Electron-electron Bound States in Parity-Preserving QED$_3$

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By considering the Higgs mechanism in the framework of a parity-preserving Planar Quantum Electrodynamics, one shows that an attractive electron-electron interaction may come out. The $e^−−e^−$ interaction potential emerges as the non-relativistic limit of the Möller scattering amplitude and it may result attractive with a suitable choice of parameters. Numerical values of the $e^−−e^−$ binding energy are obtained by solving the two-dimensional Schrödinger equation. The existence of bound states is to be viewed as an indicative that this model may be adopted to address the pairing mechanism in some systems endowed with parity-preservation.

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I. INTRODUCTION

In the latest 10 years, Planar Quantum Electrodynamics - QED$_3^−$ has shown to be an appropriate theoretical framework for discussing the low-energy limit of some Condensed Matter systems. Recent applications of this theory to underdoped high-$T_c$ superconductors [1] has again caught attention for its theoretical possibilities. The history of the relation between QED$_3$ and superconductivity goes back to the final 80’s, when the anyonic model was established by the works of Laughlin [3], and others [3]. Despite its initial success, it was afterwards demonstrated that anyonic model supports the superconducting phase only at zero temperature [3]. An alternative approach, also based on the QED$_3$ framework, began to be adopted by Kogan [15] to explain the formation of electron-electron bound states. Into the domain of the QED$_3$, there exits the necessity of yielding a mass to the gauge field in order to circumvent the appearance of a confining potential associated to the long-range Coulombian interaction. The Maxwell-Chern-Simons (MCS) term [14] is then introduced as the generator of (topological) mass for the photon, implying a screening on the Coulomb interaction. This MCS-QED$_3$ model was used by some authors [15], [17] as basic tool for evaluation of the Möller scattering amplitude at tree-level, whose Fourier transform (in the Born approximation) yields the $e^−e^−$ interaction potential. In a general way, these works furnish the same result: the $e^−e^−$ interaction comes out attractive when the topological mass ($ϑ$) surpasses the electronic mass ($m_e$), that is, $ϑ > m_e$. This condition prevents the applicability of the MCS model to solid-state systems, since the existence of a physical excitation with so large energy in the domain of a Condensed Matter system is entirely unlikely. It is possible to argue that the introduction of the Higgs mechanism in the context of the MCS electrodynamics [7], [10] brings out a negative contribution to the scattering potential that will make feasible a global attractive potential regardless the condition $ϑ > m_e$.

Some preliminary elucidations on the general character of this paper are noteworthy. In spite of dealing with electron-electron condensation (a mechanism that takes place in the realm Condensed Matter Physics), the approach adopted here does not follow the usual procedure of Solid State Physics, where one usually extracts information from the crystal lattice about the interactions of the system. Indeed, our intention is not to follow a solid-state approach, but to show instead that, starting from a quantum-field theoretical framework, one may obtain (in the non-relativistic limit) an attractive interaction that could be able to favour the electronic pairing. In this way, we really develop a typical field-theoretical approach, namely: one starts with a relativistic parity-preserving QED$_3$ Lagrangian (without MCS term) [4], [5], [6] containing fermions, scalar and vector bosons, and endowed with spontaneous symmetry breaking (SSB). After the SSB, a Higgs boson and a massive photon appear in the spectrum in the broken phase. These two particles mediate the Möller scattering, whose amplitude leads to a Bessel-$K_0$ interaction potential, that can be attractive (independent of the electron polarization) whenever the negative contribution stemming from the Higgs scalar interchange dominates over the repulsive gauge interaction. It is also important to stress that the explicit use of the Higgs-scalar interaction (generated from the SSB) as an active mediator in the Möller scattering is the key factor for the attainment of an attractive potential. Relying on the nonrelativistic approximation, the $K_0$− potential...
is inserted into the Schrödinger equation. Its numerical solution provides us with values of the $e^- e^-$ pairing energy, which are exhibited in Table I.

