The Importance of Local Kinetic Processes in Determining Proton Temperature in the Solar Wind

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ABSTRACT
We use magnetic helicity to investigate the polarisation properties of Alfvénic fluctuations at proton-kinetic scales in the solar wind as a function of $\beta_p$, the ratio of proton thermal pressure to magnetic pressure, and $\theta_{0B}$, the angle between the proton flow and local mean magnetic field, $B_0$. Using almost 15 years of Wind observations, we separate the contributions to helicity from fluctuations with wave-vectors, $k$, quasi-parallel and oblique to $B_0$, finding that the helicity of Alfvénic fluctuations is consistent with predictions from linear Vlasov theory. This result suggests that the non-linear turbulent fluctuations at these scales share at least some polarisation properties with Alfvén waves. We also investigate the dependence of proton temperature in the $\beta_p$-$\theta_{0B}$ plane, finding that it correlates with $\theta_{0B}$. The proton temperature parallel to $B_0$ is higher in the parameter space where we measure the helicity of right-handed Alfvénic fluctuations, and the temperature perpendicular to $B_0$ is higher where we measure left-handed fluctuations. This finding is inconsistent with the general assumption that solar wind fluctuations are ergodic in the sense that sampling different $\theta_{0B}$ allows us to analyse the dependence of the turbulence simply on $\theta_{0B}$, the angle between $k$ and $B_0$. We rule out both instrumental and expansion effects. Our results provide new evidence for the importance of local kinetic processes that depend on $\theta_{0B}$ in determining proton temperature in the solar wind.

Key words: Sun: heliosphere – solar wind – plasmas – turbulence – waves

1 INTRODUCTION
In-situ measurements of the solar wind provide insights into the fundamental physical processes occurring in expanding astrophysical plasmas. The couplings between large-scale dynamics and small-scale kinetic processes are central to our understanding of energy transport and heating in these plasmas. Fluctuations in the solar wind plasma and electromagnetic fields exist over many orders of magnitude in scale, linking both microscopic and macroscopic processes (see Matteini et al. 2012; Alexandrova et al. 2013; Verscharen et al. 2019, and references therein). In solar wind originating from open field lines in the corona, these fluctuations are predominantly Alfvénic (Coleman 1968; Belcher et al. 1969; Belcher & Davis Jr. 1971), with only a small compressional component (Howes et al. 2012; Klein et al. 2012; Chen 2016; Safránková et al. 2019). At scales $10^5 \lesssim L \lesssim 10^8$ m, called the inertial range, non-linear interactions between fluctuations lead to a turbulent cascade of energy towards smaller scales (Tu & Marsch 1995; Bruno & Carbone 2013; Kiyani et al. 2015). This range is characterised by fluctuations with increasing anisotropy toward smaller scales, $k_\perp \gg k_\parallel$, where $k_\parallel$ and $k_\perp$ are components of the wave-vector, $k$, in the direction parallel and perpendicular to the local mean magnetic field, $B_0$, respectively (Dasso et al. 2005; Hamilton et al. 2008; Horbury et al. 2008; MacBride et al. 2010; Wicks et al. 2010; Chen et al. 2011, 2012).

At scales close to the proton inertial length, $d_p$, and proton gyro-radius, $pp$, typically $L \sim 10^5$ m at 1 au, the
coupling of electromagnetic fluctuations and proton velocity distribution functions (VDFs) can lead to energy transfer between fluctuating fields and the particles. At these proton-kinetic scales, the properties of the fluctuations change due to Hall and finite-Larmor-radius effects (Howes et al. 2006; Schekochihin et al. 2009; Boldyrev & Perez 2012), and the non-linear turbulent fluctuations exhibit some properties that are consistent with those of kinetic Alfvén waves (KAWs; Leamon et al. 1999; Bale et al. 2005; Howes et al. 2008; Sahraoui et al. 2010). Collisionless damping of these fluctuations can lead to dissipation of the fluctuations and plasma heating (e.g., Marsch 2006). At the same time, the non-Maxwellian nature of ion VDFs in the solar wind provides sources of free energy for growing instabilities at these scales (Kasper et al. 2002b, 2008, 2013; Bale et al. 2009; Maruca et al. 2012b; Bourouaine et al. 2013; Gary et al. 2015; Alterman et al. 2018; Klein et al. 2018). These instabilities act to alter the macroscopic thermodynamics of the plasma, coupling the small-scale local processes with large-scale dynamics, typically leading to a deviation from Chew-Goldberger-Low theory (CGL; Chew et al. 1956) for double adiabatic expansion.

Measurements of the solar wind plasma by single spacecraft are restricted to the sampling of a time series defined by the trajectory of the spacecraft with respect to the flow velocity, \(\mathbf{v}_{SW}\). This limitation means that we can only resolve the component of \(\mathbf{k}\) along the sampling direction. Previous studies (e.g., Horbury et al. 2008; Wicks et al. 2010; He et al. 2011; Podesta & Gary 2011a) often assume that solar wind fluctuations are ergodic in the sense that the turbulence is independent of \(\theta_{kB}\), the angle between \(\mathbf{v}_{SW}\) and \(\mathbf{B}_0\). In other words, these studies assume that measurements of turbulence at different \(\theta_{kB}\) allow us to probe both the nature of the turbulence and its impact on the plasma as a function of \(\theta_{kB}\), the angle between \(\mathbf{k}\) and \(\mathbf{B}_0\).

In this paper, we use the polarisation properties of Alfvénic fluctuations at proton-kinetic scales in the solar wind to test how the dissipation of turbulence at these scales affects the macroscopic bulk properties of the solar wind. Following Woodham et al. (2019), we separate the magnetic helicity of fluctuations with \(k\) quasi-parallel and oblique to \(\mathbf{B}_0\). We show that these fluctuations are consistent with linear Vlasov theory (Gary 1986) and that turbulent fluctuations present at proton-kinetic scales in the solar wind share at least some properties with Alfven waves. By investigating the statistical distribution of proton temperature in the \(\beta_p-\theta_{kB}\) plane, we find that there is a clear dependence on \(\theta_{kB}\) that correlates with the magnetic helicity of Alfvénic fluctuations in the same plane, ruling out both instrumental and expansion effects. This result means that we cannot sample different \(\theta_{kB}\) to analyse the dependence of the turbulence on \(\theta_{kB}\) without considering other plasma properties. Our results provide new evidence for the importance of local kinetic processes that depend on \(\theta_{kB}\) in determining proton temperature in the solar wind.

