Compensation method for complex hysteresis characteristics on piezoelectric actuator based on separated level-loop Prandtl–Ishlinskii model

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Abstract Piezoelectric ceramic actuators show nonlinear hysteresis characteristics due to material properties. In order to modify the inverse piezoelectric effect as an ideal linear execution, the classical Prandtl–Ishlinskii (PI) model is usually used on compensation by feedforward control. The PI model performs well on the simple hysteresis characteristics. However, when the output requirements are complex, the PI model has uneven compensation accuracy on the complex hysteresis characteristics and cannot achieve the accuracy as same as the simple hysteresis. This paper proposes a simplification of the complex hysteresis: Separated level-loop PI (SLPI) model. Firstly, use a loop separation logic algorithm simplification of the complex hysteresis characteristics to obtain hysteresis single loops with loop levels and vertexes. Secondly, hysteresis characteristics of each loop are independently modeled using the PI model. Finally, the inverse model is reconstructed by the rollback method to restore a positive sequence of the feedforward voltage and then input the feedforward voltage as a compensation to achieve higher and more uniform accuracy. Experiments and discussions show that the SLPI model can effectively improve the compensation results of complex hysteresis characteristics by 50.3%, and the average compensation accuracy difference between single hysteresis loops is reduced by 53.7%.

Keywords Piezoelectric ceramics actuator · Inverse piezoelectric effect · Nonlinear characteristics · Algorithm identification · Feedforward control compensation

1 Introduction

The inverse piezoelectric effect of piezoelectric materials makes piezoelectric ceramic actuators have the advantages of nanometer precision and fast response [1, 2]. The effect is widely used in precision measurement, micro-nanoprocessing, nanopositioning and other precision mechanical engineering fields [3–6]. However, the nonlinear output exhibited by the inherent characteristics of piezoelectric materials affects the accuracy of the piezoelectric actuator [7]. The hysteresis characteristic is the most important factor leading to the nonlinear characteristic [8].
In order to correct the nonlinear characteristics to an ideal linear state, many international scholars have conducted extensive research on the piezoelectric hysteresis feedforward compensation method \[9\]. Establishing differential equations and operator models are common phenomenological modeling solutions \[10–13\]. The classical PI model is a type of operator model, which has the advantages of simple structure and few parameters in the inversion process and has a wide range of applications \[14, 15\]. The PI model is formed by the weighted superposition of the Play operators. Although the Play operator can describe the hysteresis characteristics of the hysteresis well, its symmetry limits the modeling accuracy of the PI model \[16\]. The improved classical PI models apply methods, such as generalized models and parameters increase models, to keep the hysteresis while getting rid of the symmetry, so that the compensation accuracy can be improved to a certain extent \[17\].

In previous studies, a segmented PI model was also used to divide the hysteresis characteristics into PI model segments \[18, 19\], but this method is only based on the physical properties of the material’s crystal phase change. No segmentation basis is given associated with the operator model. When facing with complex hysteresis, the characteristics data calculation is large, and the increase number of segments will easily cause the inverse model to lose connectivity, which is difficult to apply compensation for complex hysteresis characteristics. Therefore, the above research is only suitable for improving the modeling accuracy of simple hysteresis characteristics. Complex hysteresis has multiple hysteresis loops, and each loop does not have the same characteristics \[20, 21\]. Neither the classical PI model nor the improved PI model can eventually meet the requirements of uniform high-precision compensation for complex hysteresis characteristics \[22, 23\].

In order to solve the problems mentioned above, this paper proposes an SLPI model. Firstly, the complex hysteresis characteristic data are obtained by experiment under the complex working voltage of the device. Secondly, the loop separation logic algorithm is used to simplify the complex hysteresis characteristics and record the loop levels and vertexes, according to the characteristics change law: Madelung principle. Each single loop is modeled by the classical PI model, and the inverse model is calculated. Before compensating, the rollback method is used to restore the positive sequence of the hysteresis inverse model of each inner loop. The complete inverse model is reconstructed according to the vertexes and loop levels and then used for feedforward control. Experiments show that the overall modeling accuracy of the SLPI model is higher than classical PI model, and the accuracy differences between the loops at all levels are reduced, and the compensation result for complex hysteresis characteristics is well improved.

The structure of this paper is as follows. The second section designs a two-degree-of-freedom nanopositioning device based on a flexible mechanism by analyzation and optimization and measures out a set of complex hysteresis characteristic data. In the third section, the characteristics of complex hysteresis data are understood and a reasonable simplification method is derived from the Madelung principle, with the process of level-loop separation logic algorithm and rollback reconstruction for feedforward control described in detail. The fourth section presents the comparison and verification experiment of the compensation control results of SLPI model and classical PI model. The fifth section is an analogy summary of the work of this paper and similar research work.

2 Background

This section contains two parts. The first part shows the device that supports this paper: a precision machining nanopositioning device based on a flexible mechanism, and its design modeling and working principles. The other part is the voltage–displacement complex nonlinear characteristic data measured on the positioning device by experiment, including the detailed process and characteristic analysis.

