On the Min-cost Traveling Salesman Problem with Drone

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Abstract

Previously used exclusively in military domain, unmanned aerial vehicles (drones) have stepped up to become a part of new logistic method in commercial sector called "last-mile delivery". In this novel approach, small unmanned aerial vehicles (UAV), also known as drones, are deployed alongside with trucks to deliver goods to customers in order to improve the service quality and reduce the transportation cost. It gives rise to a new variant of the traveling salesman problem (TSP), called TSP with drone (TSP-D). A variant of the problem which aims to minimize the time at which the last customer is serviced (or to maximize the service quality in other words) was studied in the literature [1]. This article considers a new variant of the TSP-D where the objective is to minimize the total transportation cost. The problem is first formulated mathematically and two algorithms are proposed for the solution. The first algorithm (TSP-LS) inspired from the idea of [1] in which an optimal TSP solution is converted to a feasible TSP-D solution by local searches. The second one, a Greedy Randomized Adaptive Search Procedure (GRASP), is based on a new split procedure which optimally splits any TSP tour into a TSP-D solution. Once a TSP-D solution is generated, it is then improved through local search operators. Numerical results obtained on various instances with different sizes and characteristics are presented.

Keywords. Traveling Salesman Problem with Drone, Minimize transportation cost, Integer programming, Heuristic, GRASP

1. INTRODUCTION

Companies always tend to look for the most cost-efficient new methods to distribute goods across logistic networks [2]. Traditionally, trucks have been used to handle these tasks and the corresponding transportation problem is modelled as a traveling salesman problem (TSP). However, a new distribution method has recently arisen in which small unmanned aerial vehicles (UAV), also known as drones, are deployed to support parcel delivery. On the one hand, there are four advantages of using a drone for delivery: (1) it can be operated without a human pilot, (2) it can avoid the congestion of traditional road networks by flying over them, (3) it is faster than trucks, and (4) it has much lower per kilometre transportation costs [3]. On the other hand, because the drones are powered by batteries, their flight distance and lifting power are limited, meaning they are restricted in both maximum travel distance and parcel size. In contrast, a truck has the advantage of long range travel capability and can carry large and heavy cargo with a diversity of size, but it is also heavy, slow and cost inefficient.

Consequently, the advantages of truck offset the disadvantages of drones and—similarly—the advantages of drones offset the disadvantages of trucks. These complementary capabilities are the foundation of a novel method named "last mile delivery with drone" [4], in which the truck transports the drone close to the customer locations, allowing the drone to service customers while
remaining within its flight range, effectively increasing the usability and making the schedule more flexible for both drones and trucks. Specifically, a truck departs from the depot carrying the drone and all the customer parcels. As the truck makes deliveries, the drone is launched from the truck to service a nearby customer with a parcel. While the drone is in service, the truck continues its route to further customer locations. The drone then returns to the truck at a location different from its launch point.

From an application perspective, a number of remarkable events have occurred since 2013, when Amazon CEO Jeff Bezos first announced Amazon’s plans for drone delivery [5], termed “a big surprise.” Recently, Google has been awarded a patent that outlines its drone delivery method [6]. In detail, rather than trying to land, the drone will fly above the target, slowly lowering packages on a tether. More interestingly, it will be able to communicate with humans using voice messages during the delivery process. Google initiated this important drone delivery project, called Wing, in 2014, and it is expected to launch in 2017 [7]. A similar Amazon project called Amazon Prime Air ambitiously plans to deliver packages by drone within 30 minutes [8]. Other companies worldwide have also been testing delivery services using drones. In April 2016, Australia Post successfully tested drones for delivering small packages. That project is reportedly headed towards a full customer trial later this year [9]. In May 2016, a Japanese company—Rakuten—will launch a service named “Sora Kaku” that “delivers golf equipment, snacks, beverages and other items to players at pickup points on the golf course” [10]. In medical applications, Matternet, a California-based startup, has been testing drone deliveries of medical supplies and specimens (such as blood samples) in many countries since 2011. According to their CEO: it is “much more cost-, energy- and time-efficient to send [a blood sample] via drone, rather than send it in a two-ton car down the highway with a person inside to bring it to a different lab for testing,” [11]. Additionally, a Silicon Valley startup named Zipline International will begin using drones to deliver medicine in Rwanda starting in July, 2016 [12].

We are aware of several works in the literature that have investigated the routing problem related to the truck-drone combination for delivery. Murray and Chu [1] introduce the problem, calling it the "Flying Sidekick Traveling Salesman Problem" (FSTSP). A mixed integer programming (MIP) formulation and a heuristic are proposed. Basically, their heuristic is based on a "Truck First, Drone Second" idea, in which they first construct a route for the truck by solving a TSP problem and, then, repeatedly run a relocation procedure to reduce the objective value. In detail, the relocation procedure iteratively checks each node from the TSP tour and tries to consider whether it is suitable for use as a drone node. The change is applied immediately when this is true, and the current node is never checked again. Otherwise, the node is relocated to other positions in an effort to improving the objective value. The relocation procedure for TSP-D is designed in a "best improvement" fashion; it evaluates all the possible moves and executes the best one. The proposed methods are tested on only small-sized instances with up to 10 customers.

Agatz et al. [13], in an unofficial publication, study a slightly different problem—called the “Traveling Salesman Problem with Drone” (TSP-D), in which the drone has to follow the same road network as the truck. Moreover, in TSP-D, the drone may be launched and return to the same location, while this is forbidden in the FSTSP. This problem is also treated as an MIP formulation but solved by a “Truck First, Drone Second” heuristic in which drone route construction is based on either local search or dynamic programming. More recently, Ponza [14] extended the work of Murray and Chu [1] to solve the FSTSP in his master’s thesis, proposing an enhancement to the MIP model and solving the problem by a heuristic method based on Simulated Annealing.

All the works mentioned above aim to minimize the time at which the truck can complete the route and return to the depot, which can improve the service quality [15]. However, in every logistics or delivery operations, transportation cost also plays an important role in the overall...
business cost (see [16] and [17]). Hence, minimizing this cost by using a more cost-efficient vehicle is a vital objective of every company involved in transport and logistics activities. Recently, such an objective function was studied by Mathew et al. [18] for a related problem called the Heterogeneous Delivery Problem (HDP). However, unlike in [1], [13] and [14], the problem is modelled on a directed physical street network where a truck cannot achieve direct delivery to the customer. Instead, from the endpoint of an arc, the truck can launch a drone that will service the customers. In this way, the problem can be favourably transformed to a Generalized Traveling Salesman Problem (GTSP). The authors use the Nood-Bean Transformation available in Matlab to reduce a GTSP to a TSP, which is then solved by a heuristic proposed in the study. However, to the authors’ knowledge, the min-cost objective function has not been studied for TSP-D when the problem is defined in a more realistic way—similarly to [1], [13], and [14]. Consequently, this lack of knowledge provides a strong motivation for studying TSP-D with the min-cost objective function.

