Distributed Topology Design for Network Coding Deployed Large-scale Sensor Networks

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Abstract

In this paper, we aim to design a distributed solution to topology formation problem of large-scale sensor networks with multi-source multicast flows. We propose a distributed solution based on game-theoretic approaches in conjunction with network coding. While it is NP-hard to find an optimal topology for network coding deployed multi-source multicast flows, we propose an algorithm that requires significantly low computational complexity. We formulate the problem of distributed network topology formation as a network formation game by considering the nodes in the network as players that can take actions for making outgoing links. We show that the proposed game, which consists of multiple players and multicast flows, can be decomposed into independent link formation games played by only two players with a unicast flow. It is guaranteed that the proposed algorithm is converged, i.e., a stable network topology can be always formed. Our simulation results confirm that the computational complexity of the proposed solution is low enough for practical deployment in large-scale sensor networks.

Index Terms

network coding, game theory, topology design, distributed solution, sensor network, multi-source multicast flows

I. INTRODUCTION

Modern mobile devices such as smartphones and tablets are sophisticated computing and networking platforms with enhanced sensor capabilities (e.g., user location detection and air temperature and humidity measures). Moreover, mobile medical devices are now equipped with significantly advanced medical sensors for Electrocardiography (ECG) signals, body temperature, blood glucose, heart rates, blood oxygen saturation, etc. The popularity of mobile devices now enables instantaneous formation of large-scale ad hoc
networks by making connections among them. With the support of autonomous networking technologies (e.g., Qualcomm Wi-Fi SON [1] and Bluetooth mesh networking [2]), devices can participate in such networks by transmitting the data collected from their sensors and relaying data from other devices. Therefore, it is important to optimally form a network topology that can improve data transfer rates and data fidelity under severe energy constraints.

One of the challenges in designing a network topology is the high computational complexity involved in finding the optimal solution in a large-scale network. Because recent sensor networks contain a large number of sensor nodes, the number of potential network topologies increases exponentially with the number of nodes. Therefore, it is difficult in general to solve the optimization problem for network topology unless it is formulated as a special type of optimization problem (e.g., convex optimization problem).

Another challenge in large-scale sensor networks comes from multi-source multicast flows, which are inevitable in ad hoc networks. Multiple sources can be in the network because sensor nodes can generate data based on their own sensing operations and deliver information to a set of target destination nodes, resulting in multicast flows in the network. Examples of multi-source multicast flows in networks include a sensor grid [3], a healthcare wireless sensor network [4], and the Internet of things [5]. Multi-source multicast flows frequently overlap in network paths, but only one flow can be delivered at a time. This is the bottleneck problem that can induce delay in data delivery and degraded network throughput [6]. Therefore, the incoming rate of a node should be taken into account as a constraint for the network topology design problem, such that it should not exceed the link capacity.

In this paper, we propose a distributed network topology design that overcomes the challenges discussed above, while explicitly considering the multi-source multicast flows in large-scale sensor networks. Specifically, we adopt a game-theoretic approach to formulate a distributed network topology as a network formation game. The nodes in the network are considered as players in a game, which can decide to make connections with their neighbor nodes by considering the associated rewards and costs. The reward in the utility function represents reduced distance between source and destinations nodes, which enables to make shortcuts between sources to destinations as well as reduce connection failure ratio between them, leading to potential network throughput improvement. For this, we design the reward as a decreasing function of distance between a node and the destination, which provides an incentive for a node distant from the destination to make alternative links as it has higher path diversity but a node close to the destination to make direct links to the destination. Hence, the reward function represents the importance of locations of nodes in a network. We impose the cost associated with link formation on the utility function to prevent nodes from making redundant outgoing links. Therefore, each node can make the
optimal number of outgoing links by taking actions that maximize its utility. Because each node can determine its own actions for outgoing link formation, the proposed approach is indeed a distributed solution to the network topology formation problem. Unlike a centralized optimization solution, which must evaluate all possible potential network topologies and thereby incurs high computational complexity, the proposed solution enables each node to choose its own actions, leading to significantly lower overall complexity.

The proposed approach also adopts the network coding [7] to solve the bottleneck problem incurred by multi-source multicast flows in the network [8]. Network coding is widely known to have several advantages, such as efficient resource usage (e.g., bandwidth and power), and improved robustness and throughput [9], [10]. In this paper, we employ inter-session network coding [11] which combines multiple packets from different sources into a single packet before transmission [12]. However, it is challenging to design a low-complexity strategy for topology formation since it is an NP-hard problem to find the optimal network topology in a network with multi-source multicast flows, where network coding is blindly deployed [13].

In this paper, we propose a low-complexity solution to topology design of multi-source multicast flows by deploying network coding. Since network coding operations combine multiple incoming packets into a single packet, the outgoing rate of a node can always be fixed, so that it can eliminate the constraints on the incoming rate for a node. This enables a node to build its outgoing links without considering the link formations of other nodes, which means that decisions about link formation can be made between only two nodes. Therefore, an $n$-player network formation game that includes multicast flows can be decomposed into independent 2-player link formation games with a unicast flow, as we analytically show in this paper. Because the complexity required to solve a 2-player link formation game with a unicast flow is significantly lower than that needed for an $n$-player network formation game with multi-source multicast flow, the overall complexity can be significantly reduced. Note that if network coding is not deployed, the $n$-player network formation game cannot be decomposed into 2-player link formation games.

The main contributions of this paper can be summarized as follows.

- We formulate the problem of network topology design as a network formation game, which leads to a distributed strategy for topology formation.
- We propose to deploy network coding as a solution to the bottleneck problem inherently incurred by multi-source multicast flows.
- We design a utility function for the network formation game, where players in the game would like to maximize it by taking actions, which eventually leads to increased network throughput and
reduced number of unnecessary redundant links between nodes.

- We analytically show that network coding can decompose the network formation game into link formation games, leading to an algorithm with significantly low complexity.

Note that the focus of this paper is not on the code design for inter-session network coding, which has been extensively studied in prior works [14]–[21]. We rather focus on how to design network topologies that can lead to improved network performance (e.g., throughput and delay), which have been mostly considered as a given condition in previous literature.

The rest of the paper is organized as follows. In Section II, we briefly review the related works. The network model and detailed process of data collection and dissemination based on network coding operations are discussed in Section III. The network formation game for multicast flows and its decomposition into link formation games for a unicast flow are proposed in Section IV. Simulation results and numerical evaluations are presented in Section V, and conclusions are drawn in Section VI.

II. RELATED WORKS

Before the notion of network coding, it was infeasible to achieve upper bound of multicast capacity by conventional store-and-forward (SF) relaying architectures [22]. The Steiner tree based topology design can achieve the upper bound of multicast capacity, but solving the Steiner tree is an NP-hard problem [23]. In [7], it is firstly shown that network coding can achieve the maximum throughput via the max-flow min-cut theorem, and it is further proved that linear network coding [24] can achieve the upper bound of capacity. The optimal topology solution for a single source scenario is studied in [25], however, in multi-source scenarios which are frequently observed in sensor network scenarios, the max-flow min-cut bounds cannot fully characterize the capacity region, and thus, only loose outer bounds [26] and sub-optimal solution [27] are studied.

Network coding has been deployed in a variety of sensor network scenarios [9], [10], [28]. For example, network coding can improve the energy efficiency of a body area sensor network [28]. A robust network coding protocol is proposed for smart grids to enhance the reliability and speed of data gathering [9]. In [10], a mobile crowd-sensing scenario is considered for decentralized data collection, and network coding is deployed for energy and spectrum efficiency.

