Disorder Effects in CA-Models for Traffic Flow

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Abstract. We investigate the effect of quenched disorder in the Nagel-Schreckenberg model of traffic flow. Spatial inhomogenities, i.e. lattice sites where the braking probability is enlarged, are considered as well as particle disorder, i.e. cars of a different maximum velocity. Both types of disorder lead to segregated states.

1 Introduction

The formation of traffic jams is one of the fundamental problems of traffic flow theory. Traffic jams can form spontaneously as well as due to hindrances, e.g. road works or slow cars. Although these hindrances often cover only a small part of the road, they can cause large jams, or, in a more physical language, one observes macroscopic effects due to local defects in the system. This behaviour is one of the characteristic properties of nonequilibrium systems. In the context of driven lattice gases one can distinguish in principle two types of defects: 1) lattice defects, e.g. sites where the mobility of particles is reduced, and 2) particle defects, for example ‘slow’ particles.

For the simplest model for a driven lattice gas in one dimension, the asymmetric exclusions process (ASEP)\textsuperscript{1}, it has been shown that both types of defects can cause phase transitions. Slow particles, i.e. particles with a reduced hopping rate, determine completely the flow at small densities. Moreover, if the distribution of the hopping rates fulfills certain conditions, one can show analytically that the variance of the distance distribution has a logarithmic divergence at the transition density \textsuperscript{2,3,4,5}.

Also lattice defects in the ASEP have been investigated extensively \textsuperscript{6,7,8,9,10,11}. Implementing lattice defects one can observe three different phases: A high and a low density phase, where the average flow of the homogeneous system is recovered, and a segregated phase at intermediate densities, where the flow takes a constant value. Despite extensive effort, exact analytical results exist only for a special type of update \textsuperscript{12}, but approximative descriptions are in reasonable agreement with simulation results.
In this work we generalize the results of the ASEP to the Nagel-Schreckenberg (NaSch) model of traffic flow (for a detailed explanation of the model see [13,14]). Compared with the ASEP, the particles (cars) can hop more than one site in a single update step and the update rules are applied in parallel to all cars. Therefore slow cars can be considered in two ways. First, one can think of cars with a smaller maximum velocity and second, of cars with an enlarged braking probability. This has been done very recently and analogous results to the $v_{max} = 1$ case have been found [15]. In contrast to [15] we discuss the case of two different maximum velocities in the third chapter. In the next chapter we show simulation results for the NaSch model with defect sites, implemented as sites where the braking probability of cars is higher compared to the rest of the lattice. An alternative choice has been used in [16], where a 'speed limit’ in a part of the lattice has been considered.

2 Defect Sites

As already mentioned above, in this chapter we show simulation results of the NaSch model on a lattice with defect sites. Fig. 1 shows the fundamental diagram

![Fundamental diagram of the NaSch model with a single defect site. The model parameters are given by $v_{max} = 2$ (maximum velocity), $p = 0.25$ (braking probability), $p_d = 0.75$ (braking probability at the defect site) and $L = 3200$ (system size).](image_url)

of a system with a single defect site. Obviously we can distinguish three different phases depending on the density. In the high and low density phases the average flow of the defect systems takes the same value as in the homogeneous system. For intermediate densities the flow is constant and limited by the capacity of the defect site.

This behaviour of the average flow can be explained looking at the density profile. In the high and low density phase only local deviations from a constant profile can be observed, but at intermediate densities one observes a separation into macroscopic high and low density regions. Changing the global density
Fig. 2. Density profiles in the three phases. In the high (red, \( \rho = 0.60 \)) and low (blue, \( \rho = 0.10 \)) density phases only local inhomogenities occur near the defect site (the defect site is located at \( x_d = 1600 = L/2 \)), but for intermediate densities one observes phase separation (green, \( \rho = 0.30 \)). We used the same parameters as in Fig. 1.

within the segregated phase, the bulk densities in the high (\( \rho_h \)) and low density region (\( \rho_l \)) remain constant, only the length of the high and low density region changes. Consequently the average flow is constant in the segregated phase, because the average density in the vicinity of the defect site does not depend on the global density.

Near the average position of the shock the density profile decays exponentially from \( \rho_h \) to \( \rho_l \). Therefore one can introduce a length scale \( \xi \), which corresponds to the magnitude of the fluctuations of the shock position. For \( v_{max} > 1 \) we found numerically that \( \xi \sim \sqrt{L} \) for all densities in the segregated phase we took into account. This scaling behaviour is already known for the ASEP at \( \rho \neq 0.5 \). The modified scaling behaviour \( \xi \sim L^{1/3} \) at \( \rho = 0.5 \) was not found for \( v_{max} > 1 \). This confirms the picture that the reduction of fluctuations is a consequence of the particle-hole symmetry [6].

