Dynamic Data Compression with Distortion Constraints for Wireless Transmission over a Fading Channel

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We consider a wireless node that randomly receives data from different sensor units. The arriving data must be compressed, stored, and transmitted over a wireless link, where both the compression and transmission operations consume power. Specifically, the controller must choose from one of multiple compression options every timeslot. Each option requires a different amount of power and has different compression ratio properties. Further, the wireless link has potentially time-varying channels, and transmission rates depend on current channel states and transmission power allocations. We design a dynamic algorithm for joint compression and transmission, and prove that it comes arbitrarily close to minimizing average power expenditure, with an explicit tradeoff in average delay. Our approach uses stochastic network optimization together with a concept of place holder bits to provide efficient energy-delay performance. The algorithm is simple to implement and does not require knowledge of probability distributions for packet arrivals or channel states. Extensions that treat distortion constraints are also considered.

Categories and Subject Descriptors: ... [...]: ...

General Terms: queueing analysis, stochastic network optimization, sensor networks, data fusion, distortion, fading channel

1. INTRODUCTION

We consider the problem of energy-aware data compression and transmission for a wireless link that receives data from \( N \) different sensor units (Fig. 1). Time is slotted with normalized slot durations \( t \in \{0, 1, 2, \ldots\} \), and every timeslot the link receives a packet from a random number of the sensors. We assume that packets arriving on the same timeslot contain correlated data, and that this data can be compressed using one of multiple compression options. However, the signal processing required for compression consumes a significant amount of energy, and more sophisticated compression algorithms are also more energy expensive. Further, the data must be transmitted over a wireless channel with potentially varying channel conditions, where the transmission rates available on the current timeslot depend on the current channel condition and the current transmission power allocation. The goal is to design a joint compression and transmission scheduling policy that minimizes time average power expenditure.

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This problem is important for modern sensor networks where correlated (and compressible) data flows over power limited nodes. Compressing the data can save power by reducing the amount of bits that need to be transmitted, provided that the transmission power saved is more than the power expended in the compression operation. It is important to understand the optimal balance between compression power and transmission power. Work in [Barr and Asanović 2003] considers this question for a wireless link with fixed transmission costs, and describes practical compression issues and reports communication-to-computation energy ratios for popular algorithms. Work in [Sadler and Martonosi 2006] considers a similar static situation where the wireless channel condition is the same for all time. There, it is shown experimentally that compression can lead to a significant power savings when data is transmitted over multiple hops. The proposed algorithm of [Sadler and Martonosi 2006] uses a fixed data compression scheme, an adaptation of the Lempel-Ziv-Welch (LZW) compression algorithm for sensor networks. Techniques for distributed compression using Slepian-Wolf coding theory are considered in [Pradhan et al. 2002] [Cristescu et al. 2005]. Models of spatial correlation between data of different sensors are proposed in [Pattem et al. 2004] and used to construct and evaluate energy-efficient routing algorithms that compress data at each stage.

The above prior work has concentrated on static environments where transmission power is directly proportional to the number of bits transmitted and/or traffic rates are fixed and known, so that compression and transmission strategies can be designed in advance. Here, we focus attention on a single link, but consider a stochastic environment where the amount of data received every slot is random, as is the current channel condition for wireless transmission. Further, the transmission rate is an arbitrary (possibly non-linear) function of transmission power. Optimal policies in this stochastic context are more complex, and more care is required to ensure transmissions are energy-efficient.

In this paper, we design a dynamic compression and transmission scheduling algorithm and prove that the algorithm pushes total time average power arbitrarily close to optimal, with a corresponding tradeoff in average delay. We assume the
Our algorithm bases decisions purely on this information and does not require a-priori knowledge of the packet arrival or channel state probabilities. The algorithm is simple to implement and is robust to situations where these probabilities can change. This work is important as it demonstrates a principled method of making on-line compression decisions in a stochastic system with correlated data. Our solution applies the techniques of Lyapunov optimization developed in our previous work [Neely 2006] [Georgiadis et al. 2006], and is perhaps the first application of these techniques to the dynamic compression problem. This paper also extends the general theory by introducing a novel concept of place-holder bits to improve delay in stochastic networks with costs. Related Lyapunov optimization techniques for network flow control applications are developed in [Neely 2003] [Neely et al. 2005], and alternative fluid model approaches are developed in [Stolyar 2005] [Eryilmaz and Srikant 2005].

In the next section we describe the system model, and in Section 3 we characterize the minimum average power in terms of an optimization problem based on channel and packet arrival probabilities. In Section 4 we develop an on-line algorithm that makes simple decisions based only on current information. The algorithm achieves time average power that can be pushed arbitrarily close to optimum via a simple control parameter that also affects an average delay tradeoff. A simple improvement via place-holder bits is developed in Section 5. Extensions to systems with distortion constraints are given in Section 6. Simulations are provided in Section 7.

2. SYSTEM MODEL

Consider the wireless link of Fig. 1 that operates in slotted time and receives packets from \( N \) different sensor units. If an individual sensor sends data during a timeslot, this data is in the form of a fixed length packet of size \( b \) bits, containing sensed information. Let \( A(t) \) represent the number of sensors that send packets during slot \( t \), so that \( A(t) \in \{0, 1, \ldots, N\} \). The data from these \( A(t) \) packets may be correlated, and hence it may be possible to compress the information within the \( A(t) \) packets (consisting of \( A(t)b \) bits) into a smaller data unit for transmission over the wireless link. This is done via a compression function \( \Psi(a, k) \) defined as follows. There are \( K+1 \) compression options, comprising a set \( \mathcal{K} = \{0, 1, \ldots, K\} \). Option 0 represents no attempted compression, and options \( \{1, 2, \ldots, K\} \) represent various alternative methods to compress the data. The function \( \Psi(a, k) \) takes input \( a \in \{0, 1, \ldots, N\} \) (representing the number of newly arriving packets) and compression option \( k \in \mathcal{K} \), and generates a random variable output \( R \), representing the total size of the data after compression.

Every timeslot the link controller observes the random number of new packet arrivals \( A(t) \) and chooses a compression option \( k(t) \in \mathcal{K} \), yielding the random compressed output \( R(t) = \Psi(A(t), k(t)) \). Let \( P_{\text{comp}}(t) \) represent the power expended by this compression operation, and assume this is also a random function of the number of packets compressed and the compression option. We assume that the compressed output \( R(t) \) is conditionally i.i.d. over all slots that have the same number of packet arrivals \( A(t) \) and the same compression decision \( k(t) \). Likewise,
compression power $P_{comp}(t)$ is conditionally i.i.d. over all slots with the same $A(t)$ and $k(t)$. The average compressed output $m(a, k)$ and the average power expenditure $\phi(a, k)$ associated with $A(t) = a, k(t) = k$ are defined:

$$m(a, k) = \mathbb{E}\{\Psi(A(t), k(t)) | A(t) = a, k(t) = k\}$$ \hspace{1cm} (1)

$$\phi(a, k) = \mathbb{E}\{P_{comp}(t) | A(t) = a, k(t) = k\}$$ \hspace{1cm} (2)

We assume the values of $m(a, k)$ and $\phi(a, k)$ are known so that the following table can be constructed:

| $k$ | $\Psi(a, k)$ | $\mathbb{E}\{\Psi(a, k)\}$ | $\mathbb{E}\{P_{comp} | a, k\}$ |
|-----|--------------|----------------------------|--------------------------------|
| 0   | ab           | ab                         | $\phi(a, 0) = 0$                |
| 1   | Random       | $m(a, 1)$                  | $\phi(a, 1)$                   |
| 2   | Random       | $m(a, 2)$                  | $\phi(a, 2)$                   |
| ... | ...          | ...                        | ...                            |
| $K$ | Random       | $m(a, K)$                  | $\phi(a, K)$                   |

Note that we assume $\Psi(a, 0) = 0$ and $\phi(a, 0) = 0$, as the compression option $k = 0$ does not compress any data and also does not expend any power. We further assume that $m(a, k) \leq ab$ for all $a \in \{0, 1, \ldots, N\}$ and all $k \in K$, so that compression is not expected to expand the data.

2.1 Data Transmission and Queueing

The compressed data $R(t) = \Psi(A(t), k(t))$ is delivered to a queueing buffer for transmission over the wireless link (see Fig. 1). Let $U(t)$ represent the current number of bits (or unfinished work) in the queue. The queue backlog evolution is given by:

$$U(t+1) = \max[U(t) - \mu(t), 0] + R(t)$$ \hspace{1cm} (3)

where $\mu(t)$ is the transmission rate offered by the link on slot $t$. This rate is determined by the current channel condition and the current transmission power allocation decision, as in [Georgiadis et al. 2006]. Specifically, the channel is assumed to be constant over the duration of a slot, but can potentially change from slot to slot. Let $S(t)$ represent the current channel state, which is assumed to take values in some finite set $S$. We assume the channel state $S(t)$ is known at the beginning of each slot $t$, so that the link can make an opportunistic transmission power allocation decision $P_{tran}(t)$, yielding a transmission rate $\mu(t)$ given by:

$$\mu(t) = C(P_{tran}(t), S(t))$$

where $C(P, s)$ is the rate-power curve associated with the modulation and coding schemes used for transmission over the channel. We assume $C(P, s)$ is continuous in power $P$ for each channel state $s \in S$. Transmission power allocations $P(t)$ are restricted to some compact set $\mathcal{P}$ for all slots $t$, where $\mathcal{P}$ contains a maximum transmission power $P_{max}$. For example, the set $\mathcal{P}$ can contain a discrete set of power levels, such as the two element set $\mathcal{P} = \{0, P_{max}\}$. Alternatively, $\mathcal{P}$ can be a continuous interval, such as $\mathcal{P} = \{P | 0 \leq P \leq P_{max}\}$. We assume throughout that $0 \in \mathcal{P}$ and that $C(0, s) = 0$ for all channel states $s \in S$, so that zero transmission power yields a zero transmission rate. Further, we assume that $C(P_{max}, s) \geq$
\( C(P, s) \) for all \( s \in S \) and all \( P \in \mathcal{P} \), so that allocating maximum power yields the largest transmission rate that is possible under the given channel state.

### 2.2 Stochastic Assumptions and the Control Objective

For simplicity, we assume the packet arrival process \( A(t) \) is i.i.d. over slots with a general probability distribution \( p_A(a) = Pr[A(t) = a] \). Likewise, the channel state process \( S(t) \) is i.i.d. over slots with a general distribution \( \pi_s = Pr[S(t) = s] \).

The distributions \( p_A(a) \) and \( \pi_s \) are not necessarily known to the link controller. Every slot the link controller observes the number of new packets \( A(t) \), the current queue backlog \( U(t) \), and the current channel state \( S(t) \), and makes a compression decision \( k(t) \in \mathcal{K} \) (expending power \( P_{\text{comp}}(t) \)) and a transmission power allocation \( P_{\text{tran}}(t) \in \mathcal{P} \). The total time average power expenditure is given by:

\[
\lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} [P_{\text{comp}}(\tau) + P_{\text{tran}}(\tau)]
\]

The goal is to make compression and transmission decisions to minimize time average power while ensuring the queue \( U(t) \) is stable. Formally, we define a queueing process \( U(t) \) to be stable if:

\[
\limsup_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{U(\tau)\} < \infty
\]

This type of stability is often referred to as strong stability, as it implies a finite average backlog and hence a finite average delay. In Section 4, we shall design a class of dynamic algorithms that can drive time average power arbitrarily close to the minimum average power required for stability, with a corresponding explicit tradeoff in average queue backlog and average delay.

