Student’s scheme in solving mathematics problems

Nining Setyaningsih13, Dwi Juniati24, and Suwarsono2
1Mathematics Education, Universitas Muhammadiyah Surakarta, Indonesia
2Mathematics Education, Universitas Negeri Surabaya, Indonesia
3Email: nS259@ums.ac.id, 4dwi_juniati@yahoo.com

Abstract. The purpose of this study was to investigate students’ scheme in solving mathematics problems. Scheme are data structures for representing the concepts stored in memory. In this study, we used it in solving mathematics problems, especially ratio and proportion topics. Scheme is related to problem solving that assumes that a system is developed in the human mind by acquiring a structure in which problem solving procedures are integrated with some concepts. The data were collected by interview and students’ written works. The results of this study revealed are students’ scheme in solving the problem of ratio and proportion as follows: (1) the content scheme, where students can describe the selected components of the problem according to their prior knowledge, (2) the formal scheme, where students can explain in construct a mental model based on components that have been selected from the problem and can use existing schemes to build planning steps, create something that will be used to solve problems and (3) the language scheme, where students can identify terms, or symbols of the components of the problem. Therefore, by using the different strategies to solve the problems, the students’ scheme in solving the ratio and proportion problems will also differ.

1. Introduction
Mathematics is a science that has a relatively strict structure. So, when we will study mathematical concepts, we must go through certain sequences. It means, in the early part of the study of mathematical concepts, it generally begins with a definition that is the basis for understanding the topics of the next part which generally consists of axioms or theorems. The basic objects of mathematics in the form facts, concepts, operations or relations, and principles are mental objects or mind objects [1]. Because of facts, concepts, operations and principles are mental objects, so information processing will occur and they will be stored in memory [2].

Scheme is a collection of prior knowledge that provides a context for interpreting new information [3][4]. Howard defines the scheme as an abstract knowledge structure that organizes some information into meaningful systems [5]. From the explanation of the scheme, it can be derived that the scheme is a concept structure that is stored in memory and helps to interpret the situation or new information.

The scheme can be done through the learning process [6,7]. Students can form a new scheme of a new experience and students can add new components in the old scheme. In the learning process, students face changes to the scheme either by adding components, refining, developing or altering the old scheme [8], [9]. The scheme that students have can be developed through the giving of problems. To sharpen students’ skills in mathematics, problems need to be addressed, where the problem is not a routine problems [10]. However, students cannot solve the mathematics problem due to lack of
students in understanding the problem and low ability in computing [11][12]. Meanwhile, Silver is more interested in discussing the contents and arrangements of student knowledge (problem solver) [13]. He stated that by knowing the background information (knowledge) students can help to explain why the instruction is not effective. Based on the explanation, this study aims to investigate student’s scheme in solving mathematics problems.

2. Content knowledge scheme
Rumelhart and Norman said that Schemes are data structures for representing the generic concepts stored in memory [14,15]. There are schemes for generalized concepts underlying objects, situation, events, sequences of events, action and sequences of actions [14,15]. Schemes in some sense represent the stereotypes of these concepts. Howard explained that a scheme is a mental representation of some aspect of the world [5,16]. It has slots that are related to each other in prescribed ways and that are filled by stimuli to create an instantiation of the scheme.

Carrel, Adams and Collins define the scheme is as an abstract knowledge structure [17,18]. Furthermore, they claim that the acquired knowledge is called the knowledge background of the solver. They explain the types of schemes, namely (1) content scheme, (2) formal scheme, and (3) linguistic scheme[17,18]. Content scheme relates to the prior knowledge background of the text content. Content schemes contain conceptual knowledge or information about what is happening, and how relationships occur to each other to form a logical relationship [17,18]. Formal scheme is related to a formal and structured knowledge background [17,18]. And also, formal scheme is related to knowledge that discusses how stories or problems are built that are expected to improve understanding. Thus, the formal scheme is a process of constructing and testing of a text or problem based on the knowledge possessed. Language scheme deals with knowledge of terms, grammar, vocabulary etc. The language scheme plays an important role in understanding a text or information [17,18].

3. Content knowledge in solving mathematics problems
The part of mathematics topics which have been learned by students are ratio and proportion. The concepts of ratio and proportion is very important because ratio and proportion are commonly used in daily life [11,19,20]. However, in general, students’ achievement on this topic is not good enough [21]. The possible cause is that because students find some difficulties in learning this topic [21].

