Transport and noise properties of a normal metal–superconductor–normal metal junction with mixed singlet and chiral triplet pairings

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Abstract
We study transport and zero frequency shot noise properties of a normal metal–superconductor–normal metal (NSN) junction, with the superconductor having mixed singlet and chiral triplet pairings. We show that in the subgapped regime when the chiral triplet pairing amplitude dominates over that of the singlet, a resonance phenomena emerges out at zero energy where all the quantum mechanical scattering probabilities acquire a value of 0.25. At the resonance, crossed Andreev reflection mediating through such junction, acquires a zero energy peak. This reflects as a zero energy peak in the conductance as well depending on the doping concentration. We also investigate shot noise for this system and show that shot noise cross-correlation is negative in the subgapped regime when the triplet pairing dominates over the singlet one. The latter is in sharp contrast to the positive shot noise obtained when the singlet pairing is the dominating one.

Keywords: mesoscopic superconductivity, proximity effect, transport, shot noise

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physical properties of such superconductors with broken inversion symmetry becomes interesting due to the mixing of a spin-singlet and spin-triplet order parameter without any parity symmetry. Non-centrosymmetric superconductors [24, 25] (NCS) are examples of such superconductors where the spin-singlet and triplet pairing mixing [26] is present with time reversal symmetry but with broken inversion symmetry [27–29]. The absence of the parity symmetry in NCS may lead to several interesting properties determined by the ratio of the amplitudes of the spin-singlet to spin-triplet pair potentials [30]. Among such properties emergence of spin current in NCS superconductor [31, 32], magneto-electric effects [23], magnetism [25] etc have been reported in recent times. These interesting properties have drawn the attention of the community towards exploring the nature of the NCS superconductor and as a consequence, the list of NCS materials is growing gradually [33]. Another reason behind this attraction is that recently it has been shown that this type of superconductor characterized by time-reversal symmetry can hold an even number of Majorana Fermions [34–36]. Therefore, further investigations are required to explore the properties of NCS as well as the effect of NCS on transport phenomena through superconducting hybrid junctions.

Very recently the transport signature of NS and superconductor–normal metal–superconductor (NSN) junction with mixed singlet and chiral triplet pairing has been reported by Burset et al [30]. They obtain a zero-energy peak in the conductance in a NS junction when the triplet pairing is the dominating one over the singlet part. However, the NSN junction and the properties of CAR in the above context has never been studied so far. The latter motivated us to investigate transport and shot noise phenomena through a NSN junction in which a one-dimensional (1D) nanowire (NW) is placed in close proximity to a superconductor which contains a pair potential of mixed singlet and chiral triplet type. The NW is attached to two normal metal (N) leads. We incorporate three regimes corresponding to the amplitude of the spin-singlet part being lesser, equal to and larger than that of the spin-triplet part. We adopt Blonder–Tinkham–Klapwijk (BTK) formalism [1] to calculate the quantum mechanical scattering amplitudes through the junction and conductance, shot noise therein. We find zero-energy peak in the conductance depending on the degree of mixing of the pair potentials and the doping. We also calculate the zero frequency shot noise (auto and cross-correlation) and show that the shot noise cross-correlation becomes positive to negative as long as the triplet pairing dominates over the singlet one.

The remainder of this paper is organized as follows. In section 2 we describe our model. Section 3 is devoted to the scattering matrix (BTK) formalism by which we calculate the quantum mechanical scattering amplitudes to obtain conductance and shot noise through the NSN junction. We present our numerical results in section 4 which includes scattering probabilities, conductance and shot noise for different parameter regimes. Finally, we summarize and conclude in section 5.

2. Model

In figure 1 we present the schematic of our proposed set-up in which a 1D NW is placed in close proximity to a bulk superconducting material. Here superconductivity is induced in the NW via the proximity effect. The NW is attached to two normal metal leads. A gate voltage G can tune the chemical potential inside the NW. Instead of a conventional s-wave superconductor here we consider the pairing potential of the superconductor as a combination of spin-singlet and chiral spin-triplet states mimicking the NCS superconductor. We choose the x-axis along the direction of the NW. The two N–NW interfaces are located at $x=0$ and $x=L$ respectively. At each N–NW interface we consider an insulating barrier which is modeled by the δ-function potential given as $V(x) = (h^2k_F/m)\delta(x)$ where $k_F$ is the Fermi wave vector, $m$ denotes electron mass and Z is the strength of the barrier.

Hence the NSN junction can be described by the Bogoliubov–de Gennes (BdG) equations as,

$$H(k)\psi(k) = \epsilon\psi(k)$$

where

$$H(k) = \begin{bmatrix} [E(k) - \mu]\sigma_0 & \hat{\Delta}(k) \\ \hat{\Delta}^\dagger(k) & [\mu - E(-k)]\sigma_0 \end{bmatrix}.$$  \hspace{1cm} (2)

Here $E(k) = (h^2/2m)k_x^2 + U$ is the dispersion relation of the electronic excitation measured from chemical potential $\mu$. $U$ is the electrostatic potential in the normal region. $\sigma_0$ is the $2 \times 2$ identity matrix in spin space. We write the four component wave function in Nambu representation as $\psi(k) = [u_\sigma(k), v_\sigma(k)]^T$ where $u_\sigma(k)$ and $v_\sigma(k)$ are the electron and hole components respectively with spin $\sigma = \uparrow, \downarrow$ and $k$ is the wave vector.

