Bending analysis of skew ribbed plates with a meshfree method

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Abstract. Based on the first-order shear deformation theory (FSDT), a meshless method for solving the free vibration problem of the skew ribbed plates is proposed. The plates and ribs are discretized with a series of points to obtain a meshless model of the ribbed plate by simulating the ribs with beams. The FSDT and the moving least-squares approximation are used to establish the displacement field. The displacement fields of the stiffener can be expressed in terms of the mid-surface displacement of the plate by imposing displacement compatible conditions between the plate and the stiffener. Then the stiffness equation of the entire skew ribbed plate can be derived by superimposing the strain energy of plate and stiffener. And the boundary condition is introduced by the full transformation method. Several examples are calculated, and the results given by the proposed method are compared with those from other researchers or ABAQUS. The results show that the method can effectively analyze the bending problem of the skew ribbed plate, and it avoids redistribution of plate nodes when rib position changes.

1. Introduction

Due to its lightweight and high rigidity, ribbed plates have been widely found in civil engineering, aerospace and machinery industry, etc. Therefore, it is undoubtedly important to analyze the various mechanical properties of stiffened plates, and many scholars have conducted lots of researches on them through various methods.
In the aspect of static problem, Gong Shuguang et al.\cite{1} discussed the pros and cons of several plates locking problems based on the meshless method. Peng Linxin et al.\cite{2} applied the element-free Galerkin method to analyze the linear bending problem of corrugated plates. Yang Liu et al.\cite{3} analyzed the bending problem of the skew plate through the General Differential (GD) solution. The results show that the numerical method is a better way to solve the plate bending problem. Wang Kelin et al.\cite{4} used the Fourier series method to calculate the exact analytical solutions of the orthotropic skew plate under the action of in-plane tension and shear force. Fang Dianxin et al.\cite{5} investigated the plate bending problem by using the element-free Galerkin method.

In the aspect of dynamics problem, McGee.GO\cite{6,7} performed a literature review of the vibration analysis of skew plates and provided a solution to obtain the correct frequency for the effect of stress singularities on plate bending vibration at the obtuse angle of the skew plate. Zhou et al.\cite{8} studied the free vibration of the skew plate using moving-least square method and obtained stable convergence vibration results. Jin.C et al.\cite{9} proposed a new weak-form integral element method for the accurate analysis of the free vibration of a skew plate. The results show that this method can obtain the accurate frequency without explicitly considering the singular points of the bending moment at obtuse angles. Liu Canli et al.\cite{10} used the transfer matrix method to solve the free vibration problem of skew plates, and obtained that the method can be applied to the dynamic characteristics analysis of skew plates accurately in the range of small plate deflection. Li Xinghui\cite{11} applied integral method to solve the natural frequency problem of curved thick rectangular plates. Zeng Juncai et al.\cite{12} used the improved Fourier series method to create a vibration model of a rectangular plate, derived a matrix equation equivalent to the vibration control equation, and obtained the solution of the control equation under various boundary conditions. Modeling a rectangular plate based on the first-order shear deformation theory, Peng Linxin\cite{13} analyzed the problems of bending and vibration of ribbed plates. Liew K M et al.\cite{14} used the Rayleigh-Ritz method to analyze the dynamic behaviors of stiffened plates.

From above, it is obvious that most researches have focused on flat plates or rectangular ribbed plates, but there is no relevant literature for the research of skew ribbed plates. In addition to this, the meshless method as a new proposed by Belytschko T et al.\cite{15}, Liu WK et al.\cite{16} as an emerging numerical method does not require meshing or constructing high-order shape functions and it is simple and easy to find a good numerical solution. So, it has become one of the research hotspots in computational mechanics. In this paper, based on the advantages of meshless method, the bending problem of skew ribbed plates is analyzed based on the moving least squares approximation\cite{17} and the
first-order shear deformation theory\(^{[18]}\). The meshless column of skew ribbed plates is given in this paper. The C++ programming calculation is used to solve the skew ribbed plates with different parameters. And the presented results is verified via ABAQUS finite element. The results show that the meshless method can effectively solve the bending problem of skew ribbed plates. Because of no elements in the proposed meshless formulation, ribs are allowed to be set anywhere on plate and remeshing of the plate can be avoided when stiffener location changes or cracks propagate. Therefore, the meshless method can be applied to optimize the rib arrangement of skew ribbed plates.