This paper studies a novel variant of TSP-D and proposes the following hypotheses: (1) The truck and the drone must travel alongside each other, and both vehicles can make deliveries. (2) The drone must be launched and retrieved in two different customer locations. (3) The truck cannot return to any previously visited customer to pick up the drone (i.e., it avoids the subtour problem in TSP). Most importantly, the objective is to minimize the total transportation cost of both vehicles.

In this paper, we propose an MIP model and two heuristics to solve the considered TSP-D: a Greedy Randomized Adaptive Search Procedure (GRASP) and a heuristic adapted from the work of [1] called TSP-LS. In detail, the contributions of this paper are as follows:

- We introduce a new variant of TSP-D in which the objective is to minimize the total transportation cost. We call the problem "min-cost TSP-D".
- We propose an abstract model together with an MIP formulation.
- We develop two heuristics for the solution: TSP-LS and GRASP, which contains a new split procedure and local search operators.
- We introduce various sets of instances with different numbers of customers and a variety of options to test this problem.

This article is structured as follows: Section 1 provides the introduction. Section 2 describes the problem and the abstract model. The MIP formulation is introduced in Section 3. We describe our two heuristics in Sections 4 and 5. Section 6 presents the experiments, including instance generations and settings. We discuss the computational results in Section 7. Finally, Section 8 concludes the work and provides suggestions for future research.

2. Problem definition

In this section, we provide a description of the problem and discuss an abstract model for the min-cost TSP-D in a step by step manner. Here, we consider a list of customers to whom a truck and a drone will deliver parcels. To make a delivery, the drone is launched from the truck and later rejoins the truck at another location. Each customer is visited only once and is serviced by either the truck or the drone. Both vehicles must start from and return to the depot. When a customer is serviced by the truck, this is called a truck delivery, while when a customer serviced by the drone, this is called a drone delivery. This is represented as a 3-tuple \((i, j, k)\), where \(i\) is a launch node, \(j\) is a drone node (a customer who will be serviced by the drone), and \(k\) is a rendezvous node, as listed below:
Node $i$ is a launch node at which the truck launches the drone. The launching operation must be carried out at a customer location.

Node $j$ is a node serviced by the drone, called a "drone node".

Node $k$ is a customer location where the drone rejoins the truck. At node $k$, the two vehicles meet again; therefore, we term it a "rendezvous node". While waiting for the drone to return from a delivery, the truck can make other truck deliveries.

A tuple $⟨i, j, k⟩$ is called feasible if the drone has sufficient power to launch from $i$, deliver to $j$ and rejoin the truck at $k$. The drone can be launched from the depot but must subsequently rejoin the truck at a customer location. Finally, the drone’s last rendezvous with the truck can occur at the depot.

When not actively involved in a drone delivery, the drone is carried by the truck. Furthermore, the truck and the drone each have their own transportation costs per unit of distance. In practice, the drone’s cost is much lower than the truck’s cost because it is not run by gasoline but by batteries. We also assume that the truck provides new fresh batteries for the drone (or recharges its batteries completely) before each drone delivery begins.

The objective of the min-cost TSP-D is to minimize the total transportation cost of both vehicles. Because the problem reduces to a TSP when the drone’s endurance is 0, it is NP-Hard. Examples of TSP and TSP-D optimal solutions on the same instance in which the unitary transportation cost of the truck is 25 times more expensive than that of the drone are shown in Figure 1.

We now develop an abstract model for the problem. We first define basic notations relating to the graph, sequence and subsequence. Then, we formally define drone delivery and the solution representation as well as the associated constraints and objective.

### 2.1. The min-cost TSP-D problem

The min-cost TSP-D is defined on a graph $G = (V, A)$, $V = \{0, 1, \ldots, n, n + 1\}$, where 0 and $n + 1$ both represent the same depot but are duplicated to represent the starting and returning points. The set of customers is $N = \{1, \ldots, n\}$. Let $d_{ij}$ and $d'_{ij}$ be the distances from node $i$ to node $j$ travelled by the truck and the drone, respectively. Furthermore, $C_1$ and $C_2$ are the transportation costs of the truck and drone, respectively, per unit of distance.

Given a sequence $s = (s_1, s_2, \ldots, s_t)$, where $s_i \in V, i = 1 \ldots t$, we denote the following:

- $V(s) \subseteq V$ the list of nodes of $s$
- $pos(i, s)$ the position of node $i \in V$ in $s$
- $next_s(i)$ the next node of $i$ in $s$
- $prev_s(i)$ the previous node of $i$ in $s$
- $first(s)$ the first node of $s$
- $last(s)$ the last node of $s$
- $s[i]$ the $i^{th}$ node in $s$
- $size(s)$ the number of nodes in $s$
- $sub(i, j, s)$, where $i, j \in s$, the subsequence of $s$ from node $i$ to node $j$
- \( A(s) = \{(i, next_s(i))|i \in V(s) \setminus \text{last}(s)\} \) the set of arcs in \( s \)

As mentioned above, we define a **drone delivery** as a 3-tuple \( (i, j, k) : i, j, k \in V, i \neq j, j \neq k, k \neq i, d'_{ij} + d'_{jk} \leq \epsilon \), where \( \epsilon \) is a constant denoting the drone’s endurance. We also denote \( P \) as the set of all possible drone deliveries on the graph \( G = (V, A) \) as follows:

\[
P = \{(i, j, k) : i, j, k \in V, i \neq j, j \neq k, k \neq i, d'_{ij} + d'_{jk} \leq \epsilon\}.
\]

### 2.2. Solution representation

A **min-cost TSP-D solution**, denoted as \( \text{sol} \), is represented by two components:
- A truck tour, denoted as $TD$, is a sequence $\langle e_0, e_1, \ldots, e_k \rangle$, where $e_0 = e_k = 0, e_i \in V, e_i \neq e_j$ and $i \neq j$.

- A set of drone deliveries $DD$ such that $DD \subseteq P$.

which can also be written as

$$sol = (TD, DD).$$

2.3. Constraints

A solution $(TD, DD)$ of the min-cost TSP-D must satisfy the following constraints:

(A) Each customer must be serviced by either the truck or the drone:

$$\forall e \in N : e \in TD \text{ or } \exists \langle i, e, k \rangle \in DD.$$  

By definition, during a truck tour, a customer cannot be visited twice by the truck. The above constraint does not prevent a customer from being serviced by both the truck and the drone nor from being serviced twice by the drone.

(B) A customer is never serviced twice by the drone:

$$\forall \langle i, j, k \rangle, \langle i', j', k' \rangle \in DD : j \neq j'.$$

(C) Drone deliveries must be compatible with the truck tour:

$$\forall \langle i, j, k \rangle \in DD : j \notin TD, i \in TD, k \in TD, \text{pos}(i, TD) < \text{pos}(k, TD).$$

This constraint implies that a customer cannot be serviced by both the truck and drone.

