Abstract

In the earlier works on quantum geometrodynamics in extended phase space it has been argued that a wave function of the Universe should satisfy a Schrödinger equation. Its form, as well as a measure in Schrödinger scalar product, depends on a gauge condition (a chosen reference frame). It is known that the geometry of an appropriate Hilbert space is determined by introducing the scalar product, so the Hilbert space structure turns out to be in a large degree depending on a chosen gauge condition. In the present work we analyse this issue from the viewpoint of the path integral approach. We consider how the gauge condition changes as a result of gauge transformations. In this respect, three kinds of gauge transformations can be singled out: Firstly, there are residual gauge transformations, which do not change the gauge condition. The second kind is the transformations whose parameters can be related by homotopy. Then the change of gauge condition could be described by smoothly changing function. In particular, in this context time dependent gauges could be discussed. We also suggest that this kind of gauge transformations leads to a smooth changing of solutions to the Schrödinger equation. The third kind of the transformations includes those whose parameters belong to different homotopy classes. They are of the most interest from the viewpoint of changing the Hilbert space structure. In this case the gauge condition and the very form of the Schrödinger equation would change in discrete steps when we pass from a spacetime region with one gauge condition to another region with another gauge condition. In conclusion we discuss the relation between quantum gravity and fundamental problems of ordinary quantum mechanics.
1. Introduction

One of unsolved problems of the Wheeler - DeWitt quantum geometrodynamics is that of Hilbert space structure. The Wheeler - DeWitt quantum geometrodynamics was the first attempt of constructing a quantum theory of the Universe as a whole, however, if its Hilbert space structure is not rigorously determined, one cannot consider it as full and consistent, as well as any quantum theory.

The reasons, why this problem cannot be solved in the framework of the Wheeler – DeWitt quantum geometrodynamics, are closely connected with the fact that it was thought of as a gauge invariant theory. According to the original idea of Wheeler, a wave function of the Universe, which is a basic object in quantum geometrodynamics, must depend on 3-geometry of a manifold $\mathcal{M}$. In other words, if $\text{Riem}(\mathcal{M})$ is the space of all Riemannian metrics on $\mathcal{M}$, and $\text{Diff}(\mathcal{M})$ is diffeomorphism group, the wave function must be defined on the so-called superspace of all 3-geometries, or factor space $\text{Riem}(\mathcal{M})/\text{Diff}(\mathcal{M})$. One possible way to express this dependence would be to regard the wave function as a function of an infinite set of geometrical invariants. It is not clear, however, how to put this idea into practice. Actually, the wave function depends on a 3-metric, and it was believed that, if the wave function satisfied a quantum version of gravitational constraints, it would ensure its dependence on 3-geometry only. The very requirement for the wave function to satisfy the Wheeler – DeWitt, but not a Schrödinger, equation leads to the problem of Hilbert space, in particular, it is questionable how an inner product of state vectors should be determined (for a recent review on related problems, see [4, 5]). On the other hand, the Wheeler – DeWitt quantum geometrodynamics is based on Arnowitt – Deser – Misner (ADM) formalism, and, as some authors have emphasized [6, 7, 8], the latter is equivalent to some kind of gauge fixing, so there is the inconsistency between appealing to ADM formalism and the requirement for a wave function to be invariant under diffeomorphism group transformations.

In this work I shall discuss another approach to quantum geometrodynamics, namely, quantum geometrodynamics in extended phase space [9, 10, 11, 12]. The main features of this approach were presented on the previous PIRT conference [13]. As was shown in [13], in the “extended phase space” approach a physical part of the wave function satisfies a Schrödinger equation, whose form, as well as a measure in Schrödinger inner product, depends on a gauge condition, or a chosen reference frame (the basic formulae will be repeated in Section 2). The situation can be illustrated by the following scheme (Fig. 1). All metrics $g_{\mu\nu}$ related by gauge transformations are unified into an equivalence class representing dynamics of some 3-geometry. Two metrics $g_{\mu\nu}$ and $g'_{\mu\nu}$, which can be obtained from each other by a coordinate transforma-
Riem(M) – the space of all Riemannian metrics on a manifold M