2.1 A mechanical design based on actuators

The design of the precision machining nanopositioning device refers to a kind of nanopositioning stage that uses a rectangular cut section flexure hinge \[24\]. The working principle of the flexible hinge mechanism is shown in Fig. 1. The expansion and contraction of the piezoelectric ceramic can be amplified by the flexible hinge and transmitted to the driven part. Therefore, the output direction of displacement is perpendicular to the deformation direction of the piezoelectric ceramic.
The piezo-actuated device is used as an adjust mechanism in high-speed turning machine. In order to correct the error of the rotational motion, the real-time signal deviation is found by sensors matrix, and the piezo-actuated positioning device is required to compensate the identified problem smoothly.

The positioning device has two degrees of freedom in the \( x \)- and \( y \)- directions and supports the processing of the planar shape by the working device. When positioning, it is necessary to divide the specified displacement into \( x \)-direction displacement and \( y \)-direction displacement in advance to obtain the corresponding voltage value of each displacement and then cooperate with the voltage drive output to complete the machining nanopositioning. Figure 2 is the three-dimensional model of the working device.

Table 1 is the design stiffness calculation table, where \( k_x, k_y, k_z \) are the stiffness in the \( x \)-, \( y \)- and \( z \)-directions, and \( k_{\theta x}, k_{\theta y}, k_{\theta z} \) are the torsional stiffness around the \( x \)-, \( y \)- and \( z \)-directions, respectively [25]. The flexible hinge part adopts slow-moving wire cutting and nickel plating on the surface of the material, because these processing technologies are widely used [26, 27].

2.2 Data acquisition and characteristic analysis

The experimental system was built according to the equipment shown in Fig. 3. Renishaw’s high-precision laser interferometer and a 0-100 V range voltage controller were selected.

Perform the following steps to test the complex voltage–displacement nonlinear characteristics of the nanopositioning device in the \( x \)-direction:

1) Connect the computer, voltage controller and laser interferometer. The data are collected by the computer’s corresponding software. Adjust the laser interferometer to make the signal intensity within a reasonable range.

2) The random complex voltage is shown in Fig. 4. The initial drive voltage is 0 V and then uses the voltage controller to load the electrical signal according to Fig. 4.s

3) Use the laser interferometer to measure the displacement value and record it once every 0.8 s at an interval of 4 V. The peak and valley displacements of the complex voltage in Fig. 4 need to record additionally.

4) Calculate the average of two measurements. Check the equipment and shut down.

The obtained complex voltage-displacement nonlinear characteristics curve is shown in Fig. 5.

Table 1 is the design stiffness calculation table, where \( k_x, k_y, k_z \) are the stiffness in the \( x \)-, \( y \)- and \( z \)-directions, and \( k_{\theta x}, k_{\theta y}, k_{\theta z} \) are the torsional stiffness around the \( x \)-, \( y \)- and \( z \)-directions, respectively [25]. The flexible hinge part adopts slow-moving wire cutting and nickel plating on the surface of the material, because these processing technologies are widely used [26, 27].

The voltage-displacement nonlinear characteristics of piezoelectric ceramic materials are often caused by the following three reasons [28–31]:

1) Hysteresis characteristics: After voltage is applied to the piezoelectric ceramics, the nonlinear late-achieving characteristics shown by the displacement are called hysteresis characteristics, as shown in Fig. 6a as the voltage-displacement curve. The hysteresis of the boost phase is often shown as a concave curve, and the hysteresis of the back phase is often shown as a convex curve.
2) Creep characteristics: After the applied voltage remains unchanged, the piezoelectric ceramic material still produces additional deformation within a period of time after the immediate response. This deformation is called creep characteristics, as shown in Fig. 6b. The speed of the extra displacement value gradually decreases with time and can reach a stable value after a period of time.

3) Temperature change characteristics: The property of changing the deformation ability of piezoelectric ceramic materials caused by temperature changes is called the temperature change characteristics, as shown in Fig. 6c. Too high or too low temperature will reduce the deformability of piezoelectric ceramics. The ideal working temperature for piezoelectric ceramic materials is between 220 and 390 K.

In the complex voltage-displacement curve shown in Fig. 5, the nonlinear characteristics are almost all hysteresis. When outputting a complex displacement, the residence time at a specific voltage value is short enough, so the influence of creep characteristics can be ignored; when the working environment is room temperature, the temperature change characteristics are not a nonlinear factor. Therefore, the nonlinear characteristics are mainly caused by hysteresis.

In order to ensure the working stability, it is necessary to output linear displacement. It is a simple and effective solution to directly analyze the hysteresis nonlinear characteristics and use the correct mathematical model for compensation.

