Research on the Reducer for Polar Subglacial Bedrock Coring Drilling

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Abstract. This paper presents a new type of reducer for coring drilling for polar subglacial bedrock. The structure of the reducer is designed, and the strength of the reducer meshing parts is checked. According to the contact strength formula, the contact strength between movable teeth and meshing parts is estimated. ANSYS Workbench is used to analyze the statics of the reducer. The results show that the reducer meets the design requirements, and provides a basis for optimizing the reducer for polar subglacial bedrock coring drilling.

1. Introduction

This paper presents the application of sine movable teeth transmission in the reducer for polar subglacial bedrock coring drilling. The sine movable teeth transmission has the advantages of compact structure, light weight, small volume, stable transmission, high transmission efficiency, good lubrication performance and simple sealing mode. Planetary gear transmission and harmonic gear transmission have been used in polar subglacial core drilling [1, 2, 3, and 4]. During a drilling of CRREL electromechanical drilling tool in Greenland, the flexible wheel of harmonic gear was severely worn and forced to stop drilling [5]. This is because the flexible gear of harmonic reducer is prone to fatigue failure, the load of supporting bearing is too high, and the wear-resistant life of solid lubrication film on the reducer tooth surface is short. The fatigue wear of planetary gears of armed cable-suspended electromechanical subglacial bedrock coring drill developed by Jilin University Polar Research Center is serious. This is because each pair of planetary gear teeth in the meshing process, the impact vibration of each other is prone to fatigue damage, too high torque will also cause the damage of reducer, and life is relatively short. The sine movable teeth transmission doesn’t have the process impact of the above-mentioned gear transmission and the engagement, and has the advantages of high multi-tooth meshing bearing capacity, good working force of the movable teeth, long service life and high torque.

2. Structure design of reducer

When the input shaft and movable teeth rack rotate in the same direction, drive ratio of the reducer is i.

\[ i = \frac{Z_1 + Z_3}{Z_1} \] (1)
Where $Z_1$ is number of sine raceway cycles outside the input shaft and $Z_3$ is the number of sine raceway cycles inside the shell.

Because the movable teeth are rolling connected with the envelope surface of the sine raceway outside the input shaft and the sine raceway inside the shell. According to the condition that the theoretical tooth profile curve of sine raceway doesn’t occur top cutting [6]. When the minimum curvature radius $\rho_{\text{min}}$ of the sine raceway curve $K$ on the cylindrical surface of the inner and outer raceway is not less than the small circular radius $r_a$ of the meshing part between the movable teeth and the sine raceway, the theoretical tooth profile curve of the sine raceway is continuous. On the contrary, the theoretical tooth profile curve of sine raceway occur top cutting. The expansion diagram of the theoretical tooth profile curve of sine raceway is shown in Fig. 1.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{The expansion diagram of the theoretical tooth profile curve of sine raceway.}
\end{figure}

The equation of the theoretical tooth profile curve of the sine raceway inside the shell is as follows

\[
\begin{align*}
  x &= r_3 \cos \varphi \\
  y &= r_3 \sin \varphi \\
  z &= A \sin(Z_3 \varphi)
\end{align*}
\] (2)

Where $r_3$ is the radius of cylindrical surface inside the shell, $r_3 = R + r \cdot \sin(a_n_3)$, mm; $R$ is the radius of rotation in the circumferential direction of the center of the movable teeth, mm; $r$ is the radius of movable teeth, mm; $\varphi$ is the position angle of the center of the movable teeth in the circumferential direction, rad; $A$ is sine amplitude, mm; $Z_3$ is the number of sine raceway cycles inside the shell.

Position vector of any point on curve $K$, $r = xi + yj + zk$. According to differential geometry, the curvature radius of the theoretical tooth profile curve of the sine raceway inside the shell is as follows

\[
\rho = \left| r'' \times r''' \right| / |r''|^3
\] (3)

Where after inserting formula (2), we obtain the minimum radius of curvature is as follows

\[
\rho_{\text{min}} = r_3^2 / \sqrt{A^2 Z_3^4 + r_3^2}
\] (4)

The radius of the small circle of the meshing part between movable teeth and sine raceway is $r_a$. 
Where $b_n$ is sine raceway depth, mm. Therefore, according to the condition that the theoretical tooth profile curve of sine raceway doesn’t occur top cutting. The shell and the input shaft should satisfy the following formulas.

$$
 r_n^2 = \sqrt{r^2 - (r - b_n)^2}
$$

(5)

Where $n = 1, 3$; $r_1$ is the radius of cylindrical surface outside the input shaft, mm; $r_3$ is the radius of cylindrical surface inside the shell, mm.

