The Minimal Supersymmetric SO(10) GUT has developed into a fully realistic theory in which not only are the gauge couplings unified but the known fermion spectrum and mixing matrices could fit accurately using the latitude introduced by inclusion of quantum corrections to the GUT-effective MSSM-SM matching conditions. The fits yield predictions about the nature of the sparticle spectrum on the basis of the required threshold corrections. This indicated a necessarily large value for $A_0$ in 2008: well before Higgs discovery at 126 GeV made it a commonplace assumption. GUT scale threshold corrections to the normalization of the emergent effective MSSM Higgs ameliorate the long standing Susy GUT puzzle of fast dimension five operator mediated proton decay. Numerical investigation indicates that B-violation rates below or near the current experimental upper limits are feasible in fully realistic models. Our results imply that UV completion models with large numbers of fields, like Kaluza-Klein models or String Theory, must be able to compute threshold corrections to be considered quantitative theories and not just fables. Required improvements in the fitting procedure are discussed. A generalization of the NMSGUT by gauging the flavour symmetry of the kinetic terms, while retaining renormalizability and the successful MSGUT symmetry breaking patterns, may allow dynamical generation of the observed Yukawa structure of the MSSM via the spontaneous breaking of the full gauge symmetry down to the MSSM at the unification scale. Focus on the emergence of the MSSM Higgs from the multiple Higgs doublets in the GUT thus provides a crucial window to view the energetically remote UV dynamics specified in fully calculable and realistic MSGUTs.
1. Introduction

Fundamental Unity of Being has always loomed large in both Greek (Plotinus) and Indian (Vedanta) Metaphysics. Coming from the Punjab (fabled in Greece, I was told, since the Alexandrian conquest, as πενταποταμία) which was the meeting ground and melting pot of Greek and Indian civilizations, it was a special privilege and pleasure to participate in the Planck2015 conference at the University of Ioannina and speak on Unification: to which Indians and Greeks have alike contributed much.

Although it amounts to no more than a simple minded extension of the Standard model, Grand Unification[1] has stood the test of time as the most convincing scenario for the unification of forces and types of matter even while signals conforming to the expectations it arouses, most notably to proton decay and Supersymmetry(Susy), remain elusive. Recall that as far back as 1982 [2] it was realized that, with a larger value of the Weinberg angle than the then current experimental value (\(\sin^2 \theta_W \approx 0.215\)) and a top quark (then still undiscovered) much more massive than the then established limit (\(m_{top} > 20\) GeV), the gauge couplings of Minimal Supersymmetric SM(MSSM) with a single pair of doublets, but not the SM nor any other variant of the MSSM, and a Susy breaking scale in the TeV (up to logarithmic precision), unify accurately at \(M_{X}^0 \approx 2 \times 10^{16}\) GeV. Precision measurements at LEP eventually confirmed that the Weinberg angle had the required enhanced value (\(\sin^2 \theta_W \approx 0.23\)) and the top quark was also finally discovered in the range predicted[2].

These data also allowed an early appreciation[3] that the(tree level) Yukawa couplings of the third generation in the MSSM can actually converge at the unification scale \(M_{X}^0\): provided the ratio of MSSM Higgs doublet vevs \(<H>/<\tilde{H}> = \tan \beta\) was around 50. Such a convergence is to be expected in SO(10) GUTs since they place an entire MSSM fermion family plus a conjugate neutrino field (\(\nu_{cL}\)) within a single 16-plet of Spin(10). Spin(10)-less precisely SO(10)- unification further points towards very small neutrino masses since the gauge singlet \(\nu_{cL}\) can be expected to have a large Majorana mass from GUT scale vevs, implying that the effective light(left handed) neutrino masses will be very small. In the period 1996-1998, when the neutrino mass magnitudes were still unknown we investigated Minimal Supersymmetric Left Right models(MSLRMs)[4] with view to examining their support for neutrino masses and dark matter. In these models ad-hoc R-parity(\(R_p = (-)^{3(B-L)+2S}\)) introduced for proton longevity in the MSSM is effectively part of the gauge group since the Abelian factor of the Left Right gauge group is \(U(1)_{B-L}\). When MSLRMs are set up with the SU(2) triplets required to implement small Seesaw masses via \(SU(2)_R \times U(1)_{B-L} \rightarrow U(1)_Y\) breaking it turns out to be phenomenologically necessary and inevitable that R-parity is preserved to the lowest energies[4]. Then the lightest sparticle (LSP) must be stable, making it an excellent Dark matter(DM) candidate due to the “WIMP miracle”[5]. This implements[3] a deep connection[7] between LR symmetry, Supersymmetry, Neutrino masses, Dark matter and Susy SO(10) GUTs! The discovery of the long sought for neutrino oscillations at the Super-Kamiokande neutrino oscillation/nucleon decay telescope/microscope[8], indicating milli-eV neutrino masses, following soon after Top quark discovery, kept up the drumbeat of discoveries pointing towards Supersymmetric SO(10) Grand Unification as a highly natural and favoured framework, restrictive and ample at the same time, for Grand Unification of the known gauge forces and matter fields. Amazingly even the recent placement of the SM capstone, namely the discovery of a SM like Higgs at 125 GeV in 2012, did not produce any contradiction with the
low energy effective theory being the MSSM: although by requiring that the soft Susy breaking parameters \( \{ m_F, A_0 \} \simeq M_{\text{Susy}} \) lie in the Multi-TeV range, it did debunk the ‘papal’ claims that the \( M_S \) would only -because of ‘naturality’ - be of TeV scale. Interestingly, the detailed development of a realistic Susy SO(10) model proposed \[9\] at the beginning of the Susy Unification story and revived \[13\] in the SO(10) resurgence triggered by Super-Kamiokande as the Minimal (parameter counting) realistic Susy GUT had already (in 2008 \[11\]) indicated that \( M_{\text{Susy}} \) and more specifically the trilinear parameter \( A_0 \) would be large (as required by a 125 GeV MSSM Higgs)! Finally the determination\[11\] around 2005 that minimal SO(10) fits of the fermion mass and mixing data would only be successful if the PMNS mixing angle \( \theta_{13} \) -then quite unknown - was quite large (in the range of 6-12 degrees) was confirmed\[12\] by the results of Daya Bay reactor neutrino experiments in 2012.

