Formation of monopoles and domain walls after brane inflation

Tomohiro Matsuda

Laboratory of Physics, Saitama Institute of Technology,
Fusaiji, Okabe-machi, Saitama 369-0293, Japan

Abstract

We study cosmological defect formation that is induced by brane dynamics after brane inflation. The cosmological defects are corresponding to the branes that have less than three spacial dimensions in the uncompactified spacetime. Contrary to the previous arguments, production of monopoles and domain walls are not always negligible. Monopoles and domain walls are formed by the branes extended between mother branes.
1 Introduction

Models with more than four dimensions are interesting, because all the physical ingredients of the Universe might be unified in a higher dimensional theory. String theory is the most promising scenario where quantum gravity is included by the requirement of additional dimensions and supersymmetry. The idea of large extra dimension\(^\ddagger1\) is important, because it might solve the hierarchy problem. In this case, the observed Planck mass is obtained by the relation \(M_p^2 = M_*^{n+2} V_n\), where \(M_*\) and \(V_n\) denote the fundamental scale of gravity and the volume of the \(n\)-dimensional compact space, respectively. In the scenarios of large extra dimension, fields in the standard model are expected to be localized on a wall-like structure, while the graviton propagates in the bulk. In the context of string theory, a natural embedding of this picture is realized by brane construction. The brane models are interesting from both phenomenological and cosmological viewpoints.

Analyses of cosmological defect formation are important in brane models as well as in usual cosmological models.\(^2\) One may consider three types of brane defects.

- Defects are branes.
  
  In this case, cosmological defects are formed by the branes that have less than three spacial dimensions in the uncompactified spacetime. In the previous discussions, it was concluded that the cosmological production of monopoles and domain walls are negligible. In this paper, however, we reconsider the cosmological formation of monopoles and domain walls and show explicitly how they can be formed by the brane dynamics.

- Defects are formed by the spatial deformation of branes.
  
  In this case, cosmological defects are formed by the continuous deformation of branes. First in ref.\(^9\), and later in ref.\(^8\)\(^10\), the fields that parameterize the

\(^2\)Inflation in models of low fundamental scale is interesting\(^2\)\(^16\)\(^3\). Baryogenesis is discussed in ref.\(^4\)\(^5\)\(^6\), where defects play important roles. Non-static brane configurations such as brane defects or brane Q-balls\(^7\)\(^8\) are important, because we are expecting that future cosmological observation will reveal the cosmological evolution of the Universe, which will also reveal the physics beyond the standard model. If one wants to know what kinds of brane defect are produced in the early Universe, one needs to understand how they are formed.
positions of the branes are shown to fluctuate in spatial directions to form the cosmological defects. Cosmic strings are constructed in ref. [9], where the singularity is resolve by smearing the brane. Monopoles, strings and domain walls are discussed in [8, 10], where the relative positions between branes are utilized.

- The center of the localized matter fields are shifted in the defect.
In ref. [11], the localization of the matter fields on a fat domain wall is discussed to explain the small interactions. This idea is important when the suppression factor is required in the interaction, so that it does not violate the constraints from the proton lifetime. In ref. [12], we have constructed defect configurations, which induce the shift of the center of the localized matter fields. These defects are used to produce the baryon asymmetry of the Universe [12, 6].

In this paper, we reconsider the cosmological formation of brane defects, which are described by branes which have less than three spacial dimensions in the uncompactified space. In our model, the defect branes are extended between branes. We will also consider a cosmological brane creation induced by the excitation of a sphaleron-like brane configuration.

First we will review the essence of the previous arguments about the cosmological formation of brane defects. In the original scenario of brane inflation [13], inflationary expansion is driven by the potential between D-brane and anti-D-brane evolving in the bulk. Then the scenario of the inflating branes at a fixed angle is studied in ref. [14], where the slow-roll condition is improved by introducing a small angle. The end of brane inflation is induced by the brane collision where brane annihilation (or recombination) proceeds through tachyon condensation [15].

During brane inflation, tachyon is trapped in the false vacuum, which may result in the formation of lower-dimensional branes after brane inflation. The production of cosmological brane defects was discussed in ref. [14, 17], where it was concluded that cosmic strings are copiously produced in these scenarios, but monopoles and domain walls are negligible. In ref. [18], however, it was discussed that all kinds of defects are produced, and the conventional problems of domain walls and monopoles arise. Later in ref. [19, 20, 21, 22], the formation of cosmological defects was

---

3It is possible to construct models for brane inflation without tachyon condensation [16].
reexamined and the conclusion was different from [17] and [18]. As was noted in ref. [17], the effect of compactification is significant for the defect formation, if it is induced by tachyon condensation after brane inflation. Since the compactification radius must be small compared to the horizon size during inflation, any variation of a field in the compactified direction is suppressed. Then the daughter brane, which is the defect formed by tachyon condensation on the worldvolume of the mother brane, wraps the same compactified space as the mother brane. As a result, the codimension of the daughter brane should lie within the uncompactified space. Since the number of the codimension must be even, the cosmological defect is inevitably a cosmic string. In ref. [19], it was discussed that the analysis does not fully account for the effects of compactification, because the directions transverse to the mother brane had not been considered. In ref. [19], the effect of the RR fields, which are extended to the compactified dimensions, were discussed. Their conclusion was that the creation of the gradients of the RR fields in the bulk of the compactified space is costly in energy, so that the creation of the daughter D brane is suppressed if they do not fill all the compactified dimensions.

