We study an important contribution to the electric dipole moment (EDM) of the electron (or quarks) at the two-loop level due to the $W$-EDM in the recently proposed scenario of split supersymmetry. This contribution is independent of the Higgs mass, and it can enhance the previous estimation of the electron (neutron) EDM by $20-50\%$ ($40-90\%$). Our formula is new in its analytical form.

I. INTRODUCTION

Supersymmetry (SUSY) has been considered an extended symmetry beyond Standard Model (SM) to solve the gauge hierarchy problem, to stabilize the scalar sector, and to provide a theoretical ground for possible unification of gravity with all other fundamental forces. However, Minimum Supersymmetricly Standard Model (MSSM) predicts plethora of new superpartner particles, but none of them has been observed yet. Therefore soft terms are introduced to break SUSY but keep the feature of taming the quadratic divergence. Unfortunately, it is well known that at the electroweak (EW) scale the softly broken SUSY generates many unwanted phenomenological problems, such as flavor changing neutral currents, CP violation, and so on. Motivated by the string landscape scenario and the cosmological constant problem, Arkani-Hamed and Dimopoulos have recently proposed a scenario (dubbed Split SUSY) that SUSY is broken at an energy scale way beyond the collider search and could be even near the scale of the grand unification theory (GUT). As a result the scalar superpartners of Standard Model fermions are all super heavy. On the other hand, fermions are protected by symmetries, such as, to be more specifically, chiral symmetry, R-symmetry and PQ symmetry, so they acquire masses around electroweak to TeV or so. By doing so, the phenomenological problems of SUSY at the EW scale can be avoided. However, the existence of a CP-even SM like Higgs with mass around $100-250$ GeV requires a fine tuning in the Higgs potential. To address this fine tuning problem, they argue that extreme fine turning is required for solving the cosmological constant problem which is viewed as choosing a stringy ground state with small cosmological constant from an extreme huge pool of vacuum candidates. Admitting this kind of fine tuning, one is no longer worried about the naturalness principle and the fine turning in the Higgs sector. (Recently it was pointed out by [2] that extra fine tuning is needed to get a reasonable values for $\tan \beta$.)

Phenomenologically, the characteristics of the Split SUSY can be summarized as following. (1) All scalars, except the CP-even SM like Higgs, are super heavy $\sim 10^9$ GeV - $M_{\text{GUT}}$. (2) Gaugino masses are around the EW scale to TeV protected by R-symmetry (PQ symmetry). (3) $\mu$ parameter is around the EW scale such that the lightest neutralino can annihilate effectively to give the dark matter density. (4) Coupling unification still works, mainly due to the gauginos contributions.

There are already many discussions on detecting or testing the split SUSY by accessible and/or near future experiments [3, 4, 5, 6], on the connection with the neutrino masses [7], on the implication in cosmology and astrophysics[8]. In this note we will only focus on the inherent contributions to EDM in the Split SUSY scenario.

In split SUSY, the gaugino masses parameters, $M_{1,2,3}$ for $U(1), SU(2)$ and $SU(3)$ gauge group respectively, as well as the Higgsino mixing mass parameter $\mu$, are all around the EW scale. Consequently, the charginos and neutralinos have masses at the same scale. These parameters are generally complex with respect to each other, and their mutual phases cannot be removed by redefinition of fields. If so, the CP violation in the gaugino-Higgsino sector is genuine, and it can give rise to the EDM of an elementary particle at low energy. Nevertheless, all possible one-loop contributions to EDM are highly suppressed by the super heavy scalar mass in the loop. Thus the leading sources of the EDM starts at the two-loop level where SM particles and EW charginos and neutralinos run in the loops. A study of possible EDM due to the complex pseudo-scalar coupling of light neutral Higgs, see Fig. 1(a), has been done in [3, 9]. This type of EDM will be referred as $d^h$. However, we want to point out that the $W$ gauge boson EDM, see Fig. 1(b), can...
where the unitary matrices $C$ with $C_{R}^{\dagger}C_{L}R = \text{diag} \{ m_{\omega_{1}}, m_{\omega_{2}} \}$ and $N^{T}M_{N}N = \text{diag} \{ m_{\chi_{1}}, m_{\chi_{2}}, m_{\chi_{3}}, m_{\chi_{4}} \}$. The diagonalized masses are positive and real. We use the convention that $m_{\omega_{1}} < m_{\omega_{2}}$ and $m_{\chi_{1}} < m_{\chi_{2}} < m_{\chi_{3}} < m_{\chi_{4}}$. Notation $s_{W}(\sin \beta)$ stands for $\sin \theta_{W}(\sin \beta)$ and $\tan \beta = v_{u}/v_{d}$. The matrices $C^{L,R}$ are not uniquely defined. However the resulting EDM is basis independent.

