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Critical Slowing Down as an Early Warning Signal for Financial Crises?

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Abstract. The global impact of the recent financial crisis has once more stressed the urgency of new approaches to designing early warning signals (EWS) for financial crises. In the recent literature on constructing EWS through identifying characteristics of critical slowdown on the basis of time series observations, finance has repeatedly been coined as an important potential application area. On the one hand, this appealing idea is supported by the fact that there is ample empirical and experimental evidence to suggest that nonlinearities play a role in the expectations feedback governing market dynamics. On the other hand, financial markets differ from many natural complex systems, for which evidence of critical slowing down has been reported, in that market dynamics are not necessarily captured well by an ordinary differential equation, the fixed point of which may lose stability through a saddle-node bifurcation, as is the case for the cusp catastrophe. Also, financial time series exhibit persistent near unit root behaviour. In this paper we consider a number of historical financial crises, to investigate whether there is indeed evidence for critical slowing down prior to market collapses. The four events considered are Black Monday 1987, the 1997 Asian Crisis, the 2000 Dot.com bubble burst, and the 2008 Financial Crisis. Our analysis shows evidence for critical slowing down before Black Monday 1987, while the results are mixed and insignificant for the other financial crises.

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1 Introduction

The recent financial crisis has intensified theoretical and empirical research on the underlying instabilities of economic systems. Several papers have been concerned with the development of early warning signals that could give policymakers and market participants warnings on an upcoming financial crisis. In macroeconomics, most contributions are based on “dynamic stochastic general equilibrium” (DSGE) models. These models seek to understand the economy under the assumption that the set of prices will result from an overall equilibrium under perfectly rational behaviour. Unfortunately, when the crisis came, the serious limitations of such macro-financial models became apparent. They were unable to capture the observed large movements in financial markets and the macro economy and failed to provide predictions or even the possibility of several extreme events in financial history. A good understanding of the dynamic behaviour of macro-financial systems is still lacking. The global financial crisis, which began in 2008, may be seen as a stress test for DSGE models or even a challenge to their validity. It may be that something is missing from these conventional economic models that prevents them from describing the behaviour of financial markets. In November 2010, the then president of the European central bank, J. Trichet, addressed the ECB Central Banking Conference stating that “Macro models failed to predict the crisis and seemed incapable of explaining what was happening to the economy in a convincing manner”, and moreover “In the face of the crisis, we felt abandoned by conventional tools.” (7). The lessons of the financial crisis for macroeconomic and financial analysis are profound. They lead us to ask questions such as: What are the determinants of crises? Can crises be predicted? Can crises be avoided with sufficient early warnings? We need to develop complementary tools to improve the robustness of our overall framework.

During the last decades, a growing number of researchers have come to recognise that economic systems should be considered as complex systems (7, 8; 9, 10; 11, 12; 13, 14, 8). Unlike traditional economic theories under the assumption of general equilibrium, they describe the economic systems as dis-equilibrium processes. In such systems market crashes are mainly endogenously driven events resulting from complex interactions and nonlinear feedbacks. The complexity approach offers new ways for understanding the underlying mechanisms causing market crashes and, in principle, the potential for predicting such crashes. Moreover, inspiration can be obtained from other disciplines.
where complexity theory has already successfully been applied, for example in ecology, physics, engineering, psychology, biology and so on. Within those fields, sophisticated tools for analysing complex dynamic systems have been developed. These tools have proven to be helpful in understanding and even predicting important complex phenomena in global weather forecasting, ecology, epidemiology, crowd psychology and so on.

Several researchers have recently applied complexity tools, and already demonstrated their value in economics and finance (?, ?, ?, ?). Applying complexity approaches to financial systems can help deepen our understanding on the dynamic behaviour of financial markets and (parts of) the economy. In this paper, we will apply concepts from complexity theory to real financial data related to historical financial crises. We will also investigate, for the first time, whether the crises considered can be linked to critical transitions of the corresponding financial system. In particular, we will investigate whether in principle it is possible to develop an early warning signal based on a number of indicators for different types of historical financial crisis.

The exploration of indicators of critical transitions in complex systems has been quite fruitful in other disciplines. Important signals that have been suggested in the complexity literature as an early warning indicator are measures of “Critical slowing down”. This approach is based on the slowing down of the dynamics of a complex system approaching a critical point. Several authors developed methods to extract signals of critical slowing down from time series data. ? observed that power spectral properties changed as the earth system moved closer to a bifurcation point in a hemispheric thermohaline circulation (THC) model. ? used a trend in the decay rate of climate sub-systems as an indicator of critical slowing down. The first-order autoregressive coefficient obtained from time series was used as a measure of the decay rate of the system. This method was applied to the North Atlantic THC model, providing an early warning signal for the climate system. ? developed another way of detecting critical slowing down by using detrended fluctuation analysis (DFA). This analysis was originally developed by ? to detect DNA sequences’ long-range correlations. ? found an early warning signal for an upcoming critical transition in the North Atlantic THC system by investigating model output, as well as Greenland ice core paleotemperature data. The subsequent work by ? was the first to study critical slowing down in empirical time series. The results showed increased autocorrelation in eight climate time
series prior to critical transitions. They contributed to the early warning signal literature by offering some general guidelines for choosing the parameter settings of the analysis. They improved the robustness of ACF and DFA techniques and gave additional examples showing evidence of critical slowing down in both palaeodata and climate model output. Recently, expanded the theory of critical slowing down to a broader class of situations where a system becomes increasingly sensitive to perturbations even without catastrophic transitions. They showed that critical slowing down could even be used in a more general sense as an early warning signal.

The possibility of applying early warning tools to financial data was suggested by (, ). Encouraged by the successes of early warning signals in many complex systems, some efforts have been made to explore the possibility of constructing early warnings from financial time series. For instance, by using information dissipation length (IDL) as an indicator, detected early warning signals prior to Lehman Brothers collapse in both USD and EUR interest rate swaps (IRS). They suggested that the IDL may be used as an early warning signal for critical transitions. observed critical slowing down in the U.S. housing market. They detected strong early warning signals associated with a sequence of coupled regime shifts during the period of subprime mortgage loans transition and the sub-prime crisis. They also found weaker signals during the Asian financial crisis and technology bubble crisis. However, up until now, no evidence of early warning signals has been found in time series data of stock markets. To fill this gap, this paper is, to our best knowledge, the first attempt to apply complexity theory based on “critical slowing down” to real financial data. Four financial crises are analysed: Black Monday 1987, the 1997 Asian Crisis, the Dot-com Bubble in 2000 and the 2008 Financial Crisis. The sample lag-1 autocorrelation and the variance are considered as early warning indicators to examine whether financial systems slow down before the critical point is reached.

