Single-photon nonlinearity with intracavity electromagnetically induced transparency in blockaded Rydberg ensemble

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A scheme is presented to realize strong single-photon nonlinearity with intracavity electromagnetically induced transparency in blockaded Rydberg ensemble. In our scheme, the photons in the cavity are in the form of cavity dark-state polaritons and thus the effect of tuning the control field is equivalent to that of changing the cavity quality factors Q. Profiting from this special feature, the system behaves very strong single-photon nonlinearity, and the nonlinear strength would be three orders of magnitude larger than both the effective cavity decay rate and atomic decay rate. We also show the application of the strong nonlinearity to strong photon blockade effect.

In optical science and engineering, strong nonlinearities at the single-photon level have many important applications, such as photon blockade [1–4], single-photon quantum router [5], and quantum phase transitions of light [6]. It is thus of considerable interest to develop schemes achieving larger and larger photon nonlinearities, as these would make further quantum effects accessible in experiments. A possible route towards large nonlinearities is the use of strong, coherent interactions between light and matter in high-Q cavities. Typically, for the practical applications of the nonlinearities with negligible losses, the strong-linearity condition,

\[ \chi \gg \kappa, \gamma, \]

is required, with \( \chi \) the effective nonlinear coupling strength, \( \kappa \) the cavity decay rate, and \( \gamma \) parasitic loss rate. In cavity quantum electrodynamics (QED), one of the main approaches to achieving strong nonlinearities is increasing the quality factors Q and thus decreasing the cavity decay rate. However, increasing the quality factors Q requires sophisticated techniques. Although progress has been made towards high-Q cavities in the past 30 years, achieving strong-nonlinearity condition in Eq (1) in an optical cavity is still a challenging pursuit in the experiments [3, 4]. In this paper we propose a scheme for realizing strong single-photon nonlinearity of the three-mode mixing with intracavity electromagnetically induced transparency (EIT) in blockaded Rydberg ensemble. In our scheme, the photons in the cavity are in the form of cavity dark-state polaritons and the effective cavity decay rate is \( \kappa' = \cos^2 \theta \kappa \), where \( \kappa \) is the bare cavity decay rate without EIT medium and \( \theta \) is the mixing angle between cavity field and atoms components of dark-state polariton [7]. With a decrease of \( \cos \theta \) by tuning the control field, the effective cavity decay rate \( \kappa' \) decreases, thus the effect of tuning the control field is equivalent to that of increasing the quality factors Q. Profiting from this special feature, this system behaves very strong single-photon nonlinearity and the strong-nonlinearity condition in Eq. (1) could be achieved by tuning the control field.

An EIT medium placed in a cavity, which is known as intracavity EIT termed by Lukin et al. [8], can substantially affect the properties of the resonator system. The intracavity EIT provides an effective way to significantly narrow the cavity linewidth [9] and enhance the cavity lifetime [10, 11]. We here present a scheme to realize the nonlinear process of three-mode mixing with intracavity EIT in blockaded Rydberg ensemble. In our scheme, the photons in the cavity are in the form of dark-state polaritons and the strong nonlinear process could be achieved by tuning the control field in intracavity EIT. We show that the nonlinear strength in our scheme would be three orders of magnitude larger than both the effective cavity decay rate and atomic decay rate. We also discuss the application of the strong nonlinearity to strong photon blockade, which has been exploited as a powerful tool for quantum control of light quanta [12].

As illustrated in Fig.1(a), our model consists of an ensemble of N cold Rydberg atoms inside a two-sided optical cavity. The concrete atomic level structure and relevant transitions are shown in Fig.1(b). Each atom has a stable ground state \( |g\rangle \), an excited state \( |e\rangle \), and three high-lying Rydberg states \( |r_k\rangle \ (k = 1, 2, 3) \). The transition \( |g\rangle \leftrightarrow |e\rangle \) of is resonantly coupled by the cavity mode \( a \) with coupling strength \( g \), while a control field with Rabi

