Modelling and Simulation of Pseudo-Noise Sequence-Based Underwater Acoustic OSDM Communication System

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Featured Application: Underwater acoustic communication and underwater networking applications, especially adaptive underwater acoustic communication applications.

Abstract: Orthogonal signal division multiplex (OSDM) is an emerging signal modulation technology which has a lower peak-to-average power ratio (PAPR) and a flexible subcarrier system architecture. Particularly, it can be seen as a bridge between the single-carrier modulation and the orthogonal frequency division multiplexing (OFDM) modulation in the frequency domain. Aiming at the development trend and demand of underwater acoustic hybrid and adaptive modulation communication technology, a pseudo-noise (PN) sequence-based underwater acoustic OSDM communication system is proposed in this paper. A data frame structure with PN sequence is designed to solve the multipath and Doppler effect of underwater acoustic channel. On the basis of the PN sequence, a compressive sensing method based on the orthogonal matching pursuit (OMP) algorithm and the minimum mean square error (MMSE) algorithm is designed for channel estimation and equalization. On the basis of the system construction, the relationship among the OSDM vector length M, the OSDM subcarrier number N, and the underwater acoustic channel length is further studied for adaptive modulation of underwater acoustic communication. Finally, the proposed system is verified by simulation. The OSDM system has lower and controllable PAPR. When the OSDM vector length M is bigger than the channel length, and the system subcarrier flexibility is guaranteed, the bit error rate (BER) of the OSDM system is lower than that of the OFDM system and the single-carrier system. The PN sequence-based compressive sensing channel estimation and equalization with the OMP and MMSE algorithms has a good performance to resist the multipath effect of underwater acoustic channel.

Keywords: underwater acoustic communication; orthogonal signal division multiplex; vector length; compressive sensing; orthogonal matching pursuit; multi-carrier modulation; MMSE
1. Introduction

Underwater acoustic (UWA) communication enables a wireless link in an underwater environment by the transmission of acoustic waves. Recently, UWA communication has become an essential necessity for underwater operations in marine science and ocean engineering. However, due to the disadvantages of severe transmission loss, time-varying multipath propagation, severe Doppler spread, limited and distance-dependent bandwidth, and high propagation delay, UWA channel is considered as one of the most challenging communication media in use [1]. In the past development process, underwater acoustic communication is mainly divided into two categories that include the single-carrier underwater acoustic (SC-UWA) communication technology and the multi-carrier underwater acoustic (MC-UWA) communication technology such as the orthogonal frequency division multiplexing (OFDM). Due to the high peak-to-average power ratio (PAPR), it is difficult to use MC-UWA communication technology for long-distance transmission. On the other hand, the SC-UWA signals are more suitable to keep a wide coverage for the lower PAPR. However, in high-speed underwater acoustic communication, multi-carrier (MC) modulation technology such as OFDM is superior to single-carrier (SC) modulation technology [2].

At present, considering the communication rate and distance coverage, the hybrid modulation of single- and multi-carrier technology has attracted the attention of researchers. In radio frequency wireless communication, a hybrid single- and multi-carrier system for next-generation mobile communication system was studied in [3], and both high data rate and wide coverage were obtained in the proposed hybrid system. The generalized frequency division multiplexing (GFDM) system which can achieve the unification of single- to multi-carrier systems is suitable for low-latency scenarios foreseen for 5G networks, especially for Tactile Internet [4]. In underwater acoustic communication, the single-carrier frequency-domain equalization (SC-FDE) is a typical hybrid modulation technology for high speed and wide coverage UWA communication, using the low PAPR of a single-carrier system and the relatively simple FDE technology of the OFDM system [5]. At the same time, adaptive multi-carrier modulation technology is also a research hotspot of high performance underwater acoustic communication [6].

