Three-loop universal anomalous dimension of the Wilson operators in $\mathcal{N} = 4$ Supersymmetric Yang-Mills theory

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Abstract

We present results for the three-loop universal anomalous dimension $\gamma_{uni}(j)$ of Wilson twist-2 operators in the $\mathcal{N} = 4$ Supersymmetric Yang-Mills theory. These expressions are obtained by extracting the most complicated contributions from the three-loop anomalous dimensions in QCD. This result is in agreement with the hypothesis of the integrability of $\mathcal{N} = 4$ Supersymmetric Yang-Mills theory in the context of AdS/CFT-corrrespondence.

1 Introduction

The anomalous dimensions of the twist-2 Wilson operators govern the Bjorken scaling violation for parton distributions in a framework of Quantum Chromodynamics (QCD). These quantities are given by the Mellin transformation

$$\gamma_{ab}(j) = \int_0^1 dx \, x^{j-1} W_{b \to a}(x) \tag{1}$$

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of the splitting kernels $W_{b \to a}(x)$ for the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation which evolves the parton densities $f_a(x, Q^2)$ (hereafter $a = \lambda, g, \phi$ for the spinor, vector and scalar particles, respectively) as follows

$$\frac{d}{d \ln Q^2} f_a(x, Q^2) = \int_x^1 \frac{dy}{y} \sum_b W_{b \to a}(x/y) f_b(y, Q^2).$$

(2)

The anomalous dimensions and splitting kernels in QCD are known up to the next-to-next-to-leading order (NNLO) of the perturbation theory. The QCD expressions for anomalous dimensions can be transformed to the case of the $\mathcal{N}$-extended Supersymmetric Yang-Mills theories (SYM) if one will use for the Casimir operators $C_A, C_F, T_f$ the following values $C_A = C_F = N_c, T_f n_f = NN_c/2$. For $\mathcal{N}=2$ and $\mathcal{N}=4$-extended SYM the anomalous dimensions of the Wilson operators get also additional contributions coming from scalar particles. These anomalous dimensions were calculated in the next-to-leading order (NLO) for the $\mathcal{N}=4$ SYM. However, it turns out, that the expressions for eigenvalues of the anomalous dimension matrix in the $\mathcal{N}=4$ SYM can be derived directly from the QCD anomalous dimensions without tedious calculations by using a number of plausible arguments. The method elaborated in Ref. [4] for this purpose is based on special properties of the integral kernel for the Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation in this model and a new relation between the BFKL and DGLAP equations (see [8]). In the NLO approximation this method gives the correct results for anomalous dimensions eigenvalues, which was checked by direct calculations in Ref. [5]. Its properties will be reviewed below only shortly and a more extended discussion can be found in [4]. Using the results for the NNLO corrections to anomalous dimensions in QCD and the method of Ref. [4] we derive the eigenvalues of the anomalous dimension matrix for the $\mathcal{N}=4$ SYM in the NNLO approximation.

The obtained result is very important for the verification of the various assumptions coming from the investigations of the properties of a conformal operators in the context of AdS/CFT correspondence.

## 2 Evolution equation in $\mathcal{N}=4$ SYM

The reason to investigate the BFKL and DGLAP equations in the case of supersymmetric theories is related to a common belief, that the high symmetry may significantly simplify their structure. Indeed, it was found in the leading logarithmic approximation (LLA) that the so-called quasi-partonic operators in $\mathcal{N}=1$ SYM are unified in supermultiplets with anomalous dimensions obtained from the universal anomalous dimension $\gamma_{uni}(j)$ by shifting its argument by an integer number. Further, the anomalous dimension matrices for twist-2 operators are fixed by the superconformal invariance. Calculations in the maximally extended $\mathcal{N}=4$ SYM, where the coupling constant is not renormalized, give even more remarkable results. Namely, it turns out, that here all twist-2 operators enter in the same multiplet, their anomalous dimension matrix is fixed completely by the superconformal invariance and its universal anomalous dimension in LLA is proportional to
\[ \Psi(j-1) - \Psi(1), \text{ which means, that the evolution equations for the matrix elements of quasi-partonic operators in the multicolour limit } N_c \to \infty \text{ are equivalent to the Schrödinger equation for an integrable Heisenberg spin model } \text{[16][17]. In QCD the integrability remains only in a small sector of these operators } \text{[18] (see also [19]). In the case of } N = 4 \text{ SYM the equations for other sets of operators are also integrable [20][11][21].} \\

