Long-range attraction between particles in dusty plasma and partial surface tension of dusty phase boundary

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I. INTRODUCTION

A gas–solid phase transition observed in different dusty laboratory plasmas [1–8]. It counts in favor of presence of strong long-range attraction between similarly charged dusty particles. By now some physical mechanisms are proposed to explain formation of a regular arrangement of micron-sized particles embedded in a gas discharged plasma. A part of them is based on the account for electrostatic fields of strata and walls of discharge tubes[9]. In doing, so a generality of solution of the problem of the effective attractive potential is lost and, in particular, the situation of dusty plasma crystal in a thermal dusty plasma of gas burner [5] where there are no walls and external fields drops out of such interpretations.

The most promising approach holds that the ion streaming motion causes an attractive wake potential behind the dust particles. Originally it was developed [10, 11] for particular case of supersonic flows which is realized in the sheath of radiofrequency discharges. Later, it was extended with participation of one of the authors of Ref. [10] to the case of subsonic ion flows [12], but physics of shielding of dust charged particles was supposed to be strongly modified with mandatory regard to anisotropy and asymmetry of the ion temperature in the sheath.

Below is shown that the attraction between similarly charged particles can be resulted from a dynamical screening of the Coulomb potential remaining in the frame of a single physical mechanism for both supersonic and subsonic regimes. The distinction of the dynamical screening from the static Debye screening is due to a motion of particles relative to screening charges (countersions or electrons). From the physical point of view, this effect can be interpreted as a consequence of loss of spherical symmetry of the Debye screening cloud around the moving charged particle resulted in a space charge with the opposite sign forming in its wake. Not only do this space charge compensates the particle’s potential but may also give rise to a local wake potential of the opposite sign. In the supersonic regime the wake potential oscillates inasmuch of Cherenkov wave generation.

This effect is well known in the usual electron-ion plasma [13–15]. When the charge is moving, the static Debye screening modifies so that the potential in the oncoming flow of electrons grows approaching to the Coulomb potential with an increasing of velocity while the potential in the outward flow decreases up to alternating of its sign.

Similar effect was found [15, 16] in a system of gravitating masses where the static screening is absent and accounting for the dynamical screening gives rise to an alternating potential of gravitational interaction.

The aim of the present paper is to propose the most general model of dynamical screening of field of dusty particle charge to explain the observable interparticle attraction. It may be supposed that the distance from the particle to the attractive minimum of the wake potential determines a period of the resulted lattice of dusty particles.

Besides, the interparticle attraction is to give rise to a surface tension of a dusty phase interface. Really, this explains a sharp non-diffusion character of a particle density variation observed at the interfaces in the experiments on dusty plasma crystals. The most characteristic phenomenon of such kind are voids in a homogeneous dusty plasma. The characteristic lenticular form of the voids may be explained by strong peculiar dependence of the surface tension on the surface orientation relative to the ion flow.

The notion of the surface tension of a dusty phase was mentioned briefly by Tsitovich [17] but his estimation of its value was erroneous. Conceivably that might be the reason why the surface tension was not mentioned in the subsequent papers by Tsitovich (as a co-author) on the theory [18, 19] of spherical voids.

The outline of the paper is the following. In Sec. II, the model for effective non-potential attractive forces of in-
Interactions between dust particles will be developed. This model can explain an appearance of the lattice structure both in gas discharge dusty plasma and in thermal dusty plasma of gas burner. In Sec. III, a surface tension of a boundary of dusty phase is estimated and conditions of stability of a lenticular void in the dusty plasma are examined.

II. EFFECTIVE INTERPARTICLE INTERACTION

The system under consideration consists of negatively charged dusty particles of the charge \( Q = -Z_p e \) and concentration \( n_p \), and positively charged counter-ions of the charge \( e \) (for definiteness sake we will consider singly charged ions) and mass \( m \). For simplicity, neutral molecules of a buffer gas are neglected here and dusty particles are considered as point charges.

