A Proposed New Average Method for Solving Multi-Objective Linear Programming Problem Using Various Kinds of Mean Techniques

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Abstract: In this paper, we initiated New Average Method with various kinds of mean techniques to solve multi-objective linear programming problem. In this method multi-objective functions are converted into single objective function by using different kinds of mean techniques. Also an algorithm of New Average Method is suggested for solving multi-objective linear programming problem. We illustrate numerical problem using Chandra Sen’s Method, Average Method and New Average Method. The numerical result in this paper indicates that New Average Method gives promising result than Chandra Sen’s Method and Average Method. Also we observed that, in New Average Method, Harmonic mean technique gives better result than other mean techniques like as Quadratic mean, Arithmetic mean, Identric mean, Logarithmic mean, Geometric mean techniques.

Keywords: Multi-objective Linear Programming Problem, Average Method, New Average Method, Harmonic Mean

1. Introduction

Linear Programming problems are concerned with the efficient use or allocation of limited resources to meet desired objectives. In different sectors like design, construction, maintenance, producing planning, financial and corporate planning and engineering, decision makers have to take decisions and their ultimate goal is to minimize effort or maximize profit. Linear programming problem is formed with a cost or profit function with some constraints conditions, where a single cost or profit function need to be optimized [1]. However, in many situations, decision makers want to optimize several different objective functions at the same time under same constraints conditions. This leads to Multi-Objective concept. It is seen that if the multiple objective functions are not similar to each other, then this problem becomes more critical. There have been many methods suggested for Multi-objective linear programming problem (MOLPP).

A study of multi objective linear programming problem is introduced by Chandra Sen. In which multi-objective function are converted into single objective function with limitation that, individually each objective function optimum value must be greater than zero [2]. Using mean and median solving multi objective programming problem is studied by Sulaiman and Sadiq [3]. Sulaiman and Mustafa also used Harmonic mean to solve MOLPP [4]. A new geometric average technique is studied to solve MOLFPP by Nahar and Alim [5]. They also proposed a Statistical Average Method using Arithmetic, Geometric and Harmonic mean [6].

In order to extend this work, in this paper we propose an algorithm to solve MOLPP with New Average Method with various types of mean techniques and compare the numerical result with Chandra Sen’s Method and Average Method. New Average Method gives better result than Chandra Sen’s Method and Average Method. Among all New Harmonic mean technique gives the best result.

2. Mathematical Formulation of MOLPP

Mathematical general form of MOLPP is given as:

Max $Z_1 = C_1^T X + \alpha_1$

Max $Z_2 = C_2^T X + \alpha_2$
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Max $Z_r = C_i^r X + \alpha_r$

Min $Z_{r+1} = C_i^{r+1} X + \alpha_{r+1}$

Min $Z_{r+2} = C_i^{r+2} X + \alpha_{r+2}$

Min $Z_s = C_i^s X + \alpha_s$

Subject to:

$AX = b$

$X \geq 0$

Where, $X$ is $n$-dimensional and $b$ is $m$-dimensional vectors. $A$ is $m \times n$ matrix. $\alpha_1, \alpha_2, ..., \alpha_s$ are scalars. Here, $Z_i$ is need to be maximized for $i = 1, 2, ..., r$ and need to be minimized for $i = r + 1, ..., s$.

3. Chandra Sen’s Method

In this method, firstly all objective functions need to be maximized or minimized individually by Simplex method. By solving each objective function of equation (1) following equations are obtained:

Max $Z_1 = \varphi_1$

Max $Z_2 = \varphi_2$

... ................................. ...

Max $Z_r = \varphi_r$

Min $Z_{r+1} = \varphi_{r+1}$

Min $Z_{r+2} = \varphi_{r+2}$

... ................................. ...

Min $Z_s = \varphi_s$

Where, $\varphi_1, \varphi_2, ..., \varphi_s$ are the optimal values of objective functions.

These values are used to form a single objective function by adding (for maximum) and subtracting (for minimum) of each result of dividing each $Z_i$ by $\varphi_i$. Mathematically,

Max $Z = \sum_{i=1}^{r} \frac{Z_i}{\varphi_i} - \sum_{i=r+1}^{s} \frac{Z_i}{\varphi_i}$

Where, $|\varphi_i| \neq 0$.

Subject to the constraints are remain same as equation (1).

Then this single objective linear programming problem is optimized.

