Dark Matter Axions in Models of String Cosmology

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Abstract

Axions are produced during a period of dilaton-driven inflation by amplification of quantum fluctuations. We show that for some range of string cosmology parameters and some range of axion masses, primordial axions may constitute a large fraction of the present energy density in the universe in the form of cold dark matter. Due to the periodic nature of the axion potential energy density fluctuations are strongly suppressed. The spectrum of primordial axions is not thermal, allowing a small fraction of the axions to remain relativistic until quite late.
I. INTRODUCTION

Axions are hypothetical particles, invented to solve the strong CP problem [1,2]. Many ongoing experimental efforts aim to detect axions, but have yet to produce evidence for their existence. Cosmological and astrophysical implications of axions are well studied [3,4], in particular, axions are among the leading candidates for providing the missing dark mass in the universe.

String theory possesses many axion candidates [5,6] of the “invisible axion” type [7]. We will focus on the “model-independent axion” [5] which respects a Peccei-Quinn symmetry to all orders in perturbation theory. There are no good general arguments that QCD provides the dominant contribution to the potential energy of any of the stringy axions, including the model independent axion. However, there are some theoretical conditions under which the model-independent axion could be the axion that solves the strong CP problem. In any case, we will assume that this is indeed so.

We consider axion production in models of string cosmology which realize the pre-big-bang scenario [8,9]. In this scenario the evolution of the universe starts from a state of very small curvature and coupling and then undergoes a long phase of dilaton-driven kinetic inflation and at some later time joins smoothly standard radiation dominated cosmological evolution, thus giving rise to a singularity free inflationary cosmology. Axions are produced during the period of dilaton-driven inflation by the standard mechanism of amplification of quantum fluctuations [10].

The spectrum of relic axions depends on their potential and interactions, and on some of the string cosmology model parameters. By applying simple constraints, such as requiring that the energy density of the universe does not exceed the critical density at different stages of the evolution, we are able to constrain parameters of string cosmology models and axion potential and find a consistent parameter range in which most of the energy of the universe today is in the form of cold dark matter axions whose origin is quantum fluctuations from the pre-big-bang. This consistent range overlaps with the range in which relic gravity wave background produced during the dilaton-driven inflationary phase could be detected by planned gravity wave experiments [11]. The same parameter range could perhaps lead to formation of observable primordial black holes [12].

II. THE MODEL

We assume that the model independent axion receives the dominant part of its potential from QCD instantons, therefore, roughly speaking, the axion is massless until the universe cools down to a temperature \( T_{QCD} \gtrsim \Lambda_{QCD} \) at time \( t = t_{QCD} \). A more sophisticated estimate as in [3,4] will be used later. The axion then develops a periodic potential of overall approximate strength \( \Lambda_{QCD}^4 \) (recall that \( \Lambda_{QCD} \sim 200 \text{MeV} \)), and period which is apriori a free parameter \( f_{PQ} \). The scale \( f_{PQ} \) is typically less than \( 10^{16} \text{GeV} \) resulting in an axion mass \( m_a \gtrsim 10^{-10} eV \). Astrophysical constraints further bound \( f_{PQ} \) from below \( f_{PQ} \gtrsim 10^{9} \text{GeV} \) resulting in an axion mass \( m_a \lesssim 10^{-2} eV \). We will discuss cosmological constraints on \( f_{PQ} \) in more detail later. The explicit form of the axion potential that we will assume \( V(\psi) = \frac{1}{2} V_0 \left( 1 - \cos \left( \frac{\psi}{\psi_0} \right) \right) \), depends on two parameters, \( \psi_0 \), related to the Peccei-Quinn scale \( f_{PQ} \), and \( V_0 = m_a^2 \psi_0^2 \). For QCD axions \( V_0 \approx f_{\pi}^2 m_\pi^2 \sim \Lambda_{QCD}^4 \).
The model of background evolution we adopt in this paper is a simplified model used also in [13]. The evolution of the universe is divided into four distinct phases, the first phase is a long the dilaton-driven inflationary phase, the second phase is a high-curvature string phase of otherwise unknown properties, followed by ordinary Friedman-Robertson-Walker (FRW) radiation dominated (RD) evolution and then a standard FRW matter dominated (MD) evolution. We assume throughout an isotropic and homogeneous four dimensional flat universe, described by a FRW metric. The model is described in full detail in [13], and we reproduce here only its important features. We note in particular that the axion field is assumed to have a trivial vacuum expectation value during the inflationary phase.

