Flexible Direction-of-Arrival Simulation for Automotive Radar Target Simulators

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ABSTRACT In order to simulate realistic traffic scenarios, a radar target simulator must be able to generate multiple radar targets with different directions of arrival. The presented concept is able to generate an arbitrary amount of targets with individual directions of arrival for the radar under test. By measuring the radar channel, the novel approach enables target simulators to simulate arbitrary directions of arrival, while minimizing the required hardware. The optimum setup is derived for radars with a uniform linear receive antenna array. The compensation of placement errors for automotive chirp-sequence frequency modulated continuous wave radars is demonstrated. Finally, the calibration for the setup is provided, and the performance of the presented approach is validated.

INDEX TERMS Automotive radar, chirp-sequence modulation, direction of arrival, FMCW radar, radar target simulator.

I. INTRODUCTION

The usage of radar sensors in automotive applications is rapidly increasing [1]. The robustness offered by radar detection makes the employment for advanced driver assistance systems (ADAS) highly attractive [2], [3]. With higher levels of autonomous functionality, thorough testing is required in order to ensure safe operation. To demonstrate the safe operation of an autonomous vehicle, hundreds of millions of test kilometers would be required [4]. By simulating various traffic scenarios with the help of a radar target simulator, the costs and time for test drives can be reduced significantly. In addition, specific traffic scenarios are completely reproducible and could therefore be used as a benchmark for comparing different radar systems or development iterations thereof. In a typical traffic scenario, like an intersection, many objects at different ranges, velocities, and directions are present at the same time. A radar target simulator suitable for the validation and testing of complex ADAS must be able to reproduce those scenarios accurately. While there is a vast variety of concepts for the generation of radar targets with variable distances and velocities [5]–[8], there are only a few concepts for the generation of targets with a specific direction of arrival (DoA). The DoA of targets allows radar systems to estimate the position and orientation of other road users [9] or obstacles [10] and must be simulated as well for a realistic simulation. A test environment that is able to generate multiple targets with different DoAs for an automotive radar sensor was presented in [11] and [12]. The generation of the angular information relies on a mechanical approach that moves the target simulator antenna to the position of the intended DoA. This means the approach enables the target simulator to generate as many targets with an individual DoA at the same time, as there are movable antennas. Another approach was presented in [13], where low-cost modulated reflectors have a fixed position and therefore simulate a fixed DoA. For an entire automotive scenario that might have a field of view of 60°, this approach...
would require 30 target simulators for a resolution of 2°. Similar approaches with two-dimensional antenna grids have been proposed in [14], [15].

In this article a novel approach for the generation of targets with flexible DoA is presented. By using coherently modulated target elements placed around the radar sensor under test, a joint DoA simulation is possible. Via electrical steering, the DoA of each individual target in the simulated scenario is generated, allowing the simulation of continuously changing DoAs. Besides, the amount of different DoAs that can be simulated at the same time is not limited. Due to the electrical steering, no mechanically moving parts are employed, reducing costs and enhancing the overall reliability. Therefore, the simulation environment has a lower complexity, in comparison to mechanical solutions [11], [12]. Besides that, the restriction of a limited number of targets with individual DoA is resolved. The amount of required hardware is reduced significantly in comparison to a simulation environment that employs a target simulator for each simulated DoA [13]–[15]. The new concept is built on simple hardware and sophisticated signal processing, yielding a low-cost system.

The structure of this article is as follows. First, in Section II a general overview of the system is given. In Section III the mathematical description of the system and the simulation of targets with individual DoA is derived. The compensation of path length differences and the optimal positions for the radar and target simulator are analyzed. Afterwards, in Section IV the calibration process for the employment of the concept is presented. Simulation results of the influence of placement errors on the achievable DoA accuracy are presented in Section V. Lastly, measurements with a radar sensor and a target simulator, based on the proposed concept, are presented in Section VI, and the conclusion is given in Section VII.

