Exceptional field theories, superparticles in an enlarged 11D superspace and higher spin theories.

Igor Bandos

† Department of Theoretical Physics, University of the Basque Country UPV/EHU,
P.O. Box 644, 48080 Bilbao, Spain
‡ IKERBASQUE, Basque Foundation for Science, 48011, Bilbao, Spain

Abstract: Recently proposed exceptional field theories (EFTs) making manifest the duality $E_{n(n)}$ symmetry, first observed as non-linearly realized symmetries of the maximal $d = 3, 4, ..., 9$ supergravity ($n = 11 - d$) and containing 11D and type IIB supergravity as sectors, were formulated in enlarged spacetimes. In the case of $E_{7(7)}$ EFT such an enlarged spacetime can be identified with the bosonic body of the $d = 4$ central charge superspace $\Sigma^{(60|32)}$, the $\mathcal{N} = 8$ $d = 4$ superspace completed by 56 additional bosonic coordinates associated to central charges of the maximal $d = 4$ supersymmetry algebra.

In this paper we show how the hypothesis on the relation of all the known $E_{n(n)}$ EFTs, including $n = 8$, with supersymmetry leads to the conjecture on existence of 11D exceptional field theory living in 11D tensorial central charge superspace $\Sigma^{(528|32)}$ and underlying all the $E_{n(n)}$ EFTs with $n = 2, ..., 8$, and probably the double field theory (DFT). We conjecture the possible form of the section conditions of such an 11D EFT and show that quite generic solutions of these can be generated by superparticle models the ground states of which preserve from one half to all but one supersymmetry. The properties of these superparticle models are briefly discussed. We argue that, upon quantization, their quantum states should describe free massless non-conformal higher spin fields in D=11. We also discuss some relevant representations of the M-theory superalgebra which, in the present context, describes supersymmetry of the 11D EFT.

Keywords: Supersymmetry, U-duality, superspace, superparticle, higher spin theory, double field theory, exceptional field theories.
Contents

1. Introduction 3

2. On EFTs, their section conditions and central charge superspaces 5
   2.1 $E_{n(n)}$ EFTs with $n = 2, ..., 8$. Additional coordinates, section conditions and their classical counterparts 5
   2.2 Some differences between $E_{4(4)}$, $E_{7(7)}$ and $E_{8(8)}$ EFTs 5
   2.3 $E_{8(8)}$ EFT and maximal supersymmetry 5

3. Maximal 11D tensorial central charge superspace $\Sigma^{(528|32)}$ and uEFT conjecture 10
   3.1 $\Sigma^{(528|32)}$ geometry and maximal supersymmetry 10
   3.2 Section conditions of the hypothetical 11D EFT 10
   3.3 uEFT section conditions and $E_{n(n)}$ EFTs with $n = 5, 6, 7, 8$. 14
   3.4 Dynamical model generating (solutions of) the section conditions
      3.4.1 Preonic superparticle and conformally invariant section conditions 15
      3.4.2 Preonic superparticle with composite bosonic spinor 17
      3.4.3 Spinor moving frame variables 18
      3.4.4 A family of superparticle ‘solving’ the classical section conditions 21

4. On uEFT superparticles and 11D higher spin theories 23
   4.1 Free $D = 4, 6, 10$ conformal higher spin theory description in $\Sigma^{(m(m+1)/2|m)}$ superspace with $m = 2(D - 2) = 4, 8, 16$ 23
   4.2 $m=4,8,10$ counterparts of the preonic superparticle and conformal higher spin fields in $D=4,6,10$ 23
   4.3 On superparticle models for massless non-conformal higher spin theories in $D=6,10$ and $D=11$ 25

5. Embedding 11D supergravity into representations of the M-theory superalgebra, the supersymmetry superalgebra of uEFT 26
   5.1 A particular class of eigenstates of the generalized momentum 26
   5.2 Some unitary highest weight representations of the M-algebra 27
   5.3 Unitary highest weight representations with $r = 16$ and 11D supergravity 28
   5.4 On moduli space of (complex structures defining) the highest weight representations 31

6. Conclusions and discussion 32
1. Introduction

Recently exceptional field theories (EFTs) \(^1\), manifestly invariant under U–duality symmetry groups \(E_{n(n)}\) \(^2\) with \(n = 2, 3, 4, 5, 6, 7, 8\) and containing 11D and 10D type IIB supergravity theories as sectors were formulated in enlarged \(d=3,4,5,...,9\) spaces \(^3\) \(\{4, 5, 6, 7, 8, 9, 10, 11\}\). The value of \(d\) is related to \(n\) by \(d + n = 11\), and in this sense one can call the \(E_{n(n)}\) EFT ‘\(d\)-dimensional’\(^1\), the name which also reflects its manifest invariance under the \(d\)-dimensional Lorentz group \(SO(1, d−1)\) \(^2\). They can be regarded as M-theoretic counterparts of \(D=10\) double field theory (DFT) \(^2\) \(\{23, 24, 25, 26, 27, 28, 29, 30, 31\}\) designed to have a manifest T-duality symmetry, characteristic for string theory \(^3\). The DFT is formulated in the space with doubled number, \(2D\), of bosonic coordinates (usually \(D=10\) is assumed in this case). The number of the additional bosonic coordinates \(y^{\Sigma}\) of the \(d\) dimensional \(E_{n(n)}\) EFT is \(d\)/\(n\)- dependent: it varies from \(3\) in the recently proposed \(9\)d ‘F-theory action’ of \(^1\) \(\{18\}\) to \(56\) in \(d=4\) \(E_{7(7)}\) EFT \(^1\) \(\{4, 6\}\) and \(248\) in \(d = 3\) \(E_{8(8)}\) EFT \(^1\) \(\{14\}\). The dependence of the fields on additional coordinates is restricted by the so–called section conditions\(^4\) the strong version of which is imposed (‘by hand’) on any pair of functions of the theory.

![Table 1](image)

Table 1. Additional coordinates and section conditions of the \(E_{n(n)}\) EFTs. The notation for \(n = 7\) and \(n = 4\) cases are described below. The other cases will not be discussed and we refer to the original papers (cited at the end of the lines) for the notation.\(^5\)

\(^1\)The embedding of massive IIA requires a deformation of (section conditions of) the EFT \(^1\) \(\{21, 22\}\).

\(^2\)It is also worth commenting that, while \(E_{6(6)}\), \(E_{7(7)}\) and \(E_{8(8)}\) of EFTs with manifest \(d=5,4,3\) Lorentz symmetries are the exceptional Lie groups from the Cartan list, for lower \(n\) \(E_{n(n)}\) denote simpler groups: \(E_{5(5)} = SO(5,5)\), \(E_{4(4)} = SL(5)\), \(E_{3(3)} = SL(3) \times SL(2)\) and, as it was proposed in recent \(\{18\}\): \(E_{2(2)} = SL(2) \times \mathbb{R}^\pm\). \(^3\)See \(^2\) and refs. therein for T-duality and \(^\{23, 24\}\) for string and superstring in doubled (super)spaces. Notice also that we usually denote the number of spacetime dimensions by \(D\) when it is equal to 10 or 11, and by \(d\) when it is lower, so that \(d \leq 9\).

\(^4\)The name ‘section conditions’ was introduced in \(^1\) \(\{4\}\) developing \(E_{d(4)} = SL(5)\) (pre-)EFT formalism \(^1\) \(\{14\}\). The name EFT was introduced in \(^1\) \(\{4\}\) which starts a series of papers formulating the EFTs for exceptional U-duality groups \(E_{7,7}, E_{6(6)}\) and \(E_{8(8)}\) in its complete form, including all the differential form fields of maximal \(d = 11 − n\) dimensional supergravity.

\(^5\)Notice a partial intersection of (the ‘left hand side’ of) this Table 1 with Table 2 of \(^1\) \(\{24\}\), where a possible relation of 11D supermembrane duality transformations with \(E_{n(n)}\) duality symmetries of dimensionally reduced maximal supergravity was discussed.
In the case of DFT the solution of the strong section conditions implies that all the
physical fields depend only on D of 2D bosonic coordinates. The manifest T-duality is
provided by the freedom in choosing the set of these D of the complete set of 2D coordinates.
This is called ‘choice of the section’ (hence the name ‘section conditions’ for the equations
solved by this choice).

The structure of the EFT section conditions looks strongly d- (or n-) dependent and
much less transparent. As we will discuss below, the analysis of differences in the structure
of EFTs with different n suggests the possible existence (and makes desirable to find) a
hypothetical underlying EFT, which we call ‘11D EFT’ or ‘uEFT’, such that all the lower
d EFTs can be obtained by its reductions. 

In this paper we make same stages toward the construction of such a hypothetical 11D
uEFT. In particular, we argue that the natural basis for its construction is provided by 11D
tensorial central charge superspace Σ(528|32), propose the section conditions for this uEFT
in this superspace, and present a family of superparticle models in Σ(528|32) which produce
quite generic solutions of these section conditions. The quantum states of these models
are massless, which allows to conjecture that their quantization results in supersymmetric
theories of free massless higher spin fields in D=11. The quantization of D=10 version(s)
of the model(s) should produce a theory of free massless non-conformal higher spin field,
the tower of which includes 10D ‘graviton’.

To gain a hint about how the quantum state spectrum of some of such models might
look like, and also as an additional argument in favour of our uEFT hypothesis, we discuss
some unitary highest weight representations of M-theory superalgebra, which in our context
describes supersymmetry of Σ(528|32) and of the hypothetical uEFT, and the embedding of
11D supergravity in these representations.

The rest of this paper is organized as follows. In the beginning of next Sec. 2 we review
the structure of $E_{n(n)}$ exceptional field theories (EFTs) with $n = 2, ..., 8$ and conjecture on
their relation with the most general supersymmetry algebra. In particular, in sec. 2.1 we
discuss the additional coordinates of $E_{n(n)}$ EFTs, the section conditions, which are imposed
to restrict the dependence of EFT fields on those, and their classical counterparts. In sec.
2.2 we argue in favor of relation of additional coordinates of $E_{n(n)}$ EFTs with maximal
supersymmetry algebra in $d = 11 - n$, describe the relation of those with central charges of
such a supersymmetry algebra observed first for $n = 7$. In sec. 2.3 we discuss the extension
of this conjecture to $n = 8$ which requires involvement of also the vectorial ‘central charges’
and leads us to the most general d=3 maximal supersymmetry algebra.

The underlying EFT (11D EFT or uEFT) conjecture is formulated in Sec. 3. In
sec. 3.1 we show that the maximally extended d=3 supersymmetry superalgebra has
actually a bigger automorphism symmetry, including SO(1,10), which allows us to call it
M-algebra (or M-theory superalgebra), describe the SO(1,10) invariant Cartan forms on
the associated supergroup manifold Σ(528|32) with 528 bosonic and 32 fermionic directions,
and conjecture on the existence of underlying 11D EFT, leaving in this superspace.

---

Notice that our hypothetical uEFT is not identical but probably complementay to the E11 program of
[43, 44, 45, 46, 47, 48, 49, 50, 51, 52]. We will comment on this more in concluding Sec. 5. A discussion on
the connection of EFTs and E11 can be found in [49].
In Sec 3.2 we propose the candidate section condition of 11D EFT needed to reduce the huge number of additional bosonic coordinates and discuss the structure of their solutions. In Sec. 3.3 we consider a series of superparticle models in $\Sigma^{(528|32)}$ which produce quite generic solution of the classical section conditions as their constraints. The actions of these models involve essentially 11D spinor moving frame variables [53, 54] (see also [55, 56, 57, 58]), also called Lorentz harmonics [59] (see also [60, 61, 62, 63, 64, 65, 66]); we describe these in Sec. 3.3.3.

In Sec. 4 we argue that the quantization of these uEFT-related superparticle models should produce towers of massless 11D higher spin fields as their quantum state spectrum. We briefly describe (in Secs. 4.1 and 4.2) the relation of lower dimensional counterpart of preonic superparticle to free massless conformal higher spin fields in $D = 4, 6, 10$ dimensions and on this basis conjecture (in Sec. 4.3) that the counterpart of above mentioned generalized superparticle models with spinor moving frame variables provide the classical mechanic description of massless non-conformal higher spin fields in $D = 6, 10$. The quantization of the models in $\Sigma^{(528|32)}$ should result in a tower of massless non-conformal 11D higher spin fields; the conformal higher spin tower is not known for this case.

In Sec. 5 we discuss unitary highest weight representations of M-theory superalgebra, which are relevant in the uEFT context, and the embedding of 11D supergravity in some of these representations. We conclude in Sec. 6 where the discussion on possible relation/complimentarity of our conjectured 11D EFT and of $E_{11}$ and $E_{10}$ hypothesis can be also found.

2. On EFTs, their section conditions and central charge superspaces

2.1 $E_{n(n)}$ EFTs with $n = 2, ..., 8$. Additional coordinates, section conditions and their classical counterparts

Schematically, the $E_{n(n)}$ EFT is constructed on the basis of maximal $d = 11 - n$ dimensional supergravity (SUGRA) by allowing the field to depend, besides $d$ spacetime coordinates $x^\mu$, on additional ‘internal’ coordinates $y^\Sigma$ the number of which (i.e. the range of the index $\Sigma$), is given by dimension $N_n$ of minimal irreducible representation of $E_{n(n)}$ ($N_2 = 3, \ldots, N_7 = 56, N_8 = 248$). Besides that, the field strengths and the Lagrangian of $d$-dimensional supergravity are modified by inclusion of terms with derivatives $\partial_\Sigma = \frac{\partial}{\partial y^\Sigma}$, and the action is constructed by integrating this modified SUGRA Lagrangian $L_{EFT}^{E_{n(n)}}$ over $d$ spacetime and all the $N_n$ internal coordinates, $S_{EFT}^{E_{n(n)}} = \int d^4x d^{N_n} y L_{EFT}^{E_{n(n)}}$. This integral is usually considered as formal as far as its rigid definition meets problems related with the next ingredients of EFT which we are going to describe now.

All the fields in EFT, $F$, are subject to the so-called weak section conditions

$$Y_{\Lambda \Xi \Sigma \Pi} \partial_\Sigma \partial_\Pi F = 0,$$

where $Y_{\Lambda \Xi \Sigma \Pi}$ is an invariant tensor of $E_{n(n)}$ the explicit form of which is strongly $n$-dependent. But moreover, all the pairs of the fields $F_1, F_2$, should be subject to the
so-called strong section conditions,

\[ Y_{\Lambda \Xi} \Sigma \Pi \partial_{\Sigma} F_1 \partial_{\Pi} F_2 = 0 \tag{2.2} \]

For \( E_{7(7)} \) EFT, in which \( \Sigma, \Pi, \Lambda, \Xi = 1, \ldots, 56 \), these section conditions can be presented in a simpler form

\[ t_G \Sigma \Pi \partial_{\Sigma} F_1 \partial_{\Pi} F_2 = 0 \tag{2.3} \]
\[ \Xi \Sigma \Pi \partial_{\Sigma} F_1 \partial_{\Pi} F_2 = 0 \tag{2.4} \]

where \( \Xi^{\Pi \Lambda} = -\Xi^{\Lambda \Pi} \) is the \( Sp(56) \) symplectic ‘metric’, \( t_G \Sigma \Pi = \Xi^{\Pi \Lambda} t_G \Lambda \Sigma \) and \( t_G \Lambda \Sigma \) are \( E_{7(+7)} \) generators in 56 representation, \( G = 1, \ldots, 133 \).

To make the equations lighter, one usually writes the strong and the weak section conditions in the schematic form

\[ Y_{\Lambda \Xi} \Sigma \Pi \partial_{\Sigma} \otimes \partial_{\Pi} = 0 \tag{2.5} \]
\[ Y_{\Lambda \Xi} \Sigma \Pi \partial_{\Sigma} \partial_{\Pi} = 0 \tag{2.6} \]

It is natural to expect that the solutions of the section conditions imply independence of all the fields on some number of internal coordinates. In the above schematic notation this can be expressed as

\[ \partial_{\Sigma}(\ldots) = K_{\Sigma} r \partial_r (\ldots), \quad \partial_r = \frac{\partial}{\partial y^r}, \quad r = 1, \ldots, \tilde{n}_n \tag{2.7} \]

where \( y^r \) are \( \tilde{n}_n \) (< \( N_n \)) additional coordinates the fields are allowed to depend on. A possible choice of this latter defines a section (i.e. a particular solution of the section conditions). The freedom in choosing among the possible sections makes the construction \( E_{n(n)} \)-invariant.