This paper is organized as follows. In Section 2 we provide some theoretical background of nonlinear dynamical systems and bifurcations underlying critical transitions. We describe the data and EWS methodology in Section 3 and 4, respectively. Subsequently, we analyse how well such early warning indicators perform when applied to financial time series. The results of our analyses are presented and discussed in Section 5. Section 7 provides a summary and conclusions.
2 Theory

The mechanism driving critical transitions in complex systems mathematically is a *bifurcation*, which is an abrupt qualitative change in the dynamical systems when one or more control parameters change. reviewed different types of bifurcations in dissipative dynamical systems. Three categories are classified based on their continuous or discontinuous dependence on the control parameters. These categories consist of safe bifurcations, with continuous growth of a new stable attractor, explosive bifurcations, with a discontinuous growth to a newly enlarged attractor along with itself, and dangerous bifurcations, in which the current attractor simply disappears, forcing the system to jump in a fast dynamic transient to a remote and entirely new attractor. Figure 1 illustrates this classification visually. The critical transitions in complex systems are often considered to be dangerous bifurcations (, ; ,).

Figure 1: Classification of bifurcations according to their outcome (Source: ).

Figure 2 provides an example of a saddle-node bifurcation. With the increase of
a single control parameter, a critical point is approached. Even a small perturbation would lead to a large qualitative change of the dynamics when the system is very close to this critical point. Once this threshold is exceeded, the whole system transits toward a different attractor. Even if the control parameter is reversed, the response will typically remain close to the new attractor. The dynamics will not automatically return to the original attractor. This highlights the irreversibility of critical transitions. ? (?, ?) proposed that this bifurcation can also be used to describe the dynamical behaviour of financial systems, systematic crashes in stock markets for instance. This suggests the possible use of applying complexity theory to financial systems\footnote{See e.g. Brock et al. (2009, p.1922, Figure 1) for a critical transition in a financial market model with boundedly rational agents and heterogeneous beliefs due to saddle-node bifurcations when the number of financial hedging instruments increases beyond a critical level. In a boundedly rational world, financial innovation may thus destabilize markets and trigger critical transitions.}.

To develop tools to forecast critical transitions based on time series, it is necessary to assume that the observed time series is generated by a rather general nonlinear dynamical system, driven by white noise, with control parameter $\rho$:

\begin{equation}
X_{t} = F(X_{t-1}, \rho) + G(X_{t-1}, \rho) \epsilon_{t},
\end{equation}

where $X_{t}$ is the state of the system, $F(X_{t}, \rho)$ specifies the deterministic part of the system, while $G(X_{t}, \rho) \epsilon_{t}$ is the stochastic part with $\epsilon_{t}$ a white noise process. The equilibrium of the undisturbed system is stable in all directions while approaching the critical value. By varying the control parameter $\rho$, the system reaches a threshold. When a real eigenvalue of the Jacobian matrix $DF_{\rho}(\bar{X})$ of the steady state finally crosses +1 (with the other real eigenvalue smaller than 1 in absolute value), a saddle-node bifurcation occurs. This bifurcation is corresponding with a critical transition in a time series. This scenario is described in detail in ?, ?, and ?. It offers ways to provide early warnings before the critical transition actually happens. As long as we understand the statistical properties of the system approaching a critical transition, we may predict the time of the transition in advance, up to some estimation uncertainty.

Several earlier attempts have been made to develop an early warning system to monitor the risk of financial systems, such as binomial/multinomial logit/probit models (?; ?, ?; ?), multivariate probability models (?), Markov switching models (?; ?), binomial tree approaches (?), and so on. However, their failure to give warnings ahead of the financial crisis in 2008 makes their predictive ability questionable. At the same time,
the exploration of indicators of critical transitions in complex systems has been quite successful in other disciplines. Important signals suggested in the literature as early warning indicators are related to critical slowing down; when a dynamical system approaches a critical point, we can expect it to become increasingly slow in recovery from small perturbations. This is characterised by the linear decay rate decreasing to zero. The theory of critical slowing down used to describe this phenomenon, is illustrated in Figure 2. Panels a, b and c of Fig. 2 show the behaviour of a dynamical system approaching a saddle-node bifurcation. The local minima of the potential well represent stable attractors and the ball shows the present state of the system. While approaching the bifurcation point, the local minimum on the right becomes shallower, and the recovery of the ball in response to small perturbations is increasingly slowing down. When this local minimum finally disappears, the ball quickly rolls into the minimum on the left. This implies that the system transits into a different steady state. The mechanisms behind this behaviour can be explained in mathematical terms. Approaching a saddle node bifurcation, the maximum real part of the eigenvalue of the Jacobian matrix tends towards 1. It indicates a slower recovery from perturbations (?, ?, and ?, ?). Therefore, as long as we are able to detect a signal of slowing down, it would be possible in principle to predict future critical transitions with some accuracy.
Figure 2: A saddle-node fold bifurcation (Source: ?, 2011). Panels a, b and c describe the critical slowing down as an early warning indicator that the system lost resilience on the way to the critical point. Local minima represent stable attractors while the ball shows the present state of the system. (a.) Far from bifurcation: small variance and fast fluctuations. (b.) Approaching the bifurcation: larger but slower fluctuation with increasing variance; (c.) At the bifurcation point: irreversible transition to a new local minimum.
In what follows, by exploring critical slowing down prior to some extreme events in financial history, we would be able to assess whether such early warning signals might be possible for financial time series.

3 Data

For the analyses time series from subsystems of the economy which clearly indicate critical transitions from one state to another are needed. Since stock market prices rather than returns often show sharp transitions, the analysis focuses on (log) prices rather than the more commonly analysed (log) returns. Moreover, stock prices often are near-unit root processes, similar to a saddle-node bifurcation with eigenvalue $+1$, while returns typically have close to 0 autocorrelations. Therefore, time series of daily stock market (log) prices are taken as the dynamical subsystem to be analysed in our work. As long as we find early warning signals in the stock market price system, we are able to detect critical transitions.

Four financial crises are analysed: Black Monday (October 19, 1987), the Asian Crisis, the Dot-com Bubble and the 2008 Financial Crisis. Although the direct causes of these crisis are different, they share the common characteristic that the stock prices for these events displayed similar bubble and burst patterns. Cusp catastrophe theory may possibly be used to describe these critical transitions in financial systems.