Here it should be noted that the above arguments are not fully reliable. The most obvious example is the inconsistency of the tension of strings produced after angled brane inflation. In the original argument in ref. [17], the $\theta$-dependence of the tension of the string did not coincide with the one calculated from the effective Lagrangian. What is wrong with the above arguments? The reason is clearly described in ref. [23]. In the original argument of ref. [17], it was discussed that the daughter branes are created on the mother brane. In fact the statement is true, but we should be more careful about the process of the brane recombination. When the distance between the mother branes becomes shorter than a critical distance, tachyon starts to condensate on a mother brane. Therefore the tachyon forms strings on the mother brane when the mother branes start to recombine. However, as we have depicted in fig. 1, the daughter $D_{p-2}$-brane must be pulled out of the $D_p$-branes. As a result, the daughter $D_{p-2}$ brane does not wrap the same compactified space as the mother brane. The tension of the cosmic string is calculated in ref. [23], which depends on $\theta$ and matches to the effective action. Therefore, the daughter brane in fig. 1 satisfies the property required from analysis of the effective Lagrangian. Thus for the angled inflation model, it is obvious that the cosmic string does not wrap the same
compactified space as the mother brane. From the above discussions, it is apparent that the original argument of ref. [17] fails because there is no obvious reason that in their final state the daughter branes wrap the same compactified space as the mother brane. The idea, which supports the formation of daughter branes that are extended between mother branes, is quite important for our discussions in this paper.

Since the previous arguments cannot fully account for the generic processes of cosmological defect formation, one can hardly accept their conclusions without reconsidering the production of monopoles and domain walls paying careful attention to the brane dynamics. Is it really impossible to produce sufficient amount of cosmological monopoles and domain walls by the branes? As we have discussed in ref. [8, 23], crucial modifications are required for the previous arguments, since it is possible to produce daughter branes that do not wrap the same compactified space as the mother brane. In this paper we will reconsider the formation of cosmological monopoles and domain walls by using the new idea. For the extended branes to be produced between mother branes, the distance between mother branes are required to vanish at a moment. The obvious examples are;

- Recombination after angled brane inflation.

At the time when the recombination starts after angled brane inflation, the distance between branes vanishes as required. Thus, daughter branes can be pulled out of

---

4 We will not address any objection to the string formation in angled inflation. The defect has the same number of directions extended in the same compact space, which of course suggests that the defect is a cosmic string. The arguments in ref. [17] are incorrect because they have claimed that the daughter brane is formed on the mother brane and wraps precisely the same compact space as the mother brane even in its final configuration. Therefore, the tension of the cosmic string calculated in ref. [17] does not match to the effective action. In fact, the daughter brane in angled inflation is extended between the recombined mother brane, which cannot wrap the same compactified space as the mother brane.

5 Our idea is generic and applicable to other conventional cosmological processes. The spontaneous symmetry breaking in the effective action is sometimes described by the recombination of the branes or by the branes falling apart, which can be induced by the thermal effects. Besides the cosmological defects that are formed by a brane creation, one may consider another type of defects that are formed by the continuous deformation of the branes. The two kinds of brane defects will be produced by the same process, as we have discussed in ref. [8, 10]. Therefore the analyses of the cosmological evolution of the mixture of these defects are quite important.
splitting branes, which finally become the cosmological defect in the effective action. As a result, the cosmological defect in the effective action is the brane extended between mother branes. Two examples are already discussed in ref. [23] and ref. [8], where the formation of cosmic strings are considered.

- Two branes coincide during inflation.

In the effective action, the brane distance plays the role of a trigger field of hybrid inflation. This type of brane inflation is already discussed in ref. [24] and ref. [25], in which the formation of unstable semilocal strings are suggested. In this paper, using the model in ref. [24], we will show that a point-like object that corresponds to a monopole in the effective action can be produced as a daughter $D_3$ brane. The daughter $D_3$ brane is extended between $D_5$ branes. We calculate the tension of the extended object and show that it coincides with the mass of a monopole in the effective action.

- Branes collide or oscillate with huge kinetic energy.

After brane collision, there would be chaotic processes, which include oscillation, recombination and production/annihilation of branes. Therefore, it seems natural to expect that there could be a stage of preheating after inflation, which induce large occupation numbers for long-wavelength configurations that is necessary for the domain wall production from sphalerons. Then, extended objects could be formed efficiently between oscillating branes. As usual, these non-equilibrium processes are always important for the discussions about cosmological defect formation. As in the case of the conventional electroweak phase transition, we do not exclude the possibility that the final state of the oscillating branes are separated at a distance, which implies a spontaneous symmetry breaking in the effective Lagrangian.

The defect branes could be created by the tachyon condensation on the mother brane, or could have been existed from the beginning and have been dissolved in the mother brane. In section 2, we consider the model of brane inflation in ref. [24] and study the cosmological formation of monopoles after brane inflation. In section 3, we discuss the formation of cosmological domain walls. As expected, there are notable differences between the previous arguments and our conclusions. The cosmological formation of monopoles and
domain walls are not always negligible in models of brane cosmology. Our results are consistent with the analyses of the effective action.

2 Monopoles

In this section, we discuss the formation of monopoles in a specific example of brane inflation. We first repeat the previous arguments in ref. [17, 19], which suggest that monopoles are not produced by the usual daughter brane creation after inflation. As is noted in ref. [17], the effect of compactification is significant for the daughter brane creation. Since the compactification radius is small compared to the horizon size during inflation, any variation of a field in the compactified direction is suppressed. As a result, the daughter brane wraps the same compactified dimensions as the mother brane. The codimension of the daughter branes should lie within the uncompactified space, which suggests that the defect is inevitably a cosmic string. Seeing the above arguments, one might conclude that the monopoles are not produced after brane inflation. Is the production of monopoles really impossible? To answer this question, we consider a simple example where monopoles are produced by the daughter brane creation after brane inflation. To avoid the ambiguity of the brane products, we consider a model in which the daughter brane formation is described without such ambiguity. The schematic representation of our idea is given in fig. 2. The typical configuration of fig. 2 will also appear in generic cosmological processes where branes collide, recombine or stick together by thermal effects.