Define \( r_{\text{min}} \) and \( r_{\text{max}} \) as follows:

\[
\begin{align*}
r_{\text{min}} &\triangleq \mathbb{E}\left\{ \min_{k \in \mathcal{K}} m(A(t), k) \right\} \\
r_{\text{max}} &\triangleq \mathbb{E}\{C(P_{\text{max}}, S(t))\}
\end{align*}
\]

where the expectations are taken over the randomness of \( A(t) \) and \( S(t) \) via the distributions \( p_A(a) \) and \( \pi_s \). Thus, \( r_{\text{min}} \) is the minimum average bit rate delivered to the queueing system (in units of bits/slot), assuming the compression option that results in the largest expected bit reduction is used every slot. The value \( r_{\text{max}} \) represents the maximum possible average transmission rate over the wireless link. We assume throughout that \( r_{\text{min}} < r_{\text{max}} \), so that it is possible to stabilize the system.

Thus, there are two reasons to compress data: (i) In order to stabilize the queue, we may need to compress (particularly if \( \mathbb{E}\{A(t)\} b > r_{\text{max}} \)). (ii) We may actually

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\( ^1 \)Using the \( T \)-slot Lyapunov drift techniques described in [Georgiadis et al. 2006], our analysis can be generalized to show that the same algorithms we derive under the i.i.d. assumption yield similar performance for arbitrary ergodic arrival and channel processes \( A(t) \) and \( S(t) \), with delay bounds that increase by a constant factor related to the mixing times of the processes.
save power if the power used to compress is less than the extra amount of power that would be used transmitting the extra data if it were not compressed.

2.3 Discussion of the System Model

This simple model captures a wide class of systems where data compression is important. The $N$ sensor scenario of Fig. 1 captures the possibility of randomly arriving data that is spatially correlated. An example is when there are multiple sensors in an environment and only a random subset of them detect a particular event. The data provided by these sensors is thus correlated but not necessarily identical, as each observation can offer new information.

The case of compression because of time correlated data can also be treated in this model by re-defining $N$ to represent the time over which a frame of data samples are gathered. Indeed, suppose a timeslot $t$ is composed of $N$ mini-slots, where data can arrive on any or all of the mini-slots. The value of $A(t)$ now represents the random number of packets arriving over the $N$ mini-slots, and the compression functions $m(a,k)$ and $\phi(a,k)$ now represent averages associated with compressing the time-correlated data. This of course assumes compression is contained to data arriving within the same frame, and does not treat inter-frame compression.

Our time-varying channel model is useful for systems with mobility, environmental changes, or restrictions that create time-varying transmission opportunities. This allows for opportunistic scheduling which can help to further reduce power expenditure. We do not consider the additional power required to measure the channel conditions here. Extensions that treat this issue can likely be obtained using the techniques for optimizing measurement decisions developed in [Li and Neely 2007]. A special case of the time-varying channel model is the static channel assumption, where $S(t)$ is the same for all timeslots $t$. This special case is similar to the static assumption in [Barr and Asanović 2003] [Sadler and Martonosi 2006]. However, this static channel scenario still creates an interesting problem that is much different from [Barr and Asanović 2003] [Sadler and Martonosi 2006]. Indeed, the random packet arrivals (with raw data rate that is possibly larger than link capacity) and the potentially non-linear rate-power curve necessitate a dynamic compression strategy that is not obvious, that depends on the packet arrival distribution, and that does not necessarily use the same compression option on every slot.

Here we assume that the compression options available within the set $K$ are sufficient to ensure that the resulting data transmitted over the link has an acceptable fidelity. An example is lossless data compression, such as Huffman or Lempel-Ziv source coding, where all original data packets can be reconstructed at the destination. Alternatively, we might have some compression options $k \in K$ representing lossy compression, provided that the distortion that may be introduced is acceptable. Extensions to systems that explicitly consider distortion due to lossy compression are considered in Section 6.

3. MINIMUM AVERAGE POWER

Here we characterize the minimum time average power required for queue stability. We first define separate functions $h^*(r)$ and $g^*(r)$ that describe the minimum average power for compression and transmission, respectively, over a restricted class
of stationary randomized algorithms. These functions depend on the steady state arrival and channel distributions $p_A(a)$ and $\pi_s$. We then show that these functions can be used to define system optimality over the class of all possible decision strategies, including strategies that do not necessarily make stationary and randomized decisions.

3.1 The Functions $h^*(r)$ and $g^*(r)$

**Definition 1.** For any value $r$ such that $r_{\text{min}} \leq r \leq b E\{A(t)\}$, the minimum-power compression function $h^*(r)$ is defined as the infimum value $h$ for which there exist probabilities $(\gamma_{a,k})$ for $a \in \{0, 1, \ldots, N\}$, $k \in K$, such that the following constraints are satisfied:

$$\sum_{a=0}^{N} \sum_{k=1}^{K} p_A(a) \gamma_{a,k} \phi(a, k) = h$$  
(6)

$$\sum_{a=0}^{N} \sum_{k=1}^{K} p_A(a) \gamma_{a,k} m(a, k) \leq r$$  
(7)

$$\gamma_{a,k} \geq 0 \quad \text{for all } a, k$$  
(8)

$$\sum_{k=1}^{K} \gamma_{a,k} = 1 \quad \text{for all } a$$  
(9)

Intuitively, the $(\gamma_{a,k})$ values define a stationary randomized policy that observes the current arrivals $A(t)$ and uses compression option $k$ with probability $\gamma_{a,k}$ whenever $A(t) = a$. The expression on the left hand side of (6) is the expected compression power $E\{P_{\text{comp}}(t)\}$ for this policy. Likewise, the expression on the left hand side of (7) is the expected number of bits $E\{R(t)\}$ at the output of the compressor for this policy. The value of $h^*(r)$ is thus the smallest possible average power due to compression, infimized over all such stationary randomized policies that yield $E\{R(t)\} \leq r$. Note from (4) that it is possible to have a stationary randomized policy that yields $E\{R(t)\} = r_{\text{min}}$, and hence the function $h^*(r)$ is well defined for any $r \geq r_{\text{min}}$. Further, the following lemma shows that the infimum value $h^*(r)$ can be achieved by a particular stationary randomized algorithm.

**Lemma 1.** For any $r$ such that $r_{\text{min}} \leq r \leq b E\{A(t)\}$, there exists a particular stationary randomized policy that makes compression decisions $k^*(t)$ as a random function of the observed $A(t)$ value (and independent of queue backlog), such that:

$$E\{\phi(A(t), k^*(t))\} = h^*(r)$$  
(10)

$$E\{m(A(t), k^*(t))\} = r$$  
(11)

where the above expectations are taken with respect to the steady state packet arrival distribution $p_A(a)$ and the randomized compression decisions $k^*(t)$.

**Proof.** The proof follows by continuity of the functions on the left hand side of (6) and (7) with respect to $\gamma_{a,k}$, and by compactness of the set of all $(\gamma_{a,k})$ that satisfy (8) and (9). See Appendix A for details.

Similar to the function $h^*(r)$, we define $g^*(r)$ as the smallest possible average transmission power required for a stationary randomized algorithm to support a
transmission rate of at least \( r \). The precise definition is given below.

**Definition 2.** For any value \( r \) such that \( 0 \leq r \leq r_{\text{max}} \), the minimum-power transmission function \( g^*(r) \) is defined as the infimum value \( g \) for which there exists a stationary randomized power allocation policy that chooses transmission power \( P_{\text{tran}}(t) \) as a random function of the observed channel state \( S(t) \) (and independent of current queue backlog), such that:

\[
\mathbb{E}\{P_{\text{tran}}(t)\} = g \quad (12)
\]

\[
\mathbb{E}\{C(P_{\text{tran}}(t), S(t))\} \geq r \quad (13)
\]

The function \( g^*(r) \) is well defined whenever \( r \leq r_{\text{max}} \) because it is possible to satisfy the constraint (13). Indeed, note by (5) that the policy \( P_{\text{tran}}(t) = P_{\text{max}} \) for all \( t \) yields \( \mathbb{E}\{C(P_{\text{tran}}(t), S(t))\} = r_{\text{max}} \). Furthermore, it is easy to show that the inequality constraint in (13) can be replaced by an equality constraint, as any policy with an average transmission rate larger than \( r \) can be modified to achieve rate \( r \) exactly while using strictly less power. This can be done by independently setting \( P_{\text{tran}}(t) = 0 \) with some probability every slot, yielding a zero transmission rate in that slot.

Because the set \( P \) is compact and the function \( C(P, s) \) is continuous in power \( P \) for all channel states \( s \in S \), an argument similar to the proof of Lemma 1 can be used to show that the infimum average power \( g^*(r) \) can be achieved by a particular stationary randomized policy. Specifically, for any \( r \) such that \( 0 \leq r \leq r_{\text{max}} \), there exists a stationary randomized algorithm that chooses transmission power \( P_{\text{tran}}^*(t) \) that yields:

\[
\mathbb{E}\{C(P_{\text{tran}}^*(t), S(t))\} = r \quad (14)
\]

\[
\mathbb{E}\{P_{\text{tran}}^*(t)\} = g^*(r) \quad (15)
\]

### 3.2 Structural Properties of \( h^*(r) \) and \( g^*(r) \)

It is not difficult to show that \( h^*(r) \) is a non-increasing function of \( r \) (because less compression power is required if a larger compressor output rate is allowed), and that \( g^*(r) \) is a non-decreasing function of \( r \) (because more transmission power is required to support a larger transmission rate). Further, both functions are convex. It is interesting to note that in the special case when there is no channel state variation so that \( C(P, s) = C(P) \), and when the function \( C(P) \) is strictly increasing and concave, then \( g^*(r) = C^{-1}(r) \), i.e., it is given by the inverse of \( C(P) \). Details on the structure of the \( g^*(r) \) function in the general time-varying case are given in [Neely 2006].

### 3.3 Minimum Average Power for Stability

The following theorem establishes the minimum time average power required for queue stability in terms of the \( h^*(r) \) and \( g^*(r) \) functions. We consider all possible algorithms for making compression decisions \( k(t) \in K \) and transmission power

\[\text{More generally, the infimum can be achieved whenever } C(P, s) \text{ is upper semi-continuous in } P \text{ for every channel state } s \in S \text{ (see [Bertsekas et al. 2003] for a definition). The upper semi-continuity property is a mild property that is true of every practical curve } C(P, s). \text{ All results of this paper hold when continuity is replaced by upper semi-continuity.}\]
decisions $P_{\text{tran}}(t) \in \mathcal{P}$ over time, including algorithms that are not necessarily in the class of stationary randomized policies.

**Theorem 1.** Let $A(t)$ and $S(t)$ be ergodic with steady state distributions $p_A(a)$ and $\pi_s$, respectively (such as processes that are i.i.d. over slots, or more general Markov modulated processes). Assume that $r_{\text{min}} < r_{\text{max}}$ (defined in (4), (5)). Then any joint compression and transmission rate scheduling algorithm that stabilizes the queue $U(t)$ yields a time average power expenditure that satisfies:

$$\limsup_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E} \{P_{\text{comp}}(\tau) + P_{\text{tran}}(\tau)\} \geq P^*_{\text{av}}$$

where $P^*_{\text{av}}$ is defined as the optimal solution to the following problem:

**Minimize:** $h^*(r) + g^*(r)$  \hspace{1cm} (16)

**Subject to:** $r_{\text{min}} \leq r \leq \min[r_{\text{max}}, b\mathbb{E}\{A(t)\}]$  \hspace{1cm} (17)

**Proof.** See Appendix B. \hfill $\square$

The above theorem shows that time average power must be greater than or equal to $P^*_{\text{av}}$ for queue stability. The result can be understood intuitively by observing that if $r$ is the rate of bits arriving to the queue from the compressor, then average transmission power can be minimized while maintaining stability by pushing the time average transmission rate down closer and closer to $r$. The optimization problem corresponding to this definition of $P^*_{\text{av}}$ may be difficult to solve in practice, as it would require exact knowledge of the $h^*(r)$ and $g^*(r)$ functions, which in turn requires full a-priori knowledge of the distributions $p_A(a)$ and $\pi_s$. In the next section, we design a simple class of dynamic algorithms that stabilize the queue without this knowledge, and that push time average power arbitrarily close to $P^*_{\text{av}}$.

