The synthesis of the algorithms for adaptive control by nonlinear dynamic objects on the basis of the neural network

V E Bolnokin¹, D I Mutin¹, E I Mutina² and S V Storozhev³

¹Mechanical Engineering Research Institute of the Russian Academy of Sciences, 4, Small Kharitonyevsky lane, Moscow, Russian Federation
²Moscow State Technological University STANKIN, 1, Vadkovski lane, 127055, Russian Federation
³Donetsk National University, 24, University st., Donetsk 283001, DPR, Ukraine

E-mail: vitalybolnokin@yandex.ru, d.i.mutin@mail.ru, e.mutina@mail.ru, stvi@i.ua

Abstract. The present work is devoted to a problem of Synthesis of the algorithms for adaptive control by nonlinear dynamic objects with the incomplete mathematical description. The method of synthesis adaptive neural networks is considered on the basis of application of some positions of a method of analytical designing. The law of adaptation is defined on a condition of maintenance of stability of the closed system with the help of the second method of Lyapunov. The resulting control systems can operate in uncertain conditions caused by external and internal disturbances. The designed parameter adaptation law of the controllers admits simple implementation, thereby facilitating the on-line adaptation process.

1. Introduction

The design problem of adaptive control systems functioning in uncertain conditions is among most important ones of cybernetics and information science. Direct analysis of the recent publications indicates that a series of significant results were obtained in adaptive control theory. In addition to traditional approaches, artificial intelligence theory finds more and more applications in the field of automatic control systems (ACSs) intellectualization. For instance, some methods of embedding artificial intelligence elements in adaptive ACSs were studied in [1–8, 14]. However, their usage in control problems for complex nonlinear dynamic plants faces substantial obstacles.

2. Analysis of problem

The analytic design methods serve for constructing optimal controllers in the sense of some performance criterion only for plants with a given mathematical description and invariable parameters during system functioning. In many situations, realization of the obtained control laws is accompanied by certain difficulties due to the necessity of solving partial differential equations. In some cases, when a plant incorporates links with a nonlinear static characteristic suffering from discontinuities of the first kind, control signal calculation appears even impossible.
On the other hand, researchers develop adaptive control systems for plants with varying parameters and uncertain descriptions. Nevertheless, analysis of design principles applied in traditional adaptive ACSs reveals a series of problems appreciably hampering the synthesis procedure of such systems. The presence of an identification unit for the dynamics of plants makes the ACS structure complex. Identification errors impair control performance or even cause unstable operation of the ACS. The adaptation laws of controller’s parameters can be derived by existing methods only in special cases. In other words, the universal synthesis method of adaptation laws takes no place.

Almost all known design methods of the parameter adaptation algorithms for controllers in nonstationary plants based on a reference model proceed from the hypothesis of quasistationary plant parameters during controller tuning. In fact, most real plants do not satisfy this requirement. As a result, we possibly substantial deviations of the actual performance characteristics from the desired ones and, furthermore, the instability of parameter adaptation procedures.

The existing design methods do not yield parameter adaptation algorithms in the case when all coefficients in the differential equation of the plant dynamics vary with time. The book [12] proposed a design method of the parameter adaptation algorithms for a controller under all varying parameters of an associated plant; however, this method requires accessing a summation unit within the plant. At the same time, the synthesized adaptation algorithms have a rather cumbersome structure causing difficulties in their implementation. This and so, we acknowledge the inherent complexity of adaptation algorithms for most adaptive systems. This explains the topicality of creating analytic design methods of adaptive control systems for nonlinear dynamic plants with simpler structure and uncomplicated implementation of corresponding computational procedures.

The classical design methods of control systems employ the well-developed apparatus of integral and differential calculus (introduced by Newton about three centuries ago) and the Laplace transform. Artificial neural networks (ANNs) represent a modern branch of automatic control theory, which exists for several years and suggests an alternative solution approach to this problem [1, 3, 6, 10-13].

The key role in implementing artificial neural networks in control field belongs to S. Narendra, see the paper [9]. Artificial neural networks have found a wide application in the problems of identification and control of dynamic plants owing to the following properties:

- the capability for learning and accumulation of information;
- the approximation capability;
- the capability for parallel signal processing.

