CURVATURE COUPLING AND ACCELERATED EXPANSION OF THE UNIVERSE

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Abstract

A new exactly solvable model for the evolution of relativistic kinetic system interacting with an internal stochastic reservoir under the influence of a gravitational background expansion is established. This model of self-interaction is based on the relativistic kinetic equation for the distribution function defined in the extended phase space. The supplementary degree of freedom is described by the scalar stochastic variable (Langevin source), which is considered to be the constructive element of the effective one-particle force. The expansion of the Universe is shown to be accelerated for the suitable choice of the non-minimal self-interaction force.

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1 Introduction

Observations of supernovae at high redshift show that the expansion of the Universe is accelerating. To explain this fact in the framework of Friedmann-Lemaître-Robertson-Walker (FLRW) model, one has to require that a cosmic medium is characterized by negative pressure [1, 2]. One of the possible explanations is the existence of dark energy (see, e.g., the reviews [3, 4]). There exists a number of dark energy candidates, the best known are a cosmological constant and different quintessence scenarios ([5, 6, 7, 8, 9, 10, 11, 12]). Negative pressure may also be the consequence of self-interactions in gas models of the Universe [13, 14, 15, 16, 17]. In particular, an “antifrictional” force, self-consistently exerted on the particles of the cosmic substratum, was shown to provide an alternative explanation for an accelerated expansion of the Universe [14, 15]. This approach relies on the fact that the cosmological principle is compatible with the existence of a certain class of (hypothetical) microscopic one-particle forces, which manifest themselves as “source” terms in the macroscopic perfect fluid balance equations. These sources can be mapped on an effective negative pressure of the cosmic medium.

It is worth pointing that effective self-interaction forces can be regarded as a specific non-minimal coupling of the cosmic gas to the Ricci scalar, Ricci tensor and Riemann tensor. Generally, a force which explicitly depends on curvature quantities describes a coupling of matter with the space-time curvature which goes beyond Einstein’s theory. However, mapping the non-minimal interaction on an imperfect fluid degree of freedom admits a self-consistent treatment on the basis
of general relativity. This may be considered as a gas dynamical counterpart to the non-minimal couplings of scalar fields or those of higher-order gravity theories. A (non-minimal) fluid interaction is designed so that it results in the desired cosmic evolution. Designing the coupling for description of a specific dynamics has already been used earlier for interacting two-component models [8]. Here this idea is applied to the case of a one-component fluid, which is self-consistently coupled with the Riemann tensor. As a characteristic feature of this approach, Hubble rate and deceleration parameter explicitly enter the microscopic dynamics, giving rise to a self-consistent coupling of the latter to the gravitational field equations.

The paper is organized as follows. In section 2 the formalism of one-particle distribution function in the extended phase space associated with a homogeneous and isotropic, spatially flat Universe is established. In section 3 macroscopic properties of the ultrarelativistic kinetic system are discussed, an effective stress-energy tensor is obtained, and the conditions of accelerated expansion of the Universe for power-law and exponential scenarios are investigated in detail. A brief summary is given in section 4.

2 Formalism of One-Particle Distribution Function in the Extended Phase Space

2.1 Kinetic Equation

The idea of the phase space extension, based on the covariant formalism of Cartan’s differentiation and integration, was proposed in [18, 19]. Along a line of development of the relativistic kinetic theory (see, [20, 21, 22] and references therein) a numerous applications of that formalism have been elaborated by different authors (see, e.g., [23, 24, 25, 26, 27]).

The simplest kinetic system with supplementary degree of freedom can be described in terms of the 8+1-dimensional scalar distribution function

\[ \Phi = \Phi \left( x^i, p^k, \omega \right), \]

depending on coordinates \( x^i \), particle momentum four-vector \( p^k \), and on the random scalar variable \( \omega \), which is called random Langevin’s source. The distribution function satisfies the kinetic equation

\[ \frac{p^i}{mc} \left( \frac{\partial}{\partial x^i} - \Gamma^k_{il} \frac{\partial}{\partial p^k} \right) \Phi + \frac{\partial}{\partial p^i} \left( F^i \Phi \right) + \frac{\partial}{\partial \omega} \left( \mathcal{H} \Phi \right) = 0, \]

where \( \Gamma^k_{il} \) are the Christoffel symbols, associated with the background metric \( g_{ik} \). Characteristics equations, corresponding to (2), form three subsystems:

\[ \frac{dp^i}{ds} + \Gamma^i_{k} p^k \frac{dx^i}{ds} = F^i, \quad \frac{d\omega}{ds} = \mathcal{H}, \quad \frac{dx^i}{ds} = \frac{p^i}{mc}. \]

