Quantum control and sensing of nuclear spins by electron spins under power limitations

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(Dated: April 2, 2019)

State of the art quantum sensing experiments targeting frequency measurements or frequency addressing of nuclear spins require to drive the probe system at the targeted frequency. In addition, there is a substantial advantage to perform these experiments in the regime of high magnetic fields, in which the Larmor frequency of the measured spins is large. In this scenario we are confronted with a natural challenge of controlling a target system with a very high frequency when the probe system cannot be set to resonance with the target frequency. In this contribution we present a set of protocols that are capable of confronting this challenge, even at large frequency mismatches between the probe system and the target system, both for polarisation and for quantum sensing.

Introduction — Nuclear spins control by electrons is a ubiquitous in quantum technology setups. Control experiments of nuclei in solids were realized via defects in diamond [1, 2], especially NV centers in diamond [3–7], Silicon Carbide [8, 9] and Silicon [10, 11]. These experiments were motivated by quantum computing [12–16], quantum sensing [17–19] and dynamical nuclear polarization [20–25]. Nuclear spins control requires to work at resonance, which is manifested by the Hartmann-Hahn (HH) condition [26]. The HH condition requires to equate the Rabi frequency (RF) at which the electron is driven to the Larmor frequency (LF) of the nuclei (Fig. 1 (a)). There is, however, a strong motivation to perform experiments at high magnetic fields due to the prolonged nuclear coherence time and the improvement in single-shot readout. Such experiments are very challenging and only a few were realized successfully [27–30]. Moreover, in some experiments (e.g., in biological environments) the maximal RF is restricted by deleterious heating effects that are associated with high power. In such cases it is challenging to reach the high RF that matches the nuclear LF (Fig. 1 (b)).

In this Letter we present a few schemes that can overcome this limitation in the various regimes of the mismatch between the RF and the targeted LF. We show that by employing a detuned driving field with a constant bounded RF or a driving field with a (bounded) modulated RF or a modulated phase, it is possible to reach the HH condition (Fig. 1 (c)). Although such protocols were achieved with pulsed schemes that require high power [31], we introduce simpler continuous drive based constructions that are significantly more power-efficient [32, 33]. While we focus on the NV center, the presented schemes are general and applicable to both the optical and microwave domains, and hence to a variety of atomic and solid state systems.

The model — We consider an NV center electronic spin that is interacting with a single or several nuclei via the dipole - dipole interaction. Under an on-resonance drive, the Hamiltonian of the NV and a nuclear spin is given by [34] \( H = \frac{\gamma_0}{2} \sigma_z + \omega_0 I_z + g\sigma_x I_z + \Omega_1 \sigma_z \cos(\omega_1 t) \), where \( \omega_0 \) corresponds to the NV’s energy gap, \( \omega_l \) is the nucleus LF, \( \sigma_z \) and \( I_z \) are the Pauli operators in the direction of the static magnetic field of the NV and the nucleus respectively, \( g \) is the NV - nucleus coupling strength, and \( \Omega_1 \) is the RF of the NV drive. For sensing and control of the nucleus by the NV the HH condition, \( \Omega_1 = \omega_l \), must be fulfilled (Fig. 1 (a)) [34].

In the high magnetic field regime the nuclear LF, \( \omega_l = \gamma_l B \), where \( \gamma_l \) is the nuclear gyromagnetic ratio and \( B \) is the static magnetic field, can be as high as \( \sim 100 \) MHz. Hence, because of either technical limitations or avoidance of heating effects that occur due to the high power that is required to generate such a large RF, it is impossible to fulfil the HH condition by an on-resonance drive. Namely, we must work in the regime where \( |\Omega_1| < \omega_l \) (Fig. 1 (b)). We term the frequency difference, \( \omega_l - \Omega_1 \), as the frequency mismatch between the NV frequency (\( \Omega_1 \)) and the nuclear LF (\( \omega_l \)).

We propose a set of protocols where even though the elec-
tron is driven with a bounded RF, $|\Omega(t)| < \omega_i$, an effective
dressed electronic energy gap that is equal to the LF is ob-
tained, and hence, the resonance condition is retrieved. Most
generally, we consider the Hamiltonian $H_I = \frac{\omega_0}{2}\sigma_x + \frac{\omega_t}{2}L_z + g\sigma_z L_x + \Omega_1(t)\sigma_x \cos(\phi(t))$, where $\Omega_1(t)$ and $\phi(t)$ are the mod-
ulated RF and modulated phase of a general driving field. The
functions $\phi(t)$ and $\Omega_1(t)$ are our control tools that are used in
order to reach the resonance condition in the small and large
frequency mismatch regimes respectively, and therefore en-
able to probe the nuclei parameters and polarize it in the high
magnetic field regime.

Small frequency mismatch — In continuous dynamical de-
coupling it is more beneficial to rely on a control by a robust
phase modulation (PM) than on a control by a noisy amplitude
modulation (AM) [35]. This concept was verified experimentally [33, 36] and here we further develop it to design efficient
and robust control in the high magnetic field regime when the
frequency mismatch is small. This scenario is relevant for a
LF of $\sim 1 - 10$ MHz. For example, the LF of $^{13}C$ ($^{15}N$) at
a magnetic field of 1T (1.5T) is 10 MHz (6.5 MHz). There
are two key advantages of PM. First, PM is much more sta-
able than a noisy AM and therefore results in longer coherence
times. Second, the extra frequency that is required to fulfil the
resonance condition ($\omega_i - \Omega_1$) originates only from the PM
and therefore does not require extra power beyond the power
limit of the bounded RF $\Omega_1$ [34].

