Possible Evolutionary Transition from Rapidly Rotating Neutron Stars to Strange Stars Due to Spin-Down

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We present a scenario for the formation of strange stars due to spin-down of rapidly rotating neutron stars left after supernova explosions. In this scenario the rapid rotation plays a crucial role. We assume that the total baryon mass is conserved but that both total energy and angular momentum are lost due to emission of gravitational waves and/or magnetic braking. Under this assumption, we calculate the transition from rapidly rotating neutron stars to slowly rotating strange stars. As a result, we find that a large amount of energy, \( \sim 10^{53} \) ergs is released. The liberated energy might become a new energy source for the delayed explosion of a supernova. Furthermore, our scenario suggests that supernovas associated with gamma-ray bursts are feasible sources of observable gravitational waves.

§1. Introduction

Since the possible existence of strange quark matter in nature was pointed out,\(^1,2\) strange stars, that is, quark stars, have been examined as the most plausible sources of strange quark matter.\(^3\) Concerning the origin and/or formation of strange stars,\(^4\) many researchers have investigated the relation between neutron stars and strange stars because it is natural to consider strange stars as compact objects left after supernova explosions, such as the soft X-ray source RX J1856.5-3754.\(^5,6,7\) As one other formation process, the transition process and the timescale from neutron stars to quark stars have been studied in detail from the point of view of pulsar spin-down due to strong magnetic fields.\(^8\) If this kind of scenario is correct, there should be a process during which the maximum density of a neutron star increases to a larger value above which a strange star can be realized, i.e. the minimum density of a strange star, \( \rho_c > 10^{15} \text{ g cm}^{-3} \).\(^2\) This kind of process has already been investigated by considering accreting stars which shrink to higher maximum energy densities, while there are large uncertainties in the physical process involved.\(^9,10\)

The above described scenario could be related to supernova explosions and gamma-ray bursts. In recent years, some gamma-ray bursts have been found to be related to supernova explosions, such as the connection between SN1998bw and GRB980425.\(^11\) The total energy of an event necessary to power sources at cosmological distances is on the order of a few \( 10^{53} \) ergs for a spherical explosion.\(^12\) We

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note that to generate a jet in supernova, rapid rotation and strong magnetic fields are thought to be required before the bounce.\textsuperscript{13,14} Among many models,\textsuperscript{15} the gravitational collapse of a super massive neutron star to a black hole was presented to explain both the energy source and the small baryon contamination.\textsuperscript{16} On the other hand, the transition to a strange star has been suggested as the energy source of gamma-ray bursts.\textsuperscript{9,17–20} Huge amounts of energy could be liberated due to the phase transition from a neutron star to a strange star within a small region, $\sim 10$ km. This energy release leads to the creation of a fireball through the process $\gamma\gamma \leftrightarrow e^+ + e^-$. The transitions may occur on various time scales on the order of ms to a few years. This idea of a phase transition from a neutron star to a strange star is attractive to avoid baryon contamination, because the baryonic mass ($\leq 10^{-5} M_\odot$) of a thin crust $M_{\text{crust}}$ in a strange star is negligibly small.\textsuperscript{17} As a consequence, the resulting fireball can be accelerated to the relativistic regime with a Lorentz factor $\Gamma \sim 500$ if the energy release from the phase transition is of the order of $10^{52}$ ergs.\textsuperscript{18}

By contrast, if color superconductivity is realized at the surface of a quark star, the glueball decay causes a fireball.\textsuperscript{20} They have total durations of $2 - 81$ s; $t_p \sim E_p^{-0.5}$ where $t_p$ is the peak duration and $E_p$ is the peak energy. This relation agrees with observation.\textsuperscript{21} Furthermore, the decay of axion-like particles into $e^+ e^-$ or photons leads to an ultra-relativistic plasma at distances $10^2 - 10^4$ km, which could be relevant to type Ib/Ic supernovae associated with weak gamma-ray bursts.\textsuperscript{22} We note that one-dimensional simulations of the explosion from neutron stars to strange stars show that the explosions are too baryon rich and the energies too small to create a fireball.\textsuperscript{23}

