Particle Acceleration and Synchrotron Self-Compton Emission in Blazar Jets. I. An Application to Quiescent Emission

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Abstract

There are still some important unanswered questions about the detailed particle acceleration and escape occurring during quiescent epochs. As a result, the particle distribution that is adopted in the blazar quiescent spectral model has numerous unconstrained shapes. To help remedy this problem, we introduce an analytical particle transport model to reproduce the quiescent broadband spectral energy distribution of blazars. In this model, the exact electron distribution is solved from a generalised transport equation that contains the terms describing the first-order and second-order Fermi acceleration, the escape of particles due to both advection and spatial diffusion, and energy losses due to synchrotron emission and inverse-Compton scattering of an assumed soft photon field. We suggest that advection is a significant escape mechanism in blazar jets. We find that in our model, the advection process tends to harden the particle distribution, which enhances the high-energy components of the resulting synchrotron and synchrotron self-Compton spectra from the jets. Our model is able to roughly reproduce the observed spectra of the extreme BL Lac object 1ES 0414+009 with reasonable assumptions about the physical parameters.

Key words: acceleration of particles – BL Lacertae objects: individual: (1ES 0414+009) – radiation mechanisms: non-thermal

1. Introduction

Blazars are radio-loud active galactic nuclei (AGNs) with a nonthermal continuum emission that arises from the jet emission taking place in an AGN whose jet axis is closely aligned with the observer’s line of sight (Ghisellini et al. 1986; Urry & Padovani 1995). Their broad spectral energy distribution (SED) from the radio to the γ-ray bands are dominated by two components, appearing as humps (e.g., Fossati et al. 1998). It is believed that the SED is dominated by various emission mechanisms in different energy regimes (Böttcher 2007). The low-energy hump that extends from radio up to soft X-rays is produced by synchrotron radiation from relativistic electrons and/or positrons in the jet (Urry 1998). Alternatively, in the leptonic model scenarios, the high-energy hump that covers the hard X-ray and γ-ray energy regime is probably produced by the inverse-Compton (IC) scattering of relativistic electrons either on synchrotron photons (synchrotron self-Compton, SSC; e.g., Maraschi et al. 1992; Bloom & Marscher 1996; Mastichiadis & Kirk 1997; Konopelko et al. 2003) and/or on some other photon populations (external Compton, EC, e.g., Dermer et al. 1992; Dermer & Schlickeiser 1993; Sikora et al. 1994; Blandford & Levinson 1995; Ghisellini & Madau 1996; Böttcher & Dermer 1998; Kataoka et al. 1999; Blazevic et al. 2000; Diltz & Böttcher 2014; Zheng et al. 2017).

Most of the early models applied to describe the quiescent broadband SED of blazars adopted a phenomenological view, assuming that some unspecified mechanism is able to produce the particle distribution that is subsequently injected into the emission region (e.g., Mastichiadis & Kirk 1997; Bednarek & Protheroe 1997, 1999; Kataoka et al. 2000; Moderski et al. 2003; Finke et al. 2008; Dermer et al. 2009; Hayashida et al. 2012). The required distribution of emitting particles may be established via a variety of mechanisms, including first-order Fermi acceleration (shock acceleration) due to multiple shock crossings (e.g., Bell 1978; Blandford & Ostriker 1978, Drury 1983a; Blandford & Eichler 1987; Jones & Ellison 1991; Summerlin & Baring 2012; Marscher 2014; Zheng et al. 2018b), second-order Fermi acceleration (stochastic acceleration) due to stochastic interactions with a random field of magnetohydrodynamic (MHD) waves (e.g., Eilek & Henriksen 1984; Schlickeiser 1984a, 1989; Dung & Petrosian 1994; Miller & Roberts 1995; Dermer et al. 1996; Petrovian & Liu 2004; Katarzynski et al. 2006; Lefa et al. 2011; Zheng & Zhang 2011; Asano & Hayashida 2015; Baring et al. 2017), and electrostatic acceleration due to magnetic reconnection (e.g., Giannios et al. 2009; Giannios 2013; Petropoulou et al. 2016; Sironi et al. 2016).

In principle, constructing the particle transport equation and obtaining its solution can produce the theoretical SED in the standard blazar paradigm. Previous efforts attempted to solve the particle distribution under some certain assumptions in both an analytical way (e.g., Kardashev 1962; Schlickeiser 1984b, 1985; Park & Petrovian 1995; Kirk et al. 1998; Keshet & Waxman 2005; Becker et al. 2006; Stawarz & Petrovian 2008; Dermer & Menon 2009; Tramacere et al. 2009; Mertsch 2011; Finke 2013; Lewis et al. 2016, 2018) and a numerical way (Chaiherger & Ghisellini 1999; Katarzynski et al. 2006; Zheng & Zhang 2011). However, the behavior of the particles that are trapped in the flow is ignored, due to the escape of particles being treated only as a spatial diffusion. In order to track the possible impact on the particle distribution and then on the photon spectrum of the advection of the jet in the outward direction, in this paper, we extend the approach introduced by Kroon et al. (2016) from pulsars to the jets of blazars. We focus on a generalized transport...
equation that contains the terms describing the first-order and second-order Fermi acceleration, the escape of particles due to both advection and spatial diffusion, and energy losses due to synchrotron emission and IC scattering of an assumed soft photon field. Our main aim is to show that the particle distribution in this context is able to reproduce the multi-wavelength spectrum under reasonable assumptions about the physical parameters.

The present paper is organized as follows. In Section 2, we describe the transport equation that contains the terms of the first-order Fermi acceleration, second-order Fermi acceleration, particle escape, and energy losses. In Section 3, we compare the timescales of the first-order Fermi acceleration, second-order Fermi acceleration, particle escape, and energy losses. In Section 4, we solve the steady-state transport equation to obtain the solution of the particle Green’s function. In Section 5, we deduce the particle Green’s function in the special case of low particle momentum. In Section 6, we calculate the theoretical photon spectrum utilizing the particle’s Green’s function. In Section 7, we apply the model to quiescent-state emission from the extreme BL Lac object 1ES 0414+009, and some discussions are given in Section 8. Throughout the paper, we assume the Hubble constant \( H_0 = 75 \text{ km s}^{-1} \text{ Mpc}^{-1} \), the dimensionless number for the energy density of matter \( \Omega_M = 0.27 \), the dimensionless number of radiation energy density \( \Omega_r = 0 \), and the dimensionless cosmological constant \( \Omega_{\Lambda} = 0.73 \).

2. Basic Equations

Assuming energetic particles in a turbulent and tenuous plasma carrying a magnetic field, the momentum spectrum of particles undergoing Fermi acceleration due to irregularly moving magnetized fluid elements can be studied in terms of a diffusion equation in momentum space (Tverskoi 1967; Tsytovich 1977):

\[
\frac{\partial f(p,t)}{\partial t} = \frac{1}{p^2} \frac{\partial}{\partial p} \left[ p^2 D(p) \frac{\partial f(p,t)}{\partial p} \right] + Q(p,t),
\]

where \( f(p,t) \) is the isotropic, homogeneous phase-space density, \( p \) the particle momentum, \( D(p) \) the second-order Fermi acceleration diffusion coefficient due to scattering off MHD waves, and \( Q(p,t) \) the sources and sinks of particles. The phase-space density is related to the total number of particles, \( N_e \), via \( N_e(p,t) = \int_0^\infty 4\pi p^2 f(p,t) dp \).

