Bino Dark Matter and Big Bang Nucleosynthesis in the Constrained $E_6$ SSM with Massless Inert Singlinos

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Abstract

We discuss a new variant of the $E_6$ inspired supersymmetric standard model ($E_6$ SSM) in which the two inert singlinos are exactly massless and the dark matter candidate has a dominant bino component. A successful relic density is achieved via a novel mechanism in which the bino scatters inelastically into heavier inert Higgsinos during the time of thermal freeze-out. The two massless inert singlinos contribute to the effective number of neutrino species at the time of Big Bang Nucleosynthesis, where the precise contribution depends on the mass of the $Z'$ which keeps them in equilibrium. For example for $m_{Z'} > 1300$ GeV we find $N_{\text{eff}} \approx 3.2$, where the smallness of the additional contribution is due to entropy dilution. We study a few benchmark points in the constrained $E_6$ SSM with massless inert singlinos to illustrate this new scenario.
1 Introduction

The evidence for dark matter is now very strong. Initially proposed to allow an explanation of observed galactic rotation curves, we now also see its effects in the cosmic microwave background (CMB). The CMB baryon acoustic oscillation measurements from WMAP allow us to estimate the relative density of cold dark matter in the universe to be \( \Omega_{\text{CDM}} = (0.1099 \pm 0.0062)/h^2 \) \cite{1}, where \( h \) is the reduced Hubble parameter \( h \approx 0.73 \).

In supersymmetric (SUSY) models such as the minimal supersymmetric standard model (MSSM) \cite{2} one typically imposes a \( Z_2 \) matter parity on the superpotential in order to remove the \( B - L \) violating terms allowed by the SM gauge symmetries. This is equivalent to so called \( R \)-parity \cite{3}, a \( Z_2 \) symmetry of the Lagrangian under which the charge of a physical state is related to its spin. The scalar and fermionic components of a chiral superfield have opposite \( R \)-parity and the lightest supersymmetric (\( R \)-parity odd) particle (LSP) is stable. In the such models one therefore predicts a new stable particle that could be a dark matter candidate, motivated for reasons not \( a \ priori \) related to dark matter. In general the LSP could be either a gravitino or a neutralino, depending for example of the nature of SUSY breaking which determines the typical value of the gravitino mass.

In such theories a sub-weak-strength interacting neutralino is generally considered a good candidate for LSP dark matter \cite{4, 5}. Since such a particle is self-charge-conjugate, its relic abundance is determined by thermal freeze-out, not by matter-antimatter asymmetry as in the case of baryons. At some point in the early universe the expansion rate of the universe would have become larger than the LSP’s self-annihilation rate (or co-annihilation rate with other supersymmetric particles). At this point the number density of LSPs would no longer be able to track its equilibrium value. It would remain much larger than the equilibrium value, only diluting due to Hubble expansion. An LSP dark matter candidate with a larger cross-section would be able to stay in equilibrium longer, meaning that there would be less of it in the universe today. Thermal neutralino dark matter has been widely studied in the MSSM \cite{6, 7, 8, 9} and constrained (c)MSSM \cite{10, 11, 12, 13, 14}. Successful dark matter may be realised if the LSP is the lightest neutralino, with various regions of parameter space corresponding to different dominant annihilation mechanisms. For example the bulk region corresponds to annihilation via t-channel slepton exchange, the focus region via t-channel chargino exchange and the funnel region via s-channel Higgs exchange. There are also other regions corresponding to co-annihilation with staus or stops.

However the MSSM has certain shortcomings and it is possible that TeV scale SUSY is realised via a richer structure. It is therefore worthwhile considering alternative SUSY theories in which dark matter may be realised differently from in the MSSM, with the neutralino dark matter having a different composition and/or annihilating via different mechanisms. For example, the \( E_6 \)SSM \cite{15, 16, 17} (or the Exceptional Supersymmetric extension to the Standard Model (SM)) is a string theory inspired supersymmetric model based on an \( E_6 \) grand unification (GUT) group. The low energy gauge group contains an extra \( U(1) \), called \( U(1)_{\chi} \), under which the right-handed neutrinos that arise in the model are not charged. This means that the right-handed neutrinos may acquire large intermediate-scale Majorana masses, leading to a type-I see-saw mechanism to explain the small observed neutrino masses. This choice, that the low energy gauge group is
SM × U(1)N, defines the model. The U(1)N gauge symmetry is spontaneously broken at low energy by a SM-singlet field which we refer to as S3. This field radiatively acquires a vacuum expectation value (VEV) s naturally of order the soft SUSY breaking scale, leading to a Z'-boson with an induced mass of order the TeV scale. Automatic anomaly cancellation is ensured by allowing three complete 27 representations of E6 to survive down to the low energy scale. These three 27s contain the three generations of known matter, however they also contain the VEV acquiring Higgs doublets and SM-singlet. This means that there are two extra copies of the Higgs doublets and SM-singlet in the low energy particle spectrum. In the E6SSM only one generation of Higgs doublets and singlets, defined to be the third, acquires the required VEVs and are called “active”, namely Hd3, Hu3 and S3. The other two generations, the first and second, of Higgs doublets Hα, Huα and SM-singlets Sα, where α ∈ 1, 2, do not acquire VEVs and are called “inert”. The inert generations have suppressed Yukawa couplings to SM matter, suppressing flavour changing neutral currents (FCNCs) and in turn explaining why they do not radiatively acquire VEVs.