For all the EFTs the \( n \)-parametric and \((n-1)\)-parametric solutions of the section conditions were found and shown to describe D=11 and D=10 type IIB supergravity \[1, 8, 10, 12, 13, 15\] (in the majority of the cases the bosonic limit of SUGRA was actually discussed). From the generic String/M-theoretic perspective, one should not expected the possibility to have a solution with functions depending on more than 11 bosonic coordinates. Although for lower \( n \), e.g. for lowest \( n = 2 \) case in \[18\], this is manifest, for higher \( n \) this expectation had been just a reasonable conjecture till recent \[67\], where it has been proved for the case of d=4 \( E_{7(7)} \) EFT. Furthermore, in \[77\] it was shown that the set of 133 section conditions of this EFT, Eqs. \[2.3, 2.8\],

\[ t_G \Sigma \Pi \partial_{\Sigma} \otimes \partial_{\Pi} = 0, \quad \Sigma, \Pi = 1, \ldots, 56, \quad G = 1, \ldots, 133 \tag{2.8} \]

is reducible in the sense that one can extract such a set of 63 conditions that their solution automatically solves also the remaining relations.
To be more specific in this latter statement, it was shown in [67] that the solutions of the set of 63 relations (2.8) involving the generators of SU(8) subgroup of $E_7(+7)$,

$$t_H \Sigma \Pi \partial_\Sigma \otimes \partial_\Pi = 0, \quad \Sigma, \Pi = 1, \ldots, 56, \quad H = 1, \ldots, 63, \quad (2.9)$$

automatically solve also the remaining 70 conditions which involve the generators of the coset $E_7(+7)_{SU(8)}$ ($t_K \Sigma \Pi \partial_\Sigma \otimes \partial_\Pi = 0, K = 1, \ldots, 70$) as well as the strong section conditions (2.4) involving the symplectic metric $\Xi^\Sigma \Pi = -\Xi^\Pi \Sigma \qquad \text{7.}$

To obtain the above results, it was very useful to analyze the classical mechanic counterpart of the section conditions which reads

$$t_E \Sigma \Pi p_\Sigma p_\Pi = 0, \quad (2.10)$$

where $p_\Sigma$ and $p_\Pi$ are classical momenta of a particle model. One notices that, if we perform a straightforward ‘quantization’ of (2.10) by replacing the momentum by derivative, $p_\Sigma \mapsto -i\partial_\Sigma$, consider (2.10) as a (first class) constraint and impose its quantum version as a condition on the wave function, we clearly arrive at the weak version of (2.8) imposed on one function rather than on the pair of functions of EFT. However, as it was discussed in [67], there exists another 'first solve than quantize' way which, starting from the classical section conditions (2.10) results in the (general solution of the) strong section conditions (2.8). The key point is that such a general solution is expected to be of the form of (2.7) and hence can be reproduced by quantization of the general solution of the classical section conditions of the form

$$p_\Sigma = K_\Sigma^r p_r, \quad r = 1, \ldots, \tilde{n}_n. \quad (2.11)$$

The classical counterpart of the section conditions had been also studied in [88] devoted to development of a twistor approach to $E_n(n)$ EFTs with $n \leq 6$. In particular, in [88] it was discussed the classical section conditions of the $E_{4(4)} = SL(5)$ EFT which reads

$$p_{[\tilde{a} \tilde{b}]} p_{\tilde{c} \tilde{d}} = 0, \quad \tilde{a}, \tilde{b}, \tilde{c}, \tilde{d} = 1, \ldots, 5. \quad (2.12)$$

Here $p_{\tilde{a} \tilde{b}} = -p_{\tilde{b} \tilde{a}}$ are momenta conjugate to the additional bosonic coordinates of the spacetime of the $E_{4(4)} = SL(5)$ EFT, $y^{\tilde{a} \tilde{b}} = -y^{\tilde{b} \tilde{a}}$ which belongs to 10 representation of $SL(5)$. The simple form of this SL(5) section conditions will be suggestive for our discussion below.

### 2.2 Some differences between $E_{4(4)}$, $E_{7(7)}$ and $E_{8(8)}$ EFTs

This is the place to illustrate the differences in the structure of section conditions of $E_{n(n)}$ EFTs with different $n$.  

---

7In terms of the derivatives in 27 and 27 of SU(8), $\partial_\Sigma = (\partial_{ij}, \tilde{\partial}^{ij})$, the (formal) solution of the section conditions (2.9) can be obtained [67] by SU(8) transformations from $\partial_{ij} = G^I_j \partial_I = \tilde{\partial}^{ij}$, with real $\partial_I = \tilde{\partial}_I$ ($I = 1, \ldots, 7$) and SO(7) Gamma matrices $G^I_j$. It is easy to check that this solved the conditions (2.4) which, in its manifestly SU(8) invariant form, reads $\partial_{ij} \otimes \tilde{\partial}^{ij} - \tilde{\partial}^{ij} \otimes \partial_{ij} = 0$. To show that this solves also 70 equation with coset generators, $\partial_{[ij} \otimes \partial_{kl]} - \frac{1}{10} \epsilon_{ijklmnp} \tilde{\partial}^{mp} \otimes \tilde{\partial}^{np} = 0$, is a bit more involving [15].
First notice that, as it was shown in [7] the solution of section conditions \( \partial_{[ab} \otimes \partial_{cd]} = 0 \) corresponding to the embedding of D=11 and of type IIB supergravity in the d=7 \( E_{4(4)} \) EFT are independent in the sense that they are not connected by transformations of \( E_{4(4)} = SL(5) \) group. The same applies to the classical counterparts of this strong section conditions given in Eq. (2.12). In contrast, as it can be deduced from the results of [67], in the case of d=4 \( E_{7(7)} \) EFT the situation is opposite: the solutions describing the embedding of D=11 and type IIB supergravities into this EFT are related by transformations of the \( SU(8) \) subgroup of \( E_{7(7)} \).

Actually this distinction does not look unnatural after comparing the number of bosonic coordinates of \( E_{n(n)} \) EFTs with that of the DFT. Indeed, a unification of the 11D and type IIB solutions of a EFT implies also the unification of (low energy limits of the) type IIA and IIB superstring theories. This is reached in the frame of DFT which is defined in the space with doubled number of coordinates, \( 2D = 20 \). From this perspective one can expect the independence of 11D and type IIB solution in \( E_{n(n)} \) EFTs with \( 2 \leq n \leq 4 \), where the number of additional and spacetime coordinates is less that 20, and their unification in EFTs with \( n \geq 5 \). It will be interesting to check this hypothesis for \( n = 5, 6 \) and \( n = 8 \) cases.

One more illustrative example is in difference between \( E_{7(7)} \) and \( E_{8(8)} \) EFTs.

The first is formulated in the space with 56 additional coordinates \( y^\Sigma = (y^{ij}, \bar{y}^{ij}) \) which can be considered [67] as a bosonic body of central charge superspace \( \Sigma^{(60|32)} \) [68]. The flat version of this superspace, \( \Sigma_0^{(60|32)} \), is the supergroup manifold associated with the most general central extension of the maximal \( D = 4 \) \( \mathcal{N} = 8 \) supersymmetry algebra

\[
\{Q_i^{\alpha}, Q_j^{\dot{\alpha}}\} = \epsilon_{\alpha\dot{\beta}} Z^{ij} , \quad \{Q_i^{\dot{\alpha}}, \bar{Q}_j^{\dot{\beta}}\} = \delta^i_j \sigma^a_{\alpha\dot{\beta}} P_a , \quad \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = \epsilon_{\alpha\dot{\beta}} \bar{Z}^{ij} , \quad (2.13)
\]

\( \alpha, \beta = 1, 2 \), \( \dot{\alpha}, \dot{\beta} = 1, 2 \), \( a = 0, 1, 2, 3 \), \( i, j = 1, \ldots, 8 \).

This observation allowed us [67] to formulate a superparticle model in central charge superspace which generates the classical counterpart of the independent section conditions (2.9) as a constraint. The model is an improved version of the \( \mathcal{N} = 8 \) superparticle described by de Azcárraga and Lukiersi in [83]. In the original model the invariance of the action under \( \kappa \)-symmetry can be reached only if we allow ourselves to impose the classical counterpart of independent section conditions, (2.10) with \( E = H \),

\[
t_H \Sigma P_\Sigma P_H = 0 , \quad H = 1, \ldots, 63 , \quad (2.14)
\]

‘by hand’, while in our improved version (which is not apparently equivalent to the original model) these appear as equations of motion.

A natural wish is to find a similar superparticle model generating (an independent part of) the section conditions for \( E_{8(8)} \) EFT. But here we meet a problem already at the first stage. The number of central charges \( Z^{pq} = -Z^{qp} \) of the central extension of maximal
\[ d = 3 \text{ supersymmetry algebra} \]

\[ \{ Q^\alpha_{\tilde{\alpha}}, Q^p_{\tilde{\beta}} \} = \gamma^a_{\tilde{\alpha} \tilde{\beta}} \delta^{pq} P_a + i \epsilon^a_{\tilde{\alpha} \tilde{\beta}} Z^{pq}, \quad \tilde{\alpha}, \tilde{\beta} = 1, 2, \quad \tilde{\alpha} = 0, 1, 2, \quad p, q = 1, .., 16 \quad (2.15) \]

is 120, while the number of the additional coordinates of \( E_{8(8)} \) EFT is 248 [1]. This is the dimension of the minimal irreducible representation of \( E_{8(8)} \) which in [1] was taken to be the adjoint representation.

Thus the relation of additional coordinates of EFT with central charges of maximal central extension of the maximal \( d \)-dimensional supersymmetry algebra observed for \( E_{7(7)} \) EFT in [67] cannot be generalized straightforwardly to the \( E_8 \) case.

The idea of our study is to insist nevertheless on the beautiful relation of additional coordinates of EFT and of the maximal \( d \) dimensional supersymmetry algebra. As we will see in a moment, this leads us to the conjecture on the existence of an underlying EFT (uEFT) 'living' in the maximal tensorial central charge superspace. This can be defined at any \( d \leq 11 \), but its associated supersymmetry algebra always has a hidden symmetry including \( SO(1, 10) \) so that it can be called M-algebra or M-theory superalgebra and our uEFT can be called 11D EFT.

### 2.3 \( E_{8(8)} \) EFT and maximal supersymmetry

If we insist on relation of additional coordinates of the \( E_{n(n)} \) EFT with maximal \( d = 11 - n \) supersymmetry algebra, in the case of \( n = 8 \), \( d = 3 \) we have to allow for contributions of some additional coordinates carrying both the indices of the internal symmetry \( SO(16) \) and of the \( d=3 \) Lorentz symmetry. Namely, we need in a coordinates conjugate to 128 of possible 405 additional vectorial 'central' charges \( Y^{pq}_{a} = Y^{(pq)}_{a} \) (where double brackets imply symmetric traceless part: \( Y^{aq}_{a} = Y^{qp}_{a}, Y^{qq}_{a} = 0 \)).

With the contribution of all these generators the right hand side (r.h.s.) of the defining relation of the maximal \( d = 3 \) supersymmetry algebra,

\[ \{ Q^\alpha_{\tilde{\alpha}}, Q^p_{\tilde{\beta}} \} = \gamma^a_{\tilde{\alpha} \tilde{\beta}} (\delta^{pq} P_a + Y^{(pq)}_{a}) + i \epsilon^a_{\tilde{\alpha} \tilde{\beta}} Z^{pq}, \quad (2.16) \]

becomes the generic 528 component \( 32 \times 32 \) matrix \( (528=3+405+120) \).

The mechanism of extraction of 128(=248-120) additional coordinates of \( E_{8(8)} \) EFT of 405(=528-3-120) additional coordinates conjugate to the vectorial central charge of \( (2.16) \) should be dynamical. A search for it is beyond the scope of this paper. For our discussion here the presence of even more ('beyond the \( E_{8(8)} \) EFT') additional coordinates is not problematic but rather suggestive.

Indeed at this stage it is tempting to conjecture the existence of an underlying exceptional field theory (uEFT), which includes as a sub-sectors all the \( E_{n(n)} \) EFT with \( 2 \leq n \leq 8 \) and lives in an enlarged superspace \( \Sigma^{(528|32)} \) with 32 fermionic and 528 bosonic coordinates. In terms of the above discussed \( n = 8 \) case, these latter can be split on \( d = 11 - n = 3 \) spacetime, 120 central charge and 405 'vector central charge' coordinates. But actually the similar splitting is possible for any \( n \): the number of spacetime coordinates will be \( d = 11 - n \) while the set of additional 528 – \( d \) coordinates will be split, in an
SO(1, d − 1) invariant way, on the subsets of scalar, vector and tensorial ‘central’ charge coordinates. The reason beyond this lays in a huge hidden automorphism symmetry of the superalgebra (2.16) which we are going to discuss now.

3. Maximal 11D tensorial central charge superspace \( \Sigma^{(528|32)} \) and uEFT conjecture

3.1 \( \Sigma^{(528|32)} \) geometry and maximal supersymmetry

The manifest \( SO(1,2) \times SO(16) \) symmetry of (2.16) is related to the basis we have used to decompose the matrix of generators in r.h.s. of this relation. There exists also the manifestly \( SO(1,10) \) invariant form of the same relation,

\[
\{Q_\alpha, Q_\beta\} = i \Gamma_{a,\alpha \beta} P_a + \Gamma_{ab} Z_{ab} + i \Gamma_{a,\beta \gamma} Z_{\beta \gamma},
\]

which explains the name of M-algebra or M-theory superalgebra \([70, 71]\) often used for this most general supersymmetry superalgebra\(^9\). The generators \( Z_{ab} = Z_{[ab]} \) and \( Z_{abcde} = Z_{[abcde]} \) are called tensorial central charges.

Actually, the M-algebra possesses \( GL(32) \) automorphism symmetry which becomes manifest if we write it in the form

\[
\{Q_\alpha, Q_\beta\} = i P_{\alpha \beta}, \quad \alpha, \beta = 1, 2, ..., 32,
\]

collecting all the generators in the r.h.s. in one symmetric \( 32 \times 32 \) matrix \( P_{\alpha \beta} \) \( (528 = \frac{32 \times 33}{2}) \).

Decomposing this on the basis of 11D gamma matrices and their products,

\[
P_{\alpha \beta} = \Gamma_{a,\alpha \beta} P_a - i \Gamma_{ab} Z_{ab} + \Gamma_{a,\beta \gamma} Z_{\beta \gamma},
\]

we arrive at the form (3.1) of the M-algebra, in which only the \( SO(1,10) \) symmetry is manifest. The transformations from the \( GL(32)/SO(1,10) \) coset mixes the vector and antisymmetric tensor central charges among themselves.

If we complete the D=11 superspace by introduce the coordinates dual to every tensorial central charge generator, we arrive at superspace \( \Sigma^{(528|32)} \) which is the supergroup manifold corresponding to the maximal supersymmetry algebra (3.2) or (3.1). We denote coordinates of this superspace by

\[
Z^{3\mathbb{R}} = (X^{\alpha \beta}, \theta^\alpha) = (x^a, y^{ab}, y^{abcde}, \theta^\alpha),
\]

\[
X^{\alpha \beta} = X^{\beta \alpha} = \frac{1}{32} x^a \Gamma_{a,\alpha \beta} - \frac{i}{64} y^{ab} \tilde{\Gamma}_{ab}^{\alpha \beta} + \frac{1}{32 \cdot 5!} y^{abcde} \Gamma_{a,\alpha \beta}.
\]

The supersymmetric invariant Cartan forms of \( \Sigma^{(528|32)} \) can be collected in a simple expressions

\[
\Pi^{\alpha \beta} = dX^{\alpha \beta} - i d\theta^{(\alpha} \theta^{\beta)}, \quad \Pi^\alpha = d\theta^\alpha,
\]

---

\(^9\)Two comments are in time. Firstly, the algebra (3.1) was described much before the M-theory epoch in \([72]\) and \([73]\). Secondly, in \([70]\) the name ‘M-algebra’ was used for the superalgebra with additional fermionic generators.
which are covariant under $GL(32)$. The $SO(1,10)$ invariant decomposition of the bosonic form reads $528=11+55+462$, i.e. (see [81] for properties of 11D gamma matrices in our notation)

$$\Pi^{\alpha\beta} = \frac{1}{32} \Pi^a \Gamma^a_{\alpha\beta} - \frac{i}{64} \Pi^{ab} \Gamma^a_{\alpha\beta} + \frac{1}{32 \cdot 5!} \Pi^{abcde} \Gamma^{abcde}_{\alpha\beta}, \quad (3.7)$$

where

$$\Pi^a = dx^a - id\theta^\alpha \Gamma^a_{\alpha\beta} \theta^\beta, \quad \Pi^{ab} = dy^{ab} - d\theta^\alpha \Gamma^{ab}_{\alpha\beta} \theta^\beta,$$

$$\Pi^{abcde} = dy^{abcde} - id\theta^\alpha \Gamma^{abcde}_{\alpha\beta} \theta^\beta. \quad (3.8)$$

Our discussion above suggests to try to use the curved $D=11$ tensorial central charge superspace $\Sigma^{(528|32)}$ as an arena for constructing the 11D EFT, underlying the '$d = 11 - n$ dimensional' $E_{n(n)}$ EFTs with $n \leq 8$ (hence the name $uEFT$ which we also use for this 11D EFT).

The above described enlarged 11D superspace $\Sigma^{(528|32)}$ with additional tensorial central charge coordinates, $y^{ab}$ and $y^{abcde}$ in (3.5), was discussed in different contexts in [4, 2, 76, 77, 78, 79, 80, 81, 82]. Of course, its 528 bosonic coordinates can be considered as finite subset of the infinite set of tensorial coordinates which were introduced in [44] in the frame of $E_{11}$ proposal [43]–[52] (see concluding section 6 for more discussion on this). Notice also the relation of $\Sigma^{(528|32)}$ with hidden gauge symmetry [72, 80, 81] of 11D supergravity [91], and that in this context the $GL(32)$ symmetry of $\Sigma^{(528|32)}$ is also broken down to its $O(1,10)$ subgroup.

The useful fact for our discussion below is that the $D = 4, 6$ and 10 counterparts of this maximally enlarged superspace (3.4), $\Sigma^{(m(m+1)/2|m)}$ with $m = 4, 8, 16$, provide the arenas for constructing free massless conformal higher spin theories in $D = 4, 6, 10$ dimensional spacetimes [25] [24, 23, 22, 21, 20, 19, 18, 17, 16, 15], and that these theories do possess $GL(m)$ and, moreover, the generalized superconformal $OSp(1|2m)$ symmetries.

The pioneering contribution in this 'tensorial superspace' or 'hyperspace' approach to conformal higher spin theories was [103] by Fronsdal, where the space parametrized by 4×4 symmetric spin-tensorial coordinates (which can be decomposed on 4-vector and anti-symmetric tensor ones) was proposed as a generalization of spacetime appropriate for the description of 4D massless higher spin fields, and $Sp(8)$ was considered as a generalized conformal symmetry. In [75] Gunaydin proposed to introduce generalized spacetime coordinates by Jordan algebras. In particular, he treated $Sp(2m)$ as conformal group of Jordan algebra of real symmetric $m \times m$ matrices and $OSp(1|2m)$ as generalized superconformal symmetry of the corresponding generalized superspace. The first dynamical model formulated in $\Sigma^{(528|32)}$ superspace was the "eleven dimensional superstring" by Curtright [4] (see [76] for even more exotic superstring model in $\Sigma^{(528|32)}$).
Coming back to our 11D uEFT proposal, the following comment is in time. We appreciate that the relation $d = 11 - n$ might suggest $d = 1$ or $d = 0$ EFT to be the underlying one. However, such hypothetical EFTs should have infinite dimensional symmetry groups $E_{10}$ [83, 84, 85, 86, 87] and $E_{11}$ [43, 44, 45, 46, 47, 48, 49, 50, 51, 52], which seems to imply the necessity to introduce an infinite number of additional coordinates. In contrast a huge but finite number of unconventional coordinates in our 11D EFT (528) provides us with the resource for additional coordinates for all the $E_{n(n)}$ EFTs with $2 \leq n \leq 8$ (and also for 10D DFT) although do not make any of these U-duality symmetries manifest. More discussion on possible relation/complimentarity of our 11D EFT and $E_{11}$ proposal can be found in Sec. 6. In the next section we present the possible section conditions of the hypothetical 11D EFT.