The dilaton-driven inflationary phase lasts until time $t = t_s$, and while it lasts the scale factor $a(t)$ and the dilaton $\phi(t)$ are given by the solution of the lowest order string-dilaton-gravity equations of motion, the so-called (+) branch vacuum. The string coupling parameter $e^{\phi} = g_{\text{string}}^2$, in the models that we consider is, of course, time dependent. Both curvature and coupling $e^{\phi}$ are growing in this phase, which is expected to last until curvatures reach the string scale and the background solution starts to deviate substantially from the lowest order solution. For ideas about how this may come about see [14].

The string phase lasts while $t_s < t < t_1$. We assume that curvature stays high during the string phase. As in [13], we assume that the string phase ends when curvature reaches the string scale $M_s$, $H(t_1) \approx M_s$. We parametrized our ignorance about the string phase background, as in [11], by the ratios of the scale factor and the string coupling $g(t) = e^{\phi(t)/2}$, at the beginning and end of the string phase $z_S = a_1/a_S$ and $g_1/g_S$, where $g_1 = e^{\phi(t_1)/2}$ and $g_S = e^{\phi(t_S)/2}$, where $a_S = a(t_s)$ and $\phi_S = \phi(t_s)$. We take the parameters to be in a range we consider reasonable. For example, $z_S$ could be in the range $1 < z_S < e^{45} \sim 10^{20}$, to allow a large part of the observed universe to originate in the dilaton-driven phase, and $g_1/g_S > 1$, assuming that the coupling continues to increase during the string phase and $10^{-3} \lesssim g_1 \lesssim 10^{-1}$ to agree with the expected range of string mass (see e.g. [16]). Some other useful quantities that we will need are $\omega_1$, the frequency today, corresponding to the end of the string phase, estimated in [12] to be $\omega_1 \sim 10^{10} \text{Hz}$, and the frequency $\omega_S = \omega_1/z_S$, the frequency today corresponding to the end of the dilaton-driven phase.

Standard RD phase and then MD phase are assumed to follow the string phase. The dilaton is taken to be strictly constant, frozen at its value today.

We have presented our assumptions about background evolution and axion potential in great detail, and will use them as presented, even though many of the assumptions can be either relaxed (without affecting dramatically our results), or improved to take into account additional known effects. However, each change adds an additional level of complication by adding parameters and assumptions, and we preferred to keep the discussion as simple as possible to capture the essential physics. Nevertheless, we do mention from time to time a possible alternative or generalization.

III. PRIMORDIAL AXIONS

The spectrum of axionic perturbations produced during the dilaton-driven inflation is approximately given by [16] (see also [13,17])

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\[ \psi_k = N_k a_s^{-1} g_s^{-1} k_s^{-1/2} \left( \frac{k}{k_s} \right)^{-\sqrt{3}}, k < k_s, \]  

(3.1)

where \( N_k = 2^{\sqrt{3}-1} \Gamma(\sqrt{3})/\sqrt{\pi} \) and \( k_s = \omega_s a(t) \). The r.m.s amplitude of the perturbation in a logarithmic \( k \) interval is defined in a standard way \( \delta \psi_k \equiv k^{3/2} \psi_k \). Using the relation \( k_s/a_s = k_1/a_1 = \omega_1(t_1) \), the assumption \( \omega_1(t_1) = M_s \) and the relation between the string mass and the Planck mass in weakly coupled string theory, \( M_s = M_p g_1 \) we obtain

\[ \delta \psi_k = N_k M_p \frac{g_1}{g_s} \left( \frac{k}{k_s} \right)^{3/2 - \sqrt{3}}, \]  

(3.2)

for perturbations outside the horizon, and

\[ \delta \psi_k = N_k M_p \frac{g_1}{g_s} \left( \frac{k}{k_s} \right)^{1/2 - \sqrt{3}} \left( \frac{H(t)}{\omega_s(t)} \right), \]  

(3.3)

for perturbations that have reentered the horizon during RD, before the axion potential is generated.