2 POLARISATION PROPERTIES OF ALFVÉN WAVES

In collisionless space plasmas such as the solar wind, the linearised Vlasov equation describes linear waves and instabilities. Non-trivial solutions exist only when the complex frequency, \(\omega = \omega_r + i \gamma\), solves the hot-plasma dispersion relation (Stix 1992). Here, \(\omega_r\) is the wave frequency and \(\gamma\) is the wave growth or damping rate. One such solution is the Alfvén wave, which is ubiquitous in space plasmas. At \(k_B d_p \ll 1\) and \(k_B \rho_p \ll 1\), this wave is incompressible and propagates along \(\mathbf{B}_0\) at the Alfvén speed, \(v_A\), resulting in transverse perturbations to the field (Alfvén 1942). The fluctuations in velocity, \(\delta \mathbf{v}\), and the magnetic field, \(\delta \mathbf{B}\), exhibit a characteristic (anti-)correlation, \(\delta \mathbf{v} \propto i \delta \mathbf{B}\), for propagation (parallel) anti-parallel to \(\mathbf{B}_0\). Here, \(\mathbf{B}\) is the magnetic field in Alfvén units, \(\mathbf{B} = \mathbf{B}/\sqrt{\rho n}\), where \(\rho\) is the plasma mass density. The Alfvén wave has the dispersion relation:

\[
\omega_r(k) = k \frac{\omega_A}{\sin \theta_{kB}}. \quad (1)
\]

Approaching \(k_B \rho_p \approx 1\) or \(k_B d_p \approx 1\), the polarisation properties of Alfvén waves change, i.e., depending on the angle \(\theta_{kB}\) (Gary 1986). The dispersion relation splits into two branches: the Alfvén ion-cyclotron (AIC) wave (Gary & Borovsky 2004) and the KAW (Gary & Nishimura 2004). We define the polarisation of a wave as:

\[
P = \frac{i \delta E_x}{\delta E_y} \frac{\omega_r}{|\omega_r|}, \quad (2)
\]

where \(\delta E_x\) and \(\delta E_y\) are components of the Fourier amplitudes of the fluctuating electric field transverse to \(\mathbf{B}_0 = B_0 \hat{z}\) (Stix 1992; Gary 1993). Therefore, \(P\) gives the sense and degree of rotation in time of a fluctuating electric field vector at a fixed point in space, viewed in the direction parallel to \(\mathbf{B}_0\). A circularly polarised wave has \(P = \pm 1\), where \(+1\) (\(-1\)) designates right-handed (left-handed) polarisation. In this definition, a right-hand polarised wave has electric field vectors that rotate in the same sense as the gyration of an electron, and a left-hand polarised wave, the same sense as ions. For more general elliptical polarisation, we take the real part, \(\text{Re}(P)\).

Magnetic helicity is a measure of the degree and sense of spatial rotation of the magnetic field (Woltjer 1958a,b). It is an invariant of ideal magnetohydrodynamics (MHD) and defined as a volume integral over all space:

\[
H_m = \int_V \mathbf{A} \cdot \mathbf{B} \, d^3 \mathbf{r}, \quad (3)
\]

where \(\mathbf{A}\) is the magnetic vector potential defined by \(\mathbf{B} = \nabla \times \mathbf{A}\). Matthaeus et al. (1982) propose the fluctuating magnetic helicity, \(H'_m(k)\), as a diagnostic of solar wind fluctuations, which in spectral form (i.e., in Fourier space) is defined as:

\[
H'_m(k) = \delta \mathbf{A}(k) \cdot \delta \mathbf{B}^*(k), \quad (4)
\]

where \(\delta \mathbf{B} = \mathbf{B} - \mathbf{B}_0\) is the fluctuating magnetic field, \(\delta \mathbf{A}\) is the fluctuating vector potential, and the asterisk indicates the complex conjugate of the Fourier coefficients (Matthaeus & Goldstein 1982b). This definition removes contributions to the helicity arising from \(\mathbf{B}_0\). By assuming the Coulomb gauge, \(\nabla \cdot \mathbf{A} = 0\), the fluctuating magnetic helicity can be written:

\[
H'_m(k) = i \frac{\delta B_y \delta B_z^* - \delta B_z \delta B_y^*}{k_x}, \quad (5)
\]
where the components of $\delta B(\mathbf{k})$ are Fourier coefficients of a wave mode with $k$. This result is invariant under cyclic permutations of the three components $x,y,z$ (Howes & Quataert 2010). We define the normalised fluctuating magnetic helicity density as:

$$\sigma_m(k) = \frac{kH_m^r(k)}{[\delta B(k)]^2},$$

(6)

where $[\delta B(k)]^2 = \delta B_x^r \delta B_x + \delta B_y^r \delta B_y + \delta B_z^r \delta B_z$. Here, $\sigma_m(k)$ is dimensionless and takes values between $[-1,1]$, where $\sigma_m = -1$ indicates fluctuations with purely left-handed helicity, and $\sigma_m = +1$ purely right-handed helicity. A value of $\sigma_m = 0$ indicates no overall coherence, i.e., there are either no fluctuations with coherent handedness or there is equal power in both left-handed and right-handed components so that the net helicity averages to zero.

Gary (1986) explores the dependence of $\text{Re}(P)$ for Alfvén waves on different parameters by numerically solving the full electromagnetic dispersion relation. He shows that $\text{Re}(P)$ changes sign depending on $\theta_{kB}$ and the ratio of proton thermal pressure to magnetic pressure, $\beta_p = n_p k B_P / (B_0^2/2\mu_0)$, where $n_p$ is the proton density, and $T_p$ is the proton temperature. In the cold-plasma limit ($\beta_p \ll 1$), the Alfvén wave has $\text{Re}(P) < 0$ for all $\theta_{kB}$. However, from linear Vlasov theory, at $\beta_p \simeq 10^{-2}$, the wave has $\text{Re}(P) < 0$ for $0^\circ \leq \theta_{kB} \leq 80^\circ$, but has $\text{Re}(P) > 0$ for $\theta_{kB} \geq 80^\circ$. As $\beta_p$ increases, the wave has $\text{Re}(P) > 0$ for an increasing range of oblique angles so that at $\beta_p \simeq 10$, the transition occurs at about $40^\circ$. We show this transition in both $\text{Re}(P)$ and $\sigma_m(k)$ in Figure 1, where the black lines are isocontours of $\text{Re}(P) = 0$ and $\sigma_m(k) = 0$, respectively. To calculate these lines, we solve the linear Vlasov equation using the New Hampshire Dispersion relation Solver (NHDS: Verscharen et al. 2013; Verscharen & Chandran 2018). Here, $k = k_x \hat{x} + k_y \hat{y} + k_z \hat{z}$, and we assume a plasma consisting of protons and electrons with isotropic Maxwellian distributions, equal density and temperature, and no drifting components. We set $kd_p = 0.05$ to probe the properties of Alfvén waves close to proton-kinetic scales, where the angle $\theta_{kB}$ defines $k_x = k \sin \theta_{kB}$ and $k_y = k \cos \theta_{kB}$. We note that while $k_xd_p$ and $k_yd_p$ change throughout the $\beta_p$-$\theta_{kB}$ plane, the normalised scale of the waves remains constant. We also set $v_A/c = 10^{-4}$, which is typical for solar wind conditions where $v_A \approx 50$ km/s (Klein & Vech 2019).

