SOME LRS BIANCHI-I STRING COSMOLOGICAL MODELS WITH VARIABLE DECELERATION PARAMETER

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Abstract

The present study deals with LRS Bianchi type I cosmological model representing massive string. The energy-momentum tensor for such string as formulated by Letelier (1983) is used to construct massive string cosmological models for which we assume that the shear scalar ($\sigma$) is proportional to the expansion scalar ($\theta$). The study reveals that massive strings dominate the early Universe. The strings eventually disappear from the Universe for sufficiently large time, which is in agreement with the current astronomical observations. Some physical and geometrical behaviour of models are also discussed.

Keywords : Massive string, LRS Bianchi type I Universe, Variable deceleration parameter

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1 Introduction

The past decades has been tremendous advances in cosmology. The discovery of dark energy (Perlmutter et al [1]−[3]; Riess et al [4]−[5]) has crushed widely-held expectations that some unknown mechanism might set the cosmological constant to zero. At the same time, substantial theoretical progress in string theory has brought forth a diverse new generation of cosmological models, some of which are subject to direct observational tests. One key advance in the emergence of methods of moduli stabilization. Compactification of string theory from the total dimension D down to four dimensions introduces many gravitationally-coupled scalar fields moduli from the point of view of the four dimensional theory. Recently we have studied inhomogeneous string cosmological model formed by geometric string and use this model as a source of gravitational field [6, 7]. We had two main reason to study the above mentioned model. First, as a test of consistency, for some particular field theories based on string models and second we point out the Universe can be represented by a collection of extended galaxies. It is generally assumed that after the big bang, the Universe may have undergone a series of phase transitions as its temperature cooled below some critical temperature as predicted by grand unified theories [8]−[12]. At the very early stage of evolution of universe, it is believed that during the phase transition, the symmetry of Universe was broken spontaneously. That could have given rise to topologically-stable defects such as domain walls, strings and monopoles [12]. Among all the three cosmological structures, only cosmic strings have excited the most interesting consequence [13], because it gives rise the density perturbations which leads to the formation of galaxies. The cosmic string can be closed (like loops) and open (like a hair) which move through time and trace out a tube or a sheet, according to whether it is closed or open. These cosmic strings have stress energy and couple to the gravitational field. Therefore, it is interesting to study the gravitational effect which arises from strings by using Einstein’s equations.

The general treatment of strings was initiated by Letelier [14] [15] and Stachel [16]. Letelier [14] obtained the general solution of Einstein’s field equations for a cloud of strings with spherical, plane and a particular case of cylindrical symmetry. Letelier [15] also obtained massive string cosmological models in Bianchi type-I and Kantowski-Sachs space-times. Benerjee et al [17] have investigated an axially symmetric Bianchi type I string dust cosmological model in presence and absence of magnetic field using a supplementary condition $\alpha = a \beta$ between metric potential where $\alpha = \alpha(t)$ and $\beta = \beta(t)$ and $a$ is constant. Exact solutions of string cosmology for Bianchi type-II, -VII, -VIII and -IX space-times have been studied by Krori et al [18] and Wang [19]. Wang [20]−[23] has investigated bulk viscous string cosmological models in different space-times. Bali and Anjali [24], Yadav [25], Pradhan et al [26, 27] and Yadav et al [28] have studied string cosmological models in different
physical contexts. The string cosmological models with a magnetic field are discussed by Chakraborty [29], Tikekar and Patel [30, 31], Patel and Maharaj [32]. Singh and Singh [33] investigated string cosmological models with magnetic field in the context of space-time with $G_3$ symmetry. Singh [34, 35] has studied string cosmology with electromagnetic fields in Bianchi type-II, -VIII and -IX space-times. Lidsey, Wands and Copeland [36] have reviewed aspects of super string cosmology with the emphasis on the cosmological implications of duality symmetries in the theory. Recently Saha and Visinescu [37] and Saha et al [38] have investigated Bianchi I string cosmological model in presence of magnetic flux. They have found that the present of cosmic string does not allow the anisotropic Universe to evolve into an isotropic one [37].