4. Average Method of MOLPP

In this method, initially all optimized values of each objective functions are calculated under given constraints. Then a single objective function is constructed by adding all maximization objective functions and subtracting all minimization objective functions and divided them by different kinds of means of maximization objective functions’ absolute maximum values and different kinds of means of minimization objective functions’ absolute minimum values respectively. Then this single objective function is optimized for the same constraints. For this method, different kinds of mean techniques are discussed.

4.1. Different Kinds of Mean Techniques

Contraharmonic Mean Technique:

Max $Z = \sum_{i=1}^{r} \frac{Z_i}{CH.M.1} - \sum_{i=r+1}^{s} \frac{Z_i}{CH.M.2}$

Where, $CH.M.1 = CH.M.(|\varphi_1|, |\varphi_2|, ..., |\varphi_r|)$ & $CH.M.2 = CH.M.(|\varphi_{r+1}|, |\varphi_{r+2}|, ..., |\varphi_s|)$ and CH. M. is Contraharmonic mean.

Quadratic Mean Technique:

Max $Z = \sum_{i=1}^{r} \frac{Z_i}{Q.M.1} - \sum_{i=r+1}^{s} \frac{Z_i}{Q.M.2}$

Where, $Q.M.1 = Q.M.(|\varphi_1|, |\varphi_2|, ..., |\varphi_r|)$ & $Q.M.2 = Q.M.(|\varphi_{r+1}|, |\varphi_{r+2}|, ..., |\varphi_s|)$ and Q. M. is Quadratic mean.

Nueman-Sándor Mean Technique:

Max $Z = \sum_{i=1}^{r} \frac{Z_i}{NS.M.1} - \sum_{i=r+1}^{s} \frac{Z_i}{NS.M.2}$

Where, $NS.M.1 = NS.M.(|\varphi_1|, |\varphi_2|, ..., |\varphi_r|)$ & $NS.M.2 = NS.M.(|\varphi_{r+1}|, |\varphi_{r+2}|, ..., |\varphi_s|)$ and NS. M. is Nuemans-Sándor mean.

Arithmetic Mean Technique:

Max $Z = \sum_{i=1}^{r} \frac{Z_i}{A.M.1} - \sum_{i=r+1}^{s} \frac{Z_i}{A.M.2}$

Where, $A.M.1 = A.M.(|\varphi_1|, |\varphi_2|, ..., |\varphi_r|)$ & $A.M.2 = A.M.(|\varphi_{r+1}|, |\varphi_{r+2}|, ..., |\varphi_s|)$ and A. M. is Arithmetic mean.

Identric Mean Technique:

Max $Z = \sum_{i=1}^{r} \frac{Z_i}{I.M.1} - \sum_{i=r+1}^{s} \frac{Z_i}{I.M.2}$
Where, $1.M. = I. M. (|\varphi_1|, |\varphi_2|, \ldots, |\varphi_r|) \& I. M. = I. M. (|\varphi_{r+1}|, |\varphi_{r+2}|, \ldots, |\varphi_s|)$ and I. M. is Identric mean.

4.2. Algorithm for Average Method of MOLPP

Step 1: Use Simplex method to find the optimal value of each of the objective function.

Step 2: Check the feasibility of step 1, if it is feasible then continue with step 3 otherwise use dual simplex method to remove infeasibility.

Step 3: Assign a name to each of the optimal value of corresponding objective function. Say $Max \ Z_i = \varphi_i, i = 1,2, \ldots, r$ and $Min \ Z_i = \varphi_i, i = r + 1, r + 2, \ldots, s$.

Step 4: Calculate the values of

$CH. M. = CH. M. (|\varphi_1|, |\varphi_2|, \ldots, |\varphi_r|) \& CH. M. = CH. M. (|\varphi_{r+1}|, |\varphi_{r+2}|, \ldots, |\varphi_s|)$ and CH. M. is Heronian mean.