### 3.2 Section conditions of the hypothetical 11D EFT

In this section we propose the set of section conditions which can used to reduce the number of spacetime coordinates in the hypothetical 11D EFT.

The proposed set of additional coordinates of the hypothetical uEFT, $y^{ab}$ and $y^{abcde}$, resembles the variables $y^{ab} = -y^{ba}$ of the $E_{4(4)} = SL(5)$ EFT, with the evident difference that in our case antisymmetric tensor coordinate carry Lorentz group indices, the same as the usual vector coordinate $x^a$, $a = 0, \ldots, 9, 10$. Then the simple form of the section conditions for $E_{4(4)} = SL(5)$ EFT [4, 7], Eq. (2.12), suggests to try the following candidate section conditions for the hypothetical 11D uEFT:

$$\partial_{[a_1 \ldots a_k} \otimes \partial_{b_1 \ldots b_l]} + \partial_{b_1 \ldots b_l} \otimes \partial_{a_1 \ldots a_k]} = 0, \quad k, l = 1, 2, 5, \quad (k, l) \neq (1, 1). \tag{3.9}$$

The classical mechanic counterparts of these relations are

$$p_{[a} p_{b]} = 0, \quad p_{[a} p_{bcdef]} = 0, \quad (3.10)$$

$$p_{[ab} p_{cdf]} = 0, \quad p_{[ab} p_{c_1 \ldots c_5]} = 0. \quad (3.11)$$

One might want to add $p_{[a} p_{b]} = 0$ and $p_{[b_1 \ldots b_5} p_{c_1 \ldots c_5]} = 0$, but these are satisfied identically at the classical level.

The trivial solution of these section conditions, $p_{ab} = 0 = p_{c_1 \ldots c_5}$, should reduce the uEFT to 11D supergravity. We expect also to have solutions which correspond to embedding of $E_{n(n)}$ ETS with $n \leq 8$.

Actually it is not difficult to find the general solution of the first two equations, (3.10). It reads

$$p_{ab} = p_{[a} q_{b]} , \quad p_{abcde} = p_{[a} q_{bcde]} , \quad (3.12)$$

with arbitrary $q_b$ and $q_{bcde} = q_{[bcde]}$. This solves also the remaining part of the classical section conditions, (3.11).

The easiest way to impose this solution of the section conditions on a function on the bosonic bosonic body $\Sigma^{(528|0)}$ of $\Sigma^{(528|32)}$ (i.e. to quantize the classical section conditions using ’first solve then quantize’ method), passes through the Fourier transform with respect
The quantum version of (3.12) imposed on the (wave)function $\Phi(p, y^{[2]}, y^{[5]})$, 

$$
\partial_{ab} \Phi(p, y^{[2]}, y^{[5]}) = -ip_{[a} q_{b]} \Phi(p, y^{[2]}, y^{[5]}), \\
\partial_{abcde} \Phi(p, y^{[2]}, y^{[5]}) = -ip_{[a} q_{bcde]} \Phi(p, y^{[2]}, y^{[5]}),
$$

(3.13)

is solved by

$$
\Phi(p, y^{[2]}, y^{[5]}) = \exp \{-iy^{bc} p^{[b} q_{c]} - iy^{bcdef} p_{[b} q_{cdef]} \} \phi(p_a, q_a, q_{a1a2a3a4}).
$$

(3.14)

One can appreciate that, as it is defined in the above equations, $\phi(p_a, q_a, q_{a1a2a3a4})$ dependence on $q_a$ and $q_{abcd}$ should be such that the following redefinition of these do not change $\phi(p_a, q_a, q_{a1a2a3a4})$:

$$
q_a \sim q_a + \tilde{q} p_a, \quad q_{abcd} \sim q_{abcd} + \tilde{q}_{[bcd} p_a.
$$

(3.15)

Taking into account that in these equations the second 'symmetry' is reducible,

$$
\tilde{q}_{abc} \sim \tilde{q}_{abc} + \tilde{q}_{[abc} p_c, \quad \tilde{q}_{ab} \sim \tilde{q}_{ab} + \tilde{q}_{[ab} p_b,
$$

(3.16)

one finds that effectively $\phi(p_a, q_a, q_{a1a2a3a4})$ depends on $11 - 1 + \{\{1\} - \{1\} + \{1\} - \{1\} = 219$ additional momenta. On first glance this might look damaging for our hypothesis, as 219 is clearly less than 248, the dimension of minimal irreducible representation of $E_8(8)$. However, let us recall that also in the case of $E_8(8)$ EFT one expects the general solution of its section condition to allow dependence of the functions on not more than 8 coordinates, while a dependence on other 241 coordinates is 'unphysical' but needed to provide a freedom in choosing section and thus the $E_8$ invariance.

This suggests that the way from uEFT to $E_8(8)$ EFT might pass through generic function $\phi(p_a, q_a, q_{a1a2a3a4})$, depending in an arbitrary ('unphysical') manner on $11 + 330 = 341 > 248$ variables $q_a$ and $q_{a1a2a3a4}$, and assume that, at the intermediate stage, an independence on some part of these appears due to some (dynamical or imposed) reduction mechanism. In this paper we will not try to find such a mechanism but rather assume its existence and exploit further the consequence of the idea of possible existence of the 11D uEFT.

Below we will describe a set of superparticle models proposed in [77] and show that these produce quite generic solutions of the above section conditions as equations of motion. One of these models possess the maximal number 32 of supersymmetries and 31 local fermionic $\kappa$–symmetries so that it has a properties of BPS preon [78]. It is nevertheless different from the original 'preonic superparticle' of [76], and this difference results in breaking of the generalized superconformal symmetry $OSp(1\vert 64)$ characteristic for the model of [74].

But before, let us discuss briefly the consistency of the proposed uEFT conditions (3.9) with the (solutions of the) section conditions of $E_{n(n)}$ EFTs with $n = 5, 6, 7, 8$. 

\[ \text{\textit{...}} \]
3.3 uEFT section conditions and $E_{n(n)}$ EFTs with $n = 5, 6, 7, 8$.

Our proposition for the EFT section conditions, Eqs. (3.9), are inspired by the form of the section conditions of the $E_{4(4)} = SL(5)$ EFT,

$$\partial_{[\tilde{a} \tilde{b}] \otimes \partial_{\tilde{c} \tilde{d}}} = 0, \quad \tilde{a}, \tilde{b}, \tilde{c}, \tilde{d} = 1, \ldots, 5. \quad (3.17)$$

The natural question to ask is whether they are consistent with these of other $E_{n(n)}$ EFTs, with $n = 5, 6, 7, 8$.

The section conditions of $E_{5(5)} = SO(5, 5)$ and $E_{6(6)}$ EFTs have the form

$$\gamma^I_{I} \partial_{\Sigma} \otimes \partial_{\Pi} = 0, \quad I = 1, \ldots, 9, 10, \quad \Sigma, \Pi = 1, \ldots, 16, \quad (3.18)$$

$$d^\Lambda \Sigma \Pi \partial_{\Sigma} \otimes \partial_{\Pi} = 0, \quad \Lambda, \Sigma, \Pi = 1, \ldots, 27, \quad (3.19)$$

and the doubts in consistency of our uEFT section conditions with these might arise from the observation that their number, 10 and 27, are less than the numbers of internal components of Eqs. (3.9) with $n = 5$ and 6,

$$\partial_{[a \otimes \partial_{bc}] \otimes \partial_{d]} = 0, \quad \partial_{[\tilde{a} \tilde{b} \otimes \partial_{\tilde{c} \tilde{d} = 0}, \quad a, b, c = 1, \ldots, n, \quad n = 5, 6, \quad (3.20)$$

$$\partial_{b} \otimes \partial_{a} \otimes \partial_{d} = 0, \quad \tilde{d} := \frac{1}{5} \epsilon_{b_{1} \ldots b_{5}} \partial_{b_{1} \ldots b_{5}}, \quad n = 5, \quad (3.21)$$

15 and 36.

However, as we have already commented, Eqs. (3.9) are reducible, and the same applies to the subset of their internal components (3.20), (3.21). The general solution of these contains the branch

$$\partial_{ab} \ldots \partial_{[a} (K_{b]}) \ldots, \quad \tilde{d} \ldots = 0, \quad n = 5, \quad \tilde{d}^b \ldots = 0, \quad n = 6, \quad (3.22)$$

which implies a possible non-trivial dependence of the functions on at least $n = 11 - d$ coordinates. To see a possibility of dependence on more coordinates, let us discuss the above solution with $K_{b} = \tilde{d}_{b}$ being a derivative with respect to some additional $n$-vector coordinates $\tilde{y}^b$. This implies that the functions of uEFT obeying the internal part of uEFT section conditions may depend nontrivially, besides $x^a$ ($\mu = 0, \ldots, (d - 1), d = 11 - n$) and $x^a$, also on these $\tilde{y}^b$. This shows that (3.20), (3.21) are not more, but rather less restrictive in comparison with the standard section conditions of $E_{5(5)} = SO(5, 5)$ and $E_{6(6)}$, EFTs, (3.18), (3.19), the general solutions of which are expected to allow for dependence on not more than 5 and 6 ($n = 11 - d$) additional coordinates respectively.

\[11\] Other branches are characterized by $\partial_a = 0$ and allow possible dependence on some part of tensorial internal coordinates $y^{a_{1}a_{2}}$ and $y^{a_{1} \ldots a_{5}}$ ($y^{a_{1} \ldots a_{5}} = \epsilon^{a_{1} \ldots a_{5}} \tilde{y}_{a}$ for $n = 6$ and $y^{a_{1} \ldots a_{5}} = \epsilon^{a_{1} \ldots a_{5}} \tilde{y}_{a}$ for $n = 5$).

\[12\] The mixed sector of the uEFT section conditions (3.9), which contains equations which carry both $d = 11 - n$ spacetime and $n$ dimensional internal indices, does not put additional restrictions on the dependence of functions on purely internal coordinates. The simples way to check this passes through the classical section conditions, which in their mixed and spacetime parts are solved by $p_{ab} = \frac{1}{2} (p_{a} q_b - p_{b} q_a )$, where $q_a$ is an arbitrary $n$-vector, the 'classical' counterpart of $\tilde{d}_{b}$, and $q_a$ is an additional d-vector, $p_{ab} = n_a q_b$ etc.
This conclusion can be easily generalized also for the case of $E_{n(n)}$ EFTs with $n = 7, 8$ in which, as we have already mentioned, the solution of section conditions is also expected to allow (and in $n = 7$ case is shown [67] to allow) the dependence of the functions of EFT on not more than 11 coordinates (i.e. on not more than $n = 11 - d$ internal coordinates $y^a$ and $d$ spacetime coordinate $x^\mu$). It is not difficult to see that the uEFT section conditions (3.23) have not a stronger, but rather a weaker effect.

Indeed, the preferable branch of the general solution of (3.9) can be formally described by

$$\partial_{ab} \ldots = \partial_{[a} (K_{b]} \ldots) , \quad \partial_{a_1 \ldots a_5} \ldots = \partial_{[a_1} (K_{a_2 \ldots a_5]} \ldots) , \quad a, b = 0, \ldots, 9, 10 ,$$

which implies nontrivial dependence on at least 11 coordinates $x^a$. Again, to see the possible dependence on more coordinates, we can consider

$$K_a = \tilde{\partial}_a = \frac{\partial}{\partial y^a}, \quad K_{abcd} = \tilde{\partial}_{abcd} = \frac{\partial}{\partial y^{abcd}},$$

which implies the dependence of the wave function, besides $x^a$, also on additional vector and antisymmetric tensor coordinates, $\tilde{y}^a$ and $\tilde{y}^{abcd}$. Actually, the classical counterpart of the above described branch of the solution of the uEFT section condition has been discussed in the previous sec. 3.2.

### 3.4 Dynamical model generating (solutions of) the section conditions

Having a candidate set of section conditions, first questions to answer is whether the corresponding EFT subject to this conditions has nontrivial solutions and, if yes, whether these are meaningful in the perspective of String/M-theory. In this sec. 3.4 we address a classical counterpart of the first of these problems: we search for supersymmetric particle models in $\Sigma^{(528|32)}$ generating solutions of the classical section conditions (3.10), (3.11). The meaning of these models will be the subject of the next Sec. 4.

#### 3.4.1 Preonic superparticle and conformally invariant section conditions

The most known superparticle model in maximal tensorial central charge superspace is the ‘preonic superparticle’ of [76]. Its action

$$S = \int d\tau \lambda_\alpha \lambda_\beta \Pi^{\alpha\beta} \equiv \int d\tau \lambda_\alpha \lambda_\beta (\partial_\tau X^{\alpha\beta} - i \partial_\tau \theta^{(\alpha} \theta^{\beta)})$$

contains, besides the bosonic and fermionic coordinate functions, $X^{\alpha\beta}(\tau) = X^{\beta\alpha}(\tau)$ and $\theta^\alpha(\tau)$, also independent bosonic spinor field $\lambda_\alpha(\tau), \alpha = 1, \ldots, 32$.

Actually the model can be defined with arbitrary number $m$ of values of the indices, $\alpha, \beta = 1, \ldots, m$, and for each value of $m$ it possesses a rigid symmetry under $OSp(1|2m)$ supergroup as well as local $\frac{m(m-1)}{2}$ parametric bosonic symmetry ($b$-symmetry) and local $(m - 1)$ parametric fermionic $\kappa$-symmetry [76]. For $m = 4, 8, 16$ cases $\alpha, \beta$ can be treated as $Spin(1, D - 1)$ indices (i.e. $SO(1, D - 1)$ spinor indices) of $D = 4, 6, 10$ dimensional spacetime and the quantization of the corresponding model results in an infinite tower of free conformal higher spin fields in these dimensions [72, 78]. The role of the generalized superconformal group is played in this approach by $OSp(1|2m)$ supergroup with the bosonic body $Sp(2m)$ playing the role of generalized conformal group [103, 72, 88].
For our original case of \( m = 32 \), the action possesses 31 \( \kappa \)-symmetries and this implies that the ground state of the model preserves all but one 11D spacetime supersymmetry i.e. possess the property of BPS preon of M-theory in the terminology of \([78]\) (see \([52]\) for a review). This is the reason to apply \((a \text{ posteriori})\) the name 'preonic superparticle' to the model of \([76]\). However, the quantization of the \( m = 32 \) \((D = 11)\) preonic superparticle results in a quantum state spectrum including state vectors with an indefinite mass. The physical interpretation of such quantum states is obscure.

For the generic value of \( m \) the ground state of the model \((3.24)\) preserves \((m - 1)\) of \( m \) supersymmetries of \( \Sigma^{[m(m+1)/2]}\) superspace which allows us to apply the name ‘preonic superparticle’ also to the cases of \( m = 2,4,8,16 \) when the treatment of \( \alpha, \beta \) as spinor indices is possible.

The canonical momentum conjugate to the bosonic coordinate function of the preonic superparticle, 
\[
p_{\alpha\beta} := \frac{\partial L}{\partial \partial_{\tau} X_{\alpha\beta}},
\]
expressed through the bilinear of the bosonic spinors,
\[
p_{\alpha\beta} = \lambda_{\alpha} \lambda_{\beta}.
\]
(3.25)

This provides a general solution of a kind of \( GL(n) \) invariant counterpart of the classical section conditions:
\[
p_{\alpha[\beta} p_{\gamma]\delta} = 0.
\]
(3.26)
The corresponding counterpart of weak section condition imposed on a (wave) function
\[
\partial_{\alpha[\beta} \partial_{\gamma]} \phi(X) = 0
\]
gives the bosonic equation proposed by Vasiliev in \([93, 94]\). For \( n = 4,8,16 \) the solutions of this equation describe the tower of free massless bosonic conformal higher spin fields in \( D = 4,6,10 \) (see \([98]\) for \( D=6,10 \) cases).

However, the fact that for \( m = 32 \) the meaning of this equation and of its solutions is unclear defends us from temptation to propose its ‘strong’ generalization
\[
\partial_{\alpha[\beta} \otimes \partial_{\gamma]} \delta + \partial_{\delta[\gamma} \otimes \partial_{\beta]\delta} = 0
\]
(3.28)
as a candidate strong section condition for our hypothetical uEFT \(^{13}\).

Decomposing the symmetric 32 \( \times \) 32 matrix of the generalized momentum \((3.25)\) of the preonic superparticle model on the basis of 11D Dirac matrices (see \((3.5)\)), we find the corresponding vector momentum and its tensor counterparts read
\[
p_a = \lambda \tilde{\Gamma}_a \lambda, \quad p_{ab} = i \lambda \tilde{\Gamma}_{ab} \lambda, \quad p_{abcd} = \lambda \tilde{\Gamma}_{abcd} \lambda.
\]
(3.29)

For the generic bosonic spinor \( \lambda_a \) these do not obey the relation \((3.10)\) and \((3.11)\) which we have proposed as candidate section conditions for the hypothetical 11D uEFT. Thus we have to search for a different \( \Sigma^{[528]}\) superparticle model to generate (solutions of) these.

\(^{13}\)Notice that the solutions of Eq. \((3.27)\), the form of which can be found in which in sec. 4.2, also solve the ’strong’ condition \((3.28)\). This is a good illustration of the statement (in sec. 2.1) that ‘first solve than quantize’ approach provides a solution of strong section conditions.
3.4.2 Preonic superparticle with composite bosonic spinor

Curiously enough, a simple modification of the preonic superparticle model makes it to obey the proposed classical section conditions of uEFT, (3.10) and (3.11). To this end it is sufficient to make the fundamental bosonic spinor \( \lambda^+_{\alpha} \) composite,

\[
\lambda_{\alpha} = \lambda^+_{q \alpha} v^{-q}_{\alpha} .
\]  

(3.30)

Here \( \lambda^+_q \) is a 16 component bosonic vector (spinor of \( SO(9) \)), \( q = 1, \ldots, 16 \), and \( v^{-q}_{\alpha} \) is a set of 16 Majorana spinors of \( SO(1,10) \) \( (\alpha = 1, \ldots, 32) \) constrained by (see [53, 54] and below for more details and references)

\[
2v^{-q}_{\alpha}v^{-q}_{\beta} = u^a_{\alpha} \Gamma^a_{\alpha \beta} , \quad v^{-q}_{\alpha} \tilde{\Gamma}^\alpha_{\alpha \beta} v^{-p}_{\beta} = \delta^a_{\alpha} u^a_{\alpha} , \quad v^{-q}_{\alpha} C^{\alpha \beta} v^{-p}_{\beta} = 0 .
\]  

(3.31)

These constraints include the 11D gamma matrices \( \Gamma^a_{\alpha \beta} \) and charge conjugation matrix \( C^{\alpha \beta} \); they are both imaginary in our mostly minus notation while

\[
\Gamma^a_{\alpha \beta} = \Gamma^a_{\alpha \gamma} C^{\gamma \beta} , \quad \tilde{\Gamma}^\alpha_{\alpha \beta} = C^{\alpha \gamma} \Gamma^a_{\alpha \beta} \] 

(3.32)

are real and symmetric.

