Research on trajectory tracking of unmanned vehicle based on model predictive control

Zheng Liu¹, Haiyang Kang²

¹,²College Of Computer Science And Engineering, Chongqing University of Technology, Chongqing 400054, China

*Corresponding author’s e-mail: kanghaiyang@2018.cqut.edu.cn

Abstract. Trajectory tracking control is a key technology in the research and development of autonomous vehicles. In view of the fact that the traditional model predictive control (MPC) method does not consider the interaction of longitudinal and lateral forces in the trajectory tracking control of unmanned vehicle, a vehicle trajectory tracking control method based on model prediction is proposed. Considering the longitudinal and lateral coupling of tire force, a more accurate tire force model and dynamic model are obtained. Besides a variety of constraints such as control quantity, control increment and output are added. The co-simulation platform CarSim / Matlab / Simulink is used to verify the designed controller by taking the four-wheel front steering vehicle. The results show that the track tracking process is stable and reliable under the condition of double lane change and the tracking effect of vehicle to reference track is very ideal.

1. Introduction

In automatic vehicle control, trajectory tracking is an important part [1,2]. The trajectory tracking algorithm based on model predictive control has the advantages of predicting the future trajectory and dealing with multi-objective constraints. The trajectory tracking control of unmanned vehicles is widely used [3,4,5]. Most of the early studies were based on vehicle kinematics, and established the corresponding model predictive controller. The vehicle performance was quite stable at low speed and flat road [6]. However, when encountering slightly complex road conditions or driving at high speed, the tracking effect could not meet the ideal requirements [7]. After the model predictive controller based on vehicle dynamics is applied to trajectory tracking control, the situation is greatly improved [8,9]. On this basis, the improvement of the accuracy of the dynamic model has become the focus of the problem, and the tire force model as the basis of the dynamic model naturally becomes the key [10,11]. Moreover, some literatures have analyzed the pure sideslip and pure slip, constructed the dynamic model, and achieved good results [12,13].

Based on the above research, this paper proposes a trajectory tracking control method of driverless vehicle based on model prediction. The method uses a six degree of freedom vehicle dynamics model, takes into account the coupling of tire forces in the vertical and horizontal directions, and takes advantage of the advantages of model predictive control, rolling optimization and handling of multivariable constraints. The performance of the controller is verified by using the co-simulation platform CarSim / Matlab / Simulink. The results show that the trajectory tracking process is stable and the controller performance is stable and reliable in the multi speed simulation under the condition of double line shifting.
2. Vehicle Dynamic model
Compared with other systems, the vehicle system is a very complex nonlinear system, which needs to be properly simplified while ensuring the accuracy of the model. In this paper, the four-wheel vehicle model is used to describe the force situation of the vehicle. Assume that the height of the center of mass of the vehicle is 0, and the pitch motion, the roll motion, rolling resistance, air resistance, road shoulder and road gradient are all ignored. The parameters of the vehicle model used in this paper are shown in Table 1. Figure 1 is the simplified stress analysis diagram of the vehicle used in this paper. According to the diagram, Newton's second law is used to analyze the force balance of the vehicle on three coordinate axes, and the mechanical balance equations of X, Y and Z axes are obtained.

\[
m\ddot{x} = m\dot{y}\dot{\psi} + F_{y,f,l} + F_{y,f,r} + F_{y,r,l} + F_{y,r,r} \\
\]

\[
m\ddot{y} = F_{y,f,l} + F_{y,f,r} + F_{y,r,l} + F_{y,r,r} \\
\]

\[
\dot{\psi} = a(F_{y,f,l} + F_{y,f,r}) - b(F_{y,r,l} + F_{y,r,r})
\]

where

\[
F_{y,l,*} = F_{y,f,}\sin\delta_{,r} + F_{c,v,}\cos\delta_{,r}
\]

\[
F_{y,r,*} = F_{y,f,}\cos\delta_{,r} - F_{c,v,}\sin\delta_{,r}
\]

