Prediction of three heavy spin-0 particles in the SM

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Abstract

It is shown that in axial-vector field theory the axial-vector field is always accompanied by a spin-0 field. The perturbation theory of the axial-vector field theory is reconstructed. These results are applied to the SM. One neutral and two charged spin-0 states are predicted, whose masses are $m_{\phi^0} = m_t e^{28.4} = 3.78 \times 10^{14} \text{GeV}$ and $m_{\phi^\pm} = m_t e^{27} = 9.31 \times 10^{13} \text{GeV}$ respectively. A new perturbation theory of the SM is proposed. The propagators of $Z$ and $W$ fields in this new perturbation theory are derived in unitary gauge. They have the same expressions as the ones derived by using the renormalization gauge in the original perturbation theory of the SM. No additional ghosts are associated with the new propagators. The new propagators of $W$ and $Z$ fields show when energies are greater than $10^{14} \text{GeV}$ the problems of indefinite metric
and negative probability arise in the SM.
1 Introduction

The standard model[1] of electroweak interactions is successful in many aspects. The Lagrangian of the SM after spontaneous symmetry breaking is

\[ \mathcal{L} = -\frac{1}{4} A_{\mu\nu}^i A^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \bar{q} \{ i\gamma \cdot \partial - m_q \} q \\
+ \bar{q}_L \left\{ \frac{g}{2} \tau_i \gamma \cdot A^i + g' \frac{Y}{2} \gamma \cdot B \right\} q_L + \bar{q}_R g' \frac{Y}{2} \gamma \cdot B q_R \\
+ \bar{l} \{ i\gamma \cdot \partial - m_l \} l + \bar{l}_L \left\{ \frac{g}{2} \tau_i \gamma \cdot A^i - g' \frac{Y}{2} \gamma \cdot B \right\} l_L - \bar{l}_R g' \gamma \cdot B l_R \\
+ \frac{1}{2} m^2_Z Z^\mu Z_\mu + m^2_W W^\mu_\mu + \mathcal{L}_{Higgs}. \quad (1) \]

Summation over the fermion fields is implicated in Eq.(1).

Because of parity nonconservation in weak interactions there are both vector and axial-vector couplings between fermions and gauge bosons. These couplings are written as

\[ \mathcal{L}_{fi} = \frac{1}{4} \bar{q} \left\{ g \tau_i A^i_{\mu} + g' Y B^\mu \right\} \gamma^\mu (1 + \gamma_5) q + \frac{g'}{4} \bar{q} Y \gamma^\mu (1 - \gamma_5) q B^\mu \\
+ \frac{1}{4} \bar{l} \left\{ g \tau_i A^i_{\mu} - g' B^\mu \right\} \gamma^\mu (1 + \gamma_5) l - \frac{g'}{2} \bar{l} \gamma^\mu (1 - \gamma_5) l B^\mu. \quad (2) \]

It is well known that in QED and QCD the gauge bosons are pure vector fields. However, the W and Z fields of the SM have both vector and axial-vector components, as shown in Eq.(2). In this paper the properties of the axial-vector field is studied. New effects are found. Based on the findings in the axial-vector field theory the SM is revisited. The paper is organized as:1) introduction; 2) theory of axial-vector field; 3)necessity of revisiting SM;
4) neutral heavy spin-0 boson; 5) charged heavy spin-0 bosons; 6) reconstruct the perturbation theory of the SM; 7) effect of vacuum polarization by intermediate boson; 8) conclusion.

2 Theory of axial-vector field

In order to compare both vector and axial-vector field theories the expression of the vacuum polarization of a vector field is presented first. The Lagrangian of a vector field and a fermion (QED) is

$$\mathcal{L} = -\frac{1}{4} (\partial_{\mu} v_{\nu} - \partial_{\nu} v_{\mu})^2 + \bar{\psi} \{ i \gamma \cdot \partial + e \gamma \cdot v \} \psi - m \bar{\psi} \psi. \quad (3)$$

This Lagrangian is invariant under the gauge transformation

$$\psi \to e^{i \alpha(x)} \psi, \quad v_{\mu} \to v_{\mu} + \frac{1}{e} \partial_{\mu} \alpha. \quad (4)$$

The amplitude of the vacuum polarization is

$$<v|s|v> = i(2\pi)^4 \delta(p' - p)\epsilon_{\mu} \epsilon_{\nu} \frac{e^2}{(4\pi)^2} D\Gamma(2 - \frac{D}{2}) \int_0^1 dx \left(\frac{\mu^2}{L_0}\right)^\frac{D-2}{2} (x(1-x)p^2 g^{\mu\nu} - m^2 g^{\mu\nu}) \right)$$

$$= 2i(2\pi)^4 \delta(p' - p)\epsilon_{\mu} \epsilon_{\nu} \frac{e^2}{(4\pi)^2} D\Gamma(2 - \frac{D}{2}) (p^\mu p^\nu - p^2 g^{\mu\nu}) \int_0^1 dx x(1-x) \left(\frac{\mu^2}{L_0}\right)^\frac{D-2}{2}, \quad (5)$$

where $L_0 = m^2 - x(1-x)p^2$. The amplitude is still gauge invariant.
The Lagrangian of a model of an axial-vector field and a fermion is constructed as

\[ \mathcal{L} = -\frac{1}{4} (\partial_\mu a_\nu - \partial_\nu a_\mu)^2 + \bar{\psi} \{ i \gamma \cdot \partial + e \gamma \cdot a \gamma_5 \} \psi - m \bar{\psi} \psi. \]  

(6)

The amplitude of the vacuum polarization is obtained

\[ \langle a|s|a \rangle = i (2\pi)^4 \delta(p' - p) \epsilon_\mu \epsilon_\nu \frac{e^2}{(4\pi)^2} D\Gamma(2 - \frac{D}{2}) \]

\[ \int_0^1 dx \left( \frac{\mu^2}{L_0} \right)^2 \{ x(1 - x)(2p^\mu p^\nu - p^2 g^{\mu\nu}) + L_0 g^{\mu\nu} + m^2 g^{\mu\nu} \} \]

(7)

\[ = 2i (2\pi)^4 \delta(p' - p) \epsilon_\mu \epsilon_\nu \frac{e^2}{(4\pi)^2} D\Gamma(2 - \frac{D}{2}) \]

\[ \int_0^1 dx \{ x(1 - x)(2p^\mu p^\nu - p^2 g^{\mu\nu}) + m^2 g^{\mu\nu} \} \left( \frac{\mu^2}{L_0} \right)^2 \hat{z}. \]

(8)

In Eq.(7) the third term \( m^2 g^{\mu\nu} \) has a plus sign, while in Eq.(4) this term has minus sign. The sign difference is caused by the fact that \( \gamma_5 \) anticommutes with \( \gamma_\mu \). Because of this sign difference the mass term is cancelled in the case of vector field and Eq.(4) is obtained, while for axial-vector field the expression(8) is obtained. Comparing with Eq.(4), in Eq.(8) there is one more term which is generated dynamically.

