REMARKS ON GEOMETRIC QUANTIZATION
OF R-MATRIX TYPE POISSON BRACKETS

Alexey Kotov *

Department of Theoretical Physics, Uppsala University
P.O. Box 803, S-75108, Uppsala, Sweden

We check the Vaisman condition of geometric quantization for R-matrix type Poisson pencil on a coadjoint orbit of a compact semisimple Lie group. It is shown that this condition isn’t satisfied for hermitian symmetric spaces. We construct also some examples when Vaisman condition takes place.

*E-mail: kotov@rhea.teorfys.uu.se
‡ permanent address: Chair of Nonlinear Dynamic Systems and Control Processes,
Department of Computational Mathematics and Cybernetics, Moscow State University,
Vorob’evy Gory, Moscow 119899, Russia
IMEM, Odessa State University, Petra Velikogo 2, Odessa 270000, Ukraine
1 Introduction

Let $G$ be a semisimple Lie group and $r$ be the Drinfeld-Jimbo $R$-matrix which satisfies the modified Yang-Baxter equation.

Consider an orbit $O$ in $g^*$ of the coadjoint action of $G$. There exists a Poisson bracket $\{\ , \}_{KKS}$ (the Kirillov-Kostant-Souriau bracket) on $O$.

We can introduce another Poisson structure on $O$ by two ways.

(i) $G$ acts on $O$ and preserves the KKS-bracket, therefore there is a representation $\rho: g \to \text{Der}(C^\infty(M), \{\ , \}_{KKS})$.

The map $\{\ , \}_r: C^\infty(M)^{\otimes 2} \to C^\infty(M)$ given by $\{f_1, f_2\}_r = \langle (\rho \otimes \rho)(r), (df_1 \otimes df_2) \rangle$ defines a skewsymmetric multiplication on $C^\infty(M)$. It satisfies the Jacobi identity iff orbit $O$ is a R-matrix type. The restriction of the 3-vector $[r, r] = [r_{12}, r_{13}] + [r_{12}, r_{23}] + [r_{13}, r_{23}]$, where $r_{12} = r \otimes 1$, on $O$ is identically zero in this case. It’s true if and only if $O$ is an orbit of a semisimple element and $O$ is a symmetric space or $O$ is an orbit of nilpotent element of height 2 [3]. The brackets $\{\ , \}_r$ and $\{\ , \}_{KKS}$ are always compatible or form a Poisson pencil on this orbit.

(ii) Let $K$ be a compact real form of the semisimple complex Lie group $G$ and $O$ is an orbit in $k^*$. Let us consider $O$ as a Poisson coset space of Poisson Lie group $G$ with the coboundary Sklyanin-Drinfeld brackets defined on $G$ as follows

$$\{f_1, f_2\} = \{f_1, f_2\}_L - \{f_1, f_2\}_R = \langle (\rho^\otimes 2_L - \rho^\otimes 2_R)(r), df_1 \otimes df_2 \rangle,$$

where $\rho_R(\rho_L)$ is the representation of the corresponding Lie algebra $g$ in the space $C^\infty(M)$ by the left-(right-) invariant vector fields.

The proper structure on $O$ can be obtained as a result of Poisson reduction [9]. It is also called the Sklyanin-Drinfeld brackets and denoted as $\{\ , \}_{SD}$. The Kirillov-Kostsant-Souriau and Sklyanin-Drinfeld brackets form a Poisson pencil iff $O$ is a hermitian symmetric space [6].

It can be proved that the Poisson pairs $a\{\ , \}_{KKS} + b\{\ , \}_{SD}$ and $a\{\ , \}_{KKS} + b\{\ , \}_r$ coincide on hss.

The simple inspection of this Poisson structures associated with $r$ shows that they are quite degenerate. Recently I.Vaisman proposed the natural generalization
of the geometric quantization scheme in the case of Poisson manifolds. Though the r-matrix structures we introduce in the context of the deformation quantization and they are quasiclassical limit of the quantum group structures, it would be interesting to check the Vaisman condition in this natural setting.