Orthogonal signal division multiplex (OSDM), also known as vector OFDM, is an emerging signal modulation technology which can be seen as a bridge between the single-carrier modulation and the OFDM modulation in the frequency domain [7]. In 2001, a precoded OFDM technology, vector-OFDM (VOFDM), was proposed by Xiang-Gen Xia, which could make a traditional OFDM have good adaptability to spectral nulls [8]. Almost at the same time, in 2002, N. Suehiro first used the Kronecker product with rows of discrete Fourier transform (DFT) matrix as a signal modulation method for data transmission [9,10]. In 2007, the Kronecker product-based signal modulation method was called an orthogonal signal division multiplex (OSDM) [11]. In 2008, a novel constellation-rotated V-OFDM with improved diversity was proposed [12]. In 2009, in order to eliminate inter-carrier interference (ICI), a vector-OFDM system of non-contiguous carriers was proposed and the performance of the non-contiguous vector-OFDM (NC-vector-OFDM) was better than that of NC-OFDM [13]. In 2012, Li and Xia analyzed vector-OFDM with linear receivers [14]. In 2014, the OSDM method was introduced into the field of UWA communication by Professor T. Ebihara from Tsukuba University [15]. In 2016, an OSDM-based data transmission mode with an anti-Doppler shift of UWA channel was designed, which provided a reliable and flexible data transmission to overcome the time-frequency dual-spread channel [16]. Han et al. used space-time block code (STBC) and space-frequency block code (SFBC) to increase coding and diversity gain for the OSDM system [17,18]. In 2018, in order to improve the system performance, iterative per-vector equalization for OSDM was proposed [7]. In 2019, a low-complexity equalization of OSDM was proposed [19].

Compared with vector OFDM, the OSDM using the Kronecker product and rows of DFT matrix has a better mathematical form. When the OSDM system’s subcarrier number N increases to the maximum, that is to say, only one bit is transmitted on each subcarrier, the system becomes an OFDM system. When the OSDM system’s subcarrier number N decreases to 1, the system becomes a
single-carrier communication system. In general, the emerging OSDM technology is an interesting modulation scheme which can realize SC-FDE modulation, OFDM modulation and different vector length OSDM modulation by changing the vector length M.

The OSDM modulation technology has potential application prospects in underwater acoustic hybrid single- and multi-carrier modulations and adaptive modulation. For this reason, a pseudo-noise (PN) sequence-based underwater acoustic OSDM communication system (UWAOSDM/PN) is proposed in this paper. A data frame structure with PN sequence is designed to solve the multipath and Doppler effect of underwater acoustic channel. On the basis of the PN sequence, a compressive sensing method based on the orthogonal matching pursuit (OMP) algorithm and the minimum mean square error (MMSE) algorithm is designed for channel estimation and equalization. On the basis of the system construction, the relationship among the OSDM vector length M, the OSDM subcarrier number N, and the underwater acoustic channel length is further studied for adaptive modulation of underwater acoustic communication.

The paper is organized as follows. In Section 2, the system design of UWAOSDM/PN is described. In Section 3, the key modules are introduced. In Section 4, the simulation results are introduced and discussed. In Section 5, the system parameters and performance are compared and in Section 6 the conclusions are summarized.

2. System Design

The system structure of the proposed UWAOSDM/PN is shown in Figure 1.

Transmitter: First of all, the transmitted data are encoded by convolutional code. Then, QPSK mapping is used to modulate the coded data. After that, the data is modulated by OSDM method. Then, PN sequences are added to get data frame. Finally, the baseband OSDM signal is modulated into the frequency band by carrier modulation.

Receiver: When the maximum correlation value between the received signal and the local PN sequence reaches the threshold value, the effective system synchronization is obtained. Then, the Doppler estimate and compensation is done. After that, the baseband signal is obtained by carrier demodulation. Then, the channel estimation and equalization are done to remove the multipath effect. After that, OSDM demodulation is used to get the transmitted data. Finally, through the QPSK demodulation and convolutional code decoding, the received data is obtained.

