CP Asymmetries in $B$ Decays with New Physics in Decay Amplitudes

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Abstract

We make a systematic analysis of the effects of new physics in the $B$ decay amplitudes on the $CP$ asymmetries in neutral $B$ decays. Although these are expected to be smaller than new physics effects on the mixing amplitude, they are easier to probe in some cases. The effects of new contributions to the mixing amplitude are felt universally across all decay modes, whereas the effects of new decay amplitudes could vary from mode to mode. In particular the prediction that the $CP$ asymmetries in the $B_d$ decay modes with $b \to c\bar{c}s$, $b \to c\bar{c}d$, $b \to c\bar{u}d$ and $b \to s\bar{s}s$ should all measure the same quantity ($\sin 2\beta$ in the Standard Model) could be violated. Since the above Standard Model prediction is very precise, new decay amplitudes which are a few percent of the Standard Model amplitudes can be probed. Three examples of models where measurable effects are allowed are given: effective supersymmetry, models with enhanced chromomagnetic dipole operators, and supersymmetry without $R$ parity.

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I. INTRODUCTION

CP violation has so far only been observed in the decays of neutral K mesons. It is one of the goals of the proposed B factories to find and study CP violation in the decays of B mesons, and thus elucidate the mechanisms by which CP violation manifests itself in the low energy world. There is a commonly accepted Standard Model of CP violation, namely that it is a result of the one physical phase in the $3 \times 3$ Cabbibo Kobayashi Maskawa (CKM) matrix $[1]$. This scenario has specific predictions for the magnitude as well as patterns of CP violation that will be observed in the B meson decays $[2]$. However, since there currently exists only one experimental measurement of CP violation, it is possible that the Standard Model explanation for it is incorrect, or more likely that in addition to the one CKM phase, there are additional CP violating phases introduced by whatever new physics lies beyond the Standard Model.

In the limit of one dominant decay amplitude, the CP violating asymmetries measured in the time dependent decays of neutral B mesons to CP eigenstates depend only on the sum of the phase of the $B^0 - \bar{B}^0$ mixing amplitude and the phase of the decay amplitude. Although the CKM matrix could have up to five large phases (only one of which is independent), we know experimentally that only two of these are large. This is manifest in the Wolfenstein $[3]$ parameterization where to leading order these phases are in the two CKM matrix elements $V_{ub} (\gamma)$ and $V_{td} (\beta)$. In principle, one can determine $\beta$ and $\gamma$ from the available data on K and B decays. However, given the large theoretical uncertainties in the input parameters ($e.g.$ $B_K$, $f_B$) the size of these phases remains uncertain $[1,3]$. Thus, the currently allowed range for the CP asymmetries measurements in $B_d$ decays is very large. Based on these facts the only precise predictions concerning the CP asymmetries made by the Standard Model are the following:

(i) The CP asymmetries in all $B_d$ decays that do not involve direct $b \rightarrow u$ (or $b \rightarrow d$) transitions have to be the same.

This prediction holds for the $B_s$ system in an even stronger form.
(ii) The $CP$ asymmetries in all $B_s$ decays that do not involve direct $b \to u$ (or $b \to d$) transition not only have to be the same, but also approximately vanish.

Thus, the best place to look for evidence of new $CP$ violating physics is obviously the $B_s$ system \cite{6,7}. The $B$ factories, however, will initially take data at the $\Upsilon(4s)$ where only the $B_d$ can be studied.

New physics could in principle contribute to both the mixing matrix and to the decay amplitudes. It is plausible that the new contributions to the mixing could be of the same size as the Standard Model contribution since it is already a one-loop effect. This is why most of the existing studies on the effects of new physics on $CP$ violating $B$ meson decays have concentrated on effects in the mixing matrix, and assume the decay amplitudes are those in the Standard Model \cite{2,8,9} (in \cite{9} a more general analysis was done where they allow for new contributions to the penguin dominated Standard Model decay amplitudes). The distinguishing feature of new physics in mixing matrices is that its effect is universal, \textit{i.e.} although it changes the magnitude of the asymmetries it does not change the patterns predicted by the Standard Model. Thus, the best way to search for these effects would be to compare the observed $CP$ asymmetry in a particular decay mode with the asymmetry predicted in the Standard Model. This is straightforward for the leading $B_s$ decay modes where the Standard Model predicts vanishing $CP$ asymmetries. However, due to the large uncertainties in the Standard Model predictions for the $B_d$ decays, these new effects would have to be large in order for us to distinguish them from the Standard Model. A slightly more sensitive analysis involves looking for inconsistencies between the measured angles and sides of the unitarity triangle \cite{10,11}. In any case, the Standard Model prediction $(i)$ concerning $B_d$ decays still holds.

