The effects of solid-liquid interfacial tension on the settlement of sediment flocs

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In this paper, the effects of interfacial tension between the sediment solid particle and liquid on the settlement of sediment flocs are investigated. After a discussion of mechanical and physical chemistry, we give a settling velocity expression including such dynamical information of the floc growth as interfacial tension and primary particle size etc.. The resulting expression indicates the average settling velocity of sediment flocs increases with increasing solid-liquid interfacial tension in a form of power law and deceases with the primary particle size. We report on a general method for analyzing settling behaviors of sediment flocs under different flocculation conditions and verify the rationality of the assumption of tension-induced flocculation by fitting typical experimental data to the electrolyte concentration-dependent sedimentation model which can follow from the relationship between interfacial tension and electrolyte concentration.

KEYWORDS: Sediment; Flocculation; Electrolyte Concentration; Settling velocity; Solid-Liquid Interfacial Tension.

I. INTRODUCTION

The aggregation-sedimentation of sediment flocs, which is widely involved by many areas of industry and engineering such as mineral processing, irrigation works, and hydraulic engineering, is a basic phenomenon in the nature. Many fundamental researches have been devoted to understanding of it [1] [2] [3] [4] [5] [6]. Primary particles aggregate and form clusters or microflocs which may continue to combine the remaining single particles or other microflocs and thus become larger flocs. When the growth of flocs is induced by the Brownian motion, the diffusion limited cluster aggregation (DLCA) model or the reaction limited cluster aggregation (RLCA) model can well describe the process of this aggregation. However, when the flocs become large enough to settle under gravity, sedimentation alters the growth mechanism. If the sediment concentration φ is low, the flocs fall under gravity, grow, fracturing and restructuring; the size of the flocs reach a final value and the space-filling network connecting does not take place. Conversely, if φ is sufficiently large, the growing microflocs will form the space-filling networks which spans all the cell very quickly.

The DLVO theory [7] [8], which attributes interactions between colloidal suspensions to Van der Waals forces and electrostatic repulsion, dynamically provides a quantitative description of many facts of flocculation. In the DLVO framework, colloidal suspensions tend to coagulate due to Van der Waals attractive forces, while electrostatic repulsive forces promote stability of the system, and the repulsive barrier may be decreased through changing the physicochemical conditions, temperature, ionic strength, pH, solvent quality and so on, which induces the flocculation of colloidal particles. Although the DLVO theory can describe basic facts of flocculation of colloids, it still can not completely elucidate the cause of flocculation phenomena. For example, the DLVO theory encounter many difficulties in aggregation of charged colloidal particles in aqueous media such as fluid-crystal coexistence phenomenon [9] [10] [11], and void formation [12] [13] etc, where a long-ranged attraction is required. In addition, the DLVO theory is also reluctant to explain the transition from RLCA to DLCA [14], since the fact that high electrolyte concentrations lead to a fractal dimension arising from DLCA model indicates that Van der Waals force cannot act as such a role that may make formation of the floc attributed only to pure Brownian collisions which means the binding force between flocs should be independent of the floc itself.

To overcome discrepancies between theoretic predictions and experimental observations, different versions of modification to the DLVO theory, some of which have additionally considered non-DLVO interactions resulting from acid-base [15], steric [16], and hydrodynamic [17] interfacial forces, have been presented. However, most investigations into the effects of the interface are concentrated on interactions between particles [18] [19] [20]. For a macroscopic system of sediment flocs, whose sizes can often reach 500~1000 µm, effects of interfacial tension between the solid particle and liquid on mechanical balance of the floc are more necessary to be taken into account, and on such a large scale the DLVO force between particles becomes negligibly small in comparison with gravity and hydrodynamic forces acting on the floc, so that dynamical reason for sediment flocculation may be attributed mainly to surface tension on the floc.

In this paper, we emphasize the effects of the interfacial tension between solid particles and liquid on the settlement of sediment flocs. Few works about settling dynamics of sediment flocs are involved in the growth mechanism of flocs, and hence the aim of the paper is to introduce such dynamical information as interfacial tension, primary particle size (pps) etc. in the settling velocity expression of sediment flocs. We present a gen-
eral method for investigating sedimentation of sediment flocs under different flocculation conditions and discuss settling behaviors influenced by electrolyte concentration in relation with typical experimental data.

