Canonical Bases of \(q\)-Deformed Fock Spaces

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Abstract

We define a canonical basis of the \(q\)-deformed Fock space representation of the affine Lie algebra \(\widehat{\mathfrak{gl}}_n\). We conjecture that the entries of the transition matrix between this basis and the natural basis of the Fock space are \(q\)-analogues of decomposition numbers of the \(v\)-Schur algebras for \(v\) specialized to a \(n\)th root of unity.

1 Introduction

The Fock space representation \(\mathcal{F}\) of \(\widehat{\mathfrak{sl}}_n\) is not irreducible. Its decomposition into simple \(\widehat{\mathfrak{sl}}_n\)-modules is given by

\[
\mathcal{F} \cong \bigoplus_{k \geq 0} M(\Lambda_0 - k\delta)^{\oplus p(k)},
\]

where \(p(k)\) denotes the number of partitions \(\lambda\) of \(k\). Hence it is not obvious to apply Kashiwara’s or Lusztig’s method to define a canonical basis of \(\mathcal{F}\).

In this note, we shall rather regard \(\mathcal{F}\) as a representation of the enlarged algebra \(\widehat{\mathfrak{gl}}_n\). Indeed, \(\mathcal{F}\) is a simple \(\widehat{\mathfrak{gl}}_n\)-module, and a \(q\)-deformation \(\mathcal{F}_q\) of this representation has been described in \([11]\). One can then define a natural semi-linear involution \(v \rightarrow \overline{v}\) commuting with the action of the lowering operators of \(\mathcal{F}_q\) and leaving invariant its highest weight vector. Using this involution, one obtains in an elementary way a canonical basis \(\{G(\lambda)\}\) of \(\mathcal{F}_q\). This basis can be computed explicitly. It would be interesting to compare it with the canonical basis obtained via a geometric approach by Ginzburg, Reshetikhin and Vasserot \([3]\).

By restriction to \(U_q(\widehat{\mathfrak{sl}}_n)\), the space \(\mathcal{F}_q\) decomposes similarly as

\[
\mathcal{F}_q \cong \bigoplus_{k \geq 0} M_q(\Lambda_0 - k\delta)^{\oplus p(k)},
\]

where \(M_q(\Lambda)\) denotes the simple \(U_q(\widehat{\mathfrak{sl}}_n)\)-module with highest weight \(\Lambda\). The \(G(\mu)\) indexed by \(n\)-regular partitions \(\mu\) coincide with the elements of Kashiwara’s global crystal basis of the basic representation \(M(\Lambda_0)\). But we insist that the rest of our basis is not compatible with the decomposition \([2]\).

We conjecture that for \(q = 1\), the coefficients of the transition matrix of our canonical basis on the natural basis of the Fock space are equal to the decomposition numbers of \(v\)-Schur algebras over a field of characteristic 0 at a \(n\)th root of unity. A previous conjecture \([12, 13]\) on decomposition matrices of Hecke algebras having been recently established by Ariki and by Grojnowski, we already know that the columns of the transition matrix indexed by \(n\)-regular...
partitions contain $q$-analogues of decomposition numbers. On the other hand, we can prove by means of a $q$-analogue of Steinberg’s tensor product theorem that an infinite number of entries of the inverse transition matrix are $q$-analogues of inverse decomposition numbers.

Details and proofs will appear in a forthcoming paper.

We follow the notation of [16] for symmetric functions, and that of [18, 11] for $q$-wedges and Fock space representations, except for the replacement of $q$ by $q^{-1}$.

## 2 A $q$-analogue of the Fock space representation of $\widehat{\mathfrak{gl}}_n$

The Lie algebra $\widehat{\mathfrak{g}}_{\mu}$ can be regarded as the sum $\widehat{\mathfrak{sl}}_n + \mathcal{H}_n$ where $\mathcal{H}_n$ is a Heisenberg algebra commuting with $\widehat{\mathfrak{sl}}_n$ and such that $\widehat{\mathfrak{sl}}_n' \cap \mathcal{H}_n = \mathbb{C}c$, where $c$ is the central generator [17] and further investigated by Misra and Miwa [17] who constructed the crystal basis. Recently, Kashiwara, Miwa and Stern [11] have shown that the action of $U_q(\widehat{\mathfrak{g}}_n)$ on $\mathcal{F}_q$ is centrally simple in the basis of Schur functions $s_\lambda$. For a node $\gamma$ of the Young diagram of a partition $\lambda$, located at the intersection of the $i$th row and the $j$th column of $\lambda$, define its residue $r(\gamma) \in \{0, 1, \ldots, n-1\}$ as $r(\gamma) = j-i \mod n$.

