Abstract: We investigate the hyperon semileptonic decay constants, $f_2/f_1$, and $g_1/f_1$, within a general framework of a chiral soliton model. All relevant parameters for the SU(3) baryon wave functions were unambiguously determined by using the experimental data for the masses of the baryon octet and the decuplet. Using then the existing experimental data for the magnetic moments of the baryon octet and the decay constants of hyperon semileptonic decays, we are able to determine all the hyperon semileptonic decay constants $f_2/f_1$ and $g_1/f_1$ of the baryon octet unequivocally. In addition, we also present the results of the axial-vector transition constants from the baryon decuplet to the octet.

Keywords: hyperon semileptonic decay constants, vector constants, axial-vector constants, chiral soliton model
1 Introduction

The Cabibbo-Kobayashi-Maskawa matrix elements $|V_{ud}|$ and $|V_{us}|$ characterize the quark mixings in the process $d \to ue^-\bar{\nu}_e$ and $s \to ue^-\bar{\nu}_e$ [1, 2] in the standard model. So far the most precise determination of $|V_{ud}|$ and $|V_{us}|$ are obtained respectively from super-allowed Fermi transition together with pion decays, and from leptonic and semileptonic kaon decays [3–5]. On the other hand, hyperon semileptonic decays (HSDs) can also provide independent constraints on $|V_{ud}|$ and $|V_{us}|$ [6, 7]. For last years, new experimental results for HSD have been reported: the KTeV collaboration first announced the measurement of the $\Xi^0 \to \Sigma^+ e^-\bar{\nu}_e$ decay [8] and determined the corresponding form factors [9] as $f_2(0)/f_1(0) = 2.0 \pm 1.2_{\text{stat}} \pm 0.5_{\text{syst}}$ and $g_1(0)/f_1(0) = 1.32^{+0.21}_{-0.17_{\text{stat}}} \pm 0.05_{\text{syst}}$. The NA48/1 collaboration brought about the branching ratios for the same process with higher statistics in comparison with the KTeV experiments: $g_1(0)/f_1(0) = 1.21 \pm 0.05$ for $\Xi^0 \to \Sigma^+ e^-\bar{\nu}_e$ [10]. Now there are experimental data for the $g_1(0)/f_1(0)$ ratios in five different decay channels in HSDs, collected by the Particle Data Group (PDG) [11].

It is also of great importance to understand the HSD constants, since they provide essential information on properties of the nucleon and low-lying hyperons. The data for HSDs reveal experimentally the pattern of flavor SU(3) symmetry breaking. In exact flavor SU(3) symmetry the ratios of the axial-vector and vector constants $g_1/f_1$ are expressed only by the two constants $F$ and $D$. Similarly, the ratios of the vector constants $f_2/f_1$ are written in terms of the anomalous magnetic moments of the proton and the neutron with flavor SU(3) symmetry assumed. However, the experimental data for HSDs show that flavor SU(3) symmetry is manifestly broken. For example, the ratio $g_1/f_1 = 1.21 \pm 0.05$ for the HSD process $\Xi^0 \to \Sigma^+ e^-\bar{\nu}_e$ measured by the KTeV collaboration [8, 9] is equal to that of the neutron $\beta$ decay ($g_1/f_1 = 1.2695 \pm 0.0029$) in flavor SU(3) symmetry. Thus, complete information on HSDs will furnish the pattern of flavor SU(3) symmetry breaking in nature. It is also interesting to see that the experimental data for the HSD constants give a clue how isospin symmetry is broken. The above-mentioned ratio of the HSD constants $g_1/f_1 = \ldots$
1.20±0.05 from the KTeV collaboration should be equal to that of its isospin partner process \( \Xi^- \rightarrow \Sigma^0 e^- \bar{\nu}_e \) \( g_1(0)/f_1(0) = 1.25^{+0.14}_{-0.16} \) \[12\] even with flavor SU(3) symmetry breaking. If it is possible to measure more precisely the HSD constants for the decay \( \Xi^- \rightarrow \Sigma^0 e^- \bar{\nu}_e \), HSDs will shed light on how isospin symmetry is broken.

For this reason, there has been a great deal of theoretical works. Chiral perturbation theory (\( \chi \)PT) is used as a framework to analyze flavor SU(3) breaking effects on \( f_1(0) \) beyond the first order \[13–16\], in which other form factors of HSD have been also studied. The results for \( V_{us} \) within HSDs were discussed in Ref. \[17\] and for the large \( N_c \) expansion in Ref. \[18\]. In lattice QCD \[16\] all form factors of the \( \Sigma^- \rightarrow n\ell\nu \) decay were first time studied and the results were given as: \( f_2(0)/f_1(0) = -1.52 \pm 0.81 \) and \( g_1(0)/f_1(0) = -0.287 \pm 0.052 \). Flavor SU(3) symmetry breaking in HSDs was also investigated within the \( 1/N_c \) expansion \[19\]. In the chiral quark-soliton model (\( \chi \)QSM) with a model-independent approach considered, the HSD constants were investigated many years ago \[20\]. Being distinguished from the self-consistent \( \chi \)QSM \[21, 22\], almost all dynamical parameters were fixed by using the experimental data of the masses and the HSD constants for the baryon octet in this model-independent approach. In Ref. \[20\], however, the baryon wave functions were not completely determined and experimental information was then not enough to fix unambiguously the parameters for the HSD constants.

In the present work we will examine the HSD constants of the baryon octet and decuplet within a general framework of a chiral soliton model (\( \chi \)SM), fixing all dynamical parameters unequivocally. To derive the collective wave functions for the baryon octet, one has to take into account both flavor SU(3) symmetry and isospin symmetry breakings such that the experimental data for the masses of the baryon octet can be employed as a whole. In particular, sources of isospin symmetry breaking arise from the electromagnetic (EM) interaction as well as from the mass difference of the up and down quarks. In Ref. \[23\], the EM mass differences between baryons in the same isospin multiplet were analyzed within the \( \chi \)SM and the corresponding model parameters were fixed by the experimental data. Together with the isospin symmetry breaking from the mass difference of the up and down quarks, Ref. \[24, 25\] showed that the collective wave functions of the baryon octet and decuplet were uniquely determined. In addition, a more complete analysis of the HSD constants can be carried out, the KTeV data for the \( \Xi^0 \rightarrow \Sigma^+ e^- \bar{\nu}_e \) process considered. Since now the five experimental data for the ratios \( g_1/f_1 \), we are able to fix all dynamical parameters for the HSD axial-vector constants with the singlet axial-vector charge from the data of deep inelastic scattering employed. Similarly, all dynamical parameters for the HSD vector constants can be fixed by using the experimental data for the magnetic moments of the baryon octet. Using the fixed parameters, we compute the HSD axial-vector and vector constants for the baryon octet. In addition, we also present the results of the transition axial-vector constants for the baryon decuplet, which will be of great use in determining the meson-baryon strong coupling constants.

The present work is sketched as follows. In Section II, we explain how to fix all dynamical parameters, using the experimental data. We also recapitulate relevant formulae for the HSD coupling constants. In Section III, we show the results of the present work and discuss physical implications of them. In the final Section, we summarize the present
work and draw conclusions. Additional detailed formulae are given in Appendices.

