Multicritical behaviour of the compressible systems

S.V. Belim

Omsk State University, 55-a, pr. Mira, Omsk, Russia, 644077

(Dated: December 6, 2018)

The behaviour of uniform elastically isotropic compressible systems in critical and tricritical points is described in field-theoretical terms. Renormalization-group equations are analyzed for the case of three-dimensional systems in a two-loop approximation. Fixed points corresponding to various types of critical and multicritical behaviour under various macroscopic conditions imposed on the system are distinguished. It is shown that the effect of the elastic deformations on the critical behaviour of compressible systems is significant. It manifests itself both in a change in the critical exponents of Ising magnetic and in the appearance of multicritical points in phase diagrams at any dimension of the order parameter. It is also shown that, in a number of experimental investigations, the multicritical behaviour is not tricritical, as it has been stated, but tetracritical. The influence exerted by elastic deformations on systems with phase diagrams already containing multicritical points is analyzed. It is shown that the effect of elastic deformations leads to a change from bicritical behaviour to a tetracritical one.

PACS numbers: 64.60.-i

I. INTRODUCTION

Phase diagrams of many materials exhibit multicritical behaviour when several lines of phase transitions are crossed. The tricritical points, in which the second-order phase transition is replaced by the first-order transition, are of particular interest. Experimental studies are mostly concerned with systems subjected to external pressure, i.e. those influenced by elastic deformations. Owing to this circumstance, taking into account the relationship between the order parameter and elastic deformations is important. As first shown in [1], in the elastically isotropic case, the critical behaviour of compressible systems with quadratic striction is unstable with respect to the relationship between the order parameter and acoustic modes, and a first-order phase transition close to the second-order transition occurs in systems of this kind. However, the conclusions of [1] are only valid at low pressures. It was shown in [2] that at high pressures, a more fundamental influence is exerted on the system by deformational effects induced by the external pressure, beginning from a certain tricritical value \( P_t \). This changes the sign of the effective constant of interaction between fluctuations of the order parameter, and, as a consequence, leads to a change of the type of the phase transition. For uniform compressible systems, two types of tricritical behaviour and the existence of a tetracritical point, at which two tricritical curves meet, were predicted in [2]. Calculations made in terms of a two-loop approximation [3] confirm the existence of two types of tricritical behaviour for the Ising systems and yield the tricritical exponents.

According to the criterion obtained in [1], striction effects, regarded as additional thermodynamic parameters, change the mode of the critical behaviour only in systems with a singular behaviour of heat capacity in the absence of deformations. The specific heat exponent \( \alpha(C \sim |T - T_c|^{-\alpha}) \) is only positive for Ising-like magnetic. In the XY-model and the Heisenberg model, the specific heat exponent is positive for "rigid" systems, and, therefore, elastic deformations must have no effect on the critical behaviour. Hence follows that the critical value of the order parameter dimensionality \( n_c < 2 \).

In this study, we develop further the model of phase transformations in uniform compressible systems characterized by various dimensions of the fluctuating order parameter [4, 5]. We analyze these transformations using the renormalization-group methods in a two-loop approximation, directly in the three-dimensional space. Also, we consider the conditions for occurrence of a tricritical behaviour owing to effects of long-range interaction of the order-parameter fluctuations caused by long-wavelength acoustic modes. Since the dependence of the exchange integral on distance makes a major contribution to the striction effects in the critical region, we consider only elastically isotropic systems.

II. THEORY

We can write the Hamiltonian of the uniform Ising model, with allowance for elastic deformations, as follows:

\[
H_0 = \int d^D x \left[ \frac{1}{2} \left( \tau_1 + \nabla^2 \right) \tilde{S}(x)^2 \right] + \frac{u_0}{4!} (\tilde{S}(x)^2)^2 + \int d^D x \left[ a_1 (\sum_{\alpha=1}^3 u_{\alpha\alpha}(x))^2 + a_2 \sum_{\alpha, \beta=1}^3 u_{\alpha\beta}^2 \right] + \frac{1}{2} a_3 \int d^D x \tilde{S}(x)^2 (\sum_{\alpha=1}^3 u_{\alpha\alpha}(x)) \tag{1}
\]