A particular time series is studied for each crisis based on its characteristic. For instance, the most popular time series data of stock prices in the literature is the Standard & Poor 500 (S&P 500) index. Therefore, we employ it to detect the early warning signals prior to Black Monday 1987 and the 2008 financial crisis. The Hongkong Hangseng index is the best candidate to describe the Asian events, so we use Hangseng index to analyze the Asian Crisis. The Dot-com Bubble is an information technology crisis which is boosted by the rapid growth of equity values in the internet sector. Hence, the subsystem we choose is NASDAQ Composite, related with technology companies and growth companies. The most recent 2008 financial crisis was followed by a credit crisis. In addition with the S&P 500, we analysed the TED spread, since it is an indicator of perceived credit risk. Moreover, as the volatility index of the S&P500, the VIX index is also analysed. Detailed information on data is presented in Table 1.

Daily time series data of stock prices, such as the S&P 500 index, the NASDAQ com-
posite and the Hangseng index, were downloaded from Thomson Reuters Datastream for the period from May 1986 until May 2011. The TED spread is derived by calculating the differences between the three-month LIBOR and the three-month T-bill interest rate. These datasets are also available from Datastream. The Volatility index VIX is downloaded from the online Chicago Board Options Exchange (CBOE) database\(^2\) (http://www.cboe.com/micro/vix/historical.aspx).

Various sample sizes before the crises events are chosen at will, ranging from 80 to 500. The reason for this is that we intend to consider a fixed ‘snapshot’ of financial time series from various markets and with various sample sizes, to stay as close as possible to the methodology by ?, who also use a snapshot of (paleo-climatological) time series with various sample sizes. The idea is to perform a first empirical investigation of whether the technology can work in principle by considering a given heterogeneous set of, in our case, financial time series. Optimising the sample size or the other test parameters is not the main aim in this first study.

Since we are interested in extracting early warning signals prior to a critical transition, the time series before each transition are carefully selected. The critical transition points are determined by visual inspection according to original records in international news or articles (such as Wikipedia). For the examples that do not have information on timing, we simply choose the points with maximum value in the corresponding period. Since the random growth in stock prices data is in percentage term and not in absolute term, we take logarithms of data. Doing so also linearises the exponential growth in original series and stabilises the variance of the analysed residuals.

\(^2\)The largest U.S. options exchange and creator of listed options, which offers equity, index and ETF options, including proprietary products, such as S&amp;P 500 options (SPX) and options on the CBOE Volatility Index (VIX)
Table 1: Summary of financial time series

| Crisis                  | Collapse Date | Time Series      | Sample Size (N) |
|-------------------------|---------------|------------------|-----------------|
| Black Monday            | 19 Oct.1987   | S&P500 index     | 200             |
| Asian Crisis            | 1 Oct.1997    | Hangseng index   | 500             |
| Dot.com                 | 24 Mar.2000   | NASDAQ composite | 400             |
| 2008 Financial Crisis   | 15 Sep.2008   | S&P500 index     | 398             |
|                         |               | TED spread       | 280             |
|                         |               | VIX index        | 80              |

4 Methodology

4.1 Detrending

In order to achieve a stationary stochastic process, the first step is to remove the trend pattern from the original time series. The residuals after detrending are further analysed by using linear and nonlinear time series techniques. Subtracting a moving average is the most commonly used technique in detrending. In this paper, we introduce the weighting scheme and use gaussian kernel smoothing. This allows data near the given time point to receive larger weights.

In the analysis of many other complex systems, additional interpolation is needed to have evenly spaced time series. This may produce spurious results (\textsuperscript{?}). Because stock market system can be already considered as a time discrete dynamical system, which is with fixed time-step $\Delta t = 1$ trading days, we skip the interpolation and therefore avoid its possible adverse effects. We detrend time series using a gaussian kernel,

$$G(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{x^2}{2\sigma^2}},$$ \hspace{1cm} (2)

based on which the moving average is given by

$$MA_j = \frac{\sum_{i=1}^{N} G(i-j)z_i}{\sum_{i=1}^{N} G(i-j)},$$ \hspace{1cm} (3)

where $z$ is the logarithm of original price index with fixed time-step $\Delta t = 1$. The bandwidth is $\sigma$. It is also the standard deviation of the Gaussian kernel distribution. In
kernel-based nonparametric regression, the choice of bandwidth is important, due to the bias-variance trade-off. A large bandwidth would lead to oversmoothed, biased, estimates in which details of the trend are washed out and, the magnitude of peaks and troughs is underestimated. Conversely, if a too small bandwidth is applied, this will result in estimation based on only a few local data points, which will lead to large variance of the detrended signal. So the aim is to choose the bandwidth in such a way that we filter out the slower trends from the data while keeping the details of the fluctuations around the local equilibrium value. had obtained a considerable number of significant results by setting the bandwidths around 10 to 25. In this paper, we use bandwidth $\sigma = 10$ and later doublecheck the results over a range of $\sigma$-values in a robustness check. By subtracting moving average from the logarithm of the original time series, the residuals time series is then given by

$$y_j = z_j - MA_j, \quad (4)$$

which fluctuates around 0.

### 4.2 Leading Indicators

An increase in autocorrelation and variance is expected for critical slowing down. Whether this is the case can be estimated using a moving estimation window. Three early warning indicators are considered: AR(1), ACF(1) and Variance.

**AR(1) indicator** When approaching a critical transition, consecutive observations in the state of the system become increasingly similar to each other. The lag-1 autocorrelation is considered as the leading indicator to measure critical slowing down. It can be estimated from an autoregressive model of lag-1:

$$f(t_{j+1}) = e^{-\kappa \Delta t} f(t_j) + \theta \eta_j, \quad (5)$$

which is known as an AR(1) model. $f(t_j)$ is the residual $y_j$ in Equation (4). $\Delta t = t_{j+1} - t_j$ and $\eta_j$ is a zero mean innovation. $\kappa$ indicates the magnitude of the recovery rate. $\lambda = e^{-\kappa \Delta t}$ is the AR(1) coefficient, which is the autoregressive coefficient at lag1. In a saddle-node bifurcation scenario, $\kappa$ vanishes on the way to the bifurcation point. As $\kappa$ goes to 0, $\lambda$ goes to 1.
The random disturbances are assumed to be white noise with zero mean and variance equal to 1. Therefore, the first order autocorrelation coefficient $\lambda$ is approximated as constant in a local time window of length $w$. We estimate $\lambda$ by an ordinary least-square (OLS) fitting method of the regression

$$y_{k+1} = \lambda y_k + u_k,$$

with $u_k$ white noise, over the set of indices $k = j - w + 1, \ldots, j$. The local window slides from left to right and traces a series of AR(1) coefficients varying with respect to index. This new series can be interpreted as the time-varying AR(1) coefficient. If it increases, this indicates that the system is driven gradually closer to a bifurcation.

