Modeling of Bank Credit Risk Management Using the Cost Risk Model

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Abstract: This article deals with the issue of managing bank credit risk using a cost risk model. Modeling of bank credit risk management was proposed based on neural-cell technologies, which expand the possibilities of modeling complex objects and processes and provide high reliability of credit risk determination. The purpose of the article is to improve and develop methodical support and practical recommendations for reducing the level of risk based on the value-at-risk (VaR) methodology and its subsequent combination with methods of fuzzy programming and symbiotic methodical support. The model makes it possible to create decision support subsystems for nonperforming loan management based on the neuro-fuzzy approach. For this paper, economic and mathematical tools (based on the VaR methodology) were used, which made it possible to analyze and forecast the dynamics of overdue payment; assess the quality of the credit portfolio of the bank; determine possible trends in bank development. A scientific and practical approach is taken to assess and forecast the degree of credit problematicity by qualitative criteria using a mathematical model based on a fuzzy technology, which can forecast the increased risk of loan default at an early stage in the process of monitoring the loan portfolio and model forecasting changes in the degree of credit problematicity on change of indicators. A methodology is proposed for the analysis and forecasting of indicators of troubled loan debt, which should be implemented as software and included in the decision support system during the process of monitoring the risk of the bank’s credit portfolio.

Keywords: bank credit risks; credit portfolio; observation; simulation modeling; bank expenses; rating; default

1. Introduction

Lending is one of the main types of banking operations, which plays a crucial role in meeting the ever-growing consumer needs of the real economy and contributes to the production and socio-economic development of the country. Its dynamic development and the variety of forms and types of bank credit show that banks have a substantial interest in lending, as a source of high profit, and there is a constant demand from business entities. An increase in the lending volumes contributes to economic growth. Alongside this, the provision of bank loans is a fairly risky activity, and most of the risks associated with it are objectively inherent to the lending process. Therefore, the further development of bank lending depends, to a large extent, on the level and quality of risk management that banks are exposed to during this activity.

Over the past few years, the role of bank lending in meeting the needs of business entities has been constantly growing, which increases its influence on the financial results
of bank activities. Therefore, banks are paying more and more attention to credit risk management, and, in particular, credit and operational risks. Banks must deepen their awareness of the importance of risk management for the development of their business and develop and introduce clear procedures for granting loans using the corresponding tools to develop a risk management system with consideration of the best global experience.

Nonetheless, lending continues to develop in many banks based on extensive approaches. The most widespread method of minimizing the risk used by the commercial banks is overstating the interest rate or establishing various types of surcharges (commissions) for using the loan, as a result of which there is a transfer of credit risk onto the responsible borrowers (Wilhelmsson and Zhao 2018). Apart from that, many banks do not have an appropriate level of organization for the monitoring and forecasting of operational risks due to the fact that the risk managers do not see them as a real threat, except for fraudulent actions of the borrowers or bank staff (Drobyazko et al. 2020a; Nosratabadi et al. 2011).

In the course of their activity, commercial banks may find themselves in a situation of high credit risk, which leads to them using systematic methods and approaches to risk identification and forecasting. (Maechler et al. 2007). The lending process is associated with the actions of numerous risk factors that can cause the non-repayment of a loan by a borrower within a specified period. The lending risk of a commercial bank can be minimized by carefully analyzing these factors (Chun and Lejeune 2020).

In a previous work (Giordana and Schumacher 2017), it was noted that an appropriate and objective assessment of a borrower is extremely important in the lending process of a bank, which is determined by creditworthiness and level of lending risk. The reliability of this assessment significantly affects the results of specific loan agreements and the efficiency of the lending activity of a bank as a whole. The accuracy of the assessment is also important for a borrower, as a decision to grant a loan and the possible amount depend on it.

As indicated in a previous study (Allen and Luciano 2019), a qualitative analysis of the financial indicators of a financial institution allows the bank to obtain the necessary information on credit risk. The analysis shows exactly how the financial institution operated in the previous period, and this gives an assumption about future operations, i.e., whether borrowers will be able to repay credit obligations to a bank.

Credit risk assessment for a commercial bank takes into account the legal and economic-financial aspects of a borrower, the quality, availability, and sufficiency of credit collateral, which is a precondition for a borrower to obtain credit funds, as well as their repayment within a specified period (Richard 2006). For the bank, the purpose of a credit risk assessment is to obtain a qualitative assessment of the activity of a borrower, based on which a decision is made whether to lend or terminate credit relations. In determining the lending risk of a financial institution, the purpose of assessing the financial condition of a borrower is to recognize the possibility of the repayment of debt on the loan from internal sources.

