Impact of co-channel interference on performance of dual-hop wireless ad hoc networks over $\alpha-\mu$ fading channels

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Summary
In this work, we investigate the performance of a dual-hop cooperative network over $\alpha-\mu$ fading channels with the presence of co-channel interference (CCI) at both the relay and destination nodes. Amplify-and-forward (AF) relaying is considered in the relay node. The upper bound of the signal-to-interference-plus-noise ratio (SINR) of the dual-hop relay link is used to determine the system performance. The probability density function (PDF) and the cumulative distribution function (CDF) of the upper bound of the SINR are analyzed. The system performance is determined in terms of the outage and error probabilities. Numerical results are used to present the performance analysis of the system.

KEYWORDS
amplify-and-forward, co-channel interference, cooperative networks, dual-hop relay network, error probability, $\alpha-\mu$ fading channel, outage probability

1 | INTRODUCTION

Cooperative communication systems play a major role in improving the network coverage and throughput of wireless communication systems. A relay node is introduced to the network to create independent paths between both the user and the base-station. Among other relaying schemes which are used in cooperative communication, amplify-and-forward (AF) is considered to be the famous one due to its simplicity and low-cost implementation. Using AF relaying scheme, the relay node simply resend a scaled copy of the received noisy signal from the source to the destination node. Another technique for processing at the relay node is to decode the received signal from the source node, re-encode it and finally retransmit it to the receiver node, and this relaying technique is termed as decode-and-forward (DF) relay scheme.

In light of the increase demand on the wireless communication services, finding a practical strategy to efficiently use the radio spectrum gave a rise to the so-called co-channel interference (CCI) due to the frequency reuse in the wireless networks. To enhance the spectrum efficiency, more investigations adopted to understand the limitations at the network performance. Motivated by that, some studies which are investigating the impact of CCI on the performance of amplify-and-forward dual-hop relay systems have been investigated in the literature. For example, Ikki and Aissa
and Zhong et al.\textsuperscript{3} studied the impact of CCI on AF dual-hop relay network performance, assuming Rayleigh fading channels, while both Al-Qahtani et al.\textsuperscript{4} and Ikki and Aissa\textsuperscript{5} assumed the Nakagami-m fading channels. In Salhab et al.\textsuperscript{6} the authors investigations assumed different fading channels for the interferes, where they considered Rayleigh, Nakagami-m, and Rician fading channels. Ilhan\textsuperscript{7} investigated the dual-hop relaying system presence of co-channel interference assuming DF relaying scheme and Nakagami-m fading is adopted. Another work that investigated AF dual-hop relay network performance is represented in Nauryzbayev et al.\textsuperscript{8} where $\alpha-\mu$ fading channel is assumed and the performance investigation in terms of the ergodic capacity and ergodic outage probability; however, the interference presence where not considered in this research. Moreover, Nauryzbayev et al.\textsuperscript{9} and Amer and Al-Dharrab\textsuperscript{10} derived the outage probability for both AF and DF relaying techniques in dual-hop networks in $\alpha-\mu$ environment, without considering the interference presence at any of the nodes. In addition, the work in Magableh et al.\textsuperscript{11} investigated dual-hop AF relay network performance in terms of end-to-end capacity and outage capacity, assuming $\alpha-\mu$ environment subject to CCI interference.

Motivated by the preceding, we investigated the error and outage performance of the dual-hop AF relaying network over $\alpha-\mu$ fading channels in the presence of CCI affecting both the relay and destination nodes. The $\alpha-\mu$ fading channel is considered since it is the general fading model for small-scale fading, and it represents the multipath fading channel model, in which the received signal consists of clusters of multipath waves that propagate in a nonhomogeneous environment.\textsuperscript{12} The $\alpha$ parameter is used to represent the nonlinear behavior of the propagation medium, while the $\mu$ parameter represents the multipath-clusters number. The Nakagami-m and Rayleigh fading can be derived from the $\alpha-\mu$ fading. In this study, the derived formulas are used to derive other fading channel models such as Rayleigh, Nakagami-m, Chi, and one-sided Gaussian where the $\alpha$ value equals 2.

Moreover, we derive the probability density function (PDF) and the cumulative distribution function (CDF) of the signal-to-interference-and-noise ratio (SINR) of dual-hop transmission with AF relaying. The obtained results are used to derive an analytical expression of the error and outage probability for the considered system. The rest of the paper is organized as follows: in Section 2, the system model is introduced. Then in Section 3, the SINR analysis is introduced. The system performance in terms of outage and average error probabilities is conducted and derived in Section 4. Numerical results are presented and discussed in Section 5. Finally, conclusions are drawn in Section 6.

