Axial charges of N(1535) and N(1650) in lattice QCD with two flavors of dynamical quarks

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(Dated: June 4, 2008)

We show the first lattice QCD results on the axial charge $g_A^{N^*N'}$ of $N^*(1535)$ and $N^*(1650)$. The measurements are performed with two flavors of dynamical quarks employing the renormalization-group improved gauge action at $\beta=1.95$ and the mean-field improved clover action with the hopping parameters, $\kappa=0.1375$, 0.1390 and 0.1400. In order to properly separate signals of $N^*(1535)$ and $N^*(1650)$, we construct 2×2 correlation matrices and diagonalize them. Wraparound contributions in the correlator, which can be another source of signal contaminations, are eliminated by imposing the Dirichlet boundary condition in the temporal direction. We find that the axial charge of $N^*(1535)$ takes small values as $g_A^{N^*N'} \sim O(0.1)$, whereas that of $N^*(1650)$ is about 0.5, which is found independent of quark masses and consistent with the predictions by the naive nonrelativistic quark model.

Introduction. Chiral symmetry is an approximate global symmetry in QCD, the fundamental theory of the strong interaction; this symmetry together with its spontaneous breaking has been one of the key ingredients in the low-energy hadron or nuclear physics. Due to its spontaneous breaking, up and down quarks, whose current masses are of the order of a few MeV, acquire the large constituent masses of a few hundreds MeV, and are consequently responsible for about 99% of mass of the nucleon and hence that of our world. Thus one would say that chiral condensate $\langle \bar{q}q \rangle$, the order parameter of the chiral phase transition, plays an essential role in the hadron-mass genesis in the light quark sector. On the other hand, chiral symmetry gets restored in systems where hard external energy scales such as high-momentum transfer, temperature$(T)$, baryon density and so on exist, owing to the asymptotic freedom of QCD. Then, are all hadronic modes massless in such systems? Can hadrons be massive even without non-vanishing chiral condensate?

An interesting possibility was suggested some years ago by DeTar and Kunihiro [1], who showed that nucleons can be massive even without the help of chiral condensate due to the possible chirally invariant mass terms, which give degenerated finite masses to the members in the chiral multiplet (a nucleon and its parity partner) even when chiral condensate is set to zero: To show this for a finite-$T$ case, they introduced a linear sigma model which offers a nontrivial chiral structure in the baryon sector and a mass-generation mechanism completely and essentially different from that by the spontaneous chiral symmetry breaking. Interestingly enough, their chiral doublet model has recently become a source of debate as a possible scenario of observed parity doubling in excited baryons [2,3,4,5,6,7], although their original work [1] was supposed to be applied to finite-$T$ systems.

It is thus an intriguing problem to reveal the chiral structure of excited baryons in the light quark sector beyond model considerations. One of the key observables which are sensitive to the chiral structure of the baryon sector is axial charges [1]. The axial charge of a nucleon $N$ is encoded in the three-point function

$$\langle N|A^\mu_a|N \rangle = i\langle u|\frac{\tau^a}{2}[\gamma_\mu\gamma_5 g_A(q^2)]h(\tau)|u \rangle.$$  \hspace{1cm} (1)

Here, $A^\mu_a \equiv \bar{q}(Q\gamma_\mu\gamma_5 h)Q$ is the isovector axial current. The axial charge $g_A$ is defined by $g_A(q^2)$ with the vanishing transferred momentum $q^2 = 0$. It is a celebrated fact that the axial charge $g_A^{N^*}$ of $N(940)$ is 1.26. Though the axial charges in the chiral broken phase can be freely adjusted with higher-dimensional possible terms and cannot be the crucial clues for the chiral structure [3,4], they would surely reflect the internal structure of baryons and would play an important role in the clarification of the low-energy hadron dynamics.