The ratio of energy density in axions per logarithmic frequency interval to the critical density \( d \Omega_a / d \ln \omega \), for perturbations that reenter the horizon during RD, before the axion potential is generated, is given by [13]

\[ \frac{d \Omega_a}{d \ln \omega} = C g_1^2 \left( \frac{g_1}{g_s} \right)^2 \left( \frac{\omega}{\omega_s} \right)^{2\sqrt{3} - 3}, \quad \omega < \omega_s, \]  

(3.4)

where \( C \) is a numerical factor which we will ignore in the following. Note that the spectral index \( 3 - 2\sqrt{3} \simeq -0.46 \) is negative, and therefore most of the energy is contained in the low-frequency modes.

The total energy density within the horizon, at a given time, is dominated by the lowest frequency which is just reentering the horizon,

\[ \Omega_a(t) = \int_{H(t)}^{\omega_s(t)} \frac{d \Omega_a}{d \ln \omega} d \ln \omega \simeq g_1^2 \left( \frac{g_1}{g_s} \right)^2 \left( \frac{H(t)}{\omega_s(t)} \right)^{3-2\sqrt{3}}, \]  

(3.5)

where \( \omega_s(t) = \omega_1(t)/z_s \). If \( H(t) > \omega_s(t) \) then the total energy density in axions produced during dilaton-driven phase simply vanishes (We will discuss an estimate for the axions produced during the string phase later on). Since \( H(t) \propto T^2(t) \) and \( \omega_s(t) \propto T(t) \), and since \( H(t_1) = M_s \), then \( \frac{\omega_s(t)}{H(t)} = T_1/(T z_s) \), and therefore

\[ \Omega_a(t) \simeq g_1^2 \left( \frac{g_1}{g_s} \right)^2 \left( \frac{M_s}{T(t) z_s} \right)^{2\sqrt{3} - 3}. \]  

(3.6)

To ensure standard RD cosmology at late times we must require that the energy density in axions remains smaller than critical \( \Omega_a < 1 \). It is enough to require this at the
lowest temperature possible, i.e., at the temperature just as the axion potential is generated, \( \Omega_a(t_{QCD}) \approx g_1^2 \left( \frac{q_1}{g_S} \right)^2 \left( \frac{M_s}{T_{QCD} z} \right)^{2\sqrt{3} - 3} < 1 \). Using \( M_s = g_1 M_p \), we obtain the following condition

\[
    z_s > \frac{g_1 M_p}{T_{QCD}} \left[ \left( \frac{g_1}{g_S} \right)^2 \frac{2}{g_1^2} \right]^{1/(2\sqrt{3} - 3)}.
\]

(3.7)

Note that since \( \frac{M_p}{T_{QCD}} \sim 10^{19} \), unless \( z_s \) is large enough condition (3.7) can be satisfied only if \( g_1 \) is unacceptably small. There is, however, a reasonable range of parameters for which condition (3.7) is indeed satisfied, for example, if \( g_1 = 10^{-3}, g_S/g_1 = 1/10 \), then (3.7) implies \( z_s \gtrsim 3 \times 10^7 \) and if \( g_S \simeq g_1 \simeq 0.01 \) then \( z_s \gtrsim 3 \times 10^8 \). If \( g_1 \sim g_S \), condition (3.7) simplifies to \( z_s \gtrsim g_1^5 M_p/T_{QCD} \). When (3.7) is saturated, axions provide near closure density of the universe just before the axion potential is generated. Condition (3.7) is valid for standard adiabatic RD evolution. If some intermediate period of matter domination or entropy production is assumed, condition (3.7) is relaxed.

