Sustainable system design of electric powertrains—comparison of optimization methods

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\section*{ABSTRACT}

The transition within transportation towards battery electric vehicles can lead to a more sustainable future. To account for the development goal ‘climate action’ stated by the United Nations, it is mandatory, within the conceptual design phase, to derive energy-efficient system designs. One barrier is the uncertainty of the driving behaviour within the usage phase. This uncertainty is often addressed by using a stochastic synthesis process to derive representative driving cycles and by using cycle-based optimization. To deal with this uncertainty, a new approach based on a stochastic optimization program is presented. This leads to an optimization model that is solved with an exact solver. It is compared to a system design approach based on driving cycles and a genetic algorithm solver. Both approaches are applied to find efficient electric powertrains with fixed-speed and multi-speed transmissions. Hence, the similarities, differences and respective advantages of each optimization procedure are discussed.

\section*{1. Introduction}

The market share of Battery Electric Vehicles (BEVs) is currently increasing (Palmer \textit{et al.} 2018). Additionally, technological improvements lead to a decrease in purchase prices, which will create further pressure on the market share of conventional vehicles with combustion engines. The market transition to electric vehicles is aiming to strengthen the sustainable development goal ‘climate action’ presented by the United Nations (2015), if renewable energies are used in efficient powertrains.

Within the conceptual design phase, the key step to account for this goal is the development of energy-efficient system designs. For the comparative assessment of powertrain systems, both the component design and the operating strategy of the vehicle have to be considered simultaneously (Silvas \textit{et al.} 2017).

Therefore, the required design task can be expressed as an optimization problem, where an energy-related objective $J$ is minimized subject to design and control constraints of the given system topology. Within this design task, the following variables are used: $u$ continuous variables $x$ for the design variables and hidden state variables of each component, and $v$ binary variables $y \in \{0, 1\}$ for the assignments within the control strategy to model a multi-speed transmission explicitly. This can be

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beneficial for the system efficiency, see e.g. Bottiglione et al. (2014), Ruan, Walker, and Zhang (2016) and Zhao et al. (2021). The resulting Mixed-Integer Nonlinear Program (MINLP) is given in the following in an abstract formulation:

\[
\begin{align*}
\min_{x,y} & \quad J = f(x, y) \\
\text{s.t.} & \quad g_k(x, y) = 0 \quad \forall \ k \in 1, \ldots, N_e, \\
& \quad h_j(x, y) \leq 0 \quad \forall \ j \in 1, \ldots, N_i, \\
& \quad x \in \mathbb{R}^u, \\
& \quad y \in \{0, 1\}^v.
\end{align*}
\] (1)

Here, \( f : \mathbb{R}^u \times \{0, 1\}^v \to \mathbb{R} \) is the objective function and \( g_k : \mathbb{R}^u \times \{0, 1\}^v \to \mathbb{R} \) and \( h_j : \mathbb{R}^u \times \{0, 1\}^v \to \mathbb{R} \) are constraint functions. The \( N_e \) equality constraints and \( N_i \) inequality constraints represent both the design and control constraints.

Unfortunately, the parametrization of this optimization problem (1) is uncertain, since different driving behaviours result in different parametrizations, and therefore in different optimal solutions. Currently, this uncertainty is mastered by deriving representative driving cycles, which approximate a specific real driving dataset, using a stochastic synthesis process, cf. Silvas et al. (2016) and Esser, Zeller, et al. (2018). Commonly used examples are legislative standard driving cycles, like the Worldwide harmonized Light vehicles Test Cycle (WLTC).

The focus in this article lies on a powertrain design that consists of a battery, a single electric machine, and a transmission which is optimized solely towards energy efficiency. A general overview on powertrain designs is given in Chan, Bouscayrol, and Chen (2009). Other approaches in the literature also consider additional objectives and derive a multi-objective optimization problem, cf. Lei et al. (2019) and Schönknecht, Babik, and Rill (2016).

Multiple design approaches based on driving cycles have been proposed for the considered powertrain design of BEVs with multi-speed transmissions. Early optimization-based approaches are given in Sorniotti et al. (2010) and Di Nicola et al. (2012). Besides these, Anselma and Belingardi (2019) use a rapid brute force approach, Tan et al. (2018) particle-swarm optimization, and Walker et al. (2013), Schönknecht, Babik, and Rill (2016), Wang et al. (2017), Esser et al. (2019) and Li et al. (2020) a genetic algorithm.

Moreover, a new approach to consider the aforementioned uncertain driving behaviour, a problem-adapted state-of-the-art approach that uses a Genetic Algorithm (GA) is also presented in this article. It is called the \textit{cycle approach}.

