Formation of a cavitation zone behind a rarefaction wave front in a shock-loaded thin layer of a multiphase liquid

V K Kedrinskiy\textsuperscript{1} and E S Zhuravleva\textsuperscript{1}

\textsuperscript{1} Lavrentyev Institute of Hydrodynamics SB RAS, 15 Lavrentyev Prosp., Novosibirsk, 630090, Russia

E-mail : kedr@hydro.nsc.ru

Abstract. The evolution of bubble cavitation behind the front of a converging cylindrical rarefaction wave in distilled water containing micro inhomogeneities is investigated numerically within the framework of the mathematical model. The dynamics of the cavitation zone state is considered in a one-dimensional formulation with cylindrical symmetry. A thin liquid layer bounded by the free surface is loaded by a shock wave generated by a piston coaxial the axis of symmetry. As a result of shock wave reflection from free surface, a rarefaction wave converging toward the axis of symmetry is formed. The influence of the initial size of microbubbles, gas phase concentration, and parameters of shock wave loading on the dynamics of formation of the cavitation zone and its growth in the course of rarefaction wave focusing is analyzed.

1. Introduction

Intense investigations aimed at the development of two-phase mathematical models were started in the 1960s. The papers of Iordanskii [1], Kogarko [2], and van Wijngaarden [3] were published in 1960, 1961, and 1964, respectively. The main distinctive feature of these works was including the Rayleigh equation into the system of conservation laws. This equation is considered as a nonlinear dynamic system, which can become the determining and determined first-class fourth variable in the form of the bubble radius $R(t)$. In 1968, the three models developed in [1-3] were united in a simplified form into the Iordanskii-Kogarko-van Wijngaarden (IKvanW) system in [4]. This unification made it possible to estimate numerically shock wave (SW) separation into a precursor and an oscillating wave for the first time and to calculate an "acoustically transparent window" in the range of resonance frequencies, which was experimentally observed in [5] by means of demonstrating that good agreement with the experiment was explained by the existence of an initial spectrum of bubble concentration (despite the complete absence of dissipative losses).

Multiphase mathematical models were developed almost 20 years earlier than intense experimental investigations of the bubble size and density distributions were started, except for obtaining data on cavitation nuclei from unity to one hundred in the interval $3 - 6$ microns per cm$^3$, which were reported in [6, 7]. Some experimental results of principal importance were obtained in the Soviet Union and abroad: the distributions and sizes of cavitation nuclei were measured by methods of holography and light scattering [8-10], cavitation susceptibility meters [11], and the ABS Acoustic Bubble Spectrometer [12, 13]. The nuclei size distribution curves in cavitation water tunnels and in the ocean were presented in [14].
The IKvanW model made it possible to solve complex physical problems with allowance for viscosity, phase changes, gravity forces, and generation of cavitation nuclei. At the beginning of the 21st century, this model was used as a basis for formulating the problem of the development of explosive volcanic eruptions that occur due to almost instantaneous rupture of the diaphragm separating the magma compressed to a pressure of 150 MPa from the free atmosphere. A powerful decompression wave is formed and propagates inward the conduit. Under the action of the decompression wave, the single-phase state of the heavy magma transforms to a complex multiphase system with generation of cavitation nuclei, growing viscosity (by orders of magnitude), diffusion processes, and drastic increase in the volume concentration of the gas phase and particle velocity. Another problem of explosive eruptions is their cyclic behavior and the explosive character of the magma flow transformation into a gas-droplet system.

It will be noted that the same problems are characteristic of the open volcanic systems (like a Erebus volcano) in which the volcanic conduit ends in a bottom of lake located at the volcano crater. Laboratory experiments on the simulation of open volcano eruption were began about ten years ago. The experiments are devoted the study of the state dynamics of flow rupture – formation and collapses of quasi-empty cavity in the layer of distilled water under shock wave loading [15]. The surface of empty cavity during all time of its existence is covered by the micro bubble layer. According to experimental data the collapse of empty cavity results in a SW generation and transformation of micro bubble layer in a ring-shaped vortex [16] which begins to ascend (rather fast) towards free surface of liquid layer. The main feature of presented paper is detail numerical investigation (on the base IKvanW-model) one of the physical results - the evolution of bubble cavitation behind the front of a converging cylindrical rarefaction wave in a layer of distilled water.

