SUPPRESSION OF STELLAR TIDAL DISRUPTION RATES BY ANISOTROPIC INITIAL CONDITIONS

KIRILL LEZHNIN and EUGENE VASILIEV

1 Moscow Institute of Physics and Technology, Institutsky per. 9, Dolgoprudny, Moscow Region, 141700, Russia; klezhnin@yandex.ru
2 Lebedev Physical Institute, Leninsky prospekt 53, Moscow, 119991, Russia; eugvas@lpi.ru
3 Rudolf Peierls Centre for Theoretical Physics, 1 Keble Road, Oxford OX1 3NP, UK

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Abstract

We compute the rate of capture of stars by supermassive black holes using a time-dependent Fokker–Planck equation with initial conditions that present a deficit of stars on low-angular-momentum orbits. One class of initial conditions has a gap in phase space created by a binary black hole, and the other class has a globally tangentially anisotropic velocity distribution. We find that for galactic nuclei that are younger than \( \sim 0.1 \) the relaxation time, the flux of stars into the black hole is suppressed with respect to the steady-state value. This effect may substantially reduce the number of observable tidal disruption flares in galaxies with black hole masses \( M_\bullet \gtrsim 10^7 M_\odot \).

Key words: galaxies: kinematics and dynamics – galaxies: nuclei

1. INTRODUCTION

A star passing a supermassive black hole (SMBH) at a sufficiently small distance is captured or tidally disrupted, producing a detectable flare in multiple wavebands (Rees 1988). Estimating the rate of such events is the focus of the loss-cone theory, first developed in the 1970s for application to hypothetical intermediate-mass black holes in globular clusters (e.g., Frank & Rees 1976; Lightman & Shapiro 1977). Later, this theory was applied to SMBHs in galactic centers (Magorrian & Tremaine 1999; Syer & Ulmer 1999; Wang & Merritt 2004), typically estimating the tidal disruption rate to be in the range of \( 10^{-4} - 10^{-5} \) events per year per galaxy. Observational constraints from optical (van Velzen & Farrar 2014), UV (Gezari et al. 2008), or X-ray (Donley et al. 2002; Khabibullin & Sazonov 2014) surveys roughly fall into the same range (see also a review by Komossa 2015). Recently, Stone & Metzger (2014) raised the concern that the observationally derived event rates are systematically lower than the theoretical estimates, by as much as one order of magnitude, although the uncertainties are barely smaller than that on either side.

In this paper, we investigate a mechanism that can significantly reduce the event rates in galaxies with sufficiently long relaxation times and alleviate the tension between theory and observations, namely, the influence of tangentially anisotropic initial conditions. The source of this anisotropy could be a gap in the low-angular-momentum region of the phase space, created by a binary SMBH (Merritt & Wang 2005), or simply a mild bias toward circular orbits. We do not attempt to model individual galaxies but instead focus on a comparison of the steady-state rates typically used in the literature with the reduced rates from anisotropic initial conditions with the same density profile in order to derive the suppression factor as a function of the galaxy parameters. We only consider spherically symmetric galaxies, and therefore obtain a lower boundary for the possible event rates.

The paper is organized as follows. In Section 2, we present the Fokker–Planck equation describing the diffusion of stars in angular momentum and derive its time-dependent analytical solution. Then, in Section 3, we present modifications of the initial conditions with tangential anisotropy. In Section 4, we obtain estimates of the capture rates for our choice of initial conditions and compute the suppression factor with respect to the steady-state capture rates. We discuss the implications of our results in Section 5.

