Quantum vortex fluctuations in cuprate superconductors

Hyok-Jon Kwon

Department of Physics and Center for Superconductivity Research, University of Maryland, College Park, MD 20742-4111
(October 9, 2000)

We study the effects of quantum vortex fluctuations in two-dimensional superconductors using a dual theory of vortices, and investigate the relevance to underdoped cuprates where the superconductor-insulator transition (SIT) is possibly driven by quantum vortex proliferation. We find that a broad enough phase fluctuation regime may exist for experimental observation of the quantum vortex fluctuations near SIT in underdoped cuprates. We propose that this scenario can be tested via pair-tunneling experiments which measure the characteristic resonances in the zero-temperature pair-field susceptibility in the vortex-proliferated insulating phase.

PACS numbers: 74.40.+k, 47.32.Cc, 74.50.+r, 74.76.-w

I. INTRODUCTION

The emergence of the pseudogap phase in cuprate superconducting materials may be attributed to strong order parameter phase fluctuations. Based on the small superfluid phase stiffness (SPS) at \( T = 0 \), together with its empirical scaling with the transition temperature \( T_c \), a proposal was put forward that the normal state pseudogap phase in the underdoped regime is a superconductor whose phase coherence is destroyed by thermal phase fluctuations but with a robust gap-like feature due to a strong pairing amplitude. Some experimental evidence was provided for strong thermal fluctuations of unbound vortices over a wide range of temperatures above \( T_c \) in the pseudogap phase. This suggests that the superconducting transition is of Kosterlitz-Thouless type with a broad phase fluctuation regime.

It is then natural to explore the effects of strong \( T = 0 \) quantum phase fluctuations, since the strengths of both thermal and quantum phase fluctuations are correlated with the small magnitude of SPS. For instance, quantum phase fluctuations reduce the Debye-Waller factor \( e^{i\phi} \) where \( \phi \) is the order parameter phase, and have significant consequences for the \( c \)-axis optical conductivity, renormalization of SPS, and the pair-field susceptibility. A more dramatic effect of quantum phase fluctuations may be the quenching of phase-coherence by vortex pair proliferation in the underdoped regime, which leads to a superconductor-insulator transition (SIT). Some experimental findings are consistent with the existence of a quantum critical point near SIT controlled by the charge carrier doping. However, direct evidence for vortex pair proliferation is yet to be found, although an experiment on current-voltage (I–V) characteristic in Bi2212 shows indications of a large density of quantum vortex pairs well below \( T_c \) and far away from SIT. When searching for definitive experimental tests, it is important to construct and study reliable phenomenological theories which incorporate vortex fluctuations. The critical properties of vortex-proliferated SIT have been studied using a framework of the dual transformation. In this framework, dual vortex fields are conveniently introduced as the new order parameter of vortex proliferation and the transition is described by the 2+1-dimensional (2+1D) Ginzburg-Landau theory of the quantum vortex order parameter, although the actual critical properties are determined by the existence of long-range interactions, periodic potential, and disorder.

In this paper, we directly apply the mean-field dual formulation to two-dimensional (2D) superconductors and study its phenomenological relevance to cuprates in both superconducting and insulating states. We find that the phase fluctuation regime may be broad enough for experimental access. In vortex-proliferated insulating states, the characteristic form of the pair-correlation function is \( \langle e^{i\phi(x,t)} e^{i\phi(0,0)} \rangle \propto e^{ia} \sqrt{|t|^2 - |\mathbf{x}|^2/\sigma^2} \), with \( a > 0 \). We propose that this novel form of the pair-correlation function can be verified by pair-tunneling experiments.

