Geometric Actions for D-Branes and M-Branes

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March 28, 2022

Abstract

New forms of Born-Infeld, D-brane and M theory five-brane actions are found which are quadratic in the abelian field strength. The gauge fields couple both to a background or induced metric and a new auxiliary metric, whose elimination reproduces the non-polynomial Born-Infeld action. This is similar to the introduction of an auxiliary metric to simplify the Nambu-Goto string action. This simplifies the quantisation and dualisation of the gauge fields.
1 Introduction

In string theory, it has proved fruitful to replace the Nambu-Goto action which gives the area of the string worldsheet with a classically equivalent action involving a worldsheet metric and a local conformal symmetry \([1, 2, 3]\). The Nambu-Goto action is non-polynomial in the string coordinates, whereas the equivalent action is quadratic in the derivatives of the coordinates, greatly simplifying the analysis and allowing a covariant quantisation \([4]\). This has a generalisation for the Nambu-Goto action for \(p\)-branes (proportional to the world-volume), but the resulting theory is only conformally invariant for the string case, \(p = 1\). The purpose of this paper is to propose and investigate an action that may play a similar role for the Born-Infeld theory of electromagnetism, and its D-brane generalisations. The Born-Infeld action is non-polynomial in the field strength \(F_{\mu\nu}\), but introducing a new intrinsic auxiliary metric gives a classically equivalent action which is quadratic in \(F_{\mu\nu}\), and which has a classical conformal symmetry in four dimensions, instead of the two dimensions for the \(p\)-brane world-volume. There are similar actions for the generalisations of Born-Infeld theory governing the effective worldvolume theories of D-branes \([5, 6, 7, 8, 9, 10, 11, 12, 13]\) and M-branes \([16, 17, 18, 19]\). As the new actions are quadratic in \(F_{\mu\nu}\), integration over the gauge fields is straightforward and, just as in string theory, the focus turns to the integration over metrics. The new action can be used to dualise the Born-Infeld gauge field in all dimensions, circumventing the problems arising in other approaches. In particular, it promises to be more convenient than the action presented in ref. \([20, 21]\) which used an auxiliary tensor field consisting of a metric together with an antisymmetric part.

2 Actions

We begin with the Born-Infeld action in \(p + 1\) dimensions \([22]\)

\[
S = -T_p \int d^{p+1}\sigma \sqrt{-\det(g_{\mu\nu} + F_{\mu\nu})}
\]

where

\[
F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu
\]

is the field strength of a \(U(1)\) gauge field \(A_\mu\), \(\mu, \nu = 0, \ldots, p\) are space-time indices and \(g_{\mu\nu}\) is the space-time metric. We now show that the action \([22]\) can be rewritten in a form which is quadratic in the field strength \(F\), and is therefore simpler to analyse and quantise. The key is to use the fact that

\[
\det(g_{\mu\nu} + F_{\mu\nu}) = \det(g_{\mu\nu} - F_{\mu\nu})
\]

to write the integrand in \([22]\) in the form

\[
\left[\det(g_{\mu\nu} + F_{\mu\nu})\right]^\frac{1}{2} = \left[\det(g_{\mu\nu} + F_{\mu\nu})\right]^\frac{1}{2} \left[\det(g_{\mu\nu} - F_{\mu\nu})\right]^\frac{1}{2} = \left(-g\right)^\frac{1}{2} \left\{-\det \left( (g_{\mu\nu} + F_{\mu\nu}) g^{\rho\sigma} (g_{\rho\sigma} - F_{\rho\sigma}) \right) \right\}^\frac{1}{2} = \left(-g\right)^\frac{1}{2} \left\{-\det(g_{\mu\sigma} - g^{\nu\rho} F_{\mu\nu} F_{\rho\sigma})\right\}^\frac{1}{2},
\]

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where $g \equiv \det(g_{\mu \nu})$. The action \((\mathbb{I})\) can thus be rewritten as

$$S' = -T_p \int d^{p+1} \sigma (-g)^{\frac{1}{4}} (-G)^{\frac{1}{4}},$$

where

$$G_{\mu \nu} = g_{\mu \nu} - g^{\rho \sigma} F_{\mu \rho} F_{\sigma \nu}$$

\text{and} $G \equiv \det[G_{\mu \nu}]$. Introducing an intrinsic metric $\gamma_{\mu \nu}$ allows us to rewrite \((\mathbb{I})\) in the following classically equivalent form which is quadratic in the gauge field strength $F_{\mu \nu}$

$$S' = -T'_p \int d^{p+1} \sigma (-g)^{\frac{1}{4}} (-\gamma)^{\frac{1}{4}} [\gamma^{\mu \nu} G_{\mu \nu} - (p - 3)\Lambda]$$