The main parts of the problem to find are: what to look for?, how is the data known? and how is it? While, the problem proves to be a problem to indicate whether a statement is true or false, or not both [21]. This can be done by answering the question of whether the statement is true or false. The main part of this problem is the hypothesis and conclusion of a theorem that must be verified. Thus, a mathematical question can be a problem for a person, depending on how to solve the problem. Therefore it is necessary to think complex from the person to think about how to solve the problem right.

Problem solving is defined as the process by which individuals combine prior knowledge to deal with new situations [22]. This means problem-solving is a process that a person does in combining prior knowledge to accomplish a task that is not yet known of the settlement procedure. Mathematical problem solving is a complex mental process that requires visualization, imagination, manipulation, analysis, abstraction and unification of ideas [6,7]. So, mathematical problem solving cannot be separated from one's knowledge of the substance of the problem.

Scheme is related to problem solving in that it assumes that a system is developed in the memory by acquiring a structure in which problem solving procedures are integrated with some concept. The scheme is responsible for providing a structure for understanding a problem solving situation as well as for guiding problem solving procedures. To solve a problem, student is first assumed to encode information in the cognitive domain. A system of production of information is then applied to transform its encoded form to intermediate form of the problem solving procedures.
4. Methods
This is a qualitative study which aims to investigate students’ scheme in solving mathematics problems. The data were collected through interview and test about the ratio and proportion. The data collected by those instruments helped researcher in interpreting and enable us to make data triangulation.

This study were conducted at Junior High School Muhammadiyah Program Khusus in Surakarta, Indonesia. The subject of this study were students from grade seven. The result of student work to the two questions were analysed to identify the knowledge applied in solving the problems. This study uses data analysis with the following stages: 1) reduction, 2) presentation, and 3) the conclusions or verification [23]. Data reduction of test results and interviews conducted with students. Then the data is presented in the form of a narrative text.

5. Results and discussion
To know the scheme in solving mathematics problem, subjects were given two tasks about ratio and proportion. Response of subject for each task and interview result presented as follows.

5.1. Task: Ratio
Mr. Rahmad plans to paint the walls of the house. But the color of paint sold in the store is less in accordance with the wishes of Mr. Rahmad. To get the desired color, Mr. Rahmad will mix the blue paint and white color. The A mixture consists of 3 blue paint cans and 2 white paint cans, while the B mixture consists of 4 blue paint cans and 3 white paint cans. Each paint can is the same size. Of the two mixtures, Mr. Rahmad wants an older blue color. Among mixes of A and mix B, which mixture yields the older blue color according to the wall color that Mr. Rahmad wants? Give the reason!

The results of written answers and interviews of subject S1 are presented in Figure 1.
Q: Have you had any problems like this before?
S1: not yet
Q: Now, solve this problem
S1: (nod, pause for a moment and then start doing)
Q: Can you explain what is known and what is asked from this problem?
S1: known: A mixture of 3 blue and 2 white and mix B consists of 4 blue and 3 white.
    Asked: Which mixture gives the older blue color?
Q: What do you mean with $\frac{3}{5}$?
S1: Blue color compared with mixed colors = $\frac{3}{5}$
Q: Why not $\frac{2}{5}$?
S1: because of the blue color question.
Q: What do you mean with $\frac{4}{7}$?
S1: Blue color compared with mixed color = $\frac{4}{7}$
Q: Why $\frac{3}{5}$ to be $\frac{21}{35}$?
S1: the denominator are equated for $\frac{3}{5}$ and $\frac{4}{7}$, that is $5 \times 7 = 35$, so $\frac{3}{5} = \frac{3 \times 7}{5 \times 7} = \frac{21}{35}$
Q: why do you choose the mixture A older than the mixture B
S1: because $\frac{3}{5}$ is larger than $\frac{4}{7}$

Meanwhile, the results of written answers and interviews of subject S2 are presented in Figure 2.

Q: Have you had any problems like this before?
S2: not yet
Q: Now, try to solve it
S2: (nod, pause for a moment and then start doing)
Q: Can you explain what is meant in the question?
S2: Mr. Rahmad wants to do the house wall, but the color is not available in the store. He made the mixture A consists of 3 blue
Q: What do you mean with $\frac{3}{2}$?
S2: Blue color compared with white color = $\frac{3}{2}$
Q: Why not $\frac{2}{3}$?
S2: because the question for blue color.
Q: What do you mean with $\frac{4}{3}$?
S2: Blue color compared with white color = $\frac{4}{3}$
Q: why do you choose the mixture A has a blue color that is older than mixture B
paint cans and 2 white paint cans and the mixture B consists of 4 blue paint cans and 3 white paint cans. Which mixture gives the older blue color.