These two-loop diagrams or similar ones have been calculated many times in the literature[10, 11, 12, 13, 14]. Here we just summarize and report the essential results for the most important contribution from two gauge invariant subsets.

In Fig. 1(a), since the coupling between the SM Higgs and charged fermion is pure scalar like, only the pseudo-scalar form factor of the photon-photon-Higgs vertex in the upper loop will contribute to EDM. We denote the momenta and polarizations for the photon-photon-Higgs vertex as $h^{0}(p = q + k) \rightarrow \gamma(k, \mu) + \gamma(q, \nu)$. In this way, the pseudo-scalar part from the can be derived to be:

$$d\Gamma^{\mu, \nu} = \frac{g^{2}e}{4\sqrt{2}\pi^{2}} \sum_{i=1}^{2} \text{Im} \, O_{i}^{\prime} m_{\omega_{i}} \int_{0}^{1} d\gamma \frac{1 - \gamma}{m_{\omega_{i}}^{2} - \gamma(1 - \gamma) p^{2}} \times e^{\mu, \nu, \rho, \lambda} k_{\rho} q_{\lambda}. \quad (6)$$

FIG. 1: The 2 loop diagrams contribute to fermion EDM.
This form factor is further connected to the SM charged fermion line and the resultant EDM becomes

\[
\frac{d\mathcal{E}_f}{e} = \frac{Q_f e^2 m_e}{4\sqrt{2}\pi^2 M_W^2 s_W^4} \sum_{i=1}^2 \text{Im} \left( O_i \frac{m_{\omega_i}}{M_W} \mathcal{F} \left( \frac{m_{\omega_i}^2}{M_W^2} \right) \right),
\]

where \( Q_f \) is the charge of SM fermion \( f \) and the function \( \mathcal{F} \) is

\[
\mathcal{F}(x) = \int_0^1 d\gamma \frac{(1-\gamma)}{x-\gamma(1-\gamma)} \ln \frac{x}{\gamma(1-\gamma)}
\]

\[
= \text{Re} \left\{ \frac{1}{\sqrt{1-4x}} \left[ \ln x \ln \frac{\sqrt{1-4x} - 1}{\sqrt{1-4x} + 1} + L_i \left( \frac{2}{1 - \sqrt{1-4x}} \right) - L_i \left( \frac{2}{1 + \sqrt{1-4x}} \right) \right] \right\}.
\]

The result agrees with the analysis of [3] when \( x > 1/4 \). However, we emphasize that only the real part is taken when \( x < 1/4 \) because the imaginary part is only a mathematical artifact. Note that \( L_i(z) = -\int_0^1 \ln(1-t)(dt/t) \).

The diagonalization of the \( 2 \times 2 \) chargino mass matrix and the coupling \( O_i \) can be done analytically, see for example [8]. We note by passing that this EDM is proportional to the \( \text{Im} (\mu M_2) \). In other word, \( d^b_0 \) vanishes in the parameter space where \( \text{arg}(\mu M_2) = 0 \mod 2\pi \).

To calculate the \( d^W \), we can first integrate out the upper loop in Fig. 1(b). Following [15], the CP violating form factor \( f_6 \) for \( W^+(p = q + k, \nu) \rightarrow W^+(q, \lambda) + \gamma(\mu, \kappa) \) is defined by the effective vertex

\[
i\Gamma^{\mu, \nu, \lambda} = -i f_6 e^{\theta, \nu, \lambda} k_\rho.
\]

Another parameterization of this vertex can be found in [16], where its implication to the electron EDM was studied with a short-distance cutoff. The cutoff is unnecessary because the general interaction of Eq. (11) prescribes the \( q^2 \) dependence in \( f_6 \),

\[
f_6(q^2) = \frac{e_0}{2\pi s_W^2} \sum_{i=1}^4 \sum_{j=1}^2 \text{Im} \left( O_i \frac{m_{\omega_i}}{M_W} \mathcal{F} \left( \frac{m_{\omega_i}}{M_W} \right) \right) \int_0^1 \frac{m_{\chi_i} m_{\omega}(1-\gamma)}{(1-\gamma)m_{\omega}^2 + \gamma m_{\chi_i}^2 - \gamma(1-\gamma)q^2}.
\]

which agrees with [17, 18, 19]. The resulting EDM of charged fermion \( f \) due to \( f_6 \) is finite,

