Pentaquark Θ⁺ Mass and Width in Dense Matter

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Abstract

We investigate medium modifications of the pentaquark Θ⁺ in dense medium, taking into account different parities of the exotic Θ⁺ baryon. We find that the chemical potential of the Θ⁺ is shifted in a density-dependent way to one-loop order. We also investigated the effect of the scaled nucleon mass in dense medium on the Θ⁺ propagator. The results turn out to depend sensitively on the scaled nucleon mass and on the parity of the Θ⁺.

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Since the discovery of the pentaquark (uudd¯ s) baryon $\Theta^+$ [1, 2, 3, 4, 5, 6], which was motivated by the work of Diakonov et al. [7], its properties [8, 9, 10, 11, 12, 13, 14] and production mechanism [15] have been extensively investigated. The unique feature of the $\Theta^+$ lies in its small mass (1540 MeV) and very narrow width (< 25 MeV), which is shared by the recently found pentaquark state $\Xi_{3/2}^-$ [16]. Since the $\Theta^+$ is known to decay into a neutron and a $K^+$, its strangeness must be +1 [1]. The isospin of the $\Theta^+$ is concluded to be zero [5, 6]. However, its spin and parity have not been measured experimentally yet. In spite of the great amount of theoretical work on the $\Theta^+$, there is no agreement on its spin and parity. For example, chiral models advocate a positive parity for the $\Theta^+$ [10] whereas lattice quantum chromodynamics (QCD) and the QCD sum rule claim that its parity should be negative [13, 14].

Recent works on the $\Theta^+$ have concentrated on how to determine its parity [17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27]. In particular, Thomas et al. [18] have put forward an unambiguous method to determine the parity of the $\Theta^+$ via polarized proton-proton scattering at and just above the threshold of the $\Theta^+$ and $\Sigma^+$: If the parity of $\Theta^+$ is positive, the reaction is allowed at the threshold region only when the spin of two protons is $S = 0$. On the other hand, if the parity is negative, the reaction is allowed only when $S = 1$. Hanhart et al. [19] extended the work of Thomas et al. [18] to determine the parity of the $\Theta^+$, asserting that the sign of the spin correlation function $A_{xx}$ agrees with the parity of the $\Theta^+$ near threshold. Similarly, Rekalo and Tomasi-Gustafsson [27] proposed methods for pinning down its parity via a measurement of the spin correlation coefficients in three different reactions, i.e., $pn \rightarrow \Lambda \Theta^+$, $pp \rightarrow \Sigma^+ \Theta^+$, and $pp \rightarrow \pi^+ \Lambda \Theta^+$. Nam et al. [23, 24] found the cross sections for the $NN \rightarrow \Theta Y$ with the positive-parity $\Theta^+$ to be approximately ten times larger than those with the negative-parity one. A similar tendency was found in the photoproduction of the $\Theta^+$ [17, 27, 28]. Hence, it is of great importance to understand how the mass and the width of the $\Theta^+$ can be changed in a medium when its two different parities are considered.

There are many indications that the fundamental properties of hadrons, e.g., masses of hadrons, are modified in a medium. Dilepton experiments at Gesellschaft für Schwerionenforschung (GSI) showed a clear signature of dropping $\rho$ masses [29] even though some controversial arguments still remain [30]. Recent experiments in relativistic heavy-ion collisions give stronger support for the dropping masses of hadrons [31, 32]. The mass shifts of hadrons also play an essential role in understanding neutron stars [33, 34, 35], e.g., the masses and the sizes of neutron stars, and the cooling history of young neutron stars [36].

Recent studies of the $\Theta^+$ in the context of heavy-ion collisions suggest that the $\Theta^+$ yield may provide information on the early stage of a heavy-ion collision due to its weak interaction with other hadrons produced in the course of the collision [37]. In this regard, it is of great importance to understand how the $\Theta^+$ is modified in dense matter.

Our aim in this work is to investigate medium modifications of the $\Theta^+$ baryon, focusing on the parity of the $\Theta^+$. We will demonstrate how changes of its chemical potential in dense matter depend on its parity. We will also investigate the effect of scaled nucleon masses in dense medium.