*Electronic address: belim@univer.omsk.su
where $S(x)$ is the $n$-dimensional order parameter; $u_0$ is a positive constant; $\tau_0 \sim |T - T_c|/T_c$, where $T_c$ is the phase transition temperature; $\varepsilon_{\alpha\beta}$ is the deformation tensor; $a_1, a_2$ are the elastic constants of the crystal; and $a_3$ is the quadratic-striction parameter. Passing in equation (1) to Fourier transforms and integrating with respect to terms depending on non-fluctuating variables, which do not interact with the parameter of the $S(x)$ order, and introducing for the sake of convenience a new variable $y(x) = \sum_{\alpha=1}^{3} u_{\alpha\alpha}(x)$, we obtain the Hamiltonian of the system in the following form:

$$H_0 = \frac{1}{2} \int d^D q (\tau_0 + q^2)S_qS_{-q}$$

$$+ \frac{u_0}{4!} \int d^D q_1 S_{q_1} S_{q_2} S_{q_3} S_{-q_1 - q_2 - q_3} +$$

$$+ a_3 \int d^D q y_q S_{q_2} S_{-q_1 - q_2}$$

$$+ \frac{a^{(0)}}{\Omega} y_0 \int d^D q y_q S_{-q} + \frac{1}{2} a_3 \int d^D q y_q y_{-q} + \frac{1}{2} \frac{a^{(0)}}{\Omega} y_0^2$$

In equation (2) the terms $y_0$, which describe uniform deformations, are separated. As shown in [1], such a separation is necessary, since the nonuniform deformations $y_q$ are responsible for acoustic phonon exchange and lead to long-range effects, which are absent in the case of uniform deformations.

Let us define the effective Hamiltonian of the system, which depends only on the strongly fluctuating order parameter $S$, in the following way:

$$\exp[-H[S]] = B \int \exp[-H_R[S, y]] \prod d y_q$$

If an experiment is carried out at constant volume, $y_0$ is a constant and the integration in equation (3) is only done over nonuniform deformations, with uniform deformations making no contribution to the effective Hamiltonian. At a constant pressure, a term $P\Omega$ is added to the Hamiltonian, with the volume represented in terms of deformation tensor components as

$$\Omega = \Omega_0 [1 + \sum_{\alpha=1}^{3} u_{\alpha\alpha} + \sum_{\alpha \neq \beta} u_{\alpha\alpha} u_{\beta\beta} + O(u^3)]$$

and integration in equation (3) also performed over uniform deformations. As noted in [6], taking into account the quadratic terms in equation (4) can be important in the case of high pressures and crystals with large striction effects. The neglect of these quadratic terms in [1] restricts applicability of the results obtained by Larkin and Pikin to only the case of low pressures. As a result, we have

$$H = \frac{1}{2} \int d^D q (\tau_0 + q^2)S_qS_{-q}$$

$$+ \frac{u_0}{4!} \int d^D q_1 S_{q_1} S_{q_2} S_{q_3} S_{-q_1 - q_2 - q_3} +$$

$$+ \frac{1}{2 \Omega} (z_0 - w_0) \int d^D q y_q S_{q_1} S_{-q_1} S_{q_2} S_{-q_2}$$

$$z_0 = a_1^2/(4a_3), \quad w_0 = a_1^{(0)}/(4a_3^{(0)})$$

The effective interaction parameter $v_0 = u_0 - 12z_0$, which appears in the Hamiltonian owing to the influence of striction effects described by the parameter $z_0$, can assume not only positive, but also negative values. As a result, this Hamiltonian describes both first- and second-order phase transitions. At $v_0 = 0$, a tricritical behaviour is exhibited by the system. In its turn, the effective interaction in equation (5), which is determined by the difference of the parameters $z_0 - w_0$, may lead to a second-order phase transition at $z_0 - w_0 > 0$ and to a first-order phase transition at $z_0 - w_0 < 0$. From the given type of the effective Hamiltonian follows that there can exist a higher-order critical point at which tricritical curves intersect if the conditions $v_0 = 0$ and $z_0 = w_0$ are fulfilled simultaneously [2]. It should be noted that, under the tricritical condition $z_0 = w_0$, the Hamiltonian of the model (5) is isomorphic to the Hamiltonian of a uniform rigid system.

