Continuous sliding mode control for gun position servo system

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Abstract. With the development of military technology and changes in the war situation, modern warfare has higher requirements for the accuracy and speed of gun firing. The control performance of the gun position servo system directly affects the accuracy and speed of the gun firing. In order to improve performance of the gun, the sliding mode control is investigated for the gun position servo system. Sliding mode control has advantages of simple structure and good robustness against uncertain system parameters and disturbance. But discontinuous sign function in the classical sliding mode control method will lead to chattering of the system. To address the shortcoming of chattering in the traditional sliding mode control, a continuous sliding mode control algorithm is investigated in this paper. Finally, numerical simulations are performed to illustrate the efficiency of the theoretical results.

1. Introduction
The transformation of modern war modes and the development of military technology requirements have put forward higher requirements on the performance of gun weapon systems. The firing accuracy and response speed of gun largely depend on the tracking performance of the gun position servo system. Therefore, it is an immediate requirement for the high-performance position tracking control algorithm.

Due to the inevitable complex external interference and parameter perturbation when the gun is running, it brings great difficulty to the design of the control algorithm. The traditional PID control is increasingly difficult to meet the control performance requirements. In order to solve the above problems, many methods have been used to solve the control problem of the gun, such as adaptive control, neural network control, active disturbance rejection control, etc.[1, 2, 3, 4]. Although good control performance is achieved, there are also many shortcomings. For the adaptive control method, parameterizable part of the parameter uncertainty and uncertain nonlinearity can be effectively estimated. A model compensation can be made by the estimated parameters. However, for the non-parametric uncertain nonlinear terms, adaptive control is powerless. In case of disturbance, the adaptive control even faces the danger of divergence. The computation of neural network control is large, which affects the real-time performance.

For systems with complex disturbance, [5] studied a higher-order sliding modes control methods, where the required higher-order real-time derivatives of the outputs is achieved by arbitrary-order robust exact differentiators with finite-time convergence.

Sliding mode control has advantages of simple structure and good robustness against uncertainty of system parameters and disturbance [6, 7, 8]. But in the classical sliding mode...
control algorithm, there is a discontinuous sign function which will lead to chattering of the system and cause the degradation of system control performance.

This paper investigates the gun position servo system via sliding mode control method. To address the shortcoming of chattering in the classical sliding mode control algorithm, a continuous sliding mode control algorithm is introduced.

The rest of this paper is organized as follows. In Section 2, dynamics and tracking objective of the gun position servo system is formulated. In Section 3, the sliding manifold for the gun position servo system is established. A continuous sliding mode tracking algorithm is studied for the gun position servo system. In Section 4, numerical simulations are performed to validate the theoretical results. Finally, concluding remarks are made in Section 5.

2. System description

The gun position servo system consists of a permanent magnet synchronous servo motor, driver and position sensor. According to the control direction, it is divided into a pitch servo system and an azimuth servo system. The working process of the gun servo system can be regarded as the position tracking which is directly driven by the AC motor. As the responding speed of the electrical part of the system is much higher than that of the mechanical part, the current loop dynamics can be ignored when the system is modeled. The current loop can be regarded as a proportional part. The gun position servo system can be modeled as:

\[ J \ddot{\theta} = k u - B \dot{\theta} + f(t), \]  

where \( J \) is the system’s total load inertia converted to the moment of inertia of the azimuth motor, \( \dot{\theta} \) and \( \ddot{\theta} \) are the angular velocity and angular acceleration of the azimuth axis, respectively, \( \theta \) is the output angle of the azimuth axis, \( k \) is the voltage torque coefficient, \( u \) is the control input voltage, \( B \) is the viscous friction factor, and \( f(t) \) represents a combination of external disturbances and model errors, etc.

Define \( x = [x_1, x_2]^T = [\theta, \dot{\theta}]^T \), \( k_u = k/J \), and \( k_b = B/J \). The system model can be rewritten as the following state space equation:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= k_u u - k_b x_2 + d,
\end{align*}
\]  

where \( d = f(t)/J \).

In engineering, disturbance sources come from many aspects, such as system parameter perturbations, model modeling errors, and non-modeled non-linear friction in motor servo systems, etc. A reasonable assumption on the system disturbance is given as follow:

**Assumption 1** There exists an upper bound for the system disturbance such that \( |d| \leq \bar{d} \).

Consider that the system tracking angle command signal is \( x_d \). The angle tracking error and the velocity tracking error are expressed as \( e_1 = x_1 - x_d \) and \( e_2 = x_2 - \dot{x}_d \). Dynamics of the tracking error can be expressed as:

\[
\begin{align*}
\dot{e}_1 &= e_2 \\
\dot{e}_2 &= k_u u - k_b x_2 - \ddot{x}_d + d.
\end{align*}
\]  

The objective of the position servo system is to make the position tracking error \( e_1 \) as small as possible.
3. **Continuous sliding mode control algorithm design**

The design procedure of a sliding mode control algorithm can be summarized as the following two steps:

1. Design a sliding manifold \( s \) such that tracking error of the system approaches 0 on the sliding surface \( s = 0 \).
2. Design a sliding mode control algorithm which drives the motion of the gun position servo system reaching and maintaining on the sliding surface.

The sliding manifold is designed as

\[
s = \mu e_1 + e_2.
\]

A classical sliding mode control algorithm can be expressed as:

\[
u = \left( k_b x_2 + \ddot{x}_d - \mu e_2 - \beta \text{sgn}(s) \right)/k_u,
\]

where

\[
\text{sgn}(s) = \begin{cases} 
\frac{s}{|s|}, & \text{if } |s| \neq 0, \\
0, & \text{otherwise}.
\end{cases}
\]

In engineering, the discontinuous sign function \((6)\) will lead to chattering of the system and cause the degradation of system control performance. To address the shortcoming, the discontinuous sign function is smoothed as

\[
\psi_c(s) = \begin{cases} 
\beta \frac{s}{|s|}, & \text{if } \beta|s| \geq \xi, \\
\beta^2 \frac{s}{\xi}, & \text{if } \beta|s| < \xi,
\end{cases}
\]

where \( \xi > 0 \).