4. THE DYNAMIC COMPRESSION ALGORITHM

Our dynamic algorithm is decoupled into separate policies for data compression and transmission rate scheduling. It is defined in terms of a control parameter $V > 0$ that affects an energy-delay tradeoff.

**The Dynamic Compression and Transmission Algorithm:**

**Compression:** Every slot $t$, observe the number of new packet arrivals $A(t)$ and the current queue backlog $U(t)$, and choose compression option $k(t) \in \mathcal{K}$ as follows:

$$k(t) = \arg \min_{k \in \mathcal{K}} [U(t)m(A(t), k) + V\phi(A(t), k)]$$  \hspace{1cm} (18)

If there are multiple compression options $k \in \mathcal{K}$ that minimize $U(t)m(A(t), k) + V\phi(A(t), k)$, break ties arbitrarily.

**Transmission:** Every slot $t$, observe the current channel state $S(t)$ and the current queue backlog $U(t)$, and choose transmission power $P_{\text{tran}}(t) \in \mathcal{P}$ as follows:

$$P_{\text{tran}}(t) = \arg \max_{P \in \mathcal{P}} [U(t)C(P, S(t)) - VP]$$  \hspace{1cm} (19)

Recall that $\mathcal{P}$ is assumed to be compact and the $C(P, s)$ function is upper semi-continuous, and hence there exists a maximizing power allocation. If there are multiple power options that maximize $U(t)C(P, S(t)) - VP$, break ties arbitrarily.
The compression policy involves a simple comparison of \( K + 1 \) values found by evaluating the \( m(a, k) \) and \( \phi(a, k) \) functions for all \( k \in K \), and can easily be accomplished in real time. The transmission policy is a special case of the Energy Efficient Control Algorithm (EECA) policy developed in [Neely 2006], and typically can also be solved very simply in real time. The next theorem establishes the performance of the combined algorithm.

**Theorem 2. (Algorithm Performance)** Suppose packet arrivals \( A(t) \) are i.i.d. over slots with distribution \( p_A(a) \), and channel states \( S(t) \) and are i.i.d. over slots with distribution \( \pi_s \). For any control parameter \( V > 0 \), the dynamic compression and transmission scheduling algorithm yields power expenditure and queue backlog that satisfy the following:

\[
\mathcal{P}_{\text{tot}} \leq P_{\text{av}}^* + B/V \tag{20}
\]

\[
\mathcal{U} \leq B + V(P_{\text{max}} + \phi_{\text{max}}) \over (r_{\text{max}} - r_{\text{min}}) \tag{21}
\]

where \( \mathcal{P}_{\text{tot}} \) and \( \mathcal{U} \) are the time averages for power expenditure and queue backlog, defined:

\[
\mathcal{P}_{\text{tot}} \triangleq \limsup_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} E\{P_{\text{comp}}(\tau) + P_{\text{tran}}(\tau)\}
\]

\[
\mathcal{U} \triangleq \limsup_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} E\{U(\tau)\}
\]

and where \( B \) and \( \phi_{\text{max}} \) are constants given by:

\[
B \triangleq \frac{1}{2} \left[ \sigma^2 + E\{C(P_{\text{max}}, S(t))^2\} \right] \tag{22}
\]

\[
\phi_{\text{max}} \triangleq E\left\{ \max_{k \in K} [\phi(A(t), k)] \right\} \tag{23}
\]

where \( \sigma^2 \) is an upper bound on \( E\{R(t)^2\} \) for all slots \( t \). For example, if no compression operation expands the data, then \( R(t) = \Psi(A(t), k) \leq bA(t) \) for all \( t \), and hence \( \sigma^2 \) is defined:

\[
\sigma^2 \triangleq b^2 E\{A(t)^2\}
\]

We prove Theorem 2 in the next subsection. Note that the parameter \( V > 0 \) can be chosen to make \( B/V \) arbitrarily small, ensuring by (20) that time average power is arbitrarily close to the optimal value \( P_{\text{av}}^* \). However, the resulting average queue backlog bound grows linearly with \( V \). By Little’s Theorem, the average queue backlog is proportional to average delay [Bertsekas and Gallager 1992]. This establishes an explicit tradeoff between average power expenditure and delay.

As an implementation detail, we note for simplicity that we can use units of bits, bits/slot, and milli-Watts for \( U(t) \), \( C(P, S) \), and \( P \). However, these units are arbitrary and any consistent units will work, with performance given by (20) and (21). Indeed, any unit changes are captured in the \( V \) constant (where \( V \) has units of \( \text{bits}^2/\text{mW} \) for the units above). For example, if milli-Watts are changed to Watts, then the algorithm will make the exact same control decisions for \( k(t) \) and \( P_{\text{tran}}(t) \).
over time, and hence yields the exact same sample path of energy use and queue backlog, as long as the $V$ constant is appropriately changed by a factor of 1000. If bits are changed to kilobits, then $V$ must change by a factor of $10^6$.

4.1 Lyapunov Performance Analysis for Theorem 2

Our proof of Theorem 2 relies on the performance optimal Lyapunov scheduling techniques from [Georgiadis et al. 2006] [Neely 2006]. First define the following quadratic Lyapunov function of queue backlog $U(t)$:

$$L(U(t)) \triangleq \frac{1}{2} U(t)^2$$

Define the one-step conditional Lyapunov drift $\Delta(U(t))$ as follows:

$$\Delta(U(t)) \triangleq \mathbb{E}\{L(U(t+1)) - L(U(t)) | U(t)\}$$

The following simple lemma from [Georgiadis et al. 2006] shall be useful.

**Lemma 2.** (Lyapunov drift [Georgiadis et al. 2006]) Let $L(U(t))$ be a non-negative function of $U(t)$ with Lyapunov drift $\Delta(U(t))$ defined in (24). If there are stochastic processes $\alpha(t)$ and $\beta(t)$ such that every slot $t$ and for all possible values of $U(t)$, the conditional Lyapunov drift satisfies:

$$\Delta(U(t)) \leq \mathbb{E}\{\beta(t) - \alpha(t) | U(t)\}$$

then:

$$\limsup_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{\alpha(\tau)\} \leq \limsup_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{\beta(\tau)\}$$

The proof involves taking expectations of (25), using iterated expectations, and summing the resulting telescoping series (see [Georgiadis et al. 2006] for details).

The queue backlog $U(t)$ for our system satisfies the queue evolution equation (3). Specifically, the queue has arrival process $R(t) = \Psi(A(t), k(t))$ and transmission rate process $\mu(t) = C(P_{\text{tran}}(t), S(t))$, where the $k(t)$ and $P_{\text{tran}}(t)$ control decisions are determined by the dynamic compression and transmission algorithm of the previous sub-section. The Lyapunov drift is given by the following lemma.

**Lemma 3.** (Computing $\Delta(U(t))$) Under the queue evolution equation (3) and using the quadratic Lyapunov function $L(U(t)) = \frac{1}{2} U(t)^2$, the Lyapunov drift $\Delta(U(t))$ satisfies the following for all $t$ and all $U(t)$:

$$\Delta(U(t)) \leq B - U(t)\mathbb{E}\{\mu(t) - m(A(t), k(t)) | U(t)\}$$

where $\mu(t) = C(P_{\text{tran}}(t), S(t))$, and $B$ is given in (22). The expectation above is taken with respect to the random channels and arrivals $S(t)$ and $A(t)$, and the potentially random control actions $k(t)$ and $P_{\text{tran}}(t)$.

---

More complete notation would be $\Delta(U(t), t)$, as the drift depends on the scheduling policy which may also depend on time $t$. However, we use the simpler notation $\Delta(U(t))$ as a formal representation of the right hand side of (24). See [Georgiadis et al. 2006] for further details on Lyapunov drift.
Proof. From (3) we have:
\[
\frac{1}{2} U(t + 1)^2 = \frac{1}{2} \left( \max[U(t) - \mu(t), 0] + R(t) \right)^2 \\
\leq \frac{1}{2} \left( U(t)^2 + \mu(t)^2 + R(t)^2 \right) \\
- U(t)(\mu(t) - R(t))
\]
and hence (taking conditional expectations given \( U(t) \)):
\[
\Delta(U(t)) \leq \frac{1}{2} \mathbb{E} \{ \mu(t)^2 + R(t)^2 \mid U(t) \} \\
- U(t)\mathbb{E} \{ \mu(t) - R(t) \mid U(t) \}
\]
It is clear that the value \( \frac{1}{2} \mathbb{E} \{ \mu(t)^2 + R(t)^2 \mid U(t) \} \) is less than or equal to the constant \( B \) defined in (22), and hence:
\[
\Delta(U(t)) \leq B - U(t)\mathbb{E} \{ \mu(t) - R(t) \mid U(t) \} 
\]
Noting that \( R(t) = \Psi(A(t), k(t)) \) and using iterated expectations, we have:
\[
\mathbb{E} \{ R(t) \mid U(t) \} \\
= \mathbb{E} \{ \Psi(A(t), k(t)) \mid U(t) \} \\
= \mathbb{E} \{ \mathbb{E} \{ \Psi(A(t), k(t)) \mid U(t), A(t), k(t) \} \mid U(t) \} \\
= \mathbb{E} \{ m(A(t), k(t)) \mid U(t) \}
\]
where we have used the definition of \( m(a, k) \) given in (1). Using this equality in (27) yields the result. \( \square \)

Following the Lyapunov optimization framework of [Georgiadis et al. 2006] [Neely 2006], we add a weighted cost term to the drift expression. Specifically, from (26) we have:
\[
\Delta(U(t)) + V \mathbb{E} \{ P_{\text{comp}}(t) + P_{\text{tran}}(t) \mid U(t) \} \leq \\
B - U(t)\mathbb{E} \{ C(P_{\text{tran}}(t), S(t)) - m(A(t), k(t)) \mid U(t) \} \\
+ V \mathbb{E} \{ P_{\text{comp}}(t) + P_{\text{tran}}(t) \mid U(t) \}
\]
where we have just added an additional term to both sides of (26). Note that \( \mathbb{E} \{ P_{\text{comp}}(t) \mid U(t) \} \) can be expressed as follows (using iterated expectations):
\[
\mathbb{E} \{ P_{\text{comp}}(t) \mid U(t) \} \\
= \mathbb{E} \{ \mathbb{E} \{ P_{\text{comp}}(t) \mid U(t), A(t), k(t) \} \mid U(t) \} \\
= \mathbb{E} \{ \phi(A(t), k(t)) \mid U(t) \}
\]
Using this equality in the right hand side of (28) and re-arranging terms yields:
\[
\Delta(U(t)) + V \mathbb{E} \{ P_{\text{comp}}(t) + P_{\text{tran}}(t) \mid U(t) \} \leq \\
B - \mathbb{E} \{ U(t)C(P_{\text{tran}}(t), S(t)) - VP_{\text{tran}}(t) \mid U(t) \} \\
+ \mathbb{E} \{ U(t)m(A(t), k(t)) + V\phi(A(t), k(t)) \mid U(t) \}
\]
Now note that we have not yet used the properties of the dynamic compression and transmission policy. Indeed, the above expression (29) is a bound that holds
for any compression and transmission scheduling decisions \( k(t) \in \mathcal{K} \), \( P_{\text{tran}}(t) \in \mathcal{P} \) that are made on slot \( t \), including randomized decisions. However, note that the dynamic compression and transmission strategy is designed specifically to minimize the right hand side of (29) over all alternative decisions that can be made on slot \( t \). Indeed, the compression algorithm observes \( A(t) \) and \( U(t) \) and chooses \( k(t) \in \mathcal{K} \) to minimize \( U(t) m(A(t), k(t)) + V \phi(A(t), k(t)) \), which thus minimizes the following term over all alternative decisions that can be made on slot \( t \):

\[
\mathbb{E} \{ U(t) m(A(t), k(t)) + V \phi(A(t), k(t)) \mid U(t) \}
\]