In the superconducting region, due to the presence of both spin-singlet and chiral spin-triplet states, the pairing potential $\hat{\Delta}(k)$ ($2 \times 2$ matrix) can be written in general $\hat{\Delta}(k) = i[\Delta_{s}(k)\sigma_0 + \sum_{j=1}^{3} d_j(k)\sigma_j]e^{i\phi}$. Here, $\delta_{1,2,3}$ are Pauli spin matrices operating on spin space and $\phi$ is the superconducting phase factor. The spin singlet pairing $\Delta_{s}(k)$ characterizes the conventional $s$ wave superconducting order parameter. Here we consider only the mean-field value of

![Figure 1. Schematic of the quasi one-dimensional NSN setup in which a nanowire (NW) (pink, light grey) is placed in close proximity to a bulk superconductor (light brown, light grey) and superconductivity is induced in the NW via the proximity effect. The NW is attached to two normal (N) metal leads (blue, black). The gate G (light cyan, light grey) controls the chemical potential in the NW. Two δ-function barriers are symbolically denoted by the two yellow (light grey) rectangular barriers at each N–NW interface.](image-url)
\( \Delta_x(\mathbf{k}) \) i.e. \( \Delta_y(\mathbf{k}) = \Delta_y \). In contrast, the triplet pairing potential is described by an odd vector function as \( \mathbf{d}(\mathbf{k}) = -\mathbf{d}(-\mathbf{k}) \).

Following Burset et al [30] we take the chiral triplet state of the form,

\[
d(\mathbf{k}) = \Delta_p \frac{k_x + ik_y \hat{z}}{|\mathbf{k}|} = \Delta_p e^{i\phi} \hat{z},
\]

where \( \Delta_p \) is non-negative amplitude of the triplet pairing potential. \( \chi \) determines the orientation of the angular momentum of the Cooper pairs and it can take the \( \pm \) sign corresponding to the parallel and anti-parallel direction respectively. \( \theta \) represents the relative orientation between the singlet and chiral triplet pairing states. With this consideration pairing potential now takes the form,

\[
\hat{\Delta}(\mathbf{k}) = i[\Delta_0 \hat{\sigma}_0 + \Delta_p e^{i\phi} \hat{\sigma}_3] \hat{\sigma}_2 e^{i\theta}.
\]

This simple choice of the pairing potential takes into consideration the mixing of the spin-singlet and spin-triplet states. With this pairing potential, the band dispersion becomes [30]

\[
e_{1,2}(\mathbf{k}) = \sqrt{E^2(\mathbf{k}) + \Delta^2_0 + \Delta^2_p \pm 2\Delta_0 \Delta_p \cos \theta},
\]

which explicitly depends on the relative orientation of singlet and chiral triplet pairing components. Now the full Hamiltonian \( H(\mathbf{k}) \) can be written in block diagonal form which implies decoupling of two spin channels \( \uparrow \) and \( \downarrow \). Hence the effective pairing potentials corresponding to these two channels become [30]

\[
\Delta_{1,2}(\theta) = [\Delta_0 \pm \Delta_p e^{i\phi}] \hat{\sigma}_2 e^{i\theta}.
\]

Therefore, it will now be sufficient to consider two effective complex pair potentials \( \Delta_{1,2}(\theta) \) among which \( \Delta_0(\theta) \) vanishes for a particular choice of the \( \Delta_0(\pm \Delta_p \cos \theta) \) and also it changes sign for \( \Delta_0 > \Delta_p \).

In the quasi 1D limit, electrons can propagate only in the \( x \)-direction with the transverse component of the wave vector \( k_z \) being conserved. Hence the band energy \( E(\pm k) \) can be written as \( E(\pm k_z) \) for a particular choice of \( k_z \). We choose \( k_z = 0 \) for our analysis. Also, we assume that the band energies for the electrons moving to the left and right are equal to each other. We define right movers by \( \theta^+ = \theta \) and left movers by \( \theta^- = \pi - \theta \). After decoupling for each spin channel, the BdG equations in the \( 2 \times 2 \) form can be written as

\[
\begin{bmatrix}
E(\mathbf{\alpha k}) - \mu & s_0 \Delta_x(\theta^+ \mathbf{e}^{i\phi}) \mathbf{e}^{i\theta} \\
\bar{s}_0 \Delta_x(\theta^- \mathbf{e}^{-i\phi}) e^{-i\theta} & \mu - E(-\mathbf{\alpha k})
\end{bmatrix}
\begin{bmatrix}
u_{e}(\theta^+) \\
u_{\bar{e}}(\theta^-)
\end{bmatrix} = 
\begin{bmatrix}
u_{e}(\theta^-) \\
u_{\bar{e}}(\theta^+)
\end{bmatrix},
\]

where \( \epsilon = 0 \) is the excitation energy; \( \alpha = \pm \) corresponds to the right and left movers; \( s_0 = (-1)^{\mathbf{\sigma} - 1} \) and \( \sigma = 1, 2 \) denotes the different spin channels. Thus, the pairing potential is different for each independent spin channel as well as for direction of motion of particles. Also, the gap amplitude can be different depending on the direction of motion as argued in [2].

