Instabilities in a Mean-field dynamics of Asymmetric Nuclear Matter

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We discuss the features of instabilities in asymmetric nuclear matter, in particular the relation between the nature of fluctuations, the types of instabilities and the properties of the interaction. We show a chemical instability appears as an instability against isoscalar-like fluctuations. Then starting from phenomenological hadronic field theory (QHD), including exchange terms, we discuss the symmetry energy and the relation to the dynamical response inside the spinodal region.

1 Introduction

Nuclear matter (NM) is a binary system (neutrons and protons), this feature has stimulated a quite exciting research field: nuclei far from $\beta$-stability, dynamical effects of a large charge asymmetry, properties of neutron stars. In particular with radioactive beams (not only), NM with a high isospin asymmetry can be created transiently, offering the possibility to study also chemical instability associated with isospin asymmetry of NM. Indeed, recent experiments on nuclear reactions involving different ratios of the neutron to proton numbers have shown that products from nuclear multifragmentation depends strongly on the isospin asymmetry of the colliding nuclei.

In the first part of the paper we will focus on the behaviour of Asymmetric NM (ANM) at sub-normal densities where we can foresee scenarios for a dynamical formation of fragments with particular isotopic contents due to instabilities. To study these problems we show, by linking thermodynamics and Fermi liquid theory, the relation between the nature of fluctuations and the types of instabilities for a general binary system. In the second part we show a relativistic field model of hadrons (QHD) based on strongly interacting nucleon and meson fields. Inside the QHD model we have recently proposed a method to introduce Fock term contributions. Here we want to stress some implications of that improvement on the isospin physics discussed in the first
part. Comparisons with the QHD model in Hartree approximation and with Skyrme interactions will be addressed.

2 Chemical and mechanical instabilities

2.1 Analytical analysis

In the context of multifragmentation the instability of NM (region where the system becomes unstable against long wave length but small amplitude fluctuation) plays a crucial role. These aspects were discussed repeatedly in the past, but the binary character will induce new features for both scenarios that are absent in one-component systems. In the framework of Landau theory for two component Fermi liquids the spinodal border was determined by studying the stability of collective modes described by two coupled Landau-Vlasov equations for protons and neutrons. In terms of the appropriate Landau parameters the stability condition can be expressed as:

\[(1 + F_{0}^{nn})(1 + F_{0}^{pp}) - F_{0}^{np} F_{0}^{pn} > 0.\]

It is possible to show that this condition is equivalent to the following thermodynamical condition:

\[\left(\frac{\partial P}{\partial \rho}\right)_{T,y} \left(\frac{\partial \mu_{p}}{\partial y}\right)_{T,P} > 0.\]

discussed in, where \(y\) is the proton fraction. In Fig. 1 we show the spinodal lines obtained from eq. (1) (continuous line with dots) which for asymmetric nuclear matter is seen to contain the lines corresponding to "mechanical instability", \(\left(\frac{\partial P}{\partial \rho}\right)_{T,y} < 0\) (crosses). Therefore both eqs. (1,2) describe the whole region of instability. We want to stress, however, that by just looking at the above stability condition we cannot determine the nature of the fluctuations against which a binary system becomes chemically unstable. Indeed, the thermodynamical condition in eq. (3) cannot distinguish between two very different situations which can be encountered in nature: an attractive interaction between the two components of the mixture \((F_{0}^{np}, F_{0}^{pn} < 0)\), as is the case of nuclear matter, or a repulsive interaction between the two species.

We define as isoscalar-like density fluctuations the case when proton and neutron Fermi spheres (or equivalently the proton and neutron densities) fluctuate in phase and as isovector-like density fluctuations when the two Fermi sphere fluctuate out of phase. Then it is possible to prove, based on a thermodynamical approach of asymmetric Fermi liquid mixtures, that chemical instabilities are triggered by isoscalar fluctuations in the first, i.e. attractive,
situation and by isovector fluctuations in the second one. For the asymmetric nuclear matter case because of the attractive interaction between protons and neutrons the phase transition is thus due to isoscalar fluctuations that induce chemical instabilities while the system is never unstable against isovector fluctuations. Of course the same attractive interaction is also at the origin of phase transitions in symmetric nuclear matter. However, in the asymmetric case isoscalar fluctuations lead to a more symmetric high density phase everywhere under the instability line defined by eq. (1).

Figure 1: Spinodal lines corresponding to chemical (circles) and mechanical (crosses) instability for three value of proton fraction $y$.

