Generation of Magnetic Fields and Jitter Radiation in GRBs. I. Kinetic Theory

Mikhail V. Medvedev

Abstract. We present a theory of generation of strong (sub-equipartition) magnetic fields in relativistic collisionless GRB shocks. These fields produced by the kinetic two-stream instability are tangled on very small spatial scales. This has a clear signature in the otherwise synchrotron(-self-Compton) \( \gamma \)-ray spectrum. Second, we present an analytical theory of jitter radiation, which is emitted when the correlation length of the magnetic field is smaller then the gyration (Larmor) radius of the accelerated electrons. We demonstrate that the spectral power \( P(\nu) \) for pure jitter radiation is well-described by a sharply broken power-law: \( P(\nu) \propto \nu^1 \) for \( \nu < \nu_j \) and \( P(\nu) \propto \nu^{-(p-1)/2} \) for \( \nu > \nu_j \), where \( p \) is the electron power-law index and \( \nu_j \) is the jitter break, which is independent of the magnetic field strength and depends on the shock energetics and kinematics. Here we mostly focus on the first problem. The radiation theory and comparison with observations will be discussed in the forthcoming publications.

INTRODUCTION

There is currently no satisfactory explanation for the origin of strong magnetic fields required in GRB shocks. Compression of the ISM magnetic field in external shocks yields a field amplitude \( B \sim \gamma B_{\text{ISM}} \sim 10^{-4}(\gamma/10^2) \) gauss, which is too weak and can account only for \( \epsilon_B = (B/B_{eq})^2 \leq 10^{-11} \) (here \( \gamma \) is the Lorentz factor of the outflow). Neither a turbulent magnetic dynamo, nor the magnetic shearing (Balbus-Hawley) instability, nor any other MHD process can be so efficient to produce the required strong fields. In principle, some magnetic flux might originate at the GRB progenitor and be carried by the outflowing fireball plasma. Because of flux freezing, the field amplitude would decrease as the fireball expands. In this case, only a progenitor with a rather strong magnetic field \( \sim 10^{16} \) gauss might produce sufficiently strong fields during the GRB emission. However, since the field amplitude scales as \( B \propto R^{-4/3} \), even a highly magnetized plasma at \( R \sim 10^7 \) cm would possess only a negligible field amplitude of \( \sim 10^{-2} \) gauss, or \( \epsilon_B \leq 10^{-7} \), at a radius of \( R \geq 10^{16} \) cm, where the afterglow radiation is emitted. Here we discuss how strong magnetic fields are generated by the kinetic relativistic two-stream
instability [2] and consider their properties. We postpone the major discussion on the jitter radiation theory [3] to a forthcoming publication.

**THE TWO-STREAM INSTABILITY**

The non-relativistic instability was first discovered by Weibel [4]. It has been used by Moiseev and Sagdeev [5] to develop a theory of collisionless non-relativistic shocks in the interplanetary space.

Let us consider, for simplicity, the dynamics of the electrons only, and assume that the protons are at rest and provide global charge neutrality. The electrons are assumed to move along the $x$-axis (as illustrated in Fig. 1a) with a velocity $\mathbf{v} = \pm \hat{x} v_x$ and equal particle fluxes in opposite directions along the $x$-axis (so that the net current is zero). Next, we add an infinitesimal magnetic field fluctuation, $\mathbf{B} = \hat{z} B_z \cos(ky)$. The Lorentz force, $-e \mathbf{v} \times \mathbf{B}$, deflects the electron trajectories as shown by the dashed lines in Fig. 1a. As a result, the electrons moving to the right will concentrate in layer I, and those moving to the left – in layer II. Thus, current sheaths form which appear to increase the initial magnetic field fluctuation. The growth rate is $\Gamma = \omega_p v_y/c$, where $\omega^2_p = (4\pi e^2 n/m)$ is the non-relativistic plasma frequency [6]. Similar considerations imply that perpendicular electron motions along $y$-axis, result in oppositely directed currents which suppress the instability. The particle motions along $\hat{z}$ are insignificant as they are unaffected by the magnetic field. Thus, the instability is driven by the anisotropy of a particle distribution function and should quench for the isotropic case.

The Lorentz force deflection of particle orbits increases as the magnetic field perturbation grows in amplitude. The amplified magnetic field is random in the plane perpendicular to the particle motion, since it is generated from a random seed field. Thus, the Lorentz deflections result in a pitch angle scattering which makes the distribution function isotropic. The thermal energy associated with their random motions will be equal to their initial directed kinetic energy. This final state will bring the instability to saturation.

Here are the main properties of the instability and the produced magnetic fields:

- This instability is driven by the anisotropy of the particle distribution function and, hence, can operate in both internal and external shocks.

- The characteristic $e$-folding time in the shock frame for the instability is $\tau \approx \gamma_{sh}^{1/2}/\omega_p$ (where $\gamma_{sh}$ is the shock Lorentz factor) which is $\sim 10^{-7}$ s for internal shocks and $10^{-4}$ s for external shocks. This time is much shorter than the dynamical time of GRB fireballs.

- The characteristic coherence scale of the generated magnetic field is of the order of the relativistic skin depth $\lambda \approx 2^{1/4} c \bar{\gamma}^{1/2}/\omega_p$ (where $\bar{\gamma}$ is the mean thermal Lorentz factor of particles), i.e. $\sim 10^3$ cm for internal shocks and $\sim 10^5$ cm for external shocks. This scale is much smaller than the spatial scale of the source.
• The generated magnetic field is randomly oriented in space, but always lies in the plane of the shock front.

• The instability is powerful. It saturates only by nonlinear effects when the magnetic field amplitude approaches equipartition with the electrons (and possibly with the ions). Therefore $|B^2/8\pi|/[mc^2n(\gamma - 1)] = \eta \sim 0.01 - 0.1$. This result is in excellent agreement with direct particle simulations.

• The instability isotropizes and heats the electrons and protons.

• Random fields scatter particles over pitch-angle and, thus, provide effective collisions. Therefore MHD approximation works well for the shocks. The magnetic fields communicate the momentum and pressure of the outflowing fireball plasma to the ambient medium and define the shock boundary.

• The generated small-scale fields affect the radiation processes [3] and produce non-synchrotron spectra of radiation, as shown in Fig. 1b.

REFERENCES

1. Sari, R., Narayan, R., & Piran, T., Astrophys. J. 473, 204 (1996)
2. Medvedev, M. V., and Loeb, A., Astrophys. J. 526, 697 (1999)
3. Medvedev, M. V., Astrophys. J. 540, 704 (2000)
4. Weibel, E. S., Phys. Rev. Lett. 2, 83 (1959)
5. Moiseev, S. S., and Sagdeev, R. Z., J. Nucl. Energy C 5, 43 (1963)
6. Fried, B. D., Phys. Fluids 2, 337 (1959)

FIGURE 1. (a) — Illustration of the instability. A magnetic field perturbation deflects electron motion along the $x$-axis, and results in current sheets ($j$) of opposite signs in regions I and II, which in turn amplify the perturbation. The amplified field lies in the plane perpendicular to the original electron motion. (b) — Typical jitter spectra from small-scale magnetic fields (dashed curve is synchrotron, for comparison).