SUPERSYMMETRIC YANG-MILLS THEORIES
IN 1 + 1 DIMENSIONS

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ABSTRACT
Supersymmetric Yang-Mills theories are considered in 1 + 1 dimensions. Firstly physical mass spectra of supersymmetric Yang-Mills theories in 1 + 1 dimensions are evaluated in the light-cone gauge with a compact spatial dimension. The supercharges are constructed in order to provide a manifestly supersymmetric infrared regularization for the discretized light-cone approach. By exactly diagonalizing the supercharge matrix between up to several hundred color singlet bound states, we find a rapidly increasing density of states as mass increases. Interpreting this limiting density of states as the string behavior, we obtain the Hagedorn temperature \( \beta_H = 0.676 \sqrt{gN} \). Secondly we have examined the vacuum structure of supersymmetric Yang-Mills theories in 1 + 1 dimensions. SUSY allows only periodic boundary conditions for both fermions and bosons. By using the Born-Oppenheimer approximation for the weak coupling limit, we find that the vacuum energy vanishes, and hence the SUSY is unbroken. Other boundary conditions are also studied. The first part is based on a work in collaboration with Y. Matsumura and T. Sakai. The second part is based on a work in collaboration with H. Oda and T. Sakai.

1. Introduction

Supersymmetric theories have now become standard models for the unified theory. Both as a model for grand unified theories and as a low energy effective theory for superstring, the dynamics of supersymmetric Yang-Mills theories is a fascinating subject. The most outstanding problem in unified theories based on supersymmetry is to understand the supersymmetry breaking. Nonperturbative dynamics is expected to be essential to study the mechanisms of supersymmetry breaking.

One of the most popular models for the supersymmetry breaking is currently to assume the gaugino bilinear condensation in the supersymmetric Yang-Mills theories \(^1\). Although the condensation itself may not break supersymmetry in the supersymmetric gauge theories, it will give rise to the supersymmetry breaking if embedded in supergravity \(^2\). Since the fermion bilinear condensation is implied by the chiral symmetry breaking in QCD, one can expect a similar nonperturbative effects in supersymmetric Yang-Mills theories. Moreover, recent progress in understanding duality in supersymmetric Yang-Mills theories opened up a rich arena for studying the nonperturbative effects in supersymmetric gauge theories \(^3\).

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It has been quite fruitful to study Yang-Mills theories in 1 + 1 dimensions instead of studying directly the four dimensional counterpart. In 1 + 1 dimensions, gauge field itself has no dynamical degree of freedom as a field theory, but gives rise to a confining potential for colored particles. Many aspects of color singlet bound states can be explored by solving the theory in the large $N$ limit. Unfortunately the supersymmetric gauge multiplet contains genuine dynamical degree of freedom in the adjoint representation of the gauge group contrary to ordinary Yang-Mills theory. Therefore one cannot obtain a simple closed form for the color singlet bound states even in the large $N$ limit.

There has been progress in studying the dynamics of matter fields in the adjoint representation in ordinary Yang-Mills theories. They have used the light-cone quantization and compactified the spatial dimension to give discrete momenta. In this discretized light-cone quantization approach, one can diagonalize the mass matrix for finite number of light-cone momenta and can hope to obtain the infinite volume limit eventually.

More recently, gauge theories in 1 + 1 dimensions with matter in adjoint representations was studied focusing attention on zero modes. The zero modes are generally important in revealing nontrivial vacuum structures such as the vacuum condensate.

The Born-Oppenheimer approximation in the weak coupling region has been used to study the vacuum structure of gauge theories with adjoint fermions. Since the gauge coupling in 1 + 1 dimensions has the dimension of mass, the weak coupling is characterized by

$$gL \ll 1,$$

where $L$ is the interval of the compactified spatial dimension. The fermion bilinear was found to possess a nonvanishing vacuum expectation value which exhibits instanton-like dependence on gauge coupling. The Yang-Mills theories with adjoint fermions were also studied at finite temperature and were shown to be dominated by instanton effects at high temperatures. The Born-Oppenheimer approximation has been used to study SUSY gauge theories in four dimensions.

In spite of these investigations of Yang-Mills gauge theories with adjoint scalar and spinor matter fields, there are two points which necessitate a new analysis of physical spectra in the case of supersymmetric gauge theories. The first point is that the coexistence of spinor and scalar gives rise to a large number of new “mixed” physical states, partly consisting of spinors and partly of scalars as constituents. The second point is the presence of a specific amount of the Yukawa interaction which is a distinguishing feature of the supersymmetric Yang-Mills theory.

As for the possibility of the SUSY breaking, the Witten index of the SUSY Yang-Mills theories has been calculated recently, and was found to be nonvanishing in 1 + 1 dimensions. Although this result implies no possibility for spontaneous SUSY breaking, we feel it still worthwhile to study the vacuum of the SUSY Yang-Mills theories in 1 + 1 dimensions by a more detailed dynamical calculation, since the calculation of the Witten index involved a certain regularization of bosonic zero modes which may not be easily justified.

In view of this situation, we have studied SUSY Yang-Mills theories from two perspectives: mass spectra and vacuum structures. The mass spectra has been computed in the discretized light-cone quantization in collaboration with Y. Matsumura and T. Sakai, and the vacuum structure is studied by the Born-Oppenheimer approximation.
in collaboration with H. Oda and T. Sakai.

We construct the supercharge explicitly and specify an infrared regularization for supercharge by means of the discretized version of the principal value prescription. By using the supercharge, we succeed in overcoming ambiguities in prescribing the infrared regularization for the light-cone Hamiltonian. As a result, the regularization preserves the supersymmetry algebra manifestly. For light-cone momenta up to 8 units of the smallest momentum, we find several hundred color singlet bound states of bosons and the same number of fermions. We exactly diagonalize the supercharge instead of the Hamiltonian to obtain masses, degeneracies, and the average number of constituents in these bound states. We observe that the density of the bound states as a function of their masses tends to converge in the large volume limit. It is consistent with the rapidly increasing density of states suggested by the closed string interpretation. Since we preserve supersymmetry at each stage of our study, we naturally obtain exact correspondence between bosonic and fermionic color singlet bound states. We have also introduced a mass term for adjoint scalar and/or spinor fields. Since these fields are superpartner of gauge field which are strictly massless, these terms break supersymmetry softly. We find indeed that the degeneracy of the mass spectra for color singlet bosons and fermions is lifted.