Topology design in sensor networks has been studied in the context of self-organizing networks [29]–[31]: in [29], protocols are proposed for the self-organization of wireless sensor networks with a large number of mainly static and highly energy constrained nodes; in [30], a self-organizing routing protocol for mobile sensor nodes declares the membership of a cluster as they move and confirms whether a mobile sensor node can communicate with a specific cluster head within a time slot allocated in a time division
TABLE I: Related studies in network topology design

|                | [29], [30] | [31], [33], [34] | [27] | [25] | [35] | This Paper |
|----------------|-------------|------------------|------|------|------|------------|
| Source type    | Multi-source| Multi-source     | Multi-source | Single source | Multi-source | Multi-source |
| Flow type      | Unicast     | Unicast          | Unicast | Multicast | Multicast | Multicast |
| Solution type  | Centralized | Distributed     | Centralized | Distributed | Distributed | Distributed |
| Relaying type  | SF          | SF               | Network coding | Network coding | SF | Network coding |

multiple access schedule; in [31], distributed energy efficient deployment algorithms are proposed for mobile sensors and intelligent devices that form an ambient intelligent network.

Distributed decision making has been widely considered in the field of game theory and there have been a large number of literatures on network formation games not only in economics but also in engineering [32]. For application to wireless sensor networks, game-theoretic distributed topology control for wireless transmission power is proposed in sensor networks [33]. The purpose of topology control is to assign per-node optimal transmission power such that the resulting topology can guarantee target network connectivity. A similar study of a topology control game in [34] aimed to choose the optimal power level for network nodes in ad hoc networks to ensure desired connectivity properties. In [35], a dynamic topology control scheme that prolongs the lifetime of a wireless sensor network is provided based on a non-cooperative game.

In Table I, several representative related works are classified in terms of source, flow, solution and relaying types. In contrast to [25], [27], [29]–[31], [33], [34], this paper includes the most generalized source and flow types, i.e., multi-source multicast flows. Compared to [35], which also considers multi-source multicast flows, this paper explicitly considers network coding function in topology design problem such that previously described throughput advantage of network coding in a multicast flow can be properly utilized.

III. NETWORK CODING BASED SENSOR NETWORKS

In this section, we describe our network model and network coding based packet dissemination in sensor networks.
A. Network Model

We consider a directed graph \( G \) with a set of nodes \( V(G) \) and a set of directed links \( E(G) \). An element \( v_i \in V(G) \) can be a source node, a destination node, or a source and destination node. The number of nodes in \( V(G) \) is denoted by \( |V(G)| = N_V \), where \( |\cdot| \) denotes the cardinality of a set. The set of destination nodes for \( v_i \) is denoted by \( D_i \subseteq V(G) \), and \( D = \{ i | v_i \in \{ D_1, \ldots, D_{N_V} \} \} \) represents an index set of destinations for all network nodes.

A directed link from \( v_i \) to \( v_j \) is denoted by \( e_{ij} \in \{0, 1\} \), where the active link \( (e_{ij} = 1) \) can deliver data. Otherwise, the link is inactive, and \( e_{ij} = 0 \). Note that the link \( e_{ij} \) has direction, i.e., \( v_i \) is the tail and \( v_j \) is the head, so that \( e_{ij} \neq e_{ji} \). In this paper, \( e_{ij} \) is called an incoming link of \( v_j \) or an outgoing link of \( v_i \). \( E(G) \) includes only active links so that \( |E(G)| \) is the number of active links in \( G \).

Let \( \delta_{ij} \) be the Euclidean distance between \( v_i \) and \( v_j \). As special cases, we define that \( \delta_{ij} = 0 \) if \( i = j \) and \( \delta_{ij} = \infty \) if \( v_i \) and \( v_j \) are not reachable. The set of neighbor nodes of \( v_i \) is denoted by \( H_i = \{ v_j | 0 < \delta_{ij} \leq \Delta \} \), where \( \Delta \) denotes a connection boundary. If \( \delta_{ij} > \Delta \), then a link between \( v_i \) and \( v_j \) cannot be formed, i.e., \( e_{ij} = e_{ji} = 0 \). \( \tilde{H}_{\text{in}}^i = \{ v_j | e_{ji} = 1, \forall v_j \in H_i \} \) and \( \tilde{H}_{\text{out}}^i = \{ v_j | e_{ij} = 1, \forall v_j \in H_i \} \) denote a set of neighbor nodes of \( v_i \) with active incoming and outgoing links, respectively. An illustrative example of a sensor network topology with seven nodes is shown in Fig. 1.

For simplicity, we assume that the capacity of a link \( e_{ij} \) is one packet per unit of time slot, i.e., a node can transmit only one packet in each time slot. We also assume that each sensor always has a packet to send at each time slot (e.g., a sensor generates a packet for every time slot). Hence, if it builds an outgoing link, there always exists a packet to be transmitted through the outgoing link. If a node has multiple outgoing links, it multicasts a single packet per time slot through all the outgoing links so that all outgoing links from one node deliver the same packet at the same time slot. If a node has multiple incoming links, it can receive multiple individual packets by deploying e.g., multipacket reception techniques. Even though a node can receive multiple packets at a single time slot, however, under the conventional SF relaying architecture, a node cannot transmit more than one packet at a time because of the link capacity constraints. Hence, a node becomes a bottleneck of flows when it receives a larger number of packets than its output link capacity (i.e., one packet per unit of time slot), which is referred to as the bottleneck problem.

1 If the considered network changes over time, the network can be modeled by a directed graph \( G_t \) with a set of nodes \( V(G_t) \) and a set of directed links \( E(G_t) \) as a function of time slot \( t \). However, our focus in this paper is on the distributed solution for topology formation at each time slot, so that we omit the subscript \( t \) in the rest of this paper without loss of generality.
Fig. 1: An illustrative example of sensor network topology $\mathcal{G}$, where $\mathcal{V}(\mathcal{G}) = \{v_1, \ldots, v_7\}$ and $\mathcal{E}(\mathcal{G}) = \{e_{12}, e_{24}, e_{45}, e_{57}, e_{31}\}$. For node $v_4$, a set of neighbor nodes with active incoming and outgoing links are $\tilde{\mathcal{H}}^{in}_4 = \{v_2\}$ and $\tilde{\mathcal{H}}^{out}_4 = \{v_5, v_6\}$, respectively. All nodes in this example have at most one incoming link.

To prevent the bottleneck problem, a node may restrict the amount of incoming packets not to exceed link capacity, and such constraint can be feasible as restricting the number of incoming link to at most one, i.e.,

$$\sum_{\forall v_i \in \tilde{\mathcal{H}}^{in}_j} e_{ij} \leq 1 \text{ for all } v_j \in \mathcal{V}(\mathcal{G})$$

which is referred to as *inter-link dependency condition* in this paper.

### B. Data Collection and Network Coding Based Dissemination

Every node in $\mathcal{V}(\mathcal{G})$ plays the role of source by collecting data (i.e., sensing) and simultaneously plays the role of relay by disseminating the collected data. Since the goal of the network is to deliver all data collected by source nodes to their own destination nodes, this may incur in multi-source multicast flows.