It is well known that the formation of a strong shock can be expected in the jets of blazars at locations a few parsecs from the core (Edwards & Piner 2002; Piner et al. 2009). The particles can gain momentum by first-order Fermi acceleration off the strong shocks (Bell 1978; Axford 1981). While particles gain momentum from both shock and turbulence, they also suffer from many kinds of momentum-loss processes. Incorporating these effects with escape into Equation (1), we obtain

\[
\frac{\partial f(p,t)}{\partial t} = \frac{1}{p^2} \frac{\partial}{\partial p} \left[ p^2 D(p) \frac{\partial f(p,t)}{\partial p} \right] - \hat{p}_{\text{gain}} f(p,t) - \hat{p}_{\text{loss}} f(p,t) - \hat{p}_{\text{escape}} f(p,t) + Q(p,t),
\]

where \( \hat{p}_{\text{gain}} \) is the momentum gain rate by shock acceleration, \( \hat{p}_{\text{loss}} \) the escape of particles due to second-order Fermi acceleration diffusion coefficient, and \( \hat{p}_{\text{escape}} \) the escape of particles due to particle Green’s function.

2.1. Stochastic Acceleration

The turbulent magnetic field component gives rise to the spatial diffusion of charged plasma particles, which is described by the spatial diffusion coefficient \( \kappa(p) \). In the case of an isotropic Alfvénic turbulence with a one-dimensional power spectrum \( W(k) \propto k^{-q} \) with a spectral index \( q \) in a finite wave-vector range \( k_{\text{min}} < k < k_{\text{max}} \), the relation between the spatial diffusion coefficient and the momentum diffusion coefficient can be written as (Skilling 1975; Webb 1983; Schlickeiser 1985; Dröge et al. 1987)

\[
D(p) = \frac{v_A^2 p^2}{9 \kappa(p)},
\]

where \( v_A \) is the Alfvén velocity. In the case of particles with Larmor radius, \( r_L \), smaller than the correlation length of the field, we can introduce a dimensionless parameter

\[
\eta = \frac{B}{\delta B} \left( \frac{\lambda_{\text{max}}}{r_L} \right)^{q-1},
\]

where \( B \) is the local magnetic field strength, \( \delta B \) the turbulent component of the magnetic field, and \( \lambda_{\text{max}} = 2\pi/k_{\text{min}} \) the maximum wavelength of the Alfvén modes; to parameterize the particle mean free path, \( \ell \), relative to the Larmor radius,

\[
\ell = \eta r_L = \frac{\ell c}{eB},
\]

where \( c \) is the speed of light, and \( e \) the magnitude of the electron charge.

The spatial diffusion coefficient associated with the mean free path is calculated using (Reif 1965; Dröge & Schlickeiser 1986)

\[
\kappa(p) = \frac{e\ell}{3}.
\]

Combining Equations (3), (5), and (6), we find that the momentum diffusion coefficient can be given as (e.g., Dermer et al. 1996; Becker et al. 2006)

\[
D(p) = D_0 m_e c p,
\]

with a momentum diffusion rate constant

\[
D_0 = \frac{eB \sigma_{\text{mag}}}{3\eta m_e c} = 5.86 \times 10^{5} \sigma_{\text{mag}} \eta^{-1} \left( \frac{B}{0.1 \text{ G}} \right) \text{ s}^{-1},
\]

where \( \sigma_{\text{mag}} = v_A^2/c^2 \) is the magnetization parameter (e.g., Sironi & Spitkovsky 2014). This relation gives the stochastic momentum gain rate as (Becker et al. 2006)

\[
\dot{\hat{p}}_{\text{stoch}} = \frac{1}{p^2} \frac{\partial}{\partial p} [p^2 D(p)] = 3D_0 m_e c.
\]

2.2. Shock Acceleration

We consider a quasi-continuous momentum gain by systematic acceleration at shock waves moving through the plasma at speed \( v_s \). The momentum gain rate by shock acceleration at an isolated single shock wave is determined (e.g., Drury 1983b; Lagage & Cesarsky 1983) via

\[
\dot{\hat{p}}_{\text{sh}} = \frac{1}{3} (U_1 - U_2) \left[ \frac{\kappa_1}{U_1} + \frac{\kappa_2}{U_2} \right] p \simeq \frac{v_A^2}{4\kappa(p)} p,
\]
where \( U_1 (U_2) \) and \( \kappa_1 (\kappa_2) \) are the flow velocities and diffusion coefficients upstream (downstream) of the shock in the shock’s comoving frame. In order to simplify the model, we attribute the momentum gain rate in Equation (2) only to shock acceleration. Combining Equations (5), (6), and (10), we find that the momentum gain rate experienced by the particles due to multiple shock crossings can be given as

\[
\dot{p}_{\text{gain}} = \dot{p}_{\text{sh}} = A_0 m_c c,
\]

with a shock acceleration rate constant

\[
A_0 = \frac{3 \xi e B}{4 m_c} = 1.32 \times 10^6 \xi \left( \frac{B}{0.1 \text{ G}} \right) \text{ s}^{-1},
\]

where \( \xi = \eta^{-1} \gamma^2 \) is an efficiency factor.

### 2.3. Momentum Loss

In the presence of ambient magnetic photon fields in the dissipated region of jet, the particles also undergo synchrotron radiation and inverse-Compton scattering (ICs). The synchrotron energy-loss rates per particle, averaged over an isotropic distribution of pitch angles, are given by (e.g., Rybicki & Lightman)

\[
\gamma_{\text{syn}} m_c c^2 = \frac{4}{3} \sigma_T c u_B \gamma^2,
\]

and

\[
\gamma_{\text{ICs}} m_c c^2 = \frac{4}{3} \sigma_T c u_{\text{ph}} \gamma^2,
\]

respectively. Here, \( \sigma_T \) is the Thomson cross section, \( u_B = B^2/8\pi \) the magnetic field density, and \( u_{\text{ph}} \) the soft photon density to be upscaled. The associated synchrotron and ICs momentum-loss rate can be written as

\[
\dot{p}_{\text{syn}} = -\frac{4 \sigma_T u_B}{3 m_c} \frac{p^2}{m_c},
\]

and

\[
\dot{p}_{\text{ICs}} = -\frac{4 \sigma_T u_{\text{ph}}}{3 m_c} \frac{p^2}{m_c}.
\]

Hence, the momentum-loss rate, \( \dot{p}_{\text{loss}} \), appearing in the Equation (2) can be written as the sum

\[
\dot{p}_{\text{loss}} = \dot{p}_{\text{syn}} + \dot{p}_{\text{ICs}} = -\frac{B_0}{m_c} p^2,
\]

where the momentum-loss rate constant \( B_0 \) is given by

\[
B_0 = \frac{4 \sigma_T}{3 m_c} (u_B + u_{\text{ph}}) = 3.25 \times 10^{-8} u \text{ s}^{-1},
\]

with a constant soft photon field \( u_{\text{ph}} \).

