STEellar Proper Motion and the Timing of Planetary Transits

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ABSTRACT

Duration and period of transits in extrasolar planetary systems can exhibit long-term variations for a variety of reasons. Here we investigate how systemic proper motion, which steadily re-orients planetary orbit with respect to our line of sight, affects the timing of transits. We find that in a typical system with a period of several days, proper motion at the level of 100 mas yr^{-1} makes transit duration vary at a rate ~ 10–100 ms yr^{-1}. In some isolated systems this variation is at the measurable level (can be as high as 0.6 s yr^{-1} for GJ436) and may exceed all other transit-timing contributions (due to the general relativity, stellar quadrupole, etc.). In addition, proper motion causes evolution of the observed period between transits $P_{\text{obs}}$ via the Shklovskii effect at a rate $\gtrsim 10 \text{ ms yr}^{-1}$ for the nearby transiting systems (0.26 ms yr^{-1} in GJ436), which in some cases exceeds all other contributions to $P_{\text{obs}}$. Earth’s motion around the Sun gives rise to additional periodic timing signal (even for systems with zero intrinsic proper motion) allowing a full determination of the spatial orientation of the planetary orbit. Unlike most other timing effects, the proper motion signatures persist even in systems with zero eccentricity and get stronger as the planetary period increases. They should be the dominant cause of transit-timing variations in isolated wide-separation (periods of months) systems that will be sought by Kepler.

Key words: Astrometry – Celestial mechanics – eclipses – Planetary systems

1. INTRODUCTION

Planetary transits have provided us with a unique opportunity to get a handle on the physical properties of the extrasolar planets such as their radii and densities. Recently, it has been suggested (Miralda-Escudé 2002; Heyl & Gladman 2007) that precision timing of the moments at which transits occur can give us additional information about the transiting systems. Various physical effects cause orbit of the planet precess in space leading to the changes in transit geometry, which can be measured through the timing of transits. Among these effects are the general relativistic precession of the orbit, gravitational influence of other planets in the system or companion stars, torques due to the spin-induced quadrupole moment of the star, and due to the tidal deformations of both the star and the planet (Miralda-Escudé 2002; Heyl & Gladman 2007; Ribas et al. 2008; Pál & Kocsis 2008; Jordán & Bakos 2008).

Another obvious reason for the re-orientation of the planetary orbit with respect to an observer at Earth is the proper motion of the exoplanetary system with respect to the solar system. Stars in the solar neighborhood move at velocities of tens of km s^{-1}, and some of them exhibit proper motion at the level of 1 mas yr^{-1}. Also, the distance to stars constantly changes as a result of their relative motion with respect to the Sun and this affects transit timing because of the finite speed of light. At some level proper motion is a characteristic of any star, including those with transiting exoplanets, and it is thus important to understand its implications for transit timing.

Proper motion is well known to be important in the timing of isolated and binary radio pulsars (Shklovskii 1970; Kopeikin 1996). In these systems, proper motion affects the pulsar spin and orbital periods through the so-called Shklovskii effect (Shklovskii 1970) while the re-orientation of the binary orbit can be (and has been) measured via the variation of the projected size of the orbit (Kopeikin 1996; Arzoumanian et al. 1996). Pulsar acceleration in external gravitational field can also be important especially for pulsars in globular clusters (Edwards et al. 2006).

Of course, there are significant differences between the timing of pulsars and of planetary transits: accuracy with which some millisecond pulsars can be timed is at the $\mu$s level (Manchester 2008) while a single planetary transit can only be timed to several seconds at best (Knutson et al. 2007). Also, the whole idea of timing is different in the two cases: for binary pulsars, one is usually able to trace the whole orbit of the neutron star in time domain while in the case of planetary transits, only two narrow time windows—primary and secondary transits—are available to play with. Nevertheless, some of the ideas developed in pulsar timing may be applied to the timing of planetary transits.

Previously, Kopeikin & Ozernoy (1999) have utilized a post-Newtonian relativistic approach for the precision Doppler measurements of the binary star orbits and discussed some of the relevant effects of the proper motion. Here, we aim at investigating the role of the proper motion in the timing of planetary transit duration and period in extrasolar planetary systems. After the submission of this work we became aware of the paper by Scharf (2007), which discusses some of the timing effects calculated here (see Section 5.5). We lay out the basics of the orbital element evolution due to the proper motion in Section 2. We discuss the evolution of the transit duration in Section 3 and the evolution of the period between transits in Section 4. Comparison with other transit-timing effects and application to real systems can be found in Section 5.