Let $x_i$ be source data collected by node $v_i$, which needs to be delivered to destination nodes. We denote $X_{ij}$ as the data transmitted at the link $e_{ij}$. The *network status* of $\mathcal{G}$ is defined as the set of data included in $\mathcal{E}(\mathcal{G})$ with the inter-link dependency condition in (1), which is denoted by $J_\mathcal{G}$ and expressed as (2). If $e_{ij} \in \mathcal{E}(\mathcal{G})$, then $X_{ij} \cdot e_{ij} = X_{ij}$. Otherwise, $X_{ij} \cdot e_{ij} = 0$. Hence, $\{X_{ij} \cdot e_{ij} | \forall v_i, v_j \in \mathcal{V}(\mathcal{G}), i \neq j\}$ in (2) represents a set of data included in $\mathcal{E}(\mathcal{G})$ for the link dependent data $X_{ij}$. 
\[ J_G = \left\{ X_{ij} \cdot e_{ij} | \forall v_i, v_j \in \mathcal{V}(G), i \neq j \right\}, \left( \sum_{v_i \in \tilde{H}^n_j} e_{ij} \leq 1, \forall v_j \in \mathcal{V}(G) \right) \]. \quad (2) \]

1) Elimination of inter-link dependency by network coding: If network coding is deployed in \( G \), the resulting network status is denoted by \( \Phi(J_G) \) and is expressed as

\[ \Phi(J_G) = \langle \{ p_j \cdot e_{ij} | \forall v_i, v_j \in \mathcal{V}(G), i \neq j \} \rangle \] \quad (3)

where \( p_j \) denotes a network coded packet that flows into \( v_j \). The network coded packet \( p_j = [C_{1j}, \ldots, C_{N_{Vj}}, y_j] \) is a vector of the global coding coefficients \([C_{1j}, \ldots, C_{N_{Vj}}]^T\) as the header and \( y_j \) as the payload, which is constructed as

\[ y_j = \sum_{k=1}^{N_v} \bigoplus (C_{kj} \otimes x_k) \] \quad (4)

where \( \oplus \) and \( \otimes \) denote the addition and multiplication operations in a Galois field (GF), respectively. Hence, the network coding function \( \Phi \) combines all packets that flow into \( v_j \) and generates a single packet \( p_j \). This operation allows a node to take multiple incoming links and prevent the bottleneck problem, so that the inter-link dependency in (2) can be eliminated as in (3).

The elimination of inter-link dependency through the network coding function \( \Phi \) can be interpreted as follows. The network coding function \( \Phi \) converts the link dependent data \( X_{ij} \) into the link independent data \( p_j \). Hence, \( v_j \) receives a single of packet \( p_j \) from all incoming links no matter how many incoming links are formed. Examples of this interpretation are illustrated in Fig. 2.

In the network operation in (4), node \( v_j \) combines its data \( x_j \) and all the incoming data \( X_{ij}, \forall v_i \in \tilde{H}^n_j \) multiplied by local coding coefficients \( c_{ij}, \forall v_i \in \tilde{H}^n_j \), expressed as

\[ y_j = \sum_{v_i \in \tilde{H}^n_j} \bigoplus (c_{ij} \otimes X_{ij}) \oplus c_{jj} \otimes x_j \]

\[ = \sum_{v_i \in \tilde{H}^n_j} \bigoplus (c_{ij} \otimes y_i) \oplus c_{jj} \otimes x_j \] \quad (5)

\[ = \sum_{v_i \in \tilde{H}^n_j} \bigoplus \left( c_{ij} \otimes \left[ \sum_{k=1}^{N_v} \bigoplus (C_{ki} \otimes x_k) \right] \right) \oplus c_{jj} \otimes x_j. \] \quad (6)

Note that (3) does not mean that the packet \( p_j \) is coming to the node \( v_j \) for all incoming links \( e_{ij} \). Actual packets in \( e_{ij} \) are not all the same as \( p_j \). All incoming packets are combined into \( p_j \) based on the network coding operation in (4) and it can be interpreted as the node receives \( p_j \) from previous nodes.
In (5), \(X_{ij} = y_i\) because \(p_i = [C_{1i}, \ldots, C_{Nv_i}, y_i]\) is transmitted through \(e_{ij}\), and (6) is induced from (4).

Since the global coding coefficient \(C_{kj}\) is updated in every encoding process, (6) can be expressed as

\[
y_j = \sum_{v_i \in \tilde{H}_j^{in}} \left( \bigoplus_{k=1}^{Nv} C_{ij} \otimes C_{ki} \otimes x_k \right) \oplus c_{jj} \otimes x_j
\]

\[
= \sum_{k=1}^{Nv} \left( \sum_{v_i \in \tilde{H}_j^{in}} C_{ij} \otimes C_{ki} \right) \otimes x_k
\]

(7)

\[
= \sum_{k=1}^{Nv} (C_{kj} \otimes x_k).
\]

(8)

In (8), the global coding coefficient is updated based on \(C_{ki} = \sum_{v_i \in \tilde{H}_j^{in}} C_{ij} \oplus C_{ki}\). Thus, \(p_j = [C_{1j}, \ldots, C_{Nv_j}, y_j]\) is constructed and forwarded to the nodes in \(\tilde{H}_j^{out}\).

In this paper, we assume that all data in a network are elements in a GF with size \(2^M\), denoted GF(\(2^M\)), i.e., \(x_i, y_i \in \text{GF}(2^M)\), as the network coding operations in (4) are performed in a GF. Moreover, we use random linear network coding (RLNC) [41] so that the local coding coefficient is randomly selected in GF(\(2^M\)).

2) Conditions for perfect decoding: Let \(S_i = \{v_j | v_i \in D_j, \forall v_j \in V(G)\}\) be a set of source nodes whose destination set includes \(v_i\) and \(S_i = \{j | v_j \in S_i\}\) be an index set of source nodes for \(v_i\). Given the packets \(\tilde{p}_1, \ldots, \tilde{p}_K\) that \(v_i\) received, we can construct a vector of network coded data \(\tilde{y} = [\tilde{y}_1, \ldots, \tilde{y}_K]^T\)
and the global coding coefficient matrix $\tilde{C}$, expressed as

$$\tilde{C} = \begin{bmatrix} \tilde{C}_{11} & \cdots & \tilde{C}_{Nv,1} \\ \vdots & \ddots & \vdots \\ \tilde{C}_{1K} & \cdots & \tilde{C}_{Nv,K} \end{bmatrix} = [\tilde{c}_1 \cdots \tilde{c}_{Nv}]$$

where $\tilde{c}_j = [\tilde{C}_{j,1}, \ldots, \tilde{C}_{j,K}]^T$.

Node $v_i$ is then able to perfectly reconstruct its source data, as long as $\tilde{C}$ satisfies the following two conditions: 1) $\tilde{c}_j \neq \mathbf{0}_K$ for all $j \in S_i$, where $\mathbf{0}_K$ denotes all zero vector with length $K$, and 2) $\tilde{C}'$ is full-rank, where $\tilde{C}'$ is the matrix with all $\tilde{c}_j = \mathbf{0}_K$ for $1 \leq j \leq N_v$ removed from $\tilde{C}$. The condition 1) ensures that the received packets include all data that should be reconstructed. This greatly depends on the connection failure ratio of $\mathcal{G}$, which will be considered in the latter part of this paper (i.e., Fig. 6) and it can be controlled as a system parameter. The condition 2) guarantees that the decoding process can uniquely reconstruct data $\hat{x}_j$ for all $j \in S_i$. Since the global coding coefficient matrix can become full-rank with high probability by RLNC [22], [42], the condition 2) can be satisfied. The decoding process can then be implemented based on the well-known Gaussian elimination in a GF [43].

