Bridge strain denoising method based on variational mode decomposition

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Abstract. In the construction and maintenance of bridge, it is very important to measure the strain of bridge accurately. However, the bridge strain measured in practical engineering is often mixed with a lot of signal noise, how to effectively remove the signal noise is a problem. Variational mode decomposition (VMD) is a denoising algorithm proposed in recent years, which has been applied in some engineering fields. Based on the vehicle bridge coupling theory, the strain of the bridge is obtained by numerical simulation, and the Gauss white noise is added to the strain to simulate the actual engineering situation. Based on VMD theory, a new bridge strain denoising method is proposed, and some key parameters are analyzed. The results of numerical simulation show that the denoising algorithm is effective. For example, when the signal-to-noise ratio of bridge strain data is 10dB, it can be increased to 21.67dB after using VMD denoising algorithm.

1. Introduction
The bridge strain measurement is significant for bridge structure construction monitoring and health monitoring. However, the bridge strain signal is often affected by external environmental factors in the measurement process, and the strain data usually contains signal noise. If the bridge strain data with signal noise is used directly, it is difficult to achieve the ideal effect. Therefore, how to effectively decrease noise for measured bridge strain signal is a worthy question for studying. At present, the main methods of signal denoising include Kalman filter method\cite{1}, wavelet transform\cite{2} and empirical mode decomposition (EMD)\cite{3}. All of the above methods have their own shortcomings, among which the Kalman filter has the disadvantage that it is difficult to determine the initial value. Wavelet analysis is easily affected by wavelet basis function, decomposition scale and threshold setting, so its algorithm is not adaptive. EMD can't separate two similar frequency components of the signal, which can easily cause mode aliasing and result in the loss of physical meaning of the denoising signal. The practicability and accuracy of the current denoising methods need to be improved.

With the rapid development of time-frequency analysis method, in 2014, Dragomiretskiy et al.\cite{4} proposed a new signal processing method: variational mode decomposition (VMD). VMD is a kind of analysis method suitable for non-stationary signals It uses the process of optimal solution to find the constrained variational model, so as to realize the signal decomposition. Compared with the time-domain filtering characteristics of EMD, VMD performs adaptive filtering from frequency domain, and has better anti modal aliasing characteristics. As a new adaptive signal processing method, VMD has been gradually applied in engineering. Mohanty et al.\cite{5} compared the VMD algorithm with EMD...
algorithm, and proved that VMD algorithm converges fast and is suitable for bearing fault diagnosis. Zhao et al.[6] used VMD algorithm to separate the displacement signal of each bridge stay cable from the displacement signal of multiple stay cables. Tang et al.[7] applied VMD algorithm to improve the effectiveness of roller bearing composite fault diagnosis. Zhang et al.[8] used VMD algorithm to separate the high frequency vibration and low frequency displacement from the strain signal of Jiangyin bridge. However, there is a lack of research on the application of VMD in the noise reduction of bridge strain signal. In view of the advantages of VMD algorithm in multi-scale time-frequency analysis, it is a good choice to denoise the bridge strain signal.

In this paper, the classical vehicle-bridge coupling vibration theory is adopted and the middle bridge strain is obtained by numerical simulation. In order to make the bridge strain more consistent with the data that measured in the actual project, Gaussian white noise is added to simulate the environmental noise. Then, based on VMD theory, the bridge strain denoising model is established, and the selection of some key parameters is compared and analysed.

2. Theory

2.1. Simulation of bridge strain by vehicle-bridge system

The equation of motion of the vehicle can be expressed as follows:

\[
\begin{bmatrix}
M_{v1} & 0 \\
0 & M_{v2}
\end{bmatrix}
\begin{bmatrix}
\ddot{Y}_{v1} \\
\ddot{Y}_{v2}
\end{bmatrix}
+ \begin{bmatrix}
C_{v11} & C_{v12} \\
C_{v21} & C_{v22}
\end{bmatrix}
\begin{bmatrix}
\dot{Y}_{v1} \\
\dot{Y}_{v2}
\end{bmatrix}
+ \begin{bmatrix}
K_{v11} & K_{v12} \\
K_{v21} & K_{v22}
\end{bmatrix}
\begin{bmatrix}
Y_{v1} \\
Y_{v2}
\end{bmatrix}
= \begin{bmatrix}
0 \\
M_x - P_{int}
\end{bmatrix}
\] (1)

where \( Y \) is the response vector of the vehicle; \( M, C, K \) are the mass, damping and stiffness matrix of the vehicle respectively; the corner marker \( v \) represents the vehicle, 1 represents the vehicle body and 2 represents the vehicle wheels; \( P_{int} \) is the vehicle–bridge interaction force vector. \( M_x \) is the static load of the vehicle.

