D-brane Dynamics in RR Deformation of NS5-branes Background and Tachyon Cosmology

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Abstract

We study D-brane dynamics in the gravity dual background of ODp theories using the effective action on the worldvolume of the brane. We explore the similarities of the system with the rolling tachyon, including the exponential decrease of pressure at late times. We also consider the case where the worldvolume theory on a D3-brane is coupled to gravity and construct a cosmological model. By considering the time reversal symmetric solutions, we find that we can have both closed and open universes depending on the initial value of the radial mode on the brane. We compare the models with tachyon driven cosmologies and find limits where slow roll inflation is realized.
1 Introduction

D-brane interactions has been a subject of interest for a long time. The subject has received more attention since it was suggested that the worldvolume theory on D-branes can, in some cases, be used to construct inflationary models for cosmology (see e.g [1],[2],[3],[4]). To construct such models one requires the D-brane to go through a time dependent process in string theory. In some cases, this time dependence is provided by considering branes with tachyonic modes which cause an instability in the system. The unstable brane decays as the tachyon, which is playing the role of the inflaton, rolls down a potential. Such models are known as tachyon driven cosmologies (see [5],[6],[7]). In other cases, the time dependence is achieved by considering non BPS configurations where the D-brane moves under the influence of a potential. The inflaton is then identified with the transverse mode that specifies the location of the brane. A considerable amount of work has been done recently using this scenario [4] and its modifications [8],[9],[10].

As an example of such a non BPS configuration, one can place a D-brane in the background produced by NS5 branes in such a way that no supersymmetry is preserved. NS5 branes are special in a number of ways. An important fact about these objects is that unlike RR charged D-branes, they keep curving the space around them even when $g_s \rightarrow 0$. This can also be seen by the fact that the tension of the D-brane goes as $1/g_s$, whereas for the fivebranes it goes as $1/g_s^2$ so that the latter becomes much heavier than the former in this limit. Therefore NS fivebranes are known to be solitonic rather than perturbative objects in string theory. Another important issue is that the string coupling becomes exceedingly large as one approaches the bottom of the throat produced by these objects.

The dynamics of a D-brane, parallel to a stack of NS5-branes was studied in [11] using the effective action on the worldvolume of the D-brane, the DBI action\(^1\). Based on the above argument, the D-brane can be considered as a probe that is exploring the background without influencing it considerably. It was shown that for a certain range of energies of the D-brane, most of the dynamical process occurs when the coupling is still small and perturbative string theory is reliable. Since this configuration of branes breaks all of the supersymmetry, the probe experiences a net force from the background and becomes unstable. This is reminiscent of the rolling tachyon problem. It was shown in [11] that by a proper field redefinition, the effective action of the probe can actually be brought into the form of the DBI action for the tachyon in the open string models [13],[14]. Being related to the transverse coordinate of the brane, the defined tachyon field finds a geometric interpretation in this case. By looking at the energy momentum tensor of the worldvolume theory, it was shown that as the brane approaches the fivebranes the pressure goes exponentially to zero which is the late time behavior of tachyon matter [15]. For the tachyon, this is interpreted as the decay of the unstable brane into closed string radiation [16]. For the probe D-brane, it was argued that the pressureless fluid is a

\(^1\)see also [12] for earlier works and [22] for the D\(_p\)-brane background.
signal that the brane sheds its energy into modes living on the fivebranes. Losing almost all of its energy as it approaches the fivebranes, the D-brane will eventually form a bound state with the fivebranes.

In the present work we study the effect of RR fields in the above processes. For this purpose, we consider the deformation of NS5-brane background in presence of nonzero RR forms \[17, 18\]. We focus on the limit where the theory on the fivebranes decouples from the bulk. In the absence of these RR fields, the resulting theory is known as Little String Theory (LST) \[19\]. The RR fields deform LST to what is known as OD\(p\) (Open D\(p\)-brane) theories. We will study the dynamics of a D-brane in the background which is the gravity dual of OD\(p\) theories.

RR fields change things in two different ways. Firstly, the metric and dilaton background fields are deformed and secondly, a term is added to the effective action describing the coupling of the D-brane to the RR background fields. It will turn out that the behavior of the D-brane, including its similarities with the rolling tachyon problem, remains unchanged at late times.

We will also consider the case where the effective action on the worldvolume of a probe D\(3\)-brane, which is moving in the NS5 branes background, is coupled to gravity \(^2\) by introducing a dynamical four dimensional metric and thus making a cosmological model. For the metric, we choose it to be the FRW solution of Einstein’s equations. We also include a small positive cosmological constant term in the action to allow for nonsingular solutions. We find that if we place the brane far from the fivebranes the resulting solution will be an open universe. For small enough distances as well, such an open solution is obtained. But there is an intermediate range of distances from the fivebranes where, if the D-brane starts from, a singularity will occur in the solution and thus the universe will close.

The deformation of the background by RR fields modifies the cosmological solutions slightly. Although in the UV regime the potential for rolling is affected by the deformation, the general behavior of the solutions remains unchanged. By analyzing the equations of motion, we comment on the relative width of the initial values that the tachyon field can take in order to result in singular solutions. We support our judgments by presenting some of the values which are obtained by numerical analysis. In each case we will compare our solutions with the ones obtained for rolling tachyon and observe that although at late times the similarities of the two systems persist in the coupled to gravity theories, we can find limits where unlike the rolling tachyon, the criteria for slow roll inflation are met.

2 D-branes in the NS5-branes Background in presence of RR fields

In this section we study the dynamics of a D-brane in the noncommutative deformation of Little String Theory (LST). Let us first remind some basic facts about

\(^2\)for a related recent work see \[23\].
type II NS5-branes and its decoupling limit. The solution describing $N$ coincident NS5-branes is \[20\]
\[
ds^2 = dx_\mu dx^\mu + h(x^n) dx_m dx^m ,
\]
\[
e^{2\Phi - 2\Phi_0} = h(x^n) ,
\]
\[
H_{mn} = -\epsilon_m^{\ qn} \partial_q \Phi .
\]
\[
(2.1)
\]
where $x^\mu$ are tangent to the fivebranes and $x^m$ are transverse to them. $H_{mn}$ is the field strength of the NSNS two form and the harmonic function $h$ is given by
\[
h = 1 + \frac{N l_s^2}{r^2} ,
\]
\[
(2.2)
\]
where $r = \sqrt{x_n x^n}$. Being much lighter than the solitonic fivebranes in the small string coupling regime, the D-brane can be considered as a probe exploring the above background. The metric describes a geometry which is flat along the fivebranes and is isotropic in the transverse directions with an $SO(4)$ symmetry. There is an infinite throat with the geometry $S^3 \times R^1 \times$ a six dimensional Minkowski space. As one moves down the throat towards the fivebranes at $r = 0$, the radius of the $S^3$ approaches an asymptotic value which is proportional to $\sqrt{N} l_s$. This is the characteristic length or the radius of the solution. Due to nonrenormalization theorems, the solution \[2.1\] remains a valid string background even if its scale is the string scale i.e if $N$ is small. The dilaton grows in the throat, diverging as one approaches the fivebranes and hence a probe D-brane traveling down the throat will sooner or later move in the large dilaton region where string perturbation theory breaks down. Yet, as was mentioned in \[11\], by assuming certain energies for the brane one can postpone this to very late times.

