Strangeness Balance in HADES Experiments and the $\Xi^-$ Enhancement

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HADES data on a strangeness production in Ar+KCl collisions at 1.76A GeV are analyzed within a minimal statistical model. The total negative strangeness content is fixed by the observed $K^+$ multiplicities on event-by-event basis. Particles with negative strangeness are assumed to remain in chemical equilibrium with themselves and in thermal equilibrium with the environment until a common freeze-out. Exact strangeness conservation in each collision event is explicitly preserved. This implies that $\Xi$ baryons can be released only in events where two or more kaons are produced. An increase of the fireball volume due to application of a centrality trigger in HADES experiments is taken into account. We find that experimental ratios of $K^-/K^+$, $\Lambda/K^+$ and $\Sigma/K^+$ can be satisfactorily described provided in-medium potentials are taken into account. However, the calculated $\Xi^-/K^+$ ratio proves to be significantly smaller compared to the measured value (8 times lower than the experimental median value and 3 times lower than the lower error bar). Various scenarios to explain observed $\Xi$ enhancement are discussed. Arguments are given in favor of the $\Xi$ production in direct reactions. The rates of the possible production processes are estimated and compared.

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I. INTRODUCTION

A new stage in the study of a strangeness production in heavy-ion collisions (HICs) has been achieved with the launch of HADES experiment at Schwerionen Synchrotron (SIS) at GSI in Darmstadt. The HADES collaboration has undertaken a complete measurement of the particles containing strange quarks in the system Ar+KCl at the bombarding energy of 1.76A GeV. Production of both open and hidden strangeness was investigated. Results on kaon and $\phi$ meson productions are reported in [1,2], on hyperons in [3]. The first observation of the doubly strange hyperon $\Xi^-$ at such a low collision energy is described in [4]. These new results are complementary to previous analyses by FOPI and KaoS collaborations [5], providing new insights upon strangeness production mechanisms and dynamics in HICs. Two puzzling observations have been reported so far. The first puzzle is the strong enhancement of the $\phi$ meson yield [1], description of which, possibly, requires the inclusion of a new type of catalytic reactions [6], see also the thorough analysis in [2,5]. The second puzzle is the anomalously large $\Xi/\Lambda$ ratio [4] exceeding the predictions of the statistical model [1] and of the transport code [11]. Such an excess over predictions of the statistical model is worrisome since it may signal about a new production mechanism not yet manifested in other observed particle yields.

In this work we formulate the minimal statistical model with explicit strangeness conservation on event-by-event basis and analyze the HADES results on strangeness production within this model. The model is especially suitable for analysis of the $\Xi$ baryon yield. Application of a centrality trigger in HADES experiments and in-medium potentials acting on all particles are incorporated. Various scenarios to explain observed $\Xi$ enhancement are discussed. Arguments are given in favor of the $\Xi$ production in direct reactions. The rates of the possible production processes are estimated and compared.

The paper has the following structure. In Section II we discuss the HADES data and define the strange particle ratios. Our minimal statistical model for the strangeness production is formulated in detail in Section III. The in-medium potentials acting on nucleons and strange hadrons are introduced in Section IV. In Section V we discuss effects of centrality bias induced by the LVL1 trigger. The final results of our calculations with and without the trigger effect are collected in Table VI and compared with the experimental data. In Section VII we discuss possible mechanisms of the $\Xi$ enhancement. Conclusions are drawn in Section VIII.

II. HADES DATA

The singly strange particle multiplicities measured by HADES [3] are:

$$M_{K^+} = (2.8 \pm 0.4) \times 10^{-2},$$
$$M_{K^-} = (7.1 \pm 1.9) \times 10^{-4},$$
$$M_{\bar{K}^0} = (1.15 \pm 0.14) \times 10^{-2},$$

**References**

[1] E.E. Kolomeitsev, B. Tomášík, D.N. Voskresensky, "Strangeness Balance in HADES Experiments and the $\Xi^-$ Enhancement," arXiv:1207.5738v1 [nucl-th] (2012).

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\[ M_{\Lambda + \Sigma^0} = (4.09 \pm 0.59) \times 10^{-2}. \]  

(1d)

The doubly strange hyperons, \( \Xi^- \), were detected in the \( \Xi^- \to \Lambda \pi^- \to p\pi^-\pi^- \) channel and the \( \Xi^- \) to \( \Lambda + \Sigma^0 \) ratio was reported in [4]:

\[ R_{\Xi/\Lambda} = M_{\Xi^-}/M_{\Lambda + \Sigma^0} = (5.6 \pm 3) \times 10^{-3}. \]  

(2)

Using the measured multiplicities [1], [2] and the strangeness conservation one can write the multiplicity of unobserved \( \Sigma^+ + \Sigma^- \) baryons:

\[
M_{\Sigma^+ + \Sigma^-} = M_{K^+} + M_{K^0} - M_{\Lambda + \Sigma^0} - M_{K^-} - M_{\Xi^-} - 2M_{\Xi^-} - 2M_{\Xi^0}.
\]

(3)

Unfortunately, not all isospin species are measured in the experiment. Relying on the conservation of isospin on the time scale of a heavy-ion collision, we can assume that ratios of isospin partner multiplicities in the final state of collision reflect the isospin asymmetry of colliding nuclei. The isospin asymmetry for ArK and ArCl collisions is characterized by the coefficient \( \eta = (A-Z)/Z \simeq 1.14 \).

Hence we can write:

\[
M_{K^+}/M_{K^0} \approx M_{K^0}/M_{K^-} \approx M_{\Xi^0}/M_{\Xi^-} \approx 1/\eta
\]

and estimate the \( \Sigma \) baryon yield as:

\[
M_{\Sigma^+ + \Sigma^-}^{(iso)} = (1 + \eta)M_{K^+} - M_{\Lambda + \Sigma^0} - (1 + 1/\eta)M_{K^-} - 2(1 + 1/\eta)M_{\Xi^-} = (1.68 \pm 0.87) \times 10^{-2}.
\]

(5)

In Ref. [3], the multiplicity of \( \Sigma^+ + \Sigma^- \) baryons was calculated differently: the information about neutral kaon (\( K^0 \)) production was used explicitly via the relation:

\[
M_{K^0} = 2M_{K^0}^0 - M_{\Lambda^0},
\]

and also one assumed that:

\[
M_{\Lambda^0} \approx M_{K^-} \quad \text{and} \quad M_{\Xi^0} \approx M_{\Xi^-}
\]

as an isospin symmetric matter. Then the following relation follows:

\[
M_{\Sigma^+ + \Sigma^-}^{(Hades)} = M_{K^+} + 2M_{K^0} - M_{\Lambda + \Sigma^0} - 3M_{K^-} - 4M_{\Xi^-} = (0.71 \pm 0.61) \times 10^{-2}.
\]

(6)

Note that in the corresponding Eq. (4) in Ref. [2] the \( \Xi^- \) multiplicity enters with factor two rather than four as it should, since two isospin species of \( \Xi^- \) carry two strange quarks each. The resulting value calculated following mentioned Eq. (4) in Ref. [2] is, thus, slightly higher, \( (0.75 \pm 0.65) \times 10^{-2} \) than that given by our Eq. (6).

We should stress that the \( \Sigma^\pm \) multiplicity based on the isospin symmetry [3] is almost factor two larger than that obtained according to Eq. (6) and that quoted in Ref. [3].

The origin of this deviation is that the ratio of observed \( K^0 \) to \( K^+ \) estimated as \( 2M_{K^0}/M_{K^+} \approx 0.82^{+0.25}_{-0.19} \) is significantly smaller than that given by the isospin symmetry coefficient of the colliding nuclei, cf. Eq. [3], \( \eta \simeq 1.14 \), as one would expect. This could indicate that, perhaps, too few \( K^0 \) are observed in the experiment or the isospin symmetry is violated in strangeness production reactions.