2. Gridless Column with the Skew Ribbed Plates

2.1. Meshfree model

As shown in Figure 3, the skew ribbed plate is made of the of homogeneous material. The elastic modulus is \(E\), the Poisson's ratio is \(\nu\), the density is \(h_p\), and the height of the rib is \(h_s\). \(\xi\) is the foundation coefficient. The skew plates and their ribs are separated by a series of points. The meshfree model is shown in Figure 4 and Figure 5.

\[ H_s(x,y) = \sum B^{-1}(x,y)q(x_i,y_i)\bar{\omega}(x-x_i,y-y_i) \]

where \( B(x,y) = \sum \bar{\omega}(x-x_i,y-y_i)q(x_i,y_i)q^T(x,y) \), \( q^T(x,y) = [1, x, y, x^2, y^2, xy] \), \( \bar{\omega} \) as the power function,
take as:
\[
\varphi(r) = \begin{cases} 
\frac{2}{3} - 4r^2 + 4r^3 & r \leq 1/2 \\
\frac{4}{3} - 4r^2 - 4r^3/3 & 1/2 < r \leq 1 \\
0 & r > 1
\end{cases}
\]

Therefore, the approximate function of the function \( u(x) \) in the domain can be expressed as (2), where \( u_I \) is the displacement of the node \( I \).

\[
u^I(x) = \sum_{I=1}^{n} H_I(x, y) u_I \tag{2}
\]

The meshfree model of the rib is shown in Figure 5, and the shape function of the rib can be similarly calculated by moving the least square method.

2.3. The strain energy of slap plate

As shown in Figure 4, the degree of freedom of the slab nodes includes \( u_{p0}, v_{p0}, w_{p0}, \varphi_{px}, \varphi_{py} \) among them, \( u_{p0}, v_{p0}, w_{p0} \) respectively, which represent the translational displacements of the nodes along \( x, y \) and \( z \) directions, \( \varphi_{px}, \varphi_{py} \) respectively. Based on the first-order shear deformation theory\(^{[17]}\), the plate displacement field can be expressed as:

\[
\begin{align*}
    u_p(x, y, z, t) &= u_{p0}(x, y, t) - z\varphi_{px}(x, y, t) = \sum_{I=1}^{n} H_I(x, y) u_{p0}(t) - z \sum_{I=1}^{n} H_I(x, y) \varphi_{px}(t) \\
v_p(x, y, z, t) &= v_{p0}(x, y, t) - z\varphi_{py}(x, y, t) = \sum_{I=1}^{n} H_I(x, y) v_{p0}(t) - z \sum_{I=1}^{n} H_I(x, y) \varphi_{py}(t) \\
w_p(x, y, z, t) &= w_p(x, y, t) = \sum_{I=1}^{n} H_I(x, y) w_p(t)
\end{align*}
\]

where \( \{u_{p0}(t), v_{p0}(t), w_p(t), \varphi_{px}(t), \varphi_{py}(t)\}^T = \Delta U_p \) is the node parameter of the tablet node \( I \). For \( \varphi_{px}, \varphi_{py}, \) \( w_p \), they are mutually independent. Write equation (3) in matrix form:

\[
U_p = \begin{bmatrix} u_p \\ v_p \\ w_p \end{bmatrix} = \sum_{I=1}^{n} N_I A_{pl} \tag{4}
\]

where

\[
N_I = \begin{bmatrix} H_I & 0 & 0 & -zH_I & 0 \\
0 & H_I & 0 & 0 & -zH_I \\
0 & 0 & H_I & 0 & 0 \end{bmatrix}
\]

From the plate displacement field can be calculated at any point \((x, y)\) strain is:

\[
\begin{align*}
\kappa_p &= \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \sum_{I=1}^{n} B_{ij} A_{pl} \\
\gamma_p &= \begin{bmatrix} \gamma_{xx} \\ \gamma_{yy} \end{bmatrix} = \sum_{I=1}^{n} B_{ij} A_{pl}
\end{align*}
\]

(5)

(6)
Among which, $B_{el} = \begin{bmatrix} H_{1,s} & 0 & 0 & -zH_{1,s} & 0 \\ 0 & H_{1,y} & 0 & 0 & -zH_{1,y} \\ H_{1,y} & H_{1,s} & 0 & -zH_{1,y} & -zH_{1,s} \end{bmatrix}$, $B_{el} = \begin{bmatrix} 0 & 0 & H_{1,s} & -H_{1} & 0 \\ 0 & 0 & H_{1,y} & -H_{1} & 0 \end{bmatrix}$.