II. SYSTEM CONCEPT

In the following a chirp-sequence frequency modulated continuous wave (CS-FMCW) radar sensor is assumed to be the radar under test. While the general concept for the DoA simulation is not restricted to CS-FMCW radars, this article focuses on radars employing the CS-FMCW modulation, since this is the state-of-the-art waveform in the automotive industry [1]. The system concept is based on \( N \) distributed target elements (TE) that are placed around the CS-FMCW radar sensor. The TEs are located at different azimuth angles but at the same distance in respect to the radar under test, as shown in Fig. 1. Each TE receives the transmit signal of the radar with an antenna directed towards the radar sensor. The received signal is amplified and modulated using an IQ-mixer. The IQ-mixer multiplies the radar signal with a modulation signal comprising of low frequency complex valued sinusoids up to several MHz. The modulation signal for all target elements is provided by a multi-channel digital-to-analog converter (DAC) and thus all modulation signals have a fixed phase relation among each other. The amplitude \( A_n \) and phase \( \varphi_{\text{mod},n} \) of the modulation signal for the \( n \)-th TE can be set arbitrarily. The frequency \( f_{\text{mod}} \) of the modulation signal for all TEs is identical and determines the range and velocity information for a simulated target. By choosing an appropriate modulation frequency \( f_{\text{mod}} \), both range and velocity of a target can be simulated. A detailed description of target simulation for CS-FMCW radars by modulating the radar signal can be found in [7], [16], [17]. Following the modulation, the signal is amplified again in order to compensate for the conversion losses, and lastly transmitted by an antenna back towards the radar.

For a radar with a single transmit (TX) antenna and \( M \) receive (RX) antennas, the received signal of the \( m \)-th antenna will be the superposition of the transmitted signals of all TEs. By tuning the freely selectable amplitudes \( A_n \) and phases \( \varphi_{\text{mod},n} \) of the TEs, a target with an arbitrary DoA can be simulated, as shown later. In order to adjust the radar cross section (RCS) of a target, the amplitudes \( A_n \) can be scaled, causing an increase or decrease of the received signal power at the radar. The simulation of multiple targets with individual DoAs is possible by superimposing the modulation signals of the individual targets in the digital domain before the DAC.

In order to calculate the required amplitudes \( A_n \) and phases \( \varphi_{\text{mod},n} \) of the TEs, a calibration is performed estimating the channel between the radar under test and the target simulator. Additionally with the information provided by the calibration, placement errors are compensated for CS-FMCW radars. The compensation relaces the required placement accuracy of the TEs and the radar sensor.

The system concept is also applicable for target elements that generate the range of simulated targets using a delay
method but modulate the Doppler frequency onto the transmit signal. Therefore, the presented concept is not limited to purely modulation based target simulators. If differences for the concept occur due to the delay method architecture, they will be mentioned explicitly in the following.

III. SIGNAL MODEL
To derive the amplitudes $A_n$ and phases $\varphi_n$ of the TEs, a suitable coordinate system is introduced. Then, the relation of the received signal at the radar under test is derived with respect to the placement, amplitudes, and phases of the TEs. Subsequently, the influence and compensation of path length differences is analyzed. Eventually, the ideal placement of the TEs and the radar is derived. The signal model will assume a uniform linear array (ULA) of the radar antennas. In the following, variables for vectors are denoted as lower case bold letters and variables for matrices as upper case bold letters.

A. COORDINATE SYSTEM
The coordinate system for an arbitrary placement of the radar and the $n$-th TE is shown in Fig. 2. The antennas of the radar and the TEs are assumed to be at the same height ($y = \text{const.}$) and the Cartesian coordinate system is reduced to two dimensions without loss of generality. The positions of the antennas and TEs are described by their position vector $p$

$$p = \begin{bmatrix} x \\ R \sin(\theta) \\ R \cos(\theta) \end{bmatrix}.$$  

All positions relate to the origin of the global coordinate system. The distance between the TX antenna and the $n$-th TE is denoted as $r_{TX,n}$. The distance between the $n$-th TE and the $m$-th RX antenna is denoted as $r_{RX,m,n}$. Consequently, the distance $r_{m,n}$ that a wave travels when it is transmitted by the TX antenna, modulated by the $n$-th TE and retransmitted back to the $m$-th RX antenna of the radar is given by

$$r_{m,n} = r_{TX,n} + r_{RX,m,n} = |p_{TE,n} - p_{TX}| + |p_{RX,m,n} - p_{TE,n}|.$$  

B. RECEIVE SIGNAL
The signal transmitted by the radar is modulated by the TEs and transmitted back towards the RX antennas. The superposition of the transmit signal, modulated by all $N$ TEs, is therefore received at the $m$-th RX antenna. The received signal $s_{RX,m}(t)$ at the $m$-th RX antenna can be described by

$$s_{RX,m}(t) = \sum_{n=1}^{N} L_{m,n} s_{TX}(t - T_{m,n}) s_{mod,n}(t)$$  

with the modulation signal of the $n$-th TE

$$s_{mod,n}(t) = A_n e^{i(2\pi f_{mod,t} + \varphi_{mod,n})}.$$  

The factor $L_{m,n}$ is the loss of the transmission path, and $s_{TX}(t - T_{m,n})$ is the transmit signal of the radar delayed by the propagation time

$$T_{m,n} = \frac{r_{m,n}}{c_0}.$$  

Additive white gaussian noise (AWGN) is neglected in the signal model due to the direct line of sight between the radar and the TEs, resulting in a high signal-to-noise ratio. First, it is assumed that the path length $r_{m,n}$ and the resulting time delay $T_{m,n}$ only cause a constant phase shift $\varphi_{m,n}$ for the radar signal with the center frequency $f_0$ and resulting free-space wavelength $\lambda_0$ of

$$\varphi_{m,n} = \frac{2\pi}{\lambda_0} r_{m,n} = \frac{2\pi}{\lambda_0} f_0 T_{m,n}.$$  