Step 5: Optimize the combined objective function using same constraints as follow:

$Max \ Z = \sum_{i=1}^{r} \frac{Z_i}{CH. M._1} - \sum_{i=r+1}^{s} \frac{Z_i}{CH. M._2}$

$Max \ Z = \sum_{i=1}^{r} \frac{Z_i}{Q. M._1} - \sum_{i=r+1}^{s} \frac{Z_i}{Q. M._2}$

$Max \ Z = \sum_{i=1}^{r} \frac{Z_i}{NS. M._1} - \sum_{i=r+1}^{s} \frac{Z_i}{NS. M._2}$

$Max \ Z = \sum_{i=1}^{r} \frac{Z_i}{A. M._1} - \sum_{i=r+1}^{s} \frac{Z_i}{A. M._2}$

$Max \ Z = \sum_{i=1}^{r} \frac{Z_i}{I. M._1} - \sum_{i=r+1}^{s} \frac{Z_i}{I. M._2}$

$Max \ Z = \sum_{i=1}^{r} \frac{Z_i}{He. M._1} - \sum_{i=r+1}^{s} \frac{Z_i}{He. M._2}$

$Max \ Z = \sum_{i=1}^{r} \frac{Z_i}{AG. M._1} - \sum_{i=r+1}^{s} \frac{Z_i}{AG. M._2}$

$Max \ Z = \sum_{i=1}^{r} \frac{Z_i}{G. M._1} - \sum_{i=r+1}^{s} \frac{Z_i}{G. M._2}$

$Max \ Z = \sum_{i=1}^{r} \frac{Z_i}{H. M._1} - \sum_{i=r+1}^{s} \frac{Z_i}{H. M._2}$

5. New Average Method of MOLPP

In this method firstly all objective functions are solved simultaneously.
individually by using simplex method. Then the result of addition (maximum type) and subtraction (minimum type) of objective functions are divided by different kinds of means to build a single maximum type objective function, where means are calculated by using maximum absolute value of maximum type objective functions and minimum absolute value of minimum type objective functions. Then this single objective function is optimized for the same constraints. For this method, different kinds of mean techniques are discussed.

5.1. Different Kinds of New Mean Techniques

New Contraharmonic Mean Technique:
Let,
\[ \min (|\varphi_i|), \quad i = 1, 2, \ldots, r \]
and
\[ \max (|\varphi_i|), \quad i = r + 1, r + 2, \ldots, s \]
Max
\[ Z = \left( \sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i \right) / CHM_{av} \]

where, \( CHM_{av} = \frac{(M_1^2 + M_2^2)}{2(M_1 + M_2)} \)

New Quadratic Mean Technique:
Max
\[ Z = \left( \sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i \right) / QM_{av} \]

where, \( QM_{av} = \frac{1}{\sqrt{2}} (M_1^2 + M_2^2) \)

New Neuman-Sándor Mean Technique:
Max
\[ Z = \left( \sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i \right) / NSM_{av} \]

where, \( NSM_{av} = \frac{M_1-M_2}{2 \text{arcsinh} \left( \frac{M_1-M_2}{M_1+M_2} \right)} \)

New Arithmetic Mean Technique:
Max
\[ Z = \left( \sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i \right) / AM_{av} \]

where, \( AM_{av} = \frac{M_1+M_2}{2} \)

New Identic Mean Technique:
Max
\[ Z = \left( \sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i \right) / IM_{av} \]

where, \( IM_{av} = \left\{ \begin{array}{ll} M_1 & \text{if } M_1 = M_2 \\ \frac{M_1-M_2}{c} & \text{else} \end{array} \right\} \)

New Heronian Mean Technique:
Max
\[ Z = \left( \sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i \right) / HeM_{av} \]

where, \( HeM_{av} = \frac{1}{2} \left( M_1 + \sqrt{M_1 M_2 + M_2} \right) \)

5.2. Algorithm for New Average Method of MOLPP

Step 1: Use Simplex method to find the optimal value of each of the objective function.

Step 2: Check the feasibility of step1, if it is feasible then go to step 3 otherwise use dual simplex method to remove infeasibility.

Step 3: Assign a name to each of the optimal value of corresponding objective function. Say \( Max Z_i = \varphi_i, i = 1, 2, \ldots, r \) and \( Min Z_i = \psi_i, i = r + 1, r + 2, \ldots, s \).

Step 4: Calculate \( M_1 \) & \( M_2 \), where, \( M_1 = \max (|\varphi_i|), i = 1, 2, \ldots, r \) and \( M_2 = \min (|\psi_i|), i = r + 1, r + 2, \ldots, s \).