Left mover with spin \( \uparrow \) and \( \downarrow \) will be affected by the pairing potential \( \Delta_0(\theta^+) \) and \( -\Delta_0(\theta^-) \) respectively. On the other hand, right mover will experience the effective pairing potential \( \Delta_{\sigma}(\theta^+) \) and \( \Delta_{\sigma}(\theta^-) \) respectively. Hence the effective pairing potential now takes the form,

\[
\hat{\Delta}(\mathbf{k}) = i[\Delta_0 \hat{\sigma}_0 + \Delta_p e^{i\phi} \hat{\sigma}_3] \hat{\sigma}_2 e^{i\theta}.
\]

\( \Delta_{\sigma}(\theta^+) \) and \( -\Delta_0(\theta^-) \) corresponding to \( \uparrow \) and \( \downarrow \) spin channels respectively.

Electron and hole components of the wave functions are given by,

\[
u_{e}(\theta^+) = \frac{1}{\sqrt{2}} \left(1 + \frac{\sqrt{\epsilon^2 - |\Delta_{\sigma}(\theta^+)|^2}}{\epsilon}\right),
\]

\[
u_{\bar{e}}(\theta^-) = \frac{1}{\sqrt{2}} \left(1 - \frac{\sqrt{\epsilon^2 - |\Delta_{\sigma}(\theta^-)|^2}}{\epsilon}\right).
\]

3. The scattering matrix

In this section we present the scattering matrix obtained employing the BTK formalism [1] for our NSN geometry. The normal metallic region is described by considering \( \hat{\Delta}(\mathbf{k}) = 0 \) and also we set \( U = 0 \) there to carry out our analysis. When an incident electron coming from one of the normal metal leads with energy below the superconducting gap scatters from the NS interface, the corresponding scattering phenomena can be described by four possible quantum mechanical processes. These processes are: (a) normal reflection of electron from the NS boundary (b) AR of incident electron as a hole in the same lead (c) elastic co-tunneling (CT) in which the incident electron transmits to the other lead as an electron and (d) transmission of hole in the other lead via the CAR process. The schematic of these processes are displayed in figure 2.

In order to obtain reflection, AR, CT and CAR amplitudes through the NSN junction we write the wave functions in the three regions as follow,

\[
\psi_{e}^{a}(z) = e^{ik_{a}z} \begin{bmatrix}1 \\0\end{bmatrix} + r_{e}(\epsilon)e^{-ik_{a}z} \begin{bmatrix}1 \\0\end{bmatrix} + r_{he}(\epsilon)e^{ik_{a}z} \begin{bmatrix}0 \\1\end{bmatrix}
\]

Figure 2. Schematic for the electron (solid sphere) and hole (hollow sphere) trajectories (solid and dashed lines, respectively) corresponding to the four quantum mechanical scattering processes occurring at a NSN junction. Notations in the figure denote eR: right-moving electron; eL: left-moving electron; hR: right-moving hole; hL: left-moving hole.
We solve these equations for each regime, setting incident electron energy \( \varepsilon \) and probability therein. We divide our study into two categories such as, the undoped regime, setting incident electron energy \( \varepsilon \gg \Delta_0 \), where, \( k_F/\xi \ll 1 \), and amplitude \( \psi^{(5)}(x) \) are the wave functions for the left, right normal metal leads and the superconductor respectively. \( k_e \) and \( k_h \) are the wave vectors for the electron and hole respectively in the normal and the superconducting region. They can be expressed as,

\[
k_{e(h)0} = k_F \sqrt{\left(1 \pm \frac{\varepsilon}{\mu}\right)} \]

where, \( \Delta_e \) can be \( \Delta_0 \) or \( \Delta_0^* \) depending on the spin channel.

Here, \( r_e, r_h, t_e \) and \( t_h \) denote the normal reflection, AR, CT and CAR amplitudes respectively. They can be obtained by considering the boundary conditions for the wave functions such as, for the left boundary \( (x = 0) \)

\[
\psi^L|_{x=0} = \psi^S|_{x=0}, \quad \partial_x \psi^S|_{x=0} = k_F \psi^L(0) \]

and for the right boundary \( (x = L) \),

\[
\psi^R|_{x=L} = \psi^S|_{x=L}, \quad \partial_x \psi^S|_{x=L} = k_F \psi^R(L) \]

Numerically solving these eight equations we get the amplitudes corresponding to all scattering processes \( (r_e, r_h, t_e \) and \( t_h) \) and probability therein. We solve these equations for each spin channel, \( \sigma = 1 \) and \( \sigma = 2 \). Here we denote \( R_{e \sigma} = |r_{e \sigma}|^2 \), \( R_{h \sigma} = |r_{h \sigma}|^2 \), \( T_{e \sigma} = |t_{e \sigma}|^2 \) and \( T_{h \sigma} = |t_{h \sigma}|^2 \) as the probability for normal reflection, AR, CT and CAR respectively. All the probabilities together satisfy the unitarity relation,

\[
R_{e \sigma} + R_{h \sigma} + T_{e \sigma} + T_{h \sigma} = 1 \quad \text{for each spin channel} \quad (\sigma = 1, 2) \]

where, \( k_F/k_0 \) is introduced in order to maintain the probability current conservation.

