A Self-Calibration Method of Zooming Camera

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Abstract — In this article we proposed a novel approach to self-calibrate a camera with variable focal length. We show that the estimation of camera’s intrinsic parameters is possible from only two points of an unknown planar scene. The projection of these points by using the projection matrices in two images only permit us to obtain a system of equations according to the camera’s intrinsic parameters. From this system we formulated a nonlinear cost function which its minimization allows us to estimate the camera’s intrinsic parameters in each view. The results on synthetic and real data justify the robustness of our method in term of reliability and convergence.

Keywords — Variable Focal Length, Unknown Planar Scene, Projection Matrices, Nonlinear Cost Function.

I. INTRODUCTION

The self-calibration of camera [1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16] is a fundamental area in computer vision, it consist of estimation of camera’s parameters with no knowledge about the observed scene. The principle of self-calibration consist to formulate equations based on the camera’s intrinsic parameters by using invariant primitives (circular points, absolute conic...) in the scene space. These primitives describe the euclidean structure of the scene.

Our self-calibration method presented in this paper consist of calibrate a camera from a unknown planar scene and by considering that the camera’s intrinsic parameters are constants except to the focal length which may be variable from one view to another which is suitable with the current applications of computer vision. The highlight of our idea lies partly in the use of only two points from an unknown planar scene and the projection of this scene in only two image planes and secondly in the implementation of a relationship between images of the absolute conic in the both views and the projection matrices, to determinate, of the two points used in the scene. The procedure of our method of self-calibration described in this work can be presented in the form of the following steps:

- Acquisition of two images of an unknown planar scene.
- Detection of interest points in the two images by using the Harris detector.
- The matching of the interest points by using the ZNCC correlation measure
- Estimation of the projection matrices of the two points of the scene
- Development of a relationship between the projection matrices estimated in the previous step and the images of the absolute conic in the two images of the scene.
- Formulation of a non-linear cost function on the basis of the relation found in the previous step.
- Optimization of the cost function by the Levenberg-Marquardt

II. RELATED WORKS

In this section we present with a non-exhaustive manner some works done in the field of the camera self-calibration. In [17] through two phases:

- Initialization phase providing an initial solution by applying constraints on the two images and some intrinsic parameters of the camera.
- Minimizing of the cost function to find the optimum solution according to the intrinsic parameters of our camera in the two images of the scene.

The present article is organized as follows: the second part has a review of previous works dealing with the problem of self-calibration. The self-calibration tools used in this article are discussed in the third section then our method of self-calibration is addressed in the fourth section. Our experimental results are presented in the fifth section and the last part contains a conclusion of our work.
III. Camera model and image of absolute conic

Our camera is modeled by a model called pinhole. Denote by \( M = (X \ Y \ Z \ w)^T \) a point in homogeneous coordinates of the scene, \( m = (x \ y \ z \ w)^T \) an image point. The projection of the \( M \) in its image \( m \) is denoted by:

\[
m \sim \xi(R \ t)M
\]

(1)

With \( R \) is the rotation matrix and \( t \) the translation vector of the camera in space and \( \xi \) is the matrix of the intrinsic camera parameters defined by:

\[
\xi = \begin{pmatrix}
I & \gamma & \mu_0 \\
0 & \eta & v_0 \\
0 & 0 & 1
\end{pmatrix}
\]

(2)

\( I \): Focal length.
\( \eta \): Scale factor.
\( \gamma \): Skew factor.
\( \mu_0 \) and \( V_0 \): Coordinates of principal point.

Considering the space of our scene is a projective space. In this space the points satisfying \( w = 0 \) are called the points at infinity, these points form the plane at infinity. In the plane at infinity the points satisfying \( M^TM = 0 \) form the absolute conic denoted \( \Omega_\infty \). The absolute conic is a complex conic of the plane at infinity, the image of absolute conic \( \omega \) is given by:

\[
\omega = (\xi^T\xi)^{-1}
\]

(3)

From (3) we deduce that the image of the absolute conic depends only on the intrinsic parameters of the camera. Once \( \omega \) is estimated we can easily estimate the intrinsic parameters of the camera using the Cholesky factorization.

IV. Proposed method

The main idea of our new self-calibration method is based on the projection of two points of an unknown planar scene in only two image planes using two projection matrices which must be estimated.

