A simplified method to estimate the fundamental frequency of simple span bridges supported on lead rubber bearing

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Abstract. The impulsive load method applied on the mid-span of a girder is often used to measure the fundamental frequency of the bridge. This method commonly involves a truck that is dropped from a certain height to generate an impulsive load. In high seismicity zones, bridge structures often involve base isolators such as lead rubber bearing (LRB) to reduce the seismic force demand. However, the application of LRBs on the bridge girders may reduce its vertical fundamental frequency. This fundamental frequency may decrease further if the girder is supported on the frames or cantilever beams on high columns. In this study, the fundamental frequency of several bridge spans is computed through a simplified analytical study based on a single degree of freedom assumption. The results show that the application of LRBs does not significantly affect the fundamental frequency of the girder when compared to the identical girder supported on hinge-roller supports, due to the high vertical stiffness of the LRBs. In average, the values of fundamental frequency obtained through the simplified method and the FE modeling are lower than which obtained from the dynamic loading test. It indicates that the stiffness of the existing bridge is higher than the analyses.

1. Introduction
In the evaluation of constructed bridges, static and dynamic test are required. The static test can be performed by placing a row of trucks having an equivalent load minimum 70% of the design load. The deformation of the elements of the bridges, particularly the vertical deflection of the girders shall be less than the design deflection or which required by the code.

In the dynamic test, the simplest method is to perform an impact load on several points of the girders such that a maximum deflection is obtained. In simple span bridges, the impact load P(t) can be in the form of truck axis dropped from the height of 200 mm at the middle span, as seen in Figure 1. The jumper device is presented in Figure 1c: small white wheels are used for the transportation. This loading method presents the implementation of an impulsive ramp load that occurs in a very short duration. Immediately after the impact load, the girders exhibit a damped-free vibration response. The fundamental frequency of the structure can be computed based on the amplitude from crest to crest by means of Fast Fourier Transform (FFT), while the damping ratio can be computed based on the obtained logarithmic decrement.
In simple span bridges, in which the supports have an infinite vertical stiffness, (i.e: real hinge-roller support), the fundamental frequency can be estimated using the empiric formula 100/L to 125/L as reported in many classic literatures, where L is the bridge span [1-3].

![Dynamic loading test using jumper](image1)

**Figure 1.** Dynamic loading test using jumper: (a) Steel jumper; (b) Front wheels of the truck on jumper; (c). Model of impulsive truck load \( P(t) \) on the mid span of the girder

The above empiric formulas may not be suitable for isolated girders because the isolator device also affects the vertical frequency of the girder system. As a matter of fact, in Indonesia nowadays, the application of seismic isolators on bridges becomes much more popular due to the finding of many new active faults that increases the seismic coefficient. The application of seismic isolation system can reduce significantly the seismic base shear so that more economical design can be achieved. Investigations on the application of base isolation on the bridges have been reported in many literatures. In the experimental study on a three-continuous spans bridge having the length of 90 m, an increase of fundamental frequency was observed due to the increase of stiffness of the LRB, caused by the low temperature of \(-10^\circ\text{C}\) during the test [4]. A parametric study was conducted to evaluate the effect of LRB and ground motion on the response of isolated bridges. The study concludes that for a certain ground motion, the smaller values of maximum isolator displacement and isolator force indicate a better seismic performance [5].

Another study investigates the possibility of buckling and tensile force on high damping natural rubber (HDNR) bearings at particular condition in a three-spans bridge using simple-span I-girders having 30 m of length [6]. That unexpected condition can be avoided by selecting the suitable shape factor of the bearings.

