Dissipative preparation of entangled many-body states with Rydberg atoms

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Keywords: Rydberg atoms, entangled many-body states, Rokhsar–Kivelson State

Abstract
We investigate a one-dimensional atomic lattice laser-driven to a Rydberg state, in which engineered dissipation channels lead to entanglement in the many-body system. In particular, we demonstrate the efficient generation of ground states of a frustration-free Hamiltonian, as well as states closely related to $W$ states. We discuss the realization of the required coherent and dissipative terms, and we perform extensive numerical simulations characterizing the fidelity of the state preparation procedure. We identify the optimum parameters for high fidelity entanglement preparation and investigate the scaling with the size of the system.

1. Introduction

The use of controlled dissipation channels holds great promise for the preparation of highly entangled quantum many-body states [1, 2]. These ideas are particularly relevant for ultracold atoms due to the tremendous experimental control over these systems, combined with the availability of efficient dissipation channels in the form of optical pumping. Here, we discuss the preparation of entangled many-body quantum states in the context of strongly interacting Rydberg atoms.

So far, explicit state preparation protocols have been mostly constrained to stabilizer states [3–9], meaning that these states can be found by minimizing the energy of a stabilizer Hamiltonian of mutually commuting observables [10]. However, as shown in [1], efficient state preparation protocols should also exist for non-commuting degrees of freedom in frustration-free models, where the ground state can be found by minimizing a sum of quasilocal projection operators.

As the physical platform to realize such dissipative quantum state engineering, Rydberg atoms appear to be particularly suited due to their highly tunable interaction and dissipation properties [11]. Remarkably, Rydberg atoms in one-dimensional lattices can be approximated by a frustration-free model [12], making it especially promising to study dissipative quantum state engineering in these systems.

A crucial quantity in the investigation of strongly interacting Rydberg gases is the blockade radius $r_b$, given by

$$ r_b = \frac{C_{6}}{h \Omega}, $$

where $C_{6}$ is the van der Waals coefficient of the Rydberg interaction and $\Omega$ is the Rabi frequency associated with the driving of the Rydberg transition. Here, we will be interested both in the regime where the blockade radius is comparable to the lattice spacing and the regime where $r_b$ is larger than the system size.

This paper is organized as follows. After a brief introduction of the atomic scheme and the theoretical approach, we discuss the preparation of the ground state of a frustration-free Hamiltonian. We present our numerical results for various detuning and explore the appropriate parameters in which the steady state overlaps with the highly entangled Rokhsar–Kivelson (RK) state. Originally formulated in the context of quantum dimer models to explain high-temperature superconductivity [13], RK states have found wide applications in the context of lattice gauge theories appearing in ultracold quantum gases [14], condensed matter [15] and high-energy physics [16]. We investigate the scaling of the fidelity with increasing system size. Finally, we perform an
2. Setup of the system

2.1. Hamiltonian dynamics

We consider a one-dimensional lattice with N sites. Each site is occupied by a two-level atom with states $|0\rangle$ and $|1\rangle$, driven by a laser beam with Rabi frequency $\Omega$ and detuning $\Delta$. The Hamiltonian for this system in the rotating wave approximation is expressed as

$$\mathcal{H} = \Omega \sum_k \sigma_z^k + \Delta \sum_k n_k + V \sum_{m>k} n_m n_k / |k - m|,$$

where $n_k = \sigma^+_k \sigma^-_k$ is the Rydberg number operator in terms of the raising and lowering operators. The atoms interact via the van der Waals interaction with the interaction strength $V$. In the following, we briefly review the mapping of the Rydberg Hamiltonian onto a frustration-free Hamiltonian [12], as this mapping is crucial to understand the dissipative state preparation procedure. By applying the unitary transformation $U = \exp[-i \sum_k n_k n_{k+1}]$ and an additional rotating wave approximation, equation (2) is rewritten as

$$\mathcal{H} = E_0 + \mathcal{H}_{\text{body}} + \mathcal{H}',$$

where $E_0 = -\Omega L \xi$ is the ground state energy and $\mathcal{H}'$ a perturbation. The three-body term $\mathcal{H}_{\text{body}}$ is given by

