Comments on heat transfer efficiency in cryogenic helium turbulent Rayleigh-Bénard convection

P Urban¹, V Musilová¹, T Kráľík¹ and L Skrbek²

¹Institute of Scientific Instruments ASCR, v.v.i., Královopolská 147, Brno, Czech Republic
²Faculty of Mathematics and Physics, Charles University, Ke Karlovu 3, 12116 Prague, Czech Republic

E-mail: Urban@isibrno.cz; Skrbek@fzu.cz

Abstract. An experimental study of turbulent Rayleigh-Bénard convection (RBC) and of the influence of various experimental aspects is presented and comparison with similar experiments discussed. We use a cylindrical cell 0.3 m both in diameter and height, designed to minimize the influence of its structure on the convective flow of cryogenic ⁴He gas of Prandtl number $Pr ≈ 1$. We measure the dependence of Nusselt ($Nu$) versus Rayleigh number ($Ra$) at high Ra and discuss various corrections that might be applicable to the raw data. For $7.2 \times 10^6 ≤ Ra ≤ 10^{11}$ our data agree with suitably corrected data from similar cryogenic experiments and are consistent with $Nu / Ra^{2/7}$. On approaching $Ra ≈ 10^{11}$ our data display a crossover to $Nu / Ra^{1/2}$ that approximately holds up to $Ra ≈ 4.6 \times 10^{13}$. We show that differences in $Nu(Ra)$ scaling observed in similar RBC experiments for $Ra ≥ 10^{11}$ cannot be explained due to the difference in $Pr$ but might depend also on experimental details, some of which are pointed out and discussed.

1. Introduction

The ideal laterally-infinite Rayleigh-Bénard convection (RBC) is a model for fundamental studies of buoyancy-driven flows. It occurs in a (Boussinesq) fluid layer confined between infinite perfectly conducting plates heated from below in a gravitational field, and it is characterized by the Rayleigh, $Ra$, and the Prandtl numbers, $Pr$. The convective heat transfer efficiency is expressed by the Nusselt number, $Nu = Nu(Ra, Pr)$. These dimensionless numbers are defined as

$$Nu = \frac{LH}{λΔT} ; \quad Ra = \frac{gα}{κν} ΔTL^3 ; \quad Pr = \frac{ν}{κ},$$

where $H$ is the total convective heat flux density, $g$ stands for the acceleration due to gravity, and $ΔT$ is the temperature difference between the parallel bottom and top plates separated by vertical distance $L$. The properties of the working fluid are characterized by its heat conductivity, $λ$, and by the combination $α/(νκ)$, where $α$ is the isobaric thermal expansion, $ν$ is the kinematic viscosity, and $κ$ denotes the thermal diffusivity. Functional dependence $Nu = Nu(Ra, Pr)$ at high $Ra$, usually expressed as a scaling law $Nu \propto Ra^{2/7} Pr^{3}$, is intensively studied theoretically, numerically and experimentally; see recent review by Ahlers, Grossmann & Lohse (2009) and references therein. Of course the most important issue is to find out whether or not at very high $Ra$ the boundary layer undergoes a laminar-turbulent transition when convection enters...
the “ultimate”, “asymptotic” regime, with $Nu \propto Ra^{1/2} Pr^{-1/4} (\log Ra)^{-3/2}$ and $0.15 < Pr < 1$, as predicted by Kraichnan.

It has been demonstrated by Niemela, Skrbek, Sreenivasan & Donnelly (2000) that utilization of cryogenic $^4$He gas as a working fluid offers an outstanding possibility to achieve the so far highest possible $Ra \approx 10^{17}$ under controlled laboratory conditions, thanks to the extremely large value of its fluid-properties ratio $\alpha \nu^{-1} \kappa^{-1}$ near the critical point ($T_c = 5.1953$ K, $p_c = 227.46$ kPa, $\rho_c = 69.641$ kg/m$^3$). Moreover, this ratio can easily be tuned over a wide range in situ within a single experimental run, as was used in a number of similar cryogenic studies. Indeed, this is why several experimental groups worldwide performed experimental studies of RBC in cryogenic conditions. We shall be mostly concerned with experiments by Roche, Gauthier, Kaiser, & Salort (2010) in Grenoble and by Niemela & Sreenivasan (2003) in Trieste performed in cylindrical experimental cells with aspect ratio $\Gamma$ close to unity.

In this paper we show that the cryogenic environment is indeed highly favorable for experimental studies of RBC, but care must be taken to design the experimental cell in such a way that cryogenic conditions are utilized in an advantageous way. There are a number of subtle experimental details that could appreciably affect the studied convective heat transport and have to be taken into account, some of which we discuss here in detail.