With this stunning record of the MSSM and (Minimal Susy) SO(10)GUT(MSGUT) it was appropriate to develop the quantum structure of the MSGUT in the full detail possible due to the explicit solubility of its spontaneous symmetry breaking\[13\] and thereby of the complete superheavy spectrum\[24, 25, 14\]. Over the last decade these studies passed significant milestones \[14, 10, 15, 16, 17\]. Staying with the minimal theory without invoking additional discrete symmetries and Higgs/matter multiplets, and in particular by focussing upon the implications of retaining only a single pair of light doublets out of the plethora of doublets generically present in Susy SO(10) and other realistic (but less economical) GUTs, it has revealed that some of the recalcitrant deficiencies besetting Susy GUTs -like the lack of constraining/falsifying predictions for the plethora of soft Susy breaking parameters, the rapidity of dimension 5 mediated proton decay and the lack of insight into the fermion flavour hierarchy- might also receive new and intriguing answers. The existence of only a single pair of light Higgs doublets provides a magic microscope into the innards of the UV completion. By the nature of their parentage in the multiple Higgs multiplets of the GUT, the light Higgs are privy to the inner working of the GUT Higgs dynamics: which is swept into the observed renormalizable parameters as far as the other light fields are concerned. Due to their multiple parentage among the heavy Higgs multiplets, the superlight (weak scale) eigenmodes with MSSM Higgs doublet quantum numbers can be subject to wave function renormalization that is strong enough to take the fields close to “dissolution”\[17\] i.e. \( Z_{H,H} \simeq 0 \). Then the canonically renormalized Higgs doublets of the effective MSSM participate in Yukawa couplings to the matter fermions which are much enhanced over the values of the tree level GUT couplings on a “Higgs dissolution edge” (a high dimensional sub-manifold of the multidimensional GUT superpotential parameter space where \( Z_{H,H} \simeq 0 \)). The GUT matter Yukawa couplings required to account for the observed fermion masses are thus much diminished. Since they also enter quadratically in the proton decay amplitudes it follows that the proton lifetime is much enhanced for realistic SO(10) Yukawa couplings. Thus ‘doublet-triplet’ splitting, long regarded as the bane of GUTs, actually seems to resolve their longstanding\[18\] structural difficulties with dimension five operators by a completely novel and generic mechanism. Finally, given that a completely acceptable UV completion of the MSSM may be available in the NMSGUT, we asked whether the sublation of the MSSM data in the NMSGUT is not the optimal starting point to attack the problem of flavour. Dynamical generation of the successful MSGUT matter couplings by promotion of the flavour symmetry to a gauged \( O(N_f) \) symmetry broken by the MSSM GUTs (promoted to irreps of the flavour symmetry) and still under the crucial constraint that a single pair of Higgs doublets remain light in the effec-
Higgs is the Window

tive MSSM is feasible[20, 21]. Surprisingly, the fermion hierarchies generated tend to carry the features we associate with the MSSM: small CKM and large PMNS mixings, hierarchical fermion Yukawas and so on. In this proceeding we run through the main issues of our current work these two areas. Details may be found in [10, 17, 20, 21].

