The $\bar{p}p \to \pi^0\pi^0$ Puzzle

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According to conventional theory, the annihilation reaction $\bar{p}p \to \pi^0\pi^0$ cannot occur from a $\bar{p}p$ atomic S state. However, this reaction occurs so readily for antiprotons stopping in liquid hydrogen, that it would require 30% P-wave annihilations. Experimental results from other capture and $\bar{p}p$ annihilation channels show that the fraction of P-wave annihilations is less than 6% in agreement with theoretical expectations. An experimental test to determine whether this reaction can occur from an atomic S state is suggested. If indeed this reaction is occurring from an atomic S state, then certain neutral vector mesons should exhibit a $\pi^0\pi^0$ decay mode, and this can also be tested experimentally.

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Because of parity conservation and symmetry of identical bosons with respect to their interchange, the annihilation reaction $\bar{p}p \to \pi^0\pi^0$ cannot occur from a $\bar{p}p$ atomic S state. However, experiments dating back to the early 70’s have shown that the reaction occurs readily for antiprotons stopping in liquid hydrogen. Although this mode is allowed from atomic P-state annihilations, the measurements with the Crystal Barrel detector of the branching ratio for this reaction (when compared with the $\bar{p}p \to \pi^+\pi^-$ branching ratio) would require 30% P-wave annihilations. Experimental results from $\pi^-p, K^-p$, and $\Sigma^-p$ capture and other $\bar{p}p$ annihilation channels show that the fraction of P-wave annihilations is less than 6% in agreement with theoretical expectations. Thus, the experimental evidence strongly indicates that the annihilation into $\pi^0\pi^0$ is occurring from an atomic S state. We will discuss an experimental test involving X-rays in coincidence with the reaction, which can determine whether or not this reaction is occurring predominantly from an atomic S state and other tests involving vector meson decays. We will also consider how conventional theory could be modified to allow this reaction to occur from an atomic S state.

The eigenvalues of parity and charge parity of a fermion-antifermion pair are given by [1],

$$\omega_{\text{parity}} = (-1)^{X+1},$$
$$\omega_{\text{Charge parity}} = (-1)^{X+s},$$

(1)

where $X$ is the relative orbital angular momentum of the two particles and $s$ is the spin of the fermion-antifermion system. The eigenvalues of parity and charge parity of a two pion system are given by [1],

$$\omega_{\text{Parity}} = (-1)^Y,$$
$$\omega_{\text{Charge Parity}} = (-1)^Y,$$

(2)

where $Y$ is the relative orbital angular momentum of the two pions. For the $\pi^0\pi^0$ system there are further constraints. Because of Bose statistics the state of two identical pions must be symmetric under interchange. Thus, $Y$ must be even and both parity and charge.
parity must be +1 for the $\pi^0\pi^0$ system. Considering only low angular momentum states (i.e., $J = 0, 1$), we obtain Tables I-III for initial and final states.

### TABLE I. Initial state of $\overline{p}p$ Atom

| State   | $J$ | Parity | Charge parity |
|---------|-----|--------|---------------|
| $^1S_0$ | 0   | -1     | +1            |
| $^3S_1$ | 1   | -1     | -1            |
| $^3P_0$ | 0   | +1     | +1            |
| $^1P_1$ | 1   | +1     | -1            |
| $^3P_1$ | 1   | +1     | +1            |

Using conservation of parity, charge parity, and total angular momentum in matching the initial and final states, we obtain the allowed reactions:

$$\overline{p}p\ (^3P_0) \rightarrow \pi^0\pi^0 (S_0),$$
$$\overline{p}p\ (^3S_1) \rightarrow \pi^+\pi^- (P_1),$$
$$\overline{p}p\ (^3P_0) \rightarrow \pi^+\pi^- (S_0).$$

(3)

Thus we see that the reaction $\overline{p}p \rightarrow \pi^0\pi^0$ cannot occur from an atomic $S$ state of the $\overline{p}p$ system.