In contrast, the effects of new physics in decay amplitudes are manifestly non-universal, \textit{i.e.} they depend on the specific process and decay channel under consideration. Experiments on different decay modes that would measure the same $CP$ violating quantity in the absence of new contributions to decay amplitudes, now actually measure different $CP$ violating
quantities. Thus, the Standard Model prediction \( i \), concerning \( B_d \) decays, can be violated.

Even though the possibility of new physics in decay amplitudes is more constrained than that in mixing amplitudes, one could detect these smaller effects by exploiting the fact that now one does not care about the predicted value for some quantity, only that two experiments that should measure the same quantity, in fact, do not. It is this possibility that we wish to study in this paper.

The outline of the paper is as follows: in Sec. II we first discuss the general effects that new physics in decay amplitudes can have. We then undertake a detailed discussion of each possible decay channel, and the uncertainties in the universality predictions introduced within the Standard Model itself by sub leading effects. Sec. III contains a brief study of models of new physics that could contain new \( CP \) violating decay amplitudes, and their expected size. We present our conclusions in Sec. IV.

II. THE EFFECTS OF NEW DECAY AMPLITUDES

A. General Effects

In this sub-section we display the well known formulae for the decays of neutral \( B \) mesons into \( CP \) eigenstates [3], and highlight the relevant features that are important when more than one decay amplitude contribute to a particular process.

The time dependent \( CP \) asymmetry for the decays of states that were tagged as pure \( B^0 \) or \( \bar{B}^0 \) at production into \( CP \) eigenstates is defined as

\[
a_{fCP}(t) = \frac{\Gamma[B^0(t) \rightarrow f_{CP}] - \Gamma[\bar{B}^0(t) \rightarrow f_{CP}]}{\Gamma[B^0(t) \rightarrow f_{CP}] + \Gamma[\bar{B}^0(t) \rightarrow f_{CP}]},
\]

(2.1)

and given by

\[
a_{fCP}(t) = \frac{(1 - |\lambda|^2) \cos(\Delta M t) - 2I m \lambda \sin(\Delta M t)}{1 + |\lambda|^2},
\]

(2.2)

where \( \Delta M \) is the mass difference between the two physical states, and
\[
\lambda = \left( \frac{M_{12}^* - \frac{i}{2} \Gamma_{12}^*}{M_{12} - \frac{i}{2} \Gamma_{12}} \right) \frac{\langle f_{\text{CP}} | H | \bar{B}^0 \rangle}{\langle f_{\text{CP}} | H | B^0 \rangle} = e^{-2i\phi_M} \frac{\bar{A}}{A},
\]

(2.3)

where we have used the fact that \( M_{12} \gg \Gamma_{12} \), to replace the first fraction in Eq. (2.3) by \( e^{-2i\phi_M} \), the phase of \( B - \bar{B} \) mixing.

We now consider the case where the decay amplitude \( A \) contains contributions from two terms with magnitudes \( A_i \), \( CP \) violating phases \( \phi_i \) and \( CP \) conserving phases \( \delta_i \) (in what follows it will be convenient to think of \( A_1 \) giving the dominant Standard Model contribution, and \( A_2 \) giving the sub leading Standard Model contribution or the new physics contribution).

\[
A = A_1 e^{i\phi_1} e^{i\delta_1} + A_2 e^{i\phi_2} e^{i\delta_2}, \quad \bar{A} = A_1 e^{-i\phi_1} e^{i\delta_1} + A_2 e^{-i\phi_2} e^{i\delta_2}.
\]

(2.4)

To first order in \( r \equiv A_2/A_1 \) Eq. (2.2) reduces to \([12]\)

\[
a_{\text{FCP}}(t) = -[2r \sin(\phi_{12}) \sin(\delta_{12})] \cos(\Delta M t)
- [\sin 2(\phi_M + \phi_1) + 2r \cos 2(\phi_M + \phi_1) \sin(\phi_{12}) \cos(\delta_{12})] \sin(\Delta M t),
\]

(2.5)

and we have defined \( \phi_{12} = \phi_1 - \phi_2 \) and \( \delta_{12} = \delta_1 - \delta_2 \).

In the case \( r = 0 \) or \( \phi_{12} = 0 \) one recovers the frequently studied case where \( a_{\text{FCP}} \) cleanly measures the \( CP \) violating quantity \( \sin 2(\phi_M + \phi_1) \). In addition, if there is no new physics contribution to the mixing matrix (or if it is in phase with the Standard Model contribution), \( a_{\text{FCP}} \) cleanly measures \( CP \) violating phases in the CKM matrix.

