Control of mental representations in human planning

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One of the most striking features of human cognition is the capacity to plan. Two aspects of human planning stand out: its efficiency, even in complex environments, and its flexibility, even in changing environments. Efficiency is especially impressive because directly computing an optimal plan is intractable, even for modestly complex tasks [1], and yet people successfully solve myriad everyday problems despite limited cognitive resources [2–4]. Standard accounts in psychology, economics, and artificial intelligence have suggested this is because people have a mental representation of a task and then use heuristics to plan in that representation [5–12]. However, this approach generally assumes that mental representations are fixed. Here, we propose that mental representations can be controlled and that this provides opportunities to adaptively simplify problems so they can be more easily reasoned about—a process we refer to as construal. We construct a formal model of this process and, in a series of large, pre-registered behavioral experiments, show both that construal is subject to online cognitive control [13–15] and that people form value-guided construals that optimally balance the complexity of a representation and its utility for planning and acting. These results demonstrate how strategically perceiving and conceiving problems facilitates the effective use of limited cognitive resources.
In the short story “On Exactitude in Science,” Jorge Luis Borges describes cartographers who seek to create the perfect map, one that includes every possible detail of the country it represents. However, this innocent premise leads to an absurd conclusion: A map that includes every detail of the country must be the size of the country itself, rendering it impractical for anyone to use. Although fantasy, Borges’ allegory illustrates an important computational principle. Namely, useful representations do not simply mirror every aspect of the world, but rather pick out a manageable subset of details that are relevant to some purpose (Figure 1A). Here, we examine the consequences of this principle for how humans flexibly construe tasks during planning.

Classic theories of problem solving distinguish between representing a task and computing a plan [5, 16, 17]. For instance, Newell & Simon [18] introduced heuristic search, in which a decision-maker has a full representation of a task (e.g., a chess board, chess pieces, and the rules of chess), and then computes a plan by simulating and evaluating possible action sequences (e.g., sequences of chess moves) to find one that is likely to achieve a goal (e.g., winning the game). In artificial intelligence, the main approach to making heuristic search tractable involves limiting the computation of action sequences (e.g., only thinking a few moves into the future) [6]. Similarly, studies of human planning largely focus on how people limit, prune, or chunk action sequences to reduce computation [7–12, 19].

However, people are not necessarily restricted to a single, full, or fixed representation for a task. This matters since simpler representations can make better use of limited cognitive resources when they are tailored to specific, relevant parts or versions of a task. For example, in chess, considering the interaction of a few pieces, or focusing on part of the board, is easier than reasoning about every piece and part of the board. Furthermore, it affords the opportunity to adapt the representation, tailoring it to the specific needs of the circumstance—a process we refer to as controlling a task construal. While studies show that people can flexibly form representations to guide action (e.g., forming the ad hoc category of “things to buy for a party” when organizing a social gathering [20]), a long-standing challenge for cognitive science and artificial intelligence is explaining, predicting, and deriving such representations from general computational principles [21, 22].
Figure 1: Construal and planning. (A) A satellite photo of Princeton, NJ (top) and maps of Princeton for bicycling versus automotive use cases (bottom). Like maps and unlike photographs, a decision-maker’s construal picks out a manageable subset of details from the world relevant to their current goals. (B) Standard models assume that a decision-maker computes a plan, $\pi$, with respect to a fixed task representation, $T$, and then uses it to guide their actions, $a$. (C) In our account, the decision-maker forms a simplified task construal, $T_c$, that is used to compute a plan, $\pi_c$. This process can be understood as two nested optimizations: an “outer loop” of construal and an “inner loop” of planning. (D) Experiment 1 tested whether value-guided construal predicts awareness of obstacles. Participants navigated a series of mazes formed by obstacles and provided an awareness judgment for each obstacle.

To study human control of construal, we draw on ideas from the artificial intelligence literature on the rational use of cognitive resources [2-4]. In particular, we derive how an ideal, cognitively-limited decision-maker should form value-guided construals that balance the complexity of a representation and its utility for planning and acting. We then show that the pre-registered predictions of this account explain human awareness and memory of task elements across three experiments, even when controlling for alternative mechanisms. Our analysis and findings suggest that con-
trolled, moment-to-moment task construals play a key role in efficient and flexible planning.