Up to one loop the amplitude of the vacuum polarization of vector field is derived from Eq.(4)

\[ \Pi^\nu_{\mu\nu} = \frac{1}{2} (p_\mu p_\nu - p^2 g_{\mu\nu}) F_{v_1}(z), \]

(9)

where

\[ F_{v_1}(z) = 1 + \frac{e^2}{(4\pi)^2} \left[ \frac{1}{3} \frac{1}{D\Gamma(2 - \frac{D}{2})} \left( \frac{\mu^2}{m^2} \right)^2 - 8 f_1(z) \right], \]

(10)
\[ f_1(z) = \int_0^1 dx x(1-x)\log\{1-x(1-x)z\}, \]  

where \( z = \frac{p^2}{m^2} \).

The amplitude of the vacuum polarization of axial-vector field is obtained from Eq.(8)

\[ \Pi^a_{\mu\nu} = \frac{1}{2}(p_\mu p_\nu - p^2 g_{\mu\nu})F_{a1}(z) + F_{a2}(z)p_\mu p_\nu + \frac{1}{2}m^2 a g_{\mu\nu}, \]

where

\[
F_{a1}(z) = 1 + \frac{e^2}{(4\pi)^2}\left\{\frac{1}{3} D\Gamma(2 - \frac{D}{2})\left(\frac{\mu^2}{m^2}\right)^2 - 8f_1(z) + 8f_2(z)\right\},
\]

\[ f_2(z) = \frac{1}{z} \int_0^1 dx \log\{1-x(1-x)z\}, \]

\[ F_{a2}(z) = -\frac{4e^2}{(4\pi)^2}f_2(z), \]

\[ m^2 a = \frac{2e^2}{(4\pi)^2} D\Gamma(2 - \frac{D}{2})\left(\frac{\mu^2}{m^2}\right)^2 m^2. \]

Comparing with Eq.(9), there are two new terms in Eq.(12): the mass term of the axial-vector field and a term proportional to \((\partial_\mu a^\mu)^2\). In Ref.[2] it has been already found that mass of axial-vector field can be dynamically generated. Both of the two new terms are generated by the vacuum polarization of the axial-vector field. The mechanism generating the new terms has been named as axial-vector symmetry breaking in Ref.[2]. Eq.(12) shows that vector and axial-vector field theories are very different.

The free and interaction Lagrangians of vector field theory(3) are defined as

\[
\mathcal{L}_{v0} = -\frac{1}{4}(\partial_\mu v_\nu - \partial_\nu v_\mu)^2 + \bar{\psi}(i\gamma \cdot \partial - m)\psi,
\]

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The massless vector field has two independent degrees of freedom. After taking the vacuum polarization (4) into account, the degrees of freedom of the vector field are still two. Therefore, the perturbation theory (17, 18) of the vector field theory satisfies unitarity. As usual, the perturbation theory of axial-vector field (6) is defined by

\[ L'_{a0} = -\frac{1}{4}(\partial_{\mu} a_{\nu} - \partial_{\nu} a_{\mu})^2 + \bar{\psi} \{ i \gamma \cdot \partial - m \} \psi, \]  

(19)

\[ L'_{ai} = e \bar{\psi} \gamma_{\mu} \gamma_5 a_{\mu}. \]  

(20)

Eq. (19) shows that the axial-vector field is massless and \( \partial_{\mu} a^{\mu} = 0 \). Therefore, it has two independent degrees of freedom. However, after taking the vacuum polarization (8) into account, the axial-vector field becomes massive and the term \( F_{a2} p_{\mu} p_{\nu} \) in Eq. (12) indicates that the divergence of the axial-vector field is no longer zero. The vacuum polarization makes the \( a_{\mu} \) field of four independent degrees of freedom. Therefore, the vacuum polarization of the axial-vector field cannot be treated perturbatively and the perturbation theory of axial-vector field theory must be reconstructed.

The function \( F_{a1} \) (13) is used to renormalize the \( a_{\mu} \) field and rewritten as (to \( O(e^2) \))

\[ F_{a1}(z) = Z_a \{ 1 + (p^2 - m_a^2)G_{a1}(p^2) \}, \]  

(21)

\[ Z_a = F_{a1}(\frac{m_a^2}{m^2}), \]  

(22)
\[(p^2 - m_a^2)G_{a1}(p^2) = F_{a1}(z) - F_{a1}(m_a^2), \quad (23)\]

where \(Z_a\) is the renormalization constant of the field \(a_{\mu}\), \(G_{a1}(p^2)\) is the radiative correction of the kinetic term. Function \(F_{a2}\) is finite and rewritten as

\[F_{a2}(z) = \xi + (p^2 - m_\phi^2)G_{a2}(p^2), \quad (24)\]

where \(m_\phi^2\) is the mass of a spin-0 state whose existence will be studied below, \(G_{a2}\) is the radiative correction of the term \((\partial_\mu a^\mu)^2\), and

\[\xi = F_{a2}(m_\phi^2). \quad (25)\]

The new perturbation theory of axial-vector field theory is constructed as

\[
\mathcal{L}_{a0} = -\frac{1}{4}(\partial_\mu a_\nu - \partial_\nu a_\mu)^2 + \xi(\partial_\mu a^\mu)^2 + \frac{1}{2}m_a^2 a_\mu^2, \quad (26)
\]

\[
\mathcal{L}_{ai} = e\bar{\psi}\gamma_\mu\gamma_5\psi a_\mu + \mathcal{L}_c, \quad (27)
\]

\[
\mathcal{L}_c = -\xi(\partial_\mu a^\mu)^2 - \frac{1}{2}m_a^2 a_\mu^2. \quad (28)
\]

\(\mathcal{L}_c\) is a counter term of the Lagrangian. Eq.(26) is the Stueckelberg’s Lagrangian.

The equation satisfied by \(\partial_\mu a^\mu\) is derived from Eq.(26)

\[
\partial^2(\partial_\mu a^\mu) - \frac{m_a^2}{2\xi}(\partial_\mu a^\mu) = 0. \quad (29)
\]

\(\partial_\mu a^\mu\) is a pseudoscalar field and we define

\[\partial_\mu a^\mu = b\phi. \quad (30)\]
The equation of the new field $\phi$ is found from Eq.(29)

$$\partial^2 \phi - \frac{m_a^2}{2\xi} \phi = 0, \quad m_\phi^2 = -\frac{m_a^2}{2\xi}. \quad (31)$$

