Fairer group draw for sports tournaments with restrictions

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“Plus on vieillit, et plus on se persuade que Sa sacrée Majesté le Hasard fait les trois quarts de la besogne de ce misérable univers.”

(Frederick the Great, the King of Prussia, to Voltaire)

Abstract

Some recent sports tournaments establish restrictions in the draw of the group stage. In this case, the mechanism used by the Union of European Football Associations (UEFA) to assign the teams into groups is unevenly distributed since not all valid allocations are equally likely. It is shown by illustrative examples how a careful choice of the pot labels and randomisation in the size of the groups can reduce that unfairness. According to our Monte Carlo simulations, the draw of the European Qualifiers to the 2022 FIFA World Cup could have been closer to uniform distribution without any negative effects. Our recommendation has a good chance to be implemented in practice.

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1 “The older we get, the more we convince ourselves that His sacred Majesty Chance does three quarters of the work of this miserable universe.” (Source: Correspondance de Voltaire, 1759, Lettre 3803. https://fr.wikisource.org/wiki/Correspondance_de_Voltaire/1759/Lettre_3803)
1 Introduction

The present paper offers a reform for the draw procedure of sports tournaments with various restrictions on the allocation of the teams into groups. Examples include qualifications for the recent UEFA European Championships, the European Qualifiers to the FIFA World Cup, and the UEFA Nations League. The outcome of the classical scheme of extracting teams from pots will be shown to depend on the order of the pots. According to simulations, the draw of the European Qualifiers to the 2022 FIFA World Cup could have been closer to uniform distribution with a careful choice of the pot labels and additional randomisation.

The Union of European Football Associations (UEFA) is strongly encouraged to consider our suggestion in order to build fairer tournaments in the future. This would be especially important because, as Csató (2022) has recently revealed, the incentive incompatibility of the European Qualifiers to the 2022 FIFA World Cup can be almost eliminated by additional draw constraints—but at the price of increasing the unfairness of the draw, which needs to be addressed according to the proposal provided here.

Our study has some antecedents in the academic literature. Focusing on the FIFA World Cup, several authors have developed draw systems for sports tournaments with geographical or seeding restrictions to produce balanced groups at roughly the same competitive level (Cea et al., 2020; Guyon, 2015; Laliena and López, 2019). The draw procedure of the 1990 (Jones, 1990), 2006 (Rathgeber and Rathgeber, 2007), and 2014 FIFA World Cups (Guyon, 2015) have been proved to be unevenly distributed. The bias of the matching mechanism used in the UEFA Champions League Round of 16 draw has also been quantified (Boczoń and Wilson, 2018; Klößner and Becker, 2013). Guyon (2014, Section 3) demonstrates that the UEFA rules do not guarantee a uniformly distributed outcome for the group draw.

However, the above works neither discuss the more complicated draw constraints recently used by the UEFA nor recognise the role of pot labelling, which constitute the central contributions of the current paper. In addition, their proposals for approaching uniform distribution require either the relaxation of draw constraints or a fundamental restructuring of the draw procedure. Consequently, our recommendation that does not affect the main principles of the draw has better prospects to be implemented in practice.

The remainder of the paper is organised as follows. Section 2 describes the group draw of the European qualifiers to the 2022 FIFA World Cup. The uneven distribution of the UEFA mechanism is discussed in Section 3, and the superiority of our modified versions is verified in Section 4. Finally, Section 5 concludes.

2 The group draw of the European Qualifiers to the 2022 FIFA World Cup

The European section of the 2022 FIFA World Cup qualification is contested by the 55 national teams of the UEFA for 13 available slots. It traditionally consists of a group stage followed by a play-off phase. In the first round, the teams are drawn into five groups of five teams (Groups A–E) and five groups of six teams (Groups F–J) each. The groups are played in a home-away round-robin format. The group winners qualify for the 2022 FIFA World Cup and the runners-up progress to the second round of the qualification.

The group draw decides the composition of the qualifying groups. Since the qualification
depends on group rankings, the procedure should create groups having roughly the same competitive level. As usual, balancedness is achieved by *seeding*: the 55 teams are classified into six pots based on the FIFA World Rankings of November 2020 such that Pot \( \ell \) contains the teams ranked from \( 10^{\ell - 9} \) to \( 10^{\ell} \) if \( 1 \leq \ell \leq 5 \), whereas Pot 6 consists of the five lowest-ranked teams.

The draw starts with Pot 1 and continues with Pot 2 until Pot 6. Each pot is emptied before proceeding to the next pot. The five teams from Pot 6 are assigned to the last five groups (Groups F–J). Therefore, every group contains at most one team from each pot.

