From normal state to magnetic storms in terms of fractal dynamics

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Abstract

We show that distinctive alterations in scaling parameters of $D_{st}$ index time series occur as a strong magnetic storm approaches. These alterations reveal a gradual reduction of complexity as the catastrophic event approaches. The increase of the susceptibility coupled with the transition from anti-persistent to persistent behavior may indicate that the onset of a severe magnetic storm is imminent. The preparation of a major magnetic storm could be studied in terms of “Intermittent Criticality”. The analysis also suggests that the continuous scale invariance is partially broken into a discrete scale invariance symmetry.
Major magnetospheric disturbances are undoubtedly among the most important phenomena in space physics and also a core subject of space weather. They are relatively rare events: as in the case of atmospheric storms, earthquakes, solar flares, etc., the occurrence of geomagnetic storms rapidly decreases as their magnitude grows.

$D_{st}$ is the disturbance storm time index, computed from an average over 4 magnetic observatories around the equator, and is considered to represent the magnetospheric ring current contribution [1]. Since the development of a strong ring current is a defining feature of magnetic storms, the $D_{st}$ has been adopted as a proxy for magnetic storm severity [2].

In the context of complex systems self-organized criticality has been associated with natural hazards [3] such as earthquakes, landslides and forest-fires. The idea of using cellular automata to model the magnetospheric activity became popular as observations showed scale-invariant features. However, the debate between a forced (driven) and/or self-organized critical magnetosphere is not at all unique [4, 5, 6].

The magnetosphere as a complex system manifests itself in linkages between space and time, producing scaling patterns and the emergence of fractal structures. We show that distinctive alterations in associated scaling parameters emerge (e.g., transition from anti-persistent to persistent behavior) as large magnetospheric disturbances approach (e.g., 6/11/2001, $D_{st} \simeq -300$ nT). The analysis also reveals the existence of discrete scale invariance, a system property invariant under only a discrete set of dilatations [7, 8].

$D_{st}$ index time series were analyzed using a wavelet analysis technique [9]. In Fig. 1–2, the $D_{st}$ time series and the associated wavelet power spectrum are shown, respectively.

If a time series is a temporal fractal then a power-law of the form $S(f) \propto f^{-\beta}$ is obeyed. $S(f)$ is the power spectral density and $f$ is the frequency. The quantity $S(f)df$ may be understood as the contribution to the total power from those components of the time series, whose frequencies lie in the interval between $f$ and $f + df$. The spectral scaling exponent $\beta$ is a measure of the strength of time correlations. In a $\log S(f) - \log f$ representation the power spectrum is a line with slope $\beta$. The goodness of the fit of a time series to the power-law is assigned with the linear correlation coefficient, $r$, of this representation.

A lot of work on complexity has been focused on power-laws, which describe the scaling properties of fractal processes and structures. Here, we examine whether distinctive alterations in the associated parameters, i.e., $r$ and $\beta$, emerge as a major magnetic storm approaches. For this purpose, the $D_{st}$ time variations were divided upon successive seg-
ments of 16 days and for each of these segments the parameters \( r \) and \( \beta \) were estimated and indicated at the top of the wavelet power spectrum in Fig. 2 (16 days is the shortest time window we can view here based on the index sampling rate and the definition of the wavelet transform.)

The temporal evolution of \( r \) (always above 0.94 and reaching values of 0.99 at the end of the time interval considered here) means that the fit to the power-law is excellent. The fractal-law \( (S(f) \propto f^{-\beta}) \) observed indicates the existence of memory. This means that the current value of the geomagnetic signal is correlated not only with its most recent value but also with its long-term history in a scale-invariant, fractal manner, namely the system refers to its history in order to define its future. We observe a gradual increase of \( r \) as the main event approaches. This suggests that the fractal character of the underlying processes and structures becomes clearer with time.

The distribution of \( \beta \) exponent is also shifted to higher values. This shift reveals several features of the underlying mechanism. As \( \beta \) increases the spatial correlation in the time series also increases \[10\]. This behavior indicates a gradual increase of the memory, and thus a gradual reduction of complexity in the underlying dynamics. This suggests that the onset of a severe magnetic storm may represent a gradual transition from a less orderly state to a more orderly state (see also \[5\]).

Maslov et al. \[11\] have formally established the relationship between spatial fractal behavior and long-range temporal correlations for a broad range of critical phenomena. By studying the time correlations in the local activity, they show that the temporal and spatial activity can be described as different cuts in the same underlying fractal. In a geometrical sense, \( \beta \) specifies the strength of the signal’s irregularity as well. The fractal dimension \( D \) is calculated from the relation \( D = (5 - \beta)/2 \[12\], which, after considering the shift of \( \beta \) to higher values, leads to a decrease of the fractal dimension as the magnetospheric crisis approaches. This may reflect that the action of anisotropy inherent to the system leads to the appearance of a clear preferred direction of elementary activities just before the main shock. Theoretical and experimental evidence support the former hypothesis: throughout the entire main and most of the early recovery phase of magnetic storms the geometry of the energy flow produces a highly asymmetric ring current configuration \[1, 2\]. The emergence of strong anisotropy rationalizes a further reduction of the complexity with time.

