PRIMORDIAL BLACK HOLES AS DARK MATTER: THE POWER SPECTRUM AND EVAPORATION OF EARLY STRUCTURES

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ABSTRACT

We consider the possibility that massive primordial black holes are the dominant form of dark matter. Black hole formation adds a Poisson noise to the matter density fluctuations. We use Lyα forest observations to constrain this Poisson term, which constrains the black hole masses to be less than approximately a few times $10^4 M_\odot$. We also find that structures with less than ~$10^3$ black holes evaporate by now.

Subject headings: black hole physics — cosmology: observations — cosmology: theory — dark matter — large-scale structure of universe — quasars: absorption lines

1. INTRODUCTION

Black holes are an alternative to the most popular candidate for the dark matter: massive nonbaryonic elementary particles (see Khalil & Munoz 2002 for a recent review). If primordial black holes (PBHs) formed in the early universe, they could become the dominant form of dark matter.

There are a variety of mechanisms proposed for PBH formation: during inflationary reheating (Garcia-Bellido & Linde 1998), from a blue spectrum of primordial density fluctuations (Carr & Lidsey 1993), or during a phase transition in the early universe (e.g., Rubin, Khlopov, & Sakharov 2000). The typical mass of PBHs at the formation time can be as large as the mass contained in the Hubble volume, $M_\text{PBH}$, down to around $10^{-4} M_\odot$ (Hawke & Stewart 2002). Carr & Hawking (1974) show that there is no physical self-similar growth mode for the PBHs in a radiation gas, and so there will be no significant accretion after PBH formation. However, Bean & Magueijo (2002) speculate that the accretion of quintessence (i.e., a scalar field) into the PBHs could lead to the scaling of the PBH mass with $M_\text{H}$.

In this Letter, we consider PBHs with $10^{-5} M_\odot < M_\text{PBH} < 10^4 M_\odot$ as the constituents of cold dark matter (CDM). The dynamical constraints on such objects have been thoroughly reviewed and discussed by Carr & Sakellariadou (1999). They show that the strongest upper limit on the mass of such compact objects in the Galactic halo is ~$10^4 M_\odot$, which comes from two independent constraints: (1) the rate of globular cluster disruptions and (2) the mass of the Galactic nucleus, accumulated because of dynamical friction. However, the globular cluster disruption limit depends on the unknown initial number of globular clusters, while the Galactic nucleus limit ignores black hole ejection through gravitational slingshot which may prevent mass accumulation at the center (Xu & Ostriker 1994). Therefore, we will take a more conservative value of $10^4 M_\odot$ as our upper limit. On the lower end, the nondetection of long-lived objects in the Galactic halo is used to place an upper limit on the mass of such compact objects. In spite of being dwarfed by such a number, this limit is of great importance as our primary constraint on the lower end of the mass spectrum of PBHs.

For the following analysis we assume that PBHs form as the result of a phase transition at some temperature $T_\text{c}$ in the radiation-dominated era. Furthermore, we assume that there is no substantial accretion following their formation era and all of their masses are around ~$M_\text{PBH}$. This assumption can be generalized to an extended accretion scenario in which $T_\text{c}$ marks the end of the accretion period (Bean & Magueijo 2002). The black hole density follows

$$n_0 \sim \frac{M_\text{PBH}}{H^3},$$

where $n_0$ is the number density of PBHs, $\delta_p$ is the fluctuation in the number of PBHs due to Poisson noise, and the $T/T_c$ factor describes the dilution in PBH density due to cosmic expansion. Since the PBHs are not correlated on acausal distances, one expects that on scales larger than the Hubble radius,

$$P_\rho = \langle \delta^2 \rangle \sim n_0^2 \delta^2.$$

For linear perturbations, on large scales, equation (1) leads to

$$\delta_\text{PBH} = \delta_p + \frac{3}{4} \delta_r,$$

where the $\delta_\text{PBH}$, $\delta_p$, and $\delta_r$ are the relative overdensities of PBHs, Poisson fluctuations, and radiation, respectively. Since $\delta_p$ in equation (1) is observable and constant, one would conclude that the quantity

$$S = \delta_\text{PBH} - \frac{3}{4} \delta_r = \delta_p$$

is gauge-invariant and conserved. Indeed, this is the entropy per PBH, which should remain constant as long as the universe expands adiabatically (e.g., see Mukhanov, Feldman, & Brandenberger 1992). The associated perturbations generated in this way are isocurvature (or entropy) perturbations, as the curvature at large scales is not (immediately) affected by the formation of compact objects at small scale.
As we are assuming that PBHs are the present-day CDM, the CDM power spectrum is given by

$$P_{\text{CDM}}(k) = T_{\text{ad}}^2(k)P_{\text{CDM}} + T_{\text{iso}}^2(k)P_{\text{p}},$$

(5)

where $T_{\text{ad}}(k)$ and $T_{\text{iso}}(k)$ are the transfer functions for adiabatic and isocurvature perturbations, respectively.

