Will Skyrmions Move to the Hot or Cold Region under Temperature Gradients?

Chaofan Gong

College of Physics and Electronic Engineering, and Center for Computational Sciences, Sichuan Normal University, Chengdu, 610068, China

(Dated: August 6, 2021)

We obtain a dynamic equation at finite temperatures in the thermal bath of magnons, electrons, and phonons. We further consider effects of the angular momentum, momentum, magneto-crystalline anisotropy gradient, entropy equivalent field, dipole field, and diffusion to extend this equation to thermal gradients. Our findings are summarized as follows: skyrmions can move towards the hot or cold region; the tangential velocity of skyrmions can be reversed; two velocity components of skyrmions can have the same or different null points; the velocity in the tangential direction can be greater than the heat flow direction; the skyrmion Hall angle can be positive and negative; forces acting on skyrmions produced by equivalent magnetic fields will decrease with temperatures under uniform temperatures. Our results are consistent with experiments and beneficial for spintronics.

**Introduction.** Magnetic skyrmions in systems with the Dzyaloshinsky Moriya interaction (DMI) are nontrivial spin textures featured by the topological charge \( Q = (1/4\pi) \int (\mathbf{m} \cdot \partial_x \mathbf{m} \times \partial_y \mathbf{m}) \, dx \, dy \) (\( \mathbf{m} \) is the unit magnetization vector) [1][2]. Emerging electromagnetic fields [3] of skyrmions can deflect particle flows, resulting in the skyrmion Hall effect. So far, skyrmions are conveniently manipulated by various means such as optics [4], electricity [5], magnetism [6], heat [7], and stress [8].

Among them, the temperature gradient is one of the most cost-effective ways to manipulate skyrmions because it can make full use of the inevitable harmful heat generated by electronic devices and the cold in ultra-low-temperature quantum devices. Nevertheless, skyrmions are described by the Landau-Lifshitz-Gilbert (LLG) equation [9] at zero temperature for now. This has brought troubles to skyrmion studies at finite temperatures and temperature gradients. Existing experiments get the following results: skyrmions can move to the cold [10] or hot [11] region under thermal gradients; the Hall angle can increase [11] or remain unchanged [5] with the growth of thermal gradients; the tangential velocity can be greater than the thermal gradient direction [12]. But these conclusions are contrary to the theoretical [7][13][16]. The reason for discrepancies is that these papers may not have fully considered the interaction mechanism between transport particles (magnons and electrons) and skyrmions and indispensable fields.

In this work, we show that skyrmions at finite temperatures are in the thermal bath [16] of magnons, electrons, and phonons. This disturbs skyrmions and reduces effective forces on them. We investigate angular momentum and momentum transfers by scattering amplitudes [17][18] and impacts of the magneto-crystalline anisotropy gradient, entropy equivalent field [19][23], dipole field [24][25], and diffusion [26]. We give a universal skyrmion dynamics equation accorded with experimental data [10][11][27][28] at finite temperatures to further study skyrmion motions under thermal gradients [Fig.1]. Our findings are summarized as follows: skyrmions can move towards the hot or cold region; the tangential velocity of skyrmions can be reversed; two velocity components of skyrmions can have the same or different null points; the velocity in the tangential direction can be greater than the heat flow direction; the skyrmion Hall angle can be positive and negative; forces acting on skyrmions produced by equivalent magnetic fields will decrease with temperatures under uniform temperatures. Our results are consistent with experiments and beneficial for spintronics.

\[
\frac{\partial \mathbf{m}}{\partial t} = -\gamma \mathbf{m} \times \mathbf{H}_{\text{eff}} + \alpha \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t} + a (\mathbf{v} \cdot \nabla) \mathbf{m} - b \mathbf{m} \times (\mathbf{v} \cdot \nabla) \mathbf{m}.
\]

(1)