Moreover, observations of GRB990705 and GRB011211 that reveal the afterglow emission lines could be interpreted as events after supernova explosions with time intervals from hours to years.\textsuperscript{10} Concerning these phenomena, Berezhiani et al.\textsuperscript{10} proposed a model of the transition of a pure hadronic star to a quark star. The central density $\rho_c$ is assumed to increase due to spin-down or mass accretion, where they used the mass-radius relation of Drago and Lavagno.\textsuperscript{24} The energy release was calculated from the difference between the gravitational mass of a metastable hadronic star and that of the final stable hybrid star (or quark star) with the same baryonic mass $M_B$.\textsuperscript{9} Though the observation of the significant delay between supernova explosions and gamma-ray bursts has not been established,\textsuperscript{25} it is desirable to consider the transition from neutron stars to strange stars by taking into account elementary physics. On the other hand, it has been suggested that the emission of gravitational radiation and/or magnetic dipole radiation could cause the spin-down of compact stars.\textsuperscript{18}

However, we should note that most of these calculations/estimates concerning the transition from neutron stars to strange stars have been studied using the spherical configurations; for example, the Tolman-Oppenheimer-Volkoff equation was solved to obtain the structures of compact stars.\textsuperscript{26} In other words, the calculations have not included rotation, physical processes such as strong electric/magnetic fields on the surface of strange stars, and multi-dimensional effects of general relativity.

Although some investigations have taken into account some of the effects men-
tioned above, two-dimensional simulations of collapse-driven supernovae with updated neutrino transport fails to show the occurrence of explosions.\textsuperscript{27} Though the calculations are very sensitive to the dynamical treatment of the energy transport between the neutrino energy and the kinetic energy, it is shown that if only a small amount of energy is added to the kinetic energy, delayed explosions can appear. Furthermore, the artificial deformation of a progenitor has produced a shock that reached a radius of more than 600 km within 230 ms after a bounce. It is now believed that rotation strongly affects the development of shock propagation.

In order to make the situation clear and obtain a fireball in numerical simulations, we need to carry out three-dimensional hydrodynamical calculations in the framework of general relativity and to include physical processes such as the neutrino transport described by the Boltzmann transport equation. Since such exact simulations are impossible at present, we need other approaches to deepen our understanding of the evolution of compact stars. It is noted that the slow rotation approximation for rotating equilibrium used by Glendenning and Weber\textsuperscript{28} cannot be applied to configurations near the mass shedding limit.\textsuperscript{29}

Evolutionary sequences of rapidly rotating strange stars in hydrostatic equilibrium were investigated for uniformly rotating configurations.\textsuperscript{29} It was shown that the ratio of the rotational energy to the potential energy $T/|W|$ for rapidly rotating massive strange stars are very large, $T/|W| \sim 0.2$, which indicates the onset of triaxial instabilities due to gravitational radiation reactions. This suggests that strange stars could be sources of gravitational radiation. Numerical simulations of the nonlinear evolution of a nonaxisymmetric bar-mode perturbation of rapidly rotating stars find losses of more than 10% of the initial angular momentum through gravitational waves over a time interval corresponding to 30 times the initial rotational period, $P_0$.\textsuperscript{30} Though it depends on the strength of the gravitational radiation reaction force, they suggest that the formed ellipsoid, which is assumed to be a proto-neutron star, will emit significant amounts of gravitational waves to dissipate the energy and the angular momentum after a long period. We note that they assumed a polytropic equation of state, and $0.2 < T/|W| < 0.26$ in the initial model of a rotating star. Recently, the nucleation of quark matter in cores of rotating neutron stars has been studied by Harko et al.,\textsuperscript{8} who assumed only a sequence of Maclaurin spheroids. The spin-down time scale was estimated to be $\sim 5$ s for an angular velocity $8000$ rad s$^{-1}$ and a magnetic field $\sim 10^{16}$ G.

In this paper, we present a new scenario for the transition from a rapidly rotating neutron star to a slowly rotating strange star due to spin down through the emission of gravitational radiation and/or magnetic braking. Rapidly rotating compact stars are briefly discussed in §2 with an adopted equation of state. Our approach is qualitatively in advance to investigate the structures in the quasi-static evolution of the compact stars, because we need to obtain two-dimensional equilibrium configurations of rapidly rotating compact stars by fully taking into account general relativity up to the mass shedding limits. In §3, we show that a significant amount of energy is released during the transition and predict the gravitational radiation accompanied by a delayed explosion of a supernova. In §4, we discuss the results of our scenario.
§2. Rapidly rotating compact stars

