\( Z \to l_i^+ l_j^- \) processes in the BLMSSM* 

Xing-Xing Dong(董幸幸)\(^1\)  Shu-Min Zhao(赵树民)\(^2\)  Xi-Jie Zhan(展希杰)
Zhong-Jun Yang(杨忠军)  Hai-Bin Zhang(张海斌)  Tai-Fu Feng(冯太傅)
Department of Physics and Technology, Hebei University, Baoding 071002, China

Abstract: In a supersymmetric extension of the Standard Model (SM) where baryon and lepton numbers are local gauge symmetries (BLMSSM), we investigate the charged lepton flavor violating (CLFV) processes \( Z \to l_i^+ l_j^- \) after introducing new gauginos and right-handed neutrinos. In this model, the branching ratios of \( Z \to l_i^+ l_j^- \) are around \( (10^{-8} - 10^{-10}) \), which approach the present experimental upper bounds. We hope that the branching ratios for these CLFV processes can be detected in the near future.

Keywords: supersymmetric, charged lepton flavor violation, branching ratios

PACS: 11.30.Pb, 13.38.Dg, 14.60.-z  DOI: 10.1088/1674-1137/41/7/073103

1 Introduction

Neutrinos have tiny masses and mix with each other, as has been proved by the neutrino oscillation experiments [1–4]. This shows that lepton flavor symmetry is not conserved in the neutrino sector. A new particle around 125 GeV has been detected by the LHC [5–7], with properties close to those of the Higgs boson, a great success for the Standard Model (SM). However, due to the GIM mechanism, the expected rates for charged lepton flavor violating (CFLV) processes [8–10] are very tiny in the SM with massive neutrinos. For example, \( Br(Z \to e\mu) \sim Br(Z \to e\tau) \sim 10^{-54} \) and \( Br(Z \to \mu\tau) \sim 10^{-60} \) [11–15] are much smaller than the experimental upper bounds. CLFV is forbidden in the SM. In Table 1, we show the present experimental limits and future sensitivities for some CLFV processes. In Ref. [16], the authors consider that future sensitivities for CLFV processes may reach \( 10^{-11} \). At a future circular \( e^+e^- \) collider (such as FCC-ee (TLEP)) [17–19], it is estimated that the sensitivities could be improved up to \( 10^{-13} \). Thus, any signal of CLFV would be a hint of new physics, and the study of CLFV processes is an effective approach to explore new physics beyond the SM.

In a simple SM extension, CLFV processes are restricted strongly by the tiny neutrino masses. As an appealing supersymmetric extension of the SM, the minimal supersymmetric standard model (MSSM) [27–30] with \( R \)-parity [29] conservation has drawn physicists’ attention for a long time. However, the left-handed light neutrinos remain massless, and it cannot explain the discovery of neutrino oscillations. Therefore, more research is ongoing on the light neutrino masses and mixings with MSSM extensions [31–36]. As a supersymmetric extension of the MSSM with local gauged baryon (B) and lepton (L) numbers, the BLMSSM has been introduced [37–40]. In the BLMSSM, the local gauged B must be broken in order to account for the asymmetry of matter and antimatter in the universe. Right-handed neutrinos are introduced to explain the data from neutrino oscillation experiments, hence lepton number is also expected to be broken [39]. In Refs. [39, 41], baryon number and lepton number are local gauged and spontaneously broken at the TeV scale in the BLMSSM.

| CLFV process | present limit | future sensitivity (TESLA) |
|--------------|---------------|----------------------------|
| \( Z \to e\mu \) | \(<7.5\times10^{-7} \) [20–22] | \(~10^{-9} \) [26] |
| \( Z \to e\tau \) | \(<9.8\times10^{-6} \) [20, 23, 24] | \(~10^{-8} \) [26] |
| \( Z \to \mu\tau \) | \(<1.2\times10^{-5} \) [20, 23, 25] | \(~0.44 \) [26] |

In this work, we continue to analyze the CLFV processes \( Z \to l_i^+ l_j^- \) after introducing new gauginos and right-handed neutrinos in the BLMSSM.

* Supported by Major Project of NNSFC (11535002, 11605037, 11647120, 11275036), Natural Science Foundation of Hebei Province (A2016201010, A2016201069) and Natural Science Fund of Hebei University (2011JQ05, 2012-242), Hebei Key Lab of Optic-Electronic Information and Materials, the Midwest Universities Comprehensive Strength Promotion Project

1) E-mail: dxx_0304@163.com
2) E-mail: zhaosm@hbu.edu.cn

Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI. Article funded by SCOAP* and published under licence by Chinese Physical Society and the Institute of High Energy Physics of the Chinese Academy of Sciences and the Institute of Modern Physics of the Chinese Academy of Sciences and IOP Publishing Ltd

073103-1
the BLMSSM. Compared with the MSSM, the neutrino masses in the BLMSSM are not zero. Three heavy neutrinos and three new scalar neutrinos are introduced in this model. A new particle, the lepton neutralino $\chi^0_L$, is also introduced. These new sources enlarge the CLFV processes via loop contributions. Therefore, the expected experimental results for the CLFV processes may be obtained in the near future.

This work is organized as follows. In Section 2, we summarize the BLMSSM briefly, including its superpotential, the general soft SUSY-breaking terms, needed mass matrices and couplings. Section 3 is devoted to the decay widths of the CLFV processes $Z \rightarrow 1^+_L 1^+_R$. In Section 4, we give the corresponding parameters and numerical analysis. The discussion and conclusion are given in Section 5. An Appendix is devoted to the concrete forms of coupling coefficients in Fig. 1.

