Re-entrant localization of single particle transport in disordered Andreev wires

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We study effects of disorder on the low energy single particle transport in a normal wire surrounded by a superconductor. We show that the heat conductance includes the Andreev diffusion decreasing with increase in the mean free path $\ell$ and the diffusive drift produced by a small particle-hole asymmetry, which increases with increasing $\ell$. The conductance thus has a minimum as a function of $\ell$ which leads to a peculiar re-entrant localization as a function of the mean free path.

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Transport phenomena in mesoscopic wires with dimensions much less than the dephasing length have been studied for several decades; their mechanisms are nowadays well understood (see [1, 2, 3] for review). Depending on the ratio between the wire length $d$ and the mean free path of electrons $\ell$ one can separate different transport regimes. The ballistic regime holds for small wire lengths $d \ll \ell$, here the conductance is given by the Sharvin expression, $G = (e^2/\pi \hbar) N$, where $N \sim (k_F a)^2$ is the number of transverse modes, $a$ is the wire radius, $\ell \ll d$, and $k_F$ is the Fermi wave vector. For a shorter $\ell \ll d$, transport is diffusion controlled with the ohmic behavior $G \sim (e^2/\pi \hbar) N \ell/d$, neglecting the weak-localization interference between scattered electronic waves. With a further decrease in the ratio $\ell/d$, the ohmic dependence breaks down due to the localization effects: the conductance decays exponentially [4] when $\ell/d < N^{-1}$.

This textbook picture holds if the transverse confinement of the electronic states inside the wire is caused by an insulating gap in the material surrounding the wire, which results in elastic scattering of electrons at the wire walls with large momentum transfer (normal reflections). In the present Letter we consider another realization of a normal-metal wire conductor where the electronic states are confined by a surrounding superconducting material. The superconducting gap $\Delta$ outside the normal wire suppresses to zero the density of states (DOS) of single-particle excitations for energies $\epsilon < \Delta$, thus localizing them in the transverse direction within the wire. These states are essentially determined by the particle-hole Andreev reflections with low momentum transfer at the superconducting/normal-metal (SN) boundaries. If the normal reflection processes at the SN boundaries can be ignored, we refer to such a normal-metal conducting region inside a superconducting environment as to an “Andreev wire”. An Andreev wire can be connected through bulk normal-metal leads to an external measuring circuit. Note that our definition of Andreev wire differs from that used, e.g., in Ref. [5] where it was applied for a normal conductor in an insulating environment, connected to superconducting leads. A simple way to create Andreev wires is to introduce vortex lines in a type-II superconductor or to drive a type-I superconductor in an intermediate filamentary state by applying a magnetic field. Andreev wires can be manufactured artificially in the form of normal channels in a superconducting matrix, using modern nano-fabrication techniques employed also for producing a wider class of hybrid SN structures such as Andreev interferometers [6] and billiards [7]. Experimentally, the main distinction between Andreev wires and usual conductors is that the measurements of thermal conductance are more appropriate to probe the single electron transport in Andreev wires because the single-particle part of the charge transport is short-circuited by the supercurrent.

As shown in Ref. [8, 9] in the ballistic limit $\ell \gg d$, Andreev processes suppress the single electron transport for all quasiparticle trajectories except for those which have momenta almost parallel to the wire thus avoiding Andreev reflection at the walls. The particles confined due to Andreev reflections do also participate in the transport but through a slow drift along the transverse modes (Andreev states) with the group velocity $v_g = \hbar^{-1} \partial \epsilon_{k_F} / \partial k_z \sim \epsilon / p_F$ much smaller than $v_F$. This Landauer-type drift contribution is the lower limit of conductance reached as the contribution of freely traversing trajectories decreases with increasing $d$. In total, the electronic heat conductance due to these two mechanisms is much smaller than what could be derived from the Wiedemann–Franz law using the Sharvin conductance of superconductors in the direction of magnetic field [10, 11].

In the present paper we investigate how the low energy transport with $\epsilon \ll \Delta$ in an Andreev wire of a radius $a \ll \xi$ is affected by a weak disorder introduced by impurity scattering. We consider clean wires $\ell \gg a$ and
neglect inelastic processes assuming $\ell_* \gg \ell$. We start our analysis from a qualitative physical picture elucidating the main results of our work. Let us consider a quasiparticle propagating within the wire along a trajectory that bounces from the normal/superconducting walls at both its ends. Neglecting the slow drift, the distributions of particles and holes are equal at the wall due to the Andreev reflection. Without disorder induced scattering, the distributions remain equal throughout the wire, thus the single-particle transport associated with these trajectories vanishes. With disorder, the distributions of particles and holes deviate from each other by an amount proportional to the probability of scattering, $a/\ell$, accumulated on their way in between the two walls. In the presence of a temperature difference at the ends of the wire, the driving force on the trajectory is proportional to $(a/d)(T_1 - T_2)$, thus the thermal conductance becomes $\kappa_A = (T/h)N_A$, where the effective number of modes is

