Baryon Physics in Holographic QCD

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Abstract

In a simple holographic model for QCD in which the Chern-Simons term is incorporated to take into account the QCD chiral anomaly, we show that baryons arise as stable solitons which are the 5D analogs of 4D skyrmions. Contrary to 4D skyrmions and previously considered holographic scenarios, these solitons have sizes larger than the inverse cut-off of the model, and therefore they are predictable within our effective field theory approach. We perform a numerical determination of several static properties of the nucleons and find a satisfactory agreement with data. We also calculate the amplitudes of “anomalous” processes induced by the Chern-Simons term in the meson sector, such as $\omega \rightarrow \pi \gamma$ and $\omega \rightarrow 3\pi$. A combined fit to baryonic and mesonic observables leads to an agreement with experiments within 16%.
1 Introduction

In the large-\(N_c\) limit, strongly interacting theories such as QCD have a dual description in terms of a weakly-interacting theory of mesons [1]. In this dual description, baryons are expected to appear as solitons made of mesons fields, usually referred as skyrmions [2,3].

Skyrmions have been widely studied in the literature, with some phenomenological successes. Nevertheless, since the full theory of QCD mesons is not known, these studies have been carried out in truncated low-energy models either incorporating only pions [2,3] or few resonances [4]. It is unclear whether these approaches capture the physics needed to fully describe the baryons, since the stabilization of the baryon size is very sensitive to resonances around the GeV. In the original Skyrme model with only pions, for instance, the inverse skyrmion size \(\rho^{-1}\) equals the chiral perturbation theory cut-off \(\Lambda_{\chi PT} \sim 4\pi F_{\pi}\), rendering baryon physics completely incalculable. Other examples are models with the \(\rho\)-meson which were shown to have a stable skyrmion solution [5]. The inverse size, also in this case, is of order \(m_{\rho} \sim \Lambda_{\chi PT}\), which is clearly not far from the mass of the next resonances. Including the latter could affect strongly the physics of the skyrmion, or even destabilize it.\(^1\)

In this article we will consider a very simple five-dimensional model for QCD, which has already been shown to give a quite accurate description of meson physics [8–10]. This 5D model has a cut-off scale \(\Lambda_5\) which is above the lowest-resonance mass \(m_{\rho}\). The gap among these two scales, which ensures calculability in the meson sector, is related to the number of colors \(N_c\) of QCD. In the large \(N_c\)-limit, one has \(\Lambda_5/m_{\rho} \to \infty\) and the 5D model describes a theory of infinite mesonic resonances. We will be interested in studying the solitons of this 5D theory that will correspond to the baryons of QCD. We will find that these 5D skyrmion-like solitons have an inverse size \(\rho^{-1} \sim m_{\rho}\) smaller than the cut-off scale \(\Lambda_5\). Therefore, contrary to the 4D case, they can be consistently studied with our 5D effective theory. The expansion parameter which ensures calculability will be provided by \(1/(\rho\Lambda_5) \ll 1\).

Once calculability is established, it is meaningful to compare our predictions with experiments. We will study numerically the 5D baryons and calculate several static properties of the nucleons such as the axial coupling, magnetic moments and radii. An important ingredient of our model will be the Chern-Simons (CS) term that not only will incorporate the QCD anomaly, but will also play a crucial role to stabilize the size of the baryons. Indeed, without the CS term, we would be back to the scenario discussed in Ref. [7] in which the skyrmion, though stable and calculable, does not provide a good description of baryons. This CS term will at the same time be responsible

\(^1\)In Refs. [6, 7] it was shown how the inclusion of the full tower of isovector resonances in a \(SU(2)_L \times SU(2)_R\) 5D model can destabilize the skyrmion.
for the anomalous processes of the mesons such as $\pi \to \gamma \gamma$, $\omega \to \pi \gamma$, etc., which we will compute and compare with the experimental data.

We claim that this is the first consistent holographic approach to baryons. Previous studies, although useful to understand the AdS/CFT correspondence for baryon physics, faced several problems. The first approaches considered only a truncated theory of resonances [11], and therefore had the same problems as 4D skyrmions. Later studies [6,12,13] were performed within the Sakai-Sugimoto model [14]. It was shown, however, that baryons are not calculable in this framework as their inverse size is of the order of the string scale which corresponds to the cut-off of the theory [6]. We will show that in our case we have an expansion parameter that, although not very small in real QCD (i.e. for $N_c = 3$), allows for a perturbative approach. Once a pertubative series has been formally built up, the accuracy of the predictions one gets at a given order depends on how fast the series converges, and not on the smallness of the naive expansion parameter. As we will see, the agreement with data of our leading-order predictions is quite good, suggesting that the series could converge fast in our case. Another important difference of our analysis is that we will find the non-linear soliton solution numerically, without the need of unreliable approximations.

2 A five-dimensional model for QCD

The 5D model that we will use to describe QCD with two massless flavors is the following. We will consider an $U(2)_L \times U(2)_R$ gauge theory with metric $ds^2 = a(z)^2 (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2)$, where $x^\mu$ represent the usual 4 coordinates and $z$, which runs in the interval $[z_{UV}, z_{IR}]$, denotes the extra dimension. We will work in AdS_5 where the warp factor $a(z)$ is

$$a(z) = \frac{z_{IR}}{z},$$

and $z_{UV} \to 0$ to be taken at the end of the calculations. In this limit $z_{IR}$ coincides with the AdS curvature and the conformal length

$$L = \int_{z_{UV}}^{z_{IR}} dz.$$  

We will denote respectively by $L_M$ and $R_M$, where $M = \{\mu, 5\}$, the $U(2)_L$ and $U(2)_R$ gauge connections. These are parametrized by $L_M = L_M^a \sigma_a/2 + \tilde{L}_M^{1/2}$ and $R_M = R_M^a \sigma_a/2 + \tilde{R}_M^{1/2}$, where $\sigma_a$ are the Pauli matrices. The chiral symmetry breaking is imposed on the boundary at $z = z_{IR}$ (IR-boundary) by the following boundary conditions:

$$(L_\mu - R_\mu) \mid_{z=z_{IR}} = 0, \quad (L_{\mu5} + R_{\mu5}) \mid_{z=z_{IR}} = 0,$$

where the 5D field strength is defined as $L_{MN} = \partial_M L_N - \partial_N L_M - i[L_M, L_N]$, and analogously for $R_{MN}$. At the other boundary, the UV-boundary, we impose generalized Dirichlet conditions.
to all the fields:
\[ L_\mu |_{z=z_{UV}} = l_\mu , \quad R_\mu |_{z=z_{UV}} = r_\mu . \] (4)

We will eventually be interested in taking the 4D “sources” \( l_\mu \) and \( r_\mu \) to vanish.