The displacement and strain of the plate are shown in Equations. (4), (5) and (6), therefore, the total strain energy of the plate can be obtained as follow:

$$\Pi_p = \frac{1}{2} A_p^T K_p A_p$$  \hspace{1cm} (7)

where

$$A_p = \{ A_{p1}, A_{p2} \ldots A_{pn} \}^T,$$

$$[K_p]_{el} = \iint \frac{1}{2} B_{el}^T D B_{el} \mathrm{d}x \mathrm{d}y + \frac{h}{\alpha} \iint B_{el}^T A B_{el} \mathrm{d}x \mathrm{d}y,$$

$\alpha = 5/6$ is the shear correction coefficient$^{[18]}$

$$A_e = \begin{bmatrix} G & 0 \\ 0 & G \end{bmatrix}, \quad G = \frac{E}{2(1+\mu)}, \quad D = \frac{E}{1-\mu} \begin{bmatrix} 1 & 0 & 0 \\ \mu & 1 & 0 \\ 0 & 0 & 1-\mu/2 \end{bmatrix}.$$  

2.4. The strain energy of rib

Using local coordinates to solve the $k$ strain energy of the rib, as shown in Figure 5, suppose the total number of nodes of a single rib is $m$. The ribs are modeled by beam elements, and the degrees of freedom of the rib nodes are respectively $u_{0s}, w_{s}, \varphi_{s}$, where $u_{0s}, w_{s}$ denote the translational displacements of the rib nodes along the longitudinal axis of the rib and the $z$ direction, respectively, and $\varphi_{s}$ denotes the turning angle of the nodes around the transverse axis of the rib.

According to the first-order shear theory, the displacement field under the local coordinate ($\vec{x}, \vec{y}$) can be expressed as:

$$\begin{align*} u_s(\vec{x}, \vec{y}, t) &= u_{0s}(\vec{x}, t) - z \varphi_{s}(\vec{x}, t) + \sum_{j=1}^{m} \Phi_{j}(\vec{x}) u_{0j}(\vec{x}, t) - z \sum_{j=1}^{m} \Phi_{j}(\vec{x}) \varphi_{sj}(t) \\
w_s(\vec{x}, t) &= \sum_{j=1}^{m} \Phi_{j}(\vec{x}) w_{sj}(t) \end{align*}$$  \hspace{1cm} (8)

where $\Phi_{j}(\vec{x})$ is the shape function of the rib, $\{u_{0j}(t), w_{sj}(t), \varphi_{sj}(t)\}_{j=1}^{n} = A_{el}$ is the displacement function of the first node of the rib with time $t$ as a variable, and equation (8) is written in matrix form:

$$U_s = \begin{bmatrix} u_s \\ w_s \end{bmatrix} = \sum_{j=1}^{m} \Phi_{j}(\vec{x}) \begin{bmatrix} 0 & 0 & -z \Phi_{j}(\vec{x}) \\ \varphi_{sj}(t) \end{bmatrix} = \begin{bmatrix} u_{0j}(t) \\ w_{sj}(t) \end{bmatrix}.$$  \hspace{1cm} (9)

The strain of the point ($\vec{x}, \vec{y}$) on the rib is:
where $B_{κsI} = [Φ_I, -z 0 -zΦ_I, x_0 -zΦ_I]$, $B_{γsI} = [0 Φ_I, x -z Φ_I]$. 

From equations (9), (10) and (11), the strain energy of the rib can be calculated as:

$$\Pi_r = \frac{1}{2} A_i^T K_r A_i$$

(12)

where

$$A_i = \{A_{i1}, A_{i2}, \ldots, A_{im}\}^T,$$

$$[K_r]_{ii} = \frac{h_s}{2} \int E B_{κsI}^T B_{κsI}^T d\xi d\tau + \int \frac{G A_l}{\alpha} B_{γsI}^T B_{γsI}^T d\tau,$$

$t_s$, $A_s$ and $I_s$ are the rib section thickness, cross-sectional area and moment of inertia section, respectively.

Adding equation (7) and equation (12), the total strain energy can be obtained:

$$\Pi_{pσ} = \frac{1}{2} A_p^T K_p A_p + \sum_{i=1}^{k} \frac{1}{2} A_i^T K_r A_i$$

(13)

where $k$ is the number of ribs.

