Amplification of Local Instabilities in a Bose-Einstein Condensate with Attractive Interactions

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We study the collapse of large homogeneous Bose-Einstein condensates due to intrinsic attractive interactions. We observe the amplification of a local instability by seeding a momentum state \( p \) and suddenly switching the scattering length negative via a Feshbach resonance. As required by momentum conservation, we also observe the appearance of atoms in the conjugate momentum state. The time scale for this depletion process is found to be comparable to that for global collapse, implying that this process will be the primary decay channel for large homogeneous condensates.

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Our current understanding of the collapse of Bose-Einstein condensates (BEC’s) with attractive interactions is incomplete. While the experiments in \(^7\)Li provided many insights into the formation kinetics and stability \( 4 \), it was not until the discovery of externally induced Feshbach resonances \( 5 \) that it became possible to tune the value of the scattering length and study in detail the effects of an attractive mean field potential. This technique was used by Donley \( et \ al \) to study \(^{85}\)Rb with a negative scattering length, and they have observed rich dynamics inherent in the collapse of the condensate. Among the most intriguing observations was the formation of low energy ‘bursts’ and ‘jets’ which were ejected out of the condensate \( 3 \). While theoretically the enhancement of quantum fluctuations could give rise to such phenomena \( 4, 5, 6 \), there is currently no consensus on the exact mechanism by which it occurs.

So far, theoretical developments have been limited by having only one experimental testing ground. The experiments using \(^{85}\)Rb were done using small condensates \((\sim 15000 \) atoms\)), where the attractive mean field energy \( \mu \) is comparable to or less than the \( \hbar \omega \) level spacing of the harmonic trapping potential. In this paper, we study the collapse of large sodium condensates far in the Thomas Fermi regime \((|\mu| \gg \hbar \omega)\), where the spatial profile of the condensate is relatively homogeneous. Much of the dynamics of such a system is then described by local phenomena. When the interactions become attractive, Yurovsky \( 6 \) predicts that local instabilities with momentum on the order of the (imaginary) speed of sound will undergo exponential growth. Simultaneously, momentum conservation requires atoms to be generated in conjugate momentum states. Since amplification happens on the time scale of the chemical potential \( \hbar / \mu \), the resulting quantum evaporation of the zero momentum condensate atoms can happen faster than the global collapse for large condensates, where the whole condensate ‘implodes’.

We probe this decay channel by seeding a particular momentum state with an initial population, then suddenly switching the scattering length negative via a Feshbach resonance. At the same time, the trapping potential is turned off so all subsequent dynamics are due only to the intrinsic attractive interactions. The resulting amplification and the associated generation of atoms in the conjugate momentum state verifies the theory. We end with a discussion on the different collapse time scales of competing processes and show that for large condensates, this decay channel becomes dominant.

The theoretical basis for the amplification of local instabilities is the dispersion relation for the elementary excitations in a Bose-Einstein condensate:

\[
\epsilon(p) = \sqrt{\frac{p^2}{2m}(2n_0 U + \frac{p^2}{2m})}
\]

where \( n_0 \) is the density, \( U = 4\pi \hbar^2 a / m = \mu / n_0 \) is the contact potential, \( a \) is the scattering length and \( m \) is the mass. For an elementary excitation whose momentum satisfies \( p^2/2m < 2|\mu| \), an instability forms when \( \mu < 0 \) (i.e. \( a < 0 \)) and oscillatory behavior gives way to exponential growth or decay. A formal derivation gives the evolution of these low momentum modes as

\[
\langle \xi_p^\dagger \xi_p \rangle(t) = \frac{|U_{n_0}|^2}{\hbar^2 \lambda^2(p)} \sinh^2[\lambda(p)t]
\]

where \( \lambda(p) \) is given by \( |\epsilon(p)|/\hbar \) (Eq \( 3 \)) when \( a < 0 \) and \( \xi_p \) is the destruction operator for mode \( p \). The instability of the mode pair \( |p, -p\rangle \) results in correlated growth, where the creation of an atom in the \( +p \) state is accompanied by the creation of an atom in the \( -p \) state. A similar phenomenon is also responsible for the four wave mixing process observed in \( 6, 7 \). At higher momentum, the energy becomes real again as the excitations now have enough kinetic energy to stabilize them against the attractive interactions.