In any case, if the production of monopoles or domain walls is suggested in the analysis of the effective action, it is natural to think that it should be possible to explain how they can be formed by the cosmological brane dynamics. Moreover, even if the production of such defects is not suggested in the effective action, they might be produced by the pure brane dynamics beyond the cut-off scale of the effective action. 6

6 The properties of such cosmological brane defects might be different from the conventional defects in the effective action. For example, Q-balls in the brane world is different from the ones in the effective action [7], which suggests that the cosmological defects might provide us the proof of the underlying brane world [8].
2.1 Brane inflation on conifold

First we make a brief review of the Halyo’s idea for brane inflation on conifolds\[24\]. The important aspect of this scenario is that the location of the inflating branes coincide at the end of inflation, then the branes separate along the $S^2$ compactified space. The model is described as a D-brane inflation on fractional $D3$ branes transverse to a resolved and deformed conifold. The D-term inflation is induced by the conifold that is resolved by the blow-up of its tip. In the effective action, the model looks like a model of hybrid D-term inflation. The slow roll is described by the slow motion of the two fractional $D3$ branes approaching each other along a compactified direction, and the trigger field parameterizes the distance between the two branes along the other compactified space in the base of the conifold. For example, one can consider a conifold that is described as a cone over $S^2 \times S^3$, where a $D3$ brane, which is transverse to the conifold, is separated into two fractional $D3$ branes on the $S^3$. The conifold singularity is resolved by replacing the tip of the cone with an $S^2$ of finite size, which induces inflation. The inflaton mass arises from another deformation of the conifold towards $ALE \times T^2$ and results in a slow motion of the two fractional branes towards each other.

As we are not interested in the parameter space of inflation itself, we spare the discussions about the condition for successful inflation. For us, the important point is to understand the formation of daughter branes after inflation, which finally become extended between the separating mother branes. As was already discussed in ref.\[24\], one usually expect that the cosmic strings, which has codimension 2 in the uncompactified space, are formed after inflation. If the inflating $D3$ brane is a $D5$ brane wrapped on the $P^1$ with radius $R$, the tension of the $D3$ brane is

$$T_{D3} = T_{D5} \int_{S^2} \sqrt{\text{det}(G + B)}, \quad (2.1)$$

where $G$ and $B$ are the $P^1$ metric and NS-NS field on the $P^1$ respectively. Taking $B = 0$ for simplicity, one can find

$$T_{D3} = \frac{4\pi R^2}{(2\pi)^6 g_s l_s^6}. \quad (2.2)$$

The tension of the inflating $D3$ brane must be equal to the energy density during inflation, which corresponds to the anomalous D-term in the effective Lagrangian. Thus in this case, the relation $T_{D3} = g^2 \xi^2$ is required, where $g$ is the gauge coupling in the effective action.
The cosmic strings are produced by tachyon condensation, which are the daughter $D3$ branes on the worldvolume of the inflation brane. The daughter brane wraps around the same compactified space as the mother brane. In this case, however, the symmetry of the corresponding field in the effective action allows only semilocal strings, which is unstable to dissolve in space, as is expected from the brane dynamics.

Let us consider a configuration of fig.2. If the seed of the daughter brane fluctuate between the two coincidental mother branes, it will be pulled out of the mother branes. The branes that are extended between mother branes are point-like objects in the effective action on the mother brane. It is easy to calculate the tension of such daughter branes and compare it with the mass of the monopoles in the effective action.\(^7\) In the final state of brane inflation, two branes are separated at a distance of \(2\pi l_s^2\sqrt{\xi}\).\(^8\) The effective mass of the extended daughter brane is

\[
M_{ext} = \frac{1}{(2\pi)^4 g_s l_s^4} \times (4\pi R^2) \times (2\pi l_s^2 \sqrt{\xi})
\]

\[
= \frac{\sqrt{\xi}}{g^2},
\]

(2.3)

where the gauge coupling is given by the formula

\[
\frac{1}{g^2} = \frac{R^2}{2\pi^2 g_s l_s^2}.
\]

(2.4)

It is easy to see that the mass of the extended daughter brane coincides with the mass of the monopole in the effective action.\(^27\)

We can extend the above analysis to the cases where more than two branes are oscillating around each other (or stick together by the thermal effects) and then fall apart along a direction of the compactified space. In this case, one can construct \(N-1\) monopoles, which are the daughter branes extended between \(N\) mother branes. A schematic representation is given in fig.3.

Seeing the above arguments, we cannot help thinking about the M theory fivebrane version of QCD (MQCD), where the quantum effects are explained by the brane dynamics.\(^27\)

\(^7\)It is already discussed in ref.\(^27\) that a $D$ string ending on a $D3$ brane provides a magnetic source for the three-brane worldvolume gauge field. Our example is a simple modification of the scenario. However, it was still unclear if such extended branes are produced after brane inflation. As we have repeated above, the previous arguments were negative to the cosmological production of such monopoles.

\(^8\)Here we follow the definitions and notations of ref.\(^24\).
In the most basic model of MQCD, which is depicted in fig.4, a monopole in Type IIA string theory\textsuperscript{[28]} is a rectangular $D2$ brane with two boundaries on $D4$ branes and other two on NS brane. The mass of such monopole is proportional to the minimal area of the hole between the branes. In realistic models, such brane configuration is expected to be embedded in the compactified space. Is the cosmological formation of the $D2$ monopoles impossible? In this case, the following conditions are required for the production of the $D2$ brane monopoles.