In the formula, * ∈ {f, r} representative before and after, • ∈ {l, r} representative left and right. The plane motion equation of the center of mass in the inertial coordinate system is as follows:

\[
\dot{X} = x\cos\psi - y\sin\psi
\]

\[
\dot{Y} = x\sin\psi + y\cos\psi
\]

Table 1. Basic parameters of controlled vehicle.

| Symbol | Description                        |
|--------|-----------------------------------|
| m      | Vehicle mass (kg)                 |
| I      | Mass moment of inertia (kg · m²)  |
| δ      | Wheel steering angle (rad)        |
| a      | Distance between CG and front axle (m) |
| b      | Distance between CG and rear axle (m) |
| ψ      | Yaw angle (rad)                   |
| ψ̇     | Yaw rate (rad/s)                  |
| F_{l,r,*} | Longitudinal forces of tires (N) |
| F_{c,v,*} | Lateral forces of tires (N)      |
| F_{x,*} | Tire forces along x in vertical axis (N) |
3. Tire model
The tire model used in this paper is an extension of the classical Pacejka tire model, which considers the comprehensive slip, that is, the simultaneous transmission of longitudinal force and transverse force [14]. The longitudinal force $F_x$ is expressed by the product of the force in the case of pure longitudinal slip (such as $F_x0$) and the gain in the case of composite slip (such as $Gx\alpha$). So is the lateral force $F_y$. Therefore, they can be simply described as:

$$
\begin{align*}
F_x &= Gx\alpha(s, \alpha) \times Fx0(s) \\
F_y &= Gy\alpha(s, \alpha) \times Fy0(\alpha)
\end{align*}
$$

(8)

Tire model parameters reference [15], considering the ultimate purpose of this study and the length of the article, do not elaborate. The relationship between them can be seen from the two simulation diagrams in Fig. 2 and Fig. 3, that is, in the calculation of longitudinal force, the influence of sideslip angle is very obvious. Correspondingly, the longitudinal slip also has an obvious influence on the calculation of lateral force. Therefore, the interaction should be added to the calculation of dynamic model to obtain a more accurate dynamic model.

![Figure 2. Longitudinal tire forces with different slip ratio.](image1)

![Figure 3. Lateral tire forces with different slip angel.](image2)

The slip ratio between tire and ground can be obtained by the following formula:

$$
S_\alpha = \begin{cases} 
\frac{r_\alpha\omega_\alpha}{v_\alpha}, & -1, v_l, \omega_\alpha, v_f \neq 0 \\
1 - \frac{v_f}{r_\alpha\omega_\alpha}, & v_f, > r_\alpha\omega_\alpha, v_f \neq 0 
\end{cases}
$$

(9)

According to the geometric relationship of tire speed, the cornering angle of tire can be obtained.

$$
\alpha = \tan^{-1} \frac{v_c}{v_l}
$$

(10)

In the formula, $v_c$ and $v_l$ represents the lateral and longitudinal speed of the tire respectively, so it can be obtained from the tire speed in the vehicle coordinate system.

$$
\begin{align*}
v_f &= v_f \sin \delta + v_l \cos \delta \\
v_c &= v_l \cos \delta - v_f \sin \delta
\end{align*}
$$

(11)

In the above formula, the tire speed is divided into front and rear wheels, which can be calculated by the following formula:
4. Design of the MPC Controller

In the process of vehicle trajectory tracking, the model predictive controller obtains the state value of the vehicle at each time, predicts the state value in the prediction time domain with the prediction model, solves a series of optimal quadratic objective functions in the prediction time domain under the constraint conditions, and selects the first as the control quantity at the next time.

