Fixing E-field divergence in strongly non-linear wakefields in homogeneous plasma

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Abstract
Available analytical wakefield models for the bubble and the blow-out regime of electron-plasma acceleration perfectly describe important features like shape, fields, trapping ratio, achievable energy, energy distribution and radial emittance. As we show, for wakefields with an extremely small amplitude these models fail to describe the accelerating electric field and its divergence in the wakefield rear. Since prominent parameter regimes like the Trojan horse regime of photocathode injection exhibit this feature, it is of great importance to work out analytical models that fix this problem; one possible model is introduced in this work. Using a phenomenological theory, we are able to better describe the divergence of the electric field and the bubble shape.

Keywords: wakefield acceleration, Trojan horse regime, particle-in-cell simulations, analytical theory

(Some figures may appear in colour only in the online journal)

1. Introduction
Plasma is an ionized medium that supports electric fields that are several orders of magnitude higher than those in traditional solid-state accelerating structures (metal, warm or superconducting). The accelerating field in plasmas can be excited either by an intense laser pulse with wavelength \( \lambda_L \), duration \( \tau \) and focal spot size \( R \) [1], or by a charged particle beam with length \( \sigma_z \), radius \( \sigma_r \) and density \( n_b \) [2, 3].

In plasmas with homogeneous density \( n_p \), the wakefield breaks as soon as the laser pulse intensity reaches a certain threshold value and the normalized laser amplitude \( a_0 > 1 \). If \( a_0 > 4 \), \( \lambda_p > R > 2 \lambda_L \) and if the laser pulse perfectly fits into the first half of the plasma period, a solitary electronic cavity, called the bubble, is formed [4–8]. It is a nearly spherical region with uniform accelerating fields that propagates with almost the speed of light \( c \) [9]. In homogeneous plasma, it traps background electrons at its tail and accelerates them to high energies. The major features that characterize the bubble regime are the quasi-monoenergetic spectrum of the accelerated electrons and the quasi-static laser pulse which propagates many Rayleigh lengths in homogeneous plasma without significant diffraction.

If the wakefield is excited by a thin \( \sigma_r \approx \sqrt{2k_p^{-1}} \gg \sigma_r \) and dense \( n_b > n_p \) charged particle beam, a spherical ion column, the so-called blow-out, is created. It is a structure similar to the laser driven bubble with a comparable accelerating field \( E^+ \) in its rear part. An important parameter for the energy gain of trapped electrons is the transformer ratio \( T = E^+ / E^- \). It describes the maximum energy gain of the witness bunch at simultaneous energy loss of the driver beam in its peak.
decelerating field $E^−$ [7, 10, 11]. If the total charge of the
witness beam exceeds a certain threshold, the plasma cavity
structure both in the bubble and blow-out regime is reshaped
and the effective accelerating field is modified. This in turn
affects final beam properties like maximum energy, energy
spread, total charge, transverse emittance, the spatial extension
in length and width, but also general acceleration parameters
like the transformer ratio [8, 12–17]. To optimize all character-
istic witness-beam parameters, an effective loading technique
is necessary.

A promising method to control beam loading is the ion-
ization injection technique. This method produces electron
beams with sub-fs temporal duration, a very high peak cur-
rent (several kA), energy spreads well below 1% for beams
with energies in the multi-GeV range and an excellent trans-
verse emittance (tens of nm rad) [11, 18–22]. The ionization
injection requires a small amount of higher-Z gas, added to
the gas used for acceleration [23, 24]. The ionization process
starts as soon as the local field strength exceeds the ioniza-
tion threshold. Therefore it can be triggered by the laser pulse
exciting the wakefield [19, 25, 26], by the wakefield itself [27],
by transversely colliding laser pulses [28, 29], or by a second,
trailling, laser pulse. In the latter case, wakefield excitation
and ionization are independent processes, which allows a pre-
cise manipulation of the phase space distribution of trapped
electrons and thus a generation of ultra-low emittance elec-
tron beams [30–37]. If the driving laser pulse is replaced by
a short electron beam, the Trojan horse regime (THWFA)
of underdense photocathode plasma wakefield acceleration
(PWFA) is reached [38, 39]. It is best suited for decoupling
the electron bunch generation process from the excitation of
the accelerating plasma cavity. Since the blow-out structure
is largely immune to shot-to-shot driver bunch characteristics
variations, the acceleration process can be controlled via
plasma density modulations, while the charge of the witness
bunch can be tuned by the release laser intensity. The combina-
tion of the non-relativistic intensities required for tunnel
ionization ($10^{14}$ W cm$^{-2}$), a localized release volume as small
as the laser focus, the greatly minimized transverse momenta,
and the rapid acceleration leads to dense phase space pack-
etcs. In homogeneous plasma they can have ultra-low normal-
ized transverse emittance of the order of nm rad and a min-
imal energy spread in the 0.1% range. Achieved electron ener-
gies after acceleration in the bubble or blow-out regime are in
the range of 1 to 8 GeV [11, 40–47], while simulations and
planned experiments aim for the generation of 10 GeV beams
in a single acceleration stage [20, 48–51].