While the conditions for perfect reconstruction can be generally satisfied with high probability, some special applications (e.g., delay-sensitive applications, error-prone networks with a high packet loss rate, etc.) may cause perfect reconstruction failure, i.e., random mixing in the inter-session network coding may lead to increased decoding delay if only a subset of the coded sources of interest is arrived at the destination node. In this case, alternative decoding algorithms [44]–[46] can be deployed.

**IV. DISTRIBUTED TOPOLOGY FORMATION BASED ON GAME-THEORETIC APPROACHES**

In this section, we propose a distributed topology formation strategy in a sensor network with multi-source multicast flows. We formulate the problem of how to make decisions on link connections between nodes in the considered network as a game, referred to as network formation game. Then, we show that the network formation game can be decomposed into link formation games, which enables each node to decide which links are active or inactive. Therefore, this eventually leads to a distributed solution.

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3 It is shown in [42] that if RLNC is employed, the probability that the global coding coefficient matrix is full-rank is at least $(1 - |\mathcal{D}|/2^M)^{|E(\mathcal{G})|}$. In general settings, GF size $2^M$ is much larger than the number of destinations $|\mathcal{D}|$ in network. Hence, it is widely accepted that the global coding coefficient matrix is full-rank with high probability if RLNC is used.
A. Network Formation Game

Given a set of nodes $\mathcal{V}(\mathcal{G})$ and a destination index matrix $\mathbf{D} = [\mathbf{D}_1^T, \ldots, \mathbf{D}_{N_v}^T]$, where $\mathbf{D}_i = [j|j \in \mathcal{D}_i]$ is an index vector for destination nodes of $v_i$, a strategic form of the network formation game can be expressed as

$$
\mathbf{G}(\mathbf{D}) = (\mathcal{V}(\mathcal{G}), (\mathbf{a}_i)_{v_i \in \mathcal{V}(\mathcal{G})), (u_i(a_i, a_{-i}, \mathbf{D}_i))_{v_i \in \mathcal{V}(\mathcal{G})),}
$$

where $\mathcal{V}(\mathcal{G})$, $\mathbf{a}_i = \times_{v_i \in \mathcal{H}_i} e_{ij} = \times_{v_i \in \mathcal{H}_i} \{0,1\}$ and $u_i(a_i, a_{-i}, \mathbf{D}_i)$ denote a finite set of players, a finite set of actions for player $v_i$, and the utility function of player $v_i$, respectively. $\times$ denotes the Cartesian product.

A network node $v_i \in \mathcal{V}(\mathcal{G})$ is a player in the network formation game, which makes decisions about link formation with its neighbor nodes $\forall v_j \in \mathcal{H}_i$. The action of $v_i$ is denoted by $a_i = (e_{ij})_{v_j \in \mathcal{H}_i} \in \mathbf{a}_i$. The utility of $v_i$ is defined as a quasi-linear utility function, expressed as

$$
u_i(a_i, a_{-i}, \mathbf{D}_i) = R_i(a_i, \mathbf{D}_i) - \lambda_i(a_i, a_{-i})$$

where $a_{-i}$ denotes a set of actions taken by players other than $v_i$ in $\mathcal{V}$. Given destination nodes, the utility of a player can be determined by the reward and cost associated with its own and others’ actions. The reward for action $a_i$ taken by $v_i$ is given as

$$
R_i(a_i, \mathbf{D}_i) = \sum_{v_j \in \mathcal{H}_i} E_{ij}(a_i) (f(\delta_{j\mathbf{D}_i}) - f(\delta_{i\mathbf{D}_i}))
$$

where $E_{ij}(a_i) = e_{ij} \in \{0,1\}$ is determined by action $a_i$.

The reward $R_i(a_i, \mathbf{D}_i)$ in (11) represents the distance reduction toward the destination nodes by the action $a_i$, which induces short delay and high throughput. If all nodes in network build links with consideration of reward, they connect to the node as close as possible to the destination and it leads to draw a shortcut from sources to destinations. Let $\delta_{i\mathbf{D}_i} = (\delta_{ij})_{v_j \in \mathbf{D}_i}$ be a vector of distances from $v_i$ to destinations $v_j$ for all $j \in \mathcal{D}_i$. We define the function $f(\delta_{i\mathbf{D}_i}) : \mathbb{R}^{||\mathbf{D}_i|| \times 1} \rightarrow \mathbb{R}$ such that it is inversely proportional to $\delta_{i\mathbf{D}_i}$ thereby leading to higher rewards for nodes closer to the destinations. For example, if $v_j$ is located closer to destinations $\mathbf{D}_i$ than $v_i$, then $f(\delta_{j\mathbf{D}_i}) > f(\delta_{i\mathbf{D}_i})$ and correspondingly, $E_{ij}(a_i) (f(\delta_{j\mathbf{D}_i}) - f(\delta_{i\mathbf{D}_i})) > 0$ meaning that positive rewards are given for the formation of link $e_{ij}$. Hence, the reward function can represent the importance of a node location. Moreover, the reward function can lead to higher source-destination connectivity in the network.

\[\text{(31)-(32)}\]
closer nodes have a greater effect on connectivity than distant nodes because a distant node has higher path diversity; it may have alternative routes toward destinations.

Given the actions \((a_i, a_{-i})\) selected by players, the cost is defined as

\[
\lambda_i(a_i, a_{-i}) = \sum_{v_j \in \mathcal{H}_i} \left( \frac{E_{ij}(a_i)}{E_{ij}(a_i) + E_{ji}(a_j)} \times \Lambda \right)
\]  

where \(\Lambda\) is a unit cost for link formation. We define \(0/0 = 0\). The cost \(\lambda_i(a_i, a_{-i})\) in (12) represents the total payment required for all outgoing links that \(v_i\) makes. This can be considered as the penalty for incurring interference to neighbor nodes or as the energy consumption required to transmit a radio signal. For a link between \(v_i\) and \(v_j\), if either \(v_i\) or \(v_j\) decides to build the outgoing link, the unit cost for link formation \(\Lambda\) is solely charged to the node that builds the link. If both nodes decide to build the link, the link formation cost is equally charged to them\(^5\), i.e.,

\[
\lambda_i(a_i, a_{-i}) = \begin{cases} 
\Lambda, & \text{if } E_{ij}(a_i) = 1, E_{ji}(a_{-i}) = 0 \\
\Lambda/2, & \text{if } E_{ij}(a_i) = 1, E_{ji}(a_{-i}) = 1 \\
0, & \text{if } E_{ij}(a_i) = 0 
\end{cases}
\]  

(13)

The solution to the network formation game \(G_G(D)\) is the set of actions \((a^*_i, a^*_{-i})\), which is optimally taken by each player and determines \(\mathcal{E}(G)\) and the corresponding network topology. While the proposed solution to the network formation game \(G_G(D)\) can be obtained in a distributed way, the computational complexity required to find the solution can be significantly increased especially as \(G\) becomes large (i.e., network size grows). Hence, we show that the network formation game can be decomposed into several link formation games by deploying network coding, which enables the solution to be found with significantly lower complexity in the next.

