See-Sawless Neutrino Masses

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(September 11, 2018)

Abstract

An alternative to the famous see-saw mechanism is proposed to explain the smallness of the neutrino masses (if present). This model involves a fourth family which mixes very little with the other three. It contains one heavy neutrino \( m_N > m_Z/2 \) and three very light neutrinos whose masses are radiatively induced. In contrast with the see-saw mechanism, all neutrino masses are Dirac masses. In one particular scenario, the three light neutrinos are almost degenerate in mass and are found to be consistent with fits to the Solar and Atmospheric neutrino deficits. They might even account for the Hot Dark Matter.
The possible presence of small neutrino masses and of neutrino oscillations is believed to be a plausible explanation for a set of experimental “discrepancies” and “evidences”: the solar neutrino problem, the atmospheric neutrino problem, and the Liquid Scintillation Neutrino Detector (LSND) data \[1\]. These experimental results, naturally, will have to be confirmed in the future.

In this paper, we propose an alternative way of looking at the neutrino mass problem without resorting to the famous see-saw mechanism \[2\]. The most important difference with the see-saw mechanism is the fact that, in our model, the neutrinos have only Dirac masses.

We now list our three main assumptions.

I) There is a non-sequential fourth family. By non-sequential, we mean that this fourth family is isolated from the first three families by tiny mixing angles. One way to realize this picture is to assume an almost unbroken 3 + 1 structure under a “light” horizontal family symmetry to isolate the fourth family. By “light” horizontal family symmetry, we mean a symmetry among the first three families.

Recently, it was found \[3\] that such a fourth generation with a quark mass \(\sim 150 \text{ GeV}\) helps bring about a unification of the SM gauge couplings at a scale \(\sim 3.5 \times 10^{15} \text{ GeV}\), corresponding to a partial proton lifetime \(\sim 3.3 \times 10^{34±2} \text{ years}\), in a non-supersymmetric \(SU(5)\) model. In Ref. \[4\], a search was proposed for long-lived quarks which can arise in such a model.

II) There is one and only one right-handed neutrino, \(N_R\), which is a singlet under that horizontal symmetry. (This is in contrast with the usual picture where there is one right-handed neutrino for each species.)

III) There is an exact global L (lepton number) symmetry and that \(N_R\) carries the same lepton number as all the other leptons. (In the context of simple grand unification, it would be the B-L global symmetry of \(SU(5)\) with minimal Higgs.) This symmetry would forbid a Majorana mass term of the form \(N_R N_R\) for \(N_R\). If one is not concerned about any grand unification, then global lepton number conservation alone is sufficient to forbid such a term.

The Yukawa couplings which respect the “light” horizontal symmetry (The SM Higgs field
φ is assumed to be a singlet under that symmetry), are of the form: 1) \( G_l(\bar{l}_L \phi e_R, i + h.c.) \); 2) \( G_E(\bar{L}_L \phi E + h.c.) \); 3) \( G_N(\bar{L}_L \tilde{\phi} N_R + h.c.) \), where \( \tilde{\phi} = i\sigma_2 \phi^* \), \( i = 1, 2, 3 \) (the “light” indices), and \( L_L = (N_L, E_L) \). When \( < \phi >= (0, v/\sqrt{2}) \), the \( 4 \times 4 \) charged lepton mass matrix is diagonal and the only neutral lepton that gets a (Dirac) mass is the fourth family \( N \). At this level, there is no mixing between \( E \) and the light leptons. It is thus natural, at tree level, to have a massive Dirac fourth neutrino and three massless neutrinos. Assumption (III) (L or B-L symmetry) forbids a Majorana mass term \( N_R N_R \).

To proceed further, one needs to embed the (light) horizontal symmetry into a larger one. For this purpose, let us assume the “light” family symmetry to be described by the group \( SO(3) \) with the first three families transforming as a 3-dimensional vector representation and with the fourth family being a singlet. Let us now assume that there is a Grand Family gauge symmetry group and it is \( SO(4) \). We shall choose the following basis for the \( SO(4) \) generators: \( M_{ij} \) and \( M_{4i} \), with \( i, j = 1, 2, 3 \), which are the generators of the \( SO(3) \) subgroup and \( SO(4)/SO(3) \) factor group respectively. The gauge bosons of \( SO(3) \) couple to \( M_{ij} \) while those of \( SO(4)/SO(3) \) couple to \( M_{4i} \). When \( SO(4) \) breaks down to \( SO(3) \), it is this \( SO(3) \) that we identify with the “light” family symmetry. Under \( SO(4) \), the left and right-handed leptons would transform as \( \psi^\alpha_L = (l^i_L, L_L) \) and \( e^\alpha_R = (e^i_R, E_R) \), where the superscripts \( \alpha = 1, .., 4 \) and \( i = 1, 2, 3 \) denote the \( SO(4) \) and “light” lepton family indices respectively. Since, in our model, there is only one right-handed neutral lepton, \( N_R \), it would automatically be a singlet under the Grand Family Symmetry \( SO(4) \).

