Efficient Approximate Query Answering over Sensor Data with Deterministic Error Guarantees

Jaqueline Brito  
UC San Diego  
jabrito@cs.ucsd.edu

Korhan Demirkaya  
UC San Diego  
kdemirk@cs.ucsd.edu

Boursier Etienne  
UC San Diego  
eboursier@cs.ucsd.edu

Yannis Katsis  
UC San Diego  
ikatsis@cs.ucsd.edu

Chunbin Lin  
UC San Diego  
chunbinlin@cs.ucsd.edu

Yannis Papakonstantinou  
UC San Diego  
yannis@cs.ucsd.edu

ABSTRACT

With the recent proliferation of sensor data, there is an increasing need for the efficient evaluation of analytical queries over multiple sensor datasets. The magnitude of such datasets makes exact query answering infeasible, leading researchers into the development of approximate query answering approaches. However, existing approximate query answering algorithms are not suited for the efficient processing of queries over sensor data, as they exhibit at least one of the following shortcomings: (a) They do not provide deterministic error guarantees, resorting to weaker probabilistic error guarantees that are in many cases not acceptable, (b) they allow queries only over a single dataset, thus not supporting the multitude of queries over multiple datasets that appear in practice, such as correlation or cross-correlation and (c) they support relational data in general and thus miss speedup opportunities created by the special nature of sensor data, which are not random but follow a typically smooth underlying phenomenon.

To address these problems, we propose PlatoDB; a system that exploits the nature of sensor data to compress them and provide efficient processing of queries over multiple sensor datasets, while providing deterministic error guarantees. PlatoDB achieves the above through a novel architecture that (a) at data import time pre-processes each dataset, creating for it an intermediate hierarchical data structure that provides a hierarchy of summarizations of the dataset together with appropriate error measures and (b) at query processing time leverages the pre-computed data structures to compute an approximate answer and deterministic error guarantees for ad hoc queries even when these combine multiple datasets.

As a result of its novel architecture, PlatoDB exhibits speedups of 1-3 orders of magnitude compared to systems that use the entire sensor datasets to compute exact query answers during experiments performed on real sensor datasets.

1. INTRODUCTION

The increasing affordability of sensors and storage has recently led to the proliferation of sensor data in a variety of domains, including transportation, environmental protection, healthcare, fitness, etc. These data are typically of high granularity and as a result have substantial storage requirements, ranging from a few GB to many TB. For instance, a Formula 1 produces 20GB of data during two 90-minute practice sessions, while a commercial aircraft may generate 2.5TB of data per day.

The magnitude of sensor datasets creates a significant challenge when it comes to query evaluation. Running analytical queries over the data (such as finding correlations between signals), which typically involve aggregates, can be very expensive, as the queries have to access significant amounts of data. This problem becomes worse when queries combine in ad hoc ways multiple sensor datasets. For instance, consider a data analytics scenario, where a user wants to combine (a) a location dataset providing the location of users for different points in time (as recorded by their smartphone’s GPS) and (b) an air pollution dataset recording the air quality at different points in time and space (as recorded by air quality sensors) to compute the average quality of air inhaled by each user over a certain time period. Answering this query requires accessing all location and air pollution measurements in the time period of interest, which can be substantial for long periods. To solve this problem, researchers have proposed approximate query processing algorithms 

1. Lack of deterministic error guarantees. Most query approximation algorithms provide probabilistic error guarantees. While this is sufficient for some use cases, it does not cover scenarios where the user needs deterministic guarantees ensuring that the returned answer is within the specified error bounds.

2. Lack of support of queries over multiple datasets. Many techniques, such as wavelets, provide error guarantees only for queries over a single dataset. The errors can be arbitrarily large for queries ranging over multiple datasets, as they are unaware of how multiple datasets interact with each other.

3. Data agnosticism. The majority of existing techniques works for relational data in general and does not leverage compression opportunities that come from the fact that sensor data are not random in nature but follow typically smooth continuous phenomena.

To overcome the limitations, we design the PlatoDB system, which leverages the nature of sensor data to compress them and provide efficient processing of analytical queries over multiple sensor datasets, while providing deterministic error guarantees. In a nutshell, PlatoDB operates as follows: When initiated, it preprocesses sensor datasets. The preprocessing phase involves compressing the data into a hierarchical structure that allows for fast approximate query answering. The system is designed to handle the following shortcomings of existing approaches:

1. Lack of deterministic error guarantees.
2. Lack of support for queries over multiple datasets.
3. Data agnosticism.

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1http://www.zdnet.com/article/formula-1-racing-sensors-data-speed-and-the-internet-of-things/

2http://www.datasciencecentral.com/profiles/blogs/that-s-data-science-airbus-puts-10-000-sensors-in-every-single
cesses each time series dataset and builds for it a binary tree structure, which provides a hierarchy of summarizations of segments of the original time series. A node in the tree structure summarizes a segment of time series through two components: (i) a compression function estimating the data points in the segment, and (ii) error measures indicating the distance between the compressed segment and the original one. The lower level nodes refers to finer-grained segments and smaller errors. During runtime, PlatoDB takes as input an aggregate query over potentially multiple sensor datasets together with an error or time budget and utilizes the tree structure for each of the datasets involved in the query to obtain an approximate answer together with a deterministic error guarantee that satisfies the time/error budget.

Contributions. In this work, we make the following contributions:

- We define a query language over sensor data, which is powerful enough to express most common statistics over both single and multiple time series, such as variance, correlation, and cross-correlation (Section 3).
- We propose a novel tree structure (structurally similar to hierarchical histograms) and a corresponding tree generation algorithm that provides a hierarchical summarization of each time series independently of the other time series. The summarization is based on the combination of arbitrary compression functions that can be reused from the literature together with three novel error measures that can be used to provide deterministic error guarantees, regardless of the employed compression function (Section 4).
- We design an efficient query processing algorithm operating on the pre-computed tree structures, which can provide deterministic error guarantees for queries ranging over multiple time series, even though each tree refers to one time series in isolation. The algorithm is based on a combination of error estimation formulas that leverage the error measures of individual time series segments to compute an error for an entire query (Section 5), together with a tree navigation algorithm that efficiently traverses the time series tree to quickly compute an approximate answer that satisfies the error guarantees (Section 6).
- We conduct experiments on two real-life datasets to evaluate our algorithms. The results show that our algorithm outperforms the baseline by 1-3 orders of magnitude (Section 7).

2. SYSTEM ARCHITECTURE

Figure 1 depicts PlatoDB's architecture. PlatoDB operates in two steps, performed at two different points in time. At data import time, PlatoDB pre-processes the incoming time series data, creating a segment tree structure for each time series. At query execution time, it leverages these segment trees to provide an approximate query answer together with deterministic error guarantees. We next describe these two steps in detail.

Off-line Pre-Processing. At data import time, PlatoDB takes as input a set of time series. The time series are created from the raw sensor data by the typical Extract-Transform-Load (ETL) scripts potentially combined with de-noising algorithms, which is outside the focus of this paper.