Light-cone approach is notoriously difficult to unravel the vacuum structure. We need alternative systematic approaches to study the vacuum structure. To study the vacuum structure, we use the Born-Oppenheimer approximation in the weak coupling region. To formulate the weak coupling limit, we need to compactify the spatial direction. Since gauge fields naturally follow periodic boundary conditions, we need to require the same periodic boundary conditions for scalar and spinor fields in order not to break SUSY by hand. We have found that the ground state has a vanishing vacuum energy, suggesting that SUSY is not broken spontaneously. This result is consistent with the result on the Witten index. We also examine all four possibilities of periodic and anti-periodic boundary conditions for fermions and bosons.

2. SUSY Yang-Mills Theories in 1 + 1 Dimensions

In two-dimensions, the gauge field $A^\mu$ is contained in a supersymmetric multiplet consisting of a Majorana fermion $\Psi$ and a scalar $\phi$ in the adjoint representation of the gauge group together with gauge field itself. After choosing the Wess-Zumino gauge, we have an action

$$S = \int d^2x \ tr \left[ -\frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} D_\mu \phi D^\mu \phi + i \bar{\Psi} \gamma^\mu D_\mu \Psi - 2ig \phi \bar{\Psi} \gamma_5 \Psi \right], \quad (2)$$

where $A_\mu$, $\phi$, $\Psi$, and $\bar{\Psi} = \Psi^T \gamma^0$ are traceless $N \times N$ hermitian matrix for $U(N)$ ($SU(N)$) gauge group, $g$ is the gauge coupling constant, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i [A_\mu A_\nu]$ and $D_\mu$ is the usual covariant derivative

$$D_\mu \phi = \partial_\mu \phi + i [A_\mu, \phi], \quad D_\mu \Psi = \partial_\mu \Psi + i [A_\mu, \Psi]. \quad (3)$$

The supersymmetry dictates the presence of the Yukawa type interaction between the adjoint spinor and scalar fields with the strength of the gauge coupling. The supersymmetric Yang-Mills gauge theory in two-dimensions can be obtained by a dimensional
reduction from the supersymmetric Yang-Mills gauge theory in three dimensions. The adjoint scalar field can be understood as the component of the gauge field in the compactified dimension and the Yukawa coupling is nothing but the gauge interaction in this compactified extra dimension.

In the Wess-Zumino gauge, the remaining invariances of the action are the usual gauge invariance and a supertransformation which is obtained by combining the supertransformation and the compensating gauge transformation in the superfield formalism. The corresponding spinor supercurrent \( j^\mu \) is given by

\[
\bar{\epsilon} j^\mu = \text{tr} \left[ -\sqrt{2}e \Psi D^\mu \phi + i \frac{1}{\sqrt{2g}} \epsilon^\nu \lambda F_{\nu \lambda} \bar{\epsilon} \gamma^\mu \Psi + \sqrt{2} \bar{\epsilon} \gamma_5 \Psi e^{\mu \nu} D_\nu \phi \right].
\]  

(4)

We introduce the light-cone coordinates where the line element \( ds^2 \) is given by

\[
x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^1), \quad ds^2 = (dx^0)^2 - (dx^1)^2 = 2dx^+ dx^-.
\]  

(5)

We decompose the spinor and use gamma matrices

\[
\Psi_{ij} = 2^{-1/4}(\psi_{ij}, \chi_{ij})^T, \quad \gamma^0 = \sigma_2, \quad \gamma^1 = i\sigma_1, \quad \gamma_5 = \gamma^0 \gamma^1 = \sigma_3.
\]  

(6)

Taking the light-cone gauge \( A_- = A^+ = 0 \) and \( x^+ \) as time, we find the action

\[
S = \int dx^+ dx^- \text{tr} \left[ \partial_+ \phi \partial_- \phi + i\psi \partial_+ \psi + i\chi \partial_- \chi 
+ \frac{1}{2g^2}(\partial_- A^+)^2 + A^+ J^+ + \sqrt{2}g \phi \{\psi, \chi\} \right],
\]  

(7)

where the current \( J^+ \) receives contributions from the scalar \( J^+_\phi \) and the spinor \( J^+_\psi \)

\[
J^+ = J^+_\phi + J^+_\psi, \quad J^+_\phi = i[\phi, \partial_- \phi], \quad J^+_\psi = 2\psi \psi.
\]  

(8)

We do not need Faddeev-Popov ghosts in this gauge. Since the action contains no time derivative for the gauge potential \( A_+ \) and the left-moving fermion \( \chi \), they can be eliminated by means of constraints obtained as their Euler-Lagrange equations

\[
i\sqrt{2}\partial_- \chi - g[\phi, \psi] = 0, \quad \partial_-^2 \bar{A}_+ - g^2 J^+ = 0.
\]  

(9)

where \( \bar{A}_+ \) is the non-zero mode of \( A_+ \). The zero mode of \( A_+ \) plays the role of a Lagrange multiplier which provides a constraint

\[
\int dx^- J^+ = 0.
\]  

(10)

This constraint will give a restriction for physical states in quantum theory. After eliminating the fields \( A_+ \) and \( \chi \), we find that the action becomes

\[
S = \int dx^+ dx^- \text{tr} \left[ \partial_+ \phi \partial_- \phi + i\psi \partial_+ \psi + \frac{g^2}{2} J^+ \frac{1}{\partial_-^2} J^+ - \frac{1}{2}i g^2 [\phi, \psi] \frac{1}{\partial_-} [\phi, \psi] \right].
\]  

(11)
Let us note that the constraints give rise to non-local terms in the action.
By the Noether procedure, we construct the energy momentum tensor $T^{\mu\nu}$, and light-cone momentum and energy $P^\pm = \int dx^- T^{\pm\pm}$ on a constant light-cone time

$$P^+ = \int dx^- \text{tr} \left[ (\partial_- \phi)^2 + i \psi \partial_- \psi \right], \quad (12)$$

$$P^- = \int dx^- \text{tr} \left[ -\frac{g^2}{2} J^+ \frac{1}{\partial_-^2} J^+ + \frac{i}{2} g^2 \partial_- (\phi, \psi) \frac{1}{\partial_-} [\phi, \psi] \right]. \quad (13)$$

