Lepton Number and Lepton Flavour Violation in Left-Right Symmetric Models

Martti Raidal

Theory Group, DESY, D-22603 Hamburg, Germany

Abstract

The sources of lepton flavour and lepton number violation in the left-right symmetric models are reviewed and the most sensitive processes to these violations are discussed. Both the non-SUSY and SUSY versions of the theories are considered. Present and future experimental constraints at low as well as at high energies are discussed. Some emphasis has been put on the doubly charged particle production at future colliders.

"Talk given in the conference "Lepton-Baryon 98," April 1998, Trento, Italy."
1 Introduction

The conservations of lepton number and lepton flavours are among the most stringently tested laws of physics \[1\]. In the standard model (SM) of the weak and electromagnetic interactions all three lepton flavours are exact global symmetries and conserved separately. This is a consequence of the vanishing neutrino masses in the SM. Therefore, if the SM is the ultimate theory of Nature then lepton flavour violation will never be observed experimentally.

The SM has been extremely successful in describing the experimental data. Nevertheless, there are both theoretical and experimental problems which cannot be explained in the SM. On the theoretical side we mention just a few of them which are relevant for our further considerations: unexplained maximal parity violation of the weak interaction, massless neutrinos, CP problems, hierarchy problem etc. On the experimental side the most serious result indicating for the physics beyond the SM is the recent Super Kamiokande measurement \[2\] of the flux of atmospheric neutrinos which clearly points in the direction of neutrino masses. Similarly the solar neutrino problem \[3\], the LSND measurements \[4\] and the COBE satellite results \[5\] on the existence of the hot component of dark matter give indications of small neutrino masses.

Most of the above listed problems can be explained in the framework of the left-right symmetric models \[6\] based on the gauge group $SU_L(2) \times SU_R(2) \times U_{B-L}(1)$. In these models left- and right-handed components of the fields are treated on the same footing. The left-right symmetry was originally proposed in order to understand the origin of the parity violation as a consequence of the spontaneous symmetry breaking. It was then used to develop models of CP violation \[7\] as well as to discuss the strong CP problem \[8\]. If the Higgs sector of the models is choosen so that the right-handed symmetry is spontaneously broken by triplets then the model gives rise to small neutrino masses naturally, via the see-saw mechanism \[9\]. In this type of models there are two sources of lepton number violation, Majorana masses of the neutrinos and Yukawa interactions of the physical triplet Higgses.

Similar to the SM the non-supersymmetric left-right models suffer from the hierarchy problem. Supersymmetric left-right models \[10\] share all the important features of the non-susy left-right models, and in addition solve many problems of the minimal SUSY extension of the SM, MSSM, like automatic conservation of R-parity and SUSY and strong CP problems. These
points have been extensively discussed in the reviews [11] by R. Mohapatra. In addition, the recent works on the vacuum structure of the SUSY left-right models have shown [12, 13] that certain Higgs bosons and their SUSY partners should be relatively light giving rise to interesting low energy as well as to collider phenomenology.

In the present talk we discuss the most stringently tested lepton number and lepton flavour violating processes in the context of left-right models. First we present some necessary details of the non-SUSY as well as the SUSY left-right models and point out the sources of the lepton number violation. Then we consider in some detail low energy experiments like $\mu \rightarrow e \gamma$ and $\mu^{-} \rightarrow e$ conversion which provide the main probes of the muon number conservation. Finally we review the collider phenomenology of doubly charged Higgses and higgsinos.

2 Left-right symmetric models

2.1 Non-supersymmetric theories

In the minimal $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model with a left-right discrete symmetry each generation of quarks and leptons is assigned to the multiplets

$$Q = \begin{pmatrix} u \\ d \end{pmatrix}, \quad \psi = \begin{pmatrix} \nu \\ l \end{pmatrix},$$

with the quantum numbers $(T_L, T_R, B - L)$

$$Q_L : \left( \frac{1}{2}, 0, \frac{1}{3} \right), \quad \psi_L : \left( \frac{1}{2}, 0, -1 \right),$$

$$Q_R : \left( 0, \frac{1}{2}, \frac{1}{3} \right), \quad \psi_R : \left( 0, \frac{1}{2}, -1 \right).$$

Concerning the Higgs sector, in order to give masses to fermions, all the left-right models should contain a bidoublet

$$\phi = \begin{pmatrix} \phi_1^0 \\ \phi_2^- \end{pmatrix} = \left( \frac{1}{2}, \frac{1}{2}, 0 \right).$$

Since the bidoublet does not break the right-handed symmetry the Higgs sector has to be enlarged. This procedure is not unique but interesting
models are obtained by adding the scalar triplets,
\[ \Delta_{L,R} = \left( \begin{array}{cc} \frac{\Delta^+_{L,R}}{\sqrt{2}} & \Delta^{++}_{L,R} \\ \Delta^0_{L,R} & -\frac{\Delta^+_{L,R}}{\sqrt{2}} \end{array} \right), \]
with the quantum numbers \( \Delta_L : (1,0,2) \) and \( \Delta_R : (0,1,2) \), respectively.

In addition, we require the full Lagrangian of the model to be manifestly left-right symmetric i.e. invariant under the discrete symmetry
\[ \psi_L \leftrightarrow \psi_R , \ \Delta_L \leftrightarrow \Delta_R , \ \phi \leftrightarrow \phi^\dagger. \]
This symmetry plays a role in minimizing the Higgs potential and breaking of parity spontaneously. In general, the symmetry breaking would be triggered by the vevs
\[ \langle \phi \rangle = \left( \begin{array}{c} k_1 \\ \sqrt{2} v_L \end{array} \right) , \ \langle \Delta_{L,R} \rangle = \left( \begin{array}{c} v_{L,R} \\ 0 \end{array} \right). \]
The vev \( v_R \) of the right triplet breaks the \( SU(2)_R \times U(1)_{B-L} \) symmetry to \( U(1)_Y \) and gives masses to new right-handed particles. Since the right-handed currents have not been observed, \( v_R \) should be sufficiently large \( [14, 15] \). Further, the vevs \( k_1 \) and \( k_2 \) of the bidoublet break the SM symmetry and, therefore, are of the order of electroweak scale. The vev \( v_L \) of the left triplet, which contributes to the \( \rho \) parameter, is quite tightly bounded by experiments \( [14] \) and should be below a few GeV-s. Thus, the following hierarchy should be satisfied: \( |v_R| \gg |k_1|, |k_2| \gg |v_L| \). In principle, due to the underlying symmetry two of the vevs can be chosen to be real but two of them can be complex leading to the spontaneous \( CP \) violation \( [4] \).