Here \( \gamma \) is the gyromagnetic ratio, \( \alpha \) is the Gilbert damping, \( \mathbf{v} \) is the velocity of magnons or electrons, \( \mathbf{m} = \mathbf{M}/M_s \) is the normalized magnetization with \( \mathbf{M} \) being the magnetization vector and \( M_s \) being the saturation magnetization, \( a \) and \( b \) are respectively adiabatic (angular momentum) and non-adiabatic (momentum) strength coefficients of magnons or electrons, and

\[
\mathbf{H}_{\text{eff}} = \frac{A}{\mu_0 M_s} \nabla^2 \mathbf{m} - \frac{2D}{\mu_0 M_s} \nabla \times \mathbf{m} + \frac{K}{\mu_0 M_s} \mathbf{n} + \mathbf{H} + \mathbf{H}_e + \mathbf{H}_{\text{dd}},
\]

(2)

is the effective magnetic field. Here \( A \) is the exchange stiffness, \( D \) is the DMI constant, \( K \) is the magneto-crystalline anisotropy

*chaofan.gong@foxmail.com*
constant expressing different cell structures, $\hat{n}$ is the easy axis unit vector, $H$ is the external magnetic field in the z-axis direction, $\mu_0$ is the vacuum permeability. $H_s$ and $H_{id}$ are respectively the entropy equivalent field and the dipole field.

Above all, we consider the derivation of adiabatic and nonadiabatic intensity coefficients $a$ and $b$. Because the emerging electromagnetic field of skyrmions will deflect magnons and electrons. So only part of magnons and electrons can pass through skyrmions. Accordingly, $a$ is proportional to the transmission coefficient $\lambda_a$ expressed by the differential scattering cross-section. Using the scattering amplitude [17, 18]

$$F(\phi) = \frac{e^{-i\frac{\phi}{\lambda}}}{\sqrt{2\pi k}} \sum_{n=-\infty}^{\infty} e^{i\pi(n + |n|Q)} \left(e^{2i\Delta_n} - 1\right)e^{in\phi},$$  

(3)

of the interaction between magnons or electrons and skyrmions ($k$ is the wave vector of magnons or electrons), letting the deflection angle $\phi \in (-\pi/2, \pi/2)$ means that magnons and electrons can pass through skyrmions to transfer angular momentum. The differential cross-section $|F(\phi)|^2$ represents the number of magnons or electrons passing through the length $R_s$ ($R_s$ is the skyrmion radius) in unit time. According to this definition, the dimensionless transmission coefficient expression

$$a_a = \frac{1}{2\pi R_s k} \left| \int_0^{\frac{\pi}{2}} \sum_{n=-\infty}^{\infty} e^{i\pi(n + |n|Q)} \left(e^{2i\Delta_n} - 1\right)e^{in\phi} d\phi \right|^2,$$

(4)

can be defined by dividing by the skyrmion radius. Where $\Delta_n \approx \Delta_0 = -ik/\lambda_0 < 1$ is the nth partial wave phase shift for low energy scattering with $k_0 = \sqrt{2m |U_0|/\hbar}$ and will decrease rapidly with increases of $k$ and $n$ (especially at that time $n > R_s k$), $U_0 = Fr = \left(v_1 (h/2\pi) (m \cdot \partial_r r \times \partial_r m) \right)$ is the potential energy magnitude, $v_1 = v_m$ or $v_s$ is the velocity of magnons or electrons, $Q = \pm 1$ is the skyrmion topological charge, $k_m = 2\pi/\lambda = 2\pi/\sqrt{\hbar A/ks}$ is the magnon wave number [31], $k_s = m_s v_s / \hbar$ is the electron wave number, $m_s$ is the electron mass, $\hbar$ is the reduced Planck constant, $s = h/2l^3$ is the spin density, and $l$ is the lattice constant.