**Table 1** Stiffness calculated values $k_x, k_y, k_z$: N/mm, $k_{hx}, k_{hy}, k_{hz}$: N-mm-rad

| Stiffness | $k_x$ | $k_y$ | $k_z$ | $k_{hx}$ | $k_{hy}$ | $k_{hz}$ |
|-----------|-------|-------|-------|----------|----------|----------|
|           | $\approx 60$ | $\approx 10^4$ | $\approx 2 \times 10^4$ | $\approx 2 \times 10^7$ | $\approx 10^8$ | $\approx 7 \times 10^6$ |
3 Separated level-loop PI model

This section specifically describes the modeling ideas and steps of the SLPI model. The first part analyzes the characteristic differences of varied levels of hysteresis loops and explains the reason why the classical PI model cannot compensate for complex characteristics with high precision through the feature of the Play operator. The second part uses a logical algorithm of separation to identify the vertexes of each loop and split them according to the loop level. The third part reviews the classical PI model and uses the PI model to describe each separated loop one by one. Each loop is inverted separately after PI modeling. Therefore, the positive sequence for compensation is restored and reconstructed by a rollback method, and the total inverse model for feedforward control is finally obtained.

3.1 Analysis of complex hysteresis characteristics and Play operator

In order to establish a correct mathematical model, the complex hysteresis characteristics and PI model features are further exposed in this part.

When analyzing the characteristics of the hysteresis loop, the Madelung principle is often used as a basis [32, 33]. The Madelung principle can be summarized as the endpoint determination characteristic, the complete closed inner-loop characteristic, the erasing characteristic, the shape consistency, and other hysteresis characteristics. Among them, the shape consistency can be explained according to the Madelung principle as the inner loop with the same voltage difference in the voltage-displacement characteristic curve, the hysteresis characteristics of the step-down section and the step-up section will show the consistency of the shape. However, this also means that the shape of the inner-loop is inconsistent with different voltage differences. When a piezoelectric drive operates with a displacement output like the one in Fig. 5, the hysteresis characteristics of its different inner loops depend on the number of stages of the inner loop. Intuitively, the inclination angle is different.

If the slope is observed at similar positions in the inner loops of different stages, such as the slope of the connection between the maximum and minimum extreme points, or the slope of the midpoint of the buck-boost section. Each hysteresis loop exhibits a different hysteresis inclination angle, as indicated by the tangent line in the schematic diagram of Fig. 7. The complex hysteresis inclination angle between the inner loops greatly affects the modeling accuracy,
such as the difference between the upper level-loop of the highest hysteresis inner loop and the lower level-loop of the hysteresis outer loop.

When analyzing the feature of PI model, first analyze the Play operator that constitutes the PI model, as shown in Fig. 8. When the input is $x(t_n)$, the output Play operator expression $p(t_n)$ can be expressed as

$$
\begin{align*}
\{ p(0) &= \max\{x(0) - r, \min[x(0) + r, p_0]\} \\
\{ p(t_n) &= \max\{x(t_n) - r, \min[x(t_n) + r, p(t_{n-1})]\} \\
\end{align*}
$$

where $r$ is the operator threshold, $n \in [1, e], 0 = t_0 < t_1 < t_2 < \ldots < t_e$ are reasonably divided intervals. When the PI model describing the hysteresis of the inverse piezoelectric effect, the initial value of displacement is usually recorded as 0, So $p_0 = 0$.

When building the model, select the single-sided operator in the first quadrant. As shown in Fig. 8, the properties of the boost phase single-sided operator are

$$
\begin{align*}
\{ p(t) &= 0 \quad x(t) \leq r \\
p(t) &= x(t) - r \quad r < x(t) \leq x(t_e) \\
\end{align*}
$$

the properties of the single-sided operator on the back phase are

$$
\begin{align*}
\{ p(t) &= x(t_e) \quad x(t_e) - 2r \leq x(t) \leq r \\
p(t) &= x(t) + r \quad 0 \leq x(t) < x(t_e) - 2r \\
\end{align*}
$$

It can be seen from the nature of the operator that compared with the classical PI model, the generalized or segmented improved PI models show an increase in accuracy when describing the hysteresis loop under simple voltage, but it does not perform well in the complex hysteresis characteristics with multiple inner loops. There are two main reasons: one is that complex hysteresis data tend to have uneven hysteresis characteristics distribution, so large data segments and small data segments are not suitable for building the optimal model under the same number of iterations; the other is that the model itself increases parameters, so when facing complex hysteresis characteristics, the generalized improved model is embodied in the additional mathematical formulas that continue to be superimposed and nested, and the segmented improvement is embodied in too many segmented points. The influence of a large number of redundant parameter problems and segmental cohesion problems on compensation control cannot be ignored. In addition, the generalized and segmented improved PI model also has other problems such as a significant increase in computation.

