Distributed Compressed Wideband Sensing in Cognitive Radio Sensor Networks

Huazi Zhang¹, Zhaoyang Zhang¹, Yuen Chau²
1. Dept.of Information Science and Electronic Engineering, Zhejiang University, China.
2. Singapore University of Technology and Design (SUTD), Singapore.
Email: ning_ming@zju.edu.cn, yuenchau@sutd.edu.sg

Abstract—A novel distributed compressed wideband sensing scheme for Cognitive Radio Sensor Networks (CRSN) is proposed in this paper. Taking advantage of the distributive nature of CRSN, the proposed scheme deploys only one single narrowband sampler with ultra-low sampling rate at each node to accomplish the wideband spectrum sensing. First, the practical structure of the compressed sampler at each node is described in detail. Second, we show how the Fusion Center (FC) exploits the sampled signals with their spectrum randomly-aliased to detect the global wideband spectrum activity. Finally, the proposed scheme is validated through extensive simulations, which shows that it is particularly suitable for CRSN.

I. INTRODUCTION

Many advanced sensor network applications require that the sensed information are delivered in the form of broad-band multimedia by resource-constrained sensor nodes, or need the sensor nodes to be survivable in highly dynamic spectrum environments. This is hard to achieve in traditional wireless sensor networks (WSN), since they usually compete for limited available bandwidth on fixed unlicensed bands. A promising sensor networking solution for these applications is the newly introduced paradigm of Cognitive Radio Sensor Networks (CRSN) [1], [2], [3], [4], which incorporates cognitive radio (CR) capability into the traditional WSN. Without causing harmful interference to the primary user (PU) systems, CRSN can provide dynamic spectrum access and the bandwidth as high as possible to meet the application-specific QoS requirement.

According to the NSF Spectrum Occupancy Measurements [6], the spectrum occupation pattern is sparse. Exploiting this sparse prior, several compressed wideband sensing schemes are developed based on the literatures of compressed sensing (CS) [3]. Tian developed a distributed compressed spectrum sensing approach for wideband CR networks [7]. Her scheme can achieve high-performance at low sampling rate below the Nyquist rate. Several other works [8], [9] have also studied the application of CS on wideband spectrum sensing. However, to the best of our knowledge, their emphasis is not on the specific sampling methods, i.e. little information is provided about the structure of the sampler and their feasibility in practical implementation. Moreover, their sampling rate for individual CR should exceed the sum of the active bandwidth, which is still too high for the energy-constrained CRSN nodes.

In this paper, we propose a novel distributed compressed wideband sensing scheme for CRSN. The main contributions of this paper are two-fold. First, we propose practical wide-band sampling structure in fine detail. Second, the required sampling rate in our scheme is equivalent to the bandwidth of a single subband, much lower than existing schemes. Our sampling structure is similar to the Sub-Nyquist sampler proposed recently by Mishali and Eldar - the modulated wideband converter (MWC) [11]. The major difference is that we only deploy one sampling channel in a single CRSN node, and each node randomly mixes the signals of all subbands by spectrum aliasing. This simple structure is extremely convenient for implementation in the resource-constrained CRSN, due to its low hardware cost and power consumption.

The rest of this paper is organized as follows. In Section II, we establish basic models for the PU signal and the CRSN, and formulate our target problem. In Section III, we introduced the distributed compressed wideband sensing scheme in detail. Then, extensive simulation results are presented in Section IV to further validate our proposed algorithm. Finally, the whole paper is concluded in Section V.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider the radio environment to be an ultra-wide frequency band where different PU systems and large numbers of CRSN nodes coexist.

A. Primary User Signal

The entire W wide spectrum is centered at zero and can be divided into L non-overlapping subbands of equal bandwidth B, thus \( W = L \cdot B \). We assume there are J PUs occupying some of the subbands. According to [6], the PU signal is sparse within the range of the whole spectrum. Therefore, we can safely assume that \( J \) is much smaller than \( L \). Note that we will take this assumption as our basic prerequisite throughout this paper.

