Fate of the $\eta'$ in the Quark Gluon Plasma

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Abstract

In this paper we study the $\eta'$ in $N_f = 2 + 1 + 1$ lattice QCD simulations at finite temperature. Results are obtained from the analysis of the gluonic defined topological charge density correlator after gradient flow. Our results favour a small dip in the $\eta'$ mass around the pseudocritical temperature, associated with chiral symmetry restoration, followed by an increase above the pseudocritical temperature. The magnitude of the dip is consistent with the reduction of the $\eta'$ mass obtained by experimental analysis and suggests that $\eta'$ mass comes close to zero temperature non-anomalous contribution.

Keywords: Meson spectroscopy, $\eta'$ meson, chiral symmetry, nonzero temperature, lattice QCD, QCD topology, Quark Gluon Plasma.

1. Introduction

The realization of the chiral and axial symmetries in QCD has important phenomenological implications. The spectrum of mesons built by light quarks – up, down, strange – is nicely accounted for by considering the spontaneous breaking of chiral symmetry. The well known puzzle associated with a too heavy $\eta'$ mass found a solution once the non trivial topological structure of the vacuum is taken into account\textsuperscript{[1]}. The solution can be nicely formalized within the framework of large-$N$ QCD where it is shown that the mass matrix should include a term with the topological susceptibility. Then, the main features of the physical spectrum can be reproduced if the topological susceptibility is non-zero\textsuperscript{[2,3,4]}. These are eminently nonperturbative phenomena, posing specific challenges: a numerical approach is mandatory, and early lattice studies have indeed confirmed a non-zero topological susceptibility\textsuperscript{[5]}, providing a clear evidence for the validity of the Witten-Veneziano analysis\textsuperscript{[6]}. Moreover, the $\eta'$ mass itself has been directly measured on the lattice, with results well in agreement with experimental data\textsuperscript{[7,8]}.

A natural question then arises, concerning the fate of the symmetry patterns at high temperatures. The study of sequential restoration, or lack thereof, of the relevant symmetries helps understanding the fundamental mechanisms underlying these phenomena. The restoration of the chiral $SU(2)_L \times SU(2)_R$ symmetry at high temperatures has been the subject of detailed lattice studies: there is now consensus that such symmetry is restored in the chiral limit at a temperature of about 140 MeV, and that the explicit breaking associated with the quark masses turns the phase transition into a crossover which occurs at about 157 MeV for physical quark masses\textsuperscript{[9]}.

The fate of the $U(1)_A$ symmetry is a subtler issue, since the $U(1)_A$ anomaly provides a mechanism for explicit symmetry breaking\textsuperscript{[10]}, which, being at operator level, exists independent of temperature. However, instantons and their suppression may provide a mechanism for its effective restoration\textsuperscript{[11]}, raising the issue of the ordering of the chiral and axial symmetry restoration. Current results are inconclusive: some results indicate a coincidence of the $U(1)_A$ restoration with that of the chiral restoration, others suggest that $U(1)_A$ breaking persists up to large temperatures being effectively restored only in the high temperatures, dilute instanton gas limit.

The pattern of symmetries at high temperature has of course influence on the meson spectrum in the plasma: without breaking, the light flavor pseudo-Goldstone bosons would become (nearly) degenerate with their chiral partners. Concerning the $\eta'$, assuming a restoration of the $U(1)_A$ symmetry coincident with that of the chiral symmetry, the natural conclusion would be that $\eta'$ follows the fate of its chiral partners. The details are subtle and will be reviewed in the following, and in a short summary ab initio calculations are mandatory to reach firm conclusions.

On the experimental side, any variation of the mass of the $\eta'$ should produce interesting signatures in heavy ions collisions\textsuperscript{[11]}. In Ref.\textsuperscript{[11]} the authors argue that a small mass of the $\eta'$ in hot and dense matter should lead to an increase of the production cross section with respect to the one in pp. The most popular experimental results are those from PHENIX and STAR at 200 GEV gold-gold collisions\textsuperscript{[12]}. They determine the best value for the in-medium mass reduction according to their model, suggesting a mass reduction of about 200 MeV.
In the same work it is also noted that different initial abundances will change the result. Indeed, one can never directly measure the \( \eta' \) mass, only relative abundances with inherent ambiguities in the interpretations of the results.

