Restoration of singlet axial symmetry at finite temperature

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To accommodate recent RHIC data on $\eta'$ multiplicity, we propose a minimal modification of the Witten-Veneziano relation at high temperature. This renders a significant drop of $\eta'$ mass at high temperature signaling a restoration of the $U(1)_A$, and the Goldstone character of $\eta'$.

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1. Introduction

On the classical level, QCD with $N_f$ massless flavors enjoys a global chiral $U(N_f)_L \otimes U(N_f)_R$ symmetry. The subgroup with unit determinant, namely the $SU(N_f)_L \otimes SU(N_f)_R$, is spontaneously broken down to its diagonal part $SU(N_f)_{L+R} = SU(N_f)_V$. This gives rise to 8 pseudoscalar Goldstone bosons: $\pi$'s, $K$'s and the $\eta$.

On the other hand, the non-abelian anomaly in the $U(1)_A$ sector,

$$ \partial_\mu \tau^0_{5\mu}(x) = \frac{g_s^2}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} F^\mu_a F^\rho_a \quad (1) $$

explicitly breaks $U(1)_A$ symmetry. Coupled with non-trivial topology in the field space of QCD, it prevents the spontaneous breaking of $U(1)_A$, thus

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generating the extra mass for $\eta'$. The simplest example of such contribution to $m_{\eta'}$ is the diamond diagram in Fig. 1. Nevertheless, the fate of $U(1)\_A$ at $T > 0$ could be drastically changed. New RHIC data from central Au+Au collisions [1] on $\eta'$ multiplicity shows a drop in it’s mass by at least 200 MeV inside the fireball

$$m_{\eta'}(\text{vacuum}) = 958 \text{ MeV} \to m_{\eta'}(\text{fireball}) = 340^{+50+280}_{-60-140} \pm 45 \text{ MeV},$$

thus signaling restoration of the Goldstone character of $\eta'$.

2. Witten-Veneziano relation at finite $T$

On the theoretical side, excess in the $\eta'$ mass is inferred from the Witten-Veneziano relation (WVR) [2, 3]

$$M_{\eta'}^2 + M_{\eta''}^2 - 2M_K^2 = \frac{2N_f}{f_\pi} \chi_{YM}. \quad (3)$$

Due to the flavor content of the pseudoscalars, WVR just says that the excess in the singlet $\eta^0$ mass is coming from the glue sector, i.e. $\chi_{YM}$ is the Yang-Mills topological susceptibility.

Previous studies [4] of WVR indicated, for various lattice forms of $\chi_{YM}$, that $\eta'$ mass increases as $\chi_{YM}$ melts, in contrast to the result [2]. This is happening as $T$ approaches $T_{Ch}$, due to the fact that there $f_\pi(T)$ starts decreasing significantly.

2.1. Connection between QCD and YM topological susceptibility

In that light, we propose [5] a minimal modification of WVR first by using Leutwyler-Smilga result [6] in the vacuum

$$\chi_{YM} = \frac{\chi}{1 + \chi \Sigma m_i} \quad (4)$$
Fig. 2. The relative-temperature dependences, on $T/T_{Ch}$, of $\tilde{\chi}^{1/4}$, $\langle \bar{q}q \rangle_0^{1/3}$, $f_\pi$ and $f_{ss}$. The solid curve depicts $\tilde{\chi}^{1/4}$ for $\delta = 0$, and the short-dashed curve is $\tilde{\chi}^{1/4}$ for $\delta = 1$. At $T = 0$, both $\tilde{\chi}$’s are equal to $\chi_{YM} = (0.1757 \text{ GeV})^4$, the weighted average \[4\] of various lattice results for $\chi_{YM}$.

Just like the WVR, Leutwyler-Smilga relation connects the quantities from two different theories: $\chi$ is the QCD topological susceptibility, and Eq. \[4\] shows that it approaches $\chi_{YM}$ only for large quark masses whereas $\chi_{YM}$ and $\chi$ are very different for light quarks. $\Sigma = \langle \bar{q}q \rangle_0$ is the condensate in the chiral limit.