### 3 REDUCED SPECTRA FROM SPACECRAFT MEASUREMENTS

From a single-spacecraft time series of magnetic field measurements, it is only possible to determine a reduced form of the fluctuating magnetic helicity (Batchelor 1970; Montgomery & Turner 1981; Matthaeus et al. 1982):

$$H_m^r(\omega_x) = \frac{2\text{Im} \{ P_{T/N}(\omega_x) \}}{k_p},$$

(7)

\footnote{The scales $d_p$ and $\rho_p$ are related by: $\rho_p = d_p \sqrt{\beta_p}$.}

where $\omega_{sc}$ is the frequency of the fluctuations in the spacecraft frame, $k_r = k \cos \theta_{kx}$, is the component of the wave-vector along the flow direction of the solar wind plasma assuming $v_{sw} \simeq v_{sw} \hat{R}^2$ and $\theta_{kv}$ is the angle between $k$ and $\vec{v}_{sw}$. Here,

$$P_{ij}^r(\omega_{sc}) = \delta B_i^r(\omega_{sc}) \cdot \delta B_j(\omega_{sc}),$$

(8)

is the reduced power spectral tensor, where the $\delta B_i(\omega_{sc})$ are the complex Fourier coefficients from the Fourier transform of the time series in radial-tangential-normal (RTN) coordinates. The reduced tensor is an integral of the true spec-

Figure 1. (a) The real part of the polarisation, $\text{Re}(P)$, and (b) normalised fluctuating magnetic helicity density, $\sigma_m(k)$, of Alfvén waves with $kd_p = 0.05$ as a function of $\beta_p$ and $\theta_{kB}$, calculated using the NHDS code (see main text). The black lines indicate the isocontours $\text{Re}(P) = 0$ and $\sigma_m(k) = 0$. 

In the RTN coordinate system, $\hat{R}$ is the unit vector from the Sun towards the spacecraft, $\hat{T}$ is the cross-product of the solar rotation axis and $\hat{R}$, and $\hat{N}$ completes the right-handed triad.
We analyse magnetic field data from the MFI fluxgate magnetometer (Lepping et al. 1995; Koval & Szabo 2013) and proton data from the SWE Faraday cup (Ogilvie et al. 1995; Kasper et al. 2006) instruments on-board the Wind spacecraft from Jun 2004 to Oct 2018. Following Woodham et al. (2019), we account for heliospheric sector structure in the magnetic field measurements by calculating the Parker-spiral angle, \( \phi_B = \arctan(B_T/B_R) \), where \( B_R \) and \( B_T \) are the components of \( B \) in RTN coordinates averaged over \( \sim 92 \) s intervals, corresponding to the SWE measurement cadence. If \( \phi_B \) averaged over a two day period exceeds 45° from the radial direction, we reverse the signs of the \( B_R \) and \( B_T \) components so that sunward fields are rotated anti-sunward. This procedure removes the inversion of the sign of magnetic helicity due to the direction of the large-scale magnetic field with respect to the Sun.  

For successive proton measurements, we define a local mean field, \( \bar{B}_0 \), averaged over the SWE integration time (\( \sim 92 \) s). We transform the 11 Hz magnetic field data associated with each proton measurement into field-aligned coordinates (Equation 12). We then compute the continuous wavelet transform (Torrence & Compo 1998) using a Morlet wavelet to calculate the magnetic helicity spectra, \( \sigma_{xy} \) and \( \sigma_{yz} \), as functions of \( f_{sc} = \omega_{sc}/2\pi \) using Equation 13. We average the spectra over \( \sim 92 \) s to ensure that the fluctuations contributing to the helicity spectra persist for at least several proton gyro-periods, \( 2\pi/\Omega_p \), giving a clear coherent helicity signature at proton-kinetic scales. Following Woodham et al. (2018), we estimate the amplitudes of \( \sigma_{xy} \) and \( \sigma_{yz} \) at proton-kinetic scales by fitting a Gaussian function to the coherent peak in each spectrum at frequencies close to the Taylor-shifted frequencies, \( v_{sw}/\rho_p \) and \( v_{sw}/\rho_p \). We neglect any peak at \( f > f_{noise} \), where \( f_{noise} \) is the frequency above which instrumental noise of the MFI magnetometer becomes significant. We also reject a spectrum if the fluctu-
ations represent a spatial cut through the plasma and we can write \( \mathcal{P}'_{ij} \) and \( H''_{m} \) as functions \( k_r \) using Equation 10. However, it is not possible to determine the full wave-vector, \( k \), or \( \theta_{CB} \), from single-spacecraft measurements. Instead, previous studies (e.g., Horbury et al. 2008; Wicks et al. 2010; He et al. 2011; Podesta & Gary 2011a) use \( \theta_{CB} \) as a measure of a specific \( \theta_{CB} \) in the solar wind. Since \( v_A \ll v_{sw} \), Taylor’s hypothesis is usually well-satisfied for Alfvén waves in the solar wind with the dispersion relation given by Equation 1, as well as for the small-wavelength extensions of the Alfvén branch under the parameters considered here (see Howes et al. 2015; Klein et al. 2014b). The normalised reduced fluctuating magnetic helicity density is defined as:

\[
\sigma_m^r(k_r) = \frac{k_r H''_{m} (k_r)}{[\mathbf{B}(k_r)]^2} = \frac{2 \text{Im} \{ \mathcal{P}'_{ij}(k_r) \}}{\text{Tr} \{ H''_{m}(k_r) \}},
\]

where \( \text{Tr}\{\} \) denotes the trace. We define the field-aligned coordinate system,

\[
\hat{x} = \frac{B_0}{|B_0|}, \quad \hat{y} = -\frac{v_{sw} \times B_0}{|v_{sw} \times B_0|}, \quad \hat{z} = \hat{y} \times \hat{x},
\]

so that \( v_{sw} \) lies in the \( x-z \) plane with angle \( \theta_{CB} \) from the \( \hat{z} \) direction (Wicks et al. 2012; Woodham et al. 2019). Using this coordinate system, we separate the different contributions to magnetic helicity from fluctuations with \( k \) quasi-parallel and oblique to \( B_0 \) using our definition:

\[
\sigma_{ij}(k_l) = \frac{2 \text{Im} \{ \mathcal{P}'_{ij}(k_l) \}}{\text{Tr} \{ \mathcal{F}'(k_l) \}},
\]

where the indices \( i,j,l = x,y,z \). We provide more details on the decomposition of \( \sigma_{ij}^m(k_r) \) into the three components, \( \sigma_{ij}(k_l) \), in the Appendix.

4 DATA ANALYSIS WITH A SINGLE SPACECRAFT

We analyse magnetic field data from the MFI fluxgate magnetometer (Lepping et al. 1995; Koval & Szabo 2013) and proton data from the SWE Faraday cup (Ogilvie et al. 1995; Kasper et al. 2006) instruments on-board the Wind spacecraft from Jun 2004 to Oct 2018. Following Woodham et al. (2019), we account for heliospheric sector structure in the magnetic field measurements by calculating the Parker-spiral angle, \( \phi_B = \arctan(B_T/B_R) \), where \( B_R \) and \( B_T \) are the components of \( B \) in RTN coordinates averaged over \( \sim 92 \) s intervals, corresponding to the SWE measurement cadence. If \( \phi_B \) averaged over a two day period exceeds 45° from the radial direction, we reverse the signs of the \( B_R \) and \( B_T \) components so that sunward fields are rotated anti-sunward. This procedure removes the inversion of the sign of magnetic helicity due to the direction of the large-scale magnetic field with respect to the Sun.  

