Abstract—In this paper, we consider the delay-sensitive power and transmission threshold control design in S-ALOHA network with FSMC fading channels. The random access system consists of an access point with \( K \) competing users, each has access to the local channel state information (CSI) and queue state information (QSI) as well as the common feedback (ACK/NAK/Collision) from the access point. We seek to derive the delay-optimal control policy (composed of threshold and power control). The optimization problem belongs to the memoryless policy problem, we exploit some special structure of the collision channel and the cardinality of the CSI states. For the threshold control problem, we exploit some special structure of the collision channel and common feedback information to derive a low complexity solution. The delay performance of the proposed design is shown to have substantial gain relative to conventional throughput optimal approaches for S-ALOHA.

Index Terms—S-ALOHA, delay, Markov decision process (MDP), local channel state information (CSI), local queue state information (QSI), threshold control, power control.

I. INTRODUCTION

Random access network is a hot research topic due to its robustness in system performance. In particular, ALOHA is a popular example of random access protocol which has attracted a lot of research attention over the past two decades. One important application is the access network (such as the infrastructure mode in WiFi) where multiple nodes compete for transmission opportunity to transmit data to an access point (AP). In [1], the authors considered the design and analysis of the traditional buffered slotted ALOHA (S-ALOHA) in which finite users with infinite buffer attempt to transmit a backlogged packet according to a transmission probability in one slot, and the packet is successfully received if and only if exactly one packet is transmitted. In asymmetric network (heterogenous users), the stability region has only been obtained in two and three user cases [2]. The study of the stability region for general number of users is difficult because the transition probability of the state space of the interacting queues alters from the non-empty to empty buffer case. In [3], the authors proposed a dominant system technique to obtain a lower bound for the stability region for the general case. In symmetric ALOHA network (homogeneous users), all users are statistically identical and hence, the stability region is degenerated to one dimension. It is shown in [1], [4] that the system is stable as long as the arrival rate is less than the average throughput. As a result, stability analysis is equivalent to the throughput analysis. The authors in [4] extended the protocol to an adaptive ALOHA over the multi-packet reception (MPR) channel to maximize the system throughput. For instance, the transmission probability is a function of the local channel state information (CSI). In [5], the authors extended to the adaptive transmission rate and power control w.r.t CSI to maximize the throughput. In [6], it is shown that a simple adaptive permission probability scheme, namely binary scheduling, is throughput optimal for homogeneous users with adaptive transmission rate in collision channel. In the binary scheduling scheme, there is a transmission threshold in which user could attempt to transmit its backlogged packet only when its local CSI exceeds the threshold.

In all the above works on stability and throughput analysis and optimization, the delay performance has been ignored completely. In practice, applications are delay-sensitive and it is critical to optimize the delay performance in S-ALOHA network to support realtime applications. In [7], the authors surveyed the recent works on delay analysis of traditional S-ALOHA network in which exact delay can be obtained only in two user case. In [8], the delay performance for finite user infinite buffer is analyzed using the tagged user analysis (TUA) method. Although the channel fading is considered, adaptive transmission probability and rate with power control is not allowed. In [9], the trade-off between delay and energy in additive write Gaussian noise (AWGN) channel with no queue state information (QSI) is investigated. However, they assumed multi-access coding to ensure successful reception for each user even if all competing users transmit simultaneously. In [10], the authors proved that the longest queue highest possible rate (LQHPR) policy, which is a centralized control policy requiring perfect knowledge of global QSI and global CSI, is delay-optimal in symmetric network. While the above works deal with the delay performance of S-ALOHA network, there are still a lot of technical challenges to be solved. They are
list below.

- **Queue-aware power and threshold control for S-ALOHA**: Previous literature focused either on the power control (under a fixed and common threshold for all users) for throughput optimization, or on the delay analysis of uncontrolled S-ALOHA network. Both the transmission threshold control and power control policies are important means to optimize the delay performance of S-ALOHA. However, due to the lack of global knowledge on CSI and QSI, it is quite challenging to design delay-sensitive control schemes for S-ALOHA networks.

- **Exploiting memory in the fading channels**: Existing works have assumed memoryless adaptation in which the control actions are done independently slot by slot (assuming fading is i.i.d). While i.i.d fading could lead to simple solution, it fails to exploit the memory of the time varying fading channels, which is critical to boost the delay performance of S-ALOHA network.

- **Utilization of local QSI and common feedback information from the AP**: Existing control policy on throughput optimization only adapts to the local CSI and did not exploit the local QSI as well as common feedback information from the AP. These side information are also critical to improve the delay performance of the S-ALOHA network.

In this paper, we shall propose a delay-sensitive power and transmission threshold control algorithm for S-ALOHA network which addresses the above three important issues. We consider a S-ALOHA network with \( K \) users. The transmit power and threshold control policies adapt to the local CSI, local QSI as well as common feedback information (ACK/NAK/(Collision) from the AP. The delay-optimization problem belongs to the memoryless policy \( K \)-agent infinite horizon decentralized Markov decision process (DEC-MDP) [11]. The problem of finding the optimal policy is proved to be \( NP \)-hard [12], [13], which means that the optimal solution is computationally intractable. To obtain a feasible and low complexity solution, we recast the optimization problem into two subproblems, namely the power control and the threshold control problem. For a given threshold control policy, the power control problem is decomposed into a reduced state MDP for single user so that the overall complexity is \( O(NJ^2) \), where \( N \) and \( J \) are the buffer size and the cardinality of the CSI states. On the other hand, we solve the threshold control problem by exploiting the special structure of the S-ALOHA network and common feedback information to derive a low complexity solution. The delay performance of the proposed design is shown to have substantial gain relative to conventional solutions.

This paper is organized as follows. In section II we outline the system model of S-ALOHA network and define the delay-optimal control policy. In section III we shall formulate the delay-optimal problem and introduce the DEC-MDP model. In section IV we exploit the special structure in symmetric network. We also extend to asymmetric case in section V and illustrate the performance via simulations in section VI. A brief summary is given in section VII finally.

![The system model in symmetric S-ALOHA network.](image)

**II. SYSTEM MODEL**

In this section, we shall elaborate the system model, including source and physical layer model, as well as the control policy in symmetric network, and extend to the asymmetric case in section V. We consider a \( K \) users S-ALOHA network in this paper. The time dimension is partitioned into slots (each slot lasts \( \tau \) seconds). The \( m \)-th slot means the time interval \((m-1)\tau, m\tau]\), \( m = 0, 1, 2 \cdots \). Fig. I illustrates the top level system model in symmetric network. The \( K \) competing users are coupled together via the transmission threshold and power control policy.

**A. Source Model**

For simplicity, the arrival packet rate of all the users is assumed to follow independent Poisson distribution with arrival rates \( \lambda \) (number of packets per second). The packet length of the data source \( N_b \), follows exponential distribution with mean packet size \( N_b \) (bits per packet), and the buffer size is \( N \) (packets). The QSI of the whole system at the \( m \)-th slot is denoted by \( Q_m = \{Q_{k,m}\}_{k=1}^{K} \in \mathbb{N}^K \), where \( Q_{k,m} \) is the number of packets in the \( k \)-th user’s buffer, and \( \mathbb{N} = \{0, 1, 2, \ldots, N\} \) denotes a finite state space of local QSI for single user. When the buffer is full, i.e., \( Q_{k,m} = N \), it will not accept any potential new packets.