In this paper, we show the first unquenched lattice QCD study [8] of the axial charge $g_A^{N^*N'}$ of $N^*(1535)$ and $N^*(1650)$. We employ 16$^3 \times 32$ lattice with two flavors of dynamical quarks, generated by CP-PACS collaboration [9] with the renormalization-group improved gauge action and the mean-field improved clover quark action. We choose the gauge configurations at $\beta=1.95$ with the clover coefficient $c_{SW}=1.530$, whose lattice spacing $a$ is determined as 0.1555(17) fm. We perform measurements with 590, 680, and 680 gauge configurations with three different hopping parameters for sea and valence quarks, $\kappa_{sea}, \kappa_{val} = 0.1375, 0.1390$ and 0.1400, which correspond to quark masses of $\sim 150, 100, 65$ MeV and the related $\pi-\rho$ mass ratios are $m_{PS}/m_{\pi}=0.804(1), 0.752(1)$ and 0.690(1), respectively. Statistical errors are estimated by the jackknife method with the bin size of 10 configurations.

Our main concern is the axial charges of the negative-parity nucleon resonances $N^*(1535)$ and $N^*(1650)$ in the $\frac{1}{2}^-$ channel. We then have to construct an optimal operator which dominantly couples to $N^*(1535)$ or
\( N^* (1650) \). We employ the following two independent nucleon fields, \( N_1 (x) \equiv \varepsilon_{abc} u^a (x) (u^b (x) C \gamma_5 d^c (x)) \) and \( N_2 (x) \equiv \varepsilon_{abc} \gamma_5 u^a (x) (u^b (x) C d^c (x)) \), in order to construct correlation matrices and to separate signals of \( N^* (1535) \) and \( N^* (1650) \). (Here, \( u (x) \) and \( d (x) \) are Dirac spinor for \( u \)- and \( d \)-quark, respectively, and \( a, b, c \) denote the color indices.) Even after the successful signal separations, there still remain several signal contaminations mainly because lattices employed in actual calculations are finite systems: Signal contaminations \((a)\) by scattering states, \((b)\) by wraparound effects.

Comment to \((a)\): Since our gauge configurations are unquenched ones, the negative parity nucleon states could decay into \( \pi \) and \( N \), and their scattering states could come into the spectrum. The sum of the pion mass \( M_\pi \) and the nucleon mass \( M_N \) is however in our setups heavier than the masses of the lowest two states (would-be \( N^* (1535) \) and \( N^* (1650) \)) in the negative parity channel. We then do not suffer from any scattering-state signals.

Comment to \((b)\): The other possible contamination is wraparound effects \((10)\). Let us consider a two-point baryonic correlator \( \langle N^* (t_{\text{sink}}) N^* (t_{\text{src}}) \rangle \) in a Euclidean space-time. Here, the operators \( N^* (t) \) and \( \bar{N}^* (t) \) have nonzero matrix elements, \( \langle 0 | N^* (t) | N^* \rangle \) and \( \langle N^* | N^* (t) \rangle \), and couple to the state \( | N^* \rangle \). Since we perform unquenched calculations, the excited nucleon \( N^* \) can decay into \( N \) and \( \pi \), and even when we have no scattering state \( | N + \pi \rangle \), we could have another “scattering states”. The correlator \( \langle N^* (t_{\text{sink}}) N^* (t_{\text{src}}) \rangle \) can still accommodate, for example, the following term:

\[
\langle \pi | N^* (t_{\text{sink}}) | N \rangle \langle N | N^* (t_{\text{src}}) | \pi \rangle \times e^{-E_N (t_{\text{sink}} - t_{\text{src}})} e^{-E_\pi (t_{\text{sink}} + t_{\text{src}} - t_{\text{src}})}.
\]

(2)

Here, \( N_t \) denotes the temporal extent of a lattice. Such a term is quite problematic and mimic a fake plateau at \( E_N - E_\pi \) in the effective mass plot because it behaves as \( e^{-E_N (t_{\text{sink}} - t_{\text{src}})} \). Although these contaminations disappear when one employ enough large \( N_t \) lattice, our lattices do not have so large \( N_t \). In order to eliminate such contributions, we impose the Dirichlet condition on the temporal boundary for valence quarks, which prevents valence quarks from going over the boundary. Though the boundary is still transparent for the states with the same quantum numbers as vacuum, e.g., glueballs, such contributions will be suppressed by the factor of \( e^{-E_G N_t} \) and we neglect them in this paper. (Wraparound effects can be found even in quenched calculations \((10)\).

