THE STELLAR MASS SPECTRUM FROM NON-ISOTHERMAL GRAVOTURBULENT FRAGMENTATION

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Abstract

Identifying the processes that determine the initial mass function of stars (IMF) is a fundamental problem in star formation theory. One of the major uncertainties is the exact chemical state of the star forming gas and its influence on the dynamical evolution. Most simulations of star forming clusters use an isothermal equation of state (EOS). We address these issues and study the effect of a piecewise polytropic EOS on the formation of stellar clusters in turbulent, self-gravitating molecular clouds using three-dimensional, smoothed particle hydrodynamics simulations. In these simulations stars form via a process we call gravoturbulent fragmentation, i.e., gravitational fragmentation of turbulent gas. To approximate the results of published predictions of the thermal behavior of collapsing clouds, we increase the polytropic exponent $\gamma$ from 0.7 to 1.1 at some chosen density $n_c$, which we vary from from $4.3 \times 10^4 \text{ cm}^{-3}$ to $4.3 \times 10^7 \text{ cm}^{-3}$. The change of thermodynamic state at $n_c$ selects a characteristic mass scale for fragmentation $M_{\text{ch}}$, which we relate to the peak of the observed IMF. We find a relation $M_{\text{ch}} \propto n_c^{-0.5 \pm 0.1}$. Our investigation supports the idea that the distribution of stellar masses largely depends on the thermodynamic state of the star-forming gas. The thermodynamic state of interstellar gas is a result of the balance between heating and cooling processes, which in turn are determined by fundamental atomic and molecular physics and by chemical abundances. Given the abundances, the derivation of a characteristic stellar mass may thus be based on universal quantities and constants.
1. Introduction

Although the IMF has been derived from vastly different regions, from the solar vicinity to dense clusters of newly formed stars, the basic features seem to be strikingly universal to all determinations (Kroupa 2001). Initial conditions in star forming regions can vary considerably. If the IMF depends on the initial conditions, there would thus be no reason for it to be universal. Therefore a derivation of the characteristic stellar mass that is based on fundamental atomic and molecular physics would be more consistent.

There are many ways to approach the formation of stars and star clusters from a theoretical point of view. In particular models that connect stellar birth to the turbulent motions ubiquitously observed in Galactic molecular clouds have become increasingly popular in recent years. See, e.g., the reviews by Larson (2003) and Mac Low & Klessen (2004). The interplay between turbulent motion and self-gravity of the cloud leads to a process we call gravoturbulent fragmentation: Supersonic turbulence generates strong density fluctuations with gravity taking over in the densest and most massive regions (e.g. Larson 1981; Fleck 1982; Padoan 1995; Padoan et al. 1997; Klessen et al. 1998, 2000; Klessen 2001; Padoan & Nordlund 2002). Once gas clumps become gravitationally unstable, collapse sets in. The central density increases and soon individual or whole clusters of protostellar objects form and grow in mass via accretion from their infalling envelopes.

However, most current results are based on models that do not treat thermal physics in detail. Typically, a simple isothermal equation of state (EOS) is used. The true nature of the EOS, thus, remains a major theoretical problem in understanding the fragmentation properties of molecular clouds.

Recently Li et al. (2003) conducted a systematic study of the effects of a varying polytropic exponent \( \gamma \) on gravoturbulent fragmentation. Their results showed that \( \gamma \) determines how strongly self-gravitating gas fragments. They found that the degree of fragmentation decreases with increasing polytropic exponent \( \gamma \) in the range \( 0.2 < \gamma < 1.4 \) although the total amount of mass in collapsed cores appears to remain roughly consistent through this range. These findings suggest that the IMF might be quite sensitive to the thermal physics. However in their computations, \( \gamma \) was left strictly constant in each case. Here we study the effects of using a piecewise polytropic equation of state and investigate if a change in \( \gamma \) determines the characteristic mass of the gas clump spectrum and thus, possibly, the turn-over mass of the IMF.