Q: From the problem, can you explain what is known and what is asked of that problem?
S2: known: the mixture A consists of 3 blue and 2 white and the mixture B consists of 4 blue and 3 white.

S2: because \( \frac{3}{2} \) is larger than \( \frac{4}{3} \)

Q: why \( \frac{3}{2} \) is larger than \( \frac{4}{3} \)
S2: the denominator are equated for \( \frac{3}{2} \) and \( \frac{4}{3} \), that is \( 2 \times 3 = 6 \), so \( \frac{3}{2} = \frac{3x3}{2x3} = \frac{9}{6} \) and \( \frac{3}{2} = \frac{4x2}{3x2} = \frac{8}{6} \), so \( \frac{3}{2} > \frac{4}{3} \) or \( \frac{8}{6} > \frac{9}{6} \)

From the results of the analysis of S1 and S2’s written answers and interviews, it show that the content scheme owned by subject S1 and S2 are shown by their ability in identifying the problem, they are able to answer what components are known and what is asked. As for the formal scheme demonstrated subject S1 and S2’s ability to plan knowledge and execute knowledge. From the interview also obtained data that subject S1 is able to explain the components on the matter of the ratio problem or in other words they are able to identify the knowledge. In addition, they are also able to form a mental model of the components that have been determined or in other words they are able to elaborate his prior knowledge about ratio to solve the problem. Furthermore, they are able to make plans to solve problems and to execute the plans to answer the problem.

The scheme built by S1 and S2 is almost the same. The difference lies in the reasoning of the ratio formulation. S1 look for ratio for blue color with mixed color, while S2 looks for ratio for blue color and white paint. The scheme of subject S1 in solving ratio problem is shown in Figure 3.

![Figure 3. Subject S1’s scheme in solving ratio problem](image-url)
5.2. Task 2: Proportion

Hidayat likes cats. He has 3 cats. For 4 days, Hidayat gave food to his three cats with 4 kg of cat food. One day, Pak Anwar, his uncle Hidayat, bought 9 cats for Hidayat. Using the same size, how many kg of cat food does Hidayat need to feed all his cats for a week?

The results of written answers and interviews of subject S1 are presented in Figure 4.

From the results of the analysis of written answers and interview results, showed that the content scheme owned subject S1 shown with his ability in identifying the problem, that he is able to answer what components are known and what is asked. As for the formal scheme demonstrated S1’s ability to plan knowledge and execute knowledge. On the other hand, from the interview also obtained data that S1 able to explain the components on the problem of proportion or in other words he was able to identify knowledge. In addition, he is also able to form a mental model of the components that have
been determined or in other words he is able to elaborate knowledge. Furthermore, he is able to make plans to solve problems and simultaneously execute knowledge.

In explaining the solution of problem-solving proportions, S1 completed in three stages. The first stage, he looks for how many kg of food for 3 cats for one day by using the unit rate strategy \[24\]. The second stage he looks for how many kg of food for 12 cats for one day and the third stage he looks for how many kg of food for 12 cats for 1 week by using the cross product strategy \[24\]. Thus, he already has a content scheme, which demonstrates his ability in identifying the components in the matter. In addition, he also has a formal scheme, which demonstrated his ability to elaborate components in problems, plan and execute knowledge. He also has a language scheme, which demonstrates his ability to write the mathematical symbols used in solving problems.

Figure 5 and Figure 6 below show the schemes of S1 and S2 in solving the proportion problem.

6. Conclusion

The scheme of the research subjects (S1 and S2) in solving mathematical problems, especially the material ratios and proportions can be explained as follows: (1) the content scheme, in which the subject describes the information or components that have been selected in accordance with the initial knowledge possessed, (2) the formal scheme, in which the subject is able to explain in constructing a mental model based on the components that have been selected from the problem and using the scheme to build the planning steps, creating something that will be used to solve the problems encountered and (3) the language scheme, in which the subject is able to identify terms, symbols of the components of the ratio problem and proportion. Also informed that the research subjects using different strategies in solving the problem of ratios and proportions, so the scheme was built also differently.

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