Apart from the bandwidth, the window size is also a very important parameter. A smaller window size allows us to track short term changes in autocorrelation. However, taking a too small window size with very few observations will make the estimation of autocorrelation less reliable. Following ?, we use half the size of the analysed time series as the sliding window size.

**ACF(1) indicator** An alternative and more straightforward way to estimate autocorrelation at lag-1 is by using the first value of the autocorrelation function (ACF)

$$\rho_1 = \frac{E[(y_t - \mu)(y_{t+1} - \mu)]}{\sigma_y^2},$$

where $\mu$ is the mean of $y_t$ in the window considered, and $\sigma_y^2$ the variance. Like the AR(1) indicator, the moving window produces a proxy series of ACF(1). It also serves as an indicator to detect critical slowing down prior to a critical transition.

**Variance indicator** An increased slowing down also induces an increased amplitude on the way of approaching threshold. This amplitude corresponds with the variance and is measured by standard deviation:

$$St.Dev. = \sqrt{\frac{1}{N-1} \sum_{t=1}^{N} (y_t - \mu)^2}.$$
Biggs et al. (2009) have argued that variance measures seem to be most robust as EWS and also seem much easier to generalize to multivariate cases.

4.3 Establishing Trends

For each indicator observed across time, i.e. $\lambda$, $\rho$, or the standard deviation, we test the trend over time for significance using the nonparametric Kendall rank correlation $\tau$ between the indicator and time variable (?). It is a statistic tool used to measure the degree of concordance between two pairs of ordinal variables:

$$\tau = \frac{C - D}{N},$$

where $C$ is the number of concordant pairs, $D$ is the number of discordant pairs, and $N = n(n - 1)/2$ is the total number of different pair combinations. The quantity $\tau$ is in the range of $[-1, 1]$. If Kendall’s $\tau$ is close to 1, the agreement between the two rankings is perfect. A high Kendall’s $\tau$ suggests a strong trend. In the presence of critical slowing down, one expects to find a significant upward trend as indicated by a significantly positive value of Kendall’s $\tau$.

5 Results

The evidence for early warning signals is evaluated in two steps. Firstly, we observe the early warning signals before real critical transitions. By using six time series, we examine four well-known extreme financial events in history - Black Monday 1987, the Asian Crisis, the Dot-com Bubble and the 2008 Financial Crisis. Secondly, we examine the likelihood of spurious early warnings. The probability of obtaining similar or more extreme early warning signals by chance is estimated using bootstrap time series. In the end, we perform an extensive analysis to examine the robustness of the results with respect to the choice of user-set parameters.

5.1 Financial Time Series

First of all, we examine whether there is evidence of critical slowing down in time series data of stock prices. Four financial crisis are investigated.
Black Monday 1987  During a single day, October 19, 1987, the Dow Jones Industrial Average (DJIA) index lost almost 22%. By the end of that month, most of the major exchanges had dropped by more than 20%. Stock markets around the world crashed, beginning in Hong Kong, spreading to Europe, and hitting the United States later after other markets had declined by a significant margin. This event marked the beginning of a global stock market decline, making ”Black Monday” one of the most dramatic days in recent financial history.

Figure 3 shows the analysis of early warning indicators around half a year preceding “Black Monday” by using Standard & Poor 500 index (S&P500) time series. The original time series in Figure 3(a) is the logarithm of the daily S&P500 index. It starts from 200 days before the crash and ends 100 days after it. Stock markets raced upward during the first half of 1987, but experienced a great depreciation in the last few months. The vertical dashed line identifies the critical transition in the time series. Since we are interested in searching for early warning signals before the critical transition, analysis is based on the data before the dashed line. To facilitate explanation, we set the x-axis of the critical transition as 0 to clearly distinguish the days before and after it. The smoothed central line shows the smoothed time series used for filtering. The dashed arrow shows the width of the moving window. Half the size of the time series is taken as the moving window, following ?. Figure 3(b) shows the residuals, that is, the detrended time series used to estimate the early warning indicators.

Figure 3(c), (d) and (e) give examples of early warning indicators of AR(1), ACF(1) and Variance. They show that the great crash on “Black Monday” is preceded by an overall upward trends in these indicators. All of these positive trends are confirmed by a positive Kendall rank correlation coefficient τ. Therefore, the examples of early earning indicators showed an increase in the period preceding the critical transition, suggesting that the S&P500 time series indeed slows down before the critical transition.

The Asian Crisis  Using the same techniques, we examine the Hangseng time series. Figure 4 shows the analysis of early warning indicators around one and a half year before the Asian Crisis. Figure 4(a) is the logarithm of daily Hangseng index from November 1995 to July 1998. This time series increases in the beginning but collapses around mid-1997, which illustrates the Asian financial crisis in July 1997. The Asian Crisis is a series of currency devaluations along with stock markets declining. The currency
Figure 3: Detecting the early warning indicators for “Black Monday” using the S&P500 time series. (a) Logarithm of the daily S&P500 time series. (b) Residual time series. (c) Indicator from AR(1) function. (d) Indicator from ACF(1) function. (e) Indicator from Standard deviation. The vertical dashed line in (a) identifies the critical transition. The dashed arrow shows the width of the moving window used to compute the indicators shown in (c)(d) and (e). The red line is the smoothed time series used for filtering.
market first failed in Thailand because its government no longer pegged their local currency, the Thai Baht, to the U.S. dollar. The currency crisis rapidly caused stock market declines spreading throughout South Asia. Thailand, South Korea and Indonesia were the countries most affected by the crisis. As a result of the crisis, the stock markets in Japan and most of Southeast Asia fluctuated dramatically.

Figure 4 has a similar format as Figure 3. The smoothed central line in Figure 4(a) shows the moving average used for filtering. The dashed arrow shows the width of the moving window, which is the half size of the analyzed time series length. Figure 4(b) shows the remaining residuals used to estimate the early warning indicators.

The examples of upward trends in indicators of AR(1), ACF(1) and Variance shown in Figure 4(c), (d) and (e) indicate critical slowing down before critical transitions. All of the trends are significant as measured by Kendall’s τ. The increased slowing down before July 1997 in the Hangseng index provides evidence on early warnings before the Asian Crisis.