For the optimal calibration of combustion engines, Wasserburger, Hametner, and Didcock (2020) showed that a stochastic optimization approach is useful to generate system designs that are less sensitive to variations in the uncertain driving behaviour. Furthermore, Caillard et al. (2014) showed that a scenario-based program can be used to derive efficient system designs for BEVs.

In the following, a new model and preprocessing approach is presented to account for this uncertain driving behaviour directly within the considered optimization task with a multi-speed transmission. Therefore, a stochastic optimization program, cf. Shapiro, Dentcheva, and Ruszczyński (2014), to master the uncertainty within problem (1) is used. This problem formulation is transformed in its certainty-equivalent form by automatically generating scenarios with unsupervised learning. Hence, the approach is called the \textit{scenario approach}. The set \( S \) is used to describe all considered scenarios \( s = 1, \ldots, N \), which approximate the considered driving cycle dataset. Furthermore, each scenario \( s \in S \) has a corresponding weight \( \pi_s \), which models its importance for the objective.

The global optimization solver SCIP (Gleixner et al. 2018) is used, which allows a lower bound (dual solution) of the objective value in (1) to be derived. This allows either the global optimality of the derived solution to be proved, or the difference (duality gap) between the primal and dual
solutions to be derived, once the computational time limit is reached. With this information, the achievable optimum is quantified.

Both approaches considered (the cycle and the scenario approaches) are shown in Figure 1 and result in optimization programs with different granularities. Therefore, the main contribution and focus besides the new modelling approach is a detailed verification.

2. Sustainable system design

The cycle approach is abbreviated in the following by ‘\( P_c \)’ and the scenario approach by ‘\( P_s \)’. To compare these approaches, a range of \( R = 500 \text{ km} \) is defined, as an exemplary value for a long-range capable vehicle. To derive energy-efficient system designs, the minimization of the mean electric energy demand \( c_{\text{mean}} \) is used as the objective function, which is proportional to the required battery capacity \( c^B \) between different powertrain designs with equal \( R \):

\[
\min c_{\text{mean}}. \tag{2}
\]

For computation, the restricted MINLP is transformed in \( P_c \) in an unrestricted MINLP. Hence, the objective is

\[
\min c_{\text{mean}} + k_p. \tag{3}
\]

This objective (3) adds a penalty term \( k_p \) for a potentially infeasible parametrization, which leads to an unconstrained optimization program. The approaches are comparable, but exploit different problem specific structures to improve the solution speed.

Besides \( c_{\text{mean}} \) and \( c^B \), the variables given in Table 1 are used. These variables are modelled either as vectorized or scalar variables in both methods used. For readability, only scalar representations are presented.

Both approaches \( P_s \) and \( P_c \) rely on the same physical system models and should lead to comparable results when evaluated on the same input dataset of driving behaviour. However, as both approaches require a problem and method-specific constraint and objective generation, a thorough comparison is mandatory to derive the limitations and potentials of both approaches.
Table 1. Variables used in both optimization approaches.

| Variable | Units           | Description                  |
|----------|-----------------|------------------------------|
| $c^B$    | kWh            | Battery capacity             |
| $c_{\text{mean}}$ | kWh/100 km | Electric energy demand       |
| $i$      | –              | Transmission ratio           |
| $m$      | kg             | Total mass                   |
| $m^B$    | kg             | Battery mass                 |
| $P^M$    | kW             | Power of the EM for a specific load |
| $P^M_{\text{max}}$ | kW | Maximum power of the EM       |
| $t^M$    | Nm             | Torque of the EM             |
| $t^M_{\text{max}}$ | Nm | Maximum torque of the EM      |
| $t^W$    | Nm             | Torque at the wheel          |
| $\eta^M$ | –              | EM efficiency                |
| $\rho^e$ | kWh/kg         | Battery energy density       |
| $\omega^M$ | s$^{-1}$ | Speed of the EM              |

Note: EM – Electric machine.

Table 2. Vehicle parameters.

| Parameter | Value | Units | Description                                      |
|-----------|-------|-------|--------------------------------------------------|
| $A$       | 2.2   | m$^2$ | Vehicle reference area                          |
| $c_w$     | 0.3   | –     | Drag coefficient                                |
| $g$       | 9.81  | m/s$^2$ | Specific gravitational constant                  |
| $M^A$     | 70    | kg    | Additional component mass in powertrain          |
| $M^P$     | 75    | kg    | Mass of the considered driver                    |
| $M^0$     | 1150  | kg    | Vehicle mass                                     |
| $r$       | 0.3   | m     | Wheel radius                                     |
| $\dot{x}$ | Based on dataset | m/s | Vehicle speed                                   |
| $\ddot{x}$ | Based on dataset | m/s$^2$ | Vehicle acceleration                          |
| $\alpha$  | 0     | –     | Terrain slope                                    |
| $\eta^B$  | 0.97  | –     | Efficiency of battery charging/discharging      |
| $\eta^G$  | $\in (0.98, 0.975)$ | – | Efficiency of transmission; first value for one-speed; Second value for two-speed |
| $\lambda_i$ | 1.0   | –     | Inertia consideration factor                     |
| $\lambda_r$ | 0.008 | –     | Rolling resistance coefficient                   |
| $\rho$    | 1.2041| kg/m$^3$ | Air density at 20$^\circ$C and sea level       |
| $\Omega^M$ | 10,000 | min$^{-1}$ | Maximum speed of the selected EM |