- There was a period when the two $D4$ branes coincide.
- $D2$ brane is created on the worldvolume of the $D4$ brane.

When the area shrinks to zero, the monopole becomes the massless excitation. The $D2$ brane is produced on the worldvolume of the $D4$ branes, due to the usual mechanism of tachyon condensation. In fig.5 we show why the $D2$ brane can be extended between the separating $D4$ branes. As usual, the daughter brane does not fluctuate in any direction of the compactified space, while it fluctuate in the uncompactified directions. As a result, the $D2$ monopoles are formed when the $D4$ branes start to fall apart. The most natural situation is that the symmetry restoration is induced by the thermal effects, which break supersymmetry and glue branes together. In the effective action, the restoration of the symmetry is naturally induced by the thermal effects. The repulsive force between the two $D4$ branes are induced by the usual supersymmetry breaking. The non-equilibrium production of such monopoles is possible during brane oscillation. The massless monopoles gain mass when the branes start to fall apart.

As a result, we conclude that the production of monopoles after brane inflation is quite natural, even if the monopoles are the daughter branes created by the usual mechanism of tachyon condensation.

3 Domain Walls

In the previous arguments in ref.\textsuperscript{[17, 19]}, it was concluded that the formation of cosmological domain walls are always negligible after brane inflation, because the daughter branes must wrap the same compactified space as the mother brane. Is it impossible to
produce daughter branes that do not wrap the same compactified space as the mother brane? First, we repeat the previous discussions, which suggest that the cosmological formation of domain walls is forbidden. As is discussed in ref. [17] and [19], the effect of compactification is believed to play significant roles in the defect formation due to tachyon condensation. Since the compactification radius should be small compared to the horizon size during inflation, any variation of a field in the compactified direction must be suppressed. As a result, the daughter brane will wrap the same compactified space as the mother brane. At the same time, since the codimension of the daughter branes should lie within the uncompactified space, the defect is inevitably a cosmic string. On the other hand, as we have discussed in ref. [23] for cosmic strings and in the previous section for monopoles, it is possible to produce daughter branes that are extended between branes, which correspond to the D-term strings or monopoles in the effective action. As we have seen in the previous section, efficient production of such monopoles is allowed in generic situations. For cosmic strings, the idea of the extended daughter brane formation was used to solve the problem of the angle-dependence of the string tension in the scenario of angled brane inflation [23]. The cosmological formation of the extended branes plays important roles in the analyses of strings and monopoles. Finally, we will examine the formation of cosmological domain walls.

Our purpose in this section is to examine the cosmological formation of domain walls. Actually, it is possible to show an example of the domain wall formation, although this example is not a pure creation of branes induced by the tachyon condensation on the mother brane.

Please imagine that two coincident 5-branes are placed on top of each other. Both of them have four space-time dimensions (uncompactified) and also wrap two-dimensional compactified space. These 5-branes look like 3-branes in the limit where the wrapped space is very small. Let us assume that after a phase transition these two 5-branes finally fall apart to the true vacuum.

In this case, it is possible to add an additional 3-brane by hand, which does not wrap the same compactified space as the above 5-branes. In this case, one can assume that the latter 3-brane is not produced by the tachyon condensation on the 5-brane, but could be present from the beginning. The latter 3-brane have four (uncompactified) space-time
dimensions, and dissolves in the 5-branes. Then, it is quite natural to think that the additional 3-brane sticks to either 5-brane when the 5-branes finally fall apart. Then there appears two kinds of domains in the Universe, depending on which 5-brane the additional 3-brane sticks to. Since the domain walls cannot interact across the scale much larger than the Hubble radius, one can expect that at least one domain wall is produced in one Hubble horizon. The resulting domain wall is the 3-brane extended between the 5-branes.

We can discuss brane creation from brane sphalerons. As far as the effective action is applicable, it seems possible to create new branes that do not wrap the same compactified radius as the mother brane, even if it is initially created on the worldvolume of the mother brane.\footnote{Our argument in this section is based on the similarity between the electroweak sphalerons and the brane sphalerons in their effective action. On the other hand, the formation of sphaleron-induced domain walls requires some specific non-equilibrium mechanisms such as preheating or Parametric Resonance (PR) after inflation. The formation of such domain walls is convinced by numerical simulations and then it is used to advocate the new mechanism of electroweak baryogenesis.}

Our second example for the formation of cosmological domain walls is the domain walls produced by the excitation of sphaleron-like brane configuration.\footnote{It might be useful to note that the “throat” configuration of the \( D_{p-2} \) brane could have their boundaries on the mother \( D_p \) branes.} The most obvious difference from the scenario of tachyon condensation is that the sphaleron-like configuration is not required to wrap the same compactified space as the mother brane, even when they are first created on the worldvolume of the mother brane. In this case, for example, cosmological domain walls can be the \( D3 \) branes extended between \( D5 \) branes. The schematic picture of the domain wall is given in fig.\textit{6} If there is a non-equilibrium process that enhances the long-wavelength configurations of the sphalerons, the efficient production of the sphaleron excitations will happen. Then the sphaleron-like branes will recombine, as is shown in fig.\textit{7}. Although the production of massive sphalerons is suppressed by an exponential factor in conventional thermal processes, the production of sphalerons are not suppressed in non-equilibrium processes such as preheating after inflation. Thus, the extended branes that are formed by the sphaleron-like excitations could be produced if there was a period when the long-wavelength configurations of the...
sphalerons are enhanced. In this case, one cannot ignore the production of domain walls. The period when branes oscillate is inevitable in generic cosmological models, as we have discussed in the previous section. Since the conventional electroweak sphaleron is quite similar to the sphaleron-like brane configuration in the effective action, we start the discussion comparing the electroweak sphalerons to the brane-sphalerons.

