Mirror-induced decoherence in hybrid quantum-classical theory

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We re-analyse the optomechanical interferometer experiment proposed by Marshall, Simon, Penrose and Bouwmeester with the help of a recently developed quantum-classical hybrid theory. This leads to an alternative evaluation of the mirror induced decoherence. Surprisingly, we find that it behaves essentially in the same way for suitable initial conditions and experimentally relevant parameters, no matter whether the mirror is considered a classical or quantum mechanical object. We discuss the parameter ranges where this result holds and possible implications for a test of spontaneous collapse models, for which this experiment has been designed.

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I. INTRODUCTION

The optomechanical experiment proposed by Marshall et al. [1] presents one of the first systems where one attempts to produce and detect coherent quantum superpositions of spatially separated macroscopic states, and to test various decoherence and wave function collapse models [2–7].

The basic idea here is close in spirit to Schrödinger’s original discussion [8,9]: a microscopic quantum system (photon), for which the superposition principle is undoubtedly valid, is coupled with a macroscopic object (mirror), in order to transfer interference effects from the former to the latter, creating a macroscopic superposition state. For this goal, one employs a Michelson interferometer with a tiny moveable mirror in one arm. In this way, since the photon displaces through its radiation pressure the tiny mirror, the initial superposition of the photon being in either arms causes the system to evolve into a superposition of states corresponding to two distinct locations of the mirror.

Nevertheless, before being able to detect macroscopic superpositions, a serious obstacle has to be overcome: decoherence induced by the mirror itself on the photon. The photon, indeed, cannot be dealt with as an isolated system, because it interacts with the mirror and, hence, decoherence can occur destroying any photon coherent superposition [10,11]. In this case, no interference effects can be transferred to the mirror.

Mirror induced decoherence has been examined in Ref. [11], considering both, the mirror and the photon, as quantum objects. However, the size of the former (≈ 1 μm) far exceeds the scales which are typical of the explored quantum regime. Therefore, a classical description of the mirror should be investigated as an alternative and differences or similarities with a quantum one need to be confronted with the planned experiments.

The purpose of our paper is to study the decoherence process with the mirror treated as a classical, rather than a quantum subsystem, while the photon obviously retains its quantum nature. Thus, we have to deal with a model comprising a quantum and a classical sector, which coexist and interact. Such a situation needs a particular theoretical framework for a consistent description, namely a quantum-classical hybrid theory.

There has been much interest in hybrid theories, both for practical and theoretical reasons. From a theoretical point of view, hybrid theories have originally been devised to provide a different approach to the quantum measurement problem [12]. Furthermore, a quantum-classical hybrid theory may be employed to describe consistently the interaction between quantum matter and classical spacetime [13]. See also, for example, the related studies in Refs. [14,20].

Even if one is not inclined to modify certain ingredients of quantum theory, there is also clearly practical interest in various forms of hybrid dynamics, in particular in nuclear, atomic, or molecular physics. The Born-Oppenheimer approximation, for example, is based on a separation of interacting slow and fast degrees of freedom of a compound object. The former are treated as approximately classical, the latter as of quantum mechanical nature. Moreover, mean field theory, based on the expansion of quantum mechanical variables into a classical part plus quantum fluctuations, leads to another approximation scheme and another form of hybrid dynamics. This has been reviewed more generally for macroscopic quantum phenomena in Ref. [21]. In all these cases hybrid dynamics is considered as an approximate description of an intrinsically quantum mechanical object. Such considerations are and will become increasingly important for the precise manipulation of quantum mechanical objects by apparently and for all practical purposes classical means, especially in the mesoscopic regime.

In particular, we recall the hybrid theory elaborated in [22–25], which overcomes the known impediments found
in earlier work. For a closely related approach, see also Ref. [26]. Herein, the classical sector is described by the standard analytical classical mechanics, while the description of the quantum sector is based on Heslot’s representation [27], cf. Ref. [28], allowing to express quantum mechanics in a Hamiltonian framework. Similarly as in classical physics, it is then possible to present states of the quantum sector in terms of couples of real time-dependent functions, rather than vectors, which play the role of canonical variables. Furthermore, the observables are no longer given by self-adjoint operators, but are represented by real quadratic functions of these canonical variables. In this way, an entire hybrid system can be studied in one, uniform, scheme.

