Probing hadron structure and strong interactions with inclusive semileptonic decays of $B$ mesons

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Abstract

The study of inclusive semileptonic decays of $B$ mesons is analyzed from the viewpoint of probing hadron structure and strong interactions. General formulas for the differential decay rates are given in terms of the structure functions in arbitrary frame of reference, taking into account the finite charged lepton mass. These formulas can be used for structure function measurements. The behavior of the structure functions is shown to contain information on the constitution of the $B$ meson and the dynamics of strong interactions. Measurements of the structure functions would reveal the existence of a point-like $b$ quark in the $B$ meson, establish the spin 1/2 nature of the $b$ quark, and identify the $b$ quantum number of the $B$ meson. These measurements would determine the $b$ quark distribution function, and allow new insight into the nature of confinement. The $b$ quark distribution function can also be extracted directly through the measurements of the decay distributions with respect to the scaling variable $\xi_+$. 
I. INTRODUCTION

Coloured quarks and gluons as physical constituents of hadrons and quantum chromodynamics (QCD) as the theory of strong interactions have been generally accepted as the modern point of view with regard to the structure of hadrons and strong interactions. Asymptotic freedom of QCD explains Bjorken scaling \[1\] observed in deep inelastic electron-proton scattering \[2\] before the advent of QCD and allows perturbative calculations of strong interactions at high energy or short distance. On the other hand, the increase of the strong coupling constant of QCD at low energy or long distance provides a reason to accept quarks and gluons as physical constituents of hadrons without directly detecting free quarks and gluons. There are a variety of experimental results, which not only provide confirmations of the quark and gluon structure of hadrons, but also lead to incisive precision tests of QCD. While the evidence is compelling that QCD is a realistic theory of strong interactions, there is still a very incomplete understanding of the nonperturbative, confining, long-distance sector of QCD, which is a basic challenge in describing strong interactions, in particular how quarks and gluons bind together to form hadrons.

The $B$ meson is heavy and relatively long-lived \[3\]. This makes it advantageous to probe hadronic physics. One powerful way of experimentally investigating the structure of the $B$ meson and strong interactions is to study its inclusive semileptonic decay. The structure of the $B$ meson and the strong interaction can be studied with this underlying weak decay process because the decay rate is modified by the strong interaction, which is responsible for quark confinement and gluon radiation. The power and beauty of inclusive semileptonic decay is that the electroweak field generated during the semileptonic decay is well understood and leptons do not interact strongly. Furthermore, because of inclusive characteristics of the decay we would expect that the decay rate is mainly affected by the initial bound state effect and is insensitive to hadronization in the final states, which are summed over and possess the completeness. These permit us to probe the structure of the $B$ meson and strong interactions by means of a known charged weak current.

While much attention has been focussed on the determination of $|V_{ub}|$ and $|V_{cb}|$ by inclusive semileptonic decays of $B$ mesons, a wealth of information on the hadron structure and strong interactions available from these processes deserves intense scrutiny in experiment and theory as well. The purpose of this paper is to discuss the study of hadron structure and strong interactions by means of inclusive semileptonic $B$ decays. We would like to show that this study will provide evidence for the quark structure of the $B$ meson and allow new insight into the nature of confinement. Of course, this study will also be beneficial to precise determination of the fundamental standard model parameters $|V_{ub}|$ and $|V_{cb}|$.

The outline of this paper is as follows. We derive in Sec. II within the framework of the standard electroweak model general formulas for the differential decay rates for inclusive semileptonic $B$ meson decays in terms of the structure functions. The derivation is made in arbitrary frame of reference and the finite charged lepton mass is taken into account. These formulas are useful for measurements of the structure functions. The behavior of the structure functions is explored in Sec. III, where we also elucidate by measurements of the structure functions what can be learned on the constitution of the $B$ meson. The extraction of the $b$ quark distribution function, to which the structure functions are related due to the light-cone dominance, is discussed in Sec. IV. In Sec. V we conclude.
II. STRUCTURE FUNCTIONS