The Dot-com Bubble Figure 5 presents the analysis of early warning indicators from about one year before the Dot-com bubble collapse. Boosted by the rising of commercial growth of internet, NASDAQ composite index experienced a speculative bubble, as shown in Figure 5(a). It peaked around the year 2000, the latter part followed a typical boom and bust cycle. When the bubble “bursts”, the stock prices of dot-com companies fall dramatically. Some companies went out of business completely, such as Pets.com. Some others survived but their stocks declined by more than 80%, such as Cisco and Amazon.com.

As for Black Monday and the Asian Crisis, the analysis of early warning indicators shows evidence of increased slowing down in NASDAQ time series before the Dot-com Bubble. The bubble collapse is preceded by an overall upward trend in the examples of early warning indicators. All the results are robust based on Kendall’s τ analysis. This suggest the possibility that critical slowing down could serve as an early warning signal for Dot-com Bubble.

The 2008 Financial Crisis The financial crisis of 2008 is known as the greatest financial crisis since the Great Depression of the 1930s. It was triggered by the bursting of U.S. housing bubble, which peaked approximately 2005 - 2006. Banks began to give out
Figure 4: Detecting the early warning indicators for the Asian Crisis using time series Hangseng index. (a) Logarithm of the daily Hangseng time series. (b) Analysis of the residuals time series. (c) Indicator from AR(1) function. (d) Indicator from ACF(1) function. (e) Indicator from Standard deviation. The vertical dashed line in (a) identifies the critical transition. The dashed arrow shows the width of the moving window used to compute the indicators shown in (c)(d) and (e). The red line is the smoothed time series used for filtering.
Figure 5: Detecting the early warning indicators for the Dot-com Bubbles using NASDAQ composite index. (a) Logarithm of the daily NASDAQ time series. (b) Analysis of the residuals time series. (c) Indicator from AR(1) function. (d) Indicator from ACF(1) function. (e) Indicator from Standard deviation. The vertical dashed line in (a) identifies the critical transition. The dashed arrow shows the width of the moving window used to compute the indicators shown in (c)(d) and (e). The smoothed red line is the smoothed time series used for filtering.
more loans than ever before to potential home owners. When the housing bubble finally bursted in the latter half of 2007, the secondary mortgage market collapsed. Over 100 mortgage lenders went bankrupt during 2007 and 2008. Several major financial institutions failed, including Lehman Brothers, Merril Lynch, Washington Mutual, Citigroup and so on. The world wide economies experienced a great recession and stock markets around the world went down.

Figure 6(a) and (b) show the analysis of early warning indicators around one year before the 2008 financial crisis by using two time series, the S&P500 and the TED spread. Both of the analyses show mixed results. In the analysis of the TED spread time series, the AR(1) and ACF(1) indicators produce strong upward trends with Kendall’s tau close to 0.7. This indicates a slowing down preceding the critical transitions in TED spread time series in September 2008, around the time Lehman Brothers went bankrupt. However, for the same period, the variance indicator shows a downward trend. In Figure 6(b), the analysis of S&P500 time series gives opposite results. AR(1) and ACF(1) indicators show downward trend while the Variance indicator shows an upward trend. The mixed results suggest the importance of applying “composite” indicators. None of the indicators alone would be able to give us accurate predictions. We should also note that suggested that the variance need not necessarily increase. Therefore, the promising indicators in our examples are AR(1) and ACF(1), which present increased trend in the analysis of the TED spread.

In order to obtain more evidence, we also analyze the volatility index (VIX) in Figure 7. The VIX index is a commonly used estimated time series of the implied volatility of S&P 500 in the next 30 days. Figure 7(a) presents the logarithm of the daily S&P500 time series, while Figure 7(b) shows the logarithm of the volatility of the S&P500 index. Because we are interested in the volatility time series, the x-axis of the critical transition in volatility index in Figure 7(b) is set to 0. As shown in Figure 7(a) and (b), this critical transition in VIX is 15 days preceding the real critical transitions in the daily S&P500 time series. In Figure 7(b), the time series kept rising after the critical transition for almost a month and decreasing slightly thereafter. However, it did not shift back to the equilibrium before critical transition. The VIX measures the expectations of markets on future stock market volatility. Therefore, our analysis implies that people expected increased volatility of stock market prices already before the financial crisis actually happened. This fear of increased volatility of stock market prices quickly accumulates
Figure 6: Detecting the early warning indicators for the 2008 Financial crisis using TED spread and S&P500 index. (a) Analysis using TED spread. (b) Analysis using S&P500 index. For each of the analysis, (I) Logarithm of the daily original time series. (II) Analysis of the residuals time series. (III) Indicator from AR(1) function. (IV) Indicator from ACF(1) function. (V) Indicator from Standard deviation. The vertical dashed line in (I) identifies the critical transition. The dashed arrow shows the width of the moving window used to compute the indicators shown in (III)(IV) and (V). The red line is the smoothed time series used for filtering.
and lasts for a long time. The whole process coincides with the depression of the economy. It also shows that the early warnings on the critical transitions in volatility index can be considered as early warnings on the crash in S&P500 index.

The same early warning methodology is applied to VIX time series. The analysis shows significant upward trends in AR(1) and ACF(1) indicators preceding the critical transition in VIX time series. It demonstrates increased slowing down and early warnings before the critical transition in the volatility index. However, the variance indicator is just horizontal with a slight downward trend. Moreover, the p-value of the estimated Kendall’s tau indicates insignificance of the trend. Therefore, no trend is found for the variance indicator.

All the above results and the parameters used in the analysis are summarized in Table 2. The symbols “(+)” indicate that significant early warning signals are detected, while the symbols “(-)” indicate that the transitions are not preceded by indicators. As shown in Table 2, the window size we choose is half the sample size in each example following ?. The choice of bandwidth is 10 under the condition that we do not overly smooth the data but still have a stationary time series.