In the MSSM the neutralino mass matrix has the familiar 4 × 4 structure corresponding to the bino ˜B, the neutral wino ˜W3 and two active[neutral Higgsinos ˜Hd3 and ˜Hw3. In the E6SSM the neutralino mass matrix is greatly enlarged to a 12 × 12 structure including an additional four neutral inert Higgsinos ˜Hdα, ˜Hwα, one active singlino ˜S3, two inert singlinos ˜Sα and an extra bino B' which is the SUSY partner of the Z'. It has been observed that the six inert Higgsinos and singlinos ˜Hdα, ˜Hwα and ˜Sα tend to decouple from the rest of the neutralino spectrum and it makes sense to consider their 6 × 6 mass matrix separately [18, 19]. Moreover the lightest inert neutralino state is predominantly inert singlino in nature and only acquires mass only via mass mixing with the inert Higgsinos proportional to the electroweak VEV v. It has a suppressed mass of order v2/s, where s is the SM-singlet VEV [18]. It is therefore natural to suppose that the LSP arises from the inert neutralino sector and is a superposition of the inert singlino and inert Higgsino components. It has been shown that it is possible to reproduce the observed relic density in this model by allowing annihilations via an s-channel Z'-boson, while ensuring that the LSP, which must be similar in mass to the Z, having a mass of about 35–50 GeV, is not ruled out by LEP constraints on Z decays [18]. It has recently been observed that, in this scenario, since the LSP always couples rather strongly to the SM-like Higgs boson, it leads to “invisible” Higgs decays at the LHC and for the same reason this dark matter scenario may be discovered or ruled out by dark matter direct detection experiments in the near future [19].

In this paper we introduce a new scenario for dark matter in the E6SSM in which the dark matter candidate is just the usual bino. At first sight having a bino dark matter candidate seems impossible since, as already discussed, the inert singlinos ˜Sα naturally have suppressed masses and it is very difficult to make them even as heavy as half the Z mass. It is clear that the singlinos will always be lighter than the bino, at least in the case of gaugino unification, as assumed in this paper. To overcome this we propose

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1We shall refer to Higgsinos as being “active” or “inert” according to whether their scalar SUSY partners do or do not develop VEVs.

2Since this paper was submitted, the latest XENON100 direct detection results have been released [20]. These results severely challenge the previous dark matter scenario in [18, 19] but are consistent with the bino dark matter scenario as discussed in this paper.
that the singlinos are exactly massless and decoupled from the bino, which is achieved in practice by setting their Yukawa couplings to zero. This is easy to do by introducing a discrete symmetry $Z_2^S$ under which the inert singlinos $\tilde{S}_\alpha$ are odd and all other states are even, a scenario we refer to as the $E_6 Z_2^S$ SSM or EZSSM. In the EZSSM the inert singlinos $\tilde{S}_\alpha$ will be denoted as $\tilde{\sigma}$ in order to emphasise their different (massless and decoupled) nature. The stable dark matter candidate (DMC) is then generally mostly bino and the observed dark matter relic density can be achieved via a novel scenario in which the bino inelastically scatters off of SM matter into heavier inert Higgsinos during the time of thermal freeze-out, keeping the bino in equilibrium long enough to give the desired relic abundance. Providing the inert Higgsinos are close in mass to the bino this is always possible to arrange, the only constraint being that the inert Higgsinos satisfy the LEP2 constraint of being heavier than 100 GeV. This in turn implies that the bino must also be heavier than or close to 100 GeV. These constraints are easy to satisfy and, unlike the inert neutralino LSP dark matter scenario, we find that successful relic abundance can be achieved within a GUT-scale-constrained version of the model (the cEZSSM), assuming a unified soft scalar mass $m_0$, soft gaugino mass $M_{1/2}$ and soft trilinear mass $A_0$ at the GUT scale [21, 22, 23].

It is worth noting that studies of the constrained (c) $E_6$ SSM [21, 22, 23] have hitherto neglected to study the full $12 \times 12$ neutralino mass matrix, only considering the $6 \times 6$ mass matrix of the MSSM augmented by the active singlino $\tilde{S}_3$ and the extra bino $\tilde{B}'$, as in the so called USSM [24]. Although the question of dark matter was addressed in the USSM, the requirement of successful relic abundance was not imposed on the c$E_6$ SSM [21, 22, 23] even though the latter analysis considered the same $6 \times 6$ mass matrix as in the USSM. Here we shall consider the c$E_6$ SSM with the full $12 \times 12$ neutralino mass matrix, including both the USSM and inert neutralinos, under the assumption that the fermionic components of the inert SM-singlet superfields, the two inert singlinos, are forbidden to acquire mass by an extra $Z_2$ symmetry of the superpotential. In practice this reduces to a $10 \times 10$ neutralino mass matrix once the two massless inert singlinos are decoupled.

In summary, the main result of this paper is that bino dark matter, with nearby inert Higgsinos and massless inert singlinos, provides a simple and successful picture of dark matter in the $E_6$ SSM consistent with GUT constrained soft parameters. We shall also consider the effect of the presence of the two massless inert singlinos in the EZSSM on the effective number of neutrinos contributing to the expansion rate of the universe prior to BBN, affecting $^4$He production. Current fits to WMAP data [25] favour values greater than three, so the presence of additional contributions to the effective number of neutrinos is another interesting aspect of the EZSSM which we shall study. In practice we find that the additional number of effective neutrino species is less than two, due to entropy dilution, depending on the mass of the $Z'$ which keeps the inert singlinos in equilibrium.

The EZSSM is introduced in Section 2. Section 3 explores the neutralino and chargino sectors of the EZSSM whereas the details of the dark matter calculation are presented in Section 4. $N_{\text{eff}}$ is defined and calculations of its value in the EZSSM are presented in Section 5. Some benchmark points are presented in Section 6 and the conclusions are summarised in Section 7.
2 The EZSSM

The $E_6$ GUT group can be broken as follows:

$$E_6 \rightarrow SO(10) \times U(1)_\psi$$
$$\rightarrow SU(5) \times U(1)_\chi \times U(1)_\psi$$
$$\rightarrow SU(3) \times SU(2) \times U(1)_Y \times U(1)_\chi \times U(1)_\psi. \quad (3)$$