Then one obtains the continuous sliding mode control algorithm as follows:

\[
u = \left( k_b x_2 + \ddot{x}_d - \mu e_2 - \psi_c(s) \right)/k_u,
\]

where \( \psi_c(s) \) is expressed as \((7)\).

**Theorem 1** Suppose that system disturbance of the gun position servo system \((2)\) is bounded. Then, the ultimately bounded tracking of the gun position servo system can be achieved by using the continuous sliding mode control algorithm \((8)\) with \( \beta \geq \bar{d} + \beta_0, \beta_0 > 0 \). The upper bound of the position tracking error is \( |e_1| \leq \frac{\bar{d}}{\mu^2} \).

**Proof** The proof consists of two steps. Firstly, we prove that the sliding manifold \((4)\) reaches a boundary layer containing \( s = 0 \) in finite time.

Consider the following Lyapunov function:

\[
V = \frac{1}{2} s^T s,
\]

\[
\beta > \bar{d} + \beta_0, \beta_0 > 0.
\]

Differentiating \( V \) along the trajectory of \((4)\) leads to

\[
\dot{V} = s \dot{s} = s \left[ \mu e_2 + k_u u - k_b x_2 - \ddot{x}_d + \bar{d} \right]
= s \left[ d - \psi_c(s) \right] \leq \bar{d}|s| - s \psi_c(s),
\]
Case I: If $\beta|s| \geq \xi$, $s\psi_c(s) = \beta|s|$. From (10) it readily follows that

$$\dot{V} \leq \ddot{d}|s| - \beta|s| \leq -\beta_0|s| < 0$$

(11)

Case II: If $\beta|s| < \xi$, $s\psi_c(s) = \frac{\beta^2}{\xi} s^2$. The time derivative of $V$ is given by

$$\dot{V} \leq \ddot{d}|s| - \frac{\beta^2}{\xi} s^2.$$ 

It is easy to verify that $\dot{V} < 0$, once $|s| > (\xi \ddot{d})/\beta^2$.

Combining together the above two cases yields the following inequality:

$$\dot{V} < 0, \; \forall |s| > (\xi \ddot{d})/\beta^2.$$ 

(12)

According to the Lyapunov theory, the sliding manifold will reach the following boundary layer in finite time:

$$\Omega_1 = \left\{ |s| \leq \frac{\xi \ddot{d}}{\beta^2} \right\}.$$ 

(13)

Secondly, we will show that the ultimately bounded tracking of gun position servo system is achieved. Inside the boundary layer (13), one has

$$\dot{e}_1 = -\mu e_1 + s,$$

where $|s| \leq \frac{\xi \ddot{d}}{\beta^2}$. The derivative of $V' = \frac{1}{2} e_1^2$ satisfies

$$\dot{V}' \leq -\mu e_1^2 + e_1 s \leq -\mu e_1^2 + |e_1| \frac{\xi \ddot{d}}{\beta^2}$$

$$< 0, \; \forall |e_1| > \frac{\xi \ddot{d}}{\mu \beta^2}.$$ 

(14)

Thus, the trajectory of the gun position servo system will reach the set $\Omega_2 = \{|e_1| \leq \frac{\xi \ddot{d}}{\mu \beta^2}, \; |s| \leq \frac{\xi \ddot{d}}{\beta^2}\}$ in finite time. Due to the fact that $s = \mu e_1 + e_2$, the ultimate boundedness of $e_1$ and $s$ infers that $e_2$ is also ultimate bounded. It means that the ultimately bounded position tracking of the gun position servo system is achieved. The proof is complete.

4. Simulations

In this section, a simulation example is performed to illustrate the effectiveness of the theoretical results, where the artillery servo system uses Kollmorgen’s permanent magnet synchronous motor. Parameters of the model are $J = 1.5\text{kg} \cdot \text{m}^2$, $k = 5\text{N} \cdot \text{m}/\text{V}$, $B = 0.2\text{N} \cdot \text{m} \cdot \text{s}/\text{rad}$. From 5s, the system suffers from disturbance $d(t) = \frac{1}{\pi} \cdot \text{arctan}(900\dot{\theta}) + 2.5 \sin(2\pi t)$.

Consider the system position command signal being a sinusoidal signal with an amplitude of $7^\circ$ and a period of 2 seconds. Parameters of the controller are set to $\mu = 5$, $\beta = 20$, $\xi = 0.5$. Simulation step size is set to 1ms. Output angle of the azimuth axis and the corresponding tracking error of the output angle are depicted in figure 1. The control signal of the controller is shown in figure 2.

Simulation results show that the continuous sliding mode control method studied in this paper has a good control performance and disturbance suppression effect. Meantime, the control signal is continuous, which avoids shortcomings in the traditional sliding mode control.
Figure 1. Frequency response of the gun servo system and the corresponding tracking error.

Figure 2. Control signal of the controller.

5. Conclusion
This paper investigates the position servo control system of the gun. A continuous sliding mode control algorithm is proposed, which has advantages of simple structure and good robustness against uncertain system parameters and disturbance. The proposed continuous sliding mode control algorithm avoids chattering of the classical sliding mode control. Finally, effectiveness of the proposed algorithm is demonstrated through simulations, which provides valuable reference for design of gun position servo system.

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