Towards A Nonsingular Tachyonic Big Crunch

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We discuss an effective field theory background containing the gravitational field, the dilaton and a closed string tachyon, and couple this background to a gas of fundamental strings and D strings. Allowing for the possibility of a non-vanishing dilaton potential of Casimir type, we demonstrate the possibility of obtaining a nonsingular, static tachyon condensate phase with fixed dilaton. The time reversal of our solution provides a candidate effective field theory description of a Hagedorn phase of string gas cosmology with fixed dilaton.

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I. INTRODUCTION

It is widely believed that the ultimate theory of space, time and matter must resolve the space-time singularities which generically arise when using Einstein’s theory of General Relativity. The resolution of cosmological singularities is essential in order to be able to construct a theory of the very early universe which is genuinely predictive, and the resolution of the singularity inside of black holes is crucial in order to completely resolve the black hole information loss problem.

Over the past two years, the challenge of trying to resolve space-like singularities has been taken up by many researchers working on string theory. Various new tools have been employed, notably the AdS/CFT correspondence [1,2], matrix theory [3,4] c = 1 matrix theory [5], matrix string theory [6], and tachyon condensation [7]. Whereas the first four approaches are non-perturbative in nature, the last one is based on perturbative ideas. In this paper, we will explore the possibility that closed string tachyon condensation could lead to a quasi-static, nonsingular final state of cosmology. The time reversal of this dynamics might lead to a nonsingular initial state for string cosmology.

Closed string theories often contain states which become tachyonic when the parameters of the background space on which the perturbative string theory is set up reach certain values. For example, in bosonic string theory or in the heterotic string theory (with Scherk-Schwarz boundary conditions on the fermions), there are string winding modes which become tachyonic when the size of a spatial section decreases below some critical value. The occurrence of the tachyon signals an instability of the background. The resulting evolution towards a new ground state can be described in terms of “tachyon condensation” (see e.g. [5] for reviews on closed string tachyon condensation). The tachyonic mode condenses in a way analogous to how the Higgs field condenses during the process of spontaneous symmetry breaking. The system will evolve towards a new and stable ground state which is called the “tachyon vacuum”.

Closed string tachyon condensation in a cosmological context was initially considered in [7] (see also [8]). It was argued that fluctuating degrees of freedom disappear once the tachyon condenses, leading to the effective disappearance of space itself. It would be nice to be able to understand the onset of tachyon condensation at the level of an effective field theory. An initial attempt at analyzing this issue was made in [10], where an effective field theory containing gravity, the dilaton and a closed string tachyon was considered. In that work, vacuum solutions were considered, and it was found that, although the string frame metric after tachyon condensation was static, the dilaton was time-dependent and tended to strong coupling.

From early works on string gas cosmology [11,12] (see [13,14] for recent reviews) it is known that string matter degrees of freedom, in particular string winding modes, can play an important role in the cosmological evolution of the system. In this paper, therefore, we will couple a gas of stringy degrees of freedom as matter sources to the background action considered in [10] and investigate the possibility that the system reaches a static Hagedorn phase [34] after the universe contracts. Since we know from [10] (in the context of vacuum solutions) and [10] (in the context of string gas matter) that the dilaton increases towards the strong coupling regime in the Big Bang/Crunch region (large tachyon and Hagedorn phases respectively), and thus D-branes become lighter while the fundamental strings become heavier, we will include as matter sources both F-strings and D-strings.

We find that in the absence of a dilaton potential, it is not possible to obtain static solutions. However, if we add a potential of Casimir type which stabilizes the dilaton, it becomes possible to find a static solution which represents a candidate for a nonsingular final state of the universe after contraction.

