THEORETICAL ASPECTS OF RARE KAON DECAYS

David Greynat and Eduardo de Rafael
CPT, CNRS–Luminy, Marseille, France

Most of the analytic approaches which are used at present to understand Kaon decays, get their inspiration from QCD in the limit of a large number of colours $N_c$. After a general overview, we illustrate this with a recent application to the evaluation of the process $K_L \to \mu^+\mu^-$ in the Standard Model.

1 INTRODUCTION

In the Standard Model, the electroweak interactions of hadrons at very low energies are conveniently described by an effective chiral Lagrangian, which has as active degrees of freedom the low–lying $SU(3)$ octet of pseudoscalar particles, plus leptons and photons. The underlying theory is a $SU(3)_C \times SU(2)_L \times U(1)_W$ gauge theory which is formulated in terms of quarks, gluons and leptons, together with the massive gauge fields of the electroweak interactions and the hitherto unobserved Higgs particle. Going from the underlying Lagrangian to the effective chiral Lagrangian is a typical renormalization group problem. It has been possible to integrate the heavy degrees of freedom of the underlying theory, in the presence of the strong interactions, perturbatively, thanks to the asymptotic freedom property of the $SU(3)$–QCD sector of the theory. This brings us down to an effective field theory which consists of the QCD Lagrangian with the $u, d, s$ quarks still active, plus a string of four quark operators and mixed quark–lepton operators, modulated by coefficients which are functions of the masses of the fields which have been integrated out and the scale $\mu$ of whatever renormalization scheme has been used to carry out this integration. We are still left with the evolution from this effective field theory, appropriate at intermediate scales of the order of a few GeV, down to an effective Lagrangian description in terms of the low–lying pseudoscalar particles which are the Goldstone modes associated with the spontaneous symmetry breaking of chiral–$SU(3)$ in the light quark sector. The dynamical description of this evolution involves genuinely non–perturbative phenomena. There has been

*Talk given by E. de Rafael
recent progress in approaching this last step, using analytic methods formulated within the context of QCD in the limit of a large number of colours $N_c$ (QCD$_\infty$), (see e.g. ref$^\text{1}$ for a recent review.)

The strong and electroweak interactions of the Goldstone modes at very low energies are described by an effective Lagrangian which has terms with an increasing number of derivatives (and quark masses if explicit chiral symmetry breaking is taken into account.) Typical terms of the chiral Lagrangian are

$$L_{\text{eff}} = \frac{1}{4} F_0^2 \text{tr} \left( D_\mu U D^\mu U^\dagger \right) + L_{10} \text{tr} \left( U^\dagger F_R \mu U F^\mu L \right) + \cdots$$

(1)

$$+ \frac{\alpha}{2} C \text{tr} \left( Q_R U Q_L U^\dagger \right) + \cdots$$

(2)

$$- \frac{G_F}{\sqrt{2}} V_{ud} V_{us} \frac{2^{10}}{27} g_8 F_0^4 \left( D_\mu U D^\mu U^\dagger \right) + \cdots$$

(3)

where $U$ is a $3 \times 3$ unitary matrix in flavour space which collects the Goldstone fields and which under chiral rotations transforms as $U \rightarrow V_R U V_L^\dagger$; $D_\mu U$ denotes the covariant derivative in the presence of external vector and axial–vector sources. The first term in the first line is the lowest order term in the sector of the strong interactions$^\text{2}$, $F_0$ is the pion–decay coupling constant in the chiral limit where the light quark masses $u$, $d$, $s$ are neglected ($F_0 \simeq 90$ MeV); the second term shows one of the couplings at $O(p^4)$; the second line shows the lowest order term which appears when photons and $Z'$s are integrated out ($Q_L = Q_R = Q = \text{diag.}([2/3, -1/3, -1/3])$, in the presence of the strong interactions; the third line shows one of the lowest order terms in the sector of the weak interactions. The typical physical processes to which each term contributes are indicated under the braces. Each term is modulated by a coupling constant: $F_0^2$, $L_{10}$, ... $C$...$g_8$... which encodes the underlying dynamics responsible for the appearance of the corresponding effective term. The evaluation of these couplings from the underlying theory is the question we are interested in. The coupling $g_8$ for example, governs the strength of the dominant $\Delta I = 1/2$ transitions for $K$ decays to leading order in the chiral expansion.

