Prediction of Interpass Softening from the Strain Hardening Rate Prior to Unloading

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It is shown experimentally that the kinetics of interpass softening, normally described in terms of the strain, strain rate and temperature, can be more conveniently specified as a function of the strain hardening rate, strain rate and temperature prior to unloading. This approach significantly reduces the number of experiments required to generate sufficient data for modeling purposes. It also eliminates the need to determine the retained strain to predict the softening kinetics in multi-hit deformation and simplifies the extrapolation of laboratory data to the conditions of industrial processing.

KEY WORDS: hot deformation; interpass softening; strain hardening.

1. Introduction

Softening between deformation operations plays an important role in hot working. It is this softening that is responsible for the reduction in forming loads and therefore for the increase in hot workability of hard-to-deform materials, as well as for grain refinement in the final product. Efficient control of hot working requires highly adaptive and comprehensive modeling of the material deformation behavior, which also includes a model for softening and its sensitivity to in-bar inhomogeneities as well as to variations in rolling temperature, speed, reduction, etc.

Softening during unloading between high-temperature deformation operations is traditionally quantified in terms of the volume fraction softened $X$. The softening kinetics are usually described by the JMAK equation for $X$

$$X(t) = 1 - \exp[-B(t/t_F)^n]$$

where $t$ is the time of unloading, $F$ is the fractional measure of interest, $B = -\ln(1-F)$, and $n$ is the Avrami exponent. Normally, $F$ is taken as 0.5 (50%), so that $t_F = t_{50}$ and $B = -\ln(0.5) = 0.693$. The time $t_{50}$ required to attain 50% fractional softening depends on the conditions during and after deformation as given by

$$t_{50} = CD_b e^{-pZ/q} \exp\left(\frac{Q_s}{RT_s}\right)$$

where $\varepsilon$ is the strain, $Z$ is the Zener–Hollomon parameter pertaining to the preceding deformation, $Z = \dot{\varepsilon} \exp(Q_{def}/RT_{def})$, $\dot{\varepsilon}$ is the strain rate, $Q_{def}$ is the activation energy for deformation, $D_b$ is the initial grain size prior to preloading, $T_{def}$ and $T_s$ are the absolute temperatures of deformation and softening, respectively, and the exponents $p$ and $q$ are taken as positive here for convenience. The coefficients $C$, $n$, $p$, $b$ and $q$, as well as the activation energy for softening $Q_s$ depend on the material, the deformation conditions ($T$, $\varepsilon$, $\dot{\varepsilon}$) and the predominant softening mechanism.

Three major softening mechanisms operate during interpass intervals in hot working. First, there is conventional static recovery, which can bring about substantial softening. At relatively high temperatures, recovery is followed by conventional static recrystallization (SRX). At relatively low temperatures (below the no-recrystallization temperature $T_{nr}$), static recovery can be followed by metadynamic recrystallization (MDRX) if dynamic recrystallization (DRX) has taken place during the preceding deformation. At temperatures above $T_{nr}$ both SRX and MDRX can operate simultaneously in different parts of the material if the preceding deformation has been sufficient to initiate DRX: the dynamically nucleated grains grow due to MDRX, while in the regions that have not undergone DRX, static recrystallization can occur.

The values of $p$, $q$ and $Q_s$ are often used to identify the controlling softening mechanism. The kinetics of SRX are known to exhibit fairly weak strain rate dependence (small $q$), but are strongly influenced by the preloading strain and are also quite sensitive to temperature due to the relatively high activation energy. By contrast, MDRX does not depend on the preceding strain and initial grain size ($p, b \to 0$) and has a somewhat lower activation energy; concurrently, it is much more strain rate sensitive. Within certain ranges of processing conditions ($T$, $\varepsilon$, $\dot{\varepsilon}$), there can be a gradual transition from SRX- to MDRX-controlled softening, where the respective coefficients can assume intermediate values.

All the coefficients in Eq. (2) are usually determined in laboratory simulations. The quantity to be measured experimentally is the fractional softening $X$ as a function of the time after deformation. Using Eq. (1), the time $t_{50}$ is computed.
computed from experimental values of $X$; it is then related to the experimental variables ($e, \dot{e}, T_{\text{def}}, T_s, D_h$) so that values of the coefficients in Eq. (2) can be derived. These can subsequently be employed in process control. Since the set of Eqs. (1) and (2) involves a large number of parameters that also vary with the conditions of prestraining, laboratory simulations require a considerable amount of experimental work. Moreover, further difficulties can be faced when the experimental results are applied to industrial processing, because the sensitivity of the various softening parameters to the prestraining conditions can be different in the simulations and in the mill.