Figure 2: Time evolution of the density $\rho(x, y)$ in the plane $z=0$ for initial density $\rho^{(0)} = 0.09 fm^{-3}$, at $T=5$ MeV and asymmetry $I = 0.5$. Upper panels show contour plots of $\rho(x, y)$ and lower panels the corresponding two dimensional surface.
2.2 Numerical results: heated nuclear matter in a box

The previous discussion is based on a link between thermodynamics and Fermi liquid approaches. Then numerical approach were performed in order to follow all stages of fluctuation development in the fragment formation process. In the numerical approach we consider nuclear matter in a box of size $L=24\text{fm}$ imposing periodic boundary conditions.

We follow a phase space test particle method to solve the Landau-Vlasov dynamics (using Gaussian wave packets) so the dynamics of nucleon-nucleon collision is also included. An initial temperature is introduced by distributing the test particle momenta according to Fermi distribution. We have followed the space-time evolution of test particles for a value of the initial asymmetry $I=0.5$, at initial density $\rho(0) = 0.09\text{fm}^{-3}$ and $T=5\text{ MeV}$ in such way we start from the region of chemical instability (see Fig.1). The initial density perturbation was created automatically due to the random choice of test particle positions. We report in Fig.2 density distribution in the plane $z=0$ at three time steps $t=0,100,200\text{ fm/c}$,corresponding respectively to initial conditions, intermediate and final stages of the spinodal decomposition (SD). The contour plots delimit the region with density higher than the initial value of density.

![Figure 2: Density distribution at different time steps.](image)

Figure 3: Time evolution of neutron (thicks lines) and proton (thin lines) abundance (a) and asymmetry (b) as function of density.

We report the time evolution of neutron (thick histogram) and proton (thin histogram) abundance (Fig.3a) and asymmetry in various density bins (Fig.3b). The dashed line respectively shows the initial uniform density value $\rho(0) = 0.09\text{fm}^{-3}$ (3a) and the initial asymmetry $I=0.5$ (3b). The drive to higher density regions is clearly different for neutrons and protons: at the end of the dynamical clustering mechanism we have different asymmetries in the liquid and gas phases (see the panel at 200 fm/c). This result is the same
of ref. 10 where the dynamics of mechanical instabilities were studied, demonstrating that also in a complete dynamical calculation the kind of fluctuation associated with chemical and mechanical instabilities are the same.

3 Quantum hadrodynamics model (QHD)

3.1 Treatment of Fock terms

Phenomenological hadronic field theories (Quantum Hadrodynamics, QHD) are widely used in dense nuclear matter studies. In most of the previous works on the subject, the Relativistic Mean Field (RMF) approximation of QHD has been followed. In the RMF the meson fields are treated as classical fields and a Hartree reduction of one body density matrices is used. This implies that each meson field is introduced, with appropriated readjusted couplings, just to describe the dynamics of a corresponding degree of freedom, without mixing due to many-body effects: neutral \( \sigma \) and \( \omega \) mesons are in charge of saturation properties, isospin effects are carried by isovector \( \delta \) and \( \rho \) mesons. In a sense the model represents a straightforward extension of the One-Boson-Exchange (OBE) description of nucleon-nucleon scattering.

Our aim is to introduce explicit many-body effects just evaluating exchange term contributions. Fock terms play an essential role in symmetry breaking and consequent mixing of different degrees of freedom. In particular, in the context of the QHD model, essential properties of nuclear matter come mostly from the two neutral strong meson fields. Hence it is important to evaluate the Fock contribution associated with these fields.

Then we will start from a QHD − II model where the nucleons are coupled to neutral scalar \( \sigma \) and vector \( \omega \) mesons and to the isovector \( \rho \) meson. Self-interaction terms of the \( \sigma \)-field were originally introduced for renormalization reasons and can also be considered as a way to parametrize the density dependence of \( NN \) force. Actually they are also describing medium effects essential to reproduce important properties (compressibility and nucleon effective mass) of nuclear matter around saturation density.