In this paper we have studied locally rotationally symmetric (LRS) Bianchi type I string cosmological model with time varying deceleration parameter (DP). The paper has following structure. In section 2, the metric and field equations are described. In section 3, we introduce a few plausible solutions consistent with observations. At the end we shall summarize the findings.

2 The Metric and Field Equations

We consider the LRS Bianchi type I metric of the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 (dy^2 + dz^2)$$ (1)

where, $A$ and $B$ are functions of $t$ only. This ensures that the model is spatially homogeneous.

The energy-momentum tensor $T^i_j$ for a cloud of massive strings and perfect fluid distribution is taken as

$$T^i_j = (\rho + p)v^i v_j + pg^i_j - \lambda x^i x_j,$$ (2)

where $\rho$ is the isotropic pressure; $\rho$ is the proper energy density for a cloud strings with particles attached to them; $\lambda$ is the string tension density; $v^i = (0, 0, 0, 1)$ is the four-velocity of the particles, and $x^i$ is a unit space-like vector representing the direction of string. The vectors $v^i$ and $x^i$ satisfy the conditions

$$v^i v_i = -x^i x_i = -1, \quad v^i x_i = 0.$$ (3)

Choosing $x^i$ parallel to $\partial/\partial x$, we have

$$x^i = (A^{-1}, 0, 0, 0).$$ (4)

If the particle density of the configuration is denoted by $\rho_p$, then

$$\rho = \rho_p + \lambda.$$ (5)

The Einstein’s field equations (in gravitational units $c = 1, 8\pi G = 1$) read as

$$R^i_j - \frac{1}{2} g^i_j R = -T^i_j$$ (6)

The Einstein’s field equations (6) for the line-element (1) lead to the following system of equations

$$2 \frac{B_{44}}{B} + \frac{B_4^2}{B^2} = -p + \lambda$$ (7)

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} = -p$$ (8)

$$\frac{B_4^2}{B^2} + 2 \frac{A_4 B_4}{AB} = \rho$$ (9)

Here, and in what follows, sub-in-dices 4 in $A$, $B$ and elsewhere indicates differentiation with respect to $t$. The energy conservation equation $T^i_{j, i} = 0$, leads to the following expression:

$$\rho_4 + (\rho + p) \left( \frac{A_4}{A} + 2 \frac{B_4}{B} \right) - \lambda \frac{A_4}{A} = 0,$$ (10)
which is consequence of the field equations (7)-(9).
The average scale factor (R) of LRS Bianchi type I model is defined as
\[ R = (AB^2)^{\frac{1}{3}} \]  
(11)
The spatial volume (V) is given by
\[ V = R^3 = AB^2 \]  
(12)
We define the mean Hubble parameter (H) for LRS Bianchi I space-time as
\[ H = \frac{R}{R} = \frac{1}{3} \left( \frac{A_4}{A} + 2 \frac{B_4}{B} \right) \]  
(13)
The expansion scalar (θ), shear scalar (σ) and mean anisotropy parameter (Am) are defined as
\[ \theta = 3H = \frac{A_4}{A} + 2 \frac{B_4}{B}, \]  
(14)
\[ \sigma^2 = \frac{1}{2} \left( \sum_{i=1}^{3} H_i^2 - \frac{1}{3} \theta^2 \right), \]  
(15)
\[ A_m = \frac{1}{3} \sum_{i=1}^{3} \left( \frac{H_i - H}{H} \right)^2 \]  
(16)

3 Solutions of the Field Equations

Observations of type Ia supernovae [5] allow to probe the expansion history of Universe. In literature it is common
to use a constant deceleration parameter, as it duly gives a power law for metric function or corresponding
quantity. But at present the expansion of Universe is accelerating and decelerating in the past. Also the
transition redshift from decelerating phase to accelerating phase is about 0.5. Now for the Universe which was
decelerating in the past and accelerating at present time, the DP must show signature flipping [39]. So, in
general, DP is not constant but variable. Following, Virey et al [40], we consider the DP to be variable i.e.
\[ q = -\frac{RR_{44}}{R_{44}^2} = b \ (variable) \]  
(17)
where R is average scale factor. In this paper, we show, how the variable DP models with metric (11) behave in
presence of string fluid as a source of matter.