UEFA applies various draw conditions “to issue a schedule that is fair for the participating teams, fulfils the expectations of our commercial partners and ensures with a high degree of probability that the fixture can take place as scheduled” (UEFA, 2020a):

- **Competition-related reasons**: The four participants in the UEFA Nations League Finals 2021 should play in a group with five teams (Groups A–E).
- **Prohibited team clashes**: The UEFA Executive Committee decided that four pairs of teams cannot be drawn into the same group due to political reasons.
- **Winter venue restrictions**: At most two out of the ten countries identified as venues with risk of severe winter conditions can be placed in a group. The Faroe Islands and Iceland, having the highest risk, cannot play in the same group.
- **Excessive travel restrictions**: A maximum of one country pair out of the 18 identified with excessive travel relations can be drawn into the same group.

The constraints substantially complicate the draw procedure because some allocations become infeasible. The standard *UEFA mechanism* assigns the team drawn to the first available group in alphabetical order as indicated by the computer such that at least one allocation remains feasible for the teams still to be drawn, thus any dead end is avoided.

### 3 Potential sources of uneven distribution

Following the previous literature on constrained assignment in sports tournaments, a draw is called *fair* if all feasible allocations of the teams into groups are equally likely. Otherwise, the distortions caused exclusively by the draw procedure favour certain teams and harm others.

As demonstrated in Guyon (2014, Section 3), the UEFA mechanism is unfair. It is not a futile exercise to understand some causes of the bias through basic examples.

**Example 1.** There are three groups A–C and four teams \( T_1 \)–\( T_4 \) such that \( T_2 \) cannot be placed in group B and \( T_4 \) (the only team in the last pot) should be assigned to group C. In an evenly distributed random draw, \( T_2 \) should play in group A and group C with an equal chance of 50\%, respectively. However, the UEFA draw procedure assigns \( T_2 \) to group A with a probability of 1/3 (when \( T_2 \) is drawn first) and to group C with a probability of 2/3. On the other hand, both \( T_1 \) and \( T_3 \) have a probability of 1/3 to play in group A (when they are drawn first), 1/2 to be assigned to group B (when they are drawn second, or third and \( T_2 \) is drawn second), and 1/6 to be placed in group C.

The problem of Example 1 can be solved by additional randomisation. If the group of team \( T_4 \) is drawn randomly in the first step, both teams \( T_1 \) and \( T_3 \) play against \( T_4 \) with
Table 1: The probability that two teams play in the same group in Example 2

(a) Uniform distribution

|     | T1 | T2 | T3 | T4 | T5 | T6 | T7 | T8 | T9 |
|-----|----|----|----|----|----|----|----|----|----|
| T3  | 0  | 0  | 0  | 3/7| 3/7| 1/7| 3/7| 2/7| 2/7|
| T6  | 3/7| 3/7| 1/7| 0  | 0  | 0  | 3/7| 2/7| 2/7|
| T8/T9| 5/14| 5/14| 5/14| 2/7| 5/14| 2/7| 0  | 0  | 0  |

(b) Standard UEFA mechanism (Draw order: Pot 1, Pot 2, Pot 3)

|     | T1  | T2  | T3  | T4  | T5  | T6  | T7  | T8  | T9  |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| T3  | 0   | 0   | 0   | 1/3 | 1/3 | 1/3 | 5/9 | 2/9 | 2/9 |
| T6  | 1/3 | 1/3 | 1/3 | 0   | 0   | 0   | 5/9 | 2/9 | 2/9 |
| T8/T9| 7/18| 7/18| 2/9 | 7/18| 7/18| 2/9 | 0   | 0   | 0   |

(c) Modified UEFA mechanism (Draw order: Pot 3, Pot 2, Pot 1)

|     | T1 | T2 | T3 | T4 | T5 | T6 | T7 | T8 | T9 |
|-----|----|----|----|----|----|----|----|----|----|
| T3  | 0  | 0  | 0  | 4/9| 4/9| 1/9| 4/9| 5/18| 5/18|
| T6  | 4/9| 4/9| 1/9| 0  | 0  | 0  | 1/3| 1/3 | 1/3 |
| T8/T9| 13/36| 13/36| 5/18| 1/3| 1/3| 1/3| 0  | 0   | 0   |

A probability of $1/3 \times 1/3 + 1/3 \times 1/2 + 1/3 \times 1/6 = 1/3$ under the UEFA mechanism. This coincides with the chance of being assigned to the group of T4 under the uniform distribution.