$g'_{\mu\nu}$ can be obtained from $g_{\mu\nu}$ by a coordinate transformation

Every geometry is a point in Riem(M)/Diff(M)

Schrodinger equation II: $H_2\Psi = i \partial\Psi / \partial t$

Schrodinger equation I: $H_1\Psi = i \partial\Psi / \partial t$

The Wheeler – DeWitt equation

2. The Hilbert space in “extended phase space” version of quantum geometrodynamics

In [13] we considered a simple minisuperspace model with the action

$$S = \int dt \left\{ \frac{1}{2}v(\mu, Q)\gamma_{ab}\dot{Q}^a\dot{Q}^b - \frac{1}{v(\mu, Q)}U(Q) + \pi_0 \left( \dot{\mu} - f_a\dot{Q}^a \right) - i\omega(\mu, Q)\dot{\theta}\dot{\overline{\theta}} \right\}. \quad (2.1)$$
where $Q = \{Q^a\}$ are physical variables, $\theta, \bar{\theta}$ are the Faddeev – Popov ghosts and $\mu$ is a gauge variable, its parameterization being determined by the function $v(\mu, Q)$. In simple cases $\mu$ can be bound to the scale factor $a$ and the lapse function $N$ by the relation $a^3/N = v(\mu, Q)$.

$$w(\mu, Q) = \frac{v(\mu, Q)}{v,_{\mu}}; \quad v,_{\mu} \overset{\text{def}}{=} \frac{\partial v}{\partial \mu}. \quad (2.2)$$

We used a differential form of gauge conditions

$$\mu = f(Q) + k; \quad k = \text{const}, \quad (2.3)$$

namely,

$$\dot{\mu} = f,a \dot{Q}^a, \quad f,a \overset{\text{def}}{=} \frac{\partial f}{\partial Q^a}. \quad (2.4)$$

The wave function is defined on extended configurational space with the coordinates $\mu, Q, \theta, \bar{\theta}$. In “extended phase space” version of quantum geometrodynamics we quantize ghost and gauge gravitational degrees of freedom on an equal basis with physical degrees of freedom. The motivation for it was that it is impossible to separate gauge, or “non-physical” degrees of freedom from physical ones if the system under consideration does not possess asymptotic states, and it is indeed the case for a closed universe as well as in a general case of nontrivial topology. Then, we come to the Schrödinger equation, which is derived from a path integral with the effective action (2.1) without asymptotic boundary conditions by the standard well-defined Feynman procedure, and which is a direct mathematical consequence of the path integral.

$$i \frac{\partial \Psi(\mu, Q, \theta, \bar{\theta}; t)}{\partial t} = H \Psi(\mu, Q, \theta, \bar{\theta}; t), \quad (2.5)$$

where

$$H = -i \frac{\partial}{w \partial \theta \partial \bar{\theta}} - \frac{1}{2M} \frac{\partial}{\partial Q^\alpha} MG^{\alpha\beta} \frac{\partial}{\partial Q^\beta} + \frac{1}{v} (U - V); \quad (2.6)$$

$$M(\mu, Q) = v \frac{K^2}{2}(\mu, Q)w^{-1}(\mu, Q); \quad (2.7)$$

$$G^{\alpha\beta} = \frac{1}{v(\mu, Q)} \left( f,a f^a f,a \gamma^{ab} \right); \quad \alpha, \beta = (0, a); \quad Q^0 = \mu, \quad (2.8)$$