![Figure 1. System structure of pseudo-noise sequence-based underwater acoustic orthogonal signal division multiplex (OSDM) communication system (UWAOSDM/PN).](image)

3. Key Modules

3.1. OSDM Theory

First, the data X, in the $1 \times MN$ dimension vector, is grouped into N groups of $1 \times M$ dimension vector as follows:
\[ X_0 = (x_{00}, x_{01}, \ldots, x_{0(M-1)}) \\
X_1 = (x_{10}, x_{11}, \ldots, x_{1(M-1)}) \\
\vdots \\
X_{N-1} = (x_{(N-1)0}, x_{(N-1)1}, \ldots, x_{(N-1)(M-1)}) \tag{1} \]

As we all know, inverse discrete Fourier transform (IDFT) formula is:

\[ x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)e^{j2\pi nk/N} = \frac{1}{N} \sum_{k=0}^{N-1} X(k)W_N^{-nk} \tag{2} \]

IDFT matrix is:

\[ F_N^{-1} = \begin{bmatrix} f_0 \\
f_1 \\
\vdots \\
f_k \\
\vdots \\
f_{N-1} \end{bmatrix} \tag{3} \]

where,

\[ f_k = \frac{1}{\sqrt{N}} (W_N^0, W_N^k, W_N^{2k}, \ldots, W_N^{(N-1)k}) \tag{4} \]

Using the Kronecker product between formulae (1) and (4), we can get:

\[ S_k = f_k \otimes X_k \tag{5} \]

Formula (5) can be expanded to

\[
\begin{align*}
W_N^0(x_{00}, x_{01}, \ldots, x_{0(M-1)}), & W_N^0(x_{00}, x_{01}, \ldots, x_{0(M-1)}), \ldots, W_N^0(x_{00}, x_{01}, \ldots, x_{0(M-1)}) \\
W_N^1(x_{10}, x_{11}, \ldots, x_{1(M-1)}), & W_N^1(x_{10}, x_{11}, \ldots, x_{1(M-1)}), \ldots, W_N^1(x_{10}, x_{11}, \ldots, x_{1(M-1)}) \\
\vdots & \\
W_N^k(x_{(N-1)0}, x_{(N-1)1}, \ldots, x_{(N-1)(M-1)}), & W_N^k(x_{(N-1)0}, x_{(N-1)1}, \ldots, x_{(N-1)(M-1)}), \ldots, \\
W_N^k(x_{(N-1)0}, x_{(N-1)1}, \ldots, x_{(N-1)(M-1)}), & \ldots,
\end{align*}
\tag{6} \]

The orthogonality between any two \( S_k \) can be demonstrated [11]. By summing \( S_k \), we can get the modulated signal \( s \) which is the baseband OSDM signal. This is a good mathematical expression for the principle of OSDM modulation.

Signal modulated by Formulae (1-6) also can be modulated by grouping and inverse fast Fourier transform (IFFT). As shown in Figure 2, first, all data are divided into \( M \) groups in a matrix denoted as \( d \). Then, \( M \) times of \( N \)-point IFFT are calculated. The obtained data are stored in a data table by column and the data are taken out by row, where, \( x \) is the original data and \( s \) is IFFT results of \( x \). In this way, \( MN \) data are modulated on \( N \) subcarriers, each subcarrier carrying \( M \) data.
We define permutation matrix:

$$P_{N,M} = \begin{bmatrix}
I_N \otimes e_M^H(0) \\
I_N \otimes e_M^H(1) \\
... \\
I_N \otimes e_M^H(M-1)
\end{bmatrix}$$

(7)

To do OSDM modulation, the above modulation process can be expressed as:

$$s = P_{N,M}^H(I_M \otimes F_N^H)P_{N,M}d$$

(8)

where, matrix $P_{N,M}^H$, $I_M \otimes F_N^H$ and $P_{N,M}$ is row-wise write operations, N-point IDFT and row-wise read operations, respectively, and $d$ is the left matrix in Figure 2.

As shown in Figure 3, the data of the same group of IFFT are dispersed on the time scale. It is obvious that the OSDM modulation separates the original IFFT-transformed symbols into the time domain by (M-1) bits information in the same vector. In this way, not only the advantage of orthogonal carrier in the frequency domain is utilized, but also the time-scale diversity in the time domain is used. It can resist the influence of multipath of a certain delay on the data in the same IFFT group. Thus, OSDM can be considered as a hybrid modulation method in the time-frequency domain.

At the receiver, in the OSDM demodulation, only the inverse process of the modulation process needs to be performed, and then the modulated signal can be demodulated. The demodulation can be expressed as:

$$x = P_{N,M}^H(I_M \otimes F_N)P_{N,M}r$$

(9)

where, $r$ means the data after channel equalization.

