Survey of $J = 0, 1$ mesons in a Bethe-Salpeter approach

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(Dated: September 22, 2009)

The Bethe-Salpeter equation is used to comprehensively study mesons with $J = 0, 1$ and equal-mass constituents for quark masses from the chiral limit to the $b$-quark mass. The survey contains masses of the ground states in all corresponding $J^{PC}$ channels including those with “exotic” quantum numbers. The emphasis is put on each particular state’s sensitivity to the low- and intermediate-momentum, i.e., long-range part of the strong interaction.

PACS numbers: 14.40.-n, 12.38.Lg, 11.10.St
Keywords:

I. INTRODUCTION

Mesons offer a prime target for studies of various approaches to quantum chromodynamics (QCD), which is widely accepted as the quantum field theory of the strong interaction. While in terms of the number of constituents their appearance is simple at first glance, mesons provide a broad range of phenomena and challenges to both theory and experiment. On the theoretical side the key challenge is to understand mesons (and hadrons in general) as bound states of QCD’s elementary degrees of freedom, quarks and gluons. The various approaches used to provide direct or indirect insight regarding this problem are (relativistic) quark models (e.g. \cite{1, 2, 3, 4, 5, 6, 7, 8, 9} and references therein), reductions of the Bethe-Salpeter equation (e.g. \cite{10, 11, 12, 13} and references therein), lattice QCD (e.g. \cite{14, 15, 16, 17, 18, 19} and references therein), effective field theories (e.g. \cite{20} and references therein), and the Dyson-Schwinger approach used herein. On the experimental side, present-day challenges can be exemplified by the recent measurement of the pseudoscalar ground-state mass in the bottomonium system \cite{21}.

In the present work, mesons are studied by means of QCDs Dyson-Schwinger-equations (DSEs); for recent reviews, see \cite{22, 23}. The DSEs are an infinite set of coupled and in general nonlinear integral equations for the Green functions of a quantum field theory, which makes the approach fully nonperturbative. It can therefore be used to study prominent nonperturbative phenomena of QCD, namely dynamical chiral symmetry breaking and confinement, as well as bound states in a single framework. The latter are studied in this approach with the help of covariant equations. In particular, the Bethe-Salpeter equation (BSE) is used to describe a meson as a quark-antiquark system in QCD \cite{24}. Note that the analogous approach to baryons as systems of three spin-1/2 quarks is considerably more involved; a first realization of this problem has been achieved only recently \cite{25}. In the past, intermediate steps have been taken to allow for the same level of sophistication as in corresponding meson studies (for recent advances, see \cite{26, 27, 28, 29, 30} and references therein).

In principle one would aim at a complete, self-consistent solution of all equations, which is equivalent to a solution of the underlying theory. While this spirit can be held up in investigations of certain aspects of the theory (see, e.g. \cite{31, 32} and references therein), numerical studies of hadronic observables require a truncation of the infinite tower of equations. In practice this means the choice of a subset of equations which are solved self-consistently by neglecting or making sophisticated Ansätze for the Green functions whose DSEs are not solved explicitly.

A popular truncation for meson studies in the DSE approach, which is also used in this work, is the rainbow-ladder (RL) truncation for reasons of simplicity and for satisfying among others the axial-vector Ward-Takahashi identity (AVWTI). The AVWTI is a welcome restriction of the unknown quark-antiquark scattering kernel in the BSE and its satisfaction guarantees the correct implementation of chiral symmetry and its dynamical breaking. A possible symmetry-preserving nonperturbative truncation scheme \cite{33, 34} contains the RL truncation as the lowest order, various corrections to which can be systematically included in studies “beyond RL” \cite{35, 36, 37, 38, 39, 40, 41, 42}. Another recent approach aims at the direct construction of the symmetry-preserving kernel of the BSE from a general quark-gluon vertex \cite{43}. With chiral symmetry and its dynamical breaking correctly realized in this fashion and built into the calculation from the very beginning, one obtains a generalized Gell-Mann–Oakes–Renner relation valid for all pseudoscalar mesons and all current-quark masses \cite{44, 45, 46}. In particular, the pion becomes massless in the chiral limit.

