A gaussian model of the dynamics of an inextensible chain

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In this work an approximated path integral model describing the dynamics of a inextensible chain is presented. To this purpose, the nonlinear constraints which enforce the property of inextensibility of the chain are relaxed and are just imposed in an average sense. This strategy, which has been originally proposed for semi-flexible polymers in statistical mechanics, is complicated in the case of dynamics by the extra dependence on the time variable and by the presence of nontrivial boundary conditions. Despite these complications, the probability function of the chain, which measures the probability to pass to a given initial conformation to a final one, is computed exactly. The Lagrange multiplier imposing the relaxed condition satisfies a complicated nonlinear equation, which has been solved assuming that the chain is very long.

INTRODUCTION

The Rouse’s \textsuperscript{1} and Zimm’s \textsuperscript{2} models are able to describe satisfactorily the dynamical behavior of a polymer in a solution \textsuperscript{3}. There are however physical situations, such as for instance the case of a chain pulled by strong forces in experiments of micromanipulation of DNA \textsuperscript{4–8}, whose correct interpretation requires that the chain is inextensible. To implement the property of inextensibility, it is necessary to introduce constraints in the stochastic equations that govern its dynamics. To this purpose, one may use generalized coordinates or Lagrange multipliers. Alternatively, it is possible to add forces, which become strongly repulsive whenever the chain attempts to abandon the region of the phase space in which the constraints are satisfied. Applying these methods one arrives at equations which determine the dynamics of a chain in the presence of constraints in a very rigorous way, but are also very complicated. Their solution, as much as the practical calculation of physical quantities, require numerical simulations. An approach which leads to a simpler formulation of the problem consists in considering the chain as a system of particles held together by an elastic
potential. This potential must be chosen in such a way that in the equilibrium position the distance between two neighboring particles in the chain is equal to some constant \( a \neq 0 \). In the limit of infinite elastic constant, one obtains a discrete chain composed by particles connected together by massless segments of length \( a \). In the limit in which \( a \) goes to zero, while the number of particles becomes infinite, one obtains the probability function of a continuous chain, which measures the probability that the chain passes after a fixed time \( t_f \) from a given initial conformation to a given final conformation. It is possible to show that such probability function is equivalent to the partition function of a nonlinear field theory, which has been discussed in Ref. [9]. The appearance of nonlinearity is related to the inextensibility constraints which require that at any instant \( t \) and at each point of the chain, whose position is provided by the radius vector \( \mathbf{R}(t, s) \), the relation \( \left| \frac{\partial \mathbf{R}(t, s)}{\partial s} \right|_2^2 = 1 \) is satisfied. Here we have denoted with \( s \) the arc-length of the curve describing the spatial conformation of the chain. The nonlinear model of Ref. [9] has a relatively simple formulation in terms of path integrals and allows analytic calculations, such as for example that of the dynamical form factor in the semiclassical approximation [10]. Yet, it is important to have also a suitable approximation of that model which is able to simplify the functional Dirac delta function \( \delta \left( \left| \frac{\partial \mathbf{R}(s)}{\partial s} \right|_2^2 - 1 \right) \). This delta function appears in the model as a result of the inextensibility constraints and it is a very nonlinear term. A similar delta function has been simplified so far in the case of the statistical mechanics of a freely hinged chain, where the time variable is not present, using the substitution \( \delta \left( \left| \frac{\partial \mathbf{R}(s)}{\partial s} \right|_2^2 - 1 \right) \rightarrow e^{\frac{3}{2\pi} \int_0^L ds \left( \frac{\partial \mathbf{R}(s)}{\partial s} \right)^2} \) [11]. In the latter formula, \( L \) represents the total length of the chain. Unfortunately, the extension of the approach of Ref. [11] to polymer dynamics is not feasible. To obtain a Gaussian approximation of the functional delta function comparable to that proposed by the authors of [11] also in the case of dynamics, we use here an alternative strategy borrowed from the statistical mechanics of semi-flexible chains [13, 14]. The idea is to loose the condition that all arcs composing the chain must be inextensible and to require instead that the time averaged total length of the chain should fluctuate around a fixed average value \( L \). This means that the length of the chain is allowed to change in time as it happens in the Rouse model, but only in such a way that its average is equal to \( L \). This relaxed constraint is imposed in the probability function of the chain by introducing a Lagrange multiplier \( \lambda \), which is determined by minimizing the partition function as a function of \( \lambda \). With respect to the case of semi-flexible polymers, the situation is complicated by nontrivial boundary
conditions and the presence of the time. At the end we are able to compute exactly the explicit expression of the probability function, but for its minimization it is necessary to solve a complicated algebraic equation in \( \lambda \). We find its solution in the limit of very large values of the average total length of the chain \( L \) up to corrections of the order \( \frac{1}{L} \) included.

