In this semiclassical picture, the ionization rate of the electron is only determined by the instantaneous amplitude of the laser field and the final photoelectron momentum is equal to the vector potential of the laser field at the ionization moment with opposite sign if the influence of the ionic potential is ignored, however, this correspondence, or the accuracy of the semiclassical approximation of the photoionization process, has never been rigorously checked, or in other words, how accurate the semiclassical picture is still remains an open question. Recently, an attosecond angular streaking technique, also dubbed as “attoclock” technique, has been developed to investigate the temporal dynamics of atoms and molecules in intense laser fields [7][13]. This technique, different from the conventional attosecond measurement which relies on attosecond pulses generated using high-order harmonic generation process and is very technically demanding, uses the rotating electric-field vector of an intense close-to-circularly polarized pulse to deflect photo-ionized electrons in the radial spatial direction. Then the instant of ionization is mapped to the final angle of the momentum vector in the polarization plane according to the semiclassical picture. When a few-cycle pulse is applied, a comparison between the peaks of the measured photoelectron angular distribution and the simpleman’s prediction shows an offset angle which has caused much
debate on its underlying mechanism \cite{8 10 11 13 14}. Ionic Coulomb potential, tunneling time delay and nonadiabatic effect have been proposed to explain this offset angle. By comparing the experimental data with the semiclassical calculation including the Coulomb potential, Eckel et al. place an intensity-averaged upper limit of 12 attoseconds on the tunneling delay time in strong field ionization of a helium atom \cite{8}, Boge et al. find that the nonadiabatic effect is unimportant \cite{11}. However, by solving a three-dimensional time-dependent Schrödinger equation, Ivanov et al. \cite{14} give an opposite view against the calculations using the semiclassical model \cite{11}. Recently, Torlina et al. show that the offset angle can be attributed to the Coulomb potential effect and confirm the zero tunneling time delay by theoretical calculation \cite{15}.

It is noteworthy that the principle of the attosecond streaking technique, viz., the angle of the final momentum vector corresponds to the instant of the tunneling ionization, is based on the semiclassical picture. Therefore, prior to consideration of the Coulomb potential and nonadiabatic effects etc. \cite{8 10 13 24}, one needs to rigorously check the principle of the attoclock technique, or the accuracy of the semiclassical picture in description of the photoionization process which is an intrinsic quantum process.

In this paper, we use a Wigner-distribution-like (WDL) function \cite{24 29} to investigate the ATI process of an atom in few-cycle laser pulses in the framework of the strong-field approximation (SFA). It is worthwhile mentioning that the SFA is accurate for a short-range system, e.g., negative ion, so it is suitable to investigate the validity of the semiclassical picture or the principle of the original attoclock technique scheme in which the Coulomb potential is ignored in the first place. The WDL function enables us to calculate the time-emission angle distribution, angular distribution and ionization time distribution of the ATI process of atom in elliptically and circularly polarized laser pulses with different carrier-envelop phases (CEPs). Then we explicitly and rigorously check the validity and accuracy of the attosecond angular streaking technique by comparing the quantum distributions with results given by the semiclassical simulation. Moreover, we also calculate the time-energy distribution and ionization time distribution of atoms in linearly polarized laser pulses and compare with semiclassical simulations to further systematically check the accuracy of the semiclassical picture in description of the photoionization process. Furthermore, we consider the non-adiabatic effect by performing calculation with PPT theory. The result shows that the non-adiabatic effect indeed reduces the discrepancy between semiclassical and quantum distributions and improves the accuracy of the attoclock technique.