Eqs.(25,31) lead to that the mass of the $\phi$ boson is the solution of the equation

$$2F_a(\frac{m_\phi^2}{m^2})m_\phi^2 + m_a^2 = 0. \quad (32)$$

In order to show the existence of solution of Eq.(32) we take

$$\frac{e^2}{\pi^2} \frac{m^2}{m_a^2} = 1$$

as an example. The numerical calculation shows that $\xi < 0$ and the solution of Eq.(32) is found to be

$$m_\phi = 8.02m.$$

If

$$\frac{e^2}{\pi^2} \frac{m^2}{m_a^2} = 0.5$$

is taken we obtain

$$m_\phi = 20.42m.$$
We separate $a_\mu$ field into a massive spin-$1$ field and a pseudoscalar field

$$a_\mu = a'_\mu + c\partial_\mu \phi,$$

(33)

$$\partial_\mu a'^\mu = 0.$$  

(34)

Substituting Eq.(33) into Eq.(26), the Lagrangian is divided into two parts

$$\mathcal{L}_{a0} = \mathcal{L}_{a'0} + \mathcal{L}_{\phi0},$$

(35)

$$\mathcal{L}_{a'0} = -\frac{1}{4}(\partial_\mu a'_\nu - \partial_\nu a'_\mu)^2 + \frac{1}{2}m_a^2 a'_\mu a'^\mu,$$

(36)

$$\mathcal{L}_{\phi0} = \frac{1}{2m_\phi^2}\partial_\mu \phi \{\partial^2 + m_\phi^2\} \partial^\mu \phi.$$  

(37)

The coefficient $c$ is determined by the normalization of $\mathcal{L}_{\phi0}$

$$c = \pm \frac{1}{m_a},$$

(38)

and substituting Eq.(33) into Eq.(30), we obtain

$$b = -cm_\phi^2 = \mp \frac{m_\phi^2}{m_a}.$$  

(39)

The sign of Eqs.(38,39) doesn’t affect the physical results when $\phi$ appears as virtual particle.

From Eq.(26) the propagator of the $a_\mu$ field is derived

$$\Delta_{\mu\nu} = \frac{1}{p^2 - m_a^2}\{-g_{\mu\nu} + \left(1 + \frac{1}{2\xi}\right)p_\mu p_\nu\}.$$  

(40)
Eq. (40) can be rewritten as

$$\Delta_{\mu\nu} = \frac{1}{p^2 - m_a^2}\{ -g_{\mu\nu} \frac{p_{\mu}p_{\nu}}{m_a^2} \} - \frac{1}{m_\phi^2} \frac{p_{\mu}p_{\nu}}{p^2 - m_\phi^2}. \quad (41)$$

The first part is the propagator of the massive spin-1 field and the second part is the propagator of the pseudoscalar field. From Eqs. (33, 41), the propagator of the pseudoscalar is determined to be

$$- \frac{1}{p^2 - m_\phi^2}. \quad (42)$$

It is different from the propagator of a regular spin-0 field by a minus sign. The minus sign of Eq. (42) indicates

$$[a(k), a^\dagger(k')] = -\delta_{k k'}, \quad (43)$$

where $a(k)$ and $a^\dagger(k)$ are the annihilation and creation operators of the $\phi$ field respectively.

The energy of the $\phi$ field is derived from Eq. (37)

$$E = \int d^3x \mathcal{H} = \sum_k \omega \{N_k + \frac{1}{2}\}, \quad (44)$$

$$N_k = -a^\dagger(k)a(k), \quad (45)$$

where $N_k$ is the number operator of the particle of momentum $k$ and $\omega = \sqrt{k^2 + m_\phi^2}$. The energy (44) is positive. In order to show that Eq. (43) is understandable we take $\xi = -\frac{1}{2}$ in Eq. (26) as an example. In the case of $\xi = -\frac{1}{2}$ both the spin-1 and spin-0 fields have the
same mass. Canonical quantization leads to

\[ [a_\lambda(k), a_{\lambda'}^{\dagger}(k')] = -g_{\lambda\lambda'}\delta_{kk'} \]

Eq. (46) is Lorentz covariant. Taking \( \lambda, \lambda' = 0 \) the Eq. (43) is obtained. Eq. (43) shows that there are problems of indefinite metric and negative probability when the \( \phi \) field is on mass shell.

The coupling between the \( \phi \) field and the fermion is found to be

\[ \pm \frac{e}{m_\alpha} \bar{\psi} \gamma_\mu \gamma_5 \psi \partial^\mu \phi = \pm \frac{e}{m_\alpha} 2i m \bar{\psi} \gamma_5 \psi \phi. \]

The coupling is proportional to the fermion mass.

It is necessary to point out that the axial-vector field theory described by Eq. (6) has triangle anomaly. As the SM we can add the second fermion with \( -e \) as the coupling constant in Eq. (6), the triangle anomaly is cancelled out. Because the number of the vertices in the vacuum polarization is even the results obtained above still exist.

To summarize the results obtained in this section,

1. Two new terms, mass term of \( a_\mu \) field and a term proportional to \( (\partial_\mu a_\nu)^2 \), are generated by the diagram of vacuum polarization of the axial-vector field,

2. These two new terms make the \( a_\mu \) a field of four independent components: a massive spin-1 field and a pseudoscalar field,
3. The mass of the pseudoscalar is determined and the coupling between $\phi$ field and fermion is proportional to the mass of the fermion,

4. The vacuum polarization cannot be treated perturbatively. The new perturbation theory of the axial-vector field (6) is reconstructed (26-28),

5. It is well known that the propagator of a massive vector boson is expressed as

$$\Delta^v_{\mu\nu} = \frac{1}{p^2 - m^2} \{-g_{\mu\nu} + \frac{p_{\mu}p_{\nu}}{m^2}\}. \quad (48)$$

This propagator causes quadratic divergence. However, the propagator of massive axial-vector boson (40) is very different from the propagator of massive vector boson (48). It doesn’t cause quadratic divergence.

6. Eq.(41) shows that there is a pole at $\sqrt{p^2} = m_\phi$. On the other hand, the minus sign of Eq.(41) indicates when $\phi$ is on mass shell there are problems of indefinite metric and negative probability in the theory of axial-vector field.

3 Necessity of revisiting the SM

As shown in eq.(2) both W and Z fields of the SM have axial-vector components. Based on the results obtained in last section a revisit of the SM is necessary. The issue is how to do perturbation in the SM after spontaneous symmetry breaking. Using the unitary gauge,
after spontaneous symmetry breaking the free Lagrangian of W and Z bosons in the original perturbation theory is defined as

\[ \mathcal{L}_0 = -\frac{1}{4} (\partial_\mu Z_\nu - \partial_\nu Z_\mu)^2 + \frac{1}{2} m_Z^2 Z_\mu Z^\mu - \frac{1}{2} (\partial_\mu W_\nu^+ - \partial_\nu W^+_\mu) (\partial^{\mu} W^{-\nu} - \partial^{\nu} W^{-\mu}) + m_W^2 W_{\mu}^+ W^{-\mu}. \]

(49)

There are others

\[ \mathcal{L}_{\gamma 0} + \mathcal{L}_{f 0} + \mathcal{L}_{H 0}, \]

where \( \mathcal{L}_{\gamma 0}, \mathcal{L}_{f 0} \) and \( \mathcal{L}_{H 0} \) are the free Lagrangians of photon, fermions and Higgs respectively.