We begin by introducing the effective Lagrangian for $n\Theta K$ coupling [28]:

$$\mathcal{L}_{n\Theta K} = -\frac{g^*_A}{2f_\pi} \bar{\Theta} \gamma_\mu \gamma_5 \left( \partial^\mu K^\mp n - \partial^\mu K^0 p \right) + \text{h.c.},$$

where $g^*_A$ denotes the pseudo-vector coupling constant for the $n\Theta K$ vertex. Here, positive parity of the $\Theta^+$ is assumed. Since the parity of the $\Theta^+$ has not been determined yet from
experiments, we also need to consider the Lagrangian for a negative-parity Θ$^+$ in which there is no $i\gamma_5$ in the vertex. For numerical estimates, we used $g_A^* = 0.28$ for positive parity and $g_A^* = 0.16$ for negative parity. These coupling constants are fixed to reproduce the decay width $\Gamma_{\Theta^+} = 15$ MeV. The nucleon propagator in a dense medium with the Fermi momentum $k_F$ can be written as

$$G_0^{\alpha\beta} = \left[ (\gamma^\mu k^\mu + M)_{\alpha\beta} \right] \left[ \frac{1}{k_A^2 - M^2 + i\epsilon} \right] \left[ 1 + \frac{i\pi}{E_0^0(k)} \delta \left( k^0 - E_0^0(k) \right) \theta(k_F - |\vec{k}|) \right]$$

$$= G_F^0(k)_{\alpha\beta} + G_D^0(k)_{\alpha\beta}, \quad (2)$$

where $E_0^0(k) = \sqrt{k^2 + M^2}$, $G_F^0(k)$ indicates the nucleon propagator in free space and $G_D^0(k)$ indicates the density-dependent part of the propagator. For simplicity, we consider symmetric nuclear matter. Note that the density is given as

$$\rho_N = \frac{\gamma}{6\pi^2 k_F^3} \quad (3)$$

with a degeneracy $\gamma = 4$ for symmetric nuclear matter. We assume that the kaon propagator in dense matter becomes

$$\Delta_K^0(k) = \frac{1}{k_F^2 - m_K^2 + i\epsilon}, \quad (4)$$

where $m_K$ is the kaon mass in free space. There are many experimental and theoretical indications that the effective masses of baryons and mesons change in a medium. To be more consistent, one may have to consider the density-dependent nucleon and kaon properties. However, since we cannot treat them self-consistently in our analysis and since our aim is to understand how the properties of Θ$^+$ change in a medium, we take the free-space nucleon and kaon masses as a first-order approximation. Later, we consider the density-dependent nucleon mass as a second approximation.

The density-dependent part of the Θ self-energy can be obtained by considering the contribution of $G_D^0(k)$ to the diagram in Fig. [1]

$$\Sigma(p) = \frac{1}{i} \frac{i^4}{(2\pi)^4} \left| \frac{g_A^*}{2f_\pi} \right|^2 \int d^4k \left[ (\not{p} - \not{k})\gamma_5 \Delta_K^0(p - k)G_F^0(k)\gamma_5(\not{p} - \not{k}) \right]$$

$$= \Sigma^s(p) - \gamma^0 \Sigma^0(p) + \vec{\gamma} \cdot \vec{p} \Sigma^v(p), \quad (5)$$

where $p$ is the incident momentum of the Θ, and $k$ stands for the loop momentum carried by the nucleon in the medium. The Θ$^+$ propagator in the medium is modified as

$$G(p) = \frac{1}{\gamma^0(p_0 + \Sigma^0) - \vec{\gamma} \cdot \vec{p}(1 + \Sigma^v) - (M_\Theta + \Sigma^s) + i\epsilon}, \quad (6)$$
where \( M_\Theta \) is the \( \Theta^+ \) mass in free space. The chemical potential \( p_0 \) of the \( \Theta^+ \) can be obtained by solving the equation

\[
(p_0 + \Sigma^0(p))^2 - |\vec{p}|^2(1 + \Sigma^v)^2 - (M_\Theta + \Sigma^s(p))^2 = 0.
\]  
(7)

In the case of the negative-parity \( \Theta^+ \), we obtain a similar expression with one opposite sign in \( \Sigma^s(p) \) due to the absence of \( i\gamma_5 \) in the Lagrangian.