At zero temperature, conductance for a particular electron energy \( \varepsilon \) and a chiral angle \( \theta \) can be found by taking contributions from both the spin channel \( \sigma = 1 \) and using the following relation,

\[
G(\varepsilon, \theta) = \frac{e^2}{h} \sum_{\sigma} (|r_{e \sigma}|^2 - |t_{h \sigma}|^2). \quad (15)
\]

In this section we present our numerical results for the scattering probabilities, conductance and shot noise. Depending on the ratio of triplet to singlet phase of the superconducting pairing potential we consider three different regimes of interest: \( \Delta_p < \Delta_s < \Delta_p = \Delta_0 \). When we present all our numerical results only for the regime \( \Delta_p > \Delta_0 \), which is the interesting regime. Also, we show \( R_{e\sigma}, R_{h\sigma}, T_{e\sigma} \) and \( T_{h\sigma} \) as functions of different parameters of the system only for \( \sigma = 1 \) without loss of generality and hence we use the notation \( R_e, R_h, T_e \) and \( T_h \) in place of \( R_{e\sigma}, R_{h\sigma}, T_{e\sigma} \) and \( T_{h\sigma} \) respectively throughout our results. The length of the superconducting region and energy of the incident electron are normalized by the superconducting coherence length (\( \xi \) ) and amplitude of the pair potential \( \Delta_0 \) respectively i.e. \( L/\xi \rightarrow L, \varepsilon/\Delta_0 \rightarrow \varepsilon \). Moreover, depending on the doping in the normal metal side we divide our study into two categories such as, the undoped case where we set \( \mu = 0 \) and finite doping condition for which we fix \( \mu = 5 \). Throughout our calculation, the values of some parameters are taken as \( Z = 2, \phi = 0, e = 1, h = 1 \) and \( U = 15 \) (for the superconducting region). The chosen value of \( U \) makes the superconductor doped and creates large Fermi wave-length mismatch between the normal and superconducting regions and also fulfills the requirement of the mean field condition of superconductivity i.e. \( \mu + U \gg \Delta_0 \). We use the unit where \( \Delta_0 = 1 \).

4.1 Scattering processes

In this subsection we show our numerical results for the scattering probabilities for two different doping conditions.

4.1.1 Undoped regime (\( \mu = 0 \)). In figure 3 we show all the four possible scattering probabilities \( R_e, R_h, T_e \) and \( T_h \) as a function of the length (\( L \)) of the superconductor for \( \Delta_p \geq \Delta_s \), regime, setting incident electron energy \( \varepsilon = 0 \). With this choice of energy value we are within the superconducting sub-gapped regime. Panel (a) and (b) in figure 3 correspond to \( \theta = 0 \) and \( \theta = \pi/4 \) respectively.

It is evident from figure 3(a) that for \( \theta = 0 \), AR dominates over all other scattering processes except for very small values of \( L \). To illustrate this, we show \( R_e, R_s, T_e \) and \( T_h \) in the inset of figure 3(a), for small values of \( L \) (\( L < \xi \)). Note that all the scattering probabilities are almost identical to each other for \( L < 0.03 \xi \), i.e. they occur with almost equal probability of value \( \sim 0.25 \), which is in sharp contrast to the \( \Delta_s > \Delta_p \) regime.
where \( T_b \) (CAR) is vanishingly small. On the other hand, for \( \xi > 0.075 \), all scattering processes except AR become vanishingly small.

To investigate whether the above mentioned resonance phenomena persists for other values of \( \theta \), we show the behavior of \( R_e, R_h, T_e \) and \( T_h \) as a function of \( L \) in figure 3(b) for \( \theta = \pi/4 \). Recall that the scattering probabilities no longer remain equal to each other for the \( \xi \ll L \) regime even we set \( \Delta > \Delta_s \).

Instead, probability for CT dominates over the others and attains the maximum value \( \sim 1 \) for \( \xi \ll L \), which is illustrated in the inset of figure 3(b). Nevertheless, as soon as \( L \) becomes larger than \( \xi \) normal reflection begins to dominate over CT, as visible from figure 3(b). For \( L \gg \xi \) all processes except normal reflection die away and the junction becomes perfectly reflecting. Comparing the two cases we can say that for \( \theta = 0 \), AR dominates over the other processes when \( L \gg \xi \). On the other hand, the contribution for normal reflection process becomes dominant for \( \theta = \pi/4 \) and \( L \gg \xi \). For both the \( \theta \) values, the contribution for the two non-local processes \( T_e \) (CT) and \( T_h \) (CAR) becomes vanishingly small when \( L > \xi \).

