GUT cosmic strings and inflation

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Abstract

We show that GUT cosmic strings generically form after inflation if a non-inert symmetry breaks after inflation; they form irrespectively of the inflationary scenario and in both supersymmetric and non-supersymmetric theories.

In this note, we point out that the formation of field theory cosmic strings after inflation when Spontaneous Symmetry Breaking (SSB) takes place is generic. In Ref. [1], it was conjectured that cosmic strings always form at the end of both standard F-term and D-term inflation [2,3]. In Ref. [4], it was shown that cosmic strings always form in the case of SO(10) GUT with standard hybrid inflation [5,2]. In Ref. [6], an exhaustive study of topological defect formation in all possible SSB patterns in all GUT phenomenologically acceptable was made; it was found that indeed cosmic strings always form if standard F-term inflation is assumed [2]; the strings could be topological or embedded. In view of the revival interest in cosmic (super)-strings [7], it is important to clarify and put forward the idea of Ref. [1]. We show that the formation of cosmic strings is quite general and does not rely on the standard hybrid inflationary scenario. The only assumption is that inflation solves the monopoles problem and is followed by a phase transition associated with spontaneous symmetry breaking. We do not discuss the nature of the strings which can be topological [8] or embedded [9] and, with non minimal Higgs content, semi-local [10]. The reason why cosmic string form after inflation is a simple argument. In a nutshell, they form because the group which is broken after inflation must contain a U(1) factor, and its rank is lowered by (at least) one unit.

The paper is organized as follows. First, the GUT monopole problem is reviewed. Inflation as a solution to the monopole problem is then discussed. It is then shown that if

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1Non-topological cosmic strings may remain stable down to low energy. If the strings form but rapidly decay, they can still lead to observable effects [11]. In some circumstances, unstable strings may not form [12].
inflation is followed by (at least one) phase transition during which spontaneous symmetry breaking takes place (in the visible sector) and no unwanted defect form after inflation, cosmic strings form.

The monopole problem In a cosmological context, when a gauge group $G$ spontaneously breaks down to a subgroup $H$ of $G$, topological defects form if the vacuum manifold $G/H$ is non-trivial \[ \text{[8]}. \] Monopoles and domain walls are cosmologically catastrophic whereas numerical simulations show that the density of a cosmic string network eventually reaches a scaling solution, independently of the initial string density \[ \text{[13]}. \] The reason is that cosmic strings can lose energy by forming loops which rapidly decay via gravitational radiation and particle emission \[ \text{[13]}. \]

Undoubtedly of the initial monopole density, monopole-antimonopole pairs annihilate until the monopole-to-entropy ratio reaches its final value \[ \text{[14, 13]} \]

\[
\frac{n_M}{s} \sim \frac{1}{h^6 \beta \sqrt{g_{ss}} M_{pl}}
\]  

where $n_M$ is the monopole density and $s$ is the entropy. $h$ is the monopole magnetic charge which is given by $h = -\frac{4\pi}{g}$, where $g$ is the gauge coupling constant. $\beta \sim (1 - 5)g_s$, where $g_s$ is the effective number of degrees of freedom and $g_{ss}$ is the effective number of helicity states for particles with mass $m + p < T$. Finally, $m$ is the monopole mass and $M_{pl}$ is the Planck mass. The monopole mass is bounded from below by \[ \text{[15]} \]

\[
m \geq \frac{4\pi}{e} M_M
\]  

where $M_M$ is the scale at which the monopoles form. For monopoles forming between the GUT scale $M_{GUT} \simeq 3 \times 10^{16}$ GeV and the electroweak scale $M_Z \simeq 10^2$ GeV, the monopole density

\[
\left( \frac{n_M}{s} \right)_{\text{Theo}} \in [10^{-12} - 10^{-27}].
\]  

Observations imply a much lower density. The less stringent bound comes from neutron stars observations; the latter can trap monopoles which can catalyse proton decay and thus increase the star luminosity. Limits on the luminosity of neutron stars imply a bound on the monopole flux which translates into a bound on the monopole density given by \[ \text{[16]} \]

\[
\left( \frac{n_M}{s} \right)_{\text{Exp}} \leq 10^{-31}.
\]
Comparing theoretical predictions Eq.(3) with observations Eq.(4), we conclude that GUT monopoles have to be diluted, independently of the scale at which they form.