In terms of the field-theoretical approach [7], the asymptotic critical behaviour and the structure of phase diagrams in the fluctuation region are determined by the Callan-Symanzik renormalization-group equation for the vertex parts of irreducible Green function. In order to calculate $\beta$- and $\gamma$-functions as functions appearing in the Callan-Symanzik equation of renormalized interaction vertices $u, a_1, a_1^{(0)}$ as well as functions of complex vertices $z = a_1^2/(4a_3), w = a_1^{(0)}/(4a_3^{(0)})$, $v = u - 12z$, which are more convenient for determining the critical and tricritical behaviour of the model, we applied the standard method based on Feynman’s diagram technique and the renormalization procedure [8]. As a result, we obtained in terms of the two-loop approximation the following expressions for the $\beta$-functions

$$\beta_v = -v \left(1 - \frac{n + 8}{6} v + \frac{11n + 190}{243} v^2 \right)$$

$$\beta_z = -z \left(1 - \frac{n + 2}{3} v - 2nz + \frac{23(n + 2)}{243} v^2 \right)$$

$$\beta_w = -w \left(1 - \frac{n + 2}{3} v - 4nz + 2nw + \frac{23(n + 2)}{243} v^2 \right)$$

It is known that the perturbation-theory expansions are asymptotic and the interaction vertices of order-parameter fluctuations in the fluctuation region are sufficiently large for equations (14) to be applied directly. Therefore, in order to extract the necessary physical information from the expressions obtained, we use the Pade-Borel method extended to a three-parameter case.
Then the direct and inverse Borel transformations have the following form:

\[
    f(v, z, w) = \sum_{i_1, i_2, i_3} c_{i_1, i_2, i_3} v^{i_1} z^{i_2} w^{i_3} \\
    = \int_0^\infty e^{-t} F(vt, zt, wt) dt \\
    F(v, z, w) = \sum_{i_1, i_2, i_3} \frac{c_{i_1, i_2, i_3}}{(1 + i_1 + i_2 + i_3)!} v^{i_1} z^{i_2} w^{i_3}.
\]

To obtain an analytical continuation of the Borel transform of the function, we introduce a series in auxiliary variable \( \theta \):

\[
    \hat{F}(v, z, w, \theta) = \sum_{k=0}^\infty \theta^k \sum_{i_1, i_2, i_3} \frac{c_{i_1, i_2, i_3}}{k!} v^{i_1} z^{i_2} w^{i_3} \delta_{i_1 + i_2 + i_3, k},
\]

(7)

to which we apply the Pade \([L/M]\) approximation at the point \( \theta = 1 \). This technique was proposed and tested in [9] for describing the critical behaviour of a number of systems characterized by several interaction vertices of order-parameter fluctuations. The property of symmetry conservation of the system upon application of the Pade approximant in variable \( \theta \), revealed in [9], becomes significant in describing multivertex models.

In the two-loop approximation, we used the \([2/1]\) approximant for calculating the \( \beta \)-functions. The nature of the critical behaviour is determined by the existence of a stable fixed point that satisfies the set of equations

\[
    \beta_i(v^*, z^*, w^*) = 0 \quad (i = 1, 2, 3).
\]

The requirement that a fixed point be stable is reduced to the condition that the eigenvalues \( b_i \) of the matrix

\[
    B_{i,j} = \frac{\partial \beta_i(u^*_1, u^*_2, u^*_3)}{\partial u_j} \quad (u_i, u_j \equiv v, z, w)
\]

(9)

lie in the right-hand complex half-plane. The fixed point with \( v^* = 0 \), corresponding to the critical behaviour, is a saddle point and must be stable in the directions specified by the variables \( z, w \) and unstable in the direction specified by the variable \( v \). The tricritical fixed point is stabilized in the direction specified by the variable \( v \) as a result of taking into account in the effective Hamiltonian of the model terms of sixth order in order-parameter fluctuations. The fixed point with \( z^* = w^* \), which corresponds to the tricritical behaviour of the second type, is also a saddle point and must be stable in the directions specified by the variables \( v, z \) and unstable in the direction specified by the variable \( w \). Its stabilization may be due to anharmonic effects.