Taking the electron as a reference. Because the magnon spin size is twice that of the electron, its corresponding spin-transfer torque is twice that of the electron. Furthermore, consider effects of the skyrmion chirality [32, 33] and the spin polarization on the spin flow net density [21]. Motions of negative chirality skyrmions are in the particle flow direction after the spin angular momentum transfer, while the positive chirality is the opposite. So the final expression of the adiabatic strength coefficient is

$$a = 2D^\pm SP\lambda_a,$$

(5)

where $D^\pm = \pm 1$ is the skyrmion chirality, $S$ is the spin size, $P$ is the spin polarizability. For the electron, there is $a = D^\pm P\lambda_a$, which satisfies previous conclusions that $a$ was taken as 1 [7].

For the diabatic coefficient $b$, magnons and electrons are deflected at any angle to transfer momentum except in the case of $\phi = 0$. Consequently, the scattering coefficient is $\lambda_b = 1 - |F(0)|^2/R_s$. Moreover, the larger skyrmion radius $R_s$ will lead to the more significant emerging electromagnetic field, which can deflect more magnons and electrons. The longer wave number $k$, the more pronounced particle properties of magnons and electrons, the stronger coupling between the magnetic moment and emerging electromagnetic field, the greater the "magnetic" electric Lorentz force magnons or electrons will be subjected. As a result, more magnons and electrons can transfer momentum $2hk$ to skyrmions. Sequentially, it is natural to find that the adiabatic intensity is proportional to the relative size $R_s k/2\pi$ of the skyrmion linearity [17] and wavelength (the idea here refers to the reflection of magnons on domain walls [34]). So the adiabatic strength coefficient is

$$b = \frac{\lambda_b R_s k}{2\pi}.$$

(6)

Next, we analyze effects of the magneto-crystalline anisotropy gradient, entropy equivalent field, and thermal induction dipole field. Firstly, we replace $A$, $D$, $M_s$, and $K$ with $A_m^{1,2,3,4}$, $D_m^2$, $M_s^e$, and $K_m^2$ respectively to make these parameters as the function of temperatures [29]. As a result, the magnetocrystalline anisotropy gradient is

$$\frac{\partial H_K}{\partial r_\parallel} = \frac{K}{\mu_0 M_s} \frac{\partial m_e}{\partial T} \frac{\partial T}{\partial r_\parallel} \hat{n}.$$

(7)

Integrating it from 0 to $r_\parallel$, we can get

$$H_K = \frac{K}{\mu_0 M_s} \frac{\partial m_e}{\partial T} \frac{\partial T}{\partial r_\parallel} r_\parallel \hat{n}.$$

(8)

which varies with temperature gradients. Secondly, the entropy equivalent field expression

$$H_e = - \frac{0.9A}{R_s \mu_0 M_s m_e^{1.5}} \frac{\partial m_e}{\partial T} \frac{\partial T}{\partial r_\parallel} \hat{n}.$$

(9)
is given in the Ref.[21]. Here the skyrmion radius is expressed as \( R_s = 2\pi Am_e^{-3/8}/D \) and \( \hat{r}_r \) represents the direction from the heat source to the cold region. Thirdly, the Ref. [25] indicates that because the magnetization varies with temperatures, the thermal effect will induce the magnetic dipole charge density \( \rho_m = -\nabla \cdot m \propto \pi w^2 d (A - m) \),

\[ (10) \]
in a cylinder of volume \( \pi w^2 d \) [Fig.1(7)] (\( d \) is the film thickness). From the magnetic charge viewpoint, the magnetic field produced here is

\[ H_{td} = \frac{\rho_m}{4\pi r^2} \hat{r} = -\frac{M_s dw^2}{4r^2} \frac{\partial m_e}{\partial T} \frac{\partial T}{\partial r} \hat{r}. \]

\[ (11) \]

Because the focus is on dynamics. We only consider the thermal induction dipole field that can influence the motion of skyrmions, but not the z-axis field that can affect their stability.