2. EFFECT OF PROPER MOTION

To quantitatively evaluate the effect of stellar proper motion on the timing of planetary transits, let us consider a planet in orbit around a star with period $P$, semimajor axis $a$, and eccentricity $e$. We introduce a unit vector $\mathbf{n}$ pointing from the observer at Earth to the barycenter of the transiting system. Vector $\mathbf{n}$ varies...
in time because of the linear motion of the plane of the sky. Orientation of the binary in space is fully determined by the unit vector \( \mathbf{I} \) parallel to the orbital angular momentum \( \mathbf{L} \) of the binary (i.e., \( \mathbf{I} \) is perpendicular to the orbital plane) and the unit vector \( \mathbf{g} \) pointing from the prime focus of the planetary orbit toward its pericenter. We assume \( \mathbf{L} \) to be constant thus neglecting possibility of tidal coupling between \( \mathbf{L} \) and planetary and stellar spins, and gravitational effects of any companions. We also assume that orientation of the orbital ellipse in space is fixed, i.e., \( \mathbf{g} \) is constant too. In doing this we disregard precession of the planetary orbit caused by the general relativity, stellar oblateness, and so on. We can do this because observed changes of the orbital configuration caused by different physical mechanisms add up linearly, and here we want to concentrate on just one of them.

Orbital plane crosses the plane of the sky along the line of nodes and we introduce vector \( \mathbf{m} = (\mathbf{I} \times \mathbf{n})/\sin i \) along this line (\( \mathbf{m} = 1 \)), where \( i \) is the observed inclination of the planetary orbit given by \( \sin i = |\mathbf{n} \times \mathbf{l}| \). If \( \omega \) is the angle between \( \mathbf{m} \) and \( \mathbf{g} \) in the direction of planetary motion—the argument of pericenter—then at any moment of time

\[
\mathbf{g} = \frac{\cos \omega}{\sin i} (\mathbf{l} \times \mathbf{n}) - \frac{\sin \omega}{\sin i} [\mathbf{n} - (\mathbf{n} \cdot \mathbf{l})].
\]

(2)

Differentiating relation \( \cos i = (\mathbf{n} \cdot \mathbf{l}) \) with respect to time, we find using Equation (1)

\[
i_\mu = -\frac{(\mathbf{m} \cdot \mathbf{I})}{\sin i} = -\mu \cos \beta,
\]

(3)

where \( \mu = |\mu| \) and \( \beta \) is the angle in the plane of the sky between \( \mu \) and vector \( \mathbf{I} - \mathbf{n}(\mathbf{l} \cdot \mathbf{n}) \)—the projection of \( \mathbf{I} \) on the sky plane. We use the notation \( f \equiv df/dt \) throughout the text. Differentiating with respect to time relation \( \cos \omega = (\mathbf{g} \cdot \mathbf{m}) = (\mathbf{g} \cdot (\mathbf{l} \times \mathbf{n}))/\sin i \) and using Equations (1)–(3) we find (Kopeikin 1996)

\[
\dot{\omega}_\mu = -\frac{(\mathbf{m} \cdot (\mathbf{l} \times \mathbf{n}))}{\sin^2 i} = -\frac{\mu \sin \beta}{\sin i}.
\]

(4)

Equations (3) and (4) fully determine the evolution of the observed orientation of planetary orbit in space caused by the stellar proper motion.

3. VARIATION OF THE TRANSIT DURATION

Planet transit is characterized by an impact parameter \( p = r_a \cos i/R_p \)—minimum separation between the planetary trajectory and the stellar disk center projected onto the plane of the sky, in units of stellar radius \( R_p \). Here, \( r_a \) is the value of the spatial separation \( r \) between the planet and the center of the star at transit midpoint—moment of time when the projected separation between the planet and the center of the stellar disk is minimized. In general

\[
r = \frac{a(1 - e^2)}{1 + e \cos f},
\]

(5)

where \( f \) is the true anomaly counted from the line of apsides. Transit midpoint occurs at \( f = \pi/2 - \omega \), so that

\[
p = \frac{a \cos i(1 - e^2)}{R_p(1 + e \sin \omega)}.
\]

(6)

Clearly, for the transit to occur one needs \( p < 1 + R_p/R_p \), where \( R_p \) is the planetary radius, which translates into

\[
\cos i < \frac{R_p + R_p e \sin \omega}{a(1 - e^2)}.
\]

(7)

Transit duration calculated as the time between the crossings of the edge of the stellar disk by the center of the planetary disk is (see, e.g., Tingley & Sackett 2005)

\[
T_{tr} = \frac{2R_p(1 - p^2)^{1/2}}{v_{\varphi, tr}} = \frac{2(1 - e^2)^{1/2}}{n(1 + e \sin \omega)} \frac{R_p(1 - p^2)^{1/2}}{a},
\]

(8)

where \( v_{\varphi, tr} = n(a + e \sin \omega)/(1 - e^2)^{1/2} \) is the value of the azimuthal (transverse) component of planetary velocity at the transit midpoint and \( n = 2\pi/P \) is the planetary mean motion. In deriving Equation (8), we have neglected the curvature of projected planetary trajectory and the variation of planetary speed during the transit—this introduces only a small error.