We assume real-valued continuous-time wideband signal of the primary users to be \( x(t) \), which is band-limited within \((-\frac{W}{2}, \frac{W}{2})\) and its Fourier transform can be defined by

\[
X(f) = \int_{-\infty}^{+\infty} x(t)e^{-j2\pi ft}dt
\]  (1)

As mentioned above, the frequency domain is zero centered and divided into \( L \) subbands of equal bandwidth \( B \), \( X(f) \) can be re-written as
\[ X(f) = \sum_{l=-L_0}^{+L_0} X_l(f) \]  
where \( L = 2L_0 + 1 \) and 
\[ X_l(f) = \begin{cases} X(f), & f \in \left( (l-\frac{1}{2})B, (l+\frac{1}{2})B \right) \\ 0, & \text{otherwise} \end{cases} \] is the signal in the \( l \)th subband of the wideband spectrum. Thus, we can represent the primary signal using a vector \( \mathbf{X}(f) = [X_{L_0}(f), \cdots, X_0(f), \cdots, X_{-L_0}(f)]^T \). Since \( x(t) \) is real-valued continuous-time signal, the real part of Fourier transform \( X(f) \) is an even function, and the imaginary part is an odd function. Then, obviously \( |X_l(f)| = |X_{-l}(f)| \).

### B. Secondary User System: CRSN nodes and Fusion Center

The Secondary User (SU) system is comprised of CRSN nodes and Fusion Centers. The spatially distributed CRSN nodes can be wireless terminals such as webcams, cellphones and laptops. Their application-specific QoS requires that these CRSN nodes deliver event features reliably and timely in the form of multimedia, resulting in high bandwidth demands. Therefore, our objective is to design a resource-efficient wideband spectrum sensing scheme for the CRSN.

As shown in Fig.1, CRSN nodes located in a restricted locality form a cluster [5]. All nodes in one cluster sense the wideband cooperatively. There is a Fusion Center (FC) in each cluster, which is a special kind of node equipped with more or renewable power sources. As a result, FC have additional computation capability and longer transmission ranges. Since all the CRSN nodes in a cluster are close to each other, we assume that a common control channel can be found to exchange spectrum sensing information.

Suppose that there are \( K \) CRSN nodes engaged in the wideband spectrum sensing task. Because these \( K \) nodes are close to each other, the PU activities in their ambient radio environment are assumed to be identical. However, due to different channel fadings, the received primary signal can be different at each CRSN node. During the sensing interval, the primary signal received by the \( k \)th CRSN nodes can be expressed by

\[ X^k(f) = X(f)H^k(f) + W^k(f) \]  

Using \( \mathbf{H}^k(f) = \begin{bmatrix} H^k_{L_0}(f), \cdots, H^k_0(f), \cdots, H^k_{-L_0}(f) \end{bmatrix}^T \) and \( \mathbf{W}^k(f) = \begin{bmatrix} W^k_{L_0}(f), \cdots, W^k_0(f), \cdots, W^k_{-L_0}(f) \end{bmatrix}^T \) to represent channel fading and noise of all subbands, we can rewrite (3) in the matrix form

\[ \mathbf{X}^k(f) = \text{diag}(\mathbf{H}^k(f))\mathbf{X}(f) + \mathbf{W}^k(f) \]  

### III. Distributed Compressed Wideband Spectrum Sensing Scheme

In this section, we propose a cooperative spectrum sensing scheme that can effectively distribute the sensing task to many CRSN nodes and relieve the sensing and processing burden on a single node. We first describe the compressed aliasing sampling scheme originated from the Modulated Wideband Converter (MWC), then explain the fusion strategy and corresponding recovering algorithm.

As shown in Fig.2, our proposed sensing scheme comprises two steps.