2. Cryogenic apparatus to study RBC

We have built an experimental apparatus schematically shown in Fig. 1 to study turbulent RBC in a cylindrical cell of height 0.3 m, diameter $2R = 300$ mm; for a detailed technical description see Urban, Hanzelka, Kralik, Musilova, Skrbek & Srnka (2010). It is designed to minimize the influence of its structure on the convective flow of cryogenic $^4$He gas of Prandtl number $Pr \approx 1$, with the aim of resolving existing contradictions in $Nu(Ra)$ scaling. The $\Gamma = 1$ cell (see Fig. 2) is designed to withstand pressure up to 350 kPa. The sidewall parasitic heat fluxes into the working fluid are minimized by using very thin ($\delta = 0.5$ mm) stainless steel sidewalls of low thermal conductivity $\lambda_w$, as well as thanks to a special design of the cell corners – see lower inset of Fig. 1 in (Urban, Musilova & Skrbek, 2011).

![Figure 1. Schematic view of the cryogenic apparatus.](image1)

![Figure 2. 3D view of the experimental cell.](image2)

The top and bottom plates (surface roughness $\approx 1.6 \mu m$) are made of 28 mm thick annealed OFHC copper of thermal conductivity $\lambda_p$ at least 2 kWm$^{-1}$K$^{-1}$. This value was measured at 5 K
using a thin machined-off sample that underwent the same heat treatment (additional annealing in the process of brazing) as both plates. The upper plate is thermally connected to the liquid helium (LHe) vessel via a heat exchange chamber filled with gaseous $^4$He and of nearly the same diameter as the cell. Design of the heaters glued in the spiral grooves milled on the external sides of plates ensures better than 1 mK temperature homogeneity of the internal side of plates, under the assumption that the heat is uniformly supplied or removed. Four calibrated Lake Shore GR-200A-1500-1.4B Ge temperature sensors (5 mK absolute accuracy guaranteed by the manufacturer for two of them, additional calibration – see Urban, Hanzelka, Kralik, Musilova, Skrbek & Srnka (2010) – allows determination of $\Delta T$ within 2 mK) are imbedded in the center and near the edge of Cu plates.

The top plate temperature is roughly set by the heat exchange chamber pressure and stabilized precisely by the heater using the Lake Shore 340 temperature controller. The pressure in the cell is measured with the MKS Baratron 690A (calibration traceable to NIST) with 0.08% reading accuracy. Helium properties are gained from the NIST database, based on the actual pressure in the cell and the mean temperature $T_m$ assessed as arithmetic average of the plate temperatures. The cell is filled via a filling tube that connects it with the cold valve placed on the bottom of the LHe vessel.

![Figure 3](time_record_of_pressure.png)  
**Figure 3.** Time record of pressure in the cell due to thermal acoustic oscillations.

![Figure 4](power_spectrum.png)  
**Figure 4.** Power spectrum of pressure fluctuations due to thermal acoustic oscillations.

3. Experimental results

Both the raw and corrected $Nu$ data obtained with densities from 0.04 kg/m$^3$ to 37 kg/m$^3$ and $50 \text{ mK} \leq \Delta T \leq 1.7$ K have already been published in a Letter form: we direct the reader to Urban, Musilova & Skrbek (2011); see also Fig. 6. We stress that the measuring protocol has been chosen to stay sufficiently away from the critical point. Without applying any corrections, for $Ra \lesssim 10^{11}$ we obtain $Nu \propto Ra^{\gamma}$ with the power exponent $\gamma \approx 2/7$. For higher $Ra$ the observed $Nu(Ra)$ dependence is consistent with a power law with $\gamma \approx 1/3$. This crossover in $\gamma$ takes place together with slight increase in $Pr$, crossing the value of one. There is no sign of a transition to the ultimate Kraichnan regime.

4. Experimental problems and corrections to raw data

4.1. Thermal acoustic oscillations

When testing the newly built apparatus, we encountered problems with the thermal acoustic oscillations (TAO) in the venting tubes, as mentioned in Urban, Hanzelka, Kralik, Musilova,
Skrbek & Srnka (2010). Fig. 3 and 4 show the time record of pressure in the cell and its power spectrum, indicating TAO at frequency slightly above 20 Hz. Suppressing TAO, which among other potential problems lead to additional parasitic heat leak to the cell, represents an experimental challenge that was solved by using of additional volumes connected to the tubes at room temperature, resulting in suppression of the observed pressure oscillations by at lest two orders of magnitude.

