Resource Reduction For Distributed Quantum Information Processing Using Quantum Multiplexed Photons

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Distributed quantum information processing is based on the transmission of quantum data over lossy channels between quantum processing nodes. These nodes may be separated by a few microns or on planetary scale distances, but transmission losses due to absorption/scattering in the channel are the major source of error for most distributed quantum information tasks. Of course quantum error detection (QED) /correction (QEC) techniques can be used to mitigate such effects but error detection approaches have severe performance limitations due to the signaling constraints between nodes and so error correction approaches are preferable — assuming one has sufficient high quality local operations. Typically, performance comparisons between loss-mitigating codes assume one encoded qubit per photon. However single photons can carry more than one qubit of information and so our focus in this work is to explore whether loss-based QEC codes utilizing quantum multiplexed photons are viable and advantageous, especially as photon loss results in more than one qubit of information being lost. We show that quantum multiplexing enables significant resource reduction: in terms of the number of single photon sources while at the same time maintaining (or even lowering) the number of two-qubit gates required. Further, our multiplexing approach requires only conventional optical gates already necessary for the implementation of these codes.

There are many active approaches being pursued in the development of quantum technologies, including those associated with imaging and sensing [1, 8], communication [4, 9] and computation [10–15]. What has become clear is that many of these will be distributed in nature [3] and, as such, it will be essential to share quantum information between the remote nodes, regardless of whether those nodes are separated on the atomic or planetary scales [16–18]. This distributed nature means we are going to require both a quantum interface between matter & photonic qubits and a photonic bus to transfer such information between nodes [19]. However, real implementations will suffer from losses, which will dramatically affect the performance of the quantum protocols in which such devices are being used. Mechanisms must be developed to mitigate such detrimental effects.

There are quite a number of routes available to offset loss effects, ranging from the development of lower loss fibers to more efficient quantum information coding. The latter route is quite appealing as it can be used with current technology and is likely to be more compatible with our existing infrastructure. There is a well known set of loss based quantum detection and error correction codes that can be used in this situation. In [20] they discuss a simple quantum network scenario in which the quantum multiplexing (QMu) of photonic degrees of freedom allows one to design a single-step combined entanglement distribution and error detection protocol with improved entanglement generation rates, using fewer physical photons and quantum memories) and temporal resources. However, their performance is still limited by the probabilistic nature of the various quantum operations and the resulting necessary heralding signals.

Quantum error correction codes (ECCs) naturally avoid a heralding bottleneck, with example loss based codes including the quantum parity [21], cat [22], binomial [23], Reed-Solomon [24], surface [25] and GKP codes [26]. They allow us to approach the deterministic transmission of quantum information over a lossy channel, as long as those total losses do not exceed a certain threshold (50% at most) [27, 28]. Typical encodings use either the polarization or time bin degrees of freedom but are not particularly resource efficient as they require a large number of single photons. The creation of reliable single photon sources has proved challenging since the generation probability does not exceed 70% [24, 25], whereas, on the contrary, single-qubit gate fidelities can reach 99% [33]. Another limiting factor comes from two qubit gates, which require longer times with fidelities below 90% [30, 38]. However, single photons have the potential to carry much more information using different degrees of freedom (see Supplemental Material for further detail). Hence the natural question is whether using multiple degrees of freedom is advantageous, in terms of reducing the number of photons while maintaining the same number of two qubit gates.

Here we investigate the potential of quantum multiplexing to reduce the resources required to implement loss based error correction codes. We take as a central figure of merit the required number of single photons as well as qubits. We analyze two well known ECCs, the redundant quantum parity [21] and quantum Reed-Solomon codes [24], determining the number of photons and qubits required to reach a threshold success proba-
bility with the multiplexing method.