The decoupling limit of \[2.1\], known as the linear dilaton background, is obtained by taking the limit $g_s \to 0$ and keeping $l_s$ and $u \equiv \frac{r}{g_s}$ fixed where \[2.1\] reduces to
\[
ds^2 = dx_\mu dx^\mu + \frac{N l_s^2}{u^2} (du^2 + u^2 d\Omega_3^2) ,
\]
\[
e^{2\phi} = \frac{N l_s^2}{u^2} , \quad dB = 2N e_3 .
\]
\[
(2.3)
\]
This background provides the gravity dual for Little String Theory (LST) which lives on the worldvolume of the fivebranes when decoupled from the bulk \[19\].

One can deform LST by adding RR fields to the story. What happens in practice is that one first obtains the solution describing NS5-branes in presence of nonzero RR fields and then takes the decoupling limit of the resulting background. It turns out that in order to include an RR electric $p + 1$-form in the problem one has to add an RR magnetic $5 - p$-form as well and vice versa ($p = 0, \cdots, 5$). Hence the theory with electric $p + 1$-form is the same as that with magnetic $5 - p$-form. The
decoupling limit, though, is quite different from what we saw above (which resulted in the linear dilaton background). The resulting background provides the gravity dual description for ODp theories which live on the worldvolume of NS5-branes and the excitations of which include light open Dp-branes [21],[18].

After doing all this, the solution describing N coincident NS5-branes in presence of nonzero RR fields after taking the decoupling limit is [18]

\[
ds^2 = (1 + a^2 r^2)^{1/2} \left[ -dt^2 + \sum_{i=1}^{p} dx_i^2 + \frac{\sum_{k=p+1}^{5} dx_k^2}{1 + a^2 r^2} + \frac{N}{r^2} dx_m dx^m \right],
\]

\[
A_{0\ldots p} = \frac{1}{\tilde{g}} a^2 r^2 , \quad A_{(p+1)\ldots 5} = \frac{1}{\tilde{g}} \frac{a^2 r^2}{1 + a^2 r^2},
\]

\[
e^{2\phi} = \tilde{g}^2 \frac{(1 + a^2 r^2)^{(p-1)/2}}{a^2 r^2}, \quad dB = 2N \epsilon_3, \quad a^2 = \frac{l_{\text{eff}}}{N},\]

(2.4)

where \( p = 1, 3, 5 \) in IIB and \( p = 0, 2, 4 \) in IIA theory, \( r = \sqrt{x^m x^m} \) and \( l_{\text{eff}} \) and \( \tilde{g} \) are effective string tension and coupling after the decoupling limit respectively. In the above solution we have dropped powers of \( l_s \) that appear as an overall factor in the metric and in the definitions for the RR fields because they will play no role in the equations of motion for the probe branes.

### 2.1 Setup of the problem

We will now study the dynamics of a probe Dₚ'-brane in the background (2.4), using the effective action on the worldvolume of the D-brane. The embedding of the D-brane in the bulk space is expressed through the maps \( X^A(\xi^\mu) (A = 0, 1, \ldots, 9) \). We choose the coordinates of the probe, \( \xi^\mu (\mu = 0, 1, \ldots, p') \), to lie completely inside the directions of the original NS5-brane so that it is pointlike in the directions transverse to them, \( X^m (m = 6, 7, 8, 9) \), which we parameterize as

\[
\sum_{m=6}^{9} (dX^m)^2 = dr^2 + r^2 (d\phi^2 + \sin^2 \phi \, d\theta^2 + \cos^2 \phi \, d\psi^2) . \quad (2.5)
\]

We also assume that \( X^m \)'s depend only on the time coordinate of the worldvolume, \( \xi^0 \), such that \( X^m(\xi^\mu) = X^m(\xi^0) \). The other transverse coordinates of the D-brane (if any), we assume to be constant. One can also in general have gauge fields on the D-brane which, in this work, we always assume to be zero. Therefore the only dynamical fields of the worldvolume effective theory will be \( X^m(\xi^0) \).

The effective action of a D-brane consists of a DBI part and CS terms arising from coupling to the RR fields

\[
S_p = -\tau_p \int d^{p+1} \xi \, e^{-(\Phi - \Phi_0)} \sqrt{-\det(G_{\mu\nu} + B_{\mu\nu})} + \tau_p \int d^{p+1} \xi \, A , \quad (2.6)
\]

where \( G_{\mu\nu} \) and \( B_{\mu\nu} \) are the metric and the NS two form induced on the worldvolume respectively and \( A \) is potential form for RR fields. For the ansatz we have taken \( B \) will always be zero.
In the cases we study in the following sections, the action (2.6) will eventually lead to a Lagrangian, \( \mathcal{L} \), describing the motion of a particle moving in a central force field. Thus the motion will totally take place in a 2-plane which, without loss of generality, we take to be at \( \phi = \frac{\pi}{2} \) and hence we are left with only two dynamical fields, \( r \) and \( \theta \). There are two conserved quantities associated with the motion of the D-brane; energy \( (E) \) and angular momentum \( (L) \)

\[
E = \frac{\partial \mathcal{L}}{\partial \dot{X}^m} \dot{X}^m - \mathcal{L}, \quad L = \frac{\partial \mathcal{L}}{\partial \theta},
\]

(2.7)

where dot means differentiation with respect to \( \xi^0 \). The above relations enable us to find \( \dot{r} \) in terms of \( r, E \) and \( L \) and to define an effective potential, \( V_{\text{eff}}(r) \), which describes a particle of mass = 2 and zero energy moving in one dimension. Cases with \( L = 0 \) describe purely radial motion.