Another drawback of the official rule is the inflexible treatment of the draw constraints for the sake of simplicity. While this is not a problem in the case of prohibited clashes, it is disadvantageous for more complex restrictions such as the winter venue constraint.

**Example 2.** There are three groups A–C and nine teams T1–T9 divided into three pots of three teams sequentially such that at most two teams from the set \{T3, T6, T8, T9\} can be placed into the same group. Table 1 compares the probability that two teams play in the same group under three procedures:

- Uniform distribution;
- Standard UEFA mechanism with the draw order Pot 1, Pot 2, Pot 3;
- Modified UEFA mechanism where the draw order is Pot 3, Pot 2, Pot 1.

Teams T1 and T2, T4 and T5, as well as T8 and T9 are treated equally by the three methods.

The labelling of the groups is arbitrary, thus, there exist $3! \times 3! = 36$ different allocations of the teams, among which eight are infeasible (when T3, T6, and either T8 or T9 play in the same group). Among them, there are only four cases in which teams T3 and T6 play against each other since the third team of the group should be T7. On the other hand, the UEFA procedure does not anticipate the draw constraint affecting the teams of Pot 3, thus its standard version places T3 and T6 into the same group with a probability of 1/3. Naturally, the modified UEFA method also suffers from this shortcoming but it assigns the
Table 2: The number of draw restrictions for each pot

| Type of restriction   | Pot 1 | Pot 2 | Pot 3 | Pot 4 | Pot 5 | Pot 6 |
|-----------------------|-------|-------|-------|-------|-------|-------|
| Prohibited clash      | —     | 2     | 3     | 1     | 4     | —     |
| Winter venue          | —     | 1     | 4     | 1     | 4     | —     |
| Excessive travel      | 5     | 1     | 9     | 2     | 16    | 3     |

Note: Prohibited clash and excessive travel are taken for both pots into account. The pair Faroe Islands and Iceland is accounted as a prohibited clash.

two teams to the same group only with a probability of 1/9 as $T3$ cannot play in the group that already contains $T6$ and either $T8$ or $T9$. While both of the latter procedures are biased concerning the pair $T3$ and $T6$, the error is much smaller for the modified variant. What is more, the modified UEFA mechanism is closer to uniform distribution than the standard UEFA mechanism for all pairs of teams.

The explanation of Example 2 is intuitive: since the draw constraint applies for one team in Pots 1 and 2 but for two teams in Pot 3, it is better to start the draw with Pot 3 to minimise the chance that the allocation of the teams will be seriously restricted later.

To summarise, randomisation in the group size can reduce unfairness according to Example 1, whereas Example 2 sheds light on the non-negligible role of pot labelling.

4 The comparison of three draw procedures

Regarding the extent of the departure from even distribution, previous papers were able to derive exact results both in the case of the FIFA World Cup draw (Jones, 1990; Rathgeber and Rathgeber, 2007; Guyon, 2015) and the UEFA Champions League Round of 16 draw (Klößner and Becker, 2013). In our case, this is clearly impossible because there exist $(10!)^5 \times 5! \approx 7.55 \times 10^{34}$ different permutations of the teams in which they can be drawn. Although the draw conditions somewhat reduce this number, for instance, it is sufficient to investigate only $4! \times 6!$ cases for Pot 1 instead of $10!$ due to competition-related constraints, the degree of unfairness can be quantified only via simulations.

The official UEFA draw rule is not independent of the group labels as the teams are assigned to the first available group in alphabetical order. Hence the outcome might be closer to uniform distribution if the groups consisting of six teams are labelled randomly, see Example 1. Table 2 shows the number of draw constraints by pots. Based on Example 2, the draw is worth starting with pots containing more national teams affected by the winter venue constraint. Consequently, three options will be considered:

- Official UEFA mechanism with the standard draw order Pot 1, Pot 2, Pot 3, Pot 4, Pot 5, Pot 6;
- Modified UEFA mechanism: analogous to the official UEFA mechanism but the draw order is Pot 1, Pot 3, Pot 5, Pot 4, Pot 2, Pot 6;
- Modified random UEFA mechanism: analogous to the modified UEFA mechanism but the groups with six teams are chosen randomly at the beginning of the draw.

