THE STRUCTURE AND DARK HALO CORE PROPERTIES OF DWARF SPHEROIDAL GALAXIES

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ABSTRACT

The structure and dark halo core properties of dwarf spheroidal galaxies (dSphs) are investigated. A double-isothermal (DIS) model of an isothermal, non-self-gravitating stellar system embedded in an isothermal dark halo core provides an excellent fit to the various observed stellar surface density distributions. The stellar core scale length $a_s$ is sensitive to the central dark matter density $\rho_{0,d}$. The maximum stellar radius traces the dark halo core radius $r_{c,d}$. The concentration $c_s$ of the stellar system, determined by a King profile fit, depends on the ratio of the stellar-to-dark-matter velocity dispersion $\sigma_s/\sigma_d$. Simple empirical relationships are derived that allow us to calculate the dark halo core parameters $\rho_{0,d}$, $r_{c,d}$, and $c_s$ given the observable stellar quantities $\sigma_s$, $a_s$, and $c_s$. The DIS model is applied to the Milky Way’s dSphs. All dSphs closely follow the same universal dark halo scaling relations $\rho_{0,d} \times r_{c,d} = 75^{+85}_{-45} M_\odot \text{pc}^{-2}$ that characterize the cores of more massive galaxies over a large range in masses. The dark halo core mass is a strong function of core radius, $M_{c,d} \sim r_{c,d}^2$. Inside a fixed radius of $\sim400 \text{pc}$ the total dark matter mass is, however, roughly constant with $M_d = 2.6 \pm 1.4 \times 10^7 M_\odot$, although outliers are expected. The dark halo core densities of the Galaxy’s dSphs are very high, with $\rho_{0,d} \approx 0.2 M_\odot \text{pc}^{-3}$. dSphs should therefore be tidally undisturbed. Evidence for tidal effects might then provide a serious challenge for the CDM scenario.

Key words: dark matter – galaxies: dwarf – galaxies: formation – galaxies: kinematics and dynamics – galaxies: structure

1. INTRODUCTION

The nature of dark matter is still a mystery. Standard cosmology works with the assumption of a massive, weakly and gravitationally interacting particle (e.g., White & Negroponte 1982; Steigman & Turner 1985; Jungman et al. 1996). This cold dark matter (CDM) scenario has proven to be very successful on large galactic and extragalactic scales, from cosmic structure formation to the outer rotation curves of galaxies and the stability of galactic disks (Ostriker & Peebles 1973). The success of the CDM model, however, is also to some extent frustrating as any additional physical properties of the CDM particle remain hidden. Scientists are therefore searching for failures in CDM predictions that might lead to new insight into the nature and origin of the dark matter particle.

Several small-scale problems of the CDM model have been discussed in the literature. These include the distribution of satellite galaxies in large planar structures (e.g., Kroupa et al. 2005; Ibata et al. 2013; Goerdt et al. 2013) or the mass–luminosity problem of satellite galaxies (Kroupa et al. 2010). One of the most prominent heavily debated questions is the cusp-core problem (e.g., Flores & Primack 1994; Moore 1994; Burkert 1995; Strigari et al. 2008; Primack 2009; de Blok 2010; Boylan-Koehlin et al. 2011; Ogiba & Mori 2014; Ogiba et al. 2014; Ogiba & Burkert 2015). While simulations predict cuspy central density profiles of CDM halos with the density increasing steeply toward the center (Dubinski & Carlberg 1991; Navarro et al. 1997; Moore et al. 1999; Dekel et al. 2003a), observations often indicate a flat dark matter density core (e.g., Flores & Primack 1994; Moore 1994; Burkert 1995; de Blok et al. 2008; Gentile et al. 2009; Oh et al. 2011). The detection of cored dark matter halos is not necessarily inconsistent with CDM. Various mechanisms have been identified that can generate cores from an initially cuspy density distribution. Prominent examples are fluctuations in the galactic potential, induced by AGN feedback and galactic winds (e.g., Navarro et al. 1996; Ogiba & Mori 2011, 2014; Pontzen & Governato 2012; Teyssier et al. 2013; Amorisco et al. 2014) or gaseous and stellar clumps spiraling to the center (e.g., El-Zant et al. 2001; Ma & Boylan-Kolchin 2004; Tonini et al. 2006; Goerdt et al. 2010; Inoue & Saitoh 2011).