The obtained set of the summed-up functions contains a wide variety of fixed points. Table 1 presents fixed points for the Ising model \((n = 1)\), XY model \((n = 2)\) and the Heisenberg model \((n = 3)\), which are the most interesting for describing the critical and tricritical types of behaviour, which lie in the physical range of vertices with \( v, z, w \geq 0 \). The table also contains the eigenvalues of the stability matrix for the corresponding fixed points.

Analysis of the magnitudes of fixed points and their stability suggests the following: the Gaussian fixed points \( I_0, X_0, G_0 \) are critical and unstable with respect to the effect of elastic deformations. The critical behaviour of incompressible systems with respect to the deformation degrees of freedom is unstable for the Ising model \((I_1)\) and stable for the Heisenberg model \((G_1)\). For the \( XY \) model \((X_1)\), the eigenvalue \( b_2 < 0 \), but is comparable in order of magnitude with the accuracy of calculations; therefore, we cannot make any unambiguous conclusion concerning the stability of this fixed point. Apparently, the difficulties in description of the \( XY \) model are associated with the closeness of the critical dimensionality of the order parameter \( n_c \) to 2. According to the criterion obtained in [1], \( n_c < 2 \), whereas the two-loop approximation gives \( n_c = 2.011 \). For Ising systems, the fixed point is stable at constant pressure \((I_2)\); for Heisenberg systems, the corresponding point is unstable \((G_2)\); for the \( XY \) model, it is impossible to make any unambiguous conclusion since the critical dimensionality is also close to 2. The fixed points \( I_3, X_3 \) and \( G_3 \) describe the first type of the tricritical behaviour of compressible systems, which can be observed at constant pressure. The fixed points \( I_4, X_4 \) and \( G_4 \) are tricritical for systems investigated at constant volume. The points \( I_5, X_5 \) and \( G_5 \) are critical points of fourth order, with two critical lines intersecting at these points.

The magnitudes of vertices, obtained in the two-loop approximation for the fixed points corresponding to the critical and tricritical types of behaviour of the compressible Ising model, make it possible to calculate the critical exponent for the given systems on the basis of the expressions (summed by the PadeBorel method) for the exponents \( \nu \) and \( \eta \):

\[
    \nu = \frac{1}{2} \left( 1 + \frac{n + 2}{12} \right), \eta = \frac{2(n + 2) + 1}{243} v^{\nu^2}.
\]

(10)

The values of the other critical exponents can be obtained from scaling expressions relating them to the exponents \( \nu \) and \( \eta \).

The critical behaviour of compressible Ising systems at constant pressure \((12)\) is characterized by renormalized critical exponents according to Fishers theory of the effect of additional thermodynamic variables \([10]\)

\[
    \nu^{(f)} = 0.632, \eta^{(f)} = 0.028, \alpha^{(f)} = 0.103, \beta^{(f)} = 0.325, \gamma^{(f)} = 1.247.
\]

For the critical behaviour of the first type \((I_3, X_3, G_3)\) the Hamiltonian \((5)\) is isomorphic to the Hamiltonian of the uniform incompressible model, and, therefore, the critical exponents coincide in this case with those for the
Table 1. Magnitudes of fixed points and eigenvalues of the stability matrix.

| N | ν | z | w | b₁ | b₂ | b₃ |
|---|---|---|---|----|----|----|
| n=1 | 0 | 0 | 0 | -1 | -1 | -1 |
| I0 | 1.064472 | 0 | 0 | 0.6536 | -0.1692 | -0.1692 |
| I1 | 1.064472 | 0.089187 | 0 | 0.6536 | 0.1702 | 0.1710 |
| I2 | 1.064472 | 0.089187 | 0.089187 | 0.6536 | 0.1702 | -0.1710 |
| I3 | 0 | 0.5 | 0 | -1 | 1 | -0.16923 |
| I4 | 0 | 0.5 | 0.5 | -1 | 1 | -1 |
| I5 | 0.25 | 0.25 | -1 | 1 | -1 |

