Higgs Mass in Dark Matter Selected High-Scale
Supersymmetry

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Abstract

The prediction for the Higgs mass in dark matter selected high-scale SUSY is explored. We show the bounds on SUSY-breaking scale in models of SM + \tilde{w} and SM + \tilde{h}/\tilde{s} due to observed Higgs mass at the LHC. This study of high-scale SUSY supports the idea that the inflation and SM superpartners probably share the same origin of SUSY breaking.
1 Introduction

In particle physics the interest on supersymmetry (SUSY) is based on four main reasons: i) solution of the naturalness problem, ii) successful gauge coupling unification, iii) viable thermal dark matter (DM) candidate with mass near weak scale, iv) ingredients of string theory. The so called low-scale SUSY addresses i)-iii). However, the first run of LHC shows bad prospect for conventional low-scale SUSY. In contrast to low-scale SUSY, only iv) is addressed in the “minimal” high-scale SUSY, it is because of that in “minimal” high-scale SUSY all superpartner masses are far above the weak scale, and the connection between weak and SUSY breaking scale is lost. But iii) should be addressed in any realistic model, and this can be done in two classes of “non-minimal” high-scale SUSY.

The first one is named as “split” SUSY \(^1\), in which the scalar superpartners masses are far above the weak scale, but all fermionic superpartners including gaugino \(\tilde{g}\) and higgsinos \(\tilde{h}_u\) and \(\tilde{h}_d\) are light due to the protection of \(R\) symmetry. In this class of high-scale SUSY DM is identified as the lightest supersymmetric particle. The observed Higgs mass in high precision [2], by virtue of two-loop RGEs and one-loop threshold corrections, suggests that the scale of SUSY breaking \(\tilde{m} \leq 10^8\) GeV [3, 4] when scalar superpartner threshold corrections are not very large. Above the scale \(\tilde{m}\) the physical states are described as the minimal supersymmetric standard model (MSSM), while below it described as standard model (SM) + \(\tilde{g}\) + \(\tilde{h}\).

The other class was firstly studied in [5], in which \(R\)-symmetry breaking isn’t suppressed, and either some new parity instead of \(R\) symmetry keeps higgsino \((\tilde{h})\) and singlino \((\tilde{s})\) light or there exists a light wino \(\tilde{w}\) DM due to environmental selection. In the former case, the singlino state is needed because of that pure higgsino DM isn’t viable. So below the scale \(\tilde{m}\) the physical states are described as SM+\(\tilde{h}/\tilde{s}\) (SM+\(\tilde{w}\)) when DM is mixing state of \(\tilde{h}\) and \(\tilde{s}\) (\(\tilde{w}\)-like). The observed Higgs mass, similar to the analysis performed in the first class, can be used to constrain the scale of SUSY breaking \(\tilde{m}\). Since the region of model parameters discussed in [5] corresponds to Higgs mass of order \(127 - 142\) GeV in model SM+\(\tilde{w}\) and \(141 - 210\) GeV in model SM+\(\tilde{h}/\tilde{s}\) (see Table 4 therein) it is necessary and also interesting to revise the Higgs mass in such DM selected high-scale SUSY. This is the aim of this paper. In particular, instead of taking \(\tilde{m} = 10^{14}\) GeV and large \(\tan\beta\) limit as in [5], \(\tilde{m}\) will be considered as a free parameter in this paper, and region of small \(\tan\beta\) will be covered also.

In section 2, similar to Split SUSY [1] we discuss the two-loop RGE for Higgs quartic coupling

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\(^1\)Actually, issue ii) can be roughly addressed in no-minimal high-scale SUSY discussed here, and it is possible to achieve unification of gauge couplings in high precision.
\[ \lambda(\tilde{m})/\cos^2 2\beta \] as function of \( \tilde{m} \) for \( \delta\lambda = 0 \).

\( \lambda \) and one-loop RGEs for gauge couplings and other Yukawa couplings, and threshold correction to \( \lambda \) due to heavy SUSY particles will be parameterized for prediction for Higgs mass. In section 3, we estimate prediction for the Higgs mass \( M_h \) in models of SM+\( \tilde{w} \) and SM+\( \tilde{h}/\tilde{s} \), with uncertainty for \( M_h \) due to uncertainty of top quark mass and threshold correction. Finally, we conclude in section 4. RGEs for parameters related to Higgs mass are presented in appendix A.

