Chiral phase transitions in strong chromomagnetic fields at finite temperature and dimensional reduction

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Abstract

Dynamical fermion mass generation in external chromomagnetic fields is considered at non-zero temperature. The general features of dynamical chiral symmetry breaking ($D\chi SB$) are investigated for several field configurations in relation to their symmetry properties and the form of the quark spectrum. According to the fields, there arises dimensional reduction by one or two units. In all cases there exists $D\chi SB$ even at weak quark attraction, confirming the idea about the dimensional insensitivity of this mechanism in a chromomagnetic field.

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1. Introduction

As is well known, the high energy region (short distances) of strong interactions can be considered perturbatively in the framework of QCD due to its property of asymptotic freedom. On the other hand, in order to study nonperturbative phenomena at low energies (large distances) like the QCD vacuum with gluon and quark condensates [1] and the hadronization process analytically, various approximate methods and effective models have been proposed. One approximate possibility to simulate a realistic gluon condensate is to introduce an appropriate external (abelian or non–abelian) chromomagnetic field [2]. Exact solutions of particle equations in external field models allow in particular for explicit analytical calculations and estimates of various nonperturbative effects, demonstrating nonanalytic dependence of them on field intensities [3–6]. There exist also simple field–theoretic models like the Nambu–Jona–Lasinio (NJL) chiral quark model which has been successfully used to describe $D\chi SB$, bosonization and low energy behavior of mesons (see e.g. [7] and references therein). In particular, for a QCD–motivated NJL–model with gluon condensate and finite temperature, it was shown that a weak gluon condensate plays a stabilizing role for the behavior of the constituent quark mass, the quark condensate, meson masses and coupling constants for varying temperature [8].

The effect of dynamical chiral symmetry breaking ($D\chi SB$) under the influence of a magnetic field, resulting in a fermion mass generation, has recently attracted much attention [9, 10]. In the framework of four–fermion models it has been shown that a constant magnetic field serves as a catalyzing factor in the fermion mass generation even under conditions, when the interaction between fermions is rather weak [11–12]. Moreover, in a recent paper [13] it has been argued that $D\chi SB$ does not take place in an external axial–symmetric chromomagnetic field in $D = 3 + 1$ dimensions when attraction between fermions is weak. Thus, this result seems to contradict the idea about the dimensional insensitivity of this mechanism in a chromomagnetic field.

The purpose of the present letter is to investigate the phenomenon of $D\chi SB$ for various external chromomagnetic fields like non–abelian axial–symmetric and rotational–symmetric as well as abelian fields. In particular, we will show that in all cases even for weak coupling of quarks the $D\chi SB$ does exist confirming the idea about its dimensional insensitivity and that it
is related to an effective dimensional reduction. Furthermore, we will find a simple relation between symmetry properties of external fields, the degeneracy of quark energy spectra and the phenomenon of dimensional reduction. The latter effect leads to a nonanalytic logarithmic dependence of the quark condensate on the field strength in the strong field limit. Finally, we shall consider the effect of finite temperature and show that in the strong field limit there exists a finite critical temperature at which a phase transition takes place and chiral symmetry is restored in both abelian and non–abelian models of the gluon condensate. In particular, there arises an interesting relationship \( T_c = C m(0) \) between the critical temperature and the zero temperature fermion mass \( m(0) \), with a universal constant \( C \) for different fields.

2. Quark condensates in external fields

2.1 General definitions

Let us consider an \( SU(N_f) \) flavor–symmetric model of quarks \( q_i \) moving in an external chromomagnetic field of the color group \( SU(3)_c \). The Lagrangian takes the following form (for convenience, we choose Euclidean D–dimensional space–time with \( it = x_D \))

\[
\mathcal{L} = \sum_{i=1}^{N_f} \bar{q}_i (\gamma_\mu \nabla_\mu + m_i) q_i, \tag{1}
\]

where \( m_i = m \) are (equal) masses of quarks, \( \nabla_\mu = \partial_\mu - ig_2 \frac{1}{2} \lambda_\alpha A^\alpha_\mu \) is the covariant derivative of quark fields in the constant external fields \( F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f_{abc} A^b_\mu A^c_\nu \), determined by the potentials \( A^a_\mu (a = 1, ..., 8) \). Next, introduce the vacuum functional \( Z_q \) of quark fields,