In Section II, we will employ this hybrid theory to specify a Hamiltonian, which constitutes the starting point for the analysis of the dynamics of the Marshall et al. optomechanical system. – In Section III, we will derive and solve the corresponding equations of motions. – The solutions of these equations will be employed to calculate the off-diagonal matrix elements of the reduced density matrix for the photon and provide an evaluation of the decoherence induced by the classical mirror. The analysis of this decoherence process and the discussion of the results obtained will be presented in Section IV. – Finally, in Section V we will relate the off-diagonal matrix elements to quantities that can be determined experimentally. – In the concluding section, implications of our results shall be discussed.

II. THE QUANTUM-CLASSICAL HYBRID HAMILTONIAN

The Hamiltonian for the optomechanical interferometer system proposed by Marshall et al. [1] consists naturally of three different parts which have their counterparts in our hybrid description: terms related to the photon (the quantum sector), terms associated with the mirror (the classical sector), and the hybrid coupling of both sectors.

Since the photon is treated quantum mechanically, it is represented by a Hamilton operator \( \hat{H}_{QM} \):

\[
\hat{H}_{QM} = \hbar \omega (\hat{c}_A^\dagger \hat{c}_A + \hat{c}_B^\dagger \hat{c}_B),
\]

where \( \hat{c}_A \) and \( \hat{c}_A^\dagger \), respectively, are creation and annihilation operators for a photon in arm A, and correspondingly for arm B. – Instead, the mirror is considered here as a classical subsystem. While it was described as a quantum harmonic oscillator in Ref. [1], it is now represented by a classical one with Hamiltonian \( \hat{H}_{CL} \):

\[
\hat{H}_{CL} = \frac{p^2}{2M} + \frac{M \Omega^2}{2} x^2,
\]

in which \( x \) and \( p \) denote position and momentum of the mirror, respectively. – The hybrid coupling \( \hat{I} \), i.e. the interaction between photon and mirror, incorporates both, a quantum operator and a classical variable, for photon and mirror, respectively. Being essentially related to the radiation pressure of the photon, as shown in detail in Refs. [29] [33], we have:

\[
\hat{I} = \hbar g x \hat{c}_A^\dagger \hat{c}_A,
\]

where \( g := \omega / L \).

Following Refs. [22] [25], we obtain the full hybrid Hamiltonian \( H \) as:

\[
H = \hat{H}_{CL} + \langle \psi | (\hat{H}_{QM} + \hat{I}) | \psi \rangle,
\]

in which \( | \psi \rangle \) denotes a generic photon state, satisfying always \( \langle \psi(t) | \psi(t) \rangle = 1 \).

In order to evaluate the expectation values in Eq. (4), we consider the orthonormal basis in the Hilbert space of the photon formed by the two vectors \( |1, 0 \rangle \) and \( |0, 1 \rangle \), representing the states in which the photon is either in arm A or B of the interferometer; since we consider only one-photon states, this is also complete. Accordingly, we expand the state \( | \psi \rangle \) as follows:

\[
| \psi \rangle = \frac{1}{\sqrt{2}} (X_A + iP_A) |1, 0 \rangle + \frac{1}{\sqrt{2}} (X_B + iP_B) |0, 1 \rangle,
\]

where \( X_A, X_B, P_A, \) and \( P_B \) are real time-dependent functions, which play the role of canonical variables [22] [25]. This gives:

\[
H = \frac{p^2}{2M} + \frac{M \Omega^2}{2} x^2 + \frac{\omega}{2} \sum_{i=A,B} (X_i^2 + P_i^2) + \frac{g}{2} x (X_A^2 + P_A^2).
\]

This is the full hybrid Hamiltonian for the Marshall et al. optomechanical system, which is a real function of position and momentum of the mirror and of the canonical variables pertaining to the photon, resembling a completely classical formalism. Nevertheless, the quantum nature of the photon is described correctly according to the representation of Eq. (5).