(D) No interference between drone deliveries:

$$\forall \langle i, \cdot, k \rangle \in DD, \forall e \in sub(i, k, TD), \forall \langle i', j', k' \rangle \in DD : e \neq i'.$$

The above constraint means that when the drone is launched from the truck for a drone delivery, it cannot be relaunched before the rendezvous from that delivery. As a consequence, we also cannot have any other rendezvous during that period.

2.4. Objective

Regarding the costs, the following notations are used:

- $cost(i, j, k) = C_2(d'_{ij} + d'_{jk})$, where $\langle i, j, k \rangle \in P$ [cost of drone delivery $\langle i, j, k \rangle$]

- $cost(TD) = \sum(i, j) \in A(TD) : C_1 d_{ij}$ [cost of a truck tour $TD$]

- $cost(DD) = \sum\langle i, j, k \rangle \in DD : cost(i, j, k)$ [total cost of all drone deliveries in $DD$]

- $cost(TD, DD) = cost(TD) + cost(DD)$ [cost of a solution]

- $cost(sub(i, k, s))$ the total cost for both truck and drone (if any) in a subsequence $s \in TD$

The objective is to minimize the total transportation cost:

$$\min cost(TD, DD).$$
3. Mixed Integer Programming Formulation

The min-cost TSP-D defined in the previous section is represented here in an MIP formulation. We first define two subsets of \( V \), \( V_L = 0, 1, \ldots, n \) and \( V_R = 1, 2, \ldots, n + 1 \) to distinguish the nodes that a drone can launch from (\( V_L \)) or return to (\( V_R \)). Let \( x_{ij} \in \{0,1\} \) equal one if the truck goes from node \( i \) to node \( j \) with \( i \in V_L \) and \( j \in V_R, i \neq j \). Let \( y_{ijk} \in \{0,1\} \) equal one if (\( i,j,k \)) is a drone delivery. We can denote \( p_{ij} \in \{0,1\} \) as equalling one if node \( i \in N \) is visited before node \( j \in N, j \neq i \), in the truck’s path. We also set \( p_{ij} = 1 \) for all \( j \in N \) to indicate that the truck always starts the tour from the depot. As in standard TSP subtour elimination constraints, we denote \( 0 \leq u_i \leq n + 1 \) as the position of the node \( i, i \in V \) in the truck’s path.

The MIP model is as follows:

\[
\begin{align*}
\text{Min} & \quad C_1 \sum_{i \in V_L} \sum_{j \in V_R \atop i \neq j} d_{ij}x_{ij} + C_2 \sum_{i \in V_L} \sum_{j \in N \atop i \neq j} \sum_{k \in V_R} (d'_{ij} + d'_{jk})y_{ijk} \\
\sum_{i \in V_L} x_{ij} + \sum_{i \in V_L \atop i \neq j} \sum_{k \in V_R} y_{ijk} = 1 & \quad \forall j \in N \\
\sum_{j \in V_R} x_{0j} = 1 & \quad (1) \\
\sum_{i \in V_L} x_{i,n+1} = 1 & \quad (2) \\
u_i - u_j + 1 \leq (n + 2)(1 - x_{ij}) & \quad \forall i \in V_L, j \in \{V_R : i \neq j\} \\
\sum_{i \in V_L} x_{ij} = \sum_{k \in V_R \atop k \neq j} x_{jk} & \quad \forall j \in N \\
2y_{ijk} \leq \sum_{h \in V_L \atop h \neq i} x_{hi} + \sum_{l \in N \atop l \neq k} x_{lk} & \quad \forall i \in N, j \in \{N : i \neq j\}, k \in \{V_R : \langle i, j, k \rangle \in P\} \\
y_{0jk} \leq \sum_{h \in V_L \atop h \neq k} x_{hk} & \quad j \in N, k \in \{V_R : \langle 0, j, k \rangle \in P\} \\
u_k - u_j \geq 1 - (n + 2)(1 - \sum_{j \in N \atop \langle i, j, k \rangle \in P} y_{ijk}) & \quad \forall i \in V_L, k \in \{V_R : k \neq i\} \\
\end{align*}
\]
The objective is to minimize the total transportation cost. We now explain the constraints. The letter in parenthesis at the end of each bullet item, if any, denotes the association between an MIP constraint and a constraint described in the abstract model:

- Constraint 2 guarantees that each node is visited once by either a truck or a drone. (A)
- Constraints 3 and 4 state that the truck must start from and return to the depot. (Modelling TD)
- Constraint 5 is a subtour elimination constraint. (Modelling TD)
- Constraint 6 indicates that if the truck visits \( j \) then it must depart from \( j \). (Modelling TD)
- Constraint 7 associates a drone delivery with the truck route. In detail, if we have a drone delivery \( \langle i,j,k \rangle \), then there must be a truck route between \( i \) and \( k \). (C)
- Constraint 8 indicates that if the drone is launched from the depot, then the truck must visit \( k \) to collect it. (C)
• Constraint 9 ensures that if there is a drone delivery for \( \langle i, j, k \rangle \), then the truck must visit \( i \) before \( k \). (C)

• Constraints 10 and 11 state that each node in \( V_L \) or \( V_R \) can either launch the drone or retrieve it at most once, respectively. (B)

• Constraints 12, 13, 14 and 15 ensure that if \( i \) is visited before \( j \) in the truck route, then its ordering constraint must be maintained. (D)

• Finally, for constraint 16, if we have two drone deliveries \( \langle i, j, k \rangle \) and \( \langle l, m, n \rangle \) and \( i \) is visited before \( l \), then \( l \) must be visited after \( k \). This constraint avoids the problem of launching a drone between \( i \) and \( k \). (D)

4. A Greedy Randomized Adaptive Search Procedure (GRASP) for TSP-D

This section presents a Greedy Randomized Adaptive Search Procedure (GRASP) [19] to solve the min-cost TSP-D. In the construction step, we propose a split algorithm that builds a min-cost TSP-D solution from a TSP solution. In the local search step, new operators adapted from the traditional ones are introduced for the min-cost TSP-D. The general outline of our GRASP is shown in Algorithm 1. More specifically, it first generates \( n_{TSP} \) TSP tours using a nearest-neighbour heuristic (line 3). The heuristic starts from the depot, iteratively builds the tour by choosing a node \( v \) randomly from among the \( m \) closest unvisited nodes. Next, for each tour, we construct a min-cost TSP-D solution using the split algorithm (line 5) and then improve it by local search (line 6). The best solution found is also recorded during the processing of the tours (lines 7 to 9). The detailed implementation of the split algorithm is described in Algorithms 2 and 3.

