Constraints on dark energy dynamics and spatial curvature from Hubble parameter and baryon acoustic oscillation data

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1 INTRODUCTION

It is widely accepted that the universe is undergoing accelerated expansion today. The consensus cosmological model, $\Lambda$CDM, posits that this acceleration is driven by the spatially homogeneous, constant dark energy density $\rho_\Lambda$ of the cosmological constant $\Lambda$ (Peebles 1984). For reviews of the accelerated cosmological expansion and of the $\Lambda$CDM model, see Ratra & Vogeley (2008), Martin (2012), Brax (2018), and Luković et al. (2018). In this model, cold dark matter (CDM) is the second largest contributor to the current energy budget and, with non-relativistic baryonic matter, powered the decelerating cosmological expansion at earlier times.

The consensus $\Lambda$CDM model assumes flat spatial hypersurfaces, but observations don’t rule out mildly curved spatial hypersurfaces; observations also do not rule out the possibility that the dark energy density varies slowly with time. In this paper we examine, in addition to the general (not necessarily spatially flat) $\Lambda$CDM model, the XCDM parametrization of dynamical dark energy, and the $\phi$CDM model in which a scalar field $\phi$ is the dynamical dark energy.\textsuperscript{1} In the XCDM and $\phi$CDM cases we allow for both vanishing and non-vanishing spatial curvature. Details of the three models we study are summarized in Sec. 2, and more information can be found in Farooq (2013).

Ooba et al. (2018) have recently shown that, in the spatially flat case, the Planck 2015 CMB anisotropy data from Planck Collaboration (2016) (and some baryon acoustic oscillation distance measurements) weakly favor the $\Lambda$CDM parametrization and the XCDM model of dynamical dark energy over the $\Lambda$CDM consensus model. The XCDM case results have been confirmed by Park & Ratra (2018a) for a much bigger compilation of cosmological data, including most available Type Ia supernova apparent magnitude observations, BAO distance measurements, growth factor data, and Hubble parameter observations.\textsuperscript{2} Also, spatially flat XCDM and $\phi$CDM both reduce the tension between CMB temperature anisotropy and weak gravitational lensing estimates of $\sigma_8$, the rms fractional energy density inhomogeneity averaged over $8 h^{-1}$Mpc radius spheres, where $h$ is the Hubble constant in units of 100 km s$^{-1}$ Mpc$^{-1}$ (Ooba et al. 2018; Park & Ratra 2018a).

In non-flat models nonzero spatial curvature provides an additional length scale which invalidates usage of the power-law power spectrum for energy density inhomogeneities in the non-flat case (as was assumed in the analysis of non-flat models in Planck Collaboration 2016). Non-flat inflation models (Gott 1982; Hawking 1984; Ratra 1985) provide the only known physically-consistent mecha-
nism for generating energy density inhomogeneities in the non-flat case; the resulting open and closed model power spectra are not power laws (Ratra & Peebles 1994, 1995; Ratra 2017). Using these power spectra, Ooba et al. (2017a) have found that the Planck 2015 CMB anisotropy data in combination with a few BAO distance measurements no longer rule out the non-flat ΛCDM case (unlike the earlier Planck Collaboration (2016) analyses based on the incorrect assumption of a power-law power spectrum in the non-flat model). Park & Ratra (2018b) confirmed these results for a bigger compilation of cosmological data, and similar conclusions hold in the non-flat dynamical dark energy XCDM and φCDM cases (Ooba et al. 2017b,c; Park & Ratra 2018a).

Additionally, the non-flat models provide a better fit to the observed low multipole CMB temperature anisotropy power spectrum, and do better at reconciling the CMB anisotropy and weak lensing constraints on σ8, but do a worse job at fitting the observed large multipole CMB anisotropy temperature power spectrum (Ooba et al. 2017a,b,c; Park & Ratra 2018a,b). Given the non-standard normalization of the Planck 2015 CMB anisotropy likelihood and that the flat and non-flat ΛCDM models are not nested, it is not possible to compute the relative goodness of fit between the flat and non-flat ΛCDM models quantitatively, although qualitatively the flat ΛCDM model provides a better fit to the current data (Ooba et al. 2017a,b,c; Park & Ratra 2018a,b).