With the assumption of Rayleigh damping, the equation of motion for the bridge under the action of moving load can be written as:

\[
M_b \ddot{Y}_b + C_b \dot{Y}_b + K_b Y_b = H P_{int}
\] (2)

where \( Y_b \) is the response vector of the bridge; \( M_b\), \( C_b\), \( K_b\) are the mass, damping and stiffness matrix of the bridge; \( H \) denotes the Hermitian cubic interpolation shape functions.

Combining Eqn. (1) and (2), the vehicle-bridge’s coupling system can be written as:

\[
\begin{bmatrix}
M_{v1} & 0 & HM_{v2} \\
0 & M_{v1} & 0 \\
0 & 0 & M_{v2}
\end{bmatrix}
\begin{bmatrix}
\ddot{Y}_{v1} \\
\ddot{Y}_{v2} \\
\ddot{Y}_b
\end{bmatrix}
+ \begin{bmatrix}
C_{v1} & HC_{v11} & HC_{v12} \\
0 & C_{v11} & C_{v12} \\
0 & 0 & C_{v21} & C_{v22}
\end{bmatrix}
\begin{bmatrix}
\dot{Y}_{v1} \\
\dot{Y}_{v2} \\
\dot{Y}_b
\end{bmatrix}
+ \begin{bmatrix}
K_{v11} & HK_{v11} & HK_{v12} \\
0 & K_{v11} & K_{v12} \\
0 & 0 & K_{v21} & K_{v22}
\end{bmatrix}
\begin{bmatrix}
Y_{v1} \\
Y_{v2} \\
Y_b
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
HM_b
\end{bmatrix}
\] (3)

where \( K_r \) is the wheel stiffness; \( r \) is road surface roughness.

Equation (3) can be through the Newmark-Beta method to obtain the node response of the bridge. At time \( t \), the deflection at the point \( x \) to the left of the bridge can be expressed as:

\[
w(x,t) = H(x)Y_b(t)
\] (4)

According to equation (4), the strain at a point \( x \) and at time \( t \) can be written as follows:

\[
\varepsilon(x,t) = -h \frac{\partial^2 w(x,t)}{\partial x^2} = -h \frac{\partial^2 H(x)Y_b(t)}{\partial x^2}
\] (5)
2.2. Denoising theory of VMD

The basic principle of VMD is to decompose the intrinsic mode function (IMF) components with different bandwidth by transforming the estimation problem into a constrained variational problem. Assume that the bridge strain signal is contaminated by signal noise, as shown:

\[ f(t) = \varepsilon(t) + n(t) \]  

where \( \varepsilon(t) \) is the original bridge strain signal; \( n(t) \) is the signal noise; \( f(t) \) is the bridge strain signal polluted by noise.

The VMD method is used to separate the signal modes and determines the center frequency of each IMF through continuous iteration. Finally, we get the assemble of IMF signal components, as shown in the following equation:

\[ f(t) = \sum_{k=1}^{K} u_k(t) \]  

where \( u_k(t), \ k = 1,2,3... \) is the single IMF signal obtained by decomposing the bridge strain signal; \( K \) is the number of decomposition layers.

Each IMF signal needs to go through the following 3 steps:

1. The unilateral frequency spectrum of the analysis signal corresponding to each IMF signal is obtained:

\[ \left( \delta(t) + \frac{j}{\pi t} \right) u_k(t) \]  

2. The center frequency of each modal analysis signal is estimated, and the spectrum of each modal is modulated to the corresponding basic frequency band:

\[ \left( \delta(t) + \frac{j}{\pi t} \right) u_k(t) e^{-j\omega_k t} \]  

3. The bandwidth of each IMF signal is estimated by square \( L^2 \) regularization and \( H^1 \) Gaussian smoothing. Finally, the variational model for VMD to estimate the bandwidth of each IMF band is as follows:

\[ \min \left\{ \sum \left\| \partial_t \left[ \left( \delta(t) + \frac{j}{\pi t} \right) u_k(t) \right] e^{-j\omega_k t} \right\|_2^2 \right\} \]  

where \( \{ u_k \} = \{ u_1,\ldots,u_K \} \) and \( \{ \omega_k \} = \{ \omega_1,\ldots,\omega_K \} \) are the IMF signal and corresponding center frequencies respectively; \( \partial_t \) is the partial derivative of time \( t \) for function; \( \delta(t) \) is the pulse function.