The AdS/CFT correspondence tells us how to interpret the above 5D model in terms of a 4D QCD-like theory, whose fields we will generically denote by \( \Psi (x) \) and its action by \( S_4 \). This is a strongly coupled 4D theory that possesses an \( U(2)_L \times U(2)_R \) global symmetry with associated Noether currents \( j^\mu_{L,R} \). If the 4D theory were precisely massless QCD with two flavors, the currents would be given by the usual quark bilinear, \( (j^\mu_{L,R})_{ij} = \overline{Q}_L \gamma^\mu Q_R \). Defining \( Z[l_\mu, r_\mu] \) as the generating functional of current correlators, the AdS/CFT correspondence states that [15, 16]
\[
Z[l_\mu, r_\mu] \equiv \int D\Psi \exp \left[ iS_4 [\Psi] + i \int d^4x \text{Tr} (j^\mu_{L,L} + j^\mu_{R,R}) \right] = \int DLM DRM \exp \left[ iS_5 [L, R] \right],
\] (5)

where the 5D partition function depends on the sources \( l_\mu, r_\mu \) through the UV-boundary conditions in Eq. (4).

The correspondence Eq. (5) leads to the following implication. Under local chiral transformations, \( Z \) receives a contribution from the \( U(2)^3 \) anomaly, which is known in QCD. This implies [16–18] that the 5D action must contain a CS term
\[
S_{CS} = -i \frac{N_c}{24\pi^2} \int [\omega_5(L) - \omega_5(R)],
\] (6)

whose variation under 5D local transformations which does not reduce to the identity at the UV exactly reproduces the anomaly. The CS coefficient, which is anyhow quantized even from a purely 5D point of view, will be fixed to \( N_c = 3 \) when matching QCD. The CS 5-form, defining \( A = -iA_M dx^M \), is
\[
\omega_5(A) = \text{Tr} \left[ A(dA)^2 + \frac{3}{2} A^3(dA) + \frac{3}{5} A^5 \right].
\] (7)

When \( A \) is the connection of an \( U(2) \) group, as in our case, one can use the fact that \( SU(2) \) is an anomaly-free group to write \( \omega_5 \) as
\[
\omega_5(A) = \frac{3}{2} \hat{A} \text{Tr} [F^2] + \frac{1}{4} \hat{A} (d\hat{A})^2 + d \text{Tr} \left[ \hat{A} AF - \frac{1}{4} \hat{A} A^3 \right],
\] (8)

where \( A = A + \hat{A} \mathbb{1}/2 \) and \( A \) is the \( SU(2) \) connection. The total derivative part of the above equation can be dropped, since it only adds to \( S_{CS} \) an UV-boundary term for the sources.

\(^2\)We are not considering the \( U(1) \cdot SU(N_c)^2 \) QCD anomaly, responsible for the \( \eta' \) mass, since this is subleading in the large-\( N_c \) expansion.
The full 5D action will be given by $S_5 = S_g + S_{CS}$, where $S_g$ is made of locally gauge invariant terms. $S_g$ is also invariant under transformations which do not reduce to the identity at the UV-boundary, and for this reason it does not contribute to the anomalous variation of the partition function. Taking the operators of the lowest dimensionality, we have

$$S_g = -\int d^4x \int_{z_{UV}}^{z_{IR}} dz \, a(z) \frac{M_5}{2} \left\{ \text{Tr} \left[ L_{MN} L^{MN} \right] + \frac{\alpha^2}{2} \hat{L}_{MN} \hat{L}^{MN} + \{ L \leftrightarrow R \} \right\}. \quad (9)$$

We have imposed on the 5D theory invariance under the combined $\{ x \rightarrow -x, L \leftrightarrow R \}$, where $x$ denotes ordinary 3-space coordinates. This symmetry, under which $S_{CS}$ is also invariant, corresponds to the usual parity on the 4D side. Notice that we have normalized differently the kinetic term of the $SU(2)$ and $U(1)$ gauge bosons, since we do not have any symmetry reason to put them equal. In the large-$N_c$ limit of QCD, however, the Zweig’s rule leads to equal couplings (and masses) for the $\rho$ and $\omega$ mesons, implying $\alpha = 1$ in our 5D model. Since this well-known feature of large-$N_c$ QCD does not arise automatically in our 5D framework (as, for instance, the equality of the $\rho$ and $\omega$ masses does), we will keep $\alpha$ as a free parameter. The CS term, written in component notation, will be given by

$$S_{CS} = \frac{N_c}{16\pi^2} \int d^5x \left\{ \frac{1}{4} \epsilon^{MNOPQ} \hat{L}_M \text{Tr} \left[ L_{NO} L_{PQ} \right] + \frac{1}{24} \epsilon^{MNOPQ} \hat{L}_M \hat{L}_{NO} \hat{L}_{PQ} - \{ L \leftrightarrow R \} \right\}. \quad (10)$$

The 5D theory defined above has only 3 independent parameters: $M_5$, $L$ and $\alpha$.

From Eq. (5) we can extract the current operators through which the theory couples to the external EW bosons. These currents are obtained by varying Eq. (5) with respect to $l_\mu$ (exactly the same would be true for $r_\mu$) and then taking $l_\mu = r_\mu = 0$. The variation of the l.h.s. of Eq. (5) simply gives the current correlator of the 4D theory, while in the r.h.s. this corresponds to a variation of the UV-boundary conditions. The effect of this latter can be calculated in the following way. We perform a field redefinition $L_\mu \rightarrow L_\mu + \delta L_\mu$ where $\delta L_\mu(x, z)$ is chosen to respect the IR-boundary conditions and fulfill $\delta L_\mu(x, z_{UV}) = \delta l_\mu$. This redefinition removes the original variation of the UV-boundary conditions, but leads a new term in the 5D action, $\delta S_5$. One then has

$$i \int d^4x \text{Tr} \left[ (j^ \mu_L(x)) \delta l_\mu(x) \right] = i \int D L_M D R_M \delta S_5 [L, R] \exp [iS_5 [L, R]], \quad (11)$$

