Mathematical model of the technological vibratory unit with electromagnetic excitation

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Abstract. The mathematical model of the technological vibratory unit with electromagnetic excitation is considered to its features that are important for the dynamic design problems. The model is based on the differential equations of electrical balance and mechanical balance, taking into account the mobility degree of inertial masses with spring linkages. The vibratory unit vibration properties are analyzed concerning the power voltage and frequency coupling, interacting inertia masses mobility degree, spring linkages properties, magnetic core steel magnetizing characteristic non-linearity, leakage fluxes, electric, magnetic and mechanical energy losses. The research performed for dynamical design problem solution is distinguished by a novel approach to the estimation of the capability of changing vibration properties of the movable operating element of the technological vibratory unit with an additional mass attached to it. The model design technology based on the structural modelling method supported by Matlab Simulink is proposed. The research results can be useful for specialists in vibration engineering, machines and devices durability dynamics and vibroprotection.

1. Introduction

Vibratory machines and units with electromagnetic excitation are widely used in vibrostands, vibratory compressors, vibratory pumps and other technological plants. [1–5].

Such machines and units applied in different kinds of vibratory equipment should operate in full frequency and amplitude ranges. The vibration amplitude can achieve values from 3 to 6 mm for the frequency range from 25 to 100 Hz. If the frequency is more than 100 Hz, the vibration amplitude can reach tenths of millimetres. The vibration amplitude in such vibratory units significantly depends on an additional linked mass. The linked mass values can achieve the vibratory unit mass.

Therefore, a technological vibratory machine or unit with electromagnetic excitation is a device generating mechanical vibrations of the reactive mass (core) with linked reaction mass and additional mass. Here the required relationship between the mechanical vibration amplitude and frequency of the electromagnetic vibratory unit should be satisfied.

In comparison with other types of vibratory drives, an electromagnetic vibratory linear drive has several advantages including long service life, high reliability, simply implemented independent amplitude and frequency control, high efficiency at the resonant frequency [6–9].

The great attention is focused on the derivation of simplified analytical relations for the vibratory technological machines design that describe the interaction between elements of mechanical vibratory systems [10–12].
Even though there are many papers on the considered vibratory units design, it is important to create new mathematical models which are maximally reflecting dynamical properties of elements and their interaction. These models should help to improve vibratory units of design methods.

The research purpose is the synthesis of the mathematical model of the technological vibratory unit with electromagnetic excitation. The model takes into account the dynamical properties of the mechanical vibratory system with two degrees of freedom and energy loss.

2. Synthesis of the mathematical model

The general scheme of the technological vibratory unit with electromagnetic excitation is shown in Figure 1.

The vibratory unit has the electromagnetic drive inside the frame 1 that consists of the magnetic core 2, the excitation coils 3, 4 mounted on the magnetic core and the movable core 5. The movable core 5 and the magnetic core 2 are made of electric steel sheets. The distance between them is equal to the operating air gap width $\delta$.

![Figure 1. Technological vibratory unit with electromagnetic excitation](image1)

![Figure 2. Dynamical design diagram of the vibratory unit with two degrees of freedom](image2)

The movable core 5 is mounted on the rod 6 and rigidly connected with the elastic damper 7. When the movable core travels vertically, coaxial is provided by the diaphragm 8 made of an elastic material. There is the platform 9 mounted on the end of the rode 6. The mass 10 is connected with the platform 9 which is the operating element of the vibratory unit.

The vibroinsulators 12, 13 placed between the frame 1 and the platform mounting area 11 help to damp vertical vibrations and reduce the vibratory unit negative influence on the environment.
When the periodical current passes through the excitation coils 3, 4, the vertical vibration of the movable core 5 is produced by the interaction of the force of electromagnetic field and the elastic force of the damper 7.

The mechanical system and the magnetic system of the vibratory unit are connected by the electromagnetic force dependence on the current $i$ in the excitation coil and the operating air gap width $\delta$: $f_{em} = f(i, \delta)$. The electrical balance equation generally describes the electrical system balance:

$$u(t) = i r + \frac{d\psi(i, \delta)}{dt},$$  \hspace{1cm} (1)

where $u(t)$ is the excitation coil voltage, $\psi(i, \delta)$ is the coil flux linkage, $r$ is the Active coil resistance.

Figure 2 states the dynamical design diagram of the vibratory unit for the mentioned above connection between the mechanical system and magnetic system. Concerning the motion of interacting masses, the vibratory system has two degrees of freedom ($N = 2$).

The motion differential equations are formed by the methods based on the second type Lagrange equation [13, 14]:

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{x}_i} \right) - \frac{\partial T}{\partial x_i} = -\frac{\partial \Pi}{\partial \dot{x}_i} - \frac{\partial \Phi}{\partial \dot{x}_i} + Q_i,$$  \hspace{1cm} (2)

where $T$ is the kinetic energy of the system, $\Pi$ is the potential energy of the system, $\Phi$ is the dissipative system function, $Q_i$ is the i-th generalized external force, $x_i$ is the i-th generalized coordinate, $\dot{x}_i$ is the i-th generalized velocity.