The supercharges $Q_1$ and $Q_2$ are defined as integrals of the upper and lower components of the spinor supercurrent $j^\mu = (j_1^\mu, j_2^\mu)$ in eq.(4)

$$Q_1 \equiv \int dx^- j_1^+ = 2^{1/4} \int dx^- \text{tr} [\phi \partial_- \psi - \psi \partial_- \phi], \quad (14)$$

$$Q_2 \equiv \int dx^- j_2^+ = 2^{3/4} g \int dx^- \text{tr} \left[ J^+ \frac{1}{\partial_-} \psi \right] = 2^{3/4} g \int dx^- \text{tr} \left\{ i[\phi, \partial_- \phi] + 2 \psi \partial_- \psi \right\}. \quad (15)$$

Using the conjugate momenta $\pi_\phi = \partial L / \partial (\partial_+ \phi) = \partial_- \phi$ for adjoint scalar field $\phi_{ij}$ and $\pi_\psi = \partial L / \partial (\partial_+ \psi) = i \psi$ for adjoint spinor field $\psi_{ij}$, the canonical (anti)commutation relation are given at equal light-cone time $x^+ = y^+$ by

$$[\phi_{ij}(x), \partial_- \phi_{kl}(y)] = i \{ \psi_{ij}(x), \psi_{kl}(y) \} = \frac{1}{2} i \delta(x^- - y^-) \delta_{il} \delta_{jk}. \quad (16)$$

We expand the fields in modes with momentum $k^+$ at light-cone time $x^+ = 0$

$$\phi_{ij}(x^-, 0) = \frac{1}{\sqrt{2\pi}} \int_0^\infty \frac{dk^+}{\sqrt{2k^+}} (a_{ij}(k^+) e^{-ik^+x^-} + a_{ij}^+(k^+) e^{ik^+x^-}), \quad (17)$$

$$\psi_{ij}(x^-, 0) = \frac{1}{2\sqrt{\pi}} \int_0^\infty dk^+ \left( b_{ij}(k^+) e^{-ik^+x^-} + b_{ij}^+(k^+) e^{ik^+x^-} \right). \quad (18)$$

The canonical (anti-)commutation relations (19) are satisfied by

$$[a_{ij}(k^+), a_{lk}^+(\tilde{k}^+)] = \{ b_{ij}(k^+), b_{lk}^+(\tilde{k}^+) \} = \delta(k^+ - \tilde{k}^+) \delta_{il} \delta_{jk}. \quad (19)$$

3. Discretized Light-Cone Quantization of Supercharge

In order to prescribe the infrared regularization precisely and to evaluate the mass spectrum in spaces with finite number of physical states, we compactify spatial direction $x^-$ to form a circle with radius $2L$ by identifying $x^- = 0$ and $x^- = 2L$. In order to preserve supersymmetry, we need to impose the same boundary condition on scalars $\phi_{ij}$ and spinors $\psi_{ij}$. It is in general necessary to choose periodic boundary conditions on bosonic field and to retain zero modes, if one wishes to take into account the possibility
of vacuum condensate or spontaneous symmetry breaking. Since we are primarily interested in physical mass spectrum, we neglect the zero modes in the present work. We shall choose periodic boundary conditions for both scalars $\phi_{ij}$ and spinors $\psi_{ij}$, leaving the problem of zero modes for a further study

$$\phi_{ij}(x^-) = \phi_{ij}(x^- + 2L), \quad \psi_{ij}(x^-) = \psi_{ij}(x^- + 2L).$$

(20)

The allowed momenta become discrete and the momentum integral is replaced by a summation,

$$k^+_n = \frac{\pi}{L} n, \quad n = 1, 2, 3, \ldots, \quad \int_0^\infty dk^+ \to \frac{\pi}{L} \sum_{n=1}^\infty.$$

(21)

Then mode expansions (17) and (18) for $\phi_{ij}$ and $\psi_{ij}$ become discretized

$$\phi_{ij} = \frac{1}{\sqrt{4\pi}} \sum_{n=1}^\infty \frac{1}{\sqrt{n}} \left[ A_{ij}(n)e^{-i\pi nx^1/L} + A_{ji}^\dagger(n)e^{i\pi nx^1/L} \right],$$

(22)

$$\psi_{ij} = \frac{1}{\sqrt{4L}} \sum_{n=1}^\infty \left[ B_{ij}(n)e^{-i\pi nx^1/L} + B_{ji}^\dagger(n)e^{i\pi nx^1/L} \right],$$

(23)

$$A_{ij}(n) = \sqrt{\pi/L}a_{ij}(k^+ = \pi n/L), \quad B_{ij}(n) = \sqrt{\pi/L}b_{ij}(k^+ = \pi n/L),$$

(24)

$$\left[ A_{ij}(n), A_{lk}^\dagger(n') \right] = \left[ B_{ij}(n), B_{lk}^\dagger(n') \right] = \delta_{nn'}\delta_{il}\delta_{jk}.$$  

(25)

Let us define the supercharge in this discretized light-cone quantization. The first supercharge $Q_1$ in this compactified space is given by

$$Q_1 = 2^{1/4}i\sqrt{\frac{\pi}{L}} \sum_{n=1}^\infty \sqrt{n} \left[ A_{ij}(n)B_{ij}^\dagger(n) - A_{ij}^\dagger(n)B_{ij}(n) \right].$$

(26)