We want to compute the decay rate for the inclusive semileptonic decay of the $B$ meson to lowest order in the weak interaction in arbitrary frame of reference. In this inclusive process $B \rightarrow X\ell\bar{\nu}$, the $B$ meson of mass $M$ with four-momentum $P$ decays into any possible hadronic final state $X$, over which we sum, via the emission of a virtual $W$ boson which materializes as a charged lepton $\ell$ of mass $M_\ell$ with four-momentum $k_\ell$ and a massless antineutrino $\bar{\nu}$ with four-momentum $k_\nu$. By standard steps we find the unpolarized decay rate to be proportional to the leptonic tensor $L^{\mu\nu}$ times the hadronic tensor $W_{\mu\nu}$:

$$d\Gamma = \frac{G_F^2 |V_{qb}|^2}{(2\pi)^5 E} L^{\mu\nu} W_{\mu\nu} \frac{d^3 k_\ell d^3 k_\nu}{2E_\ell 2E_\nu},$$  \hspace{1cm} (2.1)$$

where $E$, $E_\ell$, and $E_\nu$ denote the energies of the $B$ meson, the charged lepton, and the antineutrino, respectively. $V_{qb}$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element, which is $V_{cb}$ for the $b \rightarrow c$ transition induced decay and $V_{ub}$ for the $b \rightarrow u$ transition induced decay. The leptonic tensor for the lepton pair is completely determined by the standard electroweak theory since leptons do not have strong interactions:

$$L^{\mu\nu} = 2(k_\ell^{\mu} k_\nu^{\nu} + k_\nu^{\mu} k_\ell^{\nu} - g^{\mu\nu} k_\ell \cdot k_\nu + i\varepsilon^{\mu\nu\alpha\beta} k_\ell^{\alpha} k_\nu^{\beta}).$$  \hspace{1cm} (2.2)$$

The hadronic tensor incorporates all nonperturbative QCD physics for the inclusive semileptonic $B$ decay. It is summed over all hadronic final states and can be expressed in terms of a current commutator taken between the $B$ meson states:

$$W_{\mu\nu} = -\frac{1}{2\pi} \int d^4 y e^{iq\cdot y} \langle B [j_\mu(y), j_\nu^+ (0)] | B \rangle,$$  \hspace{1cm} (2.3)$$

where $q = k_\ell + k_\nu$ stands for the momentum transfer from the $B$ meson to the lepton pair and $j_\mu(y) = \bar{q}(y)\gamma_\mu(1 - \gamma_5)b(y)$ is the charged weak current. The $B$ meson state $|B\rangle$ satisfies the standard covariant normalization $\langle B | B \rangle = 2E(2\pi)^3\delta^3(0)$.

The most general hadronic tensor form that can be contracted is a linear combination of $P_\mu P_\nu$, $P_\mu q_\nu$, $q_\mu P_\nu$, $q_\mu q_\nu$, $\varepsilon_{\mu\nu\alpha\beta} P^\alpha q^\beta$, and $g_{\mu\nu}$, with coefficients being scalar functions $W_a(q^2, \nu^2)$ of the two independent Lorentz invariants, $\nu \equiv q \cdot P/M$ and $q^2$. However, the combination $P_\mu q_\nu - q_\mu P_\nu$ does not contribute since $L^{\mu\nu}(P_\mu q_\nu - q_\mu P_\nu) = 0$. Thus the hadronic tensor must take the form

$$W_{\mu\nu} = -g_{\mu\nu} W_1 + \frac{P_\mu P_\nu}{M^2} W_2 - i\varepsilon_{\mu\nu\alpha\beta} \frac{P^\alpha q^\beta}{M^2} W_3 + \frac{q_\mu q_\nu}{M^2} W_4 + \frac{P_\mu q_\nu + q_\mu P_\nu}{M^2} W_5.$$  \hspace{1cm} (2.4)$$

Equation (2.3) shows that $W_{\mu\nu}^* = W_{\nu\mu}$, so $W_a, a = 1, \ldots, 5$, are real. The interesting physics describing the hadron structure and strong interactions is wrapped up in the five dimensionless real structure functions $W_a(\nu, q^2), a = 1, \ldots, 5$, for the unpolarized processes.