### 2.4. Escape of Particles

The escape of particles in calculations can generally be explained via the real process of particles moving into the region of a source where the magnetic field strength is significantly smaller and therefore the efficiency of the particle emission is also significantly less (e.g., Katarzynski et al. 2006). In this scenario, the particles remain in the acceleration region for mean times \( t_{\text{esc}} \) before escaping. In order to properly treat the dominant spatial transport processes on large and small scales, we introduce a momentum-dependent escape timescale, \( t_{\text{esc}}(p) \).

It is believed that the Larmor radius of a particle with a small momentum is much smaller than the size of the acceleration region. In this case, the particle is trapped in the flow, and the escape of particles from the acceleration region occurs via advection (Becker & Begelman 1986). This process is called shock-regulated escape (Steinacker & Schlickeiser 1989) with a timescale \( t_{\text{SRE}}(p) \) (e.g., Jokipii 1987; Gallant & Achterberg 1999),

\[
t_{\text{SRE}}(p) = \frac{p}{C_0 m_c c},
\]

where \( C_0 \) is the shock-regulated escape rate constant,

\[
C_0 = \frac{eB}{\omega m_c c} = 1.76 \times 10^6 \omega^{-1} \left( \frac{B}{0.1 \text{ G}} \right) \text{ s}^{-1}.
\]

Here, \( \omega \) is a dimensionless constant of order unity that accounts for time dilation and obliquity in the relativistic shock (Kroon et al. 2016).

On the contrary, in the case of a particle with a large momentum, the escape of the particles occurs via spatial diffusion. This process is called Bohm diffusive escape (Dermer & Menon 2009) with a timescale (Kroon et al. 2016),

\[
t_{\text{Bohm}}(p) = \frac{m_c c}{F_0 p},
\]

where \( F_0 \) is the Bohm diffusive escape rate constant,

\[
F_0 = \frac{\eta m_c c^3}{r_s e B} = 5.12 \times 10^{-20} \eta \left( \frac{r_s}{10^3 \text{ cm}} \right)^{-2} \left( \frac{B}{0.1 \text{ G}} \right)^{-1} \text{ s}^{-1}.
\]

Here, \( r_s \) is the size of the blob.

It can be seen that either the particles with small momentum are likely to advect away into the downstream region, or the particles with large momentum are likely to diffuse out of the acceleration region via Bohm diffusion. In order to ensure that the behavior of particles with both the small and large momentum are properly taken into account, these two escape rates can be included in the net escape rate, \( t^{-1}_{\text{esc}}(p) \), given by

\[
t^{-1}_{\text{esc}}(p) = t^{-1}_{\text{SRE}}(p) + t^{-1}_{\text{Bohm}}(p).
\]

Inserting Equations (19) and (21) into (23), we find

\[
t_{\text{esc}}(p) = \left( \frac{C_0 m_c c + F_0 p}{p} \right)^{-1}.
\]

### 2.5. Particle Injection

The injection process creates a seed population of nonthermal particles, but the particle injection mechanism is not well understood. Since the model considered here includes significant components of particle acceleration and cooling, the evolution of momentum distribution is independent of the precise form of the momentum distribution of the injected electrons (e.g., Katarzynski et al. 2006; Zheng & Zhang 2011). We can utilize this insensitivity by assuming that the injected particles have a monoenergetic distribution, with a characteristic injection
momentum, \( p_0 \),

\[
Q(p, t) = \frac{\dot{N}_0 \delta(p - p_0)}{4\pi p_0^2},
\]

(25)

where \( \dot{N}_0 \) is the continuous injection rate in units of \( \text{cm}^{-3} \text{s}^{-1} \) and \( \delta(p) \) the Dirac injection function.

### 2.6. Transport Equation

Substituting Equations (7), (11), (17), (24), and (25) into Equation (2), we find the basic transport equation given by

\[
\frac{\partial f}{\partial t}(p, t) = \frac{1}{p^2} \frac{\partial}{\partial p} \left\{ p^2 \left\{ D_0 m_e c p \frac{\partial f}{\partial p} - A_0 m_e c f(p, t) \right\} \right. \\
+ \frac{B_0 p^2}{m_e c^2} f(p, t) \left. \right\} - \left( \frac{C_0 m_e c}{p} + \frac{F_0 p^3}{m_e c} \right) f(p, t) \\
+ \frac{\dot{N}_0 \delta(p - p_0)}{4\pi p_0^2}.
\]

(26)

This equation can be used to calculate the particle distribution in the emission region of blazar jets through both analytical (e.g., Schlickeiser 1984b, 1985; Park & Petrosian 1995; Becker et al. 2006; Stawarz & Petrosian 2008; Mertsch 2011; Zheng et al. 2018a) and numerical approaches (e.g., Chaiberge & Ghisellini 1999; Katarzynski et al. 2006; Zheng & Zhang 2011).

### 3. The Timescales

The timescale determines how long it takes a particle to gain or lose momentum, allowing easy comparison between the efficiencies of different gain and loss mechanisms. When the gain or loss rate of momentum is determined, we can estimate the timescale by

\[
t(p) = \frac{p}{\dot{p}}.
\]

(27)

We show the calculated timescales in the comoving frame of the plasma as a function of the particle momentum in Figure 1. It is believed that the effective escape timescale is \( t_{\text{esc}} \to 0 \) in the limits of \( p \to 0 \) and \( p \to \infty \). In this scenario, we can find the crossover momentum with \( t_{\text{SRE}}(p) = t_{\text{Bohm}}(p) \) from

\[
p_c = m_e c \sqrt{\frac{C_0}{F_0}} \\
= 1.6 \times 10^{-4} \eta^{-1} \omega^{-1} \left( \frac{r_s}{10^{16} \text{cm}} \right) \left( \frac{B}{0.1 \text{G}} \right).
\]

(28)

It can be seen that, in the regime of \( p < p_c \), the escape of particles is dominated by advection, and in the regime of \( p > p_c \), the escape of particles is dominated by spatial diffusion. The crossover momentum can determine a critical Lorentz factor by \( \gamma_c = p_c/(m_e c) \). If we adopt the typical values of \( r_s \sim 10^{16} \text{cm} \) and \( B \sim 0.1 \text{G} \) for a blazar jet, we find \( \gamma_c \sim 10^{12} \). This is far from the Lorentz factor, which yields TeV \( \gamma \)-ray photons by SSC processes in the jet. As an open issue, we suggest that advection is a significant escape mechanism in blazar jets. This issue tends to harden the particle distribution, which enhances the high-energy components of the resulting synchrotron and SSC spectrum from jets.

On the other hand, a dynamic equilibrium is generated by a kind of competition among the acceleration, injection, escape, and cooling of particles from the shock region. Because the escape of particles is dominated by advection in blazar jets, we can expect a theoretical equilibrium momentum, \( p_e \), from \( t_{\text{Bohm}}(p) = \min[t_{\text{stoch}}(p), t_{\text{gain}}(p), t_{\text{SRE}}(p)] \). Relativistic particles with an equilibrium momentum may be responsible for the X-ray and TeV \( \gamma \)-ray photons in the SSC framework. These can provide a rough estimate for the time it takes the electron distribution to reach equilibrium.

