Further remarks on $\pi\pi$ scattering dispersion relations

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Abstract

The naive use of higher order perturbation theory leads the left–hand cut integrals in $\pi\pi$ dispersion relations [1, 2] divergent. This problem is discussed and solved. Also we point out that the Adler zero condition imposes three constraints on the dispersion relations. The $\sigma$ pole position is determined using the improved method, $M_\sigma = 483 \pm 13$ MeV, $\Gamma_\sigma = 705 \pm 50$ MeV. The scattering length parameter is found to be in excellent agreement with the experimental result.

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In Ref. [1, 2], we have established a new dispersion representation for the partial wave $\pi\pi$ scattering $S$ matrix. The key point for setting up the dispersion representation is the observation that both the real part and the imaginary part of $S$ defined in the physical region are analytic functions on the cut plane expressed in terms of poles, the left hand cut integrals and the kinematic factor, $\rho = \sqrt{1 - 4m^2_\pi / s}$. To be specific, the real part of $S$, $\cos(2\delta_\pi)$ (where $\delta_\pi$ is the scattering phase shift) and the imaginary part of the $S$ matrix, $\sin(2\delta_\pi)$ satisfy the following dispersion relations:

$$\sin(2\delta_\pi) \equiv \rho F, \quad F(s) = \alpha + \sum_i \frac{\beta_i}{2i\rho(s_i)(s - s_i)} - \sum_j \frac{1}{2i\rho(z_j^{II})(s - z_j^{II})} + \frac{1}{\pi} \int_L \frac{\text{Im}_L F(s')}{s' - s} ds' + \frac{1}{\pi} \int_R \frac{\text{Im}_R F(s')}{s' - s} ds',$$

(1)

and

$$\cos(2\delta_\pi) \equiv \bar{F} = \bar{\alpha} + \sum_i \frac{\beta_i}{2(s - s_i)} + \sum_j \frac{1}{2s'(z_j^{II})(s - z_j^{II})} + \frac{1}{\pi} \int_L \frac{\text{Im}_L \bar{F}(s')}{s' - s} ds' + \frac{1}{\pi} \int_R \frac{\text{Im}_R \bar{F}(s')}{s' - s} ds',$$

(2)

where $s_i$ denote the possible bound state pole positions and $\beta_i$ are the corresponding residues of $S$; $z_j^{II}$ denote the possible resonance pole positions on the second sheet. The integrals denote the cut contributions, $L=\mathbb{R}$ is the left hand cut (l.h.c.) and $R$ starts from the $\bar{K}K$ threshold once the $4\pi$ cut are neglected, $\alpha$ and $\bar{\alpha}$ denote...
the subtraction constants, and one subtraction to the integrals in above expressions is understood. With these dispersion relations we can then generalize the single channel unitarity relation \( S^+ S = 1 \), which is only valid in the single channel physical region when \( s \) is real, to the whole complex \( s \) plane [1]: \( \cos^2(2\delta_s) + \sin^2(2\delta_s) = 1 \). The latter is equivalent to the well known generalized unitarity condition in quantum mechanics but was firstly discussed in field theory in Ref. [1].

The above method is valid for any partial wave scattering. It is however worth pointing out that in the scattering process with a non-vanishing angular momentum, \( J \), restrictions among parameters should exist to ensure the threshold behavior, \( \delta_s(s) \propto k^{2J+1} \), where \( k = \sqrt{s-4} \). In order to make use of Eqs. (1) and (2) in phenomenological discussions, a knowledge on the l.h.c. integrals is necessary. It is not very clear how to calculate these l.h.c. integrals in the nonperturbative scheme. Predictions on the left hand cuts from nonperturbative models like the Padé approximation are not always trustworthy [3]. Therefore results from chiral perturbation theory (CHPT) are used in estimating these integrals via the following formula,

\[
\text{Im}_L F = 2\text{Im}_L \text{Re}_R T(s) = 2\text{Im}_L T - 2\text{Re}_L \text{Im}_R T, \quad (3)
\]

\[
\text{Im}_L \tilde{F} = -2\rho(s)\text{Im}_L \text{Im}_R T(s), \quad (4)
\]

since

\[
F = 2\text{Re}_R T, \quad \tilde{F} = 1 - 2\rho \text{Im}_R T. \quad (5)
\]

