Breaking rotational symmetry in two-flavor color superconductors

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The color superconductivity under flavor asymmetric conditions relevant to the compact star phenomenology is studied within the Nambu-Jona-Lasinio model. We focus on the effect of the deformation of the Fermi surfaces on the pairing properties and the energy budget of the superconducting state. We find that at finite flavor asymmetries the color superconducting BCS state is unstable towards spontaneous quadrupole deformation of the Fermi surfaces of the $d$ and $u$ quarks into ellipsoidal form. The ground state of the phase with deformed Fermi surfaces corresponds to a superposition of prolate and oblate deformed Fermi ellipsoids of $d$ and $u$ quarks.

I. INTRODUCTION

A possible outcome of the de-confinement phase transition in hadronic matter, when it is compressed in the centers of compact stars to densities several times the nuclear saturation density, is the formation of a two-flavor ($u$ and $d$) quark matter coexisting with leptons under β stability and global charge neutrality. It is believed that the strong interaction among quarks in the attractive channel(s) pairs the $u$ and $d$ quarks of two color (leaving one color unpaired) to form a Bardeen-Cooper-Schrieffer (BCS) type color superconductor (the so called 2SC phase). In the asymptotic regime of high-densities rigorous perturbative results can be obtained in the weak coupling regime; at the intermediate densities, relevant for compact stars, one has to rely upon effective-field theory models of QCD, such as the Nambu-Jona-Lasinio (NJL) model.

In compact stars the color superconductivity is likely to occur at finite isospin asymmetry, where the $u$ and $d$ quarks fill two distinct Fermi spheres. The separation of their Fermi energies is of the order of the electron chemical potential as required by the charge neutrality and the stability against weak processes $d \rightarrow u + e + \bar{\nu}$ and $u + e \rightarrow d + \nu$, where $e$, $\nu$ and $\bar{\nu}$ refer to electron, electron neutrino and anti-neutrino. Large separations of the Fermi energies (of the order of pairing gap) and corresponding Fermi momenta implies an incoherence of the phase space for the $u$ and $d$ quarks, which will eventually destroy the 2SC superconductivity.

The loss of the condensation energy due to the separation of the Fermi surfaces of the $u$ and $d$ quarks, however, might favor a non-Fermi liquid occupation of the fermionic states, where the condensation energy is maintained at the cost of extra kinetic energy. One possibility is the deformation of the Fermi spheres (DFS), to the lowest order, in the ellipsoidal form (hereafter -DFS phase). While deviations from the spherically symmetric form of the Fermi surfaces cost extra kinetic energy, the gain in the condensation energy due to an increase in the phase-space overlap between the states that pair can stabilize the system. Note that we assume that the pairing interaction is isotropic in space; such a deformation spontaneously breaks the rotational $O(3)$ symmetry of the original Lagrangian down to $O(2)$. Both LOFF and DFS superconducting phases break the global space symmetries. Another possibility that maintains these symmetries is the rearrangement of the Fermi surface of the $d$-quarks [interior gap pairing (IGP)]. The IGP assumes $d$-quark holes located in a strip around the Fermi surface of the $u$-quarks are lifted to the excited states above the Fermi surface of the $d$ quarks and the pairing occurs at the shores of the $u$-quark Fermi sea.

A robust feature of the 2SC superconductors is the appearance of the crystalline color superconducting (CCS) state (the analog of LOFF phase) in a wide range of chemical potential asymmetries. In the CCS state the Cooper-pairs carry finite center-of-mass momentum, their order parameter varies periodically in space and, hence, the CCS spontaneously breaks the global space symmetries of the original BCS state. One of the consequences of the broken space symmetries is the existence of new massless Goldstone modes (phonons) in the CCS phase: another consequence is that, the glitches in compact stars (rotational anomalies seen in the timing data from pulsars) could originate in the CCS phase, if the compact stars feature such a phase.

This paper studies numerically the color superconducting DFS phase; it extends the previous non-relativistic analysis to relativistic systems and interactions specific to the color superconductors. The main result of this analysis is that the spatially homogenous 2SC supercon-
ductor is unstable towards formation of the DFS phase with quadrupole deformed Fermi surfaces. Clearly, this observation does not identify the true ground state of a 2SC superconductor. Studies of the interplay between different non-BCS phases sketched above are needed to answer this question. For applications to compact stars these phases need to be studied under β equilibrium conditions (for example see [14, 22, 21]). 

