baller2vec++: A Look-Ahead Multi-Entity Transformer For Modeling Coordinated Agents

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Abstract
In many multi-agent spatiotemporal systems, agents operate under the influence of shared, unobserved variables (e.g., the play a team is executing in a game of basketball). As a result, the trajectories of the agents are often statistically dependent at any given time step; however, almost universally, multi-agent models implicitly assume the agents’ trajectories are statistically independent at each time step. In this paper, we introduce baller2vec++\(^1\), a multi-entity Transformer that can effectively model coordinated agents. Specifically, baller2vec++ applies a specially designed self-attention mask to a mixture of location and “look-ahead” trajectory sequences to learn the distributions of statistically dependent agent trajectories. We show that, unlike baller2vec (baller2vec++’s predecessor), baller2vec++ can learn to emulate the behavior of perfectly coordinated agents in a simulated toy dataset. Additionally, when modeling the trajectories of professional basketball players, baller2vec++ outperforms baller2vec by a wide margin.

1 Introduction and Related Work
Whether it is a team executing a play in a game of basketball, a family navigating to an attraction in a theme park, or friends posting about a birthday party on a social media platform, humans frequently coordinate their behavior in response to shared information. When this coordinating information is unobserved (which is often the case in many machine learning datasets), the individuals’ observed behaviors become correlated, i.e., the behavior of one individual at a specific moment contains information about the behavior of another individual at the same time. In the context of modeling agent trajectories in multi-agent spatiotemporal systems, this property translates to the trajectories being statistically dependent at each time step. However, nearly all multi-agent spatiotemporal models (e.g., \([1][6]\)) implicitly (through their loss functions) assume the trajectories of the agents at each time step are statistically independent given the agents’ previous locations (Figure\([1]\)).

\(^1\)All data and code for the paper are available at: https://github.com/airalcorn2/baller2vecplusplus

Preprint.
Zhan et al. [7] explicitly focused on modeling coordinated multi-agent trajectories, using “macro-intents” [8] that are shared across agents to do so. The macro-intents are generated from a separately trained recurrent neural network (RNN) that learns to predict a future, coarse, “stationary” location for each agent at each time step. The macro-intents model implicitly assumes the macro-intents for the agents at each time step are statistically independent, i.e., the macro-intent for one agent does not depend on the macro-intents of the other agents.

Further, the trajectories-generating model still implicitly assumes the trajectories of the agents at each time step are independent, which is only true if the shared macro-intent variable perfectly captures all of the unobserved information that could cause the agents’ trajectories to be correlated.

Notably, Social-BiGAT [9] does partially account for trajectory correlations through a global adversarial loss. Specifically, the trajectories for each agent are separately passed through an encoder RNN, and the final hidden states for each agent are then processed with a graph attention network (GAT) [10]. The output of the GAT is then used as an input to a discriminator that classifies whether or not the input trajectories are real or generated. This global adversarial loss, however, is only a single component of the full Social-BiGAT loss function, and other components of the loss function do implicitly make the independence assumption. Further, interestingly, adding the global discriminator to a baseline model only improved the model’s performance for one out of six pedestrian datasets.

In this paper, we describe a novel multi-agent spatiotemporal model that integrates information about concurrent actions of agents to predict statistically dependent distributions of trajectories. Specifically, we extend the recently introduced multi-entity Transformer baller2vec [6] by: (1) augmenting its input with a parallel sequence of “look-ahead” agent trajectories and (2) using a specially designed self-attention mask, which allows our model to exploit the chain rule of probability (Section 3). We find that:

1. baller2vec++ is an effective learning algorithm for modeling coordinated agents. Unlike baller2vec, baller2vec++ can learn to emulate perfectly coordinated agents from a simulated toy dataset (Section 5.1). Further, baller2vec++ outperforms baller2vec by a wide margin (8.9%) when modeling the trajectories of professional basketball players (Section 5.1).

2. baller2vec++ makes better predictions when conditioned on concurrent trajectory information from other agents, supporting our proposition that the commonly used independence assumption for agent trajectories is overly strong (Section 5.2).
Lastly, the joint probability assigned to a sequence by baller2vec++ is approximately permutation invariant with respect to the order of the agents, i.e., baller2vec++ respects the properties of the chain rule (Section 5.3).