**THE GAUSSIAN APPROXIMATION**

We consider in this work the partition function of the model [9]:

\[
Z = \int_{\mathbf{R}(0,s)=\mathbf{R}_0(s)}^{\mathbf{R}(t_f,s)=\mathbf{R}_{f}(s)} \mathcal{D}\mathbf{R}(t,s) e^{-c \int_0^{t_f} dt \int_0^{L} ds \mathbf{R}'^2(t,s) \delta(\mathbf{R}'^2(t,s) - 1)}
\]

where \( \mathbf{R}(t,s) : [0, t_f] \times [0, L] \rightarrow \mathbb{R}^d \) is a two dimensional vector field with \( d \) components. The partial derivatives of \( \mathbf{R}(t,s) \) with respect to \( t \) and \( s \) are denoted as follows:

\[
\dot{\mathbf{R}}(t,s) = \frac{\partial \mathbf{R}(t,s)}{\partial t} \quad \mathbf{R}'(t,s) = \frac{\partial \mathbf{R}(t,s)}{\partial s}
\]

The parameter \( c \) appearing in the action (1) is given by:

\[
c = \frac{M}{2L} \frac{1}{2k_B T \tau}
\]

\( k_B \) denotes the Boltzmann constant, \( T \) is the fixed temperature of the thermal bath in which the chain fluctuates and \( \tau \) is the relaxation time which characterizes the rate with which the infinitesimal beads lose their speed due to friction. Finally, \( M \) is the total mass of the chain.

It has been shown in [9, 12] that this model describes the dynamics of an inextensible continuous chain of length \( L \) obtained by performing the continuous limit of the partition function of a freely jointed chain. The dynamics of the chain is followed during the period of time \( t \in [0, t_f] \). \( s \in [0, L] \) is the arc-length measuring the distance along the chain. The vector field \( \mathbf{R}(t,s) \) denotes the positions of the infinitesimal elements of length \( ds \) composing the chain at a given instant \( t \). \( Z \) has the meaning of the probability function which measures the probability that the chain during its fluctuations passes from the initial conformation \( \mathbf{R}_0(s) \) at the instant \( t = 0 \) to the final conformation \( \mathbf{R}_{f}(s) \) at the instant \( t = t_f \). The constraints enforcing the conditions that the length of the links connecting the beads should be constant become in the continuous limit the constraint:

\[
\mathbf{R}'^2(t,s) = 1
\]
which is imposed in Eq. (1) using a functional Dirac delta function. It is easy to check that this constraint implies that the length of every part of the chain is constant in time. Let us take for instance an arc of the curve $R(t, s)$ delimited by the points $R(t, s_1)$ and $R(t, s_2)$. Its length $\ell_{12}(t)$ is given by: $\ell_{12}(t) = \int_{s_1}^{s_2} ds |R'(t, s)|$. Due to Eq. (4) it is possible to write: $\ell_{12}(t) = \int_{s_1}^{s_2} ds = s_2 - s_1$. Thus

$$\dot{\ell}_{12}(t) = 0 \quad (5)$$

The probability function $Z$ in Eq. (1) should be completed by appropriated boundary conditions. We require that at the initial and final instants $0$ and $t_f$ the chain is in the configurations $R_0(s)$ and $R_f(s)$ respectively, i.e.:

$$R(t_f, s) = R_f(s) \quad (6)$$
$$R(0, s) = R_0(s) \quad (7)$$

Moreover, we suppose that the chain is closed, so that periodic boundary conditions will be chosen with respect to $s$:

$$R(t, s) = R(t, s + L) \quad (8)$$

In the following, it will be convenient to expand the boundary conformations in Fourier series:

$$R_f(s) = \sum_{n=-\infty}^{+\infty} e^{2\pi in\frac{s}{L}} b_n \quad (9)$$
$$R_0(s) = \sum_{n=-\infty}^{+\infty} e^{2\pi in\frac{s}{L}} a_n \quad (10)$$