First, we briefly review the sphaleron interaction in the electroweak gauge theory. At zero temperature, the rate per unit volume of baryon number violating processes is exponentially suppressed by the factor

$$\Gamma(T = 0) \sim \exp(2S_E) \sim 10^{-170}. \quad (3.1)$$

In the high temperature, however, the baryon number violating processes that are mediated by sphalerons are not exponentially suppressed[30]. The sphalerons have typically the sizes given by the magnetic correlation length

$$\xi_\Delta \sim (\alpha T)^{-1}. \quad (3.2)$$

In this case, one may think that the space is divided up into cells of this size, although these massless sphalerons look nothing like sphalerons because the “barrier” does not make sense when the symmetry is restored by the thermal effects. There is another idea for the sphaleron production[31], where the non-equilibrium sphaleron transition is used to explain the baryon asymmetry of the Universe.

For the brane dynamics, sphaleron-like brane configuration is discussed in ref.[29]. The configuration is described by an appropriate excitation of the transverse coordinate field, which corresponds to the Higgs expectation value in the discussion of the electroweak sphalerons. Here we do not repeat the analysis in ref.[29], because the analogy comparing the sphaleron-like brane configuration to the electroweak sphaleron seems appropriate and sufficient for qualitative discussions, as far as the effective action is applicable. The sphaleron-like brane configuration is excited when the following conditions are satisfied.

- More than two Branes coincide during inflation.

After inflation, the branes fall apart toward the true vacuum configuration. In the effective action, this corresponds to the symmetry restoration during hybrid inflation.
• Brane inflation ends with brane collision.

If the brane collision is accompanied by the brane oscillation, sphaleron-like branes will be produced during this period. In the effective action, this corresponds to the non-equilibrium sphaleron production, which is discussed in ref. [31].

• Symmetry restoration is induced by the thermal effects.

Spontaneous breaking of the symmetry in the effective action is explained by brane separation or brane recombination. Sphalerons are excited when they are massless. Sphalerons-like brane configuration is expanded between mother branes and obtains mass when the mother branes fall apart.

In general, one cannot simply ignore the production of the sphaleron-like brane configurations, irrespective of their stability. Even if there was no non-equilibrium processes that makes the sphalerons form stable global structure of massive domain wall networks, there is a possibility that unstable defects (and their decay products) play important roles in the evolution of the Universe.  

In ref. [8], we have discussed the formation of the domain walls, which are induced by the spatial deformation of the D4 branes. The domain walls formed by the spatial deformation of the branes are, at least in principle, different from the ones that are formed by the brane creation. However, since the cosmological requirement for their formation is the same [8], the actual cosmological defects are inevitably the mixture of the two different kinds of brane defects.

---

The domain wall that is depicted in the right picture in fig. 4 is unstable, because it must shrink to a point as it obtains mass. However, in the special cases that multiple sphalerons are produced at the same time within the same horizon, multiple sphalerons will recombine into larger or smaller cells, as is depicted in fig. 4. Of course, it is not trivial if the sphalerons could interact each other to form the significant global structure of the domain wall networks. If the correlation length is short, sphalerons are small even if they are massless. If the nucleation rate of the sphalerons is too small to make them interact each other to form larger structure of the domain wall networks, sphalerons must shrink to a point after the phase transition. In this case, sphalerons cannot become stable domain walls. Therefore, it is required that a non-trivial mechanism that enhances the correlation length (or the nucleation rate of the sphalerons) so that the sphalerons interact each other to seed the domain walls. In the scenario of preheating, the domain wall formation is already convinced by simulation. In the dynamics of brane sphalerons, the brane distance is corresponding to the higgs field. In this respect, the domain wall formation is likely to occur in the brane-motivated models, when some conditions are satisfied. As a result, it is important to note that one cannot simply ignore the domain wall production after brane inflation.
4 Conclusions and discussions

It was believed that monopoles and domain walls cannot be produced by the daughter brane creation after brane inflation, because the daughter brane must wrap the same compactified space as the mother brane. As we have discussed in this paper, the above statement is not fully reliable. In the final state, daughter branes are not always expected to wrap the same compactified space as the mother brane. The most obvious example is found in the discussion about the inconsistency of the tension of the strings produced after angled brane inflation. In the original argument in ref. [17], the $\theta$-dependence of the tension of the string did not coincide with the analysis of the effective Lagrangian.

In the original argument [17], it was discussed that the daughter branes are created on the mother brane. In fact the statement is true, but we should be more careful about the process of the brane recombination. When the distance between the mother branes becomes shorter than a critical distance, tachyon starts to condensate on a mother brane. Therefore the tachyon forms strings on the mother brane and wraps the same compactified space as the mother brane. However, as we have depicted in fig. 11, the daughter brane must be pulled out if there was a recombination of the mother branes. As a result, the daughter brane does not wrap the same compactified space as the mother brane. Thus for the angled inflation model, the cosmic strings can depend on $\theta$, which is the property required from analysis of the effective Lagrangian. Therefore the original argument of ref. [17] fails because there is no obvious reason that in their final state the daughter branes wrap the same compactified space as the mother brane.

In this paper, we have reconsidered the production of monopoles and domain walls, paying special attention to the brane dynamics. As a result, we have found that monopoles are produced by daughter brane creation. We have also suggested that domain walls could be produced by sphaleron-like brane creation if there is an enhancement of long-wavelength configurations of the brane-sphalerons. Our conclusions are consistent with the analyses of the effective action.