For each such time series, PlatoDB's Segment Tree Generator creates a hierarchy of summarizations of the data in the form of a segment tree; a tree, whose nodes summarize the data for segments of the original time series. Intuitively, the structure of the segment tree corresponds to a way of splitting the time series recursively into smaller segments: The root $S_1$ of the tree corresponds to the entire time series, which can be split into two subsegments (generally of different length), represented by the root’s children $S_{1,1}$ and $S_{1,2}$. The segment corresponding to $S_{1,1}$ can in turn split further into two smaller segments, represented by the children $S_{1,1,1}$ and $S_{1,1,2}$ of $S_{1,1}$ and so on. Since each node provides a brief summarization of the corresponding segment, lower levels of the tree provide a more precise representation of the time series than upper levels. As we will see later, this hierarchical structure of segments is crucial for the query processor’s ability to adapt to a wide variety of error/time budgets provided by the user. When the user is willing to accept a larger error, the query processor will mostly use the top levels of the trees, providing a quick response. On the other hand, if the user demands a lower error, the algorithm will be able to satisfy the request by visiting lower levels of the segment trees (which exact nodes will be visited also depends on the query and the interplay of the time series in it). Leveraging the trees, PlatoDB can even provide users with continuously improving approximate answers and error guarantees, allowing them to stop the computation at any time, similar to works in online aggregation [15, 26].

Each node of the tree summarizes the corresponding segment through two data items: (a) a compression function, which represents the data points in a segment in a compact way (e.g., through a constant [21] or a line [19]), and (b) a set of error measures, which are metrics of the distance between the data point values estimated by the compression function and the actual values of the data points. As we will see, the query processor uses the compression function and error measures of the segment tree nodes to produce an approximate answer of the query and the error guarantees, respectively. Interestingly, PlatoDB’s internals are agnostic of the compression function used. As we will discuss in Section 4, PlatoDB’s query processor works independently of the employed compression functions, allowing the system to be combined with all popular compression techniques. For instance, in our example above we utilized the Piecewise Aggregate Approximation (PAA) [21], which returns the average of a set of values. However, we could have used other compression techniques, such as the Adaptive Piecewise Constant Approximation (APCA) [20], the Piecewise Linear Representation (PLR) [19], or others.

Remark. It is important to note that the segment tree is not necessarily a balanced tree. PlatoDB decides whether a segment need to be split based on how close the values derived from the compression function are to the actual values of the segment. PlatoDB splits the segment when the difference is large. Intuitively, this means that the segment tree contains more nodes for parts of the domain where the time series is irregular and/or rapidly changing, and fewer nodes for the smooth parts. PlatoDB treats the problem of finding the splitting positions as an optimization problem, splitting at positions that can bring the largest error reduction. We will present the segment tree generator algorithms in Section 4.

Example 1. Figure 1(a) shows the segment tree for a time series $T$. The root node $S_1$ of the tree (corresponding to the segment covering the entire time series) summarizes this segment through two items: a set of parameters describing a compression function $f_1$ (in this case the function returns the average $v$ of the values of the time series and can therefore be described by the single value $v$) and a set of error measures $M_1$ (the details of error measures will be presented in Section 4). This entire segment is split into two
subsegments \( S_{1.1} \) and \( S_{1.2} \), giving rise to the identically-named tree nodes. Note that the tree is not balanced. Segment \( S_{1.2} \) is not split further as its function \( f_{1.2} \) correctly predicts the values within the corresponding segment. In contrast, the segment \( S_{1.1} \) displays great variability in the time series’ values and is thus split further into segments \( S_{1.1.1} \) and \( S_{1.1.2} \).

**On-line Query Processing.** At query evaluation time, PlatoDB’s Query Processor receives a query and a time or error budget and leverages the pre-processed segment trees to produce an approximate query answer and a corresponding error guarantee satisfying the provided budget.

To compute the answer and error guarantee, PlatoDB traverses in parallel in a top-down fashion the segment trees of all time series involved in the query. At any step of this process, it uses the compression function and error measures in the current accessed nodes to calculate an approximate query answer and the corresponding error. If it has not reached yet the time/error budget (i.e., if there is still time left or if the current error is still greater than the error budget), PlatoDB greedily chooses among all the currently accessed nodes the one, whose children nodes would yield the greatest error reduction and uses them to replace their parent in the answer and error estimation. Otherwise, PlatoDB stops accessing further nodes of the segment trees and outputs the currently computed approximate answer and error. Query processing is described in detail in Sections 5 and 6.

**Remark.** It is important to note that, in contrast to existing approximate query answering systems, PlatoDB can answer queries that span across different time series, even though the segment trees were pre-processed for each time series individually. As we will see, the fact that the segment trees were generated for each time series individually, leads to interesting problems at query processing time, such as aligning the segments of different time series and reasoning about how these segments interact to produce the query answer and error guarantees. Finally, it is also important to note that PlatoDB adapts to the provided error budget by accessing different number of nodes. Larger error budgets lead to fewer node accesses, while smaller error budgets require more node accesses.

**Example 2.** Consider a query \( Q \) involving two time series \( T_1 \) and \( T_2 \) and an error budget \( \varepsilon_{\max} = 10 \). Figure 1(b) shows how the query processing algorithm uses the pre-computed segment trees of the two time series. PlatoDB first accesses the root nodes of both segment trees in parallel and computes the current approximate query answer \( \hat{R} \) and error \( \hat{\varepsilon} \), using the compression function and error measures in the root nodes. Let’s assume that \( \hat{\varepsilon} = 20 \). Since \( \hat{\varepsilon} > \varepsilon_{\max} \), PlatoDB keeps traversing the trees by greedily choosing a node and replacing it by its children, so that the error reduction at each step is maximized. This process continues until the error budget is satisfied. For instance, assume that using the yellow shaded nodes in Figure 1(b) PlatoDB obtains an error \( \hat{\varepsilon} = 6 < \varepsilon_{\max} \). Then PlatoDB stops traversing the trees and outputs the approximate answer and the error \( \hat{\varepsilon} = 6 \). Note that none of the descendants of the shaded nodes is touched, resulting in big performance savings.

As a result of this architecture, PlatoDB achieves speedups of 1-3 orders of magnitude in query processing of sensor data compared to approaches that use the entire dataset to compute exact query answers (more details are included in PlatoDB’s experimental evaluation in Section 7).

3. DATA AND QUERIES

Before describing the PlatoDB system, we first present its data model and query language.
A query expression \( Q \) is an arithmetic expression of the form \( A r \oplus A r \), where \( \oplus \) is the standard arithmetic operation (+, −, ×, ÷) and \( A r \) is either an arithmetic literal or an aggregation expression over a time series. An aggregation expression \( S u m(T, t, t) \) over a time series \( T \) computes the sum of all data points of \( T \) in the time interval \([t, t]\). Note that the time series that is aggregated could either be a base time series or a derived time series that was computed from a set of base time series through a set of time series operators. PlatoDB allows a series of time series operators, including \( P l u s(T_1, T_2) \), \( M i n u s(T_1, T_2) \), and \( T i m e s(T_1, T_2) \) (which return a time series that has data points computed by adding, subtracting, and multiplying the respective data points of the original time series, respectively), as well as \( S e r i e s G e n(v, n) \), which takes as input a value \( v \) and a counter \( n \) and creates a new time series that contains \( n \) data points with the value \( v \).

Note that the query language can be used to express many common statistics over time series encountered in practice and all the queries we encountered during the DELPHI project conducted at UC San Diego, which explored how health-related data about individuals, including large amounts of sensor data, can be leveraged to discover the determinants of health conditions and which served as the motivation for this work [18]. These include the mean and variance of a single time series, as well as the covariance, correlation, and cross-correlation between two time series. Table 1 shows how common statistics can be expressed in PlatoDB’s query language.

## 4. SEGMENT TREE

As explained in Section 2 at data import time, PlatoDB creates for each time series a hierarchy of summarizations of the series in the form of the segment tree. In this Section we first explain the structure of the tree and then describe the segment tree generation algorithm.

