Relation of Strangeness Nuggets to Strangeness Condensation and the Maximum Mass of Neutron Stars

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Abstract

The recent experimental indications of density dependence in the pion decay constant $f_\pi^*$ and $\omega$ meson mass $m_\omega^*$ and the discovery of $S^0(3115)$ and other strange “nuggets” are providing a strong support for kaon condensation in dense hadronic matter, thereby re-kindling the interest in the issue of the critical stable mass of neutron stars. The density-dependent quantities provide increases in the vector mean fields mediated by $\rho$ and $\omega$-meson exchange which increase by a factor $\sim 1.56$ the Weinberg-Tomozawa term in kaon-nucleon interactions which accounts for $\sim$ half of the binding energy of the $K^-$ meson in dense matter. Furthermore lattice gauge calculations have pinned down the value of $\Sigma_{KN}$, the explicit chiral symmetry breaking in the strangeness sector. The partial rotation out of this explicit breaking provides the other $\sim$ half of the $K^-$ binding energy. The net result is to confirm the work of Thorsson et al. that strangeness condensation takes place at $u = n/n_0 \simeq 3$, where $n_0$ is nuclear matter density, in neutron stars. We suggest in this article that a support for this scenario is provided by the recent experiments of Suzuki et al. who found tightly bound strangeness nuggets. The strangeness nuggets discovered in the experiments involve approximately the same ratio of nucleons to $K^-$ meson as in the center region of neutron stars. But whereas the latter can be described by mean fields, in which the medium effects are substantially more attractive than in the finite system, the binding in neutron star matter should be substantially (say, $\sim 20\%$) greater than that in the strangeness nugget. This would strengthen the argument by Brown and Bethe that the accompanying softening in the equation of state should limit the maximum neutron star mass to

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$M_{\text{NS}}^{\text{max}} \simeq 1.5M_\odot$. This low $M_{\text{NS}}^{\text{max}} \sim 1.5M_\odot$ has major consequences in astrophysics, especially for the merging rate of compact stellar objects.

1 Introduction

Based on the detailed numerical calculations of Thorsson et al. [1], Brown and Bethe [2] claimed that the result of kaon condensation setting in at a density $n \sim 3n_0$, where $n_0$ is nuclear matter density, gives a sufficient softening of the equation of state in neutron star matter to limit the maximum possible neutron star mass to $\sim 1.5M_\odot$. In the analysis of Supernova 1987A, which Bethe and Brown considered to have evolved into a black hole, these authors [3] were able to establish a maximum mass for the compact object of $1.56M_\odot$, based on the observed amount of nickel produced in the supernova explosion. The $M_{\text{NS}}^{\text{max}} \sim 1.5M_\odot$ was consistent with 1987A evolving into a low-mass black hole. Furthermore, the ZAMS (Zero Age Main Sequence) 18$M_\odot$ progenitor of 1987A was calculated to have an Fe core mass of $\sim 1.5M_\odot$. Brown et al.[4,5] argued that this Fe core mass was about the same as the compact object mass on the basis of calculations by Woosley showing that fallback in the supernova explosion compensated for the increased binding energy of the compact object.

We claim that the above astrophysical phenomenon involving neutron star masses is intricately connected with the softness in the equation of state coming when the electrons in the neutron star can shed their high degeneracy energy by changing into $K^-$-mesons, in a zero momentum Bose condensate. This is a particle and nuclear physics problem which in our view must incorporate the vacuum structure of dense matter involving chiral symmetry and hence in-medium properties of hadronic masses. Our focus here will therefore be on particle/nuclear physics aspect. We shall make a brief summary of the astrophysical consideration in a section below but leave the detailed discussion to a longer, technical paper [6].

The standard chiral perturbation approaches predominantly employed in the field are anchored on chiral Lagrangians whose parameters are determined in the matter-free vacuum (that we shall refer to as “zero-vacuum”) and computations are done by perturbation around the zero-vacuum in terms of a set of presumed small expansion parameters. Our approach will differ from them in a crucial way. Ours will be a mean-field approach with a Lagrangian defined in a sliding vacuum [7]. The basic idea is similar to Landau Fermi-liquid theory for many-body systems as applied to nuclear matter [8]. Here one performs “double decimation” [7]. The crucial point is that the mean field approximation with an effective chiral Lagrangian with scaling parameters is equivalent to Fermi-liquid fixed point theory. It is not known how to obtain the effective
Lagrangian with “sliding vacua” from an effective Lagrangian defined in the zero-vacuum just as it is unknown how to derive from a fundamental chiral Lagrangian the four-Fermi interaction Lagrangian effective near the Fermi surface that via Wilsonian RGE gives rise to Fermi-liquid fixed point theory [9]. In the same vein, our starting Lagrangian that is defined with “sliding vacua” is not derived in any systematic way from a zero-vacuum chiral Lagrangian. It will be the same Lagrangian that gives rise to the Fermi-liquid fixed point theory, here extended to three flavors. We will employ mean field approximation with this Lagrangian. Kaon condensation will emerge as instability against strangeness condensation like superconductivity resulting from an unstable Fermi surface triggered by the Cooper pairing. Since in this way of approaching kaon condensation one is approaching from below the QCD phase transition, namely, chiral restoration, we will call this “bottom-up” approach.

Since kaon condensation must take place before, and in the vicinity of, the chiral restoration point, a mean field approach should make a better sense if one fluctuates around the “vector-manifestation (VM)” fixed point discovered by Harada and Yamawaki [10] of hidden local symmetry Lagrangian Wilsonian matched to QCD. Within the framework of hidden local symmetry (HLS) approach to hadron physics, there are two points at which the theory is reliably known. One is the zero-vacuum around which fluctuations can be computed with confidence if vector mesons are treated on the same footing as the pions. A suitable perturbation expansion treating the vector meson mass as a small parameter in the sense of the $1/N_c$ expansion is found to give results in good agreement with experiments in matter-free space [10]. The other is the vector manifestation (VM) point at which in the chiral limit, the light-quark vector mesons become massless and the parameters of the Lagrangian controlling low-energy physics get fixed to a definite value and to which HLS flows as temperature, density or the number of flavors is dialled to the critical value for chiral restoration. If one is not too far from the VM point, it can be substantially advantageous to start from the VM fixed point at which the Lagrangian is known. We will call this “top-down” approach. An extreme case where this approach is successful even for a process taking place in matter-free space is the chiral doubler splitting of the D meson discussed in [11] \(^1\). Other examples are discussed in [12] where it has been suggested that certain processes in baryonic environment and/or particularly sensitive to the presence of vector degrees of freedom are much more effectively treated starting from the VM than from the zero-vacuum. This “top-down” approach is not yet fully formulated for the process in question. So we can only give a drastically simplified treatment below.