In equation (5), $P_{\text{CDM}} = A k^n$ with $n \approx 1$ is the adiabatic power spectrum that is produced through inflation (or an alternative method of generating scale-invariant adiabatic perturbations), while $P_{\text{p}}$ is given in equation (2).

One can easily see that the isocurvature term on the right-hand side of equation (2) contributes a constant to the power spectrum, as both $P_{\text{p}}$ and

$$T_{\text{iso}}(k) = \frac{2}{3}(1 + z_{eq})\text{ for } k \gg a_{eq}H_{eq}$$

(6)

are independent of $k$ (e.g., Peacock 1998). Note that this is the simple linear growth due to gravitational clustering, which is the same for adiabatic fluctuation. Since the power spectrum of adiabatic fluctuations decays as $k^{-3}$ at small scales, one expects to see the signature of this Poisson noise at large $k$'s. Combining equations (2), (5), and (6) gives the power offset

$$\Delta P_{\text{CDM}} \approx \frac{9M_{\text{PBH}}(1 + z_{eq})^2}{4\rho_{\text{CDM}}}$$

$$= 4.63 \frac{M_{\text{PBH}}}{10^5 M_\odot}(\Omega_{\text{CDM}} h^3)(h^{-1} \text{Mpc})^3,$$

(7)

which is also a lower bound on the matter linear power spectrum.

Figure 1 shows the linear power spectrum for different masses of the PBHs. We see the Poisson plateau (eq. [7]) at large $k$'s, which drops with decreasing mass. The impact of this plateau on the Lyα forest power spectrum is discussed in the next section.

3. SIMULATIONS OF Lyα FOREST

The lines in Figure 2 show the predicted change in the power spectrum of the Lyα forest transmitted flux, $P_{\alpha}(k)$, as $M_{\text{PBH}}$ is varied. The points with error bars are $P_{\alpha}(k)$ measured by Croft et al. (2002) using their fiducial sample ($z = 2.72$). The predictions were made using the large set of numerical simulations and the interpolation code described in P. McDonald et al. (2003, in preparation). Our result is based on hydro-PM simulations (e.g., Gnedin & Hui 1998; McDonald, Miralda-Escude, & Cen 2002; McDonald 2003). The curves that we show are smooth because the power spectra computed from the simulations have been compressed into the parameters of an analytic-fitting formula. The background cosmological model used in Figure 2 is assumed to be flat with a cosmological constant, $\Omega_{\text{CDM}} = 0.26$, $\Omega_c = 0.04$, $h = 0.72$, and $n = 0.9$ (this value of $n$ is close to the best fit found by Croft et al. 2002 for our model). The Lyα forest model assumed in the simulation is controlled by three parameters: the mean transmitted flux fraction in the forest, $\bar{F}$, and the parameters $T_{\text{14}}$ and $\gamma - 1$ of a power-law temperature-density relation for the gas in the interGalactic medium, $T = T_{\text{14}}(\Delta/14)^{-1}$, where $\Delta$ is the density of the gas in units of the mean density (see McDonald 2003 for a demonstration of the effects of these parameters on the flux power spectrum). The allowed range of each of these parameters has been constrained by independent observations. We use the measurement $\bar{F} = 0.746 \pm 0.018$ from McDonald et al. (2000) and the measurements $T_{\text{14}} = 20,500 \pm 2000 \text{ K}$ and $\gamma - 1 = 0.4 \pm 0.2$ from McDonald et al. (2001). To obtain these values at $z = 2.72$, we interpolated

![Fig. 1.—Linear power spectrum for different masses of the PBHs. Here $\sigma_8^2$ is $\sigma_8$ for the model without the PBHs, and the amplitude of the primordial adiabatic modes is the same for all models.](image1)