II. DUAL THEORY OF QUANTUM VORTICES

In this paper, we denote the space-time 3-vectors with \( x \) and \( k \) and the spatial vectors with \( \mathbf{x} \) and \( \mathbf{k} \). We use Greek indices running from 0 to 2 and Roman indices \( i,j \) from 1 to 2. We also adopt a metric tensor \( g^\mu\nu = \delta^\mu\nu (1 - 2\delta^{00}) \) to evaluate contractions of indices. For notational convenience, we set \( \hbar = 1 \). We begin with a \( T = 0 \) BCS model of a 2D superconductor coupled to the electromagnetic field.

\[
S = \int dt \int d^2x \sum_\sigma \left[ \frac{c_\sigma^+ \left( iD_t - \frac{D_\mathbf{x}^2}{2m} - \mu \right) c_\sigma}{2} \right] + \frac{1}{g} \left[ |\Delta(\mathbf{x},t)|^2 + \sum_{\mu} |A_\mu(\mathbf{x},t)|^2 \right] \]

where \( c_\sigma \) is a fermion field, \( D_\mu = \partial_\mu - iA_\mu \) is the covariant derivative, \( A_\mu \) is the electromagnetic vector potential, and \( m \) is the effective fermion mass. Here we do not specify the pairing symmetry, since it is not significant.
within the accuracy of our discussion. \( S[A_\mu] \) is an electromagnetic gauge field action which takes the following form in 2D:

\[
S[A_\mu] = \sum_k \left[ \frac{|k|}{4\pi\epsilon} A_\mu^2 - \frac{(k^2\epsilon^2 - \omega^2)d^2}{8\pi\epsilon^2} A_\mu^2 \right],
\]

where we chose the Coulomb gauge \( \nabla \cdot A = 0 \) and \( d \) is the thickness of the film. We assume that the order parameter amplitude fluctuations are negligible \( \|\Delta(x,t)\| = \Delta \), and focus on the phase degree of freedom \( \phi = -i \ln \Delta(x,t)/\Delta \). In order to decouple the \( \phi \) field from the order parameter amplitude, we perform a singular gauge transformation \( \psi_\sigma(x,t) = c_\sigma(x,t) e^{-i\phi(x,t)/2} \), with \( \psi_\sigma \) the field operators for the transformed quasiparticle. We eventually arrive at the following effective theory of the phase and the electromagnetic \( A_\mu \) fields after integrating out the fermion degrees of freedom:

\[
S[\phi, A_\mu] = \frac{1}{2} \int d^2x \, dt \left\{ \rho_\phi \left[ (2A_\mu + \partial_\mu \phi)^2 / c_s^2 \right] - (2A + \nabla \phi)^2 \right\} + \partial_\mu \phi \, n_s \} + S[A_\mu],
\]

where \( \rho_\phi \) is the SPS defined as \( \rho_\phi = n_s/4m \), \( n_s \) is the superconducting fermion density, and \( c_s \) is an analog of the phonon velocity in a superfluid. In a 2D BCS superconductor, \( c_s \) is related to the Fermi velocity \( v_F \) by \( c_s = v_F/\sqrt{2} \). We are interested in deriving the effective theory of the phase fields which describes vortices. Therefore, we separate \( \partial_\mu \phi \) into \( \partial_\mu \phi = \partial_\mu \theta + A_\mu \) where \( \theta \) is the spin-wave-type Goldstone fluctuation and \( A_\mu \) gives topologically non-trivial phase gradients generated by vortices. Then we integrate out both \( \theta \) and \( A_\mu \) fields to obtain the final effective action of \( A_\mu \):

\[
S[A_\mu] = \frac{1}{2} \sum_k \left[ \mathcal{K}_0(k)|A_\mu|^2 - \mathcal{K}_T(k)|\vec{A}|^2 \right],
\]

with a constraint that \( \nabla \cdot \vec{A} = 0 \). Here \( \mathcal{K}_0 = \rho_\phi k^2/(k^2\epsilon^2 - \omega^2 + 8\pi\epsilon^2 \rho_\phi |k|) \) and \( \mathcal{K}_T = \rho_\phi (-\omega^2 / \epsilon^2 + k^2)/\left(\lambda^2 + k^2 \right) \) where \( \lambda \) is the penetration depth. Over the time scale of our interest, \( \mathcal{K}_T \approx \rho_\phi k^2/(\lambda^2 + k^2) \). Below we will neglect the term \( \partial_\mu \phi / n_s/2 \) in Eq. (2), which plays the role of a dual magnetic field but is insignificant near SIT since \( n_s \) is renormalized and approaches zero in the insulating state.