2 | SYSTEM MODEL

Consider a dual-hop wireless communication system is shown in Figure (1), where the source node $S$ communicates with a destination node $D$ through a relay node $R$. The received signals at the relay node $R$ and destination node $D$ are corrupted by CCI signals from $N$ and $L$ co-channel interferer’s denoted as $\{x_{h_j}\}_{j=1}^{N}$ with energy of $E_{h_j}$ and $\{x_{g_k}\}_{k=1}^{L}$ with energy of $E_{g_k}$, respectively. The fading coefficients for all the links are assumed to be $\alpha-\mu$ fading channel. AF relaying scheme is considered due to its simplicity compared to other relaying schemes, where the signal received by the relay node is amplified before being forwarded to the destination node, the amplification factor of AF scheme is proportional to the inverse gain of the channel. The destination node $D$ is assumed to have perfect knowledge of the channel state information (CSI), and with reference to Ikki and Aissa,\textsuperscript{2} eq. 1 the received signal at the relay node $R$ is given by

![Figure 1](image-url) Dual-hop relay network with co-channel interference at the relay and destination nodes
\[
y_{SR} = \sqrt{E_S h_{SR} d_S} + \sum_{j=1}^{N} \sqrt{E_h h_j d_j} + n_{SR},
\]

where \(h_{SR}\) is the \(\alpha-\mu\) channel fading coefficient of the \(S \rightarrow R\) link, \(E_S\) is the energy of the transmitted signal, \(d_S\) is the desired data with unit energy, \(h_j\) is the \(\alpha-\mu\) channel fading coefficient of the \(j^{th}\) \(\rightarrow\) \(R\) link, \(E_h\) is the energy of the \(j^{th}\) interferer at \(R\), and \(d_j\) is the \(j^{th}\) co-channel interferers data with unit energy at the relay node. The additive-white-Gaussian noise (AWGN) at the relay node is denoted as \(n_{SR}\) with a zero-mean and \(N_o\) variance \(-\text{CN}(0,N_o)\).

The signal received at the destination node \(D\) from the relay node is expressed as

\[
y_{RD} = \sqrt{E_R h_{RD}} y_R + \sum_{k=1}^{L} \sqrt{E_{g_k} g_k d_k} + n_{RD},
\]

where \(E_R\) represents the energy of the relay node transmitted signal, \(h_{RD}\) is the \(\alpha-\mu\) channel fading coefficient of the \(R \rightarrow D\) link, \(E_{g_k}\) is the energy of the interference signal at the destination node, \(g_k\) is the \(\alpha-\mu\) fading coefficient of the interference channel at the destination, and \(d_k\) is the \(k^{th}\) co-channel interferers data with unit energy at the destination, \(n_{SR}\) denotes the AWGN at the destination node with a zero-mean and \(N_o\) variance \(-\text{CN}(0,N_o)\). And \(y_R\) is the relayed signal, which and can be represented as

\[
y_R = G_{AF} y_{SR},
\]

where the AF gain factor \(G_{AF}\) is set to maintain the output energy from the relay to \(E_R\). The gain factor in the presence of CCI is computed as

\[
G_{AF} = \frac{1}{\sqrt{N_o + E_S |h_{SR}|^2 + \sum_{j=1}^{N} E_h |h_j|^2}}.
\]

After substituting Equations (1) and (3) into Equation (2) then with some algebraic simplifications, the received signal at the destination node \(D\) can be expressed as

\[
y_{RD} = G_{AF} \sqrt{E_R h_{RD}} \left[ \sqrt{E_S h_{SR} d_S} + \sum_{j=1}^{N} E_h h_j d_j \right] + \sqrt{E_{g_k} \sum_{k=1}^{L} g_k d_k} + G_{AF} \sqrt{E_R h_{RD} n_{SR}} + n_{RD}.
\]

### 3 | SINR ANALYSIS

In this section, we will derive the expression for the SINR of the proposed dual-hop relaying communication system, where the SINR at the destination node \(D\) can be written as

\[
\gamma_{SRD} = \frac{E_S |h_{SR}|^2 E_R |h_{RD}|^2}{G_{AF}^2 E_R |h_{RD}|^2 N_o + G_{AF}^2 E_R |h_{RD}|^2 \sum_{j=1}^{N} E_h |h_j|^2 + N_o \sum_{k=1}^{L} E_{g_k} |g_k|^2 \left[ N_o + \sum_{j=1}^{N} E_h |h_j|^2 E_S |h_{SR}|^2 \right]}. \tag{6}
\]

The destination node can be further simplified after introducing the effective signal-to-noise ratio (SNR) expressions \(\gamma_{\text{eff}}\), as

\[
\gamma_{SRD} = \frac{\gamma_{\text{eff}} \gamma_{\text{eff}}}{\gamma_{SR} + \gamma_{RD} + 1}, \tag{7}
\]

where \(\gamma_{SR} = \gamma_{SR} \left( \left(1 + \sum_{j=1}^{N} \gamma_{h_j}\right) \right)\) and \(\gamma_{RD} = \gamma_{RD} \left( \left(1 + \sum_{k=1}^{L} \gamma_{g_k}\right) \right)\) represent the effective SINR’s at the relay and the destination node, respectively. The SNR at the relay and destinations nodes are expressed as \(\gamma_{SR} = E_S |h_{SR}|^2 / N_o\) and
The interference affecting the relay and destination nodes is denoted as 
\[ \gamma_{RD} = E\left|h_{RD}\right|^2/N_o, \]
respectively. The interference affecting the relay and destination nodes is denoted as 
\[ \gamma_{h_j} = E\left|h_j\right|^2/N_o \]
for \( j = 1, 2, ..., N \) and 
\[ \gamma_{s_k} = E\left|g_k\right|^2/N_o \]
for \( k = 1, 2, ..., L \). The statistical characteristics (i.e., the PDF and the CDF) of the SINR of the proposed dual-hop network are developed in the following sections, in addition to the CCI density function presented in the system nodes.