Though the proto-neutron star left after a supernova explosion is very hot, \( T \sim 50 \text{ MeV} \), it cools down to a cold neutron star \( (T \sim 1 \text{ MeV}) \) in some tens of seconds.\(^{31} \) Thus, during a time of a few hours after a supernova explosion, it is reasonable for a proto-neutron star to cool down to a cold neutron star. For a cold neutron star, we use a typical equation of state (EOS) derived by Wiringa,\(^{32} \) which includes a three-body force. For strange stars, we adopt an EOS of strange matter based on the MIT bag model. Considering the uncertainties of the EOS for the strange matter, we adopt the simple EOS\(^{29} \)

\[
P = \frac{1}{3} (\rho - 4B), \quad \rho = an^{4/3} + B, \tag{2.1}
\]

where \( \rho \) is the energy density, \( n \) is the baryon number density in units of fm\(^{-3} \), \( a = 952.4 \text{ MeV fm} \), and \( B \) is the bag constant in MeV fm\(^{-3} \). While we ignore both the strange quark mass and quark interactions, the quantity \( B \) is included to represent confinement effects. The quantity \( B \) is considered to be in the range \( 60 - 120 \text{ MeV fm}^{-3} \). From the observation of RX J1856.5-3754,\(^{33} \) Kohri et al.\(^{5} \) derived an upper limit on the mass of a quark star for various sets of bag-model parameters. In this paper, we choose three cases, \( B = 60, 90, \) and \( 120 \) MeV fm\(^{-3} \). The corresponding maximum gravitational masses (proper radii) for spherical strange stars are \( 2.63 \) (10.7), \( 1.94 \) (8.75), and \( 1.56 \ M_\odot \) (7.58 km), respectively. Therefore, the EOS (2.1) can cover the range of maximum masses that are obtained from more complex EOSs of strange matter.\(^{5}, 24, 26, 35} \) It should be noted that there is no crust in a strange star expressed by the EOS (2.1), and therefore there is no baryonic contamination during the transition from a neutron star to a strange star. We note also that our results can be applied to a quark star of the type investigated by Ouyed and Sannino\(^{20} \) as inner engines for gamma-ray bursts because they used almost the same EOS (2.1), with \( M_{\text{crust}} \sim 5 \times 10^{-5} \ M_\odot \).

With the same method used to investigate equilibrium configurations of rapidly rotating stars,\(^{34, 36, 37} \) we can obtain equilibrium sequences under the condition of stability for axisymmetric collapse as

\[
\frac{\partial M}{\partial \rho_{\text{max}}} \Big|_J > 0, \tag{2.2}
\]

where \( J \) is the total angular momentum.\(^{38, 39} \) An equilibrium model can be obtained by computing the equilibrium structure for a fixed \( \rho_c \) and a given ratio \( r_p/r_e \) of the polar radius \( (r_p) \) to the equatorial radius \( (r_e) \). The stability condition for secular instability due to gravitational radiation emission is \( T/|W| \gtrsim 0.14 \). The value 0.14 is an upper limit, because instability for smaller wavelengths sets in for lower values of \( T/|W| \).

The timescale \( \tau_{\text{sec}} \) of the growth of the secular instability due to gravitational radiation can be estimated.\(^{40} \) If we take the critical value for the onset of the instability, \( (T/|W|)_c = 0.14 \), we get \( \tau_{\text{sec}} \) for specific problems: \( \tau_{\text{sec}} \sim 6 \times 10^{-4} \text{ s} \) for a gravitational mass \( M \sim 1 \ M_\odot \), a proper radius \( R \sim 10 \text{ km} \), and \( T/|W| \sim 0.21 \).
Shibata and Karino\textsuperscript{30} have obtained the growth time scale of the instability as $(3.5 - 31) \times P_0$ and estimated the evolution time scale of a proto-neutron star due to the gravitational radiation emission to be $\sim 12$ s $(R/20$ km)$^3(M/1.4M_\odot)^{-3}$. Thus, we can regard the evolution of a compact star born after supernova as a sequence of equilibrium states for constant $M_B$, as studied for rotating proto-neutron stars.\textsuperscript{41} A numerical simulation\textsuperscript{30} shows that $J$ is dissipated due to gravitational waves by $\sim 10\%$ at the time $t = 30P_0$ for a neutron star of $\sim 1.4M_\odot$. For the rapidly rotating case, a neutron star formed after a supernova explosion will lose a significant amount of angular momentum in a rather short period $t < 0.03$ s. Therefore, before the transition from a neutron star to a strange star, some amount of the total energy and angular momentum could be lost. This would lead to an increase in $\rho_c$.