Now we assume a certain distribution of \( N \) point-like vortices. At a fixed time, the vortices in 2D superconductors are pancake-like. In 2+1D, however, we can consider the space-time paths of pancake vortices as 3D lines of vortices. The current density \( (\mathcal{J}_\mu) \) of the vortices are obtained from \( \mathcal{J}_\mu = e^{\mu\lambda} \partial_\mu A_\lambda \). \( \mathcal{J}_\mu \) can be expressed in term of the space-time paths of vortices in 2+1D as follows:

\[
\mathcal{J}_\mu(x) = 2\pi \sum_{l=1}^N \int du_l \, \frac{dX_\mu^l(u_l)}{du_l} \delta(3)(\vec{X}_l(u_l) - x), \quad (3)
\]

where \( u_l \) parameterizes the space-time path \( \vec{X}_l \) of the \( l \)-th vortex. Therefore, Eq. (3) describes the interactions between infinitesimal vortex segments in 2+1D (Ref.[23]) if we re-express \( S[A_\mu] \) in terms of \( \mathcal{J}_\mu \). Here we assume only vortices of one flux quantum; the ones traveling backward (forward) in time are antivortices (vortices).

The field theory of Eqs. (2) and (3) is inconvenient for description of particle-like vortices. Therefore, we first transform Eqs. (2) and (3) into particle dynamics. Later we will conveniently transform the particle dynamics into field dynamics of vortices. Before the transformation, we first separate the vortex-current interactions in Eq. (2) into contact (short-range) and long-range interactions. The contact interaction can be approximately expressed in the form of a relativistic particle action as follows:

\[
S_{\text{cont}} \approx \sum_i \int du \, m_v \left[ -\frac{e^2}{c s} \left( \frac{d\vec{X}^0}{dt} \right)^2 + \frac{1}{2} \left( \frac{d\vec{X}}{dt} \right)^2 \right],
\]

where we have deduced the rest mass from the contact interaction as following:

\[
m_v c_s^2 \delta(0)(0) = (2\pi)^2 \sum_k \mathcal{K}_T(k)/2k^2.
\]

The velocity \( v_c \) is taken as \( v_c = Cv_F \) where \( C = O(1) \) on physical grounds[23]. In addition to Eq. (4), there is contact repulsion between vortices of distinct labels. We will not estimate or discuss this repulsive interaction explicitly, except to mention that it stabilizes the non-zero expectation value of vortex fields in the insulating state which we will discuss later. In order to incorporate the long-range interactions into the particle dynamics, we first separate the dynamics of \( \mathcal{J}_\mu \) and \( A_\mu \) by introducing a Lagrange multiplier \( G_\mu \). This amounts to adding a term \( \int d^2 x \, dt \, G_\mu (e^{\mu\lambda} \partial_\mu A_\lambda - \mathcal{J}_\mu) \) to the action in Eq. (4) which enforces the relation between \( \mathcal{J}_\mu \) and \( A_\mu \). Then we integrate out \( A_\mu \) fields to obtain the following effective interactions:

\[
S_{\text{int}} = \frac{1}{2} \sum_k \left[ \frac{k^2 + \lambda^2}{\rho_s} |G_0|^2 - \frac{(k^2 c_s^2 + 8\pi\epsilon^2 \rho_\phi |k| - \omega^2)}{\rho_s} |G|^2 \right] + 2\pi \sum_l \int du_l \, G_\mu(\vec{X}_l) \frac{d\vec{X}_\mu^l}{du_l},
\]

with a Coulomb gauge constraint \( \nabla \cdot G = 0 \). Thus, combining Eqs. (4) and (6), we have obtained a theory of relativistic particles with a rest mass \( m_v \), coupled to the field \( G_\mu \) which mediates the superfluid phase modes between the vortices.