Given that \( p \) and \( \omega \) in Equation (8) evolve as a result of stellar proper motion, it is obvious that \( T_{tr} \) would not remain constant. Differentiating expression (8) with respect to time one finds

\[
\dot{T}_{tr} = -\frac{T_{tr}}{1 + e \sin \omega} \times \left[ e \omega \cos \omega - g \left( i \sin i + \dot{\omega} \cos i \frac{e \cos \omega}{1 + e \sin \omega} \right) \right],
\]

(9)

where

\[
g = \frac{a}{R_p} - \frac{p}{1 - p^2}.
\]

(10)

In Equation (9), the first term in brackets describes the variation of \( T_{tr} \) caused by the change of \( v_{\varphi, tr} \) due to the precession of the orbital ellipse while the second and the third terms embody the variation of transit geometry (change of impact parameter \( p \)) caused by the change of the inclination of the orbital plane and the precession of the orbital ellipse, respectively. The third term is normally much smaller than the second one because \( \cos i \ll 1 \) in transiting systems. Note that \( \dot{\omega} \) affects \( T_{tr} \) only if the planetary orbit is eccentric, while \( i \) causes variation of \( T_{tr} \) even for circular orbits.

Expression for \( T_{tr} \) caused by the proper motion can be written with the aid of Equations (3), (4), and (9) as

\[
T_{tr, \mu} = \frac{T_{tr} \mu \sin \beta}{1 + e \sin \omega} \times \left[ \frac{e \cos \omega}{\sin i} - g \left( \frac{\sin i}{\tan \beta} + \frac{\cos i}{\sin i} \frac{e \cos \omega}{1 + e \sin \omega} \right) \right],
\]

(11)

This equation explicitly shows how \( T_{tr} \) varies as a function of the absolute value of the stellar proper motion \( \mu \) and the orientation of \( \mu \) with respect to the projection of the orbital angular momentum onto the plane of the sky—angle \( \beta \).

4. VARIATION OF THE PERIOD BETWEEN TRANSITS

Apparent precession of the planetary orbit caused by the proper motion makes the observed period between the consecutive planetary transits \( P_{obs, \mu} \) different from the true orbital period \( P_0 \equiv 2\pi(a^3/GM) \), where \( M \) is the total mass of the

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2 This definition differs from that usually adopted in the literature (which assumes that transit lasts while stellar and planetary disks have at least some overlap) but this does not affect our results significantly.
system. Indeed, suppose that we try to measure the time lag between the successive inferior conjunctions of the planet. Every orbital period precession at a rate \( \dot{\omega}_p \) turns the orbit by an angle \( \Delta \psi = P_0 \dot{\psi} \) with respect to our line of sight which gets reflected in the length of the time interval between successive conjunctions. The extra time it takes a planet to cover this additional angle is \( \Delta \psi_0 = \Delta \psi/\dot{\psi} \), where \( \dot{\psi} = v_{p,fr}/r_{fr} \) is the angular frequency of the planet at the point of conjunction. Using Equation (5) and expression for \( v_{p,fr} \), we can write the deviation of \( P_{\text{obs,} \mu} \) from \( P_0 \) as

\[
\frac{\Delta P_{\text{obs,} \mu}}{P_0} = \frac{P_{\text{obs,} \mu} - P_0}{P_0} = -\frac{\dot{\omega}_0}{n} \left( 1 - e^2 \right)^{3/2}. \tag{12}
\]

Note that \( \Delta P_{\text{obs,} \mu} \) is nonzero even in the case of circular orbits.

Apart from \( \Delta P_{\omega} \) which owes its existence to the apparent re-orientation of the planetary orbit there is another contribution to \( P_{\text{obs,} \mu} \) related to the systematic motion: the distance to the planetary system changes, which because of the finite speed of light gives rise to a special relativistic contribution \( \Delta P_{\text{rel}} \) given by

\[
\frac{\Delta P_{\text{rel}}}{P^2} = \frac{v_r}{c}, \tag{13}
\]

where \( v_r \) is the line-of-sight velocity of the system (positive for systems moving away from us).

In practice, one cannot measure \( P_{\text{obs}} - P_0 \) directly since \( P_0 \) is not known a priori. However, one might try to measure the variation of \( P_{\text{obs}} \) over an extended period of time to get a handle on the precessional effects.

It should be kept in mind, though, that in general planetary orbit would also precess (even in the absence of the proper motion). Previously, Heyl & Gladman (2007) have shown that

\[
\dot{\omega}_0 = \frac{2\pi}{\left[ 1 - e^2 \right] \left( 1 + e \sin \omega \right)^2} \left[ 2(\omega_0)^2 - e \cos \omega \right], \tag{15}
\]

with \( D \) being the distance to the planetary system and \( v_r = \mu D \) being its transverse velocity.

