Nuclear Shadowing and the Proton Structure Function at Small x

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Abstract

A new scaling variable is introduced in terms of which nuclear shadowing in deep-inelastic scattering is universal, i.e. independent of A, Q² and x. This variable can be interpreted as a measure of the number of gluons probed by the hadronic fluctuations of a virtual photon during their lifetime. According to recent data from the H1 and ZEUS experiments, the gluon density in a proton rises steeply as x tends to zero. So nuclei which in the infinite momentum frame have a large surface density of gluons model at larger x values what we expect for the proton at much smaller x. Using experimental information on nuclear shadowing we predict the unitarity corrections to the proton structure function at small x and extract the bare Pomeron intercept.

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Nuclear shadowing at small Bjorken $x \ll 1$ is experimentally well established in deep-inelastic lepton scattering (DIS) \[1, 2, 3\] and in Drell-Yan lepton pair production \[4\]. Theoretically shadowing in the nuclear structure functions has been predicted on the basis of the old fashioned parton model \[5, 6\] and more recently in QCD calculations \[7, 8\]. In both approaches shadowing is the result of parton recombination.

Shadowing, or unitarity corrections, is supposed to exist in the proton structure function as well. In this case, however, there is no direct way to observe experimentally shadowing in the proton because there is nothing to compare to. The unitarity corrections at small $x$ are expected to be significant because the virtual photoabsorption cross section rises steeply \[11, 10\], steeper than hadronic cross sections at high energy. This fast growth is usually interpreted as being due to the Pomeron contribution to the structure function corrected for shadowing. No reliable way to calculate unitarity corrections has been proposed.

Since at small $x$ the parton clouds of different nucleons overlap \[5\], one expects larger parton densities, and consequently stronger shadowing effects, in nuclei than in protons. The question arises: can one use the data on nuclear shadowing at small $x$ in order to improve the reliability of the unitarity corrections to the proton structure function? The aim of this letter is to answer this question.

There is a widespread belief that the gluon and sea quark distribution functions experience nearly the same nuclear shadowing, because the sea, which dominates at small $x$, is supposed to be generated through gluons. This would be true, if our electromagnetic probes of the gluon distribution were really hard. We argue below that this is not obvious even at high $Q^2$.

A photon, through its quark-antiquark fluctuations, interacts with gluons. This process can also be interpreted as an electromagnetic interaction with gluonic $q\bar{q}$ fluctuations. At high $Q^2$ perturbative QCD can be applied, and it is convenient to decompose the photon wave function in a quark-gluon basis. Assuming that the transverse separation $\rho$ of the $q\bar{q}$ fluctuations in a highly virtual photon is small (see, however, the discussion below) and that the interaction cross section vanishes as $\sigma(\rho) \propto \rho^2$ \[12, 13\], one can present the total photoabsorption cross section in the following form \[14, 15\].
\[ \sigma_{\gamma^* N}^{\gamma^* N}(x, Q^2) \approx \frac{2\pi^2}{3} \alpha_s(\rho) \langle \rho^2 \rangle g_N(x, Q^2) \tag{1} \]

Here \( g_N(x, Q^2) = xG_N(x, Q^2) \) is the gluon distribution function. The averaging in (1) is weighted with the square of the wave function of the \( q\bar{q} \) fluctuation of the photon \[ \langle \rho^2 \rangle = \int \frac{d\alpha}{\alpha} \int d^2\rho |\Psi_{\gamma^*}(\rho, \alpha)|^2 \rho^2, \]
where \( \alpha \) is the fraction of the photon light-cone momentum carried by the quark; \( |\Psi_{\gamma^*}(\rho, \alpha)|^2 \propto [1 - 2\alpha(1 - \alpha)] \epsilon^2 K_1^2(\epsilon \rho) \) and \( \epsilon^2 = \alpha(1 - \alpha)Q^2 + m_q^2 \) is an important parameter which determines the transverse separation of the \( q\bar{q} \) fluctuation, \( \rho^2 \propto 1/\epsilon^2 \). It is of order \( 1/Q^2 \) except at the edges of the kinematical region, where \( \alpha \) or \( 1 - \alpha \sim m_q^2/Q^2 \), and the \( q\bar{q} \) fluctuation acquires a large transverse size, \( \rho^2 \sim 1/m_q^2 \).