Then, using the equation \( F = \int f_{00} M_m \cdot \mathbf{H} \cdot (\partial m/\partial r) dV \) [35 36], integrating in the column coordinate, bringing in \( r_r = (T_0 - T)/G \) (\( T_0 \) is the heat source temperature), and replacing the integral of the coordinate \( dr_r \) with the integral of the temperature \( (dr_r/dT) dT = dG/T \), we can convert the above equivalent fields into the forces

\[ F_e = 0.9 D d \int (T - T_0) m_e^{4.6} (\partial T/\partial m_e)^2 dG \hat{r}_r, \]

\[ F_{td} = \frac{1}{2} \mu_0 M_s^2 w^2 d^2 G \int \frac{1}{T - T_0} m_e (\partial T/\partial m_e)^2 dG \hat{r}_r, \]

\[ F_K = \frac{2\pi K d}{G} \int m_e (\partial T/\partial m_e)^2 (T - T_0) dG, \]

acting on skyrmions. Subsequently, the Brownian motion satisfies the following relation [26]

\[ \dot{r}_r = \theta T, \]

\[ (15) \]

with \( \theta \) being the constant, \( r_r \) being the velocity, and \( r_r \) being the displacement. Derivation of \( r_r \) in the Eq.14, we have

\[ r_r d\dot{v}_{ih}/dr_r + v_{ih} = (T_0 - T) d\dot{v}_{ih}/dT + v_{ih} = \theta T. \]

\[ (14) \]

Under the condition that the thermal diffusion only relates to temperature gradients and has nothing with temperatures, we get the differential equation solution is

\[ v_{ih} = \theta T. \]

\[ (16) \]

Finally, we can rewrite the Eq.(1) into the skyrmion motion equation

\[ -\hat{z} \times (v + \sum_i a_i v_i) + \eta (av - \sum_i b_i v_i) - \frac{\gamma}{4\pi Q \mu_0 M_s m_e d} F = 0, \]

\[ (17) \]

by using the collective coordinate method [35 36]. Here we ignore the skyrmion interaction potential [37] since it is small [19]. Writing the Eq.(16) in the scalar form with two orthogonal directions and adding the thermal diffusion effect, we have

\[ v_\parallel = \sum_i (a_i + b_i \alpha^2) v_i + \frac{a \gamma}{4\pi Q \mu_0 M_s m_e d} (F_\parallel + F_\perp) + \theta T, \]

\[ v_\perp = \sum_i (b_i + a_i \alpha) \eta v_i + \frac{a \gamma}{4\pi Q \mu_0 M_s m_e d} (F_\parallel + F_\perp). \]

\[ (18) \]

\[ (19) \]

Where \( a \) represents the adiabatic effect of spin angular momentum transfer, \( b \) represents the non-adiabatic effect of momentum transfer, \( i \) taking \( m \) or \( s \) respectively represents the contribution of magnons or electrons, \( v_\parallel \) and \( v_\perp \) respectively represent along and perpendicular to the heat flow direction, \( F_\parallel = F_e + F_{td} + F_K \) is the sum of equivalent forces, \( \eta \) is the damping coefficient, \( v_m = \mu B \ln P/\epsilon_m M_s \) is the magnon velocity, \( m_m \) is the magnon mass, \( v_e = \mu B \ln P/\epsilon_m M_s \) is the electron velocity, \( J_m = -\kappa_e dT/dr \) is the magnon surface density, \( J_e = \sigma S_e dT/dr \) is the current surface density, \( \mu_B \) is the Bohr magneton, \( S_e \) is the Seebeck coefficient, \( \sigma \) is the electrical conductivity, and \( \kappa_e (H, T) = De^2 k_B^2 T(2 + y y H/2A S) k^2 [\exp(-\gamma y H/k_B T)]/8 \sqrt{2}\pi^{3/2} A^2 l^3 h \) is the magnon thermal Hall conductivity [10 11 38]. Driven by non-particle flows at finite temperatures, skyrmions are also in the heat bath of magnons, electrons, and phonons. The interaction between them and skyrmions makes the equilibrium magnetization \( m_e (T) \) [59] reduce. At this time, the dynamic equation of skyrmions is

\[ v = \frac{a \gamma m_e (T)}{4\pi Q (1 + a^2 \eta^2)} \int H \cdot (\partial \mathbf{m}/\partial r) dV. \]

\[ (20) \]

Here \( H \) is the effective or equivalent magnetic field caused by external stimuli. The physical meaning of the Eq.(20) is that thermally excited particles with the larger average velocity at high temperatures have stronger disturbances to skyrmions to make effective or equivalent forces acting on skyrmions smaller.

