Modeling the behaviour of shape memory materials under large deformations

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Abstract. In this study, the models describing the behavior of shape memory alloys, ferromagnetic materials and polymers have been constructed, using a formalized approach to develop the constitutive equations for complex media under large deformations. The kinematic and constitutive equations, satisfying the principles of thermodynamics and objectivity, have been derived. The application of the Galerkin procedure to the systems of equations of solid mechanics allowed us to obtain the Lagrange variational equation and variational formulation of the magnetostatics problems. These relations have been tested in the context of the problems of finite deformation in shape memory alloys and ferromagnetic materials during forward and reverse martensitic transformations and in shape memory polymers during forward and reverse relaxation transitions from a highly elastic to a glassy state.

1. Introduction
Shape memory alloys (SMA), ferromagnetic shape memory alloys (FSMA) and shape memory polymers (SMP) refer to the class of functional (smart) materials. Constructions made of such materials may significantly change their configuration under the action of external thermal, magnetic or electric fields. SMAs have two phases, each with a different crystal structure and therefore different properties [1]. The phase transformation occurs by shear distortion of the lattice structure. On cooling, the crystal structure changes from austenite (the high symmetry parent phase) to martensite (the low symmetry product phase) and the material experiences inelastic deformation. When the material is heated being in the martensitic phase, the crystal structure transforms back to austenite and inelastic strain is recovered (the shape memory effect).

The combination of ferromagnetic properties and the structural (martensitic) phase transition makes FSMA a rather promising material in search for reliable ways of controlling the shape of a sample by varying temperature, pressure, and external magnetic field [2]. With the help of the magnetic field the phase transition process can be effectively controlled, as was shown in experimental studies [3]. If the direct or reverse phase transitions in the ferromagnetic material are realized under the action of the magnetic field, the specific temperatures of the process change according to the generalized Clausius-Clapeyron law and depend on the magnetic and stress fields. Such effect of the magnetic and stress fields allows us to vary temperatures of direct and reverse phase transformations and thereby to control the process of the austenite-to-martensite phase transition.

When SMP is cooled over the transition temperature range from the highly elastic to the glassy state, the deformation is “stored” in the material due to the formation of additional polymer chains and...
their interactions, which severely restricts the large-scale conformational motion of the polymer as long as low temperature is maintained [4, 5, 6]. Such process of deformation storage does not occur in the material when heated over the transition temperature range. In this case, the material exhibits the so-called one-way shape memory effect when the material in the temporary shape at low temperatures remembers its permanent shape at high temperatures.

2. Basic relations

2.1. Kinematic relations

The behavior of media under finite deformations can be described using, according to [7, 8], the following process characteristics: the initial undeformed \( \kappa_0 \) and current (actual) \( \kappa \) configurations, the deformation gradient \( F \), converting the initial configuration to the actual one, the Cauchy-Green deformation measure \( C = F^T \cdot F \) and tensor \( E = (C - \mathbf{g})/2 \) (\( \mathbf{g} \) is the identity tensor). Following [9], the history of any deformation process can be traced by introducing the intermediate \( \kappa^* \) configuration closely approaching the current one. To formalize this closeness, a small parameter \( \varepsilon \) is used. For the intermediate configuration the deformation gradient is written as \( F = (g + \varepsilon \mathbf{h}) \cdot F_\ast = (g + \varepsilon (\varepsilon + \mathbf{d})) \cdot F_\ast \), where \( F_\ast \) is the deformation gradient converting the initial configuration to the intermediate one, \( \mathbf{h} \) is the gradient of the small displacement vector, which connects the configuration \( \kappa^* \) and \( \kappa \), \( \varepsilon \) and \( \mathbf{d} \) are the tensors of small strains and rotations for the intermediate configuration (symmetric and skew-symmetric parts of the tensor \( \mathbf{h} \)). Thus, we obtain the following relations:

\[
C = C_\ast + 2 \varepsilon F^T \cdot \mathbf{F} \cdot F_\ast \quad \text{and} \quad E = E_\ast + \varepsilon F^T \cdot \mathbf{F} \cdot F_\ast = E_\ast + \Delta E.
\]  (1)

According to these relations, in the limit of going from the intermediate \( \kappa^* \) to the current configuration \( \kappa \), the rates of changes in \( C \) and \( E \) are given by: \( \dot{C} = 2 F^T \cdot \mathbf{D} \cdot F \), \( \dot{E} = F^T \cdot \mathbf{D} \cdot F \), where \( \mathbf{D} \) is the strain rate tensor.

