Granularity and Generalized Inclusion Functions - Their Variants and Contamination

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Abstract. Rough inclusion functions (RIFs) are known by many other names in formal approaches to vagueness, belief, and uncertainty. Their use is often poorly grounded in factual knowledge or involve wild statistical assumptions. The concept of contamination introduced and studied by the present author across a number of her papers, concerns mixing up of information across semantic domains (or domains of discourse). RIFs play a key role in contaminating algorithms and some solutions that seek to replace or avoid them have been proposed and investigated by the present author in some of her earlier papers. The proposals break many algorithms of rough sets in a serious way. In this research, algorithm-friendly granular generalizations of such functions that reduce contamination are proposed and investigated from a mathematically sound perspective. Interesting representation results are proved and a core algebraic strategy for generalizing Skowron-Polkowski style of rough mereology is formulated.

Keywords: Rough Inclusion Functions, Contamination Problem, GRIF, Subjective Conditional Probability, Rough Objects, Generalized Granular operator Spaces, Fuzzy Logic, Rough Mereology, T-Norms

1 Introduction

If \( A, B \in S \subseteq \wp(S) \), with \( S \) being closed under intersection, then the quantity

\[
\nu(A, B) = \begin{cases} 
\frac{\#(A \cap B)}{\#(B)} & \text{if } B \neq \emptyset \\
1 & \text{if } B = \emptyset
\end{cases}
\]  

(K0)

can be interpreted in multiple ways including as conditional subjective probability, relative degree of misclassification, vague inclusion, majority inclusion function and inclusion degree. In this it is possible to replace intersections with commonality operations that need not be idempotent or commutative or even
associative. Many generalizations of this function are known in the rough set, belief theory, subjective probability, fuzzy set and ML literature.

Generalized versions of rough inclusion functions have also been used as primitives for defining concepts of general approximation spaces in [1]. These are particularly significant for one of the rough mereological approaches [2]. An overview of generalizations of the concept of rough inclusion functions can be found in [3,4].

Granules or information granules are often the minimal discernible concepts that can be used to construct all relatively crisp complex concepts in a vague reasoning context. Such constructions typically depend on a substantial amount of assumptions made by the theoretical approach employed in question [5,6,7].

In the present author’s axiomatic approach to granularity [5,6,7,10], fundamental ideas of non-intrusive data analysis have been critically examined and methods for reducing contamination of data (through external assumptions) have been proposed. The need to avoid over-simplistic constructs like rough membership and inclusion functions have been stressed in the approach by her. New granular measures that are compatible with rough domains of reasoning, and granular correspondences that avoid measures have also been invented in the papers. These granular measures can be improved further with the following goals in mind:

- to improve the measures so that they can integrate seamlessly with rough and hybrid semantic domains,
- to provide a contamination free measure that goes beyond the limited heuristics of dominance based rough sets, and
- to provide a reasonable basis for translating reasoning across different approaches to vagueness, mereology and uncertainty.

In this research paper, these goals are achieved to a substantial extent. Specifically, the core of Skowron-Polkowski style mereology is generalized.

2 Background

For basics of rough sets, the reader is referred to [11,12]. The granular approach due to the present author can be found in [5,6].

Granular operator spaces and related variants are not necessarily basic systems in the context of applied general rough sets. They are mathematically accessible powerful abstractions for handling semantic questions, formulation of semantics and the inverse problem. As many as six variants of such spaces have been defined by the present author - all these can be viewed as special cases of a set theoretic and a relation-theoretic abstraction with abstract operations from a category-theory perspective.

**Definition 1.** A High General Granular Operator Space (GFSG) $S$ shall be a structure of the form $S = (S, G, l, u, P, \leq, \lor, \land, \top)$ with $S$ being a set, $G$ an admissible granulation (defined below) for $S$ and $l, u$ being operators $S \rightarrow S$ satisfying
An element $x \in \mathbb{S}$ is said to be lower definite (resp. upper definite) if and only if $x^l = x$ (resp. $x^u = x$) and definite, when it is both lower and upper definite. $x \in \mathbb{S}$ is also said to be weakly upper definite (resp weakly definite) if and only if $x^u = x^{uu}$ (resp $x^u = x^{uu}$ & $x^l = x$). Any one of these five concepts may be chosen as a concept of crispness.

