Colour-octet Contributions to $J/\psi$
Hadroproduction at Fixed Target Energies.

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Abstract

We investigate the integrated cross section for forward $J/\psi$ production in proton-proton and pion-proton collisions at centre of mass energies upto 60 GeV. We find that colour-octet contributions to the cross section are crucially important, and their inclusion produces a very good description of the data. Since the values of non-perturbative matrix elements are fixed by other experiments, these predictions are parameter-free.
The production of bound states of heavy quarks has been theoretically studied in terms of the colour-singlet model \[1\]. In this approach, the dominant contribution to quarkonium production is expected to come from quark-antiquark or gluon-gluon fusion, leading to the formation of a heavy-quark pair in a colour-singlet state with the correct spin, parity and charge-conjugation assignments projected out. By using radial wave-functions or their derivatives derived from other measurements as inputs, the colour-singlet model makes definite predictions for the production cross sections of specific quarkonium resonances.

While this model gives a reasonable description of large-\(p_T\) \(J/\psi\) production at ISR energies, it completely fails to explain the \(p_T\) integrated cross sections at fixed-target and ISR energies. This failure is easily traced back to the model assumptions. The requirement that a colour-singlet state with the correct quantum numbers be produced at the hard vertex imposes rather stringent constraints (coming from \(C\)-invariance) and rules out the zero-\(p_T\) production of the \(S\)-state quarkonia at lowest order. Thus, to leading order, the inclusive cross section for \(J/\psi\) production is obtained only from direct production of \(\chi_c\) followed by their decay to \(J/\psi\).

Recent improvements in the theory of quarkonium production have been provoked by the anomalously large cross section for \(J/\psi\) production at large \(p_T\) measured recently in the CDF experiment at the Tevatron \[2\]. For values of \(p_T\) much larger than the charm quark mass, fragmentation of gluons and charm quarks become dominant \[3\]. The CDF data has been successfully explained by taking into account both the fusion and fragmentation contributions \[4\].

The analyses of the CDF data have brought out another limitation of the colour-singlet approach, i.e., the relative velocity, \(v\), between the heavy quarks in the bound state is ignored. However, \(v\) is often not negligible in quarkonium states and corrections of order \(v\) need to be taken into account. Starting from a non-relativistic QCD Lagrangian, a systematic analysis using the factorisation method has been recently carried out \[5\]. In this formulation, the quarkonium wave-function admits of a systematic expansion in powers of \(v\) in terms of Fock-space components. For example, the wave-functions for the \(P\)-state charmonia have the conventional colour-singlet \(P\)-state component at leading order, but there exist additional contributions at
non-leading order in $v$, involving octet $S$-state components; \textit{i.e.},

$$|\chi_J\rangle = O(1)|Q\bar{Q}[^3P_J^{(1)}]\rangle + O(v)|Q\bar{Q}[^3S_1^{(8)}]g\rangle + \ldots$$  \hspace{1cm} (1)

While the octet components are necessary for a consistent perturbative description of the $P$-states \cite{3}, for $S$-states it is required in order to explain the anomalously large $\psi'$ \cite{4} and direct $J/\psi$ production at the Tevatron \cite{5}.

As a consequence of factorisation, each matrix element is split into two parts. The short-distance parts for the colour-octet process are calculable, but the long-distance non-perturbative octet matrix elements (the analogues of the wavefunctions in the colour-singlet model) are not. They have been obtained by a fit to the Tevatron data \cite{5}. More recently, the colour-octet processes which contribute to photoproduction of $J/\psi$ have been studied \cite{4}. Using both the fixed-target and HERA data on photoproduction, a different linear combination of the same octet matrix-elements has been fitted.

The time seems ripe for a reappraisal of the mass of data on hadroproduction of $J/\psi$ in proton-nucleon and pion-nucleon collisions at centre of mass energies, $\sqrt{S}$, up to about 60 GeV (see \cite{10} for a compilation of data). In this paper we carry out this investigation. We find that the low-energy data on forward ($x_F > 0$) integrated hadroproduction cross sections are well described by the inclusion of octet components. These are parameter-free predictions, because the values of the non-perturbative matrix elements are derived from other experiments.