In the present work, the parameters of post-deformation softening are evaluated by utilizing the data from conventional flow curves. This permits a significant reduction in the experimental work necessary to establish the softening kinetics and simplifies the application of the results of laboratory simulations to industrial hot working conditions. As will be seen below, the method is based on the current value of the (normalized) rate of work hardening. In a subsequent work, this approach will be extended to deformation under conditions of varying strain rate, as in the roll bite of industrial mills.

2. Experimental

The present study was concerned with a plain low C–Mn steel (wt%: 0.04% C, 0.22% Mn, 0.012% Si, 0.05% Al) that was shown earlier\(^{10}\) to undergo DRX during high-temperature, constant strain rate deformation. Therefore, various mechanisms of post-deformation softening can be expected.

Laboratory melt ingots were hot rolled to 20 mm thick plates using a laboratory rolling mill. Standard cylindrical compression specimens with initial heights of 15 mm and initial diameters of 10 mm were machined from the plates with the specimen axes parallel to the plate transverse direction.

Fractional softening was determined by means of interrupted uniaxial compression testing using a Gleeble® 1500 thermomechanical simulator. In the tests, specimens were reheated to 1100°C for 30 s and then cooled to the test temperature (900–1050°C) at 5°C/s. After 20 s of soaking at the test temperature, the specimens were compressed in a single hit to a true strain of 1.2 (70% height reduction) and in the double-hit tests to first-hit strains of $e_{01}=0.11–0.5$ (10–40%). In all the tests, constant strain rates of 0.1–10 s\(^{-1}\) were employed. Quenching was not performed immediately after deformation as the phase change made it difficult to carry out microstructural studies.

Using the offset stress method (for example\(^{29}\)), the fractional softening was evaluated from the rule of mixtures as

$$X = \frac{\sigma_{f1} - \sigma_{02}}{\sigma_{f1} - \sigma_{01}} \tag{3}$$

where the values of the first-hit “offset” yield stress $\sigma_{01}$, the flow stress $\sigma_f$ at the end of the first hit, and the yield stress at the same offset strain in the second hit $\sigma_{02}$ were determined as illustrated in Fig. 1. Due to the ram inertia of the testing machine being used, the applied force and hence the calculated stresses exhibited oscillations that compromised the precise detection of the beginning and end of deformation. The “offset” stresses were therefore defined as corresponding to the ends of the linear portions of the stress-time curves (Fig. 1(a)). This method is equivalent to using offset strains of approximately 0.003–0.006. At the end of deformation, some time was required for the ram to come to a stop. During this time, deformation continued over a strain of approximately 0.005 at decreasing stresses (Fig. 1(b)). The stress at the “end” of deformation was thereby defined as the maximum value on the flow curve before the decrease in ram speed occurred. Because of these ram inertia effects, the shortest unloading time was limited to 0.1 s.

3. Results

The single hit compression behavior of the present C–Mn steel was described earlier\(^{10}\). The continuous flow curves exhibited stress peaks indicative of DRX over the entire range of deformation conditions studied. The peak stresses and strains obeyed the following power laws:

$$\sigma_p = AZ^{0.15}, \quad \sigma_p = BD_iZ^{0.10} \quad \text{and} \quad \sigma_p = 265 \text{kJ/mol.} \tag{4}$$

In this work, the ratio of DRX critical stress $\sigma_c$ to peak stress $\sigma_p$ was found to be $\sigma_c/\sigma_p = 0.77$ and that of the corresponding strains was $\varepsilon_c/\varepsilon_p = 0.5$.

Typical double-hit compression flow curves are illustra-
ed in Fig. 2. Conventional double logarithmic plots 
$\ln [-\ln (1-X)]$ versus logt were constructed (not shown here) to determine $t_{50}$. An Avrami exponent of $n=1.1$ was defined for low preloading strains, which is consistent with the occurrence of SRX. At higher prestrains, $n=1.3$, which can be attributed to softening by MDRX.\textsuperscript{11,12} Rapid softening even at short unloading times prevented evaluation of the contribution of static recovery.

The strain exponent $p$ and strain rate exponent $q$, Eq. (2), were computed from the strain and strain rate dependencies of log $t_{50}$ shown in Fig. 3. At low preloading strains, a strong dependence of $t_{50}$ on strain and a weak dependence on strain rate were detected, while the reverse was observed after preloading to strains close to the stress peaks. The average absolute value of $p$ drops sharply from 3.3 to $\sim 0.4$ as the strain approaches the DRX critical strain $\varepsilon_c$ and then continues to decrease slowly. At the same time, the average absolute value of the strain rate exponent $q$ increases from $\sim 0$ to 0.77 when the prestrain exceeds $\varepsilon_c$ and then remains approximately constant and high.

