Viscosities in the Gluon-Plasma within a Quasiparticle Model

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Abstract

A phenomenological quasiparticle model, featuring dynamically generated self-energies of excitation modes, successfully describes lattice QCD results relevant for the QCD equation of state and related quantities both at zero and non-zero net baryon density. Here, this model is extended to study bulk and shear viscosities of the gluon-plasma within an effective kinetic theory approach. In this way, the compatibility of the employed quasiparticle ansatz with the apparent low viscosities of the strongly coupled deconfined gluonic medium is shown.

1. Introduction

Within the past years, an immense effort has been put into revealing the nature and properties of deconfined strongly interacting matter both theoretically and experimentally in relativistic heavy-ion collisions. The aim is to understand characteristics of QCD matter such as its equation of state (EoS), collective behaviour or transport properties which have a wide range of implications in cosmology and astrophysics. While the equation of state describes the system in thermal equilibrium, the transport coefficients, such as viscosities, characterize the system’s ability to relax from nonequilibrium towards equilibrium.

The success of perfect fluid hydrodynamic calculations in describing the collective flow observed in heavy-ion collisions \cite{1,2,3} suggests small viscosities of the created medium which has also lead to viewing it as being strongly coupled \cite{4}. Nonetheless, in particular the shear viscosity cannot be arbitrarily small due to unitarity \cite{5}. Besides, in weak coupling certain relations hold among different transport coefficients, cf. e. g. \cite{6}.

Apart from the first-principle calculations of bulk (\(\zeta\)) and shear (\(\eta\)) viscosities by means of lattice QCD \cite{7,8} a variety of other approaches was proposed. Among these the rigorous perturbative calculations starting either from the Boltzmann equation \cite{9,10} or from the Kubo formalism \cite{11} have to be mentioned. Besides, considerations employing different spectral functions \cite{12} or numerical transport calculations of the Boltzmann equation \cite{13} have been made.

Here, we adress viscosities by viewing the gluon-plasma as composed of quasiparticle excitations. The underlying quasiparticle model (QPM) was successfully tested to describe lattice QCD results of the EoS \cite{14,15}. In the QPM, gluon and quark quasiparticles obey dispersion relations, where the entering self-energies \(\Pi\), in general, depend on temperature \(T\) and chemical potential \(\mu\) both explicitly and also implicitly via a phenomenological effective coupling \(G^2(T,\mu)\) \cite{16}. We extend this picture to nonequilibrium systems by means of an effective kinetic theory for gluon quasiparticles, i. e. \(\mu = 0\) in the following.
2. Effective kinetic theory for quasiparticle excitations

For a system in nonequilibrium, the gluon quasiparticle dispersion relation no longer depends on an uniquely defined temperature but becomes space-time dependent, $E(x) = \sqrt{\vec{p}^2 + \Pi(x)}$. In this case, the space-time dependence of the distribution function $b(x, p)$ is governed by the Boltzmann equation

$$
\left(p^\alpha(x)\partial_\alpha + \sqrt{\Pi(x)} F_{\alpha\beta}(x) \frac{\partial}{\partial p^{\alpha}(x)}\right)b(x, p) \equiv \mathcal{Db}(x, p) = C[b(x, p)] ,
$$

where $C[b(x, p)]$ is the collision term. The force $F^\alpha = \partial^\alpha \Pi/(2\sqrt{\Pi})$ satisfies $p_\alpha F^\alpha = 0$ such that the spatial gradient of the self-energy acts as an external force changing the momenta of the quasiparticles between collisions. The scattering interaction conserves locally energy and momentum which results in a vanishing collision term when multiplied by $p^\alpha$ and integrated over three-momentum $\vec{p}$. Hence, an energy-momentum tensor

$$
T^{\mu\nu}(x) = d\int \frac{d^3 \vec{p}}{(2\pi)^3 E(x)} p^\mu(x)p^\nu(x)b(x, p) + g^{\mu\nu} B(\Pi(x))
$$

can be defined which, as a consequence of the Boltzmann equation, obeys energy-momentum conservation $\partial_\mu T^{\mu\nu}(x) = 0$ under the condition

$$
\frac{\partial B}{\partial \Pi(x)} = -\frac{1}{2} q(x) , \quad q(x) = d\int \frac{d^3 \vec{p}}{(2\pi)^3 E(x)} b(x, p) ,
$$

for the mean field $B$. Here, $q(x)$ is an auxiliary field [11], $g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$ and $d$ is the number of degrees of freedom. As a result of Eq. (3), the space-time dependence of the self-energy is determined by the auxiliary field. Thus, the Liouville operator $\mathcal{D}$ on the left hand side of Eq. (1) is a functional of the distribution function $b(x, p)$ itself.