**B. Network Coding Based Game Decomposition**

We define edge-disjoint subgraphs of \(G\) as a set of subgraphs whose links are disjoint and the union of them is \(G\). Specifically, for \(N\) edge-disjoint subgraphs \(L_1, \ldots, L_N\) of \(G\),

- \(\mathcal{V}(L_n) \subseteq \mathcal{V}(G)\),
- \(\mathcal{E}(L_n) \subseteq \mathcal{E}(G)\),
- \(\bigcup_{n=1}^{N} \mathcal{E}(L_n) = \mathcal{E}(G)\) and

\(^5\) The equal-division mechanism was first proposed in \([47]\) and it has been extensively deployed in network formation cost (e.g., \([48], [49]\)).

\(^6\) There can be maximum \(\binom{N}{2}\) edge-disjoint subgraphs in \(G\) as each link with two nodes becomes a subgraph of \(G\). Hence, it is always possible to decompose \(G\) into edge-disjoint subgraphs.
• \( \mathcal{E}(L_n) \cap \mathcal{E}(L_m) = \emptyset \) for \( 1 \leq n, m \leq N, n \neq m \).

The network formation game for a subgraph \( L_n \) with \( D \) can be expressed as

\[
G_{L_n}(D) = \langle V(L_n), (a_i)_{v_i \in V(L_n)}, (u_i(a_i, a_{-i}, D_i))_{v_i \in V(L_n)} \rangle
\]

and the network status for the resulting network from \( G_{L_n}(D) \) is denoted by \( J_{G_{L_n}(D)} \), as defined in (2).

Since the actions simultaneously determined by the players in a game are the union of the links that are active and inactive in the network, the product operation for games can be considered as the union of their network status, expressed as

\[
\prod_{n=1}^{N} G_{L_n}(D) \triangleq \bigcup_{n=1}^{N} J_{G_{L_n}(D)}.
\]

(14)

In Theorem 1, we show that network coding can decompose the network formation game \( G(D) \) into independent games \( G_{L_n}(D) \) for subgraph \( L_n \) for \( 1 \leq n \leq N \).

**Theorem 1.** The network formation game for a graph can be decomposed by network coding into independent games for edge-disjoint subgraphs.

**Proof.** To show that the network formation game for a graph can be decomposed by network coding into independent games for edge-disjoint subgraphs it should be proved that

\[
\Phi(G(D)) = \prod_{n=1}^{N} \Phi(G_{L_n}(D)).
\]

(15)

where \( \Phi \) is the network coding function defined in (3).

The network formation game for a graph \( G \) is the joint game of edge-disjoint subgraphs \( L_1, \ldots, L_N \), which can be played as sequential conditional games based on a chain rule as in (16)–(18). In here, the equality between (16) and (17) is based on (14), and (18) is based on the definition of network status in (2). Note that the network formation game expressed in (18) still includes the inter-link dependency.
\[ G_G(D) = G_{L_1, L_2, \ldots, L_N}(D) \]

\[ = G_{L_1}(D) \cdot G_{L_2|L_1}(D) \cdots G_{L_N|L_1, L_2, \ldots, L_{N-1}}(D) \]  

(16)

\[ = J_{G_{L_1}(D)} \cup J_{G_{L_2|L_1}(D)} \cup \cdots \cup J_{G_{L_N|L_1, L_2, \ldots, L_{N-1}}(D)} \]  

(17)

\[ = \left\{ X_{ij} \cdot e_{ij} | \forall v_i, v_j \in V(L_1), i \neq j \right\}, \left( \sum_{\forall v_i \in H_{i}^{n}} e_{ij} \leq 1, \forall v_j \in V(L_1) \right) \]  

\[ \cup \left\{ X_{ij} \cdot e_{ij} | \forall v_i, v_j \in V(L_2), i \neq j \right\}, \left( \sum_{\forall v_i \in H_{i}^{n}} e_{ij} \leq 1, \forall v_j \in V(L_1) \cup V(L_2) \right) \]  

\[ \cup \cdots \cup \left\{ X_{ij} \cdot e_{ij} | \forall v_i, v_j \in V(L_N), i \neq j \right\}, \left( \sum_{\forall v_i \in H_{i}^{n}} e_{ij} \leq 1, \forall v_j \in V(L_n) \right) \]  

(18)

By applying the network coding function \( \Phi \) in (18), we have

\[ \Phi(G_G(D)) \]

\[ = \left\{ \{ p_j \cdot e_{ij} | \forall v_i, v_j \in V(L_1), i \neq j \} \right\} \]

\[ \cup \left\{ \{ p_j \cdot e_{ij} | \forall v_i, v_j \in V(L_2), i \neq j \} \right\} \]

\[ \cup \cdots \cup \left\{ \{ p_j \cdot e_{ij} | \forall v_i, v_j \in V(L_N), i \neq j \} \right\} \]

(20)

\[ = \Phi(J_{L_1}(D)) \cup \Phi(J_{L_2}(D)) \cup \cdots \cup \Phi(J_{L_N}(D)) \]

(21)

\[ = \Phi(G_{L_1}(D)) \cdot \Phi(G_{L_2}(D)) \cdots \Phi(G_{L_N}(D)) \]

(22)

where the equality between (20) and (21) is based on (3), and (22) is based on (14). Therefore, the network formation game for a graph can be decomposed by network coding into independent games for edge-disjoint subgraphs, which completes the proof.

Importantly, Theorem 1 implies that \( G_G(D) \) with multiple destinations in \( D \) can be further decomposed into \( G_{L_n}(d) \) for \( 1 \leq n \leq N \) and \( d \in D \) with single destination node \( v_d \), which is shown in Theorem 2.

In order to prove this, we define a virtual subnode of \( v_i \) that has flows to be delivered to destination \( v_d \) as \( v_i(d) \). By definition, there are \( |D| \) virtual subnodes in \( v_i \). Similarly, a virtual sublink of \( e_{ij} \) with
destination \( v_d \) is denoted by \( e_{ij}(d) \), and \( e_{ij} \) includes |\( D \)| sublinks. Then, a virtual subgraph for destination \( v_d \) can be defined as \( \mathcal{L}_n(d) \), which satisfies

- \( \mathcal{V}(\mathcal{L}_n(d)) \subseteq \mathcal{V}(\mathcal{L}_n(D)) \),
- \( \mathcal{E}(\mathcal{L}_n(d)) \subseteq \mathcal{E}(\mathcal{L}_n(D)) \),
- \( \bigcup_{d \in D} \mathcal{E}(\mathcal{L}_n(d)) = \mathcal{E}(\mathcal{L}_n(D)) \) and
- \( \mathcal{E}(\mathcal{L}_n(d)) \cap \mathcal{E}(\mathcal{L}_n(d')) = \emptyset \) for \( d, d' \in D, d \neq d' \).

**Theorem 2.** The network formation game with multicast flows can be decomposed by network coding into independent games with unicast flows for edge-disjoint subgraphs.

**Proof.** In this proof, we show that \( \Phi(\mathcal{G}(\mathcal{D})) \) can be decomposed into \( \Phi(\mathcal{G}_{\mathcal{L}_n}(d)) \) for \( 1 \leq n \leq N \) and \( d \in D \).