Similarly, the transmission power allocation algorithm is designed to minimize the following term over all alternative decisions that can be made on slot \( t \):

\[
-\mathbb{E} \{ U(t) C(P_{\text{tran}}(t), S(t)) - VP_{\text{tran}}(t) \mid U(t) \}
\]

It follows that the right hand side of (29) is less than or equal to the corresponding expression with \( P_{\text{tran}}(t) \) and \( k(t) \) replaced by \( P_{\text{tran}}^*(t) \) and \( k^*(t) \), where \( P_{\text{tran}}^*(t) \) and \( k^*(t) \) are any other (possibly randomized) policies that satisfy \( P_{\text{tran}}^*(t) \in \mathcal{P} \) and \( k^*(t) \in \mathcal{K} \):

\[
\Delta(U(t)) + \mathbb{E} \{ P_{\text{comp}}(t) + P_{\text{tran}}(t) \mid U(t) \} \leq B - \mathbb{E} \{ U(t) C(P_{\text{tran}}^*(t), S(t)) - V P_{\text{tran}}^*(t) \mid U(t) \} + \mathbb{E} \{ U(t) m(A(t), k^*(t)) + V \phi(A(t), k^*(t)) \mid U(t) \}
\]

(30)

Now let \( r_1 \) be any particular value that satisfies \( r_{\text{min}} \leq r_1 \leq b \mathbb{E} \{ A(t) \} \), and let \( k^*(t) \) be the stationary randomized policy that yields:

\[
\mathbb{E} \{ \phi(A(t), k^*(t)) \} = h^*(r_1)
\]

(31)

\[
\mathbb{E} \{ m(A(t), k^*(t)) \} = r_1
\]

(32)

Such a policy exists by (10) and (11) of Lemma 1. Similarly, let \( r_2 \) be any value that satisfies \( 0 \leq r_2 \leq r_{\text{max}} \), and let \( P_{\text{tran}}^*(t) \) be the stationary randomized power allocation policy that yields:

\[
\mathbb{E} \{ C(P_{\text{tran}}^*(t), S(t)) \} = r_2
\]

(33)

\[
\mathbb{E} \{ P_{\text{tran}}^*(t) \} = g^*(r_2)
\]

(34)

Such a policy exists by (14) and (15). Further, the stationary randomized policies of (31)-(34) base decisions only on the current \( A(t) \) and \( S(t) \) states, which are i.i.d. over slots (and are hence independent of the current queue backlog \( U(t) \)). Thus, the expectations of (31)-(34) are the same when conditioned on \( U(t) \). Plugging (31)-(34) into the right hand side of (30) thus yields:

\[
\Delta(U(t)) + V \mathbb{E} \{ P_{\text{comp}}(t) + P_{\text{tran}}(t) \mid U(t) \} \leq B - U(t)(r_2 - r_1) + V (h^*(r_1) + g^*(r_2))
\]

(35)

The above inequality holds for all \( r_1 \) and \( r_2 \) that satisfy \( r_{\text{min}} \leq r_1 \leq b \mathbb{E} \{ A(t) \} \) and \( 0 \leq r_2 \leq r_{\text{max}} \). Let \( r_1 = r_2 = r^* \), where \( r^* \) is the value of \( r \) that optimizes the problem in (16) and (17) of Theorem 1, so that \( P_{av}^* = h^*(r^*) + g^*(r^*) \). Plugging into (35), we have:

\[
\Delta(U(t)) + V \mathbb{E} \{ P_{\text{comp}}(t) + P_{\text{tran}}(t) \mid U(t) \} \leq B + VP_{av}^*
\]
Using the above drift inequality in the Lyapunov Drift Lemma (Lemma 2) and defining \( \alpha(t) = VP_{\text{comp}}(t) + VP_{\text{tran}}(t) \) and \( \beta(t) = B + VP^* \) yields \( \mathcal{P}_{\text{tot}} \leq P^* + B/V \), proving equation (20) of Theorem 2.

Now choose \( r_1 = r_{\text{min}} \) and \( r_2 = r_{\text{max}} \). Plugging into (35) and noting that \( P_{\text{comp}}(t) \geq 0 \) and \( P_{\text{tran}}(t) \geq 0 \) gives:

\[
\Delta(U(t)) \leq B - U(t)(r_{\text{max}} - r_{\text{min}}) + V(h^*(r_{\text{min}}) + g^*(r_{\text{max}}))
\]

Using the above drift inequality in the Lyapunov Drift Lemma (Lemma 2) and defining \( \alpha(t) = U(t)(r_{\text{max}} - r_{\text{min}}) \) and \( \beta(t) = B + V(h^*(r_{\text{min}}) + g^*(r_{\text{max}})) \), we have:

\[
\limsup_{t \to \infty} \frac{1}{t} \sum_{r=0}^{t-1} E\{U(r)\} \leq \frac{B + V(h^*(r_{\text{min}}) + g^*(r_{\text{max}}))}{(r_{\text{max}} - r_{\text{min}})}
\]

The result of (21) follows because \( g^*(r_{\text{max}}) \leq P_{\text{max}} \) and \( h^*(r_{\text{min}}) \leq \phi_{\text{max}} \). This completes the proof of Theorem 2.

5. A SIMPLE DELAY IMPROVEMENT

Here we present a simple improvement to the transmission algorithm that can decrease queue backlog while maintaining the exact same average power performance specified in Theorem 2. First observe that the performance theorem (Theorem 2) and the Lyapunov Drift Lemma (Lemma 2) both specify time average behavior that is independent of the initial queue backlog. Indeed, the affects of the initial condition are transient and decay over time. Now suppose the transmission algorithm has the following property:

Property 1: There exists a finite constant \( U_{\text{thresh}} \geq 0 \) such that if \( U(0) \geq U_{\text{thresh}} \), then \( U(t) \geq U_{\text{thresh}} \) for all time \( t \geq 0 \).

Thus, Property 1 says that if the queue has an initial condition of at least \( U_{\text{thresh}} \) bits, then it will never fall below this threshold of bits. Clearly Property 1 always holds with \( U_{\text{thresh}} = 0 \). In any system where Property 1 holds for some constant \( U_{\text{thresh}} > 0 \), then the first \( U_{\text{thresh}} \) bits in the queue are just acting as a place holder to make \( U(t) \) large enough to properly affect the stochastic optimization.

5.1 Example Showing that \( U_{\text{thresh}} > 0 \) is Typical

Suppose there is a finite constant \( \beta_{\text{max}} \) such that:

\[ C(P, S) \leq \beta_{\text{max}} P \quad \text{for all } P \in \mathcal{P} \text{ and all } S \in \mathcal{S} \]

For example, if the transmission rate function \( C(P, S) \) is differentiable with respect to \( P \), then \( \beta_{\text{max}} \) can be defined as the largest derivative with respect to \( P \) over all possible channel states. Because the algorithm chooses \( P_{\text{tran}}(t) \) every slot as the maximizer of \( U(t)C(P, S(t)) - VP \) over all \( P \in \mathcal{P} \), it is clear that \( P_{\text{tran}}(t) = 0 \) whenever \( U(t)\beta_{\text{max}} < V \). Indeed, we have for any channel state \( S(t) \):

\[ U(t)C(P, S(t)) - VP \leq [U(t)\beta_{\text{max}} - V]P \]

which is maximized only by \( P = 0 \) if \( U(t)\beta_{\text{max}} < V \). Therefore, if \( U(t) < V/\beta_{\text{max}} \), we have \( P_{\text{tran}}(t) = 0 \) and hence \( \mu(t) = C(P_{\text{tran}}(t), S(t)) = 0 \), so that the queue
backlog cannot further decrease. Define $\mu_{\text{max}}$ as the largest possible transmission rate during a single slot (equal to the maximum of $C(P_{\text{max}}, S)$ over all $S \in S$). It follows that Property 1 holds in this example with:

$$U_{\text{thresh}} \triangleq \max \left[ 0, \frac{V}{\beta_{\text{max}}} - \mu_{\text{max}} \right]$$

(36)

The value $U_{\text{thresh}}$ determines the number of place holder bits required in the system.

5.2 Delay Improvement Via Place Holder Bits

If Property 1 holds for $U_{\text{thresh}} > 0$, performance can be improved in the following way: With $U(t)$ being the actual queue backlog, define the place-holder backlog $\hat{U}(t)$ as follows:

$$\hat{U}(t) \triangleq U(t) + U_{\text{thresh}}$$

The value $\hat{U}(t)$ can be viewed as backlog that is equal to the actual backlog plus $U_{\text{thresh}}$ “fake bits.” Now assume that $U(0) = 0$, but implement the Dynamic Compression and Transmission Algorithm using the place-holder backlog $\hat{U}(t)$ everywhere, instead of the actual queue backlog. That is, choose $k(t) \in K$ and $P_{\text{tran}}(t) \in P$ as follows:

$$k(t) = \arg \min_{k \in K} [\hat{U}(t)m(A(t), k) + V\phi(A(t), k)]$$

$$P_{\text{tran}}(t) = \arg \max_{P \in P} [\hat{U}(t)C(P, S(t)) - VP]$$

With this implementation, we have $\hat{U}(0) = U_{\text{thresh}}$, and so $\hat{U}(t) \geq U_{\text{thresh}}$ for all $t$ (by Property 1). It follows that any transmission decisions never take $\hat{U}(t)$ lower than $U_{\text{thresh}}$, which is equivalent to saying that all transmission decisions transmit only actual data (so that $U(t) \geq 0$), rather than fake data. The resulting decisions are the same as those in a system with initial backlog of $U_{\text{thresh}}$, which yields the same $O(1/V)$ energy performance guarantee as before, and yields the same $O(V)$ time average backlog guarantee for the $\hat{U}(t)$ backlog. However, the actual queue backlog is exactly $U_{\text{thresh}}$ bits lower than $\hat{U}(t)$ at every instant of time, and so the time average backlog is also exactly $U_{\text{thresh}}$ bits lower. Thus, this simple improvement yields less actual queue backlog in the system, without any loss in performance. This improvement does not change the $[O(1/V), O(V)]$ tradeoff relation (it simply multiplies the $O(V)$ congestion bound by a smaller coefficient), but can yield practical backlog and delay improvements for implementation purposes.