In granular operator spaces and generalizations thereof, it is possibly easier to express singletons and the concept of rough membership functions can
be generalized to these from a granular perspective. For details see [6,8]. Ev-
every granular operator space can be transformed to a higher granular operator space, but to speak of this in a rigorous way, it is necessary to define related morphisms and categories [6].

**Proposition 1.** The following interrelations between the different types of granular operator spaces hold:

- Every higher granular operator space (HGOS) is a high granular operator space (FSG)
- Every higher general granular operator space (HGGS) is a high general granular operator space (GFSG)
- Every higher granular operator space (HGOS) is a higher general granular operator space (HGGS).

**Rough Objects** A rough object cannot be known exactly in the rough semantic domain, but can be represented in a number of ways. The following representations of rough objects have been either considered in the literature (see [5,13,6]) or are reasonable concepts that work in the absence of a negation-like operation:

- **RD** Any pair of definite elements of the form \((a, b)\) satisfying \(a < b\)
- **RP** Any distinct pair of elements of the form \((x_l, x_u)\).
- **RIA** Interval of the form \((x_l, x_u)\).
- **RI** Interval of the form \((a, b)\) satisfying \(a \leq b\) with \(a, b\) being definite elements.

### 2.1 T-Norms, S-Norms

Triangular norms (t-norms) and s-norms (or t-conorms) are well known in the literature on fuzzy logic and multi-criteria decision making [14]. They are respectively used for expressing conjunctions and disjunctions in suitable logics.

**Definition 4.** A t-norm is a function \(t : [0,1]^2 \rightarrow [0,1]\) that satisfies all of the following four conditions:

1. \((\forall a)\ t(a, 1) = a.\ (\forall a, b)\ t(a, b) = t(b, a)\)
2. \((\forall a, b, c)\ |b \leq c\rightarrow t(a, b) \leq t(a, c)\).
3. \(t(a, t(b, c)) = t(t(a, b), c)\).

An s-norm \(s : [0,1]^2 \rightarrow [0,1]\) is a function for which there exists a t-norm \(t\) such that:

\[(\forall a, b)\ s(a, b) = 1 - t(1 - a, 1 - b)\).

In other words, t-norms are commutative monoidal order compatible operations with unit 1 on the unit interval \([0,1]\). s-norms are also commutative monoidal order compatible operations on \([0,1]\), but with unit 0. The following are popular pairs of t- and s-norms:

- \(t_M(a, b) = \min(a, b)\) (Min t-norm); \(s_M(a, b) = \max(a, b)\) (Max s-norm)
• \( t_p(a, b) = a \cdot b \) (Product t-norm); \( s_p(a, b) = a + b - a \cdot b \) (Probabilist s-norm)
• \( t_l(a, b) = \max(a + b - 1, 0) \) (Łukasiwicz t-norm); \( s_l(a, b) = \min(a + b, 1) \)

Any left continuous t-norm \( t \) can be used to define unique residual implications: \( a \implies b = \sup\{c : t(c, a) \leq b\} \)

### 3 General Rough Inclusion Functions

In this section, the different known rough inclusion functions are generalized to high granular operator spaces of the form \( S = (\mathbb{S}, \mathcal{G}, \mathbb{L}, \mathbb{U}, \mathbb{P}, \leq, \lor, \land, \top) \). RIFs are correctly defined over power sets in [1,4], and subject to \( \mathbb{P} = \leq = \subseteq, \lor = \cup \) and \( \land = \cap \).

Consider the conditions,

\[
(\forall a, b)(\kappa(a, b) = 1 \iff \mathbb{P}ab) \quad \text{(R1)}
\]

\[
(\forall a, b, c)(\kappa(b, c) = 1 \implies \kappa(a, b) \leq \kappa(a, c)) \quad \text{(R2)}
\]

\[
(\forall a, b, c)(\mathbb{P}bc \implies \kappa(a, b) \leq \kappa(a, c)) \quad \text{(R3)}
\]

\[
(\forall a, b)(\mathbb{P}ab \implies \kappa(a, b) = 1) \quad \text{(R0)}
\]

\[
(\forall a, b)(\kappa(a, b) = 1 \implies \mathbb{P}ab) \quad \text{(IR0)}
\]

\[
(\forall a)(\kappa(a, \bot) = 0) \quad \text{(RB)}
\]

\[
(\forall a, b)(\kappa(a, b) = 0 \implies a \land b = \bot) \quad \text{(R4)}
\]

\[
(\forall a, b)(a \land b = \bot \implies \kappa(a, b) = 0) \quad \text{(IR4)}
\]

\[
(\forall a, b)(\kappa(a, b) = 0 \& \mathbb{P}a \leftrightarrow a \land b = \bot) \quad \text{(R5)}
\]

\[
(\forall a, b, c)(\mathbb{P}a \& b \lor c = \top \implies \kappa(a, b) + \kappa(a, c) = 1) \quad \text{(R6)}
\]

These mostly correspond to the definition in [4]. \( \kappa_1 \) is RB, and \( \kappa_2 \) is R3 under the conditions mentioned.

