Observation of a subharmonic gap singularity in interlayer tunneling characteristics of Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$

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A subharmonic structure in Josephson junctions appears due to Andreev reflections within the junction. Here we report on experimental observation of a subharmonic half-gap singularity in interlayer tunneling characteristics of a layered high temperature superconductor Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$. The singularity is most pronounced in optimally doped crystals and vanishes with decreasing doping. It indicates existence of non-vanishing electronic density of states and certain metallic properties in the intermediate BiO layers, which grows stronger with increasing doping. This provides an additional coherent interlayer transport channel and can explain a gradual transition from an incoherent quasi-two-dimensional c-axis transport in underdoped to a coherent metallic transport in overdoped cuprates. Furthermore, due to a very small sub-gap current, the singularity allows unambiguous extraction of the superconducting gap, without distortion by self-heating.

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The mechanism of interlayer (c-axis) transport in cuprate high temperature superconductors remains an actively debated subject. A qualitative difference between metallic in-plane and non-metallic out-of-plane resistivities [1] is a strong indication for predominantly incoherent nature of c-axis transport, which is achieved by interlayer hopping or tunneling [2–7]. The tunneling nature of c-axis transport leads to appearance of the intrinsic Josephson effect between CuO$_2$ planes in layered Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ (Bi-2212) cuprates at temperatures below $T_c$ [8]. However, the electronic system in cuprates is not strictly two-dimensional. This has been demonstrated by observation of bonding-antibonding bilayer splitting of electronic bands [9]. Indications for coherent transport were obtained in strong magnetic fields [10]. Since the two dimensional superconductivity is suppressed by fluctuations [11], presence or absence of the coherent metallic transport in the c-axis direction, i.e., in the third dimension, and the mechanism of interlayer coupling remain to be important issues for understanding high temperature superconductivity.

The intrinsic Josephson effect provides an accurate way of probing weak interlayer coupling in cuprates. Due to a d-wave symmetry of the order parameter, the product of the Josephson critical current $I_c$ and the normal resistance $R_n$ in intrinsic junctions should strongly depend on the coherence (momentum conservation) upon tunneling. The $I_cR_n$ is maximum $\sim \Delta/e$ for coherent, and zero for completely incoherent tunneling [12]. Here $\Delta$ is the maximum value of the superconducting energy gap. Analysis of $I_cR_n$ in intrinsic Josephson junctions indicated that in overdoped Bi-2212 interlayer tunneling is predominantly coherent $I_cR_n \sim \Delta/e$ [13] [14]. However, $eI_cR_n/\Delta$ rapidly decreases upon opening of the pseudo-gap in the underdoped state [13] [14]. This may either indicate that interlayer tunneling becomes progressively more incoherent with decreasing doping [13], or that the Fermi surface is reconstructed by the pseudogap [14].

Intrinsic Josephson junctions are characterized by low dissipation [15]. This is often taken as evidence for Superconductor-Insulator-Superconductor (SIS) structure of Bi-2212, in which S are superconducting CuO$_2$-Ca-CuO$_2$ bilayers and I is the insulating SrO-2BiO-SrO layer. Yet, this does not preclude that some of the layers in the SrO-2BiO-SrO stack are metallic, like in case of SINIS (N - is a normal metal) or SIS'IS junctions [16] [17], provided that transparency of the I interface is sufficiently low. The metallic behavior of the intermediate layer is manifested in appearance of subharmonic gap structure [18] [21] due to Andreev reflection of quasiparticles into Cooper pairs [22] at the SIN interface.

In this work we report on observation of the subharmonic gap singularity in intrinsic tunneling characteristics of small Bi-2212 mesa structures. This indicates presence of a finite electronic density of states in BiO layers. The singularity is most pronounced in optimally doped crystals and decreases with decreasing doping. The sub-gap singularity allows evaluation of the gap value at negligible self-heating. We demonstrate that the energy gap can be confidently extracted for more than two orders of magnitude variation of the dissipation power. This provides an important self-consistency check for intrinsic tunneling spectroscopy. Finally, we discuss consequences of the metallic behavior of BiO layers, which provides an additional coherent transport channel and can explain a gradual transition from a two-dimensional incoherent to a three-dimensional coherent c-axis transport with increasing doping.