So far we have considered only axions that were produced during the dilaton-driven phase and ignored axions that were produced during the subsequent string phase. We would like to show that it is reasonable to neglect axion production during the string phase by giving an estimate based on the extrapolation used in [17], which assumes constant \( H \) and \( \dot{\phi} \) during the string phase. The resulting energy density \( \Omega_{sp}^a \) is given by

\[
    \frac{d\Omega_{sp}^a}{d\ln \omega} \approx g_1^2 \left( \frac{g_1}{g_S} \right)^2 \left( \frac{\omega}{\omega_S} \right)^{-2\zeta} \omega < \omega < \omega_1,
\]

(3.8)

where the spectral index \( \zeta = \ln(g_1/g_S)/\ln z_s \), is positive and therefore the energy density decreases with frequency. The total additional energy in axions produced during the string phase \( \int_{\omega_S(t)}^{\omega_1(t)} \frac{d\Omega_{sp}^a}{d\ln \omega} d\ln \omega \), is up to a numerical factor \( \sim g_1^2 \left( \frac{g_1}{g_S} \right)^2 \frac{1}{2\zeta} \), which is indeed negligible (for large \( z_s \) and reasonable \( g_1, g_S \)) compared with the energy density (3.4) in axions produced during the dilaton-driven phase. The same conclusion is expected as long as the spectrum of axions produced during the string phase continues to decrease.

If the model-independent axion were to remain massless its total energy just before matter radiation equality would be given by \( \Omega_a(t_{eq}) \approx g_1^2 \left( \frac{g_1}{g_S} \right)^2 \left( \frac{M_s}{T_{eq} z} \right)^{2\sqrt{3} - 3} \), (recall that \( T_{eq} \sim 1eV \)). Axions would overclose the universe and lead to an unacceptable cosmology, unless the parameters of string cosmology, and in particular \( z_s \) are pushed to uncomfortable values.

**IV. DARK MATTER AXIONS**

We turn to discuss the effects of the axion potential as it turns on when the universe cools down to QCD temperatures. If we try to approximate the axion potential by a quadratic potential, a common practice in most investigations, we encounter a puzzle. The axion energy density becomes formally divergent as soon as the axion potential turns on! (if we assume that the dilaton-driven phase lasted only a finite time then the formal divergence is
replaced by a singular dependence on the duration of the dilaton-driven phase). The relative energy density in axions, assuming a quadratic potential $\frac{d\Omega_{QP}}{d\ln \omega}$, was computed in [18] and we reproduce here its low frequency part,

$$\frac{d\Omega_{QP}}{d\ln \omega} \simeq g_1^2 \left( \frac{g_1}{g_5} \right)^2 \sqrt{M_\phi m_c} \left( \frac{\omega}{\omega_S} \right)^{3-2\sqrt{3}}, \quad \omega < \omega_m, \omega_S. \quad (4.1)$$

Once the potential is generated, all the low frequencies reenter the horizon at once, so to obtain the total energy density inside the horizon we need to integrate $\frac{d\Omega_{QP}}{d\ln \omega}$ from the minimal amplified frequency $\omega_{\text{min}}$, which is either zero, if the duration of the dilaton-driven phase is infinite, or exponentially small if the duration is finite but large, $\Omega_{QP}(t) = \int_{\omega_{\text{min}}}^{\omega_{\text{max}}} \frac{d\Omega_{QP}}{d\ln \omega} d\ln \omega$.

The lower frequency part of the spectrum yields a divergent contribution, proportional to $\omega_{\text{min}}^{3-2\sqrt{3}}$ (recall $3 - 2\sqrt{3} \simeq -0.46$). This result does not make sense.

The resolution of the puzzle depends crucially on the periodic nature of axion potential $V(\psi) = \frac{1}{2} V_0 \left( 1 - \cos \left( \frac{\psi}{g_0} \right) \right)$. This point was first understood by Kofman and Linde [18], and we have adopted their ideas to our particular situation. First, the total potential energy is limited to $V_0$ and does not continue to increase indefinitely as the axion field increases, providing a “topological cutoff” on the total axionic energy density and as important, large fluctuations in the axion field are also “topologically cutoff”, producing exponentially small energy density perturbations. Large fluctuations lead to a uniform distribution of the axion field inside the horizon, with very small statistical fluctuations.