$$\mathcal{H}_{\text{body}} = \sum_k h_k^1 h_k^2 h_k^3,$$

i.e., it is a sum of projection operators $h_k$. These projection operators have the form

$$h_k = \sqrt{\frac{1}{\xi + 1}} P_{k-1} P_{k+1} [\sigma_z^k + \xi^{-1} n_k + \xi (1 - n_k)],$$

where $P_k = 1 - n_k$. Within the phase diagram of one-dimensional Rydberg gases [17, 18], the perturbation $\mathcal{H}'$ is minimal when the blockade radius $r_b$ is given by $r_b = 2a$ with $a$ being the lattice spacing [12]. The manifold of the region that can be approximated by the three-body Hamiltonian $\mathcal{H}_{\text{body}}$ adiabatically connects to the region between crystalline phases at half and third filling [19].

A Hamiltonian that can be expressed as summation of the projection operators is known as frustration-free Hamiltonian with a RK ground state [13]

$$|\xi\rangle = \frac{1}{\sqrt{Z_\xi}} \prod_k (1 - \xi P_{k-1} P_{k+1}) |00 \cdots 0\rangle,$$

where $Z_\xi = [(1/2)(1 + \sqrt{1 + 8 \xi^2})]^N$ is a normalization constant. This state is a coherent superposition of all states that has no nearest neighbor excitations and is entangled for all finite values of $\xi$, reaching a Greenberger–Horne–Zeilinger state in the limit $\xi \to \infty$ [12]. For the case $\xi = 1$, the RK state can be understood as a superoperator $\mathcal{P}^\dagger$ acting on the antisymmetric superposition of the two atomic states, $\Pi_i |i\rangle = \Pi_{i=0} |00 \cdots 0\rangle / \sqrt{2}$, where $\mathcal{P}^\dagger$ imposes a constraint prohibiting two neighboring excitations. E.g., for a system consisting of three sites, this state is written explicitly as $\frac{1}{2} (|000\rangle - |100\rangle - |010\rangle - |001\rangle)$.

2.2. Jump operators

For the dissipative preparation, we will be interested in the coupling to an external environment, which leads to the desired quantum many-body state as a steady state of the dynamics. Here, we consider an open quantum system that is described by a quantum master equation in Lindblad form

$$\frac{d}{dt} \rho(t) = -\frac{i}{\hbar} [\mathcal{H}, \rho] + \sum_k \left( c_k \rho c_k^\dagger - \frac{1}{2} \{ c_k^\dagger c_k, \rho \} \right).$$

For the dissipative preparation of the RK state, we focus on the jump operators

$$c_k = \sqrt{\rho_k} P_{k-1} (|1\rangle \langle 0|) P_{k+1}.$$

This jump operator flips the site $k$ from the symmetric state $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ into the antisymmetric state $|-\rangle$, provided both neighboring sites $k$ and $k + 1$ are in the state $|0\rangle$. Hence, this jump operator avoids the existence of two neighboring atoms in the $|1\rangle$ state and transfers the atoms to an antisymmetric superposition. In the context of Rydberg atoms, we envision three different possibilities to realize such a correlated jump operator: (i) Combining strong Rydberg interactions with electromagnetically induced transparency (EIT) [20–22], it is possible to engineer effective jump operators of the desired type when the Rydberg interaction destroys the EIT feature. (ii) Using coherent interactions with optically pumped auxiliary atoms, allowing to realize largely

analogous analysis for the generation of entangled $\tilde{W}$ states, where we find high state preparation fidelity even for relatively large system sizes.
arbitrary many-particle jump operators [23]. (iii) Realization of a rotated version of the Hamiltonian, (equation (2)), using Rydberg dressing of hyperfine spin states [24–26], where a \( \sigma \) -jump operator in the rotated bases corresponds to the desired \( \left| -\chi_j^+ \right| \) operator. In the following, we will focus on the third option, as represents the best route towards realization of the proper jump operator. A detailed derivation is shown in appendix A.

