Exploring the quark flavor puzzle within the three-Higgs double model

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We extend the standard model with two extra Higgs doublets. Making use of a symmetry principle, we present flavour symmetries based on cycle groups \( Z_N \) that oblige each Higgs doublet to contribute to the mass of only one generation. The Higgs doublets couple to the fermions with different strengths and in this way accommodate the quark mass hierarchy. We systematically search for all charge configurations that naturally lead to the alignment in flavour space of the quark sectors, resulting in a CKM matrix near to the identity, determined by the quark mass hierarchy, and with the correct overall phenomenological features. The minimal realization is by the group \( Z_7 \). We show that only a limited number of solutions exist, and that any accidental global symmetry that may occur together with the discrete symmetry is necessarily anomalous. A phenomenological study of each class of solutions concerning predictions to the flavour changing neutral current (FCNC) phenomena is also performed: for some solutions, it is possible to obtain realistic quark masses and mixing, while the flavour violating neutral Higgs are light enough to be accessible at LHC.
I. INTRODUCTION

The discovery of the Higgs boson in 2012 at the LHC has attested the success of the standard model (SM) in describing the observed fermions and their interactions. However, there exist many theoretical issues or open questions that have no satisfactory answer. In particular, the observed flavour pattern lacks of a definitive explanation, i.e., the quark Yukawa coupling matrices $Y_u$ and $Y_d$, which in the SM reproduce the six quark masses, three mixings angles and a complex phase to account for CP violation phenomena, are general complex matrices, not constrained by any gauge symmetry.

Experimentally the flavour puzzle is very intricate. First, there is the quark mass hierarchy in both sectors. Secondly, the mixings in the SM, encoded in the Cabibbo-Kobayashi-Maskawa (CKM) unitary matrix, turns out to be close to the identity matrix. If one takes also the lepton sector into account, the hierarchy there is even more puzzling [1]. On the other hand, in the SM there is in general no connection between the quark masses hierarchy and the CKM mixing pattern. In fact, if one considers the Extreme Chiral Limit, where the quark masses of the first two generations are set to zero, the mixing does not necessarily vanish [2], and one concludes that the CKM matrix $V$ being close to the identity matrix has nothing to do with the fact that the quark masses are hierarchical. Indeed, in order to have $V \approx 1$, one must have a definite alignment of the quark mass matrices in the flavour space, and to explain this alignment, a flavour symmetry or some other mechanism is required [2].

Among many attempts made in the literature to address the flavour puzzle, extensions of the SM with new Higgs doublet are particularly motivating. This is due to fact that the number of Higgs doublets is not constrained by the SM symmetry. Moreover, the addition of scalar doublets gives rise to new Yukawa interactions and as a result it provides a richer framework in approaching the theory of flavour. On the other hand, any new extension of the Higgs sector must be very much constrained, since it naturally leads to flavour changing neutral currents. At tree level, in the SM, all the flavour changing transitions are mediated through charged weak currents and the flavour mixing is controlled by the CKM matrix [3,4]. If new Higgs doublets are added, one expects large FCNC effects already present at tree level. Such effects have not been experimentally observed and they constrain severely any model with extra Higgs doublets, unless a flavour symmetry suppresses or avoids large FCNC [5].

Minimal flavour violating models [6–11] are examples of a multiHiggs extension where FCNC are present at tree-level but their contributions to FCNC phenomena involve only off-diagonal ele-
ments of the CKM matrix or their products. The first consistent models of this kind were proposed by Branco, Grimus and Lavoura (BGL) \[12\], and consisted of the SM with two Higgs doublets together with the requirement of an additional discrete symmetry. BGL models are compatible with lower neutral Higgs masses and FCNC’s occur at tree level, with the new interactions entirely determined in terms of the CKM matrix elements.

The goal of this paper is to generalize the previous BGL models and to, systematically, search for patterns where a discrete flavour symmetry naturally leads to the alignment of the flavour space of both the quark sectors. Although the quark mass hierarchy does not arise from the symmetry, the effect of both is such that the CKM matrix is near to the identity and has the correct overall phenomenological features, determined by the quark mass hierarchy, \[13\]. To do this we extend the SM with two extra Higgs doublets to a total of three Higgs $\phi_a$. The choice for discrete symmetries is to avoid the presence of Goldstone bosons that appear in the context of any global continuous symmetry, when the spontaneous electroweak symmetry breaking occurs. For the sake of simplicity, we restrict our search to the family group $Z_N$, and demand that the resulting up-quark mass matrix $M_u$ is diagonal. This is to say that, due to the expected strong up-quark mass hierarchy, we only consider those cases where the contribution of the up-quark mass matrix to quark mixing is negligible.

If one assumes that all Higgs doublets acquire vacuum expectation values with the same order of magnitude, then each Higgs doublet must couple to the fermions with different strengths. Possibly one could obtain similar results assuming that the vacuum expectation values (VEVs) of the Higgs have a definite hierarchy instead of the couplings, but this is not considered here. Combining this assumption with the symmetry, we obtain the correct ordered hierarchical pattern if the coupling with $\phi_3$ gives the strength of the third generation, the coupling with $\phi_2$ gives the strength of the second generation and the coupling with $\phi_1$ gives the strength of the first generation. Therefore, from our point of view, the three Higgs doublets are necessary to ensure that there exists three different coupling strengths, one for each generation, to guarantee simultaneously an hierarchical mass spectrum and a CKM matrix that has the correct overall phenomenological features e.g. $|V_{cb}|^2 + |V_{ub}|^2 = O(m_s/m_b)^2$, and denoted here by $V \approx 1$.

Indeed, our approach is within the BGL models, and such that the FCNC flavour structure is entirely determined by CKM. Through the symmetry, the suppression of the most dangerous FCNC’s, by combinations of the CKM matrix elements and light quark masses, is entirely natural.