### 3.1 Co-channel interference probability density function

CCI occurs when a frequency band is used by multiple users at the same time. For cellular networks, CCI occurs by the reuse of frequency in the neighboring cells, which accordingly will affect the performance of the network. CCI may degrade the performance of the system; hence, studying its impact is very important and helps in developing mitigation techniques to reduce the performance degradation. CCI can be modeled as the sum of \( N \) independent-not-identically-distributed (INID) \( \alpha-\mu \) variates, (i.e., \( \sum_{i=1}^{N} \gamma_{h_i} \)), but in this section, the derivation of the formulas will be based on the independent-identically-distributed (IID) assumption. The SNR for a single interferer link can be expressed as

\[
\gamma_{h_i} = \frac{E|h_i|^2}{N_o}.
\]

Assuming \( \alpha-\mu \) fading channel, the \( \alpha-\mu \) PDF for a single interferer link is represented by

\[
f_{\gamma_{h_i}}(\gamma) = \frac{\alpha \mu^{\mu/2-1}}{2\Gamma(\mu)} \left( \frac{\gamma}{\mu} \right)^{\mu/2} \exp\left( -\frac{\gamma^{\mu/2}}{(\mu^{\mu/2})} \right), \tag{9}
\]

The moment-generating function (MGF) approach will be used to develop a mathematical formula that can be used to express the PDF of the \( N \) INID \( \alpha-\mu \) variates. The MGF is defined mathematically as eqs. 3.3-6 and 3.3-7 in Peebles Peyton\(^1\):

\[
M_{\gamma}(s) = E[e^{-s\gamma}] = \int_{0}^{\infty} f_{\gamma}(\gamma)e^{-s\gamma} d\gamma. \tag{10}
\]

The \( \alpha-\mu \) PDF represented in Equation (9) can then substituted in Equation (10). The next step is to use and introduce the Meijer G-Function of the exponential function which is given as in Prudnikov and Marichev,\(^1\) eq. 8.4.3.1

Finally, the resulting integral can be solved using eq. 2.24.3.1 in Prudnikov and Marichev,\(^1\) with defining the parameters: \( p = n = 0, q = m = 1, \mu = 1/2 \) and \( \gamma^* = 1/2 \) also a new fraction is to be defined as well \( \frac{n}{k} \) such that \( \left( \frac{1}{k} \right) \) with the \( \gcd(l,k) = 1 \); great common divisor) by this the noninteger values of \( \alpha \) would be included in the calculations. The MGF is formulated as

\[
M_{\gamma_{h_i}}(s) = \frac{\alpha \mu^\mu}{2\Gamma(\mu)} \sum_{l,k} G_{k,1}^{1,1} \left( \frac{\mu}{(d\gamma)^{\alpha/2}}, \frac{l}{\delta(k,k)}, \Delta\left( l, 1 - \frac{\alpha \mu}{2} \right) \right), \tag{11}
\]

where \( \Delta(k,a) = \frac{a}{k}, \frac{a+1}{k}, ..., \frac{a+k-1}{k} \), and the Meijer G-function is defined generally in Wolfram,\(^1\) eq. 07.34.02.0001.01

Applying the MGF approach, the density function of \( \left( \sum_{i=1}^{N} \gamma_{h_i} \right) \) can be derived and mathematically expressed using

\[
f_{\sum_{i=1}^{N} \gamma_{h_i}}(\gamma) = \mathcal{L}^{-1}\left\{ \left( \frac{N}{\sum_{i=1}^{N} M_{\gamma_{h_i}}(s)} \right) \right\}. \tag{12}
\]

where \( M_{\gamma_{h_i}}(s) \) represents the MGF of the \( i \)th link which is expressed in Equation (11), and \( \mathcal{L}^{-1}\{.\} \) is the inverse Laplace transformation. Substituting the derived MGF, the PDF of the sum of \( N \)-IID interferers is then can be evaluated as
Equation (13) is not analytically tractable. However, under some conditions, for example when $l = k = 1$, this results in $\alpha = 2$, and then the density function can be analytically expressed as

$$f_{\sum_{i=1}^{N} (\gamma_{hi})} = \mathcal{L}^{-1} \left\{ \left( \frac{\alpha \mu}{2} \right)^{\frac{\alpha}{2} - \frac{1}{2}} \frac{k^{\frac{\alpha}{2} - \frac{1}{2}}}{(2\pi)^{\frac{1}{2}} \Gamma(\frac{\alpha}{2})} G_{1,1}^{1,1} \left( \begin{array}{c} \mu \left( \frac{\gamma}{\gamma_{hi}} \right)^{\frac{1}{2}} \cr 0 \end{array} \right) \right\}^N, \quad (14)$$

Using the Meijer G-function expression given as in Wolfram, eq. 07.34.03.0271.01 finally, the PDF of $\sum_{i=1}^{N} \gamma_{hi}$ can be written as

$$f_{\sum_{i=1}^{N} (\gamma_{hi})} = \left( \frac{\mu_{hi}}{\gamma_{hi}} \right)^{N\mu_{hi}} \frac{\Gamma(N\mu_{hi})}{\Gamma(N)} e^{-\frac{\mu_{hi}}{\gamma_{hi}}}. \quad (15)$$

This formula can be used to generate different IID CCI models such as the Rayleigh fading model as shown in Ikki and Aissa, eq. 14 where $\alpha = 2$ and $\mu = 1$ and the Nakagami-m fading model as shown in Ikki and Aissa, eq. 14 where $\alpha = 2$.