Our goal is to verify the Vaisman condition of geometric quantization for R-matrix type Poisson brackets \( \{ , \}_{SD} + \lambda \{ , \}_{KKS} \). We are show that generalization of the quantization conditions is failed in the case of hermitian symmetric spaces. We give two simple examples when this condition is valid for the Poisson 2-matrix structures.

In the first part of this work we introduce a Poisson pencil connected with the standard modified r-matrix and formulate the problem. In the second part we review some basic results of the theory of Poisson Lie groups and coset spaces. We also check the conditions when the Poisson pencil is degenerate. In the third part we discussed the Vaisman conditions for the geometric quantization of this Poisson pencil.

2 Sklyanin-Drinfeld brackets on Hermitian Symmetric Spaces.

Definition. A Lie group \( G \) is called a Poisson Lie group if it is a Poisson manifold such that the multiplication \( G \times G \to G \) is a morphism of Poisson manifolds, where \( G \times G \) is equipped with the product Poisson structure.

Every Poisson Lie structure on a semisimple connected or a compact Lie group can be written in the coboundary form

\[
\pi(g) = l_{g^*}(r) - r_{g^*}(r)
\]

where \( r \in \Lambda^{\otimes 2} g \) satisfies the modified Yang-Baxter equation, i.e. \([r, r]\) is \( AdG \)-invariant [2].

Definition. A Lie subgroup \( H \) of a Poisson Lie group \( G \) which has its own Poisson Lie structure is called a Poisson Lie subgroup if the inclusion \( i : H \hookrightarrow G \) is a Poisson morphism.
A coset space $G/H$ with a Poisson structure is called a Poisson coset space if the natural map $G \to G/H$ is a Poisson one.

Let us assume that $H$ is connected. There exists the Poisson structure on $G/H$ if a subspace $C^\infty(G/H)$ of $H$-right invariant functions on $G$ is a subalgebra of Poisson algebra $C^\infty(G)$\[9].

The intrinsic derivative of a Poisson tensor $\pi$

$$d_e : g \to g \wedge g$$

given by $X \to (L_X \pi)(e)$, where $\bar{X}$ is any vector field on $G$ with $\bar{X}(e) = X$ define a Lie algebra structure $[\, , ]^* : \bigwedge^2 g^* \to g^*$ on $g^*$ dual to $d_e = \delta$ and hence a Lie bialgebra structure on $g \oplus g^*$.

**Proposition.** [9] $H$ is a Poisson Lie subgroup iff $h^\perp$ is an ideal in $g^*$ (where $h$ is a Lie algebra of $H$). $G/H$ is a Poisson coset space iff $h^\perp$ is a subalgebra.

Let $g$ be a semisimple Lie algebra over $\mathbb{C}$, $g_0$ be the same algebra considered over $\mathbb{R}$. Let $k$ be a compact real form of $g$, i.e. $k$ is a fixed point set of a standard Chevalley antiinvolution $\sigma$ of $g$:

$$\sigma(E_{\pm \alpha},H_\alpha) = -E_{\mp \alpha}, -H_\alpha, \sigma(\lambda X) = \bar{\lambda} \sigma(X)$$

where $E_{\pm \alpha}, H_\alpha$ is a Chevalley basis of $g$ with respect to a Cartan subalgebra $h$.

Let $h_c$ be a Cartan subalgebra of $k$ and $h = Ch_c$. If we choose the system $\Delta_+$ of positive roots of $h$ in $g$, then we have a decomposition

$$g_0 = k + a + n_+$$

where $n_+ = \sum_{\alpha \in \Delta_+} g_\alpha$, $a = i h_c$ is non-compact part of the Cartan subalgebra. This additive decomposition leads to the *Iwasawa decomposition* $K \times A \times N \to G_0$.