Credit risk is often associated with default—the inability or unwillingness of a counterparty to comply with obligations on time and/or in full volume, which leads to violation of the terms of the contract and allows the creditor to begin debt repayment procedures. A comprehensive analysis of the credit risk should not only include an assessment of the probability of bankruptcy of the counterparty, but also consider the occurrence of credit events that lead to the deterioration of the creditworthiness of the borrower or the credit properties of financial instruments. The existence of risk is not always a cause for concern. Risks are considered justified if they are clear, controlled, and can be measured, and correspond to the bank’s ability to quickly respond to negative circumstances. Unjustified risk may arise from intentional or unintentional actions. If the risks are unjustified, the risk managers should engage with the management and supervisory board of the bank, encouraging them to mitigate or eliminate these unjustified risks. The measures that the bank should implement, in this case, include the reduction in the sums at risk and an increase in the capital or strengthening of the risk management processes (Maechler et al. 2007).
The specified reasons and realia of bank lending practice demonstrate the need for considerable improvements to the processes of credit risk management in the banking sector, considering the introduction of appropriate models and granting them the status of system integrators and protectors of credit fund losses. In this context, the study of forms, methods, and tools for managing bank lending risks, and the development of methodological foundations for the control of bank lending credit risks is relevant both from a scientific and practical point of view.

The research objective is focused on the improvement and development of methodological support and practical recommendations for lowering operational risk levels in crediting commercial banks and the improvement of the value-at-risk (VaR) methodology for the assessment of credit risk.

2. Materials and Methods

Many business entities operate in the conditions of a market economy, and a commercial bank is one such example, so in terms of its activities, the goal of a bank is to maximize profits. As noted by Duffie and Pan (1997) and Michta (2005), all credit institutions are aware of the need to analyze and manage risks in the course of credit operations over time. Researchers such as Chun and Lejeune (2020) and Moore and Zhou (2013) insist that a commercial bank should develop a risk profile to identify the risks that threaten the bank and the level of risk it can tolerate. Such measures are required to balance the yield-to-risk ratio at such a level that the bank does not default.

In the works of Steiner et al. (2006), it is noted that an important step in determining the risk profile is to control the risks and to find the means that can keep them at the desired level. This is required to assess all the risks that may arise from attempts to increase profitability, the search for business line and product line expansion, as well as increase the customer base.

Segoviano and Goodhart (2009) define three fundamental stages in the risk management system: (1) risk analysis (risk identification and risk assessment); (2) risk control (credit risk monitoring); (3) risk minimization (risk mitigation).

The definition of “risk analysis” consists of the initial identification of credit risk, as well as of its subsequent evaluation. In fact, credit risk analysis is about identifying parameters that increase or directly reduce a particular type of risk when performing specific banking transactions (Turnbull 2018). It follows that credit risk assessment is the measurement of its level by qualitative and quantitative methods.

Ronald and Sundaresan (2000) note that the magnitude of credit banking risk is nothing more than an estimate of the risk in value that can be expressed as the maximum amount directly lost by a bank as a result of varying risk factors over a period of time.

After identifying the risk as a threat of bank loss, where the level of risk is determined by the size of the loss, we can analyze the probabilistic meaning of this concept. In conclusion, credit risk can be determined from a loss analysis with a sufficient degree of accuracy (Arici et al. 2019; Drobyazko 2020). As a result, the major part of credit risk assessment is based on probability theory, which is a systematic statistical method of determining the probability that an event may occur in the future, expressed as a percentage.

In the methodological aspect, the authors state the same type of scientifically based methods for assessing the creditworthiness of the borrower and the risk of repayment of loans and making decisions regarding the possibility and conditions of lending; this can lead to a deterioration in the quality of the bank’s loan portfolio. There is a range of special methods that can be used in assessing the risk of lending to financial institutions, the most common of which are: the statistical method, the method of analyzing the feasibility of expenses, the method of expert estimates, the analytical method and the method of using analogues. The assessment of the industry risk of financial institutions, assessment of client risks, and calculation of competitive risks are examples of the statistical method’s use in practice. This method enables the analysis and assessment of scenarios for the implementation of specific activity of financial institutions.
In the banking sector, the average expected value and standard deviation indicators are widely used as a criterion in the qualitative assessment of the risk of crediting financial institutions’ activities. The method of cost and benefit analysis, which are grounded in the fact that there are expenditures for each particular activity of financial institutions, as well as for their elements, has different levels of risk (Steiner et al. 2006)—that is, the degree of risk of different activities of one financial institution and the degree of risk of its individual elements of expenses within the same activity of the financial institution are different.

