Exploring the distance-redshift relation with gravitational wave standard sirens and tomographic weak lensing

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We explore how cosmological parameters can be constrained with gravitational wave standard sirens and weak gravitational lensing. In addition to auto-correlations of these observables, we take into account cross-correlations of the projected number density of gravitational wave sources and weak lensing convergence field. For weak lensing, we use tomography technique to efficiently obtain information of large-scale structure at wide ranges of redshifts. Combining all these correlations, we present a forecast of constraints on four cosmological parameters, i.e., Hubble parameter, matter density, the equation of state parameter of dark energy, and the amplitude of matter fluctuation. In the case of the upcoming surveys such as \textit{Euclid} for weak lensing and Einstein Telescope for gravitational waves, we can place a tight constraint on these parameters.

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I. INTRODUCTION

The first detection of gravitational wave (GW) signal from merging binary black holes (BH), GW150914, by Advanced Laser Interferometer Gravitational Wave Observatory (LIGO) provides us with the new probes into cosmology and astrophysics [1–2]. After four successful detections of GW signals from black hole mergers (GW151226 [3], GW170104 [4], GW170608 [5] and GW170814 [6]), the first detection of the GW signal from a neutron star (NS) binary is reported (GW170817) [7]. Several interferometers, e.g., KAGRA [8] and Advanced VIRGO [9], are in operation or under construction to aim for detection of more sources and better localization. Once this network is established, it enables us to search the GW sources for the whole sky with high sensitivity. Furthermore, more telescopes both on ground and in space, e.g., Einstein Telescope [10], eLISA [11, 12], and DECIGO [13] are planned to achieve unprecedented measurements of GW signals over wide ranges of frequency. These telescopes will enable us to detect large numbers of GW sources with accurate wave forms.

One of the important aspects of GW measurements is that from the observed wave form we can measure the amplitudes both at observer and source frames. Thus, we can infer the luminosity distance of the source (standard sirens). If the redshift of the GW source is known, we can investigate the geometry of the Universe through the distance-redshift relation. However, solely with GW observations, the redshift of the source can not be measured. One of methods to estimate the source redshift is to observe electro-magnetic (EM) counterpart of the GW event. For the NS binary event GW170817, the EM counterpart has been detected with optical imaging observation [14–16], but detecting a counterpart is still challenging due to the short time scale of GW events and large uncertainty of localization with current interferometers. On the other hand, without redshift information, the anisotropic distribution of GW sources can be used as a cosmological probe [17]. Similarly to the number density distribution of galaxies, we can naively expect that the number density of compact object binaries should reflect the large scale matter density distribution. Thus, statistics of GW source distribution such as two-point correlation functions can be used to probe into cosmology.

Though the GW source distribution itself is useful for cosmology, when combining another observable which redshift information is available, we can obtain the information about the distance-redshift relation indirectly. One of such candidates is the spatial distribution of spectroscopically observed galaxies [18]. Since the redshift of such galaxies are precisely determined, one can probe into the distance-redshift relation. However, there is a drawback of using the spectroscopic galaxy samples. In order to obtain cosmological information, we need to introduce a galaxy bias which relates the galaxy number density distribution with matter fluctuation. In most of measurements, the bias is treated as a free parameter and marginalized finally. This degrades the constraints on cosmological parameters. For better parameter determination we need another cosmological probe, where redshift information is available and robust to systematics. In this work, we focus on weak gravitational lensing (WL). One of advantages is that WL is an unbiased tracer of density fluctuation, which does not necessitate a bias parameter. However, since the observables of WL is a projected quantity, information of matter distributions at different redshifts are entangled. We can evade this problem with technique known as tomography [19]. The whole source galaxy samples can be divided into several bins according to photometric redshifts of source galaxies. Then one can construct observables of WL using galaxies in each redshift bin, and measure auto- and cross-correlations of observables. As a result, we can efficiently obtain information of matter distribution at var-

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ious redshifts.