In order to solve the constrained variational problem, both a quadratic penalty term \( \alpha \) and Lagrangian multipliers \( \lambda \) are applied. The extended Lagrangian expression is as follows:

\[ L(u_k,\omega_k,\lambda) = \alpha \sum_k \left\| \partial_t \left[ \left( \delta(t) + \frac{j}{\pi t} \right) u_k(t) \right] e^{-j\omega_k t} \right\|_2^2 + \left\| f(t) - \sum_k u_k(t) \right\|_2^2 + \left( \lambda(t), f(t) - \sum u_k(t) \right) \]  

The specific implementation process of signal decomposition by VMD is shown in the following 4 steps:

1. Initializing \( \{ u_k \} \), \( \{ \omega_k \} \), \( \lambda \) and define \( n = 0 \).

2. Setting up the cycle process, let \( n = n+1 \), update \( u_k \), \( \omega_k \), the equation is as follows:
\[ u_{i}^{n+1}(\omega) = \left[ \hat{f}(\omega) - \sum_{i=1}^{K} u_{i}^{n+1}(\omega) - \sum_{i=1}^{K} u_{i}^{n}(\omega) + \frac{\dot{\lambda}(\omega)}{2} \right] / \left[ 1 + 2\alpha(\omega - \omega_{k})^{2} \right] \]  \hspace{1cm} (12)

\[ \omega_{k}^{n+1} = \left[ \int_{0}^{\infty} \omega |u_{k}^{n+1}(\omega)| d\omega / \left[ \int_{0}^{\infty} |u_{k}^{n+1}(\omega)| d\omega \right] \right] \]  \hspace{1cm} (13)

(3) Updating the multiplication operator \( \lambda \), the equation is as follows:

\[ \lambda^{n+1}(\omega) = \lambda^{n}(\omega) + \tau [ f(\omega) - \sum_{i=1}^{K} u_{i}^{n+1}(\omega) ] \]  \hspace{1cm} (14)

(4) Judge whether the components meet the constraint conditions:

\[ \left\| \sum_{k=1}^{K} \| u_{k}^{n+1} - u_{k}^{n} \|^{2} \right\| / \left\| u_{k}^{n} \right\|^{2} < \varepsilon \]  \hspace{1cm} (15)

If the constraints are met, stop the cycle, obtain a finite number of IMF components; if the constraint condition is not satisfied, return to step (2) and repeat the above steps.

3. Numerical simulations

In this paper, the vehicle bridge coupling program is compiled by MATLAB, and the bridge strain is calculated. Then the VMD theory is used to compile the denoising program, and the numerical simulation of bridge strain denoising is realized.

3.1. Bridge and vehicle model

![Vehicle-bridge model](image)

In this study, a 20m simply-supported girder bridge is taken as the prototype bridge (figure 1). The bridge is designed according to the Chinese bridge design code[9], and its mechanical parameters are shown in table 1.

| Table 1  | Parameters of bridge models. |
|----------|-----------------------------|
| Bridge parameters | |
| \( L = 20 \text{m} \) | \( h = 1.756 \text{m} \) | \( \xi = 0.02 \) |
| \( \rho = 7850 \text{kg/m}^3 \) | \( A = 0.526 \text{m}^2 \) | \( EI = 1.978 \times 10^{11} \text{N/m} \) |

where \( L \) is the total length of the bridge; \( h \) represents the distance from the neutral axis of the beam to the bottom surface; \( \xi \) is the damping ratio, \( \rho \) is the density of bridge materials, \( A \) is the cross-sectional area of box girder and \( EI \) is bending rigidity.

