An inclusive measurement of the photon energy spectrum in $b \to s \gamma$ decays

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We report a fully inclusive measurement of the flavor changing neutral current decay $b \to s\gamma$ in the energy range $1.8 \text{ GeV} \leq E_\gamma^* \leq 2.8 \text{ GeV}$, covering 95% of the total spectrum. Using $140 \text{ fb}^{-1}$, we obtain $B(b \to s\gamma) = (3.55 \pm 0.32 \pm 0.30 \pm 0.12) \times 10^{-4}$, where the errors are statistical, systematic and from theory corrections. We also measure the first and second moments of the photon energy spectrum above $1.8 \text{ GeV}$ and obtain $\langle E_\gamma \rangle = 2.292 \pm 0.026 \pm 0.034 \text{ GeV}$ and $\langle E_\gamma^2 \rangle - \langle E_\gamma \rangle^2 = 0.0305 \pm 0.0074 \pm 0.0063 \text{ GeV}^2$, where the errors are statistical and systematic.

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The flavor changing neutral current decay $b \to s\gamma$ process is of remarkable theoretical interest. Its total branching fraction is very sensitive to physics beyond the Standard Model as it may be affected by the presence of charged Higgs or SUSY particles in the loop. Yet the present theoretical prediction for the branching fraction of $(3.79 \pm 0.36) \times 10^{-4}$ \cite{1,2}, and the average experimental value $(3.3 \pm 0.4) \times 10^{-4}$ \cite{3,4} agree well. This agreement sets a strong constraint on, e.g., models \cite{5} that accommodate the observed difference in CP asymmetries in the $B \to J/\psi K_S$ and $B \to \phi K_S$ decays \cite{6}. To obtain stronger constraints on physics beyond the Standard Model, more precise theoretical predictions and experimental measurements are needed.

On the other hand, the photon energy spectrum is almost insensitive to physics beyond the Standard Model \cite{7}. At the parton level, the photon is monochromatic with energy $E \approx m_b/2$ in the $b$ quark rest frame. The energy is smeared by the motion of the $b$ quark inside the $B$ meson and gluon emission. A measurement of the moments of this spectrum allows a determination of the $b$-quark mass and of its motion. This information can then be used to extract the CKM matrix elements $|V_{cb}|$ and $|V_{ub}|$ from inclusive semileptonic $b$ decays \cite{7,8}. However, a measurement of the low-energy tail of the photon spectrum is important in this context \cite{9}.

Belle has previously measured the $b \to s\gamma$ branching fraction with $5.8 \text{ fb}^{-1}$ of data using a semi-inclusive approach. Because we applied an effective cut $E_\gamma > 2.24 \text{ GeV}$ in the $B$ rest frame, the precision of that measurement is limited by theoretical errors due to the extrapolation to the whole energy spectrum. More recently the CLEO collaboration has reported a measurement of the branching fraction and the energy spectrum moments performed in a fully inclusive way \cite{10} for the range $E_\gamma^* > 2.0 \text{ GeV}$ in the center-of-mass frame \cite{11}. Here we present a measurement using a similar approach, but based on a much larger dataset allowing a detailed study of the backgrounds. We extend the photon energy range to $E_\gamma^* > 1.8 \text{ GeV}$, covering almost the entire spectrum.
The $b \to s\gamma$ decay is studied using the Belle detector at the KEKB asymmetric $e^+e^-$ storage ring [12]. The data consists of a sample of 140 fb$^{-1}$ taken at the $\Upsilon(4S)$ resonance corresponding to $(152.0^{+0.6}_{-0.7}) \times 10^6 B\bar{B}$ pairs. Another 15 fb$^{-1}$ sample has been taken at an energy 60 MeV below the resonance and is used to measure the non-$B\bar{B}$ background. Throughout this Letter, we refer to these data samples as the ON and OFF samples, respectively.

The Belle detector is a large-solid-angle magnetic spectrometer described in detail elsewhere [13]. The main component relevant for this analysis is the electromagnetic calorimeter (ECL) made of 16.2 radiation lengths long CsI(Tl) crystals. The photon energy resolution is about 2% for the energy range relevant in this analysis.