Step 5: Calculate the values of
\( CM_{av}, QM_{av}, NSM_{av}, AM_{av}, IM_{av}, HeM_{av}, AGM_{av}, LM_{av}, GM_{av}, GHM_{av}, HM_{av} \)

Step 6: Optimize the combined objective function using same constraints as follow:
Max
\[ Z = \left( \sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i \right) / CM_{av} \]
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\[ \text{Max } Z = \left( \sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i \right) / QM_{av} \]

\[ \text{Max } Z = \left( \sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i \right) / NSM_{av} \]

\[ \text{Max } Z = \left( \sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i \right) / AM_{av} \]

\[ \text{Max } Z = \left( \sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i \right) / IM_{av} \]

\[ \text{Max } Z = \left( \sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i \right) / HeM_{av} \]

\[ \text{Max } Z = \left( \sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i \right) / AGM_{av} \]

\[ \text{Max } Z = \left( \sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i \right) / LM_{av} \]

\[ \text{Max } Z = \left( \sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i \right) / GM_{av} \]

\[ \text{Max } Z = \left( \sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i \right) / GHM_{av} \]

\[ \text{Max } Z = \left( \sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i \right) / HM_{av} \]

6. Mathematical Example of MOLPP

\[ \text{Max } Z_1 = x_1 + 2x_2 \]

\[ \text{Max } Z_2 = x_1 \]

\[ \text{Min } Z_3 = -2x_1 - 3x_2 \]

\[ \text{Min } Z_4 = -x_2 \]

Subject to:

\[ 6x_1 + 8x_2 \leq 47 \]

\[ x_1, x_2 \geq 3 \]

\[ x_1 \leq 4 \]

\[ x_2 \leq 3 \]

\[ x_1, x_2 \geq 0 \]

The optimal values of each of the objective function are calculated by using simplex method and are given below:

Now for first objective function,

\[ \text{Max } Z_1 = x_1 + 2x_2 \]

Subject to:

\[ 6x_1 + 8x_2 + x_3 = 47 \]

\[ x_1, x_2 - x_3 = 3 \]

\[ x_1 + x_5 = 4 \]

\[ x_2 + x_6 = 3 \]

\[ x_1, x_2, x_3, x_4, x_5, x_6 \geq 0 \]

| $c_i$ | $c_j$ | $1$ | $2$ | $0$ | $0$ | $0$ | $b_i$ |
|-------|-------|-----|-----|-----|-----|-----|-------|
| $x_1$ | 6     | 8   | 1   | 0   | 0   | 0   | 47    |
| $x_4$ | -1    | -1  | 0   | 0   | 0   | 0   | -3    |
| $x_5$ | 1     | 0   | 0   | 0   | 1   | 0   | 4     |
| $x_6$ | 0     | 1   | 0   | 0   | 0   | 1   | 3     |
| $x_7$ | 1     | 2   | 0   | 0   | 0   | 0   | 0     |
| $x_8$ | 0     | 1   | 8   | 0   | 0   | 0   | 23    |
| $x_9$ | 1     | 1   | 0   | -1  | 0   | 0   | 0     |
| $x_{10}$ | 0  | 0   | 0   | 1   | 0   | 0   | 4     |
| $x_{11}$ | 0 | -1  | 0   | 0   | 1   | 0   | 0     |
| $x_{12}$ | 0  | 1   | 0   | 2   | 0   | 0   | 0     |
| $x_{13}$ | 0  | 1   | 0   | 0   | 0   | -8  | 0     |
| $x_{14}$ | 0  | 0   | 2   | 0   | 0   | 23  | 0     |
| $x_{15}$ | 0  | 0   | 0   | 1   | 0   | 0   | 1     |
| $x_{16}$ | 0  | 0   | 0   | 0   | 1   | 0   | 0     |
| $x_{17}$ | 0  | 0   | 0   | 0   | 0   | 0   | 2     |
| $x_{18}$ | 0  | 0   | 0   | 1   | 0   | 0   | 0     |
| $x_{19}$ | 0  | 0   | 0   | 0   | 0   | 0   | 23/6  |
| $x_{20}$ | 0  | 0   | 0   | 1   | 0   | 3   |
| $x_{21}$ | 0  | 0   | 0   | 0   | 0   | 1   |
| $x_{22}$ | 0  | 0   | 0   | 0   | 1   | 0   |
| $x_{23}$ | 0  | 0   | 0   | 0   | 1   |
| $x_{24}$ | 0  | 0   | 0   | 0   | 0   |

Table 1. Simplex table for 1st objective function.