In Eqs. (3), (4) and (5), \( T \) is the partial wave scattering \( T \) matrix: \( S = 1 + 2i\rho T \). The quantity \( \text{Im}_L F \) is estimated at \( O(p^4) \) in [2] and \( \text{Im}_L \tilde{F} \) vanishes at this order. A question naturally arises as how close is the \( O(p^4) \) results to the real situation. Since at \( O(p^4) \) higher resonances do not contribute to \( \text{Im}_L F \) and \( \text{Im}_L \tilde{F} \), it is argued in [1] that the contribution from the \( t \) channel resonance exchange to the l.h.c. is very small at low energies. It is confirmed by a calculation to the tree level \( \rho \) exchange diagram [4], which indicates that the contribution to the l.h.c. integral is numerically very small up to \( s = 1\text{GeV}^2 \). Since resonance contributions saturate the \( O(p^4) \) \( \pi\pi \) interaction chiral Lagrangian [5] at low energies and they contribute \( \text{Im}_L F \) and \( \text{Im}_L \tilde{F} \) at \( O(p^6) \), the above discussions may suggest that the \( O(p^4) \) results of \( \text{Im}_L F \) and \( \text{Im}_L \tilde{F} \) are good approximations to the real situations at \( s < 1\text{GeV}^2 \) despite of the ambiguity in choosing the cutoff parameter when estimating the integrals. However, even though high energy contributions to the left hand cut integrals may be small, the availability of the systematic use of perturbation theory in estimating \( \text{Im}_L F \) and \( \text{Im}_L \tilde{F} \) needs to be proved. This issue is not as trivial as it looks like at first glance.

To have an understanding to the problem occurring in using higher order perturbation theory results when estimating \( \text{Im}_L F \) and \( \text{Im}_L \tilde{F} \), let us focus on Eq. (1). Since \( \text{Im}_L \tilde{F} \) vanishes at \( O(p^4) \), the leading order contribution is of \( O(p^6) \),

\[
\text{Im}_L \text{Im}_R T(s) = 2\rho T_2 \text{Im}_L \text{Re}_R T_4 \\
= 2\rho T_2 \text{Im}_L T_4 - 2\rho^2 T_2^3. \quad (6)
\]

To obtain the second equation we have made use of the perturbative unitarity relation, \( \text{Im}_R T_4 = \rho T_2^2 \) where (and hereafter) the subscripts denote the order of the
chiral expansions. Taking into account the results from CHPT we find that when \( s \) approaches 0−, \( \text{Im}_\nu \tilde{F} \) behaves as \( O((1/\sqrt{-s})^3) \) due to the presence of the kinematic factor. The integration in the left hand integral in Eq. \( 1 \) is therefore divergent when \( s' \to 0_- \). If higher order results are used the problem is getting worse since there will be higher powers of \( 1/\sqrt{-s} \). The same situation occurs in \( \text{Im}_l F \) when using higher order results from ChPT. We will demonstrate in the following that this problem is only a deceptive artifact inherited from perturbation theory and can be corrected. In fact, from rather general considerations \( \text{Im}_l F(s) \) and \( \text{Im}_l \tilde{F}(s) \) should behave as \( O(\sqrt{-s}) \) and \( O(1/\sqrt{-s}) \), respectively, when \( s \) approaches zero. Hence the left hand integrals in Eqs. \( 1 \) and \( 2 \) are well defined quantities.

In order to understand the behavior of \( \text{Im}_l \tilde{F}(s) \) when \( s \to 0 \), we should firstly understand the behavior of \( T_f^J(s) \) and \( \text{Im}_l T_f^J(s) \) as \( s \to 0 \). Since \( s = 0 \) is a branch point for \( T_f^J(s) \) we first let \( s \) approaches 0 from the positive side along the real axis. From the partial wave projection formula

\[
T_f^J(s) = \frac{1}{32\pi(s - 4m^2_\pi)} \int_{4m^2_\pi}^0 dt \quad P_J(1 + \frac{2t}{s - 4m^2_\pi})T_f^J(s, t, u) , \quad u = 4m^2_\pi - s - t ,
\]