Under \( SO(4) \), the only invariant Yukawa coupling that can be written is \( G_L \bar{\psi}^\alpha_L \phi e_{R, \alpha} \). This alone would be unsatisfactory from a phenomenological viewpoint since it would give equal masses, \( m_0_E \), to all four charged leptons. Furthermore, from LEP2, one has \( m_E > m_W \) and since the fourth family is assumed to mix very little with the other three, one cannot use some kind of democratic mass matrix (with unity everywhere) to make \( E \) much heavier than the lighter three. An extra term is needed to give an additional mass to the fourth charged lepton.

Another important question concerns the nature of the Yukawa term \( G_N(\bar{L}_L \tilde{\phi} N_R + h.c.) \).
which gives a Dirac mass to the “heavy” 4th neutrino \( N \). It is, however, not \( SO(4) \) invariant (although it is invariant under the “light” family symmetry \( SO(3) \)). To be consistent, this Yukawa coupling should be derived from an \( SO(4) \)-invariant term.

To address the above issues, let us introduce the Higgs fields needed to break \( SO(4) \). For instance, to break \( SO(4) \) completely, one might use four Higgs fields belonging each to a vector representation. It is beyond the scope of this paper to discuss the details of such a breaking and we shall assume that it can be done. Let us call one of such 4-dimensional Higgs fields \( \Omega \) where \( \Omega = (\Sigma^i, \Theta) \), with \( i = 1, 2, 3 \). Let us assume that \( \Omega \) develops the following vacuum expectation value (VEV): \( <\Omega> = (0, <\Theta>) \), with \( <\Theta> = M \) being typically the scale of \( SO(4) \) breaking. In fact, if \( \Omega \) were the only Higgs field present for \( SO(4) \), its VEV would spontaneously break \( SO(4) \) down to the “light” family symmetry \( SO(3) \).

We now propose the following minimal set of extra “superheavy” fermions- singlets under \( SO(4) \)- whose attractive feature is to generate tree-level masses for \( N \) and \( E \). These are the fermions which can couple to \( \Omega \). They are (under \( SU(2)_L \otimes U(1)_Y \)): 1) \( F_{L,R} = (2, -1/2) \); 2) \( M_{L,R} = (1, -1) \). These extra fermions are vector-like under the SM. As a result, they can have the following gauge-invariant mass terms: \( \mathcal{M}_F \bar{F}_L F_R + h.c. \) and \( \mathcal{M}_M \bar{M}_L M_R + h.c. \), where \( \mathcal{M}_F \) and \( \mathcal{M}_M \) are assumed to be of the order of the \( SO(4) \) breaking scale. We propose the following Yukawa interactions which respect \( SO(4) \otimes SU(2)_L \otimes U(1)_Y \):

\[
\mathcal{L}_Y = G_1 \bar{\psi}^\alpha_L \Omega \alpha F_R + G_2 \bar{F}_L \tilde{\phi} N_R + G_3 \bar{F}_L \phi M_R +
\]

\[
G_4 \bar{M}_L \Omega \alpha e^\alpha_R,
\]

where \( \alpha = 1, \ldots, 4 \) is the \( SO(4) \) index. We shall endow \( \psi^\alpha_L \), \( e^\alpha_R \) and \( \Omega \) with a discrete symmetry so that \( \psi^\alpha_L \) and \( e^\alpha_R \) couple only to \( \Omega \) and not to other 4-dimensional \( SO(4) \) Higgs fields. (This discrete symmetry could be, for instance, the simultaneous change of sign of these three fields.)

