Nonlinear bifurcation diagrams of the current states of microcavity exciton-polaritons in the lattice

I Yu Chestnov¹, A V Yulin² and O A Egorov³

¹Vladimir State University named after A. G. and N. G. Stoletovs, Gorkii St. 87, 600000, Vladimir, Russia
²ITMO University, Kronverksky pr. 49, 197101, St. Petersburg, Russia
³Technische Physik, Physikalisches Institut and Wilhelm Conrad Röntgen-Center for Complex Material Systems, Universität Würzburg, Am Hubland, D-97074 Würzburg, Germany

E-mail: igor_chestnov@mail.ru

Abstract. We study nonequilibrium properties of exciton-polaritons in an incoherently driven semiconductor microcavity with an embedded weak-contrast periodic lattice. Within the framework of mean-field theory, we describe nonlinear eigenstates of coherent exciton polaritons having the structure of Bloch waves and identify nonlinear current states at the boundaries of Brillouin zone. We demonstrate a profound modification of the eigenfrequency-momenta bifurcation diagrams of the condensate evoked by exciton-exciton interactions. The discussed phenomenon occurs for the states possessing high degree of symmetry, i.e. in particular, for the states lying at the centre or at the boundary of Brillouin zone.

1. Introduction

Periodic media have been attracting attention of physicist for many decades. One of the reasons of such interest is the rich dispersion properties typical for these media. The modification and control of the dispersive properties are among the fundamental problems of current material science, photonics, condensed matter and solid state physics. In particular, manipulation of a band structure of a given physical system is a vital practical task of the design of new micro- and nanostructures possessing unusual properties [1]. Using of nonlinear properties of the medium for managing of its dispersion is one of the promising approaches for this task. For instance, it has been revealed for both atomic Bose-Einstein condensates (BECs) [2] and for solid-state polaritonic systems [3] that the periodical modulation can support self-localized states of the quantum particles known as gap solitons even under repulsive nonlinear interactions between them. So, the nontrivial nonlinear dynamics and coherent properties of the periodic media was always the fascination problem.

A semiconductor microcavity supporting exciton polaritons is a very promising platform for the investigation of the spontaneous coherence build-up in periodic lattices [3-5]. Unlike the conservative bosonic systems, in particular, atomic BEC [6], the exciton-polaritons are subject to rapid radiative decay. Thus the population of polariton state has to be maintained by an external pumping. The direct way to it is the use of nonresonant pumping implying creation of a reservoir of hot excitons, which can relax in energy to form a polariton condensate. This nonequilibrium open-dissipative behaviour represents an important intrinsic feature of the exciton-polaritons in semiconductor microcavities and imprint the nonlinear dynamics in both homogeneous [7] and periodically modulated configurations.
[8]. In the paper we consider spatially modulated exciton-polariton systems focusing our attention on the nonlinear states possessing nonzero current. For the sake of clarity and simplicity we restrict our consideration to a 1D case.

Note that the nonlinear energy renormalization of conservative atomic BEC placed in 1D optical lattice have been discussed before [9-11]. In particular, it was shown that two-body (atom-atom) interactions lead to the formation of the loops in the dispersion characteristics of the nonlinear states (so-called "swallow tails") both at the centre and at the edges of BZ. The most intriguing feature of the obtained loop structure is connected with a non-zero flow appearing at the edges of BZ. The existence of such nonlinear current-currying states was interpreted as a consequence of superfluid behaviour of a condensate in periodic potential [10]. The linear stability properties of these nonlinear Bloch waves have been intensively studied [12-14]. The regions of both the energetic and dynamical instabilities have been identified for such essentially conservative systems. The novelty of our paper is related to the open-dissipative nature of the polariton condensate which shows a number of interesting effects.

In such class of problems analytical methods are suitable only for special kinds of a potential such as elliptic [12] or Kronig-Penney [13, 14] potentials. That is why in this paper we use the numerical methods of finding nonlinear Bloch waves which are appropriate for the strongly nonlinear case when perturbative approaches fail.

