Three-dimensional interaction of a shock with a thin non-circular channel of low-density gas

O G Sutyrin, P Yu Georgievskiy and V A Levin
Institute of Mechanics of Lomonosov Moscow State University, 1 Michurinsky prosp., Moscow 119192, Russia

E-mail: sutyrin@imec.msu.ru

Abstract. Numerical simulation of three-dimensional interaction of a shock with thin circular or non-circular – elliptic or rectangular – low-density gas channel is presented. Euler’s equations for inviscid gas are solved using 5th-order finite-difference WENO scheme. Large-scaled shock wave precursor structure is described in detail. It is shown that in three-dimensional flow internal shear layer instabilities develop faster than in axisymmetric case. High-pressure jet cumulation effect is found to amplify moderately for elliptic and rectangular cases. An influence of channel cross-section shape on the precursor growth rate is studied. It is found that the duration of linear precursor growth phase is greatly increased for stretched channel cross-section shapes.

1. Introduction
The effect of the strong restructuring of the shock wave front in shock tubes was first described in 1954 [1]. Similar phenomenon was found to occur during the propagation of blast waves produced by intense explosions [2]. A shock front region adjacent to the surface outruns the main shock front which leads to the formation of an oblique shock wave that propagates in the undisturbed gas at significant distance from the wall. The whole shock-wave structure that runs ahead of the main shock front got the name “precursor”. The flow disturbance is caused by interaction of incident shock wave with thin hot-gas layer formed near the surface heated by shock front radiation. Similar effect of the large-scale flow restructuring due to the presence of a “thermal spike” underlies the idea of reducing the wave drag of blunt bodies using local energy deposition into the oncoming supersonic gas flow [3]. In [4] it is shown that the sidelong precursor shock angle depends only on gas layer temperature and the precursor growth is assumed to eventually become nearly self-similar with constant growth rate. Although later computations for larger timescale [5] have shown that linear precursor growth pattern is unstable and the growth is slowed down with time due to the “flow choking” effect and shear layer instabilities inside the precursor.

The present work presents the study of three-dimensional interaction of a shock with generalized cylindrical channel of high-temperature (low-density) gas and specifically the influence of cylinder cross-section shape on the key flow features.

2. Problem formulation and mathematical model
Unsteady three-dimensional gas flows are modelled with Euler’s equations:
Figure 1. Initial condition scheme: \textit{is} – initial shock wave, \textit{ce} – low-density gas channel edge.

\[
\frac{\partial}{\partial t} \begin{pmatrix} \rho \\ p \\ \rho u \\ \rho v \\ \rho w \\ e \end{pmatrix} + \frac{\partial}{\partial x} \begin{pmatrix} \rho u \\ p + \rho u^2 \\ \rho u v \\ \rho u w \\ (e + p) u \end{pmatrix} + \frac{\partial}{\partial y} \begin{pmatrix} \rho v \\ p + \rho v^2 \\ \rho v w \\ (e + p) v \end{pmatrix} + \frac{\partial}{\partial z} \begin{pmatrix} \rho w \\ p + \rho w^2 \\ \rho v w \\ (e + p) w \end{pmatrix} = 0,
\]

where \( \rho \) is gas density, \( p \) is pressure, \( u, v \) and \( w \) are velocity components along Cartesian axes \( x, y \) and \( z \), correspondingly, and \( e \) is total gas energy per volume unit:

\[ e = p/\gamma - 1 + \rho \left( u^2 + v^2 + w^2 \right)/2 \]

For numerical simulation, 5th-order finite-difference WENO-Z scheme [6] is used. The method is supplemented with H-correction procedure [7] for suppression of transversal shock wave instabilities (“carbuncles”) typical for high-order numerical schemes. For comparison with axisymmetric flow, the scheme has been adapted for two-dimensional axisymmetric Euler’s equations as in [8].

Interaction of a strong plane shock wave – pressure jump – with a thin channel filled with reduced-density gas is considered. For perfect gases reduced density is equivalent to elevated temperature and thus such gas channel represents a simple model of “thermal spike”. The initial condition scheme is given in figure 1. The reference frame is tied to the shock: gas containing cylindrical area of reduced density flows with supersonic velocity along the cylinder axis which is aligned normally to the shock front. Perpendicular cross-section of the cylinder has the shape of circle, ellipse or rectangle. Its geometry is set by ellipse diameter ratio (or rectangle side ratio) \( b \), so that value \( b=1 \) corresponds to round (or square) shape.