The unpolarized differential decay rate can be expressed in terms of the structure functions:

$$\frac{d^3\Gamma}{d\eta d\nu dq^2} = \frac{G_F^2 |V_{qb}|^2}{32\pi^3 E} [W_1(2q^2 - M_\ell^2) + W_2(4\eta - 4\eta^2 - q^2 + M_\ell^2) + W_3(\nu q^2 - 2\eta q^2 - M_\ell^2\nu)/M + W_4 M_\ell^2 (q^2 - M_\ell^2)/M^2 + W_5 M_\ell^2 (\nu - \eta)/M],$$  \hspace{1cm} (2.5)$$

where $\eta = M_\ell^2/(2M_\ell \nu)$. The important graph for the calculation is shown in Fig. 1.
where we define the Lorentz invariant $\eta \equiv k_\ell \cdot P/M$. Equation (2.3) shows that the $B$ meson structure functions could be separated and extracted from the measurement of the differential decay rate at different values of $\eta$ for fixed $\nu$ and $q^2$.

It would be useful to express the differential decay rate in terms of the angle $\theta$ between the charged lepton and the $B$ meson in the virtual $W$ rest frame. The angle $\theta$ is related to the Lorentz invariants $\eta$, $\nu$, and $q^2$ by

$$2\eta = (1 + \frac{M^2}{q^2})\nu - (1 - \frac{M^2}{q^2})\sqrt{\nu^2 - q^2 \cos \theta}. \quad (2.6)$$

Using Eq. (2.6), the differential decay rate in terms of $\cos \theta$, $\nu$, and $q^2$ can be obtained from Eq. (2.5):

$$\frac{d^3\Gamma}{d\cos \theta d\nu dq^2} = \frac{G_F^2|V_{qb}|^2}{32\pi^3E} (1 - \frac{M^2_{\ell}}{q^2})\sqrt{\nu^2 - q^2} \left[ W_1 2(q^2 - M^2_{\ell}) + W_2 (\nu^2 - q^2 + M^2_{\ell} - \kappa^2) \right.
+ W_3 2(q^2 - M^2_{\ell})\sqrt{\nu^2 - q^2 \cos \theta}/M + W_4 M^2_{\ell}(q^2 - M^2_{\ell})/M^2
+ W_5 2M^2_{\ell}(\nu - \kappa)/M \right], \quad (2.7)$$

where

$$\kappa = \frac{M^2_{\ell}}{q^2}\nu - (1 - \frac{M^2_{\ell}}{q^2})\sqrt{\nu^2 - q^2 \cos \theta}. \quad (2.8)$$

From Eq. (2.7) we see that the structure functions could also be separated and extracted experimentally by looking at the angular $\theta$ distribution of the charged lepton at each value of $\nu$ and $q^2$.

Integrating Eq. (2.7) over $\cos \theta$ gives

$$\frac{d^2\Gamma}{d\nu dq^2} = \frac{G_F^2|V_{qb}|^2}{32\pi^3E} (1 - \frac{M^2_{\ell}}{q^2})\sqrt{\nu^2 - q^2} \left[ W_1 \frac{1}{2} 2q^2 (1 + \frac{M^2_{\ell}}{q^2} - \frac{2M^4_{\ell}}{q^4} - 2q^2 + \frac{M^4_{\ell}}{q^2} - M^2_{\ell}) \right.
+ W_3 M^2_{\ell}(q^2 - M^2_{\ell}) + W_5 \frac{2M^2_{\ell}\nu}{M}(1 - \frac{M^2_{\ell}}{q^2}) \right]. \quad (2.9)$$

This decay distribution would also be useful to measure the structure functions except for $W_3$, whose contribution is absent.

Equations (2.3), (2.7), and (2.9) show that the contributions of the structure functions $W_4$ and $W_5$ are suppressed as they are multiplied by the square of the charged lepton mass. Therefore, the inclusive semileptonic $B$ decay with a tau lepton in the final state would be appropriate for measuring $W_4$ and $W_5$.