\begin{table}[h]
\centering
\caption{Physical quantities for $B = 60$ MeV fm$^{-3}$ and $M_B = 2.6M_\odot$.}
\begin{tabular}{cccccccc}
\hline
Model & $M$ & $n$ & $r_p/r_e$ & $r_e$ & $J$ & $T/|W|$ & $\Delta E$ \\
\hline
NS* & 2.251 & 0.72 & 0.575 & 14.5 & 3.06 & 0.309 \\
NS & 2.249 & 0.75 & 0.668 & 12.9 & 2.80 & 0.117 & 0.036 \\
SS & 1.975 & 0.76 & 0.892 & 11.7 & 1.50 & 0.036 & 5.1 \\
SS' & 1.948 & 0.95 & 1 & 10.9 & 0 & 0 & 0.31 \\
\hline
\end{tabular}
\end{table}

Here, NS* represents a neutron star of maximum rotation. NS and SS describe the situations before and after the phase transition from a rotating neutron star to a strange star. SS' represents a strange star without rotation. $\Delta E$ is the total energy released in the transitions from NS* to NS, NS to SS, and SS to SS'. The total angular momentum $J$ is in units of $10^{49}$ g cm$^2$ s$^{-1}$, and $\Delta E$ is in units of $10^{53}$ ergs. The maximum energy release, $\Delta E_{\text{max}}$ between NS* and SS' is 5.45 in these units.

\begin{table}[h]
\centering
\caption{Same as Table 1 but for $B = 90$ MeV fm$^{-3}$ and $M_B = 1.7M_\odot$. $\Delta E_{\text{max}} = 1.92$.}
\begin{tabular}{cccccccc}
\hline
Model & $M$ & $n$ & $r_p/r_e$ & $r_e$ & $J$ & $T/|W|$ & $\Delta E$ \\
\hline
NS* & 1.547 & 0.53 & 0.575 & 15.7 & 1.43 & 0.111 \\
NS & 1.533 & 0.54 & 0.706 & 13.4 & 1.30 & 0.087 & 0.25 \\
SS & 1.462 & 0.73 & 0.797 & 9.90 & 1.00 & 0.074 & 1.3 \\
SS' & 1.436 & 0.86 & 1 & 9.11 & 0 & 0 & 0.47 \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\caption{Same as Table 1 but for $B = 120$ MeV fm$^{-3}$ and $M_B = 1.5M_\odot$. $\Delta E_{\text{max}} = 0.641$.}
\begin{tabular}{cccccccc}
\hline
Model & $M$ & $n$ & $r_p/r_e$ & $r_e$ & $J$ & $T/|W|$ & $\Delta E$ \\
\hline
NS* & 1.379 & 0.49 & 0.581 & 15.8 & 1.13 & 0.105 \\
NS & 1.375 & 0.50 & 0.695 & 13.7 & 1.00 & 0.086 & 0.071 \\
SS & 1.366 & 1.19 & 0.935 & 8.11 & 0.50 & 0.026 & 0.16 \\
SS' & 1.343 & 1.33 & 1 & 7.85 & 0 & 0 & 0.41 \\
\hline
\end{tabular}
\end{table}

\section{Results}

The transition density is unknown because of our lack of understanding of the basic physics concerning quark matter in a hadronic medium.\textsuperscript{6} Therefore, we assume
that the transition due to the increase in the maximum density from a neutron star to a strange star occurs from an equilibrium state of a rapidly and uniformly rotating neutron star to another equilibrium state of a slowly rotating strange star with i) the baryon mass remaining constant and ii) the total energy and the angular momentum decreasing through gravitational radiation and/or magnetic dipole radiation. This prescription is essentially the same as that used for the evolution of an isolated rapidly rotating strange star that loses total energy and angular momentum.\footnote{42}

The results for the three values $B = 60, 90, \text{ and } 120 \text{ MeV fm}^{-3}$ are listed in Tables I – III and in Figs 1–8. We present an detailed scenario of the stellar transition based on the models of rapidly rotating compact stars. Instead of the method of Berezhiani et al.,\footnote{10} who used $M_B$ to look for equilibrium sequences of spherical stars, we the use two parameters $J$ and $M_B$ to find evolutionary sequences of uniformly rotating stars. Because there are many uncertainties in the transition mechanism, we present possible evolutionary paths.