The first equation in (3) is the well-known equation of general relativistic particle dynamics under the influence of the force \( F^i \left( x^i, p^k, \omega \right) \). The second equation is
the evolutionary equation for the scalar random variable $\omega$, which is modeling the stochastic influence of the environment on the particle. Using (3), one can write the rate of evolution of the distribution function $\Phi$ in the form

$$\frac{d\Phi}{ds} = -\left[\frac{\partial F^i}{\partial p^i} + \frac{\partial H}{\partial \omega}\right] \Phi.$$  

(4)

The moments of the distribution function (1) can be obtained by averaging over five-dimensional statistical ensemble:

$$N^i(x) \equiv \int dP d\omega \cdot \Phi \cdot p^i, \quad T^{ik}(x) = \int dP d\omega \cdot \Phi \cdot p^i p^k.$$  

(5)

Denoting the term $\int d\omega \cdot \Phi$ by $f(x, p)$, one obtains from (5) the standard formulas for the particle number density vector $N^i(x)$ and for the stress-energy tensor $T^{ik}(x)$. The standard definition for the entropy flux vector:

$$S^i(x) \equiv -k_B c \int dP d\omega \cdot \Phi \cdot p^i \left[\ln h^3 \Phi - 1\right]$$  

(6)

is used. The transport equations for the particle number, stress-energy and for the entropy have the form, respectively:

$$\nabla_i N^i(x) = 0, \quad \nabla_k T^{ik}(x) = mc \int dP d\omega \cdot \Phi \cdot F^i,$$  

(7)

$$\sigma(x) \equiv \nabla_i S^i = k_B mc^2 \int dP d\omega \cdot \Phi \left[\frac{\partial F^i}{\partial p^i} + \frac{\partial H}{\partial \omega}\right].$$  

(8)

### 2.2 Cosmological Background

The spatially flat FLRW solution of Einstein’s equations is considered:

$$ds^2 = c^2 dt^2 - a^2(t) \left[(dx^1)^2 + (dx^2)^2 + (dx^3)^2\right],$$  

(9)

$$\ddot{a} = -\frac{4\pi G}{3} a (\mathcal{E} + 3\mathcal{P}), \quad \dot{a}^2 = \frac{8\pi G}{3} a^2 \mathcal{E}. $$  

(10)

Here $\mathcal{E}$ and $\mathcal{P}$ are the effective energy density and effective pressure, respectively. These quantities are known to be the eigenvalues of the conserved effective stress-energy tensor

$$T^{ik}_{(\text{eff})} = (\mathcal{E} + \mathcal{P})U^i U^k - g^{ik}\mathcal{P}, \quad \nabla_k T^{ik}_{(\text{eff})} = 0.$$  

(11)

The velocity four-vector is equal to $U^i = \delta^i_0$, as usual.

### 2.3 Force Field Modeling

The motion of Brownian particle in the framework of classical dynamics can be modeled by the force three-vector [28]

$$\vec{F} = -\lambda (\vec{v} - \vec{U}) + \kappa \vec{\xi}.$$  

(12)
The first term of this force is the Stokes friction force, it vanishes when the particle velocity three-vector \( \vec{v} \) coincides with the medium flow three-vector \( \vec{U} \). The second term is the stochastic Langevin force with random three-vector \( \vec{\xi} \).