2. The model
To describe polariton condensate properties we use an open driven-dissipative model that accounts for the particle exchange between the condensate and the excitonic reservoir formed by a non-resonant pump [12]. This approach is based on a mean field theory and assuming the spontaneous formation of the exciton-polariton condensate, we consider an open-dissipative Gross-Pitaevskii (GP) model, that describes incoherently pumped condensate coupled to an excitonic reservoir. The polariton order parameter \( \Psi(x,t) \) is described by a GP-type equation

\[
\frac{i\hbar}{\partial_t} \frac{\partial \Psi}{\partial t} = \left[ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V_0 \cos(\beta x) + g_c |\Psi|^2 + \frac{i\hbar}{2} (Rn - \gamma_c) + g_R n \right] \Psi, \tag{1}
\]

and the exciton reservoir density \( n(x,t) \) is governed by the rate equation:

\[
\frac{\partial n}{\partial t} = P - \gamma_R n - Rn |\Psi|^2, \tag{2}
\]

Here \( \gamma_c \) and \( \gamma_R \) are the condensate and the reservoir dumping rates, \( g_c \) and \( g_R \) are coefficients of nonlinearity due to polariton-polariton and polariton-reservoir interaction. The reservoir of incoherent polaritons is formed by an external pump \( P \). The parameter of coupling between reservoir and condensate is denoted as \( R \). The value of \( V_0 \) characterizes depth of the lattice, \( \beta = 2\pi / l \) is the lattice wavevector, and \( l \) – the lattice period. We expect that nonlinearity affects polariton band structure if the term \( g_c |\Psi|^2 \) is comparable or higher than the potential depth \( V_0 \).

In this paper we use the following parameters, which agree with experimental data [15]:

\[
\gamma_c = 0.33 \text{ ps}^{-1}, \quad \gamma_R = 0.495 \text{ ps}^{-1}, \quad g_c = 6 \times 10^{-3} \text{ meV} \mu\text{m}^{-2}, \quad g_R = 2g_c, \quad R = 0.01 \text{ ps}^{-1}\mu\text{m}^{-2}, \quad m = 0.568 \text{ meV} \mu\text{m}^{-2}, \quad l = 8 \mu\text{m}.
\]

3. Steady-states and frequency-momenta bifurcation diagrams of the condensate in the lattice

3.1. Searching for the condensate stationary states
The problem of the nonlinear states in the periodic lattice is solved as follows. Imposing periodic boundary conditions and restricting ourselves with the class of solutions to the functions having the same periodicity as lattice we search the stationary solutions which are parameterized by the quasi-momentum, the index of the mode and the average particle density. The obtained solutions resemble the familiar Bloch states and formally obey the Bloch theorem [16]. However the investigated
solutions satisfy the nonlinear problem and possess several distinctive features which will be discussed latter. That is why we address these states as the nonlinear Bloch states [13].

In the case of weak nonlinearity the problem can be approached perturbatively when the solution is sought in the form of the expansion over the basis of the Bloch modes calculated for the corresponding linear problem and treating the nonlinearity as a perturbation. Naturally, within this approach the modes with quasi-momentum lying at the edges of Brillouin zone (BZ) are necessarily currentless. However this result cannot be generalized for highly nonlinear systems like exciton-polariton condensate. In this case strong Kerr-like nonlinear properties of condensate caused by many-body interactions could essentially affect the properties of the nonlinear Bloch modes.

We are looking for the solution of equations (1) and (2) in the general form

\[ \Psi(x,t) = \psi(x)e^{i\mathbf{k}_x t} e^{-\mu t}, \quad n(x,t) = f(x), \]  

(3)

where the condensate wavefunction is represented in the standard form of the structural function \( \psi \) and the exponent with the quasi-momentum \( \mathbf{k} \) and eigenfrequency (chemical potential) \( \mu \).