We now turn to the conditions under which monopoles form. Let’s consider a gauge group $G$ which spontaneously breaks down to a subgroup $H$ of $G$. The breaking can be direct, $G \to H$, or via one or more intermediate symmetry subgroups, $G \to \cdots \to K \to \cdots \to H$. Classification of topological defects is usually done by using homotopy theory. Domain walls form when $\pi_0(G/H) \neq I$; they thus form when a discrete symmetry is broken. Cosmic strings form when $\pi_1(G/H) \neq I$; they thus form when an abelian symmetry is broken. And monopoles form if $\pi_2(G/H) \neq I$. If $G$ is connected (i.e. $\pi_0(G) = I$) and simply connected (i.e. $\pi_1(G) = I$) then $\pi_2(G/H) \cong \pi_1(H)$. Monopoles which form during a phase transition during which $G \to H$ remain stable during the next phase transition during which $H \to F$ if $\pi_2(G/H) \neq I$. Note that if $G$ is not simply connected, we can always work with its universal covering group which is simply connected [8]; for example, in the case of SO(10) which is not simply connected, we work with its universal covering group, Spin(10), which is simply connected. We are only interested in the formation of monopoles, and hence we assume that $G$ does not contain any discrete symmetry, i.e. that it is connected.

Let first $G = G_{\text{GUT}}$ a unified gauge group which does not contain a U(1) factor. It can be simple or semi-simple. Most unified models fit in this category. For examples, $G_{\text{GUT}}$ can be the Georgi-Glashow model SU(5), SO(10), E(6), the Pati-Salam model $SU(4)_c \times SU(2)_L \times SU(2)_R$, the trinification $SU(3)_c \times SU(3)_L \times SU(3)_R$ or SU(6). $\pi_0(G) = I$ and $\pi_1(G) = I$ (if $G$ is not simply connected we consider its universal covering group). $G$ must be broken directly or via intermediate steps down to the Standard Model gauge group $G_{\text{SM}} = SU(3)_c \times SU(2)_L \times U(1)_Y \equiv H$. Now $\pi_1(G_{\text{SM}}) = Z$ because of its U(1) factor $(\pi_1(SU(3)) = \pi_1(SU(2)) = I)$. Hence $\pi_2(G_{\text{GUT}}/G_{\text{SM}}) = Z$ and therefore monopoles always form at some stage of the SSB breaking pattern. Furthermore, since $\pi_1(SU(3)_c \times U(1)_Q) = Z$, the monopoles are topologically stable down to low energy.

Now let’s assume that $G_{\text{GUT}}$ contains a U(1) factor which is not inert. We call an inert symmetry, a symmetry which is orthogonal to the Standard Model group. This is for example the case of flipped SU(5), $\tilde{SU}(5) \times U(1)$. In this case, $\pi_2(G_{\text{GUT}}/G_{\text{SM}}) = I$ and the formation of monopoles can be avoided [17].

In the final case, we assume that $G$ contains a U(1) factor which is inert. So $G$ can be written as $G_{\text{vis}} \times U(1)_I$ and $G_{\text{SM}}$ is fully embedded in $G_{\text{vis}}$. In that case, one should
consider the breaking of $G_{vis} \rightarrow G_{SM}$ and $U(1)_I \rightarrow I$ separately; since $\pi_2(G_{vis}/G_{SM}) = \mathbb{Z}$ topological monopoles form.

The conclusion is that as soon as the $U(1)_Y$ symmetry of the Standard Model is embedded in a non-abelian group which does not contain a $U(1)$ factor, unwanted monopoles form. This is the famous GUT monopole problem [14].