Another type of brane defects [8], which are formed by the deformation of branes, can also be produced by the same cosmological processes. Therefore, the actual cosmological relics are the mixture of the two kinds. Arguments about the evolution of the mixed
brane defects are interesting and deserve further investigation.

5 Acknowledgment

We wish to thank K. Shima for encouragement, and our colleagues in Tokyo University for their kind hospitality.

References

[1] I. Antoniadis, N. A-Hamed, S. Dimopoulos, and G. R. Dvali, New dimensions at a millimeter to a fermi and superstrings at a TeV, Phys.Lett.B436(1998)257 [hep-ph/9804398]; I. Antoniadis, A possible new dimension at a few TeV, Phys.Lett.B246(1990)377; N. A-Hamed, S. Dimopoulos and G. R. Dvali, The hierarchy problem and new dimensions at a millimeter, Phys.Lett.B429(1998)263 [hep-ph/9803315].

[2] N. Arkani-Hamed, S. Dimopoulos, N. Kaloper, and J. March-Russell, Rapid asymmetric inflation and early cosmology in theories with submillimeter dimensions, Nucl.Phys.B567(2000)189 [hep-ph/9903224]; R. N. Mohapatra, A. Perez-Lorenzana, and C. A. de S. Pires, Inflation in models with large extra dimensions driven by a bulk scalar field, Phys.Rev.D62(2000)105030 [hep-ph/0003089]; A. Mazumdar, Extra dimensions and inflation, Phys.Lett.B469(1999)55 [hep-ph/9902381]; A. M. Green and A. Mazumdar, Dynamics of a large extra dimension inspired hybrid inflation model, Phys.Rev.D65(2002)105022 [hep-ph/0201209]; D. H. Lyth, Inflation with TeV scale gravity needs supersymmetry, Phys.Lett.B448(1999)191 [hep-ph/9810320]; P. Kanti and K. A. Olive, On the realization of assisted inflation, Phys.Rev.D60(1999)043502 [hep-ph/9903524]; P. Kanti and K. A. Olive, Assisted chaotic inflation in higher dimensional theories, Phys.Lett.B464(1999)192 [hep-ph/9906331]; T. Matsuda, Kaluza-Klein modes in hybrid inflation, Phys.Rev.D66(2002)107301 [hep-ph/0209214]; T. Matsuda, Successful D term inflation with moduli, Phys.Lett.B423(1998)35 [hep-ph/9705448].
[3] T. Matsuda, *Topological hybrid inflation in brane world*, JCAP 0306(2003)007 [hep-ph/0302204]; T. Matsuda, *Q ball inflation*, Phys.Rev.D68(2003)127302 [hep-ph/0309339].

[4] G. R. Dvali, G. Gabadadze, *Nonconservation of global charges in the brane universe and baryogenesis*, Phys.Lett.B460(1999)47 [hep-ph/9904221]; A. Masiero, M. Peloso, L. Sorbo, and R. Tabbash, *Baryogenesis versus proton stability in theories with extra dimensions*, Phys.Rev.D62(2000)063515 [hep-ph/0003312]; A. Pilaftsis, *Leptogenesis in theories with large extra dimensions*, Phys.Rev.D60(1999)105023 [hep-ph/9906265]; R. Allahverdi, K. Enqvist, A. Mazumdar and A. Perez-Lorenzana, *Baryogenesis in theories with large extra spatial dimensions*, Nucl.Phys. B618(2001)377 [hep-ph/0108225]; S. Davidson, M. Losada, and A. Riotto, *A new perspective on baryogenesis*, Phys.Rev.Lett.84(2000)4284 [hep-ph/0001301].

[5] T. Matsuda, *Activated sphalerons and large extra dimensions*, Phys.Rev.D66(2002)047301 [hep-ph/0205331]; T. Matsuda, *Defect mediated electroweak baryogenesis and hierarchy*, J.Phys.G27(2001)L103 [hep-ph/0102040].

[6] T. Matsuda, *Hybridized Affleck-Dine baryogenesis*, Phys.Rev.D67(2003)127302 [hep-ph/0303132]; T. Matsuda, *Affleck-Dine baryogenesis after thermal brane inflation*, Phys.Rev.D65(2002)103501 [hep-ph/0202209]; T. Matsuda, *Affleck-Dine baryogenesis in the local domain*, Phys.Rev.D65(2002)103502 [hep-ph/0202211]; T. Matsuda, *Electroweak baryogenesis mediated by locally supersymmetry breaking defects*, Phys.Rev.D64(2001)083512 [hep-ph/0107314].

[7] T. Matsuda, *Brane Q-ball, branonium and brane inflation* [hep-ph/0402223].

[8] T. Matsuda, *Formation of cosmological brane defects* [hep-ph/0402232].

[9] G. R. Dvali, I. I. Kogan and M. A. Shifman, *Topological effects in our brane world from extra dimensions*, Phys.Rev.D62(2000)106001 [hep-th/0006213].

[10] T. Matsuda, *Incidental Brane Defects*, JHEP 0309(2003)064 [hep-th/0309266].

[11] N.Arkani-Hamed, M.Schmaltz, *Hierarchies without symmetries from extra dimensions*, Phys.Rev.D61(2000)033005 [hep-ph/9903417].
[12] T. Matsuda, Baryon number violation, baryogenesis and defects with extra dimensions, Phys.Rev.D66(2002)023508 [hep-ph/0204307]; T. Matsuda, Enhanced baryon number violation due to cosmological defects with localized fermions along extra dimensions, Phys.Rev.D65(2002)107302 [hep-ph/0202258].