In the analyses discussed above, the Planck 2015 CMB anisotropy data played the major role. Those authors found consistency between cosmological constraints derived using the CMB anisotropy data in combination with various non-CMB data sets. CMB anisotropy data are sensitive to the behavior of cosmological spatial inhomogeneities. Here we derive constraints on spatial inhomogeneities using all available CMB data as well as all available radial and transverse BAO data. Unlike the CMB anisotropy data, the $H(z)$ and these BAO data are not sensitive to the behavior of cosmological spatial inhomogeneities.

The models that we study, and the methods we use to analyze these data, are the same as those presented in Farooq et al. (2017, 2015), and we also use some of the same $H(z)$ and baryon acoustic oscillation measurements. We differ from those studies by now using all currently available $H(z)$ and baryon acoustic oscillation data.

The constraints we derive here are consistent with, but weaker than, those of the papers cited above; this provides a necessary and useful consistency test of those results. In particular, we find that the consensus flat ΛCDM model is a reasonable fit, in most cases, to the BAO and $H(z)$ data we study here. However, depending somewhat on the Hubble constant prior we use, consensus flat ΛCDM can be 1σ away from the best-fit parameter values in some cases, which can favor mild dark energy dynamics or non-flat spatial hypersurfaces.

In Sec. 2 we provide a short summary of the models we studied. Sec. 3 presents the data that we used, and in Sec. 4 we describe the methods by which we analyzed these data. Sec. 5 describes the results of our analyses, and our conclusions are given in Sec. 6.

2 MODELS

The models we examine in this paper are characterized by their expansion rate as a function of redshift $z$,

$$E(z) = \frac{H(z)}{H_0}.$$  (1)

Here $H(z)$ is the Hubble parameter and $H_0 \equiv H(0)$ is the Hubble constant.

In the ΛCDM model dark energy is a constant vacuum energy density with negative pressure, equivalent to an ideal fluid with equation of state parameter

$$w = \frac{p_b}{\rho_b} = -1.$$  (2)

Here $p_b$ and $\rho_b$ are the homogeneous parts of the pressure and energy density, respectively. The expansion rate can be written in terms of the density parameters

$$E(z) = \sqrt{\Omega_{m0}(1+z)^3 + (1-\Omega_{m0} - \Omega_{\Lambda})(1+z)^2 + \Omega_{\Lambda}},$$  (3)

where $\Omega_{m0}$ is the current value of the non-relativistic matter density parameter, $\Omega_{\Lambda}$ is the cosmological constant energy density parameter, and $\Omega_{k0} = 1 - \Omega_{m0} - \Omega_{\Lambda}$ (which is nonzero in general) is the current value of the spatial curvature energy density parameter. Here, and in the other models we study, we ignore the contributions from CMB photons and neutrinos, which are very small at the redshifts of the data we use, so the ΛCDM model is characterized by two parameters: $p = (\Omega_{m0}, \Omega_{\Lambda})$.

In the XCDM parametrization of dark energy, $w = w_X$ where $w_X$ is a negative constant (in general $w_X \neq -1$). Hence

$$E(z) = \sqrt{\Omega_{m0}(1+z)^3 + \Omega_{k0}(1+z)^2 + \Omega_{X0}(1+z)^{(1+w_X)}},$$  (4)

where $\Omega_{X0}$ is the current value of the dark energy density. In contrast to ΛCDM, the dark energy density parameter $\Omega_{X0}(1+z)^{(1+w_X)}$ varies with time. If, however, $w_X = -1$, then XCDM reduces to ΛCDM, with $\Omega_{k0} + \Omega_{\Lambda}$. In general, the XCDM parametrization has three free parameters: $p = (\Omega_{m0}, \Omega_{X0}, -w_X)$. We shall also consider spatially flat XCDM, with $p = (\Omega_{m0}, -w_X)$.

The φCDM model (Peebles & Ratra 1988; Ratra & Peebles 1988; Farooq 2013; Pavlov et al. 2013) provides a simple, physically consistent description of dynamical dark energy. In this model, the dark energy is a scalar field $\phi$ with a potential energy density given by

$$V(\phi) = \frac{1}{2}k m_n^2 \phi^\alpha.$$  (5)

Here $\alpha > 0$, $m_n^2 \equiv G^{-1}$, $G$ is the gravitational constant, and

$$k = \left(\frac{8}{3}\right)\left(\frac{\alpha + 4}{\alpha + 2}\right)^{2/3} \alpha^{\alpha/2}.$$  (6)

In the XCDM parametrization, the energy density and pressure of the dark energy fluid, $\rho_X(t)$ and $p_X(t)$, are space-independent functions of time. When $\rho_X(t)$ is negative, this is an inconsistent parametrization that is rendered consistent by assuming a constant speed of acoustic inhomogeneities (typically $c_s X = 1$). The BAO and $H(z)$ data we consider only constrain the spatially homogeneous part of the cosmological models.
respectively. 