6. DISTORTION CONSTRAINED DATA COMPRESSION

In the previous sections, we have assumed that all compression options $k \in K$ are either lossless, or that the distortion they introduce is acceptable. In this section, we expand the model to allow each compression option to have its own distortion properties. Specifically, let $D(t)$ be a non-negative real number that represents a measure of the distortion introduced at time $t$ due to compression. We assume

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4The $U_{\text{thresh}}$ value in (36) satisfies Property 1, but is not necessarily the largest value that satisfies this property. Performance can be improved if a larger value that satisfies Property 1 can be found, which is often possible for concave rate-power curves defined over a continuous interval.
that this is a random function of $A(t)$ (the number of packets compressed) and $k(t)$ (the compression option), and that this function is stationary and independent over all slots with the same $A(t)$ and $k(t)$. Define $d(a, k)$ as the expected distortion function:

$$d(a, k) \triangleq \mathbb{E}\{D(t) \mid A(t) = a, k(t) = k\}$$

We assume the maximum second moment of distortion is bounded by some finite constant $\delta^2$:

$$\delta^2 \triangleq \max_{a \in \{1, \ldots, N\}, k \in \mathcal{K}} \mathbb{E}\{D(t)^2 \mid A(t) = a, k(t) = k\}$$

Assuming that distortion is additive, the goal in this section is to make joint transmission and compression actions to minimize time average power expenditure subject to queue stability and subject to the constraint that time average distortion is bounded by a constant $d_{av}$:

$$\lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{D(\tau)\} \leq d_{av} \quad (37)$$

Let $\mathcal{K}$ now represent a set of extended compression options, which includes lossy compression options (and may also include the maximum distortion option of throwing away all data that arrives on slot $t$). Lossless compression options may still be available and yield $D(t) = 0$.

To ensure the distortion constraint (37) is satisfied, we introduce a distortion queue $X(t)$ that accumulates the total amount of distortion in excess of $d_{av}$. This is similar to the virtual power queue introduced in [Neely 2006] for ensuring average power constraints, and is an example of a virtual cost queue from [Georgiadis et al. 2006]. Specifically, the $X(t)$ queue is implemented purely in software. It is initialized to $X(0) = 0$, and is changed from slot to slot according to the following dynamics:

$$X(t + 1) = \max[X(t) - d_{av}, 0] + D(t) \quad (38)$$

where $D(t)$ is the random amount of distortion introduced on slot $t$ (observed at the end of the compression operation). Stabilizing the $X(t)$ queue ensures the time average rate of the $D(t)$ “arrivals” is less than or equal to $d_{av}$, which is equivalent to the distortion constraint (37).

6.1 The Distortion-Constrained Algorithm

The queue backlog $U(t)$ evolves as before, according to the dynamics (3). The transmission algorithm that selects $P_{\text{tran}}(t) \in \mathcal{P}$ every slot is the same as before (equation (19)), and the new compression algorithm is given as follows:

**Distortion-Constrained Compression Algorithm:** Every slot $t$, observe the number of new packet arrivals $A(t)$ and the current queue backlogs $U(t)$ and $X(t)$, and choose compression option $k(t) \in \mathcal{K}$ as follows:

$$k(t) = \arg \min_{k \in \mathcal{K}} [U(t)m(A(t), k) + X(t)d(A(t), k) + V \phi(A(t), k)]$$
After the compression operation, observe the actual distortion $D(t)$ and update $X(t)$ according to (38).

We note that the distortion-constrained dynamic compression algorithm above is useful even in cases when the average power expenditure $\phi(A(t), k)$ due to compression is negligible. This is because compression can significantly save transmission power, although intelligent compression strategies are required to meet the distortion constraints.

6.2 Minimum Average Power with Distortion Constraints

Define $h^*(r)$ as the infimum time average power over all compression strategies that make decisions $k(t) \in K$ as a stationary and random function of the observed number of packets $A(t)$, subject to a time average output rate of the compressor that is at most $r$ bits/slot, and subject to a time average distortion of at most $d_{av}$. Let $r_{d,\min}$ represent the smallest possible time average output rate of the compressor that yields a time average distortion of at most $d_{av}$, optimized over all algorithms that choose $k(t) \in K$ as a stationary and random function of $A(t)$ (and hence independently of queue backlog). We assume that $r_{d,\min} < r_{\max}$, so that it is feasible to meet the distortion constraint. We further assume that it is possible to meet the distortion constraint with strict inequality while stabilizing the system. That is, we assume there is an $\epsilon > 0$ such that it is possible to achieve a time average distortion rate of $D(t) \leq d_{av} - \epsilon$ using a policy that chooses $k(t) \in K$ as a stationary and random function of $A(t)$, such that $E\{m(A(t), k(t))\} \leq r_{\max}$.

Recall that $g^*(r)$ is the infimum time average transmission power over all stationary randomized strategies $P_{\text{tran}}(t) \in \mathcal{P}$ that yield time average transmission rate at least $r$ bits/slot. Let $P^*_{\text{av}}$ represent the minimum time average power (summed over compression and transmission powers) required to stabilize the queueing system subject to the distortion constraint (37).

**Theorem 3.** (Distortion Constrained Minimum Average Power) The distortion-constrained minimum time average power $P^*_{\text{av}}$ is equal to the solution of the following optimization problem:

$$\begin{align*}
\text{Minimize:} & \quad h^*(r) + g^*(r) \\
\text{Subject to:} & \quad r_{d,\min} \leq r \leq \min[r_{\max}, bE\{A(t)\}]
\end{align*}$$

**Proof.** The proof is similar to the proof of Theorem 1 and is omitted for brevity. \qed

6.3 Lyapunov Analysis

Let $Z(t) \triangleq [U(t), X(t)]$ be the combined queue state, and define the Lyapunov function:

$$L(Z(t)) \triangleq \frac{1}{2}U(t)^2 + \frac{1}{2}X(t)^2$$

Define the Lyapunov drift:

$$\Delta(Z(t)) \triangleq E\{L(Z(t + 1)) - L(Z(t)) | Z(t)\}$$
Lemma 4. For any constant $V \geq 0$, the Lyapunov drift $\Delta(Z(t))$ satisfies the following for all $t$ and all $Z(t)$:

$$
\Delta(Z(t)) + V \mathbb{E} \{ P_{\text{tot}}(t) \mid Z(t) \} \leq C
$$

where the constant $C$ is given by:

$$
C \triangleq \frac{1}{2} \left[ d_{\text{av}}^2 + \delta^2 + \sigma^2 + \mathbb{E} \left\{ C(P_{\text{max}}, S(t))^2 \right\} \right]
$$

Proof. The proof is similar to the proof of Lemma 3 and is omitted for brevity. 

It can be seen that the Distortion Constrained Compression and Transmission Algorithm is designed to observe current queue backlogs $X(t)$, $U(t)$ and arrival and channel states $A(t)$, $S(t)$, and take control actions $k(t) \in K$, $P_{\text{tran}}(t) \in \mathcal{P}$ to minimize the right hand side of the drift bound given in Lemma 4.

Theorem 4. (Algorithm Performance with Distortion Constraint) Suppose $A(t)$ and $S(t)$ are i.i.d. over slots, and that $r_{d,\text{min}} < r_{\text{max}}$. For any control parameter $V > 0$, the Distortion Constrained Compression and Transmission Algorithm stabilizes the network and satisfies:

$$
\mathcal{P}_{\text{tot}} \leq P_{\text{av}}^* + \frac{C}{V} \quad (41)
$$

$$
\mathcal{U} \leq \frac{C + V(P_{\text{max}} + \phi_{\text{max}})}{(r_{\text{max}} - r_{d,\text{min}})} \quad (42)
$$

$$
\mathcal{D} \leq d_{\text{av}} \quad (43)
$$

where $\mathcal{U}$, $\mathcal{P}_{\text{tot}}$, and $\mathcal{D}$ represent lim sup time average expected queue backlog, total power, and distortion, and the constant $C$ is defined in (40).

Proof. See Appendix D.

Thus, the algorithm meets the time average distortion constraint, and the parameter $V$ can be used to push total time average power expenditure within $O(1/V)$ of the optimal $P_{\text{av}}^*$, with an $O(V)$ tradeoff in average queue congestion $\mathcal{U}$ and hence average delay. We note that an improved delay performance can be achieved by using $\hat{U}(t) = U(t) + U_{\text{thresh}}$ as a replacement for $U(t)$, as described in Section 5, with $U_{\text{thresh}}$ satisfying Property 1, such as the value given in (36).

7. Simulations

For simplicity, we consider simulations of the dynamic compression and transmission algorithm of Section 4 (with the simple improvement of Section 5), without treating distortion constraints. To begin, we first consider a system where the
optimal compression decision is trivial and does not require a stochastic optimization. Specifically, suppose that we have a system where all three of the following “Singularity Assumptions” hold:

—The channel is static, so that \( S(t) \) is the same for all \( t \).
—The rate-power curve is linear in power, so that \( C(P) = \alpha P \) for all \( P \in \mathcal{P} \), for some constant \( \alpha \).
—The raw data arrival rate is less than the maximum transmission rate, that is, \( b\mathbb{E}\{A(t)\} < \alpha P_{\text{max}} \).

In this simple case, the time average transmission power is directly proportional to the time average rate of bits transmitted, and so we do not require careful transmission decisions (all transmissions are equally energy efficient). Further, compression is not required for stability. It is easy to show in this special case that the exact minimum energy expenditure is achieved by the alternative algorithm of observing \( A(t) \) every slot \( t \) and choosing a compression option \( \hat{k}(t) \in \mathcal{K} \) as follows:

\[
\hat{k}(t) = \arg\min_{k \in \mathcal{K}} \left[ \phi(A(t), k) + \frac{1}{\alpha} m(A(t), k) \right]
\] (44)

and then transmitting whenever there is a sufficient amount of backlog to achieve an efficiency of \( \alpha \) bits/unit energy. That is, we simply choose the compression option that minimizes the sum of the total energy required to compress and transmit the bits. A similar observation is used in [Barr and Asanović 2003] in the study of compression energy ratios for popular algorithms. However, this \( \hat{k}(t) \) policy is fragile, in that its optimality strongly relies on all three of the above Singularity Assumptions. Our dynamic compression and transmission algorithm is an all-purpose algorithm that should work well for any system, including systems that satisfy the above Singularity Assumptions, as well as systems that do not.

### 7.1 Scenario I: Singularity Assumptions

We first consider a scenario where all three of the “Singularity Assumptions” hold. The channel is static with \( S(t) = \text{ON} \) for all time slots \( t \). The transmit power is constrained to two options \( \mathcal{P} = \{0, 1\} \) (we used normalized units of power). The rate-power curve is given by \( C(P = 1) = 2048 \) bits/slot, and \( C(P = 0) = 0 \). It is clear that the optimal transmission decision in this scenario is to transmit only when the queue size is greater than or equal to \( \mu_{\text{max}} = 2048 \) bits, so that all transmissions have efficiency \( \alpha = \mu_{\text{max}} \) bits per unit power.

The wireless link receives data from 8 different sensor units. The packet arrival process at each sensor is i.i.d. over slots and follows a Bernoulli distribution with the probability of an arrival \( p = \frac{1}{2} \). We fix the packet size, \( b \), to 256 bits. Hence, the arrival process for the wireless link, \( A(t) \), follows a Binomial distribution with parameters \( (8, \frac{1}{2}) \) with an average arrival rate of \( b\mathbb{E}\{A(t)\} = 1024 \) bits/slot. Note that in this case we have \( b\mathbb{E}\{A(t)\} < \mu_{\text{max}} \), and so compression is not needed for queue stability.