In short, the behaviour of the \( \eta' \) at finite temperature has both theoretical and experimental significance. However, the present understanding of the behaviour of \( \eta' \) at high temperatures is incomplete. Studies have been carried out \([13, 14, 15, 16, 17, 18, 19, 20]\) within the framework of effective field theory, and we will briefly review them below. The results are inconclusive and call for a first principle analysis.

In this note we will present the first lattice results on the behaviour of the \( \eta' \) around and above the chiral transition.

2. \( \eta' \), QCD symmetries and topology

At the classical level massless QCD enjoys the symmetry \( U(N_f) \times U(N_f) = S(U(N_f)) \times SU(N_f) \times U(1) \times U(1) \). In the limit of \( N_f = 3 \) massless quarks in the normal phase of QCD \( S(U(3)) \times SU(3) \) is spontaneously broken down to \( SU(3)_V \), producing an octet of Goldstone bosons, \( \pi, s', K', s, \) and \( \eta \).

The associate quark condensate is not invariant under the \( U(1)_A \), which, again, is exact at classical level. More precisely, an anomalous \( U(1)_A \) transformation would generate a mass-independent, parity violating term proportional to \( \bar{F}_j F_j = \frac{1}{2} \eta x \bar{F} F \). However, as this term is irrelevant at all orders in perturbation theory, it was initially ignored, leading the conclusion that \( U(1)_A \) is exact in the chiral limit. Since the quark condensate, generated by the breaking of \( SU(3)_V \times SU(3)_R \) is not invariant under \( U(1)_A \), the axial symmetry would also be spontaneously broken, producing a ninth, flavor singlet Goldstone boson, the \( \eta' \).

Physical quark masses explicitly break chiral symmetry, and the masses of the (now) pseudo-Goldstone bosons may be computed in chiral perturbation theory, using \( \Lambda_{QCD} \) as a sole input, leading to the well known results: \( m_{\pi}^2 \propto (m_u + m_d) \Lambda_{QCD} \), \( m_{K}^2 \propto (m_u + m_d + m_s) \Lambda_{QCD} \), \( m_{\eta'}^2 \propto (m_u + m_d + 4m_s) \Lambda_{QCD} \). If \( U(1)_A \) were spontaneously broken, the \( \eta' \) mass would be in the same mass range. Its experimental value, \( m_{\eta'} \sim O(1 \text{ GeV}) \) is then incompatible with the chiral perturbation theory result.

As mentioned above, the solution was found once realized that \( q = \frac{1}{2\sqrt{3}} \bar{F} F \) has a topological nature, hence in principle it could be non-zero: \( \int \bar{F} F = 32\pi^2 Q \), with \( Q \) being the topological charge. Within the framework of large-\( N \) QCD \([3, 21, 22]\) the mass matrix should then include an anomalous term \( m_{\eta'}^2 = 4N_f/3f_\pi^2 \chi \), with \( \chi = \frac{4\pi^2}{3} \) being the large-\( N \), or, equivalently, the Yang-Mills topological susceptibility. In the isospin limit, in which pion and K do not mix with anything else, adopting the basis \( I \equiv \frac{1}{2\sqrt{3}} (u\bar{u} + d\bar{d}); S \equiv s\bar{s} \), the mass matrix describing the \( \eta \) complex reads \([3]\):

\[
\begin{pmatrix}
    m_{\pi}^2 + m_{\eta'}^2 & m_{\eta'}^2 / \sqrt{3} \\
    m_{\eta'}^2 / \sqrt{3} & 2m_{K}^2 - m_{\pi}^2 + 3m_{\eta'}^2 / 2
\end{pmatrix}
\]