BAO provide observers with a “standard ruler” which can be used to measure cosmological distances (see Bassett & Hlozek 2010 for a review). These distances can be computed in a given cosmological model, so measurements of them can be used to constrain the parameters of the model in question. The BAO distance measurements we use are listed in Table 1.

The transverse co-moving distance is

\[ D_M(z) = \frac{D_C}{H_0 \sqrt{\Omega_0}} \left( \frac{c}{H_0} \right) \sinh \left( \frac{D_C}{c} H_0 \right) \]  

\[ \sinh \left( \frac{D_C}{c} H_0 \right) = \begin{cases} \frac{D_C}{c} H_0, & \text{if } \Omega_0 > 0, \\ \frac{D_C}{c} H_0, & \text{if } \Omega_0 < 0, \end{cases} \]  

where \( D_C \) and \( H_0 \) are the comoving distance and the Hubble constant, respectively.

### DATA

BAO provide observers with a “standard ruler” which can be used to measure cosmological distances (see Bassett & Hlozek 2010 for a review). These distances can be computed in a given cosmological model, so measurements of them can be used to constrain the parameters of the model in question. The BAO distance measurements we use are listed in Table 1.

| \( z \) | \( \Delta z \) | \( \sigma \) | \( \text{Ref.} \) |
|---|---|---|---|
| 0.07 | 69 | 19.6 | Zhang et al. (2014) |
| 0.09 | 69 | 12 | Simon et al. (2005) |
| 0.12 | 68.6 | 26.2 | Zhang et al. (2014) |
| 0.17 | 83 | 8 | Simon et al. (2005) |
| 0.179 | 75 | 4 | Moresco et al. (2012) |
| 0.199 | 75 | 5 | Moresco et al. (2012) |
| 0.2 | 72.9 | 29.6 | Zhang et al. (2014) |
| 0.27 | 77 | 14 | Simon et al. (2005) |
| 0.28 | 88.8 | 36.6 | Zhang et al. (2014) |
| 0.352 | 83 | 14 | Moresco et al. (2012) |
| 0.3802 | 83 | 13.5 | Moresco et al. (2016) |
| 0.4 | 95 | 17 | Simon et al. (2005) |
| 0.4004 | 77 | 10.2 | Moresco et al. (2016) |
| 0.4247 | 87.1 | 11.2 | Moresco et al. (2016) |
| 0.4497 | 92.8 | 12.9 | Moresco et al. (2016) |
| 0.47 | 89 | 50 | Ratsimbazafy et al. (2017) |
| 0.4783 | 80.9 | 9 | Moresco et al. (2016) |
| 0.48 | 97 | 62 | Stern et al. (2010) |
| 0.593 | 104 | 13 | Moresco et al. (2012) |
| 0.68 | 92 | 8 | Moresco et al. (2012) |
| 0.781 | 105 | 12 | Moresco et al. (2012) |
| 0.875 | 125 | 17 | Moresco et al. (2012) |
| 0.88 | 90 | 40 | Stern et al. (2010) |
| 0.90 | 117 | 23 | Simon et al. (2005) |
| 1.037 | 154 | 20 | Moresco et al. (2012) |
| 1.3 | 168 | 17 | Simon et al. (2005) |
| 1.363 | 160 | 33.6 | Moresco et al. (2015) |
| 1.43 | 177 | 18 | Simon et al. (2005) |
| 1.53 | 140 | 14 | Simon et al. (2005) |
| 1.75 | 202 | 40 | Simon et al. (2005) |
| 1.965 | 186.5 | 50.4 | Moresco et al. (2015) |

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where

\[
D_H = \frac{c}{H(z)}
\]

(14)

\[
D_C = \frac{c}{H_0} \int_0^z \frac{dz'}{E(z')}
\]

(15)

and the volume-averaged angular diameter distance is

\[
D_V(z) = \left[ \frac{c^2 D_M^2(z)}{H_0 E(z)} \right]^{1/3}
\]

