Optimization of the nonlinear regime of self-compression at femtosecond laser pulses in silica and air

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Abstract. In the present work it is demonstrated two efficient methods of self-compression of femtosecond pulses based on suitable selection of optical elements, parameters of the medium and laser radiation. The basic idea is that the phase modulated pulses more efficiently can be compressed thorough nonlinear mechanisms. The first method can be applied for mediums with significant dispersion like fused silica, where the sign of the dispersion of the group velocity is important. We show that the combination of focusing by optical lens and a balance between anomalous dispersion and nonlinearity lead to significant compression from 100fs to ~20-30fs of optical pulse. The second method for self-compression is by using only one optical diffraction grating to obtain broadband pulses and the following self-compression in nonlinear regime. In the second case in addition is observed generation of X wave.

1. Introduction
One of the main tasks of modern laser physics is to obtain ultra short laser pulses. There are different optical schemes for self-compression based on pair diffraction grating or by other optical elements when the pulses propagate in linear regime [1]. The shortening of pulse can be obtained also by using nonlinear mechanisms such as self-phase modulation, when a pulse reaches critical power for self-focusing or self-modulation. Another modern method for compressing of pulses in time is by using of hollow optical fibers filled with noble gas. During propagation of negative chirped pulses in hollow optical fiber the self-phase modulation leads to compression. These effects have been studied in different fiber types, including single-mode fibers [1, 2], hollow fibers filled with noble gas [3, 4]. Problem of this method is in the damage of the fibers. There is also one limitation of the energy connected with the recompressing of the pulse due to plasma ionization of the gas. The most efficient schemes for compression require careful analyze for collection of possible optical elements, linear and nonlinear effects to obtain maximal compression. This work investigates two different mechanisms for self-compression of femtosecond pulses - in fused silica and in air.

In the first case we compare the self-compression of laser pulses during their nonlinear propagation in the normal and anomalous dispersion region of a glass. The pulses are spatial phase modulated initially by long focused lens. The numerical experiment is performed for 100 fs pulses at wavelength 800 nm and 1400 nm correspondingly.

In the second case we perform comparing of the nonlinear compression of no-chirped initial 25 fs pulse and the chirped by grating ones, during their propagation in air. Our numerical results show, that
after only one pass through optical diffraction grating the chirped pulse self-compress more efficiently in nonlinear regime and in addition obtain X wave deformation.

2. Nonlinear regime of narrow-band pulse in silica
The no-modulated femtosecond pulses admit relatively narrow band spectrums \( \Delta k_z << k_0 \) and their evolution can be described into the frame of nonlinear spatio-temporal nonlinear optics. The main problem in this regime is to use proper combination of optical lens and to find proper balance between dispersion and nonlinearity [5].

2.1 Basic equation
The normalized scalar paraxial (3D+1) amplitude equation describing the propagation of narrow-band optical pulse in nonlinear dispersive medium has the form:

\[
i \frac{\partial A}{\partial z} = \frac{1}{2} \Delta_\perp A - \beta \frac{\partial^2 A}{\partial t^2} + \gamma |A|^2 A
\]

Where \( A(x,y,z,t) \) is the scalar amplitude function, characterizing the pulse envelope. \( \beta = \frac{z_{\text{dif}}}{z_{\text{disp}}} \) is a dimensionless parameter giving the ration between the dispersion and the diffraction length of the pulse, \( z_{\text{disp}} = \frac{t_0^2}{k^*} \) is the dispersion length, \( z_{\text{dif}} = k_0 r_\perp^2 \) - the diffraction length. The nonlinearity of the medium is of Kerr type. It is determined by the expression \( \gamma = k_0^2 r_\perp^2 n_\perp |A_\perp|^2 \), \( k_0 \) - is the wave number, \( \Delta_\perp = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \) is the transverse Laplace operator. This equation counts effects of paraxial diffraction, second order linear dispersion and Kerr type nonlinearity. We use the initial conditions for the numerical experiments Gaussian pulse modulated by lens:

\[
A(x,y,t,0) = \exp \left\{ -\frac{x^2 + y^2 - \delta t^2}{2} \right\} \exp \left\{ jf' \left( x^2 + y^2 \right) \right\}
\]

where \( \delta = \frac{r_\perp^2}{t_0^2} \) is the relation between spatial transverse and longitudinal time durations, and \( f' = \frac{\pi r_\perp^2}{\lambda_0 f} \) is normalized length factor.