2 Baryon matrix elements of the vector and axial-vector currents

The transition matrix elements of the vector and axial-vector currents for the baryon octet are expressed respectively in terms of the six real form factors:

\[
\langle B_2 | V_\mu | B_1 \rangle = \bar{u}_{B_2} (p_2, s_2) \left[ f_{12}^{B_1 \rightarrow B_2} (q^2) \gamma_\mu - \frac{i f_{12}^{B_1 \rightarrow B_2} (q^2) \sigma_{\mu \nu} q^\nu}{M_{B_1}} \right] u_{B_1} (p_1, s_1) ,
\]

\[
\langle B_2 | A_\mu | B_1 \rangle = \bar{u}_{B_2} (p_2, s_2) \left[ g_1^{B_1 \rightarrow B_2} (q^2) \gamma_\mu \right. \left. + \frac{i g_2^{B_1 \rightarrow B_2} (q^2) \sigma_{\mu \nu} q^\nu}{M_{B_1}} \right] + \frac{g_3^{B_1 \rightarrow B_2} (q^2) q_\mu}{M_{B_1}} \gamma_5 u_{B_1} (p_1, s_1) ,
\]

where the vector and axial-vector currents are defined as

\[
V_\mu (x) = \bar{\psi} (x) \gamma_\mu \frac{1}{2} \lambda^\chi \psi (x) , \quad A_\mu^\chi (x) = \bar{\psi} (x) \gamma_\mu \gamma_5 \frac{1}{2} \lambda^\chi \psi (x) . \tag{2.2}
\]

The \( u_{B_1} (\bar{u}_{B_2}) \) denote the Dirac spinors corresponding to the initial and final baryon states, respectively. The \( \lambda^\chi \) designate flavor Gell-Mann matrices for strangeness conserving \( \Delta S = 0 \) transitions (\( \chi = 3, 8, 1 \pm i 2 \)) and for \( |\Delta S| = 1 \) ones (\( \chi = 4 \pm i 5 \)), respectively.

The \( g_1^{B_1 \rightarrow B_2} \) and \( g_3^{B_1 \rightarrow B_2} \) are real quantities due to CP-invariance, depending only on the square of the momentum transfer. We can neglect \( f_{12}^{B_1 \rightarrow B_2} \) and \( g_3^{B_1 \rightarrow B_2} \) for the reason that their contributions to the decay rate is proportional to the ratio \( m_l^2 / M_{B_1}^2 \ll 1 \), where \( m_l \) represents a mass of the lepton (\( e \) or \( \mu \)) in the final state and that of the baryon in the initial state, \( M_{B_1} \), respectively. The \( g_2^{B_1 \rightarrow B_2} \) are finite only with the effects of flavor SU(3) symmetry and isospin symmetry breakings because of its opposite \( G \) parity to the axial-vector current, so that they are very small for the baryon octet. Moreover, the Ademollo-Gatto theorem [26] does not allow \( f_{12}^{B_2 \rightarrow B_1} \) to acquire the linear-order corrections of flavor SU(3) symmetry breaking, so that \( f_{12}^{B_2 \rightarrow B_1} \) are merely expressed in terms of the SU(3) Clebsch-Gordan coefficients.

Concerning the HSD constants for the transition from the baryon decuplet to the octet, it is difficult to determine the vector HSD constants because of a lack of experimental data for the \( M1 \) and \( E2 \) transitions from the baryon decuplet to the octet. For this reason, we will consider only the axial-vector transitions in the present work. There are several different ways of decomposing the matrix elements of the axial-vector current between the baryon decuplet and the octet. We will follow here the formalism of the Alder form factors \( C_i^A (q^2) [27–30] \) to describe the axial-vector transitions of the baryon decuplet. The matrix elements between the baryon decuplet and the octet can be written as follows

\[
\langle B_8 | A_\mu^\chi | B_{10} \rangle = \bar{u}_{B_8} (p_2, s_2) \left[ C_5^A (q^2) g_{\mu \nu} + C_6^A (q^2) q_\mu q_\nu \right]
\]

- 3 -
\[
+ \left\{ C_3^A(q^2)\gamma^\alpha + C_4^A(q^2)p'^\alpha \right\} (g_\alpha g_{\mu\nu} - q_\nu g_{\alpha\mu}) \right\} u_{B_{10}}(p_1, s_1), \tag{2.3}
\]

where the \( u_{B_{10}}(p_1, s_1) \) represents the Rarita-Schwinger spinor that describes the baryon decuplet with spin \( 3/2 \). \( p' \) is defined as \( p' = (M_{B_{10}}, 0, 0, 0) \).

Since we want to determine the HSD constants within the framework of a \( \chi \)SM with all dynamical parameters in the model fixed by the experimental data, we first explain the \( \chi \)SM in brief. The detailed formalism can be found for example in Ref. [24] and references therein. The \( \chi \)SM is characterized by the following collective Hamiltonian:

\[
H = M_{cl} + H_{rot} + H_{sb}, \tag{2.4}
\]

where \( M_{cl} \) stands for the classical soliton mass. The collective Hamiltonian \( H_{rot} \) comes from the collective quantization of the chiral soliton by considering its slow rotation. We often call it the \( 1/N_c \) rotational correction and express it as

\[
H_{rot} = \frac{1}{2I_1} \sum_{i=1}^{3} \hat{J}_i^2 + \frac{1}{2I_2} \sum_{p=4}^{7} \hat{J}_p^2, \tag{2.5}
\]

where \( I_{1,2} \) denote the moments of inertia of the soliton, which depend on dynamics of specific formulations of the \( \chi \)SM. \( \hat{J}_i \) stand for the usual spin operators and \( \hat{J}_p \) represent the group generators corresponding to the right rotation in flavor SU(3) space. The last term in eq. (2.4) comes from the flavor SU(3) symmetry breaking, which are expressed as

\[
H_{sb} = (m_d - m_u) \left( \sqrt{3} \alpha D_{38}^{(6)}(R) + \beta \hat{T}_3 + \frac{1}{2} \gamma \sum_{i=1}^{3} D_{3i}^{(8)}(R) \hat{J}_i \right)
+ (m_s - \bar{m}) \left( \alpha D_{88}^{(6)}(R) + \beta \hat{Y} + \frac{1}{\sqrt{3}} \gamma \sum_{i=1}^{3} D_{8i}^{(8)}(R) \hat{J}_i \right)
+ (m_u + m_d + m_s) \sigma, \tag{2.6}
\]

where \( m_u, m_d, \) and \( m_s \) denote the up, down, and strange current quark masses, respectively. \( \bar{m} \) designates the average of the up and down current quark mass. \( D_{ab}^{(R)}(R) \) are the SU(3) Wigner \( D \) functions in a \( R \) irreducible representation. \( \hat{T}_3 \) and \( \hat{Y} \) stand for the operators corresponding to the third component of the isospin and the hypercharge, respectively. The dynamical parameters \( \alpha, \beta, \) and \( \gamma \) encode a specific dynamics of a certain chiral soliton model. They are defined as

\[
\alpha = -\left( \frac{2}{3} \frac{\Sigma_{\pi N}}{m_u + m_d} - \frac{K_2}{I_2} \right), \quad \beta = -\frac{K_2}{I_2}, \quad \gamma = 2 \left( \frac{K_1}{I_1} - \frac{K_2}{I_2} \right), \tag{2.7}
\]

where \( \Sigma_{\pi N} \) indicates the \( \pi N \) sigma term. The \( \sigma \) in eq. (2.6) is defined as

\[
\sigma = -(\alpha + \beta) = \frac{1}{3} \frac{\Sigma_{\pi N}}{\bar{m}}. \tag{2.8}
\]

In the \( \chi \)SM, the value of the eighth component of the soliton angular velocity \( J_8 \) provides a very important constraint on the collective quantization. In the Skyrme model, it has its origin in the Wess-Zumino term [31–33], whereas it is related to the presence of
the discrete valence quark level in the Dirac-sea spectrum in the SU(3) chiral quark-soliton model [34, 35]. Its presence has no effects on the chiral soliton but allows one to consider only the flavor SU(3) irreducible representations with zero triality.