The interaction Lagrangian can be found from Eq.(1). If renormalization gauge is chosen there is gauge fixing term in Eq.(49). However, ghost is associated. In Eq.(49) both W and Z are massive spin-1 fields.

According to the study of section(2) the axial-vector couplings between the intermediate bosons and fermions(2) lead to the existence of one neutral and two charged spin-0 states, which are associated with Z and W\(^\pm\) respectively. In the free Lagrangian of the intermediate bosons(49) there are no such spin-0 state. Therefore, the free Lagrangian of intermediate bosons should be redefined and the perturbation theory of the SM should be reconstructed.

Therefore, revisit of the SM is necessary.

In the SM the intermediate bosons are Yang-Mills fields. Besides the vacuum polarization by fermions the intermediate bosons contribute to the vacuum polarization too. However,
the calculation of the contribution of the intermediate bosons to the vacuum polarization can only be done after the propagators of intermediate bosons are defined by the new free Lagrangian of the intermediate bosons in the new perturbation theory. In section(7) the issue will be addressed.

4 Neutral heavy spin-0 state

The $F_{1,2}$ functions defined in section(2) can be found from the vacuum polarization of $Z$ boson by fermions.

We start from the t- and b-quark generation. From the SM(1) the Lagrangian of the interactions between $Z$-boson and t and b quarks is found

$$\mathcal{L} = \frac{g}{4} \left\{ (1 - \frac{8}{3} \alpha) \bar{t} \gamma_\mu t + \bar{t} \gamma_\mu \gamma_5 t \right\} Z^\mu - \frac{g}{4} \left\{ (1 - \frac{4}{3} \alpha) \bar{b} \gamma_\mu b + \bar{b} \gamma_\mu \gamma_5 b \right\} Z^\mu, \quad (50)$$

where $\alpha = \sin^2 \theta_W$. The S-matrix element of the vacuum polarization of t and b quark generation at the second order is obtained

$$< Z | s^{(2)} | Z > = i(2\pi)^4 \delta(p - p') \epsilon^\mu \epsilon^\nu \frac{g^2}{8} \frac{N_C}{(4\pi)^2} D \Gamma(2 - \frac{D}{2}) \int_0^1 dx \left\{ x(1 - x)(p_\mu p_\nu - p^2 g_{\mu\nu})[\left( \frac{\mu^2}{L_t} \right) \tilde{\pi} [(1 - \frac{8}{3} \alpha)^2 + 1] + \left( \frac{\mu^2}{L_b} \right) \tilde{\pi} [(1 - \frac{4}{3} \alpha)^2 + 1]]

+ m_t^2 \left( \frac{\mu^2}{L_t} \right) \tilde{\pi} g_{\mu\nu} + m_b^2 \left( \frac{\mu^2}{L_b} \right) \tilde{\pi} g_{\mu\nu} \right\}, \quad (51)$$

where $L_t = m_t^2 - x(1 - x)p^2$ and $L_b = m_b^2 - x(1 - x)p^2$. The S-matrix elements of other
two generation of quarks are obtained too. The kinetic term of Eq.(51) is generated by both
the vector and the axial-vector couplings and the mass terms originate in the axial-vector
coupling only.

The interaction Lagrangian between Z-boson and the leptons of e and $\nu_e$ is obtained from
Eq.(1)

$$L = \bar{\nu}_e \gamma_\mu (1 + \gamma_5) \nu_e Z^\mu - \bar{\nu}_e \gamma_\mu \{ (1 - 4\alpha) \bar{e} \gamma_\mu e + \bar{e} \gamma_\mu \gamma_5 e \} Z^\mu. \quad (52)$$

We obtain

$$<Z|s^{(2)}|Z> = i(2\pi)^4 \delta(p - p') \frac{\bar{\nu}_e \gamma_\mu (1 + \gamma_5) \nu_e Z^\mu - \bar{\nu}_e \gamma_\mu \{ (1 - 4\alpha) \bar{e} \gamma_\mu e + \bar{e} \gamma_\mu \gamma_5 e \} Z^\mu}{(4\pi)^2 D \Gamma(2 - \frac{D}{2})}$$

$$\int_0^1 dx \{ x(1 - x)(p_\mu p_\nu - p^2 g_\mu\nu) \left[ (\frac{\mu^2}{L_e})^2 [(1 - 4\alpha)^2 + 1] + 2(\frac{\mu^2}{L_\nu})^2 \right]$$

$$+ m_\mu^2 (\frac{\mu^2}{L_e})^2 g_\mu\nu + m_\nu^2 (\frac{\mu^2}{L_\nu})^2 g_\mu\nu \} \right \}, \quad (53)$$

There are other two lepton generations contributing to the vacuum polarization.

The amplitude of the vacuum polarization of fermions is expressed as

$$\Pi^Z_{\mu\nu} = \frac{1}{2} F_{Z1}(z)(p_\mu p_\nu - p^2 g_\mu\nu) + F_{Z2}(z)p_\mu p_\nu + \frac{1}{2} \Delta m_e^2 g_\mu\nu, \quad (54)$$

$$F_{Z1} = 1 + \frac{\bar{\nu}_e \gamma_\mu (1 + \gamma_5) \nu_e Z^\mu - \bar{\nu}_e \gamma_\mu \{ (1 - 4\alpha) \bar{e} \gamma_\mu e + \bar{e} \gamma_\mu \gamma_5 e \} Z^\mu}{64\pi^2 \{ N_C \sum_q \left[ (\frac{\mu^2}{m_q^2})^2 + \sum_l (\frac{\mu^2}{m_l^2})^2 \right]$$

$$- 2N_C \sum_q f_1(z_q) + \sum_l f_1(z_l) \} + 2 \left[ \sum_q f_2(z_q) + \sum_{l=e,\mu,\tau} f_2(z_l) \right] \},$$

$$F_{Z2} = - \frac{\bar{\nu}_e \gamma_\mu (1 + \gamma_5) \nu_e Z^\mu - \bar{\nu}_e \gamma_\mu \{ (1 - 4\alpha) \bar{e} \gamma_\mu e + \bar{e} \gamma_\mu \gamma_5 e \} Z^\mu}{64\pi^2 \{ N_C \sum_q \left[ (\frac{\mu^2}{m_q^2})^2 + \sum_l f_2(z_l) \right] \}, \quad (55)$$

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\[ \Delta m_Z^2 = \frac{1}{8 (4\pi)^2} \bar{g}^2 D \Gamma(2 - \frac{D}{2}) \{ N_c \sum_q m_q^2 \left( \frac{\mu^2}{m_q^2} \right)^{\frac{1}{2}} + \sum_l m_l^2 \left( \frac{\mu^2}{m_l^2} \right)^{\frac{1}{2}} \}. \]

where \( y_q = 1 + (1 - \frac{8}{3} \alpha)^2 \) for \( q = t, c, u \), \( y_q = 1 + (1 - \frac{4}{3} \alpha)^2 \) for \( q = b, s, d \), \( y_l = 1 + (1 - 4 \alpha)^2 \), for \( l = \tau, \mu, e \), \( y_l = 2 \) for \( l = \nu_e, \nu_\mu, \tau_\mu \), \( z_i = \frac{p^2}{m_i^2} \) and the functions \( f_{1,2} \) are defined by Eqs.(11,14) respectively. In the SM the Z boson gains mass from the spontaneous symmetry breaking and the mass term \( \Delta m_Z^2 \) has been refereed to the renormalization of \( m_Z^2 \). In Ref. 2Y the mechanism of generating \( m_Z \) and \( m_W \) is explored. Both the vector and axial-vector couplings of Eq.(2) contribute to \( F_{Z1} \). Only the axial-vector coupling of Eq.(2) contributes to \( F_{Z2} \). \( F_{Z2} \) is finite.

