Domains identification by the parameter values in multidimensional space

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Abstract. As a result of cluster and discriminant data analysis, the objects are distributed among different classes represented in the multidimensional space of parameter values. At the next stage, it is relevant to use the results obtained in various applications. The most frequently solved task is the diagnostics of a newly received object when the class of the object is determined through parameter values. In terms of a data view model, the problem can be reduced to determining a domain in parameter space, the new object belonging to this domain. The solution to this problem depends on the way the data analysis results are described and presented. Previously the graphic data model was considered to describe the domains separated by surfaces in the multidimensional space. Moreover, a constructive approach was proposed to describe an error in the representation of the boundaries, making it possible to take into account the experimental data errors. The mathematical model proposed in the given article is used to develop a domain identification algorithm by the parameter values in the multidimensional space. The proof of correctness and the estimation of computational complexity are provided for the algorithm. The results obtained make it possible to use the algorithm to calculate various characteristics of objects: state, intervals of existence, etc.

1. Introduction
The scope of applied problems solved with the domain identification algorithm is quite wide. One possible application of the algorithm is spatial scanning. In [1] the model is constructed using joint data of hydraulic testing and parameter space analysis. Response surfaces for multiple datasets are used to reduce the uncertainty of parameter values. Parameter space approaches are also used to select the relative weights of individual datasets in joint parameter estimation. A unique response surface and a corresponding confidence region result.

In [2] the problem of submarine mine water inrush in coastal gold mining operations is investigated. Using various types of water composition analysis, the main sources of water inrush are identified. The information obtained makes it possible to use the domain identification algorithm to predict the possibility of flooding and reduce the cost of mining.

An alternative method for identifying domains is used in cluster analysis methods when the convexity of domains in the parameter space is assumed [3]. Then the domain identification is carried out by calculating the distance from the centroids. The disadvantage of this approach is that it is impossible to use it with non-convex boundaries. In some cases, the centroid may be outside the boundaries of its domain.
A more flexible approach to domain identification is proposed in [4], where for each point of the parameter space the plausibility of its belonging to the cluster is calculated. In this case, there is no need to represent the boundaries. It can be determined if a point belongs to the boundary between two neighbouring clusters when the equality of the corresponding measures is achieved. A similar approach to identification is used in [5] where the location of the desired object has a probabilistic nature. This approach is often used when solving the problems of optimal search control with the given functions of the probability distribution of the target coordinates and the risk of their loss.

In paper [6], the problem of identification near the domain boundaries is solved for computer-aided design systems. Errors occur when identifying protrusions and depressions at the boundaries of areas with free-form surfaces. The main idea of the method lies in the use of the tangent continuity between the boundary segments at their intersections.

The choice of domain identification method in the parameter space depends on the choice of the data view model for the domain boundaries. In paper [7], a data model based on the representation of boundaries as functional dependencies in the multidimensional space is considered, and a constructive approach is proposed to describe the error in the boundaries representation. The mathematical model in [7] makes it possible to construct an identification algorithm for domains with a formally strict proof of its operation correctness and to estimate its computational complexity. This article explores this algorithm.

2. Preliminary information on the data model

In paper [7], a mathematical model to describe data in the multidimensional parameter space was considered. Let us briefly present the information from that article that is necessary for the further consideration of the material.

As a result of data processing, domains in the parameter space (clusters) should be obtained, which are separated by multidimensional surfaces. Let $E^n$ be a Cartesian space with orthogonal basis $e_1, e_2, ..., e_n$, ordinary scalar product and norm:

$$a, b \in E^n, \quad a = (a_1, a_2, ..., a_n), \quad b = (b_1, b_2, ..., b_n),$$

$$ (a, b) = \sum_{i=1}^{n} a_ib_i, \quad \|a\| = (a, a),$$

where $a_i, b_i$ are real numbers, $i = 1, n$.

In $E^n$ there are domains $\Theta_i, i = 1, N$, which must satisfy the following properties.