Our account builds on models of sequential decision-making expressed as Markov Decision Processes \[^{[23]}\]. Formally, a task \( T \) consists of a state space, \( S \); an initial state, \( s_0 \in S \); an action space, \( A \); a transition function \( P : S \times A \times S \to [0, 1] \); and a utility function \( U : S \to \mathbb{R} \). In standard formulations of planning, the optimality of a plan \( \pi : S \times A \to [0, 1] \) from a state \( s \) is determined by the expected, cumulative utility of using that plan \[^{[1]}\]: \( V_{\pi}(s) = \sum_a \pi(a | s) \sum_{s'} P(s' | s, a) \left[ U(s') + V_{\pi}(s') \right] \). Standard planning algorithms \[^{[6]}\] (e.g., heuristic search methods) attempt to efficiently compute plans that optimize value. Notably, such planning computations are performed with respect to a fixed task representation, \( T \), that is not subject to the decision-maker’s control (Figure 1B). Our aim is to relax this constraint and consider the process of adaptively selecting simplified task representations for planning, which we call the construal process (Figure 1C).

Intuitively, a construal “picks out” details in a task to consider. Here, we focus on construals that pick out cause-effect relationships in a task. This is motivated by the intuition that a key source of task complexity is the interaction of different causes and their effects with one another. For instance, consider interacting with various objects in someone’s living room. Walking into the couch and hitting it is a cause-effect relationship, while pulling on the coffee table and moving it might be another relationship. These individual effects can interact and may or may not be integrated into a single representation of moving around the living room. For example, imagine pulling on the coffee table and causing it to move, but in doing so, backing into the couch and hitting it. Whether or not a decision-maker anticipates and represents the interaction of multiple effects depends on what causes and effects are incorporated into their construal.

To model construals, we require a way to express the flexible composition of different cause-effect relationships. For this, we use a product of experts \[^{[24]}\], a technique from the machine learning literature for combining distributions. Specifically, we assume that the agent has \( N \) primitive cause-effect relationships (the “experts”) that each assign probabilities to state, action, and next-state transitions, \( \phi_i : S \times A \times S \to [0, 1], i = 1, ..., N \). Each \( \phi_i(s' | s, a) \) is a potential function representing, say, the local effect of colliding into the couch or pulling on the coffee table.
Then a construal is a subset of primitive cause-effect relationships, \( c \subseteq \{\phi_1, ..., \phi_N\} \), that produces a task construal, \( \mathcal{T}_c \), with the following construed transition function:

\[
P_c(s' | s, a) \propto \prod_{\phi_i \in c} \phi_i(s' | s, a).
\]

Here, we assume that task construals (\( \mathcal{T}_c \)) and the original task (\( \mathcal{T} \)) share the same state space, action space, and utility function. But, crucially, the construed transition function can be simpler than that of the task.

What task construal should a decision-maker select? Ideally, it would be one that only includes those elements (cause-effect relationships) that lead to successful planning, excluding any others so as to make the planning problem as simple as possible. To make this intuition precise, it is essential to first distinguish between computing a plan with a construal and using the plan induced by a construal. In our example, suppose the decision-maker forms a construal of their living room that includes the effect of pulling on the coffee table but ignores the effect of colliding into the couch. They might then compute a plan in which they pull on the coffee table without any complications, but when they use that plan in the actual living room, they inadvertently stumble over their couch. Assuming that stumbling is unwanted, this construal is less than optimal.

Thus, we formalize the distinction between the computed plan and actual utility associated with a construal: If the decision-maker has a task construal \( \mathcal{T}_c \), denote the plan that optimizes it as \( \pi_c \). Then, the utility of a construal \( c \) when starting at state \( s_0 \) is:

\[
U(c) = \sum_a \pi_c(a | s_0) \sum_{s'} P(s' | s_0, a) \left[ U(s') + V_{\pi_c}(s') \right].
\]

Again, the utility of a construal is determined by the consequences of using it to plan and act in the actual task.

Having established the relationship between a construal and its utility, we can define the value of representation (VOR) associated with a construal. Our formulation resembles previous models of resource-rationality [3] and the expected value of control [14] by discounting utilities with a
model-complexity cost, $C$. This cost could be further enriched by specifying algorithm-specific
time or memory costs. However, our aim is to understand value-guided construal with respect to
the complexity of the construal itself and with minimal algorithmic assumptions. To this end, we
use a cost that penalizes the number of effects considered: $C(c) = |c|$, where $|c|$ is the cardinality
of $c$. Intuitively, this cost reflects the description length of a program that expresses the construed
transition function in terms of primitive effects \cite{25}. It also generalizes recent economic models
of sparsity-based behavioral inattention \cite{26}. The value of representation for construal $c$ is then:

\[ VOR(c) = U(c) - C(c). \]  

(3)

In short, we introduce the notion of a task construal (Equation 1) that relaxes the assumption
of planning over a fixed task representation. We then define an optimality criterion for a construal
based on its complexity and its utility for planning and acting (Equations 2-3). This optimality
criterion provides a normative standard we can use to ask whether people form optimal value-
guided construals \cite{27, 28}. We note that while the question of precisely how people identify or
learn optimal construals is beyond the scope of our current aims, we believe it is a key direction
for future research that will build on the approach taken here.