![Fig. 2.—Influence of PBHs on the Lyα forest flux power spectrum, $P_{\alpha}(k)$. The black solid curve shows our prediction for $P_{\alpha}(k)$ in a standard $\Lambda$CDM model (i.e., no PBHs) in which the amplitude of the linear power spectrum, $\sigma_8^2$, was adjusted to match the data points from Croft et al. (2002). The other curves show the predicted $P_{\alpha}(k)$ when white-noise power due to PBHs with various masses is added. The Lyα forest model parameters and $\sigma_8^2$ were not adjusted to find a best fit for each mass, so the disagreement between the PBH models and the data points does not indicate that the models are ruled out.](image2)
between the redshift bins used by McDonald et al. (2000, 2001). We subtracted 50% of the potential continuum-fitting bias that they discuss from \( F \) and add the same number in quadrature to their error on \( F \). We add 2000 K in quadrature to the error bars on \( T_\alpha \) to help absorb any systematic errors. To produce Figure 2, we fixed these Ly\( \alpha \) forest parameters to their measured values and fixed the normalization of the initially adiabatic component of the linear power spectrum, \( \sigma_8 \), to the value that gives the best fit when \( M_{\text{PBH}} = 0 \). It is not surprising to see that the Ly\( \alpha \) forest power increases dramatically as the white-noise power from the PBHs becomes significant on the observed scales.

Figure 2 is not sufficient to place constraints on \( M_{\text{PBH}} \) because we have not varied any of the other parameters to see if the predicted power can be adjusted to match the observations. To obtain an upper limit on the PBH mass, we compute \( \chi^2(M_{\text{PBH}}) \), minimizing over the amplitude of the linear power and the three Ly\( \alpha \) forest parameters, subject to the observational constraints described above on \( F \), \( T_\alpha \), and \( \gamma \). We follow Croft et al. (2002) in using only \( P_j(k) \) points with \( k < 0.04 \text{ km} \text{s}^{-1} \). Defining an upper limit by \( \chi^2(M_{\text{PBH}}) - \chi^2(0) = 4 \), we find \( M_{\text{PBH}} < 17,000 M_\odot \). Figure 3 shows how this limit is obtained. The temperature-density relation parameters play no significant role, but both \( \hat{F} \) and \( \sigma_8^* \) are important.

Figure 3 may at first appear unconvincing to the reader unfamiliar with the Ly\( \alpha \) forest; however, the result is ultimately simple to understand. In Figure 1, we see that the white-noise power begins to dominate on the scales to which the Ly\( \alpha \) forest is sensitive when \( M_{\text{PBH}} \sim 10,000 M_\odot \) (note that 1 comoving Mpc \( h^{-1} = 108 \text{ km} \text{s}^{-1} \text{ at } z = 2.72 \) in our model). As \( M_{\text{PBH}} \) increases, there is simply too much power on the scale of the Ly\( \alpha \) forest to produce the observed level of fluctuations. Increasing \( \hat{F} \) can cancel some of the effect, but the size of the increase is limited because \( \hat{F} \) is directly observable.

A factor of \( \sim 2 \) relaxation in the upper bound seems unlikely but not inconceivable. For example, if we arbitrarily increase the error bar on \( \hat{F} \) to \( 0.03 \), the limit that we derive is \( M_{\text{PBH}} < 41,000 M_\odot \).

The limit is \( M_{\text{PBH}} < 37,000 M_\odot \) if we arbitrarily decrease the predicted \( P_j(k) \) by 10% for all models. The assumed value of \( n \) has no effect on the result (we obtain \( M_{\text{PBH}} < 18,000 M_\odot \) using \( n = 1 \)). Finally, we remind the reader that the Ly\( \alpha \) forest only constrains the power spectrum in units of \( \text{km}^2 \text{ s}^{-2} \text{ Mpc}^{-3} \) at \( z = 2.72 \). Equation (7) and our assumed cosmological model were used to compute \( M_{\text{PBH}} \).

4. Early Structures and Relaxation Effects

The collisional relaxation time for a gravitational system is of the order of the number of particles times the dynamical time of the system. Therefore, one expects the relaxation related effects, e.g., evaporation and core collapse, to happen faster for systems with smaller number of particles. As the structures form bottom-up in a hierarchical structure formation scenario (and even more so in the presence of PBHs as the spectrum is bluer), and the dynamical time for cosmological halos is of the order of the age of the universe, such effects may be important only for the first structures that form right after the recombination era, which have the smallest number of PBHs.

