The long-range interactions between branes in diverse dimensions

Rong-Jun Wu $^1$ and Zhao-Long Wang $^2$

Interdisciplinary Center for Theoretical Study
University of Science and Technology of China, Hefei, Anhui 230026, China

Abstract

We calculate the long-range interactions between two simple branes placed parallel at a separation in diverse dimensions via an effective field theory approach. We also compute for the first time the explicit long-range interaction between two D-branes with each carrying a world-volume non-abelian magnetic flux in three special cases, respectively. In particular, we demonstrate that the half-string creation between a D$_0$-brane and a D$_8$-brane continues to hold even in the present context, therefore lending further support to the previous assertion of this. Our computations re-raise also the issue in one case on whether so constructed (D$_0$, D$_8$) bound state is actually a marginal one.
1 Introduction

Usually, we have three methods to calculate the static interaction between two branes placed parallel at a separation: the stringy computations [1, 2] if a string description is applicable, the brane-probe computations [3, 4, 5, 6] and the effective field theory computations [2, 7, 8]. Each method has its own applicability, advantage and disadvantage. For the string-level computations and for D-branes, the lowest-order stringy interaction can be computed either as an open string one-loop annulus diagram with one end of the open string located at one D-brane and the other end at the other D-brane or as a closed string tree-level cylinder diagram with one D-brane emitting a closed string, propagating for a certain amount of time and finally absorbed by the other D-brane. The validity of this computation requires a small string coupling. For the brane-probe computations, we consider a brane probe moving in a background produced by a brane source. Obviously, the probe must not change the background, i.e., the number of the probe branes should be much smaller than that of the source branes. The interaction can be obtained by finding the potential of the probe in the fields of source brane. While in the effective field theory computations, once the effective field theories both in the bulk and on the world-volume are given, we can find the propagator for each bulk (massless) mode and the corresponding coupling with the brane, and the interaction between branes can be calculated subsequently, for example, following [8]. Here, we need neither brane to be heavier than the other nor the explicit configuration of the source brane. The first method has its advantage if the branes such as D-branes have a stringy description and the string coupling is small. The second and third methods may also be good if only the low-energy effective descriptions both for the bulk and for the branes are available such as the case for the M2-brane and the M5-brane in M-theory. In particular, for the (transverse) M5-brane, the explicit interaction between two such branes can be calculated only this way at present. For the second approach, one needs in addition the explicit configuration of the source brane which may not be always available. So the least requirement is for the third approach and the interaction computed should be good for large brane separation in general, sometime even at stringy level such as the one between a D_0-brane and a D_S-brane. This is the focus of the present paper for brane interactions in diverse dimensions.

In this paper, we calculate the long-range Coulomb-type interaction between two p-branes placed parallel at a separation via the effective field theory computations mentioned above. The p-branes in diverse dimensions follow the brane-scan given in [9, 4]. For finding the interaction, only the bosonic part of the bulk or the brane world-volume effective action is needed. For cases where the world-volume modes involve a vector such as for a D-brane
or a tensor such as for a M5-brane, we will set them vanish since these modes will in general not be excited for calculating the long-range interaction. Then the bosonic part of the effective world-volume action coupled with the bulk modes can be simply described by the usual Nambu-Goto type action plus the corresponding Wess-Zumino term, involved only the scalars in the respective multiplet which are described by the spacetime embedding coordinates. This kind of computations serves to check the “no-force” condition beyond the probe approach at large brane separation for BPS \(p\)-branes, sometime even valid at stringy level such as the case for the \(D_0\)-\(D_8\) system. For certain cases such as for M2-brane, M5-brane and NS5-brane in type IIA where a better description such as a stringy one is not available in general, this computation is particularly useful. The long-range interaction between a \(p\)-brane and an anti \(p\)-brane can be obtained from the above by switching the sign of the contribution due to the \((p+1)\)-form potential.

When specified to D-branes, we can consider cases with various constant world-volume fluxes. The effective action for the branes is now the usual DBI action plus the corresponding Wess-Zumino term or the non-abelian extension of this. Given the bulk fields as described by the low energy type II supergravities, the relevant couplings are all gauge singlets even when the effective brane action is non-abelian. When there is one abelian flux present, this has been considered in [8]. In this paper, we will consider three different cases with each involving a special world-volume non-abelian magnetic flux in the spirit of [10], which corresponds to either \((D_{p-4}, D_p)\) or \((D_{p-6}, D_p)\) or \((D_0, D_8)\) configuration, and calculate the corresponding long-range interaction. In particular, for the last system, this computation can be even valid at the stringy level for the contribution due to the \(D_0\)-brane and \(D_8\)-brane in the interaction between two such bound states since it is well-known that only the massless modes contribute to this part of interaction. Further, for the same system, the previously discovered half-string creation [11, 12, 13, 14] and the associated divergent zero-point energy [15] between a \(D_0\)-brane and a \(D_8\)-brane are found to continue to hold even in the present case, therefore lending further support to both. Moreover, when the half-string creation is considered, the net interaction between two such bound states is beautifully canceled, just like the case of \((D_{p-4}, D_p)\), showing the long-believed marginal bound-state nature of \((D_0, D_8)\). This re-raises the issue on whether so constructed \((D_0, D_8)\) bound state given in [10] and used in the present calculations is actually a marginal one.

This paper is organized as follows. In Section 2, we calculate the static long-range interaction between two simple \(p\)-branes placed parallel at a separation in diverse dimensions. We start with the relevant bosonic part of supergravity action which is the bulk low energy effective action and from this we can read the propagator for each relevant
bulk field when expressed in the canonical form. From the relevant brane world-volume action coupled with bulk fields, we can read the respective couplings. From these, the respective long-range interaction between two simple branes can be simply calculated in diverse dimensions. For BPS branes, we can check the “no-force” condition beyond the probe approach. Also, the interaction between a $p$-brane and an anti $p$-brane can also be similarly calculated. In Section 3, we specify to D-branes with special world-volume non-abelian magnetic fluxes, which corresponds to either $(D_{p-4}, D_p)$ or $(D_{p-6}, D_p)$ or $(D_0, D_8)$ configuration, and calculate the corresponding long-range interaction between two such branes in type II string theories in a similar spirit. We discuss various issues regarding the $(D_0, D_8)$ bound state such as the half-string creation and its nature as a marginal bound state. We discuss the results and conclude this paper in Section 4.

2 The interaction between two $p$-branes in diverse dimensions

In this section, we will calculate the static long-range interaction between two simple $p$-branes placed parallel at a separation in diverse dimensions. The two $p$-branes can be both (anti-) BPS ones or one is BPS and the other is anti-BPS. In the former case, a net-zero interaction is expected, while in the latter a non-vanishing result is expected. For this, we first express the relevant bosonic part of the bulk effective action, i.e., the bulk supergravity action, in the canonical form in spacetime dimension $D$. With this, we can find the couplings of the $p$-brane with the relevant bulk (massless) fields through the corresponding world-volume effective action which is taken as the Nambu-Goto one plus the Wess-Zumino term. We then calculate the long-range interaction between two such $p$-branes as described in the Introduction in diverse dimensions.

The relevant bosonic part of supergravity in spacetime dimension $D$ with $p$-brane $\sigma$-model metric $G_{\mu\nu}$ \footnote{The Greek indices $\mu, \nu, \ldots$ label the spacetime directions 0, 1, \ldots, $D$.} is [16]

$$S_D = \frac{1}{2\kappa_D^2} \int d^D x \sqrt{-G} e^{-\frac{(D-2)\alpha(p)}{2(p+1)}\Phi} \left[ R - \frac{1}{2} \left( 1 - \frac{\alpha^2(p)(D-1)(D-2)}{2(p+1)^2} \right) (\nabla \Phi)^2 - \frac{1}{2} |F_{p+2}|^2 \right], \quad (1)$$

where the $(p+2)$-form field strength $F_{p+2}$ is given by $F_{p+2} = dC_{p+1}$ with $C_{p+1}$ the $(p+1)$-form field strength.
form potential, and $\alpha(p)$ satisfies
\[
\alpha^2(p) = 4 - \frac{2(p+1)(D-p-3)}{D-2}.
\] (2)

To consider the field theory limit, it is proper to express the above action in the Einstein or canonical frame. This can be achieved through the so-called Einstein metric $g_{\mu\nu}$ which is related to the $p$-brane $\sigma$-model metric $G_{\mu\nu}$ as
\[
g_{\mu\nu} = e^{-\frac{\alpha(p)}{p+1}\phi}G_{\mu\nu},
\] (3)

where
\[
\phi \equiv \Phi - \Phi_0
\] (4)
with $\Phi_0$ the asymptotic value (or VEV) of the dilaton $^4$. In this frame, we have
\[
S_D = \frac{1}{2\kappa^2} \int d^Dx \sqrt{-g} \left[ R - \frac{1}{2} (\nabla \phi)^2 - \frac{1}{2} e^{-\alpha(p)\phi}|F_{p+2}|^2 \right].
\] (5)

In the above, we introduce the physical gravitational coupling $2\kappa^2 = 2g_0^2\kappa_D^2$ with the dimensionless parameter $g_0 = e^{\frac{(D-2)p}{2(p+1)}\Phi_0}$, which becomes the string coupling when the 10 $D$ fundamental string is considered.