and we recognize that with \( m_{\eta'} = 0 \) the \( \eta' \) is a pure \( s\bar{s} \) state with mass \( m_{\eta'} \approx 700 \text{ MeV} \), independent on the pion mass. Alternatively, \( \eta \) and \( \eta' \) may be expressed in terms of octet-singlet states \([1] = \frac{1}{\sqrt{3}} (u\bar{u} + d\bar{d} + s\bar{s}); [8] = \frac{1}{\sqrt{6}} (u\bar{u} + d\bar{d} - 2s\bar{s}) \) and a mixing angle \( \theta : [\eta] = \cos \theta [8] - \sin \theta [1] \), \( [\eta'] = \sin \theta [8] + \cos \theta [1] \). Physical value is about \( -20^\circ \) and would reduce to about \( -50^\circ \) for \( m_{\eta'} = 0 \). The mixing is then a useful diagnostic for the anomalous contribution.

Beyond leading order in \( 1/N \), there are studies which extend the DGMOR relations to the \( U(1)_A \) sector, however a precise quantitative analysis requires the knowledge of still poorly known decay constants \([24]\).

A completely ab initio calculation requires a lattice study. The standard way to investigate the meson masses in lattice simulations is based on the measurement of the correlators of corresponding bilinear quark operators \( A(x) = \bar{\psi} \Gamma \psi(x) \), where \( \Gamma \) acts on flavour and Dirac indices and depends on the meson channel under study. The \( \eta' \) mass may be computed from the decay of the correlator \( G_{\eta'} = \langle \bar{\psi} \Gamma_1 \phi(x) / \psi(\phi(y)) \rangle \), however, applying such method for extraction of the \( \eta' \) mass poses some complications \([7, 24, 25, 26, 27]\). Since \( \eta \) and \( \eta' \) are not exactly flavour eigenstates, but a mixing of flavour octet and singlet states, one has to analyze the \( 2 \times 2 \) matrix of fermionic current correlators, leading to the masses as well as to the \( \eta, \eta' \) mixing angle. Another complication comes from the fact, that the fermionic correlator corresponding to \( \eta' \) channel has also a large so-called disconnected contribution, which determination requires considerable numerical effort. Alternatively, one may consider \( G_{\eta'} = \langle 1/N^2 \langle \phi(x) \phi(y) \rangle \rangle \), where \( \phi(x) \) is the topological charge density. The correlator of the topological charge density only contains the \( \eta' \) and simply reduces to the Witten-Veneziano formula at leading order. At zero temperature with this method the \( \eta' \) mass was determined in \([28]\). A recent paper has successfully cross checked the two methods at zero temperature for \( N_f = 2 \) twisted mass Wilson fermions \([29]\). At finite temperature this approach was used for extraction of the sphaleron transition rate in gluodynamics \([30]\).

All in all, at zero temperature, in the broken phase of QCD, we have a quantitative understanding of the \( \eta \) mass complex. At finite temperature the situation is far less clear. Already by looking at the basic relations, it is clear that the fate of the \( \eta' \) at high temperature is entangled with the behaviour of the light mesons as well as with that of the topological susceptibility. In symmetry languages, with chiral and axial symmetries restoration.

Chiral and \( U(1)_A \) restoration at high temperatures and their implications on the \( \eta' \) mesons have been considered by several authors \([13, 14, 15, 16, 17, 18, 19, 20]\). A generalization of the Witten-Veneziano approach to finite temperature \([14]\) indicates a dip in the \( \eta' \) mass followed by an increase, leading to a near-degeneracy with the pion mass. This approach requires a parameterization of the temperature dependence of the anomalous contribution to the mass, which has a strong quantitative influence on the results, even if some general features – the dip, and the increase – are robust. Ref. \([16]\) used Ward identities and \( U(3) \) CHPT to study the pattern of degeneration of chiral partners and mixing angles. Also in this study the \( \eta' \) mass has a drop, and the mixing angle approaches the ideal one, consistent with the disappearance of the anomaly and the restoration...
Table 1: Parameters and statistics used in the simulations.

