Noise Intensity-Intensity Correlations and the Fourth Cumulant of Photo-assisted Shot Noise

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We report the measurement of the fourth cumulant of current fluctuations in a tunnel junction under both dc and ac (microwave) excitation. This probes the non-Gaussian character of photo-assisted shot noise. Our measurement reveals the existence of correlations between noise power measured at two different frequencies, which corresponds to two-mode intensity correlations in optics. We observe positive correlations, i.e. photon bunching, which exist only for certain relations between the excitation frequency and the two detection frequencies, depending on the dc bias of the sample.

In mesoscopic physics, a lot of effort has been put in the measurement and understanding of current fluctuations. Of particular interest are the deviations from the ubiquitous Gaussian noise, i.e. the study of high order cumulants. As a matter of fact, while the only parameter characterizing a Gaussian distribution, the variance or second cumulant of the fluctuations, contains some information about electron transport, much more could be learned by the statistical study of the fluctuations beyond their variance. These are characterized by cumulants of order three and higher, which are zero for a Gaussian distribution. For the simplest systems, such as a tunnel junction between normal metals or a quantum point contact, only the third cumulant has been measured until now. Higher order cumulants have been experimentally accessible solely in quantum dots where electrons enter only one by one. In all these measurements the system is driven out of equilibrium by a dc voltage bias. Here we address the statistics of photo-assisted shot noise, i.e. current fluctuations in the presence of an ac excitation. While the variance of such fluctuations has been well explored both theoretically and experimentally, no experiment has been performed yet that reports the existence of higher order cumulants in such conditions.

In the following we present a link between the measurement of the correlation $G_2 = \langle P_1 P_2 \rangle - \langle P_1 \rangle \langle P_2 \rangle$ between the noise powers $P_1$ and $P_2$ at two different frequencies in the GHz range, $f_1$ and $f_2$. The source of shot noise is a tunnel junction that is dc biased and excited at frequency $f_0$. Since power fluctuations of Gaussian noise are independent at all frequencies, our measurement probes only the non-Gaussian part of current fluctuations. This technique has been applied many times to the study of $1/f$ noise in glassy or disordered materials and has been proposed to measure the fourth cumulant of a quantum point contact without ac excitation. In optics, $G_2$ corresponds to intensity-intensity correlations, a usual probe of the statistical properties of light. Here we show that $G_2$ gives the fourth cumulant at frequencies $(f_1, f_2, 0)$ of photo-assisted shot noise in the tunnel junction. In the optics community, current/voltage fluctuations, which are simply another point of view of a fluctuating electromagnetic field, are rather thought of as time dependent electric and magnetic fields. Thus, a noisy electronic device is also a light source, and it is natural to apply tools developed in optics to analyze high frequency electronic noise. This point of view will also be studied.

Results

Experimental principle. The experimental setup is depicted on Fig. 1. The sample, a tunnel junction described in the Methods section, is voltage biased by a dc source and connected to microwave signal generators below 4 GHz and above 8 GHz while the noise generated by the junction at point A is amplified by a cryogenic amplifier in the 4–8 GHz frequency range. This setup allows noise to be measured in the 4–8 GHz range while the excitation frequency $f_0$ can take any value below 4 GHz or above 8 GHz. This insures that the amplifier never sees the ac
excitation of the sample, which avoids spurious signals due to nonlinearities in the amplifier at high excitation power. However, it prevents the use of an excitation frequency in the range 4–8 GHz.

The amplified noise is split into two branches: in branch 1 (respectively branch 2) the signal is bandpass filtered around \( f_1 = 4.5 \text{ GHz} \) with a bandwidth \( \Delta f_1 = 0.72 \text{ GHz} \) (resp. \( f_2 = 7.1 \text{ GHz} \), \( \Delta f_2 = 0.60 \text{ GHz} \)). In each branch a fast power detector (diode symbol on Fig. 1, bandwidth \( \Delta \omega \sim 100 \text{ MHz} \)) measures the “instantaneous” (averaged over a few ns) noise power integrated in the corresponding bandwidth, i.e. \( v_{\text{th}}(t) \propto P_k(t) \) with \( k = 1, 2 \). The dc voltage at point \( B_k \) is thus proportional to the noise spectral density, \( S(f_i) = \langle |i(f_i)|^2 \rangle \) with \( i(f) \) the Fourier component of the instantaneous current: \( \langle v_{\text{th}}(t) \rangle \propto S(f_i) \Delta f_i \). A dc block (capacitor on Fig. 1) removes the dc part of \( v_{\text{th}}(t) \) to give \( v_{\text{th}}(t) \propto P_k(t) - \langle P_k(t) \rangle = \delta P_k(t) \). Finally, voltages at points \( C_1 \) and \( C_2 \) are simultaneously digitized with a 2-channel, 14-bit, 400 MS/s acquisition system and the correlator \( G_2 = \delta P_1 \delta P_2 \) is computed in real time by a 12-core parallel computer, as are the autocorrelations of both channels \( \delta P_k(t) \).