Estimate. Because the electron wave number \( k = 3.367 G \times 10^{-8} m^{-1} \) (where \( G \) only represents the thermal gradient numerical value) is very small, it has almost no diabatic effect. Furthermore, in universal skyrmion materials, there is almost a vertical anisotropy [10] that affects the stability rather than the motion of skyrmions. So we omit their contributions here.

Discussion. To avoid confusion, we should first point out that \( H_e \) is caused by magnon entropy [22], which is characterized by the variation of exchange stiffness with temperatures. Magnons once generated will immediately transfer to the cold region, making the magnon state density bound in skyrmions higher than the hot region. So skyrmions tend to the hot to balance magnon entropy. While \( v_{ih} \) is caused by a skyrmion as a soliton, it has entropy to spread to the cold. Besides, if it is not similar to laser heating on the surface but directly on the edge, the local magnetic charge cannot form [23]. Therefore, the dipole field will not exist.

In general, we find that \( v_\parallel, v_\perp, \) and \( \theta \) can be positive and negative due to competitions among the angular momentum, momentum, entropy equivalent field, dipole field, and diffusion [Fig.2, Fig.3, and Fig.4]. We can see from Fig.2(a) and
Fig. 2. (a) The contribution of magnon adiabatic and non-adiabatic effects to $v_{||}$. The velocity here is just the magnitude. (b) The contribution of the adiabatic torque, entropy equivalent field, dipole field, and diffusion to $v_{||}$. (c) The comparison of our results with previous theories in the case of considering only different transmission coefficients $\lambda_d$. (d) The effect of skyrmion chirality on $v_{||}$.

Fig. 3. (a) $v_{||}$ and $v_{\perp}$ vary with temperatures. (b), (c), and (d) are comparisons between our results and experiments.

plain such a big disparity. However, from our viewpoint, it is easy to understand. In the experiment of Yu et al., the environment temperature and temperature difference are minimal, which shows that the wavenumber and skyrmion radius are also small. It makes the adiabatic and entropy effect dominant over others (no dipole field due to heating on edges uniformly). So the transmission coefficient increases, skyrmions move to the hot at the greater velocity [Fig. 3(c)]. They also shows that skyrmion Hall angles relate to temperatures. In this paper, the Hall angle $\theta = \arctan v_{\perp}/v_{||}$ is consist with their conclusion [Fig. 3(d)], because velocities are the function of temperatures.

The experiment of Litzius et al. [12] shows that skyrmion Hall angles are independent of temperatures. This seems to disagree with the conclusion of Yu et al.. In fact, in their experiment with the temperature range of $200K \sim 300K$, there is almost only the adiabatic effect of electrons because the electron wavenumber is too small. The expression of the hall angle at this time is $\theta = -\alpha \eta$, which is obviously a constant independent of temperatures.

We can indicate that widely used adiabatic and diabatic parameters [9], respectively defined as $a = 1$ and $b \in (0, 1)$ (a constant), are defective. For high-frequency incident waves (magnons), the adiabatic effect is weak ($a < 1$), while the diabatic effect is strong ($b > 1$). The velocity will not match the actual if we follow the original definition. This situation is not obvious for electrons (low frequencies), so many papers can get better results with the previous definition. Generally speaking, the previous spin angular momentum transfer theory is a particular case of our paper under the firm adiabatic limit ($\lambda_d = 1$ and $\lambda_d = 0$), the contribution of the diabatic strength coefficient to the velocity $v_{||}$ under the previous definition is almost negligible, as shown in Fig. 2(c).

