Determination of the wind power systems load to achieve operation in the maximum energy area

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Abstract. This paper analyses the operation of the wind turbine, WT, in the maximum power point, MPP, by linking the load of the Permanent Magnet Synchronous Generator, PMSG, with the wind speed value. The load control methods at wind power systems aiming an optimum performance in terms of energy are based on the fact that the energy captured by the wind turbine significantly depends on the mechanical angular speed of the wind turbine. The presented control method consists in determining the optimal mechanical angular speed, $\omega_{\text{OPTIM}}$, using an auxiliary low power wind turbine, WT$_{\text{AUX}}$, operating without load, at maximum angular velocity, $\omega_{\text{MAX}}$. The method relies on the fact that the ratio $\omega_{\text{OPTIM}}/\omega_{\text{MAX}}$ has a constant value for a given wind turbine and does not depend on the time variation of the wind speed values.

1. Introduction

In the current phase of harnessing wind energy, an optimum functioning is a fundamental requirement, considering the fact that the investments (in all stages: development, approval procedures, wind turbine acquisition, building & exploitation) are huge and optimum operating conditions under given wind characteristics, have to be ensured for maximizing the yield of wind power plants [1].

In the scientific literature the operation problem of the wind turbine (WT) in the maximum energy area is extensively treated, [2] but simplified, at constant wind speeds over time, using different mathematical models (MM - WT) given by the wind turbine producers and deduced under laboratory conditions, far different from those in operation. For that reason, the obtained electric energy has less value than the maximum possible provided in the Maximum Power Point (MPP) operation, obtainable at optimum angular mechanical velocity ($\omega_{\text{OPTIM}}$). At time-varying wind speeds, the operation in MPP becomes a complex problem and not always timely solvable, due to large mechanical inertia and rapid changes in wind speed over time [3].

To reach the MPP area, currently the main used calculation procedure is the so-called small perturbation method; the obtained solutions introduce power swing in the generator and do not provide an operation in MPP. The main issue is to adapt and develop control algorithms and strategies that are based on the current wind speed value and the prescription of the optimum speed (RPM) [4].

The dependence of the WT power to mechanical angular velocity, ie the $P_{\text{WT}}(\omega)$ function, at a certain constant wind speed, has a maximum at $\omega_{\text{OPTIM}}$. The optimum energy value is reached for $\omega_{\text{OPTIM}}$, value at which through the wind turbine the captured power is a maxim, the MPP from the
equivalent power characteristic. The important points from the power characteristic are the maximum power point (MPP) with \( \omega_{\text{OPTIM}} \) and the zero power point with maximum angular velocity, \( \omega_{\text{MAX}} \). The correct determination of these points, in operating conditions, provides the operation in the MPP zone.

The maximum power point (MPP) coordinate, \( \omega_{\text{OPTIM}} \) and \( P_{\text{WT-MAX}} \), directly depend on the wind velocity variation in time. The correct determination of those depends on the correct wind speed information and the knowledge of their influence over the value of the MPP coordinates [5].

Therefore, achieving optimum performance for the wind energy system in terms of energy, two fundamental issues are distinguished: the determination of the MPP coordinates based on the estimated wind speed values.

In the present paper, the knowing of the wind speed time variation is done in two ways: through direct measurement or by measuring the mechanical velocity from the auxiliary wind turbine, WT\(_{\text{AUX}}\), operating without a load. The determination of the MPP coordinates are based on the \( \omega_{\text{OPTIM}} / \omega_{\text{MAX}} \) ratio has a constant value for a given wind turbine and does not depend on the wind speed variations.

2. Mathematical models

The simulations presented in the paper, are based on the classic mathematical models of the wind turbines and the Permanent Magnet Synchronous Generator [6], based on them, the optimum angular velocity, \( \omega_{\text{OPTIM}} \), is deduced.

2.1. Mathematical model of the wind turbine (MM-WT)

We envisage a classic model for the turbine model [7], which allows determination of the optimum angular velocity so that the captured energy is maximized.

The wind turbine power is computed based on the equation:

\[
P_{\text{WT}} = \rho \pi R_b^2 C_p(\lambda) \omega^3
\]

where: \( \rho \) - air density, \( \omega \) – wind speed, \( C_p(\lambda) \) – power conversion coefficient, \( R_b \) - rotor blade radius [5].