The sign indices of spinors, \( \pm \), and vectors, \( \# \) \((=++\) and \( =--\)), indicate the weight of different variables under \( SO(1,1) \) transformations which play an important role when clarifying the group theoretical meaning of the constrained variables \( v^{-q}_{\alpha} \); we will discuss this in the next section.

It is important that the constraints (3.31) imply the vector \( u^a_{\alpha} \) is light-like,

\[
u^a_{\alpha} u^a_{\alpha} = 0 .
\]  

(3.33)

Furthermore, we can show that, as a result of these constraints, both the spacetime momentum \( p_a \) and the momenta conjugate to the tensorial central charge coordinates, \( p_{ab} \) and \( p_{abcde} \), are proportional to \( u^a_{\alpha} \),

\[
p_a = u^a_{\alpha} \rho^\# ,
\]  

(3.34)

\[
p_{ab} = u^a_{[\alpha} q^b_{\beta]} , \quad p_{abcde} = u^a_{[\alpha} q^#_{bcde]} ,
\]  

(3.35)

where

\[
\rho^\# = \lambda^+_{q \alpha} \lambda^+_{q \beta} ,
\]  

(3.36)

and \( q^#_b \) and \( q^#_{bcde} \) are also certain bilinears of \( \lambda^+_{q \alpha} \) (we describe them below). It is easy to see that (3.34), (3.35) solve the candidate uEFT section conditions (3.10) and (3.11).

Thus we have shown that a solution of the candidate section conditions (3.10) and (3.11) is generated by the generalized superparticle model with the action

\[
S = \int d\tau \lambda^+_{q \alpha} v^{-q}_{\alpha} \lambda^+_{p \beta} v^{-p}_{\beta} \Pi^{\alpha \beta} = \int d\tau \lambda^+_{q \alpha} v^{-q}_{\alpha} v^{-p}_{\beta} (\partial_{\tau} X^{\alpha \beta} - i\partial_{\tau} \theta^{(\alpha} \theta^{\beta)})
\]  

(3.37)

where \( X^{\alpha \beta}(\tau) \) and \( \theta^{\alpha}(\tau) \) are bosonic and fermionic coordinate functions describing the embedding of the superparticle worldline into 11D tensorial central charge superspace \( \Sigma^{(528|32)} \), \( \lambda^+_{q \alpha}(\tau) \) are 16-component bosonic vectors (which can be considered as spinors of \( SO(9) \)) and \( v^{-q}_{\alpha}(\tau) \) is a set of bosonic variables constrained by (3.31).
3.4.3 Spinor moving frame variables

We have seen that the constraints (3.31) are useful as due to them the momenta conjugate to the 11D spacetime coordinate and to the tensorial central charge coordinates obey the classical section conditions (3.10) and (3.11). Furthermore, they also imply that the projection of the worldline to 11D spacetime is light-like as (3.31) result in (3.34), (3.33) and, hence,

\[ p^a p_a = 0 . \]  

(3.38)

This implies that the quantum states of our dynamical systems are massless from the perspective of 11D spacetime.

However, at first glance, the meaning of the constraints (3.31) might look obscure. To clarify this, let us first notice that the above constraints have a trivial solution

\[ v^{-q}_\alpha = \delta^q_\alpha = \begin{pmatrix} 0 & 0 \\ 0 & I_{16 \times 16} \end{pmatrix} \]  

(3.39)

for which (with an appropriate representation of the 11D gamma matrices) \( u^{(0)}_a = \delta^0_a - \delta^{10}_a \) and the composed bosonic spinor (3.30) has 16 vanishing components,

\[ \lambda^0_\alpha = \lambda^+_\alpha \delta^q_\alpha = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 & \lambda^+_1 \lambda^+_2 \cdots \lambda^+_1 \end{pmatrix} . \]  

(3.40)

The solution (3.39) breaks the manifest SO(1,10) Lorentz symmetry of the constraints (3.31) to its \( SO(1,1) \otimes SO(9) \otimes K_9 \) subgroup (see below for definition of \( K_9 \)). The general solution is given by a Lorentz rotated version of (3.40) in which the parameters of the Lorentz rotations are considered as additional dynamical variables.

A Lorentz rotation of 11D spinors are described by real \( 32 \times 32 \) matrix taking values in the double covering of the 11D Lorentz group, \( Spin(1,10) \),

\[ V^{(b)}_\alpha = \left( v^{+q}_\alpha, v^{\alpha q} \right) \in Spin(1,10) , \quad \alpha = 1, \ldots, 32 \ , \quad q = 1, \ldots, 16 . \]  

(3.41)

When the elements of this matrix is considered as fields, in our case as 1–dimensional fields \( V^{(b)}_\alpha (\tau) = \left( v^{+q}_\alpha (\tau), v^{\alpha q} (\tau) \right) \) (3.41) can be called spinor moving frame matrix. This matrix and its counterpart with sign inverted, \( -V^{(b)}_\alpha (\tau) \), are in two–to–one correspondence with the moving frame matrix, which is \( SO(1,10) \) valued matrix \( U^{(b)}_a (\tau) \)

\[ U^{(b)}_a = \left( \frac{1}{2} \left( u^+_a + u^\#_a \right) , \ u^+_a , \frac{1}{2} \left( u^\#_a - u^-_a \right) \right) \in SO(1, D - 1) ; \]  

(3.42)

this describes the moving frame attached to the worldline.

The correspondence is given by the conditions of Lorentz invariance of the Gamma matrices (see (3.32))

\[ V \Gamma^{(a)} V^T = \Gamma_b^{(b)} \]  

(3.43)

\[ V^T \tilde{\Gamma}_b V = U^{(a)}_b \tilde{\Gamma}^{(a)} , \]  

(3.44)
and of the charge conjugation matrix

\[ VCV^T = C. \] (3.45)

The splitting of the Lorentz group valued matrix \( U_a^{(b)} \) in (3.42) is invariant under \( SO(1, 1) \otimes SO(9) \) subgroup of \( SO(1, 10) \), and the condition \( U_a^{(b)} \in SO(1, 10) \) implies the following conditions on the vectors forming this matrix (see \([30, 31]\))

\[
\begin{align*}
    u_a^\alpha u^a = & \ 0, \quad u_a^\alpha u^a_I = 0, \quad u_a^\alpha u^a# = 2, \quad (3.46)
    u_a^# u^a# = & \ 0, \quad u_a^# u^a_I = 0, \quad (3.47)
    u_a^I u^aJ = & \ -\delta^{IJ}. \quad (3.48)
\end{align*}
\]

With the suitable representation for 11D gamma matrices, the conditions of correspondence between moving frame and spinor moving frame variables, (3.43), (3.44) and (3.45) (equivalent to \( V_a^{(b)} \in \text{Spin}(1, 10) \) and actually defining \( U_a^{(b)} \in \text{SO}(1, 10) \)) can be split into the following set of constraints for the spinor moving frame variables (this is to say for the rectangular blocks of \( V_a^{(b)} \in \text{Spin}(1, 10) \))

\[
\begin{align*}
    2v_{a\alpha}^q v_{\beta}^q = & \ \Gamma_a^{\alpha\beta} u_a^q(a), \quad v^{-q} \tilde{\Gamma}_a v^{-p} = u_a^{-q} \delta^{qp}(b), \quad v_{a\alpha}^{-q} C^{\alpha\beta} v_{\beta}^{-p} = 0 \quad (c), \quad (3.49)
    2v_{a\alpha}^+ v_{\beta}^+ = & \ \Gamma_a^{\alpha\beta} u_a^q(a), \quad v^{+q} \tilde{\Gamma}_a v^{+p} = u_a^{+q} \delta^{qp}(b), \quad v_{a\alpha}^{+q} C^{\alpha\beta} v_{\beta}^{+p} = 0 \quad (c), \quad (3.50)
    2v_{a(a\alpha)}^+ v_{b\beta}^+ = & \ \Gamma_a^{\alpha\beta} u_a^I(a), \quad v^{-q} \tilde{\Gamma}_a v^{+p} = u_a^I \gamma_{qp}^{-I}(b), \quad I = 1, ..., 9, \quad (3.51)
    & \ v_{a\alpha}^+ C^{\alpha\beta} v_{\beta}^{+p} = i \delta_{qp} \quad (c).
\end{align*}
\]

Clearly, the relations (3.49) coincide with (3.31). Notice that just this set of relations, i.e. Eqs. (3.51), are invariant under local \( O(16) \) transformations of \( v_{a\alpha}^q \),

\[
v_{a\alpha}^{-q} \mapsto v_{a\alpha}^{-p} O_{pq}, \quad O_{pp'} O_{qq'} = \delta_{qp} \quad \Leftrightarrow \quad O_{qp} \in O(16). \quad (3.52)
\]

This symmetry is broken down to \( \text{Spin}(9) \) by the constraints (3.53a,b) which involve the \( d = 9 \) Dirac matrices

\[
\begin{align*}
    \gamma^I_{pq} = & \ \gamma^I_{pq}, \quad (\gamma^I \gamma^J + \gamma^J \gamma^I)_{qp} = \delta^{IJ} \delta_{qp}, \quad q, p = 1, ..., 16, \quad I = 1, ..., 9. \quad (3.53)
\end{align*}
\]

The manifest gauge symmetry of the complete set of constraints (3.49)–(3.51) is \( SO(1, 1) \times \text{Spin}(9), \)

\[
\begin{align*}
    v_{a\alpha}^{-q} \mapsto & \ v_{a\alpha}^{-p} S_{pq} e^{-\beta}, \quad v_{a\alpha}^{+q} \mapsto v_{a\alpha}^{+p} S_{pq} e^{+\beta}, \quad (3.54)
    u_a^\alpha \mapsto & \ u_a^\alpha e^{-2\beta}, \quad u_a^# \mapsto u_a^# e^{+2\beta}, \quad u_a^I \mapsto u_a^I O^{JI}, \quad (3.55)
\end{align*}
\]

where

\[
\begin{align*}
    SS^T = & \ I_{16 \times 16}, \quad S_{pq} \gamma^I_{pq} S_{pq'} = \gamma^I_{qp'} O^{JI} \quad \Rightarrow \quad O^{IK} O^{JK} = \delta^{IJ}, \quad (3.56)
    \Leftrightarrow & \ S_{qp} \in \text{Spin}(9), \quad O^{JI} \in \text{SO}(9).
\end{align*}
\]
These $SO(1,1) \times Spin(9)$ transformations also leave invariant the splittings (3.42) of moving frame matrix and (3.41) of the spinor moving frame matrix on rectangular blocks $v_{a}^{\pm q}$. However, if we consider a dynamical model involving only one of these two blocks, $v_{a}^{-q}$ in the case of our model, the gauge symmetry is enhanced up to $[SO(1,1) \otimes SO(9)] \otimes K_9$, where $K_9$ transformations are defined by

$$
v_{a}^{q} \rightarrow v_{a}^{-q}, \quad v_{a}^{+q} \rightarrow v_{a}^{+q} + v_{a}^{-p} \gamma^{I}_{pq} k^{#I}, \quad u_{a}^{\pm} \rightarrow u_{a}^{\pm} + 2u_{b}^{I} k^{#I} + u_{a}^{-q} k^{#I} k^{#I}, \quad u_{a}^{I} \rightarrow u_{a}^{I} + u_{a}^{-q} k^{#I},
$$

(3.57)

Thus, in a theory which is invariant under $SO(1,1) \otimes SO(9)$ transformations (3.54) and does not contain $v_{a}^{+q}$, the set of spinor variables $v_{a}^{-q}$ constrained by (3.49) (equivalent to (3.31)) can be identified with homogeneous coordinate of the coset $SO(1,10)/[SO(1,1) \otimes SO(9)] \otimes K_9$ which is isomorphic to a nine-sphere $S^9$ [63, 64, 59].

In the model where these $v_{a}^{-q}$ can be treated as spinor moving frame variable, this $S^9$ can be recognized as the celestial sphere of the 11D observer [63, 64, 59].

Using the above constraints and their consequences, such as the unity decomposition

$$\delta_{\beta}^{\alpha} = iC_{\alpha \gamma} (v_{\gamma}^{+q} v_{\beta}^{-q} - v_{\beta}^{+q} v_{\gamma}^{-q}) \quad \Leftrightarrow \quad iv_{\alpha}^{+q} v_{\beta}^{-q} - iv_{\beta}^{+q} v_{\alpha}^{-q} = C_{\alpha \beta},
$$

(3.60)

one can check that

$$v^{-q} \tilde{\Gamma}^{\alpha \beta}_{ab} v^{-p} := v_{a}^{-q} \tilde{\Gamma}^{\alpha \beta}_{ab} v_{b}^{-p} = -2i u_{[a}^{I} u_{b]}^{J} \gamma^{I} q^{p},
$$

(3.61)

$$v^{-q} \tilde{\Gamma}^{I} \Gamma_{abde} v^{-p} \equiv -4 u_{a}^{I} u_{b}^{I} u_{c}^{K} u_{d}^{L} \gamma^{IJKL} q^{p}.
$$

(3.62)

For the generalized superparticle model (3.37) the canonical momenta conjugate to the tensorial coordinate functions $y^{ab}$ and $y^{abcde}$ have the form of (3.24) with composite bosonic spinor (3.30), so that (3.61) implies

$$p_{ab} = 2 u_{[a}^{I} u_{b]}^{I} \lambda^{+} \gamma^{I} \lambda^{+},
$$

(3.63)

$$p_{abde} = -4 u_{[a}^{I} u_{b}^{I} u_{c}^{K} u_{d}^{L} \lambda^{+} \gamma^{IJKL} \lambda^{+},
$$

(3.64)

where $\lambda^{+} \gamma^{I} \lambda^{+} := \lambda_{q}^{+} \gamma^{I} q^{p} \lambda^{+}$, etc. This set of equations has the form of the general solution (3.35) of the classical section conditions (3.10), (3.11) with

$$q_{a} = u_{a}^{I} \frac{\lambda^{+} \gamma^{I} \lambda^{+}}{(\lambda^{+} \lambda^{+})}, \quad q_{abcd} = -4 u_{a}^{I} u_{b}^{I} u_{c}^{K} u_{d}^{L} \frac{\lambda^{+} \gamma^{IJKL} \lambda^{+}}{(\lambda^{+} \lambda^{+})}.
$$

(3.65)

Hence we have shown that the preonic superparticle with composite bosonic spinor (3.30), described by the action (3.37), generates a solution of the classical counterparts (3.10), (3.11) of the proposed section conditions (3.9) of the hypothetical uEFT. In this sense we can say that (3.37) is (one of the) uEFT superparticle(s).
3.4.4 A family of superparticle 'solving' the classical section conditions

The next natural question is: are there more uEFT superparticle models? In this section we present the family of superparticle models in $\Sigma^{(528|32)}$ superspace, first described in [77], and show that each of these generates a constraint solving the classical section conditions (3.10), (3.11) of the hypothetical 11D EFT. The actions of these models can be collected in the universal expression

$$S = \int d\tau \rho_{\alpha\beta}^q v_{\alpha}^{-q} v_{\beta}^{-p} \Pi_{\alpha\beta} = \int d\tau \rho_{\alpha\beta}^q v_{\alpha}^{-q} v_{\beta}^{-p} (\partial_\tau X^{\alpha\beta} - i\partial_\tau \theta^{(\alpha} \theta^{\beta)}) ,$$

in which $\rho_{\alpha\beta}^q = \rho_{\alpha\beta}(\tau)$ is a symmetric $16 \times 16$ bosonic matrix field, $X^{\alpha\beta} = X^{\alpha\beta}(\tau)$ and $\theta^{\alpha} = \theta^{\alpha}(\tau)$ are 528 bosonic and 32 fermionic coordinate functions, the same as in (3.24) and (3.37), and $v_{\alpha}^{-q} = v_{\alpha}^{-q}(\tau)$ are the spinor moving frame variables (3.59) discussed in the sec. 3.4.3.

One can consider the action (3.66) as describing a class of superparticle models the properties of which depend essentially on the rank of symmetric matrix $\rho_{\alpha\beta}$. Alternatively one can speak about dynamical system with several branches determined by this rank. Of these, let us especially notice the following particular cases preserving minimal and maximal amount of supersymmetry:

- The case of rank 16 matrix with unity eigenvalues,

$$\rho_{\alpha\beta}^q = \rho_{\alpha\beta}^q \delta_{\alpha\beta} ,$$

(3.67)

defines the massless 11D superparticle (sometimes called M0-brane), see [53, 54]. This model has 16 $\kappa$-symmetries and, correspondingly, its ground state preserves one half of 32 spacetime supersymmetries.

- The case of rank 1 matrix

$$\rho_{\alpha\beta}^q = \lambda^+_q \lambda^+_p ,$$

(3.68)

as discussed below, correspond to a preonc superparticle model (in terminology of [78]). It possesses 31 $\kappa$-symmetries and, hence, its ground state preserves all but one supersymmetries.

Generically, if we restrict the model by requiring all the eigenvalues of matrix $\rho_{\alpha\beta}^q$ of the rank $r$ to be positive, it always can be written in the form

$$\rho_{\alpha\beta}^q = \lambda^+_q \lambda^+_p , \quad s = 1, ..., r .$$

(3.69)

Thus, without loss of generality (in practical terms, i.e. if not considering a problematic models) one can describe the branch of the dynamical system (3.66) with rank($\rho_{\alpha\beta}^q$) = $r$ by

$$S^{(r)} = \int d\tau \lambda^+_q \lambda^+_p v_{\alpha}^{-q} v_{\beta}^{-p} \Pi_{\alpha\beta} , \quad s = 1, ..., r .$$

(3.70)
In this family $S^{(1)}$ is the preonic action, corresponding to (3.68), while the standard massless superparticle action is $S^{(16)}$ with $\lambda_q^+ = \sqrt{\rho^# \delta^s}$.