The paper is organized as follows. In Sec. 2, starting from the NJL Lagrangian in the chirally symmetric phase, we derive the thermodynamic potential of the flavor asymmetric 2SC phase at finite temperatures. Self-consistent equations for the gap and the partial densities of the u and d quarks are obtained. The numerical solutions of these equation and the resulting phase diagram of the color superconducting DFS phase are shown in Sec. 3. Sec. 4 contains our conclusions.

II. THERMODYNAMICS OF COLOR SUPERCONDUCTING DFS PHASE

Our treatment is based upon the Nambu-Jona-Lasinio (NJL) with two flavors \( N_f = 2 \) and three colors \( N_c = 3 \) [22]. We shall assume from the outset that the color SU(3) \(_c\) matrix acting in the SU(3) \(_c\) flavor space, \( \lambda \) and \( i, j \) are the second component of the Pauli matrix acting in the SU(2) \(_f\) flavor space, \( \lambda_A \) is the antisymmetric Gell-Mann matrix acting in the SU(3) \(_c\) color space. The coupling constant \( G_1 \) stands for the four-fermion contact interaction. For the sake of simplicity we neglect the effects of the mass gap in the quark-anti-quark channel below the critical line for the chiral phase transition on the Cooper pairing. The generalization of the Lagrangian \([1]\), which leads to coupled gap equations in the di-quark and Cooper channels, is straightforward. The common Ansatz for the order parameter in the 2SC phase is

\[
\Delta \propto \langle \psi^T(x)C\gamma_5\tau_2\lambda_2\psi(x) \rangle, \tag{2}
\]

where \( \lambda_2 \) is the second component of the Gell-Mann matrix. The Ansatz for the order parameter [Eq. (2)] implies that the color SU(3) \(_c\) symmetry is reduced to SU(2)\(_c\) since only two of the quark colors are involved in the pairing while the third color remains unpaired. More complicated Ansätze would allow for a spin 1 pairing of the quarks of the remaining color [23, 24].

As is well known, the gap equation and the partial densities of the up and down quarks can be found from the fixed points of the thermodynamic potential density \( \Omega \):

\[
\frac{\partial \Omega}{\partial \Delta} = 0, \quad \frac{\partial \Omega}{\partial \rho_f} = \rho_f; \tag{3}
\]

the flavor index \( f = u, d \) refers to up (\( u \)) and down (\( d \)) quarks. In the superconducting state the self-consistent solution of equations \( \Omega \) corresponds to a minimum of the thermodynamic potential of the system. For the Lagrangian density defined by Eq. (1) and the pairing channel Ansatz Eq. (2), the finite temperature thermodynamical potential \( \Omega \) per unit volume is

\[
\Omega(\beta \mu) = -\frac{1}{\beta} \sum_{\omega_n} \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2} Tr \ln \left[ \beta \left( S_{11}^{-1}(i\omega_n, \vec{p}) S_{21}^{-1}(i\omega_n, \vec{p}) S_{12}^{-1}(i\omega_n, \vec{p}) S_{22}^{-1}(i\omega_n, \vec{p}) \right) \right] + \frac{\Delta^2}{4G_1}, \tag{4}
\]

where \( \beta \) is the inverse temperature. The matrix structure of the inverse Matsubara propagators \( S_{kl}^{-1}(i\omega_n, \vec{p}) \), \( (l, k = 1, 2) \) reflects the Nambu-Gor’kov extension of the particle-hole space to account for the pair correlations. The elements of the Nambu-Gor’kov matrix are \( 2 \times 2 \) matrices defined as \( \text{diag} [S_{11}^{-1}] = (\hat{p} + \mu u \gamma_0, \vec{p} + \mu d \gamma_0) \), and \( \text{diag} [S_{22}^{-1}] = (\hat{p} - \mu u \gamma_0, \vec{p} - \mu d \gamma_0); \) \( \text{diag} [S_{12}^{-1}] = (\Delta^* \gamma_5 \tau_2 \lambda_2, \Delta \gamma_5 \tau_2 \lambda_2) \) and \( \text{diag} [S_{22}^{-1}] = (-\Delta^* \gamma_5 \tau_2 \lambda_2, -\Delta \gamma_5 \tau_2 \lambda_2) \) with off-diagonal elements zero. Upon carrying out the traces in the spin, flavor and color spaces and the fermionic Matsubara summation over the frequencies \( \omega_n \) one finds

\[
\Omega = -2 \int \frac{d^3 p}{(2\pi)^3} \left\{ 2p + \sum_{ij} \left[ \frac{1}{\beta} \log \left( 1 + e^{-\beta \xi_{ij}} \right) + E_{ij} + \frac{2}{\beta} \log \left( 1 + e^{-\beta s_{ij} E_{ij}} \right) \right] \right\} + \frac{\Delta^2}{4G_1}, \tag{5}
\]

