A model of Gaussian laser beam self-trapping in optical tweezers for nonlinear particles

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Abstract
The optical tweezers are used to trap the particles embedded in a suitable fluid. The optical trap efficiency is significantly enhanced for nonlinear particles which response to the Kerr effect. The optical transverse gradient force makes these particles’ mass density in trapping region increasing, and the Kerr medium can be created. When the laser Gaussian beam propagates through it, the self-focusing, and consequently self-trapping can appear. In this paper, a model describing the laser self-trapping in nonlinear particle solution of optical tweezers is proposed. The expressions for the Kerr effect, effective refractive index of nonlinear particle solution and the intensity distribution of reshaped Gaussian laser beam are derived, and the self-trapping of laser beam is numerically investigated. Finally, the guide properties of nonlinear particles-filled trapping region and guiding condition are analysed and discussed.

Keywords Nonlinear optical tweezers · Kerr effect · Self-focusing · Self-trapping · Organic dye

1 Introduction
Ashkin has shown that a dielectric particle having refractive index larger than that of its embedding fluid will be trapped in the focus of laser Gaussian beam (Ashkin 2002). The particle’s optical trap efficiency $Q$ is higher when the refractive index of embedding fluid is lower (Kim et al. 2003; Quy Ho Quang 2018a, b; MacDonald et al. 2002), or the refractive index of trapped particle is higher. Consequently, the optical tweezers are efficient to trap Kerr particles (Couris et al. 2003; Wilkes et al. 2009; Quy and Nam 2012; Quy 2019). This is easily resolved for a single particle in the thin fluid where having no influence of nonlinear particle solution on the laser beam. However, if the nonlinear particle solution is thick enough, the density of the trapped nonlinear particle will become more and more
increasing in the trapping region along the laser axis due to action of transverse optical forces, so called the trapping cylinder (Quy et al. 2010). Therefore, not only does nonlinear particle solution’s mass refractive index increase in the trapping cylinder (Quy et al. 2010) but also its nonlinearity. Consequently, the nonlinear particle solution will become the graded index (GRIN) medium. The Gaussian laser beam propagating through GRIN medium will be self-focused (Nam et al. 2013), and then self-trapped in the small trapping cylinder (Devi and De 2016). This phenomenon is similar to that the laser beam propagating through a self-written optical waveguide in a solid photopolymer material volume (Li et al. 2015), Alkaline-earth atoms (Cooper 2018), and thermophoresis (Lamhot et al. 2010). Specially, the self-trapping has been experimentally observed in the human red blood cells suspension by the propagation of laser beam, seen as nonlinear medium (Lamhot et al. 2010). Lamhot and his co-workers also investigated the optical soliton beam in nanoparticle suspension by virtue of thermophoresis (Gautam 2019). In this paper, we proposed the model of optical tweezers by using the thick nonlinear particle solution. The expressions of the reshaped laser beam and effective refractive index of nonlinear particle solution are also derived. The self-focusing and self-trapping of laser beam in the trapping region are numerically calculated by using the iteration method. Finally, the guide properties of trapping region in optical tweezers, guiding condition are analyzed and discussed.

2 Principle model for simulation

Assuming that the optical tweezers for our investigation model is illustrated in Fig. 1. An incoming laser Gaussian beam (ILGB) with peak intensity $I_0$ and radius of beam waist of $W_0$ propagates through the thick chamber of nonlinear particle solution. The nonlinear particle has radius $a$, linear index $n_p$, nonlinear refractive coefficient $n_2$ embedded in the fluid of lower refractive index $n_f$. Under the action of optical transverse gradient force $F_{\rho}$, nonlinear particles are pulled to the laser axis and hold in the trapping region with radius $W_0/2$ (Quy et al. 2010). This makes the mass density $m_p$ of nonlinear particles in the trapping region and the effective refractive index of nonlinear particle solution $n_{\text{eff}}$ increase. The refractive index of nonlinear particle is directly proportional to ILGB’s intensity. The effective refractive index of nonlinear particle solution in the trapping region is graded and reduced from laser axis, i.e., the trapping region becomes a GRIN one.

Every differential GRIN cylinder with the thickness of $d = 2a$ (Fig. 1) will operate as the nonlinear thin lens (NTL) and in contrast, NTL also reshape the ILGB. The self-focusing

![Fig. 1](image-url)
appears continuously through NTL. Consequently, the beam waist of the reshaped laser Gaussian beam (RLGB) decreases, the mass density of nonlinear particles in the differential trapping cylinder increases, and the effective refractive index of nonlinear particle solution increases. The self-focusing and increasing of divergence angle $\theta_0 = \lambda/\pi W_0$ of RLGB are simultaneously occurred by the beam waist’s decrease. If both processes are in balance, the spatial optical soliton will appear (Li et al. 2015; Lamhot et al. 2010; Quy et al. 2004), means that the RLGB will be self-trapped in the center of the trapped region.