2 Background

2.1 Multi-agent trajectory modelling

Our problem description closely follows Alcorn and Nguyen [6], whom we quote here:

Let \( A = \{1, 2, \ldots, B\} \) be a set indexing \( B \) agents and \( P = \{p_1, p_2, \ldots, p_K\} \subseteq A \) be the \( K \) agents involved in a particular sequence. Let \( C_t = \{(x_{t,1}, y_{t,1}), (x_{t,2}, y_{t,2}), \ldots, (x_{t,K}, y_{t,K})\} \) be an unordered set of \( K \) coordinate pairs such that \((x_{t,k}, y_{t,k})\) are the coordinates for agent \( p_k \) at time step \( t \). The ordered sequence of sets of coordinates \( \mathcal{C} = (C_1, C_2, \ldots, C_T) \), together with \( P \), thus defines the trajectories for the \( K \) agents over \( T \) time steps.

In multi-agent trajectory modeling, the goal is to model a joint probability of the form:

\[
p(\Delta x_{t,1}, \Delta x_{t,2}, \ldots, \Delta x_{t,K}|x_{1:t,1}, x_{1:t,2}, \ldots, x_{1:t,K})
\]

i.e., the joint probability of the \( K \) agents’ trajectories \( \Delta x_{t,k} \) at time step \( t \) given the agents’ location histories \( x_{1:t,k} \). We note here that the common practice of simultaneously predicting the trajectories for all of the agents at a specific time step is not required by theory. Using the chain rule of probability, the joint probability of the agents’ trajectories can be factorized as, e.g.:

\[
p(\Delta x_{t,1}, \Delta x_{t,2}, \ldots, \Delta x_{t,K}) = p(\Delta x_{t,1})p(\Delta x_{t,2}|\Delta x_{t,1})\cdots p(\Delta x_{t,K}|\Delta x_{t,1}, \Delta x_{t,2}, \ldots, \Delta x_{t,K-1})
\]

where we omit the conditional historical trajectories for brevity. As a result, it is perfectly acceptable to generate trajectories agent-wise, using the previously generated trajectories as additional conditioning information when generating the trajectories for later agents (see Figure 2).

Figure 2: At inference time, a model is not required to simultaneously generate the trajectories for all of the agents at a specific time step. An alternative strategy is to allow the model to generate the agents’ trajectories one at a time, and let the model use the previously generated trajectories as additional conditioning information when generating the trajectories for later agents.

2.2 baller2vec is a (conditional) generative model.

baller2vec is a recently described multi-entity Transformer that can model sequences of sets (the underlying data structure for multi-agent spatiotemporal systems), as opposed to sequences of individual inputs (like words in a sentence). When used to model the game of basketball, the input at each time step for baller2vec is a set of feature vectors where each feature vector contains information about the identity and location of a player on the court. baller2vec maps each input feature vector to an output feature vector, which is then used to “classify” the binned trajectory for that specific player at that specific time step.
Here, we provide a probabilistic interpretation of baller2vec, which establishes the theoretical grounds for using the chain rule to generate trajectories agent-wise at each time step in baller2vec++. Without loss of generality, we only consider one-dimensional trajectories for a single agent here. To briefly summarize, the outputs of the softmax function over the \( n \) binned trajectories in baller2vec can be interpreted as mixture proportions for a mixture of uniform distributions with predetermined bounds that partition the Euclidean trajectory space. Further, because \( x_{t+1} = x_t + \Delta x_t \), baller2vec is in fact a conditional generative model that assigns a probability to a sequence of trajectories given the initial position of the agent, i.e., \( p(\Delta x_1, \Delta x_2, \ldots, \Delta x_T|x_1) \).

Using the chain rule, we decompose the joint probability of the trajectories as:

\[
p(\Delta x_1, \Delta x_2, \ldots, \Delta x_T) = p(\Delta x_1)p(\Delta x_2|\Delta x_1) \cdots p(\Delta x_T|\Delta x_1, \Delta x_2, \ldots, \Delta x_{T-1})
\]

to reflect their temporal structure (we omit the conditional initial position term for brevity). Therefore, new trajectories can be generated from baller2vec with the following procedure (see Figure 3):

1. First, sample one of the \( n \) different mixture components using the mixture proportions output from the classifier \( f \) (i.e., baller2vec) conditioned on the agent’s current position, i.e., \( i \sim \text{Categorical}(\pi_1, \pi_2, \ldots, \pi_n) \) where \( \{\pi_1, \pi_2, \ldots, \pi_n\} = f(x_t) \).

2. Next, sample a trajectory from the uniform distribution associated with the sampled component, i.e., \( \Delta x_i \sim U(a_i, b_i) \).

3. Finally, add the sampled trajectory to the agent’s input position to generate the agent’s position at the start of the next time step, i.e., \( x_{t+1} = x_t + \Delta x_i \).