[13] G. R. Dvali and S. H. Henry Tye Brane inflation, Phys.Lett.B450(1999)72 [hep-ph/9812483].

[14] C. Herdeiro, S. Hirano and R. Kallosh, String theory and hybrid inflation / acceleration, JHEP0112(2001)027 [hep-th/0110271]; K. Dasgupta, C. Herdeiro, S. Hirano and R. Kallosh, D3 / D7 Inflationary model and M theory, Phys.Rev.D65(2002)126002 [hep-th/0203019], J. Garcia-Bellido, R. Rabadan and F. Zamora, Inflationary scenarios from branes at angles, JHEP 0201(2002)036 [hep-th/0112147].

[15] A. Sen, Rolling tachyon, JHEP 0204(2002)048 [hep-th/0203211]; A. Mazumdar, S. Panda and A. Perez-Lorenzana, Assisted inflation via tachyon condensation, Nucl.Phys.B614(2001)101 [hep-ph/0107058].

[16] T. Matsuda, F term, D term and hybrid brane inflation, JCAP 0311(2003)003 [hep-ph/0302078]; T. Matsuda, Nontachyonic brane inflation, Phys.Rev.D67(2003)083519 [hep-ph/0302035] T. Matsuda, Thermal hybrid inflation in brane world, Phys.Rev.D68(2003)047702 [hep-ph/0302253].

[17] N. Jones, H. Stoica, and S. H. H. Tye, Brane interaction as the origin of inflation, JHEP 0207(2002)051 [hep-th/0203163]; S. Sarangi, S. H. H. Tye, Cosmic string production towards the end of brane inflation, Phys.Lett.B536(2002)185 [hep-th/0204074]; L. Pogosian, S. H. H. Tye, I. Wasserman and M. Wyman, Observational constraints on cosmic string production during brane inflation, Phys.Rev.D68(2003)023506 [hep-th/0304188]; M. Gomez-Reino and I. Zavala, Recombination of intersecting D-branes and cosmological inflation, JHEP0209(2002)020 [hep-th/0207278].

[18] M. Majumdar and A. Christine-Davis, Cosmological creation of D-branes and anti-D-branes, JHEP 0203(2002)056 [hep-th/0202148].
[19] G. Dvali and A. Vilenkin, *Formation and evolution of cosmic D strings* [hep-th/0312007].

[20] E. J. Copeland, R. C. Myers and J. Polchinski, *Cosmic F and D strings* [hep-th/0312067].

[21] E. Halyo, *Cosmic D term strings as wrapped D3 branes* [hep-th/0312268].

[22] P. Binetruy, G. Dvali, R. Kallosh, A. Van Proeyen, *Fayet-Iliopoulos terms in supergravity and cosmology* [hep-th/0402046].

[23] T. Matsuda, *String production after angled brane inflation, to appear in PRD* [hep-ph/0403092].

[24] E. Halyo, *D-brane inflation on conifolds* [hep-th/0402155]; E. Halyo, *Inflation on fractional branes: D-brane inflation as D term inflation* [hep-th/0312042]; E. Halyo, *Models of inflation on D-branes* [hep-th/0307223].

[25] G.R. Dvali, *Infrared hierarchy, thermal brane inflation and superstrings as superheavy dark matter, Phys.Lett.B459(1999)489* [hep-ph/9905204].

[26] G. Aldazabal, S. Franco, L. E. Ibanez, R. Rabadon, A.M. Uranga, *Intersecting brane worlds, JHEP 0102(2001)047* [hep-ph/0011132]; R. Blumenhagen, D. Lust, S. Stieberger, *Gauge unification in supersymmetric intersecting brane worlds, JHEP 0307(2003)036* [hep-th/0305146]; I. R. Klebanov, E. Witten, *Proton decay in intersecting D-brane models, Nucl.Phys.B664(2003)3-20* [hep-th/0304079]; M. Cvetic, P. Langacker, J. Wang, *Dynamical Supersymmetry Breaking in Standard-like Models with Intersecting D6-branes, Nucl.Phys.B642(2002)139* [hep-th/0303208]; M. Cvetic, I. Papadimitriou, *More Supersymmetric Standard-like Models from Intersecting D6-branes on Type IIA Orientifolds, Phys.Rev.D67(2003)126006* [hep-th/0303197]; S.A. Abel, A.W. Owen, *Interactions in Intersecting Brane Models, Nucl.Phys.B663(2003)197* [hep-th/0310257]; D. Bailin, G.V. Kraniotis, A. Love, *Standard-like models from Intersecting D5-branes, Phys.Lett.B547(2002)43* [hep-th/0210219]; C. F. Doran, M. Faux, *Intersecting Branes in M-Theory and Chiral Matter in Four Dimensions, JHEP
0208(2002)024 [hep-th/0208030]; C. Kokorelis, Exact Standard Model Compactifications from Intersecting Branes, JHEP 0208(2002)036 [hep-th/0206108]; C. Kokorelis, New Standard Model Vacua from Intersecting Branes, JHEP 0209(2002)029 [hep-th/0205147]; D. Cremades, L.E. Ibanez, F. Marchesano, Standard Model at Intersecting D5-branes: Lowering the String Scale, Nucl.Phys.B643(2002)93 [hep-th/0205074].

[27] A. Giveon and D. Kutasov, Brane dynamics and gauge theory, Rev.Mod.Phys.71(1999)983 [hep-th/9802067] and references therein.

[28] A. Hanany, M. J. Strassler and A. Zaffaroni, Confinement and strings in MQCD, Nucl.Phys.B513(1998)87 [hep-th/9707244]; A. Hanany, M. J. Strassler and A. Zaffaroni, Confinement and strings in MQCD, Nucl.Phys.B513(1998)87 [hep-th/9707244].