Then,
$$e_is_\lambda = \sum s_\nu, \quad f_is_\lambda = \sum s_\mu,$$
where $\nu$ (resp. $\mu$) runs through all partitions obtained from $\lambda$ by removing (resp. adding) a node of residue $i$.

The $q$-analogue $\mathcal{F}_q$ of the Fock space representation of $\widehat{\mathfrak{g}}_n$, in the form of a Fock space representation of the quantized enveloping algebra $U_q(\widehat{\mathfrak{g}}_n)$, has been constructed by Hayashi [3] and further investigated by Misra and Miwa [17] who constructed the crystal basis. Recently, Kashiwara, Miwa and Stern [11] have shown that the action of $U_q(\widehat{\mathfrak{g}}_n)$ on $\mathcal{F}_q$ is centralized by a Heisenberg algebra $\mathcal{H}_q^n$. Let $\mathcal{U}$ be the subalgebra of End($\mathcal{F}_q$) generated by these actions of $U_q(\widehat{\mathfrak{g}}_n)$ and $\mathcal{H}_q^n$ (it would be interesting to compare $\mathcal{U}$ with the standard $q$-deformation $U_q(\widehat{\mathfrak{g}}_n)$ considered in [3, 4]). The Fock space is an irreducible $\mathcal{U}$-module, and the canonical basis constructed in this paper will be adapted to this representation.

The Fock space representation of $U_q(\widehat{\mathfrak{g}}_n)$ can be described as follows (note that our conventions are slightly different from those of [17] and [11]). Let
$$\mathcal{F}_q = \bigoplus_\lambda \mathbb{Q}(q)|\lambda\rangle$$
be the $\mathbb{Q}(q)$ vector space with basis $|\lambda\rangle$ indexed by the set of all partitions. Let $\lambda$ and $\mu$ be two partitions such that $\mu$ is obtained from $\lambda$ by adding a node $\gamma$ of residue $i$. Let $I_i(\lambda)$ be the number of indent $i$-nodes of $\lambda$, $R_i(\lambda)$ the number of its removable $i$-nodes, $I_i^f(\lambda, \mu)$ (resp. $R_i^f(\lambda, \mu)$) the number of indent $i$-nodes (resp. of removable $i$-nodes) situated to the left of $\gamma$ (not included), and similarly, let $I_i^r(\lambda, \mu)$ and $R_i^r(\lambda, \mu)$ be the corresponding numbers for nodes located on the right of $\gamma$. Set $N_i(\lambda) = I_i(\lambda) - R_i(\lambda)$, $N_i^f(\lambda, \mu) = I_i^f(\lambda, \mu) - R_i^f(\lambda, \mu)$ and $N_i^r(\lambda, \mu) = I_i^r(\lambda, \mu) - R_i^r(\lambda, \mu)$. Then,
$$f_i|\lambda\rangle = \sum_\mu q^{N_i^f(\lambda, \mu)}|\mu\rangle, \quad e_i|\mu\rangle = \sum_\lambda q^{N_i^r(\lambda, \mu)}|\lambda\rangle$$
(3)

where in each case the sum is over all partitions such that $\mu/\lambda$ is a $i$-node,
$$q^{h_i}|\lambda\rangle = q^{N(\lambda)}|\lambda\rangle \quad \text{and} \quad q^{D}|\lambda\rangle = q^{-N(\lambda)}|\lambda\rangle$$
(4)
where $D$ is the degree generator and $N^0(\lambda)$ the total number of 0-nodes of $\lambda$.