**Formulation.** We here give a brief introduction to our formulation \((10)(11)\). Let us assume that we have a set of \( N \) independent operators, \( O_{\text{sink}} \) for sinks and \( O_{\text{src}} \) for sources. We can then construct an \( N \times N \) correlation matrix \( C_{ij} (T) \equiv \langle O_{\text{sink}}^j (T) O_{\text{src}}^i (0) \rangle = C_{1i} (T) C_{i1} \). Here, \( C_{1i} (T) \) is a general matrix and \( \Lambda (T)_{ij} \) is a diagonal matrix given by \( \Lambda (T)_{ij} \equiv \delta_{ij} e^{-E_{ij} (T)} \). The optimal source and sink operators, \( O_{\text{src}}^i \) and \( O_{\text{sink}}^i \), which couple dominantly (solely in the ideal case) to \( i \)-th lowest state, are obtained as \( O_{\text{src}}^i = \sum_j O_{\text{src}}^i \langle C_{j1} (T) \rangle^{-1} O_{\text{sink}}^j \), since \( \langle C_{j1} (T) \rangle^{-1} \) is diagonal. Besides overall constants, \( C_{j1} (T) \) and \( C_{j1} (T) \) are obtained as the right and left eigenvectors of \( \bar{C} (T + 1) \), respectively.

The zero-momentum-projected point-type operators,

\[
N_1 (t) \equiv \sum_x \varepsilon_{abc} u^a (x, t) (u^b (x, t) C \gamma_5 d^c (x, t))
\]

and

\[
N_2 (t) \equiv \sum_x \varepsilon_{abc} \gamma_5 u^a (x, t) (u^b (x, t) C d^c (x, t)),
\]

are chosen for the sinks. For the sources, we employ the following wall-type operators in the Coulomb gauge,

\[
\bar{N}_1 (t) \equiv \sum_{x_1, x_2, x_3} \varepsilon_{abc} \bar{u}^a (x_1, t) (\bar{u}^b (x_2, t) C \gamma_5 \bar{d}^c (x_3, t))
\]

and

\[
\bar{N}_2 (t) \equiv \sum_{x_1, x_2, x_3} \varepsilon_{abc} \gamma_5 \bar{u}^a (x_1, t) (\bar{u}^b (x_2, t) C \bar{d}^c (x_3, t)).
\]

The parity is flipped by multiplying the operator by \( \gamma_5 \); \( N_1^+ (t) = N_1 (t) \) and \( N_1^- (t) = \gamma_5 N_1 (t) \). The optimized sink (source) operators \( N_1^+ (N_1^-) \), which couple dominantly to the \( i \)-th lowest state are constructed as

\[
N_1^+ (t) = N_1^+ (t) + \left[ \left( C_{11} \right)^{-1} \right] N_2^+ (t)
\]

and

\[
\bar{N}_1^+ (t) = \bar{N}_1^+ (t) + \left[ \left( C_{11} \right)^{-1} \right] \bar{N}_2^+ (t).
\]

Now that we have constructed optimized operators, we can easily compute the (non-renormalized) vector and axial charges \( g_{V,A} \) for the positive- and negative-parity nucleons using three-point functions with the so-called sequential-source method \((12)\). In practice, we evaluate \( g_{V,A} \) defined as

\[
g_{V,A} (t) = \frac{\text{Tr} \Gamma_{V,A} (B (t_{\text{sink}}) J_{\mu} V,A (t) B (t_{\text{src}}))}{\text{Tr} \Gamma_{V,A} (B (t_{\text{sink}}) B (t_{\text{src}}))}.
\]

and extract \( g_{V,A} \) by the fit \( g_{V,A} = g_{V,A} (t) \) in the plateau region. \( B (t) \) denotes the (optimized) interpolating field for nucleons, and \( \Gamma_{V,A} = \gamma_\mu \gamma_5 \) and \( \gamma_\mu \gamma_5 \), respectively. \( J_{\mu} V,A (t) \) are the vector and the axial vector.
currents inserted at \( t \). We show in Fig. 1 \( g_A^{-0}[\text{lat}](t) \) for \( N^+(1535) \) as a function of the current insertion time \( t \). They are rather stable around \( t_{\text{src}} < t < t_{\text{sink}} \).