The following relation determines \( C_p(\lambda) \), the power conversion coefficient:

\[
C_p(\lambda) = c_1 \left( \frac{c_2}{\lambda} - c_3 \right) e^{-\frac{c_4}{\lambda}}
\]

where: \( \lambda = \omega R_b / v \), \( c_1 \div c_4 \) – catalog constants and \( \omega \) - angular mechanical velocity.

\[
\frac{1}{\lambda} = \frac{1}{\lambda} - 0.035 = \left( \frac{v}{1.5 \omega} - 0.035 \right)
\]

Replacing equation (3) in (2), \( C_p(\lambda) \), the power conversion coefficient [5] results as:

\[
C_p(\lambda) = c_1 \left( c_2 \left( \frac{v}{1.5 \omega} - 0.035 \right) - c_3 \right) e^{-k_1 \left( \frac{v}{1.5 \omega} - 0.035 \right)}
\]

The wind turbine power is computed through (1) or

\[
P_{\text{WT}} = k_1 \left( k_2 \left( \frac{v}{\omega} - 0.035 R_p \right) - c_3 \right) e^{-k_4 \left( \frac{v}{\omega} - 0.035 R_p \right)} v^3
\]

where: \( k_1 = 1.225 R_p^2 \cdot c_1 \), \( k_2 = c_2 / 1.5 \), \( k_3 = c_3 / 1.5 \) [7].

The producers of wind turbines, WT, with \( R_p = 1.5 \text{m} \), give the experimental power characteristics \( P_{\text{WT}}(\omega, v) \) or torque \( \tau_{\text{WT}}(\omega, v) \), called the experimental mechanical characteristics:

\[
\tau_{\text{WT}}(\omega) = \frac{P_{\text{WT}}(\omega, v)}{\omega}
\]

For the reference mechanical angular speed, \( \omega_{\text{ref}} \), we obtain, through derivation, the maximum of the \( P_{\text{WT}} \) function:
\[
\frac{dP_{WT}(\omega, \nu)}{d\omega} = 0
\]  
Solving the equation (7), we obtain
\[
\omega_{\text{OPTIM}} = k_0' \nu.
\]  
(8)

The achieved result demonstrates the direct link between \( \omega_{\text{OPTIM}} \) and wind speed. Substituting the obtained results in the \( P_{WT}(\omega, \nu) \) function, we obtain:
\[
P_{WT}(\omega, \nu) = kP \cdot \nu^3.
\]  
(9)

At an operation without load, correspond to the maximum angular mechanical velocity, obtained from
\[
P_{WT}(\omega, \nu) = 0,
\]  
(10)

provides
\[
\omega_{\text{MAX}} = k_{\text{WL}} \cdot \nu,
\]  
(11)

where \( k_{\text{WL}} \) – represents the constant without load.

The ratio \( \omega_{\text{OPTIM}} / \omega_{\text{MAX}} \) has the value
\[
\frac{\omega_{\text{OPTIM}}}{\omega_{\text{MAX}}} = \frac{k_0' \nu}{k_{\text{WL}}} \quad \text{or}
\]  
\[
\frac{\omega_{\text{OPTIM}}}{\omega_{\text{MAX}}} = \frac{k_{\text{M}}}{k_{\text{WL}}},
\]  
(12)

where \( k_{\text{M}} \) – is the constructive constant of the turbine.

The maximum power of the wind turbine, at a certain wind speed \( \nu \), can be determined based on mechanical angular speed without load, \( \omega_{\text{MAX}} \); since replacing equation (11) in (9), results:
\[
P_{\text{WT-MAX}} = k_1 \cdot \nu^3 \cdot \omega_{\text{MAX}},
\]  
(14)

where \( k_{\nu,1} \) – represent constructive turbine constant.

The value of the maximum mechanical angular speed, \( \omega_{\text{MAX}} \), is given from the operation, without load, of an auxiliary, low power, turbine, \( \text{WT}_{\text{AUX}} \).

In cases where the wind speed varies significantly over time, the obtained result should be reconsidered and it imposes to define an equivalent wind speed [3].