The Fock representation of the Heisenberg algebra $\mathcal{H}_n^q$ of [14] is defined by means of Stern’s construction of semi-infinite $q$-wedges [18]. The basis vector $|\lambda\rangle$ is interpreted as the semi-infinite wedge $u_I = u_{i_1} \wedge u_{i_2} \wedge \cdots$ where the sequence $I$ is defined by $i_k = \lambda_k - k + 1$. With our conventions, the commutation relations for $q$-wedges are the following. Suppose that $\ell < m$ and that $\ell - m \mod n = i$. If $i = 0$ then $u_\ell \wedge u_m = -u_m \wedge u_\ell$ and otherwise,

$$u_\ell \wedge u_m = -q^{-1}u_m \wedge u_\ell + \left(q^{-2} - 1\right) \left(u_{m-i} \wedge u_{\ell+i} - q^{-1}u_{m-n} \wedge u_{\ell+n} + q^{-2}u_{m-n-i} \wedge u_{m+n+i} - \cdots\right)$$

where in the last sum one retains only the normally ordered terms.

Then, the generator $B_k$ of $\mathcal{H}_n^q$ acts on $u_I$ by

$$B_k u_I = \sum_{i \geq 1} u_{I+kn\delta^i}$$

where $\delta^i$ is the sequence $(\delta^i_j)_{j \geq 1}$ (Kronecker symbols). The semi-infinite wedge $u_I$ will be called the fermionic realization of the basis vector $|\lambda\rangle$. These operators verify the relations [11]

$$[B_k, B_\ell] = k \frac{1 - q^{2nk}}{1 - q^{-2k}} \delta_{k+\ell,0}. \quad (6)$$

The action of $\mathcal{H}_n^q$ in the bosonic picture is better described in terms of other operators $U_k, V_k \in U(\mathcal{H}_n^q)$ [14]. Recall that for $k > 0$, $B_{-k}$ is a $q$-analogue of multiplication by $p_{kn} = p_k(t_1^n, t_2^n, \ldots) = \psi^n(p_k)$, where $\psi_n$ is the ring homomorphism raising the variables to the $n$th power. On the graphical representation by Young diagrams, multiplication by the complete homogeneous functions $\psi_n(h_k) = h_k(t_1^n, t_2^n, \ldots)$ of $n$th powers has a simple combinatorial description. Let

$$V_k = \sum_{m_1 + 2m_2 + \cdots + km_k = k} \frac{1}{m_1!m_2!\cdots m_k!} (B_{-1})^{m_1} (B_{-2})^{m_2} \cdots (B_{-k})^{m_k} \quad (7)$$

be the $q$-analogue of the multiplication operator by $\psi^n(h_k)$. Then,

$$V_k |\lambda\rangle = \sum q^{-\mathbf{h}(\mu/\lambda)} |\mu\rangle \quad (8)$$

where the sum is over all partitions $\mu$ such that $\mu/\lambda$ is a horizontal $n$-ribbon strip of weight $k$, and

$$\mathbf{h}(\mu/\lambda) = \sum_R (\text{ht}(R) - 1)$$

where the sum is over all the $n$-ribbons $R$ tiling $\mu/\lambda$, ht $(R)$ being the height of the ribbon $R$ (see [14]).

The scalar product on $F_q$ is defined by $\langle \lambda | \mu \rangle = \delta_{\lambda\mu}$. The adjoint operator $U_k$ of $V_k$ acts by

$$U_k |\mu\rangle = \sum q^{-\mathbf{h}(\mu/\lambda)} |\lambda\rangle \quad (9)$$

where the sum is over all partitions $\lambda$ such that $\mu/\lambda$ is a horizontal $n$-ribbon strip of weight $k$.