Let \( [\Delta x_{min}, \Delta x_{max}] \) be an interval on the real line such that any trajectory \( \Delta x < \Delta x_{min} \) or \( \Delta x \geq \Delta x_{max} \) has zero density (i.e., such trajectories are humanly impossible). Let \( \{a_i, b_i\}_{i=1}^{n} \) be a set of \( n \) intervals that partition the interval \( [\Delta x_{min}, \Delta x_{max}] \) into \( n \) bins, i.e., \( \bigcup_{i=1}^{n} [a_i, b_i] = [\Delta x_{min}, \Delta x_{max}] \) and \( i \neq j \Rightarrow (a_i, b_i) \cap (a_j, b_j) = \emptyset \). Recall that the probability density function (PDF) for a uniform distribution with bounds \( -\infty < a < b < \infty \) is:

\[
p(\Delta x) = \begin{cases} \frac{1}{b-a} & \text{for } \Delta x \in [a, b) \\ 0 & \text{otherwise} \end{cases}
\]

Letting \( c_i = \frac{1}{b_i-a_i} \), the PDF for a mixture of uniforms with these bounds is thus:

\[
p(\Delta x) = \sum_{i=1}^{n} \pi_i U(\Delta x; a_i, b_i) = \sum_{i=1}^{n} \pi_i c_i (1)
\]

where \( p(\Delta x) \) is the density assigned to \( \Delta x \) by the mixture, \( \pi_i \) is the mixture proportion for the mixture component indexed by \( i \) (i.e., \( 0 \leq \pi_i \leq 1 \) and \( \sum \pi_i = 1 \)), and \( U(\Delta x; a_i, b_i) \) is the density assigned to \( \Delta x \) by the uniform distribution with bounds \( -\infty < a_i < b_i < \infty \). Because the bounds of the uniform distributions partition \( [\Delta x_{min}, \Delta x_{max}] \), Equation (1) reduces to:

\[
p(\Delta x) = \pi_i c_i
\]

Figure 3: baller2vec can be viewed as a conditional generative model that assigns a probability to a sequence of trajectories given the initial positions of the agents. Here, we show a graphical model depiction of a baller2vec model that generates a sequence of one-dimensional trajectories for a single agent. Given the initial position of the agent (the circle containing \( x_1 \)), one of \( n \) different uniform distributions (the square containing \( i_1 \)) is sampled using the mixture proportions \( (\pi_i) \) output by baller2vec \((f)\). The agent’s trajectory (the diamond containing \( \Delta x_1 \)) is then sampled from the selected uniform distribution, which has bounds \(-\infty < a_i < b_i < \infty\). At the start of the next time step, the agent’s position is \( x_2 = x_1 + \Delta x_1 \). Maximizing the likelihood of baller2vec as a classifier over the binned trajectories is thus equivalent to maximizing its likelihood when assuming the trajectories are generated from a mixture of uniform distributions that partition the Euclidean trajectory space (see Section 2.2 for details).
where $\Delta x \in [a_i, b_i)$ (because the other uniform distributions will assign a density of zero to $\Delta x$).

The likelihood for data $D$ (with $|D| = N$) is then:

$$
\mathcal{L}(D) = \prod_{j=1}^{N} p(\Delta x_j) = \prod_{j=1}^{N} \pi_{j,i'} c_{j,i'}
$$

where $\pi_{j,i'}$ is the mixture proportion assigned to the component with $\Delta x_j \in [a_i, b_i)$ and $c_{j,i'}$ is the associated density. Taking the negative logarithm of the likelihood gives:

$$
- \ln(\mathcal{L}(D)) = - \sum_{j=1}^{N} \ln(\pi_{j,i'}) - \sum_{j=1}^{N} \ln(c_{j,i'})
$$

Because the bounds are fixed, the second summation is a constant, and Equation (2) becomes:

$$
- \ln(\mathcal{L}(D)) = - \sum_{j=1}^{N} \ln(\pi_{j,i'}) + C
$$

where $C = - \sum_{j=1}^{N} \ln(c_{j,i'})$. Therefore, minimizing the loss of baller2vec as a classifier of binned trajectories is equivalent to minimizing the loss of the model when assuming the trajectories are generated from a mixture of uniform distributions as specified in Equation (1).

