Macroscopic Floquet topological crystalline steel and superconductor pump

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Abstract – The transport of a macroscopic steel sphere and a superconducting sphere on top of two-dimensional periodic magnetic patterns is studied experimentally and compared with the theory and with experiments on topological transport of magnetic colloids. Transport of the steel and superconducting sphere is achieved by moving an external permanent magnet on a closed loop around the two-dimensional crystal. The transport is topological, i.e., the spheres are transported by a primitive unit vector of the lattice when the external magnet loop winds around specific directions. We experimentally determine the set of directions the loops must enclose for nontrivial transport of the spheres into various directions. We show that the loops can be used to sort steel and superconducting spheres. We show that the topological transport is robust with respect to the scale of the system and therefore speculate on its down scalability to the molecular scale.

Introduction. – Topological nontrivial matter is a class of material, where the response of the material to external perturbations only depends on the global properties not on the local properties of the material. Such properties are called topological invariants and they change in a discrete way, i.e., a continuous change of the perturbation results in a discrete response of the material. Topological properties of matter play a fundamental role in electronic transport behavior of quantum solid state matter [1,2], in mesoscopic systems [3–8] and in macroscopic matter [9–12]. One important class of topological material are Floquet topological systems, where the material is subject to a time-periodic external perturbation that causes the pumping of excitations or quasi particles through the material. The topological pump effect [13] is usually protected by certain symmetries of the problem. Such symmetries include the point group symmetries of the lattice. As a consequence of the topologically nontrivial bulk pump process one obtains a bulk edge correspondence [14,15]. When one augments the voltage applied to a bulk insulator, transient topological pumping of bulk electrons from one edge channel to the opposite edge sets in to restore a steady state with asymmetric filling of both edges.

In the current letter we demonstrate such a bulk pumping process with an intriguingly simple setup on a macroscopic scale. We have shown similar pumping with a very different mesoscopic colloidal system, where we also outline the theory of the pumping [8]. The colloidal system requires the use of complex magnetic patterns obtained via lithographic techniques [16]. The macroscopic system presented here in contrast is so simple that it can be easily rebuilt. We believe it to be the technologically simplest macroscopic example of a topological pump. Moreover the full dynamics, i.e., that of the particle and that of the external field are easy to observe. The current work presents three macroscopic examples of topological magnetic crystals, with magnetic point symmetry protected Floquet transport properties of paramagnetic (soft magnetic) and diamagnetic (superconducting) spheres placed above the crystal. We experimentally determine regions of orientation around which we have to wind an external magnetic field to pump the spheres into certain directions. These regions turn out to strongly depend on the symmetry of the lattice.

Steel pump setup. – The system consists of a two-dimensional magnetic pattern of up- and down-magnetized domains creating a two-dimensional magnetic potential for the steel sphere above the pattern.
The potential parametrically depends on the direction of a superposed external magnetic field. The steel sphere moves in this potential when we adiabatically modulate the potential by changing the direction of the external field.