When we consider the wider temperature ranges and all effects, the Hall angle has non-monotonic behaviors: with the increase of temperatures, $\theta$ first decreases, then increases, and then decreases. The point of $\theta$ from positive to negative represents $v_{\perp} \rightarrow 0^+ \Rightarrow v_{\perp} \rightarrow 0^-$, which is not actually drawn because $v_{\perp}$ is also approaching zero. Then $\theta$ suddenly goes from negative to positive because of $v_{||} \rightarrow 0^- \Rightarrow v_{||} \rightarrow 0^+$. The non-monotonicity of Hall angles shows the positive and
negative alternations of skyrmion velocities, so Fig. 4(a) is different from Fig. 3(d).

Application. From Fig. 3(a), Fig. 4(b), Fig. 4(c), and Fig. 4(d), we can see that two velocity components of skyrmions have different or same critical points with variations of some parameters. Skyrmion motions in critical points on both sides are in opposite directions. If only one velocity direction has critical points, we can fabricate the thermal diode according to this property. Otherwise, we can make skyrmions have four motion directions to design the thermosensitive tetrode. Additionally, we can design the spin oscillator by blocking the radial motion. Compared with the current-based spin oscillator, it has advantages of adjustable rotation directions and low energy consumption. More importantly, better skyrmion manipulations by temperature gradients can promote quantum state regulations in topological insulators and topological superconductors.

Outlook. Unlike ferromagnetic materials, the spin degeneracy in antiferromagnetic materials causes magnons to have positive and negative double charges. Because of its double lattice property, the magnetization dynamic equation is also different. It is engaging to explore dynamic behaviors and applications for antiferromagnets in the future.

Conclusion. In short, we have analyzed effects of the angular momentum, momentum, entropy equivalent field, dipole field, magnetocrystalline anisotropic gradient, diffusion, and thermally excited particle bath. We give the complete skyrmion dynamic equation at finite temperatures and further solve the skyrmion dynamics problem under temperature gradients. We answer the question raised in this paper: skyrmions can move to the hot or cold region under temperature gradients. However, due to the length, we have not studied the influence of thermal activations on skyrmions. The thermal effect can make skyrmions have higher energy to better cross the pinning region in cases of quantum and classic.

Parameter values. Parameter values of Cu2OSeO3 are as follows: α = 0.01, Mₐ = 3.7 × 10⁴ A/m, A = 1.4 × 10⁻¹² J/m, D = 1.7 × 10⁻¹⁴ J/m², H = 2 × 10⁴ or 1.35 × 10⁵ A/m, μ₀ = 4π × 10⁻⁷ N/A², m = 2.06, Tc = 60 K, T₀ = 20 K, d = 10⁻⁸ m, l = 5 × 10⁻⁹ m, k_B = 1.38 × 10⁻²³ J/K, P = 1, θ = 2.41 × 10⁻¹² m²/(K·s) and μ_B = 9.2732 × 10⁻²⁴ A·m². Parameter values of Ta/CuFeB/MgO are as follows: γ = 2.2 × 10⁵ m/(A·s), α = 0.1, Mₐ = 9.03 × 10⁴ A/m, A = 1 × 10⁻¹¹ J/m, D = 2 × 10⁻¹⁰ J/m², H = 1.2 × 10⁻² A/m, Tc = 1450 K, T₀ = 350 K, σ = 7.6 × 10⁶ Ω⁻¹·m⁻¹, S = −0.5 μV/K, θ = 5.83 × 10⁻¹² m²/(K·s), d = 0.01, and l = 4 × 10⁻⁹ m.

Acknowledgement. Chaofan-Gong acknowledges the researcher Lin-Li for his help in writing and support by the Project of Sichuan Science and Technology Program (Grants No.2020YJ0136).