A. Comments

The scenario on which our work is based, resulting in the hybrid Hamiltonian [6], may be related to real experiments. We comment on two important aspects.

Assuming that the single photon within the interferometer typically will be produced by a source outside of the cavity under consideration, the interferometer should be treated as an open system. This has been pointed out in Ref. [24] and studied in a fully quantum mechanical approach (invoking the rotating wave approximation when coupling photon cavity mode inside and continuum modes outside). Depending on the bandwidth of the initial photon wave function as compared to the cavity linewidth, the photon may obtain simultaneously nonzero
amplitudes to be inside and outside of the cavity. This will not affect the hybrid coupling \([3]\), on which we have presently focussed attention. However, it introduces an additional time dependence into the dynamics. Our description of the photon sector can naturally be adapted to this, as before \([34]\), while the coupling to the considered classical mirror remains the same. Anticipating the results obtained in the following, we expect similar effects as observed in Ref. [34], which remains to be confirmed by a detailed study.

Furthermore, one wonders whether our present derivation must be generalized, in order to allow for the simultaneous presence of several photons inside the cavity, which might be experimentally of interest. Let us consider for illustration the interferometer as a closed system and assume a two-photon state inside. Each photon can be present in its arm A or B. Any resulting two-photon state can be embedded in a four-dimensional Hilbert space (e.g. spanned by Bell states) and be expanded analogously to Eq. (5) above. (This has recently been applied in a somewhat similar setting of two q-bit interacting with a classical oscillator [35].) Correspondingly, there will be a proliferation of terms in the resulting hybrid Hamiltonian. Nevertheless, the bilinearity of the hybrid Hamiltonian in the canonical coordinates, as in Eq. [6], will not be affected. The character of the eventually resulting coupled equations of motion, cf. Section [17] will only change by number. However, for high intensity of the cavity field, genuinely nonlinear multi-photon interactions are to be expected, rendering the hybrid coupling constant \(g\) in Eq. (3) effectively time and photon state dependent. This is clearly an interesting regime to be studied, concerning comparison between fully quantum mechanical and hybrid descriptions in particular.

This leads us to mention another hybrid system currently attracting much interest [36, 37], namely a massive body (considered quantum mechanical) interacting with the gravitational field (considered classical and non-relativistic), thus interchanging the role of quantum mechanical and classical degrees freedom as compared to the situation presently studied. It has recently been shown that this hybrid can be represented by the nonlinear Schrödinger-Newton equation, in the limit that the extended body is composed of many “mass-concentrating” constituents (atoms or similar) [38].

It remains to be further examined to what extent such effectively nonlinear descriptions of quantum-classical hybrids are consistent in the sense of criteria discussed earlier [14, 16, 19, 22, 20].

### III. THE HYBRID EQUATIONS OF MOTION

Having specified the hybrid Hamiltonian for the Marshall et al. optomechanical interferometer system, we are now ready to derive the corresponding equations of motion. Following the procedure defined by the appropriate Poisson bracket structure for the algebra of observables and the Hamiltonian, in particular, we will obtain the equations of motion and their solutions here, cf. Refs. [22, 25].

#### A. Derivation of the equations

We begin with the photon in arm A. Its equations of motion are formally given by:

\[
\frac{\partial X_A}{\partial t} = \{X_A, H\}_X, \quad \frac{\partial P_A}{\partial t} = \{P_A, H\}_X, \quad (7)
\]

with \(\{f, g\}_X := \{f, g\}_{CL} + \{f, g\}_{QM}\). Here the “quantum mechanical” Poisson bracket is given by \(\{f, g\}_{QM} := \sum_{i=A,B} \left( \frac{\partial f}{\partial x_i} \frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial x_i} \right)\), while the classical Poisson bracket \(\{f, g\}_{CL}\) is the usual one, of course - following the rules of the hybrid theory constructed in Ref. [22]. Inserting Eq. (6) into Eq. (7), we obtain:

\[
\frac{\partial X_A}{\partial t} = \omega P_A - gxP_A, \quad \frac{\partial P_A}{\partial t} = -\omega X_A + gxX_A. \quad (8)
\]