The key property of the open-dissipative model (1), (2) is that a nontrivial solution \( \Psi \neq 0, \ n \neq 0 \) can be formed when the pump exceeds a threshold value \( P^\text{th} \). For the spatially homogeneous system (\( V_0 = 0 \)) equations (1) and (2) possesses solution in the form of nonlinear plain waves (3) \( \Psi = \Psi_0 e^{-i\mathbf{q}_x t} \) and \( n_0 = \gamma_c / R \) above the threshold \( P > P^\text{th} = f_R \gamma_c / R \). In this case the chemical potential is \( h\mu_0 = g_c |\Psi_0|^2 + g_R n_0 \) and the condensate density \( |\Psi_0|^2 = (P - P^\text{th}) / \gamma_c \) increases linearly with the pump \( P \).

The situation is more complicated when the condensate is placed in a periodic lattice. Then we expand \( \psi(x) \) and \( n(x) \) in Fourier series on spatial harmonics of the external periodic potential:

\[ \psi(x) = \sum_{j=-N}^{N} \psi_j e^{i\beta_j x}, \quad n(x) = \sum_{j=-N}^{N} n_j e^{i\beta_j x}, \]  

(4)

with normalization condition \( I = \sum_{j=-N}^{N} |\psi_j|^2 \), where the value of \( I \) corresponds to the condensate density averaged over lattice cell, i.e. \( I = \frac{1}{L} \int_{-L/2}^{L/2} |\psi(x)|^2 dx \). We restrict ourselves with \( 2N+1 \) harmonics where \( N \) is a maximum number of spatial harmonic. This approach is similar to the method used in [11] to determine the energy band structure of atomic BEC localized in an optical lattice. Unlike the latter case the exciton-polariton condensate is strongly affected by the spatial modulation of excitonic reservoir [12], and that is accounted by the second expansion in equation (4).

Substituting (4) into (1) and (2) and collecting terms with equal exponential factors one obtains \( 4N+2 \) coupled equations with \( 4N+3 \) unknowns (including the eigenfrequency \( \mu \)). The missing equation can be found from the condition of the stationarity of condensate density:

\[ \frac{\partial |\Psi|^2}{\partial t} = 0. \]  

(5)

The used approach allows to determine condensate wavefunction and its energy in every particular point of BZ simultaneously, i.e. to determine the full bifurcation digram of the solution.

The number of energy bands described by (4) and (3) is \( 2N+1 \). However we are primarily interested in the lowest energy levels on which polariton condensation occurs in current experiments [4, 5]. In [8] the three-mode approximation \( (N=1) \) was used to describe momentum space oscillations of condensate in a lattice. This approximation adequately describes polariton dynamics for the case of weak-contrast lattices and for relatively low pumping rates. However it becomes inaccurate in a strongly nonlinear regime, i.e. far above the condensation threshold. Indeed, with the increase of condensate density \( |\Psi|^2 \) the nonlinearity leads to effective coupling between different harmonics \( \psi_j \). Thus, more Fourier components are required to provide sufficient precision of the simulations. We
have solved equations (1) and (2) for different values of $N$. Our numerical analysis has shown that for the moderate value of potential depth ($V_0 / h = 0.5 \text{ ps}^{-1}$) the shape of the solutions belonging to the low energy bands almost does not depend on $N$ for $N > 5$. It means that the major contribution to wavefunctions is provided by spatial harmonics with low wavevectors. All numerical calculations in the present paper have been made for $N = 9$ which guaranties a sufficient precision of our numerical analysis.

3.2. Stability of the nonlinear Bloch states

Stability is a crucial characteristic of the nonlinear Bloch states discussed above. To prove the dynamical stability of the solutions we use the standard Bogoliubov-de Gennes approach [9, 17], which implies perturbation of the solution (3) in the form

$$\Psi(x) = e^{i(kx + \omega t)} \sum_j [\psi_j(x) + \delta\psi_j(x)] e^{i\beta x},$$

(6)

$$n(x) = \sum_j [n_j(x) + \delta n_j(x)] e^{i\beta x},$$

(7)

where small perturbations of the condensate and the reservoir steady states are taken in the form

$$\delta\psi_j(x) = u_j e^{i\omega t + iqx} + v_j e^{-i\omega t - iqx}, \quad \delta n_j(x) = \eta_j e^{i\omega t + iqx} + \bar{\eta}_j e^{-i\omega t - iqx}.$$ 

Here $\omega$ is eigenfrequency of small excitation with quasi momentum $q$. Substituting this solution into equations (1) and (2) and keeping only those terms which are linear in respect to small perturbations we obtain the linearized system of equations for all $u_j$, $v_j$ and $\eta_j$. Stability analysis implies the solution of the eigenvalue problem for this system. The Bloch state is unstable if it possesses frequency $\omega$ with a negative imaginary part for any values of $q$. Since analytical analysis is impossible in our case, we perform linear stability analysis numerically.