| $m_\pi$ [MeV] | $N_x \times N^3_c$ | $a$ [fm] | $T$ [MeV] | # conf |
|----------------|-----------------|--------|---------|-------|
| 369            | $20 \times 48^3$| 0.0646(26) | 153(6) | 1173  |
| 369            | $18 \times 40^3$| 0.0646(26) | 170(7) | 1198  |
| 369            | $16 \times 32^3$| 0.0646(26) | 191(8) | 3879  |
| 369            | $14 \times 32^3$| 0.0646(26) | 218(9) | 3965  |
| 372            | $16 \times 32^3$| 0.0823(37) | 150(7) | 2679  |
| 213            | $24 \times 48^3$| 0.0646(26) | 127(3) | 1120  |
| 213            | $20 \times 48^3$| 0.0646(26) | 153(6) | 552   |
| 213            | $18 \times 48^3$| 0.0646(26) | 170(7) | 470   |

Table 2: The pseudo-critical temperatures associated with the mass (temperature) derivatives of the chiral order parameters. The first error is statistical, the second systematic, see text for details. Adapted from Ref. [31].

| $m_\pi$ [MeV] | $a$ [fm] | $T_c$ [MeV] | $T_\Delta$ [MeV] |
|----------------|--------|------------|-----------------|
| 369            | 0.0646(26) | 185(1)(3)  | 180(5)(1)       |
| 372            | 0.0823(37)| 189(2)(1)  | 194(2)(0)       |
| 213            | 0.0646(26) | 158(1)(4)  | 165(3)(1)       |

$m_\pi \simeq 370$ MeV. In Tab. [1] the parameters and statistics used in our simulations are summarized, while in Tab. [2] we note, for future reference, the pseudocritical temperatures for the two pion masses considered in this work. Let us remind the reader that for a non-zero quark mass the transition turns into a crossover, whose position is prescription dependent: in Tab. [2] we show the crossover temperatures associated with the maximum of the derivative with respect to the mass and with the maximum of the derivative w.r.t. the temperature of the chiral order parameter, denoted as $T_c$ and $T_\Delta$.

The mass of the $\eta'$ is extracted from the correlator of the topological charge density:

$$G(\tau) = \int d^3\bar{x}q(0)\bar{q}(\tau, \bar{x}) = \int d^3\bar{x} \frac{1}{32\pi^2} F_{\mu\nu} \bar{F}_{\mu\nu}(0) \times \frac{1}{32\pi^2} F_{\mu\nu} \bar{F}_{\mu\nu}(\tau, \bar{x})$$

(1)

To measure the topological charge density correlator we used (Wilson) Gradient Flow (GF) [32]. Starting from lattice gauge field $U(x,\mu)$, the evolution of the gauge variables in auxiliary GF time $t$ is performed:

$$\partial_t V(x,\mu) = -\partial_{\alpha} [S_W[V_t]] V(x,\mu), \quad V_0(x,\mu) = U(x,\mu).$$

(2)

This effectively smears the gauge fields over the range $\sim \sqrt{\Delta t}$.

3. Lattice details and methodology

The results have been obtained on lattice configurations with $N_f = 2 + 1 + 1$ flavors of twisted mass Wilson fermions, on a subset of parameters used in [31]. In some cases we have enlarged the statistics by performing new simulations. The mass parameters of $s$ and $c$ quarks are fixed at their physical values and several sets of light doublet masses are available, all corresponding to larger than physical values of pion mass $m_\pi$. In the following we use two values of pion mass, $m_\pi = 210$ MeV and $m_\pi \simeq 370$ MeV.