In order to estimate the chemical potential in the rest frame of \( \Theta^+ \), we consider \( \vec{p} = 0 \). There are both real and imaginary parts in the density-dependent self-energy. However, since the imaginary part is very small compared to the \( \Theta^+ \) mass in the medium, one can treat the real and the imaginary parts separately in the first approximation. Then, the chemical potential can be obtained from the linear equation

\[
p_0^\pm = M_\Theta + \text{Re}(\pm \Sigma^s(p_0) - \Sigma^0(p_0)),
\]  
(8)

where the superscript \( \pm \) indicates the parity of the \( \Theta^+ \) in the Lagrangian. The contribution to the decay width can also be obtained in this order by

\[
\Gamma^\pm(p_0) = 2 \times \text{Im}(\pm \Sigma^s(p_0) - \Sigma^0(p_0)),
\]  
(9)

where \( p_0 \) is the solution of Eq. (8). However, since the phase space for \( \Theta^+ \to KN \) disappears beyond \( k_F \approx 260 \text{ MeV} \), detailed investigations of the decay width are not required. Thus, we will concentrate on the effective masses in this work.
FIG. 3: Shift in the chemical potential of the $\Theta^+$ in symmetric nuclear matter with scaled nucleon masses. When the reduced nucleon effective mass is applied, the kink disappear, and the shift shows a monotonic behavior. The notations are the same as in Fig. 2.

In Fig. 2, the shifts $\Delta M$ in the chemical potential of $\Theta^+$ in a medium are summarized in the solid curve for the positive-parity $\Theta^+$ and the dot-dashed curve for the negative-parity one. In some $k_F$ region, there is more than factor of two difference between the positive and the negative parity states.

In the previous section, we obtained the density-dependent results in the first approximation where there are no shifts in the nucleon and the kaon masses. However, the effective masses of nucleons and kaons are well known to change in a medium. In order to investigate the effects of these density-dependent masses, one has to solve the equation of state for the nucleon more consistently. However, since our goal is to see the properties of the $\Theta^+$ not the nucleon itself, we test the possible effects of the density-dependent nucleon mass by considering its simple scaling in Eqs. (8) and (9):

$$M_N^* = \frac{1}{1 + 0.23 \rho/\rho_0} M_N.$$  \hfill (10)

The scaling for the kaon is rather uncertain, so we have assumed the kaon mass to be constant in our estimate drawn in Fig. 3. The kinks in Fig. 2 disappear when the nucleon mass is scaled, and the shift of the $\Theta^+$ mass turns out to be monotonic. Note that the parity inverts the sign in the mass shifts as $k_F$ increases because the pole (when $M_\Theta = E_N + E_K$) disappears due to the reduced nucleon mass.

In this letter, we investigated the medium modification of the $\Theta^+$ chemical potential. We also investigated the effects of the density-dependent nucleon masses. We found that
the contributions to the chemical potential were very sensitive to the scaled nucleon mass and the parity of the $\Theta^+$. While we considered the $KN$ one-loop diagrams only, possible contributions of particle-hole diagrams, such as $Yh$ and $Y^*h$, may not be negligible in describing the medium modification of the $\Theta^+$ [39].

Most of the $\Theta^+$ produced in a dense medium will decay outside of the fireball due to the very narrow decay width. Thus, the change in the chemical potential may not be easily detected in experiments. Furthermore, current experiments cannot provide good enough statistics for the $\Theta^+$ production, at the moment; thus, it may be hard to obtain meaningful information on the parity from $\Theta^+$ production in heavy ion collisions. However, if the momentum of the $\Theta^+$ is small enough, its medium modification may be detected. For example, the $\Theta^+$ produced via $\pi + N \rightarrow \Theta^+ + K$ near threshold can have a small momentum [40]. In that case, the $\Theta^+$ may be bound inside the nuclei. A recent study [39] supports a possible bound of the $\Theta^+$ inside light and medium nuclei. Hence, we believe that the present investigation will provide a guideline for future experiments and for related theories.

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