So, the optimized value, $\text{Max } Z_1 = 9.8333$

Now for second objective function,
Max $Z_2 = x_1$

Subject to:
\[
\begin{align*}
6x_1 + 8x_2 + x_3 &= 47 \\
x_1 + x_2 - x_4 &= 3 \\
x_1 + x_5 &= 4 \\
x_2 + x_6 &= 3 \\
x_1, x_2, x_3, x_4, x_5, x_6 &\geq 0
\end{align*}
\]

Table 2. Simplex table for 2nd objective function.

| $C_B$ | $C_f$ | 1 | 0 | 0 | 0 | 0 | 0 | $b_i$ |
|-------|-------|---|---|---|---|---|---|-------|
| 0     | $x_1$ | 6 | 8 | 1 | 0 | 0 | 0 | 47    |
| 0     | $x_4$ | -1| -1| 0 | 1 | 0 | 0 | -3    |
| 0     | $x_5$ | 1 | 0 | 0 | 0 | 1 | 0 | 4     |
| 0     | $x_6$ | 0 | 1 | 0 | 0 | 0 | 1 | 3     |
| $\zeta_j$ | 1 | 0 | 0 | 0 | 0 | 0 | $Z_2 = 0$ |
| 0     | $x_3$ | 0 | 2 | 1 | 6 | 0 | 0 | 23    |
| 1     | $x_1$ | 1 | 1 | 0 | -1| 0 | 0 | 3     |
| 0     | $x_5$ | 0 | -1| 0 | 1 | 1 | 0 | 1     |
| 0     | $x_4$ | 0 | 1 | 0 | 0 | 0 | 1 | 3     |
| 0     | $x_6$ | 0 | 0 | 0 | 0 | 0 | 1 | 3     |
| $\zeta_j$ | 0 | 0 | 0 | 0 | 0 | -1| 0 | $Z_2 = 4$ |

So, the optimized value, $Max Z_2 = 4$

Now for third objective function,

Min $Z_3 = -2x_1 - 3x_2$

Subject to:
\[
\begin{align*}
6x_1 + 8x_2 + x_3 &= 47 \\
x_1 + x_2 - x_4 &= 3 \\
x_1 + x_5 &= 4 \\
x_2 + x_6 &= 3 \\
x_1, x_2, x_3, x_4, x_5, x_6 &\geq 0
\end{align*}
\]

Table 3. Simplex table for 3rd objective function.

| $C_B$ | $C_f$ | -2 | -3 | 0 | 0 | 0 | 0 | $b_i$ |
|-------|-------|----|----|---|---|---|---|-------|
| 0     | $x_1$ | 6 | 8 | 1 | 0 | 0 | 0 | 47    |
| 0     | $x_4$ | -1| -1| 0 | 1 | 0 | 0 | -3    |
| 0     | $x_5$ | 1 | 0 | 0 | 0 | 1 | 0 | 4     |
| 0     | $x_6$ | 0 | 1 | 0 | 0 | 0 | 1 | 3     |
| $\zeta_j$ | -2| -3| 0 | 0 | 0 | 0 | $Z_3 = 0$ |
| 0     | $x_3$ | -2| 0 | 1 | 8 | 0 | 0 | 23    |
| -3    | $x_2$ | 1 | 1 | 0 | -1| 0 | 0 | 3     |
| 0     | $x_1$ | 1 | 0 | 0 | 0 | 1 | 0 | 4     |
| 0     | $x_6$ | -1| 0 | 0 | 1 | 0 | 0 | 1     |
| $\zeta_j$ | 1 | 0 | 0 | -4| 0 | 0 | $Z_3 = -9$ |
| 0     | $x_3$ | 6 | 0 | 1 | 0 | 0 | -8| 23    |
| -3    | $x_2$ | 0 | 1 | 0 | 0 | 0 | 1 | 3     |
| 0     | $x_2$ | 1 | 0 | 0 | 1 | 0 | 0 | 4     |
| 0     | $x_6$ | -1| 0 | 0 | 1 | 0 | 0 | 1     |
| $\zeta_j$ | -2| 0 | 0 | 0 | 0 | 3 | $Z_3 = -9$ |
| -2    | $x_1$ | 1 | 0 | 1/6| 0 | 0 | -8/6| 23/6 |
| -3    | $x_2$ | 0 | 1 | 0 | 0 | 0 | 1 | 3     |
| 0     | $x_6$ | 0 | 0 | -1/6| 0 | 1 | 8/6| 1/6   |
| 0     | $x_4$ | 0 | 0 | 1/6| 1 | 0 | -1/3| 23/6 |
| $\zeta_j$ | 0 | 0 | 1/3| 0 | 0 | 1/3| $Z_3 = -16.66667$ |