The action (3.71) is invariant under the $(32-r)$-parametric local fermionic $\kappa$–symmetry
\[
\delta \kappa X^{\alpha \beta} = i \delta \kappa \theta^{(\alpha} \theta^{\beta)} , \quad \delta \kappa v_\alpha^{-q} = 0 , \quad \delta \kappa \lambda_q^+ = 0
\]
\[
\delta \kappa \theta^\alpha = \kappa^+ q v_q^{-\alpha} + \kappa^- s w_q^+ v_q^+ \alpha , \quad \bar{s} = 1 , ..., (16 - r) .
\] (3.71)

Notice that Eqs. (3.71) describes the general solution of the equation
\[
\delta \kappa \theta^\alpha v_\alpha^{-q} \lambda_q^+ = 0 , \quad \left\{ \begin{array}{l} q = 1 , ..., 16 , \\ s = 1 , ..., r \end{array} \right. \quad \Leftrightarrow \quad \delta \kappa \theta^\alpha v_\alpha^{-q} \rho^#_{\alpha \beta} = 0 , \quad \text{rank}(\rho^#_{\alpha \beta}) = r .
\] (3.72)

To write this solution we have introduced the set of Spin(1,10) spinors $v_\alpha^\pm = \pm i C^{\alpha \gamma} v_\gamma^\pm$ which obey (see (3.60))
\[
v_\alpha^+ v_\alpha^- = \delta_{pq} , \quad v_\alpha^- v_\alpha^- = 0 ,
\] (3.73)

and a set of 16–vectors $w_\bar{s}^s$ orthogonal to $\lambda_q^+$
\[
w_\bar{s}^s \lambda_q^+ = 0 , \quad \bar{s} = 1 , ..., (16 - r) .
\] (3.74)

In other words, that are $(16 - r)$ null-vectors of the rank $r$ matrix $\rho^#_{\alpha \beta} = \lambda_q^+ \lambda_p^+ , \quad w_\bar{s}^s \rho^#_{\alpha \beta} = 0$.

Let us calculate the canonical momentum conjugate to the bosonic coordinates in (3.66). In the spin-tensor notation we obtain
\[
p_{\alpha \beta} = \rho^#_{\alpha \beta} v_\alpha^{-q} v_\beta^{-p} .
\] (3.75)

Using the constraints (3.31) we can find that this implies that the spacetime momentum of the system is a light-like 11-vector
\[
p_a = \frac{p_{qq}}{32} u_a^- = 0 .
\] (3.76)

Hence from the 11D spacetime perspective, any of the models (3.70) describes a massless particle or a set of massless particles.

Furthermore, using (3.61) it is not difficult to show that the momenta conjugate to the tensorial coordinates have the form
\[
p_{ab} = u_{[a}^\gamma q_{b]}^# \quad \text{and} \quad p_{abcd} = u_{[a}^\gamma q_{bcd]}^#
\]

with
\[
q_a = u_a^I \frac{\lambda^{+r} \gamma^{IJKL} \lambda^+}{(\lambda^+ s \lambda^+ s)^+} , \quad q_{abcd} = -4 u_a^I u_b^J u_c^K u_d^L \frac{\lambda^{+r} \gamma^{IJKL} \lambda^+}{(\lambda^+ s \lambda^+ s)}. \] (3.77)

Thus any model from the family described by a (nondegenerate) action of the form (3.66) or (3.70) generate a solution of the classical section conditions (3.10), (3.11) of the hypothetical underlining uEFT.
4. On uEFT superparticles and 11D higher spin theories

In the previous Section 3.4 we have presented a family of superparticle models which produce as constraints quite generic solutions of the section conditions proposed for the hypothetical underlying 11D EFT (uEFT) in Sec. 3.2. In this section we will argue that, curiously enough, the quantization of these uEFT superparticles should result in the theory of free massless higher spin fields in 11 dimensional spacetime.

4.1 Free $D = 4, 6, 10$ conformal higher spin theory description in $\Sigma^{(m(m+1)/2|m)}$ superspace with $m = 2(D - 2) = 4, 8, 16$

To argue in favor of the above conclusion, we begin with already mentioned relation of the original preonic superparticle model (3.24) with $m = 4, 8$ and 16 ($\alpha, \beta = 1, \ldots, m$) with free conformal massless higher spin field theories in spacetime of dimensions $D = m + 2 = 4, 6, 10$ [92, 98]. Namely, the quantization of these models of superparticle in $\Sigma^{(m(m+1)/2|m)}$ superspace with $m = 2(D - 2) = 4, 8, 16$ results in the quantum state spectrum described by an infinite tower of all $D=4, 6, 10$ massless conformal higher spin fields.

This is related to the fact that generalized superconformal symmetry $OSp(1|2m)$ can be realized on towers of the bosonic and of the fermionic massless conformal fields which can be packed into a scalar $\phi(X)$ and a 'spinor' (s-vector) field $f_\alpha(X)$ on the tensorial space (hyperspace) $\Sigma^{(m(m+1)/2|m)}$ (see [103] for $m = 4$) which obey the Vasiliev’s equations $\partial_{[\alpha} \partial_{\beta]} \phi(X) = 0$ and $\partial_{[\alpha} f_{\beta]}(X) = 0$ [93, 94]. These fields can be also collected in superfield defined on $\Sigma^{(m(m+1)/2|m)}$ superspace satisfying $D_{[\alpha} D_{\beta]} \Phi(X, \theta) = 0$ [97].

On the other hand, all the tower of the solutions of all the free conformal higher spin equations for bosonic fields in $D=4, 6$ and 10 can be described by a scalar function $\tilde{\phi}(\lambda)$ of one unconstrained real bosonic spinor $\lambda_\alpha$, $\alpha = 1, \ldots, m$, with $m = 2(D - 2)$, subject to the restriction to be even with respect to $\lambda_\alpha \rightarrow -\lambda_\alpha$, and also by specifying the class of functions $\tilde{\phi}(\lambda)$ belongs to [92, 98]. With a suitable choice of this latter the solution of the bosonic Vasiliev equation (3.27) is given by

$$\phi(X) = \int d\lambda \tilde{\phi}(X, \lambda) = \int d\lambda \tilde{\phi}(\lambda) e^{i\lambda_\alpha \lambda_\beta X^{\alpha\beta}}. \quad (4.1)$$

4.2 $m=4,8,10$ counterparts of the preonic superparticle and conformal higher spin fields in $D=4,6,10$

This $\tilde{\phi}(\lambda)$, and also its fermionic counterpart, can be obtained by quantization [92, 98] of the $m = 4, 8, 10$ versions of the superparticle model (3.24) in terms of components of orthosymplectic twistor ($\lambda_\alpha, \mu^\alpha, \eta$) related to the $\Sigma^{(m(m+1)/2|m)}$ coordinates by

$$\mu^\alpha = X^{\alpha\beta} \lambda_\beta - \frac{i}{2} \theta^\alpha \theta^\beta \lambda_\beta, \quad \eta = \theta^\alpha \lambda_\alpha. \quad (4.2)$$

The fundamental representation of $OSp(1|2m)$ acts on orthosymplectic supertwistors by left multiplication, and the above incidence relations (4.2) explain the possibility to realize $OSp(1|2m)$ as superconformal symmetry of $\Sigma^{(m(m+1)/2|m)}$.
For simplicity, we restrict our discussion here by quantization of purely bosonic limit, $\theta = 0$, of $m = 2(D - 2) = 4, 8, 16$ superparticle. Using the Leibniz rule the bosonic action in (3.24),

$$S_0 = \int d\tau \lambda_\alpha \lambda_\beta \partial_\tau X^{\alpha\beta},$$

(4.3)
can be written in the form

$$S = \int d\tau (\lambda_\alpha \partial_\tau \mu^\alpha - \partial_\tau \lambda_\alpha \mu^\alpha)$$

(4.4)
with $\mu^\alpha = X^{\alpha\beta} \lambda_\beta$ (4.2). This new variable $\mu^\alpha(\tau)$ carries all the physical degrees of freedom in $X^{\alpha\beta}(\tau) = X^{\beta\alpha}(\tau)$ (the remaining $m(m-1)$ components can be gauged away) and can be considered as a momentum conjugate to $\lambda_\alpha$ (or vice versa: coordinate conjugate to momentum $\lambda_\alpha$) and the action (4.4) can be considered as a Hamiltonian action with Hamiltonian equal to zero.

Then the quantization of the model (4.4) is trivial; its state vector can be represented by an arbitrary function of $\lambda_\alpha$, $\tilde{\phi}(\lambda)$. The spacetime treatment of this quantum state spectrum uses the relation (3.25),

$$p^{\alpha\beta} - \lambda_\alpha \lambda_\beta = 0,$$

(4.5)
which can be obtained as a primary constraint when constructing Hamiltonian approach to our dynamical system on the basis of the original action (4.3).

An alternative quantization of (4.3) with $m = 2(D - 2) = 4, 8, 16$, which passes through the stage of development of such a Hamiltonian approach and conversion of the second class constraints (4.2), results in a wavefunction dependent on both $X^{\alpha\beta}$ and $\lambda_\gamma$ and obeying the quantum counterpart of the constraint (4.5); the so-called preonic equation

$$(\partial_{\alpha\beta} + i\lambda_\alpha \lambda_\beta)\phi(X, \lambda) = 0.$$  

(4.6)
The solution of this equation is given by $\tilde{\phi}(X, \lambda) = \tilde{\phi}(\lambda) e^{i\lambda_\alpha \lambda_\beta X^{\alpha\beta}}$ and its integration with a suitable measure $d^n\lambda$ give the wavefunction in the generalized coordinate, $X^{\alpha\beta}$ representation (4.1).

On the other hand, the wavefunction in the momentum representation, $\phi(p_{\alpha\beta})$, is localized on the solutions of (4.5), which implies, in particular, that the standard $D$-vector momentum extracted from (4.5) is

$$p_a = \lambda_\alpha \bar{\Gamma}_a \lambda \equiv \lambda_\alpha \bar{\Gamma}_a \lambda_\beta.$$  

(4.7)
For $D = 4, 6, 10$ (and also for $D = 3$) this momentum is light-like $p_ap^a = 0$ and hence the quantum states of the model are massless.

Actually, the quantum state spectrum of D=4 model consists of an infinite tower of the massless fields of all possible helicities. In the case of D=6 and D=10 model, where, in contrast to $D = 4$, not all the free massless fields are conformal, we obtain a tower of massless conformal higher spin fields (see e.g. [98] for their description). The fields in the
tower are ‘enumerated’ by a set of integer numbers which can be considered as momenta conjugate to the coordinates of $S^{(D-3)}$ ($S^1$, $S^3$ and $S^7$) spheres realized as Hopf fibrations

$$S^{(n-1)}/S^{D-2} = S^{2D-5}/S^{D-2} = (S^3/S^2, S^7/S^4, S^{15}/S^8).$$

Let us describe how these appear. The space of light-like momenta in D-dimensions is

$$\{p_a|p^2 = 0\} = \mathbb{R}_+ \otimes S^{(D-2)} \quad (4.8)$$

The space of nonvanishing $n$-component bosonic spinors our wavefunction $\tilde{\phi}(\lambda)$ depends on is

$$\{\lambda_\alpha\} = \mathbb{R}^n - \{0\} = \mathbb{R}_+ \otimes S^{(n-1)}, \quad (4.9)$$

and the scale of momenta ($\mathbb{R}_+$ in (4.8)) is given by the square of the scale of the bosonic spinor ($\mathbb{R}_+$ in (4.9)). Thus, besides the light-like momenta, the wavefunction depend on $(D-3)$ coordinates of the fibrations

$$\{\lambda_\alpha\}/\{p_a|p^2 = 0\} = S^{(n-1)}/S^{(D-2)} = S^{2D-5}/S^{(D-2)}, \quad D = 4, 6, 10 \quad (4.10)$$

which are isomorphic to $S^{(D-3)}$ spheres. These spaces are compact and a momentum conjugate to a compact coordinate is quantized.

Hence passing to the momentum representation on this compact directions, we will arrive at the wave function depending on D dimensional light-like momentum $(D = 4, 6, 10)$ and characterized by $(D - 3)$ integer numbers. In D=4 one integer number obtained in such a way is the doubled helicity of a massless fields. The description of D=10 and D=6 conformal higher spin fields can be found in [98] and refs. therein.

4.3 On superparticle models for massless non-conformal higher spin theories in D=6,10 and D=11

Notice that, although the interest in a tensorial (super)space or hyper(super)space description of higher spin fields persists already more than 15 years [92, 93, 94, 95, 96, 97, 98, 99, 100, 101], the research is mainly concentrated on D=4 case. The reason beyond this, besides that the $m = 8, 16$ cases are more complicated, is that, in contradistinction to D=4, in D=6 and D=10 dimensional cases not all the massless fields are conformal, and these which are look quite exotic [98]. In particular neither the linearized equations for graviton, nor the Maxwell equations for D-vector potential are conformally invariant in D=6 and D=10 dimensions.

In the 11D case the straightforward generalization of the above derivation of free higher spin theories fails. Namely, symplectic twistor quantization is universal and results in a wavefunction $\tilde{\phi}(\lambda)$ depending on $m = 32$ component bosonic spinor in an arbitrary manner, but what fails is its spacetime interpretation: in contrast to D=4,6,10 cases, in D=11 the momentum constructed from spinor bilinear (3.25) is not light-like. Actually with this 11-momentum the mass of the quantum states remains indefinite which hamper the spacetime interpretation of D=11 (super)particle model (3.24).
In the above perspective there exist the quests for superparticle models providing the classical mechanic description of D=11 higher spin field theory and of D=6, 10 dimensional massless non-conformal higher spin theories. The \( m = 32 \) and \( m = 8, 16 \) versions of the above described superparticle models (3.66) and (3.70) are good candidates for these roles.

This conjecture is suggested by a series of observations the first of which is that, as we have described above, all these models produce the constraints \( p_a \propto u^a \) which implies \( p_a p^a = 0 \). As a result, their quantum state spectrum is formed by massless states. Then, the analogy with the above discussed \( n = 2(D - 2) = 4, 8, 16 \) version of the preonic superparticle model suggests that this quantum state spectrum provides us with a theory of free higher spin fields.

This conjecture looks especially natural in the case of preonic-type model (3.37) with composite spinor field (3.30). As far as the other models (3.66), (3.70), preserving from one half to all but two supersymmetries are concerned, this might be considered as a counterpart of the D=4 \( OSp(4|2) \) invariant models in [92] which preserve 2 of 4 supersymmetries and also describes supermultiplet of free massless higher spin fields by its quantum state spectrum.

Furthermore, as there are no traces of conformal invariance in the models (3.66), (3.70) with \( m = 8, 16, 32 \), their quantum state spectrum should not be conformal. Indeed, it is easy to see that even in the preonic-type model with composite bosonic spinor (3.37), which is included as \( r = 1 \) representative in the set of models (3.70), the presence of spinor moving frame variables \( u^a_q \) in (3.30) breaks the \( Sp(m) \) invariance down to D-dimensional Lorentz group.

Thus we have argued that the quantization of the models (3.70), (3.66) with \( m = 8, 16 \) and 32 should result in a theory of free non-conformal higher spin fields in \( D = 6, 10 \) and \( D = 11 \) dimensional spacetime.

The check of this conjecture by explicit quantization of these models and by the analysis of their quantum state spectrum will be the subject of a forthcoming paper. In the next section we present a discussion on some representation of the M-theory superalgebra and on the embedding of 11D supergravity in these representations, which actually suggests how the quantum state spectrum of some of these models might look like.

To conclude this section, we just notice that the idea on that the 11D higher spin fields are necessary ingredients of (the hypothetical) underlying 11D exceptional field theory, uEFT, is in consonance with discussions on their necessity in the context of (also hypothetical) \( E_{10} \) and \( E_{11} \) theories [45, 104].

5. Embedding 11D supergravity into representations of the M-theory superalgebra, the supersymmetry superalgebra of uEFT

One more argument in favor of relevance of 11D tensorial central charge superspace (6.1) as a basis for hypothetical underlying uEFT can be gained by discussing unitary representations of the M-theory superalgebra (3.1), the supersymmetry superalgebra of \( \Sigma^{(528|32)} \), and by showing that 11D supergravity multiplet can be included in (some of) these representations. A more complicated counterpart of such a study can be found in [105] where
Gunaydin showed that \( osp(1|32) \) admits unitary representations which contain 11D SUGRA when the contraction and reduction to the super-Poincaré is taken.

Actually, as far as M-algebra \( (3.1) \) can be also obtained by contraction of \( osp(1|32) \), the affirmative answer on the question of whether 11D SUGRA can be embedded in some of its highest weight unitary representations is guaranteed by the results of \( [105] \). However, we find suggestive to construct such an embedding explicitly, in particular because it gives us a hint about how the results of the quantization of some of the superparticle models described in section 3 might look like.

The construction of highest weight representations of M-algebra \( (3.1) \) is simpler than that of semisimple superalgebra \( osp(1|32) \): the algebra of its bosonic generators, \( P_a, Z_{ab} \) and \( Z_{abcde} \), which can be collected in \( P_{\alpha\beta} (3.2) \), is Abelian, \( [P_{\alpha\beta}, P_{\gamma\delta}] = 0 \), and, hence, they can have the basis of common eigenvectors or eigenstates. We denote such eigenstates by \( |A, p_{\alpha\beta}\rangle \), where \( p_{\alpha\beta} \) are eigenvalues of \( P_{\alpha\beta} \),

\[
P_{\alpha\beta}|A, p_{\alpha\beta}\rangle = p_{\alpha\beta}|A, p_{\alpha\beta}\rangle , \tag{5.1}
\]

and \( A \) denotes possible indices or additional variables the state depends on.