3.3. Frequency-momenta bifurcation diagrams

The results of numerical solution of equations (3)-(5) are presented below. As expected, in the vicinity of the pumping threshold, where condensate density is small and nonlinear effects are negligible, the frequency-momenta diagrams of exciton-polariton condensate, i.e. $\mu(k)$, resembles a dispersion of a single particle in periodic potential [16] – see figure 1. Note that it is characterized by zero derivatives at the BZ edge, $k = \beta / 2$, meaning that in the linear case all the Bloch waves at the zone boundary possess zero group velocity $v_g = \partial \mu / \partial k = 0$, i.e. carry no current. This phenomenon can be interpreted as the first-order Bragg scattering of the condensate on the periodic lattice. In this case the states at the boundary of BZ are separated by the energy gap which is approximately equal to $V_0$ – see [8].

![Figure 1. Frequency-momentum diagram of the exciton-polariton condensate in a one-dimensional periodic lattice ($V_0 / h = 0.1 \text{ ps}^{-1}$) in the vicinity of the condensation threshold $P_{\text{th}} = 16.335 \mu \text{m}^{-2} \text{ps}^{-1}$.](image-url)
When pumping $P$ increases, the condensate density $|\Psi|^2$ increases as well, making "extra" nonlinear solutions of equations (1) and (2) possible to appear. The properties of the solutions of a Bloch-type are governed by the periodic lattice. That is why the nonlinear Bloch states appear due to the interplay between periodic potential and nonlinearity, i.e., in particular, when the nonlinear mixing between the condensate harmonics $\psi_j$ (mediated by the term $g_c|\Psi|^2$ in equation (1)) becomes comparable with the linear scattering of them by the potential. Thus the nonlinear regime for nonlinear Bloch waves can be reached either under high pumping rates or for weak-contrast lattices (small values of $V_0$) under moderate level of pumping.

Besides the two-particle repulsive interaction between coherent polaritons the incoherent reservoir also contributes to the nonlinear renormalization of the band structure. Indeed, owing to the reservoir-polariton scattering (the term proportional to $g_R$ in equation (1)) the energy of the Bloch waves experiences an additional nonlinear blue shift. Moreover, the density-dependent saturation of the gain results in the spatial modulation of the reservoir which induces additional spatially modulated potential. All aforementioned nonlinear mechanisms affect substantially both shape and stability of the Bloch waves resulting in the modification of the energy dispersion typical for the linear system.

Note that the conception of the energy band structure is not relevant for a nonlinear dissipative system such as exciton-polaritons in a lattice. Actually, the particle number is a free parameter in the conservative system parametrizing the family of the solutions with different $k$. However in the driven dissipative case the condensate population is fixed by the balance of gain and losses which is essentially affected by the condensate distribution. Thus the relevant parameter which parameterizes solutions is the pumping strength $P$ which is a key control parameter and could be smoothly varied in experiment. In this case all the states belonging to the same band have different spatial shapes and thus possess different densities. So, here we search the Bloch solutions possessing different wavevectors $k$ for the fixed value of pumping $P$. The obtained dependences of the condensate eigenenergy versus momentum, $\mu(k)$, differ from the convenient dispersion and can be interpreted as frequency-momenta bifurcation diagram. For instance, the value $\partial\mu/\partial k$ is not a group velocity in the open-dissipative system.

Modification of the energy-momenta diagram near the BZ edge is shown in figure 2 for the case of gradual reduction of the potential contrast from $V_0/h = 0.5 \text{ ps}^{-1}$ for panel (a) down to $0.075 \text{ ps}^{-1}$ for panel (d). All the diagrams are plotted for the fixed pumping $P = 50 \text{ \mu m}^2\text{ps}^{-1}$ that is sufficiently above condensation threshold.