$$N_A = A(k_F a)^2 (a^2/\ell d). \quad (1)$$

The counter-intuitive behavior of the single-particle conductance $\kappa_A$ which increases with decreasing $\ell$ was first predicted by Andreev [12] (see also [13]). The coefficient $A$ in (1) appears to be a slow function of $\ell$: $A \sim \ln(\ell/a)$ [12]. For such “Andreev diffusion”, disorder with $a \ll \ell \ll d$ stimulates the single-particle transport as compared to that in the ballistic limit: it opens new single-particle conducting modes blocked by Andreev reflections in the ballistic limit. This differs from the disorder effects in normal-metal/insulator/superconductor systems where disorder opens two-particle tunneling processes for electrical conductance, see [3] for review. The conductance $\kappa_A$ reaches its maximum when the mean free path decreases down to $\ell \sim a$; it further transforms into $\kappa_D = (T/h)N_D$ for a dirty wire $\ell \ll a$ where [14]

$$N_D \sim v_F a^2 D/d \sim (k_F a)^2 \ell/d. \quad (2)$$

Here $v_F$ is the normal-state DOS at the Fermi level, and $D = v_F \ell/3$ is the diffusion coefficient. For a large mean free path $\ell \sim d$ the conductance $\kappa_A$ transforms into the ballistic expression [3] $\kappa \propto 1/d^2$.

We find that disorder also modifies the single-particle transport due to the drift along Andreev states with a group velocity $v_g$, which results from a small non-quasicaltistic particle-hole asymmetry. The characteristic mean free path for the drift appears to be $\ell_{\text{eff}} = v_g \tau$ which is considerably shorter than the usual mean free path $\ell$. The thermal conductance $\kappa_L = (T/h)N_L$ associated with the disorder-modified drift is found to be proportional to the mean free path for $\ell_{\text{eff}} \ll d$

$$N_L \sim v_F a^2 D_{\text{eff}}/d \sim (k_F a)^2 (v_g/v_F)^2 (\ell/d) \quad (3)$$

where $D_{\text{eff}} = v_g \ell_{\text{eff}} = v_g^2 \tau$ is effective diffusion coefficient much smaller than $D$ that appears in Eq. (2). This drift saturates at the ballistic Landauer-type expression [3]

$$N_L \sim (k_F a)^2 (v_g/v_F) \quad (4)$$

for very long $\ell$ when $\ell_{\text{eff}} \gg d$.

The total heat conductance $\kappa = (T/h)(N_A + N_L)$ includes the Andreev diffusion decreasing as $\ell^{-1}$ and the diffusive drift that increases with increasing $\ell$. Equations (1) – (4) are illustrated in Fig. 1 as functions of $\ell$. If $d \gg a(v_F/v_g)$, the number of modes has a minimum

$$N_{\text{min}} \sim (k_F a)^2 (a/d) (v_g/v_F)$$

at $\ell_{\text{min}} \sim a(v_F/v_g)$. According to the criterion of Ref. [4], a decrease in the number of modes down to $N_{\text{min}} \sim 1$ may lead to localization of the transport. Thus, varying the mean free path around $\ell_{\text{min}}$ we can obtain a peculiar reentrant localization: for a long enough Andreev wire the quasiparticles may become localized not only in a dirty limit $\ell \ll a$ but also for a quite long $\ell \gg a$ in such a way that the conduction opens again through either the quasiparticle drift for longer $\ell$ or through the Andreev diffusion for shorter $\ell$. This occurs for a wire length $d > d_c$ where $d_c \sim a(k_F a)^2 (v_g/v_F)$.

Model. – To develop a more quantitative theoretical description we use a quasiclassical approach modified taking account of the trajectory drift along the Andreev wire. The excitation spectrum is

$$\epsilon_n = \frac{h v_F}{2k} \left( n + \frac{1}{2} \right) \sqrt{1 - \frac{p_z^2}{p_F^2}}. \quad (5)$$

Here $p_z = p_F \cos \theta$ and $2x_c \sim a$ is the length of the projection of the trajectory section between two walls onto the plane perpendicular to the wire axis (the $z$ axis), see Fig. 2. Due to a $p_z$ dependence of the energy, particles perform a slow drift along $z$ with a group velocity

