Rapid Oscillatory quintessence with variable equation of state parameter in non minimal derivative coupling model

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Abstract

The rapid oscillating scalar field is considered as the quintessence in the framework of nonminimal kinetic coupling model. The scalar field behaves like a perfect fluid with a variable equation of state parameter which can be expressed as a function of the Hubble parameter. This feature enables us to study its evolution via continuity equation. The scalar field may behave as dark energy in the present epoch, while it behaves like dark matter in the future. This characteristic allows the occurrence of a decelerated expansion after the present acceleration, without invoking any interaction. The model is also studied in the presence of an interaction between dark sectors alleviating the coincidence problem. The stability of the model is investigated and the stable attractor solutions are studied.

1 Introduction

In 1998, based on astrophysical data, some evidences indicating that the expansion of the Universe has been accelerated [1], were found. Many models have been proposed to describe this odd phenomenon which is still inexplicable in the context of the general reactivity with known matters. These models may be classified in two categories: (Einstein) General theory of gravity with an exotic source with negative pressure dubbed as dark energy [2], and theories based on modification of general relativity in large scale [3]. The most natural and simple candidate for dark energy is the cosmological constant which suffers from fine tuning and coincidence problems, and may not be related to the vacuum energy of the standard particle physics model due to its extremely small value [4]. Inspired by the inflation theory in the early

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Universe, one can introduce an exotic scalar field dubbed as quintessence playing the role of dark energy [5]. The advantage of this model is its dynamics which may solve the coincidence problem and explain the evolution of dark energy density. In this framework, to describe the present acceleration of the Universe, the scalar field model with a non-minimal derivative coupling to gravity was proposed in [6]. Thereafter, the non minimal derivative coupling model has also been employed to describe the possible super acceleration in the late time [7]. Further studies about this model can be found in [8].

Although most of the scalar field models use the slow roll conditions to explain the inflation, but as was quoted in [9], the acceleration may be continued in the rapid oscillation after the slow roll. Also in [10] it was tried to introduce the quintessence as a rapid oscillating scalar field. In the rapid oscillation phase the scalar field behaves as a fluid with a constant equation of state (EoS) parameter which, by a suitable choice of the potential, becomes enough negative to result in the cosmological acceleration.

In this paper we consider the spatially flat Friedman Robertson Walker (FRW) Universe in the framework of non minimal derivative coupling model proposed in [6].

In the second section we give an overview of the model and introduce the rapid oscillating scalar field as the quintessence. As in the present time we cannot neglect the contribution of dark matter, we generalize the results obtained in [12] which was derived for a Universe consisted only of inflaton. In contrast to the minimal case, the scalar field acquires a dynamical EoS parameter which depends on the Hubble parameter. This enable us to use the continuity equation to study its evolution. The dependence of the EoS parameter on the Hubble parameter gives rise to some novel and interesting features such as the possibility that the Universe reenters a decelerated expansion without considering any energy exchange between dark matter and dark energy components. This feature is explained explicitly by deriving a series solution in the third section. In that section we also obtain the ratio of dark energy and dark matter energy densities at the beginning of the acceleration (deceleration) phase. In resemblance to the inflaton decay during its rapid oscillation, we consider also the case that the oscillatory quintessence is allowed to exchange energy but only with dark matter alleviating the coincidence problem. The stability of the model and its late time attractor solutions are discussed. At the end of the third section a short discussion is given on the instabilities of interacting dark energy model in the perturbed FRW Universe. In the fourth section, our previous results are discussed by using some illustrative examples. The evolution of dark energy and dark energy are numerically depicted in terms of the redshift parameter and behaviors of the EoS parameter of the scalar field and the decelerated parameter of the Universe are discussed.

We use units $h = c = 1$. 

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2 Rapid oscillatory scalar field with variable EoS parameter

We consider the action

\[ S = \int \left[ \frac{1}{2} M_P^2 R - \frac{1}{2} \left( g^{\mu \nu} - W G^{\mu \nu} \right) \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) \right] \sqrt{-g} d^4 x + S_m, \quad (1) \]

describing a scalar field, \( \varphi \), whose the kinetic term is coupled to the Einstein tensor \( G^{\mu \nu} = R^{\mu \nu} - \frac{1}{2} g^{\mu \nu} R \) via the coupling \( W \) which has inverse-square mass dimension \([6]\). \( S_m \) is the matter action containing components other than the scalar field. \( M_P \) is the reduced Planck mass, \( M_P = \sqrt{\frac{1}{8 \pi G}} \approx 1.2209 \times 10^{19} GeV \). The Universe is taken to be a spatially flat FRW space time, with scale factor \( a(t) \).