\text{and} substituting back into \((\mathbb{I})\) yields the action \((\mathbb{I})\), which is identical to the Born-Infeld action \((\mathbb{I})\). The constants $T_p, T'_p$ are related by

$$T'_p = \frac{1}{4} \Lambda^{\frac{p}{p-3}} T_p.$$  

For $p = 3$, the four-dimensional action \((\mathbb{I})\) is invariant under the Weyl transformation

$$\gamma_{\mu \nu} \to \omega(\sigma) \gamma_{\mu \nu}$$

and the $\gamma_{\mu \nu}$ field equation implies

$$\gamma_{\mu \nu} = \frac{1}{\Lambda} (g_{\mu \nu} - g^{\rho \sigma} F_{\mu \rho} F_{\sigma \nu})$$

\text{and} substituting back into \((\mathbb{I})\) yields \((\mathbb{I})\), which is identical to the Born-Infeld action \((\mathbb{I})\). The constants $T_p, T'_p$ are related by

$$T'_p = \frac{1}{4} \Lambda^{\frac{p}{p-3}} T_p.$$  

This can be generalised to the D-brane kinetic term

$$S = -T_p \int d^{p+1} \sigma e^{-\phi} \sqrt{-\det(g_{\mu \nu} + F_{\mu \nu})}$$

where

$$F_{\mu \nu} \equiv F_{\mu \nu} - B_{\mu \nu},$$

$\phi$, $g_{\mu \nu}$ and $B_{\mu \nu}$ are the pullbacks to the worldvolume of the background dilaton, metric and NS antisymmetric two-form fields and $F = dA$, with $A$ the $U(1)$ world-volume gauge field. This action gives the effective dynamics of the zero-modes of
the open strings with ends tethered on a D-brane when $F$ is slowly varying, so that corrections involving $\nabla F$ can be ignored, and has therefore played a central role in recent studies of D-brane dynamics and string theory duality [24]. However, the non-linearity of (13) makes it rather difficult to study. In particular, the action (13) is inconvenient for the purpose of quantisation, and its dualisation has proved rather difficult [8, 10, 25, 26, 27]. It is therefore useful to know classically equivalent, alternative forms of this action which have a more tractable dependence on the spacetime coordinates $X$ or on the field strength $F$.

As before, introducing an intrinsic metric $\gamma_{\mu\nu}$ allows us to rewrite (13) in the classically equivalent form

$$S' = -T_p \int d^{p+1} \sigma e^{-\phi} (-g)^{\frac{p}{2}} [\gamma_{\mu\nu} G_{\mu\nu} - (p - 3) \Lambda]$$

and we find

$$T_{\mu\nu} = (-g)^{\frac{p}{2}} \left\{ -T_p \int \frac{1}{T_p} \frac{1}{(-g)^{\frac{p}{2}}} \frac{\delta S}{\delta \gamma_{\mu\nu}} \right\}$$

where $T_p, T'_p$ are related as in eq. (4).

The energy-momentum tensor $T_{\mu\nu}$ can be defined from the form (15) of the D-brane kinetic term by

$$T_{\mu\nu} = (-g)^{\frac{p}{2}} \left\{ -T_p \int \frac{1}{T_p} \frac{1}{(-g)^{\frac{p}{2}}} \frac{\delta S}{\delta \gamma_{\mu\nu}} \right\}$$

and we find

$$T_{\mu\nu} = (-g)^{\frac{p}{2}} \left\{ -\frac{1}{4} \gamma_{\mu\nu} \left[ g_{\rho\sigma} (g^{\kappa\tau} F_{\rho\kappa} F_{\tau\sigma}) - (p - 3) \Lambda \right]ight.$$

This is traceless (i.e. $\gamma^{\mu\nu} T_{\mu\nu} = 0$) if $p = 3$ as a result of the Weyl invariance (10), and the equation $T_{\mu\nu} = 0$ implies the field equation of the metric (8) or (11).