### 3 Model Architecture

Figure 4: A naive strategy for learning to predict statistically dependent agent trajectories is to adapt the baller2vec self-attention mask so that baller2vec can “look ahead” at future positions of agents whose trajectories are generated prior to the agent being processed in the current time step. However, this look-ahead self-attention mask cannot be used with multi-layer Transformers because doing so necessitates “seeing the future”. For example, after the model attends to the blue agent’s position at time step $t + 1$ when processing the yellow agent at time step $t$, the yellow agent’s resultant feature vector contains information about the blue agent’s future position. As a result, when the model attends to the yellow agent while processing the blue agent at the next level, the model is seeing the future.

We motivate our baller2vec++ architecture by first highlighting an issue that arises in baller2vec when trying to model agent trajectories using the chain rule. The baller2vec self-attention mask can be adapted so that baller2vec “looks ahead” at the future positions of agents whose trajectories are generated prior to the agent being processed in the current time step (Figure 4). However, this look-ahead self-attention mask can only be used with the final layer of the Transformer; otherwise, the model needs to see the future (Figure 4). As a result, baller2vec is severely limited in the conditional distribution functions it can learn.

baller2vec++ (Figure 5) overcomes this limitation by: (1) augmenting the baller2vec input with two other sets of feature vectors and (2) using a specially designed self-attention mask. The three sets of feature vectors in baller2vec++ take the following forms:
which gives a probability distribution over the trajectory bins for agent $g_r$.

Transformer. The three sets of feature vectors consist of location feature vectors like those found in baller2vec ($z_{t,k}$), look-ahead trajectory feature vectors ($u_{t,k}$), and starting location feature vectors ($r_k$; not shown). As can be seen in these partial depictions of baller2vec++ and the baller2vec++ self-attention mask, this design allows the model to integrate information about *concurrent* agent trajectories through multiple Transformer layers without seeing the future.

**Figure 5:** To learn statistically dependent agent trajectories, baller2vec++ uses a specially designed self-attention mask to simultaneously process three different sets of features vectors in a single Transformer. The three sets of feature vectors consist of location feature vectors like those found in baller2vec ($z_{t,k}$), look-ahead trajectory feature vectors ($u_{t,k}$), and starting location feature vectors ($r_k$; not shown). As can be seen in these partial depictions of baller2vec++ and the baller2vec++ self-attention mask, this design allows the model to integrate information about *concurrent* agent trajectories through multiple Transformer layers without seeing the future.

$$
\triangledown = u_{t,k} = \text{Bin}(\Delta x_{t,k}, \Delta y_{t,k})
$$

$$
\bigcirc = z_{t,k} = g_z([c(p_k), x_{t,k}, y_{t,k}, h_{t,k}])
$$

$$
\bigcirc = u_{t,k} = g_u([c(p_k), x_{t+1,k}, y_{t+1,k}, h_{t,k}, \Delta x_{t,k}, \Delta y_{t,k}])
$$

where $g_z, g_u,$ and $g_c$ are multilayer perceptrons (MLPs), $e$ is an agent embedding layer, and $h_{t,k}$ is a vector of optional contextual features for agent $p_k$ at time step $t$. $z_{t,k}$ is the same location feature vector used in baller2vec and contains information about a specific agent’s identity and the agent’s location at time step $t$. $u_{t,k}$ is a “look-ahead” trajectory feature vector that contains information about a specific agent’s identity, the agent’s location at the next time step $t+1$, and the agent’s trajectory at time step $t$, i.e., $(x_{t+1,k} - x_{t,k}, y_{t+1,k} - y_{t,k})$. Lastly, $r_k$ is a starting location feature vector that contains information about a specific agent’s identity and the agent’s location at time step $t = 1$. The $r_k$ feature vectors are necessary so that baller2vec++ can “see” the initial locations of all the agents when processing the agents agent-wise in the first time step.

These three sets of feature vectors are combined to form a $(K + 2TK) \times F$ matrix $Z$ such that the first $K$ rows consist of the $K$ $r_k$ feature vectors, and the remaining $2TK$ rows consist of the $TK z_{t,k}$ and $TK u_{t,k}$ feature vectors interleaved with one another, i.e., each $z_{t,k}$ is followed by its corresponding $u_{t,k}$ in the matrix. This matrix is passed into the Transformer along with the specially designed self-attention mask, which encodes the following dependencies (see Figure 5):

1. When processing $r_{k_1}$, baller2vec++ is exclusively allowed to “look” at each $r_{k_2}$ (i.e., baller2vec++ cannot look at any location or look-ahead feature vectors when processing $r_{k_1}$).