$$v_g = \frac{\partial \epsilon_n}{\partial p_z} = -v_F \frac{\epsilon_n}{2E_F} \cos \theta \frac{\cos \theta}{\sin^2 \theta}. \quad (6)$$
The Boltzmann kinetic equation is region coincides with the DOS ǫ. Here \( \langle \cdots \rangle \) is the distance along the trajectory. The particle is Andreev reflected at the wall and returns back along the hole trajectory (2) with the coordinates \( x = s_+ \sin \theta_+ \), \( z = z_0 + s_+ \cos \theta_+ \), where \( s_+ \) is the distance along the trajectory. The particle is Andreev reflected at the wall and returns back along the hole trajectory (2) with the coordinates \( x = s_+ \sin \theta_+ \), \( z = z_0 + s_+ \cos \theta_+ \). Both \( s_\pm \) are measured from the wire axis. The intersection with the wall has the coordinates \( x = x_c \) and \( z = z_0 + x_c \cot \theta_+ = z'_0 + x_c \cot \theta_+ \). The drift velocity defined as \( v_g = v_F \sin \theta(z'_0 - z_0)/2x_c \) coincides with Eq. (3). This approach is valid for wide wires \( a \gg \xi \) where the drift \( z_c \) is much larger than the width \( k_F^{-1} \) of the wave packet. Since \( z_c \sim a(v_g/v_F) \), this requires \( (v_g/v_F)(k_F a) \gg 1 \) which is equivalent to the condition \( \epsilon \gg \epsilon_0 \), where \( \epsilon_0 \) is the lowest level energy in Eq. (5).

**Kinetic equations.**– For \( \epsilon \gg \epsilon_0 \) the DOS in the normal region coincides with the DOS \( v_F \) in the normal state. The Boltzmann kinetic equation is

\[
\frac{\partial n}{\partial s} = -\frac{1}{\tau} (n - \langle n \rangle) .
\]

Here \( \langle \cdots \rangle \) denotes averaging over the momentum directions. For particle and hole distributions

\[
f_+ (\epsilon, \mathbf{p}) = n_{\mathbf{p}+} , \quad f_- (\epsilon, \mathbf{p}) = 1 - n_{-\mathbf{p},-+} ,
\]

respectively, the Boltzmann equation takes the form

\[
\pm v_\pm \frac{\partial f_\pm}{\partial s} = -\frac{1}{\tau} (f_\pm - \langle f_\pm \rangle) .
\]

In both the upper-sign and lower-sign equations, the distance \( s \) is measured in the direction of \( +\mathbf{p} \).

Since all particles are Andreev reflected as holes, the single-particle current through the wire side walls vanishes. Using \( mv_\pm \sin \theta_\pm = p_\pm^z \) we put at the walls

\[
v_+ \sin \theta_+ f_+ = v_- \sin \theta_- f_- .
\]

The kinetic equation (10) on the trajectory (1) gives

\[
f_+ (s_+) = f_+ (0) e^{-s_+/\ell_+} + \ell_+^{-1} e^{-s_-/\ell_+} \int_0^{s_+} \langle f_+ (s'_+) \rangle e^{s'_+/\ell_+} ds'_+ \text{ (11)}
\]

The function \( f_+ (0) \) is taken at \( x = 0, z = z_0 \). To get the corresponding expression on trajectory (2) one substitutes \( f_+, s_+, \ell_+, \) and \( z_0 \) with \( f_-, s_-, \ell_-, \) and \( z'_0 \), respectively. Putting \( s_+ = x_c / \sin \theta_+ \) and \( s_- = x_c / \sin \theta_- \) for \( f_+ \) and \( f_- \), respectively, we insert the result into the boundary condition Eq. (10). The distributions at the trajectories (3) and (4) for \( s < 0 \) are found by replacing \( z_0 \leftrightarrow z'_0 \). The boundary condition for them is applied at \( s_+ = -x_c / \sin \theta_+ \) for \( f_+ \) and at \( s_- = -x_c / \sin \theta_- \) for \( f_- \).

Assuming that \( \langle f(x, z) \rangle \) depends only on \( z \),

\[
\langle f(s) \rangle = \langle f(0) \rangle + s \cos \theta \frac{\partial \langle f \rangle}{\partial z} ,
\]

the next step is to expand the boundary conditions in small \( v_g/v_F \) and \( s_c / \ell \) for angles \( \theta \gg a/\ell \). Neglecting small terms \( \partial f_\pm /\partial z \) we obtain at the wire axis \( x = 0 \)

\[
f_2 = \frac{v_g}{v_F \cos \theta} f_1 - \cos \theta \frac{\partial f_1}{2\ell} ,
\]

\[
f_2 = \ell \frac{v_g}{v_F} \frac{\partial f_1}{\partial z} = -(f_1 - \langle f_1 \rangle) .
\]

We denote \( s_c = x_c / \sin \theta \) and introduce

\[
f_1 = -(f_+ + f_-) , \quad f_2 = -(f_+ - f_-)
\]

in accordance with the definitions used in the theory of superconductivity [17]. The functions \( f_1 \) and \( f_2 \) are nearly constant along the trajectory within the wire. Analysis of Eq. (11) shows that for angles \( \theta \ll a/\ell \), the distinction between the usual and the Andreev diffusion disappears, and \( f_2 = -\ell \cos \theta \partial (f_1) /\partial z \) while the counterpart of the first term in Eq. (12) proportional to \( (v_g/v_F) f_1 \) decreases exponentially.