**Step 1:** \( K \) CRSN nodes sample the wideband independently at ultra-low rate equivalent to the bandwidth of a single subband, then report the aliased spectrum information to the FC.

**Step 2:** After receiving all the aliased spectrum information from the \( K \) CRSN nodes, the FC performs compressed sensing recovery algorithm, and broadcast the sensing result to all the CRSN nodes within the cluster.

#### A. Distributed Sampling based on Spectrum Aliasing

The sampling method is similar to the Modulated Wideband Converter (MWC) [11][12]. The difference is that we only deploy one channel of the MWC system in a single CRSN node. The sampling structure in the \( k \)th CRSN node is shown in Fig.3.

According to Fig.3, after the primary signal \( x^k(t) \) arrives at the \( k \)th CRSN node, it is multiplied by a high frequency mixing function \( p^k(t) \), \( p^L(t) \) is a \( T_s \)-length periodic spread-spectrum mixing function which aims to alias the spectrum...
and thus obtain a mixture of signals from all the subbands.

\[ p^k(t) = \alpha_{km} m \frac{T_s}{M} \leq t \leq (m+1) \frac{T_s}{M}, 0 \leq m \leq M - 1 \] (5)

where \( \alpha_{km} \in \{+1, -1\} \), and \( p^k(t) = p^k(t + nT_s) \) for every \( n \in \mathbb{Z} \). In practice, \( p^k(t) \) is characterized by pseudorandom sequences generated by certain seeds known at the FC. Therefore, the aliasing pattern of the spectrum is known at the FC.

Then, the mixed analog spectrum signal goes through a Low-Pass Filter (LPF) with cutoff frequency \( 1/2T_s \) and the filtered signal is sampled at rate \( f_s = 1/T_s \). For convenient spectrum access, we choose \( f_s \) to be equal to \( B \), so low that existing A/D converters are competent for the task.

We perform discrete-time Fourier transform (DTFT) on the sampled data, in order to obtain the frequency domain form of the mixed spectrum. According to [11],

\[ Y^k(e^{j2\pi f T_s}) = \sum_{l=-L_0}^{+L_0} c^k_l X^k(f) e^{-j2\pi l f T_s} \] (6)

where \( f \in \mathcal{F}_s = [-B/2, B/2] \), \( L_0 = \left\lfloor \frac{W-B}{2B} \right\rfloor \), and \( c^k_l = B^{-1/2} \int_{-B/2}^{+B/2} p^k(t) e^{-j2\pi l t} dt \) is the Fourier coefficient of \( p^k(t) \). After a closer examination, we can easily find that (6) is a weighted sum of the spectrum of all subbands, and can be rewritten in the matrix form:

\[ Y^k(f) = c^k \left( \text{diag}(H^k(f)) X(f) + W^k(f) \right) \] (7)

where \( c^k = [c^k_{-L_0}, \ldots, c^k_{L_0}]^T \).

At last, we calculate the average level of the band-limited spectrum mixture, and transmit this aliased spectrum information to the FC through the control channel.

\[ Y^k \equiv \bar{Y}^k(f) = \int_{-B/2}^{+B/2} \left| Y^k(f) \right| df \] (8)

**B. Fusion Strategy and Recovering Algorithm**

\( K \) CRSN nodes engage in the spectrum sensing task and report their spectrum mixture value to the fusion center, all the reported value can compose a \( K \)-length vector \( \mathbf{Y} = [Y^1, \ldots, Y^K]^T \).

Here, we further use the assumption that all the \( K \) CRSN nodes and the FC are close to each other and their channel fadings are approximately the same, i.e. \( H^1(f) = \cdots = H^K(f) = H(f) \). It can be estimated at the FC and broadcasted to all the CRSN nodes.

