EXCITATION OF RESONATORS BY ELECTRON BEAMS

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Abstract

Elements of a little-known vector theory of open resonators and experiments on excitation of a fundamental mode with transverse and longitudinal polarization in such resonators are discussed.

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The propagation of electromagnetic waves in vacuum is described by Maxwell equations. More simple wave equations for electromagnetic fields coupled by conditions \( \nabla \times E = \nabla \times H = 0 \) or for the potentials \( A \), \( \phi \) introduced by the equations \( H = \nabla \times A \), \( E = \nabla \phi \) and coupled by a gauge condition can be used for a simplified solution of electrodynamical problems. The following simplification using the electric and magnetic Hertz vectors \( \mathbf{E} \), \( \mathbf{H} \) is made. They are introduced by \( A = (1/c)(\mathbf{E} \times \mathbf{H}) = (\mathbf{E} \times \mathbf{H}) \) and \( \phi = \mathbf{E} \cdot \mathbf{H} \) are defined both the gauge and Maxwell equations. In this case, the electric and magnetic field strengths are

\[
E = \nabla \phi = \frac{1}{c} \nabla \left( \frac{\mathbf{E} \times \mathbf{H}}{c^2} \right) \quad \text{and} \quad H = \nabla \times E = \frac{1}{c} \nabla \left( \frac{\mathbf{E} \times \mathbf{H}}{c^2} \right). \quad \text{(1)}
\]

The vectors \( \mathbf{E} \) and \( \mathbf{H} \) fill the wave equation \( \nabla \times (\nabla \times \mathbf{E}) = 0 \). We can identify the solution of the scalar wave equation \( \nabla \times (\nabla \times \mathbf{E}) = 0 \) with one component of vectors \( \mathbf{E} \) or \( \mathbf{H} \) \( (\mathbf{E} \times \mathbf{H}) = 0 \). Substituting the vector in (1) we find the electromagnetic field strengths. Then we can identify the same solution \( \mathbf{E} \) by another component of the Hertz vector, equate the rest components to zero, and calculate the other electromagnetic field strengths. After going through all the components of components we obtain a set of six different electromagnetic wave modes [1].

The monochromatic light beam of a limited diameter related to the resonator modes can be written in the form \( \mathbf{E}(x,y,z) = e^{i(kx + tz)} \), where \( V(x,y,z) \) is the function of a coordinate slowly varying in comparison with \( \exp[i(kx + tz)] \). In a paraxial approximation \( \mathbf{E} = e^{i(kx + tz)} \), \( 2k \left| \mathbf{E} \right| = 0 \). The solution of this equation by the method of separation of variables in the cylindrical coordinates, where \( V(r) = 0 \), \( \mathbf{E}(r,z) = G(u) \exp[ikr^2 + 2q(z)] \exp[iS(z)] \), \( r = \sqrt{x^2 + y^2} \), and \( u = u = \sqrt{r^2 + z^2} \), has the form

\[
V(r,z) = \frac{C}{w(z)} \sin m \frac{\pi m}{m} \left( \frac{2r^2}{w(z)^2} \right) \quad \text{and} \quad \exp \left( i\frac{kr^2 + 2q(z)}{2q(z)} \right) \exp[iS(z)] \quad \text{(2)}
\]

where \( q(z) \) are the Laguerre polynomials \( L_n^m(\ ) = \)

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E1; L0 = 1; ..., Zk = w0k, the Rayleigh length; = 2 o1, the wavelength; C = constant; l=3(z) = _l=R(z) + 1 = w2; R(z) = z[l + (z^2=R(z)]2, the radius of the wave front of Gaussian beam; w2(z) = w02[1 + (z^2=R(z))]2; w(z), the radius of the beam, and w0(z) the radius of the beam waist.

The combinations of U(x0; z) = 0 and v = 0; y = 0; z = 0 or z = 0; x = 0; y = 0; z = 0; y = 0; x = 0; z = 0, together with the conditions v = 0, where the upper superscripts show the components of the electric Hertz vector corresponding to the transverse (xy) and longitudinal (z) polarizations. The second case can be solved as the first case by substitution of the variable x by y, and vice versa. The fields strengths are calculated by the magnetic Hertz vector E3 = B; H = E.

In the first case, the Green's beam has only the transverse electric components, whereas E3 = 0. In the second case, the electric components are described by the values E2(x = 0) E 0; y = 0; z = 0, [1], [2]. These are the fundamental transversely and longitudinally polarized TEM01 and TEM00 modes, respectively. The TEM00 modes can be excited by the transition radiation emitted on mirrors of open resonators by electrons that are homogeneously moving along their axes. Such an excitation was probably observed in the experiments published in [3]. Previous experiments on excitation of open resonators were done under conditions when the electron trajectories were directed either at some angle to the axis of a resonator [4], or on caustics of the fundimental mode of a resonator [5], where the fundamental TEM00 modes had not a longitudinal component of the electric field strength at the open resonator axis. Electron beam s of finite transverse dimensions moving along the axes of open resonators can excite non-zero and higher modes like TEM102 having small longitudinal component nears the axes of resonators [3, b].

Excitation of the TEM00 modes is possible at even harmonics of undulator radiation in free-electron lasers using the undulators with a high density of longitudinal oscillations at high R. Radiation stored at this mode in supercavity can be used for laser-driven acceleration in vacuum as well.

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