Study of the propagation of solitary waves produced by an assembly of quantum dots through optical fibers

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Abstract. In this research, the potential use of semiconductor nanostructures as optical field sources to optimize the propagation of lossless pulses along non-linear optical fibers is studied. During this investigation, we propose the external excitation of a set of semiconductor SQD points through a source of optical pulses, giving rise to the generation of solitonic pulses that propagate through an optical fiber with non-linear optical characteristics. Theoretically, soliton formation studied from the non-linear interaction between SQD and external optical excitation. In the study, the non-linear Schrödinger NLSE equation solved numerically using the Fourier Split-Step method to understand the evolution of the soliton emitted for SQD within an optical fiber with real physical parameters.

1. Introduction.
The quantum dots are semiconductor nanostructures that restrict the movement of charge carriers in the three spatial dimensions. Its discovery has motivated during the last decades to carry out theoretical-experimental research within the physics of the solid state and related sciences such as optics. The research lines that study low dimensionality systems have made great contributions in the advancement and development of the technology based on semiconductor quantum dots SQDs since these have encouraged the combination of these nanostructures with optical and photonic devices. Some of the most recent investigations indicate that this type of heterostructures can undergo abrupt changes in the spectral response with minimal variations in their size and morphology, offering important applications to optics, among which are the lasers of new generations, diodes light emitters, optical multiplexers, biosensors, spectral tuners, quantum computing, logic gates, among others. On the other hand, optical fibers are one of the most important mechanisms for the propagation of light waves with low losses, these are an indispensable tool for the propagation of electromagnetic waves in the optical, infrared and ultraviolet range, however, there are other remarkable applications for optical fibers in the area of sensors and in the study of nonlinear optical phenomena. Nowadays, it has been possible to combine nanostructures with other polymeric materials such as optical fibers, giving rise to nanocomposites, which are generally composed of several phases such as SiO2, where one or several of its dimensions are found at the nanoscale [6-8]. The SQDs embedded in polymer matrices are of great interest at present in the realization of photonic devices [9], their strong applications are in the
manufacture lasers [10] of low consumption, high optical coherence and light sources with wavelengths of emission that have narrow bandwidths. In the study of nonlinear optical phenomena, some investigations have been developed for the propagation of soliton-type optical pulses, because they can propagate along nonlinear optical media with low losses. For the understanding of the nonlinear optical phenomena that a wave can experience in an optical fiber, it is necessary to consider a theory of propagation of waves in dispersive media, in this sense, the nonlinear equation of Schrödinger NLSE provides a complete description of a variety of localized nonlinear effects that have been extensively studied in various contexts of science and whose theory can be applied directly to the propagation of intense optical pulses in nonlinear optical fibers giving rise to optical solitons. At present, different solitonic solutions of the NLSE equation have been theoretically studied, among which are the Akhmediev, Peregrine, Kuznetsov-Ma type solutions, among others, and at the same time, the observation of certain types of finite base solitons has been reported. Solitons on finite background SFB in optical fibers, such as Kuznetsov-Ma type solitons. Although there may be some experimental advances in the propagation of solitons in optical fibers, in practice it has not been possible to propagate solitonic pulses at long distances at a commercial level due to the different technical limitations [11,12].

In this research, a theoretical study is carried out to demonstrate that optical solitons can be produced by the external optical excitation of an assembly of quantum dots within a waveguide. The study allows proposing a general model that allows coupling the solitons from the assembly of quantum dots with specific parameters in an optical fiber with non-linear optical characteristics. As a result, a numerical simulation is developed to study the evolution of the soliton inside the non-linear optical fiber using the Fourier Split-Step numerical technique with the real parameters associated with the SQDs and the optical fiber.

