Solitary waves in three-dimensional crystal-like structures

Edward Arévalo
Max-Planck-Institut für Physik komplexer Systeme, Nöthnitzer Strasse 38,
D-01187 Dresden, Germany
E-mail: edward@pks.mpg.de

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Abstract. The motion of three-dimensional (3D) solitary waves and vortices in nonlinear crystal-like structures is studied. It is demonstrated that collective excitations in these systems can be tailored to move along any predetermined direction in the 3D system. The effect of the modulation instability is studied, showing that in some cases it can be delayed by using a lensing factor. Analytical results supported by numerical simulations are presented.

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1. Introduction

Recently, great progress has been made in the experimental fabrication of three-dimensional (3D) crystal-like optical nano structures, such as photonic crystals [1] and photonic metamaterials [2]. Light in 3D crystal-like optical nano structures can interact with the regular pattern of the structure, setting up resonances. These resonances can cause beams and pulses to be deflected in unconventional directions and even to slow down the speed of light. Engineered optical metamaterials with unique electromagnetic properties have become in recent years a hot research topic because of their interesting physics and exciting potential applications such as, e.g., cloaking [3, 4] or negative refractive index [5, 6] among others.

The theoretical analysis of these systems has been mostly performed with the help of numerical simulations, where the 3D spatial distribution of the effective electromagnetic properties of the material medium (such as the permittivity and permeability) are tailored to have specific electromagnetic properties in the continuum limit [3]. Other discrete effects of the system on the propagation of light waves are usually neglected. Similar approximations have also been adopted for studying cloaking of matter waves [4]. So far, most of the studies have been done for linear planewaves, so nonlinear effects have been neglected. There is, however, strong evidence from low-dimensional systems [8, 9, 11, 12] that discrete effects in combination with nonlinearities may play an important role in describing realistic 3D crystal-like structures. So, nonlinear excitations such as moving solitary waves (in short solitons) and vortices can be expected.

With respect to theoretical models, it has been shown that electromagnetic waves interacting with metamaterials in the tight binding limit [13, 14] can be described as a 3D lattice of microresonators modeled by the 3D discrete nonlinear Schrödinger equation (3D-DNLSE) [6]. In the case of 3D photonic crystals, so far, no discrete model has been proposed. However, it is well known that the dynamics of light beams in photonic waveguides can be described by the 2D-DNLSE [11, 12], where the discreteness is transversal to the light propagation. Moreover, it has also been shown that trapped light waves travelling along chains of optical resonators [7] may be described by the 1D DNLSE [8]–[10], where the discreteness is longitudinal to the light propagation. These two low-dimensional models strongly suggest that 3D photonic crystals with high index of refraction [1] may be effectively modeled in the tight-binding limit [13, 14] by a 3D lattice of optical resonators governed by the 3D-DNLSE [15]–[17].

It is important to mention that the 3D-DNLSE is an ubiquitous dynamical-lattice model, which may emerge from a variety of other important problems and has its direct physical realization in Bose–Einstein condensates (BECs) trapped in strong optical lattices [15]–[17].

The aim of the present work is to study the dynamics of moving solitons and vortices in nonlinear 3D-crystal-like structures described by the 3D-DNLSE. The effects of dissipation and inhomogeneities are neglected.

2. Theory

The general form of the 3D-DNLSE with coupling constants $J_x$, $J_y$ and $J_z$ is

$$i\partial_t \psi_{m,n,p} + J_x (\psi_{m-1,n,p} + \psi_{m+1,n,p}) + J_y (\psi_{m,n-1,p} + \psi_{m,n+1,p}) + J_z (\psi_{m,n,p-1} + \psi_{m,n,p+1}) - U \rho_{m,n,p} \psi_{m,n,p} = 0,$$  

(1)
where $\psi_{m,n,p}$ is the complex field at site $\{m, n, p\}$. In equation (1)

$$\rho_{m,n,p} = |\psi_{m,n,p}|^2$$

(2)
can be interpreted as an intensity (e.g. in crystals built of microresonators) or as a probability density (in BEC arrays). In equation (1) the nonlinear coefficient $U$ is a real constant and $t$ is the time coordinate.

