PolyFit: Polynomial-based Indexing Approach for Fast Approximate Range Aggregate Queries

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Abstract—Range aggregate queries find frequent application in data analytics. In some use cases, approximate results are preferred over accurate results if they can be computed rapidly and satisfy approximation guarantees. Inspired by a recent indexing approach, we provide means of representing a discrete point data set by continuous functions that can then serve as compact index structures. More specifically, we develop a polynomial-based indexing approach, called PolyFit, for processing approximate range aggregate queries. PolyFit is capable of supporting multiple types of range aggregate queries, including COUNT, SUM, MIN and MAX aggregates, with guaranteed absolute and relative error bounds. Experiment results show that PolyFit is faster and more accurate and compact than existing learned index structures.

I. INTRODUCTION

A range aggregate query [29] retrieves records in a data set that belong to a given key range and then applies an aggregate function (e.g., SUM, COUNT, MIN, MAX) on a attribute of those records. Range aggregate queries are used in OLAP [29], [52] and data analytics applications, e.g., for outlier detection [34], [56], data visualization [18] and tweet analysis [40]. For example, network intrusion detection systems [56] utilize range COUNT queries to monitor a network for anomalous activities. In many application scenarios, users accept approximate results provided that (i) they can be computed quickly and (ii) they are sufficiently accurate (e.g., within 5% error). We target such application and focus on approximate evaluation of range aggregate queries with error guarantee.

A recent indexing approach represents the values of attributes in a dataset by continuous functions, which then serve to enable compact index structures [20], [34]. When compared to traditional index structures, this approach is able to yield a smaller index size and faster response time. The existing studies [20], [34] focus on computing exact results for point and range queries on 1-dimensional data. In contrast, we conduct a comprehensive study of approximate range aggregate queries, supporting many aggregate functions and multi-dimensional data.

The idea that underlies our proposal for using functions to answer approximate range aggregate queries may be explained as follows. Consider a stock market index (e.g., the Hong Kong Hang Seng Index) at different time as a dataset $D$ consisting of records of the form (index value, timestamp), where the former is our measure and the latter is our key that is used for specifying query ranges—see Figure 1(a). A user can find the average stock market index value in a specified time range $[l_q, u_q]$ by issuing a range SUM query. We propose to construct the cumulative function of $D$ as shown in Figure 1(b). If we can approximate this function well by a polynomial function $P(x)$ then the range SUM query can be approximated as $P(u_q) - P(l_q)$, which takes $O(1)$ time. As another example, the user wishes to find the maximum stock market index in a specified time range. The timestamped index values in $D$ can be modeled by the continuous function in Figure 1(c). Again, if we can approximate this function well using a polynomial function $P(x)$ then the range MAX query can be answered quickly using mathematical tools, e.g., by applying differentiation to identify maxima in $P(x)$.

Given a two-dimensional dataset of tweets’ locations as shown in Figure 2(a), where each data point has a longitude (as key 1) and a latitude (as key 2), our idea works as follows. Suppose that the user wishes to count the number of tweets in a geographical region. With this dataset, we can derive the cumulative count function shown in Figure 2(b). By approximating this function with a polynomial function $P(x_1, x_2)$ (of two variables), a two-dimensional range COUNT query can be answered in $O(1)$ time, as we will explain in Section VI.

Another difference between our work and existing studies [20], [34] lies in the types of functions used. Our proposal uses piecewise polynomial functions, rather than piecewise linear functions [20], [34]. As we will show in Section IV polynomial functions yield lower fitting errors than the linear functions. Thus, our proposal leads to smaller index size and faster queries.

The technical challenges are as follows. (1) How to find polynomial functions with low approximation error efficiently? (2) How to answer range aggregate queries with error guarantees? (3) How to support multiple aggregate functions (e.g., COUNT, SUM, MIN, MAX) and data with dimensionality higher than one?

To tackle these challenges, we develop the polynomial-based indexing approach (PolyFit) for processing approximate
range aggregate queries. Our contributions are summarized as follows.

- To the best of our knowledge, this is the first study that utilizes polynomial functions to learn indexes that support approximate range aggregate queries.
- PolyFit supports multiple types of range aggregate queries, including COUNT, SUM, MIN, and MAX with guaranteed absolute and relative error bounds.
- Experiment results show that PolyFit achieves significant speedups, compared with the closest related works [20], [34], and traditional exact/approximate methods.

II. RELATED WORK

Range aggregate queries are used frequently in analytics applications and constitute an important functionality in OLAP and data warehousing. [7], [8], [14], [17], [29], [31], [42], [52]. Exact solutions are based on prefix-sum arrays [29] or aggregate R-trees [45]. Due to the need for real-time performance in some applications (e.g., response time in the µs [56]), many proposals exist that aim to improve the efficiency of range aggregate queries. These proposals can be classified as being either data-driven or query-driven.

Data-driven proposals build statistical models of a dataset for estimating query selectivity or the results of range aggregate queries. These models employ multi-dimensional histograms [30], [39], [43], [51], data sampling [25], [27], [38], [50], kernel density estimation [23], [24], [28]. Although such proposals that compute approximate results are much faster than exact solutions, e.g., achieving ms (10^{-3}) level response time [46], they still do not offer real-time performance (e.g., µs level [56]). Furthermore, these proposals do not offer theoretical guarantees on the errors between the approximate results and the exact results.

The query-driven approaches utilize query workload to build statistical models of a dataset. Typical methods include error-feedback histograms [4], [6], [37] and max-entropy histograms [41], [49]. In addition, Park et al. [46] explore the approach of using mixture probabilistic models. These methods assume that new queries follow the historical query workload distribution. However, as a study observes in [9], this assumption may not always hold in practice. Further, even when this assumption is valid, the number of queries that are similar to those used for training may be much smaller if the queries follow a power law distribution [55], which can cause poor accuracy, and render it impossible to obtain useful theoretical guarantees on the errors of the approximate results of range aggregate queries.