Recently, various works are devoted to probing the distance-redshift relation utilizing standard sirens, e.g. auto-correlation of GW source distribution \textsuperscript{[17]}, cross-correlation between standard sirens and galaxy distributions \textsuperscript{[18]}. In this paper, we address the cross-correlation between tomographic weak lensing and GW source distributions. Similarly to the measurement of galaxy clustering, forthcoming weak lensing surveys cover large areas. Therefore, combining these measurements has a possibility to place a very tight constraint on cosmological models.

This paper is organized as follows. First, we give formulation of auto- and cross-correlations of tomographic weak gravitational lensing and source distribution of GW signals. Then, we forecast how cosmological parameters can be constrained with upcoming GW and lensing measurements. We adopt flat $\Lambda$ cold dark matter model, and cosmological parameters; Hubble parameter $H_0 = 100h \text{ km/s/Mpc} = 67.27 \text{ km/s/Mpc}$, the present day density parameters of cold dark matter and baryon $\Omega_{\text{c}}h^2 = 0.1198$, $\Omega_{\text{b}}h^2 = 0.2225$, the tilt and the amplitude of the scalar perturbation $n_s = 0.9645$, $A_s = 2.2065 \times 10^{-9}$, and the total mass of neutrinos $M_\nu = 0.06 \text{ eV}$ based on the measurements of the anisotropy of temperature and polarization of cosmic microwave background (TT, TE, EE+lowP) by Planck mission \textsuperscript{[20]}. There are derived parameters which will be used later; the total matter density parameter $\Omega_m = \Omega_\text{c} + \Omega_\text{b} = 0.3153$, and the amplitude of matter fluctuation at the scale of $8 h^{-1}$ Mpc $\sigma_8 = 0.831$. We assume that the neutrino component consist of two massless and one massive neutrinos.

II. FORMULATION

In this section, we formulate how one can compute the auto- and cross-correlations of the GW source number density and WL convergence field.

A. Gravitational wave sources

In the measurements of merging binaries of compact objects, the luminosity distances can be obtained from the wave form. However, the estimated luminosity distance can deviate from the true value due to several uncertainties, e.g., degeneracy with other parameters such as the mass of the compact objects or the inclination angle, and statistical fluctuation. We assume that the inferred luminosity distance $\hat{D}$ follows the log-normal distribution where the mean is the true one $D$,

$$p(\hat{D}|D) = \frac{1}{\sqrt{2\pi}\sigma_{\ln D}} \exp[-x^2(\hat{D}, D)],$$

where

$$x(\hat{D}, D) = \ln \frac{\hat{D} - \ln D}{\sqrt{2}\sigma_{\ln D}},$$

and we assume $\sigma_{\ln D} = 0.05$ for Einstein Telescope observation. In addition, the estimate of the luminosity distance is subject to weak gravitational lensing by intervening matter in the Universe. Since the object looks brighter due to the magnification effect, the luminosity distance becomes smaller compared with the case of no lensing. This effect can be expressed as,

$$D = \hat{D}(z)\mu^{-1}(\theta, z) \simeq \hat{D}(z)[1 - \kappa(\theta, z)],$$

where $\hat{D}$ is the luminosity distance computed in the flat Friedmann-Lemaître-Robertson-Walker metric. In the weak field limit, the magnification $\mu$ is approximated as $1 + 2\kappa$, where $\kappa$ is the convergence field. The convergence corresponds to the projected matter density contrast $\delta_m$ convolved with distance kernel,

$$\kappa(\theta, \chi) = \frac{3H_0^2\Omega_m}{2c^2} \int_0^\chi d\chi' \frac{\chi' \delta_m(\chi', \chi')}{a(\chi')},$$

where $\chi$ is comoving distance from the observer and $a$ is the scale factor. Hereafter, we adopt the comoving distance as the indicator of the comic time instead of the redshift. However, we can convert each other by the relation,