We also analyse the dependence of this resonance phenomenon on the incident electron energy and show the corresponding behavior of \( R_e, R_h, T_e \) and \( T_h \) as a function of \( \epsilon \) in figure 4. It is evident from the inset of figure 4(a) that all the scattering probabilities become equal in magnitude \( \sim 0.25 \) at \( \epsilon = 0 \). Similar 1/4 resonance behavior at finite energy had been predicted earlier in a superconducting double barrier (NSNSN) structure [37] where the superconductor was considered to be a purely singlet one. However, zero energy peak (ZEP) for CAR with peak height of \( \sim 0.25 \) for a NSN geometry in the \( \Delta > \Delta_s \) regime when \( \theta = 0 \) is one of the main results of our paper. The physical reason behind the emergence of this ZEP can be attributed to the vanishing of the effective pairing gap for \( \theta = 0 \) when \( \Delta_p = \Delta_s \) and changing sign depending on whether \( \Delta_p > \Delta_s \) or \( \Delta_p < \Delta_s \) (see equation (4)). This leads to the appearance of a zero-energy Andreev bound state and zero energy resonance phenomena therein.

On the other hand, the behavior of \( R_e, R_h, T_e \) and \( T_h \) for \( \theta = \pi/4 \) is depicted in figure 4(b). We observe that normal reflection has sharp zero energy as well as finite energy peaks, while the CT process has sharp dips at those energy values. These peaks are clearly shown in the inset of figure 4(b). In this parameter regime AR and CAR probability are always vanishingly small. Note that, for this \( \theta \) value, the energy dispersion changes according to equation (4) leading to different resonance behavior.
Note that the amplitudes for different scattering processes depend on whether \( \mu > \epsilon \) or \( \mu < \epsilon \). This can be understood qualitatively from equation (12). Whether \( k_\mathrm{h} \) is real or imaginary, it completely depends on the relative strength of gate voltage and applied bias which in turn changes the scattering amplitudes. For \( \mu > \epsilon \), particles can only tunnel through the superconductor resulting in \( T_e = 1 \) as shown in figure 4(a) for \( \theta = 0 \).

The zero energy resonance phenomena also survives with the enhancement of the barrier strength \( Z \). The reason behind such resonance structure and ZEP for the AR and CAR can be attributed to the formation of Andreev bound states (ABS) inside the proximity induced superconducting region of the NW. The nature of the ABS from the shot noise point of view will be presented at a later subsection of this article.

4.1.2. Doped regime (\( \mu = 5 \)). Here, we examine the behavior of the scattering probabilities with the change of doping in the normal metal while choosing the value of the other parameters same as in the undoped case. Figure 5 depicts the variation of \( T_e, T_h, R_e \), and \( R_h \) as a function of the length of the superconductor where panel (a) and (b) correspond to \( \theta = 0 \) and \( \pi/4 \), respectively. In figure 5(a) we observe that the behavior of the scattering probabilities qualitatively remains similar to that of the undoped (\( \mu = 0 \)) case. Here also AR dominates over all other scattering processes and also the 1/4 resonance phenomena takes place below a critical value of \( L/\xi \). The latter is depicted in the inset of figure 5(a). Hence, doping has a very small effect on the scattering phenomena when \( \theta = 0 \) and \( \epsilon = 0 \).

However, if we choose a different value of \( \theta \), the effect of doping is much more visible in figure 5(b) in comparison to figure 3(b). All the scattering probabilities become oscillatory with respect to \( L/\xi \) when we set \( \theta = \pi/4 \) for finite doped regime. The only common feature between the two cases is that with the enhancement of the length of the superconducting region, normal reflection dominates over all other processes while CT and CAR become vanishingly small. These periodic variation with \( L \) can be manifested as the interference between the electron and hole wave-functions inside the superconducting region.

In figure 6 we show the variation of \( R_e, R_h, T_e, \) and \( T_h \) with incident electron energy \( \epsilon \). Here panels (a) and (b) correspond to \( \theta = 0 \) and \( \theta = \pi/4 \) while the value of the other parameters remain unchanged as in the undoped case. The inset of figure 6(a) illustrates that AR, CAR and CT processes acquire a sharp peak and all of them achieve a value \( \sim 0.25 \) at zero energy. They gradually become vanishingly small for \( |\epsilon| > 0.002\Delta_0 \). On the other hand the probability for \( R_e \) becomes close to unity and the junction becomes nearly perfectly reflecting for energy values other than zero. In contrast, for \( \theta = \pi/4 \), the ZEP no
longer exists as depicted in figure 6(b). There are two resonance points symmetrically situated around \( \approx \pm \Delta \varepsilon \) in the sub-gapped regime. Both AR and CAR have sharp peaks whereas the other two processes (Re and CT) have dips at those points (see the inset of figure 6(b)). These peaks (dips) are shifted from zero energy due to finite \( \theta \) in the doped regime.