2.5. Displacement coordination

The skew ribbed plate is shown in Figure 6, and the cross-section along the rib is shown in Figure 7. For a node $S$ on the rib, a point $P$ corresponding to it will be found on the surface of the plate (point $P$ is not necessarily a plate node), and point $C$ is the corresponding point of node $S$ and point $P$ on the rib-to-plate contact surface. The translational displacement and the angular displacement of the rib node and the plate node are all related to the angle $θ$, and the deflection of the two are equal, so the displacement relationship between the rib and the plate is as follows:

$$\begin{bmatrix} W_p \end{bmatrix}_p = \begin{bmatrix} W_s \end{bmatrix},$$

(14)

$$\begin{bmatrix} u_p \end{bmatrix}_p \cos θ + \begin{bmatrix} v_p \end{bmatrix}_p \sin θ = \begin{bmatrix} u_s \end{bmatrix},$$

(15)

$$\begin{bmatrix} φ_p \end{bmatrix}_p \cos θ + \begin{bmatrix} ϕ_p \end{bmatrix}_p \sin θ = \begin{bmatrix} ϕ_s \end{bmatrix},$$

(16)
There are \( m \) nodes on the rib, corresponding to different nodes on the plate, so there are \( m \) relations of the similar equations (14), (15) and (16):

\[
w_p(x_i, y_i) = w_i(x_i) \quad (i = 1, \ldots, m)
\]

\[
u_p(x_i, y_i, \frac{h_p}{2}) \cos \theta + v_p(x_i, y_i, \frac{h_p}{2}) \sin \theta = u_i(x_i, \frac{h_p}{2}) \quad (i = 1, \ldots, m)
\]

\[
\frac{\varphi_{ps}}{\varphi_{ps}}(x_i, y_i) \cos \theta + \varphi_{ps}(x_i, y_i) \sin \theta = \varphi_{ps}(x_i) \quad (i = 1, \ldots, m)
\]

According to the first-order shear theory and the moving least-squares approximation, we can derive from (17), (18) and (19):

\[
T_p A_p = T A
\]

\[
T_p (A_p \cos \theta + A_{ps} \sin \theta) = T A
\]

where

\[
A_n = \{w_{01}, w_{02}, \ldots, w_{0m}\}, \quad A_u = \{u_{01}, u_{02}, \ldots, u_{0m}\}
\]

\[
A_{\sigma n} = \{\varphi_{01}, \varphi_{02}, \ldots, \varphi_{0m}\}, \quad A_{\sigma u} = \{\varphi_{u1}, \varphi_{u2}, \ldots, \varphi_{um}\}
\]

\[
A_n = \{w_{p1}, w_{p2}, \ldots, w_{pm}\}, \quad A_u = \{u_{p1}, u_{p2}, \ldots, u_{pm}\}
\]

\[
A_{\sigma n} = \{\varphi_{ps1}, \varphi_{ps2}, \ldots, \varphi_{psm}\}, \quad A_{\sigma u} = \{\varphi_{ps1}, \varphi_{ps2}, \ldots, \varphi_{psm}\}
\]

\[
T_p = \begin{bmatrix}
H_1(x_1, y_1) & H_2(x_1, y_1) & \cdots & H_n(x_1, y_1) \\
H_1(x_2, y_2) & H_2(x_2, y_2) & \cdots & H_n(x_2, y_2) \\
\vdots & \vdots & \ddots & \vdots \\
H_1(x_m, y_m) & H_2(x_m, y_m) & \cdots & H_n(x_m, y_m)
\end{bmatrix}
\]

\[
T = \begin{bmatrix}
\Phi_1(x_1) & \Phi_2(x_1) & \cdots & \Phi_n(x_1) \\
\Phi_1(x_2) & \Phi_2(x_2) & \cdots & \Phi_n(x_2) \\
\vdots & \vdots & \ddots & \vdots \\
\Phi_1(x_m) & \Phi_2(x_m) & \cdots & \Phi_n(x_m)
\end{bmatrix}
\]

\[
e = \frac{h_p + h_s}{2}
\]