Table 2: Studies of early warning indicators for critical transitions in different time series. N is sample size. \(w\) and \(\sigma\) are the window size and bandwidth used in the analysis. \(\tau\) is the estimated Kendall’s tau coefficient. Symbols (+) indicate early warning signals are detected, while (-) indicates that the transitions are not preceded by indicators.

| Extreme Event | Time Series | N  | \(w\) | \(\sigma\) | \(\tau\)          |
|---------------|-------------|----|------|---------|------------------|
|               |             | AR(1) | ACF(1) | Variance |
| Black Monday  | S&P500      | 200  | 100   | 10      | 0.718***(+), 0.685***(+), 0.259***(+), |
| Asian Crisis  | Hangseng    | 500  | 250   | 10      | 0.385***(+), 0.334***(+), 0.462***(+), |
| Dot-com       | NASDAQ      | 398  | 194   | 10      | 0.501***(+), 0.478***(+), 0.343***(+), |
| 2008 Crisis   | S&P500      | 400  | 200   | 10      | -0.503***(-), -0.470***(-), 0.354***(+), |
|               | TEDspread   | 280  | 140   | 10      | 0.699***(+), 0.705***(+), -0.441***(-), |
|               | VIX         | 80   | 40    | 10      | 0.463***(+), 0.446***(+), -0.149(-), |

\* \* \* significant at 1% level, \* \* significant at 5% level, \* significant at 10% level.

The summary of the results in Table 2 suggests early warnings indicated by the ex-
Figure 7: Detecting the early warning indicators for the 2008 Financial crisis using volatility index (VIX). (a) Logarithm of the daily S&P500 index. (b) Logarithm of the volatility of the S&P500 index. (c) Analysis of the residuals time series. (d) Indicator from AR(1) function. (e) Indicator from ACF(1) function. (f) Indicator from Standard deviation. The vertical dashed line in (a) and (b) identifies the critical transitions in the time series of the S&P500 index and the volatility index. The dashed arrow shows the width of the moving window used to compute the indicators shown in (d) (e) and (f). The red line is the smoothed time series used for filtering.
amples of AR(1), ACF(1) and variance preceding the crashes of the stock market prices for Black Monday 1987, the Asian Crisis and the Dot-com Bubble. However, the signals for the 2008 Financial Crisis are mixed. They lead us to the question whether there is increased slowing down prior to the 2008 Financial Crisis. The AR(1) and ACF(1) methods in TED spread and VIX time series suggest so but they give the opposite results in S&P500 time series. Are these signals just spurious early warnings? Or were there no bifurcations underlying the 2008 financial crisis? To solve these questions, more work is still to be done.
5.2 Bootstrapped Time series

In order to test the likelihood of having the trend statistic estimation of Kendall’s $\tau$ by chance, we apply the same early warning methodology to surrogate time series. We also calculate the probability of the trend statistics in surrogate time series being at least as high as for the original records. The surrogate time series are generated by bootstrapping the financial time series of the S&P500 index, the Hangseng index, the NASDAQ composite, TED spread and the VIX index in three different ways.

**Bootstrapping Residuals** Firstly, we bootstrap the residuals after detrending following the test in ?. By resampling the order, we generate surrogate time series with similar means and variances.

**Bootstrapping Log-returns** Secondly, instead of residuals, the log-returns of the original time series are bootstrapped. Similar with the first method, we bootstrap the time series by randomly picking data with replacement. Moreover, we take the cumulative sum before detrending.

**Parametric GARCH Bootstrap** Thirdly, surrogate time series are generated by fitting a GARCH(1,1) model \(^3\) to log-returns:

\[
y_t = \sigma_t \epsilon_t, \\
\sigma_t^2 = \omega + \alpha y_{t-1}^2 + \beta \sigma_{t-1}^2, \quad t = 1, ..., T,
\]

where $\epsilon_t$ is a white noise process with $\epsilon_t \sim N(0,1)$, $\sigma_t^2$ is the volatility. $\omega, \alpha, \beta$ are parameters that satisfy $\omega > 0, \alpha \geq 0, \beta \geq 0$ to ensure the positivity of the conditional variance. The process $y_t$ is stationary for $0 < \alpha + \beta < 1$.

**Random Segments Bootstrap** Spurious early warnings can occur in the fluctuations in a single regime without transiting to a different one. In order to test this possibility, we analyse the early warnings on random segments in financial time series. Figure 8

\(^3\)Generalized autoregressive conditional heteroskedasticity, originally introduced by ?, ?, represent the dynamic evolution of conditional variances.
shows an example of the test of the early warning indicators for random segments in Hangseng time series. It follows the same format of figures as in the analysis of financial time series. The analysis is based on a randomly picked segment from a long period of Hangseng time series. However, the sample size is kept the same as it in the case of the Asian Crisis. As shown in Figure 8 based on our random segments bootstrap, there is no evidence of early warnings in this case with respect to the early warnings of AR(1), ACF(1) and Variance indicators.

Many other random segments of the financial time series are tested in the same way. However, the trends of the early warning indicators are diverse. Some of them show no early warnings while the others do. The early warning indicators obtained from 1000 random segments are analysed. Figure 9(a), (b) and (c) show the histograms of the examples of AR(1), ACF(1) and Variance indicators individually. The dashed lines are Kendall’s $\tau$ of the indicators of the real critical transitions in the Asian Crisis. The arrows indicate the subsets for which the segments trend statistics are higher than the trend statistics of the true critical transitions. The fractions of these subsets are indicated by percentages in the figure. For all three indicators, the fractions of the subsets are around 20% - 30%. Following the ideas on the analysis of bootstrapped time series in Section 5.2, Figure 9 implies that the examples of early warning indicators of AR(1), ACF(1) and variance give the early warnings in Hangseng index before critical transitions but only at 20% - 30% significance level.

**Random Walk Bootstrap** To compare with financial time series, we perform the analysis on realisations of a random walk process. A simple random walk is presented as:

$$y_t = y_{t-1} + \epsilon_t,$$

where $\epsilon_t$ is white noise.