Algorithm 1: Greedy Randomized Adaptive Search Procedure (GRASP) for min-cost TSP-D

| Result: bestSolution |
|----------------------|
| 1 bestSolution = null ; |
| 2 bestObjectiveValue = \( \top \) ; |
| 3 tours = generate \( n_{TSP} \) random TSP tours ; |
| 4 foreach tour in tours do |
| 5 tspdSolution = Split_Algorithm(tour) ; |
| 6 tspdSolution = Local_Search(tspdSolution) ; |
| 7 if \( f(tspdSolution) < bestObjectiveValue \) then |
| 8 bestSolution = tspdSolution ; |
| 9 bestObjectiveValue = \( f(tspdSolution) \) ; |
| 10 return bestSolution ; |

4.1. A Split Algorithm for min-cost TSP-D

Given a TSP tour, the split procedure algorithm selects nodes to be visited by the drone to obtain a solution for the min-cost TSP-D, assuming that the relative order of the nodes is fixed. Other split procedures are now used widely in state-of-the-art metaheuristics such as [20, 21, 22, 23] to solve many variants of VRPs. We start from a given TSP tour \( s = (s_0, s_1, \ldots s_{n+1}) \) and must convert this tour into a feasible min-cost TSP-D solution. This is accomplished by removing nodes from the truck tour and substituting drone deliveries for those nodes. There are two main steps in
the split algorithm: auxiliary graph construction and solution extraction. The pseudo code for these is listed in Algorithms 2 and 3 respectively. The most important step of the split algorithm is the construction of the auxiliary graph, in which each subsequence of nodes \((s_i, \ldots s_k)\) can be turned into a drone delivery such that \(s_i\) is the launch node, \(s_k\) is the rendezvous node and \(s_j\), where \(\text{pos}(s_i, s) < \text{pos}(s_j, s) < \text{pos}(s_k, s)\), is the drone node. We now describe the split algorithm in detail.

**Building the auxiliary graph** In Algorithm 2, we construct an auxiliary weighted graph \(H = (V', A')\) based on the TSP tour \(s\) of the graph \(G = (V, A)\). We have \(V' = V\) and an arc \((i, j)\) \(\in A'\) that represents a subroute from \(i\) to \(j\), where \(\text{pos}(s_i, s) < \text{pos}(s_j, s) < \text{pos}(s_k, s)\). If \(i\) and \(j\) are adjacent nodes in \(s\), then the cost \(c_{ij}\) is calculated directly as follows:

\[
\text{if } i \text{ and } j \text{ are adjacent nodes in } s, \text{ then the cost } c_{ij} = C_1 d_{ij}. \quad (17)
\]

However, when \(i\) and \(k\) are not adjacent and a node \(j\) exists between \(i\) and \(k\) such that \langle i, j, k \rangle \in \mathcal{P}\), then

\[
c_{ik} = \min_{(i,j,k) \in \mathcal{P}} \text{cost}(\text{sub}(i,k,s)) + C_1 \left( d_{\text{prev}(j,s)\text{next}(j,s)} - d_{\text{prev}(j,s)j} - d_{j,\text{next}(j,s)} \right) + \text{cost}(i,j,k). \quad (18)
\]

If \(i\) and \(k\) are not adjacent and no node \(j\) exists between \(i\) and \(k\) such that \langle i, j, k \rangle could be a drone delivery, then

\[
c_{ik} = +\infty. \quad (18)
\]

The arc’s cost calculation is shown in lines 1 to 20 in Algorithm 2. Moreover, in lines 19 and 20, we store the list of possible drone deliveries \(\mathcal{T}\). This list will be used in the extraction step.

The auxiliary graph is used to compute the cost \(v_k\) of the shortest path from the depot to node \(k\). Because the graph is a directed acyclic graph, these values can be computed easily using a dynamic programming approach. Moreover, an arc \((i, k)\) in the shortest path that does not belong to the initial TSP tour means that a drone delivery can be made where \(i\) is the launch node, \(k\) is the rendezvous node, and the delivery node is a node between \(i\) and \(k\) in the TSP tour. This computation ensures that no interference occurs between the chosen drone deliveries. We therefore obtain the best solution from the TSP tour while respecting the relative order of the nodes.

In detail, given \(v_0 = 0\), the value \(v_k\) of each node \(k \in V' \setminus \{0\}\) is then calculated by

\[
v_k = \min \{ v_i + c_{ik} : (i, k) \in A' \} \quad \forall k = 1, 2, \ldots, n + 1. \quad (19)
\]

We also store the shortest path from 0 to \(n + 1\) in \(P(j), where j = 1 \ldots n + 1; j\) is the node, and the value \(P(j)\) is the previous node of \(j\). These steps are described in lines 22 to 28 in Algorithm 2.
auxiliary graph for Figure 1 is shown in Figure 2.

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Algorithm 2: SplitAlgorithm, Step 1: Building the auxiliary graph

Data: TSP tour \( s \)

Result: \( P \) stores the shortest path from the auxiliary graph, \( V \) is the cost of that shortest path, and \( T \) is a list of the possible drone deliveries + costs

/* List of 3-tuple elements \((i, j, \text{cost}(i, j))\) */ arcs = \( \emptyset \);

\( T = \emptyset \);

/* Auxiliary graph construction - Arcs */

foreach \( i \) in \( s \setminus \{ \text{last}(s) \} \) do

\( k = \text{pos}(i, s) + 1 \);

arcs = arcs \( \cup \) \((i, k, \text{cost}(i, k, s))\);

foreach \( i \) in \( s \setminus \{ \text{last}(s), s[\text{pos(last}(s, s) - 1]] \} \) do

foreach \( k \) in \( s \) : \( \text{pos}(k, s) \geq \text{pos}(i, s) + 2 \) do

minValue = \( \top \);

minIndex = \( \top \);

foreach \( j \) in \( s \) : \( \text{pos}(i, s) < \text{pos}(j, s) < \text{pos}(k, s) \) do

if \( \langle i, j, k \rangle \in P \) then

\( \text{cost} = \text{cost}(\text{sub}(i, k, s)) + C_1 (d_{\text{prev}(j, s)}n_{\text{ext}(j, s)} - d_{\text{prev}(j, s)}i - d_{\text{next}(j, s)}j) + \text{cost}(i, j, k) \);

if \( \text{cost} < \text{minValue} \) then

minValue = \( \text{cost} \);

minIndex = \( \text{pos}(j, s) \);

arcs = arcs \( \cup \) \{(i, k, minValue)\} ;

if \( \text{minIndex} \neq \top \) then

\( T = T \cup \{(i, s[minIndex], k, minValue)\} \);

/* Auxiliary graph construction: calculate the cost of each vertex */

\( V[0] = 0.0 \);

\( P[0] = 0 \);

foreach \( k \) in \( s \setminus \{ 0 \} \) do

foreach \( (i, k, \text{cost}) \) in \( \text{arcs} \) do

if \( V[k] > V[i] + \text{cost} \) then

\( V[k] = V[i] + \text{cost} \);

\( P[k] = 1 \);

return \((P, V, T)\);
Extracting min-cost TSP-D solution  Given $P(j), j = 1 \ldots n + 1$ defined as above and a list of possible drone deliveries $T$, we now extract the min-cost TSP-D solution in Algorithm 3. In the first step, given $P$, we construct a sequence of nodes $S_a = 0, n_1, \ldots, n + 1$ representing the path from 0 to $n + 1$ in the auxiliary graph (lines 2 to 9). Each two consecutive nodes in $S_a$ are a subroute of the complete solution. However, they might include a drone delivery; consequently, we need to determine which node might be the drone node in the subroute, which is computed in $T$.