Of course such a component of the photon cannot be used as a probe of the gluon distribution. For instance, if one needs to estimate how many nucleons are in a nucleus, and one uses, say, a pion-nucleus inelastic interaction as a probe, one obviously will get a wrong result, i.e., that the number of nucleons is \( \sim A^{2/3} \). The source of the trouble is the softness of the probe, \( \sigma(\pi N) \) is too large. The same problem may arise in DIS. It is not clear what part of the observed nuclear shadowing comes from the reduction of the gluon density caused by recombination, and what is due to the softness of the \( q\bar{q} \) probe. The latter corresponds to the multiple scattering shadowing mechanism.

Although multiple scattering corrections violate the factorization property of eq. (1), it is natural to assume that both shadowing mechanisms solely depend upon the number of gluons involved in the interaction, rather than upon \( x, Q^2 \) or the nuclear mass number \( A \). In this paper we will test this idea on available data and use the results to predict shadowing for protons. We do not try to disentangle the two sources of nuclear shadowing, mentioned above, gluon recombination and multiple scattering.

We begin by testing the universality of the relation between the number of gluon participating in the interaction and the ratio of the nucleus and nucleon structure functions. The general expression for the nuclear photoabsorption cross section \[ \sigma_{\gamma^* A}^{\gamma^* A}(x, Q^2) = 2 \int d^2b \left( 1 - \frac{\sigma(\rho, x)T(b)}{2A} \right)^A \tag{2} \]
The averaging here is the same as defined in eq. (1). \( T(b) \approx \int_{-\infty}^{\infty} dz \rho_A(b, z) \) is the nuclear thickness function, where \( \rho_A(b, z) \) is the nuclear density, which depends upon the impact parameter \( b \) and the longitudinal coordinate \( z \). The cross section for the interaction of a \( q\bar{q} \) fluctuation with a nucleon in eq. (2) can according to standard Regge phenomenology be represented in the form, \( \sigma(\rho, x) = \sigma(\rho) x^{-\Delta_\rho(Q^2)} \).

Expanding eq. (2) one can represent nuclear shadowing effects in the form,

\[
R_{A/N}(x, Q^2) = \frac{\sigma_{tot}^A(x, Q^2)}{A \sigma_{tot}^N(x, Q^2)} = 1 - \frac{1}{4} \frac{\langle \sigma^2(\rho) \rangle}{\langle \sigma(\rho) \rangle} \langle T(b) \rangle \left( \frac{1}{x} \right)^{\Delta_\rho(Q^2)} + \ldots , \tag{3}
\]

where \( \langle T(b) \rangle = (A - 1)/A^2 \int d^2 b T^2(b) \).

Assuming \( \sigma(\rho) \approx C \rho^2 \) we obtain

\[
\frac{\langle \sigma^2(\rho) \rangle}{\langle \sigma(\rho) \rangle} = \frac{2.4C}{2m_q^2 \ln(Q^2/2m_q^2)} \tag{4}
\]

Eq. (3) looks very similar to the well known Glauber approximation [20]. It is, however, very different. In the Glauber approximation the first rescattering correction is proportional to \( \langle \sigma(\rho) \rangle \propto 1/Q^2 \). In our case it is proportional to eq. (4) and is of the order of \( 1/m_q^2 \). This comparison demonstrates that, in terms of multiple scattering theory, nuclear shadowing in DIS is dominated by inelastic shadowing [21], while the Glauber eikonal contribution [20] vanishes at high \( Q^2 \) [18].

An important ingredient of eq. (2) is the assumption that the lifetime of a photon fluctuation in the nuclear rest frame is long enough that it can be regarded as propagating through the whole nucleus with a frozen intrinsic separation \( \rho \). This is the same as saying that \( x \) is sufficiently small, \( x \ll 1/m_N R_A \), that all the parton clouds of nucleons with the same impact parameter overlap in the infinite momentum frame [4]. However most available data are in the transition region, where the lifetime, usually called the coherence time, is comparable to the nuclear radius. This can be taken into account by introducing a phase shift between \( q\bar{q} \) wave packets produced at different longitudinal coordinates, in the same way as was done in refs. [22, 18]. This is equivalent to the replacement of the mean nuclear thickness function in eq. (3) by an effective one.
\[ \langle \tilde{T}(b) \rangle = \frac{A - 1}{A^2} \int d^2b \left[ \int_{-\infty}^{\infty} dz \rho_A(b, z) e^{iqz} \right]^2 \approx \langle T(b) \rangle F_A^2(q). \quad (5) \]

We use a Gaussian form for the nuclear density in order to make a factorization of expression (5) possible. Generally we use the realistic nuclear density of ref. [23]. \( F_A(q) = \exp(-q^2R_A^2/6) \) is the nuclear longitudinal formfactor, where \( R_A^2 \) is the mean square nuclear radius. The decrease of the effective nuclear thickness function at large \( q \) can be interpreted as a result of the short path length of the hadronic fluctuation in the nucleus if we sit in the latter’s rest frame, or as an incomplete overlap of the gluon clouds of the nucleons which have the same impact parameter in the infinite momentum frame of the nucleus.