2 BLMSSM

The local gauge group of BLMSSM $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_B \otimes U(1)_S$ [38, 42, 43] enlarges the SM. In the BLMSSM, the new quark superfields $Q_L \sim (3, 2, 1/6, B_4, 0)$, $U^c_L \sim (\overline{3}, 1, -2/3, -B_4, 0)$, $D^c_L \sim (\overline{3}, 1, 1/3, -B_4, 0)$, $Q^c_R \sim (3, 2, -1/6, -(1 + B_3), 0)$, $U^c_R \sim (3, 1, 2/3, 1 + B_3, 0)$ and $D^c_R \sim (3, 1, -1/3, 1 + B_3, 0)$ are introduced to cancel the B anomaly. To break baryon number spontaneously, the model introduces Higgs superfields $\Phi_B \sim (1, 1, 0, 1, 0)$ and $\varphi_B \sim (1, 1, 0, -1, 0)$. The new lepton superfields $L_L \sim (1, 2, -1/2, L_4)$, $E^c_L \sim (1, 1, 1, 0, -L_4)$, $N^c_L \sim (1, 1, 0, 0, -L_4)$, $\bar{N}^c_L \sim (1, 2, 1/2, 0, -3 + L_4)$, $\bar{E}^c_L \sim (1, 1, -1, 0, 3 + L_4)$ and $\bar{N}^c_L \sim (1, 1, 0, 3 + L_4)$ are introduced to cancel the L anomaly. The exotic Higgs superfields $\Phi_L \sim (1, 1, 0, 0, -2)$ and $\varphi_L \sim (1, 1, 0, 0, 2)$ can break lepton number spontaneously. Here $B_4$ and $L_4$ stand for baryon and lepton numbers for a given field respectively. In our numerical calculation, we use $B_4 = 3/2$ and $L_4 = 3/2$. The exotic Higgs superfields $\Phi_B$, $\varphi_B$ and $\Phi_L$, $\varphi_L$ acquire nonzero vacuum expectation values (VEVs), then the exotic quarks and exotic leptons obtain masses. The model also includes the superfields $X \sim (1, 1, 0, 2/3 + B_4, 0)$ and $X' \sim (1, 1, 0, -(2/3 + B_4), 0)$ to make exotic quarks unstable. Furthermore, with $X$ and $X'$ mixing together, the lightest mass eigenstate can be a dark matter candidate.

The superpotential of the BLMSSM is shown as follows [44]

$$W_{BLMSSM} = W_{MSSM} + W_B + W_L + W_X,$$

with $W_{MSSM}$ representing the superpotential of the MSSM. The concrete forms of $W_B$, $W_L$ and $W_X$ can be obtained in Ref. [44].

In the BLMSSM, the soft breaking terms $L_{\text{soft}}$ are generally given by [38, 39, 44], and only the leptonic terms contribute to our study:

$$L_{\text{soft}} = -(m_{\chi^0_i})_{IJ} N^c_i \tilde{N}^c_j - m_{\Phi_L}^2 \Phi^*_L \Phi_L - m_{\varphi^* L}^2 \varphi^*_L \varphi_L - (m_L \lambda_L \lambda_L + h.c.) + A_N Y_L \tilde{L} H_u \tilde{N}^c + A_N \lambda_N \tilde{N}^c \varphi_L + B_L \mu_L \Phi_L \varphi_L + h.c.$$. (2)

Here $\lambda_L$ represents the gaugino of $U(1)_L$. The $SU(2)_L$ doublets $H_u$ and $H_d$ obtain the nonzero VEVs $v_u$ and $v_d$.

$$H_u = \left( \frac{1}{\sqrt{2}} (v_u + H^0_u + i P^0_u) \right),$$

$$H_d = \left( \frac{1}{\sqrt{2}} (v_d + H^0_d + i P^0_d) \right).$$ (3)

The $SU(2)_L$ singlets $\Phi_L$ and $\varphi_L$ acquire the nonzero VEVs $v_L$ and $\bar{v}_L$.

$$\Phi_L = \frac{1}{\sqrt{2}} (v_L + \Phi^0_L + i P^0_L),$$

$$\varphi_L = \frac{1}{\sqrt{2}} (\bar{v}_L + \varphi^0_L + i P^0_L).$$ (4)

In the BLMSSM, the mass matrices of lepton neutralinos, neutrinos, sleptons and sneutrinos are introduced as follows.

In the basis $(i \lambda_L, \psi_{\Phi_L}, \psi_{\varphi_L})$ [37, 45, 46], the mixing mass matrix of lepton neutralinos is obtained.

$$M_{LN} = \begin{pmatrix}
2M_L & 2v_L g_L & -2\bar{v}_L g_L \\
2v_L g_L & 0 & -\mu_L \\
-2\bar{v}_L g_L & -\mu_L & 0
\end{pmatrix}. \quad (5)$$

Then the three lepton neutralino masses are deduced by diagonalizing the mass matrix $M_{LN}$ by $Z_{N_L}$.

After symmetry breaking, the mass matrix of neutrinos is deduced in the basis $(\nu, N^c)$ [47, 48]

$$\begin{pmatrix}
0 & \frac{v_u}{\sqrt{2}} (Y_{\nu})_{IJ} \\
\frac{v_u}{\sqrt{2}} (Y_{\nu}^T)_{IJ} & \frac{\bar{v}_L}{\sqrt{2}} (\lambda_{N^c})_{IJ}
\end{pmatrix}. \quad (6)$$

Then diagonalizing the neutrino mass matrix by the unitary matrix $U_\nu$, we can get six mass eigenstates of neutrinos, which include three light eigenstates and three heavy eigenstates.