where the 5D path integral is now performed by taking $l_\mu = r_\mu = 0$, i.e. normal Dirichlet conditions. The explicit value of $\delta S_5$ is given by

$$\delta S_5 = \int d^4x \text{Tr} \left[ J^ \mu_L(x) \delta l_\mu(x) \right] + \int d^5x (\text{EOM}) \cdot \delta L, \quad (12)$$

where $J^ \mu_L = J^ \mu_L \sigma^a + \tilde{J}^ \mu_L l$ and

$$J^ \mu_L = M_5 (a(z) L^ \mu_5(a(z)) \big|_{z=z_{UV}}, \quad \tilde{J}^ \mu_L = \alpha^2 M_5 (a(z) L^ \mu_5) \big|_{z=z_{UV}}. \quad (13)$$
The last term of Eq. (12) corresponds to the 5D “bulk” part of the variation, which leads to the equations of motion (EOM). Remembering that the EOM always have zero expectation value, we find that we can identify $J^\mu_L$ of Eq. (13) with the current operator on the 5D side:

$$\langle j^\mu_L \rangle_{4D} = \langle J^\mu_L \rangle_{5D}.$$ 

Notice that the CS term has not contributed to Eq. (12) due to the fact that each term in $S_{CS}$ which contains a $\partial_z$ derivative (and therefore could lead to a UV-boundary term) also contains $L_\mu$ or $R_\mu$ fields; these fields on the UV-boundary are the sources $l_\mu$ and $r_\mu$ that must be put to zero.

### 3 Baryons as 5D skyrmions

#### 3.1 The static solution

The QCD baryons correspond to the solitons of the above 5D theory, also referred as 5D skyrmions. These static solutions, exactly like the ones considered in Ref. [7], have unit topological charge

$$B = \frac{1}{32\pi^2} \int d^3x \int_{z_{UV}}^{z_{IR}} dz \epsilon_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}} \text{Tr} \left[ L_{\hat{\mu}} L_{\hat{\nu}} \delta_{\hat{\rho}\hat{\sigma}} - R_{\hat{\mu}} R_{\hat{\nu}} \delta_{\hat{\rho}\hat{\sigma}} \right],$$

which we identify with the baryon number. The derivation of the solitonic configurations closely follows the one of Ref. [7]. There are, however, few differences. First, in Ref. [7] we were using time-reversal symmetry to put consistently to zero the temporal component of the gauge fields. Here, on the contrary, we have to use time-reversal $t \to -t$ combined with $\hat{L} \to -\hat{L}$ and $\hat{R} \to -\hat{R}$, under which the CS is invariant. This transformation reduces, in static configurations, to a sign change of the temporal component of $L$ and $R$ and of the spatial components of $\hat{L}$ and $\hat{R}$. We can therefore consistently put them to zero. For the remaining fields one can impose “cylindrical” symmetry, i.e. invariance under combined $SU(2)$ gauge and 3D spatial rotations, and parity invariance under the combined action $L \leftrightarrow R$ and $x \to -x$. This determines our ansatz to be

$$L^a_i(x, z) = -R^a_i(-x, z), \quad L^a_5(x, z) = R^a_5(-x, z), \quad \hat{L}_0(x, z) = \hat{R}_0(-x, z),$$

and

$$R^a_j = -\frac{1 + \phi_2(r, z)}{r^2} \epsilon_{jak} x_k + \frac{\phi_1(r, z)}{r^3} \left( r^2 \delta_{ja} - x_j x_a \right) + \frac{A_1(r, z)}{r^2} x_j x_a,$$

$$R^a_5 = \frac{A_2(r, z)}{r} x^a,$$

$$\hat{R}_0 = \frac{1}{\alpha} \frac{s(r, z)}{r}.$$ 

The above ansatz reduces our solitonic configuration to 5 real functions in 2D, $A_\mu = \{A_1, A_2\}$, $\phi = \phi_1 + i\phi_2$ and $s$, where $x^\mu = \{r, z\}$. We notice that we have a residual $U(1)$ invariance

$^3$We have actually shown this here; notice that $\delta L_\mu$ was completely arbitrary in the bulk, but the variation of the functional integral can only depend on $\delta l_\mu = \delta L_\mu(x, z_{UV})$. 

5
corresponding to \( g_L^\dagger = g_R = \exp[i\alpha(r,z)x^a\sigma_a/(2r)] \) under which \( A_\mu \) is the gauge field, \( \phi \) has charge +1 and \( s \) is neutral.

The contribution of \( S_g \) to the energy is easily computed. The contribution from the \( SU(2) \) part is given in Eq. (11) of Ref. [7], while the \( U(1) \) part only adds the kinetic energy of \( s \). We then have

\[
E_g = 8\pi M_5 \int_0^\infty dr \int_{z_{UV}}^{z_{IR}} dz a(z) \left[ |D_\mu \phi|^2 + \frac{1}{4} r^2 F_{\mu\nu}^2 + \frac{1}{2 r^2} \left( 1 - |\phi|^2 \right)^2 - \frac{1}{2} (\partial_\mu s)^2 \right].
\]

(16)

Notice that \( s \) has a negative kinetic term, since it corresponds to the temporal component of a gauge field. The CS term gives also a contribution to the energy. From the ansatz of Eqs. (14) and (15) we have \( L_0(r,z) = \hat{R}_0(r,z) = s/(\alpha r) \), that allows us to write the CS energy as a coupling of \( s \) to the topological charge density (the baryon number density), given in Eq. (12) of Ref. [7]:

\[
E_{CS} = 8\pi M_5 \frac{-\gamma L}{2} \int_0^\infty dr \int_{z_{UV}}^{z_{IR}} dz s \frac{8}{r^2} \epsilon^{\mu\nu} \left[ \partial_\mu (-i\phi^* D_\nu \phi + h.c.) + F_{\mu\nu} \right],
\]

(17)

where

\[
\gamma = \frac{N_c}{16\pi^2 M_5 L \alpha}.
\]

(18)