In the following section, we will derive the cosmological equations of motion for our dilaton-tachyon background coupled to string matter sources. In Section III we determine some solutions to these equations. We will show that, provided a suitable potential for the dilaton is introduced, it is possible to obtain a static solution with a tachyon condensate. Section IV is devoted to a discussion of possible implications for cosmology, in particular for string gas cosmology.
II. EQUATIONS OF MOTION

Our starting point is the following action \( S_g \) for the background gravitational field, the dilaton \( \Phi \) and the tachyon \( T \) (generalized from the action given in [10]):

\[
S_g = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} e^{-2\Phi} \left[ R + 4(\nabla \Phi)^2 - (\nabla T)^2 - V(T) - \tilde{V}(\Phi, \lambda) \right],
\]

where \( g \) is the determinant of the metric, \( R \) is the Ricci scalar, \( V(T) \) is the tachyon potential, and \( \tilde{V}(\Phi, \lambda) \) which also depends on the scale factor in a way which could arise if the potential comes from a Casimir-type force. We will take the metric to be homogeneous and isotropic, i.e.

\[
ds^2 = -N^2(t)dt^2 + a(t)^2(dx^2 + dy^2 + dz^2),
\]

where \( N(t) \) is the lapse function. Following the standard notation [17], we will write the scale factor \( a(t) \) in terms of the function \( \lambda(t) \) as

\[
a(t) = e^{\lambda(t)}. \tag{3}
\]

With this ansatz, the background action \( S_g \) becomes

\[
S_g = \frac{1}{2\kappa^2} \int dt \frac{1}{N} e^{3\lambda - 2\Phi} \left[ 3\lambda^2 - (3\lambda - 2\Phi)^2 + \tilde{T}^2 - N^2 V(T) - N^2 \tilde{V}(\Phi, \lambda) \right]. \tag{4}
\]

We will couple a gas of string matter to the above background action. The action for a gas of fundamental strings is

\[
S_{F1} = -\int dt NF_{F1}(\lambda, \beta N), \tag{5}
\]

where \( F_{F1} \) is the free energy density of the gas of strings and \( \beta \) is the inverse temperature. Since, in the absence of matter, the dilaton is dynamical and increases towards the large coupling regime as tachyon condensation progresses, it is important to include, in addition to the fundamental strings, a gas of D-strings which become light at large dilaton values. As can easily be derived from S-duality considerations, the action \( S_{D1} \) of a gas of D-strings is given by

\[
S_{D1} = -\int dt Ne^{-2\Phi} F_{D1}(\lambda - 2\Phi, \beta N), \tag{6}
\]

where \( F_{D1} \) is the free energy density of the gas of D-strings.

The reader should note that the objects we call D-strings are not the D1-branes of type II string theory; in particular, for the heterotic string, stable D-branes do not exist. However, for compactification on a 6D torus, an NS5-brane wrapped on a 4 dimensions of the torus is S-dual to the F-string (at tree level), just as a D-string would be in 10D type IIB string theory [18]. This occurs because the S-duality used in this paper is S-duality of the 4D dilaton \( \Phi \), which already includes the size moduli of the compactification, as opposed to the S-duality of the 10D type IIB string. For a complete thermodynamics invariant under the full \( SL(2, Z) \) duality, we should also include bound states of the F- and D-strings; that is, we should consider the gas of all type of string states.

It proves convenient to rewrite the action in terms of the shifted dilaton \( \varphi \) defined via

\[
\varphi = 2\Phi - 3\lambda. \tag{7}
\]

We obtain

\[
S_g = \frac{1}{2\kappa^2} \int dt \frac{1}{N} e^{-\varphi} \left[ 3\lambda^2 - \varphi^2 + \tilde{T}^2 - N^2 V(T) - N^2 \tilde{V} \right],
\]

where we must keep in mind that \( \tilde{V} \) depends on both \( \lambda \) and \( \varphi \).

The variational equations of motion with respect to \( N, \lambda, \varphi \) and \( T \) then become (after setting \( N = 1 \)):

\[
-3\lambda^2 + \varphi^2 - \tilde{V} - \tilde{T}^2 - V(T) = 2\kappa^2 e^{\varphi} E_{F1} + 2\kappa^2 e^{-3\lambda} E_{D1}, \tag{9}
\]

\[
\tilde{\lambda} - \varphi \lambda + \frac{1}{6} \frac{\partial \tilde{V}}{\partial \lambda} = \frac{\kappa^2}{3} e^{\varphi} P_{F1} + \frac{\kappa^2 e^{-3\lambda}}{3} P_{D1} - \frac{2\kappa^2}{3} e^{-3\lambda} P_{D1}, \tag{10}
\]