2. When processing $z_{t_2,k_2}$, baller2vec++ is allowed to “look” at: (i) each $r_{k_1}$, (ii) any $z_{t_1,k_1}$ where (a) $t_1 < t_2$ or (b) $t_1 = t_2$ and $k_1 \leq k_2$, and (iii) any $u_{t_1,k_1}$ where (a) $t_1 < t_2$ or (b) $t_1 = t_2$ and $k_1 < k_2$.

3. When processing $u_{t_2,k_2}$, baller2vec++ is allowed to “look” at: (i) each $r_{k_1}$, (ii) any $z_{t_1,k_1}$ where (a) $t_1 < t_2$ or (b) $t_1 = t_2$ and $k_1 \leq k_2$, and (iii) any $u_{t_1,k_1}$ where (a) $t_1 < t_2$ or (b) $t_1 = t_2$ and $k_1 \leq k_2$.

Each processed $z_{t,k}$ feature vector is then passed through a linear layer that is followed by a softmax, which gives a probability distribution over the trajectory bins for agent $p_k$ at time step $t$. Similar to baller2vec, the loss for each sample is:
\[
\mathcal{L} = \sum_{t=1}^{T} \sum_{k=1}^{K} -\ln(f(Z)_{t,2k-1}[v_{t,k}] ) \tag{3}
\]

where \( f(Z)_{t,2k-1}[v_{t,k}] \) is the probability assigned to the trajectory bin \( v_{t,k} \) (where \( v_{t,k} = \text{Bin}(\Delta x_{t,k}, \Delta y_{t,k}) \) is an integer from one to \( n^2 \)) by \( f \), i.e., Equation (3) is the negative log-likelihood (NLL) of the data according to the model.

Because any ordering of a chain rule decomposition of a joint probability produces the same value, e.g.:

\[
p(\Delta x_{t,1})p(\Delta x_{t,2}\mid \Delta x_{t,1})p(\Delta x_{t,3}\mid \Delta x_{t,1}, \Delta x_{t,2}) = p(\Delta x_{t,3})p(\Delta x_{t,2}\mid \Delta x_{t,3})p(\Delta x_{t,1}\mid \Delta x_{t,3}, \Delta x_{t,2})
\]

like [11], we shuffled the order of the agents in each training sequence to encourage the model to learn joint probabilities of the agent trajectories that are approximately permutation invariant with respect to the ordering of the agents.

4 Experiments

We tested baller2vec++ on two different datasets. To highlight the pathological behavior of models that assume agent trajectories are statistically independent at each time step, we trained scaled down versions of baller2vec++ and baller2vec on a toy dataset consisting of simulated trajectories for two perfectly coordinated agents. Additionally, to demonstrate the efficacy of baller2vec++ in real world settings, we trained baller2vec++ and baller2vec on a dataset of trajectories for professional basketball players [6].

4.1 Toy dataset

Each training sample was initialized with the agents starting at \((-1, 0)\) and \((1, 0)\) on a grid in random order (i.e., the first agent could be placed to either the left or the right of the origin). At each time step, one of nine actions (corresponding to the \(3 \times 3\) grid surrounding the agent) was sampled from a uniform distribution, and each of the agents was translated along this trajectory. This process was repeated for 20 time steps (see Figure 6(a) for a sample).

4.2 Basketball dataset

We used the same National Basketball Association (NBA) dataset employed by Alcorn and Nguyen [6], whom we paraphrase here:

The NBA dataset consists of trajectories from 631 games from the 2015-2016 season, which were split into 569/30/32 training/validation/test games, respectively. During training, each sequence was sampled using the following procedure: (1) randomly select a training game, (2) randomly select a starting time from the game, (3) take the following four seconds of data and downsample it to 5 Hz from the original 25 Hz, and then (4) randomly (with a probability of 0.5) rotate the court \(180^\circ\). This sampling procedure gave us access to on the order of \(~82\) million different (albeit overlapping) training sequences. For both the validation and test sets, \(~1,000\) different, non-overlapping sequences were selected for evaluation by dividing each game into \([\frac{1}{N}]\) non-overlapping chunks (where \(N\) is the number of games), and using the starting four seconds from each chunk as the evaluation sequence.