\(^1\) It is perhaps significant to note that in this process, even though the starting point is the VM, the tree contribution dominates, loop corrections making up only $1/3$ of the total.
In mean-field in the bottom-up approach on which we will be mainly focused, there are two main driving forces towards kaon condensation. One is the movement towards the restoration of the explicitly broken chiral symmetry $\Sigma_{KN}$ in the strangeness sector [13]. This is characterized in the calculations by the chiral symmetry parameter $a_3 m_s$ related to the $KN$ sigma term. We shall update the calculations of strangeness condensation here of Thorsson et al. [1] who used three different values of $a_3 m_s$. This parameter has now been calculated on the lattice [14], with central value $a_3 m_s = -231$ MeV, and quoted accuracy of $3 - 4\%$ which is only slightly greater in magnitude than the Thorsson et al. central value of $-222$ MeV which we shall use. Furthermore, Thorsson et al. used nuclear compression moduli of $K_0 = 180$ and $240$ MeV. We favor the value of $210$ MeV for which $M_{\text{NS}}^{\text{max}} = 1.5 M_\odot$. Kaon condensation sets in at $n = 3.08 n_0$, where $n$ is the density and $n_0$ is nuclear matter density and the maximum neutron star mass is $1.5 M_\odot$.

The second main driving force towards kaon condensation comes from the Weinberg-Tomozawa term treated in the mean field. This comes from vector mean fields between the nucleons and the $K^-$. Following [15], in mean field, the force mediated by the $\omega$-meson exchange between a $K^-$ and a nucleon can be related to that between two nucleons as

$$V_{K^-}(\omega) = -\frac{1}{3}V_N$$

where $V_N$ is the vector mean field from nucleons in nuclear matter. The $1/3$ is easy to understand, because there is only one nonstrange antiquark in the $K^-$, whereas there are three nonstrange quarks in the nucleon. The $\rho$ meson exchange gives

$$V_{K^-}(\rho) = \pm\frac{1}{3}V_{K^-}(\omega)$$

repulsive for neutrons, attractive for protons. In the Weinberg-Tomozawa term, the vector-meson propagator for symmetric nuclear matter is approximated by zero momentum,

$$V_{K^-}(\omega) = -\frac{3}{8 F_\pi^2} n,$$

where $n$ is the nucleon density and $x_p$ ($x_n$) is the proton (neutron) fraction, the connection with the mean fields being by way of a KSRF-type relation

$$m_{V}^2 = 2 F_\pi^2 g_V^2.$$

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In eqs. (3) and (4), $F_\pi$ is a parameter that appears in the HLS Lagrangian which is related to the pion decay constant $f_\pi$ and $g_V$ is the vector (or flavor gauge) coupling.

Two recent experimental developments provide crucial information that allows to determine the above mean field potentials in medium:

1. The "$f_\pi$" connected with the $\rho$-meson via the KSRF relation has been found in deeply bound pionic atoms to be substantially below its free space value \([16]^{3}\),

$$\left(\frac{F_\pi^*(n_0)}{F_\pi}\right)^2 = 0.65 \pm 0.05$$

extracted at tree order. Other determinations are reviewed by Brown and Rho \([7]\).

2. As what may be taken as the first direct and unambiguous verification of the scaling proposed in \([17]\), the decrease in $\omega$-meson mass with density has been measured as $m_\omega^*/m_\omega \simeq 0.84$ at $n = n_0$ \([18]\). This is somewhat larger than the 0.8, consistent with Eq. (5), which we shall use; we believe $f_\pi^*$ to be more accurately determined by the extensive data on deeply bound pionic atoms.

As shown by Harada and Yamawaki \([10]\), $g_V$ changes with scale, such that both $g_V^*$ and $m_V^*$ go to zero proportionally to the quark condensate $\langle \bar{q}q \rangle$ near the VM fixed point as $g_V^*/m_V^*$ constant. In fact there is some evidence that this behavior sets in from $n \simeq n_0$ upward \([7,19,20]\). However, there is no evidence that $g_V^*$ scales appreciably below $n = n_0$. Brown and Rho \([7]\) have roughly summarized the situation by letting $m_V^*$ scale up to nuclear matter density as $m_V^*(n_0)/m_V \simeq 0.8$, with $g_V$ constant. Beyond $n_0$ $g_V^*/m_V^*$ is taken to be constant. This is certainly a rough description, but it reproduces the main features of the Harada-Yamawaki scaling.

With the scaling in $F_\pi^2$ only up to nuclear matter density $n_0$, we find the mean fields to be increased by a factor

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We make the distinction between $F_\pi$ and $f_\pi$. It is $f_\pi$ which is an order parameter of chiral symmetry vanishing at the critical point in the chiral limit. Harada and Yamawaki show that $f_\pi \equiv f_\pi(Q^2 = 0) = F_\pi + \Delta$ where $\Delta$ is a quadratically divergent term that cancels $F_\pi$ at the critical point. Note that it is $F_\pi$ that we are concerned with here.

We should note here that it is really $F_\pi$ that is determined in this experiment, not the order parameter $f_\pi$. However at the mean field level and up to nuclear matter density, they are the same and hence the interpretation as "partial restoration of chiral symmetry" as measured in nuclei is correct. This aspect is frequently confused in the literature.
An astonishing new development has been the experimental discovery of deeply bound kaonic nuclear states [21]. The interpretation by Akaishi et al. [22] (see, for related discussions, Mares et al. [23] who studied the widths of deeply bound kaonic atoms and arrived at $100 \sim 200$ MeV binding energy in these states) is of particular interest in connection with the Thorsson et al. [1] calculation and our present work is bringing it up to date. Although the deeply bound kaonic-nuclear states are small, essentially the strangeness (anti)nuggets suggested in many papers, the players in the nuggets, two neutrons, one proton and a $K^-$-meson, are the same as those in the Thorsson et al. mean field approach, at the density beyond kaon condensation where one $K^-$ is present for every two neutrons. We show that the Thorsson et al. mean field analysis gives substantial support to the Akaishi et al. analysis. In return, since medium dependent effects, essentially the dropping meson masses, have larger effects in the mean field than in the “nugget” calculations, experimental confirmation of the latter would give strong support to the work of Thorsson et al. and consequently the low $M_{NS}^{\text{max}}$ of Brown and Bethe [2].

The plan of this note is as follows. In Sec. 2 we shall schematize the Thorsson et al. (bottom-up) calculation, carefully separating effects from scalar and vector mean fields in lowest order, showing how they get mixed up in higher order. We also discuss the range term, introduced in higher-order in [24,25] and show why the medium dependence in this effect has confounded investigators who obtain input parameters from $K^-$-nucleon scattering lengths. We then show in Sec. 3 how the mean field calculation is related to the strangeness “nugget” calculation of Akaishi et al. [22]. In Sec. 4, we discuss certain specific features associated with the VM point when the treatment is made top-down from the VM. In Sec. 5 we review some of the many other calculations of strangeness condensation and discuss why $\Sigma^-$ hyperons do not seem to figure importantly in dense matter, or low $\Sigma^-$ even if they do — they would not be expected to alter our $M_{NS}^{\text{max}}$ appreciably.

Although the detailed analysis of the evolution of compact binaries, double neutron stars and low-mass black-hole, neutron-star binaries, will be put off to a later paper [6], we point out the obvious consequences of a low $M_{NS}^{\text{max}}$ mass in Sec. 6; namely that the giant progenitors of a binary neutron star must be very close ($\sim 4\%$) in ZAMS mass. As more and more double binary neutron stars are observed, this can easily be checked. We shall point out that present observations already defy the statistics of the standard model of binary neutron star evolution, necessitating an alternative model.

In Appendix, we construct a crystalized dense system comprised of tribaryon-
Fig. 1. Projection onto the $\sigma, K$ plane. The angular variable $\theta$ represents fluctuations toward the kaon mean field.