Since we see no grounds not to believe in the isospin symmetry, we prefer to use the estimate [5] rather than [3]. Nevertheless, since the value [5] is already in use (actually the one obtained from Eq. (4) in Ref. [3]), we will compare the results of statistical model with both values [5] and [6] to see which one is better described.

Using the data [1a], [1b], [1d] and [6] we construct the ratios:

\[
R_{K^-/K^+} = M_{K^-}/M_{K^+} = 2.54^{+1.21}_{-0.91} \times 10^{-2},
\]

(7a)

\[
R_{\Lambda/K^+} = M_{\Lambda^0}/M_{K^+} = 1.46^{+0.49}_{-0.37},
\]

(7b)

\[
R_{\Sigma/K^+}^{(Hades)} = M_{\Sigma^+ + \Sigma^-}/2M_{K^+} = 0.13^{+0.15}_{-0.11},
\]

(7c)

and with the \( \Sigma \) yield [5] obtain the ratio:

\[
R_{\Sigma/K^+}^{(iso)} = M_{\Sigma^+ + \Sigma^-}/2M_{K^+} = 0.30^{+0.23}_{-0.17}.
\]

(8)

As for \( \Xi \) production, in order to reduce the dependence of \( R_{\Xi/\Lambda} \) on the total strangeness content of the fireball we will also use the double ratio:

\[
R_{\Xi/\Lambda} = M_{\Xi^-}/(M_{\Lambda + \Sigma^0} M_{K^+}) = 0.20^{+0.17}_{-0.12}.
\]

(9)

III. STRANGENESS PRODUCTION WITHIN THE MINIMAL STATISTICAL MODEL

A. A brief description of the model

We assume that for a collision at SIS energy the whole energy contained within a nucleus overlap becomes thermalized. Then, an initially prepared hot and dense nuclear fireball expands in vacuum. Simplifying, we assume the fireball to be spatially uniform and characterized by a time-dependent temperature \( T(t) \), baryon density \( \rho_B(t) \) and volume \( V(t) \). At SIS energies the fireball consists mostly of strongly interacting nucleons, \( \Delta \) isotriplets and pions, cf. [11]. It is assumed that the fireball expansion lasts till a moment of freeze-out characterized by the values \( \rho_{B,fo} \) and \( T_{fo} \). Henceforth in-medium particle thermal momentum distributions become distributions of free-streaming particles.

Strange particles and antiparticles need a special care. In the statistical model used before in Refs. [12, 13], the strange particles are assumed to be most efficiently produced at the early hot and dense stage of the nuclear fireball.

At SIS energies the fireball is baryon-rich, therefore the created kaons (\( K^+ \) and \( K^0 \)) have the longest mean free path compared with strangeness \( -1 \) hadrons. After being produced in some processes together with particles carrying \( s \) quark, they can easily move off the production point, and either leave the fireball immediately or, first, thermalize via elastic kaon-nucleon scatterings and then leave it at some intermediate stage. Since the strangeness production probability is very small, even if the kaon stays in the fireball for a while, there is little chance that it meets anti-kaon or hyperon and is absorbed by
them. Thus, in the course of collision the amount of negative strangeness of the fireball grows. The accumulated strangeness is redistributed among $K^-$, $K^0$, $\Lambda$, $\Sigma$ and $\Xi$ baryons\footnote{Particles with a higher strangeness, such as $\Omega$, and antiparticles contribute very little and can be ignored.} sustaining in the thermal equilibrium with pions, nucleons and deltas until a common freeze-out. The $K^+$ yield measured in the experiment can be used to normalize the abundance of negatively strange particles. This differs our model from the statistical model [3], which assumes that $K^+$ and $K^0$ remain in thermal and chemical equilibrium with other constituents of the fireball till the fireball freeze-out. The latter assumption may hold at much higher collision energies than at SIS energies, see [14]. For processes with a small number of produced strange particles the attention ought to be paid to exact strangeness conservation in each collision event. This means, e.g., that $\Xi$ baryons can be produced only in events involving two and more kaons.

Modification of strange hadrons in medium should be incorporated, otherwise it would be impossible to describe measured $K^-$ data satisfactorily. In our minimal statistical model, the medium effects will be described in the mean-field approximation, see Section IV.

More involved p-wave kaon-baryon interactions considered in [12 13] will be disregarded. Note that the HADES data acquisition system includes the first-level trigger (LV1.1), which selects the most central collisions, requiring more than 16 charged particles to be produced in the collision. The sensitivity of strange particle ratios to this centrality bias will be studied in Section V.

B. Kaon event classes

Strangeness production is a rare event. Typically the creation of one $s\bar{s}$ pair, from which the $\bar{s}$ quark leaves the system as the $K^+$ or $K^0$ meson, occurs only in 2–3% of collisions. We will call such events the single-kaon events. In these events $\Xi$ baryon cannot be created, because it needs at least two $s$ quarks to be produced. The much rarer events, in which two $K^+$, or two $K^0$, or $K^+$ and $K^0$, come out from the fireball we will call the double-kaon events. Depending on the number of $s$ quarks remaining in the fireball various strange hadrons and their combinations can be observed in the final state. So the chemical equilibrium conditions are different for the events with different numbers of produced kaons. To proceed further, we divide the totality of events with strangeness production into classes of events with one, two, three and so on, $s\bar{s}$ quark pairs created. We will call them as $n$-kaon events and denote the probability of creation of exactly $n$ strange quark pairs as $P_{s\bar{s}}^{(n)}$. Now we will show how this probability can be related to the observed multiplicity of $K^+$ mesons [1a].

Let $W(\rho_B, T)$ be the probability of $s\bar{s}$ pair production per unit volume and per unit time as a function of the temperature and the baryon density. The integral probability of the pair production is given by

$$W = \int_{t_0}^{t_{\text{max}}} V(t)W(\rho_B(t), T(t))\,dt.$$  \hfill (10)

The integral is taken over the fireball evolution time until the freeze-out moment $t_{\text{fo}}$. The probability of creation of exactly $n$ pairs (the $n$-kaon event) is determined by the Poisson distribution

$$P_{s\bar{s}}^{(n)} = W^n e^{-W} / n!.$$  \hfill (11)

For scale-less hydrodynamic expansion, we can express the current fireball volume through the freeze-out volume $V_{\text{fo}}$ and some scaling function, $V(t)/V_{\text{fo}}$, the fireball expansion time can be expressed as $t_{\text{fo}} = \tau V_{\text{fo}}^{1/3}$, see [10]. Hence, we can write

$$W = \bar{W} \tau V_{\text{fo}}^{4/3} \equiv \lambda V_{\text{fo}}^{4/3},$$  \hfill (12)

where $\tau$ and the averaged probability $\bar{W}$ are constants. Since the probability of the strangeness production is small, $W \ll 1$, we may expand the exponent in (11).

Keeping terms up to the third order we have

$$P_{s\bar{s}}^{(1)} = \lambda V_{\text{fo}}^{4/3} - \lambda^2 V_{\text{fo}}^{8/3} + \frac{1}{2} \lambda^3 V_{\text{fo}}^4 + O(\lambda^4),$$
$$P_{s\bar{s}}^{(2)} = \frac{1}{2} \lambda^2 V_{\text{fo}}^{8/3} - \frac{1}{2} \lambda^3 V_{\text{fo}}^4 + O(\lambda^4),$$
$$P_{s\bar{s}}^{(3)} = \frac{1}{8} \lambda^3 V_{\text{fo}}^4 + O(\lambda^4).$$  \hfill (13)

The value $\lambda$ is fixed by the total $K^+$ multiplicity observed in an inclusive collision. Each of $n$ anti-strange quarks produced in the $n$-kaon event can leave the fireball not only as $K^+$ but also as $K^0$. Since the isospin composition of the fireball dictates the ratio of $K^0$ to $K^+$ multiplicities [4], we can write

$$M_{K^+}^{(n)} = \frac{n}{1 + \eta} P_{s\bar{s}}^{(n)}.$$  \hfill (14)

Here $M_{K^+}^{(n)}$ is the multiplicity of $K^+$ mesons produced in $n$-kaon events, i.e., the number of $K^+$ mesons produced in all $n$-kaon events divided by the total number of events.