2. Theoretical Analysis.

SQDs systems are zero-dimensional nanostructures that can confine the movement of charge carriers in all spatial dimensions, which present a discrete energy spectrum. The values of the transitions of the dipole moments in this type of systems can cause that the intensity in the interaction between the SQDs and the optical pulse increases significantly in comparison with the atomic systems, which can modify some properties with interesting applications in the area of non-linear optics. At present, different SQDs have been manufactured using different growth techniques and where it has been demonstrated that the optical properties exhibited by this type of structures have a strong influence of their morphology and the materials used in their manufacturing process [5]. On the other hand, the optical solitons were observed for the first time in media that had a high absorption at specific wavelengths known as resonant optical media, but that at a certain minimum power the medium presented a transparent behavior, for this purpose it is known as self-induced transparency SIT. This effect is experienced by the resonant atoms in a medium, which can be in resonance depending on the frequency of the carrier with respect to the optical transitions given in the material and at the same time some conditions must be satisfied [11]. In the case of SQDs, these generally have extremely large dipole moments, which causes the non-linear interaction between the SQDs and the optical excitation to be greater compared to the atomic systems, so from the point of view of the effects Nonlinear optics, the SIT effect in SQDs systems will be different compared to atomic systems. As an important consequence in the interaction of intense light with SQD systems is the formation of non-linear optical waves such as optical solitons, which acquire certain characteristics in term of the incident light and the morphological characteristics of the SQDs.

We will consider a quantum dot as a three-level energy system, where we will denote $|1\rangle$ as the base state with energy $\epsilon_1 = 0$, $|2\rangle$ is the first excited state with energy $\epsilon_2$, and $|3\rangle$ is the second excited state with energy value $\epsilon_3$. These states are solutions of the Schrödinger equation:

$$\hat{H}_0 |n\rangle = \epsilon_n |n\rangle$$

(1)
The figure 1 shows a schematic of an external light laser source, composed of a linearly polarized plane wave high intensity, incident on SQDs, in the process, due to the interaction of the light with the SQDs, the light is re-emitted in the form of a non-linear wave with special characteristics that will depend on the morphology of the quantum dot and the incident wave.

In the theoretical model, we consider the Hamiltonian corresponding to the interaction of a plane wave linearly polarized with an assembly of quantum dots QDs, of the form:

\[
\hat{H} = \hbar \nu_{sl} |2\rangle\langle 2| + \hbar \nu_{ss} |3\rangle\langle 3| - \hat{P} \cdot \hat{E}
\]  

(2)

where \(\nu_{sl}\) and \(\nu_{ss}\) are the frequences excitonic and biexcitonic. The incident light is considered linearly polarized in the TE mode and has a width \(T\) and a frequency \(\omega\), with an electric field \(\hat{E}(x, y, z, t) = e\hat{E}(x, y, z, t)\) propagating along the \(z\)-axis, where \(e\) is the polarization vector in the direction \(y\). The wave equation of the plane wave that affects the assembly of quantum dots and that produces a secondary dipole field that propagates in the same direction as the incident field \(\hat{E}\) is given by:

\[
-\varepsilon^2 \frac{\partial^2 \hat{E}}{\partial z^2} + \eta^2 \frac{\partial^2 \hat{E}}{\partial t^2} - \hat{\rho} \cdot \hat{E} = -4\pi \frac{\partial^2 \hat{E}}{\partial t^2}
\]  

(3)

where \(\hat{\rho}(x, z, t) = N \int dE g(\omega) \langle \hat{\rho} \rangle + c.c.\) and \(\langle \hat{\rho} \rangle = \langle n | \hat{\rho} | m \rangle = \mu_{nn} \rho_{nn} + \mu_{nm} \rho_{nm}\), corresponds to the expected value the dipolar moments of the transitions between the levels \(|n\rangle - |m\rangle\), \(g(\omega)\) is a function of distribution of the frequencies of the transitions, which depend on the sizes of the point \(y \cdot \omega_{01} = \omega_0 - \omega\) is the detuning, \(\eta\) is the refractive index of the semiconductor, and \(N\) is the density of quantum dots, the quantities \(\rho_{nn}, \rho_{nm}\) are the elements of the density matrix which are determined by the Liouville equation.

In concordance, we can use the Liouville equation to determine the elements of the density matrix [14].

\[
\frac{i\hbar}{\partial \tau} \hat{\rho} = [\hat{H}, \hat{\rho}] = \sum_{\ell} (\langle n | H | \ell \rangle \rho_{nn} - \rho_{\ell n} \langle \ell | H | m \rangle) \quad n, m, \ell = 1, 2, 3
\]  

(4)

There are alternative methods such as the Bloch Vector and the optical Bloch equations that relate the optical phenomena that occur in this type of transitional problems with the density matrix. Substituting the perturbed Hamiltonian described in equation (2) in the equation (1) and taking into account that \(\langle n | \hat{V} | m \rangle = V_{nm} = -\mu_{nn} \cdot E\) we obtain a system of equations for the elements of the density matrix.