$K^-$ bound states. We find that the $K^-$'s get unbound for densities exceeding $2n_0$. This indicates that in dense medium kaon condensation can set in starting from $n \sim 2n_0$. Since we started with the fixed proton fraction (1/3) in this approach, however, this critical density cannot be directly compared with the critical density for kaon condensation in the mean field approach.

2 Strangeness Condensation

In this section we sketch the principal arguments for our bottom-up approach.

2.1 Scalar channel

We first discuss the contribution towards the restoration of the explicitly broken chiral symmetry, essentially considering $V$-spin [13] (see Fig. 1). This way of looking at kaon condensation does not capture all aspects of the phenomenon but it gives a clear conceptual picture of what is involved.

The Hamiltonian for explicit chiral symmetry breaking is

$$H_{\chi SB} = \Sigma_{KN} \langle \bar{N}N \rangle \cos \theta + \frac{1}{2} m_K^2 F_{\pi}^2 \sin^2 \theta$$

$$\simeq \Sigma_{KN} \langle \bar{N}N \rangle \left( 1 - \frac{\theta^2}{2} \right) + \frac{1}{2} m_K^2 F_{\pi}^2 \theta^2$$

(7)

where the last expression is obtained for small fluctuation $\theta$ in the direction of the $K$-axis. We then find
\[ m_K^* = m_K^2 \left( 1 - \frac{\Sigma_{K\bar{N}}(\bar{N}N)}{f_\pi^2 m_K^2} \right) \]  

(8)

showing that the effective kaon mass \( m_K^* \) is strongly reduced by the explicit chiral symmetry breaking \( \Sigma_{K\bar{N}} \) in the strangeness sector.

As noted above, the explicit chiral symmetry breaking has been evaluated in lattice calculations by Dong et al. [14] giving \( \Sigma_{K\bar{N}} = 362 \) MeV. Although in chiral perturbation theory the scalar nuclear density \( \langle \bar{\psi}\psi \rangle \) can be replaced by the vector density \( n = \langle \psi^\dagger \psi \rangle \) since the above expression is in the highest order calculated, we prefer to keep the scalar density because at \( \sim 3n_0 \) we may have gone over to constituent quark degrees of freedom. Looking at the behavior of the order parameter \( f_\pi^* (n) \), we see that chiral symmetry restoration can only be declared\(^4\) at \( n \sim 4n_0 \). Although one cannot take this result at its face value, we take this to suggest that the nucleon effective mass decreases linearly from \( n = 0 \) to \( n_c = 4n_0 \). This gives \( \langle \bar{\psi}\psi \rangle / \langle \psi^\dagger \psi \rangle \) for the proton as 0.82, \( \langle \bar{\psi}\psi \rangle / \langle \psi^\dagger \psi \rangle \) for neutron as 0.57 when we take 90% neutrons and 10% protons as Thorsson et al. [1] did at \( n = 3n_0 \). So, the contribution of \( \Sigma_{K\bar{N}} \) will be reduced by about 20% if we take the scalar density instead of vector density. In the arguments given below, instead of taking the scalar density self-consistently, we use vector density with smaller \( \Sigma_{K\bar{N}} = 310 \) MeV, which correspond to \( a_3 m_s = -222 \) MeV, for the comparison with the results of Thorsson et al. [1].

Now, one notable development following the Thorsson et al. work was the calculation of the range term [25]

\[ \Sigma_{K\bar{N}}^{\text{eff}} = \left( 1 - 0.37 \left( \frac{\omega_{K^-}^*}{m_{K^-}} \right)^2 \right) \Sigma_{K\bar{N}}. \]  

(9)

For self consistency the final \textit{in-medium} \( \omega_{K^-} \) should be used here although this is at odds with the standard chiral counting that we are not adhering to. If we use the Thorsson et al. \( \mu_e = 219 \) MeV at \( n_c = 3.08n_0 \) for \( a_3 m_s = -222 \) MeV (remember that the \( K^- \) mass \( m_{K^-}^* \equiv \omega_{K^-}^* \) is equal to \( \mu_e \) at the phase transition.) then

\(^4\) Whereas Nambu-Jona Lasinio theory gives chiral restoration at \( n_c \approx 2n_0 \) [26], the transition from nucleon to constituent quark degrees of freedom, which NJL starts with, should take another \( \sim 2n_0 \). With finite temperature the two transitions, one from nucleon to constituent quarks and then chiral restoration are discussed in [27]. It is possible to approach also \( n_c \approx n_0 \) from the fixed point (at which \( m_\rho^* \to 0 \) in the chiral limit) since the role of Brown-Rho scaling is known \( (m_\rho^*/m_\rho \simeq 1 - 0.2n/n_0) \) for \( n < n_0 \) (see our later eq. (20)) and then the mass drops linearly in \( \langle \bar{q}q \rangle^* \) for higher densities.
\[ \Sigma_{KN}^{\text{eff}} = 0.93 \Sigma_{KN} \]  \hspace{1cm} (10)

Without range term it follows from Eq. (8) that

\[ m_K^* = 333 \text{ MeV}, \]  \hspace{1cm} (11)

whereas with \( \Sigma_{KN}^{\text{eff}} = 0.93 \Sigma_{KN} \)

\[ m_K^* = 347 \text{ MeV}. \]  \hspace{1cm} (12)

Thus, with introduction of the range term there would be a 14 MeV correction upwards in the \( \mu_e \) necessary for kaon condensation in Thorsson et al.[1].

The use of the full \( \Sigma_{KN} \) is highly preferable to the procedure used by most research workers who obtained \( \Sigma_{KN}^{\text{eff}} \) from fitting the \( K^- \)-nucleon scattering lengths at threshold. They would have obtained

\[ \Sigma_{KN}^{\text{eff}} = 0.63 \Sigma_{KN} \quad \text{(threshold scattering)} \]  \hspace{1cm} (13)

since \( \omega_{K^-} = m_{K^-} \) is assumed in this procedure. 5

2.2 Vector channel

We now turn to the Weinberg-Tomozawa term. As noted in Eq. (1), for \( \omega \)-exchange the mean field is \( V_{K^-}(\omega) = -(1/3) V_N \), and for the \( \rho \)-meson \( V_{K^-}(\rho) = \pm (1/3) V_{K^-}(\omega) \).

We calculate first with these mean fields. With the 90% neutrons and 10% protons at \( n_c = 3.1 n_0 \), the vector mean field contribution to \( \omega_{K^-} \) would be

\[ V_{K^-}(\omega) = -\frac{1}{3} g_\omega^2 \frac{1}{m_\omega^2} \left( \frac{x_n}{2} + x_p \right) n \approx -126 \text{ MeV}, \]  \hspace{1cm} (14)

where \( g_\omega = 3 g_\rho \) with \( g_\rho \sim 5 \), and \( x_{n,p} \) are the neutron and proton fractions. Putting this together with the \( m_K^* = 347 \text{ MeV} \) of Eq. (12) gives us

\[ \omega_{K^-}^*(n = 3n_0) = 221 \text{ MeV}, \]  \hspace{1cm} (15)

5 Indeed, for the pion the range term is \( \sim -1.1 \Sigma_{nN} \) and its introduction changes a quite appreciable attractive interaction into a slightly repulsive one. The fact that the range term in pion scattering is so well known makes it inexcusable to omit it in the \( K^- \)-nucleon scattering.
essentially the same as Thorsson et al. [1]. However, from the medium dependence in hand, we increase the magnitude of the 126 MeV by 1.56 given by (6), giving an additional $\sim 70$ MeV drop in the $K^{-}$ mass. As noted, 14 MeV of this is used up in the range term, so we are left with 56 MeV more binding than Thorsson et al. [1].