The cusp-core problem is best documented in low-mass dwarf galaxies, which are characterized by low baryon fractions and therefore are ideal tracers of the underlying dark halo structure, unperturbed by the gravitational influence of the baryonic component. In addition, dwarf galaxies appear to host the highest density dark matter cores, making an analysis of the halo structure easier. Ideally, one would like to investigate $H_\alpha$ HI rotation curves (e.g., Carignan & Freeman 1985; de Blok et al. 2001; Corbelli et al. 2014), which are a clear tracer of the gravitational potential as a function of radius. One of the problems, however, is that for low-mass galaxies the stellar and gas velocity dispersions begin to exceed their rotational velocity. This is especially true for one of the smallest types of galaxy known as dwarf spheroidals (dSph), the target of this paper. Recently, Kormendy & Freeman (2014; KF14) investigated the dark halo scaling laws in late-type galaxies, including dSphs (see also Burkert 1997; Salucci et al. 2012). Interestingly, the stellar velocity dispersion $\sigma_s$ of the dSphs is of the order of $8–10 \text{km s}^{-1}$, very similar to the universal turbulent velocity of the diffuse gas component in most low-redshift star-forming disk galaxies (Dib et al. 2006). However, in contrast to more massive galaxies, the gravitational field in dSphs is small, generating rotation curves of the order of $v_{\text{rot}} \approx 10 \text{km s}^{-1}$ (see Section 4). Even if the star-forming gas would have had enough angular momentum to settle initially, it is not clear if the dark matter core would have been able to form.
into a thin disk configuration, stellar feedback processes could easily destroy the disk, ejecting gas into wind and leading to a dispersion-dominated spheroidal stellar system (Navarro et al. 1996; Maller & Dekel 2002; Teyssier et al. 2013).

KF14 consider a spherically symmetric stellar system with constant $\sigma_s$, embedded in a spherically symmetric dark matter core with constant density $\rho_{0,d}$. Solving the hydrostatic equation (see Equation (3)) and assuming that the stellar system is not self-gravitating they find that the stellar density distribution should be a Gaussian

$$\rho_s(r) = \rho_{0,s} \times \exp\left(-\frac{r^2}{a_s^2}\right)$$

with $\rho_{0,s} = \rho_s(r = 0)$ the central stellar density and

$$a_s = \left(\frac{3\sigma_s^2}{2\pi G \rho_{0,d}}\right)^{1/2}$$

the scale length of the stellar system. The projected stellar surface density distribution $\Sigma_s$ in this case is also a Gaussian with the same scale length. Remarkably, KF14 show that many dSPhs follow a Gaussian surface density distribution better than the typical exponential profile seen for galactic disks.

KF14 however also find that some dSPhs cannot be fitted by a Gaussian. In addition, dSPhs with inner Gaussian slopes also show deviations farther out. Another problem is the fact that isothermal, isotropic dark matter cores in equilibrium cannot have precisely constant densities, requiring a more detailed investigation. Finally, Equation (2) does not provide any information about the dark halo core radii, velocity dispersions, and masses. KF14 shift the observed stellar scale length $a_s$ and velocity dispersion $\sigma_s$ along lines of constant $\rho_{0,s}$ onto the core scaling relations of more massive galaxies in order to infer the halo core properties. It is, however, not clear whether dSPhs should follow the same core scaling relations as more massive galaxies.

In this paper we therefore have a more detailed look at the coupled kinematics of stars and dark matter in dSPhs, relaxing the assumption of a constant density dark matter core. Section 2 solves the hydrostatic equation of two isothermal particle systems, coupled with their joint gravitational field. We derive formulae on how to determine the dark matter core density and how to shift the observed stellar velocity dispersion $\sigma_s$ and central stellar scale length $a_s$ in order to infer the dark halo velocity dispersion $\sigma_d$ and halo core radius $r_{c,d}$. Section 3 then focuses on deviations from Gaussian profiles and the origin of King profiles in isothermal dSPhs. This section demonstrates that the stellar King concentration parameter $c_s$ is tightly related to the dark halo velocity dispersion $\sigma_d$, which also makes it possible to determine the halo core radius and core mass. These analytical results are then applied to the Milky Way’s system of dSPhs in Section 4 to investigate their halo core properties. Section 5 discusses the conjecture of Strigari et al. (2008) that dSPhs have a universal mass of the order of $10^7 M_\odot$ within a scale radius of 300 pc. Section 6 summarizes the results and concludes.

2. THE STRUCTURE OF DOUBLE-ISOTHERMAL (DIS) PARTICLE SYSTEMS

We assume that the cores of dark matter halos in dSPhs are isothermal and isotropic with a constant velocity dispersion $\sigma_{0,d}$. This assumption certainly has to break down at some point, as otherwise the dark halo mass would diverge as $M_d(r) \sim r^3$. For investigations of the outer halo regions, the Burkert profile (Burkert 1995) might therefore provide a better approximation. It combines an isothermal-like inner core with the characteristic $r^{-3}$ outer density decline seen in most CDM simulations (Navarro et al. 1997).

Observations also indicate that the stellar body of dSPhs is characterized by an almost constant velocity dispersion $\sigma_s$, well beyond the half-light radius (Evans et al. 2009; Walker et al. 2009; Salucci et al. 2012; KF14). Majewski et al. (2013) recently reported a drop in $\sigma_s$ in the heart of the Sagittarius dSPhs galaxy. A similar feature is seen in Sculptor (Breddels & Helmi 2014), indicating that some dSPhs might be more complex two-component galaxies (Amorisco & Evans 2011). The dense cores are limited to the very center, which is small compared to the half-light radius. A central decline in $\sigma_s$ might in fact be a characteristic property of dSPhs in general. Many of them show a strong increase in stellar surface density in the very center, requiring a change in their kinematics, most likely a declining velocity dispersion, in order to be in hydrostatic equilibrium. The origin of cold nuclei in dSPhs is certainly an interesting and yet unsolved problem. Here, however, we are interested in the global structure of dSPhs, which to good approximation is observed to be isothermal and will neglect their cold hearts. In addition, in order to keep the number of free parameters to a minimum, we assume that both the stellar components and the dark matter cores are isotropic with negligible anisotropy effects (Ciotti 1999; Evans et al. 2009; Salucci et al. 2012).

