A mathematical procedure to predict optical performance of CPCs

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Abstract. To evaluate the optical performance of a CPC based concentrating photovoltaic system, it is essential to find the angular dependence of optical efficiency of compound parabolic concentrator (CPC-θe) where the incident angle of solar rays on solar cells is restricted within θe for the radiation over its acceptance angle. In this work, a mathematical procedure was developed to calculate the optical efficiency of CPC-θe for radiation incident at any angle based radiation transfer within CPC-θe. Calculations show that, given the acceptance half-angle (θa), the annual radiation of full CPC-θe increases with the increase of θe and the CPC without restriction of exit angle (CPC-90) annually collects the most radiation due to large geometry (C90); whereas for truncated CPCs with identical θa and C90, the annual radiation collected by CPC-θe is almost identical to that by CPC-90, even slightly higher. Calculations also indicate that the annual radiation on the absorber of CPC-θe at the angle larger than θe decrease with the increase of θe but always less than that of CPC-90, and this implies that the CPC-θe based PV system is more efficient than CPC-90 based PV system because the radiation on solar cells incident at large angle is poorly converted into electricity.

1. Introduction

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In recent years, solar photovoltaic (PV) have been widely used for electrical generation over the world due to emerging issues of conventional energy shortage [1]. However, the application of PV system is limited due to high cost of electricity generation as compared to the conventional electrical generation technologies [2]. Therefore, to lower the cost of electricity generating from PV systems is a unique solution to expanding applications of PV technology. Apart from seeking for new materials and production techniques of solar cells, the use of cheap optical concentrator is regarded as an effective way to cost reduction of electricity generation from PV systems.

Concentrating PV systems (CPV) are generally classified into high concentrating PV system (HCPV) and low concentrating systems (LCPV). CPVs seem simple in the mechanism but are difficult to implement, especially HCPVs, which require expensive and complex sun-tracking device, cooling technique of solar cells and specially designed solar cells [3, 4]. Thus LCPV is more attractive due to no need of sun-tracking system. In the past two decades, many theoretical and experimental studies were performed to employ compound parabolic concentrators (CPC) for concentrating solar radiation on solar cells [5-9]. Compared to CPCs, the V-trough concentrator shares the advantages of easy construction and more uniform solar irradiation on the base, but the increase in power output from V-trough based CPVs is limited [10-12]. To make the irradiation on solar cells of CPC based CPV uniform, Hatwaambo tested a CPV system where semi-diffuse reflective materials were used, and found that the increase of the fill factor of the photovoltaic system was insignificant [13, 14]. However, recent study by Yu and Tang found that the use of semi-diffuse reflectors in CPVs would lead the collectible radiation decrease greatly[15]. For a CPV system, the incident angle of solar rays reflecting from the lower part of reflectors is considerably large, thus can not be efficiently converted into electrical power due to poor solar absorption [16,17]. To increase the absorption of solar radiation by solar cells of CPC based PV system, CPCs with a restricted exit angle (CPC-\( \theta_e \)) was first proposed [5,19,20], but it is in recent years that several researches on its performance are found in the literature [17,19,20,21]. For such CPC, all solar rays over its acceptance angle (\( \theta_a \)) arrive on the absorber at the angle less than the desired value\( \theta_e \), whereas for solar radiation beyond its acceptance angle, a fraction of incident radiation will arrive on the receiver at the angle larger than \( \theta_e \), but it was not considered in previous works of the authors [20,21], leading its performance underestimated. In this work, a general mathematical procedure to determine the angular dependence of optical efficiency of CPC-\( \theta_e \) with any \( \theta_e \) was developed, and effects of \( \theta_e \) on its performance was theoretical investigated in terms of annual collectible radiation.