2. ANALYTICAL SOLUTION OF THE DIFFUSION EQUATION

The two-body relaxation that changes the energies \( E \) and angular momenta \( J \) of stars can be described in terms of the orbit-averaged Fokker–Planck equation (e.g., Merritt 2013a, Chapter 5). It is commonly assumed that changes in angular momentum are much more important for computing capture rates (Frank & Rees 1976; Lightman & Shapiro 1977), and thus diffusion in energy is usually neglected; a numerical solution of the more general two-dimensional Fokker–Planck equation (Cohn & Kulrung 1978; Merritt 2015) confirms the validity of the one-dimensional approximation for the processes that occur on timescales much shorter than the relaxation time \( t_{\text{rel}} \). At fixed energy \( E \), the distribution function (DF) of stars \( f(j) \) as a function of the normalized angular momentum \( j = J/j_{\text{isc}}(E) \), where \( j_{\text{isc}} \) is the angular momentum of a circular orbit with the same energy, satisfies the diffusion equation in a cylindrical geometry:

\[
\frac{\partial f(E, j, t)}{\partial t} = D \frac{\partial}{\partial j} \left( \frac{\partial f}{\partial j} \right). \tag{1}
\]

Here, \( D(E) \propto t_{\text{rel}}^{-1} \) is the orbit-averaged diffusion coefficient (e.g., Merritt 2013b, their Equation (18)), which is assumed to be independent of \( j \), allowing an analytical solution of this equation (Milosavljević & Merritt 2003). The boundary condition at \( j = 1 \) is a Neumann-type condition: \( \partial f/\partial j = 0 \) (zero-flux condition). The presence of the black hole creates a capture boundary at \( j = j_c \), the angular momentum at which a star would be tidally disrupted at periapsis. As discussed in Lightman & Shapiro (1977), there are two limiting cases for the behavior of \( f(j) \) near \( j_c \), depending on the ratio \( q \equiv DT/j_{\text{isc}}^2 \) between the mean-square change in \( j \) due to relaxation over one orbital period \( T \) and the size of the loss cone. In the general case, one may use a Robin-type boundary condition (a linear combination of the function and its derivative), which naturally interpolates between the regimes
Figure 1. Monte Carlo simulations of a binary SMBH in a galaxy with a $\gamma = 1$ Dehnen density profile. The binary mass is $10^{-2}$ of the total stellar mass, the mass ratio $q = 1$, and the binary began on a nearly circular orbit at a separation of 0.2, roughly equal to the radius of influence. Top panel: phase space (squared angular momentum vs. energy) after the binary has cleared the low-angular-momentum region. The dashed green and dot-dashed blue lines show the definition of the gap region in this study and in Merritt & Wang (2005); the solid red line marks the angular momentum of a circular orbit. Bottom panel: approximation of the initial distribution function (Equation (6)) used in this work.

of an empty ($q \ll 1$) and full ($q \gg 1$) loss cone:

$$f(j_c) = \frac{\alpha j_c}{2} \frac{\partial f(j)}{\partial j} \bigg|_{j=j_c}, \quad \alpha(q) \approx \left(q^2 + q^4\right)^{1/4}. \quad (2)$$

In the steady state, the flux of stars toward the capture boundary $F = (D/dj)\partial f(j)/\partial j$ is nearly independent of $j$ at low $j$, and the solution of Equation (1) has a logarithmic profile: $f(j) \propto \log j + \text{const}$. Extrapolating this functional form to all $j$, one obtains the classical quasi-steady-state solution (e.g., Vasiliev & Merritt 2013, their Equation (51) with $R = f^2$). Cohn & Kulsrud (1978) defined the effective capture boundary $j_0 \equiv j_c \exp(-\alpha/2)$ at which the inward extrapolation of the logarithmic solution reaches zero; this provides the same boundary condition at the true capture boundary as Equation (2), except for their slightly different expression for $\alpha$.