THEORY AND NUMERICAL METHODS

The Wigner-distribution-like function derived from the strong-field approximation model is defined as (see \cite{24} for more details):

\[
f(t, p^2 2) = \frac{1}{\pi} \int_{-\infty}^{\infty} S^*(t + t') S'(t - t') e^{-2i \frac{p^2 2}{m} t'} dt'. \quad (1)
\]

where \( S' \) is given by

\[
S' = \frac{\sqrt{2\pi} \partial \phi_o(q)}{\sqrt{\nu}} \cdot E(t) \times \exp \left\{ i \int_{-\infty}^{t} [p \cdot A(\tau) + \frac{A^2(\tau)}{2}] d\tau + i I_p t \right\}. \quad (2)
\]

Here, \( \nu \) is normalization volume, \( E(t) \) the electric field and \( I_p \) the ionization potential of the atom. \( \phi_o(q) \) is the Fourier transform of the atomic ground state \( |\phi_0\rangle \) and \( q \equiv p + A(t) \).

In this paper, we consider a laser pulse with a \( \sin^2 \)-type temporal envelop. The vector potential is given by

\[
A(t) = -\frac{E_0}{\omega} \sin^2 \left( \frac{\omega t}{n} \right) \left[ \cos \frac{\theta}{2} \cos(\omega t + \varphi) e_x - \sin \frac{\theta}{2} \sin(\omega t + \varphi) e_y \right],
\]

where \( E_0 \) is the peak field strength, \( \omega \) the laser frequency, and \( n/2 \) the number of optical cycles contained in the laser pulse. \( \varphi \) is the initial carrier envelop phase (CEP). \( e_x \) and \( e_y \) are the unit vectors along the x and y axes, respectively. \( \epsilon = \cot \frac{\theta}{2} \) is the ellipticity. The major axis is y-axis.

For simplification, here we only consider the distribution of electrons emitted in the polarization plane for a three-dimensional system. For a fixed kinetic energy \( \frac{p^2}{2} \) in the polarization plane, the electron emission angle \( \Theta \) varies from 0 to \( 2\pi \), where \( \Theta \) is the angle between the final momentum of electrons and x axis. From Eq. (2), we can find that \( S' \) is a function of momentum \( p \), which can be written as a function of two scalar variables \( p \) and \( \Theta \) in a two-dimensional model. Here \( p \) is the absolute value of the electron momentum vector. Therefore, the formula of the Wigner-distribution-like function for a two-dimensional system can be written as following:

\[
f(t, p^2 2, \Theta) = \frac{1}{\pi} \int_{-\infty}^{\infty} S^*(t + t', \Theta) S'(t - t', \Theta) e^{-2i \frac{p^2 2}{m} t'} dt'. \quad (4)
\]

The time-angle distribution function is given by

\[
f'(t, \Theta) = \int f(t, p^2 2, \Theta) dt \left( \frac{p^2}{2} \right).
\]

(5)
Then one can find that the ionization probability as functions of time and emission angle can be given by

\[ P(t) = \int f'(t, \Theta) d\Theta. \quad (6) \]

and

\[ W(\Theta) = \int f'(t, \Theta) dt. \quad (7) \]

RESULTS AND DISCUSSIONS

In this paper, we consider H atoms ionized by a 6-cycle elliptically polarized laser field with peak intensity of \(1 \times 10^{14} \text{ W/cm}^2\) and ellipticity \(\epsilon = 0.882\). It is worthwhile mentioning that all integrations are performed numerically in our quantum calculations.

At first, we show the comparison of momentum spectra given by numerical solution of time-dependent Schrödinger equation (TDSE)(for details, see [27, 28]) and the SFA theory with the laser parameters of the CEP \(\varphi = 0.5\pi\) and \(\omega = 0.05691\) a.u. in Fig. 1. The short-range potential of \(V = -\frac{A}{e^{-\kappa r}}\) is adopted with \(\kappa = 1\) in the TDSE method, where the value of \(A\) is taken as 1.90847 in order to make the model potential have the ground state energy of -0.5 a.u. The momentum spectra in Fig. 1 show distinct ATI rings and are in good agreement with each other.