So, the optimized value, $Min Z_3 = -16.66667$

Now for fourth objective function,

Min $Z_4 = -x_2$
Subject to:

\[ 6x_1 + 8x_2 + x_3 = 47 \]
\[ x_1 + x_2 - x_4 = 3 \]
\[ x_1 + x_5 = 4 \]
\[ x_2 + x_6 = 3 \]
\[ x_1, x_2, x_3, x_4, x_5, x_6 \geq 0 \]

Table 4. Simplex table for 4th objective function.

| \( C_u \) | \( C_j \) | \( b_i \) |
|----------|----------|----------|
| 0        | \( x_3 \) | 6        |
| 0        | \( x_4 \) | -1       |
| 0        | \( x_5 \) | 1        |
| 0        | \( x_6 \) | 0        |
| \( C_j \) | -1       | 0        |
| 0        | \( x_3 \) | -2       |
| -1       | \( x_2 \) | 1        |
| 0        | \( x_5 \) | 1        |
| 0        | \( x_6 \) | -1       |
| \( C_j \) | 1        | 0        |
| 0        | \( x_3 \) | 6        |
| -1       | \( x_2 \) | 0        |
| 0        | \( x_5 \) | 1        |
| 0        | \( x_6 \) | 0        |
| \( C_j \) | 1        | 0        |
| 0        | \( x_3 \) | -1       |
| -1       | \( x_2 \) | 0        |
| 0        | \( x_5 \) | 1        |
| 0        | \( x_6 \) | 0        |

So, the optimized value, \( \min Z_4 = -16.66667 \)

Table 5. Initial Table.

| \( t \) | \( \varphi_t \) | Values of \( M_1 \& M_2 \) |
|--------|----------------|-----------------------------|
| 1      | 9.83333        | \( M_1 = 9.83333 \)         |
| 2      | 4              | \( M_2 = 3 \)               |
| 3      | -16.66667      |                             |
| 4      | -3             |                             |

Now using the values of \( \varphi_t, M_1 \& M_2 \) different types of means are calculated and given below:

Table 6. Values of Means.

| \( C.M. \_1 \) | 8.14658 | \( C.M. \_2 \) | 14.5819 | \( G.M \_av \) | 8.2359 |
|----------------|---------|----------------|---------|-----------------|-------|
| \( Q.M. \_1 \) | 7.5064  | \( Q.M. \_2 \) | 11.9745 | \( Q.M \_av \) | 7.2696 |
| \( S.S.M. \_1 \) | 7.1122 | \( S.S.M. \_2 \) | 10.5372 | \( S.S.M \_av \) | 6.6985 |
| \( A.M. \_1 \) | 6.9160  | \( A.M. \_2 \) | 9.83333 | \( A.M \_av \) | 6.4166 |
| \( l.M. \_1 \) | 6.7031  | \( l.M. \_2 \) | 8.9337  | \( l.M \_av \) | 6.0919 |
| \( H.e.M. \_1 \) | 6.7016 | \( H.e.M. \_2 \) | 8.9125  | \( H.e.M \_av \) | 6.0882 |
| \( A.G.M. \_1 \) | 6.5901 | \( A.G.M. \_2 \) | 8.3953  | \( A.G.M \_av \) | 5.9137 |
| \( L.M. \_1 \) | 6.4852  | \( L.M. \_2 \) | 7.9698  | \( L.M \_av \) | 5.7560 |
| \( G.M. \_1 \) | 6.2716  | \( G.M. \_2 \) | 7.0711  | \( G.M \_av \) | 5.4314 |
| \( G.H.M. \_1 \) | 5.9692 | \( G.H.M. \_2 \) | 5.9556  | \( G.H.M \_av \) | 4.9883 |
| \( H.M. \_1 \) | 5.6867  | \( H.M. \_2 \) | 5.0841  | \( H.M \_av \) | 4.5974 |

Now for New Harmonic mean technique,
Harmonic mean, \( H.M \_av = \frac{2}{\frac{1}{\varphi_1} + \frac{1}{\varphi_2}} = 4.5974 \)

\[
\text{Max} \ Z = \left( \frac{Z_1 + Z_2}{Z_1 + Z_3} \right) / H.M \_av \\
= \left( \frac{(x_1 + 2x_2 + x_3) - (-2x_1 - 3x_2 - x_2)}{4.5974} \right)
= \left( \frac{4x_1 + 6x_2}{4.5974} \right)
= 0.87x_1 + 1.305x_2
\]