Adopting a central density $\rho_0(r = 0) = \rho_{0,d}$, the radial density distribution $\rho_d(r)$ of an isothermal halo in hydrostatic equilibrium is determined by

$$\frac{\sigma_d^2}{2} \frac{d \ln \rho_d}{dr} = -\frac{GM_d(r)}{r^2}.$$  (3)

Here $\sigma_d$ is the dark matter velocity dispersion and $M_d(r)$ is the cumulative dark matter mass inside radius $r$. The assumption of virial equilibrium might not always be valid (Kroupa 1997), especially for tidal dwarf galaxies (Wetstein et al. 2007; Ploeckinger et al. 2015) and strongly tidally affected dSPhs like the Sagittarius dwarf spheroidal (Ibata et al. 1994; Kroupa 1997; Yang et al. 2014). Deviations from hydrostatic equilibrium might therefore help to identify those (mostly outer) regions of dwarf satellites that are tidally interacting with the host galaxy. The thick gray line in the upper left and right panels of Figure 1 shows the density distribution and logarithmic slope $d \ln \rho_d / d \ln r$, respectively, of a non-singular isothermal sphere with finite $\rho_{0,d}$. Inside the core radius that for an isothermal sphere is defined as

$$r_{c,d}^2 = \frac{9 \sigma_d^2}{4 \pi G \rho_{0,d}}.$$  (4)

the dark matter density distribution is roughly flat. Note however that $\rho_d(r)$ is not exactly constant. This is due to the fact that, in equilibrium, the pressure gradient $\sigma_d^2 d \ln (\rho_d) / dr$ has to balance the gravitational force $GM_d(r) / r^2$. One might consider this a negligible effect for $r < r_{c,d}$. It is, however, precisely this small density gradient that determines the density distribution of the embedded stellar component.
Let us therefore now include a non-self-gravitating, isothermal stellar component, as observed for dSphs. The stars then represent direct tracers of the underlying dark matter potential. It is unlikely that dSphs started that way. Obviously, the gas clouds from which the stars formed were self-gravitating (Burkert & Hartmann 2013; Nipoti & Binney 2014).

A low star formation efficiency combined with strong galactic winds and ram pressure stripping might, however, have removed most of the baryons before they could condense into stars. This conclusion is also supported by the low metallicities of dSphs (Dekel & Silk 1986). If the galactic outflow was violent enough, in addition to leaving behind a non-self-gravitating stellar system, it could also have reshaped an initially cuspy dark halo, generating a core (e.g., Navarro et al. 1996; Teyssier et al. 2013).

As both the stars and the dark matter particles move inside the same joint gravitational potential, their density distributions are coupled:

$$\sigma_*^2 \frac{d}{dr} \frac{\ln \rho_*}{r^2} = -\frac{GM_d(r)}{r^2} = \sigma_d^2 \frac{d}{dr} \frac{\ln \rho_d}{r^2}. \tag{5}$$

Here, $\rho_*(r)$ and $\sigma_*$ correspond to the density distribution and velocity dispersion of the stellar component. Equation (5) leads to

$$\rho_*(r) = A \times \rho_d^\kappa(r) \tag{6}$$

with

$$\kappa = \frac{\sigma_d^2}{\sigma_*^2} \tag{7}$$

and $A$ a constant of integration that determines the total stellar mass. Equation (6) shows that it is indeed the dark matter...
density gradient that determines \( \rho_s(r) \): \[
\frac{d \ln \rho_s}{d \ln r} = \kappa \frac{d \ln \rho_d}{d \ln r} \tag{8}
\]

The three dashed lines in the upper panels of Figure 1 show \( \rho_s(r) \) for embedded stellar systems of our DIS model with velocity dispersions \( \sigma_s/\sigma_d \) of 0.7, 0.5, and 0.25, corresponding to \( \kappa \) values of 2, 4, and 16, respectively. For \( \kappa > 1.5 \), the stellar density distribution decreases faster than \( r^{-3} \) in the outer region. In the academic limit that the DIS model holds for all \( r \), the stellar system would have a finite mass, despite the fact that it is isothermal at all radii.

The middle left panel of Figure 1 shows the logarithm of the stellar density profile \( \rho_s(r) \) and surface density distribution \( \Sigma_s(r) \) (dashed and solid lines, respectively) as a function of \( r^2 \) for \( \kappa = 2, 4, \) and 16. A Gaussian profile would be represented by a straight line and indeed fits the inner profiles in general quite well. The larger \( \kappa \), the more similar are the density and surface density distributions and the more the profiles resemble a Gaussian.