Let us begin by exploring the redundant quantum parity code \[ [21] \] in a photon transmission regime for which both the number of qubits (memories within the node) and the number of photons can be reduced using our quantum multiplexing approach, all while maintaining the near deterministic transmission of information between the two nodes. In the redundant quantum parity code the information \( \alpha, \beta \) in our encoded state \( |\psi\rangle_{(n, m)} = \alpha |+\rangle_1 \cdots |+\rangle_n + \beta |-\rangle_1 \cdots |-\rangle_n \) (each block term \( |\pm\rangle_i \) = \( |H\rangle^\otimes m \pm |V\rangle^\otimes m \) containing \( m \) photons) is successfully transmitted over the channel when at least one block of \( m \) qubits arrives intact (no losses) and each other block retains at least one photon (see Figure 1a inset). The success probability is given by \[ [20] \]

\[
P_S = [1 - (1 - p_t)^m]^n - [1 - p_t^n - (1 - p_t)^m]^n,
\] (1)

where \( p_t \) is the single photon transmission probability through the channel. Our first observation is that this concatenated code is not particularly resource efficient as the number of qubits at the first logical layer, \( m \), grows inversely with the transmission probability \( p_t \). Further, \( n \) grows inversely with \( p_t^n \) and so \( (m, n) \) grow exponentially with distance between nodes. Our quantum multiplexer is a natural solution \[ [20] \]: here we encode multiple qubits onto a single photon, meaning less photons in total need to be transmitted. More specifically, we enact a two qubit gate between the first degree of freedom (polarization) and a second photonic or matter qubit. Then, swapping the polarization of the initial photon with another degree of freedom (time bin in this case), a third system can then interact independently with the the polarization of this same photon (further details in Supplemental Material). The quantum multiplexer has many potential benefits including deterministic operations between different degrees of freedom — especially important when single photon sources are probabilistic in nature.

Let us explore this in a little more detail. In the inset of Fig. (1a.i) we illustrate a six photon redundant quantum parity code realization without the use of quantum multiplexing in which 3 blocks of 2 photons each are used. After the photons are transmitted over the lossy channel, the code is successful if at least one block contains two photons and the other two blocks each contain one or more photons. One can think of substituting those 6 photons with three quantum multiplexed photons each carrying two qubits of information. In Fig. (1a.ii) these are represented by the colored lines connected to the dots contained in the blocks (see Supplemental Material for further details). In this case, the ECC code only tolerates the loss of one photon. Therefore it would seem logical that we can reduce the number of photons by using the multiplexing approach, provided that the success probability is above the desired threshold value. This raises the question as to what the success probability \( P_S^{\text{QM}} \) will be in this quantum multiplexed approach. One can show for \( n_{\text{tot}} \) transmitted photons that

\[
P_S^{\text{QM}} = \sum_{i=0}^{n^*} \left[ \left( \binom{n_{\text{tot}}}{i} - (U_i + E_i) \right) p_t^{n_{\text{tot}}-i}(1 - p_t)^i \right],
\] (2)

where \( U_i \), \( E_i \) are the number of events in which losing \( i \) photons will leave none of the blocks with the initial number of qubits or at least one empty block and \( n^* \) is the number of lost photons the ECC code can tolerate. We need to determine both \( U_i \) and \( E_i \), which are highly dependent on how the quantum multiplexed photons are connected to the blocks (see Fig. (1b)). Different configurations lead to different success probabilities. We can also release the constraints of all blocks having to have the same number of qubits (an unbalanced configuration), which is typically not utilized in error correction schemes. This enables us to further reduce the number of qubits (and photons) even in the non-multiplexed case (see Supplemental Material).

In Figure (1b) we plot the overall success probability \( P_S \) versus \( p_t \) for two non-multiplexed (equal and unbalanced) configurations alongside one quantum-multiplexed situation with a minimum threshold success probability requirement of \( P_S = 0.995 \) (typical for many quantum computation based tasks). It is clear that our 3 photon quantum multiplexed case (3 blocks with 2 qubits / photon) dramatically outperforms the traditional 6 photon non-multiplexed case (3 blocks with 2 qubits, photons each). In the region \( 0.958 \lesssim p_t \lesssim 0.976 \) the 6 photon case does not reach our threshold target, while the 3 photon multiplexed approach does. The 7 photon configuration (with the first block containing 3 photons while the second and third blocks contain 2 photons each) performs slightly better than the multiplexed case. However both are above the threshold and the multiplexed situation uses fewer photons, qubits and two qubit gates. The multiplexed approach also halves the number of photons in the region \( 0.976 \lesssim p_t \lesssim 0.995 \). These are critical resource savings.