In the following section, we first recap fields and forces
in the bubble for various plasma configurations and bubble
shapes—including small amplitude wakefields similar to those
known from Trojan horse regime. Afterwards, in section 3,
we present an analytical model for small amplitude wake-
fields, beginning with an approximation of the electron layer.
Already available theoretical bubble models for THWFA lose
their applicability toward the bubble back, since there the elec-
tric field exhibits a divergence. We derive field approxima-
tions for homogeneous plasma, introducing an artificial trick
to model the field divergence at the bubble back phenomeno-
logically.

2. Theoretical model: Forces

In the following we derive the radial and longitudinal force
components acting on a test electron moving inside the bubble
for various cases regarding plasma density and bubble shape.
Throughout the derivation we use normalized units, where
time is normalized to the inverse plasma frequency $\omega_p^{-1} =
\sqrt{4\pi e^2 n_e/m_e}$, lengths to $k_p^{-1} = c/\omega_p$, kinetic momenta
to $m_e c$, energies to $m_e c^2$, fields to $m_e \omega_p/e$, charges to the
elementary charge $e$, masses to the electron’s rest mass $m_e$, and
potentials to $m_e c^2/e$. Throughout the simulations, we use the
plasma wavelength of $\lambda_p = 200 \mu m$.

Starting point for all of the following calculations is the quasi-static
wavefield potential [9], which is generally given as

$$\Psi = A_z - \varphi. \quad (1)$$

As the bubble is moving with velocity $V_0$ close to the speed of
light, we express all functions in dependence of the coordinate
$\xi = z - t$. In this co-moving frame with cylindrical symmetry
and distance to the symmetry axis $r$, the magnetic and electric
field are

$$B = \nabla \times A = \left( \frac{\partial A_r}{\partial \xi} - \frac{\partial A_\xi}{\partial r} \right) \hat{e}_\varphi, \quad (2)$$

$$E = \nabla \Psi - \hat{e}_z \times B. \quad (3)$$

The written-out electric field components are connected to the
wavefield potential via

$$E_z = \frac{\partial \Psi}{\partial \xi}, E_r = \frac{\partial \Psi}{\partial r} + B_\varphi. \quad (4)$$

In general, the corresponding force components become

$$F_z = -E_z \frac{p_\gamma}{\gamma} B_\varphi, \quad (5)$$

$$F_r = -\frac{\partial \Psi}{\partial r} \left( 1 - \frac{p_\gamma}{\gamma} \right) B_\varphi, \quad (6)$$

where $\gamma = \sqrt{1 + |\mathbf{p}|^2}$ and $\mathbf{p}$ is the electron’s kinetic
momentum.

For the sake of generality, we consider an arbitrary radial
plasma profile $\rho(r)$ and follow the models of [52, 53] for
the definition of the wavefield potential in the bubble or blow-out
regime in dependence on the coordinates $\xi$ and $r$:

$$\Psi(\xi, r) = \int_0^r \frac{S_l(r')}{r'} dr' + \Psi_0(\xi). \quad (7)$$
Here,

\[ S_l(x) = \int_0^x \rho(r) r \, dr > 0 \]  

is the integral source, and

\[ \Psi_0(\xi) = -\int_0^\xi \frac{S_l(r)}{r} \, dr - \frac{S_{lb}\beta(r_b)}{2}. \]

From now on, we use \( S_{lb} \) as a shorthand notation for \( S_l(r_b) \).