Since the elimination of gauge field $A_+$ introduces a singular factor $1/\partial_-$ in supercharge $Q_2$, we need to specify an infrared regularization for this factor. Following the procedure of 'tHooft, we employ the principal value prescription for the supercharge. Namely we simply drop the zero momentum mode

$$Q_2 = 2^{1/4}(-i)g\sqrt{\frac{L}{\pi}} \sum_{m=1}^\infty \frac{1}{m} \left[ B_{ij}^\dagger(m)\bar{J}_{ij}(-m) - (\bar{J}_{ij}(-m))^{\dagger}B_{ij}(m) \right]$$

$$= -i2^{-1/4}g\sqrt{L} \left( \sum_{l,n=1}^\infty \frac{l+2n}{2l}\frac{1}{\sqrt{n(l+n)}} \left[ (A^\dagger(n)B^\dagger(l) - B^\dagger(l)A^\dagger(n))_{ij} A_{ij}(l+n) \right.$$ $$- A_{ij}^\dagger(l+n)(A(n)B(l) - B(l)A(n))_{ij} \right]$$

$$+ \sum_{l=3}^\infty \sum_{n=1}^{l-1} \frac{l-2n}{2l}\frac{1}{\sqrt{n(l-l-n)}} \left[ B_{ij}(l)(A(n)A(l-n))_{ij} - (A^\dagger(n)A^\dagger(l-n))_{ij} B_{ij}(l) \right]$$

$$- \sum_{l,n=1}^\infty \frac{1}{l} \left( \frac{1}{n} \right) \left[ (B^\dagger(n)B^\dagger(l))_{ij} B_{ij}(l+n) + B_{ij}^\dagger(l+n)(B(n)B(l))_{ij} \right]$$

$$+ \sum_{l=2}^{l-1} \sum_{n=1}^{l-1} \frac{1}{l} \left( \frac{1}{n} \right) \left[ B_{ij}^\dagger(l)(B(n)B(l-n))_{ij} + (B^\dagger(n)B^\dagger(l-n))_{ij} B_{ij}(l) \right].$$

(27)
The supersymmetry algebra requires a relation between supercharges and the light-cone momentum $P^+$ and the Hamiltonian $P^-$ operators

$$\{Q_1, Q_1\} = 2\sqrt{2}P^+, \quad \{Q_2, Q_2\} = 2\sqrt{2}P^-, \quad \{Q_1, Q_2\} = 0,$$

in our choice of spinor notations (I). Infrared regularizations of $P^+$ and $P^-$ have to be done consistently with the supersymmetry algebra. It is actually difficult to guess the correct infrared regularization for the Hamiltonian unless we start from the supercharge. The Hamiltonian $P^-$ can be defined by just squaring the supercharge $Q_2$. Then the above principal value prescription for the supercharge $Q_2$ specifies uniquely the prescription for the Hamiltonian. In this way we can check that the supersymmetry algebra holds in our formulation of the discretized light-cone quantization.

Physical states take the following form

$$\frac{1}{N^{m/2}\sqrt{s}} \text{tr} [O(n_1) \cdots O(n_m)] |0\rangle, \quad m > 1,$$

where $O$ represents $A^\dagger$ or $B^\dagger$. The symmetry factor $s$ is the number of possible permutations of constituents which give the same state (II). Note that we should consider only states with two or more constituents $m > 1$ since we should discard singlet to the leading order of the $1/N$ expansion of $U(N)$ gauge theory. It is also absent in the case of $SU(N)$ gauge theory anyway. All these states satisfy the physical state condition coming from the constraint (I0). Here we note that there are both bosonic and fermionic oscillators in our supersymmetric theory. This fact gives rise to much larger number of new physical states compared to the purely fermionic or bosonic adjoint matter case.

4. Numerical Results of Supercharge Diagonalization

As we have seen, our procedure preserves supersymmetry manifestly throughout the calculation. Therefore we are naturally led to obtain supersymmetric mass spectra with exactly the same bosonic and fermionic spectra for color singlet states.

If we consider the states with finite values of the discrete momentum $K$, we have only finitely many physical states to diagonalize the mass matrix.

We have explicitly constructed bosonic and fermionic color singlet states for higher values of the cut-off momentum $K$ up to $K = 11$. We find the number of bosonic color singlet states for $K = 5, 6, 7, 8, 9, 10$, and $11$ to be $24, 61, 156, 409, 1096, 2953$, and $8052$ respectively. The number of fermionic color singlet states is exactly the same as the corresponding bosonic one with the same $K$.

After evaluating the supercharge for these subspace up to $K = 8$, we diagonalize the supercharge exactly to obtain the mass eigenvalues. In Fig.5 we plot the accumulated number of bosonic color singlet bound states as a function of mass divided by $g\sqrt{\pi}$. We can see that the number of states is approaching to a limiting value at least for smaller values of $M^2$. The present tendency seems to suggest that the density of states is increasing rapidly as the mass squared increases. This behavior is in qualitative agreement with the previous results for the adjoint scalar or adjoint spinor matter constituents in nonsupersymmetric gauge theories (II). Namely the density of states showed an exponential increase as mass squared increases in accordance with the closed string
Figure 1: The accumulated number of bound states as a function of mass for $K = 4, 5, 6, 7, 8$; there is no difference in behavior between bosonic and fermionic state.
interpretation. From this result we find numerically that the limiting (Hagedron) temperature for the string is given by

$$\beta_H = 0.676 \sqrt{\frac{\pi}{g^2 N}}$$

(30)

The fermionic color singlet bound states show the same behavior.

5. Weyl Gauge and Fundamental Domain

In this section we compactify the spatial direction to a circle with a finite radius $L/2\pi$. The gauge fields naturally follow periodic boundary conditions

$$A^\mu(x = 0) = A^\mu(x = L).$$

(31)

We shall specify boundary conditions for $\Psi$ and $\phi$ later.