In the absence of interaction between \( \varphi \) and other sectors, the equation of motion of the scalar field \( \varphi(t) \) is

\[ (1 + 3 WH^2) \ddot{\varphi} + 3H (1 + 3 WH^2 + 2 \dot{H}) \dot{\varphi} + V'(\varphi) = 0, \quad (2) \]

where \( H := \frac{\dot{a}}{a} \) is the Hubble parameter. The dark energy density, \( \rho_\varphi \), and pressure, \( P_\varphi \), are given by

\[
\begin{align*}
\rho_\varphi &= \frac{1}{2} (1 + 9 WH^2) \dot{\varphi}^2 + V(\varphi) \\
P_\varphi &= \frac{1}{2} (1 - 3 WH^2) \dot{\varphi}^2 - V(\varphi) - W \frac{d(H \dot{\varphi}^2)}{dt},
\end{align*}
\]

satisfying the continuity equation

\[ \dot{\rho}_\varphi + 3H (\rho_\varphi + P_\varphi) = 0. \quad (4) \]

In our study, \( V(\varphi) \) is an even function specified by the power law form

\[ V(\varphi) = \lambda \varphi^n, \quad (5) \]

where \( n > 0 \). We focus on (quasiperiodic) rapidly oscillating scalar field solutions characterized by

\[ H \ll \frac{1}{T}, \quad (6) \]

where \( T \) is the period of the \( \varphi \) oscillation, and \( \frac{H}{T} \ll \frac{1}{T} \). \( T \) is given by

\[ T = 2 \int_{\Phi}^{\Phi} \frac{d\varphi}{\dot{\varphi}}, \quad (7) \]

where \( \Phi \) is the amplitude of quasiperiodic oscillation of \( \varphi \). This type of solution was discussed in the literature to study the reheating era after the inflation in the minimal coupling case \([13]\) . Using the Friedman equation

\[ H^2 = \frac{1}{3M_P^2} \rho, \quad (8) \]
we find that the total energy density, like the Hubble parameter, does not change significantly during one oscillation \[14\]

\[ \left| \frac{\dot{H}}{H} \right| \approx \frac{1}{T(t)}. \]  

In the non minimal derivative coupling case \([11]\) and in high friction regime: \(WH^2 \gg 1\), the rapidly oscillating scalar field, following the same steps as \([14]\), was studied in details in \([12]\) (we do not repeat the details here) in a Universe dominated by \(\varphi\) and in high friction regime \(WH^2 \gg 1\), the solution may be represented as \(\varphi(t) = \Phi(t) \cos \left( \frac{2\pi}{T(t)} \right)\), where \(\Phi\) is the amplitude of the \(\varphi\) oscillation. Like the harmonic oscillation the energy may be approximated by the energy potential at the tuning points:

\[ \rho_\varphi = V(\Phi) = \lambda \Phi^n. \]  

The period of an oscillation is given (see \([14]\))

\[ T = 2 \int_{-\Phi}^{\Phi} d\bar{\varphi} \sqrt{\frac{1 + 9WH^2}{\rho_\varphi - V(\varphi)}} \]

\[ = 2 \sqrt{\frac{1 + 9WH^2}{\rho_\varphi}} \Phi^{1-n} \int_0^1 \frac{dx}{\sqrt{1-x^n}} \]

\[ = \frac{\Gamma \left( \frac{1}{n} \right)}{n \Gamma \left( \frac{1}{2} + \frac{1}{n} \right)} \sqrt{\frac{8\pi(1 + 9WH^2)}{\rho_\varphi}} \Phi^{1-n}. \]  

Rapid oscillation condition \([13]\) may now be rewritten as

\[ \frac{\Phi}{M_P} \ll \sqrt{\frac{3n \Gamma \left( \frac{1}{n} + \frac{1}{n} \right)}{8\pi \Gamma \left( \frac{1}{n} \right)}} \sqrt{\Omega_\varphi \frac{\Omega_\varphi}{1 + 9WH^2}}, \]  

where \(\Omega_\varphi = \frac{\rho_\varphi}{3M_P^2H^2}\). Note that \(\Omega_\varphi\) cannot exceed unity. For \(WH^2 \ll 1\) \([12]\) reduces to the result obtained in minimal case \([10]\), and for high friction regime \(WH^2 \gg 1\) we obtain

\[ \Phi \ll \left( \frac{\Gamma \left( \frac{1}{n} \right)}{8\pi \Gamma \left( \frac{1}{2} + \frac{1}{n} \right)} \right)^{\frac{2}{n+2}} \left( \frac{M_P^2\Omega_\varphi}{\sqrt{WH}} \right)^{\frac{n}{n+2}}, \]  

which is the same as what was obtained in \([12]\) for \(\Omega_\varphi = 1\). In general \([6]\) is satisfied when

\[ \lambda \gg \left( \frac{\Gamma \left( \frac{1}{n} \right)}{n \Gamma \left( \frac{1}{2} + \frac{1}{n} \right)} \right)^n \left( 8\pi(9WH^2 + 1) \right)^{\frac{n}{2}} \left( \frac{H_0}{M_P} \right)^2 \left( 3\Omega_\varphi \right)^{\frac{2+n}{2}} H^2 M_P^{4-n}, \]  

\[ 4 \]
where the dimensionless parameters are defined by \( \tilde{W} = \frac{W}{H_0^2} \) and \( \tilde{H} = \frac{H}{H_0} \).

