ANALYSIS AND SYNTHESIS OF COMPLEX
TECHNOLOGICAL SYSTEMS

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Abstract.
This article outlines the theoretical, methodological and practical problems of analysis and synthesis of complex systems. The static or dynamic property of a system can be evaluated when the system structure is known, and all its parameters are specified. The objective of system analysis is to find an exact analytical or approximate solution of equations based on a corresponding mathematical model, as well as its further research. Theoretical conclusions and results have allowed to build mathematical models applicable to the management of objects with different principles of operation, in particular, to the management of complex technical and technological objects that can be represented as nonlinear dynamic systems. Nonlinear dynamic integral models are deemed appropriate to study and design such systems in some critical application.

Keywords: technical systems, engineering systems, mathematical models, management, system analysis, complex system.

I. Introduction
Any control system can be conditionally represented as consisting of two parts: a controllable object and a controlling device or controller. The controllable object is any technical device, the operating mode of which must be maintained by the controlling device with the help of specially organized actions. Generally, the object of control is influenced by disturbances resulting from the interaction of control system with the external environment.
When studying specific processes, the control theory abstracts away from the nature and design features of the object, and considers mathematical models instead of real objects. A mathematical model is a system of mathematical relationships describing the behavior of controllable object [I,VI].

II. Methods

There are many approaches to the methodology of the study of various systems. Commonly, a system is a collection of objects with certain properties connected with each other and forming certain integrity.

The choice of one or another way of describing state spaces and external influences is critically depending on the type of the system in question.

Complexity is one of the main properties of any system that can be used only when comparing with reference system, herewith complexity estimations are divided into qualitative and quantitative [VI].

At the present stage of development of system analysis, there is no rigorous mathematical definition of a complex system that covers all intuitive notions of real systems.

A complex system possesses at least one of the following properties:

1) large number of variables and a large number of links between them and other parts of the system,
2) higher order of equations describing the system,
3) large number of nonlinear elements appearing in the system.

The parameters of complex systems are often non-numeric, which generates a significant dependence of their qualitative analysis and synthesis on the introduced topologies and metrics in the corresponding deterministic environment. Different complex systems, such as multidimensional, multivariable, multi-component, highly complex, large, multi-level systems are described in the scientific literature.

Mathematical problems of systems theory include a whole range of problems of analysis and synthesis.

As for analysis, the structure of the system is fully known, all its parameters are specified, and it is required to evaluate one of its static or dynamic properties. The problem of analyzing any system means finding an exact analytical or approximate solution of the corresponding mathematical model describing the system in question, and its further research in a given region of the state space. This problem is well investigated and is widely used in engineering practice for defining the construction of a model with known parameters by means of numerical methods [I,II,III].

III. Results

Synthesis tasks are usually considered as inverse to analysis tasks, since they need to determine as precisely as possible the structure and parameters of the initial system according to specified quality criteria and pursue a rational choice of initial parameters and basic characteristics of the system under consideration. This line of research is called design or synthesis of control and management systems. It helps to find optimal solutions for certain technical conditions of the environment in which the dynamic system under consideration is located [III,IV].

Various transformations constituting the system model play an important role in system synthesis; therefore, the system synthesis problem consists in creating
optimal software and stabilizing controls, as well as in developing processing algorithm for input data, observing and block diagrams given the specified quality criteria and perturbing factors of different nature.

A characteristic example of transformation $A: X \rightarrow Y$ in a linear dynamic system is Volterra integral operator of the form [V]:

$$y = A(x) = \int_{0}^{t} k(t - \tau)x(\tau)d\tau,$$

where $k(t)$ is the specified function equal to zero at $t < 0$, belonging to a generalized metric (in particular, normed) space of input and output signals in the system under consideration.

If the kernel of the Volterra integral operator is a superposition of two functions, and the operator itself is the product of the nonlinear Nemytsky functional operator and the linear Volterra integral operator, or it is given in the form of a graph (and problems of this type arise when one need to build a model of a dynamic system function), it is advisable to pre-approximate it with the help of a feasible function $K_{0}(t)$, using any method of approximation [V,VI,VII].

Using analytical methods for correctly posed optimization problems in the sense of Hadamard, it is possible to build equations describing optimal processes in dynamic systems. In this case, the synthesis of the system with regard to the implementation of the optimization process is highly important.

The difficulties in synthesizing optimal regulatory systems are largely associated with the type of solution obtained, as well as with the infrastructure available to the researcher. It is possible to significantly simplify the structure of optimal system regulator merely by limiting the objective to the approximate implementation of optimal processes [III,IV].

The problem of optimal and suboptimal systems synthesis is the subject of a number of theoretical and experimental studies. Most publications are related to the synthesis of optimal servosystems based on the equations for switching surface in the phase space.