The first term in Eq. (12) describes the drift along the Andreev states with the velocity \( v_g \). The second term is the Andreev diffusion [12]. Equation (17) introduces an effective mean free path \( \ell_{\text{eff}} = (v_g/v_F) \ell = v_g \tau \) much shorter than the usual \( \ell \). In the ballistic limit of very long \( \ell \) such that \( \ell_{\text{eff}} \gg d \), the distribution \( f_1 \) is constant along the wire and

\[
f_2 = \frac{v_g}{v_F \cos \theta} f_1 .
\]
In the most practical limit when $\ell_{\text{eff}} \ll d$,

$$f_1 = \langle f_1 \rangle - v_g T \frac{\partial \langle f_1 \rangle}{\partial z}$$  \hspace{1cm} (16)

and

$$f_2 = -\left[\frac{v_g^2}{v_F^2} \frac{\cos \theta}{2\ell} + \frac{s^2}{2\ell} \frac{\partial \langle f_1 \rangle}{\partial z} + \frac{v_g}{v_F} \cos \theta \langle f_1 \rangle\right].$$  \hspace{1cm} (17)

The first term in brackets describes the disorder-modified drift. Its relative magnitude with respect to the Andreev diffusion (the second term) is of the order of $(v_g/v_F)^2(\ell/s)^2$, i.e., much larger than non-quasiclassical corrections of the order $(\epsilon/E_F)$ to the usual diffusion. We neglect those corrections in what follows.

The energy current has the form \[I_E = -\nu_F v_F \int d^2r \int \frac{d\Omega_p}{4\pi} \int_{-\infty}^{+\infty} \cos \theta \, f_2 \, d\epsilon. \]  \hspace{1cm} (18)

For very long mean free path $\ell_{\text{eff}} \gg d$ the distribution is determined by Eq. (16), and we recover the Landauer formula derived in Ref. \[4\].

$$I_E = -\nu_F \int d^2r \int \frac{d\Omega_p}{4\pi} \int_{-\infty}^{+\infty} \epsilon v_g f_1 \, d\epsilon.$$  \hspace{1cm} (19)

With $f_1 = \tanh(\epsilon/2T_1)$ for $v_g > 0$ and $f_1 = \tanh(\epsilon/2T_2)$ for $v_g < 0$ we arrive at Eq. (4) for the heat conductance.

Assuming that the group velocity is independent of $\theta$ we obtain a qualitative behavior described in introduction. Indeed, for $\ell_{\text{eff}} \ll d$, the distribution obeys Eq. (17) where the last term does not contribute to the current. The current becomes $I_E = \kappa (T_1 - T_2)$ with the total thermal conductance $\kappa = (T/h)(N_A + N_L)$ given by Eqs. (1) and (3). This simplified picture has to be modified, however, due to a rapid divergence of $v_g$ at small angles. This leads to a more complicated behavior of the drift contribution to the heat conduction as a function of temperature and of the mean free path characterized by different power laws in different regions of $\ell$ and $T$. The detailed analysis of the heat conduction will be published elsewhere. Here we discuss its behavior for a long wire with $d \gg a \sqrt{E_F/T}$. For relatively short mean free path $a \ll \ell \ll a \sqrt{E_F/T}$, the angular integral of the first term in the brackets in Eq. (17) is cut off by the exponential decay of the drift term for $\theta \ll a/\ell$. Therefore,

$$N_L \sim (k_F a)^2(T/E_F)^2(\ell^3/a^2d).$$

This behavior is replaced with $N_L \sim (k_F a)^2(T/E_F)(\ell/d)$ for $a \sqrt{E_F/T} \ll \ell \ll d$ and further transforms into

$$N_L \sim (k_F a)^2(T/E_F)L$$

where $L \sim \ln(\ell/d)$ for $d \ll \ell \ll d(E_F/T)$ and $L \sim \ln(E_F/T)$ for $\ell \gg d(E_F/T)$. Therefore, the full saturation at the $\ell$-independent Landauer-type drift condition $\text{Eq. (19)}$ occurs when $\ell_{\text{eff}} \sim (T/E_F)\ell$ becomes larger than $d$. The total conduction reaches its minimum at $\ell_{\text{min}} \sim a \sqrt{E_F/T}$. The re-entrant localization is possible for wire lengths longer than $d_c \sim a(k_F a)^2(\sqrt{T}/E_F)$.

To summarize, we develop a theory of single electron transport and re-entrant localization in clean Andreev wires. Our results could stimulate experimental research of these phenomena in the mixed or intermediate state, as well as in hybrid SN structures.

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