2. Mathematical procedure to calculate the optical efficiency of CPC-\( \theta_e \)

2.1. Equation of reflectors

As seen from figure 1, the reflectors of CPC-\( \theta_e \) consist of upper parabola and lower plane mirror, and the plane mirror is tangent to the lower end of parabolic reflector. To be convenient analysis, the width of the absorber is set to be 1, thus, parabolic reflector can be expressed by:

\[
\begin{align*}
x & = \frac{\sin \theta_a + \sin \theta_e \sin \theta}{1 - \cos \theta_e} - 0.5 \\
y & = \frac{\sin \theta_a + \sin \theta_e \cos \theta}{1 - \cos \theta_e} \\
& \quad \left( \theta_e \leq \theta \leq \theta_a \right)
\end{align*}
\]  

(1)
where $\theta$ is the polar angle at the absorber end; $\theta_e$ is the maximum exit angle for radiation within its acceptance angle; and $\theta_i$ is the edge-ray angle of CPC-$\theta_e$. For full CPC-$\theta_e$, $\theta_i = \theta_a$ and $C_i = \sin \theta_e / \sin \theta_a$; whereas for truncated CPCs with a given $C_i$, $\theta_a$ and $\theta_e$, the $\theta_i$ can be calculated based on equation (1) [20]. Obviously, CPC-90, the CPC without restriction of exit angle, is a special case of CPC-$\theta_e$ for the case of $\theta_e = 90^\circ$.

It should be noted that the maximum $\theta_i$ should be less $\theta_e$ otherwise CPC-$\theta_e$ is reduced into a V-trough concentrator. The coordinate of lower end $D$ of the parabola can be determined based on equation (1) by setting $\theta = \theta_e$ as follows:

$$
\begin{align*}
    x_D &= \frac{(\sin \theta_e + \sin \theta_a) \sin \theta_e}{1 - \cos(\theta_e + \theta_a)} - 0.5 \\
    y_D &= \frac{(\sin \theta_e + \sin \theta_a) \cos \theta_e}{1 - \cos(\theta_e + \theta_a)}
\end{align*}
$$

(2)

The plane mirror is expressed by:

$$
y = c \tan \gamma_D (x - 0.5) \quad (0.5 \leq x \leq x_D)
$$

(3)

The tilt angle of plane mirrors ($\gamma_D$) relative to y-axis is given by:

$$
\gamma_D = 0.5(\theta_e - \theta_a)
$$

(4)

![Figure 1. Geometry of CPC-$\theta_e$](image1)

![Figure 2. Angle range of solar rays that arrive on the absorber after more than two reflections.](image2)

2.2. Optical efficiency of CPC-$\theta_e$
To calculate the collectible radiation of CPC-$\theta_e$, it is essential to find the angular dependence of optical efficiency. For CPC-$\theta_e$, all of radiation over its acceptance angle will arrive on the absorber at the incident angle ($\theta_i$) less than $\theta_e$ as shown in figure 1; whereas for radiation incident at $\theta_p > \theta_a$, part of radiation incident on the plane reflector (see figure 2) and upper parabolic reflector (see figure 3) will arrive on the absorber at $\theta_{in} > \theta_e$. Thus the collectible radiation on the receiver includes the radiation incident at $\theta_{in} \leq \theta_e$ (I$_1$), radiation reflecting from the plane reflector (I$_2$) at $\theta_{in} > \theta_e$ and that reflecting from the upper parabolic reflector to the opposite plane mirror first and then reflecting onto the absorber (I$_3$) at $\theta_{in} > \theta_e$. Therefore, the optical efficiency of CPC-$\theta_e$ is given by:

$$\eta(\theta_p) = \frac{I_1 + I_2 + I_3}{I_{ap}} = f_1 + f_2 + f_3$$

\[(5)\]

**Figure 3.** Fraction of radiation that arrive on the absorber after more than two reflections.

**Figure 4.** Transfer of radiation incident on the plane mirror of CPC-$\theta_i$ at $\theta_a < \theta_p < \theta_i$

**Figure 5.** Transfer of radiation incident on mirror of CPC-$\theta_e$ at $\theta_i < \theta_p < \theta_{p,c1}$
In equation (5), the $I_{ap}$ is the radiation incident on the aperture of CPC-$\theta$, $f_1$ is the optical efficiency contributed by radiation arriving on the absorber at $\theta_{in} \leq \theta_{e}$, $f_2$ is the efficiency contributed by radiation on the absorber after reflection from the plane reflectors at $\theta_{in} > \theta_{e}$, and $f_3$ is that contributed by radiation on the absorber after reflecting from the upper parabolic reflector first and then from its opposite plane reflector at $\theta_{in} > \theta_{e}$.