### 4. Stationary Particle Distribution

Despite variability, which is found from radio to TeV \( \gamma \)-ray bands, being one of the major characteristics of blazars, these sources should persist in the quiescent state throughout many epochs. In this scenario, we expect a stationary particle distribution and radiation in the emission region. In order to do so, we set \( \partial f(p, t)/\partial t = 0 \) in Equation (26) and then solve a stationary particle transport equation.

We first solve the steady-state Green’s function, \( f_G(p, p_0) \), with a given source distribution \( Q(p) \) under the proper boundary condition (e.g., Schlickeiser 1984b). Once Green’s function is determined, the steady-state density \( f(p) \) can be obtained using the convolution

\[
f(p) = \int_0^\infty f_G(p, p_0) \delta(p - p_0) dp_0.
\]

(29)

We define the dimensionless momentum, \( \nu = p/(m_e c) \), and the dimensionless time, \( \tau = D \nu \). Recalling the characteristics of Dirac’s function, we combine Equations (26) and (29). In the case of the new coordinates \((\nu, \tau)\), the steady-state transport...
At the momentum $$\nu = \nu_0$$, Equation (33) can be rewritten as a confluent hypergeometric function (Kummer 1837),

$$
\nu \frac{d^2 N_G(v, \nu_0)}{d \nu^2} + (2 \hat{B} \nu - \frac{\hat{C}}{\nu} - \hat{F} \nu) \frac{d N_G(v, \nu_0)}{d \nu} + \frac{N_0 m_e c^2 (\nu - \nu_0)}{D_0} = 0.
$$

(35)

In this case, Green’s function, $$N_G(v, \nu_0)$$, fulfills appropriate boundary conditions at both $$v \to 0$$ and $$v \to \infty$$ (Melrose 1971; Tademaru et al. 1971; Bicknell & Melrose 1982). It can be expressed in terms of the Whittaker function $$M_{\sigma, \mu}$$ and $$W_{\sigma, \mu}$$ (Abramovitz & Stegun 1970),

$$
N_G(v, \nu_0) \propto \nu^\frac{\sigma}{2} e^{-\frac{B \nu^2}{2}} \left\{ \begin{array}{ll}
M_{\sigma, \mu} \left( \frac{B \nu^2}{2} \right), & v \leq \nu_0, \\
W_{\sigma, \mu} \left( \frac{B \nu^2}{2} \right), & v > \nu_0.
\end{array} \right.
$$

(36)

where we define the parameters $$\sigma = 1 + \hat{A}/4 - \hat{F}/(2 \hat{B})$$ and $$\mu = 0.25(2 + \hat{A}^2) + 4 \hat{C}^2/\nu_0^2$$. Taking into account the continuity of Green’s function at the momentum $$v = \nu_0$$, Kroon et al. (2016) gives the particle Green’s function

$$
N_G(v, \nu_0) = \frac{N_0 m_e c^2 (\mu - \sigma + 0.5)}{B D_0 \Gamma(1 + 2 \mu) \nu_0^2} \left( \frac{v}{\nu_0} \right)^\frac{\sigma}{2} e^{-\frac{B \nu^2}{2 \nu_0} - \frac{\nu_0}{\nu}} \times M_{\sigma, \mu} \left( \frac{B \nu_0^2}{2} \right)
$$

(37)

with $$v_1 = \min[v, \nu_0]$$ and $$v_2 = \max[v, \nu_0]$$, where $$\Gamma(x)$$ is the Gamma function. This equation exhibits the particle distribution resulting from a dynamic equilibrium among the acceleration, injection, escape, and cooling of the particles.

We can utilize the relation $$E_\nu = p^2 c^2 + m_e^2 c^2$$ to connect the nonthermal particle energy $$E_\nu$$ and the momentum in general. This gives the relationship $$v = \gamma^2 - 1$$ between $$v$$ and the Lorentz factor of particles $$\gamma$$. Since the ultrarelativistic particles ($$\gamma \gg 1$$) dominate the SEDs of blazars, we can write $$v = \gamma$$ without making a significant error. In this scenario, we can rewrite Equation (37) as

$$
N_G(\gamma, \gamma_0) = \frac{N_0 m_e c^2 (\mu - \sigma + 0.5)}{B D_0 \Gamma(1 + 2 \mu) \gamma_0^2} \left( \frac{\gamma}{\gamma_0} \right)^\frac{\sigma}{2} e^{-\frac{B \gamma^2}{2 \gamma_0} - \frac{\gamma_0}{\gamma}} \times M_{\sigma, \mu} \left( \frac{B \gamma_0^2}{2} \right)
$$

(38)

The particle distribution given by Equation (38) can be used to calculate the theoretical SED produced from a population of radiating relativistic particles in the blazar jet under the combined action of stochastic acceleration, shock acceleration, particle escape, and synchrotron and ICs losses. We show the particle distribution with different parameters in Figure 2. It can be seen that (1) in the regime of $$\gamma \lessgtr \gamma_0 = p_c/(m_e c)$$, the particle distributions show a cusp centered at the injection Lorentz factor, $$\gamma_0 = p_0/(m_e c)$$, surrounded by two power-law wings with different spectral indices. The indices of the power-law wings are sensitive to the four dimensionless parameters. (2) In the regime of $$\gamma > \gamma_0$$, where the energy losses overwhelm the particle acceleration, the particle distribution terminates in an exponential cutoff.

5. The Special Case of Lower Particle Momentum

It is interesting to note that, when the particle momentum satisfies $$p < p_c$$, the processes of energy loss and Bohm diffusion escape should be ignored. In this special case, we find that Equation (33) reduces to an Euler equidimensional
equation as
\[
\frac{\nu^2 d^2 \mathcal{N}_G(\nu, \nu_0)}{d\nu^2} - (1 + \hat{A})\nu \frac{d\mathcal{N}_G(\nu, \nu_0)}{d\nu} - \hat{C}\mathcal{N}_G(\nu, \nu_0) = - \frac{\hat{N}_0 \nu \delta(\nu - \nu_0)}{D_0}.
\]  
(39)

At the momentum \( \nu = \nu_0 \), Equation (39) can be rewritten as a homogeneous Euler equidimensional equation of the form
\[
\frac{\nu^2 d^2 \mathcal{N}_G(\nu, \nu_0)}{d\nu^2} - (1 + \hat{A})\nu \frac{d\mathcal{N}_G(\nu, \nu_0)}{d\nu} - \hat{C}\mathcal{N}_G(\nu, \nu_0) = 0.
\]  
(40)