Figure 1: Correlators as a function of Euclidean time, shifted and normalized with hyperbolic fits superimposed for pion mass 210 (370) MeV, left (right). The fit extrema are $[-4 : 4]$, with the exception of the two largest temperatures for pion mass $m_\pi = 370$ MeV, which have been fitted in the $[-2 : 2]$ and $[-3 : 3]$ interval, respectively. GF time is $t/a^3 = 1.5$. The full details on the results can be seen in Figs. 2–3. In order to avoid a complete superposition of data points belonging to different temperatures we applied a tiny shift along the $\tau$ axis.
After performing GF, we measured the correlator of the topological charge density \( q(\tau, \bar{x}) = \frac{1}{V} \epsilon_{\mu\nu\rho\sigma} F_{\mu\nu}(\tau, \bar{x}) F_{\rho\sigma}(\tau, \bar{x}) \) as a function of GF time based on field variables \( V \). For field strength tensor \( F_{\mu\nu} \) we used a clover discretization.

Since Gradient Flow smears the gauge fields, at large GF times \( t \) the contribution of higher excited states to the correlator \( G(t, \tau) \) become suppressed and the statistical noise in \( G(t, \tau) \) is reduced.

However, the correlator \( G(t, \tau) \) has a large positive contact term at \( C(\tau = 0) \) which is not related to any propagating physical degree of freedom. At GF times \( t \gtrsim \tau^2/8 \) the smeared contact term significantly changes the correlator \( G(t, \tau) \) and such points are omitted in our analysis.

\[ \text{Note that the correlator (1) is negative at } \tau \neq 0 \text{ due to pseudoscalar nature of the topological charge density } q. \]

Within the discussed above restrictions the correlator \( G(t, \tau) \) should have a simple behaviour \( G(t, \tau) \sim \cosh (m(\tau - N_c/2)) \), with the parameter \( m \) giving the mass of the \( \eta' \). Our discussion ignores the width of the spectral function: we will see that the data are compatible with the simple spectral decomposition we have assumed. This indicates that, within our errors, the spectral function remains narrow enough to allow the identification of the mass of the ground state (or, more precisely, with an average value of the mass range spanned by the peak).

4. Results

To have a first feeling of the results, we select a fixed value of the GF time \( t/a^2 = 1.5 \) and in Fig. 1 we plot the normalized correlator \( G(\tau - N_c/2)/G(N_c/2) \) for various temperatures, at \( a = 0.0646(26) \) fm. We superimpose hyperbolic

![Figure 2: Mass extracted from hyperbolic fit within various ranges of Euclidean time \([\tau_{\mu}, N_c/2]\) vs GF time. Lattice step is \( a = 0.0646(26) \) fm for all plots except for the upper left plot which is for \( a = 0.0823(37) \) fm. Red horizontal lines indicate the plateau, which is used to determine the mass. Pion mass is \( m_\pi = 370 \) MeV.](image)

![Figure 3: Same as Fig. 2, but for \( m_\pi = 210 \) MeV. Lattice step is \( a = 0.0646(26) \) fm.](image)
fits $\cosh (m(\tau - N_r/2))$ (the systematic thereof will be discussed momentarily): for the lighter pion mass the data suggest a small drop around $T = 150 \text{ MeV}$, followed by an increase of the mass, for a pion mass of about 370 MeV the qualitative trend is similar, but the drop happens around $T = 170 \text{ MeV}$.

Let us go further and analyze the GF time dependence of the correlator. For each GF time $t$ we fit the correlator $G(\tau)$ with hyperbolic fit: $G(\tau) \sim A \cosh (m(\tau - N_r/2))$ within the range $\tau \in [\tau_a, N_r/2]$, where we changed the left point $\tau_a$ of the fit range. We plot the result of the fit for $m(t)$ as a function of the GF time $t$ for several fitting ranges $\tau \in [\tau_a, N_r/2]$ in Figs. 2-3. The plateau in the $\tau$-dependence corresponds to the $\eta'$ mass. We find the smallest value of the left end $\tau_a$ of the interval $[\tau_a, N_r/2]$, which still has a plateau and by the height of this plateau extracted the mass. The results are shown by horizontal lines in Figs. 2-3 and summarized in the fourth column of the Tab. 3.