The function \( F_{Z1} \) is used to renormalize the Z-field. We are interested in \( F_{Z2} \) and it is rewritten as

\[ F_{Z2}(z) = \xi_Z + (p^2 - m_{\phi^0}^2) G_{Z2}(p^2), \]  

where \( m_{\phi^0}^2 \) is the mass of a new neutral spin-0 field, \( \phi^0 \), which will be studied, \( G_{Z2} \) is the radiative correction of this term, and

\[ \xi_Z = F_{Z2}\big|_{p^2=m_{\phi^0}^2}. \]  

Now we reconstruct the perturbation theory. The free Lagrangian of the Z-field is defined as

\[ \mathcal{L}_{Z0} = -\frac{1}{4} (\partial_\mu Z_\nu - \partial_\nu Z_\mu)^2 + \xi_Z (\partial_\mu Z^\mu)^2 + \frac{1}{2} m_Z^2 Z_\mu^2. \]
The equation of $\partial_\mu Z^\mu$ is derived from Eq.(58)

$$\partial^2(\partial_\mu Z^\mu) - \frac{m_Z^2}{2\xi Z}(\partial_\mu Z^\mu) = 0.$$  \hspace{1cm} (59)

$\partial_\mu Z^\mu$ is an independent spin-0 field. Following section(2) we have

$$Z_\mu = Z'_\mu \pm \frac{1}{m_Z} \partial_\mu \phi^0,$$ \hspace{1cm} (60)

$$\partial_\mu Z'^\mu = 0,$$ \hspace{1cm} (61)

$$\phi^0 = \mp \frac{m_Z}{m_{\phi^0}} \partial_\mu Z^\mu,$$ \hspace{1cm} (62)

$$\partial^2 \phi^0 - \frac{m_Z^2}{2\xi Z} \phi^0 = 0.$$ \hspace{1cm} (63)

The mass of $\phi^0$ is obtained

$$2m_{\phi^0} F_{Z2}|_{\mu^2 = m_{\phi^0}^2} + m_Z^2 = 0,$$ \hspace{1cm} (64)

$$m_{\phi^0}^2 = -\frac{m_Z^2}{2\xi Z}.$$ \hspace{1cm} (65)

Using Eq.(55), Eq.(64) is rewritten as

$$3 \sum_q \frac{m_q^2}{m_Z^2} z_q f_2(z_q) + \sum_i \frac{m_i^2}{m_Z^2} z_i f_2(z_i) = \frac{32\pi^2}{g^2}.$$ \hspace{1cm} (66)

For $z > 4$ it is found from Eq.(14) that

$$f_2(z) = \frac{2}{z} - \frac{1}{z^2} (1 - \frac{4}{z})^{1/2} \log \frac{1 - (1 - \frac{4}{z})^{1/2}}{1 + (1 - \frac{4}{z})^{1/2}}.$$ \hspace{1cm} (67)
Because of the ratios of $m_t^2/m_Z^2$ and $m_l^2/m_Z^2$ top quark dominates the Eq.(66) and the contributions of other fermions can be ignored. Eq.(66) has a solution at very large value of $z$. For very large $z$ Eq.(66) becomes

$$\frac{2(4\pi)^2}{\bar{g}^2} + \frac{6m_t^2}{m_Z^2} = \frac{3m_t^2}{m_Z^2} \log \frac{m_{\phi^0}^2}{m_t^2}.$$ (68)

The mass of the $\phi^0$ is determined to be

$$m_{\phi^0} = m_t e^{\frac{m_t^2}{m_Z^2} \frac{16\bar{g}^2}{3\pi^2} + 1} = m_t e^{28.4} = 3.78 \times 10^{14}\text{GeV},$$ (69)

and

$$\xi_Z = -1.18 \times 10^{-25}.$$

The neutral spin-0 boson is extremely heavy.

The propagator of $Z$ boson is found from Eq.(58)

$$\Delta_{\mu\nu} = \frac{1}{p^2 - m_Z^2} \left\{ -g_{\mu\nu} + \left(1 + \frac{1}{2\xi_Z} \right) \frac{p_{\mu}p_{\nu}}{p^2 - m_{\phi^0}^2}\right\},$$ (70)

It can be separated into two parts

$$\Delta_{\mu\nu} = \frac{1}{p^2 - m_Z^2} \left\{ -g_{\mu\nu} + \frac{p_{\mu}p_{\nu}}{m_Z^2} \right\} - \frac{1}{m_Z^2} \frac{p_{\mu}p_{\nu}}{p^2 - m_{\phi^0}^2}.\tag{71}$$

The first part is the propagator of the physical spin-1 $Z$ boson and the second part is the propagator of a new neutral spin-0 meson, $\phi^0$.

It is well known that the propagator of $Z$ boson of the original perturbation theory takes the same form as Eq.(70) in the renormalization gauge. However, from physical point of
view they are different. The SM is gauge invariant before spontaneous symmetry breaking. Therefore, in general, there is a gauge fixing term in the Lagrangian of the SM. In the study presented above the gauge parameter has been chosen to be zero, unitary gauge. The differences between Eq.(70) and the propagator of renormalization gauge in the original perturbation theory of the SM are

1. In eq.(70) the $\xi_z$ is determined by Eqs.(57,69) dynamically. Physics results depend on it. The gauge parameter of the renormalization gauge is determined by choosing gauge and because of unitarity physics results are independent of the gauge parameter.

2. In renormalization gauge ghosts are accompanied. However, there are no additional ghosts associated with Eqs.(70,71).

The propagator(70) shows that when $p^2 << m^2_{\phi^0}$ it takes

$$\Delta_{\mu\nu} = \frac{1}{p^2 - m^2_Z} \{ -g_{\mu\nu} + \frac{p_{\mu}p_{\nu}}{m^2_Z} \}. \tag{72}$$

Because of the huge value of $m^2_{\phi^0}$ Eq.(72) is always a good approximation. Only for extremely high momentum the complete propagator(70) is needed.