2. Thermal Properties of Star-Forming Clouds

Observational evidence predicts that dense prestellar cloud cores show rough balance between gravity and thermal pressure (Benson & Myers 1989; Myers et al. 1991). Thus, the thermodynamical properties of the gas play an important role
in determining how dense star-forming regions in molecular clouds collapse and fragment. Observational and theoretical studies of the thermal properties of collapsing clouds both indicate that at densities below $10^{-18}$ g cm$^{-3}$, roughly corresponding to a number density of $n = 2.5 \times 10^5$ cm$^{-3}$, the temperature decreases with increasing density. This is due to the strong dependence of molecular cooling rates on density (Koyama & Inutsuka 2000). Therefore, the polytropic exponent $\gamma$ is below unity in this density regime. At densities above $10^{-18}$ g cm$^{-3}$, the gas becomes thermally coupled to the dust grains, which then control the temperature by far-infrared thermal emission. The balance between compressional heating and thermal cooling by dust causes the temperature to increase again slowly with increasing density. Thus the temperature-density relation can be approximated with $\gamma$ above unity in this regime (Larson this volume; see also Larson 1985, Spaans & Silk 2000). Changing $\gamma$ from a value below unity to a value above unity results in a minimum temperature at the critical density. As shown by Li et al. (2003), gas fragments efficiently for $\gamma < 1.0$ and less efficiently for higher $\gamma$. Thus, the Jeans mass at the critical density defines a characteristic mass for fragmentation, which may be related to the peak of the IMF.

3. Numerical Approach

To gain insight into how molecular cloud fragmentation the characteristic stellar mass may depend on the critical density we perform a series of smoothed particle hydrodynamics calculations of the gravitational fragmentation of supersonically turbulent molecular clouds using the parallel code GADGET designed by Springel et al. (2001). SPH is a Lagrangian method, where the fluid is represented by an ensemble of particles, and flow quantities are obtained by averaging over an appropriate subset of SPH particles, see Benz (1990) and Monaghan (1992). The method is able to resolve large density contrasts as particles are free to move, and so naturally the particle concentration increases in high-density regions. We use the Bate & Burkert (1997) criterion the resolution limit of our calculations. It is adequate for the problem considered here, where we follow the evolution of highly nonlinear density fluctuations created by supersonic turbulence. We replace the central high-density regions of collapsing gas cores by sink particles (Bate et al. 1995). These particles have the ability to accrete gas from their surroundings while keeping track of mass and momentum. This enables us to follow the dynamical evolution of the system over many local free-fall timescales.

We compute models where the polytropic exponent changes from $\gamma = 0.7$ to $\gamma = 1.1$ for critical densities in the range $4.3 \times 10^4$ cm$^{-3} \leq n_c \leq 4.3 \times 10^7$ cm$^{-3}$. Each simulation starts with a uniform density distribution, and turbulence is driven on large scales, with wave numbers $k$ in the range $1 \leq k < 2$. We
Figure 1. Mass spectra of protostellar cores for four models with critical densities in the range $4.3 \times 10^4 \text{cm}^{-3} \leq n_c \leq 4.3 \times 10^7 \text{cm}^{-3}$. We show two phases of evolution, when about 10% and 30% of the mass has been accreted onto protostars. The vertical solid line shows the median mass of the distribution. The dotted line serves as a reference to the Salpeter (1955) slope of the observed IMF at high masses. The dashed line indicates our mass resolution limit.

We use the same driving field in all four models. The global free-fall timescale is $\tau_{ff} \approx 10^5 \text{yr}$. For further details see Jappsen et al. (2004).
Figure 2. Plot of the median mass of the protostellar cores $M_{\text{median}}$ versus critical density $n_c$. We display results for different ratios of accreted gas mass to total gas mass $M_{\text{acc}}/M_{\text{tot}}$, and fit the data with straight lines. Their slopes take the values $-0.43 \pm 0.05$ (solid line), $-0.52 \pm 0.06$ (dashed-dotted line), and $-0.60 \pm 0.07$ (dashed line), respectively.