Based on the expert assessments, one efficiently solves the following important risk analysis tasks: identification of sources and causes of risk, identification of all possible risks, identification of areas of risk reduction, creation of scenarios in case of risk realization, forecasting of competitors’ actions, etc. The heuristic methods include widely known methods, which are used in international practice: the BERI methodology and the methodology of the Swiss Banking Corporation (Slovik and Couronné 2011). These have a global nature and enable the possibility to determine the degree of risk of the entire economy, but not a specific financial institution.

The analytical method is a kind of combination of statistical assessment and the principles of expert analysis. The work of Twala (2010) defines this method as a system of statistical assessments based on a preliminary expert selection of the key parameters for further analysis of the impact of factors on them. The analogue method is used when other risk assessment methods are unacceptable. This method is most useful when it is necessary to identify the degree of risk of any innovative activity of a financial institution in the absence of a basis for comparison, but this method usually takes into account only one activity—innovation (Yan et al. 2021). In the present study, an integrated approach will be used, which will be formed on the elements of analytical and statistical methods for expansion of the scope of the value-at-risk (VaR) methodology of assessing the credit risk of the bank and assessment of their impact on the likelihood of bankruptcy of a commercial bank.

The hypothesis of the study is that the complexity of identifying and processing different types of credit risks in the activity of a commercial bank requires a symbiotic use of modern and proven methods and techniques. However, the authors consider it necessary to introduce into the complex practice of the model risk assessment both value-at-risk (VaR) and neuro-fuzzy methods. It is their combination that can take the quality and accuracy of credit risk identification and prevention for banks to a new level and provide an opportunity to minimize both risk losses and operating costs for technologies and risk management processes.

3. Results

The authors believe that the bank’s credit risk management system (Figure 1) should ensure flexibility, integrity and integration cooperation of organizational, institutional and methodological components, as well as coordination of the activities of the corresponding responsibility centers of the bank, combined with the target and functional subsystems, under formation and support of the optimal system by the structure and quality of the bank’s loan portfolio and the performance of banking activities following the main priority objectives of the bank’s loan policy.

The authors deepened the methodological approach to classification of the borrower’s credit risk factors, according to which they are identified at the micro and macro levels with simultaneous distribution of the factors of general action and factors specific to a certain borrower. Such an approach provides for an increase in the speed of management decisions due to the formation of a two-level system for managing the bank’s credit risk; this makes it possible to differentiate management tools into: general (by all components of the loan portfolio to eliminate the negative impact of common factors) and specific (for the country; the localization region of borrowers’ business interests; the type of economic activity and industry; the borrower’s business, product or service) (Protter 1990).
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Figure 1. The system of the bank’s credit risk management. (Adapted with permission from Arici et al. 2019; Chun and Lejeune 2020).

Thus, for example, the factors specific to a certain type of economic activity or industry are found in basic properties such as (Gupta and Chaudhry 2019): (1) the borrower’s resistance to macroshocks—i.e., a change in the profitability of activities and sales volumes under the influence of negative changes in macroeconomic factors, which is determined by the technical and technological characteristics of the organization of activities in a certain
industry with the corresponding spending structure; (2) the level of dependence of the borrower’s default losses on the average liquidity of the property of enterprises in the industry with consideration of the related collateral concentration; (3) correlation between the probability of defaults of the borrowers which have the same industry affiliation, which increases when crisis events occur in the economy.

3.1. Value-at-Risk (VaR) Methodology for Conducting a Bank Credit Risk Assessment

The purpose of the study was to assess the default risk of a commercial bank credit portfolio using the VaR methodology. The VaR methodology is nothing more than an estimate of the magnitude of the losses directly expressed in the base currency, and which, with a set confidence probabilities, will not exacerbate the loss of the credit portfolio over a certain period of time:

\[ P \{ \text{Loss}_p < \text{VaR} \} = p \]  

(1)

where \( \text{Loss}_p \)—the amount of losses by a portfolio, \( p \)—a set confidence probabilities.

The confidence level as well as the length of time for which this indicator should be calculated are the main parameters in calculating the VaR estimate. The confidence level can be selected in accordance with regulations established by regulators or a direct risk attitude (Bedin et al. 2019). The time over which a commercial bank credit portfolio has not changed significantly is used as a time horizon.

To calculate the quantitative estimate of the risk arising from bank credit operations, it is necessary to first build an empirical function of distributing monetary losses for the selected credit portfolio and then to calculate the VaR directly as a quantile of the required order. The most popular methods of calculating the VaR are: Monte Carlo experiment method, analytical method, as well as historical modeling method (Cochrane 2011; Drobyakso et al. 2020b).