There are two crucial observations to be made concerning the relation of these low energy constants to the underlying theory. The low–energy constants of the Strong Lagrangian, like $F_0^2$ and $L_{10}$, are the coefficients of the Taylor expansion of appropriate QCD Green’s Functions. By contrast, the low–energy constants of the Electroweak Lagrangian, like e.g. $C$ and $g_8$, are integrals of appropriate QCD Green’s Functions. Their evaluation appears to be, a priori, quite a formidable task because they require the knowledge of Green’s functions at all values of the euclidean momenta; i.e. they require a precise matching of the short–distance and the long–distance contributions to the underlying Green’s functions. These two observations are generic in the case of the Standard Model, independently of the $1/N_c$ expansion. The large–$N_c$ approximation helps, however, because it restricts the analytic structure of the Green’s functions to be meromorphic functions; i.e., they only have poles as singularities.

The QCD$\infty$ approach that we are proposing in order to compute a specific coupling of the chiral electroweak Lagrangian consists of the following steps:

1. Identification of the relevant Green’s functions.
2. Evaluation of the short–distance behaviour and the long–distance behaviour of the relevant Green’s functions.
3. Hadronic approximation of the underlying Green's functions with a finite number of poles; i.e., the minimum number required to satisfy the leading power fall-off at short-distances, as well as the appropriate $\chi$PT long-distance constraints.

We have checked this approach with the calculation of a few low-energy observables:

i) The electroweak $\Delta m_\pi$ mass difference\textsuperscript{5}.

ii) The hadronic vacuum polarization contribution to the anomalous magnetic moment of the muon $a_\mu$\textsuperscript{6}.

iii) The $\pi^0 \rightarrow e^+e^-$ and $\eta \rightarrow \mu^+\mu^-$ decay rates\textsuperscript{7}.

These successful tests have encouraged us to pursue a systematic analysis of $K$-physics observables within the same large-$N_c$ framework. So far, the following calculations have been made:

- The $B_K$ Factor in the Chiral Limit\textsuperscript{8}.
- Weak Matrix Elements of the Electroweak Penguin Operators $Q_7$ and $Q_8$\textsuperscript{9,10}.
- Hadronic Light-by-Light Scattering Contribution to the Muon $g-2$\textsuperscript{11}.
- Electroweak Hadronic Contributions to the Muon $g-2$\textsuperscript{12}.
- Weak Matrix Elements of the Strong Penguin Operators $Q_4$ and $Q_6$\textsuperscript{13}.

It appears now to be possible to apply the same techniques to the study of rare $K$ decays as well, and we shall illustrate this here, with a first application to the process $K_L \rightarrow \mu^+\mu^-$.\textsuperscript{15}

2 RARE KAON DECAYS IN $\chi$PT

In fact, it was at the Blois Conference in '98 where some of the first applications of $\chi$PT to rare Kaon decays were reported. The predicted invariant mass spectrum of the two gammas in the mode $K_L \rightarrow \pi^0\gamma\gamma$, to lowest order in the chiral expansion\textsuperscript{17,16} was discussed prior to the earlier experimental measurements by the NA31 and E731 collaborations. Since then, $\chi$PT has been confirmed as the appropriate framework to study rare Kaon decays with many successful applications, (see e.g., ref.\textsuperscript{17} for the latest conference review on the subject, where further references can be found.)

Unfortunately, the predictive power of $\chi$PT is seriously limited at present, because of the fact that several coupling constants of the higher order terms in the electroweak chiral Lagrangian remain unknown. As a result, many predictions at present rely on models which are not clearly related to the underlying Standard Model theory. The analytic approach which we have briefly summarized in the introduction offers, however, an interesting possibility to make progress in this field.

