ABOUT THE $Q^2$ DEPENDENCE OF THE ASYMMETRY $A_1$ FROM THE SIMILARITY OF THE $g_1$ AND $F_3$ STRUCTURE FUNCTIONS

A.V. KOTIKOV$^{1\dagger}$ and D.V. PESHEKHONOV$^{2\dagger}$

$^1$ Bogoliubov Laboratory of Theoretical Physics, JINR, Dubna, 141980 Russia
$^2$ Laboratory of Particle Physics, JINR, Dubna, 141980 Russia
$\dagger$ E-mail: kotikov@thsun1.jinr.ru, peshehon@sunse.jinr.ru

Abstract

We consider a new approach for taking into account the $Q^2$ dependence of asymmetry $A_1$. This approach is based on the similarity of the $Q^2$ behavior and the shape of the spin-dependent structure function $g_1(x, Q^2)$ and spin averaged structure function $F_3(x, Q^2)$.

An experimental study of the nucleon spin structure is realized \[1\] by measuring the asymmetry $A_1(x, Q^2) = g_1(x, Q^2)/F_1(x, Q^2)$. The most known theoretical predictions on spin dependent structure function (SF) $g_1(x, Q^2)$ of the nucleon were done by Bjorken \[2\] and Ellis and Jaffe \[3\] for the so called first moment value $\Gamma_1 = \int_0^1 g_1(x)dx$.

Studying the properties of $g_1(x, Q^2)$ and the calculation of the $\Gamma_1$ value require the knowledge of SF function $g_1$ at the same $Q^2$ in the hole $x$ range. Experimentally asymmetry $A_1$ is measuring at different values of $Q^2$ for different $x$ bins. An accuracy of the modern experiments \[4\]-\[7\] allows to analyze data in the assumption \[8\] that asymmetry $A_1(x)$ is $Q^2$ independent (i.e. SF $g_1$ and $F_1$ have the same $Q^2$ dependence)

\[A_1(x, Q^2) = A_1(x)\] \hspace{1cm} (1)

But the precise checking of the Bjorken and Ellis - Jaffe sum rules requires considering the $Q^2$ dependence of $A_1$ or $g_1$. Moreover, the assumption (1) does not follow from the theory. On the contrary, the behavior of $F_1$ and $g_1$ as a functions of $Q^2$ is expected to be different due to the difference between polarized and unpolarized splitting functions.

There are several approaches (see \[9\] and references therein) how to take into account the $Q^2$ dependence of $A_1$. They are based on different approximate solutions of the DGLAP equations. Some of them have been used already by Spin Muon Collaboration (SMC) and E154 Collaboration in the last analyses of experimental data (see
In this paper we review another idea which is based on the observation in [10, 11] that the splitting functions of the DGLAP equations for the \( g_1 \) and \( F_3 \) and the SF shapes are similar in a wide \( x \) range and, thus, their \( Q^2 \) dependences have to be close as a consequence. Our approach for \( Q^2 \)-dependence of \( A_1 \) is very simple (see Eq. (3)) and leads to the results, which are very similar to ones based on the DGLAP evolution.

To demonstrate the validity of the observation, we note that splitting functions in the r.h.s. of DGLAP equations for nonsinglet (NS) parts of \( g_1 \) and \( F_3 \) and the SF shapes are similar in a wide \( x \) range and, thus, their \( Q^2 \) dependences have to be close as a consequence. Our approach for \( Q^2 \)-dependence of \( A_1 \) is very simple (see Eq. (3)) and leads to the results, which are very similar to ones based on the DGLAP evolution.

In principle, \( Q^2 \) independence of the \( A_1^* \) ratio can be violated at small \( x \) values, because the SF \( F_3(x) \) has only nonsinglet structure and the SF \( g_1(x) \) contains gluon and sea quark contributions. We note, however, that

- in polarized case the gluon and sea quark contributions are not so large even at modern small \( x \) values (see Fig. 1 which is taken from [7]): only at \( x \leq 10^{-3} \) these contributions start to be dominant.
- The double-logarithmic estimations of small \( x \) asymptotics for the SF \( g_1 \) and \( F_3 \) \( g_1^{\text{NS}}(x) \), \( F_3(x) \sim x^{-a_{\text{NS}}} \) and \( g_1^S(x) \sim x^{-a_S} \), given in [12], lead to the results: \( a_{\text{NS}} \sim 0.4 \) and \( a_S \sim 3/2a_{\text{NS}} \sim 0.6 \), that supports a similarity of nonsinglet and singlet polarized components.