While the jump operators of equation (8) have the RK state as a dark state, it is not unique, as configurations involving a large number of up spins are also dark, as there is no site on which the constraints involving the \( P_g \) operators can be fulfilled. Hence, we add a second set of jump operators, related to optical pumping of on spin state into the other via an intermediate state. The strength of this second set of decay processes is chosen to be relatively weak to the former, ensuring that the desired RK state remains an approximate dark state of the dynamics. Interestingly, the precise form of this second set is not very important. Even in the case (ii) outlined above, where the decay is actually occurring in a rotated basis, we find a very efficient decay of the undesired class of dark states. In this case, we can integrate out the intermediate state within the effective operator formalism [27], see appendix B, obtaining for these addition operators in the rotated frame

\[
\begin{align*}
    c_k' &= \frac{\Omega'}{\sqrt{\gamma}} e^{-i\frac{\Delta}{\kappa}} (|0\rangle + |1\rangle) |0\rangle e^{i\frac{\Delta}{\kappa}}, \\
    c_k'' &= \frac{\Omega'}{\sqrt{\gamma}} e^{-i\frac{\Delta}{\kappa}} (|0\rangle + |1\rangle) |1\rangle e^{i\frac{\Delta}{\kappa}}.
\end{align*}
\]

In total, the jump operators \( c_i \) in equation (7) contain the sets \( c_k, c_k', \) and \( c_k'' \), for all lattice sites \( k \).

3. Numerical results

We now perform numerical simulations of the quantum master equation based on the wave function Monte Carlo approach [28] to obtain the steady state \( \rho_{\text{st}} \) of the dynamics. As discussed in [12], the perturbation \( \mathcal{H}' \), describing the corrections to the RK Hamiltonian are minimized for \( V \approx 64 \Omega \xi \). In our simulations, we ensure that this condition is always fulfilled. Furthermore, the correction is further suppressed for \( \xi = 1 \), which is precisely the projection on the antisymmetric state created by the jump operators of equation (8). Consequently, we focus on the case \( \xi = 1 \) in most of our simulations.

Importantly, there is a third condition for which the correction terms are minimized, which manifests itself as a condition imposed on the detuning \( \Delta \). We now turn to numerical simulations to probe this condition in detail. To this end, we first compare the expectation value of the energy in the steady state of the dynamics to the ground state energy of the Hamiltonian at \( \xi = 1 \), see figure 1. Around \( \Delta = -2 \kappa \), the energies are in close agreement, indicating that this condition is optimal for the preparation of the RK state.

We can further explore the condition for the optimal detuning and hence the efficiency of our dissipative preparation scheme by considering the fidelity \( F_{\text{RK}} \) of preparing the RK state in terms of the overlap of the steady state with the RK state.
where $\rho_s$ is the steady state density matrix from the solution of the quantum master equation. Figure 2 shows the fidelity as a function of the detuning $\Delta$. Again, we find that a detuning of $\Delta = -2 \kappa$ is optimal, when we observe the fidelity approaching unity ($F_{\text{RK}} = 0.97$). The fidelity reaches a maximum value at different detunings for various values of $\xi$. While the dissipative state preparation works best for $\xi = 1$, as expected, we find that for the optimum detuning, high fidelity preparation of RK states is possible over a wider range of values for $\xi$. Extending the range of RK state preparation to an even wider range of $\xi$ values would require a modification of the correlated jump operators of equation (8).

Furthermore, we perform an evaluation of our preparation procedure as a function of the system size, focusing on the optimum choice of the detuning for the RK state preparation. We also focus on the $\xi = 1$ case, noting that other choices can be expected to behave very similar, albeit at a lower overall value of the fidelity. Generically, the fidelity is expected to decay exponentially, as small local deviations from the RK state get multiplied for larger system sizes, a phenomenon also known as 'orthogonality catastrophe.' Indeed, as shown in figure 3, we observe a decrease of the fidelity for larger system sizes. Nevertheless, we still obtain a substantial overlap with the RK state for system sizes as large as $N = 11$. Here, we focus on an odd number of sites, as they give slightly larger values for the fidelity. We attribute this difference in even and odd system sizes to the periodic boundary conditions; for even values, the number of configurations satisfying the nearest neighbor constraint is reduced compared to the odd values.

$$F_{\text{RK}} = \langle \xi | \rho_s | \xi \rangle,$$  \hspace{3cm} (10)
4. Generation of $\tilde{W}$ states