Roughly speaking, from a theoretical point of view, there are three types of $K$-decay modes:

- The golden modes of $\chi$PT
  They correspond to processes which are fixed by $\chi$PT only. The chiral loops are finite and there are no unknown counterterms at the order one is working in the chiral expansion. Examples of that are $K_S \rightarrow \gamma\gamma$ and $K_L \rightarrow \pi^0\gamma\gamma$.

- The short-distance golden modes
  They directly test the structure of the Wilson coefficients, because the relevant effective weak Hamiltonian for these processes is of the unique type: quark-current $\times$ lepton current. The Wilson coefficients have been calculated perturbatively with a sufficient degree of accuracy which makes some of these modes an excellent laboratory of possible Physics
beyond the Standard Model. The paradigm mode of this type is $K_L \rightarrow \pi^0 \nu \bar{\nu}$ (see e.g. ref.\textsuperscript{19} for a recent review on this mode)

- The mixed modes

This is, unfortunately, the largest class. Some modes are sensitive to short–distance Physics, CP violation in particular, like $K_L \rightarrow \pi^0 e^+ e^-$ and $\epsilon / \epsilon'$, but their calculation in the Standard Model brings in unknown couplings of the chiral Lagrangian. It is here where hard theoretical effort is mostly required.

3  $K_L \rightarrow \mu^+ \mu^-$

This is a decay, with both an interesting short–distance component which is particularly sensitive to $V_{td}$, and a long–distance contribution which has a large absorptive component from the dominant $\gamma \gamma$ intermediate state\textsuperscript{20}, illustrated in Fig. 1. The present experimental rates are

\begin{align}
\text{Br}(K_L \rightarrow \mu^+ \mu^-) & = (7.18 \pm 0.17) \times 10^{-9} \text{ BNL–E871} \textsuperscript{21} \quad (4) \\
\text{Br}(K_L \rightarrow e^+ e^-) & = \left(8.7 \, ^{+5.7}_{-4.1}\right) \times 10^{-12} \text{ BNL–E871} \textsuperscript{22} \quad (5)
\end{align}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Contribution to $K_L \rightarrow \bar{l}l$ from the $\gamma \gamma$ loop and local counterterms.}
\end{figure}

From a phenomenological point of view, it is convenient to normalize the $K_L \rightarrow \bar{l}l$ decay rate to the one of the $K_L \rightarrow \gamma \gamma$, which is also known experimentally $[\text{Br}(K_L \rightarrow \gamma \gamma) = (5.86 \pm 0.17) \times 10^{-4}]$. Then

$$R_{\bar{l}l} \equiv \frac{\Gamma(K_L \rightarrow \bar{l}l)}{\Gamma(K_L \rightarrow \gamma \gamma)} = 2\beta \left(\frac{\alpha}{\pi}\right)^2 \left(\frac{m_l^2}{m_K^2}\right) |A_{\bar{l}l}|^2 ,$$

with

$$\text{Im} A_{\bar{l}l} = \frac{\pi}{2\beta_l} \log \left(\frac{1 - \beta_l}{1 + \beta_l}\right), \quad \text{and} \quad \beta_l = \sqrt{1 - \frac{4m_l^2}{M_K^2}} .$$

In fact, the contribution from the absorptive amplitude $\text{Im} A_{\bar{l}l}$ almost saturates the observed experimental rate, leaving very little room for the real part

$$|\text{Re} A_{\bar{l}l}|^2 < (7.1 \pm 0.2) \times 10^{-9} .$$
The real part of $A_{\bar{\mu}}$ has a component from the $\gamma\gamma$ loop which is divergent and requires a local long–distance counterterm. The relevant couplings from the long–distance effective Hamiltonian are the following

$$
H_{\text{LD}}^{(\text{eff})} = C \left\{ \frac{\alpha N_c}{12 \pi f_\pi} \varepsilon_{\mu \nu \alpha \beta} F^{\mu \nu} A^\alpha (\partial^\beta K_L) + \left( \frac{\alpha}{\pi} \right)^2 \frac{\lambda_{\text{LD}}(\mu)}{4 f_\pi} (\partial_\mu K_L) \bar{\gamma}_\mu \gamma_5 l \right\}.
$$