2.2. State equation

Out of the available equivalent forms of writing the constitutive equations for simple materials, which satisfy the objectivity principle [7], we choose the following form: \( T = J^{-1}F \cdot \mathbf{g}(C_E, \Theta) \cdot F^T \), where \( T \) is the true stress tensor, \( \mathbf{g}(C_E, \Theta) \) is the function of the material response (second-order tensor), \( C_E \) is the Cauchy-Green measure of elastic strains, \( J = I_3(F) \) is the third invariant of \( F \) (Jacobian, determining a relative variation in the volume element) and \( \Theta \) is the absolute temperature. The tensor \( \mathbf{g} \) in this equation is known as the second (symmetric) Piola-Kirchhoff stress tensor \( \mathbf{P}_E \equiv \mathbf{g} \), which in the theory of finite elastic deformations is represented in terms of elastic potential \( W(C_E) : \)

\[
\mathbf{P}_H = 2(\partial W / \partial C_E). \]  (2)

For the intermediate configuration, this constitutive equation is written as

\[
\dot{\mathbf{P}}_H = \dot{\mathbf{P}}_H^\ast + \varepsilon L^{\ast V} \cdot \mathbf{E}_E = \dot{\mathbf{P}}_H^\ast + \tilde{L}^{\ast V} \cdot \Delta \mathbf{E}_E. \]  (2)

Here \( L^{\ast V} \) and \( \tilde{L}^{\ast V} \) are the fourth-rank tensors specifying the material responses to the small elastic strains \( \varepsilon \mathbf{e}_E \) and to the increment of elastic strain \( \Delta \mathbf{E}_E \) pertinent to the intermediate configuration. Passing to the limit yields \( \dot{\mathbf{P}}_H = L^{\ast V} \cdot \mathbf{D}_E = \tilde{L}^{\ast V} \cdot \mathbf{E}_E \). Here and below superscript “dot” differentiating with respect to time.
2.3. Phase transition in the shape memory alloys (SMA and FSMA)

Kinematics of the process is defined by the deformation gradients \( F_A \) in the austenitic state and \( F_M \) in the martensitic state and expressions for other kinematic tensors such as \( C_A, \ C_M, \ E_A, \ E_M \) are similar to those in (1). We introduce the following kinematic equation [10]:

\[
\dot{E} = (\varphi_A E_A)^* + (\varphi_M E_M)^* + \dot{E}_{p_h} + \dot{E}_p + \dot{E}_\theta,
\]

where \( \varphi_A \) and \( \varphi_M \) are the parameters that specify the volume fractions of the martensitic and austenitic phases in the material \( (\varphi_M + \varphi_A = 1) \), \( \dot{E}_\theta = \alpha \dot{\Theta} C \) is the rate of change in the temperature strains \( (\alpha \) is the linear thermal expansion factor), \( \dot{E}_p = 3 \dot{\sigma}_i / (2 H \sigma_i) (F^T : S : F) \) is the rate of change in the plastic strains \( (S \) is the deviator of \( T, \sigma \) is the stress intensity, \( H \) is the modulus of plastic strain hardening), \( \dot{E}_{p_h} \) is the rate of a change in the phase strains.

In order to approximate the dependence of \( \varphi_M \) on the temperature, we use the following relation:

\[
\varphi_M(\xi) = \left\{ \begin{array}{ll}
0, & \xi \leq 0; \\
0.5 (1 - \cos(\pi \xi)), & 0 < \xi < 1; \\
1, & \xi \geq 0.
\end{array} \right.
\]

Here \( M_s, M_f, A_s, A_f \) are the characteristic temperatures at the beginning and end of the direct and reverse martensitic transformations in the absence of the stress and magnetic fields. In the case, when the thermal phase transitions occur in the presence of the stress and magnetic fields, there is a shift in \( \Theta_c = M_c = (M_s + M_f) / 2 \) for direct martensitic transformation and \( \Theta_c = A_c = (A_s + A_f) / 2 \) for reverse martensitic transformation according to the generalized Clausius-Clapeyron law [10]:

\[
\Theta_c = \Theta_{c0} + \frac{\Theta_{c0}}{\Theta_c} (P_{II} \cdot [E_{p_h}] + \mu_0 H \cdot [M]),
\]

where \( \Theta_c \) is a new value of the average temperature, \( \Theta_{c0} \) is the value of the average temperature in the absence of the stress and magnetic fields, \( H \) is the magnetic field vector, \( [q] \), \( [E_{p_h}] \) and \( [M] \) are changes in the released or absorbed heat, phase strain and magnetization during phase transition, and \( \mu_0 \) is the magnetic permeability of vacuum.