(16)

(Hogg 1999; Farooq 2013). All of the measurements in Table 1 are scaled by the size of the sound horizon at the drag epoch \(r_d\). This quantity is (see Eisenstein & Hu 1998 for a derivation):

\[
r_s = \frac{2}{3k_{\text{eq}}} \sqrt{\frac{6}{R_{\text{eq}}} \ln \left[ \frac{1 + R_d + \sqrt{R_d^2 + R_{\text{eq}}^2}}{1 + \sqrt{R_{\text{eq}}^2}} \right]}
\]

(17)

where \(R_d \equiv R(z_d)\) and \(R_{\text{eq}} \equiv R(z_{\text{eq}})\) are the values of \(R\), the ratio of the baryon to photon momentum density,

\[
R = \frac{3p_B}{4\pi r}
\]

(18)

at the drag epoch and matter-radiation equality epoch, respectively. Here \(k_{\text{eq}}\) is the scale of the particle horizon at the matter-radiation equality epoch, and \(p_B\) and \(r\) are the baryon and photon mass densities. In analyses, where appropriate, the original data listed in Table 1 have been rescaled to a fiducial sound horizon \(r_{\text{fid}} = 147.60\) Mpc (from Table 4, column 3, of Planck Collaboration 2016). This fiducial sound horizon was determined by using the ΛCDM model, so its value is model dependent, though not to a significant degree (as can be seen by comparing the computed \(r_s\) of the Planck Collaboration 2016 baseline model to that measured using the spatially open ΛCDM and flat XCDM parametrization of Planck Collaboration 2016).

In Table 2 we list 31 \(H(z)\) measurements determined using the cosmic chronometric technique, which are the same as the cosmic chronometric \(H(z)\) data used in Yu et al. (2018) (see e.g. Morescoc et al. 2012 for a discussion of cosmic chronometers). With this method, the Hubble rate as a function of redshift is determined by using

\[
H(z) = \frac{1}{1 + z} \frac{dz}{dt}.
\]

(19)

Although this determination of \(H(z)\) does not depend on a cosmological model, it does depend on the quality of the measurement of \(dz/dt\), which requires an accurate determination of the age-redshift relation for a given chronometer. See Morescoc et al. (2012) and Morescoc (2015) for discussions of the strengths and weaknesses of this method. While their approach requires accurate knowledge of the star formation history and metallicity of massive, passively evolving early galaxies, and although the two different techniques they use give slightly different values, they also point out that the measurement of \(H(z)\) from this method is relatively insensitive to changes in the chosen stellar population synthesis model.

4 METHODS

To determine the values of the best-fit parameters, we minimized \(\chi^2(p) \equiv -2\ln L(p)\),

\[
\chi^2(p) \equiv -2\ln L(p)
\]

(20)
the number of data points. BIC and AIC provide means to compare models with different numbers of parameters; they penalize models with a higher $k$ in favor of those with a lower $k$, in effect enforcing Occam’s Razor in the model selection process.

To determine the confidence intervals $r_n$ on the 1d best-fit parameters, we computed one-sided limits $r_n^\pm$ by using

$$
\frac{\int_{p}^{r_n^+} L(p) dp}{\int_{p}^{r_n^-} L(p) dp} = \sigma_n,
$$

(28)

where $\hat{p}$ is the point at which $L(p)$ has its maximum value, such that $n = 1, 2$ and $\sigma_1 = 0.6827$, $\sigma_2 = 0.9545$. Because the one-dimensional likelihood function is not guaranteed to be symmetric about $\hat{p}$, we compute the upper and lower confidence intervals separately. In the $\Lambda$CDM model, for example, the 1-sigma confidence intervals on $\Omega_{m0}$ are computed by first integrating the likelihood function $L(\Omega_{m0}, \Lambda)$ over $\Lambda$ to obtain a marginalized likelihood function that only depends on $\Omega_{m0}$.

$$
\int_{0}^{1} L(\Omega_{m0}, \Lambda) d\Lambda = L(\Omega_{m0}),
$$

(29)

and then inserting this marginalized likelihood function into eq. (28).