In accordance with VaR methodology, credit risk is the maximum possible loss of a bank for a specific credit portfolio, with a given level of confidence probability. The maximum losses can be divided into what the bank expects (Expected Loss, \( EL_p \)), and those that are unexpected (Unexpected Loss, \( UL_p \)):

\[ \text{VaR}^a = \text{Expected} \_ \text{Loss} + \text{Unexpected} \_ \text{Loss} \]  

(2)

The calculation of expected and unexpected losses is actually the main task in the analysis and assessment of credit risk for a portfolio (Richard 2006). This implies that the expected bank losses are nothing more than the average credit loss incurred by a borrower in defaulting in full. The unexpected losses show the deviation of the expected average loss.

3.1.1. The Calculation of Expected Losses (Expected Loss, \( EL_p \))

The expected losses are nothing more than a mathematical expectation of losses in the event of the borrower defaulting on its obligations. The following formula calculates the expected portfolio losses for each borrower:

\[ EL_p = \sum_{i=1}^{N} (PD_i \times CD_i \times (1 - RR_i)) \]  

(3)

where

- \( PD_i \) (probability of default)—the probability that the default of the borrower will occur—namely, the likelihood that the counterparty will not fulfill the terms of the contract in a certain period;
- \( CD_i \) (credit exposure)—the value of assets that are at risk at the time of default;
- \( RR_i \) (recovery rate)—the level of compensation for the losses incurred and the share of debt that can be repaid in the event of borrower default through the performance of guarantees, collateral, etc.
Assessment of the likelihood of bankruptcy of each borrower is one of the biggest problems in calculating the level of expected losses. Many studies based on models of discriminant analysis, neural networks, rating systems, as well as logit and profit models are devoted to this topic.

The metric $PD_i$ can be found in two stages:

1. During the first stage, information on statistics for credit operations of a commercial bank should be collected, and a complete analysis of the factors that directly affect the likelihood that the borrower may not repay the credit should be the next step. Regression analysis is an effective research tool at this time (Frahm and Huber 2019). As a result, a specific logit model should be constructed, which shows the direct dependence of the individual borrower default case on certain available characteristics, where the default data of commercial bank borrowers over the last three years are taken as a basis.

2. The second stage is characterized by the choice of a particular method that will be the basis for assessing the probability of default of each borrower. As a result of the estimated probability of default of each borrower, the expected losses will be calculated.

3.1.2. The Calculation of Unexpected Losses (Unexpected Loss, $UL_p$)

The unexpected losses are nothing more than a direct deviation from the level of average expected losses, and also determine the degree of risk of the credit portfolio. The calculation of unexpected losses is made by Formula (4):

$$\text{Unexpected Loss} = \text{VaR}^a - \text{Expected Loss}$$

Within this article, it was decided to select the 98.8% confidence level for calculating the VaR, which satisfies the Basel Committee requirements and its recommendations. As a rule, the time horizon of calculating the VaR for commercial bank credit portfolios is one year. You cannot determine if the distribution of losses for the credit portfolio can be directly referred to one of the known classes of distributions. The losses for the credit portfolio cannot exceed 100%, and, compared to the normal distribution, their distribution can rather often have “heavy tail areas”. Such default data are aggregated into a portfolio, resulting in an aggregate estimate of portfolio losses. Initially, a large number of simulations of the level, the aggregate costs, are generated, on the basis of which the next step will be to build an empirical distribution of losses for the portfolio (Krkoska and Schenk-Hoppé 2019; Pacelli and Azzollini 2011).

3.2. Conducting a Computational Experiment and Analyzing the Results

We will conduct a risk analysis of the bank credit portfolio using the above scheme. The aggregate of outstanding balances for active bank credit transactions at a particular date is a credit portfolio. A corporate credit portfolio of a commercial bank, which is an aggregate of credits to individuals, will be considered. At the date under review, the bank credit portfolio consisted of 100 credits totaling EUR 57 million. The empirical basis of the study was the lending activities of the joint-stock commercial bank “Monobank” (Ukraine). The selection of this bank is justified by the fact that this bank uses the model of a “virtual bank” and provides banking services through a digital format of interaction with customers. The bank “Monobank” has the greatest need to develop a system for assessing and forecasting credit risks, especially on the basis of neuro-fuzzy technologies in the VaR risk model.