$$\chi(z) = \int_0^z \frac{dz'}{H(z')}.$$  

Then, let us consider the number density field of GW sources. We divide the whole sources according to the observed luminosity distance. For $i$th bin we select sources with $D_{i, \text{min}} < \hat{D} < D_{i, \text{max}}$. The number density field is obtained by projecting sources as

$$n_i^w(\theta) = \int_{D_{i, \text{min}}}^{D_{i, \text{max}}} \frac{d\chi}{d\chi} \chi^2 G_i(\chi)n_{\text{GW}}(\chi\theta, \chi),$$

where $\chi_H$ is the comoving distance to the horizon, $G_i(\chi, \theta)$ is the selection function,

$$G_i(\chi, \theta) \equiv \frac{1}{2} \left( \text{erfc}\{x(D_{i, \text{min}}, D(\chi))\} - \text{erfc}\{x(D_{i, \text{max}}, D(\chi))\} \right),$$

and $n_{\text{GW}}$ is the three-dimensional number density of GW sources. Since the modulation effect on the luminosity distance due to lensing is relatively small, one can Taylor expand the selection function as

$$G_i(\chi, \theta) \simeq G_i|_{D=\hat{D}} + \left. \frac{dG_i}{d\hat{D}} \right|_{D=\hat{D}} (D - \hat{D})$$

$$= \left[ \frac{1}{2} \text{erfc}\{x(D_{i, \text{min}}, \hat{D}(\chi))\} - \text{erfc}\{x(D_{i, \text{max}}, \hat{D}(\chi))\} \right]$$

$$+ \kappa(\theta, \chi) - \frac{1}{\sqrt{2\pi}\sigma_{\ln D}} \left\{ - \exp[-x^2(D_{i, \text{min}}, \hat{D}(\chi))] \right\} + \exp[-x^2(D_{i, \text{max}}, \hat{D}(\chi))]$$

$$\equiv S_i(\chi) + \kappa(\theta, \chi)T_i(\chi).$$
The averaged number density is expressed as
\begin{align}
\bar{n}_i^W &= \int_0^{\chi_H} d\chi \chi^2 S_i(\chi) \tilde{n}_{GW}(\chi) \\
&= \int_0^{\chi_H} d\chi \chi^2 S_i(\chi) T_{\text{obs}}(\chi) \tilde{n}_{GW}(\chi),
\end{align}
where \(T_{\text{obs}}\) is the duration of the observation and \(\tilde{n}_{GW}(\chi)\) is the rate density of detectable merger events. Since the convergence vanishes when averaged in angular space, only the first term in Eq. (8) remains.

We can construct the two-dimensional number density contrast of GW sources as
\begin{align}
\delta_i^W(\theta) &= \frac{n_i^W(\theta)}{\bar{n}_i^W} - 1 \\
&= \frac{1}{\bar{n}_i^W} \int_0^{\chi_H} d\chi \chi^2 S_i(\chi) \tilde{n}_{GW}(\chi) \delta_{GW}(\chi, \theta) \\
&\quad + \frac{1}{\bar{n}_i^W} \int_0^{\chi_H} d\chi \chi^2 T_{\chi}(\chi) \tilde{n}_{GW}(\chi) \kappa(\chi, \theta, \theta). \quad (9)
\end{align}
We can rewrite the second term and define a kernel as,
\begin{align}
\frac{1}{\bar{n}_i^W} \int_0^{\chi_H} d\chi \chi^2 S_i(\chi) \tilde{n}_{GW}(\chi) \delta_{GW}(\chi, \theta) &= \frac{1}{\bar{n}_i^W} \int_0^{\chi_H} d\chi \int_0^\chi d\chi' \chi^2 S_i(\chi) \tilde{n}_{GW}(\chi) W^\kappa(\chi; \chi') \delta_m(\chi, \theta, \chi') \\
&= \int_0^{\chi_H} d\chi W_i^G(\chi) \delta_m(\chi, \theta, \chi). \quad (10)
\end{align}