we conclude that the partial wave amplitude is regular at \( s = 0_+ \), since in the unphysical region \( s = 0, 0 \leq t, u < 4m^2_\pi \) the full amplitude \( T_f^J(s, t, u) \) contains no singularity. CHPT results are consistent with the conclusion obtained from the general analysis: actually in CHPT, at each order of the chiral expansion, the limit \( T_f^J(2n, 0_+) \) exist as demonstrated by the explicit calculation for \( n = 1, 2, 3 \) and it is reasonable to assume it exists for arbitrary \( n \). For the behavior of \( \text{Im}_l T_f^J(s) \) near \( s = 0 \), we have

\[
\text{Im}_l T_f^J(s) = \frac{1 + (-1)^{J+J}}{32\pi(s - 4m^2_\pi)} \int_{4m^2_\pi}^{4m^2_\pi - s} dt \quad P_J(1 + \frac{2t}{s - 4m^2_\pi})\text{Im}_l T_f^J(s, t) ; s \leq 0 ,
\]

from which we conclude \( \text{Im}_l T_f^J(s) \sim (\sqrt{-s})^3 \) as \( s \to 0_- \): one factor \( s \) comes from the integral interval in the above equation, another factor of \( \sqrt{-s} \) comes from the threshold behavior of \( \text{Im}_l T_f^J(s, t) \) near \( t = 4m^2_\pi \). This observation is in agreement with the \( O(p^4) \) CHPT results in Ref. \( 2 \) and can be obtained from a more careful discussion \( 6 \). Again this behavior is expected to hold for \( \text{Im}_l T_f^J(2n) \). For simplicity in the following discussion we drop out the spin and isospin indices for partial wave amplitudes.

For the asymptotic behavior of \( \text{Im}_R T(s) \) as \( s \to 0 \), in perturbation theory the unitarity condition of the partial wave amplitude is satisfied at each order of perturbation expansion on the unitary cut \( s > 4m^2_\pi \):

\[
\text{Im}_R T_2(s) = 0 , \quad \text{Im}_R T_{2n}(s) = \rho(s) \sum_{i=1}^{n-1} T_{2n-2i}(s) T^*_{2i}(s) , \quad \text{for } n \geq 2 .
\]

To be specific,

\[
\text{Im}_R T_4(s) = \rho(s) T_2^2(s) ,
\]
To summarize, the most divergent term of \( \text{Im}_R T_0(s) \) is more and more singular when we expand the amplitude to higher orders, because of the higher power of \( \rho(s) \). We denote the most divergent term of \( \text{Im}_R T_{2n}(s) \) as \( \text{Im}_R T_{2n}^{(1)}(s) \) and the next to leading divergent term as \( \text{Im}_R T_{2n}^{(2)}(s) \). By simple deduction, one obtains for \( n \geq 3 \),

\[
\begin{align*}
\text{Im}_R T_{2n}^{(1)}(s) &= (-2i\rho(s)T_2(s))^{n-2}\rho(s)T_2^2(s), \\
\text{Im}_R T_{2n}^{(2)}(s) &= (n-1)(-2i\rho(s)T_2(s))^{n-3}T_4(s)\rho(s)T_2(s).
\end{align*}
\]

From the fact that \( T_2(0) \) is only a nonzero real constant and \( \text{Im}_L T_4(s) \sim O((\sqrt{-s})^3) \) as \( s \to 0_- \), we conclude that the behaviors of \( \text{Im}_L \text{Im}_R T_{2n}^{(1)}(s) \) and \( \text{Im}_L \text{Im}_R T_{2n}^{(2)}(s) \) as \( s \to 0_- \) are,

\[
\begin{align*}
\text{Im}_L \text{Im}_R T_{2n}^{(1)}(s) &\sim \begin{cases} 
(\frac{1}{\sqrt{-s}})^{n-1}, & \text{as } s \to 0_- \text{ if } n \text{ odd}, \\
0, & \text{as } s \to 0_- \text{ if } n \text{ even},
\end{cases} \\
\text{Im}_L \text{Im}_R T_{2n}^{(2)}(s) &\sim \begin{cases} 
(\frac{1}{\sqrt{-s}})^{n-5}, & \text{as } s \to 0_- \text{ if } n \text{ odd}, \\
(\frac{1}{\sqrt{-s}})^{n-2}, & \text{as } s \to 0_- \text{ if } n \text{ even},
\end{cases} \\
\text{Re}_L \text{Im}_R T_{2n}^{(1)}(s) &\sim \begin{cases} 
0, & \text{as } s \to 0_- \text{ if } n \text{ odd}, \\
(\frac{1}{\sqrt{-s}})^{n-1}, & \text{as } s \to 0_- \text{ if } n \text{ even},
\end{cases} \\
\text{Re}_L \text{Im}_R T_{2n}^{(2)}(s) &\sim \begin{cases} 
(\frac{1}{\sqrt{-s}})^{n-2}, & \text{as } s \to 0_- \text{ if } n \text{ odd}, \\
(\frac{1}{\sqrt{-s}})^{n-5}, & \text{as } s \to 0_- \text{ if } n \text{ even}.
\end{cases}
\end{align*}
\]