\[
Z_q = \int d\bar{q}dq \exp[- \int d^Dx \mathcal{L}] = \prod_{i=1}^{N_f} \det(\gamma_\mu \nabla_\mu + m) = \\
= \prod_{i=1}^{N_f} \exp\{ Tr \ln(\gamma_\mu \nabla_\mu + m) \} = \exp W_E, \tag{2}
\]
with $W_E$ being the Euclidean effective action

$$W_E = \sum_{i=1}^{N_f} \int \frac{dp_D}{2\pi} \sum_{k,\kappa} \ln(p_D^2 + \varepsilon_k^2(i, \kappa)), \quad (3)$$

and $\varepsilon_k(i, \kappa)$ is the energy spectrum of quarks of flavor $i$ and color $\kappa$ with quantum numbers $k$ moving in the constant external field $F_{\mu\nu}^a$.

In the case of finite temperature $T = 1/\beta > 0$ the effective potential $v = -W_E/(\beta L^D)$ [5] is obtained after substituting $p_D \to 2\pi/\beta (l + 1/2)$, $l = 0, \pm 1, \pm 2, ...$

$$v = -\frac{1}{\beta L^{D-1}} \sum_{i=1}^{N_f} \sum_{k,\kappa} \sum_{l=-\infty}^{l=+\infty} \ln \left[ \left( \frac{2\pi(l + 1/2)}{\beta} \right)^2 + \varepsilon_k^2(i, \kappa) \right] =$$

$$= \frac{1}{\beta L^{D-1}} \sum_{l=-\infty}^{l=+\infty} \sum_{i=1}^{N_f} \sum_{k,\kappa} \int \frac{ds}{\lambda^2} \exp \left\{ -s \left[ \left( \frac{2\pi(l + 1/2)}{\beta} \right)^2 + \varepsilon_k^2(i, \kappa) \right] \right\} =$$

$$= \frac{N_f}{2\sqrt{\pi} L^{D-1}} \sum_{k,\kappa} \int \frac{ds}{\lambda^2} \exp \left\{ -s \varepsilon_k^2(i, \kappa) \right\} =$$

$$\times \left[ 1 + 2 \sum_{l=1}^{\infty} (-1)^l e^{-\frac{\beta^2 l^2}{4s}} \right], \quad (4)$$

where $\Lambda$ is an ultraviolet cutoff ($\Lambda \gg m$). According to (2) we then find for the quark condensate

$$\langle \bar{q}q \rangle = \int d\bar{q}dq \bar{q}q \exp \left\{ - \int d^D x L \right\} =$$

$$= -\frac{1}{Z} \frac{\partial Z}{\partial m} = -\frac{1}{\beta L^{D-1}} \frac{\partial W_E}{\partial m} = \frac{\partial v}{\partial m}, \quad (5)$$

which gives [4]

$$\langle \bar{q}q \rangle = -\frac{m N_f}{L^{D-1} \sqrt{\pi}} \sum_{k,\kappa} \int_{\frac{1}{\lambda^2}}^{\infty} \frac{ds}{\sqrt{s}} \exp \left\{ -s \varepsilon_k^2(i, \kappa) \right\} \left[ 1 + 2 \sum_{l=1}^{\infty} (-1)^l e^{-\frac{\beta^2 l^2}{4s}} \right] =$$

$$= \langle \bar{q}q \rangle_{T=0} + \langle \bar{q}q \rangle_{T \neq 0}. \quad (6)$$
Clearly, in the case of a vanishing external field \((F_{\mu\nu} = 0)\), we have
\[
\varepsilon_k^2 = \sum_{i=1}^{D-1} k_i^2 + m^2 \quad (-\infty < k_i < \infty).
\]
Then, at \(T = 0\) one obtains
\[
\langle \bar{q}q \rangle = -\frac{3mN_f}{2^{D-2}\pi^{D/2}} \int_{-\infty}^{\infty} \frac{ds}{s^{D/2}} e^{-sm^2} . \tag{7}
\]
In the following, we shall analyze three special cases of external chromo-
magnetic fields.