Elliptical and circular polarization

The semiclassical method in this paper is based on the ADK model [29–31] without taking into account the ionic Coulomb potential, which corresponds to the SFA model considered in our WDL calculation. The ionization rate in the semiclassical result is obtained using Eq. (21) of [32] with the Coulomb correction factor removed. The distribution of the initial momentum at the tunnel exit is

\[ P(p_{0\perp}, p_{0\parallel}) \propto \exp\left(-\frac{p_{0\perp}^2}{2E(t)}\right)\exp\left(-\frac{p_{0\parallel}^2}{2E(t)}\right) \quad (E(t) \text{ is the laser field amplitude at time } t, \quad p_{0\parallel} \text{ and } p_{0\perp} \text{ are the initial momenta parallel and vertical to the direction of the laser field polarization, respectively}) \quad [29]. \]

When only initial transverse momentum is considered, we have \(p_{0\parallel} = 0\). For clarity, we label the two different semiclassical methods according to the different initial momentum adopted as ADK\(_{\perp}\) with only the initial transverse momentum considered and ADK\(_{\perp,\parallel}\) with both the initial transverse and longitudinal momenta considered.

Figure 2 shows the calculation of the time-emission angle distribution for H atoms in a 6-cycle laser pulse with the CEP \(\varphi = 0.5\pi\) for different laser frequencies \(\omega = 0.182\) a.u. ((a) and (b)), 0.05691 a.u. ((c) and (d)) and 0.03502 a.u. ((e) and (f)). Peak intensity \(I = 1 \times 10^{14} \text{ W/cm}^2\) and the ellipticity \(\epsilon = 0.882\). Quantum results: (a), (c), and (e) ; calculations of the semiclassical theory with ADK\(_{\perp,\parallel}\): (b), (d), and (f).

Figure 2 shows the calculation of the time-emission angle distribution for H atoms in a 6-cycle laser pulse with the CEP \(\varphi = 0.5\pi\) for three different optical frequencies. The distributions in Fig. 2 are obtained by Eq. (5)(see Figs. 2(a), 2(c), and 2(e)) and by semiclassical theory with ADK\(_{\perp,\parallel}\) (see Figs. 2(b), 2(d), and 2(f)), respectively. For the semiclassical calculation obtained by ADK\(_{\perp,\parallel}\), all distributions with different laser frequencies look similar- two main peaks located at \([\Theta, t] \sim [0(2\pi), 2.75 \text{ o.c.}]\) and \([\pi, 3.25 \text{ o.c.}]\) (o.c. is the abbreviation of optical cycle) and two additional small peaks at \(t \sim 2.3\) and \(3.7\) o.c. (not shown in Fig. 2(d) and 2(f)) which correspond to the secondary peaks of the field amplitude. This is expectable since the ionization is independent of the frequency in the quasistatic tunneling picture. In contrast, for the quantum calculation, the time-emission
angle distributions show a clear transition with decreasing the laser frequency. For $\omega=0.182$ a.u. ($\gamma=3.4$), the distribution (Fig. 2(a)) is similar to the structure of the semiclassical calculation (Fig. 2(b)). When the laser frequency further decreases to $\omega=0.05691$ a.u. ($\gamma=1.07$) and 0.035 a.u. ($\gamma=0.66$), the quantum results move more and more mimic the semiclassical results. Therefore, Fig. 2 demonstrates a transition from the multiphoton regime to the tunneling regime in the elliptically polarized laser field, which is consistent with the case of linearly polarized laser fields [25]. It is worthwhile mentioning that the time-emission angle distribution is also calculated using ADK$_\perp$ method, however, the width is much narrower than the quantum result and is hardly dependent on the laser frequency (not shown here), indicating that the initial longitudinal momentum is an important non-adiabatic effect even in the tunneling regime.