4.1. Linear time varying model

Due to the high real-time requirements of the motion controller when the unmanned vehicle is running at high speed, the nonlinear model is difficult to meet the practical requirements. The linearized dynamic model is obtained by linearizing the model through Taylor expansion at the reference point. Nonlinear dynamic model can be written as:

\[
\ddot{\xi} = f'_{\xi}(\xi, u, t)
\]

Linearization model can be obtained by linearized:

\[
\begin{aligned}
\ddot{\xi}(t) &= A(t)\dot{\xi}(t) + B(t)u(t) \\
y(t) &= C\xi(t)
\end{aligned}
\]  

(14)

Where \(A(t)\) = \(\frac{\partial f}{\partial \xi}\), \(B(t)\) = \(\frac{\partial f}{\partial u}\), \(C = [0, 0, 0, 1, 0, 0]\).

Because the process of model predictive control is discrete, it is necessary to discretize the model before it can be applied to the model predictive controller

\[
\begin{aligned}
\dot{\xi}(k+1) &= A(k)\xi(k) + B(k)u(k) \\
y(k) &= C\xi(k)
\end{aligned}
\]  

(15)

where

\[
\begin{bmatrix}
A(k) \\
B(k)
\end{bmatrix} = \begin{bmatrix}
I + TA(t) \\
TB(k)
\end{bmatrix}
\]  

(16)

\(T\) is the sampling period; \(I\) is the identity matrix.

For the convenience of calculation and expression, set up:

\[
x(k | t) = 
\begin{bmatrix}
\xi(k | t) \\
u(k-1 | t)
\end{bmatrix}
\]

(17)

Then, we can get a new state space expression

\[
\begin{aligned}
x(k+1 | t) &= A(k)x(k) + B(k)u(k) \\
\eta(k | t) &= C(k)x(k)
\end{aligned}
\]

(18)

Where

\[
\begin{bmatrix}
A(k) \\
B(k)
\end{bmatrix} = 
\begin{bmatrix}
A(k) \\
B(k)
\end{bmatrix} = 
\begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix}
\]

\(\eta(k | t)\) represents the output of the system in the prediction time domain. It is assumed that the
The prediction time domain of the system is $N_p$, control time domain is $N_c$, and $N_c \leq N_p$. Then the output of the system at time $t$ is:

$$Y(t) = \Psi(t) \xi(t \mid t) + \Theta(t) \Delta U(t)$$

(19)

Where

$$Y(t) = \begin{bmatrix} \eta(t+1 \mid t) \\ \eta(t+2 \mid t) \\ \vdots \\ \eta(t+N_c \mid t) \\ \vdots \\ \eta(t+N_p \mid t) \end{bmatrix}$$

(20)

$$\Psi(t) = \begin{bmatrix} C(k) A(k) \\ C(k) A^2(k) \\ \vdots \\ C(k) A^{N_c}(k) \\ \vdots \\ C(k) A^{N_p}(k) \end{bmatrix}$$

$$\Theta(t) = \begin{bmatrix} C(k) B(k) & 0 & 0 & 0 \\ C(k) A(k) B(k) & C(k) B(k) & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ C(k) A^{N_r-1}(k) B(k) & C(k) A^{N_r-2}(k) B(k) & \cdots & C(k) B(k) \\ C(k) A^{N_r}(k) B(k) & C(k) A^{N_r-1}(k) B(k) & \cdots & C(k) A(k) B(k) \\ \vdots & \vdots & \ddots & \vdots \\ C(k) A^{N_p-1}(k) B(k) & C(k) A^{N_p-2}(k) B(k) & \cdots & C(k) A^{N_p-N_r+1}(k) B(k) \end{bmatrix}$$

(23)

4.2. Constraints conditions

In order to make the trajectory tracking more ideal, the control increment is constrained while the control quantity is constrained, which makes the constraints more detailed and reduces the error. In the aspect of vehicle dynamics, the sideslip angle of the vehicle mass center is constrained to ensure the stability of the vehicle in the driving process.