In our experiments, we created large cigar-shaped sodium condensates in the \( F = 1 \), \( m_F = -1 \) spin state with typical atom numbers of \( \sim 30 \) million and peak densities of \( 3 \times 10^{14} \) cm\(^{-3} \) in a Ioffe-Pritchard (IP) magnetic trap. Following this, they were adiabatically loaded into

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The beams were red-detuned from the sodium D line by 3 nm to minimize Rayleigh scattering, and had a frequency difference of $\Delta \omega \approx 2\pi \times 700 \text{ Hz}$. The images in Fig. 1 probe the radial dynamics of the condensates and provide dramatic visual verification of quantum evaporation. Fig. 1(a) shows the $+p$ excitations moving out of the condensate without any amplification. In contrast, Fig. 1(b) was taken after the condensate had been held at $a = -0.82 \text{ nm}$ for 600 $\mu$s. Not only was the number of atoms in the $+p$ momentum state significantly amplified, it was accompanied by the formation of excitations in the $-p$ momentum state, here seen moving out of the condensate in the opposite direction. These observations clearly demonstrate the instability of a condensate with negative scattering length.

Due to the large aspect ratio of our condensates, only part of the condensate could be imaged at high magnification. However, since the $+p$ and $-p$ excitations were created predominantly in the radial direction, this was not a limitation. The small ‘kinks’ that were also apparent in our condensate are most likely a result of imperfections in our optical dipole trap. Yet rather than degrade the signal, they highlight the parallel contours between the condensate and the ridge of excitations as atoms move out with a definite momentum.

In order to perform more quantitative tests of this phenomenon, we first characterized the negative scattering length dependence on the field by directly probing the strength of the attractive interactions. To do this, we prepared optically trapped condensates close to a Feshbach resonance as above. The confining infrared laser beam was then replaced with a repulsive 3 nm blue-detuned “antitrap” beam as we simultaneously jumped to negative scattering lengths. At the right intensity, the antitrap beam provided the correct amount of repulsive dipole force needed to compensate for the attractive interactions within the condensate and suppressed any...
global contraction of size. However, this is an unstable equilibrium and any sloshing of the condensate or misalignment of the laser beam caused the condensate to be repelled. Therefore the antitrap was fine-aligned to milliradian accuracy such that a condensate with \(a > 0\) was ripped apart radially into a hollow cylinder. Using this method, we were able to stabilize an attractive condensate significantly for 0.2 to 2 ms, depending on \(a(B)\), before unavoidable losses became significant. The radial dimension was used to monitor the mechanical dynamics of the condensate occurring on the 250 Hz time scale of the trap frequency. For minimal distortion of the spatial image, absorption images were taken after only 2 ms of ballistic expansion necessary for the high magnetic fields to die out.

In equilibrium, \(a \propto F_{\text{attractive}} = F_{\text{repulsive}} \propto I\). Therefore we plot the light intensity \(I\) needed for stabilization vs. magnetic field \(B\) in Fig. 2 and obtain the scattering length dependence on the field. A red-detuned laser beam with a similar detuning of 3 nm was employed to obtain the points in the positive scattering length regime. By fitting the expected Feshbach curve \(a(B) = a_0(1 + \frac{\Delta B}{B - B_0})\) to our data, we find the width \(\Delta B\) of the 1195 G Feshbach resonance to be (2.4 ± 0.4) G. Here, \(a_0 = 3.3\) nm is the triplet scattering length at high fields \([6]\). While the exact position \(B_0\) of the resonance is unimportant for our experiment, the width of the resonance is crucial as it determines the range of the magnetic field we have to work within \([10]\).

