Recent Developments In Heavy Quarkonium Physics

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Abstract

Recent developments in heavy quarkonium physics are reviewed in brief, including (i) nonrelativistic QCD (NRQCD), (ii) the importance of color-octet \((Q\bar{Q})\) components in the decay and the production of a physical heavy quarkonium state, (iii) limitation of the NRQCD factorization and (iv) the double humps in the \(\pi\pi\) spectrum in the decay \(\Upsilon(3S) \to \Upsilon(1S)\pi\pi\).

I. INTRODUCTION

Heavy quarkonium physics was one of the early applications of perturbative QCD (PQCD). In the original treatment of the decay and the production of a heavy quarkonium state \(H\), it was assumed that \((Q\bar{Q})_1\) inside \(H\) is in a definite \(2S+1L_J\) color-singlet state. And production and decay rates of a heavy quarkonium \(H\) are assumed to be factorized into (i) the short distance (SD) parts which are calculable in PQCD (in \(\alpha_s(M_Q)\)), and (ii) the long distance (LD) parts that may be parametrized in terms of the \((Q\bar{Q})_1\) wave function and its derivatives at the origin [1].

This factorization hypothesis works well for the \(S\)-wave and \(P\)-wave states to lowest order in \(\alpha_s\). For example, the \(\chi_{c0}\) state decays into light hadrons (LH) through \(\chi_{c0} \to gg\) at the parton level:

\[
\Gamma(\chi_{c0} \to \text{LH}) = \frac{18\alpha_s^2(M_c)}{M_c^4} |R_p'(0)|^2. \tag{1.1}
\]

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However, such a factorization in the CSM fails if PQCD correction is included, which is signaled by the infrared divergence in its decay rates [2]. This infrared divergence implies the failure of the factorization in terms of a single wave function of \((Q\bar{Q})_1\) in \(H\). This problem remained unsolved until 1992, when Bodwin et al. pointed out the importance of the color-octet \((c\bar{c})_8\) component of \(H\) (see (5)) [3].

In this decade, the prompt \(J/\psi\) production at the Tevatron could be measured with the development of high resolution vertex detector. The data was astonishing in that the CSM underestimates the data by a factor of \(\sim 10\) for \(J/\psi\) production, even worse for \(\psi'\) production (by \(\sim 30\)). Possible contributions in the CSM are (i) gluon-gluon fusion \((gg \to J/\psi + g)\), (ii) gluon fragmentation \((g \to J/\psi + gg)\), (iii) \(c\bar{c}\)-quark fragmentation \((c \to J/\psi + c)\), and (iv) cascade decays from \(P\)-wave charmonia \((\chi_{cJ}(1P) \to J/\psi + \gamma)\), which is the most important for the \(J/\psi\) productions. However, for the case of \(\psi'\) production at the Tevatron, one observed that \(\sigma_{\text{exp}} \sim 30 \times \sigma_{\text{th}}\) based on CSM. Since there are no known \(P\)-wave states (such as \(\chi_{cJ}(2P)\)) that can decay into \(\psi'\), two options are available: (i) hypothetical \(\chi_{cJ}(2P)\) decaying into \(\psi' + \gamma\) with a suitable branching ratio [4], and can be tested in \(B\) and \(\Upsilon\) decays [5], and (ii) color-octet gluon fragmentation \(g \to (c\bar{c})_8\) followed by \((c\bar{c})_8 \to J/\psi + \text{soft} gg\) [6]. This second idea is the main theme of this talk, and the next two sections will be devoted to this issue.

Another less understood phenomenon in the heavy quarkonium physics in 1980’s is the double humps in \(\pi\pi\) spectrum in \(\Upsilon(3S) \to \Upsilon(1S)\pi\pi\) [7]. This issue is related with chiral dynamics of pions, rather than heavy quarkonium itself, and will be separately discussed in Sec. IV.

In this talk, we review these recent developments in brief. In Section II, the NRQCD (and factorization formula) is introduced. In Section III, we discuss the color-octet mechanism for \(J/\psi\) productions in various high energy processes. In Section IV, the double peaks in \(\Upsilon(3S) \to \Upsilon(1S)\pi\pi\) is discussed, and the summary is given in Section V.