Recently, learning has been used to construct more compact and effective index structure, that hold potential to accelerate database operations. Kraska et al. [34] propose the RMI index, which incorporates different machine learning models, e.g., linear regression and deep-learning, to improve the efficiency...
of range queries. Galakatos et al. [20] develop the FITing-tree, which is a segment tree [16] like structure with absolute error guarantee. Such that it’s able to efficiently evaluate exact point queries. Wang et al. [53] extend this learning idea to the spatial domain with their learned Z-order model that aims to support fast spatial indexing. These studies all differ from our study in two important ways. First, they either support range queries [34], [53] or point queries [20], not range aggregate queries. Second, we are the first to exploit polynomial functions to build index structures for approximate range aggregate queries.

Another related area could be time series data approximation, similar to the above mentioned learning approaches, researchers represent the huge amount of time series data by a bunch of models. Typical techniques for approximation include piecewise linear approximation [33], discrete wavelet transform [17], [47], discrete fourier transform [19], [48] and high-level approaches built on top of them [32], [44]. However, such techniques focus only on uniformly-spaced records which no longer holds in our scenarios, in addition, they only limits on 1-dimensional data and provide no relative error guarantee.

III. PRELIMINARIES

First, we define range aggregate queries and its approximate versions in Section III-A. Then, we discuss the baselines for solving the exact range aggregate queries in Section III-B. Table I summarizes the symbols that will be used for the remaining sections.

Table I: Symbols

| Symbol   | Description                              |
|----------|------------------------------------------|
| D        | dataset                                  |
| n        | number of records in D                   |
| Rcount   | range COUNT query                        |
| Rsum     | range SUM query                          |
| Rmin     | range MIN query                          |
| Rmax     | range MAX query                          |
| CFsum    | cumulative function for range SUM query  |
| DFmax    | key-measure function for range MAX query |
| P(k)     | polynomial function                     |
| I        | interval                                 |
| deg      | degree of polynomial function            |

A. Problem Definition

We focus on the setting that a range aggregate query specifies a key attribute (for range selection) and a measure attribute for aggregation. We shall consider the setting of multiple keys in Section VII. As such, the dataset D is a set of (key,measure) records, i.e., D = \{(k1,m1),(k2,m2),..., (kn, mn)\}. For ease of presentation, we assume that key values are distinct and every mi is non-negative.

We proceed to define a range aggregate query.

Definition 1. Let G be an aggregate function on measures (e.g., COUNT, SUM, MIN, MAX). Given a dataset D and a key range [l,q], we express the result of a range aggregate query, in terms of relational algebra operations [5], as follows:

\[ R_G(D, [l,q]) = G(\sigma_{k \in [l,q]}(D)) \] (1)

In this paper, we aim to develop efficient methods for obtaining an approximate result of \( R_G(D, [l,q]) \) with two types of theoretical guarantees, namely (1) absolute error guarantee [21], [22] (cf. Problem 1) and (2) relative error guarantee [21], [22] (cf. Problem 2).

Problem 1. [21], [22] Given an absolute error \( \varepsilon_{abs} \) and a range aggregate query, we ask for an approximate result \( A_{abs} \) such that:

\[ |A_{abs} - R_G(D, [l,q])| \leq \varepsilon_{abs} \] (2)

Problem 2. [21], [22] Given a relative error \( \varepsilon_{rel} \) and a range aggregate query, we ask for an approximate result \( A_{rel} \) such that:

\[ \frac{|A_{rel} - R_G(D, [l,q])|}{R_G(D, [l,q])} \leq \varepsilon_{rel} \] (3)

B. Baselines: Exact Methods

We proceed to discuss the exact methods for solving two types of range aggregate queries, which are SUM and MAX. These methods can be easily extended to support COUNT and MIN queries respectively.

1) Exact method for range SUM query: First, we define the key cumulative function as \( CF_{sum}(k) \):

\[ CF_{sum}(k) = R_{sum}(D, [l, \infty)) \] (4)

With the additive property of \( CF_{sum} \), we compute the exact result of the range SUM query as:

\[ R_{sum}(D, [l, u]) = CF_{sum}(u) - CF_{sum}(l) \] (5)

It remains to discuss how to obtain the terms \( CF_{sum}(l) \) and \( CF_{sum}(u) \) efficiently. Although \( CF_{sum} \) is a continuous function, it can be expressed by a discrete data structure with finite space. For this purpose, we can presort the dataset \( D \) by the ascending key order, then construct a key-cumulative array \( KCA \) of entries \( (k, CF_{sum}(k)) \) based on the keys in \( D \), as shown in Figure 3. At query time, the terms \( CF_{sum}(l) \) and \( CF_{sum}(u) \) can be obtained by performing binary search on the above key-cumulative array \( KCA \). This step takes \( O(\log n) \) time.

As a remark, our key-cumulative array is similar to the prefix-sum array [29]. The difference is that our array allows floating-point search key, but the prefix-sum array does not.

2) Exact method for range MAX query: First, we define the following key-measure function \( DF_{max}(k) \) to capture the data distribution in the dataset \( D \). An example of function \( DF_{max}(k) \) is shown in Figure 4(a).
\[DF_{\text{max}}(k) = \begin{cases} 
  m_i & \text{if } k_i \leq k < k_{i+1} \\
  m_{i+1} & \text{if } k_{i+1} \leq k < k_{i+2} \\
  \vdots & \text{if } k_{i} \leq k < k_{i+1} \\
  0 & \text{otherwise}
\end{cases} \quad (6)\]

To support the range \(\text{MAX}\) query, we may build the aggregate max-tree \([45]\) in Figure 4(b) in advance. In this tree, each internal node covers an interval and stores the maximum measure within that interval.