Do people form construals that optimally balance complexity and utility? To answer this ques-
tion, we designed a behavioral paradigm similar to the navigation problem shown in Figure 1.
Participants navigate through two-dimensional mazes composed of different arrangements of ob-
stacles. The start, goal, and obstacles change from trial to trial, and once a participant begins
moving, they only receive a bonus if they reach the goal without stopping. As such, the task
strongly encourages participants to examine the arrangement of obstacles and plan a path to the
goal. Following each trial, participants were given probes designed to assess their awareness or
memory of each obstacle.

We assume obstacles included in a construal will have greater awareness and thereby memory;
accordingly, we probed memory for obstacles after each maze to test whether participants formed
Figure 2: Experiment 1 results. (A) Histograms of mean awareness judgments by item (maze and obstacle) for obstacles with a value-guided construal expected obstacle probability $\leq 0.5$ (grey) and $> 0.5$ (blue). (B and C) Value-guided construal predictions and participant mean awareness judgments for three of the twelve mazes used in the experiment ($S =$ start, $G =$ goal; see Extended Data Figs 2-4 for visualizations of all alternative model predictions and mean experiment responses). Participant judgments generally reflect value-guided construal of mazes.

value-guided construals of the mazes. Additionally, we can contrast these predictions with those of alternative mechanisms, such as heuristic search. Recall that heuristic search plans by simulating possible action sequences in a fixed representation, which here corresponds to a maze with all obstacles. Awareness or memory of an obstacle is then a byproduct of whether it was encountered while simulating action sequences. In contrast, value-guided construal proposes that responses result from actively forming a construal with the fewest obstacles necessary to compute a useful plan. Put another way, it predicts that participants will only be aware of or remember obstacles that are relevant to planning.

The predictions of value-guided construal and heuristic search sometimes coincide since obstacles encountered during search are often relevant. However, their predictions can also diverge. For instance, an obstacle that is near the starting state but ultimately irrelevant to planning would be noticed according to heuristic search but ignored according to value-guided construal. Conversely, an obstacle that is far but highly relevant (e.g., one that blocks a distant, narrow passageway leading to the goal) is more likely to be noticed according to value-guided construal than heuristic search.
In our first experiment, 161 participants navigated a series of 12 distinct mazes, each consisting of fixed central walls and seven obstacles that could change on each trial. After navigating a maze, they were presented with a series of awareness probes in which an image of the last maze was shown with one of the obstacles highlighted, and asked to respond to the following question using an 8-point scale: “How aware of the highlighted obstacle were you at any point?” (see Figure 1D). Responses were scaled to be between 0 and 1. Our first analysis compared mean awareness responses for each obstacle to the probability the obstacle would be included in a construal, according to our model (predictions were pre-registered; see Methods for details). Since obstacle construal probabilities predicted by the model were largely deterministic (only 2 of the 84 items were rated between .10 and .90), we split the obstacles based on whether their probabilities were above or below 0.5, and then tested whether this predicted whether the corresponding mean awareness judgements were above or below 0.5. A $\chi^2$ test of independence showed a significant relationship ($\chi^2(1, N = 84) = 23.03, p < .0001$), indicating that even at a coarse-grained level, awareness judgments were consistent with value-guided construal (see Figure 2).

In Methods, Experiment 1, we report hierarchical linear model analyses that compare the predictions of value-guided construal to six alternative predictors, including those based on heuristic search (see Extended Data Fig 1 and Table 1). Additionally, we report the results of a second pre-registered experiment in which 162 participants could not see the obstacles as they moved through the maze and trials randomly terminated early in order to control for the effects of visual information and plan execution (Methods, Experiment 2). These additional results and analyses demonstrate that value-guided construal predicts distinct patterns in participant responses.