### 3.2 Effective SINR statistical characteristics

In this section, we will derive the statistical characteristics of the effective SINR $\gamma_{SR}^{\text{eff}}$ and $\gamma_{RD}^{\text{eff}}$ of the links $S \rightarrow R$ and $R \rightarrow D$, respectively, in terms of the PDF and the CDF when both links are subject to $\alpha-\mu$ fading channel. The PDF of $\gamma_{SR}^{\text{eff}}$ can be derived using eqs. 6–60 in Papoulis and Pillai, which can be expressed as

$$f_{\gamma_{SR}^{\text{eff}}} (\gamma) = \left( y + 1 \right) \frac{\alpha_{SR} \mu_{SR}^{\alpha_{SR}}}{2 \Gamma(\mu_{SR}) \alpha_{SR}^{\mu_{SR}}} \left( \frac{\gamma}{\gamma_{sr}} \right)^{\alpha_{SR} - 1} \exp \left( -\frac{\gamma}{\gamma_{sr}} \right) \mathcal{L}^{-1} \left\{ \prod_{l=1}^{N} \frac{\alpha_{SR} \mu_{SR}^{\alpha_{SR}}}{2 \Gamma(\mu_{SR}) \alpha_{SR}^{\mu_{SR}}} \left( \frac{\gamma}{\gamma_{sr}} \right)^{\alpha_{SR} - 1} \exp \left( -\frac{\gamma}{\gamma_{sr}} \right) \right\}, \quad (16)$$

Under some conditions, where $l_{hi} = k_{hi} = 1$, $\alpha_{SR} = \alpha_{RD} = \alpha_{hi} = 2$, and using eq. 3.383.4 in Gradshteyn and Ryzhik, the effective SINR can be evaluated after mathematical simplifications as

$$f_{\gamma_{SR}^{\text{eff}}} (\gamma) = \frac{\mu_{SR}^{\alpha_{SR}}}{\Gamma(\mu_{SR})} (\gamma_{sr})^{\alpha_{SR} - 1} e^{-\frac{\gamma_{sr}}{\gamma_{sr}}} \frac{\Gamma(N\alpha_{SR})}{\Gamma(N)} e^{-\frac{\gamma_{sr}}{\gamma_{sr}}} \left( \frac{\mu_{SR}}{\gamma_{sr}} \right)^{N\alpha_{SR} - 1} e^{-\frac{\gamma_{sr}}{\gamma_{sr}}}, \quad (17)$$

where $W_{\nu}(z)$ is the Whittaker function which is defined in Wolfram, eq. 07.45.02.0001.01

Since the PDF of $\gamma_{SR}^{\text{eff}}$ is assumed to be identical for both the SR and the RD links, the effective SINR for RD link $\gamma_{RD}^{\text{eff}} (\gamma)$ can be evaluated as
\[ f_{\text{eff}}^{\gamma} (\gamma) = \frac{\mu_{\text{rd}} \left( \frac{\mu_{\text{rd}}}{\gamma_{\text{rd}}} \right)^{L} \left( \frac{\mu_{\text{rd}}}{\gamma_{\text{rd}}} \right)^{L-1}}{\Gamma(\mu_{\text{rd}})} e^{\frac{\gamma_{\text{rd}}}{\mu_{\text{rd}}}} \left( \frac{\mu_{\text{rd}}}{\gamma_{\text{rd}}} + \frac{\mu_{\text{rd}}}{\gamma_{\text{rd}}} \right)^{\frac{\gamma_{\text{rd}}+1}{\gamma_{\text{rd}}}} \gamma_{\text{rd}}^{\mu_{\text{rd}}-1} \] (18)

The CDF \( F_{\text{eff}}^{\gamma} (\gamma) \) is presented as

\[ F_{\text{eff}}^{\gamma} (\gamma) = \Pr \{ \gamma_{\text{eff}}^{\gamma} \leq \gamma \} = \int_{0}^{\gamma} f_{\text{eff}}^{\gamma} (t) dt. \] (19)

Substituting Equation (17) in Equation (19), then by using eq. 4.6 in Papoulis and Pillai\textsuperscript{17} and then applying eq. 2.19.5.12 in Prudnikov and Marichev,\textsuperscript{14} the CDF of \( \gamma_{\text{SR}}^{\text{eff}} \) is evaluated as

\[ F_{\text{SR}}^{\text{eff}} (\gamma) = 1 - \frac{1}{\Gamma(\mu_{\text{sr}})} e^{\left( \frac{\gamma_{\text{sr}}}{\mu_{\text{sr}}} \right)} \sum_{k=0}^{\infty} \frac{\left( \frac{\mu_{\text{sr}}}{\gamma_{\text{sr}}} \right)^{k}}{\mu_{\text{sr}}} \Gamma(\mu_{\text{sr}}-k) \left( \frac{\mu_{\text{sr}}}{\gamma_{\text{sr}}} + \frac{\mu_{\text{sr}}}{\gamma_{\text{sr}}} \right)^{-\frac{\gamma_{\text{sr}}^{\mu_{\text{sr}}-1}}{\mu_{\text{sr}}}} \] (20)