The possibility of small subhalos of compact objects in the Galactic halo was first studied in Carr & Lacey (1987). Various dynamical constraints on such subhalos, including the evaporation limit, are reviewed in Carr & Sakellariadou (1999). In what follows, we implement this evaporation limit with the Poisson fluctuations of § 2 as the seed for the formation of these subhalos.

Let us make a simple estimate of how evaporation of early structures sets a lower limit on the mass of virialized objects. The evaporation time of an isolated cluster can be estimated using

\[
\tau_{\text{evap}} \approx 300 \tau_{\text{rel}} \approx 300 \left[ \frac{0.14N}{\ln(0.14N)} \right] \sqrt{\frac{r_h^3}{GM}}
\]  

(8)

(see Binney & Tremaine 1987), where the subscripts “evap” and “rel” refer to the characteristic times, subsequently associated with the evaporation and relaxation of the structure, while \( N \) and \( r_h \) are the number of particles and the median radius, respectively. To relate \( r_h \) to the formation time, for simplicity, we consider a truncated singular isothermal sphere which within the spherical collapse model (e.g., Afshordi & Cen 2002) yields

\[
r_h \approx \frac{2GM}{15\sigma_8^*} \approx (2GM)^{1/3} \left( \frac{t_f}{27\pi} \right)^{2/3},
\]  

(9)

where \( t_f \) is the formation time of the object. Combining this with equation (8) gives

\[
\tau_{\text{evap}} \approx \left[ \frac{0.7N}{\ln(0.14N)} \right] t_f.
\]  

(10)

The next assumption is that the approximate formation time of the structure is when the variance of fluctuations at the mass.
scale of the structure \(\sigma(M)\) is around the critical overdensity in the spherical collapse model, \(\delta_c \approx 1.67\) (Gunn & Gott 1972).

On the basis of the calculations of § 2, \(\sigma(M)\), which is dominated by the Poisson noise at the small scales, is given by

\[
\sigma^2(M) \approx \frac{M_{\text{PBH}}}{M} \left[ \frac{3}{2} \left( \frac{1 + \epsilon_{\text{col}}}{1 + z_{\text{col}}} \right) \right]^2,
\]

(11)

neglecting a late-time cosmological constant that is not important at the formation time of the early structures. Now, combining equations (9)–(11) with

\[
\frac{t}{t_f} \approx \left( \frac{1}{1 + z_{\text{col}}} \right)^{3/2}
\]

(12)

for a flat universe gives the minimum mass for the structure not to evaporate,

\[
M_{\text{min}} \sim M_{\text{PBH}} \left[ \frac{3(1 + \epsilon_{\text{col}})}{2\delta_c(1 + z_{\text{col}})} \right]^{\frac{1}{2}} \left[ \frac{\Omega_m^{0.2} \ln (0.14N_{\text{min}})}{0.7} \right]^{\frac{2}{7}}
\]

\[
\approx 3 \times 10^3 M_{\text{PBH}} (1 + z)^{-1} \left( \frac{\Omega_m}{0.3} \right) \left( \frac{h}{0.7} \right)^{12/7},
\]

(13)

and consequently, the structures with \(M < M_{\text{min}}\) should evaporate by redshift \(z\).

5. CONCLUSIONS

We have studied the possibility of having CDM in the form of PBHs (or any other massive compact object) and its impact on the large-scale structure of the universe. We see that the simple Poisson noise, enhanced by gravitational clustering in the matter-dominated era, leads to a plateau in the power spectrum at large wavenumbers (see Fig. 1). Comparison of numerical simulations of the Ly\(\alpha\) forest with the current observational data rules out the PBH masses larger than a few times \(10^4 M_\odot\). This leaves a small window for PBHs between this limit and the gravitational lensing limit of \(30 M_\odot\). Improved Ly\(\alpha\) measurements and future microlensing surveys should be able to close this window and either detect the effects of PBHs or rule out PBHs with masses above \(0.3 M_\odot\) as a dominant component of the dark matter. The discrete nature of the PBHs can also lead to the evaporation of small (early) structures. A simple estimate shows that this puts a lower limit of about \(10^3 M_{\text{PBH}}\) on the mass of small structures.

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