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Wildness of the problem of classifying nilpotent Lie algebras of vector fields in four variables. (English) Zbl 1453.17014

Linear Algebra Appl. 568, 165-172 (2019).

Summary: Let $\mathbb{F}$ be a field of characteristic zero. Let $W_n(\mathbb{F})$ be the Lie algebra of all $\mathbb{F}$-derivations with the Lie bracket $[D_1, D_2] := D_1D_2 - D_2D_1$ on the polynomial ring $\mathbb{F}[x_1, \ldots, x_n]$. The problem of classifying finite dimensional subalgebras of $W_n(\mathbb{F})$ was solved if $n \leq 2$ and $\mathbb{F} = \mathbb{C}$ or $\mathbb{F} = \mathbb{R}$. We prove that this problem is wild if $n \geq 4$, which means that it contains the classical unsolved problem of classifying matrix pairs up to similarity. The structure of finite dimensional subalgebras of $W_n(\mathbb{F})$ is interesting since each derivation in case $\mathbb{F} = \mathbb{R}$ can be considered as a vector field with polynomial coefficients on the manifold $\mathbb{R}^n$.

MSC:

17B66 Lie algebras of vector fields and related (super) algebras
16G60 Representation type (finite, tame, wild, etc.) of associative algebras
34C14 Symmetries, invariants of ordinary differential equations

Keywords:
nilpotent Lie algebra; wild problem; vector field; derivation

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