The wave functions for the baryon octet and decuplet are derived by diagonalizing the collective SU(3) Hamiltonian of the χSM [24, 25, 34, 35] and are expressed in terms of the SU(3) Wigner D functions:

$$\langle A|R, B(Y T T_3, Y' J J_3)\rangle = \sqrt{\text{dim}(R)} (-)^{I_3+Y'/2} D^{(R)*}_{(Y,T,T_3)(-Y',J,-J_3)}(A),$$

(2.9)

where $R$ designates the allowed irreducible representations of the flavor SU(3) group, i.e. $R = 8, 10, \cdots$. $Y, T,$ and $T_3$ denote the corresponding hypercharge, isospin and its third component, respectively. The right hypercharge is constrained to be $Y' = 2J_3/\sqrt{3} = -N_c/3 = -1$ that selects a tower of allowed flavor SU(3) representations. The baryon octet and decuplet, which are the lowest representations among them, coincide with those of the quark model.

In the presence of flavor SU(3) symmetry breaking, a baryon state is no more pure state but a mixed one with higher representations. Taking into account the strange current quark mass $m_s$ as a perturbation, we obtain the collective wave functions for the baryon octet and decuplet mixed with higher representations as

$$|B_8\rangle = |8_{1/2}, B\rangle + c^B_{10} |10_{1/2}, B\rangle + c^B_{27} |27_{1/2}, B\rangle,$$

$$|B_{10}\rangle = |10_{3/2}, B\rangle + a^B_{27} |27_{3/2}, B\rangle + a^B_{35} |35_{3/2}, B\rangle,$$

(2.10)

where the spin indices $J_3$ have been suppressed. The $m_s$-dependent coefficients in eq. (2.10) are written as

$$c^B_{10} = \frac{\sqrt{5}}{\sqrt{2}} c^B_{27} = c_{27}, a^B_{27} = a_{27}, a^B_{35} = a_{35},$$

(2.11)

respectively in the basis $[N, \Lambda, \Sigma, \Xi]$ and $[\Delta, \Sigma^*, \Xi^*, \Omega]$. The mixing coefficients in eq. (2.11) that contain $m_s$ and $\overline{m}$ are written as

$$c_{10} = -\frac{I_2}{15} (m_s - \overline{m}) \left( \alpha + \frac{1}{2} \gamma \right), \quad c_{27} = -\frac{I_2}{25} (m_s - \overline{m}) \left( \alpha - \frac{1}{6} \gamma \right),$$

$$a_{27} = -\frac{I_2}{8} (m_s - \overline{m}) \left( \alpha + \frac{5}{6} \gamma \right), \quad a_{35} = -\frac{I_2}{24} (m_s - \overline{m}) \left( \alpha - \frac{1}{2} \gamma \right).$$

(2.12)

In order to determine the dynamical parameters $\alpha$, $\beta$, $\gamma$ and the mixing coefficients in eq. (2.12) by using the experimental data, we have to take into account the effects of isospin symmetry breaking, which consist of the electromagnetic interactions and the up and down quark mass differences. The corresponding formalisms and analyses in detail can be found in Ref. [23] and Ref. [24], respectively. The numerical values of the coefficients that were obtained from Ref. [24] are given as

$$c_{10} = 0.0434 \pm 0.0006, \quad c_{27} = 0.0203 \pm 0.0003.$$
Note that it is of great importance to know these mixing coefficients, since they allow one to carry out an unambiguous analysis of the vector and axial-vector HSD constants.

In a \( \chi \)SM, the collective operators for the HSD vector and axial-vector constants can be expressed in terms of the SU(3) Wigner \( D \) functions [20, 36, 37]:

\[
\begin{align*}
\hat{f}_{2B_1 \rightarrow B_2} &= w_1 D^{(8)}_{\chi 3} + w_2 d_{pq3} D^{(8)}_{\chi p} \hat{J}_q + \frac{w_3}{\sqrt{3}} D^{(8)}_{\chi 3} \hat{J}_3 + \frac{w_4}{\sqrt{3}} d_{pq3} D^{(8)}_{\chi p} D^{(8)}_{dq} \\
&+ w_5 \left( D^{(8)}_{\chi 3} D^{(8)}_{88} + D^{(8)}_{\chi 8} D^{(8)}_{83} \right) + w_6 \left( D^{(8)}_{\chi 3} D^{(8)}_{88} - D^{(8)}_{\chi 8} D^{(8)}_{83} \right), \\
\hat{g}_{1B_1 \rightarrow B_2} &= a_1 D^{(8)}_{\chi 3} + a_2 d_{pq3} D^{(8)}_{\chi p} \hat{J}_q + \frac{a_3}{\sqrt{3}} D^{(8)}_{\chi 3} \hat{J}_3 + \frac{a_4}{\sqrt{3}} d_{pq3} D^{(8)}_{\chi p} D^{(8)}_{dq} \\
&+ a_5 \left( D^{(8)}_{\chi 3} D^{(8)}_{88} + D^{(8)}_{\chi 8} D^{(8)}_{83} \right) + a_6 \left( D^{(8)}_{\chi 3} D^{(8)}_{88} - D^{(8)}_{\chi 8} D^{(8)}_{83} \right),
\end{align*}
\]

where \( w_i \) and \( a_i \) stand for the parameters encoding the specific dynamics of a chiral solitonic model as in the case of the \( \alpha, \beta, \) and \( \gamma \). For example, they can be explicitly computed by using the \( \chi \)QSM [21, 22]. Each \( w_i \) has its own dynamical meaning: \( w_1 \) (\( a_1 \)) corresponds to the leading-order contribution, \( w_2 \) (\( a_2 \)) and \( w_3 \) (\( a_3 \)) arise from the rotational \( 1/N_c \) corrections, and \( w_4 \) (\( a_4 \)), \( w_5 \) (\( a_5 \)) and \( w_6 \) (\( a_6 \)) are originated from flavor SU(3) symmetry breaking, in which the strange current quark mass \( m_s \) is included. There are two different linear \( m_s \) corrections to the HSD constants \( f_{2B_1 \rightarrow B_2} \) and \( g_{1B_1 \rightarrow B_2} \): The \( m_s \) corrections from the collective operators given in eq.(2.14) and those from the baryon wave functions because of the mixing coefficients in eq.(2.12).

3 Analysis of the HSD constants for the baryon octet

We are now in a position to determine the parameters \( w_i \) and \( a_i \) in eq.(2.14). The experimental data for the magnetic moments of the baryon octet, which are listed in Table 1, can be used for the determination of \( w_i \).

| Experimental data [11] |       |
|------------------------|-------|
| \( \mu_p \)            | 2.792847356 ± 0.000000023 |
| \( \mu_n \)            | -1.9130427 ± 0.0000005    |
| \( \mu_A \)            | -0.613 ± 0.004             |
| \( \mu_{\Sigma^+} \)   | 2.458 ± 0.010             |
| \( \mu_{\Sigma^-} \)   | -1.160 ± 0.025            |
| \( \mu_{\Xi^0} \)      | -1.250 ± 0.014            |
| \( \mu_{\Xi^+} \)      | -0.6507 ± 0.0025          |

Table 1. The experimental data for the magnetic moments of the baryon octet [11] in units of the nuclear magneton \( \mu_N \), which are used as input in the present work.

