Combined explanations of muon $g-2$ and $R_{K,K^*}$ anomalies in left-right model with inverse seesaw

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(Dated: July 14, 2022)

We investigate the possibility of combining explanations for the muon anomalous magnetic moment $g_\mu - 2$ and observations of $B$-physics anomalies in $R_{K,K^*}$ ratios in a left-right model with an inverse seesaw mechanism. We emphasize that the observed deviation from the Standard Model predictions can be accommodated in a large part of the parameter space of this class of models, where loops with massive neutrinos and charged Higgs as well as $W$-boson contribute significantly to both $g_\mu - 2$ and $R_{K,K^*}$. Stringent constraints due to lepton flavor violation $\mu \to e\gamma$ and the electron anomalous magnetic moment $g_e - 2$ are considered, and the results are compatible.

I. INTRODUCTION

Non-vanishing neutrino masses inferred from neutrino oscillation experiments$^{[1,5]}$, provided strong evidence for New Physics (NP) Beyond the Standard Model (BSM). The extension of the SM to account for neutrino masses and mixing implies a new source of lepton flavor violation, which could explain the long-standing discrepancy between the SM prediction for the muon anomalous magnetic moment $a_\mu$ and its experimental measurement, in addition to the recent $B$-meson anomalies, such as $R_K$ and $R_{K^*}$.

Recent experimental results indicate a possible 3.1$\sigma$ difference between the measured value of the anomalous magnetic moments of muons $a_\mu$ and the SM expectations$^{[4,9]}$, namely

$$\delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (2.78 \pm 0.88) \times 10^{-9}. \quad (1)$$

Furthermore, the LHCb Collaboration revealed intriguing results for the ratios $R_K = \frac{\text{BR}(B^+ \to K^+\mu^+\mu^-)}{\text{BR}(B^+ \to K^+e^+e^-)}$ and $R_{K^*} = \frac{\text{BR}(B^{0} \to K^{*0}\mu^+\mu^-)}{\text{BR}(B^{0} \to K^{*0}e^+e^-)}$. It has been reported that $R_K$ and $R_{K^*}$ are given for two dilepton invariant mass-squared bins by$^{[10,11]}

$$R_K = 0.846^{+0.060+0.016}_{-0.054-0.014} \quad \text{for} \quad 1.1 \text{GeV}^2 \leq q^2 \leq 6 \text{GeV}^2, \quad (2)$$

$$R_{K^*} = \begin{cases} 0.66^{+0.11}_{-0.07} \pm 0.03 & \text{for} \quad 0.045 \text{GeV}^2 \leq q^2 \leq 1.1 \text{GeV}^2 \\ 0.69^{+0.11}_{-0.07} \pm 0.05 & \text{for} \quad 1.1 \text{GeV}^2 \leq q^2 \leq 6 \text{GeV}^2 \end{cases} \quad (3)$$

These measurements deviate from the SM expectations, $R_{K,K^*} \simeq 1$$^{[12]}$, by about 2.5$\sigma$.

We consider the combined explanation of both $a_\mu$ and the recent $R_K$ and $R_{K^*}$ anomalies in the Left-Right (LR) Model with Inverse Seesaw mechanism (LRIS) to generate light neutrino masses and mixing at low energy scale. The salient feature of this class of models is the large neutrino Yukawa couplings, which allow for significant non-universal leptonic contributions to the $a_\mu$ and $b \to s\ell^+\ell^-$ transition, which can then account for both $R_K$ and $R_{K^*}$ anomalies via box diagrams mediated by charged Higgs bosons and right-handed neutrinos. As constraints, we impose the experimental limits of the lepton flavor violation $\mu \to e\gamma$$^{[13,14]}$ and the electron anomalous magnetic moment $g_e - 2$ as well as providing an elegant explanation for the origin of parity violation.

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in low-energy weak interactions. The LRIS that has been analyzed in Ref. [15] has a Higgs sector consisting of one scalar bidoublet and a scalar right-handed doublet only, in addition to singlet fermions that couple with right-handed neutrinos and have a small mass \( \sim O(1) \) KeV, generated radiatively. Such TeV scale LR model can be probed in current and future experiments as emphasized in Ref. [15]. Also, it was argued in Ref. [16] that the tension between the SM prediction and the experimental results of the \( R_D \) and \( R_{D^*} \) ratios, defined by \( B(\{D^*, D\}) = \frac{BR(B \rightarrow \{D^*, D\} \nu)}{BR(B \rightarrow D^{(*)} \nu)} \), where \( l = e, \mu \), can be resolved in this class of LRIS models. In fact there are several new physics scenarios have been proposed to accommodate \( \delta a_\mu, \delta a_e, R_K \) and \( K^{\ast} \) results. See Ref. [17-51].