3 Theoretical background

3.1 Effective refractive index in nonlinear particle solution

Consider a solution of nonlinear particles embedded in the fluid with certain mass density. Under the optical tweezers, a number of the nonlinear particles $m$ is trapped in the differential trapping cylinder with thickness $d = 2na$, where $n$ is the number of differential GRIN cylinders, the mass densities of particles $m_p$ and fluid $m_f$ in the differential trapping cylinder (DTC) can be derived as:

$$m_p = \frac{V_p}{V_{DTC}} \text{ and } m_f = \frac{V_f}{V_{DTC}} = \frac{V_{DTC} - V_p}{V_{DTC}}$$  \hspace{1cm} (1)

where

$$V_p = \frac{m4\pi a^3}{3}, \quad V_f = V_{DTC} - V_p \quad \text{and} \quad V_{DTC} = \frac{n\alpha W_0^2}{2}$$  \hspace{1cm} (2)

are total volume of particles, fluid and the differential trapping cylinder, respectively. Using approximation of mass refractive index for the multi-component mixtures $\phi$, the effective refractive index $n_{eff}$ of nonlinear particle solution in the differential trapping cylinder can be derived as:

$$n_{eff} = \sum_i m_i n_i = \frac{V_p}{V_{DTC}} n_p + \frac{V_{DTC} - V_p}{V_{DTC}} n_f = \frac{8m_\alpha W_0^2}{3nW_0^2} + \frac{(3nW_0^2 - 8m_\alpha W_0^2)n_f}{3nW_0^2}$$  \hspace{1cm} (3)

Consider the intensity distribution of ILGB is (Saleh and Teich 1998):

$$I(\rho, z) = \frac{I_0}{\sqrt{1 + \frac{\rho^2}{\rho_0^2}} \exp \left( - \frac{2\rho^2}{W_0^2 \left( 1 + \frac{z}{z_0} \right)} \right)}$$  \hspace{1cm} (4)

When nonlinear particle solution is irradiated by ILGB, its effective refractive index contribution will be radial-graded and Eq. (3) will be modified as:
where \( n_2 \) is the nonlinear reflective index coefficient of nonlinear particle. We consider the laser wavelength to be shorter than the radius of beam waist.

The mass density of particles in the trapping region will increase if all of particles are trapped and directly pulled to laser beam’s axis, which is called the trapping condition of optical tweezers. This condition will be always satisfied if these particles on the edge of beam’s waist \( W_0 \) (see Fig. 1) (where the laser intensity and its gradient are the smallest) are trapped. That means that the optical force acting these particles must be larger than 1pN (Ashkin 2002).

Using Eq. (4) and Eq. (5) and Eq. 6 in Refs. 3, 4 we obtained the trapped condition as follows:

\[
F_{gr,p}(W_0, 0) = -\frac{8\pi n_f I_0 a^3}{W_0 \exp(2)} \left[ \left( \frac{n_{\text{eff}, p}}{n_p} \right)^2 - 1 \right] > 1 \text{pN} \tag{6}
\]

where

\[
n_{\text{eff}, p} = n_p + \frac{n_2 I_0}{\exp(2)} \tag{7}
\]

Considering the trapped particles are pulled in the region near laser beam’s axis. This means that their positions are approximately \( \rho << W_0 \). Assuming that the differential trapping cylinder is placed at the waist of ILGB and \( d << z_0 \) (see Fig. 1), the function of intensity radial distribution in the input surface of differential trapping cylinder can be simplified as follows (Thai Dinh et al. 2016; Nguyen et al. 2015; Ho Quang et al. 2020):

\[
I(\rho) = I_0 \exp \left( -\frac{2\rho^2}{W_0^2} \right) \approx I_0 \left( 1 - \frac{2\rho^2}{W_0^2} \right) \tag{8}
\]