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We estimate the normalised cross-helicity (Matthaeus & Goldstein 1982a),

\[
\sigma_c = \frac{2 \left( \delta v \cdot \delta b \right)}{|\delta v|^2 + |\delta b|^2},
\]

where \( \delta b = b - (b)_l h \), \( \delta v = v_{sw} - (v_{sw})_l h \), and \( b = B_0/\sqrt{\text{mean}} \) is normalised to Alfvén units. Here, the mean is over a one hour window centred on the instantaneous values and we assume that \( v_{sw} = v_p \), where \( v_p \) is the proton bulk velocity. An averaging interval of one hour gives \( \sigma_c \) for fluctuations in the inertial range. The cross-helicity, \( \sigma_c \in [-1, 1] \), is a measure of the (anti-)correlation between velocity and magnetic field fluctuations, and therefore, Alfvénicity (D’Amicis & Bruno 2015; D’Amicis et al. 2019; Stansby et al. 2019). A value \( |\sigma_c| = 1 \) indicates purely unbalanced Alfvénic fluctuations with propagation in only one direction, whereas \( \sigma_c = 0 \) indicates either balanced (equal power in opposite directions) or a lack of Alfvénic fluctuations. For solar wind fluctuations dominantly propagating anti-sunward, the sign of \( \sigma_c \) depends only on the direction of \( B_0 \). For the case of \( \sigma_c = 0 \), however, we expect no coherent value of \( |\sigma_m| > 0 \) at proton-kinetic scales.

3 See Table 1 in Woodham et al. (2019).
4 See Appendix in Woodham et al. (2018).
We include only measurements of Alfvénic solar wind, \(|\sigma| \geq 0.75\), and low collisionality, \(N_C < 1\), which contain the strongest Alfvénic fluctuations with a non-zero magnetic helicity. Here, \(N_C\) is the Coulomb number (Maruca et al. 2013; Kasper et al. 2017), which estimates the number of collisional timescales for protons. We bin the data in \(\log_{10}(\beta_p)\) and \(\theta_{vB}\) using bins of width \(\Delta \log_{10}(\beta_p) = 0.05\) and \(\Delta \theta_{vB} = 5^\circ\). We restrict our analysis to \(10^{-2} \leq \beta_p \leq 10^1\) and include the full range of \(\theta_{vB} = [0^\circ, 180^\circ]\) to account for any dependence on heliospheric sector structure. In Figure 2, we plot the probability density distribution of the data,

\[
\hat{p} = \frac{n}{N\beta_p \Delta \theta_{vB}},
\]

in the \(\beta_p-\theta_{vB}\) plane, where \(n\) is the number of data points in each bin and \(N\) is the total number of data points. We overplot the isocontour of \(\sigma_m(\mathbf{k}) = 0\) from Figure 1(b) by replacing \(\theta_{vB}\) with \(\theta_{vB}\), i.e., \(\sigma_m(\theta_{vB}) = 0\). If we assume the turbulence is independent of \(\theta_{vB}\), then any dependence on \(\theta_{vB}\) exclusively reflects a dependence on \(\theta_{k,B}\) (see Horbury et al. 2008; Wicks et al. 2010; He et al. 2011; Podesta & Gary 2011a). We mirror the \(\sigma_m(\theta_{vB}) = 0\) curve around the line \(\theta_{vB} = 90^\circ\) to account for heliospheric sector structure. The distribution of data in Figure 2 shows two peaks at \(\theta_{vB} \sim 65^\circ\) and \(\theta_{vB} \sim 115^\circ\) around \(\beta_p \sim 0.5\). There are fewer data points at quasi-parallel angles, showing that the majority of data are associated with oblique angles, which we attribute to the average Parker spiral in the solar wind. There is also a clear \(\beta_p\) dependence, with the majority of the data lying in the range \(0.1 \lesssim \beta_p \lesssim 1\).

In Figure 3, we plot the median values of \(\sigma_{\parallel}\) and \(\sigma_{\perp}\) for each bin in the \(\beta_p-\theta_{vB}\) plane. We neglect any bins with fewer than 20 data points to improve statistical reliability. From Figure 1, we expect to measure KAW-like fluctuations with \(\sigma_{\parallel} > 0\) in the area of the \(\beta_p-\theta_{vB}\) plane enclosed by the two dashed lines at quasi-perpendicular angles, and measure AIC wave-like fluctuations with \(\sigma_{\parallel} < 0\) elsewhere at quasi-parallel angles. Figure 3 is consistent with this expectation; we see a strong negative helicity signal at \(0^\circ \leq \theta_{vB} \leq 30^\circ\) and \(150^\circ \leq \theta_{vB} \leq 180^\circ\), with a minimum of \(\sigma_{\parallel} \approx -0.8\) approaching \(\theta_{vB} \approx 0^\circ\), as well as a weaker positive signal of \(\sigma_{\parallel} \approx 0.4\) at angles \(60^\circ \leq \theta_{vB} \leq 120^\circ\). Both \(\sigma_{\parallel}\) and \(\sigma_{\perp}\) are symmetrically distributed about the line \(\theta_{vB} = 90^\circ\) since we remove the ambiguity in the sign of the helicity due to the direction of \(\mathbf{B}_0\). The distribution of \(\sigma_{\parallel}\) is consistent with the presence of quasi-parallel propagating AIC waves from kinetic instabilities in the solar wind (Woodham et al. 2019; Zhao et al. 2019a). Elsewhere in Figure 3(a), the median value of \(\sigma_{\parallel}\) is zero, showing that a coherent signal of parallel-propagating
fluctuations at proton-kinetic scales in the solar wind is not measured at oblique angles. 

In Figure 3(b), there are two peaks in the median $\sigma_\perp$ close to $\beta_p \sim 1$, located at $\theta_{pB} \sim 70^\circ$ and $\theta_{pB} \sim 110^\circ$. Despite these peaks, the signal is spread across the parameter space, albeit weaker at quasi-parallel angles. We interpret this spread using Taylor’s hypothesis. Due to the $k \cdot \mathbf{v}_{sw}$ term in the $\delta$-function in Equation 9, a cos $\theta_{kB}$ factor modifies the contribution of all modes to the reduced spectrum measured in the direction of $\mathbf{v}_{sw}$. If $\theta_{kB} = 0^\circ$, then cos $\theta_{kB} = 1$, and the waves are measured at their actual $k$. However, for oblique angles, $\theta_{kB} \gtrsim 60^\circ$, waves measured at a fixed $\omega_c$ correspond to a higher $k$ in the plasma frame. Since a turbulent spectrum decreases in amplitude with increasing $k$, the reduced spectrum is most sensitive to the smallest $k$ in the sampling direction. For parallel propagating fluctuations such as AIC waves, $\theta_{kB} \approx \theta_{kB}$, but for a broader $k$-distribution of obliquely propagating fluctuations, multiple fluctuations with different $k$ and therefore, different $\theta_{kB}$, contribute to a single $\theta_{pB}$ bin. The signal at $\theta_{pB} \lesssim 30^\circ$ is then likely due to fluctuations with $\theta_{pB} \gtrsim 60^\circ$, since they contribute to $\sigma_\perp$, i.e., have a significant $k_\perp$ component.