In the $E_6$SSM $E_6$ is broken at the GUT scale directly to $SU(3) \times SU(2) \times U(1)_Y \times U(1)_N$, where

$$U(1)_N = \cos(\vartheta)U(1)_\chi + \sin(\vartheta)U(1)_\psi \quad (4)$$

and $\tan(\vartheta) = \sqrt{15}$ such that the right-handed neutrinos are chargeless. Three complete 27 representations of $E_6$ then survive down to low energy in order to ensure anomaly cancellation. These 27 ($i \in \{1, 2, 3\}$) decompose under the $SU(5) \times U(1)_N$ subgroup as follows:

$$27_i \rightarrow \left(10, 1/\sqrt{40}\right)_i + \left(\bar{5}, 2/\sqrt{40}\right)_i$$
$$+ \left(\bar{5}, -3/\sqrt{40}\right)_i + \left(1/\sqrt{40}, 1\right)_i + \left(1, 0\right)_i \quad (5)$$

The first two terms contain normal matter, whereas the final term, which is a singlet under the entire low energy gauge group, contains the right-handed neutrinos. The second-to-last term, which is charged only under $U(1)_N$, contains the SM-singlet fields $S_i$. The third generation SM-singlet acquires a VEV $\langle S_3 \rangle = s/\sqrt{2}$ which, as we shall see, generates the effective $\mu$-term and spontaneously breaks $U(1)_N$ leading to a mass for the $Z'$-boson. The remaining terms contain three generations of down- and up-type Higgs doublets $H_{di}$ and $H_{ui}$ as well as exotic coloured states $\bar{D}_i$ and $D_i$. Again only the third generation of Higgs doublets acquire VEVs $\langle H_{d3}^0 \rangle = v_d/\sqrt{2} = v \cos(\beta)/\sqrt{2}$ and $\langle H_{u3}^0 \rangle = v_u/\sqrt{2} = v \sin(\beta)/\sqrt{2}$.

The low energy gauge invariant superpotential can be written as follows:

$$\mathcal{W} = \mathcal{W}_0 + \mathcal{W}_1 + \mathcal{W}_2, \quad (6)$$

where

$$\mathcal{W}_0 = \lambda_{ijk} S_i H_{dj} H_{ak} + \kappa_{ijk} S_i D_j \bar{D}_k + h^N_{ijk} N_i^c H_{uj} L_k$$
$$+ h^U_{ijk} u_i^c H_{uj} Q_k + h^D_{ijk} d_j^c H_{dj} Q_k + h^E_{ijk} e_i^c H_{dj} L_k. \quad (7)$$

$$\mathcal{W}_1 = g_{ijk}^O D_i Q_j Q_k + g_{ijk}^q D_i \bar{d}_j^c u_k^c, \quad (8)$$

$$\mathcal{W}_2 = g_{ijk}^N N_i^c D_j d_k^c + g_{ijk}^E e_i^c D_j u_k^c + g_{ijk}^D e_i^c Q_j L_j \bar{D}_k. \quad (9)$$

It is now clear that the effective $\mu$-parameter is given by $\mu = \lambda_{333} s/\sqrt{2}$ generating the term $\mu H_{d3} H_{u3}$ in the superpotential.

In order to suppress non-diagonal flavour transitions arising from the Higgs sector the superpotential obeys an approximate $\mathbb{Z}_2$ symmetry called $\mathbb{Z}_2^H$. Under this symmetry all superfields other than $S_3$, $H_{d3}$ and $H_{u3}$ are odd. It is this approximate symmetry that distinguishes between the third generation and the inert generations of Higgs doublets and
SM-singlets, with the inert generations having suppressed couplings to matter and not radiatively acquiring VEVs. This approximate symmetry suppresses $\lambda_{ijk}$ couplings of the forms $\lambda_{333}$, $\lambda_{3a3}$, and $\lambda_{3\beta\gamma}$, where $\alpha, \beta, \gamma \in \{1, 2\}$ i.e. labelling the inert generations only. Such an approximate $\mathbb{Z}_2^H$ symmetry, with a stable hierarchy of couplings, can be realised in $E_6$ SSM flavour theories such as the one proposed by Howl et al. [26]. The symmetry cannot be exact or else the lightest of the exotic coloured states would be absolutely stable which would contradict observation.

Given this, an exact $\mathbb{Z}_2$ symmetry must be imposed on the superpotential in order to avoid rapid proton decay. There are two ways to impose an appropriate $\mathbb{Z}_2$ symmetry on $\mathcal{W}$ that leads to baryon and lepton number conservation. The first option is to impose a symmetry called $\mathbb{Z}_{2\ell}$ under which only the lepton superfields are odd. In this case the superpotential is equal to $\mathcal{W}_0 + \mathcal{W}_1$ and the model is called the $E_6$ SSM-I. $U(1)_B$ and $U(1)_L$ are symmetries of the superpotential if the exotic coloured states $\tilde{D}$ and $D$ are interpreted as diquarks and antidiquarks, with ($B = \pm 2, L = 0$). The second option is to impose a symmetry called $\mathbb{Z}_2^B$ under which both the lepton superfields and the exotic $\tilde{D}$ and $D$ superfields are odd. In this case the superpotential is equal to $\mathcal{W}_0 + \mathcal{W}_2$ and the model is called the $E_6$ SSM-II. $U(1)_B$ and $U(1)_L$ are symmetries of the superpotential if the exotic coloured states are interpreted as leptoquarks, with ($B = \mp 1, L = \mp 1$).

It needs to be noted that the superpotential of the $E_6$ SSM is already automatically invariant under the usual matter parity of the MSSM, provided that the exotic $\tilde{D}$ and $D$ superfields as well as the Higgs SM-singlet superfields are interpreted as being even under matter parity, along with the Higgs doublets. The $B - L$ violating terms of the MSSM superpotential that matter parity is invoked to forbid are never present in the $E_6$ SSM superpotential of Eq. (6) since they would violate the extra surviving $U(1)_N$ gauge symmetry contained in $E_6$. We shall refer to this usual matter parity of the MSSM which automatically arises in this model as $\mathbb{Z}_2^M$. In the usual way it can be recast as $R$-parity in terms of the superfield components, with the scalar components of the superfield being assigned the same parity as the superfield, and the fermionic components being assigned the opposite parity. As in the MSSM, the states which are odd under $R$-parity are called the superpartners, with the lightest superpartner being absolutely stable.