This paper is organized as follows. In Section II we highlight relevant interactions in the LRIS, as the details of the model are given in previous papers [15, 16]. Section III is devoted for analyzing new LRIS contributions to \( b \rightarrow \nu \nu \) transitions. The analytic expressions of the Wilson coefficients due to one loop box diagrams generated by charged Higgs, \( Z, W, Z' \) gauge bosons and neutral and charged Higgs bosons. In Section IV we present the effective Hamiltonian describing \( b \rightarrow s \ell^+ \ell^- \) transition. The neutrino mass matrix \( M \) is recovered by means of low-energy weak interactions. The LRIS that has been analyzed in Ref. [15] has a Higgs sector consisting of one right-handed doublet \( \chi_R \) that breaks down left-right symmetry to the SM gauge symmetry and one bidoublet \( \phi \) that is broken down into two SM Higgs doublets. Furthermore, a \( Z_2 \) discrete symmetry is assumed, with all particles having even charges except \( S_1 \) which has an odd charge. This symmetry prevents the mixing mass term \( M S_1^2 S_2 \) from being used to allow for the IS mechanism.

The most general LRIS Yukawa Lagrangian is given by

\[
\mathcal{L}_Y = \sum_{i,j=1}^3 \tilde{L}_i (\phi y_{ij}^L + \tilde{\phi} y_{ij}^R) L_{Rj} + \tilde{Q}_i (\phi y_{ij}^Q + \tilde{\phi} y_{ij}^Q) Q_{Rj} + \tilde{L}_i \tilde{\chi}_R y_{ij}^s S_{2j}^c + \text{h.c.},
\]

where \( i, j = 1 \ldots 3 \) are family indices, \( \tilde{\phi} \) is the dual bidoublet of the scalar bidoublet \( \phi \), defined as \( \tilde{\phi} = \tau_2 \phi^* \tau_2 \), and \( \tilde{\chi}_R \) is the dual doublet of the scalar doublet \( \chi_R \), given by \( \chi_R = i \tau_2 \chi_R' \). A non-vanishing VEV of \( \chi_R, \langle \chi_R \rangle = \frac{v_R}{\sqrt{2}} \) breaks the \( B-L \) anomaly caused by \( S_1 \). The LRIS has a simple Higgs sector consisting of one right-handed doublet \( \chi_R \) that breaks down left-right symmetry to the SM gauge symmetry and one bidoublet \( \phi \) that is broken down into two SM Higgs doublets. Furthermore, a \( Z_2 \) discrete symmetry is assumed, with all particles having even charges except \( S_1 \), which has an odd charge. This symmetry prevents the mixing mass term \( M S_1^2 S_2 \) from being used to allow for the IS mechanism.

After \( B-L \) symmetry breaking and EW symmetry breaking, the following 9 \times 9 neutrino mass matrix is obtained in the basis \((\nu_L^e, \nu_R, S_2)\)

\[
M_\nu = \begin{pmatrix}
0 & M_D & 0 \\
M_D^T & 0 & M_R \\
0 & M_R^T & \mu_s
\end{pmatrix},
\]

where the 3 \times 3 matrix \( M_D = v (y^L s_\beta + \tilde{y}^L c_\beta)/\sqrt{2} \) is the Dirac neutrino mass matrix and the 3 \times 3 matrix \( M_R = y^R v_R/\sqrt{2} \). Here, we have assume that \( k_1 = v c_\beta, \ k_2 = v s_\beta \), where \( v = 246 \) GeV is the electroweak VEV, and \( s_\beta = \sin \beta, \ c_\beta = \cos \beta \) and \( t_\beta = \tan \beta \), henceforth. The neutrino mass matrix \( M_\nu \) can be diagonalized by 9 \times 9 matrix \( U \) satisfying \( U^T M_\nu U = M_\nu^{\text{diag}} \), yielding the physical light and heavy neutrino states \( \nu_{l_i}, \nu_{h_j}, \ i = 1 \ldots 3, \ j = 1 \ldots 6, \)
with the following follows light and heavy mass eigenvalues
\[ m_{\nu_i} = M_D M_R^{-1} \mu_s (M_R^T)^{-1} M_D^T, \quad i = 1 \ldots 3, \]
\[ m_{\nu_j}^2 = M_R^2 + M_D^2, \quad j = 1 \ldots 6. \]