The paper is organised as follows. In the next section, we present our model and classify the patterns allowed by the discrete symmetry in combination with our assumptions. In Sec. \[III\] we
give a brief numerical analysis of the phenomenological output of our solutions. In Sec. [IV], we
examine the suppression of scalar mediated FCNC in our framework for each pattern. Finally, in
Sec. [V] we present our conclusions.

II. THE MODEL

We extend the Higgs sector of the SM with two extra new scalar doublets, yielding a total
of three scalar doublets, as $\phi_1$, $\phi_2$, $\phi_3$. As it was mentioned in the introduction, the main idea
for having three Higgs doublets is to implement a discrete flavour symmetry, that leads to the
alignment of the flavour space of the quark sectors. The quark mass hierarchy does not arise
from the symmetry, but together with the symmetry the effect of both is such that the CKM matrix
is near to the identity and has the correct overall phenomenological features, determined by the
quark mass hierarchy.

Let us start by considering the most general quark Yukawa coupling Lagrangian invariant in
our setup

$$-L_Y = (\Omega_a)_{ij} \overline{Q}_{Li} \tilde{\phi}_a u_{Rj} + (\Gamma_a)_{ij} \overline{Q}_{Li} \phi_a d_{Rj} + h.c., \quad (1)$$

with the Higgs labeling $a = 1, 2, 3$ and $i, j$ are just the usual flavour indexes identifying the gener-
ations of fermions. In the above Lagrangian, one has three Yukawa coupling matrices $\Omega_1$, $\Omega_2$, $\Omega_3$
for the up-quark sector and three Yukawa coupling matrices $\Gamma_1$, $\Gamma_2$, $\Gamma_3$ for the down sector, cor-
responding to each of the Higgs doublets $\phi_1$, $\phi_2$, $\phi_3$. Assuming that only the neutral components
of the three Higgs doublets acquire vacuum expectation value (VEV), the quark mass $M_u$ and $M_d$
are then easily generated as

$$M_u = \Omega_1 \langle \phi_1 \rangle^* + \Omega_2 \langle \phi_2 \rangle^* + \Omega_3 \langle \phi_3 \rangle^*, \quad (2a)$$
$$M_d = \Gamma_1 \langle \phi_1 \rangle + \Gamma_2 \langle \phi_2 \rangle + \Gamma_3 \langle \phi_3 \rangle, \quad (2b)$$

where VEVs $\langle \phi_i \rangle$ are parametrised as

$$\langle \phi_1 \rangle = \frac{v_1}{\sqrt{2}}, \quad \langle \phi_2 \rangle = \frac{v_2 e^{i\alpha_2}}{\sqrt{2}}, \quad \langle \phi_3 \rangle = \frac{v_3 e^{i\alpha_3}}{\sqrt{2}}, \quad (3)$$

with $v_1$, $v_2$ and $v_3$ being the VEV moduli and $\alpha_2$, $\alpha_3$ just complex phases. We have chosen the VEV
of $\phi_1$ to be real and positive, since this is always possible through a proper gauge transformation.
As stated, we assume that the moduli of VEVs $v_i$ are of the same order of magnitude, i.e.,

$$v_1 \sim v_2 \sim v_3. \quad (4)$$

Each of the $\phi_a$ couples to the quarks with a coupling $(\Omega_a)_{ij}, (\Gamma_a)_{ij}$ which we take to be of the same order of magnitude, unless some element vanishes by imposition of the flavour symmetry. In this sense, each $\phi_a$ and $(\Omega_a, \Gamma_a)$ will generate its own respective generation: i.e., our model is such that by imposition of the flavour symmetry, $\phi_3, \Omega_3, \Gamma_3$ will generate $m_t$ respectively $m_b$, that $\phi_2, \Omega_2, \Gamma_2$ will generate $m_c$ respectively $m_s$, and that $\phi_1, \Omega_1, \Gamma_1$ will generate $m_u$ respectively $m_d$. Generically, we have

$$v_1 |(\Omega_1)_{ij}| \sim m_u, \quad v_2 |(\Omega_2)_{ij}| \sim m_c, \quad v_3 |(\Omega_3)_{ij}| \sim m_t, \quad (5a)$$
$$v_1 |(\Gamma_1)_{ij}| \sim m_d, \quad v_2 |(\Gamma_2)_{ij}| \sim m_s, \quad v_3 |(\Gamma_3)_{ij}| \sim m_b, \quad (5b)$$

which together with Eq. (4) implies a definite hierarchy amongst the non-vanishing Yukawa coupling matrix elements:

$$|(\Omega_1)_{ij}| \ll |(\Omega_2)_{ij}| \ll |(\Omega_3)_{ij}|, \quad (6a)$$
$$|(\Gamma_1)_{ij}| < |(\Gamma_2)_{ij}| \ll |(\Gamma_3)_{ij}|. \quad (6b)$$

Next, we focus on the required textures for the Yukawa coupling matrices $\Omega_a$ and $\Gamma_a$ that naturally lead to an hierarchical mass quark spectrum and at the same time to a realistic CKM mixing matrix. These textures must be reproduced by our choice of the flavour symmetry. As referred in the introduction, we search for quark mass patterns where the mass matrix $M_u$ is diagonal. Therefore, one derives from Eqs. (2a), (6a) the following textures for $\Omega_a$

$$\Omega_1 = \begin{pmatrix} x & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \Omega_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \Omega_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & x & 0 \end{pmatrix}. \quad (7)$$

The entry $x$ means a non-zero element. In this case, the up-quark masses are given by $m_u = v_1 |(\Omega_1)_{11}|, m_c = v_2 |(\Omega_2)_{22}|$ and $m_t = v_3 |(\Omega_3)_{33}|$.