In order to calculate the longitudinal momentum transfer in DIS, \( q = (Q^2 + M^2)/2\nu \), one needs to know the effective mass of the produced \( q\bar{q} \) wave packet. However a \( q\bar{q} \) state with definite separation \( \rho \) does not have a definite mass. We evaluate \( q \approx 2xm_N \) assuming \( M^2 \sim Q^2 \). Thus \( x \) is a parameter which controls the value of \( \langle \tilde{T}(b) \rangle \).

According to eqs. (3) and (5) the number of gluons which interact with the \( q\bar{q} \) fluctuation during its lifetime is

\[ n(x, Q^2, A) = \frac{1}{4} \frac{\langle \sigma^2(\rho) \rangle}{\langle \sigma(\rho) \rangle} \langle T(b) \rangle F_A^2(q) \left( \frac{1}{x} \right)^{\Delta_P(Q^2)}. \quad (6) \]

We expect nuclear shadowing to scale as a function of this new variable \( n(x, Q^2, A) \). Now we are in position to test this prediction.

First of all we fixed \( \Delta_P(Q^2) \) by a fit to recent data from the ZEUS [10] and H1 [11] experiments on the proton structure functions using a parameterization \( x^{-\Delta_P} \) at fixed \( Q^2 \). The results of the fit are plotted as circles (H1) and squares (ZEUS) versus \( Q^2 \) in Fig. 1. We also included a hadronic point which we fixed conventionally at \( \Delta_P(0) = 0.1 \) which corresponds to results from various fits of the energy dependence of hadronic cross sections.

The \( Q^2 \)-dependence of the \( \Delta_P(Q^2) \) as shown in Fig. 1 was fitted by the ansatz \( \alpha_P(Q^2) = a + b \exp(-cQ^2) \). The solid curve corresponds to \( a = 0.36 \pm 0.016, b = -0.26 \pm 0.016 \) and \( c = 0.052 \pm 0.009 \).

The expected universal dependence of the nuclear shadowing on \( n(x, Q^2, A) \) is not affected by the overall normalization of this variable. Provided that \( Q^2 \gg m_q^2 \), one can test
this scaling unambiguously despite the uncertainty in the value of the factor $C$ and the quark mass $m_q$ displayed in eq. (4).

The values of the variable $n(x, Q^2)$ have been calculated from eq. (6) using the data from the NMC [1, 2] and E665 [3] experiments, as well as the results of our fit to $\Delta_{F}(Q^2)$. The data on the ratio of the nuclear and nucleonic photoabsorption cross sections $R_{A/N}(x, Q^2)$ is plotted against the new variable $n(x, Q^2, A)$ in Fig. 2. The data demonstrate an excellent scaling in $n(x, Q^2, A)$. The concrete choice of the parameters $C$ and $m_q$ (which are strongly correlated) in eq. (4) was adjusted to fit eq. (3), the solid curve, to the data. We fixed $2m_q^2 = 0.1 \text{ GeV}^2$ and $C/2m_q^2 = 22$.

We should comment on this procedure:

(i) Our considerations are valid for small $x$, so we limit the $x$-region to $x < 0.07$. At this $x$ the nuclear structure function shows a small enhancement relative to the proton one, what results in $R_{A/N}(x, Q^2) > 1$ for $n \approx 0$. For this reason we renormalized eq. (3) by 3%.

(ii) Data points [2, 3] for $Q^2 < 0.5 \text{ GeV}^2$, were excluded from the analysis because they are in the realm of the vector dominance model, rather than DIS. They should correspond to the same nuclear shadowing experienced by a $\rho$-meson as in the real photon limit. This is the reason for the saturation of nuclear shadowing at small $x$, claimed in [3, 2]. On the contrary, nuclear shadowing in DIS at small $x$, but high $Q^2$ is not supposed to saturate. $R_{A/N}(x, Q^2)$ is expected to decrease logarithmically down to small $x$ below the real photon limit due to gluon fusion [8, 18, 14].