In the inclusive semileptonic B decay where an electron (or muon) is the final state lepton, the mass effect of it may be negligible. Neglecting the charged lepton mass, then the differential decay rates in Eqs. (2.3), (2.7), and (2.9) reduce, respectively, to

$$\frac{d^3\Gamma}{d\eta d\nu dq^2} = \frac{G_F^2|V_{qb}|^2}{32\pi^3E} [W_1 2q^2 + W_2 (4\eta \nu - 4\eta^2 - q^2) + W_3 \frac{2q^2}{M} (\nu - 2\eta)], \quad (2.10)$$
\[ \frac{d^3 \Gamma}{d \cos \theta d \nu dq^2} = \frac{G_F^2 |V_{qb}|^2}{64 \pi^3 E} \sqrt{\nu^2 - q^2} \left[ W_1 2q^2 + W_2 (\nu^2 - q^2) \sin^2 \theta + W_3 \frac{2q^2}{M} \sqrt{\nu^2 - q^2 \cos \theta} \right], \]  

(2.11)

\[ \frac{d^2 \Gamma}{d \nu dq^2} = \frac{G_F^2 |V_{qb}|^2}{48 \pi^3 E} \sqrt{\nu^2 - q^2} \left[ W_1 3q^2 + W_2 (\nu^2 - q^2) \right]. \]  

(2.12)

Note that only the structure functions \( W_1 \) and \( W_2 \) contribute to the double differential decay rate \( d^2 \Gamma / (d \nu dq^2) \) with lepton masses being neglected.

The formulas for the differential decay rates derived in this section are quite general and can be used to measure the \( B \) meson structure functions in any particular frame. However, we should point out that, in addition to the lowest order contribution in the weak interaction, there are higher order radiative processes which contribute to the decay rate. In order to extract the structure functions from experimental data, the radiative corrections to the decay rate must be implemented. The electroweak radiative corrections were discussed in Ref. [4]. The QCD radiative corrections were calculated at the parton level of quarks and gluons in Refs. [5–8].

### III. BEHAVIOR OF THE STRUCTURE FUNCTIONS

The behavior of the structure functions manifests the structure of the \( B \) meson and the dynamics of strong interactions. In this section we explore theoretically how the \( B \) meson structure functions look like. We will also discuss the previous results in Ref. [9].

In order to see the way in which the structure of the \( B \) meson and strong interactions manifest themselves in the structure functions, it will be useful to start with an investigation of the behavior of the structure functions in the free quark limit when no strong interaction takes place. The unpolarized tree-level decay rate for the free point-like quark decay \( b \rightarrow q \ell \bar{\nu} \) is given in arbitrary frame by

\[ d \Gamma_{\text{tree}} = \frac{G_F^2 |V_{qb}|^2}{(2\pi)^3 p_b^0} L_{\mu \nu, \nu} w^{\mu \nu} \frac{d^3 k_\ell d^3 k_\nu}{2E_\ell 2E_\nu}, \]  

(3.1)

where \( p_b \) stands for the four-momentum of the \( b \) quark of mass \( m_b \). \( L_{\mu \nu} \) is the leptonic tensor given in Eq. (2.2). \( w^{\mu \nu} \) is the quark tensor

\[ w^{\mu \nu} = 4\delta[(p_b - q)^2 - m_q^2] \left\{ -g^{\mu \nu}(m_b^2 - q \cdot p_b) + 2p_\mu^b p_\nu^\ell + i\varepsilon^{\mu \nu \rho \sigma} p_\rho^b q_\sigma - (p_\mu^\ell q_\nu + q_\mu^\ell p_\nu) \right\}, \]  

(3.2)

where \( m_q \) is the mass of the decay produced \( q \)-quark.