Figure 3: Key-cumulative array

\[\text{CF}_{\text{sum}}(k) = \begin{cases} 
  \text{CF}_{\text{sum}}(l_q) & \text{if } l_q \leq k < u_q \\
  \text{CF}_{\text{sum}}(u_q) & \text{otherwise}
\end{cases}\]

\[i_q \quad u_q \quad \text{CF}_{\text{sum}}(l_q) \quad \text{CF}_{\text{sum}}(u_q) \]

\[\text{HKG.Index (measure)} \quad \text{DF}_{\text{max}}(k) \quad \text{FIT}(k) \quad P(k) \quad \text{LR}(k)\]

We then discuss how to process the query \(R_{\text{max}}(D, [l_q, \text{CF}_{\text{sum}}(u_q)])\). The query range is indicated by the red line in Figure 4(a). In Figure 4(b), we start from the root of the tree and explore its children, i.e., \(N_1\) and \(N_2\). If a node (e.g., \(N_1\)) partially intersects with the query range, we visit its child nodes (e.g., \(N_3, N_4\)). When the interval of a node (e.g., \(N_4, N_5\)) is covered by the query range, we directly use its stored aggregate value without visiting its child nodes. During the traversal process, we keep track of the maximum measure seen so far. This procedure takes \(O(\log n)\) time as we check at most two branches per level.

Figure 4: Aggregate \(\text{MAX}\) tree

IV. INDEX CONSTRUCTION

Traditional index structures (e.g., B-tree \([15]\)) need to store \(n\) keys, where \(n\) is the cardinality of the dataset \(D\). Thus, the index size grows linearly with the data size. To reduce the index size dramatically, we plan to index a limited number of functions (instead of \(n\) keys).

In Section IV-A, we present our idea on polynomial fitting and compare it with existing fitting functions in \([20], [34]\). Then we introduce our indexing framework in Section IV-B. Next, we propose our method to construct the polynomial fitting for an interval (cf. Section IV-C). Finally, we discuss how to minimize our index size with respect to an error guarantee (cf. Section IV-D).

A. Why Polynomial Fitting?

Figure 5 shows the \(DF_{\text{max}}(k)\) function for the Hong Kong 40-Index in 2018 \([1]\). Observe that the shape of this \(DF_{\text{max}}(k)\) is complex, in which it is hard to use the linear functions, e.g., linear regression \([34]\) and linear segment \([20]\), to accurately approximate this function. In order to achieve better approximation, we adopt the polynomial function, which can capture the nonlinear property of \(DF_{\text{max}}(k)\). Observe from Figure 5 once we choose the degree-4 polynomial function (blue dotted line), we can achieve much better approximation, compared with all linear functions.

Figure 5: Hong Kong 40-Index in 2018 \([1]\), \(LR(k)\), \(FIT(k)\) and \(P(k)\) are linear regression, linear segment and degree-4 polynomial functions for approximating \(DF_{\text{max}}\) respectively.

B. Indexing Framework

To reduce the fitting error, we utilize multiple polynomial functions to accurately approximate the exact function \(F(k)\) (e.g., key-cumulative function/ key-measure function).

\[F(k) = \begin{cases} 
  \text{CF}_{\text{sum}}(k), & \text{if } g = \text{SUM} \\
  DF_{\text{max}}(k), & \text{if } g = \text{MAX}
\end{cases} \quad (7)\]

Our proposed framework is shown in Figure 6. In this framework, we build the index on top of these polynomial functions. Once the number \(m\) of polynomial functions is much smaller than \(n\), we can achieve significant efficiency improvement for searching, as the root to leaf path is much shorter.
C. Optimal Polynomial Fitting for an Interval

Observe from Figure 6 we need to use a polynomial function to represent a set of consecutive points in each interval, e.g., $I_1 = [l_1, u_1]$. In order to provide the good approximation, we ensure the difference between the $deg$th order polynomial function $P(k) = \sum_{j=0}^{deg} a_j k^j$ and each data point in a given interval to be as small as possible. Therefore, we can formulate the following optimization problem in Definition 4. As a remark, it is possible to use very high order polynomial function (i.e., large $deg$) to fit the points in a interval. However, with higher degree $deg$, the online evaluation cost is higher (more terms). We will discuss how to choose the suitable degree $deg$ of the polynomial function in the experimental section (cf. Section VII-D).

**Definition 4.** Let $F(k)$ be either the key-cumulative function or key-measure function and given an interval $I$ which contains a set of consecutive points $\{(k_1, F(k_1)), (k_2, F(k_2)), ..., (k_\ell, F(k_\ell))\}$ and the $deg$th order of the polynomial function, we aim to find those $deg$ coefficients, $a_0, a_1, ..., a_{deg}$ that can minimize the following error:

$$E(I) = \min_{a_0, a_1, ..., a_{deg}} \max_{1 \leq i \leq \ell} |F(k_i) - P(k_i)|$$

(8)

Based on some simple derivations, we can obtain the following linear programming problem which is equivalent to Equation 3:

\[
\begin{align*}
\text{MINIMIZE} \quad & t \\
\text{SUBJECT TO:} \quad & -t \leq F(k_1) - (a_{deg} k_1^{deg} + ... + a_2 k_1^2 + a_1 k_1 + a_0) \leq t \\
& -t \leq F(k_2) - (a_{deg} k_2^{deg} + ... + a_2 k_2^2 + a_1 k_2 + a_0) \leq t \\
& ... \\
& -t \leq F(k_\ell) - (a_{deg} k_\ell^{deg} + ... + a_2 k_\ell^2 + a_1 k_\ell + a_0) \leq t \\
& \forall a_i \in R
\end{align*}
\]

(9)

The time complexity for solving the linear programming problem (9) is in $O(2^{1.5})$ [35].