Let \((S)\) an unknown planar scene and \(A(x_A, y_A, z_A, 1)^T\) and \(B(x_B, y_B, z_B, 1)^T\) two points in homogeneous coordinates of \((S)\) and \(a_k(u_{ak}, v_{ak}, 1)^T\) and \(b_k(u_{bk}, v_{bk}, 1)^T\) for \( k = i \) or \( j \) its projections in the two images (see Fig. 1).

Assuming that the plane containing the segment \([AB]\) is in \( Z = 0 \) and \( 0 < \theta < \Pi/2 \). From the Fig. 1 we can deduce:

\[
x_a = \frac{\|AB\|}{2} \cos(\theta) \quad \text{and} \quad y_a = \frac{\|AB\|}{2} \sin(\theta)
\]

From (1) we have for the image \( i \):

\[
a_i \sim \xi_i(r_{i_1}, r_{2i}, t_i)A
\]

(4)

with \( r_{qi} \) is the column number \( q \) of \( R_i \).

The equation (4) can be rewritten as follows:

\[
a_i \sim \xi_i(r_{i_1}, r_{2i}, t_i)
\]

(5)

Let:

\[
H_i \sim \xi_i(r_{i_1}, r_{2i}, t_i)
\]

(6)

\( H_i \) is a 3x3 matrix called the homography matrix scene to image \( i \).

The equation (5) becomes:

\[
a_i \sim H_i(x_A, y_A, 1)^T
\]

(7)

Let \( H_i = (h_{i1}, h_{i2}, h_{i3}) \) with \( h_{ki} \) are column vectors of \( H_i \). The vectors \( r_{i1} \) and \( r_{2i} \) are orthogonal which permit us to write from (6):

\[
\begin{align*}
0 &= h_{i1}^T \xi_i \xi_i^{-1} h_{2i} \\
0 &= h_{i2}^T \xi_i \xi_i^{-1} h_{2i}
\end{align*}
\]

(8)

Fig. 1. Vision system used
From (7) we have for image $i$:

\[
\begin{align*}
    a_i &\sim H_i A' \\
    b_i &\sim H_i B'
\end{align*}
\]

(9)

For image $j$:

\[
\begin{align*}
    a_j &\sim H_j A' \\
    b_j &\sim H_j B'
\end{align*}
\]

(10)

From (9) and (10) we can write:

\[
\begin{align*}
    a_i &\sim H_i QW \\
    a_j &\sim H_j QW
\end{align*}
\]

(11)

And:

\[
\begin{align*}
    b_i &\sim H_i QW' \\
    b_j &\sim H_j QW'
\end{align*}
\]

(12)

The matrix $P_i$ consists of eight unknowns and the above system gives us twelve equations that allows us to estimate the matrix $P_i$. The matrix $P_j$ can be estimated from (16).

Let $P_i = \left( p_{i1}, p_{21}, p_{3i} \right)$, with the $p_{li}$ are the column vectors of $P_i$. From (13) we can deduce:

\[
\begin{align*}
    p_{li} &= \frac{AB}{2} \cos(\theta) h_{li} \\
    p_{2i} &= \frac{AB}{2} \sin(\theta) h_{2i}
\end{align*}
\]

(18)

The system of equations (18) gives us:

\[
\begin{align*}
    h_{li} &= \frac{2}{|AB|} \cos(\theta)^{-1} p_{li} \\
    h_{2i} &= \frac{2}{|AB|} \sin(\theta)^{-1} p_{2i}
\end{align*}
\]

(19)

By replacing in (9) $h_{li}$ and $h_{2i}$ by their expressions in (19) and by posing $\lambda_1 = \frac{2}{|AB|} \cos(\theta)^{-1}$ and $\lambda_2 = \frac{2}{|AB|} \sin(\theta)^{-1}$, we obtain for image $i$:

\[
\begin{align*}
    \lambda_1 \lambda_2 p_{i1}^T \omega_i p_{2i} &= 0 \\
    \lambda_1^2 p_{i1}^T \omega_i p_{li} &= \lambda_2^2 p_{2i}^T \omega_i p_{2i} \\
    \lambda_2^2 p_{2i}^T \omega_i p_{2i} &= 0
\end{align*}
\]

