Separated spin-up and spin-down evolution of degenerated electrons in two-dimensional systems: Dispersion of longitudinal collective excitations in plane and nanotube geometry

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**Abstract** – Applying the separated spin evolution quantum hydrodynamics to the two-dimensional electron gas in plane samples and nanotubes located in external magnetic fields we have found a novel type of waves in the electron gas which is called spin-electron acoustic wave. A separate spin-up and spin-down electrons’ evolution reveals the replacement of the Langmuir wave by a pair of hybrid waves. One of the two hybrid waves is a modified Langmuir wave. Another hybrid wave is a spin-electron acoustic wave. We studied the dispersion of these waves in two-dimensional structures of electrons. We also considered the dependence of dispersion properties on spin polarization of electrons in an external magnetic field.

The Langmuir wave is a fundamental process in multielectron three-dimensional and low-dimensional systems creating a background for many applications. Plasmonics is one of the most significant examples of these applications. However, considering a separate evolution of spin-up and spin-down electrons moving in an external magnetic field one can find that a group of electrons displays a new collective excitation: the spin-electron acoustic wave (SEAW) [1]. This wave has been recently predicted and studied in a three-dimensional electron gas for two regimes: the one of parallel propagation and another one of orthogonal propagation relatively to the external magnetic field [1]. The oblique propagation of the SEAWs was considered in ref. [2] where the appearance of the second branch of the SEAWs was discovered and its dispersion dependence was studied. The properties of the spin-electron acoustic wave and the modification of properties of the Langmuir wave in two-dimensional electron gases (2DEGs) with plane and cylindrical (nanotube) geometry are subject of the theoretical consideration in this paper. To underline the importance of the SEAWs we mention their recent application to the modeling of the high-temperature superconductivity [3].

Carbon nanotubes are the most known nanotubes, since they have been used in various applications. However, there are other types of nanotubes: gold [4–6], silicon-based [7], platinum [8] nanotubes, and MgO and Fe₃O₄-based nanostructures [9].

A hydrodynamic description of nanotubes can be found in refs. [10–19]. π electron-hole plasma in single-walled metallic carbon nanotubes is considered in ref. [20] in terms of two-fluid hydrodynamics. The carbon nanotubes contain two types of electrons belonging to π and σ orbitals. Their concentrations are different \( n_\sigma = 3n_\pi \). This leads to the splitting of the Langmuir wave into two waves [21,22]. This effect is described by the two-fluid hydrodynamics where electrons on π and σ orbitals are considered as different interacting species [21,22].

In contrast to the electron gas, electrons in graphite and carbon nanotubes are bound to atoms. Three of four valence electrons are in strong σ bonds, and one electron occupies the π orbital. At hybridization of orbitals, the electrons occupy all four states with different modules and projections of the orbital momentum (one s state and three p states) having the same spin projection. Thus, valence electrons in carbon nanotubes are fully polarized. Since electrons of the carbon nanotubes are fully polarized,
we cannot expect the spin-electron acoustic wave in the carbon nanotubes.

The influence of spin on nanotubes’ properties has been recently discussed in ref. [23]. Energy loss of a plasmon in a disorder-free carbon nanotube and plasmon decays into neutral bosonic excitations of the electron liquid were considered in ref. [24].

The main tool of our research is the separated spin evolution quantum hydrodynamics (SSE-QHD) derived in ref. [1]. The SSE-QHD is a generalization of the spin-(1/2) quantum hydrodynamics [25–28]. The time evolution of the two-dimensional particle densities (concentrations) $n_u$ and $n_d$ gives the continuity equations

$$\partial_t n_s + \nabla (n_s v_s) = -(1)^i T_s,$$

where $s = \{ u = \uparrow, d = \downarrow \}$, $|n_s| = cm^{-2}$, $T_z = \frac{2}{m^2 h^2} (B_z S_y - B_y S_x)$ is the $z$-projection of torque presented in Cartesian coordinates, $i_x = i_y = 2, i_z = 1$, with the spin density projections $S_x$ and $S_y$, each of them is a mix of $\psi_\uparrow$ and $\psi_\downarrow$ which are components of the wave spinor. The explicit form of $S_x$ and $S_y$ appear as

$$S_x = \psi^* \sigma_x \psi = \psi^*_\uparrow \psi_\uparrow + \psi^*_\downarrow \psi_\downarrow$$

where $\psi_\uparrow$, $\psi_\downarrow$ are components of the wave spinor.