In the same manner, we obtain the equations of motion for the photon in arm B:

\[
\frac{\partial X_B}{\partial t} = \omega P_B, \quad \frac{\partial P_B}{\partial t} = -\omega X_B. \quad (9)
\]

Finally, for the mirror, we similarly obtain:

\[
\frac{\partial x}{\partial t} = \frac{p}{M}, \quad \frac{\partial p}{\partial t} = -M\Omega^2 x + q \left( X_A^2 + P_A^2 \right). \quad (10)
\]

Remarkably, the equations of motion for our quantum-classical hybrid system have a completely classical appearance. This will be employed in the following derivation of their solutions.

#### B. Solution of the hybrid equations

The equations obtained in the previous subsection describe a set of coupled harmonic oscillators, which can be solved analytically. In particular, while the equations for the photon in arm B are completely decoupled, those associated with the mirror and the photon in arm A are coupled. This is as expected, since the photon in arm B does not interact with anything, while when it is in arm A it inevitably interacts with the mirror.

The coupling term in Eq. (10), \(\propto (X_A^2 + P_A^2)\), at first sight, complicates solving of the equations. However, this term consists in a constant of motion and, therefore, can be replaced by its initial value. We note that:

\[
\frac{1}{2} \frac{\partial}{\partial t} \left( X_A^2 + P_A^2 \right) = X_A \frac{\partial X_A}{\partial t} + P_A \frac{\partial P_A}{\partial t} = 0, \quad (11)
\]

using Eq. (3). In order to determine the value of this constant of motion, we consider its physical meaning: it
simply describes the probability to find the photon in arm A, expressed in the representation of Eq. (5). We assume a fifty-fifty beam splitter in the interferometer  |. Therefore, this probability is equal to 1/2:

\[
\frac{1}{2} = \langle \psi | e_A^\dagger e_A | \psi \rangle = \frac{1}{2\hbar} (X_A^2 + P_A^2) \langle 0, 1 | e_A^\dagger e_A | 1, 0 \rangle
\]

= \frac{1}{2\hbar} (X_A^2 + P_A^2) \langle 0, 1 | 1, 0 \rangle = \frac{1}{2\hbar} (X_A^2 + P_A^2) ,
\]

since \( \langle 0, 1 | 1, 0 \rangle = 1 \). Thus, we have: \( X_A^2 + P_A^2 = \hbar \).

Similarly, we find: \( X_B^2 + P_B^2 = \hbar \).

Making use of this in Eq. (10), the equations for the mirror become:

\[
\frac{\partial x}{\partial t} = \frac{p}{\hbar} , \quad \frac{\partial p}{\partial t} = -\hbar^2 \Omega^2 x + \hbar \phi / 2 ,
\]

which can be easily solved to give:

\[
x (t) = A \sin (\Omega t + \phi) - \frac{\hbar \phi}{2\hbar^2} , \quad p (0) = A \hbar \Omega \cos (\phi) ,
\]

and abbreviating \( x_0 \equiv x (0) \), \( p_0 \equiv p (0) \), we find:

\[
A (x_0, p_0) = \frac{p_0}{M \Omega \cos [\phi(x_0, p_0)]} , \quad \phi (x_0, p_0) = \arctan [(M \phi x_0 + \hbar \phi / 2\hbar) / p_0] .
\]

Knowing the solutions of the equations for the mirror, we can solve those for the photon in arm A, Eqs. (8). It is straightforward to obtain:

\[
X_A (t) = \sqrt{\hbar} \cos \left( \Omega t + \frac{A \phi}{\hbar} \cos (\Omega t + \phi) - \cos (\phi) \right) ,
\]

\[
P_A (t) = -\sin \left( \Omega t + \frac{A \phi}{\hbar} \cos (\Omega t + \phi) - \cos (\phi) \right) .
\]

conveniently introducing \( \Omega_+ \) and \( k_+ := \omega + k^2 \Omega := \omega + \hbar^2 / (2M \Omega^2) \).