To describe the rate of change in the phase strains (without regard to reversible shape memory effect of reverse transformation), we used the equations, which were proposed for small strains in [11]. In our study, these relations were generalized to finite strains: \( \dot{E}_{p_h} = \ddot{E}_{p_h} \phi_M \), where

\[
\ddot{E}_{p_h} = (a g + b \dot{P}_{II} + c E_{p_h}), \quad \phi_M > 0; \\
\dot{E}_{p_h} = \left( \frac{c E_{p_h}^{(0)}}{\exp(c \varphi_M^{(0)})} + c E_{p_h} \right), \quad \phi_M < 0.
\]

In these expressions \( a, b, c \) are the material parameters, \( \dot{P}_{II} \) is the deviator of \( P_{II}, \varphi_M^{(0)}, E_{p_h}^{(0)} \) are the values of the martensite phase parameter and phase strain at the starting point of the reverse transformation process.
2.4. Relaxation transition in the shape memory polymers

Kinematics of the process is defined by the deformation gradients \( \mathbf{F}_r \) in the highly elastic (rubbery) state and \( \mathbf{F}_g \) in the glassy state and the expressions for other kinematic tensors such as \( \mathbf{C}_r, \mathbf{C}_g, \mathbf{E}_r, \mathbf{E}_g \) are similar to those in (1). We introduce the following kinematic equation [12]:

\[
\mathbf{\dot{E}} = (\phi_r \mathbf{E}_r) + (\phi_g \mathbf{E}_g) + \mathbf{\dot{E}}_f + \phi_g \mathbf{\dot{E}}_\theta,
\]

where \( \phi_g \) and \( \phi_r \) are the parameters that specify the volume fractions of the glassy and the highly elastic phases in the material \((\phi_r + \phi_g = 1)\), \( \mathbf{\dot{E}}_\theta \) is the rate of change in the temperature strains and is defined by the relation same as in the previous subsection, \( \mathbf{\dot{E}}_f = \mu^{-1} (\mathbf{F}^T \cdot \mathbf{T} \cdot \mathbf{F}) \) is the rate of a change in the viscous strains \((\mu \) is the viscosity), \( \mathbf{\dot{E}}_f \) is the rate of freezing of highly elastic strains in the cooling process.

In order to approximate the dependence of \( \phi_g \) on temperature, we use the following relation [4]:

\[
\phi_g(\Theta) = \frac{1}{1 + A(\Theta_h - \Theta)},
\]

where \( A \) is the material parameter, \( \Theta_h \) is the characteristic temperature of the relaxation transition. According to [4]:

\[
\mathbf{\dot{E}}_f = \begin{cases} 
\phi_r \mathbf{E}_r, & \Theta < 0, \\
(\phi_g / \phi_r) \mathbf{E}_f, & \Theta > 0, \\
0, & \Theta = 0.
\end{cases}
\]

2.5. Description of the elastic behavior

To formulate the constitutive equation for SMA and FSMA, we use a simplified Signorini law [8] for describing both the austenitic and martensitic phase behavior:

\[
\mathbf{P}_{II} = J(\mathbf{C}_r) \left[ (k_{1r} + k_{2r}) \mathbf{C}^{-1}_r - 2k_{2r} \mathbf{C}^{-2}_r \right], \quad \gamma = A, M,
\]

\[
k_{1r} = \Lambda_r \left[ 3 - I_1(\mathbf{C}^{-1}_r) \right] / 2 + (\Lambda_r + G_r) \left[ 3 - I_1(\mathbf{C}^{-1}_r) \right]^2 / 8, \quad k_{2r} = G_r - (\Lambda_r + G_r) \left[ 3 - I_1(\mathbf{C}^{-1}_r) \right] / 2,
\]