Pradhan et al [6] and recently Yadav and Yadav [41], have obtained cosmological models with proportionality
relation between shear scalar (σ) and expansion scalar (θ). This condition leads to the following relation between
the metric potentials:
\[ A = B^n, \]  
(18)
where n is a positive constant.
From equation (17), we obtain
\[ \frac{R_{44}}{R} + \frac{b}{R^2} = 0 \]  
(19)
In order to solve equation (19), we have to assume \( b = b \ (R) \). It is important to note here that one can assume
\( b = b \ (t) = b \ (R(t)) \), as R is also a time dependent function. But this is possible only when one avoid singularity
like big bang or big rip because both t and R are increasing function.
Thus the general solution of equation (19) with assumption \( b = b \ (R) \), is given by
\[ \int e^{\frac{4}{3}bR} = t + m \]  
(20)
where m is the constant of integration.
One can not solve eq. (20) in general as b is variable. So, in order to solve the problem completely, we have to choose \(\int \frac{b}{R} dR\) in such a manner that eq. (20) be integrable with out any loss of generality, we consider
\[
\int \frac{b}{R} dR = \ln L(R)
\]
(21)
Which does not effect the nature of generality of the solution.
Hence, from equations (20) and (21), we obtain
\[
\int L(R)dR = t + m
\]
(22)
Of course, the choice of \(L(R)\), in eq. (22), is quite arbitrary but, since we are looking for a physically viable models of Universe consistent with observations. We consider the following cases

### 3.1 Solution in the polynomial form
Let us consider \(L(R) = \frac{1}{2k_1 \sqrt{R + k_2}}\), where \(k_1\) and \(k_2\) are constants.
In this case, on integrating, eq. (22) gives the exact solution
\[
R = \alpha_1 T^2 + \alpha_2 T + \alpha_3
\]
(23)
where
\(T = t + m\), \(\alpha_1 = k_1^2\), \(\alpha_2 = 2c_1 k_1^3\), \(\alpha_3 = c_1^2 k_1^2 - k_2\)
Here, \(c_1\) is constant of integration. Solving equations (11), (18) and (23), we obtain the metric function as
\[
A = \left(\alpha_1 T^2 + \alpha_2 T + \alpha_3\right)^{\frac{3}{n+2}},
\]
(24)
\[
B = \left(\alpha_1 T^2 + \alpha_2 T + \alpha_3\right)^{\frac{1}{n+2}}.
\]
(25)
Hence the metric (11) is reduced to
\[
ds^2 = -dT^2 + \left(\alpha_1 T^2 + \alpha_2 T + \alpha_3\right)^{\frac{4}{n+2}} dx^2 + \left(\alpha_1 T^2 + \alpha_2 T + \alpha_3\right)^{\frac{6}{n+2}} (dy^2 + dz^2)
\]
(26)
The expressions for the isotropic pressure \(p\), the proper energy density \(\rho\), the string tension \(\lambda\) and the particle density \(\lambda_p\) for the model (26) are obtained as
\[
p = \frac{3(3n - 2 - 5n^2)(2\alpha_1 T + \alpha_2)}{(n+2)(\alpha_1 T^2 + \alpha_2 T + \alpha_3)^2} - \frac{6\alpha_1(n+1)}{(n+2)(\alpha_1 T^2 + \alpha_2 T + \alpha_3)},
\]
(27)
\[
\rho = \frac{9(2n+1)(2\alpha_1 T + \alpha_2)^2}{(n+2)^2(\alpha_1 T^2 + \alpha_2 T + \alpha_3)^2},
\]
(28)
\[
\lambda = \frac{3(n+4-5n^2)(2\alpha_1 T + \alpha_2)^2}{(n+2)^2(\alpha_1 T^2 + \alpha_2 T + \alpha_3)^2} - \frac{6\alpha_1(n-1)}{(n+2)(\alpha_1 T^2 + \alpha_2 T + \alpha_3)},
\]
(29)
\[
\rho_p = \frac{3(5n^2 + 5n - 1)(2\alpha_1 T + \alpha_2)^2}{(n+2)^2(\alpha_1 T^2 + \alpha_2 T + \alpha_3)^2} - \frac{6\alpha_1(1-n)}{(n+2)(\alpha_1 T^2 + \alpha_2 T + \alpha_3)}.
\]
(30)
The energy conservation equation (10) is satisfied identically by the above solutions, as expected.