In the EZSSM the superpotential is also invariant under an additional exact $\mathbb{Z}_2$ symmetry called $\mathbb{Z}_2^S$. Under this symmetry only the two inert SM-singlet superfields $S_\alpha$ are odd. The couplings of the forms $\lambda_{aij}$ and $\kappa_{aij}$ are forbidden. This means that the fermionic components of $S_\alpha$, the inert singlinos $\tilde{\sigma}$, are forbidden to have mass and do not mix with the other neutralinos. They only interact via their gauge couplings to the $Z'$-boson, which exist since they are charged under the extra $U(1)_N$ gauge symmetry. The extra $\mathbb{Z}_2^S$ symmetry of the superpotential does not change the forms of the mass matrices of the the non-inert sectors of the model. In particular this means that all of the squark, slepton, gluino and non-inert Higgs scalar masses and mass matrices are the same as the ones given in [22]. The issue of $Z$-$Z'$ mixing is also the same as discussed there.

All of the exact and approximate $\mathbb{Z}_2$ symmetries of the superpotential mentioned in this section are summarised in Table 1.

It is known that the model as presented thus far leads to gauge coupling unification at too high a scale, with the GUT scale typically being higher than the Planck scale. This issue can be solved by having the $E_6$ GUT group be broken to an intermediate group before being broken finally to $SM \times U(1)_N$ as shown in [27]. In this paper, however, for
Table 1: The transformations of the superfields under the various $Z_2$ symmetries of the superpotential that are mentioned in this paper. $Z_2^H$ is an approximate flavour symmetry. Either $Z_2^L$ or $Z_2^B$ is imposed in order to avoid rapid proton decay. $Z_2^M$ matter parity is already a symmetry of the $E_6$ SSM due to gauge symmetry. $Z_2^S$ is the extra symmetry which is imposed in the EZSSM, forcing the inert singlinos to be massless.

| | $Z_2^H$ | $Z_2^L$ | $Z_2^B$ | $Z_2^M$ | $Z_2^S$ |
|---|---|---|---|---|---|
| $S_\alpha$ | - | + | + | + | - |
| $H_{d\alpha}$, $H_{u\alpha}$ | - | + | + | + | + |
| $S_3$ | + | + | + | + | + |
| $H_{d3}$, $H_{u3}$ | + | + | + | + | + |
| $Q_i$, $u_i^c$, $d_i^c$ | - | + | + | + | + |
| $L_i$, $e_i^c$ | - | - | - | - | + |
| $\bar{D}_i$, $D_i$ | - | + | - | + | + |

simplicity, we implement the usual solution where the superpotential contains a bilinear term involving extra fields from incomplete $27'$ and $\overline{27}'$ representations $W' = \mu' H' \overline{H}'$. To some extent this solution reintroduces the $\mu$ problem, but $\mu'$ is not required to be related to the electroweak symmetry breaking (EWSB) scale and in order to observe satisfactory gauge coupling unification it is only required that $\mu' \lesssim 100$ TeV.

3 The Neutralinos and Charginos of the EZSSM

In the EZSSM the chargino mass matrix in the interaction basis

$$\tilde{C}_{\text{int}} = \begin{pmatrix} \tilde{C}_{\text{int}}^+ \\ \tilde{C}_{\text{int}}^- \end{pmatrix},$$

where

$$\tilde{C}_{\text{int}}^+ = \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}^+_{u_3} \\ \tilde{H}^+_{u_2} \\ \tilde{H}^+_{u_1} \end{pmatrix}$$

and

$$\tilde{C}_{\text{int}}^- = \begin{pmatrix} \tilde{W}^- \\ \tilde{H}^-_{d_3} \\ \tilde{H}^-_{d_2} \\ \tilde{H}^-_{d_1} \end{pmatrix},$$

is given by

$$M^C = \begin{pmatrix} P \end{pmatrix} \begin{pmatrix} P^T \end{pmatrix},$$

where

$$P = \begin{pmatrix} M_2 \sqrt{2} m_W s_\beta & \sqrt{2} m_W c_\beta \\ \mu & \frac{1}{\sqrt{2}} \lambda_{332} s + \frac{1}{\sqrt{2}} \lambda_{331} s \\ \sqrt{2} \lambda_{332} s & \sqrt{2} \lambda_{331} s \\ \sqrt{2} \lambda_{331} s & \sqrt{2} \lambda_{312} s & \sqrt{2} \lambda_{311} s \end{pmatrix}.$$
off-diagonal blocks are suppressed under the approximate $Z_2^H$ and are therefore expected to be small, implying that there should not be much mixing between the MSSM and inert states.

We define the term “neutralino” not to include the massless inert singlinos which have no Yukawa couplings involving them and are decoupled. The neutralino mass matrix $M^N$ in the interaction basis

$$\tilde{N}_{\text{int}} = \left( \begin{array}{c} \tilde{B} \\ \tilde{W}^3 \\ \tilde{H}_d^0 \\ \tilde{H}_u^0 \\ \tilde{S}_3 \\ \tilde{B}' \end{array} \right) \left( \begin{array}{c} \tilde{H}_{du} \\ \tilde{H}_{u3}^0 \end{array} \right)^T$$

is then equal to

$$\begin{pmatrix}
M_1 & 0 & -\frac{1}{2}g^\prime v_d & \frac{1}{2}g^\prime v_u \\
0 & M_2 & \frac{1}{2}g^\prime v_d & -\frac{1}{2}g^\prime v_u \\
-\frac{1}{2}g^\prime v_d & \frac{1}{2}g^\prime v_d & 0 & -\mu \\
\frac{1}{2}g^\prime v_u & -\mu & 0 & -\lambda_{345} \sqrt{v_2} \\
-Q_d g_1^\prime v_d & -Q_a g_1^\prime v_u & Q_S g_1^\prime s & 0 \\
-Q_d g_1^\prime v_d & -Q_a g_1^\prime v_u & Q_S g_1^\prime s & M_1' \\
-Q_d g_1^\prime v_d & -Q_a g_1^\prime v_u & Q_S g_1^\prime s & M_1'
\end{pmatrix},$$