In the free quark limit, the \( B \) meson and the \( b \) quark in it have the same velocity: \( p_b / m_b = P / M \). By use of the delta function relation

\[ \delta[(p_b - q)^2 - m_q^2] = \frac{1}{M(m_b - M \xi_+ - m_b)} \delta(\xi_+ - \frac{m_b}{M}), \]  

(3.3)

where
\[ \xi_{\pm} = \frac{\nu \pm \sqrt{\nu^2 - q^2 + m_d^2}}{M}, \tag{3.4} \]

Comparing Eqs. (2.1) and (2.4) with Eqs. (3.1) and (3.2) allows us to identify the structure functions in the free quark limit

\[ W_1^{\text{free}} = 2\delta(\xi_+ - \frac{m_b}{M}), \tag{3.5} \]
\[ W_2^{\text{free}} = \frac{8\xi_+}{\xi_+ - \xi_-}\delta(\xi_+ - \frac{m_b}{M}), \tag{3.6} \]
\[ W_3^{\text{free}} = -\frac{4}{\xi_+ - \xi_-}\delta(\xi_+ - \frac{m_b}{M}), \tag{3.7} \]
\[ W_4^{\text{free}} = 0, \tag{3.8} \]
\[ W_5^{\text{free}} = W_3^{\text{free}}. \tag{3.9} \]

The “free” functions \( W_1^{\text{free}}(\nu, q^2)/2, (\xi_+ - \xi_-)W_2^{\text{free}}(\nu, q^2)/8, (\xi_+ - \xi_-)W_3^{\text{free}}(\nu, q^2)/4 \) exhibit the intriguing property that they are functions of only one variable \( \xi_+ \) and not of \( \nu \) and \( q^2 \) independently. It is interesting to see if this scaling property persists under the strong interaction modification, which we shall discuss below.

We go on to explore the actual behavior of the structure functions as functions of their arguments \( \nu \) and \( q^2 \), switching on strong interactions. In the inclusive semileptonic \( B \) decays, the square of the momentum transfer lies in the range \( M_f^2 \leq q^2 \leq (M - M_{X\text{min}})^2 \), where \( M_{X\text{min}} \) is the minimal hadronic invariant mass in the final state, which is the \( D \) meson (pion) mass for the \( b \to c \) \( (b \to u) \) transition. Because of heaviness of the \( B \) meson, the decays occur mostly at large momentum transfer. From Eq. (2.3) we see that at large momentum transfer the hadronic tensor is dominated by the space-time region near the light cone \( y^2 \to 0 \), where \( y \) is the space-time interval between the points at which the currents \( j_\mu(y) \) and \( j_\nu(0) \) act.

The light-cone dominance implies that the five structure functions, \textit{a priori} independent, are related to a single distribution function \[ f(\xi) = \frac{1}{4\pi M^2} \int d(y \cdot P)e^{ik_y P} \langle B|\bar{b}(0)\gamma_\mu(1 - \gamma_5)b(y)|B\rangle|_{y^2=0}. \tag{3.15} \]

The distribution function reflects the long-distance strong interaction responsible for the \( b \)-quark confinement in the \( B \) meson. The structure functions show a simplified structure at large momentum transfer. We see that on the light-cone dominance \( W_4 \) remains zero and also \( W_5 = W_3 \) just as in the free quark limit.
It is instructive to pin down the form of the $b$ quark distribution function in the free quark limit for comparison with its form in the real world. In this limit, the free Dirac field $b(y) = e^{-iy\cdot p_b}b(0)$ with the velocity equality $p_b/m_b = P/M$, so from Eq. (3.15) it follows that the distribution function becomes

$$f(\xi) = \delta(\xi - \frac{m_b}{M}).$$

(3.16)

Substituting Eq. (3.16) into Eqs. (3.10)–(3.14), we recover the structure functions in the free quark limit in Eqs. (3.5)–(3.9). Deviations from the delta function behavior would signal the strong interaction effects.

Several important properties of the distribution function can be derived from field theory [9,12]. It obeys positivity with a support $0 \leq \xi \leq 1$. The distribution function is normalized to unity:

$$\int_0^1 d\xi f(\xi) = 1.$$  

(3.17)

The normalization of $f(\xi)$ is exact and does not get renormalized as a consequence of $b$ quantum number conservation. Since the $b$ quark inside the $B$ meson behaves as almost free because of its large mass, relative to which its binding to the light constituents is weak, we expect that the distribution function is very close to its asymptotic form — the delta function in Eq. (3.16). This is supported by the analysis using the operator product expansion and the heavy quark effective theory method [13–17], which indicates that the distribution function is sharply peaked around $\xi = \frac{m_b}{M}$ with a width of order $\Lambda_{QCD}/M$ [9,10,12].

