Model of the spectral dependence of changes in the polarization state of laser radiation in magneto-optical crystal in the presence of multiple reflections from its faces

A V Seleznev¹, D A Rodionov¹, N V Kovalenko¹, R I Shaidullin², O A Ryabushkin²

1. Moscow Institute of Physics and Technology, Institutskiy per. 9, Dolgoprudnyy, Moscow Region, Russia
2. Fryazino Branch of the Kotelnikov Institute of Radio Engineering and Electronics of RAS, Vvedensky sq. 1, Fryazino, Moscow Region, Russia

E-mail: sanya.seleznyov@yandex.ru

Abstract. In this work we investigated the phenomena in the Faraday rotator that can affect the polarization state of transmitting laser radiation. We have found that multiple reflections of radiation from the faces of the terbium gallium garnet (TGG) crystal can cause deterioration of radiation extinction ratio and, therefore, isolation properties of Faraday cell. We were able to calculate the impact of this factor on polarization extinction ratio of monochromatic and quasimonochromatic laser radiation transmitting through the crystal.

1. Introduction
Faraday isolators are widely used to suppress the back reflection of laser radiation, that is necessary in some applications. The principle of operation of the Faraday isolator is based on the rotation of the plane of polarization of the optical radiation that passes through magneto-optical material placed in a magnetic field. Angle of rotation is proportional to the magnitude of the magnetic field and the length of the magneto-optical element. The proportionality coefficient, called the Verde constant, has temperature and spectral dependencies that are widely described in the literature [1]. This paper for the first time presents mathematical analysis of the spectral dependence of the extinction ratio change of the polarization state of radiation passing through a TGG crystal, taking into account the effect of multiple reflections from the crystal faces.

2. Mathematical model of change of polarization extinction of laser radiation in TGG crystal
The basis of the operation of the Faraday isolator is the non-reciprocity of the Faraday effect: the polarization plane of radiation that passed through the Faraday cell and then reflected back rotates by a double angle relative to the initial position. However, under certain conditions this property can cause significant changes in the polarization characteristics of radiation. It was found that in case of relatively high reflection from the facets of magneto-optical crystal polarization ellipse of some part of the output radiation is additionally rotated due to multiple reflections [2]. Thus, the radiation at the output of a magneto-optical crystal is the sum of an infinite number of pairwise orthogonal relative to
each other by angles, proportional to the number of reflections inside the crystal, with the equal phase delay between them and proportionally decreased amplitudes

2.1. Mathematical description of the effect of polarization state change of monochromatic light under conditions of an infinite number of re-reflections inside the crystal.

In our previous work [2] we took into account only the first several field components, neglecting the infinite number of reflections. In this work we present universal approach for calculation of polarization state of output radiation taking into account infinite number of reflected field components.

First, we have studied the influence of described effect on the monochromatic elliptical ly polarized radiation. To do this, we have used the mathematical apparatus of Jones matrices. In this approach, an arbitrary elliptic state of polarization is represented as a vector (in the orthogonal basis):

\[
\begin{pmatrix}
E_x \\
E_y
\end{pmatrix} = Ae^{i\delta} \begin{pmatrix}
\cos(\Theta)\cos(\varepsilon) - i\sin(\Theta)\sin(\varepsilon) \\
\sin(\Theta)\cos(\varepsilon) + i\cos(\Theta)\sin(\varepsilon)
\end{pmatrix}
\]

where, \(\varepsilon = \pm b/a\) - ellipticity, \(\Theta\) - azimuth of the large axis of ellipse, \(\delta\) - phase, \(A = \sqrt{a^2 + b^2}\) - amplitude.

To simplify further calculations, we will assume that initial radiation is elliptically polarized and the large half-axis of the polarization ellipse coincides with the y-axis (Fig. 1). Then the expression for the electric field vector will take the following form:

\[
\begin{pmatrix}
E_0^x \\
E_0^y
\end{pmatrix} = A \begin{pmatrix}
-i\sin(\varepsilon) \\
\cos(\varepsilon)
\end{pmatrix}
\]

Let's assume that this vector describes the state of polarization of light passing through a magneto-optical crystal once without additional reflections and denote the Jones vector corresponding to such a state as "zero" vector.