The modeling essence of the PI model is the weighted superposition of a limited number of Play operators. From the perspective of modeling features, when describing a single hysteresis loop, the boost phase operator is selected for the data increase edge, and the back phase operator is selected for the data reduction edge, which can model the hysteresis loop with high accuracy without affecting each other. From the perspective of hysteresis characteristics, a complex hysteresis often has multi-loop characteristics. Multiple single loops of different lengths contained in multiple loops correspond to different hysteresis inclination angles. There are some certain voltage values corresponding to more than one unilateral displacement value. Hence, the modeling accuracies of each segment are all affected. Judging from the above two perspectives, in order to eliminate the influence of operator modeling, different hysteresis loops should be modeled strictly according to the independent characteristics of them. Therefore, a single hysteresis level-loop separation logic algorithm is proposed.

![Fig. 8 The Play operator](image-url)
3.2 A logical algorithm for level-loop separation

The Madelung principle highly summarizes the characteristics of the complex hysteresis loop. Except for the law mentioned in the first part of this section, there are other three important laws:

1) Turning point law: the characteristic of hysteresis curve starting from the loop turning point is uniquely determined by this point.

2) Loop formation law: the trend of hysteresis curve starting from a new turning point tends to get back to the last turning point;

3) Regression law: the trend of the curve returns to the last lower level-loop after formation and exceeding the last turning point constructed by the law (2).

It can be judged from the above laws that the complex hysteresis characteristics must be composed of several hysteresis inner loops and one hysteresis outer loop. In order to eliminate the influence of Play operator modeling in the PI model, single hysteresis loops should be separated. The outer loop is defined as the 0-level loop, and the number of levels gradually increases, so the innermost hysteresis ring has the highest level. In the separation process, the priority of separating the data of the high-level loop does not affect the hysteresis characteristics of the low-level loop. To achieve the level-loop separation, it is necessary to obtain the vertex information and loop level information from the complex hysteresis characteristics.

PI model consists of operators that can separated into loops. Each part retains the characteristics of the original complex hysteresis due to the Madelung principle. In modeling, the segmented approach decouples the model features of different level loops. Similarly, different operator parameters are used in the identification between the boost phase and the back phase. It is also the advantage that the classical PI model does not have.

Hence, a logical algorithm of single level-loop separation is used to divide the complex characteristics into the single hysteresis loops at all levels, including the number of loop level and vertex information. In order to facilitate the calculation of the algorithm, the kth experimental data are converted from \(voltage_k, displacement_k\) into the format as

\[NewData(k) : [k, v_k, d_k, tr(k), lv(k), sp(k)]\]

where \(k\) is the measurement sequence when acquiring data, \(v_k\) and \(d_k\) are the \(k\) th voltage data and corresponding displacement data, respectively, \(tr(k)\) is the mark of the boost or back phase: when \(tr(k) = 1\), it is on the boost phase; when \(tr(k) = -1\), it is on the back phase, \(lv(k)\) is the value of hysteresis loop level of the \(k\) th data, \(sp(k)\) is the vertex identifier: the non-vertex data \(sp(k) = 0\); the suspected vertex data \(sp(k) = 1\); the true vertex data \(sp(k) = 2\).

Start logical judgment from the initial data in the data sequence. The outer loop level is 0 level, and the initial is the boost phase. The zero point must be the true vertex of the outer loop or 0 level loop, and hence, the zero point is defined as

\[NewData(0) : [0, 0, 0, 1, 0, 2]\]

After determining the initial new data, judge the next data in order. Due to the data changes at the turning points of the hysteresis curve, the sequence trend must be increased and decreased alternately, so the logic algorithm flow is also an alternate cycle of boost-back logic judgment. When the \(k\) th data are not a turning point in the hysteresis curve, it meets the condition of

\[
\begin{align*}
v_{k+1} - v_k & > 0 \quad tr(k) = 1 \\
v_{k+1} - v_k & < 0 \quad tr(k) = -1
\end{align*}
\]

At this time, the \(k\) th data are impossible to be a vertex, so \(sp(k) = 0\). The level value of subsequent data hysteresis loops remains unchanged: \(lv(k) = lv(k - 1)\), and the boost-back mark remains unchanged: \(tr(k) = tr(k - 1)\).

The decrease data appearing in the boost phase and the increasing data appearing in the back phase are the turning points of the hysteresis curve. When the limited influence of creep characteristics on the data being ignored, a turning point meets the condition of

\[
\begin{align*}
v_{k+1} - v_k & < 0 \quad tr(k) = 1 \\
v_{k+1} - v_k & > 0 \quad tr(k) = -1
\end{align*}
\]

The Madelung principle stipulates that two turning points form a closed loop, and one of them is the vertex of the closed loop. Therefore, when a new turning point \(NewData(k_x)\) is judged by Eq. (7), the current data should be marked as a suspected vertex: \(sp(k) = 1\). Then add one to the value of hysteresis...
loop level for the data: \( lv(k) = lv(k - 1) + 1 \) and invert the boost-back mark: \( tr(k) = -tr(k - 1) \). The suspected vertex needs to use subsequent logic to determine whether the uncertain vertex is removed from the suspicion or becomes a true vertex. The authenticity of suspected vertexes is judged in the following situations:

1. When the logic judgment flow is in the back phase \( tr(k) = -1 \), if the voltage data value judgment goes less than or equal to the last suspected vertex \( NewData(k_a) \), whose value of level is current level minus one. Then confirm \( sp(k_a) = 2 \);

2. When the logic judgment flow is in the boost phase \( tr(k) = 1 \), if the voltage data value judgment goes greater than or equal to the last suspected vertex \( NewData(k_a) \), whose value of level is current level minus one. Then confirm \( sp(k_a) = 2 \);

After confirming that \( NewData(k_a) \) is the true vertex with the hysteresis loop level of \( lv(k_a) = lv(k) - 1 \), separate all the data of this level under the hysteresis single loop starting and ending from the vertex. The separated data are sewed at the vertex to form a single hysteresis loop with level value and vertex information. Then set \( lv(k + 1) = lv(k) - 2 \) and continue to make subsequent judgments.

If the hysteresis loop separation has been performed on a higher level loop before the new separation, separation will be done on the remaining data since last separation. The remaining data are sewed together at the vertex of the previously separated single hysteresis loop to form a new hysteresis characteristic waiting for the next separation.

After operating the single loop separation logic algorithm, the original complex multi-loop hysteresis characteristics are separated into several hysteresis single loops of different levels from the highest value to 0. Figure 9 is three hysteresis single-loop images obtained from the data in Fig. 5 after the single loop separation logic algorithm. The triangle signs mark the vertexes as loop original point of each level of hysteresis loops, and the circle signs mark the position of the sewed point as recovery of the lower hysteresis loop after the higher one is separated. Figure 9a is the highest level value inner loop in the experimental data, so this single hysteresis loop is preferentially separated from the level II true vertex. Figure 9b is the level I hysteresis loop. The data are separated from the level I true vertex and sewed at the level II true vertex. Figure 9c is the remaining level 0 outer hysteresis loop, so the true vertex of the outer loop is the zero point.

In a period of complex hysteresis nonlinear characteristics, the largest data of the curve, which is also a turning point, will be defined as a suspected level I vertex when it is detected. Therefore, suspected vertexes newly appearing on the back phase of outer loop will continue to increase the level value, while the original suspected vertex still exists, until the judgment finished. The phenomenon of adding one to level value in the back phase will not affect the result of the single loop separation logic algorithm, because the purpose of the logic algorithm is to separate the hysteresis single loop, and the separation step depends only on the level values and the position of the true vertexes. Consequently, the exact number of level value is irrelevant.

In conclusion, the single loop separation logic algorithm imports the data in the order measured in the experiment and applies the sequential conditional sentence detection algorithm of the programming language. Logical judgment filters out the hysteresis loop vertexes with loop value information. Then, the obtained level value information is used for separation from highest value to 0. In the next two part of this section, high-precision modeling of complex hysteresis characteristics can be obtained by using the PI modeling on each single loop.

3.3 Prandtl–Ishlinskii model and inverse model

Each obtained single loop hysteresis is modeled by PI model independently. As shown in Fig. 8, unilateral operators are selected for the boost and back phases, respectively. The selected operators are weighted and superimposed. The PI model can be expressed as:

\[
P[x(t)] = \omega_0 \cdot x(t) + \sum_{i=1}^{n} \omega_i \cdot p_i(t) \tag{8}
\]

where \( P[x(t)] \) is the PI model output corresponding to the operator input \( x(t) \), \( \omega_0 \) is the first weight, usually a positive number. The corresponding output of the \( i \) th operator is \( p_i(t) \), which has a corresponding threshold \( r_i \) and a corresponding weight \( \omega_i \).
Figure 10 is a modeling diagram of three single hysteresis loops obtained by separation method by the PI model. It can be seen from the modeling results that the PI model has high modeling accuracy for each hysteresis single loops with different hysteresis inclination angles, which verifies that the theory of eliminating the overlapping effects of operator features is feasible.

When modeling the hysteresis in Fig. 10, a quadratic programming iteration is used to find the optimal parameter values. In this algorithm, the minimum number of iterations to obtain the optimal modeling for each single loop is different. For a single loop of length $m$, the minimum iterations $A_m$ are usually taken as:

$$A_m = 10 \sqrt{\frac{m_e}{m}} + 1$$

where $m_e$ is the total length of experimental data.

Tables 2, 3 and 4 are the parameter of Fig. 10, respectively. The threshold $r_i$ and optimal weight $\omega_i$ are shown in the tables.