The most general Yukawa Lagrangian for leptons invariant under the gauge group is given by
\[ -\mathcal{L}_Y = f_{ij} \bar{\psi}_i L \phi \psi^j_R + g_{ij} \bar{\psi}_i L \tilde{\phi} \psi^j_R + h.c. \]
\[ +i(h)_{ij} \left( \psi^{iT}_L C \tau_2 \Delta_L \psi^j_L + \psi^T_R C \tau_2 \Delta_R \psi^j_R \psi_R \right) + h.c., \]
where \( f, g \) and \( h \) are matrices of Yukawa couplings. The left-right symmetry \( [4] \) requires \( f \) and \( g \) to be Hermitian. The Majorana couplings \( h \) can be taken to be real and positive due to our ability to rotate \( \psi_L \) and \( \psi_R \) by a common phase without affecting \( f \) and \( g \).
The Lagrangian (7) is the most relevant expression for our discussion because it gives rise to lepton number violation in the left-right models. The origin of it is twofold. Firstly, triplet Higgses which carry two units of lepton number can mediate lepton number or lepton flavour violating processes. Secondly, Lagrangian (8) gives to neutrinos Majorana masses as follows.

Neutrino masses derive both from the $f$ and $g$ terms, which lead to Dirac mass terms, and from the $h$ term, which leads to large Majorana mass terms. Defining, as usual, $\psi^c \equiv C(\bar{\psi})^T$, the mass Lagrangian following from Eq.(7) can be written in the form

$$-\mathcal{L}_{mass} = \frac{1}{2}(\bar{\nu}^c_L M \nu_R + \bar{\nu}_R M^* \nu^c_L),$$

(8)

where $\nu^c_L = (\nu_L, \nu^c_R)^T$ and $\nu_R = (\nu^c_L, \nu_R)^T$ are six dimensional vectors of neutrino fields. The neutrino mass matrix $M$ is complex-symmetric and can be written in the block form

$$M = \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix},$$

(9)

where the entries are $3 \times 3$ matrices given by

$$M_L = \sqrt{2} h v_L, \quad M_D = h_D k_+, \quad M_R = \sqrt{2} h v_R.$$  

(10)

Here we have defined $h_D = (f k_1 + g k_2)/(\sqrt{2} k_+)$, where $k^2_+ = k^2_1 + k^2_2$. The masses of charged leptons are given by $M_l = (g k_1 + f k_2)/\sqrt{2}$ and, therefore, without fine tuning of $f$ and $g$ one has $M_D \simeq M_l$. Moreover, on the basis of avoiding possible fine tunings it is natural to assume that all the Yukawa couplings $h, h_D$ are of similar magnitude for a certain lepton family. In this case the mass matrix (9) has a strong hierarchy between different blocks which is set by the hierarchy of vevs. It is convenient to employ the self-conjugated spinors

$$\nu \equiv \frac{\nu_L + \nu^c_L}{\sqrt{2}}, \quad N \equiv \frac{\nu_R + \nu^c_R}{\sqrt{2}}.$$  

(11)

In a single generation case these are also the approximate mass eigenstates with masses

$$m_\nu \simeq \sqrt{2} \left( h v_L - \frac{h_D^2 k_+^2}{2 h v_R} \right),$$  

$$m_N \simeq \sqrt{2} h v_R.$$  

(12)
respectively. The mixing between them depends on the ratio of the masses as $\sin^2 \theta \sim m_\nu / m_N$.

Since the neutrino mass matrix is symmetric it can be diagonalized by the complex orthogonal transformation

$$U^T M U = M^d,$$  \hspace{1cm} (13)

where $M^d$ is the diagonal neutrino mass matrix. If we denote

$$U = \begin{pmatrix} U_L^* \\ U_R \end{pmatrix},$$  \hspace{1cm} (14)

then in the basis where the charged lepton mass matrix is diagonal (we can choose this basis without loss of generality) the physical neutrino mixing matrices which appear in the left- and right-handed charged currents are simply given by $U_L$ and $U_R$, respectively. Indeed, $U_L$, $U_R$ relate the left-handed and right-handed neutrino flavour eigenstates $\nu_{L,R}$ with the mass eigenstates $\nu_m$ according to

$$\nu_{L,R} = U_{L,R} \nu_m.$$  \hspace{1cm} (15)

2.2 Supersymmetric theories

SUSY left-right models have got a lot of attention recently. This is because they offer solutions [11] to the problems occurring in the MSSM. In SUSY versions of the left-right models all the qualitative discussion above is valid. In particular, the see-saw mechanism is active but the expression for the light neutrino mass gets its canonical form

$$m_\nu \simeq - \frac{h_D^2 k^2}{\sqrt{2} h v_R}.$$  

The important feature of a class of SUSY left-right models is that R-parity is automatically conserved as suggested by the stringent proton decay limits [16] while in the MSSM one has to tune R-parity violating couplings to be small.

The superfield content of the minimal SUSY left-right model is not identical to the particle content of the model considered in the last section. In order to cancel chiral anomalies the number of triplets should be doubled.
Table 1: Field content of the SUSY left-right model

| Fields | SU(2)_L × SU(2)_R × U(1)_{B−L} representation |
|--------|----------------------------------------------|
| Q      | (2,1,+ 1/3)                                 |
| Q^c    | (1,2,− 1/3)                                 |
| L      | (2,1,−1)                                    |
| L^c    | (1,2,+1)                                    |
| Φ_{1,2}| (2,2,0)                                     |
| Δ      | (3,1,+2)                                    |
| Δ      | (3,1,−2)                                    |
| Δ^c    | (1,3,+2)                                    |
| Δ^c    | (1,3,−2)                                    |

by adding new multiplets with the opposite $B − L$ quantum numbers. Also, to get a nontrivial quark mixing matrix the number of bidoublets should be doubled (supersymmetry forbids $\tilde{\Phi}$ in the superpotential). The superfields present in the minimal SUSY left-right model together with their quantum numbers are listed in Table 1.