The \( K \)-length vector collected from \( K \) distributed CRSN nodes to the FC can be expressed as

\[
Y = \begin{cases} 
+\frac{B}{2} & 
\int_{-\frac{B}{2}}^{+\frac{B}{2}} |C \left( \text{diag}(H(f)) X(f) + W(f) \right) | df \\
-\frac{B}{2} & 
\int_{-\frac{B}{2}}^{+\frac{B}{2}} |A(f) X(f) + W'(f) | df 
\end{cases}
\] (9)

where \( C \) is the Fourier coefficient matrix of mixing functions \( p(t) \), \( A(f) \) is a \( K \times L \) measurement matrix, \( A_{kl} = c^k_l H(f) \) and \( W' \) is also a noise vector.

Assuming that the channel fading are also almost flat within a single subband, i.e. \( H_l = H_l(f) = \int_{-\frac{B}{2}}^{+\frac{B}{2}} |H_l(f) | df \), equation (9) can be rewritten as

\[
Y = AX + W'
\] (10)

where \( A_{kl} = c^k_l H_l \), and we denote \( X = \int_{-\frac{B}{2}}^{+\frac{B}{2}} |X(f) | df \), \( W' = \int_{-\frac{B}{2}}^{+\frac{B}{2}} |W'(f) | df \) to be the average level of the primary signal and noise, respectively.

Based on the analysis in [11], \( A \) can be expressed as (11) (on the top of next page), where \( \theta = e^{-j\frac{2\pi f T_s}{W}} \) and

\[
d_l = \frac{1}{T_s} \int_{-\frac{B}{2}}^{+\frac{B}{2}} \frac{1}{\theta} e^{-j\frac{2\pi f T_s}{W}} dt = \begin{cases} 
\frac{1}{2} l & 
\text{if } \theta = 0 \\
\frac{l}{2\pi} & 
\text{if } \theta \neq 0 \neq 0 
\end{cases}
\] (12)

Considering (10), where \( X \) is an unknown sparse vector of dimension \( L \), \( \mathbf{Y} \) is the measured vector of dimension \( K \), and \( \mathbf{A} \) is the known random measurement matrix of size \( K \times L \).

Our final goal is to recover the original primary signal \( \mathbf{X} \) from the measured compressed vector \( \mathbf{Y} \) and determine the idle subbands for opportunistic spectrum access. This is exactly the compressed sensing (CS) problem [13] since \( K << L \).

It is proved that a random sign matrix, whose entries are drawn independently from \( \pm 1 \) with equal probability, has the RIP of order \( S \) if \( K \geq C \cdot S \log (L/S) \) [12][13], where \( C \) is a positive constant. Also, any fixed unitary row transformation of a random sign matrix has the same RIP [11]. From (12), we can see \( S_{K \times L} \) is a random sign matrix, this implies that \( A_{K \times L} \) also has RIP and (10) can be solved using existing CS recovery methods.

In this paper, we employ the basis pursuit (BP), a convex programming method that can solve CS problems in polynomial-time. We choose BP for the following two reasons. First, the sparsity level of the input signal is known in other OMP [14] related CS recovery algorithm. However, in our scenario, the sparsity level means the number of the active subbands and can not be estimated beforehand. Second, the BP algorithm has robust performance in the noisy scenario.

We first estimate global spectrum vector \( \hat{X} \) through \( l_1 \) minimization based noisy BP algorithm:

\[
\min \left\| \hat{X} \right\|_{l_1} \text{ subject to } \left\| A\hat{X} - Y \right\|_{l_2} \leq \sigma_W^2
\] (13)
where \( \sigma^2_w \) is the variance of the Gaussian noise \( W' \).

And then define a decision vector \( \hat{d} \) of the PU activity state by comparing the \( l \)th subband estimation \( \hat{X}_l \) with a decision threshold \( \lambda \):

\[
\hat{d} = [\hat{d}_{L_0}, \ldots, \hat{d}_1, \ldots, \hat{d}_{-L_0}]
\]

where \( \hat{d}_l = \begin{cases} 0, & X_l < \lambda \\ 1, & X_l > \lambda \\ \end{cases} \)

The decision vector serves as our compressed wideband spectrum sensing result. After this decision vector is obtained at FC, this final result is broadcasted to all the CRSN nodes within the cluster. The CRSN nodes then access the spectrum according to this sensing result.