2.2 Nonlinear pulse propagation in the positive dispersion region of silica
We investigate numerically the propagation of laser pulse in the positive dispersion regions of fused silica by using the following parameters of the laser source: time duration \( t_0 = 100 \) fs; spot \( r_\perp = 140 \) μm at carrying wave wavelength \( \lambda = 800 \) nm. The dispersion of the group velocity in the positive dispersion region is \( k^* = 3.3 \times 10^{-26} \) s^2/cm and the focus distance of the lens is \( f = 30 \) cm. The parameters of the pulse of the medium are selected to obtain the relation between diffraction and dispersion length to be \( \beta = 0.5 \). The nonlinear coefficient \( \gamma = 2.2 \) is chosen to be slightly above the critical for self-focusing. The numerical result shown in Fig. 1 is obtained by solving equations (1) under initial conditions (2).
Figure 1: Nonlinear propagation of initial modulated by lens (f=30 cm) optical pulse in medium with normal dispersion $\beta = 0.5$ and coefficient of nonlinearity $\gamma = 2.2$. The side (x, t) projection of the intensity profile is plotted. Numerical result is obtained by solving equation (1) under initial condition (2).

The numerical result shown in Fig.1 presents side (x,t) projection of the intensity profile at four distances $z = 0, \ z = z_{\text{diff}}, \ z = 2z_{\text{diff}}, \ \text{and} \ z = 3z_{\text{diff}}$. The evolution of the pulse describes well the observed in the experiments and numerical simulations separation of the temporal profile into two maximums and the significant spectral enlarging.

2.3 Nonlinear pulse propagation in the negative dispersion region of silica

Two main tasks are placed when we investigate the propagation of optical pulse in the negative region of dispersion in fused silica. The first purpose is by one long focusing optical lens in nonlinear regime to minimize the energy in the pedestal of the self-focused pulse. The second purpose is the negative frequency modulation (chirp) of the dispersion to compensate the positive phase modulation of the nonlinearity. In this way significant part of the energy will go in the nonlinear focus, which will lead to natural self-compression and self-focusing of the pulse. On Fig.2 is presented the nonlinear propagation of a pulse, spatially phase modulated by the same lens with $f=30 \, \text{cm}$, and dynamics governed by numerical solution of equation (1) under initial condition (2). The dimensionless parameters are $\beta = -0.5$ and $\gamma = 2.2$. As can be expected from the performed above qualitative analysis of the physical parameters, significant self-focusing without loss of energy in the pedestal is observed. The compression of the pulse is from 100 fs up to $\sim 20-30$ fs.

Figure 2: Propagation of initially modulated by lens (f = 30cm) optical pulse in medium with negative dispersion $\beta = -0.5$ and coefficient of nonlinearity $\gamma = 2.2$. These parameters correspond to propagation of a laser pulse on carrying wavelength $\lambda = 1400 \, \text{nm}$, with time duration $t_0 = 100$ fs and
spot \( r_\perp = 180\mu\text{m} \). The corresponding at \( \lambda=1400\text{nm} \) negative group velocity dispersion is \( k''=-3.3\times10^{-26}\text{s}^2/\text{cm} \).

3. Non-paraxial nonlinear regime of broad–band pulses in air modulated by optical grating

The spatio-temporal equation (1) can’t govern the propagation of broad-band pulses [6]. This is the reason, if we investigate nonlinear regime of propagation of pulses with time duration of order of 20-30 fs, to use non-paraxial envelope equation [6]. The pulses with such duration at short distances enlarge significantly their spectrum by self-phase modulation and reaches spectral widths of the order of \( \Delta k_z \approx k_0 \). Other reason to investigate phase modulation by grating in gases is that the group velocity dispersion in air is very small \( (k''=2.5\times10^{-31}\text{s}^2/\text{cm}) \) and practically we can’t modulate the phase of the pulse by dispersion on few diffraction lengths.