Gauge theories have a large number of redundant gauge degrees of freedom which should be eliminated by a gauge-fixing condition. In this section we quantize the system in the Weyl gauge,

$$A^0 = 0.$$ 

(32)

We can impose Gauss’ law as a subsidiary condition for the physical state $|\Phi\rangle$

$$[D_1 E^a(x) - g \rho^a(x)]|\Phi\rangle = 0, \quad \rho^a = f^{abc} \phi^b \pi^c + \frac{i}{2} f^{abc} \Psi^c \Psi^b.$$ 

(33)

where $\pi^a$ and $-E^a \equiv F^{a01}$ are the conjugate variables of $\phi^a$ and $A^{a1}$ respectively, and $\rho^a$ is the color charge density, and $f^{abc}$ are the structure constants of the Lie algebra of $SU(N)$ : $[t^a, t^b] = if^{abc}t^c$. Note that the Gauss law determines $E$, except for its constant modes $e$. One can eliminate $A^1$ by using an appropriate gauge transformation, except for the $N - 1$ spatially constant modes $a^p$ which are given by

$$\mathcal{P} \exp \left( ig \int_0^L dx A^1(x) \right) = V e^{igaL} V^\dagger, \quad (a = a^p t^p),$$

(34)

where $V$ is a unitary matrix. Hereafter we shall use the convention that $a, b, \cdots = 1, 2, \cdots, N^2-1$ represent the indices of the generators of $SU(N)$, and $p, q, \cdots = 1, 2, \cdots, N-1$ represent those of Cartan subalgebra. The commutation relation between $a^p$ and $e^q$ is given as

$$[e^p, a^q] = i\delta^{pq} \quad p, q = 1, \ldots, N - 1.$$ 

(35)

In the physical state space, we can eliminate redundant gauge degrees of freedom by solving the Gauss law constraint (33), and find the Hamiltonian

$$H = \int_0^L dx H(x) = K_a + H_\epsilon + H_b + H_t + H_{\text{int}},$$

(36)

$$K_a = \frac{1}{2L} \sum_p e^{e^p} e^p,$$ 

(37)
\[
H_c = \frac{g^2}{L} \sum_{n=-\infty}^{\infty} \sum_{ij} \int_0^L dy \int_0^L dz (1 - \delta_{ij}\delta_{n0}) \left( \frac{(\rho(y))_{ij} (\rho(z))_{ji}}{\frac{2\pi n}{L} + g(a_i - a_j)} \right)^2 e^{2\pi in(y-z)/L},
\]

\[
H_b = \int_0^L dx \left\{ \frac{1}{2} \pi^a \pi^a + \frac{1}{2} (D_1 \phi)^a (D_1 \phi)^a \right\},
\]

\[
H_I = \int_0^L dx \left\{ -i \frac{1}{2} \Psi^a \sigma_3 (D_1 \Psi)^a \right\}, \quad H_{\text{int}} = \int_0^L dx \text{tr} \left\{ ig\phi \bar{\Psi} \gamma_5 \Psi \right\}
\]

where \(a_i = a^p p_i^a\) with no summation over \(i\) implied, and \(\sum_i a_i = 0\). Here the covariant derivative \(D_1\) contains only the zero mode of \(A^1\): \(D_1 = \partial_1 - ig[a, \, \cdot]\). One should note that gauge fields \(A^a\), except the zero modes \(a^p\), are completely eliminated.

In order to investigate the vacuum structures of our model, we solve Schrödinger’s equation with respect to the Hamiltonian \([34]\)

\[
H|\Phi\rangle = E|\Phi\rangle,
\]

where \(|\Phi\rangle\) denote state vectors in the physical space. Because of hermiticity of the variables \(a\), the kinetic energy \(K_a\) is given in terms of the Jacobian \(J[a]\) of the transformation \([34] \) \([2]\)

\[
K_a = \frac{1}{2L} e^{p^a p^p} = -\frac{1}{2L} J[a] \partial_a J[a] \partial_a p^a,
\]

\[
J[a] = \prod_{i>j} \sin^2 \left( \frac{1}{2} gL(a_i - a_j) \right).
\]

In analogy with the radial wavefunctions, it is useful to define a modified wave function \n\Phi[a] = \sqrt{J[a]} \Phi[a].
\]

The kinetic energy operator for \(\Phi\) is (with the notation \(\partial_p = \partial/\partial a^p\)),

\[
K'_a \equiv \sqrt{J} K_a \frac{1}{\sqrt{J}} = -\frac{1}{2L} \partial_p \partial_p + V[^N],
\]

\[
V[^N] \equiv \frac{1}{2L} \sqrt{J} \left( \partial_p \partial_p \sqrt{J} \right) = -(gL)^2 \frac{N(N^2 - 1)}{48L}.
\]

Thus we obtain a boundary condition for the modified wavefunction,

\[
\Phi[a] = 0, \quad \text{if} \quad J[a] = 0.
\]

Let us now quantize the fields \(\Psi\) and \(\phi\). The gauge field zero modes \(a^p\) couple only to off-diagonal elements, which are parameterized as: \(\varphi_{ij} = \sqrt{2} \Psi_{ij}, \varphi_{ij}^\dagger = \sqrt{2} \bar{\Psi}_{ji}, \xi_{ij} = \sqrt{2} \phi_{ij}, \xi_{ij}^\dagger = \sqrt{2} \bar{\phi}_{ji}, \eta_{ij} = \sqrt{2} \pi_{ij}\), and \(\eta_{ij}^\dagger = \sqrt{2} \pi_{ji}\) \((i < j)\). With these conventions the Hamiltonian takes the form

\[
H_I = H_{I,\text{diag}} + H_{I,\text{off}}, \quad H_b = H_{b,\text{diag}} + H_{b,\text{off}},
\]

\[
H_{f,\text{diag}} = \frac{1}{2i} \sum_p \int_0^L dx \Psi^p \sigma_3 \partial_1 \Psi^p.
\]
\[ H_{f,\text{off}} = \sum_{i<j} \int_0^L dx \varphi_{ij}^\dagger \sigma_3 \left( \frac{1}{i} \partial_i - g(a_i - a_j) \right) \varphi_{ij}, \]  
\[ H_{b,\text{diag}} = \sum_p \int_0^L dx \left( \frac{1}{2} \pi^p \pi^p + \frac{1}{2} \left( \partial_1 \phi^p \right) \left( \partial_1 \phi^p \right) \right), \]  
\[ H_{b,\text{off}} = \sum_{i<j} \int_0^L dx \left\{ \eta_{ij}^\dagger \eta_{ij} + \left( \partial_1 \xi_{ij}^\dagger - ig(a_j - a_i) \xi_{ij}^\dagger \right) \left( \partial_1 \xi_{ij} - ig(a_i - a_j) \xi_{ij} \right) \right\}. \]  