where the \( \mu_s \lesssim O(10^{-5}) \) GeV, \( M_R \sim O(\text{few}) \) TeV and \( y' \sim O(10^{-1}) \). For these values, Eq. \(6\) shows that the light neutrino masses can be of order eV. The inverse relation of Eq. \(6\) is \( M_D = U_{\nu M S N S} \sqrt{m_{\nu i} R} (M_R^T)^{-1} M_D \), where \( R \) is an orthogonal matrix and \( U_{\nu M S N S} \) is the 3 \times 3 light neutrinos mixing matrix \[53, 54].

In the following two sections, we will study processes \((a_\mu \text{ and } R_{K,K^*})\), which are dominated by charged Higgs contributions at the loop level; thus, we provide a brief analysis for charged Higgs masses and interactions based on Ref. \[15\]. In the flavor basis \((\phi_1^+, \phi_2^+, \chi_R^+)\), the charged Higgs bosons symmetric mass matrix takes the form
\[ M_{H^\pm}^2 = \frac{\alpha_{32}}{2} \begin{pmatrix} \frac{\nu c_2}{v c_2 + v' c_2} & \frac{\nu c_2}{v c_2 + v' c_2} & \frac{\nu c_2}{v c_2 + v' c_2} \\ \frac{\nu c_2}{v c_2 + v' c_2} & -v \nu R c_\beta & -v \nu R c_\beta \\ \frac{\nu c_2}{v c_2 + v' c_2} & -v \nu R c_\beta & -v \nu R c_\beta \end{pmatrix} \]  

This matrix can be diagonalized by the unitary matrix,
\[ Z^{H^\pm} = \begin{pmatrix} \frac{\nu c_2}{v c_2 + v' c_2} & 0 & \frac{\nu c_2}{v c_2 + v' c_2} \\ \frac{\nu c_2}{v c_2 + v' c_2} & \frac{\nu c_2}{v c_2 + v' c_2} & \frac{\nu c_2}{v c_2 + v' c_2} \\ \frac{\nu c_2}{v c_2 + v' c_2} & \frac{\nu c_2}{v c_2 + v' c_2} & \frac{\nu c_2}{v c_2 + v' c_2} \end{pmatrix} \]  

Thus, the mass eigenstates are given by \((\phi_1^+, \phi_2^+, \chi_R^+)^T = (Z^{H^\pm})^T (G_1^+, G_2^+, H^+)^T\) where \(Z^{H^\pm} M_{H^\pm}^2 (Z^{H^\pm})^T = \text{diag}(0, 0, m_{H^\pm}^2)\). Here \(G_1^+\) and \(G_2^+\) represent the Goldstone massless charged bosons which are eaten by the charged gauge bosons \(W_\mu\) and \(W_\mu\) to acquire their masses and \(H^+\) is the physical massive charged Higgs boson. The charged Higgs mass \(m_{H^\pm}\) is given by
\[ m_{H^\pm}^2 = \frac{\alpha_{32}}{2} \left( \frac{\nu^2}{c_2} + v^2 c_2 \right), \]

where \(\alpha_{32}\) is defined in terms of the scalar potential parameters \(\alpha_3\) and \(\alpha_2\) \[15\] as \(\alpha_{32} = \alpha_3 - \alpha_2\). We notice from Eq. \[10\] that \(\alpha_{32} > 0\) as long as \(c_{2\beta} > 0\) (\(t_\beta < 1\)) and vice versa. Moreover, for \(v_R \sim O(\text{TeV})\) and \(\alpha_{32} \sim O(10^{-2})\), the charged Higgs boson mass can be of order hundreds GeV. The physical charged Higgs boson is defined as a linear combination of the flavor basis fields \(\phi_1^+, \phi_2^+, \chi_R^+, \text{ i.e. } (\text{corrected from } [15]),\)
\[ H^\pm = Z_{31}^{H^\pm} \phi_1^+ + Z_{32}^{H^\pm} \phi_2^+ + Z_{33}^{H^\pm} \chi_R^+. \]

It is worth noting that for \(v_R \gg v, v_R \sim O(\text{TeV})\) is enough, one gets \(Z_{33}^{H^\pm} \ll 1\) and
\[ H^\pm \approx -(s_\beta \phi_1^+ + c_\beta \phi_2^+). \]