Integrating out the heavy fields \( F \) and \( M \) below the \( SO(4) \) breaking scale and with \( <\Omega> = (0, <\Theta>) \), it is straightforward to derive the following effective Yukawa terms:
\[ G_N \tilde{L}_L \phi R \] with \( G_N = G_1 G_2 < \Theta > / M_F \), and \( G_E \tilde{L}_L \phi E_R \) with \( G_E = G_1 G_3 G_4 < \Theta >^2 / M_F M_M \). From these terms we obtain the following masses: \( m_{0N} = G_1 G_2 \frac{< \Theta >^2}{\sqrt{2}} M_F \), and \( \tilde{m}_E = G_1 G_3 G_4 \frac{< \Theta >^2}{\sqrt{2}} M_F \). The total mass of the fourth charged lepton, \( m_E \), would be the sum of \( \tilde{m}_E \) and \( m^0_E \) (the mass which is common to all four charged leptons). Phenomenologically, one could have \( m^0_E \ll \tilde{m}_E \) which would provide the desired hierarchy.

There are two steps that one could do to compute the “light” neutrino masses. These steps are depicted in Figs.1 and 2 which show the \( E - e^i \) mixing and the effective Yukawa term \( \bar{\nu}_L \phi^0 N_R \) respectively. We shall assume that \( SO(3) \) breaking will give rise to a non-diagonal mass matrix for the “light” charged lepton sector. (Its detailed form and mechanism is not essential to the arguments presented below.)

In Fig.1, we have introduced another quartet of Higgs field which we denote by \( \tilde{\Omega} = (\tilde{\Sigma}^i, \tilde{\Theta}) \). (This is one of several Higgs field needed to spontaneously break \( SO(4) \).) First, this particular quartet is prevented by the previous discrete symmetry from coupling to the fermions. Secondly, in contrast with \( \Omega \), we require that \( < \tilde{\Omega} > = ( < \tilde{\Sigma}^i >, < \tilde{\Theta} > ) \) so that one obtains an effective Yukawa coupling of the form: \( C_i \bar{e}_L \phi^0 E_R \), where \( C_i \) is a constant containing the loop inegration and various couplings. (The contribution of \( \Omega \) to \( C_i \) is zero because \( < \Omega > = (0, < \Theta >) \).) This can be easily seen because such a Yukawa mixing between \( e^i_L \) and \( E_R \) can only arise when \( SO(3) \) is itself broken. This kind of vacuum expectation value can be arranged in a general potential. It is beyond the scope of the paper to present it here. We shall assume that it can be done.

The gauge bosons in Fig.1 are the massive \( SO(4)/SO(3) \) gauge bosons. Without loss of generality, we shall assume that their masses, \( M_4 \), are of the order of \( < \tilde{\Theta} > \). With this in mind, the coefficient \( C_i \) can be computed to be: \( C_i = \tilde{\lambda} (\frac{g_F^2}{16\pi}) (\frac{\sqrt{2m_{e_i}}}{v}) \frac{< \tilde{\Sigma}^i >}{M_4} \ln \left( \frac{M_4^2 + m^2_{\tilde{\Theta}}}{m^2_{\phi^0}} \right) \), where \( g_F \), \( m_{\tilde{\phi}} \), and \( M_4 \) are the \( SO(4) \) gauge coupling, the mass of \( \tilde{\Theta} \), and the mass of the \( SO(4)/SO(3) \) gauge bosons respectively. The factor \( \tilde{\lambda} \) comes from the cross coupling \( \tilde{\lambda} \phi^i \phi \tilde{\theta}^i \tilde{\Omega} \). Also, in \( C_i, m_{e_i} \) is the mass eigenvalue of the charged lepton \( e_i \). We shall comment
below on the possible ranges for the various parameters in \( C_i \).

In the following discussion, for simplicity, we shall assume that \(< \tilde{\Sigma}^i > = M_3\), independent of \( i \). \( M_3 \) will be related to the scale of \( SO(3) \) breaking.

Using \( C_i \tilde{e}_L^i \phi^0 E_R \), one can now calculate (in the Feynman-'t Hooft gauge) the diagrams shown in Fig.2a,b. The various factors that enter the vertices of the diagrams need some explanations. For \( \tilde{\nu}_L \phi^- N_R \), one has \( \sqrt{2} m_{0N} / v \). \( C_i \tilde{e}_L^i \phi^0 E_R \) gives the \( \tilde{e}_L^i E_R \) mixing. The \( \tilde{\nu}_i L \phi^+ e_j R \) vertex contains a term \( V_{ij} \sqrt{2} m_{e_j} / v \), where \( V_{ij} \)'s are actually elements of the matrix that diagonalizes the charged lepton sector. The reason is that there is only \( \text{one} \) right-handed neutral lepton, \( N_R \), and the neutral lepton mass matrix is necessarily diagonal. We show below what the effective right-handed component for \( \text{each} \) neutrino will be.