The covariant generalization of the Stokes and the Langevin forces is well-known:

\[
F_{\text{Stokes}}^i = \lambda \left( \delta_k^i - \frac{p^j p_k}{(p^l p_l)} \right) U^k, \quad F_{\text{Langevin}}^i = \kappa \left( \delta_k^i - \frac{p^j p_k}{(p^l p_l)} \right) \xi^k. \tag{13}
\]

The projector with respect to particle four-momentum in the parentheses provides the forces to be orthogonal to \( p^i \): \( F^i p_i = 0 \).

In this paper we consider the force four-vector

\[
F^i(x^i, p^k, \omega) = \omega \frac{q}{mc^2} \left[ (p^l p_l) \delta_k^i - p^l p_k \right] U^k, \tag{14}
\]

where \( q \) is considered to be a function of cosmological time and is a subject of modeling. It was shown in [29] that the existence of the force with such a structure is compatible with the symmetry requirement

\[
\mathcal{L}_\zeta g_{ik} = 2\Psi \left( g_{ik} - \frac{\zeta_i \zeta_k}{(\zeta^l \zeta_l)} \right), \tag{15}
\]

formulated in terms of Lie derivative. The corresponding time-like vector \( \zeta^i \) is shown to exist for FLRW space-time, and the explicit example of the so-called generalized equilibrium states has been found in [29]. Starting from this fact we have studied in [14, 17] the consequences of the appearance of the antifriction force. The force (14) gives the Stokes force when \( \omega q m = \lambda \) and the Langevin force when \( \omega q m U^i = \kappa \xi^i \).

The function \( \mathcal{H} \) is modeled as follows:

\[
\mathcal{H}(x^i, p^k, \omega) = \omega \frac{\chi}{mc^2} (p^k U_k). \tag{16}
\]

The coefficients \( q \) and \( \chi \) are considered to be scalar functions, depending on the Ricci scalar \( R \), on the Hubble parameter \( H \) (which is proportional to the scalar of expansion \( H = \frac{1}{3} \nabla^k U_k \)) and on the scalar \( \tilde{R} \equiv R_{ik} U^i U^k \). The terms \( F^i \) and \( \mathcal{H} \) are linear in dimensionless random variable \( \omega \), i.e., one can indicate a stochastic Langevin’s source \( \omega \) as multiplicative one [30]. The function \( \mathcal{H} \) in the form (16) guarantees that the trivial value \( \omega \equiv 0 \) is a singular solution of the dynamic equation (3). Since

\[
\frac{\partial F^i}{\partial p^l} + \frac{\partial \mathcal{H}}{\partial \omega} = \frac{(p^k U_k)}{mc^2} (\chi - 3q\omega) \neq 0, \tag{17}
\]

the entropy production scalar \( \sigma \) (8) does not vanish.

### 2.4 Solution to the Characteristics Equations

Supposing that in the FLRW space-time (9) all the macroscopic functions depend on time only, and using the consequence of the characteristics equations

\[
\frac{cdt}{ds} = \frac{p^0}{mc}, \tag{18}
\]
one can rewrite the dynamic equations in terms of $t$

$$\dot{p}_\alpha = -q \omega p_\alpha, \quad \dot{\omega} = \chi \omega.$$  \hspace{1cm} (19)

The dot denotes the derivative with respect to time, and $\alpha = 1, 2, 3$. The solutions of (19) are the following:

$$p_\alpha = C_\alpha e^{-\Omega J(t,t_0)}, \quad J(t,t_0) \equiv \int_{t_0}^{t} dt q(t) I(t,t_0),$$  \hspace{1cm} (20)

$$\omega = \Omega \cdot I(t,t_0), \quad I(t,t_0) \equiv e^{\int_{t_0}^{t} dx(t)}.$$  \hspace{1cm} (21)

By definition $J(t_0,t_0) = 0$ and $I(t_0,t_0) = 1$, providing the relations $p_\alpha(t_0) = C_\alpha$ and $\omega(t_0) = \Omega$. Using the solution for $p_\alpha$ and the normalization condition $p^i p_i = m^2 c^2$, one can obtain the $p^0 = p_0$ component of the particle momentum:

$$p^0(t) = \sqrt{m^2 c^2 + a^{-2}(t) C^2 e^{-2\Omega J(t,t_0)}}.$$  \hspace{1cm} (22)