The axion potential is highly non-linear, therefore it is not possible to solve the perturbation equation mode-by-mode. In [18], the following strategy is suggested. Consider the axion field $\psi(\vec{r}, t)$ at the time when the axion potential is turned on. The low $k$ Fourier modes $\psi_k, k/a(t_{\text{QCD}}) < H(t_{\text{QCD}})$, provide an essentially constant field $\psi_c$ across the horizon. The value of $\psi_c$ is random, and in our case it is determined statistically by a Gaussian distribution $P(\psi_c)$ with zero average and standard deviation $\sigma_c = \sqrt{\int \frac{d\ln k}{k_{\text{min}}} |\delta \psi_k|^2}$. Since in all cases that we will be interested in, $\psi_0 < 10^{16}$ GeV, and $\delta \psi_k > M_p$ for all $k < aH$ (see eq. (3.2)), the width $\sigma_c$ is much larger than the period of the axion potential $\sigma_c \gg 2\pi \psi_0$. The constant value $\psi_c$ becomes essentially uniformly distributed among all possible values. The average energy density in the non-relativistic part of the axion field $\rho_a = \langle V(\psi(\vec{r})) \rangle$ is given by

$$\rho_a = \int \frac{1}{2} V_0 \left( 1 - \cos \left( \frac{\psi}{\psi_0} \right) \right) P(\psi) d\psi \simeq \frac{1}{2} V_0, \quad (4.2)$$

with exponentially small corrections. Note that eq. (4.2) is valid for all reasonable values of $\omega_S$ and $g_5, g_1$. Regardless of the fraction of relativistic axions which exists at $t_{\text{QCD}}$, the low momentum modes with wavelength larger than the horizon contribute a constant energy density. The procedure that we outlined above can be repeated for any scale $\ell$, separating modes of $\psi(\vec{r}, t) = \psi_c(\ell^{-1}) + \tilde{\psi}$, where $\tilde{\psi}$ contains only modes with $k < \ell^{-1}$.

The constant axion field $\psi_c$ starts to coherently oscillate around the nearest minimum of the potential. Using completely standard arguments [3,4,19], we may obtain a bound on $m_a$ (or equivalently on $\psi_0$) by requiring that the energy density in the coherent axion oscillations be subcritical at the beginning of MD epoch. This requirement leads to the
standard bounds on the axion mass, except the possibility that the axion “starts” at a special point seems less viable. We may evaluate the number of axion particles at the initial time when the potential is turned on (defined by the condition \( m(T_{QCD}) = 3H(T_{QCD}) \)), 

\[ n_a = \rho_a / m_a. \]

Using \( m_a(T)^2 = V_0(T)/\psi_0^2 \) we may estimate \( V(T_{QCD}) \sim T_{QCD}^4 \psi_0^2/M_P^2 \), leading to the standard estimate \( \Omega_a h^2 \sim 10^{-6} eV \), where \( h \) is today's Hubble parameter in units of 100 km/Mpc/sec. Requiring subcritical \( \Omega_a \) leads to the standard bound on \( \psi_0 \), \( \psi_0 < \sim 10^{12} \text{GeV} \) and \( m_a > \sim 10^{-6} \text{eV} \).

In string theory, natural values of \( \psi_0 \) are approximately \( 10^{16} \text{GeV} \), which, if taken at face value, would lead to overclosure of the universe with axions many times over. Two possible resolutions have been suggested \([20–22]\) to allow our universe to reach its old age of today. First, that somehow, perhaps involving some strong coupling string dynamics, the low energy effective \( \psi_0 \) is some orders of magnitude below \( 10^{16} \text{GeV} \), and the second is that some non-standard matter domination epoch, or some late entropy production, has occurred in between \( T_{QCD} \) and nucleosynthesis epoch. Our results cannot shed further light on this problem, but they do reinforce the need for a resolution. If the resolution of the \( \psi_0 \) problem requires strongly coupled string theory \( g_1 > 1 \), some of our assumptions should be changed but most likely our estimates are still valid, and therefore our results are probably qualitatively correct also in that case. Of course, another possible resolution is that the model independent axion is not the QCD axion.