Similarly, we also define the kernel in the first term,
\begin{align}
\frac{1}{\bar{n}_i^W} \int_0^{\chi_H} d\chi \chi^2 S_i(\chi) \tilde{n}_{GW}(\chi) \delta_{GW}(\chi, \theta) &= \int_0^{\chi_H} d\chi \left( \frac{1}{\bar{n}_i^W} \int_0^{\chi_H} d\chi \chi^2 S_i(\chi) \tilde{n}_{GW}(\chi) b_{GW} \right) \delta_m(\chi, \theta, \chi) \\
&= \int_0^{\chi_H} d\chi W_i^G(\chi) \delta_m(\chi, \theta, \chi). \quad (11)
\end{align}

Here we assume the linear bias relation \(\delta_{GW} = b_{GW} \delta_m\) and the bias is absorbed in the kernel \(W_i^G\).

**B. Tomographic weak lensing**

WL has now been measured by optical surveys and enable one to constrain cosmological models (for comprehensive reviews, see Refs. [21, 22]). It gives rich information about the large-scale structures in the Universe. WL is characterized by convergence \(\kappa\) and shears \(\gamma_1\) and \(\gamma_2\). It is possible to transform the convergence into shears and vice versa. In this paper, we focus only on the convergence field. As is shown in Eq. (4), the convergence can be described as the projection of the matter density field, but in real surveys, the redshift distribution of source galaxies has a broad shape. Then, the observable is the one convolved with the source distribution,

\begin{align}
\kappa_i^G(\theta) &= \int_0^{\chi_H} d\chi p_i(\chi) \kappa(\theta, \chi) \\
&= \int_0^{\chi_H} d\chi W_i^G(\chi) \delta_m(\chi, \theta, \chi). \quad (12)
\end{align}

We can rewrite the second term and define a kernel as,
\begin{align}
\int_0^{\chi_H} d\chi \int_0^\chi d\chi' \chi^2 S_i(\chi) \tilde{n}_{GW}(\chi) W^\kappa(\chi; \chi') \delta_m(\chi, \theta, \chi') &= \int_0^{\chi_H} d\chi W_i^G(\chi) \delta_m(\chi, \theta, \chi). \quad (13)
\end{align}

where \(p_i(\chi)\) is the comoving distance distribution of source galaxies, and the kernel is given as
\begin{align}
W_i^G(\chi) &= \int_\chi^{\chi_H} d\chi' p_i(\chi') W^\kappa(\chi'; \chi) \\
&= \frac{3H_0^2 \Omega_m}{2c^2} \int_\chi^{\chi_H} d\chi' \frac{p_i(\chi')}{a(\chi')} \frac{\chi' - \chi}{\chi}. \quad (14)
\end{align}

This distribution is normalized as unity, i.e.,
\begin{align}
\int_0^{\chi_H} d\chi W_i(\chi) = 1. \quad (15)
\end{align}

The subscript \(i\) represents the label of the source samples. According to the photometric redshifts of the source galaxies, we can divide the whole sample with different redshift distributions. Thus, we can probe the evolution of structures. This technique is called as lensing tomography [19].

In addition to weak lensing effect, the shape of the galaxy is subject to the local tidal field. Since this tidal field is correlated with the large-scale structure as well, it modulates the observed convergence field. This effect is referred to as intrinsic alignment (IA) (for reviews, see Refs. [23, 24]). We quantify this effect based on nonlinear-linear alignment model [25, 27],
\begin{align}
\kappa_i^I(\theta) &= \int_0^{\chi_H} d\chi p_i(\chi) \left(-A_{IA} C_1 \rho_{\text{cr}} \frac{\Omega_m}{D_+} \right) \delta_m(\chi, \theta, \chi) \\
&= \int_0^{\chi_H} d\chi W_i^I(\chi) \delta_m(\chi, \theta, \chi), \quad (16)
\end{align}
where \(\rho_{\text{cr}}\) is the critical density, \(D_+\) is the linear growth factor which is normalized to unity at present, \(A_{IA}\) is a free parameter which determines the amplitude and \(C_1 = 5 \times 10^{-14} h^{-2} Mpc^{-3}\). This model has been applied to real data (see, e.g., Ref. [28]), and the dependence of the amplitude on redshift and source luminosity is shown to be very weak [29]. As a result, the
convergence field is observed as the sum of two contributions,
\[ \kappa_i = \kappa_i^G + \kappa_i^I. \]  