In the BLMSSM, the slepton mass squared matrix deduced from Eqs. (1), (2) reads as

$$\begin{pmatrix}
(M_{\tilde{L}}^2)^{LL} & (M_{\tilde{L}}^2)^{LR} \\
(M_{\tilde{L}}^2)^{LR} & (M_{\tilde{L}}^2)^{RR}
\end{pmatrix}. \quad (7)$$
where,

$$(\mathcal{M}_L^{\nu})_{LL} = \frac{g_1^2-g_2^2}{8}(v_u^2-v_d^2)\delta_{ij} + g_L^2(v_{\ell u}^2-v_{\ell d}^2)\delta_{ij} + m_{\ell i}^2\delta_{ij} + (m_{\ell}^2)_{ij},$$

$$(\mathcal{M}_L^{\nu})_{LR} = \frac{\mu^\nu_{L R}}{\sqrt{2}}(Y_{\nu R})_{ij} - \frac{v_d}{\sqrt{2}}(A_{\nu})_{ij} + \frac{v_d}{\sqrt{2}}(A_{\ell})_{ij},$$

$$(\mathcal{M}_L^{\nu})_{RR} = \frac{g_L^2}{4}(v_{\nu}^2 - v_{\ell}^2)\delta_{ij} - g_L^2(v_{\nu}^2 - v_{\ell}^2)\delta_{ij} + m_{\ell i}^2\delta_{ij} + (m_{\ell}^2)_{ij}. \quad (8)$$

Through the matrix $Z_L$, the mass matrix can be diagonalized.

From the contributions of Eqs. (1),(2), we also deduce the mass squared matrix of sneutrino $\mathcal{M}_\nu$ with $\tilde{n}^\tau = (\tilde{\nu}, \tilde{N}^c)$

$$
\begin{pmatrix}
\mathcal{M}^2_{\tilde{\nu}}(\tilde{\nu})_{ij} & \mathcal{M}^2_{\tilde{N}}(\tilde{\nu})_{ij} \\
(\mathcal{M}^2_{\tilde{N}}(\tilde{\nu})_{ij})^\dagger & \mathcal{M}^2_{\tilde{N}^c}(\tilde{N}^c)_{ij}
\end{pmatrix}, \quad (9)
$$

where,

$$\mathcal{M}^2_{\tilde{\nu}}(\tilde{\nu})_{ij} = \frac{g_1^2 + g_2^2}{8}(v_u^2 - v_d^2)\delta_{ij} + g_L^2(v_{\ell u}^2 - v_{\ell d}^2)\delta_{ij} + \frac{v_d^2}{2}(Y_{\nu R})_{ij} + (m_{\ell}^2)_{ij},$$

$$\mathcal{M}^2_{\tilde{N}}(\tilde{\nu})_{ij} = \frac{\mu^\nu_{L R}}{\sqrt{2}}(Y_{\nu R})_{ij} - \frac{v_d}{\sqrt{2}}(A_{\nu})_{ij} + (m_{\ell}^2)_{ij},$$

$$\mathcal{M}^2_{\tilde{N}^c}(\tilde{N}^c)_{ij} = -g_L^2(v_{\nu}^2 - v_{\ell}^2)\delta_{ij} + \frac{v_d^2}{2}(Y_{\nu R})_{ij} + 2\pi_{\nu}^2(\lambda_{\nu L}^c, \lambda_{\nu R}^c)_{ij} + \mu_{\ell}^\nu_{L R} + m_{\ell}^2\delta_{ij}. \quad (10)$$

Then the sneutrino masses can be obtained by the formula $Z_{\nu}^{\nu,\nu}Z_{\nu}^{\nu,\nu} = \text{diag}(m_{\nu}^2, m_{\nu}^2, m_{\nu}^2, m_{\nu}^2, m_{\nu}^2, m_{\nu}^2)$. In the BLMSSM, we deduce the corrections for the couplings which exist in the MSSM due to superfields $N^c$. The corresponding couplings for W-lepton-neutrino, Z-neutrino-neutrino, charged Higgs-lepton-neutrino, Z-neutrino-sneutrino and chargino-lepton-sneutrino are introduced in Ref. [37].

From the interactions of gauge and matter multiplets $i\bar{g}\sqrt{2}T_Y^a(\lambda^c\psi_jA^c_j-\lambda^c\psi_jA_j)$, the lepton-slepton-lepton neutralino coupling is deduced here

$$L_{N,\nu}^{ij,L} = \sqrt{2}g_L\tilde{x}_{L,j}^i \left(Z_{N,\nu}^{ij}Z_{L}^{i,j}P_L - Z_{N,\nu}^{ij}Z_{L}^{i,j}P_R\right) + h.c. \quad (11)$$

### 3 The CLFV decays $Z\rightarrow l_1^{\pm}l_2^{\pm}$

In the BLMSSM, we study the CLFV processes $Z\rightarrow l_1^{\pm}l_2^{\pm}$. The corresponding Feynman diagrams can be depicted by Fig. 1, and the corresponding effective amplitudes can be written as [15, 49, 50]