Considering small fluctuations of fields with respect to the flat Minkowski background $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ and choosing the usual harmonic gauge for $h_{\mu\nu}$, we have the action
\[
S_D = \frac{1}{2\kappa^2} \int d^Dx \left[ -\frac{1}{4} \nabla h_{\mu\nu} \nabla h_{\mu\nu} + \frac{1}{8} (\nabla h)^2 - \frac{1}{2} (\nabla \phi)^2 - \frac{1}{2} |F_{p+2}|^2 \right],
\] (6)

where we keep only the lowest order terms. The above action obviously becomes canonical with the following scalings:
\[
h_{\mu\nu} \to 2\kappa h_{\mu\nu}, \quad \phi \to \sqrt{2\kappa}\phi, \quad C_{p+1} \to \sqrt{2\kappa}C_{p+1}.
\] (7)

These will help us to determine the corresponding couplings of bulk fields with the $p$-brane in the canonical form which we will turn next.

Let us consider the bosonic world-volume action of a $p$-brane
\[
S = -T_p \int d^{p+1}\sigma \sqrt{-G} + T_p \int C_{p+1},
\] (8)

where the metric $G$ and the $(p+1)$-form potential $C_{p+1}$ are the pullbacks of the corresponding bulk fields to the world-volume, and $T_p$ is the $p$-brane tension. In the above, the

\footnote{4We choose the $p$-brane $\sigma$-model metric $G_{\mu\nu}$ to be asymptotically flat.}
first term is the Nambu-Goto action and the second one is the Wess-Zumino term. Using Eq. (3), we can express the above action in Einstein frame as

\[ S = -T_p \int d^{p+1}\sigma \sqrt{-\gamma} + T_p \int C_{p+1}, \]  

(9)

We expand the above action to the leading order for the same background fluctuations and end up with

\[ S = -T_p \int d^{p+1}\sigma \left( 1 + \frac{1}{2} \eta^{\alpha\beta} h_{\alpha\beta} + \frac{\alpha(p)}{2} \phi \right) + T_p \int C_{p+1}. \]  

(10)

Using the scalings in Eq. (7) to replace the background fluctuations in the above action, we can obtain the respective coupling in the canonical form

\[ J_h^{(i)} = -n_i c_p V_{p+1} \eta^{\alpha\beta} h_{\alpha\beta} \]  

(11)

for the graviton,

\[ J_\phi^{(i)} = -\frac{\alpha(p)}{\sqrt{2}} n_i c_p V_{p+1} \phi \]  

(12)

for the dilaton, and

\[ J_{C_{p+1}}^{(i)} = \sqrt{2} n_i c_p \frac{V_{p+1}}{(p+1)!} C_{\alpha_0 \alpha_1 \cdots \alpha_p} \epsilon_{\alpha_0 \alpha_1 \cdots \alpha_p} \]  

(13)

for the \((p+1)\)-form potential \(C_{p+1}\). In the above, \(c_p \equiv T_p \kappa\), \(V_{p+1}\) is the world-volume of the \(p\)-brane, \(\epsilon_{\alpha_0 \alpha_1 \cdots \alpha_p}\) is the totally antisymmetric tensor on the \(p\)-brane world-volume, and the index \(i\) denotes the respective stack of \(p\)-branes with \(i = 1, 2\). Note that we have introduced an extra overall integral factor \(n_i\) in each couplings to count the multiplicity of \(n_i\) coincident \(p\)-branes in each stack.

Now we calculate the lowest-order contribution in momentum space to the interaction between two \(p\)-branes placed parallel to each other at a given separation due to the exchanges of massless modes, therefore representing the interaction at large separation.

The gravitational potential energy density due to the exchange of graviton is

\[ U_h = \frac{1}{V_{p+1}} J_{h}^{(1)} J_{h}^{(2)} = n_1 n_2 c_p^2 V_{p+1} \eta^{\alpha\beta} \eta^{\gamma\delta} h_{\alpha\beta} h_{\gamma\delta}, \]  

(14)

\[^5\text{The Greek indices } \alpha, \beta, \ldots \text{label the world-volume directions } 0, 1, \ldots, p \text{ along which the } p\text{-brane extends.}\]

\[^6\text{By conventions, } \epsilon^{01\cdots p} = -\epsilon_{01\cdots p} = 1.\]

\[^7\text{The corresponding potential in coordinate space can be obtained simply by Fourier transformation following, for example, [8].}\]
where the propagator is
\[ h_{\alpha\beta}h_{\gamma\delta} = \left[ \frac{1}{2}(\eta_{\alpha\gamma}\eta_{\beta\delta} + \eta_{\alpha\delta}\eta_{\beta\gamma}) - \frac{1}{D-2}\eta_{\alpha\beta}\eta_{\gamma\delta} \right] \frac{1}{k^2_\perp} \]

from Eq. (6) for the canonically normalized graviton propagating in the transverse directions, so we have
\[ U_h = n_1 n_2 c_p^2 \frac{V_{p+1}}{k^2_\perp} \left[ (p + 1) - \frac{(p + 1)^2}{D-2} \right]. \] 

Similarly, the potential energy density due to the exchange of dilaton \( \phi \) and the one due to the \((p+1)\)-form potential \( C_{01\cdots p} \) can be calculated, respectively, as
\[ U_\phi = \frac{1}{V_{p+1}} J^{(1)}_\phi J^{(2)}_\phi = \frac{\alpha^2(p)}{2} n_1 n_2 c_p^2 \frac{V_{p+1}}{k^2_\perp} \phi \phi = \frac{\alpha^2(p)}{2} n_1 n_2 c_p^2 \frac{V_{p+1}}{k^2_\perp} \] 
\[ U_{C_{p+1}} = \frac{1}{V_{p+1}} J^{(1)}_{C_{p+1}} J^{(2)}_{C_{p+1}} = 2 n_1 n_2 c_p^2 V_{p+1} C_{01\cdots p} C_{01\cdots p} = -2 n_1 n_2 c_p^2 \frac{V_{p+1}}{k^2_\perp}. \]

In the above, we have used the respective propagator for dilaton and for the \((p+1)\)-form potential as
\[ \phi \phi = \frac{1}{k^2_\perp} \]
and
\[ C_{01\cdots p} C_{01\cdots p} = -\frac{1}{k^2_\perp}. \]

So the total contribution to the energy density is
\[ U = U_h + U_\phi + U_{C_{p+1}} = n_1 n_2 c_p^2 \frac{V_{p+1}}{k^2_\perp} \left[ (p + 1) - \frac{(p + 1)^2}{D-2} + \frac{\alpha^2(p)}{2} - 2 \right] = 0, \]

where we have used Eq. (2) in the last step.

From above, we know that the contributions from graviton, dilaton and \((p+1)\)-form potential cancel among themselves exactly, so the net interaction between \(p\)-branes vanishes. This is expected. It is well-known that two parallel static BPS branes separated by a distance feel no force between them, i.e., satisfying the “no-force” condition, and this configuration preserves 1/2 of spacetime supersymmetries.
If one stack of branes in the above is replaced by the corresponding anti-branes, the contribution from the \((p+1)\)-form potential will switch sign and the resulting net interaction is no longer vanishing. It is now

\[ U = U_h + U_\phi - U_{C_{p+1}} = 4n_1n_2c_p \frac{V_{p+1}}{k_l^2}, \tag{22} \]

which shows that the interaction between \(p\)-branes and anti \(p\)-branes is attractive \(^8\). This is due to that all the components are attractive and the underlying system breaks all the supersymmetries. Note that our computations go beyond the probe approach for which we don’t need one set of branes to be much lighter than the other set. In addition, not every case considered has a stringy description, for examples, the IIA NS5-brane case and M-brane case to which we turn next.