Substituting Eq. (8) into Eq. (5) and we have

\[
n_{\text{eff}}(\rho) = \frac{(3nW_0^2 - 8ma^2)n_f + 8ma^2(n_p + n_2 I_0)}{3nW_0^2} - \frac{16ma^2 n_2 I_0}{3nW_0^4} \rho^2 \tag{9}
\]

where \( n_{\text{eff}} \) describes the radial distribution of the effective refractive index in differential trapping cylinder and to be a function of radial radius \( \rho \), beam waist \( W_0 \), i.e. particle’s mass density \( m_p \). Therefore, it is similar to the index change of thermophoresis irradiated by laser beam in work (Couris et al. 2003) as the function of temperature and particle concentration. Consequently, we can substitute \( n_{\text{eff}} \) into nonlinear paraxial wave equation in work (Lamhot et al. 2010) to calculate the optical spatial soliton beam. In this paper, we consider the trapping region of optical tweezers as a consecutive series of NTLs and use iteration method to calculate the change of each NTL’s focal length of and laser beam’s waist. The intensity distribution and self-trapping of RLGB is also shown and discussed.
3.2 The focal of NTL and intensity distribution of RLGB

The effective refractive index in Eq. (9) can be simplified as follows:

\[ n_{\text{eff}}(\rho) = N_0 \left( 1 - \frac{\alpha^2}{2} \rho^2 \right) \]  

(10)

where

\[ N_0 = \frac{3nW_0^2n_f + 8ma^2(n_p - n_f + n_2I_0)}{3nW_0^2} \]  

(11)

\[ \alpha^2 = \frac{32ma^2 n_2 I_0}{\{3nW_0^2n_f + 8ma^2(n_p - n_f + n_2I_0)\}nW_0^2} \]  

(12)

With refractive index given in Eq. (10), the differential trapping cylinder will become NTL with focal length given as follows (Saleh and Teich 1998; Ho Quang et al. 2020):

\[ f_{nl} = \frac{1}{2N_0 \alpha^2 a} = \frac{3W_0^4}{64ma^3 n_2 I_0} \]  

(13)

which is inversely proportional to the peak laser intensity \( I_0 \), nonlinear coefficient of refractive index \( n_2 \), and radius \( a \) and number \( m \) of nonlinear particles, and directly proportional to beam waist of \( W_0 \).

The ILGB will be reshaped to RLGB when it propagates through the NTL (Saleh and Teich 1998; Ho Quang et al. 2020). Its intensity distribution is given by:

\[ I_{RLGB}(\rho, z) = I_0 \frac{W_0}{W_{RLGB}(z)} \exp \left( -\frac{2\rho^2}{W_{RLGB}(z)} \right) \]  

(14)

where

\[ W_{RLGB}(z) = W_{0RLGB} \sqrt{1 + \left( \frac{z}{z_{0RLGB}} \right)^2} \]  

(15)

is the radius of laser beam at \( z \);

\[ W_{0RLGB} = \frac{W_0}{\sqrt{1 + \left( \frac{z_{0RLGB}}{f_{nl}} \right)^2}} \]  

(16)

is the radius of reshaped beam waist placed at the output surface of differential trapping cylinder given by:

\[ z_{RLGB} = \frac{f_{nl}}{1 + \left( \frac{f_{nl}}{z_0} \right)^2} \]  

(17)

and
$$z_0 = \frac{\pi W_0^2}{\lambda}$$  \hspace{1cm} (18)

is the Rayleigh range.

### 3.3 Simulation procedure

Firstly, we check the trapping condition using Eqs. 6, 7 with an collection of parameters. The self-trapping process will be numerically observed if this trapping condition is satisfied. To observe the self-trapping process in optical tweezers, we use the solution of non-linear particles embedded in fluid, the simulation scheme given in Fig. 2 above. The self-trapping is related to the change of beam waist $W_{0RLGB}$, focal length of NTL $f_{nl}$ in z. We simulated in a self-consistent manner: we use an initial prediction to find $f_{nl}$ in Eq. 13, from which we have the beam waist of $W_{0RLGB}$ in Eq. 16 after the distance $z = in 2a$, where $i$ is the order of simulation step. Using $f_{nl}$ and $W_{0RLGB}$ obtained, we find the divergence angle $\theta_d = \lambda/\pi W_{0RLGB}$ and the focusing angle $\theta_{sf} = \tan^{-1}(W_{0RLGB}/f_{nl})$, then substituting again to Eqs. 13, 16 to find the next ones. The calculating process is iterated until the divergence

![Simulation scheme for self-trapping](image)
and focusing angles are close one to each other. The simulation procedure is given in Fig. 2.

4 Results and discussion

We consider an optical tweezers using ILGB with $\lambda = 0.532 \mu m$ (second harmonic of Nd$^{3+}$ YAG laser) $I_0 = 1 \times 10^5 W/cm^2$, $W_0 = 10^{-4} cm$ to trap nanoparticle of polyacrylamide gel doped Orange G with $a = 5 \times 10^{-7} cm$, $n_p = 1.456$, $n_2 = 1 \times 10^{-6} cm^2/W$ (Badran et al. 2011; Nguyen 2014) embedded in water with $n_f = 1.333$ (Volpe and Volpe 2013). The thickness of the differential trapping cylinder is optimally chosen $d = 20a = 1 \times 10^{-5} cm$.