C. Auto- and cross-power spectra

Here, we construct power spectra of the GW source number density and WL. The angular power spectra are defined as,
\[ \langle X_{\ell m} Y_{\ell' m'} \rangle \equiv \delta_{\ell \ell'} \delta_{mm'} C_{XY}(\ell), \]
where \( X \) and \( Y \) are the coefficient of spherical harmonic expansion of either \( \delta^w_{\ell m} \) or \( \kappa_i \). The auto-spectra of GW source number density and convergence and their cross-spectra are given as
\[
C_{w, w_i}(\ell) = C_{s_i, s_i} + C_{s_i, t_j} + C_{t_i, s_i} + C_{t_i, t_j},
\]
\[
C_{l, l_i}(\ell) = C_{G_l, G_{l_i}} + C_{G_l, G_{l_i}} + C_{G_{l_i}, G_{l_i}},
\]
\[
C_{w, l_i}(\ell) = C_{s_i, G_{l_i}} + C_{s_i, G_{l_i}} + C_{t_i, G_{l_i}} + C_{t_i, t_{l_i}}.
\]

With the Limber’s approximation \[30, 31\], we can compute the spectra as
\[
C_{X, Y_i}(\ell) = \int_0^{\pi} d\chi \frac{W_X(\chi) W^Y(\chi)}{\chi^2} P_m \left( k + \ell + 1/2, \chi, \chi \right),
\]
where \( X, Y \in \{s, t, G, I\} \), kernels \( W_X \) are defined in Eqs. \[11\], \[12\], \[14\], and \[16\], and \( P_m(k, \chi) \) is the matter power spectrum. We use linear Boltzmann code \textsc{Camb} \[32\] to generate transfer function for total matter component. For our interested scales, the nonlinear evolution of the matter fluctuation is important, hence, we employ the \textsc{Halofit} scheme \[33\] to compute nonlinear matter power spectra adopting parameters in Ref. \[34\].

D. Covariance matrix

We assume the Gaussian covariance matrix,
\[
\text{Cov}[C_{UV}(\ell), C_{XY}(\ell')] = \frac{4\pi}{\Omega_a(2\ell + 1)\Delta \ell} \times [\hat{C}_{UX}(\ell) \hat{C}_{XY}(\ell) + \hat{C}_{UY}(\ell) \hat{C}_{XY}(\ell)],
\]
where \( \Omega_a \) is the area of the survey region, \( \Delta \ell \) is the width of the multipole bins and the subscripts \( U, V, X, \) and \( Y \) denote types of observables and redshift bins, i.e., \( w_i (i = 1, \ldots, N_w) \) and \( l_i (i = 1, \ldots, N_l) \). The shot noise in GW source number density and shape noise in WL are included as
\[
\hat{C}_{XY} = C_{XY} + \delta_{XY} N_X,
\]
where \( \delta_{XY} \) is the Kronecker delta which takes unity only when the types of observables and the bins of redshifts are the same and otherwise zero, and
\[
N_w_i = \frac{1}{n_i^w}, \quad N_l_i = \frac{\sigma_i^2}{n_i^w},
\]
where \( \sigma_i \) is the intrinsic variance of galaxy shape and \( n_i^w \) and \( n_i^w \) is the number density per steradian in the \( i \)th bin for weak lensing source galaxies and GW sources, respectively.

III. RESULTS

A. Surveys

Here, we characterize surveys for measurements of auto- and cross-spectra of GW source distributions and weak lensing.