The resulting beat frequency of each TE is similar, since all TEs are placed at the same distance to the radar, and the modulation frequency $f_{mod}$ is identical, justifying this assumption for the time being. The analysis of the error caused by approximating the FMCW signal as a CW signal is considered in Section III-D. With this assumption the receive signals in (3) can be expressed via a matrix vector multiplication

$$s_{RX}(t) = C \cdot s_{mod}(t) s_{TX}(t)$$  

with the receive signal vector

$$s_{RX}(t) = \begin{bmatrix} s_{RX,1}(t) \\ \vdots \\ s_{RX,M}(t) \end{bmatrix},$$  

the modulation vector

$$s_{mod}(t) = \begin{bmatrix} A_1 e^{i(2\pi f_{mod,t} + \varphi_{mod,1})} \\ \vdots \\ A_N e^{i(2\pi f_{mod,t} + \varphi_{mod,N})} \end{bmatrix}$$  

and the complex valued channel matrix

$$C = \begin{bmatrix} L_{1,1} e^{i\phi_{1,1}} & \cdots & L_{1,N} e^{i\phi_{1,N}} \\ \vdots & \ddots & \vdots \\ L_{M,1} e^{i\phi_{M,1}} & \cdots & L_{M,N} e^{i\phi_{M,N}} \end{bmatrix}.$$
The channel matrix $C \in \mathbb{C}^{M \times N}$ contains the entire phase and amplitude relation between the TX antenna and the RX antennas via the TEs. If the number of TEs and RX antennas is identical ($M = N$), the channel matrix $C$ is quadratic. As a result, the freely selectable parameters $A_n$ and $\psi_{\text{mod},n}$ of the target elements can be determined for a given receive vector using the inverse of $C$. The modulation frequency $f_{\text{mod}}$ is the same for all target elements for a given simulated target with a certain range and velocity. Since the focus of this article is the DoA, the modulation frequency $f_{\text{mod}}$ will be dropped in the further analysis. For the calculation of $f_{\text{mod}}$ see [6], [7]. In order to derive the generation of a DoA for targets by tuning the amplitude $A_n$ and the modulation phase $\psi_{\text{mod},n}$, the general influence of a DoA is of interest [18]. To simulate a target with a DoA, the phases at the receive antennas must be adjusted to the values that an incident wave reflected by a target in the far field of the radar causes. Since the estimation of the DoA is performed at the radar by evaluating the phase difference between the antennas [18], the phase difference between the receive antennas has to be simulated accordingly by the radar target simulator. The reflection of a target in the far field at the angle $\theta_{\text{sim}}$ will create a plane wave with wavelength $\lambda_0$, described by the wave vector

$$
\mathbf{k} = \frac{2\pi}{\lambda_0} \begin{bmatrix}
-\sin(\theta_{\text{sim}}) \\
-\cos(\theta_{\text{sim}})
\end{bmatrix}.
$$

(11)

The projection of the position of a receive antenna onto the plane wave yields the received phase

$$
\phi_{\text{RX},m} = \mathbf{k}^\top \cdot \mathbf{p}_{\text{RX},m} + \phi_0.
$$

(12)

The phase offset $\phi_0$ describes the phase of the wave at the origin of the coordinate system. As can be seen in Fig. 3, in the special case of a ULA, the phase difference between the first and $m$-th receive antenna in the array with spacing $d$ parallel to the $x$-axis at a free-space wavelength $\lambda_0$ can be expressed as

$$
\Delta\phi_{\text{RX},m}(\theta) = \phi_{\text{RX},m}(\theta) - \phi_{\text{RX},1}(\theta) = -\frac{2\pi}{\lambda_0}d (m-1) \sin(\theta).
$$

(13)