This paper is organized as follows. In Sec. II, we present the parity-preserving QED$_3$ model, explore some of its properties, and realize the SSB. We then follow a field-theoretical development that leads to the $e^- e^-$ interaction potential (derived in the non-relativistic limit). In Sec. III, we accomplish a numerical procedure in order to obtain numerical values for the binding energy of $e^- e^-$ pairs. In this sense, we implement the $K_o$-potential into the Schrödinger equation and solve it. The numerical data are then displayed in Table I. In Sec. IV, we present our Final Remarks.

II. PARITY-PRESERVING QED$_3$ WITH SPONTANEOUS SYMMETRY BREAKING

We start with a parity-preserving QED$_3$ action (with SSB) \cite{3}, \cite{10}, built up by two polarized fermionic fields ($\psi_+, \psi_-$) \cite{3}, \cite{10}, a gauge potential ($A_\mu$) and a complex scalar field ($\phi$):

$$S = \int d^3x \left\{ -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \overline{\psi}_+(i\partial - m_e)\psi_+ + \overline{\psi}_-(i\partial + m_e)\psi_- - y(\overline{\psi}_+\psi_- - \overline{\psi}_-\psi_+)\phi^*\phi + D^\mu \phi^* D_\mu \phi - V(\phi^*\phi) \right\},$$

(1)

with the scalar self-interaction potential, $V$, responsible for the SSB, taken as: $V(\phi^*\phi) = \mu^2 \phi^*\phi + \frac{\lambda}{2} (\phi^*\phi)^2 + \frac{\lambda}{2} (\phi^*\phi)^3$. This sixth-order potential is the most general one renormalizable in $1 + 2$ dimensions \cite{3}. The mass dimensions of the parameters $\mu$, $\zeta$, $\lambda$, $y$ are respectively 1, 1, 0, 0, and the covariant derivatives, $D_{\pm} \equiv (\partial + i e_3 A_\mu)\psi_{\pm}$, $D \phi \equiv (\partial + i e_3 A_\mu)\phi$, state the minimal coupling between $\psi_{\pm}$, $A_\mu$, and $\phi$. It is important to point out that the $U(1)-$symmetry coupling constant in $(1 + 2)$-dimensions, $e_3$, has dimension of (mass)$^\frac{1}{2}$, a relevant fact that must to be properly considered at the moment one needs to attribute numerical values to it. We are interested only on a stable vacuum, for which the following conditions on the potential parameters have to be fulfilled: $\lambda > 0$, $\zeta < 0$, $\mu^2 \leq \frac{\lambda^2}{16}$. After the breakdown, the scalar field acquires a non null vacuum expectation value (v.e.v.), $\langle \phi \rangle = v$, one readily gets: $\langle \phi^*\phi \rangle = v^2 - \zeta/2\lambda + \sqrt{(\zeta/2\lambda)^2 - \mu^2/\lambda}$. On the other hand, the condition for a minimum reads as: $\mu^2 + \zeta v^2 + \lambda v^4 = 0$. In the broken phase, the complex scalar field is parametrized by $\phi = v + H + i \theta$, where $\theta$ is the would-be Goldstone boson and $H$ is the Higgs scalar, both with vanishing v.e.v.’s ($\langle H \rangle = \langle \theta \rangle = 0$). By replacing this parametrization relation into the action \cite{1}, adopting the ’t Hooft gauge \cite{12} ($S_{gf} = \int d^3x \left\{ -\frac{1}{2\xi} (\partial^m A_m - \sqrt{2\xi M_A})^2 \right\}$), and finally taking only the bilinear and Yukawa interaction terms, one has:

$$S^{SSB} = \int d^3x \left\{ -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} M_H^2 A_\mu A_\mu + \overline{\psi}_+(i\partial - m_{\text{eff}})\psi_+ + \overline{\psi}_-(i\partial + m_{\text{eff}})\psi_- + \partial^\mu H \partial_\mu H - M_H^2 H^2 + \partial^\mu \partial^\nu \theta^2 - M_\theta^2 \theta^2 - \frac{1}{2\xi} (\partial^\mu A_\mu)^2 \right\} - 2yv(\overline{\psi}_+\psi_- - \overline{\psi}_-\psi_+)H - e_3 (\overline{\psi}_+ A\psi_+ + \overline{\psi}_- A\psi_-),$$