"0" indicates the present time. The (time) average of a quantity over an oscillation whose period is \( T \) is given by

\[
\langle A(t) \rangle = \frac{\int_t^{t+T} A(t') dt'}{T}.
\]  

(15)

The adiabatic index of the scalar field, \( \gamma_\varphi \) is defined by

\[
\gamma_\varphi := 1 + \frac{\langle P_\varphi \rangle}{\langle \rho_\varphi \rangle}.
\]

(16)

Using (2), we obtain [12]

\[
\gamma_\varphi = \frac{\langle (1 + 3WH^2)\dot{\varphi}^2 - W\dot{\varphi}(H\dot{\varphi}^2) \rangle}{\langle \rho_\varphi \rangle} = \frac{2(1 + 3WH^2)}{(1 + 9WH^2)V(\Phi)} \int_{\Phi}^{\Phi} \frac{\rho_\varphi - V(\varphi) d\varphi}{\sqrt{\rho_\varphi - V(\varphi)}}.
\]

(17)

The EoS parameter of the scalar field is then

\[
w_\varphi = \gamma_\varphi - 1 = \frac{n - 2 - 3(n + 6)WH^2}{(n + 2)(1 + 9WH^2)}.
\]

(18)

In the minimal case, \( WH^2 \ll 1 \), we recover the very well known relation [13]

\[
w_\varphi = \frac{n - 2}{n + 2}.
\]

(19)

The scalar field mimics the behavior of barotropic fluid with a constant EoS parameter. For \( WH^2 \gg 1 \), we recover our previous result [12]

\[
w_\varphi = -\frac{n + 6}{3n + 6}.
\]

(20)

For a specific \( n > 0 \), EoS parameter in high friction regime, \( WH^2 \gg 1 \), is smaller than that in \( WH^2 \ll 1 \) limit (see fig.1).

3 Evolution equations

By taking the time average of the continuity equation \( \| \) over an oscillation, and bearing in mind that \( H \) changes very slowly in one period, we obtain

\[
\langle \dot{\rho}_\varphi \rangle + 3H \langle \rho_\varphi \rangle \left( 1 + \frac{\langle P_\varphi \rangle}{\langle \rho_\varphi \rangle} \right) = 0.
\]

(21)
Figure 1: Equation of state parameter of the rapid oscillating scalar field depicted in terms of \( n \) for minimal coupling (red dashed line) and nonminimal coupling in high friction regime (black solid line). In the nonminimal case, for all positive \( n \), we have \( w_\phi \leq -\frac{1}{3} \).

This can be regraded as the continuity equation for the effective energy density \(< \rho_\phi >\) in time scales larger than the period of oscillation, where the replacement of parameters by their average values is acceptable \[13\]. By definition we have

\[
\langle \dot{\rho}_\phi(t) \rangle = \frac{\rho_\phi(t + T) - \rho_\phi(t)}{T} = \dot{\rho}_\phi(t),
\]

(22)

where the last equality is valid for \( t \gg T \). Finally the continuity equation becomes

\[
\dot{\rho}_\phi + 6H \frac{1 + 3WH^2}{1 + 9WH^2} \frac{n}{n + 2} \rho_\phi = 0.
\]

(23)

Hence the scalar field behaves as a perfect fluid with known equation of state parameter in terms of the Hubble parameter. Henceforth we assume that the Universe is nearly filled with a rapid oscillating scalar field dark energy and matter which is composed of baryonic, \( \rho_b \), and also cold dark matter \( \rho_{cd} \)

\[
\rho_m = \rho_{cd} + \rho_b \simeq \rho_{cd}.
\]

(24)

The Friedman equation is then

\[
H^2 = \frac{1}{3M_P^2}(\rho_m + \rho_\phi).
\]

(25)

\[1\] Indeed within a period, some of the parameters such as \( P_\phi \) are different from their average values.
In the absence of interaction between the dark energy and other sectors we can write

\begin{align*}
\dot{\rho}_\phi &+ \frac{1 + 3WH^2}{1 + 9WH^2} \frac{6nH}{n + 2} \rho_\phi = 0 \\
\dot{\rho}_m + 3H\rho_m &= 0.
\end{align*}

(26)

The Hubble parameter is a decreasing function of time and its infimum is 0. Therefore we expect that \( H \) asymptotes to 0 or a constant at late time.\( \lim_{t \to \infty} H(t) = C \), where \( C \) is a nonnegative constant. One can show that \( C = 0 \): Indeed putting \( H = C \) in (26) reveals that both \( \rho_\phi \) and \( \rho_m \) tends to 0 at late time. But From Friedman equation this leads to \( H \to 0 \).