The first term provides the $K_L \gamma\gamma$–vertex interaction in the loop diagram of Fig. 1, with an unknown coupling $C$ which can however be fixed (in modulus only) from the observed $K_L \to \gamma\gamma$ decay rate:

$$
\Gamma(K_L \to \gamma\gamma) = \frac{M_K^3}{64\pi} \left( \frac{\alpha}{\pi} \right)^2 \frac{1}{F_\pi^2} | C |^2.
$$

The second term is the long–distance contact term, which appears at the same order in the chiral expansion as the contribution from the $\gamma\gamma$ loop. The coupling $\chi_{\text{LD}}(\mu)$ is the quantity that one would like to evaluate in the Standard Model. In terms of this coupling, one finds

$$
\text{Re}A_{\mu}^{(\text{LD})}(\beta_\ell) = \chi_{\text{LD}}(\mu) + \frac{N_C}{3} \left[ - \frac{5}{2} + \frac{3}{2} \ln \left( \frac{m^2_\ell}{\mu^2} \right) + C(\beta_\ell) \right],
$$

where the function $C(\beta_\ell)$ corresponds to a finite three–point loop integral which is known, and can be found in refs. The divergence associated with this diagram has been renormalized within the $\overline{\text{MS}}$ minimal subtraction scheme of dimensional regularization. The logarithmic dependence on the renormalization scale $\mu$ displayed in the above expression is compensated by the scale dependence of the renormalized coupling $\chi_{\text{LD}}(\mu)$. Let us stress here that, in contrast with the usual situation in the purely mesonic sector, this scale dependence is not suppressed in the large–$N_C$ limit, since it does not arise from meson loops.

The amplitude $A_{\bar{\mu}}$ also gets a tree level contribution from the short–distance Hamiltonian

$$
H_{\text{SD}}^{(\text{eff})} = \cdots - \frac{G_F \alpha}{\sqrt{2} \pi} \frac{2}{\sin^2 \Theta_W} [\lambda_c Y_{\text{NL}} + \lambda_t Y(x_t)] \left( \bar{s} \gamma_\mu \frac{1 - \gamma_5}{2} d \right) \bar{l} \gamma_\mu \frac{1 - \gamma_5}{2} l + \text{h.c.},
$$

where

$$
\lambda_c = V_{cs}^* V_{cd}, \quad \lambda_t = V_{ts}^* V_{td}, \quad x_t = \frac{m^2_t}{M^2_W}.
$$

In the Standard Model, this is the term in the effective Hamiltonian which emerges after integrating out the heavy degrees of freedom (Z, W; t, b, and c) in the presence of the strong interactions, and which couples directly the quark current $\bar{s} \gamma_\mu \frac{1 - \gamma_5}{2} d$ to the lepton current $\bar{l} \gamma_\mu \frac{1 - \gamma_5}{2} l$. Here $Y_{\text{NL}}$ and $Y(x_t)$ are functions of the masses of the integrated particles, which can be found in ref.

The effect of the short–distance Hamiltonian in the amplitude $A_{\bar{\mu}}$ is to induce a shift in the $\chi_{\text{LD}}(\mu)$–coupling

$$
\chi_{\text{LD}}(\mu) \Rightarrow \chi_{\text{LD}}(\mu) - \chi_{\text{SD}},
$$

with $\chi_{\text{SD}}$ fixed by the relation

$$
- \frac{G_F \alpha}{\sqrt{2} \pi} \frac{2}{\sin^2 \Theta_W} \text{Re} [\lambda_c Y_{\text{NL}} + \lambda_t Y(x_t)] = \frac{\alpha}{\pi} C \times \chi_{\text{SD}}.
$$