We plot the median values of $T_{p,\perp}/\langle T_{p,\perp} \rangle$ and $T_{p,||}/\langle T_{p,||} \rangle$ for each bin in the $\beta_p-\theta_{pB}$ plane in Figure 4. Here, $\langle T_{p,\perp} \rangle$ is the average value of $T_{p,\perp}$ over all angles for each bin in log$_{10} (\beta_p)$. This column normalisation removes the intrinsic temperature-dependence in the definition of $\beta_p$. If the properties of the turbulence are truly independent of $\theta_{pB}$, then we expect the dissipation mechanisms, and therefore, proton heating to be independent of $\theta_{pB}$. However, Figure 4 shows a clear dependence of the median column-normalised $T_{p,\perp}$ and $T_{p,||}$ on both $\theta_{pB}$ and $\beta_p$. In general, we see higher $T_{p,\perp}/\langle T_{p,\perp} \rangle$ at quasi-parallel angles where $\sigma_\parallel$ is largest in Figure 3(a), associated with AIC waves driven by kinetic instabilities (Kasper et al. 2002a; Matteini et al. 2007; Bale et al. 2009; Maruca et al. 2012a; Woodham et al. 2019). We also see higher $T_{p,||}/\langle T_{p,||} \rangle$ at oblique angles where $\sigma_\parallel$ is largest in Figure 3(b), associated with KAW-like fluctuations (Leamon et al. 1999; Bale et al. 2005; Howes et al. 2008; Sahraoui et al. 2010). However, there are also enhancements in $T_{p,\perp}/\langle T_{p,\perp} \rangle$ where the $\sigma_\perp$ helicity signal is strongest, as indicated by the contours of constant $\sigma_\perp$ in Figure 4.

6 DISCUSSION: THE ERGODICITY HYPOTHESIS

It is generally assumed that the nature and properties of the fluctuations in the solar wind are independent of $\theta_{pB}$. If this assumption is true, the fluctuations are ergodic in the sense that sampling different $\theta_{pB}$ measures different components of the same background turbulence distribution. We expect no correlation between $T_{p,\perp}/\langle T_{p,\perp} \rangle$ and the magnetic helicity of Alfvénic fluctuations at proton-kinetic scales if the dissipation processes are universal. However, our results show that the proton temperature is not independent of $\theta_{pB}$ and correlates with the magnetic helicity signatures of different fluctuations at proton-kinetic scales. We see higher $T_{p,\perp}/\langle T_{p,\perp} \rangle$ in the same location of the $\beta_p-\theta_{pB}$ plane where we also measure the magnetic helicity of AIC waves and enhancements in $T_{p,||}/\langle T_{p,||} \rangle$ at oblique angles where we measure the helicity of KAW-like fluctuations. We note that it is also possible that the fluctuations still exist, and we do not measure them at different $\theta_{pB}$.

While we expect the polarisation properties of solar wind fluctuations to affect the dissipation mechanisms and thus the proton temperature, we do not measure the magnetic helicity of all fluctuations at once due to Taylor’s hypothesis, just the dominantly radial fluctuations due to the $k \cdot \mathbf{v}_{sw}$ term in Equation 9. We interpret that these fluctuations affect the distribution of $T_{p,\perp}/\langle T_{p,\perp} \rangle$ and that the nature of the fluctuations is not independent of $\theta_{pB}$, in violation of the ergodicity hypothesis. If this interpretation is correct, studies that sample many angles at $\theta_{pB}$ as the solar wind flows past a single spacecraft to build up a picture of the turbulence in the plasma, i.e., to sample different $\theta_{pB}$, need to be interpreted very carefully (e.g., Horbury et al. 2008; Wicks et al. 2010; He et al. 2011; Podesta & Gary 2011a). Therefore, it is fair to assume that the dependence of $T_{p,\perp}/\langle T_{p,\perp} \rangle$ and $T_{p,||}/\langle T_{p,||} \rangle$ on $\theta_{pB}$ and $\beta_p$ reflects the
differences in the dissipation and heating processes ongoing at proton-kinetic scales in the solar wind.

The enhancement in $T_{p,\perp}/\langle T_{p,\perp}\rangle$ at quasi-parallel angles in Figure 4(a) is likely related to the driving of AIC waves by proton temperature anisotropy instabilities (Kasper et al. 2002a; Matteini et al. 2007; Bale et al. 2009; Maruca et al. 2012a; Woodham et al. 2019). This enhancement correlates with the peak in $\sigma_\perp$ at these angles in Figure 3(a), where we measure the strongest signal of AIC waves. A large enough $T_{p,\perp}/\langle T_{p,\perp}\rangle$ can drive AIC waves unstable and the driving of these waves is enhanced by the presence of an $\alpha$-particle proton differential flow in the solar wind (Podesta & Gary 2011a,b; Woodham et al. 2019; Zhao et al. 2019b). While we are unable to observe AIC waves at oblique angles using a single spacecraft, we also measure KAW-like fluctuations at these angles using $\sigma_\perp$ in Figure 3(b). The peaks in $\sigma_\perp$ correlate with the observed enhancement in $T_{p,\perp}/\langle T_{p,\perp}\rangle$, and therefore, are consistent with the dissipation of these fluctuations. A common dissipation mechanism proposed for KAW-like fluctuations is Landau damping (e.g., Howes 2008; Shokouhi et al. 2009); however, this leads to heating parallel to $B_0$. Instead, perpendicular heating may arise from processes such as stochastic heating (Chandran et al. 2010, 2013), although more work is needed to confirm this.

The variation of $T_{p,\parallel}/\langle T_{p,\parallel}\rangle$ in the $\beta_p-\theta_{p,B}$ plane in Figure 4(b) is more difficult to interpret. This result could also be a signature of proton Landau damping of KAW-like fluctuations, although, this process is typically stronger at $\beta_p \gtrsim 1$ (Gary & Nishimura 2004; Kawazura et al. 2019). While we measure fluctuations that can consistently explain the enhancement in $T_{p,\parallel}/\langle T_{p,\parallel}\rangle$, other fluctuations may be present that we do not measure. Direct evidence of energy transfer between the fluctuations and protons is needed to confirm this result, for example, using the field-particle correlation method (Klein & Howes 2016; Howes et al. 2017; Klein 2017; Klein et al. 2017; Chen et al. 2019; Li et al. 2019). This evidence will require higher-resolution data than provided by Wind. We note that caution must be given when interpreting these results, since several other effects may also explain the temperature dependence seen in the $\beta_p-\theta_{p,B}$ plane. For example, instrumental effects and the role of solar wind expansion may result in similar temperature profiles in this plane. We now discuss these two effects in turn, showing that they cannot fully replicate our results presented in this paper.