(15)

where once again $\alpha, \beta \in \{1, 2\}$, indexing the inert generations. $Q_d = -\frac{3}{\sqrt{40}}$, $Q_u = -\frac{2}{\sqrt{40}}$ and $Q_S = \frac{5}{\sqrt{40}}$ are the $U(1)_N$ charges of down-type Higgs doublets, up-type Higgs doublets and SM-singlets respectively and $g_1^\prime$ is the GUT normalised $U(1)_N$ gauge coupling. $M_1$, $M_2$ and $M_1'$ are soft gaugino masses. Typically $g_1^\prime \approx g_1$ all the way down to the low energy scale. If the soft gaugino masses are unified at the GUT scale (universal $M_{1/2}$) then we also have $M_1' \approx M_1 \approx M_2/2$. The matrix as written neglects the small kinetic term mixing between $\tilde{B}$ and $\tilde{B}'$. The elements left empty in this matrix and similar ones are implicitly taken to be zero.

The Yukawa couplings in the off-diagonal blocks are suppressed under the approximate $Z_2^H$. Given the smallness of these couplings, the inert neutralinos in the bottom-right block are pseudo-Dirac states with an approximately decoupled mass matrix

$$-\frac{s}{\sqrt{2}} \begin{pmatrix}
\lambda_{322} & \lambda_{321} \\
\lambda_{321} & \lambda_{312} \\
\lambda_{321} & \lambda_{311}
\end{pmatrix} \text{ in the basis } \left( \begin{array}{c} \tilde{H}_d^0 \\ \tilde{H}_u^0 \\ \tilde{H}_{u3}^0 \\ \tilde{H}_{du} \end{array} \right),$$

They are approximately degenerate with the two inert chargino Dirac states.

The top-left block contains the states of the MSSM supplemented by the third generation singlino and the bino’. This is known as the USSM sector [28]. In the case where $M_1 \approx M_1'$ is small the lightest neutralino mass state will be mostly bino. The bino’ will mix with the third generation singlino giving two mixed states with masses around $Q_S g_1^\prime s$. As $M_1 \approx M_1'$ increases the bino mass will increase relative to both the third generation Higgsino mass $\mu$ and the inert Higgsino masses given approximately by the bi-unitary diagonalisation of $-\frac{s}{\sqrt{2}} \lambda_{3a, \beta} s$. At the same time the state mostly containing the third generation singlino will have a decreasing mass as $M_1'$ increases relative to $Q_S g_1^\prime s$. 


4 Dark Matter in the cEZSSM

As discussed previously, due to the automatic matter parity of the model, there is a conserved $R$-parity under which the charginos, neutralinos, inert singlinos $\tilde{\sigma}$ and exotic $\tilde{D}$ and $D$ fermions, along with the squarks and sleptons, are all $R$-parity odd i.e. all of the fermions other than the quarks and leptons are $R$-parity odd. We shall assume that the lightest neutralino $\tilde{N}_1$ is the lightest of all of the $R$-parity odd states, excluding the massless inert singlinos $\tilde{\sigma}$. However $\tilde{N}_1$ cannot decay into $\tilde{\sigma}$ via neutralino mixing since the inert singlinos are decoupled from the neutralino mass matrix. Furthermore the possible decay $\tilde{N}_1 \to \tilde{\sigma}\sigma$, allowed by the $\sigma-\tilde{\sigma}-\tilde{B}^\prime$ supersymmetric $U(1)_N$ gauge coupling, is forbidden if $\tilde{N}_1$ is lighter than the inert SM-singlet scalars $\sigma$. In fact, in this case, no kinematically viable final states exist that have the same quantum numbers as $\tilde{N}_1$. Therefore $\tilde{N}_1$ is absolutely stable and is the DMC of the model.

In the successful dark matter scenario presented in this section $\tilde{N}_1$ has a dominant bino $\tilde{B}$ component with at least one of the two pairs pseudo-Dirac inert Higgsinos expected to be close in mass, but somewhat heavier, in order to achieve the correct relic density. This is due to a novel scenario in which the DMC, approximately the bino, inelastically scatters off of SM matter into heavier inert Higgsinos during the time of thermal freeze-out, keeping it in equilibrium long enough to give a successful relic density. In this section we discuss in detail how this novel scenario comes about in this model.

4.1 The Dark Matter Calculation

Usually in supersymmetric models the evolution of the cosmological number density $n_i$ of a supersymmetric ($R$-parity odd) particle $i$ in the early universe can be expressed as

$$\dot{n}_i = -3Hn_i - \sum_j \langle \sigma_{ij} v_{ij} \rangle \left( n_in_j - n_{i}^{eq}n_{j}^{eq} \right)$$

$$- \sum_{j \neq i} \left[ \Gamma_{ij} (n_i - n_i^{eq}) - \Gamma_{ji} (n_j - n_j^{eq}) \right]$$

$$- \sum_{j \neq i} \sum_X \left[ \langle \sigma'_{Xij} v_{ij} \rangle (n_in_X - n_i^{eq}n_X^{eq}) - \langle \sigma'_{Xji} v_{ji} \rangle (n_jn_X - n_j^{eq}n_X^{eq}) \right].$$

(16)

The first term accounts for Hubble expansion and the second term accounts for annihilations with other supersymmetric particles, including self-annihilations. The third term represents the decays of supersymmetric particles $i$ into other supersymmetric species $j$ as well decays of other supersymmetric species into species $i$. The final term represents the inelastic scattering of supersymmetric particles $i$ off of $R$-parity even particles $X$ into other supersymmetric species $j$ and vice versa [29, 30].