It is convenient to redefine the structure functions as

$$F_1(\nu, q^2) = \frac{1}{2}W_1(\nu, q^2),$$

(3.18)

$$F_2(\nu, q^2) = \frac{\xi_+ - \xi_-}{8}W_2(\nu, q^2),$$

(3.19)

$$F_3(\nu, q^2) = \frac{\xi_+ - \xi_-}{4}W_3(\nu, q^2),$$

(3.20)

$$F_5(\nu, q^2) = \frac{\xi_+ - \xi_-}{4}W_5(\nu, q^2).$$

(3.21)

With these definitions, it is straightforward to get from Eqs. (3.10)–(3.14):

$$\frac{F_1 - F_3}{2} = \frac{F_2 + \xi_- F_1}{\xi_+ + \xi_-} = \frac{F_2 + \xi_- F_3}{\xi_+ - \xi_-} = f(\xi_+).$$

(3.22)

Note that on the light-cone dominance $F_5 = F_3$. Equation (3.22) shows scaling of the linear combinations of the structure functions, i.e., for large $q^2$ they depend only on the dimensionless scaling variable $\xi_+$. The normalization condition (3.17) of the distribution function implies the sum rules

$$\int_{m_q + M}^{\infty} d\xi_+ \frac{F_1 - F_3}{2} = \int_{m_q + M}^{\infty} d\xi_+ \frac{F_2 + \xi_- F_1}{\xi_+ + \xi_-} = \int_{m_q + M}^{\infty} d\xi_+ \frac{F_2 + \xi_- F_3}{\xi_+ - \xi_-} = 1.$$

(3.23)
We may go further. Assuming the $f(\xi_-)$ terms in Eqs. (3.10)–(3.14) to be negligible implies the following relations of the structure functions:

$$\xi_+ F_1 = F_2 = -\xi_+ F_3 = -\xi_+ F_5 = \xi_+ f(\xi_+), \quad (3.24)$$

where the structure functions also scale, displaying the same property of the structure functions as in the free quark limit shown above. Furthermore, the following sum rules can be obtained from the normalization condition (3.17):

$$\int_{m_q + M_\ell}^{1} d\xi_+ F_1 = \int_{m_q + M_\ell}^{1} d\xi_+ F_2/\xi_+ = -\int_{m_q + M_\ell}^{1} d\xi_+ F_3 = -\int_{m_q + M_\ell}^{1} d\xi_+ F_5 = 1. \quad (3.25)$$

The relations (3.22) and (3.24) are a consequence of the spin 1/2 nature of the $b$ quark; spin 0 partons would lead to $F_1 = 0$ or $W_1 = 0$.

The $f(\xi_-)$ term is a consequence of field theory, corresponding to the creation of quark-antiquark pairs in the final state. Here we pursue the arguments of [9] and examine more closely the assumption that the $f(\xi_-)$ terms in Eqs. (3.10)–(3.14) are negligible. The kinematic ranges of $\xi_+$ and $\xi_-$ are

$$\frac{m_q + M_\ell}{M} \leq \xi_+ \leq 1, \quad (3.26)$$

$$\frac{M_\ell - m_q}{M} \left( \frac{M_\ell + m_q}{M} \right) \Theta(M_\ell - m_q) \leq \xi_- \leq 1 - \frac{2m_q}{M}, \quad (3.27)$$

respectively, where $\Theta(x)$ is a step function. For $b \to c$ decays $\xi_- \lesssim 0.5$, the $f(\xi_-)$ terms entering Eqs. (3.10)–(3.14) can be safely neglected since the distribution function is sharply peaked near one. However, for $b \to u$ decays $\xi_-$ can be as large as one, so that the sizes of $f(\xi_-)$ and $f(\xi_+)$ may be comparable. Thus the approximate scaling and the sum rules in Eqs. (3.24) and (3.25) would hold for the inclusive charmed semileptonic $B$ decay, but would be considerably violated by the non-negligible $f(\xi_-)$ term for the inclusive charmless semileptonic $B$ decay. By contrast, the approximate scaling and the sum rules in Eqs. (3.22) and (3.23) are valid regardless of the size of $f(\xi_-)$.