In this paper, the behavior of a simple-span bridge isolated using LRB is investigated. The study focuses on the variation of the fundamental frequency due to the contribution of the LRB vertical stiffness and the stiffness of the substructure. The method used in this study is the simplified analytic method. The analytic approach assumes a single degree of freedom (SDOF) system with a lumped mass in the mid-span of the girder. The girders can have prismatic or non-prismatic shape. The investigation is performed also on the bridges supported on the abutments that have large vertical stiffness and the bridges supported on the non-prismatic cantilever beams that have low stiffness. The later bridge type is usually applied in fly-over structures. The developed analytical method is validated through the experimental dynamic test performed on several spans (60 m each) of Jakarta-Cikampek Elevated Bridges [7].
2. Materials and methods

In Figure 2, the vertical stiffness of the girder is defined as the required force to generate a deflection at the mid-span of one unit deflection. Considering that \( E_G I_G \) is the flexural stiffness of the girder, \( L \) is the span, and \( m \) is the beam mass assumed concentrated at the mid-span, the vertical stiffness of the girder can be computed through the equation (1).

\[
k = \frac{48 E_G I_G}{L^3}
\]  

Figure 2. Model for determining the stiffness and frequency of the simple span girder

While, the Fundamental frequency is expressed as the following:

\[
f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{48 E_G I_G}{m L^3}}
\]  

where \( m = m/L \) is the mass per length. In a girder supported on real hinge-roller having infinite vertical stiffness, it can be assumed that the half of the girder mass \( \frac{1}{2} m \) is concentrated at the mid-span. The rest of mass is equally distributed to the two supports. Based on this assumption, the frequency can be expressed as \( 1.56\sqrt{(E_I g/m L^4)} \). This value is close to the analytic formula for a simple span girder \( f = 1.57\sqrt{(E_I g/m L^4)} \) [8]. The value of \( m \) in the Equation (2) can be added by the mass of the truck if that additional mass is significant compared to the mass of the girder.

In the steel girder system, the girder is usually nonprismatic in order to obtain economical design, in which the stiffness in the mid-span is larger than that of in the ends of the girder. As seen in Figure 3, using the moment area method, the stiffness of the nonprismatic girder \( k_{NP} \) can be derived as expressed in the Equation (3). If the properties of the prismatic bridge \( \alpha=\beta=1 \) are substituted, the Equation (1) is obtained.

\[
k_{NP} = \frac{48 E_G I_G}{L^3} \left[ \frac{2}{\beta} \frac{1}{2-6\alpha+6\alpha^2-2\alpha^3+6\alpha-6\alpha^2+2\alpha^3} \right]
\]  

where: \( \alpha = \frac{L_A}{L} \); \( \beta = \frac{I_{GB}}{I_G} \)

Figure 3. Model for calculating stiffness of non prismatic girder

When the LRB isolation system is used, the vertical stiffness of the LRB \( k_{LRB} \) reduces the stiffness of the girder, due to the vertical deformation of the LRB, as illustrated in the dynamic model in Figure 4. In this study, two cases are observed.
2.1. Case 1: girders supported on rigid abutment

Based on the assumption that the girder is rigid and the abutment does not experience axial deformation, the stiffness of the girder decreases due to the contribution of the stiffness of LRB, \( k_{LRB} \). With the assumption that the stiffness of the girder is parallel to the stiffness of the LRB, the value of girder stiffness \( k_{eq1} \) can be derived from the unit deflection in the mid-span taking into account the stiffness of the girder and the LRBs, as expressed in the Equation (4). It can be observed that the second term of the denominator in Equation 4 causes the reduction of the girder stiffness due to the presence of the LRB. If the real hinge-roller support with infinite stiffness is used, that second term becomes zero and Equation (1) is obtained. In order to compute the fundamental frequency through the Equation (4), the mass considered is the total mass of the girder because the stiffness of LRB contributes to the equation of motion.

\[
k_{eq1} = \frac{1}{L^3} + \frac{1}{96EGIG} + \frac{1}{4k_{LRB}} \quad (4)
\]

2.2. Case 2: girders supported on flexible cantilever

In flyovers, the girders are usually supported on cantilever pierheads having non prismatic shape. The girder stiffness will decrease further due to the vertical deformation of the cantilever. The stiffness of the structure is the equivalent stiffness taking into account the stiffness of the cantilever, LRB, and girder from the Equation (4) in series form. Two conditions are investigated in this study:

1. If the mass of the truck is neglected, no contribution of the column pillar deformation is considered. Thus, the decrease of the stiffness is caused only by the stiffness of the cantilever. Using the moment area method, the stiffness of the non-prismatic cantilever \( k_{c1} \) can be computed through the Equation (5) [9]. The stiffness of the cantilever and the LRB forms a series system, so that the equivalent stiffness \( k_{eq2} \) can be computed through the Equation (7). The mass of the cantilever in the dynamic equilibrium is assumed as the half-span of the cantilever from the end.

\[
k_{C1} = \frac{2EGICB}{L_C^3} \left[ \frac{\gamma^3}{-2y+ln(1+y)+y^2} \right] ; \gamma = \frac{l_{CA}-l_{CB}}{l_{CB}} \quad (5)
\]

\[
k_{eq2} = \frac{k_{C1}k_{LRB}}{k_{C1}+k_{LRB}} \quad (6)
\]

\[
k_{eq3} = \frac{1}{L^3} + \frac{1}{96EGIG} + \frac{1}{4k_{eq2}} \quad (7)
\]

Where: \( L_C \) is length of cantilever; \( \gamma \) is coefficient of non prismatic cantilever; \( E_C \) is modulus elasticity of cantilever; \( l_{CA} \) is inertia moment of cantilever at section A; \( l_{CB} \) is inertia moment of cantilever at section B where the unit deflection is applied as shown in Fig. 5.

In addition, if more than one concentrated load \( P(t) \) are applied in the cantilever system, the value of \( k_{eq3} \) is the average values of each girder.
2. If the mass of the impulsive truck is relatively large compared to the mass of the girder, a rotation on the pillar column $\varphi$ can be generated due to the nonsymmetrical load. This rotation at the top of the column causes a similar rotation at the end of the cantilever, so that a vertical deflection takes place, as seen in Figure 6. The stiffness of the cantilever $k_{C2}$ decreases further due to the presence of the rotation at the top of the pier column. Using the moment area method, the cantilever stiffness can be computed through Equation (8). The equivalent stiffness of the series system of the LRB and cantilever $k_{eq4}$ is expressed in the Equation (9). The stiffness of the structure $k_{eq5}$ in Equation (10) can be obtained by substituting $k_{eq2}$ in Equation (7) by $k_{eq4}$. The assumed mass for dynamic equilibrium is one-half of the cantilever mass added with the mass of the truck. The mass of the column is neglected because it does not contribute to the vertical stiffness.

$$k_{C2} = \frac{(2E_CI_CB)(E_PIp)n^3}{L_C^3E_PI_P[-2n+\ln(1+n)+n^2]+L_C^2hE_CI_CB[\ln(1+n)n^2]}$$  
$$k_{eq4} = \frac{k_{C2}k_{LRB}}{k_{C2}+k_{LRB}}$$  
$$k_{eq5} = \frac{1}{\frac{L_C^3}{9E_CI_G}+\frac{1}{4k_{eq4}}}$$  

Where: $E_PIp$ is flexural rigidity of pier's column; $h$ is height of pier's column; $n$ is the modulus of elasticity ratio $E_s/E_c$; $E_s$ is the modulus of elasticity of steel; $E_c$ is the modulus of elasticity of concrete.

3. Results and Discussions
For validation, the proposed simplified method is implemented in a simple-span bridge in Jakarta-Cikampek elevated bridge. In every three spans, expansion joint connection is used. Meanwhile in the other ends of the girders, link slab connection is used. The section of the bridge is Composite Steel Box Girder having an effective span of 58 meters and supported on the single column piers with cantilever.
pierheads, as seen in Figure 7 [7]. The girders under study are the one located at the end of the cantilever and the one close to the pier’s column. The accelerometers used for loading test are indicated by red circles. The impulsive load applied is the truck load having weight of 30 tonf that is supported by two girders. Thus, a single girder is assumed to carry 15 tonf of impulsive load.

