Research on Regional Debt Risk in Hubei Province Based on Modified KMV Model

Zhiheng Xu, Wen Fan and Fan Zhu*
School of management, Wuhan university of technology, Wuhan, China

*Corresponding author e-mail: xzhglxy@whut.edu.cn

Abstract. With the increasing amount of Regional debt, how to explain and describe this phenomenon has become very important. The revised KMV model can precisely undertake this work, and quantify the level of debt risk through the description of default probability and other factors, providing a theoretical basis for managers to solve this risk.

1. Introduction

The basic principle of the KMV model is that the enterprise regards the owner's equity as a call option, the liability as a put option, and the company's assets as the underlying asset. The credit risk of the enterprise mainly depends on the market value of the enterprise's assets, the volatility and the book value of the debt.

When the future market value of the enterprise assets is lower than the face value of the debts required by the enterprise, the enterprise will default. The distance between the expected value of the future market value of the enterprise assets and the default point is the default distance DD, and can also be based on the company’s default distance database. The expected default frequency is the default probability P of the company in a certain period of time in the future.

When this idea is applied to the assessment of the repaying capability of government bonds, it can be understood that when the government bonds expire, if the fiscal revenue used for guarantees is less than the debts to be repaid, it means that the regional government defaults. According to this idea, the debt evaluation model is constructed by using the four indicators of the growth rate of debt service, the growth rate of solvency and the volatility and the amount of debts to be paid, and the default distance of the debt repayment entity is calculated, and the probability of default is obtained.

2. Establish modified KMV model

2.1. Assumptions

Suppose the Regional fiscal revenue is subject to the geometric Brownian movement, namely:

$$dM_t = g_m M_t dt + \sigma_m M_t dz_t$$  \hspace{1cm} (1)

Where $M_t$ is the regional government's solvency at time $t$, $g_m$ is the growth rate of solvency, and $\sigma_m$ is the change rate of solvency. $z_t$ represents a wiener process, namely Brownian motion.
2.2. Default distance

When the Regional debt matures, the regional government will be at risk of default if the current available financial resources $M_T$ is less than the debt payable $B_T$. Then there is the calculation formula of default distance.

\[
DD = \frac{\ln \frac{M_T}{B_T} + \frac{\ln B_T - \frac{\ln M_T^2}{2}}{\sigma MVT}}{\sigma MVT}
\]  

(2)

2.3. Probability of default

Where, $T$ is the maturity date, $\Delta t$ is the debt maturity being examined, $t$ is the current moment, that is, $t = 0$, and $T = \Delta t$ because $\Delta t = T - t$. $M_T$ is the fiscal revenue used for debt repayment, while $B_T$ is the debt payable by regional governments.

Assuming that Regional fiscal revenue obeys lognormal distribution, the default probability can be calculated from default distance [1]:

\[
P = N(-DD) = N \left[ -\frac{\ln \frac{M_T}{B_T} + \frac{\ln B_T - \frac{\ln M_T^2}{2}}{\sigma MVT}}{\sigma MVT} \right]
\]  

(3)

3. Parameter prediction

In order to predict the debt data from 2019 to 2020, this paper first calculates the growth rate and volatility of 2011-2018 solvency, debts, and debt-servicing financial resources, and based on this calculation.

3.1. Debt service resources $M_T$

The disposable ratio refers to the ratio of fiscal revenue to total fiscal revenue that can be used to repay debt. Based on the viewpoints of Ru Tao (2009) and Han Liyan (2003), the basic construction grant and enterprise technological transformation investment are set as regional fiscal revenues that can be used for guarantees, and the disposable ratio is about 30%. Expressed as:

\[
M_T = 0.3FR
\]  

(4)

Among them, FR represents the fiscal revenue of the regional government in the current period, namely the general public budget revenue.

The datas, FR and GDP in 2011-2018, are both positively correlated relationship. In order to decrease the volatility of the data, improve the accuracy of the model, this paper will take logarithm respectively. Two variables, $\ln$ (FR) and $\ln$(GDP), are founded, showing a certain linear relationship between them. Therefore, EViews software is selected in this paper to establish a linear regression equation of one variable by using the least square method, and the following equation can be obtained:

\[
\ln (FR) = -4.6619 + 1.2185 \ln(GDP)
\]  

(5)

\[
R^2 = 0.9229
\]  

(6)

\[
F = 0.0001
\]  

(7)

To determine the goodness of fit of the equation and the statistical significance of the estimated parameters, $R^2$ with $F$ as a test statistic, the fitting equation was tested for significance. $R^2$ represents the proportion of the independent variable to the dependent variable in the regression equation, in general, if $R^2$ exceeds 0.8, the good fit of the model is considered to be high, so the regression equation of this paper is overall significant. Furthermore, $F<0.01$ indicates that the dependent variable has a significant linear relationship with the independent variable at the level of 0.01 [3].

