Hedging Extreme Co-Movements *

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Based on a recent theorem due to the authors, it is shown how the extreme tail dependence between an asset and a factor or index or between two assets can be easily calibrated. Portfolios constructed with stocks with minimal tail dependence with the market exhibit a remarkable degree of decorrelation with the market at no cost in terms of performance measured by the Sharpe ratio.

Over a hundred years ago, Vilfred Pareto discovered a statistical relationship, now known as the 80-20 rule, which manifests itself over and over in large systems: “In any series of elements to be controlled, a selected small fraction, in terms of numbers of elements, always accounts for a large fraction in terms of effect.” The stock market is no exception: events occurring over a very small fraction of the total invested time may account for most of the gains and/or losses. Diversifying away such large risks requires novel approaches to portfolio management, which must take into account the non-Gaussian fat tail structure of distributions of returns and their dependence. Recent shocks and crashes have shown that standard portfolio diversification work well in normal times but may break down in stressful times, precisely when diversification is the most important: as a caricature, one could say that diversification works when one does not really need it and may fail severely when it is most needed.

Technically, the question boils down to whether large price movements occur mainly in an isolated manner or in a coordinated way. This question is vital for fund managers who take advantage of the diversification to hedge their risks. Here, we introduce a new technique to quantify and empirically estimate the propensity for assets to exhibit extreme co-movements, through the use of the so-called coefficient of tail dependence. Using a factor model framework and tools from extreme value theory, we provide novel analytical formulas for the coefficient of tail dependence between arbitrary assets, which yields an efficient non-parametric estimator. We then construct portfolios of stocks with minimal tail dependence with the market represented by

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the Standard and Poor’s 500 index and show that their superior behavior in stressed times come together with qualities in terms of Sharpe ratio and standard quality measures that are not inferior to standard portfolios.

1 Assessing large co-movements

Standard estimators of the dependence between assets are the correlation coefficient or the Spearman’s rank correlation for instance. However, as stressed by [Embrechts et al. (1999)], these kind of dependence measures suffer from many deficiencies. Moreover, their values are mostly controlled by relatively small moves of the asset prices around their mean. To cure this problem, it has been proposed to use the correlation coefficients conditioned on large movements of the assets. But [Boyer et al. (1997)] have emphasized that this approach suffers also from a severe systematic bias leading to spurious strategies: the conditional correlation in general evolves with time even when the true non-conditional correlation remains constant. In fact, [Malevergne and Sornette (2002a)] have shown that any approach based on conditional dependence measures implies a spurious change of the intrinsic value of the dependence, measured for instance by copulas. Recall that the copula of several random variables is the (unique) function which completely embodies the dependence between these variables, irrespective of their marginal behavior (see [Nelsen (1998)] for a mathematical description of the notion of copula).

In view of these limitations of the standard statistical tools, it is natural to turn to extreme value theory. In the univariate case, extreme value theory is very useful and provides many tools for investigating the extreme tails of distributions of assets returns. These new developments rest on the existence of a few fundamental results on extremes, such as the Gnedenko-Pickands-Balkema-de Haan theorem which gives a general expression for the distribution of exceedance over a large threshold. In this framework, the study of large and extreme co-movements requires the multivariate extreme values theory, which unfortunately does not provide strong results. Indeed, in contrast with the univariate case, the class of limiting extreme-value distributions is too broad and cannot be used to constrain accurately the distribution of large co-movements.

In the spirit of the mean-variance portfolio or of utility theory which establish an investment decision on a unique risk measure, we use the coefficient of tail dependence, which, to our knowledge, was first introduced in the financial context by [Embrechts et al. (2001)]. The coefficient of tail dependence between assets $X_i$ and $X_j$ is a very natural and easy to understand measure of extreme co-movements. It is defined as the probability that the asset $X_i$ incurs a large loss (or gain) assuming that the asset $X_j$ also undergoes a large loss (or gain) at the same probability level, in the limit where this probability level explores the extreme tails of the distribution of returns of the two assets. Mathematically speaking, the coefficient of lower tail dependence between the two assets $X_i$ and $X_j$, denoted by $\lambda^-_{ij}$ is defined by