3.2. Frame Structure with PN Sequence

Due to the particularity of underwater acoustic channel, the data frame structure needs to be designed carefully in order to resist the adverse effect of UWA channel multipath and Doppler effect. The design of data frames must have the functions of synchronization, channel estimation, and Doppler
estimation. To solve the above problems, the frame structure with three pseudo-noise (PN) sequences is designed.

PN sequences are set at the head, middle, and tail of the data frame, respectively. Eight OSDM symbols with cyclic prefix are inserted among three PN sequences to form the data frame. The data frame format is shown in Figure 4.

The PN sequence consists of two identical short PN sequences to overcome multipath. PN sequences have three roles in this system. The header PN sequence can be used for signal detection and synchronization. At the same time, the header PN sequence and the middle PN sequence can be regarded as a block pilot, respectively, which is used as the pilot to do channel estimation for the subsequent OSDM symbols. Last but not least, the three PN sequences are used to do a Doppler estimation by the relative position of the three correlation peaks of sliding cross-correlation operation of the received signal.

The PN sequence not only has good randomness and cross-correlation which make it easy to be separated from other signals, but also the deterministic and repeatable nature of the PN sequence makes it have good anti-interference performance. More significantly, for the pseudo randomness, when the Toeplitz method is used to construct an observation matrix, the PN sequence can be used as initial vectors to create observation matrix $\phi$. In this way, the matrix $\phi$ satisfies the important RIP metric.

| PN | OSDM#1 | OSDM#2 | OSDM#3 | OSDM#4 | PN | OSDM#5 | OSDM#6 | OSDM#7 | OSDM#8 | PN |
|----|--------|--------|--------|--------|----|--------|--------|--------|--------|----|

Figure 4. Data frame format of the UWAOSDM/PN.

Under this frame structure design, the system data rate can be calculated as:

$$Rate = \frac{r_c(N \cdot D) \log_2 R}{T_s + T_{cp} + T_{PN}}$$

where, $r_c$ represents the coding rate. $R$ means the coding rate. $N$ is the number of symbol and $D$ is the amount of each OSDM symbol, $T_s$, $T_{cp}$ and $T_{PN}$ indicates the symbol duration, the cyclic prefix duration, and PN sequence duration, respectively.

3.3. Doppler Estimation and Compensation

Due to the water flows and the relative motion between the transmitters and receivers, the system needs to correct the Doppler effect on the received signal. The replicated PN sequences are placed at the beginning, middle, and end of the data frame, respectively. When the signal is affected by Doppler, the receiver signal length will be changed. Through the correlation calculation of the PN sequence at the receiver, the relative positions of the correlation peaks are detected to estimate the Doppler [20].

From the data frame structure, three different maximum values can be obtained in the search process, and the signal length influenced by Doppler can be estimated by the position of the first and the last maximum value. Since the PN sequence is affected by Doppler, the good autocorrelation property of the PN sequence is severely damaged. Therefore, the Doppler estimation should not use the cross-correlation calculation of the local PN sequence. On the contrary, using sliding autocorrelation between the received signal and the first PN sequence that intercepts from the received signal, three distinct correlation peaks can be obtained. Both the length of the signal at the receiver $\bar{L}$ and the length of the signal at the transmitter $L$ can be calculated by the relative position of correlation peaks, respectively.

Assume that the Doppler factor is $\alpha$, by comparing $L$ with $\bar{L}$, the receiver infers how the received signal has been compressed or extended by the UWA channel.

$$\bar{L} = (1 + \alpha)L$$
The receiver uses the estimated Doppler factor $\alpha$ to resample the received signal, so that the compression or extension caused by the Doppler can be compensated.

### 3.4. Channel Estimation and Compensation

In recent years, many studies have shown that UWA channels have sparse characteristics in many situations, that is to say, most channel taps are zero or very small and the number of channel taps is far less than the maximum channel delay. Due to the sparse characteristics, using the compressive sensing (CS) method to do channel estimation is very suitable for an underwater acoustic channel. On the one hand, the minimum mean square error (MMSE) method which has superior equalization performance should be adopted. The MMSE takes noise effects into account. As compared with the zero forcing (ZF) method, the MMSE can improve system performance. Also, the MMSE method can get higher diversity gain than the ZF method.