Meson studies in such a setup have been carried out over a number of years with various levels of sophistication \cite{44, 45, 46, 48, 49, 50, 51, 52}. Among these different variants the setup of Ref. \cite{51} has been successfully applied to the properties of pseudoscalar and vector meson ground states, in particular electromagnetic form factors (see \cite{53} and references therein). Further applications include an exploratory study of hadronic meson decays \cite{54}.
calculations of diquark properties [53, 56] (since these correlations are of importance in baryon studies), and studies of radial meson excitations [40, 57, 58, 59, 60].

While early studies were conducted for light mesons, an extension to heavy-heavy mesons seemed natural [61, 62, 63], but required a change of method, since reaching the $b$-quark mass [64, 65] is only possible with proper numerical treatment.

While many aspects of mesons with $J = 0, 1$ have been investigated separately, a comprehensive collection and discussion of the corresponding spectra is still missing. In the present work, as a first step, meson masses are presented for mesons with equal mass constituents and all quantum numbers possible for $J = 0, 1$ for the cases of light, strange, charm, and bottom quark masses. The dependence of the masses on the parameters used in the interaction are explored throughout, which appears to be an important issue for any such calculation.

The paper is organized as follows: Sec. II lists the necessary ingredients for the calculation, the results are presented and discussed in Sec. III. The necessary details on the structure of the mesons’ Bethe-Salpeter amplitudes for all quantum numbers considered here are collected in an appendix. All calculations have been performed in Euclidean momentum space.

II. GAP EQUATION, BSE, AND INTERACTION

In RL truncation one studies a meson with total $q\bar{q}$ momentum $P$ and relative $q\bar{q}$ momentum $k$ by consistently solving two equations: the homogeneous, ladder-truncated $q\bar{q}$ BSE

$$\Gamma(k; P) = -\frac{4}{3} \int_q A(q) d^4q/(2\pi)^4$$

$$\Gamma(k; \eta) = -\frac{4}{3} \int_q A(q) d^4q/(2\pi)^4$$

The inverse quark propagator has the general form

$$S(p)^{-1} = \frac{4\pi^2 D}{\omega^6} e^{-s/\omega^2} + \frac{4\pi}{\Gamma/2\ln[\tau+(1+s/\Lambda^2)]} \frac{\tau}{\Lambda^2}$$

This Ansatz produces the correct perturbative limit, i.e., it preserves the one-loop renormalization group behavior of QCD for solutions of the quark DSE. As given in [67], $F(s) = [1-\exp(-s/[4m_\pi^2])]/s$, $m_\pi = 0.5$ GeV, $\tau = e^2 - 1$, $N_f = 4$, $\Lambda_{QCD}^2 = 0.234$ GeV, and $\gamma_m = 12/(33 - 2N_f)$.

$D$ and $\omega$ are in principle free parameters of the model. In [67] they were fixed together with the current-quark mass $m_q$ in Eq. (2) to the chiral condensate as well as the pion mass and leptonic decay constant. It emerges from the results presented there that the computed values for the chiral condensate, $m_\pi$, $f_\pi$, $m_\rho$, and $f_\rho$ change very little, if one varies $D$ and $\omega$ such that their product remains constant and $\omega$ lies in the range $[0.3, 0.5]$ GeV. As a result, one can interpret a setup with $D\omega = const.$ as a one-parameter model, which is essentially determined by the requirement to correctly implement chiral symmetry and its dynamical breaking as well as to obtain the chiral condensate and $f_\pi$ of the correct magnitude. To illustrate the difference in the coupling generated by the first term in Eq. (3), Fig. I shows the corresponding curves for three values of $\omega$ (and corresponding $D$ for the above interval boundaries and its center value). It is clear that the main difference between the curves lies in the low- and intermediate-momentum range, which corresponds to the long-range part of the interaction.