In Theorem 1, it is shown that

\[
\Phi(\mathcal{G}(\mathcal{D})) = \prod_{n=1}^{N} \Phi(\mathcal{G}_{\mathcal{L}_n}(D)). \tag{23}
\]

Since a subgraph \( \mathcal{L}_n(D) \) can be decomposed into virtual subgraphs \( \mathcal{L}_n(d), \forall d \in D \), the game \( \mathcal{G}_{\mathcal{L}_n}(D) \) with network coding can also be decomposed into independent games \( \mathcal{G}_{\mathcal{L}_n}(d), \forall d \in D \) for virtual subgraphs based on Theorem 1, i.e.,

\[
\Phi(\mathcal{G}_{\mathcal{L}_n}(D)) = \prod_{d \in D} \Phi(\mathcal{G}_{\mathcal{L}_n}(d)). \tag{24}
\]

Therefore, we can conclude from (23) and (24) that

\[
\Phi(\mathcal{G}(\mathcal{D})) = \prod_{n=1}^{N} \prod_{d \in D} \Phi(\mathcal{G}_{\mathcal{L}_n}(d)) \tag{25}
\]

which completes the proof.

Theorem 2 implies that network coding allows the network formation game for multicast flows (i.e., \( \mathcal{G}_{\mathcal{L}_n}(D) \)) to be decomposed into independent games with unicast flows for edge-disjoint subgraphs (i.e., \( \mathcal{G}_{\mathcal{L}_n}(d), \forall d \in D \)). Moreover, Theorem 2 enables the topology of a network with multi-source multicast flows to be determined in a distributed way, by solving independent games of edge-disjoint subgraphs with unicast flows, referred to as link formation game in this paper. More details about the link formation game are given in the next section. An illustrative example of the network formation and link formation games are shown in Fig. 3.
C. Link Formation Games and Distributed Topology Design

As discussed in Section IV-B, a link formation game consists of two players with a unicast flow. The strategic form of the link formation game can be expressed as

$$G_{\mathcal{L}}(d) = \langle V(\mathcal{L}), (a_i)_{v_i \in V(\mathcal{L})}, (u_i(a_i, a_{-i}, d))_{v_i \in V(\mathcal{L})} \rangle$$

(26)

where $V(\mathcal{L}) = \{v_i, v_j\}$ and $a_i = \{0, 1\}$ denote a player set and an action set for player $v_i$ for destination $v_d$, respectively. The utility function is expressed as

$$u_i(a_i, a_j, d) = R_i(a_i, d) - \lambda_i(a_i, a_j)$$

$$= a_i \left( f(\delta_{jd}) - f(\delta_{id}) \right) - \Lambda \cdot \frac{a_i}{a_i + a_j}$$

(27)

(28)

where $f(\delta_{id}) : \mathbb{R} \rightarrow \mathbb{R}$ is an inversely proportional function of $\delta_{id}$. For the link formation game, the cost function can be expressed as

$$\lambda_i(a_i, a_j) = \Lambda \cdot \frac{a_i}{a_i + a_j} = \begin{cases} \Lambda, & \text{if } a_i = 1, a_j = 0 \\ \Lambda/2, & \text{if } a_i = 1, a_j = 1 \\ 0, & \text{if } a_i = 0 \end{cases}$$

The corresponding normal form of the link formation game is shown in Table II.
As a solution concept for the link formation game, we adopt the pure strategy Nash equilibrium (NE). A pure strategy NE \((a_i^*, a_j^*)\) for \(v_i\) and \(v_j\) can be expressed as

\[
 u_i(a_i^*, a_j^*, d) \geq u_i(a_i, a_j^*, d) \text{ for all } a_i \in a_i
\]

and

\[
 u_j(a_i^*, a_j^*, d) \geq u_j(a_i^*, a_j, d) \text{ for all } a_j \in a_j.
\]

If multiple pure strategy NEs exist, the set of pure strategy NEs is denoted by

\[
 A^* = \{(a_i^*, a_j^*) | u_i(a_i^*, a_j^*, d) \geq u_i(a_i, a_j^*, d), \forall a_i \in a_i, \\
 u_j(a_i^*, a_j^*, d) \geq u_j(a_i^*, a_j, d), \forall a_j \in a_j\}.
\]

The pure strategy NE enables nodes \(v_i\) and \(v_j\) to decide which outgoing links are active or inactive, resulting in a stable network topology \(\mathcal{E}(L)\).

The steps for the proposed solution are described in Algorithm 1. In Algorithm 1, nodes \(\mathcal{V}(G)\) are decomposed into \(N = \binom{N_V}{2}\) sets of node pairs \(\mathcal{V}(L_n)\) for \(1 \leq n \leq N\). Then, the link formation game is formulated as \(\mathcal{G}_{L_n}(d) = \langle \mathcal{V}(L_n), (a_i)_{v_i \in \mathcal{V}(L_n)}, (u_i(a_i, a_{-i}, d))_{v_i \in \mathcal{V}(L_n)} \rangle\) given \(\mathcal{V}(L_n)\) and a destination node \(v_d\). The link formation games, \(\mathcal{G}_{L_n}(d)\) for \(1 \leq n \leq N, \forall d \in D\), are solved by finding pure strategy NE, and all the active links can eventually be included in \(\mathcal{E}(G)\).

Note that Algorithm 1 guarantees to determine at least one stable topology, as shown in Theorem 3.

**Theorem 3.** It is guaranteed that at least one topology can be determined by Algorithm 1.

**Proof.** This can be proved by showing that there exists at least one pure strategy NE for the link formation game, i.e. \(A^* \neq \emptyset\).

It is shown from Nash’s Existence Theorem [50] that every finite game has a mixed strategy NE, where pure strategies are chosen stochastically with certain probabilities. Since the link formation game \(\mathcal{G}_{L_n}(d)\) defined in [26] includes a finite number of nodes and a finite number of actions, it is a finite game. Therefore, there exists a mixed strategy NE for this game.

---

### TABLE II: The normal form of the link formation game

| \((u_i, u_j, d)\) | \(a_j = 1\) | \(a_j = 0\) |
|------------------|-------------|-------------|
| \(a_i = 1\)     | \((R_i(1, d) - \frac{\Lambda}{2}, R_j(1, d) - \frac{\Lambda}{2})\) | \((R_i(1, d) - \Lambda, 0)\) |
| \(a_i = 0\)     | \((0, R_j(1, d) - \Lambda)\) | \((0, 0)\) |
Algorithm 1 Algorithm for Distributed Topology Design Based on Link Formation Games

Require: a set of nodes $\mathcal{V}(\mathcal{G})$, sets of neighbor nodes $\mathcal{H}_i$ for $v_i \in \mathcal{V}(\mathcal{G})$, an index set of destinations $\mathcal{D}$, utility function $u_i(a_i, a_j, d)$ for $v_i \in \mathcal{V}(\mathcal{G})$

1: Initialize: $\mathcal{E}(\mathcal{G}) = \emptyset$
2: Decompose $\mathcal{V}(\mathcal{G})$ into a set of node pairs $\mathcal{V}(\mathcal{L}_n)$ for $1 \leq n \leq \binom{N}{2}$
3: for $n = 1: \binom{N}{2}$ do
4: $(v_i, v_j) \leftarrow \mathcal{V}(\mathcal{L}_n)$ //assign players
5: for $d \in \mathcal{D}$ do
6: Initialize: $(a_i^*, a_j^*) \leftarrow (0, 0)$, flag = 0
7: // begin link formation game
8: while flag = 0 do
9: $temp_a \leftarrow (a_i^*, a_j^*)$
10: $a_i^* \leftarrow \arg \max_{a_i} u_i(a_i, a_j^*, d)$
11: $a_j^* \leftarrow \arg \max_{a_j} u_j(a_i^*, a_j, d)$
12: if $temp_a = (a_i^*, a_j^*)$ then
13: flag $\leftarrow 1$
14: // $(a_i^*, a_j^*)$ is the pure strategy NE
15: $e_{ij} \leftarrow a_i^*$, $e_{ji} \leftarrow a_j^*$
16: return $\mathcal{E}(\mathcal{G})$

Suppose that $p_i$ is a strategy of player $v_i$ with the probability of taking action $a_i = 1$. The corresponding utility is given by

$$u_i(p_i, p_j, d) = p_i \left( f(\delta_{jd}) - f(\delta_{id}) - \left( \frac{1}{1 + E_{ji}(p_j) - \Lambda} \right) \right).$$

Let $p_i^*$ and $p_j^*$ be mixed strategy NE for $v_i$ and $v_j$ that satisfy $u_i(p_i^*, p_j^*, d) \geq u_i(p_i, p_j^*, d)$ and $u_j(p_i^*, p_j^*, d) \geq u_j(p_i^*, p_j, d)$ for all $v_i, v_j \in \mathcal{V}(\mathcal{L})$.