To investigate quantitatively the accuracy of the semiclassical theory and validity of the attoclock technique, we then calculate the angular distribution $W(\Theta)$ by integrating the time-emission angle distribution over time $t$ [33] and compare the quantum calculations with the semiclassical ones (see Fig. 3). All the angular distributions calculated by the semiclassical model ADK$_\perp$ (dashed dotted lines) show a double-peak structure, which is symmetrical with respect to the angle $\Theta=1.5\pi$, in accordance with those shown in Figs. 2(b), 2(d), and 2(f). However, the widths of the angular distributions become progressively broader with the increasing frequency. We also give the results obtained by ADK$_\parallel$ (dotted lines), which also display a symmetric double-peak. The distinct feature different from the distribution gained by ADK$_\perp$ is that the width of the angular distribution hardly depends on the frequency. By comparing between these two kinds of ADK calculations, we can infer that the initial longitudinal momentum distribution can cause a broader width of the angular distribution, resulting in the difference between these two ADK calculations. This difference becomes larger when the frequency increases.

Similar to Fig. 2, the quantum calculation also clearly shows a transition from the multiphoton regime to the tunneling regime with decreasing laser frequency comparing with the semiclassical calculation. For $\omega=0.182$ a.u., the angular distribution in the quantum calculation shows a double-peak pattern with peak positions close to the semiclassical distribution obtained by ADK$_\perp$ (see Fig. 3(a)). When the frequency decreases further, the quantum distribution becomes gradually close to the semiclassical calculations (see Figs. 3(b) and (c)). However, the widths of the quantum distributions are wider than the semiclassical ones and positions of peaks in the quantum and semiclassical calculations are still noticeable different (Here, we define the difference between the peak positions in the angular distributions calculated by the quantum theory and other methods (e.g. ADK models and non-adiabatic calculation mentioned below) as offset angle denoted by $\Delta \Theta$). The angular distribution gained by TDSE is also given in Fig. 3(b), which almost coincides with the curve of SFA model. The reason that the curves of the angular distributions calculated by TDSE and SFA models do not completely overlap is that the potential used in the TDSE calculation still possesses a finite range. Moreover, the effect of the initial longitudinal momentum of the photoelectron, which is considered as the non-adiabatic effect in the tunneling ionization process [10, 34, 35] can also be clearly seen in Fig. 3.

For the lowest frequency $\omega=0.03502$ a.u., the two different semiclassical calculations (ADK$_\perp$ and ADK$_\parallel$) can hardly be distinguished. For $\omega=0.05691$ a.u., the distribution of ADK$_\perp$ becomes wider than that of ADK$_\parallel$ and is closer to the quantum distribution. The difference between two semiclassical calculations becomes larger when the frequency increases to $\omega=0.182$ a.u., however, the distribution of ADK$_\perp$ becomes even wider than the quantum distribution. This is understandable since it is already in the multiphoton regime ($\gamma=3.4$), the non-adiabatic effect cannot be treated properly by the semiclassical model.

Moreover, we calculate the ionization time distribution $P(t)$ by integrating the time-emission angle distribution over angle $\Theta$ and the results are depicted in Fig. 4. For the semiclassical simulation, the ionization time distributions obtained by both ADK$_\perp$ and ADK$_\parallel$ are the same since the initial momentum distributions in two calcula-
tions are both normalized. Furthermore, it is expected that the ionization probability is only dependent on the electric field strength but independent of the laser frequency. As shown in Fig. 4, the ionization time distributions obtained by the semiclassical methods (black solid line) exhibit a double-peak structure for the time range from $t = 2.5$ to 3 o.c. and are indeed independent of the laser frequency, whose peak positions correspond to the maxima of the electric field strength ($t = 2.76746$ and $3.23254$ o.c.). For the quantum calculation, the distribution varies with the frequency. All ionization time distributions possess a double-peak structure and the widths of the peaks decrease with decreasing laser frequencies, becoming more and more close to the semiclassical results. However, the difference between the positions of peaks in the ionization time distributions calculated by the quantum method and semiclassical model still exists (for more details, see Fig. 5). We define this difference in the ionization time distribution as offset time denoted by $\Delta t$.