D. Index Size Minimization with Error Guarantee

In Section IV-C we discuss how to obtain the best fitting (with minimum error) for a set of consecutive points in an interval. However, it is generally hard to utilize only one polynomial function to fit on a whole dataset $D$ with a small error, e.g., $\delta$. As such, we impose the following bounded $\delta$-error constraint for fitting each interval (cf. Figure 7), which can be used in Section V to solve both Problems 1 and Problem 2. We formally state this constraint in Definition 5.

**Definition 5.** (Bounded $\delta$-error constraint) Given a $deg$th order polynomial function $P(k)$, this polynomial function can produce bounded $\delta$-error for interval $I$ if:

$$E(I) \leq \delta$$

(10)

Therefore, in order to build the small index for PolyFit, we aim to minimize the number of polynomials (i.e., $m$ in Figure 6). One approach is to utilize the dynamic programming (DP) method [36] to partition these $n$ keys in $D$, in which each segment can fulfill the bounded $\delta$-error constraint (cf. Definition 5). However, this method takes $O(n^2 \times l_{max}^{2.5})$.
time to achieve the optimal solution \[36\], where \(\ell_{\text{max}}\) is the maximum length of consecutive points (cf. Definition \[4\]). As such, it is not scalable to obtain the optimal solution with this method as \(n\) can be very large (e.g., million-scale).

Here, we propose the method, called greedy segmentation (GS), which adopts the greedy approach to segment the function \(F(k)\). We show that this method can achieve (1) optimal solution and (2) better worst case time complexity compared with the optimal DP solution \[36\]. Table II summarizes the performance. In this section, we first illustrate GS method (cf. Section \[IV-D1\]) and provide the theoretical analysis for this method (cf. Section \[IV-D2\]).

| Methods | Worst case |
|---------|------------|
| DP      | \(O(n \times \ell_{\text{max}})\) |
| GS      | \(O(n \times \ell_{\text{max}})\) |

1) Greedy Segmentation (GS) Method: Observe from Figure 8, GS method incrementally adds one more key into the interval \(I\) and then checks whether \(I\) can fulfill the bounded \(\delta\)-error constraint (cf. Definition \[5\]) by solving the linear programming problem (cf. Equation \[9\]). Once this algorithm inserts the first key (the first blue block in Figure 8) to \(I\), which makes Equation \[9\] bigger than \(\delta\), it reports the current interval (i.e., yellow segment) and starts finding another interval (e.g., blue segment), using the same approach.

![Figure 8: Idea of GS method for segmenting the consecutive keys (yellow, blue and purple consecutive blocks denote different segments/ intervals), where we denote the interval \(l\) as the yellow segment](image)

Since we need to solve \(O(n)\)-times linear programming problems and each of these problems takes at most \(O(\ell_{\text{max}}^2)\) time to solve \[35\], we conclude that the time complexity of GS method is \(O(n\ell_{\text{max}}^2)\). In practice, we can adopt the idea of exponential search \[10\] to further boost the efficiency performance.

2) GS is Optimal: To prove that GS can achieve optimal solution, we first illustrate the following monotonicity property (Lemma \[1\]) of our curve fitting problem (cf. Definition \[4\]).

\[\exists k, F(k) - P(k) > \delta\]
\[\forall k, F(k) - P(k) \leq \delta\]

\(\ell_{\text{max}}\)

Figure 8: Idea of GS method for segmenting the consecutive keys (yellow, blue and purple consecutive blocks denote different segments/ intervals), where we denote the interval \(l\) as the yellow segment.

\[\text{Lemma 1. Given two intervals } I_l \text{ and } I_u \text{ which contain two sets } S_l \text{ and } S_u \text{ of consecutive points respectively, if } S_l \subseteq S_u, \text{ then we have:}\]

\[E(I_l) \leq E(I_u)\]

Proof. Recall that the value of \(E(I)\) (cf. Equation \[3\]) is the same as the minimum value of the optimization problem \[9\]. Since the number of points in \(S_l\) is the subset of the number of points in \(S_u\), the set of constraints for solving \(E(I_l)\) is also the subset of the one for solving \(E(I_u)\). Observe from the optimization problem \[9\], once we have more constraints, the value \(t\) should be larger, which proves \(E(I_l) \leq E(I_u)\). \[\square\]

Lemma \[1\] implies that once the interval includes the newly added key (i.e., first blue block in Figure 8) and the absolute error for fitting this new set of consecutive points is bigger than \(\delta\), we cannot have longer segment, which covers the yellow segment, such that the error \(\delta\) can be fulfilled. Based on this property, we can then show that GS can provide the least number of intervals (cf. Theorem \[1\]), i.e., optimal solution.

\[\text{Theorem 1. GS can provide the least number of intervals.}\]

Proof. Let \(I_{\text{OPT}} = \{I_{\text{OPT}}(1), I_{\text{OPT}}(2), \ldots\}\) and \(I_{\text{GS}} = \{I_{\text{GS}}(1), I_{\text{GS}}(2), \ldots\}\) be two sets of intervals for optimal solution and our GS method respectively, where \(I_{\text{OPT}}\) and \(I_{\text{GS}}\) must cover the key domain. Without loss of generality, the intervals should fulfill the following conditions.

1. \(\max_k S_{l_{\text{OPT}}(k)}^{(1)}\) is just smaller than \(\min_k S_{u_{\text{OPT}}(k)}^{(+1)}\).
2. \(\max_k S_{l_{\text{GS}}(k)}^{(1)}\) is just smaller than \(\min_k S_{u_{\text{GS}}(k)}^{(+1)}\).

