Substantiations Use of the Specific Energy Ice Destruction for Calculation the Cyclic Ice Load Parameters to Sea Structures

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Abstract. In the known types of models breaking the ice at contact with the structures of used power approach of the theory of elasticity in which the time factor is not a parameter of stress-strain state of the compressed volume of ice and are not involved in description of its destruction. In the models, the unknown physical criterion of ice fracture is replaced by the pressure at the ice–structure contact boundary as a derivative of the ice strength when testing small samples. This paper, the energy approach is used, which made it possible to show the mechanism of ice destruction as a Converter of the kinetic energy of the ice field into the potential energy of elastic compression of the ice volume and elastic deflection of the structure base. The processes of accumulation of elastic energy in the specific volume of ice, mechanisms of nucleation of foci of destruction and development of cracks, are considered. It is shown that the condition for the onset of destruction is to achieve a critical value of the elastic energy density in a single volume and the specific energy of ice destruction is a regulator of ice load cycles on the structure.

1. Introduction

As a result of the analysis of most proposed models of contact interaction of ice floes LF with marine ice resistant structures IRS can be stated that in all known the latest developments of model calculations of ice loads, ice breaking in the process of contact interaction described specifically to the two stages that make up one complete cycle of destruction: cracking–crushing of the virgin of ice and destruction of crushed ice–extruding fragmentation products from the zone of contact. However, in the description of all models used only a forces approach of the theory of elasticity in which the time factor is not a parameter of stress-strain state of the compressed volume of ice and are not involved in mathematical description of its destruction. Therefore, in many models an unknown physical condition of destruction of ice is replaced by the pressure at the boundary of contact ice–structure as the value derived from the strength of the ice when testing small samples with use the number of "regulations" factors, which customized contact' pressure to the conventional "destructive tension" within the compressed array of ice determined from some experiments. Many models, including ISO 19999-06 [1] those using model is "P – A", do not use the concept of "failure criteria", replacing the mathematical description of process of contact interaction "ice – structure" using a deterministic empirical model of the distribution of the contact pressure with a footnote for calculation to the same power strength parameter.
As previously shown [2 – 10], we researching phenomena – cutting Ice Field (The object - IF) and oscillations of the Ice-Resistant Structures (object - IRS) associated by a single process, resulting in layer-by-layer cyclic destruction of the ice, which does not have a place in nature if to study of each of these objects by separately. That is, the process of ice destruction is an integrative (emergent) property, which is one of the main features of the System. Given this, in order to fully understand the phenomenon of cyclical ice load on the structure, it is necessary to investigate the nature of the occurrence of ice destruction cycles. This was achieved by the author in different years by the methods of phenomenological, numerical and physical modeling of the development of the process of ice destruction on contact with the model of the structure's support.

2. Phenomenological and numerical modeling

The basis for developing a phenomenological model of ice cover end face fracture was the results of observations of the nature of the destruction in horizontal plan of natural ice fields [4, 11, 12, 13], as well as the end face of nature's ice cover during indentation models of supports [12, 14 – 17] and of laboratory experiments on ice plate cutting by ice resistant structure support model [18 – 24]. These results showed a variety of types of cracks in the ice plate: vertical, horizontal, spalls off the upper and lower faces of the plate and others. Using these data and principles of elasticity theory, the task solutions of Hertz and Boussinesq about indentation of a rigid punch into an elastic half-space, a phenomenological model of contact destruction ice for the case of the introduction of the cylinder in a vertical end facet of ice field was developed [25 – 27]. The main elements of this model are: the order of occurrence of cracks of all types in the ice field end face; the sequence of their development (Fig. 1, a-f); a process crushing ice in the middle "wedge" of the ice plate.

![Figure 1](image.png)

**Figure 1.** Results of theoretical and numerical studies of the distribution contact stresses on the faces of the IF with the support of the IRS.