The fourth cumulant of noise. In order to describe and understand the meaning of the observed correlator, let us first express the noise generated by the sample in Fourier space. We define the power fluctuations \( \delta P_k(t) = P_k(t) - \langle P_k(t) \rangle \) so that \( G_2 = \delta P_1(t) \delta P_2(t) \). The average \( \langle \cdot \rangle \) is performed over time. Introducing the Fourier component of the power fluctuations \( \delta P_k(e) \) at frequency \( e \), one has: \( G_2 = \int \text{d}e \langle \delta P_1(e) \delta P_2(-e) \rangle \). Here \( e \) is a low frequency limited by the output bandwidth of the power detectors. Note that \( \delta P_k(e = 0) = 0 \), while power fluctuations at finite frequency \( e \) are related to current fluctuations by \( \delta P_k(e \neq 0) \propto \int \text{d}f \langle i(f) i(-f) + i(f) i(e + f) \rangle \) where \( i(f) \) is the fluctuating current’s Fourier component at frequency \( f \). This integral spans over the bandwidth of the bandpass filters, i.e. \( f_k \pm \Delta f_k / 2 \). Thus, \( G_2 \) is proportional to the correlator between currents at four different frequencies which, by definition, is the time averaged fourth cumulant of current fluctuations taken at frequencies \( f_1, f_2 \) and \( e \) (with \( e \rightarrow 0 \) but \( e \neq 0 \)).

\[
G_2 \propto \langle i(f_1) i(e-f_1) i(f_2) i(-e-f_2) \rangle \Delta f_1 \Delta f_2 \Delta e \quad (1)
\]

Noting \( i_b(t) \) the current after bandpass filtering around \( \pm f_0 \), one has \( G_2 \propto \langle \langle i(t) i(t + 2k \Delta t) \rangle \rangle - \langle \langle i(t) \rangle \rangle^2 - \langle \langle i(t + 2k \Delta t) \rangle \rangle^2 \rangle \), the averaging being performed over the time \( t \). Here \( \langle x^4 \rangle = \langle x^2 \rangle^2 - 3 \langle x^2 \rangle^2 \) is the fourth cumulant of the random variable \( x \) with \( \langle x \rangle = 0 \). For a Gaussian distribution, \( \langle x^4 \rangle = 0 \), so there is no information in the fourth moment that is not contained in the variance. In contrast, for a non-Gaussian distribution, the fourth moment differs from 3 \( \langle x^2 \rangle^2 \), though only very slightly for current fluctuations involving many electrons, so the fourth cumulant is non-zero.

**Frequency dependence.** The first step in the observation of \( G_2(V_{\text{dc}}, V_{\text{ac}}, f_0) \) is to determine the excitation frequency for which \( G_2 \neq 0 \). This is done by measuring \( G_2 \) at fixed bias voltage \( V_{\text{dc}} = 0 \) or \( V_{\text{dc}} = 2.4 \text{ mV} \) and fixed excitation amplitude \( V_{\text{ac}} = 1.1 \text{ mV} \) while varying the excitation frequency \( f_0 \) from 10 MHz to 3.83 GHz and from 10 GHz to 14 GHz. These results are reported in Fig. 2, where \( G_2 \) has been reduced to units of \( k^2 \) as the measured noise spectral density \( S \) of a conductor of resistance \( R \) is often given in terms of equivalent noise temperature \( T_{\text{noise}} = RS / 2k \). We observe that \( G_2 \) is large at low frequency for both bias voltages. This simply reflects that a slow oscillation of the bias voltage induces a slow modulation of the noise, i.e. a slow oscillation of both \( P_1 \) and \( P_2 \). The response disappears when the modulation frequency exceeds the output bandwidth of the power detectors.