Also worthy of noting is that there are other sources of scaling violation: logarithmic corrections due to gluon radiation and corrections of inverse powers of $q^2$ due to departures from the light cone.

**IV. EXTRACTION OF THE DISTRIBUTION FUNCTION**

The $b$ quark distribution function $f(\xi)$ defined in Eq. (3.15) contains important information on the nature of confinement. As a necessary input, its knowledge is also crucial for improving the theoretical accuracies on the determinations of $|V_{ub}|$ and $|V_{cb}|$ from the charged lepton energy spectra [10,11] and the hadronic invariant mass spectrum [18]. An experimental extraction of the distribution function will therefore be of special importance and interest. In this section we shall discuss how one could extract the distribution function from experiment.

Equations (3.10)–(3.14) imply that $f(\xi)$ can be extracted through the measurements of the structure functions, which is discussed in section II. Recently, it has been put forward
that a measurement of the differential decay rate as a function of $\xi_+$ can be also used to extract directly the $b$ quark distribution function. The expression for $d\Gamma/d\xi_+$ has been derived in [19], neglecting lepton masses. Here we generalize this result to the case where the charged lepton mass cannot be neglected. Substituting Eqs. (3.10)–(3.14) into Eq. (2.9), changing the variables from $(\nu, q^2)$ to $(\xi_+, q^2)$, and integrating over $q^2$ with the kinematic limits $M_\ell^2 \leq q^2 \leq (\xi_+ M - m_\ell)^2$, we arrive at

$$
\frac{d\Gamma}{d\xi_+} = \frac{G_F^2 |V_{tb}|^2 M^6}{192\pi^3 E} \left[ \xi_+^5 f(\xi_+) \Phi(x, z) + \frac{12}{M^2} \int_{M_\ell^2}^{(\xi_+ M - m_\ell)^2} dq^2 \sqrt{q^2 - M_\ell^2} \Omega(\xi_+, q^2) \right]
$$

(4.1)

with

$$
\Phi(x, z) = \sqrt{\lambda} [1 - 7(x^2 + z^2 + x^4 + z^4 + x^4 z^2 + x^2 z^4) + 12x^2 z^2 + x^6 + z^6]
$$

$$+ 12(2x^4 - z^4 - x^4 z^4) \ln \frac{1 + x^2 - z^2 + \sqrt{\lambda}}{2x} + 12(1 - x^4) z^4 \ln \frac{(1 - x^2)^2 - (1 + x^2) z^2 + (1 - x^2) \sqrt{\lambda}}{2x z^2},
$$

(4.2)

$$
\Omega(\xi_+, q^2) = q^2 - M_\ell^2 + \frac{2M_\ell^2 \nu}{\xi_+ M} (1 - \frac{M_\ell^2}{q^2}) + \frac{1}{3} \xi_+ [q^2 + M_\ell^2 - \frac{2M_\ell^4}{q^2} - 4\nu^2 (1 + \frac{M_\ell^2}{q^2} - \frac{2M_\ell^4}{q^4})],
$$

(4.3)

where

$$
\lambda = -4x^2 + (1 + x^2 - z^2)^2, \quad x = \frac{m_\ell}{\xi_+ M}, \quad z = \frac{M_\ell}{\xi_+ M},
$$

(4.4)

$$
\nu = \frac{\xi_+^2 M^2 + q^2 - m_\ell^2}{2\xi_+ M},
$$

(4.5)

$$
\xi_- = \frac{q^2 - m_\ell^2}{\xi_+ M^2}.
$$

(4.6)

If the charged lepton mass is neglected, Eq. (4.1) reduces to the expression given in [19].