$$
\lambda^-_{ij} = \lim_{u \to 0} \Pr\{X_i < F_i^{-1}(u) \mid X_j < F_j^{-1}(u)\},
$$

where $F_i^{-1}(u)$ and $F_j^{-1}(u)$ represent the quantiles of assets $X_i$ and $X_j$ at the level $u$. Similarly, the coefficient of upper tail dependence is

$$
\lambda^+_{ij} = \lim_{u \to 1} \Pr\{X_i > F_i^{-1}(u) \mid X_j > F_j^{-1}(u)\}.
$$

$\lambda^-_{ij}$ (respectively $\lambda^+_{ij}$) is of concern to investors with long (respectively short) positions. We refer to [Coles et al. (1999)] and references therein for a survey of the properties of the coefficient of tail dependence. Let us stress that the use of quantiles in the definition of $\lambda^-_{ij}$ and $\lambda^+_{ij}$ makes them independent of the marginal distribution of the asset returns: as a consequence, the tail dependence parameters are intrinsic dependence measures. The obvious gain is an “orthogonal” decomposition of the risks into (1) individual risks carried by the marginal distribution of each asset and (2) their collective risk described by their dependence structure or copula.
Being a probability, the coefficient of tail dependence varies between 0 and 1. A large value of \( \lambda_{ij} \) means that large losses occur almost surely together. Then, large risks can not be diversified away and the assets crash together. This investor and portfolio manager nightmare is further amplified in real life situations by the limited liquidity of markets. When \( \lambda_{ij} \) vanishes, these assets are said to be asymptotically independent, but this term hides the subtlety that the assets can still present a non-zero dependence in their tails. For instance, two normally distributed assets can be shown to have a vanishing coefficient of tail dependence. Nevertheless, unless their correlation coefficient is identically zero, these assets are never independent. Thus, asymptotic independence must be understood as the weakest dependence which can be quantified by the coefficient of tail dependence (for other details, the reader is referred to [Ledford and Tawn (1998)].

For practical implementations, a direct application of the definitions (1) and (2) fails to provide reasonable estimations due to the double curse of dimensionality and undersampling of extreme values, so that a fully non-parametric approach is not reliable. It turns out to be possible to circumvent this fundamental difficulty by considering the general class of factor models, which are among the most widespread and versatile models in finance. They come in two classes: multiplicative and additive factor models respectively. The multiplicative factor models are generally used to model asset fluctuations due to an underlying stochastic volatility (see for instance [Hull and White (1987)] and [Taylor (1994)] for a survey of the properties of these models). The additive factor models are made to relate asset fluctuations to market fluctuations, as in the Capital Asset Pricing Theory (CAPM) and its generalizations (see [Sharpe (1964), Rubinstein (1973)] for instance), or to any set of common factors as in [Ross (1976)]’s Arbitrage Pricing Theory. The coefficient of tail dependence is known in close form for both classes of factor models, which allows, as we shall see, for an efficient empirical estimation.

2 Tail dependence generated by factor models

We first examine multiplicative factor models which account for most of the stylized facts observed on financial time series. Basically, a multivariate stochastic volatility model with a common stochastic volatility factor can be written as

\[
X = \sigma \cdot Y ,
\]

where \( \sigma \) is a positive random variable modeling the volatility, \( Y \) is a Gaussian random vector and \( X \) is the vector of assets returns. In this framework, the multivariate distribution of assets return \( X \) is an elliptical multivariate distribution. For instance, if the inverse of the square of the volatility follows the \( \chi^2 \)-distribution with \( \nu \) degrees of freedom, the distribution of assets return will be the Student’s distribution with \( \nu \) degrees of freedom. When the volatility follows ARCH or GARCH processes, then the assets returns are also elliptically distributed with fat tailed marginal distributions. Thus, any asset \( X_i \) is asymptotically distributed according to a power law\(^1\): \( \Pr \{|X_i| > x\} \sim x^{-\nu} \), with the same exponent \( \nu \) for all assets, due to the ellipticity of their multivariate distribution.