A quantitative analysis of the growth in the \(+p\) and \(-p\) modes was performed by monitoring their occupation number as a function of hold time in the attractive regime (Fig. 3). Taking into account the high loss of atoms during this process due to three-body decay or inelastic two-body collisions, we normalize the number count for each mode to the number of atoms in the condensate at the end of the hold time. The maximum duration of amplification was limited by the lifetime of the condensate, which was about 600 µs for \(a = -1.35\) nm. Following Eq. 2, an exponential dependence was fitted to the data, yielding a common growth rate of 5.89 ± 0.83 ms\(^{-1}\) for both modes. This agrees well with the theoretical growth rate of 5.57 ms\(^{-1}\), estimated using our initial mean field of \(\hbar \times 5 \text{ kHz}\).

We also investigated the dependence of the growth rate on the magnitude of the negative scattering length \(|a|\). By varying the scattering length and extracting the growth rate of the excitations as above, we observed a strong increase of the growth rate with \(|a|\) (Fig. 4). A fit to the theoretical prediction \(\lambda(p) = b_1|a| - b_2\) (Eq. 3) yielded the fit parameters \(b_1 = (1.11 ± 0.14) \times 10^8\) nm\(^{1/2}\)ms\(^{-1}\) and \(b_2 = 0.078 ± 0.04\) nm, in agreement with theoretical estimates of \(1.55 \times 10^8\) nm\(^{1/2}\)ms\(^{-1}\) and 0.06 nm respectively. For this sequence of measurements, the initial mean field was \(\mu = \hbar \times 4 \text{ kHz}\). The large error bars reflect the high sensitivity of the dynamics to the magnetic field. In particular, the three-body recombination and inelastic loss rates as a function of \(a\) have not yet been well characterized, which limits the accuracy of our data.

The results presented here prove conclusively that quantum evaporation is a part of the complex dynamics that take place during the collapse of an attractive condensate. While we select a particular mode for observation, the effect is predicted to happen for all modes satisfying the condition \(|p|^2/2m < 2|\mu|\). At short times, the pairwise emission of atoms also implies that the number of atoms in the conjugate mode pairs will be exactly correlated, although as the condensate becomes increasingly depleted, higher order effects will degrade the correlation.

Since quantum evaporation is intrinsic to the condensate, a natural question to ask would be how large a part it plays in the unperturbed collapse of the condensate. We study this by comparing the observed time taken for the condensate to decay completely without initial seeding (Fig. 5), with the theoretical prediction for the...
In addition, we compare the observed collapse time to the decay time predicted for global collapse. Even for condensates initially far in the Thomas-Fermi regime, their spatial profile is not completely uniform and the inherent pressure gradient will cause the condensate to collapse inwards. As the condensate compresses, the density will increase and sharply enhance the three-body recombination loss rate, which goes like $n_0^2$. We model the radial evolution using the root mean square radius $R = \int r^2 |\psi(r)|^2 r dr$. For our cylindrical condensates with an aspect ratio of 100:1, an analytical solution for the resulting 2D dynamics exists [13, 14], given by $\dot{R} = 4(\frac{E}{m} - \omega^2 R)$ where $E$ is the total energy of the system. Since $\omega = 0$ and $a$ is constant throughout in our experiments, $E$ is conserved and obtained from initial conditions. The time taken to reach $R = 0$ is

$$t_{\text{decay}} = \frac{1}{\omega_0} \sqrt{\frac{2m}{a}}$$

Eqs. 3 and 4 are plotted in Fig. 3 and their intersection separates the graph into two domains. In the first, $a$ is small and global collapse is predicted to dominate over quantum evaporation. For higher $a$’s, the converse is true as amplification rate and the number of unstable modes increase. The comparison with the data suggests that the decay of the condensate is not dominated by three-body decay, but is caused by amplification of unstable modes. We were unable to study the condensate lifetime at even more negative scattering lengths since the condensate then decayed almost instantaneously. In order to further separate the two time scales, the global collapse time needs to be much slower compared to $t_{1/e}$. This would require a combination of a weak trap with a high number of atoms, which is currently out of reach.

In conclusion, we have shown that large condensates far in the Thomas-Fermi regime undergo amplification of local instabilities when their scattering length becomes negative. We have studied the dependence of amplification rate on the magnitude of the negative scattering length and found reasonable agreement with the theory. For our parameters, this quantum evaporation process becomes comparable or faster than the global collapse.

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We observe the resonance around 1208±10 G, which would agree with the theoretical value within the systematic error.