**Case i):**
Rotational–symmetric non–abelian chromomagnetic field
\[
A_1^1 = A_2^2 = A_3^3 = \sqrt{H/g}, \quad H_{\mu}^a = \delta_{\mu}^a H (i = 1, 2, 3), \tag{8}
\]
with all other components of \(A_{\mu}^a\) vanishing.

The energy spectrum has six branches, two of which correspond to quarks
that do not interact with the chromomagnetic field
\[
\varepsilon_{1,2}^2 = \vec{p}^2 + m^2 , \tag{9}
\]
and the other four are given as follows
\[
\varepsilon_{3,4}^2 = m^2 + \left(\sqrt{a} \pm \sqrt{\vec{p}^2}\right)^2 , \\
\varepsilon_{5,6}^2 = m^2 + \left(\sqrt{a} \pm \sqrt{4a + \vec{p}^2}\right)^2 , \tag{10}
\]
where \(a = gH/4\).

**Case ii):**
Axial–symmetric non–abelian chromomagnetic field
\[
A_1^1 = A_2^2 = \sqrt{H/g}, \quad H_{i}^a = \delta_{i}^a \delta_{3} H , \tag{11}
\]
with all other components of the potential vanishing.

The branches of the quark energy spectrum are besides \(\varepsilon_{1,2}^2\) as follows
\[
\varepsilon_{3,4,5,6}^2 = m^2 + 2a \pm \sqrt{4a^2 + 4ap_{\perp}^2 + p_{3}^2 + p_{\perp}^2} = \\
= m^2 + p_{3}^2 + \left(\sqrt{a + p_{\perp}^2} \pm \sqrt{a}\right)^2 . \tag{12}
\]

**Case iii):**
Abelian chromomagnetic field

\[ A_\mu = \delta_3^0 \delta_\mu 2 x_1 H. \]  

This time only two color degrees of freedom of quarks with charges \( \pm g/2 \) interact with the external field. The energy spectrum of quarks is now given by

\[ \varepsilon_{3,4,5,6}^2 = \varepsilon_{n,\sigma,p_3}^2 = gH(n + \frac{1}{2} + \frac{\sigma}{2}) + p_3^2 + m^2, \]  

where \( \sigma = \pm 1 \) is the spin projection on the external field direction, \( p_3 \) is the longitudinal component of the quark momentum \( (-\infty < p_3 < \infty) \),

\[ p_\perp^2 = gH(n + \frac{1}{2}) \]  

is the transversal component squared of the quark momentum, and \( n = 0, 1, 2, ... \) is the Landau quantum number.

As can be expected from (6), the form of the spectrum is essential for the quark condensate formation. Using the above three expressions of energy spectra for field configurations \( i \), \( ii \) and \( iii \), we shall next study the corresponding three types of quark condensates in the strong field limit.

2.2. Asymptotic estimates for strong fields \( \frac{gH}{m^2} \gg 1 \).

Case \( i \):

According to (9), (10) we have

\[
\langle \bar{q}q \rangle = \frac{-mN_f 4\pi a}{\sqrt{\pi}(2\pi)^3} \int \frac{dt}{\sqrt{t}} e^{-\frac{t}{a}} \int_0^\infty dx x^2 \left[ 2e^{-x^2 t} + e^{-t(1-x)^2} + e^{-t(1+x)^2} + e^{-t(1+\sqrt{x^2+4})^2 t} + e^{-t(1-\sqrt{x^2+4})^2 t} \right] \left( 1 + 2\sum_{l=1}^\infty (-1)^l e^{-\frac{t^2a^2}{4t}} \right).
\]

Taking the \( T = 0 \) term in (10), we see that the branch of the spectrum

\[ \varepsilon_4^2 = m^2 + (\sqrt{a} - \sqrt{p^2})^2 \]
plays the main role when \( m \to 0 \) (the second term in the brackets). When \( h = gH/m^2 \gg 1 \) the following asymptotics is obtained

\[
\langle \bar{q}q \rangle = -\frac{m^3 N_f}{4\pi^2} \left[ 3 \left( \frac{\Lambda^2}{m^2} - \ln \frac{\Lambda^2}{m^2} \right) + \frac{h}{2} \ln(C_1 h) - h I_1(\beta m) \right].
\]