[Hult and Lindskog (2001)] have shown that the necessary and sufficient condition for any two assets \( X_i \) and \( X_j \) to have a non-vanishing coefficient of tail dependence is that their distribution be regularly varying. Denoting by \( \rho_{ij} \) the correlation coefficient between the assets \( X_i \) and \( X_j \) and by \( \nu \) the tail index of their distributions, they obtain:

\[
\lambda_{ij}^\pm = \int_{\pi/2}^{\pi/2 - \arcsin \rho_{ij}} dt \cos^\nu t \int_0^{\pi/2} dt \cos^\nu t = 2 I_1(\rho_{ij}) \left( \frac{\nu + 1}{2}, \frac{1}{2} \right) ,
\]

\(^1\)More precisely, the \( X_i \)'s follow regularly varying distributions (see [Bingham et al. (1987)] for details on regular variations).
where \( I_{1+\rho_{ij}}(x, y) \) denotes the incomplete gamma function. This expression holds for any regularly varying elliptical distribution, irrespective of the exact shape of the distribution. Only the tail index is important in the determination of the coefficient of tail dependence because \( \lambda_{ij}^\pm \) probes the extreme end of the tail of the distributions which all have roughly speaking the same behavior for regularly varying distributions. In constrast, when the marginal distributions decay faster than any power law, such as for the Gaussian, exponential and gamma distributions, the coefficient of tail dependence is zero.

Let us now turn to the second class of additive factor models, whose introduction in finance goes back at least to the Arbitrage Pricing Theory [Ross (1976)]. They are now widely used in many branches of finance, including to model stock returns, interest rates and credit risks. Here, we shall only consider the effect of a single factor, which may represent the market for instance. This factor will be denoted by \( Y \) and its distribution by \( F_Y \). As previously, the vector \( X \) is the vector of assets returns and \( \varepsilon \) will denote the vector of idiosyncratic noises assumed independent\(^2\) of \( Y \). \( \beta \) is the vector whose components are the regression coefficients of the \( X_i \) on the factor \( Y \). Thus, the factor model reads:

\[
X = \beta \cdot Y + \varepsilon.
\]

(5)

In contrast with multiplicative factor models, the multivariate distribution of \( X \) cannot be obtained in an analytical form, in the general case. Obviously, when \( Y \) and \( \varepsilon \) are normally distributed, the multivariate distribution of \( X \) is also normal but this case is not very interesting. In a sense, additive factor models are richer than the multiplicative ones, since they give birth to a larger set of distributions of assets returns.

Notwithstanding these difficulties, it turns out to be possible to obtain the coefficient of tail dependence for any pair of assets \( X_i \) and \( X_j \). In a first step, let us consider the coefficient of tail dependence \( \lambda_{ij}^\pm \) between any asset \( X_i \) and the factor \( Y \) itself. [Malevergne and Sornette (2002b)] have shown that \( \lambda_{ij}^\pm \) is also identically zero for all rapidly varying factors, that is, for all factors whose distribution decays faster than any power law, such as the Gaussian, exponential or gamma laws. When the factor \( Y \) has a distribution which decays regularly with tail index \( \nu \), we have

\[
\lambda_{ij}^\pm = \frac{1}{\max\{1, \frac{l}{\nu}\}}, \quad \text{where} \quad l = \lim_{u \to 1} \frac{F_X^{-1}(u)}{F_Y^{-1}(u)}.
\]

(6)

A similar expression holds for \( \lambda_{ij}^- \) obtained by simply changing the limit \( u \to 1 \) by \( u \to 0 \) in the definition of \( l \). \( \lambda_{ij}^\pm \) is non-zero as long as \( l \) remains finite, that is, when the tail of the distribution of the factor is not thinner than the tail of the idiosyncratic noise \( \varepsilon_i \). Therefore, two conditions must hold for the coefficient of tail dependence to be non-zero:

1. the factor must be intrinsically “wild” so that its distribution is regularly varying;
2. the factor must be sufficiently “wild” in its intrinsic wildness, so that its influence is not dominated by the idiosyncratic component of the asset.