### 4.1 Segment Tree Structure

Let \( T = (d_1, \ldots, d_n) \) be a time series. The segment tree of \( T \) is a binary tree whose nodes summarize segments of the time series with nodes higher up the tree summarizing large segments and nodes lower down the tree summarizing progressively smaller segments. In particular, the root node summarizes the entire time series \( T \). Moreover, for each node \( n \) of the tree summarizing a segment \( S_i = (d_{i_1}, \ldots, d_{i_2}) \) of \( T \), its left and right children nodes \( n_l \) and \( n_r \) summarize two subsegments \( S_l = (d_{i_1}, \ldots, d_{i_l}) \) and \( S_r = (d_{i_{l+1}}, \ldots, d_{i_2}) \), respectively, which form a partitioning of the original segment \( S_i \). As we will see in Sections 5 and 6 PlatoDB uses at query processing time the compression function and error measures stored in each node to compute an approximate answer of the query and deterministic error guarantees, respectively. We next describe the compression functions and error measures stored within each segment tree node in detail.

### Segment Compression Function

Let \( S = (d_1, \ldots, d_n) \) be a segment. PlatoDB summarizes its contents through a compression function \( f \) used by the user. PlatoDB supports the use of any of the compression functions suggested in the literature [21][20][19][11][5][4]. Examples include but are not limited to the Piecewise Aggregate Approximation (PAA) [21], the Adaptive Piecewise Constant Approximation (APCA) [20], the Piecewise Linear Representation (PLR) [19], the Discrete Fourier Transformation (DFT) [11], the Discrete Wavelet Transformation (DWT) [4], and the Chebyshev polynomials (CHEB) [4].

To describe the function, PlatoDB stores in the segment node parameters describing the function. These parameters depend on the type of the function. For instance, if \( f \) is a Piecewise Aggregate Approximation (PAA), estimating all values within a segment by a single value \( b \), then the parameter is just a single value \( b \). On the other hand, if \( f \) is a Piecewise Linear Approximation (PLR), estimating the values in the segment through a line \( ax + b \), then the function parameters are the coefficients \( a \) and \( b \) of the polynomial used to describe the line.

In the rest of the document, we will refer directly to the compression function \( f \) (instead of the parameters that are used to describe it). Given a segment \( (d_1, \ldots, d_n) \), we will use \( f(i) \) to denote the
value for element \( d_i \) of the segment, as derived by \( f \).

**Segment Error Measures.** In addition to the compression function, PlatoDB also stores a set of error measures for each time series segment \( S = (d_1, \ldots, d_n) \). PlatoDB stores the following three error measures:

- \( L \) : The sum of the absolute distances between the original and the compressed time series (also known as the Manhattan or \( L_1 \) distance), i.e., \( L = \sum_{i=1}^{n} |d_i - f(i)| \).

- \( d^* \) : The maximum absolute value of the original time series, i.e., \( d^* = \max \{|d_i| \mid 1 \leq i \leq n \} \).

- \( f^* \) : The maximum absolute value of the compressed time series, i.e., \( f^* = \max \{|f(i)| \mid 1 \leq i \leq n \} \).

**EXAMPLE 3.** For instance, consider a segment \( S = (5.12, 5.09, 5.07, 5.04) \) summarized through the PAA compression function \( f = 5.08 \) (i.e., \( f(1) = f(2) = f(3) = f(4) = 5.08 \)). Then \( L = [5.12 - 5.08] + [5.09 - 5.08] + [5.07 - 5.08] + [5.04 - 5.08] = 0.1 \), \( d^* = \max(5.12, 5.09, 5.07, 5.04) = 5.12 \) and \( f^* = \max(5.08, 5.08, 5.08, 5.08) = 5.08 \).

As we will see in Section 4.2, the above three error measures are sufficient to compute deterministic error guarantees for any query supported by the system, regardless of the employed compression function \( f \). This allows administrators to select the compression function best suited to each time series, without worrying about computing the error guarantees, which is automatically handled by PlatoDB.

### 4.2 Segment Tree Generation

We next describe the algorithm generating the segment tree. To build the tree, the algorithm has to decide how to build the children nodes from a parent node; i.e., how to partition a segment into two non-overlapping subsegments. Each possible splitting point will lead to different children segments and as a result to different errors when PlatoDB uses the children segments to answer a query at query processing time. Ideally, the splitting point should be the one that minimizes the error among all possible splitting points. However, since PlatoDB supports ad hoc queries and since each query may benefit from a different splitting point, there is no way for PlatoDB to choose a splitting point that is optimal for all queries.

**Segment Tree Generation Algorithm.** Based on this observation, PlatoDB chooses the splitting point that minimizes the error for the basic query that simply computes the sum of all data points of the original segment. In particular, the segment tree generation algorithm starts from the root and proceeding in a top-down fashion given a segment \( S = (d_1, \ldots, d_n) \), selects a splitting point \( d_k \) that leads into two subsegments \( S_1 = (d_1, \ldots, d_k) \) and \( S_2 = (d_{k+1}, \ldots, d_n) \) so that the sum of the Manhattan distances of the new subsegments \( L_{S_1} + L_{S_2} \) is minimized.

The algorithm stops further splitting down a segment \( S \), when one of the following two conditions hold: (i) When the Manhattan distance \( L_S \) of the segment is smaller than a threshold \( \tau \) or (ii) when the size of the segment is below a threshold \( \kappa \). The choice between conditions (i) and (ii) and the values of the corresponding thresholds \( \tau \) and \( \kappa \) is specified by the system administrator.

Since the algorithm needs time proportional to the size of a segment to compute the splitting point of a single segment and it repeats this process for every non-leaf tree node, it exhibits a worst-time complexity of \( O(mn) \), where \( n \) is the size of the original time series (i.e., the number of its data points) and \( m \) number of nodes in the resulting segment tree.

**Discussion.** Note that by deciding independently how to split each individual segment into two subsegments, the segment tree generation algorithm is a greedy algorithm, which even though makes optimal local decisions for the basic aggregation query, may not lead to optimal global decisions. For instance, there is no guarantee that the \( k \) nodes that exist at a particular level of the segment tree correspond to the \( k \) nodes that minimize the error of the basic aggregation query. The literature contains a multitude of algorithms that can provide such a guarantee for a given \( k \); i.e., algorithms that can, given a time series \( T \) and a number \( k \), produce \( k \) segments of \( T \) that minimize some error metric. Examples include the optimal algorithm of [3], as well as approximation algorithms with formal guarantees presented in [14]. However, all these algorithms have very high worst-time complexity that makes them prohibitive for the large number of data points typically found in sensor datasets and are therefore not considered in this work. Though several heuristic segmentation algorithms exist, such as the Sliding Windows [13], the Top-down [22] and the Bottom-Up [23] algorithm, similar do our greedy algorithm, they do not provide any formal guarantees.

Finally, note that the tree generated by the above algorithm will in general be unbalanced. Intuitively, the algorithm will create more nodes and corresponding tree levels to cover segments that
5. COMPUTING APPROXIMATE QUERY ANSWERS AND ERROR GUARANTEES

Given pre-computed segment trees for time series \( T_1, \ldots, T_n \), PlatoDB answers ad hoc queries over the time series by accessing their segment trees. In particular, to answer a given query \( Q \) under an error/time budget, PlatoDB navigates the segment trees of the time series involved in \( Q \), selects segment nodes (or simply segments) that satisfy the budget, and computes an approximate answer for \( Q \) together with deterministic error guarantees.

We will next present the query processing algorithm. For ease of exposition, we will start by describing how PlatoDB computes an approximate query answer and the associated error guarantees assuming that the segment nodes have already been chosen, and will explain in Section 6 how PlatoDB traverses the tree to choose the segment nodes.

