Influence of the P-delta Effect and Stiffness Irregularity on the Structural Behavior of Reinforced Concrete Buildings

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Abstract. In this research, the influence of the stiffness irregularity and the p-delta effect on the structural behavior of a reinforced concrete building is analyzed. The main objective is to determine the impact of the stiffness irregularity and the p-delta effect on the structural behavior in regular and irregular buildings. First, the linear dynamic analysis procedure is performed in order to determine the structural response in terms of drifts, shear force and moments per floor. Subsequently, we proceed with the nonlinear static analysis procedure to obtain the capacity curve of the structure. The post elastic stiffness and the overall ductility of the structure are determined from the capacity curve. Finally, a comparative analysis of the responses from the linear and nonlinear analysis is carried out to determine the percentage variation of the results. When analyzing the structures that consider the stiffness irregularity and the p-delta effect, variations of up to 16.50%, 11.00% and 14.00% have been obtained in drifts, shear force and moments per floor respectively, which are considerable values. When the p-delta effect is considered in structures with the presence of stiffness irregularity, there is a variation in stiffness of up to 59.85%. With this result it is explained that the p-delta effect produces a greater degradation of the overall stiffness of the structure.

1. Introduction

Buildings are prone to deform laterally from the original position with an eccentricity during an earthquake, when the construction of structures is subjected to seismic loads causing the structure to deform, the eccentricity resulting from the total gravity load due to inclined axes of the structure makes the extra moments exercise at the base [1]. The action of vertical loads acting through structural lateral deformations is well known as the second-order effect or P-Δ effect [2]. P-Δ effect reduces lateral resistance of a system, which under strong excitation may cause partial or total loss of load carrying capacity [3]. This effect can start a pernicious circle against the structural system because the influence of gravity loads increases as the lateral displacement grows while, at the same time, the lateral displacement is magnified as a consequence of gravity loads acting on them [4]. In very flexible buildings the destabilizing effect of gravity loads may lead to a negative post-yield stiffness, and thus, the structural collapse capacity is exhausted at a rapid rate when the earthquake drives the structure into its inelastic range of deformation even for stable hysteretic component behavior [5].

The P-Delta effect varies over the height of the structure as a function of the axial force demand and the interstory drift [6]. Generally, P-Δ effects in low-rise buildings are negligible since the total lateral deflections are kept relatively small by the story drift limitation. However, in taller, high-rise buildings, where lateral deflections may be much larger, satisfying the maximum drift requirement does not ensure that the P-Δ effects will be negligibly small [7]. The importance of this effect may be negligible
when the structure responds elastically, but is very important when the structure responds inelastically. The P-Δ effect generally increases the displacement response of structures. It can even cause dynamic instability when the structure is subjected to severe seismic movements [8]. Regarding the irregularity of stiffness; Zhao, Hu y Long [9] mention that the vertical irregularity or height of a structure is introduced by abrupt changes or discontinuities in the configuration (such as dimension, soil mass, rigidity, resistance or their combination) along its vertical direction. As a consequence, large stresses or concentrations of plastic deformation can occur in the local weakness and spread to global damage or even collapse. In relation to the P-δ effect, Shehu, Angjeliu y Bilgin [10] in their research perform the Push Over analysis to estimate the ductility of a metallic structure, which simultaneously considers the effects of P-Δ and plastic deformations.

In this research, reinforced concrete structures are analyzed that consider stiffness irregularity and geometric non-linearity (P-Delta Effect) in order to determine their influence on the structural response.

2. Method of analysis

In this research it is intended that the influence of the irregularity in stiffness with the P-delta effect to medium-rise buildings on their structural behavior is estimated.

For this reason, 9 models corresponding to a 20-story structure are made, which consider in their configuration resistant to lateral loads a dual-type structural system, in the X and Y directions of analysis. The structural elements that make up the lateral load resistant system are made of reinforced concrete. It begins with the proposal of 20-story reinforced concrete structures. This approach consists of the development of models or case studies that are organized and ordered into 3 groups based on the irregularity of their structural configuration and the consideration of the P-delta effect in the structure.