For M-brane, i.e., M2-brane or M5-brane, the relevant bulk action is the bosonic part of \(D = 11\) supergravity which has no dilaton. It is

\[ S_{11} = \frac{1}{2\kappa^2} \int d^{11} x \sqrt{-g} \left( R - \frac{1}{2} |F_4|^2 \right) - \frac{1}{12\kappa^2} \int F_4 \wedge F_4 \wedge C_3, \tag{23} \]

where \(F_4\) is the 4-form field strength of the 3-form potential \(C_3\). The second term, the Chern-Simons-like term, has no play to leading order in the small fluctuations since it is a higher order term. In essence, the long-range force calculated using the propagators for the relevant bulk fields (the graviton and the 3-form potential) read from the above bulk action and the corresponding couplings read from the effective world-volume action of M-brane coupled with these bulk fluctuations is just a special case of the above general calculations from Eqs. (1) - (22) when \(D = 11\) and \(p = 2\) or \(5\) are taken. This is due to that \(\alpha(p) = 0\) in \(D = 11\) for \(p = 2\) or \(5\) and from Eq. (12) we have \(J_\phi^{(i)} = 0\). This implies that the dilaton decouples, therefore giving no contribution to the interaction. In other words, the above general calculations apply also to M2-brane or M5-brane. The above results for the case of the IIA NS5-brane or the (transverse) M5-brane are the only known ones beyond the probe approach.

3 The interactions between two D-branes with non-abelian fluxes

We now specify our discussion to D-branes. The long-range interaction between two parallel D-branes with each carrying a single abelian world-volume flux, which describes

\(^8\)We choose conventions here that \(U > 0\) means attractive and \(U < 0\) means repulsive which differ from standard ones by a sign.
the non-threshold BPS \((F, D_p)\) bound state [17, 18, 19, 20, 21, 22] or non-threshold BPS \((D_{p-2}, D_p)\) bound state [23, 24, 25], has been discussed in [8]. Some discussions regarding multiple abelian fluxes have been given in [13]. In this section, we will consider the cases when the D-branes carry special world-volume non-abelian magnetic fluxes. We will calculate the couplings of the D-branes with the bulk massless modes of the underlying type II theories through the corresponding world-volume effective action and bulk effective action of a given string theory (IIA or IIB), and use these couplings to find the long-range interaction between two such D-brane configurations. In particular, for the system of \((D_0, D_8)\), we will address issues such as the half-string creation between a \(D_0\)-brane and a \(D_8\)-brane and the associated divergent zero-point energy in the present context.

Let us first express the bulk fields in the effective action of a given string theory in canonical forms \footnote{This part follows the e-print edition on arXiv of [8].} and we only need to consider the corresponding bosonic action too. Since this works the same way in either IIA or IIB theory, we take IIA for illustration. The bosonic part of the IIA low-energy effective action in string frame is

\[
S_{\text{IIA}} = S_{\text{NS}} + S_{\text{R}} + S_{\text{CS}},
\]

\[
S_{\text{NS}} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} e^{-2\phi} \left[ R + 4(\nabla\Phi)^2 - \frac{1}{2} |H_3|^2 \right],
\]

\[
S_{\text{R}} = -\frac{1}{4\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left[ |F_2|^2 + |\tilde{F}_4|^2 \right],
\]

\[
S_{\text{CS}} = -\frac{1}{4\kappa_{10}^2} \int B_2 \wedge F_4 \wedge F_4,
\]

where NS-NS field \(H_3 = dB_2\) while the R-R fields \(F_2 = dC_1, \tilde{F}_4 = dC_3 - C_1 \wedge H_3\). The constant \(2\kappa_{10}^2\) appearing in the action is \(2\kappa_{10}^2 = (2\pi)^7 \alpha'^4\).

To express the above action in the Einstein or canonical frame, we introduce the Einstein metric \(g_{\mu\nu}\) as

\[
g_{\mu\nu} = e^{-\phi/2} G_{\mu\nu},
\]

where \(\phi\) is defined as in Eq. \(4\). In this frame, we have

\[
S_{\text{NS}} = \frac{1}{2g_s^2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left[ R - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{2} e^{-\phi} |H_3|^2 \right],
\]

\[
S_{\text{R}} = -\frac{1}{4\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left[ e^{3\phi/2} |F_2|^2 + e^{\phi/2} |\tilde{F}_4|^2 \right],
\]

while the \(S_{\text{CS}}\) remains the same. In the above, we have introduced the string coupling \(g_s = e^{\Phi_0}\) and with this the physical gravitational coupling is \(2\kappa^2 = 2g_s^2\kappa_{10}^2\).
Similarly, considering small fluctuations of fields with respect to the flat Minkowski background, we have the action

\[ S_{\text{IIB}} = \frac{1}{2\kappa^2} \int d^{10}x \left[ -\frac{1}{4} \nabla h^{\mu\nu} \nabla h_{\mu\nu} + \frac{1}{8} (\nabla h)^2 - \frac{1}{2} (\nabla \phi)^2 - \frac{1}{2} |H_3|^2 \right] \]

\[ -\frac{1}{4\kappa_{10}^2} \int d^{10}x \left[ |F_2|^2 + |F_4|^2 \right], \]  

(27)

where \( F_4 = dC_3 \). The above action obviously becomes canonical with the following scalings:

\[ h_{\mu\nu} \rightarrow 2\kappa h_{\mu\nu}, \quad \phi \rightarrow \sqrt{2}\kappa \phi, \quad B_{\mu\nu} \rightarrow \sqrt{2}\kappa B_{\mu\nu} \]  

(28)

for NS-NS fields and

\[ C_n \rightarrow \sqrt{2}\kappa_{10} C_n \]  

(29)

for rank-\( n \) R-R potential. Note that the scaling of a NS-NS field differs from that of a R-R potential by a string coupling \( g_s \) except for the graviton which has an additional factor of \( \sqrt{2} \).

To obtain the couplings of bulk fields with the D-branes which carry world-volume non-abelian magnetic fluxes, we turn to consider the bosonic world-volume action of these D-branes. Before proceeding, let us mention a few simple facts. It is well-known that the system of \( N \) coincident \( D_p \)-branes is described in the low-energy regime by a \( U(N) \) super Yang-Mills theory in \((p+1)\) dimensions, which can be obtained via the dimensional reduction of the \( D = 10 \) super Yang-Mills [26]. In the Yang-Mills theory, \( D_{p-2k} \)-branes \((p \geq 2k)\) within the \( D_p \)-branes can be described by a configuration of gauge field \( F \) on the world-volume of \( D_p \)-branes with their charge related to the topological charge proportional to the integral of \( F^{\wedge k} = F \wedge \cdots \wedge F \) where the number of wedge products is \( k-1 \) with \( k \) an integer [27, 28]. In what follows, we will consider three special world-volume non-abelian magnetic fluxes following [10] with \( k = 2, k = 3 \) and \( k = 4 \). They correspond to \( D_{p-4} \)-branes within \( D_p \)-branes, \( D_{p-6} \)-branes within \( D_p \)-branes and \( D_0 \)-branes within \( D_8 \)-branes, respectively.

The bosonic world-volume action of \( D_p \)-branes with a constant non-abelian world-volume flux \( F \) in string frame is [29]

\[ S = -T_p \int d^{p+1}\sigma \left\{ e^{-\frac{\Phi}{\sqrt{-\det(G + B + \hat{F})}}} \right\} \]

\[ + T_p \int \left\{ e^{B + \hat{F}} \wedge \sum_k C_{p+1-2k} \right\}_{p+1} \],

(30)
where the metric $G$, the NS-NS rank-2 potential $B$, and the R-R potential $C_{p+1-2k}$ are the pullbacks of the corresponding bulk fields to the world-volume. Each of these fields is a singlet under the $U(N)$ gauge group, therefore they each can be represented by their bulk field multiplying an $N \times N$ unit matrix $I_N$ in the present context. In Eq. (30), we define $\hat{F} = 2\pi\alpha' F$ with $F$ an $N \times N$ matrix under the gauge group $U(N)$, and denote ‘Tr’ the trace in this $N \times N$ space. Note that ‘det’ denotes the determinant with respect to the world-volume indices only. The subscript in the square bracket in the above Wess-Zumino term means that in expanding the exponential form one picks up only terms of total degree of $(p + 1)$. We now express the above action in Einstein frame using Eqs. (25) and (4) as

$$S = -\frac{T_p}{g_s} \int d^{p+1}\sigma \text{Tr} \left\{ e^{(p-3)\phi/4} \sqrt{-\text{det} \left[ g + (B + \hat{F}) e^{-\phi/2} \right]} \right\}$$

$$+ T_p \int \text{Tr} \left\{ e^{B+\hat{F}} \wedge \sum_k C_{p+1-2k} \right\}_{p+1}.$$  

(31)