Contribution \( \dot{P}_{\text{obs,} \mu} \) in Equation (14) represents period variation due to \( \dot{\omega}_0 \) — precession caused by anything except the proper motion. Previously, Heyl & Gladman (2007) and Pál & Kocsis (2008) have shown that

\[
\dot{P}_{\text{obs,} \mu} = \frac{4\pi (\dot{\omega}_0^2 e \cos \omega)}{n^2 (1 + e \sin \omega)^3}. \tag{17}
\]

This expression assumes that physical mechanisms causing precession \( \dot{\omega}_0 \) do not vary in time. Then, clearly, for circular orbits \( P_{\text{obs,} \mu} = 0 \) leaving only the proper motion effects as a cause of orbital period variation.

Note that in the case of apparent precession caused by the systemic motion we must take \( \dot{\omega}_0 \) caused by nonzero \( \mu \) into account, see Equation (15). Indeed, one can easily show using results of Section 2 that \( \dot{\omega}_0 \sim \mu^2 \sim (\omega_0)^2 \), so that all terms in Equation (15) for \( \dot{P}_{\text{obs,} \mu} \) must be retained. Also note that in general, \( \dot{P}_{\omega} \) is much smaller than \( \dot{T}_w \) because \( \dot{T}_w \) is a linear function of the small parameter \( P \omega \), while \( \dot{P}_{\omega} \) is quadratic.

The timing contribution \( \dot{P}_{\text{Shk}} \), which to the best of our knowledge has never been highlighted in the context of planetary transit timing, is identical to the so-called Shklovskii effect well known from pulsar timing (Shklovskii 1970): radial motion of the system changes the observed orbital period via the Doppler effect, but the radial component of the velocity (and the Doppler factor) varies if there is a nonzero transverse component of the systemic velocity, leading to nonzero \( P_{\text{obs}} \). This contribution to \( P_{\text{obs}} \) is always positive since spatial motion of the planetary system always increases \( v_r \). In Section 5, we demonstrate that in many transiting systems Shklovskii effect dominates \( P_{\text{obs}} \).

5. DISCUSSION

Here, we compare the effects caused by the proper motion with other timing contributions and discuss their observability in different types of systems.

5.1. Period Between Transits

As a fiducial system, we will take a star located 100 pc away from the Sun and moving with transverse velocity 30 km s\(^{-1}\). Such a system has proper motion \( \mu \approx 60 \) mas yr\(^{-1}\) resulting in \( i_\mu, \dot{\omega}_0 \approx 2 \times 10^{-7} \) yr\(^{-1}\) for \( \beta = 45^\circ \) and \( i = 90^\circ \).

Timescale on which planetary orbit changes its orientation is \( \sim \mu^{-1} \sim 5 \times 10^6 \) yr. We can compare \( \dot{\omega}_0 \) to the general relativistic periapsis precession rate

\[
\dot{\omega}_{GR} = \frac{3n}{1 - e^2} \left( \frac{na^2}{c} \right)^2 \tag{20}
\]

and to the rate of precession due to the rotation-induced stellar quadrupole (Miralda-Escudé 2002)

\[
\dot{\omega}_{GR} \approx n \frac{3J_2 R^2}{2a^2} \approx 9 \times 10^{-6} \left( \frac{J_2}{10^{-6}} \right) \left( \frac{10R_\odot}{a} \right)^{7/2} \text{ yr}^{-1}, \tag{19}
\]

where \( J_2 \) is the dimensionless measure of the stellar quadrupole moment (its typical value for the solar type stars is \( J_2 \approx 10^{-6} \)), and we assumed stellar mass \( M_* = M_\odot \) and stellar radius \( R_* = R_\odot \).

These estimates clearly indicate that for solar type stars with short-period \( (P = 3 - 4) \) planets \( \dot{\omega}_GR \gg \dot{\omega}_GR \gg i_\mu, \dot{\omega}_0 \). Plugging expression (18) into Equation (15), we find that a planetary system with \( M_* = M_\odot \) and \( e = 0.1 \) and \( \omega = 45^\circ \) should exhibit

\[
\dot{P}_{\text{obs,} \mu} = -\frac{36\pi e \cos \omega}{(1 - e^2)\left(1 + e \sin \omega\right)^{3/2}} \left( \frac{na}{c} \right)^4 \tag{20}
\]

Given that \( \dot{\omega}_0 \ll \dot{\omega}_{GR} \), it is clear that \( \dot{P}_{\text{obs,} \mu} \ll \dot{P}_{\text{obs,} \mu} \) so that the re-orientation of the planetary orbit caused by the stellar proper
motion does not noticeably affect $\dot{P}_\text{mas}$ (the same is true for the precession caused by the stellar quadrupole since $\dot{\omega}_S \ll \dot{\omega}_\text{GR}$).