Two compression options are available to the link controller (\( \mathcal{K} = \{0, 1\} \)). For \( A(t) = a \) and \( k(t) = 1 \), the size of the data after compression, \( R(t) \), is uniformly distributed in \( \left[ \frac{2ab}{5}, \frac{3ab}{5} \right] \) and the compression power is uniformly distributed in
Hence, the average compressed output is $m(a, 1) = \frac{ab}{2}$ with an average power of $\phi(a, 1) = 0.5$. In this scenario, compression is energy-expensive compared to transmission, and, because compression is not required for stability, it is easy to see from (44) that transmitting all data without compression is optimal. Thus, the policy of (44) has $\hat{k}(t) = 0$ for all $t$, and transmits whenever $U(t) \geq 2048$, yielding an optimal average power $P_{tot} = bE\{A(t)\}/\mu_{max} = 0.5$.

We simulate our dynamic compression and transmission algorithm over $10^6$ slots, for various choices of the $V$ parameter. Fig. 2 shows that average power indeed converges to 0.5 as $V$ is increased. Fig. 2 also shows that, as expected, incorporating the simple delay improvement of Section 5 does not affect power expenditure. Fig. 3 shows the time average queue backlog versus $V$. For simulations without the delay improvement, the queue backlog increases linearly with $V$. The delay improvement uses $U_{thresh} = \max[V/\mu_{max} - \mu_{max}, 0]$, and reduces queue backlog (maintaining a relatively constant average backlog for $V \geq 5000$). The dashed horizontal line at $U = 1920$ bits (shown in Fig. 3) is the average queue size obtained by the $\hat{k}(t)$ policy that performs no compression and transmits only when $U(t) \geq \mu_{max}$.
7.2 Scenario II: Compression for Stability

We next consider the same scenario, but increase the raw data rate beyond $\mu_{max}$, so that compression is required for queue stability (this removes the third “Singularity Assumption”). However, the proper fraction of time to compress may be different for each observed $A(t)$ value, and in general it depends on the distribution of the arrival process $A(t)$. Our dynamic algorithm optimizes without this statistical knowledge, learning the correct actions for each observed $A(t)$ value.

Fig. 4 shows the increase in average power expenditure for our algorithm (with delay improvement) as the raw arrival rate increases. This raw arrival rate is increased by adjusting the packet size $b$ from 256 to 1024 (the parameter $V$ is fixed to $V = 10$ kbits$^2$/unit power, so that $U_{\text{thresh}} = \max[V/\mu_{max} - \mu_{max}, 0] \approx 2835$ bits, and the simulation time for each data point is five million slots). Also shown is the average power when there is no compression but when the same dynamic transmission strategy is used. For arrival rates $b\mathbb{E}\{A(t)\} > 1024$, compression is required for energy efficiency, and for $b\mathbb{E}\{A(t)\} > \mu_{max} = 2048$, compression is required for both energy efficiency and stability. For $V = 10$, our dynamic algorithm yields energy efficiency within roughly 0.4% of optimal for the rate region tested. For example, when the raw arrival rate is 3400, the optimum is $P_{av}^* = 1.310$ (achievable by compressing whenever $A(t) \geq 3$), and our algorithm achieves $P_{av} = 1.314$. Because compression reduces data on average by a factor of 2, the maximum raw arrival rate that can be stably supported is $2\mu_{max}$. When $b\mathbb{E}\{A(t)\} \geq 2\mu_{max}$, our algorithm learns to compress all data, leading to an average power expenditure of 1.5 (0.5 power units for compression, plus 1 unit for transmission).

Fig. 5 shows the change in average queue size as the raw data arrival rate is increased. Without any data compression, the average queue backlog grows to infinity as the raw data rate approaches the vertical asymptote 2048 bits/slot. With the data compression option the queue size remains quite flat beyond this threshold, increasing at a new vertical asymptote at 4096 bits/slot.
7.3 Scenario III: Nonlinear Rate-Power Curve

For this scenario, the Bernoulli arrival process is the same as in Scenario I (Section 7.1), with packet size \( b = 256 \) bits. However, we make the following changes:

— The rate-power curve is non-linear in power with \( C(P) = \alpha \log(1 + \beta P) \) for transmit power \( P \), where \( P \) is any real number in the interval \( 0 \leq P \leq P_{\text{max}} \).

— The raw data arrival rate is less than the maximum transmission rate, i.e. \( \mu \mathbb{E}\{A(t)\} = 1024 < \mu_{\text{max}} \triangleq \alpha \log(1 + \beta P_{\text{max}}) \).

— The compressed data was taken from a trace of experimental data from [Paek et al. 2006], and was compressed using the zlib compression library [www].

A single compression option \( (\mathcal{K} = \{0, 1\}) \) is available at the transmitting node. For \( k = 1 \), we have \( m(a, k) = \frac{a^3}{10} \) for \( A(t) = a \leq 3 \) packets, and \( m(a, k) = \frac{a}{15} \) for \( A(t) = a > 3 \). These average compression ratios were obtained from the experimental data in [Paek et al. 2006] using the zlib data compression library [www]. The work in [Paek et al. 2006] considers a wireless sensor network where each node senses vibrations of a large suspension bridge. We assume the power expenditure during compression, \( \phi(a, 1) \), is 5 units for \( A(t) = a \leq 3 \) and 8 units for \( A(t) = a > 3 \). For transmission, we use \( P_{\text{max}} = 750 \) power units, \( \alpha = 1060 \), and \( \beta = 1/16 \), so that \( \mu_{\text{max}} = C(P_{\text{max}}) \approx 4100 \) bits/slot.

Fig. 6 shows that the average power consumption for our dynamic algorithm converges to 22.21. As expected, the power curves are almost identical with and without delay improvement. Fig. 6 also shows the average power expenditure converges to 26.042 if all data is transmitted uncompressed (but still using our transmission strategy of (19)). Our dynamic compression algorithm yields an energy savings between 15 and 25 percent across the \( V \) range tested, as compared to sending all the data uncompressed. Fig. 7 shows how the average queue backlog increases with \( V \). The plot without delay improvement uses \( U_{\text{thresh}} = 0 \) (which is also the value of \( U_{\text{thresh}} \) that would be given by (36) for the \( V \) range tested),

\[ \text{Fig. 5. Avg. Queue size (bits) vs. Raw Arrival Rate.} \]
8. CONCLUSION

This paper presents a dynamic decision technique for joint compression and transmission in a wireless node, using results of stochastic network optimization. The approach allows total average power expenditure to be pushed arbitrarily close to optimal, with a corresponding delay (and queue congestion) tradeoff. The resulting compression and transmission algorithms are simple to implement and operate well in a variety of settings. We believe this approach will also be useful for management of compression and sensing in multi-hop networks, where energy, stability, and delay issues will become increasingly important in future applications.
Appendix A – Proof of Lemma 1

Proof. (Lemma 1) The function $h^*(r)$ is defined in terms of an infimum of $\mathbb{E}\{R_{\text{comp}}(t)\}$ over all stationary randomized policies that yield $\mathbb{E}\{R(t)\} \leq r$. It follows that there exists an infinite sequence of stationary randomized policies, indexed by integers $i \in \{1, 2, \ldots\}$, having expectations $\mathbb{E}\{R^{(i)}(t)\}$ and $\mathbb{E}\{P_{\text{comp}}^{(i)}(t)\}$ that satisfy:

$$\mathbb{E}\{R^{(i)}(t)\} \leq r \text{ for all } i \in \{1, 2, \ldots\} \quad (45)$$

$$\lim_{i \to \infty} \mathbb{E}\{P_{\text{comp}}^{(i)}(t)\} = h^*(r) \quad (46)$$

However, each policy $i$ is defined in terms of a collection of probabilities $(\gamma_{a,k}^{(i)})$ for $a \in \{0, 1, \ldots, N\}$ and $k \in K$. This collection of probabilities can be viewed as a finite dimensional vector that is contained in a compact set $\Omega$ defined by the constraints (8)-(9). The compact set $\Omega$ contains its limit points, and hence the infinite sequence $\{(\gamma_{a,k}^{(i)})\}_{i=1}^{\infty}$ contains a convergent subsequence that converges to a point $(\gamma_{a,k}^*) \in \Omega$. This point is a vector of probabilities that define a stationary randomized algorithm with expectations $\mathbb{E}\{R^*(t)\}$ and $\mathbb{E}\{P_{\text{comp}}^*(t)\}$. Now recall that a general stationary randomized algorithm yields expectations $\mathbb{E}\{R(t)\}$ and $\mathbb{E}\{P_{\text{comp}}(t)\}$ that can be expressed as linear (and hence continuous) function of the probabilities $(\gamma_{a,k})$, as shown in the left hand sides of equations (6) and (7). Hence, the properties (45) and (46) are preserved in the limit, so that $\mathbb{E}\{R^*(t)\} \leq r$ and $\mathbb{E}\{P_{\text{comp}}^*(t)\} = h^*(r)$.

If $\mathbb{E}\{R^*(t)\} = r$, then we are done. Else, we have $\mathbb{E}\{R^*(t)\} < r \leq b\mathbb{E}\{A(t)\}$ (recall that $r \leq b\mathbb{E}\{A(t)\}$ by assumption in the statement of Lemma 1). Hence, $r = \theta \mathbb{E}\{R^*(t)\} + (1 - \theta)b\mathbb{E}\{A(t)\}$ for some probability $\theta$. Note that the 0-power algorithm of no compression yields an expected compression output of exactly $b\mathbb{E}\{A(t)\}$. It follows that defining $R^*(t)$ as the stationary randomized policy that chooses $R^*(t)$ with probability $\theta$ and chooses no compression with probability $(1 - \theta)$ yields $\mathbb{E}\{R^*(t)\} = r$. This policy $R^*(t)$ cannot use more power than policy $R^*(t)$, and hence $\mathbb{E}\{P_{\text{comp}}^*(t)\} \leq h^*(r)$. But we also have $h^*(r) \leq \mathbb{E}\{P_{\text{comp}}^*(t)\}$, because $h^*(r)$ is defined as the infimum average power over all stationary randomized policies that yield a compressor output rate of at most $r$. \qed

Appendix B – Proof of Theorem 1

Here we prove Theorem 1. Consider any policy that stabilizes the queue, and let $k(t)$ and $P_{\text{tran}}(t)$ be the resulting compression and transmission power decisions chosen over time (where $k(t) \in K$ and $P_{\text{tran}}(t) \in P$ for all $t$). Let $R(t) = \Psi(A(t), k(t))$ be the resulting bit output process from the compressor, and let $P_{\text{comp}}(t)$ be the resulting compression power expenditure process. Let $\mu(t) = C(P_{\text{tran}}(t), S(t))$ be the transmission rate process. We want to show that:

$$\limsup_{t \to \infty} \frac{1}{t} \sum_{\tau = 0}^{t-1} \mathbb{E}\{P_{\text{comp}}(\tau) + P_{\text{tran}}(\tau)\} \geq P_{av}^* \quad (47)$$

where $P_{av}^*$ is defined in Theorem 1. We have two preliminary lemmas.
Lemma 5. Suppose there are constants $r$ and $\mathcal{P}_c$ together with an infinite sequence of times $\{t_i\}_{i=1}^\infty$ such that:

$$
\lim_{t_i \to \infty} \frac{1}{t_i} \sum_{\tau=0}^{t_i-1} \mathbb{E}\{R(\tau)\} = r \tag{48}
$$

$$
\lim_{t_i \to \infty} \frac{1}{t_i} \sum_{\tau=0}^{t_i-1} \mathbb{E}\{P_{\text{comp}}(\tau)\} = \mathcal{P}_c \tag{49}
$$

Then $\mathcal{P}_c \geq h^*(r)$.