(2)

where $\xi$ is a dimensionless gauge parameter and the mass generated by the SSB are: $M_\theta^2 = 2\nu^2(\zeta + 2\lambda v^2)$ (Higgs mass), $m_{\text{eff}} = m_e + y v^2$(effective electron mass), and $M_H^2 = \xi M_A^2$. The latter corresponds to non-physical poles in the gauge and $\theta$-field propagator. Their effects are mutually canceled as already known from the study of the unitarity in the ’t Hooft gauge \cite{12}.

At this point, it is instructive to highlight that from now on our relevant “physical” action is the one obtained in the broken phase, that is, $S^{SSB}$. Based on this broken action, one derives the essential results of this paper. In this tree-level broken action, no register of $V$, the scalar potential, remains. In fact, we can assert that the main role of the sixth-order potential is basically to determine the occurrence of the spontaneous symmetry breaking in a planar theory, where one uses to require renormalisation. On physical grounds, one can say that the sixth-power potential can be replaced by a fourth-power form ($\lambda\phi^4$-type), also able to induce the SSB and the phase transition, without effectively changing the features of the broken phase. The reason to use a sixth-power scalar self-interacting potential is exclusively that it is the most general renormalizable potential in Planar Quantum Electrodynamics (a quantum field theory reason). Written at tree-level, the action \cite{12} exhibits no vertex with more than three legs, which in turn contribute to loop diagrams. So, at this level of approximation, loop contributions are excluded from the physical model. One knows these loop diagrams correct the classical theory, but in a perturbation perspective, their contribution may not be enough to modify the essence of a result constructed at tree-level. This fact justifies the omission of the scalar interaction terms of higher order ($H^2, H^3, H^4, \ldots$) at this level. Although, these higher-order
terms need to be considered when one desires to analyze radiative corrections or one tries to pass from this microscopic theory to a phenomenological model where the fermions and gauge fields are integrated out.  

In the low-energy limit (Born approximation), the two-particle interaction potential is given by the Fourier transform of the two-particle scattering amplitude \[^{[18]}\]. It is important to stress that in the case of the non-relativistic Möller scattering, one should consider only the t-channel (direct scattering) \[^{[18]}\] even for distinguishable electrons, since in this limit they recover the classical notion of trajectory. From the action \[^{[1]}\], there follow the Feynman rules for the interaction vertices:  

\[ V_{\pm} = \pm 2iv_{\ell} \gamma_{\ell} \]  

Evaluating now the Fourier transform of the total amplitude scattering wave functions.

A spin dependence will arise in the low-energy terms need to be considered when one desires to analyze radiative corrections or one tries to pass from this microscopic theory to a phenomenological model where the fermions and gauge fields are integrated out.

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\[ (\langle HH \rangle \text{ and } \langle A_{\mu} A_{\nu} \rangle \text{ are the Higgs and massive photon propagators. Expressions } \[^{[3]}\] \text{ and } \[^{[4]}\] \text{ represent the scattering amplitudes for electrons of equal and opposite polarizations mediated by the Higgs particle, whereas Eqs. } \[^{[3]}\] \text{ and } \[^{[4]}\] \text{ correspond to the massive photon as mediator.} \]

The spinors \( u_{\pm}(p) \) stand for the positive-energy solution of the Dirac equation \((p^2 - m) u_{\pm}(p) = 0 \). We adopt the following conventions \( \eta_{\mu\nu} = (+, -, -), \) \( [\gamma^\mu, \gamma^\nu] = 2i\epsilon^{\alpha\beta\gamma\delta}\gamma_{\alpha}, \gamma^\mu = (\sigma_z, -i\sigma_x, i\sigma_y) \), whence one obtains

\[ u_+(p) = \frac{1}{\sqrt{N}} \left[ E + m, -ip_x - p_y \right], \quad u_-(p) = \frac{1}{\sqrt{N}} \left[ -ip_x + p_y, E + m \right], \tag{7} \]