The general time-dependent solution of Equation (1), subject to the specific boundary conditions, can be expressed in a series form (Milosavljević & Merritt 2003, Equations (24)–(26)):

$$f(j, t) = \sum_{m=1}^{\infty} C_m A(\beta_m, j) \exp\left(-D_j^2 \beta_m^2 t/4\right), \quad (3)$$

$$A(\beta, j) \equiv A_0(\beta j)Y_0(\beta j) - Y_0(\beta j)A_1(\beta),$$

$$C_m \equiv \left(\frac{\pi^2}{2}\right) \frac{\beta_m^2}{J_0^2(\beta_m j_c)} - \int_{j_c}^{j_1} j^2A(\beta_m, j) / (j^2 + j_0^2) \, dj,$$  

$$\quad (4)$$

where $J_i$ and $Y_i$ are Bessel functions of the first and the second kind, $A$ is the basis function, $\beta_m$ are the roots of a certain equation which satisfy the boundary conditions for each basis function, and $C_m$ are the expansion coefficients computed from the initial conditions $f(j, 0)$. The above study derived their expressions for the limiting case of an empty loss cone, i.e., a boundary condition $f(j_c) = 0$. Taking the same inward extrapolation of the logarithmic profile as in Cohn & Kulsrud (1978), one can use their expressions with a modified capture boundary $j_0$ for the general case of a non-empty loss cone (this method was adopted for the time-dependent solution in Vasiliev & Merritt 2013); however, the validity of this approach depends on the assumption that the solution is close to the steady-state profile. Instead, in this work, we generalize the analytical solution to the arbitrary boundary conditions given by Equation (2). Namely, the coefficients $\beta_m$ that enter the expressions for the series expansion are the roots of the following equation:

$$\left[\frac{J_0(\beta_m j_c)}{\beta_m j_c} + \alpha \beta_m j_c A_1(\beta_m j_c)/2\right] Y_1(\beta_m) - \left[\frac{Y_0(\beta_m j_c)}{\beta_m j_c} + \alpha \beta_m j_c Y_1(\beta_m j_c)/2\right] A_1(\beta_m) = 0. \quad (5)$$

This modification is necessary to obtain a rigorous solution for non-trivial initial conditions, such as those considered in the next section, although it provides only a minor correction for the more approximate treatment in the case of a nearly logarithmic form of the solution (i.e., late enough into the evolution).

The series solution with a finite number of terms $m_{\text{max}}$ breaks down at small times ($t \lessapprox 10D^{-1} m_{\text{max}}^{-2}$), and so in this case we compute the solution numerically with a finite-difference scheme.

3. Tangentially Anisotropic Initial Conditions

The stationary solution with a logarithmic $j$-dependence of DF is attained at times greater than a significant fraction of the relaxation time. On the other hand, this time may well exceed the age of the universe, especially in low-density galactic nuclei. Therefore, the choice of initial conditions becomes important for determining present-day capture rates. We consider two classes of initial DFs with a deficit of stars at low angular momenta compared to the isotropic population.

The first possible reason for such a deficit is the ejection of stars by a binary SMBH that may have previously existed in a galaxy. The slingshot mechanism is responsible for the flattening of the density profile and the formation of cores (Milosavljević & Merritt 2001; Merritt 2006) which have been detected observationally (e.g., Dulko & Graham 2012; Rusli et al. 2013). More importantly, it creates a gap in the phase space, ejecting stars with angular momenta less than some critical value which corresponds to the periapsis radius comparable to the radius of a hard binary, $a_b \equiv q/\sqrt{4(1 + q^2)} r_{\text{crit}}$, where $q \equiv m_1/m_2 \lessapprox 1$ is the mass ratio of the binary and $r_{\text{crit}}$ is the SMBH radius of influence. In this work, we adopt the definition of $r_{\text{crit}}$ as the radius containing the mass of stars equal to 2$(m_1 + m_2)$ before the slingshot process begins.