In control systems these networks can be used as controllers and identifiers. Controller and identifier design mostly involve multi-layer direct propagation neural networks, where information moves between layers in the direction of signal propagation (backward motion is prohibited).

The approximation capabilities of neural networks with dynamic learning algorithms describe complex nonlinear dynamic plants in the form of direct and inverse models based on “input-output” measurements.

3. The Suggested Method of Design Adaptive Control Systems

Consider a dynamic plant with the mathematical description

$$\dot{x}_i = f_i(x), bu_i, i = 1, n; j = 1, m,$$

(1)

where $x_i$ are state variables; $b$ denotes a constant coefficient; $f_i$ represent nonlinear functions; $u_i$ mean control signals. Let us study the system with scalar control. The adaptive controller design problem is solved in two steps as follows.
Step 1. Given the plant (1), find a control law ensuring the following conditions:

a) the asymptotic stability of the closed-loop system;

b) minimization of the performance criterion

\[ J = \int_{0}^{\infty} \left[ \psi(x)^2 + \dot{\psi}(x)^2 \right] dt, \]

where \( \psi \) indicates an arbitrary differentiable or piecewise continuous function of the phase coordinates, \( \psi(0) = 0 \).

Since no restrictions apply to the function \( \psi \), the family of stable extremals must meet the Euler equation

\[ \psi + \dot{\psi} = 0 \]  

(2)

The total derivative of the function \( \psi \) has the form

\[ \frac{d\psi}{dt} = \sum_{k=1}^{n} \frac{d\psi}{dx_k} \dot{x}_k \]

or

\[ \frac{d\psi}{dt} = \sum_{k=1}^{n} \frac{d\psi}{dx_k} f_k + \frac{d\psi}{dx_n} u \]  

(3)

\[ \frac{d\psi}{dx_n} u + \sum_{k=1}^{n} \frac{d\psi}{dx_k} f_k + \psi = 0 \]  

(4)

Condition \( \frac{d\psi}{dx_n} \neq 0 \) and (4) allows defining the control signal \( u \) in the form

\[ u = -\left( \frac{d\psi}{dx_n} \right)^{-1} \left[ \sum_{k=1}^{n} \frac{d\psi}{dx_k} f_k + \psi \right]. \]

(5)

This control signal performs transition of the representation point \( x_i \) in the state space of the system from an arbitrary initial state to a certain neighborhood of the manifold \( \psi = 0 \). The control law (5) keeps the representation point within this neighborhood during its further motion along \( \psi = 0 \). Such motion is described by the system of \( n - 1 \) differential equations

\[ \dot{x}_{ip} = f_i \left( x_{1\psi}, x_{2\psi}, \ldots, x_{(n-1)\psi} \right) \]  

(6)

Hence, for achieving the asymptotic stability of the closed-loop system, one has to choose a function \( \psi \) so that the solution of the system of differential equations (6) characterizing the motion of the representation point along the manifold \( \psi = 0 \) towards the coordinate origin appears stable. Note that the performance of the control system is determined by the corresponding manifolds.

Step 2. In the absence of sufficient information on the functions \( f_i \) and the coefficient \( b \), it turns out impossible to implement the control law (5). In this case, an alternative solution consists in adaptive controller usage. As it has been mentioned, the output signal of a neural network possesses high
approximation capability. This suggests that addressing neural networks would yield a good result. A neural controller forms a control action in the form

$$u = \frac{\sum_{j=1}^{n} \mathcal{A} \left( \prod_{i=1}^{n} \mu_{j}^{i} (x_{i}) \right)}{\sum_{j=1}^{n} \left( \prod_{i=1}^{n} \mu_{j}^{i} (x_{i}) \right)} \tag{7}$$

where $\mathcal{A}$ designates a numerical value of the control signal such that $\mu_{j} \left( \mathcal{A} \right) = 1$.

