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Heaviside-Based Morphological Filters for Topology Optimization

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Abstract. To ensure manufacturability and mesh independence in density-based topology optimization schemes, it is imperative to use restriction methods. This paper proposes a family of Heaviside-based morphological filters based on the morphology-based restriction schemes, namely open, close, open-close and close-open filters. In the proposed morphological filters, Heaviside projection filter plays the role of dilation filter and modified Heaviside projection filter acts as the erosion filter. They work as principle operators and then applied sequentially to form the close, open, close-open, and open-close filters. The proposed morphological filters are tested by the compliance minimization problem. Test results show that the proposed four filters are volume preserving, and the characteristics of each filter are discussed.

1. Introduction

Topology optimization is a design tool for identifying optimal distributions of materials within a domain [1-3]. The fundamental concept of the density-based topology optimization method is to represent the geometry as a digital image where the colour of each pixel corresponds to the value of density [4]. It is by now well-known that the use of restriction methods in density-based topology optimization schemes is critical because they facilitate checkerboard free and mesh independence [5].

Density filtering method [6] is one of the commonly used restriction method in density-based topology optimization schemes. It can be used either on its own or in conjunction with other filters to eliminate the checkerboard patterns and ensure mesh independence. The morphology-based filters proposed by Sigmund [7] can be regarded as extensions of the density filtering method. They include a family of regularization schemes that are based on image morphology operators, namely dilation filter, erosion filter, open filter, close filter, open-close filter and close-open filter. The dilation and erosion filters work as the principle operators. The close filter is defined as a dilation followed by an erosion, the open filter as an erosion followed by a dilation, the close-open filter as opening followed by closing, and the open-close filter as closing followed by opening [8].

These morphological filters have been spread to solve other topology optimization problems since its introduction in the seminal paper by Sigmund [7], including robust design [9] and minimum length scale control [10], etc. Based on the principles of morphology-based filters, a series of Heaviside-based morphological filters are proposed in this paper. In the proposed filters, the Heaviside projection filter [11] plays the role of dilation filter, and the modified Heaviside projection filter [7] works as the erosion filter. The other filters are obtained by the embedded combinations of these principle filters. The next section will introduce the construction procedure of the proposed Heaviside-based filters.
2. Construction of Heaviside-Based Morphological Filters

2.1. Heaviside Projection Filter

Heaviside projection filter uses an independent design variable field that is associated with a single-phase. When a design variable magnitude is greater than zero, the associated phase is projected from its location onto nearby element space. The Heaviside step function is defined as follows:

\[ \bar{x}_e = H(\bar{x}_e) = \begin{cases} 1 & \text{if } \bar{x}_e > 0, \\ 0 & \text{if } \bar{x}_e = 0. \end{cases} \]  

where \( \bar{x}_e \) represent the densities obtained from the density filter, which is defined as:

\[ \bar{x}_e = \frac{1}{\sum_{i \in N_e} H_{ei}} \sum_{i \in N_e} H_{ei} x_i \]  

where \( x_i \) represent the design variable densities, \( N_e \) is the neighbourhood of element \( e \), which is a set of elements \( i \) for which the centre-to-centre distance \( \Delta(e, i) \) to element \( e \) is smaller than the filter radius \( r_{\text{min}} \), \( H_{ei} \) is a weight factor defined as:

\[ H_{ei} = r_{\text{min}} - \Delta(e, i) \]  

In order to allow for the use of a gradient-based optimization scheme, the Heaviside step function is replaced with the following smooth function:

\[ \bar{x}_e = 1 - e^{-\beta} + \bar{x}_e e^{-\beta} \]  

where the parameter \( \beta \) controls the smoothness of the approximation: for \( \beta \) equals to zero, the Heaviside filter is identical to the original density filter; for \( \beta \) approaching infinity, the approximation approaches the Heaviside step function.

2.2. Modified Heaviside Projection Filter

By the definition of Heaviside projection filter, any design variable with a value greater than zero will be projected to one. So the Heaviside projection filter is a max filter, that is, the volume of filtered densities is larger than the volume of original densities. While in the modified Heaviside projection filter [7], the volume of filtered densities is smaller than the volume of original densities, i.e. the modified Heaviside projection filter is a min filter. Therefore, in the proposed Heaviside-based morphological filters, the Heaviside projection filter plays the role of dilation filter and the modified Heaviside projection filter acts as the erosion filter. The modified Heaviside projection filter is defined as [7]:

\[ \bar{x}_e = e^{-\beta(1-\bar{x}_e)} - (1-\bar{x}_e)e^{-\beta} \]  

2.3. Close-Open and Open-Close Filter

Based on the principles of morphology-based filter, the max and min filters are applied sequentially to form the close, open, close-open, and open-close filters. The close filter is defined as a max followed by a min, the open filter as a min followed by a max, the close-open filter as opening followed by closing, and the open-close filter as closing followed by opening [7-8].