The parameter \( \beta \) is in turn defined as

\[ \beta(r_b) = 2 \int_0^{\infty} \frac{e\nu_0(x) + eF_1(x)}{1 + e\nu_0(x) + eF_1(x)}, \]

where \( r_b(\xi) \) is the bubble radius, \( \epsilon = \frac{\Delta}{\rho} \) is the relative sheath width (with \( \Delta \) being the thickness of the electron sheath) and

\[ F_n(x) = \int_x^{\infty} y^\alpha g(y) \, dy \]

is the generalized moment of the function \( g(y) \) which describes the shape of the electron sheath at the bubble’s boundary, and \( F_0(0) = 1 \) is assumed. For a plasma with arbitrary radial density profile \( \rho(r) \), we have the integral components are

\[ E_z = -\left( \frac{S_{lb}}{r_b} \frac{S_{lb}\beta'(r_b)}{2} + \frac{\rho(r_b)\beta'(r_b) r_b}{2} \right) r_b', \]

\[ B_\varphi(\xi, r) = -r \frac{\partial E_z}{\partial \xi} - \Lambda(r, \xi), \]

\[ E_r = \frac{\partial \Psi}{\partial r} + B_\varphi = \frac{S_l(r)}{r} + B_\varphi. \]

With the integral current density

\[ \Lambda(r, \xi) = -\frac{1}{r} \int_0^\xi J_z(\xi, r') r' \, dr' \]

this leads to the following force components in the transverse and longitudinal directions:

\[ F_z = -E_z - \frac{p_\gamma}{\gamma} B_\varphi, \]

\[ F_r = -\frac{S_l(r)}{r} - \left( 1 - \frac{p_\gamma}{\gamma} \right) B_\varphi. \]

In the case of a homogeneous \( (\rho(r) = 1, S_l(r) = r^2/2) \) plasma, the components of the electric field exhibit the form

\[ E_r = \frac{r}{2} + B_\varphi, \]

leading to

\[ F_z = -\frac{r_\nu r_\nu'}{2} \left( 1 + \beta(r_b) + \frac{r_b\beta'(r_b)}{2} \right) - \frac{p_\gamma}{\gamma} B_\varphi, \]

\[ F_r = -\frac{r}{2} - \left( 1 - \frac{p_\gamma}{\gamma} \right) B_\varphi. \]

For a spherically shaped bubble we now set \( r_b = \sqrt{R^2 - \xi^2} \) with \( r_b' = -\xi \). If we further assume that the bubble’s sheath is thin, i.e. \( \beta = \beta' = 0 \), the electric and magnetic field components reduce to the well known form

\[ E_z = \frac{\xi}{2}, B_\varphi = -\frac{r}{4} - \Lambda(r, \xi), E_r = \frac{r}{4} - \Lambda(r, \xi). \]

In this case, the accelerating and focusing forces are

\[ F_z = -\frac{\xi}{2} + \frac{r_p}{4\gamma} - \frac{p_\gamma}{\gamma} \Lambda(r, \xi), \]

\[ F_r = -\frac{r}{4} \left( 1 + \frac{p_\gamma}{\gamma} \right) - \frac{1}{2\gamma} \Lambda(r, \xi). \]

Since the particles undergo fast betatron oscillations, we can assume that \( \langle r_p \rangle = 0 \) and since \( p_\gamma \approx \gamma \) after sufficient acceleration, the terms above can be simplified to

\[ F_z \approx -\frac{\xi}{2}, F_r \approx -\frac{r}{2}. \]
for trapped electrons. These forces are already known from simple models (compare e.g. [9, 54, 55]). They do, however, require a lot of restrictions regarding the current distribution in the electron sheath, the bubble shape, the bubble symmetry, and the background plasma density profile.

We lastly want to consider the more general case of a stretched bubble with small amplitude $R \ll \lambda_p$ as it can be found in [39]. If we generalize the bubble radius $r_b = R\sqrt{1 - (\xi/b)^2}$, $b > R$ to describe a stretched bubble shape, the electric field component in $\xi$-direction becomes $E_\xi = a \xi$ with $a \in (0, 1)$. If we stay in the thin-sheath approximation $\beta = \beta' = 0$, we get

$$B_\varphi = -\frac{r}{2} a - \Lambda(r, \xi), E_\varphi = \frac{r}{2} (1 - a) - \Lambda(r, \xi). \quad (26)$$

The resulting force components are

$$F_\xi = -a \xi + \frac{r p_r}{2} a + \frac{p_r}{\gamma} \Lambda(r, \xi), \quad (27)$$
$$F_r = \frac{r}{2} \left( 1 - a + e \frac{p_r}{\gamma} \right) - \frac{1}{2\gamma} \Lambda(r, \xi). \quad (28)$$