4.3 Model

Our baller2vec++ and baller2vec models for the basketball dataset closely followed [6], and so largely resemble the original Transformer architecture [12]. Specifically, the Transformer settings...
were: \(d_{\text{model}} = 512\) (the dimension of the input and output of each Transformer layer), eight attention heads, \(d_{\text{ff}} = 2048\) (the dimension of the inner feedforward layers), six layers, no dropout, and no positional encoding. Each MLP (i.e., \(g_z, g_u,\) and \(g_r\)) had 128, 256, and 512 nodes in its three layers, respectively, and a ReLU nonlinearity following each of the first two layers. The player embeddings \([13]\) had 20 dimensions, and \(h_{t,k}\) was a binary variable indicating the side of the frontcourt for player \(p_k\) (i.e., the direction of his team’s hoop) at time step \(t\). Lastly, the \(11\) ft \(\times\) \(11\) ft 2D Euclidean trajectory space was binned into \(121\) \(1\) ft \(\times\) \(1\) ft squares.

We used the Adam optimizer \([14]\) with an initial learning rate of \(10^{-6}\), \(\beta_1 = 0.9\), \(\beta_2 = 0.999\), and \(\epsilon = 10^{-9}\) to update the model parameters, of which there were \(\sim 19\) million. The learning rate was reduced to \(10^{-7}\) after 20 epochs of the validation loss not improving. Models were implemented in PyTorch and trained on a single NVIDIA GTX 1080 Ti GPU for \(\sim 650\) epochs (seven days) where each epoch consisted of 20,000 training samples, and the validation set was used for early stopping.

For the toy dataset, we used scaled down versions of the basketball models with \(d_{\text{model}} = 128\), four attention heads, \(d_{\text{ff}} = 512\), and two layers in the Transformer. Additionally, each MLP had two layers with 64 and 128 nodes, respectively. The models were trained for 50 epochs of 500 samples per epoch (\(\sim 10.5\) minutes) using a single learning rate of \(10^{-5}\).

5 Results

5.1 baller2vec++ can effectively model coordinated agents in both simulated and real settings

For the toy dataset, the training loss for baller2vec converged to \(\sim 2.2 \approx -\ln(\frac{1}{9})\), i.e., the model was simply independently guessing the trajectories for both agents at every time step. In contrast, the training loss for baller2vec++ converged to \(\sim 1.1 \approx -\ln(\frac{1}{9}) \div 2\), which is the expected loss for a model that perfectly learns the deterministic relationship between the agents’ trajectories (because the prediction for the second agent will always contribute \(-\ln(1.0) = 0\) to the loss).

When generating trajectories with baller2vec, the agents are completely uncoordinated, with each agent following an independent random walk around the grid (Figure 6(b)). In contrast, trajectories generated by baller2vec++ display the same coordinated agent behavior as the training data (Figure 6(c)).

For the basketball dataset, baller2vec++ achieved an average NLL of 0.472 on the test set, 8.9% better than the average NLL for baller2vec (0.518) (see Figure S1 for trajectories generated by baller2vec++ and baller2vec). As was observed in \([6]\), the trajectory bin distributions for baller2vec become much more certain after observing a portion of the sequence (Figure 7), which suggests baller2vec may be inferring some of the shared hidden variables (e.g., plays) influencing the players. If that hypothesis was true, the performance gap between baller2vec++ and baller2vec should be largest at the beginning of the sequence (before any shared hidden variables can be inferred by baller2vec). Indeed, the average NLL for baller2vec++ in the first time step of each test set sequence (1.567) is 16.1% better than the average NLL for baller2vec (1.869), while the average NLL for baller2vec++ in the last time step of each test set sequence (0.420) is only 9.7% better than the average NLL for baller2vec (0.465) (see Figure 7).

Figure 6: When trained on a dataset of perfectly coordinated agent trajectories (a), the trajectories generated by baller2vec are completely uncoordinated (b) while the trajectories generated by baller2vec++ are perfectly coordinated (c). Animated versions can be found in the code repository.
Figure 7: **Left**: when modeling the trajectories of professional basketball players, the performance gap between baller2vec++ and baller2vec is largest at the beginning of the sequence, before shared unobserved variables can be inferred by baller2vec. Each bar indicates a model’s average NLL over the entire test set for that particular time step. For full sequences, baller2vec++ outperforms baller2vec by 8.9%. **Right**: the joint probability assigned to a sequence by baller2vec++ is approximately permutation invariant with respect to the order of the agents. For each point, its \(x\) value indicates baller2vec++’s average NLL for a test set sequence using the original order of the agents in the sequence, while its \(y\) value indicates baller2vec++’s average NLL for the same sequence with the order of the agents shuffled. The shuffled average NLLs are highly correlated with their corresponding unshuffled average NLLs.

