Acceleration of heavy ions in the hall accelerator

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Abstract. The process of acceleration of heavy ions in the Hall accelerator is studied. Numerical solutions of one-dimensional hydrodynamic equations describing a three-component system-neutral particles, electrons and ions-are obtained. Ions move in a collisionless manner and are accelerated by a self-consistent electric field, electrons diffuse across the magnetic field. The self-consistent field is calculated using the Poisson equation, and it is shown that there is no singularity when the ion velocity coincides with the ion sound velocity. It is also shown that there is a critical magnetic field above which it is impossible to propagate the ion flow.

1. Introduction
In connection with the interest in plasma and ion engines, it is important to study various systems of electrons and ions designed to create accelerators. One of the issues that arise in the study of the extraction of ions from plasma is the possibility of exceeding the velocity of ion sound by the flow of heavy ions. In [1] it is shown that the rejection of the use of the exact terms of neutrality i.e., in an environment where the use of the Poisson equation for the electric field, there is no feature match of the flow velocity of the ion sound.

2. System of equations
Usually, electron Coulomb collisions are neglected in comparison with collisions with neutral atoms. The following equations accurately take into account Coulomb scattering and obtain their numerical solutions.

We present equations describing the behavior of a one-dimensional beam in a longitudinal electric field $E_z = d\phi/dz$ in an axisymmetric magnetic field $H_r$ at $H_r >> H_z$. Equation of dynamics of electrons moving in diffusion mode:

$$V_{ed} = \frac{eV_{coll}}{m_0\omega_c} \left( -\frac{d\phi}{dz} + \frac{1}{eN_e} \frac{d(T_eN_e)}{dz} \right),$$

where $V_{coll} = n_0V_{Te}\sigma_{Te} + n_iV_{Te}\sigma_{ie} + n_eV_{Te}\sigma_{ee}$ is the collision frequency, $\sigma_{Te}$ the transport cross section of the scattering by neutral particles, $\sigma_{ie}$ and $\sigma_{ee}$ the cross sections of Coulomb electron-ion and electron-electron collisions respectively, the thermal velocity of electrons, the temperature, the charge, the mass of electrons and the concentration of neutral atoms, respectively, $\omega_c$ is the cyclotron frequency. The ratio (1) is executed then, when the Larmor radius is less than the free run length. Continuity equation for ions:
\[
\frac{d(n_i V_i)}{dz} = n_i n_0 \beta, \tag{2}
\]

where \(n_i, V_i\) is the density and directed velocity of ions, \(\beta = \sigma_i V_{Te}\), is the specific frequency of particle production during ionization of neutral atoms, \(\sigma_i\) is the ionization cross-section.

The equation of pulses of cold tones can be represented as:

\[
n_i \left(M V_i \frac{dV_i}{dz} + e \frac{d\varphi}{dz}\right) = -M n_0 (V_i - V_A) \beta n_e, \tag{3}
\]

where \(M\) – the mass of the ion, \(V_A\) – the flow rate of neutral atoms. The flow conservation equation which takes into account the burnout of the neutral gas has the form:

\[
\frac{Q}{AM} = (n_i V_A + n_i V_i), \tag{4}
\]

where \(Q\) – is gas flow, \(A\) – cross-sectional area. It is used further as a condition of constancy of the discharge current:

\[
\frac{I_{dis}}{Ae} = (n_i V_i + n_i V_i), \tag{5}
\]

here \(I_{dis}\) – discharge current.

In conditions close to the real experiment [2], it is possible to put \(I_{dis}/Ae = 6 \times 10^{18} \text{ cm}^{-2} \cdot \text{s}^{-1}\) mass flow rate \(Q/AM = 4 \times 10^{18} \text{ cm}^{-2} \cdot \text{s}^{-1}\), the flow rate of atoms at the input \(V_A = 10^4 \text{ cm} \cdot \text{s}^{-1}\). Let \(T_e = 6 \text{ eV}\). In this case the velocity of ion sound in the plasma of single ionized atoms Xe will be \(c_s = (2T_e/M) = 2.95 \times 10^5 \text{ cm} \cdot \text{s}^{-1}\). Next, we will Express the density of particles \(n^*\) in units \(n^* = \frac{I_{dis}/(eA c_s)}{2.04 \times 10^{13} \text{ cm}^{-3}}\) and the velocity in units \(c_s\). To determine the cross sections of collisions, we use the results of [3]. According to this work the scattering cross section of an electron on an atom has the form:

\[
\sigma_{e0}(\varepsilon) = \frac{2e^2}{1 + 0.0045e^2} \left(1 + \frac{12 \exp(-6.6e^{11})}{e^{12}}\right) \times 10^{-16} \text{ cm}^2,
\]

where \(\varepsilon\) – the energy of the electron in eV. In our case, the average energy 6 eV, so the scattering cross section \(\sigma_{e0} \approx 7.2 \times 10^{-15} \text{ cm}^2\). The ionization cross section at \(\varepsilon > I\) (\(I\) – ionization potential) have the form:

\[
\sigma_i(\varepsilon) = \frac{16(\varepsilon - I)^{1.28}}{1 + 0.0032(\varepsilon - I)^{1.77}} \times 10^{-18} \text{ cm}^2.
\]

Since in our case ionization is produced only by particles of the Maxwell tail at \(\varepsilon > I\), then for the averaged cross section at \(T_e = 6 \text{ eV}\) can be obtained:

\[
\sigma_i = \frac{\int_0^\infty \frac{-e^{\varepsilon_2}}{T_e} \sigma_i(\varepsilon) d\varepsilon}{\int_0^\infty \frac{-e^{\varepsilon_2}}{T_e} d\varepsilon} \approx 0.38 \times 10^{-16} \text{ cm}^2.
\]

Given the frequency of ionization \(\Omega^* = \beta n_e = \sigma_i V_{Te} n^* = 1.12 \times 10^5 \text{ s}^{-1}\). We also introduce the reduced scattering frequency \(\Omega^{**} = \sigma_i V_{Te} n^* = 2.13 \times 10^7 \text{ s}^{-1}\). Determine the dimensionless length:

\(s = \frac{z(\Omega^{**}/c_s)}{z/l_0}\), and \(l_0 = (c_s/\Omega^{**}) = 2.6 \text{ cm}\). The dimensionless potential \(U = e\varphi/T_e, \nu = n/n^* -\)
dimensionless density, \( \theta = V/c_s \) – dimensionless speed. We also introduce a dimensionless gas flow \( q = Q/(AMn*c_s) \). In these variables, equations (1–5) are given as:

\[
\left( \frac{\theta^2 + U}{\nu} \right)' = 2\nu_0 \frac{\nu_e}{\nu_i} (\nu_i - \nu_e),
\]

\[
\frac{\nu_e}{\nu_e} = 2\frac{m}{M} \frac{\theta \omega e}{\Omega \Omega*} (\nu_0 + \kappa \nu_0 + \kappa_e \nu_e) + U'',
\]

\[
(\nu \theta_j)' = \nu_e \nu_0 \theta_A + i = q, \nu_e \theta_e + \nu_i \theta_i = 1.
\]

In equations (6) \( \kappa_{ie} \) – the ratio of the Coulomb scattering cross section on ions and electrons to the scattering cross section on atoms. The potential \( U \) satisfies the Poisson equation:

\[
U' = \eta (\nu_e - \nu_i),
\]

where dimensionless parameter:

\[
\eta = \frac{8\pi e^2 n_e}{\Omega^2 M} = 4.26 \times 10^7.
\]

If (7) is neglected \( U''/\eta \), the system is reduced to two equations having a singularity at \( \theta = 2^{-1/2} \) (see [4, 5]). In contrast to the work [4, 5] the magnetic field will be considered constant. Then we transform the system of equations (6–7). From (7) follows:

\[
\nu_e = \nu_i + \frac{U'}{\eta} = \nu_i + \frac{F'}{\eta^{1/2}},
\]

where \( F = E/\eta^{1/2} \) – the value of the field, reduced in \( \eta^{1/2} \) times. Denote \( \nu = \theta_i, \nu_e = \nu_i, i = \nu \nu, K = 2(m/M)(eH/mc)^2[1/(\Omega \Omega*')] \). Then the system takes the form:

\[
i' = \nu_e \frac{q - i}{\theta_A}, \nu' = \nu_e \frac{q - i \nu - \theta_A}{i} + \nu_e - \frac{\eta^{1/2} F}{2 \nu},
\]

\[
\nu_e' = \eta^{1/2} F \nu_e + K \frac{q - i + \kappa_i i \nu + \kappa_e \left( \frac{i}{\nu} + \frac{F'}{\eta^{1/2}} \right)}{\theta_A},
\]

\[
F' = \eta^{1/2} \left( \nu_e - \frac{i}{\nu} \right), U'' = F\eta^{1/2}.
\]

