Active Robust Control of Elastic Blade Element Containing Magnetorheological Fluid

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Abstract. This research study proposes a new active control structure to suppress vibrations of a small-scale wind turbine blade filled with magnetorheological (MR) fluid and actuated by an electromagnet. The aluminum blade structure is manufactured using the airfoil with SH3055 code number which is designed for use on small wind turbines. An interaction model between MR fluid and the electromagnetic actuator is derived. A norm based multi-objective $H_2 / H_\infty$ controller is designed using the model of the elastic blade element. The $H_2 / H_\infty$ controller is experimentally realized under the impact and steady state aerodynamic load conditions. The results of experiments show that the MR fluid is effective for suppressing vibrations of the blade structure.

1. Introduction
The engineering flexible structures like beams, plates and shells show modal dynamical behavior with external disturbances. It is a desired characteristic for such engineering structures to have adaptive behavior for changing external conditions. The control of adaptive structures also affects the dynamics of the flexible systems. Active vibration control approaches improve the performance in terms of reduction in vibration amplitude of the structures compared with passive systems.

In recent years, magnetorheological(MR) materials have been attracted much attention for use in engineering systems. MR fluids contain ferromagnetic particles that can change its rheological properties under application of an external magnetic field. The MR fluid has distinct properties for engineering applications and can be used in vibration control studies [1-3]. In literature, many research works have investigated the behavior of sandwich structures containing MR layer under magnetic field and active vibration control [4-6]. In this study, the ferromagnetic feature of the MR fluid is employed for an electromagnet actuator. The MR fluid is filled into a flexible blade element and attracted by the electromagnet actuator to suppress the vibration of the flexible structure.

2. Elastic blade structure
The blade structure is designed based on the airfoil with SH3055 code number and characteristics and wind tunnel aerodynamic tests for this airfoil is reported in reference [7]. The blade is fixed in its one end with a clamp using a connecting piece fitted with the profile. The length of the blade after clamping is 800 mm and the width of the blade is constant in lengthwise. The cross section and main dimensions of the blade are shown in Figure 1(a). The empty profile of the blade is filled with MR 122EG fluid and both ends are closed with leak proof stick. The amount of MR fluid filled inside the
blade profile is 87 ml. Since the structure of the blade is quite complicated, an equivalent beam with MR fluid is designed for modeling purposes. While the length of the equivalent beam is taken as the length of the original blade element, the width and thickness of the equivalent beam is selected so that the geometrical moment of inertia of the beam is equal to the geometrical moment of inertia of the original blade. In the designed equivalent beam, the inner volume contains 107.52 ml MR fluid.

![Figure 1. (a) Cross section and dimensions of the blade profile (b) the equivalent beam model](image)

**2.1 Equivalent Beam Model**

The determination of the eigenfrequencies and the corresponding eigenfunctions is necessary to analyze vibration problem in a distributed system. A modal analysis is realized for the considered elastic blade system. The equivalent beam with MR material considered in this study is schematically illustrated in Figure 1(b). Hamilton principle can be used to derive the equations of motion of the elastic blade element.

The equations of motion are obtained as follow

\[
\rho \frac{\partial^2 w(x)}{\partial t^2} + 2E_f I_f \frac{\partial^4 w(x)}{\partial x^4} - G'h_2 h_2 \left( \frac{\partial^3 w(x)}{\partial x^3} - \frac{\partial \phi}{\partial x} \right) = f(x,t) \tag{1}
\]

\[
f \frac{\partial^3 \phi}{\partial t^2} - \frac{bh_1 E_f (h_1 + h_2)^2}{2} \frac{\partial^2 \phi}{\partial x^2} - G' bh_2 \left( \frac{\partial w}{\partial t} - \frac{\partial \phi}{\partial x} \right) = 0 \tag{2}
\]

where \(w(x)\) is transverse displacement of the blade and \(\phi(x)\) is the cross-sectional rotation. For the cantilever beam, the eigenfunctions are described by

\[
W_n(x) = A_1 \cosh \lambda_n x + A_2 \sinh \lambda_n x + A_3 \cos \lambda_n x + A_4 \sin \lambda_n x
\]

\[
\Phi_n(x) = C_n \left( A_1 \sinh \lambda_n x + A_2 \cosh \lambda_n x + A_3 \sin \lambda_n x + A_4 \cos \lambda_n x \right)
\]

where \(C_n\) and \(\lambda_n\) are defined as

\[
C_n = \frac{2G'h_2 \lambda_n}{E_f h_1 (h_1 + h_2)^2 \lambda_n^2 + 2G'h_2}, \quad \lambda_n = \left( \frac{2n - 1}{2} \pi + e_n \right) \frac{L}{L}
\]

The vibration response of the blade element is calculated by

\[
w(x,t) = \sum_{n=1}^{\infty} W_n(x) e^{i\omega_n t}, \quad \phi(x,t) = \sum_{n=1}^{\infty} \Phi_n(x) e^{i\omega_n t}
\]