On the other hand, the electric field of the pulse is considered in the form \(E = \sum_{\ell=1} \hat{E}_{\ell} \exp[i(kz - \omega_{\ell} t)]\), where \(\hat{E}_{\ell}\) it is the complex amplitude of slow envelope of the electric field. Substituting the equation for the electric field and to guarantee \(E\) real, we choose \(\hat{E}_{\ell} = \hat{E}_{-\ell}^*\), which is widely used in the study.
of nonlinear waves. Using the slow envelope approach, which relies on the envelopes $\hat{E}_i$ of the pulses vary smooth enough in space and time, that is, $\left| \frac{\partial \hat{E}_i}{\partial t} \right| \ll \nu \left| \hat{E}_i \right|$, $\left| \frac{\partial^2 \hat{E}_i}{\partial z^2} \right| \ll k \left| \hat{E}_i \right|$, then we can eliminate the second derivatives of the amplitudes in the wave equation 3, taking this form:

$$\sum_{i=1}^{N} e^{i(kz-\omega t)} \left[ f \left( (ck)^2 - \eta^2 \frac{\partial^2}{\partial z^2} - 2i \eta \frac{\partial}{\partial t} \right) \hat{E}_i - 4\pi i \eta^2 \frac{\partial^2 \hat{E}_i}{\partial T^2} \right] = -4\pi i \eta^2 \frac{\partial^2 \hat{E}_i}{\partial T^2} \left[ \left( \mu_{12} \hat{P}_{21} + \mu_{32} \hat{P}_{32} \right) e^{i(kz-\omega t)} \right] + c.c$$

(5)

The elements $\rho_{nm}$, wit $n=m$, give rise to the populations in the states, and non-diagonal elements $\rho_{nm}$ wit $n \neq m$ contain the relative phase between the states that describe the atomic coherence, so they can be written in terms of $e^{i(kz-\omega t)}$. Under the condition of the system in the base state, $\rho_{11} = 1$, $\rho_{21} = 0$ y $\rho_{31} = 0$, the system of equations (4) can be reduced introduced $A(z) = \frac{\mu_{12}}{\hbar} \int_{-\infty}^{\infty} E(z,t) dt$, that corresponds to the area of the nonlinear optical pulse and $\delta = \mu_{12} / \mu_{13}$ in the form:

$$\hat{P}_{21} = \left( i / 2d^3 \right) \left( \sin 2dA + 2\delta^2 \sin dA \right)$$

(6)

$$\hat{P}_{32} = \left( i\delta / 2d^3 \right) \left( \sin 2bA - 2\sin dA \right)$$

(7)

where $d = \left( 1 + \delta^2 \right)^{1/2}$. Substituting equations (6) and (7) in the wave equation (5), and taking $\delta = 0$, So we get the well-known double-Sine-Gordon equation:

$$\psi_0 + \frac{\eta}{c} \psi_0 + \frac{\pi \eta \mu_{12}^2 N_0}{\hbar \eta^2} \sin \left( 2\psi \right) = 0$$

(8)

Is possible to solve this equation by using a transformation of coordinates for the time of the form $\xi = t - z / \nu$, where $\nu$ is the velocity of the pulse. The approach is to assume that there is a solution to determine the complex amplitude of the non-linear Schrödinger NLS equation that has the form $\chi(z,t) = U(z) S(\xi) e^{i(kz-\omega t)}$ and developing this transformation and using the approximation $\chi(z,t)$ we obtain an ordinary differential equation

$$\frac{d^2 \chi}{d \xi^2} = 2\pi \hbar \eta \mu_{12}^2 N_0 \left[ \frac{c - \eta \nu}{\eta \nu} \right] \sin \chi$$

(9)

In real quantum dots the delta of frequencies between levels and between these and the pulse are different from zero, therefore it is assumed that the transition from level 1 to 2 and from level 2 to level 3 are very close to each other and the pulse frequency. Taking into account the approximation of a non-resonant excitation with a constant detuning, i.e. $\delta_{\text{solution}} - \delta_0 - \delta_1 \approx \delta_0 - \delta_1 = \delta$ y $\mu_{12} = \mu_{32}$, then solution of equation (9) is known as solitonic solution

$$\chi(z,t) = \frac{2\hbar}{\mu_{12}} \text{sech} \left( \frac{\xi}{T} \right)$$

(10)

with $T = \left( \frac{\hbar \eta^2}{2\pi \hbar \mu_{12}^2 N_0 (c - \eta \nu / (\eta \nu - 1))} \right)^{1/2}$, that corresponds to the width of the pulse reemitted by the SQD[15].