However, this does not mean that one can just ignore the effect of the proper motion on $P_{\text{obs}}$; proper motion also affects $P_{\text{obs}}$ via the Sklovskii effect and the magnitude of this contribution

$$P_{\text{SK}} = 9.6 \left( \frac{v_t}{30 \text{ km s}^{-1}} \right)^2 \frac{100}{D} \left( \frac{a}{10 R_\odot} \right)^{3/2} \mu\text{s yr}^{-1}$$

$$= 20 \left( \frac{\mu}{100 \text{ mas yr}^{-1}} \right)^2 \frac{D}{100 \text{ pc}} \frac{P}{3 \text{ d}} \mu\text{s yr}^{-1} \tag{21}$$

may be comparable to $P_{\text{mas}}$. Clearly, $P_{\text{SK}}$ can be quite important even for tight, eccentric systems for which one would normally expect $P_{\text{mas}}$ to dominate.

One also has to keep in mind that the majority of short-period transiting systems have eccentricities consistent with zero. In such systems with circular orbits $P_{\text{mas}}$ and $P_{\text{SK}}$ vanish leaving Sklovskii effect as the only source of non-zero $P_{\text{obs}}$ at the level of tens of $\mu$ s per year. In Table 1, we have summarized the properties of observed transiting systems (supplemented with two artificial systems Sys-1 and Sys-2 with the goal of illustrating transit-timing effects in long-period systems) in which proper motion effects are particularly pronounced (namely, $P_{\text{obs}} > 10 \text{ ms yr}^{-1}$), while in Table 2 we display the values of various timing contributions in these systems, including $P_{\text{mas}}$ and $P_{\text{SK}}$. From Table 2 one can see that in some nearby high proper motion systems like GJ436, $P_{\text{SK}}$ is a good fraction of ms yr$^{-1}$. Such a high rate of period change significantly exceeds $P_{\text{mas}}$ and may in principle be measurable on a timescale of tens of years assuming observing parameters typical for the Kepler photometric mission (Miralda-Escudé 2002; Jordán & Bakos 2008).

### 5.2. Duration of Transits

Variation of the transit duration $T_t$ presents another way of detecting proper motion effects in isolated star–planet systems, as described in Section 3. Assuming that all angle-dependent factors in Equation (11) are of order unity one finds

$$\dot{T}_{t,\mu} \approx g T_t \mu$$

$$\approx 50 \frac{\mu}{100 \text{ mas yr}^{-1}} \frac{a/R_\odot}{10^{-4} \text{ hr}} T_t \text{ ms yr}^{-1}, \tag{22}$$

where in evaluating $g$ we have assumed $p = 0.5$. Thus, a typical nearby exoplanetary system indeed exhibits $\dot{T}_{t,\mu} \gg P_{\text{obs}}$. A specific value of $\dot{T}_{t,\mu}$ for a particular exoplanetary system depends not only on $\mu$ but also (sinusoidally) on the angle $\beta$ between $\mu$ and the line of nodes. The maximum possible value of $\dot{T}_{t,\mu}$ for several representative systems can be found in Table 2.

At the same time, for $M_\ast = M_\odot, e = 0.1$, and $\omega = 45^\circ$ one finds from Equation (9) the following value of $T_t$ due to the general relativity:

$$\dot{T}_{t,\text{GR}} \approx 240 \left( \frac{10 R_\odot}{a} \right)^{5/2} \frac{T_t}{4 \text{ hr}} \text{ ms yr}^{-1}. \tag{23}$$

This is not much larger than $\dot{T}_{t,\mu}$ and in some high proper motion systems $\dot{T}_{t,\mu}$ may even dominate. The best example is GJ436: because system is very compact general relativity provides $T_{t,\text{GR}} \approx -0.2 \text{ s yr}^{-1}$, but the very high proper motion of the system ($\mu \approx 1.2 \text{ mas yr}^{-1}$) gives rise to max $|\dot{T}_{t,\mu}| \approx 0.6 \text{ s yr}^{-1}$. Thus, in general one cannot simply ascribe all $\dot{T}_t$ measured in eccentric systems to the general relativity—some fraction of $\dot{T}_t$ can also be contributed by the proper motion. In systems with circular orbits $\dot{T}_{t,\text{GR}} = 0$.