3. Component models

In the following, the component models are presented in more detail.

3.1. Vehicle dynamics

The underlying vehicle is modelled based on a longitudinal vehicle model (Mitschke and Wallentowitz 2014, 83ff.):

$$t^W = \left(\lambda_i m \dot{x} + mg \left(\sin(\alpha) + \lambda_r\right) + \frac{1}{2} \rho c_w A \dot{x}^2\right) r. \quad (4)$$

The parameters used in all calculations presented and their meanings are given in Table 2. The effect of rotational inertias ($\lambda_i = 1$) is omitted, because their correct depiction dramatically increases the computational effort in the optimization of powertrains with a multi-speed transmission, results only in second-order effects, and is typically beyond the scope of conceptual system-level design.

The total mass $m$ is modelled as the sum of the battery mass $m^B$, the predefined net mass $M^0$, the additional mass for further components in the powertrain $M^A$, and the mass for the passenger $M^P$:

$$m = m^B + M^0 + M^A + M^P. \quad (5)$$
In addition to the total range requirement, a minimum required speed of 160 km/h and a specific launch torque (gradeability on a slope of 30°) at low speed are considered as additional constraints in $P_s$ and $P_c$. For $P_s$, two additional scenarios are added, which must be fulfilled but do not affect the objective ($\pi_s = 0$). For $P_c$, two specific constraints are added within the penalty evaluation.

### 3.2. Battery

The capacity $c^B$ and battery mass $m^B$ are modelled as follows:

$$c^B = m^B \rho^e, \quad (6)$$

$$\rho^e = 0.0008 \text{kg}^{-1} c^B + 0.0788 \text{kWh/kg}. \quad (7)$$

Here $\rho^e$ is the energy density of the traction battery. It was modelled with an affine relationship based on the battery capacity $c^B$, which was derived in a market study (Zimmerling 2020). Additionally, a (dis-)charging efficiency $\eta^B$ of 97% and a useable depth of discharge of 95% are used.

### 3.3. Electric machine

The modelled electric machine (EM), cf. Figure 2, is based on a reference permanent-magnet synchronous motor (An and Binder 2017). To derive EMs with higher or lower power than the reference EM, the maximum torque $t^M$ and reference efficiency map $M$ are scaled accordingly, cf. Balazs (2015) and Lange (2018). This allows the EM to be sized in each optimization run based on the underlying demands considered. For each EM torque $t^M$ and EM speed $\omega^M$ pair, the efficiency map shows the resulting EM efficiency $\eta^M = \eta^M(t^M, \omega^M): \mathbb{R}^2 \rightarrow \mathbb{R}$ that is used to derive the required power that must be provided by the battery. In this study, the mass of the EM is not scaled according to its power rating owing to its relatively small changes compared to the mass changes caused by the sizing of the battery capacity. The required electrical power $p^M$ for each time step of the driving cycle or scenario can be computed by using

$$p^M = \frac{t^M \omega^M}{\eta^M} \quad (8)$$

and in recuperation by using

$$p^M = t^M \omega^M \eta^M \quad (9)$$

### 3.4. Transmission

The EM upper speed limit $\Omega^M$ and upper torque limit $t^M$ are usually different from the traction demands at the wheels of the vehicle for the driving cycle considered. Therefore, it is mandatory to use a transmission (TM) within the powertrain. Most BEVs use a single-speed transmission. The usage of more than one speed can lead to a higher overall system efficiency. In Esser et al. (2019), a reduction potential of the electric energy demand of 3% and 7% for short-ranged and long-ranged BEVs, respectively, is identified.

The transmission is modelled by the following constraints:

$$t^M i^G = t^W \quad (10)$$

or

$$t^M i^G = \eta^G t^W \quad (11)$$

for recuperation and

$$\omega^M = i^G \omega^W \quad (12)$$
Here, the \((\text{EM torque } t^M)/(\text{EM speed } \omega^M)\) is transformed to the required \((\text{torque at the wheel } t^W)/(\text{speed at the wheel } \omega^W)\) by the transmission ratio \(i\). A fixed transmission efficiency \(\eta^G = 0.975\) for a transmission with two speeds and \(\eta^G = 0.98\) for a transmission with one speed are considered (Esser, Schleiffer, et al. 2018). An efficiency-based selection is made for the control strategy.