II. THE SETTLING VELOCITY

We now consider a system of sediment particles of the identical size in still water, and focus on such the situation that the sediment concentration $\phi$ is low so that spatially networking does not happen. After a process of aggregating, large flocs fall under gravity, then average settling velocity of the system is written as

$$U = \sum_i \psi_i v(D_i),$$  \hspace{1cm} (1)

where $\psi_i$ denotes the weight of the i-th group of flocs with the same size $D_i$, and $v(D_i)$ is (constant) settling velocity of the floc belonging to the i-th group. It is necessarily emphasized that $D_i$ should be understood as diameter of sphere of the same volume as the floc including inside water. The isometric diameter is preferentially considered here since some of experiments of measuring settling velocity of sediment flocs are based on weight change determination that directly connects with the isometric size.

Stokes settling velocity formula can be employed, if the floc are not so large that effects of turbulence on it might be ignored, for expressing the size-dependent sedimentation velocity $v(D_i)$ of the floc \cite{21}, namely

$$v(D_i) = \frac{1}{18} \frac{\rho_{Ai} - \rho_L}{\rho_L} g \frac{D^2}{\nu} = \frac{1}{18} \frac{(\rho_S - \rho_L)(1 - \epsilon_i)}{\rho_L} g \frac{D^2}{\nu},$$ \hspace{1cm} (2)

where $\rho_{Ai,L}$ represents the density of the i-th floc and liquid, respectively, $\nu$ is the viscosity of the liquid, $g$ is acceleration of gravity, $\rho_S$ is density of the solid particle, and $\epsilon_i$ denotes the porosity of the i-th floc. Here the relation $\frac{\rho_S - \rho_L}{\rho_S - \rho_{Ai}} = 1 - \epsilon_i$ is used. Generally, Eq. (2) is applicable for the case that the size of the settling floc do not exceed 150 $\mu m$.

Investigations of floc structure \cite{22, 23, 24} have shown that the density (or porosity) of the floc may be evaluated from the following relationship:

$$1 - \epsilon = d_B(D_c/D_0)^{-\alpha},$$ \hspace{1cm} (3)

where $d_B$ denotes a modifying factor, $D_c$ is the diameter of collision, which denotes the size of the smallest sphere that fully encompasses the floc, $D_0$ is the diameter of the primary particle comprising the floc, and $\alpha$ is a constant. And the mass fractal dimension $d_f$ can be scaled as $d_f = 3 - \alpha$. The number $N$ of primary particles within a floc can be expressed in terms of the collision diameter $D_c$ and the isometric size $D$, respectively, namely

$$N = A_1 \left( \frac{D_c}{D_0} \right)^{d_f},$$ \hspace{1cm} (4)

where $A_1$ is the structure prefactor, and

$$N = (1 - \epsilon)(\frac{D}{D_0})^3,$$ \hspace{1cm} (5)

as a result, the relation between $D_c$ and $D$ is given by

$$D_c = \sqrt[3]{\frac{d_B A_1}{A_1}} D.$$ \hspace{1cm} (6)

Due to Eq. (3) and Eq. (6), the sedimentation velocity $v(D_i)$ can be rewritten as

$$v(D_i) = \frac{A_2}{18} \frac{(\rho_S - \rho_L) g}{\nu\rho_L} D_0^{3-d_f} \frac{d_f}{d_f - 1},$$ \hspace{1cm} (7)

where $A_2 = A_1^{3-d_f} d_f$. The size-dependent velocity expression Eq. (7) gives the information of mass fractal dimension $d_f$, hence it is also be a theoretic equation for measuring $d_f$ of settling floc \cite{25}.