Using the Iwasawa decomposition we introduce a Lie bialgebra structure for the complex semisimple Lie algebra $g$ in the following way:

$$\delta(X) = [X \otimes 1 + 1 \otimes X, r],$$

where $r$ is the Drinfeld-Jimbo R-matrix.
\[ r = \frac{i}{2} \sum_{\alpha \in \Delta_+} E_\alpha \wedge E_{-\alpha} \]  
\[ (8) \]

In the compact case, Lie bialgebra structure for \( k \) is a real form of the standard Lie bialgebra structure for \( g \), given by r-matrix

\[ r_o = \frac{1}{4} \sum_{\alpha \in \Delta_+} V_\alpha \wedge W_\alpha \]  
\[ (9) \]

Let \( P \) be a parabolic subgroup in \( G \). Every coadjoint orbit of a compact group \( K \) for a fixed point \( x \) is naturally isomorphic to a coset \( K/K_p \), where \( K_p = P \cap K \).

\( K_p \) is a Poisson subgroup of the Poisson Lie group \( K \). Hence we conclude that every coadjoint orbit \( O \) in \( K \) has the natural Poisson structure \( \pi_{SD} \) given by the Poisson reduction. The symplectic leaves of this structure are the \( B \)-orbits on \( G/P \cong K/K_p \) or Bruhat cells [8].

**Definition.** We shall call two Poisson structures compatible if every linear combination of them is also a Poisson structure. This means that Schouten-Nijenhuis bracket of the corresponding bivector fields is equal to zero.

Let \( K \) be a compact real form of a simple complex Lie group \( G \) and \( P \) be a parabolic subgroup in \( G \), \( K_p = P \cap K \), \( p \) and \( k_p \) be Lie algebras of \( P \) and \( K_p \). Denote by \( \Delta_p \) the following subset of positive roots

\[ \Delta_p = \{ \alpha \in \Delta_+ | E_{-\alpha} \notin p \} \]  
\[ (10) \]

It is useful to remark that in the case of symmetric decomposition

\[ k = k_x \oplus m_x \]  
\[ (11) \]

is only one simple root \( \alpha \) belongs to \( \Delta_p \)[5].

Let \( O_x \simeq K/K_p \) be a coadjoint orbit for \( k \) with a fixed point \( x \).

**Theorem.** [6] The Kirillov-Kostant-Souriau \( \{,\}_{KKS} \) on \( O_x \) is compatible with the Poisson Lie structure \( \{,\}_{SD} \) induced by the coboundary Poisson Lie structure on \( G \) if and only if this orbit is a hermitian symmetric space.

**Proposition.** The structure \( \pi_\lambda = \pi_{SD} + \lambda \pi_{KKS} \) on a hermitian symmetric space is degenerate iff \( \lambda \in [-2, 0] \).
Proof.

Let us complete the orthonormal system

\[ \{ V_\alpha, W_\alpha | \alpha \in \Delta_p \} \cap \{ V_\alpha, W_\alpha | \alpha \in \Delta_+ \setminus \Delta_p \} \]  

(12)

to the orthonormal basis with respect to the scalar product \(-\frac{1}{2}tr( , )\). The coadjoint action of \(K\) preserves Kartan-Killing form \(tr( , )\) on \(k\) so we are able to identify \(Adg^{-1} = (Adg)^* = (Adg)^t\).

Let us denote the corresponding matrixes of bivectors \(r_o, r_p\) and \(r_\lambda\), where

\[ r_\lambda = l_{g^{-1}}\pi_\lambda(g) = r_o - Adg^{-1}r_o + \lambda r_p \]  

(13)
as \(R_o, R_p, R_\lambda\) respectively.

If \(dim \mathcal{O}_x = 2m\) then

\[ R_p = \frac{1}{4} \begin{pmatrix} J_{2m} & 0 & \cdots \\ 0 & 0 & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}, \text{where} J_{2m} = \begin{pmatrix} 0 & 1 & \cdots \\ -1 & 0 & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} \]  

(14)

It is easy to show that the structure \(\pi_\lambda(g)\) is degenerated if and only if the left upper \(2m \times 2m\)-minor of \(R_\lambda\) is degenerate. The rank of the left upper \(2m \times 2m\)-minor is equal to the rank of \(16R_o R_p\).