We observe that all the parameters diverge at \(T = -\frac{\alpha_2^{\frac{n}{2}} \sqrt{\frac{n^2 - 4\alpha_1 \alpha_3}{2\alpha_1}}}{\frac{\alpha_2^{\frac{n}{2}}}{2\alpha_1}}\). Therefore, the model has singularity at \(T = -\frac{\alpha_2^{\frac{n}{2}} \sqrt{\frac{n^2 - 4\alpha_1 \alpha_3}{2\alpha_1}}}{\frac{\alpha_2^{\frac{n}{2}}}{2\alpha_1}}\), which can be shifted to \(T = 0\) by choosing \(\alpha_2 = \alpha_3 = 0\). This singularity is of Point Type as all the scale factors vanish at \(T = -\frac{\alpha_2^{\frac{n}{2}} \sqrt{\frac{n^2 - 4\alpha_1 \alpha_3}{2\alpha_1}}}{\frac{\alpha_2^{\frac{n}{2}}}{2\alpha_1}}\). The parameters \(p, \rho, \rho_p\) and \(\lambda\) start off with extremely large values. In particular, the large values of \(\rho_p\) and \(\lambda\) in the beginning suggest that strings dominate the early Universe. For sufficiently large time, \(\rho_p\) and \(\lambda\) become negligible. Therefore, the strings disappear from Universe for large time that is why, the strings are not observable in the present Universe. From equation (28), it is observed that the proper energy density \(\rho\) is decreasing function of time. Fig. 1 depicts the variation of rest energy density versus time. The proper energy density \(\rho\) and particle energy density \(\rho_p\) have been graphed in
From equation (29) and (30), we have \( \frac{\rho_p}{|\lambda|} > 1 \) i.e. particle energy density \( \rho_p \) remains larger than the string tension density \( \lambda \) during the cosmic expansion, especially in early Universe. This behaviour of \( \rho_p \) and \( |\lambda| \) is clearly shown in Fig. 3. According to the ref. (Letelier [15]; Krori et al [18]), when \( \frac{\rho_p}{|\lambda|} > 1 \), in the process of evolution, the Universe is dominated by massive strings, and when \( \frac{\rho_p}{|\lambda|} < 1 \), the universe is dominated by strings. From Fig. 3, we see that \( \frac{\rho_p}{|\lambda|} > 1 \). Thus in derived model, the early Universe is dominated by massive string. According to Ref. [42], since there is no direct evidence of strings in the present day Universe, we are in general, interested in constructing models of Universe that evolves purely from the era dominated by either geometric strings or massive strings and end up in the particle dominated era with or without remnants of strings. Therefore, the above model describes the evolution of the Universe consistent with the present-day observations.

The rate of expansion in the direction of \( x, y \) and \( z \) are given by

\[
H_x = \frac{A_4}{A} = \frac{3n(2\alpha_1 T + \alpha_2)}{(n + 2)(\alpha_1 T^2 + \alpha_2 + \alpha_3)}
\]

\[
H_y = H_z = \frac{3(2\alpha_1 T + \alpha_2)}{(n + 2)(\alpha_1 T^2 + \alpha_2 + \alpha_3)}
\]