$a_n$ and $b_n$ being constant vectors. The partition function of the model described by Eq. (1) has been computed in [9] in the semiclassical approximation. It is also possible to derive an expression of the dynamic structure factor of the chain always in the semiclassical approximation as shown in [10]. Despite these successes, the presence of the functional Dirac delta function in the right hand side of Eq. (1) makes the analytical treatment of the model complicated. For this reason, it would be nice to simplify it with a suitable approximation. For instance, in the case of statistical mechanics of polymers, where the time variable is not appearing, Edwards and Goodyear [11] have shown for $d = 3$ that:

$$\delta(R^2(s) - a^2) \sim e^{-\int_0^L ds \frac{a^2}{2s} R^2(s)} \quad (11)$$
While the extension of the results of [11] to dynamics is not straightforward, it is however possible to relax the constraint (4) requiring that it is satisfied only in an average sense. To this purpose, following a procedure borrowed from the statistical mechanics of semi-flexible polymers, see for instance [13] and [14, 15], we introduce a new parameter $\lambda$ and replace the probability function (1) with the following simplified one:

$$Z(\lambda) = \int_{R(0,s) = R_0(s)}^{R(t_f,s) = R_f(s)} \mathcal{D}R(t,s) e^{-\int_0^{t_f} dt \int_0^L ds [c \dot{R}^2(t,s) + \lambda (R^2(t,s) - 1)]}$$  \hspace{1cm} (12)

Next, the parameter $\lambda$ is determined by requiring that:

$$\frac{\partial Z(\lambda)}{\partial \lambda} = 0$$  \hspace{1cm} (13)

Clearly, the above equation is equivalent to:

$$\left\langle \int_0^{t_f} dt \frac{1}{L} \int_0^L ds [c \dot{R}^2(t,s) + \lambda (R^2(t,s) - 1)] \right\rangle = 1$$  \hspace{1cm} (14)

where

$$\left\langle \ldots \right\rangle = \int_{R(0,s) = R_0(s)}^{R(t_f,s) = R_f(s)} \mathcal{D}R(t,s) e^{-\int_0^{t_f} dt \int_0^L ds [c \dot{R}^2(t,s) + \lambda (R^2(t,s) - 1)]} (\ldots)$$  \hspace{1cm} (15)

In words, with (14) the functional integration over the conformations $R(t,s)$ has been extended to curves whose arcs do not longer satisfy the constant length condition of Eq. (5). Also the total length of the chain is allowed to change in time, but its average performed over all possible conformations and during the period of time $t_f$ must be equal to $L$.

**CALCULATION OF THE PROBABILITY FUNCTION $Z(\lambda)$**

The rest of this work is dedicated to the derivation of the probability function $Z(\lambda)$ and of the parameter $\lambda$, which up to now is the still unknown ingredient of Eq. (12). Unlike the case of semi-flexible polymers, we have to deal here with the nontrivial boundary conditions (6) and (7). To get rid of them, it is convenient to split the bond vectors $R(t,s)$ into two components:

$$R(t,s) = R_{cl}(t,s) + r(t,s)$$  \hspace{1cm} (16)

$R_{cl}(t,s)$ is a solutions of the classical equations of motion related to the action:

$$S = \int_0^{t_f} dt \int_0^L ds \left[ c \dot{R}_{cl}^2(t,s) + \lambda (R_{cl}^2 - 1) \right]$$  \hspace{1cm} (17)
appearing in the probability function of Eq. (12):

\[ c\dot{R}_{cl} + \lambda R''_{cl} = 0 \]  \hspace{1cm} (18)

\( R_{cl}(t, s) \) satisfies the boundary conditions (6) and (7):

\[ R_{cl}(t_f, s) = R_f(s) \quad R_{cl}(0, s) = R_0(s) \]  \hspace{1cm} (19)

\( r(t, s) \) represents instead the fluctuations around the classical conformation \( R_{cl}(t, s) \). \( r(t, s) \) satisfies the Dirichlet boundary conditions:

\[ r(t_f, s) = r(0, s) = 0 \]  \hspace{1cm} (20)