Finally, regarding the photon in arm B, we find:

\[
X_B (t) = \sqrt{\hbar} \cos (\omega t) , \quad P_B (t) = -\sqrt{\hbar} \sin (\omega t) .
\]

Here we recalled the normalization of the photon state for arm B, which follows similarly as in Eq. (12), in order to fix the amplitudes. The phase, instead, is determined by choosing: \( X_B (0) = 1 \) and \( P_B (0) = 0 \).

IV. THE MIRROR INDUCED DECOHERENCE

In this section, we report quantitative results concerning the decoherence induced by the classical mirror on the quantum photon.

The relevant information is contained in the off-diagonal elements of the reduced density matrix for the photon. Presently, these matrix elements are given by:

\[
\rho_{AB} = \langle 1, 0 | \psi \rangle \langle \psi | 1, 0 \rangle = \rho_{BA}^* ,
\]

with \( \langle 1, 0 | \psi \rangle = 0 \). Inserting Eq. (5) here, we obtain:

\[
\rho_{AB} (t) = \frac{1}{2\hbar} (X_A + iP_A) (X_B - iP_B) ,
\]

and recalling Eqs. (19), (20), and (21), this becomes:

\[
\rho_{AB} (t; x_0, p_0) = \frac{1}{2} \exp \left\{ i \omega t \right\} \cdot \exp \left\{ -i \left[ \Omega_+ t + \frac{A \phi}{\hbar} \cos (\Omega t + \phi) - \cos (\phi) \right] \right\} ,
\]

where the dependence on \( x_0 \) and \( p_0 \) is through \( A \) and \( \phi \).

Taking the modulus of the matrix element \( \rho_{AB} \), we obtain the mirror induced decoherence as a function of time, i.e. the visibility of interference of the photon. Since the result of Eq. (24) is a pure phase, its modulus is a constant:

\[
|\rho_{AB}| = \frac{1}{2} .
\]

Thus, for pointlike initial conditions in the classical phase space of the mirror, no mirror induced decoherence occurs. In this case, the photon remains in its initial pure state, the coherent superposition of being in either of the arms of the Michelson interferometer, and the corresponding interference effects are preserved.

We emphasize that this result concerns the situation where the initial conditions of the classical mirror are perfectly known and are represented by a point in phase space. It shows a clear and expected difference between the present hybrid and the purely quantum approach  |. The latter results in mirror induced decoherence, even before thermal averaging.

However, suppose that there is some loss of information consisting in somewhat imprecisely known initial position and momentum of the mirror. Instead of sharp initial conditions, we may have a probability distribution over phase space. For instance, we consider a Boltzmann distribution:

\[
f (x_0, p_0) := \frac{\beta \Omega}{2\pi} \exp \left\{ -\beta \left( \frac{p_0^2}{2M} + \frac{M \Omega^2 x_0^2}{2} \right) \right\} ,
\]

depending on the inverse temperature, \( \beta := 1/\hbar_B T \). In this case, the physically relevant matrix element is given by the thermal average of the result of Eq. (24):

\[
< \rho_{AB} > f = \int \rho_{AB} (t; x_0, p_0) f (x_0, p_0) \, dx_0 \, dp_0 .
\]
In order to calculate this integral, we rewrite it more conveniently, using the abbreviations $\kappa^2 := \hbar g^2/(2M\Omega^3)$ (previously introduced in Ref. [1]), $\theta(t) := \Omega t - \sin(\Omega t)$, and:

\[
\begin{align*}
    h_1(x_0,t) &:= \frac{\beta M \Omega^2}{2} x_0^2 + i \frac{g}{\Omega} \sin(\Omega t) x_0, \\
    h_2(p_0,t) &:= -\frac{\beta}{2M} p_0^2 - i \frac{g}{M\Omega^2} [\cos(\omega_m t) - 1] p_0.
\end{align*}
\]

Thus, we obtain from Eq. (27):

\[
|<\rho_{AB}(t)>_f| = \frac{\beta\Omega}{4\pi} \exp\{-i\kappa^2 \theta(t)\} \cdot \int_{-\infty}^{\infty} \exp\{h_1(x_0,t)\} dx_0 \int_{-\infty}^{\infty} \exp\{h_2(p_0,t)\} dp_0
\]

\[
= \frac{1}{2} \exp\{-i\kappa^2[q_\Omega - \sin(\Omega t)] - z^2_{CL}[1 - \cos(\Omega t)]\}.
\]

with $z^2_{CL} := g^2/(\beta M \Omega^4) = 2\kappa^2 k_B T/(\hbar \Omega)$.