Here $C^2 \equiv C_1^2 + C_2^2 + C_3^2$, and the distribution function $\Phi$ takes the form

$$\Phi = \Phi_0(C^2, \Omega, t_0) e^{3\Omega J(t,t_0)} I^{-1}(t,t_0) \delta(\sqrt{(p,p)} - m^2 c^2).$$  \hspace{1cm} (23)

The transformation of the generalized volume element gives the formula

$$\int dP d\omega \cdot \Phi \{ \cdot \cdot \cdot \} = \int \frac{dC_1 dC_2 dC_3}{a^3(t)\sqrt{m^2 c^2 + a^{-2}(t) C^2 e^{-2\Omega J(t,t_0)}}} d\Omega \Phi_0 \{ \cdot \cdot \cdot \}.$$  \hspace{1cm} (24)

3 Macroscopic Properties of the Kinetic System

3.1 The Structure of Macroscopic Moments

Particle number is a conserved quantity, thus, one obtains

$$N^0 a^3(t) = \text{const}, \quad N^i(t) = N(t_0) \left[ a(t_0) / a(t) \right]^3 \delta^i_0.$$  \hspace{1cm} (25)

The stress-energy tensor takes the form

$$T_k^i(t) = \int \frac{d^3C \cdot d\Omega}{a^3(t)\sqrt{m^2 c^2 + a^{-2}(t) C^2 e^{-2\Omega J(t,t_0)}}} \Phi_0(C^2, \Omega, t_0) \cdot \tau^i_k(C^2, J(t,t_0)),$$  \hspace{1cm} (26)

where

$$\tau^i_k(C^2, J(t,t_0)) \equiv a^{-2} C^2 e^{-2\Omega J} \text{diag} \left\{ \left[ m c a |C|^{-1} e^{\Omega J} \right]^2 + 1, -\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3} \right\}.$$  \hspace{1cm} (27)

In general case the stress-energy tensor can not be represented in the analytic form, thus, for the sake of simplicity the ultrarelativistic limit is used to illustrate the idea. If the term, containing $m^2 c^2$ in the function (27), is negligible in comparison with the second term, and if the initial distribution function $\Phi_0$ is
multiplicative, i.e., \( \Phi_0(C^2, \Omega, t_0) = f_0(C^2, t_0) \cdot \Psi(\Omega, t_0) \), one can obtain the stress-energy tensor in the form:

\[
T_k(t) = A(t, t_0)W(t_0) \left[ \frac{a(t_0)}{a(t)} \right]^4 \text{diag} \left\{ 1, \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right\}.
\] (28)

Here

\[
A(t, t_0) \equiv \int d\Omega \Psi_0(\Omega, t_0)e^{-\Omega J(t, t_0)} \equiv \langle e^{-\Omega J} \rangle
\] (29)

is a statistical factor. The energy density scalar

\[
W(t_0) \equiv \int \frac{d^3C}{a^3(t_0)}|C|f_0(C^2, t_0)
\] (30)

is known to be equal to

\[
W_{\text{bosons}}(t_0) = \frac{8k_B^4\alpha^5}{15\hbar^3c^3}T^4(t_0), \quad W_{\text{fermions}}(t_0) = \frac{7k_B^4\alpha^5}{30\hbar^3c^3}T^4(t_0)
\] (31)

for ultrarelativistic (massless) bosons and for ultrarelativistic (massless) fermions. The entropy production scalar

\[
\sigma(t) = k_BcN(t) \int d\Omega \Psi_0(\Omega, t_0)(\chi - 3q\Omega I)
\] (32)

happens to be proportional to particle density scalar \( N(t) \).

### 3.2 Properties of the Averaged Macroscopic Moments

Let us suppose that the distribution over \( \Omega \) can be described by the Gaussian function

\[
\Psi_0(\Omega, t_0) = \frac{1}{\sqrt{\pi D}}e^{-\Omega^2/D},
\] (33)

where \( D \) is a dispersion parameter of this distribution at the moment \( t_0 \). For this case the direct calculations give

\[
\sigma(t) = k_Bc\chi N(t)
\] (34)

and

\[
A(t, t_0) = e^{\frac{1}{4}D^2J^2(t, t_0)}.
\] (35)

Thus, the properties of the macroscopic moments of the distribution function are predetermined by the properties of the functions \( J(t, t_0) \) (20) and \( I(t, t_0) \) (21).