With respect to the variable \( s \) both \( R_{cl}(t, s) \) and \( r(t, s) \) satisfy periodic boundary conditions analogous to those of Eq. (8). After some calculations and without performing any approximation, it is possible to show that:

\[ Z(\lambda) = e^{-S(R_{cl})} e^{Lt_f \lambda} Z_{fluct}(\lambda) \]  \hspace{1cm} (21)

where the action \( S(R_{cl}) \) contains the contribution coming from the classical background conformations:

\[ S(R_{cl}) = \int_0^{t_f} dt \int_0^L ds \left( c\dot{R}_{cl}^2 + \lambda R_{cl}^2 \right) \]  \hspace{1cm} (22)

while \( Z_{fluct}(\lambda) \) is the partition function of the fluctuations:

\[ Z_{fluct}(\lambda) = \int \mathcal{D}r e^{-\int_0^{t_f} dt \int_0^L ds \left( c\dot{r}^2 + \lambda r'^2 \right)} \]  \hspace{1cm} (23)

First, we will compute \( S(R_{cl}) \). It is possible to check after some integrations by parts that

\[ S(R_{cl}) = c \int_0^L ds \left( R_f(s) \cdot \dot{R}_{cl}(t_f, s) - R_0(s) \cdot \dot{R}_{cl}(0, s) \right) \]  \hspace{1cm} (24)

with \( \dot{R}_{cl}(t_f, s) = \frac{\partial R_{cl}(t, s)}{\partial t} \bigg|_{t=t_f} \) and \( \dot{R}_{cl}(0, s) = \frac{\partial R_{cl}(t, s)}{\partial t} \bigg|_{t=0} \). The solution of the classical equations of motion (18) satisfying the boundary conditions of Eq. (19) is:

\[ R_{cl}(t, s) = \sum_{n=-\infty}^{+\infty} e^{2\pi i n \frac{s}{L}} \left( -\frac{a_n}{\sinh(\beta_n t_f)} \sinh(\beta_n(t - t_f)) + \frac{b_n}{\sinh(\beta_n t_f)} \sinh(\beta_n t) \right) \]  \hspace{1cm} (25)

where

\[ \beta_n = \sqrt{\frac{\lambda 2\pi n}{c L}} \]  \hspace{1cm} (26)
Substituting the expression of the classical solution $R_{cl}$ of Eq. (25) in Eq. (24), we obtain:

$$S(R_{cl}) = -cL \sum_{n=-\infty}^{+\infty} \beta_n \left[ -\frac{1}{\sinh(\beta_n t_f)} (b_n \cdot a_n + a_n \cdot b_n) + \frac{\cosh(\beta_n t_f)}{\sinh(\beta_n t_f)} (b_n \cdot b_n + a_n \cdot a_n) \right]$$

(27)

Next, we derive the partition function of the fluctuations $Z_{fluct}(\lambda)$. From Eq. (23) it turns out in $d = 3$ that:

$$Z_{fluct}(\lambda) = (\det'\Delta)^{-\frac{d}{2}}$$

(28)

where $\Delta$ is the differential operator:

$$\Delta = c\frac{\partial^2}{\partial t^2} + \lambda \frac{\partial^2}{\partial s^2}$$

(29)

The prime after the symbol of determinant in Eq. (28) means that the zero modes of the operator $\Delta$ should be projected out from the expression of the determinant. At this point we compute the eigenvalues of $\Delta$. To this purpose, we have to solve the equation:

$$\left( c\frac{\partial^2}{\partial t^2} + \lambda \frac{\partial^2}{\partial s^2} \right) \psi = -E\psi$$

(30)

Dirichlet boundary conditions with respect to the time variable are assumed according to Eq. (20). Moreover, $\psi$ should obey periodic boundary conditions with respect to $s$. It is possible to check that there exist solutions of Eq. (30) with such boundary conditions when $E$ takes the values:

$$E_{n,k} = 4\pi^2 \left( n^2 \frac{\lambda}{L^2} + k^2 \frac{c}{2t_f^2} \right) \quad n, k = 0, \pm 1, \pm 2, \ldots$$

(31)

These solutions are the eigenfunctions:

$$\psi_{n,k}(t, s) = \sum_{n=-\infty}^{+\infty} e^{2\pi i n s / L} \psi_{n,k}(t)$$

(32)

where

$$\psi_{n,k}(t) = A \sinh \left[ t \sqrt{\frac{1}{c} \left( \frac{4\pi^2 n^2}{L^2} + E_{n,k} \right)} \right]$$

(33)

and $A$ is a normalization constant. Excluding the trivial zero mode occurring for $n = k = 0$, we find that

$$\det'\Delta = \prod_{n,k \geq 0} \left[ 4\pi^2 \left( n^2 \frac{\lambda}{L^2} + k^2 \frac{c}{2t_f^2} \right) \right]$$