We turn now to the question of energy fluctuations. Since there are large fluctuations in the axion field \((3.2), (3.3)\), we should worry about large energy fluctuations which will cause unacceptable deviations from isotropy and homogeneity, affecting either nucleosynthesis or the cosmic microwave background. However, as explained in \([18]\), these perturbations are suppressed. Fluctuations in the axion energy density at a scale \( \ell \sim k^{-1} \) can be computed by using the relation

\[
\int_0^\infty \left( \delta \rho_a^2 \right)_k \frac{\sin kr}{kr} d \ln k = \langle V(\psi(\vec{x})) V(\psi(\vec{x}+\vec{r})) \rangle - \langle V(\psi(\vec{x})) \rangle^2 \quad (4.3)
\]

In previous expressions \( \langle \cdots \rangle \) denotes either vacuum expectation values of operators or statistical averages.

To evaluate \((4.3)\) we need

\[
\langle \cos(\psi(0)) \cos(\psi(\vec{r})) \rangle - \langle \cos(\psi(0)) \rangle^2 = e^{-\langle \psi^2(0) \rangle} [\text{Cosh} (\langle \psi(0) \psi(\vec{r}) \rangle) - 1],
\]

using it we obtain (for the case \( \langle \psi \rangle = n \pi, n = 0, \pm 1, \cdots \))

\[
\int_{k_{min}}^{1/\ell} \left( \delta \rho_a^2 \right)_k d \ln k \sim \frac{1}{4} V_0^2 e^{-\int_{k_{min}}^{1/\ell} d \ln k \frac{\delta \psi^2}{\psi_0^2}}, \quad (4.5)
\]

and, finally, using \( \langle \rho_a \rangle = \frac{1}{2} V_0 \) we obtain

\[
\frac{(\delta \rho_a)_k}{\rho_a} \sim \frac{\delta \psi_k}{\psi_0} e^{-\frac{1}{2} \int k \frac{\delta \psi^2}{\psi_0^2}}, \quad (4.6)
\]
where the upper limit on the right-hand-side of the previous equation has been changed from $\infty$ to $k_S$ since we take into account only fluctuations produced during the dilaton-driven phase. The derivation of eq. (4.6) involves some subtleties which we will not discuss. Our result agrees with the results of [18].

Because the standard deviation of fluctuations, $\sigma_k = \sqrt{\int_k^{k_S} d \ln k |\delta \psi_k|^2}$, is much larger than the period of the axion potential, $\sigma_k \gg \psi_0$, energy fluctuations at large wavelength are exponentially small, leading to the surprising conclusion that larger field fluctuations lead to smaller energy fluctuations. Note that if the spectrum of perturbation is flat as in [18], energy perturbations are only power-law suppressed. If, as generally assumed in many cases $\delta \psi_k < \psi_0$ for all $k$, then $\left(\frac{\delta \rho_a}{\rho_a}\right)_k \simeq \frac{\delta \psi_k}{\psi_0}$, and energy fluctuations actually grow as field fluctuations grow.

Finally, we have not considered any other axion production mechanisms, such as thermal production [3,4,23], or the formations of strings, black holes, and other topological objects [24,25] which are likely to appear in our model because of the large field fluctuations and could result in additional and perhaps dominant axion production leading to a modification of our constraints. We hope to discuss these interesting alternatives in the near future.