The quantities are not related to different species of electrons with different spin projections. $S_x$ and $S_y$ describe the simultaneous evolution of both species. Hence, $S_x$ and $S_y$ do not bear subindexes $u$ and $d$. We intend to apply the QHD equations to plane and cylindrical geometries, but we have presented the torque in the Cartesian coordinates only, with no representation in cylindrical coordinates. We may do so since it is a non-linear term. It does not affect the linear properties of the electron gas which are considered in this paper. We present some non-linear terms in the Euler equation below in the same manner. We note that $v = \{v_x, v_y\}$ and $n = n(x, y), v = v(x, y)$ for plane-like 2DEG, and $v = \{v_\phi, v_z\}$, and $n = n(\phi, z), v = v(\phi, z)$ for nanotubes.

The time evolution of the particle currents for each projection of spin $J_u = n_u v_u$ and $J_d = n_d v_d$ gives the following Euler equations:

$$m n_s (\partial_t + v_s \nabla) v_s + \nabla p_s = \frac{h^2}{2m} n_s \nabla \left( \frac{\Delta \sqrt{n_s}}{\sqrt{n_s}} \right)$$

where $q_e = -e, \gamma_e = -g \frac{m_e}{2\hbar}$ is the gyromagnetic ratio for electrons, and $g = 1 + \alpha/(2\pi) = 1.00116$, where $\alpha = 1/137$ is the fine-structure constant, so, we include the anomalous magnetic moment of electrons. $J_{(M)x}$ and $J_{(M)y}$ are elements of the spin current tensor $J^{\alpha \beta}$, $J_{(M)x}, J_{(M)y}$, $T_x, T_y$ are non-linear terms and they do not make any contribution to the spectrum. $p$ is the thermal pressure, or the Fermi pressure for degenerate fermions. The term proportional to the square of the Planck constant is the quantum Bohm potential.

The right-hand side of the Euler equation presents the force fields of interaction. The first group of terms on the right-hand side represents the Lorentz forces. The second group of terms describes the effect of the $z$-projection of the magnetic field on the magnetic moments (spins) of particles. The dependence on spin projection manifests itself as different signs before these terms. The third group of terms in the Euler equations contains a part of the well-known force field $F_S = M^2 \nabla B^2$ describing the action of the magnetic field on magnetic moments [25,26]. A part of this force field has been presented by previous terms $F_{S(z)} = \pm \gamma_e n_u \frac{d}{dx} B_2$. The second part of the force field $F_{S(x,y)} = \gamma_e (S_x B_2 + S_y B_2)$. A half of this force field enters each one of the Euler equations. The last group of terms is related to the non-conservation of the particle number with different spin projections. This non-conservation provides an additional mechanism for changes in the momentum density that appear in extra force fields.

At the transition to the cylindrical coordinates the inertial forces appear in hydrodynamic equations. These forces consist of three parts: the convective part (containing the velocity field $v$), the thermal part (containing pressure $p$), and the quantum part (proportional to the square of the Planck constant $\hbar^2$). If the thermal pressure is isotropic, as we assume in this paper, the thermal pressure appears in the Euler equation in the traditional form $\nabla p$, with no extra terms. The convective part of the inertial forces makes no contribution to the linear excitations on the cylindrical surface. The quantum part of the inertial forces together with the quantum part of the momentum flux tensor can be presented as the third term in eq. (2). This form coincides with the traditional form of the quantum Bohm potential in Cartesian coordinates.

The spin evolution itself leads to new collective excitations [29–34], which are, sometimes, called the spin-plasma waves, but we do not consider them here.

We do not consider either the influence of spin evolution on longitudinal waves. Thus, we do not present equations for these quantities. These equations can be found in ref. [1].

A model aimed to describe a separate evolution of spin-up and spin-down electrons was considered in ref. [35], but electrons with different spin projections were described there in the same manner as electrons with no spin separation. However, it contradicts the model directly derived...
from the Pauli equation, independently, for spin-up and spin-down electrons [1].