From Eqs. (16) and (17) we can extract the EOM. One finds

\[
\begin{align*}
D_\mu \left( a(z) D_\mu \phi \right) + \frac{a(z)}{r} \phi \left( 1 - |\phi|^2 \right) + i \gamma L \epsilon^{\mu\bar{\nu}} \partial_\mu \left( \phi^* \right) D_\nu \phi &= 0 \\
\partial_\mu \left( r^2 a(z) F_{\mu\bar{\nu}} \right) - a(z) (i\phi^* D_\nu \phi + h.c.) + \gamma L \epsilon^{\mu\nu} \partial_\mu \left( \phi^* \right) (|\phi|^2 - 1) &= 0 \\
\partial_\mu \left( a(z) \partial_\mu s \right) - \frac{7L}{2r} \epsilon^{\mu\bar{\nu}} \left[ \partial_\mu (-i\phi^* D_\nu \phi + h.c.) + F_{\mu\nu} \right] &= 0.
\end{align*}
\]

(19)

The skyrmion configurations will be the solutions to these EOM with boundary conditions enforcing a definite topological charge \( B \). For the \( B = 1 \) solution, these boundary conditions are given in Eqs. (13) and (14) of Ref. [7] for the \( SU(2) \) fields, while for the neutral scalar \( s \) we must impose \( s = 0 \) at the three boundaries, \( z = z_{UV}, r = 0 \) and \( r \to \infty \), and impose \( \partial_z s = 0 \) at \( z = z_{IR} \). The IR and UV-boundary conditions come respectively from Eqs. (3) and (4); the one at \( r = 0 \) arises from regularity, while the one at \( r \to \infty \) is necessary for the energy to be finite. The solution will be obtained numerically by the COMSOL 3.4 package [21]; a rescaling of the \( \phi_{1,2} \) fields, as explained in Ref. [7], must be performed in order to avoid singularities at \( r = 0 \).

At this point it is important to show that the 5D skyrmion configuration is stable and its properties can be consistently calculated within the 5D effective theory described above. This is easily established, along the lines of Refs. [6, 7], in the case in which the CS term is a small perturbation to the gauge kinetic term, i.e., \( \gamma \ll 1 \). In this limit, the solution is approximately

\[\text{[Footnote]}\]

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given by a small 4D instanton configuration whose size $\rho$ is determined by minimizing the energy. The AdS curvature induces a contribution to the instanton energy that grows as $\rho/L$ [7], while the CS term generates a Coulomb potential scaling as $\gamma^2 L^2/\rho^2$ [6]. Therefore the energy is minimized for $\rho \sim \gamma^{2/3} L$. By a Naive Dimensional Analysis (NDA) one gets $\Lambda_5 \lesssim 24\pi^3 M_5$ and $\gamma \sim 1/(\Lambda_5 L) \ll 1$, so we have that $\rho \gg 1/\Lambda_5$ and therefore we expect the soliton to be insensitive to higher-dimensional operators. The situation is quite different in the Sakai-Sugimoto model [6]. There the energy dependence on $\rho$ goes as $E(\rho) \sim \rho^2/L^2 + \gamma^2 L^2/\rho^2$ where the first term comes from the curvature of the 5D space and the second from the CS with now $\gamma \sim 1/(M_{\text{st}} L)^2$, being $M_{\text{st}}$ the string-scale. In this case one gets $\rho \sim \sqrt{\gamma} L \sim 1/M_{\text{st}}$; the size of the soliton is of the order of the inverse cut-off of the model. This makes baryon physics totally sensitive to string corrections and therefore unpredictable, as already remarked in Ref. [6].

### 3.2 Soliton quantization

Exactly like in the case of the 4D skyrmion [3] (see Ref. [4] for a comprehensive review), single-baryon states are described in our model as zero-mode time-dependent fluctuations around the classical static solution. Such zero modes, also called collective coordinates, are associated to the global symmetries of the theory; the situation is similar to that of the kink and the monopole [19]. The global symmetries of our static equations are $U(2)_V$ and 3-space rotations plus 3-space translations. The latter are associated with baryons moving with uniform velocity and can be simply ignored if one is only interested in static properties like magnetic moments, the axial charges and charge radii. The action of $U(1)_V$ is trivial on all our fields and then we are left with $SU(2)_V$ and 3-space rotations. The two rotations, however, have the same effect on our cylindrically symmetric solution so that rotating in 3-space would not lead to any new configuration which cannot be reached with only $SU(2)$ rotation. Therefore, as in the case of the 4D skyrmion, we only need to consider 3 collective coordinates which are encoded in a $SU(2)$ matrix $U$.

The zero-modes fluctuations we are interested in are constructed as follows. Let us perform an $SU(2)_V$ transformation $U$ on the static solution discussed in the previous section. We obtain

$$R_{\mu}(x, z; U) = U R_{\mu}(x, z) U^\dagger, \quad \hat{R}_0(x, z; U) = \hat{R}_0(x, z),$$

5 In this NDA we are not considering the $N_c$ dependence of $\gamma$. When this is included and our 5D model is matched to large-$N_c$ QCD, we will see later that one gets $\gamma \sim 1$ and $\rho \sim L$. Therefore, including the $N_c$ factors will make the size of the baryon even larger with respect to the cut-off.

6These quantities are defined in processes with very low transfer momentum during which the baryon only suffers a negligible acceleration. Our formalism, however, is non-relativistic so that uniform baryon motion cannot be ignored in a generic reference frame. The Breit frame is the correct one to compute form factors [4].
where $\hat{\mu} = 1, 2, 3, 5$; similarly for the $U(2)_L$ gauge fields. For a constant $U$, Eq. (20) is also a solution of the EOM. We now introduce a small $t$-dependence on $U$, i.e. a small rotational velocity $K^i = -i/2 \text{Tr}[\sigma^i U^t dU/dt]$. Eq. (20) is now an infinitesimal deformation of the static solution. In order for this configuration to describe a zero-mode fluctuation, it should fulfill the time-dependent EOM to linear order in $K$ and with $dK/dt = 0$. Indeed, zero-modes correspond to directions in field space in which uniform and slow motion is permitted.