Despite the fact that the dark matter core density is not necessarily constant and therefore Equation (2) is not exactly valid, we can still formally derive an approximate Gaussian scale length \( a_s = -(d \ln \Sigma_s/dr^2)^{-1/2} \) using a least-squares linear fit of \( \ln \Sigma_s \) versus \( r^2 \) within the innermost regions of the stellar component that we define as the region where \( \Sigma_s \) decreases by a factor \( e \) with respect to the central value. The middle right panel of Figure 1 shows that \( a_s/r_{c,d} \) depends strongly on \( \kappa \). Solving for \( \rho_{0,d} \) in Equation (2) and inserting it into Equation (4) we find

\[
\left( \frac{a_s}{r_{c,d}} \right) = 0.82 \left( \frac{\sigma_s}{\sigma_d} \right). \tag{9}
\]

It is not clear whether this should work, given the fact that Equation (2) was derived for a constant density core while we have argued that it is actually the dark matter density gradient that determines the stellar density distribution (Equation (8)). However, the solid black line in Figure 1 shows that Equation (9) indeed provides an excellent fit to the actual data derived from a numerical integration of the DIS model (red points).

In the lower left panel of Figure 1 we test the validity of Equation (2) as an estimation for the underlying dark halo density \( \rho_{0,d} \) given \( \sigma_s \) and \( \sigma_d \). If we write

\[
\rho_{0,d} = \eta_b \frac{3\sigma_s^2}{2\pi G a_s^2}, \tag{10}
\]

an analysis of the DIS model shows that \( \eta_b \) depends only on \( \kappa \), as shown by the solid black line. For \( \kappa \approx 1 \) a very good approximation is (red points)

\[
\eta_b \approx 1.01 \left( 1 + 0.5 \exp \left[ - \left( \kappa - 1 \right)^{0.6} \right] \right). \tag{11}
\]

As expected, for very cold stellar systems with \( \kappa \gg 1 \) the stellar system traces the innermost dark halo core with an almost constant density distribution. Here \( \eta_b \approx 1 \) and Equation (2) provides a good estimate of \( \rho_{0,d} \). For kinematically hotter stellar systems, however, Equation (2) is not valid anymore and the correction factor \( \eta_b \) has to be taken into account.

3. KING PROFILES AND APPARENT EXTRA-TIDAL COMPONENTS

Up to now we focused on the innermost regions of dSphs that are sensitive to the central dark matter density. In order to gain information about the dark halo core radii and core masses, we need to look at stellar traces farther out. According to Equation (6), very cold dSphs with \( \kappa \gg 1 \) populate regions that are deeply embedded in the dark halo core and that are therefore not good probes to explore the larger environment. One such example is Carina, shown in the upper left panel of Figure 2. Carina can be fitted well by a Gaussian over most of the stellar body with a small change in slope \( d \ln \Sigma_s/dr^2 \) in the outermost region. In contrast, the stellar distribution of the
much more extended Sculptor dSph (upper right panel of Figure 2) deviates strongly from a Gaussian. The red solid lines in both figures show the surface density profiles of DIS systems following Equations (3)–(7) with \( \kappa = 3.3 \) and \( \kappa = 1.5 \) for Carina and Sculptor, respectively. With respect to the dark matter component, the stellar system in Sculptor (\( \sigma_\bullet = 0.81 \sigma_d \)) is kinematically hotter than in Carina (\( \sigma_\bullet = 0.55 \sigma_d \)), leading to a more extended structure.

It turns out that Equations (3)–(7) lead to surface density distributions that fit all dSphs very well, even those with more complex, extended stellar components like Sculptor. This results from the fact that dSphs are in general observed to follow King profiles (King 1966; Amorisco & Evans 2011) with different concentration parameters \( c_\bullet \). King profiles are a one-parameter family, characterized by the concentration parameter \( c \), which is equal to the logarithm of the ratio of the outer cutoff radius to the core radius of a particle system. The same is true for the projected surface density distributions of stellar systems in our DIS model. As an example, the solid lines in the left panel of Figure 3 show three different stellar systems of our DIS model with \( \kappa \) values of 2.8, 1.4, and 1, respectively. The points show the corresponding best fitting King profiles, which have concentrations of 0.6, 1.25, and 3.6, respectively. Note that this excellent fit hides a fundamental difference between DIS models and King models. Our stellar systems have constant velocity dispersions. They are part of a two-component system, with the stars being embedded in a surrounding dark halo that has in general a different velocity dispersion than the stars. King models, instead, are one-component, self-gravitating particle systems that are sometimes also called truncated isothermal spheres. They are characterized by a special velocity distribution function that has been designed to fit stellar systems like globular clusters with sharp outer edges generated as a result of tidal stripping. In order for such a sharp outer edge to exist, the velocity dispersion of the stars in the King model has to decrease with radius with \( \sigma_\bullet \to 0 \) at the outer edge. Otherwise, stars would be able to move beyond it. It is therefore surprising and at first not necessarily expected that the one-component King profiles with completely different kinematics provide such a good fit to the stellar structure of our two-component DIS systems over more than 4 orders of magnitude in \( \ln \Sigma_\bullet \).