$M$ is the measure in inner product, $K$ is a number of physical degrees of freedom, $V$ is a quantum correction to the potential $U$, that depends on the chosen parameterization and gauge [13]. The Schrödinger equation (2.5) gives a gauge-dependent description of the Universe. The general solution to the equation (2.5) is

$$\Psi(\mu, Q, \theta, \bar{\theta}; t) = \int \Psi_k(Q, t) \delta(\mu - f(Q) - k) (\bar{\theta} + i\theta) dk. \quad (2.9)$$
It can be interpreted in the spirit of Everett’s “relative state” formulation: Each element of the superposition describe a state in which the only gauge degree of freedom \( \mu \) is definite, so that time scale is determined by processes in the physical subsystem through functions \( v(\mu, Q) \), \( f(Q) \) while the function \( \Psi_k(Q, t) \) describes a state of the physical subsystem for a reference frame fixed by the condition \( \mu \). It is a solution to the Schrödinger equation with a gauge-dependent physical Hamiltonian \( H_{\text{phys}} \):

\[
i \frac{\partial \Psi_k(Q; t)}{\partial t} = H_{\text{phys}} \Psi_k(Q; t), \tag{2.10}
\]

\[
H_{\text{phys}} = \left[ \frac{1}{2M} \frac{\partial}{\partial Q^a} v \hat{\gamma}^{ab} \frac{\partial}{\partial Q^b} + \frac{1}{v} (U - V) \right]_{\mu = f(Q) + k}. \tag{2.11}
\]

Solutions to Eq. (2.10) make a basis in the Hilbert space of states of the physical subsystem:

\[
H_{\text{phys}} \Psi_{kn}(Q) = E_n \Psi_{kn}(Q); \tag{2.12}
\]

\[
\Psi_k(Q, t) = \sum_n c_n \Psi_{kn}(Q) \exp(-iE_n t). \tag{2.13}
\]

As one can see, the spectrum and eigenfunctions of the operator \( H_{\text{phys}} \) will depend on a chosen gauge condition. The dependence of the measure in the physical subspace on this gauge results from the normalization condition for the wave function (2.9):

\[
\int \Psi^*(\mu, Q, \theta, \bar{\theta}; t) \Psi(\mu, Q, \theta, \bar{\theta}; t) M(\mu, Q) d\mu d\theta d\bar{\theta} \prod_a dQ^a = \int \Psi'^*(k, t) \Psi_k(Q, t) \delta(\mu - f(Q) - k) \delta(\mu - f(Q) - k') M(\mu, Q) dk dk' d\mu \prod_a dQ^a = \int \Psi'^*(k, t) \Psi_k(Q, t) M(f(Q) + k, Q) dk \prod_a dQ^a = 1. \tag{2.14}
\]

Therefore, the whole structure of the physical Hilbert space is formed in a large degree by the chosen gauge condition (reference frame). \textit{One cannot give a consistent quantum description of the Universe without fixing a certain reference frame, as well as one cannot find a solution to classical Einstein equations without imposing some gauge conditions.} The attempt to give a gauge invariant description of the Universe in the limits of the Wheeler – DeWitt quantum geometrodynamics was not successful, and the problem of Hilbert space is just the fact indicating that this theory has to be modified.

On the other hand, in the “extended phase space” approach we face another problem, that for every gauge condition we have its own Hilbert space. Is there any relation between state vectors in these Hilbert spaces, or between solutions to Schrödinger equations corresponding to various reference frames? How does the structure of Hilbert space change if one varies a gauge condition? We shall try to discuss these issues in the next sections.
3. Path integral and three kinds of gauge transformations

Let us consider a spacetime manifold $M$, which consists of several regions $\mathcal{R}_1, \mathcal{R}_2, \mathcal{R}_3, \ldots$, in each of them various gauge conditions $C_1, C_2, C_3, \ldots$ being imposed. It is naturally to think that such regions exist in a universe with a non-trivial topology. Just for simplicity, one can assume that boundaries $S_1, S_2, \ldots$ between the regions are spacelike and can be labeled by some time variables $t_1, t_2, \ldots$ (Fig. 2).