**Property 1.**

$$ \bigcup_{i=1}^{N} \Theta_i = E^n, \quad \Theta_i \neq \emptyset. \quad (2)$$

**Property 2.**

$$ \Theta_i \cap \Theta_j = \Upsilon_{ij}, \quad i, j = 1, N, i \neq j,$$

where $\Upsilon_{ij}$ are the boundaries of the domains with dimensions $n - 1, n - 2, ..., 1, 0$. Moreover, $\Upsilon_{ij}$ are such that it is impossible to move from one domain to another without crossing the boundary. Each $\Upsilon_{ij} \neq \emptyset$ is set with a certain error $\Delta_{ij}$, where $\Delta_{ij}$ is a connected $n$-dimensional closed and bounded domain in $E^n$. While, $\Upsilon_{ij} \subset \Delta_{ij}$. Let us consider the operator of the projection $W_{l_i}$ along the coordinate $x_{l_i}$. $W_{l_i}$ can be represented as a diagonal unity matrix with dimension $n \times n$, with the line element $l_i$ being zero.

Boundaries $\Upsilon_{ij}$ are divided into single-valued domains $\Upsilon_{ij}^{q_{ij}}$ relative to the selected subspace (projection along axis $x_{l_i}$), allowing each domain to be assigned function $A_{ij}^{q_{ij}}, q_{ij} = 1, k(i, j)$. These functions together define boundary $\Upsilon_{ij}$ for the separation of domains $\Theta_i$ and $\Theta_j$. Due to
the single-valuedness of \( A^{q_1}_{i,j} \), we shall use the interpretation of indices \( i \) and \( j \): \( i \) is the number of the domain with the largest value of \( x_{i1} \) relative to \( \Upsilon_{i,j}^{q_1} \), \( j \) is the number of the domain with the smallest value of \( x_{i1} \). Each boundary \( \Upsilon_{i,j}^{q_1} \) is assigned a triple \((i, j, q_1)\).

Let the given point \( x^0 \in E^n \) correspond to the sets \( S = \{ x \in E_n | W_{i_{1}} x = W_{i_{2}} x^0 \} \) and

\[
S(i_1) = \bigcup_{(i,j,q_1)} (\Upsilon_{i,j,p}^{q_1} \bigcup \Upsilon_{i,j}^{q_1''}).
\]

**Property 3.** The sets \( S \cap S(i_1) \cap S(i_2), i_1 \neq i_2 \) should have no more than one point for each \( x^0 \in E^n \).

If at another projection for boundary \( \Upsilon_{i,j,p}^{q_1,q_2,...,q_s} \), the following is satisfied:

\[
(x, y) = 0, \quad x \in \Upsilon_{i,j,p}^{q_1,q_2,...,q_s}, \quad y \in W_E^n.
\]

then the information on \( \Upsilon_{i,j,p}^{q_1,q_2,...,q_s}, s = 2, n-1 \), needs no storing in the database, since it does not agree with the minimum and maximum values of the parameters.

In terms of the data model given in [7], the following theorems were proved.

**Theorem 1.** There is one and only one domain \( \Theta_{\varrho} \) not bounded in \( E^n \) at \( n \geq 2 \).

**Theorem 2.** Domains \( \Theta_i, i = \Gamma, N \), except one \( \Theta_{\varrho} \), are compact, and domain \( \Theta_{\varrho} \) is locally compact.

3. Domain identification algorithms

When a user works with the results of cluster or discriminant analysis, one of the main tasks is to determine the domain to which the newly obtained object belongs, based on a set of parameters. In many applications, this task is called diagnostics. In terms of the model considered, one should determine domain \( \Theta_{ij} \) by fixed point \( x^0 \) in the parameter space where \( x^0 \in \Theta_{ij} \). Point \( x^0 \) can belong to several domains at the same time if it belongs to the boundary between them. The following algorithm assumes that the representation of the data analysis results satisfies the mathematical model considered in [7].

**Algorithm 1.**

1. For all triples \((i, j, q_1)\) validation is done:

\[
A_{ijp}^{q_1,q_2,...,q_n'} \leq x_{0}^{q_1,q_2,...,q_n'} \leq A_{ijp}^{q_1,q_2,...,q_n''},
\]

where one from \( q_n' \) and \( q_n'' \) equals 1, and the other equals 2, \( p' = 1, p'' = 0 \).