2.2.1. Calculation expressions of $f_1$. For radiation incident at small $\theta_p$, partial radiation undergoes multi-reflection before arriving on the absorber [22]. To make calculations accurate, the two-reflection model, in which the radiation arriving on the absorber after more than two reflections is regarded as that arrives on the absorber after just two reflections, is employed, thus $f_1$ can be expressed by:

$$f_1 = f_0 + (f_{11} - f_{12})\rho + f_{12}\rho^2$$

(6)

where $f_0$ is the fraction of radiation directly irradiating on the absorber; $f_{11}$ and $f_{12}$ stand for the fraction of radiation arriving on the absorber after more than one and two reflections, respectively. Equation (6) also can be rewritten as:

$$f_1 = f_0 + f_{11}\rho - f_{12}\rho(1 - \rho) = \eta_1 - f_{12}\rho(1 - \rho)$$

(7)

where $\eta_1 = f_0 + f_{11}\rho = f_0 + (1 - f_0)\rho$ is the $f_1$ of CPCs estimated based on the one-reflection model [20,22], and calculated by:

$$\eta_1 = \left\{ \begin{array}{ll}
\rho + (1 - \rho)/C_i & \theta_p \leq \theta_f \\
\rho + 0.5(1 - \rho)(1 + C_i)(1 - \tan \theta_p / \tan \theta_f)/C_i & \theta_f < \theta_p \leq \theta_a \\
0.5(1 + C_i)(1 - \tan \theta_p / \tan \theta_f)/C_i & \theta_a < \theta_p \leq \theta_i \\
0 & \theta_p > \theta_i
\end{array} \right. $$

(8)

In the case of $\theta_f > \theta_a$, it is given by

$$\eta_1 = \left\{ \begin{array}{ll}
\rho + (1 - \rho)/C_i & \theta_p \leq \theta_a \\
1/C_i + 0.5\rho(1 - 1/C_i) - 0.5\rho(1 + 1/C_i)\tan \theta_p / \tan \theta_{i,2} & \theta_a < \theta_p \leq \theta_f \\
0.5(1 + C_i)(1 - \tan \theta_p / \tan \theta_i)/C_i & \theta_f < \theta_p \leq \theta_i \\
0 & \theta_p > \theta_i
\end{array} \right. $$

(9)

where $\theta_f$ (see figure 1) is given by:

$$\tan \theta_f = \tan \theta_i (C_i - 1) / (C_i + 1)$$

(10)
As shown in figure 2, solar rays incident on the (right) reflector at \( \theta_{p,e} \) and \(-\theta_{p,s}\) just strike at the end (B) of the absorber after one reflection, thus solar rays incident at \(-\theta_{p,s} < \theta_p < \theta_{p,e}\) will undergo more than two reflections before arriving on the absorber (see figure 3). Based on the reflection law of light, one has:

\[
\theta_{p,e} = \theta_f - 2\gamma_{ap} \tag{11}
\]

The \( \gamma_{ap} \) is the tilt-angle of the line tangent to the upper end of the parabolic reflector relative to y-axis (see figure 4), and can be found based on equation (1) as follow:

\[
\tan \gamma_{ap} = \frac{dx}{dy}_{\theta=\theta_t} = \frac{\cos \theta_a - \cos \theta_t}{\sin \theta_a + \sin \theta_t} \tag{12}
\]

The critical angle \( \theta_{p,s} \) is subjected to following equation:

\[
\gamma_M = 0.5(\theta_{p,s} + \gamma_{MB}) \tag{13}
\]

where \( \gamma_M \) is the tilt-angle of the line tangent to M (see figure 3), \( \gamma_{MB} \) is the tilt-angle of line MB. For a given CPC, \( \gamma_M, \gamma_{MB} \) and \( \theta_{p,s} \) as the function of \( \theta_M \) can be respectively expressed by:

\[
\tan \gamma_M = \frac{\cos \theta_a - \cos \theta_M}{\sin \theta_a + \sin \theta_M} \tag{14a}
\]

\[
\tan \gamma_{MB} = (x_m - 0.5) / y_M \tag{14b}
\]