$$M_\mu = i\gamma_\mu(F_L P_L + F_R P_R)l_i, \quad (12)$$

with

$$F_{L,R} = F_{L,R}(S) + F_{L,R}(W), \quad (13)$$

where $l_{i,j}$ represent the wave functions of the external leptons. The coefficients $F_{L,R}$ can be obtained from the amplitudes of the Feynman diagrams. $F_{L,R}(S)$ correspond to Fig. 1(1)–Fig. 1(7), and stand for the contributions from chargino-sneutrino, neutralino-slepton, neutrino-charged Higgs and lepton neutralino–slepton; $F_{L,R}(W)$ correspond to Fig. 1(8) and Fig. 1(9), and stand for the contributions from W-neutrino due to three light neutrinos and three heavy neutrinos mixing together. We formulate these coefficients as follows:
Here, $H^{S,F}_{L,R}$ ... represent the corresponding coupling coefficients of the left (right)-hand parts in the Lagrangian and the concrete expressions can be found in the Appendix. $x_i = \frac{m_{X_i}}{m_{Slep}}$, with $m$ representing the mass of the corresponding particle, and $m_{Slep}$ representing the energy scale of the new physics to make the amplitudes dimensionless. The one-loop functions $G_i(x_1, x_2, x_3), i=1,2$ are given by

$$G_1(x_1, x_2, x_3) = \frac{1}{16\pi^2} \left[ \frac{x_1^2 \ln x_1}{(x_1-x_2)(x_1-x_3)} + \frac{x_1 x_2 \ln x_2}{(x_2-x_1)(x_2-x_3)} + \frac{x_2^2 \ln x_2}{(x_2-x_3)(x_2-x_3)} \right].$$

$$G_2(x_1, x_2, x_3) = \frac{1}{16\pi^2} \left[ \frac{x_1^2 \ln x_1}{(x_1-x_2)(x_1-x_3)} + \frac{x_1 x_2 \ln x_2}{(x_2-x_1)(x_2-x_3)} + \frac{x_2^2 \ln x_2}{(x_2-x_3)(x_2-x_3)} \right].$$

Then, the branching ratios of $Z \rightarrow l_i^+ l_j^-$ can be summarized as

$$Br(Z \rightarrow l_i^+ l_j^-) = \frac{1}{12\pi} \frac{m_Z}{T_Z} \left[ |F_L|^2 + |F_R|^2 \right]$$

$$= \frac{1}{12\pi} \frac{m_Z}{T_Z} \left[ |F_L(S)|^2 + |F_L(S)|^2 + |F_R(S)|^2 \right].$$

where $T_Z$ represents the total decay width of the Z-boson and we use $T_Z \approx 2.4952$ GeV [20].

4 Numerical results for the CLFV processes $Z \rightarrow l_i^+ l_j^-$

In this section, we study the numerical results, and consider the experiment constraints from the light neutral Higgs mass $m_{h^0} \approx 125$ GeV [5–7, 20] and the neutrino experiment data [20]

$$\sin^2 \theta_{13} = (2.19 \pm 0.12) \times 10^{-2}, \sin^2 \theta_{12} = 0.304 \pm 0.014,$$

$$\sin^2 \theta_{23} = 0.51 \pm 0.05, \Delta m^2_{21} = (7.53 \pm 0.18) \times 10^{-5} \text{eV}^2,$$

$$|\Delta m^2_{31}| = (2.44 \pm 0.06) \times 10^{-5} \text{eV}^2.$$ (17)

In our previous works, the neutron EDM, muon MDM and lepton EDM were studied [45–47], and those constraints are taken into account here. In Refs. [20, 37], $Br(\mu \rightarrow e + \gamma) < 5.7 \times 10^{-13}$ and $Br(\mu \rightarrow 3e) < 1.0 \times 10^{-12}$ are strict limits for our parameter space. Furthermore, the ratios for $h \rightarrow \gamma \gamma$, $h \rightarrow ZZ^*$ and $h \rightarrow WW^*$ are around $1.16 \pm 0.18$, $1.29^{+0.23}_{-0.26}$ and $1.08^{+0.16}_{-0.17}$ respectively [20], which are also considered in our parameter space. In this work, the parameters used are [20, 45, 47]:

$m_u = 0.105 \text{GeV}$, $m_t = 1.777 \text{GeV}$, $m_w = 0.8038 \text{GeV}$, $m_a = 0.51 \times 10^{-3} \text{GeV}$, $m_{a_w} = 91.1876 \text{GeV}$,
\( \alpha(m_2) = 1/128, \ s_Y^2(m_2) = 0.23, \)

\[(Y_\nu)_{11} = 1.303 \times 10^{-6}, \ (Y_\nu)_{12} = 9.0884 \times 10^{-8}, \]

\[(Y_\nu)_{13} = 6.9408 \times 10^{-8}, \ (Y_\nu)_{22} = 1.6002 \times 10^{-6}, \]

\[(Y_\nu)_{23} = 3.4872 \times 10^{-7}, \ (Y_\nu)_{33} = 1.7208 \times 10^{-6}, \]

\[L_4 = \frac{3}{2}, \ \lambda_{N^c} = 1. \]

To simplify the discussion of the numerical result, we assume the following relations

\[ (A_i)_{ii} = A_L (A_i')_{ii} = A_R, \ (A_{N^c})_{ii} = (A_N)_{ii} = A_N, \]

\[ (m^2_{\tilde{L}})_i = (m^2_{\tilde{l}})_i = S_{\alpha L}, \ (m^2_{\tilde{N}^c})_i = M_{\alpha L}^{N^c}, \]

\[ (m^2_{\tilde{L}})_{ij} = (m^2_{\tilde{l}})_{ij} = M_{L L}, i \neq j, i, j = 1, 2, 3. \]  \( 18 \)

We choose the parameters \( A_L = -2 \) TeV, \( A_{L'} = 300 \) GeV, \( m_{\alpha L} = 300 \) TeV, \( M_{\alpha L} = 1 \) TeV, \( \tan \beta_L = \tan \beta_L \) and \( V_{L} = \sqrt{v^2_L + v^2_L} \). \( m_1 \) represents the mass of the gaugino in \( U(1) \) and \( m_2 \) represents the mass of the gaugino in \( SU(2) \). Generally, the non-diagonal elements of the parameters are defined as zero unless we note otherwise.