In a final experiment, we directly distinguished value-guided construal from alternative mechanisms by systematically dissociating two potentially confounded factors: the relevance of an obstacle to a plan (and therefore its predicted inclusion in a construal) and its distance from the solution path. To do so, we designed a new set of mazes, each of which contained a critical obstacle that was placed such that it blocked an entire route to the goal but was far from an optimal path,
Figure 3: Experiment 3 design and results. (A) The mazes included critical obstacles that were highly relevant to planning but far from an optimal path. Value-guided construal predicts critical obstacles will be included in a construal while irrelevant obstacles will not, independent of distance, unlike alternative models. (B) Participants were presented with a maze and the obstacles turned invisible once the participant began moving so as to encourage planning at the beginning of the trial. Half of participants were presented with location recall probes for each obstacle, in which they had to accurately recall the location of an obstacle and provide a confidence judgment. Note that only the obstacle being queried and its copy were shown during each probe. The remaining participants received awareness probes as in the previous experiments (see Figure 1D). (C) Mean confidence and accuracy by obstacle. Each item is a relevant/far (critical), relevant/near, or irrelevant obstacle. Consistent with value-guided construal, participants generally have higher recall accuracy and confidence for critical obstacles than for irrelevant ones. Error bars are standard errors. (D) Participant accuracy compared to predicted model accuracy according to two hierarchical logistic regression models fit on all the data. The first regression model contains six alternative predictors (including heuristic search) only, while the second additionally includes the value-guided construal predictions. Value-guided construal uniquely captures participant recall accuracy on irrelevant/near obstacles versus relevant/far (critical) obstacles.

making it highly relevant but distant. Value-guided construal predicts greater attention to such critical obstacles compared to irrelevant but nearby obstacles, unlike the alternatives (Figure 3A). Furthermore, we complemented participant awareness judgments with memory probes that did not
rely on self-reports of awareness. For this, we designed the mazes to use location recall probes, for which participants recalled whether an obstacle was in one of two locations and reported their confidence (Figure 3B). Participants were randomly assigned to receive either recall probes or the original awareness probes.

Here as well, the results provided support for value-guided construal (Figure 3C-D). Our first pre-registered analysis compared critical obstacles to irrelevant obstacles, which value-guided construal predicts will not be included in a construal. We fit hierarchical models to correct recall (logistic), confidence (linear), and awareness judgments (linear). Each model included the critical/irrelevant obstacles factor as a fixed effect. These were compared with corresponding null models without critical/irrelevant effects. For all three measures, there was a significant difference, and coefficients were positive (correct recall: $\chi^2(1) = 37.02, p < .0001, \beta = 0.65, \text{S.E.} = 0.10$; confidence: $\chi^2(1) = 53.75, p < .0001, \beta = 0.093, \text{S.E.} = 0.013$; awareness: $\chi^2(1) = 45.42, p < .0001, \beta = 0.083, \text{S.E.} = 0.012$). Consistent with value-guided construal, critical obstacles are associated with greater accuracy, confidence, and reported awareness compared to irrelevant obstacles. As with our first two experiments, we compared value-guided construal to the pre-registered predictions of six alternative factors (Methods, Experiment 3). These analyses showed that when using accuracy, confidence, or awareness as measures, value-guided construal accounts for distinct variance in responses that is not explained by the alternatives.

Together, the results of these experiments provide strong support for the idea that, when people plan, they do so by constructing a useful, simplified mental representation of a problem—a process that we refer to as the control of value-guided construal. Here, we have formalized this process, describing how an ideal, cognitively-limited decision-maker should construe a task so as to balance complexity and utility. We then showed, in three experiments, that pre-registered predictions of this account can explain people’s awareness, ability to recall, and confidence in their ability to recall problem elements (obstacles in a maze), even after controlling for alternative mechanisms. These findings support the hypothesis that people make use of a controlled process of value-guided construal, and that it can help explain the efficiency of human planning. Our formalism provides a
framework for further exploring the cognitive mechanisms involved in construal. For instance, how are construal strategies learned? How is construal selection shaped by other factors like planning computation costs or time? From a broader perspective, our analysis suggests a deep connection between the control of construals and the acquisition of structured representations like objects and their parts that can be cognitively manipulated [29, 30], which can inform the development of intelligent machines. Future investigation into these and other mechanisms that interface with the control of representations will be crucial for developing a complete theory of flexible and efficient intelligence.

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Methods

Value-guided Construal Implementation

Our model assumes the decision-maker has a set of cause-effect relationships that can be combined into a task construal that is then used for planning. To derive empirical predictions for the maze tasks, we assume a set of primitive cause-effect relationships, each of which is analogous to the example of interacting with furniture in a living room (see main text). For each maze, we modeled the following effects: A default effect of movement (i.e., pressing an arrow key moves the circle in that direction with probability \(1 - \varepsilon\) and stays in place with probability \(\varepsilon\), \(\varepsilon = 10^{-5}\)), \(\phi_{\text{Move}}\); the effect of being blocked by the center, plus-shaped (+) walls, \(\phi_{\text{Walls}}\); and effects of being blocked by each of the \(N\) obstacles, \(\phi_{\text{Obstacle}_i}, i = 1, ..., N\). Since every maze includes the same movements and walls, the model only selected which obstacle effects to include. The utility function for all mazes was given by a step cost of \(-1\) until the goal state was reached.