For the time series with linear ascending trend, the second exponential smoothing method has a good predictive effect. This method has the characteristics of simple calculation, less sample
requirement and stable results. This method has been used by many scholars in the past for GDP and financial forecast of data. Therefore, this paper uses the EViews software to select the second exponential smoothing method to predict the GDP from 2019 to 2020. Bring predictive values into a one-dimensional linear equation (5) In the middle, you can get the predicted fiscal revenue from 2019 to 2020, and (4) Calculate the predicted value of the debt service. The forecast results are shown below:

| Table 1. Forecast Results. |
|---------------------------|
| Years | GDP (100 million yuan) | FR (100 million yuan) | M_{T} Predictive value |
| 2019  | 42668                   | 4141                   | 1242                   |
| 2020  | 46066                   | 4547                   | 1364                   |

3.2. Debt payable $B_T$

The following formula should be used in predicting the amount of debt payable:

$$B_T = r_T \sum BV + (1 + r_T)BV_T$$  \hspace{1cm} (8)

Among them, the $\sum BV$ on behalf of the current outstanding debt balance, $BV_T$ represent the current balance of maturing debt. The balance of the current mature debt from 2019 to 2020 can be obtained from the bond information disclosure document of provincial finance department. On this basis, the outstanding debt balance of the whole province = the balance of the whole province's debt at the beginning of the period - the principal of the whole province's matured debt in the current year + the new debt of the whole province in the current year. Referring to the report of the NPC Standing Committee on standardizing the management of regional debt, the standard interest rate was set at 3.5% of the cost of existing debt after the replacement. The calculation process is as follows:

| Table 2. Calculation Process. |
|-----------------------------|
| Years | Unexpired debt principal $B_T = r_T \sum BV + (1 + r_T)BV_T$ | Initial debt balance (2) | Debt principal (3) | New debt (4) | Debt payable $B_T$ (5)
| 2019  | 7511                    | 6676                   | 540               | 1415           | 823               |
| 2020  | 9325                    | 8416                   | 736               | 1645           | 1088              |

Note: The new debt data in 2019 and the balance of debts at the beginning of 2020 come from the budget approved by the Ministry of Finance; The new debt data in 2020 is calculated by fitting the linear equations of the previous three years.

3.3. The financial growth rate of repayment of debts $\mu$ and the volatility of solvency and financial resources $\sigma$

Han Liyan (2003) combined the Black-Scholes model with Ito's lemma to derive the formula for growth rate:

$$\mu_t = \frac{1}{n-1} \sum_{t=1}^{n-1} \ln \frac{M_{t+1}}{M_t} + \frac{1}{2} \sigma m^2$$  \hspace{1cm} (9)

The calculation of volatility $\sigma m$ formula is [2]:

$$\sigma_m = \sqrt{\frac{1}{n-2} \sum_{t=1}^{n-1} \left( \ln \frac{M_{t+1}}{M_t} - \frac{1}{n-1} \sum_{t=1}^{n-1} \ln \frac{M_{t+1}}{M_t} \right)^2}$$  \hspace{1cm} (10)

Substituting the data on debt service financial resources into the available predictable values:
Table 3. Predictable Values.

| Years | Volatility $\sigma$ | Growth rate $\mu$ |
|-------|---------------------|-------------------|
| 2019  | 0.1122              | 0.1023            |
| 2020  | 0.1045              | 0.1175            |

The measured growth rate of debt-servicing financial resources $\sigma$ and the volatility $\sigma$ is substituted into the formulas (2) and (3) to obtain the default distance DD and the default probability P.

Table 4. Values of DD & P.

| Years | DD    | P      |
|-------|-------|--------|
| 2019  | 4.5216| 0.0003%|
| 2020  | 6.0923| 0.0000%|

4. Conclusion

According to the results just analyzed, the risk of government debt default in 2020 is significantly lower than that in 2018. Companies with credit ratings above Moody's Baa3 (above Standard & Poor's BBB-) rarely default, as measured by Moody's. Because government bonds are more stable and collateralized than corporate bonds, government bonds should have a higher level of safety than corporate bonds, so government bonds should be at least as safe as Moody's Baa (or Standard & Poor's BBB+), which means the expected default probability should be within 0.4 percent.

Therefore, the default risk of the regional government in this province is small and the default probability is almost zero. According to the data of other provinces and cities calculated by the above method, except Shanxi, Qinghai, Xinjiang and other western regions where the default probability is relatively large, the default rate measured by most provinces and cities is basically within the safe range within two years. Therefore, it can be concluded that both Hubei province and most regions maintain an appropriate bond issuance scale.

In addition, the limitations of the model to be considered in practical application are as follows:

(i) The default probability under the real distribution is different from the theoretical default probability. According to Han Liyan (2002) for two kinds of default probability calculation, because in the probability density curve, the real distribution of revenue than the theoretical distribution is more wide and flat, so for the same proportion of the debt, the real distribution is measured in the case of the probability of default will be higher than the calculated according to the theoretical value of normal distribution.

(ii) Due to incomplete information disclosure and inconsistent rating caliber, it is difficult to ensure the accuracy of data prediction. To make credit ratings more objective and convincing, regional governments should provide more comprehensive information, such as specific spending items of debt funds and repayment plans.

5. Acknowledgments

This work was financially supported by National Innovation and Entrepreneurship Training Program for College Students of Wuhan university of technology (Project No. 201910497224).

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