If \( r \neq 0 \) and \( \phi_{12} \neq 0 \) we can consider 3 distinct scenarios:

(a) Direct \( CP \) violation. This occurs when \( \delta_{12} \neq 0 \) and can be measured by a careful study of the time dependence since it gives rise to a \( \cos \Delta M t \) term in addition to the \( \sin \Delta M t \) term. Such a scenario would also give rise to \( CP \) asymmetries in charged \( B \) decays.

(b) Different hadronic final states even with the same quark content could get different relative corrections, \( i.e., \) two different processes with the same \( \phi_1 \) and \( \phi_2 \), but different \( r \). For example the decays \( B_d \to D^+D^- \) and \( B_d \to \psi\rho \) both go through the same quark level process \( b \to c\bar{c}d \) and at leading order the \( CP \) asymmetries both measure the same angle \( \beta \) (we have assumed that a transversality analysis allows us to treat \( \psi\rho \) as a \( CP \) eigenstate.
However the relative correction due to the Standard Model penguins themselves is expected to be different for the two cases since the matrix elements are different. Effects of this kind are hard to estimate, and we will not study them further.

(c) Different quark level decay channels that measure the same phase when only one amplitude contributes, can measure different phases if more than one amplitude contributes, i.e. two different processes with the same $\phi_1$, but with different $r$ or $\phi_2$.

Case (a) demands a non-vanishing strong phase difference which is hard to estimate. In order to get a valuable information from Case (b) we need better theoretical understanding of hadronic matrix elements. Thus, we feel that case (c) is the most promising way to search for new physics effects in decay amplitudes, and we concentrate on it for the rest of the paper. To this end we concentrate on the $\sin \Delta M t$ term in Eq. (2.5) rewriting it as

$$ a_{CP}(t) = -\sin 2(\phi_0 + \delta \phi) \sin \Delta M t, \quad (2.6) $$

where $\phi_0$ is the phase predicted at leading order in the Standard Model, and $\delta \phi$ is the correction to it. For small $r$, $\delta \phi \leq r$. However for $r > 1$, $\delta \phi$ can take any value. Thus, when we have $\delta \phi \geq 1$ it should be understood that its value is arbitrary.

**B. The Different Decay Channels**

There are 12 different hadronic decay channels for the $b$ quark: 8 of them are charged current mediated

$$ (c1) \ b \rightarrow c\bar{c}s, \quad (c2) \ b \rightarrow c\bar{c}d, \quad (c3) \ b \rightarrow c\bar{u}d, \quad (c4) \ b \rightarrow c\bar{u}s, \quad (c5) \ b \rightarrow u\bar{c}d, \quad (c6) \ b \rightarrow u\bar{c}s, \quad (c7) \ b \rightarrow u\bar{u}d, \quad (c8) \ b \rightarrow u\bar{u}s, \quad (2.7) $$
and 4 are neutral current

$$ (n1) \ b \rightarrow s\bar{s}s, \quad (n2) \ b \rightarrow s\bar{s}d, \quad (n3) \ b \rightarrow s\bar{d}d, \quad (n4) \ b \rightarrow d\bar{d}d. \quad (2.8) $$

If only one Standard Model decay amplitude dominates all of these decay channels, i.e. $r = 0$ in Eq. (2.5), then up to $O(\lambda^2)$ (where $\lambda \approx 0.22$ is the expansion parameter in the
Wolfenstein approximation), the \( CP \) asymmetries in \( B \) meson decays all measure one of the 4 phases,

\[
\begin{align*}
\alpha & \equiv \arg \left( -\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*} \right), & \beta & \equiv \arg \left( -\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*} \right), \\
\gamma & \equiv \arg \left( -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right), & \beta' & \equiv \arg \left( -\frac{V_{cs}V_{cb}^*}{V_{ts}V_{tb}^*} \right) \simeq 0. 
\end{align*}
\]

This situation is nicely summarized, along with relevant decay modes in Table 1 of [14].

Note that \( \beta' < 2.5 \times 10^{-2} \) is very small in the SM [5], but in principle measurable. For our purpose, however, this small value is a sub-leading correction to the clean SM prediction \((ii)\). We will study corrections to this idealized limit, as well as to the \( r = 0 \) limit, in the next sub-section. We now discuss the effects that new physics in \( b \) quark decay amplitudes could have on the predictions of Eq. (2.9).