To summarize, the most divergent term of \( \text{Im}_L \text{Im}_R T_{2n}(s) \) and \( \text{Re}_L \text{Im}_R T_{2n}(s) \) are:

\[
\begin{align*}
\text{Im}_L \text{Im}_R T_{2n}(s) &\sim \begin{cases} 
(\frac{1}{\sqrt{-s}})^{n-1}, & \text{as } s \to 0_- \text{ if } n \text{ odd}, \\
(\frac{1}{\sqrt{-s}})^{n-2}, & \text{as } s \to 0_- \text{ if } n \text{ even}.
\end{cases} \\
\text{Re}_L \text{Im}_R T_{2n}(s) &\sim \begin{cases} 
(\frac{1}{\sqrt{-s}})^{n-2}, & \text{as } s \to 0_- \text{ if } n \text{ odd}, \\
(\frac{1}{\sqrt{-s}})^{n-1}, & \text{as } s \to 0_- \text{ if } n \text{ even}.
\end{cases}
\end{align*}
\]

From Eqs. (3) and (4), \( \text{Im}_L \tilde{F} \) and \( \text{Im}_L F \) contain the same order of divergence as \( \rho(s)\text{Im}_L \text{Im}_R T(s) \) and \( \text{Re}_L \text{Im}_R T(s) \), respectively, as \( s \to 0 \). For example, the
most divergent term of $\text{Im}_L \tilde{F}$ at $O(p^6)$ behaves like $O((1/\sqrt{-s})^3)$ and $\text{Im}_L F \sim O((1/\sqrt{-s}))$, when $s \to 0_-$ and higher order divergences appear as higher order perturbation expansions are used.

It is not difficult to demonstrate that the singular behavior of $\text{Im}_L \tilde{F}$ and $\text{Im}_L F$ at $s = 0$ as discussed above is just a spurious one inherited from the use of perturbation theory.\(^2\) To understand this we notice that, for the complete, non-perturbative amplitude we have

$$\text{Im}_R T(s) = \frac{\rho(s)T^2(s)}{1 + 2i\rho(s)T(s)}, \quad (14)$$

which is obtainable from the relation $T^*(s + i\epsilon) = T^{II}(s + i\epsilon) = T/S$ and the single channel unitarity relation. As $s \to 0$ we can obtain from above equation that $\text{Im}_R T(s)$ should not be singular, or more precisely, $\text{Re}_R T(0) = i\text{Im}_R T(0) = T(0)/2$. Therefore, by simple deduction using Eq. (3) and $\text{Im}_L T \sim (\sqrt{-s})^3$, one can find that $\text{Im}_L \tilde{F}(s)$ is $O((1/\sqrt{-s}))$ and $\text{Im}_L F(s)$ is $O(\sqrt{-s})$ as $s \to 0_-$. The left hand cut integrals are therefore well defined and are finite except at $s = 0$ for $\tilde{F}$.

The Eq. (14) not only indicates the correct asymptotic behavior of each quantity as $s \to 0$, but also the reason why the naive use of perturbation results leads the left cut integrals divergent – if we could sum up the perturbation series to all orders the divergence problem would have disappeared:

$$\text{Im}_R T = \sum_{n=2}^{\infty} \text{Im}_R T_{2n}$$

$$= \sum_{n=2}^{\infty} \rho(s) \sum_{i=1}^{n-1} T_{2n-2i}(s) T^*_{2i}(s)$$

$$= \rho(s) \sum_{i=1}^{\infty} \sum_{n=i+1}^{\infty} T_{2n-2i}(s) T^*_{2i}(s)$$

$$= \rho(s) \sum_{i=1}^{\infty} \sum_{n=1}^{\infty} T_{2n}(s) T^*_{2i}(s)$$

$$= \rho(s) \left( \sum_{i=1}^{\infty} T_{2i}(s) \right)^2 - 2i\rho(s) \sum_{i=1}^{\infty} \sum_{n=1}^{\infty} T_{2n}(s) \text{Im}_R T_{2i}(s)$$

$$= \rho(s) \left( \sum_{i=1}^{\infty} T_{2i}(s) \right)^2 - 2i\rho(s) \left( \sum_{n=1}^{\infty} T_{2n}(s) \right) \text{Im}_R T, \quad (15)$$

from which we can deduce

$$\text{Im}_R T = \sum_{n=2}^{\infty} \text{Im}_R T_{2n} = \frac{\rho(s) \left( \sum_{i=1}^{\infty} T_{2i}(s) \right)^2}{1 + 2i\rho(s) \left( \sum_{i=1}^{\infty} T_{2i}(s) \right)}. \quad (16)$$

Notice that in deriving Eq. (16) we do not need to use the knowledge on analytic continuation. Comparing with Eq. (14), the above result is obtained by the simple

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\(^2\)Note that this problem will occur not only in ChPT but also in all perturbation theories because Eq. (4) is correct in all perturbation theories and all deductions above are based on this equation. In Padé approximations this problem disappears.
substitution of $T$ expanded to all orders in perturbation theory into Eq. (14). As we discussed earlier, the chiral expansion of $T(0)$ is assumed to be well-defined since each $T_{2n}(0)$ is finite and $\sim O((m_{\pi}^2/4\pi f_{\pi}^2)^n)$. Therefore Eq. (10) suggests the correct way of extracting $\text{Im}_L F$ and $\text{Im}_L F$ near $s = 0$, from a finite order perturbative calculation. That is to use

$$\frac{\rho(s) (\sum_{i=1}^n T_{2i}(s))^2}{1 + 2\rho(s) (\sum_{i=1}^n T_{2i}(s))} \quad (17)$$

instead of $\sum_{i=2}^n \text{Im}_R T_{2i}$. The former expression removes the spurious divergence in the latter introduced by the kinematic factor at $s = 0$. The Eq. (17) works well in the vicinity of $s = 0$, in other places it should be considered equally well or equally bad as the naive expression of perturbation expansion. In our early studies we made the naive use of $O(\rho^4)$ CHPT to estimate $\text{Im}_L F(s')$ which gives an incorrect $O(\frac{1}{\sqrt{m_{\pi}^2}})$ behavior near $s' = 0$ even though the left hand integral in Eq. (11) is still definable except at $s = 0$. However, the numerical influence to the cut integral at $s \geq 4m_{\pi}^2$ is very small since the integral interval overwhelmed by the spurious divergence is very small. Hence numerical results in estimating the left hand integrals are only affected very little by the naive use of perturbation theory.

In Ref. [1] we have discussed the constraint of the Adler zero condition on the $\pi\pi$ scattering dispersion relation. There we approximately fix the Adler zero at $s = m_{\pi}^2/2$ in the I=J=0 channel and force the partial wave $S$ matrix being equal to 1 at $s = m_{\pi}^2/2$. However the method is not good enough since, first of all, the Adler zero position for the partial wave amplitude is not exactly located at $s = m_{\pi}^2/2$, and secondly, the Adler zero condition actually impose more constraints than what is previously considered.

In partial wave amplitudes, the position of Adler zero can not be exactly given because the Adler zero is defined at $s = u = t = m_{\pi}^2$ in the full amplitude and after the partial wave projection one cannot fix its exact position. Taking $I=J=0$ channel for example, one can find a zero at $s = m_{\pi}^2/2$ in the tree level amplitude $T_2 = \frac{2s-m_{\pi}^2}{4\pi f_{\pi}^2}$. If the perturbative amplitude is a good approximation (i.e., converges rapidly) to the real situation in the vicinity of $s = m_{\pi}^2/2$ then the Adler zero position for the partial wave amplitude, $s_{A}$, may exist and may be determined by solving the equation $0 = T(s_A) \simeq T_2(s_A) + T_4(s_A) + ...$ using the iteration method: $s_{A} \simeq m_{\pi}^2/2 - 16\pi f_{\pi}^2 T_3(m_{\pi}^2/2) + ...$. For the given perturbative amplitudes from CHPT the Adler zero position can be estimated numerically: $s_{A} = 0.419 \pm 0.058$ in unit of $m_{\pi}^2$. The error bar appeared in the estimate reflects the uncertainties of coupling constants of the chiral Lagrangian [7]. So in the following we will not fix the position of Adler zero and instead we use it as a parameter in our fit procedure.