Approximate query answering problem under given segments. Formally, let \( T_1, \ldots, T_k \) be time series, such that time series \( T_i \) is partitioned into segments \( S_{i1}, \ldots, S_{ik} \). Given \( (a) \) these segments and the associated measures as described above and \( (b) \) a query \( Q \) over the time series \( T_1, \ldots, T_k \), we will show how PlatoDB computes an approximate query answer \( \hat{R} \) and an estimated error \( \hat{\epsilon} \), such that the approximate query answer \( \hat{R} \) is guaranteed to be within \( \pm \hat{\epsilon} \) of the accurate query answer \( R \)\(^4\), i.e., \(|R - \hat{R}| \leq \hat{\epsilon}\).

For ease of exposition, we next first describe the simple case where each time series \( T_i \) contains a single segment perfectly aligned with the single segment of the other series, before describing the general case, where each time series \( T_i \) contains multiple segments, which may also not be perfectly aligned with the segments of the other time series.

5.1 Single Time Series Segment

Let \( T_1, \ldots, T_k \) be \( k \) time series with single aligned segments, i.e., \( T_i \) is approximated by a single segment \( S_i \). Also let \( f_i \) be the compression function and \( (\ell_i, d_i, f_i) \) the error measures of segment \( S_i \), respectively. To compute the approximate answer and error guarantees of a query \( Q \) over \( T_1, \ldots, T_k \) using the single segments \( S_1, \ldots, S_k \), PlatoDB employs an algebraic approach computing in a bottom-up fashion for each algebraic operator \( op \) of \( Q \) the approximate answer and error guarantees for the subquery corresponding to the subtree rooted at \( op \).

This algebraic approach is based on formulas that for each algebraic query operator, given an approximate query answer and error for the inputs of the operator, provide the corresponding query answer and error for the output of the operator. Figure 3 shows the formulas employed by PlatoDB for each algebraic query operator supported by the system. Note that the output signatures differ between operators. This is due to the different types of operators supported by PlatoDB, as explained next. Recall from Section 3 that PlatoDB’s query language consists of three types of operators: (i) time series operators, (ii) aggregation operator, and (iii) arithmetic operators. While time series operators output a time series, aggregation and arithmetic operators output a single number. As a result, the formulas used for answer and error estimation, treat these two classes of operators differently: for time series operators, the formulas return, similarly to the input time series, the compression function and error measures of the output time series. For aggregation and arithmetic operators on the other hand, which return a single number and not an entire time series, the formulas return simply a single approximate answer and estimated error. Figure 3 shows the resulting formulas.

Without going into detail into each of them, we next explain how they can be used to compute the answer and corresponding error guarantees for an entire query through an example.

Example 4. This example shows how to use the formulas in Figure 3 to compute the approximate answer and associated error.

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\(^4\) Accurate answer means running queries over raw data. But note that, in this work, we can given estimate errors without computing the accurate answers.
error for a query computing the variance of a time series $T$ consisting of single segment $S$. For simplicity of the query expression we assume that the mean $\mu$ of $T$ is known in advance (note that even if $\mu$ was not known, the query would still be expressible in PlatoDB’s query language, albeit through a longer expression). Let $f$ be the compression function and $(L, d^*, f^*)$ the error measures of $S$. The query can be expressed as $Q = \text{Sum} (\text{Times} (\text{Minus}(T, \text{SeriesGen}(\mu, n))), \text{Minus}(T, \text{SeriesGen}(\mu, n)), 1, n)$. Figure 2 shows how PlatoDB evaluates this query in a bottom-up fashion. It first uses the formula of the $\text{SeriesGen}$ operator to compute the compression function $(f = \mu)$ and error measures $(L = 0, d^* = \mu, f^* = \mu)$ for the output of the $\text{SeriesGen}$ operator. It then computes the compression function $(f = \mu)$ and error measures $(L = (d^* + \mu), (f^* + \mu))$ for the output of the $\text{Minus}$ operator. The computation continues in a bottom-up fashion, until PlatoDB computes the output of the $\text{Sum}$ operator in the form of an approximate answer $R = n(f - \mu)^2$ where $n$ is the number of data points in $T$, and an estimated error $\hat{\varepsilon} = (d^* + f^*)L$.

Importantly, the formulas shown in Figure 3 are guaranteed to produce the best error estimation out of any formula that uses the three error measures employed by PlatoDB as explained by the following theorem:

**Theorem 1.** The estimated errors produced through the use of the formulas shown in Figure 3 are the lowest among all possible error estimations produced by using the error measures described in Section 4.

The proof can be found in Appendix A.2.

### 5.2 Multiple Segment Time Series

Let us now consider the general case, where each time series $T$ contains multiple segments of varying different sizes. As a result of the varying sizes of the segments, segments of different time series may not fully align.

**Example 5.** For instance consider the top two time series $T_1 = (S_{1,1}, S_{1,2})$ and $T_2 = (S_{2,1}, S_{2,2})$ of Figure 5 (ignore the third time series for now). Segment $S_{1,1}$ overlaps with both $S_{2,1}$ and $S_{2,2}$. Similarly, segment $S_{2,2}$ overlaps with both $S_{1,1}$ and $S_{1,2}$.

One may think that this can be easily solved by creating subsegments that are perfectly aligned and then using for each of them the answer and error estimation formulas of Section 5.1.

**Example 6.** Continuing our example, the two time series $T_1$ and $T_2$ can be split into the three aligned subsegments shown as the output time series $T_3$. Then for each of these output segments, we can compute the error based on the formulas of Section 5.7.

However, the problem with this approach is that the resulting error will be severely overestimated as the error of a single segment of the original time series may be counted multiple times, as it overlaps with multiple output segments.

**Example 7.** For instance, for a query over the time series $T_1$ and $T_2$ of Figure 5 the error of $S_{2,2}$ will be double-counted, as it will be counted towards the error of the two output segments $S_{3,2}$ and $S_{3,3}$.

To avoid this pitfall, PlatoDB does not estimate the error for its segment individually but instead computes the error holistically for the entire time series. Figures 6 and 7 show the resulting answer and error estimation formulas for time series operators and the aggregation operator, respectively. The formulas of the arithmetic operators are omitted as they remain the same as in the single segment case, as the arithmetic operators take as input single numbers instead of time series and are thus not affected by multiple segments.

### 6. NAVIGATING THE SEGMENT TREE

So far we have seen how PlatoDB computes the approximate answer to a query and its associated error, assuming that the segments that are used for query processing have already been selected. In this Section, we explain how this selection is performed. In particular, we show how PlatoDB navigates the segment trees of the time series involved in the query to continuously compute better estimations of the query answer under the given error or time budget is satisfied.

**Query Processing Algorithm.** Let $T_1, ..., T_m$ be a set of time series and $S_{1,1}, ..., S_{n,n}$ the respective segment trees. Let also $Q$ be a query over $T_1, ..., T_m$ and $\varepsilon_{max}/t_{max}$ an error/time budget, respectively. To answer $Q$ under the given budget, PlatoDB first starts from the roots of $S_{1,1}, ..., S_{n,n}$ and uses them to compute the approximate query answer $\hat{R}$ and corresponding error $\hat{\varepsilon}$ using the formulas presented in Section 5. If the estimated error is greater than the error budget (i.e., if $\hat{\varepsilon} \geq \varepsilon_{max}$) or the elapsed time is smaller than the allowed time budget, PlatoDB chooses one of the tree nodes used above, replaces it with its children and repeats the above procedure using the newly selected nodes until the given error/time budget is reached. What is important is the criterion that is used to choose the node that is replaced at each step by its children. In general, PlatoDB will have to select between several nodes, as it will be exploring in which segment tree and moreover in which part of the selected segment tree it pays off to navigate further down. Since PlatoDB aims to reduce the estimated error as much as possible, at each step it greedily chooses the node whose replacement by its children leads to the biggest reduction in the estimated error. The resulting procedure is shown as Algorithm 1.