The mean Hubble’s parameter \( (H) \), expansion scalar \( (\theta) \) and shear scalar \( (\sigma^2) \) of model (26) are given by

\[
H = \frac{(2\alpha_1 T + \alpha_2)}{\alpha_1 T^2 + \alpha_2 + \alpha_3}
\]

\[
\theta = \frac{3(2\alpha_1 T + \alpha_2)}{(\alpha_1 T^2 + \alpha_2 + \alpha_3)}
\]

\[
\sigma^2 = \frac{3(n - 1)^2(2\alpha_1 T + \alpha_2)^2}{(n + 2)^2(\alpha_1 T^2 + \alpha_2 + \alpha_3)^2}
\]

The spatial volume \( (V) \), mean anisotropy parameter \( (A_m) \) and DP \( (q) \) are found to be

\[
V = (\alpha_1 T^2 + \alpha_2 + \alpha_3)^3
\]
Figure 2: Proper energy density ($\rho$) and particle energy density ($\rho_p$) vs. time ($T$).

Figure 3: String tension density ($\lambda$), particle energy density ($\rho_p$) and $\frac{\rho_p}{|\lambda|}$ vs. time ($T$).
The variation of DP versus time has been graphed in Fig. 4. It is observed that DP evolves with in the range predicted by present-day observations. From eq. (36), it can be seen that the spatial volume is zero at \( T = -\frac{\alpha_2 \pm \sqrt{\alpha_2^2 - 4\alpha_1 \alpha_3}}{2\alpha_1} \), and it increases with the cosmic time. The parameter \( H_x, H_y, H_z, H, \theta \) and \( \sigma^2 \) diverge at initial singularity. These parameters decrease with evolution of Universe and finally drops to zero at late time. For \( n = 1 \), the mean anisotropy parameter vanishes and the directional scale factors vary as

\[
A = B = (\alpha_1 T^2 + \alpha_2 T + \alpha_3)
\]

Therefore, isotropy is achieved in the derived model for \( n = 1 \). For this particular values of \( n \), we see that \( A(T) = B(T) = R(T) \), therefore, metric (11) reduces to flat FRW space-time. Thus, the derived model acquires flatness for \( n = 1 \). But in the same spirit, the string tension density (\( \lambda \)) vanishes for \( n = 1 \). Hence we can not choose \( n = 1 \) in presence of string fluid as a source of matter in LRS Bianchi I space-time.

3.2 Solution in sine hyperbolic form

We consider, \( L(R) = \frac{1}{k_3 \sqrt{1 + R^2}} \), where \( k_3 \) is an arbitrary constant.

In this case, on integrating, eq. (22) gives the exact solution

\[
R = \sinh (k_3 T)
\]

where \( T = t + m \), and the constant of integration has been omitted by assuming that \( R = 0 \) at \( T = 0 \). Solving equations (11), (18) and (39), we obtain the metric function as

\[
A = \sinh^{\frac{3n}{2 \alpha_2}} (k_3 T)
\]

\[
B = \sinh^{\frac{3n}{2 \alpha_2}} (k_3 T)
\]

Hence the metric (11) is reduced to

\[
\dot{s}^2 = -dT^2 + \sinh^{\frac{4\alpha_1}{2 \alpha_2}} (k_3 T) dx^2 + \sinh^{\frac{4\alpha_2}{2 \alpha_2}} (k_3 T) (dy^2 + dz^2)
\]
The expressions for the isotropic pressure \( p \), the proper energy density \( \rho \), the string tension \( \lambda \) and the particle density \( \rho_p \) for the model (42) are obtained as

\[
p = \frac{3(n+1)k^2}{n+2} \cosh^2(k_3T) - \frac{9(n^2 + n + 1)k^2}{(n+2)^2} \coth^2(k_3T), \quad (43)
\]

\[
\rho = \frac{9(2n+1)k^2}{(n+2)^2} \coth^2(k_3T), \quad (44)
\]

\[
\lambda = \frac{3(n-1)k^2}{n+2} \cosh^2(k_3T) - \frac{9(n^2 + n - 2)k^2}{(n+2)^2} \coth^2(k_3T), \quad (45)
\]

\[
\rho_p = \frac{9(n^2 + 3n - 1)k^2}{(n+2)^2} \coth^2(k_3T) - \frac{3(n-1)k^2}{n+2} \coth^2(k_3T), \quad (46)
\]

The energy conservation equation (10) is satisfied identically by the above solutions, as expected.