In fact, the analyses of the magnetic moments in a similar scheme to the present one were carried out in Refs. [36–38]. However, it was then not possible to determine all relevant parameters uniquely, so that the results were shown as functions either of \( m_s \) or of the \( \pi N \)
sigma term. With the mixing coefficients in eq.(2.13) completely determined, however, we are able to proceed to fix the parameters \( w_i \) unambiguously by using the experimental data for the baryon magnetic moments listed in Table 1. Since we have the seven independent experimental data as shown in Table 1, whereas we need to find the six parameters \( (w_i) \), we use the \( \chi^2 \)-fit method. Let us set the magnetic moments of the octet baryon and the parameters \( w_i \) as vectors

\[
\mathbf{\mu} = (\mu_p, \mu_n, \mu_\Lambda, \mu_{\Sigma^+}, \mu_{\Sigma^-}, \mu_{\Xi^0}, \mu_{\Xi^-}), \quad \mathbf{w} = (w_1, w_2, w_3, w_4, w_5, w_6)
\]

(3.1)

such that the relation between \( \mathbf{\mu} \) and \( \mathbf{w} \) is cast into a matrix equation

\[
\mathbf{\mu} = \mathbf{A} \cdot \mathbf{w},
\]

(3.2)

where

\[
\mathbf{A} = \begin{bmatrix}
-\frac{2}{15} - \frac{4}{45}c_{27} & \frac{1}{15} - \frac{8}{45}c_{27} & \frac{1}{3} + \frac{2}{15}c_{27} & -\frac{2}{15} & -\frac{1}{15} & -\frac{1}{30}
\\
-\frac{1}{10} + \frac{1}{3}c_{10} - \frac{2}{45}c_{27} & -\frac{1}{10} + \frac{1}{3}c_{10} - \frac{2}{45}c_{27} & \frac{1}{60} + \frac{1}{6}c_{10} + \frac{1}{15}c_{27} & \frac{7}{70} & \frac{1}{18} & \frac{1}{30}
\\
\frac{1}{15} - \frac{1}{10}c_{27} & -\frac{1}{15} - \frac{1}{10}c_{27} & \frac{1}{3} + \frac{2}{15}c_{27} & -\frac{1}{15} & 0 & 0
\\
\frac{1}{10} + \frac{1}{3}c_{10} - \frac{2}{45}c_{27} & \frac{1}{15} - \frac{4}{45}c_{27} & \frac{1}{3} + \frac{1}{15}c_{27} & -\frac{1}{15} & -\frac{1}{15} & -\frac{1}{10}
\\
\frac{1}{30} + \frac{4}{45}c_{27} & \frac{1}{30} - \frac{4}{45}c_{27} & \frac{1}{60} + \frac{1}{15}c_{27} & -\frac{1}{15} & -\frac{1}{15} & -\frac{1}{10}
\\
\frac{1}{30} - \frac{4}{45}c_{27} & \frac{1}{30} - \frac{4}{45}c_{27} & \frac{1}{60} + \frac{1}{15}c_{27} & -\frac{1}{15} & -\frac{1}{15} & -\frac{1}{10}
\\
\frac{1}{30} - \frac{4}{45}c_{27} & \frac{1}{30} - \frac{4}{45}c_{27} & \frac{1}{60} + \frac{1}{15}c_{27} & -\frac{1}{15} & -\frac{1}{15} & -\frac{1}{10}
\end{bmatrix}
\]

(3.3)

Having solved eq.(3.2), we determine the parameters \( w_i \) as listed in Table 2. We want to point out that the results of \( w_i \) in Ref. [36] suffered from an uncertainty, so that the magnetic moments of the baryon decuplet were given as a function of an unknown parameter \( p \). On the other hand, the results of \( w_i \) in the present work are uniquely determined, as shown in Table 2.

Using the results of \( w_i \) in Table 2, we obtain straightforwardly \( f_2/f_1 \) for various HSD modes:

\[
\begin{bmatrix}
\frac{f_2}{f_1} (n \rightarrow p) \\
\frac{f_2}{f_1} (\Sigma^+ \rightarrow \Lambda) \\
\frac{f_2}{f_1} (\Sigma^- \rightarrow n) \\
\frac{f_2}{f_1} (\Xi^- \rightarrow \Lambda) \\
\frac{f_2}{f_1} (\Xi^- \rightarrow \Sigma^0) \\
\frac{f_2}{f_1} (\Sigma^- \rightarrow \Lambda) \\
\frac{f_2}{f_1} (\Lambda \rightarrow p) \\
\frac{f_2}{f_1} (\Sigma^- \rightarrow \Sigma^0) \\
\frac{f_2}{f_1} (\Xi^- \rightarrow \Xi^0) \\
\frac{f_2}{f_1} (\Xi^- \rightarrow \Xi^+) \\
\end{bmatrix}
= \mathbf{M}_V
\begin{bmatrix}
w_1 \\
w_2 \\
w_3 \\
w_4 \\
w_5 \\
w_6 \\
\end{bmatrix}
\]

(3.4)
with the matrix $M_V$ expressed as

$$M_V = \begin{bmatrix}
-\frac{7}{30} - \frac{1}{8} c_{10} - \frac{5}{27} c_{27} & \frac{7}{60} - \frac{1}{8} c_{10} - \frac{4}{15} c_{27} & \frac{1}{6} - \frac{6}{15} c_{10} + \frac{11}{15} c_{27} & -\frac{11}{27} & -\frac{1}{3} & -\frac{1}{15} \\
-\frac{3}{20} - \frac{1}{4} c_{10} - \frac{1}{12} c_{27} & \frac{3}{20} - \frac{1}{4} c_{10} - \frac{1}{6} c_{27} & -\frac{1}{10} - \frac{1}{8} c_{10} + \frac{1}{8} c_{27} & -\frac{7}{180} & -\frac{1}{30} & 0 \\
\frac{1}{15} - \frac{1}{45} c_{27} & -\frac{1}{30} - \frac{2}{33} c_{27} & \frac{1}{15} + \frac{1}{30} c_{27} & -\frac{1}{5} & -\frac{1}{30} & \frac{1}{15} \\
-\frac{1}{30} - \frac{1}{30} c_{27} & \frac{1}{60} - \frac{1}{15} c_{27} & \frac{1}{20} + \frac{1}{20} c_{27} & -\frac{1}{10} - \frac{1}{30} & 0 & \frac{1}{30} \\
\frac{2}{15} + \frac{1}{6} c_{10} + \frac{1}{3} c_{27} & \frac{1}{15} + \frac{1}{6} c_{10} + \frac{1}{6} c_{27} & \frac{1}{15} + \frac{1}{3} c_{10} - \frac{1}{2} c_{27} & \frac{1}{3} & -\frac{1}{10} & \frac{1}{30} \\
-\frac{1}{45} - \frac{1}{45} c_{27} & \frac{1}{15} - \frac{1}{15} c_{27} & \frac{1}{30} - \frac{4}{45} c_{27} & \frac{1}{1} & -\frac{1}{30} & \frac{1}{15} \\
\frac{1}{15} + \frac{3}{45} c_{27} & -\frac{1}{30} + \frac{4}{45} c_{27} & \frac{1}{15} - \frac{1}{15} c_{27} & \frac{1}{30} & \frac{1}{15} & \frac{1}{45} \\
\frac{7}{30} + \frac{1}{6} c_{10} + \frac{1}{45} c_{27} & \frac{7}{60} + \frac{1}{6} c_{10} + \frac{2}{45} c_{27} & \frac{1}{6} + \frac{1}{15} c_{10} - \frac{3}{10} c_{27} & \frac{11}{40} & \frac{1}{15} & \frac{1}{30}
\end{bmatrix}
(3.5)
$$