$$F(s) = [1-\exp(-s/[4m_\pi^2])]/s, m_\pi = 0.5 \text{ GeV},$$

$$\tau = e^2 - 1, N_f = 4, \Lambda_{QCD}^2 = 0.234 \text{ GeV}, \text{ and } \gamma_m = 12/(33 - 2N_f).$$
In the present study the same parameter range is investigated for mesons with all quantum numbers corresponding to spin 0 and 1. While in [51] the current-quark mass was slightly readjusted for each value of \( \omega \) in order to achieve the exact same results for \( m_{\pi} \), this is not done here, since it obscures the influence of \( D \) and \( \omega \) on the spectrum, which is the main point of the investigation. The parameters are fixed here as follows: the abovementioned range of \( \omega \in [0.3, 0.5] \) GeV is inherited from [51] together with the value of \( D \cdot \omega = 0.372 \) GeV\(^3\) used there. The remaining parameters, four current quark masses for the flavors \( u/d \), \( s \), \( c \), and \( b \), are then each fixed to the corresponding experimental vector-meson ground-state mass. This last step is motivated by the fact that for heavy quarkonia, the vector state is the best-known experimentally. For the \( s \bar{s} \) case, the vector state is again the better choice due to ideal SU(3)-flavor mixing in the vector case, since a corresponding pseudoscalar meson state does not appear experimentally and the RL-truncated BSE kernel does not contain flavor-mixing processes. To make everything consistent, the light-quark mass is then fixed to the \( \rho \) meson instead of the pion mass.

III. RESULTS AND DISCUSSION

Equations (1) and (2) are solved consistently in RL truncation using the interaction of Ref. [51]. As laid out in the introduction, this setup has been applied to numerous observables for pseudoscalar- and vector-meson ground states individually in the literature. In the present work, the new aspect is a consistent treatment of all meson states with \( J = 0, 1 \) and a comprehensive study of the dependence of these states on the parameters \( D \) and \( \omega \) of the model, more precisely with the restriction mentioned above, namely \( D \cdot \omega = \text{const.} \). As reported earlier in connection with radially excited states of pseudoscalar mesons in the same setup [46, 58], such a study can be used to draw conclusions about the effective range and pointwise form of the long-range part of the strong interaction between, in particular, also light quarks.

This is due to the observation that the \( \omega \)-independence observed for pseudoscalar- and vector-meson ground-state properties does not survive for radial excitations: for example, meson masses can vary by several hundred MeV over the range \( \omega \in [0.3, 0.5] \) GeV (see [46, 57, 59, 67]). Basically, it would therefore be possible to use such excited states to fix all parameters in the interaction completely and attempt a quantitative study and comparison to other approaches/experimental data. However, this is not the aim here: the present study is almost purely qualitative. It focusses simply on the effect of the long-range part of the strong interaction, encoded in the parameters of the present model, on states with various quantum numbers and quark masses. Results have been obtained for ground states with equal-mass constituents of each 0\( ^{PC} \) and 1\( ^{PC} \) channel, spanning the entire range from light to heavy quarks.

To illustrate the evolution of the various meson masses with the quark/pion mass, Fig. 2 shows the corresponding results of the calculation from the chiral limit to charmonium. The curves beyond that value continue to rise linearly as expected. The results for bottomonium are provided in Fig. 3.

Two observations from Fig. 2 are noteworthy. Firstly and most prominently, a comment on meson states with “exotic” quantum numbers is in order. Meson states with such quantum numbers, which are not available for a \( \bar{q}q \) state in quantum mechanics, appear naturally in a quark-antiquark Bethe-Salpeter equation. For example, 1\( ^{--} \) states have previously been studied and discussed using a separable BSE kernel in [70] and, for light quark masses, also using the interaction of Ref. [51] in [71]. Here they are included for completeness with the immediate remark...
that corrections to the masses of these states can be expected to be at least of the same order of magnitude as those for axial-vector mesons, whose masses are underestimated by several hundred MeV. Still, for the purpose of Fig. 3, their inclusion reveals interesting analogies discussed below.

The other observation from Fig. 2 to be made here is that the (purely RL-$\bar{q}q$) scalar state lies below the vector state in the chiral limit; the two switch positions at about the strange-quark mass, beyond which they confirm with the ordering observed experimentally. Due to the complicated situation for scalar mesons as well as the simplicity of RL truncation, a meaningful quantitative comparison of the scalar result at light quark masses with experimental data is beyond the present study.