Summing up these equations yields the somewhat simpler

$$\dot{n} \equiv \sum_i \dot{n}_i = -3Hn - \sum_i \sum_j \langle \sigma_{ij} v_{ij} \rangle \left( n_in_j - n_{i}^{eq}n_{j}^{eq} \right).$$

(17)

It should be noted that, assuming all supersymmetric particles can decay into the DMC in a reasonable amount of time, after thermal freeze-out the relic number density of the DMC will subsequently becomes equal to $n$. 
In our model the DMC is not the lightest $R$-parity odd state (an inert singlino), but the lightest neutralino $\tilde{N}_1$. We would like to use Eq. (16) to describe the evolution of $R$-parity odd states other than the inert singlinos, generically $\tilde{\chi}$. In this case we should also include in Eq. (16) processes involving $\sigma$ and $\tilde{\sigma}$ particles that change the number of $\tilde{\chi}$ particles by one. Since such processes necessarily involve inert SM-singlet scalars $\sigma$, it is valid to neglect these processes in the case where these inert SM-singlets have frozen out long before the freeze-out of dark matter. We will call this condition 1 and it should be satisfied given our assumption that the inert SM-singlet scalars are heavier than the DMC, since they only interact via the heavy $Z'$-boson. As we shall see, the value of $n$ after the thermal freeze-out of $\tilde{N}_1$ depends on annihilation cross-sections involving $\tilde{N}_1$ and other $R$-parity odd states close by in mass. As long as condition 1 is satisfied, meaning that we can neglect such annihilations that also have inert singlinos in the final state during thermal freeze-out, we can neglect the inert singlinos and use Eq. (17) to calculate the number density of $R$-parity states other than inert singlinos. $n$ will eventually be equal to the number density of DMCs after other $\tilde{\chi}$ particles have decayed to $\tilde{N}_1$.

During thermal freeze-out the annihilation rates of the $\tilde{\chi}$ particles become small compared to the expansion rate of the universe and their number densities become larger than their (non-relativistic) equilibrium values. The universe expands too fast for the number densities to track their equilibrium values. Let us assume however that these states inelastically scatter off of SM states $X$ frequently enough that the ratios of the number densities of the $\tilde{\chi}$ particles do maintain their equilibrium values during the time of thermal freeze-out. We shall call this condition 2 and assuming that it is satisfied we have

$$\frac{n_j}{n_i} = \frac{n_i^{eq}}{n_j^{eq}} \Rightarrow \frac{n_i}{n} = \frac{n_i^{eq}}{n^{eq}}, \quad (18)$$

which allows us to rewrite Eq. (17) as

$$\dot{n} = -3Hn - \langle \sigma v \rangle \left(n^2 - n_{eq}^2\right), \quad (19)$$

where $n_{eq} = \sum_i n_i^{eq}$ and

$$\langle \sigma v \rangle = \sum_i \sum_j \langle \sigma_{ij} v_{ij} \rangle \frac{n_i^{eq} n_j^{eq}}{n_{eq}^2}. \quad (20)$$

Here we see that since heavier $\tilde{\chi}$ particles would have smaller non-relativistic equilibrium number densities there would be fewer of them around during the dark matter's thermal freeze-out and annihilation cross-sections involving them would be less important.

### 4.2 The cEZSSM

In order to carry out the dark matter analysis in the constrained version of the model we have extended the RGE code used by Athron et al. [22] to include the Yukawa parameters and soft masses of the inert sector of the EZSSM. The inputs are $\kappa_{3ij}$ and $\lambda_{333}$ at the GUT scale, $\lambda_{3\alpha\beta}$ at the EWSB scale, $s$ and $\tan(\beta)$, as well as the known low energy Yukawa couplings and gauge couplings. Given these inputs and the RGEs the algorithm attempts to find points with GUT scale unified soft masses $m_0$, $M_{1/2}$ and $A_0$. The low energy
\( U(1)_N \) gauge coupling \( g'_1 \) is set by requiring it to be equal to the other gauge couplings at the GUT scale.

For consistent points in the \( E_6 \)SSM the lightest non-inert (USSM sector) supersymmetric particle is typically bino dominated. For the cEZSSM we find the same thing. The masses of the inert Higgsino states depend on \( s \) and on the the Yukawa couplings \( \lambda_{3a3} \) and the in the cEZSSM the lightest neutralino can be either the bino dominated state or a pseudo-Dirac inert Higgsino dominated state. In the latter case we find that the DMC pseudo-Dirac inert Higgsino states co-annihilate with full-weak-strength interactions and lead to a too small dark matter relic density. In the former case the bino DMC normally annihilates too weakly and yields a too large dark matter relic density. If, however, there are inert Higgsino states close by in mass, they contribute significantly to \( \langle \sigma v \rangle \) allowing for the observed amount of dark matter. This relies on condition 2 being satisfied \( \text{i.e.} \) the bins being up-scattered into inert Higgsinos with a large enough rate.

Such points with consistent dark matter relic density can be found and three are presented in Section 6. Condition 1 is satisfied since the inert SM-singlet scalars are so much heavier than the DMC and the \( Z' \)-boson mass is so large compared to the regular \( Z \)-boson mass. Annihilation and scattering processes involving inert SM-singlets and singlinos must contain a virtual \( Z' \)-boson.

To test condition 2 let us compare the rate for binos up-scattering into inert Higgsinos with the inert Higgsino co-annihilation rate. We shall label the mostly bino state \( \tilde{N}_1 \) and the lightest pseudo-Dirac inert Higgsino states \( \tilde{N}_2 \) and \( \tilde{N}_3 \). The dominant up-scattering diagrams are of the following form:

\[
\begin{array}{c}
X \\
\tilde{N}_1 \\
Z \\
\tilde{N}_2, \tilde{N}_3 \\
\end{array}
\]

We define \( R_{Zij} \) couplings such that the \( Z-\tilde{N}_i-\tilde{N}_j \) coupling is equal to \( R_{Zij} \) times the \( Z-\nu-\nu \) coupling. We have

\[
R_{Zij} = \sum_{D=3,7,9} N_{i}^{D}N_{j}^{D} - \sum_{U=4,8,10} N_{i}^{U}N_{j}^{U},
\]  

(21)

where \( N_{i}^{a} \) is the neutralino mixing matrix element corresponding to mass eigenstate \( i \) and interaction state \( a \). \( D \) and \( U \) index the down- and up-type Higgsino interaction states respectively. For the pseudo-Dirac inert Higgsino states we have

\[
m_3 \approx -m_2 \quad \text{and} \quad R_{Z23} \approx 1,
\]