2. Triplles \((i, j, q_1)\), for which the ratio (5) has never been met, are removed from further consideration. Moreover, further the algorithm will consider only those boundaries whose superscript coincides with \( q_1^n, q_2^n, ..., q_{n-1}^{n-1} \). For the indices of these boundaries, the following should be fulfilled: \( q_1^n = q_1, q_2^n = q_2, ..., q_{n-1}^{n-1} = q_{n-1} \), where \( q_1, q_2, ..., q_{n-1}, q'_n \) and \( q_1, q_2, ..., q_{n-1}, q''_n \) are the indices meeting condition (5).

Step 2. The boundaries of the \((n-2)\)-th iteration to obtain projection are considered. The validation is performed for triples \((i, j, q_1)\) chosen at step 1:

\[
A_{ijp}^{q_1,q_2,...,q_{n-1}'}(x^0) \leq x_{0}^{q_1,q_2,...,q_{n-1}} \leq A_{ijp}^{q_1,q_2,...,q_{n-1}''}(x^0),
\]

where \( A_{ijp}^{q_1,q_2,...,q_{n-1}''}(x^0) - x_{0}^{q_1,q_2,...,q_{n-1}} \) is the largest negative number for all indices \( q_{n-1} \), with \( p' = 1 \); \( A_{ijp}^{q_1,q_2,...,q_{n-1}'}(x^0) - x_{0}^{q_1,q_2,...,q_{n-1}} \) is the smallest positive number for all indices \( q_{n-1} \), with \( p'' = 0 \).

Inequation (6) is considered fulfilled if there are both parts (right and left). If the inequations are not fulfilled for triple \((i, j, q_1)\) or some sequences \( q_1, q_2, ..., q_{n-2} \), the operations of step 1 are performed.
Step $n-1$. Inequations are verified:

$$A_{ijp'}^{q_1,q_2}(x^0) \leq x_{l_2}^0 \leq A_{ijp''}^{q_1,q_2}(x^0),$$

where $A_{ijp'}^{q_1,q_2}(x^0) - x_{l_2}^0$ is the largest negative value for all $q_2$, at which $p' = 1$; $A_{ijp''}^{q_1,q_2}(x^0) - x_{l_2}^0$ is the smallest positive value for all $q_2$, at which $p'' = 0$. Next, the operations corresponding to step 2 are performed.

Step $n$. Let $r$ triples $(i, j, q_1)$ remain. The following sequence is constructed for them:

$$a_{ij1}^{q_1} = A_{ijj_1}^{q_1}(x^0) - x_{l_1}^0, \ldots, a_{ijr}^{q_1} = A_{ijr}^{q_1}(x^0) - x_{l_1}^0. \tag{7}$$

If sequence (7) lacks elements equal to zero, one considers only the largest negative elements $a_{ijp}^{q_1}$, there can be several of them, and the smallest positive $a_{ijp}^{q_1}$. Let us examine the possible situations:

1. All elements of sequence (7) have only positive or only negative values, or sequence (7) has no values. Then the solution is $x^0$, i.e. unbounded (external) domain.

2. Sequence (7) has values $a_{ijp}^{q_1}$ and $a_{ijp}^{q_2}$. The values with $i_s = j_p$ are taken. The resulting number is the solution of the algorithm.

If sequence (7) has zero values, then at these values indices $i$ and $j$ shall give the algorithm solution, i.e. the numbers of the domains with the given point $x^0$ being bounded.

**End of algorithm 1.**

It should be noted that article [7] proves the property that in the inequations of type (6) there can be no two pairs of the boundaries with matching superscripts at the same time, except for the latter, when one pair satisfies (6) according to feature $p$, and the other does not.

The way of constructing the boundaries of the domains $\Theta_{ij}^{q_1\cdots q_n}$, $s = 1, n-1$, makes it possible to formulate and prove the following lemma.

**Lemma 1.** The value of $a_{ij}^{q_1}$ is the element of the sequence (7) then and only then when $W_i x^0 \in \Theta_{ij}^{q_1}$.