\[
\tan \theta_{p,s} = (0.5C_t - x_M) / (h_t - y_M) \tag{14c}
\]
By substituting $M_{\theta}$, $M_{B_{\theta}}$ and $s_{p}$, $\theta_{p}$ obtained from equation (14) into equation (13) or by iterative calculations, one obtains $M_{\theta}$, then $s_{p}$, $\theta_{p}$ is obtained from equation (14c). As shown in figure 3, the fraction of radiation arriving on the absorber after more than two reflections is given by $f_{12} = (KF + ED) / C_{i}$, and $KF$ is given by:

$$KF = \begin{cases} 0.5C_{i} - x_{M} + (h_{i} - y_{M}) \tan \theta_{p} & (-\theta_{p,s} < \theta_{p} < \theta_{p,e}) \\ 0 & \text{else} \end{cases}$$ (15)

where $x_{M}$ and $y_{M}$ as the function of $\theta_{M}$ are x-, y-coordinates of critical point M, respectively, and $\theta_{M}$ can be calculated based on equation (13), (14a)-(14b) by setting $\theta_{p,s} = -\theta_{p}$. Similarly, $ED$ can be calculated in the same way as finding $KF$ by setting $\theta_{p} = -\theta_{p}$.

2.2.2. Calculation method of $f_{2}$. As shown in figure 4, $BA'$ is the first image of the absorber formed by the plane mirror. According to the imaging principle of plane mirrors, rays pointing to the image of the absorber formed by the mirror must arrive on the absorber after reflecting from the mirror [12]. Therefore, when solar rays incident on the plane mirror at $\theta_{p} > \theta_{a}$, partial radiation will arrive
on the absorber at $\theta_m = \theta_p + \psi > \theta_e$ (for radiation incident at $\theta_p = \theta_a$, the $\theta_m = \theta_e$, thus the opening angle of V-trough formed by two plane reflectors of the CPC-$\theta_e$ is $\psi = 2\gamma_D = \theta_e - \theta_a$), the remaining will redirect to the opposite mirror and then escape from CPC-$\theta_e$ or arrive on the absorber at $\theta_m = (\theta_p + 2\psi) > \theta_e$ [12]. Solar radiation arriving on the absorber after two reflections in between two plane mirrors must satisfy:

$$\begin{cases} 
\theta_p > \theta_a \\
\theta_p + 2\psi \leq 0.5\pi 
\end{cases} \quad (16)$$

And this leads:

$$\theta_a > 2\theta_e - 0.5\pi \quad (17)$$

This means that solar rays entering the V-trough are possible to arrive on the absorber after two reflections for CPC-$\theta_e$ with $\theta_a > 2\theta_e - 0.5\pi$. Thus for CPC-$\theta_e$ with $\theta_e = 65^\circ$ and $60^\circ$, $\theta_a$ must be larger than $40^\circ$ and $30^\circ$, respectively. In practical design of CPC-$\theta_e$, $\theta_e$ is usually larger than $65^\circ$ and $\theta_a < 35^\circ$ [17], and such CPC-$\theta_e$ is subjected to $\theta_a < 2\theta_e - 0.5\pi$. In this work, the CPCs subjected to $\theta_a < 2\theta_e - 0.5\pi$ are considered for simplifying analysis, thus no radiation arrives on the absorber of CPC-$\theta_e$ after more than two reflections in between two mirrors.

To ensure solar rays reflecting from mirrors arrive on the absorber, the $\theta_p$ should be subjected to $\psi + \theta_p \leq 0.5\pi$, i.e. $\theta_p \leq 0.5\pi - \psi$; but in the other hand, $\theta_p$ should be less than $\phi_{ap}$ due to the shade of reflectors on the opposite mirrors (see figure 5-6). Therefore, the critical incident angle $\theta_{p, c1}$, solar rays incident at the angle less than which will arrive on the absorber at $\theta_m > \theta_e$, should take the smaller one of $\phi_{ap}$ and $0.5\pi - \psi$, namely:

$$\theta_{p, c1} = Min(\phi, 0.5\pi - \psi) \quad (18)$$

As shown in figure 5-6, $\phi_{ap}$ is the tilt-angle of the line linking the upper end $E$ ($F$) of parabolic reflectors and the image of absorber end $A$ ($B$), and is calculated by:

$$\tan \phi_{ap} = \frac{0.5 + \cos \psi + 0.5C_i}{h_i + \sin \psi} \quad (19)$$

Here $h_i = 0.5(C_i + 1)/\tan \theta_i$ is the height of CPCs. As shown in figure 4, the image ($BA'$) of the absorber is fully irradiated by radiation incident at $\theta_p \leq \theta_i$ and partially irradiated as $\theta_i < \theta_p < \theta_{p, c1}$.
(see figure 5), therefore, the fraction of radiation on the absorber after reflection from the plane mirrors, $\Delta x_2$, is determined by:

$$\Delta x_2 / C_i = \begin{cases} (\cos \psi - \sin \psi \tan \theta_p) / C_i, & (\theta_u < \theta_p \leq \theta_t) \\ (h_i + \sin \psi)(\tan \varphi_{ap} - \tan \theta_p) / C_i, & (\theta_t < \theta_p < \theta_{p,c1}) \\ 0, & \theta_p \geq \theta_{p,c1} \text{or} \theta_p \leq \theta_u \end{cases}$$  (20)

$$f_2 = \rho \Delta x_2 / C_i$$  (21)

2.2.3. Calculation method of $f_3$. As shown in figure 6, solar rays incident on the upper end (F) of the parabolic reflector at $\theta_{p,c2}$ will just redirect to the image $B'$ after reflection, thus, for solar rays entering CPC-$\theta_c$ at $\theta_a < \theta_p < \theta_{p,c2}$, the radiation incident on the upper parabola (FM) will redirect to the opposite mirror (AC) and then be reflected onto the absorber, and solar rays incident on lower part of parabola (MD) will finally escape from the cavity of the CPC-$\theta_c$ after multiple reflections within the CPC cavity. The polar angle $\theta_m$ of the critical point (M) on the parabola is subjected to following equation group:

$$\begin{align*}
2 \gamma_m &= \phi_m - \theta_p \\
tan \phi_m &= \frac{x_m + 0.5 + \cos \psi}{y_m + \sin \psi}
\end{align*}$$  (22)

where $x_m$ and $y_m$ as the function of $\theta_m$ are x and y coordinates of point M, respectively; $\phi_m$ as the function of $\theta_m$ is the angle formed by line $MB'$ and y-axis (see figure 6); $\gamma_m$, as the function of $\theta_m$, is the slope of the line tangent to point M and determined by equation (12). Thus, given $\theta_p$, $\theta_{a} < \theta_p < \theta_{p,c2}$, $\theta_m$ can be obtained by iterative calculations, then the fraction of solar radiation incident on the upper parabola (FM) that arrive on the absorber after reflection from the parabola first then from the opposite mirror, $\Delta x_3 / C_i$, can be calculated by:

$$\Delta x_3 / C_i = \begin{cases} 0.5 C_i - x_m + (h_i - y_m) \tan \theta_p, & (\theta_a < \theta_p < \theta_{p,c2}) \\ 0, & \text{else} \end{cases}$$  (23)

$$f_3 = \rho^2(\Delta x_3 / C_i)$$  (24)

According to the law of light reflection, the critical incident angle $\theta_{p,c2}$ (see figure 5) is given by:
\[ \theta_{p,c2} = \phi_{ap} - 2\gamma_{ap} \]  

(25)

It is noted that \( \gamma_{ap} = 0 \) and \( \theta_{p,c2} = \theta_{p,c1} \) for full CPC-\( \theta_c \). Analysis shows that, for truncated CPC-\( \theta_c \), \( \theta_{p,c2} \) decreases with the increase of \( \theta_i \), and \( \theta_{p,c1} \) is always larger than \( \theta_{p,c2} \). It is also noted that no radiation will arrive on the absorber at \( \theta_i \geq 0.5\pi - \psi \) (see figure 6) because point \( M \) must be above the line \( BA \) to ensure solar rays reflecting from \( M \) redirect to the image \( B \) of absorber end \( B \).