In the case of concentric ribs (the center axis of the rib coincides with the center of the plate), \( e \) is zero. This can be derived rib node parameters and board node parameters of the conversion equation:

\[
A_n = T_p R A_p
\]

where

\[
A = \begin{bmatrix}
u_{01}, w_{01}, \varphi_{01}, \ldots, u_{0m}, w_{0m}
\end{bmatrix}^T
\]

\[
A = \begin{bmatrix}
u_{p1}, w_{p1}, \varphi_{p1}, \ldots, u_{pm}, w_{pm}, \varphi_{pm}
\end{bmatrix}^T
\]
\[
R = \begin{bmatrix}
    Q & 0 \\
    \vdots & \ddots \\
    0 & Q
\end{bmatrix}, \quad Q = \begin{bmatrix}
    \cos \theta & \sin \theta & 0 & e \cos \theta & e \sin \theta \\
    0 & 0 & 1 & 0 & 0 \\
    0 & 0 & 0 & \cos \theta & \sin \theta
\end{bmatrix},
\]

R is a matrix of 3n × 5n, which is related to the angle \( \theta \). When \( \theta = \beta \), the ribs are parallel to the slant edges of the slab. When \( \theta = 0^\circ \) or \( 180^\circ \), the ribs are in a horizontal position. \( T_p \) is a matrix of 3m × 3n, \( T_s \) and \( T_p \) related. Since the ribs can adopt a separate coordinate system, it is only necessary to recalculate the matrix \( T_p \) if the ribs change position and the length of the ribs in the plate is constant. Therefore, the position of the rib can be arbitrarily changed without mesh reconstruction. Note: \( T = \begin{bmatrix} T_s & 0 \end{bmatrix} \)

Therefore, the position of the rib can be converted into the node parameters of the plate by equation (24) derived by the displacement coordination relationship.

2.6. Stiffness equation of the entire skew ribbed plate

Under the action of uniform load \( q \) on the skew plate, the external work is:

\[
W = A^T_p f
\]

where

\[
[f] = \begin{bmatrix} 0 & 0 \int qH_i dx dy & 0 & 0 \end{bmatrix}^T.
\]

The potential energy of the plate can be expressed as:

\[
\Pi = \Pi_p + \Pi_s - W
\]

From Eqs. (13) and (25), we can obtain:

\[
\Pi = \frac{1}{2} A^T_p K_p A_p + \sum_{i=1}^{k} \frac{1}{2} A^T_i K_i A_i - A^T_p f
\]

Submitting equation (24) into equation (13), it can be obtained:

\[
\Pi = \frac{1}{2} A^T_p K_p A_p - A^T_p f
\]

where,

\[
K = K_p + \sum_{i=1}^{k} T_i^T K_i T_i
\]

Applying the principle of minimum total potential energy \( \delta \Pi = 0 \), we can derive stiffness equation of the entire skew ribbed plate as:

\[
K A_p = f
\]

In this paper, the complete transformation method [15] is used to deal with the essential boundary conditions.

3. Results and discussion

Taking the skew ribbed plate with different parameters as examples, and calculating via C++ programming, the result of this paper is compared with the ABAQUS finite element solution and to verify the correctness of the presented method. In the following examples, the influence domain is a
parallelogram, as shown in Figure 8. The meshless scheme is chosen to be $13 \times 13$ for the plate and 13 for the stiffener. The parallelogram domain is defined as $h_x \times h_y$ ($h_x = \text{dmax} \times c_x$, $h_y = \text{dmax} \times c_y$). And $c_x$ and $c_y$ are respectively the distances between two neighbouring nodes in the horizontal direction and in the oblique direction of the parallelogram domain. The $\text{dmax}$(scaling factor) for plate and rib is taken as 4 and 2, respectively. For comparison, the plates are analyzed using commercial FEM software ABAQUS. The element S4R (four-node shell element with reduced integration) is used for the analysis. Note that SSSS is simply supported and CCCC is coupled.

**Figure 8. influence domain**

### 3.1. Skew plate with one rib

The dimensions of a skew plate with one rib are shown in Figure 9. $a=1\text{m}$, $b=1\text{m}$, $h_p = 0.01\text{m}$. $\beta = \pi/3$. The material parameters are: $\rho = 7800\text{kg/m}^3$ $E = 1.7\text{E7Pa}$ and $\mu = 0.3$. The boundary condition is SSSS. And the load conditions are shown in Figure 10. The finite element model is discrete via 880 units.