By the same token, we now expand the above action to the leading order with fixed $\hat{F}$ for small background fluctuations and have

$$S = -\frac{T_p}{g_s} \int d^{p+1}\sigma \text{Tr} \left\{ \sqrt{-\text{det} (\eta + \hat{F})} \left[ I_N + \frac{1}{2} \left( \eta + \hat{F} \right)^{-1} \right]^{\alpha\beta} (h_{\beta\alpha} + B_{\beta\alpha}) \right. $$

$$+ \frac{1}{4} \left( (p - 3) I_N - \text{tr} \left( \hat{F} \left( \eta + \hat{F} \right)^{-1} \right) \right) \phi \right\}$$

$$+ T_p \int \text{Tr} \left\{ C_{p+1} + \hat{F} \wedge C_{p-1} + \frac{1}{2!} \hat{F} \wedge \hat{F} \wedge C_{p-3} + \cdots \right\},$$

(32)

where $\cdots$ means terms with the lower rank of R-R potentials wedged with more $\hat{F}$’s and the trace ‘tr’ is with respect to the world-volume coordinate indices. From the above action we can read the respective couplings in the canonical form\footnote{We would like to point out that from now on the bulk fluctuations such as $h_{\alpha\beta}$, $\phi$, $B_{\beta\alpha}$, $C_{p+1}$ and $C_{p+1-2k}$ are just the usual ones without multiplying each with the unit matrix $I_N$.}

$$J_h = -c_p V_{p+1} \text{Tr} \left\{ \sqrt{-\text{det} (\eta + \hat{F})} \left[ (\eta + \hat{F})^{-1} \right]^{\alpha\beta} h_{\alpha\beta} \right\}$$

(33)

for the graviton,

$$J_\phi = \frac{c_p}{2\sqrt{2}} V_{p+1} \text{Tr} \left\{ \sqrt{-\text{det} (\eta + \hat{F})} \left[ (3-p) I_N + \text{tr} \left( \hat{F} \left( \eta + \hat{F} \right)^{-1} \right) \right] \phi \right\}$$

(34)
for the dilaton,

\[ J_B = -\frac{c_p}{\sqrt{2}} V_{p+1} \text{Tr} \left\{ \sqrt{-\det(\eta + \hat{F})} \left[ (\eta + \hat{F})^{-1} \right] \right\}^{\alpha\beta} B_{\beta\alpha} \]  

(35)

for the Kalb-Ramond field,

\[ J_{C_{p+1}} = \frac{\sqrt{2} N c_p}{(p + 1)!} V_{p+1} C_{\alpha_0\alpha_1\ldots\alpha_p} \epsilon^{\alpha_0\alpha_1\ldots\alpha_p} \]  

(36)

for the R-R potential \( C_{p+1} \), and

\[ J_{C_{p+1-2k}} = \frac{\sqrt{2} c_p}{2^k k! (p + 1 - 2k)!} V_{p+1} \text{Tr} \left\{ \hat{F}_{\alpha_0\alpha_1} \ldots \hat{F}_{\alpha_{2k-2}\alpha_{2k-1}} \right\} C_{\alpha_2\alpha_2\alpha_{2k+1}\ldots\alpha_p} \epsilon^{\alpha_0\alpha_1\ldots\alpha_p} \]  

(37)

for the R-R potential \( C_{p+1-2k} \). In the above, we have used \( c_p = T_p \kappa / g_s = T_{p\kappa 10} \). We will use these couplings to calculate the long-range interactions between two D-branes with the non-vanishing integral of \( \hat{F} \wedge \hat{F} \) or \( \hat{F} \wedge \hat{F} \wedge \hat{F} \) or \( \hat{F} \wedge \hat{F} \wedge \hat{F} \wedge \hat{F} \), i.e., the \( k = 2 \) or \( k = 3 \) or \( k = 4 \) case mentioned above.

3.1 The \( k = 2 \) case

When the integral of \( \hat{F} \wedge \hat{F} \) is the only non-vanishing one, we have \( D_{p-4} \) (or \( D_{p-4} \))-branes within \( D_p \)-branes uniformly delocalized along the flux directions. We can choose the constant non-abelian magnetic flux \( \hat{F} \) on the world-volume of \( D_p \)-branes the following way

\[ \hat{F} = \begin{pmatrix}
0_{2n} \\
\vdots \\
0_{2n} \\
0_{2n} & -f \cdot u \\
-f \cdot u & 0_{2n} \\
\end{pmatrix}_{(p+1) \times (p+1)} \]  

(38)

where \( 0_{2n} \) in the above matrix stands for a \( 2n \times 2n \) zero matrix, and

\[ u = \text{Diag}\{I_n, -I_n\} \]  

(39)

with \( I_n \) the \( n \times n \) unit matrix. Note that \( u \) is one of the Cartan subalgebra generators of the \( U(N) \) algebra and we consider here \( N = 2n \) (\( n \) is a positive integer). With this flux, it
is obvious that $\text{Tr}(\hat{F} \wedge \cdots \wedge \hat{F}) \neq 0$ only when the number of $\hat{F}$ is 2 in the wedge product. Therefore the only non-vanishing coupling associated with the lower rank R-R potential, according to Eq. (37), is for $k = 2$ and the corresponding R-R potential is $C_{p-3}$. This further implies the presence of $D_{p-4}$-branes within $D_p$-branes whose charge is determined by the integral of $\text{Tr}(\hat{F} \wedge \hat{F})$. This brane configuration whose energy is the sum of the energy of $D_{p-4}$-branes and $D_p$-branes is a marginally bound state and preserves $1/4$ of spacetime supersymmetries [10]. We will denote this configuration as $(D_{p-4}, D_p)$ in the following.

With this special flux, we have

$$-\det \left( \eta + \hat{F} \right) = (1 + f^2)^2 I_{2n},$$

and

$$V = \left( \eta + \hat{F} \right)^{-1} = \begin{pmatrix}
-I_{2n} & 0 & \cdots & 0 \\
0 & I_{2n} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & I_{2n}
\end{pmatrix}. \quad (41)$$

So we have the couplings $J_h$, $J_\phi$, $J_B$, $J_{C_{p+1}}$ and $J_{C_{p-3}}$ for the corresponding fields as follows,

$$J_h^{(i)} = -2n_i c_p V_{p+1}(1 + f_i^2) \tilde{V}_i^{\alpha\beta} h_{\alpha\beta};$$

$$J_\phi^{(i)} = -2n_i \frac{c_p}{2\sqrt{2}} V_{p+1}(1 + f_i^2) \left[ (p - 3) - 4 \frac{f_i^2}{1 + f_i^2} \right] \phi;$$

$$J_B^{(i)} = -2n_i \frac{c_p}{\sqrt{2}} V_{p+1}(1 + f_i^2) \tilde{V}_i^{\alpha\beta} B_{\beta\alpha} \quad (42)$$

for the NS-NS fields and

$$J_{C_{p+1}}^{(i)} = 2n_i \sqrt{2c_p} V_{p+1} C_{01 \cdots p};$$

$$J_{C_{p-3}}^{(i)} = 2n_i \sqrt{2c_p} V_{p+1} f_i^2 C_{01 \cdots p-4} \quad (43)$$

for the R-R fields. In the above, $\tilde{V}_i$ is a $(p + 1) \times (p + 1)$ diagonal matrix on the world-volume of $D_p$-branes as

$$\tilde{V}_i = \text{Diag}\{-1, 1, \cdots, 1, \frac{1}{1 + f_i^2}, \frac{1}{1 + f_i^2}, \frac{1}{1 + f_i^2}, \frac{1}{1 + f_i^2}\}. \quad (44)$$
and this immediately implies that $J_{B}^{(i)} = 0$, therefore giving zero contribution to the interaction from this coupling. In the above, the index $i = 1, 2$, denoting the respective $(D_{p-4}, D_{p})$ in the interacting system.