Finally, the relevant charged Higgs couplings with fermions families \(i, j\) are given by
\[ \Gamma_{u_{i,j}}^{H^\pm} = -\left( \sum_{a=1}^{3} V_{j,a}^* (y_{u,a}^{Q} Z_{32}^{H^\pm} + \tilde{y}_{u,a}^{Q} Z_{31}^{H^\pm}) \right) P_L - \left( \sum_{a=1}^{3} V_{j,a} (y_{u,a}^{Q} Z_{32}^{H^\pm} + \tilde{y}_{u,a}^{Q} Z_{31}^{H^\pm}) \right) P_R \]
\[ \Gamma_{d_{i,j}}^{H^\pm} = -\left( \sum_{b=1}^{3} U_{i,b}^* (y_{d,b}^{L} Z_{32}^{H^\pm} + \tilde{y}_{d,b}^{L} Z_{31}^{H^\pm}) \right) P_L - \left( \sum_{b=1}^{3} U_{i,b} (y_{d,b}^{L} Z_{32}^{H^\pm} + \tilde{y}_{d,b}^{L} Z_{31}^{H^\pm}) \right) P_R \]
the electron anomalous magnetic moment, \( a_e \), and the charged Higgs boson \( H^\pm \). We can write

\[
\delta a = a^{\text{LRIS}} + a^{W,W'} + a^Z + a^{H^\pm}.
\]

FIG. 1. LRIS one-loop Feynman diagrams contributions to lepton \( g_\ell - 2 \) via massive neutrinos, \( V^\pm = W, W' \), \( V^0 = Z, Z' \), \( S^0 = h, A \) and the charged Higgs boson \( H^\pm \).

III. LRIS CONTRIBUTIONS TO MUON ANOMALOUS MAGNETIC MOMENT

In this section we analyze new contributions from the LRIS to the muon anomalous magnetic moment, \( a_\mu \), induced by the light and heavy \( Z, W, Z', W' \) gauge bosons, as well as the neutral scalar and pseudoscalar and charged Higgs bosons \( h, A, H^\pm \). We will also consider the constraints on these contributions imposed by the experimental limits of the electron anomalous magnetic moment, \( a_e \), and charged lepton flavor violations, particularly \( \mu \rightarrow e\gamma \). In this case, we can write \( \delta a_\mu = a_\mu^{\text{LRIS}} \), where

\[
a_\mu^{\text{LRIS}} = a_\mu^W + a_\mu^{W'} + a_\mu^Z + a_\mu^A + a_\mu^H + a_\mu^{H^\pm}.
\]

The contributions of \( W, W' \) and \( H^\pm \) with light and heavy neutrinos to \( a_\mu \) are shown in Fig. 1. We exclude the minor contributions caused by neutral Higgs bosons, \( Z \) and \( Z' \) gauge bosons mediations. The relevant amplitudes are calculated and are given by, \( W - W' \) mixing is ignored:

\[
\begin{align*}
 a_\mu^W &= G_F^\ell \sum_{k=1}^9 |U_{k,|\ell|}|^2 \left( \frac{10}{3} + \mathcal{F}_2(x_{W^*}) \right), \\
 a_\mu^{W'} &= G_F^\ell x_{W^*} \frac{1}{c_w} \sum_{k=1}^9 |U_{k,3+|\ell|}|^2 \left( \frac{10}{3} + \mathcal{F}_2(x_{W'^*}) \right), \\
 a_\mu^Z &= G_F^\ell \frac{1}{3} (c_{4W} - 5), \\
 a_\mu^{Z'} &= G_F^\ell x_{Z^*} \left( \frac{1}{48s_w^2} \right) (c_{4W'} - 12c_{2W'} - 5), \\
 a_\mu^A &= G_F^\ell x_{A} \left( \frac{1}{2} \right) \left( \frac{7}{6} + \log(x_{A^*}) \right), \\
 a_\mu^{H}\end{align*}
\]

where the lepton family order \( |\ell| = 1, 2 \) for \( \ell = e, \mu \). The dimensionless coupling \( G_F^\ell = \frac{G_F m_\ell^2}{8\sqrt{2}\pi^2} \) and the mass ratio parameters \( x_b^\ell = \frac{m_b^2}{m_\ell^2} \), \( a, b = W, W', Z, Z', h_i, A, H^\pm, \ell, u_i \) where \( u_i \)'s are the up-type quarks and their contributions in \( R_{K^0} \) will appear in the next section. The neutral gauge bosons mixing angles \( \theta_{w'} \) and the Weinberg angle \( \theta_w \) are \( s_{w'} = \frac{g'}{g}, s_w = \frac{e}{g} \), where \( g' \) is the hypercharge coupling. The \( Z - Z' \) mixing angle \( \theta_{w'} \) is contrained by \( \theta_{w'} < 10^{-3} \) \( [57, 58] \). Also, the \( W' \) mass is given by \( m_{W'} = g_2 \sqrt{v^2 + v^2/2} \gtrsim \mathcal{O}(4) \) TeV \( [57, 58] \). The scalar and pseudoscalar Higgs couplings with charged leptons are