**B. Physical Layer Model and Feedback Mechanism**

We consider a block fading channel between each user and the AP. The CSI at \( m \)-th slot is denoted by \( H_{m} = \{H_{k,m}\}_{k=1}^{K} \in \mathbb{S}^K \), where \( H_{k,m} \) is the channel gain for user \( k \), and \( \mathbb{S} = \{S_j\}_{j=1}^{J} \) denote a set of \( J \) CSI states for single user. \( \{H_{k,m}\}_{m=1}^{\infty} \) is modeled as a stationary ergodic process [14], which is independent among users. Specifically, let \( P_{i,j} = \text{Pr}\{H_{k,m} = S_j | H_{k,m-1} = S_i\} \) be the state transition probability and \( \pi_j = \text{Pr}\{H_{k,\infty} = S_j\} \) be the stationary probability. All the users share a common spectrum with a bandwidth of \( W \) Hz using S-ALOHA protocol. The signal received by the AP at \( m \)-th slot is given by:

\[
y[m] = \sum_{k=1}^{K} \sqrt{H_{k,m}}x_k[m] + z[m]
\]

where \( x_k[m] \) is the transmit signal for the \( k \)-th user at \( m \)-th slot, and \( \{z[m]\}_{m=1}^{\infty} \) is the i.i.d \( \mathcal{N}(0, N_0) \) noise. Suppose that only the \( k \)-th user attempts to transmit its packet to the AP at
the $m$-th slot. The maximum achievable data rate ($b/s$) of the $k$-th user is given by:

$$R(P_{k,m}, H_{k,m}) = W \log_2 \left( 1 + \frac{P_{k,m}H_{k,m}}{N_0W} \right)$$

(2)

where $P_{k,m}$ and $H_{k,m}$ is the power and channel gain of $k$-th user at $m$-th slot.

To decouple the delay-optimal design from the detailed implementation of the modulation and coding in the physical layer, we assumed that the data rate (2) is achievable. In fact, it has been shown [15] that the Shannon’s limit in (2) can be achieved to within 0.05dB SNR using LDPC with 2K byte block size at 1% PER. We consider a collision channel for the S-ALOHA random access and hence, the AP could only decode the data successfully when there is only one user transmitting in any time slot. At the end of each slot, the AP broadcasts the ACK/NAK/Collision feedback, denoted as $Z = (1, 0, e)$ [16], to all the $K$ users in the network. For instance, ACK ($Z = 1$) means that exactly one user has transmitted the packet, and data was successfully decoded; NAK ($Z = 0$) means that none of users has transmitted and hence, no data was received; Collision ($Z = e$) means that at least two users have transmitted, and the data was corrupted.

C. Control Policy

Each user decides whether to transmit a packet at the beginning of a slot using a threshold mechanism. Due to symmetry, a user will transmit if the buffer is not empty and its local CSI exceeds a common system threshold $\gamma_m$. If there are more than one backlogged users’ local CSI exceeding the threshold, then collision will occur and none of the packets could get through. As a result, $\gamma_m$ determines the priority on the access opportunity of each user. In this paper, we shall consider an adaptive threshold control to exploit the fading memory to minimize the system delay. A stationary threshold control policy $\pi_\gamma$ is defined below:

**Definition 1 (Stationary Threshold Control Policy):** A stationary threshold control policy $\pi_\gamma : S \times Z \to S$ is defined as the mapping from the previous slot’s system threshold $\gamma_{m-1}$ and common feedback $Z_{m-1}$ from the AP to the current slot’s system threshold $\pi_\gamma(\gamma_{m-1}, Z_{m-1}) = \gamma_m$ in current slot. The set of all feasible stationary policies $\pi_\gamma$ is denoted as $\mathcal{P}_\gamma = \{\pi_\gamma : \pi_\gamma(\gamma_{m-1}, Z_{m-1}) \in S\}$.

The threshold control is adaptive to the common information for all the $K$ users and hence, each user could determine the system threshold just from the feedback from the AP.

1Since we assume strong coding is used by each user, we ignore the case with transmission error.

2In symmetric network, users are statistically identical (e.g. same fading channel, same arrival packet rate and same average power constraint) and a common threshold is reasonable for fairness consideration (achieving the same average delay performance). On the other hand, for the asymmetric network, we have considered the flexibility of different thresholds for different users (because the users are not statistically identical anymore).

3We have assumed the deterministic threshold control policy here. In fact, the same formulation and approach can be used to deal with a transmission probability approach rather than threshold approach. The users will transmit in a probability at different CSI state according to a probability function $\varphi(H) \in [0, 1]$. The transmission control policy is defined as $\pi_\varphi(\gamma_{m-1}, Z_{m-1}) = \varphi_m$, i.e. mapping from the common information to current slot’s transmission probability function.

Denote $X_m = \{Q_m, H_{k,m-1}, \gamma_m, Z_{m-1}, H_{k,m}\}$ to be the global system state at the $m$-th slot and $\chi_{k,m} = \{Q_{k,m}, H_{k,m-1}, \gamma_{m-1}, Z_{m-1}, H_{k,m}\}$ to be the local system state which is observable locally at the $k$-th user. Note that $\{Z_{m-1}, \gamma_m\}$ is the common information for all users, and $\{Q_{k,m}, H_{k,m-1}, H_{k,m}\}$ is the local information for the $k$-th user. Given the observed local system state realization $\chi_{k,m}$, the $k$-th user should adjust the transmission power according to a stationary power control policy $\pi_p$, which is formally defined below.

**Definition 2 (Stationary Power Control Policy):** The stationary power control policy for single user $\pi_p : \mathcal{N} \times S \times S \times Z \to S$ is defined as the mapping from current local system state for $k$-th user, to current slot’s transmit power $\pi_p(\chi_{k,m}) = P_{k,m}$. The set of all feasible stationary policies $\pi_p$ is defined as $\mathcal{P}_p = \{\pi_p : \pi_p(\chi_{k,m}) \geq 0\}$. Note that $P_{k,m} = 0$ for all $H_{k,m} < \gamma_m$, because current slot’s CSI is lower than the threshold.

For simplicity, let $\pi = \{\pi_\gamma, \pi_p\}$ denote the joint control policy of all the $K$ users. The corresponding set of stationary joint control policy is given by $\mathcal{P} = \{\mathcal{P}_\gamma, \mathcal{P}_p\}$. As a result, $\pi(X_m) = \{\pi_\gamma(\gamma_{m-1}, Z_{m-1}), \pi_p(\chi_{k,m}))\}_{k=1}^K = \{\pi_\gamma(\gamma_{m-1}, Z_{m-1}), \pi_p(\chi_{k,m}))\}_{k=1}^K$.

In practice, the user with empty buffer will not transmit even if its local CSI exceeds the system threshold, and this is one important technical challenge in the delay analysis of S-ALOHA network. Instead of dealing with the delay for the original S-ALOHA network, we shall utilize the technique of dominant system [3] to obtain an upper bound of the delay performance. In the dominant system, we assume users always have virtual packets to send (even if the buffer is empty) and therefore, the delay performance associated with the dominant system is always an upper bound of the actual system. Yet, the bound is asymptotically tight in the large delay regime.

III. PROBLEM FORMULATION

In this section, we shall first formulate the delay-optimal control policy problem, and then formally introduce DEC-MDP model. We show that our problem belongs to the memoryless policy case of DEC-MDP in which finding the optimal policy is computationally intractable.