Sufficiency. Let us consider the projection of the point $x' = W_i x^0 \in \Theta_{ij}^{q_1}$. Since domain $\Theta_{ij}^{q_1}$ and its boundaries are closed and bounded, the unbounded set $S = \{x \in E_{l_i}^{n-1} | W_i x = W_i x^0 \}$ overlap with the boundary of domain $\Theta_{ij}^{q_1}$ at least twice. Based on construction, $S$ is a straight line parallel to axis $x_{l_2}$, and $x' \in S$. When moving from point $x'$ along $S$ in the descending direction $x_{l_2}$, one gets the first intersection of $S$ with the boundary of domain $\Theta_{ij}^{q_1}$, having a construction feature $p' = 1$. Such intersection exists, since otherwise $\Theta_{ij}^{q_1}$ is an unbounded domain. Moving in a similar way in the ascending direction $x_{l_2}$, we get an intersection with the boundary of domain $\Theta_{ij}^{q_1}$, having a feature $p'' = 0$. The corresponding values of $A_{ijp'}^{q_1,q_2}(x^0) - x_{l_2}^0$ and $A_{ijp''}^{q_1,q_2}(x^0) - x_{l_2}^0$ will be the largest negative and the smallest positive for $\Theta_{ij}^{q_1}$. Hence, inequation (6) is satisfied, and element $a_{ij}^{q_1}$ will belong to sequence (7).

If for boundary $\Upsilon_{ij}^{q_1,q_2}$ of domain $\Theta_{ij}^{q_1}$, condition (4) is fulfilled, $\Upsilon_{ij}^{q_1,q_2}$ is not considered, since for $S$ condition (4) is met, that is boundary $\Upsilon_{ij}^{q_1,q_2}$ is parallel to set $S$. Since domain $\Theta_{ij}^{q_1}$ is bounded and its boundaries are locally compact, $S$ crosses at least one boundary $\Theta_{ij}^{q_1}$.

Since $S$ crosses the boundaries of domain $\Theta_{ij}^{q_1}$, the point $x'' = W_i W_1 x^0$ belongs to some domain $\Theta_{ij}^{q_1,q_2}$. Repeating the reasoning for the point $x'' \in \Theta_{ij}^{q_1,q_2}$, we get that conditions (6) are satisfied for the boundaries of domain $\Theta_{ij}^{q_1,q_2}$ at step $n-2$ of algorithm 1, and so on till the first step. The sufficiency of the lemma condition is proved.

Necessity. Since element $a_{ij}^{q_1}$ belongs to sequence (7), then at step 1, 2, ..., $n-1$ conditions (6) are fulfilled. Hence, boundaries $A_{ijp'}^{q_1,q_2\cdots q_n}$ and $A_{ijp''}^{q_1,q_2\cdots q_n}$, where $p' = 1$ and $p'' = 0$ are found.
and located the closest to point \( W_{s-1} \). Given the way the boundaries are formed, one can conclude that \( W_{s-1} x^0 \in \Theta_{ij}^{q_1,q_2,\ldots,q_s}, \) \( s = 1, n - 1 \), hence, \( W_{s-1} x^0 \in \Theta_{ij}^1 \). The lemma is proved.

Let us consider a new set \( S = \{ x \in E^n | W_i x = W_i x^0 \} \).

**Lemma 2.** Set \( S \cap \Upsilon_{ij}^q \) can have no more than one point, and if \( x' \in S \cap \Upsilon_{ij}^q \), then \( x'_i = A^i_{ij}(x') \).