For $\xi \in [-p_i^*, 1 - p_i^*]$ perturbation of mixed strategy NE $p_i^*$, the resulting utility of $v_i$ can be expressed
as
\[
\begin{align*}
    u_i(p_i^* + \xi, p_j, d) \\
    &= (p_i^* + \xi) \times \left( f(\delta_{jd}) - f(\delta_{id}) - \frac{1}{1 + E_{ji}(p_j) \cdot \Lambda} \right) \\
    &= u_i(p_i^*, p_j, d) + \xi \times \left( f(\delta_{jd}) - f(\delta_{id}) - \frac{1}{1 + E_{ji}(p_j) \cdot \Lambda} \right) \\
    &= u_i(p_i^*, p_j, d) + \xi \times \beta.
\end{align*}
\]

If \( \beta > 0 \), then \( \frac{\partial u_i(p_i^* + \xi, p_j, d)}{\partial \xi} > 0 \), and thus, \( v_i \) can always decrease its utility by decreasing \( \xi \). This means that the pure strategy \( p_i = 0 \) strictly dominates any strategies \( p_i^* + \xi \). For \( \beta < 0 \), on the other hand, \( \frac{\partial u_i(p_i^* + \xi, p_j, d)}{\partial \xi} < 0 \), and thus, the pure strategy \( p_i = 1 \) strictly dominates any strategies \( p_i^* + \xi \). If \( \beta = 0 \), then \( \frac{\partial u_i(p_i^* + \xi, p_j, d)}{\partial \xi} = 0 \) and \( \xi \) does not affect on the cost such that both pure strategies \( p_i = 0 \) and \( p_i = 1 \) have the same utility as the mixed strategy NE \( p_i^* \). Therefore, the pure strategy \( p_i = 0 \) or \( p_i = 1 \) can always weakly dominate mixed strategies \( p_i^* + \xi \).

Similarly, \( \xi \) perturbation of mixed strategy NE \( p_j^* \) also concludes that the pure strategy \( p_j = 0 \) or \( p_j = 1 \) can always weakly dominate mixed strategies \( p_j^* + \xi \).

In conclusion, a pure strategy NE can always weakly dominate mixed strategy NEs in the link formation game \( G_L(d) \), implying that there exists a pure strategy NE. Hence, Algorithm 1 guarantees at least one topology.

Based on Theorem 3, we confirm that Algorithm 1 can always provide at least one network topology for multi-source multicast network. In the next section, we provide performance evaluation based on simulation results.

V. SIMULATION RESULTS

In this simulation, we consider a sensor network in which all sensors collect data, and some of them are destination nodes. Sensor nodes aim to deliver their collected data to their destination nodes in \( D_i \) by making links between them.

A. Simulation Setup

We consider \( N_i \) nodes in a cell with a radius of \( R \), where the nodes are randomly located over the cell based on a uniform distribution. Since a node \( v_i \) would like to deliver its collected data to its destination nodes in \( D_i \), there are \( \sum_{i=1}^{N_i} |D_i| \) total flows in the network. The connection boundary is set as \( \Delta = R \) so that the neighbor nodes of \( v_i \) is determined as \( \mathcal{H}_i = \{ v_j | 0 < \delta_{ij} \leq R \} \). Each node makes its own decisions for outgoing link formation with neighbor nodes based on Algorithm 1.
In the simulations, we use a function \( f(\delta_{id}) \) for the utility function in (28), defined as
\[
f(\delta_{id}) = \frac{1}{\delta_{id}^2 + 1}
\] (31)
which satisfies the requirements for \( f(\delta_{id}) \) discussed in Section IV-A, i.e., it is inversely proportional to \( \delta_{id} \) and \( f(\delta_{jd}) - f(\delta_{id}) > 0 \) for \( \delta_{id} > \delta_{jd} \). Hence, if \( v_i \) makes an outgoing link to \( v_j \) which is closer to a destination node than \( v_i \), then positive rewards are given to \( v_i \). For example, consider two node pairs \((v_i, v_j)\) and \((v'_i, v'_j)\) that have 1) the same distance between them, i.e., \( \delta_{id} - \delta_{jd} = \delta_{i'd} - \delta_{j'd} \), but 2) different locations, i.e., \( \delta_{id} < \delta_{i'd}, \delta_{jd} < \delta_{j'd} \). Then,
\[
f(\delta_{jd}) - f(\delta_{id}) > f(\delta_{j'd}) - f(\delta_{i'd}) \] (32)
which implies that the nodes close to the destination \((v_i, v_j)\) receive more reward than the distant nodes \((v'_i, v'_j)\). The utility function of the link formation game in (28) can be correspondingly expressed as
\[
u_i(a_i, a_j, d) = a_i \left( \frac{1}{\delta_{jd}^2 + 1} - \frac{1}{\delta_{id}^2 + 1} \right) - \left( \frac{a_i}{a_i + a_j} \cdot \Lambda \right).
\] (33)
We finally define network utility as a measure of the resulting networks performance, expressed as
\[
U(a_i, a_{-i}, D) = \sum_{\forall v_i \in \mathcal{V}(G)} \left( \sum_{d \in D} R_i(a_i, d) - \lambda_i(a_i, a_{-i}) \right)
\]
which includes both the total rewards from all the destination nodes and the costs required to make links in the networks. Therefore, the network utility can be used to quantify how much rewards can be earned by the nodes while reducing the costs of link formation.

B. Numerical Analysis of Proposed Topology Design

In this section, we numerically analyze several aspects of the proposed algorithm for topology formation implemented by Algorithm 1. In the simulations, we consider two destination nodes, i.e., \(|D_i| = 2\) for \(1 \leq i \leq N_V\), which are randomly determined in each experiment unless otherwise stated. The connection boundary is set as \( R = 10 \) and all the experiment results are averaged from \(1,000\) independent experiments.

Fig. 4 shows the effects of node locations on the probability of link formation. In the experiment, we consider two adjacent nodes as the destination for all nodes, located at the cell edge, i.e., \( D_1 = \cdots = D_{N_V} \). The nodes in the network are classified as three types based on their distance from destination nodes: those that are close to the destinations (NEAR), far from the destinations (FAR), and in the middle of them (MID). Fig. 4 clearly confirms that the nodes closer to the destinations make more outgoing links.
This is because $f(\delta_{id})$ in the utility function enables nodes closer to the destinations to obtain more rewards by making outgoing links.

Fig. 5 shows the total number of active links in a network ($|E(V)|$) for various unit costs ($\Lambda$) and network sizes ($N_V$). It can be confirmed that a smaller number of active links is included in a resulting network topology as the network size decreases or the unit cost increases. This is because the nodes with more neighbor nodes or with lower unit cost can make larger number of active links.