Using the change of variables with \( \nu = e^\lambda \), we can obtain a power-law solution of the form
\[
\mathcal{N}_G(\nu, \nu_0) = H_0 \nu^\alpha
\]  
(41)

for Equation (40) (e.g., Cárdenas Alzate et al. 2016), where \( H_0 \) is a normalization constant and \( \alpha \) a power-law index. We can determine the power-law index using the characteristic polynomial
\[
\alpha^2 - (2 + \hat{A})\alpha - \hat{C} = 0,
\]  
where the root
\[
\alpha_1 = \frac{2 + \hat{A} + \sqrt{(2 + \hat{A})^2 + 4\hat{C}}}{2},
\]  
(43)

applies in the low-momentum regime with \( \nu \leqslant \nu_0 \), and
\[
\alpha_2 = \frac{2 + \hat{A} - \sqrt{(2 + \hat{A})^2 + 4\hat{C}}}{2},
\]  
(44)

applies in the high-momentum regime with \( \nu > \nu_0 \). Applying the derivative jump condition given by Equation (34), Kroon et al. (2016) gives the properly normalized global solution,
\[
\mathcal{N}_G(\nu, \nu_0) = \frac{\hat{N}_0 m_e c}{4D_0 \mu} \left\{ \begin{array}{ll}
\left(\frac{\nu}{\nu_0}\right)^{\alpha_1}, & \nu \leqslant \nu_0, \\
\left(\frac{\nu}{\nu_0}\right)^{\alpha_2}, & \nu > \nu_0.
\end{array} \right.
\]  
(45)
Recalling the relation between momentum and energy, we can rewrite Equation (45) as

\[ N_G(\gamma, \gamma_0) = \frac{\bar{N}_0 m_e c}{4 D_0 \mu} \left( \frac{\gamma}{\gamma_0} \right)^{a_1}, \quad \gamma \leq \gamma_0; \]

\[ N_G(\gamma, \gamma_0) = \frac{\bar{N}_0 m_e c}{4 D_0 \mu} \left( \frac{\gamma}{\gamma_0} \right)^{a_2}, \quad \gamma > \gamma_0. \]  

In Figure 3, we compare the particle distribution with energy loss and Bohm diffusion escape, described by Equation (38), and without energy loss and Bohm diffusion escape, described by Equation (46). As shown in Figure 3, the effect of energy loss is to move high-energy particles to lower energies, resulting in an increased curvature and a steeper particle distribution at high-energy regimes. Since both the shock-regulated escape and acceleration processes tend to harden the particle spectrum, we expect powerful high-energy components of the resulting synchrotron and SSC spectra from the jet.

### 6. Theoretical Photon Spectrum

Once we have solved the steady-state transport equation to determine the particle distribution in the comoving frame of the blob in the jet, we can use the solution to calculate the jet emission components, due to both synchrotron and SSC emission.

#### 6.1. Synchrotron Emission

Assuming an isotropic distribution of electrons, the theoretical synchrotron emission coefficient can be calculated by convolving the solution with the isotropic synchrotron emission power (e.g., Rybicki & Lightman 1979),

\[ j_{\text{syn}}(\nu) = \frac{\sqrt{3} e^3 B}{4 \pi m_e c^2} \int N_G(\gamma, \gamma_0) \gamma \left( \frac{4 \pi m_e c \nu}{3 eB^2} \right) d\gamma \text{ erg cm}^{-3} \text{ s}^{-1} \text{ Hz}^{-1}, \]  

where \( R(x) \) are the modified Bessel functions of order 5/3. Synchrotron emission is accompanied by absorption, in that a photon interacts with an electron and loses its energy. From the classical scheme of electron dynamics, we obtain the absorption coefficient

\[ k_{\text{syn}}(\nu) = -\frac{\sqrt{3} e^3 B}{8 \pi m_e c^2} \int \gamma^2 R \left( \frac{4 \pi m_e c \nu}{3 eB^2} \right) \times \frac{\partial}{\partial \gamma} \left[ \frac{N(\gamma, \gamma_0)}{\gamma^2} \right] d\gamma \text{ cm}^{-1}. \]

In the spherical geometry structure, the synchrotron intensity is given (e.g., Bloom & Marscher 1996; Kataoka et al. 1999) as

\[ I_\text{syn}(\nu) = \frac{j_{\text{syn}}(\nu)}{k_{\text{syn}}(\nu)} [1 - e^{-k_{\text{syn}}(\nu) r_i}] \text{ erg cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1}. \]

#### 6.2. Synchrotron Self-Compton Emission

We assume a uniform synchrotron intensity in the whole radiation region, corrected for the fact that in reality, it decreases along the blob radius (Gould 1979). Thus, the emission coefficient of ICs is obtained as

\[ j_{\text{ic}}(\nu) = \frac{h}{4 \pi} \epsilon_{\text{ic}} \varrho(\epsilon_{\text{ic}}) \text{ erg cm}^{-3} \text{ s}^{-1} \text{ Hz}^{-1}, \]

where \( \epsilon = h\nu/m_e c^2 \) is the dimensionless particle energy and \( \varrho(\epsilon_{\text{ic}}) \) the differential photon production rate with

\[ \varrho(\epsilon_{\text{ic}}) = \int n(\epsilon_{\text{syn}}) d\epsilon_{\text{syn}} \times \int N_G(\gamma, \gamma_0) \Omega(\epsilon_{\text{ic}}, \gamma, \epsilon_{\text{syn}}) d\gamma \text{ cm}^{-3} \text{ s}^{-1}. \]

Here, the number density of the synchrotron photons per energy interval, \( n(\epsilon_{\text{syn}}) \), is described by

\[ n(\epsilon_{\text{syn}}) = \frac{4 \pi}{\hbar c \epsilon_{\text{syn}} k_{\text{syn}}(\nu)} [1 - e^{-k_{\text{syn}}(\nu) r_i}] \text{ cm}^{-3}, \]

and the Compton kernel \( \Omega(\epsilon_{\text{ic}}, \gamma, \epsilon_{\text{syn}}) \) is given by (e.g., Jones 1968)

\[ C(\epsilon_{\text{ic}}, \gamma, \epsilon_{\text{syn}}) = \frac{2 \pi r_e^2 c}{\gamma^2 \epsilon_{\text{syn}}} \left[ 2 \kappa \ln \kappa + (1 + 2 \kappa)(1 - \kappa) + \frac{(4 \kappa \epsilon_{\text{syn}})^2}{2(1 + 4 \kappa \epsilon_{\text{syn}})} (1 - \kappa) \right] \text{ cm}^3 \text{ s}^{-1}, \]

where \( r_e \) is the classical electron radius, and \( \kappa \) satisfies \( \kappa = \epsilon_{\text{ic}}/4 \epsilon_{\text{syn}}(\gamma - \epsilon_{\text{ic}}) \).

For a given \( \epsilon_{\text{syn}} \) and \( \gamma \), the differential photon production rate \( \varrho(\epsilon_{\text{ic}}) \) can be performed in the range

\[ \epsilon_{\text{syn}} \ll \epsilon_{\text{ic}} \ll \frac{4 \epsilon_{\text{syn}} \gamma^2}{1 + 4 \epsilon_{\text{syn}} \gamma}. \]

Then, we can obtain the SSC emission intensity:

\[ I_{\text{ic}}(\nu) = j_{\text{ic}}(\nu) r_e \text{ erg cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1}. \]
6.3. $\gamma\gamma$ Attenuation

Since very high-energy (VHE) photons, generally $E_\gamma > 0.1$ TeV, from the source are attenuated by photons from the extragalactic background light (EBL), we should take the absorption effect into account. These scenarios give the flux density observed at Earth as follows (e.g., Zheng & Zhang 2011; Zheng & Kang 2013; Zheng et al. 2018b):

$$F_{\text{obs}}(\nu) = \frac{\pi \delta^2 (1 + z) r_s^2}{d_L^2} [I_{\text{syn}}(\nu) + I_{\text{e}}(\nu)] \times e^{-\tau(\nu, z)},$$

where $d_L$ is the luminosity distance, $\delta$ the Doppler factor (e.g., Rybicki & Lightman 1979), and $\tau(\nu, z)$ the absorption optical depth due to the interaction of VHE photons with the photons from the EBL (Kneiske et al. 2004; Dwek & Krennrich 2005).