Identifying $F_q$ with $\mathbb{Q}(q) \otimes \text{Sym}$ by setting $|\lambda\rangle = s_\lambda$, one can define a linear operator

$$\psi^n_q : F_q \rightarrow F_q$$

by specifying the image of the basis $(h_\lambda)$ as

$$\psi^n_q(h_\lambda) = V_{\lambda_1} V_{\lambda_2} \cdots V_{\lambda_n} |\emptyset\rangle. \quad (10)$$

Then, the image $\{\psi^n_q(g_\lambda)\}$ of any basis $\{g_\lambda\}$ of symmetric functions will be a basis of the space of $U_q(\hat{a}^*_n)$-highest weight vectors in $F_q$. 

\[3\]
An involution of the Fock space

Let $\mathcal{J}$ be the set of decreasing sequences $I = (i_1, i_2, \cdots)$ such that $i_k = -k + 1$ for $k$ large enough. Then $\{u_I | I \in \mathcal{J}\}$ is the standard basis of $\mathcal{F}_q$. For $m \geq 0$, denote by $\mathcal{J}_m$ the subset of $\mathcal{J}$ consisting of those $I$ such that $\sum_k (i_k + k - 1) = m$. Let $I \in \mathcal{J}_m$, and let $u_I$ be the associated basis vector of $\mathcal{F}_q$. We denote by $\alpha_{n,k}(I)$ the number of pairs $(r,s)$ with $1 \leq r < s \leq k$ and $r - s \not\equiv 0 \mod n$.

**Proposition 3.1** For $k \geq m$, the $q$-wedge

$$\overline{u_I} = (-1)^{\binom{k}{2}} q^{\alpha_{n,k}(I)} u_{i_k} \wedge u_{i_{k-1}} \wedge \cdots \wedge u_{i_1} \wedge u_{i_{k+1}} \wedge u_{i_{k+2}} \wedge \cdots$$

is independent of $k$.

Define a semi-linear map $v \mapsto \overline{v}$ in $\mathcal{F}_q$ by

$$\sum_{I \in \mathcal{J}} \varphi_I(q) u_I = \sum_{I \in \mathcal{J}} \varphi_I(q^{-1}) \overline{u_I}. \quad (11)$$

**Theorem 3.2**

(i) $v \mapsto \overline{v}$ is an involution of $\mathcal{F}_q$.

(ii) $f_i v = \overline{f_i \overline{v}}$ and $B_{-k} v = B_{-k} \overline{v}$, $(v \in \mathcal{F}_q, \ i \in \{0, \ldots, n - 1\}, k > 0)$.

We note that there is a unique semi-linear map satisfying (ii) and $|\emptyset\rangle = |\emptyset\rangle$. This implies that the restriction of the involution $v \mapsto \overline{v}$ to the subspace $M(\Lambda_0)$ of $\mathcal{F}_q$ coincides with the usual involution in terms of which the global crystal basis of $M(\Lambda_0)$ is defined.

Let $\mu \vdash m$. Set

$$|\mu\rangle = \sum_{\lambda \vdash m} a_{\lambda \mu}(q) |\lambda\rangle.$$

**Theorem 3.3**

(i) $a_{\lambda \mu}(q) \in \mathbb{Z}[q, q^{-1}]$.

(ii) $a_{\lambda \mu}(q) = 0$ unless $\lambda \leq \mu$ and $\lambda, \mu$ have the same $n$-core.

(iii) $a_{\lambda \lambda}(q) = 1$.

(iv) $a_{\lambda \mu}(q) = a_{\mu' \lambda'}(q)$.

For $n = 2$, the matrices $A_m(q) = [a_{\lambda \mu}(q)]_{\lambda, \mu \vdash m}$ for $m = 2, 3, 4$ are

\[
\begin{bmatrix}
1 & 0 \\
q^{-1} & 1
\end{bmatrix}
\quad \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
q^{-1} & 0 & 1 & 0 & 0 \\
q^{-1} & 0 & 0 & 1 & 0 \\
q^{-1} & 0 & 0 & 0 & 1
\end{bmatrix}
\quad \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
q^{-1} & 1 & 0 & 0 & 0 \\
-1 + q^{-2} & q^{-1} & 1 & 0 & 0 \\
0 & q^{-2} - 1 & q^{-1} & 1 & 0 \\
q^{-2} - 1 & 0 & -1 + q^{-2} & q^{-1} & 1
\end{bmatrix}
\]

(partitions are ordered in reverse lexicographic order, e.g., for $m = 4$, $(4), (31), (22), (211), (1111)$).
4 Canonical bases

Let $L$ (resp. $L^-$) be the $\mathbb{Z}[q]$ (resp. $\mathbb{Z}[q^{-1}]$)-lattice in $\mathcal{F}_q$ with basis $\{\lambda\}$. Using Theorems 3.2 and 3.3 one can construct “IC-bases of $\mathcal{F}_q$”, in the terminology of Du [6] (see also [15], 7.10).