### 3.3 Effective Stress-Energy Tensor

The relation (7) demonstrates that the energy and the momentum of the kinetic system do not conserve, since the balance equation has the source term in the right-hand side. Calculating this source term using the expression (14) for the force \( F^i \) one obtains

\[
\nabla_k T^{ik} = qI(t, t_0) \int \frac{d^3C \Omega d\Omega \Phi_0}{ca^3(t)\sqrt{m^2c^2 + a^{-2}(t)C^2e^{-2\Omega J}}}[m^2c^2U^i - p^i(U_k p^k)].
\] (36)
For the ultrarelativistic model with multiplicative distribution function this expression is simplified:

\[ \nabla_k T^{ik} = -\delta_0^i qI(t, t_0) \left( \frac{a(t_0)}{a(t)} \right)^4 \int \Omega d\Omega \Psi_0(\Omega, t_0) e^{-\Omega J} \int \frac{d^3C}{ca^4(t_0)} |C| f_0(C^2, t_0). \tag{37} \]

For the Gaussian function (33) the integrals in the balance equation take the form

\[ \nabla_k T^{ik} = \delta_0^i qI(t, t_0)W(t_0) \left( \frac{a(t_0)}{a(t)} \right)^4 \frac{D^2}{2c} J(t, t_0) e^{\frac{D^2}{2} J^2(t, t_0)}. \tag{38} \]

For \( i = 1, 2, 3 \) these equations are trivial, the only informative equation is the one for \( i = 0 \). This equation (the so-called scalar balance equation) may be written as follows:

\[ \dot{W}(t) + 3H[W(t) + P(t)] = qI(t, t_0)W(t_0) \left( \frac{a(t_0)}{a(t)} \right)^4 \frac{D^2}{2} J(t, t_0) e^{\frac{D^2}{2} J^2(t, t_0)}, \tag{39} \]

where \( W(t) = T^{00} \). The equation (39) takes the form of conservation law

\[ \dot{W} + 3H(W + P) = 0, \tag{40} \]

if we introduce an effective pressure \( P \equiv P + \Pi \), where

\[ \Pi \equiv -qI(t, t_0)W(t_0) \left( \frac{a(t_0)}{a(t)} \right)^4 \frac{D^2}{6H(t)} J(t, t_0) e^{\frac{D^2}{2} J^2(t, t_0)}. \tag{41} \]

is considered to be non-Pascal pressure. The reconstruction of the effective stress-energy tensor (11) is possible if we put

\[ E = W(t) = W(t_0) \left( \frac{a(t_0)}{a(t)} \right)^4 e^{\frac{D^2}{2} J^2}, \tag{42} \]

\[ P = \frac{1}{3} W(t_0) \left( \frac{a(t_0)}{a(t)} \right)^4 e^{\frac{D^2}{2} J^2} \left[ 1 - qI(t, t_0) \frac{D^2}{2H(t)} J(t, t_0) \right]. \tag{43} \]

The formulas (42) and (43) manifest the following feature. The coefficient \( \left( \frac{a(t_0)}{a(t)} \right)^4 \) describes the decreasing of the average energy and pressure, which corresponds to the standard law of evolution of ultrarelatistic FLRW model. The coefficient \( e^{\frac{D^2}{2} J^2} \) describes the increase of the mentioned quantities due to the gas (fluid) stochastic self-interaction. The interplay between these two processes could clarify the question whether \( E \) is the increasing or decreasing quantity. The sign of \( P \) can be both positive or negative during the different time intervals. The expansion of the Universe happens to be accelerated (\( \ddot{a} > 0 \))(see, (10)), when

\[ E + 3P = 2W + 3\Pi < 0. \tag{44} \]

It is possible when

\[ q(t)I(t, t_0) \frac{D^2}{4H(t)} J(t, t_0) > 1. \tag{45} \]