A good estimate for the plateau value of the flow for the case of \( v_{max} = 1 \) can be obtained using the assumption that the system is separated into two regions of constant density, where the results from the homogeneous system can be used. Using an argumentation similar to [10] we obtain the plateau value of the average flow:

\[
J_p = \frac{1}{2} \left( 1 - \frac{1}{q + q_d} \sqrt{(q + q_d)^2 - 4q^2q_d} \right)
\]  

(1)

The bulk value of the density in the high (low) density region the density is given by \( \rho_h = \frac{q}{q + q_d} \) (\( \rho_l = \frac{q_d}{q + q_d} \)) with \( q = 1 - p \) and \( q_d = 1 - p_d \). The shock is located at \( r = x_d - \frac{\rho - \rho_h}{p - \rho} L \) if \( r \geq 0 \) or at \( r' = L - r \) if \( r < 0 \). This approximative treatment of the defect system is in a resonable agreement with the simulation
results for large system sizes (for a more detailed discussion see [15]). In principle one could treat higher velocities in the same way, but unfortunately no exact analytical description of the homogeneous system has been found by so far.

3 Particle defects

In order to study the effect of slow cars on the throughput of single lane traffic, we discuss a system which consists of fast cars with maximum velocity $v_f = 3$ and one slow car with $v_s = 2$.

![Fundamental diagram of a system with one slow car (dots) compared with fundamental diagrams of the corresponding homogeneous systems.](image)

Fig. 3. Fundamental diagram of a system with one slow car (dots) compared with fundamental diagrams of the corresponding homogeneous systems. The maximum velocity of the slow car is given by $v_s = 2$ and of the other cars by $v_f = 3$. The braking probability of all cars is $p = 0.5$.

In Fig. 3 the fundamental diagram of this system is compared with the fundamental diagram of the analogous homogeneous systems. Obviously for low densities the flow is given by $J_l(\rho) = \rho(v_s - p)$ in agreement with the homogeneous system with $v_{max} = 2$. Compared to the homogeneous system one observes the linear density dependence also for larger values of the average density. The average flow depends linearly on the global density if $J_l(\rho) \leq J_{v_f}(\rho)$ holds, where $J_{v_f}(\rho)$ denotes the stationary flow of the homogeneous system with fast cars. Deviations from this form that are observable near the intersection point of $J_l(\rho)$ and $J_{v_f}(\rho)$, are due to the finite size effects.

The reason for the stability of the free flow regime in the mixed system is the gap in front of the slow car in the stationary state. In Fig. 3 the density profile relative to the position of the slow car is shown. Obviously a large jam behind the slow car appears and the density in front of the slow car vanishes. Therefore the slow car can move with its free flow velocity $v_s - p$. This velocity determines the velocity of the whole jam and therefore the average velocity of the cars is
Fig. 4. Density profile relative to the position of the slow car. The data are obtained for the following set of parameters: \( v_s = 1, v_f = 2, p = 0.5 \) and, \( \rho = 0.1 \).

simply given by \( J_l(\rho) \). This is one possible simple scenario for the occurrence of ‘moving’ jams that are well known from measurements.

In addition to the macroscopic form, two details of the density profile have to be discussed. Behind the slow car one observes an oscillating amplitude of the density profile. These oscillations are well known from the spatial correlation function of the homogeneous system for small \( p \) and densities near \( \rho_c = 1/(v_{\text{max}} + 1) \) [19]. Obviously the slow car synchronizes the motion of the faster cars. Furthermore we want to discuss the decay of the density profile at the end of the jam. For finite values of the braking probability the jam length is fluctuating. These fluctuations are responsible for the finite size effects discussed above, because temporarily the gap in front of the slow car vanishes due to fluctuations of the jam length. Measurements of the jam length show that the fluctuations scale according to \( \Delta L_J \sim \sqrt{L} \). Due to the subextensive scaling of the fluctuations the segregated states are observable up to \( \rho = \rho_J \) if \( L \to \infty \), where \( \rho_J \) denotes the bulk density of the jam.

4 Summary

The investigation of local defects in the NaSch model has shown that these defects can change the macroscopic properties of the model. In the case of lattice defects three phases occur analogous to the ASEP (i.e. \( v_{\text{max}} = 1 \)) with continuous time update. The segregated phase also exists for higher maximum velocities, but some details of the density profile are changed. In the low density regime one observes an oscillating profile as a consequence of the parallel update. The anomalous scaling of the fluctuations of the shock position found for the ASEP at \( \rho = 0.5 \) is absent for \( v_{\text{max}} > 1 \), for all parameter combinations that have been taken into account. This result confirms the picture that the particle-hole symmetry is essential for the reduction of fluctuations.
Particle defects also produce phase separated stationary states at low densities. These states consist of a large jam behind the slowest vehicle and a large gap in front of the slowest car. The velocity of the ‘moving’ jam is completely determined by the free flow velocity of the slow car. The formation of large clusters due to slow cars can also be observed in two-lane traffic, but due to lane changing the lifetime of the large clusters is finite (at least for low densities). Nevertheless it has been shown, that already a small concentration of slow cars changes leads to a drastic reduction of the average flow at low densities [20].

Our results show that the system properties can change completely, if local disorder is taken into account. These results might also be important for an understanding of the behaviour of real traffic which are often determined by ‘imperfections’. Such imperfections are not only the defects considered here, but also other deviations from an ideal system, e.g. ramps or a finite system size.

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