Similarly to our reasoning above, we still can assume that $\langle r p_r \rangle = 0$ and $p_r \approx \gamma$, thus

$$F_\xi \approx -a \xi, F_r \approx -\frac{r}{2}. \quad (29)$$

This shows that the elongation of the bubble does not change the focusing force, as long as the electrons are already trapped and fulfill the conditions of performing betatron oscillations and having high energy. The accelerating force, however, is dependent on the bubble elongation, as can be seen by the factor $a$ in the equation above.

In the following section we derive the connection between the bubble elongation $b$ and the field modulation factor $a$.

3. Theoretical model: Electron sheath

In this section we first find an analytical approximation to the ordinary differential equation that describes the electron sheath approximation function $r_b$. Afterwards, we take a look into field approximations for homogeneous plasma.

3.1 General electron layer approximation

In general, we solve the ODE [52]

$$A(r_b) r_b'' + B(r_b) (r_b')^2 + C(r_b) = \frac{\Lambda(r_b(\xi), \xi)}{r_b} \quad (30)$$

to find the electron sheath approximation function $r_b$. Here, the coefficient functions are

$$A(r_b) = 1 + \frac{S_{l,b}}{2} + \left( \frac{\rho(r_b)}{4} + \frac{S_{l,b}}{2} \right) \beta + \frac{S_{l,b} r_b}{4} \beta', \quad (31)$$
$$B(r_b) = \frac{\rho(r_b) r_b}{2} + \left[ 3 \rho(r_b) r_b + \rho'(r_b) r_b^2 \right] \frac{\beta}{4}$$

$$+ \left[ S_{l,b} + \rho(r_b) r_b^2 \right] \frac{\beta^2}{2} + \frac{S_{l,b} r_b}{4} \beta^3. \quad (32)$$

$$C(r_b) = \frac{S_{l,b}}{2 r_b} \left[ 1 + \left( 1 + \frac{S_{l,b} \beta}{2} \right)^{-2} \right]. \quad (33)$$

If $\Delta \ll r_b$ and $\Delta \gg 2 r_b/S_{l,b}$, these functions can be simplified dramatically, so that

$$A(r_b) \approx \frac{S_{l,b}}{2 r_b}, B(r_b) \approx \frac{\rho(r_b)}{2}, \quad (34)$$

$$C(r_b) \approx \frac{S_{l,b}}{2 r_b}, \Lambda(\xi) = \Lambda(r_b(\xi), \xi) \quad (35)$$

for usual bubbles with rather large radii in the range of one plasma wavelength ($\lambda_p = 2\pi k_p^{-1}$). Unfortunately, this approximation is not valid for cases similar to the Trojan Horse regime [38], where the radius is in the order of $0.25 \lambda_p \approx 1.5 k_p^{-1}$ and thus $S_{l,b}/2$ is in the order of unity. To compensate this circumstance, we modify the coefficient functions phenomenologically to

$$A(r_b) \approx 1 + \frac{S_{l,b}}{2 r_b}, B(r_b) \approx \frac{\rho(r_b)}{2}, \quad (36)$$

$$C(r_b) \approx \frac{S_{l,b}}{2 r_b}, \Lambda(\xi) = \Lambda(r_b(\xi), \xi) \quad (37)$$

An analytical approximation to the ODE solution can be found for non-loaded ($\Lambda = 0$) bubbles and small $|\xi|$ near the bubble center (also compare [12]). Here,

$$r_b \approx R, r_b' \approx 0, r_b'' \approx -\frac{C}{A} \approx -\frac{S_{l,b}}{(2 + S_{l,b}) R} \quad (38)$$

and

$$S_{l,b} = \frac{R^2}{2} \quad (39)$$

for homogeneous plasma, so that

$$r_b \approx R - \frac{1}{1 + 4 R^{-2}} \frac{\xi^2}{2 R} \quad (40)$$

Comparing the solutions to equations (31)–(33) with the PIC results would show that they coincide with the simulations. The analytical expression would be sufficient near the bubble middle but could not describe the bubble back.