The superpotential for the theory containing these fields can be given by (we have suppressed the generation index):

$$W = g_q^{(i)} Q^T \tau_2 \Phi_i \tau_2 Q^c + f_l^{(i)} L^T \tau_2 \Phi_i \tau_2 L^c$$
$$+ i (h L^T \tau_2 \Delta + h_c L^c \tau_2 \Delta^c L^c)$$
$$+ M_\Delta [\text{Tr}(\Delta \Delta) + \text{Tr}(\Delta^c \Delta^c)] + \mu_{ij} \text{Tr}(\tau_2 \Phi_i^T \tau_2 \Phi_j).$$

However, studies of the ground state of the SUSY left-right model have shown that keeping only the renormalizable terms in the superpotential, and with the given field content (authors of Ref. [12] actually consider a model with one additional singlet) the ground state of the model breaks the electric charge unless sneutrinos obtain nonzero vevs $< \tilde{\nu} >$. These vevs break R-parity spontaneously and give rise to lepton number violating processes. Such a situation is not as dangerous as the explicit R-parity breaking because baryon number is now conserved and proton is stable. Therefore the most stringent constraints on the R-parity violation are satisfied. The non-observation of
lepton number violating processes constrain also such a scenario quite stringently. Nevertheless interesting new SUSY phenomenology at high energy collider experiments is allowed \[17\].

An important consequence of the scenario with sneutrino vevs is that the mass of the right-handed gauge boson $W_R$ should be of the order of SUSY breaking scale $\mathcal{O}(1)$ TeV. The most stringent experimental constraint coming from the $K_L - K_S$ mass difference is $M_{W_R} \gtrsim 1.6$ TeV \[14\] while at LHC the right-handed gauge boson masses as high as 7 TeV will be probed. Therefore this scenario with non-zero sneutrino vevs will be probed in the near future. At the moment there is no experimental evidence in favour of it.

Another way to ensure the conservation of the electric charge by the ground state of the SUSY left-right model is to add to the superpotential non-renormalizable terms

$$W_{NR} = A[\text{Tr}(\Delta^c \bar{\Delta}^c)]^2 + B \text{Tr}(\Delta^c \Delta^c) \text{Tr}(\bar{\Delta}^c \bar{\Delta}^c),$$  \hspace{1cm} (17)

arising from higher scale physics such as grand unified theories or Planck scale effects ($A$ and $B$ are of the order $1/M_{\text{Planck}}$). It has been shown \[13\] that in this case the right-handed breaking scale and thus the mass of the right-handed gauge bosons must be very high, $v_R \gtrsim 10^{10}$ GeV. At such a scale it is natural that higher scale operators may play a role. In this scenario the right-handed currents have no importance for the low energy phenomenology. However, the following phenomenological consequences make this scenario very attractive.

1. R-parity is an exact symmetry. No lepton number violation from this sector.

2. The masses of the light neutrinos obtained by the see-saw mechanism by choosing the mass matrix Dirac entries to be of the order of the charged lepton masses and $M_N = 10^{10}$ GeV are exactly in the correct range to provide a solution to the solar and atmospheric neutrino problems \[2\] \[3\]. Neutrinoless double beta decay is possible.

3. There are light doubly charged Higgses and higgsinos in the model whose masses may be of the order of $10^2$ GeV. Interesting new lepton number violating collider phenomenology possible.
The latter point deserves some explanation. Due to the vacuum structure of SUSY theories there may be light fields in the models despite of the very high right-handed breaking scale. If SUSY is unbroken there are flat directions in the vacuum and the particles corresponding to these flat directions are massless. When SUSY gets broken the flat directions are lifted and the previously massless fields obtain masses of the order $v_R^2/M_{\text{Planck}}$.

Motivated by these results we will concentrate in the following on the phenomenological aspects of the Majorana neutrinos and doubly charged Higgses and higgsinos.

3 Low and high energy phenomenology

3.1 Neutrinoless double beta decay and its inverse

Neutrinoless double beta decay violates the lepton number by two units \[18\]. Its observation would necessarily imply that neutrinos are Majorana particles \[19\]. In the left-right symmetric models the neutrinoless double beta decay receives contribution from three sources \[20, 21\]: from the $t$-channel exchange of the light and heavy Majorana neutrinos and from the $s$-channel exchange of the doubly charged Higgs bosons. The latter contribution is a necessary ingredient to preserve the unitarity of the cross section \[22\]. Due to the nuclear effects the heavy and light particle contributions factorize. Therefore the Heidelberg-Moscow experiment searching for the neutrinoless double beta decay in $^{76}$Ge puts the upper limit on the effective mass of the light Majorana neutrino as

$$< m_{\nu} > \lesssim 0.46 \text{ eV}.$$ \[18\]

The heavy right-handed particle contribution to the process has been studied by several groups. The latest analyses \[21\] constrain the mass of the right-handed gauge boson to be

$$M_{WR} \gtrsim 1.1 \text{ TeV}, \quad \text{(19)}$$

for the fixed effective right-handed neutrino mass $M_N = 1 \text{ TeV}$ and $M_\Delta = \infty$. In the case of relatively light, TeV scale, doubly charged Higgses the bound may be increased by a few TeV.
The inverse of the neutrinoless double beta decay \[23, 24\], the process
\[ e^- e^- \rightarrow W^-_{L,R} W^-_{L,R} , \] (20)
can be studied in the future $e^- e^-$ options of the linear colliders. The cross section for the right gauge boson production is proportional to the heavy neutrino mass $M_N$ and therefore large \[24\]. However, most likely the collision energy of the future linear colliders will not allow to produce on-shell $W_R$-s. Therefore one has to look for the ordinary gauge boson $W_L$ pair production which may proceed through the heavy-light neutrino mixing. This topic has been extensively studied in last years \[23\]. For a single neutrino generation case the non-observation of the neutrinoless double beta decay suppresses the cross section below the observable limit. In the case of three generations, and allowing for the opposite CP parities of the heavy neutrinos, there is still some part of the parameter space left for which the observable cross section at linear colliders is allowed. This conclusion is essentially independent of the further improvements of the sensitivity of the double beta decay experiments (GENIUS) \[25\] and should be tested at future colliders.

