Renormalization of a modified gravity with a quadratic Riemann tensor term

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Abstract

We consider a modified form of gravity, which has an extra term quadratic in the Riemann tensor. This term mimics a Yang-Mills theory. The other defining characteristic of this gravity is having the affine connection independent of the metric. (The metricity of the metric is rejected, too, since it implies a Levi-Civita connection.) It is then shown that, in the low density limit, this modified gravity does not differ from the General Theory of Relativity. We then point out that its Lagrangian does not contain partials of the metric, so that the metric is not a quantum field, nor does it contribute propagators to the Feynman diagrams of the theory. We also point out that the couplings of this theory (that determine the topological structure of the Feynman diagrams) all come from the term quadratic in the Riemann tensor. As a result of this situation, the diagrams of this theory and the diagrams of a Yang-Mills theory all have the same topology and degree of divergence, up to numerical coefficients. Since Yang-Mills theories are renormalizable, it follows that this theory should also be renormalizable.

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1 INTRODUCTION.

There has been much interest in the last few years on the subject of modifications of the action of the General Theory of Relativity (GTR)

\[ S_{\text{GR}} = \int \sqrt{-g} \left( \frac{2\kappa^2}{\kappa^2} R + \mathcal{L}_M \right) d^4x. \]
where $\kappa = \sqrt{32\pi G}$ is the gravitational coupling constant and $\mathcal{L}_M$ is the matter and radiation Lagrangian.\textsuperscript{[1]} The main motivation for this interest comes from the questions posed by astronomical observations that suggest the existence of dark matter and dark energy.\textsuperscript{[2]} Here we are going to consider a modification to the Lagrangian of the GTR, but our interest lies in improving its renormalization prospects. We use a particular combination of old ideas that result, taken together, in a new approach to this subject.

There are two fundamental difficulties with attempts at renormalizing the GTR. The first difficulty is that the gravitational coupling constant $\kappa \sim 1/E_{\text{Planck}}$ has units of reciprocal energy $E^{-1}$. This should lead to a quantum field theory that cannot be renormalized since, as the number of loops increases, the order of the divergence must increase, too, and then there has to be an infinite number of distinct renormalization counterterms. The second difficulty has to do with the fact that, since the Levi-Civita connection is expressed as a sum of partials of the metric, then there are going to be diagrams in the theory that contain powers of first- and second-order partials of the metric.\textsuperscript{[3]} This situation leads to double quadratic poles in the propagators and the combinations of these propagators can involve negative probabilities which make the theory meaningless.\textsuperscript{[4]}

One way of addressing the first difficulty is to use a theory for gravity that does not use a coupling constant with units. Since the Riemann tensor has units of $E^{-2}$ a term in the Lagrangian involving some squared form of the Riemann tensor would have units of $E^{-4}$ and thus would require a coupling constant with no units.\textsuperscript{[5]} Different possibilities for a squared Riemann tensor Lagrangian\textsuperscript{[6]} are of the form $R^2$, $R_{\mu\nu}R^{\mu\nu}$, and $R_{\mu\nu\xi\rho}R^{\mu\nu\xi\rho}$ (for purposes of inclusion in an action these terms are not independent since they are related by the Gauss-Bonnet topological invariant). Unfortunately, having a coupling constant with no units does not solve all the quantization problems. Powers of the first- and second-order partials of the metric still appear, and new types of diagrams keep showing up at higher orders, that require the inclusion of an infinite number of counterterms not reducible to a finite set of primitive diagrams.

String theory does contain, as an effective quantum field theory, an infinite number of terms of these type. In principle, string theory can be replaced by an infinite number of local fields. By integrating out of the path integral the massive fields it should be possible to achieve an effective non-local field theory. Not surprisingly in practice only approximate methods can be used for this purpose. One such method is to construct the quantized string theory on background fields, and then use conformal invariance to build up constraints between the background fields, which are then identified with the $\beta$ functions of the corresponding $\sigma$ model.\textsuperscript{[7]} Alternatively, it is possible from string theory to calculate the scattering amplitudes of the massless particles in tree approximation, and from there guess what the effective Lagrangian for the massless particles can be. The higher-order terms in the Lagrangian can be constructed by iteration from the first Lagrangian, using higher-order string interactions and unitarity considerations.\textsuperscript{[8]} The resulting effective theory includes metrics, antisymmetric tensors, dilaton fields and fermion fields, as well as gauge background fields in the case of the heterotic string.