We consider the following Hamiltonian of the NV and the
nucleus, $H = \frac{\omega_0}{2}\sigma_x + \frac{\omega_t}{2}L_z + g\sigma_z L_x + (\Omega_1 + \delta\Omega_1(t))\sigma_x \cos(\omega_i t + 2\Omega_1/\omega_i \sin(\Omega_1 t))$, where $\delta\Omega_1(t)$ is
the magnetic noise, $\Omega_1$ is the RF of the drive, which
defines the PM according to $\phi(t) = 2\Omega_1/\omega_i \sin(\Omega_1 t)$, and $\delta\Omega_1(t)$ is
the amplitude noise in $\Omega_1$. The NV dynamics is modu-
lated by two frequencies, $\Omega_1$ and $\Omega_2$, and thus we may ex-
pect transitions to occur whenever the resonance condition,
$\Omega_1 + \Omega_2 = \omega_i$ is met. Indeed, this Hamiltonian results in
double-dressed NV states for which we have that [34] $H_{II} \approx \frac{\omega_0}{2}\sigma_x + \frac{\omega_t}{2}L_z - \Omega_1^2 \left( e^{i\Omega_1 t} - e^{-i\Omega_1 t} \right) \sigma_x + \Omega_2 \left( e^{i\Omega_1 t} - e^{-i\Omega_1 t} \right) L_z$, where $H_{II}$ is the Hamiltonian in the second interaction pic-

ture (IP) and in the basis of the double-dressed states. From
this expression it is seen that a resonance condition appears
when $\Omega_1 + \Omega_2 = \omega_i$ (or when $\Omega_1 - \Omega_2 = \omega_i$). Even though
the power of the driving field is $\omega_i - \Omega_1^2$ and is independent of
$\Omega_2$, higher Larmor frequencies than what is available by the
peak power in a common HH scheme are reachable. While
the modulation by the frequency $\Omega_1$ originates from AM and
requires a power of $\omega_i - \Omega_1^2$, the second modulation by the
frequency $\Omega_2$ originates from the PM and as such it is not asso-
ciated with extra power. Specifically, for $\Omega_1 = \Omega_2$ the ratios
of the peak power (the maximal instantaneous power value)
and the cycle power (the power that is required for a complete
energy transfer (flip-flop) between the NV and the nucleus)
between a common HH drive and a phase modulated drive are
4 and 2 respectively [34]. Moreover, PM may result in sig-
nificantly prolonged coherence times due to the precise phase
control of microwave sources, and the elimination (to first or-
der) of amplitude fluctuations in $\Omega_1$ [34].

The above procedure is correct in the limit of $\Omega_2 \ll \Omega_1$.

However, we aim to increase $\Omega_2$ as much as possible with-
out reducing the sensitivity. To this end, we have to take
into account the Bloch-Siegent Shift (BSS) due to the counter-
rotating terms of the second modulation $\Omega_2$, which induces a
shift of the resonance. In addition, this decreases the coupling
to the nucleus, and more importantly, the coherence time of
the NV as the decoefficient of the drive is not effective any
more (Fig. 3 (blue)). To improve this, we suggest to correct
the BSS when adjusting the frequency $\Omega_2$ in the PM $\phi(t) = 2\Omega_2/\omega_i \sin(\Omega_1 t)$ and modify it to $\Omega_2 = \frac{1}{2} \Omega_1 + \sqrt{4\Omega_1^2 + 3\Omega_2^2}$.

In this case, the resonance frequency is $\Omega_1 + \Omega_2 = \omega_i$, where
$\Omega_2 = \frac{\Omega_1}{2} \left( 1 + \frac{\Omega_1 + \Omega_2}{\sqrt{\Omega_1^2 + (\Omega_1 + \Omega_2)^2}} \right)$ [34].

In Fig. (2) we show simulation results [34] for the nucleus
polarization as function of $\Omega_2$. In the main figure we con-
sider the strong coupling regime, where the polarization time
t is $2\pi/g$ is much shorter than the decoherence time of the
NV center and hence, decoherence effects are neglected. In
the inset we consider the weak coupling regime where noise
decreases the polarization rate [34]. We define the nuclear
spin polarization, $P_N$, as the probability of the nuclear spin to
be in its initial state $|\uparrow_z\rangle$. Specifically, we initialize the NV-
Nucleus state to $|\psi_i\rangle = |\downarrow_z\rangle_N |\uparrow_z\rangle_N$ and calculate the polarization according to $P_N = |\langle \uparrow_z | \psi \rangle|^2 + |\langle \downarrow_z | \psi \rangle|^2$, where $|\psi\rangle$ is the joint NV-Nucleus state at the optimal polarization
time. Hence, $P_N = 0$ corresponds to optimal polarization
and $P_N = 1$ corresponds to no polarization at all. While
in the strong coupling regime the correction always results in
better polarization rates, in the weak coupling regime the ad-
vantage of correction is lost at $\Omega_2 \approx 1.5\Omega_1$. In Fig. (3) we
show the expected coherence times, $T_2$, of the NV as function
of $\Omega_2/\Omega_1$ [34]. Without the correction the optimal coherence time

![Figure 2. Polarization as a function of $\Omega_2$ in the strong and weak (inset) coupling regimes without BSS correction (blue) and with BSS correction (green). Strong coupling regime: without correction the polarization rate begins to sharply decrease at $\Omega_2 \approx \Omega_1$. The correction enables to maintain good polarization rates up to $\Omega_2 \approx 1.5\Omega_1$. Weak coupling regime: The analysis takes noise into account. The polarization is effective up to $\Omega_2 \approx 1.4\Omega_1$.](image-url)
is sharply peaked at \( \Omega_2 \approx 0.125 \) with \( T_2 \approx 330 \mu s \) (not shown). The coherence time is reduced when \( \Omega_2 \) is increased due to an amplitude mixing of \( \frac{\Omega_2}{\Omega_1} \) between the dressed states, which introduces back a first order contribution of the drive noise \( \approx \frac{\Omega_2}{\Omega_1} \delta \Omega_1 \). This decoherence is greatly mitigated by the correction of the BSS up to \( \Omega_2 \approx \Omega_1 \), which results in an improvement of one order of magnitude in the coherence times. With the correction the optimal coherence time is peaked at \( \frac{\Omega_2}{\Omega_1} \approx 0.4 \) with \( T_2 \approx 1000 \mu s \). In this case, the coherence time is mainly limited by the second order contribution of the drive noise \( \sim \frac{\delta \Omega_1^2}{\Omega_1^2} \). The BSS correction enables to further increase \( \Omega_2 \) and results in prolonged NV’s coherence times and higher polarization rates.

**Large frequency mismatch** — The natural way to compensate for the frequency mismatch is to introduce a detuning (\( \delta \)) to the drive. This detuning induces an extra modulation that creates an effective frequency of \( \sqrt{\Omega_1^2 + \delta^2} \), which in principle, can be as high as needed \( \left( \sqrt{\Omega_1^2 + \delta^2} \gg \Omega_1 \right) \). When the effective frequency \( \sqrt{\Omega_1^2 + \delta^2} \) is equal to the LF, the HH condition is fulfilled and the electron-nucleus interaction is enabled [34]. This however, comes with a price; the electron-nucleus coupling strength is decreased by a factor of \( \sim \frac{\Omega_1}{\delta} \) [34] (Fig. 1 (c)). Here the decoupling effect of a resonant drive vanishes and the NV’s coherence time approaches \( T_2^* \). In [34] we show how to circumvent this by adding a second drive. This scheme, however, could be extremely power efficient, e.g., for \( \delta = 100 \Omega_1 \) the ratios of the peak power and the cycle power between a common HH drive and a detuned drive are 101 and 10.1 respectively [34].

