Nanostructured optical waveguide with a highly confined mode: supplement

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For the cylindrical waveguide under study, the longitudinal components of the electric $E$ and magnetic $H$ fields of the $HE_i$ mode are determined by the equations [1]

\[
E_{z}^{(in)} = A_c \alpha \left( \sqrt{\varepsilon_{c}\kappa_{z}^{2} - k_{r}^{2}} \right) \exp[i \kappa_{z} z + i \phi],
\]
\[
E_{z}^{(out)} = A_c \beta \left( \sqrt{\varepsilon_{c}\kappa_{z}^{2} - k_{r}^{2}} \right) \exp[i \kappa_{z} z + i \phi],
\]
\[
H_{z}^{(in)} = A_r \alpha \left( \sqrt{\varepsilon_{r}\kappa_{z}^{2} - k_{r}^{2}} \right) \exp[i \kappa_{z} z + i \phi],
\]
\[
H_{z}^{(out)} = A_r \beta \left( \sqrt{\varepsilon_{r}\kappa_{z}^{2} - k_{r}^{2}} \right) \exp[i \kappa_{z} z + i \phi],
\]

where $b = a + d_{c}a$ is the external radius of the waveguide, $\varepsilon_{m,cladd,out}$ are the permittivities of the waveguide core, cladding, and outer medium, respectively, $I_{1}, K_{1}$ are the modified Bessel function of the first and second kind, $A_{c}, B_{c}, C_{c}, D_{c}$ are the amplitudes of longitudinal field components, $n_{z} = k_{z} / k_{0}$ is the mode effective index. The transverse components are connected with longitudinal components via the relations following from the Maxwell equations [1]

\[
E_{r}^{(i)} = \frac{1}{\varepsilon_{r}\kappa_{z}^{2} - k_{r}^{2}} \left( i k_{r} \frac{\partial E_{z}^{(i)}}{\partial r} - \frac{k_{0}}{r} H_{z}^{(i)} \right),
\]
\[
E_{\phi}^{(i)} = \frac{1}{\varepsilon_{r}\kappa_{z}^{2} - k_{r}^{2}} \left( -i k_{0} \frac{\partial E_{r}^{(i)}}{\partial r} - \frac{k_{r}}{r} H_{z}^{(i)} \right),
\]
\[
H_{r}^{(i)} = \frac{1}{\varepsilon_{r}\kappa_{z}^{2} - k_{r}^{2}} \left( \frac{\kappa_{0}}{r} E_{z}^{(i)} + i k_{r} \frac{\partial H_{z}^{(i)}}{\partial r} \right),
\]
\[
H_{\phi}^{(i)} = \frac{1}{\varepsilon_{r}\kappa_{z}^{2} - k_{r}^{2}} \left( i k_{r} \frac{\partial E_{z}^{(i)}}{\partial r} - \frac{k_{0}}{r} H_{z}^{(i)} \right).\]
where \( i = \text{in}, \text{clad}, \text{out} \). The amplitudes \( A_{E,H}, B_{E,H}, C_{E,H}, D_{E,H} \) are found from the boundary conditions at \( r = a \) and \( r = b \). Their values are normalized by the condition
\[
\max \left( \frac{\partial (\varepsilon \omega)}{\partial \omega} |E|^2 + |H|^2 \right) = 1.
\]

**References**

1. C. Yeh and F. I. Shimabukuro, *The essence of dielectric waveguides* (Springer, Berlin, 2008).