In Ref. [1] we discussed the role of the Adler zero condition in the global fit. What we did is to enforce $S$ in Eq. (9) of Ref. [1] being unity at the zero. Which, however, did not make the full use of the Adler zero condition. According to Eqs. (5), (14) and $\text{Re}_R T(s) = T(1+S)/(2S)$ it is easy to realize that

$$F(s_A) = 0, \quad \tilde{F}(s_A) = 1, \quad \tilde{F}'(s_A) = 0 \quad , \quad (18)$$
which therefore impose three constraints on the \( \pi\pi \) scattering dispersion relations. The Eq. (18) is helpful in the phenomenological discussions. For example, it can be used to make one more subtraction to the dispersion integrals in Eq. (1) and two more subtractions to the dispersion integrals in Eq. (2). This is appreciable since more subtractions can in principle reduce the uncertainties when estimating those integrals, which mainly come from high energies. The Eqs. (1) and (2) can for example be recast as:

\[
F(s) = (s - s_A) \frac{2a_0^0}{4m_\pi^2 - s_A} + (s - 4m_\pi^2)(s - s_A) \sum_j \frac{i/(2\rho(z_j^{II})S'(z_j^{II}))}{(z_j^{II} - 4m_\pi^2)(z_j^{II} - s_A)(s - z_j^{II})}
\]

\[
\tilde{F}(s) = 1 + (s - s_A)^2(s - 4m_\pi^2) \sum_j \frac{1}{2S'(z_j^{II})(z_j^{II} - s_A)^2(z_j^{II} - 4m_\pi^2)(s - z_j^{II})}
\]

\[
\frac{(s - 4m_\pi^2)(s - s_A)^2}{\pi} \int_{L+R} ds' \frac{\text{Im}\tilde{F}(s')}{(s' - 4m_\pi^2)(s' - s_A)(s' - s)},
\]

where \( a_0^0 \) denotes the scattering length parameter. Here \( a_0^0 \) is no longer a free parameter since it is determined by \( F(s_A) = 0 \) where \( F \) is the one originally defined in Eq. (1).

Using the improved dispersion relations as described above we repeat the fit made in Ref. [1]. The fit procedure is the same as before (for example, here we also take \( \epsilon = 0.02 \) which constrains the violation of unitarity ) except that \( s_A \) is no longer held fixed. Taking into account all the uncertainties and variations of parameters we arrive at the following results,

\[
M_\sigma = 483 \pm 13 \text{MeV} , \quad \Gamma_\sigma = 705 \pm 50 \text{MeV} ;
\]

\[
a_0^0 = 0.223 \pm 0.006 , \quad s_A \simeq (0.268 - 0.309)m_\pi^2.
\]

Note that the scattering length parameter is now in excellent agreement with the result of Ref. [8, 9]. The above results should replace those given in Ref. [1]. Notice that the error bars given above only represent the uncertainties from our theoretical input and does not have a statistical meaning. The position of the Adler zero is about \( 0.3m_\pi^2 \) according to our fit, which is not very far from the result from chiral perturbation theory. The results on \( f_0(980) \) are also similar to those previously obtained. It is worth noticing that the new results are very close to the results given in Table 2 of Ref. [2] where the scattering length is constrained by hand using the result of Ref. [8]. It was found that the \( \sigma \) pole position is rather sensitive to the scattering length parameter [2], the correct use of the Adler zero condition is therefore crucial in obtaining the correct scattering length parameter and the \( \sigma \) pole position.

\[3\] In a previous version, the fit was performed by inappropriately taking \( a_0^0 \) being free, and the results were given in Phys. Lett. B549 (2002)362 (Erratum). The fit results are nevertheless not very sensitive to the different treatment of the \( a_0^0 \) parameter.
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