Let us discuss this inequality in detail.
3.4 Modeling of the $q$ and $\chi$ Functions

The functions $q$ and $\chi$ can be modeled using $R$, $\dot{R}$ and $H$ functions:

$$R = -6 \left[ \frac{\ddot{a}}{a} + \left( \frac{\dot{a}}{a} \right)^2 \right] = -6(\dot{H} + 2H^2), \quad (46)$$

$$\dot{R} = R_{00} = -3\frac{\ddot{a}}{a} = -3(\dot{H} + H^2), \quad H \equiv \frac{\dot{a}}{a}. \quad (47)$$

Taking into account these relations, $q$ and $\chi$ can be rewritten as functions of $H$ and $\dot{H}$. Since $q$ and $\chi$ have a dimension of the inverse time, it is proposed to use the following simplest structures:

$$\chi(t) = \chi_0 H(t) + \chi_1 \frac{\dot{H}}{H}, \quad q(t) = q_0 H(t) + q_1 \frac{\dot{H}}{H}. \quad (48)$$

Here $\chi_0$, $\chi_1$, $q_0$ and $q_1$ are dimensionless constants. From (21) one obtains explicitly

$$I(t, t_0) = \left[ a(t) \right]^{\chi_0} \left[ \frac{H(t)}{H(t_0)} \right]^{\chi_1}. \quad (49)$$

The function $J(t, t_0)$

$$J(t, t_0) = \int_{t_0}^{t} dt \left[ a(t) \right]^{\chi_0} \left[ \frac{H(t)}{H(t_0)} \right]^{\chi_1} \left[ q_0 H(t) + q_1 \frac{\dot{H}}{H} \right] \quad (50)$$

can be calculated explicitly for two well-known cases. These results are presented briefly in the following subsections 3.4.1. and 3.4.2.

3.4.1 Power-Law Expansion

When

$$\frac{a(t)}{a(t_0)} = \left( \frac{t}{t_0} \right)^\gamma, \quad H(t) = \frac{\gamma}{t}, \quad \dot{H} = -\frac{\gamma}{t^2}, \quad (51)$$

we obtain immediately the following functions:

$$J(t, t_0) = \left( \frac{\gamma q_0 - q_1}{\gamma \chi_0 - \chi_1} \right) [I(t, t_0) - 1], \quad I(t, t_0) = \left( \frac{t}{t_0} \right)^{\gamma \chi_0 - \chi_1}. \quad (52)$$

The predictions concerning the accelerated expansion depend on the sign of the parameter $\gamma \chi_0 - \chi_1$. Let us consider three standard cases.

First case: $\gamma \chi_0 > \chi_1$.

The inequality (45) is satisfied when

$$\left( \frac{t}{t_0} \right)^{\gamma \chi_0 - \chi_1} \geq \frac{1}{2} + \frac{\sqrt{1 + \frac{4\gamma(\gamma \chi_0 - \chi_1)}{D^2(\gamma q_0 - q_1)^2}}}{1 + \frac{4\gamma(\gamma \chi_0 - \chi_1)}{D^2(\gamma q_0 - q_1)^2}}. \quad (53)$$

Second case: $\gamma \chi_0 < \chi_1$. 

When the discriminant is positive, i.e., \(16\gamma|\gamma\chi_0 - \chi_1| < D^2(\gamma q_0 - q_1)^2\), the inequality (45) takes place, if

\[
\left[\frac{1}{2} - \frac{1}{4} - \frac{4\gamma|\gamma\chi_0 - \chi_1|}{D^2(\gamma q_0 - q_1)^2}\right] < \left(\frac{t}{t_0}\right)^{-|\gamma\chi_0 - \chi_1|} < \left[\frac{1}{2} + \frac{1}{4} - \frac{4\gamma|\gamma\chi_0 - \chi_1|}{D^2(\gamma q_0 - q_1)^2}\right].
\]

**Third case: \(\gamma\chi_0 = \chi_1\).**

For this special case the formulas (52) give

\[
I(t, t_0) = 1, \quad J(t, t_0) = (\gamma q_0 - q_1) \cdot \log\left(\frac{t}{t_0}\right).
\]