**Theorem 4.1** There exist bases $\{G(\lambda)\}$, $\{G^-(\lambda)\}$ of $\mathcal{F}_q$ characterized by:

(i) $G(\lambda) = G(\lambda)$, $G^-(\lambda) = G^-(\lambda)$,

(ii) $G(\lambda) \equiv |\lambda\rangle \mod qL$, $G^-(\lambda) \equiv |\lambda\rangle \mod q^{-1}L^-$.

Set

$$G(\mu) = \sum_\lambda d_{\lambda\mu}(q)|\lambda\rangle, \quad G^-(\lambda) = \sum_\mu e_{\lambda\mu}(q)|\mu\rangle.$$ 

Then $d_{\lambda\mu}(q) \in \mathbb{Z}[q]$, $e_{\lambda\mu}(q) \in \mathbb{Z}[q^{-1}]$, and these polynomials are nonzero only if $\lambda, \mu$ have the same $n$-core. Moreover, $d_{\lambda\lambda}(q) = e_{\lambda\lambda}(q) = 1$, and $d_{\lambda\mu}(q) = 0$ unless $\lambda \preceq \mu$, $e_{\lambda\mu}(q) = 0$ unless $\mu \preceq \lambda$. Also, $\{G(\lambda) \mid \lambda \text{ n-regular}\}$ coincides with the lower crystal basis of the basic representation $M(\Lambda_0)$ of $U_q(\hat{\mathfrak{sl}}_n)$.

Let $\{G^\dagger(\lambda)\}$ be the adjoint basis of $\{G(\lambda)\}$. It follows from Theorem 3.3 (iv) that

$$G^\dagger(\lambda)' = G^-(\lambda'),$$

where $v \mapsto v'$ denotes the semi-linear involution of $\mathcal{F}_q$ defined by $|\lambda\rangle' = |\lambda\rangle'$.

We set

$$G^\dagger(\lambda) = \sum_\mu c_{\lambda\mu}(q)|\mu\rangle,$$

so that $c_{\lambda\mu}(q) = e_{\lambda\mu'}(q^{-1})$ and $[c_{\lambda\mu}(q)] = [d_{\lambda\mu}(q)]^{-1}$.

For $n = 2$ and $m \leq 6$, the matrices $D_m(q) = [d_{\lambda\mu}(q)]_{\lambda, \mu \vdash m}$ are

\[
\begin{array}{cccc}
2 & 1 & 0 & 3 & 1 & 0 & 0 \\
11 & q & 1 & 21 & 0 & 1 & 0 \\
& & & & & & \\
4 & 1 & 0 & 0 & 0 & 0 & 0 \\
31 & q & 1 & 0 & 0 & 0 & 0 \\
22 & 0 & q & 1 & 0 & 0 & 0 \\
211 & q & q^2 & q & 1 & 0 & 0 \\
1111 & q^2 & 0 & 0 & q & 1 & 0 \\

d & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 1 & 0 & 0 & 0 & 0 & 0 \\
31 & q & 1 & 0 & 0 & 0 & 0 \\
22 & 0 & q & 1 & 0 & 0 & 0 \\
211 & q & q^2 & q & 1 & 0 & 0 \\
1111 & q^2 & 0 & 0 & q & 1 & 0 \\
\end{array}
\]
5 A \( q \)-analogue of Steinberg’s tensor product theorem

Let \( \alpha \) be a partition of \( r \). Define an element \( S_\alpha \) of \( \mathbb{U} \) by

\[
S_\alpha = \sum_{\beta \vdash r} \frac{\chi_\alpha^\beta}{z_\beta} B_{-\beta_1} B_{-\beta_2} \cdots B_{-\beta_k}
\]

where \( \beta = (\beta_1, \ldots, \beta_k) = (1^{m_1} 2^{m_2} \cdots r^{m_r}) \), \( z_\beta = 1^{m_1} m_1! \cdots r^{m_r} m_r! \) and \( \chi_\alpha^\beta \) is the value of the irreducible character \( \chi_\alpha \) of the symmetric group on a permutation of cycle type \( \beta \).