Eq.(71) shows that there is a pole at $p^2 = m^2_{\phi^0}$. On the other hand, the minus sign of Eq.(71) indicates that the annihilation and creation operators of free $\phi^0$ field obey Eq.(43). Therefore, the Fock space has indefinite metric and there is problem of negative probability when $\phi^0$ is on mass shell.
Substituting Eq.(60) into Eqs.(50,52), the couplings between $\partial_\mu \phi^0$ and $t,b,e,\nu_e$ fermions are found

\[
L = \pm \frac{\bar{g}}{m_Z} \left\{ (1 - \frac{8}{3} \alpha) \bar{t} \gamma_\mu t + \bar{t} \gamma_\mu \gamma_5 t \right\} \partial_\mu \phi^0 \\
\pm \frac{\bar{g}}{m_Z} \left\{ -(1 - \frac{4}{3} \alpha) \bar{b} \gamma_\mu b - \bar{b} \gamma_\mu \gamma_5 b \right\} \partial_\mu \phi^0 \\
\pm \frac{\bar{g}}{m_Z} \left\{ \bar{\nu}_e \gamma_\mu (1 + \gamma_5) \nu_e - (1 - 4\alpha) \bar{e} \gamma_\mu e - \bar{e} \gamma_\mu \gamma_5 e \right\} \partial_\mu \phi^0. \tag{73}
\]

The couplings with other generations of fermions take the same form. Using the equations of fermions, it can be found that the couplings between $\phi^0$ and fermions are

\[
\pm \frac{1}{m_Z} \bar{g} 2i \sum_i m_i \bar{\psi}_i \gamma_5 \psi_i \phi^0, \tag{74}
\]

where $i$ stands for the type of fermion. Eq.(74) shows that the coupling between $\phi^0$ and fermion is proportional to the mass of the fermion. There are other couplings obtained from Eq.(73). The couplings between $\phi^0$ and intermediate bosons can be obtained too.

5 Two heavy charged spin-0 fields

In the SM the fermion-W vertices are

\[
L = \frac{g}{4} \bar{\psi} \gamma_\mu (1 + \gamma_5) \tau^i \psi W^{i\mu}, \tag{75}
\]

where $\psi$ is the doublet of fermion and summation over all fermion generations is implicated.
Using the vertices (75), the expression of the vacuum polarization of fermions is obtained

\[
\Pi^W_{\mu\nu} = F_{W1}(p^2)(p_\mu p_\nu - p^2 g_{\mu\nu}) + 2F_{W2}(p^2)p_\mu p_\nu + \Delta m^2_W g_{\mu\nu},
\]  

(76)

where

\[
F_{W1}(p^2) = 1 + \frac{g^2}{32\pi^2} D\Gamma(2 - \frac{D}{2}) \int_0^1 dx x(1 - x) \{N_C \sum_{iq} (\frac{\mu^2}{L_i^q})^\frac{x}{2} + \sum_{il} (\frac{\mu^2}{L_i^l})^\frac{x}{2} \} - \frac{g^2}{16\pi^2} \{N_C \sum_{iq} f_{1q}^i + \sum_{il} f_{1l}^i \},
\]  

(77)

\[
F_{W2}(p^2) = -\frac{g^2}{32\pi^2} \{N_C \sum_{iq} f_{2q}^i + \sum_{il} f_{2l}^i \},
\]  

(78)

\[
\Delta m^2_W = \frac{g^2}{4 (4\pi)^2} D\Gamma(2 - \frac{D}{2}) \int_0^1 dx \{N_C \sum_{iq} L_i^q (\frac{\mu^2}{L_i^q})^\frac{x}{2} + \sum_{il} L_i^l (\frac{\mu^2}{L_i^l})^\frac{x}{2} \}.
\]  

(79)

where

\[
L_i^1 = m_0^2 x + m_1^2 (1 - x), \quad L_i^2 = m_s^2 x + m_c^2 (1 - x), \quad L_i^3 = m_d^2 x + m_u^2 (1 - x),
\]  

(80)

\[
L_i^1 = m_e^2 x, \quad L_i^2 = m_\mu^2 x, \quad L_i^3 = m_\tau^2 x,
\]

(81)

\[
f_{1q}^i = \int_0^1 dx x(1 - x)log[1 - x(1 - x)\frac{p^2}{L_i^q}],
\]  

(82)

\[
f_{1l}^i = \int_0^1 dx x(1 - x)log[1 - x(1 - x)\frac{p^2}{L_i^l}],
\]  

(83)

\[
f_{2q}^i = \frac{1}{p^2} \int_0^1 dx L_i^q log[1 - x(1 - x)\frac{p^2}{L_i^q}],
\]  

(84)
Function $F_{W_1}(p^2)$ is used to renormalize the W-field. In the SM W boson gains mass from spontaneous symmetry breaking and the additional mass term (79) has been treated by renormalization. Function $F_{W_2}$ leads to the existence of two charged spin-0 states, $\phi^\pm$, in the SM. $F_{W_2}$ is rewritten as

$$F_{W_2} = \xi_W + (p^2 - m_{\phi_W}^2)G_{W_2}(p^2),$$  

(85)  

$$\xi_W = F_{W_2}(p^2)|_{p^2=m_{\phi_W}^2},$$  

(86)  

where $G_{W_2}$ is the radiative correction of the term $(\partial_\mu W^\mu)^2$ and $m_{\phi_W}^2$ is the mass of the charged spin-0 states, $\phi^\pm$, whose existence will be shown below.

The free part of the Lagrangian of W-field is redefined as

$$L_{W_0} = -\frac{1}{2} (\partial_\mu W^+ - \partial_\nu W_\mu^+) (\partial_\mu W^\nu - \partial_\nu W_\mu^-) + 2\xi_W \partial_\mu W^+ \partial_\nu W^- + m_W^2 W^+ W^-.$$  

(87)  

From Eq.(87) the equation satisfied by the divergence of the W-field is derived

$$\partial^2 (\partial_\mu W^{\pm\mu}) - \frac{m_W^2}{2\xi_W} (\partial_\mu W^{\pm\mu}) = 0.$$  

(88)  