Proof. The proof is given in Appendix C. $\square$

Lemma 6. Suppose there are constants $\mu$ and $\mathcal{P}_t$ together with an infinite sequence of times $\{t_i\}_{i=1}^\infty$ such that:

$$
\lim_{t_i \to \infty} \frac{1}{t_i} \sum_{\tau=0}^{t_i-1} \mathbb{E}\{P_{\text{tran}}(\tau)\} = \mathcal{P}_t \tag{50}
$$

$$
\lim_{t_i \to \infty} \frac{1}{t_i} \sum_{\tau=0}^{t_i-1} \mathbb{E}\{\mu(\tau)\} = \mu \tag{51}
$$

Then $\mathcal{P}_t \geq g^*(\mu)$.

Proof. The proof is given in Appendix C. $\square$

Now define $\mathcal{P}_{\text{tot}}$ as the limsup total power expenditure given by the left hand side of inequality (47). Let $t_i$ be an infinite subsequence of times over which the limsup is achieved, so that:

$$
\lim_{t_i \to \infty} \frac{1}{t_i} \sum_{\tau=0}^{t_i-1} \mathbb{E}\{P_{\text{comp}}(\tau) + P_{\text{tran}}(\tau)\} = \mathcal{P}_{\text{tot}} \tag{52}
$$

Now define:

$$
\overline{R}(t) = \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{R(\tau)\} \quad , \quad \overline{P}_{\text{comp}}(t) = \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{P_{\text{comp}}(\tau)\}
$$

$$
\overline{P}_{\text{tran}}(t) = \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{P_{\text{tran}}(\tau)\} \quad , \quad \overline{\mu}(t) = \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{\mu(\tau)\}
$$

and note that for all timeslots $t$ we have:

$$
0 \leq \overline{R}(t) \leq b\mathbb{E}\{A(t)\} \quad , \quad 0 \leq \overline{P}_{\text{comp}}(t) \leq \phi_{\max}
$$

$$
0 \leq \overline{P}_{\text{tran}}(t) \leq P_{\max} \quad , \quad 0 \leq \overline{\mu}(t) \leq r_{\max}
$$

It follows that $(\overline{R}(t_i), \overline{P}_{\text{comp}}(t_i), \overline{P}_{\text{tran}}(t_i), \overline{\mu}(t_i))$ can be viewed as an infinite sequence contained in a four dimensional compact set, and thus has a convergent subsequence. Let $\{t_i\}$ represent the convergent subsequence of times, so that there
exist constants \( r, \mathcal{P}_c, \mathcal{P}_t, \) and \( \mathcal{P}_g \) such that:

\[
\lim_{t_i \to \infty} \frac{1}{t_i} \sum_{\tau=0}^{t_i-1} \mathbb{E}\{R(\tau)\} = r
\]

\[
\lim_{t_i \to \infty} \frac{1}{t_i} \sum_{\tau=0}^{t_i-1} \mathbb{E}\{P_{\text{comp}}(\tau)\} = \mathcal{P}_c
\]

\[
\lim_{t_i \to \infty} \frac{1}{t_i} \sum_{\tau=0}^{t_i-1} \mathbb{E}\{P_{\text{tran}}(\tau)\} = \mathcal{P}_t
\]

\[
\lim_{t_i \to \infty} \frac{1}{t_i} \sum_{\tau=0}^{t_i-1} \mathbb{E}\{\mu(\tau)\} = \mathcal{P}_g
\]

Furthermore, because \( \{t_i\}_{i=1}^\infty \) is an infinite subsequence of the original sequence \( \{\tilde{t}_i\} \), we have from (52) that \( \mathcal{P}_c + \mathcal{P}_t = \mathcal{P}_{\text{tot}} \). From Lemmas 5 and 6 we must have the following:

\[
\mathcal{P}_c \geq h^*(r)
\]

\[
\mathcal{P}_t \geq g^*(\mathcal{P}_g)
\]

Therefore:

\[
\mathcal{P}_{\text{tot}} \geq h^*(r) + g^*(\mathcal{P}_g)
\]  

We now use the fact that queue \( U(t) \) is stable. It is known that a stable queue must satisfy (see [Neely 2006] [Georgiadis et al. 2006]):

\[
\lim_{t \to \infty} \frac{\mathbb{E}\{U(t)\}}{t} = 0
\]  

However, note that for all times \( t_i \) we have:

\[
U(t_i) \geq \sum_{\tau=0}^{t_i-1} R(\tau) - \sum_{\tau=0}^{t_i-1} \mu(\tau)
\]

This is true because the total unfinished work in the system at time \( t_i \) is no more than the total bit arrivals minus the maximum possible bit departures during the interval from 0 to \( t_i - 1 \). Therefore (taking an expectation and dividing by \( t_i \)):

\[
\frac{\mathbb{E}\{U(t_i)\}}{t_i} \geq \frac{1}{t_i} \sum_{\tau=0}^{t_i-1} \mathbb{E}\{R(\tau)\} - \frac{1}{t_i} \sum_{\tau=0}^{t_i-1} \mathbb{E}\{\mu(\tau)\}
\]

Taking a limit of the above expression as \( t_i \to \infty \), and using (54) yields \( 0 \geq r - \mathcal{P}_g \). Therefore, queue stability implies that \( r \leq \mathcal{P}_g \). Because the function \( g^*(r) \) is non-decreasing, it follows that \( g^*(r) \leq g^*(\mathcal{P}_g) \). Using this fact in (53) yields:

\[
\mathcal{P}_{\text{tot}} \geq h^*(r) + g^*(r)
\]

Furthermore, it is not difficult to show that the values of \( r \) and \( \mathcal{P}_g \) must satisfy \( r_{\min} \leq r \leq b\mathbb{E}\{A(t)\} \) and \( 0 \leq \mathcal{P}_g \leq r_{\max} \). Because \( r \leq \mathcal{P}_g \), it follows that:

\[
r_{\min} \leq r \leq \min\{r_{\max}, b\mathbb{E}\{A(t)\}\}
\]  

\[
26
\]
Therefore, the value of \( h^*(r) + g^*(r) \) is greater than or equal to the minimum value of this quantity, minimized over all \( r \) that satisfy the constraint (55), which is the definition of \( P^*_\text{av} \). Therefore:

\[
\mathbf{P}_{\text{tot}} \geq h^*(r) + g^*(r) \geq P^*_{\text{av}}
\]

This proves Theorem 1.

Appendix C – Proof of Lemmas 5 and 6

Proof. (Lemma 5) Here we prove Lemma 5. Suppose that (48) and (49) hold. For all timeslots \( t \), we have (by iterated expectations):

\[
\mathbb{E}\{R(t)\} = \mathbb{E}\{\mathbb{E}\{R(t) \mid A(t), k(t)\}\}\]

\[
= \mathbb{E}\{m(A(t), k(t))\}
\]

Similarly, we have \( \mathbb{E}\{P_{\text{comp}}(t)\} = \mathbb{E}\{\phi(A(t), k(t))\} \) for all \( t \). The equations (48) and (49) thus become:

\[
\lim_{t_i \to \infty} \frac{1}{t_i} \sum_{\tau=0}^{t_i-1} \mathbb{E}\{m(A(\tau), k(\tau))\} = r
\]

\[
\lim_{t_i \to \infty} \frac{1}{t_i} \sum_{\tau=0}^{t_i-1} \mathbb{E}\{\phi(A(\tau), k(\tau))\} = \mathbf{P}_c
\]

For any time \( t \) we have:

\[
\frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{m(A(\tau), k(\tau))\}
\]

\[
= \sum_{a=0}^{N} \sum_{k \in \mathcal{K}} \frac{1}{t} \sum_{\tau=0}^{t-1} m(a, k)p_A(a)Pr[k(\tau) = k \mid A(\tau) = a]
\]

\[
= \sum_{a=0}^{N} \sum_{k \in \mathcal{K}} m(a, k)p_A(a)\gamma_{a,k}(t)
\]

where we define probabilities \( (\gamma_{a,k}(t)) \) as follows:

\[
\gamma_{a,k}(t) \Delta \frac{1}{t} \sum_{\tau=0}^{t-1} Pr[k(\tau) = k \mid A(\tau) = a]
\]

Similarly, for any time \( t \) we have:

\[
\frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{\phi(A(\tau), k(\tau))\} = \sum_{a=0}^{N} \sum_{k \in \mathcal{K}} \phi(a, k)p_A(a)\gamma_{a,k}(t)
\]
It follows from (56) and (57) that:

\[
\lim_{t_i \to \infty} \sum_{a=0}^{N} \sum_{k \in K} p_A(a)m(a,k)\gamma_{a,k}(t_i) = r \tag{58}
\]

\[
\lim_{t_i \to \infty} \sum_{a=0}^{N} \sum_{k \in K} p_A(a)\phi(a,k)\gamma_{a,k}(t_i) = P_c \tag{59}
\]

It is straightforward to show that the probabilities \((\gamma_{a,k}(t))\) satisfy the following constraints for all \(t\):

\[
\gamma_{a,k}(t) \geq 0 \quad \text{for all } a, k \tag{60}
\]

\[
\sum_{k \in K} \gamma_{a,k}(t) = 1 \quad \text{for all } a \tag{61}
\]

The above constraints imply that \((\gamma_{a,k}(t))\) can be viewed as a vector of values contained in a finite dimensional compact set for all \(t\). It follows that \(\{(\gamma_{a,k}(t_i))\}\) forms an infinite sequence of probability vectors contained in a compact set, and so there must exist a convergent subsequence of times \(\{t'_i\}\) for which \(\{(\gamma_{a,k}(t'_i))\}\) converges to a point \((\gamma^*_{a,k})\) contained in the set. Therefore:

\[
\gamma^*_{a,k} \geq 0 \quad \text{for all } a, k \tag{62}
\]

\[
\sum_{k \in K} \gamma^*_{a,k} = 1 \quad \text{for all } a \tag{63}
\]

\[
\sum_{a=0}^{N} \sum_{k \in K} p_A(a)m(a,k)\gamma^*_{a,k} = r \tag{64}
\]

\[
\sum_{a=0}^{N} \sum_{k \in K} p_A(a)\phi(a,k)\gamma^*_{a,k} = P_c \tag{65}
\]

where (62) and (63) follow because \((\gamma^*_{a,k})\) is a limit point of the compact set defined by (60) and (61) and hence is an element of that set. Equalities (64) and (65) follow because \(\{t'_i\}\) is an infinite sequence of the original sequence of times \(\{t_i\}\), and hence the same limits in (58) and (59) are preserved when taken over this subsequence.

Because \((\gamma^*_{a,k})\) and \(P_c\) satisfy (62)-(65), these values define a particular solution for the constraints (6)-(9) of the optimization problem of Definition 1 in Section 3. Therefore, \(P_c\) is greater than or equal to the infimum value of power, infimized over all solutions that satisfy these constraints, which is defined as \(h^*(r)\). That is, \(P_c \geq h^*(r)\). This completes the proof of Lemma 5. \(\square\)

Proof. (Lemma 6) Here we prove Lemma 6. Suppose that (50) and (51) hold.
Similar to the proof of Lemma 5, we can show that for any timeslot $t$:

$$\frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{\mu(\tau)\} = \sum_{s \in \mathcal{S}} \pi_s \overline{\mu}_s(t)$$  (66)

$$\frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{P_{\text{tran}}(\tau)\} = \sum_{s \in \mathcal{S}} \pi_s \overline{P}_s(t)$$  (67)

where $\overline{\mu}_s(t)$ and $\overline{P}_s(t)$ are defined for all $s \in \mathcal{S}$ as follows:

$$\overline{\mu}_s(t) \triangleq \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{C(P_{\text{tran}}(\tau), s) \mid S(t) = s\}$$

$$\overline{P}_s(t) \triangleq \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{P_{\text{tran}}(\tau) \mid S(t) = s\}$$

For each timeslot $t$ and each channel state $s \in \mathcal{S}$, the values $(\overline{\mu}_s(t), \overline{P}_s(t))$ defined above are in the convex hull of the set $\Omega_s$ defined below:

$$\Omega_s \triangleq \{(\mu, p) \mid p \in \mathcal{P} \text{ and } \mu = C(p, s)\}$$

The set $\Omega_s$ is 2-dimensional. It follows by Carathéodory’s theorem [Bertsekas et al. 2003] that any element $(\overline{\mu}_s(t), \overline{P}_s(t))$ contained in the convex hull of $\Omega_s$ can be expressed as a convex combination of at most 3 elements of $\Omega_s$. Thus, there exist powers $P_{s,z}(t) \in \mathcal{P}$ and probabilities $\alpha_{s,z}(t)$ such that:

$$(\overline{\mu}_s(t), \overline{P}_s(t)) = \sum_{z=1}^{3} \alpha_{s,z}(t)(C(P_{s,z}(t), s), P_{s,z}(t))$$  (68)

where $\sum_{z=1}^{3} \alpha_{s,z}(t) = 1$ for all $s, t$.

