**Strength Tortuosity - Porosity Relation in Locally Types of Porous Media (Experimental Model)**

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**Abstract.** The experimental model of five types of porosity media (fine sand, gravel, sand, porcelanite and composed media) was conducted to determine the relationship between tortuosity and porosity. The red colour (Carmoisine E122) was used as a tracer. The results have good agreement with an empirical relation based on performing regression analysis on test result through the use of the trial version 9 of data fit software of Oakdale Engineering computer programs. The proposed empirical equation was correlated as a function of three basic variables of media characteristics; porosity (n), shape factor (ϕ) and sphericity factor (P) by

\[ \tau = 1 + C_1 \cdot \phi \cdot (1 - n^2) \]

**Keywords:** Strength Tortuosity, sphericity factor and porosity media

1. **Introduction**

Tortuosity is one of the fundamental parameters characterizing the geometry and transport characteristic of porous media [1]. The tortuosity is shown in the different theoretical transport equation (electrical conductivity, permeability and diffusion). The definition of which is the ratio of actual flow path length average \( L_e \) to the length \( L \) of the porous media in the direction of macroscopic flow. In the presence of a fluid or electrical current flow through a porous media, fluid particles do not travel along a straight path as in pipe [8].

Tortuosity depends on other porous media characteristics; this includes pore diameter, porosity, sphericity factor and particle shape [19]. This means that is not a physical constant.

Mathematically, it is expressed a

\[ \tau = \frac{L_e}{L} \]  

(1)

It should be noted that there are a couple of definitions of tortuosity more frequently used in literature by researchers \( L_e/L_0 \) and \( (L_e/L_0)^2 \), or \( (L_0/L_e)^2 \).

\( L_e/L_0 \) and \( (L_e/L_0)^2 \), or \( (L_0/L_e)^2 \). Particularly if tortuosity is treated as a pure geometric variable to define the difference between the length of the flow path and the bed depth \( L_e/L_0 \) is appropriate. However, if the flow velocity is also examined and \( L_0/L_e \) is employed as the average cosine between the equivalent flow path and the bulk flow direction, then \( (L_0/L_e)^2 \) should be applied to account for both flow path length (and hydraulic gradient,) and the velocity. [20].

Since the study deals with the geometric length of flow path, the more simplistic definition of \( L_e/L_0 \) was selected.
The relationship between Tortuosity and porosity were studied by the researcher, Maxwell (1873) studied the association between tortuosity and porosity and submitted an empirical relationship given in equation below as [15]

\[ \tau = 1.5 - 0.5 n \]  

(2)

Berryman (1981). Concluded the theoretical relation between tortuosity and porosity factor as: [5]

\[ \tau = (1 + n^{-1})/2 \]  

(3)

Pape et al. (1998) obtained based on the assumption of fractal pore geometry relations as:[7]

\[ \tau = 0.67 n^{-1} \]  

(4)

Koponen et al. (1998) investigated the tortuosity of the flow in a random two-dimensional porous medium and the following correlation for the tortuosity as a function of the porosity, concluding his study with the following correlation: [12]

\[ \tau = 0.8(1 - n) + 1 \]  

(5)

Boving and Grathwohl (2001) derived from laboratory contaminant diffusion experiments the empirical equation as [6]

\[ \tau = n^{-1.2} \]  

(6)

A. Nabovati and A. C. M. SOUSA (2007) simulated at pore level using the single relaxation time Lattice Boltzmann Equation (LBE), to find tortuosity as a function of the porosity. The following formula developed: [16]

\[ \tau = -0.519 n^3 + 0.879 n^2 - 1.1657 n + 1.8058 \]  

(7)

Mohammed N.A. (2011) revised the Maxwell equation into form as [15]

\[ \tau = 1.5312 - 0.5268 n^{0.8984} \]  

(8)

Firstly the objectives of this study measure the tortuosity factor from the tracer method. The second objective is to produce propose equations to estimate the tortuosity factor from the experimental test.

2. Experimental Facilities

2.1 Sample Preparation and Sieve Analysis

Five types of porous media were tested. The first four samples consisted of (sand, Gravel, fine sand and Porcilinaite) porous media and one was composed by mixing four porous media with equal ratios (composed media). The sand, Gravel and Porcilinaite samples were carefully washed with distilled water in order to purify them from any chemical impurities. Then samples were dried in the oven at 105 °C for 48 hours. Grain-size Distribution Analysis of the five media samples was studied.

2.2 Experimental Procedure

An experimental system made of an acrylic material was used to measure the tortuosity experimentally as shown in Figure (1). The system consists of two small tanks, one is used an inlet tank with dimensions of (400mm *400mm *800mm), while the other is used as an outlet tank with dimensions of (200mm*200mm *200mm). Two small tanks are joined with each other by a circular cross-section pipe with (50mm) in diameter and (800 mm) in length. Tracer was injected through 16 holes distributed along the pipe cross-section from the inlet to the outlet of the pipe respectively. Different velocities were measured by changing the head at the inlet tank which was achieved by different head levels.

Levels of the different head were controlled through different small valves. Each valve can give a certain level of water in the inlet tank. Each level can be different from the previous one by (10 cm).
2.3. Tortuosity Test
The procedure of conducting the experiment was as follows, the First injection the Carmoisine (E-122) tracer is made firstly without media. The injection was made firstly through the first inlet tank level, where time is measured from the moment of injection the Carmoisine till the colour reached the outlet section of the pipe. Injection is then made through the second hole of the inlet tank and time is also measured in the same way and measurements continue for all holes and time (t) is determined. Then the process was used again with deferent velocity. The pipe was filled with a porous media and then the process was used again and measured the effective time (te). The actual length (Li) in each hole was measured by multiple the velocity by the effective time (te) and then calculated the equivalent length (Le) from the form as:

\[ L_e = \frac{1}{N} \sum L_i \]  

(9)

Where N: is the number of streamlines.