Generically, the down-quark Yukawa coupling matrices must have the following indicative tex-
\[ \Gamma_1 = \begin{pmatrix} x & x & x \\ x & x & x \\ x & x & x \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ x & x & x \\ x & x & x \end{pmatrix}, \quad \Gamma_3 = \begin{pmatrix} 0 & 0 & 0 \\ x & x & x \\ x & x & x \end{pmatrix}. \] (8)

We distinguish rows with bold x in order to indicate that it is mandatory that at least one of matrix elements within that row must be nonvanishing. Rows denoted with x may be set to zero, without modifying the mass matrix hierarchy. These textures ensure that not only is the mass spectrum hierarchy respected but it also leads to the alignment of the flavor space of both the quark sectors [13] and to a CKM matrix \( V \approx 1 \). For instance, if one would not have a vanishing, or comparatively very small, \((1,3)\) entry in the \( \Gamma_2 \), this would not necessarily spoil the scale of \( m_s \), but it would dramatically change the predictions for the CKM mixing matrix.

In order to force the Yukawa coupling matrices \( \Omega_a \) and \( \Gamma_a \) to have the indicative forms outlined in Eqs. (7) and (8), we introduce a global flavour symmetry. Since any global continuous symmetry leads to the presence of massless Goldstone bosons after the spontaneous electroweak breaking, one should instead consider a discrete symmetry. Among many possible discrete symmetry constructions, we restrict our searches to the case of cycle groups \( Z_N \). Thus, we demand that any quark or boson multiplet \( \chi \) transforms according to \( Z_N \) as

\[ \chi \rightarrow \chi' = e^{i Q(\chi) \frac{2\pi}{N}} \chi, \] (9)

where \( Q(\chi) \in \{0, 1, \ldots, N\} \) is the \( Z_N \)-charge attributed for the multiplet \( \chi \).

We have chosen the up-quark mass matrix \( M_u \) to be diagonal. This restricts the flavour symmetry \( Z_N \). We have found that, in order to ensure that all Higgs doublet charges are different, and to have appropriate charges for fields \( Q_{Li} \) and \( u_{Ri} \), we must have \( N \geq 7 \). We simplify our analysis by fixing \( N = 7 \) and choose:

\[ Q(Q_{Li}) = (0, 1, -2), \] (10a)
\[ Q(u_{Ri}) = (0, 2, -4), \] (10b)

In addition, we may also fix

\[ Q(Q_{Li}) = Q(\phi_i) \] (11)
It turns out that these choices do not restrict the results, i.e. the possible textures that one can have for the $\Gamma_i$ matrices. Other choices would only imply that we reshuffle the charges of the multiplets.

With the purpose of enumerating the different possible textures for the $\Gamma_i$ matrices implementable in $Z_7$, we write down the charges of the trilinears $Q(\overline{Q}_{L_i}\phi_\alpha d_{R_j})$ corresponding to each $\phi_\alpha$ as

$$Q(\overline{Q}_{L_i}\phi_1 d_{R_j}) = \begin{pmatrix} d_1 & d_2 & d_3 \\ d_1 - 1 & d_2 - 1 & d_3 - 1 \\ d_1 + 2 & d_2 + 2 & d_3 + 2 \end{pmatrix},$$

(12a)

$$Q(\overline{Q}_{L_i}\phi_2 d_{R_j}) = \begin{pmatrix} d_1 + 1 & d_2 + 1 & d_3 + 1 \\ d_1 & d_2 & d_3 \\ d_1 + 3 & d_2 + 3 & d_3 + 3 \end{pmatrix},$$

(12b)

$$Q(\overline{Q}_{L_i}\phi_3 d_{R_j}) = \begin{pmatrix} d_1 - 2 & d_2 - 2 & d_3 - 2 \\ d_1 - 3 & d_2 - 3 & d_3 - 3 \\ d_1 & d_2 & d_3 \end{pmatrix},$$

(12c)

where $d_i \equiv Q(d_{R_i})$. One can check that, in order to have viable solutions, one must vary the values of $d_i \in \{0, 1, -2, -3\}$.

We summarise in Table I all the allowed textures for the $\Gamma_\alpha$ matrices and the resulting $M_d$ mass matrix texture, excluding all cases which are irrelevant, e.g. matrices that have too much texture zeros and are singular, or matrices that do not accommodate CP violation. It must be stressed, that these are the textures obtained by the different charge configurations that one can possibly choose. However, if one assumes a definite charge configuration, then the entire texture, $M_d$ and $M_u$ and the respective phenomenology are fixed. As stated, the list of textures in Table II remains unchanged even if one chooses any other set than in Eqs. (10), (11). As stated, that all patterns presented here are of the Minimal Flavour Violation (MFV) type [6–11].

Pattern I in the table was already considered in Ref. [14] in the context of $Z_8$. We discard Patterns IV, VII and X, because contrary to our starting point, at least one of three non-zero couplings with $\phi_1$ will turn out be of the same order as the larger coupling with $\phi_2$ in order to meet
the phenomenological requirements of the CKM matrix.

Notice also, that the structure of other \( M_d \)'s cannot be trivially obtained, e.g. from Pattern I, by a transformation of the right-handed down quark fields.

Our symmetry model may be extended to the charged leptons and neutrinos, e.g. in the context of type one see-saw. Choosing for the lepton doublets \( L_i \) the charges \( Q(L_i) = (0, -1, 2) \), opposite to the Higgs doublets in Eq. (11), and e.g. for the charges \( Q(e_{Ri}) = (0, -2, 4) \) of the right-handed fields \( e_{Ri} \), we force the charged lepton mass matrix to be diagonal. Then for the right-handed neutrinos \( \nu_{Ri} \), choosing \( Q(\nu_{Ri}) = (0, 0, 0) \), we obtain for the neutrino Dirac mass matrix a pattern similar to pattern I. Of course, for this case, the heavy right-handed neutrino Majorana mass matrix is totally arbitrary. In other cases, i.e. for other patterns and charges, in particular for the right-handed neutrinos, we could introduce scalar singlets with suitable charges, which would then lead to certain heavy right-handed neutrino Majorana mass matrices.