But the most important step in this model is using an energy approach to describe the phenomenon of ice breaking as an elastic-brittle body during interaction of IF and IRS. From analogy with the destruction of the half-space by of explosion [28], as result the solution of the energy balance equation it was shown that a specific energy of destruction of a unit volume $ε_c$ when it is elastic-brittle fracture is the failure criterion of ice [27, 29, 30]. This model describes only one process in one volume of a solid body, and thus destruction can occur in it by one type, for example, by the development of shear
cracks – exactly as the "energy" theory of strength in the theory of elasticity suggests to describing the energy of changing the shape of the body.

We considered the model gave the possibility to formulate a hypothesis of volume destruction ice, is based on the fact that in reality the process contact’s destruction ice is a complex of several complex processes consisting of several simple processes that develop simultaneously (in parallel) in several volumes of the deformable array of ice. We therefore consider this parameter as a unique "energy characteristic of strength" for ice, which in the contact zone destructing several types of fracture: a cracks of normal separation (cleavage) and spall cracks (chipping); the crushing of ice wreckages and turning them into crumbs; extruding of destructed ice on periphery of the contact zone and melting the ice directly on the surface support. It should be noted that all these processes and phenomena have been repeatedly observed by many researchers, so do not require additional analysis. The essence of the proposed hypothesis is that this characteristic of \( \varepsilon_{cr} \) can be considered as an integral parameter that takes into account all types of ice destruction during not only the active phase of the contact, when the ice is destroyed, but also during the extrusion of its destruction products from the contact zone, i.e. \( \varepsilon_{cr} \) is the effective specific energy of destruction, which can be used to calculate the ice load to structure.

The developed model received qualitative verification by numerical method and with the help of physical experiment. Numerical verification of the fracture model was implemented in the ANSYS 2D and 3D software package by creating numerical models of the phenomena of interaction of the ice field end face with the vertical cylindrical support of the structure using FEM. Calculations are made for ice of different strength with varying ratios of the diameter of the structure and the thickness of the ice \( (D/h) \). On Fig.1, \( a_1-f_1 \) examples of the obtained numerical simulation data confirming the order of origin and development of vertical main cracks \( (a, a_1, b, b_1) \), horizontal cracks in the ice field thickness \( (e, e_1) \), as well as cracks of spall in the contact zone with the support \( (f, f_1) \) are given. The imposition of a "theoretical" pictures of crack growth in relevant areas of the array of ice in zones of tensile stress concentration (area +B), and arising under the action of pressure in the volumes of compression (area E) on the pattern of stress distribution obtained by the numerical method shows a complete coincidence, which confirms the authenticity and the uniqueness pictures of ice destruction.

3. The physical modelling

The detailed results of physical modelling of the process interaction model of a cylindrical support with a full-scale ice cover in [25]. The main purpose of the physical experiment was a detailed study of the mechanism of ice destruction for use for the development of a mathematical model of contact interaction taking into account the cyclic nature of the ice load. Therefore using of these experiment's results in this paper is to validation the numerical and phenomenological models that predicted the mechanisms of ice destruction by the development of a system of cracks in the end face of an ice plate of different thickness when exposed on it's edge by cylindrical support of structure.

The experiments were carried out on a specially manufactured stand with ballistic pendulum mounted with the necessary mechanisms and devices on the ice cover of the Amur Bay, had thick \( b \approx 0.9 \) m (Fig. 3). A model of cylindrical support \( D = 0.56 \) m and weighing up to 425 kg (indenter) was penetration into the ice at a different rate from 0,1 to 2,1 m/s by one-time exposure on to a plate ice measuring a \( x b \approx 1x2 \) m, 0,4~0,6 m thick at its temperature from -2°C to -12°C. Exposure was regulated by the mass of the indenter and lift height of the indenter (deflection pendulum angle), the parallelism of contact of block ice edge facet and line of support's model both was provided by the ballistic suspension.
Figure 2. Studies of the mechanisms of the ice cover edge destruction. A block of natural sea ice on the experimental stand.