Similar results related to the integrability of the multi-colour QCD were obtained earlier in the Regge limit [22]. Moreover, it was shown [8], that in the \( N = 4 \) SYM there is a deep relation between the BFKL and DGLAP evolution equations. Namely, the \( j \)-plane singularities of anomalous dimensions of the Wilson twist-2 operators in this case can be obtained from the eigenvalues of the BFKL kernel by their analytic continuation. The NLO calculations in \( N = 4 \) SYM demonstrated [4], that some of these relations are valid also in higher orders of perturbation theory. In particular, the BFKL equation has the property of the hermitian separability, the linear combinations of the multiplicatively renormalized operators do not depend on the coupling constant, the eigenvalues of the anomalous dimension matrix are expressed in terms of the universal function \( \gamma_{uni}(j) \) which can be obtained also from the BFKL equation [4]. The results for \( \gamma_{uni}(j) \) were checked by direct calculations in Ref. [5].

3 Method of obtaining the eigenvalues of the AD matrix in \( N = 4 \) SYM

In the \( N = 4 \) SYM theory [23] one can introduce the following colour and \( SU(4) \) singlet local Wilson twist-2 operators [4][5]:

\[
\begin{align*}
O^a_{\mu_1, \ldots, \mu_j} &= \hat{S} C^a_{\rho \mu_1} D_{\mu_2} D_{\mu_3} \ldots D_{\mu_{j-1}} G^a_{\rho \mu_j}, \\
\tilde{O}^a_{\mu_1, \ldots, \mu_j} &= \hat{S} G^a_{\rho \mu_1} D_{\mu_2} D_{\mu_3} \ldots D_{\mu_{j-1}} \tilde{G}^a_{\rho \mu_j}, \\
O^\lambda_{\mu_1, \ldots, \mu_j} &= \hat{S} \lambda^a_{\rho \mu_1} \gamma_{\mu_2} D_{\mu_3} \ldots D_{\mu_{j-1}} \lambda^a_i, \\
\tilde{O}^\lambda_{\mu_1, \ldots, \mu_j} &= \hat{S} \tilde{\lambda}^a_{\rho \mu_1} \gamma_{\mu_2} D_{\mu_3} \ldots D_{\mu_{j-1}} \lambda^a_i, \\
O^\phi_{\mu_1, \ldots, \mu_j} &= \hat{S} \tilde{\phi}^a_{\rho \mu_1} D_{\mu_2} D_{\mu_3} \ldots D_{\mu_j} \phi^a_r,
\end{align*}
\]

where \( D_{\mu} \) are covariant derivatives. The spinors \( \lambda_i \) and field tensor \( G_{\rho \mu} \) describe gluinos and gluons, respectively, and \( \phi_r \) are the complex scalar fields. For all operators in Eqs. (3)-(7) the symmetrization of the tensors in the Lorentz indices \( \mu_1, \ldots, \mu_j \) and a subtraction of their traces is assumed. Due to the fact that all twist-2 operators belong to the same supermultiplet the eigenvalues of anomalous dimensions matrix can be expressed through one universal anomalous dimension \( \gamma_{uni}(j) \) with shifted argument\(^\dagger\).

As it was already pointed out in the Introduction, the universal anomalous dimension can be extracted directly from the QCD results without finding the scalar particle con-

\(^\dagger\)Non-diagonal elements of the anomalous dimensions matrix are related with non-forward anomalous dimensions by means of superconformal Ward identities [24] and can be expressed also through non-forward universal anomalous dimension [20].
tribution. This possibility is based on the deep relation between the DGLAP and BFKL dynamics in the $\mathcal{N} = 4$ SYM \[3\].