Next, we address an important issue of the model, namely, whether accidental \( \mathbb{U}(1) \) symmetries may appear in the Yukawa sector or in the potential. One may wonder whether a continuous accidental \( \mathbb{U}(1) \) symmetry could arise, once the \( Z_7 \) is imposed at the Lagrangian level in Eq. (1). This is indeed the case, i.e., for all realizations of \( Z_7 \), one has the appearance of a global \( \mathbb{U}(1)_X \). However, any consistent global \( \mathbb{U}(1)_X \) must obey to the anomaly-free conditions of global symmetries [15], which read for the anomalies \( SU(3)^2 \times \mathbb{U}(1)_X, SU(2)^2 \times \mathbb{U}(1)_X \) and \( \mathbb{U}(1)^2 \times \mathbb{U}(1)_X \) as

\[
A_3 \equiv \frac{1}{2} \sum_{i=1}^{3} \left( 2X(Q_{Li}) - X(u_{Ri}) - X(d_{Ri}) \right) = 0, \quad (13a)
\]

\[
A_2 \equiv \frac{1}{2} \sum_{i=1}^{3} \left( 3X(Q_{Li}) + X(\ell_{Li}) \right) = 0, \quad (13b)
\]

\[
A_1 \equiv \frac{1}{6} \sum_{i=1}^{3} \left( X(Q_{Li}) + 3X(\ell_{Li}) - 8X(u_{Ri}) - 2X(d_{Ri}) - 6X(e_{Ri}) \right) = 0, \quad (13c)
\]

where \( X(\chi) \) is the \( \mathbb{U}(1)_X \) charge of the fermion multiplet \( \chi \). We have properly shifted the \( Z_7 \)-charges in Eq.(10) and in Table I so that \( X(\chi) = Q(\chi) \), apart of an overall \( \mathbb{U}(1)_X \) convention. In general to test those conditions, one needs to specify the transformation laws for all fermionic fields. Looking at the Table 1, we derive that all the cases, except the first case corresponding
to \( d_i = (0, 0, 0) \), violate the condition given in Eq. (13) that depends only on coloured fermion multiplets. In the case \( d_i = (0, 0, 0) \), if one assigns the charged lepton charges as \( X(\ell_L)_i = X(Q_L)_i \) one concludes that the condition given in Eq. (13) is violated. One then concludes that the global \( U(1)_X \) symmetry is anomalous and therefore only the discrete symmetry \( Z_7 \) persists.

We also comment on the scalar potential of our model. The most general scalar potential with three scalars invariant under \( Z_7 \) reads as

\[
V(\phi) = \sum_i \left[ -\mu_i^2 \phi_i^\dagger \phi_i + \lambda_i (\phi_i^\dagger \phi_i)^2 \right] + \sum_{i<j} \left[ + C_i (\phi_i^\dagger \phi_i)(\phi_j^\dagger \phi_j) + \bar{C}_i \left| \phi_i^\dagger \phi_j \right|^2 \right],
\]

where the constants \( \mu_i^2, \lambda_i, C_i \) and \( \bar{C}_i \) are taken real for \( i, j = 1, 2, 3 \). Analysing the potential above, one sees that it gives rise to the accidental global continuous symmetry \( \phi_i \rightarrow e^{i\alpha_i} \phi_i \), for arbitrary \( \alpha_i \), which upon spontaneous symmetry breaking leads to a massless neutral scalar, at tree level. Introducing soft-breaking terms like \( m_{ij}^2 \phi_i^\dagger \phi_j + \text{H.c.} \) can erase the problem. Another possibility without spoiling the \( Z_7 \) symmetry is to add new scalar singlets, so that the coefficients \( m_{ij}^2 \) are effectively obtained once the scalar singlets acquire VEVs.
TABLE I: The table shows the viable configurations for the right-handed down-quark $d_{R_i}$ and their corresponding $\Gamma_1$, $\Gamma_2$, $\Gamma_3$ and $M_d$ matrices. It is understood that, for each pattern and coupling, the parameters expressed here by the same symbol, are in fact different, but denoting he same order of magnitude, (or possibly smaller). E.g. in pattern I, coupling $\Gamma_1$, the three $\delta$, $\delta$, $\delta$, stand for $\delta_1$, $\delta_2$, $\delta_3$. The same applies to the $\varepsilon$'s and $c$'s. For patterns IV, VII, and X, which will be excluded, one of the couplings in $\Gamma_1$ turns out to be much larger.