Merritt & Wang (2005) considered a time-dependent solution for the one-dimensional Fokker–Planck equation, using the expressions of Milosavljević & Merritt (2003), i.e., an empty-loss-cone boundary condition, and taking the initial distribution.
in angular momentum as a Heaviside step function:  
\[ f(E, J) \propto \Theta(J - J_{\text{gap}}(E)). \]  
They defined the gap width as  
\[ J_{\text{gap}}(E) \equiv K a_b \sqrt{2[E - \Phi(K a_b)]} \]  
with a dimensionless constant  \( K \approx 1. \) We used a modified version of the Monte Carlo code RAGA (Vasiliev 2015; Vasiliev et al. 2015) to determine the angular momentum distribution of stars in a galaxy with a binary SMBH; our results are better fit by an energy-independent gap width,  \( J_{\text{gap}} \equiv K' G (m_1 + m_2) a_h \)  with  \( K' \approx 3, \) and a more gradual drop toward smaller  \( J \) (Figure 1):

\[
\begin{equation}
    f(E, J, 0) = f(E) \cdot \min \left( 1, \left( \frac{J}{J_{\text{gap}}} \right)^6 \right). 
\end{equation}
\]

Another possible choice of initial conditions involves a DF that has a tangential anisotropy at all  \( J, \) not just a gap at small  \( J. \) The simplest possibility is to consider DF in factorized form:  
\[ f(E, J) = (1 - \beta)[J/J_{\text{rc}}(E)]^{-2} \tilde{f}(E), \]  
where  \( \tilde{f}(E) \) is the counterpart of the usual isotropic DF and is computed from a given density profile using the method of Cuddeford (1991). Such a DF corresponds to a constant velocity anisotropy coefficient  \( \beta \) (Binney & Tremaine 2008, their Equation (4.61)); the case of weak tangential anisotropy  \( \beta = -1/2 \) is consistent with some observationally based models of galactic centers (e.g., Thomas et al. 2014, their Figure 2), and results in a very simple expression for \( f: \)

\[
\tilde{f}(E) = \frac{J_{\text{circ}}(E)}{3 \pi^2} \frac{d^2}{d\Phi^2} \frac{\rho(\Phi)}{r(\Phi)}. 
\]

4. RESULTS

We have considered a set of Dehnen (1993)  \( \gamma \)-models with a black hole mass of  \( M = 10^9 \) of the total mass in stars. The energy-dependent part of DF  \( f(E) \) or  \( \tilde{f}(E) \) and the diffusion coefficient  \( D(E) \) are computed numerically from the given density profile using the Eddington inversion formula or its Cuddeford’s generalization. We explore several values of the power-law index  \( \gamma \) of the central density profile, and for each value of  \( \gamma \) we chose to consider a one-parameter family of models by scaling  \( M, \) and  \( r_{\text{inf}} \) simultaneously, according to the following relation (Merritt et al. 2009):

\[
    r_{\text{inf}} = n_0 \left( M/10^8 M_\odot \right)^{0.56}. 
\]

As our default normalization, we set  \( n_0 = 30 \) pc, but we also consider values of  \( n_0 = 20 \) and  \( 45 \) pc. The ratio of the influence radius to the scale radius of the Dehnen profile is \( \{0.091, 0.047, 0.016\} \) for  \( \gamma = \{0.5, 1, 1.5\}. \)

It is natural to express our results in dimensionless units: the flux normalized to the steady-state capture rate and the time measured in units of relaxation time (Binney & Tremaine 2008, their Equation (7.106)) at  \( r_{\text{inf}}. \) For  \( 10^6 \leq M/10^8 M_\odot \leq 10^8 \) and  \( 0.5 \leq \gamma \leq 1.5, \) these scale approximately as

\[
\begin{align*}
    \log \left[ \frac{\tilde{F}_{\text{d}}}{(M_\odot \text{yr}^{-1})} \right] &\approx -4.6 - 1.5 \log(n_0/30 \text{ pc}) \\
    &+ 0.2(1 - \gamma) \log(M/10^8 M_\odot),
\end{align*}
\]

\[
\begin{align*}
    \frac{\tilde{F}_{\text{d}}}{(M_\odot \text{yr}^{-1})} &\approx 13 + 0.4(1 - \gamma) + 1.5 \log(n_0/30 \text{ pc}) \\
&+ 1.28 \log(M/10^8 M_\odot).
\end{align*}
\]

The steady-state flux is comparable to  \( M/\mu_{\text{rel}}. \) Figure 2, in the top panel, shows the ratio of time-dependent to steady-state capture rate as a function of time for two representative models with a gap. We also plot the same quantity measured at  \( 10^{10} \) years for various choices of  \( \gamma, M, \) and  \( q; \) in this manner, the abcissa corresponds to the black hole mass according to our scaling (10).