(34)
In the above equation the prime in the product $\prod'_{n,k \geq 0}$ means that the case $n = k = 0$, which corresponds to the excluded zero mode, should be omitted. Now we rewrite $\det'\Delta$ as follows:

$$\det'\Delta = \left( \prod_{n,k \geq 0} \frac{\lambda}{L^2} \right) \prod'_{n,k \geq 0} \left[ 4\pi^2 \left( n^2 + \frac{cL^2k^2}{2t_f^2\lambda} \right) \right]$$ (35)

Using the $\zeta$–function regularization it is possible to show that $\prod'_{n,k \geq 0} \lambda L^2 = \left( \frac{\lambda}{L^2} \right)^{\frac{5}{4}}$. Thus:

$$\det'\Delta = \left( \frac{\lambda}{L^2} \right)^{\frac{5}{4}} \prod'_{n,k \geq 0} \left[ 4\pi^2 \left( n^2 + \frac{cL^2k^2}{2t_f^2\lambda} \right) \right]$$ (36)

The remaining semi-infinite product has been evaluated by several authors, see for instance [16–20]. Here we present just the result of the calculations:

$$\prod'_{n,k \geq 0} \left[ 4\pi^2 \left( n^2 + \frac{cL^2k^2}{2t_f^2\lambda} \right) \right] = \exp \left( \ln \tau - \frac{\pi \tau}{12} + \sum_{n>0} \ln (1 - e^{-2\pi n\tau}) \right)$$ (37)

where

$$\tau^2 = \frac{2t_f^2\lambda}{cL^2}$$ (38)

Summarizing, the partition function of the fluctuations given in Eq. (28) becomes:

$$Z_{\text{fluct}}(\lambda) = \exp \left[ -\frac{15}{8} \ln \left( \frac{\tau^2 c}{2t_f^2} \right) - \frac{3}{2} \ln \tau + \frac{\pi \tau}{8} - \frac{3}{2} \sum_{n>0} \ln (1 - e^{-2\pi n\tau}) \right]$$ (39)

Putting together the results of Eqs. (27) and (39), the total probability function $Z(\lambda)$ of Eq. (21) may be rewritten as follows:

$$Z(\lambda) = e^{F(\lambda)}$$ (40)

where

$$F(\lambda) = -S(R_{cl}) + Lt_f \lambda - \frac{15}{8} \ln \left( \frac{\tau^2 c}{2t_f^2} \right) - \frac{3}{2} \ln \tau + \frac{\pi \tau}{8} - \frac{3}{2} \sum_{n>0} \ln (1 - e^{-2\pi n\tau})$$ (41)

Clearly, the condition (13) that determines $\lambda$ is equivalent to the condition $\frac{\partial F(\lambda)}{\partial \lambda} = 0$, i.e.:

$$\frac{\partial S(R_{cl})}{\partial \lambda} - Lt_f + \frac{21}{8} \frac{1}{\lambda} - \frac{\pi t_f}{8L} \sqrt{\frac{1}{2c}} \frac{1}{\sqrt{\lambda}} + \frac{3}{2} \sum_{n>0} \frac{2\pi n}{1 - e^{-2\pi n\tau}} e^{-2\pi n\tau} \frac{t_f}{L} \sqrt{\frac{1}{2c}} \frac{1}{\sqrt{\lambda}} = 0$$ (42)

As it stands, the above equation is too difficult to be solved analytically with respect to $\lambda$. For this reason, we assume that the chain is very long, i.e. $L >> 1$, so that it is possible
to expand $\lambda$ and the coefficients appearing in Eq. (42) in powers of $L^{-1}$. In doing that, we suppose that $\lambda$ remains finite in the limit $L \to +\infty$, so that:

$$\lambda \sim \lambda_0 + \frac{1}{L}\lambda_1 + \ldots$$  \hspace{1cm} (43)

This hypothesis on $\lambda$, which will be checked a posteriori, has a physical motivation. In fact, if $\lambda$ becomes infinite with the average chain length $L$, then conformations for which

$$\frac{1}{t_f} \int_0^{t_f} dt \frac{1}{L} \int_0^L ds \, R'^2 < 1$$

will be strongly preferred in the probability function of Eq. (12), a fact which is clearly unphysical.