The low-energy effective action for an open type I string includes the terms given by (13) with $p = 9$, but with $g_{\mu\nu}, B_{\mu\nu}$ the space-time metric and anti-symmetric tensor gauge field (rather than their pull-backs) [26], and can be rewritten in the equivalent form (13) with $p = 9$. The dimensional reduction of the type I string action (13) to $p + 1$ dimensions gives the action for a D-p-brane in static gauge (and with vanishing RR gauge fields), with the 9+1 vector field $A$ giving rise to a vector and $9 - p$ scalars $X_i$ on reduction. The reduction of the form (13) of the action then gives a useful form of the static-gauge D-p-brane action which is quadratic in $A, X$.

We now turn to the reduction of (13) from 9+1 to $p+1$ dimensions. We use the notation that hatted quantities are ten-dimensional, so $\hat{\mu} = 0, \ldots, 9$, while $\mu = 0, \ldots, p$ and $i = p + 1, \ldots, 9$. Then the vector field $A_{\hat{\mu}} = (A_{\mu}, X_i)$ gives a vector and $9 - p$ scalars $X_i$. We choose (for simplicity) a flat space-time metric $\hat{g}_{\hat{\mu}\hat{\nu}} = \eta_{\hat{\mu}\hat{\nu}}$ and vanishing 2-form $\hat{B}_{\hat{\mu}\hat{\nu}}$, and make the following Ansatz for the metric $\hat{\gamma}_{\hat{\mu}\hat{\nu}}$:

$$\hat{\gamma}_{\hat{\mu}\hat{\nu}} = \begin{pmatrix} \gamma_{\mu\nu} + C_i^\mu \gamma_{ij} & C_i^\mu \gamma_{ij} \\ C_i^\nu \gamma_{kj} & \gamma_{ij} \end{pmatrix}$$

(18)
Then the metric $\hat{\gamma}_{\hat{\mu}\hat{\nu}}$ gives, as usual, a $p + 1$-dimensional metric $\gamma_{\mu\nu}$, 9 - $p$ vector fields $C^i_\mu$ and $(9 - p) (10 - p)/2$ scalar fields taking values in the coset $GL(9 - p, \mathbb{R})/SO(9 - p)$. The inverse of (18) is

$$\hat{\gamma}^{\hat{\mu}\hat{\nu}} = \left( \begin{array}{cc} \gamma^{\mu\nu} & -C^{\mu i} \\ -C^{\nu i} & \gamma_{ij} + C^i_\rho \gamma^{\rho\sigma} C^j_\sigma \end{array} \right)$$

(19)

and its determinant is

$$\det \hat{\gamma}_{\hat{\mu}\hat{\nu}} = \det \gamma_{\mu\nu} \det \gamma_{ij}.$$

(20)

Setting $F_{ij} \equiv 0$ and $F_{\mu i} \equiv \partial_\mu X_i$, this gives the following static gauge D-$p$-brane action which is quadratic in both $F$ and $\partial X^i$:

$$S' = -T_p' \int d^{p+1}\sigma e^{-\phi} \left[ -\det \gamma_{\mu\nu} \det \gamma_{ij} \frac{\gamma^{\mu\nu}(\eta_{\mu\nu} + \eta^{ij} \partial_\mu X_i \partial_\nu X_j + \eta^{\rho\sigma} F_{\mu\rho} F_{\nu\sigma})}{\sqrt{-g(\partial a)^2}} \right. \nonumber$$

$$+ 2C^{\mu i} F_{\mu\rho} \partial_\sigma X_i \eta^{\rho\sigma} + \left( \eta^{ij} + \eta^{\rho\sigma} \gamma^{\rho\tau} C^j_\tau \right) (\eta_{ij} + \eta^{\rho\sigma} \partial_\rho X_i \partial_\sigma X_j) \nonumber$$

$$- (p - 3)\Lambda \bigg].$$

(21)

This quadratic action should be a convenient starting point for the study of D-$p$-brane dynamics, taking into account the Born-Infeld corrections.