The strategy to extract the signal $b \to s\gamma$ spectrum is to collect all high-energy photons, vetoing those originating from $\pi^0$ and $\eta$ decays to two photons. The contribution from continuum $e^+e^- \to q\bar{q}$ ($q = u,d,s,c$) events is subtracted using the OFF sample. The remaining backgrounds from $B\bar{B}$ events are subtracted using Monte-Carlo (MC) distributions scaled by data control samples.

Photon candidates are selected from ECL clusters of 5 × 5 crystals in the barrel region ($-0.5 \leq \cos \theta \leq 0.84$, where $\theta$ is the polar angle with respect to the beam axis). They are required to have an energy $E_\gamma^*$ larger than 1.5 GeV, and 95% of the energy has to be deposited in the central 3 × 3 crystal array. We require isolation cuts to veto photons from bremsstrahlung and interaction with matter. The center of the cluster has to be displaced from any other ECL cluster with $E > 20$ MeV by at least 30 cm at the surface of the calorimeter, and from any reconstructed track by 3 cm, or by 50 cm for tracks with a measured momentum above 1 GeV/c. Moreover, the angle between the photon and the highest energy lepton in the event has to be larger than 0.3 radians at the interaction point. We veto candidate photons from $\pi^0$ and $\eta$ decays to two photons by combining them with any other photon. We reject the pair if the likelihood of being a $\pi^0$ or $\eta$ is larger than 0.1 and 0.2, respectively. These likelihoods are determined from MC and are functions of the laboratory energy of the other photon, its polar angle $\theta$ and the mass of the two-photon system.

In order to reduce the contribution from continuum events, we use two Fisher discriminants. The first exploits the spherical shape of $B\bar{B}$ events and is built using ten event-shape variables. These variables are calculated using either all tracks and showers in the event or excluding the photon candidate. The event shape variables include Fox-Wolfram moments, thrust, and the angles of the thrust axis with respect to the beam and photon direction. The second discriminant exploits the topology of $b \to s\gamma$ events and combines three energy flows around the photon axis. These energy flow variables are obtained using all particles, except for the photon candidate, whose direction lies in the three regions defined by $\alpha^* < 30^\circ$, $30^\circ \leq \alpha^* \leq 140^\circ$, $\alpha^* > 140^\circ$, where $\alpha^*$ is the angle to the candidate photon.

To optimize these selection criteria, we use a Monte Carlo simulation [14] containing large samples of $B\bar{B}$, $q\bar{q}$ and signal weighted according to the luminosities of the ON and OFF samples. The signal MC is generated as a weighted sum of $B \to K^*\gamma$ decays, where $K^*$ is any known spin-1 resonance with strangeness $S = 1$. The relative weights are obtained by fitting the total photon spectrum to a theoretical model [15]. The signal MC is normalized to the average measured branching fraction [16]. To improve the understanding of the photon energy spectrum at low energies, the selection criteria are optimized to maximize the sensitivity to the signal in the energy bin $1.8 \text{ GeV} < E_\gamma^* < 1.9 \text{ GeV}$.

After these selection criteria we observe $1.2 \times 10^6$ photon candidates in ON and $1.1 \times 10^5$ in OFF data. The spectrum measured in OFF data is scaled by luminosity to the expected number of non-$B\bar{B}$ events in ON data and subtracted. To take into account the effect of the 60 MeV (0.5%) energy difference, the measured OFF energies are scaled by an empirical factor of 1.004 obtained from a MC study. The ON and scaled OFF spectra and their difference are shown in Fig. 11.

We then subtract the backgrounds from $B$ decays from the obtained spectrum. Five background categories are considered: (i) photons from $\pi^0 \to \gamma\gamma$, which account for more than half of the background in the 1.8–2.8 GeV range; (ii) photons from $\eta \to \gamma\gamma$; (iii) other real photons (mainly decays of $\omega$, $\eta'$, and $J/\psi$, and bremsstrahlung); (iv) ECL clusters not due to single photons (mainly electrons interacting with matter, $K_S^0$ and $\bar{n}$) beam background. For each of these categories we take the predicted background from MC and scale it according to measured yields wherever possible. The inclusive $B \to \pi^0X$ and $B \to \eta X$ spectra are measured in data.
using pairs of photons with a well-balanced energy and applying the same ON–OFF subtraction procedure. The yields obtained are 5 to 15% larger than MC expectations depending on the photon energy range. Since there is good agreement between MC and data for all features of the GEANT simulation for photons and electrons, we believe that the observed discrepancy between the measured and simulated π⁰ spectrum is due to the generator. Beam background is measured using a sample of randomly triggered events and added to the BB MC.