n=2

| X0 | 0 | 0 | 0 | -1 | -1 | -1 |
| X1 | 0.934982 | 0 | 0 | 0.6673 | -0.0017 | 0.1053 |
| X2 | 0.934982 | 0.000439 | 0 | 0.6673 | 0.0017 | 0.1087 |
| X3 | 0.934982 | 0.000439 | 0.000439 | 0.6673 | 0.0017 | -0.1053 |
| X4 | 0 | 0.25 | 0 | -1 | 1 | 1 |
| X5 | 0.25 | 0.25 | -1 | 1 | -1 |

n=3

| G0 | 0 | 0 | 0 | -1 | -1 | -1 |
| G1 | 0.829620 | 0 | 0 | 0.6813 | 0.1315 | 0.2173 |
| G2 | 0.829620 | 0.022909 | 0 | 0.6813 | -0.1311 | -0.0518 |
| G3 | 0.829620 | 0.022909 | 0.022909 | 0.6813 | -0.1311 | -0.2170 |
| G4 | 0 | 1/6 | 0 | -1 | 1 | 1 |
| G5 | 0 | 1/6 | 1/6 | -1 | 1 | -1 |

Incompressible model:

\[ \nu^{(I)} = 0.708, \eta^{(I)} = 0.028, \alpha^{(I)} = -0.125, \]
\[ \beta^{(I)} = 0.364, \gamma^{(I)} = 1.397, \]
\[ \nu^{(XY)} = 1, \eta^{(XY)} = 0, \alpha^{(XY)} = -1, \]
\[ \beta^{(XY)} = 0.5, \gamma^{(XY)} = 2, \]
\[ \nu^{(G)} = 1, \eta^{(G)} = 0, \alpha^{(G)} = -1, \]
\[ \beta^{(G)} = 0.5, \gamma^{(G)} = 2. \]

The tricritical behaviour of the second type (I₄, X₄, G₄) corresponds to the critical behaviour of the spherical model and is determined by the corresponding exponents:

\[ \nu^{(I)} = 1, \eta^{(I)} = 0, \alpha^{(I)} = -1, \]
\[ \beta^{(I)} = 0.5, \gamma^{(I)} = 2, \]
\[ \nu^{(XY)} = 1, \eta^{(XY)} = 0, \alpha^{(XY)} = -1, \]
\[ \beta^{(XY)} = 0.5, \gamma^{(XY)} = 2, \]
\[ \nu^{(G)} = 1, \eta^{(G)} = 0, \alpha^{(G)} = -1, \]
\[ \beta^{(G)} = 0.5, \gamma^{(G)} = 2. \]

The fixed points of the fourth order (I₄, X₄, G₄) are characterized by the mean-field values of the critical exponents:

\[ \nu^{(I)} = 0.5, \eta^{(I)} = 0, \alpha^{(I)} = 0.5, \]
\[ \beta^{(I)} = 0.25, \gamma^{(I)} = 1, \]
\[ \nu^{(XY)} = 0.5, \eta^{(XY)} = 0, \alpha^{(XY)} = 0.5, \]
\[ \beta^{(XY)} = 0.25, \gamma^{(XY)} = 1, \]
\[ \nu^{(G)} = 0.5, \eta^{(G)} = 0, \alpha^{(G)} = 0.5, \]
\[ \beta^{(G)} = 0.25, \gamma^{(G)} = 1. \]

Thus, the system may exhibit a multicritical behaviour under the influence of striction effects. This poses the question as to how elastic deformations affect systems whose phase diagrams already contain multicritical points of bi- or tetracritical nature. Two lines of second-order phase transitions and a line of a first-order phase transition intersect at the multicritical point in the first case, and four lines of second-order phase transitions, in the second. In the immediate vicinity of the multicritical point, the system manifests a specific critical behaviour characterized by the competition of ordering types. In this case, one critical parameter is replaced by another at the bicritical point, whereas the tetracritical point allows existence of a mixed phase with coexisting types of ordering. Systems of this kind [13] can be described by introducing two order parameters transformed in accordance with different irreducible representations.