7 ANALYSIS CAVEATS

7.1 Instrumentation & Measurement Uncertainties

The SWE Faraday cups on-board Wind measure a reduced VDF that is a function of the average direction of $B_0$ over the measurement interval (Kasper 2002). As the spacecraft spins every 3 s in the ecliptic plane, the Faraday cups measure the current due to ions in several angular windows. The Faraday cups repeat this process using a different voltage (energy) window for each spacecraft rotation, building up a full spectrum every ~92 s. By fitting a bi-Maxwellian to the reduced proton VDF, the proton thermal speeds, $w_{p,\parallel}$ and $w_{p,\perp}$, are obtained and converted to temperatures via

$$T_p \approx m_p w_p^2 / 2k_B,$$

where $m_p$ is the proton mass. Due to the orientation of the cups on the spacecraft body, the direction of $B_0$ with respect to the axis of the cups as they integrate over the proton VDF can cause inherent uncertainty in $w_{p,\parallel}$ and $w_{p,\perp}$. For example, if $B_0$ is radial, then measurements of $w_{p,\parallel}$ have a smaller uncertainty compared to when the field is perpendicular to the cup, i.e., $B_0$ is orientated out of the ecliptic plane by a significant angle, $\theta_{p,B} \gtrsim 60^\circ$ (Kasper et al. 2006). In Figure 5, we plot the percentage uncertainty in $w_{p,\parallel}$ and $w_{p,\perp}$.

$$U(w_p) = \frac{\Delta w_p}{w_p} \times 100\%,$$

Figure 5. Median percentage uncertainty in (a) $w_{p,\perp}$ and (b) $w_{p,\perp}$ across $\beta_p-\theta_{p,B}$ space. The black lines indicate the isocurves of $\sigma_m(\theta_{p,B}) = 0$ mirrored about the line $\theta_{p,B} = 90^\circ$. We also include contours of constant $\sigma_\perp$ from Figure 3(b).

$T_p \approx m_p w_p^2 / 2k_B$, where $m_p$ is the proton mass. Due to the orientation of the cups on the spacecraft body, the direction of $B_0$ with respect to the axis of the cups as they integrate over the proton VDF can cause inherent uncertainty in $w_{p,\parallel}$ and $w_{p,\perp}$. For example, if $B_0$ is radial, then measurements of $w_{p,\parallel}$ have a smaller uncertainty compared to when the field is perpendicular to the cup, i.e., $B_0$ is orientated out of the ecliptic plane by a significant angle, $\theta_{p,B} \gtrsim 60^\circ$ (Kasper et al. 2006). In Figure 5, we plot the percentage uncertainty in $w_{p,\parallel}$ and $w_{p,\perp}$.

$$U(w_p) = \frac{\Delta w_p}{w_p} \times 100\%,$$
quasi-parallel angles in Figure 5(a), which is almost independent of $\beta_p$. While the median $T_{p,\perp}/T_{p,\parallel}$ in Figure 4(a) is larger at these angles, there is a clear dependence on $\beta_p$. Therefore, increased uncertainty in the temperature measurements alone cannot completely account for the observed enhancement in $T_{p,\perp}/T_{p,\parallel}$ at these angles in the $\beta_p$-$\theta_{pB}$ plane. At quasi-perpendicular angles, the uncertainty in $w_{p,\perp}$ is less than 10%, suggesting that the enhancements in $T_{p,\perp}/T_{p,\parallel}$ in Figure 4(a) at $\beta_p \approx 1$ and $40^\circ \leq \theta_{pB} \lesssim 140^\circ$ are unlikely to result from instrumental uncertainties. From Figure 5(b), the uncertainty in $w_{p,\parallel}$ is largest at $\theta_{pB} \approx 90^\circ$, although there is a larger spread to $60^\circ \lesssim \theta_{pB} \lesssim 120^\circ$ at $\beta_p \gtrsim 0.3$. By comparing with Figure 4(b), the enhancement in $T_{p,\parallel}/T_{p,\perp}$ over the entire $\beta_p$ range does not coincide exactly with the regions of $\beta_p$-$\theta_{pB}$ space where these measurements have increased uncertainty. We also expect that any increased uncertainty in the $w_{p,\parallel}$ measurements would lead to increased noise that destroys any coherent median signal in this space, weakening the enhancement seen in Figure 4(b). Therefore, we conclude that the increased uncertainty in $w_{p,\parallel}$ at oblique angles is not the sole cause of the observed enhancement in $T_{p,\parallel}/T_{p,\perp}$.

Another source of uncertainty from the SWE measurements arises from the changing magnetic field direction over the course of the $\sim 92$ s measurement interval (Maruca & Kasper 2013). We quantify the angular fluctuations in B using:

$$\psi_B = \frac{N}{N} \sum_{i=1}^{N} \arccos \left( \hat{B}_i \cdot \hat{B}_{02} \right) / N,$$

(17)

where $N$ is the number of spacecraft rotations in a single measurement, $\hat{B}_{02}$ is the average magnetic field direction over the whole measurement interval, and $\hat{B}_i$ is the magnetic field unit vector averaged over each 3 s rotation. A large $\psi_B$ can lead to the blurring of anisotropies in the proton thermal speeds. In other words, the fluctuations in B over the integration time result in a broadening of the reduced VDFs, increasing uncertainty in these measurements (e.g., see Verscharen & Marsch 2011). To reduce this blurring effect, we remove SWE measurements with angular deviations $\psi_B > 15^\circ$. Maruca (2012) provides an alternative dataset of proton moments from SWE measurements to account for large deviations in the instantaneous magnetic field, using an average $\hat{B}_0$ over each voltage window scan (i.e., one rotation of the spacecraft, $-3$ s) to calculate $w_{p,\perp}$ and $w_{p,\parallel}$. Maruca & Kasper (2013) show that the Kasper (2002) dataset often underestimates the temperature anisotropy of proton VDFs. Our comparison with this alternative dataset (not shown here) reveals that both $T_{p,\perp}/T_{p,\parallel}$ and $T_{p,\parallel}/T_{p,\perp}$ show a similar, albeit slightly reduced, dependence on both $\beta_p$ and $\theta_{pB}$. This result suggests that the temperature dependence we see in the $\beta_p$-$\theta_{pB}$ plane is unlikely caused by the blurring of proton temperature anisotropy measurements.