In order to proceed we shall consider a traveling wave ansatz for an envelope complex function expanded into harmonics, i.e.

$$\psi_{m,n,p} = \sum_{\mu,v,\xi}^\infty \chi_{\mu,v,\xi}(S_{\mu,v,\xi}) \exp(i\Theta_{\mu,v,\xi}),$$

(3)

where

$$S_{\mu,v,\xi} = r - v_{\mu,v,\xi}t$$

(4)

and

$$\Theta_{\mu,v,\xi} = k_{\mu,v,\xi} \cdot r - \Omega_{\mu,v,\xi}t.$$  

(5)

In equations (4) and (5), $r = \{m, n, p\}$ is the position vector. In equation (4) $v_{\mu,v,\xi} = \{v_{\mu,v,\xi}^{(1)}, v_{\mu,v,\xi}^{(2)}, v_{\mu,v,\xi}^{(3)}\}$ is the velocity, where $v_{\mu,v,\xi}^{(1)}, v_{\mu,v,\xi}^{(2)}$ and $v_{\mu,v,\xi}^{(3)}$ are the components in the $m, n$ and $p$ directions. In equation (5) $k_{\mu,v,\xi} = \{\mu k_x, \nu k_y, \xi k_z\}$ is the quasi-momentum vector, where the indices $\mu, \nu$ and $\xi$ give the harmonic order and $k_x, k_y$ and $k_z$ are independent parameters. These parameters are allowed to take values within the first Brillouin zone ($|k_x| \leq \pi, w = x, y, z$). Finally, $\Omega_{\mu,v,\xi} = \Omega_0(\mu \omega_x + \nu \omega_y + \xi \omega_z)$ in equation (5) is the frequency (chemical potential in BEC arrays), where $\omega_x, \omega_y$ and $\omega_z$ are functions of the quasi-momentum components to be determined later.

In the following, we apply the quasi-continuum approximation [12, 18, 19]. Here, in equation (1) we use the full Taylor expansion of the functions $\{\psi_{m\pm n,p}, \psi_{m,n\pm p}, \psi_{m,n,p\pm 1}\} = \{\exp(\pm \theta_m), \exp(\pm \theta_n), \exp(\pm \theta_p)\}\psi_{m,n,p}$, where $m, n,$ and $p$ are regarded as continuous variables. So, equation (1) is transformed into an operator equation [12, 18, 19], which, after inserting the ansatz (3) and Fourier transforming, becomes

$$\tilde{W}_{\mu,v,\xi}(q) = a_{\mu,v,\xi}(q) \tilde{\psi}_{\mu,v,\xi}.$$  

(6)

Here,

$$a_{\mu,v,\xi}(q) = q \cdot v_{\mu,v,\xi} + \Omega_{\mu,v,\xi} + 2J_x \cos(\mu k_x + q_x) + 2J_y \cos(\nu k_y + q_y) + 2J_z \cos(\xi k_z + q_z),$$

(7)

where $q = \{q_x, q_y, q_z\}$. In equation (6), tilde marks the Fourier transformed functions, e.g.

$$\tilde{W}_{\mu,v,\xi}(q) = \frac{1}{(2\pi)^{3/2}} \int \int \int_{-\infty}^{\infty} W_{\mu,v,\xi}(S_{\mu,v,\xi}) \exp(iq \cdot S_{\mu,v,\xi}) \exp[i(\mu \omega_x + \nu \omega_y + \xi \omega_z)] dS_{\mu,v,\xi}^{(1)} dS_{\mu,v,\xi}^{(2)} dS_{\mu,v,\xi}^{(3)}.$$  

(8)

The function $W_{\mu,v,\xi}$, in the right-hand side of equation (8), collects all products of envelope functions of the nonlinear term of equation (1) that belong to the same harmonic $\exp(i\Theta_{\mu,v,\xi})$, i.e.

$$\sum_{\mu,v,\xi} W_{\mu,v,\xi} \exp(i\Theta_{\mu,v,\xi}) = U \rho_{m,n,p} \psi_{m,n,p}.$$  

(9)

Note that in equation (8) $S_{\mu,v,\xi}^{(1)}, S_{\mu,v,\xi}^{(2)}$ and $S_{\mu,v,\xi}^{(3)}$ are the components of the vector $S_{\mu,v,\xi}$ defined in equation (4).
In order to solve equation (6) we expand \( a_{\mu,v,\xi}(q) \) for small \( q \), i.e.

\[
a_{\mu,v,\xi}(q) = \sum_{j,l,s} \alpha_{\mu,v,\xi,j,l,s} q_j^i q_l^i q_s^i. \tag{10}
\]