First, we specify survey parameters for GW observation with Einstein Telescope. Based on the first observing run and first detection of the binary NS event by Advanced LIGO, the inferred binary BH merger rate density is \( 9-240 \text{ Gpc}^{-3} \text{ yr}^{-1} \) \[2\] and binary NS merger rate density is \( 320-4740 \text{ Gpc}^{-3} \text{ yr}^{-1} \) \[2\]. The merger rate density has a possibility to evolve with time \[35\]. For simplicity we assume the event rate density is \( \dot{n}_{GW} = 5 \times 10^{-6} \text{ h}^3 \text{ Mpc}^3 \text{ yr}^{-1} \) regardless of redshifts and the duration of observation is \( T_{\text{obs}} = 1 \text{ yr} \). For bias parameter, we parametrize it based on Refs. \[36, 37\], as
\[
b_{GW}(z) = b_{w1} + \frac{b_{w2}}{D_p(z)},
\]
where \( b_{w1} \) and \( b_{w2} \) are free parameters and marginalized in the analysis. For binning of luminosity distances, equivalently redshifts, we adopt the number of bins as \( N_z = 6 \) and equally spaced bins with respect to redshifts in the range of \( 0 < z < 2.7 \).

Next, let us consider weak lensing surveys. The survey area of weak lensing with \textit{Euclid} is taken as \( \Omega_s = 150000 \text{ deg}^2 \) and intrinsic variance of galaxy shape is \( \sigma_g = 0.22 \) \[35\]. The functional form of the source number density is assumed to be
\[
n(z) \propto \left( \frac{z}{z_0} \right)^2 \exp \left[ - \left( \frac{z}{z_0} \right)^{1.5} \right],
\]
where \( z_0 = 0.64 \), which roughly corresponds to the mean redshift \( z_{\text{mean}} = 0.9 \) \[35\]. This distribution is normalized as
\[
\int_{z_{\text{min}}}^{z_{\text{max}}} n(z) dz = n_0,
\]
where \( n_0 = 30 \text{ arcmin}^{-2} \) is the total source density, and the minimum (maximum) redshift is set as \( z_{\text{min}} = 0.1 \) \( (z_{\text{max}} = 2.5) \) \[35\]. Since \textit{Euclid} provides accurate photometric redshift, we ignore the scatters of photometric redshifts. Then, the number density in the \( i \)th lensing bin is given as,
\[
p_i(z) \propto \begin{cases} n(z) & (z_{i,\text{min}} < z < z_{i,\text{max}}) \\ 0 & \text{(otherwise)} \end{cases},
\]
TABLE I. Redshift binning.

| Bin | GW source distribution | Weak lensing |
|-----|------------------------|--------------|
| 1   | 0.3 < z < 0.7          | 0.10 < z < 0.52 |
| 2   | 0.7 < z < 1.1          | 0.52 < z < 0.72 |
| 3   | 1.1 < z < 1.5          | 0.72 < z < 0.90 |
| 4   | 1.5 < z < 1.9          | 0.90 < z < 1.11 |
| 5   | 1.9 < z < 2.3          | 1.11 < z < 1.39 |
| 6   | 2.3 < z < 2.7          | 1.39 < z < 2.50 |

Note that \( p_i(z) \) should be normalized as in Eq. (15). Here, we consider six lensing bins \((N_l = 6)\). We determine the bin configuration so that each bin contains the same number of source galaxies. Figure 1 and Table I show the binnings of GW source distribution and weak lensing.

Finally, let us define the binning of multipoles for auto- and cross-spectra. We fix the minimum multipole as \( \ell_{\text{min}} = 10 \) and consider two different cases for maximum multipoles, \( \ell_{\text{max}} = 100, 300 \). The bins are logarithmically equally spaced and the number of bins is 30. We summarize parameters which characterize the surveys in Table II.