(17)

Here

\[
I_1(\beta m) = -\sum_{l=1}^{\infty} (-1)^l \int_0^\infty \frac{dx}{x} \exp \left[ -\left( x + \frac{l^2 m^2 \beta^2}{4x} \right) \right] = -2 \sum_{l=1}^{\infty} (-1)^l K_0(\beta ml),
\]

where \( K_0(y) \) is the Macdonald’s function and \( C_1 \) is a certain numerical constant.

It is well-known that \( D\chi SB \), signalled by a nonvanishing quark condensate, is the origin of dynamical quark masses. The underlying mechanism can be most simply illustrated by considering an NJL model with four-fermion interactions,

\[
L_{\text{int}} = 2G \sum_{i=0}^{N_f^2-1} \left\{ \frac{1}{2} \bar{q} \lambda^F_i q \right\}^2 + \left( \bar{q} \gamma^5 \lambda^F_i q \right)^2,
\]

\[
\text{Tr} \lambda^F_i \lambda^F_j = 2\delta_{ij}, \quad \lambda^F_0 = \sqrt{2/N_f} \mathbf{1},
\]

with \( G \) being a universal coupling constant and \( \lambda^F_i \) being flavor generators.

After applying the bosonization procedure in the \( N_c \to \infty \) limit (\( N_c G = \text{const} \)), the one–loop expression for the corresponding meson functional integral is dominated by the stationary phase, and we obtain the gap equation \[7\]

\[
m = -\frac{2G}{N_f} \langle \bar{q}q \rangle,
\]

(19)

or, according to (17):

\[
\Lambda^2 (\frac{1}{g} - 1) = -m^2 \ln \frac{\Lambda^2}{m^2} + m^2 \frac{h}{6} \ln C_1 h - h \frac{m^2}{3} I_1,
\]

(20)

The limit \( N_c \to \infty \) is here only needed for technical reasons. In subsequent applications, one can set afterwards again \( N_c = 3 \) in final expressions.
where $\tilde{g} = \frac{3\Lambda^2 g}{2\pi^2}$.

For $gH \ln \frac{m}{m^2} \gg m^2 \ln \frac{\Lambda^2}{m^2}$ ($gH < \Lambda^2$) we have a solution of (20) even for weak coupling $\tilde{g} \ll 1$

$$m^2(T) = C_1 gH \exp \left[ -\frac{4\pi^2}{GgH} - 2I_1(\beta m(T)) \right]. \tag{21}$$

In particular, for $T = 0$,

$$m^2(0) = C_1 gH \exp \left( -\frac{4\pi^2}{GgH} \right). \tag{22}$$

The critical temperature can now be found from the condition $m(T_C) = 0$, which gives

$$T_C = \pi^{-1} e^\gamma m(0) \simeq 0.5669 m(0), \tag{23}$$

where $\gamma$ is Euler’s constant. Let us emphasize that the result (17) with the logarithmic term $\frac{h}{2} \ln h$ demonstrates the effect of dimensional reduction $D = 3 + 1 \rightarrow D = 1 + 1$. Indeed, integration of the main term in (16) gives

$$\langle \bar{q}q \rangle \simeq -\frac{mN_f a}{2\pi^2} \int_0^\infty ds \frac{e^{-sm^2}}{s} \approx -\frac{mN_f a}{2\pi^2} \ln \frac{a}{m^2}. \tag{24}$$

which corresponds to (7) with $D = 2$ and $\Lambda^2$ replaced by $a$.