A. System Delay

Due to the nature of random access, the queues of the $K$ users are coupled together via the control policy. When the system threshold is small, there will be a high probability of having more than one users sending packet, leading to collision and wastage of power resource. On the other hand, when the system threshold is high, there is non-negligible probability of having no user sending packet, leading to wastage of idle time. Similarly, individual user may want to increase the transmit
power when the local CSI is good but if there is collision, the transmitted power is wasted. In this paper, we seek to find an optimal stationary control policy to minimize the average delays of the $K$ competing users subject to average transmit power constraint for single user. Specifically, the average delay for the $k$-th user is

$$T_k(\pi) = \limsup_M \frac{1}{M} \mathbb{E} \left[ \sum_{m=1}^{M} Q_{k,m} \right] \quad \forall k \in \{1, \ldots, K\} \quad (3)$$

and average transmit power constraint is given by:

$$P^r(\pi) = \limsup_M \frac{1}{M} \mathbb{E} \left[ \sum_{m=1}^{M} P_{k,m} \right] \leq P_0 \quad (4)$$

where $P_{k,m}$ is the transmitted power determined by $\pi(\chi_{k,m})$, and $P_0$ is the average power constraint for single user. The delay-optimal control problem can be formally written as:

**Problem 1 (Delay Optimal S-ALOHA Control Policy):**
Find a stationary control policy $\pi$ that minimizes

$$J^r(\chi_1) = \sum_k T_k(\pi) + \xi P^r$$

$$= \limsup_M \frac{1}{M} \sum_{k} \mathbb{E} \left[ g_k(\chi_{k,m}, \pi(\chi_{k,m})) \right]$$

where $g_k(\chi_{k,m}, \pi(\chi_{k,m})) = Q_{k,m} + \xi P_{k,m}$ is the per-stage system price function and $\xi > 0$ is the Lagrange multipliers corresponding to the average power constraints in (4).

**B. DEC-MDP Model**

Problem 1 in (5) in fact belongs to the class of infinite horizon DEC-MDP, which is formally defined below [11]:

**Definition 3 (DEC-MDP):** An $K$-agent DEC-MDP is given as a tuple

$$\{I, S, A, P(s'|s, a), R(s, a), p_0\}$$

where $I = \{1, \ldots, K\}$ is a set of agents, $S = \{S_k\}$ is a finite set of states, $A = \{A_k\}$ is a set of joint actions, $S_k$ and $A_k$ is available to agent $k$, $P(s'|s, a)$ is the transition probability that transits from state $s$ to $s'$ given joint action $a$ taken, $R(s, a)$ is the price function given in state $s$ and joint action $a$ taken, $p_0$ is the initial state distribution of the system.

The association between Problem 1 and DEC-MDP is as follows: We have $s_k = \chi_{k,m}$, $a_k = \pi, P(s'|s, a)$ can be easily obtained from local system state transition $P(s_k'|s_k, a_k)$ given in lemma 1 and $R(s, a) = \sum_{k=1}^{K} \mathbb{E} \left[ g_k(\chi_{k,m}, \pi_k(\chi_{k,m})) \right]$. When the policy is given by a mapping from current local system state $s_k$ to actions $a_k \in A_k$, the problem is undecidable [21]. When the policy is given by a mapping from current local system state $s_k$ to actions $a_k \in A_k$, it is called memoryless or reactive policy. In that case, the problem is NP-hard [12], [13]. As a result, it is very difficult to obtain the optimal solution for the Problem 1. Instead of brute-force solution, we shall try to exploit the special structure of our problem to obtain low complexity solutions.

**IV. DELAY-OPTIMAL CONTROL PROBLEM IN SYMMETRIC NETWORK**

In this section, we will focus on exploiting the special structure of the symmetric network. We shall first solve an optimal power control policy by a reduced state MDP for any given threshold control policy. To solve the threshold control problem, we utilize the collision channel mechanism and derive a low complexity solution.

**A. Embedded Markov Chain under a Given Threshold Control Policy**

For a given threshold control policy, the observed local system state for single user is actually evolved as a Markov chain. Specifically, the transition probability conditioned on the power control policy $\pi_P$ is given in the following lemma.

**Lemma 1 (Transition Probability of Local System State):**
At $m$-th slot, the current state of the $k$-th user is $\chi_{k,m} = \{Q_{k,m}, H_{k,m-1}, \gamma_{m-1}, Z_{m-1}, H_{k,m}\}$. Conditioned on $\pi_P$, the transition probability to the next slot is given by:

$$\text{Pr}(\chi_{k,m+1} | \chi_{k,m}, \pi_P(\chi_{k,m})) = \mathbb{I}(\gamma_m = \pi, \gamma_{m-1} = \gamma_{m-1}) \times \text{Pr}(H_{k,m+1} | H_{k,m}) \text{Pr}(Z_{m} | Z_{m-1}, H_{k,m})$$

$$\times \text{Pr}(H_{k,m} | H_{k,m}) \text{Pr}(\chi_{k,m+1} | \chi_{k,m}, Z_{m}, \pi_P(\chi_{k,m}))$$

where $\mathbb{I}(X)$ is an indicator function, which is equal to 1 when event $X$ is true and 0 otherwise.

**Proof:** Please refer to appendix A.

**B. Reduced State MDP Formulation and Optimal Power Control Policy**

For a given threshold control policy in (5), we seek to find an optimal power control policy to minimize

$$J^\pi P(\chi_1) = \limsup_M \frac{1}{M} \sum_{k,m} \mathbb{E} \left[ g_k(\chi_{k,m}, \pi_P(\chi_{k,m})) \right] \quad (7)$$

Note that, power control policy is a function of local system state, and for the $k$-th user, its local system state transition probability is given in (6). The optimal power control policy in (7) could be decoupled into $K$ single-user optimization problems, which can be modeled as a MDP and summarized as following lemma.

**Lemma 2 (Power Control Optimization for Single User):**

The optimal power control policy $\pi^*_P$ minimizing the whole system delay can be modeled as a single user MDP problem, with state space given by local system state $\chi_m$ (ignoring user index $k$). The transition probability is given by

$5$In [17], it is named price, yet called cost in [18]. If it is called a reward, then the problem is then maximized to maximize the reward.

$6$More details about the infinite horizon DEC-MDP is provided in [19] and the references therein.

$7$Undecidability is a formal term in the computational complexity theory used to address the computability and complexity issue on decision problems. A decision problem is called (recursively) undecidable if no algorithm can decide it, such as for Tursing halting problem. It has nothing to do with whether an optimal solution of an optimization problem exist or not (or have multiple solutions), because that depends fundamentally on the structure of the problem. Yet, even if an optimal solution of an undecidable problem exists theoretically, there is no algorithm (iterative) to obtain the optimal solution and terminates [20].

$8$The power action set is compact, due to finite transmit power in practice. By Theorem 8.4.7 in [17], there exists a stationary and deterministic policy that is average optimal. Thus, it is no loss of optimality for this power control policy.
\[ \Pr\{\chi_{m+1}|\chi_m, \pi_P(\chi_m)\} \]

from lemma \[1\] and average price is given by:

\[ J_{\pi_P}(\chi_1) = \lim_{M} \frac{1}{M} \sum_{m=1}^{M} \mathbb{E}[g(\chi_m, \pi_P(\chi_m))] \quad (8) \]

For the infinite horizon MDP, the optimal policy can be obtained by solving the bellman equation recursively w.r.t \( (\theta, \{V(\chi)\}) \) as below:

\[ V(\chi_m) + \theta = \inf_{a(\chi_m)} \left\{ g(\chi_m, a(\chi_m)) + \sum_{\chi_{m+1}} \Pr\{\chi_{m+1}|\chi_m, a(\chi_m)\} V(\chi_{m+1}) \right\} \quad (9) \]

where \( a(\chi_m) = \pi_P(\chi_m) \) is the power allocation when state is \( \chi_m \). If there is a \( (\theta, \{V(\chi)\}) \) satisfying (8), then \( \theta \) is the optimal average price per stage \( J_{\pi_p}(\chi_1) \) and the corresponding optimizing policy is given by \( a^*(\chi_m) \), the optimizing action of (8) at state \( \chi_m \).