In the first group, it is a regular structure that does not present any type of structural irregularity and does not consider geometric non-linearity. The second group, made up of 4 cases or structural models, present stiffness irregularity in a defined floor. Finally, in the last group formed by 4 cases or structural models, they take into account the stiffness irregularity and the geometric non-linearity.

Next, the stability index (Q) of each case is estimated and analyzed, this index is presented in equation (1) according to the Eurocode / ASCE. Subsequently, in the cases described, linear and non-linear analysis procedures are developed.

First, the linear analysis is carried out, which consists of performing the linear dynamic analysis procedure to determine the structural response in terms of drifts, shear force and moments per floor.

\[ 0.10 < Q < 0.25 \]  

Next, we proceed with the non-linear analysis of the research, which implies performing a non-linear static analysis in order to obtain the capacity curve. Previously, the plastic hinges must be defined and assigned to the structural elements.

For the definition of the shape and behavior of the plastic hinges, the modeling parameters established by the ASCE 41-13 standard are used. Figure 1 shows a moment-rotation diagram that is constructed using the modeling parameters.

![Figure 1. Moment - rotation diagram.](image-url)
Subsequently, a bilinear representation of the capacity curve is developed according to the following procedure. A line is drawn upward from the origin with a slope equal to the initial stiffness of the structure; then a second line is drawn back from the last point $V_u, d_u$, so that when the first line intersects at point $V_y, d_y$, the area designated as $A_1$ is equal to the area $A_2$. Figure 2 shows the bilinear representation of a capacity curve.

\[ K_p = (V_u - V_y)(D_u - D_y)^{-1} \]  \hspace{1cm} (2)

\[ \mu \delta = \delta_u (\delta_y)^{-1} \]  \hspace{1cm} (3)

Finally, a comparative analysis of the responses from the linear and nonlinear analysis of the models or case studies is developed in order to determine the percentage variation of the results.

3. Results

Our study will be valid for structures of irregularity in stiffness, according to the ASCE / SEI 41-13 standard where this type of configuration is identified by equation (4).

\[ K_i < 0.70 K_{i+1} \]  \hspace{1cm} (4)

For the confirmation of our method, five buildings were chosen, one regular building and four buildings with the presence of stiffness-type irregularity in height, considering the type of office use. (See Figure 3, 4, 5)
Table 1 shows the verification of the stiffness irregularity for case 2, in the x-x direction.

| Story | SIDE RIGIDITY | CHECK N°1 70% | CHECK N°2 80% |
|-------|---------------|---------------|---------------|
| 20    | 105559.150    | 73891.405     | REGULAR       |
| 19    | 213993.963    | 149795.774    | REGULAR       |
| 18    | 309265.722    | 216486.012    | REGULAR       |
| 17    | 408320.758    | 285824.531    | REGULAR       |
| 16    | 539674.638    | 377772.247    | REGULAR       |
| 15    | 765438.544    | 535806.981    | REGULAR       |
| 14    | 7976182.000   | 558327.400    | REGULAR       |
| 13    | 875488.000    | 6149140.900   | REGULAR       |
| 12    | 9528250.000   | 6669775.000   | REGULAR       |
| 11    | 10315070.000  | 7150304.900   | REGULAR       |
| 10    | 10852193.000  | 7597935.100   | REGULAR       |
| 9     | 11451919.500  | 8016343.650   | IRREGULAR     |
| 8     | 1200798.750   | 840559.125    | REGULAR       |
| 7     | 1257193.300   | 876255.310    | REGULAR       |
| 6     | 1297753.450   | 908414.815    | REGULAR       |
| 5     | 1341977.950   | 939384.365    | REGULAR       |
| 4     | 1374920.050   | 962444.035    | REGULAR       |
| 3     | 1401011.850   | 980708.295    | REGULAR       |
| 2     | 1413622.450   | 989535.715    | REGULAR       |
| 1     | 1418016.650   | 992611.655    | REGULAR       |
Stability indices were calculated for all cases; where it is shown that for all cases of irregularity in stiffness, the application of the P-Delta effect is necessary, because its stability index at one level in each case, exceeds 0.1, established by the ASCE.