2. Minimal Susy SO(10) Grand Unified Model

The model in question was introduced long ago[9]. Besides the 3 Spin(10) 16-plets containing matter fields and the 45-plet of gauge vector multiplets the MSGUT/NMSGUT utilizes antisymmetric $\Phi_{ijkl}(210), \Sigma_{ijklm}(126), \Sigma_{ijklm}(\overline{126})$ as Higgs multiplets to break the Susy SO(10) symmetry in one step (as is necessary to avoid pseudo-goldstone problems[2, 22]) to the MSSM. In addition to these “AM” multiplets which perform the GUT symmetry breaking and generate neutrino masses($\overline{126}$), one has also the “FM” multiplets $10, 120$ responsible only for charged fermion masses. Interestingly this set makes up the full complement of antisymmetric irreps possible in SO(10) ! The superpotential is built from mass terms (SO(10) contractions are indicated by c-dots or powers)

$$m : 210^2 ; \quad M : 126 \cdot \overline{126} ; \quad M_H : 10^2 ; \quad m_\Theta : 120^2$$

and trilinear couplings :

$$\lambda : 210^3 ; \quad \eta : 210 \cdot 126 \cdot \overline{126} ; \quad \rho : 120 \cdot 120 \cdot 210 ; \quad k : 10 \cdot 120 \cdot 210$$

$$\gamma \oplus \bar{\gamma} : 10 \cdot 210 \cdot (126 \oplus \overline{126}) ; \quad \zeta \oplus \bar{\zeta} : 120 \cdot 210 \cdot (126 \oplus \overline{126})$$

$$16_A \cdot 16_B : (h_{AB} 10 + f_{AB} 126 + g_{AB} 120)$$

The couplings $h, f(g)$ are complex (anti)-symmetric matrices flavour space because of the properties of the SO(10) Clifford algebra. Either $h$ or $f$ can be chosen real and diagonal using the $U(3)$ flavor symmetry of the matter kinetic terms. Five phases say of $m, M, \lambda, \gamma, \bar{\gamma}$ may be set to zero by phase conventions. Matter Yukawa couplings contain 21 real parameters. To implement the crucial “one light Higgs pair ” consistency condition (a.k.a. doublet-triplet splitting by fine tuning) $M_H$ is fine tuned to ensure that the $6 \times 6$ Higgs doublet mass matrix $\mathcal{H}$ has zero determinant(and thus a pair of null left and right eigenvectors) keeping two Higgs doublets ($H, \overline{H}$) of the effective MSSM light. This leaves 23 magnitudes and 15 phases as parameters. $H, \overline{H}$ are a mixture of the (6 pairs of the MSSM type) doublet fields in the GUT. The mixture parameters(“ Higgs fractions” $\alpha_a, \bar{\alpha}_a, a = 1..6$ [13, 14]) are functions of the NMSGUT superpotential parameters which enter the left and right null eigenvectors of $\mathcal{H}$. The symmetry breaking vevs

$$( (15, 1, 1))_{210} : a \quad ( (15, 1, 3))_{210} : w \quad ( (1, 1, 1))_{210} : p$$

$$( (10, 1, 3))_{126} : \sigma \quad ( (10, 1, 3))_{126} : \overline{\sigma}$$

preserve SUSY from D Terms violation by $|\sigma| = |\overline{\sigma}|$, while the 4 coupled F term equations $F_{a,p,w,\sigma/\overline{\sigma}} = 0$ are then analytically soluble[13]! The GUT scale vevs( in units of $m/\lambda$) are known functions of a complex variable $x$ which solves the cubic ($\tilde{\xi} = \frac{\lambda M}{\eta m}$)

$$8x^3 - 15x^2 + 14x - 3 + \xi (1 - x)^2 = 0$$

(2.3)
592 Higgs Chiral and 33 Majorana heavy gauge supermultiplets occur in 22 complex (pairs) and 4 real MSSM representation types. Explicit solution of SSB allows explicit determination of their mass matrices and eigenvalues. The complete spectrum of the 26 different MSSM irrep types and tree level effective Superpotential is available [10] and extends the MSGUT result [23, 14, 24, 25]. This allows calculation of the superheavy threshold correction to the gauge and MSSM Yukawa couplings due to circulation of super heavy fields in the self energy diagrams for the light fields and the consequent finite renormalizations.

From the point of view of the consistency of the GU picture, the mass matrix $H$ of the six pairs of MSSM type doublet irreps carrying $[1,2,\pm1]$ of $SU(3) \times SU(2)_L \times U(1)_Y$ is most crucial. These doublets come from all the Higgs representations present in the theory and mix together thoroughly. As a result of being woven so intimately into the innards of the superheavy spectra and couplings and yet being constrained by the necessity that Supersymmetric gauge unification yield the super light Higgs doublet pair which is the $sine qua non$ of Susy unification, the light MSSM Higgs provides a unique vantage point to view the inner workings of the GUT. In other words the composition of the Higgs is the portal of irruption of the superheavy GUT world into our low energy reality. Pursuing this line of thought further we are led to a detailed investigation of the effects of superheavy fields on the couplings of the superlight fields.