To simulate a target at the DoA $\theta_{\text{sim}}$ with the target simulator setup, the required steering vector containing all the phases $\Delta\phi_{\text{RX},m}$ is multiplied with the inverse of the channel matrix $C$ to determine the amplitude $A_n$ and phase $\psi_{\text{mod},n}$ parameters of each target element. A target with a unity amplitude and DoA $\theta_{\text{sim}}$ can be simulated by the modulation signal phasor vector $\mathbf{s}_{\text{mod}}$ of

$$
\mathbf{s}_{\text{mod}}(\theta_{\text{sim}}) = \begin{bmatrix}
A_1 e^{j\psi_{\text{mod},1}} \\
\vdots \\
A_N e^{j\psi_{\text{mod},N}}
\end{bmatrix} = \left( C \cdot C_{\text{comp}} \right)^{-1} \begin{bmatrix}
e^{j\Delta\phi_{\text{RX},1}(\theta_{\text{sim}})} \\
\vdots \\
e^{j\Delta\phi_{\text{RX},N}(\theta_{\text{sim}})}
\end{bmatrix}.
$$

(14)

The compensation matrix $C_{\text{comp}}$ is applied to compensate for path length differences and is derived in subsection III-D. If no compensation is applied, this matrix is the identity matrix $I_N$. By multiplying the resulting signal phasor vector $\mathbf{s}_{\text{mod}}$ with a scalar so that all modulation amplitudes $A_n$ scale identically, the RCS of the target can be adjusted. Since the channel matrix $C$ has to be invertible, its properties are analyzed in the following.

**C. THE CHANNEL MATRIX**

In order to analyze the channel matrix $C$ and its invertibility ($M = N$), the channel matrix is calculated for a scenario with the TEs positioned in the far field. Without loss of generality, the path loss is neglected in the following, i.e., $L_{m,n} = 1$. The phase of the channel matrix is split into two matrices:

$$
C = C_{\text{RX}} \cdot C_{\text{TX}}.
$$

(15)

The matrix $C_{\text{TX}}$ accounts for the phase originating from the phase between the TX antenna and the target elements $r_{\text{TX},n}$. The phase shift caused by the paths between the TX antenna and the TEs is for each receive antenna path identical. This results in $N$ phases

$$
\phi_{\text{TX},n} = \frac{2\pi}{\lambda_0} r_{\text{TX},n}.
$$

(16)

Therefore,

$$
C_{\text{TX}} = \begin{bmatrix}
e^{j\phi_{\text{TX},1}} & 0 & \cdots & \cdots \\
0 & e^{j\phi_{\text{TX},2}} & \cdots & \cdots \\
\vdots & \vdots & \ddots & \ddots \\
0 & 0 & \cdots & e^{j\phi_{\text{TX},N}}
\end{bmatrix}.
$$

(17)

The phase relation between a TE and each receive antenna differs in the far field by the phase progression related to the angle $\theta_{\text{TE}}$. Consequently, the resulting receive channel matrix is

$$
C_{\text{RX}} = \begin{bmatrix}
c_{\text{RX},1,1} & \cdots & c_{\text{RX},1,N} \\
\vdots & \ddots & \vdots \\
c_{\text{RX},N,1} & \cdots & c_{\text{RX},N,N}
\end{bmatrix}
$$

(18)
with the elements
\[ c_{RX,m,n} = \exp\left[j \left( \phi_{RX,1,n} + \Delta \phi_{RX,m} \left( \theta_{TE,n} \right) \right) \right]. \tag{19} \]
This matrix is closely related to a Vandermonde matrix. When factoring out the constant phase term \( \phi_{RX,1,n} \) in (19), which describes the phase shift that results from the path length between the \( n \)-th TE and the first RX antenna \( r_{RX,1,n} \), from \( C_{RX} \), a Vandermonde \( C_{RX,V} \) matrix remains:
\[ C_{RX} = C_{RX,V} \cdot \hat{C}_{RX} \tag{20} \]
with
\[ \hat{C}_{RX} = \begin{bmatrix} e^{j \phi_{RX,1,1}} & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & e^{j \phi_{RX,1,N}} \end{bmatrix} \tag{21} \]
and the Vandermonde matrix
\[ C_{RX,V} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ v_1 & v_2 & \cdots & v_N \\ v_1^2 & v_2^2 & \cdots & v_N^2 \\ \vdots & \vdots & \cdots & \vdots \\ v_1^{N-1} & v_2^{N-1} & \cdots & v_N^{N-1} \end{bmatrix} \tag{22} \]
with
\[ v_n = e^{j \Delta \phi_{RX,z}(\theta_{TE,n})}. \tag{23} \]
The phase difference between the first and second RX antenna \( \Delta \phi_{RX,z}(\theta_{TE,n}) \) is calculated by (13). A Vandermonde matrix has a non-zero determinant if the values \( v_n \) are distinct \([19]\). This is the case if every \( \theta_{TE,n} \) is distinct and in the unambiguous region of the antenna array. Ambiguity can cause identical \( v_n \) entries, even though the corresponding placement angles \( \theta_{TE,n} \) are different. This occurs for ULAs if the antenna spacing of the RX array fulfills \( d > \lambda_0/2 \). Therefore only angles should be chosen, which cause the phase difference to be \( \Delta \phi_{RX,z}(\theta_{TE,n}) < \pi \), the unambiguous region. The matrices \( C_{TX} \) and \( C_{RX} \) are also always invertible since they are diagonal matrices with full rank. Therefore, the channel matrix \( C \) is invertible as long as two TEs are not placed at the same azimuth angle \( \theta_{TE,n} \) towards the radar. In practice, not only identical but also similar angles of the TEs have to be avoided, since similar placement angles \( \theta_{TE,n} \) cause the Vandermonde matrix \( C_{RX,V} \) to become ill-conditioned. Therefore, in sub-Section III-E the angular placement of the TEs is presented to achieve a well-conditioned channel matrix.