We therefore need to solve the time-dependent EOM. Due to the invariance under the transformation $\{L \rightarrow R, x \rightarrow -x\}$, we can restrict the configurations to $L_i(x, z, t) = R_i(-x, z, t)$, $L_{5,0}(x, z, t) = R_{5,0}(-x, z, t)$ and analogously for $\dot{L}$, $\dot{R}$. The EOM for the $R$ fields, after separating temporal from spatial coordinates, read

$$
\begin{align*}
D_\rho \left(a(z) R^\rho_0\right) + \frac{\gamma_{\rho \sigma}}{4} \epsilon^{\rho \omega \sigma \hat{\mu}} R_{\rho \omega \hat{\mu}} \hat{R}_{\rho \hat{\mu}} &= 0 \\
\alpha \partial_\omega \left(a(z) \hat{R}_0^\rho\right) + \frac{\gamma_{\rho \sigma}}{4} \epsilon^{\rho \omega \sigma \hat{\mu}} \left[\text{Tr} \left(R_{\rho \omega \hat{\mu}} R_{\rho \hat{\mu}}\right) + \frac{1}{2} \hat{R}_{\rho \omega \hat{\mu}} \hat{R}_{\rho \hat{\mu}}\right] &= 0 \\
D_\rho \left(a(z) \hat{R}_0^\rho\right) - a(z) D_\rho R_0^\hat{\mu} + \frac{\gamma_{\rho \sigma}}{2} \epsilon^{\rho \omega \sigma \hat{\mu}} \left[R_{\rho \omega \hat{\mu}} \hat{R}_{\rho \hat{\mu}} + R_{\rho \hat{\mu}} \hat{R}_{\rho \hat{\mu}}\right] &= 0 \\
\alpha \partial_\omega \left(a(z) \hat{R}_0^\rho\right) - \alpha a(z) \partial_0 \hat{R}_0^\mu - \gamma L \epsilon^{\rho \omega \sigma \hat{\mu}} \left[\text{Tr} \left(R_{\rho \omega \hat{\mu}} R_{\rho \hat{\mu}}\right) + \frac{1}{2} \hat{R}_{\rho \omega \hat{\mu}} \hat{R}_{\rho \hat{\mu}}\right] &= 0
\end{align*}
$$

(21)

where Euclidean metric is used to raise the spatial indices. We immediately see that, once the time-dependent ansatz in Eq. (20) has been chosen for the fields $R_\hat{\mu}$ and $\hat{R}_0$, the other components $R_0$ and $\hat{R}_0$ cannot be put to zero as in the static case. The time-dependence of $U$ in Eq. (20) acts as a source for the latter components, as can be seen by looking at the first and the fourth EOM. As a result, $R_0$ and $\hat{R}_0$ must be turned on. The same situation occurs in the case of the 4D skyrmion of Ref. [4] in which the temporal and spatial components of the $\rho$ and $\omega$ mesons are turned on in the skyrmion quantization (also in the case of the magnetic monopole for the temporal component of the gauge field).

One can show that the second and the third EOM of Eq. (21) are solved, to linear order in $K$, by the ansatz in Eq. (20) if the fields $R_0$ and $\hat{R}_0$ are chosen to be linear in $K$. Under this assumption, $\hat{R}_{\rho \sigma}$ is equal to $\partial_\rho \hat{R}_0$ up to terms proportional to $dK/dt$ or quadratic in $K$ that can be ignored in the approximation of slow motion discussed above. Furthermore, $R_{\rho \sigma}$ and $\hat{R}_{\rho \sigma}$ are of order $K$ and $D_0 R_{\rho \sigma} = 0$. Using this, the second and the third EOM of Eq. (21) reduce to the static equations of sec. 3.1.

We are left with the first and fourth equations of Eq. (21), which are 7 elliptic equations for the 7 fields $R_0$ and $\hat{R}_0$; those are the analog of the Gauss law constraint for dyons in the case of monopoles [19]. To solve such equations we can again make a 2D ansatz following a generalization of the cylindrical symmetry we used in the static case in which the rotational velocity $K$ also rotates together with the 3-coordinates $x$. The resulting 2D equations can be solved numerically, but we leave this for future work.
The quantization of the collective coordinates, from this point on, exactly proceeds as for the 4D skyrmion [3, 4]. First, one plugs the zero-mode configuration into the action and obtains a Lagrangian
\[
\mathcal{L} = -M + \lambda \text{Tr} \left[ \partial_0 U \partial_0 U^\dagger \right],
\]
where \( M = E_g + E_{CS} \) is the classical mass of the soliton obtained from the static solution, while \( \lambda \) depends also on the solutions for \( R_{\bar{\mu}} \) and \( \bar{R}_0 \). Now, the collective coordinates \( U \) are treated as quantum mechanical variables, reducing the problem to the one of quantizing a spherical rigid rotor with momentum of inertia \( \lambda \). The quantization is therefore performed in a standard way and the energy eigenstates are interpreted as baryon states. One finds, as expected in the large-\( N_c \) limit, an infinite tower of baryons with increasing spin/isospin. More precisely, baryons are in the \((p/2, p/2)\) representation of the spin/isospin \( SU(2) \times SU(2) \) group; the first two levels \( p = 1, 2 \) are interpreted, respectively, as the nucleons and the \( \Delta \) multiplet. We do not need to repeat this procedure in detail here, but only give the following useful relation that can be derived from quantization rules [3]:
\[
\text{Tr} \left[ U \sigma^b U^\dagger \sigma^a \right] = -\frac{8}{3} S^b I^a,
\]
(22)
where \( S^a \) and \( I^a \) are respectively the spin and isospin operators.

3.3 Static properties of baryons

There are several static baryon observables which are independent of \( R_0 \) and \( \bar{R}_{\bar{\mu}} \) and therefore can be computed by knowing only the static solution of sec. 3.1 together with the anstaz in Eq. (20). By looking at Eq. (13) we see that the spatial components of the vector and axial currents, and the temporal component of the scalar one are independent of \( R_0 \) and \( \bar{R}_{\bar{\mu}} \) up to order \( K^2 \) or \( dK/dt \) terms. By plugging Eq. (20) into these currents one finds
\[
J_{Ai} = -M_5 a(z_{UV}) \left[ \delta_{ai} \frac{D_z \phi_1}{r^2} + \frac{x_i x_a}{r^2} \left( F_{zr} - \frac{D_z \phi_1}{r^2} \right) \right] z_{UV} U \sigma^a U^\dagger,
\]
\[
J_{Vi} = -M_5 a(z_{UV}) \left[ \epsilon_{ai k} x_k \frac{D_z \phi_2}{r^2} \right] z_{UV} U \sigma^a U^\dagger,
\]
\[
\bar{J}_{V0} = -2M_5 a(z_{UV}) \alpha \left[ \frac{\partial_z s}{r} \right] z_{UV}.
\]
(23)
The axial current is defined as \( J_{A\mu} = J_{R\mu} - J_{L\mu} \), the vector as \( J_{V\mu} = J_{R\mu} + J_{L\mu} \) and the scalar vector current (1/2 of the baryon number current) is \( \bar{J}_{V\mu} = 1/3 \left( \bar{J}_{R\mu} + \bar{J}_{L\mu} \right) \). From Eq. (23), using Eq. (22), we can extract the baryon axial coupling \( g_A \), the isovector magnetic moment \( \mu_V \),
and the isoscalar electric charge radius \( r_{E,S}^2 \):