### TABLE II. Final state of $\pi^0\pi^0$ System

| State | $J$ | Parity | Charge parity | Comment |
|-------|-----|--------|---------------|---------|
| $S_0$ | 0   | +1     | +1            | Y=0     |

We will briefly review what happens when an antiproton or some other negatively charged particle is slowed down in liquid hydrogen and is captured in a Bohr orbit by a proton. This is illustrated in Fig. [1]. Typically, the incoming negatively charged particle is initially captured in an orbit with principle quantum number $n \approx 30$ and with high orbital angular momentum, $X$. Collisional deexcitations and radiative transitions transform the atom to lower $n$ and $X$ values. The electrically neutral atom can then penetrate neighboring atoms and experience the electric field of the protons. This causes Stark effect transitions between the degenerate orbital angular momentum states. The rates for radiative transition and nuclear absorption (or annihilation) from P states are small in comparison with the rate that the Stark effect populates the S state. Since S-state absorption (or annihilation) can happen from high $n$ values, the atom is unlikely to deexcite to low $n$ values for which P state nuclear absorption (or annihilation) is more important.

### TABLE III. Final state of $\pi^+\pi^-$ System

| State | $J$ | Parity | Charge parity | Comment |
|-------|-----|--------|---------------|---------|
| $S_0$ | 0   | +1     | +1            | Y=0     |
| $P_1$ | 1   | -1     | -1            | Y=1     |
FIG. 1. Levels of atomic orbital states for a negatively charged particle and a proton with principle quantum number $n$ and orbital angular momentum $X$. It shows the effect of Stark transitions on different $X$ states, radiative deexcitations, and levels from which nuclear absorption or annihilation are likely.

Thus, according to theory [2,3] absorption will occur predominantly from S states for $\pi^-$ and $K^-$. In 1960, Desai [4] concluded, “Rough calculations indicate that for protonium also the capture will take place predominantly from S states.”

There is strong experimental evidence that S-state capture dominates in liquid $H_2$. The
reactions $\pi^-p[3]$, $K^-p[4]$, and $\Sigma^-p[5]$ have been studied. Since these negatively-charged particles decay, one can determine the nuclear absorption time by observing the fraction which decay. The cascade times are about two orders of magnitude shorter than would be required for radiative deexcitation. Because the antiproton does not decay, such a measurement is not possible. Since the short cascade times for $\pi^-, K^-$, and $\Sigma^-$ cannot be explained without recourse to the Stark effect, the Stark effect must also play a role in the $\overline{p}p$ case.

There is some direct evidence of S-state domination in $\overline{p}p$ reactions. It has been determined [8] that $\overline{p}p \rightarrow KK < 6\%$ P-wave with a 95% confidence level. From the $\rho$ decay angular distribution, it has been determined [9,10] that $\overline{p}p \rightarrow \pi^+\pi^-\pi^0 < 5\%$ P-wave. Thus the experimental evidence strongly supports S-state domination for $\overline{p}p$ reactions.

We will now look at the experimental results for $\overline{p}p \rightarrow \pi\pi$. In liquid hydrogen the branching ratio for $\overline{p}p \rightarrow \pi^+\pi^-$ is $(31 \pm 1) \times 10^{-4}$ [11], while measurements of the branching ratio for $\overline{p}p \rightarrow \pi^0\pi^0$ are given in Table IV.

The experimental results obtained with the Crystal Barrel detector are likely to be the most accurate. As they noted in their paper [17], “Owning to our large detection efficiency and small background our result is least likely influenced by undetected systematic errors. The reliability of the result is strengthened by the internal consistency of a large set of two-body branching ratios measured with the Crystal Barrel detector and their agreement with previous determinations, especially with bubble chamber data.”

