A heat equation with memory: large-time behavior

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We study the large-time behavior in all $L^p$ norms and in different space-time scales of solutions to the Cauchy problem for a heat equation with a Caputo $\alpha$-time derivative. The initial data are assumed to be integrable, and, when required, to be also in $L^p$. A main difficulty in the analysis comes from the singularity in space at the origin of the fundamental solution of the equation when $N > 1$.

In the characteristic scale $|x| \approx t^{\alpha/2}$, dictated by the scaling invariance of the equation, solutions behave, when properly scaled to kill their decay, like $M$ times the fundamental solution, where $M$ is the integral of the initial datum. In compact sets they converge to the newtonian potential of the initial datum if $N \geq 3$, one of the main novelties of the paper, and to a constant if $N = 1, 2$, with a logarithmic correction in the decay rate for the critical dimension $N = 2$. In intermediate scales, going to infinity more slowly than the characteristic one, solutions approach a multiple of the fundamental solution of the laplacian if $N \geq 3$, and a constant in low dimensions, again with logarithmic corrections for the critical dimension.

The asymptotic behavior in scales that go to infinity faster than the characteristic one depends strongly on the behavior of the initial datum at infinity. We give results for certain initial data with specific decays.

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