Three two-dimensional magnetic patterns are built from an arrangement of NbB-magnets. The first crystal consists of magnetic cubes of side length $d_1 = 2\text{ mm}$ and remanence $\mu_0 M_1 = 1.35\text{ T}$ arranged in a fourfold symmetric $C_4$ checkerboard square lattice of alternating up- and down-magnetized cubes (fig. 1(a)). The second, a hexagonal lattice (fig. 1(b)), consists of cylindrical magnets of diameters $d_3 = 3\text{ mm}$ and $d_2 = 2\text{ mm}$, height $h = 2\text{ mm}$ and remanence $\mu_0 M_2 = 1.19\text{ T}$ and $\mu_0 M_3 = 1.35\text{ T}$ respectively. The larger size $d_2 = 3\text{ mm}$ magnets are magnetized upwards and they are surrounded by six smaller size $d_3 = 2\text{ mm}$ magnets that are magnetized downwards and that touch the larger magnet. The primitive unit cell of the lattice is a sixfold symmetric $C_6$ hexagon with corners centered within the smaller magnets. Each unit cell thus contains one large magnet and two smaller magnets. The third lattice (fig. 1(c)) is built from a hexagonally closed-packed arrangement of $d_2 = 3\text{ mm}$ diameter cylinders. The central cylinder (blue) is nonmagnetic brass, and the surrounding cylinders are NbB magnets of alternating upward and downward magnetization creating an improper sixfold $S_6$ symmetry. All two-dimensional lattices have two primitive lattice vectors of the same length $a_1 = a_2 = a = 2.82\text{ mm}$ (square lattice), $a = 4.33\text{ mm}$ ($C_6$ lattice), and $a = 5.2\text{ mm}$ ($S_6$ lattice) and are metastable (the ground-state configuration of the magnet ensemble is a magnetic rod of magnets aligned along one axis) in zero external magnetic field. We fix the arrangement with an epoxy resin placed in the voids and the lateral surroundings of the pattern. The pattern then is stable also in the presence of an external field. The crystals are put on a support and covered with a transparent PMMA spacer of thickness $z = 1–1.5\text{ mm}$ (fig. 1(d)). The potential energy of the steel sphere can be decomposed into a discrete Fourier series of contributions from reciprocal lattice vectors. The Fourier series of the potential right above the pattern is the square of the Fourier series of the magnetization of the pattern augmented by the external field. As a function of the elevation the higher Fourier coefficients are attenuated more than Fourier coefficients with lower reciprocal vectors. At the experimental elevation only the universal contributions to the potential from the lowest nonzero reciprocal lattice vectors remain relevant. The purpose of the spacer is thus to render the potential universal such that only the symmetry of the pattern, not the details of the pattern are important. We place a steel sphere of diameter $2r = 1\text{ mm}$ on top of the spacer and create a closed but transparent compartment around the steel sphere. The topological magnetic crystal with the steel sphere on top is placed in the center of a goniometer set up at an angle of 45 degrees to ensure that relevant motion is not affected by the restrictions of motion of the goniometer (fig. 1(e) and (f)) caused by the support. The goniometer holds two NbB-magnets of diameter $d_{\text{ext}} = 60\text{ mm}$, thickness $t_{\text{ext}} = 10\text{ mm}$ and remanence $\mu_0 M_{\text{ext}} = 1.28\text{ T}$ aligned parallel to each other at a distance $2R = 120\text{ mm}$ and creating an external magnetic field $\mu_0 H_{\text{ext}} = 45\text{ mT}$ penetrating the two-dimensional crystal and the steel sphere. The magnetic field gradients $\nabla H_{\text{ext}} \approx M_{\text{ext}} d_{\text{ext}}^2 / R^4$ of the external field at the position of the steel sphere is at least two orders of magnitude smaller than the field gradients of the magnetic field of the crystal $\nabla H_{\text{int}} \approx M/a$. The two external magnets can be oriented to produce an arbitrary
direction of the external magnetic field with respect to the crystal. A laser pointer pointing along $\mathbf{H}_{\text{ext}}$ is mounted on the goniometer creating a stereographic projection of the instantaneous external magnetic field direction on a recording plane.

**Topologically nontrivial transport loops.** – We reorient the external magnets by moving along a closed reorientation loop that starts and ends at the same initial orientation. The steel sphere responds to the reorientation loop with a motion that starts at one position of the lattice and ends at a final position. A topological trivial motion of the steel sphere is a motion where the steel sphere responds to a closed reorientation loop with a closed loop on the lattice. Not every closed reorientation loop causes a trivial response of the steel sphere. There are topologically nontrivial trajectories, where the steel sphere trajectory ends at a position differing from the initial position by one vector of the lattice.

The theory developed in [8] sorts the modulation loops on the sphere of external magnetic field orientation into classes that transport into different directions. Additionally the theory distinguishes adiabatic and ratchet modulation loops. For adiabatic loops the speed of the particle is enslaved to the adiabatic speed of modulation during the entire modulation loop. For ratchet modulation loops the particles perform jumps with an intrinsic speed that is uncorrelated with the speed of modulation at particular orientations of the external field. In the adiabatic motion the particle hence moves with the potential minimum at all times, while in a ratchet it jumps to a new minimum if the old minimum disappears at a critical external magnetic field. On the sphere of orientation these different modulation loops can be distinguished by their winding number around specific objects (points, lines or areas). The dimension and position of these objects depends on the symmetry of the lattice.

Experimentally we choose a collection of different non-self-intersecting reorientation loops and measured the corresponding displacement of the steel sphere. Each non-self-intersecting reorientation loop cuts the sphere of orientations that we call the control space into two areas. One of the areas is circulated by the loop in the positive sense, the other in the negative sense. We define the intersection of all positive areas of loops causing the net transport of the steel sphere as the positive common area of this transport direction. Similarly we can define the negative common area of the same transport directions. If we find a loop that cuts through the common area and reproducibly transports into the same direction, we have found a smaller common area. By performing experiments with lots of different loops we eventually approach the smallest common area. In this way we can map the common areas without relying on the theoretical predictions.