At this point, we are ready to expand all the terms entering in Eq. (42) in powers of $\frac{1}{L}$. This expansion is almost trivial apart from the case of the two terms

$$I_1 = \frac{\partial S(R_{cl})}{\partial \lambda}$$  \hspace{1cm} (44)

and

$$I_2 = \frac{3}{2} \sum_{n>0} \frac{2\pi n}{1 - e^{-2\pi n\tau}} \sqrt{\frac{1}{c}} \frac{1}{L} \sqrt{\frac{c}{2} \frac{1}{\lambda_0^2}}$$  \hspace{1cm} (45)

$I_1$ depends on $\lambda$ implicitly through the coefficients $\beta_n$, while $I_2$ has an explicit dependence on $\lambda$ together with an implicit dependence through the coefficient $\tau$. Provided Eq. (43) is fulfilled \[21\], we note that both parameters $\beta_n$ and $\tau$, being proportional to $\frac{1}{L}$, are going to zero when $L$ goes to infinity. Using the above considerations and neglecting contributions of order $\frac{1}{L}$ or higher, we obtain:

$$I_1 \sim 0$$  \hspace{1cm} (46)

$$I_2 \sim \frac{3}{8\pi t_f} \frac{L}{\sqrt{\frac{c}{2} \frac{1}{\lambda_0^2} - \frac{3}{4\lambda_0} - \frac{9}{16\pi t_f} \sqrt{\frac{c}{2} \frac{\lambda_1}{\lambda_0^2}}}}$$  \hspace{1cm} (47)

After completing the expansion in powers of $\frac{1}{L}$ of the remaining terms, Eq. (42) becomes:

$$L \left( \frac{3}{8\pi t_f} \sqrt{\frac{c}{2} \frac{1}{\lambda_0^2} - t_f} \right) + \left( \frac{15}{8} \frac{1}{\lambda_0} - \frac{9}{16\pi t_f} \sqrt{\frac{c}{2} \frac{\lambda_1}{\lambda_0^2}} \right) + O\left(\frac{1}{L}\right) = 0$$  \hspace{1cm} (48)

Equating to zero separately the coefficients accompanying different powers of $L$ we obtain two relations which are able to determine both $\lambda_0$ and $\lambda_1$:

$$\lambda_0 = \sqrt{\frac{9c}{128\pi^2 t_f^4}}$$  \hspace{1cm} (49)

$$\lambda_1 = \frac{5}{4t_f}$$  \hspace{1cm} (50)
As a result it is possible to write down the expansion of $\lambda$ which is valid for large values of $L$ up to the order $\frac{1}{L}$:

$$\lambda \sim \sqrt{\frac{9c}{128\pi^2 t_f^4}} + \frac{5}{L^4 t_f}$$

(51)

This, together with Eqs. (40–41) concludes the evaluation of the probability function (12).

**CONCLUSIONS**

In this article a gaussian approximation of the nonlinear model (1) has been presented, which describes the probability of a chain to pass from a given conformation to another. The approximation consists in replacing the local constraint (4) with a milder one, that requires only that during the fluctuations of the chain, its average length must be equal to $L$. To enforce this new constraint, the Lagrange multiplier $\lambda$ has been introduced. $\lambda$ has been determined exploiting the condition that, as a function of $\lambda$, the probability function $Z(\lambda)$ of Eq. (12) has a minimum. Eq. (14) shows that this condition is equivalent to impose the relaxed constraint. While it is possible to compute the probability function $Z(\lambda)$ exactly despite the complications due to the presence of nontrivial boundary conditions, its minimization requires the solution with respect to $\lambda$ of the complicated algebraic equation given in Eq. (42). This equation has been solved in the limit of very large values of $L$ up to the second order included, see Eq. (51). It is shown that $\lambda$ goes to zero for increasing values of the evolution time $t_f$. This means that the deviations of the length of the chain from the expected value $L$ increase with the increase of the time in which the chain is allowed to fluctuate.

The present approach avoids the difficulties involved with the functional delta function of the original probability function (1) and it is still able to take into account the inextensibility of the chain, at least in an approximated way. The probability function $Z(\lambda)$ of Eq. (12) with the relaxed constraint has been computed here exactly, see Eqs. (40–41), while the analogous probability function with the full constraint (1) could be derived only in semiclassical approximation. The approximated model of Eq. (12) may be used as a basis for further computations of physical observables, such as for instance the dynamic form factor of a chain.
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[21] More in general provided that $\lambda$ does not grow with increasing values of $L$ with power law $\lambda \propto L^{2+\epsilon}$ with $\epsilon \geq 0$. 