Next, the "first" Jones vector describes a pair of vectors that have experienced two additional reflections from the facets of a magneto-optical crystal (which means that it is rotated by an angle equal to 2\(\theta\) relative to the "zero" vector, where \(\theta\) - the angle of Faraday rotation for a single pass through the crystal). The geometric representation of the "first" and "zero" Jones vectors is shown in Figure 1. In addition, the "first" vector acquires an additional phase shift 2\(\phi\) relative to the "zero" vector because of the difference in optical paths, where \(\phi\) - phase shift for a single pass through the crystal, and is reduced by \(r^2\) times, where \(r\) - the electric field amplitude reflection coefficient. Thus, such a vector can be described as:

\[
\mathbf{E}_1 = \mathbf{\hat{T}}E_0 = r^2 e^{-2i\phi} \begin{pmatrix}
\cos(2\theta) & \sin(2\theta) \\
-\sin(2\theta) & \cos(2\theta)
\end{pmatrix} \begin{pmatrix}
E_{0x} \\
E_{0y}
\end{pmatrix}
\]

Figure 1. Vector diagram of the radiation components. The axis is directed perpendicular to the plane of the drawing. The first pair of components \((E_{x0}, E_{y0})\) refers to the radiation passed without additional reflections. The second pair of components \((E_{x1}, E_{y1}) = r^2(E_{x0}, E_{y0})e^{i2\phi}\) refers to the radiation experiencing two reflections inside the crystal.
Obviously, the n-th Jones vector will be represented as $\hat{E}_n = T_n E_{n-1} = T_n^* E_0$, in this case, the total vector is represented as

$$
\vec{E} = \sum_{n=0}^{\infty} T_n \vec{E}_0 = \sum_{n=0}^{\infty} r^{2n} e^{-2\alpha n} \begin{pmatrix}
\cos(2n\theta) & \sin(2n\theta) \\
-\sin(2n\theta) & \cos(2n\theta)
\end{pmatrix} \begin{pmatrix}
E_{0x} \\
E_{0y}
\end{pmatrix} = \sum_{n=0}^{\infty} r^{2n} e^{-2\alpha n} \begin{pmatrix}
\cos(2n\theta) & \sin(2n\theta) \\
-\sin(2n\theta) & \cos(2n\theta)
\end{pmatrix} \begin{pmatrix}
E_{nx} \\
E_{ny}
\end{pmatrix}
$$

(4)

Since $r$ is less than 1, then this expression can be simplified as the sum of elements of an infinitely decreasing geometric progression. Thus, the polarization state at the exit of the crystal will be described as

$$
\vec{E}_\Sigma = \sum_{n=0}^{\infty} e^{-2\alpha n} \begin{pmatrix}
1 - r^2 e^{-2\alpha} \cos(2\theta) \\
-2 r^2 e^{-2\alpha} \cos(2\theta) + r^4 e^{-4\alpha} \\
r^2 e^{-2\alpha} \sin(2\theta) \\
1 - 2 r^2 e^{-2\alpha} \cos(2\theta) + r^4 e^{-4\alpha}
\end{pmatrix} \begin{pmatrix}
E_{0x} \\
E_{0y} \\
E_{nx} \\
E_{ny}
\end{pmatrix}
$$

(5)

The resulting expression (5) allows us to describe the final state of polarization of monochromatic light, knowing the initial arbitrary elliptical state (1), since during calculations (3-5) we did not directly use the assumption (2).