There are three distinct cases that we consider. In the first two cases the \( \xi^\mu \)'s lie inside the \( (t, x^i) \) part of the spacetime; in the first case \( p' < p \) whereas in the second one \( p' = p \). In the third case the \( \xi^\mu \)'s fill some or all of the directions \( (t, x^i, x^k) \).

The reliability of this analysis requires small proper acceleration for the scalar fields on the worldvolume of the D-brane and a small string coupling at its location.

2.2 Case 1

In this case the \( \xi^\mu \) fill some but not all of the directions \( (t, x^i) \) such that \( p' < p \). We should necessarily have \( 2 \leq p \leq 5 \). The reparametrization invariance of the worldvolume theory allows us to choose \( \xi^\mu = x^\mu \). There will be no coupling to the RR fields in this case and the effective action will read

\[
S_{p'} = -a \tau_{p'} \int d^{p'+1} \xi f(r)^{\frac{4}{j}} r \sqrt{1 - \frac{N}{r^2} \dot{X}^m \dot{X}^m},
\]

(2.8)

where \( f(r) = 1 + a^2 r^2 \), \( j = p' - p + 2 \) and dot now stands for \( t \) derivatives.

Energy and angular momentum densities will read

\[
E = \frac{a \tau_{p'} r f^{\frac{j}{4}}}{\sqrt{1 - \frac{N}{r^2} (\dot{r}^2 + r^2 \dot{\theta}^2)}},
\]

\[
L = \frac{a \tau_{p'} N r \dot{\theta} f^{\frac{j}{4}}}{\sqrt{1 - \frac{N}{r^2} (\dot{r}^2 + r^2 \dot{\theta}^2)}}.
\]

(2.9)

Defining \( u \equiv a \ r \) we have

\[
\dot{u}^2 = \frac{u^2}{N} \left( 1 - \frac{L^2}{E^2 N} - \frac{u^2}{\lambda_{p'}^2} (1 + u^2)^{\frac{j}{2}} \right),
\]

\(^31 \leq i \leq p \) and \( p + 1 \leq k \leq 5 \)
\[ e^\Phi = \frac{\tilde{g}}{u}(u^2 + 1)^{\frac{p-1}{4}}, \]  

(2.10)

where \( \lambda_{\rho'} = \frac{E}{\tau_{\rho'}} \). One should note that for both IIA and IIB theories \( j = -2, 0 \). As the D-brane goes from large values of \( u \) towards \( u = 0 \), the coupling increases for \( p = 2, 3 \) monotonically whereas for \( p = 4, 5 \) it decreases until \( u = u_g \equiv \sqrt{\frac{2}{p-3}} \) and then increases as it gets closer to \( u = 0 \).

The effective potential is now defined as \( V_{\text{eff}}(u) = -\dot{u}^2 \). Qualitative description of motion can be achieved by finding the general shape of the potential, for which, we look at its behavior in small and large values of \( u \) for each value of \( j \).

**(A):** \( j = -2, L \neq 0 \)

\[
V_{\text{eff}} \rightarrow \frac{u^2}{N}(\frac{L^2}{E^2 N} - 1) \quad u \rightarrow 0 ,
\]

\[
V_{\text{eff}} \rightarrow \frac{u^2}{N}(\frac{1}{\lambda_{\rho'}^2} + \frac{L^2}{E^2 N} - 1) \quad u \rightarrow \infty .
\]

(2.11)

There are three regimes for energy:

**(A.1):** \( \tau_{\rho'}^2 + \frac{L^2}{N} < E^2 \); the D-brane explores all values of \( u \). It can either start at \( u = 0 \) at early times and escape to infinity at late times or vice versa.

**(A.2):** \( \frac{L^2}{N} < E^2 < \tau_{\rho'}^2 + \frac{L^2}{N} \); the brane can get at most as far as \( u_{\text{max}} = \lambda_{\rho'}' \equiv \lambda_{\rho'}\sqrt{1 - \frac{L^2}{E^2}} \) from \( u = 0 \). If the brane starts at \( u = u_{\text{max}} \) at \( t = 0 \), it will take it an infinite amount of time to fall towards the origin. If \( \lambda_{\rho'}' \ll 1 \), the brane always remains at small values of \( u \). So one can use the approximation of \( V_{\text{eff}} \) in this region to solve for \( u \) with the result

\[
\frac{1}{u} = \frac{1}{\lambda_{\rho'}'} \cosh \left( \sqrt{1 - \frac{L^2}{E^2}} \frac{t}{\sqrt{N}} \right) .
\]

(2.12)

The string coupling in this region is approximated as

\[ e^\Phi \approx \frac{\tilde{g}}{u} , \]

(2.13)

therefore if

\[ \tilde{g} \ll \lambda_{\rho'}' \ll 1 , \]

(2.14)

the solution (2.12) is reliable for a considerably long period of time. By a calculation quite similar to what is done in [II], one can see that the brane is in a spiraling motion as it falls towards the origin.

**(A.3):** \( E^2 < \frac{L^2}{N} \); the D-brane remains at \( r = 0 \) for all times.

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(B): $j = -2$, $L = 0$.
In terms of $\lambda_{p'}$, there are two cases:

(B.1): $\lambda_{p'} > 1$; the case is just like (A.1). The solution describes either an incoming brane towards $u = 0$ or an outgoing one escaping to infinity.

(B.2): $\lambda_{p'} < 1$; the situation is qualitatively that of case (A.2). Here the D-brane can get no further than $u_{\text{max}} = \frac{\lambda_{p'}}{\sqrt{1 - \lambda_{p}'}}$ from the origin. If $\lambda_{p'} \ll 1$, this maximum distance is still in the small $u$ region and the following solution will be a good approximation

$$\frac{1}{u} = \frac{1}{\lambda_{p'}} \cosh\left(\frac{t}{\sqrt{N}}\right),$$

(2.15)

where we have assumed that the brane is at rest at $t = 0$. Here again we require that the string coupling remains small. So the solution (2.15) is reliable for a long period of time if $\tilde{g} \ll \lambda_{p'} \ll 1$.

(C): $j = 0$, $L \neq 0$

$$V_{\text{eff}} \rightarrow \frac{u^2}{N} \left(\frac{L^2}{E^2 N} - 1\right) \quad u \rightarrow 0,$$

$$V_{\text{eff}} \rightarrow \frac{u^4}{\lambda_{p'}^2 N} \quad u \rightarrow \infty. \quad (2.16)$$

There are two cases to consider

(C.1): $E^2 \leq \frac{L^2}{N}$; the brane remains at $u = 0$ at all times.