3. Mathematical method to calculate daily collectible radiation

It is assumed that CPC-\( \theta_c \) is oriented in the east-west direction with the aperture being tilted at \( \beta \) from the horizon, and radiation reflected from the ground is not considered. Thus, the collectible radiation on absorber at any moment for isotropic sky diffuse radiation is given by:

\[
I = C_I I_b g(\theta_{ap}) \eta(\theta_p) \cos \theta_p + C_I \int_0^{0.5\pi} iu(\theta_p) \cos \theta_p d\theta_p
\]

(26)

where \( I_b \) is the intensity of beam radiation; \( i = 0.5I_d \) is the directional intensity of sky diffuse radiation on the cross-section of CPC-troughs [12, 22]. Equation (26) can be rewritten as

\[
I = C_I I_b g(\theta_{ap}) \eta(\theta_p) \cos \theta_p + 0.5I_d (C_{d,1} + C_{d,2})
\]

(27)

where \( I_d \) is sky diffuse radiation on the horizon, and \( C_{d,1} \) and \( C_{d,2} \) are calculated as follows:

\[
C_{d,1} = C_I \int_0^{0.5\pi} \eta(\theta_p) \cos \theta_p d\theta_p
\]

(28)

\[
C_{d,2} = C_I \int_0^{0.5\pi - \beta} \eta(\theta_p) \cos \theta_p d\theta_p
\]

(29)

For a given CPC-\( \theta_c \), \( C_{d,1} \) is a constant but \( C_{d,2} \) is dependent on \( \beta \), and both can be obtained by numerical calculations. Similarly, radiation on the absorber of CPC-\( \theta_c \) at \( \theta_i \leq \theta_e \) is expressed by:

\[
I(\theta_i \leq \theta_e) = C_I I_b g(\theta_{ap}) f_1 \cos \theta_p + 0.5C_I I_d \int_{-0.5\pi - \beta}^{\beta} f_1 \cos \theta_p d\theta_p
\]

(30)

The daily radiation on the absorber can be calculated by:

\[
H_{day} = C_I \int_{-\infty}^{\infty} I_b g(\theta_{ap}) \eta(\theta_p) \cos \theta_p dt + 0.5H_d (C_{d,1} + C_{d,2})
\]

(31)

\[
H_{day}(\theta_i \leq \theta_e) = C_I \int_{-\infty}^{\infty} I_b g(\theta_{ap}) f_1(\theta_p) \cos \theta_p dt + 0.5C_I H_d \int_{-0.5\pi - \beta}^{\beta} f_1 \cos \theta_p d\theta_p
\]

(32)
where $H_d$ is the daily sky diffuse radiation on the horizon, $t_0 = \frac{2\tau_{\text{day}}}{2\pi}$ is the sunset time on the horizon, $\omega_0$ is the hour angle of sunset on the horizon [5]. Given the geometry of CPC-$\theta_e$ and tilt-angle $\beta$, the last term of right hand in equation (32) can be numerically calculated. The daily radiation on the absorber with $\theta_{in} > \theta_e$, $H_{\text{day}}(\theta_{in} > \theta_e)$, can be simply calculated by subtracting $H_{\text{day}}(\theta_{in} \leq \theta_e)$ from $H_{\text{day}}$. The incidence angle of solar rays on the aperture of CPC-$\theta_e$ ($\theta_{ap}$) and projection incident angle ($\theta_p$) at any moment of a day can be found based solar geometry [5, 22, 23]. The $g(\theta_{ap})$ in equation (26-27) is a control function, being 1 for $\cos \theta_{ap} > 0$ otherwise zero. On knowing $\theta_p$ and $\theta_{ap}$, and $f_1$, $f_2$ and $f_3$ can be determined based mathematical procedure suggested in this work, then the $H_{\text{day}}$ and $H_{\text{day}}(\theta_{in} > \theta_e)$ can be numerically calculated, finally summing $H_{\text{day}}$ and $H_{\text{day}}(\theta_{in} > \theta_e)$ in all days of a year obtain the annual radiation on the absorber of CPC-$\theta_e$ ($S_{\text{CPCS}}$-$\theta_e$) and annual radiation on the absorber with $\theta_{in} > \theta_e$ ($S(\theta_{in} > \theta_e)$). The annual radiation on the absorber of CPC-90 with $\theta_{in} > \theta_e$ can be obtained according to the method presented in the previous work of authors [20].