where \( \Lambda_r \) and \( G_r \) are the material parameters, which coincide with the Lame parameter and the shear modulus of the linear theory of elasticity. Hence, in accordance with (2) we obtain for SMA and FSMA

\[
\mathbf{\mathbf{\dot{L}}}_{II} = J(\mathbf{C}_r) \left[ (k_{1r} + k_{2r}) \mathbf{C}^{-1}_r \mathbf{C}^{-1}_r - 2k_{2r} \mathbf{C}^{-2}_r \mathbf{C}^{-2}_r - (\Lambda_r + G_r - 2k_{2r}) \mathbf{C}^{-2}_r \mathbf{C}^{-2}_r - 2(k_{1r} + k_{2r}) \mathbf{C}^{-2}_r \mathbf{C}^{-2}_r \right]
\]

\[
- 4k_{2r} \left[ \mathbf{C}^{-2}_r \mathbf{C}^{-2}_r \mathbf{C}^{-2}_r + 4k_{2r} \left[ \mathbf{C}^{-2}_r \mathbf{C}^{-2}_r \mathbf{C}^{-2}_r \right] \right].
\]

Here \( \mathbf{C}_H \) is the second isotropic fourth-order tensor [8] and the operation \( \mathbf{B}^{IV} \cdot \mathbf{A} \) means a positional scalar multiplication (from the right) of the second-order tensor \( \mathbf{A} \) by the second basic vector of the fourth-rank tensor \( \mathbf{B}^{IV} \).

As the elastic potential for SMP, we use the Peng–Landel potential for describing both the highly elastic and the glassy state behavior:

\[
W_{\gamma} = G_r \left[ 3 I_1(\mathbf{C}_r) J^{-2/3}(\mathbf{C}_r) - 3 \right] / 2 + B_r \left[ J(\mathbf{C}_g) - 1 \right]^2 / 2, \quad \gamma = r, g,
\]

\[
\text{II}
\]

4
where \( G \) and \( B \) are the material parameters, which coincide with the shear modulus and the bulk modulus of the linear theory of elasticity. Hence, in accordance with (2) we obtain for SMP

\[
\tilde{L}_\gamma = -\frac{2}{3} J^{-2/3} (C_\gamma) C_\gamma \left[ g C_\gamma^{-1} + C_\gamma^{-1} g - \frac{1}{3} I_i (C_\gamma) C_\gamma^{-1} - I_i (C_\gamma) C_\gamma^{-1} \cdot C_{II} \cdot C_\gamma^{-1} \right] +
\]

\[
+ 2 J (C_\gamma) B_\gamma \left[ (J(C_\gamma) - \frac{1}{2}) C_\gamma^{-1} C_\gamma^{-1} - (J(C_\gamma) - 1) C_\gamma^{-1} \cdot C_{II} \cdot C_\gamma^{-1} \right].
\]

(12)

3. Verification of the model

3.1. Boundary-value problem

The variational formulation of the boundary value problem in the initial configuration in the absence of a magnetic field (for SMA and SMP) can be written as

\[
\int_{V_0} J \sqrt{n : C^{-1} \cdot n} \mathbf{Q} \cdot \mathbf{U} dS_0 + \int_{\partial V_0} \rho_0 \mathbf{K} \cdot \mathbf{U} dV_0 = 0,
\]

(13)

where \( \mathbf{n} \) is the external unit normal to the surface \( S_0 \) of the body with mass density \( \rho_0 \) and with volume \( V_0 \) in the initial configuration, \( \mathbf{U} \) is the displacement vector connecting the initial and current configurations, \( \mathbf{Q} \) and \( \mathbf{K} \) are the surface and mass forces, \( \delta \) is the symbol of variation.

Under the action of the magnetic field (for FSMA) it is necessary to solve the connected magnetomechanical problem for the magnetizable body of finite dimensions located in the space. In this case, one more variational equation is considered:

\[
\int_{V_0} J \mathbf{H}_0 \cdot \nabla \cdot \delta \psi dV_0 = \int_{V_0} J (1 + \chi) (\mathbf{F}^{-T} \cdot \nabla \psi)(\mathbf{F}^{-T} \cdot \delta(\nabla \psi))dV_0 +
\]

\[
+ \int_{V_0} J (\mathbf{F}^{-T} \cdot \nabla \psi)(\mathbf{F}^{-T} \cdot \delta(\nabla \psi))dV_0.
\]

(14)

Here \( V_0 \) is the environmental space volume, \( \mathbf{H}_0 \) is the applied external magnetic field, \( \psi \) is the scalar magnetic potential \((\mathbf{H} = \mathbf{H}_0 - \nabla \psi)\), \( \nabla \) is the Hamilton operator in the actual configuration while \( \nabla \) is the Hamilton operator relative to the initial configuration and \( \chi = \phi_M \chi_M + \phi_A \chi_A \) is the magnetic susceptibility in the mixed austenitic-martensitic state.

