Comment on “Determination of the chiral coupling constants $c_3$ and $c_4$ in new $pp$ and $np$ partial-wave analyses”

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In a recent study [M.C.M. Rentmeester et al., Phys. Rev. C 67, 044001 (2003)], the Nijmegen group reports on the determination of the chiral low-energy constants (LEC), $c_3$ and $c_4$, involved in the $2\pi$-exchange part of the $NN$ amplitude at next-to-next-to-leading order (NNLO) of chiral perturbation theory. This analysis does not apply the uniquely-determined and model-independent $NN$ amplitudes at NNLO and uses, instead, amplitudes that are up 90% smaller. We point out that this flaw produces a large systematic error, rendering the Nijmegen method unsuitable for a reliable determination of the LEC.

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In a recent paper [1], the Nijmegen group reports on a study of the chiral two-pion exchange interaction which contains the low-energy constants (LEC) $c_1$, $c_3$, and $c_4$. The long-range part of this interaction is applied in an analysis of the world $pp$ and $np$ data below 500 MeV laboratory energy. The authors state that, based upon this analysis, they were able to determine the LEC $c_3$ and $c_4$ with an accuracy of 2-5%. This recent Nijmegen analysis [1] is an update of an earlier one [2], and the present note applies to both.

It is the purpose of this Comment to point out that the Nijmegen determination of the LEC appears to have basic flaws which can be estimated to cause a large systematic error.

The traditional approach for describing nuclear forces has been the meson-exchange model in which mesons of increasing mass are used to generate contributions of decreasing range. This expansion is then truncated at a certain range which is believed to be unimportant for traditional nuclear physics purposes.

The new approach to nuclear forces, that was initiated by Weinberg [3] and pioneered by Ordóñez [4], Ray, and van Kolck [5,6], has a foundation and philosophy which are quite different from meson phenomenology. Based upon the chiral symmetry of QCD, an expansion is made in powers of the nucleon momenta. This is known as chiral perturbation theory (\chiPT).

The crucial point to realize is that—in contrast to meson phenomenology—\chiPT is a theory rather than a model, i.e., it makes exact predictions at each order (if the constants of the theory are known).

Physical constants are to be determined in a model-independent way, if by all means possible. While model-independent determinations are per se impossible if one is dealing with a model or phenomenology, model-independence is inherent to the framework of \chiPT.

At next-to-next-to-leading order (NNLO), the theory consists of one-pion-exchange (OPE), two-pion-exchange (TPE) at $O(Q^3)$ [7], where $Q$ denotes a momentum or pion mass, and contact terms of order $Q^0$ and $Q^2$. These contributions define uniquely the $NN$ amplitude at NNLO. A problem is that the parameters of the contact terms are not known. However, since these contacts contribute only to $S$ and $P$ wave amplitudes, the $NN$ amplitudes for partial waves with orbital angular momentum $L \geq 2$ do not receive any contact contributions [8]. Consequently, at NNLO, the $NN$ amplitudes for $D$ and higher partial waves are determined by OPE and TPE at $O(Q^3)$ in an unique and completely model-independent way [7,8]. These two contributions depend (apart from the nucleon and pion mass) on five constants: the axial-vector coupling constant, $g_A$, the pion decay constant, $f_\pi$, and three LEC that appear in the dimension-two $\pi N$ Lagrangian and are, conventionally, denoted by $c_1$, $c_3$, and $c_4$. The constants $g_A$ and $f_\pi$ are known from other sources (we use $g_A = 1.29$ and $f_\pi = 92.4$ MeV and the same is used in Ref. [1]) and for $c_3$ the educated estimate $c_3 = -0.76$ GeV$^{-1}$ can be made [1,2]. Now only two constants are open, namely, $c_3$ and $c_4$. Since the LEC are also involved in the $\pi N$ amplitude, these constants have been determined by calculating the $\pi N$ amplitude up to a certain order of \chiPT and adjusting the constants such that the empirical $\pi N$ information is fit [9,10]. Similarly, the $NN$ amplitudes of partial waves with $L \geq 2$ can be compared with the corresponding empirical amplitudes to extract $c_3$ and $c_4$.

Once all five constants are fixed, then there exists a unique prediction for all $NN$ amplitudes with $L \geq 2$. Using

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$c_3 = -4.78$ GeV$^{-1}$ and $c_4 = 3.96$ GeV$^{-1}$, these uniquely determined $NN$ amplitudes are shown in Fig. 1 by the solid lines for one $D$ and one $F$ wave case (see Refs. [7,11] for the details of how these amplitudes are calculated perturbatively; conventions as in Ref. [11]).

It must be stressed again that we are dealing here with a theory and that a theory makes exact predictions. Thus, the solid line in Fig. 1 is the unique result of $\chi$PT at NNLO for the $NN$ system (using the above constants). Using the same constants, there is no other result at NNLO and every theoretical physicist will independently reproduce this result.

However, when we compare this result with the one the Nijmegen group obtains and uses in their analysis (dashed line in Fig. 1), we notice a discrepancy of up to 90%. When such amplitudes are applied in an analysis, a large systematic error is produced and the extracted constants will carry such large error.

The Nijmegen amplitudes are obtained as a result of representing the chiral OPE and TPE at NNLO in terms of an $r$-space potential that is cut off at $r = 1.6$ fm. The motivation for doing this may be a confusion between the old range argument (that applies to meson-model based potentials) and the fundamentals of $\chi$PT which is an expansion in powers of nucleon momenta, and not in terms of ranges. As pointed out before, $\chi$PT has, by principle, no model-dependence, whereas model-dependence is clearly introduced in the analyses of Refs. [1,2].

Another point of concern is that the Nijmegen group uses $\chi$PT at NNLO up to 500 MeV. It is well-known that $\chi$PT at NNLO is appropriate only up to 50 MeV in $D$ waves and about 150 MeV in $F$ waves [7,11].

In conclusion, the method of determination of the chiral LEC, $c_3$ and $c_4$, applied in the recent as well as the earlier Nijmegen analyses [1,2] may be unreliable.

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\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig1}
\caption{On-shell $NN$ amplitudes up to 500 MeV laboratory energy for the $^1D_2$ and $^3F_4$ states. The solid lines represent the correct amplitudes for OPE plus TPE at NNLO (using the constants given in the text). The dashed lines show the amplitudes used in the Nijmegen analysis for OPE plus TPE at NNLO (using the same constants). Thick lines represent the real parts of the amplitudes and thin lines the imaginary parts. (Note that the thin lines are $\approx 0$ on the scale of the figures and hardly distinguishable.)}
\end{figure}