Figure 1 depicts the evolutionary transition with $B = 60 \text{ MeV fm}^{-3}$ and $M_B = 2.6 M_\odot$ in the $[\text{gravitational mass } M] - [\text{baryon number density } n]$ plane. The two curves that decrease in the direction of increasing $n$ represent evolutionary tracks with constant $M_B$. Our scenario is as follows. First, a neutron star with maximum rotation (NS*) spins down due to gravitational radiation (or magnetic dipole radiation) emission ($J = 3.1 \rightarrow 2.8, \Delta E = 3.6 \times 10^{51} \text{ ergs}$). Second, the NS changes into a slowly rotating SS ($J = 1.5, \Delta E = 5.1 \times 10^{53} \text{ ergs}$). Finally, the SS becomes a non-rotating SS' ($J = 0, \Delta E = 3.1 \times 10^{52} \text{ ergs}$). Though other evolutionary paths are possible, any path must lie between NS* and SS'. The total energy release, $\Delta E$, is calculated as the difference between the gravitational masses of the two states. We note that the amounts of $\Delta E$ depend on $M_B$. We look for the largest value of $\Delta E$ by considering a large baryonic mass of $M_B \geq 1.5 M_\odot$, as suggested by the evolutionary calculation of massive stars.\footnote{43} Physical quantities at the transition points are tabulated in Table I.

The decrease in $\Delta E$ due to the angular momentum loss $\Delta J$ is approximately

$$\Delta E = \overline{\Omega} \Delta J,$$

\hfill (3.1)

where $\overline{\Omega}$ is the averaged angular velocity. Let us assume that $J$ is lost through gravitational radiation. For the transition between neutron stars, if we adopt the value $\Delta J \sim 10^{49} \text{ g cm}^2 \text{ s}^{-1}$ and $\overline{\Omega} \sim 10^3 \text{ s}^{-1}$, then we have $\Delta E \sim 10^{52} \text{ ergs}$. On the other hand, the relativistic energy loss rate due to the spin-down of the magnetar of a proto-neutron star is estimated to be $\geq 10^{51} \text{ erg s}^{-1}$, and the emitted energy is $10^{51} - 10^{52} \text{ ergs on a time scale of } 10-100 \text{ s.}$\footnote{44} This energy release may not be sufficient to account for the gamma-ray burst energetics. By contrast, in the transition $[\text{neutron stars}] \rightarrow [\text{strange stars}]$, an enormous amount of energy is released, as seen in Tables I – III $\Delta E = (0.16 - 5.1) \times 10^{53} \text{ ergs}$. The nucleation time for $B = 90 \text{ MeV fm}^{-3}$ and $M = 1.53 M_\odot$ is estimated to be 1 yr,\footnote{4},\footnote{6} during which a metastable neutron star changes into a strange star. Harko et al.\footnote{8} calculated the transition time to be $0.01 - 1 \text{ yr for a transition temperature of } T \sim 0.01 - 1 \text{ MeV.}$. If the transition takes a long time, the energy release occurs in a clean environment without baryons. Since the energy release is radiated through gravitational radiation
by at most 10%, a significant amount of energy may be carried away by neutrinos. However, some amount of energy could be transferred to an explosion constituting of a supernova and/or produce a fireball that is contaminated by a small amount of baryons ($M_{\text{crust}} \leq 10^{-5}M_\odot$). The time scale of gravitational radiation from a realistic neutron star is estimated to be a few minutes,$^{30}$ and that of magnetic dipole radiation is $5\text{--}1000$ s.$^8$ Though these estimates are based on dimensional analysis, they are consistent with the gamma-ray burst duration.

§4. Discussions

We have shown that transitions from a rapidly rotating neutron star to a slowly rotating strange star can be the energy source for a supernova explosion due to a large energy release of $\Delta E > 10^{53}$ ergs if $B \leq 90$ MeV fm$^{-3}$, as seen in Tables I and II. While Bombaci et al.$^6$ obtained a liberated energy in the range $(0.5 - 1.7) \times 10^{53}$ ergs, we obtain a maximum liberated energy of $5.1 \times 10^{53}$ ergs. We find that the transition energy between rotating stars exceeds that for non-rotating stars. This is because the gravitational masses of a rotating stars exceed those of non-rotating stars.