We denote by $\tilde{\chi}$ the whole right hand side of Eq. \[4\], and use it at finite temperature.

Importance of this relation comes from the fact that $\chi$ is driven by the chiral quark condensate in the leading order of expansion in small quark masses

$$\chi = -\frac{\Sigma m}{N_f} + C_m, \quad \frac{N_f}{m} = \sum_f \frac{1}{m_f}. \quad (5)$$

This is the di Vecchia-Veneziano result \[6, 7\]. The next term in this expansion, $C_m$, is essential as it keeps $\chi_{YM}$ from blowing up. We fix its value at $T = 0$ by demanding $\tilde{\chi}(0) = \chi_{YM}$.

### 2.2. Exploring the thermal dependence

For the thermal dependence of $\chi$ we use an ansatz

$$\chi(T) = -\frac{\Sigma(T)m}{N_f} + C_m(0) \left[ \frac{\Sigma(T)}{\Sigma(0)} \right]^\delta.$$

This gives
\[ \tilde{\chi}(T) = \frac{\Sigma(T)m}{N_f} \left\{ 1 - \frac{1}{C_m(0)} \frac{\Sigma(T)m}{N_f} \left[ \frac{\Sigma(0)}{\Sigma(T)} \right]^\delta \right\} \] (6)

The interesting window for \( \delta \) is then \( 0 < \delta < 1 \) since the lower limit gives no thermal dependence for the correction term, and a quadratic one in the last equation for \( \tilde{\chi} \). With \( \delta = 1 \) there is an enhancement of the \( \eta' \) mass [5], and therefore this is the upper limit of interest.

The interesting window for \( \delta \) is then \( 0 \leq \delta \leq 1 \), since \( \delta = 0 \) gives the \( T \)-independent correction term, while \( \delta = 1 \) already leads to precursors of the unwanted mass enhancement in the \( \eta' - \eta \) complex. Therefore, \( \delta \sim 1 \) is the upper limit of interest, although the results for the \( T \)-dependence of the meson masses is quantitatively not much different for \( \delta = 1 \) than from the case \( \delta = 0 \), as it is depicted in Figs. 3 and 4 respectively.

3. Results and discussion

Mesons are constructed as \( q \bar{q} \) bound states via the Bethe-Salpeter equation in the ladder approximation. Dynamical quarks are build up from the Dyson-Schwinger equation in rainbow approximation. We use the successfull rank-2 separable model [8] for the gluon propagator, which was also used in Ref. [4]. Rainbow-ladder approximation is the simplest symmetry preserv-
Fig. 4. The relative-temperature dependences, on $T/T_{Ch}$, of the pseudoscalar masses for $\delta = 1$.

The bound state approach enables access only to the non-anomalous part of the meson masses, since the ladder Bethe-Salpeter kernel does not include diagrams like in Fig. 1. Therefore, the anomalous part is inferred from Eq. (3). The strategy is to use flavor mass matrices to extract $\eta$ and $\eta'$ masses from the calculated non-anomalous sector. This is presented in detail in Ref. [4].

Our main result is presented in Figs. 3 and 4. The reduction in the $\eta'$ mass is around 200 MeV, which is in quantitative agreement with RHIC data. This is possible only due to the proposed modification of the WVR relation at finite $T$. Topological susceptibility of pure glue, $\chi_{YM}$, is just too resistant, with the characteristic melting temperature being $T_{YM} = 260$ MeV (see e.g. [9, 10]).

In contrast, the pseudocritical temperatures for the chiral and deconfinement transitions in the full QCD are lower than $T_{YM}$ by some 100 MeV or more (e.g., see Ref. [11]) due to the presence of the quark degrees of freedom. In that regard, Eq. (5) is essential as it couples chiral restoration to $U(1)_{A}$ restoration, allowing for $\chi(T)$ to melt away even sooner than $f_{\pi}(T)$. In the separable model used here we have $T_{Ch} = 128$ MeV which is admittedly lower than the so far accepted value around 160-170 MeV. It has been shown [12] that the same model coupled with the gluon degrees of freedom in the form of the Polyakov loop cures this discrepancy.
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