(C.2): $\frac{L^2}{N} \leq E^2$; the brane has a turning point at $u_{\text{max}} = \lambda_{p'}$. The solution is the same as (2.12) but this time it is valid for all values of $\lambda_{p'}$. To have a small coupling for a long time period we must have $\tilde{g} \ll \lambda_{p'} \ll 1$.

(D): $j = 0$, $L = 0$.
There is only one solution here describing a brane moving in $0 \leq u \leq u_{\text{max}} = \lambda_{p'}$. The solution is (2.12) with $L = 0$ which is valid for all values of $\lambda_{p'}$. Here again we assume that $\tilde{g} \ll \lambda_{p'} \ll 1$.

### 2.3 Case 2

In this case the $\xi^\mu$ fill all of the directions $(t, x^i)$ and $p' = p$. The worldvolume theory is still reparametrization invariant and one can set $\xi^\mu = x^\mu$. This time there
is a coupling to one of the RR fields and the effective action reads $(f = 1 + a^2 r^2)$

$$S_p = -a \tau_p V_p \int dt \left( r f^{\frac{1}{2}} \sqrt{1 - \frac{N}{r^2} \dot{X}^m \dot{X}^m - ar^2} \right).$$  \hspace{1cm} (2.17)

The conserved quantities are

$$E = \frac{a \tau_p \sqrt{f^2 r^4}}{\sqrt{1 - \frac{N}{r^2}(\dot{r}^2 + r^2 \dot{\theta}^2)}},$$

$$L = \frac{a \tau_p r f^{\frac{1}{2}}}{\sqrt{1 - \frac{N}{r^2}(\dot{r}^2 + r^2 \dot{\theta}^2)}} - a^2 \tau_p r^2.$$  \hspace{1cm} (2.18)

Then, in terms of $u$, the equation of motion becomes

$$\ddot{u} = \frac{u^2}{N} \left( 1 - \frac{L^2}{E^2(1 + \frac{u^2}{\lambda_p^2})^2} - \frac{u^2 f}{\lambda_p^2(1 + \frac{u^2}{\lambda_p^2})} \right).$$  \hspace{1cm} (2.19)

Now again we find the general behavior of the effective potential by looking at its form for small and large values of $u$. There are two distinct cases to consider, $L \neq 0$ and $L = 0$.

(E): $L \neq 0$

$$V_{eff} \rightarrow \frac{u^2}{N} \left( \frac{L^2}{N E^2} - 1 \right) \quad u \rightarrow 0,$$

$$V_{eff} \rightarrow \frac{1}{N} \left( 1 - 2 \lambda_p \right) \quad u \rightarrow \infty.$$  \hspace{1cm} (2.20)

There are four regimes for energy:

(E.1): $\frac{\tau_p}{2} < E < \frac{L}{\sqrt{N}}$, if the brane starts at $u = 0$ it remains there at all times, otherwise, it can get no closer to the origin than

$$u_m^2 = \frac{\lambda_p^2 \left( 1 - \frac{L^2}{E^2 N} \right)}{(1 - 2 \lambda_p)}.$$  \hspace{1cm} (2.21)

The brane is then deflected and escapes back to infinity.

(E.2): $\lambda_p < 1$ and $E < \frac{L}{\sqrt{N}}$; the brane remains at $r = 0$ at all times.

(E.3): $\lambda_p > 1$ and $\frac{L}{\sqrt{N}} < E$; the brane goes from $r = 0$ to infinity or vice versa.

(E.4): $\frac{\tau_p}{\sqrt{N}} < E < \frac{\tau_p}{2}$; the brane moves between $u = 0$ and a maximum, $u_m$, which is given by (2.21). If $\lambda_p \ll 1$ then $u_m \ll 1$ and the approximate solution for $u$
and the condition for small coupling for a long period of time are respectively (2.12) and (2.14) with \( p' = p \).

\((F):\) \( L = 0 \).

There are two cases:

\((F.1):\) \( \lambda_p < \frac{1}{2} \); the brane moves between \( u = 0 \) and a maximum \( u_{\text{max}} = \frac{\lambda p}{\sqrt{1 - 2 \lambda_p}} \).

If \( \lambda_p \ll 1 \) then the approximate solution for \( u \) and small coupling condition are found to be respectively (2.12) and (2.14) with \( L = 0 \) and \( p' = p \).

\((F.2):\) \( \frac{1}{2} < \lambda_p \); the brane goes from \( u = 0 \) to infinity or vice versa.

### 2.4 Case 3

In this case the brane is extended in some or all of the \((t, x^i, x^k)\) directions. The worldvolume theory no longer has reparameterization invariance and hence the D-brane dynamics does depend on the way it is embedded in the bulk. Here we consider the simple ansatz for the embedding such that \( X^A \) with \( 0 \leq A \leq p \) are independent from those with \( p < A \leq 5 \). The brane thus fills \( s+1 \) of the \((t, x^i)\) and \( q-p \) of the \( x^k \) directions with \( s + q - p = p' \). One can then use the reparameterization invariance in the two subspaces to set \( x^\mu = \xi^\mu (\mu = 0, \ldots, s, p+1, \ldots, q) \).

The effective action consists only of the \( DBI \) part

\[
S'_{\psi'} = -\alpha \tau_{\psi'} \int d^{p'+1} \xi \ r f(r)^\frac{j}{4} \sqrt{1 - \frac{N}{r^2} \dot{X}^m \dot{X}^m}, \tag{2.22}
\]

where \( j = s - q + 2 \). It is interesting to note that \( j \) can only take the values 0, −2. Hence we are back to the action (2.8) and each solution in this case corresponds to one of the solutions found in case 2.

### 2.5 Comments on the solutions

The above results show that the D-brane can have four possible qualitative behaviors. The first one is staying at the origin. The second one is moving between the origin and a maximum distance. The third one is moving between the origin and infinity and finally the forth possibility is to move between a minimum distance from \( u = 0 \) and infinity. The first and fourth of these can only happen for a brane with a nonzero angular momentum. As was found in [11], all four possibilities can be experienced by a D-brane in the background (2.1). As a result of deformation, one finds some cases that even when the brane has a nonzero angular momentum, not all of the four behaviors are observed. For example, the fourth possibility is excluded in case (A) or in case (C) the brane can only have the first two behaviors. Case (E) is the only case with all four possibilities. We also see that quite like the
solutions obtained in [11], here again we find no solution describing the brane in a stable orbit around the origin.