2 RGEs and Threshold Corrections

Inspired by the case of split SUSY [3], the two-loop RGEs for SUSY model parameters in SM+\( \tilde{w} \) and SM+\( \tilde{h}/\tilde{s} \) can be similarly derived, and the results are presented in appendix A. In appendix A, we show the results in SM+\( \tilde{w} \), SM+\( \tilde{h}/\tilde{s} \) and split SUSY simultaneously, in order to illustrate the differences among them. A few comments are in order. i), As the Higgs mass is directly related to \( \lambda \), we consider the RGE for \( \lambda \) at two-loop order but for the others only at one-loop order. ii), The one-loop beta functions for gauge couplings can be derived by either following [7,8] or taking the insights of Weinberg [9]. iii), An additional parameter \( g_\lambda \) appears in SM+\( \tilde{h}/\tilde{s} \) (see Eq.(3.1)), in compared with split SUSY and SM+\( \tilde{w} \), but it affects RGEs of SM gauge (Yukawa) couplings only at two-loop (one-loop) order. Hence, we include one-loop effects due to Yukawa \( g_\lambda \) in RGE for SM Yukawa \( g_t \).

The value of Higgs quartic coupling at scale \( \tilde{m} \), \( \lambda(\tilde{m}) \), is determined by the SUSY boundary
\[ \lambda(\tilde{m}) = \frac{1}{4} \left[ g_2^2(\tilde{m}) + \frac{3}{5} g_1^2(\tilde{m}) \right] \cos^2 2\beta + \delta\lambda(\tilde{m}). \] (2.1)

up to threshold correction $\delta\lambda$ arising from heavy scalar and fermionic superpartners. Solving the RGE for $\lambda$, one obtains the electroweak (EW) scale Higgs mass $M_h = 2\lambda v^2$, $v = 174$ GeV. Threshold corrections related to our models at one-loop level have been considered in \cite{4, 6} and at the two-loop level in \cite{6}. In the next section, we consider this model parameter in the range $|\delta\lambda| \leq 0.03$, which is sufficient to cover the uncertainty of theoretic value of $\delta\lambda$ in high-scale SUSY.

Fig.1 shows the relative value $\lambda$ defined in Eq.(2.1) for split SUSY, SM+$\tilde{w}$ and SM+$\tilde{h}/\tilde{s}$, by using the one-loop RGEs for SM gauge couplings (see appendix A). It indicates that in split SUSY $\lambda$ and the prediction for Higgs mass at EW scale is roughly the largest among the three models for same $\tan\beta$ and $\delta\lambda$, except that the correction due to RGE effects is large enough to violate above expectation.

3 Higgs Mass

In this section, we estimate the prediction for the Higgs mass. Similar to the case of split SUSY \cite{4}, we use the updated experimental values of top quark mass $M_t = 173.3 \pm 0.76$ GeV \cite{11} and QCD coupling $\alpha_3(M_Z) = 0.1184 \pm 0.0007$ \cite{12} for our analysis. The measured value for the Higgs mass, $M_h = 125.15 \pm 0.25$ GeV is obtained from a naive average of the ATLAS and CMS results \cite{2}. As the Higgs mass is rather sensitive to top Yukawa, the dominant one-loop QCD corrections to top Yukawa $\delta g_t \simeq -0.065$ \cite{3,10} will be applied to the prediction.

3.1 SM+$\tilde{w}$

In model SM+$\tilde{w}$ the model parameters are wino mass $m_{\tilde{w}}$ and $\tilde{m}$. Parameter $m_{\tilde{w}}$ is constrained by the DM relic abundance, from which $m_{\tilde{w}} \sim 2$ TeV \cite{5}. Parameter $\tilde{m}$ relates to the boundary value for the Higgs mass at high energy scale, and thus the measured Higgs mass is sensitive to it. Fig. 2 shows the prediction for Higgs mass as function of $\tilde{m}$ for $\tan\beta = 1, 2, 4, 50$, with the solid lines correspond to the central values $M_t = 173.3$ GeV and $\alpha_3(M_Z) = 0.1184$ and the dotted lines represent to the uncertainty of the prediction due to top quark mass. The horizontal band indicates the measured value for the Higgs mass. With $\tan\beta$ fixed, the uncertainty of Higgs mass at high
εnergy scale shrinks to be about ~ 1 – 2 GeV at EW scale. When threshold corrections are small, \( \delta \lambda \sim 0 \), the model can be allowed in the wide range \( 10^4 \text{ GeV} \leq \tilde{m} \leq 10^{16} \text{ GeV} \).