We present data for three batches of single crystals: the Y-substituted Bi$_2$Sr$_2$Ca$_{1-x}$Y$_x$Cu$_2$O$_{8+\delta}$, Bi(Y)-2212), with the maximum $T_c \approx 94.5$K; the lead-doped Bi$_2-2x$Pb$_x$Sr$_2$Ca$_1$Cu$_2$O$_{8+\delta}$, Bi(Pb)-2212, with the maximum $T_c \approx 93$K, and the pure Bi-2212 with the maximum $T_c \approx 86$K. Small mesa structures were made on freshly...
cleaved crystals using micro/nano-fabrication techniques. Details of mesa fabrication and characterization are described elsewhere [23-26]. All presented measurements are performed at ambient magnetic field. The doping state of mesas was determined from a systematic study of all the characteristics, including (but not only) the $T_c$. Details, including the raw experimental data for different doping states can be found in Refs. [13, 14, 16]. In particular, in Ref. [14] oxygen-doped mesas from the same batch were studied. It was shown that the critical current of the mesas strongly (almost exponentially) depends on doping. This provides an accurate way of determination of doping close to optimal doping where $T_c$ vs. doping is flat.

Figure 1 (a) shows the current-voltage ($I$-$V$) characteristics of a near optimally doped Bi(Y)-2212 mesa with $T_c \approx 93$ K and $N = 9$ intrinsic Josephson junctions, at $T = 4.9$, 40 and 67 K. A pronounced current step occurs at the sum-gap voltage $V_{sg}/N = 2\Delta/e$, followed by the ohmic and almost $T$-independent resistance [23]. Such a behavior is typical for superconducting tunnel junctions [24]. Fig. 1 (b) demonstrates that the $I$-$V$ curves at different $T$ merge in one when both the voltage and the current scales are normalized by $\Delta(T)$. This is expected for SIS junctions, in which not only the voltage, but also the current scale is proportional to the superconducting gap, as seen from theoretical $I$-$V$ curves in Fig. 3 (a). Fig. 1 (c) shows normalized $dI/dV(V)$ characteristics for the same mesa. A sharp sum-gap peak occurs at $eV_{sg}/N\Delta = 2$. Simultaneously, we notice an additional bump at a half of the peak voltage, $eV/N\Delta \approx 1$. This subharmonic gap feature will be in focus of this work. The subharmonic feature is rapidly smeared out with increasing temperature, but can be traced using higher derivatives $d^2I/dV^2$, Fig. 1(d), and $d^3I/dV^3$, Fig. 1(e).

With decreasing doping the sum-gap peak is decreasing in amplitude [13, 14]. Fig. 1(f) shows a $dI/dV$ characteristics of a moderately underdoped Bi(Y)-2212 mesa with $T_c \approx 91$ K at $T = 16$ K (note the semi-logarithmic scale). It is seen that for the underdoped mesa both the sum-gap peak and the subharmonic bump have significantly smaller amplitudes than for the near optimally doped case, Fig. 1(c).

The sum-gap and the subharmonic singularities are not the only spectroscopic features in intrinsic tunneling characteristics. A double-arrow in Fig. 1(f) points at a small dip in conductance, which occurs at approximately twice the sum-gap peak voltage, i.e., $eV/N \sim 4\Delta$. The inset in Fig. 1(f) shows a zoom-in on this feature. As discussed in Refs. [30, 31], this dip is caused by reabsorption of nonequilibrium bosons generated upon relaxation of injected electrons.

Figure 2 demonstrates doping dependence of the subharmonic singularity. Fig. 2 (a) shows sub-gap parts
of differential conductance $dI/dV$ vs voltage per junction for an overdoped Bi(Pb)-2212 mesa. It is seen that in overdoped crystals the sub-gap conductance is dominated by strong phonon resonances [27,29], which appear at temperature-independent voltages. Presence of phonon resonances makes it difficult to analyze the sub-harmonic gap structure. Fig. 2 (b) shows $dI/dV(V/N)$ curves for optimally doped and underdoped mesas. It is seen that with decrease of doping the subharmonic feature is strongly decreased and gets completely washed away in moderately underdoped crystals [23] [24], while the sum-gap peak is still clearly visible. This may indicate changes in interlayer transport mechanism with doping [23].

A pronounced subharmonic gap structure at $V_n = 2\Delta/e(n+1)$, $n = 1,2,...$ has been observed in superconducting point contacts [32] and SIS junctions with pinholes [20]. The subharmonic structure is usually attributed to multiple Andreev reflections in quasi one-dimensional quantum channels [21]. The $d$-wave symmetry of the order parameter in cuprates should not destroy the subharmonic structure, but can affect its shape. In particular, it can cause an asymmetry between odd and even $n$ singularities as well as certain smearing due to angular dependence of the gap [33]. The sub-gap structure has indeed been observed in cuprate junctions [24, 27], although not all of it could be ascribed to Andreev reflections. For Bi-2212 cuprates the subharmonic gap structure has not been observed so far neither in point contacts [38], nor in intrinsic junctions, although some excess sub-gap noise was reported for the latter [39], which might be related to Andreev reflections [33].