III. THE ADDITIONAL PRESSURE OF SURFACE OF THE FLOC

Mechanism of growth of the floc affects sedimentation of it, hence we have to take into account effects of the mechanism of the growth on settling floc. It is pointed out in this paper that the interfacial tension $\gamma$ on the solid-liquid interface (or the Gibbs dividing surface) between sediment solid particle and liquid plays a key role in aggregation and sedimentation of particles or flocs. Obviously, the interfacial tension $\gamma$ balances with the additional pressure difference $p$ between both sides of the interface. The additional pressure difference $p$, which is associated directly with stress which the floc is suffering, is an important quantity determining the structure and thus settlement of the floc. For a regular surface, $p$ is in inverse proportion to radius of curvature of the surface, and then, for a floc surface with the fractal feature, what properties does it have?

We assume, to expand the floc volume an increment $dV$, the additional pressure needs to overcome the surface work of the interface area adding $dS$, that is,

$$p = \frac{\gamma dS}{dV},$$ \hspace{1cm} (8)

and then from Eq. (5) we have

$$dS = \pi \beta_s D_0^3 dN = (\pi \beta_s / D_0)d[(1-\epsilon)D^3]; \hspace{0.5cm} dV = \frac{1}{2} \pi D^2 dD,$$ \hspace{1cm} (9)

where $\beta_s$ is the proportion factor denoting the ratio of solid surface contributing to external surface of the floc.
to total solid surface, which is associated with the shape of the floc. Combining Eq. (3), (8), and (9), we derive the expression of the additional pressure difference,

\[ p = \frac{2\gamma d_f \beta_2 A_2 D_0^{2-d_f}}{D^{3-d_f}}. \]  

(10)

It is clearly seen that the additional pressure of the floc surface is inversely proportional to the size of the floc since the fractal dimension \( d_f \) is generally less than 3, and it decays following the power-law form, which is a reflection of abnormal specific surface area of the floc, and also the additional pressure may connect with electrolyte concentration through \( \gamma \) according to Gibbs adsorption law.

The reaction force of the additional pressure, acting on the floc, provides a binding force that can stick particles together. However, the inverse dependence on the floc size of the binding force characterized by \( p \) implies that not all positions on the surface of a certain floc can combine the other specific floc due to the impact of external forces such as gravity, shear force, etc. on the specific floc. In other words, under the given mechanical conditions, a floc would have a combinable region relative to the other floc colliding with it, where both of them may satisfy the mechanical condition of balance, as a result, a firm agglomeration has to require many collisions of the two flocs, which indicates a sticking probability less than 1 between flocs. In particular, the sticking probability varies as different collision pairs since it is associated with the ratio of the combinable region to the total surface area of either floc in a pair. Therefore, interfacial tension-induced flocculation dynamics may result in a growth mechanism ruled by the relative sticking probability during settling.

It is worthwhile to note that the pressure \( p \) should be understood as an average effect resulting from different solid surfaces contributing to the external surface of the floc, which can be seen from Eq. (8). In other words, for different solid surface elements on the external surface of the floc, the pressure \( p \) would be fluctuating around the average \( p \) deriving from Eq. (8).

IV. THE MAXIMAL SIZE OF THE FLOC

The interfacial tension-induced mechanism of flocculation implies that there is a maximal diameter of the floc, since size of the floc becoming large will abate the magnitude of the additional pressure of the surface, which characterizes an ability of the floc’s capturing free particles or flocs. When sedimentation dominates over Brownian motion, the floc sweeps up smaller flocs underneath and grows faster, and then, when can the floc’s size attain the maximum?

Let us assume that the floc is a sphere, then during settling, the upper hemisphere will receive few smaller flocs due to gravity, conversely the lower hemisphere can combine large numbers of smaller flocs. And we find that the pressure required to equilibrate a primary particle stuck on the floc, which is exerted by the interface between the external surface of the floc and the liquid, is minimum when the primary particle is enveloped on the bottom of the sphere (See Fig. 1). Assuming the actual additional pressure, thus the actual pressure acting on the primary particle, is averagely the same down the external surface of the floc and is determined by the size of the floc, we may know that the floc can not stick other flocs together any more if it is not able to adhere to the primary particle on its bottom. In other words, for the floc of certain a size, the nearer the particle or floc approaches the bottom of the main floc, the easier it is fixed, hence shape of the final floc generally is of cobble or strip. The case of the minimum pressure, corresponding to that of the largest floc, is exactly what we focus on.