3.2.1. The Calculation of Expected Losses (Expected Loss, $EL_p$)

The first step was the task of analyzing statistics of direct credit operations of a commercial bank for previous periods for the analysis and assessment of credit risk. In the course of the study, the data on credits issued by a commercial bank to borrowers over a
one-year period were processed. The sample size was 100 issued credits (Maechler et al. 2007). We had the following information on each specific borrower:

- the amount of credit received;
- the internal credit rating of a borrower;
- the presence/absence of credit history;
- data on the financial statements of a borrower.

Additionally, information about defaults on obligations was provided. In this study, the following events were considered a default: (1) credits with overdue principal debts and/or interest arrears exceeding 30 days; (2) credits to borrowers for which bankruptcy proceedings have been initiated or the process of liquidation is in progress; (3) credits to borrowers known for the facts of significant defaults on obligations to their counterparties.

In the commercial bank, which is considered as an example, the rating system of analysis and assessment is used as a basis, making it possible to check the creditworthiness of a borrower. This methodology is based on both qualitative and quantitative characteristics of customers (Twala 2010). These include financial statements as well as the credit history of a borrower and more. You cannot disclose the actual list of parameters and weight ratios that make up the rating. At the end of the analysis, each borrower is assigned a certain credit rating and is included in a particular risk group (Mishura 2008). Totally, there were five credit risk groups defined: A, B, C, D and E, where group A is made up of the most reliable borrowers and group E is made up of the riskiest ones. It is indicated in Slovik and Cournede’s work (2011) that credit history is a history of financial relations between the borrower and banks, which directly indicates the repayment of obligations the former has assumed.

By using the default rate of a particular credit risk group, you can calculate the probability of default (Angelini et al. 2008). For example, if we consider the group of borrowers with the highest rating A, and also assume that within the group of borrowers $N_A$ there are the ones $ND_A$ that have not repaid their obligations to the bank, then as a result we will have an estimate of the probability of default of the group of borrowers by the formula of Protter (1990):

$$P(D)_A = \frac{ND_A}{N_A}$$

where $P(D)_A$—estimation of the probability of default of borrowers with a certain rating; $ND_A$—the number of defaults of borrowers in the group; $N_A$—the total number of companies in the group.

After carrying out the procedure for each risk group, we directly obtained a table of the ratio of the level of default and the ratings of borrowers.

At the next stage, based on the data obtained, we solved the problem of calculating the expected losses of the analyzed credit portfolio (Allen and Luciano 2019). The expected losses are calculated by the formula:

$$EL_p = \sum_{i=1}^{N} (PD_i \times CD_i \times (1 - RR_i)), i = 1 \ldots \ldots 100$$

where:

- $PD_i$ (probability of default)—the probability that the borrower will not fulfill the terms of the credit agreement in due time. Each borrower is given a credit in accordance with the rating he has;
- $CD_i$ (credit exposure)—the value of assets at risk at the time of default. However, due to the lack of more detailed data, in this study, only the amount of current debt owed by the $i$-th borrower was used;
- $RR_i$ (recovery rate)—the rate of recovery, or the proportion of debt that, in the event of default of the borrower, can be repaid by executing guarantees, collateral, etc. A certain rate of recovery has been set for each category by way of peer review (Table 1; Table 2).
Table 1. The ratio of the level of default and the ratings of borrowers.

| Rating | Probability of Default |
|--------|------------------------|
| A      | \( p_A = 0.08 \)       |
| B      | \( p_B = 0.13 \)       |
| C      | \( p_C = 0.17 \)       |
| D      | \( p_D = 0.21 \)       |
| E      | \( p_E = 0.33 \)       |

Table 2. The structure of the credit portfolio of a commercial bank.

| Bank Credit Portfolio |
|-----------------------|
| Credit Rating | Number of Borrowers | Number of Defaults | Amount of Credit, UAH | Probability of Default |
| A            | 12                   | 1                  | 6,200,000             | 0.083333333            |
| B            | 23                   | 3                  | 10,900,000            | 0.130434783            |
| C            | 42                   | 7                  | 2,500,000             | 0.166666667            |
| D            | 17                   | 3                  | 13,500,000            | 0.176470588            |
| E            | 6                    | 2                  | 2,400,000             | 0.333333333            |
| Total       | 100                  | 15                 | 35,500,000            |                         |

The expected losses were calculated for each borrower in the above portfolio \( EL_i \) and for the credit portfolio \( EL_p \) as a whole. The expected losses for portfolio \( EL_p \) amounted to EUR 7.35 million or 13.1% of the portfolio volume.