### 5.2 baller2vec++ makes better predictions when conditioned on concurrent trajectory information from other agents

Implicit in much of our discussion has been the intuition that providing a model with additional (relevant) information will improve its performance. To empirically test this conjecture, we compared the performance of baller2vec++ when predicting the trajectory of a specific basketball player placed in the first position of the player order (i.e., when \(k = 1\)) vs. predicting the trajectory for that same player placed in the last position (i.e., when \(k = 10\)). Specifically, for each player in each test sequence, we calculated the NLL of the player’s trajectory in the first time step with the player in the first position of the player order. Next, we moved the player to the last position of the player order, and then randomly shuffled the remaining nine players 10 times, calculating the NLL for the player in the last position each time. Finally, we calculated the average percent change in the last position NLLs relative to their corresponding first position NLLs. On average, moving a player from the first to the last position improved the NLL for the player’s trajectory by 14.6%.

### 5.3 The joint probability assigned to a sequence by baller2vec++ is approximately permutation invariant with respect to the order of the agents

To determine whether or not baller2vec++ respects the fact that any ordering of a chain rule decomposition of a joint probability produces the same value, we measured how much the average NLL for each test sequence in the basketball dataset varied when the order of the agents changed. Specifically, for each test set sequence, we shuffled the order of the agents 10 times. Then, for each permuted sequence, we calculated the percent error in the average NLL relative to the original, unshuffled sequence. Across all test sequences, the average percent error was only \(\pm 1.5\%\). Further, as can be seen in Figure 7, the shuffled average NLLs are highly correlated with their corresponding unshuffled average NLLs (Pearson correlation coefficient = 0.997), i.e., the joint probability assigned to a sequence by baller2vec++ is approximately permutation invariant with respect to the order of the agents.

### 6 Conclusion and Future Work

In this paper, we have shown how the commonly used independence assumption of many multi-agent spatiotemporal models can severely limit their ability to learn to emulate coordinated agents. By relaxing this independence assumption in baller2vec, baller2vec++ was able to more accurately

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5Because, as previously discussed, the benefits of baller2vec++ were most pronounced in the first time step.

6See: [https://en.wikipedia.org/wiki/Approximation_error#Formal_Definition](https://en.wikipedia.org/wiki/Approximation_error#Formal_Definition)
model the trajectories of professional basketball players. Models for other multi-agent spatiotemporal environments, such as pedestrian traffic (see [15] for a survey) and vehicle traffic (e.g., [16–19]), may also benefit from the look-ahead approach used by baller2vec++. Additionally, the interleaved input design of baller2vec++ could be useful when modeling other systems involving many entities interacting through time, such as social media platforms (e.g., [20, 21]). However, confronting the quadratic complexity of the Transformer attention mechanism as the number of entities grows large in these datasets is an open problem, but recent work in sparse Transformers (e.g., [22–27]) shows encouraging progress.

7 Author Contributions

MAA conceived and implemented the architecture, designed and ran the experiments, and wrote the manuscript. AN partially funded MAA, provided the GPUs for the experiments, and commented on the manuscript.

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References

[1] Panna Felsen, Patrick Lucey, and Sujoy Ganguly. Where will they go? predicting fine-grained adversarial multi-agent motion using conditional variational autoencoders. In Proceedings of the European Conference on Computer Vision (ECCV), pages 732–747, 2018.

[2] Agrim Gupta, Justin Johnson, Li Fei-Fei, Silvio Savarese, and Alexandre Alahi. Social gan: Socially acceptable trajectories with generative adversarial networks. In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition, pages 2255–2264, 2018.

[3] Amir Sadeghian, Vineet Kosaraju, Ali Sadeghian, Noriaki Hirose, Hamid Rezatofighi, and Silvio Savarese. Sophie: An attentive gan for predicting paths compliant to social and physical constraints. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition, pages 1349–1358, 2019.

[4] Raymond A Yeh, Alexander G Schwing, Jonathan Huang, and Kevin Murphy. Diverse generation for multi-agent sports games. In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition, pages 4610–4619, 2019.

[5] Cunjun Yu, Xiao Ma, Jiawei Ren, Haiyu Zhao, and Shuai Yi. Spatio-temporal graph transformer networks for pedestrian trajectory prediction. In Proceedings of the European Conference on Computer Vision (ECCV), August 2020.

[6] Michael A. Alcorn and Anh Nguyen. baller2vec: A multi-entity transformer for multi-agent spatiotemporal modeling. arXiv preprint arXiv:2102.03291, 2021.

[7] Eric Zhan, Stephan Zheng, Yisong Yue, Long Sha, and Patrick Lucey. Generating multi-agent trajectories using programmatic weak supervision. In International Conference on Learning Representations, 2019. URL https://openreview.net/forum?id=rkxw-hAcFQ.