(22)

allowing for full-weak-strength co-annihilations of the following form:
The ratio of the rate for the mostly bino state up-scattering into the mostly inert Higgsino state to the inert Higgsino co-annihilation rate is given approximately by

$$\Upsilon = \frac{\langle \sigma'_{X'2} v_{X'} \rangle_{n_{X'}^\text{eq}}}{\langle \sigma_{23} v_{23} \rangle_{n_{23}^\text{eq}} \langle n_{1}^\text{eq} \rangle \langle n_{3}^\text{eq} \rangle} \quad (23)$$

To give an idea of the size of this ratio, if the SM particle $X$ is relativistic and $m_1 \sim m_2 \approx m_3$ then

$$\Upsilon \sim \left( \frac{R_{Z12}}{R_{Z23}} \right)^2 \frac{T^3}{\langle m_1 \rangle \exp(-|m_1|/T)}$$

$$\approx R_{Z12}^2 \left( \frac{1}{x} \right)^{3/2} e^x, \quad (24)$$

where $x = |m_1|/T$ and $T$ is the temperature. This ratio is expected to be large because of the overwhelming abundance of the relativistic SM particle $X$, but it also depends on $R_{Z12}$. The value of $R_{Z12}$ depends on the $Z^H$-breaking couplings that mix the top-left block of the neutralino mass matrix in Eq. (15), the USSM states including the bino, with the inert Higgsino states in the bottom-right block. Since this symmetry is not exact we expect these couplings to be large enough such that we can still assume $\Upsilon \gg 1$. Explicit examples of this parameter are included in Table 3 in Section 6.

With the two conditions satisfied we use micrOMEGAs [31] to calculate the dark matter relic density for low energy spectra consistent with the GUT-scale-constrained scenario. The CalcHEP model files for the EZSSM are produced using LanHEP [32]. The observed relic density of dark matter can arise in this model and examples are shown in Section 6.

The most critical factor is the mass splitting between the bino and the lightest inert Higgsinos. Too large and there would not be enough inert Higgsinos remaining at the time of the bino’s thermal freeze-out to have a significant enough effect. Too small and $\langle \sigma v \rangle$ would be dominated by inert Higgsino co-annihilations leading to a too small dark matter relic density.

Since in this scenario the DMC is predominantly bino, the spin-independent DMC-nucleon cross-section $\sigma_{SI}$ is not expected to be in the range that direct detection experiments are currently sensitive too. The spin-independent cross-section of a pure bino is suppressed by the squark masses, but is also sensitive to the squark mixing angles [33]. For each flavour the cross-section vanishes for zero squark mixing. Since in practice the DMC will also have non-zero (but small) active Higgsino components, there are also contributions to $\sigma_{SI}$ from t-channel active Higgs scalar exchange, via the bino-Higgs-Higgsino supersymmetric gauge coupling. These contributions, though dominant, are quite small
due to the overwhelming bino nature of the DMC. Estimates of $\sigma_{SI}$, using the same proton $f_d$, $f_u$ and $f_s$ parameters used by Gogoladze et al. [34], are included in Table 3 in Section 4.

5 The Inert Singlinos and Their Contribution to the Effective Number of Neutrinos prior to BBN

In the standard theory of BBN, which happens long after the thermal freeze-out of dark matter, the resultant primordial abundances of the light elements depend on two parameters—the effective number of neutrinos contributing to the expansion rate of the radiation dominated universe $N_{\text{eff}}$ and the baryon-to-photon ratio $\eta$.

Whilst the primordial abundance of $^4\text{He}$ is not the most sensitive measure of $\eta$, it is much more sensitive to $N_{\text{eff}}$ than the other light element abundances. This is because prior to nucleosynthesis when the equilibrium photon temperature is of order 0.1 MeV the number of neutrons remaining, virtually all of which are subsequently incorporated into $^4\text{He}$ nuclei, is sensitive to the expansion rate of the universe, which depends on $N_{\text{eff}}$. The greater the expansion rate, the less time there is for charged current weak interactions to convert neutrons into protons.

The analysis by Izotov et al. [35] using the more recent neutron lifetime measurement by Serebrov et al. [36] gives $N_{\text{eff}} = 3.80^{+0.80}_{-0.70}$ at 2-sigma, implying a more-than-2-sigma tension between the measured $^4\text{He}$ abundance and the Standard Model prediction for $N_{\text{eff}}$ (about 3). Although Aver et al. [37] suggest that these errors may be larger, similar results are also obtained for the effective number of neutrinos contributing to the expansion rate of the universe from fits to WMAP data [25].

In the EZSSM the two massless inert singlinos would have decoupled from equilibrium at an earlier time than the light neutrinos, but nevertheless would have contributed to $N_{\text{eff}}$. Exactly when the inert singlinos would have decoupled from equilibrium with the photon depends on the mass of the $Z'$-boson which determines the strength of an effective Fermi-like 4-point interaction vertex that would have been responsible for keeping the inert singlinos in equilibrium. The various values for $N_{\text{eff}}$ that can be achieved in this model all fit the data better than the SM value.

The implications of extra neutrino-like particles present in the early universe have long been studied and the methods used in following analysis rely on relatively simple physics [38]. The cosmological energy density of a relativistic boson, fermion $i$ with number of degrees of freedom $g_i^i$ and temperature $T_i$ is given by

$$\rho^i = (1, 7/s)g^i\frac{\pi^2}{30}(T_i)^4. \quad (25)$$

The total radiation dominated energy density is then defined to be

$$\rho = g^{\text{eff}}\frac{\pi^2}{30}T^4, \quad (26)$$

where $T = T^\gamma$ is the photon temperature. The effective number of degrees of freedom $g^{\text{eff}}$ takes into account the factor of $7/s$ for fermions and also takes into account the fact that
some species no longer in equilibrium with the photon may have a different temperature. In this radiation dominated universe the expansion rate is then given by

$$H^2 = \frac{8\pi G}{3} \rho$$

$$= \frac{1}{M_{\text{Planck}}^2} g_{\text{eff}} \frac{4\pi^3}{45} T^4$$

$$\equiv k_1^2 g_{\text{eff}} T^4,$$

where we define the constant \(k_1\) for future convenience.