It is well known that when the effects of SU(3) symmetry breaking are turned off, the vector HSD constants $f_2(0)/f_1(0)$ are expressed in terms of the anomalous magnetic moments of the proton and the neutron as listed in Table 3. Note that the constants $f_1(0)$ do not have any linear $m_s$ corrections because of the Ademollo-Gatto theorem [26]. However, one has to use the values of $\kappa_p$ and $\kappa_n$ in the exact SU(3) symmetry to compute

| Decay mode $\rightarrow$ | $f_1(0)$ | $f_2(0)$ | $f_2(0)/f_1(0)$ |
|-------------------------|----------|----------|-----------------|
| $n \rightarrow p$      | 1        | $\frac{1}{3} (\kappa_p - \kappa_n)$ | $\frac{1}{3} (\kappa_p - \kappa_n)$ |
| $\Sigma^- \rightarrow \Lambda$ | 0        | $-\frac{1}{2} \sqrt{\frac{3}{2}} \kappa_n$ | $\sqrt{\frac{3}{2}} \kappa_n$ |
| $\Sigma^0 \rightarrow \Sigma^+$ | $\sqrt{2}$ | $\frac{1}{\sqrt{2}} (\kappa_p + \frac{1}{2} \kappa_n)$ | $\frac{1}{\sqrt{2}} (\kappa_p + \frac{1}{2} \kappa_n)$ |
| $\Sigma^- \rightarrow \Sigma^0$ | $\sqrt{2}$ | $\frac{1}{\sqrt{2}} (\kappa_p + \frac{1}{2} \kappa_n)$ | $\frac{1}{\sqrt{2}} (\kappa_p + \frac{1}{2} \kappa_n)$ |
| $\Xi^- \rightarrow \Xi^0$ | 1        | $\frac{1}{2} (\kappa_p + 2 \kappa_n)$ | $\frac{1}{2} (\kappa_p + 2 \kappa_n)$ |
| $\Lambda \rightarrow p$ | $\sqrt{\frac{3}{2}}$ | $\frac{1}{\sqrt{2}} \sqrt{\frac{3}{2}} \kappa_p$ | $\frac{1}{\sqrt{2}} \kappa_p$ |
| $\Sigma^0 \rightarrow p$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}} \sqrt{\frac{3}{2}} (\kappa_p + 2 \kappa_n)$ | $\frac{1}{\sqrt{2}} (\kappa_p + 2 \kappa_n)$ |
| $\Sigma^- \rightarrow n$ | 1        | $\frac{1}{2} (\kappa_p + 2 \kappa_n)$ | $\frac{1}{2} (\kappa_p + 2 \kappa_n)$ |
| $\Xi^0 \rightarrow \Sigma^+$ | 1        | $\frac{1}{2} (\kappa_p - \kappa_n)$ | $\frac{1}{2} (\kappa_p - \kappa_n)$ |
| $\Xi^- \rightarrow \Lambda$ | $\sqrt{\frac{3}{2}}$ | $\frac{1}{\sqrt{2}} \sqrt{\frac{3}{2}} (\kappa_p + \kappa_n)$ | $\frac{1}{\sqrt{2}} (\kappa_p + \kappa_n)$ |
| $\Xi^- \rightarrow \Sigma^0$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}} \sqrt{\frac{3}{2}} (\kappa_p - \kappa_n)$ | $\frac{1}{\sqrt{2}} (\kappa_p - \kappa_n)$ |

Table 3. The expressions of $f_1(0)$, $f_2(0)$, and $g_1(0)$ in exact SU(3) symmetry. The $\kappa_p$ and the $\kappa_n$ denote the anomalous magnetic moments of the proton and the neutron, respectively.

$f_2(0)$ in SU(3) symmetry, since the experimental data for the nucleon anomalous magnetic moments already include the effects of SU(3) symmetry breaking. The numerical values of anomalous magnetic moments of proton and neutrons for the contributions of the order
we list the present results for the vector HSD constants

\[ \kappa_p^{sym} \left[ \mathcal{O}(m_s^0) \right] = 1.363 \pm 0.069, \]
\[ \kappa_n^{sym} \left[ \mathcal{O}(m_s^0) \right] = -1.416 \pm 0.049, \]
\[ \kappa_p \left[ \mathcal{O}(m_s^0) + \mathcal{O}(m_s^1) \right] = 1.793 \pm 0.087, \]
\[ \kappa_n \left[ \mathcal{O}(m_s^0) + \mathcal{O}(m_s^1) \right] = -1.913 \pm 0.069, \]  

(3.6)

where \( \kappa_p^{sym} \) and \( \kappa_n^{sym} \) represent the anomalous magnetic moments of the proton and the neutron in the exact SU(3) symmetry. We can estimate the effect of SU(3) symmetry breaking from eq.(3.6), which is approximately 25%.

| Decay mode | \((f_2/f_1)^{sym}\) | \((f_2/f_1)^{br}\) | Experimental data [11] |
|------------|----------------------|----------------------|----------------------|
| \(n \to p\) | 1.389 ± 0.042 | 1.853 ± 0.056 | |
| \(\Sigma^- \to \Lambda\) | 1.062 ± 0.037 | 1.435 ± 0.052 | |
| \(\Sigma^- \to \Sigma^0\) | 0.328 ± 0.037 | 0.418 ± 0.047 | |
| \(\Xi^- \to \Xi^0\) | -0.734 ± 0.060 | -1.017 ± 0.082 | |
| \(\Lambda \to n\) | 0.681 ± 0.035 | 0.896 ± 0.043 | |
| \(\Sigma^- \to \Lambda\) | -0.734 ± 0.060 | -1.017 ± 0.082 | -0.97 ± 0.14 |
| \(\Xi^0 \to \Sigma^+\) | 1.389 ± 0.042 | 1.853 ± 0.056 | 2.0 ± 1.2 ± 0.5 |
| \(\Xi^- \to \Sigma^0\) | -0.026 ± 0.042 | -0.060 ± 0.056 | |
| \(\Xi^- \to \Xi^0\) | 1.389 ± 0.042 | 1.853 ± 0.056 | |

Table 4. Numerical results for the ratios of the vector HSD constants \(f_2/f_1\) of the baryon octet. The superscripts “sym” and “br” denote the contribution in exact SU(3) symmetry and the total results with flavor SU(3) symmetry breaking, respectively. Note that the values for the \(\Sigma^- \to \Lambda\) mode are given for \(\sqrt{3}/2f_2\) instead of \(f_2/f_1\).

In Table 4 we list the present results for the vector HSD constants \(f_2(0)/f_1(0)\). There are only two existing data for the decay processes \(\Sigma^- \to ne^-\bar{\nu}_e\) and \(\Xi^0 \to \Sigma^+e^-\bar{\nu}_e\) from the PDG. The present results are in agreement with these experimental data, though the experimental uncertainties of \(f_2(0)/f_1(0)\) for the \(\Xi^0 \to \Sigma^+e^-\bar{\nu}_e\) decay are rather large. Note that there are old data for the \(\Lambda \to pe^-\bar{\nu}_e\) decay [12, 39, 40]. In Ref. [39], the vector HSD constant for \(\Lambda \to pe^-\bar{\nu}_e\) decay is given as \(f_2(0)/f_1(0)(\Lambda \to p) = 0.15 \pm 0.30\). However, this value needs to be scrutinized experimentally, because firstly its uncertainty is very large and secondly its value itself is quite different from most theoretical predictions. In fact, Cabbibo et al. [6] reviewed the old experimental data [41, 42] and extracted the following values: \(f_1 = 1.238 \pm 0.024\) and \(f_2 = 1.34 \pm 0.20\), which is in line with the old data: \(f_1 = 1.229 \pm 0.035\) [42]. The prediction from the present work is \(f_2 = 1.098 \pm 0.053\), which is closer to that from Ref. [6]. As shown in Table 4, the effects of SU(3) symmetry breaking on the HSD vector constants appear consistently to be more than 20%.