Compare our prediction for Higgs mass in Fig. 2 with previous result in [5, 6]. \( M_h \) approaches to ~ 140 GeV for large value of \( \tan \beta \), which is consistent with the prediction of \( M_h \approx 141 – 142 \text{ GeV} \) in [5, 6]. On the other hand, the prediction for Higgs mass should be similar to the minimal high-scale SUSY studied in [4], because the deviation from SM is smaller than split SUSY.

Fig. 3 shows the case for threshold correction taken into account. The solid line in each color corresponds to the central values of \( M_t(M_Z) \) and \( \alpha_3(M_Z) \) and \( \delta \lambda = 0 \). The dotted lines in each color represent the deviation from above due to the uncertainty of \( M_t(M_Z) \) and \( \delta \lambda \). This figure has shown that the uncertainty of \( M_h \) is about ~ 10 GeV for \( \delta \lambda | \approx 0.03 \) in compared with Fig. 2 and \( \tilde{m} \leq 10^7 \text{ GeV} \) is still allowed.

### 3.2 SM + \( \tilde{h} / \tilde{s} \)

In model SM + \( \tilde{h} / \tilde{s} \) three new parameters enter in the effective Lagrangian below \( \tilde{m} \) [5],

\[
\mathcal{L} = \mathcal{L}_{SM}(q, u, d, l, e, h) + \mu \tilde{h}_a \tilde{h}_d + \frac{m^2}{2} s^2 + g_\lambda \tilde{h}_d \tilde{sh} + h.c. \tag{3.1}
\]
Figure 3: Same as Fig. 2 but with threshold corrections $|\delta \lambda | \leq 0.03$.

Figure 4: Higgs mass as function of SUSY-breaking scale in SM $+\tilde{h}/\tilde{s}$, for $\tan \beta = 1, 2, 4, 50$ and $g_{\lambda}(M_Z) = 0.2$ (left) and $g_{\lambda}(M_Z) = 0.8$ (right). Here each solid curve corresponds to the central values $M_t = 173.3$ GeV and $\alpha_3(M_Z) = 0.1184$, while the dotted curves in each color show uncertainty of Higgs mass due to experimental uncertainty in $m_t$. Threshold corrections are ignored.
Figure 5: Same as Fig. 4 but with threshold corrections $|\delta \lambda| \leq 0.03$. The solid line in each color corresponds to the central values of $M_t(M_Z)$ and $\alpha_3(M_Z)$ and $\delta \lambda = 0$. Dotted lines in each color represent deviation from above due to the uncertainty of $M_t(M_Z)$ and $\delta \lambda$.

Ref. [5] has shown that the observed DM relic abundance can be explained in the wide range $0 < g_\lambda < 0.9$. As $\lambda(\tilde{m})$ is sensitive to $g_\lambda(M_Z)$, we choose $g_\lambda = 0.2$ in the small $g_\lambda$ region ($\leq 0.4$) and $g_\lambda = 0.8$ in the large $g_\lambda$ region ($\geq 0.7$) for comparison.

Fig. 4 shows that the uncertainty of prediction for the Higgs mass at high energy scale is suppressed at EW scale, and there is only about $\sim 1 - 2$ GeV uncertainty due to uncertainty of $m_t$. For $g_\lambda = 0.2$ (left panel) it shows that the model is excluded for $\tilde{m} \geq 10^{13}$ GeV, while the model is excluded for $\tilde{m} \geq 10^9$ GeV instead for $g_\lambda = 0.8$ (right panel). This obviously differs from the case for SM $+ \tilde{w}$. It is because that the deviation from SM in this model is larger than in SM $+ \tilde{w}$, especially in the large $g_\lambda$ region.