The transition from the strong strain dependence of $t_{50}$ to the strong strain rate dependence clearly seen in Fig. 3 occurs when the preloading strain is varied in the vicinity of $\varepsilon_c$. The variation in $t_{50}$ within the transition range can be interpreted as resulting from the heterogeneous microstructure\textsuperscript{12} that develops during the initial stages of DRX: due to the limited extent of DRX associated with preloading to strains just above $\varepsilon_c$, two post-deformation softening mechanisms, SRX and MDRX, operate concurrently.

The dependence of $t_{50}$ on temperature is shown in Fig. 4. Two distinct temperature dependencies with different slopes can be seen. At preloading strains below the critical strain for DRX, the slope $\kappa_t = \partial \ln t_{50}/\partial (1/T)|_{\varepsilon, \dot{\varepsilon}}$ is quite steep. Since this slope is equal to

$$\kappa_t = -q \frac{Q_{\text{SRX}}}{R} + \frac{Q_s}{R}$$

the activation energy for softening $Q_s$ is related to the slope by

$$Q_s = R \kappa_t + qQ_{\text{SRX}}$$

At the small preloading strains where $q$ is low and the contribution from the second term in Eq. (5) is small, $Q_s$ is fairly high (283 kJ/mol) and close to reported values of the activation energy for SRX of low C–Mn steels.\textsuperscript{7,9,11–14} Once the preloading strain exceeds the DRX critical strain ($\varepsilon_p > 0.5 \varepsilon_c$) and approaches the peak strain, the log $t_{50}$–1/T plot levels off ($\kappa_t \rightarrow 0$) and attains a sort of steady state. This corresponds to softening by MDRX, as can be expected from the occurrence of DRX during preloading. The activation energy $Q_s$ obtained from Eq. (5) under these conditions is somewhat lower and equal to 210 kJ/mol. This observation is again consistent with literature data regarding the change in softening mechanism from SRX to MDRX as the
preloading strain is increased. As follows from Eq. (5), the activation energy for softening by MDRX is determined by the activation energy for deformation since, at $k_t=0$, $Q_s=qQ_{\text{def}}$.

4. Discussion

4.1. Effect of Preloading Strain Hardening Rate on the Time for 50\% Softening

The present results indicate that the values of $p$, $q$ and $Q_s$ depend strongly on the preloading conditions and especially on the strain. (This is consistent with numerous data reported in the literature.) These dependencies call for specifying the ranges of preloading strain, strain rate and temperature over which the coefficients must be evaluated, including the range over which the transition from SRX to MDRX takes place. The variations in the dependencies lead to major difficulties in extrapolating laboratory results to industrial processing, as the latter is usually carried out under conditions that differ markedly from those in the simulations.

In the present analysis, this problem is treated by rewriting Eq. (2) as follows:

$$\tau_{50} = \tau_{50}' \exp \left( -\frac{Q_s}{RT} \right) = CD_0 e^{-kZ^{-q}}$$

The Arrhenius term on the left hand side pertains to the kinetics of post-deformation softening and is associated with a thermally activated rate controlling mechanism. This term is analogous to the “temperature-compensated time” proposed by Sellars and Whiteman. The right hand side is related only to the parameters of the prestrain. Since the experiments in this work were designed so as to keep $D_0$ constant, incorporation of the grain size into the prestrain term does not alter the final conclusions at this point.

As follows from Eqs. (4) and (6), when the preloading strain is equal to the peak strain, the temperature-compensated time $\tau_{50}$ is solely a function of $Z$, i.e., the strain dependence of $\tau_{50}$ vanishes when $\varepsilon_i=\varepsilon_p$. The same applies when preloading is interrupted at the beginning of steady state flow. Consequently, it is possible to introduce the temperature-compensated time for softening at the stress peak (or similarly, at the beginning of the steady state) as

$$\tau_{50} = \tau_{50}' \exp \left( -\frac{Q_0}{RT} \right) = CD_0 e^{-kZ^{-q}}$$

where $\tau_{50}', Q_0, q_0$ can, in general, differ from the respective quantities that pertain to other strains. Since the Zener–Hollomon parameter $Z$ is uniquely related to the peak stress and to the peak strain, the time for 50\% softening $\tau_{50}'$ and the temperature-compensated time $\tau_{50}$ must depend solely on the peak stress.