The methods above can also be applied to the M-theory five-brane action [16, 17]. In the PST formulation, the kinetic part of the action is

$$S = -T_5 \int d^6\sigma \sqrt{-\det \left( g_{\mu\nu} + i \frac{g_{\mu\rho} g_{\nu\lambda}}{\sqrt{-g(\partial a)^2}} \tilde{K}^{\rho\lambda} \right)} \nonumber$$

$$- T_5 \frac{1}{4} \int d^6\sigma \frac{1}{(\partial a)^2} \tilde{K}^{\mu\nu} H_{\mu\nu\rho} g^{\rho\lambda} \partial_\lambda a$$

(22)

where

$$\tilde{K}^{\mu\nu} \equiv \frac{1}{6} \epsilon^{\mu\nu\rho\lambda\sigma} H_{\rho\lambda\sigma} \partial_\tau a$$

(23)

with $H_{\mu\nu\rho} = 3 \partial_{[\mu} b_{\nu\rho]}$ the field strength of the self-dual two-form tensor gauge field $b_{\mu\nu}$ propagating on the world-volume, $a$ denotes the PST scalar [16] and

$$(\partial a)^2 \equiv g^{\mu\nu} \partial_\mu a \partial_\nu a.$$ (24)

Introducing an intrinsic metric $\gamma_{\mu\nu}$ as before, the action (22) can be rewritten in the classically equivalent form

$$S = -T_5' \int d^6\sigma (-g)^{\frac{3}{2}} (-\gamma)^{\frac{3}{4}} \left[ \gamma^{\mu\nu} \left( g_{\mu\nu} - \frac{g_{\mu\sigma} g_{\nu\lambda} \tilde{K}^{\rho\lambda} \tilde{K}^{\tau\sigma}}{g(\partial a)^2} \right) - 2\Lambda \right] \nonumber$$

$$- T_5 \frac{1}{4} \int d^6\sigma \frac{1}{(\partial a)^2} \tilde{K}^{\mu\nu} H_{\mu\nu\rho} g^{\rho\lambda} \partial_\lambda a$$

(25)

This is quadratic in the field strength $H$ and thus is more convenient for gauge field quantisation in the background $g_{\mu\nu}$ than (22).
3 Dual Actions

The dualisation of the form (2) of the Born-Infeld action can be achieved via the addition of a Lagrange multiplier term imposing eq. (2). Consider the action

\[ S = -T_p \int d^{p+1} \sigma \left\{ \left( -g \right)^{\frac{1}{4}} \left( -\gamma \right)^{\frac{1}{4}} \left[ \gamma^{\mu \nu} (g_{\mu \nu} - g^{\rho \sigma} F_{\mu \rho} F_{\sigma \nu}) - (p - 3) \Lambda \right] 
+ 2 \tilde{H}^{\mu \nu} \left( F_{\mu \nu} - \partial_{[\mu} A_{\nu]} \right) \right\}, \tag{26} \]

where \( \tilde{H}^{\mu \nu} \) is a tensor density and \( F \) is regarded as an independent field. Integrating out \( \tilde{H}^{\mu \nu} \) sets \( F = dA \) and yields the original action (7). Alternatively, integrating out \( A_\mu \) imposes the constraint

\[ \partial_\mu \tilde{H}^{\mu \nu} = 0 \tag{27} \]

which can be solved in terms of a \((p - 2)\)-form \( \tilde{A} \),

\[ \tilde{H}^{\mu \nu} = \frac{1}{(p - 1)!} \varepsilon^{\mu \nu \rho_1 \cdots \rho_{p-2}} \partial_{[\rho_1} \tilde{A}_{\rho_2} \cdots \rho_{p-2]} \right] \tag{28} \]

where \( \varepsilon^{\mu \nu \rho_1 \cdots} \) is the alternating tensor density. Now \( F \) is an auxiliary two-form occuring quadratically in the action and can be integrated out. The field equation for \( F_{\mu \nu} \) is

\[ \left( -g \right)^{\frac{1}{4}} \left( -\gamma \right)^{\frac{1}{4}} \left[ \gamma^{\mu \rho} g^{\nu \sigma} + \gamma^{\rho \sigma} g^{\mu \nu} \right] F_{\sigma \rho} = 2 \tilde{H}^{\mu \nu} \tag{29} \]

where \( \tilde{H}^{\mu \nu} \) is given by the solution (28), and the Gaussian integration amounts to solving this for \( F_{\mu \nu} \) and substituting the solution \( F[g_{\mu \nu}, \gamma_{\mu \nu}, \tilde{H}^{\mu \nu}] \) in the action (15). This gives the dual action \( S[g_{\mu \nu}, \gamma_{\mu \nu}, \tilde{H}^{\mu \nu}] \). In principle, an equivalent dual action \( S_D[g_{\mu \nu}, \tilde{H}^{\mu \nu}] \) can then be obtained by integrating out the auxiliary metric \( \gamma_{\mu \nu} \), but in practice this procedure is difficult to carry out explicitly because of the non-linearity in the worldvolume metric of eq. (29) and of the action \( S[g_{\mu \nu}, \gamma_{\mu \nu}, \tilde{H}^{\mu \nu}] \).