Given that $\omega_{t,\mu} \ll \omega_{t,\text{GR}}$, the magnitude of the effect of the proper motion on $T_t$ may seem disproportionately large compared to $\dot{T}_{t,\text{GR}}$. The reason for this lies in the amplifying factor $g$ in Equation (9) which propagates into $\dot{T}_{t,\mu}$, see Equation (11). According to the Equation (6), the magnitude of $g$ is determined by the transit impact parameter $p$ and the ratio $a/R_\odot$, which is usually quite large, $\sim 10$ even for rather short-period ($P = 3–4 \text{ d}$) systems. For grazing transits (such as those occurring in GJ436, see Table 2), when $1 - p < 1, g$ gets additionally boosted up because then $\dot{T}_t$ becomes a very sensitive function of $p$ and $i$, see Ribas et al. (2008). At the same time factor $g$ $\geq 1$ does not greatly affect $\dot{T}_{t,\text{GR}}$ since for precession induced by the general relativity $\dot{i} = 0$ and $g$ enters the expression for $\dot{T}_{t,\text{GR}}$ only in combination $g \cos i$, while $\cos i \ll 1$ in transiting systems ($\cos i \lesssim R_\ast/a$ so that $g \cos i \approx 1$, see Equations (6) and (10)). This explains why $\dot{T}_{t,\text{GR}} \approx \dot{T}_{t,\mu}$ even though $\dot{\omega}_{t,\text{GR}} \gg \dot{\omega}_{t,\mu}$.

Inclination of the planetary orbit with respect to our line of sight may also change because of the spin-induced quadrupole, if the stellar spin axis is misaligned with the orbital angular momentum vector. The spin-induced $T_{t,\text{S}}$ is amplified by factor $g$ in a way analogous to the amplification of $\dot{T}_{t,\mu}$. Given that in some systems $\dot{\omega}_S$ can be 1–2 orders of magnitude larger than $\dot{\omega}_{t,\mu}$ (see Table 2 where $\dot{\omega}_S$ is computed for $J_2 = 10^{-7}$) one may expect $T_{t,\text{S}} \gg T_{t,\text{GR}}$ in these systems. However, in reality it will often be the case that $i_\ast \ll i_S$ since it can be demonstrated that $i_\ast = C i_S \sin \lambda$ (Lai et al. 1995), where $C \sim 1$ is the angle-dependent factor and $\lambda$ is the angle between the stellar spin axis and the orbital angular momentum vector. Misalignment angle $\lambda$ has been measured in several systems via the Rossiter–McLaughlin effect (Rossiter 1924; McLaughlin 1924) and in the majority of measured cases $\lambda$ is close to zero, as expected from the planet formation theories. Among the systems in Table 1 for which $\lambda$ has been measured this angle was found to be small in HD189733 ($\lambda = 1^\circ \pm 1^\circ$; Winn et al. 2006) and HAT-P-1 ($\lambda = 3^\circ \pm 2^\circ$; Johnson et al. 2008) while in HD17156 misalignment may be significant ($\lambda = 62^\circ \pm 25^\circ$, Narita et al. 2008), although Cochran et al. (2008) have found $\lambda = 9^\circ \pm 9^\circ$ in this system. In HD189733 $i_\ast$ end up being $\ll i_\mu$ so that $T_{t,\text{S}}$ likely makes negligible contribution to $T_t$, which should be dominated by the proper motion. In HAT-P-1, we find $i_\ast \approx i_\mu$ and $T_{t,\text{S}} \approx \max|\dot{T}_{t,\mu}|$ with $\dot{T}_{t,\text{GR}}$ providing a non-negligible contribution. Finally, in HD17156, if we adopt a larger value of $\lambda$ found by Narita et al. (2008), $\sin \lambda \sim 1$ but $i_\ast$ is still comparable to $i_\mu$ because the semimajor axis of the system is quite large which greatly reduces $\omega_S$. As a result, $T_{t,\text{S}} \approx \max|\dot{T}_{t,\mu}|$ in this system, and both are somewhat smaller than $\dot{T}_{t,\text{GR}}$. Thus, at least in the systems presented in Table 1 the spin-induced quadrupole orbital precession does not strongly exceed the proper motion effects in timing of transit duration.

Note that the tidal deformations induced on the star and the planet by each other (Lai et al. 1995; Wu & Goldreich 2001) affect $T_t$ in a way different from that of the spin-induced quadrupole—similar to the general relativity, the tidal bulges do

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3 Thus, monitoring systems with large $p$ tend to increase the chances of measuring $T_t$. 

4 $T_{t,\nu}$ is determined by the transit impact parameter

$$T_{t,\nu} = \frac{T_t}{\cos i}$$

and in some high proper motion systems $T_{t,\nu}$ may be comparable to $T_{t,\mu}$.
not generate nonzero $\dot{i}$. Given that tidal $\dot{\omega}$ is typically smaller than $\dot{\omega}_{\text{GR}}$ (Jordán & Bakos 2008), we may conclude that tidally induced $T_{tr}$ is lower than $T_{tr,\text{GR}}$ and is thus $\lesssim T_{tr,\mu}$.