At much higher excitation frequencies, \( G_2 \) strongly depends on \( V_{\text{dc}} \). For \( V_{\text{dc}} = 2.4 \text{ mV} \), \( G_2 \) shows peaks at \( f_0 = 2.6 \text{ GHz} \) and \( f_0 = 11.6 \text{ GHz} \), which correspond to \( f = f_1 + f_2 \), where \( f_1 \) and \( f_2 \) are the signal observation frequencies. At zero bias, \( G_2 \) peaks at \( f_0 = 1.3 \text{ GHz} \), or \( f_0 / 2 \). Choosing \( f_0 = f_1 / 2 \) was not possible with this experimental setup since it lies in the 4–8 GHz range.

To understand why \( G_2 \) is nonzero at high excitation frequency, let us first consider the correlator \( \langle i(f) i(f') \rangle \). The latter is non-vanishing only for frequencies such that \( f + f' = n f_0 \), with \( n \) an integer. The case \( n = 0 \) corresponds to photo-assisted noise whereas \( n \neq 0 \) describes the noise dynamics, characterized by the correlator \( \rho_n(f, f_0) = \langle i(f) i(n f_0 - f) \rangle^{2n-2} \). In order to detect the fourth cumulant and not the fourth moment of current fluctuations, it is crucial that the four frequencies involved in \( G_2 \) (see Eq. 1 where \( c \neq 0 \)) be different so that correlators \( \langle i(f) i(-f) \rangle \) are never involved. Experimentally, the separation of the signal into two branches with non-overlapping bandwidths insures that \( f_1 \neq \pm f_2 \), while the presence of the dc blocks, by imposing \( e \neq 0 \), prevents all other possible occurrences of such correlators. In our experimental setup, relevant frequencies are close to \( \pm f_1 \) and \( \pm f_2 \), so this condition becomes \( f_1 \pm f_2 \neq n f_0 \), i.e. \( f_0 = f_1 / n \). In such cases, the fourth order correlator of Eq. (1) is dominated by the terms \( \langle i(f_1) i(f_2) \rangle \langle i(-f_1) i(-f_2) \rangle \). This product is zero unless \( f_1 \neq f_2 \), as we observe on Fig. 2.

**Voltage dependence.** We now consider the variation of \( G_2 \) as a function of the dc voltage for various ac excitation amplitudes at fixed excitation frequency \( f_0 = f_1 / n \). Data on Fig. 3 correspond to an excitation at \( f_0 = f_1 \). We observe that \( G_2 \) is maximal at high dc bias and diminishes at \( V_{\text{dc}} = 0 \). We obtained identical results for \( f_0 = f_2 \) (data not shown). Data on Fig. 4 correspond to \( f_0 = f_1 / 2 \). Here \( G_2 \) peaks at 0 dc bias and decays when \( |V_{\text{dc}}| \) increases. Moreover, the
amplitudes at frequency \( f_0 \) is an order of magnitude larger than that of \( G_2(f_0 = f_{..}/2) \).

The voltage dependence of the signal can be explained by expressing \( G_2 \) in terms of the correlators \( X_n(f, f_0) \):

\[
G_2(f_0 = f_{..}/n) = K |X_n(f, f_0)|^2
\]

with \( K \approx 4\Delta f_1\Delta f_2 \). The exact value of \( K \) involves integrals of the gain of the setup over the actual bandwidth of the filters as well as the coupling coefficient between the sample and the detection setup, which depends on the impedance of the sample. The correlators \( X_n \) have been calculated and measured in the quantum regime at very low temperature\(^{17,28,29} \). In the high temperature, classical regime that corresponds to the present experiment, \( X_n \) reduces to:

\[
X_n = \int_{-\pi}^{\pi} S_0(V_{dc} + V_{ac}\cos\theta)\exp(in\theta) d\theta
\]

Here \( S_0(V) = 2 eGV \coth(eV/2k_B T) \) is the noise of the junction at zero frequency in the absence of ac excitation. Eq. 3 is interpreted as follows: in the classical regime, the noise responds instantaneously to the time-dependent voltage \( V(t) = V_{dc} + V_{ac}\cos\theta \) with \( \theta = 2nf_0t \), so it oscillates at frequency \( f_0 \) and its harmonics. \( X_n \) is the amplitude of the \( n \)th harmonics. \( X_n \) is independent of \( f \) and \( f_0 \) as long as \( hf_0 \ll k_B T \), so \( G_2(f_0 = f_{..}/n) = K X_n^2 \) depends only on \(|n|\).