\[
g_A = -\frac{N_c}{9\pi\alpha\gamma} \frac{L}{L} \int_0^\infty dr \left[ a(z) \left( 2D_z \phi_1 + r F_\pi \right) \right]_{zUV},
\]

\[
\mu_V = \frac{N_c M_N L}{9\pi\alpha\gamma} \frac{1}{L^2} \int_0^\infty dr \left[ r^2 a(z) D_z \phi_2 \right]_{zUV},
\]

\[
r_{E,S}^2 = -\frac{L^2}{\pi\gamma} \frac{1}{L^3} \int_0^\infty dr \left[ r^3 a(z) \partial_z s \right]_{zUV}.
\]

(24)

For the definition of these observables we follow the conventions used in Ref. [4]. It should be noticed that Eq. (24) has the right scaling in \( N_c \) [20] if, in accordance with the AdS/CFT expectation, the 5D parameters \( \gamma, \alpha \) and \( L \) scale like \( N_c^0 \). We have numerically calculated all the quantities of Eq. (24) together with the mass of the baryon which is given by the classical mass \( M \). The 3 parameters of our 5D model, \( M_5, L \) and \( \alpha \), are determined from (1) the pion decay constant \( F_\pi^2 = 8M_5 \int dz/a(z) \) [10], (2) the mass of the \( \rho, m_\rho \simeq 3\pi/(4L) \) [8, 9] and (3) the decay constant ratio \( F_\omega/F_\rho = \alpha \). The results are given in Table 1 where we compare them with the experimental values. We restricted our comparison to nucleon observables (including the mass, even though all baryons are degenerate at the classical level), since we expect higher spin/isospin states (like the \( \Delta \)-multiplet) to receive larger corrections. \footnote{In our model \( S^a = \lambda K^a/2 \). Since we are performing a small \( K \) expansion, small spin (and isospin) states are predicted more reliably.} We recall that these predictions are obtained in the semiclassical (large-\( N_c \)) limit, and therefore they should be valid up to corrections \( \sim 1/N_c \). Consistently with this picture, Table 1 shows agreement with experiments within 30%.

| Experiment | AdS5 |
|------------|------|
| \( M_N \) (MeV) | 940 | 1140 |
| \( \sqrt{\langle r_{E,S}^2 \rangle} \) (fm) | 0.79 | 0.94 |
| \( g_A \) | 1.25 | 1.0 |
| \( \mu_p - \mu_n = 2\mu_V \) | 4.7 | 3.9 |

Table 1: Predictions for the baryon static quantities where \( M_5, L \) and \( \alpha \) have been determined from the experimental values of \( F_\pi = 87 \) MeV, \( m_\rho = 775 \) MeV and \( F_\omega/F_\rho = 0.88 \).

In any model with exact spontaneously broken chiral symmetry the axial coupling \( g_A \) is related to the pion-nucleon coupling \( g_{\pi NN} \) by the Goldberger-Treiman relation

\[
g_A = \frac{g_{\pi NN} F_\pi}{M_N}.
\]

(25)

This relation holds in our 5D model and can be used to derive \( g_{\pi NN} \) from the value of \( g_A \) and \( M_N \) obtained above. We get \( g_{\pi NN} \simeq 13.1 \) that is quite close to the experimental value \( g_{\pi NN}|_{\text{exp}} \simeq 13.5 \).
4 Meson anomalous couplings from the CS term

The CS term of Eq. (10) is responsible for the anomalous couplings of the mesons to the photon and among themselves. In order to compute amplitudes involving a (real or virtual) photon at leading order in the electric charge $e$, one could proceed “holographically” and use directly Eq. (5) and Eq. (13) to compute matrix elements of the electromagnetic current $J_{\text{em}}^\mu = \hat{J}_V^\mu / 3 + J_V^3$. Looking at the explicit form of this current, one easily realizes that electromagnetic interactions only proceed, in this “holographic basis”, through the single exchange of resonances. Our model implements the Vector Meson Dominance (VMD) hypothesis, in which the photon only couples to hadrons by the mixing with the $\rho$ and the $\omega$.

A second way to perform the same calculations, which in some cases is simpler, is to use a Kaluza-Klein (KK) expansion of the 5D gauge fields, in which the photon mixing matrix is automatically diagonalized. This can be done by giving a dynamics to the photon, which is the source associated to the electromagnetic current in Eq. (5). Making the source dynamical means integrating also over it in the path integral and this is the same as changing from Dirichlet to Neumann the UV-boundary condition of the corresponding 5D field $\hat{V}_\mu / 3 + V_\mu^3$, where $V = L + R$. The electric charge is fixed to its experimental value $e$ by adding a localized kinetic term for the photon of the form $-(1/e^2 - \text{c.t.}) F^2 / 4$ where the “counterterms” c.t. are needed to cancel the charge renormalization induced by QCD (i.e. bulk) effects. The latter are finite as long as $z_{\text{UV}}$ is finite. At leading order in the charge $e$, however, the localized kinetic term is infinite (as $e \to 0$), but the KK decomposition in the presence of a localized kinetic term is well known. One has a massless zero-mode with flat wave function for $\hat{V}_\mu / 3 + V_\mu^3$, that we identify with the “diagonalized” photon $A^{(\gamma)}_\mu$, which is now a mass eigenstate. One also has massive KK’s which obey, in the limit $e \to 0$, Dirichlet conditions at the UV. One basically returns to the original theory with Dirichlet 5D fields, but with an extra photon zero-mode $A^{(\gamma)}_\mu$ which decouples as $e \to 0$. The other relevant states are the pions $\pi^a$, which are identified with the zero modes of the fifth component of the axial gauge bosons, and the $\rho$ and $\omega$ resonances which are respectively the first isosinglet and isotriplet vector KK-states. We then have

$$A_5^a(x, z) = \frac{1}{\sqrt{M_5 L}} f_\pi(z) \pi^a(x) + \ldots ,$$

$$\hat{V}_\mu(x, z) = \frac{\sqrt{2}}{3} A^{(\gamma)}_\mu + \frac{1}{\alpha \sqrt{M_5 L}} f_V(z) \omega_\mu + \ldots ,$$

$$V_\mu^3(x, z) = \sqrt{2} A^{(\gamma)}_\mu + \frac{1}{\sqrt{M_5 L}} f_V(z) \rho_\mu + \ldots ,$$

where $f_\pi(z)$ and $f_V(z)$ are respectively the 5D wave-function of the pions and vectors. We have