Note that all the results presented here is for symmetric barriers placed at the two N–NW interfaces. However, our results remain qualitatively unchanged even for asymmetric barrier strengths at the two interfaces.

4.2. Conductance

In this subsection, we study the angle averaged normalized conductance \((\tilde{G}/G_0)\) as a function of incident electron energy \(\varepsilon\) using equation (17). The results are presented in figure 7 where, panels (a) and (b) correspond to the undoped (\(\mu = 0\)) and doped (\(\mu = 5\)) case, respectively. Here we have averaged over all possible orientations between the singlet and triplet pair potentials. For the undoped case, conductance increases almost linearly with energy as shown in figure 7(a) irrespective of the pairing potential amplitudes. At \(\varepsilon = 0\), averaged conductance exactly becomes zero for all the three regimes of the pairing potentials. In contrast, in the doped regime, conductance behavior is non-monotonic. There are peaks at \(\varepsilon = \pm 0.5\Delta_0\) for all the three regimes (\(\Delta_p < \Delta_s, \Delta_p = \Delta_s, \Delta_p > \Delta_s\)) and these peaks correspond to the density of states at the two boundaries of the superconducting gap. In the scattering probability profiles we obtain ZEP for CAR and CT processes in the regime \(\Delta_p > \Delta_s\). However in the conductance profile, we obtain a ZEP when \(\Delta_p = \Delta_s\) for the finite doping condition. There is only finite average conductance (no ZEP) for the other two regimes i.e. \(\Delta_p < \Delta_s\) or \(\Delta_p > \Delta_s\) (see figure 7(b)).

The absence of the ZEP in the orientation averaged conductance profile corresponding to \(\Delta_p > \Delta_s\) regime can be explained as follows. At \(\varepsilon = 0\), we have zero conductance corresponding to the regime \(\Delta_p > \Delta_s\) for the undoped case. This happens because at \(\varepsilon = 0\) either both CT and CAR probabilities have the same magnitude describing the resonance condition or they are vanishingly small as depicted in figures 4(a)–(b). Taking into account contributions due to all possible orientations \(\theta\) between the singlet and triplet pairings we finally obtain zero conductance for the undoped case. Although for non-zero \(\varepsilon\), the contribution in the conductance can arise due to entirely CT \((T_e)\) process whose probability is one for \(\varepsilon = 0\). On the other hand, for the doped case we have finite conductance at \(\varepsilon = 0\) after averaging over all possible \(\theta\)s. The reason behind this feature can be attributed to the finite \(T_e\) contribution for \(\theta = \pi/4\) (see figure 6(b)).

4.3. Shot noise

This subsection is devoted to exploring the shot noise properties mediated through our NSN junction. Our aim is to investigate the nature of zero energy resonance as mentioned before via the noise. In general shot noise in a mesoscopic
system arises due to the quantization of the electric charge [38, 39]. Measurement of shot noise can even be utilized to probe the nature of superconducting wavefunction [13]. Here we neglect thermal noise as throughout our calculation we set the temperature to zero.

The correlation function of the current in the two leads labeled by $i$ and $j$, is defined as [38],

$$S_{ij}(t - t') = \langle \Delta \dot{I}(t) \Delta \dot{I}(t') + \Delta \dot{I}(t') \Delta \dot{I}(t) \rangle$$  \hspace{1cm} (18)

in terms of the operator,

$$\Delta \dot{I}(t) = \dot{I}(t) - \langle \dot{I}(t) \rangle.$$  \hspace{1cm} (19)

After performing the Fourier transform equation (18) becomes,

$$S_{ij}(\omega) = \frac{1}{2\pi} \langle \Delta \dot{I}(\omega) \Delta \dot{I}(\omega') + \Delta \dot{I}(\omega') \Delta \dot{I}(\omega) \rangle$$  \hspace{1cm} (20)

with

$$\Delta \dot{I}(\omega) = \dot{I}(\omega) - \langle \dot{I}(\omega) \rangle.$$  \hspace{1cm} (21)

We find the expression for zero frequency ($\omega = 0$) shot noise cross correlation in terms of the scattering amplitudes following [40]. The general expression for current fluctuation in two different leads $i$ and $j$ in presence of an external bias is given by [40],

$$S_{ij} = \frac{2e^2}{h} \sum_{k,\ell,\alpha,\beta,\gamma \in e, h} \text{sgn}(\alpha) \text{sgn}(\beta) \int dE A_{k,\ell,\alpha}(i\alpha, E) A_{\alpha,\beta,\gamma}(j\beta, E) f_{\alpha,\gamma}(E)[1 - f_{\beta,\gamma}(E)]$$  \hspace{1cm} (22)

where $A_{k,\ell,\alpha}(i\alpha, E) = \delta_{\alpha\beta} \delta_{k\ell} \delta_{\gamma\beta} - 2 \delta_{k\ell}^* (E) \delta_{\alpha\gamma}$. Here $\text{sgn}(\alpha) = +1$ corresponds to $\alpha = e$ (electron) and $\text{sgn}(\alpha) = -1$ refers to $\alpha = h$ (hole). $\delta_{k\ell}^*$ represents the scattering amplitude for a particle of type $\gamma$ incident from lead $k$ being transmitted to lead $i$ as a particle of type $\alpha$ $(\alpha, \gamma \in e, h)$. Equation (22) is valid for current fluctuations in mesoscopic hybrid junctions when the superconductor region is maintained at a fixed potential [40]. Also, we consider the zero frequency limit to neglect the capacitive component in order to avoid displacement currents due to charging.