If enough particles are supplied to make the floc largest, then for a primary particle attached critically to the bottom of the floc, mechanical equilibrium gives:

\[ p' = -p, \quad G = p' \Delta s, \]

(11)

where \( p' \) is the pressure acting on the primary particle from the interface, \( G \) is net weight of the primary particle in water. Therefore the maximal diameter \( D_m \) of the floc can be derived from Eq. (10) and Eq. (11):

\[ D_m = \left( \frac{\pi d_f \beta_2 A_2 \gamma}{c_B \Delta \rho g} \right)^{\frac{1}{1-d_f}} D_0^{\frac{1-d_f}{3-d_f}}. \]

(12)

Here we apply \( G = c_B (\rho_s - \rho_L) g D_0^3 = c_B \Delta \rho g D_0^3 \), where \( c_B \) is the shape factor, and \( \Delta s = \frac{4}{3} \pi D_0^3 \). It is clearly seen that the maximal size \( D_m \) of the floc increases in a form of power law as the interfacial tension \( \gamma \) grows, while it drops with the size \( D_0 \) of the primary particle since the fractal dimension \( d_f \) is generally greater than 1 and less than 3.

The expression of the maximal size Eq. (12) is a dynamical result which excludes some geometrical factors such as competition between the flocs in particles and spatial distribution of particles or flocs. In other words, only if such dynamical parameters as \( \gamma, D_0 \), etc. can satisfy the critical condition of balance, the maximal size of the floc may be uniquely determined. However, actual average size of flocs in the system can not attain the maximal size because of clustering competition, spatial distribution, and sufficiency of particles. Therefore, the maximal size \( D_m \) is an imaginary quantity dependent only on dynamical factors.

V. THE TENSION-DEPENDENT SETTLING VELOCITY

In order to examine the effect of the interfacial tension on settling velocity of sediment flocs, according to Eq. (1) and Eq. (7), we write:

\[ U = \frac{\lambda A_2}{18 \nu \rho_L} D_0^{3-d_f} D_m^{-1}. \]

(13)
where \( \lambda = \frac{\sum_i \phi_i D_i^{d_f-1}}{D_m^{d_f-1}} = \frac{(D^{d_f-1})}{D_m^{d_f-1}}. \)

We actually provide a general method for investigating settling velocity of sediment flocs, since the dimensionless number \( \lambda \) may remove the effect of the dynamical factors such as the interfacial tension and the primary particle size, and is associated only with geometrical factors like sediment concentration and spatial distribution of particles, provided the growth of the average size \((D^{d_f-1})\) indeed dynamically results from those reasons which lead to the maximal size of the floc. In other words, the reason why the average size of flocs cannot reach the maximal could be that multi-body competition in particles under certain conditions reduces chances of the growth of each floc, which exactly arises from geometrical factors.

Then the average velocity of sediment flocs can be expressed through replacing \( D_m \) in Eq. (13) with Eq. (12) as follows:

\[
U = \frac{1}{18} \frac{\lambda}{\nu \rho_L} \frac{\pi d_f \beta_\gamma}{c_B} \frac{d_f-1}{d_f} A_2 \frac{2}{d_f} \frac{(\Delta \rho g)}{D_0} \frac{2(1-d_f)}{d_f} \frac{4(2-d_f)}{3(2-d_f)}.
\]

We thus derive a setting velocity expression of sediment flocs including dynamical information, which shows that the average settling velocity of sediment flocs increases as solid-liquid interfacial tension becomes large in a form of power law, and has the decreasing relation with the primary particle size, since Vladimir Nikora et al. have proved that the fractal dimension \( d_f \) can affect relationship between velocity and size of the floc only when it is greater than 2. In addition, because of high electrolyte concentrations resulting in larger surface tensions, which is correct for inorganic salt solutes from the knowledge of physical chemistry, setting velocity would also increase with the electrolyte concentration. We may also obtain the expression of flocculation factor \( F \):

\[
F = \lambda \left( \frac{\pi d_f \beta_\gamma}{c_B} \right) \frac{d_f-1}{d_f} A_2 \frac{2}{d_f} \frac{(\Delta \rho g)}{D_0} \frac{2(1-d_f)}{d_f} \frac{4(2-d_f)}{3(2-d_f)} ,
\]