\[
\begin{align*}
 \Gamma_{h\ell} &= \frac{v}{\sqrt{2}m_\ell} \left( Z_{12} y_{\mu,\ell}^L + Z_{12} y_{e,\ell}^L \right), \\
 \Gamma_{A\ell} &= \frac{v}{\sqrt{2}m_\ell} \left( -Z_{32} y_{\mu,\ell}^L + Z_{32} y_{e,\ell}^L \right),
\end{align*}
\]
where $Z^H$, $Z^A$ are the scalar and pseudo-scalar Higgs mixing matrices \[15, 59\].

Finally, the charged Higgs $H^\pm$ contribution is given by

$$a^H_{\ell} = G_F^\ell \xi_W \left( \frac{1}{12 s_w^2} \sum_{k=1}^{9} \left| \xi_{\nu_\ell, k} \right|^2 F_2(x^\nu_{H^\pm}) + 2 \text{Re}[\xi_{\nu_\ell, k} \xi^*_{k, \ell}] F_1(x^\nu_{H^\pm}) \right),$$

(24)

where the dimensionless couplings

$$\xi_{k, \ell} = \frac{v}{m_\ell} \left( Z^H_{31} \sum_{i=1}^{3} U_{k, i + 3 y_{\ell, i}^L} - Z^H_{32} \sum_{i=1}^{3} U_{k, i + 3 y_{\ell, i}^L}^* \right),$$

(25)

$$\xi_{\nu_\ell, k} = \frac{v}{m_{\nu_\ell}} \left( Z^H_{32} \sum_{i=1}^{3} U_{k, i} y_{\ell, i}^L - Z^H_{31} \sum_{i=1}^{3} U_{k, i} y_{\ell, i}^L + Z^H_{33} \sum_{i=1}^{3} U_{k, i + 6 y_{\ell, i}^L} \right),$$

(26)

$$\xi_W = \frac{v}{M_W} \left( (Z^H_{31})^2 + (Z^H_{32})^2 (g_{21} U^\gamma Z Z' + g_R U^R_{31} Z Z') + (Z^H_{33})^2 (g_{BL} U^\gamma_{11} Z Z' + g_R U^\gamma_{31} Z Z') \right),$$

(27)

where $U^\gamma Z Z'$ is the neutral gauge bosons mixing matrix and $g_R$ is the $SU(2)_R$ coupling \[15\]. The loop functions $F_k$ ($k = 1, 2$) in Eqs. \[16, 17\] and \[24\] are given by

$$F_k(y) = \frac{y P_k(y)}{(y-1)^{k+1}} - \frac{6 y^{k+1} \log(y)}{(y-1)^{k+2}}, \quad k = 1, 2,$$

(28)

$$P_1(y) = 3 y + 3,$$

(29)

$$P_2(y) = 2 y^2 + 5 y - 1.$$  

(30)

It is understood that for $y \to 1$, the values of the loop functions $F_k$ ($k = 1, 2$) are given by their limits and $F_1(1) = 1$ and $F_2(1) = \frac{1}{2}$. This happens when some heavy neutrinos are degenerate in mass with the charged Higgs boson as in Fig. 2, Fig. 3 and Fig. 5 below.

The charged Higgs also contribute to the $\text{BR}(\mu \to e\gamma)$. The stringent experimental limits on this decay should be regarded as constraints on the charged contribution to $a_{\mu}$. The charged Higgs mediation for $\mu \to e\gamma$ leads to

$$\text{BR}(\mu \to e\gamma)_{\text{LRIS}} = \frac{\alpha_s^3 s_w^2 m_\mu}{256 \pi^2 \Gamma_\mu} |x^\mu_{\nu_\ell}|^2 \sum_{k=1}^{9} |\xi_{\nu_\ell, k} \xi_{k, \mu} F_2(x^\nu_{H^\pm}) + (\xi_{\nu_\ell, k} \xi_{k, \mu} + \xi_{k, \mu} \xi^*_{\nu_\ell, k}) F_1(x^\nu_{H^\pm})|^2.$$  