\section*{D. Analysis and Compensation of Path Length Differences}
The assumption made in (6) that the path length \( r_{m,n} \) only creates a constant phase shift \( \phi_{m,n} \) is only correct for continuous wave (CW) signals. The frequency modulation of a CS-FMCW radar will cause different beat frequencies for each TE if the path lengths \( r_{m,n} \) are different. The inversion of the channel matrix and the subsequent vector-matrix multiplication presume phasors of the same frequency. This assumption is therefore not correct, and the impact of the frequency shift is analyzed.

The frequency difference \( \Delta f \) caused by a path length difference of \( \Delta r \) for a CS-FMCW radar with bandwidth \( B \) and chirp duration \( T_c \) is
\[ \Delta f = \Delta r \frac{B}{c T_c}. \tag{24} \]
The data acquisition, performed by the radar and consequently the sampling of each ramp, only occurs during the chirp duration \( T_c \), and the frequency difference \( \Delta f \) will cause a continuously increasing phase difference of the beat frequency starting from zero up to
\[ \Delta \phi_{t=T_c} = 2 \pi \Delta r \frac{B}{c T_c} = 2 \pi \Delta f T_c. \tag{25} \]
at the end of a ramp. The effect of this phase drift is best visible when adding two signals with a small frequency difference \( \Delta f < 1/T_c \) that have a phase difference of \( \pi \) at \( t = 0 \) and the same amplitude. Considering phasors with no frequency difference, the superposition of these signals results in a signal of zero. But due to the frequency difference, the superposition will cause a beat signal with increasing amplitude in the time domain. Compensating the frequency difference by using different modulation frequencies is not possible, since this would violate the constant phase relation between the TEs. Instead, to reduce the error, which means reducing the energy of the superposition signal as close to zero as possible, the phase difference is instead chosen as
\[ \Delta \psi = \pi - \pi \Delta f T_c = \pi + \psi_{comp}. \tag{26} \]
The compensation phase \( \psi_{comp} \) enables calculations as close as possible to phasors with identical frequencies and approximates the CW case. As can be seen in Fig. 4, the compensation phase \( \psi_{comp} \) shifts the minimum of the superposition into the center of the time interval and thus minimizes its energy. If the radar target simulator employs a delay-based method for creating the range information, the differing range can also be compensated by adjusting the time delay.
In Fig. 5 the energy of the superposition of two TEs with a path length difference of $\Delta r$ and a resulting frequency difference of $\Delta f$ of the form

$$E(\Delta f, \phi_{\text{diff}}) = \int_0^{T_c} \left| e^{j2\pi ft} + e^{j(2\pi f + \Delta f)t + \phi_{\text{diff}} + \varphi_{\text{comp}}} \right|^2 dt.$$  \hspace{1cm} (27)

is compared with the energy that results from phasors with the same frequency

$$E_{\text{ideal}}(\phi_{\text{diff}}) = \int_0^{T_c} \left| e^{j2\pi ft} + e^{j2\pi ft + \phi_{\text{diff}}} \right|^2 dt.$$ \hspace{1cm} (28)

The resulting energy error $\Delta E = |E - E_{\text{ideal}}|$ is depicted in Fig. 5 with and without the application of the compensation phase $\varphi_{\text{comp}}$. The resulting energy error can be reduced significantly by applying the compensation phase $\varphi_{\text{comp}}$. This means that with the application of $\varphi_{\text{comp}}$ the approximation of the superposition of the TEs by phasors causes a smaller error.

