A new screening function for Coulomb renormalization

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We introduce a new screening function which is useful for the few-body Coulomb scattering problem in “screening and renormalization” scheme. The new renormalization phase factor of the screening function is analytically shown. The Yukawa type of the screening potential has been used in several decades, we modify it to make more useful. As a concrete example, we compare the proton-proton scattering phase shifts calculated from these potentials. The numerical results document that high precision calculations of the renormalization are performed by the new screening function.

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When one considers the system of few charged particles, one faces, as is well known, serious difficulties in the calculation of scattering processes[1]. This is caused by the long range nature of the Coulomb potential in coordinate space, or, equivalently, its singularity in momentum space.

In order to overcome these difficulties, we would like to mention two approaches. One approach is a modified time-dependent scattering theory[2] and another approach is a “screening and renormalization” method. Many investigations have been done in few-body systems by the latter approach. For example, Alt et al.[3] have calculated three-body scattering with charged particles by the screening and renormalization scheme. Their calculations worked out successfully. However, the screening radius \( R \) used in their calculation is about 600 fm. At such a large \( R \)-value the screened Coulomb potential is no longer smooth requiring a careful treatment and increased computer resource in memory and elapsed time. Therefore, it would be desirable to work with a smaller \( R \)-value. The purpose of this letter is to investigate a new screening function different from the one used in Alt et. al which leads to precise results even at small \( R \)-values.

Before we discuss the new screening function, we would like to point out the significance of the renormalization. If one considers a bound state numerical calculations can be performed by the screening method with an appropriately large \( R \)-value and the result will be independent of the choice of \( R \). However, for a scattering state the situation is completely different. The limit of solutions achieved with screened Coulomb potentials for increasing \( R \)-values will not agree with the solutions for a pure Coulomb potential. The reason lies in the wrong asymptotic boundary conditions going with a screened Coulomb potential. A renormalization method, like the one introduced by Taylor[4] is necessary.

Alt et. al. used the Yukawa type screened Coulomb potential with the screening radius \( R \):

\[
V^R(r) = e^2 \exp\left(-\frac{r}{R}\right). \tag{1}
\]

Here in this paper we have proton-proton scattering in mind, therefore, \( e \) represents the charge of the proton. Now we introduce the new screening functions:

\[
V^R_n(r) = e^2 \exp\left(-\frac{r}{R^n}\right). \tag{2}
\]

Note that eq. (2) reduces to the Yukawa type potential for \( n = 1 \) and is also going to a sharply truncated potential for \( n \to \infty \),

\[
\lim_{n \to \infty} V^R_n(r) = \frac{e^2}{r} \cdot \theta(R - r), \tag{3}
\]

where \( \theta(r) \) is the \( \theta \)-function. The pure and the screened Coulomb potentials for \( n = 1 \) to 5 at \( R = 50 \) fm are shown in Fig. 1. This figure reveals that with large \( n \) there develops a sharper cut-off and the pure Coulomb potential is better represented in the inner region.

Next, we evaluate the phase shift for pp scattering in the state \( ^1S_0 \). As a typical NN force we take the Reid

FIG. 1: Pure and screened Coulomb potentials for \( n = 1 \) - 5.
Soft Core potential in addition to the Coulomb potential. There exists the following relation among various phase shifts, which can be gained by regarding the asymptotic behavior of the wave function:

\[ \delta^C = \delta^R - \sigma_0 + \phi_R + 0(1/R) \]  

(4)

Here \( \delta^C \) is the phase shift to the strong and pure Coulomb potential, \( \delta^R \) to the strong and screened Coulomb potential, \( \sigma_0 \) the standard Coulomb phase shift obtained from arg \( \Gamma(1+i\eta) \) and \( \phi_R \) the renormalization phase. Further, \( \eta = e^2 m / 2p \) is Sommerfeld parameter with \( m \) the nucleon mass and \( p \) the relative momentum. According to Taylor the renormalization phase \( \phi_R (p) \) is given as

\[
\phi_R (p) \equiv -\eta \int_0^{(2p)^{-1}} \exp\left[-(r/R)^n\right] \frac{dr}{r} \\
= -\eta \int_0^{(2p)^{-1}} \exp\left[-(r/R)^n\right] \frac{1}{r} \exp\left[-\eta \ln(2pR)\right] \\
+ 0(1/R^n) \\
= -\eta \left[ \ln(2pR) - \gamma \right] + 0(1/R^n),
\]

(5)

where \( \gamma \) is the Euler number \((0.5772 \ldots)\). The last line in Eq. (5) results using the substitution \((r/R)^n = s/R\). For \( n=1 \) this is the result used by Alt et al.. For \( n \to \infty \) one obtains \( \phi_R = -\eta \ln(2pR) \), which is the expression related to a sharp cut-off. Note that the definition of \( \phi_R (p) \) suffer from the uncertainty of \( 0(1/R) \) in eq. (4).

