Constraining the $g_1'$ coupling in the minimal $B - L$ Model

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Abstract

We have combined perturbative unitarity and renormalisation group equation arguments in order to find a dynamical way to constrain the $g_1'$ coupling of the minimal $B - L$ extension of the Standard Model. We have analysed the role of the $g_1'$ coupling evolution in the perturbative stability of the two-to-two body scattering amplitudes of the vector boson and scalar sectors of the model and we have shown that perturbative unitarity imposes an upper bound on the $B - L$ gauge coupling. We have made a comparison between this criterion and the triviality arguments, showing that our procedure substantially refines the triviality bounds.

1 Introduction

Nowadays the phenomenological importance of Beyond the Standard Model (BSM) physics at the TeV scale is recognised by the global experimental effort at the Large Hadron Collider (LHC).

The minimal $B - L$ (baryon minus lepton number) extension of the Standard Model (SM) is considered as one of the candidates in the description of a promising and simple BSM scenario, containing a significant set of particles and interactions whose existence could be proven both at the LHC (see [2, 3]) and future Linear Colliders (LCs) [5].

This model is based on the gauging of the $B - L$ symmetry: one obtains said extension of the SM by augmenting the gauge groups with an additional $U(1)_{B-L}$: $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$.

For consistency, three generations of heavy neutrinos in the fermion sector to cure anomalies and a complex singlet scalar field must be included, the latter giving rise to an extra Higgs boson after the spontaneous symmetry breaking of the new gauge group.
In all generality, the two $U(1)$ gauge groups will mix together, giving raise to a set of two new gauge couplings, $g'_1$ and $\tilde{g}$. While the former appears in the covariant derivative as purely related to the $B-L$ charge, the latter (coupling the new B-L gauge field to the hypercharge) controls the mixing between the two neutral massive gauge bosons at the tree level.

However, from LEP analysis [4] it is known that once a Tera-scale $Z'$ is considered (as it is the case of this paper), the small mixing observed between $Z$ and $Z'$ could be realised only by means of a small $\tilde{g}$ coupling. Hence, as a reasonable approximation, we decided to switch off the $\tilde{g}$ coupling, concentrating on the “pure” $B-L$ model only. This choice allows to perform an analytic analysis, otherwise precluded when full $Z-Z'$ mixing is taken into account.

The parameter space arising from the $B-L$ extension is bounded by both experimental (mainly precision tests at LEP, see [6]) and theoretical arguments. For the latter, a recent set of works (see [7] and [8]) has been devoted to constrain the scalar sector and the $g'_1$ coupling, that is, the only new gauge coupling of the minimal $B-L$ model at the EW scale (since, as intimated, no mixing is allowed between the SM $Z$ and $B-L$ $Z'$ boson at tree-level at such a scale).

The purpose of this paper is to show that the renormalisation group equation (RGE) based techniques as well as the perturbative unitarity criterion can be combined to give a dynamical way to constrain the $g'_1$ coupling.

To this end, we propose a detailed study of the vector boson and Higgs sectors of the model with a view to extract the most stringent bound on the (evolving) $g'_1$ coupling. We will make a comparison between this method and triviality arguments, showing that calling for perturbative unitarity stability allows us to obtain a stronger constraint on $g'_1$ with respect to traditional triviality assumptions.

This work is organised as follows: in section 2 we describe the theoretical methods adopted to constrain the $g'_1$ coupling, in section 3 we present our numerical results while in the last section we give our conclusions.

2 Constraining the $g'_1$ of the minimal $B-L$ model

The model under study is the so-called “pure” or “minimal” $B-L$ model (see [8] for conventions and references) since it has vanishing mixing between the two $U(1)_Y$ and $U(1)_{B-L}$ gauge groups. In the rest of this paper we refer to this model simply as the “$B-L$ model”. In this scenario the classical gauge invariant Lagrangian, obeying the $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ gauge symmetry, can be decomposed as:

$$\mathcal{L} = \mathcal{L}_{YM} + \mathcal{L}_s + \mathcal{L}_f + \mathcal{L}_Y,$$

where $\mathcal{L}_{YM}$, $\mathcal{L}_s$, $\mathcal{L}_f$ and $\mathcal{L}_Y$ are the Yang-Mills, scalar, fermionic and Yukawa sectors, respectively. Since it has been proven that perturbative unitarity violation at high energy
occurs only in vector and Higgs bosons elastic scatterings, our interest is focused on the corresponding sectors.