In the UWAOSDM/PN system, channel estimation is conducted in the time domain and equalization is calculated in the frequency domain. We assume that the UWA channel is sufficiently stable to allow prediction. After synchronizing the received signal using Doppler estimation and compensation, channel estimation is conducted according to the above principle. Figure 5 shows the channel estimation and equalization process. First, the PN sequence is used as the initial vector and the Toeplitz format is employed to construct the observation matrix. Because of the pseudo randomness of the PN sequence, the constructed matrix meets the RIP principle \[21\]. Then, the orthogonal matching pursuit (OMP) method is used as the recovery method to estimate the sparse channel in the time domain, and then, transforming the time domain $h$ to the frequency domain by FFT. Last, calculating the channel equalization coefficients $C_n$ according to the MMSE criterion.

![Figure 5. Channel estimation and equalization process.](image)

#### 3.4.1. Orthogonal Matching Pursuit Channel Estimation

The UWA channel is a time domain sparse channel whose length is $L$ and sparsity is $K$, so we can use the OMP algorithm of compressive sensing to do channel estimation.

After adding the cyclic prefix, the linear convolution will be changed to circular convolution, and the circular convolution can be expressed as:

\[
r = \begin{bmatrix}
  r(0) \\
  r(1) \\
  r(2) \\
  \vdots \\
  r(N-1)
\end{bmatrix} = \begin{bmatrix}
  P(0) & P(N-1) & P(N-2) & \cdots & P(N-L) \\
  P(1) & P(0) & P(N-1) & \cdots & P(N-L+1) \\
  P(2) & P(2) & P(0) & \cdots & P(N-L+2) \\
  \vdots & \vdots & \vdots & \cdots & \vdots \\
  P(N-1) & P(N-2) & P(N-1) & \cdots & P(N-L-1)
\end{bmatrix} \begin{bmatrix}
  h(0) \\
  h(1) \\
  h(2) \\
  \vdots \\
  h(L-1)
\end{bmatrix} + \begin{bmatrix}
  w(0) \\
  w(1) \\
  w(2) \\
  \vdots \\
  w(N-1)
\end{bmatrix}
\]  

\[ \text{(12)} \]
We ignore the noise $w$ and Equation (12) can be written as

$$r = P \cdot h$$  \hspace{1cm} (13)

where, $P$ means the pilot matrix, $h$ represents the sparse acoustic channel, and $w$ is Gaussian noise.

Equation (13) conforms to the general form of the OMP algorithm. The $h$ is a sparse vector and the observation matrix satisfies the RIP criteria [21]. The orthogonal matching pursuit algorithm is shown in Algorithm 1.

**Algorithm 1 Orthogonal matching pursuit**

1: Initial residual $r_k = y$, Observation matrix $\Phi = P$, Label set $\Lambda_0 = O$, Sparsity $K$
2: for each $t=1:K$ do
3:   Find the number of columns of the strongest correlation column with the residual $r$ in the observation matrix $\phi_j$:
4:      $\lambda_t = \arg \max_{j=1,2,\ldots,N} |\langle r - \Phi \theta, \phi_j \rangle|$
5:      Update the index set $\Lambda_t = \Lambda_{t-1} \cup \{ \lambda_t \}$, reconstruct collection $\Phi_t = [\Phi_t, \phi_{\lambda_t}]$
6:      Compute $\hat{\theta}_t = \text{argmin}_{\theta} \| y - \Phi_t \theta \|_2$ by least squares method.
7:      Update residuals $r_t = y - \Phi_t \hat{\theta}_t$
8: end for
9: return the channel estimation result in $\hat{\theta}$

3.4.2. Minimum Mean Square Error Channel Equalization

The minimum mean square error function can be expressed as:

$$x_n^{\text{MMSE}} = \arg\min_{x_n} \| y_n^{\text{MMSE}} - \Phi_n x_n \|^2$$  \hspace{1cm} (14)

Derivation of Equation (14) and let the partial derivative be 0 and the equalization coefficient is shown as:

$$C_n = H_n^* (\delta^2 I + H_n^2)^{-1}$$  \hspace{1cm} (15)

where, $H_n$ is estimated by OMP that have been convert into frequency domain, $\delta$ means the channel noise.