The phase shifts \( \delta^C, \delta^R \) and \( \phi_R \) are shown in Fig. 2.

In the evaluation of the phase shifts one has to study the dependencies of the c.m. energy \( E_{cm} \), the screening radius \( R \) and the power \( n \) in our new screening function. As a measure for the quality of the new screening function we introduce

\[ |\Delta \delta| \equiv |\delta^C - \delta^R - \eta [\gamma / n - \ln(2pR)]| \]  

(6)

This quantity is plotted in Figs. 3 and 4. In Fig. 3 and Fig. 4 we show the \( |\Delta \delta|\)-dependence of \( E_{cm} \) for \( R = 50 \text{ fm} \) and \( 500 \text{ fm} \), respectively. \( |\Delta \delta| \) decreases strongly with \( E_{cm} \). These figures clearly show that the calculations for \( n \geq 2 \) are more precise than for \( n=1 \). In view of Fig. 1 we have to conjecture that this is due to the better approach of the pure Coulomb potential by the screened Coulomb potential in the inner region if \( n \) is larger than 1.

In Figs. 5 - 6, we illustrate the \( R\)-dependence of \( |\Delta \delta| \) for two fixed \( E_{cm} \)-values of 10 and 100 MeV. Trivially, \( |\Delta \delta| \) decreases with increasing \( R \) but again the error is significantly smaller if \( n \) is larger than 1 in comparison to \( n=1 \).

Finally Figs. 7 and 8 give the \( n\)-dependence of \( |\Delta \delta| \) for fixed \( E_{cm} \) and various \( R \)'s at \( E_{cm} = 10 \text{ and } 100 \text{ MeV} \), respectively. It is seen that \( R = 50 \text{ fm} \) and \( n=2 \) is a good enough choice to perform the calculations.

FIG. 2: The comparison of the phase shifts \( \delta^C, \delta^R \) and \( \phi_R \) for \( R = 50 \text{ fm} \).

FIG. 3: \( |\Delta \delta| \) for \( R = 50 \text{ fm} \) against \( E_{cm} \) for various \( n\)-values.

FIG. 4: The same as in Fig. 3 for \( R = 500 \text{ fm} \).

In summary, we generalized the Yukawa type screening potential adopted by Alt et al. to the form given in Eq. (2). Already for \( n=2 \) the screened Coulomb potential is closer to the pure Coulomb potential in the inner region. The new expression for the renormalization phase \( \phi_R \) is just a bridge between the Yukawa type and sharp cut-off potential. The numerical results document that high precision calculations can be performed by the new screening function for \( n \geq 2 \) and a screening radius as small as \( R=50 \text{ fm} \).

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FIG. 5: $|\Delta\delta|$ for $E_{cm} = 10$ MeV against $R$ for various $n$.

FIG. 6: The same as in Fig. 5 for $E_{cm} = 100$ MeV.

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[1] E. O. Alt and W. Sandhas, in *Coulomb Interactions in Nuclear and Atomic Few-Body Collisions*, edited by F. S. Levin and D. A. Micha (Plenum Press, New York, 1996).
[2] J. D. Dollard, Rocky Mount. J. Math. 1, 5 (1971); J. Math. Phys. 5, 729 (1964).
[3] E. O. Alt, A. M. Mukhamedzhanov, M. M. Nishonov and A. I. Sattarov Phys. Rev. C65, 064613, (2002).
[4] J. R. Taylor, Nuovo Cimento 23B, 313, (1974); M. D. Semon and J. R. Taylor, *ibid.* 26A, 48, (1975).
[5] R. V. Reid, Ann. Phys. 50, 411, (1968).
FIG. 7: $|\Delta \delta|$ for $E_{cm} = 10$ MeV against $n$ for various $R$-values.

FIG. 8: The same as in Fig. 7 for $E_{cm} = 100$ MeV.