3. Results
The theoretical results of equation solitonic that was obtained from the double-Sine-Gordon equation, show that the excitation of SQDs from an intense nonlinear wave can produce optical solitons as a
consequence of the interaction of light with the SQDs. During the non-linear interaction process, the SQDs are considered as a three-level energy quantum system, in which the optical transitions are given from the ground state to the excitonic and biexcitonic states. The allowed transitions between the ground state and the excitonic and biexcitonic states have a much lower dipole moment than the transition between the ground state and the background of the exciton band. On the other hand, the characteristics of the light reemitted by the assembly of quantum dots in the form of optical solitons will depend on the intensity of the incident light, whose minimum value to form the optical solitons could be determined specifically depending on the nature of the SQDs. In this way, the soliton remitted will depend on the refractive index of the semiconductor \( \eta \), the dipole moment corresponding to the transitions between the base state and the excitonic state or between the excitonic state and the biexcitonic state \( \mu_{12} \), taking into account that we have considered an energy system of three equidistant levels. On the other hand, there is also a dependence on the external excitation frequency \( f \) and the density of quantum dots \( N_e \). On the other hand, for the study of the propagation of short optical pulses through non-linear optical fibers, the non-linear Schrödinger NLSE equation is used, which takes into account the effects of the length of the fiber, the dispersion effect of group speed and non-linear optical effects as a consequence of the high intensity of light. The Fourier Split-Step method is a pseudo-spectral technique that is extremely useful due to its rapid and good accuracy in calculations. In general, this method obtains an approximate solution of the propagation equation, assuming that dispersion effects and non-linear effects act independently along the fiber in very small steps [16]. This technique was used to simulate the evolution of solitaries that are re-emitted by the SQD. In the proposal that we propose in this research, we assume that the assembly of quantum dots is coupled to the optical fiber, which allows the light to re-emit by the QDs as a consequence of non-linear interaction between the light and the SQDs is directly coupled to the sea the core of the fiber and in this way the light will be guided by total internal reflection on the interior of the fiber, which can be clearly seen in figure 1. For the simulation, we have considered. In the figure 2, the profile of the soliton re-emitted by the SQDs system for different concentrations of quantum dots is observed. In the results, it is observed that the peak intensity decays at higher density values. An explanation to this effect can realize that it can increase the QDs per unit of volume, it can increase the effects of the absorption of the SQD, re-emitting with a lower intensity. On the other hand, we must bear in mind that the mathematical modeling proposed requires that the density \( N_e \) of quantum dots is lower, so that the interactions of QDs in the Hamiltonian are omitted. For the simulation we have proposed SQDs with pyramidal morphology of InAs/GaAs manufactured with a cylindrical symmetry with the parameters: \( \eta = 3.3 \), \( \mu_{12} = 1.9 \times 10^{-28} \text{Cm} \), \( \lambda = 850 \text{nm} \), \( \nu = 1.7 \times 10^8 \text{m/s} \).

**Figure 2:** Simulation of the soliton generated by the SQDs system with different densities of QDs. a) \( 8 \times 10^5 \text{cm}^{-3} \). b) \( 8 \times 10^{10} \text{cm}^{-3} \). c) \( 8 \times 10^{15} \text{cm}^{-3} \).

For the simulation of the soliton evolution in the optical fiber we have used the split-step Fourier method on a non-linear standard optical fiber with the parameters: fiber attenuation \( \alpha = 0.1 \text{dB/km} \), nonlinear fiber parameter \( \gamma = 0.3 \text{W/m} \) and second order dispersion \( \beta_2 = -20 \times 10^{-27} \text{cm}^2/\text{p} \). From the simulation results, figure 3a shows the input pulse in the fiber while in figure 3b the pulse is observed at a distance traveled of 1000.0 m.
4. Conclusions
Through the present investigation, it can be concluded that it is possible to produce solitonic pulses in quantum dots embedded in nonlinear optical fibers with the purpose of propagating fields without losses over long distances. The analytical treatment formulated in this work, would guarantee the generation of solitonic optical pulses, which would be modulated according to the amplitude of the pulse and its morphology. Likewise, it is proposed the manufacture of this type of nanostructures with different morphologies inserted in optical fibers that allow the propagation of light without losses over long distances.

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