A characteristic property of King models is that the surface density structure changes strongly for concentrations \( 1.2 \leq c_\bullet \leq 2 \) from a core with a steeply decreasing outer edge to a more extended structure. The DIS models follow this trend nicely with kinematically hotter stellar systems characterized by smaller values of \( \kappa \) and corresponding to King models with larger \( c_\bullet \). In the transition regime, however, the best fitting King profiles are somewhat steeper than the stellar systems for \( \ln \Sigma_\bullet / \Sigma_{0,\bullet} < -4 \) (see the \( c_\bullet = 1.25 \) profile in the left panel of Figure 3). Interpreting such an extended population of stars as extra-tidal, in this case, would be misleading. These stars are still deeply embedded and strongly bound to their dark halo. On the other hand, an extra-tidal component detected at that level in \( \ln \Sigma_\bullet / \Sigma_{0,\bullet} \) in systems with concentrations \( c_\bullet < 1 \) or \( c_\bullet > 2 \) cannot be explained within the framework of our model and therefore might indeed represent a separate hot halo or even an extra-tidal component.

The right panel of Figure 3 shows that there exists a tight correlation between the \( \kappa \) value of the DIS model and the King concentration parameter \( c_\bullet \) (solid black line). The kinematically hotter the stellar system, i.e., the smaller \( \kappa \), the more extended the stellar system and the larger \( c_\bullet \). The red points show the empirical relation

\[
\log \kappa = 1.25 \exp(-1.72c_\bullet)
\]

which is an excellent fit to the data. Given \( a_\bullet, \sigma_\bullet, \) and \( c_\bullet \), one can now use Equations (9)–(12) and calculate the central density \( \rho_{0,\text{tr}} \), velocity dispersion \( \sigma_d \), and core radius \( r_{c,d} \) of the
Table 1

Physical Properties of the Stellar and Dark Halo Component of the Eight Classical Milky Way dSphs (KF14; IH95; McConnachie 2012)

| Stellar Component | r_c (pc) | σ_h (km s⁻¹) | c_h | Dark Matter Component | r_c (pc) | σ_L (km s⁻¹) | η_0,M (M⊙ pc⁻³) | M_c,d (10⁶ M⊙) | M_0,10⁶ (10⁷ M⊙) |
|-------------------|---------|---------------|-----|-----------------------|---------|---------------|----------------|--------------|----------------|
| Carina            | 202     | 6.6           | 0.51|                       | 450     | 12.0          | 0.13           | 2.6          | 0.9            |
| Draco             | 176     | 9.1           | 0.50|                       | 397     | 16.7          | 0.33           | 4.4          | 2.1            |
| Leo I             | 221     | 9.2           | 0.58|                       | 460     | 15.6          | 0.22           | 4.6          | 1.6            |
| Leo II            | 174     | 6.6           | 0.48|                       | 400     | 12.4          | 0.18           | 2.4          | 1.2            |
| UMi               | 211     | 9.5           | 0.51|                       | 470     | 17.3          | 0.25           | 5.6          | 1.9            |
| Fornax            | 705     | 11.7          | 0.72|                       | 1310    | 17.7          | 0.035          | 17.3         | 0.4            |
| Sextans           | 400     | 7.9           | 0.98|                       | 641     | 10.3          | 0.053          | 3.0          | 0.5            |
| Sculptor          | 189     | 9.2           | 1.12|                       | 286     | 11.4          | 0.33           | 1.7          | 1.5            |

dark halo:

\[
\log \left( \frac{r_c}{\text{pc}} \right) = 0.088 + \log \left( \frac{a_*}{\text{pc}} \right) + 0.625 \exp \left( -1.72 c_h \right)
\]

\[
\log \left( \frac{\sigma_d}{\text{km s}^{-1}} \right) = 0.625 + \log \left( \frac{\sigma_h}{\text{km s}^{-1}} \right) + 0.625 \exp \left( -1.72 c_h \right)
\]

\[
\log \left( \frac{\rho_0,d}{M_\odot \text{ pc}^{-3}} \right) = 2.04 + 2 \log \left( \frac{\sigma_h}{\text{km s}^{-1}} \right) - 2 \log \left( \frac{a_*}{\text{pc}} \right)
\]

(13)

4. THE DARK HALO CORE PROPERTIES OF LOCAL MILKY WAY dSphs

As an application, we investigate the dark halo core properties of the eight classical Milky Way dSphs (Table 1) observed by Irwin & Hatzidimitriou (1995, hereafter IH95; see also Carignan & Freeman 1985). IH95 determined their stellar surface density distribution with high enough resolution to derive King concentration parameters and determine the central scale lengths. IH95 also provide stellar velocity dispersions. The central Gaussian scale lengths \(a_{maj}\) were determined from major axis surface brightness profiles, shown in Figure 2 of IH95. These values are very close to the King core radii, summarized in Table 4 of IH95. Following IH95, \(a_*\) was then derived as the geometrical mean along the major and minor axis with \(a_* = a_{maj} \times r_c,III/r_c,II\) where \(r_c,III\) and \(r_c,II\) are the stellar system’s geometric mean and major axis core radius, respectively, as determined by IH95.