2.2. A mathematic model of the permanent magnet synchronous generator (MM-PMSG)

To analyze at time-varying wind speeds the operation of the system (WT + PMSG), the orthogonal mathematical model for the permanent magnet synchronous generator is used, given through [8]:

\[
\begin{cases}
- U \sqrt{3} \sin \theta = R_l I_d - \omega L_q I_q \\
U \sqrt{3} \sin \theta = R_l I_q + \omega L_d I_d + \omega \Psi_{PM} \\
\tau_G = p_1 (L_d - L_q) I_d I_q + I_q \Psi_{PM} \\
P_G = R(I_d^2 + I_q^2)
\end{cases}
\]  
(15)

where: \( I_d, I_q \) – stator currents, \( U \) – stator voltage, \( \theta \) - load angle, \( \tau_G \) – electromagnetic torque, \( R_l \) – stator winding resistance, \( \Psi_{PM} \) – flux of the permanent magnet, \( L_d \) – own inductance of the stator winding on d axis, \( L_q \) – own inductance of the stator winding on q axis, \( p_1 \) – number of pole pairs. The two functions \( P_G(R, \omega) \) the electric power debited by the generator and the electromagnetic torque, \( \tau_G(R, \omega) \), depends on \( R \) – the load resistance and \( \omega \) - mechanical angular velocity. At constant \( R \), the torque [9] depends on linearly of \( \omega \) and the useful power depends on the quadratic \( \omega \).

From catalog data for a PN = 10 [kW] permanent magnet synchronous generators, following values for the parameters are obtained: \( R_l = 1.6 \Omega, L_d = 0.07 \text{H}, L_q = 0.08 \text{H}, \Psi_{PM} = 1.3 \text{Wb} \), we obtained the torque, \( \tau_G \),
\[
\tau_G = -845 \omega (5R + 8) \left( \frac{4\omega^2 + 625R^2 + 2000R + 1600}{(1250R^2 + 4000R + 3200 + 7\omega^2)^2} \right)
\]  
(16)
and the generator power, $P_G$,

$$P_G = 4225 R \omega^2 \left(\frac{4\omega^2 + 625 R^2 + 2000 R + 1600}{(1250 R^2 + 4000 R + 3200 + 7\omega^2)}\right)^2$$  \hspace{1cm} (17)

The transient analysis of the specific phenomena at variable wind speed is done by simulation, based on the movement equation:

$$J \frac{d\omega}{dt} = \tau_{WT} - \tau_{MPSG}$$  \hspace{1cm} (18)

where: $J$ – equivalent inertia moment, $\tau_{WT}$ – wind turbine torque and $\tau_{MPSG}$ – permanent magnet synchronous generator torque.

3. Control algorithm verification at different wind conditions

3.1. Analysis at constant wind speed

At a wind speed value of $v=11$[m/s], the power characteristics have the form as

$$P_{WT}(\omega, v) = 1191.5 \cdot (v/\omega - 0.02) e^{-98.02(v/\omega)} \cdot v^3$$  \hspace{1cm} (19)

$P_{WT}(\omega, 11)$, and by cancelling the derivative power,

$$\frac{dP_{WT}(\omega, v)}{d\omega} = 0$$  \hspace{1cm} (20)

we obtain $\omega_{OPTIM}=364.26$[rad/s], the maximum power being $P_{WT}(\omega_{OPTIM}, 11)=837.04$[rad/s].

Considering that initially, the wind power system operates in point $P_1$, Figure 1, at a mechanical angular speed of $\omega=469$[rad/s], the power captured by the wind turbine results, from (19) as $P_{WT}=549.25$[W].

Equaling the power from the wind turbine and the MPSG, $P_{WT}=P_{MPSG}$, at $\omega=469$[rad/s], results

$$549 = 845\omega^2 (5R+8) \cdot \frac{4\omega^2 + 625 R^2 + 2000 R + 1600}{(1250 R^2 + 4000 R + 3200 + 7\omega^2)^2}$$  \hspace{1cm} (21)

and the resistance value from the point $P_1$ becomes $R=673.64\Omega$.

![Figure 1. Power characteristic of the wind turbine, at $v=11$[m/s]](image)

Remark 1. At a stable operation in point $P_1$, the power and equivalent load resistance of the PMSG is determined from measurements. To reach the maximum power point (MPP), in a imposed time interval, e.g $\Delta t = t_k - t_{k-1} = 2000$[s], the PMSG has to take over, both, the captured wind energy by the wind turbine as well as the kinetic energy, $\Delta W_{KINETIC}$, of the rotating masses:

$$\Delta W_{KINETIC} = J \left(\frac{\omega_k^2 - \omega_{k-1}^2}{2}\right) = \frac{364.26^2 - 469^2}{2}$$  \hspace{1cm} (22)
where \( J \) – equivalent inertia moment, \( \omega_{k+1} \) – mechanical angular speed at \( t_{k+1} \) and \( \omega_k \) – mechanical angular speed at \( t_k \).