The CDF of \( \gamma_{\text{RD}}^{\text{eff}} \) is evaluated as

\[ F_{\text{RD}}^{\text{eff}} (\gamma) = 1 - \frac{1}{\Gamma(\mu_{\text{rd}})} e^{\left( \frac{\gamma_{\text{rd}}}{\mu_{\text{rd}}} \right)} \sum_{k=0}^{\infty} \frac{\left( \frac{\mu_{\text{rd}}}{\gamma_{\text{rd}}} \right)^{k}}{\mu_{\text{rd}}} \Gamma(\mu_{\text{rd}}-k) \left( \frac{\mu_{\text{rd}}}{\gamma_{\text{rd}}} + \frac{\mu_{\text{rd}}}{\gamma_{\text{rd}}} \right)^{-\frac{\gamma_{\text{rd}}^{\mu_{\text{rd}}-1}}{\mu_{\text{rd}}}} \] (21)

This result can be used to generate other special cases such as when \( \mu_{\text{SR}} = 1 \) and \( \mu_{\text{hi}} = 1 \), the CDF of \( \gamma_{\text{SR}}^{\text{eff}} \) is given in Ikki and Aissa,\textsuperscript{2} eq. 17 while setting \( \mu_{\text{RD}} = 1 \) and \( \mu_{\text{li}} = 1 \) can generate the CDF given in Ikki and Aissa,\textsuperscript{2} eq. 19.

### 3.3 Dual-hop relay SINR PDF and CDF

To derive the SINR for the relay link in Equation (7), we have used a tight upper bound to simplify the analysis of the network performance, that is,

\[ \gamma_{\text{SRD}} \leq \gamma_{\text{up}} = \min \left( \gamma_{\text{SR}}^{\text{eff}}, \gamma_{\text{RD}}^{\text{eff}} \right), \] (22)

hence, \( f_{\gamma_{\text{up}}} (\gamma) \) can be expressed as

\[ f_{\gamma_{\text{up}}} (\gamma) = f_{\gamma_{\text{SR}}} (\gamma) \left[ 1 - F_{\gamma_{\text{SR}}}^{\text{eff}} (\gamma) \right] + f_{\gamma_{\text{RD}}} (\gamma) \left[ 1 - F_{\gamma_{\text{RD}}}^{\text{eff}} (\gamma) \right]. \] (23)

Assuming identical fading links, that is, \( N = L = l \) interferes at the relay and destination nodes, the PDF of \( \gamma_{\text{up}} \) is represented by
applying the Whittaker properties in Olver et al.\textsuperscript{19} eq. 13.14.31 and the Whittaker functions and links and after applying the Lagrange's identity with some mathematical manipulation, the CDF is derived as

\[
F_{\gamma_{up}}(\gamma) = F_{\gamma_{SR}}(\gamma) + F_{\gamma_{RD}}(\gamma) - F_{\gamma_{SR}}(\gamma)F_{\gamma_{RD}}(\gamma).
\]

Assuming identical links, and substituting both Equations (20) and (21) into Equation (26), the CDF of the dual-hop relay network is expressed as

\[
F_{\gamma_{up}}(\gamma) = 1 - \frac{1}{\Gamma(\mu)} \frac{\mu^\mu}{\gamma^\mu} \left( \frac{\mu_1}{\gamma_1} + \frac{\gamma_1}{\gamma} \right)^{-\mu_1} \left( \sum_{k=1}^{\mu_1} \Gamma(\mu-k) \left( \frac{\gamma_1}{\gamma} \right)^{k} \left( \frac{\mu_1}{\gamma_1} + \frac{\gamma_1}{\gamma} \right)^{-k} \right)^2.
\]

after applying the Lagrange's identity with some mathematical manipulation, the CDF is derived as

\[
F_{\gamma_{up}}(\gamma) = 1 - \frac{1}{\Gamma(\mu)} \frac{\mu^\mu}{\gamma^\mu} \left( \frac{\mu_1}{\gamma_1} + \frac{\gamma_1}{\gamma} \right)^{-\mu_1} \sum_{k=1}^{\mu_1} \Gamma(\mu-k) \left( \frac{\gamma_1}{\gamma} \right)^{k} \left( \frac{\mu_1}{\gamma_1} + \frac{\gamma_1}{\gamma} \right)^{-k} \left( \frac{\mu_1}{\gamma_1} + \frac{\gamma_1}{\gamma} \right)^{-2\mu_1 - n - t}.
\]

This formula can be used to generate different CDF such as Rayleigh fading channels where \( \mu = 1 \). For identical links and \( N = L \), the CDF can be expressed as

\[
F_{\gamma_{up}}(\gamma) = 1 - e^{-\frac{2\gamma}{\gamma + \Lambda}} \left( \frac{\Lambda}{\gamma + \Lambda} \right)^{2L}.
\]
where $\Lambda = \frac{\bar{\mu}_w}{\bar{\gamma}_{th}}$ is the average SIR at the Relay node. For non-identical links, the CDF can be expressed as:

$$F_{\gamma_{rt}}(\gamma) = 1 - e^{-\gamma} \left( \frac{1}{\gamma + \Lambda} \right)^N \left( \frac{\gamma + Y}{\gamma + Y} \right)^L,$$

(30)

where $\Lambda = \frac{\bar{\mu}_w}{\bar{\gamma}_{th}}$ is the average signal-to-interference ratio (SIR) at the Relay node and $Y = \frac{\bar{\mu}_w}{\bar{\gamma}_{rd}}$ is the average SIR at the destination node.