Value or policy iteration can be used to solve the bellman equation (9) \[17\], \[18\]. The challenge of the two iteration algorithm lies in the size of the local state space. To reduce the complexity, we shall recast the original MDP in lemma \[2\] into a reduced state MDP. Let’s partition the policy \( \pi_P \) into a collection of actions, the above MDP could be further reduced to a simpler MDP over a reduced state \( \hat{\chi}_m = \{Q_m, H_m-1, \gamma_m-1, Z_m-1\} \) only. Specifically, we have the following definition:

**Definition 4 (Conditional Action):** Given a policy \( \pi_P \), we define \( \pi_P(\chi_m) = \{\pi_P(\chi_m) : \chi_m = (\chi_m, H_m)\} \) as the collection of actions under a given reduced state \( \hat{\chi}_m \) for all possible current slot’s CSI \( H_m \). The policy \( \pi_P \) is therefore equal to the union of all conditional actions, i.e., \( \pi_P = \bigcup\pi_P(\hat{\chi}) \).

Taking conditional expectation (conditioned on \( \hat{\chi} \)) on both sides of (9), and letting \( \bar{V}(\hat{\chi}_m) = \mathbb{E}[V(\chi_m)|\chi_m] = \sum_{H_m} \Pr\{H_m|H_m-1\} V(\chi_m) \), the Bellman equation becomes:

\[ \bar{V}(\hat{\chi}_m) + \theta = \inf_{a(\chi_m)} \left\{ \sum_{H_m} \Pr\{H_m|H_m-1\} \left[ g(\chi_m, a(\chi_m)) + \sum_{\chi_{m+1}} \Pr\{\chi_{m+1}|\chi_m, a(\chi_m)\} \bar{V}(\hat{\chi}_{m+1}) \right] \right\} \]

\[ = \inf_{a(\chi_m)} \left\{ \sum_{H_m} \Pr\{H_m|H_m-1\} g(\chi_m, a(\chi_m)) + \sum_{\chi_{m+1}} \Pr\{\chi_{m+1}|\chi_m, a(\chi_m)\} \bar{V}(\hat{\chi}_{m+1}) \right\} \]

\[ = \inf_{a(\chi_m)} \left\{ \frac{\sum_{H_m} \Pr\{H_m|H_m-1\} g(\chi_m, a(\chi_m))}{\Pr\{\chi_{m+1}|\chi_m, a(\chi_m)\}} \bar{V}(\hat{\chi}_{m+1}) \right\} \]

\[ = \inf_{a(\chi_m)} \left\{ \bar{g}(\hat{\chi}_m, a(\hat{\chi}_m)) + \sum_{\chi_{m+1}} \Pr\{\chi_{m+1}|\chi_m, a(\chi_m)\} \bar{V}(\hat{\chi}_{m+1}) \right\} \]

where \( a(\chi_m) = \pi_P(\chi_m) \) is a single power allocation action at state \( \chi_m \) and \( a(\chi_m) = \pi_P(\hat{\chi}_m) \) is the collection of power allocation actions under a given reduced state \( \hat{\chi}_m \).

Furthermore, \( g(\chi_m, a(\chi_m)) \) is the conditional per-stage price function given by:

\[ g(\hat{\chi}_m, a(\hat{\chi}_m)) = \mathbb{E}[g(\hat{\chi}_m, H_m, a(\chi_m))|\chi_m] \]

\[ = Q_m + \xi \left( \sum_{H_m} \Pr\{H_m|H_m-1\} P_m \right) \quad (11) \]

As a result, the original MDP is equivalent to a reduced state MDP, which is summarized in the following lemma.

**Lemma 3 (Equivalent MDP on a Reduced State Space):**

The original MDP in lemma \[2\] is equivalent to the following reduced state MDP with state space given by \( \hat{\chi}_m \), average price given by:

\[ J_{\pi_P}(\hat{\chi}_1) = \lim_{M} \frac{1}{M} \sum_{m=1}^{M} \mathbb{E}[g(\hat{\chi}_m, a(\hat{\chi}_m))] \quad (12) \]

\[ \Pr\{\hat{\chi}_{m+1}|\hat{\chi}_m, a(\hat{\chi}_m)\} \]

\[ = \sum_{H_m} \Pr\{H_m|H_m-1\} \Pr\{\chi_{m+1}|\chi_m, a(\chi_m)\} \]

\[ (13) \]

The Bellman equation for reduced state MDP is given in (10). Note that while the reduced state MDP is defined over the partial state \( \hat{\chi} \), the power allocation is still a function of the original complete local system state. In fact, for realization of the reduced state \( \hat{\chi}_m \), the solution of the reduced MDP gives the conditional actions for different realization of \( H_m \).

### C. Delay-Optimal Power Control Solution

Value or policy iteration can be used to solve the bellman equation (10). and the convergence of the iteration algorithms is ensured by the following lemma.

**Lemma 4 (Decidability of the Unichain of Reduced State):**

The unichain \[19\] of the reduced state MDP in lemma \[3\] is decidable under all power control policy.

**Proof:** Please refer to appendix \[B\].

The number of unichains of the reduced state MDP in \[3\] depends on the number of recurrent classes of local system state (excluding the queue state \( Q \)) in \( \hat{\chi}_m \), i.e., \( \Phi_m = \{H_i, \gamma_i, Z_i\}_{i=m-1} \). The value or policy iteration could be applied to different unichains respectively, while the convergence and unique solution is ensured \[17\]. Specifically, the bellman equation (10) could be elaborated in an offline manner as follows:

\[ \bar{V}(\hat{\chi}_m) + \theta = \inf_{\pi_P(\hat{\chi}_m)} \left\{ \bar{g}(\hat{\chi}_m, \pi_P(\hat{\chi}_m)) + \sum_{\Phi_m+1} \Pr\{\Phi_m+1|\Phi_m\} \left[ \tau \bar{V}((q_m + 1)\Lambda, N, \Phi_{m+1}) + \tau \mu \bar{V}((q_m - 1)^+, \Phi_{m+1}) + (1 - \tau \lambda - \tau \mu) \bar{V}(q_m, \Phi_{m+1}) \right] \right\} \]

\[ (14) \]

where \( \mu = \mu(\chi_m, Z_m, \pi_P(\chi_m)) \) is the mean packet service rate in \[24\], \( x_N = \min(x, N) \), and let \( \tilde{P}(\chi_m) = \pi_P(\chi_m) \).

