THE DISTRIBUTION OF DARK MATTER IN SPIRALS

Paolo Salucci

Abstract. In the past years a wealth of observations allowed to unravel the structural properties of the Dark Matter Halos around spirals. First, their rotation curves follow an Universal profile (URC) that can be described in terms of an exponential thin stellar disk and a dark halo with a constant density core, whose relative importance increases with galaxy luminosity. Careful studies of individual objects, from dwarfs to giants, reveal that dark halos have a core, whose size $r_0$ correlates with the central density $\rho_0$. These properties are in serious discrepancy with the cuspy density distribution predicted by N-body simulations in collisionless $\Lambda$CDM Cosmology.

1 Introduction

Rotation curves (RC’s) of disk galaxies are the best probe for Dark Matter (DM) on galactic scale since its discovery. However, only recently we discovered some crucial aspects of their mass distribution by a large number of high-quality RC’s and by improvements in the techniques of the RC mass-modeling. The DM distribution is usually assumed according to one of following different approaches.

An empirically one (Persic, Salucci & Stel, 1996, PSS) adopts the simplest halo velocity profile that (in combination with the stellar disk) reproduces the Universal Rotation Curve of Spirals (out to 3-4 $R_D$, $R_{\text{opt}} \equiv 3.2$ disk scale-lengths $R_D$):

$$V_{R,\text{URC}}^2(x) = V_{\text{opt}}^2 (1 - \beta) (1 + a^2) \frac{x^2}{(x^2 + a^2)}$$ (1.1)

where $x \equiv R/R_{\text{opt}}$, $a$ is the halo velocity core radius, in units of $R_{\text{opt}}$ and $\beta \equiv (V_d/V_{\text{opt}})^2$ is the fractional contribution to the circular velocity of the stellar disk at $R_{\text{opt}}$. This URC-based profile has the advantage of simplicity and, with suitable choices for $\beta$ and $a$, it can represent a variety of mass models, including the NFW one. Of course, it cannot be extrapolated beyond the region traced by the kinematics.

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The cosmological approach relies on high–resolution N–body simulations, according to which Cold Dark Matter (CDM) halos achieve the equilibrium density profile (Navarro, Frenk % White, 1996, NFW): 

$$\rho_{\text{NFW}}(r) = \rho_s \left(\frac{r}{r_s}\right) \left(1 + \frac{r}{r_s}\right)^{-2}$$

where $r_s$ and $\rho_s$ are the characteristic inner radius and density, usually expressed in terms of virial mass and radius $r_{\text{vir}}$, $M_{\text{vir}}$. By setting $c \equiv r_{\text{vir}}/r_s$, since from simulations: $c \simeq 21(M_{\text{vir}}/(10^{11}M_\odot))^{-0.13}$ (Bullock et al. 2001), we have: $V_{\text{NFW}} = V_{\text{NFW}}(R, M_{\text{vir}}, c(M_{\text{vir}}))$, and in detail:

$$V_{\text{NFW}}^2(R) = V_{\text{vir}}^2 \frac{c \cdot A(x)}{x}$$

where $x \equiv R/r_s$ and $A(x) \equiv \ln(1 + x) - x/(1 + x)$

The third approach adopts the Burkert profile in order to account for the observational evidence at inner radii and to converge to the NFW profile at outer
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Fig. 2. Central halo density $\rho_0$ (in g/cm$^3$) vs. disk mass (in solar units); bottom) vs. core radii (in kpc). Details in Salucci & Burkert, 2001

radii (Salucci & Burkert, 2000): $\rho_B(R) = \rho_0 R^3 / (R + r_0) (R + r_c^2)$ with $\rho_0$ and $r_0$ being the central DM density and the scale radius. Then ($M_0 = 4.16 \rho_0 r_0^3$):

$$V_B^2(R) = G M_0 R \left[ \ln(1 + R/r_0) - \arctan(R/r_0) + 0.5 \ln[1 + (R/r_0)^2] \right]$$  \hspace{1cm} (1.3)