We finally reach the renormalized charges \( g_{A,V}^{\pm} = \tilde{Z}_{A,V} g_{A,V}^{\pm[\text{lat}]} \) with the prefactors \( \tilde{Z}_{A,V} \equiv 2\kappa u_0 Z_{V,A} (1 + b_{V,A} \frac{m_{V,A}}{m_{N^*}}) \), which are estimated with the values listed in Ref. [9].

We show the fitted values of \( L_{1,2}^\pm \) and \( R_{1,2}^\pm \) in Table I. \( L(T) \) and \( R(T) \) are rather stable and show a plateau from relatively small value of \( T (T \sim 2) \), which is the same tendency as that found in Ref. [11]. We plot in Fig. 2 \( L_{1}^\pm(T) \) and \( R_{1}^\pm(T) \) obtained at \( \kappa = 0.1390 \), for the purpose of reference. The energies \( E_{1,2}^\pm \) are extracted from two-point correlation functions by the exponential fit as \( \langle \mathcal{N}_T^2(t_{\text{src}} + T)\mathcal{N}_T^2(t_{\text{src}}) \rangle = C \exp(-E_{1,2}^\pm T) \) in the large-\( T \) region. The value at each hopping parameter is found to coincide with that in the original paper by the CP-PACS collaboration [3], with deviations of 0.1% to 1%. We here perform simple linear chiral extrapolations for \( E_{1,2}^\pm \). The chirally extrapolated values as well as those at each hopping parameter for \( E_{1,2}^\pm \) in lattice unit are listed in Table I. Although the mass \( E_{1}^+ \) of the ground-state positive-parity nucleon at the chiral limit is overestimated in our analysis (\( \kappa^{-1} = 1.267 \text{ GeV} \)), this failure comes from our simple linear fit.

**Results.** We first take a stock of the vector charges \( g_V^{0,\pm} \) of the ground-state positive- and negative-parity nucleons as well as the axial charge \( g_A^{0,\pm} \) of the ground-state positive-parity nucleon, which are well known and can be the references. We show \( g_V^{0,\pm} \), the vector charges of the positive- and the negative-parity nucleons obtained with three hopping parameters \( \kappa = 0.1375 \), 0.1390 and 0.1400, in the upper panel in Fig. 3 where the vertical axis denotes \( g_V^{0,\pm} \) and the horizontal one the squared pion masses. (These values are also listed in Table I.) The vector charges should be unity if the charge conservation is exact, whereas we can actually find about 10% deviations in Table I or in the upper panel in Fig. 3. Such unwanted deviations are considered to arise due to the discretization errors: The present lattice spacing is about 0.15 fm, which is far from the continuum limit. In fact, the decay constants obtained with the same setup as ours deviate from the continuum values by \( \mathcal{O}(10\%) \). We should then count at least 10% ambiguities in our results. The axial charge \( g_A^{0,\pm} \) of the positive parity nucleon is also shown in the lower panel in Fig. 3. As found in the...
TABLE I: **upper table:** The fitted values of $L_{1,2}^+$ and $R_{1,2}^+$ for the ground and the 1st excited states in positive- and negative-parity channels are listed. $M_s$ and $E_{1,2}^+$ denote the pion mass and the ground- and the 1st excited-state energies for the positive- and the negative-parity channels for each $\kappa$ in the lattice unit. $(a = 0.1555$fm and $a^{-1} = 1.267$GeV.) The row “C.L.” shows the values at the chiral limit. **lower table:** The non-renormalized vector and axial charges for $n$-th positive- and negative-parity nucleons $g_{V,A}^{n\pm [u,d]}[\text{lat}]$ are listed. The superscripts (u) and (d) denote the u- and d-quark contributions, respectively. The total axial and vector charges, $g_{V,A}^{n\pm} \equiv g_{V,A}^{n\pm [u]} - g_{V,A}^{n\pm [d]}$ and $g_{V,A}^{n\pm} \equiv g_{V,A}^{n\pm [u][\text{lat}] - g_{V,A}^{n\pm [d][\text{lat}]}}$, as well as the renormalization factor and the improvement coefficients $\tilde{Z}_{V,A} \equiv 2\kappa g_{V,A} (1 + b_{V,A} \frac{m_0}{\kappa})$ [9] for vector and axial currents are also listed. $g_{V,A}^{n\pm [u,d]}[\text{lat}]$ denote the renormalized charges.