Seven experiments were carried out, the most important results of which were the data that confirmed the adequacy of the phenomenological model of the system of cracks in the contact area of the support with the edge of the ice field. The purpose of a set of statistics of indicators of force of contact interaction was not set. A system of vertical cracks dividing the slab into sectors with a central angle of 45 degrees (Fig. 3, a) and horizontal cracks, (Fig. 3) which provided to the destruction of the ice field edge in the experiment, fully confirmed our conclusions from theoretical studies and from most of the other authors. Figure 3 shows our photographs of the cross-sections of the ice plate (by type "A" in Fig. 3, a). Here the cracks in the vertical plane - which is clearly visible, in addition to the "correct" horizontal arrangement can have also, as in the horizontal plane, a "radial" orientation. The reason that many cracks begin to develop in the direction "from the centre" is the highest concentration in this point of contact of elastic stresses, upon reaching the limit values of which avalanche cracking occurs.

Figure 3. The pictures of the destruction of the plate natural ice during impact of the model a cylindrical support on its edge for dynamic fracture as for a cold (a -10°C) ice and warm (b –3°C):
reducing the "effective thickness" chipped (1, 2, 3) of the side portions of the ice cover; the crushing of middle "wedge" part of the plate(4).

4. Analysis of results and discussion
The results of the semi-natural experiment, in addition to the goals achieved, two important conclusions also given. The first is: the nature of the process of accumulation of potential elastic energy in the volume of ice during its elastic deformation remains identical in the range of real rates of interaction between IF and IRS. It is following from comparison of the nature of two charts: of our experience chart for high velocity interaction with the experience chart of Japanese researchers on the slow (speed is 2 orders of magnitude lower) pressing of the model of support into the ice as shown on Fig. 4. As a second conclusion, as a result of a comparative study, it should be noted, that the rate of interaction of ice with an obstacle directly affects the rate of achievement of critical values of this value. Consequently, the period of ice destruction depends on the speed of IF directly proportional.

The analysis of the phenomenological features of ice destruction by the results of all experimental works in this area, taking into account the description of the published models currently suggests that the cyclical ice load is the result of periodic "reset" on the surfaces of new cracks (due to an avalanche-like development of a system of cracks in this moment) of the accumulated potential energy of elastic deformation of ice in the zone of its contact with the surface of the structure. Consequently, in the energy balance equation in this zone of the IF – IRS system, the key role should be played by the energetical ice fracture criterion, which should describe the limiting energy state of a single stress-strain volume.

The scientific explanation of this phenomenon for an idealized material with homogeneous and continuous properties is described by the energy theory of solid strength (Maxwell, Beltrami, Huber, Hencky and von Mises), which uses the apparatus of the theory of elasticity in the form of combinations of principal stresses and strains at a specific point of the body. According to this theory, the dangerous state of the material for this point occurs when the specific potential energy of the change in the shape of the unit volume at this point reaches its maximum. The classical approach of the theory of elasticity cannot be applied directly to sea ice, because this body is not homogeneous and not continuous; the ice has a very complex structure and texture, where many pores of different forms, salt crystals, etc.; all this makes it transversal anisotropic material.

A large number of defects dramatically affects the processes of ice destruction, but theoretically it is not possible to take into account their functional dependence on the external load on the nature of ice destruction. Nevertheless, the apparatus of elasticity theory is easily used in the finite element method (Fig. 1) to determine the stress-strain zones in the region of contact of the ice field and the
support structure. This method makes it possible to determine such areas in the exposed to external loads body, including sea ice, where it is possible to develop cracks and sometimes determine the mechanism of development of the future crack – normal separation (splitting), shear, or spall. But the theory of elasticity cannot give a description of the fracture criterion, it must be determined by the method of fracture mechanics and verified by experimental studies.