It should be noted that, for such wide frequency regime considered here which covers from tunneling to multiphoton regime, a theory including non-adiabatic effect should be used for calculations of both the angular and ionization time distributions. These calculations will be shown in Fig. 6 but here we only show the results of the adiabatic theory for comparison with the quantum result to show clearly approach of the quantum result to the semiclassical calculation with decreasing the frequency.

In order to show the offset angle and offset time more clearly, we zoom in one of two peaks in the angular and ionization time distributions. Figs. 5(a)-5(c) show the angular distributions for $\omega = 0.182$ a.u., $0.0569$ a.u. and $0.03502$ a.u., respectively. As shown in Figs. 5(a)-5(c), it can be clearly seen that the widths of the angular distributions calculated by ADK$_{\perp \parallel}$ (red dashed dotted lines) are broader than those gained by ADK$_\perp$ (green dotted lines) and the peak position for ADK$_{\perp \parallel}$ is a little farther from the quantum peak position comparing with ADK$_\perp$ calculation. These differences between two ADK calculations result from the initial longitudinal momentum as mentioned above. When the frequency decreases, these differences also decrease, which can be attributed to relatively larger momentum of the electron acquired from the laser field with lower frequency comparing with the wavelength-independent initial momentum of the electron at the tunnel exit [29–31]. It is well-known that the semiclassical model is only valid in the quasistatic tunneling regime. Here, we also consider the non-adiabatic effect using PPT theory [36, 37]. The ionization rates for PPT are obtained by Eqs. (3.4)-(3.6) of [36]. The pre-exponential factor is taken from Eq. (59) of [37]. The initial momentum at the tunnel exit is also taken into account in the same way as that in the semiclassical model as mentioned above for an explicit comparison with the semiclassical calculation. The PPT calculations including the only initial transverse momentum and both the initial transversal and longitudinal momenta at tunnel exit are denoted as PPT$_\perp$ and PPT$_{\perp \parallel}$, respectively. From Figs. 5(a)-5(c), we can find that the peak width of PPT$_{\perp \parallel}$ (dashed line) is also broader than that ob-
the multiphoton regime. It should be noted that for quantum theory, which can also be seen clearly in Fig. 4, but also the peak position varies with frequency, which becomes more and more close to those obtained by the semiclassical theory (see Fig. 5(d)) when the frequency decreases. For the PPT theory, although the peak widths of the ionization time distributions for three frequencies are different and become more and more broad with increasing frequencies, the peak positions are the same as those of the semiclassical theory and are independent of the laser frequency, which means that the value of the offset time between the quantum and ADK calculations is the same as that between the quantum and PPT calculations for a given laser frequency. As shown in Fig. 5(d), when the frequency decreases, the ionization time distribution approaches the semiclassical results, which can be attributed to that the non-adiabatic effect is less and less important with decreasing Keldysh parameter.

Figure 6 shows the values of the offset angles (Fig. 6(a)) and offset times (Fig. 6(b)) extracted from Fig. 5. The offset angles or times are positive and negative, corresponding to the left and right peaks exhibited in Figs. 3 and 4, respectively. In order to investigate the correspondence between the offset angle and the offset time, we also give the corresponding offset angle calculated from the offset time $\Delta t$ by the relation $\omega t = \Theta$. Here we define this angular offset as $\Delta \Theta_{\text{time}} = 2\pi \Delta t$ ($\Delta t$ in units of a.u.). Similarly, we define the corresponding offset time transformed from the offset angle $\Delta \Theta$ in the angular distribution as $\Delta t_{\text{angle}}$ ($\Delta t_{\text{angle}} = \frac{\Delta \Theta}{2\pi}$). Fig. 6(a) shows the offset angles $\Delta \Theta$ and $\Delta \Theta_{\text{time}}$ for four different frequencies (the result of $\omega = 0.1$ a.u. is also shown here to exhibit the frequency dependence of the offset time and angle more clearly). One can find that the offset angles $\Delta \Theta$ of ADK$_{\perp}$, ADK$_{\parallel}$, PPT$_{\perp}$ and PPT$_{\parallel}$ are always larger than $\Delta \Theta_{\text{time}}$ for all frequencies except $\Delta \Theta$ of PPT$_{\perp}$ for $\omega = 0.182$ a.u. Moreover, the offset angle of PPT$_{\perp,NZ}$, which is smaller than the other four offset angles as mentioned before, is still larger than $\Delta \Theta_{\text{time}}$ in the tunneling ($\gamma \ll 1$) and nonadiabatic regimes ($\gamma \approx 1$) but becomes almost equal to or even smaller than $\Delta \Theta_{\text{time}}$ in the multiphoton regime.