An alternative way to reach the resonance is to modulate the amplitude of the drive. This AM generates higher harmonics of the modulation frequency that can be tuned to be on-resonance with the LF. We start with the Hamiltonian \( H = \frac{\omega_0}{2} \sigma_z + \frac{\omega_2}{2} I_z + g \sigma_z I_z + \Omega(t) \sigma_x \cos(\omega_0 t) \) and set \( \Omega(t) = \Omega_0 + \Omega_1 \cos(\omega_1 t) \). Moving to the IP with respect to \( H_0 = \frac{\omega_0}{2} \sigma_z \) and making the rotating-wave-approximation (RWA) \( (\omega_0 \gg |\Omega(t)|) \) we obtain

\[
H_1 = \frac{\Omega(t)}{2 \omega_0} \sigma_z + \frac{\omega_2}{2} I_z + g \sigma_z I_z, \quad \text{which in the basis of the NV dressed states (} x \rightarrow z, \ z \rightarrow -x, \ y \rightarrow y \text{) is given by}
\]

\[
H_1 = \frac{\Omega(t)}{2 \omega_0} \sigma_z + \frac{\omega_2}{2} I_z - g \sigma_z I_z. \quad \text{We continue by moving to the second IP with respect to } H_0 = \frac{\Omega(t)}{2 \omega_0} \sigma_z + \frac{\omega_2}{2} I_z, \quad \text{which results in}
\]

\[
H_{II} = -g \left( \sigma_x e^{i \left( \Omega_0 t + \frac{\Omega_1}{\Omega_0} \sin(\Omega_1 t) \right)} + h.c. \right) \left( I_z e^{i \omega_0 t} + I_- e^{-i \omega_0 t} \right).
\]

The exponent \( e^{i \left( \Omega_0 t + \frac{\Omega_1}{\Omega_0} \sin(\Omega_1 t) \right)} \) contains the higher harmonics of \( \Omega_2 \), i.e., \( n \Omega_2 \), where \( n \) is an integer. This can be seen by the equality \( e^{i \left( \Omega_0 t + \frac{\Omega_1}{\Omega_0} \sin(\Omega_1 t) \right)} = e^{i \Omega_0 t} \sum_{n=-\infty}^{\infty} \left( \tilde{p}^2 \right)_{\frac{\Omega_1}{\Omega_0}} e^{in \Omega_2 t} + h.c. \). We can therefore set the resonance condition to \( \Omega_0 + \Omega_2 = \omega_0 \). Assuming the RWA \( (\Omega_2 \gg \omega) \) we get that \( H_{II} \approx g J_1 \left( \frac{\Omega_1}{\Omega_0} \right) (i \sigma_+ I_0 - i \sigma_- I_0) \) when the resonance condition is fulfilled. In the regime of \( \Omega_2 \gg \Omega_1 \), \( J_1 \left( \frac{\Omega_1}{\Omega_0} \right) \approx \frac{\Omega_1}{\Omega_0} \). Hence, the coupling strength is similar to the one in the previous method, however, this scheme is robust to magnetic noise. Numerical analysis of this method is shown in Fig. 4. With a single AM the method suffers from amplitude fluctuations in \( \Omega_0 \), which could be eliminated by realising this as a second drive from a PM [34]. This scheme is also power efficient, e.g., for \( \Omega_2 = 9 \Omega_0 \) the ratios of the peak power and the cycle power between a common HH drive and an amplitude modulated drive are 25 and 3.7 respectively [34].

*Quantum sensing — Addressability* — The ability of a probe to individually address and control nuclear spins, which was discussed above. However, addressability is not necessary for quantum sensing where, e.g., one is only interested in estimating the LF, as in nano-NMR experiments. The resolution of addressability is defined by the ability to control a nucleus with a given frequency \( \omega_0 \), while leaving nuclei with different frequencies outside of a frequency width \( \Delta \omega \) (centered at \( \omega_0 \)) unaffected. As shown in Fig. 4 in blue, the addressability resolution is limited by the coupling strength. This is because that all frequencies within a width of the coupling strength from the resonance will couple to the probe. Hence, the stronger the coupling the worse the resolution is and a larger band of frequencies will be addressed by the probe. However, when the NV is used to estimate the LF, one would expect that the stronger the coupling the more information would be acquired; an increased coupling strength should improve the resolution and not limit it. The addressability resolution limit could be overcome by designing the Hamiltonian differently. In cases that control is not necessary, and one is just interested in frequency estimation of the nuclei, methods that are not limited by the coupling strength could be designed. The difference between the methods is analogous to the difference between Rabi and Ramsey spectroscopy. While power is a limiting factor in the first method (necessitating
weak pulses), it poses no limitation in the second method.

The addressability resolution problem occurs as the NV coupling operator term is a $\sigma_\pm$ operator that is in charge of energy transfer. This is crucial for control, however, it is not needed for sensing. An interaction of the addressability type, $g (\sigma_+ I_x + \sigma_- I_-)$, transfers excitations between the two spins as long as their frequency difference is smaller than the coupling strength $g$. Thus, the target frequencies within a spread of $g$ are addressed by the probe. However, an interaction of the type $g \sigma_\pm (I_+ + I_-) = g \sigma_\pm I_x$ could be utilized to estimate the frequencies of the target spins with a resolution that is not limited by the coupling strength [37–39]. This can be achieved by transforming the $\sigma_+, \sigma_-$ operators into a $\sigma_y$ (or $\sigma_x$) operator, which is doable as $\sigma_\pm = \sigma_y \pm i \sigma_z$ and $\sigma_y$ could be eliminated with a suitable control, for example, by adding a strong $\sigma_y$ drive that will eliminate the $\sigma_\pm$ part. For the case of the low frequency mismatch this can be achieved by adding an extra drive on the NV, which rotates at $\Omega_2$ (this amounts to $\Omega_2 \cos(\omega t) \cos(\Omega_2 t) \sigma_z$). In [34] we explicitly show that this results in an Hamiltonian that can be used for sensing the LF, i.e., $H_t \approx \frac{\delta}{2} \sigma_z (I_x \cos(\delta t) - I_y \sin(\delta t))$, where $\delta = \Omega_1 + \Omega_2 - \omega_l$. As the extra term acts as a spin locking at $\Omega_l$, the robustness of the methods is preserved. The classical version of this Hamiltonian was used in [37–46] where it was shown that the resolution is only limited by the clock and signal coherence times. The resolution obtained by this Hamiltonian, which is the generic sensing Hamiltonian, is only limited by the coherence time of the nuclei and the sensitivity is improved with the coupling strength [47].