The criterion establishing such a connection between external and internal force factors and size of defect was proposed by A. Griffiths. Despite the fact that this scientist considered the energy balance of the process of accumulation of internal elastic energy on 2 surfaces of only one defect, he was the first to demonstrate the energetical approach to deciding problem and give a theoretical method for calculating the energy criterion for the destruction of a solid having an internal defect. But, he considered the problem in a linear formulation, so both the specific surface energy \( G1C \), required to initiate the development of a single microdefect, as well as the stress concentration coefficient \( K1C \) – as the fracture criteria have limited application in 2D geometry. To solve the problem of volumetric destruction of such a phenomenologically complex material as sea ice, the Griffiths criterion has no practical application. At the same time, thermodynamic and kinetic approaches to the process of solid deformation have been developed. Models of real materials described in research Ekobory T., Zhurkov S. N., Meshkov Yu. Ya., Morozov E. M., Panasuyk V. V., Parton V. Z., Rabotnov Yu. N., Slepyan, L. I., Fedorov V. V., Cherepanov G. P., Schreiner, L. A. and several foreign researchers.

5. Energy hypothesis of regulation of cyclic ice destruction

Thus, on the basis of all the considered aspects of the problem of describing the process of occurrence of the phenomenon of cyclic ice load on structures, the following hypothesis about the mechanism of formation of cyclic ice load on the IRS is proposed. It consists in the fact that the source of cyclicity of the ice force impact on the structure is the mechanism of cyclic ice destruction due to the periodic achievement of the critical stress-strain state in a somewhat volume of ice in the process of contact interaction between IF and IRS. As a criterion for the fracture of sea ice, integrally taking into account and describing all the types of his destruction, with its phenomenological properties in the real physical condition and the actual conditions under which the process of destruction, it is necessary to take critical density specific elastic internal energy of the ice per unit of its mass as an effective specific energy of mechanical destruction of ice \( \varepsilon_{cr} \).

The accepted hypothesis is based on the law of conservation of energy, it just explains the development of all basicals processes and phenomena of destruction of the crystal structure and the complex texture of the ice and does not contradict existing knowledge of mechanics of deformation and fracture of solids. The truth of the hypothesis requires experimental confirmation, the following three main consequences, taking into account the characteristics of sea ice.

5.1. The physical state of the ice determines the type of destruction

In the process of dynamic loading of array ice in the contact zone of the IF with the support of IRS, the contact force \( F \) acts simultaneously on the structure, performing work on its deviation from the equilibrium position with overcoming the forces of elasticity, and to ice array in contact zone of the ice field. In the loaded volume of ice \( -Wcr \), having a certain depth \( Lcr \) (Fig. 5,a), the accumulating of potential elastic energy is occurring. During it process to partial dissipation (spending, "drop") elastic energy onto the plastic deformations, micro-breaks in volumes of ice, etc. energy internal consumption, melting.

Structural-energy interpretation of the process of elastic-plastic deformation of the stressed ice volume shows that the flow of kinetic energy of the ice field in the process of interaction with a rigid fixed support creates an influx of potential energy \( U(t) \) into the ice mass adjacent to the contact zone. Subject to illustrated the phenomenological features of the structure of sea ice, the accumulation of potential elastic energy in internal its structure when the deformation is always non-equilibrium: even under slight elastic compressing in the body, the part of the incoming energy will be jumping dissipate as plastic micro-shifts in the structure of ice and its compaction, recrystallization, merger or extrusion.
defects, pores, etc. (like other materials). Therefore, the resistance energy \( U(t) \) of the stressed volume of ice (1 – in Fig. 5) during penetration of the structure support in ice at each time is distributed in the form of its two components, reflecting the characteristic features of the ice deformation process (plastic -2 and elastic 3 in Fig. 5 b-d).

\[ U(t) = U_e(t) - U_d(t), \]

(1)

where \( U_e(t) \) is accumulated in the structural elements of ice elastic energy(energy of interatomic bonds of the ice crystal; the surface energy in the pores at the boundaries of crystals, the structural defects along the banks of dormant cracks); \( U_d(t) \) – is the dissipation of energy in the form of plastic deformations in the crystal lattice, and in the texture of the ice and the heat transfer in the surrounding body of ice as a result of plastic deformation. The nature of the process of energy accumulation in the ice depends entirely as on its physical state (determines from its temperature, Korzhavin, 1962) by velocity of external load acting.