which is defined as \( F = U/U_0 \), where \( U_0 \) denotes the classical Stokes relationship for solid spherical particles, namely \( U_0 = \frac{1}{18} \frac{\Delta \rho g D_0^3}{\nu} \). It is obvious that the remarkable effect on flocculation does not lie in electrolyte concentration but the size \( D_0 \) of the primary particle comprising sediment system, since the pps indicates the essential factors affecting flocculation such as density of sediment floc and binding force inducing flocculation etc., while electrolyte concentration is only a reflection of influences of solid adsorption action on flocculation. However, flocculation factor expression Eq. (15) shows that \( F \) varies inversely as \( \Delta \rho g \) and \( g \). It is not surprising since smaller \( \Delta \rho g \) will lead to larger average size of flocs when binding force is fixed, yet this still needs further experimental proof.

VI. COMPARISON WITH EXPERIMENTAL DATA

A. The salinity-dependent settling velocity

The effect of electrolyte concentrations on the settling behaviors of sediment flocs may be derived from the relationship between the surface tension and electrolyte concentration \( C \). The relationship between \( \gamma \) and \( C \) is exhibited in the Gibbs adsorption equation:

\[
\Gamma_{2,1} = -\frac{C_2 \; d\gamma}{RT \; dC_2},
\]

where \( \Gamma_{2,1} \) is surface excess on the Gibbs’ dividing surface, \( C_2 \) is solute equilibrium concentration, \( R \) is mole gas constant and \( T \) temperature. Therefore, if the isothermal adsorption property \( \Gamma_{2,1}(C_2) \) of the interface is given, then the relationship between \( \gamma \) and \( C_2 \) may be obtained through integrating Eq. (16). The solid-liquid interface possesses more complex adsorption properties and accordingly experimental investigations on the relationship between \( \Gamma \) and \( C \) could be more reliable to study the salinity-dependent settling behavior of sediment flocs. Nevertheless, many physicochemical experiments of sediment and soil show that the adsorption curve of some solid-liquid interfaces can be expressed by Langmuir adsorption isotherm, namely

\[
\Gamma_{2,1} = \frac{bkC_2}{1 + kC_2},
\]

where \( b \) is the saturation adsorptive capacity, and \( k \) is adsorption constant.

However, for the interface between solid and liquid, we are more interested in surface force relative to the solid rather than relative to the bulk liquid. It is known that the addition of salt to water will result in larger gas-liquid surface tension. This is because attraction of the salt ion in the bulk liquid for water molecules can draw more water molecules inside the bulk liquid so that more work would be needed to add a surface area, which shows that the adsorption quantity of the dividing surface relative to the bulk liquid is negative. In the same light, the adsorption of the solid to inorganic ions will also give rise to a negative adsorption quantity of the dividing surface relative to the bulk part between the dividing surface and
solid surface. Although the magnitude of the adsorption relative respectively to two different bulk parts, the bulk part between the dividing surface and solid surface, and the bulk liquid, is the same (the dividing surfaces in the two cases are different), the two quantities have opposite sign with each other, and this distinction will play a key role in determining properties of the interface tension.

Thus, combing Eq. (16) with Eq. (17) and noticing sign of the adsorption quantity $\Gamma_{2,1}$, we get

$$d\gamma = \frac{bkRT}{1 + kC_2} dC_2. \quad (18)$$

respectively. It is apparent that both settling velocity of sediment flocs and flocculation factor increase with increasing electrolyte concentration. However, real adsorption of solid in the solution is quite complex, even transitions between positive and negative adsorption often occurs for some cases. Therefore it would be more reliable that experimental methods determine the relationship between $\gamma$ and $C$ to predict the effects of electrolyte concentration on settling velocity of sediment flocs. The $\gamma$-$C$ form of Eq. (19) only reflects a simple case. In addition, average size of flocs dependent on the maximal size $D_m$ can not be smaller than the pps $D_0$, hence the flocculation factor $F$ is always greater than or equal to 1, which indicates a cutoff concentration $C_c$ which guarantees $(D) = \lambda D_m \geq D_0$. The cutoff concentration $C_c$ actually is the minimum salt concentration causing flocculation of particles of the $D_0$ size in the present $\gamma$-$C$ form. If $\gamma$ is a decreasing function of $C$, then $C_c$ which satisfies $F = 1$ should represent a minimum salt concentration giving rise to maximal dispersion of flocs.