For the ionization time distribution (see Fig. 5(d)), the peak position of the semiclassical theory (black solid line) corresponds to the maximal field strength. For the quantum calculations, not only the peak width (shown clearly in Fig. 4) but also the peak position varies with frequency, which becomes more and more close to those obtained by the semiclassical theory (see Fig. 5(d)) when the frequency decreases. For the PPT theory, although the peak widths of the ionization time distributions for three frequencies are different and become more and more broad with increasing frequencies, the peak positions are the same as those of the semiclassical theory and are independent of the laser frequency, which means that the value of the offset time between the quantum and ADK calculations is the same as that between the quantum and PPT calculations for a given laser frequency. As shown in Fig. 5(d), when the frequency decreases, the ionization time distribution approaches the semiclassical results, which can be attributed to that the non-adiabatic effect is less and less important with decreasing Keldysh parameter.

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Figure 6(b) depicts the offset time $\Delta t$ and $\Delta t_{\text{angle}}$ in units of attosecond (as). Some interesting features can be found in Fig. 6(b). The offset time hardly depends on the frequency. When the frequency decreases from 0.182 a.u. to 0.03502 a.u., the offset time $\Delta t$ only changes from about 1.8 as to 1.5 as. However, the corresponding offset time $\Delta t_{\text{angle}}$ varies in a much large range from 0.44 as to 26 as. Generally, the offset time $\Delta t_{\text{angle}}$ of the PPT theory is considerably smaller than that of the ADK theory, indicating that the non-adiabatic effect will indeed reduce the discrepancy between the quantum and semiclassical
FIG. 7: Time-angle distributions calculated by the quantum theory (a) and the semiclassical theory with ADK⊥∥(b) for ω=0.05691 a.u.. Angular distribution (c) and ionization time distribution (d) obtained by both the quantum and semiclassical methods. The parameters are the same as those of Fig. 3(c) except CEP ϕ = 0.

Calculations. However, it can be clearly seen that, except for the calculations of PPT⊥ and PPT⊥NZ in the multiphoton regime (ω = 0.1 and 0.182 a.u.), the offset times Δt are noticeably smaller than the corresponding offset times Δt_{angle} transformed from the offset angle in the angular distribution. This result implies that the offset angle ΔΘ in the angular distribution does not correspond to the offset time Δt in the ionization time distribution. So the “attoclock” measurement [8] which relies on the correspondence between the offset angle and time is in principle inaccurate. It is noteworthy that there is a debate on whether the non-adiabatic effect is responsible for the offset angle measured for He in the experiment [11, 14]. Our calculation shows that although the non-adiabatic effect taken into account in the PPT theory can reduce the value of the offset angle, the offset angle between the quantum and PPT calculations still does not correspond to the offset time between these two calculations, which indicates that the discrepancy between the two kinds of offsets cannot be attributed to the non-adiabatic effect [15, 18].