In the right hand side of (14), \( P(\chi_m) \) only influence \( \mu \) and...
\( \bar{g} \) is in \((11)\). Specifically, \( \Pr\{\Phi_{m+1} | \Phi_m\} \) is presented simply as \( \Pr\{H_m | H_{m-1}\} \Pr\{Z_m | Z_{m-1}\} \). Hence, the optimal power control policy for a system state \( \chi_m \) is thus given by:

\[
P(\chi_m) = \arg \min_{P(\chi_m)} \left\{ \sum_{H_m=\gamma_m}^\gamma \Pr\{H_m | H_{m-1}\} \left[ \delta(\mu_m, H_m, \gamma_m) + \Pr\{Z_m = 1 | Z_{m-1}\} \right] \right\}
\]

\[
= \left(-W \tau \Pr\{Z_m = 1 | Z_{m-1}\} \delta(\mu_m, H_m, \gamma_m) / (N_0 W \ln 2) \right)
\]

where \( \delta(\mu_m, H_m, \gamma_m) = \bar{V}((\mu_m - 1)^+, H_m, \gamma_m, Z_m = 1) - \bar{V}(\mu_m, H_m, \gamma_m, Z_m = 1) \). Note that the optimal power control action depends on the local CSI via the standard water-filling form. On the other hand, it also depends on the local QSI and common feedback \( Z \) through the water-level \[4\]. Using the optimal power allocation policy, the transition probability of reduced state is \( \Pr\{\chi_{m+1} | \chi_m\} = \Pr\{Q_{m+1} | \chi_m, Z, \mu(\chi_m)\} \Pr\{\Phi_{m+1} | \Phi_m\} \). The stationary distribution of \( \bar{\chi} \), denoted \( \omega(\bar{\chi}) \), could be found by the linear equations \( \omega(\bar{\chi}) = \sum_j \omega(\bar{\chi}_j) \Pr\{\bar{\chi}_j | \bar{\chi}\} \). Finally, the Lagrange multiplier \( \xi \) is chosen to satisfy the average power constraint per user \( P_0 \):

\[
P_0 = \omega(\bar{\chi}_m) \sum_{H_m} \Pr\{H_m | H_{m-1}\} P(\chi_m)
\]

**D. Threshold Control Policy**

Threshold control policy is determined based on the common information \( \{\gamma_{m-1}, Z_{m-1}\} \). The full exploitation of the known information is critical to improve the delay performance of the system. In fact, the common information \( \{\gamma_{m-1}, Z_{m-1}\} \) could be used to exploit the memory of all the \( K \) competing users’ fading channels, and predict their transmission events at the current slot. Specifically, in the collision channel, data will be successfully received by the AP in the S-ALOHA network, if and only if exactly one user transmits at one slot. Consequently, the known information shall be chosen to ensure the user with the largest CSI will transmit alone with the highest probability. Based on this observation, we propose a larger CSI higher priority (LCSIHP) threshold control policy as follows:

\[
\gamma^*_m = \pi(\gamma_{m-1}, Z_{m-1})
\]

\[
= \arg \max_{\gamma_m} \Pr\{\text{only 1 user transmits} | \gamma_{m-1}, Z_{m-1}\}
\]

\[\text{11As a sanity check, when the CSI are i.i.d and the the control policies are not function of QSI (i.e., } \pi(\chi, H) : \mathcal{S} \rightarrow \mathcal{S}, \pi(\mu, H) : \mathcal{S} \rightarrow \mathbb{R})\text{, using similar reduced state MDP technique, the optimal power control policy is represented as: } (-W \tau \sum_{S_i \leq \gamma} \pi(\gamma) / (N_0 W \ln 2) - N_0 W / H_m)^{+}\text{. Where } \xi = (\bar{V}(\mu_m) - \bar{V}(\mu_m - 1)^+) / \xi \text{ is the new Lagrange Multiplier, and considered as a constant since the QSI influence is ignored. Then optimal threshold } \gamma \text{ can be obtained. It is the same as the binary scheduling with power control w.r.t the CSI studied in the [5] called Variable-Rate Algorithms.}
\]

The complexity of the online procedure is negligible because it is simply a table looking up. The complexity of the offline procedure depends mostly on the solution of power control policy, which contains an iteration algorithm to solve the bellman equation in \((14)\). Specifically, the complexity of the reduced state MDP is given in following theorem.

**Theorem 1 (Complexity of the Reduced State MDP):** The worst case complexity of the reduced state MDP is \( O(f(K)) \),
where \( f(K) \) is a monotonic decreasing function of number of users \( K \). Furthermore, there exists a constant \( K_0 > 0 \) such that for all \( K > K_0 \), the complexity is reduced to \( O(NJ) \).

Proof: Please refer to Appendix C.

Theorem 1 implies that when \( K \) is large enough, there is no need to exploit the memory of the fading channels. The threshold is fixed to \( S_J \) regardless of the common feedback. This is reasonable because the more competing users we have, the smaller the chance for single user to transmit. Hence, for sufficiently large \( K \), the users are only allowed to transmit when local CSI reaches the largest state \( S_J \), so as to reduce the intensive collision. Note that, the complexity of the offline procedure is substantially reduced, compared to the complexity \( O(NJ^3) \) of the brute-force solution in the original MDP in lemma 2.

V. EXTENSION TO ASYMMETRIC NETWORK

In this section, we shall extend the delay control framework to asymmetric S-ALOHA network, in which heterogeneous users have different fading channels. Specifically, let \( S_k = \{ S_i \}_{i=1}^{J_k} \) denote a set of \( J_k \) CSI states, \( p_{ki,j}^{\gamma} \) denote the state transition probability and \( \pi_k^{\gamma} \) denote the stationary probability for user \( k \). The common threshold for all users is not applicable for the heterogeneous users and hence, the system threshold \( \gamma_m \) is extended to \( \Gamma_m = \{ \gamma_{k,m} \}_{k=1}^{K} \), where \( \gamma_{k,m} \) is the threshold for user \( k \). As a result, the threshold control policy is extended to \( \Gamma_m = \pi_{m}(\Gamma_{m-1}, Z_{m-1}) \), and power control policy for user \( k \) is denoted as \( \pi_k(\chi_{k,m}) \).

A. Optimal Power Control Policy under a Given Threshold Control Policy

For a given threshold control policy, Lemma 1 still holds. Due to the extension of single threshold \( \gamma_m \) to system threshold \( \Gamma_m \), the transition probability of local system state of the \( k \)-th user should be rewritten as:

\[
\Pr\{\chi_{k,m+1}|\chi_{k,m}, \pi_k^{\gamma}(\chi_{k,m})\} = \prod_{i \neq k} \Pr\{H_{i,m+1}|H_{i,m}\} \prod_{i \neq k} \Pr\{H_{i,m+1}|H_{i,m}\}
\]

(19)

where the transition probability of the feedback state \( Z \) is not as simple as the symmetric case shown in appendix A. For instance, the memory of channel fading of other \( (K-1) \) users should also be exploited through the known information \( \Psi_{m-1} = \{ H_{i,m-1}, \Gamma_{m-1}, Z_{m-1} \} \) of user \( k \). Hence, the joint probability of CSI for other users at \( m \)-th slot is given by:

\[
\Pr\{H_{-k,m}|\Psi_{k,m-1}\} = \prod_{i \neq k} \Pr\{H_{i,m}|H_{i,m-1}\}
\]

(20)

where \( H_{-k,m} = \{ H_{i,m}\}_{i=1,i \neq k}^{K} \) is the set of all users’ CSI at the \( m \)-th slot, excluding the \( k \)-th one, and \( \Pr\{H_{-k,m}|\Psi_{k,m-1}\} = \prod_{i \neq k} \Pr\{H_{i,m}|H_{i,m-1}\} \prod_{i \neq k} \Pr\{H_{i,m}|H_{i,m-1}\} \prod_{i \neq k} \Pr\{H_{i,m}|H_{i,m-1}\} \).
where \( \{v_k, \zeta_k\} \) are obtained in (27), \( \rho_k = Pr\{A_{k,m-1}|\Gamma_{m-1}, Z_{m-1}\} \) is the conditional probability that user \( k \) transmits at the \((m-1)\)-th slot, and \( \overline{\eta}_k = (1 - \rho_k) \).