Set the movable tooth radius $r = 6$mm, and the sine raceway amplitude $A = 6$mm. According to the condition of correct movement of the sine tooth drive. When the radius of cylindrical surface outside the input shaft is $r_1 \geq 6.9$mm, and the radius of cylindrical surface inside the shell is $r_3 \geq 33.7$mm, the movable tooth and the sine raceway don’t undergo undercutting. Structural parameters of reducer are shown in Table 1.

### Table 1. Structural parameters of reducer.

| Basic Parameters                                      | Numerical number |
|-------------------------------------------------------|------------------|
| Drive ratio $i$                                       | 7                |
| Number of sine raceway cycles outside the input shaft, $Z_1$ | 1                |
| Number of sine raceway cycles inside the shell, $Z_3$  | 6                |
| Radius of movable teeth, $r$/mm                       | 6                |
| Sine amplitude, $A$/mm                                | 6                |
| Sine raceway radius, $r'$/mm                          | 6.2              |
| Radius of rotation in the circumferential direction of the center of the movable teeth, $R$/mm | 29.5              |

Based on the formulas and structural parameters derived above, the reducer of polar subglacial bedrock coring drilling is designed as shown in Fig. 2.
3. Strength Estimation of Reducer Meshing Parts

When the reducer transmit power, under the action of alternating stress, the contact between movable teeth and meshing parts is a curved surface elastomer, which will cause contact fatigue and wear, and affect the life and performance of reducer. In this paper, strength estimation of movable teeth and meshing parts is needed. The main curvature and other parameters are needed to estimate the contact strength of meshing parts, so the calculation of the main curvature of sine movable teeth transmission is studied first.

3.1. Calculation of principal curvature

According to the definition in differential geometry, the formula for calculating the sum of principal curvatures is as follows

$$\Sigma k = k_{11} + k_{12} + k_{21} + k_{22}$$  \hspace{1cm} (7)

It is known that the normal curvature of the movable tooth surface along the tangent direction at the contact point is the same, then the two principal curvatures of the movable tooth surface are $k_{11} = k_{12} = 1/r$. In theory, the direction of the instantaneous contact line is one of the main directions of the tooth surface of the sinusoidal raceway. The corresponding principal curvature is $k_{21} = 1/r'$. The other principal direction should be in the normal direction of the instantaneous contact line [7]. In order to obtain the principal curvature $k_{22}$ of the tooth surface of the sinusoidal raceway in this direction, the total curvature $K$ of the sinusoidal tooth surface at the contact point should be calculated. The definition of the total curvature in differential geometry is as follows [8, 9]

$$K = k_{21}k_{22} = \frac{(LN - M^2)}{(EG - F^2)}$$  \hspace{1cm} (8)

Where $E = \mathbf{\bar{r}}_u^2; F = \mathbf{\bar{r}}_v^2; G = \mathbf{\bar{r}}_v^2; L = -\mathbf{\bar{n}}_u \cdot \mathbf{\bar{r}}_u; M = -\mathbf{\bar{n}}_u \cdot \mathbf{\bar{r}}_v; N = -\mathbf{\bar{n}}_v \cdot \mathbf{\bar{r}}_v;

$\mathbf{\bar{r}}_u = \{-(R + r'\cos v)\sin(\phi)\phi_{u}' + r'\sin v \sin u \sin \phi - r'\sin v \cos u \cos(\phi)\phi_{u}'\}

\cos(\phi)\phi_{u}' - r'\sin v \sin u \cos \phi - r'\sin v \cos u \sin(\phi)\phi_{u}' - (R + r'\cos v)\cos(\phi)\phi_{u}'\}

\mathbf{\bar{r}}_v = \{\cos v \sin(\phi)\phi_{u}' + \sin v \sin u \sin(\phi) - \sin v \cos u \cos(\phi)\phi_{u}'\}

