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Longitudinal magnetoresistance in Co-doped BaFe$_2$As$_2$ and LiFeAs single crystals: Interplay between spin fluctuations and charge transport in iron-pnictides

F. Rullier-Albenque, D. Colson, and A. Forget

1 Service de Physique de l’Etat Condensé, Orme des Merisiers, CEA Saclay (CNRS URA 2464), 91191 Gif sur Yvette cedex, France

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I. INTRODUCTION

The proximity and/or coexistence of antiferromagnetism and superconductivity (SC) in the iron-pnictides has been taken as an indication that the magnetic fluctuations may play a decisive role in the SC pairing mechanism. In the undoped compounds, the spin density wave (SDW) ordering is widely attributed to the antiferromagnetic (AF) wave vector. It was argued very early that SC could be also mediated by spin fluctuations at the same wave vector, resulting in an extended s-wave pairing with sign change of the order parameter between electron and hole sheets.

Indeed spin excitations were detected around $Q_{AF}$ by Inelastic Neutron Scattering (INS) measurements in the paramagnetic and superconducting (PM) states of several superconducting iron-pnictides. On the other hand, nuclear magnetic resonance (NMR) measurements have evidenced a strong increase of the nuclear spin-lattice relaxation rate at low $T$ that results from the growth of AF spin fluctuations with decreasing $T$. Surprisingly there is not so far any identified signatures of spin fluctuations in the transport properties of iron pnictides, which opened questions about the interplay between magnetism and charge carriers. Depending on the compounds, different power laws for the temperature dependence of the resistivity have been observed, including linear $T$ dependence of the resistivity for some materials. However, the multiband character of these compounds makes it difficult to extract the $T$ dependence of the scattering rates straightforwardly. Even if transport is dominated by one type of carriers, as in the Co substituted BaFe$_2$As$_2$, one has to care about a possible $T$ dependence of the number of carriers, as anticipated from the analysis of the transport properties and observed recently by Angle Resolved Photoemission (ARPES) measurements. Whether or not the spin fluctuations play a role in the transport properties of pnictides thus remains an interesting and open issue.

In this paper, we show that high resolution measurements of the longitudinal magnetoresistance (LMR) provide a useful probe for studying the coupling between the charge carriers and the spin degrees of freedom in prototypical iron arsenic materials. While a large positive LMR component is found in the AF phase of the pure BaFe$_2$As$_2$, a negative and quite small component is measured in the PM state of a series of Co-substituted BaFe$_2$As$_2$ and of the stoichiometric LiFeAs. The dependence of the LMR coefficient with $T$ and Co content mimics the behavior of the NMR relaxation rates both for Ba(Fe$_{1-x}$Co$_x$)$_2$As$_2$ and LiFeAs. This leads us to interpret the negative LMR by the suppression of AF spin fluctuations by the magnetic field. We propose a qualitative analysis of the LMR in an itinerant nearly antiferromagnetic Fermi liquid model which allows us to reveal different $T$ dependences for the magnetic and non-magnetic Ba(Fe$_{1-x}$Co$_x$)$_2$As$_2$ samples. Our findings uncover a subtle modification of the interaction between the conduction electrons and the fluctuating magnetic moments in the paramagnetic state, which occurs simultaneously with the loss of long range antiferromagnetic order at lower temperatures when $T_c$ is optimal. Understanding these charge transport properties should give useful keys for elucidating the role of magnetic interactions in the physics of pnictides.

II. Co-DOPED BaFe$_2$As$_2$.

A. Transversal and longitudinal magnetoresistances

The single crystals of Ba(Fe$_{1-x}$Co$_x$)$_2$As$_2$ were grown using the self-flux method as detailed in ref. The seven
different Co concentrations studied, spanning the phase diagram from $x = 0$ up to the overdoped $x \sim 0.15$, are indicated in the inset of Fig. 2. Transport measurements were performed on samples which have been cleaved from larger crystals to thicknesses lower than 30 $\mu$m. Low resistivity contacts were done using silver epoxy in a four-probe configuration geometry. The respective values of the antiferromagnetic and superconducting transition temperatures are summarized in Table I.

| $x$  | 0     | 0.047 | 0.06  | 0.075 | 0.1   | 0.12  | 0.154 |
|------|-------|-------|-------|-------|-------|-------|-------|
| $T_N$ (K) | 137.5 | 63    | 33(1) | –     | –     | –     | –     |
| $T_c$ (K)  | –     | 13.7  | 23    | 24.7  | 20.5  | 16.2  | 2.5   |

The magnetoresistance (MR) measurements were done in fixed temperature by sweeping the magnetic field from -14T to 14T and taking the symmetric part of the signal in order to eliminate any spurious Hall effect component due to the misalignment of the contacts. As the MR is rather small, usually less than 0.05% at 14T for $T > 100$K, it is absolutely necessary to ensure that the temperature remains constant during the magnetic field ramp. We have used a cernox sensor which has been calibrated in magnetic field and the temperature is then regulated by compensating the effect of field. The in-plane resistivity measurements were performed in three different geometrical configurations, with $H$ either perpendicular or parallel to the ab-plane. In this latter configuration we have compared the data obtained for both $H$ parallel and perpendicular to the electrical current $J$ at selected temperatures.