The ranges over which we marginalized the parameters of the $\Lambda$CDM model were $0 \leq \Omega_\Lambda \leq 1$ and $0.01 \leq \Omega_{m0} \leq 1$. For the spatially flat XCDM parametrization, we used $-2 \leq w_X \leq 0$ and $0.01 \leq \Omega_{m0} \leq 1$, and for the spatially flat $\phi$CDM model we used $0.01 \leq \alpha \leq 5$ and $0.01 \leq \Omega_{m0} \leq 1$. For 3-parameter XCDM, we used $-0.7 \leq \Omega_{\phi} \leq 0.7$, $0.01 \leq \Omega_{m0} \leq 1$, and $-2.00 \leq w_X \leq 0$. For the 3-parameter $\phi$CDM model we considered $-0.5 \leq \Omega_{\phi} \leq 0.5$, $0.01 \leq \Omega_{m0} \leq 1$, and $0.01 \leq \alpha \leq 5$.  

We analyzed the data with two independent Python codes,
written by S.D. and J.R., that produced almost identical results in the 2-parameter cases and the 3-parameter XCDM parametrization, and results that agreed to within 1% in the 3-parameter ΩCDM case.

5 RESULTS

The confidence contours for the models considered are shown in Figs. 1, 2, and 3. The solid black contours indicate the $H_0 = 68 ± 2.8$ km s$^{-1}$ Mpc$^{-1}$ prior constraints, the dashed black contours indicate the $H_0 = 73.24 ± 1.74$ km s$^{-1}$ Mpc$^{-1}$ prior constraints, and the red dots indicate the best-fit point in each prior case. Our results for the parameter values of the un marginalized and marginalized red dots indicate the best-fit point in each prior case. For the parameter limits listed in Tables 5 and 6, we list the 1σ and 2σ confidence intervals on the parameters of each of the 2-parameter (3-parameter) models. We obtained these by marginalizing the 2-parameter (3-parameter) likelihood function as described in Sec. 4. The best-fit points in these tables correspond to the maximum value of the relevant one-dimensional marginalized likelihood function. Table 3 (4) lists the corresponding two-dimensional (three-dimensional) best-fit points.

From the figures and tables, we see that the spatially flat ΛCDM model is a reasonable fit to the $H(z)$ and BAO data we use (although the flat XCDM parametrization and flat ΩCDM model provide slightly better fits in the $H_0 ± 3σH_0 = 68 ± 2.8$ km s$^{-1}$ Mpc$^{-1}$ case). In particular, from the figures, for the $H_0 ± 3σH_0 = 68 ± 2.8$ km s$^{-1}$ Mpc$^{-1}$ prior, flat ΛCDM is always within about 1σ of the best-fit value. However, the $H_0 ± 3σH_0 = 73.24 ± 1.74$ km s$^{-1}$ Mpc$^{-1}$ case favors some larger deviations from flat ΛCDM. For example in the middle panel of Fig. 1 for the flat XCDM parametrization it favors a phantom model over flat ΛCDM at a little more than 1σ, while in the center and right panels of Fig. 3 for the non-flat ΩCDM case it also favors a closed model at a little more than 2σ. Similar conclusions may be drawn from the parameter limits listed in Tables 5 and 6.

When both dynamical dark energy and spatial curvature are present (as opposed to cases with only dynamical dark energy or only spatial curvature) it is not as easy to constrain both parameters simultaneously. This can be seen by comparing the center and right panels of Fig. 1 to the left panels of Figs. 2 and 3, respectively. When spatial curvature is allowed to vary, the confidence contours in the 3-parameter XCDM parametrization and the ΩCDM model expand along the $\omega_X$ and $\alpha$ axes (these are the parameters that govern the dynamics of the dark energy).