4. Simulation and Result Analysis

4.1. Simulation Conditions

In this section, simulation results are provided to illustrate the performance of the proposed UWAOSDM/PN system over UWA channels. In the simulation, the duration $T_s$ of each OSDM symbol is 213 ms, the data bits $D$ of each OSDM symbol is a constant 1024, that is to say, the product of $M$ and $N$ is a fixed value $D$.

Table 1 shows the simulation parameters of the UWAOSDM/PN system.

| Parameters                  | Values     | Parameters        | Values     |
|-----------------------------|------------|-------------------|------------|
| Band frequency              | 21–27 kHz  | Channel coding    | Convolutional |
| Sampling rate               | 96 kHz     | Bandwidth         | 6 kHz      |
| Relative velocity           | 5 m/s      | Mapping           | QPSK       |
| Code rate                   | 1/2        | Up-sample coefficient | 16         |
| $T_{cp}$                    | 42.7 ms    | $T_{PN}$          | 213 ms     |
As shown in Equation (16), the time-domain model of the UWA channel is composed of the attenuating component and the time-delay component. Assuming that the channel response is wide-sense stationary within one data frame.

\[ h(\tau) = \sum_p A_p \delta(\tau - \tau_p) \]  
\[ (16) \]

where,\( p \) represents the number of multipaths, \( A_p \) denotes the attenuation coefficient of each path, \( \delta \) means impulse response function and \( \tau_p \) stands the delay time of the \( p \)th path.

Figure 6 shows two UWA channel models adopted in this paper. Both models have been normalized. As shown in the figure, the horizontal axis is delay time, the vertical axis is normalized amplitude, and Figure 6a shows the channel model 1 of BELLHOP [22] which contains five paths. The first path is direct path. Its maximum delay spread is about 6.9 ms. Figure 6b shows the channel model 2 which contains seven paths [23]. The first path is a direct path. Its maximum delay spread is about 12.5 ms.

**Figure 6.** Time domain impulse response of the underwater acoustic channel: (a) the channel model 1 (b) the channel model 2.

### 4.2. System Parameters Analysis of UWAOSDM/PN

Figure 7 shows the performance comparison of the UWAOSDM/PN system with different vector lengths under different UWA channels. According to the channel model adopted in this paper, the maximum delay time of the channel model 1 is 6.9 ms and that of the model 2 is 12.5 ms. The system sampling rate is 96 kHz and the up-sample coefficient is 16. With these parameters, we can calculate the vector duration of different length vectors, such that the vector duration is 5.34 ms when the vector length is 32, the vector duration is 10.67 ms when the vector length is 64, and the vector duration is 21.33 ms when the length is 128. The simulation results in Figure 7a,b show that the system bit error rate (BER) of OSDM is lower than that of OFDM when the vector length \( M \) of OSDM system is larger than the maximum channel delay spread. With the increase of the vector length, the system can obtain more diversity gain. Simultaneously, the simulation results in Figure 7a,b show that the performance of a single-carrier system is also lower than that of the OSDM system. Therefore, the increase of vector length is limited in a scope. The vector length can be expressed as:

\[ M = 2^\left\lceil \log_2 \left( \frac{1}{S} \right) \right\rceil \]  
\[ (17) \]

where, \( D \) represents channel maximum delay spread, \( S \) is system sampling frequency, \( \lambda \) is up-sample coefficient and \( \lceil \rceil \) means ceil the number.

According to Equation (17), the vector length of the OSDM system should be larger than the maximum channel delay and keep as many subcarriers as possible to ensure the system carrier flexibility.
4.3. System Performance Validation

4.3.1. System Peak-to-Average Power Ratio

The peak-to-average power ratio is defined as:

$$PAPR = 10 \log_{10} \left( 10 \cdot E \left( \frac{\max(s)^2}{E(s)^2} \right) \right) \quad (18)$$

Figure 8 shows the complementary cumulative distribution function (CCDF) curves of the PAPR of the UWAOSDM/PN system with different vector lengths. As shown in the figure, the peak-to-average power ratio (PAPR) decreases with the increase of system vector length.