In the SLPI model, the minimum number of iterations can be selected according to the length of the separated data, so that the modeling of each hysteretic single loop can reach the highest accuracy as well as avoid redundant calculations. However, PI model for the whole complex hysteresis characteristics does not have the existence condition of the minimum iterations, so the uniform number of iterations leads to modeling error.
Fig. 10  Single loop PI modelings. a Level II single loop modeling, b Level I single loop modeling, c Level 0 single loop modeling

| Table 2 | Parameter values of Fig. 10a |
|---------|-----------------------------|
| $i$     | $r_i$ | $\omega_i$ |
| 1       | 0     | 0.02943    |
| 2       | 5     | 0.01351    |
| 3       | 10    | 0.00040    |
| 4       | 15    | 0.00206    |
| 5       | 20    | 0.00230    |
| 6       | 25    | 0.00265    |
| 7       | 30    | 0.00249    |
| 8       | 35    | 0.00550    |
| 9       | 40    | 0.02802    |
| 10      | 45    | 0.09411    |

| Table 3 | Parameter values of Fig. 10b |
|---------|-----------------------------|
| $i$     | $r_i$ | $\omega_i$ |
| 1       | 0     | 0.03151    |
| 2       | 5     | 0.01298    |
| 3       | 10    | 0.01092    |
| 4       | 15    | 0.00445    |
| 5       | 20    | 0.00648    |
| 6       | 25    | 0.00000    |
| 7       | 30    | 0.00148    |
| 8       | 35    | 0.00016    |
| 9       | 40    | 0.00051    |
| 10      | 45    | 0.00100    |
The PI model has an analytical inverse model. The SLPI model is composed of a combination of independent single-loop PI models. Therefore, the inverse model equation of each separated single loop is consistent with the classical PI inverse model. The formula is expressed as

$$P^{-1}[p(t)] = \omega_0^{-1} \cdot p(t) + \sum_{i=1}^{n} \omega_i^{-1} \cdot x_i(t)$$  \hfill (10)$$

where $P^{-1}[p(t)]$ is the output corresponding to the input $p(t)$ of the classical PI inverse model operator, and $x_i(t)$ is the output of the $i$th operator. To correspond to the PI positive model, $x_0 = 0$. The threshold $r_i^{-1}$ and weight coefficient $\omega_i^{-1}$ of the PI inverse model are expressed as

$$\begin{align*}
    r_i^{-1} & = \sum_{j=1}^{i} \omega_j \cdot (r_i - r_j) \\
    \omega_0^{-1} & = \frac{1}{\omega_0} \\
    \omega_i^{-1} & = -\frac{\omega_i}{(\omega_0 + \sum_{j=1}^{i-1} \omega_j)(\omega_0 + \sum_{j=1}^{i} \omega_j)}
\end{align*}$$  \hfill (11)$$

On the basis of dealing with each single loop inverse model, a complete SLPI inverse model can be established and the feedforward control method can be used to compensate the complex hysteresis characteristics.

### 3.4 Compensation theory and inverse SLPI model

The method of feedforward control collects information through pre-experiment, observes the law to estimate the trend, and applies pre-control methods in front of the system to correct the compensation result. It is one of the commonly used compensation control methods in control engineering [34–36].

The nonlinear hysteresis characteristics shown by the voltage-displacement curve of the inverse piezoelectric effect follow Madelung principle, and there is no occasional variability and no need of feedback in the control. The PI model used in the modeling is a phenomenological model that describes the characteristics directly without explaining the internal reasons. Therefore, the feedforward control is the most effective compensation method for the piezoelectric complex hysteresis characteristics. The schematic diagram of feedforward compensation control is shown in Fig. 11.

It can be seen from Fig. 11 that the PI model of hysteresis single loops should be sorted out to construct the SLPI inverse model for the feedforward control compensation voltage. After obtaining the PI model of single loops, the reconstruction steps are:

1) Single loop model inversion: The analytical inverse model of each hysteretic single loop is obtained through the PI inverse model equation. The level value information of the hysteretic single loop is retained, as well as the true vertex modeling position and the sewed connection point position.

2) Single-loop inverse model restoration: The sewed point on a single hysteretic loop is the vertex of a higher-level valued hysteresis loop starting from this loop. The inverse model of the hysteresis single loop must be clockwise. Therefore, starting from the level 0 outer loop, the sewed point on the current hysteresis loop is changed into a break point. The inverse model of current loop is divided into two parts. The outer loop inverse model is linked to the inverse...
model of new loop at the break position of the inverse model of the level I inner loop. The new inverse model sequence is obtained in two loops which has two clockwise direction. Repeat this step to reconstruct the higher level hysteresis single loop until all the inverse models of the single inner loops are connected at break points. Finally, all inverse model restoration is completed.

In the sequence restoration process, the vertexes or the break points of the same level should be divided and spliced one by one in the order of initial separation level value, without affecting the reconstruction result of the achieved inverse model restoration.

**Fig. 13** The SLPI inverse model

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**Fig. 12** Single loop inverse model. a Level II single loop inverse model, b level I single loop inverse model, c level 0 single loop inverse model
Figure 12 is the inverse model corresponding to the three hysteresis single loop modeling in Fig. 10, respectively. The star sign is the position of the break point on the level I inverse model and the level 0 inverse model. Figure 13 shows the inverse model of SLPI after sorting and reconstructing according to the restoration method above.

The SLPI inverse model is the displacement-voltage correspondence curve. The obtained compensation voltage can be directly used for feedforward control. The compensation result of feedforward control needs to be further verified by experiments and discussion.