The time required to establish the steady-state profile at a given energy is  \( \sim (J_{\text{gap}}/J_{\text{circ}})^2 t_{\text{rel}} \) (e.g., Merritt 2013b, their Equation (34)); as the maximum of the total flux arrives from energies corresponding to  \( r_{\text{inf}}, \) the time to refill the gap is roughly  \( t_{\text{refill}} \sim (a_b/r_{\text{inf}}) t_{\text{rel}}(r_{\text{inf}}), \) or

\[
    t_{\text{refill}} \approx 10^{13} \text{ year} \times \frac{q}{4(1 + q)^2} \left( M/10^8 M_\odot \right)^{1.28} \left( \frac{n_0}{30 \text{ pc}} \right)^{1.5}.
\]

For  \( t \lesssim t_{\text{refill}} \) the flux is reduced compared to the stationary value, which is commonly used in calculations of tidal disruption rates. The maximum value of the capture rate reached at  \( t \approx t_{\text{refill}} \) is somewhat lower than the steady-state value, due to the fact that the  \( J \)-averaged DF is also depleted at high binding energies (where  \( J_{\text{gap}} \gtrsim J_{\text{circ}} \)) with respect to the value used in the steady-state calculation. The flux reaches half of its maximum value at  \( t_{1/2} \approx 0.1 t_{\text{refill}}. \) Moreover, at  \( t \gtrsim t_{\text{refill}} \)
it starts to decline in the absence of energy diffusion. Merritt (2015) performed a numerical integration of a two-dimensional \((E, J)\) Fokker–Planck equation restricted to the region inside \(r_{\text{inf}}\), also using initial conditions with a gap at \(J < J_{\text{gap}}\), and found qualitatively similar behavior if the diffusion in energy was artificially switched off. On the other hand, taking this diffusion into account modifies the solution at \(t > 0.1 t_{\text{rel}}\) so that it tends to a steady-state profile. Therefore, we may trust our calculations roughly up to a time when the flux reaches its maximum.

We also explore the effect of changing the normalization in the \(r_{\text{inf}} - M\) relation (8). This, of course, modifies both the time-dependent flux and the relaxation time at \(r_{\text{inf}}\) (9) and (10), but the normalized values still stay on the same curve for each \(\gamma\) and \(q\). Finally, the second class of models with globally tangentially anisotropic initial conditions (Figure 2, bottom panel) produce a more mild decline in the capture rate at \(t \lesssim 0.1 t_{\text{rel}}\).

5. DISCUSSION AND CONCLUSIONS

We have considered the question of whether the capture rate of stars by SMBHs can be substantially lowered with respect to the steady-state value by a suitable modification of the initial conditions. We used an analytical time-dependent solution of the Fokker–Planck equation to compute the capture rate for the given initial conditions as a function of time. Two classes of initial conditions were analyzed: a gap in the low-angular-momentum region of phase space, created by a binary SMBH, and a separable DF with a mild tangential velocity anisotropy.

The results of our study can be summarized as follows. If the relaxation time in the galaxy center is sufficiently short \((\lesssim 10^{11} \text{ yr})\), then any difference between initial conditions is erased quite rapidly and the capture rate approaches the steady-state value. On the other hand, for \(t_{\text{rel}} \gtrsim 10^{12} \text{ yr}\), anisotropic initial conditions may substantially reduce the capture rate (in the intermediate range of \(t_{\text{rel}}\), their effect is moderate). In our series of models with a phase-space gap, for SMBH masses \(M \gtrsim 10^{7} M_{\odot}\), the suppression factor (the ratio of time-dependent to steady-state capture rates) drops quite rapidly, plunging below \(10^{-4}\) for \(M \gtrsim 10^{8} M_{\odot}\). Note that for even more massive black holes, visible flares constitute a small fraction of the captured stars (e.g., MacLeod et al. 2012, their Figure 15). In models with mild tangential anisotropy, the suppression factor is not as extreme but can still reduce the flux by a factor of a few for \(M \gtrsim 10^{8} M_{\odot}\).

