Consistency bounds on the Higgs-boson mass

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In this talk we consider the modifications induced by heavy physics on the triviality and vacuum stability bounds on the Higgs-boson mass. We parameterize the heavy interactions using an effective Lagrangian and find that the triviality bound is essentially unaffected for weakly-coupled heavy physics. In contrast there are significant modifications in the stability bound that for a light Higgs boson require a scale of new physics of the order of a few TeV.

a. Introduction  The recent LEP bounds on the Higgs-boson mass \[ m_H > 113.2\text{GeV} \] together with the standard model (SM) upper limit \[ m_H < 220\text{GeV} \] (which is highly model-dependent) suggest the existence of a light Higgs boson. Should this be the case, the SM stability and triviality bounds strongly favor the appearance of new physics at scales \[ \lesssim 100\text{TeV} \]. In this talk we review the modifications to these bounds generated by new physics at scales below 50TeV.

b. Triviality and Stability  It is known \[ \cite{4} \] that some theories (e.g. QED and \( \Phi^4 \)) can be defined at all energy scales in \( \geq 4 \) dimensions only if the bare couplings are zero, i.e. they are trivial; interacting versions can be defined only by assuming an ultraviolet cutoff \( \Lambda \). In perturbation theory this corresponds to the appearance of Landau poles in the running couplings. The SM has this property, so that, for each choice of the Higgs-boson mass \( m_H \) there is a cutoff scale \( \Lambda \) beyond which the perturbation expansion breaks down. For fixed \( \Lambda \) this leads to an upper bound on \( m_H \) \[ \cite{5} \] with the corresponding conclusions: the SM is weakly coupled for all scales below a cutoff only if the Higgs-boson is sufficiently light.

A lower bound on \( m_H \) can also be derived by a different consistency argument, namely, that the SM vacuum is stable, i.e. \( V_{\text{eff}}(v) < V_{\text{eff}}(\bar{v}) \) for all \( |\bar{v}| < \Lambda \), where \( v \sim 246\text{GeV} \) is determined (for example) by the Fermi constant. This constraint is satisfied only if \( m_H \) is sufficiently large leading to a lower bound on \( m_H \) \[ \cite{4} \].

These calculations are done assuming there are no new-physics effects below \( \Lambda \). In this talk we extend these results \[ \cite{7} \] using an effective Lagrangian we parameterize the effects of the new physics at scales below \( \Lambda \) and use this parameterization to determine the modifications in the stability and triviality bounds described above. We will assume that the scale of new physics \( \Lambda \) is \( \gg v \), and that the heavy interactions are decoupling and weakly coupled. Finally we assume that chiral symmetry is natural \[ \cite{6} \]. With these constraints on the new physics, the terms in the effective Lagrangian \[ \cite{7} \] that affect the bounds on \( m_H \) are generated by the gauge-invariant operators \[ \cite{8} \] (\( O_{qt}^{(1)} \) affects \( V_{\text{eff}} \) only through RG mixing and its effects are small; other similar operators were not included for this reason): \[ O_\phi = \frac{1}{3} |\phi|^6 \quad O_{\partial \phi} = \frac{1}{2} (\partial |\phi|^2)^2 \quad O_\phi^{(1)} = |\phi|^2 |D\phi|^2 \]
\[ O_\phi^{(3)} = |\phi^3 D\phi|^2 \quad O_{\phi \phi} = |\phi|^2 \left( \bar{q} \phi t + h.c. \right) \quad O_{qt}^{(1)} = \frac{1}{2} |qt|^2 \]
where \( \phi \) denotes the SM scalar doublet, \( q \) the left-handed top-bottom isodoublet and \( t \) the right-handed top isosinglet. The Lagrangian we use is then \( \mathcal{L}_{SM} + \sum_i \alpha_i O_i/\Lambda^2 \) with the coefficients \( \alpha_i \) parameterizing the new-physics effects. We also define \( \eta \equiv \lambda v^2/\Lambda^2 \).