Then, the amplitude of \( \lambda_{ij}^\pm \) is determined by the trade-off between the relative tail behaviors of the factor and the idiosyncratic noise.

As an example, let us consider that the factor and the idiosyncratic noise follow Student’s distribution with \( \nu_Y \) and \( \nu_{\varepsilon_i} \) degrees of freedom and scale factor \( \sigma_Y \) and \( \sigma_{\varepsilon_i} \) respectively. Expression (6) leads to

\[
\begin{align*}
\lambda_i &= 0 & \text{if} & \nu_Y > \nu_{\varepsilon_i}, \\
\lambda_i &= \frac{1}{\max\{1, \frac{l}{\nu}\}} & \text{if} & \nu_Y = \nu_{\varepsilon_i} = \nu, \\
\lambda_i &= 1 & \text{if} & \nu_Y < \nu_{\varepsilon_i}.
\end{align*}
\]

(7)

\(^2\)In fact \( \varepsilon \) and \( Y \) can be weakly dependent. See [Malevergne and Sornette (2002b)] for details.
The tail dependence decreases when the idiosyncratic volatility increases relative to the factor volatility. Therefore, $\lambda_i$ decreases in period of high idiosyncratic volatility and increases in period of high market volatility. From the viewpoint of the tail dependence, the volatility of an asset is not relevant. What is governing extreme co-movement is the relative weights of the different components of the volatility of the asset.

Figure 1 compares the coefficient of tail dependence as a function of the correlation coefficient for the bivariate Student’s distribution (expression (4)) and for the factor model with the factor and the idiosyncratic noise following student’s distributions (equation (7)). Contrary to the coefficient of tail dependence of the Student’s factor model, the tail dependence of the elliptical Student’s distribution does not vanish for negative correlation coefficients. For large values of the correlation coefficient, the former is always larger than the latter.

Once the coefficients of tail dependence between the assets and the common factor are known, the coefficient of tail dependence between any two assets $X_i$ and $X_j$ with a common factor $Y$ is simply equal to the weakest tail dependence between the assets and their common factor:

$$\lambda_{ij} = \min\{\lambda_i, \lambda_j\}. \quad (8)$$

This result is very intuitive: since the dependence between the two assets is due to their common factor, this dependence cannot be stronger than the weakest dependence between each of the asset and the factor.

3 Practical implementation and consequences

The two mathematical results (4) and (6) have a very important practical impact for the estimation of the coefficient of tail dependence. As we already pointed out, its direct estimation is essentially impossible since the number of observations goes to zero by definition as the quantile goes to zero. In contrast, the formulas (4) and (6-8) tell us that one just has to estimate a tail index and a correlation coefficient. These estimations can be reasonably accurate because they make use of a significant part of the data beyond the few extremes targeted by $\lambda$. Moreover, equation (6) does not explicitly assume a power law behavior, but only a regularly varying behavior which is far more general. In such a case, the empirical quantile ratio $l$ in (4) turns out to be stable enough for its accurate non-parametrically estimation, as shown in figure 2.

As an example, the table 1 presents the results obtained both for the upper and lower coefficients of tail dependence between several major stocks and the market factor represented here by the Standard & Poor’s 500 index, over the last decade. The technical aspects of the method are given in Appendix A. The coefficient of tail dependence between any two assets is easily derived from (8). It is interesting to observe that the coefficients of tail dependence seem almost identical in the lower and the upper tail. Nonetheless, the coefficient of lower tail dependence is always slightly larger than the upper one, showing that large losses are more likely to occur together compared with the case of large gains.