One can calculate the fraction of P-wave annihilations for $\overline{p}p \rightarrow \pi\pi$ as follows,

$$\text{Fraction } P\text{-wave} = \frac{(\overline{p}p \rightarrow \pi^+\pi^-)_P + (\overline{p}p \rightarrow \pi^0\pi^0)_P}{(\overline{p}p \rightarrow \pi^+\pi^-)_{S&P} + (\overline{p}p \rightarrow \pi^0\pi^0)_P}.$$  \hfill (4)

By assuming charge independence,

$$(\overline{p}p \rightarrow \pi^+\pi^-)_P = 2 \times (\overline{p}p \rightarrow \pi^0\pi^0)_P,$$  \hfill (5)

we obtain the % P wave given in Column 2 of Table IV. The 55% P-wave, shown in Table IV for the Crystal Barrel collaboration, is obviously too high, and they decided to use another method [17], involving the P-wave fraction for $\overline{p}p \rightarrow \pi^+\pi^-$ determined from a measurement with gaseous hydrogen at NTP, giving 29% P wave for their result.

| Measured value | % P-wave | Year | Reference |
|----------------|----------|------|-----------|
| $(4.8 \pm 1.0) \times 10^{-4}$ | 40% | 1971 | Devons et. al. [12] |
| $(1.4 \pm 0.3) \times 10^{-4}$ | 13% | 1979 | Bassompierre et. al. [13] |
| $(6 \pm 4) \times 10^{-4}$ | 49% | 1983 | Backenstoss et. al. [14] |
| $(2.06 \pm 0.14) \times 10^{-4}$ | 19% | 1987 | Adiels et. al. [15] |
| $(2.5 \pm 0.3) \times 10^{-4}$ | 22% | 1988 | Chiba et. al. [16] |
| $(6.93 \pm 0.43) \times 10^{-4}$ | 55% | 1992 | Crystal Barrel [17] |
| $(2.8 \pm 0.4) \times 10^{-4}$ | 25% | 1998 | Obelix [18] |
Since even the 29\% P-wave fraction is much too high, Batty [13], trying to reduce this value, modified the earlier calculations of Reifenr"other and E. Klemp [20] using the Borie and Leon model [21]. He fitted the experimental data for \( \bar{p}p \rightarrow \pi\pi \) and \( \bar{p}p \rightarrow KK \) at several target densities. Without enhancement factors he obtained 27\% for the P-wave fraction at liquid hydrogen density. With enhancement of annihilations from fine structure states over that expected from a statistical population and some adjustment of the parameters, he could fit the data (including the Crystal Barrel result) with a P-wave fraction of just 13\% at liquid hydrogen density. Even with this heroic effort, the 13\% P-wave fraction is still too high when compared with the upper limit of 6\% P-wave obtained from the results of other \( \bar{p}p \) annihilations [8–10].

There are two other experiments indicating anomalously large fractions of P-wave (75\%) involve antiproton annihilation at rest in liquid deuterium [22,23], although results from a third experiment were consistent with 0\% P wave [24]. Again the percentage P-wave calculation is based on charge independence and the theoretical argument that \( pd \rightarrow n\pi^0\pi^0 \) cannot occur from an atomic S state.

How can one explain these anomalies? Since the results of experiments from all other capture and annihilation reactions point to S-state domination, the most reasonable conclusion is that \( \bar{p}p \rightarrow \pi^0\pi^0 \) is occurring from an atomic S state. This would have been the conclusion long ago if it did not violate conventional theory.

Although seven different groups have measured the branching ratio for \( \bar{p}p \rightarrow \pi^0\pi^0 \), showing the importance of this unexpectedly large branching ratio, no direct test has been performed to determine whether the reaction could be occurring from an atomic S state. Such an experimental test can be made by setting up an initial \( \bar{p}p \) atomic S state and looking for the \( \pi^0\pi^0 \) final state. The method is illustrated in Fig. 3. As discussed earlier, in liquid \( H_2 \) the Stark effect causes transitions to S states at high n-values, where annihilation occurs more readily than deexcitation. One can decrease the effect of Stark transitions by using \( H_2 \) gas at NTP, and thereby observe the deexcitation radiation.

The coincidence of L and K X-rays from protonium, shows that the atom is in the 1S state. The energy of the K X-rays is between 9.4 KeV (\( K_\alpha \)) and 12.5 KeV (\( K_\infty \)), while energy of the L X-rays is between 1.7 KeV and 3.1 KeV. The energy of M X-rays is between 0.5 KeV and 1.3 KeV. Thus, the X-rays from the different transitions tend to be separated.