Proof. Suppose that \( x^{(1)}(1) \in S \cap \Upsilon_{ij}^q \), \( x^{(2)}(2) \in S \cap \Upsilon_{ij}^q \) and \( x^{(1)}(1) \neq x^{(2)}(2) \). Since \( S \): \( W_i x^{(1)}(1) = W_i x^{(2)}(2) \), then \( x^{(1)}(1)_i = x^{(2)}(2)_i \). Hence, \( x^{(1)}(1)_i \neq x^{(2)}(2)_i \). This contradicts the condition that boundary \( \Upsilon_{ij}^q \) is represented by single-valued function \( x_i = A^i_{ij}(x) \), where \( x \in E^n_{ij} \). Hence, the intersection \( S \cap \Upsilon_{ij}^q \) can have no more than one point, and if \( S \cap \Upsilon_{ij}^q \neq \emptyset \), then \( x_i' = A^i_{ij}(x') \) and \( x' \in S \cap \Upsilon_{ij}^q \).

**Theorem 3.** The number of domain \( i \) is the solution of algorithm 1 then and only then, when \( x^0_0 \in \Theta_i \).

Sufficiency. Let \( x^0_0 \in \Theta_i \). Two versions are possible.

1. \( S \) does not cross the bounded domains.
2. \( S \) crosses the bounded domain.

In the first case, \( S \) is in unbounded domain \( \Theta_{i_0} \), since \( x^0_0 \in S \), then \( x^0_0 \in \Theta_{i_0} \). \( S \) has no common points with boundaries \( \Upsilon_{ij}^q \) for any triple \((i, j, q_1)\). If there is \( x \in S \cap \Upsilon_{ij}^q \), then \( x \in \Theta_j, j \neq i \), which does not satisfy the case considered. Suppose that \( W_i x^0_0 = \bigcup_{(i,j,q_1)} \Theta_{ij}^q \), then there are \( \Upsilon_{ij}^q \) and \( x \in \Upsilon_{ij}^q \), for which \( W_i x = W_i x^0_0 \). However, from the definition of set \( S \), \( x \in \Upsilon_{ij}^q \cap S \). This proves that \( W_i x^0_0 \not\in \bigcup_{(i,j,q_1)} \Theta_{ij}^q \) and the sought-for number is \( x^0_0 \).

Since \( S \) is, in fact, an unbounded straight line, it has an intersection with at least one domain, since \( S \) cannot be completely included in the bounded domain. From property 2 there are \( x \) and \( \Upsilon_{ij}^q \) such that \( x \in \Upsilon_{ij}^q \cap S \). Then, \( W_i x^0_0 = W_i x = \Theta_{ij}^q \), and set (7) is not empty.

Let \( x^0_0 \in \Theta_{i_0} \) and \( S \) cross the bounded domain. Two versions are possible.

1. \( S \) has an intersection with \( \Upsilon_{ij}^q \) only in the positive direction of the straight line \( x_i \), relative to \( x^0_0 \), or only in the negative direction, with \( x^0_0 \not\in \Upsilon_{ij}^q \) for any triple \((i, j, q_1)\).
2. \( S \) crosses boundaries \( \Upsilon_{ij}^q \) in both directions: positive and negative.

From lemma 2 it follows that in the first version all elements of sequence (7) are positive or negative, respectively. Therefore, in algorithm 1 the number of unbounded domain \( i^0 \) will be obtained as a solution. The second version will be considered later.

Let \( x^0_0 \) belong to the bounded domain \( \Theta_i \) and \( x^0_0 \) not belong to the boundary \( \Upsilon_{ij}^q \) for a random triple \((i, j, q_1)\). In this case, \( S \) has a non-empty intersection with the boundaries \( \Upsilon_{ij}^q \) in both directions along the axis \( x_i \) relative to point \( x^0_0 \). Hence, due to property 2 \( S \) should cross \( \Upsilon_{ij}^q \) in both directions. From algorithm 2 it follows that in sequence (7) there are positive and negative elements. In the second version, we have a similar situation.