Value-guided construal posits a bilevel optimization procedure involving an “outer loop” of construal and an “inner loop” of planning. Here, we can exhaustively calculate potential solutions to this nested optimization problem by enumerating and planning with all possible construals (i.e., subsets of obstacle effects). We solved the inner loop of planning for each construal using dynamic programming [31] and then used policy evaluation to calculate the value of the computed plan in the actual task (i.e., Equation 2). The general procedure for calculating the value of construals is outlined in Algorithm [1]. To be clear, our account makes no claims about whether humans use this specific procedure when evaluating construals; we include it here to clarify our methods.

Given a value of representation function, VOR, that assigns a value to each construal, we model participants as selecting a construal according to a softmax decision-rule:

\[
P(c) \propto \exp \left\{ \alpha^{-1} \text{VOR}(c) \right\},
\]

where \(\alpha > 0\) is a temperature parameter (for our pre-registered predictions \(\alpha = 0.1\)). We then
Algorithm 1 Calculate value of representation function for construals, VOR, given states \( S \), initial state \( s_0 \), actions \( A \), transition function \( P \), state utility function \( U \), and primitive cause-effect relationships \( \{ \phi_1, \ldots, \phi_N \} \).

**require**: \textsc{DynamicProgramming} and \textsc{PolicyEvaluation} functions

1: function \textsc{ConstrualValues}(\( S, s_0, A, P, U, \{ \phi_1, \ldots, \phi_N \} \))
2: Initialize value of representation map VOR
3: for \( c \subseteq \{ \phi_1, \ldots, \phi_N \} \) do
4: for \( s \in S, a \in A, s' \in S \) do
5: \( P_c(s' \mid s, a) = \prod_{\phi \in c} \phi(s' \mid s, a) \quad \triangleright \text{Multiply primitive effects in construal} \)
6: end for
7: for \( s \in S, a \in A, s' \in S \) do
8: \( P_c(s' \mid s, a) = P_c(s' \mid s, a) / \sum_z P_c(z \mid s, a) \quad \triangleright \text{Normalize construed transitions} \)
9: end for
10: \( \pi_c = \text{DynamicProgramming}(S, A, P_c, U) \quad \triangleright \text{Compute Plan} \)
11: \( u_c = \text{PolicyEvaluation}(P, U, s_0, \pi_c) \quad \triangleright \text{Utility of construal} \)
12: \( x_c = |c| \quad \triangleright \text{Complexity of construal} \)
13: \( \text{VOR}[c] = u_c - x_c \quad \triangleright \text{Value of representation} \)
14: end for
15: return VOR
16: end function

calculated a marginalized probability for each obstacle being included in the construal, from the
initial state, \( s_0 \), corresponding to the expected awareness of that obstacle:

\[
P(\text{Obstacle}_i) = \sum_c [\phi_{\text{Obstacle}_i} \in c] P(c),
\]

where, for a statement \( X \), \([X]\) evaluates to 1 if \( X \) is true and 0 if \( X \) is false \cite{32}.

The pre-registered predictions for all experiments are visualized in Extended Data Figs 2-4.

**Heuristic Search and Other Alternative Predictors**

We considered two general classes of heuristic search algorithms from the computer science literature. The first, a variant of Real-Time Dynamic Programming (RTDP) \cite{33, 34}, is a trajectory-based search algorithm that simulates sequences of states and actions in an order that they could be physically realized. Specifically, it simulates trajectories from an initial state using a greedy policy and terminates once an absorbing state is reached (in our mazes, the only absorbing state is
the goal). The algorithm then updates the value function (initialized to a heuristic) at states as they are visited using a Bellman update [33]. Our implementation was based on the Labeled RTDP algorithm of Bonet & Geffner [34], which additionally includes a labeling scheme that marks states where the estimate of the value function has converged, which enables faster convergence.

To derive the pre-registered obstacle awareness predictions for RTDP on the maze task, we ran the algorithm on each maze with all obstacles and initialized it with a heuristic corresponding to the optimal value function assuming there are plus-shaped walls but no obstacles. This models the background knowledge participants have about distances, while also providing a fair comparison to the initial information provided to the value-guided construal implementation. Additionally, if at any point the algorithm had to choose actions based on estimated value, ties were resolved randomly, making the algorithm stochastic. Thus, for each maze, we ran 50 simulations of the algorithm to convergence and examined which states were visited by the algorithm over all simulations. We calculated the mean number of times each obstacle was hit by the algorithm, where a hit was defined as a visit to a state adjacent to an obstacle such that the obstacle was in between the state and the goal. For each maze, we generated 50 planning episodes. Because the distribution of hit counts has a long tail, we used the mean natural log of hit counts +1 (an additional hit count was added to handle log(0)) as the obstacle hit score. The reason why the raw hit counts have a long tail is due to the particular way in which RTDP calculates the value of regions where the heuristic value is much higher than the actual value (e.g., deadends in a maze). Specifically, RTDP explores such regions until it has confirmed that it is no better than an alternative path, which can take many steps. More generally, a limitation of trajectory-based algorithms is that they only simulate trajectories that are physically realizable starting from the initial state.