Using (68) and (66), (67), the limit equations of (50) and (51) become:

$$\lim_{t_i \to \infty} \sum_{s \in \mathcal{S}} \sum_{z=1}^{3} \pi_s \alpha_{s,z}(t_i) C(P_{s,z}(t_i), s) = \overline{\mu}$$

$$\lim_{t_i \to \infty} \sum_{s \in \mathcal{S}} \sum_{z=1}^{3} \pi_s \alpha_{s,z}(t_i) P_{s,z}(t_i) = \overline{P}_t$$

It follows that for any $\epsilon > 0$, there exists a stationary randomized policy for choosing $P_{\text{tran}}(t)$ as a random function of the observed channel state $S(t)$ such that:

$$\mathbb{E}\{C(P_{\text{tran}}(t), S(t))\} \geq \overline{\mu} - \epsilon$$

$$\mathbb{E}\{P_{\text{tran}}(t)\} \leq \overline{P}_t + \epsilon$$

This stationary policy can be modified to create another stationary randomized policy that has average transmission rate greater than or equal to $\overline{\mu}$ simply by independently choosing $P_{\text{tran}}(t) = P_{\text{max}}$ every timeslot with some small probability. Thus, for any value $\delta > 0$, there exists a stationary randomized policy for choosing
$P_{\text{tran}}(t)$ that yields:

$$
\mathbb{E}\{C(P_{\text{tran}}(t), S(t))\} \geq \bar{P} \\
\mathbb{E}\{P_{\text{tran}}(t)\} \leq \bar{P}_t + \delta
$$

It follows that $\mathbb{E}\{P_{\text{tran}}(t)\}$ in the above policy satisfies $\mathbb{E}\{P_{\text{tran}}(t)\} \geq g^*(\bar{P})$, because $g^*(\bar{P})$ is defined as the smallest average power over the class of stationary randomized algorithms that support an average transmission rate of at least $\bar{P}$ (see Definition 2 in Section 3). Therefore:

$$
g^*(\bar{P}) \leq \mathbb{E}\{P_{\text{tran}}(t)\} \leq \bar{P}_t + \delta
$$

The above inequality holds for all $\delta > 0$, and so $g^*(\bar{P}) \leq \bar{P}_t$, which completes the proof of Lemma 6.

Appendix D – Proof of Theorem 4

Here we prove that the Distortion Constrained Compression and Transmission Algorithm yields performance as given in Theorem 4. Because the algorithm observes the current network state and makes control decisions $k(t) \in \mathcal{K}$, $P_{\text{tran}}(t) \in \mathcal{P}$ that minimize the right hand side of the drift bound given in Lemma 4, we have:

$$
\Delta(Z(t)) + V \mathbb{E}\{P_{\text{tot}}(t) \mid Z(t)\} \leq C \\
- U(t) \mathbb{E}\{C(P_{\text{tran}}^*(t), S(t)) \mid Z(t)\} \\
+ U(t) \mathbb{E}\{m(A(t), k^*(t)) \mid Z(t)\} \\
- X(t) \mathbb{E}\{d_\text{av} - d(A(t), k^*(t)) \mid Z(t)\} \\
+ V \mathbb{E}\{P_{\text{tran}}^*(t) + \phi(A(t), k^*(t)) \mid Z(t)\} \\
$$

(69)

where $k^*(t) \in \mathcal{K}$ and $P_{\text{tran}}^*(t) \in \mathcal{P}$ are any other feasible control actions for slot $t$. We obtain bounds on $\mathcal{U}$, $\mathcal{P}_{\text{tot}}$, and $\mathcal{D}$ using three different $k^*(t)$ and $P_{\text{tran}}^*(t)$ policies.

$-(\mathcal{U})$ Analysis: Let $P_{\text{tran}}^*(t) = P_{\text{max}}$, and let $k^*(t)$ be the stationary randomized policy that makes decisions independently of the current queue state, and yields the minimum output rate $r_{d_{\text{min}}}$ from the compressor, subject to the distortion constraint:

$$
\mathbb{E}\{m(A(t), k^*(t))\} = r_{d_{\text{min}}} \\
\mathbb{E}\{d(A(t), k^*(t))\} \leq d_{\text{av}}
$$

Plugging this into (69) yields:

$$
\Delta(Z(t)) + V \mathbb{E}\{P_{\text{tot}}(t) \mid Z(t)\} \leq \\
C - U(t)[r_{\text{max}} - r_{d_{\text{min}}} + V[P_{\text{max}} + \phi_{\text{max}}]
$$

and hence:

$$
\Delta(Z(t)) \leq C - U(t)[r_{\text{max}} - r_{d_{\text{min}}} + V[P_{\text{max}} + \phi_{\text{max}}]
$$

Using this drift inequality directly in the Lyapunov Drift Lemma (Lemma 2) yields the bound on $\mathcal{U}$ given in (42).
(Analysis): Let \( P_{\text{tran}}^*(t) \) and \( k^*(t) \) be the stationary randomized algorithms that choose actions independently of queue backlog and yield:

\[
\begin{align*}
\mathbb{E}\{C(P_{\text{tran}}^*(t), S(t))\} &= r^* \\
\mathbb{E}\{P_{\text{tran}}^*(t)\} &= g^*(r^*) \\
\mathbb{E}\{m(A(t), k^*(t))\} &= r^* \\
\mathbb{E}\{\phi(A(t), k^*(t))\} &= h^*_d(r^*) \\
\mathbb{E}\{d(A(t), k^*(t))\} &\leq d_{av}
\end{align*}
\]

where \( r^* \) is the optimal solution of problem (39), satisfying:

\[
h^*_d(r^*) + g^*(r^*) = P_{av}^*
\]

Plugging this into (69) yields:

\[
\Delta(Z(t)) + V \mathbb{E}\{P_{tot}(t) | Z(t)\} \leq C + VP_{av}^*
\]

Using this drift inequality in the Lyapunov Drift Lemma (Lemma 2) yields the \( P_{tot} \) bound of (41).

(Analysis): Let \( P_{\text{tran}}^*(t) = P_{\text{max}} \) and let \( k^*(t) \) be any stationary randomized policy that is independent of queue backlog and that yields:

\[
\begin{align*}
\mathbb{E}\{m(A(t), k^*(t))\} &\leq r_{\max} \\
\mathbb{E}\{d(A(t), k^*(t))\} &= d_{av} - \epsilon
\end{align*}
\]

for some value \( \epsilon > 0 \). Plugging this into (69) yields:

\[
\Delta(Z(t)) + V \mathbb{E}\{P_{tot}(t) | Z(t)\} \leq C - X(t)\epsilon + V[P_{\max} + \phi_{\max}]
\]

and hence:

\[
\Delta(Z(t)) \leq C - X(t)\epsilon + V[P_{\max} + \phi_{\max}]
\]

Using this drift inequality directly in the Lyapunov Drift Lemma (Lemma 2) yields:

\[
\limsup_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\{X(\tau)\} \leq \frac{C + V[P_{\max} + \phi_{\max}]}{\epsilon}
\]

It follows that the virtual queue \( X(t) \) is strongly stable. Because it has a finite maximum departure rate \( d_{av} \), the time average expected arrival rate to \( X(t) \) (given by \( D \)) is less than or equal to the time average expected transmission rate (given by \( d_{av} \)) [Neely 2006] [Georgiadis et al. 2006]. This proves (43).

Appendix E – Derivation of \( U_{\text{thresh}} \) for the Logarithmic Rate-Power Curve Model

The logarithmic model has \( C(P) = \alpha \log(1+\beta P) \) (using a natural log), with \( \mu_{\max} = \alpha \log(1 + \beta P_{\max}) \). The dynamic transmission algorithm solves:

Maximize: \( U(t) = \alpha \log(1 + \beta P) - V P \) \hspace{1cm} (70)

Subject to: \( 0 \leq P \leq P_{\max} \)
The largest value of $U_{\text{thresh}}$ that satisfies Property 1 is given by $U_{\text{thresh}} = \max[0, \theta]$, where $\theta$ is the minimum value for the following optimization problem:

\[
\begin{align*}
\text{Minimize:} & \quad \theta = U - \mu^*(U) \\
\text{Subject to:} & \quad \frac{V}{\alpha \beta} \leq U \leq \frac{V}{\alpha \beta} + \frac{V P_{\text{max}}}{\alpha}
\end{align*}
\]

where $\mu^*(U) = \alpha \log(1 + \beta P^*(U))$ and $P^*(U)$ is the optimum solution to (70) for $U(t) = U$, given by:

\[
P^*(U) = \left[ \frac{U \alpha}{V} - \frac{1}{\beta} \right]_{0}^{P_{\text{max}}}
\]

where the operator $[x]_{y}^{z}$ is defined $[x]_{y}^{z} = \max[0, \min[x, y]]$. This can be understood as follows: The value $U - \mu^*(U)$ is the queue backlog after transmission when $U(t) = U$. If $U(t) \leq V/(\alpha \beta)$ then $P(t) = 0$ and $\mu(t) = 0$ (so queue backlog cannot further decrease) while if $U(t) \geq V/(\alpha \beta) + V P_{\text{max}}/\alpha$ then $P(t) = P_{\text{max}}$ and $\mu(t) = \mu_{\text{max}}$, so the queue cannot drop below the value it would drop to if $U(t) = V/(\alpha \beta) + V P_{\text{max}}/\alpha$.

The problem (71) reduces to:

\[
\begin{align*}
\text{Minimize:} & \quad U - \alpha \log(U \alpha \beta/V) \\
\text{Subject to:} & \quad \frac{V}{\alpha \beta} \leq U \leq \frac{V}{\alpha \beta} + \frac{V P_{\text{max}}}{\alpha}
\end{align*}
\]

The critical points of the above problem appear at the two endpoints of the interval and at the point $U = \alpha$ (if this point is inside the interval). We thus have:

\[
U_{\text{thresh}} = \begin{cases} 
\max[0, \alpha - \alpha \log(\alpha^2 \beta/V)] & \text{if } \frac{V}{\alpha \beta} \leq \alpha \leq \frac{V}{\alpha \beta} + \frac{V P_{\text{max}}}{\alpha} \\
\max \left[0, \min \left[ \frac{V}{\alpha \beta}, \frac{V}{\alpha \beta} + \frac{V P_{\text{max}}}{\alpha} - \mu_{\text{max}} \right] \right] & \text{else}
\end{cases}
\]

(72)

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