### 3.2 $\mu - e$ conversion, $\mu \rightarrow e \gamma$ and other low energy processes

The precision reached in the low experiments looking for the violation of the muon number exceeds considerably the experimental precision in the searches for the tau lepton number violation \[26\]. In this talk we consider the most stringent experimental bounds on the muon number violation only. Also, we do not consider the possibility of having SUSY particles in the loops (for these studies see \[27\]).

In the left-right models the muon flavour violating processes $\mu \rightarrow e \gamma$ and $\mu - e$ conversion in nuclei can occur at one loop level. There are two types of loop diagrams contributing to these processes: with neutrinos and charged gauge bosons in the loops and with singly and doubly charged triplet Higgses and leptons in the loops. Let us first discuss the first possibility.

In the case of massive neutrinos in the loops the muon flavour can be violated due to the mixings of neutrinos of different generations. Because of the very small masses of the light neutrinos they contribution to the processes is suppressed and the main contribution comes from the exchange of the
heavy neutrinos and $W_L$. Therefore both $\mu \to e\gamma$ and $\mu - e$ conversion are proportional to the quantity

$$|U_L|^\mu_e = \left( \sum_i U_{L}^{ei} U_{L}^{i\mu} \right)^{1/2}. \quad (21)$$

Here the summation goes over the heavy neutrinos only which means that $U_L$ bounds the light-heavy mixings.

The general $\bar{e}\mu\gamma$ vertex can be parametrized in terms of form factors as follows

$$j^\rho = \bar{e} \left[ \gamma_\mu (f_{E0} + f_{M0} \gamma_5) \left( g^{\mu\rho} - \frac{q^\mu q^\rho}{q^2} \right) + (f_{M1} + f_{E1} \gamma_5)i\sigma^{\rho\nu} \frac{q_\nu}{m_\mu} \right] \mu, \quad (22)$$

where $q$ is the momentum transferred by the photon. While $\mu \to e\gamma$ is induced only by the form factors proportional to $\sigma^{\rho\nu}$ term then the photonic
Table 2: Upper bounds on the diagonal doubly charged Higgs couplings for $M_\Delta = 1$ TeV.

| Process   | Combination of couplings | Upper bound |
|-----------|--------------------------|-------------|
| Møller    | $h^{ee}$                 | 4           |
| $(g-2)_\mu$ | $h^{\mu\mu}$           | 10          |
| $M-M$     | $h^{ee}h^{\mu\mu}$      | 0.2         |

$\mu - e$ conversion rate is proportional to $(|f_{E0} + f_{M1}|^2 + |f_{M0} + f_{E1}|^2)$ \[28\]. On the other hand, $\mu - e$ conversion can also be induced by the non-photonic conversion mechanism, for example by the effective vertex $\bar{\epsilon}\mu Z_L$. This mode is somewhat suppressed by the mass of $Z_L$ but, on the other hand, it is proportional to the heavy neutrino mass. Using the present experimental bounds on the branching ratios of the processes $B(\mu \to e\gamma) < 4.9 \cdot 10^{-11}$ and $B(\mu - e) < 4 \cdot 10^{-12}$ and the results of Ref. \[29\] we plot in Fig. 4 the constraints on $|U_{L\mu}|$ from $\mu \to e\gamma$ and $\mu - e$ conversion against the heavy neutrino mass. The $\mu - e$ conversion bound becomes more stringent for larger neutrino masses but it is orders of magnitude smaller that the constraints predicted by the see-saw mechanism (in the see-saw the heavy-light mixing is given by $U^2 \sim m_\nu/M_N$). Therefore these results are meaningful only in the models where neutrino masses are generated by other mechanisms than the see-saw \[29, 30\].

The situation is quite different for the couplings of the triplet Higgs bosons $\Delta$. Due to large masses and unknown couplings to leptons one can presently only constrain their effective couplings of a generic form $G = \sqrt{2}h^2/(8M_\Delta^2)$. Most stringent constraints on the diagonal couplings of the triplet Higgses \[31\] are summarized in Table 2. These are obtained from the Møller scattering, $(g-2)_\mu$ studies as well as from the searches for the muonium-antimuonium conversion. Here and in the following the masses are always taken in units of TeV. As can be seen, for the scale of new physics $\mathcal{O}(1)$ TeV only the muonium-antimuonium conversion experiment constrains diagonal couplings in a meaningful way. To date there is no constraints on $h^{\tau\tau}$ without involving the off-diagonal elements.

Most stringent constraints on the triplet couplings $h$ from the muon num-
Table 3: Upper bounds on the doubly charged Higgs couplings for $M_\Delta = 1$ TeV from various low energy leptonic processes.

| Process   | Combination of couplings | Upper bound |
|-----------|--------------------------|-------------|
| $\mu \to 3e$ | $h^{\mu e} h^{ee}$       | $4 \cdot 10^{-5}$ |
| $\mu \to e\gamma$ | $(hh)^{\mu e}$        | $3 \cdot 10^{-3}$ |
| $\mu - e$    | $(hh)^{\mu e}$         | $6 \cdot 10^{-4}$ |

Number violating processes are summarized in Table 3. As expected, the strongest limit derives from $\mu \to 3e$ since it can occur at tree level. However, it constraints only a particular combination of the couplings. On the other hand, $\mu \to e\gamma$ and $\mu - e$ conversion limits apply on a sum of the coupling constant products and, therefore, provide complementary information. One should note here that the $\mu - e$ conversion is more sensitive to the Higgs interactions than $\mu \to e\gamma$. This is because the form factors $f_{E0}$ and $f_{M0}$ in Eq. (22) are enhanced by large $\ln(m_\ell^2/M_\Delta^2) \sim \mathcal{O}(10)$ while the form factors $f_{E1}$ and $f_{M1}$ are not [32]. Consequently $\mu - e$ conversion is enhanced while $\mu \to e\gamma$ is not. The enhancement arises from the diagrams in which the doubly charged Higgs $\Delta^{++}$ runs in the loop and the the photon is attached to the charged lepton line.