**Proposition 2.** The following implications between the properties are easy to verify.

- If a FSG satisfies R1, then R3 and R2 are equivalent.
- R1 if and only if R0 and IR0 are satisfied.
- R0 and R3 imply R2.
- IR0 and R2 imply R3.
- IR4 implies R3.
- R5 if and only if R4 and IR4.
- When complementation is well defined then R0 and R6 imply IR4.
- When complementation is well defined then IR0 and R6 imply R4.
- When complementation is well defined then R1 and R6 imply R5.

**Definition 5.** By a general rough inclusion function (RIF) on a FSG \( S \) will be meant a function \( \kappa : (\mathbb{S})^2 \rightarrow [0, 1] \) that satisfies R1 and R2. A general quasi rough inclusion function (qRIF) will be a map \( \kappa : (\mathbb{S})^2 \rightarrow [0, 1] \) that satisfies R0 and R3. While a general weak quasi rough inclusion function (wqRIF) will be a map \( \kappa : (\mathbb{S})^2 \rightarrow [0, 1] \) that satisfies R0 and R2.
Proposition 3. In a FSG $S$, every RIF is a qRIF and every qRIF is a wqRIF.

### 3.1 Specific Weak Quasi-RIFs

The following functions have been studied in [4] and have been used to define concepts of approximation spaces

\[ \nu_1(A, B) = \begin{cases} \frac{\#(B)}{\#(A \cup B)} & \text{if } A \cup B \neq \emptyset \\ 1 & \text{otherwise} \end{cases} \quad (K1) \]

\[ \nu_2(A, B) = \frac{\#(A^c \cup B)}{\#(1)} \quad (K2) \]

If $0 \leq s < t \leq 1$, and $\nu : mS^2 \rightarrow [0, 1]$ is a RIF, then let $\nu_{s,t}^\nu : mS^2 \rightarrow [0, 1]$ be a function defined by

\[ \nu_{s,t}^\nu(A, B) = \begin{cases} 0 & \text{if } \nu(A, B) \leq s \\ \frac{\nu(A,B) - s}{t - s} & \text{if } s < \nu(A, B) < t, \\ 1 & \text{if } \nu(A, B) \geq t \end{cases} \quad (Kst) \]

Proposition 4. In general, $\nu_{s,t}^\nu$ is a weak quasi RIF and $\nu_{s,1}^\nu$ is a RIF.

### 4 Granular Rough Inclusion Functions

To capture rough reasoning, it is important to avoid using objects and operations that are actually accessible only in an exact perspective of the context. Every concept of a weak quasi rough inclusion function considered in the previous section is flawed relative to this perspective.

The concept of contamination relates to the contexts under consideration and many levels of contamination reduction can be of interest in practice. If all objects permitted in a domain are approximations of objects in the classical semantic domain, then the domain in question would be referred to as a weak rough domain. One way of reducing contamination can be through the use of approximations or representations of rough objects instead of sets that are not perceived as such in a weaker rough or rough semantic domain respectively. But this can be done in a number of ways because an object of the classical domain can be expressed by a number of approximations. Therefore, it is reasonable to assume that measures include all or most important possibilities.

Over a general granular operator space or a higher order variant thereof the following conceptual variants of rough inclusion functions can be defined.

Definition 6. If $S$ is a set HGOS, $A, B \in S$, $\sigma, \pi \in \{l, u\}$ and the denominators in each of the expressions is non zero, then let

\[ \nu_{\sigma\pi}(A, B) = \frac{\#(A^\sigma \cap B^\pi)}{\#(B^\pi)} \quad (\sigma\pi\text{-grif1}) \]
If \( \#(B^\tau) = 0 \), then set the value of \( \nu_{\sigma\tau} \) at the point to 1.