Gas flow in front of the shock \( x < 0 \) is uniform in respect to velocity and pressure and fully uniform behind the shock \( x \geq 0 \). At the initial time moment \( t=0 \) cylinder end face touches the shock front. Gas parameters behind the shock are calculated using Rankine-Hugoniot conditions.

Pressure and density of undisturbed gas in front of the shock are assumed equal to unity. Only one parameter with length dimension – channel diameter – is present in the problem formulation. Thus it is convenient to set it as the length scale \( l_0 \). For non-circular cylinder cases, \( l_0 \) is the smallest section diameter. The ratio of \( l_0 \) to the speed of sound in undisturbed gas is used as a time scale: \( t_0 = l_0/a_0 \), \( a_0^2 = \gamma p_0/\rho_0 = \gamma \). All length-dimensioned parameters below (coordinates and precursor length) are normalized by \( l_0 \) and time – by \( t_0 \).
Four parameters govern the problem in total: heat capacity ratio $\gamma$, Mach number $M$ of the oncoming gas, gas density inside the channel to the undisturbed gas density ratio $\omega$ and the cylinder cross-section diameter ratio $b$. For all simulations below parameters are taken as follows: $\gamma = 1.4$, $M = 3$, $\omega = 1/3$, and $b$ equals 1, 2 or 4 for both elliptic and rectangular channel cases. Computational domain is a cube formed by 400x400x400 equal cubic grid cells and the smallest channel diameter contains 20 cells. Axisymmetric simulations are performed using equivalent grid consisting of 400x200 square cells.

### 3. Flow structure

For $b=1$, two-dimensional axisymmetric and three-dimensional flows are completely identical at the early flow stage. Density contours shown in figure 2a belong to later time moment $t = 7.6$ which correspond to the next (intermediate) flow stage, but the main flow features typical for early stage are still present. Near the symmetry axis a local flow originating from one-dimensional Riemann problem takes place: leading shock $ls$ propagates upstream in the channel, rarefaction wave travels downstream and contact surface $me$ stays between them. A sidelong precursor shock $ps$ is formed between the

---

**Figure 2.** Axial half-sections of the precursor at time moment $t = 7.6$: density contours in the increments of 0.1. Coordinate line $y = z = 0$ is at the bottom edge. (a) two-dimensional axisymmetric flow, (b) three-dimensional flow for $b = 1$. $ls$ – leading shock; $ps$, $hs$ – lateral and “hanging” internal precursor shocks; $tp$ – triple point; $se$ – shocked channel edge; $je$ – high-pressure jet edge; $sj$ and $rj$ – front and reverse axial jets; $ca$ – high-pressure jet cumulation area; $me$ and $ms$ – contact surface and shock wave that bound subsonic reverse flow area.
leading shock and an undisturbed part of the initial shock. Tangential discontinuity $se$, “hanging” shock $hs$ and high-pressure stream bounded by shear layer $je$ are formed inside the precursor.

The start of the intermediate stage is marked by the high-pressure jet cumulation at the symmetry axis: high pressure and density area $ca$ is formed with two supersonic jets $sj$ and $rj$ originating from it. Subsonic reverse flow area bounded by contact surface $me$ is separated from axial jet $sj$ by shock $ms$. Such flow structure persists for up to $t \approx 4.5$ and during that precursor grows linearly with time. Later the “flow choking” effect (the collision of contact surface $me$ with shear layer $se$) and development of instabilities inside the precursor lead to nonlinear slowdown of its growth process [5].

Differences between three-dimensional and axisymmetric flows start to manifest themselves at $t \approx 6$ : shock $ms$ and contact and tangential discontinuities inside the precursor undergo additional distortion and an extra triple point is formed on the front of hanging shock $hs$. To the time moment $t = 7.6$ presented in figure 2b, the flow inside the precursor becomes significantly chaotic, showing that instabilities in the three-dimensional flow develop faster that in axisymmetric case. Instability growth details in different axial sections vary to some extent (not presented in figures): discontinuity fronts distortions and small-scaled vortexes are located closer to the symmetry axis in sections that are slanted with respect to computational grid lines. The front of sidelong shock $ps$ is almost exactly straight in all the sections.