### 4.1 \( Z \rightarrow e\mu \)

The experimental upper bound for the branching ratio of \( Z \rightarrow e\mu \) is around \( 7.5 \times 10^{-7} \). The parameter \( m_1 \) is related to the mass matrix of the neutralino, which means the contributions from neutralino-slepton can be influenced by the parameter \( m_1 \). With \( g_L = 0.3, \ S_m = 1 \) TeV, \( AN = 500 \) GeV, \( m_2 = 1 \) TeV, \( M_{L'_L} = 1 \times 10^5 \) GeV, \( m_{\alpha L} = 1 \) TeV, \( \tan \beta_L = 15, \) we plot the results versus \( m_1 \) in Fig. 2. As \( m_1 > 0, \) the results decrease with increasing \( m_1 \). However, the results are in the region \( (3.0 \times 10^{-9} \sim 3.5 \times 10^{-9}) \) and the effect of \( m_1 \) is small.

Fig. 2. With \( g_L = 0.3, S_m = 1 \) TeV, \( AN = 500 \) GeV, \( m_2 = 1 \) TeV, \( M_{L'_L} = 1 \times 10^5 \) GeV, \( m_{\alpha L} = 1 \) TeV, \( \tan \beta_L = 15, \) the contributions to \( Br(Z \rightarrow e\mu) \) versus \( m_1 \) are plotted by the solid line.

As a more sensitive parameter, \( m_2 \) not only presents in the mass matrix of neutralino, but also in the mass matrix of the chargino. This parameter affects the numerical results through the neutralino-slepton and chargino-sneutrino contributions. In Fig. 3, we show the effects from \( m_2 \) with \( g_L = 0.2, \ S_m = 1 \) TeV, \( AN = -500 \) GeV, \( \tan \beta = 15 \) and \( M_{L'_L} = 1 \times 10^5 \) GeV. We plot the result with \( m_1 = 500 \) GeV, 1000 GeV and 1500 GeV by the solid, dotted and dashed lines respectively. The three lines all become small quickly with increasing \( m_2 \). This implies that \( m_2 \) is a relatively sensitive parameter to the numerical results.

Fig. 3. With \( g_L = 0.2, S_m = 1 \) TeV, \( AN = -500 \) GeV, \( \tan \beta = 15, \) \( M_{L'_L} = 1 \times 10^5 \) GeV, \( Br(Z \rightarrow e\mu) \) versus \( m_2 \) is plotted for \( m_1 = 500 \) GeV (solid line), 1000 GeV (dotted line), and 1500 GeV (dashed line).

The parameters \( g_L, \tan \beta_L \) and \( V_{L} \) are all present in the mass squared matrices of sleptons, sneutrinos and lepton neutralinos. Therefore, these three parameters affect the results through slepton-neutrino, sneutrino-chargino and slepton-lepton neutralino contributions. We choose the parameters \( m_1 = 500 \) GeV, \( m_2 = 1 \) TeV, \( S_m = 1 \) TeV, \( AN = 500 \) GeV and \( \tan \beta = 15. \) As \( V_{L} = 3 \) TeV, we plot the allowed results with \( \tan \beta_L \) versus \( g_L \) in Fig. 4. Obviously, when the value of \( g_L \) is large
enough, the value of $\tan \beta_L$ approaches 1. When $g_L \leq 0.3$, the parameter $\tan \beta_L$ can vary in the region of 0–2. This implies that $g_L$ is a sensitive parameter to the numerical results. For $\tan \beta_L = 2$, $g_L$ versus $V_{Lt}$ is scanned in Fig. 5. We find that the allowed scope of $V_{Lt}$ shrinks and the value of $V_{Lt}$ decreases with increasing $g_L$. Therefore, the value of $g_L$ should not be large. Generally, we take $0.05 \leq g_L \leq 0.3$ and $V_{Lt} \sim 3$ TeV in our numerical calculations.

4.2 $Z \rightarrow \ell \tau$

In a similar way, the CLFV process $Z \rightarrow \ell \tau$ is numerically studied and its experimental upper bound is around $9.8 \times 10^{-6}$. As discussed in the previous section, $g_L$ can affect the contribution strongly through the masses of sleptons, sneutrinos and lepton neutralinos. $S_m$ is the diagonal element of $m^2_L$ and $m^2_R$ in the slepton mass matrix, which can affect slepton-neutralino and slepton-lepton neutralino contributions in the CLFV process. Using the parameters $m_1 = 500$ GeV, $m_2 = 1$ TeV, $AN = 500$ GeV, $\tan \beta = 12$ and $M_{Lj} = 1 \times 10^5$ GeV$^2$, we study the branching ratio versus $S_m$ with $g_L = 0.1 (0.15, 0.2)$ in Fig. 6, with the results plotted by the solid line, dotted line and dashed line respectively. These three lines decrease quickly with $S_m$ increasing from 1000 GeV to 2500 GeV, which indicates that $S_m$ is a very sensitive parameter for the numerical results. When $S_m > 2500$ GeV, the results decrease slowly and the branching ratios are around $(10^{-5} \sim 10^{-10})$.