Fig. 5 shows the case for threshold correction taken into account. The solid line in each color corresponds to the central values of $M_t(M_Z)$ and $\alpha_3(M_Z)$ and $\delta \lambda = 0$. The dotted lines in each color represent the deviation from above due to the uncertainty of $M_t(M_Z)$ and $\delta \lambda \neq 0$. In comparison with Fig. 4, it shows that for threshold corrections $\delta \lambda_{\max} = 0.03$, the model is still allowed for $\tilde{m} \leq 10^7$ GeV.

4 Conclusions

Inspired by the present LHC results on SUSY, the prediction for the Higgs mass in high-scale SUSY with weak interacting massive DM is explored in this paper. Similar to well known split SUSY, models of SM $+ \tilde{w}$ and SM $+ \tilde{h}/\tilde{s}$, in which wino and mixing state of higgsino and singlino serves as DM respectively, are studied in detail. The main results in this study include: i), In model of SM $+ \tilde{w}$, the SUSY-breaking scale $\tilde{m}$ is allowed in the whole range of $10^4$ GeV $\leq \tilde{m} \leq 10^{16}$ GeV.
GeV for vanishing threshold correction, and $\tilde{m} \leq 10^7$ GeV is still allowed for $\delta \lambda_{max} = 0.03$. ii) in model of SM $+\tilde{h}/\tilde{s}$ with vanishing threshold correction, $\tilde{m} \leq 10^{13}$ GeV ($10^9$ GeV ) is allowed in the small (large) $g_\lambda$ region. For threshold corrections $\delta \lambda_{max} = 0.03$, the model is still allowed for $\tilde{m} \leq 10^7$ GeV.

Although high-scale SUSY loses its connection to EW scale and has no promising prospect for discovery at the LHC, the prediction for Higgs mass in this paper encourages relating it to cosmology of early universe, especially the inflation physics. It supports the idea [13] that inflation and SM superpartners probably share the same origin of SUSY breaking of order $\sim 10^{16}$ GeV.

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A RGE

Given a coupling $g_i$, its RGE can be written as,

$$\frac{dg_i}{d\ln Q} = \beta^{(1)}(g_i) \left(\frac{4\pi}{2}\right)^2 + \beta^{(2)}(g_i) \left(\frac{4\pi}{4}\right)^4$$

(A.1)

The relevant coupling in SM $+\tilde{w}$ include three SM gauge couplings $g_i$, the third-generation Yukawa couplings $(g_t)$, and the Higgs quartic $(\lambda)$, while an extra Yukawa $g_\lambda$ must also be included in SM $+$ $\tilde{h}/\tilde{s}$. Compare with split SUSY, the three gaugino couplings defined in [3] disappear either in SM $+\tilde{w}$ or SM $+\tilde{h}/\tilde{s}$.

At one-loop order the RGEs for gauge couplings $g_i$ are given by,

$$\beta^{(1)}(g_i) = b_i g_i^3.$$  

(A.2)

where the coefficients $b_i$ are shown in table 1.

| $b_i$ = $(b_1, b_2, b_3)$ | Split SUSY | SM $+$ $\tilde{w}$ | SM $+$ $\tilde{h}/\tilde{s}$ |
|---------------------------|------------|-------------------|--------------------------|
| $(\frac{9}{2}, -\frac{7}{6}, -5)$ | $(\frac{41}{10}, -\frac{11}{6}, -7)$ | $(\frac{9}{2}, -\frac{19}{6}, -7)$ |

Table 1: One-loop beta function coefficients for SM gauge couplings.
As more fermionic SUSY states should be integrated below scale $\tilde{m}$ in our models, in compared with split SUSY, the one-loop RGEs for $g_t$ in both SM+$\tilde{w}$ and SM+$\tilde{h}$/$\tilde{s}$ can be directly reduced from those in split SUSY by taking the vanishing gaugino couplings $\tilde{g}_{u,d} = 0$,

$$\beta^{(1)}(g_t) = g_t \left( g_\lambda^2 + \frac{9}{2}g_t^2 - \frac{17}{20}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2 \right) \quad (A.3)$$

where the $g_\lambda$ term should be included in SM+$\tilde{h}$/$\tilde{s}$ but ignored in SM+$\tilde{w}$.