That the enhancement factor (6) is called for is clearly indicated in the structure of nuclear matter. Were we to take the free pion decay constant $f_\pi \simeq 93$ MeV in the $K^{-}$ vector potential (3), it would give, at nuclear matter density,

$$V_{K^{-}} = \frac{1}{3} V_N \simeq 58 \text{ MeV}$$

(16)

which would mean that the $\omega$-mean field for the nucleon would be $V \simeq 174$ MeV. This is much weaker than the Walecka mean fields which are employed at nuclear matter density $n = n_0$. They are more like $V(\text{Walecka}) \gtrsim 270$ MeV; i.e., $\gtrsim 50\%$ higher. Thus, the phenomenology favors the mean field growing with density.

In fact, the vector mean field is proportional to $g_V^2/m_\omega^2$ and a recent experimental study shows that $m_\omega^*$ decreases with density [18]. Much more study has been made of the density dependence of $f_\pi^*$ in deeply bound pionic atoms which arrive at

$$\left( \frac{F^*_\pi(n_0)}{F_\pi} \right)^2 = 0.65 \pm 0.05$$

(17)

extracted at tree level [16].

A similar

$$\frac{F^*_\pi(n_0)}{F_\pi} \simeq 0.8$$

(18)

can be obtained [28] from the Gell-Mann-Oakes-Renner relation 6

$$f^2 \pi m^2 \pi = -\hat{m} \langle \bar{q}q \rangle$$

(19)

under the assumption that $m_\pi$ and $\hat{m}$ do not scale with density, giving $(f^*_\pi/f_\pi)^2 = \langle \bar{q}q \rangle^*/\langle \bar{q}q \rangle$, and then using the model independent

6 Note that here and in the next two equations, there is no difference between $f_\pi$ and $F_\pi$ since we are going up to $n \simeq n_0$. 10
\[
\frac{\langle \bar{q}q \rangle^*}{\langle q\bar{q} \rangle} = 1 - \frac{\Sigma_{p,n} n}{f^2_\pi m^2_\pi}.
\] (20)

Thus by now we have ample evidence of scaling in \(m^*_\rho\) and \(m^*_\omega\). By taking \(f^*_\pi/f_\pi \simeq 0.8\) in our formulae we assume them to drop from their bare value by 20% at nuclear matter density \(n_0\). This gives our factor 1.56 increase in the vector mean fields.

3 (Anti) Strange Nugget

Recent experiments \[21,29\] and theoretical interpretation \[22\] involve the same players, e.g., two neutrons, one proton and a \(K^-\) in the strange tribaryon \(S^0(3115)\) as in the neutron star matter. For example, in Table 4 of Thorsson et al. \[1\] which refers to \(a_3m_s = -222\) MeV for \(n \sim 3.8n_0\) the proportion of \(K^-\) is \(x_{K^-} \sim 0.5\), the \(K^-\) having replaced the electrons, so that the ratio of protons to neutrons is \(\sim 0.5\) for overall charge neutrality. For this value of \(x\) the system is well into kaon condensation (with threshold \(u = n/n_0 = 3.08\); note the similarity to mean density of \(S^0(3115)\) of \(u = 3.1\)). Thus, if one cuts out a piece of the strangeness condensed star (which is on its way into a black hole) that contains two neutrons, it will have one proton and one \(K^-\).

First of all, in our description, the star should undergo strangeness condensation at a lower value of \(u\) than the nugget. In the description of the star, mean fields are used; e.g., the Weinberg-Tomozawa term is

\[
V_{\text{ave}} = -\frac{1}{3} \frac{g^2 V}{m^2_V} n_z.
\] (21)

for the above composition of two neutrons for each proton. Note that since \(V_{\text{ave}}\) is a mean field, it is evaluated for zero momentum transfer. This maximizes the medium effect in terms of the dropping vector meson mass \(m_V \rightarrow m^*_V\).

In fact, Brown et al. \[30\] have argued that in the GSI experiments with nucleon and kaon momenta \(|P_N| \sim 444\) MeV and \(|P_K| \sim 322\) MeV, form factors of \(f_V(p) \sim 0.82\) must be employed in order to take into account the finite size of the nucleon. There is a partial decoupling of the vector interaction. Thus, the interactions in the nugget should be somewhat weaker than in the infinite

\^7 The parametric masses of the \(\omega\) and \(\rho\) mesons should scale in the same way in medium in HLS theory if one assumes \(U(N_f)\) symmetry but the pole masses can differ a bit due to medium-dependent loop corrections. Here we are focusing on the former since we are working in the mean field approximation.
system, both because of the decoupling of the vector interaction and because medium effects are maximum for mean fields.

The same interactions enter into the neutron star and strangeness nugget calculations, but the neutron star is well and truly strangeness-condensed, well beyond threshold for the composition at which the nugget is formed. This seems reasonable in terms of the stronger mean field interactions.

Akaishi et al. [22] formulate their strangeness nugget problem in terms of potentials. Whereas the Weinberg-Tomozawa term, which is a vector potential, gives the main attraction between the two neutrons and the proton, we prefer to formulate the other main attraction as coming from the partial restoration of the explicitly broken chiral symmetry in the strangeness sector, essentially a scalar mean field. This would require something like a local density approximation in $\langle \bar{\psi} \psi \rangle$ in the nugget problem, which would be of doubtful accuracy.

We agree with Akaishi et al. [22] that the $\sim 50$ MeV difference between the binding energy (143 MeV) they obtained and the observed one (194 MeV) comes from the medium effects (“Similar to the case of observed pionic bound states” [22] as we discussed in $f_\pi$ being replaced by $f_\pi^*$) and it should be noted that in our description – which is different in spirit from theirs, we get about the same $\sim 50$ MeV as Akaishi et al.

In other words, we believe that there is a mapping of the neutron-star problem onto the strangeness nugget problem, or vice versa, and that the discovery of the nugget strongly supports the former.

4 a = 1 At and Away From the Fixed Point

The behavior of the various quantities we work with should be better determined at the fixed point of Harada and Yamawaki [10] where $m_\rho^*$ and $g_V^*$ both go to zero, linearly with $\langle \bar{q} q \rangle^*$ which goes to zero (as does $f_\pi^*$) and the quantity $a$, which we now define $\to 1$. Mean field approximations are expected to get better the closer we are to the VM fixed point [10]. This suggests the top-down approach to kaon condensation as well as to the strangeness nugget problem. Here we present in the simplified form what we can say about kaon condensation.