### 3.3 Collider phenomenology

The most distinctive signatures of the new physics at collider experiments would be produced by the decays of the doubly charged Higgses $\Delta^{++}_{L,R}$ and higgsinos $\tilde{\Delta}^{++}_{L,R}$. If these particles are light enough to be produced at future colliders then the most stringent constraints on their couplings will be obtained from these experiments. The production of $\Delta^{++}$ at linear and muon colliders as well as at the LHC is already extensively studied [33, 34, 35, 36, 37, 38]. The production of their supersymmetric partners, $\tilde{\Delta}^{++}$, has received somewhat less attention [39, 40]. We shall discuss the $\tilde{\Delta}^{++}$, production later in this Subsection and start with the Higgs mediated processes.

While $\Delta^{++}_{L,R}$ do not couple to quarks they still can be produced at LHC in pairs (Drell-Yan) or singly ($WW$ fusion). The latter process is kinematically
more favoured but it depends on the unknown model parameters: $M_{W_R}$ for the $\Delta^{++}_R$ production and $v_L$ for the $\Delta^{+-}_L$ production. On the other hand, the Drell-Yan pair production is almost model independent due to the $\Delta^{++}_L$ couplings to the photon and $Z_L$. Using the results of Ref. [34] we plot in Fig. 2 the production cross sections of the Drell-Yan and $WW$ fusion processes at LHC for the parameters indicated in the figure. In the pair production the doubly charged Higgses with masses below 600 GeV can be discovered at LHC. For the optimistic choice of the unknown model parameters the $WW$ fusion may extend the discovery reach up to 1-2 TeV or so.

At the future linear and muon colliders the doubly charged Higgses with masses $M_\Delta \lesssim \sqrt{s}/2$ can be pair produced in $e^+e^-$ collisions due to their couplings to photon and $Z$ [35]. However, the most appropriate for studying $\Delta^{++}_{L,R}$s are $e^-e^-$ and $\mu^-\mu^-$ collision modes in which the resonant $s$-channel production of them via the process

$$e^-e^-(\mu^-\mu^-) \rightarrow \ell^-_i \ell^-_i, \quad (23)$$

$i = e, \mu, \tau$, is possible. It has been shown that for the realistic machine and beam parameters sensitivity of

$$h^{ij} \lesssim 5 \cdot 10^{-5} \quad (24)$$
can be achieved \[36\].

Despite of this extraordinary sensitivity it could happen that due to small \( h \)'s or high Higgs masses no positive signal will be detected at future colliders. However, this may not be the case if neutrinos are massive enough. If the sum of light neutrino masses exceeds \( \sim 90 \text{ eV} \) at least one of them has to be unstable. In order not to overclose the Universe the lifetime and mass of such an unstable neutrino \( \nu_l \) must satisfy the requirement \[11\]

\[
\tau_{\nu_l} \lesssim 8.2 \cdot 10^{31} \text{ MeV}^{-1} \left(\frac{100 \text{ keV}}{m_{\nu_l}}\right)^2.
\]

(25)

The radiative decay modes \( \nu_l \rightarrow \nu_f \gamma, \nu_f \gamma \gamma \) are highly suppressed \[42\] and cannot satisfy the constraint (25). The same is also true for \( Z' \) contribution to \( \nu_l \rightarrow 3\nu_f \) decay \[13\]. The only possibility which lefts over is the decay \( \nu_l \rightarrow 3\nu_f \) due to neutrino mixings induced by the neutral component of triplet Higgs \( \Delta^0_L \).

Clearly, since \( \Delta^0_L \) and \( \Delta^-_L \) belong to the same multiplet the reaction (23) can be related to decay \( \nu_l \rightarrow 3\nu_f \). Since the latter decay rate is bounded from below by the constraint (23) and limits on neutrino mixings also the cross section of (23) has a lower limit. Let us consider numerically the most conservative case, \( \nu_\tau \rightarrow 3\nu_e \). From the constraints presented above one obtains the following bound on \( G_{ee} \) which induces the process \( e^- e^- \rightarrow \tau^- \tau^- \) \[37\]

\[
G_{\tau\tau} > 2 \cdot 10^{-3} \frac{h_{\tau\tau}}{h_{ee} + h_{\tau\tau}} \text{ TeV}^{-2},
\]

(26)

where \( G_{\tau\tau} = \sqrt{2} h_{ee} h_{\tau\tau}/(8 M_A^2) \). On the other hand, studies of the process (23) at linear colliders give that far off the resonance the minimal testable \( G^{ij} \) are \( G^{ij}(\text{ min}) = 1.4 \cdot 10^{-4}/s \text{ TeV}^{-2} \), approximately the same for all relevant \( i, j \). Therefore, the process \( e^- e^- \rightarrow \tau^- \tau^- \) should be detected at the 1 TeV linear collider unless \( h_{\tau\tau}/h_{ee} < 10^{-1} \). On the other hand, if this is the case then

\[
G_{ee} > 2 \cdot 10^{-3} \text{ TeV}^{-2},
\]

(27)

and the excess of the electron pairs due to the s-channel Higgs exchange will be detected. Note that the positive signal should be seen if \( \sqrt{s} > 0.3 \text{ TeV} \) which is below the planned initial energy of the linear collider.
Figure 3: Achievable constraints of the triplet Yukawa couplings from the process (28) as functions of the Higgs mass. The present bound from $\mu \to 3e$ is also shown.

Similar argumentation applies to all possible neutrino decays with even higher lower limits. Therefore, if one of the neutrino mass, indeed, exceeds 90 eV one should detect at least one of the processes (23) at future colliders.

If the $e^-e^-$ option of the linear collider will not be available then single production of $\Delta^{++}_{L,R}$ in $e^+e^-$ and $e^-\gamma$ collisions allows one to probe masses up to $\sqrt{s}$ and test all the couplings $h^{ij}$ more sensitively than any of the present low energy experiments [38]. In $e^-\gamma$ mode the production reaction is

$$e^-\gamma \rightarrow l^+\Delta^{--}$$

(28)

and in $e^+e^-$ mode

$$e^+e^- \rightarrow e^+l^+\Delta^{--}.$$  

(29)

Let us first consider the reaction (28). The primary lepton created in the process will remain undetected as it is radiated almost parallel to the beam.
axis. One cannot tell whether this particle is a positron, antimuon or antitau. Therefore, the quantity which one can test in the reaction is actually the sum $h_{ee}^2 + h_{e\mu}^2 + h_{e\tau}^2$. The upper bound obtained for this sum is, of course, the upper bound of each individual term of the sum separately.