We now use the above couplings to calculate the long-range interaction energy density in momentum space between two parallel $(D_{p-4}, D_{p})$ separated by a transverse distance. The gravitational potential energy density due to the exchange of graviton is

$$U_{h} = \frac{1}{V_{p+1}} f^{(1)}_{h} f^{(2)}_{h}$$

$$= c_{p}^{2} V_{p+1} 4 n_{1} n_{2} \left(1 + f_{1}^{2}\right) \left(1 + f_{2}^{2}\right) \tilde{V}_{1}^{\alpha \beta} \tilde{V}_{2}^{\gamma \delta} h_{\alpha \beta} h_{\gamma \delta}$$

$$= c_{p}^{2} V_{p+1} 4 n_{1} n_{2} \left[\frac{-p^{2} + 6 p + 7}{8} + \frac{-p^{2} + 10 p - 21}{8} f_{1}^{2}ight.$$  

$$+ \frac{-p^{2} + 10 p - 21}{8} f_{2}^{2} + \frac{-p^{2} + 14 p - 33}{8} f_{1}^{2} f_{2}^{2}\bigg), \quad (45)$$

where in the last equality we have used the graviton propagator Eq. (15). With the dilaton propagator Eq. (19), the contribution to the interaction due to the exchange of dilaton can be calculated as

$$U_{\phi} = \frac{1}{V_{p+1}} f^{(1)}_{\phi} f^{(2)}_{\phi}$$

$$= \frac{1}{8} c_{p}^{2} V_{p+1} 4 n_{1} n_{2} \left(1 + f_{1}^{2}\right) \left(1 + f_{2}^{2}\right)$$

$$\left[\frac{(p - 3) - 4 f_{1}^{2}}{1 + f_{1}^{2}}\right] \left[\frac{(p - 3) - 4 f_{2}^{2}}{1 + f_{2}^{2}}\right] \phi \phi$$

$$= c_{p}^{2} V_{p+1} 4 n_{1} n_{2} \left[\frac{p^{2} - 6 p + 9}{8} + \frac{p^{2} - 10 p + 21}{8} f_{1}^{2}\right.$$  

$$+ \frac{p^{2} - 10 p + 21}{8} f_{2}^{2} + \frac{p^{2} - 14 p + 49}{8} f_{1}^{2} f_{2}^{2}\bigg). \quad (46)$$

The total energy density from the NS-NS sector is

$$U_{NS-NS} = U_{h} + U_{\phi} = 8 n_{1} n_{2} \left(1 + f_{1}^{2} f_{2}^{2}\right) c_{p}^{2} V_{p+1} \frac{k_{\perp}}{k_{\perp}^{2}}, \quad (47)$$

We now turn to the calculations of the contributions from R-R fields. The contribution from the exchange of R-R potential $C_{01...p}$ is

$$U_{C_{p+1}} = \frac{1}{V_{p+1}} f^{(1)}_{C_{p+1}} f^{(2)}_{C_{p+1}}$$

$$= 2 c_{p}^{2} V_{p+1} 4 n_{1} n_{2} C_{01...p} C_{01...p}$$

$$= -8 n_{1} n_{2} c_{p}^{2} V_{p+1} \frac{k_{\perp}}{k_{\perp}^{2}}, \quad (48)$$

$$14$$
where the rank-\((p + 1)\) R-R potential propagator is Eq. (20). Similarly we have

\[
U_{C_{p-3}} = \frac{1}{V_{p+1}} J_{C_{p-3}}^{(1)} J_{C_{p-3}}^{(2)}
\]
\[
= 2c^2_p V_{p+1} 4n_1n_2 f_1^2 f_2^2 C_{p-3} C_{01...p-4}
\]
\[
= -8n_1n_2 f_1^2 f_2^2 c^2_p V_{p+1} \frac{1}{k^2_\perp}, \tag{49}
\]

where the propagator for the rank-\((p - 3)\) R-R potential has the same form as Eq. (20), i.e.,

\[
C_{01...p-4} C_{01...p-4} = -\frac{1}{k^2_\perp}. \tag{50}
\]

So the total energy density from the R-R sector is

\[
U_{R-R} = U_{C_{p+1}} + U_{C_{p-3}} = -8n_1n_2 \left(1 + f_1^2 f_2^2\right) c^2_p V_{p+1} \frac{1}{k^2_\perp}. \tag{51}
\]

From Eqs. (47) and (51), we know the total energy density from both sectors is

\[
U = U_{NS-NS} + U_{R-R} = 0. \tag{52}
\]

This shows that the interaction between two \((D_{p-4}, D_p)\) vanishes. This can be understood from the well-known fact that there is no interaction between two constituent \(D_p\)-branes or between two constituent \(D_{p-4}\)-branes or between one \(D_p\)-brane and one \(D_{p-4}\)-brane. This net-zero interaction indicates that the underlying system preserves also 1/4 of the spacetime supersymmetries.

- The long-range interaction between \((D_{p-4}, D_p)\) and \((\bar{D}_{p-4}, \bar{D}_p)\)

The charge of a \(\bar{D}\)-brane, i.e., anti D-brane, has the opposite sign to that of a D-brane. The configuration of \((\bar{D}_{p-4}, D_p)\) can be obtained with the following flux \(\hat{F}\)

\[
\hat{F} = \begin{pmatrix}
0_{2n} & f \cdot u & & \\
& & & \\
& & & \\
& & & \\
& & & \\
& & & \\
& & & \\
& & & \\
0_{2n} & -f \cdot u & & \\
-f \cdot u & 0_{2n} & & \\
f \cdot u & 0_{2n} & & \\
\end{pmatrix}_{(p+1) \times (p+1)}. \tag{53}
\]
With this flux, $J_h$, $J_\phi$, $J_B$ and $J_{C_{p+1}}$ remain the same as before while $J_{C_{p-3}}$ changes its sign. So the total energy density is

\[ U = U_h + U_\phi + U_{C_{p+1}} - U_{C_{p-3}} = 16n_1n_2 f_1^2 f_2^2 c_p^2 \frac{V_{p+1}}{k_\perp}, \] (54)

which implies the expected attractive interaction between $(D_{p-4}, D_p)$ and $(\bar{D}_{p-4}, \bar{D}_p)$. This attractive interaction is actually due to that between $D_{p-4}$-branes and $\bar{D}_{p-4}$-branes.

- The long-range interaction between $(D_{p-4}, D_p)$ and $(D_{p-4}, \bar{D}_p)$

The configuration $(D_{p-4}, \bar{D}_p)$ can be realized via the flux $\hat{F}$ given in Eq. (53) and with the change of sign of the corresponding Wess-Zumino term in Eq. (30). With these, $J_h$, $J_\phi$, $J_B$ and $J_{C_{p-3}}$ don’t change while $J_{C_{p+1}}$ changes sign. So the total energy density is

\[ U = U_h + U_\phi - U_{C_{p+1}} + U_{C_{p-3}} = 16n_1n_2 c_p^2 \frac{V_{p+1}}{k_\perp}, \] (55)

which again implies an attractive interaction between $(D_{p-4}, D_p)$ and $(D_{p-4}, \bar{D}_p)$. It is now due to that between $D_{p}$-branes and $\bar{D}_{p}$-branes.

- The long-range interaction between $(D_{p-4}, D_p)$ and $(\bar{D}_{p-4}, \bar{D}_p)$

The state $(\bar{D}_{p-4}, \bar{D}_p)$ can be obtained simply by the change of sign of the corresponding Wess-Zumino term in Eq. (30). With this, $J_h$, $J_\phi$ and $J_B$ remain the same as before while $J_{C_{p+1}}$ and $J_{C_{p-3}}$ both change their signs. So the total energy density is

\[ U = U_h + U_\phi - U_{C_{p+1}} - U_{C_{p-3}} = 16n_1n_2 (1 + f_1^2 f_2^2) c_p^2 \frac{V_{p+1}}{k_\perp}, \] (56)

which is the sum of Eqs. (54) and (55) as expected. This indicates that this net interaction is due to that between $D_p$-branes and $\bar{D}_p$-branes and that between $D_{p-4}$-branes and $\bar{D}_{p-4}$-branes but there is no interaction between $D_p$-branes and $\bar{D}_{p-4}$-branes or between $D_{p-4}$-branes and $\bar{D}_p$-branes, again as expected. This property is just a consequence of $(D_{p-4}, D_p)$ as a marginal bound state.
3.2 The \( k = 3 \) case

The calculations of the long-range interaction for this case follow basically the same steps as in the previous subsection. The configuration of \( D_{p-6} \)-branes within \( D_p \)-branes can be realized with the following constant non-abelian magnetic flux \( \hat{F} \)

\[
\hat{F} = \begin{pmatrix}
0_{4n} & \ldots & 0_{4n} \\
\ldots & \ddots & \ldots \\
0_{4n} & -f \cdot u_1 & 0_{4n} \\
f \cdot u_1 & 0_{4n} & \ldots \\
0_{4n} & -f \cdot u_2 & 0_{4n} \\
f \cdot u_2 & 0_{4n} & \ldots \\
0_{4n} & -f \cdot u_3 & 0_{4n} \\
f \cdot u_3 & 0_{4n} & \ldots 
\end{pmatrix}, \quad (57)
\]

where the \( 0_{4n} \) stands for \( 4n \times 4n \) zero matrix, and

\[
\begin{align*}
  u_1 &= \text{Diag}\{I_n, I_n, -I_n, -I_n\}, \\
  u_2 &= \text{Diag}\{I_n, -I_n, -I_n, I_n\}, \\
  u_3 &= \text{Diag}\{I_n, -I_n, I_n, -I_n\}. 
\end{align*} \quad (58)
\]

Note that \( u_1, u_2 \) and \( u_3 \) are three of the Cartan subalgebra generators of the \( U(N) \) algebra with now \( N = 4n \) (\( n \) is a positive integer). With this flux, \( \text{Tr}(\hat{F} \wedge \cdots \wedge \hat{F}) \neq 0 \) only when the number of \( \hat{F} \) is 3 in the wedge product. Therefore from Eq. (37), we know that the only non-vanishing coupling with the lower rank R-R potential is for \( k = 3 \) and the corresponding R-R potential is \( C_{p-5} \). This implies that we have \( D_{p-6} \)-branes within \( D_p \)-branes with their charge proportional to the integral of \( \text{Tr}(\hat{F} \wedge \hat{F} \wedge \hat{F}) \). This brane configuration whose energy exceeds the sum of the energy of \( D_{p-6} \)-branes and \( D_p \)-branes is not a bound state in the usual sense but a relative stable state, and breaks all spacetime supersymmetries [10]. We will denote this configuration as \( (D_{p-6}, D_p) \) in the following.