The same can be done in the large frequency mismatch regime. The interaction should be changed from the flip - flop interaction $g (\sigma_+ I_+ + \sigma_- I_-)$ to $g \sigma_\pm I_x$ by adding, for example, a $\sigma_y$ drive to the modulation. In this case the Hamiltonian is transformed to $H \approx g I_x \frac{\omega_l}{2} \sigma_z (I_x \cos(\delta t) - I_y \sin(\delta t))$. The result of using this Hamiltonian for estimating the nuclei’s frequencies is shown in Fig. 4. The yellow line is the Fourier transform of the time series of NV measurements for a scenario in which a few nuclei are present at the three frequencies $\Omega_0, \Omega_1, \Omega_2$. The width of these peaks (one over the total experiment time) is narrower than the peaks of the control method (blue line), which is limited by the coupling strength.

The challenge of controlling and sensing high-frequency nuclei under power limitations of the driving fields was addressed both in the small and large frequency mismatch regimes. We have designed schemes that are robust both to magnetic field fluctuations and RF noise. The presented protocols could potentially allow for the realization of experiments in an important regime which is currently out of reach and could considerably simplify state of the art experiments.

We would like to note that during the preparation of this manuscript we became aware of a related independent work by Casanova et al. [48].

Acknowledgements A. R. acknowledges the support of ERC grant QRES, project No. 770929, grant agreement No 667192(Hyperdiamond), the MicroQC, the ASTERIQS and the DiaPol project.

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I. SUPPLEMENTARY MATERIAL

II. THE MODEL

We consider an NV center electronic spin that is interacting with a single or several nuclei via the dipole - dipole interaction. As we are interested in the regime in which the energy gap due to the Zeeman splitting of the NV is orders of magnitude larger than the energy gap of the nucleus, only the $T_{1} + T_{-1} = \frac{3}{2} \sin \theta \cos \theta \sigma_{z} (I_{x} \cos \phi + I_{z} \sin \phi)$, (see, for example, [49]). Complement $B_{11}$ term of the dipole - dipole interaction is significant, where $\theta, \phi$ are the polar angles representing the vector joining the NV and the nucleus. All other terms of the dipole-dipole interaction are fast rotating and thus can be neglected to leading order. In most cases it is the above term that is used for polarization and for sensing, in particular, in the high-field NMR experiments. Because the energy gaps of the ground state sub-levels of the NV are much larger than the RF, only two levels are addressed by the microwave driving fields and thus, the NV could be approximated as a two-level system.

III. THE HARTMANN-HAHN CONDITION

Under an on-resonance drive of the NV, the Hamiltonian of the NV center spin and the nuclear spin is given by

$$H = \frac{\omega_{0}}{2} \sigma_{z} + \frac{\omega_{l}}{2} I_{z} + g \sigma_{z} I_{x} + \Omega_{1} \sigma_{z} \cos (\omega_{0} t),$$

where $\omega_{0}$ corresponds to the energy gap of the NV center spin, $\omega_{l}$ is the Larmor frequency of the nuclear spin, $\sigma_{z}$ and $I_{z}$ are the Pauli operators in the direction of the static magnetic field of the NV center and the nucleus respectively, $g$ is the NV - nucleus coupling strength, which depends on the distance between the two, and where we have simplified the $T_{1} + T_{-1}$ term to $g \sigma_{z} I_{x}$, and $\Omega_{1}$ is the Rabi frequency of the on-resonance driving field of the NV center. By moving to the interaction picture (IP) with respect to the first term, $H_{0} = \frac{\omega_{0}}{2} \sigma_{z}$, and making the rotating-wave-approximation (RWA) assuming that $\omega_{0} \gg \Omega_{1}$, we obtain

$$H_{I} = \frac{\Omega_{1}}{2} \sigma_{z} + \frac{\omega_{l}}{2} I_{z} + g \sigma_{z} I_{x}.$$  \hspace{1cm} (2)

In the basis of the dressed NV center states ($x \rightarrow z, z \rightarrow -x$, and $y \rightarrow y$) we have that

$$H_{I} = \frac{\Omega_{1}}{2} \sigma_{z} + \frac{\omega_{l}}{2} I_{z} - g \sigma_{z} I_{x}.$$  \hspace{1cm} (3)

Moving now to the second IP with respect to $H_{0} = \frac{\Omega_{1}}{2} \sigma_{z} + \frac{\omega_{l}}{2} I_{z}$, we arrive at

$$H_{II} \approx -g \left( \sigma_{+} I_{x} e^{i(\Omega_{1} - \omega_{l}) t} + \sigma_{-} I_{x} e^{-i(\Omega_{1} - \omega_{l}) t} \right),$$

where fast rotating terms have been neglected. It is clear from eq. 4 that for sensing and control of the nuclear spin by the NV it is necessary to fulfill the resonance condition, $\Omega_{1} = \omega_{l}$, which is the Hartmann-Hahn condition.

IV. PHASE MODULATION - THE BASIC SCHEME

We consider the following Hamiltonian of the NV center and the nucleus,

$$H = \frac{\omega_{0}}{2} \sigma_{z} + \delta B(t) \sigma_{z} + \frac{\omega_{l}}{2} I_{z} + g \sigma_{z} I_{x}$$

$$+ (\Omega_{1} + \delta \Omega_{1}(t)) \sigma_{z} \cos \left( \omega_{0} t + 2 \Omega_{1} \frac{\omega_{l}}{\Omega_{1}} \sin (\Omega_{1} t) \right),$$

where $\delta B(t)$ is the noise in the magnetic field, $\Omega_{1}$ is the RF of the driving field, which defines the phase modulation according to $\phi(t) = 2 \frac{\omega_{l}}{\Omega_{1}} \sin (\Omega_{1} t)$, and $\delta \Omega_{1}(t)$ is the amplitude noise in the drive amplitude $\Omega_{1}$.