3. GUT scale threshold corrections and Baryon decay rate

Superpotential parameters renormalize only by wave function corrections. This implies [26] that the threshold corrections to the matching conditions between gauge and Yukawa couplings can be calculated simply by considering the (finite) wavefunction renormalization of the light fields due to loops containing (at least) one heavy supermultiplet. Since the complete decomposition of the NMSGUT trilinear vertices by MSSM quantum numbers was already obtained [13, 10] when calculating the mass spectra, the one loop effects are straightforward to compute. Threshold corrections to the gauge coupling matching conditions depend upon the ratios of masses. There is a freedom to choose $M_X$ within a small range around the MSSM unification scale $M_0 X = 2 \times 10^{16}$ GeV as well as the SO(10) coupling $\alpha_G(M_X)$. The spread of mass eigenvalues allows cancellation among threshold corrections and thus a sensible result: belying [14] the expectation [27] that such corrections make Susy SO(10) unification meaningless.

The threshold corrections to the Yukawa couplings of the MSSM are much more tedious and run [17] to some 1500 sums each of which runs over one or more multiplicity indices of the 26 superheavy MSSM irrep types which mix.

For a generic light field $\Phi_i$ the one loop wave function constants $(Z = 1 - H)$ have form:

\[
\mathcal{H}_i^j = - \frac{g_{10}^2}{8\pi^2} \sum_{\alpha,k} Q_{ik}^\alpha Q_{kj}^\alpha F(m_\alpha, m_k) + \frac{1}{32\pi^2} \sum_{kl} Y_{ikl}^l Y_{jkl}^l F(m_k, m_l) \tag{3.1}
\]

here $A_{10}^\mu$ is a SO(10) heavy gauge boson $Q^\alpha$ the associated generator $(g_{10}$ is the SO(10) gauge coupling. $m_{\alpha,k}$ are heavy gauge and chiral multiplet masses). The generic Yukawa couplings are defined by the superpotential $W = \frac{1}{6} Y_{ijk} \Phi_i \Phi_j \Phi_k$. The sums in eqn. (3.1) run over pairs of intra-loop fields with at least one member superheavy and $F(m_\alpha, m_k)$ is a standard 1-loop Passarino-Veltman
function:
\[ F_{12}(M_A, M_B, Q) = \frac{1}{(M_A^2 - M_B^2)}(M_A^2 \ln \frac{M_A^2}{Q^2} - M_B^2 \ln \frac{M_B^2}{Q^2}) - 1 \] (3.2)
which reduces to just \[ F_{11}(M_A, Q) = F_{12}(M_A, 0, Q) = \ln \frac{M_A^2}{Q^2} - 1 \] when one field is light \( (M_B \to 0) \).

The simplest contribution to a Higgs line correction is in the \( W[6, 3, 2/3] \otimes \bar{Y}[6, 2, 13] \) channel (see [4] for the alphabetical notation for MSSM irreps which occur in the MSGUT):
\[ K_{\bar{Y}} = \left| -\gamma U_{11}^H + \frac{2\eta}{\sqrt{3}} U_{21}^H - \zeta U_{51}^H + \frac{i\zeta}{\sqrt{3}} U_{61}^H \right|^2 F_{12}(m^W, m^Y, Q) \] (3.3)

Here \( U^H \) are matrices that participate in the diagonalization of the Higgs doublet mass matrices. When multiple copies of the running heavy multiplets mix there are diagonalization matrices for those mass matrices present as well. An example of a gauge correction (for the matter light line \( \bar{u} \), \( m_\lambda \) are gaugino masses) is
\[
(16\pi^2)\mathcal{X}^\bar{u} = -2\varepsilon_{10}^2(0.05F_{11}(m_{\lambda_g}, Q) + F_{11}(m_{\lambda_\mu}, Q) + F_{11}(m_{\lambda_\mu}, Q) + 4F_{11}(m_{\lambda_e}, Q) \\
+ 2F_{11}(m_{\lambda_e}, Q))
\] (3.4)

The (over 1500) other terms in the the dressing of the light lines entering an MSSM vertex are considerably more involved and we refer the reader to the original work for details [17].

The dressing of light by heavy lines implies [26] a finite wave function renormalization in the fermion and Higgs Kinetic terms
\[ \mathcal{L} = \sum_{A,B} \left[ f_A^\dagger(Z_f)A^B f_B + f_A^\dagger(Z_f)A^Bf_B \right] + H^T Z_H H + \bar{T}^T Z_R T |_D + .. \] (3.5)

Diagonalizing the matter wavefunction dressing matrix by Unitary matrices \( U_{Z_1}, \bar{U}_{Z_1} (U^\dagger ZU = \Lambda_Z) \), one defines a new basis to put the light Kinetic terms in canonical form So the MSSM Yukawa match the dressed Yukawa couplings of the new (canonical Kinetic term) light fields \( \bar{Y}_f \):
\[
\bar{Y}_f = \Lambda_{Z_1}^{\dagger} U_{Z_1}^T \frac{Y_f}{\sqrt{Z_H}} \bar{U}_{Z_1} \Lambda_{Z_1}^{\dagger} = U_{Z_1}^T \frac{Y_f}{\sqrt{Z_H}} \bar{U}_{Z_1}
\] (3.6)

and not to the original SO(10) tree level ones.