(iii) The data in Fig. 2 show that $R_{A/N}(x, Q^2)$ depends to a good accuracy linearly upon $n(x, Q^2, A)$ for $n < 0.3$. On the one hand, this implies that higher order terms in the expansion in eq. (3) are small. On the other hand it confirms the validity of the small-$\rho^2$ approximation for $\sigma(\rho)$ used in eq. (1) and eq. (4). Data for heavier nuclei and/or smaller $x$ will probably demand higher order terms in eq. (3).

The shadowing described by eq. (3) depends entirely on the gluon thickness function of the target. It should, therefore, be possible to apply the same formalism to the shadowing in DIS off protons. The nuclear shadowing fixes the parameter $C/m_q^2$ of eq. (4), and thus it reduces ambiguities faced by our evaluation of the unitarity corrections to the proton
structure function. For numerical calculations we used the mean thickness function of proton $\langle T \rangle_{\beta 1}/4\pi B$. The value of the slope parameter $B \approx 6 \text{GeV}^{-2}$ was estimated using data on $pp$ elastic scattering, assuming the Pomeron factorization.

The predicted ratio of the measured proton structure function and the bare Pomeron contribution, $F_p^2(x, Q^2)/[F_p^2(x, Q^2)]_P$, is presented in Fig.3 as function of $x$ versus $Q^2$. For each value of $Q^2$ we have put constraints on $n \leq 0.3$ where nuclear data exist and we are sure that the linear approximation (3) is valid.

After subtraction of the unitarity corrections to the proton structure function we can determine the bare Pomeron intercept. We refitted the data [11, 10] and deduce the new values of $\Delta_0^P(Q^2)$. The new interpolating dashed curve corresponds to the parameters defined above: $a = 0.45 \pm 0.017$, $b = -0.35 \pm 0.017$ and $c = 0.0525 \pm 0.008$. The function $\Delta_0^P(Q^2)$ which we obtained is quite close at large $Q^2$ to the intercept of the BFLK Pomeron [9].

Another assumption, which we quietly made above, should be commented upon. As was mentioned above, there are two contributions to the shadowing in DIS, one comes from the suppression of the gluon density as a consequence of gluon fusion $gg\beta g$, which corresponds to the triple Pomeron graph in the framework of standard Regge phenomenology. Another contribution to the shadowing comes from the Glauber-like rescattering of the $q\bar{q}$ fluctuation off gluons. This process can also be represented as a parton fusion, but as a fusion of gluons into a $q\bar{q}$ pair, $gg\beta q\bar{q}$. In Regge-model language this process corresponds to the Pomeron-Pomeron-Reggeon graph. Both mechanisms show the same $x$-dependence $\propto 1/x^{2\Delta P}$, except for logarithmic corrections. The logarithmic corrections for the triple-Pomeron contribution differ for nuclei and the nucleon. Our estimate shows, however, that, in the $x$ and $A$ domain investigated here, this difference is small and it decreases with $x\beta 0$.

To summarize: we have found a new variable $n(x, Q^2, A)$ which all available data on nuclear shadowing in DIS scale with at small-$x$. This variable gives a measure of the number of gluons which a hadronic fluctuation of the virtual photon interacts with during its lifetime.

The data on nuclear shadowing allowed us to fix the important parameter $C/m_q^2$ which
determines the screening of the virtual photon, and this reduced the uncertainties in the estimations of the unitarity corrections to the proton structure function. A value of the bare Pomeron intercept was evaluated from the data on the proton structure function corrected for shadowing. This intercept is still an effective one, valid in the $x$ region reached by HERA.

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Figure capture

Fig. 1 The Pomeron intercept in DIS off a proton, $\alpha_P(Q^2) = 1 + \Delta_P(Q^2)$. The data points are results of a fit to the data on $x$-dependence of the proton structure function measured by the H1 [11] and ZEUS [10] experiments at HERA. The solid curve is the interpolation of the results of the fit to the effective intercept. The dashed curve corresponds to the bare Pomeron contribution after unitarity corrections are extracted from the data.

Fig. 2 Data on nuclear shadowing at small $x$ from the NMC [1, 2] and E665 [3] experiments versus the scaling variable $n(x, Q^2, A)$ defined in eq. (6). The straight line corresponds to $F - 2^A/F_D^2 = 1 - n(x, Q^2, A)$.

Fig. 3 The unitarity/shadowing correction to the bare Pomeron contribution to the proton structure function. Each curve corresponds to the straight line in Fig. 2, plotted as a function of $x$ versus $Q^2$.