It’s evident to see that

\[ \det \left( 16Adg^t R_o Adg R_p + (\lambda + 1)E_p \right) = 0 \]  

(15)

where \(E_p = -16R_o R_p = -16R_p^2\)

We also see that

\[ \det \left( 16Adg^t R_o Adg R_p + (\lambda + 1)E \right) = 0. \]  

(16)

This is a characteristic equation for matrix \(16Adg^t R_o Adg R_p\). Therefore

\[ |\lambda + 1| \leq \| 16Adg^t R_o Adg R_p \| \leq 1, \text{where} \| A \| = \sup_{\|v\|=1} \langle Av, v \rangle. \]  

(17)
Now we have to prove the necessary condition.

Let us consider a subgroup $K_\alpha \subset K$ with Lie subalgebra $k_\alpha = RH_\alpha \oplus RV_\alpha \oplus RW_\alpha$, where $\alpha$ is a single root from $\Delta_p$. It is well-known that such global subgroup exists.

The orbit of $K_\alpha$-action on $O_x$ is naturally isomorphic to the ordinary sphere. Moreover, symplectic leaves of the induced structure on a "small" orbit coincide with the intersection of symplectic leaves on $O_p$ and this orbit (if we consider it as a Poisson coset space of the Poisson Lie subgroup $K_\alpha$. $\pi_\lambda$ on $CP^1$ is degenerate for all $\lambda \in [-2, 0]$. Thus we conclude that our structure $\pi_\lambda$ is degenerate for all $\lambda \in [-2, 0]$ too. □

Let $w$ be an element of the normalizer $N(H)$ of the maximal length ih Wejl group $W(\Delta_+)$. Then $Adw(r_o) = -r_o$ because $Adw|_{\Delta_+} : \Delta_+ \to -\Delta_+$.

**Proposition.** $l_w \pi_\lambda(g) = -\pi_{-(\lambda+2)}(g)$.

**Proof**

$$(l_w) \pi_\lambda(g) = l_g(\pi_\lambda(w^{-1}g)) = l_g(l_{w^{-1}g}) \pi_\lambda(w^{-1}g) = l_g(r_o - Adg^{-1}(w^{-1}g)r_o + \lambda r_p) = l_g(-r_o + Adg^{-1}r_o + (\lambda + 2)r_p) = -l_g(\pi_{-(\lambda+2)}(g)).$$

That is why the topological properties (i.e. topological structure of symplectic leaves, quantization condition) of $\pi_\lambda$ and $\pi_{-(\lambda+2)}$ are equivalent. If $\lambda = 0$ then $\pi_o = \pi_{DS}$ is equivalent to $\pi_{-\lambda}$.

## 3 On Geometric Quantization of R-type Poisson Pencil

**Definition.** The geometric quantization of a Poisson structure is a map of some subset of smooth functions on smooth manifold $M$ to the set of hermitian operators which acts in the space of the global cross sections of the hermitian line bundle over $M$. Moreover, the following equation must be satisfied

$$\{\hat{f}_1, \hat{f}_2\} = \frac{2\pi i}{\hbar}[\hat{f}_1, \hat{f}_2],$$

where $f_1, f_2 \in C^\infty(M)$ and $\hat{f}$ is the image of $f$ (operator), $\hbar$ is the Planck constant.
In the symplectic case some fundamental results were obtained:

\[ \hat{f} = f + \frac{\hbar}{2\pi i} \nabla c(f). \]  

(19)

Here \( c(f) \) is a vector field which acts as \( c(f)g = \{f,g\} \) for all \( g \in C^\infty(M) \), \( \nabla \) is a covariant derivative which set a linear connection with the curvature form \( \Omega \) and \( \Omega = \frac{1}{2\pi} \omega \), where \( \omega \) is a symplectic form on \( M \). It was shown that \( \Omega \) is a curvature form of some complex line bundle if and only if the cohomology class of \( \Omega \) is integer. The hermitian structure on the line bundle which endows the space of global sections with Hilbert space structure exists and is preserved iff this form \( \Omega \) is real, i.e. \( \omega \in \hbar H^2(M,\mathbb{Z}) \).