A single sub-gap feature reported here is hard to reconcile with one-dimensional pinholes in the barrier, for which one would expect to see a series of subharmonic singularities [20]. Rather it closely resembles the characteristics of homogeneous two-dimensional SINIS junctions [19, 38]. In SINIS junctions quasiparticles can travel at arbitrary angles with respect to the interface. The corresponding angular averaging leads to a significant smearing of the subharmonic structure so that only a leading $n = 1$ singularity remains distinguishable. Thus, observation of a single subharmonic feature may indicate presence of a finite metallic conductivity in BiO layers.

This, however, is not the only possible interpretation. A similar single subharmonic feature at $\epsilon V = \Delta$ occurs also in SIS junction when $S$ is a gapless superconductor with a finite electronic density of states at the Fermi level. The gaplessness can originate from nodes in the gap in combination with partly incoherent (momentum non-conserving) tunneling and from impurity scattering.

Fig. 3 presents numerical simulations for a gapless SIS junction, in which the finite density of states at the Fermi level was introducing adding a depairing factor (inverse quasiparticle lifetime) $\Gamma = 2$ meV to the conventional BCS density of states (for details see e.g., the Supplementary material to Ref. [39]). Fig. 3 (a) shows calculated $I$-$V$ curves at $T = 4.9K$ and $40K$, with both $I$- and $V$-scales normalized by $\Delta(T)$. Gaps and temperatures are the same as in Fig. 1 (a) to facilitate a direct comparison. A single subharmonic singularity can be seen as an approximately linear upturn of the quasiparticle current at $V \sim \Delta/e$, indicated by dashed lines in Figs. 1 (b) and 3 (a). The corresponding $dI/dV(V)$, $d^2I/dV^2$ and $d^3I/dV^3$ curves, normalized by $\Delta(T)$, are shown in Figs. 3 (b), (c) and (d), respectively. Those theoretical curves are consistent with experimental data from Figs. 1 (c-e).

The subharmonic feature in this case is entirely due to gaplessness and has the same origin as the singularity at $\epsilon V = \Delta$ in SIN junctions.

Thus, a single half-gap singularity is expected both in SINIS junctions with finite density of states in the intermediate N-layer and in gapless SIS junctions with finite density of states in the S-layers. For our intrinsic junctions those two cases would correspond
to a finite metallic density of states in BiO layers or to a
gapless case with a finite density of states in CuO₂ lay-
ers. It is difficult to discriminate those scenarios just by
looking at the shapes of $dI/dV$ curves because the
latter look very similar in both cases. However, a cer-
tain discrimination between those two scenarios can be
made from the analysis of doping evolution of the singu-
larity. As seen from Fig. 2 (b), the subharmonic singular-
ity is rapidly decreasing with decreasing doping. From
angular resolved photoemission [40, 41] and surface tun-
neling spectroscopy [42] it is known that the depairing
factor $\Gamma$ is increasing with decreasing doping. There-
fore, for the gapless SIS junction scenario the residual
density of states at the Fermi level should not decrease
with decreasing doping and the subharmonic singular-
ity should still remain visible in underdoped junctions,
which is not the case. To the contrary, for the SINIS sce-
nario with BiO being the N-layer, it is expected that the
density of states in the BiO layer will gradually decrease
with decreasing doping and will eventually vanish upon
approaching the insulating state. Consequently, the ob-
served disappearance of the subharmonic singularity in
moderately underdoped intrinsic junctions is consistent
with SINIS case and implies presence of finite metallic
properties in BiO layers.

Observation of the subharmonic singularity allows ac-
curate extraction of the genuine gap value, not affected
by self-heating. Indeed, due to a smallness of the sub-
gap current, the subharmonic singularity corresponds to
a very small dissipation power and, therefore, is free from
self-heating. For example, for the small BiO(2212) mesa
from Fig. 1 (f), $P = 0.01$ mW at the sub-gap, 0.21 mW
at the sum-gap and 1.21 mW at the four gap singular-
ities. Fig. 4 (a) and (b) show $T-$ dependencies of the
sum-gap voltage (solid squares) and the sub-gap singu-
larities for (a) an optimally doped Bi(Y)-2212 mesa and
(b) for an underdoped Bi-2212 mesa. Solid lines represen-
t the subharmonic half-gap singularity, rather than
phonon resonances, which are $T$-independent [28, 29],
as seen from Fig. 2 (a).

Figures 4 (a) and (b) show $T-$ dependencies of the
sum-gap voltage (solid squares) and the sub-gap singu-
larities for (a) the optimally doped Bi(Y)-
2212 mesa from Fig. 1 and (b) for a moderately un-
derdoped Bi-2212 mesa with $T_c \simeq 80 K$, studied in Ref.
[29]. The solid lines represent half of the sum-gap voltage
and demonstrates that the sub-gap feature indeed indi-

cates the subharmonic half-gap singularity, rather than
phonon resonances, which are $T$-independent [28, 29],
as seen from Fig. 2 (a).