### 3.5. Additional model assumptions

The consideration of secondary energy demands, \(e.g.\) for heating, ventilation and air conditioning, is important to depict realistic vehicle energy demands, see Jardin et al. (2019). Incorporation of secondary demands would result in a constant energy demand offset for both approaches, depending on assumptions regarding \(e.g.\) ambient temperatures and the driving cycle. It would hence diminish systematic differences for the verification of both optimization approaches. For this reason, secondary demands are not considered within this analysis.

### 4. Scenario approach

The physical relations mentioned previously in Sections 3.1–3.4 are used as constraints for the set \(S\) of predefined scenarios. A more detailed description of the model used is given in the online supplemental data. The speed selection for the transmission with two speeds is modelled with the help of binary variables \(b_{g,s}\). Here, the index \(g\) represents the selected speed based on a given set of speeds \(G = \{1, 2\}\) and \(s\) represents the considered scenario, \(s \in S\). This approach uses a quasi-stationary model representation and omits the explicit time dependence of the standardized driving cycles used.

#### 4.1. Scenario generation

Within preprocessing of \(\mathcal{P}_s\), cf. Figure 1, it is important for verification to derive scenarios that represent the underlying dataset of the given driving cycle. Therefore, a preprocessing procedure is presented that is based on the \(k\)-means algorithm (MacQueen 1967)—an unsupervised learning
algorithm. As shown by Löhndorf (2016), this method is capable of approximating the sample distribution by a smaller discrete distribution with a predefined number of scenarios. Up to 400 iterations are used with different starting conditions to derive a more robust local optimum of each cluster assignment in the implementation of the $k$-means algorithm used (Pedregosa et al. 2011).

The underlying driving cycle is mapped to the $(t^W_s, \omega^W_s) \in \mathbb{R}^2$ domain by using the longitudinal vehicle model shown in Equation (4) and the parameters in Table 2. The only unknown is the total mass $m$, since it is derived within the optimization. A total mass reference of 1500 kg is used to generate the scenarios, which are then rescaled in the following step by an inverse model of Equation (4) to derive the required constraints.

This heuristic neglects sign-changes based on a changing mass and the given relationships between the velocity and acceleration, but captures the most important domain relations in common driving cycles.

The result of this scenario generation procedure for the WLTC is shown in Figure 3. The center operating points $(t^W_s, \omega^W_s)$ of the clusters are used as scenarios $s \in S$ within the optimization. Furthermore, a weight $\pi_s$ is assigned to each cluster by using the total number of measurements and the average power.

As these reference torques and powers depend on the total mass of the vehicle $m$, they are scaled within the optimization program. The longitudinal vehicle model is linear in the total mass $m$. Hence, it is possible to rewrite the model in Equation (4) as follows:

$$t^W_s = m \left( r \lambda_1 \ddot{x}_s + r g \left( \sin (\alpha) + \lambda_\tau \right) \right) + \frac{1}{2} \rho c_w A \dot{x}_s^2 r, \quad (13)$$

which results in

$$t^W_s = m \alpha_{1,s} + \alpha_{2,s}. \quad (14)$$

The parameters $\alpha_{1,s}$ and $\alpha_{2,s}$ are computed based on the underlying longitudinal vehicle model and the derived acceleration and speed values given by the precomputed scenarios $(t^W_s, \omega^W_s), s \in S$.

Furthermore, all scaled corner operating points on the convex hull of each cluster are used, as shown in Figure 3, within additional constraints to ensure a more robust solution. They must be located within the feasible EM domain for the selected transmission ratios.

### 4.2. Efficiency map approximation

The measured efficiency map $\mathcal{M}$, Figure 2, is approximated with an outer polyhedral approximation for each quadrant to derive a faster solution time for $\mathcal{P}_s$. A direct consideration of $\mathcal{M}$ would result in highly nonlinear constraints, which would slow down the exact solver considerably. The benefit of the present approach is the usage of only affine constraints without the requirement of the definition of further binary variables, as for instance required in piecewise-linear approximations, cf. Vielma, Ahmed, and Nemhauser (2010) and Leise, Simon, and Altherr (2020). This polyhedral approximation is possible since the efficiency map is almost concave, if both quadrants are considered separately. The polyhedral approximation of the given EM used for the first quadrant is shown in Figure 4(a), while Figure 4(b) shows the difference between the approximation and the reference efficiency map model. Figure 4(c) shows the approximation in the fourth quadrant and Figure 4(d) the difference according to the reference.