(31)

Unlike the SM where all particles acquire their masses via the VEV of only one degree of freedom, in LRIS, as in any other left-right model, there are many sources of particles masses. In the SM, couplings are thus fixed by
particle masses. But in LRIS, gauge couplings, Yukawa elements and scalar potential parameters are in general free parameters. So, VEVs can be varied while particle masses are kept fixed. Neutrino masses and their mixing matrix $U_{\text{PMNS}}$ are fixed in terms of Yukawa couplings $y^L, \bar{y}^L$ and $y^s$ and the electroweak VEVs ratio $t_\beta$ and $v_R$ of Eq. (4). Also the charged Higgs mass is varied versus $t_\beta$ and the scalar potential parameter $\alpha_{32}$ as in Eq. (10). The charged Higgs mixing is given in Eq. (9) in terms of $t_\beta$ and $v_R$.

Fig. 2 (left) depicts the muon (electron) $g_{\mu(e)} = 2$ anomalies $\delta a_{\mu(e)}$ in LRIS, as given in Eq. (16)–Eq. (24), resulting from the charged Higgs contribution versus the heavy neutrino-charged Higgs mass ratio. The green (red) borders indicate the 1σ (2σ) level of accuracy around the average $a_\mu$. The electron anomaly $a_e$ is guaranteed to be within the allowed uncertainty limit. Furthermore, Fig. 3 shows that the leptonic violating process $\text{BR}(\mu \rightarrow e\gamma)$ in Eq. (31) satisfies the upper bound for the same set of parameter values as in Fig. 2. For light neutrinos, Eq. (28) shows that the loop functions $F_k(x_{H^\pm}^p)$ ~ 0 ($k = 1, 2, i = 1 \ldots 3$) and thus their contributions in Eq. (24) vanish. It is worth noting that the two Yukawa matrices $y^Q, \bar{y}^Q$ are obtained inversely in terms of the quark masses and $t_\beta$. Similarly, $y^L, \bar{y}^L$ are given in terms of charged lepton masses and light neutrino masses $m_{\nu_{li}}$ ($l = 1, 2, 3$), in addition to the two $M_R (y^s)$ and $\mu^s$ free matrices. Finally, $\alpha_{32}$ determines the charged Higgs mass $m_{H^\pm}$ and $t_\beta$ controls the mixing $Z^{H^\pm}$. We scanned over the parameter space of the following ranges with $v_R \sim \mathcal{O}(6.5)$ TeV

$$\alpha_{32} \sim [-0.050, 0.005], \quad t_\beta \sim [1, 10], \quad (y^s)_{ij} \sim [0.01, 0.50] \delta_{ij}, \quad (\mu^s)_{ij} \sim [10^{-9}, 10^{-5}] \delta_{ij} \text{ GeV} \quad (32)$$

| Quantity | $\delta a_\mu$ | $-\delta a_e$ | $\text{BR}(\mu \rightarrow e\gamma)$ | $R_K$ |
|----------|----------------|---------------|--------------------------------|------|
| Value    | $2.5 \times 10^{-9}$ | $8.1 \times 10^{-17}$ | $3.4 \times 10^{-13}$ | $0.89$ |

TABLE II. Results of muon and electron $g_{\mu(e)} = 2$, $\text{BR}(\mu \rightarrow e\gamma)$ and $R_{K, K^*}$ of the BP given in Tab. III

We checked that all our benchmark points are validated to satisfy the usual HiggsBounds and HiggsSignals limits confronted with the latest LEP, Tevatron and LHC data [63, 61]. They provide important tests for compatibility of any model beyond the SM. In our analysis, the LRIS model was first built in the SARAH package, then it was passed to SPHENO [62, 63] for numerical spectrum calculations.
FIG. 4. Feynman box diagrams of LRIS contributions to $b \to s \ell^+ \ell^-$ from $W$ bosons, heavy neutrinos and charged Higgs.