II. NRQCD

A. NRQCD Lagrangian

A heavy quarkonium state \(H\) is associated with several different scales with the following hierarchical structure [7]:

\[
\Lambda_{QCD} \simeq M_Q v^2 <\ll M_Q v <\ll M_Q.
\] (2.1)

In brief, \(M_Q\) is the heavy quark mass scale that is a typical energy scale for decay and production of \(H\), and PQCD in \(\alpha_s(M_Q)\) becomes applicable. The typical momentum scale of \(Q\) inside \(H\) is \(p \sim M_Q v \sim 1/\text{size}\), and the typical kinetic energy scale of \(Q\) inside \(H\) is \(K.E. \sim M_Q v^2 \sim \text{the level splittings}\). Finally, the typical energy scale for nonperturbative dynamics of light quarks and gluons is characterized by \(\Lambda_{QCD}\). From the observed spectra of charmonium and upsilon families, one gets \(v^2 \sim 0.3\) for the \(\psi = (c\bar{c})\) system, and \(\sim 0.1\) for

\footnote{Although this puzzle has nothing to do with the limitation of the CSM}
the $\Upsilon = (b\bar{b})$ system. For sufficiently heavy $Q$, one has $v \sim \alpha_s(M_Q v) > \alpha_s(M_Q)$. Formally, the $v^2$ expansion is more important than $\alpha_s$ correction [4].

In order to obtain the NRQCD lagrangian, one integrates out the modes with momentum $> M_Q$ (no $Q/\bar{Q}$ creation/annihilation possible), and expand the resulting effective action in terms of heavy quark velocity. Thus NRQCD lagrangian possesses $SU(3)_c$ gauge symmetry, $P$ (parity) and $C$ (charge conjugation) symmetry, rotational symmetry, and approximate heavy quark spin symmetry. It is written in terms of heavy quark ($\chi$), and their covariant derivatives:

$$\mathcal{L}_{\text{NRQCD}} = \bar{\psi}(iD_0 + \frac{D^2}{2M_Q})\psi + \chi^\dagger(iD_0 - \frac{D^2}{2M_Q})\chi + \mathcal{L}_{\text{light}} + \delta\mathcal{L}$$  \hfill (2.2)

Gluon exchange with $k \sim p \sim M_Q v$ is included in terms of static potential, $A^0$, whereas soft gluons with $k \sim M_Q v^2$ are treated in QCD multipole expansion. $\delta\mathcal{L}$ includes the correction terms of $O(M_Q v^4)$ and higher. One can determine the coefficients of operators in NRQCD lagrangian by calculating the same process in QCD and NRQCD, and expand both results in powers of $k/M_Q \sim v$, and match to the desired order in $v$ and $\alpha_s$.

In NRQCD, a physical heavy quarkonium state is represented as a superposition of various ($Q\bar{Q}$)$_{1,8}$ and dynamical gluons: $|J/\psi\rangle = O(1)[|(c\bar{c})_{1}[^3S_1]\rangle + O(v)[|(c\bar{c})_{8}[^3S_1|g\rangle + O(v^2))[|(c\bar{c})_{8}[^1P_0]\rangle + ...$. Relative importance of various Fock states in the above equation (and NRQCD matrix elements in factorization formulae discussed in the next subsection) are determined by velocity scaling laws that can be derived from field equation of motion of NRQCD [4].

### B. NRQCD factorization

Decays of a heavy quarkonium into light hadrons are described in terms of local 4-fermion operators in NRQCD lagrangian by optical theorem:

$$\Gamma(H \rightarrow LH) = 2 \text{Im} \langle H|\delta\mathcal{L}_{\text{4-fermion}}|H\rangle = \sum_n \frac{2\text{Im}f_n(\Lambda)}{M_{Q_n}^{d-4}} \langle H|O_n(\Lambda)|H\rangle \hfill (2.3)$$

This is a double expansion in $v^2$ (Non Pert.) and $\alpha_s(M_Q^2)$ (Pert.).