\[
2\tilde{\varphi} - 3\lambda^2 - \varphi^2 - \tilde{T}^2 + V(T) + \tilde{V} - \frac{\partial \tilde{V}}{\partial \varphi} = -2\kappa^2 e^{-3\lambda} F_{D1} + 2\kappa^2 e^{-3\lambda} P_{D1}, \tag{11}
\]

\[
\tilde{T} - \tilde{T} \tilde{\varphi} + \frac{1}{2} \frac{\partial V}{\partial t} = 0, \tag{12}
\]

where \( E \) and \( P \) denote the total energy and pressure, respectively, and the subscripts indicate whether the terms refer to the contributions of the fundamental strings or the D-strings, respectively. Note that in the second equation, the partial derivative \( \partial \tilde{V} / \partial \varphi \) indicates that the derivative is to be taken at constant value of \( \varphi \). Rewritten in terms of partial derivatives with respect to the original field \( \Phi \), we have

\[
\frac{\partial \tilde{V}}{\partial \lambda} = \frac{\partial \tilde{V}}{\partial \lambda} + \frac{3}{2} \frac{\partial \tilde{V}}{\partial \Phi}. \tag{13}
\]

III. SOLUTIONS OF THE EQUATIONS

In the absence of a tachyon and of D-strings, and without a dilaton potential, our equations of motion [9,12] reduce to the ones used in the context of string gas cosmology and which were first discussed in [17, 19]:

\[
-3\lambda^2 + \varphi^2 = 2\kappa^2 e^{\varphi} E_{F1}, \tag{14}
\]

\[
\tilde{\lambda} - \varphi \lambda = \frac{\kappa^2}{3} e^{\varphi} P_{F1}, \tag{15}
\]

\[
2\tilde{\varphi} - 3\lambda^2 - \varphi^2 = 0. \tag{16}
\]
Adding (14) to (16) we obtain
\[ \ddot{\varphi} - 3\lambda^2 = \kappa^2 e^\varphi E_{F1}, \quad (17) \]

From these equations it follows that, if the pressure of the string gas vanishes, the source term in the equation of motion (15) for the scale factor \( \tilde{a} \) vanishes. If we consider the region of phase space in which \( \dot{\varphi} < 0 \), the running dilaton acts as friction, and the scale factor approaches a constant. This situation is realized if we consider a gas of closed strings on a compact space with stable winding modes [11, 17]. In this case (in the context of an expanding universe), as we go back in time and the temperature approaches the Hagedorn temperature, the contributions of the string winding and string momentum modes combine to give vanishing pressure, and the string frame scale factor is static. However, from (17) it follows that the dilaton is running, and for the branch of solutions of interest in string gas cosmology [11, 13, 14], the dilaton increases without bound as we go back in time, and thus rapidly approaches the strong string coupling regime.

In [10], another limiting case of our basic equations [9,12] was studied, the limit in which there is no string matter - only the background fields and the tachyon are present. In addition, the dilaton potential is set to zero. In this case, we obtain the following equations:

\[ -3\lambda^2 + \dot{\varphi}^2 - \ddot{T}^2 - V(T) = 0, \quad (18) \]
\[ \dddot{\varphi} - \dot{\varphi} = 0, \quad (19) \]
\[ 2\dot{\varphi} - 3\lambda^2 - \ddot{T}^2 + V(T) = 0, \quad (20) \]
\[ \dddot{\varphi} - \dot{T} \ddot{\varphi} + \frac{1}{2} \frac{dV}{dT} = 0. \quad (21) \]

The authors of [11] were interested in the dynamics which occurs once tachyon condensation sets in. Thus, they considered a tachyon potential \( V(T) \) for which Minkowski with fixed dilaton and vanishing value of the tachyon is an unstable point, i.e.,
\[ V(T) = -\frac{1}{2} m^2 T^2 + O(T^3). \quad (22) \]

It was shown that, during the phase of tachyon condensation, there are solutions in which the string frame metric is static. However, as can be seen from the dilaton equation of motion, the dilaton runs off to the strong coupling regime, and in fact reaches a strong coupling singularity in finite proper time. The solutions were interpreted in [10] as corresponding to a big crunch occurring in finite proper time.