The experimentally measured multiplicity of kaons [11] is expressed through (13) using Eq. (11) as

$$M_{K^+} = \sum_n \langle M_{K^+}^{(n)} \rangle = \frac{(W)}{1 + \eta}. $$  \hfill (15)

The brackets indicate that we are dealing with quantities averaged over the collision impact parameter. This averaging means

$$\langle \ldots \rangle = \frac{\int_0^{b_{\text{max}}} db \langle \ldots \rangle}{\int_0^{b_{\text{max}}} db},$$  \hfill (16)
where the integration over the impact parameter runs from 0 up to the maximal possible value \( b_{\text{max}} \). The Ar+KCl collision studied in [4] is nearly symmetrical with the number of nucleons in each colliding nucleus \( A = 39.5 \). Thus, simplifying we may take \( b_{\text{max}} = 2 r_0 A^{1/3} \) with \( r_0 \approx 1.12 \text{ fm} \). This accuracy is sufficient to calculate averaged characteristics of the fireball.

Since we neglect the nuclear surface effects, the initial temperature and density of the fireball do not depend on the impact parameter. Then from Eq. (15) and (12) we find

\[
\lambda = (1 + \eta) \mathcal{M}_K^+/\langle V_0^{4/3}\rangle. \tag{17}
\]

Now the probabilities (13) averaged over the impact parameter can be expressed through the experimental \( K^+ \) multiplicity (1a) up to \( O(M_{K^+}^4) \) as follows

\[
\langle I_{ss}^{(1)} \rangle = (1 + \eta) \mathcal{M}_K^+ \left[ 1 - (1 + \eta)\langle (2)\mathcal{M}_K^+ \right.
+ \frac{1}{2}(1 + \eta)^2 \langle (3)\mathcal{M}_K^2 \rangle,
\]

\[
\langle I_{ss}^{(2)} \rangle = \frac{1}{2}(1 + \eta)^2 \mathcal{M}_K^+ \left[ \langle (2) - (1 + \eta)\langle (3)\mathcal{M}_K^+ \right],
\]

\[
\langle I_{ss}^{(3)} \rangle = \frac{1}{2}(1 + \eta)^3 \langle (3)\mathcal{M}_K^3 \rangle. \tag{18}
\]

Here we introduced the numerical coefficients

\[
\zeta^{(n)} = \langle V_0^{4+n}\rangle/\langle V_0^{4/3}\rangle^n. \tag{21}
\]

The fireball volume at freeze-out can be expressed as

\[
V_{\text{fo}}(b) = 2 A F(b/b_{\text{max}})/\rho_{B,\text{fo}}, \tag{22}
\]

where the function \( F(x) \) describes the overlap of two colliding nuclei. For the overlap function we take the parameterization from Appendix of Ref. [17], which for the symmetrical collision is

\[
F(x) = (x - 1)^2 [1 + (3/\sqrt{2} - 1)x],
\]

\[
\int_0^1 dx x F(x) = (1 + \sqrt{2})/20.
\]

The inclusive fireball volume equals to \( \langle V_{\text{fo}} \rangle \approx A/(2\rho_{B,\text{fo}}) \). Using Eq. (23) in averaging (10), we obtain from (21)

\[
\zeta^{(1)} = 1, \quad \zeta^{(2)} = 2.51, \quad \zeta^{(3)} = 8.11. \tag{24}
\]

We see that the volume dependence of the \( ss \) production probability \( W \) leads to an enhancement, \( \zeta^{(n)} > 1 \), of the multi-pair production probability \( \langle I_{ss}^{(n+1)} \rangle \).

After all \( K^+ \) and \( K^0 \) mesons have left the fireball in the given \( n \)-kaon event, the fireball becomes negatively strange with the total strangeness multiplicity

\[
M_s^{(n)} = n I_{ss}^{(n)}. \tag{25}
\]

With the help of the experimental kaon multiplicity (1a) we estimate

\[
\langle M_s^{(1)} \rangle = 5.2 \times 10^{-2}, \quad \langle M_s^{(2)} \rangle = 8.8 \times 10^{-3},
\]

\[
\langle M_s^{(3)} \rangle = 8.7 \times 10^{-4}. \tag{26}
\]

Thus, \( \langle M_s^{(2)} \rangle/(1 + \eta)\mathcal{M}_{K^+} \simeq 15\% \) of kaons are produced pairwise, and \( \langle M_s^{(3)} \rangle/(1 + \eta)\mathcal{M}_{K^+} \simeq 1\% \) of kaons are produced triplewise.

It is interesting to compare the negative strangeness concentrations in the fireball averaged over the impact parameter for the different classes of events

\[
\rho_S^{(n)} = \langle M_s^{(n)}/V_0 \rangle. \tag{27}
\]

From Eq. (13) we have

\[
\rho_S^{(1)} = (1 + \eta) \frac{\mathcal{M}_{K^+}^+}{\langle V_0 \rangle} [\zeta^{(1)} - (1 + \eta)\zeta^{(2)} \mathcal{M}_{K^+}^+ \frac{1}{2}(1 + \eta)^2 \zeta^{(3)} \mathcal{M}_{K^+}^3], \tag{28a}
\]

\[
\rho_S^{(2)} = (1 + \eta) \frac{\mathcal{M}_{K^+}^2}{\langle V_0 \rangle} \left[ \zeta^{(2)} - (1 + \eta)\zeta^{(3)} \mathcal{M}_{K^+} \right], \tag{28b}
\]

\[
\rho_S^{(3)} = \frac{1}{2}(1 + \eta)^3 \frac{\mathcal{M}_{K^+}^3}{\langle V_0 \rangle} \zeta^{(3)}. \tag{28c}
\]

The numerical values of the coefficients \( \zeta^{(n)} \) are

\[
\zeta^{(1)} = 0.693, \quad \zeta^{(2)} = 1.04, \quad \zeta^{(3)} = 2.85. \tag{30}
\]

Using these values we estimate

\[
\frac{10^3 \rho_S^{(1)}}{\rho_{B,\text{fo}}} \simeq 1.9, \quad \frac{10^5 \rho_S^{(3)}}{\rho_{B,\text{fo}}} \simeq 1.6. \tag{31}
\]

C. Strangeness statistical probability

The statistical probability that strangeness will be released at freeze-out in a hadron of type \( a \) with the mass \( m_a \) is given by the standard Gibbs’ formula

\[
P_a = \frac{1}{z_{sa}} V_0 p_a = \frac{1}{z_{sa}} V_0 \nu_a e \frac{B_{sa}}{2 T} f(m_a, T_0), \tag{32}
\]

\[
f(m, T) = \frac{d^3 p}{(2\pi)^3} e^{-\frac{m^2}{2T}} = \frac{m^2 T}{2\pi^2} K_2 \left( \frac{m}{T} \right) \tag{33}
\]

where \( B_{sa} \) is the baryon number of the hadron, the degeneracy factor \( \nu_a \) is determined by the hadron’s spin \( I_a \) and isospin \( G_a \) as \( \nu_a = (2 I_a + 1) (2 G_a + 1) \), \( K_2 \) is the MacDonald function. The baryon chemical potential at freeze-out is determined by

\[
\mu_{B,\text{fo}} \simeq -T_0 \ln \left( 4 \left[ f(m_N, T_0) + 4 f(m_{\Delta}, T_0) \right] /\rho_{B,\text{fo}} \right), \tag{34}
\]

where \( m_N \) is the nucleon mass. \( \Delta \) isobars are also treated as stable particles with the mass \( m_\Delta = 1232 \text{ MeV} \), and small contribution of heavier resonances, hyperons and anti-particles is neglected.