Hence, we have

\[
\rho_k = \left\{ \begin{array}{ll}
\eta_k \prod_{i \neq k} \overline{\eta}_i / \sum_k \eta_k \prod_{i \neq k} \overline{\eta}_i & \text{if } Z_{m-1} = 1 \\
\eta_k (1 - \prod_{i \neq k} \overline{\eta}_i) / (1 - \prod_i \overline{\eta}_i - \sum_i \eta_i \overline{\eta}_i) & \text{if } Z_{m-1} = 0
\end{array} \right.
\]

(26)

where \( \eta_k = Pr\{A_{k,m-1}|\gamma_{k,m-1}\} = \sum S_j \geq \gamma_{k,m-1} \pi_j^k \) is the transmission probability of user \( k \), given the threshold is \( \gamma_{k,m-1} \), and \( \overline{\eta}_k = 1 - \eta_k \).

C. Summary of the Solution in Asymmetric Network

The overall solution of the control policy in asymmetric network also consists of an offline procedure and an online procedure. Compared with the symmetric case, the optimal power control policy \( \pi_{Pk}(\chi) \) is not the same for all the heterogenous users. In the offline procedure, \( \pi_{Pk}(\chi) \) should be calculated and stored in corresponding user’s table for online looking up.

Similarly, the online procedure is a table looking up and hence, the complexity is negligible. Since the threshold control policy is decoupled to one dimensional optimization problem for single user, the complexity of the offline procedure still depends mostly on the iteration algorithm. Due to the extension of the system threshold, the number of reduced state \( \check{\chi}_{k,m} \) is \( O(N \prod_j J_j) \). However, theorem 1 still holds in the asymmetric network. For sufficiently large \( K \), the threshold control policy will increase the threshold of each user so as to avoid intensive collision. As a result, the number of possible \( \Gamma \) states is substantially reduced and the asymptotic complexity of user \( k \) becomes \( O(N J_k) \) as in the symmetric case.

VI. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we shall illustrate the delay performance of the proposed control policy via numerical simulations. We set the time of a slot \( \tau = 1 \) ms, bandwidth \( W = 1 \) KHz. We model the packet arrival and CSI event follows the assumption in the system model (Section IID). With different simulation scenarios, we calculate the optimal policies in offline. In the online application, the users simply implement the policy at each slot corresponding to the system state observed in that slot. The packet will stay in the buffer until it is successfully serviced, and the performance is evaluated with sufficient realizations.

Fig. 2 compares the LCSIHP threshold control policy (corresponding optimal power control policy) in symmetric network with three reference baselines. Baseline 1 corresponds to the binary scheduling algorithm in [6]. Baseline 2 corresponds to the LCSIHP control policy without power control. Baseline 3 corresponds to the variable-rate algorithm with power control proposed in [5]. We observe that there is a significant gain in both delay and throughput of the proposed policy over these three baselines. Fig. 3 compares packet dropping probability (packet arrives when the buffer is full \( Q = N \)). It shows that packet dropping performance is also improved by the proposed policy. This scenario can also be inferred from the optimal power control policy, which will
potentially put more power on the node with larger QSI to reduce the delay.

Fig. 5. Comparison of delay performance between BSP (power control w.r.t CSI additionally) and proposed Asymm policy in asymmetric network with two heterogenous users, and their CSI models are listed in Table (user1/user2). Specifically, BSP-user2 denotes the delay performance for user 2 under BSP policy, while BSP-user1 is denoted for user 1. BSP-network denotes the average delay performance of the two users under BSP policy. Correspondingly, the notation started with Asymm denotes the delay performance under Asymm policy.

Fig. 6. Comparison of delay performance between BSP (power control w.r.t CSI additionally) and proposed Asymm policy in asymmetric network with 10 heterogenous users. Every group has two homogeneous users.

metric network. There are 10 heterogenous users which are divided into 5 groups. In each group, there are two homogeneous users. Furthermore, we assume a larger buffer size $N = 10$, and $\lambda = 0.4$. It can be observed that in a larger network, the fairness improvement is less obvious. This is because the threshold is increased to avoid the intensive collision both under Asymm or BSP policy, and the freedom of the improvement for Asymm policy is reduced. However, the delay performance is obviously guaranteed due to the additional dimension in QSI for the Asymm policy.

We set $\beta = 0.9$ to leave margin

In our original formulation, we have set the transmit data rate according to the instantaneous mutual information of the channel, i.e., $R_k = W \log_2(1 + \frac{P_k H_k}{\lambda N})$ (see )}. As a result, the transmitted packet could be decoded only when there is exactly one user transmits. In order to allow for possibility of capture, we set the data rate to be $R_k = \beta W \log_2(1 + \frac{P_k H_k}{\lambda N})$ in the simulation, where $\beta < 1$. As a result, we leave some margin in the transmit data rate so that when there is collision, the transmit data rate may still be smaller than the instantaneous mutual information $C_k(\text{collision}) = W \log_2(1 + \frac{P_k H_k}{\sum_{j \neq k} P_j H_j + \lambda N})$ and packet detection is possible. The criteria to determine the success of capture is based on comparing the $R_k$ and $C_k$(collision). If $R_k \leq C_k(\text{collision})$, then the packet from the $k$-th user can be successfully decoded. Otherwise, it will be corrupted.
for the possibility of capture in case of collision. It can be observed that there is significant performance gain of the proposed scheme when there is capture.

VII. SUMMARY

We considered delay-sensitive transmit power and threshold control design in S-ALOHA network. The users adaptively adjust their transmission threshold and power, to achieve the minimal delay of the network. The jointly optimal policy is revealed to be computationally intractable and hence brute force solution is simply infeasible. However, for a given threshold control policy, we decompose the optimal power control policy into a reduced state MDP for single user, in which the overall complexity is \( O(NJ) \). Threshold control policy is proposed by exploiting the special structure of the collision channel and the common feedback to derive a low complexity solution, which is a one dimensional optimization problem in symmetric and asymmetric networks. The delay performance of the proposed design is illustrated to have substantial gain relative to conventional random access approaches in both networks.

APPENDIX A

PROOF OF LEMMA 11: TRANSITION PROBABILITY OF LOCAL SYSTEM STATE

Note that the transition event is from \( \chi_{k,m} \) to \( \chi_{k,m+1} = \{Q_{k,m+1}, H_{k,m}, \gamma_{m}, Z_{m}, H_{k,m+1}\} \). Specifically, the system threshold \( \gamma_{m} \) is given by the threshold control policy, i.e., \( \gamma_{m} = \pi_{\gamma}(\gamma_{m-1}, Z_{m-1}) \) with certainty, and \( \Pr\{H_{k,m+1} = S_{j} | H_{k,m} = S_{i}\} = p_{i,j} \), independent of other states. The transition probability of feedback and queue state is given below.