Two clusters of assets clear stand out: those with a tail dependence of about 10% (or more) and those with a tail dependence of about 5%. These stocks offer the interesting possibility of devising a prudential portfolio which can be significantly less sensitive to the large market moves. Figure 3 compares the daily returns of the Standard & Poor’s 500 index with those of two portfolios $P_1$ and $P_2$: $P_1$ is made of the four stocks (Chevron Corp., Philip Morris Cos Inc., Pharmacia Corp. and Texaco Inc.) with the smallest $\lambda$’s while $P_2$ is made with the four stocks (Bristol-Meyer Squibb Co., Hewlett-Packard Co., Schering-Plough Corp. and Texas Instruments Inc.) with the largest $\lambda$’s. For each set of stocks, we have constructed two portfolios, one in which each stock have the same weight $1/4$ and the other with asset weights chosen to minimize the variance of the resulting portfolio. We find that the results are almost the same between the equally-weighted
Table 1: This table presents the coefficients of lower and of upper tail dependence with the Standard & Poor’s 500 index for a set of 12 major stocks traded on the NYSE during the time interval from January 1991 to December 2000. The numbers within the parentheses gives the estimated standard deviation of the empirical coefficients of tail dependence.

| Company                      | Lower Tail Dependence | Upper Tail Dependence |
|------------------------------|-----------------------|-----------------------|
| Bristol-Myers Squibb Co.     | 0.16 (0.03)           | 0.14 (0.01)           |
| Chevron Corp.                | 0.05 (0.01)           | 0.03 (0.01)           |
| Hewlett-Packard Co.          | 0.13 (0.01)           | 0.12 (0.01)           |
| Coca-Cola Co.                | 0.12 (0.01)           | 0.09 (0.01)           |
| Minnesota Mining & MFG Co.   | 0.07 (0.01)           | 0.06 (0.01)           |
| Philip Morris Cos Inc.       | 0.04 (0.01)           | 0.04 (0.01)           |
| Procter & Gamble Co.         | 0.12 (0.02)           | 0.09 (0.01)           |
| Pharmacia Corp.              | 0.06 (0.01)           | 0.04 (0.01)           |
| Schering-Plough Corp.        | 0.12 (0.01)           | 0.11 (0.01)           |
| Texaco Inc.                  | 0.04 (0.01)           | 0.03 (0.01)           |
| Texas Instruments Inc.       | 0.17 (0.02)           | 0.12 (0.01)           |
| Walgreen Co.                 | 0.11 (0.01)           | 0.09 (0.01)           |

and minimum-variance portfolios. This makes sense since extreme tail dependence should not be controlled by the variance, which accounts only for the price moves of moderate amplitudes.

Figure 3 presents the results for the equally weighted portfolios generated from the two groups of assets. Observe that only one large drop occurs simultaneously for \( P_1 \) and for the Standard & Poor’s 500 index in contrast with \( P_2 \) for which several large drops are associated with the largest drops of the index. Quite a few of the largest drops of \( P_2 \) occur desynchronized with the index. This is probably due to the idiosyncratic contributions \( \varepsilon \) in (5) which are in reality not completely independent of the index. They contain in particular the effect of other factors that have been left out of this analysis.

Figure 3 shows an almost circular scatter plot for \( P_1 \) compared with a rather narrow ellipse for \( P_2 \): the small tail dependence between the index and the four stocks in \( P_1 \) automatically implies that their mutual tail dependence is also very small, according to (8); as a consequence, \( P_1 \) offers a good diversification with respect to large drops. This effect already quite significant for such small portfolios will be overwhelming for large ones. The most interesting result stressed in figure 3 is that optimizing for minimum tail dependence automatically diversifies away the large risks.

These advantages of portfolio \( P_1 \) with small tail dependence compared to portfolio \( P_2 \) with large tail dependence with the Standard & Poor’s 500 index comes at almost no cost in terms of the Sharpe ratio, equal respectively to 0.058 and 0.061 for the equally weighted and minimum variance \( P_1 \) and to 0.069 and 0.071 for the equally weighted and minimum variance \( P_2 \).