The electric and magnetic fields in the Euler equation (2) have the following explicit relation to the sources of fields:

$$E = -q_{e} \nabla \int \frac{n_{u} + n_{d} - n_{0}}{r - r'} \, dr', \quad (5)$$

and

$$B = \int [(M \nabla) \nabla - M \Delta] \frac{1}{r - r'} \, dr', \quad (6)$$

where $dr'$ is a differential of the two-dimensional surface: $dr' = dx dy$ for planes, and $dr' = Rd\xi dz$ for cylinders. $n_{0}$ in eq. (5) presents motionless ions. $\mathbf{B} = \mathbf{B}(x, y)$, or $\mathbf{B} = \mathbf{B}(\varphi, z)$, and $\mathbf{B} = \{B_{x}, B_{y}, B_{z}\}$ or $\mathbf{B} = \{B_{r}, B_{\varphi}, B_{z}\}$, the structure of $\mathbf{M}$ is similar to the one of $\mathbf{B}$. The motionless ions are presented in formula (5) by their equilibrium concentration $n_{0}$.

The equation of state for the pressure of spin-up $p_{u}$ and spin-down $p_{d}$, degenerate electrons, appears as $p_{s} = \pi \hbar^{2} n_{s}^{2}/2m$.

The pressures of spin-up electrons and spin-down electrons are different due to the external magnetic field which alters the equilibrium concentration of each species $n_{0s} \neq n_{0u}$. In the pressure $p_{s}$ we have assumed that it is only one particle with a chosen spin direction which can occupy one quantum state. As a consequence, the coefficient in the equation of state is twice larger than the one in the 2D Fermi pressure.

The equilibrium condition is described by non-zero concentrations $n_{0\uparrow}$, $n_{0\downarrow}$, $n_{0} = n_{0\uparrow} + n_{0\downarrow}$, and the external magnetic field $\mathbf{B}_{\text{ext}} = B_{0}\mathbf{e}_{z}$. Other quantities are equal to zero: $\mathbf{v}_{0\uparrow} = \mathbf{v}_{0\downarrow} = 0$, $\mathbf{E}_{0} = 0$, $S_{\text{ex}} = S_{y} = 0$. If we consider plane-like 2DEGs, we place them in the plane $z = 0$, perpendicular to the external magnetic field. Hence, the waves propagate perpendicularly to the external magnetic field. The perturbations of physical quantities are presented as $\delta f = F(\omega, k_{x}, k_{y})e^{-i\omega_{t}+ik_{x}x+ik_{y}y}$ and $k^{2} = k_{x}^{2} + k_{y}^{2}$, with $\delta = \{\delta n_{u}, \delta n_{d}, \delta u, \delta v\}$. If we consider nanotubes we place the cylinder axis parallel to the external magnetic field. We present perturbations in the following form: $\delta f = \int \sum_{k=0}^{\infty} F_{1}(k, \omega)e^{-iw(\xi+k_{\text{ex}}+\xi_{\varphi}+\xi_{\text{rad}})}dkd\omega$.

The representation of perturbations via exponents leads to sets of linear algebraic equations relatively to $N_{Au}$, $N_{Ad}$, $V_{Au}$, and $V_{Ad}$. For instance, for the plane geometry we find $-\omega_{s} N_{As} + n_{0u} k V_{As} = 0$ from the continuity equations (1) and $-\omega_{s} n_{0u} V_{As} + k m u_{s}^{2} N_{As} = -e^{2} n_{0u} k(2\pi/k)(N_{Au} + N_{Ad}) - e n_{0u} B_{0}(V_{Ay} e_{x} - V_{Ax} e_{y})/c$ from the Euler equations (2), where $U_{s}^{2} = \frac{2\hbar^{2}}{m n_{0u}} + \frac{k^{2}mc^{2}}{2\omega_{s}^{2}}$ presents a combined contribution of the Fermi pressure and the quantum Bohm potential, and we have used the linearization of formula (5) to get the first term on the right-hand side. The condition of existence of nonzero solutions for amplitudes of perturbations gives us a dispersion equation. The hydrodynamic model reproduces the results of the kinetic theory in the small wave vector approximation. To derive this limit case from the kinetic model one needs to take into account the minor powers of the wave vector. No such analysis is required for the hydrodynamic model as it already complies with this case.