### 5.3. Proper Motion Effects in Long-Period Systems

It is obvious from the preceding discussion that the proper motion can have an appreciable (if not dominant in many cases) effect on transit timing in the short-period systems. This statement becomes much more robust when we go to systems with wider separations. It is obvious from Equations (18)–(20), and (23) that $\dot{\omega}_{\text{GR}}$, $\dot{\omega}_S$, and all contributions to $\dot{P}_{\text{obs}}$ and $T_{tr}$ caused by the effects of the general relativity, stellar quadrupole and tidal deformations are rapidly decreasing functions of $a$. At the same time, $\dot{\omega}_{\mu}$, $\dot{i}_{\mu}$ are independent of $a$ while both $\dot{P}_{\text{Shk}}$ and $T_{tr}$ increase quite rapidly with $a$, see Equations (21) and (22). This means that the proper motion should completely dominate transit-timing variations in isolated (i.e., containing no other planets) wide-separation systems. For example, a transiting planet in a 30 d orbit around a solar type star would exhibit $\dot{P}_{a,\text{GR}} \approx 0.6 \mu s\,\text{yr}^{-1}$ and $\dot{T}_{tr,\text{GR}} \approx 7.5\;\text{mas yr}^{-1}$ if $e = 0.1$ and $T_{tr} = 4\;\text{hr}$. If this system is located 100 pc away from the Sun and has proper motion $\mu = 100\;\text{mas yr}^{-1}$ then one finds $\dot{P}_{\text{Shk}} \approx 200\mu s\,\text{yr}^{-1}$ and $\dot{T}_{tr,\mu} \approx 200\;\text{mas yr}^{-1}$, so that both $\dot{P}_{a,\text{GR}} < \dot{P}_{\text{Shk}}$ and $\dot{T}_{tr,\mu} < \dot{T}_{tr,\mu}$. To additionally illustrate the importance of proper motion for wide-separation systems, we introduce two artificial systems (Sys-1 and Sys-2) in Table 1 and calculate their timing parameters in Table 2. Such wide-separation systems are one of the primary goals of photometric missions like *Kepler*. It is clear that if such systems are found to exhibit transit-timing variations then these variations must be caused by the systemic proper motion, provided that the influence of possible additional companions is proven to be negligible. In this case according to Equation (11), the measurement of $T_{\mu}$ would serve as a measurement of angle $\beta$ giving us information on the full three-dimensional orientation of the transiting system. However, one must remember that if additional planets in external orbits are present in these systems then their influence may not be disregarded (Ribas et al. 2008) since their effect on transit timing grows with a faster than $T_{tr,\mu}$ and $\dot{P}_{\text{Shk}}$ do.

Note that the rapid increase of $\dot{P}_{\text{Shk}}$ and $\dot{T}_{tr,\mu}$ with $a$ does not immediately imply that their actual detection is facilitated as $P$ increases. Even though the timing signal increases with $P$, the number of transits, which determines the timing error, decreases as $P^{-1}$ for a given time interval over which the system is being monitored. Using the results of Ford et al. (2008) and Heyl & Gladman (2007) on transit-timing precision, we find that the time $\Pi_P$ one needs to monitor the transiting system to detect $\dot{P}_{\text{obs}}$ induced by the proper motion scales as $^4 \Pi_P \propto P^{4/15}$, i.e., it increases with $P$ but not very rapidly; it takes 3.6 times longer for $P = 1\;\text{yr}$ system to get the same $\Sigma/N$ for $\dot{P}_{\text{Shk}}$ due to proper motion as for the 3 d system. The uncertainty in $T_{\mu}$ decreases with the number of observed transits slower than the uncertainty in $T_{\text{obs}}$. As a result, the time $\Pi_{\mu}$ one needs to monitor the transiting system to detect $\dot{T}_{tr}$ caused by the proper motion decreases with $P$ as $^5 \Pi_{\mu} \propto P^{-2/19}$. Thus, it is easier to measure $\dot{T}_{tr,\mu}$ in wide-separation transit systems.

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4 This dependence is found by equating $\dot{P}_{\text{Shk}} \Pi_P$ to $\sigma_{\dot{P}}/N$ where $N = \Pi_P/P$ is the number of observed transits and $\sigma_{\dot{P}}$ is the uncertainty in measurement of $\dot{T}_{tr}$, which is given by Equation (6) of Ford et al. (2008).

5 This follows from equating $\dot{T}_{tr,\mu} \Pi_{\mu}$ to $\sigma_{\dot{T}}$ given by Equation (6) of Ford et al. (2008).
5.4. Measurability of the Proper Motion Effects

Based on the results of Ford et al. (2008), it was estimated by Jordán & Bakos (2008) that the Kepler mission should be able to achieve a timing precision of $\sim 1.5$ s in 1 yr observation of a 12th magnitude solar type star transited by a Jupiter-like planet with $P = 5$ d. One can easily deduce from this that a 3$\sigma$ detection of $\dot{T}_{\text{tr,GR}} = 100$ ms yr$^{-1}$ (which is not unreasonable for proper motion) should take $\Pi_{\text{tr}} \approx 10$ yr of observations. Measurement error of $P_{\text{obs}}$ drops very rapidly with time but it would still take $\Pi_{\text{tr}} \approx 70$ yr to achieve a 3$\sigma$ detection of $P_{\text{obs}} = 100$ $\mu$s yr$^{-1}$ caused by the Shklovskii effect. Thus, while one might hope to measure $T_{\text{tr,GR}}$ in some nearby, high proper motion systems (like GJ436) on timescale of $\sim 10$ yr, the measurement of $P_{\text{obs}}$ would likely require next generation facilities with photometric precision much higher than that of the Kepler mission.