The deflection $w$ (displacement in $z$-direction) along edge ($y=b/2=0.5\text{m}$) of the stiffened plate subjected to a uniformly distributed load are listed in Table 1 and shown in Figure 11, respectively. And the results of the plate subjected to patch load are listed in Table 2 and shown in Figures 12, respectively. The displacement contours of the plates with different load conditions are shown in Figure 13 and 14, respectively. It shows that the present results are close to the FEM results, and the relative error is within the allowable range of the project, which verifies the effectiveness of the proposed method.

**Figure 9. Skew ribbed plates with one rib**

**Figure 10. Load distribution of plate**

Table 1. displacement $w$ ($z$-direction displacement) along edge ($y=b/2=0.5\text{m}$) of the skew ribbed plate subjected to a uniformly distributed load

| Node coordinates $(x,y)$ | Present results ($\times 10^4\text{m}$) | FEM results ($\times 10^4\text{m}$) | relative error(%) |
|--------------------------|----------------------------------------|------------------------------------|-------------------|
|                          |                                        |                                    |                   |
Figure 11. displacement w (z-direction displacement) along edge \((y=b/2=0.5\text{m})\) of the skew ribbed plate subjected to a uniformly distributed load

Table 2. displacement w (z-direction displacement) along edge \((y=b/2=0.5\text{m})\) of the skew ribbed plate subjected to patch load

| Node coordinates \((x,y)\) | Present results \(\times 10^{-4}\text{m}\) | FEM results \(\times 10^{4}\text{m}\) | relative error(%) |
|---------------------------|---------------------------------|-----------------|-----------------|
| (0,0.5)                   | 0                               | 0               | 0               |
| (0.15,0.5)                | 3.4140                          | 3.38694         | 0.7989          |
| (0.3,0.5)                 | 6.7374                          | 6.57145         | 2.5256          |
| (0.45,0.5)                | 9.5068                          | 9.37953         | 1.3568          |
| (0.6,0.5)                 | 11.7158                         | 11.6453         | 0.6054          |
| (0.75,0.5)                | 13.8691                         | 13.2593         | 4.5987          |
| (0.9,0.5)                 | 14.5431                         | 14.1491         | 2.7845          |
| (1.05,0.5)                | 14.7075                         | 14.2774         | 3.0125          |
| (1.2,0.5)                 | 13.9118                         | 13.6839         | 1.6658          |
| (1.35,0.5)                | 12.8676                         | 12.2602         | 4.9546          |
| (1.5,0.5)                 | 10.6485                         | 10.2009         | 4.3874          |
| (1.65,0.5)                | 7.8818                          | 7.56005         | 4.2561          |
| (1.8,0.5)                 | 4.5819                          | 4.4764          | 2.3562          |
| (1.95,0.5)                | 1.1921                          | 1.17235         | 1.684           |
| (2,0.5)                   | 0                               | 0               | 0               |
3.2. Reset rib position

A skew ribbed plate with two ribs is considered. As shown in Figure 15, rib I and rib II are located at a/2, b/2, respectively. The dimensions of the skew ribbed plate are a=2m, b=1m, t = 0.01m, hs = 0.1m and hp = 0.01m. The elastic properties are $E = 1.7E7$Pa and $\mu = 0.3$. The density is $\rho = 7800$ kg / m$^3$. Both $\beta$ and $\theta$ are $\pi/3^\circ$. The boundary condition is SSSS. And the load conditions are shown in Figure 16. This example also performs three-dimensional study for this plate using FEM software ABAQUS and the finite element model is discrete via 920 units. The deflection $w$ (displacement in z-direction) along edge ($y=b/2=0.5m$) of the stiffened plate subjected to a uniformly distributed load are listed in Table 3 and shown in Figure 17, respectively. Then, we change the position of rib II, as shown by the dotted line position in Figure 15 (note rib II'), and the results are listed in Table 4 and shown in Figure 18. The displacement contours of the plates with different position of rib are shown in Figures 19 and 20, respectively.