The Asterix Collaboration has detected \( K_\alpha \) X-rays in coincidence with L X-rays [25]. The experiment we are proposing is very similar, but requires the triple coincidence of L and K X-rays from protonium and the \( \pi^0\pi^0 \) annihilation mode. The detection of such events can prove that the annihilation reaction is occurring from an atomic S state.

As discussed above, even without this crucial experiment, there is significant evidence that the standard theory concerning this reaction may not be valid. We will now look at what could be wrong with the theory. To match the initial \(^3S_1\) state of \( \bar{p}p \), the \( \pi^0\pi^0 \) system needs a \( P_1 \) state with parity = -1 and charge parity = -1 as shown on the second line of Table III for the \( \pi^+\pi^- \) system. In conventional theory, the \( \pi^0\pi^0 \) system has parity = +1 and charge parity = +1 for all states.

The big difference in those eigenvalues between the \( \pi^+\pi^- \) system and the \( \pi^0\pi^0 \) system in conventional theory is caused by the two \( \pi^0 \)'s being identical while the \( \pi^+ \) and \( \pi^- \) are not. Thus, it must be possible for one \( \pi^0 \) to exist in one internal states while the other \( \pi^0 \) is in another internal state. This would result in the two \( \pi^0 \)'s not always being identical.
FIG. 2. Levels of atomic orbital states for protonium with principle quantum number $n$ and orbital angular momentum $X$. It shows radiative cascades to the 1S state which involve K and L X-rays. Detection of triple coincidences of K and L X-rays and annihilation into $\pi^0\pi^0$ can prove that the reaction occurs from an atomic S state.
Assume the $\pi^0$ field is given by,

$$
\phi(x) = \sum_p f(\omega_p) \left\{ \left[ \chi(p) - \eta(p) \right] e^{ip \cdot x} + \left[ \chi^\dagger(p) - \eta^\dagger(p) \right] e^{-i p \cdot x} \right\},
$$

(6)

where $\chi^\dagger(p)$ and $\eta^\dagger(p)$ are creation operators for two different $\pi^0$ states.

If the $\pi^0$'s are in different states, there is no requirement that the state of two $\pi^0$'s must be symmetric under interchange. Therefore, $Y$ can be odd for some $\pi^0\pi^0$ states just as for the $\pi^+\pi^-$ system, leading to a $P_1$ state with parity of $-1$.

Next we will consider what is required for a state of two $\pi^0$'s to have charge parity of $-1$. Consider a particular state of two $\pi^0$'s in the center of mass system,

$$
\Phi = \sum_K e^{i K \cdot R} \chi^\dagger(K) \eta^\dagger(-K) \Phi_0,
$$

(7)

where $\Phi_0$ is the vacuum state, $K$ is the momentum of each particle, and $R = r_1 - r_2$ is the relative coordinate. Applying the charge parity operator,

$$
C \Phi = \sum_K e^{i K \cdot R} C \chi^\dagger(K) C^{-1} \eta^\dagger(-K) C^{-1} \Phi_0.
$$

(8)

If $\chi^\dagger(p)$ and $\eta^\dagger(p)$ were to each change into themselves under $C$, then this state would be unchanged and the eigenvalue of charge parity would still be $+1$. We need,

$$
C \chi^\dagger(K) C^{-1} = -\eta^\dagger(K),
$$

$$
C \eta^\dagger(K) C^{-1} = -\chi^\dagger(K),
$$

(9)

for then,

$$
C \Phi = \sum_K e^{i K \cdot R} \eta^\dagger(K) \chi^\dagger(-K) \Phi_0.
$$

(10)

But $\chi^\dagger(p)$ and $\eta^\dagger(p)$ commute. Thus, after letting $K \rightarrow -K$ we obtain,

$$
C \Phi = \sum_K e^{-i K \cdot R} \chi^\dagger(K) \eta^\dagger(-K) \Phi_0.
$$

(11)

Applying the coordinate exchange operator $P_r$ results in,

$$
P_r C \Phi = \sum_K e^{i K \cdot R} \chi^\dagger(K) \eta^\dagger(-K) \Phi_0 = \Phi.
$$