3.2.2. The Calculation of Unexpected Losses (Unexpected Loss, \( UL_p \))

In order to estimate the level of unexpected losses for a portfolio, it is necessary to calculate \( VaR \). Let us consider the algorithm of credit risk assessment of the portfolio using the Monte Carlo method (Ghodselahi and Amirmadhi 2011). The modeling algorithm consists of the following steps:

1. For every borrower of each class, random variables evenly distributed in the interval from 0 to 1 are generated (Table 3):

\[
\xi_k \in R, 0, 1, i = 1, \ldots, NA
\]  

where

- \( NA \)—the number of borrowers with a certain rating in a bank credit portfolio;
- \( k \)—number of repetitions of algorithm steps, = 1, \ldots, 10,000.

Table 3. Expected losses for a credit portfolio of a commercial bank.

| Amount of Expected Losses for a Portfolio |
|------------------------------------------|
| Portfolio under review | Amount of expected losses | \( R(i) \) |
| EL(A)                    | 2,930,000.4               | 0.46       |
| EL(B)                    | 1,500,000                 |            |
| EL(C)                    | 4,050,000                 |            |
| EL(D)                    | 2,412,000.9               |            |
| EL(E)                    | 77,800                    |            |
| EL(P)                    | 73,500,000                |            |
| EL(P)=                   | 13.1%                     |            |

(2) Based on the dependency of defaults of a borrower on the rating assigned to him, the level of losses for each borrower belonging to a particular group is calculated (Han and Kamber 2006). In the model, it is considered that a default occurrence is an exceedance
of the probability by a randomly generated value, which adds up to 1, the probability of default of the respective rating group (Tables 4 and 5):

\[
L_k^i = \begin{cases} 
CE_i \text{ if } \frac{\xi_i^k}{\xi_i^k} \geq 1 - P(D)_{\text{risk group}} \\
0 \text{ if } \frac{\xi_i^k}{\xi_i^k} < P(D)_{\text{risk group}} 
\end{cases}
\]  

(8)

where \(L_k^i\) — the level of losses of the borrower.

**Table 4.** Example of calculation using the Monte Carlo method.

| N(A) | k = 1 | k = 2 | k = 3 | k = 4 | k = 5 |
|------|-------|-------|-------|-------|-------|
| 1    | 0.91  | 0.71  | 0.17  | 0.88  | 0.65  |
| 2    | 0.99  | 0.43  | 0.51  | 0.04  | 0.2   |
| 3    | 0.28  | 0.85  | 0.4   | 0.117 | 0.89  |
| 4    | 0.31  | 0.39  | 0.21  | 0.93  | 0.5   |
| 5    | 0.05  | 0.31  | 0.2   | 0.35  | 0.43  |
| 6    | 0.9   | 0.096 | 0.84  | 0.76  | 0.6   |
| 7    | 0.29  | 0.302 | 0.151 | 0.97  | 0.93  |
| 8    | 0.87  | 0.135 | 0.6   | 0.29  | 0.29  |
| 9    | 0.15  | 0.81  | 0.43  | 0.9   | 0.94  |
| 10   | 0.94  | 0.98  | 0.22  | 0.98  | 0.91  |
| 11   | 0.45  | 0.53  | 0.41  | 0.09  | 0.08  |
| 12   | 0.43  | 0.21  | 0.705 | 0.95  | 0.5   |

**Table 5.** An example of applying the system to a risk group.

|   | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 700,000 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 700,000 | 0 |

(3) The aggregate losses for the borrowers of each group are calculated by summing up the losses for each client from that group (Torgo 2011):

\[
L_{risk \_ group}^k = \sum_{i=1}^{N} N_A \times L_i^k
\]

(9)

For borrowers with other ratings, a similar procedure is carried out and the aggregate losses for the credit portfolio \(L_p^k\) is calculated (Formula (10)):

\[
L_p^k = L_A^k + L_B^k + L_C^k + L_D^k + L_E^k
\]

(10)

The first four steps of Algorithm 1–4 are repeated a large number of times, and using sample \(L^k\), an empirical function of loss distribution for the credit portfolio is built (Sawik 2012). The results of 100 Monte Carlo experiments allowed us to build an empirical function of loss distribution (Figure 2).

An empirical distribution function makes it possible to assess the credit risk of a portfolio using VaR methodology. Using the established confidence level \(P_l = 0.98\) we found \(P_l < VaR = 0.01\). The found value of VaR 0.98% with the time horizon of one year for the analyzed portfolio was EUR 92,000,000.