\[
\frac{d^W}{e} = \pm \frac{\alpha^2 m_f}{8\pi s_W^4 M_W^2} \sum_{i=1}^2 \sum_{j=1}^2 \frac{m_{\chi_i} m_{\omega}}{M_W^2} \text{Im} \left( O_i \frac{m_{\omega}}{M_W} \mathcal{G} \left( r_i^0, r_j^{\pm}, r_f \right) \right),
\]

\[
\mathcal{G} \left( r_i^0, r_j^{\pm}, r_f \right) = \int_0^\infty dz \int_0^1 \frac{d\gamma}{\gamma} \int_0^1 \frac{dy}{y z} \frac{y z (y+z/2)}{(z + R)^3 (z + K_{ij})}
\]

\[
= \int_0^1 \frac{d\gamma}{\gamma} \int_0^1 \frac{dy}{y} \left[ \left( -\frac{3 K_{ij}}{R} + 2 (K_{ij} + R) y \right) + \frac{K_{ij} (K_{ij} - 2 y R^2)}{2 R (K_{ij} + R)^3} \right] \right].
\]

The plus-minus sign in front the right-handed side of Eq. (11) corresponds to the fermion \( f \) with weak isospin \((-)1/2 \).

The short-hand symbols \( K, R, r_s \) are defined as

\[
R = y + (1-y) r_f, \quad K_{ij} = \frac{r_i^0}{1-\gamma} + \frac{r_j^{\pm}}{\gamma}, \quad r_i^0 = \frac{m_{\omega_i}}{M_W^2}, \quad r_j^{\pm} = \frac{m_{\chi_i}}{M_W^2}, \quad r_f = \frac{m_{\omega_f}}{M_W^2}.
\]

Here \( r_f \) is the electroweak \( SU(2) \) partner of \( f \). In the large \( K_{ij} \) limit, the leading expansion result agrees with [18]. However we emphasize that our Eq. (11) is an exact formula which does not appear previously. For example, our result is numerically few percents larger than those given in [18, 19] when the chargino and neutralino masses are around the EW scale.

We diagonalize the \( 4 \times 4 \) neutralino mass matrix directly by the numerical method.

III. NUMERICAL RESULTS

In previous study [3, 5], the EDM \( d^b_0 \) was shown as the function of the ratio of \( m_{\omega_2}/m_{\omega_1} \). However a general scheme is represented by any points in a space of seven parameters mentioned above. Therefore we evaluate both \( d^b_0 \) and \( d^W \) by randomly scanning the following parameter space, 200 GeV < \( M_1, M_2, \mu < 1.0 \) TeV, 120 GeV < \( M_H < 170 \) GeV.
FIG. 2: The total EDM and the ratio of $d^W/d^h_0$.

and all the three CP phases vary within $[0, 2\pi]$. The above range of the Higgs mass was suggested by [4]. However, some variants allow the light Higgs to be as heavy as 400 GeV [5]. The numerical result of five hundred randomly selected points are shown in Fig. 2 for $\tan \beta = 0.5, 5.0$, and 50 respectively. The current upper limit on the electron EDM, $<1.7 \times 10^{-27}$ e-cm 95% CL [20], is shown as the dash line in the graphs.

From these plots, we notice that for electron:

1. contributions of $d^h_0$ and $d^W$ share the same sign and the ratio $d^W/d^h_0$ lies around $0.2 - 0.5$ for a light Higgs mass within 120 - 170 GeV. Indeed there are very few points within the scanned range not appear in the plots. For those rare cases, the reason can be identified as $\arg(\mu M_2) \sim 0$ or $d^h_0 \ll d^W$ and the total possibility is less than 1%.

2. The electron EDM is around $10^{-28} - 10^{-27}$ e-cm for $\tan \beta = 0.5$ and it decreases to $10^{-30.5} - 10^{-29.5}$ e-cm when $\tan \beta = 50$.

The electron EDM versus $\tan \beta$ is shown in Fig. 3. Based on the parameter scan, it seems very promising in the observation of the electron EDM by experiments with the sensitivity of $10^{-29}$ e-cm [21].

As the lightest neutral Higgs becomes heavier, the $d^W$ contribution to the EDM of the charged SM fermion turns out to be increasingly important. The values of $d^h_0$ and $d^W$ are roughly compatible when $M_H \sim 600$ GeV, see Fig. 4 and $d^W$ dominates over $d^h_0$ for larger $M_H$. In the extreme case of a super heavy Higgs, $d^W$ is the sole contribution to the EDM of SM fermions. In Fig. 5 we show the only contribution $d^W$ in this extreme limit, where the EDM is roughly half order of magnitude smaller than that of a light Higgs mass within 120 - 170 GeV already illustrated in Fig. 3. Nevertheless, an electron EDM around $10^{-29}$ e-cm predicted in the extreme case of a super heavy Higgs is still probably detectable in the future experiment.