For calculating the compensation phase terms, the path length differences are determined during the calibration process. Due to the amount of path lengths $r_{m,n}$, there will be $MN$ compensation phase terms $\varphi_{\text{comp},m,n}$. However, the compensation phase caused by unaccurate placement of the TEs (differing distance from the center point), will be similar for each receive antenna. A single compensation phase per TE has to be used to preserve the difference between the received phases that are described by $C_{RX,V}$ in (22). Hence, the arithmetic mean of the compensation phases for each TE

$$\varphi_{\text{comp},n} = \frac{1}{M} \sum_{m} \varphi_{\text{comp},m,n}$$ \hspace{1cm} (29)

forms the channel compensation matrix

$$C_{\text{comp}} = \begin{bmatrix} e^{j\varphi_{\text{comp},1}} & 0 \\ \vdots & \ddots \\ 0 & e^{j\varphi_{\text{comp},N}} \end{bmatrix}. \hspace{1cm} (30)$$

E. OPTIMAL PLACEMENT

Since path length differences impact the performance of the system, the optimum position of the radar is determined which reduces the occurring path length differences. Besides, the optimum placement of the TEs should yield the most robust DoA simulation regardless of placement errors. The optimal placement for a radar sensor with four RX antennas is shown in Fig. 6. The distance of the TEs with respect to the origin of the coordinate system is identical, hence

$$R_{TE,1} = R_{TE,n} \hspace{1cm} n = 1 \ldots N.$$ \hspace{1cm} (31)
To achieve the most robust angular placement of the TEs $\theta_{TE,n}$, the possible steering vectors of the radar must be covered uniformly by the TE positions. Due to the $\sin(\theta)$ component of the phase difference in (13), the DoA resolution at DoAs close to $\theta = \pm 90^\circ$ is lower than at DoAs close to $\theta = 0^\circ$. Consequently, the TEs are placed accordingly to replicate the angular dependency of the resolution. The angles of the TEs $\theta_{TE,n}$ are chosen in order to create uniformly distributed receive differences $\Delta \phi_{RX,2}(\theta_{TE,n}) \in [-\pi, \pi]$. The TE angle spacing is a function of the radar antenna spacing $d$. The optimum angular spacing for simulating DoAs within the entire unambiguous region of the radar is

$$\theta_{TE,n} = \sin^{-1}\left(\left(-1 + \frac{2n - 1}{N}\right) \frac{2 \sin^{-1}\left(\frac{\lambda}{2f}\right)}{\pi}\right). \quad (32)$$

Choosing this spacing turns the Vandermonde Matrix $C_{RX,V}$ in (22) into a Discrete Fourier Transform (DFT) related matrix, because $u_n$ are equally spaced complex numbers on the unit circle, and $C_{RX,V}$ is therefore ideally conditioned [20]. If only a field of view is required to be simulated that is smaller than the placement of the outer TEs, with $\theta_{max} = \max |\theta_{sim}| < \theta_{TE,N}$, the alternative spacing of

$$\delta_{TE,n} = \sin^{-1}\left(\left(-1 + \frac{2n - 2}{N - 1}\right) \sin(\theta_{max})\right) \quad (33)$$

should be used. In practice the radiation pattern of a single element of the radar antenna array will determine $\theta_{max}$, since it limits the effective field of view. Using the spacing of (33) reduces high losses due to target elements being placed outside the effective field of view of the radar. The spacing is constructed by placing the outer target elements (TE1 and TEN) at $\pm \theta_{max}$. The other elements are then placed to create uniformly distributed receive differences $\Delta \phi_{RX,2}(\theta_{TE,n})$ within this region.

Since the placement of the RX- and TX antennas influences the resulting path lengths of the individual TEs, the position of the radar is chosen to minimize the longest occuring path length difference

$$\Delta l_{max} = \max_{m,n,h} |r_{m,n} - r_{m,h}| \quad m, n, h = 1, 2, \ldots N. \quad (34)$$

As discussed in subSection III-D, path length differences alter the superposition of the retransmitted TE signals for a single target. In Fig. 7 a simulation of the maximum resulting path length difference $\Delta l_{max}$ for different positions of the radar is depicted. The simulation result is achieved under the assumption that the TX antenna and RX antennas have a fixed spacing $l_{TX,RX}$, set by the radar’s manufacturer. Thus, the position of the radar is determined by the position of the TX antenna. The minimum path length difference occurs at an $x$-offset of $l_{TX,RX}/2$ and a $z$-offset of zero and therefore, the center should be chosen halfway between the position of the TX antenna and the center of the RX antenna array. In Fig. 7 it can be also seen that the placement in $x$-direction is more crucial than in $z$-direction.