For each selection criterion and each background category we determine the $E_\gamma^*$-dependent selection efficiency in OFF-subtracted ON data and MC using appropriate control samples. We then scale the MC background sample according to the ratio of these efficiencies. The efficiencies of the π⁰ and η vetoes for non-π⁰, non-η photons are measured in data using one photon from a well reconstructed π⁰ applying the veto without using the other photon of the pair. The π⁰ veto efficiency is measured using a sample of photons coming from measured π⁰ decays. We use partially reconstructed $D^+ \to D^0 \pi^+$, $D^0 \to K^- \pi^+ \pi^0$ decays where the π⁰ is replaced by the candidate photon in the reconstruction. The η veto efficiency for photons from π⁰’s and event-shape criteria efficiencies are measured using a π⁰ anti-veto sample. It is made of photons passing all selection criteria except the π⁰ veto, which are combined with another photon in the event to give a π⁰-likelihood larger than 0.75. Other efficiencies are measured using the signal sample.

The ratios of data and MC efficiencies versus $E_\gamma^*$ are fitted using first or second order polynomials, which are used to scale the background MC. Most are found to be statistically compatible with unity. An exception is the efficiency of the requirement that 95% of the energy be deposited in the central nine cells of the $5 \times 5$ cluster, which is found to be poorly modelled by our MC for non-photon backgrounds. We estimate the efficiency for data using a sample of candidate photons in OFF-subtracted ON data after subtracting the known contribution from real photons. This increases the yield of background (iv) by 50%. The yield from the five background categories, after having been properly scaled by the above described procedures, are subtracted from the OFF-subtracted spectrum. The result is shown in Fig. 1.

The spectrum contains 24100 ± 2140 ± 1260 events in the 1.8–2.8 GeV energy range, where the two errors are the statistical error of the OFF-subtracted ON data and of the BB background subtractions, and the systematic error related to the data/MC efficiency ratio fits used in the BB background scaling. We correct this spectrum for the signal selection efficiency function obtained from signal MC, applying the same data/MC correction factors as for the generic photon background category (iii). The average signal selection efficiency is 23%.

The efficiency-corrected spectrum is shown in Figure 2. The two error bars for each point show the statistical and the total error, including the systematic error which is correlated among the points. As expected, the spectrum above the endpoint for decays of B mesons from the $\Upsilon(4S)$ at about 3 GeV, is consistent with zero. Integrating this spectrum from 1.8 to 2.8 GeV, we obtain a partial branching fraction of $(3.51 \pm 0.32 \pm 0.29) \times 10^{-4}$.

The sources of systematic error are listed in Table I. They are added in quadrature. The largest sources are the errors of the data/MC efficiency ratio fits (5.9% of the signal yield). For the error related to the choice of the polynomial functions in the data/MC efficiency ratio fits, we perform the same fit increasing the polynomial order by one. The number of BB events is determined from the number of hadronic events in ON and OFF data. The relative luminosities of the two samples are determined from radiative Bhabhas and $e^+e^- \to \mu^+\mu^-$ events. The errors on the OFF data subtraction are estimated using

![Figure 2](image-url)  
**FIG. 2:** Efficiency-corrected photon energy spectrum. The two error bars show the statistical and total errors.

| Source of systematic error | $\times 10^{-4}$ |
|----------------------------|-----------------|
| Raw branching fraction     | 3.51 ± 0.32     |
| Data/MC efficiency ratio fits | $\pm 0.208$    |
| Choice of fitting functions | $\pm 0.048$     |
| Number of $B\bar{B}$-events | $\pm 0.139$    |
| ON-OFF data subtraction    | $\pm 0.026$     |
| Other $B\bar{B}$ photons   | $\pm 0.054$     |
| $\eta$ veto efficiency on $\eta$ | $\pm 0.008$ |
| Signal MC                  | $\pm 0.089$     |
| Photon detection efficiency | $\pm 0.072$    |
| Energy leakage             | $\pm 0.035$     |
| Total error for partial $\mathcal{B}(b \to \gamma)$ | $\pm 0.282$ |

**TABLE I:** Overview of systematic errors.
the result of the fit to the spectrum above the endpoint. We integrate the resulting function in the 1.8–2.8 GeV range and obtain a yield of +40 ± 160. We add ±200 (0.8%) to the systematic error. As we do not measure the yields of photons from sources other than η’s and η’s in $B\bar{B}$ events, we vary the expected yields of these additional sources by ±20%. For the model dependence of signal selection efficiency we use an alternate signal MC that favors high-mass resonances decaying into high-multiplicity final states. Using this MC to correct for the efficiency changes the branching fraction by ±2.5%.