FIG. 3: (Color online). Numerically calculated characteristics for a gapless SIS junction with a finite depairing factor $\Gamma = 2$
meV and with parameters of optimally doped Bi(Y)-2212 mesa from Fig. 1 (a) $I-V$ characteristics at $T = 4.9$ and 40 K scaled
by $\Delta(T)$. (b), (c) and (d) first, second and third derivatives (curves for different $T$ are shifted vertically for clarity). Note appearance of the sub-harmonic singularity at $eV = \Delta$ due to the gaplessness.

FIG. 4: (Color online). (a) and (b) Temperature dependencies of the sum-gap (squares) and the sub-gap singularities for (a)
an optimally doped Bi(Y)-2212 mesa from Fig. 1 and (b) for an underdoped Bi-2212 mesa. Solid lines represents half the
sum-gap voltage. The dashed line in (a) shows the conventional BCS $T$-dependence of the superconducting gap. (c) Dissipation
power dependence of the gap extracted from the sub-harmonic (circle) sum-gap (square) and four-gap (rhombus) singularities
for a small underdoped Bi(Y)-2212 mesa from Fig. 1 (f). Solid line represents the BCS gap at the effective mesa temperature.

Due to self-heating [24, 43] the gap is gradually decreas-
ing with decreasing power and, therefore, is free from
self-heating. For example, for the small Bi(Y)-2212 mesa from
Fig. 1 (f), $P = 0.01$ mW at the sub-gap, 0.21 mW
at the sum-gap and 1.21 mW at the four gap singular-
ities. Fig. 4 (c) shows gap values obtained from those
three singularities as a function of the dissipation power.

Theory: $T_{\text{eff}} = 26.5$ K/mW
ing with increasing $P$ because the effective temperature of the mesa $T_{eff} = T + R_{th}P$ is elevated above the base temperature $T$. Here $R_{th}$ is the thermal resistance of the mesa. The solid line in Fig. 4 (c) represents the BCS $\Delta$ vs $T_{eff}$ dependence (top axis) obtained using $R_{th}$ as a fitting parameter. The corresponding $R_{th} = 26.5$ K/mW is in good agreement with the values obtained by in-situ measurements of self-heating \[23\] and from the analysis of size-dependence of intrinsic tunneling characteristics \[24\] on similar mesas.

From Fig. 4 (c) it is seen that the energy gap can be confidently obtained from intrinsic tunneling spectroscopy on small Bi-2212 mesas within more than two orders of magnitudes of the dissipation power. In all cases, reported here, self-heating at the subharmonic singularity is negligible. Therefore, this singularity provides a decisive self-consistency test and confirms accurate extraction of the energy gap by intrinsic tunneling spectroscopy made on small mesas \[23\].

To conclude, we reported on observation of subharmonic half-gap singularity in interlayer tunneling characteristics of Bi-2212 cuprates. The subharmonic singularity allows unambiguous determination of the energy gap because it occurs at a very small sub-gap current and negligible self-heating. This is an important step for development of intrinsic tunneling spectroscopy of cuprates. We have argued that the subharmonic singularity is a manifestation of the finite electronic density of states at the Fermi level in the superconducting state. The observed doping dependence indicated that the phenomenon is most likely brought forward by metallic behavior of intermediate BiO layer. The latter may have a significant influence on properties of layered cuprates. For example, it is well established that the anisotropy in cuprates is strongly doping dependent \[1\]. For Bi-2212 it changes from about a million in underdoped to a hundred in overdoped state. For YBa$_2$Cu$_3$O$_{7-x}$ it changes from about a hundred in underdoped (which even exhibit the intrinsic Josephson effect \[45\]) to about five in a slightly overdoped case. Such a behavior can be explained by a gradual enhancement of metallic properties of intermediate layers with increasing doping, which adds a coherent mechanism to interlayer transport, and provides a way for establishing a fully coherent three-dimensional $c$-axis transport in the strongly overdoped metallic (Fermi liquid) state. To the contrary, in the underdoped state the metallic behavior of BiO layers becomes weak. When the sheet resistance exceeds the quantum resistance, $h/e^2$, a metal-insulator transition takes place due to Coulomb blocking of transport \[44\]. The corresponding Coulomb energy can represent one of the contributions to the $c$-axis pseudogap phenomenon and is consistent with recent observation of an additional "dressed" electron energy in interlayer tunneling characteristics of underdoped Bi-2212 \[46\].

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