| Pattern | $Q(d_{R_i})$ | $\Gamma_1$ | $\Gamma_2$ | $\Gamma_3$ | $M_d$ |
|---------|-------------|------------|------------|------------|-------|
| I       | (0, 0, 0)   | $\begin{pmatrix} \delta & \delta & \delta \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} \varepsilon & \varepsilon & \varepsilon \\ 0 & 0 & 0 \\ c & c & c \end{pmatrix}$ | $\begin{pmatrix} \varepsilon & \varepsilon & \varepsilon \\ 0 & 0 & 0 \\ c & c & c \end{pmatrix}$ |
| II      | (0, 0, 1)   | $\begin{pmatrix} \delta & \delta & 0 \\ 0 & 0 \varepsilon & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} \varepsilon & \varepsilon & \varepsilon \\ 0 & 0 & 0 \\ c & c & 0 \end{pmatrix}$ | $\begin{pmatrix} \varepsilon & \varepsilon & \varepsilon \\ 0 & 0 & 0 \\ c & c & 0 \end{pmatrix}$ |
| III     | (0, 0, -3)  | $\begin{pmatrix} \delta & \delta & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} \varepsilon & \varepsilon & \varepsilon \\ 0 & 0 & 0 \\ c & c & 0 \end{pmatrix}$ | $\begin{pmatrix} \varepsilon & \varepsilon & \varepsilon \\ 0 & 0 & 0 \\ c & c & 0 \end{pmatrix}$ |
| IV      | (0, 0, -2)  | $\begin{pmatrix} \delta & \delta & 0 \\ 0 & 0 & 0 \\ 0 & 0 \varepsilon & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} \varepsilon & \varepsilon & \varepsilon \\ 0 & 0 & 0 \\ c & c & 0 \end{pmatrix}$ | $\begin{pmatrix} \varepsilon & \varepsilon & \varepsilon \\ 0 & 0 & 0 \\ c & c & 0 \end{pmatrix}$ |
| V       | (0, 1, 0)   | $\begin{pmatrix} \delta & 0 & \delta \\ 0 & 0 \varepsilon & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} \varepsilon & 0 & \varepsilon \\ 0 & 0 & 0 \\ c & c & 0 \end{pmatrix}$ | $\begin{pmatrix} \varepsilon & 0 & \varepsilon \\ 0 & 0 & 0 \\ c & c & 0 \end{pmatrix}$ |
| VI      | (0, -3, 0)  | $\begin{pmatrix} \delta & 0 & \delta \\ 0 & 0 & 0 \\ 0 & 0 \varepsilon & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} \varepsilon & 0 & \varepsilon \\ 0 & 0 & 0 \\ c & c & 0 \end{pmatrix}$ | $\begin{pmatrix} \varepsilon & 0 & \varepsilon \\ 0 & 0 & 0 \\ c & c & 0 \end{pmatrix}$ |
| VII     | (0, -2, 0)  | $\begin{pmatrix} \delta & 0 & \delta \\ 0 & 0 & 0 \\ 0 & 0 \varepsilon & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} \varepsilon & 0 & \varepsilon \\ 0 & 0 & 0 \\ c & c & 0 \end{pmatrix}$ | $\begin{pmatrix} \varepsilon & 0 & \varepsilon \\ 0 & 0 & 0 \\ c & c & 0 \end{pmatrix}$ |
| VIII    | (1, 0, 0)   | $\begin{pmatrix} 0 & \delta & \delta \\ \delta & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & \varepsilon & \varepsilon \\ 0 & 0 & 0 \\ c & c & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & \varepsilon & \varepsilon \\ 0 & 0 & 0 \\ c & c & 0 \end{pmatrix}$ |
| IX      | (-3, 0, 0)  | $\begin{pmatrix} 0 & \delta & \delta \\ 0 & 0 & 0 \\ 0 & 0 \varepsilon & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & \varepsilon & \varepsilon \\ 0 & 0 & 0 \\ c & c & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & \varepsilon & \varepsilon \\ 0 & 0 & 0 \\ c & c & 0 \end{pmatrix}$ |
| X       | (-2, 0, 0)  | $\begin{pmatrix} 0 & \delta & \delta \\ 0 & 0 & 0 \\ \varepsilon & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & \varepsilon & \varepsilon \\ 0 & 0 & 0 \\ c & c & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & \varepsilon & \varepsilon \\ 0 & 0 & 0 \\ c & c & 0 \end{pmatrix}$ |
III. NUMERICAL ANALYSIS

In this section, we give the phenomenological predictions obtained by the patterns listed in Table I. Note that, although these patterns arrive directly from the chosen discrete charge configuration of the quark fields, one may further perform a residual flavour transformation of the right-handed down quark fields, resulting in an extra zero entry in $M_d$. Taking this into account, all the parameters in each pattern may be uniquely expressed in terms of down quark masses and the CKM matrix elements $V_{ij}$. This follows directly from the diagonalization equation of $M_d$:

$$V^\dagger M_d W = \text{diag}(m_d, m_s, m_b) \implies M_d = V \text{diag}(m_d, m_s, m_b) W^\dagger$$ (15)

with $V$ being the CKM mixing matrix, since $M_u$ is diagonal. Because of the zero entries in $M_d$, it is easy to extract the right-handed diagonalization matrix $W$, completely in terms of the down quark masses and the $V_{ij}$. Thus, all parameters, modulo the residual transformation of the right-handed down quark fields, are fixed, i.e., all parameters in each pattern may be uniquely expressed in terms of down quark masses and the CKM matrix elements $V_{ij}$, including the right-handed diagonalization matrix $W$ of $M_d$. More precisely, all matrix elements of $V$ are written in terms of Wolfenstein real parameters $\lambda$, $A$, $\bar{\rho}$ and $\bar{\eta}$, defined in terms of rephasing invariant quantities as

$$\lambda \equiv \frac{|V_{us}|}{\sqrt{|V_{us}|^2 + |V_{ud}|^2}}, \quad A \equiv \frac{1}{\lambda} \frac{|V_{cb}|}{|V_{us}|},$$

$$\bar{\rho} + i \bar{\eta} \equiv -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$$ (16a)

and $\text{diag}(m_d, m_s, m_b)$ in Eq. (15)

$$\sqrt{\frac{m_d}{m_s}} = \sqrt{\frac{k_d}{k_s}} \lambda \quad \implies \quad m_d = k_d \lambda^4 m_b$$

$$\frac{m_s}{m_b} = k_s \lambda^2 \quad \implies \quad m_s = k_s \lambda^2 m_b$$ (17)
with phenomenologically, \(k_d\) and \(k_s\) being factors of order one. Writing \(W^\dagger\) in Eq. (15) as \(W^\dagger = (v_1, v_2, v_3)\), with the \(v_i\) vectors formed by the \(i\)-th column of \(W^\dagger\), we find e.g. for pattern II,

\[
v_3 = \frac{1}{n_3} \begin{pmatrix} \frac{m_d}{m_b} V_{11} \\ \frac{m_d}{m_b} V_{12} \\ V_{13} \end{pmatrix} \times \begin{pmatrix} \frac{m_d}{m_b} V_{31} \\ \frac{m_s}{m_b} V_{32} \\ V_{33} \end{pmatrix}
\]