In this work, monthly horizontal radiation in Beijing ($\lambda = 39.95^\circ$), the capital of China, was used for the analysis [24], the daily sky diffuse radiation, $H_d$, and beam radiation ($I_b$) at any moment of a day are estimated based on the empirical correlations proposed by Collares-Pereira and Rabl [25]. The angle step for calculating $C_{d,1}$ and $C_{d,2}$ is take to be 0.005°; the time step to calculate daily radiation on the absorber is taken to be 1 min; and the $\theta_a$ of CPCs is set to 26° [26]. To investigate the optical performance of CPC-$\theta_e$, two cases with the $\beta$ being yearly fixed and yearly adjusted four times at three tilts are considered. For CPCs with $\beta$ being yearly fixed (1T-CPC), the $\beta$ is taken to be site latitude ($\lambda$) [26]; whereas for CPCs with $\beta$ being yearly adjusted four times at three tilts (3T-CPC), the $\beta$ is set to be $\lambda$ during periods of 22 days around both equinoxes, and adjusted to $\lambda + 23^\circ$ and $\lambda - 23^\circ$ in winters and summers, respectively [12].

4. Results and discussions

4.1. Optical efficiency comparison between CPC-$\theta_e$ and CPC-90

As seen from figure 7-8, $f_2$ and $f_3$ is zero for radiation within the acceptance angle but not for radiation beyond the acceptance angle as expected. It is also seen that the $f_3$ is larger than $f_2$ for full CPC-65 and less than $f_2$ for truncated CPC-65. This means that for full CPC-$\theta_e$, the contribution of upper parabola together with the opposite plane mirror to $S(\theta_{in} > \theta_e)$ is larger than that of plane mirror alone; whereas for truncated CPC-$\theta_e$, the situation is reversed. Comparisons of optical efficiency between CPC-90 and CPC-65 are presented in figure 9 and 10. It is found that the optical efficiency of both CPCs is almost identical for radiation within the acceptance angle; whereas for radiation incident at $\theta_p > \theta_a$, the $f$ of CPC-65 is always larger than that of CPC-90 for full CPCs, but for truncated CPCs with identical $C_i$ and $\theta_a$, the $f$ of CPC-90 is larger than that of CPC-65 in
the case of \( \theta_p > 31^\circ \) due to the large edge-ray angle \( (\theta_l) \) of CPC-90 and zero \( f_3 \) of CPC-65 (see figure 6)

![Graph](image1)

**Figure 7.** Angular variations of optical efficiency of full CPC-65

![Graph](image2)

**Figure 8.** As in Fig.7 but for truncated CPC-65
4.2. Comparison of annual collectible radiation between CPC-θc and CPC-90

Figure 11 presents the ratio of annual radiation concentrated by CPC-θc to that by CPC-90 in the case of β being yearly fixed (1T-CPCs). It is seen that, for full CPCs with identical θa, the annual radiation collected by CPC-90 is always larger than that by CPC-θc due to large geometric concentration of CPC-90; whereas for truncated CPCs with identical θa and Cγ, the annual radiation collected by CPC-θc are almost identical to that by CPC-90, and even slightly higher in the case of low Cγ. The same results as in figure 11 are also found in figure 12 in which β is yearly adjusted four times at three tilts (3T-CPCs).
4.3. Comparison of $S(\theta_{in} > \theta_{e})$ between CPC-$\theta_{e}$ and CPC-90

The annual radiation on the absorber of CPCs with $\theta_{in} > \theta_{e}$ are presented in figure 13-14. It is seen that $S(\theta_{in} > \theta_{e})$ of both CPC-$\theta_{e}$ and CPC-90 decreases with the increase of $\theta_{e}$, and $S(\theta_{in} > \theta_{e})$ collected by CPC-$\theta_{e}$ is much less than that by CPC-90 in the case of $\theta_{e}$ less than 70°, this implies...
that the CPC-$\theta_e$ based PV system should be more efficient than CPC-90 based PV system because the radiation on solar cells with $\theta_m > 65^\circ$ is poorly absorbed by solar cells.