We would emphasize that the path integral approach allows us to examine this situation without any generalization of the formalism. The path integral over the manifold $M$ is

$$
\int \exp \left( iS \left[ g_{\mu\nu} \right] \right) \prod_{x \in M} M \left[ g_{\mu\nu} \right] \prod_{\mu, \nu} dg_{\mu\nu}(x) =
\int \exp \left( iS_{(eff)} \left[ g_{\mu\nu}, C_1, \mathcal{R}_1 \right] \right) \prod_{x \in \mathcal{R}_1} M \left[ g_{\mu\nu}, \mathcal{R}_1 \right] \prod_{\mu, \nu} dg_{\mu\nu}(x) \times
\times \exp \left( iS_{(eff)} \left[ g_{\mu\nu}, C_2, \mathcal{R}_2 \right] \right) \prod_{x \in \mathcal{R}_2} M \left[ g_{\mu\nu}, \mathcal{R}_2 \right] \prod_{\mu, \nu} dg_{\mu\nu}(x) \times
\times \prod_{x \in S_1} M \left[ g_{\mu\nu}, S_1 \right] \prod_{\mu, \nu} dg_{\mu\nu}(x) \times \ldots
$$

(3.1)

Here $S_{(eff)} \left[ g_{\mu\nu}, C_1, \mathcal{R}_1 \right]$ is the effective action in the region $\mathcal{R}_1$ with gauge conditions $C_1$, which
includes gauge fixing and ghosts terms, etc.

From the viewpoint of gauge invariant approach, the path integral is not to depend on gauge conditions, in other words, we could write

\[ \int \exp \left( i S \left[ g_{\mu \nu} \right] \right) \prod_{x \in M} M \left[ g_{\mu \nu} \right] \prod_{\mu, \nu} dg_{\mu \nu}(x) = \]

\[ = \int \langle g_{\mu \nu}^{(0)}, S_0 | g_{\mu \nu}^{(1)}, S_1 \rangle \langle g_{\mu \nu}^{(1)}, S_1 | g_{\mu \nu}^{(2)}, S_2 \rangle \prod_{x \in S_1} M \left[ g_{\mu \nu}, S_1 \right] \prod_{\mu, \nu} dg_{\mu \nu}(x) \times \ldots \] (3.2)

In this case the initial state \(| g_{\mu \nu}^{(0)}, S_0 \rangle\), as well as intermediate states \(| g_{\mu \nu}^{(1)}, S_1 \rangle\), \(| g_{\mu \nu}^{(2)}, S_2 \rangle\), etc. are supposed to be gauge invariant (i.e. independent on ghosts and gauge conditions). This assumption would be justified only if all the states were asymptotic, but it cannot be true at least for the intermediate states. Moreover, the path integral \((3.2)\) needs to be regularized, that implies imposing gauge condition on the surface \(S_1\) (see also \([17]\)). Eq.\((3.2)\) is a generalization of the well-known quantum mechanical operation when one inserts “a full set of states” at some moment \(t_1\). But in the present consideration we should bear in mind that the states in the regions \(R_1\) and \(R_2\) belong to different Hilbert spaces.

Within the region \(R_1\) the evolution of the physical subsystem is determined by a unitary operator \(\exp \left[ -i H_{1(\text{phys})} (t_1 - t_0) \right]\), where \(H_{1(\text{phys})}\) is a physical Hamiltonian in the region \(R_1\) with gauge conditions \(C_1\). Let at initial time \(t_0\) on the surface \(S_0\) the state of the system is given by a vector \(| g_{\mu \nu}^{(0)}, S_0 \rangle\). Then the state on the boundary \(S_1\) reads

\[ | g_{\mu \nu}^{(1)}, S_1 \rangle = \exp \left[ -i H_{1(\text{phys})} (t_1 - t_0) \right] | g_{\mu \nu}^{(0)}, S_0 \rangle. \] (3.3)