We observe that all the parameters diverge at \( T = 0 \). Therefore, the model has big bang singularity at \( T = 0 \). This singularity is of Point Type as all the scale factors vanish at \( T = 0 \). The parameters \( p, \rho, \rho_p \) and \( \lambda \) start off with extremely large values.

The proper energy density \( \rho \), string tension density \( \lambda \) and particle energy density \( \rho_p \) have been graphed versus time \( T \) in Fig. 5. It is evident that string tension density becomes negligible for sufficient large time. Therefore, the strings disappear from Universe at late time that is why, the strings are not observable in present universe. From Fig. 5, we see that \( \frac{\rho_p}{\lambda} \gg 1 \). Therefore, the early Universe was dominated by massive string.

The rate of expansion in the direction of \( x, y \) and \( z \) are given by

\[
H_x = \frac{3nk_3}{n+2} \coth(k_3T), \quad (47)
\]

\[
H_y = H_z = \frac{3k_3}{n+2} \coth(k_3T), \quad (48)
\]

The mean Hubble's parameter \( H \), expansion scalar \( \theta \) and shear scalar \( \sigma^2 \) of model (42) are given by

\[
H = k_3 \coth(k_3T), \quad (49)
\]

\[
\theta = 3k_3 \coth(k_3T), \quad (50)
\]
Figure 6: The plot of DP (q) vs. time (T).

\[
\sigma^2 = \frac{3(n-1)^2k_3^2}{(n+2)^2} \coth^2 (k_3T) \tag{51}
\]

The spatial volume \(V\), mean anisotropy parameter \(A_m\) and DP \(q\) are found to be

\[
V = \sinh^3 (k_3T), \tag{52}
\]

\[
A_m = \frac{2(n-1)^2}{(n+2)^2}, \tag{53}
\]

\[
q = -\tanh^2 (k_3T) \tag{54}
\]

The variation of DP versus time has been graphed in Fig. 6. It is observed that DP evolves with in the range predicted by SN Ia [1]–[5] and CMBR [43] observations. We observe that at \(T = 0\), the spatial volume vanishes and it increases with cosmic time. For \(n = 1\), the mean anisotropy parameter vanishes and the directional scale factors vary as

\[
A(T) = B(T) = \sinh (k_3T)
\]

Therefore, \(n = 1\), turns out to condition of isotropy but in the same spirit, the string tension density \(\lambda\) vanishes. Therefore, the presence of cosmic string does not allow to choose \(n = 1\).

4 Concluding Remarks

In this paper, we have studied LRS Bianchi type I string cosmological models in general relativity. The Einstein’s field equations have been solved exactly with suitable physical assumptions and the solutions satisfy the energy conservation equation identically. Therefore, exact and physically viable LRS Bianchi I string cosmological models have been obtained. The derived models have singular origin i. e. the Universe starts expanding with a big bang singularity.

The main features of the models are as follows:

- The models are based on exact solution of Einstein’s field equations for LRS Bianchi I space-time in presence of string fluid as a source of matter.
• It has been found that massive strings dominate the early Universe, which is eventually disappear from the Universe for sufficiently large time. This is in agreement with current astronomical observations.

• In the derived models, \( n = 1 \), turns out to be a condition of isotropy and flatness of Universe. It is important to mention here that for \( n = 1 \), the string tension density (\( \lambda \)) vanishes in both cases. So, we conclude that presence of cosmic string does not allow to choose \( n = 1 \). Thus, the anisotropic LRS Bianchi I Universe may not evolve into isotropic one in presence of cosmic string. The same is predicted by Saha and Visinescu [37] with different approach in Bianchi I space-time.

• The DP \((q)\) is evolving with negative value and the existing range of \( q \) is in nice agreement with SN Ia data and CMBR observations. Thus the derived models are realistic.

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