The second step is to construct a min-cost TSP-D solution. To do that, we first initialize two empty sets: a set of drone deliveries $S_d$ and a set representing the truck’s tour sequence $S_t$ (lines 11 and 12). We now build these sets one at a time.

For drone delivery extractions, we consider each pair of adjacent positions $i$ and $i + 1$ in $P_{new}$ and determine the number of in-between nodes. If there is at least one $j$ node between the $i$ and $i + 1$ positions in the TSP tour, we will choose the drone delivery in $T$ with the minimum value, taking its drone node $j$ as the result (lines 15 to 17).

To extract the truck’s tour, we start from the depot 0 in $S_a$. Each pair $i, i + 1 \in S_a$ is considered as a subroute in the min-cost TSP-D solution by taking the nodes from $i$ to $i + 1$ in the TSP solution. However, in cases where $i$ and $i + 1$ are launch and rendezvous nodes of a drone
Algorithm 3: Split_Algorithm step 2: Extract_TSPD_Solution

Data: P stores the path in the auxiliary graph, V is the cost of the path in P, T is the list of drone deliveries + costs, and tspTour is the truck-only TSP tour

Result: tspdSolution

1 /* Construct the sequence of nodes representing the path stored in P */
2 j = n + 1 ;
3 i = ⊤ ;
4 S_a = j ;
5 while i ≠ 0 do
6   i = P[j] ;
7   S_a = S_a ∪ i ;
8   j = i ;
9   S_a = S_a.reverse() ;
10 /* Create a min-cost TSP-D solution from S_a */
11 S_d = ∅ ;
12 S_t = ∅ ;
13 /* Drone deliveries */
14 for i = 0; i < S_a.size - 1; i++ do
15   if between S_a[i] and S_a[i + 1] in tspTour, there is at least one node then
16     n_drone = obtain the associated drone node in tuples T ;
17     S_d = S_d ∪ ⟨S_a[i], n_drone, S_a[i + 1]⟩ ;
18 /* Truck tour */
19   currentPosition = 0 ;
20 while currentPosition ≠ n + 1 do
21   if currentPosition is a launch node of a tuple t in S_d then
22     S_t = S_t + all the nodes from the currentPosition to the return node of t in tspTour except the drone node ;
23     currentPosition = the return node of t ;
24 else
25     S_t = S_t ∪ currentPosition currentPosition = tspTour[indexOf(currentPosition) + 1] ;
26 tspdSolution = (S_t, S_d) ;
27 return tspdSolution ;

4.2. Local search operators

Two of our local searches are inspired from the traditional move operators Two-exchange and Relocation [24]. In addition, given the characteristics of the problem, we also develop two new move operators, namely, "drone relocation," which is a modified version of the classical relocation operator, and "drone removal," which relates to the removal of a drone node. In detail, from a min-cost TSP-D solution (TD, DD), we denote the following:

- N_T(TD, DD) = {e : e ∈ TD, ⟨e, ·, ·⟩ ∉ DD, ⟨·, ·, e⟩ ∉ DD} is the set of truck-only nodes in the solution (TD, DD) that are not associated with any drone delivery

- N_D(TD, DD) = {e : ⟨·, e, ·⟩ ∈ DD} is the set of drone nodes in the solution (TD, DD)

We now describe each operator.

Relocation: This is the traditional relocation operator with two differences: (1) We consider only truck-only nodes; (2) we only relocate into a new position in the truck’s tour. An example is
shown in Figure 3. In detail, we denote

\[
\text{relocate}_{T}((TD, DD), a, b), a \in N_{T}(TD, DD), b \in TD, b \neq a, b 
\neq 0
\]  

(20)

as the operator that—in effect— relocates node \(a\) before node \(b\) in the truck tour.

Drone relocation: The original idea of this operator is that it can change a truck node to a drone node or relocate an existing drone node so it has different launch and rendezvous locations. The details are as follows: (1) We consider both truck-only and drone nodes; (2) each of these nodes is then relocated as a drone node in a different position in the truck’s tour. This move operator results in a neighbourhood that might contain more drone deliveries; hence, it has more possibilities to reduce the cost. An example is shown in Figure 4. More precisely, we denote

\[
\text{relocate}_{D}((TD, DD), a, i, k)
\]  

(21)

\[a \in N_{T}(TD, DD) \cup N_{D}(TD, DD), i, k \in TD \setminus \{a\}, i \neq k,
\]

\[\text{pos}(i, TD) < \text{pos}(k, TD), (i, a, k) \in P
\]

as the operator procedure, where \(a\) is the node to be relocated and \(i\) and \(k\) are two nodes in \(TD\). There are two possibilities for effects: (1) If \(a\) is a truck-only node, this move creates a new drone delivery \((i, a, k)\) in \(DD\) and removes \(a\) from \(TD\); (2) if \(a\) is a drone node, the move changes the drone delivery \((\cdot, a, \cdot)\) to \((i, a, k)\).

Drone removal: In this move operator, we choose a drone node \(j \in TD\) and replace the drone delivery by a truck delivery. An example is shown in Figure 5. In detail, we denote

\[
\text{remove}_{D}((TD, DD), j, k), j \notin TD, (\cdot, j, \cdot) \in DD, k \in TD, k \neq \{0\}
\]  

(22)

as the operator procedure, where \(j\) is the drone node to be removed and \(k\) is a node in \(TD\) such that \(j\) will be inserted before \(k\). As a result, we have a new solution in which the number of nodes in \(TD\) has been increased by one and \(DD\)’s cardinality has been decreased by one.

Two-exchange: We exchange the position of two nodes. There are three possibilities: (1) When the exchanged nodes are both drone nodes, we make the change in the drone delivery list; (2) when the exchanged nodes are both truck nodes, we first exchange their positions in the truck sequence and then apply changes in the drone delivery list; and (3) when the exchanged nodes are a truck node and a drone node, we remove the old tuple and create a new one with the exchanged node. Next, we update the truck sequence and apply changes to the tuples if the truck node is associated with any drone delivery. An example is shown in Figure 6. In detail, we denote

\[
\text{two\_exchange}((TD, DD), a, b), a, b \in V \setminus \{0, n + 1\}, a \neq b
\]  

(23)

as the operator procedure, where \(a\) and \(b\) are the two nodes to be exchanged. We then swap their positions. The three swap possibilities are as follows: (1) a drone node with a node in \(TD\); (2) two drone nodes; and (3) two nodes in \(TD\).