3.1. Linear dynamic analysis
A comparison of the drifts, shear force and moments per floor was made; with each case applying the P-Delta Effect.
The percentages of variation of drifts of the Regular are 4.40%, of Case 1, they are 16.50%; of Case 2, 10.4%; Case 3 is 14.4%; Case 4 is 13.60%; as evidenced in Figure 6; therefore, it can be said that in the different cases, the most critical case is floors 3 and 5, due to the fact that it generates a higher percentage of drifts; in addition, a considerable increase is seen with respect to the regular model, when adding the irregularity in stiffness in different floors.

The percentages of variation of shear per floor, of Regular are 3.50%, of Case 1, they are 11.0%; of Case 2, 9.4%; Case 3 is 9.8%, Case 4 is 8.60%, as evidenced in Figure 7; therefore, it can be said that in different cases, the most critical case is floors 3 and 5 together, because it generates a higher percentage of shear force at the base; in addition, a considerable increase is seen with respect to the regular model, when adding the irregularity in stiffness in different floors.

The percentages of variation of moment per floor, of the Regular are 6.60%, of the Case 1, they are 14.0%; of Case 2, 11.5%; Case 3 is 11.0%, Case 4 is 12.86%, as evidenced in Figure 8; therefore, it can be said that in different cases, the most critical case is floor 9, because it generates a higher percentage of moments; in addition, a considerable increase is seen with respect to the regular model, when adding the irregularity in stiffness in different floors.

3.2. Nonlinear static analysis

Capacity curve. With the nonlinear static analysis procedure, the capacity curves of the study models are obtained in the directions of analysis, this is shown in Figure 9 and Figure 10.
Ductility. In Figure 11 the ductility values are shown for the x-x direction as a function of the consideration of the P-delta effect. In cases 2, 3 and 4, an increase in ductility is observed when the P-delta effect is considered in the structural models in the non-linear analysis. In case 1 its ductility is not affected by the P-delta effect.

![Figure 11. Ductility of cases without PΔ effect vs with PΔ effect – x-x direction.](image)

In Figure 12 the ductility values are shown for the y-y direction as a function of the consideration of the P-delta effect. In case 1 there is a slight increase in ductility caused by the P-delta effect, while in cases 2 and 4 there is a moderate increase in ductility.

![Figure 12. Ductility of cases without PΔ effect vs with PΔ effect – y-y direction.](image)

Stiffness. Table 2 shows the results of the post elastic stiffness that varies according to the incorporation of the stiffness irregularity in a specific floor. In the regular model it is observed that in the x-x direction there is post-elastic stiffness of 20068.05 ton / m and in the y-y direction there is a post-elastic stiffness of 21846.64 ton / m.

In case 1, the post-elastic stiffness in the x-x direction of 13178.28 ton / m is less than the post-elastic stiffness in the other cases. In the y-y direction, case 2 has a post-elastic stiffness of 18626.71 ton / m, which is lower than the other cases.

Table 2. Post-elastic stiffness - no PΔ effect.

| Direction | Case 1     | Case 2     | Case 3     | Case 4     | Case regular |
|-----------|------------|------------|------------|------------|--------------|
| X-X       | 13178.28   | 14477.22   | 14057.88   | 15473.52   | 20068.05     |
| Y-Y       | 19043.00   | 18626.71   | 18975.01   | 19504.17   | 21846.64     |
Table 3 shows the results of the post elastic stiffness that varies according to the incorporation of the stiffness irregularity in a specific floor and the P-Delta effect. In case 2, the post-elastic stiffness in the x-x direction of 12641.97 ton/m is less than the post-elastic stiffness in the other cases. In the y-y direction, case 1 has a post-elastic stiffness of 14903.95 ton/m, which is lower than the other cases.