7. Application of the Theoretical Photon Spectrum

In this section, we apply the theoretical photon spectrum to attempt to understand the nature of the quiescent-state emission from blazar jets. In order to do so, we first determine the model parameters and show the effects on the theoretical photon spectrum for various parameter changes. We then apply the theoretical photon spectrum to the quiescent-state emission from PKS 0414+009.

7.1. Determination of Model Parameters

Applying the theoretical photon spectrum requires the specification of both the particle spectral parameters including $N_0$, $D_0$, $\gamma_0$, $A$, $B$, $\zeta$, and $\hat{F}$, and the jet parameters including $B$, $\delta$, and $r_s$. To conveniently determine the model parameters, we expect to relate the dimensionless theory parameters $\hat{A}$, $\hat{B}$, $\hat{\zeta}$, and $\hat{F}$ to some special physical quantity. From Equations (8), (12), (18), (20), and (22), we find

$$\hat{A} \simeq \frac{3 \eta \xi}{\hat{\sigma}_{\text{mag}}^2},$$

$$\hat{B} = 5.54 \times 10^{-13} \frac{\eta}{\hat{\sigma}_{\text{mag}}} \left( \frac{B}{0.1 \text{ G}} \right)^{-1},$$

$$\hat{\zeta} = \frac{3 \eta}{\omega \hat{\sigma}_{\text{mag}}},$$

and

$$\hat{F} = 8.74 \times 10^{-26} \eta \sigma_{\text{mag}}^{-1} \left( \frac{r_s}{10^{17} \text{ cm}} \right)^2 \left( \frac{B}{0.1 \text{ G}} \right)^2.$$ 

In our approach, we treat $N_0$, $\gamma_0$, $\eta$, $\xi$, $\hat{A}$, $\hat{B}$, and $\hat{\zeta}$ as free particle spectral parameters. In these scenarios, we can deduce the magnetization parameter, $\sigma_{\text{mag}}$, and dimensionless timescale constant, $\omega$,

$$\sigma_{\text{mag}} = \frac{3 \eta \xi}{\hat{A}},$$

$$\omega = \frac{\hat{A}}{\hat{\zeta} \hat{\zeta}}.$$ 

Once the value of $\sigma_{\text{mag}}$ has been obtained, we can calculate the parameters $D_0$ and $\hat{F}$, and both the magnetic field and soft photon field energy density $u$ using Equations (8) and (60) and the relation

$$u = 1.8 \times 10^{12} \hat{B} \sigma_{\text{mag}}^{-1} \left( \frac{B}{0.1 \text{ G}} \right) \text{ erg cm}^{-3}$$

by adding two jet parameters $B$ and $r_s$.

The model presented in this work uses an exact electron distribution that is solved from a generalized transport equation that contains the terms describing first-order and second-order Fermi acceleration, the escape of particle due to both advection and spatial diffusion, and energy losses due to synchrotron emission and IC scattering of an assumed soft photon field. Since it specifies the physical processes, instead of an assumed electron distribution, we have to introduce more parameters to control formation on the electron spectrum. In principle, the model requires 10 free parameters ($N_0$, $\gamma_0$, $\eta$, $\xi$, $\hat{A}$, $\hat{B}$, $\hat{\zeta}$, $B$, $\delta$, $r_s$) to calculate the theoretical photon spectra. More free parameters greatly increase the uncertainty of the model spectrum.

To help alleviate these problems, we establish the following constraints on the parameters: (1) since the parameter $\eta$ is valid for the case of particles with gyroradii smaller than the correlation length of the field, this scenario implies that $\eta \leq 1$; (2) the maximum efficiency factor $\xi$ is determined by ensuring that the speed of the shock wave $v_s$ is less than the light speed $c$. Hence, we obtain the constraint $\xi \leq 1$; (3) the size of the emission region is constrained by the variability timescales $\tau_{\text{var}}$ with $r_s \sim c \tau_{\text{var}}/(1 + z)$.

7.2. Effects of Changes in the Parameters

In order to penetrate the variety of spectral behaviors observed from blazar jets, it is important to investigate how the particle spectral parameters and/or jet parameters in the emission region affect the theoretical photon spectrum. To highlight the effects caused by the changes in individual parameters, we change only one parameter while keeping the other parameters fixed. We adopt $N_0 = 8.0 \times 10^{21} \rho^{-1} \text{ cm}^{-3} \text{ s}^{-1}$, $\gamma_0 = 100$, $\eta = 1.0$, $\xi = 0.1$, $\hat{A} = 30$, $\hat{B} = 5.54 \times 10^{-8}$, $\hat{\zeta} = 66$, $\hat{B} = 0.1 \text{ G}$, $\delta = 21$, and $r_s = 5.0 \times 10^{15} \text{ cm}$ as a baseline of the theoretical SED.

The changes in synchrotron and SSC spectra by varying the free parameters are shown in Figure 4. We note that (1) the intensity of the spectrum becomes higher when $N$ and $r_s$ increase, because the injected power depends on the continuous injection rate $\dot{N}$, and the total number of particles is proportional to the volume of the blob. The change in the flux proportional to $\delta^4$ and the blueshift of the frequency proportional to $\delta$ are clearly seen; (2) the shape of the theoretical photon spectrum is dominated by the characteristic Lorentz factor of the injection particle $\gamma_0$, dimensionless parameter $\eta$, shock acceleration efficiency factor $\xi$, dimensionless parameter $\hat{A}$, and dimensionless parameter $\hat{\zeta}$, because these parameters determine the particle distribution; (3) due to the peak frequency of the synchrotron component proportional to the magnetic field $B$, the peak frequencies of the synchrotron and SSC components increase when the magnetic field strengthens. On the contrary, the dimensionless parameter $\hat{B}$ increases, resulting in the decrease of the equilibrium energy of the particle; the peak frequencies of the synchrotron and SSC components decrease when the dimensionless parameter $\hat{B}$ increases.
Figure 4. Theoretical photon spectrum for various parameter changes. (1) Continuous injection rate $\dot{N}$; (2) characteristic Lorentz factor of the injection particle $\gamma_0$; (3) dimensionless parameter $\eta$; (4) shock acceleration efficiency factor $\xi$; (5) dimensionless parameter $\eta$; (6) dimensionless parameter $\xi$; (7) local magnetic field strength $B$; (8) beaming factor $\delta$; and (9) size of the blob $r_s$. 
Table 1.