Of course, all the moves must satisfy the constraints of the problem.

5. TSP-LS Heuristic

The TSP-LS algorithm is adapted from the work of [11]. The algorithm starts by calculating a TSP tour and then repeatedly relocates customers until no more improvement can be reached. The outline is shown in Algorithm 4. Lines 1–8 define the global variables, which are Customers = \([1, 2, \ldots, n]\), the sequence of truck nodes—truckRoute, and an indexed list truckSubRoutes of
smaller sequences that represent the subroutes in truckRoute. The distinct combination of elements in truckSubRoutes must be equal to truckRoute. We define \( i^*, j^*, k^* \), where \( j^* \) is the best candidate for relocation and \( i^* \) and \( k^* \) denote the positions between which \( j^* \) will be inserted. We also store maxSavings, which is the cost improvement value of this relocation. The two Boolean variables isDroneNode and Stop respectively determine whether a node in a subroute is a drone node and whether TSP-LS should terminate. These global variables are updated during the iterations, and
Figure 6: A two-exchange move operator in which a drone node is exchanged with a truck node: $\text{two\_exchange}((TD, DD), 3, 2)$

the heuristic terminates when no more positive $\text{maxSavings}$ can be achieved ($\text{maxSavings} = 0$).

**Algorithm 4: TSP-LS heuristic**

| Data: | truck-only sequence $\text{truckRoute}$ |
|-------|---------------------------------------|
| Result: | TSP-D solution $\text{sol}$ |

1. Customers = $N$;
2. $\text{truckRoute} = \text{solveTSP}(N)$;
3. $\text{truckSubRoutes} = \{\text{truckRoute}\}$;
4. $\text{sol} = (\text{truckRoute}, \emptyset)$;
5. $i^* = -1$;
6. $j^* = -1$;
7. $k^* = -1$;
8. $\text{maxSavings} = 0$;
9. $\text{isDroneNode} = \text{null}$;
10. $\text{Stop} = \text{false}$;
11. repeat
12. \hspace{1em} foreach $j \in \text{Customers}$ do
13. \hspace{2em} $\text{savings} = \text{calcSavings}(j)$;
14. \hspace{2em} foreach $\text{subroute} \in \text{truckSubRoutes}$ do
15. \hspace{3em} if $\text{drone}(\text{subroute}, \text{sol})$ then
16. \hspace{4em} $(\text{isDroneNode}, \text{maxSavings}, i^*, j^*, k^*) = \text{relocateAsTruck}(j, \text{subroute}, \text{savings})$;
17. \hspace{3em} else
18. \hspace{4em} $(\text{isDroneNode}, \text{maxSavings}, i^*, j^*, k^*) = \text{relocateAsDrone}(j, \text{subroute}, \text{savings})$;
19. \hspace{2em} if $\text{maxSavings} > 0$ then
20. \hspace{3em} $(\text{sol}, \text{truckRoute}, \text{truckSubRoutes}, \text{Customers}) = \text{applyChanges}(\text{isDroneNode}, i^*, j^*, k^*, \text{sol}, \text{truckRoute}, \text{truckSubRoutes}, \text{Customers})$;
21. \hspace{3em} $\text{maxSavings} = 0$;
22. \hspace{2em} else
23. \hspace{3em} $\text{Stop} = \text{true}$;
24. until $\text{Stop}$;
25. return $\text{truckSubRoutes}$;

For an additional notation used in Algorithm 4, line 15, given a solution $(TD, DD)$, we denote...
drone(s, (TD, DD)) ∈ \{True, False\} as True if the subsequence s in TD is associated with a drone:

\[
\text{drone}(s, (TD, DD)) = \begin{cases} 
  \text{True} & \text{if } \exists j \in V(s), j \neq \text{first}(s), j \neq \text{last}(s) : (\text{first}(s), j, \text{last}(s)) \in DD; \\
  \text{False} & \text{if } \forall j \in V(s), j \neq \text{first}(s), j \neq \text{last}(s) : (\text{first}(s), j, \text{last}(s)) \notin DD.
\end{cases}
\]

In detail, each iteration has two steps: (1) Consider each customer in Customers to determine the best candidate for relocation along with its new position and the cost savings. (2) If the candidate relocation can improve the current solution, then relocate the customer by updating truckRoute and truckSubRoutes and remove it from Customers so it will not be considered in future iterations; otherwise (when the candidate relocation cannot improve the current solution), the relocation terminates. We now explain each step and its implementations in Algorithms 4, 5, 6, 7, 8.

Step 1 of the iteration is presented from lines 12 to 18 in Algorithm 4. It first considers each customer j (line 12) and then calculates the cost savings by removing j from its current position (line 13). The calculation is shown in Algorithm 5. Next, line 14 considers each subroute in truckSubRoute as a possible target for the relocation of j. When the current considered subroute is a drone delivery (line 15), we then try to relocate j into this subroute as a truck node (line 16); otherwise, we try to relocate j as a drone node to create a new drone delivery (line 18). The relocation analyses of j as a truck node or a drone node are presented in Algorithms 6 and 7, respectively.

Algorithm 5: calcSavings(j)

Data: j : a customer currently assigned to the truck 
Result: Solution
1. \( i = \text{prev}_{\text{truckRoute}}(j) \); 
2. \( k = \text{next}_{\text{truckRoute}}(j) \); 
3. \( \text{savings} = (d_{ij} + d_{jk} - d_{ik})C_1 \); 
4. return savings;

In Algorithm 5 we aim to find the best position in subroute s to insert the current customer under consideration j by checking each pair of adjacent nodes i and k in s (line 3). If the cost of inserting j in this position is less than the current savings, then relocating j here results in some savings (line 5). Furthermore, because this subroute has a drone delivery, we need to check whether inserting j into it still lies within the drone’s power limit so that the truck can still pick up the drone (line 6). Finally, if the cost saved is below the best known maxSavings, we apply the changes to this location by updating the values of isDroneNode, i*, j*, k*, and maxSavings (lines 7
Algorithm 6: relocateAsTruck(j, subroute, savings)—Calculates the cost of relocating the customer j into a different position in the truck’s route

Data:
- j: current customer under consideration
- s: current subroute under consideration
- savings: savings that occur if j is removed from its current position