**Algorithm Optimality.** Given its greedy nature, one may wonder whether the query processing algorithm is optimal. To answer this question, we have to first define optimality. Since the aim of the query processing algorithm is to produce the lowest possible error in the fastest possible time (which can be approximated by the number of nodes that are accessed), we say that an algorithm is optimal if for every possible query, set of segment trees, and error budget $\varepsilon_{max}$ it answers the query under the given budget accessing the.

*Note that the algorithm is shown for both error and time budget case. In contrast to the case when a time budget is provided, in which the algorithm has to always keep a computed estimated answer $\hat{R}$ to return it when the time budget runs out, in the case of the error budget this is not required. Thus, in the latter case, it suffices to compute $\hat{R}$ only at the very last step of the algorithm, thus avoiding its iterative computation during the whole loop.*
Time Series Operators

| Operator | Comp. func. | Output Error Measures |
|----------|-------------|-----------------------|
| $\text{SeriesGen}(v,n)$ | $t$ | $L$ | $d^*$ | $f^*$ |
| $\text{Plus}(T_a,T_b)$ | $\left\{ \langle u,i \rangle \mid f_{c,i} = f_{a,u} + f_{b,v} \text{ if } i \in [1,k] \right\}$ | $\sum_{i=1}^{q} L_{a,i} + \sum_{j=1}^{q} L_{b,j}$ | $\max \left\{ d_{c,i} \mid d_{c,i} = d_{a,u} + d_{b,v} \text{ if } i \in [1,k] \right\}$ | $\max \left\{ f_{c,i} \mid f_{c,i} = f_{a,u} + f_{b,v} \text{ if } i \in [1,k] \right\}$ |
| $\text{Minus}(T_a,T_b)$ | $\left\{ \langle u,i \rangle \mid f_{c,i} = f_{a,u} - f_{b,v} \text{ if } i \in [1,k] \right\}$ | $\sum_{i=1}^{q} L_{a,i} + \sum_{j=1}^{q} L_{b,j}$ | $\max \left\{ d_{c,i} \mid d_{c,i} = d_{a,u} - d_{b,v} \text{ if } i \in [1,k] \right\}$ | $\max \left\{ f_{c,i} \mid f_{c,i} = f_{a,u} - f_{b,v} \text{ if } i \in [1,k] \right\}$ |
| $\text{Times}(T_a,T_b)$ | $\left\{ \langle u,i \rangle \mid f_{c,i} = f_{a,u} \times f_{b,v} \text{ if } i \in [1,k] \right\}$ | $L_{T,c}$ | $\max \left\{ d_{c,i} \mid d_{c,i} = d_{a,u} \times d_{b,v} \text{ if } i \in [1,k] \right\}$ | $\max \left\{ f_{c,i} \mid f_{c,i} = f_{a,u} \times f_{b,v} \text{ if } i \in [1,k] \right\}$ |

Figure 6: Formulas for estimating answer and error for time series operators (multiple segments). For each output time series segment $S_{c,i}$, let $S_{a,u}$ and $S_{b,v}$ be the input segments that overlap with $S_{c,i}$.

Aggregation Operator

| Operator | Approximate Output | Estimated Error |
|----------|-------------------|-----------------|
| $\text{Sum}(T_a,T_c)$ | $\sum_{i=1}^{u} \sum_{j=1}^{v} f_i(j)$ | $\sum_{i=1}^{u} L_i$ |

Figure 7: Formulas for estimating answer and error for the aggregation operator (multiple segments).

Algorithm 1: PlatoDB Query Processing

Input: Segment Trees $T_1, \ldots, T_m$, query $Q$, error budget $\epsilon_{\text{max}}$ or time budget $t_{\text{max}}$.
Output: Approximate answer $R$ and error $\hat{\epsilon}$

1. Access the roots of $T_1, \ldots, T_m$;
2. Compute $R$ and $\hat{\epsilon}$ by using the compression functions and error measures of the currently accessed nodes (see Section 5 for details);
3. while $\hat{\epsilon} > \epsilon_{\text{max}}$ or elapsed time $< t_{\text{max}}$ do
   4. Choose a node maximizing the error reduction;
   5. Update the current answer $R$ and error $\hat{\epsilon}$ using the compression functions and error measures of the currently accessed nodes;
4. Return $(R, \hat{\epsilon})$;

Theorem 2: There is no optimal algorithm in $A$.

Proof: Consider the following segment trees of two time series $T_1$ and $T_2$. The segment tree of $T_1$ is shown in Figure 8 and the segment tree of $T_2$ is a tree containing a single node. Now consider a query $Q$ over these two time series and an error budget $\epsilon = h - 1$ where $h > 1$ is the height of the $T_1$’s tree. Assume that the query error using the tree roots is $\epsilon_{\text{root}} = 2h$. Also assume that whenever the query processing algorithm replaces a node by its children, the error for the query is reduced by $\frac{1}{2^h}$ with the exception of the shaded node, which, when replaced by its children, leads to an error reduction of $h + 1$. This means that the query processing algorithm can only terminate after accessing the children of the shaded node, as the query error in that case will be at most $2h - (h + 1) = h - 1$. Otherwise, the error estimated by the algorithm will be at least $2h - \left( \frac{1}{2^h} \right) = 2h - 1 > h - 1$, which exceeds the error budget and thus does not allow the algorithm to terminate. Since the shaded node can be placed at an arbitrary position in the tree, for every given deterministic algorithm, we can place the shaded node in the tree, so that the algorithm accesses the children of the shaded node only after it has accessed all the other nodes in the tree. However, this is suboptimal, as there is a way to access the children of the shaded node with fewer node accesses (i.e., by following the path from the root to the shaded node). Therefore, no algorithm in $A$ is optimal. □

As a result of the above theorem, PlatoDB’s query processing algorithm cannot be optimal in general. However, we can show that it is optimal for segment trees that exhibit the following property: For every pair of nodes $N$ and $N'$ of the segment tree, such that $N'$ is a descendant of $N$, the error reduction $\epsilon_{\Delta}(N)$ achieved by replacing $N$ with its children is greater or equal to the error reduction $\epsilon_{\Delta}(N')$ achieved by replacing $N'$ with its children. Such a tree is called fine-error-reduction tree and intuitively it guarantees that any node leads to a greater or equal error reduction than any of its descendants. If all trees satisfy the above property, PlatoDB’s query processing algorithm is optimal:

Theorem 3: In the presence of segment trees that are fine-error-reduction trees, PlatoDB’s query processing algorithm is optimal.