IV. CALIBRATION PROCESS

In order to estimate the losses $L_{m,n}$, phase shifts $\phi_{m,n}$, and path length differences $\Delta l_{m,n}$, a calibration must be performed. The calibration estimates the channel matrix $C$ and the path length differences for the calculation of the compensation matrix $C_{comp}$. To determine the path loss $L_{m,n}$ and phase $\phi_{m,n}$, the magnitude and phase of targets simulated by the TEs in the range-Doppler (RV) domain are evaluated. All estimated parameters require accurate phase, frequency, and magnitude information. Using a maximum likelihood approach a low frequency mismatch is achieved by maximizing the magnitude level of the target peaks.

The calibration is performed stepwise by using two TEs simultaneously with different modulation frequencies. The path length differences are determined relative to TE1. Therefore, TE1 is always active, and the second active TE is changed between the steps, beginning with TE2 and ending at TEN. If the calibration process is started at an arbitrary point in time $t_0$ and a single target is generated by TE1 with the modulation frequency $f_{mod}$, the resulting phase at the $m$-th receive antenna is described by

$$\phi_{cal,m,1}(t_0) = \phi_{m,1} + 2\pi f_{mod}t_0 + \psi_{mod,1}. \quad (35)$$

Thus, the direct estimation of the phase $\phi_{m,1}$ is not possible due to the additional phase term $2\pi f_{mod}t_0$ caused by the arbitrary point in time. To enable the estimation of the phase terms, a second TE $(n$-th element) is used at the same time with the modulation frequencies $2f_{mod}$ and $3f_{mod}$. The two TEs combined will simulate three targets with three corresponding peaks in the range-Doppler evaluation of the radar. The beat frequencies of the simulated targets at the radar are denoted as $f_{p1}$, $f_{p2}$, and $f_{p3}$. All other TEs are turned off during this calibration step. In Fig. 8 an idealized RV-plot is shown with the three target peaks, generated by two active TEs. The phases of the target peaks created by the $n$-th TE can be described as

$$\phi_{cal,m,n}(t_0) = \phi_{m,n} + 4\pi f_{mod}t_0 + \psi_{mod,n}. \quad (36)$$
By forming the difference of $\phi^I_{cal,m,n}(t_0)$ and $\phi^I_{cal,m,n}(t_0)$, the part of the phase which is caused by the random time instance $t_0$ is calculated:

$$2\pi f_{mod} t_0 = \phi^I_{cal,m,n}(t_0) - \phi^I_{cal,m,n}(t_0)$$

Therefore, the phase information of the channel matrix $C$ for the two TEs if $\varphi_{mod,1} = \varphi_{mod,n} = 0$ can be calculated via

$$\phi_{m,1} = \phi_{cal,m,1}(t_0) + \phi^I_{cal,m,n}(t_0) - \phi^I_{cal,m,n}(t_0)$$

$$\phi_{m,n} = 3\phi^I_{cal,m,n}(t_0) - 2\phi^I_{cal,m,n}(t_0).$$

The second quantity that needs to be determined is the occurring path-length differences for each receive antenna to calculate the compensation phase via (26). The path-length difference can be estimated from the $R_v$-plot as well. The two target points created by the $n$-th TE with the frequencies $f_{p2}$ and $f_{p3}$ will have a path-length difference corresponding exactly to $f_{mod}$. The frequency difference between the first and the second target peak $\Delta f_{p1,p2}$ is however

$$\Delta f_{p1,p2} = f_{p2} - f_{p1} = f_{mod} + \Delta f_{m,n},$$

with $\Delta f_{m,n}$ being the frequency shift, caused by the path-length difference

$$\Delta r_{m,n} = r_{m,n} - r_{m,1}.$$
FIGURE 10. Resulting RMS DoA error $\Delta \theta$ of a simulated target for misplacements range ($\Delta R_{TE,2}$) direction of TE2. The resulting RMS DoA error $\Delta \theta$ with the application of $\varphi_{comp}$.

FIGURE 11. Photograph of the measurement setup. The radar is at the bottom of the photo. The TEs are positioned around the radar at the angles $\theta_{TE} = -33^\circ$, $-10.5^\circ$, $10.5^\circ$, and $33^\circ$.