Defining the matrices

\[ f_{\mu}^{\space \nu} = F_{\mu \rho} g^{\rho \nu}, \quad h_{\mu}^{\space \nu} = \left( -g \right)^{-\frac{1}{4}} \left( -\gamma \right)^{-\frac{1}{4}} g_{\mu \rho} \tilde{H}^{\rho \nu}, \quad \beta_{\mu}^{\space \nu} = 2 \left( g_{\mu \rho} \gamma^{\rho \nu} - \delta_{\mu}^{\nu} \right), \tag{30} \]

the equation (29) can be written as

\[ h = f + \{ \beta, f \} \equiv (1 + L_\beta) f \tag{31} \]

where for any matrices \( X, Y \), the operator \( L_X \) is defined by

\[ L_X Y \equiv \{ X, Y \}. \tag{32} \]

Then (31) can be inverted to give

\[ f = (1 + L_\beta)^{-1} h = \left( 1 - L_\beta + L_\beta^2 - L_\beta^3 + \ldots \right) h \]
\[ = h - \{ \beta, h \} + \{ \beta, \{ \beta, h \} \} - \{ \beta, \{ \beta, \{ \beta, h \} \} \} + \ldots. \tag{33} \]
where the equation (39) can be written as

\[
S = -T_p \int d^{p+1} \sigma \left\{ (-g)^{\frac{1}{2}} (-\gamma)^{\frac{1}{2}} [\gamma_{\mu\nu} g_{\mu\nu} - (p - 3) \Lambda] \\
+ 2(-g)^{\frac{1}{2}} (-\gamma)^{\frac{1}{2}} \text{tr} \left[ h (1 + L_\beta)^{-1} h \right] \right\}
\]

\[
= -T_p \int d^{p+1} \sigma \left\{ (-g)^{\frac{1}{2}} (-\gamma)^{\frac{1}{2}} [\gamma_{\mu\nu} g_{\mu\nu} - (p - 3) \Lambda] \\
+ 2(-g)^{-\frac{1}{2}} (-\gamma)^{-\frac{1}{2}} H^{\mu\sigma} M_{\mu\nu\sigma} \tilde{H}^{\nu\rho} \right\},
\]

(34)

where the tensor \( M_{\mu\nu\rho\sigma} \) is defined by

\[
\text{tr} \left[ h (1 + L_\beta)^{-1} h \right] = h^{\mu\sigma} M_{\mu\nu\sigma\nu} h^{\nu\rho}
\]

(35)

(where \( h^{\mu\sigma} = g^{\mu\tau} h_{\tau\sigma} \)) and is given to lowest orders by

\[
M_{\mu\nu\rho\sigma} = \gamma_{\mu\nu} g_{\rho\sigma} \left[ \delta^\delta_\rho \delta^\epsilon_\sigma - \Sigma^\rho\sigma \Sigma^\epsilon\delta \delta^\epsilon_\sigma - \Sigma^\rho\sigma \Sigma^\epsilon\delta \Sigma^\epsilon\delta \delta^\epsilon_\sigma - \Sigma^\rho\sigma \Sigma^\epsilon\delta \Sigma^\epsilon\delta \Sigma^\epsilon\delta \delta^\epsilon_\sigma - \ldots \right],
\]

(36)

where

\[
\Sigma^\mu_\nu \equiv g_{\nu\rho} \tilde{f}^{\rho\mu}
\]

(37)

and \( \Sigma^\mu_\nu \) denotes the inverse of the matrix \( \Sigma^\mu_\rho \). The auxiliary metric \( \gamma_{\mu\nu} \) occurs algebraically and can in principle be eliminated using its equation of motion, giving \( \gamma_{\mu\nu} \) as a function of \( g_{\mu\nu} \) and \( \tilde{H}^{\mu\nu} \). Although this is hard to do explicitly, it can be done perturbatively, giving \( \gamma_{\mu\nu} \) to any desired order in \( \tilde{H}^{\mu\nu} \).