After the integration of Eq. (18), a tension-concentration relationship can be obtained:

$$\gamma = bRT \ln(1 + kC_2) + \gamma_0. \quad (19)$$

Here $\gamma_0$ denotes the interface tension between solid and pure solvent. Finally, the salinity-dependent settling velocity $U$ and flocculation factor $F$ can be expressed as follows,

$$U = \frac{1}{18} \frac{\nu p L}{\rho g} \left( \frac{\pi df}{cB} \right) \left( \frac{bRT \ln(1 + kC_2) + \gamma_0}{\lambda} \right)^{\frac{d_f-1}{2}} A_2^{\frac{3-d_f}{3}} (\Delta \rho g)^{\frac{d_f-1}{2}} D_0^{\frac{4(2-d_f)}{3}}$$

and

$$F = \lambda \left( \frac{\pi df}{cB} \right) \left( \frac{bRT \ln(1 + kC_2) + \gamma_0}{\lambda} \right)^{\frac{d_f-1}{2}} A_2^{\frac{3-d_f}{3}} (\Delta \rho g)^{\frac{d_f-1}{2}} D_0^{\frac{4(2-d_f)}{3}}$$

$$\quad (20)$$

respectively. It is apparent that both settling velocity of sediment flocs and flocculation factor increase with increasing electrolyte concentration. However, real adsorption of solid in the solution is quite complex, even transitions between positive and negative adsorption often occurs for some cases. Therefore it would be more reliable that experimental methods determine the relationship between $\gamma$ and $C$ to predict the effects of electrolyte concentration on settling velocity of sediment flocs. The $\gamma$-$C$ form of Eq. (19) only reflects a simple case. In addition, average size of flocs dependent on the maximal size $D_m$ can not be smaller than the pps $D_0$, hence the flocculation factor $F$ is always greater than or equal to 1, which indicates a cutoff concentration $C_c$ which guarantees $(D) = \lambda D_m \geq D_0$. The cutoff concentration $C_c$ actually is the minimum salt concentration causing flocculation of particles of the $D_0$ size in the present $\gamma$-$C$ form. If $\gamma$ is a decreasing function of $C$, then $C_c$ which satisfies $F = 1$ should represent a minimum salt concentration giving rise to maximal dispersion of flocs.

**B. The fitting of experimental data**

In order to examine the rationality of the assumption of tension-induced flocculation, we fit the typical experimental data which come from the report on sedimentation experiments of the coastal silt at the Bohai Bay in China by Wu Deyi to the electrolyte concentration-dependent sedimentation model Eq. (21). Wu Deyi et al. utilize the sedimentation balance to measure the electrolyte ($Na^+$) concentration-dependent settling velocity in still water of silt flocs after analyzing the particle size of salt-free silt sample by means of particle size analyzer. The salt-free silt sample can be gained through washing silt with distilled water and stirring and dispersing them in distilled water. They report an empirical formula between settling velocity and the salinity as well as the pps as follows,

$$\frac{U_s}{U_0} = h \cdot D_0^{-3.1} \cdot (S/1000)^0.4,$$  

where $D_0$ is the average size of the salt-free silt sample, $S/1000$ is the mass ratio concentration of $Na^+$, and the

![FIG. 2: The fitting of experimental data to the model Eq. (23). The experiments were performed with coastal silt at the Bohai Bay in China of sediment concentration 4~5 kg/cm³, and the experimental temperature was about 293K. Solid lines denote fitting curves and $D_{0\tau}$ is the cumulative average size of the salt-free silt samples.](image-url)
would attain better fits, nevertheless, percentage differ-
ence between the two values of experiments, for example, Ref [29, 30, 31, 32, 33], etc. In addition, the fact that values of the fitted pa-
rameter $b$ in different measures are approximately equal indicates that the parameter $b$ may indeed represent such a physical quantity as saturation adsorptive capacity.