In order to see how the Hamiltonian of Eq. 5 results in the resonance condition $\Omega_{1} + \Omega_{2} = \omega_{l}$, we start by moving to the first IP in which the drive is time independant, i.e., with respect to $H_{0} = \frac{\omega_{0}}{2} \sigma_{z} + \frac{\omega_{l}}{2} I_{z}$, this results in

$$H_{I} = \frac{(\Omega_{1} + \delta \Omega_{1}(t))}{2} \sigma_{z} + \delta B(t) \sigma_{z} - \Omega_{2} \cos (\Omega_{1} t) \sigma_{z}$$

$$+ \frac{\omega_{l}}{2} I_{z} + g \sigma_{z} I_{x},$$

(6)

which is similar to a concatenated double-drive Hamiltonian, this time, however, with a very stable second drive, $\Omega_{2}$. Because the magnetic noise is perpendicular to the basis of the dressed states robustness to the magnetic noise (in first order) is achieved. From here on, we neglect the magnetic noise whose leading (second order) contribution is $\sim \frac{\delta B(t)^{2}}{\Omega_{1}}$.

We continue by rotating to the basis of the dressed states

![Figure 5. Small frequency mismatch scheme. (a) The electron spin is driven with a bounded RF $\Omega_{1}$ that is smaller than the nuclear LF $\omega_{l}$ ($\Omega_{1} < \omega_{l}$), but with a phase modulation $\phi(t)$ (see text). (b) This results in electronic dressed states with an energy gap of $\Omega_{1}$ that are driven on-resonance by a second drive with a RF of $\Omega_{2}$. The second drive originates only from the phase modulation which does not require additional power beyond the power of $\sim \Omega_{1}^{2}$ that is required for the bounded RF of $\Omega_{1}$. (c) The second drive $\Omega_{2}$ results in double dressed states of the electron that match the resonance condition with $\Omega_{1} + \Omega_{2} = \omega_{l}$. Doted lines indicate resonance frequencies and solid lines indicate driving fields.](image-url)
(as in section III) such that

\[
H_I = \left( \Omega_1 + \delta \Omega_1(t) \right) / 2 \sigma_z + \Omega_2 \cos(\Omega_1 t) \sigma_x
\]

\[
+ \frac{\omega_1}{2} I_z - g \sigma_x I_z.
\]  

(7)

It is now clear that the phase modulation results in a second drive that drives the dressed states on-resonance with a RF of \( \delta \Omega_2 \). The double-dressed states are obtained by moving to the second IP with respect to \( \Omega \).

By setting \( \Omega_1 + \Omega_2 = \omega_0 \) (or when \( \Omega_1 - \Omega_2 = \omega_0 \)).

Even though the power of the driving field is \( \propto \Omega_1^2 \) and is independent of \( \Omega_2 \), Larmor frequencies which are higher than what is available by the peak power in a common HH scheme are reachable (Fig. 5).

\[ \dot{\Omega}_1 = \frac{1}{3} \left( \Omega_1 + \sqrt{4 \Omega_1^2 + 3 \Omega_2^2} \right), \]

(14)

In this case the amplitude-mixing between the dressed states is greatly diminished and hence, the coherence time of the NV center may be significantly prolonged compared to the scenario without the correction. Hence, we modify the frequency \( \Omega_1 \) in the phase modulation \( \phi(t) = \frac{2 \Omega_1}{\omega_0} \sin(\Omega_1 t) \) in Eq. 5 to

\[ \dot{\tilde{\Omega}}_1 = \frac{1}{3} \left( \Omega_1 + \sqrt{4 \Omega_1^2 + 3 \Omega_2^2} \right). \]

(15)

\[ \Omega_1 + \Omega_2 = \omega_0, \]

(16)

VI. NUMERICAL ANALYSIS

A. Strong coupling regime

In the strong coupling regime we consider the scenario in which the polarization time \( t = 2\pi/g \) is short enough such that the effect of noise on the polarization is minor, that is, the polarization time is much shorter than the decoherence time of the NV center. Hence, we neglect decoherence effects.

Because we consider the regime of high magnetic fields, we have that \( \omega_0 \gg \Omega_1 \) and the RWA is valid with respect to the first drive \( \Omega_1 \). Hence, in the numerical analysis we simulated the Hamiltonian in the first IP, which is given by Eq. 6. Since here we neglect decoherence effects we omitted the terms of the magnetic and drive noise. The simulations were performed with \( \Omega_1 = 2\pi \times 3.3 \text{ MHz} \) and \( g = 0.04\Omega_1 \), where the value of \( \Omega_2 \) was varied. For each value of \( \Omega_2 \) we scanned the
resonance frequency around the ideal value of \( \omega_l = \Omega_1 + \Omega_2 \) or \( \omega_l = \Omega_1 + \Omega_2 \) (without and with the correction of the Bloch-Siegert shift respectively) and found the additional shift in the resonance frequency \( \delta \omega_l \), which results from the effect of the fast-rotating terms (Fig. 6). At the resonance frequency, \( \omega_l = \Omega_1 + \Omega_2 + \delta \omega_l \) or \( \omega_l = \Omega_1 + \Omega_2 + \delta \omega_l \) with the correction), the maximal polarization is obtained. We define the nuclear spin polarization, \( P_N \), as the probability of the nuclear spin to be in its initial state \( |\uparrow\rangle \). Specifically, we initialize the NV-Nucleus system to \( |\psi_i\rangle = |\downarrow\rangle_N V|\uparrow\rangle = |\downarrow\rangle_N |\uparrow\rangle \) and calculate the polarization according to \( P_N = |\langle \uparrow | \psi \rangle|^2 + |\langle \downarrow | \psi \rangle|^2 \), where \( |\psi\rangle \) is the joint NV-Nucleus state at the optimal polarization time. Hence, \( P_N = 0 \) corresponds to optimal polarization and \( P_N = 1 \) corresponds to no polarization at all. Note that since the \( \hat{z} \) basis is the basis of the double-dressed states and also the measurement axis, the Rabi frequencies \( \Omega_1 \) and \( \Omega_2 \) are invisible to the population measurement of \( P_N \). In Fig. (7) we show the nuclei polarization as function of the ratio \( \frac{\Omega_2}{\Omega_1} \) both with (green) and without (blue) the correction of the Bloch-Siegert shift. It is clear that in the strong coupling regime the correction results in better polarization rates, especially when \( \Omega_2 \gtrsim \Omega_1 \).