In total, 226 points are used for each quadrant, where the gradient information is evaluated based on the given data on $\mathcal{M}$. All affine outer approximations are then added to $\mathcal{P}_s$ as constraints to restrict the efficiency depending on the torque and speed values.
Figure 3. Exemplary clustering of the WLTC for the reference vehicle given in Table 2 with a total mass of 1500 kg. All values with zero speed are omitted. The centre operating points (⋆) are used as scenarios in the optimization: (a) 12 clusters for the first quadrant; (b) 5 clusters for the fourth quadrant. Additionally, the points on the convex hulls (dashed lines) of each cluster are used as further constraints to ensure a more robust solution.

5. Cycle approach

As mentioned in Section 1, one common optimization approach concerning the design of powertrains is the application of a GA. In addition to the design problem, a control strategy has to be determined for a specific powertrain parametrization. Often the control strategy for the respective design parameter set is derived as a nested, inner optimization problem for the particular driving cycle and is not simultaneously adjusted by the GA—cf. e.g. Silvas et al. (2017). By splitting the design and control problem, the GA does not need to address discrete choices in each time step of a particular driving cycle. This results in a nonlinear program for the design.
Figure 4. Outer polyhedral approximation of the EM efficiency map with 226 evaluation points in each quadrant: (a) normalized approximation of the first quadrant; (b) difference between the approximation used in the first quadrant and the reference efficiency map model, cf. Figure 2; (c) normalized approximation of the fourth quadrant; (d) difference between the approximation used in the fourth quadrant and the reference efficiency map, cf. Figure 2.

Based on the works of Meier (2013), Eghtessad (2014) and Schleiffer and Rinderknecht (2017), an optimization framework using a GA for the identification of the ecological potential of various powertrain concepts was derived in a publicly funded project at TU Darmstadt by Esser, Schleiffer, et al. (2018), which is the reference for the analysis presented in this section. In this study, a backward-facing implementation of the longitudinal vehicle dynamics is applied to ensure a time-efficient optimization process.

In the cycle approach $\mathcal{P}_c$, a detailed driving cycle simulation of the vehicle is embedded into an optimization framework based on a GA.

The traction demand to follow a driving cycle is calculated with the vehicle dynamics model presented in Equation (4) and depends on the parametrization of the powertrain, e.g. the mass of the battery $m^B$. For the two-speed transmission, an operating strategy determines the optimal transmission speed in every time step of the driving cycle simulation regarding the efficiency in the respective operating points of the EM. In this way, the temporal sequence of optimal transmission speeds is preserved and restrictions on the frequency or feasibility of transmission speed shifts could be introduced. However, no restrictions are defined in this work to preserve comparability with $\mathcal{P}_s$. The efficiency of a specific operating point is estimated directly through bilinear interpolation of the scaled EM efficiency map nodes without further approximation. Ultimately, the cycle approach
yields time-resolved results of the determined operating points and thus the State Of Charge (SOC) of the battery and electric energy demand over the whole driving cycle considering the models of the powertrain components presented in Sections 3.1–3.4.

BEVs with specific powertrain parametrizations are defined as the phenotypes of the GA optimization framework, represented by sets of powertrain parameters as their genetic representations. The genome of a vehicle comprises the battery capacity $c_B^a$, the maximum torque of the EM $t_M^a$ and the transmission ratio $i$.

The other variables shown in Table 1 are derived from the genome according to the model equations of the powertrain components.

The objective of the approach is to find the genome with the best fitness. The fitness of different vehicle parametrizations is evaluated with the unconstrained objective function (Equation (3)) in which the mean electric energy demand of the vehicle $c_{\text{mean}}$ results from the driving cycle simulation and a penalty term $k_p$ for a potentially infeasible parametrization to incorporate the constraints.

Infeasibility may occur, for instance, when the vehicle is not able to meet the traction demands imposed by design constraints and the driving cycle itself, or the battery capacity is too low to meet the required range of the vehicle. This definition ensures that the fittest individual is the powertrain parametrization that yields the lowest electric consumption, while still satisfying all design constraints.

Lower and upper bounds for the powertrain parameters as well as a number of 20 stall generations as a termination criterion are defined. Here, the relative change of fitness values of the best individuals does not exceed a given function tolerance of $\epsilon_{\text{stall}} = 10^{-5}$. A randomly generated starting population of $n_{\text{pop}} = 600$ individuals is iteratively evolved through a rank-based selection regarding their fitness values and through crossover and mutation of the genotypes, until the termination criterion is met. The default settings for crossover and mutation of the GA from the MATLAB® 2018b Global Optimization Toolbox (MathWorks 2018), are used. An elitism count of $n_{\text{elit}} = 30$ ensures that the best individuals of a generation are directly passed on to the next generation to preserve the best current parametrizations.