Given these position on Fig. 5 the interpretation of equation (1) is presented, where it is shown that in the process of loading the volume of ice the plastic deformation is always, as a result of the expenditure of elastic energy that feeds increase of defects. And the volume of plastic processes varies in proportion to the value of elastic deformation. Therefore, the second term in (1) will always be positive. And for warm ice, for example, for slow loading of its, almost all transmitted his energy has time to dissipate in the array of ice, almost not accumulates in the form of elastic deformation and then \( U_d(t) \approx 0 \), and the ice resistance to the introduction of the body in such case \( U(t) \rightarrow 0 \).

In this case, the ice can work according to the scheme of viscous deforming or fluidity, simply disintegrating into crystals (Fig. 5, b), i.e. "not destructed" – without the characteristic development of cracks. But at low temperatures and high load speed \( U_d(t) \rightarrow 0 \) then \( U(t) \rightarrow U_e(t) \), and at achievement the maximum value of elastic energy \( \varepsilon_{cr} \) in the unit ice volume, it can destruct as brittle body (Fig. 5, d), with the formation of blocks between the cracks (Fig. 1).

The analysis shows that the energy equilibrium equation (1) describes all the characteristic types of ice destruction in the range from viscous to brittle and it can be the basis of the energy model of ice destruction. The key to this model, which will determine the real value of the maximum ice load can be only the ice temperature.

5.2. The limit value of specific energy of ice destruction is corresponding its structure and physical state

Considering that the ice in its array always has some level of specific density of internal energy \( \varepsilon_0 \) corresponding to its structure and physical state. Exceeding this level by a certain value \( \Delta \varepsilon_0 \) to the limit density of elastic energy in a single volume of ice (Fig. 6, a) is a fracture condition, and the critical
value of value $\varepsilon_{cr}$ is called the specific energy of destruction. Given body volume and dynamics of the process, this condition can be written in the form:

$$\varepsilon_e(\mathbf{r}_*, t) = \varepsilon_o(\mathbf{r}_*, t_0) + \Delta\varepsilon_e(\mathbf{r}_*, t_*) \geq \varepsilon_{cr}; \quad \varepsilon_{cr} = \text{const},$$

(2)

where $\varepsilon_o(\mathbf{r}_*, t_0)$ – density of internal energy of the ice in the initial state ($t = 0$); $\Delta\varepsilon_e(\mathbf{r}_*, t_*)$ – an increase of the density of internal energy in ice local volume at period time of deformation $t*$; $\mathbf{r}_*$ – parameter coordinate of the local volume of ice.

According to the undertaken hypothesis, the increase of the specific energy density of elastic deformation in the loaded volume should be proportional to the time interval $\Delta t$ and the difference between the limit value of the elastic energy density $\varepsilon_{cr}$ and the elastic energy density $\varepsilon_e$ in the loaded volume at the current time:

$$\Delta\varepsilon_e = (\varepsilon_{cr} - \varepsilon_{et})\Delta t.$$  

(3)

In the form of a differential equation, this law will take the form:

$$\frac{d\varepsilon_e}{dt} = k(\varepsilon_{cr} - \varepsilon_{et}).$$

(4)

In this equation, $k$ – is the parameter of ice "rigidity", which depends on its structure and physical state and shows the ratio of elastic and viscous deformations of ice under its loading (Fig. 5, b-d). Integrating this equation after separating the variables under the initial condition $\varepsilon_e(0) = 0$ will give a function of the form:

$$\varepsilon_e = \varepsilon_{cr} \left(1 - e^{-kt}\right).$$

(5)

The graphical representation of this equation is a "graph of the saturation function" of the elastic energy of the stressed ice volume to the level $\varepsilon_{cr}$, which will be reached at time $t_{crd}$ (Fig. 6, a, b).