To date, MR measurements in undoped and Co-doped BaFe$_2$As$_2$ have been exclusively focused on the magnetic and the signal in the paramagnetic phase has been ignored as being barely measurable (less than 0.05% at 14T $\pm 0.2$). The $T$ evolution of the resistivity increase measured at 14T is reported for pure BaFe$_2$As$_2$ in Fig. 1 in the magnetic and paramagnetic phase for $H \perp ab$ and $H \parallel ab$. Both the transverse (TMR) and longitudinal (LMR) magnetoresistances undergo a discontinuity at $T_N$ and indeed the signals become very weak in the paramagnetic phase, albeit still measurable as shown in the inset of Fig. 1. One can see there that, while the TMR remains positive and vanishes very rapidly, the LMR becomes negative and has a weaker $T$ dependence.

Similar LMR and TMR behaviors are observed for all the samples in the paramagnetic phase. This is illustrated for the 7.5%Co-substituted sample in Fig. 2 that shows the field dependence of the in-plane magnetoresistance normalized to the zero field value $\Delta \rho/\rho_0$ for $T = 45$K up to 120K. As in the PM state of the undoped compound, the TMR is positive and the LMR is negative. Both components show a quadratic dependence with the applied magnetic field with comparable absolute values which are quite small even at low $T$ ($|\Delta \rho/\rho_0|$ less than 0.2% at 45K and 14T) and decrease with increasing temperatures. We can notice in the inset of Fig. 2 that the negative LMR does not depend on the respective orientations of $H$ and $I$. Very comparable LMR data are found for the underdoped and overdoped samples as shown in Appendix A. As no magneto-orbital effect is expected to occur as long as the applied magnetic field is strictly parallel to the FeAs planes in these nearly two-dimensional compounds, this is a strong indication that the negative longitudinal components originate from a spin scattering effect.

### B. Variation of the LMR with $T$ and Co-doping

The natural explanation for the negative LMR is a suppression of the spin fluctuations by the magnetic field, which results in the decrease of the magnetic part of the resistivity $\rho_{sf}(T)$. The small value of the LMR measured here suggests that this magnetic term plays a minor role in the total temperature dependence of the resistivity. This is in agreement with our previous analysis of the transport properties of Ba(Fe$_{1-x}$Co$_x$)$_2$As$_2$ in which we have shown that the electron scattering rate displays a very similar $T^2$ dependence for all $x$ between 0.04 and 0.2, with no apparent incidence of the spin fluctuations.

In order to compare the variation of $\rho_{sf}(T,H)$ in the different samples, we have reported in Fig. 2 the coefficient $\gamma(T)$ defined as

$$\Delta \rho(T,H) = -\gamma(T)H^2$$

While $\gamma$ is barely measurable in the most doped 15.4%Co sample with $T_c \simeq 0$, it increases with underdoping and reaches its maximal value for the 4.7%Co sample near the

![FIG. 1: (color on line) In-plane resistivity increase measured at 14T for BaFe$_2$As$_2$ in two different configurations (TMR:triangles, LMR:squares). A discontinuity of the two components occurs at the magnetic transition temperature $T_N$ above which both the TMR and the LMR becomes very small. Their evolution in the paramagnetic state is better visualized in the inset where the $\Delta \rho$ scale was multiplied by a factor $\sim 35$.](image)
magnetic transition. For all the superconducting samples, one can notice that $\gamma$ displays a maximum value around 40K before decreasing when approaching $T_c$. This might be due to the suppression of the superconducting fluctuations by the magnetic field, which would add an additional positive contribution to the LMR below 40K. One can also see a net change of behavior between the 6%Co magnetic sample and the 7.5%Co non-magnetic sample, $\gamma$ vanishing more rapidly with increasing $T$ in the latter case.

The evolution of $\gamma$ with doping and temperature bears striking similarities with the behavior of the NMR nuclear spin-lattice relaxation rate $1/T_1T$ for the same series of Ba(Fe$_{1-x}$Co$_x$)$_2$As$_2$ samples. In particular, the temperatures at which upturns of $1/T_1T$ are detected in ref. indicate full arrows at the bottom of fig.3 correspond very well to the onset temperature for observing a LMR coefficient larger than $\sim 2 \times 10^{-4}$ $\mu\Omega$cm/$T^2$. This strongly supports the relation between the negative LMR and the AF spin fluctuations. This also confirms that the superconductivity may be linked to the growth of AF spin fluctuations in these iron pnictides.