For small \( V_{ac} \) and \( f_0 \approx f_{..} \), the noise oscillates with an amplitude given by \( X_1 \approx V_{ac} \frac{dS_0}{dV} \), so \( G_2 = KX_1^2 \) computes to zero at \( V_{dc} = 0 \) and is maximal at large \( V \), which corresponds to the shape observed on Fig. 3. For \( f_0 = f_{..}/2 \), \( G_2 \) is given by the amplitude of the noise that oscillates at \( 2f_0 \) which is given by \( X_2 \approx \frac{d^2S_0}{dV^2} V_{ac}^2 \) for small \( V_{ac} \). Therefore \( G_2 = KX_2^2 \) is maximal at \( V_{dc} = 0 \) and decays at finite dc bias, as observed on Fig. 4. Solid lines on Fig. 3 and 4 represent the theoretical predictions of Eqs. (2,3) and agree very well with the measurements.

**Photon bunching.** Current fluctuations generated by a tunnel junction are known to be non-Gaussian, and thus should exhibit a non-zero fourth cumulant even in the absence of ac excitation\(^{30} \). This contribution, together with potential environmental effects\(^{31,32} \), are negligible as compared to the signals we report here. For example, at \( V_{dc} = 2 \) mV and \( V_{ac} = 0 \), the correlator would correspond to \( G_2 \sim 10^{-5} \) K. Thus, by adding an ac excitation on the sample we have been able to boost the fourth cumulant by 5 orders of magnitude.

Furthermore, our experiment corresponds to intensity-intensity correlation, which is the usual way used to differentiate classical from quantum light by showing evidence of photon bunching or antibunching. More precisely, in order to make a connection with experiments performed in optics, let us define the dimensionless correlator:

\[
g_2 = \frac{(P_1P_2)}{(P_1)(P_2)} = 1 + \frac{G_2}{(P_1)(P_2)}
\]

which is usually referred to as the same-time two-mode second order correlator of the electromagnetic field radiated by the junction.

The variations of \( g_2 \) as a function of \( V_{dc} \) are depicted in Fig. 5 for fixed \( V_{ac} = 1.0 \) mV for both \( f_0 = f_{..} \) (red triangles) and \( f_0 = f_{..}/2 \) (green circles). It is clear from these data that we always observe a positive correlation between the power fluctuations, i.e. photon bunching \((g_2 > 1)\). Each acquisition performed by the digitizer (integrated over \( \tau = 2.5 \) ns) corresponds to an averaged number of \( \langle n_2 \rangle = P_2 \tau/\hbar f_2 \sim 21 \) photons at 7.2 GHz emitted by the sample at \( V_{dc} = V_{ac} = 1 \) mV and \( f_0 = f_{..} \), plus \sim 40 photons from the amplifier. Thus our experiment does not measure correlations at the single photon level, but is very far from that limit, which can be reached by lowering the temperature and the ac power.

![Figure 3](image-url) Reduced \( G_2 \) vs dc bias voltage for various ac excitation amplitudes at frequency \( f_0 = f_{..}/2 = 1.3 \) GHz. Symbols are experimental data and solid lines theoretical expectations of Eq. (2) and (3).

![Figure 4](image-url) Reduced \( G_2 \) vs dc bias voltage for various ac excitation amplitudes at frequency \( f_0 = f_{..}/2 = 1.3 \) GHz (red triangles) or \( f_0 = f_{..}/2 \) (green circles). The dashed line at \( g_2 = 1 \) represents independent photons, i.e. chaotic light, while \( g_2 > 1 \) corresponds to photon bunching.