Firstly, substituting given parameters into Eqs. 6, 7, we found: $F_{gr}(W_0, 0) \approx 2.4 \times 10^{-11} N = 24 pN \gg 1 pN$, that means the trapping condition of optical tweezers to be satisfied and all particles locating inside the beam waist $W_0$ are also trapped and pulled in beam’s axis. These make the mass density of particles inside beam waist increase and the self-focusing appears. Trapping and self-focusing make the mass density of particles increase and the self-focusing more powerfully. Consequently, the laser beam will be trapped inside the cylinder of solution with nonlinear particles.

Secondly, we numerically calculate the laser self-trapping process. Simulation process was done until $i = 16$ at which $\theta_{sf} = 22.56^\circ \approx \theta_d = 22.31^\circ$ (Fig. 4) by using the Maple software. The obtained results are presented in Table 1.

The self-trapping simulation in nonlinear particle solution are shown in Fig. 3. We see that the ILGB (Fig. 3a) is self-trapped (Fig. 3b) in trapping region at $z = 1.5 \mu m$ where its divergence and focusing (angles) effects are in balance (see Fig. 4). When the balance of divergence and focusing effects occurs, the longitudinal gradient of laser intensity reduces, and then the distribution of the longitudinal gradient force also reduces, that has been shown in works (Quy Ho Quang 2018b; Quy 2019; Devi and De 2016; Jiang et al. 2010). Moreover, the spatial optical soliton appears and propagates continuously. Our result in Fig. 3 is similar to that obtained by (Li et al. 2015) for the self-trapping of optical beam in the self-written optical waveguide in a solid photopolymer material.

Figure 5 shows RLGB’s beam waist shortening along the trapping region. Since the particle mass density increases, the effective refractive index in nonlinear particle solution also increases (Fig. 6).

Due to trapping and focusing effects, the trapping region is seem as the optical fiber with increasing of its refractive index change $\Delta = (n_{eff} - n_f)/n_{eff}$ (Fig. 7). The $\Delta(\rho = 0)$

| i | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $z$ (10$^{-5}$ cm) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| $W_{0RLGB}$ (10$^{-6}$ cm) | 100 | 98 | 96.2 | 94.2 | 92 | 89.7 | 87.2 | 84.5 |
| $f_{\text{eff}}$ (10$^{-4}$ cm) | 30.4 | 28.3 | 26.1 | 24 | 21.8 | 19.7 | 17.6 | 15.5 |
| i | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| $z$ (10$^{-5}$ cm) | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| $W_{0RLGB}$ (10$^{-6}$ cm) | 81.6 | 78.3 | 74.7 | 70.6 | 65.8 | 60.1 | 53 | 43.7 |
| $f_{\text{eff}}$ (10$^{-4}$ cm) | 13.5 | 11.5 | 9.5 | 7.5 | 5.7 | 3.9 | 2.4 | 1.1 |
and $\Delta (\rho = W_{0RLGB}/2)$ increases from 0.002 to 0.0095 and from 0.00095 to 0.0068, respectively, and larger than a predicted low limit $\Delta_{\text{min}} = 0.001$ (Saleh and Teich 1998) or measurement limit $\Delta_{\text{min}} = 0.000475$ (Ahmed et al. 2019) for conventional optical fiber. This means that the deeper the laser beam propagates into trapping region, the
more effectively it guides. This consideration can be explained and proved by increasing of the critical angle \( \theta_c = \cos^{-1}(n_f/n_{\text{eff}}(W_{\text{0RLGB}}/2)) \) at boundary between trapping region and fluid, and acceptance angle \( \theta_a = \sin^{-1}(n_{\text{eff}}(0)\sqrt{2\Delta(0)}) \) at boundary among differential trapping cylinders shown in Figs. 8, 9, respectively.
5 Conclusion

We have theoretically shown that the Gaussian laser beam used for optical tweezers can be self-trapped in trapping region by Kerr effect in nonlinear particle solution. The self-trapping of Gaussian beam in nonlinear particle solution has been numerically calculated, the guide properties of the trapping region in optical tweezers is also analyzed and discussed. These results show that the Gaussian beam for optical tweezers not only traps nonlinear particles, but also be reshaped and self-trapped. Finally, our results may hint a new study for increasing and stabilizing the density of nonlinear biomedical cells in experimental observation (Quy et al. 2004).

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