The low energy theorem of the hidden gauge theory valid to all orders of chiral perturbation theory [10] is

$$m_\rho^{*2} = a^* F_\pi^* g_V^{*2},$$  \hspace{1cm} (22)
defining $a$ in terms of the other variables we use. Earlier, when we talked about integrating out the $\rho$-mass, we used the KSRF-type relation

$$m_{\rho}^2 = 2F_\pi^2 g_V^2$$

(23)

which follows from vector dominance in free space, $a$ being then equal to 2 at the scale of $m_{\rho}$. However, in turns out [31] that the vector dominance at $a = 2$ is on an unstable trajectory of RG flow of the HLS theory with no connection to the trajectory that leads to the Harada and Yamawaki vector manifestation and that the fact that in nature the vector dominance model seems to work in matter free space is merely an accident. In fact, apart from the pionic form factor at zero temperature and zero density, $a$ near 1 provides a highly satisfactory phenomenology. In particular, $a = 1$ provides a good description for the coupling of the photon to the nucleon. [7]. Other evidences for $a \simeq 1$ in nature including chiral doubling in $D$ mesons are discussed in [12].

Thus, we find it more appropriate to use

$$\frac{g_V^*}{m_{\rho}^*}^2 = \frac{1}{a^* F_\pi^2}$$

(24)

with $a^*$ not far from 1, as compared with the

$$\frac{g_V}{m_{\rho}}^2 = \frac{1}{2F_\pi^2}$$

(25)

which pertained to matter-free space in which the vector meson was integrated out to get the Weinberg-Tomozawa relation. As can be seen, the vector mean field interaction is thus found to be increased by a factor of $2/a^*$, or a factor of 2 as the fixed point is reached.

Our next objective is to estimate (in the chiral limit) at which density $n_c$ the fixed point is reached. We do this by finding out at which density $m_{\rho}^*$ goes to zero.

Although initially $m_{\rho}^*$ decreases as $\sqrt{\langle \bar{q}q \rangle}$ following the scaling of $F_\pi^*$ (see argument above following Eq. (20)) the Harada and Yamawaki work shows that once it starts dropping it scales as $\langle \bar{q}q \rangle$. (See the empirical verification of this in Koch and Brown [32] who showed that the entropy matched that in LGS if the meson masses were allowed to scale as $\langle \bar{q}q \rangle$, which was referred to

\footnote{It is shown in [10] that at the matching scale $\Lambda \sim 4\pi f_\pi \sim 1$ GeV, HLS Lagrangian is given by $a \simeq 1.3$. Since this Lagrangian is expected to hold in the large $N_c$ limit, one can think of $a \simeq 1.3$ as representing the large $N_c$ limit. It turns out however that as far as phenomenology is concerned, $a \simeq 1$ gives as good a fit as $a \simeq 1.3$.}
as “Nambu scaling.”) As noted earlier, $g_V$ does not seem to scale up to nuclear matter density $n_0$, but then Nambu scaling sets in. Nambu scaling is $\sqrt{2}$ times faster than the initial scaling of $m_\rho^*$ from $n = 0$ to $\sim n_0$, which decreases $m_\rho^*$ by 20%. Thus, we believe in the interval $n_0$ to $2n_0$, $m_\rho^*$ will decrease $\sqrt{2}$ times 20%, or $\sim 28\%$, and the same from $2n_0$ to $3n_0$, and from $3n_0$ to nearly $4n_0$ where $m_\rho^* = 0$ in the chiral limit. Thus, the fixed point at $n_c$ is at $n \lesssim 4n_0$.

From our earlier argument that $g_V^*$ scales as $m_\rho^*$ for $n > n_0$, but up to $n_0$, $g_V$ remains constant, whereas $m_\rho^*$ scales, we find that Eq. (24) can be expressed as

$$\frac{g_V^*}{m_\rho^*} = \frac{1}{a^*} \left( \frac{1}{0.8F_\pi} \right)^2 \tag{26}$$

and we know that $a^* = 1$ at $n_c$, where it, together with $m_\rho^*$ and $g_V^*$, has a fixed point. Compared with the matter-free expression Eq. (25), which is the KSRF relation, we see that

$$\frac{[g_V^2/m_\rho^2]_{\text{fixed point}}}{[g_V^2/m_\rho^2]_{\text{zero density}}} \approx \frac{2}{0.8^2} \approx 3.13. \tag{27}$$

Thus, the mean field felt by the $K^-$ is increased by the factor 3.13 when the scaling of both $F_\pi^*$ and $a$ are included, at the fixed point at $n_c$, the final doubling coming from the scaling in $a^*$. At the VM fixed point, the condensate is zero (in the chiral limit), so the scalar contribution from the rotation of $\Sigma_{KN}$ is gone. Thus seen from the VM point, the essential doubling of the attractive vector interaction replaces the attraction given by the rotating out of the $\Sigma_{KN}$ term. In fact within the range of $a$ relevant to the problem [12], say, $1 \lesssim a \lesssim 1.3$, the coefficient of the density $n$ needed to bring the $m_{K^-}^*$ down, is relatively constant up to $n_c$. And furthermore the scenario is close to the bottom-up scenario discussed above.

We emphasize that our above argument connects strangeness condensation with chiral restoration, in that with the large increase in vector mean fields in going to the fixed point, strangeness condensation certainly takes place at an $n$ below $n_c$.

Following Eq. (15) it was noted that because of medium effects, we had an additional 56 MeV binding of the $K^-$ as compared with Thorsson et al. Rather than $a\mu_{c} = m_{K^-}(n_c) \simeq 219$ MeV of these authors, we would find $221 - 56 = 165$ MeV.

Now in our calculation about the fixed point, with the same 90% neutrons and 10% protons at $n_c$, we would have the factor of 3.13 times 126 MeV giving 394
MeV binding, just 100 MeV in magnitude less than the kaon mass in matter-free space. However, if we took \(a = 1.3\) as suggested by large \(N_c\) considerations, we would have 302 MeV binding, with \(\mu_{e,c} = m^*_{K^-} = 192\) MeV. Thus, within the possible range of \(a\) that could be arrived at by the RG flow from the fixed point (we believe \(\approx 1.3\) to be maximal), we find the \(\mu_{e,c}\) of Thorsson et al., corrected by medium effect, to be midway in the range of \(\mu_{e,c}\)'s that could be reached starting from the fixed point. A similar attraction was obtained for \(n_P/n \approx 0.25\) by Tsushima et al. [33] in a different approach that takes into account the scaling behavior.

We see from Thorsson et al., for \(a = 1 - 1.3\), the necessary electron chemical potential is easily reachable by \(n = 3n_0\) so strangeness condensation seems assured by \(3n_0\). In fact, the mass of the Hulse-Taylor pulsar is \(1.44M_\odot\) and since it must be stabilized, it will give a lower bound on \(n_c\).

The Kaplan-Nelson term involving \(\Sigma_{KN}\) that is important in the bottom-up approach to kaon condensation in an infinite matter would be difficult to include reliably in the strangeness nugget calculation, so it would be far more advantageous to carry it out top-down from the VM point. Since they do not deal with mean fields, but rather with relatively high vector meson momenta in

\[
\frac{q_V^2}{m^*_V^2 + q^2}
\]

they will not benefit as much as the kaon condensation calculation from the change in \(a\) from 2 in free space to \(\sim 1\) in medium. However, the factor 1.56 for \((F_\pi/F^*_\pi)^2\) remains. Thus, we might expect the binding there to be \(\sim 1.56 \times 126\) MeV = 147 MeV. In fact 126 MeV was the original vector mean field for 90% neutrons and 10% protons whereas Akaishi et al. have two neutrons for each proton, so the number would be 160 MeV at \(n = 3n_0\). Hence the binding would be \(1.56 \times 160\) MeV = 250 MeV.