The case shown on the panel (a) corresponds to the deep potential $V_0/h = 0.5 \text{ ps}^{-1}$ when the influence of nonlinear effects should be suppressed. However even in this case the energy-momenta diagrams substantially differs from the linear case shown in figure 1. In particular, the chemical potential of the condensate at the center of the first band ($k = 0$) is larger than at BZ edge. Such a contraintuitive result should be attributed to the driven-dissipative nature of exciton-polariton system. Actually, the average population of the particular Bloch state is determined by the balance of losses and effective gain which is determined by the spatial overlap of the condensate distribution $|\Psi(x)|^2$ with the distribution of the hot excitons $n(x)$. Since all the states of the same band have different spatial structures they possess different densities and thus gain different blueshifts. In particular, the steady-state distribution of reservoir $n(x) = P/\left(\gamma_n + R|\Psi|^2\right)$ is noted for the local depletion which implies that the value of $n$ is minimal at the maxima of the condensate density. Therefore, the overlap integral $\int_{-1/2}^{1/2} |\Psi(x)|^2 n(x) dx$ responsible for the effective gain (see equation (1)) is lower for the Bloch
states with high density contrast. Thus, these states are less populated than the states with a smooth density profile.

Figure 2. (a)-(d) eigenfrequency-momenta bifurcation diagrams, \( \mu(k) \), near the boundary of BZ (vertical dashed line) in the extended zone representation. For all panels the pumping rate is \( P = 50 \mu m^2 ps^{-1} \mu m^2 ps^{-1} \) whereas the depth of periodic potential gradually decreases for (a)-(d). In particular, (a) - \( V_0 / h = 0.5 \) ps\(^{-1} \); (b) - \( V_0 / h = 0.3 \) ps\(^{-1} \); (c) - \( V_0 / h = 0.2 \) ps\(^{-1} \); (d) - \( V_0 / h = 0.1 \) ps\(^{-1} \); (e) - \( V_0 / h = 0.075 \) ps\(^{-1} \). Red (thick) sections correspond to the regions of dynamical stability while \( k_{\text{inst}} \) denotes momentum above which the states of the lower band becomes dynamically unstable. The inset to the panel (e) shows the magnified region of the closed loop below it. Panel (f) the averaged current \( \langle j \rangle \) distribution for the parameters corresponding to panel (d). The black curve corresponds to the upper band. The blue curve (with the red sections) corresponds to the first band.
In particular, the lower state at the BZ boundary (labeled as '1' in figure 1(a)) possesses cosine-like shape and its density vanishes at the potential maxima. At the same time the state at the zone centre has a more smooth density distribution. Its density minima also locate under the potential maxima but are always above zero since the condensates located at the neighboring lattice cites are in-phase (cf. with [18]). So, the state at the BZ centre is usually more populated than the state at the zone boundary. Thus it possesses the stronger blueshift. Besides the state '1' acquires a strong redshift for the case high-contrast lattice since the energy gap is proportional to $V_0$. So, the interplay of the aforementioned effects leads to the unusual shape of the energy band shown in figure 2(a).

The shape of the first band modifies for the smaller potential depth. Unlike the case of conservative BEC system [9-11] the energy loops emerge from the first band not at the BZ edge but at some $k$ located inside Brillouin zone – see panel (b) of figure 2. Reduction of the potential depth $V_0$ allows nonlinearity to increase its impact. The loops move towards each other and tie into a knot of a rather complicated shape at the zone boundary – panel (c). With the further decrease of the potential depth the edges of this knot stretch towards the second band forming a typical loop structure shown in panel (d) of figure 2. The shape of this loop resembles the "swallow tail" structure [13-18] but with the twisted upper edge.