Assuming the integrated luminosities of $e^-\gamma$ collisions to be $L = 5, 10, 20, 40 \text{ fb}^{-1}$ and that for the discovery of $\Delta_{R}^{-}$ one needs ten events, we obtain the upper bounds plotted in Fig. 3. As one can see from the figure, the sensitivity of the linear collider on the quantity $(h_{ee}^2 + h_{e\mu}^2 + h_{e\tau}^2)^{1/2}$ is on the level of $10^{-3}$ almost up to the threshold value of the $\Delta_{R}^{-}$ mass. In other words,

$$h_{ee}, h_{e\mu}, h_{e\tau} \lesssim 10^{-3}$$

for $M_{\Delta^{-}} \lesssim \sqrt{s_{\gamma\gamma}}$. Among the present constraints only the one from the $\mu \rightarrow 3e$ competes with these bounds and does so only at the low mass values. For the coupling $h_{e\tau}$ no bounds exist from the present experiments.

![Figure 4: Achievable constraints of the triplet Yukawa couplings from the process (29) as functions of the Higgs mass. The present bound from $\mu \rightarrow 3e$ is also shown.](image)

17
For the same $\Delta_{R}^{--}$ mass, the cross section of the process (29) is roughly two orders of magnitude smaller than the cross section of the process (28), implying that the constraints obtained for $(h_{ee}^{2} + h_{e\mu}^{2} + h_{e\tau}^{2})^{1/2}$ are correspondingly weaker, although the higher luminosities $L = 20, 50, 100, 200$ fb$^{-1}$ of $e^{+}e^{-}$ slightly compensate the lack in cross section. The resulting bounds are presented in Fig. 4.

The production of SUSY partners of the doubly charged Higgses, $\tilde{\Delta}^{++}$, at linear colliders has been previously studied in Ref.[39]. Since in the model we consider the R-parity is conserved then the SUSY particles can be produced in pairs only. Therefore the simplest process to study is the $\tilde{\Delta}^{++}, \tilde{\Delta}^{--}$ pair production in $e^{+}e^{-}$ collisions. This reaction is mediated by the $s$-channel photon and $Z$ exchange and $t$-channel selectron exchange. Therefore the process is sensitive to the triplet coupling $h$. In Fig. 5 we plot the pair production cross section of $\tilde{\Delta}^{++}$ against the collision energy. The choice of parameters and beam polarizations are indicated in figure.

$$e^{+}e^{-} \rightarrow \tilde{\Delta}^{+} \tilde{\Delta}^{-}$$

Figure 5: The polarized cross sections for the pair production of doubly charged higgsino $\tilde{\Delta}^{++}$ against the collision energy. The choice of parameters and beam polarizations are indicated in figure.
of the parameters is indicated in the figure. The cross section is large and depends strongly on the coupling $h$.

The produced higgsinos will decay fast. We assume here that their dominant decay modes are $\tilde{\Delta}^{++} \to l^+ l^+ \tilde{\chi}^0$ which are mediated by the selectron. Since the $\tilde{\Delta}^{++}$ lifetime is very small, only the correlated production and decay can be observed experimentally:

$$e^+ e^- \to \tilde{\Delta}^{--} \tilde{\Delta}^{++} \quad \xrightarrow{\tilde{\chi}^0 + (l^+ l^+)} \quad \xrightarrow{\tilde{\chi}^0 + (l^- l^-)}$$

The analysis is complicated as the two invisible neutralinos in the final state do not allow for a complete reconstruction of the events. In particular, it is not possible to measure the $\tilde{\Delta}^{++}$ production angle $\Theta$; this angle can be determined only up to a two-fold ambiguity. Therefore one cannot measure the fundamental interactions of the theory directly from the primary production process.

The way out of this problem is to study the spin correlations of the final state higgsinos which will be reflected in the distributions of the final state leptons. Indeed, one has shown [40] that by measuring the final state lepton distributions one can actually measure the total cross section $\sigma$ and the following combinations of the helicity amplitudes of the primary process:

$$\mathcal{P} = \frac{1}{4} \int d \cos \Theta \sum_{\pm} \left[ |\langle \pm; ++ \rangle|^2 + |\langle \pm; +\rangle|^2 - |\langle \pm; -+ \rangle|^2 - |\langle \pm; -- \rangle|^2 \right],$$

$$\mathcal{Q} = \frac{1}{4} \int d \cos \Theta \sum_{\pm} \left[ |\langle \pm; +++ \rangle|^2 - |\langle \pm; +-- \rangle|^2 - |\langle \pm; --+ \rangle|^2 + |\langle \pm; --- \rangle|^2 \right],$$

$$\mathcal{Y} = \frac{1}{2} \int d \cos \Theta \sum_{\pm} \text{Re} \left\{ \langle \pm; -- \rangle \langle \pm; ++ \rangle^* \right\}. \quad (31)$$

In the helicity amplitudes $\langle \pm; \pm \rangle$ the first $\pm$ denotes the helicity of the electron and the last ones the helicities of $\tilde{\Delta}^{++}$ and $\tilde{\Delta}^{--}$, respectively. The measurements of the cross section at an energy $\sqrt{s}$, and the ratios $\mathcal{P}^2/\mathcal{Q}$ or $\mathcal{Y}/\mathcal{Q}$ give enough information on the interactions of the doubly charged particles. Thus the quantum numbers of the doubly charged higgsino and its coupling $h$ to lepton-slepton pair can be determined from the experiment.
To demonstrate that let us assume that at $\sqrt{s} = 500$ GeV the experimentally "measured" cross section $\sigma$, $P^2/Q$ and $Y/Q$ are

$$\sigma(e^+e^- \rightarrow \tilde{\Delta}^{++} \tilde{\Delta}^{--}) = 0.106 \text{ pb}, \quad \frac{P^2}{Q} = -2.0, \quad \frac{Y}{Q} = 0.26. \quad (32)$$