Since both GS and OPT must cover the key domain, we have:

\[\min_k S_{l_{\text{GS}}(k)}^{(1)} = \min_k S_{l_{\text{OPT}}(k)}^{(1)}\]

Recall that GS method includes the key into the segment one by one until it reaches the error threshold \(\delta\). Based on Lemma \[1\], we cannot have longer interval which covers \(S_{l_{\text{GS}}(k)}^{(1)}\) such that this interval has error smaller than \(\delta\). Therefore, as shown in Figure 9, we have:

\[\max_k S_{l_{\text{GS}}(k)}^{(1)} \geq \max_k S_{l_{\text{OPT}}(k)}^{(1)}\]  \(11\)

Based on the conditions (1) and (2), we have:

\[\min_k S_{l_{\text{GS}}(k)}^{(2)} \geq \min_k S_{l_{\text{OPT}}(k)}^{(2)}\]  \(12\)

Now, we assume that:

\[\max_k S_{l_{\text{GS}}(k)}^{(2)} < \max_k S_{l_{\text{OPT}}(k)}^{(2)}\]  \(13\)

Since GS method finds the largest interval such that \(I_{l_{\text{GS}}}^{(2)}\) fulfills the error guarantee \(\delta\), i.e., \(E(I_{l_{\text{GS}}}^{(2)}) \leq \delta\), we cannot find another interval which covers the set \(S_{l_{\text{GS}}(k)}^{(2)}\) of points but this interval still fulfills the error guarantee \(\delta\) (cf. Lemma \[1\]). However, based on Equations \[12\] and \[13\], we have \(S_{l_{\text{GS}}(k)}^{(2)} \subset S_{l_{\text{OPT}}(k)}^{(2)}\). However, based on the above argument, we have \(E(I_{\text{OPT}}) > \delta\), which contradicts to the correctness property of \(I_{\text{OPT}}\).
As such, Equation 13 is the wrong assumption (dashed red interval in Figure 9) and therefore:

$$\max_k S_{l^{(2)}_{OPT}} \geq \max_k S_{l^{(2)}_{GS}}$$ (14)

Based on Equations 11 and 14 and the similar argument, we can conclude, for any integer $\ell$, that:

$$\max_k S_{l_{OGS}}^{(\ell)} \geq \max_k S_{l_{OPT}}^{(\ell)}$$

which means GS method can cover more keys, compared with the optimal solution, given the same number of intervals.

Therefore, the number of intervals of $I_{GS}$ should be at most the same as the number of intervals of $I_{OPT}$. Based on the optimality condition of $I_{OPT}$, we can prove that GS method can achieve the least number of intervals.

V. QUERYING METHODS

Once we have built the PolyFit (cf. Figure 6), we can utilize this index structure and follow the querying framework (cf. Figure 10) to answer different types of approximate range aggregate queries with theoretical guarantee (i.e., Problems 1 and 2). In this section, we first discuss how to support the approximate SUM query in Section V-A. After that, we discuss how to support the approximate MAX query in Section V-B. As a remark, we can easily support both approximate SUM and MIN queries, which are the simple extension of the techniques for supporting approximate COUNT and MAX queries respectively.

A. Approximate range SUM Query

Recall from Section IV-D1, we ensure GS method can produce polynomial functions in each of which of them satisfies the bounded $\delta$-error constraint (cf. Definition 5) for each interval (cf. Figure 10). Here, we discuss how to specify this parameter $\delta$ in order to achieve the absolute error (cf. Problem 1) and relative error guarantee (cf. Problem 2) for $A_{abs}$ and $A_{rel}$ respectively.

**How to solve Problem 1?**

Based on Definition 5, we can conclude:

$$|CF_{sum}(l_q) - P_{l_q}(l_q)| \leq \delta$$

$$|CF_{sum}(u_q) - P_{l_q}(u_q)| \leq \delta$$

where $I_l$ and $I_u$ are two intervals which contain $l_q$ and $u_q$ respectively.

Once we let $A_{abs} = \mathbb{P}_{l_q}(u_q) - \mathbb{P}_{l_q}(l_q)$ and set $\delta = \frac{\varepsilon_{abs}}{2}$, we can solve Problem 1 (cf. Lemma 2).

**Lemma 2.** If we set $\delta = \frac{\varepsilon_{abs}}{2}$, we can satisfy $|A_{abs} - R_{sum}(D, [l_q, u_q])| \leq \varepsilon_{abs}$.

**Proof.** Based on some simple algebraic operations, we have:

$$CF_{sum}(u_q) - CF_{sum}(l_q) - 2\delta \leq A_{abs} \leq CF_{sum}(u_q) - CF_{sum}(l_q) + 2\delta$$

Recall from Equation 5, we have:

$$R_{sum}(D, [l_q, u_q]) - 2\delta \leq A_{abs} \leq R_{sum}(D, [l_q, u_q]) + 2\delta$$

Therefore, once we set $\delta = \frac{\varepsilon_{abs}}{2}$, we can achieve the absolute error guarantee $\varepsilon_{abs}$.

Since the absolute error $\varepsilon_{abs}$ is known in advance, we can build the PolyFit with $\delta = \frac{\varepsilon_{abs}}{2}$ (cf. Figure 9). As such, we can always pass the error condition $\varepsilon_{abs}$ in this case (cf. Figure 10).

**How to solve Problem 2?**

To achieve the relative error guarantee $\varepsilon_{rel}$, we can adopt the similar concept of solving Problem 1. We let $A_{rel} = \mathbb{P}_{l_q}(u_q) - \mathbb{P}_{l_q}(l_q)$. Therefore, with the similar concept of the proof in Lemma 2, we also have:

$$|A_{rel} - R_{sum}(D, [l_q, u_q])| \leq 2\delta$$ (15)

Based on some simple algebraic operations, we can also achieve:

$$R_{sum}(D, [l_q, u_q]) \geq A_{rel} - 2\delta$$ (16)

By dividing Equation 15 with Equation 16, we can achieve the following relative error.

$$\frac{|A_{rel} - R_{sum}(D, [l_q, u_q])|}{R_{sum}(D, [l_q, u_q])} \leq \frac{2\delta}{A_{rel} - 2\delta}$$

As such, once we ensure $\frac{2\delta}{A_{rel} - 2\delta} \leq \varepsilon_{rel}$, we can solve Problem 2 which is stated in Lemma 2. We omit the proof as this is trivial.