**Case ii:**

In this case we have

$$\langle \bar{q}q \rangle = -\frac{mN_f}{2\pi^2} \int_0^\infty ds \int \frac{dp}{a^2} \int_0^\infty dp_\perp \times$$

$$\times \left[ e^{-s p^2_\perp} + e^{-s(\sqrt{a + p^2_\perp} - \sqrt{a})^2} + e^{-s(\sqrt{a - p^2_\perp} + \sqrt{a})^2} \right] \left[ 1 + 2 \sum_{l=1}^\infty (-1)^l e^{-\beta^2 l^2} \right].$$

The gap equation for $h \gg 1$ now takes the form

$$\Lambda^2 \left( \frac{1}{\tilde{g}} - 1 \right) = -m^2 \left( \ln \frac{\Lambda^2}{m^2} - \frac{h}{3} - \frac{\sqrt{\pi} h}{3} I_2(\beta m) \right). \tag{25}$$
where

\[ I_2(\beta m) = \sum_{l=1}^{\infty} (-1)^l \int_0^\infty \frac{dx}{x^{3/2}} \exp \left[ - \left( x + \frac{\beta^2 l^2 m^2}{4x} \right) \right] = \]

\[ = 2 \sqrt{\frac{\pi \beta m}{\beta m}} \sum_{l=1}^{\infty} (-1)^l e^{-\sqrt{\beta ml}/\sqrt{l}}. \]

Note that the \( T \to 0 \) limit of expression (25) differs from the corresponding one in [13] by the sign of the term \( h/3 \). A nontrivial solution of (25) is then possible for

\[ \tilde{g} > \frac{1}{1 + (gH/(3\Lambda^2))}. \]

and it exists even for \( \tilde{g} < 1 \), i.e. for weak quark attraction. It has the form

\[ m^2 = \Lambda^2 \exp \left[ - \frac{\Lambda^2}{m^2} \left( 1 - \frac{1}{\tilde{g}} \right) - \frac{h}{3} - \frac{\sqrt{\pi h}}{3} I_2(\beta m) \right], \]

(26)

and demonstrates the possibility of chiral symmetry breaking in a non-abelian chromomagnetic field at \( D = 3 + 1 \) for \( \tilde{g} < 1 \) (weak attraction).

The dependence on \( h \) in (25) is found from the dominating term in (3) arising from the branch

\[ \varepsilon^2 = m^2 + p_3^2 + (\sqrt{p_\perp^2 + a} - \sqrt{a})^2. \]

Then we have for \( a \to \infty \)

\[ \langle \bar{q}q \rangle \sim \int_{\frac{1}{a}}^{\infty} \frac{ds}{s} e^{-sm^2} \int_0^{\infty} dp_\perp e^{-sp_\perp^2/4a} \sim \sqrt{a} \int_{\frac{1}{a}}^{\infty} \frac{ds}{s^{3/2}} \sim a \]

corresponding to (4) with \( D = 3 \), which demonstrates the \( 3 + 1 \to 2 + 1 \) dimensional reduction in this type of field.

**Case iii:**

For the abelian chromomagnetic field with the spectrum (14) we obtain

\[ \langle \bar{q}q \rangle = -\frac{m^3 N_f}{4\pi^2} \left\{ h \ln \frac{h}{2\pi} + 3 \left( \frac{\Lambda^2}{m^2} - \ln \frac{\Lambda^2}{m^2} \right) - 2hI_1(\beta m) \right\}, \]

(27)
which is similar to (17), but differs by an overall factor 2 in field-dependent terms. This difference is simply due to the fact, that the main term \( h \ln h \) is obtained from two colors in the spectrum (14), while in the non-abelian case only one branch of the spectrum contributes to (17).

For \( gH \ln \left( \frac{gH}{m^2} \right) \gg m^2 \ln \left( \frac{\Lambda^2}{m^2} \right) \) \( (gH < \Lambda^2) \) we obtain for \( m^2(T), m^2(0) \) and \( T_C \) the same equations (21)–(23) as in the non-abelian case \( i \), but with the obvious replacements \( C_1 \to \frac{1}{2\pi} \) and \( 4\pi^2 \to 2\pi^2 \) in the exponents.