Using the set of Equations (13) we now can calculate the halo core parameters \(r_c,\), \(σ_d,\), and \(ρ_0,d\). The results are summarized in Table 1. The upper left panel of Figure 4 shows \(σ_*\) versus \(a_*\) (gray triangles) and \(σ_d\) versus \(r_c,d\) (red points with error bars) for the eight dSphs. Since IH95, updated stellar velocity dispersion measurements have been published (e.g., Walker et al. 2009; McConnachie 2012). Here we adopt the values of \(σ_*\) as given in the regularly updated McConnachie database (McConnachie 2012). Dark halo cores are hotter than their stellar systems with velocity dispersions between 10 and 18 km s⁻¹ and, on average, \(σ_d ≈ 1.6 σ_*\), corresponding to \(κ ≈ 2.56\). The halo core radii lie in the range of 290 pc–1.3 kpc. On average, \(r_c,d ≈ 1.9 a_*\). Donato et al. (2004) analyzed a sample of high-resolution rotation curves of 25 disk galaxies and determined independently the disk scale lengths \(r_{disk}\) and dark matter core radii. They found that both radii are strongly correlated with \(r_{disk} ≈ 2.4 r_c,d\). It is remarkable that the ratio between the stellar scale length and the dark halo scale length is the same (of the order of 2) in very different galactic systems over a large range of masses. The origin is still unclear and might provide further insight into the mechanisms that lead to dark matter cores.

The upper right panel of Figure 4 shows the dark halo central surface densities \(ρ_0,d\) as a function of \(r_c,d\). Typical values are 0.2 \(M_\odot \text{ pc}^{-3}\) with a range of 0.03–0.3 \(M_\odot \text{ pc}^{-3}\). It has been argued that dark halo cores follow a universal scaling relation with constant core surface density \(ρ_0,d\) (e.g., Athanassoula et al. 1987; Burkert 1995; Salucci & Burkert 2000; Kormendy & Freeman 2004, 2014; de Blok et al. 2008; Donato et al. 2009; Gentile et al. 2009; Cardone & del Popolo 2012; Saburova & del Popolo 2014). KF14 find that this scaling relation holds over more than 18 mag in \(M_B\). The black triangles in the upper right panel and in both lower panels of Figure 4 show the core properties of galaxies compiled from the literature by KF14 (see the list of references for the original data in Table 1 of KF14). The core surface densities of all galaxies lie in a narrow range of 30 \(M_\odot \text{ pc}^{-2} ≤ (ρ_0,d \times r_c,d) ≤ 160 M_\odot \text{ pc}^{-2}\) (dashed lines). The eight Milky Way dSphs fall precisely into this regime, despite the fact that their \(r_c,d\) are on average a factor of 6 smaller, with \(ρ_0,d\) being a factor of 6 larger. The lower left panel of Figure 4 shows the dSph core masses, which for non-singular isothermal spheres are

\[
M_c,d \equiv 2.17 \times ρ_0,d r_c,d^3 = 162.75 \left( \frac{(ρ_0,d \times r_c,d)}{75 M_\odot \text{ pc}^{-2}} \right) \left( \frac{r_c,d}{\text{pc}} \right)^2 M_\odot.
\]

(14)

The core masses cover a range of one order of magnitude with masses between (see Table 1) \(1.7 \times 10^6 M_\odot ≤ M_c,d ≤ 1.7 \times 10^8 M_\odot\). The expected correlation between \(M_c,d\) and \(r_c,d\) (Equation (14)) is drawn for \((ρ_0,d \times r_c,d) = 30, 75,\) and \(160 M_\odot \text{ pc}^{-2}\), together with the core masses of more massive galaxies. The dSphs follow the same core mass scaling relations as massive galaxies with the same spread. Finally, the lower right panel shows again \(σ_d\) versus \(r_c,d\). Now we compare the dSphs with the more massive galaxies. Both follow the same universal scaling relation \(σ_d^2 \propto r_c,d^{−2.5}\) (km s⁻¹)² pc⁻¹ (see also de Vega et al. 2014), again with precisely the same spread.
5. THE ORIGIN OF A COMMON MASS AND LENGTH SCALE FOR DARK MATTER CORES

Strigari et al. (2008) proposed that all dSphs of the Milky Way have the same total dark matter mass \( \log(M_{\odot, d}/M_\odot) = 7.0 \pm 0.3 \) contained within a radius of 300 pc. The origin of a universal and constant mass might at first appear surprising given the fact that the core masses are a strong function of core size (Equation (14)). Note however that \( M_{\odot, d} \) is measured within \( r_{cd} \) whereas \( M_{300, d} \) is the mass within a fixed radius 300 pc that can be smaller or larger than \( r_{cd} \). Questions still arise of why there should exist such a universal radius \( r_{cd} \), inside which halo cores have the same mass \( M_{\odot, d} \), and what determines this radius. In addition, adding Andromeda dSphs, Collins et al. (2014) find outliers that are not consistent with the Strigari et al. mass, which indicates that the situation could be more complex.

Ogiya et al. (2014) discussed a possible connection between the existence of a universal mass scale and the universal core surface density of dSphs. Following Ogiya et al. (2014), let us now explore this question within the context of the DIS model. We start with a population of dSphs that has a common core surface density \( \langle \rho_{0, d} \rangle = 75 M_\odot \text{ pc}^{-2} \) that fits all galaxies very well. The two dashed lines show the observed spread with the upper and lower limit corresponding to core surface densities of 30 and 160 \( M_\odot \text{ pc}^{-2} \), respectively.