The consensus model, spatially flat ΛCDM, is consistent with current $H(z) +$ BAO data, but these data allow some nonzero spatial curvature. In particular, we find that the best-fit values of the parameters in the ΛCDM model imply a curvature energy density parameter of $\Omega_{k0} = 0.03$ for the $H_0 ± 3σH_0 = 68 ± 2.8$ km s$^{-1}$ Mpc$^{-1}$ prior case, and $\Omega_{k0} = -0.07$ for the $H_0 ± 3σH_0 = 73.24 ± 1.74$ km s$^{-1}$ Mpc$^{-1}$ prior case. More precisely, using the $\Omega_{m0}$ and $\Omega_{\lambda}$ best-fit values and error bars for flat ΛCDM from Table 5, and combining the errors in quadrature, an approximate estimate is $\Omega_{k0} = 0.03(1 ± 1.8)$ and $\Omega_{k0} = -0.07(1 ± 0.59)$ for the $H_0 ± 3σH_0 = 68 ± 2.8$ km s$^{-1}$ Mpc$^{-1}$ and $H_0 ± 3σH_0 = 73.24 ± 1.74$ km s$^{-1}$ Mpc$^{-1}$ priors, with the data favoring a closed model at a little over 1σ in the second case. The 3-parameter models, in both prior cases, favor closed spatial hypersurfaces, but the error bars are so large that these results only stand out in the $H_0 ± 3σH_0 = 73.24 ± 1.74$ km s$^{-1}$ Mpc$^{-1}$ prior case of the ΩCDM model (see the center and right panels of 3). While not very statistically significant, we note that these results are not inconsistent with those of Ooba et al. (2017a,b,c) and Park & Ratra (2018a,b), who found that CMB anisotropy data, in conjunction with other cosmological data, were not inconsistent with mildly closed spatial hypersurfaces.

The current data are also not inconsistent with some mild dark energy dynamics, although the size of the effect varies depending on the choice of $H_0$ prior and whether or not $\Omega_{k0}$ is allowed to vary as a free parameter. In the flat ΩCDM model, for instance, $\alpha$ can be different from zero only in the $H_0 ± 3σH_0 = 68 ± 2.8$ km s$^{-1}$ Mpc$^{-1}$ prior case, whereas $\alpha$ can be different from zero in both prior cases if $\Omega_{k0}$ is allowed to vary (see the right panel of 1 and the left panel of 3).

6 CONCLUSIONS

We analyzed a total of 42 measurements, 31 of which consisted of uncorrelated $H(z)$ data points, with the remainder coming from BAO observations (some correlated, some not), to constrain dark energy dynamics and spatial curvature, by determining how well these measurements can be described by three common models of dark energy: ΛCDM, the XCDM parametrization, and ΩCDM.

The consensus flat ΩCDM model is in reasonable accord with these data, but depending on the model analyzed and the $H_0$ prior used, it can be a little more than 1σ away from the best-fit model. These data are consistent with mild dark energy dynamics as well as non-flat spatial hypersurfaces. While these results are interesting and encouraging, more and better data are needed before we can make definite statements about the spatial curvature of the universe and about dark energy dynamics.

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Figure 3. Confidence contours for 3-parameter $\phi$CDM. Solid (dashed) 1, 2, and 3$\sigma$ contours correspond to $H_0 \pm 3\sigma_{H_0} = 68 \pm 2.8$ (73.24 ± 1.74) km s$^{-1}$ Mpc$^{-1}$ prior, and the red dots indicate the location of the best-fit point in each prior case. Left: $\Omega_{m0}$ marginalized. The horizontal $\alpha = 0$ axis corresponds to the $\Lambda$CDM model. Center: $\Omega_{\phi0}$ marginalized. The vertical $\alpha = 0$ axis corresponds to the $\Lambda$CDM model and the horizontal blue dashed line here and in the next panel correspond to the spatially flat $\phi$CDM case. Right: $\alpha$ marginalized. Color online.

Table 3. Best-fit values for 2-parameter models. $\Delta \chi^2$ is evaluated relative to $\chi^2$ of $\Lambda$CDM for each $H_0$ prior.

| $H_0$ prior (km s$^{-1}$ Mpc$^{-1}$) | Model | $\Omega_{m0}$ | $\Omega_{\phi0}$ | $w_X$ | $\alpha$ | $\chi^2$ | $\Delta \chi^2$ | AIC | BIC |
|-------------------------------------|-------|--------------|----------------|-------|---------|---------|----------------|-----|-----|
| 68 ± 2.8                           | $\Lambda$CDM | 0.29 | 0.68 | - | - | 25.35 | 0.00 | 29.35 | 32.83 |
|                                     | flat $\phi$CDM | 0.29 | -0.94 | - | 0.16 | 25.05 | -0.30 | 29.05 | 32.53 |
| 73.24 ± 1.74                       | $\Lambda$CDM | 0.30 | 0.77 | - | - | 26.92 | 0.00 | 30.92 | 34.40 |
|                                     | flat $\phi$CDM | 0.29 | -1.13 | - | 0.01 | 28.26 | 1.34 | 32.26 | 35.74 |