3.1 Accelerated expansion

Obtaining an analytical solution for (26) is not feasible. But one can examine the ability of the model to describe the deceleration to acceleration phase transition and its converse (which is salient feature of this model), by presenting a series solution near the possible transition time. When the sign of \( \dot{H} + H^2 \) changes from negative to positive, the decelerated expansion of the Universe becomes accelerated. By assuming that the Hubble parameter is an analytic function of time, in the neighborhood of the transition time, which for simplicity we take as \( t = 0 \), the Hubble parameter has the Taylor expansion \( H = h_0 + h_1 t^\alpha + h_2 t^{\alpha + 1} + \mathcal{O}(t^{\alpha + 1}) \). \( \alpha \) is the order of the first nonzero derivative of \( H \) with respect to \( t \) at transition time. \( \dot{H} + H^2 = 0 \) at \( t = 0 \), leads us to take \( \alpha = 1 \) and \( h_1 = -h_2 \). Hence near the transition time we have

\[ H = h_0 - h_2^0 t + h_2 t^2 + \mathcal{O}(t^3). \]

(27)

In this way \( \dot{H} + H^2 = 2(h_2 - h_0^2) t \). For \( h_2 > h_0^2 \) the transition occurs from deceleration to acceleration phase at \( t = 0 \). The opposite transition occurs for \( h_2 < h_0^2 \). To see whether our model admits such a solution, we examine (27) in the continuity and Friedman equations. With the Hubble parameter taken as (27), the series solution of the \( \rho_m \) is given by

\[ \rho_m(t) = \rho_m(0) - 3h_0 \rho_m(0) t + 6h_2^2 \rho_m(0) t^2 + \mathcal{O}(t^3). \]

(28)

In the same way

\begin{align*}
\rho_\phi(t) &= \rho_\phi(0) - \frac{6n(1 + 3W h_0^2) h_0 \rho_\phi(0)}{(n + 2)(1 + 9W h_0^2)} t \\
&\quad + \frac{3n h_0^2 (7n + 36n W h_0^2 + 81n W^2 h_0^4 + 54W^2 h_0^4 + 2) \rho_\phi(0)}{(1 + 9W h_0^2)^2(n + 2)^2} t^2 \\
&\quad + \mathcal{O}(t^3).
\end{align*}

(29)
By inserting (27), (28), and (29) in the Friedman equation and equating the coefficients of each power of $t$, we deduce that this equation, up to $O(t^3)$, is satisfied provided that:

$$h_0^2 = \frac{1}{3M_P^2}(\rho_\varphi(0) + \rho_m(0)),$$

(30)

$$h_0^2 = \frac{1}{2M_P^2}\left(\rho_m(0) + \frac{2n(1 + 3W h_0^2)}{(n+2)(1 + 9W h_0^2)}\rho_\varphi(0)\right),$$

(31)

and

$$2h_2 = \frac{1}{3M_P^2}\left(\frac{3n h_0(7n + 36nW h_0^2 + 81nW^2 h_0^4 + 54W^2 h_0^4 + 2)}{(n+2)^2(1 + 9W h_0^2)^2}\rho_\varphi(0)
+ 6h_0\rho_m(0)\right) - h_0^3$$

(32)

hold. Using these equations we obtain

$$h_2 - h_0^3 = \frac{-n^2 - 3n - 2 + 9(n^2 + 4n - 4)W h_0^2 - 27(n + 6)W^2 h_0^4}{(n+2)(1 + 9W h_0^2)}(-n + 2 + 3(n + 6)W h_0^2)$$

(33)

In the minimal model $W = 0$, $h_2 - h_0^3 > (<)0$ reduces to $-\frac{n-1}{n+2} > (<)0$. So in this case there may be only one transition. In the nonminimal case, the situation changes and $h_2 - h_0^3 > (<)0$ is equivalent to

$$-\frac{n^2 + 3n - 2 + 9(n^2 + 4n - 4)W h_0^2 - 27(n + 6)W^2 h_0^4}{(n+2)(-n + 2 + 3(n + 6)W h_0^2)} < (>0).$$

(34)

Therefore, in principle, we may have both deceleration to acceleration and acceleration to deceleration phase transitions.

The ratio of energy densities at transition time is given by

$$r = \frac{\rho_\varphi(0)}{\rho_m(0)} = \frac{n + 2 + 9nW h_0^2 + 18W h_0^2}{4(9W h_0^2 - n + 1)}$$

(35)

For $Wh_0^2 \gg 1$, $r \simeq \frac{1}{2} + \frac{n}{4}$ while for $Wh_0^2 \ll 1$ we have $r = \frac{n+2}{4(1-n)}$.