Let us now discuss the range of the variables \( a^p \). Eq. (54) shows that the \( gLa \) are angular variables which are defined only in modulo \( 2\pi \). If the parameterization of \( a \) is one-to-one and permutations of the eigenvalues are contained in a single domain, the domain is called the elementary cell. For example, in the SU(2) case, two eigenvalues of the matrix \( a \) are \( a_1 = a^3/2 \) and \( a_2 = -a^3/2 \). Then, the elementary cell is the interval \(-\pi \leq \frac{a^3}{2} \leq \pi \), with the end points identified. If \( a^3 \) is negative in the elementary cell, the Weyl reflection \( a^3 \rightarrow -a^3 \) maps the interval \(-\frac{2\pi}{gL} < a^3 < 0 \) onto the interval \( 0 < a^3 < \frac{2\pi}{gL} \) (simultaneously, \( \varphi^{12} \leftrightarrow \varphi^{21} \)). In the \( SU(N) \) case, similarly, the elementary cell is divided into \( N! \) domains by the Weyl group since the Weyl group of \( SU(N) \) is the permutation group \( P_N \). These \( N! \) domains are called fundamental domains. Boundaries of the fundamental domains consist of the hypersurfaces where two of the eigenvalues match. If two of the eigenvalues have the same value, the Jacobian \( J[a] \) vanishes. In the case of \( SU(2) \), we take the following interval as the fundamental region

\[ 0 \leq a^3 \leq \frac{2\pi}{gL}. \]  

The Jacobian \( J[a] = \sin^2 \left( \frac{1}{2} gLa^3 \right) \) vanishes at \( a^3 = 0 \), \( \frac{2\pi}{gL} \). Note that the modified wavefunction \( \tilde{\Phi}[a] \) vanishes at these points.

6. Vacuum Structures of SUSY SU(2) Yang-Mills Theories

In this section, we determine the wave function of the vacuum state in the fundamental domain by using the Born-Oppenheimer approximation. If \( gL \ll 1 \), the energy scale of the system of \( a^p \) is given by \( (gL)^2/L \), while that of non-zero modes of \( \Psi \) and \( \phi \) is in general of order \( 1/L \). Therefore we can integrate the non-zero modes of \( \Psi \) and \( \phi \) to obtain the effective potential for \( a^p \). We will retain the zero modes of \( \Psi \) and \( \phi \), since their spectrum is continuous. By solving the Schrödinger equation with respect to the resulting effective potential, we obtain the wavefunction \( \tilde{\Phi}[a] \), which describes the vacuum structures of our model. In these procedures we must pay attention to the boundary conditions for \( \tilde{\Phi}[a] \) resulting from the Jacobian.

To calculate the effective potential as a function of the gauge zero modes \( a^p \), we have to find the ground state of fermion \( \Psi \) and boson \( \phi \) for a fixed value of \( a^p \). Here, we must take care with regards to the boundary conditions for \( \Psi(x) \) and \( \phi(x) \). Since spinors and scalars are superpartners of gauge fields which obey the periodic boundary condition, the spinors \( \Psi(x) \) and scalars \( \phi(x) \) should be periodic in order for the boundary conditions to maintain supersymmetry

\[ \Psi(x = 0) = \Psi(x = L), \quad \phi(x = 0) = \phi(x = L). \]
Hereafter we refer to this boundary condition as the \((P,P)\) case. In this section we investigate the vacuum structures for the gauge group \(SU(2)\).

### 6.1. Born-Oppenheimer Approximation

For \(gL \ll 1\), the Coulomb energy \((38)\) and the Yukawa interaction \((40)\) can be neglected. In this limit, the relevant parts of the Hamiltonian are, for \(SU(2)\),

\[
\hat{H} = K'_a + H_{b,\text{diag}} + H_{b,\text{off}} + H_{f,\text{diag}} + H_{f,\text{off}}.
\]

\[
K'_a = -\frac{1}{2L} \frac{\partial^2}{\partial a^2} + V_{[N=2]},
\]

\[
H_{b,\text{diag}} = \frac{1}{2} \int_0^L dx \left\{ \pi^3 \pi^3 + (\partial_1 \phi^3)(\partial_1 \phi^3) \right\},
\]

\[
H_{b,\text{off}} = \int_0^L dx \left\{ \eta^\dagger \eta + (\partial_1 \xi^\dagger + i ga \xi^\dagger)(\partial_1 \xi - i ga \xi) \right\},
\]

\[
H_{f,\text{diag}} = \frac{1}{2i} \int_0^L dx \Psi^3 \sigma_3 \partial_1 \Psi^3, \quad H_{f,\text{off}} = \int_0^L dx \varphi^\dagger \sigma_3 \left( \frac{1}{i} \partial_1 - ga \right) \varphi,
\]

\[
\varphi \equiv \varphi_{12}, \quad \xi \equiv \xi_{12}, \quad \eta \equiv \eta_{12}, \quad a \equiv a^3 = a_1 - a_2.
\]

A remnant of large gauge transformations becomes a discrete symmetry \(S [3]\)

\[
S : \quad a \rightarrow -a + \frac{2\pi}{gL},
\]

\[
\varphi \rightarrow e^{2i\pi x/L} \varphi^\dagger, \quad \xi \rightarrow e^{2i\pi x/L} \xi^\dagger, \quad \eta \rightarrow e^{2i\pi x/L} \eta^\dagger,
\]

\[
\Psi^3 \rightarrow -\Psi^3, \quad \phi^3 \rightarrow -\phi^3, \quad \pi^3 \rightarrow -\pi^3.
\]

This operator can be chosen to satisfy \(S^2 = 1\) and \([S, H] = 0\). \(SYM_2\) has a topologically nontrivial structure \(\pi_1[SU(N)/Z_N] = Z_N\). The symmetry \(S\) corresponds to a nontrivial element of this \(Z_{N=2}\) group for \(SU(2)\).