To begin with, the eigenvalues of the BFKL kernel turn out to be analytic functions of the conformal spin $|n|$ at least in two first orders of perturbation theory \[4\]. Further, in the framework of the DR-scheme \[26\] one can obtain from the BFKL equation (see \[8\]), that there is no mixing among the special functions of different transcendentality levels $i$, i.e. all special functions at the NLO correction contain only sums of the terms $\sim 1/j^i$ $(i = 3)$. More precisely, if we introduce the transcendentality level $i$ for the eigenvalues $\omega(\gamma)$ of integral kernels of the BFKL equations in an accordance with the complexity of the terms in the corresponding sums

$$
\Psi \sim 1/\gamma, \quad \Psi' \sim \beta(2) \sim 1/\gamma^2, \quad \Psi'' \sim \beta''(3) \sim 1/\gamma^3,
$$

then for the BFKL kernel in the leading order (LO) and in NLO the corresponding levels are $i = 1$ and $i = 3$, respectively.

Because in $\mathcal{N} = 4$ SYM there is a relation between the BFKL and DGLAP equations (see \[8\]), the similar properties should be valid for the anomalous dimensions themselves, i.e. the basic functions $\gamma^{(0)}_{uni}(j)$, $\gamma^{(1)}_{uni}(j)$ and $\gamma^{(2)}_{uni}(j)$ are assumed to be of the types $\sim 1/j^i$ with the levels $i = 1$, $i = 3$ and $i = 5$, respectively. An exception could be for the terms appearing at a given order from previous orders of the perturbation theory. Such contributions could be generated and/or removed by an approximate finite renormalization of the coupling constant. But these terms do not appear in the DR-scheme.

It is known, that at the LO and NLO approximations (with the SUSY relation for the QCD color factors $C_F = C_A = N_c$) the most complicated contributions (with $i = 1$ and $i = 3$, respectively) are the same for all LO and NLO anomalous dimensions in QCD \[2\] and for the LO and NLO scalar-scalar anomalous dimensions \[5\]. This property allows one to find the universal anomalous dimensions $\gamma^{(0)}_{uni}(j)$ and $\gamma^{(1)}_{uni}(j)$ without knowing all elements of the anomalous dimensions matrix \[4\], which was verified by the exact calculations in \[5\].

Using above arguments, we conclude, that at the NNLO level there is only one possible candidate for $\gamma^{(2)}_{uni}(j)$. Namely, it is the most complicated part of the QCD anomalous dimensions matrix (with the SUSY relation for the QCD color factors $C_F = C_A = N_c$). Indeed, after the diagonalization of the anomalous dimensions matrix its eigenvalues should have this most complicated part as a common contribution because they differ each from others only by a shift of the argument and their differences are constructed from less complicated terms. The non-diagonal matrix elements of the anomalous dimensions matrix contain also only less complicated terms (see, for example, anomalous dimensions exact expressions at LO and NLO approximations in Refs. \[2\] for QCD and \[5\] for $\mathcal{N} = 4$ SYM) and therefore they cannot generate the most complicated contributions to the eigenvalues of anomalous dimensions matrix.

Thus, the most complicated part of the NNLO QCD anomalous dimensions should coincide (up to color factors) with the universal anomalous dimension $\gamma^{(2)}_{uni}(j)$.

\[\dagger\]Note that similar arguments were used also in \[27\] to obtain analytic results for contributions of some complicated massive Feynman diagrams without direct calculations.
4 NNLO anomalous dimension for $\mathcal{N} = 4$ SYM

The final three-loop result for the universal anomalous dimension $\gamma_{uni}(j)$ for $\mathcal{N} = 4$ SYM is

$$\gamma(j) \equiv \gamma_{uni}(j) = \hat{a} \gamma_{uni}^{(0)}(j) + \hat{a}^2 \gamma_{uni}^{(1)}(j) + \hat{a}^3 \gamma_{uni}^{(2)}(j) + \ldots, \quad \hat{a} = \frac{\alpha N_c}{4\pi},$$