The error on the photon detection efficiency in the ECL is measured to be 2.3% using radiative Bhabha events. This error also affects the estimation of photons from $B\bar{B}$ and contributes ±2.0% to the systematic error. Due to the low-energy tail in the photon energy measurement, some part of the spectrum may lie below the range of integration. We estimate this fraction to be smaller than 1%. As this value is shape-dependent, we do not correct the measured branching fraction for it but instead add a (+0.1, −0.0)% systematic error.

In order to obtain the total $b \to s\gamma$ branching fraction we apply corrections for the contribution from Cabibbo suppressed $b \to d\gamma$ decays and for the invisible part of the spectrum below 1.8 GeV. The ratio of the $b \to s\gamma$ and $b \to d\gamma$ branching fractions is assumed to be $R_{d/s} = (3.8 \pm 0.6)\%$. The selection efficiency for $b \to d\gamma$ is found to be equal to the efficiency for $b \to s\gamma$ within 10%, which we include in the systematic error. The fraction of the spectrum above 1.8 GeV is assumed to be $R_{1.8} = 0.952 \pm 0.013$ from Gambino and Misiak. As a cross-check we also use the value from Kagan and Neubert $R_{1.8} = 0.958 \pm 0.013$, and $R_{1.8} = 0.95 \pm 0.01$ from Bigi and Uraltsev. We combine the errors on $R_{d/s}$, $R_{1.8}$ and the difference between the $R_{1.8}$ values into the theoretical error. With these two corrections, we obtain

$$B(b \to s\gamma) = (3.55 \pm 0.32 ^{+0.30 +0.11} _{-0.31 -0.07}) \times 10^{-4}$$

for the total $b \to s\gamma$ branching fraction. This result is in good agreement with theoretical expectations and with previous experimental measurements.

We also measure the first two moments of the energy spectrum in the $B$ rest frame. We extract the raw moments from the distribution shown in Fig. 2 in the range 1.8 GeV $\leq E_\gamma^* \leq$ 2.8 GeV and correct them for the effect of the boost of the $B$ meson in the $\Upsilon(4S)$ frame, for the energy resolution and for the 100 MeV binning. We do not correct the moments for the missing low-energy tail. We obtain the following moments for $E_\gamma^* > 1.8$ GeV (corresponding to $E_\gamma > 1.815$ GeV in the $B$ rest frame):

$$\langle E_\gamma^* \rangle = 2.292 \pm 0.026 \pm 0.034 \text{ GeV}$$

$$\langle E_\gamma^* \rangle ^2 - \langle E_\gamma \rangle ^2 = 0.0305 \pm 0.0074 \pm 0.0063 \text{ GeV}^2,$$

where the errors are statistical and systematic.

The systematic error contains the errors related to the moments corrections and the error sources already mentioned for the branching fraction extraction. For the first moment the systematic error is dominated by the data/MC efficiency ratio fits (±0.9%) and the shape of the energy resolution (±1.0%). The error on the second moment is dominated by the data/MC efficiency ratio fits (±17%). These results agree within 1σ with the only previous measurement, done by the CLEO collaboration. However, it should be noted that the CLEO results are obtained for $E_\gamma^* > 2.0$ GeV.

In conclusion, we have measured the branching fraction and photon energy spectrum of $b \to s\gamma$ in the energy range 1.8 GeV $\leq E_\gamma^* \leq$ 2.8 GeV in a fully inclusive way. For the first time 95% or more of the spectrum is measured, allowing the theoretical uncertainties to be reduced to a very low level. Using 140 fb$^{-1}$ of data taken at the $\Upsilon(4S)$ and 15 fb$^{-1}$ taken below the resonance, we obtain $B(b \to s\gamma) = (3.55 \pm 0.32 ^{+0.30 +0.11} _{-0.31 -0.07}) \times 10^{-4}$, where the errors are systematic, and theoretical, respectively. This result is in good agreement with the latest theoretical calculations.

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