In thermal equilibrium, characterized by a local distribution function $b^0(x, p) = (e^{\mu u_\alpha / T} - 1)^{-1}$, where the fluid four-velocity $u_\alpha$ satisfies $u_\alpha u^\alpha = 1$, one has to demand that $\Pi(q(x))|_{p=0} \equiv \Pi(T)$ to recover equilibrium results from the energy-momentum tensor (2). In fact, by comparing Eq. (2) evaluated for $b^0(x, p)$ in the local rest frame with $T^{\mu\nu}_{(0)} = \epsilon u^\mu u^\nu - P(g^{\mu\nu} - u^\mu u^\nu)$, the quasiparticle model expressions for energy density $\epsilon$ and pressure $P$ are recovered, cf. [14, 16]. Moreover, Eq. (3) represents the nonequilibrium generalization of the stationarity condition ensuring thermodynamic self-consistency and also implying that the principles of statistical mechanics assure the physical meaning of $b^0(x, p)$ [17]. In this way, $T^{\mu\nu}(x)$ in Eq. (2) represents the general form for an isotropic fluid composed of quasiparticle excitations including only $g^{\mu\nu}$ and $u^\mu$ which, in addition, satisfies thermodynamic self-consistency in equilibrium.

3. Bulk and shear viscosities

The calculation of transport coefficients from $T^{\mu\nu}(x)$ as the first-order corrections to thermal equilibrium is straightforward. Assuming small deviations from equilibrium, $b(x, p) = b^0(x, p) + \delta b(x, p)$ with $\delta b \ll b^0$, $T^{\mu\nu}(x)$ can be decomposed into $T^{\mu\nu} = T^{\mu\nu}_{(0)}[b^0] + \delta T^{\mu\nu}[\delta b]$. Expanding $B(\Pi(q(x)))$ in terms of small deviations from its equilibrium value and approximating $E(x)$ to lowest order by $E = \sqrt{\vec{p}^2 + \Pi(T)}$, $\delta T^{\mu\nu}$ to lowest order in $\delta b$ can be written as

$$
\delta T^{\mu\nu} = d\int \frac{d^3 \vec{p}}{(2\pi)^3 E} \delta b(x, p) \left( p^\mu p^\nu - \frac{1}{2} g^{\mu\nu} \left[ q \frac{\partial \Pi}{\partial q} \right]_{\mu\nu} \right) .
$$
Assuming that collisions always result in an exponentially fast restoration of local equilibrium with the relaxation time $\tau$, all the complexity of $\mathcal{C}(b)$ is encoded in $\tau$. Correspondingly, $\delta b$ can be approximated by $\delta b = -\mathcal{C}(b)/\tau$, which in turn can be expressed by the Boltzmann equation (1). Thus $\delta b$ in Eq. (4) is a functional of $\mathcal{D}$ acting on $b$ reading to lowest order

$$
\delta b(x, p) = -\frac{\tau}{E} \left( p^a \partial_a - \frac{1}{2} \tilde{\nabla} \Pi(T) \frac{\partial}{\partial \tilde{p}} \right) b^0(x, p). \quad (5)
$$

Considering gluonic quasiparticles with $\Pi(T) = \frac{1}{4} T^2 G^2(T)$, we note first that for recovering the equilibrium QPM one needs to identify $\Pi(q) = \tilde{q}/(\beta_0) \log[1/(\sqrt{q^2} - T_0/T_c)^2]$, where $\tilde{q}_{[0]} \equiv (N_q)_{[0]} \equiv T^2$ and $\beta_0 = 11/(8\pi^2)$. Furthermore, one finds $q (\partial \Pi/\partial q)_{[0]} \equiv T^2 (\partial \Pi(T)/\partial T)$. When evaluating Eq. (3) supplemented by Eq. (5), it is appropriate to replace the convective derivatives of $T$ and $\tilde{u}$ by spatial gradients. In line with the Chapman-Enskog strategy, the space-time dependence of $b$ is assumed to be determined only by $T$ and $\tilde{u}^4$, as well as their gradients. Then, the conservation equations are given in first approximation by the equations of motion and energy of an ideal fluid. To make the decomposition of $b(x, p)$ unique, we employ the Landau-Lifshitz condition (19), i.e. $u_i T^{\mu\nu} \equiv u_i T_{\mu\nu}^{\ast} = \epsilon u^4$, implying $\delta T^{00} = 0$ in the local rest frame.