We are now in a position to transform the particle dynamics into a field theory. Since we are interested in vortex-antivortex pair creation (annihilation), we now specialize into distribution of 2+1D vortex loops by requiring that the paths \( \vec{X}(u_l) \) are closed trajectories. Here we follow Ref.[23] and use the particle-field correspondence to re-express the total action \( S_{\text{cont}} + S_{\text{int}} \) in terms of a
relativistic complex scalar field $\Phi$ coupled to the gauge field $G_\mu$:

$$S[\Phi, \Phi^*, G] = \int d^2x \, dt \left[ -(\partial_\mu - 2\pi i G_\mu)\Phi^*(\partial^\mu + 2\pi i G^\mu)\Phi - m_c^2|\Phi|^2 \right] + S[G_\mu],$$

(7)

where $G_\mu$ represents the local U(1) gauge symmetry of $\Phi$ fields. Here $S[G_\mu]$ is the part quadratic in $G_\mu$ from Eq. (3) and we set $\nu_c = 1$ for notational convenience. This is the well-known dual form of the theory of superconductivity where the roles of magnetic (vortices) and electric (Cooper pairs) charges are interchanged. It is easy to show that the vortex current can be related to $\Phi$ as $J_\mu/2\pi = i(\Phi\partial_\mu\Phi^* - \Phi^*\partial_\mu\Phi) + 4\pi G_\mu|\Phi|^2$, which automatically satisfies the necessary conservation condition $\partial_\mu J^\mu = 0$.

III. STRENGTH OF QUANTUM VORTEX FLUCTUATIONS IN CUPRATES

Next, we wish to explore the feasibility of experimental observation by studying the width of the phase fluctuation regime where we expect to observe strong phase fluctuation effects. We assume a linear scaling of SPS with charge-carrier doping, and search for the magnitude of SPS (doping) at the most likely point of SIT in the underdoped cuprates. The action in Eq. (7) can be viewed as the effective 3D Ginzburg-Landau functional of $\Phi$. Then the SIT occurs upon ordering [U(1)-symmetry breaking] of the $\Phi$ fields when $m_c^2 < 0$, so that $\langle \Phi \rangle \neq 0$. In this case, $\langle \Phi(x)\Phi^*(y) \rangle \approx \Phi_0^2$, where $\Phi_0^2$ is a positive constant in the mean-field approximation, independent of $|x - y|$. The correlation function $\langle \Phi(x)\Phi^*(y) \rangle$ can be considered as the expectation value of the number of vortex paths that connect $x$ and $y$. Therefore, $\langle \Phi(x)\Phi^*(y) \rangle = constant$ implies a non-zero and constant probability of arbitrarily long 2+1D vortex loops. The non-zero expectation value of $\Phi$ leads to recovery of the U(1)-symmetry of the superconducting order parameter, and hence the Meissner effect is absent even in the presence of a non-zero bare SPS. Below we discuss the renormalization of $m_c^2$ to one loop expansion, assuming that the loop expansions are reliable.

We use following parameters of optimally doped cuprates: $\Delta \approx 20$ meV, $\lambda \approx 200$ nm, the coherence length $\xi \approx 2$ nm, and the film thickness $d \approx 1.5$ nm which roughly corresponds to a monolayer film. We assume a vortex core size of $\xi$ which provides a momentum (short-range) cutoff at $\Lambda_c = \pi/\xi$ and a frequency (short-time) cutoff at $\nu_c\Lambda_c$. With this prescription, we avoid ultraviolet divergences and obtain $\delta^{(3)}(0) = \nu_c^2\Lambda_c^3/(2\pi^3)$. We can then estimate $m_c$ and $\nu_c$ from Eq. (6) in terms of the above parameters. The result is $m_c\nu_c^2 \approx \rho_o\pi\ln\kappa$ where $\kappa = \Lambda\Lambda_c$. We find that $m_c \approx 0.36$ eV in optimally doped cuprates. For simplicity, we assume that the charge-carrier doping only affects magnitudes of the bare SPS, and we keep the other parameters constant. From Eq. (6) we can calculate one-loop corrections to $m_c^2$ as shown in Fig. (a) and find the point where the correction is of the same order of magnitudes as the bare value. In fact, the only correction comes from Fig. (a)(a); Fig. (b) does not contribute because the $\Phi$ loop vanishes, and Fig. (c) has already included as the contact interaction. The correction $\delta m_c^2/m_c^2$ roughly behaves as $-8(v_c\Lambda_c)^3/3\rho_o^2\pi^4\ln^3\kappa$, whose magnitude becomes large when the bare SPS is smaller. Eventually, we find that $m_c^2 - \delta m_c^2 \rightarrow 0$ (breakdown of Ginzburg-Landau theory) occurs when $\rho_o \approx 0.3\Delta/\ln\kappa \approx 2$ meV for cuprate films, which corresponds to about 10% of that of the optimal doping [See Fig. (d)]. This implies that, using the empirical scaling between $\rho_o$ and $T_c$, the phase fluctuation regime begins when $T_c$ is less than 10 K or so. In terms of charge-carrier (hole) doping concentration $p$, this corresponds to $p \approx 0.35\rho_o$ where $\rho_o$ is the optimal doping concentration. This is from the empirical relation between the critical temperature and doping, $T_c/T_{CO} \approx 1 - 2.1(p/p_o - 1)^2$, where $T_{CO}$ is the optimal critical temperature and $T_c \approx 0$ for $p \approx 0.3\rho_o$. Therefore, there may be a broad enough doping range ($0.3\rho_o < p < 0.35\rho_o$) for experimental studies on critical vortex fluctuations such as non-linear $I-V$ characteristic. For instance, the underdoped YBCO used in the anomalous proximity effect experiment performed by Decca et al. fall within this parameter regime, which indicates strong phase fluctuations. The critical properties of the SIT are beyond the scope of this paper. Instead, we will simply assume a non-zero expectation value of $\Phi$ and discuss definitive experimental tests of vortex-proliferated insulating states.
IV. PAIR-FIELD SUSCEPTIBILITY