Coupling to RR fields only happens when $p' = p$ i.e. in case 2. The effects of nonzero RR fields also appear in the dependence of the solutions on the value of $p$. In the approximate solutions which are obtained for $u \ll 1$, this dependence vanishes. This is consistent with the fact that the effects of nonzero RR fields become unimportant in the infrared regime [18]. As a result, the solutions we find in these cases are quite similar to the ones found in [11] with the general behavior $r \sim \frac{1}{\cosh t}$ for small values of $r$. We also found two solutions in cases (C) and (D), where $j = 0$, which are exact regardless of the initial conditions. This was not possible in the background (2.1).

As was mentioned in [11], for purely radial motions of a D-brane in the NS5-branes background one can, by a proper field redefinition, write the effective action as a DBI action for the tachyon. In our problem, for example in case 1 and for $L = 0$, defining

$$T = \sqrt{N} \ln u,$$  \hspace{1cm} (2.23)

one can write the action as

$$S_T = -\int d^{\nu + 1} \xi V_{p'}(T) \sqrt{1 - \dot{T}^2},$$  \hspace{1cm} (2.24)

where

$$V_{p'}(T) = \tau_{p'} e^{\frac{p'}{\sqrt{N}}} (1 + e^{\frac{2T}{\sqrt{N}}})^{rac{1}{4}}.$$  \hspace{1cm} (2.25)

As $u \to 0$, $T \to -\infty$ and as $u \to \infty$, $T \to \infty$. We now look at the behavior of the potential in these two limits

$$\frac{1}{\tau_{p'}} V_{p'}(T) \approx e^{\frac{p'}{\sqrt{N}}} \quad T \to -\infty.$$  \hspace{1cm} (2.26)

As expected, in the limit $T \to -\infty$ ($u \to 0$), the situation is quite similar to the one studied in [11] and the potential goes exponentially to zero which is the late time behavior of the tachyon potential for unstable D-branes. In the opposite limit, due to the nonzero RR fields, we expect to see deviations from the long range gravitational attraction between the D-brane and the fivebranes, $V(T) \approx -1/T^2$, observed in the background (2.1). This is actually the case

$$\frac{1}{\tau_{p'}} V_{p'}(T) \approx e^{\frac{p'}{\sqrt{N}}} \quad T \to \infty (j = 0),$$

$$\frac{1}{\tau_{p'}} V_{p'}(T) \approx 1 - \frac{1}{2} e^{\frac{2T}{\sqrt{N}}} \quad T \to \infty (j = -2).$$  \hspace{1cm} (2.27)

For very small values of $u$, the potential governing the dynamics of the D-brane is that for a tachyonic field at late times. It is known that the pressure of a tachyonic field drops exponentially to zero at late times [15]. In [11] it was shown that this
is also the case for a probe D-brane near NS5-branes. For our problem also, as expected, this happens. To see this, we just need to find the energy momentum tensor associated with (2.8) and read the pressure

\[ P \sim -u(1 + u^2)^{3/4}\sqrt{1 + Nu^2\dot{u}^2}. \]  

(2.28)

Plugging \( u \) from (2.15) in the above expression results in the following behavior for pressure at late times

\[ P \sim e^{-2t/\sqrt{N}}. \]  

(2.29)

In the next section, we will ask if this similarity persists when we use the world-volume theory to construct a cosmological model. Motivated by this, we couple the worldvolume theory of our probe D\(_3\)-brane to gravity by introducing a four dimensional dynamical background metric \( g_{\mu\nu} \) which we choose to be a FRW solution of Einstein’s equations. We will then compare the resulting models with the ones arising from tachyon driven cosmologies. We will do this analysis both before and after the NS5-branes background is deformed by RR fields.

### 3 Coupling to gravity

The worldvolume theory on a D\(_5\)-brane moving in the NS5 branes background or its deformation can be coupled to gravity by truncating the space to a finite range of values for the radial coordinate. The region is then smoothly glued to a proper compact manifold to the outside space \( M^4 \). As a result of this truncation, the effective theory will include an Einstein-Hilbert term in the action \( S \). The resulting theory will describe a six dimensional cosmology. If we insist on having a four dimensional model, we must start with the background where two of the six noncompact directions of the NS5 branes are wrapped around a compact manifold.

For example in [10] a D\(_5\)-brane has been considered in the presence of NS5-branes where both are wrapped around a 2-sphere and therefore the potential on the D-brane is that induced by Maldacena-Nunez background [24]. One could as well consider a D\(_3\)-brane in the MN background as a system which can couple to four dimensional gravity. We will discuss this scenario in section 3.3.

Now let us see if and how the RR deformation of NS5-branes background that we considered in the previous sections can couple to four dimensional gravity. For this purpose we must note that, as stated in [18], one can find this background by considering M5-branes of 11 dimensional SUGRA in the presence of a \( C \) field where the branes are smeared in a transverse direction. If one reduces this direction then the IIA NS5-branes background with an electric 3 form is obtained. The other solutions with different forms can then be found by T-duality in the directions where the magnetic forms are defined namely in the directions \( p + 1, \cdots, 5 \) of (2.4). Upon taking the decoupling limit the background of our interest is obtained.
Therefore it is implicit in this construction that the magnetic directions are considered on a torus. This can also be understood by the fact that the magnetic part of the metric shrinks by a factor of \(1/ar\) for large values of \(a\). As a result, one might suspect that a probe \(D_3\)-brane in the electric directions \(0, \cdots, 3\) can couple to four dimensional gravity\(^4\). For other cases this reasoning clearly breaks down. A full account of this problem and the way the effective potential on the probe brane is modified, due to compactifying the two transverse directions of the \(D_3\)-brane, requires a more careful study which we postpone to a future work.