In systems with low proper motion, $T_{\text{tr,GR}}$ can be viewed as an irreducible systematic uncertainty to which quantities like $T_{\text{tr,GR}}$ can be measured. This is because even if $\mu$ is precisely known one still does not know a priori the angle $\beta$ (but see below) which determines $T_{\text{tr,GR}}$. In reality, $\mu$ itself is going to have some measurement error increasing the systematic timing uncertainty. Thus, proper motion limits to some extent our ability to interpret the measurement of $T_{\text{tr}}$ in terms of the physical parameters of the system (e.g., $J_2$, etc.). Measurement of $P_{\text{obs}}$ does not suffer from this uncertainty since $P_{\text{sh}}$ is independent of $\beta$ and can thus be fully accounted for once $\mu$ is known from astrometric measurements.

5.5. Effect of the Earth Motion

Previous discussion has implicitly assumed that a transiting system moves at a constant speed with respect to observer. In reality, observer is located at Earth, which orbits the Sun. Thus, while one might hope to measure $T_{\text{tr,GR}}$ in some nearby, high proper motion systems (like GJ436) on timescale of $\sim 10$ yr, the measurement of $P_{\text{obs}}$ would likely require next generation facilities with photometric precision much higher than that of the Kepler mission.

In systems with low proper motion, $T_{\text{tr,GR}}$ can be viewed as an irreducible systematic uncertainty to which quantities like $T_{\text{tr,GR}}$ can be measured. This is because even if $\mu$ is precisely known one still does not know a priori the angle $\beta$ (but see below) which determines $T_{\text{tr,GR}}$. In reality, $\mu$ itself is going to have some measurement error increasing the systematic timing uncertainty. Thus, proper motion limits to some extent our ability to interpret the measurement of $T_{\text{tr}}$ in terms of the physical parameters of the system (e.g., $J_2$, etc.). Measurement of $P_{\text{obs}}$ does not suffer from this uncertainty since $P_{\text{sh}}$ is independent of $\beta$ and can thus be fully accounted for once $\mu$ is known from astrometric measurements.

The maximum magnitude of this apparent proper motion is $\mu_E \approx \pi E / D \approx 60$ mas yr$^{-1}$, where $\pi E \approx 30$ km s$^{-1}$. According to Equations (11) and (22), $\mu_E$ gives rise to periodically varying $T_{\text{tr,GR}}$ with an amplitude dependent on the orientation of the orbital plane of the transiting system with respect to the ecliptic, potentially providing a method of measuring angle $\beta$. The maximum possible value of such timing signal is about

$$T_{\text{tr,GR}} \sim 30 \frac{\mu}{100 \text{ mas yr}^{-1}} \frac{a/R_\star}{10} \frac{T_{\text{gr}}}{4 \text{ hr}} \text{ ms yr}^{-1}. \quad (24)$$

It is also obvious that the terrestrial orbital motion produces a periodic contribution to $P_{\text{obs}}$, even if the transiting system has zero intrinsic proper motion.

Such annual variations in $\dot{T}_{\text{tr}}$ and $\dot{P}_{\text{obs}}$ can arise only as a result of the proper motion effects. One can hope to measure them on decade-long baselines by properly combining the data on transit duration measured at different orbital phases of the Earth. If such variations are detected then this periodic part of the timing signal can be used to constrain angle $\beta$ (another way of measuring $\beta$ was proposed by van Belle 2008), allowing one to remove the aforementioned systematic uncertainty in measuring other timing contributions.

6. CONCLUSIONS

We investigated the effect of the proper motion on timing of transiting exoplanetary systems in which the gravitational effect of other possible companions can be neglected. Proper motion re-orient the planetary orbit with respect to our line of sight and changes the distance to the system. Short-period transiting systems having proper motion at the level of $100$ mas yr$^{-1}$ should exhibit variation of the transit duration at the level of $\sim 100$ ms yr$^{-1}$, which may be comparable to or exceed the timing signatures produced by the general relativity or stellar quadrupole and which should not be hard to detect. Proper motion also causes variation of the observed orbital period through the Shklovskii effect which dominates $\dot{P}$ for high proper motion systems. Orbital motion of the Earth around the Sun gives rise to periodically varying transit-timing signal even in systems having zero intrinsic proper motion. Timing effects induced by the proper motion become especially important in systems with zero eccentricity and in wide-separation systems with periods longer than a month which should be discovered by Kepler.

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