Incremental Error Update

Table 2: Incremental update of estimated errors for time series operators. $p_{a,i} \in \{d_{a,i}, f_{a,i}\}$.

| Operator | Incremental Error Update |
|----------|--------------------------|
| $\text{Plus}(T_a,T_b)$ | $\epsilon' = \epsilon - (L_a - (L_{a,1} + L_{a,2}))$ |
| $\text{Minus}(T_a,T_b)$ | $\epsilon' = \epsilon - (L_a - (L_{a,1} + L_{a,2}))$ |
| $\text{Times}(T_a,T_b)$ | $\epsilon' = \epsilon - \max(p_{b,1}, \ldots, p_{b,k})L_a + \max(p_{a,1}, \ldots, p_{a,h})L_{a,1}$ | $\max(p_{a,1}, \ldots, p_{a,h})L_{a,1}$ | $\max(p_{a,1}, \ldots, p_{a,h})L_{a,2}$ |

Incremental Error Update. Having proven the optimality of the algorithm for fine-error-reduction trees, we will next discuss an optimization that can be employed to speedup the algorithm. By
studying the algorithm, it is easy to observe that as the algorithm moves from a set \( N = \{N_1, \ldots, N_a\} \) of nodes to a set \( N' = \{N_1, \ldots, N_{a-1}, N_{a+1}, \ldots, N_a\} \) of nodes (by replacing node \( N_a \) by its children \( N_{a_1} \) and \( N_{a_2} \)), it recomputes the error using all nodes in \( N' \), although only the two nodes \( N_{a_1} \) and \( N_{a_2} \) have changed from the previous node set \( N \).

This observation led to the incremental error update optimization of PlatoDB’s query processing algorithm described next. Instead of recomputing from scratch the error of \( N' \) using all nodes, PlatoDB incrementally updates the error of \( N' \) using the new error measures of the newly replaced node \( N_a \) and the newly inserted nodes \( N_{a_1} \) and \( N_{a_2} \). Let \((L_a, d_a, f_a^*)\), \((L_{a_1}, d_{a_1}, f_{a_1}^*)\), and \((L_{a_2}, d_{a_2}, f_{a_2}^*)\) be the error measures of nodes \( N_a \), \( N_{a_1} \), and \( N_{a_2} \), respectively. Assume that the segments \( S_{b_1}, \ldots, S_{b_k} \) overlap with the segment of node \( N_a \), the segments \( S_{b_1}, \ldots, S_{b_k} \) overlap with the segment of node \( N_{a_1} \), and the segments \( S_{b_1}, \ldots, S_{b_k} \) overlap with the segment of node \( N_{a_2} \). Then the estimated error \( \hat{\varepsilon} \) using nodes \( N_a, N_{a_1} \) and \( N_{a_2} \) can be incrementally computed from the error \( \varepsilon \) using node \( N_a \) through the incremental error update formulas shown in Table 3.

### Probabilistic Extension

While PlatoDB provides deterministic error guarantees, which as we discussed above are in many cases required, it is interesting to note that it can be easily extended to provide probabilistic error guarantees if needed. Most importantly this can be done simply by changing the error measures computed for each segment from \((L, d', f')\) to \((\sigma_r, \varepsilon^*, f^*)\), where \(\sigma_r\) is the variance of \(d_i - f(i)\), and \(\varepsilon^*\) is the maximal absolute value of \(d_i - f(i)\). Then we can employ the Central Limit Theorem (CLT) \[10\] to bound the accurate error \(\varepsilon\) by \(Pr(\varepsilon \leq \hat{\varepsilon}) \geq 1 - \alpha\), where \(\alpha\) can be adjusted by the users to get different confidence levels. It is interesting that the rest of the system, including the hierarchical structure of the segment tree and the tree navigation algorithm employed at query processing time do not need to be modified. In our future work we plan to further explore this probabilistic extension and compare it to existing approximate query answering techniques with probabilistic guarantees.

### 7. EXPERIMENTAL EVALUATION

To evaluate PlatoDB’s performance and verify our hypothesis that PlatoDB is able to provide significant savings in the query processing of sensor data, we are conducting experiments on real sensor data. We present here early data points that we have discovered.

#### Datasets

For our preliminary experiments, we used two real sensor datasets:

1. **Intell Lab Data (ILD)**[^ILD]: Smart home data (humidity and temperature) collected at 31-second intervals from 54 sensors deployed at the Intel Berkeley Research Lab between February 28th and April 5th, 2004. The dataset contains about 2.3 million tuples (i.e., 4.6 million sensor readings in total).

2. **EPA Air Quality Data (AIR)**[^AIR]: Air quality data collected at hourly intervals from about 1000 sensors from January 1st 2000 to April 1st 2016. The dataset contains about 133 million tuples (i.e., 266 million sensor readings in total).

[^ILD]: The *SeriesGen* operator is omitted, since its input is not a time series and as a result there is no segment tree associated with its input.

[^AIR]: To make a fair comparison, the raw data size refers only to the combined size of the attributes used in the time series and does not include other attributes that exist in the original dataset (such as location codes etc).

From each dataset we extracted multiple time series, each corresponding to a single attribute of the dataset: Humidity and Temperature for ILD and Ozone and SO\(_2\) for AIR. We then used PlatoDB to create the corresponding segment tree for each time series and to answer queries over them.

#### Experimental platform

All experiments were performed on a computer with a 4th generation Intel i7-4770 processor (4 × 32 KB L1 data cache, 4 × 256 KB L2 cache, 8 MB shared L3 cache, 4 physical cores, 3.6 GHz) and 16 GB RAM, running Ubuntu 14.04.1. All the algorithms were implemented in C++ and compiled with g++ 4.8.4, using -O3 optimization. All data was stored in main memory.

### 7.1 Experimental Results

In our preliminary evaluation, we measured two quantities: First, the size of the segment tree created by PlatoDB, since this segment tree is stored in main memory, and second, the query processing performance of PlatoDB compared to a system that answers queries using the entirety of the raw sensor data. In our future work, we will be conducting a more thorough evaluation of the system. We present our preliminary results:

| Dataset | # Tuples | Raw Data | Segment Tree |
|---------|----------|----------|--------------|
| ILD     | 2,313,153| 35.29 MB | 908 MB       |
| AIR     | 133,075,510| 1.98 GB | 4.37 MB       |

**Table 3:** Raw data and segment tree sizes.

#### Segment tree size

Table 3 shows the size of the raw data and the combined size of the segment trees built for all the time series extracted from the ILD and AIR datasets[^ILD][^AIR]. We experimented with two different compression functions, resulting in different segment tree sizes; a 0-degree polynomial (corresponding to the Piecewise Aggregate Approximation[^PAA]), where each value within a segment is approximated through the average of the values in the segment) and a 1-degree polynomial (corresponding to the Piecewise Linear Approximation[^PLA]), where each segment is approximated through a line). As shown, the segment trees are significantly smaller than the raw sensor data (about 0.40% − 1.90% and 0.22% − 0.40% smaller for the ILD and AIR datasets, respectively). As a result, the segment trees of the time series can be easily kept in main memory, even when the system stores a large number of time series.

![Query processing performance for correlation query (time shown in ms).](http://db.lcs.mit.edu/labdata/labdata.html)

(a) ILD (b) AIR
processing performance of PlatoDB against a baseline, which is a custom in-memory algorithm that computes the exact answer of the queries using the raw data. To compare the systems, we measured the time required to process a correlation query between two time series (i.e., correlation(Humidity, Temperature) in ILD and correlation(Ozone and SO2) in AIR) with a varying error budget (ranging from 5% to 25%). Figure 9 shows the resulting times for each of the two datasets. Each graph depicts the performance of three systems; Exact, which is the baseline method of answering queries over the raw data, and PlatoDB-0, PlatoDB-1, which are instances of PlatoDB using the 0-degree and 1-degree polynomial compression functions, as explained above.

By studying Figure 9 we can make the following observations:

- Both instances of PlatoDB outperform Exact by one to three orders of magnitude, depending on the provided error budget.
- In contrast to Exact which always uses the entire raw dataset to compute exact query answers, PlatoDB allows the user to select the appropriate trade-off between time spent in query processing and resulting error by specifying the desired error budget. The system adapts to the budget by providing faster responses as the allowed error budget increases;
- Notably, PlatoDB remains significantly faster than Exact even for small error budgets. In particular, PlatoDB is over $9 \times$ and $37 \times$ faster than Exact when the error is 5% in ILD and AIR, respectively.