It is important to stress the following points: a) the mass in spirals is distributed according to the Inner Baryon Dominance (IBD) regime (PSS, Salucci and Persic, 1999): there exists a "transition radius" $R_{IBD} \approx 2 R_d M_d^{1/4}$, with $M_d$ the disk mass in units of $10^{11} M_\odot$. For $R \leq R_{IBD}$, (Salucci et al. 2000), the luminous matter, in an exponential thin disk of length-scale $R_d$, accounts for the whole gravitating mass. For $R > R_{IBD}$, instead, an additional dark component, distributed unlike the stellar disk, emerges and it rapidly becomes dominant. b) the HI and the bulge contribution to $V(R)$ do not play a relevant role in the subject of this paper.

2 Dark Halos Properties from the Universal Rotation Curve

PSS have derived, from $\sim 20000$ velocity measurements, relative to $\sim 900$ rotation curves, $V_{syn}(R_{opt}; M_I)$, the synthetic circular velocity of spirals, binned in intervals of I-magnitudes (see Fig. 1). At a fixed luminosity and normalized radius, the spiral RC’s show a cosmic variance, with respect to $V_{syn}$, that is much smaller than their radial variations and than their luminosity dependence. As result,
spirals sweep a very narrow locus in the RC-profile/amplitude/luminosity space. The whole set of synthetic RC’s define the Universal Rotation Curve (URC), that we analytically represent as the sum of two terms: a) the exponential thin disk:

\[ V_d^2(x) = 1.28 \beta V_{opt}^2 x^2 (I_0 K_0 - I_1 K_1)|_{1.6x} \] (2.1)

and b) the spherical halo given by eq (1.1). Then: \( V_{URC}^2(x) = V_{h,URC}^2(x, \beta, a) + V_{d,URC}^2(x, \beta) \), with \( a \) and \( \beta \) as free parameters, reproduces the synthetic curves \( V_{syn}(R) \) up to their rms (i.e. within 2%) when \( \beta = \beta(\log V_{opt}) \) and \( a = a(\beta) \) as given in PSS and in Salucci and Burkert, 2000. Notice that also (bulge-free) individual high-quality RC are generally well fit by eq. (1.1). Inside \( R_{opt} \) smaller objects have larger dark-to-stellar mass ratio: \( M_*/(M_*/2 \times 10^{11} M_\odot)^{0.75} \). Scaling relationships relate the halo and disk mass parameters implying that the densest halos harbor the least massive disks and the most inefficient ones to transform the original HI content in present day stars. Notice that the velocity core radius \( a \) suggests, but does not prove, a flat core in the DM density distribution, that can be unambiguously revealed only by proper individual RC’s.