The overtaken wind energy by the wind turbine in the \( \Delta t \) time interval is computed by integrating the power, under the form of:

\[
W_{WT} = \int_{t_k}^{t_{k+1}} P_{WT} dt
\]

and depends on the current angular mechanical velocity, \( \omega \), that changes in the time interval \( \Delta t \) in the field of \( \omega_{\text{OPTIM}} + \omega(P_1) = 364.26 \text{[rad/s]} \pm 469\text{[rad/s]} \).

For this reason, the power integration is not able to be computed and the captured wind energy is done approximating a linear power variation between the points \( P_1 \) and MPP, Figure 1.

Considering an average value for the wind turbine power,

\[
P_{WT} = \frac{P_{WT-P1} + P_{WT-MPP}}{2} = \frac{594.25 + 837.04}{2} = 693.15W
\]

we obtain the value of the captured wind energy through the wind turbine, in the given time interval \( \Delta t \), as:

\[
W_{WT} = \int_{t_k}^{t_{k+1}} P_{WT} dt = 693.15 \cdot 2000 = 1.386 \cdot 10^6 J
\]

By summing the two values, we obtain the necessary energy, \( W_{G-\text{necessary}} \), what the generator should produce in the time interval \( \Delta t \):

\[
W_{G-\text{necessary}} = W_{WT} + \Delta W_{\text{KINETIC}} = (1.3863+1.937) \cdot 10^6.
\]

Reaching the optimum value of the mechanical angular speed is achieved in condition in that the power of the generator has the value \( P_{G-\text{necessary}} = 3.35 \cdot 10^6/2000 = 1675[W] \), at an average value of the mechanical velocity of \( \omega_{\text{average}} = (\omega_{\text{OPTIM}} + \omega(P))/2 = (364.26+469)/2 = 416.63\text{[rad/s]} \), resulting the value of the system load resistance, \( R \), by replacing \( \omega_{\text{average}} \) and PG-necessary in the PPMSG expression, as \( R=168.64[\Omega] \).

**Verification by solving the movement equation.** At \( R=168.64[\Omega] \) and \( \omega(0)=469 \), the movement equation becomes:

\[
45 \frac{d\omega}{dt} = 1191.5(11/\omega-0.02)e^{0.02(11/\omega)11^3} - 845\omega^2(5R+8) - \frac{4\omega^2 + 625R^2 + 2000R + 1600}{\left[1250R^2 + 4000R + 3200 + 7\omega^2\right]},
\]

and the resulted value of the mechanical velocity at the end of the interval, becomes \( \omega(2000)=372.95\text{[rad/s]} \), compared to the optimum, \( \omega_{\text{OPTIM}}=364.26\text{[rad/s]} \), the recorded difference being worth 2.38%, therefore acceptable.

### 3.2 Analysis at variable wind speed

At a time variation of the wind speed [10] as in Figure 2 [5], the optimal values have been obtained, \( P_{WT-MAX} = 2690[W] \) at \( \omega_{\text{OPTIM}} = 579.7\text{[rad/s]} \).

It’s considered that the wind power system operates initially in the point \( P_2 \), Figure 3, at the angular mechanical velocity of \( \omega=669\text{[rad/s]} \). At this value of the angular mechanical velocity, the power of the wind turbine is given from the captured wind energy and has the value \( P_{WT} = 62849/35 = 1795.7[W] \).

Equaling the power from the MPSG and the wind turbine, \( P_{MPSG} = P_{WT} \), at \( \omega=669\text{[rad/s]} \), results in the value of the load resistance in point \( P_2 \): \( R=414.45[\Omega] \).

**Remark 2.** At a stable operation in \( P_2 \), the equivalent power and load resistance from the PMSG is determined from measurements. To reach the maximum power point (MPP), in an imposed time interval, e.g \( \Delta t = t_k - t_{k-1} = 2000[s] \), the PMSG has to take over, both, the kinetic energy of the rotating
masses (substituting the values in equation (22)), \( \Delta W_{\text{KINETIC}} = 2509 \times 10^6 \text{[J]} \) as well as the captured wind energy by the wind turbine.

\[
\Delta W_{\text{KINETIC}} = 2509 \times 10^6 \text{[J]}
\]

The wind energy taken over by the wind turbine in this given time interval is computed based on the power integration and depends on the current mechanical angular speed, \( \omega \), that changes in \( \Delta t \) between \( \omega_{\text{OPTIM}} = 579.7 \text{[rad/s]} \) and \( 669 \text{[rad/s]} \).