This is confirmed in Fig. 5, where values of $\tau_{50}$ are plotted against the preloading stress $\sigma_i$. All the values of $\tau_{50}$ that correspond to preloading strains close to stress peaks fall on the same straight line (note the logarithmic scale in Fig. 5). Using Eq. (4), this line can be described as

$$\ln \tau_{50}' = K - P \ln \sigma_i = K - P \ln A - mP \ln Z$$

where $K$ and $P$ are constants. From Fig. 5, for the steel studied here, $K=7.2$, $P=5.13$, and $q_0=mP=0.77$, which is the value determined above for softening at high prestrains. From Eq. (7), it also follows that $K=\ln CD_0^b$.

Figure 5 reveals that Eq. (6) (and hence Eq. (2)) calls for the strain dependence of $\tau_{50}$ to vanish when the strain hardening rate $\dot{\varepsilon}=(\partial \sigma/\partial \varepsilon)\dot{\varepsilon}_{\text{ref},D_0}$ drops to zero during preloading. This generalization remains valid, regardless of the operating softening mechanism, and is therefore independent of whether DRX has taken place or not. It can further be assumed that $\tau_{50}$ (and $\tau_{50}'$) should depend, not only on $T$, $\varepsilon$ and $\dot{\varepsilon}$, but should also be related to the strain hardening rate during preloading, since the latter is also dependent on $T$, $\varepsilon$ and $\dot{\varepsilon}$. Indeed, the strain dependence of $\log \tau_{50}$ displays the same trend as the strain dependence of the strain hardening rate $\theta$, as exemplified in Fig. 6. Both $\log \tau_{50}$ and $\theta$ decrease with strain, with the decrease being rapid at small strains but becoming quite slow at higher strains.

These considerations are further illustrated in Fig. 7, in which the dependence of $\log \tau_{50}/\tau_{50}$ on strain hardening rate $\theta_i$ at the end of preloading is displayed. The ratio $\log(\tau_{50}/\tau_{50})=\log(\tau_{50}'-\log \tau_{50})$ appears to be a function of strain hardening rate only, and is independent of $Z$. Moreover, Fig. 7 reveals a linear relationship between $\log \tau_{50}$ and $\theta_i$:

$$\ln \tau_{50} = \ln \tau_{50}' + a\theta_i$$

where $a$ is a constant. From Fig. 7, $a=0.011$.

The combination of Eqs. (8) and (9) leads to

$$\tau_{50} = K - q_0 \ln Z + a\theta$$

or

$$\tau_{50} = Z^{-q_0} \exp (K+a\theta)$$

Equation (10) allows for a radical reduction in the number of tests required to characterize the rate of post-deformation softening, provided the data regarding the deformation behavior are in the form of Eq. (4) and the strain hardening behavior is obtained beforehand by analytical or numerical differentiation of the flow curves. This procedure can be summarized as follows.

First, it is necessary to construct a new version of Fig. 5, that is, to find the $Z$-dependence of $\tau_{50}'$. For this purpose, several tests are carried out by preloading to the stress peak ($\varepsilon_p$) at various values of $Z$; various unloading times are then...
employed to define \( q_0 \). These tests will give the value of \( q_0 \) and hence \( Q_s = q_0 Q_{0s} \), as well as the grain size term \( K \). The temperature-compensated time \( \tau_{50} \) can be computed for any strain once the strain hardening rate during preloading \( \theta_{0s} \) is known and the values of \( \tau_{50} \) and \( a \) have been determined. Then, a few further tests interrupted at strains that differ from \( \varepsilon_p \) are needed to evaluate the constant \( a \) in Eq. (9) using the values of \( \theta \) obtained beforehand from the single-hit flow curves. Since the constant \( a \) is independent of the preloading conditions (Fig. 7), the latter can be selected randomly, with the only requirement being that \( Q_s \) be evaluated at prestrains that differ from \( \varepsilon_p \). To determine the effect of grain size, it is sufficient to produce several plots similar to Fig. 5 using prestraining to the peak strain at different values of \( D_0 \). In this way, the strain dependent term \( Z = e^\gamma \) in Eq. (2) is effectively replaced by the strain hardening term \( \exp(K + a\theta) \) in Eq. (10). The test sequence can be further simplified by employing the considerations presented below in Sec. 4.4.

4.2. “Strain Accumulation” Due to Incomplete Softening

So far, the discussion has been limited to inter-hit softening during double hit deformation. The situation becomes significantly more complicated during multi-hit deformation if softening during the intervals of unloading is incomplete. As can be seen from Fig. 2, in the case of partial softening, the reloading flow curve cannot be fitted onto the preloading curve. The reloading and preloading curves match each other only in the two extreme cases where either no softening occurs at all between hits (\( X = 0 \)) or where softening goes to completion (\( X = 1 \)) without significant change in grain size. For partial softening (0 < \( X < 1 \)), neither the strain nor the strain hardening rate can be taken directly from the initial (preloading) flow curve for use in Eq. (2) or Eq. (9), respectively, to determine the softening kinetics after unloading and subsequent reloading to a strain of \( \varepsilon_{f2} \).