In the Standard Model the \( CP \) asymmetries in the decay modes \((c1)\) \( b \to c \bar{c} s \) (e.g. \( B_d \to \psi K_S, B_s \to D_s^+D_s^- \)), \((c2)\) \( b \to c \bar{c} d \) (e.g. \( B_d \to D^+D^-, B_s \to \psi K_S \)), and \((c3)\) \( b \to c \bar{u} d \) (e.g. \( B_d \to D_0^{CP}\rho, B_s \to D_0^{CP}K_S \)) all measure the angle \( \beta \) in \( B_d \) decay and \( \beta' \) in \( B_s \) decays. \[(c5) b \to u \bar{c} d \) acts as a correction to \((c3)\) and will be addressed later. In the presence of new contributions to the \( B - \bar{B} \) mixing matrix, the \( CP \) asymmetries in these modes would no longer be measuring the CKM angles \( \beta \) and \( \beta' \). However, they would all still measure the same angles \((\beta + \delta_{md}, \beta' + \delta_{ms})\), where \((\delta_{md}, \delta_{ms})\) are the new contributions to the \( B_{(d,s)} - \bar{B}_{(d,s)} \) mixing phase. In contrast, new contributions to the \( b \) quark decay amplitudes could affect each of these modes differently, and thus they would each be measuring different \( CP \) violating quantities.

Several methods [13] have been proposed based on the fact that the two amplitudes \((c4) \ b \to c \bar{u} s \) and \((c6) \ b \to u \bar{c} s \) (e.g. \( B_d \to D_{CP}K_S, B_s \to D_{CP}\phi \)) are comparable in size, and contribute dominantly to the \( D^0 \) or \( \bar{D}^0 \) parts of \( D_{CP} \) respectively to extract the quantity

\[
\arg(b \to c \bar{u} s) + \arg(c \to d \bar{d} u) - \arg(b \to u \bar{c} s) - \arg(\bar{c} \to \bar{d} \bar{d} u) \equiv \gamma 
\]

(2.10)
This measurement of $\gamma$ is manifestly independent of the $B - \bar{B}$ mixing phase.\footnote{We emphasize that $CP$ asymmetries into final states that contain $D_{CP}$ cannot be affected by possible new contributions to $D - \bar{D}$ mixing. One identifies $D_{CP}$ by looking for $CP$ eigenstate decay products like $K^+K^-$, $\pi\pi$ or $\pi K_S$. As $(\Delta\Gamma/\Gamma)_D$ is known to be tiny, the mass eigenstates cannot be identified. The relevant quantity that enters in the calculation of the $CP$ asymmetry is the $D$ meson decay amplitude and not the $D - \bar{D}$ mixing amplitude. Thus, the only new physics in the $D$ sector that could affect the standard analysis are new contributions to the $D$ decay amplitudes.}

The mode $(c7)\ b \to u\bar{u}d$ (e.g. $B_d \to \pi\pi$, $B_s \to \rho K_s$) measures the angles $(\beta + \gamma, \beta' + \gamma)$ in the Standard Model. We can combine this measurement, with the phase $(\beta, \beta')$ measured in the $(c1)\ b \to c\bar{c}s$ mode to get another determination of $\gamma$ that is independent of the phase in the $B - \bar{B}$ mixing matrix e.g. comparing $a_{CP}(t)[B_d \to \psi K_S]$ to $a_{CP}(t)[B_d \to \pi\pi]$ allows us to extract

$$\arg(b \to c\bar{c}d) - \arg(b \to u\bar{u}d) \equiv \gamma.$$ \hfill (2.11)

Since both of the above evaluations of $\gamma$, Eqs. (2.10) and (2.11) are manifestly independent of any phases in the neutral meson mixing matrices, the only way they can differ is if there are new contributions to the $B$ or $D$ meson decay amplitudes.

The remaining charged current decay mode $(c8)\ b \to u\bar{u}s$ suffers from large theoretical uncertainty since the tree and penguin contributions are similar in magnitude and we will not study it here.

For the neutral current modes we will first assume that the dominant Standard Model contribution is from a penguin diagram with a top quark in the loop, and discuss corrections to this later. Since these are loop mediated processes even in the Standard Model, $CP$ asymmetries into final states that can only be produced by flavor changing neutral current vertices are likely to be fairly sensitive to the possibility of new physics in the $B$ meson decay amplitudes. The modes $(n3)\ b \to s\bar{d}d$ and $(n4)\ b \to d\bar{d}d$ however, result in $CP$ eigenstate final states that are the same as for the charged current modes $(c8)\ b \to u\bar{u}s$ and...
(c7) $b \to u\bar{u}d$ respectively. Hence they cannot be used to study $CP$ violation, but rather act as corrections to the charged current modes.

In the Standard Model the mode $(n1) b \to s\bar{s}s$, (e.g. $B_d \to \phi K_S$, $B_s \to \phi\eta'$) measures the angle $\beta$ or $0$ in $B_d$ and $B_s$ decays. We can once again try and isolate new physics in the decay amplitudes by comparing these measurements with the charged current measurements of $\beta$. Finally, $(n2) b \to d\bar{s}s$, e.g. $(B_d \to K_S K_S$, $B_s \to \phi K_S$) measures the angle $0$ and $\beta$ for Standard Model $B_d$ and $B_s$ decays.