It is clear that the lower \( p_t \) is, the more qubits (two qubit gates) and photons we will need to reach \( P_S \). It is important, in reducing the total numbers of these resources, to also explore unbalanced quantum multiplexing configurations. In Figure (2) we plot the minimal number of qubits, \( N_{\text{min}} \), and photons, \( n_{\text{min}} \), versus \( p_t \) for resource-optimal configurations with 2 and 4 qubits per photon. Quantum multiplexed systems utilize fewer photons, however the number of qubits is either the same or slightly higher, except in a small region near \( p_t \sim 0.97 \) (Fig. (1b)). In fact we can almost halve the number of photons being transmitted over the channel — quite an advantage, especially as single photon sources are currently not as efficient as quantum gates or measurements.

The number of qubits can be maintained equal to the non multiplexing case, while reducing the number of photons, with a mixed strategy, in which each photon can carry an arbitrary number of qubits (from 1 to 4). Table 1 shows the total number of photons and the total...
3 photons, 2 qubits per photon (2 q/p)
7 photons (no QMu)
6 photons (no QMu)

Success probability
0.999
0.998
0.997
0.996
0.995
1

0.96 0.965 0.97 0.975 0.98 0.985 0.99 0.995 1

0.5 0.55 0.6 0.65 0.7 0.75 0.8 0.85 0.9 0.95 1

Figure 1. (a) Plot of the overall success probability versus photon transmission probability $p_t$ of the redundant quantum parity code with (blue curve) and without (red and yellow curves) quantum multiplexed photons. The inset of (a) depicts a schematic illustration of a particular instance of the six-qubit quantum redundancy parity code in which each photon carries one qubit (i) and three photons carry two qubits of information each (2 q/p) (ii). Similarly in (b), we show the success probability $P_{S}^{QMu}$ versus $p_t$ for three different configurations of a quantum multiplexed system. Here 6 photons carry 3 qubits each, distributed over 6 blocks (each block containing 3 qubits).

Figure 2. Plot of the minimum number of qubits (solid lines) and photons (dotted lines) for the (a) redundant quantum parity and (b) Reed-Solomon codes required to reach a threshold success probability of $P_{S} = 0.995$ versus the photon transmission probability $p_t$ using a quantum multiplexed encoding of $2 - 5$ qubits/photon (q/p), respectively. Also shown is the non-multiplexed situation of 1 qubit/photon (q/p) for all codes including the hexagonal GKP code (black curves).

| Total number of photons | Total number of qubits |
|-------------------------|------------------------|
| 15 (1q/p)               | 15                     |
| 14, 13, 12* (mixed)     | 15                     |
| 11 (2q/p)               | 22                     |
| 11, 10, 9, 8 (mixed)    | 21                     |
| 7 (3q/p)                | 21                     |

Table I. Minimum number of photons and qubits required to reach our overall information transfer success probability threshold of $P_{S} = 0.995$ with $p_t = 0.916$. Similar results are seen for most values of $p_t$. The star corresponds to the optimal case, in which, by using the mixing strategy, for a given $N_{min}$ we reach the lowest $n_{min}$ for a specific value of $p_t$.

The number of qubits needed for reaching $P_{S}$ at $p_t = 0.916$ using the pure and the mixed strategies. We observe that we can reach the required $P_{S}$ with a lower number of photons (12) given the same number of qubits (15) when we apply the mixed strategy. The number of two qubit gates required is therefore the same as for the non-multiplexing case, even for bigger codes. This further highlights the potential advantages of quantum multiplexing. Can these improvements be generalized to other loss based quantum error correction codes?

In the quantum Reed–Solomon $[[d, 2k - d, d - k + 1]]_d$ code information is encoded in $d$ qudits, with the code failing on the loss of $d - k + 1$ out of $d$ qudits. For comparative purposes, we will express the degree of multiplexing as $q$ qubits of information per photon. When we encode the qudits in these $q$ degrees of freedom of quantum multiplexed photons, any qudit of information depends upon the successful transmission of $\left\lceil \log_2(d)/q \right\rceil$ photons [24]. The probability of failure is therefore