### 3.2 Interacting case and attractor solutions

We can generalize the model by allowing energy exchange between dark components. In the presence of an interaction $Q$, the continuity equations read

$$\dot{\rho_\varphi} + \frac{1 + 3WH^2}{1 + 9WH^2}6nH \rho_\varphi = Q$$

$$\dot{\rho_m} + 3H \rho_m = -Q.$$

(36)
Inspired by the models proposed for reheating in which the inflation decays during coherent rapid oscillation [13], we choose the source term as \( Q = -\Gamma \dot{\phi}^2 \), \( \Gamma > 0 \), but here instead of radiation (ultra relativistic particles) the scalar field is assumed to decay to dark matter. So the equation of motion of the scalar field becomes

\[
(1 + 3WH^2)\ddot{\phi} + 3H(1 + 3WH^2 + 2WH)\dot{\phi} + V'(\varphi) = -\Gamma \dot{\phi},
\]

which in the minimal case reduces to the well known equation for inflaton decay [13]

\[
\ddot{\phi} + 3H\dot{\phi} + V'(\varphi) = -\Gamma \dot{\phi}.
\]

(38)

With the same method used to compute \( \gamma_\phi \) in (17), we arrive at

\[
<Q> = -\Gamma <\dot{\phi}^2> = -\frac{2n\Gamma}{(n+2)(1+9WH^2)}\rho_\phi.
\]

(39)

An interesting property of this interaction is that it depends on \( H \) such that it is inoperative for \( \Gamma \ll (1 + 9WH^2) \). But, as \( \dot{H} < 0 \), \( H \) decreases during Universe expansion and the decay becomes more relevant. This may alleviate the coincidence problem, i.e. why the matter and dark energy densities are at the same order today. This energy exchange, postpones the beginning of the acceleration phase of the Universe and also expedites the possible dark matter domination at late time. Let us now study the late time attractor solutions of the model and investigate the fate of the Universe.

By defining dimensionless parameters \( \tau := tH_0, \tilde{H} := \frac{H}{H_0}, \tilde{W} := WH_0^2, \tilde{\Gamma} := \frac{\Gamma}{H_0}, \) and also dimensionless fractional energy densities \( \Omega_m := \frac{\rho_m}{3M_p^2H^2}, \Omega_\varphi := \frac{\rho_\varphi}{3M_p^2H^2} \), the continuity equations and the Friedman equation after some computations reduce to a system of autonomous differential equations

\[
\Omega'_\varphi = -\frac{2n\tilde{\Gamma}}{(n+2)(1+9\tilde{W}\tilde{H}^2)}\Omega_\varphi - \left( \frac{6n\tilde{H}}{n+2} \right) \Omega_m - \left( \frac{1 + 3\tilde{W}\tilde{H}^2}{1 + 9\tilde{W}\tilde{H}^2} \right) \Omega_\varphi
\]

\[
+3\tilde{H} \left( \Omega_m + \left( \frac{2n}{n+2} \right) \Omega_\varphi \right) \left( \frac{1 + 3\tilde{W}\tilde{H}^2}{1 + 9\tilde{W}\tilde{H}^2} \right) \Omega_\varphi
\]

\[
\Omega'_m = -3\tilde{H}\Omega_m + 3\tilde{H} \left( \Omega_m + \left( \frac{2n}{n+2} \right) \Omega_\varphi \right) \left( \frac{1 + 3\tilde{W}\tilde{H}^2}{1 + 9\tilde{W}\tilde{H}^2} \right) \Omega_\varphi
\]

\[
+\frac{2n\tilde{\Gamma}}{(n+2)(1+9\tilde{W}\tilde{H}^2)}\Omega_\varphi
\]

\[
\tilde{H}' = -\frac{3}{2} \tilde{H}^2 \left( \Omega_m + \left( \frac{2n}{n+2} \right) \Omega_\varphi \right) \left( \frac{1 + 3\tilde{W}\tilde{H}^2}{1 + 9\tilde{W}\tilde{H}^2} \right) \Omega_\varphi.
\]

(40)

Prime denotes derivative with respect to the time \( \tau \). Note that as the EoS parameter of the scalar field and the interaction term are \( H \) dependent, we
cannot simply eliminate $H$ in our equations by redefinition of time parameter, hence the evolution of the Hubble parameter is included in the three dimensional autonomous system.

By setting $\Omega^\prime_\varphi = 0$, $\Omega^\prime_m = 0$, $\dot{H} = 0$ we can find the fixed points $(\Omega_m^*, \Omega_\varphi^*, H^*)$. The only fixed point consistent with $\Gamma \neq 0$ is $(\Omega_m^* = 1, \Omega_\varphi^* = 0, H^* = 0)$. To study the stability of this fixed point we consider small homogenous variations about the fixed point: $\Omega_\varphi^* + \delta \Omega_\varphi$, $\Omega_m^* + \delta \Omega_m$, $H^* + \delta H$. Inserting these into (40) we obtain

$$
\frac{d}{d\tau} \begin{pmatrix} \delta \Omega_\varphi \\ \delta \Omega_m \\ \delta H \end{pmatrix} = \mathcal{M} \begin{pmatrix} \delta \Omega_\varphi \\ \delta \Omega_m \\ \delta H \end{pmatrix}
$$

(41)

where $\mathcal{M} = \begin{pmatrix} -\frac{2n}{n+2} \tilde{\Gamma} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. The Jordan form of $\mathcal{M}$ is given by $\begin{pmatrix} 0 & 0 & 0 \\ 0 & -\frac{2n}{n+2} \tilde{\Gamma} & 0 \\ 0 & 0 & 0 \end{pmatrix}$, therefore as $n > 0$ by definition, the critical point is unstable only for $\tilde{\Gamma} < 0$. So in our model there is a late time stable attractor associated to cold matter dominated epoch. In this framework, the present acceleration of the Universe must be considered as a transient era.