Linearity in parameters and stationarity are two qualities inherent in models of a many control systems. They are characterized by proportionality between the causes and consequences of an object dynamics, equality between aggregate effect of all factors and the sum total of individual factors effects, and the uniqueness of dynamic equations solutions. These advantages should remain when describing nonlinear objects.

In particular, one can use the opportunity to clearly express the relationship between the input and output of a dynamic system, the simplicty of describing the state space, the interpretation of linear objects as a subclass of nonlinear ones, etc.

Examples include various nonlinear inertial models, non-linear in parameters, in which the Hammerstein-Uryson integral operators play a large role; Volterra-Picard integrated functional series with non-linear superposition operators, as well as systems of the above equations. In particular, for some classes of nonlinear radio engineering systems, which can be represented as a combination of a linear dynamic subsystem and a non-inertial nonlinear part, it is sometimes possible to directly apply identification using an adaptive model [V,VI,VII,VIII].
The authors of recent publications covering the synthesis of dynamic systems pay much attention to the properties and role of the so-called dominant linear and nonlinear operators in the model of system under consideration. If an operator is dominated by another operator, called a dominant, then the properties of the latter significantly affect the properties of the former. This suggests an idea of using the technique of dominant operators in nonlinear models.

At present, problems of dynamical systems have been explored in a detailed way taking into account their interaction with the environment, where either linear or linearized models were considered.

Another equally important problem is creating a space describing the environment in which the system is located. This would allow to reduce the qualitative picture of the original dynamic system behavior to the study of corresponding mathematical model, in which the input-output operator would have desirable properties [I,II,III,IV].

The study of nonlinear dynamical systems is simplified if viewed from the position of operator approach. Integral systems play an important role in multidimensional processes, such as television systems, radar systems, geophysics, control and energy-saving systems; they can serve as a typical model example. That is why the function space should be involved when solving this type of problems.

Building an exact or approximate analytical solution and an algorithm for processing input observations and block diagrams within the specified quality criteria is of no less importance.

Let us briefly cover some nonlinear operators in dynamic systems prevalent in engineering practice. The structure of a non-linear system model is often represented as a combination of simpler links: linear and functional, suggesting that in a functional link the duration of transition process is negligible. Nonlinear operators included in the integral model are[VIII]:

- Nonlinear superposition operator
  \[ y(\tau) \equiv \Phi(\tau, x(\tau)) \]  
  \[ (2) \]

- Nonlinear integral Hammerstein operator:
  \[ H(\tau(\tau)) : = \int_{0}^{T} K(\tau, \tau_1) \Phi[\tau_1, x(\tau_1)] \, d\tau_1 ; \]  
  \[ (3) \]

- Nonlinear integral Urysohn operator:
  \[ U(\tau(\tau)) : = \int_{0}^{T} K[\tau, \tau_1, x(\tau_1)] \, d\tau_1 ; \]  
  \[ (4) \]

- Lichtenstein-Lyapunov operator:
  \[ L(\tau(\tau)) : = \sum_{p,q_2,..,q_k} \int_{0}^{T} \int_{0}^{T} K_{p,q_2,..,q_k}(\tau, \tau_1, \tau_2,..,\tau_k) \cdot x^{p}(\tau_1)x^{q_2}(\tau_2)\ldots x^{q_k}(\tau_k) \, d\tau_1d\tau_2\ldots d\tau_k. \]  
  \[ (5) \]

Naturally, not all nonlinear systems can be described by models mentioned above with the help of operators linking the input of the system with its output in an explicit form. If, for example, there is a feedback in the considered nonlinear dynamic
system, then this system is described by an operator equation of the following form:

\[ y(\tau) = B[x(\tau), y(\tau)] + x(\tau), \quad (6) \]

Where \( B \) is a nonlinear operator. Thus, in dynamic electricity monitoring and metering systems (which are servo systems), operator \( B \) is often represented as a non-linear Hammerstein integral operator \( H \).

As it turned out, the above nonlinear integral operators are H-operators, subject to the condition:

\[ \|T(x) - T(x + y); E_2\| \leq \sum_{k=1}^{n} H(\|T(x) - T(x + \chi_{A_k} \cdot y); E_2\|), \]

Where the function \( H \in \Phi(L) \), for each element \( y \in E \) and for all pairwise disjoint (disjunctive) measurable subsets \( A_k \subset \Omega, (k = 1, n), \chi_{A_k} \) is an indicator of \( A_k \) subset.