In this case, the model Hamiltonian of the system is as follows:

\[
H_0 = \int d^2x \left[ \frac{1}{2} (\tau_1 + \nabla^2) \Phi(x)^2 + \frac{1}{2} (\tau_1 + \nabla^2) \Psi(x)^2 + \frac{u_{10}}{4!} (\Phi(x))^2 + \frac{u_{20}}{4!} (\Psi(x))^2 + \frac{2u_{30}}{4!} (\Phi(x)\Psi(x))^2 + g_1 y(x)\Phi(x)^2 + g_2 y(x)\Psi(x)^2 + \beta y(x)^2 \right],
\]

where \( \Phi(x) \) and \( \Psi(x) \) are fluctuating order parameters; \( u_{10} \) and \( u_{20} \) are positive constants; \( \tau_1 \sim |T - T_{c1}|/T_{c1} \), \( \tau_2 \sim |T - T_{c2}|/T_{c2} \), where \( T_{c1} \) and \( T_{c2} \) are the phase-transition temperatures for, respectively, the first and
This Hamiltonian leads to a wide variety of multicritical points. As in the case of incompressible systems, both tetracritical \((v_3 + 12(z_1 z_2 - w_1 w_2))^2 < (v_1 + 12(z_1^2 - w_1^2))(v_2 + 12(z_2^2 - w_2^2))\) and bircritical \((v_3 + 12(z_1 z_2 - w_1 w_2))^2 \geq (v_1 + 12(z_1^2 - w_1^2))(v_2 + 12(z_2^2 - w_2^2))\) types of behaviour are possible. In addition, the striction effects may lead to multicritical points of higher order.

In order to calculate functions as functions appearing in the Callan-Symanzky equation of renormalized interaction vertices \(u_1, u_2, u_3, g_1, g_2, g_1^{(0)}, g_2^{(0)}\) or as functions of complex vertices \(z_1, z_2, w_1, w_2, v_1, v_2, v_3\), which are more convenient for determining the multicritical behaviour of the model, we applied the standard method based on Feynman diagram technique and on a renormalization procedure [8]. As a result, we obtained in terms of the two-loop approximation the following expressions for the functions:

\[
\begin{align*}
\beta_{v_1} &= -v_1 + \frac{3}{2} v_1^2 + \frac{1}{6} v_1^3 - \frac{77}{81} v_1^4 - \frac{23}{243} v_1 v_3^2 - \frac{2}{27} v_3^3, \\
\beta_{v_2} &= -v_2 + \frac{3}{2} v_2^2 + \frac{1}{6} v_2^3 - \frac{77}{81} v_2^4 - \frac{23}{243} v_2 v_3^2 - \frac{2}{27} v_3^3, \\
\beta_{v_3} &= -v_1 + \frac{2}{3} v_3^2 + \frac{1}{2} v_1 v_3 + \frac{1}{2} v_2 v_3^2, \\
\beta_{v_1^2} &= -\frac{41}{243} v_3^3 - \frac{23}{162} v_1^2 v_3 - \frac{23}{162} v_2^2 v_3 - \frac{1}{3} v_1 v_3^2 - \frac{1}{3} v_2 v_3^2, \\
\beta_{v_1^3} &= -\frac{7}{243} v_3^2 z_1 - \frac{2}{27} v_3^2 z_2, \\
\beta_{w_1} &= -w_1 + \frac{1}{3} v_1 w_3 + 4 z_1^2 w_1 - 2 w_3^3 + 4 z_1 z_2 w_2, \\
\beta_{w_2} &= -2 w_1 w_2 + \frac{1}{3} v_3 w_3^2 - \frac{23}{81} v_3^2 w_1 - \frac{7}{243} v_3^3 w_1 - \frac{2}{27} v_3^2 w_2, \\
\beta_{w_3} &= -w_2 + v_2 w_2 + 4 z_1^2 w_2 - 2 w_3^3 + 4 z_1 z_2 w_1, \\
\beta_{w_1^2} &= -2 w_1 w_2 + \frac{1}{3} v_3 w_3^2 - \frac{23}{81} v_3^2 w_2 - \frac{7}{243} v_3^3 w_2 - \frac{2}{27} v_3^2 w_1.
\end{align*}
\]

The obtained set of summed-up functions contains a wide variety of fixed points, which lie in the physical range of vertices with \(v_i \geq 0\).