Diagram (2a) is proportional to \( m^2_{e_i} m^2_E \) while Diagram (2b) is proportional to \( m^3_{e_i} m_E \) with similar coefficients in front. Therefore Diagram (2a) is larger than (2b) by a factor \( m_E / m_{e_i} \), which is much larger than unity even for \( m_{e_i} = m_\tau \). Because of this we list below the dominant contribution to Fig.2, namely Fig.2a. We obtain:

\[
\frac{m_{0i}}{m_{0N}} = K \sum_{j=1}^{3} V_{ij} \frac{m^2_{e_j}}{v^2},
\]

where

\[
K = \tilde{\lambda} \frac{g^2 F}{16 \pi^2} M_3 \frac{1}{M_4} \frac{m^2_E}{4 \pi^2} \ln \left( \frac{M^2_W + m^2_{\tilde{\phi}}}{m^2_{\tilde{\phi}}} \right) \left\{ \ln \left( \frac{M^2_W + m^2_{\tilde{\phi}}}{m^2_{\tilde{\phi}}} \right) \right\} + \frac{m^2_E}{M^2_W + m^2_{\tilde{\phi}}}. \]

We first show how \( m_{0i} \) and \( m_{0N} \) are related to the Dirac masses of the neutrinos. We then discuss their relative magnitudes.

Let us write the part of the Lagrangian containing the kinetic and mass terms for the four neutrinos. For simplicity, we shall omit the gauge part in the kinetic term. One has:

\[
\mathcal{L}^{(\nu)} = \sum_{j=1}^{3} i \tilde{\nu}_j L \phi \nu_j L + i \tilde{N}_L \phi N_L + i \tilde{N}_R \phi N_R - m_{0N}(\tilde{N}_L N_R + \text{h.c.}) - \sum_{j=1}^{3} m_{0i}(\tilde{\nu}_j L N_R + \text{h.c.}).
\]

Let us introduce the following 4-component Dirac spinors: \( \tilde{\nu}_i = (\nu_{iL}, \alpha N_R) \); \( \tilde{N} = (N_L, \alpha N_R) \). By writing \( \mathcal{L}^{(\nu)} \) in terms of the above Dirac spinors and comparing it with the previous expressions, it is easy to see that
one has to have $\alpha = 1/2$. It means that the Dirac masses, $m_{\nu_i}$ and $m_N$, coming from terms like $m_{\nu_i}\bar{\nu}_i\nu_i$ and $m_N\bar{N}N$, are now given by: $m_{\nu_i} = 2m_{0\nu_i}; m_N = 2m_{0N}$. The ratio, however, remains the same, i.e. $m_{\nu_i}/m_N = m_{0\nu_i}/m_{0N}$.

To know the relative magnitude of $m_{\nu_i}$ compared with $m_N$, one needs to have an estimate for various parameters which appear in $K$. To be more specific, let us take a definite example. For instance, let us assume that $m_E \sim 2m_W$, $M_3 \sim M_4$, $g_F \sim g_{\text{weak}}$. A crude estimate then gives $K \sim 1.5 \times 10^{-3}\tilde{\lambda}$. Now let us recall that $\tilde{\lambda}$ comes from $\tilde{\lambda}\phi^\dagger\phi\tilde{\Omega}^\dagger\tilde{\Omega}$. In order to prevent the SM Higgs field from acquiring a large mass, it is easy to see that the constraint on $\tilde{\lambda}$ is approximately $\tilde{\lambda} < v^2/M_4^2$ where we have set the SM quartic coupling $\lambda \sim O(1)$. In consequence, if $v \ll M_4$, $\tilde{\lambda}$ can be very small. (This is the familiar statement of gauge hierarchy encountered in the construction of Grand Unified theories.) For example, if the Grand Family breaking is in the TeV region so that, say, $v^2/M_4^2 \sim 10^{-4}$, one then has $\tilde{\lambda} < 10^{-4}$ implying $K < 1.5 \times 10^{-7}$. Since $|V_{ij}| \lesssim 1$ and the heaviest charged lepton mass is $m_\tau = 1.784$ GeV, the heaviest “light” neutrino can have a mass of roughly 1.3 eV for $m_N \sim 160$ GeV. It could, of course, be heavier or lighter depending on, e.g. the ratio $v^2/M_4^2$, among other things.