2. Experimental setup
The flow rupture process is modeled in a transparent explosive chamber with a conducting bottom in two electromagnetic hydrodynamic shock tubes (EM HSTs) with pulsed loading of the distilled water layer by using two capacitor batteries with energies up to 100 J and 5 kJ. As was demonstrated in [9], distilled water is a multiphase system containing free bubbles approximately 1.5 μm in size. The density of these microbubbles together with solid microparticles is approximately 10^6 per cm^3. When the high-voltage circuit is closed, the capacitor battery is discharged onto a flat helical coil located directly under the membrane bottom. Owing to the skin effect, the impact of the membrane onto the liquid layer generates a shock wave in the liquid, and then the membrane returns to its initial place approximately in 10 μs. Here we note that the parameters of the high-voltage circuit were chosen to ensure that the discharge process was aperiodic. Due to inertia, the liquid continues its upward motion, forming flow rupture as quasi-empty cavity [15].

Figure 1 demonstrates the final part of the process of cavity-rupture implosion (frame Nos. 50 – 100) in a liquid layer 4 cm high on a membrane 12 cm in diameter with the stored energy of 0.8 kJ and filming frequency of 10^4 frames per second. The following phenomena are observed in the course of cavity implosion (theoretically, down to zero): SW formation and reflection from the free surface, cavitation zone formation (frame No. 83), and transformation of the micro–bubble layer (on rupture surface) into a bubble cluster. These phenomena are followed by a situation whose mechanism is yet to be understood: the cluster transforms into a
ring-shaped vortex (torus) and fairly rapidly ascends to the free surface (frame No. 100) [16]. The analysis of the state dynamics of cavity-rupture structure made it possible to discover the complex structure of the flow at the stage of cavity implosion: formation of a converging one-dimensional cylindrical flow along the membrane under the cavity-rupture. This allows one to assume that the implosion at the final stage acquires a cylindrical, close to one-dimensional character (Figure 2).

![Figure 2. Dynamics of the external structure of the cavity surface profile in the course of its closure](image)

This assumption is confirmed by the experiment illustrated above (see frame Nos. 75 and 83).

3. **IKvanW model**

The model is a combination of conservation laws for the mean pressure \( p \), density \( \rho \), and particle velocity \( v \), equations of state for the liquid (Tait form) and gas components, equation of state of the bubble mixture \( \rho(k) \), replace \( k \cdot R \) and Rayleigh equation of the bubble dynamics:

\[
\frac{dp}{dt} + p \text{div} \ v = 0 \quad \frac{d\rho}{dt} + \nabla p = 0
\]

\[
p = B \cdot [(\frac{\rho_m}{\rho_\text{liq}})^\gamma - 1] , \quad p_g \cdot v_g = \text{const} \quad \rho = (1 - k) \cdot \rho_\text{liq} , \quad k = k_0(R/R_0)^3
\]

\[
R \frac{d^2R}{dt^2} + \frac{3}{2} \left( \frac{dR}{dt} \right)^2 = \rho_\gamma \left[ p_g - p \right]
\]

Here index \( \text{liq} \) (l) means liquid medium, \( g \) – gas medium, \( 0 \) – initial state, \( \gamma \), \( n \) – index of adiabats for gas and liquid.

4. **Calculated results**

Finite difference scheme of second order and Runge-Kutta method (fourth order) are used in the processes of calculation. Boundary conditions: free surface (right side) - pressure is set equal constant (0.1 MPa), in the vicinity of the axis (left side) – mass velocity is set by the co-axis piston. The initial state of distilled water can

![Figure 3. Dynamics of the concentration distribution at 80 µs for the amplitudes of 40 (violet), 80 (red), 120 (green), 160 (yellow), and 200 m/s (blue) and at 120 µs - for of 40 (violet), 80 (yellow), 120 (green), 160 (red), and 200 m/s](image)
be described with the following parameters: cavitation nucleus size $R_0 = 1.5 \, \mu \text{m}$, volume concentration $k_0 = 10^{-5}$, and channel radius 100 mm (the channel is closed by the free surface). The dynamics of the cavitation zone evolution along the radius from the free surface to the piston surface is considered. The zone is calculated for different values of the $U_{\text{max}} = 40, 80, 120, 160,$ and 200 m/s and for value 10 $\mu$s of the exponent constant.