In summary, our preliminary results show that PlatoDB shows significant potential for speeding up query processing of ad hoc queries over large amounts of sensor data, as it outperforms exact query processing algorithms in many cases by several orders of magnitude. Moreover, it can provide such speedups, while providing deterministic error guarantees, in contrast to existing sampling-based approximate query answering approaches that provide only probabilistic guarantees, which may not hold in practice. Despite the difference in guarantees, in our future work we will be conducting a more thorough evaluation of the system comparing it also against sampling-based systems.

8. RELATED WORK

Approximate query answering has been the focus on an extensive body of work, which we will summarize next. However, to the best of our knowledge, this is the first work that provides deterministic guarantees for aggregation queries over multiple time series.

Approximate query answering with probabilistic error guarantees. Most of the existing work on approximate query processing has focused on using sampling to compute approximate query answers by appropriately evaluating the queries on small samples of the data [17, 1, 37, 2, 26, 25]. Such approaches typically leverage statistical inequalities and the central limit theorem to compute the confidence interval or variance of the computed approximate answer. As a result, their error guarantees are probabilistic. While probabilistic guarantees are often sufficient, there are not suitable for scenarios where one wants to be certain that the answer will fall within a certain interval.

A special form of sampling-based methods are online aggregation approaches, which provide a continuously improving query answer, allowing users to stop the query evaluation when they are satisfied with the resulting error [15, 7, 26]. With its hierarchical segment tree, PlatoDB can support the online aggregation paradigm, while providing deterministic error guarantees.

Approximate query answering with deterministic error guarantees. Approximately answering queries while providing deterministic error guarantees has so far received only very limited attention [31, 24, 30]. Existing work in the area has focused on simple aggregation queries that involve a single relational table. In contrast, PlatoDB provides deterministic error guarantees on queries that may involve multiple time series (each of which can be thought of as a single relational table), enabling the evaluation of many common statistics that span tables, such as correlation, cross-correlation and others.

Approximate query answering over sensor data. Moreover, PlatoDB is one of the first approximate query answering systems that leverage the fact that sensor data are not random but follow a usually smooth underlying phenomenon. The majority of existing works on approximate query answering looked at general relational data. Moreover, the ones that studied approximate query processing for sensor data, focused on the networking aspect of the problem, studying how aggregate queries can be efficiently evaluated in a distributed sensor network [25, 8, 9]. While these works focused on the networking aspect of sensor data, our work focuses on the continuous nature of the sensor data, which it leverages to accelerate query processing even in a single machine scenario, where historical sensor data already accumulated on the machine have to be analyzed.

Data summarizations. Last but not least, there has been extensive work on creating summarizations of sensor data. Work in this area has come mostly from two different communities; from the database community [16, 30, 27, 35] and the signal processing community [21, 20, 19, 5, 11, 11].

The database community has mostly focused on creating summarizations (also referred to as synopses or sketches) that can be used to answer specific queries. These include among others histograms [16, 30, 12, 29] (e.g., EquiWidth and EquiDepth histograms [29], V-Optimal histograms [16], Hierarchical Model Fitting (HMF) histograms [39], and Compact Hierarchical Histograms (CHH) [22]), as well as sampling methods [14, 6], used among other for cardinality estimation [16] and selectivity estimation [39]. In contrast to such special-purpose approaches, PlatoDB supports a large class of queries over arbitrary sensor data.

The signal processing community on the other hand, produced a variety of methods that can be used to compress time series data. These include among others the Piecewise Aggregate Approximation (PAA) [21], the Adaptive Piecewise Constant Approximation (APCA) [20], the Piecewise Linear Representation (PLR) [19], the Discrete Wavelet Transform (DWT) [5], and the Discrete Fourier Transform (DFT) [11]. However, it has not been concerned on how such compression techniques can be used to answer general queries. PlatoDB’s modular architecture allows the easy incorporation of such techniques as compression functions, that are then automatically leveraged by the system to enable approximate answering of a large number of queries with deterministic error guarantees.

9. CONCLUSION
In this paper, we proposed the PlatoDB system that allows users the efficient computation of approximate query answers to queries over sensor data. By utilizing the novel segment tree data structure, PlatoDB creates at data import time a set of hierarchical summarizations of each time series, which are used at query processing time to not only enable the efficient processing of queries over multiple time series with varying error/time budgets but to also provide error guarantees that are deterministic and are therefore guaranteed to hold, in contrast to the multitude of existing approaches that only provide probabilistic error guarantees. Our preliminary results show that the system can in real use cases lead to several or more orders of magnitude improvements over systems that access the entire dataset to provide exact query answers. In our future work, we plan to perform a thorough experimental evaluation of the system, in order to both study the behavior of the system in different datasets and query workloads, as well as to compare it against systems that provide probabilistic error guarantees.

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APPENDIX

A. PROOFS

A.1 Error measures for the Times operator (Single Segment)

Let $f^{(1)}$ and $f^{(2)}$ be the compression functions of $T_1 = (d_1^{(1)}, ..., d_n^{(1)})$ and $T_2 = (d_2^{(2)}, ..., d_n^{(2)})$ respectively. Let $(L_1, d_1^{(1)}, f_1^{(1)})$ and $(L_2, d_2^{(2)}, f_2^{(2)})$ be the error measures for time series $T_1$ and $T_2$. For $\text{Times}(T_1, T_2) \rightarrow T$ operator, the compression function $f$ and the error measures $(L, d^*, f^*)$ for the output time series $T = (d_1, ..., d_n)$ are computed as follows:

- $f = f^{(1)} \times f^{(2)}$, i.e., the product of two compression functions.
- $L = \sum_{i=1}^n |d_i - f(i)|$, i.e., the error of the compression function $f$ over the time series $T$.

There are two options to transform this expression:

1. $*$

2. $\times$
Option 1: $L = \sum_{i=1}^{n-1} |d_i^{(1)} - d_i^{(2)}| = \sum_{i=1}^{n-1} |f^{(1)}(i) - f^{(2)}(i)|$ \(\leq f_2 T_1 + f_1 T_2\).

Option 2: $L = \sum_{i=1}^{n} |d_i^{(1)} - d_i^{(2)}| = \sum_{i=1}^{n} |f^{(1)}(i) - f^{(2)}(i)|$ \(\leq d_2 L_1 + f_1 L_2\).

Thus, we choose the minimal one between these two options.

That is $L = \min\{f_2 L_1 + d_2 L_2, d_2 L_1 + f_1 L_2\}$.

- $d^* = \max\{|d_i| \mid 1 \leq i \leq n\} = \max\{|d_i^{(1)} - d_i^{(2)}| \mid 1 \leq i \leq n\} \leq d_2 \times d_2$.
- $f^* = \max\{|f(i)| \mid 1 \leq i \leq n\} = \max\{|f_i^{(1)} - f_i^{(2)}| \mid 1 \leq i \leq n\} \leq f_2 \times f_2$.

A.2 Proof of the optimality of the error estimation formulas of Figure 3

Aggregation operator. Depending on whether it is a single segment time series, there are two cases.

Case 1. A time series $T$ (with $n$ data points) contains only one single segment. There are two subcases depending on whether $T$ is entirely used in the query or not. That is $\ell_e - \ell - n \neq 0$.