Subject to:

\[ 6x_1 + 8x_2 + x_3 = 47 \]
\[ x_1 + x_2 - x_4 = 3 \]
\[ x_1 + x_5 = 4 \]
\[ x_2 + x_6 = 3 \]
\[ x_3, x_4, x_5, x_6 \geq 0 \]

Table 7. Simplex table for New Harmonic mean techniques objective function.

| \( C_i \) | \( C_j \) | 0.87 | 1.305 | 0 | 0 | 0 | 6 | \( b_j \) |
|---|---|---|---|---|---|---|---|---|
| Basis | \( x_1 \) | \( x_2 \) | \( x_3 \) | \( x_4 \) | \( x_5 \) | \( x_6 \) |   |
| 0 | \( x_1 \) | 6 | 8 | 1 | 0 | 0 | 0 | 47 |
| 0 | \( x_4 \) | -1 | -1 | 0 | 1 | 0 | 0 | -3 |
| 0 | \( x_5 \) | 1 | 0 | 0 | 1 | 0 | 4 |   |
| 0 | \( x_6 \) | 0.5517 | 0.8275 | 0 | 0 | 0 | 1 | 3 |
| 0 | \( C_j \) | 0.5517 | 0.8275 | 0 | 0 | 0 | 1 | 3 |
| 0.87 | \( x_3 \) | 6 | 0 | 1 | 0 | 0 | 0 | -8 |
| 1.305 | \( x_2 \) | 0 | 1 | 0 | 0 | 0 | 1 | 23 |
| 0 | \( x_3 \) | 1 | 1 | 0 | 1 | 0 | 4 |   |
| 0 | \( x_6 \) | -0.2276 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | \( C_j \) | 0.5517 | 0.8275 | 0 | 0 | 0 | 1 | 3 |
| 0.5517 | \( x_1 \) | 1 | 0 | 1/6 | 0 | 0 | 0 | 23/6 |
| 0 | \( x_2 \) | 0 | 1 | 0 | 0 | 0 | 1 | 3 |
| 0 | \( x_5 \) | 0 | 0 | -1/6 | 1 | 0 | 0 | 1/6 |
| 0 | \( C_j \) | 0 | 0 | -1/6 | 1 | 0 | 0 | 1/6 |

So, the optimized value, \( \text{Max } Z = 7.2504 \)

Now using Simplex method, \( Z \) is optimized for different mean techniques and result are shown below:

Table 8. Final Table.

| Techniques | \( x_i \) | Value of \( Z \) |
|---|---|---|
| Chandra Sen | (3.83333, 3) | 3.9534 |
| Contraharmonic Mean | (3.83333, 3) | 3.0263 | New Average Method |
| Quadratic Mean | (3.83333, 3) | 3.4631 | 4.0473 |
| Neuman-Sándor Mean | (3.83333, 3) | 3.7879 | 4.9762 |
| Arithmetic Mean | (3.83333, 3) | 3.9761 | 5.1948 |
| Identic Mean | (3.83333, 3) | 4.2402 | 5.4717 |
| Heronian Mean | (3.83333, 3) | 4.2459 | 5.4750 |
| Arithmetic-geometric Mean | (3.83333, 3) | 4.4164 | 5.6366 |
| Logarithmic Mean | (3.83333, 3) | 4.5750 | 5.7911 |
| Geometric Mean | (3.83333, 3) | 4.9604 | 6.1372 |
| Geometric-harmonic Mean | (3.83333, 3) | 5.5917 | 6.6823 |
| Harmonic Mean | (3.83333, 3) | 6.2715 | 7.2504 |

It is seen that new proposed method is better than Chandra Sen’s Method and Average Method, and new Harmonic Mean technique gives the best result.

7. Conclusion

In this paper, we have used different types of method for solving MOLPP such as Chandra Sen’s Method, Average Method and New Average Method. New Average Method is compared with Average Method and Chandra Sen’s Method. It is seen that Chandra Sen’s Method is better than some mean (Contraharmonic mean, Quadratic mean, Neuman-Sándor mean) techniques in Average Method and New Average Method is better than Chandra Sen’s Method and Average Method for all mean techniques. Among all the techniques, new Harmonic mean technique gives the best result.

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