Analysis of the magnitudes of fixed points and of their stability suggests the following. The bircritical fixed point of incompressible systems \((v_1 = 0.934982, v_2 = 0.934982, v_3 = 0.934982, z_1 = 0, z_2 = 0, w_1 = 0, w_2 = 0)\) is unstable with respect to the effect of uniform deformations \((b_1 = 0.090, b_2 = 0.523, b_3 = 0.667, b_4 = -0.521, b_5 = -0.002, b_6 = -0.521, b_7 = -0.002)\). Striction effects stabilize the tetracritical fixed point for compressible systems \((v_1 = 0.934982, v_2 = 0.934982, v_3 = 0.934982, z_1 = 0, z_2 = 0, w_1 = 0, w_2 = 0, b_1 = 0.090, b_2 = 0.523, b_3 = 0.667, b_4 = 2.144, b_5 = 0.267, b_6 = 5.223, b_7 = 0.882)\).

The question of whether or not other multicritical points exist cannot be resolved in terms of the model described here since the calculations lead to a degenerate system of equations. The degeneration is lifted when terms of higher order in deformation tensor components and in fluctuating order parameters are taken into account in the Hamiltonian.

The investigation we performed revealed a significant influence of elastic deformations on the critical behaviour of compressible systems, which is manifested both in a change in the magnitudes of critical exponents for the Ising systems and in the appearance of multicritical points in the phase diagrams for all the three models. The boundary value of the order parameter dimensionality is close to 2, as in the case of the influence of frozen defects. This conclusion is in agreement with the criterion obtained in [1] on the basis of the mean-field theory.
The critical exponents for compressible systems are in good agreement with Fishers theory [10] of the influence exerted by additional thermodynamic variables. All the experimental studies of the multicritical behaviour we are aware of have been carried out for compressible systems [11]. All of them lead to a conclusion that the critical behaviour is characterized by a Gaussian fixed point with mean-field magnitudes of the critical exponents. In a number of papers, an opinion has been expressed that the system demonstrates a tricritical behaviour [12], while in others, it has been stated that this behaviour is bicritical. The critical exponents found in this study led us to conclude that a tetracritical behaviour determined by a fourth-order critical point was observed in the studies mentioned above. In systems, described by two fluctuating order parameters, the striction interaction with elastic deformations leads to a change of the bicritical behaviour for a tetracritical one.

The work is supported by Russian Foundation for Basic Research N 04-02-16002.

[14] Larkin A I and Pikin S A 1969 JETP 56 1664
[15] Imry Y 1974 Phys. Rev. Lett. 33 1304
[16] Belim S V and Prudnikov V V 2001 Phys. Solid State 45 1353
[17] Laptev V M and Skryabin Yu N 1979 Phys. Stat. Soli'di B91 K143
[15] Skryabin Y N and Shechonov A B 197 Phys. Lett. A234 147
[6] Bergman D J and Halperin B I 1976 Phys. Rev. B13 2145
[7] Amit D 1976 Field Theory the Renormalization Group and Critical Phenomena (New York: McGrawHill)
[8] Zinn-Justin J 1989 Quantum Field Theory and Critical Phenomena (Oxford: Clarendon Press)
[9] Sokolov A I and Varnashev K B 1999 Phys. Rev. B59 8363
[10] Fisher M E 1976 Phys. Rev. 176 257
[11] arland C W, Bruins D E and Greytak T J 1975 Phys. Rev. B12 2759 Frederichs G E 1971 Phys. Rev. B4 911
[12] Schwartz P 1971 Phys. Rev. B4 920 Steinbrener S 1981 Phys. Rev. B23 162
[13] Prudnikov V V, Prudnikov P V and Fedorenko A A 2000 Phys. Tverd. Tela 42 158
[14] S. V. Belim, Pisma Zh. ksp. Teor. Fiz. 77, 118 (2003) [JETP Lett. 77, 112 (2003)].
[15] V. V. Prudnikov, P. V. Prudnikov, and A. A. Fedorenko, Fiz. Tverd. Tela (St. Petersburg) 42, 158 (2000) [Phys. Solid State 42, 165 (2000)].
[16] D. Amit, Field Theory the Renormalization Group and Critical Phenomena (McGraw-Hill, New York, 1976).
[17] J. Zinn-Justin, Quantum Field Theory and Critical Phenomena (Clarendon Press, Oxford, 1989).