In the following we take the first steps towards this problem and construct a four dimensional model by considering the effective action on a probe \(D_3\)-brane moving in the NS5 or its deformed background where the flat metric is replaced by a dynamical one. We also assume that by a KKLT like mechanism\(^3\) a nonzero cosmological constant is included in the action. In fact we ”assume” that there is a certain string compactification that results in such a model. We also take all the moduli to be fixed. For the dynamical metric, we take the following form

\[
\begin{aligned}
    ds^2 &= -dt^2 + a(t)^2 \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right) , & k &= 0, \pm 1 , \\
    \end{aligned}
\]

where \(r\) is the radial coordinate inside the brane. To make contact with the tachyon problem, by proper field redefinition we will introduce a tachyon field \(T\), in terms of which the action is brought into the general form

\[
S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[ R - \int d^4x \sqrt{-g} \left( V(T) \sqrt{1 - \dot{T}^2} + \Lambda \right) \right] ,
\]

where \(G \sim g_s^2\) is the four dimensional gravitational coupling and \(V \sim 1/g_s\). The potential \(V(T)\) is determined in each case and is to be compared with the potential responsible for the decay of unstable D-branes\(^{13,14}\)

\[
V_{\text{tac}}(T) = \frac{V_0}{\cosh \frac{T}{\sqrt{2}}} ,
\]

where we assume \(V_0 \gg \Lambda\). In each of the following cases we will compare the results with the tachyon problem studied in\(^5\) whose steps we follow closely. For this purpose, we first write down the equations of motion for \(a(t)\) and \(T(t)\) arising from

\[
\begin{aligned}
    \ddot{a} &= \frac{8\pi G}{3} \left[ \Lambda + \frac{V(T)}{\sqrt{1 - \dot{T}^2}} (1 - \frac{3}{2} \dot{T}^2) \right] , \\
    \dot{T} &= -(1 - \dot{T}^2) \left[ \frac{V'(T)}{V(T)} + 3 \frac{\dot{T}}{a} \frac{\dot{a}}{a} \right] . \\
    \end{aligned}
\]

There is a constraint equation, known as Friedman equation, which will read

\[
\frac{(\dot{a}/a)^2}{a^2} = -\frac{k}{a^2} + \frac{8\pi G}{3} \left[ \frac{V(T)}{\sqrt{1 - \dot{T}^2}} + \Lambda \right] .
\]

\(^4\)We thank M. Alishahiha for useful comments on this point.
In the above equation, \( k/a^2 \) is known as the curvature term and \( V(T)/\sqrt{1 - \dot{T}^2} \) is the energy density associated with the field \( T \). Out of the four initial conditions, \( a_0, \dot{a}_0, T_0 \) and \( \dot{T}_0 \) (evaluated at \( t = 0 \)), required to solve the equations of motion, one, which we choose to be \( a_0 \), is obtained in terms of the other three through (3.35). We can set \( \dot{T}_0 = 0 \) by choosing a proper time origin and thus we are left with a two parameter family of solutions which are labeled by \( T_0 \) and \( \dot{a}_0 \). Motivated by the decay of an unstable D-brane which is a time reversal symmetric process, we will only consider a one parameter subspace of the possible solutions labeled by \( \dot{a}_0 = 0 \) for which we have the time reversal symmetry. But this is not the most general solution of the equations of motion and obtaining the full solutions is of course an interesting generalization.

The above initial conditions require that \( k = 1 \). for \( k = -1 \), \( a_0 \) is imaginary and for \( k = 0 \), \( a_0 = 0 \) and the model starts with singularity. Therefore for \( k = 1 \) we have

\[
a_0 = \left[ \frac{8\pi G}{3} (V(T_0) + \Lambda) \right]^{-1/2}.
\] (3.36)

3.1 NS5 geometry

In this case as was mentioned in \([11]\), \( T(r) \) is introduced by the relation

\[
\frac{dT}{dr} = \sqrt{h(r)},
\] (3.37)

where \( h(r) = 1 + \frac{N}{r^2} \). The tachyon potential \( V(T) \) is defined by

\[
V(T) = \frac{V_0}{\sqrt{h(r(T))}},
\] (3.38)

where \( V_0 = \tau_3/g_s \). As \( r \to 0 \), \( T \to -\infty \) and as \( r \to \infty \), \( T \to \infty \). In these limits the potential behaves as

\[
V(T) \approx V_0 e^{\frac{r}{\sqrt{N}}} \quad T \to -\infty,
\]

\[
V(T) \approx V_0(1 - \frac{N}{2T^2}) \quad T \to \infty.
\] (3.39)

This potentials has the same behavior as \( V_{\text{tac}} \) at late times (\( T \to -\infty \)) but looks quite differently in the large \( T \) limit.

Despite this difference, the analysis of the solutions of (3.33) and (3.34) goes in close analogy with the discussion presented in \([5]\), having in mind that \( |T_0| \ll 1 \) for negative values of \( T_0 \) in the tachyon problem corresponds to \( T_0/\sqrt{N} \gg 1 \) in our problem. This is expected because in this limit the potential \( V(T) \) goes

\[^{5}\text{for simplicity we will take } l_s = 1 \text{ hereafter.}\]
asymptotically to the value $V_0$ which is the top of the potential. If it were that $V$ could acquire this value, $T$ would not evolve and the solution for $a(t)$ would read

$$a(t) = a_0 \cosh \left( \frac{t}{a_0} \right). \quad (3.40)$$

But the potential can get at most very close to $V_0$ for very large $T_0$ and becomes an approximation. A quantity of interest is $V'/V$ at $t = 0$. This term that appears on the right hand side of equation (3.34), is the characteristic of how flat the potential for rolling is. The smaller is $V'/V$, the better an approximation (3.40) becomes at the beginning of the process. For large values of $T$, $V'/V \sim 1/T^3$.

By the same reasoning of [5], as $T$ evolves and $\dot{T}$ approaches its limiting value of $-1$, which is the attractive fixed point of the equations (for small enough $g_s$, the negative term $\dot{T} \dot{a}/a$ is small compared to $V'/V$ and the right hand side of (3.34) remains negative), the relative magnitude of the cosmological constant term $\Lambda$ and the curvature term at the point where the energy density and curvature become of the same order (the cross-over point), determines whether the solution describes an open or a closed universe. If $\Lambda$ is large compared to the curvature at this point the expansion continues and if not it will eventually stop. For larger values of $T_0$, the exponential like expansion of $a(t)$ is a better approximation and the curvature term will be smaller at the cross-over point. Therefore for a given $\Lambda$, there is a critical value for the scalar field $T_{c1}$ such that for $T_0 > T_{c1}$ the universe is open.

For sufficiently large negative values of $T_0$, the initial value of the curvature term becomes of the same order as the cosmological constant term. As $T$ evolves, the curvature becomes smaller (although not very rapidly because the potential is not flat enough and the near exponential expansion is not a good approximation) and thus the right hand side of (3.35) always remains positive and expansion continues. Therefore there is a second critical value for the scalar field $T_{c2}$ such that for $T_0 < T_{c2}$ the universe is again open and for the intermediate values $T_{c2} < T_0 < T_{c1}$, the universe is closed. By time reversal symmetry, the closed universe solution describes both a big bang and a big crunch whereas the other one describes neither.