where

$$\begin{align*}
\frac{1}{4} \gamma_{uni}^{(0)}(j+2) &= -S_1, \\
\frac{1}{8} \gamma_{uni}^{(1)}(j+2) &= \left( S_3 + 3\overline{S}_{-3} \right) - 2\overline{S}_{-2,1,1} - 2S_1 \left( S_2 + \overline{S}_{-2} \right), \\
\frac{1}{32} \gamma_{uni}^{(2)}(j+2) &= 2\overline{S}_{-3} S_2 - S_5 - 2\overline{S}_{-2} S_3 - 3\overline{S}_{-5} + 24\overline{S}_{-2,1,1,1} \\
&+ 6 \left( \overline{S}_{-4,1,1} + \overline{S}_{-3,2,2} + \overline{S}_{-2,3,2} \right) - 12 \left( \overline{S}_{-3,1,1,1} + \overline{S}_{-2,1,1,2} + \overline{S}_{-2,2,1,1} \right) \\
&- \left( S_2 + 2S_1^2 \right) \left( 3\overline{S}_{-3} + S_3 - 2\overline{S}_{-2} \right) - S_1 \left( 8\overline{S}_{-4} + \overline{S}_{-3,2}^2 \right) \\
&+ 4S_2 \overline{S}_{-2} + 2S_2^2 + 3S_4 - 12\overline{S}_{-3,1,1} - 10\overline{S}_{-2,2} + 16\overline{S}_{-2,1,1,1}
\end{align*}$$

and $S_a \equiv S_a(j)$, $S_{a,b} \equiv S_{a,b}(j)$, $S_{a,b,c} \equiv S_{a,b,c}(j)$ are harmonic sums

$$S_a(j) = \sum_{m=1}^{j} \frac{1}{ma}, \quad S_{a,b,c,\ldots}(j) = \sum_{m=1}^{j} \frac{1}{ma} S_{b,c,\ldots}(m),$$

$$S_{-a}(j) = \sum_{m=1}^{j} \frac{(-1)^m}{ma}, \quad S_{-a,b,c,\ldots}(j) = \sum_{m=1}^{j} \frac{(-1)^m}{ma} S_{b,c,\ldots}(m),$$

$$\overline{S}_{-a,b,c,\ldots}(j) = (-1)^j S_{-a,b,c,\ldots}(j) + S_{-a,b,c,\ldots}(\infty) \left( 1 - (-1)^j \right).$$

The expression (13) is defined for all integer values of arguments (see [28, 30, 29]) but can be easily analytically continued to real and complex $j$ by the method of Refs. [30, 31, 29].

5 Integrability and the AdS/CFT-correspondence

The investigation of the integrability in $\mathcal{N} = 4$ SYM for a BMN-operators [31] gives a possibility to find the anomalous dimension of a Konishi operators [11], which has the anomalous dimension coinciding with our expression (8) for $j = 4

$$\gamma_{uni}(j)|_{j=4} = -6 \hat{a} + 24 \hat{a}^2 - 168 \hat{a}^3 = -\frac{3\alpha N_c}{2\pi} + \frac{3\alpha^2 N_c^2}{2\pi^2} - \frac{21\alpha^3 N_c^3}{8\pi^4}. \quad (14)$$

Note, that in an accordance with Ref. [7] our normalization of $\gamma(j)$ contains the extra factor $-1/2$ in comparison with the standard normalization (see [2]) and differs by sign in comparison with one from Ref. [3].

Note, that $\gamma_{uni}^{(1)}(j)$ was obtained also by direct calculations in Ref. [5].
It is confirmed also by direct calculation in two \cite{10,5} and three-loop \cite{12} orders.

A very interesting result comes from the consideration of the factorized S-matrix \cite{13}, which based on the investigation of the both side of AdS/CFT-correspondence \cite{31,20,11,21,32,33} and gives a possibility to find three-loop anomalous dimension from the Bethe ansatz for arbitrary values of the Lorenz spin. The resulting Bethe ansatz reproduces our results for universal anomalous dimension $\gamma_{uni}(j)$ Eq. \cite{8} and, then, confirm the hypotheses on integrability in $\mathcal{N} = 4$ SYM.

6 Conclusion

We found the NNLO anomalous dimension $\gamma_{uni}(j)$ in the $\mathcal{N} = 4$ SYM \cite{9}. This universal anomalous dimension at $j = 4$ was used to calculate the anomalous dimension of Konishi operator up to 3-loops. It is remarkable, that our results coincide with corresponding expression obtained from dilatation operator approach and integrability \cite{11,32}. Moreover, these results for the universal anomalous dimension was used for a verification of the S-matrix approach to AdS/CFT-correspondence \cite{13}, which is based on the integrability of the corresponding dual theories at large-$N_C$ limit.

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