Finally, we wish to discuss how the dissipation preparation procedure can be generalized to cover other classes of entangled quantum many-body states. Here, we consider the case where the blockade radius is larger than the system size, $r_b \gg N_0$, corresponding to a fully blockaded ensemble [29]. In our simulations, we realize this condition by working in the regime $V \gg \Omega$. In such a situation, the dynamics is mostly confined to the manifold of states containing at most a single Rydberg excitation. In this regime, the jump operator is defined as

$$c_k = \sqrt{\kappa} \prod_{j=k}^{l-1} P_g^{l} \left( | - \rangle \langle + | \right)^{k}$$

$$= \sqrt{\kappa} P_g^{l} \otimes P_g^{l} \otimes P_g^{l-1} \otimes \left( | - \rangle \langle + | \right)^{k} \otimes P_g^{l+1},$$

i.e., the quantum jump can only occur if there are no Rydberg excitations located at other sites. This is a natural generalization of the jump operators used for RK state preparation, in which the projection operators $P_g$ are not restricted to nearest neighbors as in equation (8), but are extended over the entire system. Then, the dark state of interest is closely related to a $W$ state, i.e.,

$$|\tilde{W}\rangle = \frac{1}{\sqrt{N+1}} ( |\downarrow \downarrow \cdots \downarrow \rangle - |\uparrow \downarrow \cdots \downarrow \rangle - |\downarrow \uparrow \cdots \downarrow \rangle - \cdots).$$

As in the investigation of the dissipative preparation of the RK state, we now turn to analyzing the fidelity of the $W$ state preparation, defined as

$$F_\tilde{W} = \langle \tilde{W}|\rho_\text{eq}|\tilde{W}\rangle.$$

Figure 4 shows the fidelity for various detunings $\Delta$ and for different number of the atoms in the lattice.

As a first observation, we note that the maximum overlap with the $\tilde{W}$ state occurs at different detunings for different number of the atoms. This behavior can be understood with the framework of dark state pumping [30]. In dark state pumping, a pure state $|\psi\rangle$ arises as the steady state, when it is an eigenstate of the Hamiltonian and when it is annihilated by all jump operators, i.e., $c_i|\psi\rangle = 0$. To explain the observed behavior of the detuning, it is important to understand when the dark state becomes an eigenstate of the Hamiltonian. For $N > 1$, the number of Rydberg excitations in the state $|\tilde{W}\rangle$ is balanced towards the ground state, requiring a finite value of the detuning to realize this state as an eigenstate of the Hamiltonian. Furthermore, when increasing the system size, the collective Rabi frequency between the state without any Rydberg excitations and the symmetric combination of single Rydberg excitation will increase as $\sqrt{N} \Omega$. This increase in the Rabi frequency has to be balanced by an increase in the detuning to maintain an eigenstate of the system, explaining the observed dependence of the detuning.

Remarkably, we also find that the fidelity does not significantly decrease for larger system sizes, potentially allowing to efficiently prepare large ensembles of entangled atoms. Performing numerical simulations based on matrix product states [31–34] or the variational principle for open quantum systems [35] will allow to test the precise scaling of the fidelity with the size of the system for much larger ensemble sizes.
5. Summary

We have investigated the dissipative preparation of entangled states in one-dimensional atomic lattices in which the atoms interact via a long-range Rydberg interaction. We find that we can efficiently prepare moderately sized systems in a highly entangled RK state, as well as larger ensembles of atoms in a state closely related to a $W$ state. Our results underline the strengths of the dissipative quantum state preparation paradigm. In particular, our approach allows to realize quantum states that lie outside the class of stabilizer states. It is an interesting question whether future generalizations of our approach will also be able to reach states that cannot be represented by ground states of frustration-free Hamiltonians.

Acknowledgments

This work was funded by the Volkswagen Foundation and by the DFG within SFB 1227 (DQ-mat) and SPP 1929 (GiRyd).