Next, we defined the major and minor axes of the polarization ellipse described by expression (5). To do this, we restored the time dependence by multiplying the expression (5) by $e^{i\varepsilon}$, where $\varepsilon$ - is phase parameter, corresponding to the position of the field vector on the ellipse and changing between 0 and 2$\pi$, and considered the intensity function as $I(\varepsilon) = E_\Sigma e^{i\varepsilon} E_\Sigma^*$. By looking for an extremum of the expression $E_\Sigma e^{i\varepsilon}$ by varying $\varepsilon$, we can find the direction on one of the axes. Omitting cumbersome calculations, we obtained a final expression for the $\delta_{ext}$ and found the value of extinction ratio Ext:

$$
\delta_{ext} = \frac{1}{2} \arctan \left( \frac{2 r^2 (\cos(2\theta) - r^2 \cos(2\phi)) \sin(2\phi)}{1 - 2 r^2 \cos(2\theta) \cos(2\phi) + r^4 \cos(4\theta)} \right)
$$

(6)

$$
\text{Ext} = 20 \log \left( \left| \frac{e^{i\delta_{ext} E_\Sigma}}{e^{i\delta + \frac{\pi}{2}} E_\Sigma} \right| \right)
$$

(7)

In addition, substituting $\delta_{ext}$ for $\delta$ in the expression $\text{Re}[E_\Sigma e^{i\delta}]$ allowed us to calculate the components of the vector corresponding to one of the semi-axes of polarization ellipse, which allows determining of how much is the resulting polarization state rotated relative to a single Faraday rotation (further denoted as angular deflection $\alpha$).

Figures 2 and 3 show the results of the phase delay dependence of extinction ratio for the same parameters of laser radiation and TGG crystal that we used in [2]. The calculation results are in good agreement with [2], although we used a different and much more universal model considering infinite number of radiation reflections.

As can be seen, the dependence of extinction ratio on the phase delay between different Jones vectors can be very strong. Since real magneto-optical elements can have large size, and, therefore, long optical path, this value can vary greatly for different spectral components of non-monochromatic laser radiation. So, it is very important to consider the case of quasimonochromatic radiation.
2.2. The effect of multiple reflections of radiation in a magneto-optical element in the case of quasimonochromatic radiation.

Consider the spectral dependence of polarization state change of radiation in realistic Faraday cells. Consider a 2 cm long Faraday cell made of TGG, with Fresnel reflection coefficient from the facets \( r = \sqrt{0.1} \), Faraday rotation angle \( \theta = \pi / 4 \), ellipticity of radiation \( \varepsilon = \arctan(0.01) \). For such length of crystal, the dependence of the phase delay on the wavelength becomes very sharp, so let’s consider the narrow spectrum 1069.99-1070.01 nm. Spectrum of such width can be achieved by using for example a DFB-laser. The spectral dependence of the refractive index of TGG \( n(\lambda) \) from [3] was used for the calculation. Figures 4-5 show the dependences of extinction ratio and radiation extinction ratio Ext and angular deflection \( \alpha \) on the radiation wavelength.

Thus, the radiation at the exit from such system can be significantly depolarized. In order to evaluate the share of depolarized radiation, we have used mathematical apparatus of the coherence matrix (8) and Stokes parameters introduced in [4]:

\[
J = \begin{pmatrix}
\langle E_x(t)E_x^*(t) \rangle & \langle E_y(t)E_y^*(t) \rangle \\
\langle E_y(t)E_x^*(t) \rangle & \langle E_x(t)E_y^*(t) \rangle
\end{pmatrix}
\]

(8)

where \( \langle E_{x,y}(t)E_{x,y}^*(t) \rangle \) - time averaging of the Jones vector components in the time representation.
However, in order to reduce the complexity we preferred to make calculations in the spectral representation by taking the inverse Fourier transform:

\[ \langle E_{x,y}(t)E_{x,y}^*(t) \rangle = \lim_{T \to \infty} \frac{1}{T} \int_{-T}^{T} E_{x,y}(t)E_{x,y}^*(t)dt = \lim_{T \to \infty} \frac{1}{T} \int_{-T}^{T} dt \int \int \hat{E}_{x,y}(\omega)\hat{E}_{x,y}^*(\omega')e^{i(\omega-\omega')t}d\omega d\omega' = \lim_{T \to \infty} \frac{1}{T} \int \int \hat{E}_{x,y}(\omega)\hat{E}_{x,y}^*(\omega-\Omega)\sin\left(\frac{T}{2}(\omega-\omega')\right)d\omega d\omega = 2\pi \int \int \hat{E}_{x,y}(\omega)\hat{E}_{x,y}^*(\omega-\Omega)\delta(\Omega)d\omega d\Omega = 2\pi \int \hat{E}_{x,y}(\omega)\hat{E}_{x,y}^*(\omega)d\omega \]