In order to perform the Born-Oppenheimer approximation, we first expand the spinor fields \(\varphi\) and \(\Psi^3\), and impose a canonical anticommutation relation

\[
\varphi (x) = \frac{1}{\sqrt{L}} \sum_{k=-\infty}^{\infty} \begin{pmatrix} a_k \ b_k \end{pmatrix} e^{i2\pi kx/L}, \quad \{a_k, a_{k'}^\dagger\} = \{b_k, b_{k'}^\dagger\} = \delta_{k,k'},
\]

\[
\Psi^3 (x) = \frac{1}{\sqrt{L}} \sum_{k=-\infty}^{\infty} \begin{pmatrix} c_k \ d_k \end{pmatrix} e^{i2\pi kx/L}, \quad c_{-k} = c_k^\dagger, \quad d_{-k} = d_k^\dagger,
\]

\[
\{c_k, c_{k'}^\dagger\} = \{d_k, d_{k'}^\dagger\} = \delta_{k,k'}, \quad k, k' \geq 0.
\]

The Hamiltonian \(H_{f,\text{off}}\) in \((59)\) takes the form

\[
H_{f,\text{off}} = \sum_{k=-\infty}^{\infty} \left( a_k^\dagger a_k - b_k^\dagger b_k \right) \left( \frac{2\pi k}{L} - ga \right).
\]

In the Born-Oppenheimer approximation, the vacuum state for the off-diagonal part of the fermion is obtained by filling the Dirac sea for the fermion \(\varphi\). We assume the \(a_k\)
modes to be filled for \( k < M \). The Gauss law constraint (53) dictates that the \( b_k \) modes should be filled for \( k \geq M \). Denoting the vacuum state for the fermion as \(|0_\varphi; M\rangle\), the vacuum energy can be written as

\[
H_{\varphi, \text{off}}|0_\varphi; M\rangle = \left[ \sum_{k=-\infty}^{M-1} \left( \frac{2\pi k}{L} - ga \right) - \sum_{k=M}^{\infty} \left( \frac{2\pi k}{L} - ga \right) \right] |0_\varphi; M\rangle \\
\equiv V_{\varphi, \text{off}}(a; M)|0_\varphi; M\rangle.
\]

Notice that \( S \) acts on the state \(|0_\varphi; M\rangle\) according to

\[
S|0_\varphi; M\rangle = e^{i\alpha M}|0_\varphi; 2 - M\rangle. 
\]

In addition, the phase factor \( e^{i\alpha M} \) is constrained by \( S^2 = 1 \), or in other words, \( e^{i\alpha M} = e^{-i\alpha - M+2} \).

For diagonal part of the fermion, we obtain the Hamiltonian from (59)

\[
H_{\varphi, \text{diag}} = \sum_{k \geq 1} \frac{2\pi k}{L} \left( c_k^\dagger c_k + d_k^\dagger d_k - 1 \right). 
\]

On the vacuum \(|0_\psi\rangle\) defined by \( c_k|0_\psi\rangle = d_k^\dagger|0_\psi\rangle = 0, \ k \geq 1 \), we find

\[
H_{\varphi, \text{diag}}|0_\psi\rangle = -\sum_{k \geq 1} \frac{2\pi k}{L} |0_\psi\rangle \equiv V_{\varphi, \text{diag}}|0_\psi\rangle. 
\]

Next we expand the scalar fields \( \xi, \eta, \phi^3 \), and \( \pi^3 \), and impose canonical commutation relations

\[
\xi(x) = \sum_{k=-\infty}^{\infty} \frac{1}{\sqrt{2L}E_k} (e_k + f_k^\dagger) e^{i2\pi kx/L}, \quad E_k = \left| \frac{2\pi k}{L} - ga \right|, \quad k \text{ even} \\
\eta(x) = \sum_{k=-\infty}^{\infty} i \sqrt{\frac{E_k}{2L}} (-e_k + f_k^\dagger) e^{i2\pi kx/L}, \\
\phi^3(x) = \sum_{k=-\infty}^{\infty} \frac{1}{\sqrt{2L}E_k} (g_k + g_k^\dagger) e^{i2\pi kx/L} + \phi_{\text{zero}}, \quad F_k = \left| \frac{2\pi k}{L} \right|, \quad k \text{ odd} \\
\pi^3(x) = \sum_{k=-\infty}^{\infty} i \sqrt{\frac{F_k}{2L}} (-g_k + g_k^\dagger) e^{i2\pi kx/L} + \frac{1}{L} \pi_{\phi_{\text{zero}}}. 
\]

\[
[e_k, e_{k'}^\dagger] = [f_k, f_{k'}^\dagger] = [g_k, g_{k'}^\dagger] = \delta_{k,k'}, \quad [\phi_{\text{zero}}, \pi_{\phi_{\text{zero}}}] = i. 
\]

The Hamiltonian \( H_{\varphi, \text{off}} \) in (58) is given by

\[
H_{\varphi, \text{off}} = \sum_{k=-\infty}^{\infty} E_k \left( e_k^\dagger e_k + f_k f_k^\dagger \right) \\
= \sum_{k=-\infty}^{\infty} E_k \left( e_k^\dagger e_k + f_k f_k^\dagger \right) - \sum_{k=-\infty}^{N-1} \left( \frac{2\pi k}{L} - ga \right) + \sum_{k=N}^{\infty} \left( \frac{2\pi k}{L} - ga \right),
\]
where $N$ is an integer satisfying
\[
\frac{2\pi N}{L} - ga \geq 0, \quad \frac{2\pi(N - 1)}{L} - ga < 0. \tag{75}
\]
On the vacuum state $|0_\xi\rangle$ defined by $e_k|0_\xi\rangle = f_k|0_\xi\rangle = 0$, for all $k$, we find the vacuum energy
\[
H_{b,\text{off}}|0_\xi\rangle = \left[- \sum_{k=-\infty}^{N-1} \left(\frac{2\pi k}{L} - ga\right) + \sum_{k=N}^{\infty} \left(\frac{2\pi k}{L} - ga\right)\right]|0_\xi\rangle \\
\equiv V_{b,\text{off}}(a)|0_\xi\rangle. \tag{76}
\]
We find that the zero mode Hamiltonian $H_0$ is separated as
\[
H_{b,\text{diag}} = \sum_{k \geq 1} \frac{2\pi k}{L} \left(g_k^+ g_k + g_{-k}^+ g_{-k} + 1\right) + H_0, \tag{77}
\]
\[
H_0 = \frac{1}{2L} \pi_{\phi_{\text{zero}}} \pi_{\phi_{\text{zero}}}. \tag{78}
\]
On the vacuum for the nonzero modes of $\phi^3$ satisfying $g_k|0_\phi\rangle = g_{-k}|0_\phi\rangle = 0, k \geq 1$, we find the vacuum energy
\[
V_{b,\text{diag}} = \sum_{k \geq 1} \frac{2\pi k}{L}. \tag{79}
\]