The main logarithmic term in (27) is obtained from the \( n = 0, \sigma = -1 \) contribution in the sum over quantum states in (3)

\[
\langle \bar{q}q \rangle \sim -\int_{1/gH}^{\infty} \frac{ds}{s} \int_{-\infty}^{+\infty} dp_3 \sum_{n=0}^{\infty} (2 - \delta_{n0}) \exp[-gH ns - sm^2 - p_3^2 s] \sim \\
\sim -\int_{1/gH}^{\infty} \frac{ds}{s} e^{-sm^2} \approx -\ln \frac{gH}{m^2}.
\]

Obviously, this corresponds to (7) with \( D = 2 \), which demonstrates the dimensional reduction in this case \( 3 + 1 \to 1 + 1 \), similar to the non-abelian case \( i \). (The replacement \( \Lambda^2 \to gH \) follows here from the requirement \( 1/\Lambda^2 < 1/gH \ll s \) for the integration region.)

### 3. Summary and discussions

As shown in this letter, the phenomenon of \( D\chi SB \) does exist for all the field configurations considered here even for weak coupling. This effect is accompanied by an effective lowering of dimensionality in strong chromomagnetic fields, where the number of reduced units of dimensions depends on the concrete type of the field. It should also be noted that there is no contradiction with the Mermin–Wagner–Coleman (MWC) theorem [15], when the effective dimensionality reduces to the value \( 1 + 1 \), since only charged channels are affected by this reduction, while Nambu–Goldstone bosons are neutral [12]. It is interesting to mention that our result (21), being effectively \( 1 + 1 \) dimensional, corresponds to considerations of paper [16], where a similar expression for the dynamical mass has been obtained directly in the \( 1 + 1 \) dimensional Gross–Neveu model.
The physical reason of dimensional reduction is quite transparent on the basis of the symmetry properties of the external fields and the corresponding degeneracy of the quark spectra. If the field potentials are rotationally symmetric as in (8), the spectrum depends on $|\vec{p}|$, being twice degenerated with respect to azimuthal and polar angles of the momentum vector $\vec{p}$. The branch of the spectrum $\varepsilon_1^2 = m^2 + (\sqrt{a} - \sqrt{p^2})^2$ providing the main contribution to the condensate formation (17) has its minimum $\varepsilon_{\text{min}}^2 = m^2$ at nonzero value of $|\vec{p}| = \sqrt{a}$. In this case the dimensionality is lowered from $3+1$ to $1+1$, i.e. by 2 units, which is equal to the degeneracy of the quark energy spectrum. For the axially symmetric potential (11) the spectrum of quarks is a function of longitudinal $p_3$ and transversal $p_\perp$ components of the momentum vector $\vec{p}$, while $\varepsilon^2 = \varepsilon_{\text{min}}^2 = m^2$ for $p_\perp = p_3 = 0$. Now dimensions are reduced by 1 unit ($3+1 \rightarrow 2+1$) which is again equal to the degeneracy of the quark spectrum. Finally, for the abelian potential (13) the main contribution for strong fields comes from the small region around the minimum value $\varepsilon_{\text{min}}^2 = m^2$ for $p_3 = 0, n = 0, \sigma = -1$. In this case $(p^2_\perp)_{\text{min}} = gH/2 \gg m^2$ and a quark is localized in a small region $(\Delta r)^2_{\text{min}} \sim 2/(gH) \ll \frac{1}{m^2}$ in the $xy$–plane which clearly demonstrates $3+1 \rightarrow 1+1$ effective dimensional reduction by 2 units.

We note in passing that the Dirac equation in both the abelian and non–abelian chromomagnetic fields possesses the property of supersymmetry [14], which provides for the lowest possible value of the energy $\varepsilon_{\text{min}}^2 = m^2$. This is crucial for the formation of the quark condensate in the infrared region $m \rightarrow 0$ ($gH \gg m^2$) in the strong field limit. In the opposite case of weak chromomagnetic fields, $gH \ll m^2$, the mass generation is possible only in the strong coupling regime, when $\tilde{g} > 1$. In this case the gluon condensate has a stabilizing effect in contrast to the strong field case. Here the dynamical mass $m$ and $T_C$ increase with the field [8,9].

Finally, we remark that it is interesting to study also the effect of the temperature dependence of the external chromomagnetic field [5] not only in the weak field limit [9] but also for the case of strong fields.

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