6. Data generation for verification

To compare and verify both approaches, it is mandatory to use the same underlying dataset. Therefore, the same specific driving cycles are used as inputs for $P_s$ and $P_c$.

The evaluations are performed for a variety of driving cycles, cf. Figure 5, and the given vehicle parameters in Table 2 to ensure a wide verification basis. Note that the focus lies on the comparison of the approaches $P_s$ and $P_c$ based on different datasets, thus it is not intended to derive general optimal solutions for a combined dataset in this article.

The first six driving cycles are composed of well-known standardized cycles and Artemis cycles (André 2004). The remaining cycles are based on driving data that have been recorded during the use of two vehicles at TU Darmstadt. The cycles for the pool vehicle (overall and city-only) are based on a plug-in hybrid vehicle that is used by various different employees for business duties. The institute vehicle cycle is based on the combined driving data of the pool vehicle and a further plug-in hybrid vehicle, which is only driven by a single person. These cycles have been synthesized applying the method of Esser, Zeller, et al. (2018) and are included so as also to consider driving cycles representing real driving.

The well-known first six driving cycles do not contain any slope information. Even though the synthesizing method used in Esser, Zeller, et al. (2018) allows incorporating slope information based on the real driving data, the slope is set to zero for all cycles in order to preserve comparability between all cycles considered.
7. Optimization results and discussion

Both optimization approaches are compared for the given driving cycles. The scenario-based program $P_s$ is modelled using PyScipOpt (Maher et al. 2016) and the mixed-integer nonlinear programming solver SCIP 6.0.2\(^1\) (Gleixner et al. 2018), with the termination criteria (i) a duality gap limit of 0.25%, (ii) a time limit of 30 minutes, and (iii) a RAM limit of 12 GB. In total, 27 scenarios (including speed and gradeability requirements) are used for vehicles with single-speed transmission and 19 scenarios (including speed and gradeability requirements) for vehicles with two-speed transmission. Furthermore, the genetic algorithm implementation from the MATLAB\(^\text{®}\) toolbox (MathWorks 2018) is used to solve $P_c$.

7.1. Results

The results for both problem formulations, $P_c$ and $P_s$, with single-speed transmission are shown in Table 3. The differences in the results between the optimization approaches for each respective driving cycle are relatively small. For the single-speed BEV, the transmission ratio $i$ is mostly set to 7.07 by both optimization approaches, since this is the highest value that still enables the required maximum speed of 160 km/h to be reached. Lower transmission ratios would require increased electrical power to satisfy the launch torque requirement. This would be non-optimal for the electric energy demand since the driving cycles mostly demand driving at partial load with non-optimal efficiencies, which would be further stressed with higher EM power. The power of the EM is dimensioned
Table 3. Rounded computational results of $P_c$ and $P_s$ and a single-speed transmission.

|      | $p^M$ (kW) | $i$ | $m$ (kg) | $c^B$ (kWh) | $c_{\text{mean}}$ (kWh/100km) |
|------|------------|-----|---------|-------------|-------------------------------|
|      | $P_c$ | $P_s$ | $P_c$ | $P_s$ | $P_c$ | $P_s$ | $P_c$ | $P_s$ | $P_c$ | $P_s$ | $P_c$ | $P_s$ |
| WLTC | 157  | 160  | 7.07  | 7.07  | 1841 | 1847 | 76.5 | 78.0 | 14.5 | 14.8 |
| NEDC | 155  | 157  | 7.07  | 7.07  | 1818 | 1814 | 70.9 | 70.0 | 13.5 | 13.3 |
| Art. Urban | 162  | 161  | 7.07  | 7.07  | 1859 | 1856 | 81.0 | 80.3 | 15.3 | 15.3 |
| Art. Rural | 153  | 157  | 7.07  | 7.07  | 1799 | 1807 | 66.5 | 68.3 | 12.6 | 13.0 |
| Art. Motorway | 168  | 170  | 7.00  | 7.07  | 1946 | 1954 | 107.0 | 109.9 | 20.3 | 20.9 |
| FTP75 | 153  | 156  | 7.07  | 7.07  | 1795 | 1798 | 65.7 | 66.4 | 12.5 | 12.6 |
| Pool vehicle | 158  | 162  | 7.07  | 7.07  | 1863 | 1871 | 82.0 | 84.1 | 15.6 | 16.0 |
| Pool veh. city | 156  | 157  | 7.04  | 7.07  | 1814 | 1798 | 69.8 | 66.3 | 13.0 | 12.6 |
| Inst. vehicle | 160  | 163  | 7.07  | 7.07  | 1874 | 1878 | 85.1 | 86.2 | 16.2 | 16.4 |
| Mean dev. | 2.44 | 0.01 | 6.67 | 1.69 | 0.29 |
| Median dev. | 3.00 | 0.00 | 6.00 | 1.50 | 0.30 |
| Max. dev. | 4.00 | 0.07 | 16.00 | 3.50 | 0.60 |

Note: The first sub-column in each column group shows the result for $P_c$, while the second shows the minimum results of 10 evaluations each for the objective of $P_s$. All deviation values are absolute.