The scalar non-linear non-paraxial envelope equation in Galilean frame is given by:

\[
2i\alpha \delta^2 \frac{\partial A}{\partial t'} = \Delta_z A - \beta \frac{\partial^2 A}{\partial z'^2} + \left( \delta^2 + \beta^2 \right) \left( \Delta - \frac{\partial^2 A}{\partial t' \partial z'} \right) + \gamma \left( A^2 A + A^3 e^{i2\alpha(z-z')} \right),
\]

where \( \beta \) and \( \gamma \) are the same normalized numbers and in addition the following normalized constants appear: \( \alpha = k_0 z_0 \); \( z_0 = v_{gr} t_0 \); \( \delta = r_\perp / z_0 \) and \( \hat{v}_n = (v_{ph} - v_{gr}) / v_{gr} \) is the normalized group-phase velocity delay. The number \( \alpha \) with precise \( 2\pi \) counts the number of cycles under the envelope and in this way we can control the spatial (and temporal) shape of the initial pulse in respect to optical period \( \lambda_0 \). In gases the standard way to obtain large phase modulated spectrum is through a diffraction grating. In our numerical experiment we will modulate the chirp parameter by the complex number \( ia \) in the initial conditions [1]. That why our initial Gaussian pulse takes the form:

\[
A(x, y, z, 0) = A_0 e^{-\frac{x^2+y^2+(z+ia z'_0)^2}{2}}
\]

In this paper we investigate the following two basic cases:

1) White (no-phase modulated) initial Gaussian pulse \( (a = 0) \).
2) Broad-band phase modulated Gaussian pulse with chirp parameter \( a = 8 \).

In the both cases the initial width of the pulse is selected to by 25 femtoseconds.

3.1 Propagation of white (no-phase modulated) initial Gaussian pulse \( (a=0) \).

To investigate the nonlinear propagation of no-modulated 25 fs pulse we solve numerically the envelope equation (4) under the initial condition (5) with chirp parameter \( a = 0 \). As the group velocity dispersion in air is of order of \( k''=2.5\times10^{-31}\text{s}^2/\text{cm} \) the relation between the diffraction and the dispersion lengths is a very small number - \( \beta \approx 10^{-5} \). Thus, practically the spatial evolution of the pulse appears as balance between non-paraxial diffraction and nonlinearity. The nonlinear coefficient \( \gamma =1.8 \) again is chosen to be slightly above the critical for self-focusing. The numerical result shown in Fig. 1 presents side \((x,z)\) projection of the intensity profile at four distances \( z = t = 0 \), \( z = z_{\text{diff}} \), \( z = 2z_{\text{diff}} \), and \( z = 3z_{\text{diff}} \). As it can be expected, the typical separation of the pulse on two maximums with significant enlarging of the spectral width is observed. The main part of the energy is situated in the pedestal and the self-compression is not so effective.
Fig. 3. Propagation of spectrally limited (narrow-band) 25fs pulse (\(a=0\)) governed by Eq. (4). The typical self-compression with two maximums is observed. Side \((x,z)\) projection of the intensity profile at four distances \(z = t = 0; \quad z = z_{\text{diff}}; \quad z = 2z_{\text{diff}}; \quad z = 3z_{\text{diff}}\) is plotted. The main part of the energy is situated in the pedestal and the self-compression is not so effective.

3.2 Propagation of broad-band pulse with chirp parameter \(a=8\);

Fig. 4. Propagation of broad-band 25fs pulse (with chirp parameter \(a=8\)) governed by Eq. (4).

On Fig 4 self-compression with decreasing maximums is clearly seen. In addition, typical X-wave formation in the nonlinear focus is observed. In Galilean frame we have a real deformation of the X-wave, while in earlier works performing in Local time coordinate system the inversed pictures of the deformation are presented.

4. Conclusions
In the present work two efficient methods for self-compression of femtosecond pulses based on suitable selection of optical elements, parameters of the medium and laser radiation are demonstrated. We explore the basic idea, that the phase modulated pulses more efficiently can be compressed through nonlinear mechanisms. The first method is applied for mediums with significant dispersion like fused silica, where the sign of the dispersion of the group velocity is important. We show that the combination of focusing by optical lens for relative reduction of energy in the pedestal and balance between anomalous dispersion and nonlinearity lead to significant compression from 100fs to \(~20-30\)fs of optical pulse in silica. The second method for self-compression is applied to gases, where the dispersion is negligible. In this case we use optical diffraction grating to obtain broadband pulses and
the following self-compression in nonlinear regime. In the second case in addition is observed
generation of X wave.

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