Figure 3 shows the concentration distributions $k(r)/k_0$ for two times: initial time (80 $\mu$s, 80 mm from the axis) and the time corresponding to the smallest distance from the focusing axis (120 $\mu$s, 20 mm from the axis). One can see the essential difference of the $k/k_0$ distributions for times 80 and 120 $\mu$s (lower curve for 40 m/s up to upper curve for 200 m/s). Figure 4 shows the radial distributions of the curves at the time instant $t = 120 \mu$s for each value of $U_{\text{max}}$ (the same as before - lower curve for 40 m/s up to upper curve for 200 m/s) from the free surface (100 mm coordinate) to the near by axis - 20 mm coordinate. The cavitation intensity is smoothly enhanced in the course of focusing, approximately in the interval from 100 up to 40 mm, drastically increases when approaching the focusing center, and reaches 3500 units of the normalized concentration for the most intense wave (200 m/s) in the system in the interval of 40 – 20 mm (see figure 4).

Figure 4. Radial distributions of concentration for 120 $\mu$s for the system with 40, 80, 120, 160, and 200 m/s

Figure 5. Radial distributions of concentration for $U_{\text{max}}=100$ m/s, $R_0 = 1.5 \, \mu \text{m}$, $k_0 = 10^{-5}$, and constants of exponent of 10, 20, 30, and 40 $\mu$s

Figure 5 shows the concentration distributions over the radius $r$ from the free surface toward the axis of symmetry. The calculations are performed for $U_{\text{max}} = 100$ m/s, core radius of 1.5 $\mu$m, and volume concentration $k_0 = 10^{-5}$ for a system of prescribed constants of exponent of the initiated wave on the piston equal to 10, 20, 30, and 40 $\mu$s, increasing the positive phase of loading. The conclusion is interesting because it is difficult to distinguish the results for the constants of exponent of 30 and 40 $\mu$s in the entire calculation.

Figure 6. Linear and point interpretation of one calculation performed for four values of the exponent constants: 10, 20, 30, and 40 $\mu$s.
interval, which suggests that it is sufficient to use the value of exponent constant not more than 30 µs in the calculations, in any case, for the channel with the free surface with a radius of 100 mm.

Figure 6 shows the calculated distributions of concentration over the radius with the same constants of exponent of the initiated wave on the piston (10, 20, 30, and 40 µs), but for the greater sizes of cavitation nuclei (7 µm and volume concentration of 10⁻³). It is of interest to note that the state parameters significantly affect the changes in the concentration distributions: the concentration smoothly increases toward the focus. The distance of 20 mm from the focus corresponds to the time instant of 120 µs when the decompression wave front reaches this point from the beginning of initiation of a positive SW pulse in the vicinity of the focus. Both fronts (of SW and DW) move with the same velocity of sound equal to 1.5 km/s. Therefore, the horizontal linear scale of 100, 80, 60, 40 and 20 mm corresponding the horizontal time scale of 80, 90, 100, 110, and 120 µs can be substituted if it will be interesting.

5. Conclusions

The dynamics of cavitation evolution behind the decompression wave front in the case of its focusing is calculated within the framework of the IKvanW mathematical model for different amplitudes of the maximum particle velocity $U_{\text{max}}$ ranging from 40 to 200 m/s. The distributions of the concentration $k(r)/k_0$ are presented for two time instant: at the initial time and at the time of the smallest distance from the focusing axis. The latter case (120 µs) shows that the cavitation process is intensified for each value of $U_{\text{max}}$ in the interval from 100 – 40 mm in the course of focusing, then it drastically increases when approaching the focusing center, and reaches 3500 units of the concentration for the most intense wave (200 m/s) in the system in the interval of 40 – 20 mm. The total value of the concentration is close to 6 %. The calculations with different values of the constants of exponent (10, 20, 30, and 40 µs) of the initiated wave reveal a significant effect of the initial state of the system. If the initial state of the system is preserved ($R_0$, $k_0$), it is next to impossible to distinguish the results obtained for the constants of exponent of 30 and 40 µs in the entire calculation domain, which is important for the purpose of limiting the costs of exponent from above. The total value of the concentration is slightly higher than 2 %. It is of interest to note a significant effect produced by the changes in the initial state parameters (nuclei radius 7 µm and concentration 10⁻³). For these parameters, the calculated concentration distributions for the same system of precursors become noticeably different: it smoothly increases as the focus is approached. The total value of concentration is higher than 15 %.

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