It is well known that zero-frequency current fluctuation between two different normal metals leads is always negative for fermions [38]. Nevertheless in the presence of a singlet superconductor shot noise cross-correlation can be positive depending on the parameter values [13, 14, 41, 42]. The expression for shot noise in terms of transmission and reflection co-efficients are given in the appendix.

In figures 8(a)–(b) we show the behavior of shot noise cross correlation ($S_{ij}$) as a function of the incident electron energy $\epsilon$ in the regime $\Delta_p > \Delta_s$ for the undoped ($\mu = 0$) case. Here panels (a) and (b) correspond to $\theta = 0$ and $\theta = \pi/4$ respectively. We observe that $S_{ij}$ gradually reduces to $-1$ for a very small range around $\epsilon = 0$, but sharply reverts back to zero exactly at $\epsilon = 0$ as illustrated by the inset of figure 8(a). This sign change of $S_{ij}$ from $-1$ to $0$ reflects the presence of the zero energy resonance where all the scattering probabilities have equal magnitude of 1/4. Moreover, for energy values other than zero below the subgapped regime, $S_{ij}$ is exactly zero since $T_e$ is 1 and $R_e = R_h = T_h = 0$ (see figure 4(a)). On the other hand, there are sharp positive peaks in the $S_{ij}$ profile for $\theta = \pi/4$, as shown in figure 8(b). For $\epsilon = 0$ and $\epsilon \approx 0.5$, $R_e = 1$ and $R_h = T_e = T_h = 0$.
as depicted in figure 4(b). Also $T_c = 1$ for the other values of energy. Hence, shot noise cross correlation $S_{ij}$ is vanishingly small ($\sim 10^{-5}$) compared to the $\theta = 0$ case.

Similar to the undoped case, we also calculate the zero frequency shot noise cross-correlation for the doped system. Our results are shown in figures 9(a)–(b). We qualitatively obtain the similar behavior for $S_{ij}$ as in the undoped case. As depicted in figure 6(a), at $\epsilon = 0$ all the scattering probabilities have the same value of 0.25 resulting in zero $S_{ij}$ (see figure 9(a)). On the other hand, for $\theta = \pi/4$, $S_{ij}$ becomes $-1$ in a small range around $\epsilon \approx \pm 0.5\Delta_p$. Another interesting feature is that $S_{ij}$ changes sign from negative to positive for both the cases $\theta = 0$ (around $\epsilon = 0$) and $\theta = \pi/4$ (around $\epsilon \approx \pm 0.5\Delta_0$) as depicted in the insets of figures 9(a) and (b) respectively. This transition of the shot noise cross-correlation from negative to positive value is in contrast to that case of purely singlet superconductor where shot noise cross correlation is only positive [13, 14, 41].

So far, the behavior of shot noise cross-correlation has been discussed for particular values of $\theta$ which is the orientation between the triplet to singlet amplitude of the superconductor. To obtain the angle averaged shot noise we integrate over all possible orientations as,

$$\tilde{S}_{ij} = \int_{-\pi/2}^{\pi/2} S_{ij} \cos \theta \, d\theta. \quad (23)$$

The behavior of angle averaged shot noise $\tilde{S}_{ij}$ is presented as a function of the energy of the incident electron in figure 10. For $\mu = 0$, $\tilde{S}_{ij}$ vanishes at $\epsilon = 0$ irrespective of the ratio of the pairing potential amplitudes. Moreover, the feature of $\tilde{S}_{ij}$ is monotonic similar to the conductance (see figure 7(a)) for $\epsilon = 0$.

On the other hand, there are crossovers from positive to negative and vice-versa for the doped case corresponding to all the three regimes of the pairing amplitudes. Nevertheless, $\tilde{S}_{ij}$ is always negative for $\Delta_p > \Delta_0$ in the subgapped regime i.e. $\epsilon \ll \Delta_0$. However when the $s$ wave pairing amplitude dominates over that of the $p$ wave ($\Delta_s > \Delta_p$), $\tilde{S}_{ij}$ remains positive over the major range of the subgapped regime and changes sign around $\epsilon \approx 0.75\Delta_0$. The emergence of negative shot noise cross correlation ($\tilde{S}_{ij}$) in the $\Delta_p > \Delta_s$ regime in contrast to the positive $\tilde{S}_{ij}$ in the $\Delta_p < \Delta_s$ regime is another main result of our article.