In this model, a $J/\psi$ can be obtained either directly, or through radiative decays of $\chi_c$ states. Any of these can be produced through a colour-singlet channel or through the octet channel. Quantum number arguments show that the cross sections for direct $J/\psi$ or $\chi_1$ production in the singlet channel are zero (to lowest order in $\alpha_s$). As a result, we can write the cross section

$$\sigma_{J/\psi}(s) = \sigma_{J/\psi}^8(s) + \sigma_{\chi_1}^8(s) \ BR_{\chi_1} + \left(\sigma_{\chi_0}^1(s) + \sigma_{\chi_0}^8(s)\right) \ BR_{\chi_0}$$

$$+ \left(\sigma_{\chi_2}^1(s) + \sigma_{\chi_2}^8(s)\right) \ BR_{\chi_2}, \hspace{1cm} (2)$$

where $BR_{\chi_J}$ denotes the branching ratio for the decay of $\chi_J$ into $J/\psi$. The individual terms are all known. We write down the forward integrated cross sections below.
The singlet channel cross sections \([1]\) are given in terms of the strong coupling \(\alpha_s\) and the charm quark mass \(m\) by

\[
\sigma_{\chi_0}^1 = G \frac{\alpha_s^2 \pi^3}{48m} \langle O_1^{\chi_0}(3P_0) \rangle, \\
\sigma_{\chi_2}^1 = G \frac{\alpha_s^2 \pi^3}{180m} \langle O_1^{\chi_2}(3P_2) \rangle.
\] (3)

The quantity \(G\) is defined in terms of the gluon momentum densities in the projectile (P) and target (T) by

\[
G = \int_{1/\sqrt{\tau}} dx \frac{g_P(x)g_T(\tau/x)}{x}.
\] (4)

The lower limit of the integral, \(\sqrt{\tau} = 2m/\sqrt{S}\), is written for the forward cross section. If all values of \(x_F\) were to be considered, the limit would have changed to \(\tau\).

The two matrix elements are related to the derivative of the wavefunction at the origin by

\[
\langle O_{\chi J}(3P_J) \rangle = \frac{9(2J+1)}{2\pi} |R'(0)|^2.
\] (5)

The octet cross section for \(J/\psi\) has been calculated recently \([11]\). It is given by

\[
\sigma_{J/\psi}^8 = G \frac{5\alpha_s^2 \pi^3}{48m^5} \left[ \langle O_{8J/\psi}(1S_0) \rangle + \frac{3}{m^2} \langle O_{8J/\psi}(3P_0) \rangle + \frac{4}{5m^2} \langle O_{8J/\psi}(3P_2) \rangle \right] \\
+ Q \frac{\alpha_s^2 \pi^3}{54m^5} \langle O_{8J/\psi}(3S_1) \rangle.
\] (6)

The quantity \(Q\) is defined in terms of the quark momentum densities in the projectile and target by

\[
Q = \sum_f \int_{1/\sqrt{\tau}} dx \frac{q_f'(x)\bar{q}_f'(\tau/x)}{x} + (P \leftrightarrow T),
\] (7)

where the sum is over flavours. The combination of non-perturbative matrix elements in the coefficient of \(G\) is precisely that needed for photoproduction \([9]\).

There are also the octet contributions for the \(\chi_c\) states. These can be easily obtained by noting the following facts. The leading contribution comes
from the $^3S_1$ colour-octet states. The gluon fusion contribution to this channel vanishes \cite{11}. Hence, the only cross section required is

$$
\sigma_{^8\chi J}^S = Q\frac{\alpha_s^2 \pi^3}{54m^2} \langle O_{^8\chi J}^{^3S_1} \rangle.
$$ (8)

The singlet matrix elements of eq. (5) are extracted from data on hadronic decays of the $\chi_c$ states. For a recent compilation see \cite{12}. The colour-octet matrix elements $\langle O_{^8\chi}^{^3S_1} \rangle$ and $\langle O_{^8J/\psi}^{^3S_1} \rangle$ have been extracted from the hadroproduction rates at the Tevatron. The different resonances are identified in these experiments, allowing separate fits to these matrix elements. We use the tabulation of \cite{13}. We require all the matrix elements $\langle O_{^8\chi J}^{^3S_1} \rangle$.

These are related by

$$
\langle O_{^8\chi J}^{^3S_1} \rangle = (2J + 1) \langle O_{^8\chi 0}^{^3S_1} \rangle.
$$ (9)

The remaining combination of matrix elements has been extracted from photoproduction data \cite{9}. We use this value

$$
\left[ \langle O_{^8J/\psi}^{^1S_0} \rangle + \frac{3}{m^2} \langle O_{^8J/\psi}^{^3P_0} \rangle + \frac{4}{5m^2} \langle O_{^8J/\psi}^{^3P_2} \rangle \right] = 0.020 \pm 0.001 \text{ GeV}^3
$$ (10)

in the calculations reported here.