7.2 CGL Spherical Expansion

Another possible source of proton temperature dependence on $\theta_{pB}$ is the expansion of the solar wind as it flows out into the heliosphere. The double adiabatic closure presented by Chew et al. (1956) predicts the evolution of $T_{p,\perp}$ and $T_{p,\parallel}$ assuming no collisions, negligible heat flux, and no local heating:

$$\frac{d}{dt} \left( \frac{T_{p,\perp}}{B} \right) = 0 \quad \text{and} \quad \frac{d}{dt} \left( \frac{T_{p,\parallel}B^2}{n_p} \right) = 0,$$

(18)

where $d/dt$ is the convective derivative. Under the assumption of steady-state spherical expansion, which is purely transverse to the radial direction with a constant radial velocity, $v_{sw} = \hat{r}R$, the continuity equation gives $n_p \propto 1/r^2$, where $r$ is the radial distance from the Sun. We assume that the radial evolution of the magnetic field in the equatorial plane follows the Parker spiral (Parker 1958),

$$B \propto \frac{\cos^2 \phi_0 + r^2 \sin^2 \phi_0}{r^2},$$

(19)

which gives a radial dependence of $B \propto 1/r^2$ when $\phi_0 = 0^\circ$ and $B \propto 1/r$ when $\phi_0 = 90^\circ$. Here, $\tan \phi = B_\parallel/B_r$ is the azimuthal angle of B in the equatorial plane and is a function of r. The initial value of this angle, $\phi_0$, at a distance $r_0$ is:

$$\phi_0 = \arctan \left( \frac{\Omega_\odot}{v_{sw}} \right),$$

(20)

where $\Omega_\odot = 2.85 \times 10^{-6}$ rad/s is the constant solar angular rotation rate. Therefore, a value of $v_{sw}$ sets the value of $\phi_0$ at a given radius, $r_0$. The two angles are related by $\tan \phi = R \tan \phi_0$, where $R = r/r_0$. From Equations 18, 19 and the radial dependence of $n_p$, we obtain:

$$\frac{T_{p,\perp}}{T_{\perp,0}} = \frac{\cos^2 (\phi_0) + R^2 \sin^2 (\phi_0)}{R^2},$$

(21)

and

$$\frac{T_{p,\parallel}}{T_{\parallel,0}} = \frac{1}{\cos^2 (\phi_0) + R^2 \sin^2 (\phi_0)},$$

(22)

where $T_{\perp,0}$ and $T_{\parallel,0}$ are the perpendicular and parallel proton temperatures at $r_0$, respectively. We use Equations 21 and 22 to investigate the dependence of proton temperature on $\phi$ at $r = 215R_\odot \approx 1$ au. Since the solar wind velocity is radial, the angle $\phi$ is approximately equal to $\theta_{pB}$. We set $T_{\perp,0} = 10$, $T_{\parallel,0} = 1$, and $r_0 = 20R_\odot$, giving $R = 10.75$. We create a distribution of angles $\phi_0$ using Equation 20 by selecting a range of wind speeds: $100 \lesssim v_{sw} \lesssim 1000$ km/s. This range of $\phi_0$ gives $20^\circ \lesssim \phi \lesssim 80^\circ$ at 1 au. In Figure 6, we show the variation of $T_{p,\perp}$ and $T_{p,\parallel}$ with $\phi$. We choose a larger $T_{\perp,0}$ to show more clearly the variation in $T_{p,\perp}$. We see that there is a lack of cooling for $T_{p,\parallel}$ at small $\phi$, which decreases towards zero at $\phi \gtrsim 70^\circ$. On the other hand, $T_{p,\perp}$ is largest at $\phi \gtrsim 60^\circ$ and approaches 0.1 for $\phi \lesssim 30^\circ$. This dependence of $T_{p,\perp}$ and $T_{p,\parallel}$ is opposite to what we observe in Figure 4, which in general shows larger $T_{p,\perp}$ at $\theta_{pB} \approx 90^\circ$ and larger $T_{p,\perp}$ at $\theta_{pB} \approx 0^\circ$. Therefore, spherical expansion alone cannot explain our results.

MNRAS 000, 1–12 (2020)
8 CONCLUSIONS

We use magnetic helicity to investigate the polarisation properties of Alfvénic fluctuations with finite $k_{r}$ at proton-kinetic scales in the solar wind. Using almost 15 years of Wind observations, we separate the contributions to helicity from fluctuations with wave-vectors quasi-parallel and oblique to $B_{0}$, finding that the helicity of Alfvénic fluctuations is consistent with predictions from linear Vlasov theory. In particular, the peak in magnetic helicity signature at proton-kinetic scales shown in Figure 3 and its variation with $\beta_{p}$ and $\theta_{vB}$ are in agreement with the dispersion relation of linear Alfvén waves (Gary 1986). This result suggests that the non-linear turbulent fluctuations at these scales share at least some polarisation properties with Alfvén waves.

We also investigate the dependence of proton temperature in the $\beta_{p}$-$\theta_{vB}$ plane to probe whether the turbulence is truly independent of $\theta_{vB}$. In Figure 4, we find that both $T_{p,\perp}$ and $T_{p,\parallel}$, when normalised to their average value in each $\beta_{p}$-bin, show a clear dependence on $\theta_{vB}$. The temperature parallel to $B_{0}$ is generally higher in the parameter-space where we measure a coherent helicity signature associated with KAW-like fluctuations, and perpendicular temperature higher in the parameter-space where we measure a signature arising from AIC waves. We also see small enhancements in the perpendicular temperature where we measure the strongest helicity signal of KAW-like fluctuations. However, we note that a lack of a wave signal is not the same as a lack of presence of waves. These findings are inconsistent with the general assumption that solar wind fluctuations are ergodic in the sense that sampling different $\theta_{vB}$ allows us to sample different parts of turbulence that is otherwise unaltered in its statistical properties. Therefore, studies that sample different $\theta_{vB}$ in order to sample different $\theta_{vB}$ need to be interpreted very carefully.

Our results suggest that the nature of fluctuations at proton-kinetic scales in the solar wind depends on the angle $\theta_{vB}$. We therefore interpret that the dissipation mechanisms and proton heating also depend on $\theta_{vB}$, leading to the enhancements in proton temperature we observe in Figure 4. Our results are consistent with the role of wave-particle interactions in determining proton temperature at a fixed distance from the Sun. For example, whenever we measure the helicity of AIC waves or KAWs, then we also measure enhancements in proton temperature. However, the inverse is not necessarily true. We suggest that heating mechanisms associated with KAWs lead to both parallel (Howes 2008; Schekochihin et al. 2009) and perpendicular (Chandran et al. 2010, 2013) heating. We rule out both instrumental and large-scale expansion effects, finding that neither of them alone can explain the observed temperature profile in the $\beta_{p}$-$\theta_{vB}$ plane. In summary, our results show that the properties of Alfvénic fluctuations with $k_{r} \neq 0$ depend on $\theta_{vB}$ (and $\beta_{p}$), suggested by a dependence of the proton temperature on $\theta_{vB}$. This observation is consistent with localised heating in the solar wind and suggests that the properties of these fluctuations at proton-kinetic scales determine the level of proton heating. Therefore, we provide new evidence for the importance of local kinetic processes in determining proton temperature in the solar wind. We emphasise that our conclusions do not invoke causality, just correlation. For example, we cannot rule out a lack of cooling rather than heating. However, while the adiabatic expansion of the solar wind will cause the temperature to vary with $\theta_{vB}$, this cannot explain the observed temperature profiles in the $\beta_{p}$-$\theta_{vB}$ plane. Further work is ongoing in order to confirm these results and develop a theory for the processes associated with the polarisation properties of Alfvénic fluctuations that lead to the observed temperature profiles.