5. Summary and conclusions

To summarize, we have explored the conductance and shot noise phenomena through a NSN junction where the superconductor is characterized by a mixture of both the spin-singlet and spin-triplet pairings. Our NSN set up comprises of a 1D NW placed in close proximity to a bulk superconductor with mixed pairing of singlet and triplet type (e.g. NCS superconductor). The NW is also coupled to two normal metal leads. Depending on the ratio of their pairing amplitudes and the doping concentration in the normal metal region we study the behavior of scattering amplitudes, conductance and zero frequency shot noise for three different regimes ($\Delta_p > \Delta_s$, $\Delta_p = \Delta_s$, $\Delta_p < \Delta_s$). The main feature we obtain in this geometry is the appearance of a zero energy resonance. At the resonance, probability for all the four possible scattering processes (reflection, AR, CT and CAR) acquire same magnitude (1/4) when chiral triplet pairing amplitude dominates over the singlet one. The angle averaged conductance also exhibits a zero energy peak in the doped regime. Moreover, for a chosen orientation ($\theta$) between the singlet and triplet pairing, zero frequency shot noise cross correlation exhibits positive to negative transition in the chiral triplet pairing dominated regime. However, for the doped regime, angle averaged shot noise remains negative in the subgapped regime when $\Delta_p > \Delta_s$ and becomes positive in the opposite regime ($\Delta_s > \Delta_p$). Very recently, transition from positive to negative shot noise cross-correlation also has been reported in the context of Majorana bound states [43–45].

As far as practical realization of our NSN structure is concerned, a NW may be possible to fabricate in close proximity to a NCS superconductor for e.g. MoAlC, BiPd etc [33, 46]. Such superconductors posses a coherence length $\xi \approx 10–20$ nm for critical magnetic field $H_c(0) \approx 1.2–1.5$ T as reported in [33, 46]. Hence, our findings for the angle averaged conductance and shot noise cross-correlation may be realizable in a proximity induced NW where the length of the superconducting region can be $L = 0.75\xi \approx 5–15$ nm. Our setup may also be used for making future generation entangler devices with unconventional superconductor [7, 19, 47].
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Appendix. Expression for the shot noise

We study the current cross-correlation in our NSN geometry at zero temperature and zero frequency limit following [40]. The shot noise contributions arising from different scattering amplitudes can be separated in terms of the normal reflection, AR, CT and CAR amplitudes as follows.

\[ S_y^0(\epsilon) = -\frac{2e^2}{\hbar} \left[ (r_1(\epsilon) r_2^* (\epsilon) + r_2(\epsilon) r_1^* (\epsilon)) + (t_1(\epsilon) t_2^* (\epsilon) + t_2(\epsilon) t_1^* (\epsilon)) \right] \]

\[ S_y^h(\epsilon) = \frac{4e^2}{\hbar} |r_1(\epsilon) t_2^* (\epsilon) + t_1(\epsilon) r_2^* (\epsilon)|^2. \quad (A.1) \]

Hence the total shot noise reads,

\[ S_y(\epsilon) = S_y^0(\epsilon) + S_y^h(\epsilon) \quad (A.2) \]

where, \( S_y^0, S_y^h \) represent the cross-correlation corresponding to the phenomenon of CT and CAR respectively. Also, due to particle-hole symmetry we can write

\[ S_y^0(\epsilon) = S_y^h(-\epsilon) \]

\[ S_y^h(\epsilon) = S_y^0(-\epsilon). \quad (A.3) \]

Here we scale \( r_k \) and \( t_k \) by \( \sqrt{\frac{k}{\epsilon}} \) (i.e. after scaling we have \( r_k \equiv r_k \sqrt{\frac{\epsilon}{k}} \) and \( t_k \equiv t_k \sqrt{\frac{\epsilon}{k}} \) in order to maintain the probability conservation (unitarity) as discussed earlier. All these scattering amplitudes and hence can be expressed as follow,

\[ r_k = |r_k|e^{i\theta_1}, \quad r_\epsilon = |r_\epsilon|e^{i\phi_1}, \]

\[ t_k = |t_k|e^{i\phi_2}, \quad t_\epsilon = |t_\epsilon|e^{i\phi_2}, \]

where \( \theta_1, \phi_2, \phi_1, \phi_2 \) are the phase factors of the corresponding complex scattering amplitudes. They play crucial role in determining the nature of the shot noise which can be realized from equation (A.1). We emphasize the scenario where all the scattering probabilities are equal in magnitude (0.25) i.e. the zero energy resonance condition. As mentioned earlier, we obtain this zero energy resonance for both undoped (\( \mu = 0 \)) and doped (\( \mu \neq 0 \)) condition. Thus, at zero energy, the expression for shot noise cross-correlation takes the form.

\[ S_y(\epsilon) = -\frac{e^2}{2\hbar} \cos^2(\phi_2 - \theta_2) + \cos^2(\phi_1 - \theta_1) + \frac{e^2}{2\hbar} \left[ 1 + \cos(\theta_1 + \theta_2 - \phi_1 - \phi_2) \right]. \quad (A.4) \]

From equation (A.4) it is evident that shot-noise correlation depends only on the phases of different scattering amplitudes at resonance. If the phases cancel out among each other then \( S_y \) becomes zero which we obtain at the resonance.

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