It shows that the present results are closed to those that are obtained with ABAQUS, the relative errors between the two are less than 5%, which has verified the correctness of the method in this paper. In addition, because of no elements, in the proposed meshless formulation, the ribs are allowed be set anywhere on the flat plate and remeshing of the flat plate can be avoid when rib location changes. Therefore, when the rib location changes, only the $T$ matrix in equation (24) needs to be recalculated.
Figure 15. Skew ribbed plates with two rib

Table 3. displacement $w$ (z-direction displacement) along edge ($y=b/2=0.5m$) of the skew ribbed plate subjected to a uniformly distributed load

| Node coordinates $(x,y)$ | Present results ($\times 10^{-4}m$) | FEM results ($\times 10^{-4}m$) | relative error(%) |
|--------------------------|--------------------------------------|-------------------------------|------------------|
| (0,0.5)                  | 0                                    | 0                             | 0                |
| (0.1,0.5)                | 0.6510                               | 0.627998                      | 3.6584           |
| (0.25,0.5)               | 1.5104                               | 1.46277                       | 3.2561           |
| (0.4,0.5)                | 2.1609                               | 2.1044                        | 2.6845           |
| (0.6,0.5)                | 2.6349                               | 2.57849                       | 2.1863           |
| (0.75,0.5)               | 2.7653                               | 2.66381                       | 3.8095           |
| (0.9,0.5)                | 2.6457                               | 2.60266                       | 1.6542           |
| (1,0.5)                  | 2.6280                               | 2.56075                       | 2.6254           |
| (1.1,0.5)                | 2.7801                               | 2.66381                       | 4.3652           |
| (1.25,0.5)               | 2.6996                               | 2.57849                       | 4.6987           |
| (1.4,0.5)                | 2.3192                               | 2.2657                        | 2.3621           |
| (1.55,0.5)               | 1.7292                               | 1.70056                       | 1.6849           |
| (1.7,0.5)                | 0.9423                               | 0.923954                      | 1.9874           |
| (1.85,0.5)               | 0.3477                               | 0.34429                       | 0.9854           |
| (1.95,0.5)               | 0                                    | 0                             | 0                |

Figure 16. Load distribution of plate

Table 4. displacement $w$ (z-direction displacement) along edge ($y=b/2=0.5m$) of the skew ribbed plate subjected to a uniformly distributed load

Figure 17. displacement $w$ (z-direction displacement) along edge ($y=b/2=0.5m$) of the skew ribbed plate subjected to a uniformly distributed load
for reset rib position

| Node coordinates (x,y) | Present results \(\times10^{-4}\) m | FEM results \(\times10^{-4}\) m | relative error(%) |
|------------------------|------------------------------------|---------------------------------|-----------------|
| (0.05,0.5)             | 0.6265                             | 0.62214                         | 0.6999          |
| (0.2,0.5)              | 2.4656                             | 2.43516                         | 1.2512          |
| (0.35,0.5)             | 4.0958                             | 4.01503                         | 2.0125          |
| (0.55,0.5)             | 5.6510                             | 5.5425                          | 1.9584          |
| (0.7,0.5)              | 6.3528                             | 6.15214                         | 3.261           |
| (0.8,0.5)              | 6.3511                             | 6.28661                         | 1.0256          |
| (0.95,0.5)             | 6.2233                             | 6.09191                         | 2.1566          |
| (1.05,0.5)             | 5.9050                             | 5.72108                         | 3.2151          |
| (1.15,0.5)             | 5.2928                             | 5.18841                         | 2.0125          |
| (1.25,0.5)             | 4.7392                             | 4.53217                         | 4.5684          |
| (1.35,0.5)             | 3.9425                             | 3.80204                         | 3.6954          |
| (1.5,0.5)              | 2.7367                             | 2.7089                          | 1.0256          |
| (1.6,0.5)              | 2.1359                             | 2.11506                         | 0.9854          |
| (1.7,0.5)              | 1.6054                             | 1.57924                         | 1.6584          |
| (1.85,0.5)             | 0.8197                             | 0.79852                         | 2.6584          |

Figure 18. displacement \(w\) (z-direction displacement) along edge \((y=b/2=0.5\text{ m})\) of the skew ribbed plate for reset rib position

Figure 19. Displacement contours of the plate plates subjected to a uniformly distributed load for reset rib position

Figure 20. Displacement contours of the plates subjected to a uniformly distributed load for reset rib position

3.3. The applicability of the present method to different boundary conditions

In order to analyze applicability of the present method to different boundary condition, we change the boundary condition in section 3.1 to CCCC and CCSS, respectively. The deflection \(w\) (displacement in
z-direction) along edge \((y=b/2=0.5\text{m})\) of the stiffened plate subjected to a uniformly distributed load are listed in Table 5–6 and shown in Figures 21–22.

The results show that the solution of this paper is very close to finite element solution, and the relative error is within the allowable range of the project. So, it can be concluded that the proposed method has no limitations on the change of boundary conditions.