In the split SUSY models, the charged lepton EDMs follow the simple mass scaling law and the muon EDM is given by the electron EDM scaled up by the factor of $m_\mu/m_e$, which is quite different from some models, for example see [22]. Therefore, models can be distinguished by comparing the electron and the muon EDMs. However, split SUSY predicts the $d_\mu$ to be roughly $10^{-24.5} - 10^{-27}$ e cm, which is 6 to 7 orders of magnitude lower than the current limit [23].
and it will be a great challenge for the newly proposed $d_{\mu}$ measurement \cite{24}.

Now we turn our attention to the neutron EDM.

In MSSM, usually the chromo dipole moment is the dominant contribution to the neutron EDM due to the large $\alpha_s$ of the strong interaction. However, in the split SUSY models, the CP phases associated with gluinos can always be shuffled off upon the squarks mass matrix by phase redefining of the gluino field. The chromo dipole moment therefore vanishes because all the squarks are decoupled from the low energy physics and $d_{h^0}$ and $d_W$ become the leading contribution to the neutron EDM.

Given the nonperturbative nature of hadron physics, it is not clear how to make reliable theoretical prediction on the neutron EDM under control. However, as an order of magnitude estimation, the quark model prediction $d_n = (4d_u - d_d)/3$ can be used to give a rough estimation of the neutron EDM. By trivially scaling up the fermion masses and replacing the fermion charge accordingly, we can express the neutron EDM as

$$d_{n}^{h^0} = -\frac{8m_u + m_d}{9m_e} d_e^{h^0}, \quad d_{n}^{W} = -\frac{4m_u + m_d}{3m_e} d_e^{W}. \quad (14)$$

In arriving at the last expression of $d_{n}^{W}$, we have ignored masses of $SU(2)$ doublet partners, the $u$ and $d$ quarks, in the loop. Since the light quark masses are much less than $M_W$, this approximation is quite safe.

As in the electron case, the relative sign between the two contributions is also positive.

The estimation of the resulting neutron EDM is displayed in Fig. 5, where the current quark masses, $m_u = 3$ MeV and $m_d = 6$ MeV, have been used. Here, the same range of the parameter space is scanned as in Fig. 3. Note that the proper choice of quark masses is still controversial, however such a question is beyond the scope of this article. The readers should keep in mind that our estimation of neutron EDM is conservative and the prediction could receive substantial enhancement due to the unknown nature of hadronic physics.

The current upper limit of neutron EDM, $< 6.3 \times 10^{-26}$ e-cm at 90\% CL \cite{25}, is also displayed as the dash line in the graph. In comparison with the electron EDM, the $d_W$ becomes more important in the neutron EDM study due to the naive enhancing factor in Eq. (14). While the sum of $d_W$ and $d^{h^0}$ is still below the current upper limit, $d_W$ plays an indispensable role in the neutron EDM.
This article studies the EDM in the scenario of Split SUSY. We point out that an overlooked but important two-loop contribution, Fig. (b), due to the $W$ gauge boson EDM, has to be included together with others given by previous EDM study where only Fig. (a) type diagram was considered.

(1) For most of the parameter space, the $W$-EDM diagram enhances the previous estimation of the electron EDM by 20 – 50%.

(2) For some special circumstances that $\arg(M_2\mu) \sim 0$ or the neutral Higgs are super heavy, the EDM contribution from $d^{h_0}$ vanishes, and the fermion EDM will be dominated by $d^W$.

(3) Combining these two EDM contributions, we have scanned the whole parameter space and found that the electron EDM is likely to be seen in next run of EDM experiments.

(4) We estimate the neutron EDM by using the naive quark model. With typical current quark masses $m_u = 3$ MeV and $m_d = 6$ MeV, the numerical result indicates that the contribution from $d^W$ is about the same size of $d^{h_0}$. However, the overall result is about an order of magnitude or more below the current experimental bound.

FIG. 6: The neutron EDM v.s tan $\beta$. 

IV. CONCLUSION

This article studies the EDM in the scenario of Split SUSY. We point out that an overlooked but important two-loop contribution, Fig. (b), due to the $W$ gauge boson EDM, has to be included together with others given by previous EDM study where only Fig. (a) type diagram was considered.

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