Table 4. Rounded computational results of $P_c$ and $P_s$ and a two-speed transmission.

|      | $p^M$ (kW) | $i_1$ | $i_2$ | $m$ (kg) | $c^B$ (kWh) | $c_{\text{mean}}$ (kWh/100km) |
|------|------------|-------|-------|---------|-------------|-------------------------------|
|      | $P_c$ | $P_s$ | $P_c$ | $P_s$ | $P_c$ | $P_s$ | $P_c$ | $P_s$ | $P_c$ | $P_s$ |
| WLTC | 70  | 67  | 16.61 | 16.61 | 5.99 | 5.76 | 1804 | 1808 | 67.7 | 68.5 |
| NEDC | 69  | 67  | 20.30 | 16.16 | 6.34 | 5.70 | 1759 | 1764 | 58.2 | 59.2 |
| Art. Urban | 69  | 66  | 20.09 | 19.49 | 6.60 | 7.07 | 1745 | 1758 | 55.4 | 58.0 |
| Art. Rural | 80  | 67  | 14.21 | 16.23 | 6.18 | 5.88 | 1773 | 1768 | 61.0 | 60.0 |
| Art. Motorway | 111 | 82  | 13.19 | 14.56 | 5.48 | 5.14 | 1931 | 1928 | 102.1 | 101.2 |
| FTP75 | 68  | 65  | 18.57 | 18.93 | 6.30 | 5.70 | 1733 | 1737 | 53.1 | 53.9 |
| Pool vehicle | 76  | 75  | 18.27 | 17.28 | 5.88 | 5.56 | 1829 | 1832 | 73.4 | 74.2 |
| Pool veh. city | 68  | 67  | 19.23 | 21.31 | 6.68 | 7.07 | 1718 | 1721 | 50.3 | 50.9 |
| Inst. vehicle | 70  | 67  | 18.61 | 18.38 | 5.78 | 5.79 | 1843 | 1846 | 76.8 | 77.6 |
| Mean dev. | 6.44 | 1.31 | 0.37 | 4.78 | 1.03 | 0.20 |
| Median dev. | 3.00 | 0.99 | 0.34 | 4.00 | 0.80 | 0.20 |
| Max. dev. | 29.00 | 4.14 | 0.64 | 13.00 | 2.60 | 0.50 |

Note: The first sub-column in each column group shows the result for $P_c$, while the second shows the minimum result for $P_s$, based on 10 computations each. All deviation values are absolute.

almost similarly for all cycles, since the maximum power requirements are not imposed directly by the cycles. Instead, the EM must be able to fulfil the launch torque requirement with different vehicle weights, which also leads to a further variation in the EM powers. Furthermore, the modelling differences between the two approaches affect the solution values and result in the shown minor deviations.

Additionally, the results for a two-speed transmission vehicle are shown in Table 4. A significant reduction of the required EM power $p^M$ is observed for both optimization approaches. This is a result of the two-speed transmission, which enables the required launch torque requirement to be fulfilled with a high first transmission ratio and low EM torque, while the second transmission ratio enables the maximum required speed to be achieved. This leads to an increased overall efficiency for two reasons. Firstly, the multi-speed transmission enables choosing between two operating points. Secondly, the downsizing effect reduces partial-load driving situations and enables a higher specific utilization of the EM. The results from both optimization approaches show that the two-speed transmission can lead to a significant reduction of electrical energy demand $c_{\text{mean}}$, which also leads to a possible reduction of the battery capacity $c^B$ for the predefined range of 500 km (see also Section 3.4). For low-speed cycles like Artemis Urban or the pool vehicle city cycle, the improvements compared to the single-speed BEV are especially significant. The transmission ratios vary between the cycles, since
the optimization process tries to match frequent driving situations of each cycle to the EM efficiency map.

Furthermore, it is important to mention that the shown transmission ratios and EM powers of $\mathcal{P}_s$ are sensitive to the scenarios used. For the two-speed transmission, $\mathcal{P}_s$ leads to a variation of transmission ratios and EM maximum power caused by slight changes in the clustering preprocessing step. Hence, the minimum value of the objective of 10 evaluations of each driving cycle is used. The maximum duality gap when reaching the computational time limit in all cases shown is less than 6%.