C. Analysis of the negative magnetoresistance

In order to explain the magnetic order at $Q_{AF}$, both local-moment and itinerant scenarios have been invoked. However the observation that the local susceptibility around $Q_{AF}$ displays a Curie-Weiss behavior, while the uniform susceptibility does not, seems to favor a description in terms of itinerant electrons. The analysis of the spin dynamics of the parent and Co-doped 122 compounds as determined by INS or NMR have shown that the imaginary part of the dynamic spin susceptibility $\chi(q, \omega)$ can be very well described in the framework of a 2D itinerant nearly AF metals theory. In particular, the NMR ($1/T_1T$) has been interpreted as being directly related to the AF spin fluctuations and expressed in terms of the static staggered susceptibility which displays the Curie-Weiss $T$ dependence $\chi_Q \propto 1/(T - \theta)$, with $\theta \sim T_N$ for the magnetic samples. Negative values of $\theta$ were found for the non magnetic samples with short-range AF spin fluctuations.

The electrons that are more scattered by spin fluctuations are the ones near the hot spots of the Fermi surface, i.e., those connected by the ordering wave vector $Q_{AF}$. The resistivity contribution due to spin fluctuations $\rho_{sf}$ is thus directly expressed in terms of the dynamical spin susceptibility $\chi(q, \omega)$. When the spin fluctuation spectrum is two-dimensional, as observed in the PM state of the iron pnictides, one finds that $\rho_{sf}(T) \sim T^{20,21}$. The effect of a magnetic field then arises through the coupling between the uniform mode at $q = 0$ and the modes with $q \approx Q_{AF}^{23}$. This results in a negative MR when this mode-mode coupling is positive. In a first approximation, the MR value can be obtained by considering the effect of $H$ only on the dynamical susceptibility as $\chi_Q(q, \omega) = \frac{\chi_0(q, \omega) \left[1 - a(T)H^2\right]}{\chi_0(q, \omega)}$ where $a(T)$ measures the mode-mode coupling effectiveness. So the magnetoresistivity due to the suppression of spin fluctuations...
The similarity between the behavior of $\gamma(T)$ displayed in fig.4 and that of $1/T_\varphi T$ suggests that $a(T)$ is directly connected to the $T$ dependence of the staggered susceptibility. In order to test this assumption we have plotted in the right part of fig.4 the quantity $T/\gamma$ versus $T$ for the lower Co dopings (from 0 to 7.5%). For all the magnetic samples, we observe a very good linear $T$ dependence, which indicates that indeed $\Delta\rho(T, H)/H^2 \propto T/(T - \theta)$ with $\theta \sim T_N$ for $x = 0.047$ and 0.06 ($\theta = 57$ and 27K compared to $T_N = 63$ and 33K as determined from the sharp anomaly in the $\rho(T)$ curves - see Tab.II). For the undoped compound, the first order of the magnetic transition prevents a direct comparison.

However, the data for the non-magnetic 7.5% clearly deviates from this linear behavior. Instead, we rather find that the MR curves can be very well scaled by $H^2/T$ from 45 to 130K. This is visualized in the left part of fig.4 in which the quantity $\sqrt{T/\gamma}$ is now plotted versus $T$ for the non-magnetic samples. It is seen that the LMR can be written as $\Delta\rho(T, H)/H^2 \propto T/(T - \theta)^2$ where $\theta$ decreases slightly from 0 in the optimally doped sample to $\sim -50$K in the most doped 15.4%Co one. These negative values of $\theta$ that reveal the existence of short-range AF spin fluctuations are in relatively good agreement with those extracted from INS or NMR experiments, albeit a little smaller.

So this analysis establishes that the negative LMR measured in the Ba(Fe$_{1-x}$Co$_x$)$_2$As$_2$ single crystals can be rather well interpreted in terms of suppression of the spin fluctuations by magnetic field in an itinerant electrons approach. More importantly, we also evidence a change of the $T$ dependence of the LMR coefficient at the boundary between magnetic and non-magnetic compounds, which suggests that the coupling between the charge carriers and the spin fluctuations must be different in these two types of samples.