4. Dependency of the Characteristic Mass

To illustrate the effects of varying the critical density, we plot in Fig. 1 the resulting mass spectra at different times when the fraction of of mass accumulated in protostellar objects has reached approximately 10% and 30%. This range of efficiencies corresponds roughly to the one expected for regions of clustered star formation (Lada & Lada 2003). In the top-row model, the change in $\gamma$ occurs below the initial mean density. It shows a flat distribution with only few, but massive cores. These reach masses up to $10 M_\odot$ and the minimum mass is about $0.3 M_\odot$. The mass spectrum becomes more peaked for higher $n_c$ and shifts to lower masses.

We find closest correspondence with the observed IMF (Scalo 1998; Kroupa 2002; Chabrier 2003) for a critical density $n_c$ of $4.3 \times 10^6 \text{ cm}^{-3}$ and for stages of accretion around 30%. For high masses, the distribution exhibits a (Salpeter 1955) power-law behavior. For masses about the median mass the distribution has a small plateau and then falls off towards smaller masses.

The change of median mass $M_{\text{median}}$ with critical density $n_c$ is quantified in Fig. 2. As $n_c$ increases $M_{\text{median}}$ decreases. We fit our data with straight lines. The slopes take values between $-0.4$ and $-0.6$. 

5. Discussion and Summary

Using SPH simulations we investigate the influence of a piecewise polytropic EOS on the gravoturbulent fragmentation of molecular clouds. We study the case where the polytropic index \( \gamma \) changes from 0.7 to 1.1 at a critical density \( n_c \), and consider the range \( 4.3 \times 10^4 \text{ cm}^{-3} \leq n_c \leq 4.3 \times 10^7 \text{ cm}^{-3} \).

A simple scaling argument based on the Jeans mass \( M_J \) at the critical density \( n_c \) leads to \( M_J \propto n_c^{-0.95} \) (see Jappsen et al. 2004). If there is a close relation between the average Jeans mass and the gravoturbulent fragmentation spectrum, a similar relation should hold for the characteristic mass \( M_{ch} \) of protostellar cores. Our simulations qualitatively support this hypothesis, however, with the weaker density dependency \( M_{ch} \propto n_c^{-0.5 \pm 0.1} \). So indeed, the density at which \( \gamma \) changes from below unity to above unity defines a preferred mass scale. Consequently, the peak of the resulting mass spectrum decreases with increasing critical density. The distribution not only shows a pronounced maximum but also a power-law tail towards higher masses, similar to the observed IMF.

Altogether, supersonic turbulence in self-gravitating molecular gas generates a complex network of interacting filaments. The overall density distribution is highly inhomogeneous. Turbulent compression sweeps up gas in some parts of the cloud, but other regions become rarefied. The fragmentation behavior of the cloud and its ability to form stars depend on the EOS. However, once collapse sets in, the final mass of a fragment depends not only on the local Jeans criterion, but also on additional processes. For example, protostars grow in mass by accretion from their surrounding material. In turbulent clouds the properties of the gas reservoir are continuously changing. And in addition protostars may interact with each other, leading to ejection or mass exchange. These dynamical factors modify the resulting mass spectrum, and may explain why the characteristic stellar mass depends on the EOS more weakly than expected from simple Jeans-mass scaling arguments.

Our investigation supports the idea that the distribution of stellar masses depends, at least in part, on the thermodynamic state of the star-forming gas. If there is a low-density regime in molecular clouds where the temperature \( T \) sinks with increasing density \( \rho \), followed by a higher-density phase where \( T \) increases with \( \rho \), fragmentation seems likely to be favored at the transition density where the temperature reaches a minimum. This defines a characteristic mass scale. The thermodynamic state of interstellar gas is a result of the balance between heating and cooling processes, which in turn are determined by fundamental atomic and molecular physics and by chemical abundances. The theoretical derivation of a characteristic stellar mass may thus be based on quantities and constants that depend mostly on the chemical abundances in the star-forming cloud.
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