4.2. Corrections to parasitic heat leak and adiabatic temperature gradient
The radiation heat load is reduced by a cell radiation shield thermally anchored to the LHe vessel. The shield temperature does not exceed 7 K, thus the total external parasitic heat leak to the cell (both radiative and conductive) is suppressed to less than 1% of the lowest convective heat flux used in the experiment, measured within 0.5% accuracy. The design of the cell leads to correction to the adiabatic temperature gradient of less than 1 mK. Within our experimental protocol (ΔT > 50 mK) this correction therefore hardly matters.

4.3. Sidewall corrections
One way to estimate the influence of the sidewall is via the wall parameter \( W = 2\lambda_w\delta^{-1}R^{-1} \) introduced by Roche, Castaing, Chabaud, Hébral & J. Sommeria (2001). For our cell \( W \approx 0.16 \), while estimated values for the Grenoble \( (W \approx 0.48) \) and Oregon/Trieste \( (W \approx 0.58) \) cells are higher, leading to larger sidewall corrections. Moreover, as we already argued in Urban, Musilova & Skrbek (2011), the thick stainless steel flanges adjacent to the copper plates in the Oregon/Trieste cell require introduction of an effective wall thickness \( \delta \approx 6 \text{ mm} \), leading to even higher effective value of \( W \). We have shown that for \( 7.2 \times 10^6 \leq Ra \leq 10^{11} \) our sidewall corrected data agree with suitably corrected data from these similar cryogenic experiments and are consistent with \( Nu \sim Ra^{2/7} \). On approaching \( Ra \approx 10^{11} \) our data display a crossover to \( Nu \sim Ra^{1/3} \) (as predicted by analytical theory) that approximately holds up to \( Ra \approx 4.6 \times 10^{13} \). Differences in \( Nu(Ra) \) scaling observed in similar RBC experiments for \( Ra > 10^{11} \) cannot be attributed to sidewall corrections, as they do not play a significant role any more here, and cannot be explained due to the difference in \( Pr \) either.

4.4. Comments on finite heat conductivity and heat capacity of plates
Unlike the sidewalls, the bottom and the top plates of thickness \( a \) are expected to influence the observed convection at high values of \( Ra \), due to their finite heat conductivity and heat capacity. Based on the numerical simulations of Verzicco (2004) and the experiments with water by Funfschilling, Brown, Nikolaenko & Ahlers (2005) and with SF6 by Ahlers, Funfschilling & Bodenschatz (2009) it is expected that finite conduction of plates will suppress the \( Nu \) values. Taking into account the dynamics of plumes, Chilla, Rastello, Chaumat & Castaing (2004) derived approximate criteria to account for the “plates effect”. According to their “thin plates criterion” (which for our plates is more stringent than their “thick plates criterion”), the condition

\[
Cr = \left( \frac{\pi}{\Gamma} \right)^2 \frac{a\lambda_p}{L\lambda} \frac{1}{Re Pr} > 1
\]

means negligible “plates effect” in RBC experiments with cryogenic He. Fig. 5 displays the \( Cr \) values evaluated for our cell. We have also estimated values of \( Cr \) for the cryogenic Oregon/Trieste and the Grenoble cells of aspect ratio \( \Gamma \) about unity; for the same \( Ra \) they are similar in magnitude but lower than ours. We conclude that, on the basis of this criterion (again confirming our cell design as suitable for cryogenic RBC studies), due to the finite conductivity of plates, the observed differences between the cryogenic RBC experiments at high \( Ra > 10^{11} \) cannot be interpreted as “effect of plates”.
Figure 5. The $Cr$ parameter evaluated for our cell using equation 2. The data points above the dotted horizontal line satisfy the criterion $Cr > 1$.

Figure 6. Spurious increase of Nu as observed for high $T_m$ as indicated; corrected data points (yellow filled circles) are also shown.

4.5. Influence of the “chimney effect” in venting tubes

Two stainless steel tubes are used for venting of our RBC cell; their inlets positioned in the cell sidewall at 2/3 of the cell height (see Fig. 2 and 1). To reduce conductive parasitic heat leak both tubes are thermally anchored to the LHe vessel. As shown in Fig. 6, the test measurements at deliberately substantially higher mean temperature of the RBC cell $T_m$ reveal a spurious increase in $Nu$ that can be explained by convection inside these venting tubes between the cell inlet (at $\approx T_m$) and the thermal anchor at the LHe vessel 140 mm above. We have attempted to correct these data points by assessment of the convection in the filling tubes using the same $Nu(Ra)$ scaling as observed in the cell. We assume the existence of a parallel heat transfer channeling the heat directly to the LHe vessel, thus bypassing the upper plate. Quantitatively, we use only a half of this parallel heat channel (i.e., convective heat transfer in one of the two identical venting tubes only), as this process does not affect the heat transfer in the bottom part of the cell. We understand that justifying any realistic quantitative model for this “chimney effect” is hardly possible, nevertheless, Fig. 6 shows that such a correction works surprisingly well.