The expansion (10) is done up to second order in \( q \), forcing the first terms of the expansion to be zero \([12, 18, 19]\), so, transforming back to the position space, we find a second-order differential equation for the first (\( F \)) harmonic \( \chi_{1,1,1}(\mu = \nu = \xi = 1) \), which reads as

\[
\alpha_0 \partial_x^2 \chi_F + \alpha_y \partial_y^2 \chi_F + \alpha_z \partial_z^2 \chi_F - \alpha_0 \chi_F + 3U|\chi_F|^2 \chi_F = 0, \tag{11}
\]

where \( \chi_F = \chi_{1,1,1} \), and

\[
\alpha_0 = 2[J_x \cos(k_x) + J_y \cos(k_y) + J_z \cos(k_z)](1 - \Omega_0),
\]

\[
\alpha_x = -J_x \cos(k_x),
\]

\[
\alpha_y = -J_y \cos(k_y),
\]

\[
\alpha_z = -J_z \cos(k_z). \tag{12}
\]

In equation (11) the coordinates \( \{x, y, z\} = S_{1,1,1} \), where \( S_{\mu,v,\xi} \) is given in equation (4). In equation (12) the constants \( \Omega_0, k_x, k_y, \) and \( k_z \) can be chosen as independent parameters.

Since the coordinates \( \{x, y, z\} \) are mutually independent, equation (11) can be integrated. So, finally, approximate soliton solutions of equation (1) read as

\[
\psi_{m,n,p} = e^{i\Theta_{1,1,1}} \frac{\sqrt{\alpha_0}}{3U} \prod_{w=x,y,z} \left[ \delta_{j,w} 2^{1/6} \sech^{1/3} \left( \frac{w}{L_{Bw}} \right) + \delta_{j,w,-1} \tanh^{1/3} \left( \frac{w}{L_{Dw}} \right) \right], \tag{13}
\]

where \( \delta_{j,w,+1} \) is the Kronecker delta with \( \beta_w = \text{sign}(\gamma_w \alpha_0 / \alpha_w) \) and \( w = x, y \) and \( z \). In equation (13) soliton widths read as

\[
L_{Bw} = \sqrt{\gamma_w \alpha_0 / \alpha_w} \tag{14}
\]

and

\[
L_{Dw} = \sqrt{-\gamma_w \alpha_0 / (2\alpha_w)}, \tag{15}
\]

where the relation \( \gamma_x + \gamma_y + \gamma_z = 1 \) should be satisfied. Besides,

\[
\Theta_{1,1,1} = k_x m + k_y n + k_z p - \Omega_{1,1,1} t, \tag{16}
\]

\[
\Omega_{1,1,1} = -2\Omega_0 [J_x \cos(k_x) + J_y \cos(k_y) + J_z \cos(k_z)], \tag{17}
\]

and

\[
\nu_{1,1,1} = 2 \{ J_x \sin(k_x), J_y \sin(k_y), J_z \sin(k_z) \}. \tag{18}
\]

For completeness we note that other soliton solutions with embedded vorticity can be calculated by using spherical coordinates \((r, \theta, \phi)\). In that case equation (11) can be written as

\[
\beta r^2 R(|R|^2 - 1) + \partial_r (r^2 R) + \partial_\theta ((1 - \zeta^2) \partial_\theta R) + \partial_\phi^2 R / (1 - \zeta^2) = 0, \tag{19}
\]

where

\[
R = \sqrt{3U/\alpha_0 \chi_F}, \tag{20}
\]

\[
r^2 = \alpha_0 (x^2 / \alpha_x + y^2 / \alpha_y + z^2 / \alpha_z), \tag{21}
\]
Figure 1. Solutions of equation (25) for the cases $\beta = 1$ ($\epsilon = 0$, (black solid line), $\epsilon = 1$ (red dashed line), $\epsilon = 2$ (blue dotted line and green dot-dash line)), and $\beta = -1$ ($\epsilon = 1$ (orange double-dot-dash line)).

\[ \zeta = \cos(\theta), \quad (22) \]
\[ \theta = \tan^{-1} [\sqrt{\alpha_x x} / (\sqrt{\alpha_y y})], \quad (23) \]
\[ \phi = \cos^{-1} [\sqrt{\alpha_0 z} / (\sqrt{\alpha_r r})]. \quad (24) \]

By using the ansatz $R = F(r)Y^\sigma_\epsilon(\theta, \phi)$, equation (19) can be reduced to
\[ \frac{d}{dr} \left( r^2 \frac{d}{dr} f \right) + \beta r^2 f (f^2 - 1) - \epsilon(\epsilon + 1) f = 0. \quad (25) \]

Here, $f = |Y^\sigma_\epsilon|F(r)$ and $Y^\sigma_\epsilon$ is a spherical harmonic function of degree $\epsilon$ ($\epsilon \geq 0$) and order $\sigma$ ($|\sigma| \leq \epsilon$). In equation (25) the parameter $\beta = 1$ ($\beta = -1$) for $\alpha_0 \alpha_w > 0$ ($\alpha_0 \alpha_w < 0$) with $w = x, y, z$.