Credit VaR = 0.99% − \(EL_p = 92,000,000 − 73,500,000 = 18,500,000\) euros or 20.1% from the amount of the credit portfolio. In percentage terms, the level VaR of the credit portfolio is 13.1% + 20.1% = edit VaR 33.2% from the amount of all credits of the portfolio.
4. The Neuro-Fuzzy Technologies in the VaR Model

To build a model for assessing the degree of problematicity of a bank loan, one used the neuro-fuzzy technologies, grounding on two fundamentally different mathematical constructions: neural networks and fuzzy logic. The mixture of the two mentioned technologies manifests itself as a combination of the abilities to learn neural networks with the clarity and understandable interpretation of the fuzzy rules “IF—THEN.” The neuro-fuzzy technologies enable the possibility to expand the capabilities of modeling complex objects and processes, which is very relevant in real conditions in the absence of reliable data, availability of incomplete and fuzzy statistical data about the object, and complex nonlinear dependencies of the system outputs on its inputs.

To build a model for assessing the degree of the loan’s problematicity, we used the method of linguistic identification, realized using two phases (Pacelli and Azzollini 2011): (1) structural identification: formation of a fuzzy knowledge base that roughly reproduces the dependence of the output (estimating the degree of problem of a loan) on the inputs (factors of its problem) using the linguistic rules “IF—THEN”, which are generated from experimental data on completed risky credit agreements; (2) parametric identification: search of such parameters of the fuzzy knowledge base, which will minimize the deviation of modeled and experimental results. As the value of integral assessment of the loan risk level (y), we selected the linguistic values (“low” (L), “medium” (M), “high” (H), and “default” (D)) depending on the result of fulfillment of the loan agreement by the debtor. For example, \( y = M \) if the debtor repaid 50% or more of the principal debt as a result of restructuring or settlement of the nonperforming loan.

The neuro-fuzzy model of dependency of the level of the loan problematicity on the factors of the borrower \( x_1, x_2, \ldots, x_{10} \) was reviewed in the following form for the VaR model:

\[
y = f(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10})
\] (11)

Thus, to develop model (11), we built a fuzzy knowledge base as a system of fuzzy linguistic expressions of the type “IF—THEN”, which link linguistic estimates (“low”, “medium”, “high”) of the incoming variables with an outgoing variable y. To build model (1), the linguistic estimates “low”, “medium”, and “high” are formalized using membership functions. We set these functions in the following form:

\[
\mu^T(x) = \frac{1}{1 + (\frac{x-b}{c})^2},
\] (12)
where $\mu_T(x)$—the number in the range $[0, 1]$, which characterizes the subjective measure of compliance of the value of $x$ with the fuzzy term $T$ (“low”, “medium”, “high”);

$b$ and $c$—parameters that are first selected by an expert and then adjusted to the experimental data: $b$—the coordinate of the maximum of the function $\mu_T(x)$, with $\mu_T(b) = 1$, and $c$—the coefficient of concentration-extension of the function $\mu_T(x)$.

The fuzzy knowledge base corresponds to the following submission of object (3) in the form:

$$y = \frac{\sum_{k=1}^{4} d_k \mu(y_k)}{\sum_{k=1}^{4} \mu(y_k)}$$  \hspace{1cm} (13)$$

where $y_k$—the value of the variable $y$ (L, M, H or D); as the centers of classes, one selected the values $d1 = 1, d2 = 2, d3 = 3$ and $d4 = 4$.

The functions $\mu(y_k)$ depend on the functions of independence of the factors and $w_i$ (the number in the range $[0, 1]$, which characterizes the subjective measure of confidence of the expert regarding the expression with the number and with the knowledge base—that is, it is the weight of the second rule). The type of this dependency is determined by the built credit history. Using model (13), the effect of changes in feature factors on the degree of credit problem can be estimated, while the adequacy of the model to experimental data will depend on the successful initial selection of parameters $b, c, w$ for the VaR model.

The model can be used for: (1) calculation of the forecasted value of the loan problematicity degree for any totality of factors of influence; (2) determination of the ranges of change of each of the borrower’s performance indicators for which the degree of credit problematicity remains high (Figure 3; Figure 4). The model makes it possible to create decision support subsystems on nonperforming loan management based on a neuro-fuzzy approach.

**Figure 3.** The dynamics of credit problematicity level relative to the changes of the information and legal factors. Source: author’s study.