Case 1.1. $\ell_e - \ell - n$. In this case, the error $\varepsilon = \sum_{i = \ell_e}^{\ell} |d(i) - f(i)| = \sum_{i = \ell_e}^{\ell} |d(i) - f(i)|$. And we have $L = \sum_{i = \ell_e}^{\ell} |d(i) - f(i)|$.

Therefore, we can get $\varepsilon = L$. That means that by using $L$ we are able to get the accurate error (the optimal error estimation). As desired.

Case 1.2. $\ell_e - \ell - n < 0$. Assume there exists an estimator $A$ that gives an approximate error $\tilde{\varepsilon}$, where $\tilde{\varepsilon} = L - \alpha$, where $\alpha > 0$ is a small value. Let $T$ be a segment series with length $n$ such that $d(i) = f(i)$ for $i \in \{1, n - (\ell_e - \ell) - 1\}$ and $d(i) = f(i) + 1$ for $i \in \{n - (\ell_e - \ell), n\}$. Thus $L = \sum_{i = \ell_e}^{\ell} |d(i) - f(i)| = \ell_e - \ell_s$.

For a query range with $[1, \ell_e - \ell_s]$, $A$ gives the approximate error as $\ell_e - \ell_s - \alpha$, which is correct as the accurate error is 0. Now we switch the points $d(i)$ with $d(i)$ (as well as $f(i)$ with $f(i)$) for $i \in [1, \ell_e - \ell_s], j \in [n - (\ell_e - \ell_s), n]$ to generate a new segment $T'$. Note that $T'$ and $T$ have the same $L$. Now the accurate error is also $L = \ell_e - \ell_s$. However, $A$ still gives the approximate error as $\ell_e - \ell_s - \alpha$, which is incorrect as it produces smaller error than the optimal one. Therefore, there does not exist an estimator that produces approximate errors less than that of our estimator, which means our estimator achieves the lower bound. As desired.

Case 2. A time series contains multiple segments $S_1, \ldots, S_n$. Note that, segments $S_{1}, \ldots, S_{n-1}$ are all always entirely used by the query. According to case 1, our estimator gives the optimal error estimation. For the left-most and right-most segments, i.e., $S_1$ and $S_n$, our estimator achieves the lower bound of error estimation according to case 1.2. As desired.

Plus and Minus operators. Similar to the proof presented above, it is easy to see our estimator achieves the lower bound. Otherwise, there must exist an incorrect error estimation.

Times operator. We distinguish between two cases.

Case 1. Time series $T_1$ with $n$ data points (resp. $T_2$) only contains one segment. Depending on whether $T_1$ and $T_2$ are entirely used. There are two subcases.

Case 1.1. $T_1$ and $T_2$ are entirely used. The accurate error is $\varepsilon = \sum_{i=1}^{n} |d_i^{(1)} - d_i^{(2)} - f^{(1)}(i) \times f^{(2)}(i)|$. Let the data point in $T_1$ have the following features $d_i^{(1)} = d_i^{(2)} - 1, f^{(1)}(i) = f^{(1)}(i+1) - 1$ and $f^{(2)}(i) = f^{(2)}(i) - 1$ for $i \in [1, n]$. Let $T_2$ have the same data. Thus, $L = \sum_{i=1}^{n} |d_i^{(1)} - f^{(1)}(i)| = n$ and $L = \sum_{i=1}^{n} |d_i^{(1)} - f^{(2)}(i)| = n$. So the estimated error of our estimator is $\hat{\varepsilon} = \min\{f_2 L_1 + d_2 L_2, d_2 L_1 + f_1 L_2\}$.

Assume there exists an error estimator $A$ that produces an approximate error $\hat{\varepsilon}'$, where $\hat{\varepsilon}' = \hat{\varepsilon} - \alpha$, where $\alpha > 0$ is a small value. Since the accurate error $\varepsilon = \sum_{i=1}^{n} |2f(1)(i) + 1| < \varepsilon$, $A$ returns the correct estimation. Then we make $d_i^{(1)} = d_i^{(2)} + f(i + 1)| < f^{(1)}(i) + 1 | i \in [1, n]$ in both two time series. Then the error measures of both time series stay the same. Now, the accurate error is $\varepsilon = \sum_{i=1}^{n} (2f^{(1)}(i) + 1) < \hat{\varepsilon}$. That is our estimator produces the optimal error (meaning the error is equal to the accurate one). But $A$ still gives the same estimation $\hat{\varepsilon} - \alpha$, which is incorrect. So $A$ does not exist. As desired.

Case 1.2. $T_1$ and $T_2$ are not entirely used. The proof is similar to the case 1.2. In this case, $A$ also produces an approximate error $\hat{\varepsilon}'$, where $\hat{\varepsilon}' = \hat{\varepsilon} - \alpha$, where $\alpha > 0$ is a small value. Since the accurate error $\varepsilon = \sum_{i=1}^{n} |2f^{(1)}(i) + 1| < \varepsilon$, $A$ returns the correct estimation. Then we make $d_i^{(1)} = d_i^{(2)} + f(i + 1)| < f^{(1)}(i) + 1 | i \in [1, n]$ in both two time series. Then the error measures of both time series stay the same. Now, the accurate error is $\varepsilon = \sum_{i=1}^{n} (2f^{(1)}(i) + 1) < \hat{\varepsilon}$. That is our estimator produces the optimal error (meaning the error is equal to the accurate one). But $A$ still gives the same estimation $\hat{\varepsilon} - \alpha$, which is incorrect. So $A$ does not exist. As desired.

Case 2. $k$ segments $S_1^{(1)}, \ldots, S_1^{(k)}$ in time series $T_1$ overlapping one segment $S_2^{(2)}$ in time series $T_2$ are used in the query with range $[a,b]$. The proof is similar to that in Case 1.1 and Case 1.2.

Case 3. $k_1$ segments $S_1^{(1)}, \ldots, S_1^{(k_1)}$ in time series $T_1$ overlapping $k_2$ segments $S_2^{(1)}, \ldots, S_2^{(k_2)}$ in time series $T_2$ are used in the query. The proof is similar to that in Case 2.

Arithmetic Operators For arithmetic operator $A_1 \odot A_2$, there are three cases depending on the number of approximate answers of $A_1$ and $A_2$.

Case 1. Zero $A_1$, i.e., both $A_1$ and $A_2$ are numbers. $A_1 \odot A_2$ can be transformed as number $\odot$ number. Then the answers are accurate answers. As desired.

Case 2. One $A_1$, i.e., $A_1 \odot A_2$ can be transformed as $A_1 \odot$ number. Let $A_1$ and $\hat{\varepsilon}$ be the output approximate answer of $A_1$ and the estimated error by PlatioDB. Therefore, we know that $\hat{\varepsilon}$ is the lower bound of $[A_1 - A_2]$. For $A_1$ number, the error is $|A_1 + number - (A_1 + number)| = |A_1 - A_2|$. Thus, $\hat{\varepsilon}$ is also the lower bound of $A_1 + number$. Similarly, we can prove the lower bound property of the operators for $\times, \div, \odot$ operators.

Case 3. Two $A_1, A_1 \odot A_2$ can be transformed as $A_1 \odot A_2$. For $A_1 \odot A_2$, we have $|A_1 \times A_2| \\leq A_1 \times \hat{\varepsilon_2} + \hat{\varepsilon_1} \times A_2$. We can prove it by the lower bound property of constructing the query, the accurate error is equals to this one, which means there does not exist a better estimation. Similarly, for $A_1 \odot \hat{\varepsilon_2}$, we can prove $\hat{\varepsilon_2} \times \hat{\varepsilon_1} - \hat{\varepsilon_2} \times \hat{\varepsilon_1} \times \hat{\varepsilon_1}$ is lower bound error.