---

For a study of these processes in the Sakai-Sugimoto model see Ref. [23].
Table 2: Prediction of the anomalous partial decay widths in MeV where $M_5$, $L$ and $\alpha$ have been determined from the experimental values of $F_\pi = 87$ MeV, $m_\rho = 775$ MeV and $F_\omega/F_\rho = 0.88$.

| Decay Width    | Experiment | AdS5 |
|----------------|------------|------|
| $\Gamma(\omega \rightarrow \pi \gamma)$ | 0.75       | 0.86 |
| $\Gamma(\omega \rightarrow 3\pi)$     | 7.6        | 6.1  |
| $\Gamma(\rho \rightarrow \pi \gamma)$ | 0.068      | 0.072|
| $\Gamma(\omega \rightarrow \pi \mu \mu )$ | $8.2 \cdot 10^{-4}$ | $7.9 \cdot 10^{-4}$ |
| $\Gamma(\omega \rightarrow \pi e e)$   | $6.5 \cdot 10^{-3}$ | $7.8 \cdot 10^{-3}$ |

The value of $\alpha$ turns to be very close to 1; for AdS we find $\alpha \approx 1.18$. We will understand later why this is the case. We also want to remark that Eq. (27) shows an interesting relation between the $\omega \gamma \pi$ (and $\rho \gamma \pi$) coupling and $g_{\rho \pi \pi}$, the coupling of the $\rho$ to two pions. This relation is fulfilled for any five-dimensional space.

From Eq. (27) we can calculate several meson partial decay widths. The first term of Eq. (27) leads to the decay of the $\pi^0$ to two photons in accordance with the anomaly prediction. The decay widths $\Gamma(\omega \rightarrow \pi \gamma)$ and $\Gamma(\rho \rightarrow \pi \gamma)$ arise from the second term of Eq. (27), while $\Gamma(\omega \rightarrow 3\pi)$ proceeds through virtual rhos, $\omega \rightarrow \rho^{(n)\ast} \pi \rightarrow 3\pi$. This latter process is dominated by the lowest state, the $\rho$, whose $\omega \rho \pi$ coupling is given by the third term of Eq. (27):

$$A[\omega_{\mu}(p) \rightarrow \pi^0(q_0) + \pi^+(q_+) + \pi^-(q_-)] = \frac{N_c g^3_{\rho \pi \pi}}{4\pi^2 F_{\pi} m_\rho^2} \epsilon_{\mu \nu \rho \sigma} q_0^\nu q_+^\rho q_-^\sigma \left[ D((q_+ + q_-)^2) + D((q_+ + q_0)^2) + D((q_- + q_0)^2) \right],$$

(29)

where $D(p^2) = m_\rho^2/(m_\rho^2 - p^2)$. The predictions for these partial decay widths are given in Table 2 showing a very good agreement with the experimental data.

The CS term also contributes to different pion form factors. For calculating form factors, however, it is more suitable to work in the holographic basis. We have seen that in this basis the
model exhibits the property of VMD. For example, the decay $\omega \rightarrow \pi \gamma$ proceeds as $\omega \rightarrow \pi \rho^{(n)} \rightarrow \pi \gamma$ and similarly for rho decays. As in VMD models [24], this allows us to derive the following sum-rule that relates the $\omega \gamma \pi$ coupling, $g_{\omega \gamma \pi}$, and the $\omega \rho \pi$ coupling, $g_{\omega \rho \pi}$:

$$g_{\omega \gamma \pi} = \sum_{n} \frac{g_{\omega \rho^{(n)} \pi}}{m_{\rho^{(n)}}} \frac{F_{\rho^{(n)}}}{m_{\rho}},$$

where $F_{\rho^{(n)}}$ are the rho’s decay constants that can be found in Ref. [8, 9]. It is easy to verify Eq. (30). Using Eq. (27), Eq. (30) gives $xg_{\rho \pi \pi}F_{\rho} \simeq m_{\rho}$ that, since $x \simeq 1$, implies $g_{\rho \pi \pi}F_{\rho} \simeq m_{\rho}$. This latter equation was derived in Ref. [9] valid for any five-dimensional model. We can now easily calculate form factors. The $\pi^{0} \gamma \gamma^{*}$ form factor, $F_{\pi \gamma}(q^{2})$, at low Euclidian momentum is dominated by the exchange of the $\omega$. We then have, as in VMD, $F_{\pi \gamma}(q^{2}) = m_{\omega}^{2}/(m_{\omega}^{2} + q^{2})$, where we have normalized the form factor as $F_{\pi \gamma}(0) = 1$. We obtain

$$F'_{\pi \gamma}(0) = \frac{a}{m_{\pi}}^{2}, \quad a = \frac{m_{\pi}^{2}}{m_{\omega}^{2}} \simeq 0.03,$$

in perfect agreement with the experimental value [22]: $a|_{\text{exp}} \simeq 0.032 \pm 0.004$. This form factor was previously studied in Ref. [25]. We can also calculate the partial decay width $\Gamma(\omega \rightarrow \pi \mu \mu(ee))$ that proceeds through a virtual photon. This process is proportional to the $\omega \pi \gamma^{*}$ form factor that in our model is simply given by $F_{\omega \pi}(p^{2}) = A(\omega \rightarrow \pi \gamma)D(p^{2})$, where $A(\omega \rightarrow \pi \gamma)$ is the on-shell $\omega \rightarrow \pi \gamma$ amplitude. The prediction for this partial decay width is given in Table 2.

Let us finalize this section with the following comment. In five-dimensional models arising from string theory, the effective gauge theory consists of the DBI term and the CS term. From the DBI term arises not only the kinetic term of the gauge bosons, but also higher-dimensional operators suppressed by the string scale. Since the anomalous couplings discussed in this section can only arise from the CS and not from the DBI, these couplings will not receive corrections from higher-dimensional operators.