However, if we gone from the region \(R_1\) to \(R_2\), we would find ourselves in another Hilbert space with a basis formed from eigenfunctions of the operator \(H_{2(\text{phys})}\). The transition to a new basis is not a unitary operation, as follows from the fact that a measure in the physical subspace depends on gauge conditions \([13, 18]\) (in our minisuperspace model it is demonstrated by \((2.14)\)). Denote the operation of the transition to a new basis in the region \(R_2\) as \(P (S_1, t_1)\). Then the initial state in the region \(R_2\) is

\[ P (S_1, t_1) \exp \left[ -i H_{1(\text{phys})} (t_1 - t_0) \right] | g_{\mu \nu}^{(0)}, S_0 \rangle \] (3.4)

and

\[ | g_{\mu \nu}^{(3)}, S_3 \rangle = \exp \left[ -i H_{3(\text{phys})} (t_3 - t_2) \right] P (S_2, t_2) \exp \left[ -i H_{2(\text{phys})} (t_2 - t_1) \right] \times \]

\[ \times P (S_1, t_1) \exp \left[ -i H_{1(\text{phys})} (t_1 - t_0) \right] | g_{\mu \nu}^{(0)}, S_0 \rangle. \] (3.5)

So, at any border \(S_i\) between regions with different gauge conditions unitary evolution is broken down. The operators \(P (S_i, t_i)\) play the role of projection operators, which project states.
obtained by unitary evolution in a region $\mathcal{R}_i$ on a basis in Hilbert space in a neighbour region $\mathcal{R}_{i+1}$.

We now turn to different types of gauge transformations. It is conventionally believed that gauge conditions

$$F^\mu \left[ g^{\lambda \rho}(x), \theta^\nu(x) \right] = 0 \quad (3.6)$$

should be chosen to fix completely gauge transformation parameters. Meanwhile, one knows that, in general, these conditions fix gauge parameters up to residual transformations satisfying the equations which are consequence of (3.6):

$$\delta F^\mu \left[ g^{\lambda \rho}(x), \theta^\nu(x) \right] = 0 \Rightarrow A^\mu_\nu(x) = \frac{\delta F^\mu}{\delta g^{\lambda \rho}} \frac{\delta g^{\lambda \rho}}{\delta \theta^\nu} \theta^\nu(x) = 0. \quad (3.7)$$

However, we should not worry about this kind of transformations since they do not change the conditions (3.6) and not affect the structure of Hilbert space.

More interesting are the transformations whose parameters can be related by homotopy. Consider two gauge conditions

$$F^1 \left[ g^{\lambda \rho}(x), \theta^\nu_1(x) \right] = 0; \quad (3.8)$$

$$F^2 \left[ g^{\lambda \rho}(x), \theta^\nu_2(x) \right] = 0, \quad (3.9)$$

fixing points on a gauge orbit in which a group element is parameterized by $\theta^\nu_1(x)$ and $\theta^\nu_2(x)$, correspondingly. Let us assume that there exists continuous functions $L^\nu(r, x)$, so that

$$L^\nu(r, x) : \quad L^\nu(0, x) = \theta^\nu_1(x), \quad L^\nu(1, x) = \theta^\nu_2(x), \quad (3.10)$$

or, more generally,

$$L^\nu(r, x) : \quad L^\nu(r_1, x) = \theta^\nu_1(x), \quad L^\nu(r_2, x) = \theta^\nu_2(x). \quad (3.11)$$