$\partial_\mu W^{\pm\mu}$ are spin-0 fields. Therefore, the W field of the SM has four independent components. The W-field is decomposed as

$$W^\pm_\mu = W^{\mu\pm}_\mu \pm \frac{1}{m_W} \partial_\mu \phi^\pm,$$  

(89)  

$$\partial_\mu W^{\mu\pm} = 0,$$  

(90)  

$$\phi^\pm = \mp \frac{m_W}{m_{\phi_W}^2} \partial_\mu W^{\pm\mu}.$$  

(91)
The equation of $\phi^\pm$ is derived from Eqs.(88,91)

$$\partial^2 \phi^\pm - \frac{m_W^2}{2\xi_W} \phi^\pm = 0. \quad (92)$$

It is the same as Eq.(64) the mass of $\phi^\pm$ is determined by the equation

$$2m_{\phi_w}^2 F_{W2}(p^2)|_{p^2=m_{\phi_w}^2} + m_W^2 = 0 \quad (93)$$

and

$$m_{\phi_w}^2 = -\frac{m_W^2}{2\xi_W}. \quad (94)$$

Numerical calculation shows that top quark is dominant in $F_{W2}$. Keeping the contribution of top quark only, Eq.(78) becomes

$$\frac{p^2}{m_W^2} F_{W2} = -\frac{3g^2}{32\pi^2} \frac{m_t^2}{m_W^2} \left\{ \frac{3}{4} + \frac{1}{2z} + \left[ \frac{1}{2} - \frac{1}{z} + \frac{1}{2z^2} \right] \log(z - 1) \right\}, \quad (95)$$

where $z = \frac{p^2}{m_t^2}$. Eq.(93) has a solution at very large $z$. At very large $z$ Eq.(95) becomes

$$\frac{p^2}{m_W^2} F_{W2} = -\frac{3g^2}{64\pi^2} \frac{m_t^2}{m_W^2} \log z. \quad (96)$$

The mass of $\phi^\pm$ is determined to be

$$m_{\phi_w} = m_t e^{\frac{16\pi^2 m_W^2}{3g^2 m_t^2}} = m_t e^{27} = 9.31 \times 10^{13} GeV, \quad (97)$$

and

$$\xi_W = -3.73 \times 10^{-25}. \quad (98)$$
The charged $\phi^\pm$ are very heavy too.

The propagator of W-field is derived from Eq.(87)

$$\Delta^W_{\mu\nu} = \frac{1}{p^2 - m_W^2} \{-g_{\mu\nu} + \frac{1}{2\xi_W} \frac{p_\mu p_\nu}{p^2 - m_\phi^2}\},$$  \hspace{1cm} (98)

and it can be separated into two parts

$$\Delta^W_{\mu\nu} = \frac{1}{p^2 - m_W^2} \{-g_{\mu\nu} + \frac{p_\mu p_\nu}{m_W^2}\} - \frac{1}{m_W^2} \frac{p_\mu p_\nu}{p^2 - m_\phi^2}. \hspace{1cm} (99)$$

The first part of Eq.(99) is the propagator of physical spin-1 W-field and the second part is related to the propagator of the $\phi^\pm$ field. The same as mentioned in section(5) the physics of the propagator(98) is different from the one derived by using renormalization gauge in the original perturbation theory of the SM. When $p^2 << m_{\phi^\pm}^2$ we have

$$\Delta^W_{\mu\nu} = \frac{1}{p^2 - m_W^2} (-g_{\mu\nu} + \frac{p_\mu p_\nu}{m_W^2}). \hspace{1cm} (100)$$

In most cases Eq.(100) is a very good approximation.

Eq.(99) shows that at very high energy there is a pole at $\sqrt{p^2} = m_\phi$. On the other hand, the minus sign in Eq.(99) indicates that there are problems of indefinite metric and negative probability when $\phi^\pm$ are on mass shell.

The Lagrangian of interactions between fermions and $\partial_\mu \phi^\pm$ field is found from Eqs.(75,89)

$$\mathcal{L}_{q\phi} = \pm \frac{1}{m_W^2} \frac{g}{4} \sum_j \bar{\psi}_j \gamma_\mu (1 + \gamma_5) \tau^i \psi_j \partial^\mu \phi^i, \hspace{1cm} (101)$$
where \( j \) is the type of the fermion and

\[
\phi^1 = \frac{1}{\sqrt{2}}(\phi^+ + \phi^-) \quad \phi^2 = \frac{1}{\sqrt{2i}}(\phi^+ - \phi^-).
\] (102)

Using the dynamical equations of fermions of the SM, for t and b quark generation we obtain

\[
L'_{q\phi} = \pm \frac{i}{m_W} \frac{g}{4}(m_t + m_b) \{ \bar{\psi}_t \gamma_5 \psi_b \phi^+ + \bar{\psi}_b \gamma_5 \psi_t \phi^- \}.
\] (103)

The coupling is proportional to the fermion mass. The couplings between other generations of fermion and \( \phi^\pm \) field are the same as Eq.(103).

6 Reconstruct the perturbation theory of the SM

We have learned from sections(4,5) that when the vacuum polarization of W and Z fields by fermions are treated nonperturbatively three heavy spin-0 states, \( \phi^0 \) and \( \phi^\pm \), are revealed. Therefore, the W and Z of the SM(1) are the fields of four independent components. They are composed of spin-1 and spin-0 states. In the original perturbation theory of the SM there are no such spin-0 states. Therefore, the perturbation theory of the SM must be reconstructed. The changes are made in the free Lagrangian of W and Z fields and related counter terms. All other parts of the Lagrangian are kept the same as in the original perturbation theory of the SM. The new perturbation theory of the SW is constructed as
1. Based on the study of sections(4,5) the new free Lagrangian of the W and Z fields is constructed as

\[
\mathcal{L}_0 = -\frac{1}{4}(\partial_\mu Z\nu - \partial_\nu Z\mu)^2 + \xi Z(\partial_\mu Z^\mu)^2 + \frac{1}{2}m_Z^2 Z\mu Z^\mu \\
-\frac{1}{2}(\partial_\mu W^+\nu - \partial_\nu W^+\mu)(\partial_\mu W^-\nu - \partial_\nu W^-\mu) + 2\xi W\partial_\mu W^+\mu\partial_\nu W^-\nu + m_W^2 W^+\nu W^-\mu. 
\]

(104)

2. Counter terms must be added to the interaction part of the Lagrangian of the SM

\[
\mathcal{L}_c = -\xi Z(\partial_\mu Z^\mu)^2 - 2\xi W\partial_\mu W^+\mu\partial_\nu W^-\nu.
\]

(105)

3. The propagators of Z and W fields are expressed as Eqs.(70,98). There are no ghosts which associate with Eqs.(70,98). When momentum is much less than the mass of the spin-0 state the propagators(70,98) go back to Eqs.(72,100). Because of the values of \(m_{\phi_0}^2\) and \(m_{\phi_W}^2\) are so large. Eqs.(72,100) are good approximations. However, for loop diagrams in which propagators of intermediate bosons are involved the propagators(70,98) must be taken.

4. The existence of three heavy spin-0 states are predicted. The masses of \(\phi^0\) and \(\phi^\pm\) are determined by Eqs.(69,97). The interactions between \(\phi^0\) and \(\phi^\pm\) and fermions are proportional to the masses of fermions(74,103). The interactions of \(\phi^0\) and \(\phi^\pm\) with Z' and W' can be obtained too.
5. The free Lagrangian(104) is in the unitary gauge. Renormalization gauge can be used too. However, the expression of the propagators(70,98) show that the choice of renormalization gauge is not necessary.