4.2.1. Constraints of control quantity and control increment

According to the actual physical performance of the vehicle, the control quantity and increment of the front wheel angle are set. The constraint expression of control quantity is as follows:
\[
U_{\text{min}}(k + t) < u(k + t) < U_{\text{max}}(k + t), k = 0,1, \cdots, N - 1
\]  
(24)

The expression of control increment constraint is as follows:
\[
\Delta U_{\text{min}}(k + t) < \Delta u(k + t) < \Delta U_{\text{max}}(k + t), k = 0,1, \cdots, N - 1
\]  
(25)

Both of them are constrained at the same time to ensure that the control output calculated by the controller is physically stable and feasible.

4.2.2. Dynamic constraints

In order to ensure the safe and smooth driving of the unmanned vehicle, the dynamic constraints of the vehicle, the centroid yaw angle constraints and the vehicle attachment conditions are added as constraints.

a. Centroid sideslip angle constraint

The sideslip angle of vehicle centroid has a great influence on driving stability. On the ideal road surface, the limit of centroid deflection angle can reach plus or minus 12 degrees. When the road condition is poor and the wheel ground friction coefficient is reduced, the limit value of the sideslip angle of the center of mass is about plus or minus 2 degrees. Considering the normal situation in real life, when the vehicle is running on the normal road, the sideslip angle of the mass center is restricted to 
\[-10^\circ \leq \beta \leq 10^\circ.\]

b. Attachment constraints

The dynamic performance of vehicle is also sensitive to the change of road adhesion coefficient. When the road adhesion condition is good, it has little influence on the vehicle driving, but when the road adhesion condition is bad, it will have a certain impact on the vehicle dynamic performance and passenger comfort. However, if the constraint value is set too small, the controller will fail. Therefore, the road adhesion coefficient is set as soft constraint. The relationship between the lateral acceleration and the road adhesion coefficient is as follows
\[
a_y \leq \mu g
\]  
(26)

Therefore, the constraint of road surface adhesion condition can be written as follows:
\[
a_{y,\text{min}} - \varepsilon \leq a_y \leq a_{y,\text{max}} + \varepsilon
\]  
(27)

4.3. Design controller

Even after a lot of reasonable simplification and corresponding linearization, the vehicle dynamic model is still far more complex than the kinematic model with geometric constraints. With a lot of constraints, the actual situation is more complex. Therefore, in the actual implementation process, the controller may be unable to get the optimal solution or the control quantity may have a sudden change in the specified calculation time. In order to avoid this situation, a relaxation factor is added to the objective function
\[
J_{H\rho}(\xi(t),u(t-1),\Delta U_i) = \frac{H^2}{2} \sum_{t=1}^{N_j} \| \hat{\eta}_{(t+i,t)} - \eta_{\text{ref}(t+i,t)} \|^2_2
\]  
\[
+ \sum_{t=1}^{N_i-1} \| \Delta u_{(t+i,t)} \|^2_2 + \rho \varepsilon^2
\]  
(28)

s.t. \[
\Delta U_{\text{min}} \leq \Delta U_i \leq \Delta U_{\text{max}}
\]
\[
U_{\text{min}} \leq U_i + A \Delta U_i \leq U_{\text{max}}
\]
\[
\beta_{\text{min}} \leq \beta \leq \beta_{\text{max}}
\]
\[
y_{hc,\text{min}} \leq y_{hc} \leq y_{hc,\text{max}}
\]
\[ y_{hc,\min} + \varepsilon \leq y_{sc} \leq y_{hc,\max} + \varepsilon (\varepsilon > 0) \]

Where, \( y_{hc} \) is the output hard constraint, that is, it cannot exceed the constraint range; \( y_{sc} \) is the soft constraint quantity, the so-called soft constraint refers to the dynamic and small value constraint range adjustment through the effect of relaxation factor; \( y_{hc,\min} \), \( y_{hc,\max} \) are the limit values of hard constraint under corresponding simulation conditions, \( y_{sc,\min} \), \( y_{sc,\max} \) are the limit values of soft constraint under corresponding simulation conditions. Each time, the above equation is solved to obtain a series of continuous control input increments in the control time domain.