The primordial axion spectrum is not thermal, and may consists of a fraction of relativistic axions even after their potential is generated. Our understanding of the dynamics of the relativistic part of the spectrum is not quite complete because after the axion potential is generated the problem becomes an essentially non-linear problem. We believe that a better treatment of the relativistic axions is interesting and should be done using numerical simulations and tools similar to those used in the theory of topological defects. But we can nevertheless reach a few conclusions. First, a necessary condition for a relativistic tail to exist after the axion potential is fully developed is that $\omega/S(T \sim \Lambda_{QCD}) > m_a$, otherwise it can be shown that all modes are non-relativistic. This condition leads to the condition $z_S \lesssim \psi_0/\Lambda_{QCD}$. If $z_S \gtrsim \psi_0/\Lambda_{QCD} \simeq 10^{10}(\psi_0/10^9 GeV)(100 MeV/\Lambda_{QCD})$, then all axions are massive. Whether this $z_S$ range can be consistent with condition (3.7) depends on $\psi_0$, $g_1$, $g_S$. For $g_1 \sim g_S$ the condition becomes $1g_1^2 M_p/\Lambda_{QCD} \lesssim z_S \lesssim \psi_0/\Lambda_{QCD}$, requiring $1g_1^2 M_p \gtrsim \psi_0$, pushing parameters into a relatively narrow region. Modes for which $\omega/\omega_S > \Lambda_{QCD} z_S/\psi_0$ are relativistic at $T \sim \Lambda_{QCD}$.

The spectrum of axionic perturbations inside the horizon after the generation of the potential is quite complicated. A full treatment of these perturbations is outside the scope of this paper, and may even result in the conclusion that for the particular case we are considering it is not allowed to have any relativistic axions after the potential is generated. However, there is a range of frequencies for which we can nevertheless draw definite conclusions. This is the upper end of the frequency range, for which the perturbation is small $\delta \psi_k < \psi_0$, and relativistic $\omega > m_a$. As the universe expands, kinetic energies redshift and more axions become non-relativistic. We may evaluate their energy density by calculating their number just after the onset of the potential, and, using number conservation, calculate their energy density at later times and in particular at matter-radiation equality time. This was done in [13],

$$d \Omega_a/d \ln \omega \simeq g_1^2 \left(\frac{g_1}{g_S}\right)^2 \left(\frac{\omega}{\omega_S}\right)^{2-2\sqrt{3}} \sqrt{m_a^2 + \omega^2} \omega_S^{-\omega}.$$ Evaluating the axion energy density at matter-radiation equality, assuming that all particles have become non-relativistic by then, we obtain

8
\[
\frac{d\Omega_a}{d \ln \omega} \simeq g_1^2 \left( \frac{g_1}{g_S} \right)^2 \left( \frac{\Lambda_{QCD} \psi_0}{\psi_0} \right)^{3-2\sqrt{3}} \frac{\Lambda_{QCD}}{T_{eq}}.
\]

(4.7)

Since \( \frac{\Lambda_{QCD}}{T_{eq}} \sim 10^8 \) we see that this region of parameter space gives an uncomfortably large energy density, leading to a seemingly favorable region of parameter space \( z_S \gtrsim \psi_0/\Lambda_{QCD} \). However, since our estimates are quite rough, and a small amount of entropy production during the evolution of the universe may relax this condition we would not like at this moment to completely rule out this interesting possibility. Note that in this case even if \( \psi_0 \sim 10^9 GeV \) and the axion mass gets pushed towards its upper limit \( m_a \sim 10^{-2}eV \) axions can provide closure density. This is important for their possible detection.

In general, for modified spectra, and other relic particles produced by amplification of quantum fluctuations during the dilaton-driven phase, it may well be that a fraction of relativistic particles remains at \( t_{eq} \) and therefore it is possible that a single species provides simultaneously hot and cold dark matter.

V. CONCLUSIONS

We have shown that relic axions are produced by amplification of quantum fluctuations with a specific spectrum. In some range of string cosmology model parameters it is predicted that most of the energy in our universe today is in the form of cold dark matter axions, with suppressed energy density fluctuations at large wavelengths. Axions could provide closure density if their masses lie in the allowed range \( 10^{-6}eV \lesssim m_a \lesssim 10^{-2}eV \), depending on parameters of string cosmology. The spectrum of primordial axions is not thermal, and could contain a relativistic tail until quite late times.

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