7 Effects of vacuum polarization by intermediate bosons

Besides the vacuum polarization by fermions the intermediate bosons contribute to the vacuum polarization too. Before we proceed to study the effects of the vacuum polarization by intermediate bosons it is necessary to restate the theoretical approach exploited in this paper. In the SM there are two kinds of vacuum polarization of W and Z: vacuum polarization by fermions and by intermediate bosons themselves. As shown in sections(4,5) the free Lagrangian of the intermediate bosons needs to be reconstructed. At the lowest order the propagators of fermions are not affected and the vacuum polarization of W and Z by fermions are calculated in sections(4,5). However, in the new perturbation theory the vacuum polarization by the intermediate bosons can be calculated only after the propagators of W and Z are defined.

After the propagators of Z and W are defined(70,98) we can proceed to study the effects of the vacuum polarization by intermediate bosons. The interaction Lagrangian of intermediate
bosons is obtained from the SM

\[
\mathcal{L}_i = i\bar{g}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)W^{-\mu}W^{+\nu} + \bar{g}Z^\mu\{(\partial_\mu W^\nu_+ - \partial_\nu W^\mu_+)W^-_\nu - (\partial_\mu W^-_\nu - \partial_\nu W^-_\mu)W^+_\nu\} - \bar{g}^2\{Z_\mu Z^\mu W^{-\nu}W^+ - Z_\mu Z_\nu W^{+\mu}W^{-\nu}\}. \tag{106}
\]

We calculate the contribution of W bosons to the vacuum polarization of Z boson. The same as Eq.(54) in the amplitude of the vacuum polarization of Z boson there are three parts: kinetic term, mass term, and the term proportional to \(p_\mu p_\nu, F'_Z p_\mu p_\nu\). We are interested in the last term. The calculation shows that only the second term of the Lagrangian(106) contributes to \(F'_Z\). Using the propagator of W boson(98) and Eq.(106), we obtain

\[
F'_Z = \frac{2\bar{g}^2}{(4\pi)^2}\left\{\frac{1}{4}\Gamma(2 - \frac{D}{2})(\frac{\mu^2}{m_W^2})^\frac{1}{2} - \frac{1}{12} + \frac{3}{2}\int_0^1 dx\left[\frac{1}{z}x(1 - x) - x(5 - 7x)\right]\log[1 - x(1 - x)z]\right\} + \frac{2\bar{g}^2b}{(4\pi)^2}\left\{-\frac{13}{40} + \frac{3}{2}\int_0^1 dx x^3(1 - x)[x(m_{\phi W}^2 - m_W^2) + m_W^2]\left[\left(x(m_{\phi W}^2 - m_W^2) + m_W^2\right) - x(1 - x)p^2\right]^{-1}\right\} - \frac{3}{bm_W^2}\int_0^1 dx x^2(2x - \frac{3}{2})[m_W^2 - x(1 - x)p^2]\log\left\{\left[m_W^2 - x(1 - x)p^2\right]\right\} \right\} \tag{107}

\[
[x(m_{\phi W}^2 - m_W^2) + m_W^2 - x(1 - x)p^2]^{-1} \right\} - \frac{9}{2p^2}\int_0^1 dx\int_0^x dy[y(m_{\phi W}^2 - m_W^2) + m_W^2]\log\left\{\left[y(m_{\phi W}^2 - m_W^2) + m_W^2 - x(1 - x)p^2\right]\right\} \right\} \tag{107}

\[
[y(m_{\phi W}^2 - m_W^2) + m_W^2]^{-1} \right\},
\]

where \(z = \frac{\mu^2}{m_W^2}\) and \(b = 1 + \frac{1}{2\xi_W}\).
There is divergence in Eq.(107), therefore, renormalization of the operator $(\partial_{\mu} Z^\nu)^2$ is required. As Eq.(56) $F'_{Z2}$ is written as

$$F'_{Z2} = F'_{Z2}(m_{\phi^0}^2) + (p^2 - m_{\phi^0}^2)G'_{Z2}(p^2).$$ (108)

$F'_{Z2}$ is divergent and $G'_{Z2}$ is finite and another term of radiative correction of $(\partial_{\mu} Z^\nu)^2$. We define

$$\xi_Z + F'_{Z2}(m_{\phi^0}^2) = \xi_Z Z_Z,$$ (109)

$$Z_Z = 1 + \frac{1}{\xi_Z} F'_{Z2}(m_{\phi^0}^2).$$ (110)

$Z_Z$ is the renormalization constant of the operator $(\partial_{\mu} Z^\nu)^2$. The renormalization constant $Z_Z$ defined in Eq.(110) guarantees the Z boson the same propagator(70).

In the same way, the contribution of Z and W bosons to the vacuum polarization of W boson can be calculated and the renormalization constant $Z_W$ can be defined. After renormalization of $\partial_{\mu} W^{+\nu} \partial_{\nu} W^{-\mu}$ we still have the same propagator of W boson(98).

**8 Conclusion**

To summarize the results obtained in this paper,

1. After taking vacuum polarization of one loop into account, a spin-0 state is revealed
from a axial-vector field theory. The perturbation theory of axial-vector field theory is reconstructed.

2. The W and Z fields of the SM are different from photon and gluon fields. Besides vector components they have axial-vector components. After treating the vacuum polarization of W and Z by fermions nonperturbatively, the W and Z of the SM are composed of spin-1 and spin-0 fields.

3. One neutral and two charged spin-0 states are revealed from the SM. They are very heavy, \( m_{\phi^0} = 3.78 \times 10^{14} \text{GeV} \), \( m_{\phi^W} = 9.31 \times 10^{13} \text{GeV} \). The energy scale of Grand Unification Theories is \( m_{\text{GUT}} \sim 10^{14} - 10^{16} \text{GeV} \) and the scale of quantum gravity is \( m_{\text{PL}} \sim 10^{19} \text{GeV} \). The strengths of the couplings between these spin-0 fields and fermions are proportional to the masses of the fermions.

4. Due to the existences of three spin-0 states in the SM new perturbation theory of the SM is constructed. The propagators of W and Z bosons are derived. They have the same expressions as the ones in renormalization gauge in the original perturbation theory. However, there are no additional ghosts. In the range of practical energies Eqs. (72,100) are good approximations.

5. The vacuum polarization by W and Z fields contribute to renormalizations and radia-
tive corrections.

6. The effects of the spin-0 states can be found in loop diagrams in which propagators of intermediate bosons are involved. Studies of measurable effects of spin-0 states are beyond the scope of this paper.

7. The interactions between $\phi^0, \phi^\pm$, fermions, and intermediate bosons are shown in Eqs. (73,101,106).

8. In the new propagators of W and Z fields there are poles of $\phi^{0,\pm}$ at very high energies $\sim 10^{14} GeV$. On the other hand, the minus signs of Eqs. (71,99) show that if these spin-0 states are on mass shell there are problems of indefinite metric and negative probability in the SM. These problems arise at energy scale of $10^{14} GeV$.

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