### III. SPIN FLUCTUATIONS IN LiFeAs

The LiFeAs crystal with $T_c = 16.5$K at the midpoint of the transition and a residual resistivity of 4.5$\mu\Omega$cm has been grown as detailed in ref.24. The LMR is reported for $H \parallel J$ in fig.5 from 20K to 50K. Although the value of the LMR coefficient is relatively large at low $T$ and similar to that found in optimally doped Co-BaFe$_2$As$_2$, it decreases much more rapidly with increasing $T$ and becomes nearly unmeasurable above 50K, as found in ref.23. Let us notice that, contrary to what is found for the Co-122, the spin fluctuation contribution to the MR is very small with respect to the transverse MR. Consequently it was quite legitimate to neglect the spin contribution and assimilate the orbital MR to the TMR as done in ref.24.

Even though it is not possible to discriminate between different $T$ dependences with the very few data points obtained here, it seems natural to try to fit the data in the same way as in the optimally and overdoped Co samples.25. This is done in the inset of fig.5 which shows that the LMR can be expressed as $\Delta\rho(H, T) \propto -H^2/T$ with a Curie-Weiss temperature $\theta \sim 0$, similarly to what is found for the 7.5%Co sample. So this can be taken as the sign that LiFeAs is located very near the boundary with the magnetic phase and that AF spin fluctuations could play a significant role in the superconductivity of this compound. This is in agreement with the INS observation of a peak in the imaginary part of the susceptibility at an incommensurate AF wave vector despite the poor nesting between the electron and hole pockets observed by ARPES27.
IV. DISCUSSION AND CONCLUSION

We demonstrated here that high sensitivity measurements of the longitudinal magnetoresistance are an efficient tool to reveal the effect of spin fluctuations on the transport properties of electron doped iron pnictides. The small and similar magnitude of the LMR signals found in the Co substituted BaFe$_2$As$_2$ compounds and in LiFeAs supports the idea that the effect of spin fluctuations cannot be directly deduced from the $T$ dependences of the resistivity, which are very different in these two families. Our study through the whole phase diagram of the Co substituted BaFe$_2$As$_2$ compounds allowed us to show that the coupling between charge carriers and spin fluctuations in the PM state is changing concomitantly with the loss of long range magnetic ordering.

An important feature of the iron pnictides is their multiorbital nature. In particular it has been pointed out early on that Hund’s rule interaction plays a prominent role in the physics of these compounds by allowing a strong orbital differentiation between the 3$d$ Fe orbitals. As a result, a description in terms of coexisting itinerant and localized electrons was proposed in different theoretical models. The situation appears then reminiscent of that encountered in $s$-$d$ AF metals where the electrons in the conduction $s$ band are the charge carriers while those in the narrow $d$ band contribute to the spin fluctuations. In this case, the spin susceptibility $\chi_s(T)$ of the conduction electrons can be considered to be Pauli-like and does not contribute to the $T$ dependence of the resistivity.

Beyond optimal doping, our experimental finding suggests an additional $1/(T - \theta)$ contribution to the LMR upon Co doping. A possible explanation might be that $\chi_s(T)$ has now acquired the same $T$ dependence as the staggered susceptibility. This could be linked to the weakening of the orbital differentiation with electron doping predicted theoretically, which results in a strong hybridization between local moment and itinerant electrons. A salient feature is that this effect happens at the very boundary between magnetic and non magnetic samples where $T_c$ is optimal. This should trigger more theoretical work, with a precise consideration of the characteristics and nesting properties of the different Fermi sheets, in order to explain the quantitative evolution of the LMR and clarify the exchange interactions between charge carriers and magnetic fluctuations in these multi-orbital systems.

As for LiFeAs, the observation of a $T$ dependence of the LMR very similar to that found in the optimally Co doped compound suggests that this compound is not so far from a hypothetical magnetic phase.

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Appendix A: LMR in the underdoped 4.7%Co and overdoped 10%Co substituted samples

The LMR data normalized to their zero field values are plotted versus $H^2$ in Fig for $x=0.047$ and $x=0.10$. As in the 7.5%Co sample, a very good matching of the curves is observed in the overdoped sample for two respective orientations of $H$ and $I$, indicating that the LMR is isotropic in the (ab) plane. For the 4.7%Co sample, the LMR data are found systematically larger in absolute value for $H \parallel I$ than for $H \perp I$. We do not have any clear explanation for this discrepancy, but as the ratio (of the order of 1.2) between these two configurations remains constant with temperature, this will not result in different temperature dependences of the LMR components. So we have only considered here the LMR component measured for $H \parallel I$ for which only the spin effect is expected to contribute to the magnetoresistivity.

FIG. 6: (color in line) Longitudinal $\delta \rho/\rho_0$ values plotted versus $H^2$ for $H \parallel I$ (full symbols) or $H \perp I$ (empty symbols) for (a) the underdoped 4.7%Co sample and (b) the overdoped 10%Co sample.
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