In the limit of small but not vanishing potential depths ($V_0 / \hbar = 0.075 \text{ ps}^{-1}$, panel (e)) the first and the second bands merge. In this case the nonlinear effects completely dominate over the contribution of the external periodic potential in the sense that the energy gap, which is a key feature of a linear periodic potential, disappears. Finally, the $\mu(k)$ dependence approaches a parabolic shape implying that the lattice becomes effectively screened by the nonlinear interactions.

The stability analysis of the nonlinear Bloch states was also performed. The domains of dynamical stability are shown by thick red curves in figure 2. The region of existence of the stable states grows with the reduction of $V_0$. Besides as the influence of the nonlinearity increases the second energy band also stabilizes – see figure 2(d). Finally, all the nonlinear Bloch states stabilize as the bands merge and approach parabolic shape.

A remarkable feature of the observed loops of frequency-momenta bifurcation diagrams (see figures 2(c) and 2(d)) is the existence of states possessing nonzero current

$$j = \frac{i \hbar}{2m} \left( \frac{\partial \Psi^*}{\partial x} \Psi - \frac{\partial \Psi}{\partial x} \Psi^* \right)$$

even at the edges of BZ. The state is currentless if the value of current averaged over the lattice period

$$\langle j \rangle = \frac{1}{l} \int_{-l/2}^{l/2} j(x) dx$$

is zero. Figure 2(f) illustrates the current-momenta diagram corresponding to the typical loop structure shown in the panel (d). The red curve corresponds to the nonlinear Bloch states belonging to the first band while the black one corresponds to the second band. Note that the red curve crosses BZ boundary five times. There are four current states but only two of them are dynamically stable – points '3' in figure 3(f). These states correspond to equivalent oppositely directed polariton flows and thus they are frequency degenerate – point '3' in figure 3(d). Our numerical simulations reveal that nonlinear current states possessing smaller absolute value of the average current are always unstable (see [19, 20] for the details).
Figure 3. (a) the typical shape of a frequency-momenta bifurcation diagram with a closed gap between the second and the third band. (b) the magnified region framed with the black square on the panel (a). The parameters are: $P = 25 \mu m^2 ps^{-1}$ and $V_0 / h = 0.1 \text{ ps}^{-1}$.

Besides these "fluid states" the conventional currentless Bloch solutions at the BZ boundaries also survive even in the strongly nonlinear regime. For instance, these nonlinear Bloch modes belong to the closed curve on the dispersion relation shown in the inset to figure 2(d). The lower and the upper states of this curve [marked by '1' and '2'] are direct counterparts of the highly symmetric Bloch waves known for the single-particle band structure, which nest on the top of the first and on the bottom of the second Bloch bands respectively – see figures 1 and 2(a). We have found that all the states on this closed curve are unstable and thus they do not exhibit in condensate dynamics.

The shape of the second energy band in the vicinity of $k = 0$ also needs to be discussed. In the atomic BEC systems the emergence of swallow tails was predicted from the top of the second band. Our simulations reveal that in our case the nonlinear Bloch states appear only in the very narrow region close to the zone centre, see figure 3. Like at the BZ edges the gap between the second and the third bands closes in the strongly nonlinear limit resulting in formation of the closed loop shown in figure 3(b). Note that all the states belonging to this loop are unstable.

4. Conclusion

We have performed the theoretical analysis of the properties of the nonlinear Bloch states of exciton-polariton condensate confined in 1D periodic potential. The interplay between the periodic potential and the nonlinear interactions results in essential modification of the energy landscape of the condensate states possessing the same periodicity as the lattice. The relevant eigenfrequency-momenta bifurcation diagrams of the nonlinear Bloch states were plotted for the fixed values of pumping rate $P$ which is a key control parameter of a driven-dissipative polariton system. The main peculiarity of these diagrams is the loop-like structures emerging at the first and the second bands. We have shown that the main manifestations of the periodic potential, which are typical for the linear problem, i.e. energy gap and currentless states at the Brillouin zone boundary, becomes suppressed in the limit of strong nonlinearity. Formation of the nonlinear current states at the BZ boundary was also described. Such nonlinear current of polariton condensate in a lattice can be experimentally observed either with the specifically designed localized pumping [19] or via spontaneous formation in the presence of complex periodic potential [20].

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