VI. MEASUREMENTS

The measurement setup consists out of four TEs. Each TE can realize a maximum gain of 13 dB at the highest modulation amplitude $A_n$ between the antenna connections. A side band suppression of 30 dB is achieved by the IQ-mixer, used for the modulation. A CS-FMCW radar was used to verify the approach for the simulation of arbitrary DoAs $\theta_{sim}$. The antenna array of the radar used for the verification has a 6 dB beamwidth of $66^\circ$ for the combined radiation patterns of the TX- and RX-antennas. A limited simulated field of view of $66^\circ$ was therefore chosen. The ideal positions for the TEs are according to (33) $-33^\circ$, $-10.5^\circ$, $10.5^\circ$, and $33^\circ$. The measurement setup is shown in Fig. 11 and its parameters are listed in Table I.

In Fig. 12 the magnitude of the Bartlett beamformer DoA estimation, implemented by a 1024-point DFT, is shown. The DoA estimation is performed for the target peak in the range-Doppler plot with the highest magnitude. The DoA estimation result for three simulated DoA values that are between the azimuth positions of the TEs $\theta_{TE}$ is shown exemplarily in Fig. 12.

Simulated targets with a DoA $\theta_{sim}$ property in $5^\circ$-steps were created consecutively to validate the achievable accuracy. In order to confirm the impact of the compensation phase $\varphi_{comp}$, a second measurement was performed where TE2 was moved 4 cm towards the radar to create an additional path length difference due to inaccurate placement ($\Delta R_{TE,2}/\delta R_{res} = 0.5$).

In Fig. 13 the error between the intended simulated DoA $\theta_{sim}$ and the estimated DoA $\theta_{est}$ via a Bartlett beamformer is depicted. For accurately placed TEs, the compensation phase has no significant impact and the deviation from the intended DoA is less than $1.5^\circ$. If TE2 is placed with a 4 cm offset.

TABLE I. Overview of Measurement Setup

| Parameter                  | Value          |
|----------------------------|----------------|
| Ramp duration $T_e$        | 200 $\mu$s     |
| Bandwidth $B$              | 1.8 GHz        |
| Range resolution $\delta R_{res}$ | 83 mm          |
| Start frequency            | 77.2 GHz       |
| Number of RX channels      | 4              |
| RX antenna spacing $d$     | 1.95 mm = 0.5$\lambda_0$ |
| Radar RX-TX 6-dB beamwidth | $66^\circ$     |
| Simulated field of view    | $66^\circ$     |
| TE positions $\theta_{TE}$ | $\pm 33^\circ$, $\pm 10.5^\circ$ |
| TE maximum gain            | 13 dB          |
| TE sideband suppression    | 30 dB          |
| Distance to target simulator $R_{TE,sim}$ | 2.1 m         |
| Simulated distance of target | 6.3 m        |
| Simulated speed of target  | 3 m s$^{-1}$   |
error $\Delta R_{\text{TE},2}$, the accuracy of the generated DoA without the compensation phase will suffer greatly with a deviation up to $7^\circ$ between intended and estimated DoA, while the compensation phase can increase the accuracy to a level close to the accurately placed setup. In order to emphasize the tremendous advantages of the here presented setup for simulating complex scenarios, 41 point scatterers were simulated. In Fig. 14, the resulting $Rv$-plot and the Cartesian $xz$-plot are depicted. The estimation of the DoA is performed on targets with a velocity $v \neq 0 \text{m/s}$. The corresponding targets between the $Rv$-plot and the Cartesian $xz$-plot are marked by the orange and green borders. For $Rv$-cells encircled by the green border up to three targets per $Rv$-cell were simulated. The number of targets per $Rv$-cell increases with greater distances. This shows that the approach allows the simulation of multiple targets with identical range and velocity but different DoAs. The targets, marked by the orange border, display the continuously changeable DoA capability of the setup between $-33^\circ$ and $33^\circ$. The signal used to simulate the scenario is the superposition of the modulation signals for the 41 individual targets, calculated in the digital domain. For each simulated target the required amplitudes and phases of the TEs for the intended DoA is calculated using (14).

VII. CONCLUSION

In this article, a DoA generation approach is presented suitable for target simulators with a modulation capability. With distributed, coherently modulated target elements, a variable DoA generation is achieved solely by tuning the amplitude and phase of the modulation signals employed in the target elements. The ideal positioning and the compensation of placement errors are discussed and verified with simulations and measurements. A calibration is presented in order to estimate the phase and amplitude relation between the radar and the target elements. With low hardware costs and a high accuracy, arbitrary DoAs for an indefinite amount of targets can be simulated, while maintaining a high reliability due to electrical steering. Besides, several targets can be simulated within a single $Rv$-cell with differing DoAs. The DoA generation approach exhibits a measured accuracy better than $\pm 1.5^\circ$ for a radar sensor with four receive antennas.

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