Figure 4. Gray triangles in the upper left panel show the stellar velocity dispersion \( \sigma_* \) vs. the central stellar scale length \( a_* \) of the eight classical Milky Way dSphs. Red points with error bars depict the corresponding dark halo velocity dispersion \( \sigma_d \) vs. the halo core radius \( r_{cd} \). The red points in the upper right and lower left and right panels show the correlation of \( r_{cd} \) with the dark halo central densities \( \rho_{0, d} \), core masses \( M_\odot \), and velocity dispersions \( \sigma_d \), respectively. Black triangles depict the core properties of more massive galaxies. Typical error bars for these data points are shown in the upper right or left corners of each plot. The thick gray line in all four panels corresponds to the dark halo core scaling relation \( \langle \rho_{0, d} \rangle = 75 M_\odot \text{ pc}^{-2} \) that fits all galaxies very well. The two dashed lines show the observed spread with the upper and lower limit corresponding to core surface densities of 30 and 160 \( M_\odot \text{ pc}^{-2} \), respectively.
dark halo core with $r_{cd} = r_{ad}$ (Equation (14)):

$$\log r_{cd} = \frac{1}{N} \sum_{i=1}^{N} \log r_{c,d,i}$$

$$M_{ad,d} = 2.17 (\rho_{0,d} \times r_{cd})^2$$

(Equation (15))

with $N$ the number of dSphs and $r_{c,d}$, $r_{ad}$ the core radius of galaxy $i$. The origin of a maximum for $r_{cd} = r_{ad}$ can be easily understood. For $r_{cd} > r_{ad}$ the core radius is larger than the region sampled by $r_{ad}$ and the density is roughly constant with a value of $\rho_{0,d} \sim 1/r_{cd}$ due to the assumption of a constant core surface density. The enclosed mass with a given fixed radius $r_{ad}$ is then $M_{ad,d} \sim \rho_{0,d} \sim 1/r_{cd}$, leading to $M_{ad,d}$ decreasing with increasing $r_{cd}$. For $r_{cd} < r_{ad}$ the region is larger than the core and extends out to radii where the dark matter density distribution begins to decrease. To first order we can approximate $M_{ad,d}$ now as the mass of a constant density core $\rho_0(r) = \rho_{0,d}$ for $r \leq r_{cd}$ plus the mass of an envelope with a power-law density distribution $\rho_0 = \rho_{0,d} (r_{cd}/r)^2$. As $\rho_{0,d} \sim 1/r_{cd}$ we get $M_{ad,d} \sim \rho_{0,d} (r_{cd}/r_{cd} - 2/3)$ which is a continuously increasing function with increasing $r_{cd}$.

Applying Equation (15) to our sample of dSphs we find $r_{ad,d} = 400 \pm 100$ pc, and with $\rho_{0,d}(r_{cd})=75M_\odot pc^{-2}$ we get $M_{ad,d} = 2.4 \pm 1.4 \times 10^7 M_\odot$, in good agreement with Strigari et al. (2008). If the population has a large spread in $r_{cd}$ we also expect to find outliers populating the wings of the distribution farther away from the maximum with smaller masses $M_{ad,d}$. This might explain the detection of outliers in Andromeda’s system of dSphs (Collins et al. 2014).

6. SUMMARY AND CONCLUSIONS

Motivated by the conjecture that dSphs are isothermal stellar systems (e.g., Evans et al. 2009) embedded in isothermal dark matter cores, we investigated the structure of two particle systems with constant but different velocity dispersions in virial equilibrium within their joint gravitational potential. Note that here the main objective was not to demonstrate that dark matter cores are isothermal and isotropic, and it certainly has to break down outside of some radius (Burkert 1995). We worked with this assumption because it is the most simple model of a halo core with the least number of free parameters. Of course, the fact that we find solutions that fit the observations very well is promising. However, it is not a proof due to the fact that a fine tuned radial distribution of anisotropy, coupled with a properly chosen gradient in velocity dispersion, could always lead to similar core density profiles.

We demonstrate that the surface density distributions of the non-self-gravitating stellar component of a DIS galaxy can show a rich variety of profiles. They can formally be fitted by King profiles despite the fact that the DIS model consists of two isothermal components in contrast to the one-component, non-isothermal King model. We find that the stellar systems in the DIS models have steeply decreasing outer edges, especially for high values of $\kappa$, not because they are tidally limited but because they are deeply embedded within the inner core regions of their dark halo. This is in contrast to the real one-component King model which is characterized by a tidal radius, the Roche radius, where the gravitational potential of the host.
The galaxy begins to dominate and where stars are unbound to the satellite. In addition, DIS systems have projected stellar velocity dispersion profiles that remain constant all the way to their outermost radius. One-component King systems, on the other hand, show outer velocity dispersion profiles that decrease approaching \( \sigma_{\text{e}} = 0 \) at the tidal radius. Measurements of the stellar velocity dispersion of dSphs close to their cutoff radius could therefore help to distinguish tidally truncated one-component systems without confining dark halos (Yang et al. 2014) from those where the maximum radius is determined by a strong dark matter confinement. This is also important, as the outer radii of satellite galaxies, interpreted as tidal radii, have been used in order to gain information about the satellites’ orbital parameters or the dark halo mass distribution of the host galaxy (e.g., Pasetto et al. 2011). Our analysis instead shows that the maximum radius of the satellites measure the core radii of their own dark halos and therefore do not contain information about the tidal radius and the strength of the host galaxy’s tidal field.