Following the Becchi-Rouet-Stora (BRS) invariance (see [9]), the amplitude for emission or absorption of a “scalarly” polarised gauge boson becomes equal to the amplitude for emission or absorption of the related would-be-Goldstone boson, and, in the high energy limit ($s \gg m^2_{W^\pm, Z, Z'}$), the amplitude involving the (physical) longitudinal polarisation (the dominant one) of gauge bosons approaches the (unphysical) scalar one, the so-called Equivalence Theorem (ET), see [10]. Therefore, the analysis of the perturbative unitarity of two-to-two particle scatterings in the gauge sector can be performed, in the high energy limit, by exploiting the Goldstone sector. (Further details of this formalism can be found in [7].)

Moreover, since we want to focus on $g'_1$ limits, we assume that the two Higgs bosons of the model have masses such that no significant contribution to the spherical partial wave amplitude will come from the scalar four-point and three-point functions, according to [7]). Such upper value is usually referred as the Lee-Quigg-Tacker (LQT) limit [11] on the Higgs boson mass, evaluated in the ET framework. Taking Higgs boson masses smaller than the LQT limit is therefore a way to exclude any other source of unitarity violation different from the largeness of the $g'_1$ gauge coupling.

In the search for the $g'_1$ upper limits, we will assume that we can neglect the other gauge couplings in the covariant derivative:

$$D_\mu \simeq \partial_\mu + ig'_1 Y^{B-L} Z'_\mu.$$  

(2)

In order to have a consistently gauge invariant theory, in this particular model we must choose $Y^{B-L}_H = 0$ and $Y^{B-L}_\chi = 4$, and this leads us to a relatively small set of Feynman Rules (FRs) for the Higgs and Goldstone sector of the theory.

The scalar Lagrangian and its FRs have been thoroughly studied in [7], where it was shown that the inclusion of $g'_1$ in the covariant derivative gives rise to two new FRs, i.e.

$$Z'h_1 z' = -i Y^{B-L}_\chi g'_1 \sin \alpha (p^\mu_{h_1} - p^\mu_{z'}) ,$$  

(3)

$$Z'h_2 z' = i Y^{B-L}_\chi g'_1 \cos \alpha (p^\mu_{h_2} - p^\mu_{z'}) ,$$  

(4)

where all the momenta are considered incoming and $z'$ is the would-be-Goldstone boson associated with the new $Z'$ gauge field.

Finally, it is important to recall the relation between the $Z'$ mass and the $B-L$ Higgs singlet Vacuum Expectation Value (VEV) $x$, that is,

$$M_{Z'} = Y^{B-L}_\chi g'_1 x .$$  

(5)

\footnote{In other versions of the $B - L$ model this quantum number could change: for example, in [12] one has $Y^{B-L}_{\chi} = 1$.}
Now that the background is set, we focus on the techniques that we have used to obtain the aforementioned unitarity bounds in combination with the RGE analysis.

Firstly, it is crucial to define the evolution of the gauge couplings via the RGEs and their boundary conditions. As already established in [8], the RGEs of $g_1$, $g'_1$ and $\tilde{g}$ are:

$$\frac{d(g_1)}{d(\log \Lambda)} = \frac{1}{16\pi^2} \left[ \frac{41}{6} g_1^3 \right],$$

$$\frac{d(g'_1)}{d(\log \Lambda)} = \frac{1}{16\pi^2} \left[ \frac{32 + (Y_B - L)^2}{3} g_1^3 + 2 \frac{16}{3} g_1^2 \tilde{g} + \frac{41}{6} g'_1 \tilde{g}^2 \right],$$

$$\frac{d(\tilde{g})}{d(\log \Lambda)} = \frac{1}{16\pi^2} \left[ \frac{41}{6} \tilde{g}^2 \right] + \frac{32 + (Y_B - L)^2}{3} g'_1 \tilde{g}^2 \right],$$

where $g_1(EW) \simeq 0.36$ and $\tilde{g}(EW) = 0$ (in the minimal $B-L$ model there is no mixing at the EW scale). This fully fixes the evolution of $g'_1$ with the scale.