Figure 3 provides the essential information for the conclusions presented from this study. It contains all results for meson states with $0^{PC}$ and $1^{PC}$ for the four relevant values of the quark mass as functions of the model parameter $\omega$. As has been mentioned above and is apparent from the figure, the vector-meson masses have been fixed to the corresponding experimental values for the central value of $\omega$. In accordance with the original observation in [51], pseudoscalar and vector masses depend on $\omega$ only very slightly. It should be stressed again here that the quark masses were not refitted for different values of $\omega$ in order not to obscure the $\omega$ dependence of the state. In a quark-model interpretation, with the quark-antiquark orbital angular momentum denoted by $L$, pseudoscalar

FIG. 3: (Color online) dependence of meson masses on $\omega$. Dotted lines correspond to experimental data [21, 69]. Note that there is no flavor mixing in RL truncation, i.e. there is no experimental number for an $\bar{s}s$ pseudoscalar meson.

FIG. 4: (Color online) dependence of meson leptonic decay constants on $\omega$. Dotted lines correspond to experimental data [69, 72]. Note that there is no flavor mixing in RL truncation, i.e. there is no experimental number for an $\bar{s}s$ pseudoscalar meson decay constant. Furthermore, measurements for the $\eta_b$ decay constant have not yet been reported.
and vector states correspond (mainly) to $L = 0$ states, whereas all other states under consideration here have higher $L$ and are therefore orbital excitations; the exotics are constructed using additional gluonic degrees of freedom and thus correspond to excitations as well. It is therefore not surprising that all other states in Fig. 3 share the characteristic $\omega$ dependence of radial excitations demonstrated earlier [46, 57, 59, 67]. An additional aspect of exotic states is helpful in understanding this: they are in fact radial excitations of their non-exotic counterparts in a more general setup where the restriction on a particular $C$ parity is lifted.

With this overall picture in mind, it is clear that quantitative studies in this approach must explore the parameter dependence of the results on their respective interaction. This, in turn, can lead to conclusions about the pointwise form of that interaction.

As a further illustration, Fig. 4 shows the $\omega$ dependence of the pseudoscalar- and vector-meson leptonic decay constants for spins $J = 0, 1$ for a sophisticated model of QCD using a rainbow truncation of the quark DSE and the ladder quark-antiquark BSE. Beyond various studies of aspects of meson properties in this setup the results presented existent in the literature, the results presented should be interpreted cautiously and in the context of the present qualitative study.

IV. CONCLUSIONS AND OUTLOOK

This paper presents a qualitative overview of meson spectra and decay constants for spins $J = 0, 1$ for a sophisticated model of QCD using a rainbow truncation of the quark DSE and the ladder quark-antiquark BSE. Beyond various studies of aspects of meson properties in this setup existent in the literature, the results presented here are comprehensive over the whole range of quark masses from the chiral limit up to the bottom quark and for all quantum numbers possible for $J = 0, 1$ including so-called “exotic” ones, which appear naturally in a Bethe-Salpeter treatment of the quark-antiquark system. Furthermore, the results’ dependence on the model parameters is explored systematically.

The results show a pattern where states corresponding mainly to $L = 0$ in the quark model show only a slight sensitivity to the long-range part of the strong interaction, whereas those corresponding to $L = 1$ or with exotic quantum numbers show a clear dependence, which is analogous to that discovered previously for radial meson excitations. As a consequence, all excitations can be related to the long-range details or form of the interaction under consideration, which in principle can allow a pointwise investigation of that interaction and makes these states a prime object for further studies.

Here only states with equal-mass constituents were studied. Subsequent investigations in this approach will, among other things, deal with higher spin states as well as states with unequal-mass constituents; in particular heavy-light systems are of interest, since they combine the two regimes of heavy and light quarks in a unique fashion, thus offering an opportunity to also study in the same detail the interaction between quarks of light and heavy masses.

APPENDIX: STRUCTURE OF THE BSA

The BSA $\Gamma^{(\mu)}$ of a meson as a bound state of a quark-antiquark pair depends on their total momentum $P$ as well as the relative momentum $q$. The appearance of the Lorentz index $\mu$ is related to the spin $J$ of the meson: for $J = 0$, the amplitude has no index, for $J = 1$ there is $\mu$.