(18)

where \(n_3\) is the norm of the vector obtained from the external product of the two vectors. Taking into account the extra freedom of transformation of the right-handed fields, we may choose \(M_{31}^d = 0\), corresponding to \(c_1 = 0\) in Table I, and we conclude that

\[
v_1 = \frac{1}{n_1} \begin{pmatrix} \frac{m_d}{m_b} V_{31} \\ \frac{m_s}{m_b} V_{32} \end{pmatrix} \times v_3^* 
\]

(19)

Obviously, then \(v_2 = \frac{1}{n_2} v_1^* \times v_3^*\). This process is replicated for all patterns. Thus, \(V\) and \(W\), are entirely expressed in terms of Wolfenstein parameters and \(k_d\) and \(k_s\) of Eq. (17). These two matrices will be later used to compute the patterns of the FCNC’s in Table III. Indeed, in this way, we find e.g. for pattern II, in leading order order,

\[
M_d = m_b \begin{pmatrix} -k_d \lambda^3 (\bar{\rho} - i \bar{\eta}) \lambda^3 & 0 \\ -k_d \lambda^2 & \lambda^2 & -k_s \lambda^3 \\ 0 & 1 & 0 \end{pmatrix}
\]

(20)

which corresponds to the expected power series where the couplings in \(\Gamma_1\) to the first Higgs \(\phi_1\) are comparatively smaller than then couplings in \(\Gamma_2\), and these smaller to the couplings in \(\Gamma_3\). Similar results are obtained for all patterns in Table II, except for patterns IV, VII and X, where e.g. for pattern IV, we find that the coupling in \((\Gamma_1)_{33}\) is proportional to \(\lambda\), which is too large and contradicts our initial assumption that all couplings in \(\Gamma_1\) to the first Higgs \(\phi_1\) must be smaller than the couplings in \(\Gamma_2\) to the second Higgs \(\phi_2\). Therefore, we exclude Patterns IV, VII and X.

We give in Table II a numerical example of a Yukawa coupling configuration for each pattern.

We use the following quark running masses at the electroweak scale \(M_Z\):

\[
\begin{align*}
m_u &= 1.3^{+0.4}_{-0.2} \text{ MeV}, & m_d &= 2.7 \pm 0.3 \text{ MeV}, & m_s &= 55^{+5}_{-3} \text{ MeV}, \\
m_c &= 0.63 \pm 0.03 \text{ GeV}, & m_b &= 2.86^{+0.05}_{-0.04} \text{ GeV}, & m_t &= 172.6 \pm 1.5 \text{ GeV}.
\end{align*}
\]

(21a)
TABLE II: A numerical example of a Yukawa coupling configuration for each pattern that gives the correct hierarchy among the quark masses and mixing.

| Pattern | $v_1Y_1$ | $v_2Y_2$ | $v_3Y_3$ | $M_d$ |
|---------|----------|----------|----------|-------|
| I       | $(0.00277, 0.0124, 0.0101 e^{1.307 i})$ | $(0, 0, 0)$ | $(0, 0.0537, 0.119)$ | $(0, 0, 2.86)$ |
| II      | $(0.0123, 0.0101 e^{-1.23 i}, 0)$ | $(0, 0, 0)$ | $(0, 0, 0)$ | $(0, 0, 0.0123, 0.0101 e^{-1.23 i}, 0)$ |
| III     | $(0.0127, 0.0102 e^{-1.23 i}, 0)$ | $(0, 0, 0)$ | $(0, 0, 0)$ | $(0, 0, 0)$ |
| IV      | $(0.0127, 0.0101 e^{-1.23 i})$ | $(0, 0, 0)$ | $(0, 0, 0)$ | $(0, 0, 0)$ |
| V       | $(0.0127, 0.0102 e^{-1.23 i})$ | $(0, 0, 0)$ | $(0, 0, 0)$ | $(0, 0, 0)$ |
| VI      | $(0.0127, 0.0102 e^{1.23 i}, 0)$ | $(0, 0, 0)$ | $(0, 0, 0)$ | $(0, 0, 0)$ |
| VII     | $(0.0117, 0.0102 e^{1.307 i})$ | $(0, 0, 0)$ | $(0, 0, 0)$ | $(0, 0, 0)$ |
| VIII    | $(0.0127, 0.0101 e^{1.23 i})$ | $(0, 0, 0)$ | $(0, 0, 0)$ | $(0, 0, 0)$ |
| IX      | $(0.0127, 0.0101 e^{1.23 i})$ | $(0, 0, 0)$ | $(0, 0, 0)$ | $(0, 0, 0)$ |

which were obtained from a renormalisation group equation evolution at four-loop level \[16\], which, taking into account all experimental constrains \[17\], implies:

\[
\lambda = 0.2255 \pm 0.0006, \quad A = 0.818 \pm 0.015, \quad \lambda = 0.2255 \pm 0.0006, \quad A = 0.818 \pm 0.015, \\
\bar{\rho} = 0.124 \pm 0.024, \quad \bar{\eta} = 0.354 \pm 0.015.
\] \[22a\] \[22b\]