The quantity \( z_{sa} \) in (32) is a normalization factor. It is related to a probability to find one \( s \)-quark in the hadron.
We assume that this factor is the same for all types of strange hadrons. This is equivalent to the assumption that all strange hadrons carrying $s$-quarks are in chemical equilibrium. For multi-strange hadrons $z_S$ enters Eq. (52) as $z_S^{(n)}$, where $n$ is the number of strange quarks in the hadron. The factor $z_S^{(n)}$ follows from the requirement that the sum of probabilities for production of different strange species and their combinations, which are allowed in the final state, equals to one. The factor $z_S^{(n)}$ depends on how many strange quarks are produced. Hence, it is different in single-, double- and triple-kaon events. Therefore, we introduce the notation

$$P_a^{(n)} = (z_S^{(n)})^a V_0 p_a,$$

where the superscript $n$ indicates to which class of events this probability and $z_S$ factor belong.

Consider now the ensemble of nucleus-nucleus collisions with the fixed impact parameter. In a single-kaon event one $s$-quark can be released as $\bar{K}$, $\Lambda$ or $\Sigma$. Hence, the normalization condition for the probabilities reads

$$P_K^{(1)} + P_\Lambda^{(1)} + P_\Sigma^{(1)} = z_S^{(1)} V_0 (p_K + p_\Lambda + p_\Sigma) = 1. \quad (36)$$

The multiplicity $M_a^{(1)}$ of strange hadrons of type $a = \{\bar{K}, \Lambda, \Sigma\}$ produced in such single-kaon events is given then by

$$M_a^{(1)} = g_a M_S^{(1)} p_a^{(1)} = g_a M_S^{(1)} z_S^{(1)} V_0 p_a,$$

(37)

where $M_S^{(1)}$ is the multiplicity of strange quarks in the fireball at freeze-out in a single-kaon event given by (25) and (18). The isospin factor $g_a$ takes into account the asymmetry in the yields of particles with various isospin projections induced by the global isospin asymmetry of the collision, $\eta \neq 1$. It depends on the baryon number, $B_a$, strangeness $s_a$ of the hadron $a$, and its third component of isospin, $t_{3a}$,

$$g_a = \frac{\eta^{-t_{3a} + (B_a + s_a)/2}}{\sum_{t_{3a}} \eta^{-t_{3a} + (B_a + s_a)/2}},$$

(38)

the sum here is taken over all possible values of $t_{3a}$ . The combination $t_{3a} + (B_a + s_a)/2$ is, of course, nothing else than a charge of the hadron $a$. For $\eta = 1$ this factor reduces to the standard one $1/(2G_a + 1)$.

In double-kaon events there can be $\Xi$ baryons besides all possible combinations of kaon and hyperon pairs. For double-kaon events the normalization condition to (30) is to be replaced by the following one

$$(P_K^{(2)} + P_\Lambda^{(2)} + P_\Sigma^{(2)})^2 + P_\Xi^2$$

$$= z_S^{(2)} V_0^2 (p_K + p_\Lambda + p_\Sigma)^2 + z_S^{(2)} V_0 p_\Xi = 1, \quad (39)$$

In Eq. (39) we assumed validity of the classical statistics for bosons and fermions, i.e., the probability of two hadron event, $P_{2h}$ with $h = \{\bar{K}, \Lambda, \Sigma\}$, is assumed to be equal to the probability of single hadron event squared, $P_{h}^2$. Quantum effects for bosons and fermions make $P_{2\bar{K}} > P_{2\bar{K}}^2$ and $P_{2\Xi < P_{2\Xi}^2}$, but the differences are tiny for the temperatures under consideration. The factors 2, which appear at the cross terms after opening the brackets, reflect the number of combinations with which two $s$ quarks can be released as a given combination of hadrons, e.g., $\bar{K}\Lambda, \bar{K}\Sigma$ and $\Sigma\Lambda$.

The multiplicity of the particle $a$ with one $s$-quark produced in the double-kaon events is equal to

$$M_a^{(2)} = g_a 2 P_s^{(2)} p_a^{(2)} \sum b P_b^{(2)} = g_a M_S^{(2)} P_a^{(2)} \sum b P_b^{(2)} \quad (40)$$

where $a, b = \{\bar{K}, \Lambda, \Sigma\}$, and $M_S^{(2)}$ is given by Eqs. (25) and (18). We take here into account that the hadron $a$ can be produced in pair or in various combinations with other strange hadrons. In both cases the quark combinatoric factor 2 is due, as we discussed in Eq. (39). The multiplicity of produced $\Xi$ baryons is

$$M_\Xi^{(2)} = g_\Xi P_s^{(2)} \sum_{a} P_a^{(2)} = \frac{1}{2} g_\Xi M_S^{(2)} P_\Xi^{(2)}, \quad (41)$$

Note the absence of factor 2 in the first equality.

For completeness now consider a vary rare event class when three $K^+$ mesons are produced. For triple-kaon events the normalization condition is

$$(P_K^{(3)} + P_\Lambda^{(3)} + P_\Sigma^{(3)})^3 + 3 P_\Xi^3 (P_K^{(3)} + P_\Lambda^{(3)} + P_\Sigma^{(3)})$$

$$+ P_\Omega^3 = 1 = z_S^{(3)} V_0^3 (p_K + p_\Lambda + p_\Sigma)^3$$

$$+ 3 z_S^{(3)} V_0^2 (p_K + p_\Lambda + p_\Sigma) p_\Xi + z_S^{(3)} V_0 p_\Omega. \quad (42)$$

Factors 3 appearing in this relation show in how many different ways three $s$ quarks can be distributed between three hadrons, e.g., $\bar{K}\Lambda, \bar{K}\bar{K}, \bar{K}\bar{K}$, or two hadrons, e.g., $\Xi K, \Xi\Lambda$. The multiplicity of hadrons with one $s$-quark produced in a triple-kaon event is

$$M_a^{(3)} = g_a M_S^{(3)} P_a^{(3)} \left[(P_K^{(3)} + P_\Lambda^{(3)} + P_\Sigma^{(3)})^2 + P_\Xi^{(3)}\right], \quad (43)$$

For hadrons with two $s$-quarks we find

$$M_\Xi^{(3)} = g_\Xi M_S^{(3)} P_\Xi^{(3)} \left[(P_K^{(3)} + P_\Lambda^{(3)} + P_\Sigma^{(3)})\right], \quad (44)$$

and for the $\Omega$ baryon

$$M_\Omega^{(3)} = \frac{1}{3} M_S^{(3)} P_\Omega^{(3)} = \frac{1}{3} M_S^{(3)} z_S^{(3)} V_0 p_\Omega, \quad (45)$$

with $M_S^{(3)}$ given by Eqs. (25) for $n = 3$ and (20).

We can easily write the solutions of Eqs. (44), (45) and (46) with respect to $z_S^{(n)}$:

$$z_S^{(1)} \approx 1/V_0 (p_K + p_\Lambda + p_\Sigma), \quad (46)$$

$$z_S^{(2)} \approx 1/(V_0 (p_K + p_\Lambda + p_\Sigma)^2 + V_0 p_\Xi), \quad (47)$$

$$z_S^{(3)} \approx \left[1 - \frac{p_\Xi}{V_0 (p_K + p_\Lambda + p_\Sigma)^2}\right]^{-1}, \quad (48)$$

$$z_S^{(3)} \approx \left[1 - \frac{p_\Xi}{V_0 (p_K + p_\Lambda + p_\Sigma)^2}\right]^{-1} - \frac{p_\Xi}{V_0^2 (p_K + p_\Lambda + p_\Sigma)^3}. \quad (49)$$
We keep here only the terms up to the first order in \( p_\Xi \) and \( p_\Omega \), which constitute correction contributions in the square brackets in (47) and (48) of the order of 4% or less for temperatures and densities of interest, see below. The neglected higher order terms are still strongly suppressed since \( p_\Xi, p_\Omega \ll p_\Lambda + p_\Sigma \) and \( (V_{\text{fo}}/p_\Lambda + p_\Sigma) > 1 \).