FIG. 1: (Color online) (a) Schematic setup for intracavity EIT in blockaded Rydberg ensemble. This system behaves very strong single-photon nonlinearities, by tuning the control field in intracavity EIT. (b) The relevant atomic level structure and transitions.
frequency $\Omega$ resonantly drives the transition $|e\rangle \leftrightarrow |r_2\rangle$. Thus they form the three-level EIT configuration \cite{12}, in which the interaction Hamiltonian is described by $H_I = \sum_{j=1}^{N} (g |e\rangle_j \langle g| a + \Omega |r_2\rangle_j \langle e| + H.c.$ If the probe field is a weak field, we would assume that almost all atoms are in the ground state $|G\rangle = \prod_{j=1}^{N} |g_j\rangle$ at all times, and define a dark-state polariton, $m_D = \cos \theta a - \sin \theta C_{r_2}$, and a bright-state polariton, $m_B = \sin \theta a + \cos \theta C_{r_2}$, with $\cos \theta = \sqrt{\sqrt{N}g^2 + \Omega^2}$ and $C_{r_2} = (N/\sqrt{N}) \sum_{j=1}^{N} |e\rangle_j \langle g|$ ($\epsilon = e, r_2$). In terms of these polaritons the Hamiltonian $H_I$ can be represented by \cite{12}

$$H_I = \sqrt{N}g^2 + \Omega^2 (C_{r_2}^\dagger m_B + C_e m_D^\dagger),$$

(2)

where only the bright-state polariton $m_B$ resonantly couples to the excited state. Taking into account the cavity mode $a$ interacting with external fields through input–output processes, two polaritons $m_D$ and $m_B$ couple to the external fields described by the Hamiltonian \cite{12}

$$H_{in-out} = \sum_{\alpha, \beta} \int_{-\infty}^{+\infty} \omega d\omega \Theta^\dagger(\omega) \Theta(\omega) + \sum_{\alpha, \beta} \int_{-\infty}^{+\infty} \omega d\Theta^\dagger(\omega) (\sqrt{\frac{k}{2\pi}} m_D + \sqrt{\frac{k}{2\pi}} m_B) + H.c.,$$

where $\omega$ is the frequency of the external field, $\epsilon$ is the bare cavity decay rate without EIT medium, and $\Theta(\omega)$ with the standard relation $[\Theta(\omega), \Theta^\dagger(\omega')] = \delta(\omega - \omega')$ denotes the one-dimensional free-space mode, $\kappa = \cos^2 \theta \kappa_c$ ($\kappa_c = \sin^2 \theta \kappa$) denotes the effective cavity decay rate of the dark-state (bright-state) polariton. When the bright-state polariton $m_B$ resonantly couples to the excited state in the so-called strong coupling regime, $\sqrt{N}g^2 + \Omega^2 \gg \kappa, \gamma_c$, this coupling will induce an effect known as vacuum Rabi splitting \cite{16}, i.e., the splitting of the cavity transmission peak for the bright-state polariton $m_B$ into a pair of resolvable peaks at $\omega = \omega_0 \pm \sqrt{\sqrt{N}g^2 + \Omega^2}$, here $\gamma_c$ is the spontaneous-emission rate of the excited state $|e\rangle$ and $\omega_0$ is the resonant frequency of cavity mode. Assuming that the external field inputted to the cavity is inside of the bandwidth $[\omega_0 - \omega_b, \omega_0 + \omega_b]$ with $\omega_b \ll \sqrt{N}g^2 + \Omega^2$, the coupling of the external fields to the bright-state polariton $m_B$ can be neglected, leading to

$$H_{in-out}^\prime = \sum_{\alpha, \beta} \int_{\omega_0 - \omega_b}^{\omega_0 + \omega_b} \omega d\omega \Theta^\dagger(\omega) \Theta(\omega) + \sum_{\alpha, \beta} \int_{\omega_0 - \omega_b}^{\omega_0 + \omega_b} d\omega \sqrt{\frac{\kappa}{2\pi}} \Theta^\dagger(\omega) m_D + H.c.,$$

(3)

We note that the polariton $m_D$ corresponds to the well-known “dark-state polariton” in EIT in free space \cite{7}.

Ref \cite{7}, the mixing angle $\theta$ determines the group velocity of dark-state polariton, whereas the mixing angle $\theta'$ here determines the effective cavity decay rate. As shown below, this feature of the polariton $b_0$ plays a vital role in achieving strong-nonlinearity condition in Eq. (1).