As an alternative method, we fit the correlator $G(t, \tau)$ simultaneously for all points with the restriction: $\tau/a \geq 4$, $\tau > b \sqrt{8t}$ by the function:

$$G(t, \tau) = a(t) \cosh (m(\tau - N_r/2)).$$

(3)

Note that $m$ is the same for all values of GF time $t$, while the coefficient $a(t)$, in principle, depends on $t$. By varying the number $b$ we change the region of the fitting, thus we can balance between statistical and systematic error (see the discussion in Sec. 5). Since the characteristic radius of smearing is $\sim \sqrt{8t}$, one should take $b \sim 2$. In the following we used $b = 2$ as well as smaller numbers $b = 1.6$ and $b = 1.8$. We present the value of the $\eta'$ mass extracted by this method, along with the mass, extracted by previous method, in Fig. 4. From these figures one sees that the results extracted by various methods, as well as the results for different lattice steps are in agreement with each other. If one increases $b$, that is, decreases the fitting range, the errorbars grow larger, however the results for various ranges $b$ are in agreement with each other. As the final estimation we take the results of the fit for the smaller lattice step $a = 0.0646(26) \text{ fm}$ and $b = 1.8$, which are also presented in the Tab. 3. For comparison, we quote in the Tab. 3 the results reported in [8] for the $\eta'$ mass at low temperatures for $N_f = 2 + 1 + 1$ flavors of twisted mass Wilson fermions and the same setup which we have used here.

### 5. Discussion

The final plot for the dependence of $m_{\eta'}$ as a function of temperature is shown in Fig. 5. In the same plot we show the low temperature result of Ref. [8]. We note that the $\eta'$ mass is rather robust against temperature, with a suggested small dip around the pseudocritical temperature, followed by an increase at larger temperature. In the same plot we also indicate, as a horizontal line, the zero temperature non-anomalous component of the $\eta'$ mass.
computed at leading order in $1/N, m_{\pi} = 700$ MeV. Clearly more statistics will be needed to assess the quantitative reduction in mass. At the same time, the trend of the propagators gives some confidence in this observation. Indeed, and interestingly, the reduction, and subsequent increase, seems correlated with a signal and the light flavor sector: in Fig. 6 we superimpose our results for the $\eta'$ mass with those for the renormalized chiral condensate $R\langle \bar{\psi}\psi \rangle$ obtained on the same lattices in Ref. [31]. Also, the pseudocritical temperature associated with the inflection point of the chiral condensate, or with the peak of the disconnected susceptibility can be read off Tab. 2 again suggesting a correlation with the region of the dip of the $\eta'$ mass. The reduction of the $\eta'$ mass observed at finite temperature may be due to a vanishing anomalous contribution, which, if confirmed, could indicate that the ideal mixing has been reached.

In conclusion, we have studied the $\eta'$ for two different pion masses, and varying temperatures. We have observed a correlation between the behaviour of the $\eta'$ at high temperature and that of the chiral observables. We confirm that the $\eta'$ increases above the pseudocritical temperature associated with the light degrees of freedom. The results suggest a modest dip correlated with the same pseudocritical temperature. The magnitude of the dip is not incompatible with the 200 MeV reduction indicated by experimental analysis, and of confirmed would bring the $\eta'$ mass close to the zero temperature non-anomalous result, implying ideal mixing in the $\eta$ sector.

These results are open to different interpretation: they may be linked with the $SU(2) \times SU(2) \times U(1)_A$ restoration, or they may be simply a manifestation of the complicated $SU(N_f)$ \times $SU(N_f)$, $N_f = 2$ or $N_f = 3$, dynamics in the light sector. A more complete set of observables, including the light meson spectrum, the $\eta$ mass and the mixing angle, is needed in order to draw conclusive results on these issues.

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Figure 6: The ratio of the temperature-dependent mass $m_\eta(T)$ to its zero-temperature value $m_\eta(0)$ superimposed with the results for the renormalized chiral condensate. $a = 0.08464(26)\,\text{fm}$. In order to avoid a complete superposition of data points belonging to different pion masses we applied a tiny shift along the $T$ axis.

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