### 3.3 Interacting dark energy in perturbed FRW Universe

Cosmological perturbations in the inflationary era in a model where the rapid oscillating scalar field is considered as the inflaton (the only ingredient of the early Universe), is discussed in details in [12]. Here we are only concerned about the late time acceleration where other ingredients are also present. Requiring stability against perturbations puts some constraints on the parameters. In this part based on [15] [16] we study briefly this subject for some special cases.

Consider the perturbed FRW space time

$$
ds^2 = a^2 \left( -(1 + 2\xi) \eta^2 + 2 \partial_i B \eta dx^i + (1 - 2\psi) \delta_{ij} + 2 \partial_i \partial_j E \right) dx^i dx^j.
$$

(42)

$\eta$ is the conformal time and $\xi, B, \psi, E$ are scalar functions. The $n$th fluid four vector velocity is $u_n^* = a^{-1}(1 - \xi, \vec{\nabla} v_n)$ where $v_n$ is the peculiar velocity potential. Evolution of the density perturbation of the $n$th component, $\delta_n :=$
\[ \frac{\delta \rho_n}{\bar{\rho}_n} \] and velocity perturbation \( \theta_n = -k^2(B + v_n) \) are given by \[ 15 \]

\[ \delta_n' + 3H(c_{sn}^2 - w_n)\delta_n + \gamma_n \theta_n + 3H \left( 3H\gamma_n(c_{sn}^2 - w_n) + w_n' \right) \frac{\theta_n}{k^2} \]

\[ + k^2 \gamma_n(B - E') - 3\gamma_n \psi' = \frac{aQ_n}{\bar{\rho}_n} \left( \xi - \delta_n + 3H(c_{sn}^2 - w_n) \frac{\theta_n}{k^2} \right) \]

\[ + \frac{a}{\bar{\rho}_n} \delta Q_n, \] (43)

\[ \theta_n' + 3H(1 - 3c_{sn}^2)\theta_n - \frac{k^2}{\gamma_n} c_{sn} \delta_n + \frac{2k^4 \pi_n}{3a^2 \gamma_n \rho_n} - k^2 \xi \]

\[ = \frac{a}{\bar{\gamma}_n \bar{\rho}_n} (Q_n(\theta - (1 + c_{sn}^2)\theta_n) - k^2 f_n) \] (44)

Prime denotes derivative with respect to the conformal time and \( H = \frac{\rho \dot{a}}{a \eta} \). Bar denotes the unperturbed value of a parameter on the background, e.g. \( \rho_n = \bar{\rho}_n + \delta \rho_n \). \( \pi_n \) is related to anisotropic stress tensor \( \pi_{ni} = (\partial_i \partial_j - \frac{1}{3} \delta_{ij} \nabla^2)\pi_n \). \( c_{sn} = \frac{\delta P_n}{\delta \rho_n} \) is the sound speed in the rest frame of \( n \)th fluid. \( f_n \) is the momentum transfer potential. Each fluid has its own evolution equation \( \rho_n' + 3H\gamma_n \rho_n - aQ_n = 0 \). For a system of interacting dark energy \( \rho_\phi \) and dark matter \( \rho_m \), \( Q_m = -Q_\phi = -Q \). \( \theta = -k^2(v + B) \), where \( v \) is the total four velocity potential.

For \( w_n \to -1 \), the equation (44) exhibits an instability. For the interaction \( Q = \Gamma \dot{\varphi}^2 \), we have \( \frac{Q}{\gamma_n} = \frac{1}{1 + 3WH^2} \). In large scale \( \xi \) and \( \delta \) terms may be dropped and for \( (3WH^2 + 1) \gg \Gamma \) the interaction term may be neglected and there is no instability \[ 16 \]. For \( WH^2 \ll 1 \), solving (44), one obtains \[ 16 \]

\[ \theta_\varphi = \theta_\varphi(\Gamma = 0)e^{\beta(\tau - \tau_0)} \] (45)

\( \beta = 2(1) \), when the transfer of the energy momentum is parallel to dark matter (dark energy) and \( \tau_0 \) is the time at which \( \theta_\varphi = \theta_\varphi(\Gamma = 0) \). \( \Gamma > 0 \) leads to an instability which has significant fingerprints in the present era provided that the growth rate of velocity perturbation satisfies \( \beta \Gamma \gtrsim H_0 \).