Figure 7. Elevated steel box girder bridge with span of 60 meter (a). Plan and elevation; (b). Cross section

3.1. Case 1
The location of the observed girders is between piers P349-P350 and the box girders are located at the end of the cantilever. The simple-span composite nonprismatic girder is divided into five segments consisting of two end-segments having web thickness of 16 mm, length of 11,000 mm made of JIS SM520 steel, and three intermediate and middle segments having web thickness of 20 mm, length of 3x12,000 mm made of JIS SM570 steel. The dimension of the steel box girder can be seen in Figure 8a. The cross section of the bridge as presented in Figure 8b indicates the average concrete slab thickness of 318 mm and width of 11,500 mm (the effective width of a single box girder is 4895,5 mm). With the concrete compressive strength $f_{c'}$ of 35 MPa and elastic-section assumption, the moment inertia of the composite girder is obtained $I_G = 3.3177E+11$ mm$^4$. The girder stiffness $k_{NP}$ can be computed through Equation (3) resulting in $16272$ N/mm.

The mass of the bridge of 204.2 N sec$^2$/mm is assumed one half of the total span and concentrated at the mid-span. Using Equation (2) the bridge frequency is obtained 1.422 Hz (with assumption of real hinge-roller support) as described in Tabel 1. If the girder is assumed prismatic, a frequency of 1.424 is obtained. Thus, only 0.14% of different is observed. From the technical specification, the vertical stiffness of the LRB is 1489540 N/mm. Thus, using Equation (3) the values of $k_{eq1} = 32455$ N/mm and fundamental frequency = 1.420 Hz (taking into account the LRB stiffness) are obtained, as reported in Table 1, column 2. That obtained frequency is only 0.14% lower than that of using real hinge-roller assumption. It is cause by the large vertical stiffness of the LRB, which is 91.5 times larger than that of the girder. If the truck mass 15 tonf is taken into account, the frequency of 1.392 Hz is obtained, as seen in Tabel 1, column 3. This value is only 1.97% lower than the frequency without considering the truck mass.

Concrete cracking is known to have a knock-on effect on the durability performance and long-term service life of reinforced concrete structures [1, 2]. This phenomenon is primarily attributed to complex loading conditions that may arise from external loading under day-to-day service in addition to environmental exposures, creating even more complex physical and chemical changes in concrete properties [3]. This will cause not only localised distress and stiffness degradation of structural members but also poor resistance of penetration from aggressive agents (e.g. alkali-silica reaction and chloride transport) which will result in further significant deterioration [4, 5].
In order to analyze the cantilever stiffness, two conditions are considered:

### 3.2. Case 2a

The values of the frequency in Table 1, column 4, are computed as the following: with $I_{CA} = 5.1992E+12$ mm$^4$ and $I_{CB} = 9.60097E+11$ mm$^4$; from Equation (4) it is obtained that $\gamma = 4.415$ and $k_{C1} = 1209641$ N/mm. The stiffness of the system without the truck mass $k_{eq2}$ through Equation (5) and $k_{eq3}$ through Equation (6) are obtained 667539 N/mm and 32151 N/mm, respectively. The frequency computed through Equation (2) is obtained 1.303 Hz. Furthermore, the frequency of the box girder near the pier column is obtained 1.307 Hz and the averaged values is 1.305 Hz as seen in Table 1, column 4. This value is 8.10% lower than the frequency without considering the cantilever stiffness as seen in Table 1, column 2. It indicates that the contribution of the cantilever stiffness is significant to reduce the fundamental frequency of the structure.