This situation is normally handled in the literature by employing the “accumulated”, or “retained”, strain. The latter is based on the Taylor “isostrain” approach to fractional softening, which assumes that in preloading the strain rate and hence the strain are uniform throughout the material.16 Strain uniformity allows for the determination of fractional softening using the rule of mixtures for stresses, Eq. (3). In the Taylor model, it is further implicit that, during post-deformation softening, the “strain” recovers uniformly over the entire volume, so that a partially softened material retains a certain amount of uniformly distributed strain. Clearly, this “retained strain” is not a strain in the conventional sense (i.e. a measure of local displacements or changes in the workpiece dimensions and shape), but a quantity related to the internal state of the material (stored strain energy). This is because the external dimensions and shape of the workpiece do not change during unloading. For this reason, the retained strain is frequently specified in the literature8, 14, 17–19 as a function of the preloading strain \( \varepsilon_{f1} \) and the fractional softening \( X \)

\[
\varepsilon_e = \lambda (1 - X) \varepsilon_{f1} \quad \text{..........(11)}
\]

The empirical parameter \( \lambda \) is considered to range from 0.5 to 1 and is also a function of steel composition, the preloading conditions and the fractional softening \( X \). To evaluate the softening kinetics during the next unloading, \( \varepsilon_e \) is added to the reloading strain \( \varepsilon_{f2} \) and the result is input into Eq. (2). The kinetics of softening during a given unloading interval therefore depend on the fractional softening during the previous unloading and this effect accumulates in multi-hit deformation.

Estimation of the “retained strain” presents a formidable experimental task because, in addition to the softening kinetics, knowledge of the effect of the processing variables

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**Fig. 6.** Strain dependencies of (a) the strain hardening rate (obtained after smoothing the experimental flow curves) and (b) the temperature-compensated time for softening.

**Fig. 7.** Dependence of the temperature-compensated time for softening on strain hardening rate within the ranges of studied deformation conditions.
If the offset strain of the testing machine is neglected and the reloading has changed upon reloading. In particular, if the ram inerstrain, as long as neither the stress state nor the strain path form strain. The flow stress, given by a mixture rule analogous to Eq. (9) allows for differentiation of the flow stress at any uniform strain. The flow stress, given by a mixture rule analogous to Eq. (3), can also be differentiated at any reloading strain, as long as neither the stress state nor the strain path has changed upon reloading. In particular, if the ram inertia of the testing machine is neglected and the reloading offset strain (i.e. strain at “yielding”) is assumed equal to the preloading strain \( \varepsilon_{\text{f1}} \) (strain at the moment of unloading), then

\[
\frac{\partial}{\partial \varepsilon} \sigma_{02} = \frac{\partial}{\partial \varepsilon} \sigma_{01} X + \frac{\partial}{\partial \varepsilon} \sigma_{11}(1-X) = \theta_{02} = \theta_{01} + \theta_{11}(1-X) \quad \text{..........}(12)
\]

That is, the strain hardening rate at the end of each reloading stage can be described with the rule of mixtures using the partial fractional softening \( X \) attained during the previous unloading. (Note that the value of \( X \) in Eq. (12) is that pertaining to the previous pass and is already known. Here, it is used to predict the softening in the next pass.) Furthermore, within the framework of the Taylor model, the rule of mixtures in differential form, Eq. (12), can be applied to any reloading strain since the fractional softening \( X \) during prior unloading is independent of the strain during subsequent reloading. To compute the required strain hardening rates, the values of \( \theta \) for the “prestrained” and “fully softened” constituents can be taken from single-hit tests (Fig. 8). Neither superposition of the flow curve nor back extrapolation of the reloading curve to zero stress (which is sometimes employed to evaluate \( \varepsilon_{\text{a}} \)) is any longer necessary.

Thus the strain hardening rate at the end of each loading stage expressed in terms of the rule of mixtures can be used to evaluate the softening kinetics after this stage, and so forth (instead of the “accumulated” strain).