Some applications of this scheme were proposed by Vaisman [10] in a degenerated case.

Let \( M^m \) be a smooth manifold with a Poisson structure defined by a bivector \( \pi \). Let \( \mathcal{A} \) is a Poisson algebra, where \( \mathcal{A} = (C^\infty(M), \{ , \}) \).

We say that \( M \) permits a prequantization over a complex line bundle \( L \to M \) if the following identities are satisfied:

\[ \{\hat{f}_1,\hat{f}_2\} = 2\pi i \frac{\hbar}{h} \hat{[f_1,f_2]}, \]

(20)

where \( \hat{f}_j = f_j + \frac{\hbar}{2\pi i} \nabla f \).

The differential operator \( \nabla_f \) acts in the space of global sections as

\[ \nabla_f gs = \{f,g\} s + g \nabla f s \]

(21)

for all \( f, g \in C^\infty(M), s \in \Omega^o(L) \).

Let us consider an operator \( \delta : \Upsilon^*(M) \to \Upsilon^{*+1}(M) \) of Poisson differential in the following way:

\[ \delta f = c(f), \delta X = [[\pi,X]], \]

(22)

where \([ [ , ] \) is the Schouten-Nijenhuis bracket, \( f \in C^\infty(M), X \in \Upsilon^*(M) \) is a polivector field. It is easy to show that

\[ \mathcal{P}(\lambda)(\alpha_1,\ldots,\alpha_k) = (-1)^k \lambda(\mathcal{P}(\alpha_1)\cdots\mathcal{P}(\alpha)) \]

for all \( \alpha_1,\ldots,\alpha_k \in \Omega^1(M) \), where \( \beta(\mathcal{P}(\alpha)) = \pi(\alpha,\beta) \forall \alpha,\beta \in \Omega^1(M), \lambda \in \Omega^k(M) \) and \( \mathcal{P}(\lambda) \in \Upsilon^k(M) \).
\( \mathcal{P} \) is a Hamiltonian operator defined by \( \pi \).

It was shown that the Jacobi condition for the Poisson bracket \( \{ \cdot, \cdot \} \) is equivalent to the condition \( \delta^2 = 0 \) which is satisfies iff \( [[\pi, \pi]] = 0 \). Therefore \((\Upsilon^*(M), \delta)\) is a complex which is named the Poisson complex with Poisson cohomologies \([7]\).

\[
H^k(A) = \frac{Ker\{\delta^k : \Upsilon^k(M) \to \Upsilon^{k+1}(M)\}}{Im\{\delta^{k-1} : \Upsilon^{k-1}(M) \to \Upsilon^k(M)\}} \tag{23}
\]

**Theorem.** \([10]\) The Poisson manifold \((M, \pi)\) has a quantization bundle if and only if there exist a vector field \(X\) and a closed 2-form \(\omega\) that represents an integer cohomology class of \(M\), such that the relation

\[
\pi + L_X \pi = \mathcal{P}(\omega) \tag{24}
\]

holds.

In that case a class \(\frac{1}{2\pi}[c] \in H^2(A)\) called the Poisson-Chern class \(c_1(\pi)\) is well-defined. It’s image of integral class from \(H^2(M, \mathbb{Z})\).

**Example 1.** Let \((p, q) \in \mathbb{R}^2\) is standard symplectic space with such brackets correlation:

\(\{p, q\} = 1\).