The Lagrangian density for this model is given by:

\[
L = \bar{\psi} [\gamma_{\mu} (i \partial^{\mu} - g_V V^{\mu} - g_\rho B^{\mu} \cdot \tau) - (M - g_s \phi)] \psi + \frac{1}{2} (\partial_\mu \phi \partial^\mu \phi - m_S^2 \phi^2) - \frac{3}{4} \phi^3 - \frac{b}{4} \phi^4 - \frac{1}{4} W_{\mu\nu} W^{\mu\nu} + \frac{1}{2} m_V V_\rho V^\rho - \frac{1}{4} L_{\mu\nu} \cdot L^{\mu\nu} + \frac{1}{2} m_\rho^2 B_\nu B^\nu \quad (3)
\]

where \( W^{\mu\nu}(x) = \partial^{\mu} V^{\nu}(x) - \partial^{\nu} V^{\mu}(x) \) and \( L^{\mu\nu}(x) = \partial^{\mu} B^{\nu}(x) - \partial^{\nu} B^{\mu}(x) \). Here \( \psi(x) \) generally denotes the fermionic field, \( \phi(x) \) and \( V^{\nu}(x) \) represent neutral
scalar and a vector boson fields, respectively. $B^{\nu}(x)$ is the charged vector field and $\tau$ denotes the isospin matrices.

We have treated Hartree-Fock (HF) terms in EOS and transport equation at the same level, for this reason we performed the many-body calculations in the quantum phase space introducing the Wigner transform of the one-body density matrix of the fermion field. This method has the advantages of a direct derivation of dynamical transport equations. In ref. we introduce Fock terms is discussed in detail, here we want to discuss some results for EOS of ANM.

![Figure 4: Symmetry energy per nucleon vs. baryon density. Long dashed line: NLH with $\rho$ meson. Solid line: NLHF results. Dashed line: NLH with $\rho$ and $\delta$ mesons.](image)

3.2 Asymmetric Nuclear Matter

We obtain, thanks to Fock terms, scalar and vector isovector contributions to symmetry energy, generally associated respectively with $\delta$ and $\rho$ mesons, even without isovector mesons. We show the comparison between our Non Linear Hartree-Fock (NLHF) and those of the Non linear Hartree (NLH), including the isovector $\rho$ and $\delta$ mesons, with parameters fitted in order to give the same saturation properties and a symmetry energy at $\rho_0$ of 31 MeV. The region in $(T,\rho)$ plane at different asymmetry of chemical and mechanical region are quite similar to the one of Skyrme interaction (see Fig.1).

In Fig.4 the symmetry energy obtained in NLHF (solid line) in comparison with the result of NLH including both $\rho$ and $\delta$ mesons (dashed line). Actually the kinetic contribution is subtracted (anyway it is the same in all the models), so we will refer to potential symmetry energy $E_{sym}^\text{pot}$. For reference the most common result, including only $\rho$ as isovector meson, is plotted (long dashed
line). In all these relativistic models a quite repulsive density dependence of the symmetry term is obtained, but we notice that the density dependence of $E_{pot}^{sym}$ in the complete NLH+$\rho+\delta$ model is quite different respect to our result. This is due to the fact that in NLHF the coupling in the isovector channels become density dependent. This implies a "softer" behaviour of the potential symmetry term below the saturation density in the NLHF case and a "stiff" behaviour more similar to NLH+$\rho$ above.

![Figure 5: Ratio of isovector and isoscalar amplitudes as function of asymmetry $I$ at half saturation density. The lines have the same meaning of Fig.4](image)

A transport equation can be consistently derived to be used for the study of dynamical evolution of nuclear matter far from normal conditions. We expect that the dynamical evolution can point out the important difference between NLH and NLHF by comparison with experimental data for isospin of IMF (in multifragmentation or neck events) and for isospin flows. We have just found as the latter are affected by density behaviour of $E_{pot}^{sym}$ and the nucleon effective mass splitting. In this view we have performed the study of collective response in ANM by means of a relativistic kinetic equation in linear approximation respect to the fluctuation. In Fig.5 we present the unstable isoscalar-like solution of the dispersion relation for low density NM (the density is $\rho = 0.4\rho_0$), in particular the ratio $\delta\rho_3/\delta\rho_0$ as function of the initial asymmetry. We notice that the NLHF tendency to restore the isotopical symmetry in the spinodal decomposition of neutron rich NM is different respect to NLH result (both dashed and long dashed result). In particular we stress that the discrepancy is equal to the one obtained with Skyrme interaction, in a non relativistic approach, comparing an Asy-soft with an Asy-stiff term for symmetry energy. However at variance with Skyrme results the behaviour of $E_{sym}$ in NLHF appear to be Asy-soft at subnuclear density but it’s Asy-
stiff above saturation density. A mixed behaviour is predicted with expected interesting effects on experimental observable depending on the probed barion density region of the interacting nuclear matter.

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