### 5.1 A particular class of eigenstates of the generalized momentum

Let us discuss a particular class of such states, \( |A, v_{aq}, \lambda^+^q\rangle \), for which the eigenvalue matrices \( p_{\alpha\beta} \) have rank \( r \leq 16 \) and can be presented in the form (cf. \( (3.75) \) with \( (3.69) \))

\[
p_{\alpha\beta} = \lambda^+_q \lambda^+^p v_{\alpha}^{-q} v_{\beta}^{-p} , \tag{5.2}
\]

where \( v_{\alpha}^{-q} \) form a rectangular \( (32 \times 16) \) block of a spin group valued matrix (see \( (3.41) \)), and hence obeys \( (3.31) \), and \( \lambda^+_q \) is \( 16 \times r \) matrix of maximal rank;

\[
P_{\alpha\beta} |A, v_{aq}^{-}, \lambda^+_q\rangle = \lambda^+_q \lambda^+^p v_{\alpha}^{-q} v_{\beta}^{-p} |A, v_{aq}^{-}, \lambda^+_q\rangle . \tag{5.3}
\]

Let us observe that \( (5.2) \) and \( (3.31) \) imply that the eigenvalue of the 11–momentum operator \( P_a, p_a \propto \Gamma_a^{\alpha\beta} p_{\alpha\beta} \propto \lambda^+_q \lambda^+^p u_{a}^{-q} \) is light-like, \( p_a p^a = 0 \),

\[
P_a |A, v_{aq}^{-}, \lambda^+_q\rangle = \lambda^+_q \lambda^+^s u_{a}^{-q} u_{a}^{-p} |A, v_{aq}^{-}, \lambda^+_q\rangle , \quad u_{a}^{-q} u_{a}^{-p} = 0 . \tag{5.4}
\]

Furthermore, as \( (5.2) \) is invariant under \( SO(1,1) \times SO(9) \) transformations (if we allow these to act also on \( \lambda^+_q \)), using this symmetry as an identification relation, we can consider \( v_{\alpha}^{-q} \) as a kind of homogeneous coordinates of celestial sphere (cf. \( (3.53) \))

\[
\{v_{\alpha}^{-q}\} = S^9 . \tag{5.5}
\]

One can see that the described algebraic properties of \( v_{\alpha}^{-q} \) are the same as that of spinor moving frame variables \( v_{\alpha}^{-q}(\tau) \) used in the generalized superparticle models of sec. 3. In this section we do not use this name (neither its shorter version ’spinor frame variables’ \( [107] \)) as it might be confusing in the context of superalgebra representations. Notice however, that the similarity of variables marking states in this section with 1d fields of sec. 3 is not occasional: the quantization of the superparticle models of sec. 3 should result
in the multiplet of quantum states transforming under representations which we discuss in this section.

The space of states $|A, v_{aq}^-, \lambda^+_q; \rho^\# >$ splits into the sectors $\{ |A, v_{aq}^-, \lambda^+_q; \rho^\# > \}$ with different ranges of the values of index $s$: $s = 1, ..., r \leq 16$. In the sector with $r = 16$ a special role is played by the states with $\lambda^+_q = \sqrt{2} \rho^# \delta^s_q$, $|A, v_{aq}^-, \rho^# > := |A, v_{aq}^-, \sqrt{2} \rho^# \delta^s_q; 16 >$.

(5.6)

On such states the eigenvalues of the generalized momenta $Z_{ab}$ and $Z_{abcde}$ vanish and only (super)Poincaré generators are realized nontrivially,

$$P_{\alpha \beta} |A, v_{aq}^-, \rho^# > = 2 \rho^# v_{\alpha}^- v_{\beta}^- |A, v_{aq}^-, \rho^# > = p_a \Gamma_{\alpha \beta} |A, v_{aq}^-, \rho^# > ,$$

(5.7)

with eigenvalues determined by $p_a \Gamma_{\alpha \beta} = 2 \rho^# v_{\alpha}^- v_{\beta}^-$ in terms of constrained spinors (5.5) and densities $\rho^#$ (cf. (3.34), (3.49)).

### 5.2 Some unitary highest weight representations of the M-algebra

Representations of the supersymmetry generators on the states $|A, v_{aq}^-, \lambda^+_q >$ can be characterized by equation

$$Q_{\alpha} = v_{\alpha}^- \lambda^+_q \mathcal{C}_s , \quad q = 1, ..., 16 , \quad s = 1, ..., r \leq 16 ,$$

(5.8)

where $\mathcal{C}_s$ are generators of $r$-dimensional Clifford algebra

$$\{ \mathcal{C}_s, \mathcal{C}_t \} = 2 \delta_{st} , \quad s, t = 1, ..., r .$$

(5.9)

In the context of the above discussion, a more rigorous way is to write

$$Q_{\alpha} |A, v_{aq}^-, \lambda^+_q > = v_{\alpha}^- \lambda^+_q \mathcal{C}_s |A, v_{aq}^-, \lambda^+_q > ,$$

(5.10)

where the action of Clifford operator $\mathcal{C}_s$ on the state is still to be defined. The representation of M-algebra is described by (5.10) completed by (5.11) which in a more schematic form, similar to (5.8), reads (cf. (5.2))

$$\mathcal{P}_{\alpha \beta} = \lambda^+_q \lambda^+_p \sqrt{2} \rho^# v_{\alpha}^- v_{\beta}^- .$$

(5.11)

Notice that (5.10) implies that only $r(\leq 16)$ of 32 supersymmetries are realized nontrivially on the M-algebra representations under consideration. Thus we will be working with short or BPS multiplets of states; all the states of such supermultiplet preserve $32 - r$ supersymmetries and only $r$ of the supersymmetry generators mix the different states. If choosing $r = 1$, we would be dealing with preonic multiplets preserving all but one supersymmetries [78, 79]. Here we will be interested mainly in a more conventional type of multiplets, with $r = 16$, all the states of which preserve one half, i.e. 16 of 32 supersymmetries.

In the case of even $r$, to construct unitary highest weight representations of the M-algebra, following the line of [105] and using the above type of the eigenstates of generalized
momenta, we have to introduce a kind of complex structure and to split the set of \( r \) Hermitian generators of Clifford algebra, \( \mathfrak{C}_s \), on two conjugate sets of \( r/2 \) generators, \( \mathfrak{B}_A \) and \( \mathfrak{B}^\dagger_A \) obeying

\[
\{ \mathfrak{B}_A, \mathfrak{B}_B \} = 0, \quad \{ \mathfrak{B}^\dagger_A, \mathfrak{B}_B \} = \delta_B^A, \quad \{ \mathfrak{B}^\dagger_A, \mathfrak{B}^\dagger_B \} = 0, \quad A, B = 1, \ldots, \frac{r}{2}. \tag{5.12}
\]

(An explicit form of the relation between \( \mathfrak{C}_s \) and \( \mathfrak{B}_A \) and \( \mathfrak{B}^\dagger_A \) will be discussed below).

Then we can define the highest weight state \( |v^{-q}_\alpha, \lambda^+_q \rangle \) by

\[
\mathfrak{B}_A |v^{-q}_\alpha, \lambda^+_q \rangle = 0, \quad A = 1, \ldots, \frac{r}{2} \tag{5.13}
\]

and construct the states of the unitary representation of the M-algebra by acting on that by (products of) \( \mathfrak{B}^\dagger_B \) operators.

For the case of \( r = 16 \), which is of our main interest here, in such a way we arrive at the representation with \( 128(=1+28+70+28+1) \) bosonic states

\[
\mathfrak{B}^\dagger_A |v^{-q}_\alpha, \lambda^+_q \rangle \quad , \quad \mathfrak{B}^\dagger_A \mathfrak{B}^\dagger_B |v^{-q}_\alpha, \lambda^+_q \rangle \quad , \quad \mathfrak{B}^\dagger_A \ldots \mathfrak{B}^\dagger_A |v^{-q}_\alpha, \lambda^+_q \rangle \quad , \quad \mathfrak{B}^\dagger_A \ldots \mathfrak{B}^\dagger_A |v^{-q}_\alpha, \lambda^+_q \rangle \quad \tag{5.14}
\]

and \( 128(=8+56+56+8) \) fermionic states

\[
\mathfrak{B}^\dagger_A |v^{-q}_\alpha, \lambda^+_q \rangle \quad , \quad \mathfrak{B}^\dagger_A \mathfrak{B}^\dagger_B \mathfrak{B}^\dagger_C |v^{-q}_\alpha, \lambda^+_q \rangle \quad , \quad \mathfrak{B}^\dagger_A \ldots \mathfrak{B}^\dagger_A |v^{-q}_\alpha, \lambda^+_q \rangle \quad , \quad \mathfrak{B}^\dagger_A \ldots \mathfrak{B}^\dagger_A |v^{-q}_\alpha, \lambda^+_q \rangle \quad \tag{5.15}
\]

We claim that, when \( \lambda^+_q = \sqrt{2\rho^\# \delta_q^s}, \ s = 1, \ldots, 16 \), the states of the above described unitary highest weight representation of the M-algebra can be identified with degrees of freedom of the eleven-dimensional supergravity \[91\]. Then, in the case of generic \( \lambda^+_q \), the states (5.14), (5.15) can be associated to the fields of 11D supergravity multiplet depending, besides 11-vector coordinate or momenta, on a set of 135 additional variables. These latter can be described by

\[
\varphi_q^s = \frac{\lambda^+_q}{\sqrt{\lambda^+_{16} \lambda^+_{16}}} \tag{5.16}
\]

defined modulo \( O(16) \) transformations:

\[
\varphi_q^s \approx \varphi_q^t O_t^s, \quad O O^T = I_{16 \times 16}. \tag{5.17}
\]

**5.3 Unitary highest weight representations with \( r = 16 \) and 11D supergravity**

One of the way to see the above claimed relation of \( r = 16 \) unitary highest weight representation of the M-algebra with 11D supergravity starts form a seemingly different representation of the generators on the set of 128 bosonic states, 44 of which are enumerated by the symmetric traceless pair of SO(9) vector indices, \( A = IJ = ((IJ)) \), and remaining 84 - by the set of three antisymmetric SO(9) vector indices, \( A = IJK = [IJK] \), and
128 fermionic states enumerated by the gamma-traceless set of 9-vector and SO(9) spinor indices, $A = Is$,

$$128 = 84 + 44 : |IJK, v_{aq}^{-}, \lambda_q^{+s} >= |IJK|, v_{aq}^{-}, \lambda_q^{+s} >,$$

$$|IJ, v_{aq}^{-}, \lambda_q^{+s} > = |IJ|, v_{aq}^{-}, \lambda_q^{+s} >, |II, ... >\equiv 0 , \quad (5.18)$$

$$128 = 144 - 16 : |Is, v_{aq}^{-}, \lambda_q^{+s'} > , \quad \gamma_{st}^I It, ... >\equiv 0 , \quad (5.19)$$

where $\gamma_{st}^I$ are 9d Dirac matrices $(s, t = 1, ..., 16)$. This representation is described by Eqs. (3.46), (3.51) and (3.73), one can easily check that solve the linearized field equations of 11D SUGRA in the momentum representation, essentially the same representation had been obtained in the light-cone quantization [106]. The bosonic and fermionic fields corresponding to the basic moving frame formulation.

It is not difficult to check that the Rarita-Schwinger equation, \( \Gamma_{\alpha}^{\beta} \Psi_\beta = 0 \), follows from last three equations in (5.26).

The M-algebra representations with generic $\lambda_q^{+s}$ are characterized by nonvanishing eigenvalues of tensorial momentum generators $Z_{ab}$ and $Z_{abce}$. The basic states of such
a representation can be represented by on-shell fields of 11D supergravity depending on additional variables,

\[ A_{IJK}(v_{aq}, \rho^#, \varphi_q^s) \leftrightarrow |IJK, v_{aq}, \lambda_q^+ > , \quad h_{IJ}(v_{aq}, \rho^#, \varphi_q^s) \leftrightarrow |IJ, v_{aq}, \lambda_q^+ > , \]

\[ A_{IJK} = A_{[IJK]} , \quad h_{IJ} = h_{JI} , \quad h_{II} = 0 , \quad (5.28) \]

\[ \Psi_{Is}(v_{aq}, \rho^#, \varphi_q^s) \leftrightarrow |Is, v_{aq}, \lambda_q^+ > , \quad \gamma_{st}^I \Psi_{It} \equiv 0 , \quad (5.29) \]

where \( \rho^# = \lambda_q^+ \lambda_q^+ \) and \( \varphi_q^s \) is defined in (5.16), (5.17).

To show that the representation of M-algebra which is described by (5.8), (5.11), (5.20), (5.21) can be identified with the highest weight representation (5.14), (5.15), we have to introduce a complex structure which breaks the natural SO(9)(\( \subset SO(16) \)) symmetry of our construction down to Spin(7)(\( \subset SU(8) \)). This is achieved by introducing a complex null vector \( U_I \), obeying

\[ U_I U_J = 0 , \quad \bar{U}_I \bar{U}_J = 0 , \quad U_I U_J = 2 , \quad \bar{U}_I = (U_I)^* , \quad (5.30) \]

and the rectangular 16×8 complex conjugate matrices \( w_s^A \) and \( \bar{w}_s^A = (w_s^A)^* \) obeying

\[ \bar{w}_s^B w_s^A = \delta_B^A , \quad w_s^A w_s^B = 0 , \quad \bar{w}_s^A \bar{w}_s^B = 0 \quad (5.31) \]

and

\[ U_I \gamma_{st}^I = 2 \bar{w}_s^A \bar{w}_{tA} , \quad \bar{U}_I \gamma_{st}^I = 2 w_s^A w_t^A \quad (5.32) \]

involving 9d gamma matrices \( \gamma_{st}^I \).

Then the relation of the complex and hermitian generators of Clifford algebra is

\[ \mathfrak{B}_A = \bar{w}_s^A \mathcal{C}_s , \quad \mathfrak{B}_A^+ = w_s^A \mathcal{C}_s , \quad A = 1, ..., 8 , \quad s = 1, ..., 16 , \quad (5.33) \]

and the highest weight vector is defined by

\[ |v_{aq}, \lambda_q^+ > : = U_I U_J |IJ, v_{aq}, \lambda_q^+ > . \quad (5.34) \]

Indeed, using (5.20), (5.31) and (5.32) it is easy to check that this vector obeys (5.13) with \( \mathfrak{B}_A \) from (5.33).

This completes the proof of the equivalence of highest weight representation (5.14), (5.15) and the representation defined by (5.8), (5.20), (5.21), which contains 11D supergravity by construction [106, 54].

5.4 On moduli space of (complex structures defining) the highest weight representations

Some comments concerning new objects (5.31) and (5.30) defining the complex structures characterizing the above discussed highest weight representations might be useful. As a simplest possibility, one can think about fixed null-vector \( U_I = \delta_I^8 + i \delta_I^9 \) and chose \( w_{sA} \) to be an arbitrary factorization of complex nilpotent (rank 8) matrix \( \gamma^8 + i \gamma^9 \),

\[ (\gamma^8 + i \gamma^9)_{st} = 2 \bar{w}_{sA} \bar{w}_{tA} , \quad (\gamma^8 - i \gamma^9)_{st} = 2 w_s^A w_t^A , \quad (I - i \gamma^8 \gamma^9)_{st} = 2 \bar{w}_{sA} w_s^A . \]
A generic complex structure on 16-dimensional Clifford algebra can be defined by \((\bar{w}_s A, w_s A)\) formed from the columns of Spin(9) valued matrix \(w_s^{(t)}\). Then the light-like vector \(U_I\) is formed from the columns of the SO(9) valued matrix

\[
U_I^{(J)} = \left( U_I^J, U_I^{(8)}, U_I^{(9)} \right) = \left( U_I^J + \bar{U}_I, \frac{1}{2} \left( U_I - \bar{U}_I \right) \right) \in SO(9)
\]

(5.35)

related to \(w_s^{(t)} \in Spin(9)\) by \(U_I^{(J)} \gamma_{qp} = w_q^{(q')} \gamma^{(J)} (q) w_p^{(p')}\) (see [107] for details).

Thus, starting from one 11D supergravity-related representation (5.28), we can construct a family of unitary highest weight representations, elements of which are characterized by complex structures described by \((\bar{w}_s A, w_s A) \in Spin(9)\) defined modulo \(Spin(7) \times U(1)\) transformations, i.e. parametrizing the coset \(Spin(7) \times U(1)\). In this sense we can say that the moduli space of (complex structures defining) the highest weight representations constructed on the basis of one 11D supergravity-related representation is

\[
\mathcal{M}_{hw} = \frac{Spin(9)}{Spin(7) \times U(1)}.
\]

(5.36)

It is natural to expect that the above discussed unitary highest weight representations as well as the equivalent, explicitly 11D SUGRA-related representations (5.21), can be obtained as a result of quantization of generalized superparticle models described by (5.37) with (5.69) and \(r = 16\). Such a quantization and the analysis of quantum state spectra are still to be done and we hope to address this problem in a forthcoming publication.

6. Conclusions and discussion

In this paper we conjectured the existence of hypothetical underlying exceptional field theory, which we abbreviated as 11D EFT or uEFT, defined in the maximal tensorial central charge superspace

\[
\Sigma^{(528|32)} = \{ x^a, y^{ab}, y^{abcde}, \theta^\alpha \}, \quad a, b, c, d, e = 0, 1, \ldots, 9, 10, \quad \alpha = 1, \ldots, 32,
\]

(6.1)

which is the group manifold associated to the M-theory superalgebra (3.1). We have presented some arguments in favor of this conjecture, based on the hypothesis that the additional coordinates of all the \(E_{n(n)}\) EFTs with \(n \leq 8\) should be related to the maximally extended \(d = 11 - n\) dimensional supersymmetry algebra, and have proposed the candidate section conditions for this hypothetical uEFT

\[
\partial_{[a} \otimes \partial_{bc]} + \partial_{[bc} \otimes \partial_{a]} = 0, \quad \partial_{[a} \otimes \partial_{bcdf]} + \partial_{[bcdef} \otimes \partial_{a]} = 0,
\]

(6.2)

\[
\partial_{[ab} \otimes \partial_{bc]} = 0, \quad \partial_{[ab} \otimes \partial_{cdefg]} + \partial_{[cdefg} \otimes \partial_{ab]} = 0.
\]

(6.3)

To check that these section condition are reasonable, i.e. that their general solution is not trivial, we have discussed a series of superparticle models in \(\Sigma^{(528|32)}\) and show that they produce quite generic solutions of (the classical counterparts of) these hypothetical section
conditions as constraints on their generalized momenta. Of course, the next question was what is the physical meaning of these superparticle models. To address it we have presented some arguments that these superparticle models should produce free massless 11D higher spin field theories as their quantum state spectrum.