**Fourfold symmetric pattern.** In fig. 2(a) we show the common area (yellow) determined in this way for the transport into the $n \times Q_1$-direction for the fourfold symmetric pattern. The common area is a rectangle centered around the primitive reciprocal vector $-Q_1$ of the lattice. Whenever we wind the modulation loop around the common area in a way that does not touch the area, the result is the same nontrivial transport as shown in fig. 2(b). Two modulation loops with similar winding number around the common area are shown in fig. 2(a). The resulting transport over a period is the same for both loops which shows that the transport is protected against perturbations of the modulation. Entering the common area leads to a statistical trivial or nontrivial response transport direction of the steel sphere (the transport direction is no longer reproducible). The sphere passes from the up-magnetized region toward the down-magnetized regions or vice versa when the loop crosses the gates (dark and bright green circles). In the experiments we observe a hysteresis, i.e., the gate in control space is positioned at the bright green circles of the southern hemisphere when the external field moves from north toward the south and at the dark green circles of the northern hemisphere for the opposite direction. The region of the hysteresis is shown as the green area in control space. Note that similar common areas repeat every $2\pi/4$ along the equator because of the $C_4$-symmetry. In previous work [8] we have computed the theoretical position of the common area as well as the position of the gates. Theoretically the common area is just one point, the $-Q_1$-direction, and the gate is a (green) line on the equator showing no hysteresis. In fig. 2(b) we show the trajectory of the steel sphere on the fourfold lattice subject to the purple loop in fig. 2(a) encircling the reciprocal vector $-Q_1$ in the mathematical positive sense. Movies of the motion to both fig. 2 and fig. 3 can be found in the supplementary material (SM) as videos Fig2b.avi, Fig2d avi and Fig2f.avi corresponding to figs. 2(b), (d) and (f). The first two movies were done by following a predesigned path on a screen with the laser pointer of the goniometer. The last movie is done by moving the external magnet by hand showing the robustness of the motion with respect to deviations of the modulation path. Videos Fig3b.avi, Fig3d.avi and Fig3f.avi corresponding to figs. 3(b), (d) and (f) show the different behavior of steel and superconducting spheres when immersed into liquid nitrogen.

**Improper sixfold symmetric pattern.** In fig. 2(c) we show the control space of the $S_6$-symmetric pattern. Non-trivial transport into the $n \times Q_1$-direction occurs if we wind the loop around the yellow common area. We call the borders of the yellow area the fence. A new feature of the $S_6$-symmetric pattern is that the transport is still predictable if we enter and exit the yellow area with a loop through fence segments marked in blue and red. A loop exiting the common area in the north (south) has the same result as a common-area-avoiding loop with the same winding number around the cusp point joining the two blue (red) segments of the fence. Although the transport direction of those common-area-passing loops is the same...
Fig. 2: (Color online) (a) Control space of the $C_4$-symmetric lattice. The $-Q_1$-direction of control space is opposite to the reciprocal lattice vector direction $Q_1$ in (b). (a) Theoretical fence points are shown in yellow, gates as green lines. The fence points that one must wind around to achieve nontrivial transport enlarges to the fence area (yellow) in the experiment. Gates are shown as dark and light green circles and show a hysteresis (green area) when winding around the fence area in different directions. We depict a purple loop encircling the yellow area as an example loop inducing nontrivial colloidal transport into the $n \times Q_1$-direction by exactly one lattice vector $a_2$. This transport direction is protected against perturbations of the driving and the dotted purple loop results in exactly the same net transport because it shares the winding number around the yellow common area with the unperturbed solid modulation loop. (b) Trajectory (blue and red) of the steel sphere subject to the solid purple loop shown in (a). Red segments correspond to faster motion than blue segments. The adiabatic motion smoothly changes from fast to slower. The yellow arrow corresponds to the primitive unit vector pointing into the transport direction. (c) Control space of the $S_6$-symmetric lattice. Theoretical fences for paramagnets are shown in red and blue and for diamagnets in black. The experimentally determined fence for the steel sphere lies further outside with two separate regions (blue and red) of instability when leaving the yellow area toward the north or south. We depict a palindrome modulation loop in purple that cycles through the common area back and forth in control space but causes an open trajectory (see (d)) with ratchet jumps of the steel sphere above the lattice. For the part of the loop moving in the mathematical positive sense the winding number around the upper cusp joining the two blue segments is nonzero and causes nontrivial transport, while for the same loop traveling in the mathematical negative sense the winding number around the lower cusp joining the red segments is zero and causes trivial motion. (d) Trajectory (blue and red) of the steel sphere subject to the purple loop shown in (c). The ratchet motion discontinuously changes from slow adiabatic to fast ratchet jumps. (e) Control space of the $C_6$-symmetric lattice. Theoretical fences as blue and red lines with the experimental fence shown as crosses of the same color. The purple example loop causes adiabatic transport in the $(Q_2 - Q_3)$-direction. (f) Trajectory (blue and red) of the steel sphere subject to the purple loop (e). Red segments correspond to faster motion than blue segments. The adiabatic motion smoothly changes from fast to slower. Movies of the motion are provided in the SM.

as that of the common-area–avoiding loops, their character is that of a ratchet. A ratchet jump of the steel sphere from one point on the lattice to a different point occurs when we exit the common area. Loops avoiding the common area cause a smooth quasi adiabatic transport of the steel sphere. Entering or exiting the common area in regions where there are no blue or red fence segments yields statistical results for the steel transport direction. Two further common areas exist at the location turned by $\pm 2\pi/3$ along the equator. The theoretical prediction is in topological agreement with the experiments, however, the theoretical common area enclosed between the red and blue line fence segments is smaller than the experimentally measured area and the northern and southern fence segments form a closed line around it with no statistical segments of the common area border. The theoretical
common areas repeat when we turn the control space by $2\pi/3$ around the normal vector. The cyan common areas correspond to theoretical areas supposed to cause nontrivial transport of diamagnetic (superconducting) spheres.