In order to establish that underdoped cuprates are under strong quantum vortex fluctuations, a direct probe into the pair-fluctuations is necessary. Here we propose a pair-tunneling experiment to measure the pair-field susceptibility which contains information about pair-correlation functions. We first give a brief summary of the desired experimental setup. We consider a c-axis tunnel junction of a thickness $d$ between two cuprate samples, where one of them is optimally doped with a c-axis penetration depth $\lambda_c$, and the other is an underdoped insulating film at $T = 0$ with a thickness $d$. Since one of the electrodes is insulating, the usual Josephson current oscillating at a frequency of $2eV/h$ is absent. However, an excess current will flow due to Josephson coupling of the superconducting pair-field of the superconducting electrode to the fluctuating pair-field of the insulator. Neglecting vortex fluctuations in the optimally doped electrode, the excess current can be related to the pair-field susceptibility of the insulator as $I_{\text{ex}} = (eE J^2/4\hbar^2)\Im D^R(q, \omega)$, where $D^R(x, t) = -i\theta(t) [e^{i\phi(x)} e^{i\phi(0)}]$ is the retarded pair-correlation function, $E_J$ is the Josephson coupling energy of the junction, and $S$ is the junction contact area. Here $\omega$ is a frequency $2eV/h$ and $q$ is a wave vector $q(H) = 2e\hbar(\lambda_c + d/2 + \delta)/hc$ which is determined by a small magnetic field $H$ applied parallel to the junction [9]. Therefore, the excess current can provide information about the spectrum of phase fluctuations. Below we obtain the form of the pair-correlation function of vortex-proliferated insulators.

First, we re-express the pair-correlation function as $D(x, t) = -i\exp[i \int_{0}^{t} d\mu(x)\delta^{(3)}(y-x)]$, ignoring contributions from $\theta$ fields since they are sub-leading in long length scales. Here we assume that $2eV < \Delta$ to avoid the effect of order parameter amplitude fluctuations. Then we introduce a source function $f_{\mu}(x) = \int_{0}^{t} dy_{\mu}(\delta^{(3)}(y-x))$ where we take $y$ to be a straight line which connects $(0,0)$ and $(x,t)$. Now we can re-express $D(x,t)$ as follows:

$$iD(x,t) = \frac{\int D\Phi D\Phi^* D\mu \ e^{i \int d^3x A_{\mu}(x) - IS[\Phi,\Phi^*,G_{\mu}]} \ \int D\Phi D\Phi^* D\mu \ e^{i S[\Phi,\Phi^*,G_{\mu}]} }{\int D\Phi D\Phi^* D\mu \ e^{i S[\Phi,\Phi^*,G_{\mu}]} }$$

where the second line is obtained by the method of functional integration (see Appendix [3]). In order to obtain the vortex-current correlation functions, we first define the polarization functions:

$$\Pi_{\mu}(x-y) = \langle \Phi^2 \gamma^{(3)}(x-y) - P_{\mu}(x) P_{\mu}^*(y) \rangle / 2,$$