We then study the process with the parameters $M_{Lj}$ and $m_2$. For $S_m = \sqrt{2}$ TeV, $g_L = 0.2$, $m_1 = 500$ GeV, $AN = 500$ GeV, and $\tan \beta = 12$, we study the results versus $M_{Lj}$ with $m_2 = 1$, 1.5, and 2 TeV in Fig. 7, shown by the solid line, dotted line and dashed line respectively. As $M_{Lj} = 0$, the branching ratio for $Z \rightarrow \ell \tau$ is almost zero, but the results increase sharply when $|M_{Lj}| > 0$.

We deduce that non-zero $M_{Lj}$ is a sensitive parameter and has a strong effect on lepton flavor violation.

4.3 $Z \rightarrow \mu \tau$

The experimental upper bound for the CLFV process $Z \rightarrow \mu \tau$ is $1.2 \times 10^{-5}$, which is about one order of magnitude larger than the process $Z \rightarrow e\mu$. The parameter $AN$ is present in the sneutrino mass matrix and affects sneutrino-chargino contributions. Supposing $m_1 = 500$ GeV, $m_2 = 1$ TeV, $g_L = 0.2$, $S_m = 1$ TeV, $M_{Lj} = 1 \times 10^5$ GeV$^2$ and $\tan \beta = 1(2,3)$, we plot the results with the $AN$ in Fig. 8. For $AN \leq 4$ TeV, the branching ratios are around $4 \times 10^{-9}$. For $AN > 4$ TeV, these three lines increase quickly and $AN$ has an obvious influence on the numerical results.

Fig. 5. For $Br(Z \rightarrow e\mu)$, the allowed parameter space in the plane of $g_L$ versus $V_{Lt}$ with $m_1 = 500$ GeV, $m_2 = 1$ TeV, $S_m = 1$ TeV, $AN = 500$ GeV, $\tan \beta = 15$ and $\tan \beta_L = 2$.

Fig. 6. With $m_1 = 500$ GeV, $m_2 = 1$ TeV, $AN = -500$ GeV, $\tan \beta = 12$, and $M_{Lj} = 1 \times 10^5$ GeV$^2$, the contributions to $Br(Z \rightarrow \ell \tau)$ versus $S_m$ for $g_L = 0.1$ (solid line), 0.15 (dotted line), and 0.2 (dashed line).

Fig. 7. With $S_m = \sqrt{2}$ TeV, $g_L = 0.2$, $m_1 = 500$ GeV, $AN = 500$ GeV, and $\tan \beta = 12$, the contributions to $Br(Z \rightarrow \ell \tau)$ versus $M_{Lj}$ for $m_2 = 1$ TeV (solid line), 1.5 TeV (dotted line), and 2 TeV (dashed line).

Fig. 8. For $AN = 1(2,3)$, we plot the results with the AN in Fig. 8. For $AN \leq 4$ TeV, the branching ratios are around $4 \times 10^{-9}$. For $AN > 4$ TeV, these three lines increase quickly and $AN$ has an obvious influence on the numerical results.
Finally, the effects from the parameter \( \tan \beta \) are studied. \( \tan \beta \) is related to \( v_u \) and \( v_d \), and appears in almost all mass matrices of CLFV processes. With \( m_1 = 500 \text{ GeV}, m_2 = 1 \text{ TeV}, S_m = 1 \text{ TeV}, g_L = 0.3, M_{L_f} = -1 \times 10^5 \text{ GeV}^2 \) and \( AN = 500 \text{ GeV}, Br(Z \to \mu \tau) \) versus \( \tan \beta \) is plotted by the solid line.

In this model, three new scalar neutrinos are introduced in our work. The numerical results can be influenced by the slepton-lepton neutralino contributions. As the non-diagonal elements, \((M_L^2)_{LR}\) are not small and can obviously improve the lepton flavor violation effects. Furthermore, the parameters \((m_{\tilde{T}_1}^2)_{JJ}\) and \((m_{\tilde{T}_2}^2)_{JJ}\) respectively exist in \((M_L^2)_{LL}\) and \((M_L^2)_{RR}\). It indicates that the non-diagonal element \(M_{L_f}\) of \((m_{\tilde{T}_1}^2)_{JJ}\) and \((m_{\tilde{T}_2}^2)_{JJ}\) leads to strong mixing for sleptons. Therefore, \((M_L^2)_{LR}\) and \(M_{L_f}\) influence our results strongly.

In our used parameter space, the numerical results show that the rates for \( Br(Z \to l^+_1 l^-_1) \) can almost reach the present experimental upper bounds. The numerical analyses indicate that parameters \( m_1, m_2, g_L, M_{L_f}, S_m, AN \) and \( \tan \beta \) are important. The sensitive parameters are \( g_L, M_{L_f} \) and \( S_m \) and they affect the results strongly. We hope that experimental results for \( Z \to l^+_1 l^-_1 \) can be detected in the near future.

Appendix A

The concrete forms of coupling coefficients corresponding to Fig. 1(1)–Fig. 1(9) are shown as:

- Fig. 1(1): \[ S_1 = \bar{\nu}_l, S_2 = \bar{\nu}_m, F = \chi^c \]
- \[ H_L^{S_1J}\bar{F}J(1) = \frac{g_{e}}{s_{u}} Z_{l}^{J+ k} Z_{\nu}^{J+ n} + Y_{\nu}^{J+ n} Z_{\nu}^{J+ (l+3) n} \]
- \[ H_R^{S_1J}\bar{F}J(1) = \frac{e}{2 s_{u} c_{w}} Z_{l}^{J+ k} Z_{\nu}^{J+ n} \]
Fig.1 (2): \( S_1 = \hat{L}_m, S_2 = \hat{L}_m, F = \chi^0 \)

\[
H_{L}^{S F_{1}^{(2)}}(2) = -\sqrt{2} e \ \delta_{L}^{j(\pm 3)m} Z_{N}^{k} \chi \delta_{L}^{j(\pm 3)k} + Y_{l}^{j} \hat{Z}_{L}^{lm} \ \delta_{l}^{j(\pm 3)k},
\]

\[
H_{R}^{S F_{1}^{(2)}}(2) = \frac{e}{\sqrt{2} \delta_{s}^{j(\pm 3)k} \ \delta_{L}^{j(\pm 3)k} + Y_{l}^{j} \hat{Z}_{L}^{lm} \ \delta_{l}^{j(\pm 3)k}},
\]

\[
\begin{align*}
H_{L}^{S F_{1}^{(2)}}(2) & = \frac{e}{\sqrt{2} \delta_{s}^{j(\pm 3)k} \ \delta_{L}^{j(\pm 3)k} + Y_{l}^{j} \hat{Z}_{L}^{lm} \ \delta_{l}^{j(\pm 3)k}}, \\
H_{R}^{S F_{1}^{(2)}}(2) & = \frac{e}{\sqrt{2} \delta_{s}^{j(\pm 3)k} \ \delta_{L}^{j(\pm 3)k} + Y_{l}^{j} \hat{Z}_{L}^{lm} \ \delta_{l}^{j(\pm 3)k}}, \quad (A2)
\end{align*}
\]

Fig.1 (3): \( S_1 = \hat{L}_m, S_2 = \hat{L}_m, F = \chi_L \)

\[
H_{L}^{S F_{1}^{(3)}}(3) = -\sqrt{2} g_{i} \delta_{L}^{j} Z_{N}^{k} \chi \delta_{L}^{j} Z_{N}^{k},
\]

\[
H_{R}^{S F_{1}^{(3)}}(3) = \sqrt{2} g_{i} \delta_{L}^{j} Z_{N}^{k} \chi \delta_{L}^{j} Z_{N}^{k},
\]

\[
H_{L}^{S F_{1}^{(3)}}(3) = -\sqrt{2} g_{i} \delta_{L}^{j} Z_{N}^{k} \chi \delta_{L}^{j} Z_{N}^{k},
\]

\[
H_{R}^{S F_{1}^{(3)}}(3) = \sqrt{2} g_{i} \delta_{L}^{j} Z_{N}^{k} \chi \delta_{L}^{j} Z_{N}^{k}, \quad (A3)
\]

Fig.1 (4): \( S_1 = H^\perp (G^\perp), S_2 = H^\perp (G^\perp), F = \nu \)

\[
H_{L}^{S F_{1}^{(4)}}(4, H) = -\sin \beta Y_{l}^{j} U_{l}^{i(\pm 3)k},
\]

\[
H_{R}^{S F_{1}^{(4)}}(4, H) = -\sin \beta Y_{l}^{j} U_{l}^{i(\pm 3)k},
\]

\[
H_{L}^{S F_{1}^{(4)}}(4, H) = -\sin \beta Y_{l}^{j} U_{l}^{i(\pm 3)k},
\]

\[
H_{R}^{S F_{1}^{(4)}}(4, H) = -\sin \beta Y_{l}^{j} U_{l}^{i(\pm 3)k}, \quad (A4)
\]

Fig.1 (5): \( F_1 = \chi^0_1, F_2 = \chi^0_2, S = \nu \)

\[
H_{L}^{S F_{1}^{(5)}}(5) = -Y_{l}^{j} U_{l}^{i(\pm 3)k},
\]

\[
H_{R}^{S F_{1}^{(5)}}(5) = \left[ \frac{e}{\sqrt{2} \delta_{s}^{j(\pm 3)k} \ \delta_{L}^{j(\pm 3)k} + Y_{l}^{j} \hat{Z}_{L}^{lm} \ \delta_{l}^{j(\pm 3)k}} \right],
\]

\[
H_{L}^{S F_{1}^{(5)}}(5) = \frac{e}{\sqrt{2} \delta_{s}^{j(\pm 3)k} \ \delta_{L}^{j(\pm 3)k} + Y_{l}^{j} \hat{Z}_{L}^{lm} \ \delta_{l}^{j(\pm 3)k}},
\]

\[
H_{L}^{S F_{1}^{(5)}}(5) = \left[ \frac{e}{\sqrt{2} \delta_{s}^{j(\pm 3)k} \ \delta_{L}^{j(\pm 3)k} + Y_{l}^{j} \hat{Z}_{L}^{lm} \ \delta_{l}^{j(\pm 3)k}} \right],
\]

\[
H_{R}^{S F_{1}^{(5)}}(5) = \left[ \frac{e}{\sqrt{2} \delta_{s}^{j(\pm 3)k} \ \delta_{L}^{j(\pm 3)k} + Y_{l}^{j} \hat{Z}_{L}^{lm} \ \delta_{l}^{j(\pm 3)k}} \right], \quad (A5)
\]

Fig.1 (6): \( F_1 = \chi^0_1, F_2 = \chi^0_2, S = \bar{L} \)

\[
H_{L}^{S F_{1}^{(6)}}(6) = -\sqrt{2} e \ \delta_{L}^{j(\pm 3)k} \chi,\]

\[
H_{R}^{S F_{1}^{(6)}}(6) = \frac{e}{\sqrt{2} \delta_{s}^{j(\pm 3)k} \ \delta_{L}^{j(\pm 3)k} + Y_{l}^{j} \hat{Z}_{L}^{lm} \ \delta_{l}^{j(\pm 3)k}},
\]

\[
H_{R}^{S F_{1}^{(6)}}(6) = \frac{e}{\sqrt{2} \delta_{s}^{j(\pm 3)k} \ \delta_{L}^{j(\pm 3)k} + Y_{l}^{j} \hat{Z}_{L}^{lm} \ \delta_{l}^{j(\pm 3)k}},\]

\[
H_{R}^{S F_{1}^{(6)}}(6) = \frac{e}{\sqrt{2} \delta_{s}^{j(\pm 3)k} \ \delta_{L}^{j(\pm 3)k} + Y_{l}^{j} \hat{Z}_{L}^{lm} \ \delta_{l}^{j(\pm 3)k}}, \quad (A6)
\]