The inequality (45) is satisfied when

\[
\log\left(\frac{t}{t_0}\right) > \frac{4\gamma}{(\gamma q_0 - q_1)^2 D^2}.
\]

### 3.4.2 Exponential Expansion

When

\[
a(t) = e^{H_0(t-t_0)}, \quad H(t) = H(t_0) = H_0, \quad \dot{H} = 0,
\]

the integration gives

\[
J(t, t_0) = \frac{q_0}{\chi_0} [I(t, t_0) - 1], \quad I(t, t_0) = e^{\chi_0 H_0(t-t_0)}.
\]

**First case: \(\chi_0 > 0\).**

The inequality (45) is satisfied when

\[
(t - t_0) > \frac{1}{\chi_0 H_0} \log \left[\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{4\chi_0}{D^2 q_0^2}}\right].
\]

**Second case: \(\chi_0 < 0, |\chi_0| < \frac{D^2 q_0^2}{16}\).**

The inequality is satisfied when

\[
\left[\frac{1}{2} - \frac{1}{4} - \frac{4|\chi_0|}{D^2 q_0^2}\right] < e^{-|\chi_0| H_0(t-t_0)} < \left[\frac{1}{2} + \frac{1}{4} - \frac{4|\chi_0|}{D^2 q_0^2}\right].
\]

**Third case: \(\chi_0 = 0\)**

The formulas (58) give

\[
I(t, t_0) = 1, \quad J(t, t_0) = q_0 H_0(t-t_0).
\]

The inequality (45) is satisfied when

\[
(t - t_0) > \frac{4}{q_0^2 H_0 D^2}.
\]
4 Discussion

Here the model of evolution of the self-interacting gas (fluid) Universe is presented. The model is based on the suggestion that the expansion of the Universe gives rise to the specific non-equilibrium self-interaction in the kinetic system. The force effecting the particle can be classified as a Stokes friction force, since it disappears when particle co-moves with the system as a whole. The force under consideration can be indicated as a Langevin force, since it is proportional to the random scalar variable, and the particle motion can be classified as a sort of Brownian motion. Finally, this force can be called tidal or curvature induced force, since it depends on Hubble rate and its derivative, which are known to form the Riemann tensor, Ricci tensor, and Ricci scalar. In this sense one can say that we deal with curvature induced Stokes-Langevin force.

In the subsection 3.4. the time intervals are found, during which the Universe expands with acceleration. The explanation of such a behaviour may be the following. The self-interaction in the gas (fluid) produces the growth of the averaged energy and pressure in the system, in contrast to the decreasing of these parameters due to the expansion. The interplay of such conflicting tendencies provides non-monotonic behaviour of the $\dot{a}$ function, and the model admits the existence of both periods of evolution: expansion with acceleration and expansion with deceleration. The model under consideration requires to investigate the next step - the estimation of the parameters, which have been phenomenologically introduced into the force term. This task is planned to be fulfilled in the next paper [31].

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References

[1] Turner, M. S., and Riess, A. Do SNe Ia provide direct evidence for past deceleration of the Universe? (astro-ph/0106051).

[2] Riess, A.G. et al. (1998). Astron. J. 116, 1009; Schmidt, B. et al. (1998). Astrophys. J. 507, 46; Perlmutter, S. et al. (1999). Astrophys. J. 517, 565; Riess, A.G. (astro-ph/0005229); de Bernardis, P. et al. (2000). Nature 404, 955; Hanany, S. et al. (2000). Astrophys. J. Lett. 545, 5.

[3] Sahni, V., and Starobinsky, A.A. (2000). Int. J. Mod. Phys. D 9, 373.

[4] Ellis, J. (2003). Phil.Trans.Roy.Soc.Lond. A361, 2607.