Without loss of generality, we can fix $\mu$ at the $\rho$ mass and define the parameter

$$
\chi_{\text{eff}} = \chi_{\text{LD}}(M_\rho) - \chi_{\text{SD}}
$$

The branching ratio $R_{\mu}$ in Eq. (6) is then a function of only the unknown parameter $\chi_{\text{eff}}$, and the predicted shapes for $l = \mu, e$ are as in Fig. 2. From the comparison with the experimental value of $R_{\mu\mu}$ (the horizontal band in Fig. 3a) we conclude that

$$
3.9 \leq \chi_{\text{eff}} \leq 6.5.
$$
The minimum of the predicted parabola in Fig. 2a corresponds to the lower bound when

\[
\text{Re} \mathcal{A}_{\bar{\mu} \mu} = 0 \quad \Rightarrow \quad \chi_{\text{eff}} = 5.2. \quad (18)
\]

**Fig. 2a** The predicted branching ratio \( R_{\bar{\mu} \mu} \) as a function of \( \chi_{\text{eff}} \). The horizontal lines correspond to the present experimental result with errors.

Because of the small electron mass, the term \( \log \frac{m^2_{\text{e}}}{M_\rho} \) dominates the r.h.s. of Eq. (11), and there is very little we can learn on \( \chi_{\text{eff}} \) from the, otherwise, quite remarkable experimental determination of \( R_{\text{ee}} \).

**Fig. 2b** The predicted branching ratio \( R_{\text{ee}} \) as a function of \( \chi_{\text{eff}} \). The horizontal lines correspond to the present experimental result with errors.
3.1 EVALUATION OF $\chi_{LD}(\mu)$ IN THE LARGE-$N_c$ QCD FRAMEWORK

The relevant Green’s function for the process we are concerned with is the four–point function

$$W_{\mu\nu}(q, p) = \lim_{t \to 0} \int d^4x e^{iq \cdot x} \int d^4y e^{ip \cdot y} \int d^4z e^{i\xi \cdot z} \langle 0 | T \left\{ J_\mu(x) J_\nu(y) \right\} \mathcal{P}(0) \mathcal{L}_{\text{eff}}^{S=1}(z) | 0 \rangle \]$$

$$= \frac{2}{3}\epsilon_{\mu\nu\alpha\beta} q^\alpha p^\beta \frac{1}{(q + p)^2 - M_K^2} W[q^2, p^2, (q + p)^2],$$

where, in the first line, $J_\mu$ is the electromagnetic current; $\mathcal{P} = \bar{q} i\gamma_5 \frac{\lambda(\nu)}{2} q$ the interpolating density current operator of the $K_L$–state and $\mathcal{L}_{\text{eff}}^{S=1}$ the strangeness changing chiral effective Lagrangian. The function $W[q^2, p^2, (q + p)^2]$ governs both the $K_L \to \mu^+\mu^-$ and the $K_L \to \gamma\gamma$ decay rates. In particular, the relation to the constant $C$ introduced in Eqs. (3) and (10) is as follows

$$W[0, 0, M_K^2] = \frac{N_c}{8\pi^2} \frac{|\langle \bar{\psi}\psi \rangle|}{F_0^2} C,$$

while the evaluation of the $K_L \to \mu^+\mu^-$ amplitude requires the knowledge of the function $W[q^2, q^2, M_K^2]$ at all values $0 \leq -q^2 \leq \infty$.

It is well known that the lowest order evaluation of the constant $C$ in $SU(3)_L \times SU(3)_R$ $\chi$PT gives a contribution which is proportional to the lowest order Gell-Mann Okubo mass relation: $3M_\eta^2 - 4M_K^2 + M_\pi^2 = 0$. The first non–trivial contribution to $C$ comes from chiral loops and a combination of undetermined couplings of the effective $\mathcal{L}_{\text{eff}}^{S=1}$ Lagrangian at $O(p^4)$. This is why it has become so difficult to evaluate $K_L \to \gamma\gamma$, and hence $K_L \to \mu^+\mu^-$ in the Standard Model. We do not attempt to do a calculation of $C$, but we shall use a crucial observation which, to leading non–trivial order in the chiral expansion, relates the residue of the short–distance behaviour ($-q^2 \to \infty$) of the function $W[q^2, q^2, M_K^2]$ to the constant $C$, known in modulus from the observed $K_L \to \gamma\gamma$ decay rate. The relation is the following

$$\lim_{-q^2 \to \infty} W[q^2, q^2, M_K^2] = \frac{|\langle \bar{\psi}\psi \rangle|}{-q^2} \left( C + \frac{8}{9} 4L_5 \frac{M_K^2}{F_0^2} \frac{M_\eta^2}{M_K^2 - M_\pi^2} \right),$$

and is based on the following facts:

- The first non–trivial contribution to the constant $C$ in $\chi$PT and to leading order in the $1/N_c$ expansion, comes from the bosonization of the $T$–product in Eq. (19), with the contribution to $J_\mu(x)J_\nu(y)$ induced by the local one–Goldstone contribution from the $O(p^4)$ Wess–Zumino Lagrangian, the $O(p^2)$ bosonization of $\mathcal{P}(0)$ and the problematic $O(p^4)$ contribution from $\mathcal{L}_{\text{eff}}^{S=1}(z)$.

- When performing the operator product expansion of the two electromagnetic currents, the leading contribution is governed by the local operator: $\bar{q}Q^2\gamma^\beta q$, which also has to be bosonized in terms of one–Goldstone states and produces the same combination of fields as the one emerging from the Wess–Zumino $J_\mu(x)J_\nu(y)$ product discussed in the previous item. The resulting $T$–product of this $\bar{q}Q^2\gamma^\beta q$–operator, with the remaining $\mathcal{P}(0)$ and $\mathcal{L}_{\text{eff}}^{S=1}(z)$ operators, has to be evaluated at long–distances. To first non–trivial order, there are two pieces which contribute to its bosonization: the one from $\bar{q}Q^2\gamma^\beta q$ at $O(p^2)$.

<sup>a</sup>It is in this sense that we differ from the large–$N_c$ inspired $U(3)_L \times U(3)_R$ phenomenological analysis of ref. 23.

<sup>b</sup>Notice the the short–distance behaviour obtained here is in disagreement with the hadronic ansatz proposed in ref. 24.

<sup>c</sup>We have identified the combination of $O(p^4)$ couplings which contribute 15.
with $\mathcal{P}(0)$ at $\mathcal{O}(p^2)$ and $\mathcal{L}_{\text{eff}}^{\Delta S=1}(z)$ at $\mathcal{O}(p^4)$ produces a term proportional to the same contribution discussed in the previous item; the other one from $\bar{q}Q^2\gamma^\beta q$ at $\mathcal{O}(p^4)$, which is proportional to the $L_5$ coupling, times $\mathcal{P}(0)$ at $\mathcal{O}(p^2)$ and $\mathcal{L}_{\text{eff}}^{\Delta S=1}(z)$ also at $\mathcal{O}(p^2)$. Fortunately, this second contribution brings in known couplings of the chiral Lagrangian.

- The other possible contributions to $\mathcal{W}[0,0,M_K^2]$ and $\lim_{q^2 \to \infty} \mathcal{W}[q^2,q^2,M_K^2]$, at the same order in the chiral expansion, either give vanishing contributions, or are proportional to the Gell-Mann Okubo combination of masses. However, higher order contributions in the chiral expansion may very likely destroy this dynamical symmetry observed at the first non–trivial order. This is why our main result in Eq. (22) is only valid to first non–trivial order in the chiral expansion. Furthermore, the effect of chiral loops, subleading in the $1/N_c$ expansion are also neglected.

The procedure to evaluate the effective coupling $\chi_{\text{LD}}(\mu)$ is now entirely similar to the one discussed in ref. [7] with the result

$$\chi_{\text{LD}}(M_V) = \frac{11}{12} N_c - 4\pi^2 \frac{F_0^2}{M_V^2} \left( 1 \pm \frac{8}{3} \frac{M_K^2 - m_\pi^2}{F_0^2} \frac{4L_5}{1.19 \pm 0.16} \right), \quad (23)$$

where the first $\pm$ sign comes from the fact that the constant $C$ is only known in modulus. This implies a two–fold result for the effective coupling $\chi_{\text{eff}}$ in Eq. (16) which, using the short–distance evaluation $\chi_{\text{SD}} = \pm (1.8 \pm 0.1)$, results in

$$\chi_{\text{eff}} = \begin{cases} 
2.33 \pm 0.9 - (1.8 \pm 0.1) = 0.5 \pm 0.9 \\
2.03 \pm 0.9 + (1.8 \pm 0.1) = 3.8 \pm 0.9
\end{cases} \quad (24)$$

The corresponding branching ratios for the two solutions are shown in Figure 3 below

**Fig. 3** Branching ratio versus $\chi_{\text{eff}}$. The horizontal band corresponds to the experimental value with errors. The predicted solutions are the two solid vertical bands with vertical error bars.