Introduce the vector function

$$\zeta \left( \mathcal{A} \right) = \left( \zeta^{1} \left( \mathcal{A} \right), \zeta^{2} \left( \mathcal{A} \right), \ldots, \zeta^{n} \left( \mathcal{A} \right) \right)^{T}, \tag{8}$$

where $\zeta^{j} \left( \mathcal{A} \right)$ defined by

$$\zeta^{j} \left( \mathcal{A} \right) = \frac{\prod_{i=1}^{n} \mu_{j}^{i} (x_{i})}{\sum_{j=1}^{n} \left( \prod_{i=1}^{n} \mu_{j}^{i} (x_{i}) \right)} \tag{9}$$

Having in mind formula (9), rewrite the expression (7) as

$$u = \mathcal{A}^{T} \zeta \left( \mathcal{A} \right). \tag{10}$$

According to (10), the control signal represents a function of the state variables. However, in some cases without direct measurement of all state variables, the control signal can be specified depending on the error vector.

4. The Adaptive Law for the Parameters of a Neural Controller

The values $\mathcal{A}^{j}$ are unknown during solution of the design problems. Formulate the evaluation problem for $\mathcal{A}^{j}$ based on the stabilization condition of the control system. For the sake of simple exposition, consider the following second-order system:

$$\begin{aligned}
\dot{x}_{1} &= f_{1} \left( x \right) \\
\dot{x}_{2} &= f_{2} \left( x \right) + u,
\end{aligned} \tag{11}$$

Here $f_{i}$, $i = 1, 2$ are nonlinear functions; $x = [x_{1}, x_{2}]^{T}$ means the state vector; $y = x_{1}$ stands for the output signal. By assumption, some coordinates of the state vector are unmeasurable. In this case, it appears impossible to implement a desired control law via the state vector. The discussed problem can be solved by constructing control actions based on the control error vector.

To define the control law, choose the function $\Psi \left( \varepsilon \right) = a_{1} \varepsilon_{1} + a_{2} \varepsilon_{2}$, where $\varepsilon = [\varepsilon_{1}, \varepsilon_{2}]^{T}$ denotes the error vector; $\varepsilon_{1} = y_{m} - y = x^{3} - x_{1}$ is the control error; $\varepsilon_{2} = \dot{\varepsilon}_{1}$ corresponds to the rate of change of the error; $a_{1}$, $a_{2}$ specify constant coefficients. Find the derivatives of the function $\Psi$ with respect to the state vector coordinates:
The solution of the differential equation (16) is defined by
\[ \frac{d\Psi}{dx_1} = \alpha_1 \frac{df_1}{dx_1} + \alpha_2 \frac{df_2}{dx_2} = -\alpha_1 - \alpha_2 \frac{df_1}{dx_1}. \]  
(12)

According to (11), (12) and (13), the control law (5) takes the form
\[ \ddot{u} = \left[ \alpha_2 \frac{df_2}{dx_2} \right]^{-1} \left[ \frac{1}{T} \Psi + \left( -\alpha_1 - \alpha_2 \frac{df_1}{dx_1} \right) \dot{f}_1 - \alpha_2 \frac{df_2}{dx_2} \right]. \]  
(14)

Let us demonstrate that the control law (14) ensures the zero-control error. For this, take into account the expression (14) and rewrite the system of equations (11) as
\[ \dot{x}_i = \frac{df_i}{dx_i} \dot{x}_i + \frac{df_i}{dx_i} \dot{x}_2 = \frac{\alpha_i}{\alpha_2 T} (y_m - x_i) - \left( 1 + \frac{\alpha_i}{\alpha_2 T} \right) \dot{x}_i, \]
\[ \dot{x}_i + \left( 1 + \frac{\alpha_i}{\alpha_2 T} \right) \dot{x}_i + \frac{\alpha_i}{\alpha_2 T} (x_i - y_m) = 0. \]  
(15)

Introduce the following notation:
\[ z = x_i^3 - x_i; a_1 = 1 + \frac{\alpha_1}{\alpha_2}; a_0 = \frac{1}{T \alpha_2}. \]

Therefore, equation (15) can be transformed into
\[ \ddot{z} + a_1 \dot{z} + a_0 z = 0. \]  
(16)