Let us also assume that all the particle masses are known ($M_\Delta$ can be measured from the threshold cross section). In Fig. 6 we plot the contour lines

![Contour lines](image)

Figure 6: Contour lines from the measurements of $\sigma$, $P^2/Q$ and $Y/Q$ determining the coupling constant $h$ and the weak isospin $I_3$ of the doubly charged higgsino.

in $(h, I_3)$ plane corresponding to the measured $\sigma$, $P^2/Q$ and $Y/Q$ keeping $I_3$, the isospin, as a continuous parameter. Since $I_3$ can be only an integer or a half integer we see from Fig. 6 that the cross section measurement alone determines $I_3 = 1$. Because the negative interference between $s$– and $t$–channel contributions does not allow to determine $h$ unambiguously from one measurement then $f$ may still have two solutions, $h = 0.25$ or 0.31. However,
the three contour lines derived from the three measurements meet at a single point \((I_3 = 1, h = 0.25)\) which gives the physical values of \(I_3\) and \(h\). Since the \(\mathcal{Y}/Q\) curve crosses the \(\sigma\) curve at angle almost \(\pi/2\) then the measurement can be carried out with high precision. In fact, the third \(P^2/Q\) curve does not give any new information in this case and can be used just for the cross check.

4 Conclusions

In this talk we have discussed the sources of lepton number and lepton flavour violation in the non-SUSY and SUSY left-right symmetric models. After reviewing the basics of the models we have considered low and high energy phenomenology of the massive neutrinos and doubly charged Higgses and higgsinos. Recent works have shown that in SUSY models the doubly charged particles should be relatively light. In particular we have discussed neutrinoless double beta decay, \(\mu \rightarrow e\gamma\), \(\mu - e\) conversion in nuclei and future experiments at LHC and linear and muon colliders. The present constraints on the new particle masses and interactions allow for the lepton number violating signals in all these experiments. We emphasise the role of the future colliders for testing the left-right models.

Acknowledgments

I would like to thank organizers for creating pleasant atmosphere during the conference and P. Zerwas and U. Sarkar for several discussions. I am greatful for the A. von Humboldt Foundation for the grant.

References

[1] C. Caso et al., Particle Data Group, Eur. Phys. J. C 3 (1998) 1.

[2] Y. Fukuda et al., Super-Kamiokande Collaboration, [hep-ex/9807003](http://arxiv.org/abs/hep-ex/9807003).

[3] For the recent analyses see, J.N. Bahcall, P.I. Krastev and A.Yu. Smirnov, [hep-ph/9807216](http://arxiv.org/abs/hep-ph/9807216).
[4] A. Athanassopoulos et al., LSND Collaboration, Phys. Rev. Lett. 77 (1996) 3082.

[5] G.F. Smoot et al., Ast. J. 396 (1992) L1.

[6] J.C. Pati and A. Salam, Phys. Rev. D10 (1974) 275; R.N. Mohapatra and J.C. Pati, Phys. Rev. D11 (1975) 566, 2558; G. Senjanović and R.N. Mohapatra, Phys. Rev. D12 (1975) 1502; R.N. Mohapatra and G. Senjanović, Phys. Rev. D 23, (1981) 165.

[7] G. Senjanović, Nucl. Phys. B153, 334 (1979) 334; G. Senjanović and P. Senjanović, Phys. Rev. D 21, 3253 (1980); D. Chang, Nucl. Phys. B214, 435 (1983); H. Harari and M. Leurer, Nucl. Phys. B233, 221 (1984); G. Ecker and W. Grimus, Nucl. Phys. B258, 328 (1985); M. Leurer, Nucl. Phys. B266, 147 (1986); J.-M. Frère et al., Phys. Rev. D 46, 337 (1992); G. Barenboim, J. Bernabéu and M. Raidal, Nucl. Phys. B478, 527 (1996); B. Barenboim, J. Bernabéu and M. Raidal, Nucl. Phys. B511, 577 (1998); B. Barenboim, J. Bernabéu and M. Raidal, Phys. Rev. Lett. 80 (1998) 4625.

[8] R. Kuchimanchi, Phys. Rev. Lett. 76 (1996) 3486; R.N. Mohapatra and A. Rasin, Phys. Rev. Lett. 76 (1996) 3490; R.N. Mohapatra, A. Rasin and G. Senjanović, Phys. Rev. Lett. 79 (1997) 4744. For a review see, e.g., R.N. Mohapatra, hep-ph/9801235.

[9] M. Gell-Mann, P. Ramon and R. Slansky, in Supergravity, ed. P. van Niewenhuizen and D. Freedman (North-Holland, Amsterdam, 1979), p. 315; T. Yanagida, in Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe, ed. O. Sawada and A. Sugamoto (KEK, Tsukuba, Japan, 1979); R.N. Mohapatra and G. Senjanović, Phys. Rev. Lett. 44 (1980) 912.

[10] For the early studies of SUSY left-right models see, M. Cvetic and J. Pati, Phys. Lett. B135 (1984) 57; Y. Ahn, Phys. Lett. B149 (1984) 337; R.M. Francis, M. Frank and C.S. Kalman, Phys. Rev. D43 (1991) 2369; R.M. Francis, C.S. Kalman and H.N. Saif, Z. Phys. C59 (1993) 655; K. Huitu, J. Maalampi and M. Raidal, Nucl. Phys. B420 (1994) 449.
[11] R.N. Mohapatra, hep-ph/9707518; hep-ph/9806520.

[12] R. Kuchimanchi and R.N. Mohapatra, Phys. Rev. D48 (1993) 4352; Phys. Rev. Lett. 75 (1995) 3989; K. Huitu and J. Maalampi, Phys. Lett. B344 (1995) 217.

[13] C.S. Aulakh, K. Benakli and G. Senjanović, Phys. Rev. Lett. 79 (1997) 2188; C.S. Aulakh, A. Melfo and G. Senjanović, Phys. Rev. D57 (1998) 4174; Z. Chacko and R.N. Mohapatra, hep-ph/9712359; C.S. Aulakh, A. Melfo, A. Rašin and G. Senjanović, hep-ph/9712551; Z. Chacko and R.N. Mohapatra, hep-ph/9802388.