Writing \( V_\mu = V_{\mu_1} V_{\mu_2} \cdots V_{\mu_r} \), one has

\[
S_\alpha = \sum_{\mu \vdash r} \kappa_{\alpha \mu} V_\mu,
\]

where the \( \kappa_{\alpha \mu} \) denote the entries of the inverse Kostka matrix. Hence, using [8], one can describe the action of \( S_\alpha \) on Young diagrams.

The operator \( S_\alpha \) is a \( q \)-analogue of the multiplication by the plethysm \( \psi^n(s_\alpha) \) in the ring of symmetric functions.

Let \( \lambda \) be a partition such that \( \lambda' \) is \( n \)-singular. We can write \( \lambda = \mu + n\alpha \) where \( n\alpha = (n\alpha_1, n\alpha_2, \ldots) \) and \( \mu' \) is \( n \)-regular.

**Theorem 5.1** \( G^-(\lambda) = S_\alpha (G^-(\mu)) \).

This reduces the computation of \( \{ G^-(\lambda) \} \) to that of the subfamily \( \{ G^-(\mu) \mid \mu \text{ \( n \)-regular} \} \).

Let \( D_m \) denote the decomposition matrix of the \( v \)-Schur algebra \( S_m \) over a field of characteristic 0, for \( v \) a primitive \( n \)-th root of unity. We use the notational convention of James [9], that is, the rows and columns of \( D_m \) are indexed in such a way that \( D_m \) is the matrix \( \Delta_m \) of [9] for big \( p \).

**Conjecture 5.2** The matrix \( D_m(1) \) is equal to the decomposition matrix \( D_m \).

This conjecture, which generalizes Conjecture 6.9 of [13], is already verified to a large extent. Indeed, on the one hand Ariki [1] and Grojnowski have verified independently our previous conjecture, which means that the \( d_{\lambda\mu}(1) \) for \( \mu \text{ \( n \)-regular} \) are equal to the corresponding decomposition numbers of \( S_m \). On the other hand, it follows from results of James [9] that the entries of the inverse matrices \( D_m^{-1} \) satisfy the same properties as those deduced from Theorem 5.1 for the coefficients \( c_{\lambda\mu}(1) \). Since we have verified, using the tables of [9], that \( D_m^{-1} = [c_{\lambda\mu}(1)] \) for
m \leq 10 \text{ (any } n), \text{ we deduce that an infinite number of } c_{\lambda\mu}(1) \text{ coincide with the corresponding entries of } D^{-1}_m. \]

The following refined conjecture has also been checked for small \( m \). It generalizes the conjecture of Section 9 of [13], due to Rouquier. Let \( (W(\lambda)^i) \) be the Jantzen filtration of the Weyl module \( W(\lambda) \) for the \( v \)-Schur algebra \( S_m \), and let \( L(\mu) \) be the irreducible module corresponding to \( \mu \).

**Conjecture 5.3** Let \( \lambda, \mu \) be partitions of \( m \). Then,

\[
\begin{align*}
\sum_{i \geq 0} W(\lambda)^i W(\lambda)^{i+1} : L(\mu) q^i.
\end{align*}
\]

Finally, we note the following combinatorial description of some polynomials \( e_{\lambda\mu}(q) \) in the case \( n = 2 \), which proves that they are equal (up to sign and the replacement of \( q \) by \( q^{-2} \)) to the \( q \)-analogues of Littlewood-Richardson coefficients introduced in [2].

**Theorem 5.4** Let \( n = 2 \). One has

\[
\begin{align*}
e_{2\lambda,\mu}(q) &= \varepsilon_2(\mu) \sum_{T \in \text{Yam}_2(\mu,\lambda)} q^{-2\text{spin}(T)}
\end{align*}
\]

where \( \text{Yam}_2(\mu, \lambda) \) denotes the set of Yamanouchi domino tableaux of shape \( \mu \) and weight \( \lambda \), and \( \varepsilon_2(\mu) \) is the 2-sign of \( \mu \).

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