### 3.4 Dual-hop relay SINR MGF

The MGF is very useful for deriving the performance metrics of the network, which is given as

$$M_{\gamma_{rt}}(s) = \int_0^\infty f_{\gamma_{rt}}(\gamma) e^{-s\gamma} d\gamma,$$

(31)

substituting the PDF of the dual-hop given in Equation (24) while assuming identical fading links, and using eq. 2.3.6.9 the MGF can be expressed after some mathematical manipulation as

$$M_{\gamma_{rt}}(s) = \frac{2}{\Gamma(\mu)} \sum_{j=0}^{\mu} \sum_{k=0}^{\mu-j} \sum_{n=0}^{k-1} \Gamma(\mu-k+1) \Gamma(\mu+k-1) \left( \frac{\mu_j}{\nu_{\gamma}} \right) \left( \frac{\mu_k}{\nu_{\gamma}} \right) \left( \frac{\mu_n}{\nu_{\gamma}} \right) e^{-s\gamma} \left( \frac{\gamma^{j} + \nu_{\gamma} \gamma^{k} + \nu_{\gamma} \nu_{\gamma} \gamma^{n} + \nu_{\gamma} \nu_{\gamma} \nu_{\gamma} \gamma^{m-n}}{\nu_{\gamma} \nu_{\gamma} \nu_{\gamma} \gamma^{m-n}} \right),$$

(32)

where the $U(a, b, x)$ is the confluent hypergeometric function of the second kind and is defined generally as in Wolfram, eq. 07.33.02.0001.01

### 4 DUAL-HOP RELAY PERFORMANCE ANALYSIS

#### 4.1 Outage probability analysis

In this section, the outage probability of the dual-hop relay network will be analyzed. The outage probability is an important performance measure, as the analysis of the outage probability is essential to characterize the error performance and reliability. Outage probability is defined as the probability at which the SINR falls below a threshold value ($\gamma_{th}$), or mathematically,

$$P_{out} = P_{\gamma}(\gamma_{th}) = \int_0^{\gamma_{th}} f_{\gamma_{rt}}(\gamma) d\gamma,$$

(33)

which can be evaluated after substituting the PDF and evaluating the integration as

$$P_{out} = 1 - \left( \frac{\mu_1}{\nu_{\gamma}} \right)^{N_{\gamma}} \left( \frac{\mu_2}{\nu_{\gamma}} \right)^{N_{\gamma}} e^{-\nu_{\gamma} \gamma_{th}} \sum_{k=0}^{\mu_{\gamma} - 1} \frac{\nu_{\gamma} \gamma_{th}}{\nu_{\gamma} \gamma_{th}} \frac{\mu_{\gamma} \gamma_{th}}{\nu_{\gamma} \gamma_{th}} k \left( \Gamma(\mu_{\gamma} - k) \right)
$$

$$\Gamma(\mu_{rd} - j) \left( \frac{\mu_2}{\nu_{\gamma}} \gamma_{th} + \frac{\mu_2}{\nu_{\gamma}} \right) \sum_{j=0}^{\gamma_{rd} - \gamma_{th} - 1} \left( \frac{\mu_{\gamma} \gamma_{th}}{\nu_{\gamma} \gamma_{th}} + \frac{\mu_{\gamma} \gamma_{th}}{\nu_{\gamma} \gamma_{th}} \right) \Gamma(\mu_{rd} - j) \left( \frac{\mu_2}{\nu_{\gamma}} \gamma_{th} + \frac{\mu_2}{\nu_{\gamma}} \right).$$

(34)
Equation (34) can be used to generate many different formulas such as Ikki and Aissa, eq. 21 which represents identical fading channel with \( \mu = 1 \), and as in Ikki and Aissa, eq. 23 for non-identical fading channel with \( \mu = 1 \), and many other formulas for other fading channels such that with \( \mu = m \) which corresponds to the Nakagami-\( m \) fading model.