There are precisely 26 different combinations of the 26 MSSM representation types that run in the loops on the Higgs lines in the MSSM matter fermion Yukawa vertices. The SO(10) Yukawa couplings \( (h, f, g)_{AB} \) which enter the light field Yukawa couplings to the light Higgs also enter the coefficients of the \( d = 5 \) baryon decay operators in the effective dimension 4 superpotential obtained by integrating out the heavy chiral supermultiplets that mediate baryon decay [23, 14, 10]. These operators do not have external light Higgs lines and as a result are not boosted by the Higgs dressing. Thus suppression of matter field Yukawas by Higgs on the dissolution edge \( (Z_{H, B} \sim 0) \) self-consistently lowers the SO(10) matter Yukawas \( (f, h)_{AB} \) causing their dimension 5 proton decay effect to be much suppressed while simultaneously ensuring that the dressing of matter fields by heavy fields is ultraweak and therefore no enhancement of Baryon violation due to matter field...
dressing. It is an interesting and open question as to whether higher dimensional superexotic (B-L violating etc) operators containing a light Higgs field can receive a boost to observable levels due to Higgs wave function enhancement. Such operators are specially interesting from the point of view of novel scenarios of Baryogenesis and B violation except that they tend to be badly suppressed by the additional powers of $M_X^{-1}$ incurred due to their higher dimension. The light field dressing also has effects on the soft breaking terms that may eventually prove important in precision fits of MSSM data in terms of GUT couplings.

Using these corrections we were able to show [17] that prima facie a solution of the dimension 5 proton decay problem, outstanding for long, was feasible in a realistic Susy GUT. This was done using an iterated fitting at high scales (of the RG extrapolated low energy Yukawa couplings of the MSSM) and at low scales of the SM Yukawas in terms of the Susy threshold corrected GUT generated Yukawas. Large $\tan \beta$ driven (H-Hbar mixing) threshold corrections to down type fermion yukawas prove crucial towards achieving realistic down type quark masses in the first and second generations. We can fit charged fermion masses $y_t \simeq y_b \simeq y_\tau (M_X)$ and $\tan \beta \simeq 50$ if MSSM radiative corrections raise $Y_{d,s}^{GUT}$ by 3-4 times while $Y_b^{GUT}$ is lowered by around 5%. GUT scale threshold corrections which repair fast proton decay rate also loosen stringent tree level constraints on $10^{\oplus} 120$ generated charged fermion masses. The iterative procedure has drawbacks such as occurrence of large SO(10) gauge couplings and ambiguities regarding which couplings to use at successive stages. Moreover the light sparticle masses remained to be loop corrected on the fly. These improvements have been incorporated and a non-iterative search for fits, based upon a random choice in the large ($>44$) dimensional parameter space followed by threshold corrections and running all quantities down to the Electroweak scale, before trying the fit the known low energy data, is underway, but exhibits slow convergence due to the plethora of local minima encountered by the Nelder-Mead amoeba as it traverses the high dimensional parameter landscape. Calculation of the off diagonal 1-loop corrections in the MSSM, generalizing [29] are also required for a fully satisfactory treatment and are underway. Our searches show it is difficult to suppress the coefficient of the dimension 4 B-violation operator in the effective superpotential below about $10^{-21.5}$ GeV$^{-1}$. The resultant value of the proton lifetime also depends on the soft susy spectrum, so that it may be necessary [19] to optimize both GUT scale and $O(M_{susy})$ parameters to achieve acceptable B-violation. This case has the upside that fully realistic fits will both constrain Soft susy couplings and predict a proton life time upper bound near to the current limits. Minimal Susy SO(10) may thus prefigure an upcoming discovery of proton decay.