If we are interested in the big crunch in cosmology, it is not justified to neglect matter. If the dilaton has the tendency to run towards the strong coupling regime, it is also not justified to focus on fundamental strings as the only matter source. Hence, we need to add both fundamental strings and their s-duals, namely D-strings, as matter sources to the system, as we have done in Section [11]. Since the D-strings have a coupling strength to the gravitational background which depends inversely on the dilaton compared to the coupling of F-strings, there is the hope that by adding both D-strings and F-strings, the dilaton might be naturally stabilized, even in the absence of a stabilizing dilaton potential. This idea has already been investigated by [20, 21]. However, at the level of the effective field theory equations we have derived [9,12], this hope is not realized. This can be seen most easily in the following way. We combine the equation (9) and (11) to obtain (for fixed value of \( \lambda \))
\[ \ddot{\varphi} - \dot{T}^2 = \kappa^2 e^\varphi E_{F1} + \kappa^2 e^{-3\lambda} E_{D1} - \kappa^2 e^{-3\lambda} P_{D1} + \kappa^2 e^{-3\lambda} P_{D1}. \quad (23) \]

On the other hand, from the equation (10) it follows that, for vanishing fundamental string pressure, the relation
\[ F_{D1} = \frac{2}{3} P_{D1} \quad (24) \]

is required to be consistent with a static solution. Inserting this relation into (23) it follows that, for an equation of state of D-strings with \( P > -E \), the right hand side of the equation is the sum of two positive terms, and thus is inconsistent with the left hand side being negative. Hence, without the dilaton potential, the dilaton cannot be time-independent.

We will now look for solutions to (9) in the presence of a non-trivial dilaton potential which have fixed scale factor and fixed dilaton. This ansatz is consistent provided that
\[ T^2 = V - \dot{V} - 2 \kappa^2 e^\varphi E_{F1} - 2 \kappa^2 e^{-3\lambda} E_{D1}, \quad (25) \]
\[ \frac{1}{6} \frac{\partial V}{\partial \lambda} = \frac{\kappa^2}{3} e^\varphi P_{F1} + \kappa^2 e^{-3\lambda} E_{D1} - \frac{2\kappa^2}{3} e^{-3\lambda} P_{D1}, \quad (26) \]
\[ T^2 = V + \dot{V} - \frac{\partial \dot{V}}{\partial \varphi} + 2 \kappa^2 e^{-3\lambda} F_{D1} - 2 \kappa^2 e^{-3\lambda} P_{D1}, \quad (27) \]
\[ \dddot{V} + \frac{1}{2} \frac{dV}{dT} = 0. \quad (28) \]

It is straightforward in this case to integrate (28) to find
\[ T^2 = V_0 - V(T), \quad (29) \]
where \( V_0 \) is the value of the potential at the point when \( \dot{T} = 0 \). So the tachyon rolling is undamped and unaffected by any other expectation values in this case.

Adding (25) and (27) yields
\[ - \frac{\partial \dot{V}}{\partial \varphi} = 2T^2 + 2 \kappa^2 e^\varphi E_{F1} + 2 \kappa^2 e^{-3\lambda} E_{D1} - 2 \kappa^2 e^{-3\lambda} F_{D1} + 2 \kappa^2 e^{-3\lambda} P_{D1}. \quad (30) \]
IV. CONNECTION TO COSMOLOGY

The picture of tachyon condensation in a collapsing universe is as follows: initially, the universe is decreasing in size and no tachyon is present. Once the size has reached some critical value, the tachyonic modes begin to condense, and the field $T$ begins to roll away from $T = 0$, the local maximum of its effective potential. In the context of our effective action, we expect that a nonsingular tachyon condensate phase should have constant scale factor and dilaton. We have only been able to find this behavior by introducing a dilaton potential with additional Casimir-like dependence on the scale factor.