The difference between spin-up and spin-down concentrations of electrons $\Delta n = n_{0\uparrow} - n_{0\downarrow}$ is caused by external magnetic field. Since electrons are negatively charged, their spins get a preferable direction opposite to the external magnetic field $\eta \equiv \Delta n / n_{0} = \tan \left(\frac{\gamma_{B}}{\xi_{F_{c}, 2D}}\right) = -\tan \left(\frac{\gamma_{B}}{\xi_{F_{c}, 2D}}\right)$, where $\xi_{F_{c}, 2D} = \pi n_{0} h^{2}/m$, and $n_{0} = n_{0u} + n_{0d}$.

If we consider plasmas in the uniform constant external magnetic field, we can see that in the linear approach the numbers of electrons of each species are conserved.

Let us present the results for the wave dispersion. We start from the plane-like 2DEG. We assume that the external magnetic field is orthogonal to the plane where the electron gas is located. The dispersion dependences appear in the following form:

$$\omega^{2} - \Omega^{2} = \frac{1}{2} \left( \omega_{L, u}^{2} + \omega_{L, d}^{2} + (U_{u}^{2} + U_{d}^{2})k^{2} \right)$$

$$+ \left[ \left( \omega_{L, u}^{2} + \omega_{L, d}^{2} + (U_{d}^{2} - U_{u}^{2})k^{2} \right)^{2} + 2k^{2}(U_{u}^{2} - U_{d}^{2})(\omega_{L, u}^{2} - \omega_{L, d}^{2}) \right]^{1/2}, \quad (7)$$

where $\omega_{L, s}^{2} = 2e^{2} n_{0s} k/m$ is the two-dimensional Langmuir frequency for species $s$ of electrons located in a plane, $\omega_{L, u}^{2} + \omega_{L, d}^{2}$ is the full Langmuir frequency, $\Omega = q_{e} B_{0}/(\pi e)$ is the cyclotron frequency.

Dropping the quantum Bohm potential and passing to dimensionless variables we obtain

$$\xi_{pl} = \frac{1}{2} \left( K + \Lambda K^{2} \pm \sqrt{K^{2} + 2\Lambda K^{3}} \right), \quad (8)$$

where $\xi_{pl} = (\omega_{L}^{2} - \omega_{ch}^{2})^{1/2}$, $K = k_{x}^{2}/\lambda_{x}^{2}$, $\Lambda = r_{B}/\sqrt{\lambda_{x}}$, with $\omega_{ch}^{2} = 2\pi^{2} n_{0}^{2} /m$, $r_{B} = h^{2}/(me^{2})$ is the Bohr radius. The numerical analysis of formula (8) is presented
The corresponding analytical solution is presented by formula (8) with minus before the square root.

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The linearity of QHD equations contains terms up to the second degree if we drop the quantum Bohm potential contribution. Formulae (7) and (8) represent the general solution of the dispersion equation due to the square root which contains all the degrees of \( k \). To be more explicit we present formula (7) in the long-wavelength limit including terms up to \( k^2 \). We find \( \omega^2_f = \omega^2_{L,cyl} + \varepsilon_{Fe,2D}(1 + \eta^2)k^2/m + \Omega^2 \) from the solution with plus in front of the square root and \( \omega^2_f = \varepsilon_{Fe,2D}(1 - \eta^2)k^2/m + \Omega^2 \) from the solution with minus in front of the square root. The first solution \( \omega_f(k) \) describes the Langmuir wave presented in fig. 1. In the three-dimensional magnetized plasmas it is called upper hybrid frequency. The second solution \( \omega_f(k) \) presents the SEAW and demonstrates a high dependence on the spin polarization via multiplier 1. At the increase of the spin polarization \( \eta \) the frequency of the SEAWs decreases as is shown in numbers in fig. 2. The appearance of the gap in the spectrum (7) is related to the cyclotron motion of electrons in the plane orthogonal to the external magnetic field under the Lorentz force action \( \mathcal{F}_L = q_e\mathbf{n}_{ds}[v, B_0]/c \). Hence, circular orbits of electrons are located in the plane of motion. This affects the spectrum of waves propagating orthogonally to the external magnetic field. The SEAWs in three-dimensional systems of waves propagating orthogonally to the external magnetic field are considered in refs. [1] and [2]. In refs. [36,37], a solution to the SEAW was found for the plane 2DEG. The authors of refs. [36,37] call their solution the spin plasmon. It has the following dispersion dependence: \( \omega^2_{pl} = 8\varepsilon_F(1 - \eta)\varepsilon_F^2/3m_n \), where \( \varepsilon_F \) is the Fermi energy and \( m_n \) is the band mass (see formula (9) in refs. [36,37] and notations to it). The spin plasmon spectrum found in ref. [37] is gapless, while our solution has a gap due to the cyclotron motion. This difference is related to the consideration of the in-plane magnetic field in ref. [37].