That \( r_c \) traces the dark halo’s core radii follows from the fact that the dSphs on average have \( \kappa \approx 2.6, r_c \) is close to the point where the logarithmic density gradient \( d \ln \rho / d \ln r \) begins to decrease faster than \( -3 \) which, according to Equation (8), then corresponds to a dark halo density gradient of \( d \ln \rho / d \ln r = -3/\kappa = -1.2 \). The upper right panel of Figure 1 shows that this slope is close to the core radius \( r_c \) of the dark matter halo. It is not clear yet whether this is a coincidence or whether the processes that generated the dark halo cores and their non-self-gravitating stellar tracer component naturally lead to such a configuration (Dekel & Woo 2003b).

For DIS systems, the central Gaussian scale length \( a_* \), the velocity dispersion \( \sigma_* \), and the concentration \( c \) of the stellar component completely specify the dark halo core parameters \( \rho_{0,d}, \sigma_d, \text{ and } r_{c,d} \). We determined these parameters for eight dSphs of the Milky Way and find that their dark halos have the same core surface densities \( \rho_{0,d} \times r_{c,d} = 75_{-45}^{+65} M_{\odot} \text{ pc}^{-2} \) as more massive galaxies, with exactly the same spread. This is very puzzling, as dSphs have a different structure and live in very different environments. At the moment it is not clear whether this result is also true for M31’s dSphs and whether it can be extended to the ultra-faint satellite population, which, due to their smaller radii, should have even higher dark halo core densities. An analysis similar to what was presented in this paper would require deeper observations of their stellar surface density distributions that are accurate enough to make King profile fits.

The origin of dark matter cores is still not well understood. Suggestions range from gravitational interaction with the baryonic component (e.g., Navarro et al. 1996; El-Zant et al. 2001; Goerdt et al. 2010; Inoue & Saith 2011; Ogiya & Mori 2011, 2014; Governato et al. 2012; Pontzen & Governato 2012; Gritschneder & Lin 2013; Teysier et al. 2013; Ogiya et al. 2014) to a non-standard primordial power spectrum (Zentner & Bullock 2002; Polisensky & Ricotti 2014), warm dark matter (e.g., Lovell et al. 2014), other intrinsic properties of dark matter like self-interaction and self-annihilation (e.g., Burkert 2000; Spergel & Steinhardt 2000; Loeb & Weiner 2011; Elbert et al. 2014), or modifications of Newtonian dynamics (e.g., Milgrom 1983; Kroupa 2012). Whatever the mechanisms, considerable fine tuning is required in order to generate a universal core scaling relation over more than 18 orders of magnitudes in the blue magnitude \( M_B \) with exactly the same spread.

Adopting a constant core surface density, \( M_{c,d} \) depends strongly on \( r_c \). Focusing, however, on a fixed radius \( r_{c,d} \), the enclosed mass \( M_{c,d} \) shows a different dependence on the halo core radius, with a maximum at \( r_{c,d} = r_{e,d} \). All halos with core radii in the vicinity of this maximum should therefore show similar values of \( M_{c,d} \), which could explain the observations of Strigari et al. (2008). The best choice of \( r_{e,d} \) is therefore dependent on the dSph’s distribution of \( r_{c,d} \). However, there does not exist a universal mass scale \( M_{c,d} \) that is independent of \( r_{c,d} \). Smaller values of \( M_{c,d} \) are expected for outliers with core radii that are very different from \( r_{c,d} \). Turning this argument around, if such a universal mass would exist independent of \( r_{c,d} \), it would be a clear signature that dark halo cores are not isothermal.

As the core radii of dSphs are small, their core densities have to be high in order for the core surface density to remain constant. This should shield dSphs efficiently against the tidal forces of their host galaxies. Adopting a constant rotation curve \( v_{\text{rot}} \), the Milky Way’s mean density within a given radius \( r \) is

\[
\langle \rho_{\text{MW}} \rangle = \frac{3 v_{\text{rot}}^2}{4 \pi G r^2} = 2.7 \left( \frac{v_{\text{rot}}}{220 \text{ km s}^{-1}} \right)^2 \left( \frac{\text{pc}}{r} \right)^2 M_\odot \text{ pc}^{-3}.
\]

The stellar system in dSphs would be tidally affected if \( \langle \rho_{\text{MW}} \rangle > \rho_{0,d} \approx 0.2 M_\odot \text{ pc}^{-3} \), which requires orbital pericenters of the order of a few kpc, which is very unlikely. dSphs therefore should be strongly shielded from any tidal affects by their deep dark matter potential wells and should survive as satellites of the Milky Way for a long time to come. However extra-tidal debris has been reported in some dSphs (IH95; Walcher et al. 2003; Majewski et al. 2005; Battaglia et al. 2012). We discussed in Section 3 that the DIS model in a certain concentration regime indeed leads to profiles that are somewhat more extended than the best fitting King profiles. This could be misinterpreted as a tidal component. Strong evidence for tidal interactions would, however, represent a real challenge for the existence of a shielding dark halo, opening the door for alternative ideas (Milgrom 1983; Yang et al. 2014).

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