[14] G. Beall, M. Bander and A. Soni, Phys. Rev. Lett. 48, (1982) 848; G. Barenboim, J. Bernabéu, J. Prades and M. Raidal, Phys. Rev. D 55, (1997) 4213.

[15] P. Langacker and S. Uma Sankar, Phys. Rev. D 40, (1989) 1569.

[16] For reviews see, e.g., G. Bhattacharyya, Nucl. Phys. Proc. Suppl. 52A (1997) 83; H. Dreiner, hep-ph/9707433.

[17] K. Huitu, J. Maalampi and K. Puolamäki, hep-ph/9705406, hep-ph/9708491.

[18] For the review of new physics in neutrinoless double beta decay see, e.g., R.N. Mohapatra, hep-ph/9808284, and references therein.

[19] J. Schechter and J.W. Valle, Phys. Rev. D 25 (1982) 2951.

[20] R.N. Mohapatra, Phys. Rev. D 34 (1986) 909; M. Doi and T. Kotani, Progr. Theor. Phys. 89 (1993) 139.

[21] M. Hirsch, H.V. Klapdor-Kleingrothaus and O. Panella, Phys. Lett. B374 (1996) 7.

[22] T.G. Rizzo, Phys. Lett. B116 (1982) 23.

[23] C.A. Heusch and P. Minkowski, Nucl.Phys.B416 (1994) 3; J. Gluza and M. Zralek, Phys.Rev.D52 (1995) 6238; J. Gluza and M. Zralek, Phys. Lett.B362 (1995) 148; G. Belanger, F. Boudjema, D. London and H. Nadeau, Phys.Rev. D53 (1996) 6292; T. G. Rizzo, Int.J.Mod.Phys.
[24] P. Helde, K. Huitu, J. Maalampi and M. Raidal, Nucl. Phys. B437 (1995) 305.

[25] H.V. Klapdor-Kleingrothaus, these proceedings and hep-ex/9802003.

[26] For the experimental prospects see, e.g., A. Czarnecki, hep-ph/9710425.

[27] M. Frank and H. Hamidian, Phys. Rev. D54 (1996) 6790; G. Couture, M. Frank and H. König Phys. Rev. D56 (1997) 4219; G. Couture, M. Frank, H. König and M. Pospelov, hep-ph/9701299.

[28] H.C. Chiang et al., Nucl. Phys. A559 (1993) 526; A. Czarnecki, W. Marciano and K. Melnikov, hep-ph/9801218.

[29] G. Barenboim and M. Raidal, Nucl. Phys. B484 (1997) 63.

[30] D. Wyler and L. Wolfenstein, Nucl. Phys. B218 (1983) 205; R.N. Mohapatra and J.W.F. Valle, Phys. Rev. D34 (1986) 1642; E. Witten, Nucl. Phys. B268 (1986) 79; J. Bernabeu et al., Phys. Lett. B187 (1987) 303; J.L. Hewett and T.G. Rizzo, Phys. Rep. 183 (1989) 193; P. Langacker and D. London, Phys. Rev. D38 (1988) 886,907; W. Buchmüller and C. Greub, Nucl. Phys. B363 (1991) 345; E. Nardi, Phys. Rev. D48 (1993) 3277; D. Tommasini, G. Barenboim, J. Bernabeu and C. Jarlskog, Nucl. Phys. B444 (1995) 451; J. Gluza, J. Maalampi, M. Raidal and M. Zrałek, Phys. Lett. B407 (1997) 45.

[31] F. Cuypers, S. Davidson, Eur. Phys. J. C2 (1998) 503.

[32] W.J. Marciano and A.I. Sanda, Phys. Rev. Lett. 38 (1977) 1512; M. Raidal and A. Santamaria, Phys. Lett. B421 (1998) 250; and references therein.

[33] T. Rizzo, Phys. Rev. D25 (1982) 1355, Phys. Rev. D27 (1983) 657; M. Lusignoli and S. Petrarca, Phys. Lett. B226 (1989) 397; M.D. Swartz, Phys. Rev. D40 (1989) 1521; J.A. Griñols, A. Mendez and G.A. Schuler, Mod. Phys. Lett. A4 (1989) 1485; J.F. Gunion, J.A. Griñols, A. Mendez,
B. Kayser and F. Olness, Phys. Rev. D40 (1989) 1546; J. Maalampi, A. Pietilä and M. Raidal, Phys. Rev. D48 (1993) 4467; N. Lepore et al., Phys. Rev. D50 (1994) 2031; E. Accomando and S. Petrarca, Phys. Lett. B323 (1994) 212; S. Chakrabarti, D. Choudhuri, R.M. Godoble and B. Mukhopadhyaya, hep-ph/9804297.

[34] K. Huitu, J. Maalampi, A. Pietilä and M. Raidal, Nucl. Phys. B487 (1997) 27.

[35] J.F. Gunion, C. Loomis and K.T. Pitts, hep-ph/9610327.

[36] J.F. Gunion, Int. Jour. Mod. Phys. A11 (1996) 1551; F. Cuypers and M. Raidal, Nucl. Phys. B501 (1997) 3.

[37] M. Raidal, Phys. Rev. D57 (1998) 2013.

[38] G. Barenboim, K. Huitu, J. Maalampi and M. Raidal, Phys. Lett. B394 (1997) 132.

[39] K. Huitu, J. Maalampi and M. Raidal, Nucl. Phys. B420 (1994) 449, Phys. Lett. B328 (1994) 60; B. Dutta and R.N. Mohapatra, hep-ph/9804277.

[40] M. Raidal and P. Zerwas, in preparation.

[41] E. Kolb and M. Turner, Phys. Rev. Lett. 67 (1991) 5; P. Herczeg and R.N. Mohapatra, Phys. Rev. Lett. 69 (1992) 2475.

[42] A. De Rujula and S.L. Glashow, Phys. Rev. Lett. 45 (1980) 942; P.B. Pal and L. Wolfenstein, Phys. Rev. D 25 (1982) 766.

[43] Y. Hosotani, Nucl. Phys. B 191 (1981) 411; J. Schechter and J.W. Valle, Phys. Rev. D 25 (1982) 774.