## 5 Global fit to mesonic and baryonic observables

In the previous sections we have computed several properties of baryons and mesons, and we have shown that they agree with the experimental values within 30%. Nevertheless, in order to gain a better insight on the statistical significance of this approach, we will perform in this section a combined fit to many (baryonic and mesonic) physical observables. Our list of observables is presented in Table 3, we have taken the physical quantities calculated in this article, together with other mesonic observables calculated in Refs. [8–10, 18]. Our global fit is carried out by minimizing
Table 3: Global fit of mesonic and baryonic physical quantities. Masses, decay constants and widths are given in MeV. The RMS error of the fit is 16%. Physical masses have been used in the kinematic factors of the partial decay widths.

|    | Experiment | AdS$_5$ | Deviation |
|----|------------|---------|-----------|
| $m_\rho$ | 775       | 850     | +10%      |
| $m_{a_1}$ | 1230      | 1390    | +13%      |
| $m_\omega$ | 782       | 850     | +9%       |
| $F_\rho$ | 153       | 175     | +14%      |
| $F_\rho/F_\omega$ | 0.88   | 0.90    | +2%       |
| $F_\pi$ | 87        | 91      | +5%       |
| $g_{\rho\pi\pi}$ | 6.0    | 5.4     | -10%      |
| $L_9$ | $6.9 \cdot 10^{-3}$ | $6.2 \cdot 10^{-3}$ | -10% |
| $L_{10}$ | $-5.5 \cdot 10^{-3}$ | $-6.2 \cdot 10^{-3}$ | +12% |
| $M_N$ | 940       | 1180    | +25%      |
| $\sqrt{\langle r_{E,S}^2 \rangle}$ (fm) | 0.79 | 0.87 | +21% |
| $g_A$ | 1.25      | 0.98    | -21%      |
| $\mu_p - \mu_n$ | 4.7     | 3.7     | -22%      |
| $\Gamma(\omega \rightarrow \pi \gamma)$ | 0.75  | 0.82    | +10%      |
| $\Gamma(\omega \rightarrow 3\pi)$ | 7.5    | 7.1     | -6%       |
| $\Gamma(\rho \rightarrow \pi \gamma)$ | 0.068 | 0.072   | +5%       |
| $\Gamma(\omega \rightarrow \pi \mu \mu)$ | $8.2 \cdot 10^{-4}$ | $7.4 \cdot 10^{-4}$ | -9% |
| $\Gamma(\omega \rightarrow \pi e e)$ | $6.5 \cdot 10^{-3}$ | $7.4 \cdot 10^{-3}$ | +14% |

The statistical meaning of this procedure is the following. All our predictions are obtained at leading order in the $(16\pi^2 M_5 L)^{-1} \sim 1/N_c$ expansion, so that quite large quantum corrections are expected, which translate in quite large theoretical errors. Let us assume that all the observables have the same relative theoretical error $\xi$, i.e. $\Delta O^i_{\text{pre}} = \xi O^i_{\text{exp}}$; by looking at Eq. (32) one immediately sees that $\text{RMSE}(M_5, L, \alpha) = \xi \sqrt{\chi^2/N_p}$, where $\chi^2$ is the usual chi-squared variable constructed with errors $\Delta O^i_{\text{pre}}$. The minimum of Eq. (32) therefore represents the minimal value of the relative corrections $\xi$ for which our model would successfully pass the $\chi^2$ test, i.e. for which $\chi^2 = N_p$. All the observables included in the fit have an experimental error $\lesssim 10\%$, which we can neglect because it is smaller, as we will see, than the final RMS error of the fit.

The global fit gives $1/L \simeq 350$ MeV, $M_5 L \simeq 0.017$ and $\alpha \simeq 0.9$ ($\gamma \simeq 1.27$). As expected, the
value of $\alpha$ from the fit is close to 1, as predicted in the large-$N_c$ limit. Also it is worth noticing that the value of $M_5L$ comes to be quite close to the prediction arising from matching the current-current vector correlator to QCD at large momentum [8, 9], $M_5L = N_c/(24\pi^2) \simeq 0.013$. The RMS error of the fit is 16%. From the fit one can see that mesonic quantities are better predicted than the baryonic ones. Indeed, mesonic quantities alone give a fit with a RMS error around 10%, of order of the experimental errors.

6 Conclusions

Using a simple holographic model for QCD, we have been able to compute the properties of the baryons. We have seen that the presence of the CS term, needed to reproduce the QCD anomaly, is crucial to stabilize the size of the baryon at around the GeV. The contribution of this CS term to the mass of the baryon is as important as the leading $F^2$ term, implying the need for a fully numerical analysis of the non-linear solitonic configuration, not carried out before in the literature.

We have calculated the axial coupling, the vector magnetic moment and the isoscalar electric charge radius of the nucleus as a function of the 3 free parameters of the model. These predictions have shown a good agreement with data -see Table 1. Our approach can be extended to calculate other baryonic physical quantities that we leave for future work.

We have also seen that the CS term is responsible for anomalous processes involving an odd number of pions, some of which we have explicitly calculated (see Table 2) and showed to have an excellent agreement with the experimental data. Finally, we have done a combined fit of the mesonic and baryonic quantities predicted by this holographic model (Table 3) that have shown an agreement with experiments of 16%.

Some final comments on the important issue of calculability, i.e. on the dependence of our results on higher-dimensional operators. These operators are suppressed by the cut-off of the theory $\Lambda_5$. Naive dimensional arguments say that the maximal value of $\Lambda_5$ is determined by the scale at which loops are of order of tree-level effects. Computing loop corrections to the $F^2$ operator which arise from the $F^2$ term itself, one gets $\Lambda_5 \sim 24\pi^3 M_5$. Nevertheless, one gets a lower value for $\Lambda_5$ from the CS term due to the $N_c$ dependence of $\gamma$. Indeed, at the one-loop level, the CS term gives a contribution of order $M_5$ to the $F^2$ operator for $\Lambda_5 \sim 24\pi^3 M_5/N_c^{2/3}$. Even though the cut-off scale lowers due to the presence of the CS term, we can still have, in the large-$N_c$ limit, a 5D weakly coupled theory where higher-dimensional operators are suppressed. For $M_5 \sim N_c/(16\pi^2 L)$, as expected from the AdS/CFT correspondence, one has in fact $\Lambda_5 L \sim N_c^{1/3} \to \infty$. The same factor protects the physics of the 5D skyrmions which have size $L$. 

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