In this low-energy interpretation given to the string theory, the terms in the metric and
partials of the metric are grouped to form Riemann tensors constructed from Levi-Civita connections. The problems involved in the quantization of an effective theory of this type are basically the same as those encountered in a direct quantization of Einstein’s gravity. This situation obviously it does not constitute evidence of any fundamental incorrectness of string theory, since we are dealing only with an approximation to the much better behaved worldsheet, but, be it as may, we cannot quantize directly this string-suggested theory of gravity, either.

In order to avoid the problems presented by the presence of powers of the metric and its partials we resort to a metric-affine setup, also sometimes also called Einstein-Palatini. We take the metric and the connection to be completely independent. (The connection is taken to be symmetric and there is no torsion.) Thus the connection is not Levi-Civita. The metric-affine setup is often used as a prelude to establishing a gauge theory of gravity, but we will not proceed this way here. We are assuming invariance only under coordinate diffeomorphisms.

As we shall see, the connection will still turn out to be Levi-Civita, but only in the low-density limit, and as a result of the equations of motion. By low-density it is meant that the energy and momentum densities are roughly of the order of the average density of our universe or less.

Consider the modified action

$$S_{\text{mod}} = \int \sqrt{-g} \left( \frac{1}{4} R^\rho_{\sigma\mu\nu} g^{\mu\tau} g^{\nu\rho} R^\sigma_{\rho\tau\nu} + \frac{2}{\kappa^2} R + \mathcal{L}_M \right) \, d^4x,$$  

(2)

which has an extra term with respect to (1). The new term mimics the curvature term in a Yang-Mills Lagrangian, with the group indices of the gauge field replaced by coordinate indices; this reason for this choice over the other quadratic forms will eventually become clear. This action generates two equation of motion, one due to a first variation with respect to the connection, and another due to a first variation with respect to the metric.

In the remainder of this paper we shall show that, while for low densities this action cannot be distinguished from the GTR, it does have good perspectives of renormalization.

## 2 The metricity of the metric.

The Levi-Civita connection depends on the metric:

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\zeta} (g_{\mu\zeta,\nu} + g_{\nu,\mu} - g_{\mu,\nu,\zeta}).$$  

(3)

This relation between the connection and the metric follows from the so-called metricity of the metric,

$$g_{\mu\nu,\lambda} = 0.$$  

(4)

Since we are not taking the connection to be Levi-Civita, the metric has no metricity, that is, (4) does not hold. The metricity of the metric clearly holds in the 3-dimensional world
of our experience, and, theoretically, Einstein-Cartan theories usually respect metricity. So let us study for a moment the implications of non-metricity.

The point of view on metricity in Riemannian geometry becomes clear in the following representative definition, standard in textbooks on the subject. The metric is an inner symmetric product $\langle \ , \ \rangle$ defined on the tangent space $T_{p}^{\mathbb{R}}$ that is associated with each point $p$ of a real differentiable manifold of dimension $n$. A Riemannian manifold is a manifold with a metric. In terms of a coordinate basis this product has components

$$g_{\mu\nu} = \langle \frac{\partial}{\partial x^{\mu}}, \frac{\partial}{\partial x^{\nu}} \rangle = \langle \partial_{\mu}, \partial_{\nu} \rangle.$$ 

Thus two vectors $u$ and $v$ have the metric or inner product

$$\langle u, v \rangle = \langle u^{\mu} \partial_{\mu}, v^{\nu} \partial_{\nu} \rangle = u^{\mu} g_{\mu\nu} v^{\nu}.$$ 

Let us assume a covariant differentiation operator $\nabla$. Then the standard definition for the covariant differentiation of the inner product of two vector fields $u$ and $v$, both defined along a parametrized curve $x(t)$, is given by:

$$\frac{d}{dt} \langle u, v \rangle = \langle \nabla_{u} \frac{dt}{d\tau}, v \rangle + \langle u, \nabla_{v} \frac{dt}{d\tau} \rangle.$$ (5)