Let \( E_1 \) and \( E_2 \) be two \( \Omega \)-measurable functions, the \( A \)-operator: \( E_1 \rightarrow M, \forall x \in E_1 \), Let the \( A \)-operator be \( H \)-operator, if for \( \forall v \in E_1 \) the following condition is met:

\[ \|A(v) - A(v + \lambda x); E_2\| \leq H(\|A(v) - A(v + \lambda \chi_{D_1} x); E_2\|) + H(\|A(v) - A(v + \lambda \chi_{D_2} x); E_2\|), \]

Where the function \( H: \mathbb{R}^n \rightarrow \mathbb{R} \) is in the class \( \Phi(L) \), for each element \( x \in E_1 \) and any measurable subsets \( D_1 \) and \( D_2 \) from \( \Omega \) such that \( D_1 \cup D_2 = \Omega \) and \( D_1 \cap D_2 = \emptyset \), \( \chi_{D_i} \) is the indicator of set \( D_i \), \( i = 1, 2 \) and \( \lambda \in \mathbb{R} \).

The superposition operator, nonlinear atomic integral Hammerstein and Urysohn operators, integro-power Volterra series, normal integrants and locally-defined operators are particular instances of \( H \)-operator \( A \).

Various model transformations play an important in the synthesis of systems. Examples include nonlinear inertia dynamical systems in the case when minimum information on physical system is available. In such models, non-linear Hammerstein-Uryson integral equations, Voltaire-Picard integro-power series with essentially non-linear input operators, equations with internal superposition operators are widely used.

Example. Let there be any system given by input/output mapping with the help of a matrix-function with impulse response \( H(t, \tau) \) in the form of

\[ y(t) = \int_{0}^{t} H(t, \tau) u(\tau) d\tau. \]

There is an implementation of an arbitrary causal linear input-output mapping, given by state equations of the form:

\[ \dot{\chi}(t, \xi)/\dot{\tau} = \dot{\chi}(t, \xi)/\dot{\tau} + H_u(t + \xi, t) u(t), \]

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One of our goals was to consider the synthesis of a system with superlinear $H_\lambda$-operator. We consider the operator as superlinear if

$$y(t) = \int_{t_0}^t H_\lambda(t, t + \xi) x(t, \xi) d\xi; \quad x(0, \xi) = 0.$$  

where $H_\lambda \in \mathbb{R}$, $t \in \mathbb{R}$, $u, th \in E_0$.

Examples of superlinear operators are:

1. Every continuous convex functional $\Gamma$. It is enough to choose as small $t_0$ as small that the number $\Gamma(u + th)$ has the same sign as $\Gamma(u)$ at $|t| \leq t_0$. Then the convexity of the functional $\Gamma$ implies:

$$|\Gamma(u + th) - \Gamma(u)| \leq |\Gamma(u) + \Gamma(th) - \Gamma(u)| = |\Gamma(th)|$$

at all $|t| \leq t_0$.

2. Nonlinear integral Urysohn operator

$$Tu(\tau) = \int_{\Omega_0} K[\tau, s, u(s)] d\mu(s)$$

In the pair of $F$-quasinormed spaces $(E_0(\Omega_0), E_1(\Omega_1))$ will be superlinear, if its kernel is subject to the condition:

a) $K(\tau, s, \lambda u) = \lambda K(\tau, s, u)$ at $\lambda \in \mathbb{R}$.

b) $|K(\tau, s, u + th) - K(\tau, s, u)| \leq |K(\tau, s, th) + o(th)|$.

3. Sufficient condition for superlinearity of the Nemytsky operator $f[u(s) = f[s, u(s)]]$, where $f$ is a normalintegrant (in particular, Caratheodory function), is homogeneity of the function $f[s, u]$ on unit each satisfying of constraint

$$|f(s, u + th) - f(s, u)| \leq |f(s, th) + o(th)|$$

Nonlinear dynamic systems described by nonlinear ordinary differential equations should be reduced to the corresponding nonlinear integral equations.

IV. Conclusions

As is known, the sequence of stages for solving the problem of complete synthesis may be different. As a rule, firstly, the algorithmic structure of the system should be defined by mathematical methods, and then the appropriate structural elements should be selected, which is quite difficult, since these elements could not be available. Therefore, the functionally necessary elements, that form together with the controllable object an unalterable element of the system, are selected based on the requirements to the designation of the system and its operating conditions. [VI, VIII].

At the next stage, the alterable element including conversion unit and correcting elements is defined based on the requirements for static and dynamic
properties of the system. Thus, the processes of determining the algorithmic and functional structures of the entire system are interlinked.

The final stage of system synthesis is parametric optimization: calculation of settings for controller’s parameters. Having solved the problem of synthesis, it is necessary to perform the analysis of synthesized system that is to check whether the system has the necessary precision factor and indicators of stability and quality.

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