We consider the blockade interaction via Rydberg level “hopping” described by the Hamiltonian $H_R = \lambda_{ij} \sum_{j=0}^{N} |r_2\rangle_j \langle r_1| \langle r_1| + H.c.$ \cite{17}, where $\lambda_{ij} \sim \varphi_{r_2r_1} \varphi_{r_2r_2}/r^3_{ij}$, $\varphi_{r_2r_1}$ ($\varphi_{r_2r_2}$) is the dipole matrix element for the corresponding transition and $r_{ij}$ is the distance between the two atoms. In general this interaction does not affect the singly excited Rydberg state $|r_2\rangle$ but leads to a splitting of the levels when two or more atoms are excited to the state $|r_2\rangle$. The manifold of doubly excited states of atomic ensemble trapped in a finite volume $V$ has an energy gap of order $\lambda = \varphi_{r_2r_1} \varphi_{r_2r_2}/V$ \cite{17}. Using the collective atomic operators and the expression of $C_{r_2}$ in the dark-state and bright-state polaritons: $C_{r_2} = \cos \theta m_B - \sin \theta m_D$, the Hamiltonian of $H_R$ can be represented by $H_R = \lambda C_{r_2}^\dagger C_{r_1}^\dagger (\sin^2 \theta m_B^2 + \cos^2 \theta m_B^2 - 2 \sin \theta \cos \theta m_B m_D) + H.c.$ Since the bright-state polariton $m_B$ is not excited here, we disregard these terms including the polariton $m_B$, and obtain

$$H_R'' = \chi C_{r_2}^\dagger C_{r_1}^\dagger m_B^2 + H.c.,$$

(4)

with $\chi = \lambda \sin^2 \theta$. Equation (4) describes the nonlinear process of three-mode mixing \cite{18}. Before describing the detailed applications and supporting numerical calculations, next we briefly discuss the features and estimate the potential parameters for intracavity EIT with blockaded Rydberg ensemble.

The dominant loss associated with the leakage of the dark-state polariton $m_D$ through the mirrors is reflected by the cavity quality factor

$$Q = \frac{\omega_0}{2\kappa} = \frac{1}{\cos \theta} \frac{\omega_0}{2\kappa} = \frac{Q_0}{\cos^2 \theta},$$

(5)

with $Q_0$ the quality factor of the bare cavity without the EIT medium. From Eq. (5), we see that $Q$ is determined by not only $Q_0$ but also the added factor $1/\cos^2 \theta$. With a decrease of $\cos \theta$ by tuning the control field, the quality factor $Q$ increases. A second dissipative channel is the spontaneous emission of Rydberg states, luckily the Rydberg states have long coherence time and small decay rate $\gamma \approx 2\pi kHz$ \cite{19}.

In order to have the strong-nonlinearity effects, the nonlinear strength $\chi$ should be much larger than the rate at which the decoherences occur. When $\cos \theta \ll 1$, the nonlinear strength $\chi = \lambda \sin^2 \theta = \lambda(1-\cos^2 \theta) \approx \lambda$, which has a large value for the high Rydberg state \cite{19}. Hence one could achieve the strong-nonlinearity condition, i.e., $\chi \gg \kappa, \gamma_c$, by decreasing $\cos \theta$. For example, a 400-atom Bose-Einstein condensate (BEC) is trapped in an optical cavity. For the high Rydberg states $n \geq 100$, the blockade interaction strength over the atomic sample is
The cavity EIT is governed by the effect. Based on above analysis, the dynamics of intra-cavity light quanta [6]. Now we show that the strong nonlinearities for both the intracavity and the transmitted light fields are strongly modified by nonlinear processes. This is known as photon blockade effect [1], which has been exploited as a powerful tool for quantum control of Rydberg atomic decay rate $\gamma$. For the relevant cavity parameters, typically $(\kappa, \gamma, 2\pi) \approx (8, 5.2) \text{MHz}$, and $g/2\pi \approx 25 \text{MHz}$ [20]. With the choice $\cos \theta = 0.1$, we have $\sqrt{Ng^2 + \Omega^2} \approx 2\pi \times 505 \text{MHz}$, $\chi = \lambda \sin^2 \theta = 2\pi \times (49 \text{MHz} \sim 198 \text{MHz})$, $\kappa' = \cos^2 \theta \kappa \approx 2\pi \times 0.08 \text{MHz}$. Thus the condition $\sqrt{Ng^2 + \Omega^2} \gg \kappa, \gamma$ is well satisfied, and the nonlinear strength $\chi$ would be more than three orders of magnitude larger than both the effective cavity decay rate $\kappa$ and Rydberg atomic decay rate $\gamma$.