**Theorem 1.** In a set HGOS \( S \), all of the following hold (\( \alpha \) being any one of \( \ll, \lu, \ul \) or \( \uu \)):

\[
\begin{align*}
(\forall A, B) \nu_{\ll}(A, B) &\leq \nu_{\ul}(A, B) & \text{(ul1)} \\
(\forall A, B) \nu_{\lu}(A, B) &\leq \nu_{\uu}(A, B) & \text{(lu2)} \\
(\forall A, B, E) (A \subset E \rightarrow \nu_{\alpha}(A, B) &\leq \nu_{\alpha}(E, B)) & \text{(mo)} \\
(\forall A) \nu_{\lu}(A, A) &\leq \nu_{\ll}(A, A) = 1 = \nu_{\uu}(A, A) = \nu_{\ul}(A, A) & \text{(refl)} \\
(\forall A) \nu_{\alpha}(\bot, A) & = 1 & \text{(bot)} \\
(\forall A) (\top = \top^1 = \top^u \rightarrow \nu_{\alpha}(A, \top) & = 1) & \text{(top)}
\end{align*}
\]

**Proof.**
- \( \text{ul1} \) follows from \( (\forall A \in S) A^\ll \subseteq A^\ul \).
- Proof of \( \text{lu2} \) is similar.
- Since both \( \ll \) and \( \uu \) are monotonic and the denominator is invariant in \( \nu_{\alpha}(A, B) \leq \nu_{\alpha}(E, B) \), \( \text{mo} \) follows.
- Proof of \( \text{refl} \) is direct.

\( \square \)

This leads to

**Definition 7.** In a set HGOS \( S \), by the granular rough inclusion function of type-1 (GRIF-1) \( \zeta^\nu \) we will mean a function \( \zeta^\nu : S^2 \rightarrow \mathbb{M}_Q \) (\( \mathbb{M}_Q \) being the set of \( 2 \times 2 \) matrices over the set \( Q^+ \cap [0, 1] \), of positive rationals less than 1) defined for any \( (A, B) \in S^2 \) as below:

\[
\zeta^\nu(A, B) = \begin{pmatrix}
\nu_{\ll}(A, B) & \nu_{\lu}(A, B) \\
\nu_{\lu}(A, B) & \nu_{\uu}(A, B)
\end{pmatrix}
\]

The set of \( 2 \times 2 \) matrices over the field of rationals forms a noncommutative ring, but a direct interpretation of the ring operations is not possible in the context. Denoting an arbitrary t-norm by \( \otimes \) on the set \( Q^+ \cap [0, 1] \) and its dual s-norm by \( \oplus \), the following operations can be defined for any \( (a_{ij}), (b_{ij}) \in \mathbb{M}_Q \):

\[
\begin{align*}
(a_{ij}) \otimes (b_{ij}) &:= (a_{ij} \oplus b_{ij}) & \text{(disjunction)} \\
(a_{ij}) \otimes (b_{ij}) &:= (\bigoplus_k a_{ik} \otimes b_{kj}) & \text{(conjunction)}
\end{align*}
\]

More generally,

**Definition 8.** In a high granular operator space \( S \), if \( \tau \) is a wqRIF, then the granular weak quasi rough inclusion function \( \zeta^\tau \) (GwqRIF) induced by \( \tau \) will be a mean a function \( \zeta^\tau : S^2 \rightarrow \mathbb{M}_Q \) defined for any \( (A, B) \in S^2 \) as below:

\[
\zeta^\tau(A, B) = \begin{pmatrix}
\tau(A^\ll, B^\ll) & \tau(A^\ll, B^\ul) \\
\tau(A^\ul, B^\ll) & \tau(A^\ul, B^\ul)
\end{pmatrix}
\]
In general, these definitions do not necessarily involve definite objects.

**Proposition 5.** In a high granular operator space \( S \) if \( \tau \) is a weak quasi RIF and \( A \) is a definite element then

\[
\zeta^\tau(A, B) = \left( \frac{\tau(A, B^l)}{\tau(A, B^l)} \right) \tau(A, B^u)
\]

**Proposition 6.** If \( A^l = A = A^u \) and \( B^l = B = B^u \), then

\[
\zeta^\tau(A, B) = \left( \frac{\tau(A, B)}{\tau(A, B)} \right)
\]

**Proposition 7.** If \( B^l = B = B^u \), then

\[
\zeta^\tau(A, B) = \left( \frac{\tau(A^l, B)}{\tau(A^u, B)} \right)
\]

The above three propositions motivate the following definition:

**Definition 9.** In a high granular operator space \( S \) if \( \tau \) is a weak quasi RIF and

- if \( A \) is a definite element, then let \( \xi^\tau(A, B) = (\tau(A, B^l), \tau(A, B^u)) \) and
- if \( B \) is a definite element, then let \( \omega^\tau(A, B) = (\tau(A^l, B), \tau(A^u, B)) \)

\( \xi \) and \( \omega \) will respectively be referred to as the 1-certain GRIF (1GwqRIF) and 2-certain GRIF (2GwqRIF) induced by the weak quasi RIF \( \tau \).