Figure 1 shows the dimensionless shift of the dispersion dependence of the Langmuir wave \( \xi_{pl} = K - F \) from the Langmuir frequency square for 2DEG in plane \( \omega^2_L \). We note that the 2D Langmuir frequency \( \omega^2_L \) is the linear function of the wave vector \( k = K/\sqrt{n_0} \). Figure 1 depicts the dependence of the shift \( \xi_{pl} = K - F \) on the dimensionless wave vector \( K \) and the spin polarization \( \eta \). We see that \( \xi_{pl} = K \), as well as \( \xi_{pl} \), increases with the increase in the wave vector. It happens due to the pressure of degenerate electron gas. Figure 1 also shows that the growth of \( \xi_{pl}(K) \) increases with the increase in the spin polarization. This effect appears due to the dependence of the pressure contribution on the spin polarization via different occupations of quantum states by spin-up and spin-down electrons. We take account of this effect via the difference in pressures of spin-up and spin-down electrons. Nevertheless, this effect can be included in a single-fluid model of electrons via the corresponding equation of state (see ref. [38] for more details).

The dispersion dependence of the SEAW \( \xi_{pl}(K) \) at various spin polarizations \( \eta \) is depicted in fig. 2. We see that the frequency square of the SEAW \( \xi_{pl}(K) \) is by three orders smaller than the Langmuir frequency. We also see that the frequency of the SEAW increases with the increase in the wave vector \( K \). The rate of the increase slows down as the spin polarization grows.

For the experimental observation one needs to use a two-dimensional electron gas formed in a ferromagnetic material, applying the external magnetic field to form a single domain. Hence, the domain walls and difference of the spin directions in different domains would not affect the observations. In this case one will have a partially spin-polarized electron gas. Its spin polarization depends on the material. Therefore, spin polarization is a free parameter of the our model and it should be measured experimentally.

Now we present the results for 2DEG on the cylindrical surface. The external magnetic field is parallel to the axis of the cylinder (nanotube), where the electron gas is located. The corresponding dispersion dependence appears as

\[
\omega^2 = \frac{1}{2} \left\{ \omega^2_{L,cyl} + \frac{l^2}{R^2} \left[ v_u^2 + v_d^2 + \frac{h^2}{2m}(k^2 + \frac{l^2 + 2}{R^2}) \right] \right. \\
+ k \left[ v_u^2 + v_d^2 + \frac{h^2}{2m} \left( k^2 + \frac{l^2}{R^2} \right) \right] \\
\left. \pm \left[ \left( k^2 + \frac{l^2}{R^2} \right) \left( v_u^2 - v_d^2 + \frac{e^2Rc(n_0u - n_0d)}{m} \right)^2 \right]^{1/2} \right\},
\]

(9)

where

\[
\omega^2_{L,cyl,s} = \frac{Ge^2n_{0s}}{m} R\left( k^2 + \frac{l^2}{R^2} \right)
\]

(10)

is the Langmuir frequency of the electron gas on the cylinder for species \( s = \{ u = \uparrow, d = \downarrow \} \) of electrons, \( \omega^2_{L,cyl} = \omega^2_{L,cyl,u} + \omega^2_{L,cyl,d} \) is the full Langmuir frequency,
the area of small wave numbers and large spin polarization.

The analytical expression of the dispersion dependence is given by formula (11) with the minus before the square root.

$$v_0^2 = 2\pi \hbar^2 n_0 / m^2$$ presents the contribution of the Fermi pressure, and 
$$G = G(R,k,l) = 4\pi I_l(kR)K_l(kR),$$ with 
$$I_l(x), K_l(x)$$ are modified Bessel functions.

The dimensionless form of the dispersion dependence (9) can be written down as

$$\xi_{cyl} = \frac{1}{2} \left[ K^2 + l^2 Y^2 \right] \left( \frac{G}{2\pi Y} + \Lambda \right)$$
$$\pm \sqrt{ \left( \frac{G}{2\pi Y} \right)^2 + 2\eta^2 \frac{G}{2\pi Y} \Lambda + \eta^2 \Lambda^2 },$$

where $$\xi_{cyl} = \omega^2 / \omega_{ch}^2$$, 
$$K = k / \sqrt{n_0}$$, 
$$\Lambda = r_B \sqrt{n_0}$$, 
$$Y = 1/(R \sqrt{n_0})$$.