In analogy with the phenomenon of coulomb blockade, when photons transport through a nonlinear cavity, the properties for both the intracavity and the transmitted light fields are strongly modified by nonlinear processes. This is known as photon blockade effect [1], which has been exploited as a powerful tool for quantum control of light quanta [6]. Now we show that the strong nonlinearities in our model would lead to strong photon blockade effect. Based on above analysis, the dynamics of intracavity EIT in blockaded Rydberg ensemble is governed by the Hamiltonian

$$H = H_{\text{in-out}} + H_{R} - \frac{i\gamma}{2} m_D^\dagger m_D$$

$$= \sum_{\theta = \alpha, \beta} \int_{\omega_{\text{hi}} - \omega_b}^{\omega_{\text{lo}} + \omega_b} \omega d\omega \Theta^\dagger(\omega) \Theta(\omega) - \frac{i\gamma}{2} m_D^\dagger m_D$$

$$+ \sum_{\theta = \alpha, \beta} \int_{\omega_{\text{hi}} - \omega_b}^{\omega_{\text{lo}} + \omega_b} d\omega \sqrt{\frac{\kappa}{2\pi}} \Theta^\dagger(\omega) m_D + \chi C_{r1}^\dagger C_{r3}^\dagger m_D^2 + \text{H.c.},$$

(6)

where we have incorporated the effect of radiative decay out of Rydberg states by using the non-Hermitian terms.

The Hamiltonian $H_{\text{in-out}}$ alone induces a very narrow transmission peak. The Hamiltonian $H_{R}$ is responsible for the interaction of the polaritons and leads to photon blockade effect. If they are acting together, we anticipate a new regime in which the photons pass through the cavity one by one in a very narrow-bandwidth transmission window. Thereby this system acts as a narrow-band single-photon turnstile.

Without loss of generality, we assume the external probe field is a weak continuous-wave (cw) coherent field. Thus the effective non-Hermitian Hamiltonian for intracavity EIT is given by

$$H' = (E m_D^\dagger + \chi C_{r1}^\dagger C_{r3}^\dagger m_D^2 + \text{H.c.}) - \frac{i(\kappa' + \gamma)}{2} m_D^\dagger m_D,$$

(7)

here $E = \sqrt{2\kappa' \beta}$ is the driving strength which is proportional to the external probe field amplitude $\beta$. To confirm the validity of the blockade effect, we perform numerical simulations with Hamiltonian $H'$. Our investigation relies on the calculations of photon correlations based on the normalized intensity correlation function $g^2(\tau)$ for the forward-propagating transmitted light. The incident field (which approximates a coherent state) has $g_0^2(\tau) = 1$, corresponding to a Poisson distribution for photon number independent of time delay $\tau$. The ideal photon blockade would achieve $g^2(0) = 0$, in correspondence to the state of a single photon. More generally, $g^2(0) < 1$ represents a nonclassical effect with the variance in photon number reduced below that of the incident field. In Fig. 2 (a), we plot numerical simulations results of $g^2(0)$ as a function of $\cos \theta$. From Fig. 2(a), we see that $g^2(0)$ is very sensitive to $\cos \theta$. The decrease of $\cos \theta$ leads to stronger photon blockade effects. When $\cos \theta = 0.1$, $g^2(0)$ would be in the order of $10^{-4}$. Figure 2(b) shows the polariton number $\langle n \rangle = \langle m_D^\dagger m_D \rangle$ versus the normalized time $t/\kappa$. We clearly see that the number of cavity polaritons varies between 0 and 1, but never exceeds unity. These numerical simulations results show that the system behaves strong nonlinearities at the single-photon level.

In summary, we have theoretically investigated intracavity EIT with blockaded Rydberg ensemble and show that this system would provide an effective approach to strong single-photon nonlinearity in an optical cavity. In the system, the photons in the cavity are in the form of dark-state polaritons and the effective cavity decay rate $\kappa$ is determined by not only the bare cavity decay rate $\kappa$ but also the added factor $\cos^2 \theta$. With a decrease of $\cos \theta$ by tuning the control field, the effective cavity decay rate $\kappa$ decreases, thus the effect of tuning the control field is equivalent to that of increasing the quality factors $Q$. Profiting from this especial feature, we forecast the nonlinear strength $\chi$ in the system would be three orders of magnitude larger than both the effective cavity decay rate $\kappa$ and atomic decay rate $\gamma$ under current experiment technology. The strong nonlinearity could be used for strong photon blockade. By storing a single photon in a Rydberg state [21] and subsequently transmitting a second photon, single-photon quantum router [3] could be created with the strong photon blockade. Furthermore, we anticipate their interesting applications.
to exploration of the rich physics promised by strongly-correlated quantum systems. For example, it can be used for realization of a Mott insulator to superfluid quantum phase transition in coupled arrays of cavities. Remarkably, Mücke et al. have recently succeeded in demonstrating intracavity EIT with a single atom and observing a narrow transmission window corresponding to the dark state. Thus the present scheme would be realized in the near future.

Acknowledgments: This work was supported by the National Natural Sciences Foundation of China (Grants No. 11204080, No. 11274112, No. 61275215), the Fundamental Research Funds for the Central Universities (Grants No. WM1214019, No. WM1114024, No. WM1313003).

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