This motivates our use of a second class of algorithms that are graph-based. We used LAO* [35], a version of the classic A* algorithm [36] generalized to be used on Markov Decision Processes. Unlike trajectory-based algorithms, graph-based algorithms like LAO* maintain a graph of states that have been previously simulated. LAO* specifically works by building a graph starting from an initial state and performing dynamic programming updates over that graph. If an update results in
an optimal action that leads to a state at the frontier of the graph, it expands the graph to include that state and restarts the update cycle. Otherwise, the algorithm terminates. By maintaining a graph and performing dynamic programming updates, the algorithm is able to intelligently explore the most promising and relevant (according to the heuristic) regions of the state space without being constrained to physically realizable sequences. Among other benefits, this helps with handling dead ends without excessive simulation.

To derive obstacle awareness predictions for LAO*, we used the same heuristic as with RTDP to initialize the algorithm as well as a similar random scheme for handling ties. We then calculated the total number of times an obstacle was hit during graph expansion phases only, using the same definition of a hit as above. For each maze, we generated 50 planning simulations and used the raw hit counts as the hit score.

Along with the heuristic search measures, we considered several additional predictors based on low-level visual cues and participants’ observed behavior. These included the minimum Manhattan distance from an obstacle to the start location, the goal location, and any of the locations visited by the participant in a trial (navigation distance). We also considered the timestep at which participants were closest to an object as a measure of how recently they were near an object.

**Experimental Procedure**

All experiments used a variant of a maze-navigation task in which participants moved a circle from a starting location on a grid to a goal location using the arrow keys. Mazes were constructed of plus-shaped walls (in black) and tetromino-shaped obstacles (blue) that blocked movement. Experiments 1 and 2 used the same set of twelve 11 × 11 mazes containing seven obstacles and center walls arranged in a +. Each maze appeared once per participant, but mazes were randomly rotated or flipped. Experiment 3 used a different set of four 13 × 13 mazes each containing five obstacles that each participant encountered twice (for a total of 8 trials). For all experiments, the order and orientation of mazes were randomized. Visualizations of all mazes and pre-registered model predictions are in **Extended Data Figs 2-5**

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For all experiments, after consenting to participate, participants were introduced to the mechanics of the paradigm by navigating several simple mazes with only two obstacles. They were then given instructions that included all details about the main part of the experiment and asked comprehension questions.

On each trial in the main part of each experiment, participants were first shown a screen showing only the walls in the maze. When they pressed the spacebar, the circle they controlled, the goal, and the obstacles appeared, and they could begin moving immediately. In addition, to ensure that participants remained focused on moving, we placed a green square on the goal that shrank and would disappear after 1000ms but reset whenever an arrow key was pressed, except at the beginning of the trial when the green square took longer to shrink. For Experiment 1, the square took 5000ms to shrink initially, while for Experiments 2 and 3 it took 60000ms to shrink to provide participants ample time to plan given the obstacles would no longer be visible once they began to move. Participants then received $0.10 ($0.15 in Experiment 3) for reaching the goal without the green square disappearing. In total, participants could earn a bonus of up to $1.20 in addition to a base pay of $0.98.

Following navigation, participants were presented with probes for each obstacle, randomly ordered. Awareness responses and confidence responses were given on an 8-point scale, but were rescaled to range from 0 to 1 for the analyses. None of the probes were associated with a bonus. At the end of the experiment, participants were asked to provide free response answers about how they answered the awareness probes as well as their age, gender, and how often they play video games (results summarized in Table 2).