We generate 1000 random walk process realisations and calculate the early warning indicators based on them. Figure 10 shows the histograms of Kendall’s $\tau$ and $p$-values of AR(1) and Variance indicators. Due to gaussian detrending in the methodology, $-1 < \tau < 1$ is expected as shown in the figure. Moreover, if it is the methodology which creates spurious trends in Kendall’s $\tau$, the distribution would left skewed and clustered near 1. However, Figure 10 shows all the Kendall’s $\tau$ are distributed evenly between -1 and 1, which rejects the hypotheses above.
Figure 8: Testing the early warning indicators for random segments in Hangseng time series. (a) Logarithm of the random segments in Hangseng time series. (b) Analysis of the residuals time series. (c) Indicator from AR(1) function. (d) Indicator from ACF(1) function. (e) Indicator from Standard deviation. The dashed arrow shows the width of the moving window used to compute the indicators shown in (c)(d) and (e). The red line is the smoothed time series used for filtering.
Figure 9: Histograms of Kendall’s tau by Random segments in the Hangseng time series from 1988 to 2009. The number of time series is N=1000. a, b and c show the histograms of the examples of AR(1), ACF(1) and Variance indicators individually. The blue dashed lines are the Kendall’s tau of the indicators of the real critical transitions in Asian Crisis. The blue arrow indicates the subsets which the segments trend statistic is higher than the trend statistic of the true critical transition. The blue percentage numbers indicate the fractions higher than original time series.
Figure 10: Histograms of Kendall’s tau and $p$-value in the examples of AR(1) and Variance indicators in 1000 random walk processes. (a) and (c) show the histograms of the Kendall’s tau in the examples of AR(1) and Variance indicators while (b) and (d) show the distributions of $p$-values.
For the time series data of the S&P500 index, the Hangseng index, the NASDAQ composite, the TED spread and the VIX index, 1000 surrogate time series are estimated under each bootstrap method. The trend statistics of Kendall’s $\tau$ coefficients and $p$-values are represented by histograms. Figure 11 shows one of the examples of the analysis. It presents the analysis of the Kendall’s $\tau$ coefficients of the examples of AR(1) indicator in 1000 surrogate S&P500 time series, under the bootstrap method of fitting the GARCH(1,1) model. Figure 11(a) presents the probability distributions of the $p$-values while Figure 11(b) shows the histogram of the Kendall’s $\tau$ coefficients. The dashed line represents the trend statistic of the original $\tau^*$ in the Black Monday example.

The likelihood of obtaining trend statistic estimates by chance is estimated by the subsets that the surrogate trend statistics are higher than the trend statistics of the original time series, $p(\tau \geq \tau^*)$. This subset is indicated by the arrow in Figure 11(b). It refers to the probability that surrogate Kendall’s $\tau$ lie on the right hand side of the original Kendall’s $\tau$. The $p$-values as $p(\tau \geq \tau^*)$ would evaluate the significances of the trend statistic estimations of the early warnings in financial examples. The results for the examples of AR(1) indicators are shown in Table 3.

Table 3 shows the likelihood of obtaining trend statistic estimates by chance, which is estimated by the probability of the surrogate estimated trend statistic Kendall’s $\tau$ larger than the trend statistic of the original residual records. Only the results for the examples of AR(1) indicators are shown here. The probabilities vary from case to case, but the differences between the three bootstrap methods are small. This suggests the reliability of the bootstrap results. As shown in Table 3, the results are mixed. The probability of having the observed trends by chance are less than 10% in the examples of the Black Monday (S&P500) and the 2008 Financial Crisis (TED spread) in all three bootstrap methods. It implies that the early warning signals we found in both examples are significant, at the 10% significance level. In particular, the performance of the example of the Black Monday (S&P500) is fairly good. Its results are significant at the 5% significance level. It highlights the reliability of the early warning signals preceding the Black Monday crash in the S&P500 time series. In other cases, the probability of obtaining the trends by chance are around 20% - 30%. The significance levels of the early warnings we found in the Asian Crisis, the Dot-com Bubble and the 2008 Financial Crisis(VIX) are only around 30%.

$^{4}\tau < -\tau^*$ for 2008 Financial Crisis case using S&P500
Figure 11: Analysis of the Kendall’s $\tau$ coefficients of the examples of AR(1) indicator in 1000 surrogate S&P500 time series, under the bootstrap method three which is to fit GARCH(1,1) model. (a) presents the bootstrap density of the $p$-values; (b) shows the histogram of the Kendall’s $\tau$ coefficients. The Red dashed line represents the trend statistic of the original $\tau^+$ in the Black Monday example. The Red arrow indicates the subset which the surrogate trend statistic is larger than the trend statistic of the original residual records. From this subset, only values equal to or higher than the original record are used to estimate the likelihood of acquiring trend statistic estimates this far in the tail by chance.
Table 3: The likelihood of obtaining trend statistic estimates by chance, estimated by the probabilities of the surrogate estimated trend statistic (Kendall’s $\tau$) larger than the trend statistic of the original residual records. Only the results for the examples of AR(1) indicators are shown here. Symbol (+) indicates early warning signals are detected. (-) indicates the transitions are not preceded by indicators. Surrogate set $N = 1000$.

| Extreme Events   | Time Series | Bootstrap $P(\tau > \tau^*)$ | $\tau^*$ |
|------------------|-------------|------------------------------|----------|
|                  | I.          | II.                         | III.     | IV.      | V.      |
|                  | Per. Res.   | Per. Ret.                   | GARCH    | Ran. t.s.| Ran. wal.|
| Black Monday     | S&P500      | 0.055*                      | 0.045**  | 0.060*   | 0.025** | 0.074*  | 0.718(+)|
| Asian Crisis     | Hangseng    | 0.300                       | 0.287    | 0.309    | 0.268   | 0.210   | 0.385(+)|
| Dot-com          | NASDAQ      | 0.354                       | 0.335    | 0.350    | 0.133   | 0.174   | 0.501(+)|
| 2008 Crisis      | S&P500      | 0.205                       | 0.200    | 0.183    | 0.202   | 0.175   | -0.503(-)|
| TEDspread        | 0.072*      | 0.061*                      | 0.074*   | 0.098*   | 0.060*  | 0.699(+)|
| VIX              | 0.311       | 0.300                       | 0.288    | 0.289    | 0.323   | 0.463(+)|

** 5% level, * 10% level

6 Robustness of parameters

The early warning analysis in this paper is influenced by two key parameters: bandwidth and moving window size. Bandwidth size is a very important parameter when filtering out long term trends from the original time series. There is a trade-off when making the choice. A too narrow bandwidth would not only remove the long run trends but also the short run fluctuations we intend to study; a too wide bandwidth would not remove enough long run trends. There would be still some slow trends left which may lead to spurious trends of the indicators. A similar trade-off also affects the window size. A smaller window size is good to track short run changes, but a too small window size with too few sample points would make the estimations less reliable.

In order to check the robustness of the parameters in our analysis, we perform an additional analysis by using rolling window and rolling bandwidth. The contour plots in Figure 12 and 13(a) show the influence of parameters on the observed trends of AR(1) indicators for Black Monday 1987 and the Asian crisis. The black dot indicates the combination of window size and bandwidth size that shows the strongest positive
trends. The white dot indicates the parameters used in our early warning analysis. The Kendall’s tau distributions in Figure 12 and 13 confirm the strong positive trends of Kendall’s tau in the contour plots. This robustness analysis indicates that the results are quite robust with respect to the parameters chosen. It also shows that even more significant trends could be obtained by moving parameters in the direction of the black dot, which in turn confirms the robustness of the results with respect to changes in the parameters in this paper.