IV. LRIS CONTRIBUTIONS TO $R_{K,K^*}$

As mentioned in the introduction, the LHCb collaboration recently reported hints of new physics (NP) in lepton flavor non-universal observables $R_K$ and $R_{K^*}$. The decay $B \to K/K^* \mu^+ \mu^-$ is generated at the quark level by the flavor changing neutral current (FCNC) transition $b \to s \ell^+ \ell^-$. These processes are especially intriguing because they are highly suppressed in the SM and many extensions of the SM are capable of producing measurable effects beyond the SM [64] and references therein for related studies. The effective Hamiltonian of $b \to s \ell^+ \ell^-$ transition is given by

$$H_{\text{eff}} = \sum_i (C_i(\mu_b)Q_i(\mu_b) + \tilde{C}_i(\mu_b)\tilde{Q}_i(\mu_b)) + \text{h.c.},$$

where $Q_i(\mu_b)$ are the $\Delta B = 1$ transition operators, evaluated at the renomalization scale $\mu_b \simeq O(m_b)$. Fig. 4 depicts the Feynman box diagrams that contribute to the $R_{K,K^*}$ in LRIS. The relevant operators for this process are given by

$$O_9 = (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma_\mu \ell)$$

$$O_{10} = (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma_\mu \gamma_5 \ell)$$

The operators $\tilde{Q}_i$ and Wilson coefficients $\tilde{C}_i$ are obtained from $Q_i$ and $C_i$, respectively, by replacing $L \leftrightarrow R$. The Wilson coefficients $C_i(\mu)$ at a lower scale $\mu_b = O(m_b)$ can be extrapolated by the corresponding ones at high scale $\mu_W = O(m_W)$ as

$$C_i(\mu_b) = \sum_j \hat{U}(\mu_b, \mu_W)_{ij} C_j(\mu_W),$$

where the evolution matrix $\hat{U}(\mu_b, \mu_W)$ is given in Ref. [65, 66]. Based on the numerical values of the evaluation matrix elements, we find that the Wilson coefficients $C_{9,10}$ at scale $m_b$ can be extrapolated by the corresponding ones at high scale $m_W$ [66, 67]. With the addition of New Physics (NP) effects in $b \to s \ell^+ \ell^-$, $R_K$ and $R_{K^*}$ can be written as follows [36]:

$$R_K \simeq 1 + \Delta_+,$$

$$R_{K^*} \simeq 1 + \Delta_+ + p(\Delta_- - \Delta_+),$$

where $\Delta_\pm$ are defined by

$$\Delta_\pm = \frac{2}{|C_9^{\text{SM}}|^2 + |C_{10}^{\text{SM}}|^2} \left[ \text{Re} \left( C_9^{\text{NP},\mu} (C_9^{\text{NP},\mu} + \tilde{C}_9^{\mu})^* \right) + \text{Re} \left( C_{10}^{\text{NP},\mu} (C_{10}^{\text{NP},\mu} + \tilde{C}_{10}^{\mu})^* \right) - (\mu \to e) \right].$$

The parameter $p$ is function of $q_{\text{min}}^2$ and $q_{\text{max}}^2$, such that $p(1\text{GeV}^2, 6\text{GeV}^2) \sim 0.86$ [39]. As is customary, we assume that $C_i^{\text{NP}} \ll C_i^{\text{SM}}$, so only linear terms of $C_i^{\text{NP}}/C_i^{\text{SM}}$ are retained in the expressions of $R_K$ and $R_{K^*}$. In the case of $\tilde{C}_9 = \tilde{C}_{10} = 0$, as shown below, we have $\Delta_+ = \Delta_-$ and $R_{K^*} = R_K = 1 + \Delta_+$. 

[Note: The formula for $\Delta_\pm$ is slightly misprinted in the text; the correct formula is included here.]
The $W$ contribution with massive neutrinos of Fig. 4(a) is

$$C_{10}^W = -C_{9}^W = -\frac{4G_F}{\sqrt{2}} \frac{\alpha_{\text{em}}}{4\pi} \frac{1}{s_W^2} \sum_{i=1}^{3} \sum_{j=1}^{9} V_{di} V_{sj}^* |U_{ij}|^2 \left[ \left( 1 + \frac{1}{16} x_W^i x_W^j \right) B_2(x_W^i, x_W^j) - \frac{1}{4} x_W^i x_W^j B_1(x_W^i, x_W^j) \right]$$

(40)

where $m_i$ are the up-type quarks’ masses, and the loop functions

$$J_k(x) = \frac{1}{1-x} + \frac{x^k \log(x)}{(1-x)^2},$$

$$B_k(x, y) = \frac{J_k(x) - J_k(y)}{x-y}, \quad k = 1, 2$$

(41)