C. Standard Model Pollution

In all of the preceding discussion, we have considered the idealized case where only one Standard Model amplitude contributes to a particular decay process and we worked to first order in the Wolfenstein approximation. We would now like to estimate the size of the sub-leading Standard Model corrections to the above processes, which then allows us to quantify how large the new physics effects have to be in order for them to be probed, and what are the most promising modes to study.

There is a Standard Model penguin contribution to $(c1) b \to c\bar{c}s$. However, as is well known, this contribution has the same phase as the tree level contribution (up to corrections of order $\beta'$) and hence $\delta\phi = 0$ in Eq. (2.6). Thus in the absence of new contributions to decay amplitudes, the decay $B_d \to \psi K_S$ cleanly measures the phase $\beta + \delta m_d$ (where $\delta m_d$ denotes any new contribution to the mixing phase). The mode $(c2) b \to c\bar{c}d$ also has a penguin correction in the Standard Model. However, in this case $\phi_{12} = \mathcal{O}(1)$ and we estimate the correction as

$$
\delta\phi_{SM}(b \to c\bar{c}d) \simeq \frac{V_{tb}V_{td}^*}{V_{cb}V_{cd}^*} \frac{\alpha_s(m_b)}{12\pi} \log\left(\frac{m_b^2}{m_t^2}\right) \lesssim 0.1,
$$

(2.12)

where the upper bound is obtained for $|V_{td}| < 0.02$, $m_t = 180$ GeV and $\alpha_s(m_b) = 0.2$. The mode $(c3) b \to c\bar{u}d$ does not get penguin corrections, however there is a doubly Cabibbo suppressed tree level correction coming from $(c5) b \to u\bar{c}d$. Thus $B_d \to D_{CP}\rho$ gets a second
contribution with different CKM elements. While in general $\delta\phi$ can be a function of hadronic matrix elements, here we expect this dependence to be very weak [17]. In the factorization approximation, the matrix elements of the leading and sub-leading amplitude are identical, as are the final state rescattering effects. Moreover, both these cases get contributions from only one electroweak diagram, thus reducing the possibility of complicated interference patterns. We then estimate

$$\delta\phi_{SM}(b \to c\bar{u}d) = \frac{V_{ub}V_{cd}^*}{V_{cb}V_{ud}^*} r_{FA} \leq 0.05. \quad (2.13)$$

where $r_{FA}$ is the ratio of matrix elements with $r_{FA} = 1$ in the factorization approximation. We have used $|V_{ub}/V_{cb}| < 0.11$, and used what we believe is a reasonable limit for the matrix elements ratio, $r_{FA} < 2$, to obtain the upper bound.

The technique proposed to extract $\gamma$ using the modes $(c4) b \to c\bar{u}s$ and $(c6) b \to u\bar{c}s$ is manifestly independent of any “Standard Model pollution”. Finally $(c7) b \to u\bar{d}d$ suffers from significant Standard Model penguin pollution, which we estimate as [16,11]

$$\delta\phi_{SM}(b \to u\bar{d}d) \approx \frac{V_{tb}V_{td}^* \alpha_s(m_b)}{12\pi} \log(m_t^2/m_b^2) \lesssim 0.4, \quad (2.14)$$

where the upper bound is for $|V_{td}| < 0.02$, $|V_{ub}| > 0.002$, $m_t = 180$ GeV and $\alpha_s(m_b) = 0.2$. The effects of the Standard Model penguin can be removed by an isospin analysis [18]. However, this technique would then also rotate away any new physics contributions to the gluonic penguin operator.

For the neutral current modes $(n1) b \to s\bar{s}s$ the sub-leading Standard Model contribution is in phase with the dominant contribution. However, in the absence of new decay amplitudes, the $CP$ asymmetry in $B_d \to \phi K_S$ will measure the angle $\beta - \beta' + \delta m_d$ and, $\delta\phi_{SM} = \beta' \leq 0.025$. Another source of uncertainty comes from $SU(3)_{flavor}$ mixing. The $\phi$ also contains a small part of $u\bar{u}$, and thus $B_d \to \phi K_S$ can also be mediated via the tree level $b \to u\bar{d}s$ decay that has a different weak phase than the leading penguin diagram. From the data [13] we can conservatively estimate that this extra uncertainty is about 1%. Combining these two sources of uncertainty we conclude
\[ \delta \phi_{SM}(b \to s\bar{s}s) \leq 0.04. \] 

(2.15)

This uncertainty can be reduced once \( \beta' \) is measured, using e.g. \( B_s \to D_s^+D_s^- \).

Finally, \( n2 \) \( b \to d\bar{s}s \) suffers from an \( \mathcal{O}(30\%) \) correction due to Standard Model penguins with up and charm quarks \[20\].