VII. CONCLUSION

This paper proposes a theoretic model which attributes the dynamical reason of flocculation of flocs to the interfacial tension between solid particles and liquid, and provides a general method of analyzing sedimentation of sediment flocs. The resulting expression of settling velocity of sediments flocs can give a good description of the relationship between flocculation factor and the electrolyte concentration without loss of physical sense of fitted parameters. In fact, actual interaction is so complex that more me-
chanical factors need to consider. For example, during aggregation ruled by Brownian motion while interacting flocs being effectively small, other forces have to be intro-
duced into balance conditions. Further study should be aimed at direct confirmation of the present model with real experiments.

TABLE I: Comparison of Fitted Values of $F$ with Experimental Data as well as Values of Related Parameters of Best Fit

| $D_{055} = 2\mu m$ | $D_{050} = 1.35\mu m$ | $D_{010} = 0.55\mu m$ |
|-------------------|-------------------|-------------------|
| Salinity $[1/1000]$ (experimental) | $F$ (fitted) | $F_s$ | $%$Difference | $B$(best fit) | $b$(best fit) | $k$(best fit) |
| 1.3 | 32.4 | 26.74 | 17.5% | $1.79 \times 10^{-21}$ | 0.99 | 62.49 |
| 9.3 | 65.4 | 85.04 | 23.1% | | |
| 12.9 | 113.5 | 97.69 | 13.9% | | |
| 39.7 | 146.0 | 146.18 | 0.12% | | |
| 1.3 | 63.4 | 57.62 | 9.1% | $8.42 \times 10^{-22}$ | 0.997 | 98.8 |
| 9.3 | 129.0 | 159.3 | 19.02% | | |
| 12.9 | 208.0 | 180.36 | 13.3% | | |
| 39.7 | 258.0 | 259.89 | 0.73% | | |
| 1.3 | 260.0 | 285.42 | 8.9% | $7.53 \times 10^{-23}$ | 0.998 | 1634.75 |
| 9.3 | 400.0 | 488.93 | 18.2% | | |
| 12.9 | 700.0 | 526.29 | 24.8% | | |
| 39.7 | 1460.0 | 1461.80 | 0.12% | | |

$F = \frac{U_0}{\sqrt{\rho_{055}}} \frac{U_0}{\sqrt{\rho_{050}}}$, and $\frac{U_0}{\sqrt{\rho_{010}}}$ corresponding respectively to different $D_{055}$.  

A numerical value of $h$ is estimated as $4 \times 10^{-6}$. Due to $F \sim D_0^{1-1-d_f}$, we get the fractal dimension $d_f$ in an average sense $d_f \simeq 2.3$ which is empirically reasonable according to a large number of results of experimental investigations on the fractal dimension of dispersions.

Let us rewrite Eq. (21) as

$$F = B \cdot [bRT \ln(1 + k \frac{S}{23}) + \gamma_0^{d_f-1} (\Delta \rho g)^{\frac{d_f-1}{d_f}} D_0^{\frac{2(1-d_f)}{d_f}}],$$  

and take $R = 8.31 \times 10^7$(dyne-cm·mol$^{-1}$·K$^{-1}$), $T = 293$(K), $\gamma_0 = 72.92$(dyne·cm$^{-1}$), $\Delta \rho = 1.5$(g·cm$^{-3}$), and $g = 1000$(cm·s$^{-2}$). Here the unit of $k$ is [L/mol] and $B$ is a dimensionless parameter. We use the least-squares fit of the model Eq. (23) to experimental data reflecting the relationship between $F$ and $S$ to estimate values of the parameters $B$, $b$, and $k$. The results are shown in Fig. 2 and Table. I.

It can be seen from the results of fit that the flocculation factors obtained from the model Eq. (23) are close to those from real experiments. Therefore the assumption of the tension-induced flocculation of sediment particles are reasonable. The more experimental data would attain better fits, nevertheless, percentage difference between $F$ obtained by fits and experiments may still provide a verification of the rationality of the model proposed. In particular, the magnitude of the fitted pa-
rameter $k$ which possesses definite physical sense accords with our understanding of the adsorption of metal ions on sediment particles which is generated from large num-
bers of experiments, for example, Ref [29, 30, 31, 32, 33], etc. In addition, the fact that values of the fitted parameter $b$ in different measures are approximately equal indicates that the parameter $b$ may indeed represent such a physical quantity as saturation adsorptive capacity.