As we can see in figure 3, the correct choice of $\Delta$ and $R$ leads to a perfect fit of the bubble border. The electric field in the separatrix however is not described well using this ansatz (compare figure 1). This is due to the fact that $E_\xi$ is proportional to $r_b''$, but $r_b''$ is very flat. The strong field in the separatrix leads to these problems of description. This is why we introduce a new methodology in order to fix this effect.
The analytical model with repair function (red dashed lines) is able to describe the fields well in the bubble mid and even toward its back, in the region around $\xi \leq -100 \mu m$.

3.2. Field modulation near bubble back for homogeneous plasma

If we stay within the thin sheath approximation, it is sufficient to use the simplified expression

$$E_z = -S_{1,b} \frac{r_b'}{r_b},$$

(41)

for the longitudinal electric field inside the bubble. If we approximate $E_z$ by its lowest order for small $\xi$ in homogeneous plasmas we see that

$$E_z \approx E_0 \xi,$$

(42)

where

$$E_0 = \frac{1}{2 + 8 R^{-2}}.$$  

(43)

In this form $|dE_z/d\xi|$ cannot exceed $1/2$, which is the upper limit for very large ($R \gg 1$) bubbles. In the same way, $|dE_z/d\xi|$ is limited by zero, which is the lower limit for very small amplitude ($R \ll 1$) bubbles.

If we compare $E_z$ from (12) to the actual form from PIC simulations in figure 1 for homogeneous plasma we see that (12) does not describe the progression correctly in the rear bubble back. However, the model is in quite good accordance near the bubble mid. The most likely reason for this deviation is the fact that the predicted bubble radius $r_b$ does not decrease strongly enough and that at the bubble rear part so far ignored electron currents become important for the field configuration.

To incorporate the behavior of the $E_z$-field near the bubble stern, we assume that the field diverges at $L = \xi (r_b = 0)$. The magnitude of the divergence cannot be reproduced from the current bubble model so we search for a function $f(\xi)$ that bears the divergence but does not change $E_z$ near the bubble mid. A good choice for such a function is

$$f(\xi) = \left( \frac{L}{L - \xi} \right)^{1/m} = \frac{1}{\sqrt{1 - \xi / L}}$$

(44)

which has the desired features

$$\lim_{\xi \to L} f(\xi) = \infty, f(\xi \approx 0) \approx 1.$$

(45)

Since $E_z$ from (42) is acceptable in a whole range near the bubble mid, we should choose a parameter $m > 1$ to reduce the influence of $f(\xi)$ on $E_z$ for too small $\xi$. A rather rough fit to PIC simulations shown in figure 2 yields $m \in [3, 4]$ being a good choice. Finally, the quasi-analytical longitudinal electrical field in a stretched bubble is

$$E_z = \frac{E_0 \xi}{\sqrt{1 - \xi / L}}.$$  

(46)
If we follow the argumentation for the forces occurring in homogeneous plasma, we get

\[ B_\varphi = -\frac{r}{2} \frac{E_0(4L - 3\xi)}{4L\sqrt{1 - \frac{\xi}{L}}}, \]  

(47)

\[ E_\varphi = \frac{r}{2} \left( 1 - \frac{E_0(4L - 3\xi)}{4L\sqrt{1 - \frac{\xi}{L}}} \right), \]  

(48)

\[ F_\zeta \approx -\frac{E_\varphi}{\sqrt{1 - \frac{\xi}{L}}}, \]  

(49)

\[ F_r = -\frac{r}{2} + \left( 1 - \frac{\nu_r}{\gamma} \right) \frac{E_0(4L - 3\xi)}{4L\sqrt{1 - \frac{\xi}{L}}}. \]  

(50)

4. Conclusion

Current analytical models of the bubble or blow-out regime of plasma wakefield—while being very successful at describing the boundary of the bubble—fail to describe the divergence of the accelerating electric field at the wakefield rear in the bubble for extremely small amplitude wakefields which are relevant for Trojan horse injection regime of PWFA. In this paper, we have derived a phenomenological theory that fixes this problem using a repair function. The repair function was chosen to correspond to the results of PIC simulations for bubbles of different size in the case of homogeneous plasmas. This model can be used to study the acceleration of electrons in more physically sound accelerating and focusing plasmas. This model can be used to study the acceleration of electrons in more physically sound accelerating and focusing plasmas. This model can be used to study the acceleration of electrons in more physically sound accelerating and focusing plasmas. This model can be used to study the acceleration of electrons in more physically sound accelerating and focusing plasmas.

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