$$P_{Fail} = \sum_{j=d-k+1}^{d} \binom{d}{j} \left(1 - p_t^{\frac{\log_2(d)}{q}}\right)^j p_t^{j \frac{\log_2(d)}{q}(d-j)}.$$  \hspace{1cm} (3)
In this code the block is given by the total number of photons encoding a single qudit, and if a block is incomplete, the qudit is not successfully transmitted. Therefore, the performance can be improved by maintaining independence between these blocks, and by reducing the chances for loss events within any single block. Adding additional quantum multiplexing will help so long as it preserves independence between qudit loss events. For the quantum Reed-Solomon code we can also determine the lowest number of qubits and photons required to reach $T_S$, as shown in Fig. 2b. Here, the advantage of using quantum multiplexed photons is evident in terms of a reduction of the number of qubits, two qubit gates and photons compared to the no quantum multiplexing case. In particular, the higher is the quantum multiplexing degree the less qubits and photons we require. For instance, at $p_t = 0.85$, we have that for $q = 4$, $N_{\min} \simeq 40$ and $n_{\min} \simeq 10$, whereas when no quantum multiplexed photons are in use, we have that both $N_{\min}$ and $n_{\min}$ are over 1000. As $p_t$ gets lower, the number of photons and qubits increases considerably, hence, we need to use higher degrees of quantum multiplexing. Furthermore, by comparing Fig. 2a with Fig. 2b, we infer that there is always a specific value of $q$ for which the Reed-Solomon code requires a lower number of resources compared to the parity code (for $q = 4$, at $p_t = 0.85$, $N_{\min}(n_{\min})$ is 72% (75%) lower for the Reed-Solomon code than the parity code). For other error correction codes based on the transmission of qudits we expect the same reduction in the number of qubits, two qubit gates and photons when quantum multiplexing is in use.

There are other loss codes based on encoding information in superposition of photon number (bosonic 25 and GKP 26 for instance), in which quantum multiplexing is ineffective. In these cases this would correspond to the assignment of information about multiple excitation to the various degrees of freedom of a single mode. However, any quantum multiplexed photon mode is equivalent, in this case, to a no quantum multiplexed mode. There is always therefore a code using fewer excitations and a higher number of modes than the original that will perform as well as the quantum multiplexed case.

It is essential to compare these quantum multiplexed codes to the best loss codes currently known - namely the GKP codes 40, 41. In particular, in 41 the authors show that the hexagonal GKP code is the optimal among all single-mode bosonic codes against loss errors expressed as a Gaussian displacement channel. In Fig. 2a, b) we plot (black curves) the average number of photons for the hexagonal GKP code 42. This suggests that there are regions where the GKP code has a better performance and other regions where this is reversed. The multiplexed codes operate better in the higher loss regimes. Further, a critical consideration has to be the near deterministic implementation of the code itself. Our quantum multiplexing approach requires the same basic two qubit/qudit gates needed for quantum logic (and the original codes themselves) with the addition of high efficiency optical switches to swap state between the different degrees of freedom. On the other hand, the initialization of the GKP code is quite demanding to achieve in a near-deterministic way and necessitates a more complex continuous variable procedure, though Gaussian operations are sufficient for subsequent qubit control. Generating such codes in a heralded but probabilistic fashion has been achieved but unfortunately increases the resources required 43, 44 (see Supplemental Material). This indicates that additional resources will be required at the end nodes to process the quantum data transmitted over the communication channel. We note that a proof-of-principle demonstration of deterministic preparation was performed in 44, and look forward to the development of this promising approach going forward.

To summarize, we have shown how quantum multiplexed loss codes have the potential to significantly decrease the resources required to transfer quantum information between two adjacent nodes. This is achieved while maintaining or even lowering the required number of two-qubit gates. Two primary error correction codes were considered: the redundant quantum parity code and the quantum Reed-Solomon code. For the former, we found that the total number of single photons that need to be transmitted through the channel can be dramatically reduced (near 50 percent) without significantly increasing the number of qubits. Further, we found it advantageous for individual photons to have different degrees of quantum multiplexing, as well as for blocks to contain different numbers of qubits. The quantum Reed-Solomon code significantly outperforms the redundant quantum parity code and, using quantum multiplexed qudits, has the potential to reduce simultaneously the number of photons, qubits and gates used. These improvements should be possible in many (but not all) of the other loss based error correction codes when quantum multiplexing is used. Quantum multiplexing has the potential to be a new resource saving tool especially for near term implementations. Our findings can be applied to any communication system that needs error correction to improve its efficiency, such as in quantum repeaters, quantum computation and quantum sensing.

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