### B. Weak coupling regime

In the weak coupling regime the polarization time \( t = 2\pi/\gamma \) is long enough such that decoherence effects must be taken into account. Hence, the terms of the magnetic noise and the driving amplitude noise in Eq. 6 are not omitted. In Fig. (8) we show the nuclei polarization as function of the ratio \( \frac{\Omega_2}{\Omega_1} \) both with (green) and without (blue) the correction of the Bloch-Siegert shift. The simulations were performed with \( \Omega_1 = 2\pi \times 3.3 \) MHz and \( g = 0.01\Omega_1 \). While in the strong coupling regime the correction always results in better polarization rates, in the weak coupling regime the advantage of correction is lost at \( \Omega_2 \approx 1.5\Omega_1 \).

The numerical simulations of the polarization rates and coherence times were performed under the assumption that the pure dephasing time of the NV is \( T_2^* = 3\mu s \), which results from a magnetic noise, \( B(t) \), that is modeled by an Ornstein-Uhlenbeck (OU) process [50, 51] with a zero expectation value, \( \langle B(t) \rangle = 0 \), and a correlation function \( \langle B(t) B(t') \rangle = \frac{1}{\tau} e^{-|t-t'|/\tau} \), where \( \tau \) is the diffusion constant and \( \tau = \frac{1}{\gamma} = 25\mu s \) is the correlation time of the noise. An OU process was also used to realize driving fluctuations. Here we used a correlation time of \( \tau = 500\mu s \), and a relative amplitude error of \( \delta \Omega_1 = 1\% \).

## VII. LARGE FREQUENCY MISMATCH

### A. Method I - Detuned driving field

In this section we provide the derivation of the the detuned driving method. The calculation goes as follows. By introducing a detuning \( \delta \) to the driving field in Eq. 1 we obtain the Hamiltonian

\[
H = \frac{\omega_0}{2} \sigma_z + \frac{\omega_2}{2} I_z + \Omega_1 \cos((\omega_2 - \delta)t) \sigma_z + g \sigma_x I_z. \tag{17}
\]

In order to analyze this scenario it is advantageous to move to the IP in which the drive is time independent and the problem can be analyzed via the resulting dressed states. Hence, we choose to move to the IP with respect to \( \frac{\omega_0 - \delta}{2} \sigma_z \), which results

![Figure 6. The polarization, which is defined as the population of the nuclei in the initial \( |\uparrow\rangle \) state (see text), as a function of the resonance frequency shift \( \delta \omega_l \) in units of \( \Omega_2 \). Here we consider polarization with the correction of the Bloch-Siegert shift with \( \Omega_2 = 1.2\Omega_1 \). The optimal polarization is obtained for \( \frac{\delta \omega_l}{\Omega_2} = -0.015 \) and is equal to \( P_N = 0.0015 \). (Inset) The polarization as function of time for the optimal value of \( \delta \omega_l \). The time is in units of \( \mu s \).](image)

![Figure 7. The polarization as a function of \( \Omega_2 \) in units of \( \Omega_1 \) in the strong coupling regime. First method without correction of the Bloch-Siegert shift (blue) - the polarization rate begins to sharply decrease at \( \Omega_2 \approx \Omega_1 \). Second method with correction of the Bloch-Siegert shift (green) - the correction enables to maintain good polarization rates up to \( \Omega_2 \approx 1.8\Omega_1 \).](image)
in

\[ H_I = \frac{\delta}{2} \sigma_z + \frac{\Omega_1}{2} \sigma_+ + \frac{\omega_I}{2} I_z + g \sigma_x I_x \]

\[ = \frac{\sqrt{\delta^2 + \Omega_1^2}}{2} \sigma_0 + \frac{\omega_I}{2} I_z + g (\sigma_0 \cos \theta + \sigma_0 \sin \theta) I_x, \]

where \( \sigma_0 \) is a Pauli matrix in the direction \( \theta = \arctan \left( \frac{\Omega_1}{\delta} \right) \), which is the angle in the \( x-z \) plane from the \( z \) axis and \( \sigma_{0 \pm} \) is the Pauli matrix in the orthogonal direction.

Close to resonance, when \( \sqrt{\delta^2 + \Omega_1^2} = \omega_I \), we can expect excitation transfer between the electron and the nucleus due to the last term, i.e., \( \sigma_0, \sin(\theta) I_x \). Since for large detunings \( \sin \theta \approx \frac{\Omega_1}{\delta} \), the effective coupling strength is reduced from \( g \) to \( \approx \frac{\Omega_1}{\delta} g \), which is depicted in Fig. 1 (c) of the main text.

This method, however, suffers from the fact that the decoupling effect due to the resonant drive vanishes and the coherence time of the NV approaches the \( T_2^* \) time. Specifically, when moving to the first IP, the magnetic noise \( \delta B(t) \sigma_z \) is modified to \( \delta B(t)(\sigma_0 \cos \theta + \sigma_0 \sin \theta) \), which results in a first order contribution as long as has \( \cos \theta \neq 0 \). In order to circumvent this issue and to prolong the coherence time we suggest to introduce a second drive by adding an extra term in the Hamiltonian, namely,

\[ H = \frac{\omega_I}{2} \sigma_z + \frac{\omega_I}{2} I_z + g \sigma_x I_x + \Omega_1 \cos((\omega_0 - \delta)t) \sigma_x \]

\[ + \Omega_2 \cos((\omega_0 - \delta)t + \pi/2) \cos \left( \sqrt{\delta^2 + \Omega_1^2} \right) \sigma_x. \]  

(19)

Moving to the first IP with respect to \( \frac{\omega_0 - \delta}{2} \sigma_z \) and the to the basis of the dressed states as above, we obtain

\[ H_I = \frac{\sqrt{\delta^2 + \Omega_1^2}}{2} \sigma_0 + \frac{\omega_I}{2} I_z + g(\sigma_0 \cos \theta + \sigma_{0 \pm} \sin \theta) I_x \]

\[ + \frac{\Omega_2}{2} \cos \left( \sqrt{\delta^2 + \Omega_1^2} \right) \sigma_y. \]  

(20)

We continue by moving to the second IP with respect to \( H_0 = \frac{\sqrt{\delta^2 + \Omega_1^2}}{2} \sigma_0 \), which leads to

\[ H_{II} = \frac{\Omega_2}{4} \sigma_y + \frac{\omega_I}{2} I_z \]

\[ + g \sin \theta (\sigma_{0 \pm} e^{i\sqrt{\delta^2 + \Omega_1^2} t} + \sigma_0 e^{-i\sqrt{\delta^2 + \Omega_1^2} t}) I_x, \]  

(21)

where \( \sigma_{0 \pm} \) and \( \sigma_0 \) are the raising and lowering operators in the basis of \( \sigma_0 \) respectively. Similar to Eq. 8, we see that the resonance condition is fulfilled when \( \sqrt{\delta^2 + \Omega_1^2} + \Omega_2^2 = \omega_I \).