Let us now turn to a more complete discussion of all neutrino masses. From $C_i$, one can see that, for the range of parameters mentioned above, the mixing between $E$ and the “light” charged leptons is very small, i.e. $\sim 10^{-5}$. In consequence, the $3 \times 3$ mixing submatrix, among the “light” charged leptons, whose elements are $V_{ij}$ will be approximately unitary. Without loss of generality, we shall use the standard CKM parametrization for such a matrix, neglecting any possible CP violation effect. The weak charged current can be written, in terms of mass eigenstates, as: $J_\mu = 2\sum_{i=1}^{3} \bar{l}_i V_{ij} \nu_{Lj}$. Here $l_{Li} = (e, \mu, \tau)$ and $\nu_{Lj}$ are the mass eigenstates. Also $V_{ij} = U^\dagger_i U_j$, where $U_i$ and $U_j$ are the matrices which diagonalize the charged and neutral lepton sectors respectively. As we have stated earlier, the neutral lepton mass matrix is diagonal because there is only one right-handed neutrino. As a result, $U_\nu = 1$ and $V_{ij} = U^\dagger_i$. The neutrino masses computed below are directly related to the matrix that diagonalizes the charged lepton sector. The “light” neutrino mass eigenvalues can now be
written in terms of the \( \text{\textquoteleft light\textquoteright} \) charged lepton mass eigenvalues as:

\[
m_{\nu_1} = \frac{K m_N}{v^2} \left\{ (c_\omega c_\phi) m_e^2 + \left( -s_\omega c_\psi + c_\omega s_\psi s_\phi \right) m_\mu^2 + (s_\omega s_\psi - c_\omega c_\psi s_\phi) m_\tau^2 \right\}
\]

(4)

\[
m_{\nu_2} = \frac{K m_N}{v^2} \left\{ (s_\omega c_\phi) m_e^2 + \left( c_\omega c_\psi - s_\omega s_\psi s_\phi \right) m_\mu^2 + (-c_\omega s_\psi - s_\omega c_\psi s_\phi) m_\tau^2 \right\}
\]

(5)

\[
m_{\nu_3} = \frac{K m_N}{v^2} \left\{ s_\phi m_e^2 + s_\psi c_\phi m_\mu^2 + c_\psi c_\phi m_\tau^2 \right\}
\]

(6)

where \( c \equiv \cos \) and \( s \equiv \sin \). The angles \( \omega, \phi, \psi \) are used in Ref. [6]. The hierarchy, if any, among the \( m_{\nu_i} \)'s depends on several factors: the relative magnitudes of the charged lepton masses and the magnitude of the coefficients appearing in front of these masses. In the discussion of neutrino oscillation, the angles that appear there come from the equation that expresses the flavour eigenstates in terms of the mass eigenstates, namely \( \nu^0_L = \sum_{j=1}^3 V_{ij} \nu^0_L \), where \( \nu^0_L = (\nu_e, \nu_\mu, \nu_\tau) \). Therefore, the neutrino masses are intrinsically linked to the oscillation angles. Next, one observes that: \( m_\mu^2/m_\tau^2 \sim 3.5072 \times 10^{-3} \) and \( m_e^2/m_\tau^2 \sim 10^{-7} \), \( m_e^2/m_\mu^2 \sim 2.3 \times 10^{-5} \). Unless the coefficients appearing in front of \( m_\mu^2 \) and \( m_\tau^2 \) are extremely small- a rather unusual scenario- it is safe to say that the main contributions to the masses come from \( m_\mu^2 \) and \( m_\tau^2 \) in Eqs. (4,5,6). We thus neglect the \( m_e^2 \) terms from hereon.

The mass differences relevant to neutrino oscillation are customarily written as: \( \Delta m^2_{ij} = |m^2_{\nu_i} - m^2_{\nu_j}| \). From Eqs. (4,5,6), one can compute the following relevant ratios: \( \frac{\Delta m^2_{e\mu}}{m^2_{\nu_3}} \) and \( \frac{\Delta m^2_{e\tau}}{m^2_{\nu_3}} \). These ratios depend only on the three angles and on the known masses, \( m_\mu \) and \( m_\tau \). The actual values of \( \Delta m^2_{ij} \) will also depend on \( m_{\nu_3} \) which is a free parameter since it depends on various factors such as, for example, the breaking scale of the Grand Family symmetry, among others. Its magnitude could have interesting implications concerning the physics of the family symmetry.