Using coordinates this definition immediately implies (4). To see this let us write this equation in components, using a coordinate frame. Then (5) would have the following equivalent form in the open coordinate patch that contains path $x(t)$:

$$\frac{\partial}{\partial x^{\lambda}}(u^{\mu} g_{\mu\nu} v^{\nu}) = (u^{\mu} \lambda \Gamma_{\lambda\epsilon}^{\mu} u^{\epsilon}) g_{\mu\nu} v^{\nu} + u^{\mu} \Gamma_{\lambda\epsilon}^{\mu} g_{\mu\nu} v^{\nu} + u^{\mu} g_{\mu\nu} (v^{\nu} \lambda + \Gamma_{\lambda\epsilon}^{\nu} v^{\epsilon}).$$ (6)

Applying the rule for partial differentiation of a product on the left-hand side of this equation, which results in three terms, we see metricity follows immediately.

But one can argue differently. One can say that the original definition (5) is actually missing a term on the right-hand side of the equation, because it is ignoring the possibility that the metric can also be affected by the differentiation process. This becomes evident using components. In this case the definition should include a covariant differentiation of the metric, too, and the differentiation of the scalar $u^{\mu} g_{\mu\nu} v^{\nu}$ results in

$$\frac{\partial}{\partial x^{\lambda}}(u^{\mu} g_{\mu\nu} v^{\nu}) = u^{\mu} \lambda g_{\mu\nu} v^{\nu} + u^{\mu} g_{\mu\nu} \lambda v^{\nu} + u^{\mu} g_{\mu\nu} v^{\nu} \lambda.$$ 

After we expand the covariant derivatives, all terms cancel and we are left with the tautology $0 = 0$, so that metricity is not implied. Thus definition (5) is really equivalent to equation (4).

In physics metricity is derived from Einstein’s Equivalence Principle. According to the Equivalence Principle, given a small region in spacetime, it is always possible to find there a coordinate system that approximates a locally inertial frame. Thus in the small region
the metric is approximately a Minkowski metric \( \eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1) \), with null first-order partial derivatives, but not necessary null second-order partial derivatives. Therefore \( \eta_{\mu\nu;\lambda} = 0 \). We can transform this equation to a different coordinate system, following the rules for transforming tensors, and the result is then that, in all possible coordinate systems, \( g_{\mu\nu;\lambda} = 0 \). Thus if we do not want metricity we also have to do away with the Equivalence Principle. This principle gives conceptual unity to the GTR and helps derive, up to a point, its equations, but, since we are questioning the GTR, it is only to be expected that we should also question its guiding principle. In this paper metricity turns out to be true only as a low density limit.

3 The low-density limit of the modified action.

In this section we look at the low-density limit of Lagrangian (2). This limit is the state most of the Universe we can observe is in.

Let us compare, in this limit, the two terms which appear in (2) that involve the Riemann tensor. The first goes as \( \bar{R}^2 \), where \( \bar{R} \) is the average value of the curvature scalar in our universe, and the second as \( \bar{R}L_{\text{Planck}}^{-2} \). Since \( |\bar{R}| \ll L_{\text{Planck}}^{-2} \), in this limit the inequality

\[
\frac{1}{4} R^\rho_{\sigma\mu\nu} g^{\mu\tau} g^{\nu\rho} R^\sigma_{\rho\tau\nu} \ll \frac{2}{\bar{R}^2} |\bar{R}|
\]

holds, and Lagrangian (2) can be approximated by the Lagrangian that appears in (1), that is, the GTR Lagrangian, except that we are still considering the connection and the metric as independent. The first-order variation of (1) with respect to the connection results in Einstein’s equation, and the first-order variation with respect to the metric results in metricity, as is well-known from the Einstein-Palatini variation. Thus the resulting theory is very similar to the GTR.

4 Improved renormalization prospects of the Lagrangian.

In this section we study Lagrangian (2), where the Riemann tensor is a functional of the connection given by

\[
R^\rho_{\sigma\mu\nu} = \Gamma^\rho_{\nu\sigma,\mu} - \Gamma^\rho_{\mu\sigma,\nu} + \Gamma^\rho_{\mu\eta} \Gamma^\eta_{\nu\sigma} - \Gamma^\rho_{\nu\eta} \Gamma^\eta_{\mu\sigma}.
\]

We are interested in showing that the theory associated with (2) is renormalizable.