We have extended the work of Merritt & Wang (2005) into the range of smaller SMBH masses \((\lesssim 10^{6} M_{\odot})\), since they are more numerous in the universe and are expected to dominate the overall tidal disruption rates. Moreover, smaller \(M\) residing in more compact galactic nuclei are not well described by the empty-loss-cone regime adopted in that paper. In this study, we have derived the solution for the general case; we checked that assuming an empty-loss-cone boundary condition does not substantially change the results for \(M \gtrsim 10^{7} M_{\odot}\), but overestimates the flux by a factor of a few for the least massive SMBHs. We used a somewhat different initial DF inside the gap than in the above paper, but confirmed that adopting their initial conditions changes the results only marginally. However, our estimates of the time required for the capture rate to reach half of its steady-state value, \(t_{\text{rel}} \approx 0.1 t_{\text{rel}}\) with the latter given by Equation (11), are about an order of magnitude longer than is shown in Figure 3 or Equation (14) of Merritt & Wang (2005) for the same galaxy parameters. We believe that this discrepancy might be due to a calibration error in that paper, as our expressions for \(t_{\text{rel}}\) agree with Equation (7) in Merritt & Szell (2006) and Equation (36) in Merritt (2013b).

We deliberately made a number of simplifying assumptions that drive our capture rates toward lower values. First, we assumed a spherical geometry, although it is known that non-spherical torques can result in a higher efficiency of loss-cone repopulation (Magorrian & Tremaine 1999; Merritt & Poon 2004; Holley-Bockelmann & Sigurdsson 2006). Naturally, the difference between spherical and non-spherical geometry starts to manifest itself above the same threshold value of \(M\), as the influence of initial conditions (Figure 4 in Vasiliev 2014), again underlining the distinction between relaxed and non-relaxed galactic nuclei. Note that here by relaxed we mean those systems which had enough time to establish a nearly steady-state logarithmic profile in angular momentum distribution; this does not mean they were able to relax in energy space as well and develop a Bahcall & Wolf (1976) cusp. Second, we neglected non-classical phenomena which may increase the relaxation rate (e.g., Alexander 2012), such as mass segregation (Freitag et al. 2006), massive perturbers (Perets et al. 2006), or resonant relaxation (e.g., Merritt 2013a, Chapter 5.6); the latter, however, is typically not very effective in boosting the capture rates (Hopman & Alexander 2006). A number of other refinements have been shown by Stone & Metzger (2014) to have little impact on the final values. Binary SMBHs that have not yet coalesced also suppress the capture rates (Chen et al. 2008), although they may demonstrate brief episodes of increased encounter rates at the early stages of evolution (Ivanov et al. 2005; Chen et al. 2011; Wegg & Bode 2011). Thus, our findings can be regarded as robust lower limits to the capture rates for single SMBHs.

The effect investigated in this paper is unlikely to substantially reduce the rate of tidal disruption events for black hole masses smaller than \(~ 10^{7} M_{\odot}\). Although the volumetric rate of tidal disruption events is dominated by galaxies with the smallest \(M\) (e.g., Figure 8 in Stone & Metzger 2014), the rate of observable events (Figure 10 in that paper) may be significantly affected by the suggested mechanism if either (1) the SMBH mass function is suppressed at the low-mass end, or (2) flare emission mechanisms are inefficient for low-mass SMBHs. These factors are still the main sources of uncertainty in the estimates of the observable event rate (Stone & Metzger 2014); however, it is notable that the distribution of observed flares peaks around \(M = 10^{7} M_{\odot}\) (Figure 11 in that paper). Thus, the effect considered here may potentially be important in comparing the theoretical predictions of tidal disruption rates with observations.

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