As an example, the open-close filter is discussed. The filtered densities obtained with the open-close filter are denoted by \( \bar{x}_{oc} \). They are computed in four steps:

\[ \bar{x}_e = 1 - e^{-\beta_{oc}} + \bar{x}_e e^{-\beta} \]  

\[ \bar{x}_e = e^{-\beta(1-\bar{x}_e)} - (1-\bar{x}_e) e^{-\beta} \]
\[ \bar{X}_e = e^{-\beta(1-x_e)} - (1 - \bar{X}_e)e^{-\beta} \]  
(8)

\[ \bar{X}_e = 1 - e^{-\beta \bar{x}_e} + \bar{X}_e e^{-\beta} \]  
(9)

The sensitivity \( \frac{\partial f}{\partial x_i} \) of a function \( f(\bar{X}_e) \) with respect to the \( i \)-th design variable \( x_i \) is obtained by applying the chain rule:

\[ \frac{\partial f}{\partial x_i} = \sum_e \sum_p \sum_q \sum_r \sum_s \frac{\partial f}{\partial \bar{X}_e} \frac{\partial \bar{X}_e}{\partial \bar{x}_e} \frac{\partial \bar{x}_e}{\partial x_p} \frac{\partial x_p}{\partial x_q} \frac{\partial x_q}{\partial x_s} \frac{\partial x_s}{\partial x_i} \]  
(10)

3. Case study

3.1. Problem Formulation

The classic compliance minimization problem is investigated in this section. The MBB beam is employed as research object, and the settings are shown in Figure 1. The load is applied vertically in the upper left corner and there is symmetric boundary condition along the left edge \( \Gamma_D \), the structure is supported horizontally in the lower right corner \( \Gamma_E \).

![Figure 1. Settings of MBB beam.](image)

The optimization problem for the MBB beam with the SIMP scheme may be written as:

\[ \min \rho \mathbf{f}(\mathbf{x}) = \mathbf{U}^T \mathbf{K} \mathbf{U} = \sum_{e=1}^{N} \mathbf{u}_e^T \mathbf{k}_e \mathbf{u}_e \]  
\[ \text{s.t.} \; \mathbf{K} \mathbf{U} = \mathbf{F} \]  
\[ g = V(\mathbf{x}) / V^* - 1 = \sum_{e=1}^{N} v_e x_e / V^* - 1 \leq 0 \]  
\[ 0 \leq x \leq 1 \]  
(11)

where \( f \) is the compliance, \( \mathbf{K} \), \( \mathbf{U} \) and \( \mathbf{F} \) are the global stiffness matrix, displacement vector and force vector, respectively, \( \mathbf{u}_e \) is the element displacement vector, \( \mathbf{k}_e = x_e^p \mathbf{k}_0 \) is the element stiffness matrix, \( \mathbf{k}_0 \) is the element stiffness matrix for unit Young’s modulus, \( \rho \) is the penalty factor, \( N \) is the number of elements used to discretize the design domain \( \Omega \), \( V \) is the material volume and \( V^* \) is the material resource constraint, \( v_e \) is the volume of element \( e \).
3.2. Test Examples

For filters that apply continuation schemes to update filter parameters, volume preserving is critical because this property helps to avoid oscillations in optimization iterations when using the continuation scheme, thereby improving the stability and convergence of the optimization process [12].

As mentioned above, the Heaviside projection filter is a max filter and the modified Heaviside projection filter is a min filter, thus both of them are not volume preserving. While the embedded filter, i.e. the open, close, open-close and close-open filter are volume preserving. Figure 2 displays the optimization results of MBB beam. Parameter settings are as follows: the design domain $\Omega$ is discretized with 120 by 40 bi-linear quadrilaterals, the material resource constraint is $V^* = 0.5$, the penalty power is $p = 3$, and the filter radius is $r_{\text{max}} = 3.5$.

![Figure 2](image)

**Figure 2.** Results for the MBB test example with Heaviside-based morphological filters. For each filter, two images are shown. The upper one shows the original design variable field and the lower one shows the filtered density field. (a) Heaviside projection filter, (b) modified Heaviside projection filter, (c) open filter, (d) close filter, (e) open-close filter and (f) close-open filter.

Figure 2(a) and (b) display the optimization results of the Heaviside projection filter and modified Heaviside projection filter, respectively. As seen from the figures, the filtered volumes are different from the original density volumes, these two filters are not volume preserving. The remaining four filters, open filter, close filter, open-close filter and close-open filter (shown in figure 2(c) to figure 2(f)), however, are volume preserving. This is the advantage of the morphological filters.

In the Heaviside projection filter, some small-scale holes appear in the result, which may make the end design unmanufacturable, and the topologies exhibit sharp corners that may cause stress concentrations. In the modified Heaviside projection filter, the topologies exhibit round corners and no small-sized holes appear in the optimization result.

The open filter exhibits the characteristics of Heaviside projection filter, that is, the topologies formed with sharp corners, and a small hole appear in the lower right corner of the structure. The close filter keeps the characteristics of modified Heaviside projection filter. In both the open-close and close-open filters, no small-scale holes appear in the final topologies. The open-close filter exhibits the characteristics of open filter and the close-open filter exhibits the characteristics of close filter.

4. Conclusion

Based on the principles of morphology-based restriction schemes, this paper proposes a series of Heaviside-based morphological filters. In the proposed morphological filters, the Heaviside and
modified Heaviside filter work as the principle operators. The open, close, open-close and close-open filters are obtained by applying the two principle operators sequentially.

Test results show that the open, close, open-close and close-open filter are volume preserving. This property facilitates stability and convergence as it helps to avoid oscillations in optimization iterations when using continuation scheme. The analysis indicates that the open filter exhibits the characteristics of Heaviside projection filter, and the close filter keeps the characteristics of modified Heaviside projection filter. The open-close filter exhibits the characteristics of open filter and the close-open filter exhibits the characteristics of close filter.

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