The colour-type behavior of the power spectrum density \((\beta > 0)\) means that the spec-
trum manifests more power at low frequencies than at high frequencies. The increase in
the spectral exponent $\beta$ with time indicates the gradual enhancement of lower frequency
fluctuations. This observation is consistent with the following physical picture: the acti-
vated substorms interact and coalesce to form larger fractal structures, i.e., the events are
initiated at the lowest level of the hierarchy, with the smallest elements merging in turn
to form larger and larger ones. This sign may be considered as candidate precursor of the
forthcoming shock.

The $\beta$ exponent is related to the Hurst exponent, $H$, by the formula $\beta = 2H + 1$, with
$0 < H < 1$ ($1 < \beta < 3$) for the fractional Brownian motion (fBm) random field model [12].
The exponent $H$ characterizes the persistent / anti-persistent properties of the signal [13].
The range $0 < H < 0.5$ ($1 < \beta < 2$) during the normal period (0 – 80 days) indicates anti-
persistency, reflecting that if the fluctuations increase in a period, it is likely to decreasing in
the interval immediately following and vice versa. Physically, this implies that fluctuations
tend to induce stability within the system (negative feedback mechanism). The observed
systematic increase of the $H$ ($\beta$) exponent during this stage indicates that the fluctuations
become more correlated with time [14].

We pay attention to the fact that the time series appear persistent properties, $0.5 < H < 1$
($2 < \beta < 3$), at 80 – 112 days. This means that if the amplitude of fluctuations increases
in a time interval it is likely to continue increasing in the interval immediately following.
In other words, the system tends toward irreversibility (positive feedback mechanism) [14].
$H = 0.5$ ($\beta = 2$) suggests no correlation between the repeated increments. Consequently,
this particular value takes on a special physical meaning: it marks the transition between
persistent and anti-persistent behavior in the time series.

In Fig. 1 the $D_{st}$ cumulative square amplitudes are also shown. A significant increase in
the rate of energy release as the main geomagnetic storm approaches is observed. This may
show that during the persistent epoch the system is not only near the peak of the magnetic
storm in the sense of having power-law correlations, but also in terms of exhibiting high
susceptibility.

Fractals have dimensions that are in general real numbers. The generalization from the
set of integers to the set of real numbers embodies the transition from the symmetry of
translation invariance to symmetry of scale invariance. Fractals are also described by fractal
dimensions that belong to the complex numbers. In the context of critical phenomena, the
complex fractal dimension is associated with a discrete scale invariance (DSI), i.e., to the invariance of the system or of its properties only under magnifications that are integer powers of a fundamental ratio. Interestingly, the appearance of DSI signifies a partial symmetry breaking of a continuous scale invariance and the emergence of DSI on characteristic scales.

DSI manifests itself in data by log-periodic corrections to scaling \[ E(t) = A + B(t_f - t)^m \{ 1 + C \cos[\omega \log(t_f - t) + \phi] \} \], where \( E(t) \) is the cumulative energy released, \( t_f \) is the time of the main shock (storm peak), \( \omega \) is the frequency and \( \phi \) is just an offset.

We focus on Fig. 1 and in the energy oscillations observed prior to the main magnetic event (32 – 96 days). One can observe a trace of oscillations modulating the main power-law behavior in the energy density. In Fig. 3 we fit a power-law with log-periodic oscillations to the cumulative square \( D_{st} \) amplitudes. It is clear that a law of this form can adequately describe the observations.

As expected, the log-periodic oscillations are modulated in frequency with a geometric increase of the frequency on the approach to the time \( t_f \): the intermittent accelerations and quiescences of geomagnetic activity around the power-law acceleration become more closely spaced as the main event is approached. The aforementioned behavior reflects a preparation stage for major geomagnetic storms, in which pre-storm activities occur at particular discrete times and not in a continuous fashion: these discontinuities in turn mirror the localized and threshold nature of the underlying mechanism. It is this “punctuated” physics which gives rise to the scaling precursors modeled mathematically by the log-periodic correction to scaling.