[5] Ratra, B., and Peebles, P.J.E. (1988). Phys. Rev. D 37, 3406; Wetterich, C. (1988). Nucl. Phys. B 302, 668.
[6] Frieman, J.A., Hill, C.T., Stebbins, A., and Waga, I. (1995). Phys. Rev. Lett. 75, 2077; Caldwell, R.R., Dave, R., and Steinhardt, P.J. (1998). Phys. Rev. Lett. 80, 1582; Zlatev, I., Wang, L., and Steinhardt, P.J. (1999). Phys. Rev. Lett. 82, 986; Faraoni, V. (2000). Phys. Rev. D 62, 023504.

[7] Amendola, L. (2000). Phys. Rev. D 62, 043511; Amendola, L., and Tocchini-Valentini, D. (2001). Phys. Rev. D 64, 043509.

[8] Zimdahl, W., Pavón, D., and Chimento, L.P. (2001). Phys. Lett. B 521, 133; Zimdahl, W., and Pavón, D. [astro-ph/0210484].

[9] Baccigalupi, C., Matarrese, S., and Perrota, F. (2000). Phys. Rev. D 62, 123510.

[10] Uzan, J.-Ph. (1999). Phys. Rev. D 59, 123510; Amendola, L. (1999). Phys. Rev. D 60, 043501; Chiba, T. (1999). Phys. Rev. D 60, 083508; de Ritis, R., Marino, A.A., Rubano, C., and Scudellaro, P. (2000). Phys. Rev. D 62, 043506; Boisseau, B., Esposito-Farese, G., Polarski, D., and Starobinsky, A.A. (2000). Phys. Rev. Lett. 85, 2236.

[11] Capozziello, S. (2002). Int. J. Mod. Phys. D 11, 483.

[12] Frolov, A., Kofman, L., and Starobinsky, A. (2002). Phys. Lett B 545, 8.

[13] Zimdahl, W., and Balakin, A.B. (2001). Phys. Rev. D 63, 023507.

[14] Zimdahl, W., Schwarz, D.J., Balakin, A.B., and Pavón, D. (2001). Phys. Rev. D 64, 063501; Zimdahl, W., Balakin, A.B., Schwarz D.J., and Pavón, D. (2002). Grav. Cosmol. 8(Suppl.II), 158.

[15] Zimdahl, W., and Balakin, A.B. (2002). ENTROPY 4, 49. [gr-qc/0109081]

[16] Schwarz, D.J. [astro-ph/0209584].

[17] Balakin, A.B., Pavó’n, D., Schwarz, D.J., and Zimdahl, W. (2003). New J. Phys. 5, 85.1 - 85.14. [astro-ph/0302150].

[18] Laptev, B.L. (1958). In Geometrija i teoriya otositel’nosti, Kazan University Press, Kazan, p.75 (in Russian).

[19] Vlasov A.A. (1966). Statistical Distribution Functions, Nauka, Moscow, (in Russian).

[20] Ehlers, J. (1971). In General Relativity and Cosmology, Sachs, B.K. (ed.), Academic Press, New York, pp. 1-70.

[21] Stewart, J.M. (1971). Non-equilibrium Relativistic Kinetic Theory, Springer, New York.

[22] de Groot, S.R., van Leeuwen, W.A., and van Weert, Ch. G. (1980). Relativistic Kinetic Theory, North Holland, Amsterdam.

[23] Hakim, R. (1968). J. Math. Phys. 9(1), 116-130.
[24] Israel, W. (1978). *Gen. Relat. Grav.* **9**(5), 451-468.

[25] Feldman, Y., and Tauber, G.E. (1980). *Gen. Relat. Grav.* **12**(10), 837-856.

[26] Elze, H.T., and Heinz, U. (1989). *Phys.Rep.* **183**, 81.

[27] Litim, D.F., and Manuel, C. (2002). *Phys.Rep.* **364**, 451.

[28] Balescu, R. (1975). *Equilibrium and Non-Equilibrium Statistical Mechanics*, Wiley, New York.

[29] Zimdahl, W., and Balakin, A.B. (1998). *Phys.Rev.D* **58** 063503 (1-10).

[30] Landa, P.S., and McClintock, P.V.E. (2000). *Phys. Rep.* **323**, 1-80.

[31] Zimdahl, W., and Balakin, A.B. (in preparation).