5.3. The frequency of ice destruction is determined by the speed of the ice field
The condition (2) will lead to the destruction of most strain elementary volume of ice in its array and further – to the spontaneous process of "chain reaction" destruction of some macrovolume to depth $L_{cr}$. But the time of occurrence of the fracture condition is due to the fact that for ice, as for other materials, there is a certain limit value of the absorption rate (dissipation power) of the elastic energy $\varepsilon_d$ in its volume. The excess growth rate of the elastic energy in unit volume of ice $- W_0$ over the dissipation its power coming from the external influence of elastic energy $\mathbf{E}_d$ will change the energy balance of all the elements of the ice structure and to it will collapse. The time of the beginning fracture of ice volume $W_{cr}$ can written so:

$$\frac{\Delta\varepsilon_e(\mathbf{r}_*, t)}{dt} = \left(\frac{dU_e(t)}{dt} - \frac{dU_d(t)}{dt}\right) < 0.$$  

(6)
Inequality (3) integrally describes the processes of energy accumulation and dissipation in a stressed volume of ice on contact with a support and can be reasonably considered as a "typical element of chain" of cyclic ice load, as demonstrated by its graphical interpretation in Fig. 6, d. Here shows the growth of the elastic energy density \( \varepsilon_t \) in during loading volume \( W_{cr} \) created by the influx of energy \( U/W_{cr} \) in the form difference of accumulated specific elastic energy \( U_0(t) \) and the energy density \( U_0(t) \), dissipated in the volume of ice. Physically this means, that the excess of elastic energy growth rate over absorption power it by the structure of ice is impossible, since such otherwise would exceed the allowable level of surface energy in crystals (dislocations, cells of brine) and on their boundaries (surface defects, interlayer brine) and as a result – to growth of number and size of microcracks, their coalescence into macrocracks, i.e., a compressed ice volume will be destruction.

Therefore, at the time of \( t\alpha \) the inequality (3) will cease to be satisfied, since its right part turns to zero due to the equalization of the elastic energy growth rate in the stressed ice volume with the power of its dissipation in ice array (Fig. 6, b point Y). This will correspond to the maximum value of the elastic energy growth rate \( \Delta \varepsilon_t \) (due to the reversal of its derivative in zero: point Z in Fig. 6, b) and density of elastic energy at time \( t\alpha \), according to (2) reaches a criterion value destruction ice: \( \varepsilon_0 + \Delta \varepsilon_t = \varepsilon_{cr} \) (Fig. 6, a: point X). Then the load-fracture cycle will repeat for the next volume of ice.

Here the time parameter is one of the main parameters of the process, not only the defining the beginning of destruction of ice moment (\( t_{cr} \)) but release of elastic energy accumulated in its array, but also characterizing the type of deformation and fracture of sea ice in unit volume of ice by the unique character curve of the speed of the increase of elastic energy depending in time.

Theoretically, the \( t_{cr} \) fracture cycle time of the stressed ice volume \( W_{cr} \) is defined as the quotient of the division of the perfect work (equivalent to the energy spent on its destruction \( A_{cr} = F \cdot L \cdot \varepsilon_{cr} \) - \( W_{cr} \)) by the average power of the specific elastic energy accumulation until it reaches the critical value \( \varepsilon_{cr} \) and the fracture frequency \( f_{cr} \) is the inverse of the process end time:

\[
t_{cr} = \frac{A_{cr}}{\varepsilon_{cr}}, \quad f_{cr} = 1/t_{cr}
\]  

6. Discussion of the energy hypothesis consequentials

In many previous studies and attempts to describe the process of interaction between IF and IRS, a number of authors [8, 9, 12, 14, 22, 31] in their models of ice load on the support also proposed to apply the rate of loading of ice. But, in contrast to the energy approach presented here, which in the elastic formulation takes into account the rate of inflow of elastic energy into the stressed volume of ice \( d\varepsilon_t/dt \), these authors operated on the rate of growth of contact pressures \( d\sigma/dt \) for example, in this form [4]:

\[
\dot{\sigma} = \frac{d\sigma}{dt} = \frac{2DpV_{ice}}{\pi L^2}
\]  

Here the symbols coincide with those in Fig. 1, a.