Rather than justifying various corrections to the observed data, we decided to improve the design of the cell to eliminate this “chimney effect” completely. For this purpose we have mounted electrical resistance heaters and thermometers to both venting tubes at the level of their thermal anchor to the LHe vessel (see Fig. 1) that allow adjusting and controlling the local temperature to $T \gtrsim T_m$ during measurements.

5. Conclusions

We have designed and constructed a new RBC apparatus containing a cell designed to minimize the influence of its structure on the convective flow of cryogenic $^4$He gas and re-measured the $Nu(Ra)$ dependence for $7.2 \times 10^6 \leq Ra \leq 4.6 \times 10^{13}$ at $0.67 < Pr < 2.4$. High $Ra$ are attained with cryogenic helium gas sufficiently far away from its critical point. The measured $Nu$ (both corrected and uncorrected values) obey, at least approximately, $Nu(Ra)$ power law scaling with exponent $\gamma \approx 2/7$ in the region $7.2 \times 10^6 \leq Ra \lesssim 10^9$ where $Pr < 1$; within the next 2-3 decades of $Ra$ the power exponent slowly increases and approaches $\gamma \approx 1/3$ which, on slightly increasing $Pr$, holds approximately up to $Ra \approx 4.6 \times 10^{13}$. By applying suitable sidewall corrections, we show full agreement among $\Gamma \approx 1$ cryogenic experiments for $Ra$ up to about $10^{11}$, while at higher $Ra$ all these sets of data differ considerably.
In order to explain distinctly different $Nu(Ra)$ scaling as observed in various high $Ra$ experiments for $Ra > 10^{11}$, we have performed additional test measurements and discussed various experimental issues that likely affect the RBC experiments as well as corrections that might be applied to the raw data. It follows that experimental details such as influence of the parasitic heat leak, additional corrections to finite heat conductivity and heat capacity of plates (including perhaps adjacent parts of sidewall) and/or rather more physical reasons such as various quantitative measures of Boussinesq conditions (not discussed in this paper) will have to be carefully considered.

We improved our apparatus in various ways in order to take advantage of the cryogenic environment and aim to use it for detailed investigations of heat transport by turbulent convection up to at least $Ra \approx 10^{15}$.

Acknowledgments

We thank P. Hanzelka and A. Srnka for technical help during the experiments. This work was supported by the Grant Agency of the Academy of Sciences GAAV under KJB200650902 and by research plans AV0Z20650511 and MS 0021620834 of the Czech Republic.

References

Ahlers, G. Grossmann, S. & Lohse, D. 2009 Heat transfer and large scale dynamics in turbulent Rayleigh-Benard convection, *Rev. Mod. Phys.* 81, 503–537.

Ahlers, G. Funfschilling, D. & Bodenschatz E. 2009 Transitions in heat transport by turbulent convection at Rayleigh numbers up to $10^{15}$, *New J. Phys.* 11, 123001.

Chilla, F. Rastello, M. Chaumat, S. & Castaing, B. 2004 Ultimate regime in Rayleigh-Bénard convection: The role of plates. *Phys. Fluids* 16, 2452–2456.

Funfschilling, D. Brown E., Nikolaenko A. & Ahlers G. 2005 Heat transport by turbulent Rayleigh-Benard convection in cylindrical samples with aspect ratio one and larger. *J. Fluid Mech.* 536, 145–154.

Niemela, J. J. Skrbek, L. Sreenivasan, K. R. & Donnelly, R. J. 2000 Turbulent convection at very high Rayleigh numbers. *Nature* 404, 837–840.

Niemela, J. J. & Sreenivasan, K. R. 2003 Confined turbulent convection. *J. Fluid Mech.* 481, 355–384.

Roche, P.-E. Gauthier, F. Kaiser, R. & Salort, J. 2010 On the triggering of the Ultimate Regime of convection. *New J. Phys.* 12, 085014.

Roche, P.-E. Castaing, B. Chabaud, B. Hébral, B. & Sommeria, J. 2001 Side wall effects in Rayleigh Benard experiments. *Eur. Phys. J. B* 24, 405–408.

Urban, P. Musilová, V. & Skrbek, L. 2011 On Efficiency of Heat Transfer in Turbulent Rayleigh-Bénard Convection. *Phys. Rev. Lett.*, 107, in print.

Urban, P. Hanzelka, P. Kralik, T. Musilova, V. Skrbek L. & Srnka, A. 2010 Helium cryostat for experimental study of natural turbulent convection. *Rev. Sci. Instrum.* 81, 085103.

Verzicco, R. 2004 Effect of nonperfect thermal sources in turbulent thermal convection. *Phys. Fluids* 16, 1965–1979.