Result: Updated i*, j*, k*, isDroneNode

1. \( a = \text{first}(s) \)
2. \( b = \text{last}(s) \)
3. \( \text{foreach } (i, k) \in A(s) \) do
4. \( \Delta = (d_{ij} + d_{jk} - d_{ik})C_1 \)
5. \( \text{if } \Delta < \text{savings} \) then
6. \( \text{if } \text{dist}_T(s) + (d_{ij} + d_{jk} - d_{ik}) < \text{Drone endurance} \) then
7. \( \text{if } \text{savings} - \Delta > \text{maxSavings} \) then
8. \( \text{isDroneNode} = \text{False}; \)
9. \( j^* = j; i^* = i; k^* = k; \)
10. \( \text{maxSavings} = \text{savings} - \Delta; \)
11. return (isDroneNode, maxSavings, i*, j*, k*);

In Algorithm 7, we consider the relocation of a customer j in a subroute s that does not have drone delivery. The objective is simple: try to make j become the drone node of this subroute to reduce cost. Hence, we consider each pair of i and k in s, where i precedes k (line 1), and check whether \( (i, j, k) \) could be a viable drone delivery (line 2). Next, we check whether the relocation is better than the best known maxSavings in line 5. Finally, we update the relocation information in lines 6–8 as in Algorithm 6.

Algorithm 7: relocateAsDrone(j, subroute, savings) - Calculates the cost of relocating customer j as a drone node

Data:
- j: current considered customer
- s: current considered subroute
- savings: current savings if j is removed from its position

Result: Updated i*, j*, k*, isDroneNode

1. for i = 0 to size(s) - 2 do
2. \( \text{for } k = i + 1 \text{ to size}(s) - 1 \) do
3. \( \text{if } (s[i], j, s[k]) \in \mathcal{P} \) then
4. \( \Delta = (d'_{s[i],j} + d'_{s[k]})C_2 \)
5. \( \text{if } \text{savings} - \Delta > \text{maxSavings} \) then
6. \( \text{isDroneNode} = \text{True}; \)
7. \( j^* = j; i^* = s[i]; k^* = s[k]; \)
8. \( \text{maxSavings} = \text{savings} - \Delta; \)
9. return (isDroneNode, maxSavings, i*, j*, k*);

In step 2 of the iteration in Algorithm 8 when any cost reduction exists (maxSavings ≠ 0), we apply the changes based on the current values of i*, j*, k*, and isDroneNode. If isDroneNode = True,
we relocate $j^*$ between $i^*$ and $k^*$ as a drone node, forming a drone delivery (line 1 to 5). Otherwise, $j^*$ is inserted as a normal truck node (line 6 to 8). More specifically, these changes take place on the truckRoute and truckSubRoutes.

Returning to Algorithm 4, after the changes have been applied in line 18, we reset the value of maxSavings to 0 to prepare for the next iteration. Moreover, the algorithm terminates when maxSavings = 0 (line 21).

Algorithm 8: applyChanges function

Data: isDroneNode, $i^*$, $j^*$, $k^*$, sol, truckRoute, truckSubRoutes, Customers

Result: Updated truckRoute, truckSubRoutes, t

1 if isDroneNode == True then
2 The Drone is now assigned to $i^* \rightarrow j^* \rightarrow k^*$;
3 Remove $j^*$ from truckRoute and truckSubRoutes;
4 Append a new truck subroute that starts at $i^*$ and ends at $k^*$;
5 Remove $i^*, j^*, k^*$ from Customers;
6 else
7 Remove $j^*$ from its current truck subroute;
8 Insert $j^*$ between $i^*$ and $k^*$ in the new truck subroute;
9 Update sol using truckRoute and truckSubRoutes;
10 return (sol, truckRoute, truckSubRoutes, Customers);

6. Experiment settings

For the experiments, we generate customer locations on a plane satisfying the triangle inequality. We consider graphs with 10, 50 and 100 customers. These customers are created in regions with three different areas: 20 km$^2$, 100 km$^2$ and 500 km$^2$. In total, 35 instances were generated; their characteristics are shown in Table 1:

| Instances | Customers size | Region size (km$^2$) |
|-----------|----------------|----------------------|
| A1 to A5  | 10             | 20                   |
| B1 to B5  | 50             | 20                   |
| B6 to B10 | 50             | 100                  |
| B11 to B15| 50             | 500                  |
| C1 to C5  | 100            | 20                   |
| C6 to C10 | 100            | 100                  |
| C11 to C15| 100            | 500                  |

Table 1: Instances of min-cost TSP-D

For all tests, the speed of the drone and truck are both set to 40 km/h. Moreover, distance $d_{ij}$ is calculated using Manhattan distance, while $d'_{ij}$ is in Euclidean distance, so that we can partially simulate the road network of the truck and the shortest path travel of the drone, respectively. Let $\epsilon$ be the drone’s endurance, which can be varied to investigate the relationship between the drone’s endurance and the distance among customers (or the size of the region). By default, $\epsilon$ is set to 13 km, which equals 20 minutes of flight time. The truck’s cost $C_1$ is set to 25 times the drone’s cost $C_2$. 
For the results, we denote $\gamma$, $T$, and $\rho$ as the objective value, running time in seconds and performance ratio, respectively, defined as follows:

$$\rho = \frac{\text{value}}{\text{referenceValue}} \times 100,$$

(24)

where $\text{value}$ is the objective value obtained by the considered algorithm and $\text{referenceValue}$ is the objective value obtained by a reference algorithm. We will specify these algorithms for each experiment. Because we are dealing with a minimization problem, a ratio $\rho$ less than 100 % means that the considered algorithm provides a better solution than the reference algorithm.

CPLEX 12.6.2 is used whenever the MIP model needs to be solved, and optimal TSP tours are obtained with the state-of-the-art Concorde solver [25]. During the local search, we consider the neighbourhood of the drone relocation operator $\text{relocate}_d((TD, DD), a, i, k)$, where $\text{pos}(k, TD) - \text{pos}(i, TD) = 1$, because the transportation cost of the drone is significantly lower than that of the truck. Hence, in the min-cost TSP-D optimal solution, the truck never services a customer while waiting for the drone.

7. Results

In this section, we discuss the computational results obtained by the proposed methods. All the algorithms are implemented in Scala and run on an Intel Core i7-5500U 2.40 GHz low-voltage processor. Different comparisons are provided to evaluate the performance of each method: the TSP solution (i.e., no drone delivery), the optimal approach for the min-cost TSP-D based on the MIP model (if possible), GRASP and TSP-LS.