**Figure 4.** The dynamics of credit problematicity level relative to the changes in the legal and market factors. Source: author’s study.
To assess the quality of the portfolio of nonperforming loans based on the results of the study for the VaR model, one can offer the main indicators of the portfolio of problem credit debt using the developed methodology, including the use of the economic and mathematical tools in the following stages:

1. Analysis of the dynamics of the overdue debt on loans in the banking system (by region, type of economic activity, etc.) and their comparison with the indicators of a specific bank, which reflects the efficiency of the bank’s implemented credit policy;

2. Analysis of the overdue debt in the bank’s loan portfolio (upon expiry terms) and forecasting of its dynamics, which will reflect the effectiveness of actions on the regulation of problematic questions of the borrower, taken by the bank at the beginning stage, and the aggravation of the need for changing approaches to the choice and use of methods for managing nonperforming loans or making changes to the bank’s credit policy;

3. The determination of the most probable trends in the bank’s development under the currently formed level of indicators of nonperforming loans; proposal of the need for measures on stabilization of the credit institution activities in accordance with a long-term development strategy.

5. Discussion

In terms of practical implementation of the task of improving the efficiency of nonperforming loans management, the best efforts of each individual bank in the identification of potentially nonperforming loans, optimal use of methodologies for managing them and reforming the credit structure of banks (i.e., actions at the micro level) may not be sufficient in a situation where there is a big number of obstacles to the effective regulation of delinquent loans at the institutional level and when there are no effective legal and tax fields for dealing with problematic loans.

The seeker considered the main obstacles to effective management of nonperforming loans at the national level and systematized the institutional, regulatory, and tax aspects of successful implementation of methodologies and strategies by banks, selected using a scientific approach, as proposed by the author.

The given recommendations relate primarily to:

(1) The institutional aspect: the introduction of the institution of bankruptcy of the individuals; improvement of the procedure for the recognition of insolvent legal entities with the main objective of accelerating the liquidation of the unsustainable companies; encouragement of the use of legal extrajudicial restructuring methods; introduction of a time management system for planning the efficient allocation of expenditures and human resources in the judicial processes; specialization of judges on the cases about recognition of legal persons as unable to pay;

(2) The tax aspect: the review of the question about recognition of additional benefits (in particular, the amount of debt forgiveness) in tax accounting, which do not form a cash flow, income; the maximum approximation of determination and recognition of received losses on loans in tax and financial accounting; the review of the order of recognition of the loan write-offs at the expense of the previously created reserves for covering of the possible losses.

6. Conclusions

The credit portfolio risk assessment process was based on VaR methodology. In accordance with this concept, credit risk implies the maximum possible losses for a portfolio at the established confidence probability, which in turn are subdivided into expected and unexpected losses. One justified a system of quantitative and qualitative indicators for assessing the risk of lending for commercial banks based on the expansion of the models based on value-at-risk. The proposed indicators are appropriate for integral assessment, they consider the sectoral specifics of the commercial banks’ activity, they do not have a high functional relationship among themselves, and provide the possibility to characterize
factors of formation of the risk of the borrowers’ activity crediting in a complex, under optimization of the time spent on the implementation of the assessment procedures.

It is proven that with each structural change in a credit portfolio, there appears to be a need to regularly repeat the entire credit risk assessment for the portfolio. Additionally, a database with customer information must be constantly updated. For example, the amount of unexpected losses is a determining parameter that shows the reliability of a particular loan portfolio, as well as a commercial bank as a whole. The point of this is that the level of reliability of a bank is characterized by the adequacy of the capital that the bank has to cope with possible unexpected losses. The protection of a bank against defaulting and bankruptcy is one of the main functions of the bank capital, which is a “safety cushion”, which in the case of very large, unexpected losses can allow both depositors and creditors to withdraw their finances.

To select an effective method of problem loan management, the VaR model was introduced, which reduces the complex process of such a selection to solve a number of simple problems of pairwise comparisons of different factors and provides an opportunity to update calculations with each change of priority according to the lending policy of a bank. Given the priority of portfolio profitability and capital adequacy in the strategic goals of a bank, the most effective are restructuring and methods of additional financing. The separate stages of using methods of inertial (indirect) strategy are described in detail: factoring and additional financing of the investment project, using an example of comparative calculation advantages for all parties in the process of settlement of the credit problem, are provided.

In the scientific work, the model for assessing the degree of problematicity of credit as a component of the nonperforming credit management system is built using neuro-fuzzy technologies, which enable the expansion of the possibilities of modeling complex objects and processes, which is very relevant in real conditions under the absence of reliable data, availability of incomplete and fuzzy statistical data about the object, and complex nonlinear dependencies of the system outputs on its inputs. The model makes it possible to create decision support subsystems on nonperforming loan management based on a neuro-fuzzy approach. To analyze credit quality at the portfolio level, one used economic and mathematical tools (based on the VaR methodology), which enable the analysis and forecasting of the dynamics of the overdue payment, assessment of the quality of the credit portfolio of the bank, and determine possible trends in the bank’s development.

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