The results of both optimization approaches are again comparable. They result in almost identical values for the battery size $c^B$, electric energy demand $c_{\text{mean}}$, and mass $m$. The most significant deviation in the choice of design parameters can be observed in Artemis Motorway. $\mathcal{P}_c$ selects a substantially higher EM power. Although the results suggest that a better solution can be found with aid of $\mathcal{P}_s$, the optimal parameter set of $\mathcal{P}_s$ actually leads to a slightly inferior solution in the environment of $\mathcal{P}_c$. Due to a higher electric energy demand in the complete cycle dataset, this solution is infeasible in $\mathcal{P}_c$, as the corresponding battery size of 101.2 kWh is less than 2% too small to meet the range requirement of 500 km.

### 7.2. Discussion

Both shown programs, $\mathcal{P}_c$ and $\mathcal{P}_s$, result in comparable designs for each driving cycle used for the two optimization approaches. The modelling approach $\mathcal{P}_s$ can estimate the underlying results for the single-speed transmission comparable to $\mathcal{P}_c$. For the two-speed design all variables besides the transmission ratios, $i_1$ and $i_2$, and motor power, $\overline{P}^M$, can be estimated with the reduced information given by the derived scenarios with minor differences. To reduce the differences between $\mathcal{P}_c$ and $\mathcal{P}_s$ even further, the local optimal clustering mechanism in the preprocessing, the non-smooth outer approximation of the EM efficiency map and the computational burden of the global non-convex optimization approach should be improved. Furthermore, using the example of the Artemis Motorway cycle in the optimization for a two-speed transmission, it was shown that the parameters of the global optimal solution of $\mathcal{P}_s$ do not necessarily represent the best solution in the environment of $\mathcal{P}_c$, and vice versa.

The solution process used based on the mixed-integer nonlinear programming solver SCIP allows the global optimality of the solution found to be proved by generating a duality certificate. Of course, this certificate only holds for the assumptions and approximations made in the optimization model considered, $\mathcal{P}_s$.

The extensibility of the given modelling and solving approach $\mathcal{P}_s$ is not yet as user-friendly as $\mathcal{P}_c$. Furthermore, the underlying highly nonlinear model parts, such as the efficiency map, have to be approximated to allow for solvability in an exact nonlinear optimization framework. Therefore, the solution process greatly depends on the approximation strategies used for the sub-models.

The GA used in $\mathcal{P}_c$ is an heuristic optimization method that does not rely on gradient calculations. Therefore, a precise modelling of every sub-model with arbitrary nonlinear functions is possible. Since there is no requirement for a specific mathematical form, the fitness evaluation is not limited to a single function, but can consist of multiple inner iteration loops. This method is for example used in Esser et al. (2019) to solve the control problem from the operating strategy separately in every evaluation of the fitness function of the design problem.

When synthesizing driving cycles, a minimum cycle length is required to preserve the physical properties of the underlying driving profile, cf. Esser et al. (2019). Additionally, the computational time depends noticeably on the hyper-parameters of the GA, in particular on the population size and the termination criterion chosen, i.e. in the present case the number of stall generations. Nonetheless, the solutions found with the genetic algorithm have been shown to be robust towards the choice of hyper-parameters. Differences in the results between the two approaches arise rather from the systematic modelling differences described above.
8. Conclusion

A new stochastic approach to designing energy-efficient multi-speed powertrains for battery electric vehicles is proposed in this article. It uses multiple scenarios, which were derived by using an unsupervised learning algorithm, instead of a complete driving cycle. The explicit consideration of a multi-speed transmission leads to a mixed-integer nonlinear program, which is solved using exact optimization. This approach allows global optimality to be proved; however, the underlying electric machine efficiency map model is based on an outer polyhedral approximation to allow a faster solution speed.

To verify the abilities of this new approach, a problem-specific optimization approach based on a genetic algorithm was developed as well.

The two approaches were compared and verified on equivalent test datasets derived from commonly used legislative and synthesized driving cycles and showed comparable results, especially for single-speed transmissions. The scenario-based results were partially sensitive to parameter changes based on the local-optimal clustering in the preprocessing. This should be reduced with further improved preprocessing and modelling approaches in the future.

Additionally, if a GA-based driving cycle approach is chosen, the scenario-based approach can be used to gain insights into further optimization potential of solutions derived by the GA-based approach, since a GA can potentially fall into local optima. Combining both approaches in the conceptual design can therefore be useful by benefiting from the respective advantages of both approaches.

Note
1. With SoPlex v. 4.0.2 and Ipopt v. 3.12.

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