7.1. Comparison with the min-cost TSP-D optimal solution

The tests show that the MIP model cannot solve to optimality instances with more than 10 nodes under a time-limit constraint of 1 hour. Therefore, in this subsection, we use only the 10-customer instances to compare the solutions obtained by GRASP, TSP-LS and TSP with the optimal solution of the min-cost TSP-D computed through the MIP model. The $\text{referenceValue}$ used to compute the ratio $\rho$ is the optimal min-cost TSP-D solution. The number of TSP tours generated in the GRASP algorithm, $n_{\text{TSP}}$, is set to 10. The comparison results reported in Table 2 show that GRASP-10 can find all optimal solutions using much less computation time than the MIP model. Although TSP-LS is faster at providing a solution, it cannot find any optimal solution and is clearly outperformed by GRASP-10 in term of solution quality. From the column "TSP", we observe that adding the drone saves more than 60 % of the transportation cost. Next, we focus on analysing the performance of GRASP and TSP-LS on the larger instances.

| Instance | TSP | MIP Model | GRASP-10 | TSP-LS |
|----------|-----|-----------|----------|--------|
|          | $\gamma$ | $\rho$ | $\gamma$ | $T$ | $\rho$ | $\gamma$ | $T$ | $\rho$ | $\gamma$ | $T$ | $\rho$ |
| A1       | 4570 | 160.37    | 2849.66  | 188 | 100    | 2849.66  | 0.4  | 100    | 3452.93  | 0.4  | 121.17 |
| A2       | 4910 | 164.76    | 2979.99  | 176 | 100    | 2979.99  | 0.5  | 100    | 3538.00  | 0.3  | 118.73 |
| A3       | 4883 | 164.70    | 2964.78  | 96  | 100    | 2964.78  | 0.7  | 100    | 3935.54  | 0.2  | 132.74 |
| A4       | 5251 | 165.90    | 3164.99  | 213 | 100    | 3164.99  | 0.5  | 100    | 4130.59  | 0.2  | 130.51 |
| A5       | 4601 | 162.69    | 2827.92  | 125 | 100    | 2827.92  | 0.7  | 100    | 3066.49  | 0.2  | 108.44 |

Table 2: Comparison with the TSP-D optimal solution
7.2. Performance of heuristics on larger instances

In this subsection, we aim to analyse the performance of GRASP and TSP-LS on the larger instances—those with 50 and 100 customers. The TSP, TSP-LS and two versions of GRASP called GRASP-fast and GRASP-slow, corresponding to two values for the parameter \( n_{\text{TSP}} \) (20 and 100), are tested and compared with each other. The reference value used to compute the ratio \( \rho \) is the objective function of the TSP optimal solution. Tables 3 and 4 show the results for the instances with 50 and 100 customers, respectively.

As can be observed, both versions of GRASP outperform TSP-LS. With only 20 TSP tours generated, the GRASP-fast algorithm shows remarkable dominance over the TSP-LS algorithm. More specifically, it provides better solutions. In terms of running time, its average is slightly slower on the 50-customer instances but is three times faster on the 100-customer instances. Another observation is that TSP-LS runs more slowly on higher-density instances, which are generated in larger regions. However, this is not true for GRASP. This can be explained as follows. In a larger area, given a fixed drone endurance, the number of possible drone deliveries is significantly lower than in smaller areas. In Algorithm 7, TSP-LS considers every pair \( i, i+1 \) in every subroute to relocate \( j \) as a drone node between them (line 1 to 3). For each possible drone delivery, it must compute and analyse the possible cost savings associated with a relocation (lines 4 to 8). When the number of possible drone deliveries \( P \) is large, this process is quite time consuming. On the other hand, GRASP is not dependent on the size of \( P \) because it considers only the existing drone deliveries, which is a fairly small number compared with the cardinality of \( P \). Therefore, the running time of GRASP is quite insensitive to the size of the instance region.

In addition, it is obvious that increasing the parameters \( n_{\text{TSP}} \) of GRASP leads to higher-quality solutions. GRASP-slow gives the best found solutions, which are reported in Column \( \gamma \). Regardless of slow speed, its computational time is still acceptable (approximately 10 minutes on average for the 100-customer instances). The results obtained once again prove the effectiveness of using the drone for delivery. GRASP-slow gives solutions with a cost saving of more than 30 % compared with the TSP optimal solutions, which do not use any drone delivery.

7.3. Impact of cost ratio

In this experiment, we explore the impact of the drone/truck cost ratio on the objective values of the solutions provided by the GRASP-fast, GRASP-slow and TSP-LS algorithms. By default, this quantity is set to 1:25; therefore, we added two more ratios, 1:10 and 1:50. Table 5 shows the average values of \( \rho \) for the three heuristics. The reference value used to compute the ratio \( \rho \) is the objective value of the TSP optimal solution.

Logically, the value of \( \rho \) should decrease as the ratio increases. However, it does not reduce proportionally. More specifically, for GRASP-slow, when the ratio changes from 1:10 to 1:25, the average \( \rho \) decreases by approximately 7 % for the 50-customer instances and approximately 10 % for the 100-customer instances. In contrast, as the ratio changes from 1:25 to 1:50, the \( \rho \) decreases by only approximately 1 % for both cases. The same phenomenon is observed for TSP-LS and GRASP-fast. Consequently, when constructing distribution networks for drone/truck combinations, overestimating the transportation cost of the drone does not always improve the results. This means that the efficiency of investment in improving the cost ratio should be carefully considered because such an investment may prove more expensive than the savings in transportation costs.

Varying the cost ratio does not significantly impact the relative performance among the heuristics. The GRASP-slow algorithm still provides the best solutions, and the GRASP-fast
Algorithm outperforms the TSP-LS algorithm in almost all cases. However, on 100-customer instances where the ratio is 1:10, TSP-LS is slightly better.
Table 5: Performance of heuristics with different cost-ratio settings

| Cost ratio | N = 50 |          | N = 100 |          |
|------------|--------|----------|---------|----------|
|            | GRASP-fast | GRASP-slow | TSP-LS | GRASP-fast | GRASP-slow | TSP-LS |
| 1:10       | 77.28  | 72.75    | 83.54   | 83.45    | 79.76     | 83.06  |
| 1:25       | 72.35  | 65.69    | 78.82   | 75.38    | 69.76     | 78.04  |
| 1:50       | 67.06  | 64.88    | 75.94   | 73.16    | 68.48     | 75.19  |

8. Conclusion

This paper introduces a new variant of the Traveling Salesman Problem with Drone (TSP-D) whose objective is to minimize the total transportation cost. We propose an abstract model, a mixed integer programming formulation and two heuristic methods—GRASP and TSP-LS—to solve the problem. TSP-LS is adapted from an existing heuristic, while GRASP is based on our new split algorithm and local searches. Numerous experiments conducted on a variety of instances show the performance of our GRASP algorithm. Overall, its faster version outperforms TSP-LS in terms of both solution quality and running time, while its slower version provides better solutions in an acceptable computational time. In future work, we intend to use our new split algorithm to build more complex metaheuristics, e.g., genetic algorithms, Variable Neighbourhood Search (VNS) to solve the problem as well as test other variants of TSP-D. It could also prove interesting to study more general problems that involve multiple trucks and multiple drones.

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