IV. PREDICTIONS OF FLAVOUR CHANGING NEUTRAL CURRENTS

In the SM, flavour changing neutral currents (FCNC) are forbidden at tree level, both in the gauge and the Higgs sectors. However, by extending the SM field content, one obtains Higgs Flavour Violating Neutral Couplings \[5\]. In terms of the quark mass eigenstates, the Yukawa couplings to the Higgs neutral fields are:
\[-\mathcal{L}_{\text{Neutral Yukawa}} = \frac{H_0}{v} (d_L D_d d_R + \bar{u}_L D_u u_R) + \frac{1}{v' d_L N_1^d (R_1 + i I_1) d_R}
\quad + \frac{1}{v' d_L N_1^u (R_1 - i I_1) u_R + \frac{1}{v' d_L N_2^d (R_2 + i I_2) d_R}
\quad + \frac{1}{v' d_L N_2^u (R_2 - i I_2) u_R + h.c.}\] (23)

where the $N_i^{u,d}$ are the matrices which give the strength and the flavour structure of the FCNC,

\begin{align}
N_1^d &= \frac{1}{\sqrt{2}} V^\dagger \left( v_2 \Gamma_1 - v_1 e^{i \alpha_2} \Gamma_2 \right) W, \quad (24a) \\
N_2^d &= \frac{1}{\sqrt{2}} V^\dagger \left( v_1 \Gamma_1 + v_2 e^{i \alpha_2} \Gamma_2 - \frac{v_1^2 + v_2^2}{v_3} e^{i \alpha_3} \Gamma_3 \right) W, \quad (24b) \\
N_1^u &= \frac{1}{\sqrt{2}} \left( v_2 \Omega_1 - v_1 e^{-i \alpha_2} \Omega_2 \right), \quad (24c) \\
N_2^u &= \frac{1}{\sqrt{2}} \left( v_1 \Omega_1 + v_2 e^{-i \alpha_2} \Omega_2 - \frac{v_1^2 + v_2^2}{v_3} e^{-i \alpha_3} \Omega_3 \right). \quad (24d)
\end{align}

Since in our case the $N_i^u$ are diagonal, there are no flavour violating terms in the up-sector.

Therefore, the analysis of the FCNC resumes only to the down-quark sector. One can use the equations of the mass matrices presented in Eq. (2) to simplify the Higgs mediated FCNC matrices for the down-sector:

\begin{align}
N_1^d &= \frac{v_2}{v_1} D_d - \frac{v_2}{\sqrt{2}} \left( \frac{v_2}{v_1} + \frac{v_1}{v_2} \right) e^{i \alpha_2} V^\dagger \Gamma_2 W - \frac{v_2 v_3}{v_1 \sqrt{2}} e^{i \alpha_3} V^\dagger \Gamma_3 W \quad (25a) \\
N_2^d &= D_d - \frac{v_2}{v_3 \sqrt{2}} e^{i \alpha_3} V^\dagger \Gamma_3 W \quad (25b)
\end{align}

In order to satisfy experimental constraints arising from $K^0 - \bar{K}^0$, $B^0 - \bar{B}^0$ and $D^0 - \bar{D}^0$, the off-diagonal elements of the Yukawa interactions $N_1^d$ and $N_2^d$ must be highly suppressed \[18\] \[19\].

For each of our 10 solutions in Table II we summarize in Table III all FCNC patterns, for each solution, and for $v_1 = v_2 = v_3$ and $\alpha_2 = \alpha_3 = 0$. These patterns are of the BGL type, since in Eq. (25) all matrices can be expressed in terms of the CKM mixing matrix elements and the down quark masses. As explained, to obtain these patterns, we express the CKM matrix $V$ and the matrix $W$ in terms of Wolfenstein parameters.

The tree level Higgs mediated $\Delta S = 2$ amplitude must be suppressed. This may allways be achieved if one chooses the masses of the flavour violating neutral Higgs scalars sufficiently heavy. However, from the experimental point of view, it would be interesting to have these masses
TABLE III: For all allowed patterns, we find that the matrices $N_1^d - D_d$ and $N_2^d$ are proportional to the following patterns, where $\lambda$ is the Cabibbo angle.

| Pattern | $(N_1^d - D_d) \sim$ | $N_2^d \sim$ |
|---------|-----------------------|---------------|
| I       | $\begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^5 & \lambda^2 & \lambda^2 \\ \lambda^7 & \lambda^4 & 1 \end{pmatrix}$ | $\begin{pmatrix} \lambda^4 & \lambda^7 & \lambda^3 \\ \lambda^9 & \lambda^2 & \lambda^2 \\ \lambda^7 & \lambda^4 & 1 \end{pmatrix}$ |
| II      | $\begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^5 & \lambda^4 & 1 \end{pmatrix}$ | $\begin{pmatrix} \lambda^4 & \lambda^7 & \lambda^3 \\ \lambda^9 & \lambda^2 & \lambda^2 \\ \lambda^7 & \lambda^4 & 1 \end{pmatrix}$ |
| III     | $\begin{pmatrix} \lambda^3 & \lambda^3 \\ \lambda & \lambda^2 & 1 \end{pmatrix}$ | $\begin{pmatrix} \lambda^4 & \lambda^5 & \lambda^3 \\ \lambda & \lambda^2 & 1 \end{pmatrix}$ |
| IV      | $\begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & \lambda^2 & 1 \end{pmatrix}$ | $\begin{pmatrix} \lambda^4 & \lambda^5 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & \lambda^2 & 1 \end{pmatrix}$ |
| V       | $\begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^5 & \lambda^4 & 1 \end{pmatrix}$ | $\begin{pmatrix} \lambda^4 & \lambda^5 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & \lambda^2 & 1 \end{pmatrix}$ |
| VI      | $\begin{pmatrix} \lambda^3 & \lambda^3 \\ \lambda & \lambda^2 & 1 \end{pmatrix}$ | $\begin{pmatrix} \lambda^4 & \lambda^5 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & \lambda^2 & 1 \end{pmatrix}$ |
| VII     | $\begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & \lambda^2 & 1 \end{pmatrix}$ | $\begin{pmatrix} \lambda^4 & \lambda^5 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & \lambda^2 & 1 \end{pmatrix}$ |
| VIII    | $\begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^5 & \lambda^4 & 1 \end{pmatrix}$ | $\begin{pmatrix} \lambda^4 & \lambda^5 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & \lambda^2 & 1 \end{pmatrix}$ |
| IX      | $\begin{pmatrix} \lambda^3 & \lambda^3 \\ \lambda & \lambda^2 & 1 \end{pmatrix}$ | $\begin{pmatrix} \lambda^4 & \lambda^5 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & \lambda^2 & 1 \end{pmatrix}$ |
| X       | $\begin{pmatrix} \lambda^3 & \lambda^3 \\ \lambda & \lambda^2 & 1 \end{pmatrix}$ | $\begin{pmatrix} \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & \lambda^2 & 1 \end{pmatrix}$ |