In order to evaluate the mirror induced decoherence, we calculate the modulus of this averaged matrix element (commonly referred to as “visibility”):

\[
|<\rho_{AB}(t)>_f| = \frac{1}{2} \exp\{-z^2_{CL}[1 - \cos(\Omega t)]\}.
\]

This shows the decoherence induced by the classical mirror on the quantum photon, taking into account that the mirror initial conditions are thermally distributed.

The Fig. 1 shows the temporal behaviour of the visibility of interference. As in the purely quantum case, the visibility has a maximum at the initial instant and then decreases, due to the mirror induced decoherence. However, after half a period of the mirror oscillation, we observe a revival of coherence of the photon and the visibility returns to its maximum value exactly at $t = 2\pi/\Omega$.

It is important to realize that the result of Eq. (31), incorporating the thermal average over classical initial conditions, has exactly the same form as the earlier one obtained in Ref. [1] for a quantum mechanical mirror as part of the interferometer. In particular, we find the same time dependence. The only difference resides in that our parameter $z_{CL}$, defined after Eq. (30), has to be replaced by the corresponding parameter $z_{QM}$ given by:

\[
\begin{align*}
    z^2_{QM} &= 2\kappa^2\left(\bar{n}(\hbar\Omega/k_BT) + 1/2\right) \\
    &= z^2_{CL}(\hbar\Omega/k_BT)\left(\bar{n}(\hbar\Omega/k_BT) + 1/2\right),
\end{align*}
\]

with the Bose-Einstein distribution $\bar{n}(x) := (\exp(x) - 1)^{-1}$. Here we incorporated the appropriate finite temperature correction indicated (but not explicitly given) in Ref. [1] for the quantum mechanical mirror.

Thus, we find that in the high-temperature limit, with $\hbar\Omega/k_BT \ll 1$, both parameters coincide,

\[
z^2_{QM} = z^2_{CL}\left(1 + \frac{1}{12}[\hbar\Omega/k_BT]^2 + \mathcal{O}([\hbar\Omega/k_BT]^4)\right),
\]

and, consequently, the visibilities given by the right-hand side of Eq. (31), with either $z_{CL}$ or $z_{QM}$ inserted, become equal as well, for all times!

More generally, considering the ratio $\eta$ of the result of Eq. (31) divided by the quantum mechanical result from [1], $\eta := |<\rho_{AB}(t)>_f|/|<\rho_{AB}(t)>_{QM}|$, we find numerically that for experimentally relevant temperatures $10^{-5}K < T < 10^{-3}K$ and mirror frequency $\Omega = 2\pi \cdot 500Hz$ – the deviation of both results can be correspondingly bounded by $10^{-2} > \eta - 1 > 10^{-6}$, indeed a surprising result. Furthermore, due to identical time dependence, $\propto 1 - \cos(\Omega \tau)$, in the exponent on the right-hand side of Eq. (31) and the corresponding quantum mechanical result, the deviation of both visibilities goes to zero always when $\Omega\tau$ approaches $2\pi$ times an integer, which is the experimentally interesting region close to maximal visibility, cf. Fig. 1.