One can determine the NRQCD matrix elements either from experimental data [8] or lattice QCD [9]. For $\chi_{c0}$ decay, the CMS prediction, (1), is modified (in the NRQCD formalism) into

$$\Gamma(\chi_{c0} \rightarrow LH) = \pi\alpha_s^2(M_c) \frac{M_c^5}{12M_c^3} \langle O_1(3P_0) \rangle + \frac{N}\frac{\pi\alpha_s^2(M_c)}{12M_c^3} \langle O_8(3S_1) \rangle$$  \hfill (2.4)

where

$$\langle O_1 \rangle = |\langle 0|\chi(-\frac{i}{2}D\cdot\sigma)\psi_{\chi_{c0}}\rangle|^2 \hfill (2.5)$$

$$\langle O_8 \rangle = \langle \chi_{c0}|\psi T^a\chi^\dagger T^a\psi|\chi_{c0}\rangle. \hfill (2.6)$$

Second term comes from the annihilation of the color-octet $(c\bar{c})_{8}$ ($S$–wave) in the physical $\chi_{c0}$ state. IR div. in the first term at $O(\alpha_s^3)$ cancelled by the IR div. in the color-octet matrix.
Inclusive cross section for a heavy quarkonium ($H$) production

$$\Sigma_X d\sigma(1 + 2 \rightarrow H(P) + X) = \frac{1}{4E_1E_2v_{12}} \frac{d^3P}{(2\pi)^3 2E_P} \sum_{mn} C_{mn} \langle 0|O^H_{mn}|0 \rangle$$

(2.7)

or,

$$d\sigma = \sum_n d\hat{\sigma}(Q\bar{Q}[n] + X) \langle 0|O^H(n)|0 \rangle$$

(2.8)

$C_{mn}$ takes into account the SD of order 1/$M_c$ or less, and therefore are calculable using PQCD in $\alpha_s(M_c)$. The matrix element $\langle 0|O^H_{mn}|0 \rangle$ is the VEV of a four-ferion operators of NRQCD, and is proportional to the inclusive transition probability of the perturbative $Q\bar{Q}[n]$ state into the physical heavy quarkonium state $H$. In terms of NRQCD operators,

$$O^H_{mn} = \chi^+ K_{m}^{+\psi} P_H \psi^+ K_n \chi,$$

(2.9)

where $K$’s are product of a spin matrix, a color matrix and a polynomial in the covariant derivative, $D$, and the projection operator $P$ is defined as

$$P_H \equiv \sum_S |H(P = 0, S)\rangle \langle H(P = 0), S|,$$

(2.10)

where the sum is over soft hadron states $S$ with $E < \Lambda$.

**III. COLOR-OCTET MECHANISM IN $J/\psi$ PRODUCTIONS**

**A. $\psi'$ anomaly at the Tevatron**

As discussed in the introduction, the color-octet gluon fragmentation $g \rightarrow (c\bar{c})_8[^3S_1]$ followed by $(c\bar{c})_8[^3S_1] \rightarrow J/\psi + \text{(soft gluons)}$ may explain the large excess of the $\psi'$ at the Tevatron. Qualitatively, one has

$$\text{CSM} \sim \alpha_s^3 v^3 (\text{SD suppressed, LD enhanced})$$

(3.1)

$$\text{COM} \sim \alpha_s v^7 (\text{SD enhanced, LD suppressed})$$

(3.2)

Cho and Leibovich included the color-octet $^1S_0$ and $^3P_J$ contributions as well as the color-octet $^3S_1$ in terms of three NP parameters, and obtained the following constraints on these parameters from the Tevatron data:

$$\frac{\langle 0|O^{J/\psi}(^1S_3)|0 \rangle}{M_c^2} = (6.6 \pm 2.1) \times 10^{-3} \text{ GeV}^3$$

$$\frac{\langle 0|O^{J/\psi}(^3P_0)|0 \rangle}{3} = (2.2 \pm 0.5) \times 10^{-2} \text{ GeV}^3$$