This suggests that GwqRIFs may be viewed as semilinear transformations of 2GwqRIFs and 1GwqRIFs. Clearly there is much to be fixed for this view that pairs of inclusion measures of objects in crisp objects correspond to inclusion measures of objects in other not necessarily crisp objects or that pairs of inclusion measures of crisp objects in not necessarily crisp objects correspond to inclusion measures of objects in other objects.

**Theorem 2.** \( M_Q \) along with the operations \( \otimes, \oplus \) and neutral elements, 0 and 1 forms a semiring with unity when \( \otimes \) coincides with the maximum s-norm or when \( \oplus \) is the minimum t-norm operation.

**Proof.**
- \( \oplus \) distributes over \( \otimes \) if and only if \( \otimes \) is the maximum s-norm.
- Dually \( \otimes \) distributes over \( \oplus \) if and only if \( \oplus \) is the minimum t-norm.
- For any three elements \( (a_{ij}), (b_{ij}) \) and \( (c_{ij}) \) in \( M_Q \),

\[
\bigotimes \left( \bigoplus_{k} [a_{ik} \otimes (b_{kj} \oplus c_{kj})] \right) = \bigoplus_{k} (a_{ik} \otimes b_{kj}) \bigoplus (a_{ik} \otimes c_{kj})
\]

the last step holds whenever distribution of \( \otimes \) over \( \oplus \) holds.

The dual part of the theorem can be proved in a similar way. □
Discussion  The converse of this result holds under suitable constraints on the domain. Distributivity is not essential for the central purpose of this paper. The theorem provides yet another way of deciding on when a fuzzy strategy can possibly mimic a granular rough set approach in a transparent way because

- representation of GwqRIFs through 2GwqRIFs need not hold in general,
- but such representation can possibly be approximated through choice of t-norms and s-norms,
- choice of t-norms and s-norms correspond to a fuzzy reasoning strategy, and
- all this is reasonable from a general inclusion function perspective.

4.1 Extended Set-Theoretic Mereology

Definition 10. By the natural partial order on the set $M_Q$ will be meant the relation $\preceq$ defined by

$$(a_{ij}) \preceq (b_{ij})$$

if and only if $$(\forall i, j) \ a_{ij} \leq b_{ij}$$

Definition 11. In a high granular operator space $S$, an element $A$ will be $\tau$-included in $B$ ($A \subseteq_{\tau} B$) if and only if $\tau \preceq \zeta^\tau(A, B)$ for a GwqRIF $\zeta$.

Theorem 3. In a high granular operator space $S$ with $\tau$ being a RIF, all of the following hold:

$$(\forall A, B, C)(A \subseteq_{\tau} B \land B \subseteq_q C \rightarrow (\exists h) \ h \preceq \tau \land h \preceq q \land A \subseteq_h C)$$

$$(\forall A, B)(A \subseteq_h B \neq \bot \land 0 \prec h \rightarrow (\exists q) \ B \subseteq_q A \land 0 \prec q)$$

$$(\forall A, B)(PAB \land C \subseteq_h A \rightarrow (\exists r) \ h \preceq \tau \land C \subseteq_r B)$$

Proof.  
- The first property is a consequence of the definition of $\zeta^\tau$ and the properties of the approximations assumed. It includes the case with $h = \bot$. So the property holds always and even when $\tau$ is a wqRIF and not a RIF.
- $0 \prec h$ yields at least one of the entries in the matrix is non zero. This means some nonempty granules are part of the upper approximations of $A$ and $B$. This in turn yields the existence of a $q$ satisfying $0 \prec q$ and $B \subseteq_q A$. Condition R1 is assumed in this.
- The third property holds if condition R0 is satisfied by $\tau$. This happens because it restricts the possible values of $\zeta^\tau(A, B)$.