### 4.2 Average error probability analysis

The average error probability is given by

\[
P_b(e) = \int_0^\infty P_e(\gamma) f_{\text{TSS}}(\gamma) d\gamma, \tag{35}\]

where \( P_e(\gamma) \) is the conditional error probability for a given \( \gamma \). For coherent binary-phase-shift-keying (BPSK) modulation, the average bit error rate (BER) is expressed as

\[
P_b(e) = \int_0^\infty Q\left(\sqrt{2\gamma}\right) f_{\text{TSS}}(\gamma) d\gamma, \tag{36}\]

using the alternative form of the Q-function expressed in terms of the MGF as in Goldsmith, eq. 6.44 and applying the upper bound given in Simon and Alouini, eq. 9.27 we have developed several formulas for BER of BPSK modulation for different cases as

- For identical fading channel coefficient with \( \alpha = 2 \), \( \mu = 1 \) \( L = N \), \( \mu_{\text{sr}} = \mu_{\text{rd}} = \mu_{\text{ln}} = \mu_{\text{lg}} = 1 \), \( \bar{\gamma}_{\text{sr}} = \bar{\gamma}_{\text{rd}} = \bar{\gamma} \) and \( \bar{\gamma}_{\text{ln}} = \bar{\gamma}_{\text{lg}} = \bar{\gamma}_{\text{1}} \), the average error probability is as
  \[
P_b(e) = NE_{2N+1} \left( \frac{2\Lambda}{\bar{\gamma}} \right) e^{\left( \frac{2\Lambda}{\bar{\gamma}} \right) \left( \gamma + \frac{1}{\gamma} \right)} + \frac{\Lambda}{\bar{\gamma}} E_{2N} \left( \frac{2\Lambda}{\bar{\gamma}} \right) e^{\left( \frac{2\Lambda}{\bar{\gamma}} \right) \left( \gamma + \frac{1}{\gamma} \right)}, \tag{37}\]

where \( \Lambda = \bar{\gamma}_{\text{39a}} = \bar{\gamma}_{\text{39a}} \) is the average SIR at the relay or the destination node.

- For non-identical fading channel links with \( L \neq N \), \( \mu_{\text{sr}} = \mu_{\text{rd}} = \mu_{\text{ln}} = \mu_{\text{lg}} = 1 \), \( \bar{\gamma}_{\text{sr}} \neq \bar{\gamma}_{\text{rd}} \) and \( \bar{\gamma}_{\text{ln}} \neq \bar{\gamma}_{\text{lg}} \), the error probability is expressed as
  \[
P_b(e) = \frac{\Lambda N^{L}}{4} e^{\left( \gamma + \Lambda \right) \left( \frac{1}{\gamma} \right)} \left[ \sum_{i_1=1}^{L} Na_{i_1} E_{i_1} \left( \gamma \left( \frac{1}{\gamma} \right) + \frac{1}{\gamma} + 1 \right) \right] + \sum_{i_2=1}^{N+1} Nb_{i_2} E_{i_2} \left( \frac{1}{\gamma} + \frac{1}{\gamma} + 1 \right) + \sum_{i_3=1}^{L+1} La_{i_3} E_{i_3} \left( \frac{1}{\gamma} + \frac{1}{\gamma} + 1 \right) + \sum_{i_4=1}^{N} Lb_{i_4} E_{i_4} \left( \frac{1}{\gamma} + \frac{1}{\gamma} + 1 \right) \left[ \sum_{i_5=1}^{L} a_{i_5} E_{i_5} \left( \gamma \left( \frac{1}{\gamma} \right) + \frac{1}{\gamma} + 1 \right) \right] + \sum_{i_6=1}^{N} b_{i_6} E_{i_6} \left( \frac{1}{\gamma} + \frac{1}{\gamma} + 1 \right) \left[ \sum_{i_7=1}^{L} a_{i_7} E_{i_7} \left( \gamma \left( \frac{1}{\gamma} \right) + \frac{1}{\gamma} + 1 \right) \right]. \tag{38}\]

The coefficients are defined as

\[
a_{i_1} = \left. \frac{1}{(L-i_1)!} \frac{\partial^{L-i_1}}{\partial \gamma^{L-i_1}} \left[ \frac{1}{\left( \gamma + \Lambda \right)^{N+1}} \right] \right|_{\gamma = -1}, \tag{39a}\]

\[
b_{i_2} = \left. \frac{1}{(N-i_2+1)!} \frac{\partial^{N-i_2+1}}{\partial \gamma^{N-i_2+1}} \left[ \frac{1}{\left( \gamma + \Lambda \right)^{L}} \right] \right|_{\gamma = -1}. \tag{39b}\]
\[ a_i = \frac{1}{(L-i_3+1)! \partial \gamma^{L-i_3+1}} \left[ \frac{1}{(\gamma + \Lambda)^N} \right]_{\gamma = -\tilde{\gamma}}, \]  
(39c)

\[ b_i = \frac{1}{(N-i_4)! \partial \gamma^{N-i_4}} \left[ \frac{1}{(\gamma + \bar{\gamma})^{L+1}} \right]_{\gamma = -\Lambda}, \]  
(39d)

\[ a_i = \frac{1}{(L-i_5)! \partial \gamma^{L-i_5}} \left[ \frac{1}{(\gamma + \Lambda)^N} \right]_{\gamma = -\tilde{\gamma}}, \]  
(39e)

\[ b_i = \frac{1}{(N-i_6)! \partial \gamma^{N-i_6}} \left[ \frac{1}{(\gamma + \bar{\gamma})^{L+1}} \right]_{\gamma = -\Lambda}, \]  
(39f)

and \( \Lambda = \frac{\tilde{\gamma} \bar{\gamma}}{\tilde{\mu} \bar{\mu}} \) is the average SIR at the relay node, and \( (\bar{\gamma} = \frac{\tilde{\mu} \bar{\gamma}}{\tilde{\mu} \bar{\mu}}) \), is the average SIR at the destination node.