The dualisation of the action (13), which is classically equivalent to the D-brane kinetic term (13), proceeds in a similar way. Consider the action

\[
S = -T_p \int d^{p+1} \sigma \left\{ e^{-\phi} (-g)^{\frac{1}{2}} (-\gamma)^{\frac{1}{2}} [\gamma_{\mu\nu} (g_{\mu\nu} - g^{\rho\sigma} F_{\mu\rho} F_{\sigma\nu}) - (p - 3) \Lambda] \\
+ 2\tilde{H}^{\mu\nu} \left( F_{\mu\nu} - 2\partial_{[\mu} A_{\nu]} \right) \right\}.
\]

(38)

Integrating out \( \tilde{H}^{\mu\nu} \) yields the original action (15). Alternatively, integrating over \( A_\mu \) imposes the constraint (27), which is solved in terms of a \((p - 2)\) form \( \tilde{A} \) as in (28). Now \( F \) is an auxiliary two-form occuring algebraically. The field equation for \( F_{\mu\nu} \) is

\[
(-g)^{\frac{1}{2}} (-\gamma)^{\frac{1}{2}} (\gamma^{\mu\rho} g^{\nu\sigma} + \gamma^{\nu\sigma} g^{\mu\rho}) (F_{\sigma\rho} - B_{\sigma\rho}) = 2\tilde{H}^{\mu\nu},
\]

(39)

where \( \tilde{H}^{\mu\nu} \) is given by the solution (28).

Defining the matrix

\[
\tilde{f}_\mu^\nu \equiv (F_{\mu\rho} - B_{\mu\rho}) g^{\rho\sigma}
\]

(40)

the equation (39) can be written as

\[
h = \tilde{f} + \{ \beta, \tilde{f} \} = (1 + L_\beta) \tilde{f}
\]

(41)

where the matrices \( h \), \( \beta \) and the operator \( L_\beta \) are defined as in (31) and (32). This can be inverted to give

\[
\tilde{f} = (1 + L_\beta)^{-1} h = h - \{ \beta, h \} + \{ \beta, \{ \beta, h \} \} - \{ \beta, \{ \beta, \{ \beta, h \} \} \} + \ldots
\]

(42)
Substituting this solution for $\mathcal{F}$ back in (26) gives

$$S = -T'_p \int d^{p+1}\sigma \left\{ (-g)^{1/2} (-\gamma)^{1/2} \left[ \gamma^{\mu\nu} g_{\mu\nu} - (p - 3) \Lambda \right] + 2 \bar{H}^{\mu\nu} B_{\mu\nu} ight. \\
+ \left. 2(-g)^{-1/2} (\gamma)^{-1/2} \bar{H}^{\mu\sigma} M_{\mu\rho\sigma\nu} \bar{H}^{\nu\rho} \right\},$$  \hspace{1cm} (43)

with the tensor $M_{\mu\rho\sigma\nu}$ defined as in (36).

4 Conclusion

In this paper, we have presented new forms of Born-Infeld as well as D-brane and M theory five-brane kinetic terms which are quadratic in the abelian gauge field strength. The gauge fields couple both to a background or induced metric $g_{\mu\nu}$ and to a new intrinsic metric $\gamma_{\mu\nu}$, and both of these world-volume metrics appear in the action in a remarkably symmetric way. These actions could play an important role in the quantisation of Born-Infeld theory and of the static gauge effective world-volume theories of D-Branes and M-Branes, similar to the role played in string theory by the actions of ref. [1, 2, 3, 4].

The dualisation of the $U(1)$ gauge fields is achieved by adding a Lagrange term imposing the constraint (2), and the dual action is quadratic in the field strength of the appropriate dual potential. The dual action involves an infinite power series in the auxiliary intrinsic metric, which can be eliminated perturbatively.

The four dimensional action (7) with $p = 3$ has a classical Weyl invariance (10), which is closely related to that of the string. Quantum mechanically, this will be anomalous [29, 30]. We hope to return to a discussion of this anomaly elsewhere, but we note here that it is trivial to generalise our action to that for a theory of $N$ abelian vector fields and $Nd$ scalars $X_i$, and it is intriguing that it may be possible to choose the numbers $N, d$ to take critical values that give a cancellation of the conformal anomaly, generalising the critical dimension of string theory.

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