Fig.1 (7): \( F_1 = \nu_1, F_2 = \nu_2, S = H^\perp (G^\perp) \)

\[
H_{L}^{S F_{1}^{(7)}}(7, H) = -\sin \beta Y_{l}^{j} U_{l}^{i(\pm 3)k},
\]

\[
H_{R}^{S F_{1}^{(7)}}(7, H) = -\sin \beta Y_{l}^{j} U_{l}^{i(\pm 3)k},
\]

\[
H_{L}^{S F_{1}^{(7)}}(7, H) = -\sin \beta Y_{l}^{j} U_{l}^{i(\pm 3)k},
\]

\[
H_{R}^{S F_{1}^{(7)}}(7, H) = -\sin \beta Y_{l}^{j} U_{l}^{i(\pm 3)k}, \quad (A7)
\]

Fig.1 (8): \( W_1 = W_1, W_2 = W_2, F = \nu \)

\[
H_{L}^{W_{1}^{(8)}}(8) = \frac{e}{\sqrt{2} \delta_{s}^{j(\pm 3)k} \ \delta_{L}^{j(\pm 3)k} + Y_{l}^{j} \hat{Z}_{L}^{lm} \ \delta_{l}^{j(\pm 3)k}},
\]

\[
H_{R}^{W_{2}^{(8)}}(8) = \frac{e}{\sqrt{2} \delta_{s}^{j(\pm 3)k} \ \delta_{L}^{j(\pm 3)k} + Y_{l}^{j} \hat{Z}_{L}^{lm} \ \delta_{l}^{j(\pm 3)k}}, \quad (A8)
\]

Fig.1 (9): \( F_1 = \nu_1, F_2 = \nu_2, W = W \)

\[
H_{L}^{W_{1}^{(9)}}(9) = \frac{e}{\sqrt{2} \delta_{s}^{j(\pm 3)k} \ \delta_{L}^{j(\pm 3)k} + Y_{l}^{j} \hat{Z}_{L}^{lm} \ \delta_{l}^{j(\pm 3)k}},
\]

\[
H_{R}^{W_{2}^{(9)}}(9) = \frac{e}{\sqrt{2} \delta_{s}^{j(\pm 3)k} \ \delta_{L}^{j(\pm 3)k} + Y_{l}^{j} \hat{Z}_{L}^{lm} \ \delta_{l}^{j(\pm 3)k}}, \quad (A9)
\]
References

1. K. Abe et al (T2K Collab), Phys. Rev. Lett., 107: 041801 (2011)
2. P. Adamson et al (MINOS Collab), Phys. Rev. Lett., 107: 181802 (2011)
3. Y. Abe et al (DOUBLE-CHOOZ Collab), Phys. Rev. Lett., 108: 131801 (2012)
4. F. An et al (DAYA-BAY Collab), Phys. Rev. Lett., 108: 171803 (2012)
5. ATLAS Collaboration, Phys. Lett. B, 716: 1 (2012)
6. CMS Collaboration, Phys. Lett. B, 716: 30 (2012)
7. CMS Collaboration, JHEP, 06: 081 (2013)
8. A. Abada, D. Das, A. Vicente, and C. Weiland, JHEP, 09: 015 (2012), arXiv:hep-ph/1206.6497
9. A. Abada, V. De Romeri, and A. M. Teixeira, JHEP, 02: 083 (2016), arXiv:hep-ph/1510.06657
10. M. Lindner, M. Platscher, F. S. Queiroz, arXiv: hep-ph/1610.06587
11. J.I. Illana, M. Jack and T. Riemann, arXiv:hep-ph/0001273
12. J. I. Illana and T. Riemann, Phys. Rev. D, 63: 053004 (2001)
13. G. Mann and T. Riemann, Annalen Phys., 40: 334 (1984)
14. O. Ilhan and I. Turan, Phys. Rev. D, 65: 013001 (2002)
15. A. F. Tlalpaa, J. M. Hernandezb, G. T. Velascoa and J. J. Toscanob, Phys.Rev. D, 65: 073010 (2002)
16. CEPC-SPPC Study Group, "CEPC-SPPC Preliminary Conceptual Design Report", 389 (2015)
17. J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, Phys. Rev. D, 53: 2442-2459 (1996)
18. H. B. Zhang, T. F. Feng, S. M. Zhao and F. Sun, Phys. Rev. D, 92: 115016 (2015)
19. J. M. Arnold, P. F. Perez, B. Fornal, and S. Spinner, Phys. Rev. D, 85: 115024 (2012)
20. T. F. Feng, S. M. Zhao, H. B. Zhang et al, Nucl. Phys. B, 871: 223 (2013)
21. J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, Phys. Rev. D, 53: 2442-2459 (1996)
22. J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, Phys. Rev. D, 53: 2442-2459 (1996)
23. J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, Phys. Rev. D, 53: 2442-2459 (1996)
24. J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, Phys. Rev. D, 53: 2442-2459 (1996)
25. J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, Phys. Rev. D, 53: 2442-2459 (1996)