The one-loop beta function for $g_\lambda$ in SM+$\tilde{h}$/$\tilde{s}$ is given by,

$$\beta^{(1)}(g_\lambda) = g_\lambda \left( \frac{5}{2}g_\lambda^2 + 3g_t^2 - \frac{9}{4}g_2^2 - \frac{9}{20}g_1^2 \right). \quad (A.4)$$

The two-loop RGE for Higgs quartic $\lambda$ in SM+$\tilde{w}$ is given by

$$\beta^{(1)}(\lambda) = \lambda(12\lambda + 12g_t^2 - \frac{9}{5}g_1^2 - 9g_2^2) - 12g_t^4 + \frac{27}{100}g_1^4 + \frac{9}{10}g_2^2g_4^2 + \frac{9}{4}g_4^4 \quad (A.5)$$

and

$$\beta^{(2)}(\lambda) = -78\lambda^3 + \lambda^2 \left( \frac{54}{5}g_1^2 + 54g_2^2 - 72g_t^2 \right) + \lambda \left[ \frac{1887}{200}g_1^4 + \frac{117}{20}g_1^2g_2^2 - \frac{73}{8}g_2^4 - 3g_t^4 + g_t^2 \left( \frac{17}{2}g_1^2 + \frac{45}{2}g_2^2 + 80g_3^2 \right) \right] + 60g_t^6 - \frac{3411}{1000}g_1^6 + \frac{305}{8}g_2^6 - \frac{289}{40}g_1^2g_2^4 - \frac{1773}{200}g_4^4g_2^2 - 64g_3^4g_t^4 - \frac{16}{5}g_2^4g_t^4 - \frac{9}{2}g_2^4g_t^4 + \frac{3}{5}g_2^4g_t^2 \left( -\frac{57}{10}g_1^2 + 21g_2^2 \right) + g_4^2 \left( -12g_2^2 + 15\lambda - \frac{12}{5}g_1^2 \right) \quad (A.6)$$

where we have included the two-loop effects due to $\tilde{w}$ in $\mathcal{O}(g_4^2)$ terms of the last line in Eq. (A.6).

The two-loop RGE for Higgs quartic $\lambda$ in SM+$\tilde{h}$/$\tilde{s}$ differs from SM+$\tilde{w}$ as,

$$\beta^{(1)}(\lambda) = \lambda(12\lambda + 12g_t^2 - \frac{9}{5}g_1^2 - 9g_2^2) - 12g_t^4 + \frac{27}{100}g_1^4 + \frac{9}{10}g_2^2g_4^2 + \frac{9}{4}g_4^4 + 4\lambda g_\chi^2 - 4g_\lambda^4 \quad (A.7)$$

and

$$\beta^{(2)}(\lambda) = -78\lambda^3 + \lambda^2 \left( \frac{54}{5}g_1^2 + 54g_2^2 - 72g_t^2 \right) + \lambda \left[ \frac{1887}{200}g_1^4 + \frac{117}{20}g_1^2g_2^2 - \frac{73}{8}g_2^4 - 3g_t^4 + g_t^2 \left( \frac{17}{2}g_1^2 + \frac{45}{2}g_2^2 + 80g_3^2 \right) \right] + 60g_t^6 - \frac{3411}{1000}g_1^6 + \frac{305}{8}g_2^6 - \frac{289}{40}g_1^2g_2^4 - \frac{1773}{200}g_4^4g_2^2 - 64g_3^4g_t^4 - \frac{16}{5}g_2^4g_t^4 - \frac{9}{2}g_2^4g_t^4 + \frac{3}{5}g_2^4g_t^2 \left( -\frac{57}{10}g_1^2 + 21g_2^2 \right) + 20g_\chi^4 - 2g_\lambda^2 \left( -\frac{9}{100}g_1^4 - 12\lambda^2 - \frac{3}{4}g_4^2 - \frac{3}{10}g_1^2g_2^2 + \frac{3}{4}g_1^2 + \frac{15}{4}g_2^2 \right) \quad (A.8)$$

where $\mathcal{O}(g_\lambda)$ terms in the last line of Eq. (A.8) refer to contribution due to Yukawa $g_\lambda$. 


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