![Figure 5](image-url) Measured \( g_2 \) vs dc bias voltage at \( V_{ac} = 1.0 \) mV for an excitation frequency \( f_0 = f_{..} \) (red triangles) or \( f_0 = f_{..}/2 \) (green circles).
**Discussion**

It follows from the observed behaviour that by choosing the excitation frequency, dc bias and ac excitation, we can control how non-Gaussian the shot noise of the junction can be. The level of non-Gaussianity of the signal is characterized here by the fourth cumulant, which is directly linked to the measured correlator.

It should be noted that the power detector, which measures the square of the electric field, cannot differentiate absorption from emission of photons by the sample. This has to be taken into account when comparing the data with theories such as \(24,26,27\) which consider emission of photons by the sample. This has to be taken into account when comparing the data with theories such as \(24,26,27\) which consider emission of photons by the sample.

The tunnel junction behaves as a source of white noise whose amplitude is instantaneously modulated by the bias voltage. This description holds only because the temperature is large in the present experiment. At very low temperature, the noise can no longer be considered as white and no longer responds adiabatically to an ac excitation, so Eq.(3) will no longer be valid. In particular, \(X\) depends on \(f\) and \(f_0\), so that excitations at \(f_1 + f_2\) or \(f_1 - f_2\) will no longer correspond to the same \(G_2\). Still the present analysis, and in particular the link between \(G_2\) and \(X\) given by Eq.(2), will remain valid. Our measurements open the way to the study of the fourth cumulant of current fluctuations in the quantum regime at very low temperature, where the same setup can be used to detect correlations at the single photon level.

**Methods**

**Sample.** We have chosen to perform the measurement on the simplest system that exhibits well understood shot noise, the tunnel junction. The sample is a \(\sim 1 \mu m \times 15 \mu m\) Al/Al oxide/Al tunnel junction made by photolithography, similar to that used for noise thermometry\(^9\), cooled at \(T = 3.0\) K so the aluminium remains a normal metal. The resistance of the junction at that temperature, \(R = 22\Omega\), is close enough to the \(50 \Omega\) impedance of the microwave circuitry to ensure a good coupling. The capacitance of the junction corresponds to an RF frequency cutoff of \(\sim 6\) GHz, so it influences the amplitude of the noise we measure and the amplitude of the ac excitation experienced by the junction. Both effects are calibrated out as explained below.

**Calibration.** In order to have a quantitative measurement of \(G_2\), it is necessary to calibrate the ac excitation voltage reaching the sample and the overall gain of the detection. The calibration of the ac voltage across the sample is performed by measuring the usual photo-assisted noise, i.e. \(S v_{\text{det}}\) in the presence of a microwave excitation for various excitation voltages\(^10\). The temperature is large enough \((k_BT \gg h_f f_0)\) to approximate the noise measured in each branch by its value at zero frequency. In the absence of an excitation, the noise spectral density is given by \(S_0(v_{\text{det}}) = eG_\text{VAC} \coth(h_f v_{\text{det}}/2k_BT)\) with \(G = 1/R\), the sample’s conductance. Since the excitation frequency \(f_0\) is always such that \(h_f f_0 \ll k_BT\), the photo-assisted noise can be approximated by its time-average value as if the junction was responding instantaneously to the time-dependent voltage. To calibrate the gain of the setup, we consider the single channel autocorrelations \((\Delta P_0^2)\). Those are related to fourth order current correlations \((i(t) i(-f) i(t') i(-f'))\) \((t \sim t')\). However, here \(f\) and \(f'\) belong to the same frequency band, so the correlator is dominated by terms \(f \sim -f'\), and \((\Delta P_0^2) \propto (P_0^2)\). The fourth cumulant \(G_2\) is only a very small correction to this. Thus, power correlations within the same frequency band are totally dominated by Gaussian fluctuations, as \(^{11}\). Note that since the amplifier noise dominates \((P_0)\), it also dominates \((\Delta P_0^2)\), but only the fourth cumulant of the amplifier’s noise contributes to \(G_2\) (here a very small contribution).

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Author contributions
J.-C.F. and F.B.S. performed the measurements and data analysis. S.B. designed and implemented the real-time digital correlator. L.S. fabricated the samples. C.L. designed and programmed the control of the experiments. B.R. designed the experiment and performed the theory. B.R. and C.L. supervised the measurements. The article was mainly written by J.-C.F. and B.R.

Additional information
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