Substituting equations (5) into (1), the following equation is obtained as

\[
\sum_{n=1}^{\infty} \left[ -\rho \omega_n^2 W_n(x) + 2E_f I_f \frac{d^4 W_n(x)}{dx^4} - G'h_2 \left( \frac{d^3 W_n(x)}{dx^3} - \frac{d \Phi_n(x)}{dx} \right) \right] = 0 \tag{6}
\]

In this study, the blade structure contains MR fluid and does not fit this condition but the effect of the MR fluid is negligible due to low stiffness effect of fluid. The natural frequencies of the beam containing MR fluid are calculated as follows

\[
\omega_n = \sqrt{\frac{2E_f I_f \lambda_n^4 - G'h_2 (\lambda_n C_n - \lambda_n^2)}{\mu}} \tag{7}
\]
2.2 State Space Model

For each vibration mode of the cantilever blade structure, the separated equation of motion is given by

\[ \ddot{x}_n(t) + 2\zeta\omega_n\dot{x}_n(t) + \omega_n^2x_n(t) = f(t)\psi_n(x_n) \]  

where \( \omega_n \) is the mode natural frequency, \( \zeta \) is the damping coefficient and \( \psi_n(\cdot) \) is the mode shape function. The state space equation for each modal behavior is obtained as follows

\[ \dot{x}_n(t) = A_nx_n(t) + B_nu(t) + D_n\dot{d}(t) \]  

where \( x_n(t) \) is the state vector, \( A_n \) is the system matrix, \( B_n \) is the control input matrix and \( u(t) \) is the control input. Also, \( \dot{d}(t) \) shows the disturbance input. The structure of the state vector and matrices are as follows

\[ x_n = \begin{bmatrix} x_n(t) \\ \dot{x}_n(t) \end{bmatrix}, A_n = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix}, B_n = \begin{bmatrix} 0 \\ \psi_n(x_n) \end{bmatrix}, D_n = \begin{bmatrix} 0 \\ \psi_n(x_n) \end{bmatrix} \]  

A reduced order state space model for the control design study can be obtained by considering the first two modes. The reduced order state space equation is written as

\[ \dot{x}_r(t) = A_rx_r(t) + B_ru(t) + D_r\dot{d}(t) \]

\[ y_r(t) = C_rx_r(t) \]

2.3 Force characterization

The interaction between the blade element with MR fluid and electromagnet is illustrated in Figure 2. The electromagnet is positioned over the MR blade element with an air gap (Figure 2(a)). When the current is applied to the electromagnet and a magnetic field is created, the iron particles in the MR fluid become ordered and an attractive force on the blade is generated (Figure 2(b)). At this stage, the blade stands at the nominal position and still not moving. When the current increases the attractive force also increases and the blade moves to the actuator side as shown in (Figure 2(c)). The nonlinear magnetic force is derived as

\[ f(t) = k\frac{(i_0 + i(t))^2}{w_0 - w(t)} \]

where \( k = 1/4\mu_0N^2A \). The electromagnetic force is linearized around \((i_0, w_0)\) values as follows.

\[ f(t) = K_ww + K_ii_c \]

where \( K_w = 2k(i_0^2 / w_0^3) \) and \( K_i = 2k(i_0 / w_0^2) \).

![Figure 2. Illustration of the MR fluid electromagnetic actuator interaction](image-url)

It is important to show how much electromagnetic force is transmitted to the MR blade. To understand the force variation at the MR blade side a load cell is installed under the blade as shown in Figure 3(a). The variations of the experimental forces are shown in Figure 3(b). In these experiments, the air gap between the electromagnet and MR blade is set to 1 mm. At large coil currents, the loss is increasing as seen in Figure 3(b).
Figure 3. (a) Experimental setup (b) Variation of the electromagnetic force and the response force

3. Multi-objective $H_2/H_\infty$ control

In norm based robust control, $H_2$ and $H_\infty$ norm performances are two important specifications. While $H_\infty$ performance is convenient to enforce robustness against model uncertainty, $H_2$ performance is useful to handle stochastic aspects such as measurement noise and control cost [8-9].

The control design block structure is shown in Figure 4(a). In this block, $P(s)$ is the reduced order plant model and $K(s)$ is the controller that will be designed. In the blade system, the input $n$ shows the aerodynamic disturbances and sensor noise. Also, the input $d$ represents the disturbance caused by unstructured uncertainty or unmodeled high frequency dynamics in the system. The controlled variables $z_\infty$ and $z_2$ are outputs of the transfer functions obtained for each external input. Consider the points $p_1$, $p_2$ and $p_3$ in Figure 4(a). The transfer functions from $n$ to $p_1$, $p_2$ and $p_3$ are obtained as

$$G_{p,n}(s) = -\frac{K(s)\varepsilon}{I + P(s)K(s)} = -\varepsilon K(s)S(s), \quad G_{p,n}(s) = \frac{\varepsilon}{I + P(s)K(s)} = \varepsilon S(s), \quad G_{p,n}(s) = G_{p,n}(s)$$

where $S(s) = (I + P(s)K(s))^{-1}$ is the sensitivity transfer function. Moreover, the transfer functions from $d$ to the points $p_1$, $p_2$ and $p_3$ are given as

$$G_{p,d}(s) = -\frac{P(s)K(s)}{I + P(s)K(s)} = -T(s), \quad G_{p,d}(s) = \frac{P(s)}{I + P(s)K(s)} = T_y(s), \quad G_{p,d}(s) = G_{p,d}(s)$$