Table 4. Best-fit values for 3-parameter models. $\Delta \chi^2$, $\Delta$AIC, and $\Delta$BIC are evaluated relative to $\chi^2$, AIC, and BIC of $\Lambda$CDM for each $H_0$ prior.

| $H_0$ prior (km s$^{-1}$ Mpc$^{-1}$) | Model | $\Omega_{m0}$ | $\Omega_{\phi0}$ | $w_X$ | $\alpha$ | $\chi^2$ | $\Delta \chi^2$ | AIC | BIC |
|-------------------------------------|-------|--------------|----------------|-------|---------|---------|----------------|-----|-----|
| 68 ± 2.8                           | XCDM | 0.31 | -0.18 | -0.76 | - | 23.65 | -1.70 | 29.65 | 0.30 | 34.86 | 2.03 |
|                                     | $\phi$CDM | 0.31 | -0.22 | -0.96 | 0.96 | 23.82 | -1.53 | 29.82 | 0.47 | 35.03 | 2.20 |
| 73.24 ± 1.74                       | XCDM | 0.32 | -0.21 | -0.84 | - | 26.48 | -0.44 | 32.48 | 1.56 | 37.69 | 3.29 |
|                                     | $\phi$CDM | 0.32 | -0.26 | -0.62 | 0.62 | 26.30 | 0.95 | 32.30 | 1.38 | 37.51 | 3.11 |

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### Table 5. 1σ and 2σ parameter intervals for 2-parameter models.

| H_0 prior (km s^{-1}Mpc^{-1}) | Model | Marginalization range | Best-fit | 1σ                  | 2σ                  |
|--------------------------------|-------|-----------------------|----------|---------------------|---------------------|
| 68 ± 2.8                       | ΛCDM  | 0 ≤ Ω_{Λ0} ≤ 1       | Ω_{m0} = 0.29 | 0.27 ≤ Ω_{m0} ≤ 0.31 | 0.26 ≤ Ω_{m0} ≤ 0.32 |
|                                |       | Ω_{Λ0} = 0.68        |          | 0.63 ≤ Ω_{Λ0} ≤ 0.73 | 0.58 ≤ Ω_{Λ0} ≤ 0.77 |
| flat XCDM 0 ≤ w_X ≤ 0          |       | Ω_{m0} = 0.29        | 0.28 ≤ Ω_{m0} ≤ 0.31 | 0.26 ≤ Ω_{m0} ≤ 0.33 |
| flat XCDM 0.01 ≤ Ω_{m0} ≤ 1    |       | w_X = -0.94          | -1.02 ≤ w_X ≤ -0.87 | -1.10 ≤ w_X ≤ -0.80 |
| flat φCDM 0.01 ≤ α ≤ 5         |       | Ω_{m0} = 0.29        | 0.28 ≤ Ω_{m0} ≤ 0.31 | 0.26 ≤ Ω_{m0} ≤ 0.33 |
| flat φCDM 0.01 ≤ Ω_{m0} ≤ 1    |       | α = 0.16             | 0.06 ≤ α ≤ 0.43    | 0.02 ≤ α ≤ 0.72     |
| 73.24 ± 1.74                   | ΛCDM  | 0 ≤ Ω_{Λ0} ≤ 1       | Ω_{m0} = 0.30 | 0.29 ≤ Ω_{m0} ≤ 0.32 | 0.27 ≤ Ω_{m0} ≤ 0.33 |
|                                |       | Ω_{Λ0} = 0.77        |          | 0.73 ≤ Ω_{Λ0} ≤ 0.81 | 0.69 ≤ Ω_{Λ0} ≤ 0.84 |
| flat XCDM 0 ≤ w_X ≤ 0          |       | Ω_{m0} = 0.29        | 0.28 ≤ Ω_{m0} ≤ 0.31 | 0.26 ≤ Ω_{m0} ≤ 0.32 |
| flat XCDM 0.01 ≤ Ω_{m0} ≤ 1    |       | w_X = -1.13          | -1.20 ≤ w_X ≤ -1.06 | -1.27 ≤ w_X ≤ -1.00 |
| flat φCDM 0.01 ≤ α ≤ 5         |       | Ω_{m0} = 0.31        | 0.29 ≤ Ω_{m0} ≤ 0.32 | 0.28 ≤ Ω_{m0} ≤ 0.34 |
| flat φCDM 0.01 ≤ Ω_{m0} ≤ 1    |       | α = 0.01             | 0.01 ≤ α ≤ 0.09    | 0.01 ≤ α ≤ 0.20     |