We introduce a new structure

\[
\{f_1, f_2\}_{\text{new}} = \{f_1, h_1\}\{f_2, h_2\} - \{f_1, h_2\}\{f_2, h_1\}, \text{ where } h_1 = p, h_2 = pq.
\]

Here \(\pi = p \frac{\partial}{\partial p} \wedge \frac{\partial}{\partial q}\) and \(\{p, q\} = p\). The Poisson structure is degenerated if \(p = 0\). Because of \(H^*(\mathbb{R}^2) = 0\) we can get a trivial complex bundle \(\mathbb{R}^2 \times C \to \mathbb{R}^2\) and the set of global cross sections is equal to \(C^\infty(M) \otimes C\). That’s why \(\omega = 0\).

Our equation \(\pi + [[X, \pi]] = 0\) has a nontrivial solution \(X = q \frac{\partial}{\partial q}\). This bracket was also quatized by means of the Drinfeld series \([4]\).

**Example 2.** Let us consider an orbit of a nilpotent element in \(g^*\), where \(g = sl(2, \mathbb{R})\).

We shall identify \(g^*\) with \(g\) using the scalar product \(tr \ < \ >\). Let us introduce a coordinate system \((x_1, x_2, x_3)\) in \(g\), such that every matrix can be written in the
The equation $x_1^2 + x_2^2 = x_3^2$ corresponds to the singular orbit of the nilpotent element.

In spherical coordinates

\[
\begin{align*}
x_1 &= r \cos \phi \sin \theta \\
x_2 &= r \sin \phi \sin \theta \\
x_3 &= r \cos \theta
\end{align*}
\]

this orbit is given by $\theta = \frac{\pi}{4}$.

The Drinfeld-Jimbo R-matrix from $\Lambda^2 g$ defines the \(r\)-brackets

\[\{r, \phi\}_r = -\frac{r}{2} \sin \phi\]  

which are degenerated if $\sin \phi = 0$.

We construct a geometric prequantization in a trivial complex line bundle as above:

\[
\hat{F} = F + \frac{\hbar}{2\pi i} c(F) + X(F),
\]

where $X = -r \ln r \frac{\partial}{\partial r}$ is a solution of the equation (24).

Let us study the geometric quantization of R-matrix type Poisson family $\pi_\lambda$ on $O_x$-orbit of a compact semisimple Lie group $K$.

In particular case $K = SU(2), O_x = CP^1$. $\pi_\lambda$ can be written in the form

\[
\pi_\lambda = -\frac{i}{2}(1 + |z|^2)(\lambda + (\lambda + 2)|z|^2) \frac{\partial}{\partial z} \wedge \frac{\partial}{\partial \bar{z}}.
\]

For $\lambda \in [-2, 0]$ the Poisson bivector is degenerate on the set

\[
\Xi_\lambda = \{z||z|^2 = -\frac{\lambda}{\lambda + 2}\}.
\]

The geometric quantization condition is $\pi_\lambda + \delta X = \mathcal{P}\omega$, where $[\omega] \in H^2(CP^1, Z)$

This is equal to $\mathcal{P}^{-1}(\pi_\lambda) + d(\mathcal{P}^{-1}X) = \omega$ everywhere except $z \in \Xi_\lambda$. In other words
\[-\frac{2}{i} \frac{1}{1 + |z|^2} \frac{1}{(\lambda + (\lambda + 2)|z|^2)} dz \wedge d\bar{z} = \omega - d \left( \frac{1}{\lambda + (\lambda + 2)|z|^2}\sigma \right), \quad (30)\]

where \(\sigma \in \Omega^1(CP^1)\).