where the indices \( i, j = (+, -) \) sum over the four branches of the paired and unpaired quasiparticle spectra defined, respectively, as \( \xi_{\pm} = (p \pm \mu) \pm \delta \mu \) and \( E_{\pm} = \sqrt{(p \pm \mu)^2 + |\Delta|^2} \pm \delta \mu \), where \( \delta \mu = (\mu_u - \mu_d) / 2 \) and \( \mu = (\mu_u + \mu_d) / 2 \) with \( \mu_u \) and \( \mu_d \) being the chemical potentials of the up and down quarks; \( s_{+} = 1 \) and \( s_{-} = \text{sgn}(p - \mu) \). Expressions analogous to \( \Omega \) were derived in refs. [23, 20, 28, 29] for finite/zero temperature and flavor symmetric/asymmetric.
cases. The variations of the thermodynamic potential [1] provide the gap equation

$$
\Delta = 8 G_1 \int \frac{d^3 p}{(2\pi)^3} \left\{ \frac{\Delta}{E_{+-} + E_{++}} \left[ \tanh \left( \frac{\beta E_{++}}{2} \right) + \tanh \left( \frac{\beta E_{+-}}{2} \right) \right] + \frac{\Delta}{E_{++} + E_{--}} \left[ \tanh \left( \frac{\beta E_{--}}{2} \right) + \tanh \left( \frac{\beta E_{++}}{2} \right) \right] \right\},
$$

and the partial densities of the up/down quarks

$$
\rho_{u/d} = \int \frac{d^3 p}{(2\pi)^3} \left\{ \frac{2 f(\xi_{+-}) - 2 f(\xi_{++})}{E_{++} + E_{--}} \left[ 1 \pm \frac{\xi_{--} + \xi_{++}}{E_{--} + E_{++}} \right] \tanh \left( \frac{\beta E_{--}}{2} \right) \pm \left[ 1 \mp \frac{\xi_{--} + \xi_{++}}{E_{--} + E_{++}} \right] \tanh \left( \frac{\beta E_{++}}{2} \right) \right\},
$$

where \( f(\xi_{ij}) \) are the Fermi distribution functions and the upper/lower sign corresponds to the \( u/d \)-quarks. The changes in the thermodynamic potential due to the phase transition to the superconducting phase at constant \( \beta \) and \( \mu_f \) are equivalent to the changes in the free energy at constant \( \beta \) and \( \rho_f \) when the potentials are expressed in terms of appropriate thermodynamical variables. The free energy \( F \) is related to the thermodynamic potential \( \Omega \) by the relation \( F = \Omega + \mu_r \rho_r + \mu_d \rho_d \). Below, we choose to minimize the free energy of the system at constant temperature and density of the matter as a function of the flavor asymmetry parameter defined as \( \alpha \equiv (\rho_d - \rho_u)/(\rho_d + \rho_u) \). Thus, fixing the net density of the quark matter and the flavor asymmetry, fixes the chemical potentials \( \mu_f \) by means of Eqs. (6) and (7). The chemical potentials are isotropic if the Fermi surfaces of the species are spherical. To accommodate the possibility of their deformation we expand the chemical potentials in Legendre polynomials with respect to an (arbitrary) axis of spontaneous symmetry breaking of the spherical symmetry of the Fermi surfaces:

$$
\mu_f = \sum_{l=0}^{\infty} \mu_{f,l} P_l(\cos \theta).
$$

The \( l = 0 \) terms are constants which merely renormalize the chemical potentials from their values for the case of vanishing pairing interactions. Since the interactions are translationally invariant, the \( l = 1 \) terms can not contribute, since they manifestly break the translational symmetry. (The breaking of translational symmetry can be caused by the kinetic energy contribution, as is the case for the LOFF/CCS phases.) We shall keep below the leading order \( l = 2 \) terms which break the rotational symmetry of the system by deforming the Fermi surfaces into an ellipsoidal form. The corresponding expansion coefficients are treated below as variational parameters to minimize the free energy of the color superconductor. With the leading order in the deformation terms kept, the chemical potentials of the \( u \) and \( d \) quarks can be cast in the simple form

$$
\mu_f = \bar{\mu}_f \left[ 1 + (\varepsilon_S \pm \varepsilon_A)(\cos \theta)^2 \right],
$$

