\(\beta\)-decay half-lives including first-forbidden contributions for neutron-rich Zn isotopes in the extended QRPA with neutron-proton pairing

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Abstract. The \(\beta\) decays of neutron-rich Zn isotopes are investigated within the extended quasiparticle random-phase approximation, where neutron-neutron, proton-proton, and neutron-proton pairing correlations are considered in the similar manner. The Brückner \(G\)-matrix obtained with the charge-dependent Bonn nucleon-nucleon force is used for the residual particle-particle and particle-hole interactions in addition to the pairing interactions. Contributions from both allowed Gamow-Teller and first-forbidden transitions are considered, and \(\beta\)-decay half-lives together with \(\beta\)-delayed neutron emission probabilities are calculated. The calculated results are found to agree well with the available experimental data.

1. Introduction
Beta decay plays an important role in understanding nuclear structure and astrophysics [1, 2, 3]. Continual attention has been paid from both experimental and theoretical sides. One of the popular theoretical models is the quasiparticle random-phase approximation (QRPA). Within the QRPA framework, pairs of like nucleons, proton with proton (pp) or neutron with neutron (nn), have been widely considered, but treatments of neutron-proton (np) pairing correlations are less documented. It is usually expected that spin-orbit and Coulomb forces become stronger and restrain np pairing in heavy nuclei with \(N > Z\). But the recent theoretical study [4] suggests that enhanced np interactions occur in heavy nuclei along a trajectory of approximately equal numbers of valence protons and neutrons. Such a phenomenon is understood in terms of parameter-free spatial overlaps of proton and neutron Nilsson orbitals with \(\Delta K|\Delta N, \Delta n_z, \Delta \Lambda|\rangle=0[110]\).

There are two types of np pairing, namely, the isoscalar \(T = 0\) and isovector \(T = 1\) pairing interactions. Some QRPA calculations deal with the isoscalar np pairing interaction in the QRPA particle-particle channel by finite-range or zero-range forces [5, 6, 7, 8, 9]. Some deal with the isovector np pairing interaction in the Hartree-Fock-Bogoliubov (HFB) equations besides pp and nn pairing interactions and the isoscalar np pairing contributions are added by renormalizing the pairing potential gaps [10, 11, 12]. Based on the second approach, we have recently investigated
allowed Gamow-Teller (GT) transitions of $r$-process waiting-point nuclei around the neutron magic numbers within the extended QRPA with np pairing [2]. As a further improvement, the present study reports on the investigation of both GT and first-forbidden (FF) transitions for neutron-rich Zn isotopes within the extended QRPA. In particular, very neutron-rich Zn isotopes are in the vicinity of the $N = 50$ shell closure and of great interest to describe the rapid neutron-capture ($r$-process). For example, $^{82}$Zn has been produced and separated at the ISOLDE-CERN facility and its $\beta$-decay half-life and $\beta$-delayed neutron emission probability have been remeasured with high precision [13].

2. Theoretical framework

The partial half-life $t_{1/2}$ for $\beta$ transitions is obtained by the expression $f t_{1/2} = 6170$ s with [14, 15, 16, 17]

$$f = \int_{W_0}^W dWC(W)F(Z, R, W)(W_0 - W)^2W\sqrt{W^2 - 1},$$

where $W_0$ and $W$ are the maximum energy and the total energy (including the rest energy) of the $\beta$ particle, $Z$ and $R$ are separately the atomic number and the nuclear radius of the daughter nucleus, $C(W)$ is the so-called shape factor depending on nuclear transition matrix elements, and $F(Z, R, W)$ is the Fermi function which accounts for the Coulomb interaction between the charged $\beta$ particle and the residual daughter nucleus. For GT transitions, the shape factor has the simple form without dependence on the $\beta$ energy, $C(W) = B(GT) = (g_A/\hbar c^2)\langle f|\vec{\sigma}\vec{\tau}|i\rangle^2/(2J_i + 1)$. In contrast, the shape factor of FF transitions depends on the $\beta$ energy $W$. If only dominant terms are considered, it can be written as $C(W) = k(1 + aW + b/W + cW^2)$. The details of $k$, $ka$, $kb$ and $kc$ can be found in [14, 15, 16, 17] according to the treatment by Behrens and Bühring [18].

To manipulate the dynamics of the decaying system, the main focus lies on the calculation of the shape factor $C(W)$. Our approach is based on the extended QRPA with np pairing [2, 10, 11, 12]. The first step of our calculations is to deal with the $T = 1$, $J = 0$ pp, nn, and np pairing correlations using the HFB equations, which turn the simple single-particle mean-filed description to the quasiparticle picture for the decaying ground state. The quasiparticle states are formed by the admixture of proton and neutron single-particle states. Then the particle-hole and particle-particle excitations are considered using the extended QRPA equations, which mix the pure quasiparticle states for the excitation states of residual daughter nuclei. It should be noted that the two-body interaction matrix elements are evaluated based on the Brückner $G$-matrix with charge-dependent Bonn nucleon-nucleon forces. The details of the calculations are described in [19, 20].