(12)

Since $P_r^{-1} = P_r$, $C \Phi = P_r \Phi$. The eigenvalues of $P_r = \pm 1$ depending upon the relative angular momentum of the two particles. Thus, the charge parity becomes the same as in the $\pi^+\pi^-$ case, given by Eq. (2). If the $\pi^0$ field satisfies Eqs. (6) and (9), then the annihilation reaction,

$$
\bar{p}p (^3S_1) \rightarrow \pi^0\pi^0 (P_1)
$$

(13)
is allowed.

According to the Standard Model, the pion is composed of up and down quarks,

\[
\pi^+ = (u \bar{d}), \quad \pi^- = (\bar{u} d), \quad \pi^0 = \frac{1}{\sqrt{2}}(u \bar{u} - d \bar{d}).
\]  

(14)

Although the \(\pi^0\) has two different states (i.e., \(u \bar{u}\) and \(d \bar{d}\)), the different states do not change into each other under charge parity as required by Eq. (9). Thus some change to the pion model is needed. We cannot make any suggestions in that regard other than to note that composite states, such as \(g \bar{h}\) and \(h \bar{g}\), where \(g\) and \(h\) are two different fermions, satisfy Eq. (4).

If the neutral pions are not identical just as the charged pions, then the branching ratio of \(\bar{p}p \rightarrow \pi^0\pi^0\) from S states should be half that of \(\bar{p}p \rightarrow \pi^+\pi^-\) by charge independence. However, since the neutral pions have two different states they will only be non-identical half the time. Therefore, assuming all annihilations are from S states, the branching ratio of \(\bar{p}p \rightarrow \pi^0\pi^0\) should equal \(25 \times (31 \times 10^{-4}) = 7.8 \times 10^{-4}\) which is roughly in agreement with the Crystal Barrel collaboration’s result of \((6.93 \pm 0.43) \times 10^{-4}\).

A referee has pointed out that some of vector mesons could decay into two \(\pi^0\)'s if they are not identical, providing another test of this theory. As discussed above non-identical \(\pi^0\)'s can be in a \(P_1\) state with \(J = 1\), parity = \(-1\), and charge parity = \(-1\) which matches the values for some of the vector mesons.

We considered the \(\pi^0\pi^0\) decay mode of the \(\rho(770)^0\), \(\omega(782)\), \(\phi(1020)\), and \(J/\psi(1S)\). These reactions are forbidden by isospin conservation, but they can proceed electromagnetically. In estimating the branching ratios, we assumed that the \(\pi^0\pi^0\) mode would occur at about the same rate as the \(\pi^+\pi^-\) mode, but reduced by a factor of two because the two \(\pi^0\)'s are only non-identical half the time. However, the \(\rho(770)^0\) has \(I = 1\), so the reaction \(\rho(770)^0 \rightarrow \pi^+\pi^-\) is allowed by isospin conservation because the \(\pi^+\pi^-\) system can have \(I = 0, 1, \text{ or } 2\), while the reaction \(\rho(770)^0 \rightarrow \pi^0\pi^0\) is forbidden because the \(\pi^0\pi^0\) system can only have \(I = 0\) or \(2\). Occurring electromagnetically, its branching ratio would be reduced by the factor \(\alpha^2\).

Our estimated branching ratios are,

\[
\begin{align*}
BR(\rho(770)^0 \rightarrow \pi^0\pi^0) &= 5 \times 10^{-5}, \\
BR(\omega(782) \rightarrow \pi^0\pi^0) &= 1 \times 10^{-2}, \\
BR(\phi(1020) \rightarrow \pi^0\pi^0) &= 4 \times 10^{-5}, \\
BR(J/\psi(1S) \rightarrow \pi^0\pi^0) &= 7 \times 10^{-5}.
\end{align*}
\]

(15)

The only measured upper limit \cite{20}, \(BR(\phi(1020) \rightarrow \pi^0\pi^0) < 4 \times 10^{-5}\), is just in the expected range.

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