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APPENDIX A: DECOMPOSITION OF FLUCTUATING MAGNETIC HELICITY

Previous studies have calculated the total reduced fluctuating helicity, $\sigma_m(k_r)$, as a function of $\theta_B$, showing a broad right-handed signature at oblique angles and a narrow left-handed signature at quasi-parallel angles (He et al. 2011, 2012a,b; Podesta & Gary 2011a; Klein et al. 2014a; Bruno & Telloni 2015; Telloni et al. 2015). These signatures are associated with KAW-like fluctuations from the turbulent cascade and proton-kinetic instabilities, respectively (Telloni & Bruno 2016; Woodham et al. 2019). In Figure A1, we plot the median value of the peak in $\sigma_m^l(k_r)$ across the $\beta_B-\theta_B$ plane, showing two helicity signatures of opposite handedness. Here we present a mathematical derivation that decomposes $\sigma_m^l(k_r)$ into the different contributions, $\sigma_l(k_r)$, calculated using Equation 13 (see also Wicks et al. 2012). This technique allows us to separate the helicity signatures of different fluctuations at proton-kinetic scales in the solar wind, as we show in Figure 3.

Let us consider that a spacecraft samples a single wave-mode with wave-vector,

$$\mathbf{k} = k_\perp \cos \alpha \hat{x} + k_\perp \sin \alpha \hat{y} + k_\parallel \hat{z},$$  \hspace{1cm} (A1)

where $\alpha$ is the azimuthal angle of $\mathbf{k}$ in the $x$-$y$ plane. The full signal from turbulence will simply be the linear superposition of the signals from the each of the wave-modes, so considering a single wave-mode is sufficient to understand how the components $\sigma_l(k_r)$ are related to $\sigma_m^l(k_r)$. Without loss of generality, we take the solar wind velocity to be in the $x$-$z$ plane,

$$\mathbf{v}_{SW} = v_{SW} \sin \theta_B \hat{x} + v_{SW} \cos \theta_B \hat{z},$$  \hspace{1cm} (A2)

and the local mean magnetic field to be $\mathbf{B}_0 = B_0 \hat{z}$. We can use the relation, $2 \Im \{a^* b\} = i (a^* b - a b^*)$, to rewrite $H_m^\prime(k_r)$ in RTN coordinates from Equation 7 into the form:

$$H_m^\prime(k_r) = i \frac{\delta B_T \delta B_N - \delta B_r \delta B_N}{k_r},$$  \hspace{1cm} (A3)

The normalised reduced fluctuating magnetic helicity, $\sigma_m^l(k_r)$, is then given by Equation 11. We define the relation between the RTN and the field-aligned (Equation 12) coordinate systems using the unit vector along the sampling direction, $\hat{\chi}_{SW} = \mathbf{v}_{SW}/|\mathbf{v}_{SW}|$.

$$\hat{R} = \hat{\chi}_{SW} = \sin \theta_B \hat{x} + \cos \theta_B \hat{z},$$
$$\hat{T} = \hat{B}_0 \times \hat{\chi}_{SW} / |\hat{B}_0 \times \hat{\chi}_{SW}| = \hat{y},$$
$$\hat{N} = \hat{R} \times \hat{T} = -\cos \theta_B \hat{x} + \sin \theta_B \hat{z},$$  \hspace{1cm} (A4)
and the non-reduced fluctuating magnetic helicity (Equation 5) as:

\[
H'_m(k) = \frac{\delta B_x \delta B_y - \delta B_x \delta B_y}{k_z} = \frac{\delta B_y \delta B_x - \delta B_y \delta B_x}{k_y} = \frac{\delta B_x \delta B_y - \delta B_x \delta B_y}{k_y}.
\]

We can equate each of the terms between the two forms in Equation A6 to obtain the following direct relations between \(\sigma_{xy}^m(k_x)\) and \(\sigma_{xy}(k_x)\), \(\sigma_{xz}(k_y)\) and \(\sigma_{yz}(k_y)\):

\[
\sigma_{xy}(k_x) = \frac{H'_m(k_x)}{|\delta B(k_x)|^2} k_x = \sigma_{xy}^m(k_x) \frac{k_z}{k_y}, \quad (A8)
\]

\[
\sigma_{xz}(k_y) = -\frac{H'_m(k_x)}{|\delta B(k_x)|^2} k_y = -\sigma_{zm}^m(k_x) \frac{k_y}{k_x}, \quad (A9)
\]

\[
\sigma_{yz}(k_x) = H'_m(k_x) \frac{H'_m(k_x)}{|\delta B(k_x)|^2} k_x = \sigma_{zm}^m(k_x) \frac{k_z}{k_y}. \quad (A10)
\]

As the solar wind velocity is confined to the \(x-z\) plane, we have no information about \(k_y\), so the contribution \(\sigma_{xz}(k_y)\) is not useful in a practical sense. These relations show clearly that \(\sigma_{xy}(k_x)\) will dominate when \(k_x \gg k_z\), and so we denote \(\sigma_{xy} \equiv \max_{k_x} \sigma_{xy}(k_x)\) to diagnose the helicity of the modes with nearly parallel wavevectors. Similarly, \(\sigma_{yz}(k_x)\) will dominate when \(k_y \gg k_x\), i.e., for modes that have \(k\) at large angles relative to the direction of \(B_0\). Since the anisotropic Alfvénic turbulent cascade leads to the generation of such nearly perpendicular wavevectors, we denote \(\sigma_{yz} \equiv \max_{k_x} \sigma_{yz}(k_x)\) to diagnose the helicity of the modes with \(k_y \gg k_x\).

To highlight the separation of different fluctuations in the solar wind using this technique, we show in Figure A2(a) a time series of magnetic helicity spectra, \(\sigma_{zm}^m\), measured by Wind on 01/07/2012. We plot the spectra as a function of frequency in the spacecraft frame, \(f_{sc} = \omega_{sc}/2\pi\). In panels (b)-(d), we also plot the three components \(\sigma_{ij}\), showing the decomposition of \(\sigma_{zm}^m\) into its different components. We can see that the two coherent signatures of opposite handedness at \(f_{sc} \approx 1\) Hz in panel (a) are completely separated into the components \(\sigma_{xy}\) and \(\sigma_{yz}\) in panels (b) and (c). In panel (d), we see only small enhancements close to 0.33 Hz, which corresponds to the spin frequency of the spacecraft. Besides this spacecraft artefact, there is no coherent helicity signature in \(\sigma_{xz}\), as expected.