The retained strain can still be determined if needed, for instance, for applications other than evaluation of the softening kinetics. This can be exemplified by determining \( \varepsilon_{\text{a}} \) in a double-hit test using the fractional softening \( X \) during unloading. Eqs. (2) and (10) can be solved for \( \theta \), which leads to the following relation:

\[
\theta = \ln(Z^{(q_e-\theta)/p} e^{-p/\kappa_e}) \quad \text{.....................}(13)
\]

If \( \theta = \theta_{01} \) is the reloading “offset” strain hardening rate in the above equation, then \( \varepsilon \) is the desired retained strain. With \( \theta_{01} \) given by the rule of mixtures (Eq. (12)),

\[
\varepsilon_{\text{a}} = Z^{(q_e-\theta)/p} \exp \left[ \frac{a}{p} \left( \theta_{01} + \theta_{11}(1-X) \right) \right] \quad \text{..........}(14)
\]

where \( q \), \( p \), and \( \kappa_e \) pertain to the unloading interval of interest. In Eq. (14), \( \varepsilon_{\text{a}} \) is the (uniformly distributed) strain that corresponds to the amount of stored energy retained after softening to the specific value of \( X \). It is this stored energy that is responsible for the strain hardening rate \( \theta_{02} \) upon reloading. Using Eq. (11), the \( \lambda \) parameter can also be derived:

\[
\lambda = \frac{Z^{(q_e-\theta)/p}}{\varepsilon_{\text{f1}}(1-X)} \exp \left[ \frac{a}{p} \left( \theta_{01} + \theta_{11}(1-X) \right) \right] \quad \text{..........}(12)
\]

To determine \( \varepsilon_{\text{a}} \) and \( \lambda \), however, the strain exponent \( p \) and the strain rate exponent \( q \) are still needed. This step can be based on Eq. (6), from which the incremental change in the temperature-compensated time due to variations in the processing parameters \( (\varepsilon, \dot{\varepsilon}, T) \) can be given as

\[
d \ln T_{\text{def}} = -pd \ln \varepsilon - qd \ln \dot{\varepsilon} - \kappa_e d \left( \frac{1}{T_{\text{def}}} \right) \quad \text{..........}(15)
\]

where all the coefficients are again taken as positive. They are defined by the following partial derivatives:

\[
p = -\frac{\partial \ln T_{\text{def}}}{\partial \ln \varepsilon} \quad ; \quad q = -\frac{\partial \ln T_{\text{def}}}{\partial \ln \dot{\varepsilon}}
\]

\[
\kappa_e = -\frac{\partial \ln T_{\text{def}}}{\partial (1/T_{\text{def}})} \quad \text{.........................}(16)
\]

Using Eq. (8) to define \( \ln T_{\text{def}} \) the coefficient \( p \) (strain sensitivity of \( T_{\text{def}} \)) becomes

\[
p = -\frac{\partial \ln T_{\text{def}}}{\partial \ln \varepsilon} \quad = -a \varepsilon_{\text{f1}} \left[ \frac{\partial \theta_{01}}{\partial \varepsilon} \right] \quad \text{..........}(17)
\]

because \( \tau_{\text{def}} \) does not depend on the strain. It should be noted that \( p=0 \) only at steady state, where the derivative

\[\lambda \geq 1/(1-X)\]

*It should be noted that the view that \( l \) falls between 0.5 and 1.7,19) is an oversimplification. For example, when there is partial softening in a double-hit test, the retained strain cannot exceed the strain of the first hit, that is the product \( \lambda(1-X) \) in Eq. (11) cannot exceed 1. This leads to the following upper bound for \( l \):

\[l \leq 1/(1-X)\]

Thus the \( l \) parameter can very well be greater than unity for any \( X>0!\)
\( \partial \theta / \partial \varepsilon = 0 \). At the stress peak, \( p \neq 0 \) since \( \partial \theta / \partial \varepsilon \neq 0 \), as can be seen from Fig. 6.

The coefficient \( q \) (strain rate sensitivity of \( \tau_{50} \)) is

\[
q = - \frac{\partial \ln \tau_{50}}{\partial \ln \dot{\varepsilon}} \bigg|_{\tau_{50}, \dot{\varepsilon}} = - \frac{\partial \ln \tau_{50}}{\partial \ln \dot{\varepsilon}} - a \frac{\partial \theta}{\partial \ln \dot{\varepsilon}}
\]

\[
= q_0 - a \frac{\partial \ln \tau_{50}}{\partial \ln \dot{\varepsilon}} \left( \frac{\partial \sigma_{\text{eff}}}{\partial \varepsilon} \right) = q_0 - a \frac{\partial}{\partial \varepsilon} \frac{\sigma}{\partial \ln \dot{\varepsilon}}
\]

so that

\[
q = - \frac{\partial \ln \tau_{50}}{\partial \ln \dot{\varepsilon}} \bigg|_{\tau_{50}, \dot{\varepsilon}} = q_0 - a \frac{\partial \ln \tau_{50}}{\partial \ln \dot{\varepsilon}} \left( \frac{\partial \sigma_{\text{eff}}}{\partial \varepsilon} \right) = q_0 - a \frac{\partial}{\partial \varepsilon} \frac{\sigma}{\partial \ln \dot{\varepsilon}}
\]