**Inflation as a solution to the monopole problem** A solution to the monopole problem is inflation. - It is often forgotten that inflation was originally invented for this purpose [18]. - Inflation solves the monopole problem if it takes place after the phase transition which leads to their formation. Alternatively the phase transition can take place during inflation, but in the case of GUT scale monopole it must be at least some 19 e-folds before the end of inflation [19]. Another solution to the monopole problem would be to have a Langacker-Pi type phase transition [20]. We focus on inflation.

**GUT cosmic strings are produced after inflation** We now show that cosmic strings are always produced after inflation if the following assumptions are satisfied:

1. The Standard Model is embedded in a semi-simple grand unified gauge group $G_{GUT}$
2. Inflation solves the monopole problem
3. Spontaneous Symmetry Breaking (SSB) in the visible sector takes place after inflation\(^2\) (apart from Standard Model breaking).

The grand unified gauge group $G_{GUT}$ must break down to $G_{SM}$. The breaking can be direct, $G_{GUT} \rightarrow G_{SM}$, or via intermediate symmetry subgroups, $G_{GUT} \rightarrow \ldots \rightarrow G_{SM}$. Whereas direct breaking is allowed in the supersymmetric case, at least one intermediate symmetry group is needed for unification of the coupling constants if there is no supersymmetry. Particle physics can hardly constrain the spontaneous symmetry breaking patterns\(^3\). Here, we use cosmology [11 14]. Since SSB is assumed to take place after inflation, $G_{GUT}$ cannot break directly down to $G_{SM}$, otherwise monopoles would form after

\(^2\)This is always true in the case of standard hybrid inflation [2].

\(^3\)Accelerator experiments can constrain the intermediate groups if they break at very low scale. Some intermediate breakings are excluded because they lead to rapid proton decay. In some cases, neutrinos constrains can also be used. From theoretical side, patterns with large number of intermediate steps are disfavored.
inflation. Therefore there must be at least one intermediate symmetry group, let’s call it $K$, such that $G_{GUT}$ breaks down to $K$ before inflation and the monopoles form \(^4\), and no unwanted defect form when $K$ breaks down to $G_{SM}$ after inflation \(^5\). $G_{GUT} \supset K \supset G_{SM}$. There can be intermediate breaking(s) between $G_{GUT}$ and $K$ and between $K$ and $G_{SM}$:

$$G_{GUT} \rightarrow \cdots \rightarrow K \rightarrow \cdots \rightarrow G_{SM}. \tag{5}$$

For example, $K$ cannot contain any discrete symmetry otherwise unwanted domain walls would form (unless this discrete symmetry remains unbroken down to low energy).

Since monopoles form when $G_{GUT}$ breaks down to $K$, the latter contains at least one $U(1)$ factor, which is not inert, otherwise monopole would again form when $K$ breaks down to $G_{SM}$. We call it $U(1)_X$. $K$ can contain more abelian factor(s) (inert or not); $\pi_1(K) = Z^n$ if $K$ contains $n$ $U(1)$ factors. $K$ is thus of the form

$$K = J \times U(1)_X \times U(1) \times \cdots \times U(1) \tag{6}$$

where $J$ a semi-simple group and $n \geq 1$.

We now consider the various possibilities for $n$ and $J$ and the consequences for defect formation.

Case 1: $n > 1$

In this case, the first homotopy group $\pi_1(K/G_{SM}) = Z^{n-1} \neq I$ and therefore topological cosmic strings always form. They form regardless of the embedding of flavor, color and electric charge in $K$. The rank of $K$ is strictly greater than the rank of the Standard Model gauge group, i.e. $\text{rank}(K) \geq 5$, and $\text{rank}(G_{GUT}) \geq 5$.

Case 2: $n = 1$

In this case, the intermediate symmetry group $K = J \times U(1)_X$, $J \supset SU(3)_c \times SU(2)_L$ and $G_{SM}$ is fully embedded in $J \times U(1)_X$. There are then two possibilities.

\(^4\)The breaking must happen at least 19-efolds before the end of inflation.