With the above flux, by the same token, we have the following couplings as

\[
\begin{align*}
J_{h}^{(i)} &= -4n_i c_p V_{p+1} (1 + f_i^2)^{3/2} \tilde{V}_i^{\alpha \beta} h_{\alpha \beta}, \\
J_{\phi}^{(i)} &= -4n_i c_p V_{p+1} (1 + f_i^2)^{3/2} \left[ (p - 3) - 6 \frac{f_i^2}{1 + f_i^2} \right] \phi, \\
J_{B}^{(i)} &= -4n_i c_p V_{p+1} (1 + f_i^2)^{3/2} \tilde{V}_i^{\alpha \beta} B_{\alpha \beta} \quad (59)
\end{align*}
\]
for the NS-NS fields and

\[ J_{C_p+1}^{(i)} = 4n_i \sqrt{2c_p V_{p+1} C_{01..p}}, \]
\[ J_{C_p-5}^{(i)} = -4n_i \sqrt{2c_p V_{p+1} f_3^2 C_{01..p-6}} \]

(60)

for the R-R fields. In the above, we have the diagonal matrix

\[ \tilde{V}_i = \text{Diag}\{-1, 1, \ldots, 1, \frac{1}{1 + f_1^2}, \frac{1}{1 + f_2^2}, \frac{1}{1 + f_1^2}, \frac{1}{1 + f_2^2}, \frac{1}{1 + f_1^2}\}, \]

(61)

which immediately implies \( J_B^{(i)} = 0 \). Using Eqs. (59) - (61), we then have the respective long-range interaction energy density in momentum space due to the exchange of the corresponding massless field as

\[
U_h = c_p^2 \frac{V_{p+1}^2}{k_\perp^2} 16n_1 n_2 \sqrt{(1 + f_1^2)(1 + f_2^2)} \left( \frac{-p^2 + 6p + 7}{8} + \frac{-p^2 + 12p - 35}{8} f_1^2 \right. \\
+ \left. \frac{-p^2 + 12p - 35}{8} f_2^2 + \frac{-p^2 + 18p - 65}{8} f_1^2 f_2^2 \right),
\]

\[
U_\phi = c_p^2 \frac{V_{p+1}^2}{k_\perp^2} 16n_1 n_2 \sqrt{(1 + f_1^2)(1 + f_2^2)} \left( \frac{p^2 - 6p + 9}{8} + \frac{p^2 - 12p + 27}{8} f_1^2 \\
+ \frac{p^2 - 12p + 27}{8} f_2^2 + \frac{p^2 - 18p + 81}{8} f_1^2 f_2^2 \right),
\]

\[
U_B = 0
\]

(62)

for the NS-NS fields and

\[
U_{C_{p+1}} = -32n_1 n_2 c_p^2 \frac{V_{p+1}^2}{k_\perp^2},
\]

\[
U_{C_{p-5}} = -32n_1 n_2 f_1^3 f_2^3 c_p^2 \frac{V_{p+1}^2}{k_\perp^2}
\]

(63)

for the R-R fields. The total contribution to the energy density from the NS-NS sector is

\[
U_{\text{NS-NS}} = U_h + U_\phi + U_B \\
= 16n_1 n_2 \sqrt{(1 + f_1^2)(1 + f_2^2)} \left( 2 - f_1^2 - f_2^2 + 2f_1^2 f_2^2 \right) c_p^2 \frac{V_{p+1}^2}{k_\perp^2},
\]

(64)

while the total one from the R-R sector is

\[
U_{\text{R-R}} = U_{C_{p+1}} + U_{C_{p-5}} = -32n_1 n_2 (1 + f_1^3 f_2^3) c_p^2 \frac{V_{p+1}^2}{k_\perp^2}.
\]

(65)

So the total energy density from both sectors is

\[
U = U_{\text{NS-NS}} + U_{\text{R-R}} \\
= c_p^2 \frac{V_{p+1}^2}{k_\perp^2} 16n_1 n_2 \left[ \sqrt{(1 + f_1^2)(1 + f_2^2)} \left( 2 - f_1^2 - f_2^2 + 2f_1^2 f_2^2 \right) - 2 (1 + f_1^3 f_2^3) \right].
\]

(66)
We can show \(^{(11)}\) that \(U \leq 0\) if \(f_1 f_2 \geq 0\) and the equality holds only if \(f_1 = f_2\). This indicates that the interaction between two \((D_{p-6}, D_p)\) is in general repulsive and vanishes only if the two fluxes are identical. Note that, as pointed out in \([30]\), the \((D_{p-6}, D_p)\) system itself doesn’t preserve any supersymmetry and is unstable, so the \(U = 0\) case doesn’t imply any supersymmetry preservation of the interacting system under consideration, unlike the other cases such as the non-threshold BPS \((D_{p-2}, D_p)\) bound states.

By the same token, the force nature for other cases as in \(k = 2\) can also be analyzed and discussed.

### 3.3 The \(k = 4\) case

This case corresponds to the configuration of \(D_{p-8}\)-branes within \(D_p\)-branes. The only relevant case is for \(p = 8\), i.e., \(D_0\)-branes within \(D_8\)-branes. The brane system can be realized with \(D_8\)-branes carrying the following constant non-abelian magnetic flux \(\hat{F}\)

\[
\hat{F} = \begin{pmatrix}
0_{8n} & 0_{8n} & -f \cdot u_1 & 0_{8n} \\
0_{8n} & -f \cdot u_2 & 0_{8n} \\
-f \cdot u_1 & 0_{8n} & 0_{8n} & -f \cdot u_3 \\
0_{8n} & -f \cdot u_2 & 0_{8n} & f \cdot u_4 \\
\end{pmatrix}, \tag{67}
\]

where the \(0_{8n}\) stands for \(8n \times 8n\) zero matrix, and

\[
\begin{align*}
\mathbf{u}_1 &= \text{Diag}\{I_n, I_n, I_n, I_n, -I_n, -I_n, -I_n, -I_n\}, \\
\mathbf{u}_2 &= \text{Diag}\{I_n, I_n, -I_n, -I_n, I_n, I_n, -I_n, -I_n\}, \\
\mathbf{u}_3 &= \text{Diag}\{I_n, -I_n, I_n, -I_n, I_n, -I_n, -I_n, -I_n\}, \\
\mathbf{u}_4 &= \text{Diag}\{I_n, -I_n, -I_n, I_n, -I_n, I_n, I_n, -I_n\}. \tag{68}
\end{align*}
\]

\(^{11}\)If \(2(1 + f_1^2 f_2^2) \leq f_1^2 + f_2^2\), this is obviously true. So we need to check that this remains so for \(2(1 + f_1^2 f_2^2) > f_1^2 + f_2^2\) with \(f_1 f_2 \geq 0\). For this, we need to show that

\[
\sqrt{(1 + f_1^2)(1 + f_2^2)} \left[2(1 + f_1^2 f_2^2) - f_1^2 - f_2^2\right] \leq 2(1 + f_1^2 f_2^2)\left[2(1 + f_1^2 f_2^2) - f_1^2 - f_2^2\right]^2 \leq 4(1 + f_1^2 f_2^2)^2 
\]

since the left and right of the inequality are both positive. This latter inequality can be simplified to

\[
(f_1 - f_2)^2 \left[3 + 3 f_1^2 f_2^2 - f_1^2 - f_2^2\right] (f_1 + f_2)^2 - 4 f_1^2 f_2^2 \geq 0.
\]

Note that \((f_1 + f_2)^2 \geq 4 f_1 f_2\), then we have the term in the square bracket greater than or equal to zero if \(f_1 f_2 \geq 0\). Therefore we have \(U \leq 0\) if \(f_1 f_2 \geq 0\).
Note that $u_1$, $u_2$, $u_3$ and $u_4$ are four of the Cartan subalgebra generators of the $U(N)$ algebra with now $N = 8n$ ($n$ is a positive integer). With this flux, $\text{Tr}(\hat{F} \wedge \cdots \wedge \hat{F}) \neq 0$ only when the number of $\hat{F}$ is 4 in the wedge product. Therefore from Eq. (37), we know that the only non-vanishing coupling with the lower rank R-R potential is for $k = 4$ and the corresponding R-R potential is $C_1$. This implies that we have $D_0$-branes within $D_8$-branes with the $D_0$-brane charge proportional to the integral of $\text{Tr}(\hat{F} \wedge \hat{F} \wedge \hat{F} \wedge \hat{F})$. We will denote this configuration as $(D_0, D_8)$ in the following.