Appendix A. Correlated jump operators in Rydberg-dressed systems

To realize the desired jump operators of equation (8), we turn to a situation where a highly excited Rydberg state is weakly admixed to two hyperfine ground states. If the coupling to the Rydberg state is realized in a Raman-type configuration, then one obtains within degenerate fourth-order perturbation theory the effective Hamiltonian

$$\mathcal{H} = \Omega \sum_{k} n_k \sigma_z^k + \Delta \sum_{k} n_k \sigma_x^k + V' \sum_{m<k} |k - m|^2 n_m n_k^\dagger,$$

where the projection operator $n_k^\dagger = (1 + \sigma_z^k)/2$ has been used [26]. Hence, the Hamiltonian has exactly the same functional form as the original Hamiltonian of equation (2), except the role of the $x$ and $z$ axes of the spins are swapped. In the following, we assume that the primed coupling constants are chosen to exactly match the unprimed ones.

In the limit $V \gg \Omega$, which is well respected in all our calculations, the interaction term can be used to control the dissipation channels in the system. If we consider an optical pumping process turning the spin-up state into the spin-down state, the excitation process within the optical pumping is only resonant if the condition $n_n = 0$ holds on the neighboring sites; otherwise the interaction shifts the excitation laser out of resonance. As a result, the dynamics of the optical pumping has to be described by a correlated jump operator of the form

$$c_k = \sqrt{\gamma} P_{k-1}^\dagger \sigma_z^k P_{k+1},$$

where $\gamma$ is the rate of the optical pumping process and $P_k^\dagger$ corresponds to the projection operator $P_k^\dagger = 1 - n_k$. After a unitary transformation, which again swaps the $x$ and the $z$ axes of the spins, thus turning equation (A1) back into equation (2), we obtain the desired jump operators of the form equation (8).

Experimentally, the model can be realized, for instance, using the Rydberg state $31P_{1/2}$ in $^{87}\text{Rb}$. Driving via the Rydberg state in a Raman configuration [26] with a Rabi frequency $\Omega_r = 2\pi \times 20$ MHz, and a detuning from the Rydberg state of $\Delta_r = 2\pi \times 40$ MHz leads to an interaction strength of $V = 1.2$ MHz at a distance of $a = 1 \mu m$. In such a Raman configuration, the effective two-level model will feature additional interaction terms of the form $n_n n_m$, which limit the RK state preparation fidelity to $F = 0.92$. However, these additional terms can be canceled by employing a third Rydberg laser, resulting in the realization of the effective two-level model of equation (A1).

Appendix B. Jump operators in the effective operator formalism

One approach which eliminates the fast decaying excited states manifold is via effective quantum jump operators [27], which reduces the system dynamics to the non-decaying states.

Here, we will neglect the decay of the Rydberg state and focus only on the decay of intermediate state $|p\rangle$, see figure B1. This procedure gives rise to an effective Hamiltonian

$$\mathcal{H}_{\text{eff}} = -\frac{1}{2} V \left[ \mathcal{H}_{\text{NH}}^{-1} + (\mathcal{H}_{\text{NH}}^{-1})^\dagger \right] V + \mathcal{H}_{\text{g}},$$

where $V_+/V_-$ are the excitation/de-excitation operators coupling to the decaying states. $\mathcal{H}_{\text{NH}}$ is the non-Hermitian Hamiltonian of the decaying states manifold.
where $\mathcal{H}_e$ is the Hamiltonian in the manifold of the decaying states and $\mathcal{L}_k$ describes the quantum jump operators. The effective quantum jump operators are given by

$$\mathcal{L}_k^{\text{eff}} = \mathcal{L}_k \mathcal{H}_{\text{NH}}^{-1} \mathcal{V}_r.$$ (B3)

Applying these definition to a process that describes optical pumping from the $|1\rangle$ state into the $|0\rangle$ state via an intermediate level, we obtain

$$\mathcal{L}_0^{\text{eff}} = \frac{i \Omega'}{\sqrt{\gamma}} (|0\rangle \langle 0| + |0\rangle \langle 1|_k),$$

$$\mathcal{L}_1^{\text{eff}} = \frac{i \Omega'}{\sqrt{\gamma}} (|1\rangle \langle 1| + |1\rangle \langle 1|_k),$$ (B4)

where $\Omega'$ is the Rabi frequency describing the driving of the transition between $|1\rangle$ and the intermediate state and $\gamma$ is its spontaneous decay rate. After transformation into the rotating frame, we obtain the jump operators given in equation (3).

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