We note that the estimations in [12] have been given at small \( Q^2 \) values. However, as it was shown earlier in [13] and [14], respectively, the values of \( a_{\text{NS}} \) and \( a_S \) should be nearly \( Q^2 \)-independent, that is supported also by recent fits (see [13] and references therein).

To apply the proposed approach we use the SMC [5] data (the SLAC [6, 7] data have been considered in [11]). We parameterize CCFR data on \( F_2(x, Q^2) \) and \( xF_3(x, Q^2) \) [10]...
and take also the SLAC parameterization of $R(x, Q^2)$ \[13\] to obtain structure function $F_1(x, Q^2)$. We use in Eq.\((3)\) parameterizations of CCFR data \[14\] for both SF $xF_3(x, Q^2)$ and $F_2(x, Q^2)$ to avoid systematic uncertainties and nucleon correlation in nuclei.

The SF $g_1(x, Q^2)$ is calculated using the asymmetry $A_1(x, Q^2)$ as

$$g_1(x, Q^2) = A_1(x, Q^2) \cdot F_1(x, Q^2),$$  \(4\)

where the spin average SF $F_1$ has been calculated using NMC parameterization of $F_2(x, Q^2)$ \[17\]. The results are presented in Fig. 2. Our results are in very good agreement with the calculations based on direct DGLAP evolution.

To make another comparison with the theory we have calculated also the first moment value of the structure function $g_1$ at different $Q^2$. Using Eq.\((3)\), we recalculate the SMC measured asymmetry of the proton and deuteron and E154 one of neutron at $Q^2 = 100 \text{ GeV}^2$, $30 \text{ GeV}^2$, $Q^2 = 10 \text{ GeV}^2$ and $3 \text{ GeV}^2$ and get the value of $\int g_1(x)dx$ through the measured $x$ ranges. To obtain the first moment values $\Gamma_1^p(\bar{d})$ we use an original estimations of SMC and E154 for unmeasured regions. In Table we present the results for the mean values of $\Gamma_1^p - \Gamma_1^n$, because the errors coincide with the errors of original analyses \[14-16\]. The value of $\Gamma_1^p - \Gamma_1^n$ at $Q^2 = 10 \text{ GeV}^2$ obtained by direct DGLAP evolution are taken from article \[18\].

| $Q^2 (\text{GeV}^2)$ | 100 | 30 | 10 | 3 |
|---------------------|-----|----|----|---|
| SMC proton and deuteron data |
| $A_1$-scaling | 0.297 | 0.226 | 0.202 | 0.170 |
| Evolution | | 0.183 | |
| $A_1^*$-scaling | 0.210 | 0.201 | 0.191 | 0.176 |
| SMC proton and E154 neutron data |
| $A_1$-scaling | 0.221 | 0.209 | 0.194 | 0.170 |
| $A_1^*$-scaling | 0.194 | 0.190 | 0.185 | 0.175 |
| Theory | 0.194 | 0.191 | 0.187 | 0.180 |

Table: The mean values of $\Gamma_1^p - \Gamma_1^n$.

Let us now present the main results, following from the Table and the Fig. 2.

- The results are in very good agreement with $g_1(x, Q^2)$ data of SMC, based on direct DGLAP evolution. So, the suggestion about $Q^2$ independence of the $A_1^*$ ratio leads to correct results in simplest way.

- Our method allows to test of the Bjorken sum rule in a simple way with a good accuracy. Obtained results on the $\Gamma_1^p - \Gamma_1^n$ show that used experimental data well confirm the Bjorken sum rule prediction.

- A violation of $A_1^*$-scaling \[2\] in (future) experimental data will demonstrate large gluon and/or sea quark contributions to SF $g_1(x, Q^2)$. Then, the check of $Q^2$ dependence of $A_1^*$ ratio should be very useful for an indication of large contributions of gluons and sea quarks in future polarized experiments.

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