5 The Lack of \(\Sigma^-\) Role

The mass of the \(\Sigma^-pn\) system lies at \(\sim 3075\) MeV, well below that of the \(S^0(3115)\), and one might think that this system plays a role in strangelets. Similarly, in neutron stars, the \(\Sigma^-\) will replace both a nucleon and an electron. This consideration led many investigators to consider \(\Sigma^-\) condensation, most recently by Kolomeitsev and Voskresensky [34].

We do not think that the \(\Sigma^-\) obstructs our principal argument. Batty et al. [35] noted that “for \(K^-\) atoms, the fitted potential becomes repulsive inside the
nucleons, implying that Σ hyperons generally do not bind in nuclei.” One can see from Fig. 4 in Brown et al. [36] that in the density dependencies favored by Batty et al. [35], the Σ− optical potentials are highly repulsive. Furthermore there are now direct experimental nuclear data (to be distinguished from atomic data) that show that the repulsion increases with the neutron excess [37,38]. A DWIA analysis [39] of these (π−, K+) data confirms the repulsion which was first predicted from Σ− atoms by Batty et al. [40] and extended by Mares et al. [41] in a relativistic mean field approach.

In Kolomeitsev and Voskresensky [34], although the S-wave K− condensate occurs only at ρc > 4ρ0 in neutron star matter, it is preceded by a long, essentially mixed phase of hyperon condensation which begins at u ∼ 2. During a mixed phase the pressure is constant; even with Gibbs construction the gradient in pressure is low, so that the force, given by gradient in pressure, is low. Thus, the neutron star should compress substantially in this phase and it seems unlikely that the maximum mass will be greater than our 1.5M⊙.

6 Astrophysical Consequences

In this Section we briefly touch on astrophysical implications relegating details to a future publication [6]. The important astrophysical consequence of the low-mass Mmax NS is that the standard scenario for binary neutron star evolution [42] does not result in a double neutron star, but in a black-hole, neutron-star binary. In this scenario, after the first born neutron star is formed, it goes into common envelope evolution with the companion giant as the latter expands in red giant stage. During this common envelope evolution it accretes a substantial amount of matter from the hydrogen envelope of the giant companion, as it removes this envelope. Bethe and Brown [43] estimated this amount to be ∼ 1M⊙ for a 1.4M⊙ neutron star, whereas with more accurate calculation [44] found ∼ 3/4M⊙ for this mass neutron star. The accretion is ∼ 0.5M⊙ for a 1.1–1.2M⊙ neutron star. Obviously these will be sent into black holes and the result will be a black-hole, neutron star binary.

As shown in [45], there is a special way in which the neutron star evolution in common envelope could be avoided. If the two giant progenitors are < 4% different in mass, they will expand in red giant and burn helium at the same time. Their hydrogen envelopes are then removed in the double helium star common envelope evolution. There is no time for the hydrogen to cross the helium molecular weight barrier [46] so the helium star remains very close in mass. The result is that the two resulting neutron stars have very nearly equal masses.

In the Table 1 we show the presently measured double neutron star binary
Table 1
Compilation of the compact objects in binaries by Lattimer and Prakash (2004). References are given in their paper. We have added, following the comma, the recent measurement of Van der Meer et al.[50].

| Object            | Mass ($M_\odot$) | Object            | Mass ($M_\odot$) |
|-------------------|------------------|-------------------|------------------|
| **X-ray Binaries**|                  |                   |                  |
| 4U1700−37         | 2.44$^{+0.27}_{-0.27}$ | Vela X-1         | 1.86$^{+0.16}_{-0.16}$ |
| Cyg X-1           | 1.78$^{+0.23}_{-0.23}$ | 4U1538−52        | 0.96$^{+0.19}_{-0.16}$ |
| SMC X-1           | 1.17$^{+0.16}_{-0.16}$ | XTE J2123−058    | 1.53$^{+0.30}_{-0.42}$ |
| LMC X-4           | 1.47$^{+0.22}_{-0.19}$ | Her X-1          | 1.47$^{+0.12}_{-0.18}$ |
| Cen X-3           | 1.09$^{+0.30}_{-0.26}$ | 2A 1822−371      | > 0.73            |
| **Neutron Star - Neutron Star Binaries** |                  |                   |                  |
| 1518+49           | 1.56$^{+0.13}_{-0.44}$ | 1518+49 companion | 1.05$^{+0.45}_{-0.11}$ |
| 1534+12           | 1.333$^{+0.0010}_{-0.0010}$ | 1534+12 companion | 1.3452$^{+0.0010}_{-0.0010}$ |
| 1913+16           | 1.4408$^{+0.0003}_{-0.0003}$ | 1913+16 companion | 1.3873$^{+0.0003}_{-0.0003}$ |
| 2127+11C          | 1.349$^{+0.40}_{-0.040}$ | 2127+11C companion | 1.363$^{+0.040}_{-0.040}$ |
| J0737−3039A       | 1.337$^{+0.005}_{-0.005}$ | J0737−3039B      | 1.250$^{+0.005}_{-0.005}$ |
| J1756−2251        | 1.40$^{+0.02}_{-0.03}$ | J1756−2251 companion | 1.18$^{+0.03}_{-0.02}$ |
| **Neutron Star - White Dwarf Binaries** |                  |                   |                  |
| B2303+46          | 1.35$^{+0.06}_{-0.10}$ | J1012+5307       | 1.68$^{+0.22}_{-0.22}$ |
| J1713+0747        | 1.54$^{+0.007}_{-0.008}$ | B1802−07         | 1.26$^{+0.08}_{-0.17}$ |
| B1855+09          | 1.57$^{+0.12}_{-0.11}$ | J0621+1002       | 1.70$^{+0.32}_{-0.29}$ |
| J0751+1807        | 2.20$^{+0.20}_{-0.20}$ | J0437−4715       | 1.58$^{+0.18}_{-0.18}$ |
| J1141−6545        | 1.30$^{+0.02}_{-0.02}$ | J1045−4509       | < 1.48             |
| J1804−2718        | < 1.70            | J2019+2425       | < 1.51             |
| **Neutron Star - Main Sequence Binaries** |                  |                   |                  |
| J0045−7319        | 1.58$^{+0.34}_{-0.34}$ |                  |                  |

masses. In the Hulse-Taylor pulsar 1913+16 the pulsar mass is 1.44$M_\odot$, the most massive neutron star in the double neutron star binaries. It has been evolved from an $\sim 20M_\odot$ giant [47]. Since the number of giants go as

$$N(M) \propto \frac{1}{M^{2.35}}$$  \hspace{1cm} (29)

according to the Salpeter mass function, if there were no correlation in pulsar
and companion masses, its companion would be most likely to come from an 
$M \sim 10M_\odot$ giant, with neutron star mass $\sim 1.2M_\odot$.

As can be seen from the accurately measured 1534+12, 1913+16, and 2127+11C there is a close correlation in the masses of the pulsar and companion.

In the case of the lower-mass binaries, the double pulsar J0737−3039A,B and J1756−221 equality of masses has been disturbed by the helium star evolution. In the chain of evolutions, the first born neutron star has a helium star companion, and low-mass helium stars expand in their own red giant stage. In so doing they transfer $\sim 0.1 - 0.2M_\odot$ to the neutron star, but since these are of low mass, this is not enough to send them into a black hole.