In addition, the PAPR reflects the advantage of the OSDM from another perspective. High PAPR has low signal energy utilization and short propagation range, while low PAPR has high signal energy utilization and long propagation range. Therefore, the relationship between the vector length and propagation range can be shown in Figure 9. The appropriate vector length can be selected according to propagation range, that is to say, OSDM is a PAPR tunable modulation method.
4.3.2. System Power of ICI

The UWA multi-carrier system is more susceptible to Doppler frequency offset. The OSDM system signal is similar to a traditional OFDM [24]. Therefore, the power of ICI caused by the Doppler spread can be bounded by:

$$P_{\text{ICI}} \leq \frac{\alpha_1}{12} (2\pi f_d T_s)^2$$

(19)

where, $T_s = \frac{1}{\Delta f}$, $\alpha_1 \leq 1$ [25], $f_d = \frac{v}{c}$.  

The formula (19) can be expressed as:

$$P_{\text{ICI}} \leq \frac{1}{12} (2\pi \frac{v f_c}{B_c} N)^2$$

(20)

The relationship between $P_{\text{ICI}}$ and relative velocity of transmitter and receiver is shown in Figure 10, low $P_{\text{ICI}}$ is achieved by reducing the subcarrier number $N$. However, the spectrum flexibility decreases with the subcarrier number decreasing.

![Figure 9. The relationship between the vector length and propagation range.](image)

4.3.3. System Channel Estimation and Equalization

Figure 11a shows the system performance with different equalization methods and different vector lengths. The diversity gain obtained by the ZF method is smaller than the MMSE method. With the increase of SNR, the diversity gain of the ZF method will decrease. On the contrary, the MMSE method can achieve good diversity gains with different vector lengths.

Figure 11b shows the system performance with the MMSE equalization method and different channel estimation methods. Compared with least squares (LS) and DFT-based least squares (LS-DFT), the system using the OMP channel estimation method can obtain the lowest BER. It is worth noting that
OMP and LS-DFT have similar BER performance. In the OMP method, the signal recovery and signal denoising is combined. LS-DFT needs to know the noise threshold to achieve better performance.

![Figure 11](image)

**Figure 11.** The system performance comparison: (a) comparison of zero forcing (ZF) and minimum mean square error (MMSE) equalization methods with different vector lengths; (b) comparison of least squares (LS), discrete Fourier transform-based least squares (LS-DFT), and orthogonal matching pursuit (OMP) estimation methods with MMSE equalization method and different vector lengths.

### 4.3.4. System Complexity

For the SC-FDE, OFDM, and OSDM systems, system complexity is mainly determined by IFFT and FFT points. The modulation complexity of SC-FDE, OFDM, and OSDM are compared in Table 2. At the transmitter, D-point IFFT which complexity is \( O(D\log D) \) is used to modulate the OFDM signal, and M times N-point IFFT which complexity is \( O(MN\log N) \) is used for the OSDM modulation. Given that the product of M and N is a fixed value D, the OSDM and OFDM modulation complexity is merely influenced by logarithmic term. With the increase of vector length M (the decrease of subcarriers N), the modulation complexity of the OSDM system decreases with fewer subcarriers. Generally speaking, the transmitter complexity is sorted from low to high as

\[
SC\text{-}FDE < OSDM < OFDM
\]

### Table 2. The system complexity of single-carrier frequency-domain equalization (SC-FDE), orthogonal frequency division multiplexing (OFDM) and OSDM.

| Modulation Scheme | Transmitter Complexity | Receiver Complexity |
|-------------------|-------------------------|---------------------|
| SC-FDE            | -                       | \( O(2D\log D + DLK) \) |
| OSDM              | \( O(MN\log N) \)       | \( O(2D\log D + MN\log N + DLK) \) |
| OFDM              | \( O(D\log D) \)        | \( O(D\log D + DLK) \) |