Or:

\[
\begin{align*}
    p_{i1}^T \omega_i p_{2i} &= 0 \\
    \lambda_1^2 p_{i1}^T \omega_i p_{li} &= \lambda_2^2 p_{2i}^T \omega_i p_{2i} \\
    \lambda_2^2 p_{2i}^T \omega_i p_{2i} &= 0
\end{align*}
\]

(20)

And for image $j$:

\[
\begin{align*}
    \lambda_1 \lambda_2 p_{j1}^T \omega_j p_{2j} &= 0 \\
    \lambda_1^2 p_{j1}^T \omega_j p_{1j} &= \lambda_2^2 p_{2j}^T \omega_j p_{2j}
\end{align*}
\]

Or:

\[
\begin{align*}
    p_{j1}^T \omega_j p_{2j} &= 0 \\
    \lambda_1^2 p_{j1}^T \omega_j p_{1j} &= \lambda_2^2 p_{2j}^T \omega_j p_{2j}
\end{align*}
\]

(21)

From (20) and (21) we can deduce that:

\[
\begin{align*}
    p_{i1}^T \omega_i p_{2i} &= \frac{p_{2i}^T \omega_i p_{2j}}{p_{i1}^T \omega_i p_{1j}} \\
    p_{i1}^T \omega_i p_{1i} &= \frac{p_{1i}^T \omega_i p_{2j}}{p_{i1}^T \omega_i p_{1j}}
\end{align*}
\]
Or:
\[ p_{i}^{T} \omega_{i} p_{2j}^{T} = p_{i}^{T} \omega_{i} p_{2j} p_{i}^{T} \omega_{i} p_{i} \]  
(22)

Let \( H_{y} = \begin{pmatrix} \tilde{h}_{y_{1}} \\ \tilde{h}_{y_{2}} \\ \tilde{h}_{y_{3}} \end{pmatrix} \) with \( \tilde{h}_{y_{1}}, \tilde{h}_{y_{2}}, \tilde{h}_{y_{3}} \) are the row vectors of \( H_{y} \).

From (16) we can deduced that the first two rows and columns of the matrices \( p_{i}^{T} \omega_{i} p_{2j}^{T} \) and \( p_{j}^{T} \omega_{j} p_{j}^{T} \) are identical, which permit us to write:

\[
\begin{align*}
\begin{bmatrix}
\epsilon_{1i} e_{12i} - e_{11i} e_{12i} = 0 \\
\epsilon_{1i} e_{22i} - e_{11i} e_{22i} = 0
\end{bmatrix}
\]  
(29)

From (20), (21), (22), (24) and (29) we can deduce the following system:

\[
\begin{align*}
p_{i}^{T} \omega_{i} p_{2i} &= 0 \\
p_{i}^{T} \omega_{i} p_{2j} &= 0 \\
p_{j}^{T} \omega_{j} p_{2i} &= 0 \\
p_{j}^{T} \omega_{j} p_{2j} &= 0 \\
\epsilon_{1i} e_{12j} - e_{11i} e_{12i} &= 0 \\
\epsilon_{1i} e_{22j} - e_{11i} e_{22i} &= 0
\end{align*}
\]  
(30)

In the present article we are assumed in the one hand that the intrinsic camera parameters are unknown and constant except for the focal length which can be vary from one view to another and in the second hand we are used two images only of an unknown planar scene which gives us six unknown, the system of equations (30) contains six equations where the possibility to estimate the intrinsic parameters of camera in the both views.

The equations of the system (30) are nonlinear therefore to solve it we minimize the cost function given by the following expression:

\[
\min_{\Delta_{y}} \sum_{j=i+1}^{n} \sum_{i=1}^{n-1} \left( \beta_{i}^{2} + \beta_{j}^{2} + \beta_{3ij}^{2} + \beta_{4ij}^{2} + \beta_{5ij}^{2} + \beta_{6ij}^{2} \right)
\]  
(31)

with:
\[
\begin{align*}
\beta_{i} &= p_{i}^{T} \omega_{i} p_{2i} \\
\beta_{j} &= p_{j}^{T} \omega_{j} p_{2j} \\
\beta_{3ij} &= p_{3i}^{T} \omega_{3i} p_{3j} - p_{3i}^{T} \omega_{3i} p_{2i} \\
\beta_{4ij} &= p_{4i}^{T} \omega_{4i} p_{4j} - p_{4i}^{T} \omega_{4i} p_{2i} \\
\beta_{5ij} &= e_{1i} e_{12j} - e_{11i} e_{12i} \\
\beta_{6ij} &= e_{1i} e_{22j} - e_{11i} e_{22i}
\end{align*}
\]

And \( \Delta_{y} \) is the vector of the intrinsic parameters of the camera in the two images \( i \) and \( j \) and \( n \) is the number of images used.