Hence, Larmor frequencies that are much higher than what is available by the power limitation, which here is \( \approx \Omega_2^2 + \Omega_1^2 \), are reached.

Because the second drive \( \Omega_2 \) is along the \( y \) axis, which is perpendicular to the basis of the dressed states that is in the \( x-z \) plane, the second drive achieves robustness to (first order) magnetic noise and amplitude noise in \( \Omega_1 \). Hence, the second drive prolongs the coherence time of the NV, and thus the resolution, while shifting the resonance to \( \sqrt{\delta^2 + \Omega_1^2} + \Omega_2^2 = \omega_I \). The disadvantage of this method is that the second drive \( \Omega_2 \), which results in a drive along the \( y \) direction, cannot be generated by a phase modulation, which results in a drive along the \( z \) direction, and thus it is not robust against amplitude fluctuations of \( \Omega_2 \). Meaning, this decoupling limit is as good as the coherence time achieved in regular spin locking which is roughly an order of magnitude longer than \( T_2^* \).

For some scenarios (weak coupling regime) the coherence time is reached.

**B. Method II - Amplitude modulation**

With only a single amplitude modulation the method suffers from amplitude fluctuations in \( \Omega_0 \). These fluctuations could be eliminated by creating this drive as a second drive from a phase modulation as in the small frequency mismatch method. Specifically, the driving Hamiltonian of the NV center is given by

\[ H = \frac{\omega_0}{2} \sigma_z + \Omega_0 \cos (\omega_0 t + \varphi(t)), \]

(22)

where

\[ \varphi(t) = 2 \left( \frac{\Omega_1}{\Omega_0} \sin (\omega_0 t) \right) \]

\[ + \left( \frac{\Omega_2}{\Omega_0^2 - \Omega_3^2} \right) \left[ \Omega_0 \cos (\omega_3 t) \sin (\omega_0 t) \right. \]

\[ - \left. \Omega_3 \cos (\omega_0 t) \sin (\omega_3 t) \right) \]  

(23)
Moving to the IP with respect to \( H_0 = \frac{\omega_0}{2} \left( \Omega_1 + \Omega_2 \cos(\Omega_1 t) \right) \Omega_2 \), we obtain that
\[
H_1 = \frac{\Omega_0}{2} \sigma_z - \left( \Omega_1 + \Omega_2 \cos(\Omega_1 t) \right) \cos(\Omega_2 t) \sigma_z.
\] (24)
Hence, robustness to amplitude fluctuation in \( \Omega_0 \) is achieved. Moreover, due to the utilization of phase modulation, increasing either \( \Omega_1, \Omega_2 \), or \( \Omega_3 \) is not associated with an increased power consumption.

VIII. POWER CONSUMPTION

In this section we consider the difference in power consumption of the proposed schemes in comparison to the common Hartmann-Hahn method. For a given driving field, the magnitude of the magnetic field is proportional to the Rabi frequency, \( B(t) = \Omega(t) \), and since the magnitude of the electric field is proportional to the magnitude of the magnetic field we have that the power density \( P(t) = \frac{1}{\mu_0} E \times B \), where \( \mu_0 \) is the vacuum permeability, is \( P(t) \propto \Omega^2(t) \). Because we are only interested in the ratio between the power consumption of the Hartmann-Hahn method, \( P_{HH}(t) \), and the power consumption of a proposed method \( m \), \( P_m(t) \), we have that \( \frac{P_{HH}(t)}{P_m(t)} = \frac{\Omega_{HH}^2(t)}{\Omega_m^2(t)} \), and hence, it is not necessary to calculate the exact power density.

We consider two figures of merits for the comparison of power consumption. The first is the peak power of a drive (the maximal instantaneous power value), which we denote by \( p_{\text{peak}} \), and the second is the total cycle power that is required for a complete energy transfer (flip-flop) between the NV spin and the nucleus, which is denoted by \( p_{\text{cycle}} = \int_0^T P(t) \, dt \), where \( T \) is the cycle time.

The Rabi frequency of an on-resonance Hartmann-Hahn drive is given by \( \Omega_{HH} = \Omega \cos(\omega_0 t) \), where \( \Omega = \omega_0 \). Denoting the NV-nucleus coupling rate by \( g \), the Hartmann-Hahn cycle time is given by \( T_{HH} = \frac{2\pi}{\delta} \). For the Hartmann-Hahn drive we therefore have that
\[
p_{\text{peak}} \propto \Omega^2, \quad p_{\text{cycle}} \propto \frac{1}{2} \Omega^2 T_{HH},
\] (25)
where for \( p_{\text{cycle}} \) the above expression is approximately correct in the limit of \( \omega_0 \gg \Omega \).

A. Phase modulation

The Rabi frequency of a phase modulated driving field is given by \( \Omega_1 \cos \left( \omega_0 t + \frac{\Omega_2}{\Omega_1} \sin(\Omega_1 t) \right) \). In the case of phase modulation the effective NV-nucleus coupling rate is reduced by a factor of 2 (see Eq. (9)) and hence the cycle time is increased by a factor of 2, \( T_{PM} = \frac{4\pi}{g} \), so we have that
\[
p_{\text{peak}} \propto \Omega_1^2, \quad p_{\text{cycle}} \propto \frac{1}{2} \Omega_1^2 T_{PM}.
\] (26)
This results in
\[
\frac{p_{\text{peak}}}{p_{PM}} = \frac{\Omega_1^2}{\Omega_1^2} = \frac{p_{\text{cycle}}}{p_{PM}} \approx \frac{\Omega_1^2 T_{HH}}{\Omega_1^2 T_{PM}}.
\] (27)
The phase modulation scheme is relevant for the small frequency mismatch regime, where good polarization rates can be achieved for \( \Omega_2 = \Omega_1 \). In this case \( \Omega = \omega_0 = 2\Omega_1 \) and hence
\[
\frac{p_{\text{peak}}}{p_{PM}} = \frac{\Omega_1^2}{\Omega_1^2} = 4, \quad \frac{p_{\text{cycle}}}{p_{PM}} \approx \frac{\Omega_1^2 T_{HH}}{\Omega_1^2 T_{PM}} = 2.
\] (28)