| Physical Parameters | 2005–2009 Data | Archival Data |
|---------------------|----------------|---------------|
| \( N_0 \) (p\(^{-3}\) cm\(^{-3}\) s\(^{-1}\)) | \( 1.2 \times 10^{11} \) | \( 1.0 \times 10^{10} \) |
| \( \gamma_0 \) | 300 | 100 |
| \( \xi \) | 0.2 | 0.2 |
| \( \eta \) | 1.0 | 1.0 |
| \( \hat{B} \) | 60 | 60 |
| \( \hat{C} \) | \( 9.4 \times 10^{-10} \) | \( 4.44 \times 10^{-9} \) |
| \( B \) (G) | 174 | 126 |
| \( \delta \) | 0.15 | 0.15 |
| \( r_s \) (cm) | 1.7 \times 10^{16} | 3.8 \times 10^{16} |
| \( D_B \) (s\(^{-1}\)) | 1.34 \times 10^{-22} | 2.69 \times 10^{-23} |
| \( \sigma_{\text{mag}} \) | \( 8.79 \times 10^{5} \) | \( 8.79 \times 10^{3} \) |
| \( \omega \) | 1.72 | 2.38 |
| \( u \) (erg cm\(^{-3}\)) | 25.38 | 119.88 |
| \( F_{\text{inj}} \) (erg s\(^{-1}\)) | \( 1.6 \times 10^{50} \) | \( 5.14 \times 10^{49} \) |

7.3. Application to 1ES 0414+009

1ES 0414+009 resides in an elliptical host galaxy at a redshift of \( z = 0.287 \) (Halpern et al. 1991), with absolute magnitude \( M_B = -23.5 \) (Falomo et al. 2003). Both the original radio, optical, and X-ray observations (Ulmer et al. 1983) and the polarization measurements (Impey & Tapia 1988) confirmed the classification of this source as a BL Lac object. The archival observations of 1ES 0414+009 in X-ray bands show the synchrotron peak above a few keV (Giommi et al. 1990; Brinkmann et al. 1995; Kuo et al. 1998; Costamante et al. 2001; Sambruna et al. 1994; Beckmann et al. 2002). As an extreme source, the spectrum of the BL Lac object 1ES 0414+009 was measured extending up to 0.1 TeV in the multiwavelength observation campaign in the 2005–2009 epoch (Abramowski et al. 2012). Because the particle distribution in this context is solved from the stationary particle transport equation, what is important for the data is that the observations were made during quiescence or averaged over some epochs. In this scenario, we compile the archival data from Costamante & Ghisellini (2002) and the average spectra in the 2005–2009 epoch from Abramowski et al. (2012).

As mentioned above, in order to check whether the scenario in this context can explain the multiwavelength emission, we apply the results of the simulation to the extreme BL Lac object 1ES 0414+009. In order to do that, we first establish the value of the model parameters. Our approach for reproducing the multiwavelength spectrum from 1ES 0414+009 sets \( \eta = 1 \) and \( \xi = 0.2 \) in all of the numerical calculations. The other model parameters \( N_0, \gamma_0, \hat{A}, \hat{B}, \hat{C}, B, \delta, \) and \( r_s \) are varied until a reasonable qualitative fit to the multiwavelength spectral data is obtained. That is, assuming a continuous injection rate \( N_0 \) with an injected Lorentz factor \( \gamma_0 \), we calculate the electron distribution with the dimensionless parameters \( \hat{A}, \hat{B}, \) and \( \hat{C} \). Therefore, we can reproduce the multiwavelength spectrum with the magnetic field strength \( B \), the Doppler factor \( \delta \), and the size of the emission region \( r_s \). We report the physical parameters of both the average spectra in the 2005–2009 epoch and the archival data in Table 1. In Figure 5, we compare the theoretical multiwavelength spectrum with archival data and the average spectra in the 2005–2009 epoch from the BL Lac object 1ES 0414+009. We also show the particle distributions reproducing the multiwavelength spectra. It can be seen that (1) the analytical particle transport model considered here is able to roughly reproduce the observed spectra, and (2) despite the model requiring higher injection power, the particle distribution in this context is able to reproduce the multiwavelength spectrum with reasonable assumptions about the physical parameters.

8. Discussion

As an open issue, determining the jet physics from the SED is a tricky problem of inversion. The present paper introduces an analytical particle transport model to reproduce the quiescent broadband SED of blazars. In the model, the exact electron distribution is solved from a generalized transport equation that contains the terms describing the first-order and
second-order Fermi acceleration, escape of particle due to both advection and spatial diffusion, and energy losses due to synchrotron emission and IC scattering of an assumed soft photon field. We do not take into account the modification of the electron distribution in the Klein–Nishina regime (e.g., Moderski et al. 2005; Nakar et al. 2009). Furthermore, we do not include nonlinear synchrotron (Schlickeiser & Lerche 2007) and SSC (Schlickeiser 2009) cooling of relativistic electrons. Assuming suitable model parameters, we apply the results of the simulation to the extreme BL Lac object 1ES 0414+009. It is clear that the particle injection rate, $N$, and the Lorentz factor of injected electrons, $\gamma_0$, play an important role in determining the emission intensity. Assuming isotropic emission, the power associated with the injected particles is given by $P_{\text{inj}} = 4\pi r_0^3 \gamma_0 m_e^2 c^3 N / 3$. The model presented in this work suggests an extreme injection power with $P_{\text{inj}} \sim 10^{50}$ erg s$^{-1}$. This value exceeds the Eddington luminosity of a supermassive black hole $\sim 2 \times 10^9 M_\odot$ by two orders of magnitude. Despite equilibrium between the radiation pressure acting outward and the gravitational force acting inward being assumed for a spherically symmetric geometry (Eddington 1916), if the photons are trapped inside the accretion flows and are advected into the black holes (e.g., Abramowicz et al. 1988; Beloborodov 1998; Wang et al. 1999; Mineshige et al. 2000; Chen & Wang 2004), above which the radiation force dominates the gravity of the central black hole, the radiation luminosity can exceed the Eddington luminosity. In this paper, we do not propose an explanation for why the injection power exceeds the Eddington luminosity. However, it is interesting to speculate that this might be a result of the shock front overrunning a region in the jet in which the local plasma density is enhanced (e.g., Kirk et al. 1998; Zheng & Zhang 2011). In this scenario, we expect that the number of injected particles increases as an avalanche occurs in the jet. Incidentally, the particle injection rate, $N$, increases significantly and results in extreme injection power into the emission region. We also noted that the model suggests the magnetization parameter $\sigma_{\text{mag}} = 0.01$. This result is within the range from $\sigma_{\text{mag}} \sim 0.001$ in the MHD models (e.g., Kennel & Coroniti 1984) to $\sigma_{\text{mag}} \sim 1$ in the striped wind models (e.g., Komissarov 2013).

The present work differs from earlier efforts that only assumed that the escape of particles occurs via spatial diffusion (e.g., Stawarz & Petrosian 2008; Tammi & Duffy 2009; Lewis et al. 2016, 2018). Here, we concentrate on both shock-regulated escape and Bohm diffusive escape. We suggest that advection is a significant escape mechanism in blazar jets. Because advection tends to harden the particle distribution, which enhances the high-energy components of the resulting synchrotron and SSC spectra from blazar jets, we argue that the model can likely be used to understand the origin of hard spectra, although there are some other interpretations based on observed hard spectra from distant blazars (Lefa et al. 2011; Zheng & Kang 2013; Cerruti et al. 2015; Zheng et al. 2016).