For the considered gluonic system, the bulk and shear viscosities are the only independent transport coefficients characterizing the fluid. In case of small deviations from equilibrium, they are obtained from the spatial part of the nonequilibrium energy-momentum tensor $\delta T^{ij} = -\zeta \tilde{\delta}^{ij} \partial_k \tilde{u}^k + \eta W^{ij}$ as coefficients of the scalar and the traceless part $W^{ij} = \partial^i \tilde{u}^j + \partial^j \tilde{u}^i - \frac{2}{3} \tilde{g}^{ij} \partial_k \tilde{u}^k$. In the local rest frame one finds

$$
\zeta = \frac{d}{3 \mathcal{T}} \int \frac{d^3 \tilde{p}}{(2\pi)^3 E} \tau b^0 \left( 1 + b^0 \right) \left( \frac{\tilde{p}^2}{3E} - \left[ E - T \partial E / \partial T \right] \frac{\partial P}{\partial E} \right) \left\{ 2T^2 \frac{\partial \Pi(T)}{\partial T^2} - \Pi(T) \right\}, \quad (6)
$$

$$
\eta = \frac{d}{15 \mathcal{T}} \int \frac{d^3 \tilde{p}}{(2\pi)^3 E} \tau b^0 \left( 1 + b^0 \right) \frac{\tilde{p}^4}{E}. \quad (7)
$$

From Eqs. (6) and (7) it is clear, that only $\zeta$ is significantly influenced by the medium-dependent quasiparticle dispersion relations $E$, cf. also (20), while in $\eta$ the mean field contributions vanish.

To quantify $\eta$, for instance, the parameters of the effective coupling, $T_c$ and $\lambda/T_c$, are first adjusted to lattice QCD results of the EoS, for example, of the scaled interaction measure $(\epsilon - 3P)/T^4$ (left panel of Fig. 1). In addition, for $\tau$ required in Eq. (7), we employ an ansatz inspired by previous work (22), $\tau^{-1} = a_\eta/(32\pi^2) T G^4 \log(a_\eta G^2)$, where the QCD running coupling is replaced by our effective coupling $G^2(T)$. The corresponding result for the shear viscosity to entropy density ratio $\eta/s$ is depicted in Fig. 1(right panel).

4. Conclusion

We discuss the bulk and shear viscosities by means of an effective kinetic theory for gluon quasiparticle excitations in the relaxation time approximation. Thermodynamic self-consistency necessary in such a quasiparticle description turns out to be solely a consequence of energy-momentum conservation. Our numerical results for $\eta/s$ are in agreement with available lattice QCD results (23) and with other approaches (12). Moreover, $\eta/s$ exhibits the expected minimum close to $T_c$ similar to classical fluids (24), which here is mostly driven by $\tau$. Contrary to the popular view, where a large quasiparticle mean free path implies large $\eta/s$ (24), our results suggest that a quasiparticle description is still admitted for the strongly coupled gluon-plasma.
Figure 1: Left: Comparison of QPM results for the scaled interaction measure with lattice QCD results for pure SU(3) \([21]\) (boxes). The adjusted model parameters read \(d = 16, T_s = 0.53 T_c, \lambda = 2.88\), where in line with lattice QCD results we set \(T_c = 271\) MeV . Right: Corresponding QPM result for \(\eta/s\), employing \(\alpha = 6.8\) in the ansatz for the relaxation time \(\tau\), compared with lattice QCD results (boxes from \([7]\), diamonds and triangles from \([8]\)). In addition, the unitarity limit \([5]\) \(\eta/s = 1/(4\pi)\) (dotted curve) is depicted.

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