While forming of the negative charges of dusty particles the electrons are condensed on particles’ surfaces and their concentration in plasma decreases sufficiently. As a result, in the case of great \( Z_p n_p \) their density becomes so little that the electron Debye radius of screening of the ion charge appears to be much greater the ion Debye radius of screening. Thus, we can restrict our consideration to the simplified model of one-component ion plasma in which the negatively charged dust particles are immersed.

If the negative test point charge \( Q \) is moving in the system of ions and dusty particles with the velocity \( \mathbf{u} \) it gives rise to some perturbations of the system state. Due to great difference in masses and concentrations of ions and dusty particles we can restrict our consideration to a perturbation of the ion component only (just as a perturbation of an electron component is taken into account only, as a rule, in the problem of screening of an ion charge in the electron-ion plasma). Such perturbation of ionic subsystem is described by the set of the Vlasov equation for a distribution function over coordinates and velocities of ions and Poisson equation for an effective potential induced by the perturbation of ion density and moving test charge. Interactions between ions and neutral molecules of the buffer gas are neglected here [20].

Such simplified model is well studied in the test-particle approach to the electron-ion plasma theory [13–15] so we can omit intermediate calculations and write down at once the resulted form of the effective potential of the moving test charge

\[
\Phi(r, t) = \frac{Q}{2\pi^2} \int d^3 k \exp\left\{i \mathbf{k} \cdot (r - ut)\right\} \frac{1}{k^2 \varepsilon(k, \mathbf{k} \cdot \mathbf{u})} \tag{1}
\]

where

\[
\varepsilon(k, \mathbf{k} \cdot \mathbf{u}) = 1 + \frac{k^2}{k^2} W \left( \frac{\mathbf{k} \cdot \mathbf{u}}{k \tilde{v}} \right) \tag{2}
\]

is the dynamical permittivity of the ion subsystem. Here \( \kappa = (4\pi e^2 n_i/(k_B T_i))^{1/2} \) is the ionic Debye wave number, \( \tilde{v} = (2k_B T_i/m)^{1/2} \) is the mean heat velocity of the ions and

\[
W(t) = 1 - \sqrt{\pi} t e^{-t^2} \text{erfi}(t) + i\sqrt{\pi} t e^{-t^2} \tag{3}
\]

Let us choose an axis \( z \) along \( \mathbf{u} \) and introduce dimensionless variables \( Z = (z - ut)\kappa, \ X = x\kappa, \ K = k/\kappa \) and the Mach number \( M = u/\tilde{v} \) relative to the ion heat velocity. Then, in the test particle’s frame accounting for the cylindrical symmetry, we get

\[
\Phi(r) = \frac{Q\kappa}{2\pi^2} \int_0^\pi d\phi \int_0^\pi d\theta \int dKK^2 \sin\theta \frac{\exp\{iK\Delta\}}{K^2 + W(M \cos\theta)} = Q\kappa \varphi(X, Z; M), \tag{4}
\]

\[
\varphi(X, Z; M) = \frac{1}{(X^2 + Z^2)^{1/2}} - I(X, Z; M),
\]

where

\[
I(X, Z; M) = \frac{1}{2\pi^2} \int_{-\pi}^\pi d\phi \int_0^\pi d\theta \int dK W(M \cos\theta) \sin\theta \frac{\exp\{iK\Delta\}}{K^2 + W(M \cos\theta)}, \tag{5}
\]
This integral determines the departure of the effective potential from the Coulomb potential to which the first member of \( \varphi(X, Z; M) \) corresponds. When the Cauchy integral over \( K \) is taken, we find

\[
I(X, Z; M) = \frac{1}{4\pi^2} \int_{-\pi}^{\pi} d\phi \int_{0}^{\pi} d\theta \sin \theta \sqrt{W} \left[ 2i \sinh(\Delta \sqrt{W}) \text{Ci}(-i \Delta \sqrt{W}) + \cosh(\Delta \sqrt{W}) \left( \pi + 2i \text{Shi}(\Delta \sqrt{W}) \right) \right].
\]

(6)

It is not difficult to calculate numerically the value of this integral in an arbitrary range of the parameter \( M \) variations at any point \( X, Z \).