where the lower/upper sign corresponds to up/down quarks; \( \varepsilon_{S/A} = (3/2)(\mu_{2d}/\bar{\mu}_{0d} \pm \mu_{2u}/\bar{\mu}_{0u}) \) and \( \bar{\mu}_f = \mu_f - \mu_f/2 \). The parameters \( \varepsilon_S \) and \( \varepsilon_A \) describe the deformations of the Fermi surfaces; the symmetric part \( \varepsilon_S \) is the measure of the conformal expansion/contraction of the Fermi surfaces; the antisymmetric part \( \varepsilon_A \) describes the relative deformations of the Fermi spheres of \( u \) and \( d \) quarks. Note that the expansion need not conserve the volume of a Fermi sphere; the isotropic expansion parameters are self-consistently found form the normalization to the same matter density as in the case for the undeformed state.

### III. THE PHASE DIAGRAM

As the NJL model is non-renormalizable, the momentum integrals in the gap equation [1] need to be regularized by a cut-off; we employ a three-dimensional momentum space cut-off \( |p| < \Lambda \). The phenomenological value of the coupling constant \( G_1 \) in the \( \langle q\bar{q} \rangle \) Cooper channel is related to the coupling constant in the \( \langle \bar{q}q \rangle \) di-quark channel by the relation \( G_1 = N_c/(2N_c - 2)G \); the latter coupling constant and the cut-off are fixed by adjusting the model to the vacuum properties of the system [23, 24, 27]. We employ the parameter set \( G_1 = 3.10861 \) GeV⁻² and \( G_1 \Lambda^2 = 1.31 \) from Ref. [24]. For this parameter set the value of the gap is 40 MeV at the baryonic density \( \rho_B = 0.31 \text{ fm}^{-3} \) which corresponds to the chemical potential value \( \mu = 320 \text{ MeV} \) in the flavor symmetric state at \( T = 0 \). Figure 1 summarizes the main features of the DFS phase in the physically relevant regime of flavor asymmetries \( 0.1 \leq \alpha \leq 0.3 \) likely to occur in the charge-neutral matter under \( \beta \) equilibrium. The color superconducting gap (left panel) and the free energy difference between the superconducting state and normal state (right panel) are shown as a function of deformation parameter.
$\varepsilon_A$ for several flavor asymmetries at the baryonic density $\rho_B = 0.31 \text{ fm}^{-3}$ and temperature $T = 2 \text{ MeV}$. The conformal deformation has been constrained to $\varepsilon_S = 0$. The gap is normalized to its value in the flavor symmetric and undeformed state $\Delta_0 \equiv \Delta(\alpha = 0, \varepsilon_A = 0) = 40 \text{ MeV}$. The free-energy difference $\delta F = F_S - F_N$ is likewise normalized to its value in the flavor symmetric and undeformed state $\delta F(\alpha = 0, \varepsilon_A = 0) = 37 \text{ MeV fm}^{-3}$. The $\beta$ equilibrated quark matter requires an excess of the $d$ over $u$ quarks, therefore the range of the flavor asymmetry is restricted to the positive values. The deformation parameter $\varepsilon_A$ assumes both positive and negative values. The main features seen in Fig. 1 are: (i) for a fixed $\alpha \neq 0$ and $\varepsilon_A > 0$, the gap is larger in the DFS state than in the ordinary BCS state ($\varepsilon_A = 0$); (ii) the minimum of the free energy corresponds to the DFS state with $\varepsilon_A \simeq 0.25$, and its position weakly depends on the value of $\alpha$.

The behavior of the gap as a function of $\alpha$ and $\varepsilon_A$ is best understood in terms of the symmetric and anti-symmetric combinations of the quasiparticle spectra $E_S = (E_{++} + E_{--})/2$ and $E_A = (E_{++} - E_{--})/2$. Since we assume that $\varepsilon_S = 0$ the symmetric combination of the spectra is weakly affected by the deformation [$\sim O(\varepsilon_A \delta \mu)$]. In the case $E_A = 0$ one recovers the BCS result with perfectly overlapping Fermi surfaces. The effect of finite $E_A$ at $\varepsilon_A = 0$ is to induce a phase-space decoherence in the kernel of the gap equation; this blocking effect reduces the magnitude of the gap.

As the chemical potential shift $\delta \mu$ and $\varepsilon_A$ contribute to $E_A$ with different signs, switching on finite $\varepsilon_A > 0$ acts to reduce $E_A$ and the magnitude of the gap increases due to the restoration of the phase-space overlap. Once the condition $\varepsilon_A > \delta \mu$ is satisfied further increase in the deformation acts to increase the de-coherence and, thus, decrease the magnitude of the gap.