For each of the three experiments, we recruited 200 participants with IP addresses in the United States through the Prolific online experiment platform. After exclusions based on pre-registered criteria, we analyzed 13,342 awareness responses across 161 participants in Experiment 1, 13,321 awareness responses across 162 participants in Experiment 2, and 3,070 location recall responses/confidence judgments across 78 participants and 3,080 awareness responses across 78 participants in Experiment 3.
**Experiment 1**

In addition to the analysis discussed in the main text, we used pre-registered hierarchical linear models to assess whether value-guided construal captures unique variance in awareness responses. All seven predictors (value-guided construal probability, trajectory-based search hit score, graph-based search hit score, start distance, goal distance, navigation distance, and navigation distance timestep; see Value-guided Construal Implementation and Heuristic Search and Other Alternative Predictors) were included as fixed effects in a hierarchical linear model that also included by-participant intercepts, by-participant trial number slopes, and by-maze intercepts (see Extended Data Table[1] for estimated coefficients for models used in all experiments). We compared a model that included all the predictors as fixed effects with one that included all but value-guided construal probability. A log-likelihood ratio test revealed that including value-guided construal probability significantly increased model fit ($\chi^2(1) = 581.57, p < .0001$). Thus, value-guided construal predicts distinct variance in responses not captured by the alternatives considered collectively.

To assess value-guided construal in relation to each alternative individually, we fit several models with each predictor as a single fixed effect and compared each to a model that included the original predictor and value-guided construal probability. Log-likelihood tests for each model showed that value-guided construal probability significantly increased model fit in all cases (for all models, $\chi^2(1) > 1085.54, p < .0001$). Additionally, the Aikaike Information Criterion (AIC) scores for each of these models are plotted in Extended Data Fig[1A] and show that including value-guided construal improves the AIC score for every model by over 1000. These analyses indicate that value-guided construal predicts patterns of awareness responses that are distinct from all of the six alternatives.

While the results reported above suggest that value-guided construal makes a significant contribution to human planning, the post-trial measurement of obstacle awareness could have been influenced by plan execution. As such, it may not directly reflect the construal process, which is hypothesized to occur prior to execution. Experiment 2 was designed to evaluate this possibility and provide a stronger test of our account.
**Experiment 2**

To confirm that participant responses reflect a construal process that occurs prior to executing actions, we made two changes to the original paradigm in Experiment 1. First, immediately after executing the first action on a trial, the obstacles (but not the walls, goal, or agent) became invisible but could still block movement. This modification provided additional incentive to plan before acting and removed visual information about obstacles during execution. Second, on half of the trials, participants were given the awareness probes after taking two steps—that is, without executing most of their plan, and prior to navigating in proximity to obstacles. This variation allows us to assess the extent to which models predicted obstacle awareness as a result of planning, prior to plan execution (early termination trials), as compared to the influence of executing a plan (full trials).

For our first analysis, we confirmed that the hierarchical analyses of Experiment 1 replicated in the absence of visual information about obstacles during execution. We fit a hierarchical linear model with value-guided construal probability and the six alternative predictors as fixed effects and by-participant and by-maze random intercepts to awareness responses associated with full trials only. Comparison with a model that did not include value-guided construal probability was significant ($\chi^2(1) = 233.66, p < .0001$).

We additionally analyzed whether early termination interacted with each of the predictors. We fit a hierarchical “main-effects model” to both early termination and full trials with by-participant and by-maze random intercepts. Value-guided construal probability, the six alternatives, and whether a trial terminated early (sum-coding: early = 1, full = −1) were set as main fixed effects. Note that for minimum navigation distance, we replaced the empirical distances with the mean distance from 50 trajectories generated from the optimal policy on a maze since some trials terminated early.

We compared the main-effects model to models that each additionally included an interaction term between early termination and the following predictors: value-guided construal probability, trajectory-based search hit score, graph-based search hit score, and minimum navigation distance.
If including an interaction term increases the fit, this indicates that execution affects how a predictor influences awareness judgments. Comparisons with value-guided construal probability and graph-based hit score interactions showed no significant difference ($\chi^2(1) = 1.57, p = .21$ and $\chi^2(1) = 1.89, p = .17$, respectively). This finding confirms that processing reflecting value-guided construal occurs before plan execution.

The trajectory-based search hit score interaction resulted in a greater fit ($\chi^2(1) = 25.99, p = 3.43 \times 10^{-7}$), but note that since the corresponding main effect has a negative coefficient ($\beta = -0.19, \text{S.E.} = .014$) and the interaction has a positive coefficient ($\beta = 0.04, \text{S.E.} = .008$), this means early termination decreased the association with judgments. Finally, including the navigation distance interaction led to a greater fit ($\chi^2(1) = 12.25, p = 4.7 \times 10^{-4}$). The corresponding main effect coefficient is negative ($\beta = -0.40, \text{S.E.} = 0.01$) and interaction is positive ($\beta = 0.03, \text{S.E.} = 0.01$), which similarly indicates that not executing a plan attenuates the association with navigation distance.