Note that the above discussions are true in the domain of validity of \( WH^2 \gg 1 \) and \( WH^2 \ll 1 \). Other interactions may also be taken for this theory, such as \( Q \propto \rho_m \) which was considered in \[ 17 \]. The large scale non adiabatic instability in this case may be cured by allowing dark energy EoS parameter to vary \[ 17 \]. In our model, this variation is not an ad hoc assumption but a characteristic of our solution.
4 Numerical illustrations and discussions

The continuity equations of energy densities, in terms of the redshift parameter \( z \) defined by \( a = \frac{1}{1+z} \), are

\[
-(1+z)\frac{d}{dz}\tilde{\rho}_\psi + 3\gamma_\psi \tilde{\rho}_\psi = -\left(\frac{2n}{n+2}\right)\frac{\tilde{\Gamma}}{H(1+9WH^2)}\tilde{\rho}_\psi
\]

\[
-(1+z)\frac{d}{dz}\tilde{\rho}_m + 3\tilde{\rho}_m = \left(\frac{2n}{n+2}\right)\frac{\tilde{\Gamma}}{H(1+9WH^2)}\tilde{\rho}_\psi
\]

\[-(1+z)\frac{d}{dz}\tilde{\rho}_b + 3\tilde{\rho}_b = 0.
\]

\( \gamma_\psi \) is given by eq.(17). The energy exchange is considered only between dark sectors. The Hubble parameter is given by \( \tilde{H}^2 = \tilde{\rho}_\psi + \tilde{\rho}_cd + \tilde{\rho}_b \). We have employed dimensionless parameters specified by a tilde character: \( \tilde{\Gamma} = \Gamma H_0 \), \( \tilde{W} = WH_0^2 \), \( \tilde{H} = H H_0 \), \( \tilde{\rho}_i = \frac{\rho_i(0)}{\rho_c(0)} \) where the present critical energy density is \( \rho_c(0) = 3M_P^2H_0^2 \), and "0" as before denotes the present time characterized by \( a = 1 \) or \( z = 0 \). Therefore the initial conditions are \( \tilde{H}(0) = 1 \) and \( \tilde{\rho}_i(0) = \Omega_i(0) \), where \( \Omega_i = \frac{\rho_i}{3M_P^2H_0^2} \).

In the absence of an analytical solution for the system (46), let us use numerical methods to illustrate the evolution of the model and justify our previous results. In our analysis we neglect the contribution of the radiation.

In fig.(2), dark energy and cold dark matter densities are depicted in terms of the redshift parameter for \( \{\tilde{W} = 1000, w_\psi(0) = -0.9, \Omega_\psi(0) = 0.685, \Omega_cd(0) = 0.266, \Omega_b(0) = 0.049\} \), in the absence of interaction. The values of fractional energy densities and the nearly constant EoS parameter of dark energy at the present time are picked from Planck 2013 data [11]. In this case the potential is \( V(\psi) = \lambda |\psi|^n \) [9]. For \( z > 0.5 \) the matter was dominated but as the EoS parameter of the scalar field is negative, dark energy becomes dominant later.
Figure 2: $\rho_m$ (red dashed line) and $\tilde{\rho}_\varphi$ (black solid line) depicted in terms of the redshift $z$ for the parameters $\{\tilde{W} = 1000, n = 0.352, \Gamma = 0, \Omega_{\varphi}(0) = 0.685, \Omega_{cd}(0) = 0.266, \Omega_b(0) = 0.049\}$. $\varphi$ has a nearly constant EoS parameter, therefore $\tilde{\rho}_\varphi$ decreases very slowly.

The EoS parameter of $\varphi$ is approximately a constant: $w_\varphi \simeq -1$, therefore as it can be seen in fig. (2), in the absence of interaction $\rho_\varphi$ changes very slowly.

In the intermediate regime where $WH^2 \gg 1$ does not hold, $\varphi$ has a dynamical EoS parameter. In this case the behaviors of $\tilde{\rho}_\varphi$ and $\tilde{\rho}_{cd}$, are plotted in fig. (3), for $\tilde{W} = 1$ and a quadratic potential.

Figure 3: $\rho_{cd}$ (red dashed line) and $\tilde{\rho}_\varphi$ (solid black line) in terms of $z$ and for $\{\tilde{W} = 1, n = 2, \Gamma = 0, \Omega_{\varphi}(0) = 0.685, \Omega_{cd}(0) = 0.266, \Omega_b(0) = 0.049\}$. $\varphi$ has not a constant EoS parameter, and ultimately behaves as pressureless matter. $\rho_\varphi$ is diluted faster than the high friction case in the previous figure.
In this example, the EoS parameter shows a good agreement with the Chevallier-Polarski-Linder parametrization \cite{18},

$$w_\phi = w_\phi(0) + w_\phi a(1 - a).$$ \hspace{1cm} (47)

The EoS parameter given by the approximation (47) and also what was obtained from our numerical solutions are shown in fig. (4).

Figure 4: EoS parameter of oscillatory dark energy in our model (black solid line) with the same initial conditions as in fig.(3), and Chevallier-Polarski-Linder parametrization $w_\phi = w_\phi(0) + w_\phi a(1 - a)$ (red dashed line), with \{\omega[0] = -0.6, \omega[a] = -0.066\} depicted in terms of $z$.