| $\kappa$ | $L_1^+$ | $R_1^+$ | $L_1^-$ | $R_1^-$ | $L_2^+$ | $R_2^+$ | $L_2^-$ | $R_2^-$ | $M_s$ | $E_1^+$ | $E_1^-$ | $E_2^+$ | $E_2^-$ |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 0.1375   | -0.4341  | -0.4573  | 0.0355   | 0.0126   | -1.353   | -314.1   | -1.432   | -1.302   | 0.8958(5) | 1.699(1)  | 2.137(10) | 2.524(53) | 2.141(14) |
| 0.1390   | -0.4526  | -0.4552  | 0.1115   | -0.2036  | -845.9   | -228.1   | -2.729   | -1.084   | 0.7351(5) | 1.459(1)  | 1.854(13) | 2.162(44) | 1.908(17) |
| 0.1400   | -0.1605  | -0.3552  | 0.0990   | -0.0151  | -408.9   | -143.6   | -1.510   | -1.038   | 0.6024(6) | 1.270(2)  | 1.665(15) | 2.046(67) | 1.733(25) |
| C.L.     | -        | -        | -        | -        | -        | -        | -        | -        | -        | 0.936(3)  | 1.277(25) | 1.570(109)| 1.411(38) |

| $\kappa$ | $g_{V}^{0\pm [u][\text{lat}]}$ | $g_{V}^{0\pm [d][\text{lat}]}$ | $g_{A}^{- [u][\text{lat}]}$ | $g_{A}^{- [d][\text{lat}]}$ | $g_{A}^{0\pm [u][\text{lat}]}$ | $g_{A}^{0\pm [d][\text{lat}]}$ | $g_{A}^{- [u][\text{lat}]}$ | $g_{A}^{- [d][\text{lat}]}$ | $Z_V$ | $Z_A$ |
|----------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|-------|-------|
| 0.1375   | 4.208(8)            | 3.844(76)           | 3.852(42)           | -1.073(49)          | 4.925(24)           | 1.045(1)            | 0.989(19)           | 1.247(8)            | 0.2530| 0.2576|
| 0.1390   | 4.492(10)           | 4.152(160)          | 3.978(94)           | -1.244(44)          | 5.222(126)          | 1.089(11)           | 1.036(60)           | 1.261(7)            | 0.2446| 0.2491|
| 0.1400   | 4.663(9)            | 4.380(206)          | 3.952(136)          | -1.150(55)          | 5.102(145)          | 1.115(2)            | 1.048(111)          | 1.261(8)            | 0.2390| 0.2434|

previous lattice studies, the axial charge of the positive parity nucleon shows little quark-mass dependence, and they lie around the experimental value 1.26.