The two fields to quantize are the metric \( g_{\mu\nu} \) and the connection \( \Gamma^\rho_{\mu\nu} \), considered independent of each other. A fundamental point is to notice that Lagrangian (2) does not contain partials of the metric, and that therefore there is no canonical momentum field conjugate to the metric in this theory. Thus the metric is not a quantum field, just a classical background field. One natural way to proceed is to integrate it out of the effective action path integral; probably it should be possible to do it. Here we will follow for now another procedure.
The elements involved in the diagrams pertaining the renormalization process of a theory are basically vertices and propagators. In the theory we are studying the connections have propagators *but the metrics do not*, since they are not quantum excitations. There are no partials of the metric available in the action to allow the construction of a propagator. This means that metrics, in the Feynman diagrams of the theory, will have no lines associated with them. The vertices of the theory do involve both connections and metrics. However, the metrics in the vertices serve only a bookkeeping purpose, to keep track of the covariance or contravariance of an index, but no line is connected to them.

In the Feynman diagrams of a theory, some of the diagrams involve closed loops, and some of these closed loops are divergent. If the more complicated divergent diagrams can be built up out of primitive subdiagrams (where all the primitive subdiagrams are members of a finite set), and the resulting diagrams are then non-divergent (discounting the divergent primitive subdiagrams, for which there are counterterms available in the Lagrangian) we then call the theory renormalizable. We proceed to show that that is precisely the case for this theory of gravity.

Let us write below the Lagrangian of action (2) with the Riemann tensor expressed in terms of the connection:

\[
L_{\text{mod}} = \frac{1}{4} \left( \Gamma^\rho_{[\nu,\sigma,\mu]} + \Gamma^\rho_{[\mu\eta]} \Gamma^\eta_{\nu][\sigma] \right) g^{\mu\tau} g^{\nu\nu'} \left( \Gamma^\sigma_{[\nu,\rho,\tau]} + \Gamma^\sigma_{[\tau,\eta]} \Gamma^\eta_{\nu][\rho] \right) + 2 \kappa^{-2} \left( g^{\sigma\nu} \Gamma^\rho_{[\nu,\sigma,\rho]} + g^{\sigma\nu} \Gamma^\rho_{[\rho\eta]} \Gamma^\eta_{\sigma][\nu] \right) + L_M. \tag{8}
\]

Notice the similarity of the first term with a Yang-Mills Lagrangian. The Yang-Mills field is a vector field \( A_{\mu a} \), where \( a \) takes value in the adjoint representation of a Lie algebra, while \( \Gamma^\rho_{\mu\sigma} \) is an affine connection invariant under diffeomorphisms.

Notice, too, that the second term of (8) contains only one- and two-point coupling vertices. Remember here that the only quantum field of this theory is the connection, and that this field appears linearly and quadratically in the second term of (8). Let us ignore this second term, and deal with it at the end of this section. If we keep in our theory of gravity only the first term in (8), plus the matter and radiation term \( L_M \), then the couplings that determine the topology of a Feynman diagram are very similar to the ones of a Yang-Mills theory.

We can make transparent the similarity between both Lagrangians (gravity and Yang-Mills), by using a trick. It consists in writing the affine connection so that it looks like a Yang-Mills field. Consider the connection \( \Gamma^\rho_{\sigma\nu} \) as four matrices \( \Gamma_\sigma \), \( \sigma = 0, 1, 2, 3 \), each one with 16 components classified by two indices \( \rho \) and \( \nu \), the first contravariant and the second covariant:

\[
\Gamma^\rho_{\sigma\nu} = (\Gamma_\sigma)^\rho_{\nu}. \tag{9}
\]

Using this convention we can write, for instance,

\[
\Gamma^\rho_{\mu\eta} \Gamma^\eta_{\nu\sigma} \delta^\sigma_\rho = (\Gamma_\mu \Gamma_\nu)^\rho_\sigma \delta^\sigma_\rho = \text{Tr}(\Gamma_\mu \Gamma_\nu). \tag{10}
\]

Keeping these ideas in mind we can express the first term of (8) as follows:

\[
L_{1^\nu \text{ term}} = \frac{1}{4} \text{Tr} \left( (\Gamma_{[\nu,\mu]} + [\Gamma_{\mu}, \Gamma_{\nu}]) g^{\mu\tau} g^{\nu\nu'} (\Gamma_{[\nu,\rho,\tau]} + [\Gamma_{\tau}, \Gamma_{\nu}]) \right), \tag{9}
\]
in complete analogy with the Yang-Mills case.