IV. PERFORMANCE EVALUATION

To evaluate the accuracy of our proposed compressed sensing recovery algorithm, we define the normalized root mean-square estimation error (MSE):

\[
\text{MSE} = \mathbb{E}\left( \frac{\| \hat{X} - X \|_2}{\| X \|_2} \right)
\]

Just like any cognitive radio network, we choose the probability of detection \( P_d \) and the probability of false alarms \( P_f \) as our wideband sensing performance metrics. In the multi-band scenario, they can be expressed as

\[
P_d = \mathbb{E}\left( \frac{\text{Num}_{l=0}^{L_0} (d_{l}=1 \text{ and } \hat{d}_{l}=1)}{\text{Num}_{l=0}^{L_0} (d_{l}=1)} \right)
\]

\[
P_f = \mathbb{E}\left( \frac{\text{Num}_{l=0}^{L_0} (d_{l}=0 \text{ and } \hat{d}_{l}=1)}{\text{Num}_{l=0}^{L_0} (d_{l}=0)} \right)
\]

where \( d_{l} = \begin{cases} 0, & \text{the } l \text{th subband is idle} \\ 1, & \text{the } l \text{th subband is busy} \end{cases} \)

For the following Monte Carlo experiments, we repeat a hundred thousand times to compute the target value.

2) Performance of Compressed Sensing

In the first experiment, we examine the effect of measurement noise and \( K \) on the recovery accuracy in terms of MSE. As shown in Fig.4, we adjust the \( K \) from 25 to 50, and find that the MSE drops as \( K \) increases. It shows that, if too few measurements are collected, the performance of the compressed sensing algorithm will deteriorate.

3) Performance of Distributed Wideband Sensing

In the second experiment, we evaluate the performance of distributed wideband sensing in terms of \( P_d \) and \( P_f \) versus the number of engaged CRSN nodes \( K \). We adjust \( K \) from 25 to 50, and find that \( P_d \) increases quickly, while \( P_f \) drops.
A COMPARISON OF SAMPLING RATE REQUIRED BY DIFFERENT SCHEMES

| System Parameters                  | Conventional Nyquist Sampling | Existing Compressed Spectrum Sensing | Distributed Compressed Wideband Sensing |
|------------------------------------|-------------------------------|-------------------------------------|----------------------------------------|
| Total Bandwidth                    | 6 GHz                         | 3.6 GHz                              | 30 MHz                                 |
| Subband Bandwidth                  |                               |                                     |                                        |
| Occupied Bandwidth                 |                               |                                     |                                        |
| Total Subbands                     |                               |                                     |                                        |
| Occupied Subbands                  |                               |                                     |                                        |
| Occupation Ratio                   |                               |                                     |                                        |

Comparison of Sampling Rate at Individual Node

- Conventional Nyquist Sampling: 6 GHz
- Existing Compressed Spectrum Sensing: 3.6 GHz
- Distributed Compressed Wideband Sensing: 30 MHz

This implies that as more CRSN nodes engaged in the sensing task, the detecting performance of the cluster improves. The numerical results are shown in Fig.5.

We also computed the Receiver Operating Characteristic (ROC) curve under different $K$. As shown in Fig.6, the closer the curve is to the upper left corner, the better the performance is. The four curves represent the cases when CRSN nodes numbers are 25, 30, 40 and 60. The performance improves as the engaging CRSN nodes number $K$ grows. Interestingly, we observe that as $K$ outnumbers $J$ by four times, the detector’s performance is nearly optimal.