Because no numerical simulations has shown a supernova explosion, a small amount of energy release is helpful to recover decaying shocks. Even if the available energy for a supernova is overestimated by a factor of 10,$^{23}$ it is enough to explain hypernovae. Though the exact timescale concerning the angular momentum loss is unknown, the rapid rotation of compact stars would cause instabilities. However, we stress that this transition is accompanied by gravitational radiation and/or magnetic dipole radiation. In particular, the strength of the gravitational radiation determined from the rotation period/law may be related to the observed short gamma-ray bursts.$^{45}$ Since gamma-ray bursts related to supernova explosions originate from jet-like explosions, nonspherical configurations would result in gravitational radiation.$^{13}$ Therefore, gamma-ray bursts associated with supernovae are feasible sources for the detection of gravitational waves.

Finally, we note that we have examined uniformly rotating configurations. In fact, after a supernova explosion, a compact star would rotate differentially. For $r_p/r_e = 0.23$, a moderate differential rotation (rotation parameter $A = 1.0$ in the system studied by Komatsu et al.$^{36}$), and $\rho_c \sim 10^{15}$ g cm$^{-3}$, the gravitational mass is increased by a factor of 2.5 compared to the case of uniform rotation. Thus, the energy release obtained from differentially rotating neutron stars is larger than the values of $\Delta E$ given in Tables II – III. On the other hand, differential rotations lead to significant deformations which are not only sources of gravitational radiation but are responsible for jet formation.$^{14}$ Therefore, even if pre-expelled supernova ejecta exist, a collimated jet could reach regions at large distances; a jet accelerated by rapid rotation can also accelerate the pre-expelled material and leave an exit for the jet. It is suggested that a strong magnetic field related to the dipole radiation inherent in a rapid rotation creates a hole inside the supernova ejecta. The duration of gamma-ray bursts in the observer’s frame due to external shocks is estimated to be $10$ s for typical values of the initial energy, the mass loss rate, and the wind
velocity with $\Gamma \sim 100$.\footnote{15,18} It is inferred that a newly born neutron star moves outward at a kick velocity of $\sim 10^3$ km s$^{-1}$. If a phase transition occurs around the front of the supernova ejecta, the jet does not suffer from the contamination of the ejecta.\footnote{45}

Considering the statics of high speed neutron stars (or strange stars), the detectable rate is estimated to be $10^{-4} - 10^{-6}$ per galaxy per year. Though the differential rotation law describing jet formation is not known, it is worthwhile to investigate that amount that differential rotation affects our transition scenario. Hydrodynamical calculations coupled with the dynamics of the phase transition from a neutron star to a strange star are necessary to elucidate the effects presented in the present investigation.