The kinetic energy of the connection is given by some of the terms of (8):

\[ L_{KE} = \frac{1}{4} \Gamma_{\rho\nu\sigma} g^{\mu\nu} \Gamma_{\rho\nu\sigma} g_{\mu\tau} \Gamma_{\sigma\nu} \Gamma_{\tau\rho}. \]

There is another very interesting and useful similarity between both types of theories. In Yang-Mills theories frequently the following term is added to the Lagrangian:

\[ L_{YM \text{ fix}} = -\frac{1}{2} \xi (\partial_\mu A_{a\mu})^2. \]

It is evident that this term is not invariant under a Yang-Mills gauge transformation and thus serves to partially fix the gauge and make the propagator matrix non-singular so it can be inverted. In the theory of gravity discussed here a similar term can be introduced for the same purpose. It is

\[ L_{\text{grav fix}} = -\frac{1}{2} \xi \text{Tr} \left( (\partial^\mu \Gamma_\mu)^2 \right) = -\frac{1}{2} \xi (g^{\lambda\mu} \partial_\lambda \Gamma_\mu)(g^{\tau\nu} \partial_\tau \Gamma_\mu). \] (10)

The remarkable point is that this term is not invariant under diffeomorphisms since \( \partial_\lambda \Gamma_\mu \) is not a tensor; that is why it serves to fix the gauge.

We can use (9) and (10) to write the action of the kinetic energy of the connection:

\[
\int (L_{KE} + L_{\text{grav fix}}) \ d^4 x = \frac{1}{2} \int \Gamma_{\mu\nu\sigma} \left[ g^{\mu\nu} \partial^2 - \left( 1 - \frac{1}{\xi} \right) \partial^\mu \partial^\nu \right] \Gamma_{\nu\sigma} \ d^4 x \\
= -\frac{1}{2} \int \Gamma_{\mu\nu\sigma}(x)D(x, y)^{\sigma\mu}_{\rho} |^{\nu\tau} \Gamma_{\nu\sigma}(y) \ d^4 x d^4 y,
\]

where the matrix \( D(x, y)^{\sigma\mu}_{\rho} |^{\nu\tau} \) is defined by

\[
D(x, y)^{\sigma\mu}_{\rho} |^{\nu\tau} \equiv \delta^\tau_\sigma \delta^\nu_\rho \left( g^{\mu\nu} \partial^2 - \left( 1 - \frac{1}{\xi} \right) \partial^\mu \partial^\nu \right) \delta(x - y) = \frac{1}{(2\pi)^4} \int \left[ \delta^\tau_\sigma \delta^\nu_\rho \left( g^{\mu\nu}(p^2 - i\epsilon) - \left( 1 - \frac{1}{\xi} \right) p^\mu p^\nu \right) \right] e^{ip \cdot (x-y)} \ d^4 p.
\]

The propagator is the reciprocal of the matrix in the square brackets in the equation above. This reciprocal actually exists since the matrix in the square brackets is not singular due to the addition of the diffeomorphism-breaking term (10).

Let us compare this result with the one for a Yang-Mills theory. The terms in the Yang-Mills action for the propagator are

\[
\frac{1}{2} \int A_{\nu a} \left[ g^{\mu\nu} \partial^2 - \left( 1 - \frac{1}{\xi} \right) \partial^\mu \partial^\nu \right] A_{\nu a} \ d^4 x = \frac{1}{2} \int A_{\mu a}(x) D(x, y)^{\mu}_{\nu a} |^{\nu\tau} A_{\tau b}(y) \ d^4 x d^4 y,
\] (11)

\[
= -\frac{1}{2} \int A_{\mu a}(x) D(x, y)^{\mu}_{\nu a} |^{\nu\tau} A_{\tau b}(y) \ d^4 x d^4 y,
\] (12)
where the indices $a$ and $b$ take values in the adjoint representation of the Lie group of the Yang-Mills theory. The matrix $D$ is given by

$$D(x, y)^{\mu a}_{\nu b} \equiv \delta_{ab} \left( g^{\mu \nu} \partial^2 - \left(1 - \frac{1}{\xi}\right) \partial^\mu \partial^\nu \right) \delta^4(x - y)$$

$$= \frac{1}{(2\pi)^4} \int \left[ \delta_{ab} \left( g^{\mu \nu} (p^2 - i\epsilon) - \left(1 - \frac{1}{\xi}\right) p^\mu p^\nu \right) \right] e^{ip \cdot (x - y)} d^4p.$$ 

