The punching mechanism of holes in plates made of fragile material

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Abstract. The shock loading of a plate made of fragile material is considered. The hypothesis is accepted that destruction of a plate can happen at the expense of a multiple splitting off of its back part because of action of the reflected pulling stresses and a simultaneous cut of a front part in the form of a stopper because of action of tangential stresses generated by a wave of compression. It is shown that the destruction of a ceramic plate can happen at the expense of a multiple splitting off of its back part because of action of the reflected pulling stresses and a simultaneous cut of a front part in the form of a stopper because of action of tangential stresses generated by a wave of compression. Necessary and sufficient conditions a punching of a plate are formulated and probable coordinates of its sections in which there are destructions of the type of a splitting off are determined. The results of numerical calculations are presented. The received formulas can help to pick up the characteristics of the striker providing punching of holes of the necessary diameter in plates made of friable material with beforehand given properties.

1. Introduction
Realization of holes is one of the necessary technological operations, for example, at statement of bolts, rivets etc. There is a number of ways of receiving holes, but one of the most efficient is the dynamic way of punching holes. At any way of manufacturing of holes, the problem significantly increases in solid materials. As a rule, solid materials at the same time are also friable. For such materials it is expedient to realize mechanisms of brittle failure which are the destructions because of the action of tension of tensile arising after reflection of a wave of compression from a dorsum and tension of longitudinal shift (cut) on the cylindrical surface corresponding to diameter of the punched hole. In this article the mechanism of punching of holes because of a back splitting off, the subsequent formation in the rest of a plate of a so-called stopper and its promotion for a dorsum is modelled.

2. Problem definition
Many models of assessment of parameters of a penetrative action assume existence of the software product allowing to solve wave problems about longitudinal waves of rated tensions [1], radial waves of pressure [2] and radial waves of longitudinal shift and torsion [3] and from interface of their decisions to define the necessary characteristics of the punching [4]. However for the whole class of the tasks connected with a shock interaction and the destruction of contacting materials there is no need of a numerical integration of wave sets of equations. In particular, in fragile materials (and with
particular assumptions it is possible to consider also some metal materials friable) an analytical description of their intense strained state at a shock loading and the corresponding model operation of destruction can be performed [5].

Let's consider the creation of such model on the example of ceramics. High hardness and thermal resistance of ceramics in combination with rather not big density do it by a perspective technological material. In this regard the problem of receiving holes in it is relevant at high-speed influence of tough strikers. Below the mechanism of a punching of a ceramic plate is analysed by thickness $h$ the rigid cylindrical tool - the striker weight $M$ for a case of normal shock with the speed $V_0$.

3. Theory

At the time of shock from the front face of a plate, the wave of pressure loads begins to be generated [4]. As in ceramics owing to its brittleness the considerable plastic strains even do not develop at destruction, the leading edge of the wave front carries the stress:

$$
\sigma_0 = \frac{E_1 \rho_1 E_2 \rho_2}{E_1 \rho_1 + E_2 \rho_2} V_0 ,
$$

where $E_1$, $E_2$ - are elastic moduli of materials of a plate and a striker; $\rho_1$ and $\rho_2$ - are density of materials of a plate and a striker respectively.

The set of equations describing a task includes an equation of motion, a condition of a continuity and a defining ratio [6]:

\begin{equation}
\begin{aligned}
\frac{\partial \sigma(z,t)}{\partial z} &= \rho \frac{\partial V(z,t)}{\partial t} \\
\frac{\partial V(z,t)}{\partial z} &= \frac{\partial \varepsilon(z,t)}{\partial t} \\
E \frac{\partial \varepsilon(z,t)}{\partial z} &= \frac{\partial \sigma(z,t)}{\partial t}
\end{aligned}
\end{equation}

where $\rho$ – is plate material density; $V(z,t)$ - is speed of particles of a plate.

As starting conditions, we accept that in an initial instant material is not intense, it is not deformed and is not mobile:

$$
\sigma(Z,0) = \varepsilon(Z,0) = V(Z,0) = 0
$$

The system (2) represents the system of quasi-linear differential equations in partial derivatives of the first order of a hyperbolic type and therefore, it is expedient to solve it by the method of characteristics [6]. For this case of a ratio between differentials of required functions along the characteristic directions in the phase plane $z0t$ will register so [6]:

– along the characteristic direction $dz = 0$:

$$
d\sigma(z,t) - E d\varepsilon(z,t) = 0 .
$$

– along the characteristic direction $dz = adt$:

$$
d\sigma(z,t) + a\rho dV(z,t) = 0 ;
$$

– along the characteristic direction $dz = -a\rho dt$:

$$
d\sigma(z,t) - a\rho dV(z,t) = 0 .
$$

It is offered to use an approximate solution of the problem of longitudinal stress waves in rods in a convenient analytical form. In this case, the hypothesis is applied that the nature of the stress change
along the positive characteristic direction of the characteristic grid is similar to the corresponding change on the leading edge of the compression stress wave. In this case, the initial condition is the condition on the loaded end of the rod at the time of the beginning of shock.