### Table 6. 1σ and 2σ parameter intervals for 3-parameter models.

| H_0 prior (km s^{-1}Mpc^{-1}) | Model | Marginalization range | Best-fit | 1σ                  | 2σ                  |
|--------------------------------|-------|-----------------------|----------|---------------------|---------------------|
| 68 ± 2.8                       | XCDM  | -0.7 ≤ Ω_{k0} ≤ 0.7  | Ω_{m0} = 0.31 | 0.28 ≤ Ω_{m0} ≤ 0.33 | 0.25 ≤ Ω_{m0} ≤ 0.36 |
|                                |       | w_X = -0.70           | -0.93 ≤ w_X ≤ -0.62 | -1.27 ≤ w_X ≤ -0.57 |
|                                |       | Ω_{k0} = -0.11        | -0.36 ≤ Ω_{k0} ≤ 0.06 | -0.59 ≤ Ω_{k0} ≤ 0.19 |
|                                |       | w_X = -0.70           | -0.93 ≤ w_X ≤ -0.62 | -1.27 ≤ w_X ≤ -0.57 |
| flat φCDM -2 ≤ w_X ≤ 0         |       | Ω_{m0} = 0.31        | 0.28 ≤ Ω_{m0} ≤ 0.33 | 0.25 ≤ Ω_{m0} ≤ 0.36 |
| flat φCDM 0.01 ≤ Ω_{m0} ≤ 1    |       | Ω_{k0} = -0.11        | -0.36 ≤ Ω_{k0} ≤ 0.06 | -0.59 ≤ Ω_{k0} ≤ 0.19 |
| flat φCDM 0.01 ≤ α ≤ 5         |       | α = 1.12             | 0.51 ≤ α ≤ 1.59    | 0.11 ≤ α ≤ 1.97     |
| flat φCDM 0.01 ≤ Ω_{m0} ≤ 1    |       | Ω_{k0} = -0.22        | -0.38 ≤ Ω_{k0} ≤ 0.07 | -0.48 ≤ Ω_{k0} ≤ 0.03 |
|                                |       | α = 1.16             | 0.53 ≤ α ≤ 1.61    | 0.12 ≤ α ≤ 2.01     |
| 73.24 ± 1.74                   | XCDM  | -0.7 ≤ Ω_{k0} ≤ 0.7  | Ω_{m0} = 0.32 | 0.29 ≤ Ω_{m0} ≤ 0.33 | 0.27 ≤ Ω_{m0} ≤ 0.35 |
|                                |       | w_X = -0.82           | -1.08 ≤ w_X ≤ -0.71 | -1.44 ≤ w_X ≤ -0.63 |
|                                |       | Ω_{k0} = -0.11        | -0.33 ≤ Ω_{k0} ≤ 0.03 | -0.55 ≤ Ω_{k0} ≤ 0.14 |
|                                |       | w_X = -0.82           | -1.08 ≤ w_X ≤ -0.71 | -1.44 ≤ w_X ≤ -0.63 |
| flat φCDM -2 ≤ w_X ≤ 0         |       | Ω_{m0} = 0.32        | 0.29 ≤ Ω_{m0} ≤ 0.34 | 0.26 ≤ Ω_{m0} ≤ 0.36 |
| flat φCDM 0.01 ≤ Ω_{m0} ≤ 1    |       | Ω_{k0} = -0.11        | -0.33 ≤ Ω_{k0} ≤ 0.03 | -0.55 ≤ Ω_{k0} ≤ 0.14 |
| flat φCDM 0.01 ≤ α ≤ 5         |       | α = 0.76             | 0.30 ≤ α ≤ 1.14    | 0.06 ≤ α ≤ 1.41     |
| flat φCDM 0.01 ≤ Ω_{m0} ≤ 1    |       | Ω_{k0} = -0.25        | -0.40 ≤ Ω_{k0} ≤ -0.14 | -0.48 ≤ Ω_{k0} ≤ -0.07 |
|                                |       | α = 0.79             | 0.32 ≤ α ≤ 1.16    | 0.06 ≤ α ≤ 1.43     |
|                                |       | Ω_{k0} = -0.24        | -0.40 ≤ Ω_{k0} ≤ -0.14 | -0.48 ≤ Ω_{k0} ≤ -0.07 |
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