3 The DM halo density

The mass structure of spirals is well probed in objects with both HI and H\( \alpha \) high-quality high-resolution RC. The existence of the URC and the Universal properties of cosmological halos allows us to concentrate only in a reasonable number of test cases. The first "absolutely safe" determination of DM halos density profiles in Spirals was obtained in Gentile et al 2004, where 5 spirals with HI and H\( \alpha \) RC’s were studied by means of "state of the art" observational, data analysis and modeling techniques. In each object HI and H\( \alpha \) rotation curves agree very well (where they coexist) and the combined H\( \alpha \) + HI RC is smooth, symmetric and extended out to 6-8 disk length-scales. The mass distribution (i.e.\( V(R) \)) is modeled as the sum of three components: two stellar/gaseous disks and a spherical dark halo: \( V_{model}^2 = V_{disk}^2 + V_B^2 + V_{gas}^2 \), with the halo contribution represented by the Burkert profile given by eq (1.3). Light traces the stellar mass via a radially constant mass-to-light ratio. The gas contribution \( V_{gas}(R) \) is obtained from HI surface brightness and the distance of the object. For each galaxy, we determine the values of the structural parameters \( \beta, r_0, \rho_0 \) by means of a \( \chi^2 \)-minimization of the velocity model: \( V^2_{model}(R; \rho_0, \beta, r_0) = V_d^2(R; \beta) + V_B^2(R; \rho_0, \beta, r_0) + V_{gas}^2(R) \) to the (measured) circular velocity \( V(R) \), subject to the constraints: \( V_{model}(at \ R_{opt}) = V(R_{opt}) \) and \( |dV/dR - dV_{model}/dR| < 0.1|dV/dR| \) (see Gentile et al. 2004 for details). This halo profile (+ the exp disk) fits the rotation curves extremely well, with no systematic deviation. None of the \( \sim 100 \) data points of the five RC’s is discrepant at the 3 \( \sigma \) level (\( \sigma \) is the observational error). The stellar I-band mass-to-light ratios lie between 0.5 and 1.8 and are consistent with population synthesis models. The presence of cores is clear: \( r_0 = (0.7 - 2.3) \times R_{opt} \) and \( \rho_0 = (0.4 - 3) \times 10^{-24} g/cm^3 \). It is worth noticing the existence of the relationship among the halo structural parameters \( \rho_0 = 5 \times 10^{-24} r_0^{-2/3} exp[\ln(r_0/2\sigma)]^2 g/cm^3 \).
4 High-quality RC’s and NFW halos

The same objects, when modeled by means of the stellar/gas disks + NFW halo components fail to reproduce the shape of the observed rotation curve. Moreover, they show systematic discrepancy in the predicted velocities: these, in the central parts, are too high. In detail: 10% of the measurements cannot be matched in any way, the difference between them and the predictions exceeding $3\sigma$ (i.e. three times the observational error). An other 10% of data suffer of a poor match, the offset is at the level of 2-3 $\sigma$. This is a huge model-data discrepancy: since 1980, every mass model without a central cusp has predicted circular velocities that resulted within, at worst, 2 $\sigma$ the observed ones.

Let us notice that, also if we leave $c$ as a free parameter, there is no appreciable improvement in the model fits, and that, at galactic scales, the actual value of the density inner slope is around -1.3, (Navarro et al 2004) making things even more difficult for standard $\Lambda CDM$. A further shortcoming is that the resulting NFW disk mass-to-light ratio turn out to be unacceptably lower than the values we estimate from the galaxy color (Gentile et al, 2004).

Finally, we draw attention on the further evidence provided by de Blok and Bosma, 2002, Simon et al 2003, Weldrake et al, 2003, Bolatto et al 2002 about the serious theory vs observations discrepancy in the DM density distribution, we show in Fig 3 a test case.

Fig. 3. Best-fit models (solid line). Also shown the DM (long-dashed), stellar (dotted) and HI (short-dashed) contributions.
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Fig. 4. The density of the dark halo of 116–G12, and the CDM predictions right

5 Conclusions

The mass distribution in spirals underlies tight relationships among their structural quantities, likely, as the results of strong feedbacks occurred during early stages of galaxy formation. DM halos around galaxies have an inner constant-density region, whose size exceeds the stellar disk length-scale and emerge as an one-parameter family. The order parameter (either the central density or the core radius) correlates with the stellar mass. There is no evidence that the density profile converges, at large radii, to a \( \rho \sim r^{-2} \) (or steeper) profile. The DM distribution is determined by physical parameters, the central core density and the core radius, that have no counterpart in the gravitational instability/hierarchical clustering picture.

Solutions for the existence of a region of “constant” density include a) DM "interacted" with baryons. Original "cuspy" halos have been smoothed out b) DM has a different power spectrum/perturbations evolution than the current Standard Picture c) Dark Matter is a "field", that mimics the effects of a cored halo of particles. d) the actual dynamical evolution of DM halos, including their baryonic content, is more complex than that presently emerging in simulations.

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(r₀ in kpc, Salucci and Burkert, 2001), crucial for its implications on the nature of dark matter.

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