**B. Spectra with fiducial parameters**

In Figures 2, 3, and 4, auto- and cross-power spectra are shown. We compute these spectra with fiducial parameters listed in Table II. For weak lensing, we can cross-correlate \( N_l (N_l + 1)/2 = 21 \) pairs of lensing bins and all of them have appreciable signals. Though we can take cross-correlation for \( N_w (N_w + 1)/2 = 21 \) pairs for GW source distributions, correlation between different bins is suppressed because the deviation of luminosity distance from true one is assumed to be small in Eq. (2). Therefore auto-correlations contain most of information for GW source distributions. For cross-spectra between GW source distribution and weak lensing, there are \( N_l \times N_w = 36 \) spectra. In total there are 78 spectra used in the analysis. In Figure 3, we show spectra where the redshift ranges of two bins are overlapped. In this case, the contribution due to IA is appreciable because the support of IA kernel is confined contrast to wide support of lensing kernel. When GW source distribution bin is located farther than lensing bin, the resultant spectrum is close to zero.

**C. Fisher forecast**

In this Section, we present forecast of parameter constraints based on Fisher matrix approach [39]. Since we assume that the covariance matrix does not depend on parameters and there are no correlations between different multipoles, the Fisher matrix can be simplified as

\[
F_{\alpha\beta} = \sum_\ell \sum_{U,V,X,Y} \frac{\partial C_{UV}(\ell)}{\partial p_\alpha} \text{Cov}[C_{UV}(\ell), C_{XY}(\ell)]^{-1} \frac{\partial C_{XY}(\ell)}{\partial p_\beta},
\]

where \( p_\alpha \) denotes a cosmological or nuisance parameter. The marginalized error for the parameter \( p_\alpha \) is given as

\[
\sigma(p_\alpha) = \sqrt{(F^{-1})_{\alpha\alpha}}.
\]

We consider the parameter space of \((h, \Omega_m, w_{de}, \sigma_8, b_{w1}, b_{w2}, A_{IA})\), where the first four parameters are our interested cosmological parameters and the latter three are nuisance parameters. When varying matter density \( \Omega_m \), we fix baryon density \( \Omega_b \) and vary only cold dark matter density \( \Omega_c \). For nuisance parameters, we always marginalize them in this analysis. We show marginalized errors for cosmological parameters in Table III and projected 68% level confidence regions with auto- and cross-spectra between GW distributions and weak lensing for two different cases of maximum multipoles \( \ell_{\text{max}} = 100, 300 \). The results show one can place a tight constraint on cosmological parameters with three different types of spectra. Especially, in addition to the dark energy parameter \( w_{de} \), we can constrain the amplitude of matter fluctuation \( \sigma_8 \), which is degenerate with galaxy bias when galaxy clustering measurement is used.

**IV. CONCLUSIONS**

The discovery of GW signals from BH binary merger by Advanced LIGO opened a new window into astrophysics and cosmology. From the observed wave forms, we can infer the absolute luminosity of GW and then measure the luminosity distance of the sources. If the redshifts of the sources are available, we can probe into
### TABLE II. Summary of parameters.

| Symbol   | Value                  | Explanation                                                                 | Reference |
|----------|------------------------|-----------------------------------------------------------------------------|-----------|
| $\sigma_{ln D}$ | 0.05                  | Standard deviation of the luminosity distance distribution.                 | Eq. (2)   |
| $T_{obs}$  | 1 yr                   | Duration of GW observation.                                                | Eq. (9)   |
| $\dot{n}_{GW}$ | $5 \times 10^{-6} h^3 \text{Mpc}^{-3} \text{yr}^{-1}$ | Mean number density of GW events per unit time.                           | Eq. (9)   |
| $\Omega_s$ | 15000 deg$^2$          | Area of the survey region.                                                 | Eq. (23)  |
| $z_0$    | 0.64                   | Redshift parameter of lensing source distribution.                         | Eq. (27)  |
| $n_0$    | 30 arcmin$^{-2}$       | Lensing source number density.                                             | Eq. (28)  |
| $\sigma_\gamma$ | 0.22                  | Intrinsic variance of shapes of source galaxies.                           | Eq. (25)  |
| $C_1$    | $5 \times 10^{-14} h^{-2} \text{M}_{\odot}^{-1} \text{Mpc}^3$ | Normalization of intrinsic alignment.                                     | Eq. (16)  |