The time reversal of this nonsingular tachyon big crunch would be a candidate for a nonsingular Hagedorn phase with fixed dilaton. In the context of this background, one would then be able to realize the recently suggested string gas cosmology structure formation scenario in which thermal fluctuations of a gas of closed strings during the Hagedorn phase generate a nearly scale-invariant spectrum of adiabatic cosmological fluctuations with a slight red tilt and a spectrum of gravitational waves with a slight blue tilt. In the following, we first present a brief review of the new structure formation scenario. In particular, we comment on why our new background satisfies the criteria required to make the scenario work.

String gas cosmology is an attempt to construct a toy model of the early universe incorporating key symmetries and new degrees of freedom which distinguish string theory from point particle field theory. The key symmetry which has been made use of extensively in string gas cosmology is T-duality, the key new stringy degrees of freedom are string oscillatory modes and closed string winding modes, neither of which are present in field theories. The presence of string oscillatory modes leads to the existence of a limiting temperature, the Hagedorn temperature for a gas of strings in thermal equilibrium.

In the following, we will imagine that all spatial sections are compact, admitting stable one cycles. In this case, string modes which wind the spatial cycles are stable. String states are thus characterized by their center of motion momentum quantum number, an integer $n$, the winding number $m$ (another integer), and the quantum numbers describing string oscillations. The T-duality symmetry is the invariance of the spectrum of string states under the interchange $R \rightarrow 1/R$, where $R$ is the radius of the spatial sections (in units of the string length), provided that the quantum numbers $n$ and $m$ are also interchanged. Based on this symmetry, a universe with radius $R$ is equivalent, at least in terms of string thermodynamics, to a universe of radius $1/R$.

Based on the above string thermodynamic analysis, it thus appears reasonable to expect that string theory should lead to a nonsingular cosmology, where $R = 1$ is a pivot point of the dynamics between a branch of cosmological expansion and another branch of cosmological...
contraction which, in the correct units, to an observer looks identical to that of an expanding universe [11]. The state of the system at this pivot point should correspond to a dense gas of strings at a temperature close to the Hagedorn temperature. We call this the “Hagedorn phase”.

It is obvious that the correct description of the Hagedorn phase will require the knowledge of non-perturbative string theory. In the absence of such a theory, we are forced to develop toy models of a classical background space-time coupled to a gas of strings. This is what we mean by “string gas cosmology”.

It is clear from the outset that our background cannot be described by General Relativity (GR) since GR violates the T-duality symmetry. A key requirement for a string gas cosmology background is that it be able to describe the transition from a static Hagedorn phase to the expanding radiation phase of standard cosmology without requiring matter which violates the weak energy condition [22]. In a lot of the work on string gas cosmology (beginning with [17]), dilaton gravity was used as a better justified background since it includes the dilaton at the same level as the graviton, as should be the case in a string theory model.

As we have already seen in Section III in dilaton gravity, whereas the string frame scale factor is indeed constant in the Hagedorn phase, the dilaton is not constant. In fact, following the radiation phase of standard cosmology into the past in the context of the dilaton gravity equations, we find that the dilaton begins to diverge towards strong coupling when the string gas approaches the Hagedorn temperature. Thus, in dilaton gravity the expected symmetry between large $R$ and small $R$ is not realized. Another way to put this problem is the following: the dilaton gravity description of the Hagedorn phase is in conflict with another key symmetry of string theory, namely S-duality.

A background in which both the dilaton and the scale factor are constant in the Hagedorn phase (and, in consequence, the string frame and the Einstein frame coincide), will yield a better description of what we believe the true stringy description of the Hagedorn phase will look like. Therefore, we have been motivated to include a potential that stabilizes the dilaton near the S-duality fixed point.