[8] Stephan Zheng, Yisong Yue, and Jennifer Hobbs. Generating long-term trajectories using deep hierarchical networks. Advances in Neural Information Processing Systems, 29:1543–1551, 2016.

[9] Vineet Kosaraju, Amir Sadeghian, Roberto Martín-Martín, Ian Reid, Hamid Rezatofighi, and Silvio Savarese. Social-bigat: Multimodal trajectory forecasting using bicycle-gan and graph attention networks. In H. Wallach, H. Larochelle, A. Beygelzimer, F. d’Alché-Buc, E. Fox, and R. Garnett, editors, Advances in Neural Information Processing Systems, volume 32. Curran Associates, Inc., 2019. URL https://proceedings.neurips.cc/paper/2019/file/d09bf41544a3385a46c9077ebb5e35c3-Paper.pdf.
[10] Petar Veličković, Guillem Cucurull, Arantxa Casanova, Adriana Romero, Pietro Liò, and Yoshua Bengio. Graph Attention Networks. *International Conference on Learning Representations*, 2018. URL [https://openreview.net/forum?id=rJXMpiK3Z](https://openreview.net/forum?id=rJXMpiK3Z): accepted as poster.

[11] Zhilin Yang, Zihang Dai, Yiming Yang, Jaime Carbonell, Ruslan Salakhutdinov, and Quoc V Le. Xlnet: Generalized autoregressive pretraining for language understanding. *arXiv preprint arXiv:1906.08237*, 2019.

[12] Ashish Vaswani, Noam Shazeer, Niki Parmar, Jakob Uszkoreit, Llion Jones, Aidan N Gomez, Lukasz Kaiser, and Illia Polosukhin. Attention is all you need. In *Advances in Neural Information Processing Systems*, pages 5998–6008, 2017.

[13] Michael A Alcorn. (batter|pitcher)2vec: Statistic-free talent modeling with neural player embeddings. In *MIT Sloan Sports Analytics Conference*, 2018.

[14] Diederik P Kingma and Jimmy Ba. Adam: A method for stochastic optimization. In *International Conference on Learning Representations*, 2015.

[15] Nachiket Deo and Mohan M Trivedi. Convolutional social pooling for vehicle trajectory prediction. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition Workshops*, pages 1468–1476, 2018.

[16] Rohan Chandra, Uttaran Bhattacharya, Aniket Bera, and Dinesh Manocha. Traphic: Trajectory prediction in dense and heterogeneous traffic using weighted interactions. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR)*, June 2019.

[17] Manzil Zaheer, Guru Guruganesh, Kumar Avinava Dubey, Joshua Ainslie, Chris Alberti, Santiago Ontanon, Philip Pham, Anirudh Ravula, Qifan Wang, Li Yang, and Amir Ahmed. Big bird: Transformers for longer sequences. In H. Larochelle, M. Ranzato, R. Hadsell, M. F. Balcan, and H. Lin, editors, *Advances in Neural Information Processing Systems*, volume 33, pages 17283–17297. Curran Associates, Inc., 2020. URL [https://proceedings.neurips.cc/paper/2020/file/c8512d142a2d849725f31a9a7a361ab9-Paper.pdf](https://proceedings.neurips.cc/paper/2020/file/c8512d142a2d849725f31a9a7a361ab9-Paper.pdf).

[18] Iz Beltagy, Matthew E Peters, and Arman Cohan. Longformer: The long-document transformer. *arXiv preprint arXiv:2004.05150*, 2020.
[26] Jonathan Ho, Nal Kalchbrenner, Dirk Weissenborn, and Tim Salimans. Axial attention in multidimensional transformers. *arXiv preprint arXiv:1912.12180*, 2019.

[27] Gedas Bertasius, Heng Wang, and Lorenzo Torresani. Is space-time attention all you need for video understanding? *arXiv preprint arXiv:2102.05095*, 2021.
Supplementary Materials

baller2vec++: A Look-Ahead Multi-Entity Transformer For Modeling Coordinated Agents

Figure S1: While baller2vec occasionally generates realistic trajectories for the red defender (b), it also makes egregious errors (c). In contrast, the trajectories generated by baller2vec++ often seem plausible (d and e). For both the baller2vec and baller2vec++ generated trajectories, the ground truth trajectories (a) for all of the non-red players were used as input at each time step. The red player was placed last in the player order when generating his trajectory with baller2vec++. Animated versions can be found in the code repository.