D. Observables

Having the normalization factors and the chemical potential from Eq. (34) at our disposal, we can calculate the multiplicity ratios (7), (8), (9) as functions of the freeze-out density and temperature.

Consider, first, the \( K^-/K^+ \) ratio. Keeping terms up to order \( M_{K^+} \) we include contributions from the \( K^- \) yield from single-kaon and double-kaon events

\[
R_{K^-/K^+} = \frac{\eta p_K}{(1 + \eta) M_{K^+}} = \frac{\eta p_K}{p_\Lambda + p_\Sigma} \frac{\langle M_{K^+}^{(1)} + M_{K^+}^{(2)} \rangle}{1 + \eta} \frac{\langle M_{K^+}^{(3)} \rangle}{M_{K^+}} + \frac{\eta p_K}{p_\Lambda + p_\Sigma} \frac{\langle V_{\text{fo}} M_{K^+}^{(2)} \rangle}{(1 + \eta) M_{K^+}}
\]

(49)

Note that we deal here with quantities observable in inclusive experiments, therefore, averaging over the collision impact parameter is performed following Eq. (10).

Using Eqs. (18), (19), (20), and (27) we write

\[
R_{K^-/K^+} = \frac{\eta p_K}{p_\Lambda + p_\Sigma} \frac{\langle M_{K^+}^{(1)} + M_{K^+}^{(2)} \rangle}{1 + \eta} \frac{\langle M_{K^+}^{(3)} \rangle}{M_{K^+}} + \frac{\eta p_K}{p_\Lambda + p_\Sigma} \frac{\langle V_{\text{fo}} M_{K^+}^{(2)} \rangle}{(1 + \eta) M_{K^+}}
\]

(50)

The second term in (50) can be expressed through \( p_\Sigma^2 \) using Eq. (27). With the help of Eq. (28) the \( K^-/K^+ \) ratio can be finally cast in the form

\[
R_{K^-/K^+} = \frac{\eta p_K}{p_\Lambda + p_\Sigma} Y_1.
\]

(51)

Here the auxiliary function \( Y_1 \) is given by

\[
Y_1 = 1 - \frac{(1 + \eta) M_{K^+} \zeta(2)}{\langle V_{\text{fo}} \rangle (p_\Lambda + p_\Sigma)^2}
\]

(52)

The second term in (52) proves to be small. For values of freeze-out temperatures and densities, which we exploit, it is of the order of 2%.

Similar calculations yield

\[
R_{\Lambda/K^+} = \frac{1}{M_{K^+}} \left( \frac{\langle M_{\Lambda}^{(1)} + M_{\Lambda}^{(2)} + \eta M_{\Lambda}^{(3)} \rangle}{\eta^2 + \eta + 1} \right)
\]

\[
= (1 + \eta) \frac{\eta p_\Lambda + \eta p_\Sigma}{p_\Lambda + p_\Sigma} Y_1,
\]

(53)

\[
R_{\Omega/K^+} = \frac{\eta^2 + 1}{2(\eta^2 + \eta + 1)} \frac{M_{K^+}}{M_{K^+}}
\]

\[
= \frac{(\eta^2 + 1)(\eta + 1)}{2(\eta^2 + \eta + 1)} \frac{p_\Sigma}{p_\Lambda + p_\Sigma} Y_1.
\]

(54)

Now let us consider the double ratio \( R_{\Xi/\Lambda/K^+} \). In order to keep the terms of the order \( M_{K^+} \), as those included above in the coefficient \( Y_1 \), we should take into account that the \( \Xi \) yield is determined by the double- and triple-kaon events, while single-kaon and double-kaon events contribute to the \( \Lambda \) yield. Using Eqs. (11) and (14) we have

\[
R_{\Xi/\Lambda/K^+} = \frac{\eta}{1 + \eta} \left( \frac{\langle M_{\Xi}^{(2)} \rangle + \eta M_{\Xi}^{(3)} \rangle}{\eta^2 + \eta + 1} \right) \frac{M_{K^+}}{M_{K^+}}
\]

\[
= \frac{\langle \frac{1}{2} M_{\Xi}^{(2)} + M_{\Xi}^{(3)} \rangle}{1 + \eta} \frac{M_{K^+}}{M_{K^+}} \frac{\langle V_{\text{fo}} \rangle (p_\Lambda + p_\Sigma + p_\Xi) V_{\text{fo}}^2 \rangle}{(p_\Lambda + \eta p_\Sigma + p_\Xi)^2 (1 + \eta) M_{K^+}}
\]

(55)

With the help of Eqs. (46), (47), (48) and (18), (19), (20) we obtain

\[
R_{\Xi/\Lambda/K^+} = \frac{\eta}{1 + \eta} \frac{\langle M_{\Xi}^{(2)} + M_{\Xi}^{(3)} \rangle}{\eta^2 + \eta + 1} \frac{M_{K^+}}{M_{K^+}} \frac{\langle V_{\text{fo}} \rangle (p_\Lambda + p_\Sigma + p_\Xi) V_{\text{fo}}^2 \rangle}{(p_\Lambda + \eta p_\Sigma + p_\Xi)^2 (1 + \eta) M_{K^+}}
\]

(56)

with an auxiliary function

\[
Y_2 = \frac{1}{2} \zeta(2).
\]

(57)

Taking numerical values from Eq. (30) we estimate \( Y_2 \approx 0.52 \). The correction from the terms of the higher order in \( M_{K^+} \) is below 1%.

For completeness let us now consider the \( \Omega^- \) baryon production. The \( \Omega^- \) baryons can be identified through their weak decays \( \Omega^- \to \Lambda K^- \) and \( \Omega^- \to \Xi^- \pi^0 \) or \( \Omega^- \to \Xi^0 \pi^+ \). The first decay mode is most simple for detection. Depending on the detection channel it is convenient to define the following ratios of the multiplicity of reconstructed \( \Omega^- \) baryons to the total multiplicities of decay products

\[
R_{\Omega/\Lambda/K^+} = \frac{M_{\Omega}}{M_{\Lambda} M_{K^+}}
\]

(58)

\[
R_{\Omega/\Xi/K^+} = \frac{M_{\Omega}}{M_{\Xi} M_{K^+}}
\]

(59)

Keeping only the leading terms we can write the following
expressions for these ratios

\[ R_{\Omega/\Lambda/K^-/K^+} = \frac{(1 + 1/\eta)(M^{(3)}_\Omega + M^{(4)}_{\Lambda})}{(M^{(1)}_\Lambda + M^{(2)}_{\Lambda})(M^{(1)}_K + M^{(2)}_K)M^{(4)}_{K^+}} \]

\[ = \frac{\eta^2 p_{0\Lambda}/2 \eta}{p_{0\Lambda}(p_K + p_{\Lambda} + p_\Sigma)/(V_{fo})^2} Y_3, \quad (60) \]

\[ R_{\Omega/\Xi/K^+} = \frac{(1 + 1/\eta)(M^{(3)}_\Omega + M^{(4)}_{\Xi})}{(M^{(2)}_\Xi + M^{(3)}_{\Xi})M^{(4)}_{K^+}} \]

\[ = \frac{(1 + 1/\eta)^2 p_{0\Xi}/2 \eta}{p_{0\Xi}(p_K + p_{\Lambda} + p_\Sigma)/(V_{fo})} Y'_3, \quad (61) \]

where we used that \( \zeta(4) = z(1) \) and introduced notations

\[ Y_3 = \frac{(\frac{1}{3}M^{(3)}_S + M^{(4)}_{\Xi})/(V_{fo}^2)}{(M^{(1)}_S + M^{(2)}_S)(1 + \eta)M^{(4)}_{K^+}} \]

\[ \approx \frac{(V_{fo}^2)}{(V_{fo}^4/3)^5} \approx 0.19, \quad (62) \]

\[ Y'_3 = \frac{Y_3}{Y_2} \approx \frac{(V_{fo}^2)/(V_{fo}^5)}{3(V_{fo}^4/3)/(V_{fo}^5/3)} \approx 0.36. \quad (63) \]

In derivations of Eqs. (51), (53), (54), and (56) we exploited exact strangeness conservation in each class of events with the n created \( s\bar{s} \) pairs. This constraint leads to the end factors \( Y_{1,2,3} \) and \( Y'_3 \). If we put these \( Y \)'s equal unity, we recover the results of the conventional canonical statistical approach, where the strangeness is conserved only on average. In practice the latter means, e.g., that \( \Xi \) baryons can come out in events when only one \( K^+ \) or \( K^0 \) meson is produced. We see that the effect of the forced strangeness conservation is very important and hinders the production of multi-strange hadrons.