with \( N = 2m(E + m) \) being the normalization constant that assures \( \overline{u}_{\pm}(p)u_{\pm}(p) = \pm 1 \). Working in the center-of-mass frame \[^{[5]}\], \[^{[6]}\], the scattering amplitudes \( M_{\text{higgs}} = -2v^2 y^2 \left( k^2 + M_H^2 \right)^{-1} \), \( M_{\text{gauge}} = +e_3 \left( k^2 + M_A^2 \right)^{-1} \) reveal to be independent of the spin polarization. Evaluating now the Fourier transform of the total amplitude scattering (\( M_{\text{total}} = M_{\text{higgs}} + M_{\text{gauge}} \)), the following interaction potential comes out:

\[ V^{CM}(r) = -\frac{1}{2\pi} \left[ 2v^2 y^2 K_0(M_H r) - e_3^2 K_0(M_A r) \right]. \tag{8} \]

Considering equal Higgs and Proca masses \( (M_H = M_A \leftrightarrow e_3^2 = \zeta + 2\lambda v^2) \), the potential \[^{[8]}\] takes the form

\[ V^{CM}(r) = CK_0(M_A r), \text{ with: } C = -\frac{1}{2\pi} \left[ 2v^2 y^2 - e_3^2 \right]. \tag{9} \]

It becomes attractive whenever \( C < 0 \), that is, \( 2v^2 y^2 > e_3^2 \). Now, it is necessary to point out that this result is independent of the spin-polarization of the scattered electrons, whereas the potential enclosed in Ref. \[^{[8]}\] derived at the same physical conditions, shows an erroneously inversion of sign (for the gauge interaction between antiparallel-spin electrons). A spin dependence will arise in the low-energy \( e^- e^- \) potential only when one considers the presence of the Chern-Simons term, resulting in a Maxwell-Chern-Simons-Proca model \[^{[9]}\].

### III. WAVEFUNCTION PROPERTIES AND NUMERICAL ANALYSIS

Having determined the interaction potential, one must now look for the numerical evaluation of the binding energy associated to the \( e^- e^- \) pairs. In the non-relativistic limit, the complete two-dimensional Schrödinger equation (supplemented by the Bessel-\( K_0 \) potential)

\[ \frac{\partial^2 \varphi(r)}{\partial r^2} + \frac{1}{r} \frac{\partial \varphi(r)}{\partial r} - \frac{l^2}{r^2} \varphi(r) + 2\mu_{\text{eff}} [E - CK_0(M_A r)] \varphi(r) = 0 \]

yields the energy of the two interacting particles. Here, \( \mu_{\text{eff}} = \frac{1}{2} m_{\text{eff}} \varphi(r) \) is the effective reduced mass of the \( e^- e^- \) system and \( \varphi \) represents the (relative) spatial part of the complete antisymmetric 2-electron wavefunction: \( \Psi(r_1, s_1, r_2, s_2) = \psi(R) \varphi(r) \chi(s_1, s_2) \), while \( \psi(R), \chi(s_1, s_2) \) stand for the center-of-mass and the spin wave functions.
For a numerical solution of the Schrödinger equation, we employ the variational method. In this respect, we take as starting point the choice of a wave function that stands for the generic features of the $e^- e^-$ state: the trial function, whose definition must observe some conditions, such as the asymptotic behavior at infinity, the analysis of its free version and its behavior at the origin. With the help of the transformation $\varphi(r) = \frac{1}{\sqrt{\nu}} g(r)$, Eq.(10) is transformed into

$$
\frac{\partial^2 g(r)}{\partial r^2} - \frac{l^2 - \frac{1}{4}}{r^2} g(r) + 2\mu_{\text{eff}} [E - CK_o(M_{Ar})] g(r) = 0,
$$

whose free version ($V(r) = 0$) for zero angular momentum ($l = 0$) state simplifies to

$$
\left[ \frac{\partial^2}{\partial r^2} + \frac{1}{4r^2} + k^2 \right] u(r) = 0.
$$