where $P_{\mu}(x) = \Phi(x) \partial_x \Phi^*(x) - \Phi^*(x) \partial_x \Phi(x)$. We also define the longitudinal ($\Pi_0$) and transverse ($\Pi_T$) polarization functions as the $\mu$-component of the polarization functions where $\mu$ is in the longitudinal (0) and transverse ($T$) direction respectively. Then the vortex-current correlation functions can be written as follows:

$$i\langle J_0(k) J_0(-k) \rangle \approx \frac{8\pi^2 k^2 \Pi_0(k)}{|k^2 + 8\pi^2 \gamma_\omega(k) \Pi_0(k)|},$$

$$i\langle J_T(k) \cdot J_T(-k) \rangle \approx \frac{-8\pi^2 k^2 \Pi_T(k)}{|k^2 + 8\pi^2 \gamma_\omega(k) \Pi_T(k)|}.$$

Using Eq. (8) and assuming $\Pi_{\mu}(k) \approx \text{constant} > 0$, which holds in the vortex proliferated state, we obtain the asymptotic behavior of $D(x,t)$ in the large $|x|, |t|$:

$$iD(x,t) \propto \exp \left[ 2\pi i \ln(L/\xi) \Pi_0 \sqrt{v^2|t|^2 - |x|^2} \right],$$

where $L$ is the size of the sample. Upon Fourier transformation, we find that $D(q, \omega) \propto \Pi_0(\omega^2 - E^2)^2$ where $E^2 = [2\pi \ln(L/\xi) \Pi_T]^2 + v^4 c^2 q^2$. Accordingly, the excess current behaves as $I_{\text{ex}}(q,2eV) \propto 1/[4eV^2 - E^2]$, resembling the imaginary part of resonance peaks located near $2eV \approx \pm 2\pi \ln(L/\xi) \Pi_T$. The resonance peak heights are determined by the normal-state junction resistance and $\Pi_0(\omega^2)$. The apparent logarithmic divergence of the exponent in Eq. (10) and of the resonance energy is due to the fact that the energy of dual vortices (Cooper pairs) is logarithmically divergent as the system size, similar to vortices in superfluid helium. This weak divergence does not pose a serious problem in realistic samples which have finite sizes, however. This excess current qualitatively differs from that due to fluctuations at finite-temperature superconducting transitions where $I_{\text{ex}} \propto \omega/(\omega^2 + \Gamma^2)$ (See Fig. 2). Therefore, it is possible to confirm the existence of phase-fluctuation driven insulating states by detecting characteristic resonances in the pair-field susceptibility. The pair-correlation function oscillates at large $|t|$ due to the presence of vortex condensates. In the phase-coherent state of the vortex fields $\Phi$, the trajectories of Cooper pairs act as dual vortex paths in 2+1D, and the pair-correlation function is the probability of a dual vortex of length $\sqrt{2c^2|t|^2 - |x|^2}$. Therefore $D(x,t)$ is determined by the action of a vortex line which connects $(0,0)$ and $(x,t)$. This explains the time-like length-
dependence of the exponent of $D(x,t)$. In fact, due to dissipative processes that we have not considered, the pair-correlation function decreases in magnitude at large $|t|$ in addition to the pure oscillation in $|t|$ shown in Eq. (10). Accordingly, the resonance peaks in the pair-correlation function will be broadened depending on the strength of the dissipation.

V. SUMMARY AND CONCLUSIONS

We discussed the possibility of observing quantum vortex fluctuations in underdoped cuprate superconductors near SIT. Using the dual theory of vortices, we showed that cuprate superconductors are subject to strong vortex fluctuations so that it is possible to experimentally access the fluctuation regime near SIT. As a definitive test of the phase fluctuation scenario, we proposed an experiment to measure the pair-field susceptibility to probe the form of the pair-correlation function in the insulating regime. We expect that the pair-field susceptibility in vortex-proliferated insulating states is qualitatively different from that observed in the normal-state fluctuation regime, and shows characteristic resonance peaks. The result can be generalized to any phase-fluctuation driven SIT of superconducting films. We anticipate that a more realistic dual theory of vortex fluctuations in superconducting films can be obtained by including the effects of normal fluids or $d$-wave nodal quasiparticles.