In summary, the cleanest modes are \( b \to c\bar{c}s \) and \( b \to c\bar{u}s \) since they are essentially free of any sub-leading effects. The modes \( b \to c\bar{u}d \) and \( b \to s\bar{s}s \) suffer only small theoretical uncertainty, less than 0.05. For \( b \to c\bar{c}d \) the uncertainty is larger, \( \mathcal{O}(0.1) \), and moreover cannot be estimated reliably since it depends on the ratio of tree and penguin matrix elements. Finally, the \( b \to u\bar{u}d \) and \( b \to d\bar{s}s \) modes suffer from large uncertainties.

III. MODELS

In this section we discuss three models that could have experimentally detectable effects on \( B \) meson decay amplitudes, and violate the Standard Model predictions (i) and (ii). We also discuss ways to distinguish these models from each other.

(a) Effective Supersymmetry: This is a supersymmetric extension of the Standard Model that seeks to retain the naturalness properties of supersymmetric theories, while avoiding the use of family symmetries or ad-hoc supersymmetry breaking boundary conditions that are required to solve the flavor problems generic to these models \[21,22\]. In this model, the \( \tilde{t}_L, \tilde{b}_L, \tilde{t}_R \) and the gauginos are light (below 1 TeV), while the rest of the super-partners are heavy (\( \sim 20 \) TeV). The bounds on the squark mixing angles in this model can be found in \[23\]. Using the formulae in \[24\] we find that for \( \tilde{b}_L \) and gluino masses in the \( 100 - 300 \) GeV range, this model generates \( b \to sq\bar{q} \) and \( b \to dq\bar{q} \) transition amplitudes via gluonic penguins that could be up to twice as large as the Standard Model gluonic penguins, and with an unknown phase. Thus this model could result in significant deviations from the predicted patterns of \( CP \) violation in the Standard Model. We estimate these corrections to be

\[ \delta \phi_A(b \to c\bar{c}s) \lesssim 0.1, \quad \delta \phi_A(b \to c\bar{c}d) \lesssim 0.2, \quad \delta \phi_A(b \to u\bar{u}d) \lesssim 0.8, \]
(b) Models with Enhanced Chromomagnetic Dipole Operators: These models have been proposed to explain the discrepancies between the $B$ semi-leptonic branching ratio, the charm multiplicity in $B$ decays and the Standard Model prediction for these quantities. These enhanced chromomagnetic dipole operators come from gluonic penguins that arise naturally in TeV scale models of flavor physics \[25\]. In order to explain the above discrepancies with the Standard Model, these models have amplitudes for $b \to sg$ that are about 7 times larger than the Standard Model amplitude. The $b \to sq\bar{q}$ transition in this model is dominated by the dipole operator for $b \to sg$ through the chain $b \to sg^* \to sq\bar{q}$. This interferes with the Standard Model $b \to sq\bar{q}$ amplitude. For the $B \to X_s \phi$ the net result is that the new amplitudes can be up to a factor of two larger than the Standard Model penguins and with arbitrary phase \[20\]. It is thus plausible that similar enhancements can be present in the exclusive $b \to c\bar{c}s$ transitions as well. In addition, $b \to dg$ can be as large as $b \to sg$. However in the Standard Model the $b \to d$ penguins are Cabbibo suppressed compared to the $b \to s$ penguins. Thus in this model the corrections to the $b \to d\bar{q}q$ modes could be much larger than the corrections to the $b \to s\bar{q}q$ modes. In the explicit models that have been studied, the relative corrections to the $b \to dg$ Standard Model amplitude are up to 3 times larger than those to the Standard Model $b \to sg$ amplitude \[26\]. We estimate the following corrections to the dominant Standard Model amplitudes

$$
\delta \phi_A(b \to s\bar{s}s) \lesssim 1,\quad \delta \phi_A(b \to d\bar{s}s) \lesssim 1,\quad (3.1)
$$

$$(b) \text{ Models with Enhanced Chromomagnetic Dipole Operators: }$$

(c) Supersymmetry without R-parity: Supersymmetric extensions of the Standard Model usually assume the existence of a new symmetry called $R$-parity. However, phenomenologically viable models have been constructed where $R$-parity is not conserved \[27\]. In the absence of $R$-parity, baryon and lepton number violating terms are allowed in the superpotential. Here we assume that lepton number is conserved in order to avoid bounds