As a boundary condition, we will accept the shock by rigid weight on a core end face:

\[ M \frac{dV(0,t)}{dt} = -\sigma(0,t) S. \]

The solution of the last equation for the change of the tension on a contact surface in time registers so:

\[ \sigma(0,t) = \sigma_0 \exp\left[-\frac{\sqrt{E \cdot p}}{M} \cdot S \cdot t \right] \]

As it was noted earlier, the leading edge of the compression stress extends in ceramics without changes, then pressure load \( \sigma_{c}(z) \) in ceramics material in section in an instant of arrival to it of a back wave of tension of tensile \( t = (2h - z)/a \) will register so [6]:

\[ \sigma_{c}(z) = \sigma_0 \exp\left[-2p_1 S \frac{h-z}{M} \right]. \]  \hspace{1cm} (3)

where \( a = \sqrt{E_1/p_1} \) – is the rate of propagation of an elastic wave in ceramics; \( S \) – is the sectional area of the striker.

Let us consider an instant \( t_1 = (2h - z)/a \) (figure 1). The material of the plate is more left than section \( z_1 \) is in a compressed state, more to the right – in tensile. Let in section \( z_1 \) tensile tension \( \sigma_{t}(z_1) \) be equal to the allowable one. Then in it the crack is generated, the continuity is broken – both the left-hand and right parts will behave further independently. In addition, in the left-hand part, the distribution of a wave front of tensile tension will continue, and in right, there will be a reflection of a wave of tensile tension and its transformation to a wave of tensile tension. At the same time the corresponding amplitudes of waves of tension will be equal in each part of a plate modulo and twice less than initial tension \( \sigma_{t}(z) \).

Let us consider the right half of a plate. The tensile tension in section \( z_1 \) equally [4]:

\[ \sigma_{t}(z_1) = \sigma_0 \left(1 - \exp\left[-\frac{2p_1 S}{M} (h - z) \right] \right) = [\sigma], \]

from where the coordinate of the probable formation of the right crack in a plate at a hole punching:

\[ z_1 = h + \frac{M}{2p_1 S} \ln \left[ \frac{[\sigma]}{\sigma_0} \right]. \]  \hspace{1cm} (4)

[3]

[4]

[6]
Figure 1. The scheme of calculation of a penetrative action for fragile barriers

A formation of a crack in section $z_1$ is a necessary, but not sufficient condition of formation of destructions of splitting off type in it. Besides, the striker needs to be checked for the longitudinal stability [7]. Intensive pressure loads in the combination with big hardness of fragile materials can lead to a buckling of the tool [8]. An additional condition for destruction of type of a splitting off is the existence of cracks and on a lateral area of a cylinder of the diameter $d_0$ [9, 10]. Owing to the lack of developed plastic strains in a wave, the wave of tension of tensile extending in the right part of a barrier has constant amplitude $\sigma_c(z) = [\sigma]$, and it also generates on the lateral area of a made mention cylinder the tangential stresses intensity:

$$\tau = \frac{[\sigma]}{2\pi d_0(h-z_1)}.$$  

When performing a condition $\tau \geq [\tau]$ in a barrier the back splitting off is implemented and the corresponding value $z_1$ will be defined so:

$$z_1 \geq h - \frac{d_0}{8} \frac{[\sigma]}{[\tau]}.$$  

(5)
Let us consider the left-hand half of a barrier now. At the time of the formation of a crack, this part is loaded with the squeezing tension determined by the formula (3). The wave front of tension of tensile extending in this part with an amplitude \(0.5 [\sigma]_t\) (figure 1) will create total tension \(\sigma^\vee(z)\), determined by the formula:

\[
\sigma^\vee(z) = \sigma_0 \exp \left[ -\frac{2\rho_1 S}{M} (h - z) - \frac{1}{2} [\sigma]_t \right].
\] (6)

Under the influence of this tension on a cylindrical surface \(d_0\) the tangential stresses [3] which can lead to the formation of a stopper under certain conditions will be generated. The given tangential stress considering no uniformity of distribution of a total rated tension (6) longwise of the left-hand part can be defined so:

\[
\tau_{st.} = \frac{S}{\pi d_0} \int_0^{z_1} \frac{\partial \sigma(z,t)}{\partial z} dz.
\] (7)

Now the condition of formation of a stopper in the left-hand part of a plate has an appearance:

\[
\tau_{st.} \geq \tau,
\]

or, considering (6) and (7):

\[
\frac{1}{4} d_0 \sigma_0 \exp \left[ -\frac{2\rho_1 S h}{M} \right] \left[ \exp \left( \frac{2\rho_1 S z_1}{M} \right) - 1 \right] \geq z_1 [\tau].
\] (8)

Realization of the condition (8) in total with (4) and (5) logically means the full of punching the plate tool in the zone II of the phase plane \(z0\) (figure 1).