In the search for the maximum $g'_1(EW)$ allowed by theoretical constraints, the contour condition

$$g'_1(\Lambda) \leq k,$$

also known as the triviality condition, is the assumption that enables to solve the system of eqs. and gives the traditional upper bound on $g'_1$ at the EW scale.

It is usually assumed either $k = 1$ or $k = \sqrt{\frac{4}{\pi}}$, calling for a coupling that preserves the perturbative convergence of the theory. Nevertheless, we stress again that this is an “ad hoc” assumption. Our aim, instead, is to extract the boundary condition by perturbative unitarity arguments, showing that under certain conditions it represents a stronger constraint on the domain of $g'_1$.

For this, we exploit the theoretical techniques that are related with the perturbative unitarity analysis, since they can be used to provide constraints on the theory, with a procedure that is not far from the one firstly described in detail by [11].

The well known result is that, by evaluating the tree-level scattering amplitude of longitudinally polarised vector bosons $V_L$ ($V = W^\pm, Z, Z'$), in the limit $m_V^2 \ll s$, by substituting each one of them with the related Goldstone boson $v = w^\pm, z, z'$, and its general validity has been proven in [10]. Schematically, if we consider a process with four longitudinal vector bosons, we have that $M(V_L V_L \rightarrow V_L V_L) = M(\nu \nu \rightarrow \nu \nu) + O(m_V^2/s)$.
While in the search for the Higgs boson mass bound it is widely accepted to assume small values for the gauge couplings and large Higgs boson masses, for our purpose we reverse such argument with the same logic: we assume that the Higgs boson masses are compatible with the unitarity limits and we study the two-to-two scattering amplitudes of the whole scalar sector, pushing the largeness of $g'_1$ to the perturbative limit.

This limit is a consequence of the following argument: given a tree-level scattering amplitude between two spin-0 particles, $M(s, \theta)$, where $\theta$ is the scattering (polar) angle, we know that the partial wave amplitude with angular momentum $J$ is given by

$$a_J = \frac{1}{32\pi} \int_{-1}^{1} d(\cos \theta) P_J(\cos \theta) M(s, \theta),$$

where $P_J$ are the Legendre polynomials, and it has been proven (see [13]) that, in order to preserve unitarity, each partial wave must be bounded by the condition

$$|\text{Re}(a_J(s))| \leq \frac{1}{2}.$$  

By direct computation, it turns out that only $J = 0$ (corresponding to the spherical partial wave contribution) leads to some bound, so we will not discuss the higher partial waves any further.

Assuming that the Higgs boson masses do not play any role in the perturbative unitarity violation ($m_{h_{1,2}} < 700$ GeV, according to the LQT limit), we have proven that the only divergent contribution to the spherical amplitude is due to the size of the coupling $g'_1$ in the intermediate $Z'$ vector boson exchange contributions. Hence, the only relevant channels are: $z'z' \to h_1h_1$, $z'z' \to h_1h_2$, $z'z' \to h_2h_2$.

As an example, we evaluate the $a_0$ partial wave amplitude for $z'z' \to h_1h_1$ scattering in the $s \gg M_{Z'}, m_{h_{1,2}}$ limit.

Firstly, we know that

$$M(s, \cos \theta) \simeq (Y_{X}^{B-L} g'_1 \sin \alpha)^2 \left(1 - \frac{4s}{s \cos \theta + 2 M_{Z'}^2 \cos \theta}\right),$$

by the integration proposed in equation (8), we then extract the $J = 0$ partial wave:

$$a_0(z'z' \to h_1h_1) = \frac{(Y_{X}^{B-L} g'_1)^2}{16\pi} \left(1 + 2 \log \left(\frac{M_{Z'}^2}{s}\right)\right) \sin^2 \alpha.$$  

It is important to notice that the mass of the $Z'$ acts as a natural regularisor that preserves both the amplitude and the spherical partial wave from any collinear divergence.