(9)

Knowing the elements of the coherence matrix, it is possible to find the degree of polarization \( P \) (10).

\[ P = \sqrt{1 - \frac{4\det(J)}{(Sp(J))^2}} \]

(10)

Using the parameters \( r = \sqrt{0.1}, \theta = \frac{\pi}{4}, \varepsilon = \text{ArcTan}(0.01) \) for calculation, we have found that polarization degree is equal to 0.98. This indicates that the radiation at the output of the crystal under such conditions can be considered mostly polarized.

Stokes vector \( S \) was obtained from the coherence matrix (11), using which we can calculate the ellipticity (12) and the angular deflection of fully polarized component (13):

\[ S = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & -i & i & 0 \end{bmatrix} \begin{bmatrix} J_1 \\ J_2 \\ J_3 \end{bmatrix} \]

(11)

\[ \varepsilon = \frac{1}{2} \text{ArcSin} \left( \frac{S_1}{\sqrt{S_1^2 + S_2^2 + S_3^2}} \right) \]

(12)

\[ \alpha = \frac{1}{2} \text{ArcTan} \left( \frac{S_2}{S_1} \right) \]

(13)

We have found that \( \varepsilon \) was equal to 0.016 (corresponding to 36 dB extinction ratio) and \( \alpha = 0.03 \) rad.

As a result, quasimonochromatic radiation passed through such system will consist of 2 % (in terms of intensity) of depolarized radiation and 98% of the polarized component with 35 dB extinction ratio.

Total angle of rotation of polarization ellipse will slightly differ relative to radiation passed without any additional re-reflections. In this case, the extinction ratio measured at the exit from such a crystal in a real measurement with an output polarizer component cannot exceed 20 dB due to the presence of a depolarized radiation.

It should be noted that these effects strongly depend on the reflection coefficient. For example, for \( r = \sqrt{0.5} \) with the same other parameters the polarization degree \( P \) is reduced to 0.67 and the extinction ration of polarized component becomes equal to 14.6 dB. Figures 6 and 7 show the dependence of the degree of polarization and extinction ratio of the polarized part of the quasimonochromatic radiation on the reflection coefficient from the facets of the TGG crystal.
2.3. Modification of the spectrum

In addition to the change of extinction ratio and the angle of polarization ellipse rotation, considered effect can also lead to a change in the spectrum of transmitted laser radiation. If we will place a linear polarizer in the "zero" direction at the exit of the magneto-optical crystal, then the transmittance coefficient of each spectral component will depend on the angle between the axis of the polarizer and the main axis of the polarization ellipse for this spectral component. Hypothetically, this effect can be used to modify the spectrum, depending on the selection of appropriate parameters. Figure 8 shows the spectrum of passed radiation, which initially had a rectangular shape.

3. Conclusion

In this paper, we have presented a universal method for describing the change in the polarization state at the output of a magneto-optical crystal in the presence of multiple reflections from the crystal faces, which allows us to consider a wide range of parameters such as the reflection coefficient, angle of polarization rotation and radiation wavelength. Using this method, we calculated the change in the polarization characteristics of monochromatic and quasimonochromatic radiation passing through such a system. We also suggested that this effect can be used to modify the spectrum of transmitted laser radiation.

The work was carried out within the framework of the state task.
References

[1] E A Khazanov, “Thermooptics of magnetoactive media: Faraday isolators for high average power lasers”, Physics Uspekhi 59 (9), pp, 886 - 909 (2016).

[2] A V Seleznev et al 2019 “Model of the polarization extinction ratio change due to multiple reflection of laser radiation from the faces of the terbium-gallium garnet crystal in Faraday rotator” J. Phys.: Conf. Ser. 1391 012142

[3] R. Yasuhara et al, J. Kawanaka, Temperature dependence of thermo-optic effects of single crystal and ceramic TGG, Opt. Express. 21 (2013)

[4] R. M. A. Azzam, N. M Bashara, Ellipsometry and polarized light North-Holland publishing company (1977)