The vehicle model adopts half-vehicle model (figure 1) with two axles, and mechanical parameters of the vehicle[10] are as shown in table 2.
Table 2 Parameters of vehicle models.

| Vehicle parameters | $a = 0.65$ | $m_1 = 800\text{kg}$ | $m_2 = 710\text{kg}$ |
|--------------------|------------|----------------------|----------------------|
| $b = 0.35$         | $K_{\alpha_1} = 3.99 \times 10^4 \text{N/m}$ | $C_{\alpha_1} = 2.32 \times 10^4 \text{N/m/s}$ |                      |
| $L_s = 2.66\text{m}$ | $K_{\alpha_2} = 3.99 \times 10^4 \text{N/m}$ | $C_{\alpha_2} = 5.18 \times 10^4 \text{N/m/s}$ |                      |
| $m_s = 1460\text{kg}$ | $K_{\alpha_3} = 3.51 \times 10^5 \text{N/m}$ | $C_{\alpha_3} = 8.00 \times 10^5 \text{N/m/s}$ |                      |
| $I_s = 1.516 \times 10^{-4}\text{m}^4$ | $K_{\alpha_4} = 3.51 \times 10^5 \text{N/m}$ | $C_{\alpha_4} = 8.00 \times 10^5 \text{N/m/s}$ |                      |

3.2. Simulation Results

It is necessary to use an effective evaluation standard to evaluate whether the denoising algorithm is effective. Therefore, this paper uses signal-to-noise ratio (SNR) to evaluate the performance of the proposed denoising method. Generally speaking, the larger the SNR is, the better the denoising effect is. The equation formula of SNR is as follows:

$$\text{SNR} = 10 \times \log_{10} \left( \sum_{n} f^2(t) / \sum_{n} \left[ f(t) - \hat{f}(t) \right]^2 \right)$$  (16)

where $f(t)$ is the bridge strain signal polluted by noise, $\hat{f}(t)$ is the bridge strain signal after denoising.

In this paper, the strain of bridge midspan is taken as an example, and add 10dB Gaussian white noise into it, as shown in figure 2. It is obvious from figure 2 that the fluctuation of the bridge strain curve becomes larger after being polluted by noise.

![Figure 2 Adding noise to the original bridge strain.](image)

Two key factors, decomposition mode number $K$ and quadratic penalty factor $\alpha$, need to be determined for VMD denoising. Because our main purpose is to remove the high-frequency noise in the strain signal, the final decomposed component should be two: strain and noise. Therefore, the value of decomposition mode number $K$ should be 2. Thus, we only need to discuss the influence of quadratic penalty factor $\alpha$ on the denoising effect.

In order to analyse the influence of quadratic penalty factor $\alpha$ on VMD denoising effect, for comparison, we observed the SNR under different quadratic penalty factor $\alpha$ (100: 100: 3000). As
shown in figure 3, the SNR first increases and then decreases with $\alpha$, when $\alpha = 1500$, the maximum value of SNR is 21.67.

![Figure 3](image3.png)

Figure 3 The value of SNR when $\alpha$ takes different values.

In order to demonstrate the denoising effect of VMD algorithm, letting $K=2$ and $\alpha = 1500$, using VMD theory to denoise the bridge strain. The results of bridge strain denoising and the corresponding spectrum are shown in figure 4.

![Figure 4](image4.png)

Figure 4 The results of bridge strain denoising and the corresponding spectrum.

According to the figure 4, after denoising by VMD algorithm, the fluctuation of bridge strain curve is obviously reduced and the SNR increases from 10dB to 21.67dB. Therefore, the VMD algorithm has a significant effect on the denoising of bridge strain with noise, it can effectively separate noise from strain according to the difference of the central frequency between the strain and the noise.

4. Conclusion

It is of great significance for practical engineering to denoise the bridge strain to obtain more accurate strain data. In this paper, a simply supported beam bridge and a half car model are used to simulate and calculate the bridge strain by establishing a vehicle bridge coupling system, and Gaussian white noise is added to the strain. Based on VMD theory, a bridge strain denoising algorithm is proposed. These conclusions can be obtained by numerical simulation:

(1) The denoising method is suitable for bridge strain and effective, when the SNR of bridge strain signal is 10dB, it will be increased to 21.67dB after using VMD denoising algorithm.
(2) The VMD theory can separate the bridge strain and noise according to different center frequencies of strain and noise.

(3) For the two key parameters: decomposition mode number $K$ and quadratic penalty factor $\alpha$, after research and comparison, the recommended values are $K=2$ and $\alpha=1500$ respectively.

In order to avoid the distortion of bridge strain due to excessive denoising, the value of preset variables used in VMD algorithm needs to be further studied in the future.

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