Black hole remnants due to GUP or quantum gravity?

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Based on the micro-black hole gedanken experiment as well as on general considerations of quantum mechanics and gravity the generalized uncertainty principle (GUP) is analyzed by using the running Newton constant. The result is used to decide between the GUP and quantum gravitational effects as a possible mechanism leading to the black hole remnants of about Planck mass.

PACS numbers: 03.65.-w, 04.60.-m, 04.70.-s, 04.70.Dy

The cogent argument for the black hole to evaporate entirely is that there are no evident symmetry or quantum number preventing it. Nevertheless, the heuristic derivation of the Hawking temperature with the use of GUP prevents a black hole from complete evaporation, just like the prevention of hydrogen atom from collapse by the uncertainty principle \[1\]. The generalized uncertainty relation takes into account the gravitational interaction of the photon and the particle being observed. This consideration relies on classical gravitational theory \[2, 3\]. The quantum corrected Schwarzschild space-time obtained with the use of running Newton constant also indicates that the black hole evaporation stops when its mass approaches the critical value of the order of Planck mass \[6\]. On the other hand the quantum corrected gravity modifies this GUP as well. It is fair to ask whether the halt of black hole radiation is provided by GUP or it is due to quantum gravitational effects. Let us give a critical view of this problem.

Let us briefly discuss the modification of Heisenberg uncertainty principle due to gravitational interaction. The main conceptual point concerning GUP is that there is an additional uncertainty in quantum measurement due to gravitational interaction. We focus on consideration of this problem presented in \[2, 3\], (\[h = c = 1\] is assumed in what follows). The approach proposed in this paper relying on classical gravity is to calculate the displacement of electron caused by the gravitational interaction with the photon and add it to the position uncertainty. The photon due to gravitational interaction imparts to electron the acceleration given by \[\alpha G_0 \Delta E/r^2\] (\[G_0\] is experimentally observed value of Newton’s constant for macroscopic values of distances). Assuming \[r_0\] is the size of the interaction region the variation of the velocity of the electron is given by \[\Delta v \sim G_0 \Delta E/r_0\] and correspondingly \[\Delta x_G \sim G_0 \Delta E\]. Therefore the total uncertainty in the position is given by

\[
\Delta x \geq \frac{1}{2\Delta E} + \alpha G_0 \Delta E, \tag{1}
\]

where \[\alpha\] is the factor of order unity in respect with the stringy induced GUP \[4\]. The Eq. (1) exhibits the minimal observable distance. However, the minimal observable distance is determined rather due to collapse of \[\Delta E\] than merely by the Eq. (1), because it puts simply the bound on the measurement procedure. In this way one gets that for \[\alpha > 2\] the \[\Delta x_{\text{min}} = \sqrt{2\alpha G_0}\] while for \[\alpha < 2\] the minimal observable distance is given by \[\Delta x_{\text{min}} = \sqrt{8G_0/(4 - \alpha)}\]. In the framework of this discussion one can obtain the GUP in higher dimensional case as well as on the brane \[5\].

The heuristic derivation of black hole evaporation proceeds as follows \[4\]. (In paper \[1\] \[\alpha = 1\] is assumed). The black hole is modelled as an object with linear size equal to the two times the gravitational radius \[2r_g\] and the minimum uncertainty condition is assumed for the radiation. Then the lower value of \[\Delta E\]

\[
\Delta E = \frac{r_g - (r_g^2 - G_0)^{1/2}}{2G_0}, \tag{2}
\]

that comes from uncertainty relation is identified to the characteristic temperature of the black hole emission with the constant of proportionality \[1/2\pi\]. As one sees from Eq. (2) the GUP amended Hawking temperature becomes complex if the mass of the black hole is less than \[1/2G_0^{1/2}\], leading thereby to the nonzero minimal black hole mass \[\tilde{\Omega}\].

As it is evident the GUP assumes two \[\Delta E\] values for a given \[\Delta x\]. But the choice of the lower value is well motivated physically because for relatively small values of \[\Delta E\] the gravitational uncertainty becomes negligible in comparison with the standard term and therefore in the limit \[\Delta x \gg \sqrt{G_0}\] one has to recover the standard uncertainty relation. On the other hand such a choice is motivated by the correct asymptotic dependance of Hawking temperature on the black hole mass in the framework of this heuristic approach.

However for the black hole with a radius not too far above the Planck length the quantum fluctuations of the metric play an important role. The effective Newton constant obtained by means of the Wilson-type effective action has the form \[6\]

\[
G(r) = \frac{G_0 r^3}{r^3 + 2.504G_0 (r + 4.5G_0 M)}, \tag{3}
\]

where \[M\] is the mass of the source. The effective Newton
constant Eq. 3 depends on the mass of source and correspondingly the mass of the test particle is implied to be negligibly small in comparison with the source mass. Since the gravitational uncertainty becomes appreciable when the energy of photon approaches the Planck scale, which in turn exceeds very much the mass of the standard model particles, one can safely use the Eq. 3. An interesting observation made in Ref. 6 is that for standard model particles, one can safely use the Eq.(3). An interesting observation made in Ref. 6 is that for standard model particles, one can safely use the Eq.(3).