3. Solving a system of equations

We present the results of the solution of the system (8) under the following initial conditions: \( i(0) = 10^5, \nu(0) = \theta_i, F(0) = F' (0) = U(0) = 0 \). Coulomb collisions can be taken into account by means of summands \( \kappa_{ie} = i/\nu \). The Coulomb section \( \sigma_e = 4\pi (e^4/c^3) \cdot \ln \Lambda \), where \( \ln \Lambda \approx 20 \). Numerical calculations show that the deviation from the neutrality condition exists only in a narrow region near the beam entry into the system and is small. Therefore, in (8) electron scattering will be taken into account by the total coefficient \( \kappa = \kappa_i + \kappa_e \).

In accordance with the above formula \( T_e = 6 \text{ eV}, \sigma_e = \times1.4\times10^{-13} \text{ cm}^2 \), the ratio \( \sigma_e/\sigma_T \approx 20 \). Taking into account the scattering by electrons can be put the total coefficient \( \kappa = \kappa_i + \kappa_e = 50 \). The
coefficient $K = 2(m/M)(eH/mc)^2[1/(\Omega \cdot \Omega')]$, at $H = 132.5$ Oe, $K \approx 35$. The same time the condition $eH/mc >> \nu_{\text{coll}}$ is obviously fulfilled.

The current dependence on the coordinate is shown in figure 1. Curve 1 – for the discharge current $1$ A, curve 2 – for the discharge current $4$ A, curve 3 – for the discharge current $1$ A taking into account the Coulomb scattering of electrons. If in the case of discharge current $\approx 1$ A, the ion saturation current is rapidly reached, then in the case of a larger discharge current ($\approx 4$ A) and after reaching a relatively large ion current, its slow increase in dependence is close to linear.

\textbf{Figure 1.} The dependence of the current coordinates: Curve I – for the discharge current $I_R = 1$ A, curve II – for the discharge current $I_P = 4$ A, curve III is for discharge current $I_P = 1$ A when accounting for the Coulomb scattering of electrons.

\textbf{Figure 2.} The dependence of the velocity for different values $\nu(s)$ of discharge current.

Figure 2 shows the dependence of the rate $\nu(s)$. Note that in all cases there is no ion-sound feature. Figure 3 shows the dependence of the charge density on the coordinate. The figure shows that the condition of quasineutrality is noticeably violated only in a narrow region. Let us note one more property of system solutions (6), (7). An increase in $H$ results in slightly larger current values, but the value of $H$ cannot exceed some critical value.

\textbf{Figure 3.} Type of charge density dependence on the coordinate.

\textbf{Figure 4.} Dependence $i(s)$ at $s < 0.01$ and at $K = 35.175$.

For values of the parameter $K < 35$, this system has reasonable solutions for such values of the parameter $s$, which significantly exceed one. At $K > 35.2$ at $s < 0.001$ there is a singularity, which is explained by the presence of non-monotonic flow velocity behavior in the region of small $s$. Figure 4 shows the dependence of $i(s)$ at $s < 0.01$, $K = 35.175$ at neglect of Coulomb collisions and at $K = 35.176$ the minimum speed reaches zero, which leads to the singularity of the solution. The
locking of ions occurs on its own charge, the charge of electrons, unlike ions, varies monotonically – slowly growing in this region of $s$ values.

4. Conclusion

One of the main results of this work is the absence of a feature of the "sound point" type that appears when using the exact condition of neutrality (see [4], [5]). In this case, the real difference from this condition is small. The presence of a critical magnetic field above which the ion beam is locked is also revealed.

References

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