[29] C. G. Callan,Jr. and J. M. Maldacena, Brane dynamics from the Born-Infeld action, Nucl.Phys.B513(1998)198 [hep-th/9708147], K.Hashimoto, Dynamical decay of brane anti-brane and dielectric brane, JHEP0207(2002)035 [hep-th/0204203].

[30] See for example, Mark Trodden, Electroweak baryogenesis, Rev.Mod.Phys.71(1999)1463 [hep-ph/9803479] and references therein.

[31] J. Garcia-Bellido, D. Yu. Grigoriev, A. Kusenko and M. E. Shaposhnikov, Non-equilibrium electroweak baryogenesis from preheating after inflation, Phys.Rev.D60(1999)123504 [hep-ph/9902449].
Figure 1: Upper row: schematic recombination of two $D_p$-branes with $(\pi - \theta) \ll 1$. The dashed line on the $D_p$-brane represents the $D_{p-2}$-brane that might appear on the worldvolume of the $D_p$-brane when the tachyon condenses. To be more precise, a careful treatment of the effective action shows that the eigenfunction of the tachyonic mode is localized on the intersection. Since the mechanism of this localization is different from the Kibble mechanism, the “seed” for the $D_{p-2}$ brane can be localized on the intersection. As the recombination proceeds, the $D_{p-2}$ brane is pulled out from the mother brane, and finally becomes extended between the mother brane. In this case, the problem of the RR field is avoided since the length of the extended $D_{p-2}$ brane vanishes at the time when it is pulled out from the mother brane. Of course, it costs energy to pull $D_{p-2}$ branes out from the mother branes, however in this case the cost is paid by the repulsive force between the splitting mother branes. Second row: schematic recombination of two $D_p$-branes with $\theta \ll 1$. As a result, the daughter $D_{p-2}$ brane does not wrap the same compactified space as the mother brane. Thus, for the angled inflation model, the cosmic strings can depend on $\theta$, which is the property required from analysis of the effective Lagrangian.
Figure 2: The initial configuration is depicted in the left picture, where two $D_p$ branes are located on top of each other. Daughter $D_{p-2}$-brane is formed when tachyon condenses. The daughter brane is denoted by the dotted line. Obviously, the previous arguments are correct so far. The crucial difference appears when the $D_p$ branes start to fall apart. The location of the $D_{p-2}$ brane fluctuates between the two $D_p$ branes, along the spatial directions of the four-dimensional space time. It should be noted that conventional cosmological strings are not formed in this case, because spatial fluctuations are inevitable. Thus in the final state, which is given in the right picture, the daughter $D_{p-2}$ brane becomes monopoles and are extended between the $D_p$ branes. Monopoles could be connected to anti-monopoles by fluxes. We stress that the above mechanism works in generic cases of brane collision, when branes oscillate and produce another branes on their worldvolume.
Figure 3: The initial configuration is depicted in the left picture, where $n$ $D_p$ branes are located on top of each other. The $D_{p-2}$ brane that is created on the world volume of the mother $D_p$ branes is denoted by the dotted line. The location of the $D_{p-2}$ brane fluctuate in the uncompactified directions. When $n$ branes fall apart from each other, the $D_{p-2}$ brane is pulled out of the mother branes. In the effective action, the extended $D_{p-2}$ branes are seen as the point-like monopoles.
Figure 4: $N = 2 \ SU(2)$ supersymmetric Yang-Mills theory realized by stretching two $D_4$ branes between two NS branes. The monopole is a rectangular $D_2$ brane with two boundaries on $D_4$ and two on NS branes, which looks like a soap-bubble that fills the “hole” between the branes.
Figure 5: $D2$ brane is created on the worldvolume of a $D4$ brane due to tachyon condensation. In the initial configuration, the $D2$ brane wraps the same compactified space as the mother $D4$ brane. Thus the previous arguments are correct so far. However, as we have discussed, a crucial difference appears when the stacked mother branes start to fall apart. Although there is no fluctuation in the $x_5$ compactified direction, the $D2$ brane can fluctuate between the coincidental $D4$ branes along the directions of the uncompactified space. Thus the daughter $D2$ brane is pulled out of $D4$ branes, which finally becomes the $D2$ brane monopoles. Of course our argument is consistent with a thermal history of the effective action if a soft mass is included. It should be noted that the previous arguments were inconsistent with the thermal history of the effective action.
Figure 6: $D_{p-2}$ brane is extended between $D_p$ branes. In the left picture, the sphaleron-like brane is massless because the distance between the two $D_p$ branes vanishes. As in the cases of the conventional electroweak sphalerons, the production of the massless sphaleron is viable and not suppressed by the exponential factor. On the other hand, if the distance is so large that the mass of the sphalerons is not negligible, the excitation of sphalerons is exponentially suppressed. In our case, the produced sphaleron-like branes gains mass as the distance grows. The configuration becomes unstable if there is no other sphalerons in the horizon. It might be useful to note that the sphaleron that we are considering in this paper can be considered as the pair creation of brane anti-brane with their boundaries on the mother branes. The “throat” solution that is constructed from $D_{p-2}$ brane anti-brane could have its boundaries on the mother $D_p$ branes. It seems obvious that the mother branes are not necessarily the $D_{p-2}$ brane anti-branes. Of course our argument is consistent with the sphaleron production in the effective action.
Figure 7: Sphalerons will recombine into larger (or smaller) pieces, if there is a viable mechanism that enhances the long-wavelength configuration of the sphalerons.