In fig. 2(d) we show the trajectory of a palindrome modulation loop for the $S_6$-symmetric lattice shown in fig. 2(c). The palindrome loop consists of two sub-loops that are the inverse of each other. The first sub-loop crosses the common area by entering the left southern fence segment (red) and exiting at the northern right segment (blue) and returning to the initial orientation by winding around the cusp joining the two blue fence segments and not winding around the red cusp joining the two red fence segments. Immediately afterwards the second sub-loop retraces the path of the first sub-loop in the opposite direction. The trajectory of the steel sphere is of the ratchet type and does not close because of irreversible jumps that happen when the modulation loop leaves the common area at different fence segments during the forward and backward period. This nontrivial ratchet motion is in contrast to trivial adiabatic motion where adiabatic palindrome loops always cause the trivial motion back and forth on the same path.

**Proper sixfold symmetric pattern.** In fig. 2(e) we show the control space of the $C_6$-symmetric pattern. The transport is adiabatic if a loop enters and exits via segments that have the same blue or red color. If both segment colors differ, the loop causes a ratchet motion. Nontrivial transport into the $\sigma(Q_i - Q_j)$-direction ($\sigma = \pm 1$, $i, j = 1, 2, 3$) occurs when the modulation loop enters the yellow area via a neighbor segment of the reciprocal unit vector $\sigma Q_i$ and exits via a nearest or next nearest neighbor segment of the reciprocal lattice vector $\sigma Q_j$. The sign $\sigma$ of the nearest or next nearest exit reciprocal vector $\sigma Q_j$ must be the same as that of the nearest reciprocal vector.
\[ \sigma Q_i \] of the entry. The experimental position of the fence (blue and red) has been determined from the irreversible jumps of the steel sphere when the external field exits the yellow area through a segment of opposite color than that of the entry. The match between experiment and theory here is almost perfect. In fig. 2(f) we depict the adiabatic trajectory of the steel sphere above a \( C_6 \)-symmetric lattice for a loop passing through the yellow area via the red fence segments in fig. 2(e).

**Transport of paramagnets and diamagnets.** If we open the compartment, fill it with liquid nitrogen and place a steel sphere and a high-temperature superconducting sphere (YBCO) we may study the transport of both paramagnetic and diamagnetic spheres. For the fourfold pattern we observe parallel transport of the steel and superconductor spheres with spheres of different character separated by an odd multiple of \( (a_1 + a_2)/2 \). On the \( C_6 \) and \( S_6 \) patterns the steel and superconductor sphere can be transported independently, because the yellow and cyan common areas are lying in different opposing locations of the control space. In fig. 3(b), (d), (f) we show the simultaneous transport of a steel and a superconductor sphere subject to example loops depicted in fig. 3(a), (c), (e).

**Discussion and conclusion.** — From the measurements we see that the experiments are in topological agreement with the theory [6–8]. The most striking difference between experiment and theory is the existence of a hysteresis visible in the \( C_4 \)- and \( S_6 \)-symmetric patterns. We explain the shift of the experimental fence with respect to the theoretical predictions as well as the hysteresis by solid friction that lets the steel particle move only when the magnetic potential exceeds a certain slope. Slopes for a forward and backward jump will have opposite sign explaining the splitting of the closed theoretical fence in the \( S_6 \)-pattern into two separate blue and red fences. The asymmetry of the hysteresis in fig. 2(a) is an indicator for the influence of noise in the magnetization of the pattern and the noise of the solid friction. Note that without solid friction there should not be any hysteresis. Hydrodynamic friction vanishes in the adiabatic limit and can, without solid friction, be made arbitrary small by reducing the speed of modulation.

Let us note that the topological protected transport theory has been developed for colloidal particles not for steel spheres. The scale invariance of the theory (the scales differ by a factor \( 10^3 \)) demonstrates the robustness of the topological concept. Presumably it is also possible to down scale the experiment from the colloidal toward molecular scales, which would provide a transport mechanism for molecular magnets above magnetic nano structures that could be sorted according to their magnetic properties. There instead of solid friction thresholds with hysteresis thermal fluctuations (the influence of which was studied in detail in ref. [7]) will be relevant, the topological properties will however probably remain robust enough to dominate the dynamics.

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