In terms of Lorentz-invariant variables, the amplitudes depend on $P^2$, $q^2$, and $q \cdot P$. The spin structure is taken into account by the fact that $\Gamma$ is a $4 \times 4$ matrix in spinor space [24]. The corresponding basis can be built from products of Dirac-$\gamma$ matrices. The general dependence of the BSA on these variables can be written in terms of $N$ covariant structures $T_i$ and scalar amplitudes $F_i$; $i = 1, \ldots, N$:

$$\Gamma^{(\mu)}(P; q) = \sum_{i=1}^{N} T_i^{(\mu)}(P; q; \gamma) F_i(q^2, q \cdot P, P^2).$$

Note that for a bound-state amplitude — the solution of a homogeneous BSE — the the on-shell condition requires $P^2 = -M^2$ to be fixed. However, one varies $P^2$ in the solution process of the homogeneous BSE to find the above condition. In the corresponding inhomogeneous BSE one has $P$ and therefore also $P^2$ as a completely independent variable.

For $J = 0$, i.e. scalar or pseudoscalar mesons, the BSA contains four independent covariant structures. For a scalar, on can choose

$$T_1 = 1, \quad T_2 = i \gamma \cdot P, \quad T_3 = i \gamma \cdot q, \quad T_4 = i \sigma^{q \cdot P}$$

where $\sigma^{q \cdot P} := i/2 [\gamma \cdot q, \gamma \cdot P]$. Note that these covariants are in general neither orthogonal nor normalized in terms of the two four-momenta $q$ and $P$ (for example, one could have $T_3 = \frac{1}{\sqrt{q^2}} \gamma \cdot q$). The scalar product for general covariants is defined via the Dirac trace

$$\sum_{\langle \mu \rangle} \text{Tr}[T_i^{(\mu)} T_j^{(\mu)}] = t_{ij} f(i, j),$$

where for an orthogonal basis one has $t_{ij} = \delta_{ij}$ and $f(i, j)$ are functions of $q^2$, $P^2$, and $q \cdot P$, and the sum over $\mu$ is carried out only for $J = 1$. 
The scalar amplitudes \( F_i(q^2, q \cdot P, P^2) \) depend on two independent variables, since the total momentum-squared \( P^2 \) is fixed for an on-shell meson. The relative-momentum squared \( q^2 \) corresponds to a radial variable in Euclidean momentum space, while the scalar product \( qP \) can be understood in terms of the cosine \( z \) of an angle between \( q \) and \( P \) by writing \( q \cdot P = \sqrt{q^2 P^2} z \). The amplitudes \( F_i \) can be decomposed by a Chebyshev expansion as

\[
F_i(q^2, q \cdot P, P^2) = \sum_{j=0}^{\infty} j F_i(q^2, P^2) U_j(z), \quad (A.3)
\]

where \( U_j(z) \) are the Chebyshev polynomials of the second kind and the \( j F_i \) are the corresponding Chebyshev moments, which effectively only depend on \( q^2 \). An illustration of ground- and excited-state pseudoscalar meson Chebyshev moments can be found in [73].

A scalar meson with equal-mass constituents has the quantum numbers \( J^{PC} = 0^{++} \). The BSA as defined above with the covariants for the scalar case has \( J^L = 0^+ \), but is not restricted \( a \ priori \) to either \( C = +1 \) or \( C = -1 \). When using a momentum partitioning parameter of \( \eta = 1/2 \), one can obtain an amplitude with positive C-parity by restricting the C-parity for each \( F_i \) according to the value of \( C \) for the corresponding \( T_i \) (see also [52, 53]). For \( T_i \) odd under \( C \) one has to ensure that \( F_i \) is odd as well. This is done via the dependence on \( q \cdot P \), which is odd under \( C \). In the Chebyshev expansion defined in Eq. (A.3), this means that for an even/odd \( F \) one would keep only even/odd terms in the expansion. In this way, it is immediately clear how to construct an amplitude with the opposite, in some cases called “unnatural” charge-parity, namely by making all odd \( F \)s even and vice versa. In this way, the BSE can be used to study mesons with “exotic” quantum numbers (see also [54, 71]).