The number of active links in a network topology can influence the successful connection from source nodes to destination nodes. In order to evaluate the impact of the number of active links on successful
network formation, we define the connection failure ratio as the number of disconnected flows over the total number of flows ($\sum_{i=1}^{N_V} |D_i|$). In Fig. 6, connection failure ratios for unit costs are presented.

As shown in Fig. 5, the number of active links decreases rapidly as $\Lambda$ increases in the range of a small $\Lambda$ (e.g., $0 \leq \Lambda \leq 0.2$). However, this does not significantly affect the connection failure ratios as shown in Fig. 6. On the other hand, small number of links can be active in the range of a large $\Lambda$ (e.g., $0.8 \leq \Lambda \leq 1$), which significantly increases the connection failure ratio, thereby resulting in high probability of unsuccessful data delivery. Hence, a network can be sustainable in terms of successful data delivery only if the number of active links is large enough to take advantage of path diversity. Note that Algorithm 1 is scalable to the network size in terms of the connection failure ratio (or network topology formation) since the impact of network size on the connection failure ratios is limited as shown in Fig. 6. Therefore, the proposed algorithm can be deployed in large-scale networks.

We next evaluate the performance of the proposed algorithm in terms of the network goodput, measured by the number of packets successfully delivered to the destinations per time slot. The results are shown in Fig. 7 and Fig. 8. In this experiments, the connection boundary is set as $\Delta = 1.1 R$, and all nodes in the network generate and deliver packets toward two destination nodes. They make decisions on link formation based on Algorithm 1. It is obvious that network goodput decreases as unit cost increases because connection failure ratio also increases as confirmed in Fig. 6. Fig. 7 shows that higher network goodput can be achieved by deploying network coding, which means that the proposed algorithm successfully
builds the network while taking advantages of network coding. In Fig. 7 and Fig. 8, it is observed that network goodput decreases as network size increases. This is because the number of hops required for a packet to arrive at the destination increases as the network size is enlarged, which takes longer time for packet delivery. The results are well aligned with [51] which theoretically proves that the goodput scales as $O(1/\sqrt{N_V\log N_V})$ in random networks, and $O(1/\sqrt{N_V})$ in the optimal networks.

In Fig. 9, the network utilities for various network sizes and unit costs are shown. The network utility increases as $N_V$ increases (i.e., more rewards can be achieved) or $\Lambda$ decreases (i.e., cost for link
formation is lowered). In the experiments, for example, network nodes decide not to build any links if \( \Lambda = 1 \), achieving no network utility.

In the next section, we compare the network utilities achieved by topology formation strategies including the proposed algorithm.

C. Performance Comparison

In this section, we evaluate the performance of the proposed algorithm in terms of network utility and computational complexity. The performance of the proposed algorithm is compared with existing network formation strategies shown below.

1) Non-NC Centralized: A centralized solution for an optimal network topology without deploying network coding. Hence, the solution should be found by explicitly considering the inter-link dependency condition. This strategy can be formulated as

\[
(a^*_i, a^{-i}_i) = \arg \max_{(a_i, a_{-i})} U(a_i, a_{-i}, D)
\]

subject to \( \sum_{\forall v_j \in H_i} E_{ij}(a_i) \leq 1, \forall v_i \in V(G) \).

2) NC Centralized: A centralized solution for an optimal network topology with network coding. Hence, the inter-link dependency condition cannot be considered. It can be formulated as

\[
(a^*_i, a^{-i}_i) = \arg \max_{(a_i, a_{-i})} U(a_i, a_{-i}, D).
\]
3) **TCLE [35]**: A distributed solution to topology formation based on a non-cooperative game. For fair comparison, we deploy network coding and no inter-link dependency condition is considered. In this strategy, a node chooses its transmission power by balancing the target network connectivity redundancy $\epsilon$ against transmission energy dissipation. The number of actions available for a node is denoted by $\eta$.

Fig. 10 shows the network utility from each network formation strategy including the proposed algorithm. The NC-Centralized strategy can achieve the highest network utility because the optimal network topology can be chosen from all possible topologies that can be formed from other strategies. Hence, this can be considered as the upper bound of network coding based strategies. The proposed strategy provides higher network utility than the Non-NC Centralized strategy and TCLE. Unlike the proposed approach, which can consider more topologies to maximize rewards by making multiple outgoing links, the Non-NC Centralized strategy can find the optimal topology that is allowed only by the inter-link dependency condition. Among the considered strategies, TCLE provides the lowest network utility because the focus of TCLE is not on construction of successful connections between source nodes and destination nodes when it forms a network topology. Instead, it considers overall network connectivity in terms of algebraic connectivity [52].

We next investigate the complexity required to deploy the network formation strategies, which is measured by the size of the search space. The search space is determined by the number of actions
available for link formation. The size of network considered in the complexity analysis is $n$. For the Non-NC Centralized strategy, each node can make at most one outgoing link, so that a node has at most $n$ link formation choices. Given $n$ nodes in the network, the maximum number of choices is thus $n^n$, and the complexity is $\mathcal{O}(n^n)$. For the NC Centralized strategy, each node has a maximum $2^{n-1}$ choices because each node has a maximum of $n-1$ neighbor nodes to make a link, and each link can be either active or inactive. Given $n$ nodes in the network, the maximum number of choices is thus $2^{n(n-1)}$ and the complexity becomes $\mathcal{O}(2^{n^2})$. The proposed algorithm has a maximum of $\binom{n}{2}$ neighbor node pairs, and each pair has four actions. Because each node pair chooses an action independently, the search space becomes $\binom{n}{2} \times 4$, so that the complexity becomes $\mathcal{O}(n^2)$. The complexity in TCLE is $\mathcal{O}(\eta \cdot n^3)$, because there are $n$ nodes and each one has the worst case complexity $\mathcal{O}(\eta \cdot n^2)$. Therefore, the proposed strategy requires the lowest complexity to find the optimal network topology, so that it can be deployed in practice into a large-scale network. The complexity required for the four network formation strategies is summarized in Table III and presented in Fig. 11.

TABLE III: Theoretical complexity with network size $n$

| Strategy                  | Non-NC Centralized | NC Centralized | TCLE ($\eta=100$) | TCLE ($\eta=5$) | Proposed |
|---------------------------|--------------------|----------------|-------------------|----------------|----------|
| Complexity (The Worst Case)| $\mathcal{O}(n^n)$ | $\mathcal{O}(2^{n^2})$ | $\mathcal{O}(\eta \cdot n^3)$ | $\mathcal{O}(n^2)$ |          |

Fig. 11: Complexity required for four strategies (worst case scenarios)
VI. CONCLUSION

In this paper, we propose an algorithm for a distributed topology formation in a network coding enabled large-scale sensor networks with multi-source multicast flows. The distributed topology formation problem is formulated as a network formation game in which the players (nodes in the network) decide whether to make links with neighbor nodes by considering a reward for distance reduction and the cost required for link formation. We show that the network formation game can be decomposed into independent link formation games by deploying network coding. Network topologies can thus be determined based on the solution to individual link formation games played by only two nodes, leading to a distributed algorithm with low complexity. We also show that the proposed algorithm guarantees to find at least one network topology. The simulation results confirm that the proposed algorithm can achieve high network utility with significantly low complexity and is scalable to any network sizes. Therefore, it can be deployed in large-size networks.

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