In ratios (51), (53), (54), and (56) all probability densities, \( p_{0\eta} \), are calculated at the freeze-out moment \( t_{fo} \), i.e. at the freeze-out temperature \( T_{fo} \) and density \( p_{0\eta,fo} \). The ratios (51), (53), and (54) are completely determined by the probability densities given by Eq. (52). The ratio \( R_{\Xi/\Lambda/K^+} \) is additionally dependent on the mean fireball freeze-out volume \( (V_{fo}) \).

We note that the ratios (53), (54), and (56) depend very weakly on the baryon density. Indeed, since \( p_{0\Lambda,\Sigma} \gg p_K \), as follows from Eq. (52), the factor \( e^{\mu_B/T} \) cancels out in Eqs. (53), (54). The density dependence of the ratio (56) is also very weak, since \( (V_{fo}) \propto 1/\rho_B \), cf. Eq. (22), and for baryons \( p_\eta \propto e^{\mu_B/T} \), cf. Eq. (22), the ratio is approximately proportional to the combination

\[ \rho_B e^{-\mu_B/T} = 4 f(m_N, T) + 16 f(m_\Delta, T), \quad (64) \]

which depends only on the temperature and the nucleon and \( \Delta \) masses. The density-dependent correction terms are small as \( p_{0\Lambda}/p_{0\Lambda,\Sigma} \ll 1 \).

In Fig. 1 we depict by dashed lines the strange particle ratios as functions of the freeze-out temperature. Following Refs. [11, 12, 13] a narrow interval of \( \rho_{0\eta} = (0.5–0.7) \rho_0 \), where \( \rho_0 = 0.17 \text{ fm}^{-3} \) is the nuclear saturation density, has been obtained from the analysis of pion, proton and \( K^- \) yields in nucleus-nucleus collisions at SIS and Bevalac energies. We take here the middle values \( \rho_{0\eta} \approx 0.6 \rho_0 \). From Fig. 1 we see that the best description of the \( K^-/K^+ \) ratio is achieved for \( T_{fo} \approx 80 \text{ MeV} \), whereas for the \( \Lambda/K^+ \) we would need a lower temperature \( T_{fo} \approx 63 \text{ MeV} \). The \( \Sigma/K^+ \) ratio depends weakly on the temperature and lies 30% above the upper experimental error-bar of the value (63) obtained using the HADES estimate of the \( \Sigma \) multiplicity (6). Compared with the ratio (51), being constructed using only the information on \( K^+ \) mesons and the isospin asymmetry of the collision, the calculated \( \Sigma/K^+ \) ratio lies 30% above the central value and within the empirical error bars. The \( \Xi/\Lambda/K^+ \) ratio is 6.4 times smaller than the central value of the ratio following from HADES measurements and lies significantly below the lower error bar.

IV. IN-MEDIUM POTENTIALS

It is well known that the medium effects are important for description of particle production in HIC at SIS energies [5, 11]. Particularly, strange particle yields are strongly influenced by them [12, 19, 22]. The in-medium modification of the energy spectrum of particle \( \alpha \) is effectively parameterized in terms of scalar \( S_\alpha \) and vector \( V_\alpha \) potentials

\[ E_\alpha(p) = \sqrt{m^2_\alpha + p^2} + V_\alpha, \quad m^*_\alpha = m_\alpha + S_\alpha, \quad (65) \]

provided one disregards more involved effects of the \( p \)-wave interactions. The scalar potential enters the spectrum through the effective mass \( m^*_\alpha \). Description in terms of the \( S_\alpha \) and \( V_\alpha \) potentials is typical for relativistic mean-field (RMF) models, cf. [21, 23]. Inclusion of the potentials \( S_\alpha \) and \( V_\alpha \) leads to the replacement of the function \( f(m_\alpha, T) \) in Eq. (52) as

\[ f(m_\alpha, T) \rightarrow f(m^*_\alpha, T) \exp(-V_\alpha/T). \quad (66) \]

For nucleons we use the parameters of the RMF model [23], \( S_N \approx -190 \text{ MeV} \rho_B/\rho_0 \) and \( V_N \approx +130 \text{ MeV} \rho_B/\rho_0 \), which produce the equation of state close to the microscopic Urbana-Argonne equation of state [24]. The same potentials are assumed to be valid also for \( \Delta \): \( S_\Delta \approx S_N, V_\Delta \approx V_N \). The vector potentials of hyperons can be related to \( V_N \) as \( V_\Lambda = 2V_\Xi = \frac{2}{3}V_N \), according to the number of non-strange quarks in the hyperon. The scalar potentials follow then as \( S_\alpha = [U_\alpha - V_\alpha(p_\alpha)] \rho_B/\rho_0 \), where the optical potential \( U \) acting on a hyperon in an atomic nucleus, \( S(p_\alpha) + V(p_\alpha) = U \), can be constrained from analysis of hypernuclei: we take \( U_\Lambda = -27 \text{ MeV} \) from [25], \( U_\Sigma = +24 \text{ MeV} \) from [26], \( U_\Xi = -14 \text{ MeV} \) from [27].

For \( K \) mesons we can use the effective scalar potential as in Ref. [21]: \( V_K = 0, S_K = U_K \rho_B/\rho_0 \). The \( K^- \) optical potential extracted from kaonic atoms is estimated as \( U_K = -(70–150) \text{ MeV} \). The transport code
FIG. 1: The ratios (51), (53), (54), (56) as functions of the freeze-out temperature for the freeze-out density 0.6ρ0 in comparison with the empirical ratios (7) shown by dotted lines. The shaded regions are the experimental error intervals. Dashed curves are calculated in the absence of in-medium potentials, whereas solid lines, with the scalar and vector potentials. For the effect is due to the presence of the numerator in Eq. (51) becomes smaller. However, the main parameter. There are no direct experimental tools to take into account.