[1] Estuarine cohesive sediment dynamics, edited by A. J. Mehta (Springer, Berlin, 1986).
[2] K. S. Black, T. Tollhurst, D. M. Paterson, and S. E. Hagerhey, J. Hydraul. Eng. 128(2002)2.
[3] J. C. Winterwerp, J. Hydraul. Res. 36(1998)300.
[4] J. C. Winterwerp, Cont. Shelf Res. 22(2002)1339.
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[5] Fine sediment dynamics in the marine environment, edited by J. C. Winterwerp and C. Kranenburg (Elsevier, Amsterdam, 2002).
[6] C. Allain and M. Cloitre, Adv. Colloid Interface Sci. 46(1993)129.
[7] B.V. Derjaguin and L.D. Landau, Acta Physicochim. URSS 14(1941),733.
[8] E.J.W. Verwey and J.T.G. Overbeek, Theory of the Stability of Lyophobic Colloids, (Elsevier, Amsterdam, 1948).
[9] W. C. K. Poon and P. N. Pusey, Observation, Prediction and Simulation of Phase Transitions in Complex Fluids, edited by M. Baus et al. (Kluwer, Dordrecht, 1995).
[10] P. D. Kaplan, J. L. Rouke, A. G. Yodh and D. J. Pine, Phys. Rev. Lett. 72(1994)582.
[11] A. D. Dinsmore, A. G. Yodh and D. J. Pine, Phys. Rev. E 52(1995)4045.
[12] B. A. Movchan and A. V. Demchishin, Phys. Met. Metallogr. 28(1969)83.
[13] J. A. Thornton, Annu. Rev. Mater. Sci. 7(1977)239.
[14] M. Carpineti and M. Giglio, Phys. Rev. Lett. 70(1993)3828.
[15] C. J. Van Oss, Colloids Surf. A Physicochem. Eng. Aspects 78(1993)1.
[16] P. O. De Gennes, Adv. Colloid Interface Sci. 27(1987)189.
[17] S. L. Goren, J. Colloid Interface Sci. 69(1973)78.
[18] P. A. Kralchevsky and I. B. Ivanov, Chem. Phys. Lett. 121(1985)116.
[19] R. R. Mazzoco and P. C. Wayner Jr., J. Colloid Interface Sci. 214(1999)156.
[20] A. M. Freitas and M. M. Sharma, J. Colloid Interface Sci. 233(2001)73.
[21] P. Y. Julien, Erosion and sedimentation, (Cambridge University Press, Cambridge, U.K., 1995).
[22] A. L. lagvankar, and R. S. Gemmell, J. Amer. Water Works Assoc. 60(1968)1040.
[23] N. Tambo, and V. Watanabe, Water Res. 13(1979)409.
[24] P. Meakin, J. Colloid Interface Sci. 102(1984)491.
[25] G. C. Bushell, Y. D. Yan, D. Woodfield, J. Raper, and R. Amal, Adv. Colloid Interface Sci. 95(2002)1.
[26] V. Nikora, J. Aberle and M. Green, J. Hydraul. Eng. 130(2004)1043.
[27] The manual book of sediment, edited by Chinese Hydraulic Engineering Society (CHES), (China Environmental Science Press, Beijing, 1992) (in chinese).
[28] Wu. Deyi et al. Laboratory Report on settling velocity of silt flocs at the Bohai Bay (unpublished), (China Institute of Water Resources and Hydropower Research, Beijing 1979) (in chinese).
[29] Stumm, Schweiz. Z. Hydrol. 21(1960).
[30] P. A. Smith, The distribution of trace metal in the surficial sediment surrounding Keweenaw Point, Upper Michigan, Proc. 15th Conf. Great Lake Res., 1972.
[31] Chen. Jingsheng, Hydrobiologia. 176/177(1987)159-170.
[32] Gao. Kwanshen, Environmental Chemistry, 2(1984)4.
[33] Zin. Xianchan, China Environmental Science, 7(1987)2.