6.2. Vacuum Structure

The vacuum energies obtained in the previous section are divergent. By regularizing them with the heat kernel, we obtain the following finite effective potential as a function of $a$
\[
U_{M,N}(a) = V_{f,\text{off}}(a; M) + V_{b,\text{off}}(a) + V_{f,\text{diag}} + V_{b,\text{diag}} + V^{[N=2]} \\
= \frac{2\pi}{L} \left(M - \frac{gL_0}{2\pi} - \frac{1}{2}\right)^2 - \frac{2\pi}{L} \left(N - \frac{gL_0}{2\pi} - \frac{1}{2}\right)^2 + V^{[N=2]}. \tag{80}
\]
In the fundamental region $0 < \frac{gL_0}{2\pi} < \pi, N = 1$ from (75). By requiring that the vacuum energy $U_{M,N}(a)$ be minimal, we can fix $M$ to obtain $M = 1$. We then find that the total vacuum energy in the fundamental domain is independent of $a$
\[
U_{M,N}(a) = V^{[N=2]}. \tag{81}
\]
Consequently we obtain the Hamiltonian which describes the vacuum structures for the periodic boundary condition
\[
\check{H} = K_{a'} + H_0 = -\frac{1}{2L} \frac{\partial}{\partial a} \frac{\partial}{\partial a} + V^{[N=2]} + \frac{1}{2L} \pi_{\phi_{\text{zero}}} \pi_{\phi_{\text{zero}}}. \tag{82}
\]
We also have the zero modes of the fermion, which form a Clifford algebra

\[ \Psi^3_{\text{zero}} = \frac{1}{\sqrt{L}} (c_0, d_0), \quad c_0 = c_0^\dagger, \quad d_0 = d_0^\dagger, \quad (83) \]

\[ \{ \lambda, \lambda^\dagger \} = 1, \quad \{ \lambda, \lambda \} = \{ \lambda^\dagger, \lambda^\dagger \} = 0, \quad \lambda \equiv \frac{1}{\sqrt{2}}(c_0 + id_0). \quad (84) \]

Let us now solve the Schrödinger equation

\[ \tilde{H}\tilde{\Phi}(a) = e\tilde{\Phi}(a). \quad (85) \]

Because of the boundary condition (47) we get the wavefunction \( \tilde{\Phi}(a) \) and the energy eigenvalue \( e \) of the ground state as

\[ \tilde{\Phi}(a) = \sqrt{gL\pi} \sin \left( \frac{gLa}{2} \right), \quad e = 0. \quad (86) \]

It is interesting to note that the vacuum energy associated with the nontrivial zero mode wavefunction (86) cancels precisely the contribution \( V^{[N=2]} \) from the Jacobian in (46). Therefore we have shown explicitly that the SUSY is not broken spontaneously. Also note that our result is consistent with the previous calculation of the nonvanishing Witten index \( [7] \). The calculation, however, ignores the Jacobian (43), which is an important ingredient in our present attempt to define the gauge field zero modes properly \( [13] \). Therefore the above explicit demonstration of the vanishing vacuum energy using the Born-Oppenheimer approximation can be regarded as another independent proof of the unbroken SUSY in SUSY Yang-Mills theories in 1 + 1 dimensions.

We define the vacuum state \( \ket{\Omega} \) to be the Clifford vacuum annihilated by \( \lambda \) and \( \ket{\tilde{\Omega}} = \lambda^\dagger \ket{\Omega} \). Note that the zero modes belong to the two-dimensional representation of the Clifford algebra (84). The field \( \phi^3 \) can take unbounded values, the zero mode spectrum is continuous. This fact makes the Witten index ill-defined. The previous attempt to compute the Witten index employed a regularization by putting a cut-off on the \( \phi^3 \) zero mode space. In that case, the Witten index can be defined and obtains \( \text{tr}(-1)^F = 1 \) \( [7] \). In spite of this complication, we can choose the wave function to be constant in the \( \phi^3 \) zero mode as the vacuum: \( H_0 \ket{\omega} = 0 \).

Let us now examine the transformation property under the discrete gauge transformation \( S \). The non-zero mode vacuum \( \ket{0; \xi; M = 1} \) turns out to be an eigenstate of \( S \)

\[ S\ket{0; \xi; M = 1} = \pm \ket{0; \xi; M = 1} \quad (87) \]

because of eq.(55) and \( S^2 = 1 \). Similarly \( \ket{0; \phi}, \ket{0; \xi}, \ket{0; \phi} \) and \( \ket{\omega} \) are eigenstate of \( S \) with eigenvalues \( \pm 1 \). For the fermion zero mode, \( S\ket{\Omega} = \pm \ket{\Omega} \) and \( S\ket{\tilde{\Omega}} = \mp \ket{\tilde{\Omega}} \). Since we should construct the full vacuum state as an eigenstate with eigenvalue \( \pm 1 \) for \( S \)

\[ \ket{0; \Omega} \equiv \ket{\tilde{\Phi}(a)}\ket{0; \phi; M = 1}\ket{0; \phi}\ket{0; \xi}\ket{0; \omega}\ket{\Omega}, \]

\[ \ket{0; \tilde{\Omega}} \equiv \ket{\tilde{\Phi}(a)}\ket{0; \phi; M = 1}\ket{0; \phi}\ket{0; \xi}\ket{0; \omega}\ket{\tilde{\Omega}}. \quad (88) \]

We find the vacuum condensate \( \left| \langle 0|\bar{\Psi}^a\Psi^a|0 \rangle \right| = \frac{1}{L} \) for both \( \ket{0} = \ket{0; \Omega} \) and \( \ket{0; \tilde{\Omega}} \). One can see that this condensate is due to the finite spacial extent \( L \).
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