**Lemma 3.** If $A_{rel} \geq 2\delta(1 + \frac{1}{\varepsilon_{rel}})$, we can achieve the relative error $\varepsilon_{rel}$.

However, unlike the above method for solving Problem 1 even though we preknow $\varepsilon_{rel}$ in advance, we cannot ensure whether $A_{rel} = \mathbb{P}_{l_q}(u_q) - \mathbb{P}_{l_q}(l_q) \geq 2\delta(1 + \frac{1}{\varepsilon_{rel}})$ can be fulfilled or not. Once it cannot fulfill this condition, this $A_{rel}$ may not fulfill the relative error guarantee (i.e., Fail in Figure 10). For this case, we can adopt the exact method (cf. Section III-B1) to obtain the exact value of COUNT query.

B. Approximate range MAX Query

Since we utilize the same index structure (PolyFit) to answer the MAX query, we can have similar results (cf. Lemma 2 and 3) for $A_{abs}$ (cf. Lemma 4) and $A_{rel}$ (cf. Lemma 5) respectively. We omit the proof of these two lemmas as we can utilize the similar idea from Lemma 2 and 5 to obtain the results.
Lemma 4. If we set \( \delta = \varepsilon_{abs} \), we can satisfy \(|A_{abs} - R_{max}(D, [l_q, u_q])| \leq \varepsilon_{abs}\).

Lemma 5. If \( A_{rel} \geq \delta(1 + \frac{1}{\varepsilon_{rel}}) \), we can achieve the relative error \( \varepsilon_{rel} \).

During the traversal of the tree, we also adopt the same method as Section III-B2 for the internal nodes and update the current maximum measure \( M_{max} \). However, instead of scanning all the (key,measure)-pairs in two leaf nodes (one of the leaf node includes \( l_q \) and another one includes \( u_q \)) for the traditional index structures, we need to find the largest values for \( P_{l_q}(k) \) and \( P_{u_q}(k) \) in regions \([l_q, U_{l_q}]\) and \([L_{u_q}, u_q]\), as shown in Figure 11.

Figure 11: The maximum measure values (red dots) for two leaf nodes, which include \( l_q \) and \( u_q \)

Here, we formulate the optimization problem (17) for finding the maximum value in the interval \([L_{u_q}, u_q]\). However, we can easily modify this optimization problem for the case \([l_q, U_{l_q}]\).

\[
\begin{align*}
\text{MAXIMIZE} & \quad P_{l_q}(k) \\
\text{SUBJECT TO} & \quad L_{u_q} \leq k \leq u_q
\end{align*}
\]

(17)

By adopting some simple calculus operations, we can obtain the global optimal solution (i.e., red dot) for (17).

VI. EXTENSIONS FOR QUERIES WITH MULTIPLE KEYS

In previous sections, we mainly focus on range aggregate queries, in which each element only contains one single key (cf. Definition [1]). However, existing works [23, 24, 28, 50] also support range aggregate queries with two keys for each element. In this section, we discuss how to extend our techniques in this setting (cf. Definition [6]). As a remark, our techniques can be also extended to the setting of multiple keys. Here, we only focus on COUNT query. However, we can also adopt our methods for other types of range aggregate queries.

Definition 6. Given \( D \) as a set of elements \((u, v, w)\), where \( u, v \) and \( w \) are the first key, second key and the measure respectively, and the ranges of keys are \([l_q^{(1)}, u_q^{(1)}]\) for \( u \) and \([l_q^{(2)}, u_q^{(2)}]\) for \( v \), we define the COUNT query as:

\[
R_{count}(D, [[l_q^{(1)}, u_q^{(1)}]][l_q^{(2)}, u_q^{(2)}]) = \text{count}(\sigma_{u \in [l_q^{(1)}, u_q^{(1)}], v \in [l_q^{(2)}, u_q^{(2)}]}(D))
\]

Instead of building the key-cumulative array (cf. Figure 2) for solving the COUNT query, we build the following key-cumulative function to represent the surface (cf. Figure 12), which are formally stated in Definition 7.

Figure 12: Tweet dataset [13] with longitude and latitude as the keys

Definition 7. The key-cumulative function with two keys for COUNT query is defined as \( CF_{count}(u, v) \), where:

\[
CF_{count}(u, v) = R_{count}(D[\infty, u][\infty, v])
\]

(18)

Therefore, we can solve the COUNT query \( R_{count}(D[l_q^{(1)}, u_q^{(1)}][l_q^{(2)}, u_q^{(2)}]) \), using the following equation.

\[
R_{count}(D[l_q^{(1)}, u_q^{(1)}][l_q^{(2)}, u_q^{(2)}]) = CF_{count}(l_q^{(2)}, u_q^{(2)}) - CF_{count}(l_q^{(2)}, u_q^{(1)}) - CF_{count}(l_q^{(1)}, u_q^{(2)}) + CF_{count}(l_q^{(1)}, u_q^{(1)})
\]
Then, we follow the similar idea in Section IV-C and utilize the polynomial surface $P(u, v)$ to approximate the key cumulative function $CF_{\text{count}}(u, v)$ with two keys, where:

$$P(u, v) = \sum_{i=1}^{\text{deg}} a_{ij} u^i v^j$$

By replacing $F(k_i)$ and $P(k_i)$ in Equation 8 by $F(u_i, v_i)$ and $P(u_i, v_i)$ respectively, we can obtain the similar linear programming problem for obtaining the best parameters $a_{ij}$. However, unlike the one-dimensional case, it takes at least $O(n^2)$ to obtain the minimum number of segmentations by using the GS method (cf. Section IV-D1), which is infeasible even for small-scale dataset (e.g., 10,000 points). As such, we propose to utilize the Quad-tree based solution to obtain the segmentation. Observe from Figure 13 once the segment does not fulfill the error guarantee $\delta$ (e.g., white rectangles), we continuously break each of these white rectangles into four rectangles in each iteration. Our method terminates when all rectangles fulfill the error guarantee $\delta$.