Axial sections along the smallest and largest diameters of elliptic channel for $b = 2$ are shown in figure 3. The comparison with round channel simulations shows that “flow choking” effect and internal precursor flow chaotization starting time moments are determined by the smallest channel

![Figure 3](image-url). Three-dimensional flow with elliptic channel for $b = 2$: axial half-sections along the smallest (a) and the largest (b) channel diameters.
diameter, but the leading shock advances slightly farther than for $b=1$ case. Despite the absence of axial symmetry in the problem formulation, jet cumulation in $ca$ area is more intense: at the time moment $t=7.6$ peak density and pressure are 10% higher than for the round cross-section case. There are much more qualitative and quantitative dissimilarities in precursor density contour patterns in different axial sections than for $b=1$ case, and thus the flow effects described above may be assumed to be caused mainly by the cross-section elongation rather than by development of circumferential instabilities.

![Figure 4](image.png)

**Figure 4.** Dependence of precursor length $d$ on time: $1D$ – one-dimensional Riemann problem (infinite channel diameter) for reference; $C1$, $C2$, $C4$ – two-dimensional axisymmetric flows for the corresponding channel diameters; $b=1$, $b=2$, $b=4$ are three-dimensional flows with the specified elliptic (dashed lines) or rectangular (dotted lines) channel diameter ratios; thin solid straight lines extrapolate linear precursor growth areas.

### 4. Precursor growth rate

Figure 4 illustrates the dependence of precursor length $d$ – the normalized distance between the leading and initial shocks along the cylinder axis – on time $t$ for different channel cross-section shapes. Solid lines represent axisymmetric simulations for channel diameters of 1, 2 and 4. These flows are equivalent with each other up to the coordinate scale, but are useful for comparison with three-dimensional simulations. Three-dimensional flows for $b=1$, $b=2$ and $b=4$ are shown by dashed (elliptic case) and dotted (rectangular case) lines and linear precursor growth areas are extrapolated with thin solid lines. Cross-section shape stretching does not influence precursor growth rate in the linear phase, but shifts it in time and significantly increases its duration. This effect is probably caused by slower flow choking effect development along the longer channel diameter and is even more pronounced for rectangular cross-section cases. Such observation allows the suggestion that linear precursor growth phase duration and high-pressure jet cumulation intensity are determined mainly by the largest cross-section linear dimension i.e. by larger ellipse diameter or rectangle diagonal.

For evaluation of grid effects influence on flow features, additional simulations were performed with half and double resolutions and different orientations of non-circular cylinder cross-section with respect to grid lines. Precursor length start to depend of grid resolution and channel orientation at late time moments (approximately $t>10$) while the internal instability development becomes grid-sensitive around $t=8$. Thus the effects described above may be assumed to be of purely gasdynamic
nature while proper viscosity and turbulence terms inclusion is necessary for correct simulation of the flow in its later stage.

5. Conclusion
Based on numerical simulation, it is shown that the shape of the cross-section of low-density gas channel significantly influences quantitative flow features – jet cumulation intensity and long-term linear precursor growth phase stability – during the interaction with a shock wave.

Acknowledgments
This study was performed according to the research plan of Institute of Mechanics of Lomonosov Moscow State University using the equipment of the shared research facilities of HPC computing resources of MSU with partial financial support of the Russian Foundation for Basic Research (projects no. 16-29-01092 and 18-01-00793).

References
[1] Shreffler R and Christian R 1954 *J. Appl. Phys.* 25(3) 324-331
[2] Taylor G. 1950 *Proc. R. Soc. Lond. A.* 201(1065) 175-186
[3] Georgievsky P and Levin V 1988 *Pisma v Zhurn. Teh. Phiz.* 14(8) 684-687
[4] Artem’ev V, Markovich I, Nemchinov I and Sulyaev V 1987 *Dokl. Akad. Nauk SSSR* 293(5) 1082-1084
[5] Georgievskii P, Levin V and Sutyrin O 2016 *Fl. Dyn.* 51(5) 696-702
[6] Castro M, Costa B and Don W 2011 *J. Comp. Phys.* 230(5) 1766-1792
[7] Sanders R, Morano E and Druguet M 1998 *J. Comp. Phys.* 145(2) 511-537
[8] Wang S and Johnsen E 2017 *arXiv.org preprint*: 1701.04834