A background with constant dilaton is also crucial in order to implement the string gas structure formation scenario of [25]. In this scenario, thermal fluctuations of a gas of closed strings induces the cosmological perturbations. During the Hagedorn phase, the Hubble radius is infinite, and thus there are no obstacles to the gradual development of thermal equilibrium. At the end of the Hagedorn phase string winding modes decay into radiation and the pressure of the string gas increases from zero to the value it has in the radiation phase. Thus, at the end of the Hagedorn phase, the Hubble radius rapidly decreases to a microscopic value, and fixed comoving scales quickly exit the Hubble radius. Matter fluctuations established in the Hagedorn phase induce metric fluctuations (see [24] for a comprehensive review of the theory of cosmological perturbations, and [30] for a pedagogical summary). Provided that the gas of strings contains stable winding modes, the specific heat of a region scales as $R^2$ with the radius $R$ of the region [28]. If the metric fluctuations can be inferred from the matter fluctuations at Hubble radius crossing making use of the Einstein constraint equations, then a roughly scale-invariant spectrum of cosmological perturbations with a small red tilt results. However, as discussed in detail in [10], in dilaton gravity the dilaton fluctuations cannot be neglected and lead to a blue spectrum (see also [28]). This emphasizes the need to stabilize the dilaton.

In Section III we have constructed a background which indeed features both constant scale factor and dilaton, and which thus is a candidate for an improved description of the Hagedorn phase. A key question to ask, therefore, in order to make contact with the work of [25, 26] is how the dynamics described in Section III can be realized in the context of an expanding cosmology.

We imagine starting the universe in the tachyon condensate phase discussed in this paper, but with a tachyon which is climbing up its potential instead of rolling down as it does when we are considering a collapsing universe. One can then imagine that when the tachyon reaches the local maximum of its potential at $T = 0$, the tachyon disappears. If the dilaton is fixed by its potential, we will then be emerging into an expanding universe. In this context, the tachyon phase can provide the strong coupling Hagedorn phase postulated in [10].

V. CONCLUSIONS

In this paper we have constructed an effective field theory background which leads to a nonsingular big crunch. Our background includes usual gravity, the dilaton, and a tachyon condensate. Matter is described by a gas of closed fundamental strings and also includes a gas of D-strings. We have shown that, provided we introduce a non-vanishing dilaton potential with Casimir-type dependence on the volume, and tune both the tachyon and dilaton potential to appropriate values, a phase with constant scale factor and constant dilaton is possible. (The presence of the tachyon itself neither helps nor hinders the stabilization of the dilaton.) Such a state can be obtained via tachyon condensation from a contracting phase of standard cosmology.

Our construction may appear rather baroque. However, we should keep in mind that, if we start with a gravitational action which is based on Einstein or dilaton gravity, we are starting from a point far removed from one in which all of the symmetries expected from string theory are manifest. Our work is therefore intended more as an “existence proof” which might shed some light on what the key ingredients of the ultimate description of the Hagedorn phase might be [30].
We have speculated that the time reversal of our big crunch construction could provide a good description of an initial Hagedorn phase of string gas cosmology with fixed scale factor and dilaton in which the recently proposed string gas structure formation scenario can be realized. A concern at this point is that in it is assumed that the cosmological fluctuations can be described using the Einstein equations. What ensures that this is the case given our modified dynamics?

A concrete toy model in which this question can be analyzed is the higher derivative, ghost-free and asymptotically free higher derivative gravity model of Cosmological solutions of this theory lead to a bouncing cosmology. The dilaton is fixed in this model. The dynamics away from the bounce point rapidly approaches that of standard cosmology. As discussed in , this model provides a background in which the string gas structure formation scenario can be realized. A concern at this point is that in it is assumed that the strings (for example triggered by the decay of string winding modes into string loops), then the scale factor will begin to increase, thus providing the cosmological background required for the structure formation scenario of . Since the dilaton is fixed, it is justified to use the usual equations of motion of the theory of cosmological perturbations in the Einstein frame to follow the evolution of the fluctuations.

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[34] The Hagedorn phase is characterized by a gas of strings in thermal equilibrium close to the maximal temperature, the Hagedorn temperature $T_H$.

[35] Note that we are working in the string frame.

[36] Note that this scenario provides an alternative to inflation in explaining the currently observed cosmological data on matter inhomogeneities and cosmic microwave anisotropies.

[37] Thus we disagree with the criticisms expressed in [28].

[38] The precise thermodynamical calculation confirming this behavior awaits future work.

[39] See [31] for an interesting attempt at constructing a background for string gas cosmology making use of a theory with two metrics in which the symmetries of string theory are more manifest.