The minimization of (31) is given by the Levenberg-Marquardt algorithm which requires an initialization step, for this purpose we assume the following assumptions:

- The main point is in the center of the image, so \( \mu_{i0}, V_{i0}, \mu_{j0} \) and \( V_{j0} \) are known.
- The pixels are square so \( \eta_{i} = \eta_{j} = 1 \) and \( Y_{i} = Y_{j} = 0 \).

By substituting these parameters in the system of equations (30) we can determine the focal lengths \( l_{i} \) and \( l_{j} \) for the both images \( i \) and \( j \).
V. EXPERIMENTAL RESULTS

To test the effectiveness of our method two types of data are used synthetic and real.

A. Simulation

In this simulation a sequence of ten 512×512 images of a calibration target is used to test the effectiveness of our self-calibration method presented in this paper. After the detection of interest points by the Harris detector [25] the corresponding points between images are determined using the correlation measure ZNCC [26]. The images are perturbed by adding a Gaussian noise of deviation σ to the pixels of images. The projection of the scene in image planes allows us to obtain linear equations whose solution allows to determine the projection matrices. The self-calibration equations are then built to formulate a nonlinear cost function which its minimization by the Levenberg-Marquardt algorithm helps us to estimate the intrinsic parameters of our camera in all views. To evaluate the performance of our approach we compared the results obtained with those given by a well-known calibration method [6] published by Z. Zhang and those given by [27]. The figures (2) and (3) consecutively present the relative error of the focal length estimated by our method and those of Z. Zhang and C-R.Huang according to the number of images used and added noise.

B. Real images

In this second experiment, two 512 × 512 images of a real planar scene are acquired by a CCD camera having constant intrinsic parameters except to the focal length. (See Fig. 4).

The both figures above show that the method developed in this paper and that of Z.Zhang gives the approximate results. By taking into account the robustness of the Z. Zhang’s method we can say that our method is effective and efficient. On the other hand, the previous figures prove the preference of our method compared with that presented in [27].

1. Detection of interest points

The interest points in the two images are detected by using the Harris detector (Fig. 5).

2. Matching of interest points

The interest points previously identified are matched between the two images by using the ZNCC algorithm (Fig. 6).

3. Estimation of intrinsic parameters

This step consists of estimating the intrinsic parameters of the camera in each view by applying our method. The following table shows the results obtained.
TABLE I
THE RESULTS OBTAINED BY OUR METHOD

|   | ι   | η   | γ   | μ₀  | ν₀  |
|---|----|----|----|----|----|
| Image 1 | 1687 | 0.95 | 0.09 | 258 | 260 |
| Image 2 | 1683 | 0.97 | 0.05 | 257 | 255 |

The above table presents the results obtained by our method of self-calibration discussed in this article. To verify the efficacy of the above results, we rectified the two images using a robust approach of image rectification [28] based on a calibrated camera. The following figure shows the rectified images obtained by applying [28] and using the values of table 1.

Fig. 7 The obtained rectified images.

The goal of image rectification is to obtain the epipolar lines horizontal and parallel to the x-axis, according to Fig. 7 we can deduce that our approach presented in this paper gives satisfactory results and this by taking into account that our method is able to self-calibrate a camera with a variable focal length from two images only based on two points only of an unknown planar scene.

VI. CONCLUSION

In this work, we have proposed a new method of self-calibration of a CCD camera with constant intrinsic parameters except to the focal length which can vary freely from one view to another from an unknown planar scene. We have shown that the determination of the intrinsic parameters is possible by using only two points of the scene. The projections of the two points in the two image planes permit us to determine their projection matrices, these matrices are used to formulate a nonlinear cost function. The minimization of the cost function obtained allowed us to estimate the intrinsic parameters of the camera. The experimental results on the synthetic and real data prove the robustness of our method.

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