as well as retaining the initial field energy due to all scales $l \gtrsim L$ in the submaximally helical case, and conserving helicity density in the maximally helical case. Complete results on $L$ and $B$ for all eras may be found in \[ \mathcal{B}. \] Fields which remain still submaximally helical (i.e. $L(T_0) \lesssim L_{c,\text{max}}^{\text{max}}$) at the present epoch have for coherence length and field strength

$$B(T_0) \approx 1.65 \times 10^{-6} G \times (T_0/\text{100 MeV})^{-n/(n+2)}$$

(3)

$$L(T_0) \approx L_{\text{gc}} x^{-2/(n+2)} \left( \frac{T_0}{\text{100 MeV}} \right)^{2/(n+2)},$$

(4)

where $x = 2.30 \times 10^{-9}$ is a small factor and $r_g = (g_B/s_t^{4/3})_g$ is a convenient measure of magnetic energy density $g_B$ in terms of radiation entropy density $s_r = (4/3) g_r (\pi^2/30) T^3$ at the magnetogenesis epoch assumed to occur at temperature $T_g$. Note that, somewhat optimistically $r_g = 0.083$, when magnetogenesis results in magnetic energy density equivalent to the photon energy density shortly after a QCD-transition with $g_0 \approx 10.75$. The comoving coherence length $L_{\text{gc}}$ at the magnetogenesis epoch is given by

$$L_{\text{gc}} \approx 0.45 \text{pc} \sqrt{n} \left( \frac{r_g}{0.083} \right)^{1/2} \left( \frac{T_0}{\text{100 MeV}} \right)^{-1}. \quad (5)$$

This yields, for example, for $r_g = 0.083$, $T_0 = 100$ MeV, $n = 3$ to the appreciable field strength of $B_c \approx 1.1 \times 10^{-11}$ G. If we were to apply the simulation observed instead of theoretically predicted decay exponent $\gamma$ the result would increase approximately to $B \sim 5 \times 10^{-11}$ G. On the other hand, fields which have reached a maximally helical (i.e. $L(T_0) \gtrsim L_{c,\text{max}}^{\text{max}}$) state at present have

$$B_c \approx 4.69 \times 10^{-12} \text{ G \ y}$$

(6)

$$L_c \approx 550 \text{ pc \ y} \sqrt{n}.$$ 

(7)

with

$$y = \left( \frac{r_g}{0.083} \right)^{1/2} \left( \frac{h_g}{10^{-8}} \right)^{1/3} \left( \frac{T_0}{\text{100 MeV}} \right)^{-1/3},$$

where $h_g$ is the fractional helicity of the maximal one $\mathcal{H}_{\text{max},g}$ at the generation epoch.
Through on small scales, it is seen that surprisingly strong fields may survive an early Universe magnetogenesis epoch to the present (cf. Fig. 3). This is interesting in light of recent simulations on the formation of clusters of galaxies from slightly overdense and pre-magnetized regions via the gravitational instability [8]. The authors find, that fields of strength $B_0 \approx 4 \times 10^{-12}$ G (corresponding to their quoted $B \approx 10^{-9}$ G at simulation starting redshift $z = 15$) yield to clusters whose Faraday rotation measures are essentially indistinguishable from those observed in real clusters [23]. Furthermore, the authors arrive at the intriguing conclusion, that this result is virtually independent of whether a homogeneous field is assumed initially, or a field whose energy contribution is dominated by fluctuations on the very smallest scales in their simulations [24]. It is not clear if such an erasure of memory of initial conditions, possibly related to an interplay between the development of shear flows and small-scale turbulence during the course of gravitational collapse, pertains if initial field coherence lengths in the cluster simulations are reduced by a further factor $\sim 100$ (due to the comparison between typical coherence lengths in Eqs. 4, 7 and the spatial resolution, $\sim 100$ kpc comoving, of the simulations). Nevertheless, if so, cosmological magnetic fields generated during early eras, either of moderate magnetic helicity, or generated fairly late, could account for present-day observed cluster magnetic fields, and as such in the absence of any further dynamo amplification. This holds true even if as recently claimed such fields have very blue spectra [25]. We conclude that this interesting possibility seems to deserve further investigation.

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[22] Here indices “c” refer to comoving values, i.e. $L_c(T) = L(T)(a(T_0)/a(T))$ and $B_c(T) = B(T)(a(T)/a(T_0))^2$ for lengths and field strengths, respectively.
[23] Fields of this final strength may also be shown to be too weak at redshift $z \approx 10^6$ to induce significant spectral distortions in the cosmic microwave background radiation [26].
[24] It is important to note that employed initial magnetic field spectra in Ref. 8, $P(k) \sim k^{-5/3}$, correspond to a blue spectral index $n = 4/3$ in our notation, as opposed to Kolmogoroff $n = -2/3$ [27].
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