By monitoring the temporal evolution of the fractal spectral characteristics in \( D_{st} \) we find that distinctive alterations in the associated scaling parameters indicate a transition from the normal state to an abnormal state (major magnetic storm) as following: (i) Emergence of long-range correlations, i.e., appearance of memory effects. This implies a multi-time-scale cooperative activity of numerous activated geomagnetic events. (ii) Increase of the spatial correlation in the time series with time. This indicates a gradual transition from a less orderly state to a more orderly state. (iii) Decrease of the fractal dimension of the variations with time, i.e., appearance of strong anisotropy in elementary activities. (iv) Existence of strong anti-persistent behavior in the first epoch of the geomagnetic activity, i.e., prior to the severe magnetic storm. (v) Decrease of the anti-persistent behavior with time. (vi)
Emergence of persistent properties in the “tail” of the time series. (vii) Predominance of large geomagnetic events with time. (viii) Significant acceleration of the energy release as the main shock approaches, i.e., increase of the susceptibility of the system. (ix) Gradual appearance of higher frequencies in the spectrum with simultaneous increase of the amplitudes at each emission rate as the magnetic storm peak approaches, mainly characterizing lower emission rates.

A question that arises is whether the evolution towards global instability is inevitable after the appearance of distinctive symptoms in the geomagnetic variations. The emergence of persistent behavior, the increase of the susceptibility of the system, the predominance of large geomagnetic events, the coherent fluctuations at all scales, may indicate that the generation of a very strong magnetic storm becomes, indeed, unavoidable.

The aforementioned crucial footprints (including temporal alterations in associated scaling parameters) distinguish the dynamics of a complex system close to its final instability. These may indicate the following scenario for the generation of a severe magnetic storm. During the normal period, the system is in an anti-persistent state, with a restricted and systematically fluctuating correlation length. Long-range correlations gradually build up through local interactions until they extend throughout the entire system. The smaller geomagnetic events are the agents by which longer correlations are established. A population of small events will advance the correlation length by an amount depending on its magnitude and magnetosphere state, triggering very strong magnetic storms only if the condition is right: in a anti-persistent regime a population of small events leads to a decaying activity, always dying out. In the persistent state is just able to continue “indefinitely”. This explains in a natural way why not every geomagnetic event can induce geomagnetic activity. A large geomagnetic event destroys long correlations on its associated network, creating a new normal period during which the process repeats by rebuilding correlation lengths towards the next large event. Thus a large shock may act as a sort of “critical point” dividing the magnetic storm into a period of growing correlations before the great event and a relatively uncorrelated phase after.

The aforementioned evolution may be overall characterized as “Intermittent Criticality”[16] that predicts a time-dependent variation in the activity as the “critical point” is approached, implying, in contrast to self-organized criticality (SOC), a degree of predictability.

The analogies with the dynamics of the SOC model for the magnetosphere have been
realized by numerous authors \[5, 6\]. Characteristically, the scale-free structure of the aurora is argued to come from a scale-free structure of a SOC magnetosphere \[6\]. However, numerous authors are cautious with this suggestion. A relevant question, emphasized by Consolini and Chang \[17\], is how well the assumption of the SOC model is met in magnetosphere. We also recall the debate between a driven and an internal origin for intermittent scale-free dynamics in magnetosphere. Work continues on this issue, which is an example of a generic problem of complex systems coupled to complex drivers \[6\]. We think that the results of this study may help to approach the real dynamics of magnetosphere. In any case the present study suggests that it can be important to distinguish between SOC and intermittent criticality in the study of the magnetic storm cycle. A proper recognition and understanding of tuning parameters may lead to the development of improved magnetospheric models having higher performance reliability. One of the main features in the complexity is the role that the topological disorder plays in such systems. The range of size scales characterizing heterogeneities of the thresholds might be acted as a tuning parameter of the underlying final magnetic storm dynamics.

We bear in mind that our analysis reveals an interesting transition from the anti-persistent to the persistent regime. The anti-persistent behavior characterizes the magnetosphere during substorms, while the catastrophic events are in reasonable agreement with persistent models. We note that Sitnov et al. \[5\] have also suggested two different regimes: while the substorm activity resembles second-order phase transitions, the largest substorms avalanches are shown to reveal the features of first-order non-equilibrium transitions. The corresponding pictures in each regime are not in contradiction.

It is important to stress the practical consequence of log-periodic structures. For forecasting purpose, it is much more constrained and thus reliable to fit a part of an oscillating data than a simple power-law which can be quite degenerate especially in the presence of noise. This idea has been noted and is vigorously investigated in several applied domains, such as earthquakes, rupture and financial crashes \[8\].

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FIG. 1: $D_{st}$ time series and corresponding cumulative square amplitudes. Red star denotes the peak of the magnetic storm of 6/11/2001 with $D_{st} = -292$ nT.

FIG. 2: Wavelet power spectrum of the $D_{st}$ time series, shown in Fig. 1. Linear correlation coefficients, $r$, and spectral exponents, $\beta$, calculated every 16 days are indicated on the top of each segment. Red star denotes the peak of the magnetic storm of 6/11/2001 with $D_{st} = -292$ nT.
FIG. 3: A fit of the cumulative square amplitudes, shown in Fig. 1, to a power-law with log-periodic oscillations.