VI. ACKNOWLEDGEMENT

The author gratefully acknowledges stimulating discussions with A. T. Dorsey, H. D. Drew, C. J. Lobb, K. Sengupta, A. Sudbø, and V. M. Yakovenko. This work was supported by the NSF DMR-9815094 and the Packard Foundation.

APPENDIX A: PAIR-CORRELATION FUNCTIONS

Here we give a brief derivation of Eq. (3). From the first line of Eq. (3), the first term in the exponent can be rewritten as

$$ S_C = i \int d^3x \int d^3y \langle \mathcal{J}^\lambda(x) \mathcal{J}^\mu(y) \rangle h_\lambda(x) h_\mu(y)/2 \quad (A1) $$

In the Coulomb gauge, the Fourier transform of Eq. (A1) can be expressed as

$$ S_C = i \sum_{k,\mu} j_\mu(k) j_\mu(-k) \langle \mathcal{J}_\mu(k) \mathcal{J}_\mu(-k) \rangle / (2k^2) \quad . $$

1. Y. J. Uemura et al., Phys. Rev. Lett. 62, 2317 (1989); S. Doniach and M. Imui, Phys. Rev. B 41, 6668 (1991); V. J. Emery and S. A. Kivelson, Nature (London) 374, 434 (1995).
2. See, for example, M. Takigawa et al., Phys. Rev. B 43, 247 (1991); A. V. Puchkov et al., Phys. Rev. Lett. 77, 1853 (1996); G. Blumberg et al., Science 278, 1427 (1997); H. Ding et al., Nature (London) 382, 51 (1996).
3. M. Franz and A. J. Millis, Phys. Rev. B 58, 14572 (1998); H.-J. Kwon and A. T. Dorsey, ibid 59, 6438 (1999).
4. J. Corson et al., Nature (London) 398, 221 (1999); Z. A. Xu et al., ibid. 406, 486 (2000).
5. L. B. Ioffe and A. J. Millis, Science 285, 1241 (1999); Phys. Rev. B 61, 9077 (2000).
6. A. Parameskanthi, M. Randeria, T. V. Ramakrishnan, and S. S. Mandal, Phys. Rev. B 62, 6786 (2000); H.-J. Kwon, A. T. Dorsey and P. J. Hirschfeld, cond-mat/0006290.
7. L. Balents, M. P. A. Fisher, and C. Nayak, Int. J. Mod. Phys. B 12, 1033 (1998); Phys. Rev. B 60, 1654 (1999).
8. S. Sachdev, Phys. Rev. B 59, 14054 (1999).
9. K. Semba, A. Matsuda, M. Mukaida, Physica B 281&282, 904 (2000).
10. R. S. Decca et al, Phys. Rev. Lett. 85, 3708 (2000).
11. J. Chiaverini et al., cond-mat/0007479.
12. See, for example, M.P.A. Fisher and D.H. Lee, Phys. Rev. B 39, 2756 (1989); H. Kleinert, Gauge Fields in Condensed Matter (World Scientific, Singapore, 1989) Vol.1.
13. K. Bardakci and S. Samuel, Phys. Rev. D 18, 2849 (1978).
14. M. P. A. Fisher and G. Grinstein, Phys. Rev. Lett. 60, 208 (1988); M. P. A. Fisher, P. B. Weichman, G. Grinstein, and D. S. Fisher, Phys. Rev. B 40, 546 (1989).
15. The resulting quasiparticle fermions would be multi-valued upon encircling a quantum of vortex [1], but we ignore such effects since we do not discuss the single-particle properties.
16. V. N. Popov, Zh. Eksp. Teor. Fiz. 64, 672 (1973) [Sov. Phys. JETP 37, 341 (1973)]; D. P. Arovas and J. A. Freire, Phys. Rev. B 55, 1068 (1997).
17. G. Deutscher, Nature (London) 397, 410 (1999).
18. R. V. Carlson and A. M. Goldman, J. Low Temp. Phys. 25, 67 (1976).
19. D. J. Scalapino, Phys. Rev. Lett. 24, 1052 (1970); B. Jankó, et al., Phys. Rev. Lett. 82, 4304 (1999).