Finally we obtain the solutions of equation (1) with embedded vorticity $\sigma$, reading as
\[ \psi_{m,n,p} = \frac{\alpha_0}{\sqrt{3U}} Y^\sigma_\epsilon(\theta, \phi) f(r) \exp(i\Theta_{1,1,1}). \quad (26) \]

In equation (26) the behavior of the function $f$ is governed by equation (25). Note that for $\epsilon = 0$, a solution of equation (25) is $f = 1$. For given $\epsilon$ and $\beta$ values in equation (25), other unbounded and bounded solutions can be numerically calculated with a shooting method. Some of these $f$ solutions are plotted in figure 1.

3. Results

The split-step Fourier method has been used to solve numerically equation (1) for a lattice size $\max(m \times n \times p) = 64 \times 64 \times 64$ with periodic boundary conditions and integration step size $\Delta t = 5 \times 10^{-3}$.

3.1. Soliton motion

In figures 2 and 3, superpositions of snapshots at different time values of a soliton–soliton–soliton and a vortex–soliton–soliton collision are presented. The plots consist...
Figure 2. Superposition of snapshots of $\rho_{m,n,p}$ isosurfaces with $\kappa_0 = 0.5$ at different time values of a soliton–soliton–soliton collision. At $t = 0$ (labels A, B and C (red color)) the initial conditions follow from equation (13), where (A) $k = 0.95\frac{\pi}{2}(0, 0, -1)$, $r_{m,n,p} = (32, 32, 52)$, $\Omega_0 = 1.05$, (B) $k = 0.95\frac{\pi}{2}(-1, -1, 0)$, $r_{m,n,p} = (52, 52, 0)$, $\Omega_0 = 1.1$ and (C) $k = 0.95\frac{\pi}{2}(1, 1, 1)$, $r_{m,n,p} = (12, 12, 12)$, $\Omega_0 = 1.5$. At $t = 10$ (label D (green color)) the collision can be observed. At $t = 24$ (labels A’, B’ and C’ (blue color)) the solitons after the collision can be observed. Other parameters are $U = -1$, $J_x = J_y = J_z = 1$, $\gamma_x = \gamma_y = \gamma_z = 1/3$. Here, $\beta_x = \beta_y = \beta_z = +1$, so only sech-type solutions in equation (13) are considered.

In order to guide the eye and show the path and direction of motion of the solitons and vortices, arrows (in yellow) are plotted in figures 2 and 3. The initial conditions in figures 2 and 3 (labels A, B and C (color red)) follow from equations (13) and (26), respectively. The direction of motion of solitons and vortices is determined by the normalized velocity vector,

$$d = v_{1,1,1}/|v_{1,1,1}|,$$

where $v_{1,1,1}$ is given by equation (18).

In figures 2 and 3 we observe that solitons and vortices when propagating undergo an instability. This instability follows from the interplay between nonlinearity and discreteness. The nonlinearity tends to localize the wave, while the discrete diffraction tends to spread it out. For moving solitons this instability corresponds to the case when the diffraction effect is stronger than the nonlinear localization effect. In the following, we shall call this instability the ‘self-defocusing process’. This self-defocusing process manifests itself as a soliton-shape broadening, accompanied by exponential-like amplitude decay. An example of this amplitude decay can be observed in figure 4, where the evolution of the absolute $\rho_{m,n,p}$ maximum ($\rho_{\text{max}}$) for the collision is plotted in figure 2.
Figure 3. The same as in figure 2 but with $\kappa_0 = 0.2$ and solutions from equation (26). At $t = 0$ (labels A, B and C (red color)) the initial conditions are (A) $k = \{\pi, \pi, -1.05 \frac{\pi}{2}\}$, $\Omega_0 = 1.05$, $\epsilon = \sigma = 1$, (B) $\Omega_0 = 1.01$, $\epsilon = \sigma = 0$ and (C) $\Omega_0 = 1.05$, $\epsilon = \sigma = 0$. At $t = 10$ (label D (green color)) the collision can be observed. At $t = 24$ (labels A', B' and C' (blue color)) the vortex and solitons after the collision can be observed. Other parameters are given in figure 2.