Also, in most experimental works [6–8, 14, 22–25], their authors explain the "sawtooth" nature of the contact force recording in time or along the length of the destroyed ice section by the manifestation of the mechanism of ice blocks cleavage, and it initiates sharp local destruction, and this leads to the loss of a significant contact area and a sharp decline in the contact force.

Figure 7 shows the data of contact force records when the ice field is cut by the structure support under natural conditions of the Bohai Bay (Fig. 7, a); the spall of a block of ice in the massif (Fig. 7, b) and tests of ice samples by cracking as a result of the development of cracks in the shear plane of one part of the sample relative to the other (Fig. 7-d). Diagrams of ice destruction in all the examples are almost identical in their form of active loading areas and fully confirm the concept of energy criterion of ice destruction considered in this study: each next act of destruction occurs when the elastic energy accumulated in the ice volume reaches its critical value. There is no doubt that the phenomenas of ice destruction, shown in Fig. 7, have a single nature and spalling is one of the" standard " mechanisms for the release of the elastic energy accumulated in the ice of its deformation by an external load.
Figure 7. Cyclic release of the elastic energy of a compressed volume of ice by chipping its individual volumes under different load conditions: (a) - when the ice field is cut by a structure according to Yue et al., 2009 [8]; b) – chip unit in the array of ice as per Frederking et al., 1990 [31]; (c) – record of the fracture force of a sea ice sample by shear in Ji et al., 2013 [9].

It should also be noted that, according to the energy theory of strength, the destruction of the material of the body will occur when the amount of full specific energy or of specific energy of changes of form of single volume will grow to critical values [28]. It is well known that the internal energy in the solid is created by forces applied to it from the outside. For example, the relation of the internal potential elastic energy in the body for the case when such a bulk load is generated by counter-directed along one axis pressures on the outer faces of the elementary volume, is written by the formula from the theory of elasticity:

\[ u_0 = \frac{1+\mu}{3E} \cdot [\sigma]^2 \]  

(9)

Not even an in-depth analysis of the power and energy approaches shows the unity of their physical essence, but the advantages of the energy approach are that energy takes into account in its balance all the processes that are occurring simultaneously and only together determining the phenomenon of ice destruction.

7. Finals provisions

Thus, the consequences of the accepted hypothesis of the formation of the cyclic ice load on the IRS theoretically show, that in the proposed sea ice fracture model the failure criterion is the effective specific energy of destruction of \( \varepsilon_{cr} \). At a constant rate of movement of the ice field \( V_{IF} \) this criterion is a phenomenological frequency control cycles destruction of ice and the cause of the phenomenon of generation of vibrations of the structure. That is, the process of transfer of external energy into an array of ice is continuous, and its expenditure on process of plastic deformation as the load is gradually in ice volumes and future destruction of these volumes occur cyclically with a period of cycles, determined primarily by the speed of the IF \( V_{IF} \), the rigidity of the ice (function of temperature) and geometric ratios of ice thickness \( h \) and width of the structure \( D \). The graph of this process \( \varepsilon_{cr} = f(V_{IF}; D/h; t) \) has a sawtooth shape and is depicted as a dashed red in Fig. 6, a.

Expression (7) for the cycle time simply describes the final act of a very complex mechanical process of elastic-plastic deformation of the material within the framework of fracture mechanics. But there is not a single parameter of cyclic ice load that can be determined analytically, even if instead of
magnitudes in the formula (7) to substitute real values of the sizes of IF and IRS, as well as the physical parameters of ice.

However, the problem of determining the parameters of the cyclical ice load (in of destruction of ice) can be solved uniquely for known dimensions of the designed facilities in a particular region of its construction. This can be done using by of minimum number of real physics – mechanical characteristics of ice described by the theoretical dependencies (4) and (7), namely: the effective specific energy of its destruction $\varepsilon_{cr}$ and the frequency of destruction $f_{cr}$. The numerical values of these calculated physical parameters of the ice fracture cycle for real conditions of interaction between IF and IRS are elementary determines from experiments on dynamic fracture of large ice samples [32].

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