V. TRIGGER EFFECTS

In the HADES experiments one uses the LVL1 trigger to select more central collisions [4]. The trigger affects averaging over the impact parameter, therefore the averaged fireball volume in Eq. (65) and the numerical factors ξ(n) and ξ(n) change. The trigger effect can be incorporated with the help of an additional weight function T_{LVL1}(b) embedded in any integration over the impact parameter. There are no direct experimental tools to constrain this weight function, and we have to rely on a modeling with the help of some transport code. Obtained from the BUU transport code in [2, 3], this function for
TABLE I: The strange particle ratios \([\Xi/\Lambda\), (\Xi/K)\] calculated with the inclusion of in-medium potentials (for \(U_K = -75\) MeV), at the freeze-out baryon density \(\rho_{B,fo} = 0.6 \rho_0\) and freeze-out temperature \(T_{fo} = 68.8\) MeV in comparison with the available experimental values. The columns “inclusive/triggered” are results of calculations with/without inclusion of the LVL1 trigger effects discussed in Section VI.

| ratio | exp. values inclusive triggered |
|-------|---------------------------------|
| \((K^-/K^+) \times 10^2\) | 2.54\textsuperscript{+1.21}_{-0.93}, 2.55, 2.55 |
| \(\Lambda/K^+\) | 1.46\textsuperscript{+0.49}_{-0.37}, 1.50, 1.50 |
| \(\Sigma/K^+\) (Hades) | 0.13\textsuperscript{+0.16}_{-0.12}, 0.290, 0.290 |
| \(\Sigma/K^+\) (iso) | 0.50\textsuperscript{+0.23}_{-0.17}, 0.290, 0.290 |
| \(\Xi/\Lambda/K^+\) | 0.20\textsuperscript{+0.16}_{-0.11}, 0.047, 0.026 |
| \((\Omega/\Xi/K^+) \times 10^2\) | ---, 0.85, 0.26 |
| \((\Omega/\Xi/K^+) \times 10^2\) | ---, 0.42, 0.23 |

VI. DISCUSSION

As we found above the experimental ratio \(\Xi/\Lambda\) measured by HADES cannot be explained within the minimal statistical model, which is based on strangeness conservation and on the assumption that the negatively strange particles \(K, \Lambda, \Sigma, \) and \(\Xi\) sustain in thermal equilibrium. The inclusion of in-medium potentials enlarges the \(\Xi\) ratio but not enough to accommodate the experimental data. Let us discuss now other possible sources of the \(\Xi\) enhancement.

(i) More attractive \(\Xi\) in-medium potential. One could try to explain the \(\Xi\) enhancement by introduction of a more attractive \(\Xi\) in-medium potential than that we used. Within our model we find that the value of the potential \(U_\Xi\) at the saturation nuclear density should be \(U_\Xi \lesssim -120\) MeV to increase the ratio \(\Xi^-/\Lambda/K^+\) up to the lowest end of the empirical error bar. Such a strong attraction comparable with the nucleon optical potential is unrealistic. It would imply that \(\Xi\) baryon is bound in nucleus stronger than two \(\Lambda\’s, \) since \(2m_\Lambda + U_\Lambda - (m_\Xi + m_\Sigma + U_\Xi) \sim 100\) MeV > 0. This would influence the description of doubly strange hypernuclei \(^{27}_9\). The leading-order analysis of the hyperon and nucleon mass shifts in nuclear matter performed using the chiral perturbation theory \(^{29}_9\) shows that the \(\Xi\) shift is much smaller than nucleon and \(\Lambda\) shifts. Recent analysis \(^{30}_9, 31\) confirm the relative smallness of \(\Xi N\) scattering lengths. Nevertheless, for completeness, we should note that there exist some potential models, which predict rather strong \(\Xi N\) interaction, see \(^{32}_9, 33\) and critical discussion in \(^{31}_9\).

(ii) Variations of the freeze-out density. We could take somewhat larger value of the freeze-out baryon density. For instance, had we taken \(\rho_{B,fo} = \rho_0\), the calculated ratio \(R_{\Xi/\Lambda/K^+}\) would increase but only modestly, from 0.026 to 0.034. The latter value is by factor of 3-10 smaller than the experimental values.

(iii) Variations of the \(K^+\) multiplicity. If we vary the \(K^+\) multiplicity within the experimental error bars and take the maximal possible value \(M_{K^+} = 3.2 \times 10^{-2}\), the ratio \(R_{\Xi/\Lambda/K^+}\) becomes equal to 0.175\textsuperscript{+0.16}_{-0.11} instead of the value 0.20\textsuperscript{+0.16}_{-0.11} presented in Table I. The calculated value is still significantly below the experimental range.

Could it be that the number of produced \(K^+\) mesons is underestimated in the experiment? There are two independent measurements of the \(K^+\) yield by KaoS collaboration at the beam energy of 1.8A GeV for C+C collisions \(^{34}_9\) and for Ni+Ni collisions \(^{35}_9\). If we scale these results down by \(A^2\) factor with the corresponding value of \(A\) (see argumentation in \(^{6}_9, 12\)) we find very similar results: \(\sigma_{c+c}/A^2 = (2.1 \pm 0.2) \times 10^{-2}\) mb and \(\sigma_{Ni+Ni}/A^2 = (1.7 \pm 0.45) \times 10^{-2}\) mb. Taking the median of 0.02 mb we obtain that for the Ar+KCl collision the total inclusive \(K^+\) production cross section would be equal to \(\sigma_{K^+} = 31\) mb. Dividing the production cross section by the geometrical cross section \(\sigma_{geom} = \pi b_{max}^2\), we estimate the \(K^+\) multiplicity for the HADES Ar+KCl
experiment as $\mathcal{M}_{K^+} = \sigma_{K^+}/(\delta_{\text{geom}} a_{\pi K^+})$. Here the coefficient $a_{\pi K^+}$ takes into account a decrease of the geometrical cross section because of triggering off peripheral collisions

$$a_{\pi K^+} = \int_0^{b_{\text{max}}} \frac{b T_{\pi K^+}(b)}{\int_0^{b_{\text{max}}} db} \sim 0.45. \quad (72)$$

Thus, we find that the kaon multiplicity expected for the HADES experiment would be $\mathcal{M}_{K^+} = 3.8 \times 10^{-2}$, which is larger than the actually observed value and lies beyond the experimental error bar, cf. Eq. (1a). The $\Xi/\Lambda/\Lambda^+$ ratio recalculated with such a kaon multiplicity would be

$$R_{\Xi/\Lambda/\Lambda^+} = 0.14^{+0.10}_{-0.09}. \quad (73)$$

As we see, the discrepancy between the theory and experiment is not overcome. (The lower experimental limit $R_{\Xi/\Lambda/\Lambda^+} = 0.05$ is still larger than the maximum theoretical estimation $R_{\Xi/\Lambda/\Lambda^+} \sim 0.03$ obtained by us.)

(iv) Earlier freeze-out for $\Xi$s. The main assumptions of our model are that the negatively strange sub-system is in thermal equilibrium with a non-strange sub-system (pions, nucleons, $\Delta$s, etc.) and that negatively strange particles are in chemical equilibrium with each other. The former condition for $\Lambda$ and $\Sigma$ hyperons is supported by the presence of efficient reactions $\Lambda N \leftrightarrow \Lambda N, \Sigma N \leftrightarrow \Sigma N$ and $\Lambda N \leftrightarrow \Sigma N$. The cross sections of these reactions vary between 80 and 25 mb [33] for the relative momenta between $p_T$ and $2p_T$, where $p_T \sim 300$ MeV is the thermal momentum of a baryon at a typical temperature $\sim 70$ MeV. The magnitude of these cross sections is large enough for the rapid equilibration. For the $K$, the thermal equilibrium is maintained by the reactions $K^- N \leftrightarrow \pi \Lambda(\Sigma)$, which are also responsible for the chemical equilibration among negatively strange particles. Going through intermediate resonance states $\Sigma(1385), \Lambda(1405), \Lambda(1520)$, these reactions have large cross sections, which can be even further enhanced in medium [37]. Thus, one may hope that the model assumptions hold at least for the strangeness $S = -1$ particles.