It is very important to check the idea of the color-octet mechanism and NRQCD factorization in other processes. In view of this, it is crucial to observe that the color-octet matrix elements $\langle 0|O^H_8[^{2S+1}L_J]|0 \rangle$ are universal, i.e. process-independent. Therefore, one can determine these matrix elements from a (set of) process(es), and then apply to other processes, and test the NRQCD factorization. A lot of works have been done in this line. To name a few, hadroproduction of $h_c(^1P_1)$, $B \rightarrow J/\psi + X$, $Z^0 \rightarrow J/\psi + X$, $e^+ e^- \rightarrow J/\psi + X$ at CLEO, to name only a few that were discussed at this workshop.
B. $B \to J/\psi + X$

The relevant effective Hamiltonian for this decay is

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cd}^* \left[ C_1(\bar{c}c)_{V-A} (\bar{q}b)_{V-A} + C_2(\bar{c}t^a c)_{V-A} (\bar{q}t^a b)_{V-A} \right],$$

with $C_1(m_b) \approx 0.13$ and $C_2(m_b) \approx 2.21$ in the leading logarithmic approximation. The CSM prediction to the lowest order in $\alpha_s$ is

$$\Gamma(B \to J/\psi + X)_{\text{CSM}} = \frac{3 \langle 0|O_{1}^{J/\psi}(3S_1)|0 \rangle}{3M_c^2} C^2_1 \left(1 + \frac{8M_c^2}{M_b^2}\right) \hat{\Gamma}_0,$$

with $\hat{\Gamma}_0 = |V_{cb}|^2 G_F^2 \frac{M_b^3 M_c}{14\pi} \left(1 - \frac{4M_c^2}{M_b^2}\right)^2$. Using $\langle 0|O_{1}^{J/\psi}(3S_1)|0 \rangle$ determined from $J/\psi \to l^+ l^-$, we get $B(B \to J/\psi + X)_{\text{CSM}} = 0.23\%$ compared with the most recent CLEO data $B(B \to J/\psi + X) = (0.80\pm 0.08)\%$. Higher order corrections in $\alpha_s$ and relativistic corrections are not that important [8]. Color-octet $3S_1$ contribution

$$\Gamma(b \to (\bar{c}c)_{S_1} + s \to J/\psi + X) = \frac{\langle 0|O_{8}^{J/\psi}(3S_1)|0 \rangle}{2M_c^2} C^2_2 \left(1 + \frac{8M_c^2}{M_b^2}\right) \hat{\Gamma}_0$$

is enhanced because of large Wilson coefficients [8], $C_2 \approx 17C_1$. Thus, we get moderate increase in the branching ratio to 0.58%. Color-octet $1S_0$ and $3P_J$ also contribute at the same order of $v^2$ expansion, dependent on $\langle 0|O_{8}^{J/\psi}(1S_0)|0 \rangle$ and $\langle 0|O_{8}^{J/\psi}(3P_J)|0 \rangle$. $J/\psi$ polarization in $B \to J/\psi + X$ [14], which may provide another useful constraint on the color-octet matrix elements. Similar analyses have been done for $B \to \chi_c + X$, and $B$ decays into $D-$wave charmonium state [15]. Here the available phase space is rather small so that the parton model description may be a poor approximation.

C. $J/\psi$ Photoproduction

The $J/\psi$ photoproduction ($\gamma + p \to J/\psi + X$) is described as a parton level subprocess $\gamma + g \to J/\psi + g$ in the PQCD and CSM [10]. This process is advocated as a nice probe of gluon distribution function inside proton.

PQCD corrections (in the CSM) to the leading order results have been done by M. Krämer [7]. The impacts of this radiative correction is that the scale dependence of $\alpha_s$ and structure functions is reduced. However, PQCD is out of control for $z > 0.8$ and $P_T^2 < 1$ GeV$^2$ at HERA energy ($\sqrt{s_{ep}} = 100$ GeV. Therefore one needs cuts, $z < 0.8$ and $P_T^2 > 1$ GeV$^2$, when one employs the PQCD correction to the $J/\psi$ photoproduction.