where \( m \) (conventional strain rate sensitivity of the flow stress) is assumed to be strain independent. Finally, the softening kinetics (Eq. (9)), the extrapolation to higher stresses. These stresses, as well as the corresponding strains for a given initial grain size \( D_0 \), are known to be solely functions of \( Z \), \( \sigma_p = \phi(Z) \) and \( \varepsilon_p = \psi(Z) \). For the steel studied here, these functions are given in Eq. (4). When the temperature sensitivity of \( \tau_{50} \) is straightforward:

\[
\kappa_T = - \frac{\partial \ln \tau_{50}}{\partial (1/T)} = - q \frac{Q_{\text{def}}}{R} \bigg|_{\tau_{50}, \dot{\varepsilon}} \bigg|_{\tau_{50}, \dot{\varepsilon}} = - q_0 \frac{Q_{\text{def}}}{R} + a \frac{\partial \ln \tau_{50}}{\partial \ln \dot{\varepsilon}} \left( \frac{\partial \sigma_{\text{eff}}}{\partial \varepsilon} \right) = q_0 - a \frac{\partial}{\partial \varepsilon} \frac{\sigma}{\partial \ln \dot{\varepsilon}}
\]

Thus, with \( q_0 \) and \( a \) known (for example, from a few interrupted tests as described above), all the coefficients can be derived directly from the extrapolation flow curve differentiated to provide the strain hardening rate \( \theta_{1f} \) at any arbitrary preloading strain.

### 4.4. Extrapolation to High Strain Rates

The extrapolation of data from low strain rate laboratory simulations to high industrial strain rates is usually based on Eq. (2) under the assumption that \( p, q \) and \( Q_p \) do not depend on strain rate. This assumption can be valid for \( Q_p \), which is indicative of the predominant softening mechanism. The above results, however, indicate that the parameters \( p \) and \( q \) are sensitive to \( \dot{\varepsilon} \) through the strain rate dependence of the strain hardening rate, which increases with \( \dot{\varepsilon} \) for any given strain and temperature. Moreover, a strain that corresponds to MDRX softening at low \( \dot{\varepsilon} \) can be insufficient to induce DRX at a higher \( \dot{\varepsilon} \) so that softening in this case occurs by SRX instead. For these reasons, extrapolations performed by simply increasing \( \dot{\varepsilon} \) (or \( Z \)) in Eq. (2) are likely to lead to erroneous results.

By accounting for the effect of strain hardening rate on the softening kinetics (Eq. (9)), the extrapolation to higher \( \dot{\varepsilon} \)'s becomes significantly easier and more accurate. For this, it is sufficient to know the strain rate dependence of the strain hardening rate. The latter can be readily obtained using the method of normalized flow curves.\(^{10}\) The essence of this technique is that plastic deformation is viewed as the evolution of a material towards a stationary, strain independent steady state that corresponds to zero strain hardening rate. This applies to both the peak \( \sigma_p \) and steady state \( \sigma_s \) stresses. These stresses, as well as the corresponding stresses for a given initial grain size \( D_0 \), are known to be solely functions of \( Z \), \( \sigma_s = \phi(Z) \) and \( \varepsilon_s = \psi(Z) \). For the steel studied here, these functions are given in Eq. (4). When the flow stress is represented as a function of the fraction of the total strain \( \varepsilon_p \) required to attain the stationary state, the flow curve can be described by a constitutive equation of the type

\[
\sigma(Z, w) = \phi(Z) f(w)
\]

where the values of the “normalized” plastic strain, \( w = \dot{\varepsilon} \varepsilon_p \), run from 0 to 1 and are independent of \( Z \). In a similar way, the flow stress at any given normalized strain \( w \) can be viewed as the degree to which the material has approached its stationary state in the course of deformation:

\[
u = \frac{\sigma(Z, w)}{\sigma_p} = \frac{\sigma(Z, w)}{\phi(Z)} = f(w)
\]

The “normalized” flow stress \( \nu = \sigma/\sigma_p \) is independent of \( Z \) and also takes values from 0 to 1. Thus, the evolution of the material towards its strain independent stationary state \( \phi(Z) \) proceeds during deformation along the same path within the range of \( Z \) for which the hardening law does not change. The \( \nu - w \) plots obtained from the initial stress–strain curves for different \( Z \) values fall on a single curve (Fig. 9), which has been shown to be valid for the present steel under the conditions studied here.\(^{10}\)