The straight lines are bonus: they show that there is significantly less linear correlations between \( P_1 \) and the index (correlation coefficient of 0.52 for both the equally weighed and the minimum variance \( P1 \)) compared with \( P_2 \) and the index (correlation coefficient of 0.73 for the equally weighed \( P_2 \) and of 0.70 for the minimum variance \( P_2 \)). Theoretically, it is possible to construct two random variables with small correlation coefficient and large \( \lambda \) and vice-versa. Recall that the correlation coefficient and the tail dependence coefficient are two opposite end-members of dependence measures, the correlation coefficient weighting the dependence between relatively small moves while the tail dependence coefficient weighting the dependence during extreme events. The finding that \( P_1 \) comes with both the smallest correlation and the smallest tail dependence coefficients suggests that there is a continuous of interlaced dependence structures between assets.
as a function of the “depth” (or quantile) in the tail of the distribution. This intuition is in fact explained and encompassed by the factor model since the larger $\beta$ is, the larger is the correlation coefficient and the larger is the tail dependence. Diversifying away extreme shocks may provide a useful diversification tool for less extreme dependences, thus improving the potential usefulness of a strategy of portfolio management based on tail dependence proposed here.

As a final remark, the almost identical values of the coefficients of tail dependence for negative and positive tails has the following consequence: minimizing the large concomittant losses between the stocks and the market comes with renouncing to the potential concomittant large gains. This point is well examplified by our two portfolios (see figure 3): $P_2$ obviously underwent severe negative co-movements but it also enjoyed large gains coming together with the large positive movements of the index. In contrast, $P_1$ is almost completely decoupled from the large negative movements of the market but this comes also with its insensitivity with respect to the large positive movements of the index. Thus, a good dynamical strategy seems to be the following one: invest in $P_1$ during bearish or trend-less market phases and prefer $P_2$ in a bullish market.

### A Empirical estimation of the coefficient of tail dependence

This appendix shows how to estimate the coefficient of tail dependence between an asset $X$ and the market factor $Y$ related by the relation (5) where $\varepsilon$ is an idiosyncratic noise uncorrelated with $X$.

Given a sample of $N$ realizations $\{X_1, X_2, \cdots, X_N\}$ and $\{Y_1, Y_2, \cdots, Y_N\}$ of $X$ and $Y$, we first estimate the coefficient $\beta$ using the ordinary least square estimator. Let $\hat{\beta}$ denote its estimate. Then, using Hill’s estimator, we obtain the tail index $\hat{\nu}$ of the factor $Y$:

$$\hat{\nu}_k = \left[ \frac{1}{k} \sum_{j=1}^{k} \log Y_{j,N} - \log Y_{k,N} \right]^{-1}, \quad (9)$$

where $Y_{1,N} \geq Y_{2,N} \geq \cdots \geq Y_{N,N}$ is the ordered statistics of the $N$ realizations of $Y$. The constant $l$ is non-parametrically estimated with the formula

$$l = \lim_{u \to 1} \frac{F_X^{-1}(u)}{F_Y^{-1}(u)} \simeq \frac{X_{k,N}}{Y_{k,N}}, \quad (10)$$

for $k = o(N)$, which means that $k$ must remain very small with respect to $N$ but large enough to ensure an accurate determination of $l$. The figure 2 presents $\hat{l}$ as a function of $k/N$.

Finally, using equation (6), the estimated $\hat{\lambda}$ is

$$\hat{\lambda}^+ = \frac{1}{\max\{1, \frac{l}{\hat{\beta}}\}^{\nu}}. \quad (11)$$
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Figure 1: Evolution as a function of the correlation coefficient $\rho$ of the coefficient of tail dependence for an elliptical bivariate Student’s distribution (solid line) and for the additive factor model with Student’s factor and noise (dashed line).
Figure 2: Empirical estimate $\hat{l}$ of the quantile ratio $l$ in (4) versus the empirical quantile $k/N$. We observe a very good stability of $\hat{l}$ for quantiles ranging between 0.005 and 0.05.
Figure 3: Daily returns of two equally weighted portfolios $P_1$ (made of four stocks with small $\lambda \leq 0.06$) and $P_2$ (made of four stocks with large $\lambda \geq 0.12$) as a function of the daily returns of the Standard & Poor’s 500 over the period January 1991 to December 2000.