as low as possible. Therefore, we also estimate the lower bound of these masses, by considering the contribution to $B^0 - \bar{B}^0$ mixing. We choose this mixing, since for our patterns, the $(3, 1)$ entry of the matrix $N_1^d$ is the less suppressed in certain cases and would require very heavy flavour violating neutral Higgses. The relevant quantity is the off-diagonal matrix element $M_{12}$, which
connects the B meson with the corresponding antimeson. This matrix element, \( M_{12}^{NP} \), receives contributions \([18]\) both from a SM box diagram and a tree-level diagram involving the FCNC:

\[
M_{12} = M_{12}^{SM} + M_{12}^{NP},
\]

(26)

where the New Physics (NP) short distance tree level contribution to the meson-antimeson contribution is:

\[
M_{12}^{NP} = \sum_{i=1}^{2} \frac{f_B^2 m_B}{96 v^2 m_i^2} \left\{ \left[ 1 + \left( \frac{m_B}{m_d + m_b} \right)^2 \right] \left( a_i^R \right)_{12} - \left[ 1 + 11 \left( \frac{m_B}{m_d + m_b} \right)^2 \right] \left( b_i^R \right)_{12} \right\} + \sum_{i=1}^{2} \frac{f_B^2 m_B}{96 v^2 m_i^2} \left\{ \left[ 1 + \left( \frac{m_B}{m_d + m_b} \right)^2 \right] \left( a_i^I \right)_{12} - \left[ 1 + 11 \left( \frac{m_B}{m_d + m_b} \right)^2 \right] \left( b_i^I \right)_{12} \right\}
\]

(27)

with \( v^2 = v_1^2 + v_2^2 + v_3^2 \) and

\[
\begin{align*}
(a_i^R)_{12} &= \left[ (N_i^d)^*_{31} + (N_i^d)^*_{13} \right]^2, \\
(a_i^I)_{12} &= -\left[ (N_i^d)^*_{31} - (N_i^d)^*_{13} \right]^2, \\
(b_i^R)_{12} &= \left[ (N_i^d)^*_{31} - (N_i^d)^*_{13} \right]^2, \\
(b_i^I)_{12} &= -\left[ (N_i^d)^*_{31} + (N_i^d)^*_{13} \right]^2, \\
i &= 1, 2
\end{align*}
\]

(28)

In order to obtain a conservative measure, we have tentatively expanded the original expression in \([18]\) and, for the three Higgs case, included all neutral Higgs mass eigenstates.

Adopting as input values the PDG experimental determinations of \( f_B, m_B \) and \( \Delta m_B \) and considering a common VEV for all Higgs doublets, we impose the inequality \( M_{12}^{NP} < \Delta m_B \). The following plots show an estimate of the lower bound for the flavour-violating Higgs masses for two different patterns. We plot two masses chosen from the set \((m^R_1, m^R_2, m^I_1, m^I_2)\), while the other two are varied over a wide range. In Fig. 1, we illustrate these lower bounds for Pattern III, which are restricted by the \((3, 1)\) entry of \( N_i^d \) matrix and suppressed by a factor of \( \lambda \). For Pattern VIII, in Fig. 2 we find the flavour violating neutral Higgs to be much lighter and possibly accessible at LHC.

V. CONCLUSIONS

We have presented a model based on the SM with 3 Higgs and an additional flavour discrete symmetry. We have shown that there exist flavour discrete symmetry configurations which lead to the alignment of the quark sectors. By allowing each scalar field to couple to each quark generation with a distinctive scale, one obtains the quark mass hierarchy, and although this hierarchy does not arise from the symmetry, the effect of both is such that the CKM matrix is near to the
identity and has the correct overall phenomenological features. In this context, we have obtained 7 solutions fulfilling these requirements, with the additional constraint of the up quark mass matrix being diagonal and real.

We have also verified if accidental $U(1)$ symmetries may appear in the Yukawa sector or in the potential, particularly the case where a continuous accidental $U(1)$ symmetry could arise, once the $Z_7$ is imposed at the Lagrangian level. This was indeed the case, however we shown that the anomaly-free conditions of global symmetries are violated. Thus, the global $U(1)_X$ symmetry is anomalous and therefore only the discrete symmetry $Z_7$ persists.

As in this model new Higgs doublets are added, one expects large FCNC effects, already present at tree level. However, such effects have not been experimentally observed. We show that, for certain of our specific implementations of the flavour symmetry, it is possible to suppress the FCNC effects and to ensure that the flavour violating neutral Higgs are light enough to be detectable at LHC. Indeed, in this respect, our model is a generalization of the BGL models for 3HDM, since the FCNC flavour structure is entirely determined by CKM.

(a) Estimate of the lower bound for the flavour-violating Higgs masses for $R_1$ and $I_1$.  
(b) Estimate of the lower bound for the flavour-violating Higgs masses for $R_2$ and $I_2$.

FIG. 1: Lower bound for the flavour-violating Higgs masses for case III.

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(a) Estimate of the lower bound for the flavour-violating Higgs masses for $R_1$ and $I_1$.

(b) Estimate of the lower bound for the flavour-violating Higgs masses for $R_2$ and $I_2$.

FIG. 2: Lower bound for the flavour-violating Higgs masses for case VIII.

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