Here $T(s)$ is the complimentary sensitivity transfer function and $T_y(s)$ is the settling function. In a control system design, shaping with $T(s)$ transfer function is preferable for noise attenuation and tracking. Also, $T(s)$ transfer function is important for robust stability with respect to multiplicative uncertainties at the system output. In control block structure, the control cost is adjusted both $H_2$ and $H_\infty$ norm objectives. The objective of the control design configuration is to minimize

$$\begin{bmatrix}
W_e(s)\varepsilon S(s) \\
W_e(s)\varepsilon K(s)S(s) \\
W_e(s)T(s) \\
W_e(s)T(s)
\end{bmatrix}
< \gamma_0 \quad \text{and} \quad 
\begin{bmatrix}
W_e(s)\varepsilon K(s)S(s) \\
W_e(s)T(s)
\end{bmatrix}
< \gamma_0$$

(16)
4. Experimental system
The photo and layout of the experimental system setup is shown in Figure 5. The elastic blade test system is studied for the aims of vibration suppression. In the experimental system, a current driver is used for the electromagnetic actuator. Vibration analysis of the blade is performed with a Bruel&Kjaer 3053 device. The designed controller is realized using dSpace 1104 control card. The controller is discretized and compiled in the state space form using a Matlab/Simulink file and installed on dSpace control card.

4.1. Experimental Results
The closed loop frequency responses of the beam obtained in experiments with $H_2 / H_\infty$ controller is shown in Figure 6. As seen in Figure 6(a) responses, the uncontrolled modes are not excited by the controller.

The designed multi-objective controller is tested in different conditions under the steady state aerodynamic load. Experimental time history responses of the closed loop system for a continuous...
control case from a starting time is shown in Figure 7(a). Moreover, the repeated controlled and uncontrolled tests are realized to understand the response characteristics of the controllers as given in Figure 7(b). A robustness test is performed in the case of parameter variation by attaching an extra mass on the surface of the blade. The results are shown in Figure 8 when an additional mass of 12% of the blade mass is added. The control system is quite robust against parameter uncertainty and vibration attenuation with additional mass.

Figure 7. Experimental results of the closed loop system under steady state aerodynamic load

Figure 8. Robustness test results for additional mass, (a) Steady state case, (b) Reference point case

5. Conclusions
In this research study, vibration of a small-scale wind turbine blade is suppressed using a MR blade-electromagnetic actuator under the effect of steady state aerodynamic disturbance. A force based interaction model between MR blade and electromagnet is derived and some characterization works are presented. An $H_2 / H_\infty$ controller is designed to attenuate the vibration of the blade structure. Some experiments are realized to show the effectiveness of the proposed MR blade electromagnetic actuator for the transient and steady state aerodynamic loads. The experimental frequency response and the time history of the closed loop system showed significant vibration reduction in the blade element.

References
[1] Sun, Q., Zhou, J. X., & Zhang, L. (2003). An adaptive beam model and dynamic characteristics of magnetorheological materials. Journal of Sound and Vibration, 261(3), 465-481.
[2] Yalcintas, M., & Dai, H. (1999). Magnetorheological and electrorheological materials in adaptive structures and their performance comparison. Smart Materials and Structures, 8(5), 560.
[3] Weiss, K. D., Carlson, J. D., Nixon, D. A., 1994. Viscoelastic properties of magneto-and electro-rheological fluids, Journal of Intelligent Material Systems and Structures 5, p. 772-775.
[4] Niu, H., Zhang, Y., Zhang, X., & Xie, S. (2010). Active vibration control of plates using electro-magnetic constrained layer damping. International Journal of Applied Electromagnetics and Mechanics, 33(1, 2), 831-837.
[5] Valevate, A. V. (2004). Semi-active Vibration Control of a Beam Using Embedded Magneto-rheological Fluids (Doctoral dissertation, Wright State University).

[6] Rajamohan, V., Sedaghati, R., & Rakheja, S. (2011). Optimal vibration control of beams with total and partial MR-fluid treatments. *Smart materials and structures, 20*(11), 115016.

[7] Selig, M. S., and McGranahan, B. D. (2004) Wind tunnel aerodynamic tests of six airfoils for use on small wind turbines. *Journal of Solar Energy Engineering (Transactions of the ASME), 126*(4): 986-1001.

[8] Sivrioglu, S., Nonami, K. (1997) Active Vibration Control by Means of LMI-Based Mixed $H_2/H_\infty$ State Feedback Control, *JSME International Journal Series C 40*(2): 239-244.

[9] Madoyan A. (2009) Design and Comparison of Mixed $H_2/H_\infty$ controller for AMB System, Master Thesis.