The analytical particle transport model is based on the development of exact analytical solutions to the linear transport equation. A potential drawback of the model is that SSC losses cannot be included when establishing the particle distribution, because they are inherently nonlinear. To render the nonlinear effect of the SSC process, we assume a constant soft photon field instead of a synchrotron emission field. On the basis of the model results, we can also calculate the synchrotron emission field $u_{\text{syn}} = 3.45 \times 10^{-4}$ erg cm$^{-3}$ for 2005–2009 data and $u_{\text{syn}} = 1.64 \times 10^{-4}$ erg cm$^{-3}$ for archival data. If we directly include the synchrotron emission field into the transport equation, we find $u = u_{\text{sh}} + u_{\text{syn}} \sim 10^{-3}$ erg cm$^{-3}$. This value is four orders of magnitude less than the constant soft photon fields of the model assumed. It is believed that the electron populations where the energy is around the equilibrium energy produce the X-ray spectra. The observation shows the SED of a source with a synchrotron peak energy located at 0.1 keV (Abramowski et al. 2012). In this scenario, we can consider that the synchrotron emission of the electron populations with equilibrium energy contribute most of the intensity around the synchrotron peak. These issues imply that the equilibrium Lorentz factor satisfies $\gamma_e \sim (v_{\text{syn}} \gamma / 3.7 \times 10^6 B_0)^{1/2} \sim 10^5$. A smaller soft photon field results in a larger equilibrium Lorentz factor. The calculated synchrotron emission field induces the equilibrium Lorentz factor $\gamma_e$ to be around $10^7$ to $10^9$. This is far from the equilibrium Lorentz factor that is required by the observed synchrotron peak energy. We argue that, despite the assumed IC losses being unable to sufficiently approximate this condition, these losses ensure that the analytical particle transport model can obtain a suitable equilibrium Lorentz factor through a kind of competition between cooling and acceleration in the case where a reasonable magnetic field parameter is adopted. Leaving out the particle escape, we expect a large soft photon field to generate the suitable equilibrium.

Actually, electrostatic acceleration is of some interest, because it characterizes the strength of the magnetic reconnection that provides the source of acceleration to leptons. As proposed in Kroon et al. (2016), electrostatic acceleration in an electric field of strength $E$, generated in the magnetic reconnection region around the shock, results in a constant momentum gain rate given by $P_{\text{gain}} = eE$. Combining the momentum gain rate with shock acceleration in Equation (11), we can establish the first-order momentum gain rate, $P_{\text{gain}} = P_{\text{sh}} + P_{\text{elec}} = A_0 m_e c$ with the first-order momentum gain rate $A_0 = A_{\text{sh}} + A_{\text{elec}}$ in units of s$^{-1}$, appearing in Equation (2), where $A_{\text{elec}} = eE/(m_e c) = 1.76 \times 10^8 (E/B)(B/0.1\ G)$ s$^{-1}$. From the process that resulted in the dimensionless model parameter $A$, we obtain the relation $E/B = A_{\text{mag}} / (3\gamma - \xi)$. It is convenient to find the relative contribution from the shock acceleration and electrostatic acceleration in the emission region from the relation $P_{\text{elec}} / P_{\text{sh}} = \xi E/B$. It can be seen that, because the model simulations suggest $\xi < 1$, if it satisfies $E/B > 1$, then the electrostatic acceleration dominates the first-order momentum gain in the region. The efficient electrostatic acceleration can result in a larger equilibrium Lorentz factor and a harder particle spectrum. Conversely, if the ambient magnetic field exceeds the electric field, the shock acceleration dominates the first-order momentum gain in the region. The model presented in this work requires increasing the parameter $u_{\text{ph}}$ as a free parameter to calculate the value of $E/B$. In order to simplify the model, we attempted a study using the model where one does not take the electrostatic acceleration into account.

We note that in the regime of the Lorentz factor far below the equilibrium Lorentz factor, the particle distribution is well represented by a broken power law. Of particular interest is the case without random motions of the MHD waves, wherein the equation is left with only the contribution due to shock acceleration. This scenario corresponds to the limit of the...
momentum diffusion rate constant $D_0 \to 0$. After some algebra, we conveniently deduce the power-law index of the electron distribution for the case of shock acceleration as $\alpha_{sh} = \lim_{m_0 \to 0} \frac{\gamma_0}{\gamma} = -\frac{\bar{C}}{\bar{A}}$ (e.g., Kroon et al. 2016). The particle-in-cell simulations in the regions of magnetic reconnection near the termination shock give the high-energy power-law index $\alpha_{sh}$ in the range of $\alpha_{sh} \in [3, -2]$ (Cerutti et al. 2014). Our model fits suggest that $\bar{C}/\bar{A} = 2.9$ and 2.1 in the 2005–2009 epoch and archival data, respectively, providing an important shock acceleration diagnostic.

Recalling the condition resulting in the steady-state Green’s function, we endeavor to propose the following paradigm for the broken power law in the blazar region (e.g., Zheng et al. 2018b): in the case of a non-relativistic and parallel shock, we assume the particle distributions satisfy $N_2(\gamma) \propto \delta(\gamma)\sqrt{\gamma} - \gamma_0$ in the upstream region, where $\gamma_0$ is the characteristic energy of the particles. If the size of the shocked flow is limitless, using the zero flux boundary condition, we could obtain a steady particle spectrum in the downstream flow $N_2(\gamma) = N_0\gamma^{-\alpha}$ with the spectrum index $\alpha$ (e.g., Dermer & Menon 2009). As for the above scenarios resulting in harder spectra than the distribution with energy loss and Bohm diffusion escape, we leave the details of the application and discussion to future work.

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References

Abramowiz, M., & Stegun, I. A. 1970, Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables (New York: Dover)

Abramowicz, M. A., Czerny, B., Lasota, J. P., & Szuszkiewicz, E. 1988, ApJ, 332, 646

Abramowski, A., Acero, F., Aharonian, F., et al. 2012, A&A, 538, 103

Asano, K., & Hayashida, M. 2015, ApJL, 808, L18

Axford, W. I. 1981, in IAU Symp. 94, Origin of Cosmic Ray, ed. G. Setti, G. Spada, & A. M. Wolfendale (Dordrecht: Reidel), 339

Baring, M. G., Böttcher, M., & Summerlin, E. J. 2017, MNRAS, 464, 4875

Bähr, A. R. 1978, MNRAS, 182, 147

Beloborodov, A. M. 1998, MNRAS, 297, 739

Bicknell, G. V., & Melrose, D. B. 1982, ApJ, 262, 511

Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433

Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433

Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433

Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433

Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433

Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433

Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433

Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433

Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433

Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433

Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433

Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433

Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433

Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433

Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433

Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433

Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433

Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433

Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433

Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433

Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433

Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433

Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433

Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433

Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433

Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433

Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433

Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433

Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433

Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433

Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433

Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433

Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433