Then tortuosity was calculated by applied equation (1). The process was used again by using varies grain sizes to change the porosity and surface area of all selected porous media that were experimented at a temperature which ranged from 200C to 250C. The results are correlated in the table (1).

| media       |  n     | 0.30 | 0.32 | 0.33 | 0.34 | 0.37 | 0.41 | 0.43 | 0.46 |
|-------------|--------|------|------|------|------|------|------|------|------|
| Sand        |   τ    | 1.80 | 1.78 | 1.71 | 1.65 | 1.63 | 1.58 | 1.53 | 1.49 |
| Fine Sand   |   n    | 0.32 | 0.33 | 0.35 | 0.37 | 0.39 | 0.40 | 0.42 | 0.45 |
|             |   τ    | 1.67 | 1.64 | 1.63 | 1.61 | 1.68 | 1.56 | 1.54 | 1.50 |
| Composite   |   n    | 0.36 | 0.38 | 0.4  | 0.42 | 0.43 | 0.45 | 0.47 | 0.49 |
|             |   τ    | 2.38 | 2.29 | 2.13 | 2.01 | 1.97 | 1.91 | 1.83 | 1.75 |
| Gravel      |   n    | 0.38 | 0.39 | 0.40 | 0.41 | 0.42 | 0.43 | 0.44 | 0.46 |
|             |   τ    | 2.41 | 2.39 | 2.37 | 2.35 | 2.30 | 2.11 | 1.94 | 1.76 |
| porcilinaite|   n    | 0.52 | 0.55 | 0.60 | 0.65 | 0.68 | 0.72 | 0.75 | 0.78 |
|             |   τ    | 2.79 | 2.49 | 1.96 | 1.78 | 1.73 | 1.65 | 1.54 | 1.45 |

3. Analytical Models
Numerical simulations reproducing the experiment were performed with use of regression analysis on test results through the use of the trial version 9 of data Fit software of Oakdale Engineering computer programs.

The tortuosity factor is a function of three variables as shown in the following expression:
\[ \tau = f(n, \phi, P) \]

Where: \( P, \phi \) and \( n \) are shape factor, spherisity factor and porosity.

Based on experimental results obtained from this paper, the empirical equation of tortuosity factor can be correlated as:

\[
\begin{align*}
\tau &= 1 + 52.46 \times P \phi (1 - n^{0.0135}) & \text{For sand} & (10) \\
\tau &= 1 + 1.807 \times P \phi (1 - n^{0.434}) & \text{For silt} & (11) \\
\tau &= 1 + 83.39 \times P \phi (1 - n^{2.392}) & \text{For composed} & (12) \\
\tau &= 1 + 54.63 \times P \phi (1 - n^{0.533}) & \text{For gravel} & (13) \\
\tau &= 1 + 54.63 \times P \phi (1 - n^{0.533}) & \text{For porcinaite} & (14)
\end{align*}
\]

4. Results and Discussion

Figures (2) to (6) show the experiment values versus proposed values of tortuosity factor using equations (10) to (14)

**Figure 2.** Experiment Values versus Proposed Values of Tortuosity for Sand

**Figure 3.** Experiment Values versus Proposed Values of Tortuosity Factor for Fine Sand
Comparisons with experimental result indicate that the proposed equations (10) - (14) are properly estimate the tortuosity factor of porous media as shown in figures (2) to (6). The proposed equations of tortuosity factors in the present study and the previous researchers applied to the experimental data of this study as shown in Figures (7) to (11).
Figure 7. A Comparison among the Experiment Data and the Proposed Equation in the Present Study with Proposed Equation of Previous Studies.

Figure 8. A Comparison among the Experiment Data and the Proposed Equation in the Present Study with Proposed Equation of Previous Studies.
Figure 9. A Comparison among the Experiment Data and the Proposed Equation in the Present Study with Proposed Equation of Previous Studies.

Figure 10. A Comparison of the Experiment Data and the Proposed Equation in the Present Study with Proposed Equation of Previous Studies.
Figure 11. A Comparison of the Experiment Data and the Proposed Equation in the Present Study with Proposed Equation of Previous Studies

Tortuosity coefficient increases when porosity decreases, however, the rate of reduction depends on the type and structure of porous media, which is expected as the flow path becomes longer and more tortuous within pore volume fraction (21). Figure (7) shows the present model for sand media successfully with the Berryman A. Nabovati and Koponen et al. models with error (10%, 22 %,) respectively, while the models proposed by Boving and Pape et al. give the highest value for the present model. For fine sand media, figure (8) show a very good agreement between the present model and Koponen et al., and A. Nabovati models with high correlation coefficients (0.9959), while results from Boving & Grathwohl equation are far away from the present model. It was also found that figures(9) and(10) are shown that the proposed equation for non-uniform Composed and gravel media seem to give the best fitting with Berryman model(R2 = 0.9987, 0.997 ) respectively. For porcilinaite media; figure (11) shows that the proposed model successfully with Boving& Grathwohl models with average percentage error to be (3.9%). It can also be seen the proposed equation value was higher than all proposed equation of researchers.

5. Conclusions
1- Based on experimental results obtained from this investigation, the empirical equation of tortuosity factor can be correlated as $\tau = 1 + C_1 \times \phi (1 - n^{C_2})$
2- Tortuosity factor increases for low porosity which is expected as the flow path becomes longer and more tortuous with decrease see in pore volume fraction.

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