We use the step-wise procedure for numerical realization of equations (13) and (14).

3.2. Shape memory alloys

To verify the model for SMA we consider the following problems on the deformation of a plate with one rigidly clamped end (the left end in figure 1 where the results of solving these problems are shown).

Problem 1. Two plates of the same length made of beryllium bronze and SMA (nickel titanium) fastened together along the length without tension. The sample is first cooled to the temperature \( \Theta_M \) corresponding to the martensitic state of the SMA and then reheated to the temperature \( \Theta_A \) corresponding to the austenitic state. Thus, the SMA first undergoes the forward martensitic transformation (this leads to bending of the two-layer plate) and then the reverse martensitic transformation (the plate takes the same shape as it has in the initial state).

Problem 2. Before the two plates are fastened together (see Problem 1), the SMA plate at the temperature \( \Theta_A \) corresponding to the fully austenitic state of the material is subjected to uniaxial
uniform tension along under the action of tensile stress 100 MPa. Then, the plate is cooled to the temperature $\Theta_M$ corresponding to a fully martensitic state of the SMA, after which the load is removed. The plates having the same length are fastened along the length without tension. This two-layer plate is heated to the temperature $\Theta_A$ (the bending of the plate) and then cooled to the temperature $\Theta_M$ (return to the original position).

Problem 3. The plate made of SMA is bended by a shear stress of 20 MPa applied to the free (right) end at the temperature $\Theta_A$ and then is cooled to the temperature $\Theta_M$, after which the load is removed (the plate remains in the bent state). After that the plate is reheated to the temperature $\Theta_A$ (the plate takes the same shape as that in the initial state).

![Figure 1](image)

**Figure 1.** Bending of the plates for Problem 1 (a), Problem 2 (b) and Problem 3 (c): the initial and final states of the plates are denoted by the dashed and solid lines, respectively. In figures (a) and (b) the SMA plate is situated at the bottom.

3.3. Ferromagnetic shape memory alloys

To verify the model for FSMA we used the experimental data of work [3], where the possibility of magnetic control of the shape memory effect was analyzed. The austenite-to-martensite phase transition in the specimen made of polycrystalline Ni-Mn-Fe-Ga alloy was realized at a fixed external temperature due to a shift of the specific phase transition temperature induced by the magnetic field. An approximately linear dependence of the martensite and austenite start temperatures on the applied magnetic field (in the absence of stress) was obtained experimentally in paper by [3].

We consider the following boundary-value problem corresponding to the above mentioned experiment. At the temperature of the austenitic state $\Theta_0$ the straight cantilevered bar-shaped specimen (figure 2 (a)) was subjected to a follower tangential force 5 MPa uniformly distributed along the surface of the free end of the bar (the specimen was bent). Then the bar was cooled down to the martensitic state temperature and unloaded. This process proceeded in the absence of the magnetic field. Then the bar was exposed to the external magnetic field $H_0 = 80 \cdot 10^5$ A/m, which led to a significant increase in the characteristic temperature of the reverse phase transition. After this the bar was heated up to the temperature $\Theta_0$, $A_f(H = 0) < A_f(H = 0) < \Theta_0 < A_f(H = H_0) < A_f(H = H_0)$, which was kept constant, the magnetic field was gradually removed and the specimen underwent the phase transition to the austenitic state at a fixed temperature $\Theta_0$.