The contribution of the $f(\xi_-)$ term to the specific spectra $d\Gamma/d\xi_+$ in Eq. (4.1) is expected to be negligible since (1) the distribution function is very close to its asymptotic form, $\delta(\xi - m_\ell/M)$, in the free quark limit as discussed in the last section and in this limit $f(\xi_-)$ vanishes and (2) the dominant contribution to the $f(\xi_-)$ integral at a given $\xi_+$ results from the large $\xi_-$ region, corresponding to the neighbourhood of the upper integration limit for $q^2$, which is suppressed by $\nu^2 - q^2$. The numerical studies carried out in [19] have confirmed this expectation. The smallness of the $f(\xi_-)$ term has already been found in the parton model [20]. As a result, $d\Gamma/d\xi_+$ is proportional to $f(\xi_+)$, so the measurement of the decay distribution can be used to extract directly the nonperturbative distribution function. In
other words, the decay distribution with respect to the scaling variable $\xi_+$ probes the QCD dynamics in a most sensitive manner.

In practice, as the measurements of the structure functions, radiative corrections must be implemented in order to extract the distribution function from the measured raw yields using the above expressions.

The decay distribution as a function of $\xi_+$ is a measurable quantity with many other remarkable uses. New hadronic uncertainty insensitive methods have been proposed [19] for precise determinations of $|V_{ub}|$ and $|V_{cb}|$ through the measurements of the $b \rightarrow u$ and $b \rightarrow c$ decay distributions as a function of $\xi_+$. Large theoretical uncertainties associated with nonperturbative strong interactions can be avoided with these methods. Moreover, in these measurements rare $b \rightarrow u$ events can be separated efficiently from $b \rightarrow c$ events. These methods would enable us to determine especially $|V_{ub}|$ in a model independent way. The $b \rightarrow u$ spectrum $d\Gamma(B \rightarrow X_u\ell\nu)/d\xi_+$ could be also used to measure reliably the inclusive charmless semileptonic branching fraction of the $B$ meson.

V. CONCLUSIONS

Inclusive semileptonic $B$ decay is a very powerful means of probing hadron structure and strong interactions. There are clear similarities between inclusive semileptonic $B$ decay and deep inelastic lepton-nucleon scattering. Inclusive semileptonic $B$ decay can be used to pursue the QCD as deep inelastic scattering. We have presented general formulas for the differential decay rates for the inclusive semileptonic decays of the $B$ meson in terms of the structure functions with the inclusion of charged lepton masses. These formulas are suitable for measuring the $B$ meson structure functions in any particular frame.

Measurements of the $B$ meson structure functions would provide evidence for the quark structure of the $B$ meson. The scaling behavior of the structure functions at large momentum transfer as in Eqs. (3.22) and (3.24) would reveal the existence of a point-like $b$ quark in the $B$ meson. The experimental verification of nonvanishing $W_1$ would indicate that the $b$ quark has spin 1/2. The sum rules as in Eqs. (3.23) and (3.25) would provide a basic element in identifying the $b$ quantum number of the $B$ meson.

The light-cone dominance simplifies the nonperturbative description of inclusive semileptonic $B$ decays and permits the structure functions to be related to a single distribution function, leading to the testable theoretical predictions. The distribution function $f(\xi)$ is defined as the Fourier transform of the matrix element of the bilocal and on-light-cone operator in $b$-quark fields taken between the $B$ meson states, which gives the probability of finding a $b$ quark with momentum $\xi_P$ inside the $B$ meson. This function depends only on the initial $B$ meson state and is therefore universal. The nonperturbative distribution function can be extracted by the measurements of either the structure functions or the decay distribution as a function of $\xi_+$.

In addition to testing the theoretical predictions discussed here, advancing our understanding of hadron structure and strong interactions, and providing new observations on the nature of confinement, measurements of the $B$ meson structure functions and the universal $b$ quark distribution function would be beneficial to precise determinations of the fundamental standard model parameters $|V_{ub}|$ and $|V_{cb}|$. 

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The study of hadron structure and strong interactions by exploiting inclusive semileptonic $B$ decays will considerably extend the $B$ physics reach. Measurements of the $B$ meson structure functions and the $b$ quark distribution function should prove rewarding.

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