Figure 4. Evolution of the maximum $\rho_{\text{max}}(t) = \max_{m,n,p} \rho_{m,n,p}(t)$ for the collision in figure 2 (black solid line). For comparison, $\rho_{\text{max}}(t)$ of the individual solitons (cases where the solitons do not collide) plotted in figure 2 is presented (red, blue and green dashed lines; lines lie very near each other).

We observe in figure 4 that $\rho_{\text{max}}$ peaks when the collision in figure 2 (label D (green color)) occurs. After collision, in figure 2, the soliton shapes further broaden (labels A', B' and C' (blue color)) and their amplitudes, in figure 4, further decay. For comparison in figure 4 the amplitude $\rho_{\text{max}}$ of the individual solitons in the absence of collision is also plotted. We observe that the $\rho_{\text{max}}$ value coincides with that of individual solitons before and after collision. It is worth remarking that in figure 4 the peak amplitude during collision is much higher than the sum of the three

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soliton amplitudes. This is because during collision the nonlinear effect becomes strong, so a
transitory self-focusing process takes place.

The self-defocusing process in solitons depends on their form and direction of motion, which in turn are governed by the quasi-momentum vector

\[ \mathbf{k} = k_{1,1,1} = \{k_x, k_y, k_z\}. \]  

(28)

As in 2D systems \([11, 12]\), solitons moving along the diagonal directions,

\[ d = \begin{cases} 
\{\pm 1, \pm 1, \pm 1\}/\sqrt{3}, \\
\{\pm 1, \pm 1, 0\}/\sqrt{2}, \\
\{\pm 1, 0, \pm 1\}/\sqrt{2}, \\
\{0, \pm 1, \pm 1\}/\sqrt{2}.
\end{cases} \]  

(29)

are less prone to self-defocusing than those moving along the main-axes directions,

\[ d = \begin{cases} 
\{\pm 1, 0, 0\}, \\
\{0, \pm 1, 0\}, \\
\{0, 0, \pm 1\}.
\end{cases} \]  

(30)

Note that it is possible to control the defocusing process so that solitons moving in different
directions and velocities can have the same decay rate. For example, solitons in figure 2 have
similar decay rates, as shown in figure 4. This can be easily achieved by first choosing with the
help of \(\mathbf{k}\) the wanted directions and velocities of motion and then with the help of \(\Omega_0\), included
in \(\alpha_0\) in equations (12), the same initial amplitudes. Here, we use the fact that the amplitude of
a soliton is proportional to its decay rate \([12, 19]\). Note that for similar decay rates we obtain
solitons with different sizes and velocities, as shown figures 2 and 3.

In figure 3, we plot the initial soliton and vortex solutions (labels A, B and C (color
red)) following from equation (26). In particular, the motion of a vortex shell \((A \rightarrow D \rightarrow A' \rightarrow \) 
in figure 3) along its azimuthal axis \((p\text{-axis})\) with vorticity \(\sigma = 1\) is shown. This is a dark
vortice (vortex solution plotted in figure 1 (orange double-dot-dash line)), which in figure 3 has
been softly truncated with the help of the function \(0.5(1 - \tanh(r - 4))\). The broadening and
breaking apart of the vortex in figure 3 are due to the modulation instability and can be also
observed in the absence of collisions. This instability effect on vortices is more pronounced for
higher values of the vorticity \((\sigma \geq 2)\). On the other hand, the soliton dynamics in figure 3 is
similar as in figure 2; however, small perturbations of the soliton shape are observed after the
collision (labels B’ and C’ in figure 3).

We remark that the center of mass of both solitons and vortices moves with a constant
velocity whose magnitude measured in simulations is very well predicted by equation (18). On
the other hand, although not seen in figures 2 and 3, the presence of low-intensity radiation tails
can be detected for both soliton and moving vortices.

3.2. Instability mitigation

A question that emerges from the above results is how to mitigate the self-defocusing effect.
In order to tackle this problem we investigate the effect of a magnification in the form of a 3D
thin-lens phase, i.e.

\[ \psi_{m,n,p} \rightarrow \psi_{m,n,p} \exp(-ir^2/(4T)). \]  

(31)
Figure 5. Superposition of snapshots of $\rho_{m,n,p}$ isosurfaces with $\kappa_1 = 0.5$ at different time values of three solitons with different thin-lens-phase factors. At $t = 0$ (labels A, B and C (red color)) the initial conditions follow from equation (13), where (A) $T = \infty$, (B) $T = 0.2$ and (C) $T = 0.1$. At $t = 12$ (labels $A'$, $B'$ and $C'$ (green color)) and $t = 24$ (labels $A''$, $B''$ and $C''$ (blue color)) solitons move with different maxima. Other parameters: $k = 0.95\frac{\pi}{2}(1, 0, 0)$, $\Omega_0 = 1.001$ and $\epsilon = \sigma = 0$.