The triviality constraints are then obtained using the evolution equations for the various couplings:
\[ \frac{d\lambda}{dt} = 12\lambda^2 - 3f^4 + 6\lambda f^2 - \frac{3\lambda}{2} \left( 3g^2 + g'^2 \right) + \frac{3}{16} \left( g'^4 + 2g^2g'^2 + 3g^4 \right) \]
\[ - 2\eta \left[ 2\alpha_\phi + \lambda \left( 3\alpha_{\phi \phi} + 4\alpha_{\phi}^{(3)} \right) \right] \]
\[ \frac{d\eta}{dt} = 3\eta \left[ 2\lambda + f^2 - \frac{1}{4} \left( 3g^2 + g'^2 \right) \right] - 2\eta^2 \alpha \]
\[ \frac{df}{dt} = \frac{9f^2}{4} - \frac{f}{2} \left( \frac{8g^2}{4} + \frac{9}{4} g^2 + \frac{17}{12} g'^2 \right) - \frac{f\eta}{2} \left( -\frac{\alpha_{\phi \phi}}{f} + \tilde{\alpha} + 3\alpha_{qt}^{(1)} \right) \]
where $\kappa = M_Z \exp(8\pi^2 t)$ is the renormalization scale, and $\bar{\alpha} = \alpha_{\bar{\theta} \phi} + 2\alpha_{\phi}^{(1)} + \alpha_{\phi}^{(3)}$. The evolution of the gauge couplings $g$, $g'$ and $g_s$ (for the strong interactions) is unaffected by the $\alpha_i$'s. These equations are solved using the following boundary conditions: $\alpha_i(\Lambda) = O(1)$ (with various sign choices); $\langle \phi \rangle = 0.246/\sqrt{2}\text{TeV}$ (at $\kappa = \nu$) and, finally, that the $W$, $Z$, $t$, $H$ masses have their physical values. Requiring that the couplings never leave the perturbative regime for $\kappa < \Lambda$ then yields the triviality bound for this extension of the SM. The plots of the running coupling constants and the triviality bounds are given in Fig.1.

The triviality results are indistinguishable from the SM due to our requirement that the model remains weakly coupled; if this is relaxed our conclusions need not hold [10].

The effective potential at one loop is easily obtained from the above Lagrangian. The result is

$$V_{\text{eff}}(\bar{\phi}) = -\eta \Lambda^2 |\phi|^2 + \lambda |\phi|^4 - \frac{\alpha_{\phi}}{3\Lambda^2} |\phi|^6 + \frac{1}{64\pi^2} \sum_{i=0}^{5} c_i R_i^2 \ln(R_i/\kappa^2) - \nu_i + O(1/\Lambda^4)$$

where $c_0 = -4$, $c_1 = 1$, $c_{2,4} = 3$, $c_6 = 6$, $c_5 = -12$, $\nu_{0,1,2,5} = 3/2$, $\nu_{3,4} = 5/6$, $R_0 = \eta \Lambda^2$ and

$$
\begin{align*}
R_1 &= \lambda(6|\bar{\phi}|^2 - \nu^2) \left[1 - (2\alpha_{\bar{\theta} \phi} + \alpha_{\phi}^{(1)} + \alpha_{\phi}^{(3)})|\bar{\phi}|^2/\Lambda^2\right] - \alpha_{\phi} |\bar{\phi}|^4/\Lambda^2 \\
R_2 &= \lambda(2|\bar{\phi}|^2 - \nu^2) \left[1 - (\alpha_{\phi}^{(1)} + \alpha_{\phi}^{(3)})|\bar{\phi}|^2/\Lambda^2\right] - \alpha_{\phi} |\bar{\phi}|^4/\Lambda^2 \\
R_3 &= (g^2/2)|\bar{\phi}|^2 \left(1 + |\bar{\phi}|^2 (\alpha_{\phi}^{(1)} + \alpha_{\phi}^{(3)})/\Lambda^2\right) \\
R_4 &= [(g^2 + g'^2)/2] |\bar{\phi}|^2 \left(1 + |\bar{\phi}|^2 (\alpha_{\phi}^{(1)} + \alpha_{\phi}^{(3)})/\Lambda^2\right)
\end{align*}
$$