V. CONCLUSION

In this paper, we proposed a distributed compressed wideband sensing scheme for the resource-constrained Cognitive Radio Sensor Networks (CRSN). First, we described the considered scenario by modeling the primary user signal and the clustered CRSN structure. Second, we provided a specific and practical structure for the sampler, which has very low sampling rate at individual node. Then, we introduced the sensing information fusion scheme, and described the compressed sensing recovery algorithm at the fusion center. Finally, we carried out several numerical simulations to validate our proposed scheme.

VI. ACKNOWLEDGEMENT

This work was supported by the National Hi-Tech Research and Development Program (863 program) of China (No. 2007AA01Z257), National Basic Research Program (973 Program) of China (No. 2009CB320405) and National Natural Science Foundation Program of China (No. 60802012, 60972057, 61001098). It is also partly supported by the Singapore University of Technology and Design (No. SRG-EPD-2010-005).

REFERENCES

[1] O. Akan, O. Karli, O. Ergul, and M. Haardt, “Cognitive radio sensor networks,” in IEEE Network, vol.23, no.4, pp.34–40 July 2009.
[2] H. Zhang, Z. Zhang, X. Chen and R. Yin, “Energy Efficient Joint Source and Channel Sensing in Cognitive Radio Sensor Networks,” in Proc. IEEE ICC, June 2011.
[3] A.S. Zahmati, S. Hussain, X. Fernando, and A. Grami, “Cognitive Wireless Sensor Networks: Emerging topics and recent challenges,” in Proc. IEEE TIC-STH, pp.593-596 Sept. 2009.
[4] G. Vijay, E. Bidra, and M. Ibeakaha, “Cognitive approaches in Wireless Sensor Networks: A survey,” Proc. QBSC, pp.177-180, May 2010.
[5] H. Zhang, Z. Zhang, H. Dai, R. Yin and X. Chen, “Distributed spectrum-aware clustering in cognitive radio sensor networks, in Proc. IEEE GlobeCom, Dec. 2011.
[6] M.A. McHenry, “NSF Spectrum Occupancy Measurements Project Summary,” in *Shared Spectrum Company Report*, Aug. 2005.

[7] Z. Tian, Blasch E., W. Li, G. Chen, X. Li, “Performance evaluation of distributed compressed wideband sensing for cognitive radio networks,” in *Proc. IEEE Information Fusion*, pp.1-8, July 2008.

[8] X. Chen, L. Zhao, J. Li, “A modified spectrum sensing method for wideband cognitive radio based on compressive sensing,” in *Proc. IEEE ChinaCOM*, pp.1-5, Aug. 2009.

[9] J. Liang, Y. Liu, W. Zhang, Y. Xu, X. Gan, X. Wang, “Joint Compressive Sensing in Wideband Cognitive Networks,” in *Proc. IEEE WCNC*, pp.1-5, April 2010.

[10] F. Deng, F. Zeng, R. Li, “Clustering-Based Compressive Wide-Band Spectrum Sensing in Cognitive Radio Network,” in *Proc. IEEE MSN*, pp.218-222, Dec. 2009.

[11] M. Mishali, Y.C. Eldar, “From Theory to Practice: Sub-Nyquist Sampling of Sparse Wideband Analog Signals,” in *IEEE Journal of Selected Topics in Signal Processing*, vol.4, no.2, pp.375-391, April 2010.

[12] M. Mishali, Y.C. Eldar, “Expected RIP: Conditioning of The modulated wideband converter,” in *Proc. IEEE ITW*, pp.343-347, Oct. 2009.

[13] E.J. Candes, M.B. Wakin, “An Introduction To Compressive Sampling,” in *IEEE Signal Processing Magazine*, vol.25, no.2, pp.21-30, March 2008.

[14] J.A. Tropp, A.C. Gilbert, “Signal Recovery From Random Measurements Via Orthogonal Matching Pursuit,” in *IEEE Trans. Info. Theory*, vol.53, no.12, pp.4655-4666, Dec. 2007.