Let us suppose that among the chosen numbers \( i_s \) there is no number matching \( i \). Let us consider the segment of the straight line \( S' = [x^0_0, x'] \subset S \), where \( x'_i = x^0_i + a_{i,s_j}^{q_1} \). The end of segment \( x' \) belongs to boundaries \( \Upsilon_{ij}^q \). In \( S' \) there should be \( x \in \Theta_i \) and \( x \neq x' \). If this condition is not fulfilled, then due to theorem 2 \( x' \) also belongs to \( \Theta_i \), i.e. \( S' \subset \Theta_i \). Hence, \( x' \in \Upsilon_{ij}^q \), where \( \Upsilon_{ij}^q \) is the boundary of domain \( \Theta_i \). Therefore, among boundaries \( \Upsilon_{ij}^q \) there is such boundary for which \( i_s = i \). Let us suppose that the point \( x \in S' \) is found and \( x \in \Theta_i \), then segment \([x, x']\) belongs to different domains and should cross boundary \( \Upsilon_{ij}^q \). In this case in (7) there is element \( a_{i,s_j}^{q_1} \) such that \( a_{i,s_j}^{q_1} \leq 0 \) and \( a_{i}^{q_1} > a_{i,s_j}^{q_1} \). It follows from the contradiction obtained that among \( a_{i,s_j}^{q_1} \) there is such element for which \( i_s = i \). By analogy it can be shown that among \( a_{i,s_j}^{q_1} \) there is \( j_p = i \).

Let us suppose that there is another pair \( i'_s = j'_p \neq i \) among \( a_{i,s_j}^{q_1} \) and \( a_{i,s_j}^{q_1} \). Let us consider
points \( x' \in S \) and \( x'' \in S \), where \( x'_i = x^0_1 + a^{q_1}_{i,j_s} \) and \( x''_i = x^0_1 + a^{q_1}_{i,j_p} \). Since \( a^{q_1}_{i,j_s} < a^{q_1}_{i,j_p} \), then \( x' \neq x'' \). This means that points \( x' \) and \( x'' \) belong to set 

\[
S \cap \left( \bigcup_{i,j \in S} Y_{ij}^{q_{ij}} \cup \bigcup_{i,j \in S} Y_{ij}^{q_{ij}} \right)
\]

which contradicts property 3. Hence, if \( x^0 \in \Theta_i \) and \( x^0 \notin \Theta_{ij}^{q_{ij}} \) for any triple \((i, j, q_1)\), the only solution of the algorithm is \( i \).

Let us suppose that \( x^0 \in Y_{ij}^{q_{ij}} \) and \( Y_{ij}^{q_{ij}} \) is the boundary of domain \( \Theta_i \). Then, one can find boundary \( \Theta_{ij}^{q_{ij}} \) of domain \( \Theta_i \), such that \( x^0 \in \Theta_{ij}^{q_{ij}} \). According to lemma 1, element \( a_{ij}^{q_{ij}} \) will be in sequence \((7)\), and it will be zero following lemma 2. Therefore, \( i \) is present in the algorithm solution.

Necessity. Let number \( i \) be the solution of the algorithm. It is necessary to show that \( x^0 \in \Theta_i \).

In this case, the following cases are possible.

1. All triples \((i, j, q_1)\) are removed till the end of the algorithm.
2. In sequence \((7)\) all the elements have the same sign.
3. In \((7)\) there are non-zero elements \( a^{q_{ij}}_{i,j_s} \) and \( a^{q_{ij}}_{i,j_p} \). In this case the solution is \( i = i_s = j_p \).
4. Sequence \((7)\) has an element corresponding to reaching the boundary \( a_{ij}^{q_{ij}} = 0 \), where \( i \) is one of the subscripts.

In the first case, \( W_i x^0 \notin \bigcup_{(i,j,q_1)} \Theta_{ij}^{q_{ij}} \). This follows from lemma 1, therefore \( S \) does not cross any boundary \( \Theta_{ij}^{q_{ij}} \). Since \( S \) is not bounded, then always \( S \cap \Theta_{i_0} \neq \emptyset \), where \( i_0 \) is the number of the unbounded domain. Since \( S \cap \bigcup_{(i,j,q_1)} \Theta_{ij}^{q_{ij}} = \emptyset \), then \( S \cap \Theta_j = \emptyset, j = 1, 2, \ldots, N, j \neq i_0 \). Hence \( S \subset \Theta_{i_0} \). Since \( x^0 \in S \) then \( x^0 \in \Theta_{i_0} \).