Since substantial mass is transferred in the neutron-star, hydrogen-envelope common envelope evolution for the more massive stars, the above correlation in neutron star masses within a binary does not give an accurate limit on $m_{\text{NS}}^{\text{max}}$. However, Brown’s [45] special double helium star scenario is an order of magnitude less likely than the standard scenario, giving $\sim 10$ times fewer double neutron star binaries. This translates into 10 times more black-hole, neutron-star binaries than double neutron-star binaries, which, with the higher masses of the former because of accretion, should give a factor of $\sim 20$ increase in the gravitational waves from merging to be detected by LIGO [43].

Since Table 1 contains three masses, those of 4U 1700−37, Vela X-1 and J0751+1807 which exceed our 1.5$M_\odot$ maximum neutron star mass, we should comment briefly.

4U 1700−37: Although this compact object has the same accretion history as the other high-mass X-ray binaries, it doesn’t pulse like the others. Brown, Weingartner and Wijers [5] evolve the compact object as a low-mass black hole.

Vela X-1: J. van Paradijs et al. [48] pointed out that in this binary with floppy B-star companion, the apparent velocity can in some cases increase by up to 30% (from the surface elements of the companion swinging around faster than the center of mass) “thereby increasing the apparent mass of the compact object by approximately the same amount”. In any case, Barziv et al.[49] from which the Vela X-1 neutron star mass in our table comes, say “The best value of the mass of Vela X-1 is 1.86$M_\odot$. Unfortunately, no firm constraints on the equation of state are possible, since systematic deviations in the radial-velocity curve do not allow us to exclude a mass around 1.4$M_\odot$ as found for the other neutron stars.”

J0751+1807: We are unable to give a critical discussion of this binary because thus far only preliminary results have been published. The sum of neutron-star and white-dwarf mass are determined by the period change from gravitational
radiation. There is an indication of a Shapiro shift and a white dwarf mass of 0.188 ± 0.012\(M_\odot\) is arrived at. Details of this are not given (of course it could be obtained from the measurement of the Shapiro shift, but none is given.) With a white dwarf mass of 0.24\(M_\odot\), the neutron star mass would come down to 1.5\(M_\odot\).

We should mention that the existence of a 2.2\(M_\odot\) neutron star would wreak havoc with our scenario of nearly equal masses in the more massive of the binary pulsars. The most abundant first-born neutron stars of mass 1.1 – 1.2\(M_\odot\) from \(\sim\) ZAMS 10\(M_\odot\) giant progenitors would accrete \(\sim\) 0.5\(M_\odot\) in hydrogen red giant phase and another 0.1 – 0.2\(M_\odot\) in helium red giant. They would then end up with pulsar mass \(\sim\) 1.8\(M_\odot\), with most likely low-mass 1.1 – 1.2\(M_\odot\) companion. Such massive pulsars should be strong in the radio and should predominate in number. None such have been observed.

7 Discussion

Although our discussion based on the bottom-up approach supports the Akaishi et al. scenario of strangeness nuggets, we have emphasized the Kaplan and Nelson [51] scalar attraction from the movement towards restoration of the explicitly broken chiral symmetry in the strange sector, Eqs. (7) and (8). Of course, this is more straightforward for us to handle in our mean field approach, than in the strangeness nugget, although the ratio of \(\langle \bar{\psi}\psi \rangle\) to vector density \(n\) is model dependent. On the other hand, the vector mean fields such as \(V_{K^-}(\omega) = -\frac{1}{3}V_N\) of Eq. (1) are straightforward, the \(K^-\) having one non-strange antiquark, and the constituent quark model should be adequate at \(n = 3n_0\) to describe the \(\frac{1}{3}\) ratio of \(V_{K^-}\) to \(V_N\). The vector mean field \(V_N\) at \(n = n_0\) has been much used in Walecka theory in nuclear physics. A value of \(V_N(n_0) = 270\) MeV is quite modest in nuclear physics. It should be noted that this value of \(3V_{K^-}(\omega)\) only reaches this after the factor \((F_\pi/F_\pi^*)^2\) is included. There is a clear indication, not least of all directly from experiment [18] that a sliding vacuum; i.e., Brown-Rho scaling must be operative. The vector meson masses continue to drop, until they go to zero in the chiral limit, but as the scale (density in this case) increases, the coupling \(g_{V}\) also decreases. Near the fixed point \((g_V^*/m_V^*)^2\) goes as a constant. We have handled the scaling crudely by assuming that \(g_V\) does not scale up to \(n_0\), for which there is substantial basis [7], and then neglect any further scaling in \(g_V^*/m_V^*\) above \(n_0\). This gives the factor \((F_\pi/F_\pi^*)^2 = 1.56\) in the medium dependence we used. In the above way we achieve a rough consistency with the Harada and Yamawaki [10] hidden local symmetry. Since all of our important results here refer to \(n > n_0\), for these the sliding in coupling cancels that in meson mass, and we can carry out the calculations with an effective \(f_\pi^*\) such that \((F_\pi/F_\pi^*)^2 = 1.56\) replacing \(f_\pi\). This was also foreseen in the Akaishi et al. work. In short, Brown-Rho
scaling essentially increases the zero-density parameters of mean fields up to the couplings which have already been used for a long time in Walecka type mean field approaches.

We suggested that mean field would be much more reliable when treated starting from the VM fixed point and sketched a highly simplified calculation that reproduces the bottom-up mean field calculation for kaon condensation. When further developed, the top-down approach will prove to be more powerful for the strangeness nugget problem.

It must be admitted that standard chiral perturbation theory calculations fluctuating from the zero-vacuum have not been carried out to the order necessary to get from the input lowest-order $f_\pi$ taken to be equal to $f_K$ in the necessary places. It is not clear to us that one can obtain a reliable result from such an endeavor. It seems that such a calculation must begin in-medium, from $F^*_\pi \simeq 0.8 F_\pi$. Thus, the in-medium $f_\pi^*$ would not be expected to be larger than the $f_\pi$ we have used in $m_K^*/m_K$.

An interesting calculation in chiral perturbation theory was carried out by Waas and Weise [52] and did go to high enough order to include the range term (they did not however employ the medium modified $f_\pi^*$). They used $\Sigma_{KN} = 230$ MeV, substantially smaller than that found in LGS. Waas and Weise found that iterating the Weinberg-Tomozawa term increased the attraction, lowering $m_K^*/m_K$ appreciably. In this way, their approach was midway between our mean field and the Akaishi et al. approach, which iterated the attraction by solving in a potential.

In our discussion of the melting of the soft glue in LGS [53] we found that nucleons changed over to constituent quarks at $T \sim 120$ MeV whereas the constituent quarks became current quarks, having lost their dynamically generated masses, at $T_c$ (unquenched) $= 175$ MeV. Although we do not know the value of $n_c$, and as we noted earlier, in Nambu Jona-Lasinio $\langle \bar{\psi}\psi \rangle$ doesn’t really go to zero until well above the $n_c$ for zero bare quark masses; we suggest that at $u = n/n_0 \sim 3$ constituent quarks may be more appropriate variables than nucleons, although they will still be held together by the soft glue in nucleons to some extent.