\(^5\)The breaking of $K$ is a dynamical problem which usually depends upon the coupling (direct or indirect) between the inflaton and the GUT Higgs field(s), but not necessarily on the reheating temperature. For example in the case of hybrid inflation, if the Higgs field which triggers the end of inflation is the Higgs field used to break $K$, the breaking of $K$ occurs independently of the value of the reheating temperature.
(a) \( U(1)_X = U(1)_Y \)

\( SU(3) \times SU(2) \) can only be embedded in a group with rank \( \geq 4 \). Thus the rank of \( J \) is \( \geq 4 \) and the rank of \( K \) is \( \geq 5 \). There are a priori five possibilities for \( J \).

i. \( J = SU(3)_c \times SU(2)_L \times F \) with \( F \) simple or semi simple. \( F \) is inert.
   \[ \text{rank}(G_{GUT}) \geq 6. \]
   Cosmic strings do not form. There are transient strings connecting monopole-antimonopole pairs if \( F \) breaks down to the identity in more than one step; for ex \( F \to U(1) \to I. \)

The remaining ones are

ii. \( J = SU(3)_c \times F \) with \( SU(2)_L \supset F \)

iii. \( J = F \times SU(2)_L \) with \( SU(3)_c \supset F \)

iv. \( J = F \times P \) with \( SU(3)_c \supset F \) and \( SU(2)_L \supset P \)

v. \( J = SU(3)_c \times SU(2)_L \supset F \)

with \( F \) and \( P \) simple or semi simple (do not contain a \( U(1) \) factor). These cases which, if at all allowed, correspond to very non-standard embeddings of color, flavor and electric charge and very non-standard GUTs. There is no string.

(b) \( U(1)_X \neq U(1)_Y \)

In this case, the hypercharge is a linear combination of \( X \) and of (at least) one diagonal generator of \( J \). The orthogonal generator is broken and embedded strings form \([9]\).

**From embedded to topological strings**  Discrete \( Z_N \) symmetries are commonly left unbroken in realistic SSB; this depends upon the Higgs representation which is used to do the breaking\(^6\). If a discrete \( Z_N \) symmetry is left unbroken after the breaking of \( K \), in cases 2.(b) topological strings form instead of embedded strings, and in cases 2.(a), \( Z_N \) strings form.

\(^6\) The most common example is the case of GUT gauge groups which contain gauged \( B-L \) and predict neutrinos masses via the see-saw mechanism. If \( B-L \) if broken with Higgs fields in “safe” representations of \( G_{GUT} \), a discrete \( Z_2 \) symmetry (which plays the role of matter parity) subgroup of \( B-L \) is left unbroken, and the proton decays at an acceptable rate. If it is broken, R-parity has to be imposed by hand.
In conclusion, we have considered generic GUT models where SSB takes place after inflation. There is no assumption made about the inflationary scenario. The model can be supersymmetric or not. Inflation solves the monopole problem if there is at least one intermediate symmetry group between $G_{GUT}$ and $G_{SM}$. We have shown that the rank of the intermediate group must be greater or equal to 5 and is lowered at the end of inflation; hence the rank of $G_{GUT}$ must also be greater or equal to 5. We have also shown that there must be at least one intermediate symmetry group of the form $J \times \underbrace{U(1) \times \cdots \times U(1)}_{n}$ where $J$ is a semi-simple group and $n \geq 1$. We found that cosmic strings always form after inflation if the symmetry which breaks after inflation is not inert. The strings could be topological or embedded. When a discrete symmetry is left unbroken after the breaking of the intermediate symmetry group, the strings are always topological. In some particular very non standard embeddings of color and flavor, which do not apply to any known GUT, cosmic strings do not form. If the strings are stable, they contribute to primordial fluctuations [22]. However their contribution may be too low for detection via temperature anisotropies of the CMB. For example, in the case of hybrid inflation, scalar perturbations are dominated by scalar perturbations from inflation for most of the parameter space [23]. On the other hand, cosmic strings may well be detected via B-type polarization of the CMB [24]. In models with hybrid inflation for example, tensor perturbations are negligible and vector and tensor perturbations are dominated by perturbations from cosmic strings. If the strings decay before nucleosynthesis, they will not be detected via CMB experiments. They may however have interesting cosmological consequences [25].

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