Its general solution is given by $u(r) = B_1 \sqrt{r} J_0(kr) + B_2 \sqrt{r} Y_0(kr)$, with $B_1$ and $B_2$ being arbitrary constants and $k = \sqrt{2\mu_{\text{eff}} E}$. In the $r \to 0$ limit, the solution to Eq.(12) approaches $u(r) \to \sqrt{r} + \lambda \sqrt{r \ln r}$. Since the second term in Eq.(12) behaves like an attractive potential, $-1/4r^2$, this implies the unphysical possibility of obtaining a bound state ($E < 0$) even for $V(r) = 0$ [11]. Among the infinite number of self-adjoint extensions of the differential operator $-d^2/dr^2 - 1/4r^2$, the only physical choice corresponds to the Friedrichs extension ($B_2 = 0$), which behaves like $\sqrt{r}$ at the origin, indicating this same behavior for $u(r)$. The complete equation, $V(r) \neq 0$, will preserve the self-adjointness of free Hamiltonian, if the potential is “weak” in the sense of the Kato condition: $\int_{-\infty}^{\infty} r(1 + |\ln(r)|) |V(r)| dr < \infty$. This condition also sets up a finite number of bound states (discrete spectrum) and the semi-boundness of the complete Hamiltonian. Provided that the Bessel-$K_0$ potential, given by Eq. (9), satisfies the Kato condition, the self-adjointness of the total Hamiltonian is assured and the existence of bound states is allowed. On the other hand, at infinity, the trial function must vanish asymptotically in order to fulfill square integrability. Therefore, a good and suitable trial function (for $l = 0$) could be taken by

$$
g(r) = \sqrt{r} \exp(-\beta r),
$$

where $\beta$ is a free spanning parameter to be numerically fixed in order to minimize the binding energy.

Once the trial function is already known, it still lacks a discussion on the physical parameters ($\nu^2, e_3^2, y^2$) that compose the proportionality constant, $C$, of the Bessel potential, in such a way that numerical values may be attributed to them. The vacuum expectation value, $\nu^2$, indicates the energy scale of the spontaneous breakdown of the $U(1)$-local symmetry. This is a free parameter, being usually determined by some experimental data associated to the phenomenology of the model under investigation, as occurs in the electroweak Weinberg-Salam model, for example. On the other hand, the $y$ parameter measures the coupling between the fermions and the Higgs scalar, working in fact as an effective constant that embodies contributions of all possible mechanisms of electronic interaction via Higgs-type (scalar) excitations, as the spinless bosonic interaction mechanisms: phonons, plasmons, and other collective excitations. This theoretical similarity suggests an identification of the field theory parameter with an effective electron-scalar coupling (instead of an electron-phonon one): $y \to \lambda_m$. Specifically, in QED3, the electromagnetic coupling constant squared, $e_3^2$, has dimension of mass, rather than the dimensionless character of the usual four-dimensional QED4 coupling constant. This fact might be understood as a memory of the third dimension that appears (into the coupling constant) when one tries to work with a theory intrinsically defined in three space-time dimensions. This dimensional peculiarity could be better implemented through the definition of a new coupling constant in three space-time dimensions [13], [14]: $e \to e_3 = e/\sqrt{l_{\perp}}$, where $l_{\perp}$ represents a length orthogonal to the planar dimension. The smaller is $l_{\perp}$, smaller is the remnant of the frozen dimension, larger is the planar character of the model and the coupling constant $e_3$, what reveals its effective nature. In this sense, it is instructive to notice that the effective value of $e_3^2$ is always larger than $e^2 = 1/137$ whenever $l_{\perp} < 1973.23 \, \text{Å}$, since $1 \, \text{Å}^{-1} = 1973.26 \, \text{eV}$. This particularity broadens the repulsive interaction for small $l_{\perp}$ and requires an even stronger Higgs contribution to account for a total attractive interaction.