A comprehensive, neat and predictive fit or else a falsification of the NMSGUT as it stands is thus feasible and on the horizon. At this stage, rather than any particular fit we wish to emphasize that this serious treatment of the quantum dynamics of the only fully realistic and calculable GUT available has made it clear that any serious UV completion must be able to carry out a corresponding estimation of heavy field effects on light field couplings or abandon any pretense at a quantitatively valid unification. Since several pre-eminent types of Unification prone to making ambitious claims are yet far from even being able to consistently specify a separation of light from heavy modes, leave alone their dressing in terms of heavy mode quantum effects separately from light field ones, it is clear that the bar for candidate models has significantly been raised by our studies.
4. Yukawon Flavour Models

The (MS)SM Fermion kinetic terms carry a large global symmetry \( U(3)_Q \otimes U(3)_u \otimes U(3)_d \otimes U(3)_L \otimes U(3)_l \) broken only by the Yukawa couplings. To understand the origin of the observed flavour hierarchy, it is natural to ask whether the breaking terms are not the residue of spontaneous breaking of flavour symmetry by some unknown high scale dynamics. Such “Yukawon” models based on the SM/MSSM must normally live with the uncomfortable necessity of non-renormalizable dynamics since the Yukawa terms are already of dimension 4 and the couplings promoted to scalar field vevs raise the operator from marginal to irrelevant. Model building with recourse to generation of structurally crucial marginal operators via irrelevant operators is always arbitrary and unconvincing except when directly motivated by phenomenology. One should rather first ask whether there is any sign of of flavour unification in RG evolution into the UV. The only known hint available is the GUT scale [3] unification of third generation couplings in the high tan\( \beta \) MSSM motivated by Susy SO(10) GUTs! Thus it is natural to ask whether the GUT Yukawa couplings found in NMSGUT fits of all the SM data might arise from field vevs. In such NMSGUT based models, however, the Higgs and Yukawon functions can be comfortably combined, without sacrificing renormalizablity, and the flavour symmetry broken at the only scale where there is a hint of flavour unification : the GUT scale. In [21, 21] we showed that this economical scenario is actually realizable in the context of models where a gauged \( O(N_g) \) flavour symmetry is appended to the NMSGUT without disturbing the structure of NMSGUT apart from the promotion of Higgs from bland to flavoured and the inverse demotion of SO(10) matter Yukawas from flavoured to to bland. We note that the freedom to gauge flavour achieved in these models is in line with the philosophy we have consistently abided by, first in MSLRMs[8] and then in MSGUTs[13, 14, 10, 16, 17] : invoke no ad hoc global symmetries whether continuous or discrete and shelter always under the sole reliable dynamical principle revealed by 20th century physics : local gauge invariance.

Although the global symmetry of the Spin(10) GUT matter kinetic terms is \( U(N_g) \), gauging just an \( O(N_g) \) subgroup seems workable. Gauging this symmetry can ensure that no Goldstone bosons arise when it is spontaneously broken. With a unitary family gauge symmetry, complex representations introduce anomalies and require irrep doubling to cancel gauge anomalies and to allow formation of holomorphic invariants for the superpotential. Moreover one finds that a Unitary flavour gauge group implies half the effective MSSM matter Yukawa couplings vanish. An \( O(N_g) \) flavour gauge group is free of these defects We emphasize that in contrast with previous models (see e.g. [3]) our model is renormalizable and GUT based.

The Yukawon GUT superpotential has the MSGUT form :

\[
W_{GUT} = \text{Tr}(m\Phi^2 + \lambda \Phi^3 + M\Sigma\Sigma + \eta \Phi \Sigma \Sigma) + \Phi . H . (\gamma \Sigma + \bar{\gamma} \bar{\Sigma}) + M_H H . H \\
W_F = \Psi_A . ((hH) + (f\Sigma) + (g\Theta))_{AB} \Psi_B
\]

(4.1)

The manner of insertion of the \( 120 \)-plet is indicated in \( W_F \) but so far we have have studied only MSGUTs (i.e with \( 10, 126 \)). Anyway \( 120 \)-plet does not take part in GUT scale symmetry breaking since it has no SM singlets. The main novelty is that MSGUT Higgs fields now carry a symmetric representation of the \( O(N_g) \) family symmetry : \( \{ \Phi, \Sigma, \Sigma, H \}_{AB}, A, B = 1, \ldots, N_g \) (the matter \( 16 \)-plets \( \psi_A \) are vector \( N_g \)-plets). Now the couplings \( h, f, g \) are no longer matrices but just (complex) numbers while the Yukawons carry symmetric \( \{ H, \Sigma \}_{AB} = \{ H, \Sigma \}_{BA} \) and anti-symmetric \( \Theta_{AB} =
- $\Theta_{BA}$ representations of $O(N_g)$ as required by the properties of Spin(10) 16-plet bilinears. Thus, for $N_g = 3$, the number of (real) matter Yukawa parameters come down from 15 ($Re[h_{AA}], f_{AB}$) to just 3 ($Re[h], f$) without the 120-plet (6 additional to just 2 additional with the 120-plet). Such renormalizable flavour unified “Yukawon” GUTs should thus be called Yukawon Ultra-Minimal GUTs (YUMGUTs).