For this reason, the power integration is not possible to be computed, the captured wind energy is calculated, only approximate, in two ways:

- Considering a linear variation of \( \omega \) between the points \( P_2 \) and MPP: \( \omega = \omega(P_2) - \left(\frac{\omega(P_2) - \omega_{\text{OPTIM}}}{2000}\right) t \), we obtain \( W_{\text{WT}} = 3.9112 \times 10^6 \text{[J]} \).

- Considering a linear variation of the power between the points \( P_2 \) and MPP, Fig. 3, and considering an average value for the power, \( P_{\text{WT-\text{AVERAGE}}} = \frac{P_{\text{WT}} - P_{\text{MPP}} + P_{\text{WT}} - P_2}{2} = \frac{1795.7 + 2060.9}{2} = 1928.3 \text{[W]} \).

Summing the two energy values, \( W_{\text{WT}} + \Delta W_{\text{KINETIC}} \), we get the necessary energy that the generator has to debit in the \( \Delta t \) time interval: \( W_{\text{G-necessary}} = (3.356 + 2.509) \times 10^6 = 6.36 \times 10^6 \text{[J]} \).

The optimum value of the mechanical angular speed is achieved when the generator power has the value \( P_{\text{G-necessary}} = 6.36 \times 10^6 / 2000 = 3182.8 \text{[W]} \), at an average of the angular mechanical velocity of \( \omega_{\text{average}} = \left( \omega_{\text{OPTIM}} + \omega_{P_2} \right) / 2 = (579.7 + 669) / 2 = 624.35 \text{[rad/s]} \). The system load resistance, \( R \), results through replacing \( \omega_{\text{average}} \) and \( P_{\text{G-necessary}} \) in the \( P_{\text{PMSG}} \) expression, as \( R = 195.9 \text{[\Omega]} \).

Achieving the optimal mechanical angular velocity, based on the link between \( \omega_{\text{OPTIM}} \) and \( \omega_{\text{MAX}} \), is done through measurements of the generators energy and mechanical velocity, in steps, in \( \Delta t = t_k - t_{k-1} \) time intervals, based on the following control algorithm, synthetized in Figure 4 and reflected in Figure 5.
4. Conclusions

Through measuring the mechanical angular speed $\omega_{\text{MAX}}$, from the operation without the load of the auxiliary turbine, WT$_{\text{AUX}}$, we obtain the maximum power point coordinates: $\omega_{\text{OPTIM}}$, using the value of the ratio $\omega_{\text{OPTIM}}/\omega_{\text{MAX}}=0.6623$ and the maximum power, $P_{\text{WT-MAX}}$, using the expression $P_{\text{WT-MAX}}=0.6288(\omega_{\text{MAX}}/50)^3$.

By analysing several cases we were able to determine the sizes that lead to an optimum performance in terms of energy and maintain the wind energy system in the MPP area, by measuring the angular mechanical velocity and the generator’s energy. The correct determination of the maximum energy area is essential; the captured energy through the wind turbine has maximum values.
in the MPP area. The power of WT, for different values of the mechanical velocity, is estimated by computing the variations of the kinetic energies of the masses in the rotation and counting the debited electricity. By the presented simulation, we were able to identify the PMSG loads so the system reaches the optimum in terms of energy in the shortest time.

The proposed method is based on the determination of $\omega_{\text{OPTIM}}$ using an auxiliary operating WT, operating without load by the fact that $\omega_{\text{OPTIM}}$ is directly proportional with $\omega_{\text{MAX}}$. This avoids the use of mathematical models in the control process, models that are valid only in certain circumstances. By measuring $\omega_{\text{MAX}}$ using an auxiliary WT, $\omega_{\text{OPTIM}}$ can be determined at any time, regardless the time variation of the wind speed. Calculating the variations of the kinetic energy and measuring the electricity debited by the generator, the value of the power is prescribed, a value which is achieved by the command of the power converter element, interposed between generator and network.

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