The following Table, constructed for zero angular momentum state ($l = 0$), has as input data the three parameters ($\nu^2, l_{\perp}, y$), while the output parameters are: $\beta$—the minimization parameter, $E_{e^- e^-}$—the $e^- e^-$ binding energy, and $\langle r \rangle$—the average-length of the wavefunction.
| $v^2$ (meV) | $l_\perp$ (Å) | $y$ | $C_y$ (meV) | $M_H$ (meV) | $\beta$ | $E_{e-e}$ (meV) | $\langle r \rangle$ (Å) |
|------------|------------|-----|----------|----------|-------|---------------|--------------|
| 120.0      | 15.0       | 2.1 | -15.7    | 480.0    | 63.1  | -74.7        | 15.6         |
| 120.0      | 14.0       | 2.1 | -4.8     | 496.8    | 35.8  | -19.7        | 27.6         |
| 120.0      | 13.0       | 2.2 | -8.6     | 515.6    | 47.2  | -37.8        | 20.9         |
| 100.0      | 12.0       | 2.6 | -24.2    | 489.9    | 81.1  | -120.2       | 12.2         |
| 100.0      | 12.0       | 2.5 | -8.0     | 489.9    | 45.9  | -35.2        | 21.5         |
| 100.0      | 10.0       | 2.7 | -2.9     | 536.6    | 27.8  | -11.0        | 35.5         |
| 100.0      | 10.0       | 2.8 | -20.4    | 536.6    | 72.1  | -97.6        | 13.7         |
| 100.0      | 6.0        | 3.5 | -8.0     | 692.8    | 45.9  | -32.5        | 21.5         |
| 80.0       | 6.0        | 3.9 | -5.4     | 619.6    | 37.6  | -21.4        | 26.2         |
| 70.0       | 4.0        | 5.1 | -6.6     | 709.9    | 41.7  | -26.2        | 23.6         |
| 60.0       | 8.0        | 3.9 | -4.0     | 464.7    | 33.1  | -16.7        | 29.8         |

| TABLE I. Input data ($v^2, l_\perp, y$) and output data ($E_{e-e}, \langle r \rangle$) for the Schrödinger Equation |
IV. FINAL REMARKS

The numerical data of Table I show that the attractive Bessel potential, derived for a non-relativistic regime, may effectively promote the formation of $e^-e^-$ bound states. The procedure here carried out puts in evidence that, by properly fitting the free parameters of the model, one can obtain bound states of the order of 10 – 100 meV and wavefunction average-length in the range 10 – 30 Â, which may reveal the suitability of the framework here adopted to address the issue of electron-electron condensation in the realm of parity-preserving planar systems. Finally, we can assert that the photon Proca mass, generated by the SSB, plays the same role of the topological mass $(\vartheta)$ in that it determines the Coulomb interaction screening and the Meissner effect, without breaking parity-symmetry. The data exhibited in Table I concern an s-wave state: $l = 0$ and spin singlet $(\uparrow\downarrow, S = 0)$. According to the results of this paper, we conclude by stressing the fundamental role played by the Higgs mechanism in QED$_3$ as essential for the appearance of an attractive $e^-e^-$ potential.

Final comments on the general procedure here employed are still necessary. We conceive this paper as the first part of a two-stage project, described as follows. At the first stage, we have a microscopic model, where the presence of all degrees of freedom (relative to fermions, vector and scalar bosons) is necessary. In this moment, our purpose is to exhibit a microscopic mechanism, at tree level, able to yield the electronic pairing. Since we are bound to the tree-approximation, higher powers in $H$ need not be considered whenever computing the transition amplitude from which we read off the inter-particle potential. This is exactly what we have done here. In a second stage of development, still to be performed, one should take into account the high-order terms in $H$ (stemming from the Higgs potential and from the electronic coupling). A functional integration on the fermions and vector bosons must be carried out, yielding an effective functional with dependence only on the Higgs-scalar field. We think this functional may exhibit a Ginzburg-Landau-like form, if the sixth-order potential is replaced by a quartic-order potential. To our mind, the Higgs field, which represents the scalar excitations (relative to the vacuum expectation value), will play (after the functional integration) the same role as the order parameter plays in the G-L model: that of a fluctuation field in the context of a mean-field theory.

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