Figure 2 (b) demonstrates the specimen configuration and the distribution of the axial component of the phase strain calculated in compliance with the above relations at the beginning of the magnetic field removal. Red and blue lines correspond to stretching and compressing strains in the bar. The dashed line is the median surface of the bar. When the magnetic field vanishes the phase strains vanish, too, and the bar regains its initial configuration.
3.4. Shape memory polymers

The model for SMP was verified using the experimental data presented in [5, 6]. In experiments [5], the sample of a shape memory polymer (epoxy polymer) was initially in the unstrained and unstressed state at a temperature $\Theta_h$ (high elasticity). In the first stage, the specimen was subjected to uniaxial force applied along its axis at a constant temperature until certain axial strain $E_*$ was reached. After that, the material was cooled to a temperature $\Theta_g$ (glassy state) at a fixed strain. This was accompanied by an increase in the stress. When the SMP in the glassy state was unloaded to zero stresses, the axial strains in the material remained frozen. Heating of the material to a temperature $\Theta_h$ led to unfreezing of the strains, and the polymer returned to the initial state. Figure 3 (a, b, c) shows curves of the nonzero component of the true stress tensor $T$ versus relative temperature, obtained for elastic material behavior (solid curves), viscoelastic material behavior (dot-and-dashed curves) and in experiment [5] (dashed curves) at the initial axial strain $E_* = 9.1\%$, $E_* = 0\%$ and $E_* = -9.1\%$.

![Figure 2. The specimen (a) and distribution of axial component of phase strain (b).](image)

![Figure 3. True stress in the sample [5] during its cooling (a), heating after cooling (b), heating after cooling and unloading (c) and Cauchy–Green axial strain in the sample [6] (d).](image)
Figure 3 (a) shows the true stress in the sample during its cooling, figure 3 (b) shows true stresses in the sample when heated in the state, corresponding to completion of the cooling process and figure 3 (c) shows the true stresses in the sample when heated in the state, corresponding to completion of the cooling process and unloading (with the final axial strain $E = 8.8\%$, $E = 0\%$ and $E = -9.3\%$).

In [6], the uniaxial loading experiments were performed with a sample of Veriflex™ shape memory polymer. Figure 3 (d) shows the Cauchy–Green axial strain versus relative temperature curves, which were plotted using the developed model for elastic material behavior (solid curves) and the data from experiments [6] (dashed curves). At a temperature $\Theta_h$ (high elasticity) the sample was stretched by the axial force to different axial strains $E_\ast = 163\%$, $E_\ast = 66.8\%$ and $E_\ast = 9.4\%$, and was cooled to a temperature $\Theta_g$ (glassy state). After that the sample was unloaded to zero axial stress and heated to a temperature $\Theta_h$.

4. Conclusion
The models, describing the behavior of shape memory alloys, ferromagnetic materials and polymers have been constructed, using a formalized approach to the development of constitutive equations for complex media under large deformations. A comparison between the results obtained in the framework of the constructed models and the available experimental data has shown that the proposed models can describe with acceptable accuracy the behavior of shape memory materials.

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References
[1] Otsuka K and Wayman C M 1998 Shape Memory Materials (Cambridge: Cambridge University Press)
[2] Vasil’ev A N, Buchel’nikov V D, Takagi T, Khovailo V V and Estrin E I 2003 Phys. Usp. 46 559–588
[3] Cherechukin A A, Dikshtein I E, Ermakov D I, Glebov A V, Koleдов V V, Kosolapov D A, Shavrov V G, Tulaikova A A, Krasnoperov E P and Takagi T 2001 Phys. Lett. A 291 175–183
[4] Baghani M, Naghdabadi R and Arghavani J 2013 J. Intelligent Material Systems and Structures 24 21–32
[5] Liu Y, Gall K, Dunn M L, Greenberg A R and Diani J 2006 Int. J. Plasticity 22 279–313
[6] Volk B L, Lagoudas D C, Chen Yi-Chao 2010 Smart Materials and Structures 19 75006–16
[7] Truesdell C 1972 A First Course in Rational Continuum Mechanics (Baltimore: J. Hopkins University)
[8] Lur’e A I 1980 Nonlinear Elasticity Theory (Moscow: Nauka)
[9] Rogovoy A A 2012 Contin. Mech. Thermodyn. 24 81–114
[10] Rogovoy A A and Stolbova O S 2016 Int. J. Plasticity 85 130–155
[11] Movchan A A, Shelymagin P V and Kazarina S A 2001 Appl. Mech. Tech. Phys. 36 864–871
[12] Rogovoy A A and Stolbova O S 2015 Appl. Mech. Tech. Phys. 56 1059–70