Once an evolution function has been established, at least empirically in the form of a \( \nu - w \) plot, the flow stress at any strain, strain rate and temperature can be calculated from the single curve. For this purpose, the functions \( \sigma_p = \phi(Z) \) and \( \varepsilon_s = \psi(Z) \) must be known from experiment or be able to be computed using Eq. (4). Furthermore, the \( \nu - w \) plot can be differentiated analytically or numerically to obtain the evolution rate \( \theta \) (normalized strain hardening rate, Fig. 10)

\[
\theta = \frac{\partial \mu}{\partial w} = \frac{\partial \sigma_p}{\partial w} \theta \quad \text{(20)}
\]

As can be seen from Fig. 10, the normalized strain hardening rate \( \theta \) does not depend on \( Z \); all the \( \theta \) curves fall on a single curve for the various deformation conditions. From Eq. (20), the true strain hardening rate \( \theta \) can be converted into its normalized value at any given strain using Eq. (4)

\[
\Theta = \frac{BD_p Z^k}{AZ^w \theta} = A' Z^{-n} \theta \quad \text{(21)}
\]

where for the present steel \( A' = BD_p/4 = 9.4 \times 10^{-2} \) (cf. Eq. (4)). Conversely, for any normalized strain \( w \), the true strain

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**Fig. 9.** Normalized flow curves for the present steel.
strain hardening rate can be computed from the normalized value
\[ \theta = (A')^{-1} \theta Z^{w-\xi} \] ...........................(22)

The normalized \( \theta - w \) plot can be obtained from single-hit mechanical tests. Extrapolation of the temperature-compensated softening time is then straightforward. For that, it is necessary to determine the normalized strain \( w = \varepsilon_{w}/\varepsilon_q \) corresponding to the prestrain of interest \( \varepsilon_{ei} \) at the strain rate of extrapolation. From the experimental \( \theta - w \) plot, the value of \( \theta \) is obtained and then converted into the true strain hardening rate \( \theta \) at the strain rate of interest using Eq. (22). This result can be further employed in Eq. (8) to evaluate the softening kinetics, the coefficients \( p, q, \) and \( \kappa_z \) by means of Eqs. (17)–(19), and the retained strain from Eq. (14), if necessary.

As shown earlier, the initiation of DRX corresponds to a single point \( \theta _i \) in the \( \theta - w \) plot, Fig. 10, which is independent of \( Z \). During the extrapolation outlined above, the onset of DRX can be detected for multi-hit deformation and for partial softening between hits without calculation of the retained strain. This can simply be done by converting the strain hardening rate \( \theta \) into the normalized strain hardening rate \( \theta \) and then comparing the latter with the critical value \( \theta _c \). For the steel studied here, \( \theta _c \approx 0.8 \) (Fig. 10).

It is often claimed that Eq. (2) is convenient for modeling the softening that takes place between hot working operations because it utilizes simple processing variables—the strain and strain rate. By contrast, the strain hardening rate cannot be regarded as a processing variable and therefore modeling using Eq. (9) may seem less convenient. However, the considerations presented here show that modeling based on the strain hardening rate requires much less information than would otherwise be needed if the strain is employed instead. This is because much of the data required for Eq. (9) can be obtained from simple single-hit mechanical tests and many fewer interrupted simulations are necessary as opposed to those required to evaluate all the unknowns in Eq. (2). According to this method, the retained strain of Eq. (2) is effectively replaced by the value of the strain hardening rate prior to unloading (Eqs. (8) and (10)) and this itself can be readily corrected for the high strain rates applicable to mill processing using Eq. (22).

The approach outlined above pertains to the hot deformation and softening of a single phase solid solution. The strain hardening behavior during reloading can become more complicated if deformation and softening are interacting with phase transformations, or with the precipitation of carbonitrides, such as occurs for example in microalloyed steels. In such cases, the normalized reloading flow curve and hence the normalized strain hardening rate will deviate from that for single phase austenite. Furthermore, non-monotonic softening can even take place, making analysis significantly more difficult.

5. Conclusions

Using a plain low C–Mn steel, the kinetics of softening after hot deformation have been shown to depend on the strain hardening rate at the end of preloading. The analysis presented here indicates that the effects of the preloading conditions (strain, strain rate and temperature) on the post-deformation softening kinetics can be described in terms of their effects on the strain hardening rate prior to unloading. This allows for: 1) a radical reduction in the number of experiments required to generate sufficient data for modeling the post-deformation softening behavior; and 2) significant simplification of the extrapolation of laboratory results to industrial high strain rate processing. The strain accumulation due to incomplete post-deformation softening can also be described in terms of the strain hardening rate within the framework of the Taylor (isostRAIN) model by utilizing the dependence of the softening kinetics on strain hardening rate.

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