FIG. 1: **Left panel:** (a) $V_{\text{eff}}$ at the scale $\kappa = \phi$ as a function of the field strength. The running of $\lambda$ (b) and $\alpha_{\phi}$ (c) when $\alpha_i(\Lambda) = -1$, $m_t = 175\text{GeV}$, for $\Lambda = 5.1\text{TeV}$, $m_H = 140.4\text{GeV}$ (curves (1)) and $\Lambda = 48.9\text{TeV}$, $m_H = 148.7\text{GeV}$ (curves (2)). **Right panel:** Triviality (a) and stability (b) bounds on $m_H$ for $m_t = 175\text{GeV}$. Stars correspond to solutions (1) and (2).
\[ R_5 = f|\varphi|^2 \left( f + 2 \alpha_\phi |\varphi|^2 / \Lambda^2 \right). \]

This has the same form as in the SM, but with modified \( R_i \). Note that \( V_{\text{eff}} \) is gauge dependent\(^\text{[11]}\) but the effects of this gauge dependence are small since the RG-improved tree-level effective potential is gauge-invariant. This leads to a variation in the Higgs-boson mass limit: \( \Delta m_H \lesssim 0.5 \text{GeV} \)\(^\text{[12]}\). A plot of the effective potential for some representative values of the parameters is presented in Fig.\(^\text{[1]}\). Using the anomalous dimension for the scalar field, \( \gamma = 3f^2/2 - 3(3g^2 + g'^2)/8 - \eta \bar{\alpha}/2 \), and a careful definition of \( V_{\text{eff}}(0) \)\(^\text{[13]}\), one can verify that \( V_{\text{eff}} \) is scale invariant.

In order to insure the stability of the SM vacuum we demand \( V_{\text{eff}}(\varphi = 0.75 \Lambda)|_{\kappa = 0.75 \Lambda} \geq V_{\text{eff}}(\varphi = \nu_{\text{phys}}/\sqrt{2})|_{\kappa = \nu_{\text{phys}}/\sqrt{2}} \). The boundary of the stability region corresponds to those values of \( m_H \) and \( \Lambda \) that saturate the above inequality. These boundary values are plotted in Fig.\(^\text{[1]}\), it is noteworthy that in contrast with the triviality bounds the presence of the effective operators \( \alpha \) as a significant impact on the stability bounds.

For example for a Higgs-boson mass of 115GeV, \( \Lambda \lesssim 4 \text{TeV} \) for \( |\alpha_i| = 0.5 \). We also find that the main effects on the stability bound are generated by \( \alpha_\phi \), \( \alpha_{t\phi} \). For example, for \( \alpha_\phi \) large and positive the potential has no minimum for fields below \( 0.75 \Lambda \); more precisely, there is a region in the \( \alpha_\phi - \alpha_{t\phi} \), given in Fig.\(^\text{[2]}\), where the SM vacuum is either absent or unstable for \( \kappa \sim 0 \; 75 \Lambda \).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2}
\caption{The unshaded region corresponds to the values of \( \alpha_\phi(\Lambda) \), \( \alpha_{t\phi}(\Lambda) \) where the effective potential has no SM minimum for fields below 0.75\( \Lambda \), for any choice of 0.5 TeV \( < \Lambda < 50 \text{ TeV} \).

\textit{c. Conclusions} \; The SM triviality upper bound remains unmodified for weakly coupled heavy physics, while the stability bound increases by \( \sim 50 \text{GeV} \) depending on \( \Lambda \) and \( \alpha_i(\Lambda) \). For \( m_H \) close to its lower LEP limit the constraint on \( \Lambda \) could be decreased dramatically even for modest values of the \( \alpha_i \). These results complement the ones obtained within specific models\(^\text{[4]}\).

Note that, strictly speaking, our expression for \( V_{\text{eff}} \) is not valid at points where it changes curvature\(^\text{[15]}\). Still we can make an arguments similar to the one above slightly below the inflection point \( |\varphi| \sim 0.75 \Lambda \); the resulting bounds are essentially unchanged due to the precipitous drop of \( V_{\text{eff}} \) beyond this point (see fig.\(^\text{[1]}\)).

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