One would say that $\theta^\nu_1(x)$ and $\theta^\nu_2(x)$ belong to the same homotopy class. Further, we could introduce a set of gauge conditions

$$F^\mu \left[ g^{\lambda \rho}(x), \theta^\nu_r(x); r \right] = 0 : \quad \theta^\nu_r(x) = L^\nu(r, x), \quad (3.12)$$

and identify $r$ with a time variable $t$. Then, time-dependent conditions (3.12) could be interpreted as describing a smooth transition from the gauge (3.8) to (3.9). Our ability to impose the set of conditions (3.12) depends on the structure of group and related to the possibility of introducing some special coordinates in group space [2]. In our simple minisuperspace model before gauge fixing the action is invariant under one-parametric Abelian group of transformations

$$\delta t = \theta(t); \quad \delta \mu = w(\mu, Q)\dot{\theta} - \dot{\mu}\theta; \quad \delta Q^a = -\dot{Q}^a\theta, \quad (3.13)$$
so that any time-dependent gauge condition

\[ \mu = f(Q, t) + k; \quad k = \text{const}, \quad (3.14) \]

would satisfy the above assumption.

Any canonical time-dependent gauge constrained physical variables and their momenta

\[ \chi(Q, P, t) = 0 \quad (3.15) \]

can be reduced by Dirac-like procedure to the form similar to (3.14). In the canonical approach, choosing a simple parameterization \( v(\mu, Q) = \frac{1}{\mu} \), one would find that the canonical Hamiltonian of the system \( H \) is proportional to the secondary constraint \( T \):

\[ H = \mu T = \mu \left[ \frac{1}{2} P_\alpha P^\alpha + U(Q) \right]. \quad (3.16) \]

From the requirement of the conservation of (3.15) in time \( [19] \) one obtains

\[ \frac{d\chi}{dt} = \frac{\partial \chi}{\partial t} + \mu \{ \chi, T \} = 0; \quad (3.17) \]

\[ \mu = -\frac{\partial \chi}{\partial t} \{ \chi, T \}^{-1} = \tilde{f}(Q, P, t), \quad (3.18) \]

the letter can be presented in a differential form. We would like to emphasize here that, though quantization schemes using canonical time-dependent gauges (3.15) are believed to be equivalent to gauge invariant Dirac quantization \( [19] \), from the viewpoint of our approach imposing such gauge conditions implies gauge-dependent structure of physical Hilbert space.

The formalism developed in \( [9, 10, 11, 12] \) can be generalized to gauge conditions explicitly depending on time. The pass integral approach includes some skeletonization procedure, which implies approximation of the gauge on each time interval \([t_i, t_{i+1}]\). In the simplest situation, we could assume that the change of gauge condition in each time interval is given by a function

\[ \delta f_i(Q) = \alpha f_i(Q), \quad (3.19) \]

\( \alpha \) is a small parameter, so that the gauge condition is a step-like function

\[ \mu = f(Q) + \sum_i \alpha f_i(Q) \theta(t - t_i) + k \quad (3.20) \]

in the sense that in each interval \([t_n, t_{n+1}]\) the gauge condition does not depend on time:

\[ [t_n, t_{n+1}]: \quad \mu = f(Q) + \sum_{i=0}^{n-1} \alpha f_i(Q) + \delta f_n(Q) + k. \quad (3.21) \]
Thus, we have come to the case of a small variation of gauge condition that was discussed in [18]. As was shown in [18], this small variation gives rise to additional terms in a physical Hamiltonian, these terms being non-Hermitian in respect to original physical subspace before variation. In our time-dependent case it means that at every moment of time we have a Hamiltonian, which acts in its own “instantaneous” Hilbert space. The instantaneous Hamiltonian is a Hermitian operator at each moment, but one should think of it as a non-Hermitian operator in respect to a Hilbert space one had at a previous moment. The situation is different from what we have in ordinary quantum mechanics for a time-dependent Hamiltonian that acts at every moment in the same Hilbert space whose measure does not change in time. An analogy can be drawn between our situation and particle creation in nonstationary gravitational field when we also have an instantaneous Hamiltonian and instantaneous Fock basis [20].