Color-octet contributions were considered by various groups [18] [19]. It was shown that $d\Gamma/dz$ spectrum in the high $z > 0.9$ region blows up, where $z$ is defined as $E_{J/\psi}/E_\gamma$ in the proton rest frame. This phenomenon was originally taken to be a signal that the determination of color-octet matrix elements from the Tevatron data on $J/\psi$ productions are inconsist with the $J/\psi$ photoproduction.

However, this may not be the case because of breakdown of NRQCD near $z = 1$ as discussed by Beneke et al. [20]. In the lowest order in $v^2$, one ignores $\Delta M \equiv M_H - 2M_Q$,
i.e. set $M_{J/\psi} \approx 2M_c$. This is not valid near the phase space boundary, since one probes the dynamics in detail: $\Delta E < \Delta M$, especially, when the matrix element has mainly a support near the phase space boundary. This is the case for $J/\psi$ photoproduction at large $z(> 0.8 \sim 0.9)$. In such cases, predictions become sensitive to the $\Delta M$, or momentum carried away by light hadrons during the hadronization. One can summarize this effect in terms of a universal shape function, as $b \to s\gamma$ in HQET. Shape function shifts the unphysical partonic boundary of phase space to the hadronic one that is physically more sensible. Therefore, the problem with $J/\psi$ photoproduction at $z > 0.9$ is probably less serious.

IV. $\pi\pi$ SPECTRUM IN $\Upsilon'' \to \Upsilon\pi\pi$

The $M_{\pi\pi}$ spectra in $\psi' \to J/\psi\pi\pi$ and $\Upsilon' \to \Upsilon\pi\pi$ can be understood in terms of QCD multipole expansion and the low energy theorem for pions, which dictates the following amplitude:

$$\mathcal{M} = A \varepsilon \cdot \varepsilon' \left[ q^2 + CM_{\pi}^2 \right], \quad (4.1)$$

with $q^2 \equiv (p_1 + p_2)^2 = M_{\pi\pi}^2$. This amplitude predicts a peak at high $M_{\pi\pi}$ region in agreement with the data from ARGUS and CLEO. However, the double humps in $\Upsilon(3S)(k, \epsilon) \to \Upsilon(1S)(k', \epsilon')\pi(p_1)\pi(p_2)$ could not be understood with the above amplitude. There are several proposals to this phenomenon, but none of them were successful. In Refs. [21], the above amplitude was modified as

$$\mathcal{M} = A \varepsilon \cdot \varepsilon' \left[ q^2 + BE_1E_2 + CM_{\pi}^2 \right], \quad (4.2)$$

and added the phase shift informations for $I = 0$, $S$ and $D$ waves. Then, the authors of Refs. [21] could fit the double humps, and predict various angular distributions. The predictions agree with the newest CLEO data except the $\cos \theta^*_\pi$ distribution. It turns out that inclusion of terms higher in pion momenta, such as $(E_1 + E_2)q^2$ and so on, improves the agreement with the data. More systematic study in ChPT is in progress [22].

V. CONCLUSION

NRQCD provides a theoretical framework in which one can study the perturbative (in $\alpha_s(M_Q)$) and nonperturbative (in $\nu^2$) aspects of heavy quarkonium physics. In particular, the role of the $|(Q\bar{Q})_{\text{g}}\rangle$ Fock state in the heavy quarkonium production and its decay can be rigorously formulated in the NRQCD. One can fit the $J/\psi$ and $\psi'$ production rates at the Tevatron via the color-octet mechanism. In order to check this idea at other processes, PQCD corrections are essential. This part has been calculated only recently by Petrelli et al. [23]. The complete phenomenological analysis including this new result has not been done yet, however. Finally, Double hump in $M_{\pi\pi}$ spectrum can be explained in terms of an amplitude that satisfies the low energy theorem for pions, once the $D$–wave dipion amplitude is properly included.
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