For the scalar basis, \( T_2 \) is odd under \( C \), the others are even. As a result, \( F_3 \) is an odd function of \( q \cdot P \), the other are even. With the above choice of covariants (and as mentioned above \( \eta = 1/2 \)), the Chebyshev moments of \( F_2 \) are purely imaginary, the others are real. One could unify all covariants’ behavior by making the modification

\[
T_2 = i \gamma \cdot P \rightarrow i q \cdot P \gamma \cdot P, \quad (A.4)
\]

after which all covariants are even and all amplitudes real. It should be noted here that in a more general setup, which is e.g. needed in the case of unequal-mass constituents, a restriction like the above is not possible, since such a state is not a C-parity eigenstate. In this case, both odd and even Chebyshev moments will contribute in general, i.e. the \( F_i \) will be complex. However, in the limiting case of equal constituent masses, the real/imaginary pattern described above is recovered also numerically.

The basis used for a pseudoscalar \( J^P = 0^- \) meson is

\[
T_1 = i \gamma_5 \quad T_2 = \gamma_5 \gamma \cdot P \quad T_3 = \gamma_5 \gamma \cdot q \quad T_4 = \gamma_5 \sigma q \cdot P \quad (A.5)
\]

where \( T_3 \) is odd under \( C \) and the others are even. As a result, for a state with \( J^{PC} = 0^- \), \( F_3 \) must be odd and the others even, and the opposite for the exotic \( J^{PC} = 0^+ \).

For \( J = 1 \) one has 12 independent covariant structures in the BSA. Since an on-shell (axial-)vector meson is transverse (i.e. \( P_2 \Gamma^\mu(P; q) = 0 \)), \( 8 \) (transverse) covariants remain. With the definitions of the transversely projected

\[
\gamma^\mu := \gamma^\mu - \frac{q^\mu}{P^2} P^\mu \quad (A.6)
\]

\[
\gamma^\mu := \gamma^\mu - \frac{q^\mu}{P^2} P^\mu \quad (A.7)
\]

one arrives at a simple basis for a \( J^P = 1^- \) state by choosing

\[
T_1^\mu = \gamma^\mu \quad T_2^\mu = q^\mu \gamma \cdot q \quad T_3^\mu = q^\mu \gamma \cdot P \quad T_4^\mu = -i \gamma^\mu \sigma q \cdot P - q^\mu \gamma \cdot P \quad T_5^\mu = i q^\mu \quad T_6^\mu = i (\gamma^\mu \gamma \cdot q - \gamma \cdot q \gamma^\mu) \quad (A.8)
\]

\[
T_7^\mu = i \gamma^\mu \gamma \cdot P \quad T_8^\mu = q^\mu \sigma q \cdot P \quad T_9^\mu \quad T_10^\mu \quad T_11^\mu \quad T_12^\mu \quad (A.9)
\]

For the axialvector \( J^P = 1^+ \) meson, the basis used can be easily constructed from the vector case above by multiplication with \( \gamma_5 \) in analogy to the \( J = 0 \) case. One has

\[
T_1^\mu = i \gamma_5 \gamma^\mu \quad T_2^\mu = i \gamma_5 q^\mu \gamma \cdot q \quad T_3^\mu = i \gamma_5 q^\mu \gamma \cdot P \quad T_4^\mu = (\gamma_5 \gamma^\mu \sigma q \cdot P - i \gamma_5 q^\mu \gamma \cdot P) \quad T_5^\mu = \gamma_5 q^\mu \quad T_6^\mu = \gamma_5 (\gamma^\mu \gamma \cdot q - \gamma \cdot q \gamma^\mu) \quad (A.9)
\]

For the axialvector case, both sets \( J^{PC} = 1^{+-} \) and \( J^{PC} = 1^{++} \) are “non-exotic”, i.e. they can be constructed in a constituent-quark model as a pure quark-antiquark state. \( T_3, T_5, T_7, \) and \( T_8 \) are odd under \( C \); to obtain \( J^{PC} = 1^{++} \) one needs the corresponding amplitudes to be odd functions of \( q \cdot P \), the others even. Again, for \( J^{PC} = 1^{+-} \) the situation is reversed.

**ACKNOWLEDGMENTS**

The author would like to acknowledge valuable discussions with M. Blank, G. Eichmann, C. D. Roberts, and M. Schwinzerl. This work was supported by the Austrian Science Fund FWF under project no. P20496-N16 and benefited from the computing resources of the Argonne National Laboratory LCRC.
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