One can easily notice that for any one of up-type quarks and heavy neutrino mass $m_{\nu_i} \lesssim \mathcal{O}(\text{few}) \, \text{TeV}$, the $W$-Goldston contribution is either suppressed by the quark mass, neutrino mass, or by the CKM or neutrino mixing matrix elements $U_{ij}$. Namely, $V_{di} V_{sj}^* |U_{ij}|^2 x_W^i x_W^j \lesssim 10^{-9}$. Thus, we can neglect the $W$-Goldston contribution and approximate Eq. (40) to the $W$ contribution in the unitary gauge

$$C_{10}^W = -C_{9}^W = -\frac{4G_F}{\sqrt{2}} \frac{\alpha_{\text{em}}}{4\pi} \frac{1}{s_W^2} \sum_{i=1}^{3} \sum_{j=1}^{9} V_{di} V_{sj}^* |U_{ij}|^2 B_2(x_W^i, x_W^j)$$

(43)

We parametrize the charged Higgs couplings with fermions in Eqs. (13,14) as

$$\Gamma_{H^\pm u, d_j}^{H^\pm} = C_{ij}^{H^\pm} P_R + D_{ij}^{H^\pm} P_L$$

$$\Gamma_{H^\pm e_j}^{H^\pm} = A_{ij}^{H^\pm} P_R + B_{ij}^{H^\pm} P_L$$

(44)

(45)

The coefficients of the loop contribution in Fig. 4(b) is

$$C_{10}^{W-H^\pm} = -C_{9}^{W-H^\pm} = -\frac{G_F}{8\sqrt{2}\pi} x_H^\pm \sum_{i=1}^{3} \sum_{j=1}^{9} \left( A_{ij}^{H^\pm} D_{ij}^{H^\pm} + A_{ij}^{H^\pm} D_{ij}^{H^\pm} \right) \sqrt{x_H^i x_H^j} B_0(x_H^i, x_H^j, x_H^i, x_H^j)$$

(46)

where

$$J_0(x, y, z) = \frac{x \log(x)}{(1-x)(x-y)(x-z)}$$

$$B_0(x, y, z) = J_0(x, y, z) + J_0(y, z, x) + J_0(z, x, y)$$

(47)

(48)

The coefficients of the loop contribution in Fig. 4(c) is

$$C_{9}^{H^\pm} = -\frac{1}{128\pi^2 m_W^2} x_H^\pm \sum_{i=1}^{3} \sum_{j=1}^{9} |D_{ij}^{H^\pm}|^2 \left( |B_{ij}^{H^\pm}|^2 + |A_{ij}^{H^\pm}|^2 \right) B_2(x_H^i, x_H^j)$$

$$C_{10}^{H^\pm} = -\frac{1}{128\pi^2 m_W^2} x_H^\pm \sum_{i=1}^{3} \sum_{j=1}^{9} |D_{ij}^{H^\pm}|^2 \left( |B_{ij}^{H^\pm}|^2 - |A_{ij}^{H^\pm}|^2 \right) B_2(x_H^i, x_H^j)$$

(49)

(50)

Fig. 5 shows scatter plots over the same parameters mentioned before. They show the values of $R_K$ with $m_{\nu}/m_{H^\pm}$, where it can reach to $\mathcal{O}(0.7)$ where the heavy neutrino is about twice the charged Higgs. We also see that a large subspace of the LRIS parameter space gives values of $R_K$ which are consistent with both the $a_\mu$ and $\text{BR(}\mu \rightarrow e\gamma\text{)}$ due to the loop contribution of heavy neutrinos and charged Higgs as in Fig. 4. The 1\sigma (2\sigma) level of accuracy around the average $a_\mu$ is determined by the the green (red) borders.

V. CONCLUSION

We have analyzed the muon anomalous magnetic moment $a_\mu$ alongside with the semileptonic $K, K^*$-meson anomalies in a minimal left-right symmetric model with neutrino masses inverse seesaw mechanism. We found that a large
FIG. 5. $R_{K,K^*}$ versus BR($\mu \rightarrow e\gamma$) and $\delta a_\mu$ and the mass ratio parameter $x_{H^\pm} = m_{H^\pm}/m_{H^\pm}$. The BP of Tab. I is encircled.

region of the parameter space of the model is consistent with the observed anomalies. We emphasized that in this type of models, only the $H^\pm$ loop explains $a_\mu$ and $R_{K,K^*}$ significantly, in agreement with the BR($\mu \rightarrow e\gamma$) and the $g_e - 2$ measured limits.

ACKNOWLEDGEMENTS

The work of M. A. is partially supported by Science, Technology & Innovation Funding Authority (STDF) under grant number 33495, and the work of K.E. and S.K. is partially supported STDF under grant number 37272.

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