![Quad-tree based approach for obtaining the segmentation](image)

Figure 13: Quad-tree based approach for obtaining the segmentation

After we have built this index structure PolyFit, we can utilize the similar approach in Section V to answer the range aggregate queries with theoretical guarantee (cf. Lemma 4 and 7).

**Lemma 6.** If we set $\delta = \frac{\epsilon_{\text{abs}}}{4}$, we can satisfy $|A_{\text{abs}} - R_{\text{count}}(D, [[u^{(1)}, v^{(1)}], [u^{(2)}, v^{(2)}]])| \leq \epsilon_{\text{abs}}$.

**Lemma 7.** If $A_{\text{rel}} \geq 4\delta(1 + \frac{1}{\epsilon_{\text{rel}}})$, we can achieve the relative error $\epsilon_{\text{rel}}$.

The proofs of Lemma 6 and 7 are similar with both Lemma 2 and 3 respectively.

### VII. Experimental Evaluation

We first introduce the experimental setting in Section VII-A. Then, we compare the response time of different range-aggregate-queries-methods (RAQ-methods) which fulfill the $\epsilon_{\text{abs}}$ (cf. Problem 1) and $\epsilon_{\text{rel}}$ (cf. Problem 2) error guarantee in Section VII-B. Next, we measure the memory space of different index structures, which fulfill the $\epsilon_{\text{abs}}$ error guarantee in Section VII-C. Later, we report the trade-off between the response time and practical error with some heuristic methods in Section VII-D. Lastly, we vary the degree deg of PolyFit and test the efficiency performance of both query and index construction stages in Section VII-E.

#### A. Experimental Setting

We have collected three real large-scale datasets (0.9M to 100M records) to evaluate the performance, which are summarized in Table III. For each dataset, we randomly generate 1000 queries. In single-key case, e.g., HKI and TWEET datasets, we choose the start and end points of each interval (i.e., query) from the keys in the datasets. In two-key case, e.g., OSM dataset, we randomly sample the rectangles, based on the uniform distribution. In our experiments, we only focus on both COUNT and MAX queries. However, we can extend our methods to other types of range aggregate queries, e.g., SUM and MIN queries.

| Name   | Size   | Selected key(s) | Selected value | Used for |
|--------|--------|-----------------|----------------|---------|
| HKI[1] | 0.9M   | timestamp       | index value    | MAX     |
| TWEET[13]| 1M     | latitude        | # of tweets    | COUNT   |
| OSM[3] | 100M   | latitude, longitude | # of records  | COUNT   |

Table IV summarizes different methods for range aggregate queries. We classify these methods based on five categories, which are listed as follows.

- Can this method provide the absolute error guarantee $\epsilon_{\text{abs}}$ (cf. Problem 1)?
- Can this method provide the relative error guarantee $\epsilon_{\text{rel}}$ (cf. Problem 2)?
- Can this method support multiple keys?
- Can this method support COUNT query?
- Can this method support MAX query?

Hist[51] adopts the entropy-based histogram for answering the COUNT query. S-tree[2] prebuilds the STX B-tree on top of the sampled subset of each dataset. Compared with other methods, both Hist and S-tree are the heuristic methods, which cannot fulfill the error guarantee for answering the range aggregate queries. Both aR-tree[45] and aMax-tree[2] are the traditional tree-based methods for answering the exact COUNT and MAX queries respectively. In addition, they can also support the setting of multiple keys. S2[26] is the sampling-based approach which can provide the error guarantee. However, instead of providing the deterministic error guarantee (e.g., $\epsilon_{\text{rel}} = 0.01$), they provide the probabilistic error guarantee (e.g., $\epsilon_{\text{rel}} = 0.01$ with probability = 0.9). By default, we set the probability as 0.9 in our experiment. The learned-index methods, including RMI[34] and FITting-tree[20], can support the absolute error and relative error guaranteed3. However, they cannot support multiple keys and MAX queries. As a remark, there are many parameters for RMI method[34], we adopt the random search[11] to obtain the most suitable parameters. Unlike these methods, our method PolyFit can support all these five properties. By default, we set the degrees of polynomial functions as two and one for COUNT and MAX queries respectively. We use the form PolyFit-deg to represent the degree deg of PolyFit in later

3In original works, they do not support the range aggregate queries with relative error guarantee. However, we can support this property with some modification of their algorithms.
sections. In addition, we adopt $\delta = 100$ and $\delta = 1000$ in PolyFit for the experiments with one key and two keys respectively by default. We implemented all methods in C++ and conducted experiments on an Intel Core i7-8700 3.2GHz PC using Windows 10.