### Varied parameters

| Symbol   | Fiducial value | Explanation                                                                 | Reference |
|----------|----------------|-----------------------------------------------------------------------------|-----------|
| $b_{w1}, b_{w2}$ | 1, 1             | Bias parameters for GW source number density distribution.                  | Eq. (26)  |
| $A_{IA}$  | 1              | Amplitude of intrinsic alignment.                                           | Eq. (16)  |
| $\Omega_m$ | 0.3153         | Matter density at the present Universe normalized by critical density.     |           |
| $h$      | 0.6727         | Hubble parameter in the unit of 100 km/s/Mpc.                              |           |
| $w_{de}$ | $-1$           | Equation of state parameter of dark energy.                                |           |
| $\sigma_8$ | 0.831          | The amplitude of matter fluctuation at the scale of $8 h^{-1}$ Mpc.        |           |

![Fig. 2. The auto-power spectra of GW source distributions. The numbers in parenthesis denote the bins.](image)
FIG. 3. The cross-power spectra of GW source distributions and tomographic weak lensing. The numbers in parenthesis denote the bins. Note that cross-correlations with IA term is always negative. Since the total spectra can be positive or negative, we show the absolute values for the spectra.
the geometry of the Universe via the distance-redshift relation. Although it has already been reported that the source redshift is identified from the EM counterpart for the NS binary merger event GW170817, measuring the source redshift is still challenging especially for BH binary merger. However, without redshift information, we can explore the distance-redshift relation by combining another observable which redshift information is accessible.

In this work, we focus on weak gravitational lensing. WL is an unbiased tracer of matter distribution in the Universe and one of main observational targets for upcoming imaging surveys. We employ tomographic technique, where the whole source galaxy samples are divided according to their photometric redshifts. Thus we can efficiently extract information of the large-scale structures in different redshifts. We show that auto- and cross-correlations of GW source distributions and WL enable us to obtain tight constraints on cosmological parameters.

FIG. 4. The auto-power spectra of tomographic weak lensing. The numbers in parenthesis denote the bins. Note that cross-correlations between lensing and IA is always negative.

| Maximum multipole | $\sigma(h)$ | $\sigma(\Omega_m)$ | $\sigma(w_{de})$ | $\sigma(\sigma_8)$ |
|-------------------|----------|-------------------|-----------------|-----------------|
| $\ell_{\text{max}} = 100$ | 0.0084  | 0.031  | 0.17  | 0.055  |
| $\ell_{\text{max}} = 300$ | 0.0033  | 0.014  | 0.086 | 0.021  |
Thus we can place a tight constraint without being de-
gressed by nuisance parameters like galaxy bias.

Finally, we would like to discuss future prospects for
standard sirens. Recently, several works present predic-
tions of angular power spectrum of GW energy distribu-
tion \cite{10, 41}. Though auto-spectra of GW energy distri-
bution contain information about cosmology and astro-
physics, by combining with other observables such as
WL, we can obtain more information and evade system-
effect like intrinsic alignments. Another topic which
should be addressed is three dimensional correlations of
GW source distributions. In this work, we focused only
on projected quantities. Since projection mixes Fourier
modes of small and large scales, we can efficiently obtain
independent information from three dimensional corre-
lations. There is a possibility that three dimensional
clustering of GW sources and cross-correlation between
GW source distributions and other observables, e.g., the
spatial distribution of spectroscopically detected galaxies
can enable us to probe into the geometry of the Universe.
We leave it for future work.

FIG. 5. Projected confidence regions at 68% level of cosmolog-
cal parameters \((h, \Omega_m, w_{de}, \sigma_8)\) from the Fisher matrix. The
red dashed (blue solid) line corresponds to the result with the
maximum multipole \(\ell_{\text{max}} = 100\) \((\ell_{\text{max}} = 300)\). The black
dashed lines show fiducial values.

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