We have also discussed some unitary highest weight representations of M-theory superalgebra and the embedding of 11D supergravity in such representations. Besides giving an additional argument in favour of relevance of $\Sigma^{(528|32)}$ superspace, and of our uEFT hypothesis, this provides us with a hint about how the quantum state spectrum of some of the generalized superparticle models might look like. Namely, the representations preserving one half (16 of 32) supersymmetries, presumably related with some special class of $\Sigma^{(528|32)}$ superparticle models, can be equivalently described by on-shell fields of 11D supergravity depending on additional variables.

By passing, we have also argued that 10D ($m = 16$) and 6D ($m = 8$) counterparts of these superparticle models, defined in $\Sigma^{(m(m+1)/2|m)}$ superspaces with $m = 2(D-2)$, provides a classical mechanics description of free non-conformal massless higher spin theories in $D=10$ and $D=6$. These are of interest because the most interesting massless fields, like $D$-dimensional graviton and photon, are not conformal in $D \neq 4$.

The above observations suggest that the hypothetical underlying uEFT or 11D EFT should contain the 11D higher spin theory as an important sector. Interestingly enough, 11D Higher spin fields were recently considered as a probably necessary ingredients of completion of 11D supergravity till a $E_{10}$ or even $E_{11}$ invariant theories [104].

Even before, the relation of $E_{11}$ with higher spin theories was discussed in [15] where the action of $[76]$ supplemented by the condition of light-likeeness of the bilinear $\lambda \Gamma^a \lambda$, is proposed as a candidate for low level (three level) approximation of a hypothetical point particle model based on a non-linear realization of $E_{11} \ltimes l_1$. Here $l_1$ is the fundamental representation associated with the ‘far end’ of the $E_{11}$ Dynkin diagram, usually called ‘node 1’ (see e.g. [40]), the (infinite) set generators of which contains, at lowest level, $P_a, Z^{ab}, Z^{abcde}$, which can be identified with the bosonic generators of M-algebra (3.1).

Such a three level approximation to hypothetical $E_{11}$ superparticle can be identified with the ‘preonic’ representative (3.37) of the family of the action of uEFT superparticles (3.70), (3.66). It would be interesting to understand a possible role of other representatives of this family in an $E_{11}$ perspective.

Thus 11D higher spin theories are probably common ingredients of the uEFT and of the $E_{11}$ and $E_{10}$ theories. On the other hand, these are not identical, but rather complementary. As it is seen in Table 1, $E_{n(n)}$ EFTs, making manifest the the rigid $E_{n(n)}$ duality symmetry, keep only the $SO(1, 10-n)$ subgroup of the 11D $SO(1,10)$ Lorentz group manifest. A part of Lorentz symmetry is nailed for an increased of the manifest rigid symmetry. In this consequence $E_{10}$ and $E_{11}$ clearly correspond to $d = 1$ and $d = 0$ (!), thus not leaving any part of the $SO(1,10)$ Lorentz symmetry manifest, while our uEFT corre-

---

As $E_{n(n)}$ EFT is an extension of $d$ dimensional supergravity, the Lorentz $SO(1,d-1)$ symmetry is one of its manifest gauge symmetries while $E_{n(n)}$ is the rigid symmetry. Although our discussion here used the models in flat superspaces where the $SO(1,D-1)$ is not local, our terminology in this section is borrowed from the complete description of EFTs.
spond to $D=11$ and $n=0$, thus leaving all the Lorentz symmetry but no rigid symmetry manifest. Together with the Lorentz symmetry this hypothetical underlying theory should possess manifest supersymmetry the generators of which form the M-theory superalgebra, the maximally enlarged supersymmetry algebra in 11D. The uEFT is defined in the 11D superspace enlarged by coordinate conjugate to the tensorial central charge generators of this superalgebra, (6.1), which provides the resource for all the additional coordinates of the lower $d$/higher $n$ EFTs with $n < 9$.

As a potentially suggestive speculation, let us to try to incorporate the hypothetical $E_{10}$ and $E_{11}$ theories and uEFT in the Table 1. Clearly, the first two should be put on the top and the last - at the bottom of the Table 1, see Table 2 below.

If doing just this, we would have the holes at $n = 9$ ($d = 2$) and $n = 1$ ($d = 10$). The first of this positions should clearly correspond to some hypothetical $E_9$ EFT having manifest symmetry under the infinite dimensional Katz-Moody algebra $E_9$ [83]. The second, $n = 1$ hole we have filled in Table 2 by putting the doubled field theory, DFT, in this line. The main motivation is that this case clearly correspond to $d=10$, so that its association with $n = 1$ comes just from $n = 11 - d$. The increasing of the number of additional coordinates till 10, in contrast to their decreasing from 248 to 3 when $d$ increased from 3 to 9, might mark the change of tendency which is then continuing on the stage of passing to 11D uEFT, which is defined in (super)space with $517=528-11$ additional coordinates. Our hypothesis does not associates the T-duality group $O(10,10)$ with a hypothetical $E_{1(1)}$ as far as the 10d Lorentz group $SO(1,9)$ is a subgroup of $O(10,10)$. Rather $E_{1(1)}$ should be associated with the generators of the coset $O(10,10)/SO(1,9)$ so that, although a big number of duality symmetries is present, they do not form a closed algebraic structure themselves, but together with Lorentz group only.
| $E_n(n)$ of the EFT | n | $d=11-n$ | $N_n$=# of $y^\Sigma$ | Section condition |
|---------------------|---|---------|----------------|-----------------|
| $? E_{11}$          | 11| $d=0$  | $\infty$      | $? \text{ see recent [110]}$ |
| $? E_{10}$          | 10| $d=1$  | $\infty$      | $?$ see [108]   |
| $E_9$               | 9 | $d=2$  | $\infty$      |                  |
| $E_{8(8)}$          | 8 | $d=3$  | 248            | $Y_{\Lambda^\Sigma} \Sigma^\Pi \partial_{\Sigma} \otimes \partial_{\Pi} = 0$, [10] |
| $E_{7(7)}$          | 7 | $d=4$  | 56             | $t_\Sigma G^\Sigma \partial_{\Sigma} \otimes \partial_{\Pi} = 0$, [9] |
| $E_{6(6)}$          | 6 | $d=5$  | 27             | $\sigma^A \Sigma^\Pi \partial_{\Sigma} \otimes \partial_{\Pi} = 0$ [1] |
| $E_{5(5)}$          | 5 | $d=6$  | 16             | $\gamma^A \Sigma^\Pi \partial_{\Sigma} \otimes \partial_{\Pi} = 0$ [18] |
| $E_{4(4)} = SL(5)$  | 4 | $d=7$  | 10 ($y^{\widetilde{ab}} = y^{[\widetilde{a}\mid \widetilde{b}]})$ | $\partial_{[\widetilde{a}\mid \widetilde{b}]} \otimes \partial_{[\widetilde{a}\mid \widetilde{b}]} = 0$ [1], [7] |
| $E_{3(3)} = SL(3) \times SL(2)$ | 3 | $d=8$  | 6 ($y^{\alpha i}$) | $\epsilon^{ijk} e^{\alpha \beta} \partial_{\alpha i} \otimes \partial_{\beta j} = 0$ [14] |
| $E_{2(2)} = SL(2) \times \mathbb{R}^+$ | 2 | $d=9$  | 3 ($y^{\alpha, z}$) | $\partial_z \otimes \partial_{\alpha} + \partial_{\alpha} \otimes \partial_z = 0$ [18] |
| DFT: $SO(10, 10)$  | 1?| $d=10$ | 10 ($\tilde{x}_\mu$) | $\partial_{\mu} \otimes \tilde{\partial}^\mu + \tilde{\partial}^{\mu} \otimes \partial_{\mu} = 0$ [23], [24] |
| uEFT: only $SO(1, 10)$ | 0?| $d=11$ | 528 ($y^{ab}, y^{abde}$) | $\partial_{[a} \otimes \partial_{bc]} + \partial_{[bc} \otimes \partial_{a]} = 0$, $\partial_{[a} \otimes \partial_{bcde}] + \partial_{[bcde] \otimes \partial_{a]} = 0$, $\partial_{[ab} \otimes \partial_{cd]} = 0$, etc. (see (6.3)) |

Table 2. Known and hypothetical EFTs. Conjectured places of the underlying EFT and of the hypothetical EFTs for infinite dimensional groups in Table 1.

The above discussion suggests the origin of doubled spacetime coordinate of DFT in the uEFT superspace $\Sigma^{(528|32)}$, and furthermore, that our 11D uEFT provides a unification of DFT with $n=2, ..., 8$ $E_n(n)$ exceptional field theories.

We hope that our underlying 11D exceptional field theory (11D uEFT) conjecture will be useful for deeper understanding of dualities and of the structure of/beyond the M-theory. It will be interesting to elaborate further on its interrelation/complementary with the $E_{11}$ and $E_{10}$ proposal. One of the important directions of further development of our approach is related to the quantization of the family of the generalized superparticle models [3.71]. In particular this should provide us with a description of towers of massless non-conformal higher spin theories in $D=11$, the possible fundamental role of which was also a subject of thinking in the Higher Spin community [10].

Acknowledgements

This work has been supported in part by the Spanish Ministry of Economy, Industry and Competitiveness grants FPA 2012-35043-C02-01 and FPA 2015-66793-P, partially financed with FEDER funds, by the Basque Government research group grant IT-979-10, and by the Basque Country University program UFI 11/55. The author is thankful to Dima Sorokin, Mikhail Vasiliev, to David Berman, Martin Cederwall, Jakob Palmkvist, Jeong-Hyuck Park, Henning Samtleben for useful discussions, to the Munich Institute for Astro-
and Particle Physics (MIAPP) of the DFG cluster of excellence "Origin and Structure of the Universe" (Munich, Germany), to the Galileo Galilei Institute for Theoretical Physics and the INFN (Florence, Italy) and to BIRS (Banff International Research Station for Mathematical Innovation and Discovery, Banff, Alberta, Canada) for the hospitality and partial support of his visits at certain stages of this work.

**Notice added.**

After this paper have been completed, the $E_{11}$ EFT was constructed in [110]. It is based (and develops) the theory non-linear realization of $E_{11} \otimes \ell_1$ [13], [18], [22], in which the fields depend on infinitely many coordinates (of $\ell_1$), but also presents a set of section conditions, which results in the dependence of fields on the finite number of coordinates. (This has allowed us to replace $\ell \mapsto \ell'$ in the first line of Table 2). An even more extended scheme based on infinite–dimensional tensor hierarchy superalgebra is also proposed in [111].

**References**

1. O. Hohm and H. Samtleben, “Exceptional Form of D=11 Supergravity,” Phys. Rev. Lett. 111 (2013) 231601 [arXiv:1308.1673 [hep-th]].
2. C. M. Hull and P. K. Townsend, “Unity of superstring dualities,” Nucl. Phys. B 438 (1995) 109 [hep-th/9410167].
3. C. M. Hull, “Generalised Geometry for M-Theory,” JHEP 0707 (2007) 079 [hep-th/0701203].
4. D. S. Berman, H. Godazgar, M. Godazgar and M. J. Perry, “The Local symmetries of M-theory and their formulation in generalised geometry,” JHEP 1201 (2012) 012 doi:10.1007/JHEP01(2012)012 [arXiv:1110.3930 [hep-th]].
5. D. S. Berman, M. Cederwall, A. Kleinschmidt and D. C. Thompson, “The gauge structure of generalised diffeomorphisms,” JHEP 1301 (2013) 064 [arXiv:1208.5884 [hep-th]].
6. A. Coimbra, C. Strickland-Constable and D. Waldram, “Supergravity as Generalised Geometry II: $E_{d(d)} \times \mathbb{R}^+$ and M theory,” JHEP 1403 (2014) 019 [arXiv:1212.1586 [hep-th], arXiv:1212.1586].
7. C. D. A. Blair, E. Malek and J. H. Park, “M-theory and Type IIB from a Duality Manifest Action,” JHEP 1401 (2014) 172 doi:10.1007/JHEP01(2014)172 [arXiv:1311.5109 [hep-th]].
8. O. Hohm and H. Samtleben, “Exceptional Field Theory I: $E_6(6)$ covariant Form of M-Theory and Type IIB,” Phys. Rev. D 89 (2014) 066016 [arXiv:1312.0614 [hep-th]].
9. O. Hohm and H. Samtleben, “Exceptional field theory. II. $E_7(7)$,” Phys. Rev. D 89 (2014) 066017 doi:10.1103/PhysRevD.89.066017 [arXiv:1312.4542 [hep-th]].
10. O. Hohm and H. Samtleben, “Exceptional field theory. III. $E_8(8)$,” Phys. Rev. D 90 (2014) 066002 [arXiv:1406.3348 [hep-th]].
11. H. Godazgar, M. Godazgar, O. Hohm, H. Nicolai and H. Samtleben, “Supersymmetric $E_{7(7)}$ Exceptional Field Theory,” JHEP 1409 (2014) 044 doi:10.1007/JHEP09(2014)044 [arXiv:1406.3235 [hep-th]].
[12] E. Musaev and H. Samtleben, “Fermions and supersymmetry in E_{6(6)} exceptional field theory,” JHEP 1503 (2015) 027 [arXiv:1412.7286 [hep-th]].

[13] F. Ciceri, B. de Wit and O. Varela, “IIB supergravity and the E_{6(6)} covariant vector-tensor hierarchy,” JHEP 1504 (2015) 094 doi:10.1007/JHEP04(2015)094 [arXiv:1412.8297 [hep-th]].

[14] O. Hohm and Y. N. Wang, “Tensor hierarchy and generalized Cartan calculus in SL(3) SL(2) exceptional field theory,” JHEP 1504 (2015) 050 doi:10.1007/JHEP04(2015)050 [arXiv:1501.01600 [hep-th]].

[15] A. Abzalov, I. Bakhmatov and E. T. Musaev, “Exceptional field theory: SO(5,5),” JHEP 1506 (2015) 088 [arXiv:1504.01523 [hep-th]].

[16] M. Cederwall and J. A. Rosabal, “Es geometry,” JHEP 1507 (2015) 007 doi:10.1007/JHEP07(2015)007 [arXiv:1504.04843 [hep-th]].

[17] E. T. Musaev, “Exceptional field theory: SL(5),” JHEP 1602 (2016) 012 doi:10.1007/JHEP02(2016)012 [arXiv:1512.02163 [hep-th]].

[18] D. S. Berman, C. D. A. Blair, E. Malek and F. J. Rudolph, “An action for F-theory: SL(2)R+ exceptional field theory,” Class. Quant. Grav. 33 (2016) no.19, 195009 doi:10.1088/0264-9381/33/19/195009 [arXiv:1512.06115 [hep-th]].

[19] M. Cederwall, “Double supergeometry,” JHEP 1606 (2016) 155 doi:10.1007/JHEP06(2016)155 [arXiv:1603.04684 [hep-th]].

[20] A. Baguet and H. Samtleben, “E_{8(8)} Exceptional Field Theory: Geometry, Fermions and Supersymmetry,” JHEP 1609 (2016) 168 doi:10.1007/JHEP09(2016)168 [arXiv:1607.03119 [hep-th]].

[21] F. Ciceri, A. Guarino and G. Inverso, “The exceptional story of massive IIA supergravity,” JHEP 1608 (2016) 154 doi:10.1007/JHEP08(2016)154 [arXiv:1604.08602 [hep-th]].

[22] D. Cassani, O. de Felice, M. Petrini, C. Strickland-Constable and D. Waldram, “Exceptional generalised geometry for massive IIA and consistent reductions,” JHEP 1608 (2016) 074 doi:10.1007/JHEP08(2016)074 [arXiv:1605.00563 [hep-th]].

[23] W. Siegel, “Two vierbein formalism for string inspired axionic gravity,” Phys. Rev. D 47 (1993) 5453 [hep-th/9302036].

[24] C. M. Hull, “A Geometry for non-geometric string backgrounds,” JHEP 0510 (2005) 065 [hep-th/0406102].

[25] C. Hull and B. Zwiebach, “Double Field Theory,” JHEP 0909 (2009) 099 [arXiv:0904.4664 [hep-th]].

[26] A. Coimbra, C. Strickland-Constable and D. Waldram, “Supergravity as Generalised Geometry I: Type II Theories,” JHEP 1111 (2011) 091 [arXiv:1107.1733 [hep-th]]; “Generalised Geometry and type II Supergravity,” Fortsch. Phys. 60 (2012) 982 [arXiv:1202.3170 [hep-th]].

[27] I. Jeon, K. Lee and J. H. Park, “Incorporation of fermions into double field theory,” JHEP 1111 (2011) 025 [arXiv:1109.2035 [hep-th]]; “Supersymmetric Double Field Theory: Stringy Reformulation of Supergravity,” Phys. Rev. D 85 (2012) 081501 [Phys. Rev. D 86 (2012) 089903] [arXiv:1112.0069 [hep-th]].
[28] J. H. Park, “Comments on double field theory and diffeomorphisms,” JHEP 1306 (2013) 098 doi:10.1007/JHEP06(2013)098 [arXiv:1304.5946 [hep-th]]; K. Lee and J. H. Park, “Covariant action for a string in ”doubled yet gauged” spacetime,” Nucl. Phys. B 880 (2014) 134 doi:10.1016/j.nuclphysb.2014.01.003 [arXiv:1307.8377 [hep-th]].

[29] O. Hohm, D. Lüst and B. Zwiebach, “The Spacetime of Double Field Theory: Review, Remarks, and Outlook,” Fortsch. Phys. 61 (2013) 926 [arXiv:1309.2977 [hep-th]].

[30] M. Hatsuda, K. Kamimura and W. Siegel, “Superspace with manifest T-duality from type II superstring,” JHEP 1406 (2014) 039 [arXiv:1403.3887 [hep-th]]; “Ramond-Ramond gauge fields in superspace with manifest T-duality,” JHEP 1502 (2015) 134 [arXiv:1411.2206 [hep-th]].

[31] M. Hatsuda, K. Kamimura and W. Siegel, “Type II chiral affine Lie algebras and string actions in doubled space,” JHEP 1509 (2015) 113 [arXiv:1507.03061 [hep-th]].