The three alternatives will be compared on the basis of $n = 10$ million randomly generated draw orders. As the error of a simulated probability $p$ is $\sqrt{p(1-p)/n}$ and the lowest $p$
Table 3: Measures of unfairness for three draw mechanisms

| Measure                          | Official UEFA | Modified UEFA | Modified random UEFA |
|---------------------------------|--------------|--------------|---------------------|
| Maximum of positive (+) biases  | 0.5561       | 0.3128       | 0.3448              |
| Mean of the 20 largest (+) biases | 0.2799     | 0.1767       | 0.1772              |
| Maximum of negative (−) biases  | 0.1861       | 0.1857       | 0.1856              |
| Mean of the 20 largest (−) biases | 0.1603     | 0.1096       | 0.1121              |
| Sum of absolute biases          | 26.652       | 18.311       | 16.241              |
| Sum of squared biases           | 3.2153       | 1.3155       | 1.2831              |

equals approximately 3.95% (Wales and Kazakhstan are assigned to the same group), 10 million iterations imply that the relative standard error of the calculated probabilities remains below 0.002. Therefore, all distortions detailed below are highly significant.

The extant literature has considered the uneven distribution of the UEFA mechanism only for the Champions League Round of 16 draw, where quantifying the deviations is straightforward as the clubs are paired. We think a reasonable metric of distortions should be based on the probability with which two teams should play against each other. Therefore, the degree of unfairness is quantified by the similarity between the number of simulation runs in which a country pair is placed in the same group under the draw procedure considered and an evenly distributed random draw among all feasible allocations. In the comparison of method \( A \) and the uniform distribution, the bias for teams \( i \) and \( j \) is

\[
\frac{\# \text{ when teams } i \text{ and } j \text{ are assigned to the same group under method } A}{\# \text{ when teams } i \text{ and } j \text{ are assigned to the same group under uniform distribution}} - 1
\]

if the value of the fraction is above one and the reciprocal of the fraction minus one otherwise. The bias equals one if teams \( i \) and \( j \) are twice or half as likely to be placed into the same group by method \( A \) compared to a fair draw.

Table 3 presents the performance of the three draw procedures. The proposed change in the order of the pots massively reduces positive deviations, the cases when a country pair has a higher probability to be assigned to the same group in the draw by the UEFA rule. Even though the worst negative deviation cannot be avoided, the average of the most extreme values can be decreased in this case, too. According to both aggregated measures of distortions, the modified UEFA draw procedure would be substantially fairer. The highest biases are not treated by randomisation in the group labels, however, it improves fairness for most pairs.

These findings are reinforced by focusing only on the direction of changes. The modified UEFA mechanism is better for 665 and the official rule is more favourable for 560 country pairs. Analogously, the modified random UEFA procedure is less (more) distorted in 735 (490) cases than the modified UEFA method.

Figure 1 plots the bias of the draw procedures separately for each national team. Our proposal is fairer for Pots 2, 3, and 4 (teams ranked between 11th and 40th), especially regarding three countries that have been identified with risk of severe winter conditions among others: Ukraine (16), Iceland (27), and Belarus (38). The effects for the other three pots are more ambiguous, however, teams drawn from Pots 5 and 6 have a marginal chance to qualify for the FIFA World Cup. Hence, on balance, the increased unfairness of the modified UEFA mechanism for Kosovo (46) and Kazakhstan (47) is more than...
compensated for. Compared to the modified UEFA method, the modified random UEFA procedure is beneficial for 48 (47) nations according to absolute (squared) distortions.

5 Conclusions

We have proposed reasonable changes in the draw procedure of sports tournaments where the groups contain a different number of teams and there exist various draw restrictions. It has been highlighted through an illustrative example that relabelling of the pots can substantially reduce the gap of the mechanism used by the UEFA to the uniform distribution. An initial random choice of larger groups is also worth considering. Our recommendation is not the philosopher’s stone, it is not able to eliminate all distortions. However, it avoids the most severe biases at almost no price. Changing the order of the pots can even contribute to the excitement of the draw: as Pot 2 contains the strongest teams outside Pot 1, the fans of the best teams cannot relax until the event is almost finished.

Analogous systems are currently used by UEFA. For instance, in the draw of the
European Qualifiers for the 2020 UEFA European Championship, competition-related draw conditions were addressed without randomisation, namely, the four Nations League finalists were drawn into Groups A–D (UEFA, 2018) rather than into four randomly chosen groups from the first five as in the draw of the European Qualifiers to the 2022 FIFA World Cup. Similarly, the pots were ordered in a reversed order of strength for the 2020/21 UEFA Nations League draw, that is, the lowest-ranked teams were drawn first (UEFA, 2020b). Our proposals require only one more step in that direction and are exactly in line with the UEFA policy of ensuring the fairness of the draw (UEFA, 2019).

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