Table IV: Methods for range aggregate queries

| Method   | Hist | S-tree | S2 | R-tree | aR-tree | RMI | FITing-tree | PolyFit (ours) |
|----------|------|--------|----|--------|---------|-----|-------------|----------------|
| $\epsilon_{abs}$ | $\times$ | $\times$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $\epsilon_{rel}$ | $\times$ | $\times$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\times$ | $\checkmark$ | $\checkmark$ |
| $\geq 2$ keys | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\times$ | $\times$ | $\checkmark$ | $\checkmark$ |
| COUNT | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\times$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| MAX | $\times$ | $\times$ | $\times$ | $\times$ | $\times$ | $\times$ | $\times$ | $\checkmark$ |

B. Response Time for Different RAQ-methods with Error Guarantee

In this section, we test the response time of different methods for answering COUNT and MAX queries in which these methods can fulfill the absolute and relative error guarantee. COUNT query with absolute error guarantee. We investigate how the absolute error $\epsilon_{abs}$ affects the response time of different methods. In this experiment, we choose five absolute error values, which are 50, 100, 200, 500 and 1000. Observe from Figure [14], due to the better approximation with nonlinear polynomial function (e.g., degree 2), PolyFit can normally achieve at least 1.5x to 6x speedup, compared with existing learned-index structures, including RMI and FITing-tree. In addition, PolyFit can achieve five-order-of-magnitude speedup, compared with the sampling-based method S2, as shown in Figure [14]. Figure [14] illustrates the response time of COUNT query with two keys. Since the state-of-the-art learned index structures (RMI and FITing-tree) can only support the query with single key, we omit these two methods in this figure. Observe that PolyFit can normally achieve at least one-order-of-magnitude speedup compared with the existing methods S2 and aR-tree, due to its compact index structure.

COUNT query with relative error guarantee. We proceed to test how the relative error $\epsilon_{rel}$ affects the response time of different methods. In this experiment, we choose five relative error values, which are 0.005, 0.01, 0.05, 0.1 and 0.2. Based on the nonlinearity of polynomial function, our method PolyFit normally can achieve better efficiency performance, compared with both learned-index and sampling-based methods (cf. Figures [15] and b). For the COUNT query with two keys, our method PolyFit can significantly outperform the existing methods aR-tree and S2 by at least one-order-of-magnitude (cf. Figure [15]).

MAX query with error guarantee. In this experiment, we proceed to investigate how the absolute error $\epsilon_{abs}$ and relative error $\epsilon_{rel}$ affect the efficiency performance of different methods. Observe from Figure [16] our method PolyFit normally can significantly outperform the existing method aMax-tree, even though the selected error is small.

Scalability to the dataset size. We proceed to test how the dataset size affects the efficiency performance of both PolyFit and other methods. In this experiment, we choose the largest dataset OSM (with 100M) for testing. Here, we focus on solving the Problem 2 for COUNT query in the single key case, in which we fix the relative error to be $\epsilon_{rel} = 0.01$ and choose the latitude attribute as key. To conduct this experiment, we choose five dataset sizes, which are 1M, 10M, 30M and 100M. Observe from Figure [17] all methods are insensitive to the dataset size.

C. Memory Space for Index Structures

In this section, we proceed to investigate the space consumption of the index structures with different bounded $\delta$-error. Since our method can construct the index structure with minimum size (cf. Section IV-D) and the polynomial function can normally provide better approximation to $F(k)$, our method can provide smaller index size, compared with other methods, e.g., RMI and FITing-tree, as shown in Figure [18].

D. Comparisons with Other Heuristic Methods

In this section, we compare the response time of PolyFit with other heuristic methods, e.g., Hist and S-tree, which do not provide any theoretical guarantee. In this experiment, we vary the bin size and sampling size for Hist and S-tree respectively and report the measured relative error and query response time. Observe from Figure [19] our PolyFit can normally provide faster response time with similar relative error.

E. How does the degree $deg$ affect the performance of PolyFit?

Recall that we need to select the degree $deg$ in order to build the PolyFit. In this section, we investigate how this parameter can affect the response time of both query stage and index construction stage.

For the query stage, once we choose the larger degree $deg$, this polynomial function can provide better approximation for $F(k)$, which can reduce the number of nodes in the index and create more compact index structure. This can reduce the response time for each query. However, the larger the degree $deg$, the larger the computation time for each node in PolyFit. Therefore, as shown in Figure [20] we can observe that the response time reduce for using $deg = 2$ polynomial function, compared with the case for $deg = 1$. However, the improvement also reduces once we change from $deg = 2$ to $deg = 3$.

For the construction stage, once we adopt the higher degree of polynomial function, the construction time can be higher (cf. Figure [21]), since the number of terms in Equation 9 becomes larger.

VIII. Conclusion

In this paper, we study the range aggregate queries with two types of approximate guarantee, which are (1) absolute error guarantee (cf. Problem 1) and (2) relative error guarantee (cf. Problem 2). Unlike the existing methods, this is the first work that can support all types of range aggregate queries (SUM,
COUNT, MIN, MAX), fulfill the error guarantee and support the setting of multiple keys.

In order to achieve the efficiency performance for these queries, we utilize several polynomial functions to fit the data points and then build the compact index structure PolyFit on top of these polynomial functions. Our experiment results show that PolyFit can achieve 1.5-6x speedup compared with existing learned-index methods and nearly six-order-of-magnitude speedup compared with other approximate methods in COUNT query with single key. For other settings (COUNT query with two keys and MAX query with single key), PolyFit can achieve 3x to one-order-of-magnitude speedup, compared with the state-of-the-art methods.

In the future, we plan to investigate how to utilize the idea of PolyFit to further improve the efficiency performance for other types of statistics and machine learning models, e.g., evaluating the kernel density estimation, kernel clustering and training the support vector machines.

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Figure 17: Response time for COUNT query in OSM dataset (using latitude attribute as single key), varying the dataset size.

Figure 18: Memory space for different index structures for COUNT query (single key) in TWEET dataset.

Figure 19: Response time between PolyFit and the heuristic methods in TWEET dataset.

Figure 20: Response time for COUNT query (single key) in the query stage in TWEET dataset, varying the degree of PolyFit.

Figure 21: Construction time for COUNT query (single key) in TWEET dataset in the construction stage, varying the degree of PolyFit.

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