The higher solution is in perfect agreement with the experimental determination, the lower one is in slight difficulty.
It is very likely that considerations similar to the ones illustrated here in the case of \(K_L \to \mu^+\mu^-\), may also apply to other rare \(K\)-decay processes and open, therefore, a new theoretical perspective in the field of rare \(K\) decays.

**Acknowledgments**

We wish to thank Samuel Friot, Marc Knecht and Santi Peris for helpful discussions. This work has been partially supported by the TMR, EC-Contract No. HPRN-CT-2002-00311 (EURIDICE).

**References**

1. E. de Rafael, *Analytic Approaches to Kaon Physics* in Proc. of the Lattice Conference at MIT, June 2002.
2. S. Weinberg, Physica A96 (1979) 327.
3. J. Gasser and H. Leutwyler, Ann. of Phys.(N.Y.) 158 (1984) 142.
4. J. Gasser and H. Leutwyler, Nucl. Phys. B250 (1985) 465.
5. M. Knecht, S. Peris and E. de Rafael, Phys. Lett. B443 (1998) 255.
6. M. Perrottet and E. de Rafael, *unpublished*.
7. M. Knecht, S. Peris, M. Perrottet and E. de Rafael, Phys. Rev. Lett. 83 (1999) 5230.
8. S. Peris and E. de Rafael, Phys. Lett. B490 (2000) 213, *erratum* arXiv:hep-ph/0006146 v3.
9. M. Knecht, S. Peris and E. de Rafael, Phys. Lett. B457 (1999) 227.
10. M. Knecht, S. Peris and E. de Rafael, Phys. Lett. B508 (2001) 117.
11. M. Knecht and A. Nyffeler, Phys. Rev. D65 (2002) 073034.
12. M. Knecht and A. Nyffeler, M. Perrottet and E. de Rafael, Phys. Rev. Lett. 88 (2002) 071802.
13. M. Knecht, S. Peris, M. Perrottet and E. de Rafael, arXiv:hep-ph/0205102.
14. T. Hambye, S. Peris and E. de Rafael, *in preparation*.
15. D. Greynat and E. de Rafael, *in preparation*.
16. E. de Rafael, in *CP VIOLATION in particle Physics and Astrophysics*, edited by J. Tran Thanh Van, Editions Frontieres.
17. G. Ecker, A. Pich and E. de Rafael, Phys. Lett. B189 (1987) 363.
18. G. D’Ambrosio, in Proceedings of *KAON 2001* Editors: F. Costantini, G. Isidori, M. Sozzi; FRASCATI PHYSICS SERIES.
19. L Littenberg, in Proceedings of *KAON 2001* Editors: F. Costantini, G. Isidori, M. Sozzi; FRASCATI PHYSICS SERIES.
20. B. Martin, E. de Rafael and J. Smith, Phys. Rev. D2 (1970) 179 [Erratum: D4 (1971) 272].
21. D. Ambrose *et al*., Phys. Rev. Lett. 84 (2000) 1389.
22. D. Ambrose *et al*., Phys. Rev. Lett. 81 (1998) 4309.
23. D. Gómez-Dumm and A. Pich, Phys. Rev. Lett. 80 (1998) 4633.
24. M. Knecht, S. Peris and E. de Rafael, Nucl. Phys. (Proc. Suppl.) B86 (2000) 279.
25. G. Buchalla and A.J. Buras, Nucl. Phys. B412 (1994) 106.
26. G. D’Ambrosio, G. Isidori and J. Portolés, Phys. Lett. B423 (1998) 385.