The spontaneous symmetry breaking at the GUT scale due to the above superpotential is largely analogous to the MSGUT case although algebraic complications require numerical solution of the F term conditions\cite{20}. The contribution of the MSGUT type Higgs fields to the flavour group D terms cannot vanish and this breaks supersymmetry unless additional chiral multiplets charged under flavour but not under SO(10) and free to cancel the MSGUT type Higgs contribution in the flavour D-Terms are introduced. Remarkably this can be done\cite{21} using “Bajc-Melfo” type metastable supersymmetry breaking which -when generalized by flavour gauging- leaves some Chiral multiplet vevs free to be determined by the flavour D term conditions. Thus a remarkable connection between spontaneously broken gauged flavour and supersymmetry breaking emerges. This strange and novel relationship in the UV theory leaves a clear relic in the effective theory consisting of very light ‘moduli’ type fields (the fermionic components get masses only radiatively while Bosonic components get the usual soft susy breaking scale masses) which may be very light(sub-GeV to sub-Electro-weak) Dark matter candidates of a novel type not ordinarily found in SO(10) GUTs. In these models the hard parameters of the effective MSSM and the soft supersymmetry breaking parameters are determined by the two parameters of the hidden sector superpotential and the Planck scale together with the (reduced) parameter space of the NMSGUT sector superpotential.. What we wish to emphasize here is that the constraint $\text{Det} [\mathcal{M}] = 0$ imposed on the generalization of the $4 \times 4$ doublet matrix in the MSGUT to a $2N_g(N_g + 1)$ dimensional Higgs mass matrix in the YUMGUT and, correspondingly, of the $6 \times 6$ doublet matrix in the NMSGUT to a $N_g \times (3N_g + 1)$ dimensional matrix is sufficient to generate solutions with all the observed features of the fermion hierarchy. See\cite{20, 21} for details.

5. Discussion

In this talk we focussed on the crucial role of the consistency condition which ensures a single pair of Light Higgs doublets in the effective MSSM arising from the fully calculable Minimal Supersymmetric GUT introduced long ago\cite{9} and developed over the years into a fully consistent theory. We have shown how the calculability of quantum effects not only allows implementation of the fermion hierarchy and fitting of the full MSSM gauge and fermion data data, including neutrinos, but also ameliorates the longstanding and generic problem\cite{18} with dimension 5 B violation operators by showing an easy approach via inescapable quantum effects (even with small couplings, due to the large number of heavy fields) to the “Higgs dissolution edge” in parameter space. On this "edge" the wavefunction renormalization constants of the light Higgs multiplets arising from the NMSGUT nearly vanish, thus amplifying operators (like the MSSM Yukawa couplings) where Higgs fields enter while suppressing the troublesome B-violation operators by several orders of magnitude. The full implications of this mechanism, both for the achievable $d = 5$ B-violation operator rates and for higher dimension rates are still under investigation. However it is likely that fully consistent fits, if found, will still predict proton lifetimes close to current lower limits. Thus
the NMSGUT indicates proton decay may be visible in the next generation of neutrino tele-/nucleon micro- scope detectors currently under construction or planning.

References

[1] J. C. Pati and A. Salam, Phys. Rev. D 10 (1974) 275 [Erratum-ibid. D 11 (1975) 703]; H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32 (1974) 438; H. Georgi, H. R. Quinn and S. Weinberg, Phys. Rev. Lett. 33 (1974) 451.

[2] W. J. Marciano and G. Senjanovic, Phys. Rev. D 25 (1982) 3092.

[3] M.S. Carena, M. Olechowski, S. Pokorski and C. E. M. Wagner, Nucl. Phys. B 426, 269 (1994) [arXiv:hep-ph/9402253]. B. Ananthanarayan, Q. Shafi and X. M. Wang, [arXiv:hep-ph/9311225]; R. Rattazzi, U. Sarid and L.J. Hall, [arXiv:hep-ph/9405313];

[4] C. S. Aulakh, K. Benakli and G. Senjanovic, Phys. Rev. Lett. 79 (1997) 2188 [hep-ph/9703434]; C. S. Aulakh, A. Melfo and G. Senjanovic, Phys. Rev. D 57 (1998) 4174, [hep-ph/9707256];

[5] For discussion and references see : G. Jungman, M. Kamionkowski and K. Griest, Phys. Rept. 267 (1996) 195 [hep-ph/9506380].

[6] C. S. Aulakh, B. Bajc, A. Melfo, A. Rasin and G. Senjanovic, Phys. Lett. B 460 (1999) 325; [hep-ph/9904352]; C. S. Aulakh, A. Melfo, A. Rasin and G. Senjanovic, Phys. Lett. B 459 (1999) 557; [hep-ph/9902409]; C. S. Aulakh, A. Melfo, A. Rasin and G. Senjanovic, Phys. Rev. D 58 (1998) 115007, [hep-ph/9712551];

[7] C. S. Aulakh, Pramana 54 (2000) 639, [hep-ph/9903309]. C. S. Aulakh, Pramana 55 (2000) 137, [hep-ph/0008331].