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**References**

1. A. R. Bodmer, Phys. Rev. D 4 (1971), 160.
2. E. Witten, Phys. Rev. D 30 (1984), 272.
3. C. Alock, E. Farhi and A. Olinto, Astrophys. J. 310 (1986), 261.
4. K. Iida and K. Sato, Phys. Rev. C 58 (1998), 2538.
5. K. Kohri, K. Iida and K. Sato, Prog. Theor. Phys. 109 (2003), 765.
6. I. Bombaci, I. Parenti and I. Vidaña, Astrophys. J. 614 (2004), 314.
7. F. Weber, Prog. in Particle and Nucl. Phys., in press, \footnote{astro-ph/0407155}.
8. T. Harko, K. S. Cheng and P. S. Tang, Astrophys. J. 608 (2004), 945.
9. I. Bombaci and B. Datta, Astrophys. J. 530 (2000), L69.
10. Z. Berezhiani, I. Bombaci, A. Drago, F. Frotera and A. Lavagna, Astrophys. J. 586 (2003), L1250.
11. T. Galama et al., Nature 395 (1998), 670.
12. S. R. Kulkarni et al., Nature 398 (1999), 389.
13. K. Kotake, H. Sawai, Yamada and K. Sato, Astrophys. J. 608 (2004), 391.
14. S. Yamada and H. Sawai, Astrophys. J. 608 (2004), 907.
15. P. Mészáros, Ann. Rev. Astron. Astrophys. 40 (2002), 137.
16. M. Vietri and L. Stella, Astrophys. J. 507 (1998), L45.
17. K. S. Cheng and Z. G. Dai, Phys. Rev. Lett. 77 (1996), 1210.
18. X. Y. Wang, D. G. Dai, T. Lu, D. M. Wei and Y. F. Huang, Astron. Astrophys. 357 (2000), 543.
19. X. R. Xu, Z. G. Dai, B. H. Hong, and G. J. Qiao, Pacific Rim Conference of Stellar Astrophysics in Hong Univ., in press, \footnote{astro-ph/9908262}.
20. R. Ouyed and F. Sannino, Astron. Astrophys. 387 (2002), 725.
21. E. E. Fenimore, 1995, Astrophys. J. 448 (1995), L101.
22. Z. Berezhiani, and A. Drago, Phys. Lett. B 473 (2000), 281.
23. C. L. Fryer, C. L., and S. E. Woosley, Astrophys. J. 501 (1998), 780.
24. A. Drago, and A. Lavagna, Phys. Lett. B 511 (2001), 229.
25. P. Luigi, Nature 426 (2003), 397.
26. A. Drago, A. Lavagna and A. Pagliara, Phys. Rev. D 69 (2004), 057505.
27. H.-Th. Janka, ed. J. M. Marcaide and K. M. Weiler, 2004, IAU Coll. 192, in press, \footnote{astro-ph/0401461}.
28. N. K. Glendenning and F. Weber, Astrophys. J. 400 (1992), 647.
29. E. Gourgouliou et al., Astron. Astrophys. 349 (1999), 851.
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30) M. Shibata and S. Karino, 2004, Phys. Rev. D, in press, astro-ph/0408016.
31) M. Prakash, J. M. Lattimer, J. A. Pons, A. W. Steiner and S. Reddy, Lecture Notes in Phys. 578 (2001), 364.
32) R. B. Wiringa, Rev. Mod. Phys. 65 (1993), 231.
33) J. J. Drake et al., Astrophys. J. 572 (2002), 996.
34) T. Nozawa, M. Hashimoto, K. Oyamatsu and Y. Eriguchi, Phys. Rev. D 53 (1996), 1845.
35) J. Madsen, 1998b, Lecture Notes in Physics, Springer Verlag, ed. Cleymans, J. in press, astro-ph/9809032.
36) H. Komatsu, Y. Eriguchi and I. Hachisu, MNRAS 237 (1989), 355.
37) M. Hashimoto, K. Oyamatsu and Y. Eriguchi, Astrophys. J. 436 (1994), 257.
38) A. Hayashi, Y. Eriguchi and M. Hashimoto, Astrophys. J. 492 (1998), 286.
39) A. Hayashi, Y. Eriguchi and M. Hashimoto, Astrophys. J. 521 (1999), 376.
40) J. L. Friedman and B. F. Schutz, Astrophys. J. 199 (1975), L157.
41) L. Villain, A. J. Pons, P. Cerdá-Durán and E. Gourgoulhon, 2003 Astron. Astrophys. in press, astro-ph/0310875.
42) D. Gondek-Rosińska, E. Gourgoulhon and P. Haensel, Astron. Astrophys. in press, astro-ph/0311128.
43) M. Hashimoto, Prog. Theor. Phys. 94 (1995), 663.
44) T. A. Thompson, P. Chang and E. Quataert, submitted to ApJ, astro-ph/0401555.
45) Y. F. Huang et al., Astrophys. J. 594 (2003), 919.
Fig. 1. An example of evolutionary transitions from NSs (upper thick and dashed curves) to SSs (lower thick and dashed curves, for which $B = 60$ MeV fm$^{-3}$) with a fixed baryonic mass of $M_B = 2.6 M_\odot$ (two curves decreasing in the direction of increasing $n$). The total angular momentum $J$ (in units of $10^{49}$ g cm$^2$ s$^{-1}$) is assumed to decrease due to gravitational radiation during the transition starting from the $J = 3.06$ state, via 2.8 and 1.5 states, and finally reaching the $J = 0$ state. The arrow indicates the transition from NS to SS. Although other transition paths are possible, the path must be between the points indicated by $\circ$ and $\bullet$. 
Evolutionary transition from rapidly rotating neutron stars to strange stars

Fig. 2. Same as Fig. 1 except with $B = 90$ MeV fm$^{-3}$, and $M_B = 1.7M_\odot$, and the transition from the $J = 1.43$ state, via 1.3 and 1.0 states, and reaching the $J = 0$ state.

Fig. 3. Same as Fig. 1 except with $B = 120$ MeV fm$^{-3}$ and $M_B = 1.5M_\odot$, and the transition from the $J = 1.13$ state, via 1.0 and 0.5 states and reaching the $J = 0$ state.