If the masses $m_1$, $m_2$ and $m_3$ move on the selected coordinates system (Figure 2) then the system kinetic energy:

$$T = \frac{m_1 \dot{x}_1^2}{2} + \frac{m_2 \dot{x}_2^2}{2},$$  \hspace{1cm} (3)

where $\dot{x}_1$, $\dot{x}_2$ are the generalized velocities;

– the system potential energy

$$\Pi = \frac{k_1(x_1 - x_2)^2}{2} + \frac{k_2 x_2^2}{2},$$  \hspace{1cm} (4)

where $k_1$, $k_2$ are the rigidity factors of spring linkages;

– the system dissipative function

$$\Phi = \frac{b_1(x_1 - \dot{x}_1)^2}{2} + \frac{b_2 \dot{x}_2^2}{2},$$  \hspace{1cm} (5)

where $b_1$, $b_2$ are the spring linkages viscous friction factors;

– the generalized forces

$$Q_{i1} = -f_{em}(i, \delta), \quad Q_{i2} = f_{em}(i, \delta),$$  \hspace{1cm} (6)

where $f_{em}(i, \delta)$ is the electromagnetic force.

The partial derivatives of the kinetic energy (3) have the form:

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{x}_1} \right) = (m_1 + m_3) \ddot{x}_1, \quad \frac{d}{dt} \left( \frac{\partial T}{\partial \dot{x}_2} \right) = m_2 \ddot{x}_2, \quad \frac{\partial T}{\partial x_1} = \frac{\partial T}{\partial x_2} = 0.$$  \hspace{1cm} (7)

The partial derivatives of the potential energy (4) have the form:

$$\frac{\partial \Pi}{\partial x_1} = k_1(x_1 - x_2), \quad \frac{\partial \Pi}{\partial x_2} = -k_1(x_1 - x_2) + k_2 x_2.$$  \hspace{1cm} (8)
The partial derivatives of the dissipative function (5) have the form:
\[
\frac{\partial \Phi}{\partial \dot{x}_1} = b_1 (\dot{x}_1 - \dot{x}_2), \quad \frac{\partial \Phi}{\partial \dot{x}_2} = -b_1 (\dot{x}_1 - \dot{x}_2) + b_2 \dot{x}_2.
\] (9)

Substitution of (6) – (9) into (2) gives the equations describing the dynamical status of the mechanical system:
\[
(m_1 + m_i) \ddot{x}_i = -k_i (x_1 - x_2) - b_1 (\dot{x}_1 - \dot{x}_2) - f_{em} (i, \delta),
\] (10)
\[
m_2 \ddot{x}_2 = k_1 (x_1 - x_2) - k_2 x_2 + b_1 (\dot{x}_1 - \dot{x}_2) - b_2 \dot{x}_2 + f_{em} (i, \delta).
\] (11)

The mathematical model of the technological vibratory unit with electromagnetic excitation (Figure 1) to the electrical balance equation (1) and the mechanical system dynamical balance equations (10), (11) takes the form:
\[
\begin{cases}
\dot{u}(t) = i_r + \frac{d\psi (i, \delta)}{dt}; \\
(m_1 + m_i) \frac{d^2 x_1}{dt^2} + b_1 \left( \frac{dx_1}{dt} - \frac{dx_2}{dt} \right) + k_1 (x_1 - x_2) = -f_{em} (i, \delta); \\
m_2 \frac{d^2 x_2}{dt^2} - b_2 \left( \frac{dx_1}{dt} - \frac{dx_2}{dt} \right) - k_1 (x_1 - x_2) + k_2 x_2 = f_{em} (i, \delta).
\end{cases}
\] (12)

The mathematical model (12) makes it possible to estimate the capability of changing the vibration properties of the operating element of the technological vibratory unit and an additional mass.

The mathematical model can be implemented in Matlab Simulink using structural modelling [15–20].

The array of static flux linkage and electromagnetic force values can be calculated with the finite-element FEMM software [21–24].

Examples of the magnetic field parameters FEMM calculation are stated in [28–28].

3. Conclusion
The mathematical model of the technological vibratory unit with electromagnetic excitation with two degrees of freedom has been developed. The Matlab Simulink model using the structural modelling method has been obtained from the derived system of differential equations. The model accuracy has been improved by taking into account inertial masses mobility and energy dissipation processes because of spring linkages viscous friction forces. The research results can be applied in the simulation of vibratory units with electromagnetic excitation, machines and devices durability dynamics and vibroprotection.