For negative values of $\varepsilon_A$ the deformation and the shift in the chemical potentials contribute to $E_A$ with the same sign, therefore the deformation acts to increase the phase-space de-coherence which reduces the magnitude of the gap. These features are seen in Fig. 3, where we show the pairing gap as a function of the flavor asymmetry $\alpha$ and the relative deformation of the Fermi surfaces $\varepsilon_A$ for the same density and temperature as in Fig. 1. [We assume, as for Fig. 1, that the conformal expansion/contraction of the Fermi surfaces is absent, $\varepsilon_S = 0$].

Figure 4 shows the difference between the free energies of the superconducting and normal states $\delta F$ normalized to its value in the flavor-symmetric BCS state $\delta F_{00} = \delta F(\alpha = 0, \varepsilon_A = 0)$. Numerically, the contribution from the entropy difference between the normal and superconducting phases is negligible. Small $\varepsilon_A > 0, \alpha \neq 0$ perturbations from the flavor symmetric/undeformed state increase $\delta F$ signaling an instability of the BCS state towards deformations of the Fermi sur-

![FIG. 2: The deformed Fermi surfaces of the $d$ and $u$ quarks for the flavor asymmetry $\alpha = 0.3$ and deformation parameter $\varepsilon_A = 0.25$. The right part of this figure refers to the Fermi surface of the $d$-quarks, exhibiting a prolate deformation, while the oblate shape on the left corresponds to the Fermi surface of the $u$-quarks.](image-url)
FIG. 3: The color superconducting gap as a function of flavor asymmetry $\alpha$ and the relative deformation parameter $\varepsilon_A$ at density $\rho_B = 0.31$ fm$^{-3}$ and temperature $T = 2$ MeV. The pairing gap is in units $\Delta(\alpha = 0, \varepsilon_A = 0) = 40$ MeV.

FIG. 4: The difference between the free energies of the superconducting and normal phases as a function of flavor asymmetry $\alpha$ and the relative deformation parameter $\varepsilon_A$. The free energy scale is normalized to its value $\delta F(\alpha = 0, \varepsilon_A = 0) = 37$ MeV fm$^{-3}$.

faces.

For flavor asymmetries $\alpha > 0.3$ and positive values of the deformation parameter the superconducting state can exists only in the DFS phase. For negative values of $\varepsilon_A$ there is no energy gain related to the deformation of the Fermi surfaces (except a marginal effect in the limit $\alpha \to 0$ and $\varepsilon_A \to 0$). Quite generally the shape of the free energy surface reflects the functional behavior of the gap as a function of the parameters $\alpha$ and $\varepsilon_A$, which is due to the dominant contribution from the condensation energy (cf. Fig. 1).

IV. CONCLUDING REMARKS

Color superconducting 2SC phase becomes unstable towards formation of a superconducting phase with deformed Fermi surfaces of up and down quarks for finite separations of their Fermi levels. Small asymmetries in the populations of the of the up/down quarks are sufficient to achieve the bifurcation point where the form of the Fermi surfaces changes from the spherical to the ellipsoidal form. Although the departures from the spherical form of the Fermi surfaces costs extra kinetic energy, there is net energy gain because the deformation increases the phase space coherence between the fermions forming Cooper pairs and, therefore, the gain in the potential energy. As mentioned in the introduction, the present study does not establish the true ground state of the 2SC superconductor under flavor asymmetry. This can be done by a careful comparison of the condensation energies of the DFS and CCS phases. With the form of the lattice of the CCS phase established in the Ginzburg-Landau regime, it becomes feasible to carry out a combined study of the DFS and CCS phases.

For applications to the compact stars, our treatment needs an extension which will include the $\beta$ equilibrium and charge neutrality among the quarks and leptons. The onset of the DFS phase (or its combinations with non-BCS superconducting states) will affect the properties of the compact stars in a number of ways. One straightforward implication is the modification of the specific heat of the superconducting phase which affects the thermal cooling of compact stars: we anticipate that the specific heat will be suppressed linearly in the DFS phase as compared to the exponential suppression in the ordinary BCS-state. The spontaneous breaking of the rotational symmetry implies emergence of massless bosonic modes (Goldstone’s theorem). The deformation of the Fermi surfaces opens new channels for neutrino radiation via the bremsstrahlung process of the type $u \to u + \nu + \bar{\nu}$ and $d \to d + \nu + \bar{\nu}$ since the phase-space probability for a transition of a quasi-particle from one point on the deformed Fermi surface to another (say, from the equator to the pole) is non-zero.

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