### Table 1. Fundamental frequencies of the bridges estimated by the simplified method

| Bridge Location | Span L (m) | Frequency (Hz) |
|-----------------|-----------|----------------|
|                 |   | Real | With LRB | LRB+Truck Load | Including Pier's Stiffness | FEM | Loading Test |
|                 |   | Eq. (3) | Eq. (4) | Eq. (4)+truck mass | Eq. (7) | Eq. (10) |
|                 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| P349-P350-a     | 60 | 1.422 | 1.420 | 1.392 | 1.305 | 1.283 | 1.400 | 1.454 |
| P196-P197-b     | 60 | 1.422 | 1.420 | 1.392 | 1.305 | 1.283 | 1.360 | 1.410 |
| P469-P470-c     | 60 | 1.422 | 1.420 | 1.392 | 1.305 | 1.283 | 1.346 | 1.340 |
| P586-P587-d     | 60 | 1.422 | 1.420 | 1.392 | 1.305 | 1.283 | 1.357 | 1.495 |
| P194-P195-e     | 60 | 1.422 | 1.420 | 1.392 | 1.305 | 1.283 | 1.362 | 1.405 |
| P32-P34-f       | 60 | 1.422 | 1.420 | 1.392 | 1.305 | 1.283 | 1.340 | 1.430 |

Note: a-One girder's edge connected with link slab; b-One bearing supported on steel frames; c-Two girders supported on steel frames; d-One girder supported on one cantilever; e-Four girders supported on steel frames; a-One girder's edge connected with link slab

### 3.3. Case 2b

If the additional mass of truck of 15 tonf is taken into account, having $n=7.2$, $E_pJ_p=1.56406E+17$ N-mm$^2$ and height of the column $h=11806$ mm, it is obtained through Equation (8), (9), and (10) that $k_{C2}$, $k_{eq5}$, $k_{eq6}$ are 673590, 463837, 31982 N/mm, respectively. The frequency of structure, taking into account the rotation of the pier column due to unsymmetrical load, is obtained 1.280 Hz as reported in Table 1, column 5.

Using the similar method, for the second box girder (near the pier column), the frequency of the structure is obtained 1.286 Hz. Thus, the average frequency is 1.283 Hz, as seen in table 1, column 5. This value indicates that 1.69% reduction of the frequency is resulted if the additional mass of truck 15 tons is considered.
In Table 1, column 6, the results from Finite Element Model (FEM) are reported: a fundamental frequency of 1.400 Hz is obtained. This value comes from the analyses performed by the designer, without considering the mass of the truck, based on the 23rd mode in vertical direction and mass participation of 46%. For the other five spans, P196-P197, P469-P470, P586-P587, P194-P195, and P32-P34, in which the other ends of the girders are supported on steel portal frames having various stiffness, their frequencies are reported in Table 1, column 6 (obtained from FEM) and column 7 (obtained from loading test). The average difference of the results between the simplified method (column 4) and FEM is 4.10%. It may be caused by several aspects that cannot be satisfied by the simplified method: for instance the link slab connection at the other end of the girder (even though its stiffness is only 3.95% of the girder stiffness), and the stiffness of the other girder ends because they are supported on the steel portal frames instead of cantilever structure.

As seen in Table 1, column 7, the average frequency obtained from the loading test is 1.422 Hz. This value is based on the observation of the accelerometer which is then post-processed through FFT as presented in Figure 9. The frequency obtained from the loading test is 1.305 Hz and 1.361 Hz larger than which obtained from the simplified analytical method and FEM, respectively. This result indicates that the constructed bridge structure is stiffer than the designed structure.

![Figure 9](image-url). (a) Time domain acceleration history obtained in the loading test; (b) Frequency domain acceleration obtained through Fast Fourier Transform

4. Conclusions
From the above discussions, several conclusions can be drawn:
1. The difference of frequency between prismatic and nonprismatic simple-span girders is not significant. In this study, with box composite section and 60 m of length, the average difference is only 0.14%
2. The application of LRB as the bridge support results in the frequency that is close to the girder supported on real hinge-roller supports. The difference is only 0.14% that can be neglected. Thus, for simple span bridge using LRB isolation, the fundamental frequency can be computed through the empirical formula for simple span beam supported by real hinge-roller supports.
3. The simplified method to compute the fundamental frequency results in the values that 4.10% lower than which obtained from FEM analyses. As a matter of fact, the simplified method does not take into account the link slab stiffness and assumes that all girders are supported on identical cantilever structures.
4. In the loading test, the fundamental frequency is obtained larger than which obtained from the simplified method and FEM analyses, indicating that the bridge is stiffer than the estimated design.

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