At the receiver, block equalization is considered in this paper. OFDM equalization and demodulation can be implemented at the same time, which complexity is \( O(D\log D + DLK) \). But both OSDM and SC-FDE are needed to be transformed by IFFT and FFT for equalization which complexity is \( O(2D\log D + DLK) \). Besides that, \( M \times N \)-point FFT which complexity is \( O(MN\log N) \) is needed for OSDM demodulation. The receiver complexity is sorted from low to high as

\[
OFDM < SC\text{-}FDE < OSDM
\]
5. Comparison with Related Systems

Table 3 shows a performance comparison of the proposed UWAOSDM/PN system with the other UWA OSDM systems. The comparison is based on simulation results. As shown in the comparison, the UWAOSDM/PN has a relatively high communication rate, a lower system BER and, a low receiver complexity. The effectiveness of the proposed PN sequence-based compressive sensing channel estimation and equalization with the OMP and MMSE algorithm is demonstrated.

Table 3. Comparison of the UWAOSDM/PN and the other underwater acoustic (UWA) OSDM systems.

| Source          | Coding     | Code Rate | Pilot Sequence | Digital Modulation | Equalization Method | Receiver Complexity | Bandwidth | BER (10^-5) | Rate (kbps) |
|-----------------|------------|-----------|----------------|-------------------|---------------------|---------------------|-----------|-------------|-------------|
| UWAOSDM/PN      | Convolutional | 1/2       | PN             | QPSK              | OMP-MMSE            | Low                 | 6 kHz     | 10^-5       | 5.09        |
| [7]             | Convolutional | 1/2       | LFM            | QPSK              | SID/PID-MMSE        | High                | 4 kHz     | 10^-5       | 2.965       |
| [16]            | RS         | 4/5       | QPSK           | 16QAM             | -                   | -                   | 5 kHz     | 10^-4       | 2.3         |
| [17]            | Turbo      | 1/3       | LFM            | QPSK              | -                   | -                   | -         | 10^-4       | -           |
| [18]            | SFBC       | -         | Chu            | QPSK              | CE-BEM              | Low                 | -         | 10^-4       | -           |

The main contributions of this paper are as follows: (1) Provide a solution of combining PN sequence, OMP, and MMSE for underwater acoustic OSDM communication. Compared with the other UWA OSDM systems, the proposed UWAOSDM/PN has a relatively high communication rate, a lower system BER, and a low receiver complexity. (2) PN sequences are specially selected and designed for the system. PN sequences have three roles in this system. The header PN sequence can be used for signal detection and synchronization. The header PN sequence and the middle PN sequence can be regarded as a block pilot, respectively, which is used as the pilot to do channel estimation for the subsequent OSDM symbols. In particular, PN sequence is suitable for the OMP algorithm. The Toeplitz method is used to construct an observation matrix, and the PN sequence can be used as initial vectors to create observation matrix $\phi$ for the pseudo randomness. In this way, the matrix $\phi$ satisfies the important RIP metric. The three PN sequences are used for the Doppler estimation by the relative position of the three correlation peaks of sliding cross-correlation operation of the received signal. (3) For the potential application prospects in the underwater acoustic hybrid single- and multi-carrier modulations and adaptive modulation, the relationship among the OSDM vector length $M$, the OSDM subcarrier number $N$, and the underwater acoustic channel length is further studied.

6. Conclusions

In this paper, an UWA communication system based on OSDM technology is proposed and designed, and a special data frame format based on PN sequence is designed. The OSDM signals with different vector lengths are analyzed. For the problems of long delay, strong multipath, and large-scale Doppler frequency shift in underwater channels, OMP-based compressive sensing channel estimation, MMSE equalization, sliding correlation Doppler estimation and resampling compensation methods are adopted. As shown in the simulation results, compared with a traditional OFDM system, the proposed UWAOSDM/PN system has low PAPR and ICI. When the OSDM vector length $M$ is bigger than the channel length, the system subcarrier flexibility is guaranteed, and the bit error rate of the OSDM system is lower than that of the OFDM system and the single-carrier system. The PN sequence-based compressive sensing channel estimation and equalization using the OMP and MMSE algorithms has a good performance to resist the multipath effect of underwater acoustic channel.
The OSDM modulation technology has potential application prospects in underwater acoustic hybrid single- and multi-carrier modulations and adaptive modulation. In the future, on the basis of the UWA OSDM system studied in this paper, an adaptive multimode UWA communication technology study based on the OSDM modulation will be carried out.

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