In order to determine the dynamical parameters \(a_i\) in eq. (2.14), we use the experimental data for the HSD constants \(g_1/f_1(B_1 \to B_2)\) listed in Table 5. Since there exist only five experimental data for them as listed in Table 5 but six unknown parameters \(a_i\), we need at least one more data. There are three known diagonal axial-vector constants, that is, \(g_A^{(0)}, g_A^{(3)}, \text{ and } g_A^{(8)}\). However, the triplet axial-vector constant \(g_A^{(3)}\) is the same as
that for the neutron $\beta$ decay $g_1/f_1(n \rightarrow p)$, we have to avoid it. The octet one $g^{(8)}_A$ is usually determined by using the HSD constants with SU(3) symmetry assumed, it is also not suitable for the present purpose. The rest is the singlet axial-vector constant $g^{(0)}_A$, which implies the quark content of the nucleon spin and is determined by the data for polarized electron-proton deep inelastic scattering. Thus, in addition to the experimental data for the five known HSD constants, we utilize $g^{(0)}_A$ to fix $a_i$.

| Experimental data | References |
|-------------------|------------|
| $g_1/f_1 (n \rightarrow p)$ | PDG [11] |
| $g_1/f_1 (\Lambda \rightarrow p)$ | 0.718 ± 0.015 PDG [11] |
| $g_1/f_1 (\Sigma^- \rightarrow n)$ | −0.340 ± 0.017 PDG [11] |
| $g_1/f_1 (\Xi^- \rightarrow \Lambda)$ | 0.25 ± 0.05 PDG [11] |
| $g_1/f_1 (\Xi^0 \rightarrow \Sigma^+)$ | 1.21 ± 0.05 PDG [11] |
| $g^0_A$ | 0.36 ± 0.03 Bass et al. [43] |

**Table 5.** The experimental data for the HSD constants from the Particle Data Group [11].

With these six input data from experiments at hand, the dynamical parameters $a_i$ can be determined by solving the matrix equation similar to eq.(3.2). The parameters $a_i$ determined by eq. (3.7) are listed in Table 6. Having fixed $a_i$, we can straightforwardly solve the following matrix equation to find other HSD constants and $g^8_A$:

\[
\begin{bmatrix}
  g_1/f_1 (n \rightarrow p) \\
  g_1/f_1 (\Sigma^+ \rightarrow \Lambda) \\
  g_1/f_1 (\Lambda \rightarrow p) \\
  g_1/f_1 (\Sigma^- \rightarrow n) \\
  g_1/f_1 (\Xi^- \rightarrow \Lambda) \\
  g_1/f_1 (\Xi^- \rightarrow \Sigma^0) \\
  g_1/f_1 (\Sigma^- \rightarrow \Lambda) \\
  g_1/f_1 (\Sigma^- \rightarrow \Sigma^0) \\
  g_1/f_1 (\Xi^- \rightarrow \Xi^0) \\
  g_1/f_1 (\Xi^0 \rightarrow \Xi^+) \\
  g^0_A(p) \\
  g^3_A(p) \\
  g^8_A(p)
\end{bmatrix}
= \mathbf{M}_A
\begin{bmatrix}
a_1 \\
a_2 \\
a_3 \\
a_4 \\
a_5 \\
a_6
\end{bmatrix},
\]

\[(3.7)\]

**Table 6.** The dynamical parameters of axial-vector transitions
where the matrix $M_A$ is expressed as

$$M_A = \begin{pmatrix}
-\frac{7}{30} - \frac{1}{3} & -\frac{2}{15} & -\frac{1}{15} & -\frac{2}{15} & -\frac{1}{15} & -\frac{11}{15} & -\frac{1}{9} & -\frac{1}{15} \\
-\frac{3}{20} - \frac{1}{4} & -\frac{1}{12} & -\frac{1}{12} & -\frac{1}{12} & -\frac{1}{12} & -\frac{1}{12} & -\frac{1}{12} & -\frac{1}{12} \\
-\frac{2}{15} & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\
\frac{1}{15} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \\
-\frac{1}{30} & -\frac{2}{15} & -\frac{1}{15} & -\frac{1}{15} & -\frac{1}{15} & -\frac{1}{15} & -\frac{1}{15} & -\frac{1}{15} \\
\frac{1}{60} & \frac{1}{15} & \frac{1}{15} & \frac{1}{15} & \frac{1}{15} & \frac{1}{15} & \frac{1}{15} & \frac{1}{15} \\
\frac{1}{15} & -\frac{1}{30} & -\frac{2}{15} & -\frac{1}{15} & -\frac{1}{15} & -\frac{1}{15} & -\frac{1}{15} & -\frac{1}{15} \\
\frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

(3.8)

In the exact SU(3) symmetry, all axial-vector decay amplitudes are given in terms of two reduced matrix elements $F$ and $D$ as follows:

$$g_{1(B_1\rightarrow B_2)}(0) = C_F^{(B_1\rightarrow B_2)} F + C_D^{(B_1\rightarrow B_2)} D. \quad (3.9)$$

Here $C_F^{(B\rightarrow B')}$ and $C_D^{(B\rightarrow B')}$ are SU(3) Clebsch-Gordan coefficients that appear when the axial-vector collective operator $g_{1(B_1\rightarrow B_2)}$ in eq.(2.14) is sandwiched between octet states. Thus, the $F$ and $D$ are determined in terms of $a_1$, $a_2$, and $a_3$

$$F = -\frac{1}{12} \left( a_1 - \frac{1}{2} a_2 \right) + \frac{1}{24} a_3 = 0.461 \pm 0.002, \quad (3.10)$$

$$D = -\frac{3}{20} \left( a_1 - \frac{1}{2} a_2 \right) - \frac{1}{40} a_3 = 0.769 \pm 0.003. \quad (3.10)$$

The results for the $F$ and $D$ are in good agreement with the empirical ones extracted from the experiments [44], given as $F = 0.463 \pm 0.008$ and $D = 0.804 \pm 0.008$.

In Table 8, the results for the axial-vector HSD constants $g_1/f_1$ are listed. In contrast to the results for $f_2/f_1$, the effects of flavor SU(3) symmetry breaking on the axial-vector HSD constants are rather small and stable. However, when it comes to the singlet axial charge, they are not negligible. Note that though $g_A^{(0)}$ is used as an input, we still can estimate the effects of SU(3) symmetry breaking, which turn out to be approximately 40%. The reason why the contribution of SU(3) symmetry breaking is large in the case the singlet axial charge can be found in eqs.(3.7, 3.8). The expression for $g_A^{(0)}$ is written as

$$g_A^{(0)} = a_3 - \frac{1}{5} (a_5 - a_6). \quad (3.11)$$

The singlet axial charge does not have any contribution from the leading order in the $1/N_c$ expansion. Indeed, this explains why the Skyrme model predicts the smallness of $g_A^{(0)}$. 

-- 11 --
the quark content of the nucleon spin. Large. Thus, the effects of SU(3) symmetry breaking play an essential role in describing the total results with flavor SU(3) symmetry breaking, respectively. Note that the values for the Table 8. Numerical results for the ratios of the axial-vector HSD constants $g_{1}/f_{1}$ of the baryon octet. The superscripts “sym” and “br” denote the contribution in exact SU(3) symmetry and the total results with flavor SU(3) symmetry breaking, respectively. Note that the values for the $\Sigma^{-} \to \Lambda$ mode are given for $\sqrt{3/2} g_{1}$ instead of $g_{1}/f_{1}$. The experimental data are taken from the Particle Data Group [11], which are used for the input.

Because of this fact, there is no wavefunction corrections to $g_{A}^{(0)}$. Moreover, the relative signs of $a_{5}$ and $a_{6}$ are different, so that the linear $m_{s}$ corrections turn out to be rather large. Thus, the effects of SU(3) symmetry breaking play an essential role in describing the quark content of the nucleon spin.