B. Detuned driving field

The Rabi frequency of a detuned driving field is given by \( \Omega_1 \cos (\omega_0 t - \delta) \). Recall that the effective NV-nucleus coupling rate is reduced by a factor of \( \sin \theta \approx \frac{\Omega_1}{\delta} \) from \( g \) to \( \approx \frac{\Omega_1}{\delta} g \) so the cycle time is increased to \( T_{det} = \frac{2\pi}{\delta \Omega_1} \). For a detuned driving field
\[
p_{\text{peak}} \propto \Omega_1^2, \quad p_{\text{cycle}} \propto \frac{1}{2} \Omega_1^2 T_{det}.
\] (29)
Assuming, for example, that \( \delta = 10\Omega_1 \) so \( \omega_0 = \sqrt{101}\Omega_1 \), which corresponds to the large frequency mismatch, we have that
\[
\frac{p_{\text{peak}}}{p_{det}} = \frac{\Omega_1^2}{\Omega_1^2} = 101, \quad \frac{p_{\text{cycle}}}{p_{det}} \approx \frac{\Omega_1^2 T_{HH}}{\Omega_1^2 T_{det}} = 10.1.
\] (30)

C. Amplitude modulation

The Rabi frequency of an amplitude modulated driving field is given by \( \Omega_0 + \Omega_1 \cos(2\Omega_2 t) \). Recall that the effective NV-nucleus coupling rate is reduced by a factor of \( J_1(\Omega_1) \approx \frac{\Omega_1}{\Omega_1^2} \) from \( g \) to \( \approx \frac{\Omega_1}{\Omega_1^2} g \) so the cycle time is increased to \( T_{det} = \frac{4\Omega_1^2}{\Omega_1^2} \). For an amplitude modulated driving field
\[
p_{\text{peak}} \propto (\Omega_0 + \Omega_1)^2, \quad p_{\text{cycle}} \propto \frac{1}{2} \left( \Omega_0^2 + \frac{1}{2} \Omega_1^2 \right) T_{AM}.
\] (31)
Assuming, for example, that \( \Omega_2 = 9\Omega_1 \) so \( \omega_0 = 10\Omega_1 \), which corresponds to the large frequency mismatch, we have that
\[
\frac{p_{\text{peak}}}{p_{AM}} = \frac{(\Omega_0 + \Omega_1)^2}{(\Omega_0 + \Omega_1)^2} = 25, \quad \frac{p_{\text{cycle}}}{p_{AM}} \approx \frac{\Omega_1^2 T_{HH}}{(\Omega_0^2 + \Omega_1^2) T_{AM}} = 3.7.
\] (32)

IX. QUANTUM SENSING

In this section we show how the \( \sigma_- \), \( \sigma_+ \) operators can be transformed into a \( \sigma_x \) (or \( \sigma_y \)) operators. For the case of the low
frequency mismatch this can be achieved by adding an extra drive on the NV in the $y$ or $z$ direction in Eq. 9 which rotates at $\Omega_2$ (this amounts to $\Omega_2 \cos(\omega_0 t) \cos(\Omega_2 t) \sigma_z$). Specifically, consider the following Hamiltonian,

$$ H = \frac{\omega_0}{2} \sigma_z + \frac{\omega_l}{2} I_z + g \sigma_z I_x 
+ \Omega_1 \sigma_z \cos(\omega_0 t) - \Omega_2 \sigma_z \cos(\Omega_1 t) 
+ \Omega_s \cos(\omega_0 t) \cos(\Omega_2 t) \sigma_z. \quad (33) $$

We proceed in a similar manner as in the previous sections. Moving to the IP with respect to $H_0 = \frac{\omega_0}{2} \sigma_z$ and to the basis of the dressed states we have that

$$ H_I = \frac{\Omega_1}{2} \sigma_z + \frac{\omega_l}{2} I_z - g \sigma_z I_x 
+ \Omega_2 \sigma_z \cos(\Omega_1 t) + \frac{\Omega_s}{2} \cos(\Omega_2 t) \sigma_z. \quad (34) $$

Moving to the second IP with respect to $H_0 = \frac{\Omega_1}{2} \sigma_z$ and to the basis of the double-dressed states results in

$$ H_{II} = \frac{\Omega_2}{2} \sigma_z + \frac{\omega_l}{2} I_z - g (\cos(\Omega_1 t) \sigma_z - \sin(\Omega_1 t) \sigma_y) I_x 
- \frac{\Omega_s}{2} \cos(\Omega_2 t) \sigma_z. \quad (35) $$

Moving to the third IP with respect to $H_0 = \frac{\Omega_2}{2} \sigma_z$ and to the basis of the triple-dressed states results in

$$ H_{III} \approx \frac{\Omega_1}{2} \sigma_z + \frac{\omega_l}{2} I_z 
+ g \sin(\Omega_1 t) \sin(\Omega_2 t) \sigma_z \cos(\Omega_2 t) \sigma_x I_x. \quad (36) $$

where we have omitted fast rotating terms. Thus, in the fourth IP, with respect to $H_0 = \frac{\Omega_1}{2} \sigma_z + \frac{\omega_l}{2} I_z$ we get a Hamiltonian which can be used for sensing the nucleus frequency, i.e.

$$ H_{IV} \approx \frac{g}{4} \sigma_z (I_x \cos(\delta t) - I_y \sin(\delta t)), \quad (37) $$

where $\delta = \Omega_1 + \Omega_2 - \omega_l$. As the extra term acts as a spin locking term at $\Omega_s$, the robustness of the methods is not decreased.

The same can be done in the large frequency mismatch regime. Thus, the interaction should be changed from the flip-flop interaction $g (\sigma_z I_x + \sigma_y I_y)$ to $g \sigma_z I_x$ by adding, for example, a $\sigma_x$ drive to the modulation. In this case the Hamiltonian is transformed to (the derivation is similar to the derivation in the small frequency mismatch, Eq. 33 - 37)

$$ H \approx g J_1 \left( \frac{\Omega_1}{\Omega_2} \right) \sigma_z (I_x \cos(\delta t) - I_y \sin(\delta t)). \quad (38) $$