### 3.2 Deformed NS5 geometry

The background [24] produces effective theories on a probe D$_3$-brane which were discussed in previous sections. We now couple these theories to gravity for the two cases of section 2.

**Case 1**: In this case, as was mentioned in section (2.5), the potential is

$$V(T) = V_0 \ e^{T/\sqrt{N}} \left( 1 + e^{2T/\sqrt{N}} \right)^{j/4}, \quad (3.41)$$

where $V_0 = \tau_3/\tilde{g}$ and $j = 0, -2$. For $T \to -\infty$ the potential goes as $e^{T/\sqrt{N}}$ regardless of $j$ and hence our discussion of the previous section remains unchanged.

---

\[\text{\footnotesize \textsuperscript{6}} j = -2 \text{ only occurs in case 3 of section 2 with (p,s,q) given by (1,0,4) or (3,1,5).}\]
in this limit. However, for large values of \( T \) and as seen in \((2.27)\), depending on the value of \( j \), the potential takes two different forms both of which look differently from what we saw in the previous section.

For \( j = 0 \) we see that \( V' / V = 1 \). So even for large values of \( T_0 \), the near exponential growth of \( a(t) \) is not a good approximation and the cross-over happens very rapidly. By choosing larger values for \( T_0 \), \( a_0 \) will be smaller and therefore the curvature term decreases with a bigger slope immediately after the evolution starts. Hence we expect \( T_c^1 \) to become large. As a result, the range of values of \( T_0 \) for which the solution becomes singular is much wider in this case. For \( j = -2 \) and for large values of \( T \), \( V' / V \sim e^{-2T} \).

**Case 2:** In this case the effective action contains an additional term coming from RR coupling and the action \((3.31)\) and the equations resulting from that are modified. The action is

\[
S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left( R \right) - \int d^4x \sqrt{-g} \left( V_1(T)\sqrt{1 - \dot{T}^2} - V_2(T) + \Lambda \right),
\]

where

\[
V_1(T) = V_0 e^{T/\sqrt{N}} (1 + e^{2T/\sqrt{N}})^{1/2},
\]

\[
V_2(T) = V_0 e^{2T/\sqrt{N}},
\]

and \( V_0 = \tau_3 / \tilde{g} \). The equations of motion for \( a(t) \) and \( T(t) \) are

\[
\ddot{a} / a = \frac{8\pi G}{3} \left[ \Lambda + \frac{V_1(T)}{\sqrt{1 - \dot{T}^2}} \left( 1 - \frac{3}{2} \dot{T}^2 \right) - V_2(T) \right],
\]

\[
\ddot{T} = -(1 - \dot{T}^2) \left[ \frac{V_1'(T)}{V_1(T)} + 3 \dot{T} \frac{\dot{a}}{a} - \sqrt{1 - \dot{T}^2} \frac{V_2'(T)}{V_1(T)} \right].
\]

The Friedman equation is

\[
\left( \frac{\dot{a}}{a} \right)^2 = -\frac{k}{a^2} + \frac{8\pi G}{3} \left[ \frac{V_1(T)}{\sqrt{1 - \dot{T}^2}} - V_2(T) + \Lambda \right].
\]

Here, the combination \((V_1' - V_2') / V_1\) determines how slowly \( T \) evolves at the beginning of its rolling. For large values of \( T \) it goes as \( e^{-4T/\sqrt{N}} \) and thus in this limit the potential for rolling is flat to a good approximation. Furthermore, as \( T \) evolves, the combination \( V_1 / \sqrt{1 - \dot{T}^2} - V_2 \) as compared to the energy density of the above cases remains almost constant for a longer period of time. On the whole, in comparison with the previous cases, on the one hand \( a(t) \) grows almost exponentially to a better approximation and on the other, the cross-over is postponed to a later time resulting in a smaller value for \( T_{c1} \).

We also note that for large negative values of \( T \), the initial curvature and cosmological constant terms become of the same order for smaller magnitudes of \( T_0 \) in comparison with the other cases thus resulting in a larger value for \( T_{c2} \). Therefore \( T_0 \) can take a narrower range of values to make a singular solution.
3.3 MN Geometry

In this subsection we study a probe D$_3$-brane in the MN background (see Appendix A) which is parallel to the flat directions of the metric.

For a purely radial motion of the probe it is easy to verify that the effective action in terms of the tachyon field $T \equiv \sqrt{N}r$ is written in the form of (2.24) with $p' = 3$ and

$$
\frac{1}{\tau_3} V(T) = \frac{\sinh^{\frac{1}{2}} \frac{2T}{\sqrt{N}}}{\left(4 \frac{T}{\sqrt{N}} \coth \frac{2T}{\sqrt{N}} - 4 \frac{T^2}{N \sinh^2 \frac{T}{\sqrt{N}}} - 1 \right)^{\frac{1}{4}}}.
$$

(3.47)

Note however that the tachyon field varies between zero and infinity this time. We then obtain the following asymptotic behaviour for the potential

$$
\frac{1}{\tau_3} V(T) \approx 1 + \frac{4}{9N} T^2 \quad T \sim 0,
$$

$$
\frac{1}{\tau_3} V(T) \approx \frac{1}{2} \left( \frac{T}{\sqrt{N}} \right)^{-\frac{1}{4}} e^{\frac{T}{\sqrt{N}}} \quad T \to +\infty.
$$

(3.48)

For large values of $T$ we see that the behavior is more or less similar to some of the results obtained so far namely case 1 with $j = 0$. In this region one obtains $V'/V \sim 1$ and, as stated before, exponential expansion is not a good approximation. For small values of $T$, which would correspond to $T \to -\infty$ of the previous cases, although the potential does not become zero its derivative does which is similar to the late time behaviour of tachyon potential. In fact $V'/V \sim T$ for small $T$ and therefore $a(t)$ grows near exponentially with a very good approximation in this region (see (3.33) and (3.34)). This causes $T_{c2}$ to become larger in comparison with the previous cases and the singular window to become narrow. A numerical analysis of the case confirms these arguments completely.