It is worthwhile mentioning that for pulses with CEP ϕ = 0.5π, because the two peaks in the angular (ionization time) distribution are symmetric, the absolute values of the offset angles (times) for the two peaks are equal. For other CEPs, the two peaks in the angular (ionization time) distribution become asymmetric and the absolute values of the offsets for these two peaks are not equal. Another special case is CEP ϕ = 0 (see Fig. 7). In this case, the maximal ionization rate occurs at the center of the laser pulse at t = 3 o.c. as shown in Fig. 7. Figs. 7(a) and 7(b) are the time-emission angle distributions calculated by the quantum theory and the semiclassical theory with ADK⊥∥, respectively. Both distributions show one main peak located at [Θ, t] ∼ [0(2π), 3 o.c.] and two other peaks at t ∼ 2.536 and 3.464 o.c. Moreover, the angular distribution and the ionization time distribution are given in Figs. 7(c) and 7(d), respectively. The non-adiabatic effect is also considered by PPT theory and the corresponding results are given in Figs. 7(c) and 7(d). As displayed in Fig. 7(c), all the angular distributions obtained by the quantum theory, semiclassical theory and PPT theory show two asymmetric peaks: one higher peak (namely one main peak) at emission angle Θ = 2π and one lower peak at emission angle Θ = π(3π). The peak positions of the two peaks calculated by these three methods coincide with each other, which means that the offset angle for the both higher peak and lower peak are all zero ΔΘ = 0. However, the ionization time distributions show a three-peak structure. For the highest peak at t = 3 o.c. which corresponds to the maximal field of the laser pulse, the offset time Δt is zero. For the other two peaks on the two sides of the highest peak, these peaks are symmetric with respect to the center of the pulse t = 3 o.c. and the absolute values of the offset times are equal |Δt| = 0.004 o.c. (see Fig. 7(d)). As shown in Figs. 7(c) and 7(d), when the non-adiabatic effect is considered, the widths of both the angular distribution and the ionization time distribution are closer to the quantum distributions than the ADK results, however, the positions of the peaks remain the same as those of the ADK calculations. In addition, the calculations for circularly polarized laser pulses are shown in Fig. 8. In this case, there is only one peak in both the angular and ionization time distributions and
the peak positions both coincide with the ADK (solid line) and PPT (dotted line) calculations, in agreement with the results of Torlina et al. [15] and Ni et al. [38].

Linear polarization

FIG. 9: Time-energy distribution of photoelectron in linearly polarized laser pulse. Here positive and negative energies indicate the emission directions of the photoelectron. (a): CEP $\varphi = 0.5\pi$; (b): CEP $\varphi = 0$. The other parameters are the same as those of Fig. 7. The prediction of the simpleman’s model is also shown (see text).

To systematically investigate the accuracy of the semiclassical model, we also calculate photoionization process of atoms in linearly polarized laser pulses. The time-energy distributions of H atoms in an 800 nm 6-cycle laser field with peak intensity of $1 \times 10^{14}$ W/cm$^2$ and CEPs of $0.5\pi$ and 0 are depicted in Fig. 9 For comparison, the prediction of the simpleman’s model is also shown in Fig. 9. In agreement with Fig. 2 of [25], the correspondence between the instant of ionization and the final kinetic energy (represented by strips in Fig. 9) is just qualitatively consistent with the simpleman’s prediction. Although the maximum of the strips is close to the semiclassical calculation near zero energy, two calculations diverge with increasing energy still.

In Fig. 10, we show the ionization time distributions of both quantum and semiclassical calculations. The results are similar to the elliptical polarization case. For CEP $\varphi = 0.5\pi$, two peaks of both calculations are symmetric and the offset times are also symmetric ($\Delta t = \pm 32$ as). While for $\varphi = 0$, the higher central peaks of two calculations coincide with each other but the lower side peaks show noticeable offsets ($\Delta t = \pm 29$ as). The PPT calculations are also given in Fig. 10 (dotted lines). The peak positions for PPT theory are the same as those for semiclassical calculations and the peak widths are broader than the semiclassical distribution, which is similar to the case of the elliptical polarization. It is noteworthy that the offset time in the linearly polarized case is considerably larger than that in the elliptically polarized laser field.