In the presence of the interaction (39), we plot the solutions of the system in figs. (5) and (6).
Figure 5: \( \tilde{\rho}_{cd} \) (red dashed line) and \( \tilde{\rho}_\varphi \) (black solid line) in terms of \( z \) for \( \{W = 1000, n = 0.352, \tilde{\Gamma} = 10000, \Omega_\varphi(0) = 0.685, \Omega_{cd}(0) = 0.266, \Omega_b(0) = 0.049\} \). Due to the interaction term, dark energy decreases faster than figure (2) and dark matter becomes again dominant in the future.

Figure 6: \( \tilde{\rho}_{cd} \) (red dashed line) and \( \tilde{\rho}_\varphi \) (black solid line) in terms of \( z \) and for \( \{\tilde{\Gamma} = 3, n = 2, \tilde{W} = 1, \Omega_\varphi(0) = 0.685, \Omega_{cd}(0) = 0.266, \Omega_b(0) = 0.049\} \). The Presence of interaction term expedites the redomination of dark matter with respect to the figure (3).

We have still used the same initial conditions at \( a = 1 \) as before. By comparing these figures with the previous ones, we find that in the presence of \( \Gamma \), the dark energy domination occurs at higher redshift \( z \). This is related to the fact that in the presence of the energy exchange, the dark matter decreases with a smaller slope while the dark energy decreases with a greater slope.
A feature of our model, which is shown clearly in the figures (5) and (6), is that the matter may become dominant in the future and the accelerated expansion ceases. We expect that the dark matter is diluted faster than dark energy which has a more negative EoS parameter, so at first glance, it seems that the cease of acceleration or the dominance of dark matter at late time is related to the positive decay rate of $\varphi$ to dark matter, but this is not the whole story: $H$ is a decreasing function of time, and the EoS parameter of $\varphi$, evolves from $w_\varphi = -\frac{n+6}{3n+6}$ at early time where $H^2 \gg 1$, to $w_\varphi = \frac{n-2}{n+2}$ corresponding to $WH^2 \ll 1$ at late time. So the EoS of $\varphi$ increases and this component may even behave as dark matter at late time (for $n=2$). In the era where the Universe is nearly filled with dark energy, pressureless dark matter and baryonic matter, the Universe EoS is be given by

$$w = \frac{P_\varphi}{\rho_\varphi + \rho_{cd} + \rho_b} = \Omega_\varphi w_\varphi.$$ (48)

$w > (<) -\frac{1}{3}$ specifies deceleration (acceleration). This can also be expressed in terms of the deceleration parameter $q = \frac{1}{2}(1 + 3w)$: for $q < (>) 0$ the expansion is accelerating (decelerating). In fig.(7) the deceleration parameter is depicted in terms of the redshift. Employed parameters are $\{ \tilde{\Gamma} = 7, n = 2, W = 1, \Omega_\varphi(0) = 0.685, \Omega_{cd}(0) = 0.266, \Omega_b(0) = 0.049 \}$.

Figure 7: Deceleration parameter of the Universe in the presence (black solid line) and absence (red dashed line) of interaction with the same parameters and initial conditions as figs.(3) and (6). Even in the absence of interaction, the Universe enters a deceleration phase in the future. The presence of interaction expedites this procedure.

This figure shows that even for $\Gamma = 0$ the acceleration ceases in the future and the presence of interaction term only expedites this procedure.
Note that in the aforementioned examples, if one takes $\Lambda \gg 10^{-5} eV$, where $\lambda = \Lambda^{4-n}$, the condition required for rapid oscillation, i.e. (14), is satisfied. To obtain this, one can evaluate numerically the right hand side of (14) by taking $H_0 = 67.3km s^{-1} Mpc^{-1}$ [11].

5 Summary

The non-minimal derivative coupling model proposed in [6 8], was employed to describe the recent acceleration of the Universe. In this context, instead of the well known slow roll scalar field model, we considered a rapid oscillating scalar field with a power law potential. After some preliminaries and generalizing the results of [12] to the quintessence case, we obtained required condition for such an oscillation and depending on the strength of the coupling we divided our study into two categories. In the high friction regime, we obtained a constant EoS parameter for dark energy which was different from what was obtained in the minimal case permitting the acceleration for a wider range of potentials. In the intermediate regime, a dynamical EoS parameter was derived allowing acceleration to deceleration phase transition in the future without invoking interaction terms. This transition was discussed by obtaining a series solution for the system. We also took into account a possible energy exchange between dark components (inspired by inflaton decay during its coherent rapid oscillation), alleviating the coincidence problem. In the high friction regime, and the minimal model, where the EoS parameter of dark energy is a constant the interaction term is responsible for the possible future acceleration to deceleration phase transition. We showed also that when the dark energy decays to dark matter, there exists a late time stable attractor for the system. A short discussion was given on the instabilities of interacting dark energy model in the perturbed FRW Universe. This discussion was restricted to high friction regime and the minimal model where the dark energy has a constant EoS parameter. An outlook is to investigate this problem in intermediate regime where the dark energy has a variable EoS parameter and also considering more general interactions terms proposed in the literature.

At the end, using numerical methods, the evolution of dark sectors were depicted and discussed in details.

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