In the present analysis $\psi_{m,n,p}$ solutions follow from equation (26), where the radial coordinate $r$ is given by equation (21). In equation (31), due to the discreteness of the system, the parameter $T$ does not correspond exactly to the focal length, as it happens in continuous systems.

In figure 5, we consider three solitons moving in the positive direction of the $m$ axis. The transversal separation distance between the solitons was chosen large enough to avoid soliton–soliton interactions. The initial soliton forms, $\psi$ (labels C, A, B (red color) in figure 5), are identical to each other but distinct from the $T$ value in the imposed thin-lens phase. As in figures 2 and 3, arrows (in yellow) are plotted to show the path and direction of motion of the solitons. In order to compare the soliton behavior, the level value of the isosurfaces in figure 5 was chosen to be fixed during the whole evolution to some initial value given by the expression

$$
\rho_{\text{Level}} = \kappa_1 \max_{m,n,p} \rho_{m,n,p}(t = 0),
$$

where $0 < \kappa_1 < 1$. In figure 6 the individual $\rho_{\text{max}}$ values of the solitons in figure 5 are plotted as well as the level $\rho_{\text{Level}}$ (straight red line). So, solitons with $\rho_{\text{max}}$ value above $\rho_{\text{Level}}$ appear plotted in figure 5. Note, e.g., that in the soliton evolution $A \rightarrow A' \rightarrow A''$ in figure 5 the soliton shape vanishes at $A''$ because its $\rho_{\text{max}}$ value (black line in figure 6) decays below the level value $\rho_{\text{Level}}$ (straight red line in figure 6) of the isosurface for $t > 12$.

For comparison three different $T$ values have been considered in figure 5, namely $T = \infty$, 0.2 and 0.1. The value $T = \infty$ ($A \rightarrow A' \rightarrow A''$) corresponds to the case where magnification is negligible. The value $T = 0.2$ ($B \rightarrow B' \rightarrow B''$) corresponds to a case where the soliton amplitude remains nearly constant (see figure 6, blue dashed line) for the scale of time considered. However, it is to be remarked that for larger time scales a self-defocusing process is observed and unavoidable. The value $T = 0.1$ ($C \rightarrow C' \rightarrow C''$) corresponds to a case where a focusing due to the thin-lens phase is immediately followed by a self-focusing effect, as can be observed in figure 6 (purple dotted line). Besides, a strong radiation tail can be also observed in figure 5 ($C''$).

For further comparison in figure 6, the case of $T = 0.05$ (green dot-dashed line) is also plotted. This case is similar to the case $T = 0.1$ where the first peak corresponds to a focusing
Figure 6. Evolution of $\rho_{m,n,p}$ maximum of individual solitons. $T = \infty$ (black solid line), $T = 0.2$ (blue dashed line), $T = 0.1$ (purple dotted line) and $T = 0.05$ (green dot-dashed line). The straight red line corresponds to the isosurface level of the plots in figure 5.

due to the imposed thin-lens phase and the second peak is due to a self-focusing effect. Note that after the second peak the soliton amplitude strongly decays and a long radiation tail, more in the fashion of figure 5(C’’), can be also observed.

4. Conclusions

We have studied, for the first time, the motion of solitons and vortices in the 3D-DNLSE. In the tight-binding limit this is the most simple model for studying the effect of nonlinearities in 3D crystal-like structures, such as 3D photonic crystals, metamaterials and 3D BEC arrays. The analytical results, which are supported by simulations, suggest that these moving excitations can appear or be excited for any finite value of the model parameters. This implies that solitary waves may play an important role in the design of 3D crystal-like structures. On the other hand, we have observed that solitons survive collisions and their modulation instability can be delayed. This supports the idea that photonic crystals may be used as a 3D compact routing [1] for optical pulses. From the practical standpoint, the exact generation of those solitons studied here remains a challenge. However, it does not preclude the fact that moving excitations observed in those systems can be analyzed as a superposition of solitons and/or vortices.

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