Figure 12: Analysis of the robustness of parameters in the example of Black Monday: window size and bandwidth. (a) Contour plots of the rolling window size and bandwidth size. Black dot indicates the assemble of window size and bandwidth size that shows the strongest positive trends. The white dot indicates the parameters used in the early warning analysis (b) Histogram of the Kendall’s tau in (a)
Figure 13: Analysis of the robustness of parameters in the Asian Crisis: Window size and bandwidth. (a) Contour plots of the rolling window size and bandwidth size. Black dot indicates the assemble of window size and bandwidth size that shows the strongest positive trends. The white dot indicates the parameters used in our early warning analysis (b) Histogram of the Kendall’s tau in (a)
7 Summary and Discussion

In the theory of “Critical slowing down”, an increase of the first order autocorrelation coefficients towards +1, is considered to be an early warning signal for an upcoming critical transition. This theory is applied to the analysis of time series by ? and ?. They used the slowing down before critical transitions to identify the bifurcations in climate systems. The method has been applied first to the North Atlantic thermohaline circulation and Greenland ice core paleotemperature using climate model output. ? developed this methodology further and applied it to real climate data for the first time. Their analysis provides robust empirical evidence. The first order autocorrelation coefficient indeed increases as the system approaches a critical transition.

Our paper investigates for the first time whether there is evidence of “critical slowing down” in financial time series. Four financial crisis are analysed by using six time series. The results suggest increases in AR(1), ACF(1) and variance indicators preceding the crashes of the stock market prices in Black Monday 1987, the Asian Crisis and the Dot-com Bubble. However, the signals for the 2008 Financial Crisis are mixed. In order to estimate the likelihood of spurious early warnings, the same methodology is applied to various surrogate time series. The analysis of the bootstrapped time series confirms the reliability and significance of the early warning signals preceding the Black Monday crash in the S&P500 time series, but suggests that the possibility of spurious early warnings in the Asian Crisis, the Dot-com Bubble and the 2008 Financial Crisis(VIX) are around 30%. Analysis of random financial time series also shows the real p-value to detect the critical transitions in financial time series are around 20% - 30%. The random walk analysis rejects the hypothesis that the methodology we apply may create increasing trend which gives rise to false early warnings. An additional analysis on the robustness of parameters is performed as well. It shows that the results are fairly robust with respect to the choices of the parameters.

This paper fills the gap between the theory of “critical slowing down” and its application to financial data. It explores the forecasting ability of the critical slowing down indicators in financial time series. The results detect fairly good early warnings before the crash of Black Monday 1987. The early warnings before the Asian Crisis, the Dot-com Bubble and the 2008 financial crisis are only detected at 20% - 30% significance level. It suggests that critical slowing down can serve as one of the complementary
early warning indicators of financial crisis. However, in order to increase the accuracy of the predictions, more sophisticated methods are still desired.

There are a number of possible reasons why the results are mixed. Firstly, so far the tools used for detecting critical slowing down are based on linear approximation. With the increase of the complexity of the nonlinear dynamics, the prediction of bifurcations may become harder. ? tried to extend the techniques using nonlinear features. However, they did not find discernible trends in the nonlinear case. The methodology used in this paper is also based on a first order linear approximation of the dynamics. Therefore, due to the complexity of financial dynamic systems, false early warnings can be expected. Secondly, the transitions in complicated financial systems may happen far from local bifurcations and do not necessarily correspond to cusp catastrophe transitions. For instance, there may be an early escape from a stable equilibrium due to exogenous shocks — a so-called noise induced transition (?). In particular, the emergence of new technology and financial instruments nowadays makes the financial markets more complex. This could explain the failure to detect the 2008 financial crisis, despite their use of advanced economic and financial models. Thirdly, the catastrophe theory approach is based on one dimensional systems with only one control variable, while the situation in financial systems is far more complicated. Perhaps a multivariate approach is required to capture the sophisticated dynamic behaviours in financial systems. It is known, for instance, that in periods of market stress returns on individual stocks are more correlated than in more tranquil periods. Fourthly, asset pricing models are usually based on the assumption that the fundamental price follows a geometric random walk process. Bottom-up heterogeneous agent models where agents are boundedly rational typically describe endogenous fluctuations around this fundamental price, where the latter is typically allowed to be a unit-root process (?). This means that (log) prices are naturally characterised by having an eigenvalue close to 1, so that a trend of the dominating eigenvalue from below one towards one may not be easy to observe in near unit root (log) price time series data. Finally, a fifth reason why the standard critical slowing down approach may not be suitable for financial/economic time series data, or more generally social systems, is that these systems differ from most natural systems due to the presence of smart agents (“atoms that can think”) whose main activities are based on their expectations on future price developments. Agents learn and adapt their behaviour and may respond to news announcements, and in particular to early
warning signals of a crisis. Their expectations and adaptive behaviour in response to the risk of an upcoming market devaluation may actually trigger or accelerate a self-fulfilling crisis. This touches upon the issue whether being able to detect an upcoming financial crisis could cause a panic reaction of the agents and thus trigger an upcoming crisis even earlier. These nonlinear expectations feedback mechanisms and adaptive behaviour may make detection of early warning signals for social and economic systems much harder than for complex natural systems.

In this paper we have deliberately focused on EWSs based on critical slowing down using a narrow set of techniques that have been successfully applied in other fields. We therefore have not addressed the issue of optimising the algorithm in any way by fine-tuning the test parameters, sample size, or data frequency. This is considered to be out of the scope of this preliminary study, and left for future research. Neither did we discuss which alternative techniques might be used to extract EWSs from financial time series data. In recognition that economic/financial systems may behave differently than many natural systems, and that critical slowing down may not be a typical phenomenon prior to market collapses, we conclude with a number of remarks regarding possible future approaches towards developing EWSs for financial crises. There are many possibilities still open here. For instance forecasting techniques based on pattern recognition might be exploited to extract signals from financial time series data. Such signals might be based, for example, on machine learning algorithms for forecasting prices. One could also imagine the use of not one but a number of different machines, the forecasts of which could be combined into a single signal. In view of the increased correlations between stock returns in periods of market stress, also multivariate approaches, for instance based on increased cross-sectional correlations between individual stocks, are promising for future EWS studies based on financial time series data. Finally, the use of complex networks techniques to monitor the evolving structure of financial-economic networks also has potential as early warning indicators for crises.
References