$$
\delta \phi_B(b \to c\bar{c}s) \lesssim 0.1,\quad \delta \phi_B(b \to c\bar{c}d) \lesssim 0.6,\quad \delta \phi_B(b \to u\bar{u}d) \lesssim 1,\quad \delta \phi_B(b \to s\bar{s}s) \lesssim 1,\quad \delta \phi_B(b \to d\bar{s}s) \lesssim 1.\quad (3.2)
$$
from proton decay and study the effects of possible baryon number violating terms. The relevant terms in the superpotential are of the form \( \lambda''_{ijk} \bar{u}_i \bar{d}_j \bar{d}_k \), where antisymmetry under \( SU(2) \) demands \( j \neq k \). The tree-level decay amplitudes induced by these couplings are then given by

\[
A(b \to u_i \bar{u}_j d_k) \approx \frac{\lambda''_{i3j} \lambda''_{jkl}}{2m^2_q}, \quad A(b \to d_i \bar{d}_j d_k) \approx \frac{\lambda''_{i3j} \lambda''_{jik}}{2m^2_q}. \tag{3.3}
\]

Note that due to the requirement \( i \neq k \) in the neutral current mode, the decay \( b \to s \bar{s}s \) will not be corrected. If we use, \( m_q \sim M_W \) for the squark masses, and assume that there are no significant cancellations between the (possibly several) terms that contribute to a single decay, then the bounds for the relevant coupling constants are \( \lambda''_{ibs} \lambda''_{ids} \lesssim \sim 5 \times 10^{-3}, \lambda''_{ibd} \lambda''_{isd} \lesssim 4.1 \times 10^{-3}, \lambda''_{ubs} \lambda''_{eds} \lesssim 2 \times 10^{-2}. \) \( \tag{3.4} \)

(We have imposed the last bound in Eq. (3.4) by demanding that the new contribution to the \( B \) hadronic width be less than the contribution from the Standard Model \( b \to c \bar{u}d \) decay mode). These lead to the following corrections to the dominant Standard Model amplitudes

\[
\delta \phi_C(b \to c \bar{c}s) \lesssim 0.1, \quad \delta \phi_C(b \to c \bar{c}d) \lesssim 0.6,
\]

\[
\delta \phi_C(b \to c \bar{u}d) \lesssim 0.5, \quad \delta \phi_C(b \to d \bar{s}s) \lesssim 1. \tag{3.5}
\]

The observed pattern of \( CP \) asymmetries can also distinguish between different classes of new contributions to the \( B \) decay amplitudes. Here we list a few examples:

1. In model \((a)\) the maximum allowable relative corrections to the \( b \to s \) and the \( b \to d \) Standard Model amplitudes are similar in size. While in model \((b)\) the relative corrections to the \( b \to d \) amplitude can be much larger.

2. In both models \((a)\) and \((b)\), the neutral current decay \( b \to s \bar{s}s \) can get significant \([O(1)]\) corrections. In model \((c)\) however, this mode is essentially unmodified.

3. The fact that the \( b \to c \bar{u}d \) channel can be significantly affected in model \((c)\) is in contrast with the other two models. In those models the new decay amplitudes were penguin induced, and required the up-type quarks in the final state to be a flavor singlet \((c \bar{c} \text{ or } u \bar{u})\), thus giving no correction to the \( b \to c \bar{u}d \) decay.
IV. DISCUSSION AND CONCLUSIONS

Table 1 summarizes the relevant decay modes with their Standard Model uncertainty, and the expected deviation from the Standard Model prediction in the three models we gave as examples. New physics can be probed by comparing two experiments that measure the same phase $\phi_0$ in the Standard Model [see Eq. (2.6)]. A signal of new physics will be if these two measurements differ by an amount greater than the Standard Model uncertainty (and the experimental sensitivity) i.e.

$$|\phi(B \rightarrow f_1) - \phi(B \rightarrow f_2)| > \delta\phi_{SM}(B \rightarrow f_1) + \delta\phi_{SM}(B \rightarrow f_2).$$  \hfill (4.1)$$

Where $\phi(B \rightarrow f)$ is the angle obtained from the asymmetry measurement in the $B \rightarrow f$ decay.

The most promising way to look for new physics effects in decay amplitudes is to compare all the $B_d$ decay modes that measure $\beta$ in the Standard Model (and the $B_s$ decay modes that measure $\beta'$ in the Standard Model). The theoretical uncertainties among all the decays considered are at most $O(10\%)$, and they have relatively large rates. The best mode is $B_d \rightarrow \Psi K_S$ which has a sizeable rate and negligible theoretical uncertainty. This mode should be the reference mode to which all other measurements are compared. The $b \rightarrow c\bar{u}d$ and $b \rightarrow s\bar{s}s$ modes are also theoretically very clean. In both cases the conservative upper bound on the theoretical uncertainty is less than 0.05, and can be reduced with more experimental data. Moreover, the rates for the relevant hadronic states are $O(10^{-5})$ which is not extremely small. Thus, the two “gold plated” relations are

$$|\phi(B_d \rightarrow \psi K_S) - \phi(B_d \rightarrow D_{CP}\rho)| < 0.05,$$  \hfill (4.2)$$

and

$$|\phi(B_d \rightarrow \psi K_S) - \phi(B_d \rightarrow \phi K_S)| < 0.04.$$  \hfill (4.3)$$