With the above flux, by the same token, we have the following couplings\footnote{Note that in what follows for convenience we express most of quantities in terms of $p$ unless explicitly specified but it should be understood that $p = 8$ always.} as

$$
J_h^{(i)} = -8n_i c_p V_{p+1}(1 + f_i^2) \tilde{V}_i^{\alpha \beta} h_{\alpha \beta},
$$

$$
J_{\phi}^{(i)} = -8n_i \frac{c_p}{2\sqrt{2}} V_{p+1}(1 + f_i^2)^2 \left[ (p - 3) - 8 \frac{f_i^2}{1 + f_i^2} \right] \phi,
$$

$$
J_B^{(i)} = -8n_i \frac{c_p}{\sqrt{2}} V_{p+1}(1 + f_i^2)^2 \tilde{V}_i^{\alpha \beta} B_{\beta \alpha}
$$

for the NS-NS fields, and

$$
J_{C_{p+1}}^{(i)} = 8n_i \sqrt{2} c_p V_{p+1} C_{01 \cdots 8},
$$

$$
J_{C_{p-7}}^{(i)} = 8n_i \sqrt{2} c_p V_{p+1} f_i^4 C_0
$$

for the R-R fields. In the above, we have the diagonal matrix

$$
\tilde{V}_i = \text{Diag}\{-1, \frac{1}{1 + f_1^2}, \frac{1}{1 + f_2^2}, \frac{1}{1 + f_3^2}, \frac{1}{1 + f_4^2}, \frac{1}{1 + f_5^2}, \frac{1}{1 + f_6^2}, \frac{1}{1 + f_7^2}, \frac{1}{1 + f_8^2}\},
$$

which immediately implies $J_B^{(i)} = 0$. Similarly, using Eqs. (69) - (71), we then have the respective long-range interaction energy density in momentum space due to the exchange of the corresponding massless field as

$$
U_h = c_p^2 \frac{V_{p+1}}{k^2} 64n_1 n_2 (1 + f_1^2) (1 + f_2^2) \left( \frac{-p^2 + 6p + 7}{8} + \frac{-p^2 + 14p - 49}{8} f_1^2 \right.
+ \frac{-p^2 + 14p - 49}{8} f_2^2 + \frac{-p^2 + 22p - 105}{8} f_1 f_2^2 \right),
$$

$$
U_\phi = c_p^2 \frac{V_{p+1}}{k^2} 64n_1 n_2 (1 + f_1^2) (1 + f_2^2) \left( \frac{p^2 - 6p + 9}{8} + \frac{p^2 - 14p + 33}{8} f_1^2 \right.
+ \frac{p^2 - 14p + 33}{8} f_2^2 + \frac{p^2 - 22p + 121}{8} f_1^2 f_2^2 \right),
$$

$$
U_B = 0
$$

(72)
for the NS-NS fields and

$$U_{C_{p+1}} = -128n_1n_2c_pV_{p+1}\frac{V_{p+1}}{k_L^2},$$

$$U_{C_{p-7}} = -128n_1n_2f_1^4f_2^4c_pV_{p+1}\frac{V_{p+1}}{k_L^2}$$

(73)

for the R-R fields. For this particular system, there is an additional coupling in the R-R sector between the one-form potential $C_1$ and the nine-form potential $C_9$ because of the duality relation for their components as $C_0 = -C_{01...8}$ [14, 15]. This coupling can also be interpreted as arising from the half-string creation between a $D_0$-brane and a $D_8$-brane [11, 12, 13] in the present context as we will demonstrate in the following. The corresponding contribution to the energy density can be calculated as

$$U_{C_{p+1}/C_{p-7}} = \frac{1}{V_{p+1}} \left( J^{(1)}_{C_{p+1}} J^{(2)}_{C_{p-7}} + J^{(1)}_{C_{p-7}} J^{(2)}_{C_{p+1}} \right)$$

$$= 128c_p^2V_{p+1}n_1n_2 \left( f_1^4 + f_2^4 \right) C_{01...8}C_0$$

$$= 128n_1n_2 \left( f_1^4 + f_2^4 \right) c_p^2\frac{V_{p+1}}{k_L^2},$$

(74)

where we have used the above mentioned duality to give

$$C_{01...8}C_0 = -C_0C_0 = -C_{01...8}C_{01...8} = \frac{1}{k_L^2}.$$  

(75)

In the above, we actually have two pieces with each positive and coming from either the coupling between $J^{(1)}_{C_{p+1}}$ and $J^{(2)}_{C_{p-7}}$ or between $J^{(1)}_{C_{p-7}}$ and $J^{(2)}_{C_{p+1}}$, therefore implying an attractive contribution. For the above mentioned purpose, we obtain the corresponding interaction in coordinate space using Fourier transformation as

$$U_{C_{p+1}/C_{p-7}}(Y) = \int \frac{d^dk_L}{(2\pi)^{\frac{1}{2}}} e^{-ik_L\cdot Y} U_{C_{p+1}/C_{p-7}} = 128n_1n_2 \left( f_1^4 + f_2^4 \right) c_p^2V_{p+1} \left( I_\infty - \frac{Y}{2} \right).$$

(76)

In the above, we have used the following relation

$$\int \frac{d^dk_L}{(2\pi)^{\frac{1}{2}}} e^{-ik_L\cdot Y} = I_\infty - \frac{Y}{2}$$

(77)

for one transverse direction. Note that the $I_\infty$ in Eq. (76) is positively infinity and independent of the separation $Y$, representing the energy when $Y = 0$, and its divergence actually reflects the $D_8$-brane nature of non-existence as an independent object as discussed in [15]. The corresponding attractive force acting on the $D_0$-branes per unit $D_0$-brane world-volume can be obtained as

$$F_{C_{p+1}/C_{p-7}} = -\frac{1}{V_{p-7}} \frac{dU_{C_{p+1}/C_{p-7}}(Y)}{dY} = 64n_1n_2 \left( f_1^4 + f_2^4 \right) c_p^2\frac{V_{p+1}}{V_{p-7}}.$$

(78)
For clearly demonstrating the half-string creation, we express the above force as

\[ F_{C_{p+1}/C_{p-7}} = F_{C_{p+1}/C_{p-7}}^{(1)} + F_{C_{p+1}/C_{p-7}}^{(2)} \]  

(79)

with

\[ F_{C_{p+1}/C_{p-7}}^{(i)} = 64n_1n_2f_1^4c_p^2V_{p+1} \]

(80)

where \( i = 1 \) or 2. From Eq. (70) or the integral of \( \text{Tr}(\hat{F} \wedge \hat{F} \wedge \hat{F} \wedge \hat{F}) \), we have the quantization condition

\[ 8n_if_1^4c_pV_{p+1} = m_ic_{p-8}V_{p-7}, \]

(81)

where \( m_i \) is an integer and represents the total number of D\(_0\)-branes within D\(_8\)-branes. We have then, taking \( i = 1 \) for example,

\[ F_{C_{p+1}/C_{p-7}}^{(1)} = 8m_1n_2c_0c_8 = 8m_1n_2\frac{1}{4\pi\alpha'}, \]

(82)

where we have used the exact value of \( c_p \) for D\(_p\)-brane as \( c_p = \sqrt{\pi} (2\pi\sqrt{\alpha'})^{3-p} \) [14, 8]. Note that \( N_2 = 8n_2 \) is the total number of D\(_8\)-branes in the second (D\(_0\), D\(_8\)) bound state and the \( m_1 \) is the total number of D\(_0\)-branes in the first bound state, therefore the force between a D\(_0\)-brane and a D\(_8\)-brane is given from the above as

\[ \frac{F_{C_{p+1}/C_{p-7}}^{(1)}}{8m_1n_2} = \frac{1}{4\pi\alpha'} = \frac{T}{2}, \]

(83)

where \( T \) is the tension of a fundamental string. This demonstrates that a string with its tension one half of a fundamental string is created between a D\(_0\)-brane and a D\(_8\)-brane, following the same spirit of [11, 12, 13, 15]. This interpretation is in line with [31] for a D\(_0\)-brane in the presence of a D\(_8\)-brane and is also consistent with the Hanany-Witten effect [32] for a D\(_0\)-brane crossing a D\(_8\)-brane. The same is true if \( i = 2 \) is taken in the above. So we demonstrate that a string with its tension one half of a fundamental string is created between a D\(_0\)-brane and a D\(_8\)-brane even in the non-abelian context, lending further support to this assertion.