We finally show the axial charges of the negative-parity nucleon resonances in the lower panel in Fig. 3. One finds at a glance that the axial charge $g_A^{-}$ of $N^{*}(1535)$ takes quite small value, as $g_A^{-} \sim O(0.1)$ and that even the sign is quark-mass dependent. While the wavy behavior might come from the sensitiveness of $g_A^{-}$ to quark masses, this behavior may indicate that $g_A^{-}$ is rather consistent with zero. These small values are not the consequence of the cancellation between u- and d-quark contributions. The u- and d-quark contributions to $g_A^{-}$ are in fact individually small, which one can find in the columns named as $g_A^{- [u]}[\text{lat}]$ and $g_A^{- [d]}[\text{lat}]$ in Table I. We additionally make some trials with lighter u- and d-quark masses at $\kappa=0.1410$. Since we have less gauge configurations and the statistical fluctuation is larger at this kappa, we fail to find a clear plateau in the effective mass plots of the two-point correlators and the extracted mass $E_1^{-}$ of the negative-parity state cannot be reliable. Leaving aside these failures, we try to extract $g_A^{-}$. The result is added in the lower panel in Fig. 3 as a faint-colored symbol, which is consistent with those obtained at other $\kappa$’s. On the other hand, the axial charge $g_A^{1-}$ of $N^{*}(1650)$ is found to be about 0.55, which has almost no quark-mass dependence. The striking feature is that these axial charges, $g_A^{0-} \sim 0$ and $g_A^{1-} \sim 0.55$, are consistent with the naive nonrelativistic quark model calculations [12, 14], $g_A^{0-} = -\frac{1}{7}$ and $g_A^{1-} = \frac{5}{7}$. Such values are obtained if we assume that the wave functions of $N^*(1535)$ and $N^*(1650)$ are $l = 1, S = \frac{1}{2}$ and $l = 1, S = \frac{3}{2}$, neglecting the possible state mixing. (Here, $l$ denotes the orbital angular momentum and $S$ the total spin.)

In the chiral doublet model [1, 2], the small $g_A^{N^*N^*}$ is realized when the system is decoupled from the chiral condensate $\langle \bar{\psi}\psi \rangle$. The small $g_A^{0-}$ of $N^*(1535)$ then does not contradict with the possible and attempting scenario, the chiral restoration scenario in excited hadrons [2]. If this scenario is the case, the origin of mass of $N^*(1535)$ (or excited nucleons) is essentially different from that of the positive-parity ground-state nucleon $N(940)$, which mainly arises from the spontaneous chiral symmetry breaking. However, the non-vanishing axial charge of $N^*(1650)$ unfortunately gives rise to doubts about the scenario.

In order to reveal the realistic chiral structure, studies with much lighter u,d quarks will be indispensable. A study of the axial charge of Roper, as well as the inclusion of strange sea quarks could also cast light on the low-energy chiral structure of baryons and the origin of mass.

Conclusions. In conclusion, we have performed the first lattice QCD study of the axial charge $g_A^{N^*N^*}$ of $N^*(1535)$ and $N^*(1650)$, with two flavors of dynam-
ical quarks employing the renormalization-group improved gauge action at \( \beta=1.95 \) and the mean-field improved clover quark action with the hopping parameters, \( \kappa=0.1375, 0.1390 \) and 0.1400. We have found the small axial charge \( g_A^0 \) of \( N^*(1535) \), whose absolute value seems less than 0.2 and which is almost independent of quark mass, whereas the axial charge \( g_A^1 \) of \( N^*(1650) \) is found to be about 0.55. These values are consistent with the naive nonrelativistic quark model predictions, and could not be the favorable evidences for the chiral restoration scenario in (low-lying) excited hadrons. Further investigations on the axial charges of \( N^*(1535) \) or other excited baryons will cast light on the chiral structure of the low-energy hadron dynamics and on where hadronic masses come from.

All the numerical calculations were performed on NEC SX-8R at RCNP and CMC, Osaka University, on SX-8 at YITP, Kyoto University, and on BlueGene at KEK. The unquenched gauge configurations employed in our analysis were all generated by CP-PACS collaboration \( \text{[9]} \). We thank L. Glozman, D. Jido, S. Sasaki, and H. Suganuma for useful comments and discussions. This work is supported by a Grant-in-Aid for Scientific research by Monbu-Kagakusho (No. 17540250), the 21st Century COE “Center for Diversity and University in Physics”, Kyoto University and Yukawa International Program for Quark-Hadron Sciences (YIPQS).

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