Any deviation from these two relations will be a clear indication for new physics in decay amplitudes.
Although not as precise as the previous predictions, looking for violations of the relation

$$|\phi(B_d \rightarrow \psi K_S) - \phi(B_d \rightarrow D^+ D^-)| < 0.1,$$

is another important way to search for new physics in the $B$ decay amplitudes. The advantage is that the relevant rates are rather large, $BR(B_d \rightarrow D^+ D^-) \approx 4 \times 10^{-4}$. However, the theoretical uncertainty is large too, and our estimate of 10% should stand as a central value of it. As long as we do not know how to calculate hadronic matrix elements it will be hard to place a conservative upper bound.

New physics can also be discovered by comparing the two ways to measure $\gamma$ in the Standard Model, *i.e.* from $b \rightarrow c \bar{c} d$ combined with $b \rightarrow u \bar{u} d$, and $b \rightarrow c \bar{u} s$ combined with $b \rightarrow u \bar{c} s$. This is not so promising since the rates are relatively small, and the theoretical uncertainties are larger. Thus one would require larger effects in order for them to be observable. Moreover, isospin analysis that would substantially reduce the Standard Model uncertainty in the $b \rightarrow u \bar{u} d$ would simultaneously remove the isospin invariant new physics effects from this mode, thus requiring effects in the $b \rightarrow c \bar{u} s$ mode (which were not found in the three models studied here).

To conclude, we have argued in this paper that new physics in the decay amplitudes of $B$ mesons could lead to observable deviations from the patterns of $CP$ violation in $B_d$ decays predicted by the Standard Model. This is because the small Standard Model uncertainties in these predictions make even $\mathcal{O}(5\%)$ effects potentially observable. This is in contrast to the more commonly studied case of new physics contributions to the $B_d - \bar{B}_d$ mixing amplitudes, where the uncertainty in the Standard Model predictions requires effects of $\mathcal{O}(1)$ in order to be observable. We gave as examples three models where measurable effects are allowed.

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| Mode    | SM angle ($\phi_0$) | $\delta \phi_{SM}$ | $\delta \phi_A$ | $\delta \phi_B$ | $\delta \phi_C$ | BR   |
|---------|---------------------|---------------------|------------------|-----------------|-----------------|------|
| $b \to c \bar{c}s$ | $\beta$            | 0                   | 0.1              | 0.1             | 0.1             | $7 \times 10^{-4}$ |
| $b \to c \bar{c}d$ | $\beta$            | 0.1                 | 0.2              | 0.6             | 0.6             | $4 \times 10^{-4}$ |
| $b \to c \bar{u}d$ | $\beta$            | 0.05                | 0                | 0.5             | 10^{-5}         |
| $b \to s \bar{s}s$ | $\beta$            | 0.04                | 0                | 0.1             | 0               | $10^{-5}$          |
| $b \to u \bar{u}d$ | $\beta + \gamma$   | 0.4                 | 0.4              | 1               | 0               | $10^{-5}$          |
| $b \to u \bar{c}s$ | $\gamma$           | 0                   | 0                | 0               | 0               | $10^{-6}$          |
| $b \to d \bar{s}s$ | 0                   | 0.3                 | 1                | 1               | 1               | $10^{-6}$          |

**TABLE I.** Summary of the useful modes. The “SM angle” entry corresponds to the angle obtained from $B_d$ decays assuming one decay amplitude and to first order in the Wolfenstein approximation. The angle $\gamma$ in the mode $b \to u \bar{c}s$ is measured after combining with the mode $b \to c \bar{u}s$. New contributions to the mixing amplitude would shift all the entries by $\delta m_d$. $\delta \phi$ (defined in Eq. (2.1)) corresponds to the (absolute value of the) correction to the universality prediction within each model: $\delta \phi_{SM}$ – Standard Model, $\delta \phi_A$ – Effective Supersymmetry, $\delta \phi_B$ – Models with Enhanced Chromomagnetic Dipole Operators and $\delta \phi_C$ – Supersymmetry without R-parity. 1 means that the phase can get any value. The BR is taken from [29] and is an order of magnitude estimate for one of the exclusive channels that can be used in each inclusive mode. For the $b \to c \bar{u}d$ mode the BR stands for the product $BR(B_d \to \bar{D} \rho) \times BR(\bar{D} \to f_{CP})$ where $f_{CP}$ is a $CP$ eigenstate.