If the condition (8) is not satisfied, then in the left-hand part of a plate a multiple repeated splitting off is possible, and the complete punching of a plate can occur, for example, in the zone IV of the phase plane. It should be noted that in the zone III destruction can't arise as there is no splitting off there and amplitude, smaller in comparison with the zone II, of the squeezing tension at the fixed area of a cut can't lead to the formation of a stopper. Time of the probable formation of the second (and the subsequent) splitting off \(t_2\) (figure 1), and the section corresponding to it \(z_2\), is possible to define without the basic difficulties by the same equations and with the starting conditions corresponding to them.

4. Discussion of results

In the figure 2 dependences maximal (the solid line) and minimum (the dash-dotted line) sizes of a back splitting off, determined by formulas (4) and (5) respectively depending on the shock speed are presented. As input data were used: sheet gage \(h=10\) mm, mass of the tool \(M=0.025\) kg, its diameter (corresponds to the diameter of the punched hole) \(d_0=10\) mm, plate material density (in an example \(Al_2O_3\)) \(\rho_1=3900\) kg/m\(^3\), tool material density (Steel 9260, or G92600) \(\rho_2=7800\) kg/m\(^3\), plate material elastic modulus \(E_1=350\) GPa, tool material elastic modulus \(E_2=210\) GPa, permissible stresses of a plate material on tensile \([\sigma]_t=140\) MPa, and on a cut \([\tau]=56\) MPa.

It is visible that conditions of a back splitting off are implemented at a shock speed \(V_0 \geq 90\) m/s. At the same time size of the back splitting off \(z_1 \approx 7\) mm. In the case considered earlier the condition (8) takes the form: \(6.27 \times 10^5 \geq 3.748 \times 10^5\), that is the condition of formation of a stopper in the plate rest is also implemented.
Figure 2. The dependences of the maximum and minimum sizes of a back splitting off on shock speed.

In figure 3 influence of thickness sheet on the section of the first splitting off are investigated. The solid line – the maximum value of size of the first splitting off, the dotted one – the minimum necessary value for its formation. The analysis of the drawing allows the conclusion that the section of the first splitting off does not depend on the thickness sheet, and is defined by the distance from a dorsum to a point of distribution of the leading edge of front of a back tensile wave in which the total tension becomes equal to ultimate strength.

Figure 3. The influence of the thickness sheet on the section of the first splitting off.

In the figure 4 the influence of diameter of the punched hole on the maximum and the minimum necessary sizes of a back splitting off is estimated. Continuous and dash-dotted lines are, respectively, the maximal and the minimum necessary sizes of the first splitting off with the diameter of the punched hole \( d_0 = 10 \) mm. Dot and dotted lines are, respectively, the maximal and minimum necessary sizes of the first splitting off with a diameter of the punched hole \( d_0 = 15 \) mm.

It is visible that with the increase in diameter of the punched hole the speed of shock, necessary for receiving the first back splitting off, decreases (in the case under consideration twice), but at the same
time the size of the first splitting off decreases from 8.2 mm to 7.26 mm. This results from the fact that speed determines the maximum amplitude of a contact stress, and, therefore, and the size of the first splitting off. The condition (8) in both cases is satisfied.

![Figure 4](image.png)

**Figure 4.** The influence of diameter of the punched hole at the maximum and minimum sizes of a back splitting off

Besides, the carried-out calculations showed that the back splitting off happens to increase in the speed of shock quicker, but at the same time length of a stopper increases and the condition (8) is not satisfied. Similar results were received earlier [11]. In many cases it is expedient to allow a reusable splitting off when choosing speed and mass of the striker.

5. **Conclusions**

It is shown that destruction of a ceramic plate can happen at the expense of a multiple splitting off its back part because of action of the reflected pulling stresses and a simultaneous cut of a front part in the form of a stopper because of action of the tangential stresses generated by a wave of compression. Necessary and sufficient conditions a punching of a plate are formulated and probable coordinates of its sections in which there are destructions of type of a splitting off are determined. The received formulas allow picking up the characteristics of the striker (first of all the weight and speed of shock) providing punching of holes of the necessary diameter in plates made of fragile material with beforehand given properties.

6. **References**

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