The total contribution to the energy density from the NS-NS sector is

\[ U_{\text{NS-NS}} = U_h + U_\phi + U_B = 128n_1n_2 \left( 1 - f_1^4 - f_2^4 + f_1^4f_2^4 \right) c_p^2 \frac{V_{p+1}}{k_\perp^2}, \]

(84)

while the total one from the R-R sector is

\[ U_{\text{R-R}} = U_{C_{p+1}} + U_{C_{p-7}} + U_{C_{p+1}/C_{p-7}} = -128n_1n_2 \left( 1 - f_1^4 - f_2^4 + f_1^4f_2^4 \right) c_p^2 \frac{V_{p+1}}{k_\perp^2}. \]

(85)
So the total energy density from both sectors is

\[ U = U_{\text{NS}} - U_{\text{R}} = 0. \]  

(86)

If the \((D_0, D_8)\) is as expected a marginal bound state, the above indicates that the underlying system preserves 1/4 of spacetime supersymmetries. Note that the contribution \(U_{C_{p+1}/C_{p-7}}\) is independent of the nature of the constituent branes, i.e., being branes or anti-branes, in the respective bound state. But this is not the case for the \(U_{C_{p+1}}\) or \(U_{C_{p-7}}\).

In analogue to the case of \((D_{p-4}, D_p)\), we can analyze the following three cases.

- **The long-range interaction between \((D_0, D_8)\) and \((\bar{D}_0, D_8)\)**

  The presence of \(\bar{D}_0\)-branes changes the sign of \(U_{C_{p-7}}\), so the total energy density is

  \[ U = U_h + U_\phi + U_{C_{p+1}} - U_{C_{p-7}} + U_{C_{p+1}/C_{p-7}} = 256n_1n_2f_1^4f_2^4c^2p \frac{V_{p+1}}{k^2}, \]  

  (87)

  which implies the expected attractive interaction between \((D_0, D_8)\) and \((\bar{D}_0, D_8)\). This attractive interaction is actually due to that between \(D_0\)-branes and \(\bar{D}_0\)-branes.

- **The long-range interaction between \((D_0, D_8)\) and \((D_0, \bar{D}_8)\)**

  The presence of \(\bar{D}_8\)-branes changes the sign of \(U_{C_{p+1}}\), so the total energy density is

  \[ U = U_h + U_\phi - U_{C_{p+1}} + U_{C_{p-7}} + U_{C_{p+1}/C_{p-7}} = 256n_1n_2c^2p \frac{V_{p+1}}{k^2}, \]  

  (88)

  which also implies an attractive interaction due to that between \(D_8\)-branes and \(\bar{D}_8\)-branes.

- **The long-range interaction between \((D_0, D_8)\) and \((\bar{D}_0, \bar{D}_8)\)**

  The presence of \(\bar{D}_0\)-branes and \(\bar{D}_8\)-branes changes the signs of \(U_{C_{p-7}}\) and \(U_{C_{p+1}}\), so the total energy density is

  \[ U = U_h + U_\phi - U_{C_{p+1}} - U_{C_{p-7}} + U_{C_{p+1}/C_{p-7}} = 256n_1n_2 (1 + f_1^4f_2^4) c^2p \frac{V_{p+1}}{k^2}, \]  

  (89)

  which is the sum of Eqs. (87) and (88) as expected. This indicates that this net interaction is due to that between \(D_0\)-branes and \(\bar{D}_0\)-branes and that between \(D_8\)-branes and \(\bar{D}_8\)-branes.

  The above computations further imply that, in analogue to the case of \((D_{p-4}, D_p)\), the interaction between \((D_0, D_8)\) and \((\bar{D}_0, D_8)\), or between \((D_0, D_8)\) and \((D_0, \bar{D}_8)\), or
between $(D_0, D_8)$ and $(\bar{D}_0, \bar{D}_8)$, is always attractive. It is entirely due to the interaction between $D_0$-branes and $\bar{D}_0$-branes or between $D_8$-branes and $\bar{D}_8$-branes, or the sum of the interaction between $D_0$-branes and $\bar{D}_0$-branes and that between $D_8$-branes and $\bar{D}_8$-branes, in the respective bound states.

The above computations strongly suggest that the bound state $(D_0, D_8)$ so constructed is a marginal one but it was argued in [10] based on the relative scaling of $\hat{F}^2$ and $\hat{F} \wedge \hat{F} \wedge \hat{F} \wedge \hat{F}$ while keeping the $D_0$-brane charge invariant and on the energy argument that it is not. While this confusion cannot be settled down for the time being, we would like to point out that the peculiar nature of $D_8$-brane being unable to be an independent object by itself [30] and the associated divergent self-energy of the $D_8$-brane may indicate that the scaling and the energy arguments mentioned above may be a bit too simple and further re-examination of this is needed. Nevertheless, we have demonstrated that the half-string creation picture still holds even in the present context, lending further support to this.

4 Summary

We have calculated the long-range interactions between two simple $p$-branes in diverse dimensions without any world-volume flux turned on. We also compute the interaction either between two $(D_{p-4}, D_p)$ or between two $(D_{p-6}, D_p)$ or between two $(D_0, D_8)$, respectively, where the lower dimensional branes in the respective state can be represented by the corresponding special world-volume non-abelian magnetic flux.

For the simple $p$-brane case, the static net interaction vanishes. So the “no-force” condition holds and we have preserved $1/2$ of the spacetime supersymmetries. If we replace one set of coincidental branes by its corresponding anti-branes, all supersymmetries are broken and the long-range interaction between one set of coincidental branes and one set of coincidental anti-branes with a separation is attractive and is explicitly given. For the case of IIA NS5-brane or (transverse) M5-brane, this is the interaction which may only be computed at the present.

For $(D_{p-4}, D_p)$, i.e., a marginally bound state, our calculations confirm the well-known fact that the interaction between two such brane configurations is the sum of two contributions: one is due to the two sets of $D_{p-4}$-branes in the two configurations, respectively, and the other is due to the two sets of $D_p$-branes. There is no contribution from the $D_{p-4}$-branes in one configuration and the $D_p$-branes in the other one. When one or both the constituent branes in one bound state are taken as the anti-ones, the corresponding explicit interaction potential computed in this paper is believed to be given
the first time.

The case of \((D_{p-6}, D_p)\) is a bit more complicated. This system itself doesn’t preserve any supersymmetry and the interaction between two such systems is in general repulsive just like that between two constituents in the state \((D_{p-6}, D_p)\). The long-range interaction can still vanish but this doesn’t imply any preservation of supersymmetry for the interacting system under consideration. Once again, the general explicit interaction potential in this case is computed the first time.

For the case of \((D_0, D_8)\). First the long-range interaction between a \(D_0\)-brane and a \(D_8\)-brane can also be calculated by the same token if a key duality relation between the R-R potential \(C_0\) associated with the \(D_0\)-brane and the R-R potential \(C_{01...8}\) associated with the \(D_8\)-brane, namely \(C_0 = -C_{01...8}\) found in [14], is employed. The counter-intuitive R-R contribution was calculated via an effective field approach by one of the present authors in [15] and the NS-NS contribution can be trivially calculated via the method described in this paper. One peculiar feature of this system is that only the massless modes rather than the full string spectrum contribute to the lowest-order stringy interaction and therefore it can be calculated via an effective field theory approach. We also demonstrate that the half-string creation in the present context continues to hold, therefore lending further support to the previous assertion of this. Non-vanishing potentials for variants of such system are computed explicitly.

Our computations indicate that the interaction between two \((D_0, D_8)\) follows the same line as the case of \((D_{p-4}, D_p)\), therefore strongly suggesting that the bound state so constructed with the special constant non-abelian magnetic flux is an expected marginal bound state. This may indicate that a further re-examination of the analysis given in [10] for this bound state is needed along with the consideration of the \(D_8\)-brane nature that this co-dimensional one brane cannot exist by itself as an independent object.

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