\[ \Delta U_i^* = \begin{bmatrix} \Delta u_{t+1}^* \Delta u_{t+1}^* \cdots \Delta u_{t+N_c-1}^* \end{bmatrix} \] (29)

The first increment obtained from the above formula is applied to the system, and the control quantity at the current time is obtained by adding it to the control quantity at the previous time

\[ u_t = u_{t-1} + \Delta u^*_t \] (30)

5. Results of Simulation

So far, we have completed the whole theoretical analysis. Next, in order to verify the control effect of the designed controller in the process of vehicle trajectory tracking control, we implemented the double lane shifting simulation of different speed conditions on the joint vehicle simulation platform of CarSim / Matlab / Simulink for verification and comparison.

5.1. Design of simulation platform

In this co-simulation platform, the vehicle model is completely provided by CarSim. CarSim software provides the choice of various vehicles, and can set the inherent parameters of the vehicle and related performance and simulation conditions. The controller is mainly composed of control function in MATLAB, and its plug-in Simulink is used to build the overall simulation architecture. The system structure of the co-simulation platform is shown in Figure 4.

Figure 4. Simulink/CarSim co-simulation platform.
5.2. Results and analysis

In this paper, the double moving line is selected as the target track, and the road adhesion coefficient is selected as 0.85. In the selection of controller prediction time domain and control time domain, through theoretical analysis and experimental analysis, it is respectively selected as, \( N_p = 20 \), \( N_c = 5 \) and then the experimental speed is selected as 5m/s, 10m/s and 15m/s. This group of speed can represent the typical low speed, medium speed and medium high speed conditions. The parameters of the controller are shown in Table 2.

Table 2. Basic parameters of controlled vehicle.

| Parameter | Value |
|-----------|-------|
| \( N_p \) | 20    |
| \( N_c \) | 5     |
| \( T(s) \) | 0.02  |
| \( \delta \)(deg) | \(-10^\circ \sim 10^\circ\) |
| \( A\delta \)(deg) | \(-0.8^\circ \sim 0.8^\circ\) |
| \( \beta \) | \(-10^\circ \sim 10^\circ\) |
| \( Y/m \) | \(-3m \sim 5m\) |
| \( \rho \) | 1000  |
| \( R \) | 1.1\times10^5 |
| \( Q \) | \[
\begin{bmatrix}
200 & 0 & 0 \\
0 & 100 & 0 \\
0 & 0 & 100 \\
\end{bmatrix}
\] |

Through co-simulation, the following simulation results can be obtained. From the trajectory tracking effect diagram in Figure 6, it can be seen that the vehicle can track the target trajectory well under three speed conditions, and the fluctuation at the corner is very small. Figure 7 and figure 8 are the comparison between the changes of the front wheel angle and the increment of the front wheel angle and the reference value respectively. It can be seen that the change of the control quantity is within the constrained range, which can ensure the handling stability of the vehicle. Figure 9 and figure 10 show the change of vehicle yaw angle and mass center sideslip angle respectively. The change fluctuation is within the normal range, especially the mass center sideslip angle, which is far less than the constraint range, and there will be no large fluctuation during driving.
To sum up, the controller designed in this paper can meet the requirements of handling stability and robustness in the case of low, medium and high speed, and it is stable and reliable in the target trajectory tracking control.

6. Conclusion
Aiming at the situation that the traditional model predictive control (MPC) method only uses magic formula under pure cornering and pure slip to calculate the tire force in the trajectory tracking control of unmanned vehicle. This paper considers the mutual influence of the two, and adds the constraints of control quantity, control increment and cornering angle, and then proposes a vehicle trajectory
tracking control method based on model predictive control. After a series of simulation and data analysis, it shows that the proposed control method performs well in the process of trajectory tracking. It not only meets the vehicle stability control, but also has good tracking effect at several different speeds, and has good speed robustness.

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