Considering the three aforementioned scattering channels, their spherical partial wave (in the high energy limit $s \gg M_{Z'}, m_{h_{1,2}}$) is represented by the following matrix:

$$a_0 = f(g'_1, s; Y_{X}^{B-L}, x) \begin{pmatrix}
0 & \frac{1}{2} \sin^2 \alpha & \frac{1}{2} \sin \alpha \cos \alpha & \frac{1}{2} \cos^2 \alpha \\
\frac{1}{2} \sin^2 \alpha & 0 & 0 & 0 \\
\frac{1}{2} \sin \alpha \cos \alpha & 0 & 0 & 0 \\
\frac{1}{2} \cos^2 \alpha & 0 & 0 & 0
\end{pmatrix},$$

(12)
where, according to equation (5),

\[
f(g'_1, s; Y^B_L, x) = \frac{(Y^B_L g'_1)^2}{16 \pi} \left( 1 + 2 \log \left( \frac{(Y^B_L g'_1 x)^2}{s} \right) \right),
\]

and the elements of the matrix are related to the four channels system consisting of \( \frac{1}{\sqrt{2}} z' z' \), \( \frac{1}{\sqrt{2}} h_1 h_1 \), \( h_1 h_2 \), \( \frac{1}{\sqrt{2}} h_2 h_2 \).

The most stringent unitarity bound on the \( g'_1 \) coupling is derived from the requirement that the magnitude of the largest eigenvalue combined with the function \( f(g'_1, s; Y^B_L, x) \) does not exceed \( 1/2 \).

If we diagonalise the matrix in equation (12), we find that the maximum eigenvalue and the corresponding eigenvector are:

\[
\frac{1}{2} \Rightarrow \frac{1}{2} \left( z' z' + h_1 h_1 \sin^2 \alpha - h_1 h_2 \sin (2\alpha) + h_2 h_2 \cos^2 \alpha \right).
\]

(14)

Combining the informations of equations (13)-(14), together with the perturbative unitarity condition in equation (9), we obtain:

\[
|\text{Re}(a_0)| = \left| \frac{(Y^B_L g'_1)^2}{32 \pi} \right| 1 + 2 \log \left( \frac{(Y^B_L g'_1 x)^2}{s} \right) \leq \frac{1}{2}.
\]

(15)

In the last equation, \( s \) represents the scale of energy squared at which the scattering is consistent with perturbative unitarity, i.e. \( s = \Lambda^2 \), where \( \Lambda \) is the evolution energy scale cut-off.

Finally, if we consider the contour of this inequality, we find exactly the boundary condition that solve the set of differential equations in (6), giving us the upper limit for \( g'_1 \) at the EW scale. In the next section we will combine all these elements to present a numerical analysis of the allowed domain of \( g'_1 \).

3 Results

The set of differential eqs. (6) has been evaluated with the well-known Runge-Kutta algorithms and the unitarity condition has been imposed as a two-point boundary value with a simple shooting method, that consisted in varying the initial conditions in dichotomous-converging steps until the unitarity bound was fulfilled.

Moreover, in order to make a fruitful comparison with the ordinary triviality arguments, we have evaluated the evolution of \( g'_1 \) with the two boundary conditions, equations (7)-(15), for several values of \( x \), the Higgs singlet VEV, and two choices of the \( B - L \) charge of the \( \chi \) field, corresponding to the basic model \( (Y^B_L = 2) \) and the so-called “inverse see-saw” version \( (Y^B_L = 1) \) proposed in [12]: the results are plotted in Figure (1).
Figure 1: Triviality (assuming $k = 1$, dotted-dashed line) and unitarity (continuous and dashed lines) limits on the $g'_1$ coupling of the minimal $B - L$ model, plotted against the energy cut-off $\Lambda$ in log$_{10}$-scale, for several choices of the singlet VEV ($x = 3.5$ TeV, black lines; $x = 10$ TeV, red (dark grey) lines; $x = 35$ TeV, green (light grey) lines) and the $B - L$ charge of the $\chi$ field ($Y_{B-L}^{\chi} = 2$, continuous-line; $Y_{B-L}^{\chi} = 1$, dashed-line).