4 Experiment and discussion

The fourth section shows the experimental verification and result discussion. Two fundamental comparisons are shown by diagrams. One is to compare the modeling description of the classical PI model and the SLPI model, and the other is to compare the final compensation results.

In addition, this section adds another two sets of special verification experiments. One is to verify whether the inner loop starting from the back phase hysteresis characteristics causes error in the modeling. The other is to verify whether the overlapped outer loop data affect the compensation method.

4.1 Model comparison

The modeling description determines the compensation result. Observing the modeling error between different models and the actual hysteresis characteristics curve can directly reflect the comparison of accuracy. The modeling of the classical PI model is shown in Fig. 14a. The modeling of the three hysteretic single loops is drawing on the same diagram as SLPI modeling, as shown in Fig. 14b.

Table 5 is the parameter of Fig. 14a. The threshold $r_i$ and optimal weight $w_i$ are shown in the tables.

By comparing the modeling diagram in Fig. 14, it can be seen that the SLPI model has a higher modeling accuracy and shows the adaptive feature of the inherent material characteristics of the outer loop. Different from the phenomenon that the accuracy gets worse in higher-level value loop modeling, the SLPI model has always maintained the accuracy.

In order to verify the comparison of the actual compensation results, the experiments are performed using the compensation voltage of both the classical PI inverse model and the SLPI inverse model as follow:

(1) Place experiment equipment such as piezoelectric driver, laser interferometer and reflector on the horizontal test bench and connect it with voltage driver and computer.

(2) Load the compensation voltage obtained from the classical PI inverse model through the controller at the ordinary control speed and record the displacement. Similarly, load the compensation voltage obtained from the SLPI
inverse model at the ordinary control speed and record the displacement of the experiment.

(3) Reduce the control speed of voltage loading and record displacements for the two model compensation results again.

(4) Deal with the displacement data and calculate the average value of the actual displacement output according to the compensation results of the different models measured above.

(5) Check the equipment and turn off the power.

Due to little influence produced by the creep characteristics, the experiment used the measurement method of changing the compensation control speed to obtain the average value of the compensation result. The comparison results are as follows, Fig. 15a is the compensation result curve of the classical PI inverse model, and Fig. 15b is the compensation result of the SLPI inverse model under compensation voltage.

Taking the value on the straight line passing through the zero point and the maximum value of the experimental data as the expected linear output value, the compensation error is the difference between the compensation result and the expected result. The error ε and the average error $\bar{ε}$ are:

$$\epsilon_i = |\delta_i - c_i|$$

$$\bar{\epsilon} = \frac{1}{\lambda} \sum_{i=1}^{\lambda} |\delta_i - c_i|$$

where $\lambda$ is the total number of experimental samples for compensation control, $\tau \in [0, \lambda]$, $\delta_i$ is the expected displacement output, and $c_i$ is the real displacement output.

As predicted by the comparative modeling description, the compensation results of the SLPI inverse model are better than the classical PI model. Specifically embodied as:

(1) Improvement of inner loop compensation: The compensation effect of high-level inner loop hysteresis is significantly improved, and it effectively solves the problem that the higher level loop has lower accuracy in the complex hysteresis characteristics. Meanwhile, the problem that the obvious compensation error difference between hysteresis loops is also corrected.

(2) The stability of the outer loop compensation: Based on the compensation of the SLPI model, the compensation results on the material inherent characteristics of the compensation outer loop do not show lower accuracy than the classical PI model. The SLPI model’s error is even smaller, and the accuracy is higher.

(3) Self-elimination of compensation error: When using the SLPI inverse model for hysteresis compensation, the cumulative error and the new error will erase each other more often than they overlap each other. This increase in accuracy may occur accidentally due to the position of the vertex of the hysteresis loop, but it does not deny the possibility that the single loop modeling itself may eliminate the asymmetry features, which is a phenomenon beneficial to compensation control.

Reference [19] uses an MSPI model. The MSPI model is a very good method. It is fitted together by each segment to further improve the fitting accuracy of the outer-loop hysteresis nonlinearity. SLPI’s inverse modeling method is better than MSPI model. Specifically embodied as:

The SLPI model that only separates the inner-loop can achieve the same modeling accuracy and compensation accuracy as the MSPI model segmented everywhere, while significantly reducing the number of discontinuities.

In the calculation result of the percentage index, the SLPI inverse model compensation reduces the maximum error by 10.0%, and the accuracy of the highest level inner loop is increased by 97.5%. The error difference between single hysteresis loops is reduced by 53.7% compared with the classical PI inverse

| Table 5 Parameter values of Fig. 14a |
|---|---|---|
| $i$ | $r_i$ | $\omega_i$ |
| 1 | 0 | 0.03040 |
| 2 | 5 | 0.01116 |
| 3 | 10 | 0.01516 |
| 4 | 15 | 0.00478 |
| 5 | 20 | 0.00000 |
| 6 | 25 | 0.00000 |
| 7 | 30 | 0.00823 |
| 8 | 35 | 0.00057 |
| 9 | 40 | 0.00000 |
| 10 | 45 | 0.00000 |
model, and the overall average error is greatly reduced by 50.3%.