Corresponding results of numerical calculations for \( M = 3 \) are illustrated in Fig.1. It is seen here that there is well defined negative minimum of the potential along the axis \( Z \) at a distance of order of five Debye radius, which has been formed as a result of deformation of screening Debye cloud in the vicinity of the moving particle.

In gas discharges, where forming of crystal structure of dusty particles was observed, a diffusion ion velocity \( u \) is determined by their mobility and an electric field strength. To define the mobility, we are to take into account a tendency of ions to unite with molecules and atoms into complexes of the types \( N^+_4, O^+_4, Ne^{+}_2, He^{+}_2 \), and recharge effect also. According to [22], the diffusion velocity of the complex \( Ne^{+}_2 \) is of order of 50 mc\(^{-1}\) in a characteristic for glow-discharge field \( E/p = 1 \) V cm\(^{-1}\)torr\(^{-1}\) and the heat velocity \( \tilde{v} = 400 \) mc\(^{-1}\) (at \( T = 300 \) K). Then \( M = 0.125 \). For the ion \( Ne^{+} \), the diffusion velocity decreases near-threefold as a consequence of the resonant recharge, then \( M \approx 0.05 \). In this case of strong field, the ion drift velocity may not only approach to the heat velocity but surpass it [22]. Because of this, a wide range of values of the parameter \( M = u/\tilde{v} \) will be considered below (asymptotic case \( M \gg 1 \) was discussed in [10, 11]).

Observed quasi-crystal structure in a thermal plasma, that is, in the dusty flame of gas burner [5, 8, 23], can be treated also as a consequence of movement of charged particles relative to ions of the flame. To obtain quantitative estimations of the effect, the additional data on a gap of velocities of particle and ion flows are necessary. This gap is determined by conditions of injection of the dusty particles into the flame, and in the steady state limit it is likely to go to a sedimentation velocity of charged particles in the rising flow.

The graphs of \( \varphi(X = 0, Z < 0; M) \) for different \( M \) in the wake of charged particle are illustrated in Figs. 2, 3. As can be seen, the depth of the negative minimum \( |\varphi_{\text{min}}| \) increases with an increasing of \( M \) from 0.01 to 1 (see Fig.4), and its location \( Z_{\text{min}} \) shifts to the particle. At low \( M \) \( |Z_{\text{min}}| \) depends on \( M \) as \( |Z_{\text{min}}| = 1.6 - 1.7 \ln M \) (see Fig.5). When further increasing of \( M \), the potential becomes oscillating and its first minimum moves away from the particle. For \( M > 2 \) the graph in Fig. 5 can be well approximated by the expression \( |Z_{\text{min}}| = 1.8\sqrt{M^2 - 1} \), which is close to the experimental data [1–4] on the period \( L \) of a crystal lattice in a dusty plasma if it is identified as \( L \approx |Z_{\text{min}}| \). It should be noted here that experimental detection of the minimum of \( L(M) \) corresponding to the minimum of the graph in Fig. 5 would be a good evidence in favor of the approach under discussion.

The values \( M > 1 \) correspond to a supersonic movement when Cherenkov radiation of ion-akoustic waves takes a place that explains an appearance of space oscillations of the wake potential. It should be noted that in the range of values of \( M \) from 1 to \( 3.3 \) an increasing of the depth of the first minimum takes a place but at further gain of \( M \) the depth of the first minimum decreases (see Fig.4).

In the incoming flow \( (Z > 0) \) the effective potential varies continuously with a gain of \( M \) from the Debye potential \( \sim \exp(-Z)/z \) to Coulomb one \( \sim 1/z \). This indicates that the forward part of the Debye screening cloud is not pressed by the incoming flow to the particle but blew away from it.

The negative minimum of the wake potential points to presence of a space charge of the opposite sign. Thus, the large charged particles moving relative ion subsystem together attendant space charges constitute a system of dipoles oriented along the direction of the relative motion. This gives rise to formation of hierarchies of particles-dipoles along lines of the ion flow.

