Implicit vs Explicit renormalization of the $NN$ force

An S-wave toy model

Abstract

We use an S-wave toy model for the two-nucleon system to show that the implicit renormalization of a contact theory matches the explicit renormalization through a flow equation which integrates out the high momentum components. By fitting the low-momentum interaction with a new contact theory, we show that the running of the contact strengths in both original and fitted contact theories match over a wide cutoff range.

Keywords

Effective Field Theory · Nuclear force · Renormalization

1 Introduction

The pioneering works by Epelbaum, Glöckle and Meissner have opened a new approach for effective field theories based on low-momentum interactions obtained from unitary transformations [1; 2; 3]. One of the schemes that has emerged is the Similarity Renormalization Group (SRG), which has been intensively applied to $NN$ interactions to produce soft momentum-space potentials that are more suitable to be used as input for nuclear structure calculations [4].

The similarity renormalization group has also been studied from the point of view of long distance symmetries and it was found that the evolved interactions exhibit the Wigner symmetry at a particular renormalization scale [5]. Also, the SRG was applied to the leading order $NN$ interaction within the subtractive renormalization framework [6].

In this work we want to compare the implicit renormalization of a contact theory at NLO to the explicit renormalization of a toy model. The implicit renormalization of the contact theory is performed by constraining the strength of the contact interactions with low-energy parameters like the two-body scattering length and effective range. The explicit integration of the high momentum components can be achieved with the evolution of the $NN$ potential with the similarity renormalization group flow equation with the Block-Diagonal (BD) generator.
2 SRG with the Block-Diagonal generator

The SRG approach is based on a non-perturbative flow equation that governs the evolution of a hamiltonian $H = T_{rel} + V$ through an unitary transformation. In operator space, the flow equation is written as

$$\frac{dH_s}{ds} = [\eta_s, H_s]$$

(1)

where $s$ is the flow parameter that ranges from 0 to $\infty$ and

$$\eta_s = [G_s, H_s]$$

(2)

is the generator of the unitary transformations.

Here we use the Block-Diagonal (BD) generator [7] as the unitary version of the $V_{low \, k}$ to explicitly integrate out the high momentum components of the Toy Model presented recently in Ref. [8]. The BD generator is given by

$$G_s = H_s^{BD} = \begin{pmatrix} PH_s P & 0 \\ 0 & QH_s Q \end{pmatrix},$$

(3)

where $P$ and $Q = 1 - P$ are the projection operators for the low-momentum and the high-momentum spaces respectively. The flow parameter $s$ can be related to a momentum scale called the similarity cutoff, $\mu = s^{-1/4}$, so that when $s$ is evolved towards infinity, the similarity cutoff $\mu$ approaches zero.

In the infra-red limit of the similarity cutoff, $\mu \to 0$ ($s \to \infty$), the evolved interaction acquires a block-diagonal form

$$\lim_{\mu \to 0} V_\mu = \begin{pmatrix} V_{low \, k} & 0 \\ 0 & V_{high \, k} \end{pmatrix},$$

(4)

and the phase shifts produced by the evolved interaction are also separated according to the low and high momentum components:

$$\lim_{\mu \to 0} \delta_\mu(p) = \delta(p)_{low \, k} + \delta(p)_{high \, k}.$$

(5)

The difference between the block-diagonal SRG and the standard $V_{low \, k}$ approach is that the SRG transformation is unitary and the phase shifts corresponding to the high-momentum components, $\delta(p)_{high \, k}$, are also preserved. This is not the case in the $V_{low \, k}$ prescription, where only the low-momentum part of the phase shifts, $\delta(p)_{low \, k}$ is preserved.

3 Numerical Results and Discussion

The question we want to answer is the following: how close the low-momentum interaction obtained by integrating out the high-momentum components is to a contact theory with its low-energy constants constrained by the two-body scattering length and effective range?

In order to answer this question, we proceed as follows. First, we use a pionless theory at NLO regulated with a smooth gaussian function, $R(\Lambda) = \exp[-p^2/\Lambda^4]$, and adjust the low-energy constants $C_0^{(2)}$ and $C_2^{(2)}$ to obtain the correct scattering length and effective range in the $^1S_0$ and $^3S_1$ channels for several values of the cutoff scale $\Lambda$.

Then we perform the SRG evolution of our $S$-wave toy model with the block-diagonal generator. In Figs. [4] and [5] we show the evolution of our toy potential in the $^1S_0$ and $^3S_1$ channels towards the infra-red limit of the similarity cutoff where we observe a complete separation of the low-momentum and high-momentum components as the similarity cutoff $\mu$ approaches zero.

Next, we take the low-momentum components of the evolved interaction up to $\mu = 0.1 \, fm^{-1}$ and fit their diagonal parts at very low momenta with a contact theory at next-to-leading order to determine the new low-energy constants $C_0^{(2)}$ and $C_2^{(2)}$. Finally, we compare the running of the low-energy constants in both the implicit renormalization and explicit renormalization approaches.

The result of this procedure can be seen in Figs. [6] and [7], where we display the running of the contact strengths with the cutoff scale $\Lambda$ in the $^1S_0$ and $^3S_1$ channels. Considering the simplicity of our toy model for the $NN$ interaction, the match of the implicit and explicit runnings over a wide cutoff range is impressive.
3

Concluding Remarks

We made a direct comparison of the implicit and explicit renormalization approaches by analyzing the running of the low-energy constants obtained in both ways and verified that they match over a wide cutoff range. In order to overcome the numerical difficulties that appear when the SRG evolution is pushed towards the infrared region of the similarity cutoff, we use a toy model which gives a good description the S-waves with few discretization points. With the simple model we are able to reduce the numerical effort in the solution of the SRG flow equation.

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Fig. 3 Running of the contact strengths in the $^1S_0$ channel with the $V_{\text{low-k}}$ cutoff $\Lambda$ in the original (black) and in the fitted (red) contact theories at the infrared limit $\lambda = 0.1\,\text{fm}^{-1}$. The blue arrows indicate the range in which they match.

Fig. 4 Running of the contact strengths in the $^3S_1$ channel with the $V_{\text{low-k}}$ cutoff $\Lambda$ in the original (black) and in the fitted (red) contact theories at the infrared limit $\lambda = 0.1\,\text{fm}^{-1}$. The blue arrows indicate the range in which they match.

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