4.2 Other verifications

Past experiments of inverse piezoelectric effect show that external disturbances such as frequency of control voltage rarely affect the application of the PI model. Therefore, whether the compensation of the SLPI inverse model keeps excellent effect depends on the characteristics of its own modeling, so two sets of verification experiments are designed.

The first experiment used the double inner loop hysteresis data of a nanopositioning stage device, as shown in Fig. 16. It can be seen from the data that one of the true vertexes is located on the back phase of the outer loop. It will be verified by observing the modeling diagram whether the hysteresis characteristics starting from the back phase of outer loop can also be compensated by the SLPI model.

Figure 17a is the optimal modeling diagram of the classical PI model of double inner loop hysteresis data. It can be seen that the classical PI model will share the error on each hysteresis loop in order to maintain the overall optimal modeling, which results in a large number of low fitting modeling. This phenomenon is more obvious in the case of multiple inner loops, especially when there are vertexes for both the boost and back phase. Figure 17b is a gathered diagram of each hysteresis single loop modeling in the SLPI

Fig. 15 Compensation result diagrams. a Classical PI compensation, b SLPI compensation, c partially enlarged Fig. 15a, d partially enlarged Fig. 15b
model. Although the description of material inherent nonlinear characteristics cannot be improved by single outer loop modeling, the influence of the vertex on the back phase is almost eliminated.

The second set of experiments is the hysteresis data of another piezoelectric actuator device, as shown in Fig. 18. It can be seen from the diagram that there are overlapped data of the inner hysteresis loop and the outer hysteresis loop. Therefore, it is verified by modeling whether the repeated hysteresis characteristics on different hysteresis single loops are also compensated well by the SLPI model.

Figure 19a is the optimal modeling diagram of the classical PI model, and Fig. 19b is the gathered diagram of each hysteresis single loop modeling by the SLPI model. Through comparison, it can be seen that the two single loop modelings are not affected by the overlapped outer loop hysteresis. The maximum errors of classical PI model still appear in the inner loop, and SLPI model effectively corrects this error.

In conclusion, in the first set of verification experiments, the modeling error difference between the two inner loops of the SLPI model is smaller than classical PI model, and both loops have high accuracy. Compared with the large error jump of classical PI modeling, the SLPI modeling of the material inherent characteristics of the outer loop is closer to the nonlinear curve, and hence, the overall accuracy is
also improved. In the second set of control experiments, the SLPI modeling is not affected by repeated hysteresis characteristics. The modeling description of SLPI model is better. Therefore, the SLPI model has high fitting accuracy for all kinds of complex hysteresis characteristics, and the improved modeling accuracy indirectly supports the enhancement in the compensation control.

5 Conclusion

The SLPI model generally improves the compensation control accuracy of complex piezoelectric nonlinear characteristics. Experiments have verified that the SLPI model not only has higher modeling accuracy than the classic PI model, but also performs better in the compensation result. The use of the SLPI model reduces the maximum error by 10.0% and increases the accuracy of the highest-level inner loop by 97.5%. Compared with the classical PI inverse model, the error difference between single loops is reduced by 53.7%, so the overall average error is also reduced by 50.3%. There is no error calculation in the two additional experiments, but it can be observed from the images that the SLPI model has a good modeling effect on all kinds of inverse piezoelectric effect nonlinear characteristics data.

In the establishment of the SLPI model, no new operators were introduced, and feedforward control was used as the same compensation method as the previous principle. Through the analysis of the Play operator and the PI model, the hysteresis advantage of the single-side Play operator is used, and the shortcomings of the PI model corresponding to multiple segments of one voltage are avoided. A single-loop independent hysteresis can be separated from the complex hysteresis characteristics. The segmented basis for modeling is to simplify the complex hysteresis nonlinearity to single loops model. Compared with the past segmented PI model, a reasonable separation basis is added to reduce the number of discontinuities. The SLPI model not only solves the phenomenon that the higher level of the inner ring leads to the lower precision of the modeling, but also enhances the adaptability of the inherent characteristic curve of the outer ring material. Compared with the MSPI model, it adds a reasonable separation basis for multi-loop complex hysteresis characteristics, reduces the number of discontinuities, and can achieve similar compensation accuracy.

In this paper, it is still the inverse PI model that used to solve the compensation control problem, which is a continuation of the previous work. The progress of the research in this paper provides a train of modeling thought and theoretical basis for simplifying the complex characteristics. The new concepts such as the endpoint and series of each hysteresis single loop proposed for nonlinear characteristic data provide

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**Fig. 19** Comparison of overlapped hysteresis modeling. **a** Classical PI modeling, **b** SLPI modeling
sufficient reference value for subsequent research on real-time compensation.

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Data availability The datasets during the current study are available from the corresponding author on reasonable request.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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