A. Feedback State Transition

From the position of user \( k \), common feedback \( Z_{m-1} \) and \( \{H_{k,m-1}, \gamma_{m-1}\} \) could provide the information how many other \( (K - 1) \) users have transmitted at the previous slot. It can be utilized to improve the prediction of their transmission behavior at current slot. Moreover, whether user \( k \) transmits at current slot will influence the realization of \( Z_{m} \) and hence, the feedback transition is determined only by \( \{Z_{m-1}, \{H_{k,i}, \gamma_{i}\}_{i=m-1}^{m}\} \). Next we shall find \( \Pr\{Z_{m} | Z_{m-1}, \{H_{k,i}, \gamma_{i}\}_{i=m-1}^{m}\} \) (denote \( \Pr\{Z_{m} | Z_{m-1}\} \) for simplicity) given in (6).

In fact, the common feedback information could modify the stationary probability of CSI states. For instance, \( Z_{m-1} = 0 \) is equal to \( \bigcup_k H_{k,m} \geq \gamma_{k,m-1} \). Given \( H_{k,m} < \gamma_{k,m} \), the stationary probability \( \Pr\{H_{k,m} = S_{j}\} \) should be modified as \( \tilde{\pi}_{j}^{k}(\gamma_{m-1}) = \frac{\pi_{j}^{k}(\gamma_{m-1})}{\sum_{\gamma_{j} < \gamma_{m-1}} \pi_{j}^{k}(\gamma_{m-1})} \). Similarly, Given \( H_{k,m} \geq \gamma_{k,m} \), the stationary probability \( \Pr\{H_{k,m} = S_{j}\} \) should be modified as \( \tilde{\pi}_{j}^{k}(\gamma_{m}) = \frac{\pi_{j}^{k}(\gamma_{m})}{\sum_{\gamma_{j} \geq \gamma_{m}} \pi_{j}^{k}(\gamma_{m})} \). Specifically, we introduce following definition for user \( k \), where \( \gamma_{k,m} \) is the threshold for \( k \)-th user, utilized in section V.

Definition 5 (Transmission Event of the \( k \)-th User): Let \( A_{k,m} \) denote the event that user \( k \) attempts to transmit at the \( m \)-th slot, i.e., \( H_{k,m} \geq \gamma_{k,m} \), while \( \overline{A}_{k,m} \) denote the complimentary event, i.e., \( H_{k,m} < \gamma_{k,m} \). Furthermore, let \( B_{k,m} \in \{A_{k,m}, \overline{A}_{k,m}\} \).

As a result, the probability of the transmission event is given by:

\[
\Pr\{A_{k,m} | \gamma_{k,m}, \gamma_{k,m-1}, B_{k,m-1}\} = \begin{cases} 
\nu_{k} = \sum_{\gamma_{k,m-1}} \sum_{\gamma_{j} \geq \gamma_{k,m}} \tilde{\pi}_{j}^{k}(\gamma_{m-1})p_{i,j}^{k} & \text{if } B_{k,m-1} = \overline{A}_{k,m-1} \\
\zeta_{k} = \sum_{\gamma_{j} \geq \gamma_{k,m}} \tilde{\pi}_{j}^{k}(\gamma_{m-1})p_{i,j}^{k} & \text{if } B_{k,m-1} = A_{k,m-1}
\end{cases}
\]

(27)

For simplicity, let \( \varpi_{k} = 1 - \nu_{k} \), and \( \zeta_{k} = 1 - \zeta_{k}. \) Note that, in symmetric network, \( \bigcup_k \nu_{k} = \nu \) and \( \bigcup_k \zeta_{k} = \zeta. \) Therefore, we ignore the user index \( k \) in the symmetric network.

- **Feedback transits from \( Z_{m-1} = 0 \):** All the other \((K - 1)\) users did not transmit at the previous slot, and transition probability is given by:

\[
\Pr\{Z_{m} | Z_{m-1} = 0\} = \begin{cases} 
\varpi_{K-1}^{k}I(A_{k,m}) & \text{if } Z_{m} = 0 \\
(1 - \varpi_{K-1}^{k})I(A_{k,m}) & \text{if } Z_{m} = 1
\end{cases}
\]

(28)

- **Feedback transits from \( Z_{m-1} = 1 \):** Only one user’s CSI exceeded \( \gamma_{m} \) at the previous slot, which could be divided into two cases. If \( H_{k,m} \geq \gamma_{k,m} \) (\( A_{k,m} \) happens), all the other users did not transmit at the previous slot. The CSI information of other users are the same as \( Z_{m} = 0 \) case, so the transition probability is \( \Pr\{Z_{m} | Z_{m-1} = 1, A_{k,m} \} = \Pr\{Z_{m} | Z_{m-1} = 0\}. \)

If \( H_{k,m} < \gamma_{m} \) (\( \overline{A}_{k,m} \) happens), only one of other users transmitted at the previous slot. Then the transition
Feedback transmits from $Z_{m-1} = e$: At least two users transmitted at the previous slot, which should also be divided into two cases. We first find the probability of exact users involved in the transmission. Specifically, given threshold $\gamma$, the probability of $k$ out $K$ users will transmit is $p_{(k)} = \binom{K}{k} \left( \sum_{j \geq \gamma} \pi_j \right)^k \left( \sum_{j < \gamma} \pi_j \right)^{K-k}$. Given additional information that at least $n$ users will transmit, the probability is improved as

$$p_{(k)}^{(K,n)} = \frac{p_{(k)}^{(K)}}{\sum_{i=0}^{n-1} \left( 1 - p_{(k)}^{(K,i)} \right)}, \forall k \geq n \quad (30)$$

If $H_{k,m-1} \geq \gamma_{m-1}$ ($A_{k,m-1}$ happens), at least one of other users transmitted, i.e.,

$$\Pr\{Z_{m-1} = e, A_{k,m-1}\} = \begin{cases} \sum_{k=1}^{K-1} p_{(K,k)}^{(K-1)} \left( \frac{\zeta}{\tau} \frac{\zeta}{\tau} \right)^{(K-1)-k} I(A_{k,m}) & \text{if } Z_m = 0 \\ \sum_{k=1}^{K-1} p_{(K,k)}^{(K-1)} \left( \frac{\zeta}{\tau} \frac{\zeta}{\tau} \right)^{(K-1)-k} I(A_{k,m}) + \sum_{k=1}^{K-1} p_{(K,k)}^{(K-1)} \left( \frac{\zeta}{\tau} \frac{\zeta}{\tau} \right)^{(K-1)-k} I(A_{k,m}) & \text{if } Z_m = e \end{cases} \quad (31)$$

If $H_{k,m-1} < \gamma_{m-1}$ ($A_{k,m-1}$ happens), at least two of other users transmitted, i.e.,

$$\Pr\{Z_{m-1} = e, A_{k,m-1}\} = \begin{cases} \sum_{k=1}^{K-1} p_{(K,k)}^{(K-1)} \left( \frac{\zeta}{\tau} \frac{\zeta}{\tau} \right)^{(K-1)-k} I(A_{k,m}) & \text{if } Z_m = 0 \\ \sum_{k=1}^{K-1} p_{(K,k)}^{(K-1)} \left( \frac{\zeta}{\tau} \frac{\zeta}{\tau} \right)^{(K-1)-k} I(A_{k,m}) + \sum_{k=1}^{K-1} p_{(K,k)}^{(K-1)} \left( \frac{\zeta}{\tau} \frac{\zeta}{\tau} \right)^{(K-1)-k} I(A_{k,m}) & \text{if } Z_m = e \end{cases} \quad (32)$$

### B. Queue State Transition

The queue state transition is correlated with the feedback. For instance, if $Z_m \neq 1$, the probability of decreased queue state should be zero, because of successful data receipt. To obtain simple solution, we consider the case the same as [22], where the time slot duration $\tau$ is substantially smaller than the average packet inter-arrival time and average packet service time $\frac{1}{\mu}$ ($\tau \ll \frac{1}{\mu}$ and $\tau \ll \frac{1}{\mu}$), where $\mu$ is the average packet service rate defined later.