In the second case, the version is considered where all the elements are negative. Let \( x^0 \in \Theta_i \), where \( \Theta_i \) is a bounded domain and \( S' = \{ x \in S | x_i = x^0_1 \} \). By the construction \( S' \) is not bounded, there is \( x' \in S' \cap \Theta_{i_0} \). Closed segment \([x^0, x']\) crosses domains \( \Theta_i \) and \( \Theta_{i_0} \). Hence, there is \( \Theta_{ij}^{q_{ij}} \), with \( \Theta_{ij}^{q_{ij}} \cap \Theta_{i_0} = \emptyset \). From lemma 1, lemma 2 and the previous considerations it follows that element \( a_{ij}^{q_{ij}} \geq 0 \) belongs to \((7)\). A contradiction is obtained; therefore, \( x^0 \in \Theta_{i_0} \).

When all elements of sequence \((7)\) are positive, the proof is similar.

In the third case, let us consider \( S' = [x', x''] \subset S \), where \( x'_i = x^0_1 + a^{q_{ij}}_{i,j_s} \) and \( x''_i = x^0_1 + a^{q_{ij}}_{i,j_p} \). Since \( S' = [x', x''] \cup [x^0, x''] \), it results that \( S' \subset \Theta_i \) by analogy with the proof of sufficiency for the second case. Since \( x^0 \in S' \), then \( x^0 \in \Theta_i \).

In the fourth case, let us suppose that \( x^0 \in \Theta_i \) and sequence \((7)\) has no index \( i \). According to lemma 2, for \( x^0 \) the following is fulfilled: \( x^0 \in Y_{ij} S \), where \( Y_{ij} \) is boundary \( \Theta_i \). Hence, in \((7)\) there is an element \( a_{ij}^{q_{ij}} = 0 \), which contradicts the supposition. **The theorem has been proved.**

4. Estimation of the computational complexity of the domain identification algorithm

Let \( N_i \) be the number of elements (triples) of level \( i \), representing the domains in the multidimensional space. For example, \( N_1 \) is the number of triples \((i, j, q_1)\). During the estimation, the identity of the operations performed in the algorithm 1 is used. The computational costs will be analysed for the \( i \)-th step of the algorithm, \( i = 1, \ldots, n \).

1. Let \( c_i \) be the number of operations to calculate the functions \( A^{q_{ij}}_{ij} (x^0) \). At the \( i \)-th step of the algorithm, one can calculate no more than \( N_i \) functions; therefore, \( c_i N_i \) is the maximum number of the operations needed to calculate the functions.

2. Let \( d \) be the number of the operations necessary to subtract the values of the coordinates \( x^0_1 \) using the values of functions \( A^{q_{ij}}_{ijp} (x^0) \). Maximum number of subtractions at the \( i \)-th step equals \( N_i \). Therefore, \( N_i d \) is an upper boundary of the number of operations for subtraction.
3. Let $f$ be the costs of carrying out the comparison operation: more, less or equal. At the $i$-th step of algorithm 1, the required number of comparisons is no more than the value of $2N - 1$. Hence, the quantity of the operations needed to compare the numbers is no more than the value of $2N_i f$.

The final number of the operations for all steps of algorithm 1 satisfies the ratio:

$$N_{op} < \sum_{i=1}^{n} N_{n-i+1} (c_{n-i+1} + d + 2f) =$$

$$= \sum_{i=1}^{n} N_i (c_i + d + 2f) = \sum_{i=1}^{n} N_i h_i \leq h \sum_{i=1}^{n} N_i,$$

where $h_i = c_i + d + 2f$, $h = \max\{h_i\}$, $i = 1, n$.

Therefore, the upper boundary of the computational complexity estimate of algorithm 1 is in linear dependence on the number of all elements in the data view.

**Conclusion**

The presented algorithm of domain identification by the total parameter values in a multidimensional space is the basis for solving the problem of diagnosing the state of the object under study by a set of parameter values. Besides the algorithm underlies various algorithms used in design, exploration, etc. [8, 9, 10]. To represent the domain boundaries, one should use the methods of mixed approximation of the surfaces [11] with the adaptation of the optimization criterion to the specifics of the problem.

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