Smooth changing of a gauge condition in time implies that solutions to the Schrödinger equation for physical part of wave function also change in a continuous and smooth manner. Another situation we face when gauge conditions in two regions fix gauge parameters which belong to different homotopy classes, and, as a rule, spacetime coordinates in these regions being related by a singular transformation. Then the gauge condition and the very form of the Schrödinger equation change in discrete steps when one passes from a spacetime region with some gauge condition to a region with another gauge condition. This case is of the most interest from the viewpoint of changing the Hilbert space structure and the most difficult to treat. In any case, an initial state in a region \( R_i \), resulting from its preceding evolution, should be written as a superposition of states in a new Hilbert space in \( R_i \). There arise a number of questions, like: Will this superposition of states be stable? Could the breakdown of unitarity give rise to some kind of irreversibility? Could we define the change of entropy of the physical system when going to a region with different gauge conditions? Possible answers depend on a chosen model and require new non-perturbation methods.

4. Conclusion: the problem of wave function reduction and Quantum Gravity

As was pointed out by von Neumann [21], in quantum mechanics one deals with two different processes, namely, unitary evolution of a physical system in time described by the Schrödinger equation, and reduction of wave function of the physical system under observation. The whole evolution of the system can be presented by the formula

\[
|\Psi(t_N)\rangle = U(t_N, t_{N-1}) \mathcal{P}(t_{N-1}) U(t_{N-1}, t_{N-2}) \ldots \times \\
\times \ldots U(t_{3}, t_{2}) \mathcal{P}(t_{2}) U(t_{2}, t_{1}) \mathcal{P}(t_{1}) U(t_{1}, t_{0}) |\Psi(t_0)\rangle, \tag{4.1}
\]
where $P(t_i)$ are projection operators corresponding to observation at moments $t_1, t_2, t_3, \ldots, t_{N-1}$ (see, for example, [22]). There arises an analogy between the formulae (3.5) and (4.1): Indeed, we interpret any reference frame as a measuring instrument representing the observer in quantum geometrodynamics. Gauge conditions define interaction between the measuring instrument (reference frame) and the physical subsystem of the Universe. Changing the interaction with the measuring instrument makes us go to another basis in a Hilbert space and, even more, to another Hilbert space.

It enables us to hope to throw a new look to the central quantum mechanical problem of wave function reduction. Roger Penrose pointed out time and again that a solution of this problem, as well as understanding of irreversibility of physical processes, must be closely related with the progress in constructing quantum theory of gravity. In his books [14, 15] Penrose proposed a mechanism anticipating a choice among spacetime geometries, each of them corresponding to an element of quantum superposition. Details of the mechanism had not been elaborated enough, and this proposal was strongly criticized by Hawking [23]. However, the main idea that quantum gravity may help in deeper understanding of quantum mechanics seems to be fruitful. In our “extended phase space” approach we face the situation when the breakdown of unitary evolution of a physical system naturally follows from the very structure of the theory – we do not need to introduce “by hands” some special interaction, which would result in the breakdown of unitarity. In its turn, it is connected to the irreversibility of measuring processes. According to the opinion of another famous scientist, Ilya Prigogine, symmetric in time quantum dynamics described by the Schrödinger equation should be generalized to involve irreversible processes. To do it, one has to extend the class of admissible quantum operators beyond Hermitian operators and include non-unitary transformations of state vectors or density matrices ([16]; see also his Nobel prize lecture [24]). On the other side, in quantum mechanics one could examine models of interaction with a measuring instrument in which coordinates of a physical system are bound to coordinates of the instrument by means of some constraints, the latter ones are, in a sense, “gauge conditions” like those we have considered in our model with finite number degrees of freedom. Similar models of interaction with the instrument had been explored yet by von Neumann [21]. In future, some general points in quantum mechanical and quantum gravitational models of interaction may be revealed.

It may seem that there are more questions than answers in this report. However, we have discussing physical interpretations of Relativity Theory already for a hundred years. So it is not surprisingly that attempts of its unification with quantum theory pose even more fundamental and intriguing questions, which still have been waiting for their resolution.
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