The solution of the differential equation (16) is defined by
\[ z(t) = C_1 \exp \left[ -\frac{1}{2} \left( a_1 - \sqrt{a_1^2 - 4a_0} \right) t \right] + C_2 \exp \left[ -\frac{1}{2} \left( a_1 + \sqrt{a_1^2 - 4a_0} \right) t \right]. \]  
(17)

We can see: \( \lim_{t \to 0} | \dot{x}_i | = \lim_{t \to 0} | x_i^3 - x_i | = \lim_{t \to 0} | z | = 0. \)

To construct the adaptation law, reduce the system of equations (11) to
\[ \begin{cases} \dot{x}_i = f_i (x) \\ \dot{x}_2 = f_2 (x) + u + \ddot{u} \end{cases} \]  
(18)

Using the expression (14) and the \( \ddot{u} = \tilde{z}^T \zeta (z) \), rewrite the system of equations (18) in the form
\[ \begin{cases} \dot{x}_i = f_i (x) \\ \dot{x}_2 = \left[ \frac{\alpha_2}{dx_2} \right]^{-1} \left[ \frac{1}{T} \Psi + \left( -\alpha_1 - \alpha_2 \frac{df_1}{dx_1} \right) \dot{x}_i + \left( \tilde{z}^T (x) \right) \zeta (\zeta) \right] \end{cases} \]
Choose the Lyapunov function in the form

\[ V = \frac{1}{2} \dot{\theta}^T \dot{\theta} + \frac{1}{2} \gamma \dot{\theta}^T \dot{\theta}, \]

where \( \dot{\theta} = \dot{\theta} - \dot{\theta}^* \) and \( \gamma \) is a positive coefficient. The total derivative of the Lyapunov function is described by

\[ \dot{V} = \Psi \dot{\Psi} + \frac{1}{\gamma} \dot{\theta}^T \dot{\theta} = \frac{1}{T} \dot{\theta}^T \dot{\theta} + \dot{\theta}^T \left[ \frac{1}{\gamma} \dot{\theta} - \Psi \alpha \frac{df_i}{dx_n} \zeta(\epsilon) \right]. \]

The total derivative can be transformed into

\[ \dot{V} = -\frac{1}{T} \dot{\theta}^T \dot{\theta} \leq 0, \]

where

\[ \dot{\dot{\theta}} = \gamma \Psi \zeta(\epsilon) = \gamma \alpha \frac{df_i}{dx_n} (\alpha_1 e_1 + \alpha_2 e_2) \zeta(\epsilon), \]

which guarantees the global stability of the closed-loop control system.

Without loss of generality, the presented method allows designing adaptive control systems for n-order nonlinear plants obeying the system of equations (1). In this case, the adaptation law acquires the form

\[ \dot{\alpha} = \gamma \alpha \frac{df_i}{dx_n} \Psi(\epsilon) \zeta(\epsilon). \]

The error vector is

\[ \epsilon = [e_1, e_2, ..., e_n]^T = [\hat{e}, \hat{e_{(n-1)}}] \text{ and } \Psi(\epsilon) = \sum_{k=1}^{n} \alpha_k e_k. \]

5. Conclusion

The suggested method possesses the following advantages: (1) controller synthesis for plants with an inaccurate mathematical description and (2) proper consideration of the accumulated knowledge and experience of human operators during control law definition. Owing to their adaptive capability, the resulting control systems can operate in uncertain conditions caused by external and internal disturbances. The designed parameter adaptation law of the controllers admits simple implementation, thereby facilitating the on-line adaptation process.

At the same time, analysis of the considered examples and control systems with neural networks described in numerous publications (see the overview in [10]) identifies a series of specific features in their application:

• there exist no analytic methods for choosing the structure of neural networks and the number of neurons;
• during control laws formation, the framework of the theory of neural networks disregards the knowledge and experience of experts; this suggests that the approaches of the theories of fuzzy algorithms and neural networks should be integrated to develop perfected control systems for complex nonlinear dynamic plants;
• still, an open issue concerns the stability of a control system with a neural controller;
• as a rule, the learning procedure of a neural network consumes much time depending on the number of neurons.

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