**FIGURE 2** Dual-hop outage probability for \( \alpha = 2, \mu = 1, \) INR = 3dB. SNR, signal-to-noise ratio

**FIGURE 3** Dual-hop outage probability for \( \alpha = 2, \mu = m = 2, \) INR = 12dB. SNR, signal-to-noise ratio
For identical fading channel links with $L = N$, $\mu_{sr} = \mu_{rd} = m$, $\mu_{ih} = \mu_{Ig} = m_I$, $\gamma_{sr} = \gamma_{rd} = \tilde{\gamma}$ and $\gamma_{ih} = \gamma_{Ig} = \tilde{\gamma}_I$, the error probability for BPSK modulation is given as

$$P_s(e) \approx \frac{1}{\Gamma(m)^2} \sum_{j=0}^{m} \sum_{k=1}^{m-1} \sum_{n=0}^{k-1} \binom{m}{j} \binom{k-1}{n} (m_I)_j (m-I)_n \Gamma(m-k+1) \Gamma(m+k-1) \left( \frac{m_I}{\tilde{\gamma}_I} \right)^{m+k-1-j-n} U \left( m+k-1, m+k-2, lm_I-j-n; \frac{m_I \tilde{\gamma}_I}{m_I} + \frac{2m_I}{m_I} \right).$$

\begin{equation}
(40)
\end{equation}

## 5 | NUMERICAL RESULTS

In this section, we illustrate the expressions developed in Section 4 for the error and outage performance of the considered dual-hop AF relaying system, and illustrate the effect of interference on system performance. We consider different values of the parameter $\mu$ of the assumed $\alpha-\mu$ fading channel model while $\alpha$ is fixed with the value of $\alpha = 2$, and in these results, the fading parameters are assumed equal for all the dual-hop links and for the interferers links. The dual-
hop performance characteristics are plotted in terms of the outage and error probability as a function of the average SNR in decibel. Figures (2), (3), and (4) depict the outage performance of a dual-hop AF relaying system where $\alpha-\mu$ fading channels are assumed with the presence of CCI interference at the relay and destination nodes. In Figure (2), the outage performance is illustrated for fading parameters $\alpha = 2$ and $\mu = 1$, with equal power interferers $INR = 3$ dB for different but equal number of interferers at the relay and destination nodes. While in Figure (3) the outage is characterized with fading parameters $\alpha = 2$ and $\mu = 2$ with equal power interferers $INR = 12$ dB and different but equal number of interferers at the relay and destination nodes. Figure (4) shows the outage with fading parameters of $\alpha = 2$ and $\mu = 2$ with different values of power interferers. Moreover, the system performance degradation due to the increase in number of interferers is obvious in these curves.

**FIGURE 6** Average error probability for identical fading channel with $\alpha = 2$, $\mu = m = m_i = 2$ and $N = L = 4$ interferers

**FIGURE 7** Average error probability for identical fading channel with $\alpha = 2$, different fading parameters $\mu = m = 1,2,3,4,5$. SNR, signal-to-noise ratio
The error performance of a dual-hop AF relaying system is shown in Figures (5), (6), and (7), assuming coherent BPSK to develop the BER. It can be seen from the curves that the floor in all cases is caused by the interference which clearly affects the system performance accordingly. In Figure 5, the average error probability is plotted for fading parameters $\alpha = 2$ and $\mu = 1$ and power interferers equally with the value of $INR = 30$ dB but the number of interferers are different at the relay and destination nodes, while in Figure 6, the fading parameters $\alpha = 2$ and $\mu = 2$. The fading parameters are set the same for the relay links and the interferers. It is clear the interferers number get larger, the performance gets worst and becomes more degraded in other words. Finally, in Figure 7, the error probability is characterized for different values of fading parameters of the interferers' links that the relay and destination nodes are subject to, where it's shown that the performance of the system is getting degraded by decreasing the $\mu$ values for both the interferers' links and the dual-hop links.

6 | CONCLUSION

In this paper, we have investigated the error and outage performance of dual-hop AF relaying network over $\alpha-\mu$ fading channels in the presence of co-channel interferences. We developed the PDF and the CDF of the upper bound of the SINR. Moreover, we have developed the PDF of N-IID interferers using the MGF approach. The derived expressions for the outage and error probability were also introduced to investigate the performance of the dual-hop AF relaying systems over other fading models such as Rayleigh ($\alpha = 2$ and $\mu = 1$), Nakagami-m ($\alpha = 2$ and $\mu = m$), and other fading models where the value $\alpha = 2$ with different values of $\mu$. The developed expressions matched exactly the previously derived expression in the literature related to investigating the performance of dual-hop AF relaying systems over Rayleigh and Nakagami-m fading channels. Furthermore, as shown in the results, there is a major impact of interference at the nodes on the system performance. Where the increase in number of interferers, the outage probability increases as a result, and that degrades the system performance accordingly. The same finding holds for the error probability, where the system performance degrades by increasing the number of interferers introduced to the system nodes, also its found that the presence of interference caused the error probability curve to floor.

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