Displacement \(w\) (z-direction displacement) along edge \((y=b/2=0.5\text{m})\) of the skew ribbed plate subjected to a uniformly distributed load

**Table 5.** Displacement \(w\) (z-direction displacement) along edge \((y=b/2=0.5\text{m})\) of the skew ribbed plate with CCCC.

| Node coordinates \((x,y)\) | Present results \((\times 10^{-4}\text{m})\) | FEM results \((\times 10^{-4}\text{m})\) | Relative error(%) |
|----------------------------|---------------------------------|---------------------------------|-------------------|
| \((0,0.5)\)               | 0                               | 0                               | 0                 |
| \((0.15,0.5)\)            | 1.2273                          | 1.20298                         | 2.0195            |
| \((0.3,0.5)\)             | 2.5477                          | 2.49465                         | 2.1264            |
| \((0.45,0.5)\)            | 3.7260                          | 3.65458                         | 1.9548            |
| \((0.6,0.5)\)             | 4.7689                          | 4.60253                         | 3.6142            |
| \((0.75,0.5)\)            | 5.3850                          | 5.2795                          | 1.9987            |
| \((0.9,0.5)\)             | 5.7807                          | 5.65323                         | 2.2556            |
| \((1.05,0.5)\)            | 5.7954                          | 5.70709                         | 1.5468            |
| \((1.2,0.5)\)             | 5.5604                          | 5.43895                         | 2.23653           |
| \((1.35,0.5)\)            | 4.9098                          | 4.86009                         | 1.0235            |
| \((1.5,0.5)\)             | 4.0381                          | 3.99822                         | 0.9984            |
| \((1.65,0.5)\)            | 2.9179                          | 2.89884                         | 0.6584            |
| \((1.8,0.5)\)             | 1.6789                          | 1.64437                         | 2.1026            |
| \((1.95,0.5)\)            | 0.3048                          | 0.30119                         | 1.2112            |
| \((2,0.5)\)               | 0                               | 0                               | 0                 |

**Figure 21.** Displacement \(w\) (z-direction displacement) along edge \((y=b/2=0.5\text{m})\) of the skew ribbed plate with CCCC.

**Table 6.** Displacement \(w\) (z-direction displacement) along edge \((y=b/2=0.5\text{m})\) of the skew ribbed plate with SSCC.

| Node coordinates \((x,y)\) | Present results \((\times 10^{-4}\text{m})\) | FEM results \((\times 10^{-4}\text{m})\) | Relative error(%) |
|----------------------------|---------------------------------|---------------------------------|-------------------|
| \((0,0.5)\)               | 0                               | 0                               | 0                 |
(0.15,0.5)  2.2862  2.24095  2.0195  
(0.3,0.5)    4.7757  4.67628  2.1264  
(0.45,0.5)  7.0368  6.90188  1.9548  
(0.6,0.5)    9.0686  8.75231  3.6142  
(0.75,0.5)  10.2933 10.0916  1.9987  
(0.9,0.5)    11.0818 10.8374  2.2556  
(1.05,0.5)  11.1145 10.9452  1.5468  
(1.2,0.5)    10.6418 10.4093  2.2365  
(1.35,0.5)  9.3549  9.26014  1.0235  
(1.5,0.5)    7.6449  7.56931  0.9984  
(1.65,0.5)  5.4830  5.44716  0.6584  
(1.8,0.5)    3.1334  3.06889  2.1026  
(1.95,0.5)  0.5638  0.557098 1.2112  
(2.0,0.5)   0        0        0

Figure 22. displacement w (z-direction displacement) along edge (y=b/2=0.5m) of the skew ribbed plate with SSCC.

4. Conclusions
A linear bending analysis on skew ribbed plates is presented with an improved meshless model for skew ribbed plates. In the model, ribs and flat plate are considered separately with independent coordinate system, and the ribs are taken as beams. The flat plate and ribs are combined by applying the displacement compatibility conditions. The stiffness equation of the skew ribbed plate is given by supposing the external work and the strain energy of the ribs and the flat plate and invoking the minimum potential energy principle. Because of no elements in the proposed meshless formulation, ribs are allowed to be set anywhere on plate and remeshing of the skew ribbed plate can be avoided. When rib location changes, only the T matrix in equation (24) needs to be recalculated. The proposed formulation is tested with different numerical examples, and the results display good consistency with the solutions given by FEM analyses using ABAQUS.

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