Figures 3 and 4 reflect the behavior of the Langmuir wave dispersion in 2DEG on the cylindrical surface. In the numerical analysis we assume that the radius of the cylinder is equal to 30 nm. At $$l = 0$$ the frequency square of the Langmuir wave $$\xi$$ depends almost linearly on the wave vector $$k$$, but the frequency shift $$\Delta \xi = \xi - \omega_{ch,cyl}^2 (k,l) / \omega_{ch}^2$$ shows a small difference from the linear growth. This difference is related to the pressure of the degenerate gas. We also see that the pressure contribution depends on the spin polarization $$\eta$$. At $$l \neq 0$$, $$\xi(k)$$ shows almost parabolic dependence due to the structure of the Langmuir frequency square for electron gas on the cylindrical surface.

The increase in the discrete wave number $$k_{cyl} = l/R$$ leads to the increase in the whole dispersion surface. At the same time, the increase in $$l$$ leads to an increase in the shift $$\Delta \xi(k,\eta)$$. The form of the surfaces showing the shift $$\Delta \xi(K,\eta)$$ also changes with the increase of $$l$$: the area of small wave numbers and large spin polarization grows up faster than other areas.

Figures 3 and 4 show that the pressure contribution depends on the spin polarization $$\eta$$. Moreover, the area of small wave vectors and small spin polarization increases relatively faster. Modifications of $$\xi_{cyl}$$ for the SEAW and $$\Delta \xi_{cyl}$$ for the Langmuir wave that occur with the change of $$l$$ are different. Different areas of

Fig. 3: (Color online) The figure shows the dispersion of the Langmuir waves on a cylindrical surface $$\xi = \xi_{cyl}$$ and the shift of the frequency square of the Langmuir waves $$\Delta \xi$$ from the Langmuir frequency square $$\omega_{cyl}^2$$ for $$l = 0$$ and $$l = 1$$. The analytical expression of the dispersion dependence is given by formula (11) with the plus before the square root.

Fig. 4: (Color online) The figure shows the dispersion of the Langmuir waves on a cylindrical surface $$\xi = \xi_{cyl}$$ and the shift of the frequency square of the Langmuir waves $$\Delta \xi$$ from the Langmuir frequency square $$\omega_{cyl}^2$$ for $$l = 2$$ and $$l = 3$$. The analytical expression of the dispersion dependence is given by formula (11) with the plus before the square root.

Fig. 5: (Color online) The figure presents the spectra of the spin-electron acoustic wave at different values of discrete wave number $$k_{cyl} = l/R$$. We present the dispersion surfaces for $$l = 0, 1, 2, 3$$. The analytical expression of the dispersion dependence is given by formula (11) with the minus before the square root.

Figure 5 describes the SEAW in the cylindrical 2DEG for $$l = 0, 1, 2, 3$$. Its general behavior shows a resemblance to the dispersion of the SEAW in the plane-like 2DEG: on an increase in $$\xi(k)$$, we have a decrease in $$\xi(k)$$ with an increase in the spin polarization $$\eta$$. An increase in $$l$$ reveals itself in the modification of the dispersion surface of the SEAW. The values of $$\xi$$ increase with the increase in $$l$$. Moreover, the area of small wave vectors and small spin polarization increases relatively faster. Modifications of $$\xi_{cyl}$$ for the SEAW and $$\Delta \xi_{cyl}$$ for the Langmuir wave that occur with the change of $$l$$ are different. Different areas of
these surfaces show a relative growth. In both cases these areas are located at small wave vectors, however, they are located at different ranges of the spin polarization $\eta$. A relative increase in $\Delta \xi$ for the Langmuir wave is at large $\eta$, whereas, the relative increase in $\xi$ for the SEAW is at small spin polarization $\eta$.

In this paper we have discussed the increase in frequency of the 2D Langmuir excitations due to different occupations of spin-up and spin-down states in the electron gas located in the external magnetic field. We have demonstrated the existence of the SEAW in 2D structures (planes and nanotubes). We have described the properties of the SEAW. These results have been obtained by means of the SSE-QHD.

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