References

[1] S. Muhlbaier, B. Binz, F. Jonietz, C. Pfleiderer, A. Rosch, A. Neubauer, R. Georgii, and P. Boni, Science 323, 915 (2009).
[2] N. Nagaosa and Y. Tokura, Nat. Nanotechnol. 8, 899 (2013).
[3] T. Schulz, R. Ritz, A. Bauer, M. Halder, M. Wagner, C. Franz, C. Pfleiderer, K. Everschor, M. Garst, and A. Roche, Nat. Phys. 8, 301 (2012).
[4] Y. Jiang, H. Y. Yuan, Z.-X. Li, Z. Wang, H. W. Zhang, Y. Cao, and P. Yan, Phys. Rev. Lett. 124, 217204 (2020).
[5] K. Litzius, J. Lelaiert, P. Bassirian, D. Rodrigues, S. Kromin, I. Lemesh, J. Zavorka, K.-J. Lee, J. Mulkers, N. Kerber, D. Heinze, N. Keil, R. M. Reeve, M. Weigand, B. Van Waeyenberge, G. Schütz, K. Everschor-Sitte, G. S. D. Beach, and M. Kluij, Nat Electron 3, 30 (2020).
[6] H. Wang, J. Chen, T. Liu, J. Zhang, K. Baumgaertl, C. Guo, Y. Li, C. Liu, P. Che, S. Tu, S. Liu, P. Gao, X. Han, D. Yu, M. Wu, D. Grundler, and H. Yu, Phys. Rev. Lett. 124, 027203 (2020).
[7] S.-Z. Lin, C. D. Batista, C. Reichhardt, and A. Saxena, Phys. Rev. Lett. 112, 187203 (2014).
[8] N. S. Gusev, A. V. Sadovnikov, S. A. Nikitov, M. V. Sapozhnikov, and O. G. Udalov, Phys. Rev. Lett. 124, 157202 (2020).
[9] M. Lakshmanan, Phil. Trans. R. Soc. A. 369, 1280 (2011).
[10] Z. Wang, M. Guo, H.-A. Zhou, L. Zhao, T. Xu, R. Tomassello, H. Bai, Y. Dong, S.-G. Je, W. Chao, H.-S. Han, S. Lee, K.-S. Lee, Y. Yao, W. Han, C. Song, H. Wu, M. Carpentieri, G. Finocchio, M.-Y. Im, S.-Z. Lin, and W. Jiang, Nat. Electron. 3, 672 (2020).
[11] X. Yu, F. Kagawa, S. Seki, M. Kubota, J. Masell, F. Yasin, K. Nakajima, M. Nakamura, M. Kawasaki, N. Nagaosa, and Y. Tokura, PREPRINT (Version 1) available at Research Square https://doi.org/10.21203/rs.3.rs-156692/v1 (2021).
[12] M. Mochizuki, X. Z. Yu, S. Seki, N. Kanazawa, W. Koshieba, J. Zang, M. Mostovoy, Y. Tokura, and N. Nagaosa, Nat. Mater. 13, 241 (2014).
[13] L. Kong and J. Zang, Phys. Rev. Lett. 111, 067203 (2013).
[14] A. A. Kovalev, Phys. Rev. B 89, 241101 (2014).
[15] G.-B. Liu, D. Li, d. C. P. F. J. Wang, W. Liu, and Z.-D. Zhang, Chin. Phys. B 25, 067203 (2016).
[16] M. Weißenhofer, L. Rözza, and U. Nowak, Phys. Rev. Lett. 127, 047203 (2021).
[17] J. Iwasaki, A. J. Beeckman, and N. Nagaosa, Phys. Rev. B 89.
[18] C. Schütte and M. Garst, Phys. Rev. B 90, 094423 (2014).
[19] A. Donges, N. Grimm, F. Jakobs, S. Selzer, U. Ritzmann, U. Atxitia, and U. Nowak, Phys. Rev. Research 2, 013293 (2020).