Moreover, this interaction between dust particles is

\[
\Delta = X \sin \theta \cos \phi + Z \cos \theta.
\]

FIG. 2: The same as in Fig. 1 along the axis \( Z \) at \( M = 0.1, 0.2, 0.3 \), correspondingly.
asymmetric in such a way that attractive force is communicated only downstream the ion flow. This situation was clearly demonstrated in the experiments \[24, 25\] where upper or lower particle was pushed by the laser beam and it was just lower dust particle that fitted its position when the upper particle was shifted and not vice versa. So heavily non-conservative character of the interparticle interactions excludes any possibility to introduce an effective potential of the interaction.

Besides, as it is seen in Fig.1, the resulted negative potential is long-range in the transverse direction \(X\), also, that provides an attraction between particles from neighbor hierarchies and, in its turn, causes the particles from neighbor hierarchies to shift relative one another along the ion flow for a distance of order of a half dipole length. Because of this interaction is much weaker then along the flow, the situations can take place when the lengthwise attraction appears quite strong for the hierarchies formation but too weak for transverse ordering \[8\].

Thus, the movement of the charged particle relative ion flow gives rise to anisotropic potential field in which an energy of another similar particle of the charge \(Q\) is \(U(r) = Q\Phi(r) = Q^2\kappa\varphi(r)\). Then, the effective force of interaction between the particles is

\[
F(r) = \frac{\partial U(r)}{\partial r} = -Q^2\kappa \frac{\partial \varphi(r)}{\partial r} \tag{7}
\]

For the formation of the crystal structure, it is necessary that the maximum deep of the attraction energy \(|U_{\text{min}}| = |Q\Phi_{\text{min}}| = Q^2\kappa|\varphi_{\text{min}}|\) surpass the heat motion energy, that is,

\[
\frac{|U_{\text{min}}|}{k_B T_p} > 1, \quad \text{or} \quad |\varphi_{\text{min}}| > \frac{k_B T_p}{Q^2\kappa} \tag{8}
\]

where \(T_p\) is the kinetic temperature of the particle. According both experimental data \[26\] and theoretical estimations \[27\] \(T_p\) may sufficiently surpass the ion temperature. For definiteness sake, we take \(T_p = 1000\),

\[
Z_p = 10^4, \quad \kappa = 400 \text{ cm}^{-1}, \quad \text{then the condition of the hierarchy stability becomes} \quad |\varphi_{\text{min}}| > 10^{-5}. \quad \text{According to the approach under discussion (see the graph in Fig. 4), this condition is fulfilled when} \quad M > 0.01.
\]

The long-range attraction between likely charged dusty particles resting in an equilibrium plasma \((M = 0)\) was found in \[28\]. It is possible that this kind of attraction can give rise to forming of crystal structure but it is irrelevant to the observed \[1–4\] dependence of the lattice period on the ion flow velocity and forming (at small \(M\)) of hierarchies along the flow non-interacting with which other.

III. SURFACE TENSION OF DUSTY PHASE

One of characteristic feature of a dispersed phase in dusty plasma is presence of sharp (non-diffusion) boundary surface of a dusty cloud (“dusty drop”). Considerable recent attention has been focused on the problem of formation of voids (“dusty bubbles”) of the dispersed
charged particles. It is reasonable to suppose that both phenomena are of the same nature related with presence of a surface tension resulted from the attractive interaction between charged particles.

In general, the surface tension coefficient of an interphase boundary is defined as the integral of a difference between transverse \( p_\perp \) and longwise \( p_\parallel \) pressures over an interphase layer of thickness \( l \)

\[
\gamma = \int_{-l/2}^{l/2} (p_\perp(z_1) - p_\parallel(z_1))dz_1
\]

(9)

where the axis \( z_1 \) directed along the normal to the layer.