**Experiment 3**

As for Experiments 1 and 2, we compared hierarchical logistic/linear models that included value-guided construal probabilities and six alternative predictors with ones that did not include value-guided construal. The models were fit to all the data. For all three measures, the full models accounted for significantly more variance than the ones that excluded value-guided construal, using a log-likelihood ratio test (correct recall: $\chi^2(1) = 131.14, p < .0001$; confidence: $\chi^2(1) = 258.51, p < .0001$; awareness: $\chi^2(1) = 588.78, p < .0001$). Thus, when using accuracy, confidence, or awareness as measures, value-guided construal accounts for variance in responses that is not explained by the alternatives.

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**Extended Data**

**Extended Data Figure 1:** Experiment 1 - Contribution of value-guided construal relative to alternative predictors. (A) Hierarchical linear models with different combinations of predictors were fit to awareness responses. Bars are grouped by alternative predictor and whether the model also includes value-guided construal. Using the Aikaike Information Criterion (AIC) as a measure of fit, results show that value-guided construal adds additional predictive power when included with each individual alternative. (B) Mean awareness by item and planning model (value-guided construal, trajectory-based heuristic search, graph-based heuristic search). Red lines and $r^2$ values are from simple linear regressions fit to the points in each plot.
Extended Data Figure 2: Mean awareness responses for Experiments 1 and 2 (mazes 0 to 5) alongside pre-registered planning model predictions.
Extended Data Figure 3: Mean awareness responses for Experiments 1 and 2 (mazes 6 to 11) alongside pre-registered planning model predictions.
Extended Data Figure 4: Recall accuracy, mean confidence, and mean awareness responses for Experiment 3 mazes alongside pre-registered planning model predictions.
**Extended Data Figure 5:** Experiment 3 mazes. Blue obstacles are the location of obstacles during the navigation part of the trial. Orange obstacles with corresponding number are copies that were shown during location recall probes. During recall probes, participants only saw an obstacle paired with its copy.
Extended Data Table 1: Estimated coefficients and standard errors for hierarchical linear/logistic models fit to participant responses from Experiments 1-3. All models are linear regressions, except for Experiment 3 accuracy, which is a logistic regression. For Experiment 2, full/early refer to fits to full versus early termination trials. For the latter, the navigation distance measure used was based on trajectories sampled from the optimal policy for a maze. All models included by-participant and by-maze random intercepts. Experiment 1 additionally included a by-participant random slope for trial number (this random effects structure was pre-registered and that removing this random slope does not change the pattern of results).

|                                | Exp 1 | Exp 2 (full) | Exp 2 (early) | Exp 3 (accuracy) | Exp 3 (confidence) | Exp 3 (awareness) |
|--------------------------------|-------|--------------|---------------|------------------|--------------------|------------------|
|                                | $\beta$ | S.E.     | $\beta$ | S.E. | $\beta$ | S.E. | $\beta$ | S.E. | $\beta$ | S.E. | $\beta$ | S.E. |
| Intercept                      | 0.45  | 0.02       | 0.40  | 0.03 | 0.37  | 0.02 | -1.40  | 1.16 | 1.34  | 0.16 | 0.89  | 0.18 |
| Value-guided Construal         | 0.15  | 0.01       | 0.15  | 0.01 | 0.13  | 0.01 | 1.18   | 0.10 | 0.20  | 0.01 | 0.34  | 0.01 |
| Trajectory-based Search        | -0.12 | 0.01       | -0.19 | 0.02 | -0.10 | 0.02 | -1.93  | 0.36 | -0.26 | 0.05 | -0.54 | 0.06 |
| Graph-based Search             | 0.17  | 0.01       | 0.17  | 0.02 | 0.14  | 0.02 | 1.75   | 0.33 | 0.10  | 0.04 | 0.31  | 0.05 |
| Start Distance                 | 0.03  | 0.02       | 0.22  | 0.02 | 0.11  | 0.02 | 2.15   | 1.09 | -0.64 | 0.15 | -0.28 | 0.17 |
| Goal Distance                  | 0.04  | 0.01       | 0.10  | 0.02 | 0.11  | 0.02 | 1.96   | 1.15 | -0.74 | 0.16 | -0.27 | 0.18 |
| Navigation Distance            | -0.45 | 0.01       | -0.49 | 0.02 | -0.38 | 0.02 | -0.90  | 0.19 | -0.10 | 0.02 | -0.39 | 0.02 |
| Nav. Dist. Timestep            | 0.15  | 0.03       | -0.41 | 0.05 | -0.98 | 0.47 | -0.21  | 0.05 | -0.64 | 0.06 |
Extended Data Table 2: Summary of responses to post-task questions for Experiments 1-3. Free responses to *Gender* were coded as Male, Female, or Neither.