It is beyond the scope of this paper to do a detailed analysis of various possible scenarios using Eqs. (4,5,6). We choose to illustrate the predictive power of these equations by taking the following example. We ask the question: Given \( m_{\nu_3} \) (input) and the three angles \( \phi, \omega \) and \( \psi \), would one obtain consistent values for \( \Delta m^2_{ij} \) and the oscillation angles? What would that
imply as far as the three neutrino masses are concerned? After a quick scan of various angles, we choose: \( \phi = 2.9^0 \), \( \omega = 47.04325^0 \), \( \psi = 55.01^0 \), and \( m_{\nu_3}^2 = 2.5 \text{ eV}^2 \). Plugging these values into the ratios \( \frac{\Delta m_{12}^2}{m_{\nu_3}^2} \) and \( \frac{\Delta m_{23}^2}{m_{\nu_3}^2} \), we obtain: \( \Delta m_{12}^2 = 8.65 \times 10^{-6} \text{ eV}^2 \); \( \Delta m_{23}^2 = 0.0233 \text{ eV}^2 \); \( \sin^2 2\theta_{e\mu} = 4s_\phi^2c_\phi^2s_\psi^2 = 6.85 \times 10^{-3} \); and \( \sin^2 2\theta_{\mu\tau} = 4s_\psi^2c_\psi^4c_\phi^4 = \sin^2 2\psi c_\phi^4 = 0.878 \).

These values are consistent with the small-angle MSW solution to the Solar neutrino deficit, and with the sub-GeV and multi-GeV data for the solution to the Atmospheric neutrino deficit [1]. Furthermore, from the relationship between the neutrino masses, we deduce the values for \( m_{\nu_1} \) and \( m_{\nu_2} \), given the above values of the angles and of \( m_{\nu_3} = 1.5811388 \text{ eV} \).

We get: \( m_{\nu_1} = 1.5884876 \text{ eV} \) and \( m_{\nu_2} = 1.5884904 \text{ eV} \). Interestingly, one has here a case of almost degenerate neutrinos: \( m_{\nu_1} \approx m_{\nu_2} \approx m_{\nu_3} \). Notice that \( m_{\nu_1} + m_{\nu_2} + m_{\nu_3} \approx 4.78 \text{ eV} \), which seems to be a preferred value for the hot dark matter in a mixed hot-cold scenario [1,8]. The neutrino masses in our scenario are of a Dirac nature and, as a consequence, they should not give rise to neutrinoless double beta decay. Their masses can be \( \sim 1.6 \text{ eV} \) and are not subject to the upper limit (for a Majorana mass) of \( \sim 0.56 \text{ eV} \) from the search for neutrinoless double beta decay by the Heidelberg-Moscow \( ^{76}\text{Ge} \) experiment [7].

Note, in passing, that, for the values of \( \Delta m_{ij}^2 \) presented above, even if \( m_{\nu_i} \sim 1.6 \text{ eV} \), there appears to be no problem with the so-called r-process in supernova nucleosynthesis of heavy elements.

One last (but not least) remark: with \( m_{\nu_3} \sim 1.6 \text{ eV} \) and assuming, e.g. \( m_N \sim 160 \text{ GeV} \), we obtain \( M_4 \sim M_3 \sim 18 \text{ TeV} \), where the expression for \( K \) has been used. The Family scales \( M_{3,4} \) can vary, depending on a number of factors contained in \( K \). If \( m_{\nu_3} \) were to be much smaller, these scales will be correspondingly much larger than the previous rough estimate. Nevertheless it is interesting to see, in this scenario, the deep connection between neutrino masses and Family scales.

I would like to thank Qaisar Shafi for an insightful comment. This work is supported in parts by the US Department of Energy under grant No. DE-A505-89ER40518.
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FIGURES

FIG. 1. Diagram showing the computation of the effective Yukawa term: $C_i \bar{e}_L^i \phi^0 E_R$.

FIG. 2. Diagrams showing the computation of the effective Yukawa term: $\bar{\nu}_L^i \phi^0 N_R$, in the Feynman-'t Hooft gauge. The large dot between $e_L^i$ and $E_R$ represents the mixing computed from the diagram shown in Fig. 1.