[32] M. B. Green, J. H. Schwarz and E. Witten, “Superstring Theory. Vol. 1: Introduction,”, 484pp., CUP 1988

[33] A. A. Tseytlin, “Duality Symmetric Formulation of String World Sheet Dynamics,” Phys. Lett. B 242 (1990) 163; “Duality symmetric closed string theory and interacting chiral scalars,” Nucl. Phys. B 350 (1991) 395.

[34] D. S. Berman, N. B. Copland and D. C. Thompson, “Background Field Equations for the Duality Symmetric String,” Nucl. Phys. B 791 (2008) 175 [arXiv:0708.2267 [hep-th]].

[35] N. B. Copland, “A Double Sigma Model for Double Field Theory,” JHEP 1204 (2012) 044 doi:10.1007/JHEP04(2012)044 [arXiv:1111.1828 [hep-th]].

[36] C. D. A. Blair, E. Malek and A. J. Routh, “An $O(D, D)$ invariant Hamiltonian action for the superstring,” Class. Quant. Grav. 31 (2014) no.20, 205011 doi:10.1088/0264-9381/31/20/205011 [arXiv:1308.4829 [hep-th]].

[37] L. De Angelis, G. Gionti S.J., R. Marotta and F. Pezzella, “Comparing Double String Theory Actions,” JHEP 1404 (2014) 171 [arXiv:1312.7367 [hep-th]]; F. Pezzella, “Some Aspects of the T-Duality Symmetric String Sigma Model,” arXiv:1512.08825 [hep-th]; C. T. Ma and F. Pezzella, “Supergravity with Doubled Spacetime Structure,” arXiv:1611.03690 [hep-th].

[38] I. Bandos, “Superstring in doubled superspace,” Phys. Lett. B 751 (2015) 408 doi:10.1016/j.physletb.2015.10.081 [arXiv:1507.07779 [hep-th]]; “Type II superstring in doubled superspace,” Fortsch. Phys. 64 (2016) 361. doi:10.1002/prop.201500055

[39] C. D. A. Blair, “Doubled strings, negative strings and null waves,” JHEP 1611 (2016) 042 doi:10.1007/JHEP11(2016)042 [arXiv:1608.06818 [hep-th]].

[40] J. H. Park, “Green-Schwarz superstring on doubled-yet-gauged spacetime,” JHEP 1611 (2016) 005 doi:10.1007/JHEP11(2016)005 [arXiv:1609.04265 [hep-th]].

[41] D. S. Berman and M. J. Perry, JHEP 1106 (2011) 074 doi:10.1007/JHEP06(2011)074 [arXiv:1008.1763 [hep-th]].

[42] M. J. Duff and J. X. Lu, “Duality Rotations in Membrane Theory,” Nucl. Phys. B 347 (1990) 394. doi:10.1016/0550-3213(90)90565-U

[43] P. C. West, “E(11) and M theory,” Class. Quant. Grav. 18 (2001) 4443 doi:10.1088/0264-9381/18/21/305 [hep-th/0104081].
[44] P. C. West, “E(11), SL(32) and central charges,” Phys. Lett. B 575 (2003) 333 [hep-th/0307098].

[45] P. C. West, “E(11) and higher spin theories,” Phys. Lett. B 650 (2007) 197 doi:10.1016/j.physletb.2007.03.061 [hep-th/0701026].

[46] P. West, “Generalised space-time and duality,” Phys. Lett. B 693 (2010) 373 doi:10.1016/j.physletb.2010.08.054 [arXiv:1006.0893 [hep-th]].

[47] M. Henneaux, B. L. Julia and J. Levine, “E_{11}, Borcherds algebras and maximal supergravity,” JHEP 1204 (2012) 078 doi:10.1007/JHEP04(2012)078 [arXiv:1007.5241 [hep-th]].

[48] P. West, “Generalised Space-time and Gauge Transformations,” JHEP 1408 (2014) 050 doi:10.1007/JHEP08(2014)050 [arXiv:1403.6395 [hep-th]].

[49] A. G. Tumanov and P. West, “E11 and exceptional field theory,” Int. J. Mod. Phys. A 31 (2016) no.12, 1650066 doi:10.1142/S0217751X16500664 [arXiv:1507.08912 [hep-th]].

[50] A. G. Tumanov and P. West, “E_{11} must be a symmetry of strings and branes,” Phys. Lett. B 759 (2016) 663 doi:10.1016/j.physletb.2016.06.011 [arXiv:1512.01644 [hep-th]].

[51] A. G. Tumanov and P. West, “E11 in 11D,” Phys. Lett. B 758 (2016) 278 doi:10.1016/j.physletb.2016.04.058 [arXiv:1601.03974 [hep-th]].

[52] A. G. Tumanov and P. West, “E11, Romans theory and higher level duality relations,” arXiv:1611.03369 [hep-th].

[53] I. A. Bandos, “Spinor moving frame, M0-brane covariant BRST quantization and intrinsic complexity of the pure spinor approach,” Phys. Lett. B 659 (2008) 388 doi:10.1016/j.physletb.2007.10.048 [arXiv:0707.2336 [hep-th]].

[54] I. A. Bandos, “D=11 massless superparticle covariant quantization, pure spinor BRST charge and hidden symmetries,” Nucl. Phys. B 796 (2008) 360 doi:10.1016/j.nuclphysb.2007.12.019 [arXiv:0710.4342 [hep-th]].

[55] I. A. Bandos and A. A. Zheltukhin, “Green-Schwarz superstrings in spinor moving frame formalism,” Phys. Lett. B 288 (1992) 77. doi:10.1016/0370-2693(92)91957-B

[56] I. A. Bandos and A. A. Zheltukhin, “Twistor-like approach in the Green-Schwarz D=10 superstring theory,” Phys. Part. Nucl. 25 (1994) 453 [Preprint IC-92-422, ICTP, Dec. 1992]

[57] I. A. Bandos and A. A. Zheltukhin, “Eleven-dimensional supermembrane in a spinor moving repere formalism,” Int. J. Mod. Phys. A 8 (1993) 1081. doi:10.1142/S0217751X93000424

[58] I. A. Bandos, “On Pure Spinor Formalism for Quantum Superstring and Spinor Moving Frame,” Class. Quant. Grav. 30 (2013) 235011 doi:10.1088/0264-9381/30/23/235011 [arXiv:1207.7300 [hep-th]].

[59] A. S. Galperin, P. S. Howe and P. K. Townsend, “Twistor transform for superfields,” Nucl. Phys. B 402 (1993) 531. doi:10.1016/0550-3213(93)90651-5

[60] E. Sokatchev, “Light Cone Harmonic Superspace and Its Applications,” Phys. Lett. B 169 (1986) 209. doi:10.1016/0370-2693(86)90652-0

[61] E. Sokatchev, “Harmonic Superparticle,” Class. Quant. Grav. 4 (1987) 237. doi:10.1088/0264-9381/4/2/007

[62] I. A. Bandos, “Superparticle in Lorentz harmonic superspace,” Sov. J. Nucl. Phys. 51 (1990) 906 [Yad. Fiz. 51 (1990) 1429 (In Russian)].
[63] A. S. Galperin, P. S. Howe and K. S. Stelle, “The Superparticle and the Lorentz group,” Nucl. Phys. B 368 (1992) 248 doi:10.1016/0550-3213(92)90527-I [hep-th/9201020].

[64] F. Delduc, A. Galperin and E. Sokatchev, “Lorentz harmonic (super)fields and (super)particles,” Nucl. Phys. B 368 (1992) 143. doi:10.1016/0550-3213(92)90201-L

[65] I. A. Bandos and A. A. Zheltukhin, “N=1 superp-branes in twistor-like Lorentz harmonic formulation,” Class. Quant. Grav. 12 (1995) 609 doi:10.1088/0264-9381/12/3/002 [hep-th/9405113].

[66] D. V. Uvarov, “On covariant kappa symmetry fixing and the relation between the NSR string and the type II GS superstring,” Phys. Lett. B 493 (2000) 421 doi:10.1016/S0370-2693(00)01151-5 [hep-th/0006185].

[67] I. Bandos, “On section conditions of E7(+7) exceptional field theory and superparticle in \( \mathcal{N} = 8 \) central charge superspace,” JHEP 1601 (2016) 132 doi:10.1007/JHEP01(2016)132 [arXiv:1512.02287 [hep-th]].

[68] P. S. Howe and U. Lindstrom, “Higher Order Invariants in Extended Supergravity,” Nucl. Phys. B 181 (1981) 487. doi:10.1016/0550-3213(81)90537-X

[69] P. Howe and J. Palmkvist, “Forms and algebras in (half-)maximal supergravity theories,” JHEP 1505 (2015) 032 doi:10.1007/JHEP05(2015)032 [arXiv:1503.00015 [hep-th]].

[70] P. K. Townsend, “P-brane democracy,” In ”The world in eleven dimensions” (Ed. *Duff, M.J.), pp. 375-389 [hep-th/9507048].

[71] D. P. Sorokin and P. K. Townsend, “M Theory superalgebra from the M five-brane,” Phys. Lett. B 412 (1997) 265 doi:10.1016/S0370-2693(97)01075-7 [hep-th/9708003].

[72] R. D’Auria and P. Fre, “Geometric Supergravity in \( d = 11 \) and Its Hidden Supergroup,” Nucl. Phys. B 201 (1982) 101 [Nucl. Phys. B 206 (1982) 496]. doi:10.1016/0550-3213(82)90376-5

[73] J. W. van Holten and A. Van Proeyen, “N=1 Supersymmetry Algebras in D=2, D=3, D=4 MOD-8,” J. Phys. A 15 (1982) 3763. doi:10.1088/0305-4470/15/12/028

[74] T. Curtright, “Are There Any Superstrings in Eleven-dimensions?,” Phys. Rev. Lett. 60 (1988) 393 [Phys. Rev. Lett. 60 (1988) 1990].

[75] M. Gunaydin, “Generalized conformal and superconformal group actions and Jordan algebras,” Mod. Phys. Lett. A 8 (1993) 1407 doi:10.1142/S0217732393001124 [hep-th/9301050].

[76] I. A. Bandos and J. Lukierski, “ Tensorial central charges and new superparticle models with fundamental spinor coordinates,” Mod. Phys. Lett. A 14 (1999) 1257 [hep-th/9811022].

[77] I. A. Bandos and J. Lukierski, “New superparticle models outside the HLS supersymmetry scheme,” Lect. Notes Phys. 539 (2000) 195 [hep-th/9812074].

[78] I. A. Bandos, J. A. de Azcarraga, J. M. Izquierdo and J. Lukierski, “BPS states in M theory and twistorial constituents,” Phys. Rev. Lett. 86 (2001) 4451 doi:10.1103/PhysRevLett.86.4451 [hep-th/0101113].

[79] I. A. Bandos, J. A. de Azcarraga, M. Picon and O. Varela, “Supersymmetric string model with 30 kappa symmetries and extended superspace and 30—32 BPS states,” Phys. Rev. D 69 (2004) 085007 doi:10.1103/PhysRevD.69.085007 [hep-th/0307106].
[80] I. A. Bandos, J. A. de Azcarraga, J. M. Izquierdo, M. Picon and O. Varela, “On the underlying gauge group structure of D=11 supergravity,” Phys. Lett. B 596 (2004) 145 doi:10.1016/j.physletb.2004.06.079 [hep-th/0406020].

[81] I. A. Bandos, J. A. de Azcarraga, M. Picon and O. Varela, “On the formulation of D = 11 supergravity and the composite nature of its three-form gauge field,” Annals Phys. 317 (2005) 238 doi:10.1016/j.aop.2004.11.016 [hep-th/0409100].

[82] I. A. Bandos, “BPS preons in supergravity and higher spin theories. An Overview from the hill of twistor approach,” AIP Conf. Proc. 767 (2005) 141 [hep-th/0501115], and refs therein.

[83] B. Julia, “Kac-moody Symmetry Of Gravitation And Supergravity Theories,” LPTENS-82-22, C82-07-06, Talk at the AMS-SIAM Summer Seminar on applications of Group Theory in Physics and Mathematical Physics, University of Chicago, July 1982. Published in: in ”Lectures in Applied Mathematics, AMS-SIAM, vol 21 (1985), p.355.

[84] R. W. Gebert and H. Nicolai, “On E(10) and the DDF construction,” Commun. Math. Phys. 172 (1995) 571 doi:10.1007/BF02101809 [hep-th/9406175].

[85] T. Damour and M. Henneaux, “E(10), BE(10) and arithmetical chaos in superstring cosmology,” Phys. Rev. Lett. 86 (2001) 4749 doi:10.1103/PhysRevLett.86.4749 [hep-th/0012172].

[86] T. Damour, M. Henneaux and H. Nicolai, “E(10) and a ‘small tension expansion’ of M theory,” Phys. Rev. Lett. 89 (2002) 221601 doi:10.1103/PhysRevLett.89.221601 [hep-th/0207267].

[87] M. Henneaux, E. Jamsin, A. Kleinschmidt and D. Persson, “On the E10/Massive Type IIA Supergravity Correspondence,” Phys. Rev. D 79 (2009) 045008 doi:10.1103/PhysRevD.79.045008 [arXiv:0811.4358 [hep-th]].

[88] M. Cederwall, “Twistors and supertwistors for exceptional field theory,” JHEP 1512 (2015) 123 doi:10.1007/JHEP12(2015)123 [arXiv:1510.02298 [hep-th]].

[89] J. A. de Azcarraga and J. Lukierski, “Supersymmetric Particle Model With Additional Bosonic Coordinates,” Z. Phys. C 30 (1986) 221-227.

[90] E. Sezgin, “The M algebra,” Phys. Lett. B 392 (1997) 323 doi:10.1016/S0370-2693(96)01576-6 [hep-th/9609086].

[91] E. Cremmer, B. Julia and J. Scherk, “Supergravity Theory in Eleven-Dimensions,” Phys. Lett. B 76 (1978) 409.

[92] I. A. Bandos, J. Lukierski and D. P. Sorokin, “Superparticle models with tensorial central charges,” Phys. Rev. D 61 (2000) 045002 doi:10.1103/PhysRevD.61.045002 [hep-th/9904109].

[93] M. A. Vasiliev, “Relativity, causality, locality, quantization and duality in the S(p)(2M) invariant generalized space-time,” In *Olshanetsky, M. (ed.) et al.: Multiple facets of quantization and supersymmetry* 826-872 [hep-th/0111119].

[94] M. A. Vasiliev, “Conformal higher spin symmetries of 4-d massless supermultiplets and osp(L,2M) invariant equations in generalized (super)space,” Phys. Rev. D 66 (2002) 066006 doi:10.1103/PhysRevD.66.066006 [hep-th/0106149].

[95] M. Plyushchay, D. Sorokin and M. Tsulaia, “Higher spins from tensorial charges and OSp(N|2n) symmetry,” JHEP 0304 (2003) 013 doi:10.1088/1126-6708/2003/04/013 [hep-th/0301067].
[96] O. A. Gelfond and M. A. Vasiliev, “Higher rank conformal fields in the Sp(2M) symmetric generalized space-time,” Theor. Math. Phys. 145 (2005) 1400 [Teor. Mat. Fiz. 145 (2005) 35] doi:10.1007/s11232-005-0168-9 [hep-th/0304020].

[97] I. Bandos, P. Pasti, D. Sorokin and M. Tonin, “Superfield theories in tensorial superspaces and the dynamics of higher spin fields,” JHEP 0411 (2004) 023 doi:10.1088/1126-6708/2004/11/023 [hep-th/0407180].

[98] I. Bandos, X. Bekaert, J. A. de Azcarraga, D. Sorokin and M. Tsulaia, “Dynamics of higher spin fields and tensorial space,” JHEP 0505 (2005) 031 doi:10.1088/1126-6708/2005/05/031 [hep-th/0501113].

[99] I. Florakis, D. Sorokin and M. Tsulaia, “Higher Spins in Hyper-Superspace,” Nucl. Phys. B 890 (2014) 279 doi:10.1016/j.nuclphysb.2014.11.017 [arXiv:1408.6675 [hep-th]].

[100] O. A. Gelfond and M. A. Vasiliev, “Symmetries of higher-spin current interactions in four dimensions,” arXiv:1510.03488 [hep-th].

[101] E. Skvortsov, D. Sorokin and M. Tsulaia, “Correlation Functions of Sp(2n) Invariant Higher-Spin Systems,” arXiv:1605.08498 [hep-th].

[102] Y. O. Goncharov and M. A. Vasiliev, “Higher-spin fields and charges in the periodic spinor space,” arXiv:1611.09102 [hep-th].

[103] C. Fronsdal, “Massless Particles, Orthosymplectic Symmetry And Another Type Of Kaluza-klein Theory,” In: ”Essays On Supersymmetry” (Fronsdal, C., Ed.), pp. 163-265 [Preprint Calif. Univ. Los Angeles - UCLA-85-TEP-10 (85, REC. Jun.) 111 pp.].

[104] M. Henneaux, “Twisted Self-Duality and SO(2) Electric-Magnetic Duality for Gravity and Higher Spin Gauge Fields”, talk at MIAPP workshop ”Aspects of Higher Spin Theory.”, Munich, May 25.

[105] M. Gunaydin, “Unitary supermultiplets of OSp(1/32,R) and M theory,” Nucl. Phys. B 528 (1998) 432 doi:10.1016/S0550-3213(98)00393-9 [hep-th/9803138].

[106] M. B. Green, M. Gutperle and H. H. Kwon, “Light cone quantum mechanics of the eleven-dimensional superparticle,” JHEP 9908 (1999) 012 doi:10.1088/1126-6708/1999/08/012 [hep-th/9907155].

[107] I. Bandos, “An analytic superfield formalism for tree superamplitudes in D=10 and D=11,” arXiv:1705.09550 [hep-th].

[108] M. Cederwall and J. Palmquist, work in progress. Talk by M. Cederwall at BIRS Workshop 17w5018 ”String and M-theory geometries: Double Field Theory, Exceptional Field Theory and their Applications”, BIRS, Banff, Canada, January 2017.

[109] M. V. Vasiliev, Private communication, MIAPP, Munich, May 2026.

[110] G. Bossard, A. Kleinschmidt, J. Palmkvist, C. N. Pope and E. Sezgin, “Beyond E11,” arXiv:1703.01305 [hep-th].