It is noted that, given $\theta_e$, the annual radiation on the absorber of CPC-90 is independent on its geometric concentration ($C_t$) and $S(\theta_m > \theta_e)$ keeps constant as a result of the fact that the $S(\theta_m > \theta_e)$ comes from the lower part of reflectors of CPC-90 [20], but $S(\theta_m > \theta_e)$ collected by CPC-$\theta_e$ is sensitive to $C_t$ because critical angles $\theta_{p,c1}$ and $\theta_{p,c2}$ are sensitive to $\theta_i$ as shown in figure 15. In fact, $S(\theta_m > \theta_e)$ collected by CPC-$\theta_e$ in a site depends on the angular dependence of $(f_2 + f_3)$ and tilt-angle adjustment strategy, and in turn the angular dependence of $(f_2 + f_3)$ is dependent on the edge-ray angle ($\theta_i$) or $C_t$ as shown from figure 16. As shown in figure 15, for 1T-CPC-65, $S(\theta_m > 65^\circ)$ increases with $C_t$ due to high $f_2 + f_3$ for $\theta_p < 30^\circ$ (see figure 15) and large $C_t$; whereas for 3T-CPC-65, $S(\theta_m > 65^\circ)$ decreases with $C_t$ as a result of the fact that the sun is almost within the acceptance angle of CPCs (i.e. $\theta_p < \theta_a$) over the daytime in any day of a year, and $S(\theta_m > 65^\circ)$ mainly originates from the sky diffuse radiation, thus $S(\theta_m > 65^\circ)$ increases with $\theta_i$ or decreases with $C_t$. Figure 14 also indicates that $S(\theta_m > 65^\circ)$ of 1T-CPC-65 is much larger than that of 3T-CPC-65, showing that the use of 3T-CPCs facilitates the improvement of photovoltaic performance CPC-$\theta_e$ based PV systems due to lower $S(\theta_m > \theta_e)$.

![Figure 13](image1.png)

**Figure 13.** Annual radiation on the absorber of 1T-CPCs at $\theta_m > \theta_e$

![Figure 14](image2.png)

**Figure 14.** As in figure 13 but for 3T-CPCs
Figure 15. Effects of geometric concentration on $S(\theta_m > 65)$ of CPC-65

Figure 16. Optical efficiency for radiation arriving on the absorber at $\theta_m > 65$

Figure 17. Effects of acceptance half-angle on $S(\theta_m > 65)$ of CPCs

Figure 17 shows the effect of acceptance angle ($\theta_a$) on $S(\theta_m > \theta_e)$. It is seen that, given $\theta_e$ (such as $65^\circ$), $S(\theta_m > \theta_e)$ collected by CPC-90 and CPC-$\theta_e$ decreases with the increase of $\theta_a$. This indicates that $\theta_a$ should be large as possible in the practical design of CPCs based on requirements of least daily operation hours, geometric concentration and strategy of tilt-angle adjustments [27].
5. Conclusions
The analysis in this work shows that part of radiation beyond the acceptance angle of CPC-\(\theta_e\) would arrive on the absorber at \(\theta_m > \theta_e\), the annual radiation on the absorber with \(\theta_m > \theta_e\) depends on the acceptance angle, geometric concentration as well tilt-angle adjustment strategy of CPC-\(\theta_e\). Calculations indicate that, for full CPCs with identical \(\theta_a\), the annual radiation collected by CPC-\(\theta_e\) is less than that by CPC-90 due to the large geometric concentration (\(C'_r\)) of CPC-90; whereas for truncated CPCs with identical \(\theta_a\) and \(C'_r\), the annual radiation collected by CPC-\(\theta_e\) is almost identical to that by CPC-90, and even slightly higher. Calculations also show that the annual radiation on the absorber of CPC-\(\theta_e\) with \(\theta_m > \theta_e\) decrease with the increase of \(\theta_e\) but always less than that of CPC-90, and this implies that the CPC-\(\theta_e\) based PV system is more efficient than CPC-90 based PV system because the radiation on solar cells incident at large angle is poorly converted into electricity.

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