In the first place, we verified that the choice of $Y_{\chi}^{B-L}$ (i.e. the choice of model) does not significantly affect the triviality bounds, so we display them for the case $Y_{\chi}^{B-L} = 2$ only.
By direct comparison of the two formulae, it is easy to see that the unitarity bounds become more important than the triviality bounds when

\[
\frac{\Lambda}{x} \simeq \exp \left( \frac{16\pi + (kY_{\chi}^{B-L})^2}{(2kY_{\chi}^{B-L})^2} \right),
\]

with the assumption that \( M_{Z'} \sim x \).

From this equation, it is straightforward to see that the choice of both the \( B - L \) charge of the \( \chi \) field and the “ad hoc” triviality parameter \( k \) is crucial for establishing which limit is the most stringent one.

In the basic version of the model, the choice \( Y_{\chi}^{B-L} = 2 \) is necessary to preserve the gauge invariance and we also embrace the widely accepted assumption \( k = 1 \) as triviality condition. If we then choose a value of the VEV \( x \) that is compatible with the experimental limits and still in the TeV range, \( x \in [3.5, 35] \) TeV according to [6], we find that the unitarity bounds are more stringent than the triviality ones when the energy scale is greater than a critical value of \( \Lambda_c \simeq 10^6 \) GeV, and this is consistent with the results in Figure (1). In a different version of the \( B - L \) model, for example the “inverse see-saw” one [12], where \( Y_{\chi}^{B-L} = 1 \), we find that \( \Lambda_c = 10^9 - 10^{10} \) GeV, and this is again confirmed by the plot in Figure (1).

In order to summarise these results, in Table (1) we present a comparison between the triviality and the unitarity bounds for several energy scales and \( B - L \)-breaking VEVs \( x \).

| \( \text{Log}_{10}(\Lambda/\text{GeV}) \) | 7   | 10  | 15  | 19  |
|---------------------------------|-----|-----|-----|-----|
| TB, \( g'_1(k = 1) \)          | 0.595 | 0.501 | 0.407 | 0.357 |
| UB, \( g'_1(x = 3.5 \text{ TeV}) \) | 0.487 | 0.360 | 0.269 | 0.230 |
| UB, \( g'_1(x = 10 \text{ TeV}) \) | 0.510 | 0.368 | 0.273 | 0.232 |
| UB, \( g'_1(x = 35 \text{ TeV}) \) | 0.542 | 0.379 | 0.277 | 0.234 |

Table 1: Triviality bounds, equation (7) with \( k = 1 \), and unitarity bounds, equation (15) with \( x = 3.5, 10, 35 \) TeV, for \( g'_1 \) in the standard \( B - L \) model for several values of the energy scale \( \Lambda \).

Though these results are scale dependent, we see that, if \( \Lambda \gg \Lambda_c \), our method basically refines the triviality bound by an absolute value of \( \simeq 0.1 \), that represents a correction of (at least) 20% on the results that have recently appeared in the literature (see [8]).

4 Conclusions

In this paper, we have shown that, by combining perturbative unitarity and RGE methods, one can significantly constrain the \( g'_1 \) coupling of the minimal \( B - L \) extension of the SM, by imposing limits on its upper value that are more stringent than standard triviality...
bounds. (Also notice that, as unitarity is more constraining than triviality, the stability of the perturbative solution obtained through the former is already guaranteed by the latter.)

We presented a full set of analytical results, plus a significative comparison between the type-I see-saw and the "inverse" see-saw (neutrino mass generation mechanisms) implementation, that turned out to be analytically accessible in the minimal \( B - L \) (i.e., no mixing between \( Z_{SM} \) and \( Z_{B-L} \) at tree-level) extension of the \( SM \).

Finally, we have verified by direct computation that even if a reasonably small \( \bar{g} \) (e.g., \( |\bar{g}| < 0.05 \)) is switched on, our conclusions are unchanged.

The present work, alongside Refs. [7] and [8], enables one to ultimately define the combined experimental and theoretical limits on the Higgs sector of the minimal \( B - L \) model, in view of its exploration at present and future colliders [13].

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