After solving the extended QRPA equation, one can obtain the forward and backward-going amplitudes $X$, $Y$, as well as the QRPA energies $\omega$. The amplitude for $\beta^-$ transitions from the ground state $|0\rangle$ of an even-even nucleus to the $m$th phonon state $|mJ^*\rangle$ of the neighboring odd-odd nucleus is expressed by [2, 10, 11, 12]

$$\beta^- \equiv \langle mJ^*|\beta^-|0\rangle = \sum_{a\alpha'b\beta'} \langle p|T_J|n\rangle \left[ X_{(a\alpha'b\beta')}^{m}(u_{a\alpha'p}v_{b\beta'p} - u_{b\beta'p}v_{a\alpha'p}) + Y_{(a\alpha'b\beta')}^{m}(v_{a\alpha'p}w_{b\beta'p} - v_{b\beta'p}w_{a\alpha'p}) \right],$$

where the one-body operators $T_J$ are associated with the GT transition operator $(\vec{\sigma})$ and the FF transition operators $(\vec{r} \times \vec{p})^\lambda$ with $\lambda = 0$, 1, 2 and $\vec{r}$, as well as relativistic vector operator $\vec{\alpha}$ and axial charge operator $\gamma_5$. The detailed expressions of $\beta^-$ can be found in [17].
3. Numerical results and discussion

Using the formalism described above, we have performed a detailed investigation on $\beta$-decay half-lives of neutron-rich Zn isotopes. The parameters used in the calculations are clearly explained in [2] and here we do not repeat it. Attention should be paid to the quenching effect which is usually taken into account by the quenching of the axial vector coupling constant. In view of the fact that the inclusion of FF transitions reduces calculated $\beta$-decay half-lives, there is a slightly difference between the quenching factors in the previous study of GT transitions [2] and in this contribution of both GT and FF transitions. Here, the quenching factor $g_A/g_A^{\text{free}} = 0.75$ is used for both GT and FF transitions except for the $0^-$ FF case. For the case of $0^-$ FF transitions, the tensor part of the transition operator is enhanced due to meson exchange current effects [15]. Therefore, following the analysis of [15], the enhancement factor $\epsilon = g_A/g_A^{\text{free}} = 2.0$ is employed for the tensor part while the factor $\epsilon$ is fixed at 1.0 for the scalar part. In our calculations, the experimental data of pairing gaps, $\beta$-decay energies, and neutron separation energies are taken either from [21, 22] when available, or from the KUTY mass formula [23]. To estimate our theoretical results, the experimental $\beta$-decay half-lives are taken from [21] and some new data with high precision are taken from [13].

First, let us focus on the calculation of $\beta$-decay half-lives. Figure 1(a) shows the numerical results of the calculated half-lives, compared with the experimental data (labeled by Expt). In 2003, Möller et al investigated GT transitions within the FRDM plus QRPA and included FF transitions by the gross theory instead of microscopic calculations [1]. For comparison, the theoretical results of Möller et al (labeled by FRDM + QRPA) are also shown. Our calculations are separately performed with and without np pairing (denoted by Calc with np and Calc without np). One can see that the calculations without np pairing yield longer half-lives as compared with the experimental data except for $^{82}$Zn. And this behavior is particularly evident for the long-lived isotopes near the $\beta$-stability line. After np pairing correlations are taken into account, the calculations give shorter half-lives and reproduce the experimental half-lives well. The slightly large deviation from the experimental data occurs at $^{82}$Zn, for which the recent
half-life measurement is known as 178(12) ms [13] while the calculated half-life is obtained as 70 ms. This may be due to large uncertainties of the theoretical \( \beta \)-decay energy used in the calculations [22, 23] and also because it is not good enough for the Woods-Saxon mean-field model to accurately describe the single-particle levels in exotic nuclei far from the \( \beta \)-stability line. In addition, the FRDM+QRPA results have a clear tendency to overestimate the experimental half-lives and yield longer half-lives as compared with the calculations with np pairing. But the FRDM+QRPA result for \(^{82}\text{Zn}\) is close to the experimental data [1].

In order to discern the competition between GT and FF transitions, we also illustrate the contributions of FF transition to the decay width in figure 1(b). For the isotopes with \( A \leq 78 \), the contributions of FF transitions are small. However, there is a strong increase in the branching ratios across \(^{78}\text{Zn}\). As a result, for \(^{80,82}\text{Zn}\) the calculated half-lives presented here are shorter by a factor of roughly two as compared with the previous study for GT transitions [2]. Moreover, \( \beta \)-delayed neutron emission probabilities \( P_n \) are calculated for the neutron-rich \(^{78}\text{Zn}\) isotopes as well. For the \( \beta \) decays of \(^{74-78}\text{Zn}\), the \( \beta \)-decay energies are generally smaller or comparable with respect to the neutron separation energies of the daughter nuclei, which leads to a low degree of neutron emission [21]. Both the theoretical calculations and the experimental measurements suggest \( P_n = 0 \) for them. For the \( \beta \) decays of \(^{80,82}\text{Zn}\), the experimental \( P_n \) values are known as 1.0(0.5)\% and 57.57(12.34)\%, respectively [13, 21]. The present calculations with np pairing yield the values of 4.8\% and 37.0\%, which are comparable with the FRDM+QRPA results 6.2\% and 44.8\% [1].

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