3.4 Comments on the solutions

The solutions we found for the NS5 geometry and its deformations show qualitatively the same behavior. For all the cases two critical values, $T_{c1}$ and $T_{c2}$ are defined such that for $T_0 > T_{c1}$ and $T_0 < T_{c2}$ the solutions are nonsingular whereas for $T_{c2} < T_0 < T_{c1}$ we will have singular solutions. Having in mind that we are dealing only with time reversal symmetric solutions, we note that the solutions with singularity describe a big bang in the past and a big crunch in the future and for the nonsingular solutions neither of the two exists. We also note that we should necessarily have a nonzero cosmological constant in order to allow for nonsingular solutions (see [7] for a supergravity analysis of a similar model where all the solutions are singular in 4D).

While the potentials for the different cases look almost the same way as $T \to -\infty$, they behave rather differently in the opposite limit. As a result, $T_{c2}$ has almost the same value for all the cases but the values for $T_{c1}$ depend rather considerably on the
case. Hence, the RR deformation affects the range of values $T_0$ can take in order to result in a singular solution.

For the values of $\frac{8\pi GA}{3} = .001$ and $\frac{8\pi G\tau_3}{3g_s} = .05$ we have found by numerical analysis that $(T_c^2/\sqrt{N}, T_c/\sqrt{N})$ for the above cases of NS5 geometry, $j = -2, j = 0$ and case 2 are respectively $(-2.7, 1.3), (-3.0, 0.9), (-3.0, 2.6)$ and $(-2.8, -0.1)$. We have also plotted $a(t)$ for $j = 0$ and for the initial values of $T_0 = -4, 1$ and 3 in figures 1 to 3 respectively. In figure 4, $T$ as a function of time is plotted for the three different initial values. For the MN geometry, we take a smaller values for the above constants in our numerical calculation in order to make the singular window visible (see figure 5).

As an important point one should note that unlike the potential for rolling tachyon $V \sim 1/\cosh T$, the potentials we have been dealing with meet the criteria for slow roll inflation in the large $T$ limit. In this limit, where $\dot{T}$ and $\ddot{T}$ are small and for $V_0 \gg \Lambda$, the slow roll parameters $\epsilon$ and $\eta$ are written as

$$\epsilon = \frac{m_p^2 V'}{2 V^3}, \quad \eta + \epsilon = \frac{m_p^2 V''}{2 V^2},$$

(3.49)

where $m_p^2 = 1/\sqrt{8\pi G}$. It is easy to see that for sufficiently large values of $T$, the above parameters are small for the cases of NS5 geometry, $j = -2$ and $j = 0$. For case 2 however, the parameters are found as

$$\epsilon = \frac{m_p^2 V'^2}{2 V_1 V_2}, \quad \eta = \frac{m_p^2 V''}{V V_1} - \frac{V V_1'}{V' V_1},$$

(3.50)

where $V = V_1 - V_2$. Here again the slow roll conditions are satisfied for large $T$.

It is important to note that all the above solutions and results are only reliable if the string coupling remains small during the rolling. Therefore, for a given $g_s$ or $\tilde{g}$, not all the initial values of $T_0$ are allowed. Even if this requirement is met in the beginning of the rolling, all the solutions will sooner or later violate the condition. We can at best postpone this violation to late times like what we did in section 2. For this purpose, we require the value of the coupling to be extremely small at $t = 0$. For the RR deformed cases this will read

$$\tilde{g} \left(1 + e^{2T_0/\sqrt{N}}\right)^\frac{1}{e^{T_0/\sqrt{N}}} \ll 1.$$  

(3.51)

For large negative values of $T_0$ the above condition, including the NS5 geometry case, becomes

$$g \ll e^{T_0/\sqrt{N}} \ll 1,$$

(3.52)

where $g$ is $g_s$ for the NS5 geometry and $\tilde{g}$ for the deformed one. This relation is the analogue of the condition $g \ll \lambda \ll 1$ of section 2.
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A Maldacena-Nunez background

The MN background for the metric, the NS three form and the dilaton is given by

\[
\begin{align*}
ds^2 &= dx_4^2 + N \left( d\rho^2 + e^{2g(\rho)} d\Omega_2^2 + \frac{1}{4} \sum_a (\omega_a - A_a)^2 \right), \\
H_3 &= N \left[ -\frac{1}{4} (\omega_1 - A_1) \wedge (\omega_2 - A_2) \wedge (\omega_3 - A_3) + \frac{1}{4} \sum_a F_a \wedge (\omega_a - A_a) \right], \\
e^{2(\phi-\phi_0)} &= \frac{2e^{g(\rho)}}{\sinh 2\rho}, \quad e^{2g(\rho)} = \rho \coth 2\rho - \frac{\rho^2}{\sinh^2(2\rho)} - \frac{1}{4}
\end{align*}
\]  

where \( \Omega_2^2 \) is a two sphere parametrized by \((\tilde{\theta}, \tilde{\phi})\) where \(NS5\) branes are wrapped and \(\omega_a\) are the \(SU(2)\) left-invariant one forms on the three sphere parametrized by \((\theta, \phi, \psi)\),

\[
\begin{align*}
\omega_1 &= \cos \phi \, d\theta + \sin \phi \sin \theta d\psi \\
\omega_2 &= -\sin \phi \, d\theta + \cos \phi \sin \theta d\psi \\
\omega_3 &= d\phi + \cos \theta d\psi.
\end{align*}
\]

and \(A_a\) are the \(SU(2)_R\) gauge fields on the two sphere

\[
\begin{align*}
A_1 &= a(\rho) \, d\tilde{\theta}, \quad A_2 = a(\rho) \sin \tilde{\theta} \, d\tilde{\phi}, \quad A_3 = \cos \tilde{\theta} \, d\tilde{\phi}, \\
F_a &= dA_a + \frac{1}{2} \varepsilon_{abc} A_b \wedge A_c, \quad a(\rho) = \frac{2\rho}{\sinh 2\rho},
\end{align*}
\]

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Figure 1: $\frac{8\pi G\Lambda}{3} = .001$ and $\frac{8\pi G\tau_s}{3g_s} = .05$. For $T_0 = -4$ the universe is open.

Figure 2: $T_0 = 1$ results in a big bang and a big crunch.

Figure 3: $T_0 = 3$ results in an open universe.
Figure 4: $T(t)$ for different initial conditions.

Figure 5: $a(t)$ for MN background with $\frac{8\pi G \Lambda}{3} = .00001$ and $\frac{8\pi G \tau}{3g_s} = .0005$. For $T_0 = 5.5$ the universe is closed.