The octet axial-vector constant $g_{A}^{(8)}$ was usually extracted from the HSD data with flavor SU(3) symmetry assumed and its value was obtained to be $g_{A}^{(8)} = 0.58 \pm 0.03$ [45].
On the other hand, the present result for $g_A^{(8)}$ in the SU(3) symmetric case is given as $g_A^{(8)} = 0.354 \pm 0.003$. With the linear $m_s$ corrections taken into account, the value of $g_A^{(8)}$ is reduced to $g_A^{(8)} = 0.325 \pm 0.004$, which is not much different from that with SU(3) symmetry. However, compared with the extracted value from Ref. [45], the present result for $g_A^{(8)}$ is approximately 40% smaller than that. Note that the octet axial charge is expressed in terms of $F$ and $D$ as follows:

$$g_A^{(8)} = \frac{1}{\sqrt{3}}(3F - D) = \frac{1}{\sqrt{3}} \left[ -\frac{1}{10}(a_1 - \frac{1}{2}a_2) + \frac{1}{20}a_3 \right]. \quad (3.12)$$

Inserting the values of $F$ and $D$ given in Ref. [44], we find $g_A^{(8)} = 0.338 \pm 0.015$, which is quite different from that of Ref. [45] but is compatible with the present value.

4 Analysis for the transition from the baryon decuplet to the octet

In this Section, we want to present the results for the axial-vector transition constants of the baryon decuplet. Since the baryon decuplet except for $\Omega^-$ decays into the baryon octet strongly, there are no direct experimental data for the axial-vector HSD constants for the baryon decuplet to date. However, there are at least two evident reasons why the axial-vector transition constants for the baryon decuplet are interesting. Firstly, the baryon decuplet can be produced by exclusive neutrino-nucleon scattering such as $\nu + N \rightarrow \mu + \Delta$ and $\nu + N \rightarrow \mu + B_{10} + M_8$ processes, where $M_8$ stands for the pseudo-scalar meson octet. These reactions are also interesting, since they may bring out information on weak generalized parton distributions [46]. The axial-vector transition constants for the baryon decuplet might be extracted from these reactions. Secondly, using the Goldberger-Treiman relation, one can relate the axial-vector transition constants to the strong coupling constants for vertices of the baryon decuplet-octet and the meson octet. Because of these reasons, it is of great interest to understand the axial-vector transition constants for the baryon decuplet.

There are four different axial-vector form factors or Adler form factors $C_i^A(q^2)$ ($i = 3, 4, 5, 6$) as given in eq.(2.3). However, $C_3^A$ and $C_4^A$ are often neglected, since the partially conserved axial-vector current (PCAC) relation yields an expression, where the coefficients in front of $C_3^A$ and $C_4^A$ are proportional to the mass difference of a decuplet baryon and an octet one divided by the mass sum, and its squares, respectively [47]. Thus, their contribution is negligible in the transition amplitude of $\nu N \rightarrow \mu B_{10}$ scattering. In the chiral limit, $C_6^A$ can be related to $C_5^A$, so that we will concentrate on $C_5^A$ in this work.

Before we present the results, we want to discuss possible relations and sum rules for the axial-vector transition constants $C_5^A$. With the linear $m_s$ corrections turned on, we
find the following eight relations for $C_5^A$

\[
(\Delta^0 \rightarrow p) = \frac{1}{\sqrt{3}} (\Delta^- \rightarrow n) = - \frac{1}{\sqrt{3}} (\Delta^{++} \rightarrow p) = - (\Delta^+ \rightarrow n),
\]

\[
(\Sigma^{*0} \rightarrow \Sigma^+) = (\Sigma^{*+} \rightarrow \Sigma^0) = (\Sigma^{*0} \rightarrow \Sigma^0) = (\Sigma^{*+} \rightarrow \Sigma^-),
\]

\[
(\Sigma^{*0} \rightarrow \Lambda) = (\Sigma^{*+} \rightarrow \Lambda),
\]

\[
(\Xi^{*0} \rightarrow \Xi^0) = (\Xi^{*0} \rightarrow \Xi^-),
\]

\[
(\Delta^{++} \rightarrow \Sigma^+) = \sqrt{3} (\Delta^+ \rightarrow \Sigma^0) = \sqrt{3} (\Delta^0 \rightarrow \Sigma^-),
\]

\[
(\Sigma^{*0} \rightarrow \Xi^0) = \sqrt{2} (\Sigma^{*0} \rightarrow \Xi^-),
\]

\[
(\Xi^{*0} \rightarrow \Sigma^0) = \sqrt{2} (\Xi^{*0} \rightarrow \Sigma^-). \tag{4.1}
\]

These relations come from isospin symmetry. We also obtain the following six sum rules for the $C_5^A$

\[
(\Delta^0 \rightarrow p) = - (\Xi^* \rightarrow \Xi^0) - \frac{1}{\sqrt{2}} (\Xi^* \rightarrow \Sigma^0) + \frac{3}{\sqrt{2}} (\Xi^* \rightarrow \Lambda) + \frac{2}{\sqrt{3}} (\Omega^- \rightarrow \Xi^0),
\]

\[
(\Sigma^{*0} \rightarrow \Sigma^+) = \frac{1}{\sqrt{2}} (\Xi^* \rightarrow \Xi^0) + (\Xi^* \rightarrow \Sigma^0) - \frac{1}{\sqrt{6}} (\Omega^- \rightarrow \Xi^0),
\]

\[
(\Sigma^{*0} \rightarrow \Lambda) = - \frac{1}{\sqrt{2}} (\Xi^* \rightarrow \Xi^0) + (\Xi^* \rightarrow \Lambda) + \frac{1}{\sqrt{2}} (\Omega^- \rightarrow \Xi^0),
\]

\[
(\Xi^* \rightarrow \Xi^0) = \frac{1}{\sqrt{3}} (\Xi^* \rightarrow \Lambda) - \frac{1}{\sqrt{3}} (\Sigma^{*0} \rightarrow \Lambda) + \frac{1}{\sqrt{3}} (\Omega^- \rightarrow \Xi^0),
\]

\[
(\Sigma^{*0} \rightarrow p) = - \frac{1}{2} (\Xi^* \rightarrow \Sigma^0) + \frac{3}{2} (\Xi^* \rightarrow \Lambda) + \frac{1}{\sqrt{6}} (\Omega^- \rightarrow \Xi^0),
\]

\[
(\Delta^{++} \rightarrow \Sigma^+) = - \sqrt{6} (\Xi^* \rightarrow \Sigma^0) + \sqrt{6} (\Sigma^{*0} \rightarrow \Xi^-) + (\Omega^- \rightarrow \Xi^0). \tag{4.2}
\]

Though we do not expect that the sum rules in eq.(4.2) will be confirmed by the experimental data in near future, they provide a consistency check of the present results.

In Tables 9 and 10, we list the results for the axial-vector transition constants of the baryon decuplet decaying into the octet in the strangeness-conserving cases ($\Delta S = 0$). Table 9 presents those for the positive charge difference between the initial state and the final state ($\Delta Q = +1$), and Table 10 for the negative charge difference ($\Delta Q = -1$). Tables 11 and 12 list the results for $C_5^A$ in the strangeness-changing cases ($\Delta S = 1$). Note that all the results presented in Tables 9, 10, 11 and 12 satisfy the isospin relations given in eq.(4.1) and also the sum rules in eq.(4.2). The effects of flavor SU(3) symmetry breaking turn out to be rather small in the case of $C_5^A$. 