[20] S. K. Kim and Y. Tserkovnyak, Phys. Rev. B 92, 020410 (2015).
[21] F. Schliekeiser, U. Ritzmann, D. Hinzke, and U. Nowak, Phys. Rev. Lett. 113, 097201 (2014).
[22] X. S. Wang and X. R. Wang, Phys. Rev. B 90, 014414 (2014).
[23] P. Yan, Y. Cao, and J. Sinova, Phys. Rev. B 92, 100408 (2015).
[24] S. Moretti, V. Raposo, E. Martinez, and L. Lopez-Diaz, Phys. Rev. B 95, 064419 (2017).
[25] Y. A. Shokr, O. Sandig, M. Erkovan, B. Zhang, M. Bernien, A. A. Ünal, F. Kronast, U. Parlak, J. Vogel, and W. Kuch, Phys. Rev. B 99, 214404 (2019).
[26] S. Miki, Y. Jibiki, E. Tamura, M. Goto, M. Oogane, J. Cho, R. Ishikawa, H. Nomura, and Y. Suzuki, J. Phys. Soc. Jpn. 90, 083601 (2021).
[27] O. Chubykalo-Fesenko and P. Nieves, in Handbook of Materials Modeling (Springer International Publishing, 2018) pp. 1–28.
[28] A. Mook, B. Göbel, J. Henk, and I. Mertig, Phys. Rev. B 97, 140401 (2018).
[29] S. Schlotter, P. Agrawal, and G. S. D. Beach, Appl. Phys. Lett. 113, 092402 (2018).
[30] S. Zhang and Z. Li, Phys. Rev. Lett. 93, 127204 (2004).
[31] U. Güngördü and A. A. Kovalev, Phys. Rev. B 94, 020405 (2016).
[32] W. Wang, M. Albert, M. Beg, M.-A. Bisotti, D. Chernyshenko, D. Cortés-Ortuño, I. Hawke, and H. Fangohr, Phys. Rev. Lett. 114, 087203 (2015).
[33] X.-G. Wang, L. Chotorlishvili, G.-H. Guo, A. Sukhov, V. Dugaev, J. Barnaš, and J. Berakdar, Phys. Rev. B 94, 104410 (2016).
[34] X.-g. Wang, G.-h. Guo, Y.-z. Nie, G.-f. Zhang, and Z.-x. Li, Phys. Rev. B 86, 054445 (2012).
[35] A. A. Thiele, Phys. Rev. Lett. 30, 230 (1973).
[36] A. A. Thiele, J. Appl. Phys. 45, 377 (1974).
[37] R. Brearton, G. van der Laan, and T. Hesjedal, Phys. Rev. B 101, 134422 (2020).
[38] Y. Onose, T. Ideue, H. Katsura, Y. Shiomi, N. Nagaosa, and Y. Tokura, Science 329, 297 (2010).
[39] R. Moreno, R. F. L. Evans, S. Khmelevskiy, M. C. Muñoz, R. W. Chantrell, and O. Chubykalo-Fesenko, Phys. Rev. B 94, 104433 (2016).
[40] L. Shen, J. Xia, G. Zhao, X. Zhang, M. Ezawa, O. A. Tretiakov, X. Liu, and Y. Zhou, Appl. Phys. Lett. 114, 042402 (2019).
[41] S. Zhang, F. Kronast, G. van der Laan, and T. Hesjedal, Nano Lett. 18, 1057 (2018).
[42] E. Khalaf, S. Chatterjee, N. Bultinck, M. P. Zaletel, and A. Vishwanath, Sci. Adv. 7, eabf5299 (2021).
[43] P. Baláž, V. K. Dugaev, and J. Barnaš, Phys. Rev. B 85, 024416 (2012).
[44] A. Fert, N. Reyren, and V. Cros, Nat. Rev. Mater. 2, 17031 (2017).
[45] D. Yu, S. Luo, Y. Li, V. Koval, and C. Jia, Phys. Rev. B 100, 104410 (2019).