A different linear combination of the same matrix elements enters the large-$p_T$ hadroproduction cross sections relevant to the Tevatron experiments. A consistency check between these two linear combinations can be made if one assumes a relation between $\langle O_{^8J/\psi}^{^3P_2} \rangle$ and $\langle O_{^8J/\psi}^{^3P_0} \rangle$. Such a relation reduces the number of unknown matrix elements by one and permits evaluation of the individual matrix elements. With the assumption that the first matrix element is 5 times the second, the Tevatron and photoproduction fits together yield a negative value of $\langle O_{^8J/\psi}^{^3P_0} \rangle$ \cite{9}. This may just indicate that the assumption has to be modified. The issue is, however, irrelevant to our computation, since we require precisely the linear combination in eq. (10).

We use the MRS D$'$ and the GRV LO sets of parton densities for the proton. Since we are interested in values of $\sqrt{S} \leq 60$ GeV, the lowest values of $x$ entering the parton distributions are $\sqrt{x} \approx 0.05$. For such large values of $x$ the differences between structure function parametrisations for the proton
are rather small. The pion structure functions are not known as accurately. We have used the parametrisations SMRS 1, SMRS 3 and GRV. All the structure functions are taken from the PDFLIB package [14].

As always in lowest order QCD cross sections, there is a choice of scale to be considered. In addition, the charm quark mass, $m$, is not precisely known. From open charm production, the limits are $1.2 \text{ GeV} \leq m \leq 1.8 \text{ GeV}$ [3]. Our results are shown for the two choices $m = 1.7 \text{ GeV}$ and a scale of $2m$, as well as $m = 1.6 \text{ GeV}$ and a scale of $4m$.

With the inputs discussed here, we find that the $\sqrt{S}$ dependence of the integrated forward $J/\psi$ production rates, for both $pp$ and $\pi p$ collisions, are described rather well by the model (see Figure[1]). We would like to emphasise that there are no free parameters in this calculation.

We find that the total $J/\psi$ hadroproduction rate in $pp$ collision is dominated by the gluon-gluon fusion part of the colour-octet direct $J/\psi$ cross section. Although $\chi_0$ is produced copiously, its contribution to the $J/\psi$ cross section is suppressed by a small branching ratio. The contribution of the $\chi_1$ is small because the singlet contribution is zero and the octet is small. The $\chi_2$ state gives the major contribution among the $P$-channel resonances, but nevertheless gives less than 5% of the inclusive forward cross section. In $\pi p$ collisions the $P$-channel contributes slightly more to the total cross section. This is due to the enhancement of the $q\bar{q}$ initiated octet $\chi$ contributions.

Since the cross section is dominated by the gluon fusion process giving a colour-octet contribution to direct $J/\psi$, the dominant uncertainty in the non-perturbative matrix elements comes from the linear combination in eq. (1). This gives a 5% uncertainty in the predictions. Although the uncertainty in the non-perturbative matrix elements for colour-octet contributions to the $\chi$ states are larger, they have a much smaller effect on the result. Similairly, the assumption stated in eq. (9) has very little impact on the result.

Larger uncertainties come from the choice of scale in QCD and the charm quark mass. The scale uncertainty can only be controlled by taking into account the order $\alpha_s^3$ contributions to the cross section. When this is done, it might be possible to control the uncertainty in $m$.

In conclusion, the world data on forward inclusive $J/\psi$ hadroproduction cross sections are well described by the inclusion of octet contributions. The calculation is completely parameter-free since the non-perturbative inputs are fixed from other experiments. Our results on other aspects of the phenomenology, such as rapidity and $x_F$ distributions, will be reported elsewhere.
Figure 1: The colour-octet model predictions for integrated forward $J/\psi$ hadroproduction cross sections as a function of the CM energy. For $pp$ collisions, the two curves for each choice of $m$ and scale $Q$ are for the structure functions MRS D$-^{'}$ and GRV LO. For $\pi p$ collisions the three sets of structure functions are MRS D$-^{'}$ for proton and SMRS 1 for $\pi$, MRS D$-^{'}$ for proton and SMRS 3 for $\pi$, GRV LO for both. Note that the colour-singlet model predictions lie far below the data.
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