[8] Y. Fukuda et al. [Super-Kamiokande Collaboration], Phys. Rev. Lett. 81 (1998) 1562, [hep-ex/9807003].

[9] T.E. Clark, T.K. Kuo, and N. Nakagawa, Phys. lett. B 115, 26(1982); C. S. Aulakh and R. N. Mohapatra,CCNY-HEP-82-4 April 1982, CCNY-HEP-82-4-REV, Phys. Rev. D 28 (1983) 217;

[10] C.S. Aulakh and S.K. Garg, [arXiv:hep-ph/0807.0917v2]; Nucl. Phys. B 857, 101 (2012) [arXiv:hep-ph/0807.0917v3].

[11] H.S. Goh, R.N. Mohapatra, S.P. Ng, Phys. Lett. B 570 (2003) 215, arXiv:hep-ph/0303055, Phys. Rev. D 68 (2003) 115008, arXiv:hep-ph/0308197; K.S. Babu, C. Macesanu, Phys. Rev. D 72 (2005) 115003, arXiv:hep-ph/0505200

[12] F. P. An et al. [Daya Bay Collaboration], Phys. Rev. Lett. 108 (2012) 171803, [arXiv:1203.1669 [hep-ex]].

[13] C.S. Aulakh, B. Bajc, A. Melfo, G. Senjanovic and F. Vissani, Phys. Lett. B 588, 196 (2004) [arXiv:hep-ph/0306242].

[14] C.S. Aulakh and A. Girdhar, Nucl. Phys. B 711, 275 (2005), [arXiv:hep-ph/0405074].

[15] C.S. Aulakh, From germ to bloom, [arXiv:hep-ph/0506291].

[16] C.S. Aulakh and S.K. Garg, Nucl. Phys. B 757, 47 (2006) [arXiv:hep-ph/0512224].

[17] C. S. Aulakh, I. Garg and C. K. Khosa, Nucl. Phys. B 882 (2014) 397, [arXiv:1311.6100 [hep-ph]].

[18] N. Sakai, T. Yanagida, Nucl. Phys. B 197 (1982) 533; S. Weinberg, Phys. Rev. D 26 (1982) 287. J. Hisano, H. Murayama, T. Yanagida, Nucl. Phys. B 402 (1993) 46, hep-ph/9207279.
[19] B. Bajc, P. Fileviez Perez and G. Senjanovic, Phys. Rev. D 66, 075005 (2002) hep-ph/0204311 and hep-ph/0210374.

[20] C. S. Aulakh and C. K. Khosa, Phys. Rev. D 90 (2014) 4, 045008, [arXiv:1308.5665 [hep-ph]].

[21] C. S. Aulakh, Phys. Rev. D 91 (2015) 055012, [arXiv:1402.3979 [hep-ph]].

[22] C. S. Aulakh, B. Bajc, A. Melfo, A. Rasin and G. Senjanovic, Nucl. Phys. B 597 (2001) 89, [hep-ph/0004031].

[23] C. S. Aulakh and A. Girdhar, Int. J. Mod. Phys. A 20 (2005) 865, [hep-ph/0204097].

[24] B. Bajc, A. Melfo, G. Senjanovic and F. Vissani, Phys. Rev. D 70, 035007 (2004) [arXiv:hep-ph/0402122].

[25] T. Fukuyama, A. Ilakovac, T. Kikuchi, S. Meljanac and N. Okada, Eur. Phys. J. C 42, 191 (2005) [arXiv:hep-ph/0401213v1,v2]; T. Fukuyama, A. Ilakovac, T. Kikuchi, S. Meljanac and N. Okada, J. Math. Phys. 46 (2005) 033505 [arXiv:hep-ph/0405300].

[26] S. Weinberg, Phys.Lett.B 91,(1980)51; L.J. Hall, Nucl. Phys. B178, 75 (1981); B.D. Wright, [arXiv:hep-ph/9404217] (1994).

[27] V.V. Dixit and M. Sher, Phys. Rev. D40,3765(1989).

[28] K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. 109 (2012) 091803 doi:10.1103/PhysRevLett.109.091803 [arXiv:1207.5771 [hep-ph]].

[29] D. M. Pierce, J. A. Bagger, K. T. Matchev and R. J. Zhang, Nucl. Phys. B 491, 3 (1997) [arXiv:hep-ph/9606211].

[30] C. S. Aulakh, [arXiv:hep-ph/0602132]; L. Lavoura, H. Kuhbock and W. Grimus, Nucl. Phys. B 754 (2006) 1 [arXiv:hep-ph/0603259].

[31] Y. Koide, Phys. Rev. D 78, 093006 (2008) [arXiv:0809.2449 [hep-ph]]; Phys. Rev. D 79, 033009 (2009) [arXiv:0811.3470 [hep-ph]]; Phys. Lett. B 665, 227 (2008).