Thus, although $f_K$ may enter in the Kaplan-Nelson term Eq. (8), the vector mean fields are connected only to the constituent $\bar{u}$ quark in the $K^-$, and we believe that all of these calculations can be handled at the constituent quark level, but with Brown-Rho scaling of the constituent quark masses. This suggests working around the VM fixed point as stressed in [12]. Whereas the lowest-order Weinberg-Tomozawa term brings $m_K^*/m_K$ down to 0.65 in nuclear matter (it would not be brought down so far in neutron stars) with a factor of 1.56 it would be brought down more than half way to zero. The
second order terms are necessarily attractive, and will be increased by a factor of $(F_x/F_\pi^*)^4 \simeq 2.4$. There may be saturation in the complete coupled channels calculation, which includes Pauli blocking, but the contribution of the Weinberg-Tomozawa terms will obviously be substantially increased by the medium dependence. On the other hand, the $\langle \bar{\psi} \psi \rangle$ for constituent quarks may be smaller than the value Thorsson et al. [1] calculated for nucleons. We believe that the additional attraction from medium effects on the vector mean fields would be adequate to compensate for this.

The strangeness nugget presents a remarkable scenario in which once the $K^-$ meson is introduced into the three remaining nucleons in $^4$He, after a proton has been kicked out, the three nucleons collapse around the $K^-$ into the small nugget system. What else can they do? All interactions, scalar from the Kaplan-Nelson term and vector mean fields from Weinberg-Tomozawa terms are highly attractive, the latter having only a small isospin dependence through the $\rho$ mean field, so the system collapses down to an average density $n \sim 3n_0$. (It is however difficult to imagine how this can be achieved in standard chiral perturbative approaches.) The $K^-$ is in a potential about 600 MeV deep at the center, with binding energy of $\sim 250$ MeV, even more after strangeness condensation in our neutron stars. Luckily the collapse is halted in the nugget formation, chiefly by the finite sizes of the interacting objects. In the case of neutron stars, there is collapse into black holes and we will not “see” the black hole, neutron-star binaries until gravitational waves from their mergers are detected.

One issue we have not addressed in this paper is whether the collapse to black holes occurs without the kaon condensed state going into a color superconducting quark matter or after such a transition. At the moment one can say practically nothing about this matter. Should the collapse occur from a quark matter rather than from a kaon-condensed state, then the question as to how the kaon-condensed state which we predict must occur before the chiral transition and which may not be a normal Fermi liquid state develops the pairing instability that turns the matter to a (color) superconducting state discussed in the literature. As far as we know this question has not been studied in the literature.

8 Conclusions

We have argued that strangeness condensation in neutron stars can be related to the strangeness nuggets found experimentally by Suzuki et al. [21,29]. In both cases, binding of the $K^-$ meson is responsible for the phenomenon. In the neutron star matter the applicable mean field method gives $\sim 50$ MeV (or $\sim 20\%$) more binding to the $K^-$ than in the nugget, where $m^*_K \sim \frac{1}{2} m_K$. 21
In both cases, the medium dependence in the vector meson mass $m_\rho^*$ and $m_\omega^*$ provide $\sim 50$ MeV of the binding. Our arguments are admittedly far from rigorous. If this relation can be put on a more rigorous basis, it will supply a firm support for out limit of $1.5M_\odot$ as maximum neutron star mass. It is remarkable that in Table 1 out of $\sim 40$ measured masses of compact objects at most two or three of the masses seem to violate our upper limit. These and other astrophysical issues will be discussed in depth in a paper in preparation [6].

Dedication

We would like to dedicate this paper to Hans Bethe who fearlessly coauthored “A Scenario for a large number of low-mass black holes in the Galaxy” [2], the large number resulting from the low maximum neutron star mass of $1.5M_\odot$. Gerry Brown was explaining the idea of kaon condensation to Hans Bethe on a Saturday morning walk on a trail near Santa Barbara, California. Hans immediately understood it, “You mean that you squeeze electrons into $K^-$mesons?”

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A Appendix: Dense system with tribaryon-$\bar{K}$ crystals

In this Appendix, we construct a crystalized dense system comprised of tribaryon-$\bar{K}^-$ bound states. The assumptions we make in the construction of the crystal potential are:

1. The effective potential of the single tribaryon-$\bar{K}$ (ppn-$\bar{K}$, Isospin=1) proposed by Akaishi et al. [22] is used:

$$V(r) = (V_0 + W_0) \exp\left[-(r/a)^2\right]$$

with $V_0 = -702$ MeV, $W_0 = -13$ MeV and $a=0.923$ fm. By solving the Klein-Gordon equation,
Table A.1
Binding Energy and rms radius of a single tribaryon-$\bar{K}$ system in medium for various densities. Beyond $2n_0$, the bound tribaryon-$K^-$ system “dissolves” indicating that the crystallized system is unstable.

| $n/n_0$ | 1.09 | 1.37 | 1.74 | 1.97 | 2.02 |
|---------|------|------|------|------|------|
| B.E.(MeV) | -217.7 | -238.5 | -275.4 | -304.3 | -310.7 |
| $V_{\text{max}}$ | -136.6 | -188.3 | -258.1 | -302.2 | -311.9 |
| rms Radius(fm) | 0.816 | 0.842 | 0.862 | 0.868 | unbound |

\[(E - V(r))^2 - m_{\bar{K}}^2 - p^2|\psi = 0, \quad (A.2)\]

Akaishi et al. obtained the binging energy and rms radius of the tribaryon-$K$ as 194 MeV and 0.74 fm, respectively. In this estimate, the enhanced core energy of the tribaryon from the core shrinkage, $\delta E \simeq 110$ MeV, is assumed. Akaishi et al. show the average density of the tribaryon-$\bar{K}$ bound system to be $n \simeq 3.1n_0$.

(2) We put these tribaryon-$\bar{K}^-$ systems in a cubic lattice by assuming that the potential generated by the tribaryon is the same as that given by Akaishi et al.

(3) In a dense medium, a tribaryon-$\bar{K}$ bound system is affected by the surrounding tribaryon cores. In order to take this effect into account, we make an effective potential of a single site by making the average over all solid angle. Finally we have an effective potential with spherical symmetry.

With the above assumptions, we obtain the binding energies and the rms radii of the tribaryon-$K^-$ system. In order to take into account the density effects, we change the distances between these tribaryon-$K^-$ systems.

In Figure A.1, we see that the minimum and the threshold of the effective potential of a tribaryon-$K^-$ system are lowered as the baryon density increases due to the overlap of the potential given by the neighboring systems. As one can see from the figure, when the binding energy is equal to the maximum value of the potential, the bound state of the tribaryon-$\bar{K}$ system dissolves.

As in Table A, the bound state of the tribaryon-$\bar{K}$ disappears at $n/n_0 \simeq 2.0$. This indicates that for densities greater than $2n_0$, the system becomes continuous, losing the crystal structure. We interpret this as kaon condensation taking place from $2n_0$. Since we started with the fixed proton fraction (1/3) in this approach, the density at which the crystal dissolves cannot be directly compared with the critical density for kaon condensation in the mean field approach. However, our results provide a way to link tribaryon-$K^-$ bound states (nuggets) to kaon condensation.
Fig. A.1. Effective potentials for various baryon densities.

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