On the molecular level, a pressure can be written in the form of a virial equation of state and expressed via a number density \( n(z_1) \), virial of a force of intermolecular interaction \( F(r) \) and radial correlation function \( g(r) \), where \( r = r_2 - r_1 \) is the distance between particles. For a bulk equilibrium fluid the function \( g(r) \) is calculated as a rule with the use of the Born–Green equation. At an interphase surface the situation is much more complex; this being so, notably rough approximations are used here. One of them is Fowler’s step approximation, when the number density \( n(z_1) \) is regarded as constant \( n_0 \) within the liquid phase and zero outside the liquid, and radial function \( g(r) \) within the surface layer is assumed to be the same as in the bulk liquid. Then the surface tension coefficient is determined by the Fowler formula [29, 30]

\[
\gamma = -\frac{\pi n_0^2}{8} \int_0^\infty r^4 F_r(r)g(r)dr, \quad z > 0.
\]

(10)

It is implied here that the \( z \)-component of the vector \( r \) is normal to the surface. Kirkwood and Buff [31] estimated the surface tension of liquid argon on the base of this formula and obtained quite satisfactory value (error \( \sim 25\% \)).

It is supposed in the Fowler’ formula that the intermolecular interaction is spherically symmetric that does not allow to apply this formula as such to the boundary of dusty phase.

Rerderiving the Fowler formula with due regard for the anisotropy of interparticle interaction we obtain

\[
\gamma = -\frac{n_0^2}{2} \int [z^2F_z(r) - xzF_x(r)] g(r)dr, \quad z > 0.
\]

(11)

where \( F_z \) and \( F_x \) are \( z \) and \( x \) components of the force (7).

When applying to the boundary of dusty phase, the pressures \( p_\perp \) and \( p_\parallel \) are to be considered as partial pressures of dusty component. Then, the surface tension of dusty phase should be interpreted as partial property too.

To estimate its value some simplification will be introduced. Firstly, the radial distribution will be used in the form of the Boltzmann factor

\[
g(r) = \exp\left(-\frac{U(r)}{k_BT}\right),
\]

that is typical for dilute systems. Secondly, when estimating the surface tension, the interactions within hierarchies will be regarded only as it is much stronger interactions between particles from different hierarchies. Then, for surfaces oriented across and along the ion flow we get

\[
\gamma_\perp = -\frac{n_p^2}{2} \int z^2F_r(r)g(r)dr, \quad z > 0.
\]

(12)

\[
\gamma_\parallel = \frac{n_p^2}{2} \int xzF_r(r)g(r)dr, \quad z > 0.
\]

(13)

The axis \( z \) in both equations directed along the normal to the surface and the forces \( F_z(r) \) in the equation for \( \gamma_\perp \) and \( F_x(r) \) in the equation for \( \gamma_\parallel \) are determined by interactions along hierarchies.

A numerical estimation of these expressions for \( n_p \sim 10^6 \text{ cm}^{-3} \) and \( M = 0.01 \) leads to \( \gamma_\perp \sim 10^5 \text{ din cm}^{-1} \) and \( \gamma_\parallel \sim -10^4 \text{ din cm}^{-1} \). Both of these coefficients in absolute magnitude surpass sufficiently the typical surface tension coefficient of a liquid at normal conditions \( \gamma_{liq} \sim 10^2 \text{ din cm}^{-1} \) (as an example, for water \( \gamma_{liq} = 70 \text{ din cm}^{-1} \)). Such great difference is quite natural. Really, the negative minimum \( U_m \) of the potential \( U \) at given conditions is of order of \( -10^{-12} \text{ erg} \) while the depth of the potential well of, for example, the Lennard-Jones potential for noble gases is of order of \( -10^{-14} \text{ erg} \). Inasmuch as the potential \( U \) enters (with sign minus) into the exponent of the radial distribution function, the surface tension coefficient appears to be very sensitive to its value. As a result, we have \( |\gamma_\perp| \gg \gamma_{liq} \) in spite of \( \gamma \propto n^2 \) and \( n_p \ll n_{liq} \).