Since the visibility of Eq. (31) shows, for sufficiently short times ($\tau := \Omega \ll 1$, cf. Fig. 1), a Gaussian decay, we may define the characteristic decoherence time $t_{CL}$ by:

\[
z^2_{CL}[1 - \cos(\Omega t)] \approx z^2_{CL}((\Omega t)/2)^2 = \left(t/t_{CL}\right)^2/2,
\]

and, correspondingly, for the case of a quantum mechanical mirror. This gives us the relevant decoherence times $t_{CL} = (z_{CL}\Omega)^{-1}$ and $t_{QM} = (z_{QM}\Omega)^{-1}$. Thus, we obtain
In analogy to Eq. (33), we conclude here that the decoherence times coincide in the high-temperature limit. For experimentally relevant parameters $[1]$, i.e., frequencies around $\Omega = 2\pi \cdot 500\text{Hz}$, while maintaining $\kappa \approx 1$, and temperatures in the interval $10^{-9}\text{K} \lesssim T \lesssim 10^{-3}\text{K}$, such that $2.4 \cdot 10^{-2} \lesssim \hbar \Omega / k_B T \lesssim 2.4 \cdot 10^{-5}$, we have that the discriminating factor $z_{QM}/z_{CL}$ in Eq. (35) deviates from 1 by less than $10^{-4}$. Therefore, the decoherence times $t_{CL}$ and $t_{QM}$ are the same to such accuracy that they will be difficult to distinguish experimentally, at present.

For all practical purposes, the similitude of the features of mirror induced decoherence is a robust result, as we have demonstrated, considering both, either a classical mirror plus photon described by quantum-classical hybrid theory or a mirror plus photon described as fully quantum mechanical system $[1]$.

V. THE PROBABILITY TO DETECT A PHOTON

The mirror induced decoherence has been evaluated in terms of an off-diagonal matrix element of the reduced density operator for the photon. In this section, we relate this quantity to the experimentally accessible probability to find a photon, respectively, in one of the two detectors situated in the interferometer.

They are given by:

$$ P_i (t) = \text{Tr} \left( < \hat{\rho} (t) > f \hat{P}_i \right), \quad i = 1, 2, \quad (36) $$

with the averaged density matrix given by:

$$ < \hat{\rho} (t) > f = \left( \frac{1}{2} \right) \frac{1}{2} \exp \left\{ -i \left[ \frac{1}{2} \exp \left\{ -i k^2 \left[ \Omega t - \sin (\Omega t) \right] - z^2 [1 - \cos (\Omega t)] \right\} \right\} \right. $$

$$(37)$$

and where $\hat{P}_1$ and $\hat{P}_2$ are projectors related to the two interferometer arms where the detectors are located $[1]$; “c.c.” denotes the complex conjugate of the upper off-diagonal matrix element. In the basis of $< \hat{\rho} (t) > f$ chosen here, the projectors are represented by:

$$ \hat{P}_1 = \frac{1}{2} \left( \begin{array}{cc} 1 & 1 \\ 1 & 1 \end{array} \right), \quad \hat{P}_2 = \frac{1}{2} \left( \begin{array}{cc} 1 & -1 \\ -1 & 1 \end{array} \right). \quad (38) $$

Inserting Eqs. (37) and (38) into Eq. (36), we obtain:

$$ P_{1,2} (t) = \frac{1}{2} \left[ 1 + 2 \text{Re} \left( < \rho_{AB} (t) > f \right) \right] $$

$$ = \frac{1}{2} \left\{ 1 \pm \cos \left[ k^2 (\Omega t - \sin (\Omega t)) \right] \right\} \cdot \exp \left[ -z^2 (1 - \cos (\Omega t)) \right\} \right. $$

$$(39)$$

This presents an important relation, because it connects $< \hat{\rho} (t) > f$, the central quantity to learn about mirror induced decoherence, with the probability of observing a photon in one of the two detectors. In this way, in principle, we could learn about the former from experimental measurements of the latter.

This result is independent of whether we consider the mirror as a classical or a quantum object, i.e. independent of whether we apply quantum theory to the whole interferometer set-up or the quantum-classical hybrid theory presently studied.

VI. CONCLUSIONS AND PERSPECTIVES

In this paper we have studied the optomechanical interferometer experiment of Marshall et al. in the hybrid
quantum-classical theory of Refs. [22–26]. Here, the mirror is considered as a perfectly classical rather than a quantum mechanical object and the quantum nature of the photon is preserved in a formally consistent framework.