Clearly $\subseteq_\tau$ is more general than the parthood predicate of RIF based rough mereology and its fuzzy variants [12]. The associated logics that differ substantially from [15] will appear separately for reasons of space.
5 Aspects of Application Contexts

In this section aspects of two distinct application contexts are considered. This example shows that granular RIFs can be way better than RIFs perceived in the Skowron-Polkowski style mereological approach.

For broad overviews of computational linguistics, the reader is referred to [16,17]. Natural languages expressed in a written script can be viewed as a set of strings in alphabets that are classifiable into words, sentences, clauses, phrases and other linguistic categories with the help of a rule set based on occurrence of particular distinguished symbols like white space and punctuation marks. In probabilist approaches these are represented through linear n-grams subject to units being characters, words, or through syntactic n-grams. For example, a linear 5-gram of words would be a sequence of five words. Syntactic n-grams, include those based on dependency relations among parts and part of speech n-grams (that are defined as subsequences of contiguous overlapping part-of-speech sequences with text size n).

Many problems of computational linguistics involve some method of identifying similar expressions in the form of n-grams in the context in question. For example, the problem of referring expression generation (REG) [18] concerns the production of a description of an entity (from a dynamic dataset) that enables the hearer to identify that entity in a given context. The first step towards solving such problems concern the selection of a suitable form of referring expression. Often it is about descriptions like the white four wheeler or a big cat or the walking bovine divinity.

RIFs can be used to reduce statements of the form A is similar to B to A is roughly included to the degree r in B, or to the form the degree of inclusion of A in B is r. A far better idea would be to use GRIFS in the scenario because it is easier to reason with relatively definite approximations of A and B.

Functional approximations of the predicate includes the meaning of and those having the form all meanings of A are included in B are also of interest. These can be handled through choice inclusive lower and upper approximations. Given a set of potential synonyms of a word, it is easy to see that even the condition \( A^{\uparrow L} = A^{\uparrow L} \) can fail.

5.1 Weights and Orders

The act of regarding all attributes as having equal value in the constructive description of objects is known to be problematic. In dominance based rough sets [19], this is addressed partially through orders and order based ranking of attributes. The approach is also related to theory of pairwise comparison. When specific covers of the attribute set are of interest, multiple weights may be assigned to attributes based on the element of the cover they belong to. For example, if attribute x belongs to A and B that are in the cover \( \delta \), then it can be reasonable to assign distinct numeric weights \( w(x, A) \) and \( w(x, B) \) to the attribute x. In these scenarios, the weights corresponding to each element of the
cover form a chain. GwqRIFs can be easily extended to handle such weighted chain decomposition of the attribute set.

The present author is also involved in the analysis of a UN agency sponsored project on health care access of women in Kolkata urban conglomerate. A few health care indices (at different stages of integration) that rely on two levels of weighting and chain decomposition of the attribute set have been proposed by her in a forthcoming paper.

5.2 Connection with Generalized Probability Theories

While the function mentioned in the introduction can be read as something analogous to conditional probability and Bayesian methods using RIFs justified (see for example [20], it is known that many theoretical concepts cannot be translated between general rough set and subjective probability theories [7]. In all kinds of probability theories, the concept of an exact object is subjective. If they are fixed in advance, then concepts of upper approximation are actualizable. If the generalized probability takes real values (or in a suitable ordered structure), then it is possible to define upper, lower and other approximations through constraints based on cut-off values [21]. As no proper granularity is admissible in a probability theory, a reasonable analogy remains elusive.

6 Remarks

In this research, the following have been achieved by the present author:

- the concept of generalized rough inclusion functions have been generalized to granular rough domains of discourse,
- specifically these have been generalized to higher granular operator spaces,
- an ideal representation has been proposed for the granular RIFs,
- the gap between theory and algorithms used in practice is reduced implicitly,
- a direction for generalizing the Polkowski-Skowron style of mereology to granular rough domains has been indicated, and applicability of the measures have been explored in brief.

This paves the way for generalizations of various rough set algorithms from the perspective rough reasoning. The ideal representation proposed motivates a number of existential and mathematical problems. In particular, the best t-norm and s-norm that attain the ideal representation can be interpreted as a fuzzy perspective that ensures the representation. These and connection of the approach with subjective probability and belief theory will be part of a forthcoming paper by the present author. New mereological logics, and few algorithms are also considered in the same paper by her.
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