Hydrodynamic stability of a spherical gas bubble or liquid drop is determined by value of a Weber number...
\( We = \rho v^2 a / \gamma \), which is the ratio of a dynamic head \( \rho v^2 \) to a surface tension pressure \( \gamma / a \), where \( a \) is the radius of the bubble or drop. For the case of the void in dusty plasma, the dynamic head \( \rho v^2 \) is determined by the flow of ions and neutral gases with the velocity \( v = u = \bar{v} M \approx 5 \cdot 10^4 M \text{ cm}^{-1} \). We can put \( \gamma \sim \gamma_{\perp} \) for the slightly curved surface of a lenticular void with the radius of curvature \( \alpha \sim 2 \text{ cm} \) sufficiently greater its size. Then at characteristic pressure of order of 0.5 torr we have \( We \approx 10^4 M^2 / \gamma_{\perp} \). The coefficient \( \gamma_{\perp} \) increases with increasing of \( M \), so that the Weber number \( We \) attains its the most great value at the minimal from considered here values of \( M \), i.e. at \( M = 0.01 \) when \( \gamma_{\perp} \approx 10^5 \). Then \( We_{\text{max}} \approx 10^{-4} \). When the Weber number is so small the surfaces of the lenticular void are stable, certainly. Their deviations from a spherical form can be due to variations of \( \gamma \) relative to \( \gamma_{\perp} \) as a consequence of inevitable (although small) distortions of the condition of orthogonality to the flow on the curved surface.

Stability of the lenticular void ensures possibility of its existence. The problem of its appearance remains to be solved. It may be suggested that a phase transition of the first kind takes a place in a homogeneous dusty plasma, and strong anisotropic surface tension can give rise to a peculiar nucleation process.

IV. CONCLUSIONS AND DISCUSSIONS

Attractive interactions between likely charged colloidal or dust particles in plasma have been discussed in this effort in a view to explain the observed crystal structure formed from these particles. The crucial point of the model under discussion is the presence of the counter-ion flow. Its velocity \( u \) relative the dust particles is not necessary to exceed the heat velocity \( \bar{v} \) of counter-ions (or ion–acoustic velocity) as it was supposed in ref. [10]. Even at small \( u \) when \( u/\bar{v} \approx 10^{-2} \), the effective attractive interaction appears to be sufficiently strong to ensure stability of regular hierarchies of particles oriented along the ion flow.

Another consequence of strong attractive interparticle interactions is the partial surface tension \( \gamma \) of the dusty phase boundary.

This concept was briefly discussed by Tsytovich [17] who estimated \( \gamma \) as a work necessary for construction of a bulk liquid of the unit surface and height \( h \). As a result, he obtained \( \gamma = U_m n_p h \), then he took \( |U_m| = 100 \text{ eV} \) that correspond to our result at \( M = 1 \) and found \( \gamma \approx 10^{-2} \text{ din cm}^{-1} \) neglecting the negative sign of \( U_m \). If he accounted for \( U_m < 0 \) he would get \( \gamma < 0 \), that is natural as the work for construction of the coupled state is to be negative.

Here, the surface tension is estimated on the base of a generalization of the Fowler’s formulae taking into account the anisotropy of interparticle interactions. As a result, the strong dependence of \( \gamma \) on orientation of the dusty interface is found. In particular, when \( u/\bar{v} \approx 10^{-2} \) we get \( \gamma_{\perp} \approx 10^5 \text{ din cm}^{-1} \) and \( \gamma_{\parallel} \approx -10^4 \text{ din cm}^{-1} \) for surfaces oriented across and along the ion flow, correspondingly. So great positive and negative value results in the characteristic lenticular form of the voids in dusty plasma having no surfaces along the ion flow. Such a form was observed both under micro-gravity condition (see Fig. 6) and in terrestrial experiments by Samsonov and Goree (see Fig. 5d in Ref.[32]). They observed also an appearance of a void mode as a penetration of a finger-shaped (in a vertical section) dust free region through the side boundary of the gas discharge (see Fig. 5c in Ref.[32]). A sharp end of the finger and its fast travel across the gas discharge volume count in favor of negative surface tension of the interface at its end. It means that an account for non-spherical form of the void is necessary for any theoretical model of the void.

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