References
[1] Lavendel E E 1981 Vibrations in the Technique: Reference Book. Vibrational Processes and Machines (Moscow: Mashinostroenie)
[2] Despotovic Z and Ribic A 2012 The Increasing Energy Efficiency of the Vibratory Conveying Drives with Electromagnetic Excitation International Journal of Electrical and Power Engineering 6(1) 38–42
[3] Azin A V, Bogdanov E P, Ponomarev S V and Rikkonen S V 2017 Calculation of energy parameters of submerged vibrating confuser of an electromagnetic vibrator Bulletin of the Tomsk Polytechnic University, Geo Assets Engineering 328(5) 16–23.
[4] Bogdanov E, Nomokonova Y, and Rikkonen S 2014 Oscillatory system of the jet electromagnetic vibrator IOP Conf. Ser.: Mater. Sci. Eng. 66 012017
[5] Ugarov G G and Neiman V Yu 1996 Evaluation of operating conditions for electromagnetic impactors Journal of Mining Science 32(4) 305–312
[6] Cherno A A 2014 Control of Resonant Electromagnetic Vibrational Drive Using a Digital Filtering Algorithm Based on Discrete Fourier Transform Journal of Automation and Information Sciences 46(7) 53–68
[7] Lanets A S 2008 High Efficiency Interresonance Vibrating Machines with Electromagnetic Drive (Theoretical Foundations and Building Practice) (Lviv: NULP)
[8] Despotovic Z and Ribic A 2012 The Increasing Energy Efficiency of the Vibratory Conveying Drives with Electromagnetic Excitation International Journal of Electrical and Power Engineering 6(1) 38–42
[9] Singh S N and Blazek K E 1974 Heat transfer and skin formation in a continuous-casting mold as a function of steel carbon content Open hearth Conference, Proceedings (Atlantic City)
[10] Maksimov N P anl Baimatov K K 2006 Fundamentals of calculating the power of electromagnetic vibrators of a rotary-vibration mill News of higher educational institutions. Non-ferrous metallurgy (4) 15–18.
[11] Torregrossa D, Fahimi B, Peyraut F, and Miraoui A 2012 Fast computation of electromagnetic vibrations in electrical machines via field reconstruction method and knowledge of mechanical impulse response IEEE Trans. Ind. Electron. 59(2) 839–847
[12] Finley W R, Hodowanec M M and Holter W G 2000 An analytical approach to solving motor vibration problems IEEE Trans. on Industry Applications 36(5) 1467–1480.
[13] Cherno A A 2014 Dynamic model of electromagnetic vibration drive // Tekhnichnaya Elektrodynamika (2) 37–43
[14] Neyman L A, Neyman V Y and Obukhov K A 2017 New method of the synchronous vibratory electromagnetic machine mechatronic module control The 18 International Conference of Young Specialists on Micro/Nanotechnologies and Electron Devices (Novosibirsk: NSTU) pp 516–519
[15] Neyman L A and Neyman V Y 2016 Dynamic model of a vibratory electromechanical system with spring linkage 11th International Forum on Strategic Technology (Novosibirsk: NSTU) 2 pp 23–27
[16] Neyman L A, Neyman V Y and Shabanov A S 2017 Vibration dynamics of an electromagnetic drive with a half-period rectifier The 18 international conference of young specialists on micro/nanotechnologies and electron devices (Novosibirsk : NSTU) pp 503–506.
[17] Klee H 2007 Simulation of Dynamic Systems with MATLAB and Simulink (CRC Press)
[18] Chaturvedi D K 2009 Modeling and Simulation of Systems Using MATLAB and Simulink (CRC Press)
[19] Chen Y et al. 2013 System Simulation Techniques with MATLAB and Simulink (John Wiley & Sons)
[20] Dabney J B and Harman T L 2004 Mastering Simulink® Pearson Education, Inc.
[21] Meeker, D 2004 Finite Element Method Magnetics: User’s Manual, 4th ver
[22] Volakis J L, Chatterjee A and Kempel L C 1998 Finite Element Methods for Electromagnetics (IEEE Press)
[23] Baltzis K B 2008 The FEMM Package: A Simple, Fast, and Accurate Open Source Electromagnetic Tool in Science and Engineering Journal of Engineering Science and Technology Review 1 83–89
[24] Simonov B F, Neyman V Y and Shabanov A S 2017 New conception of an electromagnetic drive for a vibration source in hole The 18 International Conference of Young Specialists on Micro/Nanotechnologies and Electron Devices (Novosibirsk: NSTU) pp 507–510
[25] Baltzis K B 2007 On the usage and potential applications of the finite element method magnetics (FEMM) package in the teaching of electromagnetics in higher education (CBLIS) 408–420
[26] Saraiva E, Chaves M L R and Camacho J R 2008 Three-phase transformer representation using FEMM, and a methodology for air gap calculation  in Proc. of the Int. Conf. on Electrical Machines Vilamoura 1–6.

[27] Simonov B F, Neiman V Y and Shabanov A S 2017 Pulsed Linear Solenoid Actuator for Deep-Well Vibration Source  Journal of Mining Science 53(1) 117–125

[28] Zakaria Z et al. 2010 Simulation of Magnetic Flux Leakage (MFL) Analysis Using FEMM Software  In Proceedings of the 2010 IEEE Symposium on Industrial Electronics and Applications, (Penang, Malaysia) 481–486