\cos v \cos(\phi)\phi_{u}' + \sin v \sin u \sin(\phi)\}

\mathbf{\bar{n}}_u = \{-\cos v \sin(\phi)\phi_{u}' + \sin v \sin u \sin(\phi) - \sin v \cos u \cos(\phi)\phi_{u}'\}

\cos v \cos(\phi)\phi_{u}' - \sin v \cos u \sin(\phi)\}

\mathbf{\bar{n}}_v = \{-\cos v \cos(\phi) - \sin v \cos u \sin(\phi)\}

\phi_{u}' = -\left(R^2 + A^2Z^2 \cos(\phi)\right)^{1/2} / ARZ^2; A is sine amplitude; Z is number of sine raceway cycles; R is the radius of rotation in the circumferential direction of the center of the movable teeth; u is directional angle of instantaneous contact line; v is contact angle; $\phi$ is the position angle of the center of the movable teeth in the circumferential direction.

3.2. Estimation of Contact Strength of Meshing Parts

The contact between movable teeth and raceway can be regarded as the elastic contact of two free-form surfaces. According to Hertz stress theory, the contact stress calculation formula of two free-form surface elastomers under total load $F$ is as follows

$$\sigma = \frac{3F}{2\pi ab}$$  \hspace{1cm} (9)

Where $a = m_a \sqrt{\frac{3F}{\sum k \frac{1 - v_i^2}{E_i} + \frac{1 - v_j^2}{E_j}}}$; $b = m_b \sqrt{\frac{3F}{\sum k \frac{1 - v_i^2}{E_i} + \frac{1 - v_j^2}{E_j}}}$ . The coefficients of $m_a$ and $m_b$ related to elliptic eccentricity can be obtained by looking up tables according to $\Sigma k$ and
principal curvature function $F(k)$. $F(k) = \frac{|(k_{11} - k_{12}) + (k_{21} + k_{22})|}{\sum k}$, where $k_{11}, k_{12}$ is two principal curvatures of movable teeth; $k_{21}, k_{22}$ is two principal curvatures of the tooth surface of sine raceway. Because the material of the two contact surfaces of the meshing pair is the same, the elastic modulus $E_1 = E_2 = E$ and Poisson's ratio $\nu_1 + \nu_2 = \nu$ are chosen. Through the force analysis of cylindrical sine movable teeth transmission, it can be seen that the contact force and stress between movable teeth and raceway reach the maximum near the inflection point of sine raceway. Therefore the maximum contact stress formula between movable teeth and the tooth surface of sine raceway can be written as follows

$$\sigma_{\text{max}} = \frac{1}{(\pi n_m m_p)} \cdot \sqrt{0.375F_{\text{max}} (\sum k)^2 E^2 / (1 - \nu^2)^2} \quad (10)$$

The movable tooth, the shell and the input shaft are all made of material ZGD650-830, the tensile strength $\sigma_b = 830\text{Mpa}$, and the yield strength $\sigma_s = 650\text{Mpa}$. The strength of the reducer meets the design requirements after the above strength check.

4. Static analysis of reducer

In this paper, the three-dimensional software Inventor is used to model the reducer, and the structure of the reducer is simplified correspondingly [11, 12]. The three-dimensional solid model is imported into the finite element ANSYS Workbench for static analysis. The static analysis nephogram of the reducer is shown in Fig. 2.
The material used in the simplified reducer is ZGD650-830, and the maximum yield strength of the material is 650 MPa. According to Fig. 3, the maximum equivalent stress of the reducer is 22 MPa, and the maximum stress occurs at the contact point between the movable tooth and the input shaft. The results show that the maximum equivalent stress of the reducer is far less than the yield strength of the material, and the overall structure of the reducer and the strength of its components meet the transmission requirements.

5. Conclusion
In this paper, a new type of reducer for polar subglacial bedrock coring drilling is proposed, and the structure of the reducer is designed. The contact strength of the meshing parts of the reducer is analyzed. According to the basic parameters, the three-dimensional solid model of the reducer is established and the finite element analysis is carried out. The analysis results show that the reducer meets the application requirements of the polar subglacial bedrock coring drilling, and lays a foundation for optimizing the research of the reducer of the polar subglacial bedrock coring drilling. It is suggested that the performance and reliability of the reducer in the driving circulation system of the polar subglacial bedrock coring drilling will be studied experimentally in the future.

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