...the measurement). One has to define the gravitational radius and standard position uncertainty as

\[
\Delta x \geq \begin{cases} 
1/2 \Delta E & \text{if } \Delta E \leq 3.503 G_0^{-1/2} \\
 r_g(\Delta E) & \text{if } \Delta E > 3.503 G_0^{-1/2} 
\end{cases}, \tag{5}
\]

where \( r_g(\Delta E) \) is given by Eq. 4. From Eq. 4 one gets the following minimal length \( \Delta x_{min} = 0.143 G_0^{1/2} \). The linear combination

\[
\Delta x \geq \frac{1}{2 \Delta E} + r_g(\Delta E), \tag{6}
\]

does not make any sense for \( \Delta E < \Delta E_{cr} \) since for these values of energy there is no horizon at all. (Let us comment that the idea of paper 2 to determine the total uncertainty for \( \Delta E > \Delta E_{cr} \) by Eq. 3 is not quite clear because the collapse of \( \Delta E \) puts simply the limitation on the measurement). One has to define the gravitational disturbance of the electron position directly. Taking the quantum corrected potential around the photon to be \(-\Delta E G(r)/r \) then the acceleration imparted to the electron is

\[
a = \frac{\Delta E 2 G r}{r^3 + 2.504 G_0 (r + 4.5 G_0 \Delta E)} - \frac{\Delta E G r^2}{[r^3 + 2.504 G_0 (r + 4.5 G_0 \Delta E)]^2}, \tag{7}
\]

The characteristic time and length scale for the interaction when one uses the energy \( \Delta E \) for the measurement is given by \( \Delta E^{-1} \). Thus for the GUP when \( \Delta E \leq \Delta E_{cr} \) one gets

\[
\Delta x = \frac{1}{2 \Delta E} + \alpha \frac{[22.503 G_0 \Delta E^5 + 2.504 G_0 (\Delta E^2)^5 - G_0 \Delta E]}{[1 + 2.504 G_0 (\Delta E^2 + 4.5 G_0 \Delta E^4)]^2}. \tag{8}
\]

Assuming \( \alpha = 1 \) it is easy to check that the minimal distance that comes from Eq. 5 is \( \Delta x_{min} = \Delta x(\Delta E_{cr}) = 0.147 G_0^{1/2} < r_g(\Delta E_{cr}) = 4.484 G_0^{1/2} \). But the object with the size \( \Delta x_{min} \) can not be black hole at all for the quantum corrected Schwarzschild space-time does not admit the black hole with the size less than \( r_g(\Delta E_{cr}) \). In general the parameter \( \alpha \) is of order unity, but this numerical factor can not change the result because it should be about 1000 the \( \Delta x_{min} \) to be comparable to the \( r_g(\Delta E_{cr}) \). So, one concludes that black hole evaporation is halted due to quantum gravitational effects. To be strict the disappearance of the horizon beneath the mass scale \( M_{cr} \) results in the subtle question what actually is the object left behind the evaporation, whether it is unambiguously black hole or the classical remnant is also allowed in general. But in the real situation this question does not arise because when the mass approaches \( M_{cr} \) the emission temperature becomes zero and due to absorption of the background radiation this limit is simply unattainable.

Another very interesting issue that comes from effective average action and its associated exact renormalization group equation is the fractal structure of spacetime on sub-Planckian distances with effective dimensionality 2. It is of interest to know if presence of minimum uncertainty in the position considered above (\( \Delta x_{min} = 0.147 G_0^{1/2} \)) allows one to observe the "ripples" of spacetime at sub-Planckian distances.

Acknowledgements

The author is greatly indebted to Z. Berezhiani, M. Makhviladze and R. Percacci for useful conversations. The work was supported by the grant FEL. REG. 980767.
[1] R. Adler, P. Chen and D. Santiago, Gen. Rel. Grav. 33 (2001) 2101, gr-qc/0106080
[2] R. Adler and D. Santiago, Mod. Phys. Lett. A14 (1999) 1371, gr-qc/9904026
[3] F. Scardigli, Phys. Lett. B452 (1999) 39, hep-th/9904025
[4] G. Veneziano, Europhys. Lett. 2 (1986) 199; D. Amati, M. Ciafaloni and G. Veneziano, Phys. Lett. B197 (1987) 81; ibid B216 (1989) 41; D. Gross and P. Mende, Phys. Lett. B197 (1987) 129; Nucl. Phys. B303 (1988) 407; K. Konishi, G. Paffuti and P. Provero, Phys. Lett. B234 (1990) 276.
[5] M. Makhviladze, M. Maziashvili and D. Nozadze, gr-qc/0512044
[6] A. Bonanno and M. Reuter, Phys. Rev. D62 (2000) 043008, hep-th/0002196
[7] L. Landau and E. Lifshitz, Quantum Mechanics,(Moscow, Nauka, 1989).
[8] O. Lauscher and M. Reuter, Phys. Rev. D65 (2002) 025013, hep-th/0108040, O. Lauscher and M. Reuter, Phys. Rev. D66 (2002) 0250026, hep-th/0205062, O. Lauscher and M. Reuter, JHEP 0510 (2005) 050, hep-th/0508202.