Thus the difference between the connection propagator and the Yang-Mills field propagator is just in the coefficients $\delta^\sigma_\tau \delta^\varsigma_\rho$ and $\delta_{ab}$. The type of diagrams one can construct in either theory are the same, and the only differences between them are the coupling constants and the statistical factor (whose value is due to the coefficients $\delta^\sigma_\tau \delta^\varsigma_\rho$ and $\delta_{ab}$). That is, both the topological structure as well as the algebraic structure of the propagators of each diagram in each theory are the same: diagrams of each theory only differ in their coefficients. For each diagram in the gravity theory there is a similar diagram in a Yang-Mills theory, with the same topological structure and degree of divergence. In particular, the counterterm diagrams in the Yang-Mills theories have counterparts in the gravity theory. We conclude that the gravity theory we are discussing should contain a counterterm for every divergent term (using only a finite set of primitive diagrams) since Yang-Mills theories do. They should be renormalizable.

We have to go back and explain why we ignored the second term of (8):

$$\frac{2}{\kappa^2} \left( g^{\sigma \nu} \Gamma^\rho_{[\sigma \rho]} + g^{\sigma \nu} \Gamma^\rho_{[\rho \eta]} \Gamma^\eta_{\sigma \nu} \right).$$

This term is made up of two possible vertices. Quantity $2\kappa^{-2} g^{\sigma \nu} \Gamma^\rho_{[\sigma \rho]}$ is a tadpole diagram. Let us assume that the theory of gravity that we have constructed with the first term of (8) is renormalizable. Now, using this one-point vertex it would be possible to take a connection line (that is, a connection propagator) and cut it, so it becomes two tadpoles. There is an old (fairly evident) theorem in renormalization theory that says that, given a renormalized diagram, the diagram obtained by impeding the flow of momentum through one of the lines is also renormalized. So new diagram we have obtained using the tadpoles is renormalized if the first one was. Quantity $2\kappa^{-2} g^{\sigma \nu} \Gamma^\rho_{[\rho \eta]} \Gamma^\eta_{\sigma \nu}$ is a two-point vertex and it gives a correction on a line, in a way similar to a mass correction. If one takes a renormalizable diagram and applies this vertex to one of the lines, it cuts this line and makes into two, so that the connection propagator is squared, but no extra loop integration is added. Since the diagrams of this theory only diverge logarithmically and linearly, adding an extra propagator that goes as $q^{-2}$, where $q$ is the transferred momentum, will reduce the divergence by two powers, and make the resulting integration non-divergent. So the new diagram does not diverge.

5 Comments.

We introduced a version of gravity characterized by a term quadratic in the Riemann tensor in addition to the usual Hilbert-Einstein term, and an affine connection that is taken not to
be Levi-Civita, but a field independent of the metric. See (2). It is then shown that for low densities of matter and energy the model is equivalent to the GTR.

We then studied the renormalization of this theory. It is pointed out that the resulting action does not contain partials of the metric, a fact that implies that the metric is a classical field and that there are no propagator lines due to the metric in the Feynman diagrams of the theory. Thus the only quantum field in this theory is the connection $\Gamma^\rho_{\mu\sigma}$; the metric $g_{\mu\nu}$ is just a background classical field. It is also pointed out that the vertices of more than two points of this theory appear only on the term quadratic in the Riemann tensor of the Lagrangian, and not in the linear one. (Remember the connection is the only quantum field present.)

We conclude that this theory of gravity and Yang-Mills theories have the same topological and algebraic structure for their diagrams, and that their respective diagrams differ only on numerical factors due to the different constants at the couplings (their respective Kronecker deltas and coupling constants). Since Yang-Mills theories are renormalizable, this theory of gravity theory should be renormalizable.

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