**Table 3. Post-elastic stiffness - with PΔ effect.**

| Direction | Case 1     | Case 2     | Case 3     | Case 4     | Case regular |
|-----------|------------|------------|------------|------------|--------------|
| X-X       | 13172.90   | 12641.97   | 12837.76   | 15355.20   | 20068.05     |
| Y-Y       | 14903.95   | 16999.09   | 19342.29   | 16856.24   | 21846.64     |

Table 4 shows the variations of the global stiffness of the study models as a function of the stiffness irregularity in a specific floor. In the x-x direction, cases 1, 2, 3 and 4, which have stiffness irregularity in a specific floor, have experienced a variation in stiffness greater than the regular structure. In case 1 there is a greater variation in its lateral stiffness compared to the other cases. In the y-y direction three different circumstances occur. In the first place, the variation of stiffness in case 2 and 4 of values 45.24% and 45.35% respectively, barely exceeds the regular case. On the other hand, we have case 3 that has a lower stiffness variation response compared to the regular case. And finally, the variation in stiffness of 48.70% that corresponds to case 1 is higher than the other cases.

**Table 4. Variation of the overall stiffness of the structure - no PΔ effect.**

| Direction | Case 1     | Case 2     | Case 3     | Case 4     | Case regular |
|-----------|------------|------------|------------|------------|--------------|
| X-X       | 56.30%     | 50.62%     | 53.06%     | 50.15%     | 42.06%       |
| Y-Y       | 48.70%     | 45.24%     | 44.32%     | 45.20%     | 45.20%       |

Table 5 shows the variations of the global stiffness of the study models as a function of the stiffness irregularity in a specific floor and the consideration of the P-delta effect. When the P-delta effect is considered in the case studies, a greater reduction in stiffness is generated. In the x-x direction, there is an increase in the variation of stiffness in cases 2, 3 and 4 caused by the P-delta effect. Regarding the analysis in the y-y direction, there is a similar response from the case studies. When considering the P-delta effect and the stiffness irregularity in the structures, there is an increase in the degradation of stiffness in cases 1, 2 and 4.

**Table 5. Variation of the overall stiffness of the structure - with PΔ effect.**

| Direction | Case 1     | Case 2     | Case 3     | Case 4     |
|-----------|------------|------------|------------|------------|
| X-X       | 54.40%     | 56.88%     | 57.13%     | 50.53%     |
| Y-Y       | 59.85%     | 50.03%     | 43.24%     | 52.77%     |

4. Conclusions

According to Eurocode 8, the 20-story building should take into consideration the P-Delta effect, because the stability index is greater than 0.1, but when considering the effect for cases of stiffness-type irregularity, its most critical variation percentages are 16.50%, 11.0%, 14.00%, of drifts, shear force, and moments per floor respectively; which are considerable. In a structure of 20 floors or more, a seismic performance analysis must be carried out considering the P-Delta effect obligatorily; because the design variation when incorporating this effect plus the irregularity in stiffness is considerable. When the stiffness irregularity occurs on floors 3 and 5; they are more considerable, so while the floors are affected, they are of a lower level; the greater the P delta effect. The presence of the stiffness irregularity in tall buildings leads to a degradation of the stiffness and resistance of the structure. This degradation of stiffness and strength is greater in a structure with stiffness irregularity than a regular structure. In the non-linear static analysis procedure, in the cases that consider stiffness irregularity, variations in stiffness of up to 56.30% have been obtained. Therefore, the stiffness irregularity directly influences the degradation of the stiffness of the structure.
In structures that consider the stiffness irregularity and the P-delta effect, a greater number of plastic hinges are formed in structural elements, consequently the rigidity of the structural elements decreases and the overall rigidity of the structure is degraded.

With the incorporation of the stiffness irregularity and the consideration of the P-delta effect in tall structures, variations in stiffness of up to 59.85% have been obtained, that is, with the P delta effect there is a greater degradation of the overall stiffness of the structure.

5. References

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