On the other hand, the $\Xi$-nucleon interaction, as we discussed above, is expected to be smaller than the $\Lambda(\Sigma)$-nucleon interaction. Calculations of [31] show that $\sigma(\Xi^- p \rightarrow \Xi^- p) \sim 15$ mb, $\sigma(\Xi^0 p \rightarrow \Xi^0 p) \sim 15$ mb, and $\sigma(\Xi^- p \rightarrow \Lambda \Lambda) \sim 10$ mb. These values are indeed smaller than those for $\Lambda(\Sigma)$N reactions. Scattering of $\Xi$’s on pions for nearly isospin symmetrical matter is also considerably weaker than the $\pi N$ scattering. Indeed, according to the chiral effective field theory [33], the isospin averaged scattering length $a_{\Xi N}^+ = (2 a_{\Xi N}^{1/2} + a_{\Xi N}^{1/2})/3$ is of a sub-leading order in the chiral expansion, $a_{\Xi N}^+ \sim O(m_\pi^2)$, similarly to the small value of $a_{\pi N}^+$ scattering length. The numerical value of $a_{\Xi N}^+$ derived in [33] is $a_{\Xi N}^+ = -(0.02 - 0.04)$ fm. In the $p$-wave, where the $\pi N$ scattering is dominated by the broad $3/2$ $\Delta$ resonance (the width is 120 MeV), there is only a narrow

![FIG. 2: Ratio (73) plotted as function of the $\Xi$ baryon freeze-out temperature $T_{fo}(\Xi)$. The corresponding $\Xi$ baryon freeze-out density is calculated according to the polytropic relation $\rho_{B,fo} \propto T_{fo}^{(\Xi)}$, see text for details. Two curves are drawn for assumed values of the maximal temperature and density.](image_url)
two values for $\alpha$: one, that was advocated by the analysis in Ref. [11] with $T_{\text{max}} \simeq 120$ MeV and $\rho_{B_{\text{max}}} = 2.8 \rho_0$, is shown by line 1, and the other one with somewhat smaller initial values $T_{\text{max}} = 110$ MeV and $\rho_{B_{\text{max}}} = 2 \rho_0$ is shown by line 2.

As we see from Fig. 2, the assumption of the earlier freeze-out of $\Xi$ baryons may lead only to a minor enhancement of the $\Xi/\Lambda/K^+$ ratio, $h_{\Xi} \lesssim 1.2$, provided $\Xi$ baryons have stayed in chemical equilibrium with other strange particles right up to the moment of their freeze-out.

(v) Direct reactions and sources of $\Xi$ baryons. From the performed analysis we conclude that to get any substantial increase in the number of $\Xi$’s we have to assume that these baryons are not absorbed after being produced and their number is determined by the rate of direct production reactions, as, e.g., for di-leptons. This rises up, however, a new question, if there are sufficiently strong sources of $\Xi$ baryons and enough time to produce the required number of them. There are two types of reactions: strangeness creation reactions and strangeness recombination reactions. The former ones are endothermic and require a large deposit of kinetic energy from colliding particles

$$\bar{K}N \rightarrow K\Xi - 380 \text{ MeV},$$
$$\pi\Sigma \rightarrow K\Xi - 480 \text{ MeV},$$
$$\pi\Lambda \rightarrow K\Xi - 560 \text{ MeV}.$$  \hspace{1cm} (76)

At SIS energies, when the fireball temperature does not exceed $\sim 120$ MeV, these processes have very small probability. Strangeness recombination processes are secondary processes involving two strange particles.

In Fig. 3 we show the mass spectrum of strangeness $-2$ states in channels with different baryon numbers $B$. We see that in both single and double baryon channels the states with the $\Xi$ baryon are on the bottom of the spectrum. Only the double-$\Lambda$ state has a smaller mass. This means that $\Xi$s play a role of a strangeness $-2$ reservoir being filled with a decrease of the temperature. The only sink is the reaction $\Xi N \rightarrow \Lambda\Lambda$ having, however, a relatively small cross section 30.

One type of the strangeness recombination reactions is anti-kaon-induced reactions

$$\bar{K}\Lambda(\Sigma) \rightarrow \Xi\pi + 154(232) \text{ MeV}.$$  \hspace{1cm} (77)

Their cross sections were calculated in 39 and routinely included in transport codes, e.g., see Ref. 10. The cross section has a week dependence on collision energy, of the order of 10 mb. As suggested in Ref. 40 these reactions can be enhanced in medium because of the attractive potential acting on $\bar{K}$ mesons. The potential decreases the threshold in the entrance channel of the reaction (77) so that at some density the threshold drops below the $\Xi^*(1530)$ resonance. The appearance of the latter narrow resonance above the reaction threshold leads to a strong (but local in energy) enhancement of the cross section, see Fig. 3 in Ref. 40. For the choice of the kaon potential $U_{K^-}(\rho_0) = -75 \text{ MeV}$ 7 the drop of the threshold $\bar{K}\Lambda$ for $\rho_B \sim \rho_0$ is shown in Fig. 3 by arrow; the $\Xi^*$ resonance is crossed at density $\sim \rho_0$. Thus, at some stage of the fireball evolution $\Xi$ production in reactions (77) can be stronger than what was supposed before.

The recombination reactions of the other type proposed in Ref. 40 are the double-hyperon reactions,

$$\Lambda\Lambda \rightarrow \Xi N - 26 \text{ MeV},$$
$$\Lambda\Sigma \rightarrow \Xi N + 52 \text{ MeV},$$
$$\Sigma\Sigma \rightarrow \Xi N + 130 \text{ MeV}.$$  \hspace{1cm} (78)

The yields of hyperons are an order of magnitude higher than those of anti-kaons, so we expect a higher contribution from these processes. The parameterizations of cross sections for some double-hyperon reactions based on the results of calculations 30 can be found in 40. The calculations 30 include the solution of the coupled-channel Lippmann-Schwinger equation with the potential constrained by the chiral SU(3) symmetry and parameters fixed by empirical data of nucleon-nucleon and hyperon-nucleon interactions. Recently the double-hyperon processes have been implemented in transport code 41 with the cross sections calculated within the Born approximation, being factor 5 or more higher than the cross sections presented in Ref. 30 depending on a cutoff parameter employed. With so large cross sections the double-hyperon reactions become the main source of $\Xi$ baryons and the experimental $\Xi/\Lambda$ ratio could be explained. The large cross section in Ref. 41 could be an artifact of the Born approximation and the results of Ref. 30 seem to be more realistic.

Summarizing, it seems to us possible that the enhanced yield of $\Xi^-$ baryons observed by HADES collaboration can be explained by their production in the direct reactions, provided the in-medium enhancement of the kaon-induced reactions (77) and new hyperon-induced reactions (78) are taken into account.
VII. CONCLUSIONS

We analyzed the recent HADES data on strangeness production in Ar+KCl collisions at 1.76 A GeV in the framework of the minimal statistical model for strangeness. The latter assumes that the total strange charge of the fireball created in a certain event is constrained by the number of $K^+$ mesons produced in this event, and that negatively strange particles remain in thermal and chemical equilibrium during the fireball evolution until a common freeze out. Inclusion of realistic in-medium potentials for nucleons and anti-kaons allows to describe satisfactorily $K^-/K^+$, $\Lambda/K^+$, and $\Sigma/K^+$ ratios. However, the ratio $\Xi/K^+$ comes out by factor $\sim 3$ smaller than the experimental lower error bar and by factor $\sim 8$ smaller than the median experimental value. Two effects contribute to this discrepancy: the strangeness conservation constraint, which requires that $\Xi$ are created only in the events with two or more produced $K^+$ or $K^0$ mesons, reduces the $\Xi/K^+$ ratio by factor $\sim 2$ and a centrality bias due to the LVL1 trigger in the HADES setup leads to a further reduction of the calculated ratio by factor $\sim 1.8$, see Eqs. (63), (69).

Variation of parameters of the model, such as potentials, freeze-out density and the $K^+$ yield within the experimental error bars, does not allow to accommodate the data. The assumption, that the created $\Xi$ baryons reach the chemical equilibrium with other strange particles but then leave the fireball (at their own freeze-out moment) earlier than $S = -1$ strange particles, does not allow to produce the sufficient $\Xi$ enhancement.

Thus, to overcome the contradiction we suggest that the $\Xi$ baryons do not equilibrate chemically with other strange particles and the $\Xi$ yield is determined by the direct production reactions. Various $\Xi$ production reactions are discussed.

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