- **Packet arrival:** Since packet arrival follows Poisson distribution with mean arrival rate $\lambda$, the transition probability of the queue state related to packet arrival is given by:

$$p_{q,q+1} = \Pr\{Q_{k,m+1} = q + 1|Q_{k,m} = q\} = \lambda \tau \quad (33)$$

- **Packet departure:** The packet length follows exponential distribution with mean packet size $N_0$, so the packet service time also follows exponential distribution. Conditioned on the state $(\chi_{k,m}, Z_m)$ and data rate given in (2), the mean packet service rate is:

$$\mu(\chi_{k,m}, Z_m, \pi(\chi_{k,m})) = \frac{W}{N_0} \log_2(1 + \frac{P_{m,H_{k,m}}}{N_0W})$$

where $P_{m,m} = \pi(\chi_{k,m})$ is the power transmitted at current slot determined by power control policy. Furthermore, $Z_m \neq 1$ will lead to zero service rate. Another case leads to zero service rate is $H_k < \gamma_m$, in which the power control policy will set $P_{m,m} = 0$. Hence, the probability for packet departure is given by:

$$p_{q,q-1} = \Pr\{Q_{k,m+1} = (q-1)^+|Q_{k,m} = q, \chi_{k,m}, Z_m, \pi(\chi_{k,m})\} = \mu(\chi_{k,m}, Z_m, \pi(\chi_{k,m})) \tau \quad (35)$$

- **No change in the $k$-th user:** The transition probability corresponding to no change in queue state is given by:

$$p_{q,q} = \Pr\{Q_{k,m+1} = q|Q_{k,m} = q, \chi_{k,m}, Z_m, \pi(\chi_{k,m})\} = (1 - p_{q,q-1} - p_{q,q+1}) \quad (36)$$

Since $\lambda \tau \ll 1$ and $\mu \tau \ll 1$, the probability of multiple packet arrivals or packet departures is negligible and hence $p_{q,p} = 0$ for $|p - q| > 1$. Thus the transition probability of queue state is given by $\Pr\{Q_{k,m+1}|\chi_{k,m}, Z_m, \pi(\chi_{k,m})\}$, which completes the proof.

### APPENDIX B

#### PROOF OF LEMMA

**Decidability of the Unichain of Reduced State**

Denote the state (excluding $Q$) in $\chi$ as $\Phi = (H, \gamma, Z)$, whose transition probability has been given in lemma 1 independent of $Q$ and power control policy. Specifically, $Pr\{\Phi_{m+1}|\Phi_m\} = Pr\{H_m|H_{m-1}\} Pr\{Z_m|\Phi_m, H_m, \gamma_m\}$, where $\gamma_m$ is determined from the given threshold control policy. Then, the recurrent classes of $\Phi$ could be found. Furthermore, the queue state evolves as a birth-death process.
under every power control policy, forming an unichain itself. As a result, the unichain of the reduced state $\chi = (Q, \Phi)$ is decidable.

**APPENDIX C**

**Proof of Theorem 1**

**Complexity of the Reduced State MDP**

$\{\xi_k, v_k\}$ in (27) are functions of $\{\gamma_{m-1}, \gamma_m\}$. Specifically, we assume $\xi_k \geq v_k$ for the same $\{\gamma_{m-1}, \gamma_m\}$. This is a practical assumption for fading channels, because the CSI states will not change fast [14]. Then, we have following lemma about the threshold control policy $\gamma_m$ in (17).

Lemma 5 (Monotonic Increasing Function of $\gamma_m$ w.r.t $K_2$): Given $\{\gamma_{m-1}, Z_{m-1}\}$, if $K_2 \geq K_1$, $\gamma_m(\gamma_{m-1}, Z_{m-1}, K_2) \geq \gamma_m(\gamma_{m-1}, Z_{m-1}, K_1)$. Specifically, if $\gamma_m(\gamma_{m-1}, Z_{m-1}, K_1) < \gamma_m(\gamma_{m-1}, Z_{m-1}, K_2)$, then for a sufficiently large $K_2$, $\gamma_m(\gamma_{m-1}, Z_{m-1}, K_2) > \gamma_m(\gamma_{m-1}, Z_{m-1}, K_1)$.

Proof: Given $\{\gamma_{m-1}, Z_{m-1}\}$, $\gamma_m$ just influence the $\{\zeta, v, \gamma, Z\}$ parameter in (17), and from (27), $\{\zeta, v\}$ are monotonic decreasing ($\{\zeta, v\}$ are monotonic increasing) functions of $\gamma_m$. As a result, lemma 5 is obvious when $Z_{m-1} \neq e$. If $Z_{m-1} = e$, when $K_1$ is increased to $K_2$, by comparing each term of the same $k$ case and in additional $k = (K_1 + 1) \cdots K_2$ case, using the assumption of $\zeta \geq v$ for the same $\{\gamma_{m-1}, \gamma_m\}$, the monotonic increasing characteristic is also obvious.

As the reduced state is $\chi = \{Q, H, \gamma, Z\}$, the worst case complexity is corresponding to the total number of states of $\chi$, i.e., $O(N J^2)$. On the other hand, since the QSI and CSI states are recurrent, the least number of states in a recurrent class is $O(N J)$. Next we will show that the number of states of the system threshold $\gamma$ decreases as $K$ increases, and $\gamma = S_J$ regardless of the feedback when $K$ is large enough, which completes the proof.

Given $K_1$, let $\gamma_{\min}(K_1)$ be the minimal threshold in a recurrent reduced state class. Specifically, $\gamma_{\min}(K_1) = \gamma^*_m(\gamma_{K_1}, Z_{K_1}, K_1)$, where $\gamma_{K_1} > \gamma_{\min}(K_1)$. By lemma 5 for a sufficiently large $K_2 > K_1$, $\gamma_{\min}(K_2) > \gamma_{\min}(K_1)$ and hence, the minimal threshold in the recurrent class is increased. Following the argument, the minimal threshold will increase to the largest CSI state $S_J$, when $K$ is increased to a large number $K_0$.

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Huang Huang received the B.Eng. and M.Eng. (Gold medal) from the Harbin Institute of Technology (HIT) in 2005 and 2007 respectively, all in Electrical Engineering. He is currently a PhD student at the Department of Electrical and Computer Engineering, The Hong Kong University of Science and Technology. His recent research interests include cross layer design and performance analysis via game theory in random access network, and embedded system design.
Vincent Lau obtained B.Eng (Distinction 1st Hons) from the University of Hong Kong (1989-1992) and Ph.D. from Cambridge University (1995-1997). He was with HK Telecom (PCCW) as system engineer from 1992-1995 and Bell Labs - Lucent Technologies as member of technical staff from 1997-2003. He then joined the Department of ECE, Hong Kong University of Science and Technology (HKUST) as Associate Professor. His current research interests include the robust and delay-sensitive cross-layer scheduling of MIMO/OFDM wireless systems with imperfect channel state information, cooperative and cognitive communications, dynamic spectrum access as well as stochastic approximation and Markov Decision Process.