In Section [I] we presented the hybrid Hamiltonian for the whole system, composed of the classical mirror and the quantum photon. The Hamiltonian encodes all dynamical information regarding the system and has been employed to derive the corresponding equations of motion. In Section [II] we solved the equations analytically.

In Section [III] the solutions of the equations of motion have been used to obtain the off-diagonal elements of the reduced density matrix for the photon, which forms the starting point for our quantitative evaluation of the decoherence process induced by the classical mirror on the quantum photon. As in the fully quantum approach, this decoherence destroys interference effects and is detrimental to the formation of spatially separated coherent superposition states of the mesoscopic mirror.

We emphasize that, according to the hybrid interaction scheme, the photon and the classical mirror presently do not become entangled. Thus, the mirror is at each moment of time in a classical pure state, unless thermal (or some other) fluctuations are explicitly introduced. Such classical fluctuations play an analogous role here to the quantum fluctuations in the mirror state induced by entanglement if both parts are treated as quantum systems.

More precisely, we have to distinguish two different cases. – First, if the classical initial conditions of the mirror, namely its initial position and momentum, are exactly known, then no decoherence is observed: the photon remains in its initial pure state, the coherent superposition of being in either arm of the interferometer, and related interference effects are sustained. This clearly differs from the original quantum approach [1], where also without thermal averaging over initial coherent oscillator states of the mirror, one finds mirror induced decoherence.

This result leads us to conjecture that the absence of decoherence, when the initial conditions of the classical subsystem are completely fixed, is a general feature of a composite quantum-classical hybrid. A proof has to await future studies of these phenomena.

Secondly, however, we have also examined the more realistic situation where some information about the classical initial conditions is lost and only a phase space probability distribution, instead, can be assumed or be experimentally prepared.

In particular, we have considered a thermal Boltzmann distribution specifying the mirror initial conditions. This leads to correspondingly averaged matrix elements of the reduced density matrix of the photon. Analyzing these, we find the surprising result that the mirror induced decoherence according to the hybrid theory essentially equals the one found in a fully quantum mechanical treatment [1].

This is nicely reflected in the corresponding decoherence timescales that we defined and discussed in Section [IV] in particular for the experimentally relevant range of temperatures. We pointed the stability of this equality with respect to variations of the physical parameters of the system (temperature T, mirror frequency Ω, photon frequency ω, cavity length L, and mirror mass M). We have found that the near-equality in the behaviour of the interferometer, whether treated as a quantum-classical hybrid or fully quantum mechanical system, is stable against such variations within the experimentally accessible regime.

This extends to the experimentally measurable probability of finding a photon in one of the two detectors of the original interferometer arrangement [1]. In Section [IV] we have related the off-diagonal matrix elements of the photon reduced density operator, from which mirror induced decoherence has always been calculated, to the probabilities to detect the photon in one of the two detectors.

An interesting study, which can also be performed on the basis of our formalism and results, will be to consider (thermally averaged) squeezed initial states for a quantum mirror and correspondingly deformed (Boltzmann like) initial phase space distributions for a classical mirror. Will quantum theory and quantum-classical hybrid theory remain essentially indistinguishable also in this case, concerning mirror induced decoherence?

In any case, our discussion may have implications for the interpretation of planned experiments, see, for example, Refs. [27, 29, 40] and further references therein. In fact, the optomechanical system of Marshall et al., first of all, has been proposed to test various spontaneous wave function collapse models [26]. As indicated by our results, however, the system might not be suitable to discern a quantum from a classical mirror, given the accessible experimental parameters. In this case, the observation of “anomalous decoherence” (i.e., when common sources of environmentally induced decoherence can be controlled) cannot unambiguously be attributed to a rapid collapse mechanism, perhaps the mirror has been classical from the start and yet produces a similar decoherence signal.

We conclude that applications of quantum-classical hybrid theory to describe presently considered experiments at the quantum-classical border, in particular when “macroscopic” components play a role, deserve further study. Last not least, since it is still thoroughly unknown whether, where, and what kind of border to expect.

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