We obtain using Stocks formula

\[\int \int_{|z| \geq \xi} P^{-1}(\pi_\lambda) = \int \int_{|z| \geq \xi} [-d \left( \frac{1}{\lambda + (\lambda + 2)|z|^2}\right) \sigma + \omega] \quad (31)\]

\[-2\pi \ln \left( \frac{\lambda + (\lambda + 2)|z|^2}{(\lambda + 2)(1 + \xi^2)} \right) = \phi(\xi) - \frac{1}{\lambda + (\lambda + 2)|z|^2}\psi(\xi), \quad (32)\]

where

\[\phi(\xi) = \int \int_{|z| \geq \xi} \omega \quad \text{and} \quad \psi(\xi) = \int_{|z| = \xi} \sigma, \phi, \psi \in C^\infty([\xi_0, +\infty]) \quad (33)\]

If \(\xi \to +\xi_0\) the left part of the equation contains a logarithmic singularity versus polar one of the right part. Hence the geometric quantization condition is not satisfied contrary to assumption. \(\Box\)

**Proposition.** For all \(\lambda \in [-2, 0]\) the geometrical condition for \(\pi_\lambda\) is not satisfied.

**Proof.** Let us consider again a subgroup \(K_\alpha \subset K\) with Lie subalgebra \(k_\alpha = RH_\alpha \oplus RV_\alpha \oplus W_\alpha, \alpha \in \Delta_+\) and \(\alpha\) is single.

The orbit \(O_\alpha\) of it’s action is a Poisson submanifold with respect to Poisson structure \(\pi_{SD}\). Thus if \(\lambda \neq 0\) \(\pi_\lambda(z, \bar{z})\) is degenerate iff the corresponding structure on ”small” orbit with the same \(\lambda\) is degenerate. Hence the equation \(\pi_\lambda + \delta X = \mathcal{P}(\omega)\) for all nonsingular \(z \in O_\alpha\) can be reduced to the equation \((\mathcal{P}^{-1}\pi_\lambda) + d(\mathcal{P}^{-1}(X)) = \omega\) discussed above.

This equation on \(CP^1\) has no solution. We show it using the straightforward calculations.

If \(\lambda = 0\) then \(\pi_\lambda = \pi_0 = \pi_{SD}\) is degenerate everywhere on the ”small” orbit of \(K_\alpha\) because it is a Poisson submanifold. But \(l_{w^*}\pi_\lambda = -\pi_{-(\lambda+2)}\). So for \(\lambda = 0\) the structure does not allow geometric quantization. \(\Box\)
4 Acknowledgments

I am deeply indebted to V.Rubtsov for pointing out the problem and stimulating discussions. I would also to thank V.Fock for useful suggestions and comments.

This work was finished at the Institute of Theoretical Physics of Uppsala University (Sweden). I am thankful to professor A.Niemi for hospitality and excellent conditions during my visit to Uppsala.

References

[1] Donin,J. , Gurevich,D. , Majid,S. Quantization of R-matrix type brackets , Prepr. DANPT, Cambridge Univ., March 1992

[2] Drinfeld,V. Quantum groups , Proc. ICM , Berkeley, vol.1 Am. Math. Soc., 1986, pp.789-920

[3] Gurevich,D , Panyushev,D. On Poisson pairs associated to modified R-matrixes, Duke. Math. Journ. vol.73 , n.2, 1994, pp.249-255

[4] Gurevich,D. , Rubtsov,V. , Zobin,N. Quantization of Poisson pairs : the R-matrix approach , Journ. Geom. Phys., vol.9 , n.1, 1992

[5] Helgason,S. Differential geometry , Lie groups and symmetric spaces, New York, Academic Press, 1978

[6] Khoroshkin,S. , Radul,A. , Rubtsov,V. A family of Poisson structures on hermitian symmetric spaces , CMP. 152, 1993 , pp.299-315

[7] Lichnerovicz,A., J.Diff.Geom. 12, 1977, pp.253-300

[8] Lu,J.H. , Weinstein,A. Poisson-Lie groups , dressing transformations and Bruhat decomposition , J.Diff.Geom. 31, 1990 , pp.501-526
[9] Semenov-Tyan-Shansky, M. *Dressing transformations and Poisson-Lie group actions*, Publ. Res. Inst. Math. Sci. **21**, 1985, pp. 1237-1260

[10] Vaisman, I. *On the geometric quantization of Poisson manifolds*, J. Math. Phys. vol. **32**, n. **2**, 1991