Discussion

FIG. 11: (a): Square of electric field of elliptically and circularly polarized laser pulses. (b): Electric field of linearly polarized laser pulses for $\varphi = 0$ and $0.5\pi$.

In the Wigner-distribution-like function, to obtain the ionization time distribution in the quantum theory, correlation effect must be taken into account. The ionization probability at moment $t$ is integration of the correlation between the ionization at $t + t'$ and $t - t'$ where $t'$ goes over the whole laser pulse, which means that the quantum process is temporally nonlocalized. In contrast, the ionization process in the semiclassical picture is tempo-
rally localized, viz., ionization at one moment is independent of ionization at another moment. Therefore, the semiclassical theory is an approximate description of the photoionization process and, as shown by our calculation, is usually not quantitatively consistent with the quantum calculation. The coincidence of the two calculations in some specific cases is actually accidental and can be attributed to the time-reverse invariant symmetry of the laser field. For an elliptically laser pulse with CEP $\varphi = 0$ or a circularly polarized laser pulse with any CEP, the laser field possesses time-reverse invariant symmetry with maximum at the center of the laser pulse as shown in Fig. [11]. Therefore, the ionization time and angular distributions are symmetrical and the positions of the peaks (the higher peak in the elliptically polarized case with $\varphi = 0$), which are generated at the maximum of the laser field, are independent of the calculation method. For CEP $\varphi = 0.5\pi$, though the laser field is also time-reverse invariant, this symmetry cannot ensure the coincidence between the peaks of different calculation methods but guarantee that the offsets of the two symmetrical peaks in both the ionization time and angular distributions are symmetrical (positive and negative but the absolute values are the same, see. Figs. [3] [6]). Moreover, as mentioned before, the lower peaks in the angular distribution for two calculations also coincide with each other in the $\varphi = 0$ case (see Fig. [7(c)]). This is because that the lower peak in the angular distribution is actually composed of two halves symmetrical with respect to $\Theta = \pi$ which are generated by two symmetrical peaks with respect to $t = 3$ o.c. in the ionization time distribution (see Fig. [7(d)]) and this symmetry is independent of the calculation method. In the linearly polarized cases, the offset time in the linearly polarized case is considerably larger than that in the elliptically polarized laser field. This can be attributed to stronger correlation effect in the linearly polarized laser field since in this case, the emitted photoelectrons concentrate more in one direction (laser field direction) than that in the elliptically polarized field. For other CEP, the time-reverse invariant is broken, hence the distributions are asymmetrical and the offsets are also asymmetrical. Moreover, our analysis indicates that neither the offset time nor the offset angle in the elliptically polarized laser field can be interpreted as the tunneling time delay. These offsets, together with the widths of the peaks in the distributions, however, could be taken as effective measures for the validity of the semiclassical theory in description of the ionization process in the laser field.

CONCLUSION

The WDL function enables us to obtain the time-emission angle distribution, angular distribution and ionization time distribution of atoms in the elliptically polarized laser fields and time-energy distribution and ionization time distribution of atoms in the linearly polarized laser fields. Comparison shows that these distributions are usually not in quantitative agreement with the semiclassical model calculation except in some specific cases. Discrepancy between two calculations can be attributed to correlation effect or temporal nonlocalization characteristic of quantum mechanics and coincidence between two calculations in some specific cases is due to temporal symmetry of the laser pulse. Moreover, we find that the offset angle are generally not consistent with the offset time even when the non-adiabatic effect is taken into account, indicating that the attosecond angular streaking technique is in principle inaccurate. Our result clearly shows the applicability and limit of the technique, which is important for interpretation and understanding on relevant experimental and theoretical results.

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