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\[\text{---} 14 \text{---}\]
\begin{table}
\centering
\begin{tabular}{ccc}
\hline
$B_{10}^{X=1+i2}$ & $B_8$ & $C_5^A$ (sym) & $C_5^A$ (br) \\
\hline
$\Delta^0 \rightarrow p$ & $-0.954 \pm 0.003$ & $-1.040 \pm 0.005$ \\
$\Delta^- \rightarrow n$ & $-1.653 \pm 0.006$ & $-1.801 \pm 0.008$ \\
$\Sigma^{*0} \rightarrow \Sigma^+$ & $0.675 \pm 0.002$ & $0.614 \pm 0.004$ \\
$\Sigma^{*+} \rightarrow \Sigma^0$ & $0.675 \pm 0.002$ & $0.614 \pm 0.004$ \\
$\Sigma^{*+} \rightarrow \Lambda$ & $-1.169 \pm 0.004$ & $-1.231 \pm 0.005$ \\
$\Xi^{*0} \rightarrow \Xi^-$ & $0.954 \pm 0.003$ & $0.903 \pm 0.006$ \\
\hline
\end{tabular}
\caption{Axial-vector transition constants of the baryon decuplet to octet with $X = 1 + i2$, i.e. in the strangeness-conserving ($\Delta S = 0$) case. The charge difference between the initial state and the final state is $\Delta Q = +1$.}
\end{table}

\begin{table}
\centering
\begin{tabular}{ccc}
\hline
$B_{10}^{X=1-i2}$ & $B_8$ & $C_5^A$ (sym) & $C_5^A$ (br) \\
\hline
$\Delta^{++} \rightarrow p$ & $1.653 \pm 0.006$ & $1.801 \pm 0.008$ \\
$\Delta^+ \rightarrow n$ & $0.954 \pm 0.003$ & $1.040 \pm 0.005$ \\
$\Sigma^{*+} \rightarrow \Sigma^0$ & $0.675 \pm 0.002$ & $0.614 \pm 0.004$ \\
$\Sigma^{*+} \rightarrow \Lambda$ & $1.169 \pm 0.004$ & $1.231 \pm 0.005$ \\
$\Sigma^{*0} \rightarrow \Sigma^-$ & $0.675 \pm 0.002$ & $0.614 \pm 0.004$ \\
$\Xi^{*0} \rightarrow \Xi^-$ & $0.954 \pm 0.003$ & $0.903 \pm 0.006$ \\
\hline
\end{tabular}
\caption{Axial-vector transition constants of the baryon decuplet to octet with the flavor transition operator of $X = 1 - i2$, i.e. in the strangeness-conserving ($\Delta S = 0$) case. The charge difference between the initial state and the final state is $\Delta Q = -1$.}
\end{table}

\begin{table}
\centering
\begin{tabular}{ccc}
\hline
$B_{10}^{X=4+i5}$ & $B_8$ & $C_5^A$ (sym) & $C_5^A$ (br) \\
\hline
$\Sigma^{*0} \rightarrow p$ & $-0.675 \pm 0.002$ & $-0.755 \pm 0.004$ \\
$\Sigma^{*-} \rightarrow n$ & $-0.954 \pm 0.003$ & $-1.067 \pm 0.005$ \\
$\Xi^{*0} \rightarrow \Xi^+$ & $0.954 \pm 0.003$ & $0.896 \pm 0.004$ \\
$\Xi^{*-} \rightarrow \Xi^0$ & $0.675 \pm 0.002$ & $0.633 \pm 0.003$ \\
$\Xi^{*-} \rightarrow \Lambda$ & $-1.169 \pm 0.004$ & $-1.266 \pm 0.005$ \\
$\Omega^- \rightarrow \Xi^0$ & $1.653 \pm 0.006$ & $1.612 \pm 0.007$ \\
\hline
\end{tabular}
\caption{Axial-vector transition constants of the baryon decuplet to octet with $X = 4 + i5$, i.e. in the strangeness-changing ($\Delta S = 1$) case. The charge difference between the initial state and the final state is $\Delta Q = +1$.}
\end{table}

In the SU(6) quark model, the $C_5^A(n \rightarrow \Delta^+)$ can be related to the axial-vector HSD constant $g_1/f_1(n \rightarrow p)$ [27, 48] as

$$C_5^A(\Delta^+ \rightarrow n) = \frac{2\sqrt{3}}{5} g_1/f_1(n \rightarrow p). \hspace{1cm} (4.3)$$

Equation (4.3) has been often used to determine the $\pi N \Delta$ coupling constant. Thus, it is interesting to examine it based on the present result. Using the experimental data for $g_1/f_1(n \rightarrow p)$, we obtain $C_5^A(\Delta^+ \rightarrow n) = 0.88$, whereas the present result from Table 10 is
Table 12. Axial-vector transition constants of the baryon decuplet to the octet with $X = 4 - i5$, i.e. in the strangeness-changing ($\Delta S = 1$) case. The charge difference between the initial state and the final state is $\Delta Q = -1$.

\[
\begin{array}{ccc}
B_{10}^{X=4-i5} & B_8 & C_5^{A \text{ (sym)}} & C_5^{A \text{ (br)}} \\
\Delta^{++} \rightarrow \Sigma^+ & -1.653 \pm 0.006 & -1.547 \pm 0.007 \\
\Delta^+ \rightarrow \Sigma^0 & -1.350 \pm 0.005 & -1.263 \pm 0.005 \\
\Delta^+ \rightarrow \Lambda & 0 & 0 \\
\Delta^0 \rightarrow \Sigma^- & -0.954 \pm 0.003 & -0.893 \pm 0.004 \\
\Sigma^{*+} \rightarrow \Xi^0 & -0.954 \pm 0.003 & -0.928 \pm 0.004 \\
\Sigma^{*0} \rightarrow \Xi^- & -0.675 \pm 0.002 & -0.656 \pm 0.003 \\
\end{array}
\]

$C_5^{A \Delta^+ \rightarrow n} = 0.95$. Thus, the value from the SU(6) relation is deviated from the present one approximately by 10%.

5 Summary and outlook

In the present work, we aimed at investigating the hyperon semileptonic decay constants of the baryon octet within the framework of a chiral soliton model, determining all the dynamical parameters by the experimental data. We first reviewed the analysis of the mass splittings of the baryon octet and decuplet in brief. Then, we studied the vector hyperon semileptonic decay constants $f_2/f_1$ with the dynamical parameters $w_i$ fixed in an unambiguous manner by the experimental data for the magnetic moments of the baryon octet. We found that the effects of flavor SU(3) symmetry breaking contributed to the $f_2/f_1$ approximately by 25%.

The dynamical parameters $a_i$ were fixed by using the experimental data for the axial-vector hyperon semileptonic decay constants as well as the singlet axial charge $g_A^{(0)}$ unequivocally. We first derived the $F$ and $D$ in exact SU(3) symmetry and the results were consistent with the empirical data. We also predicted other $g_i/f_1$ which are not measured yet. The octet $g_A^{(8)}$ was obtained to be $g_A^{(8)} = 0.325 \pm 0.004$. The contribution of flavor SU(3) symmetry breaking turned out to be rather small in the case of the axial-vector hyperon semileptonic decay constants. Finally, we predicted the axial-vector constants for the transitions from the baryon decuplet to the octet. In addition, we derived the isospin relations and their six different sum rules.

The present results for the axial-vector transition constants for the baryon decuplet can be used to determine the meson-baryon Yukawa coupling constants by the Goldberger-Treiman relations. The whole analysis including the baryon decuplet-decuplet-meson coupling constants is under way. Last but not least, the vector transition constants for the baryon decuplet are also interesting and important. However, experimental information on the $E_2/M_1$ ratio for the baryon decuplet is essential but only that of the $\Delta$ isobar is known. On the other hand, if one assumes SU(3) symmetry, it might be possible to study the vector transition constants for the baryon decuplet. The corresponding investigation will appear elsewhere.
Acknowledgments

The present work was supported by Basic Science Research Program through the National Research Foundation of Korea funded by the Ministry of Education, Science and Technology (Grant Numbers: NRF-2013R1A1A2063590 and NRF-2012R1A1A2001083).

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