The effective number of degrees of freedom contributing to the expansion rate of the universe during the run-up to nucleosynthesis is defined to be

$$g_{\text{eff}}^0 = g_\gamma + \frac{7}{8} g_\nu N_{\text{eff}} \left(\frac{4}{11}\right)^{4/3}.$$  \(\text{(30)}\)

$$= 2 + \frac{7}{8} N_{\text{eff}} \left(\frac{4}{11}\right)^{4/3}$$  \(\text{(31)}\)

Here \(g_\gamma = 2\) is the number of degrees of freedom of the photon and \(g_\nu = 2\) is the number of degrees of freedom of a light neutrino. The three SM neutrinos are expected to decouple from equilibrium with the photon at a temperature above the electron mass whereas nucleosynthesis does not happen until the temperature is below the electron mass. When the photon/electron temperature is around the electron mass the electrons and positrons effectively disappear from the universe\(^3\). Their disappearance heats the photons to a higher temperature then they would otherwise have had, but the neutrinos, having already decoupled, would continue to cool at the full rate dictated by Hubble expansion. Because of the neutrinos’ lower temperature at nucleosynthesis they would contribute less to \(g_{\text{eff}}^0\) per degree of freedom. Eq. \((30)\) is defined such that in the SM \(N_{\text{eff}} = 3\), for the three neutrinos decoupling above the electron mass (as we shall see). Extra particles, such as the EZSSM inert singlinos, decoupling above the muon mass would have even lower temperatures at the time of nucleosynthesis and would therefore contribute to \(g_{\text{eff}}^0\) even less than light neutrinos per degree of freedom.

### 5.1 The Calculation of \(N_{\text{eff}}\)

In the cEZSSM there is a typical scenario in which the massless inert singlinos \(\tilde{\sigma}\) decouple at a temperature above the colour transition temperature (when the effective degrees of freedom are quarks and gluons rather than mesons) and above the strange quark mass, but below the charm quark mass. This has to do with the strength of the interactions that keep the inert singlinos in equilibrium which depend heavily on the mass of the \(Z'\)-boson mass. If the inert singlinos do decouple in this range, this leads to a definite prediction for \(N_{\text{eff}}\). We shall explain why the inert singlinos typically decouple in this temperature range in the next subsection. For now we derive the value of \(N_{\text{eff}}\) in this scenario as an example.

We shall use the subscript \(0\) to denote quantities at some temperature \(T_0\) below the electron mass and the subscript \(e\) to denote quantities at some temperature \(T_e\) above the electron mass and where all light neutrino species are still in equilibrium. We shall

\(^3\)A much smaller number of electrons remains due to the small lepton number asymmetry.
use the subscript $s$ to denote quantities at some still higher temperature $T_s$ above the
colour transition and the strange quark mass and where the inert singlinos are still in
equilibrium.

At $T_s$ the effective number of degrees of freedom contributing to the expansion rate is

$$g^{\text{eff}}_s = g^\gamma + g^d + \frac{7}{8}(g^\nu + g'^d + g^u + g^d + g^s + 3g'^\nu + 2g^\nu)$$

$$= 2 + 16 + \frac{7}{8}(4 + 4 + 12 + 12 + 12 + 6 + 4) = 65\frac{1}{4}$$

and at $T_e$ it becomes

$$g^{\text{eff}}_e = 2 + \frac{7}{8}(6 + 4)$$

and at $T_0$ it becomes

$$g^{\text{eff}}_0 = 2 + \frac{7}{8}(3 + 4)$$

taking into account that the neutrinos and inert singlinos now have different temperatures.

The entropy within a given volume $V$ due to a relativistic boson, fermion $i$ with number of degrees of freedom $g^i$ is given by

$$S^i = (1, \frac{7}{8})g^i\frac{2\pi^2}{45}(T^i)^3V.$$ 

Since we are assuming that the inert singlinos decouple before the strange quark threshold, in going from $T_s$ to $T_e$ we conserve the entropy in the co-moving volume separately for the inert singlinos and for everything else. Specifically for the inert singlinos

$$T_s^3V_s = (T_e^3)^3V_e$$

and for everything else

$$[g^\gamma + g^d + \frac{7}{8}(g^\nu + g'^d + g^u + g^d + g^s + 3g'^\nu)]T_s^3V_s = [g^\gamma + \frac{7}{8}(g^\nu + 3g'^\nu)]T_e^3V_e$$

$$\Rightarrow 61\frac{3}{4}T_s^3V_s = 10\frac{3}{4}T_e^3V_e.$$ 

This allows us to write

$$\frac{T_s^3V_s}{T_e^3V_e} = \left(\frac{T_s}{T_e}\right)^3 = \frac{10\frac{3}{4}}{61\frac{3}{4}} = \frac{43}{247}$$

In going from $T_e$ to $T_0$ we conserve the entropy separately for the neutrinos, for the inert singlinos again, and for everything else

$$[g^\gamma + \frac{7}{8}g^\nu]T_e^3V_e = g^\gamma T_0^3V_0,$$

$$T_s^3V_e = (T_0^3)^3V_0,$$

$$T_e^3V_e = (T_0^3)^3V_0.$$
This gives us
\[
\left( \frac{T_0^\gamma}{T_0^0} \right)^3 = \frac{g^\gamma}{g^\gamma + 7/8 g_e} = \frac{4}{11} \tag{42}
\]
and
\[
\left( \frac{T_0^\tilde{\sigma}}{T_0} \right)^3 = \frac{43}{247} g^\gamma \frac{1}{g^\gamma + 7/8 g_e} = \frac{43\cdot 4}{247\cdot 11}. \tag{43}
\]
In this case the effective number of neutrinos contributing to the expansion rate prior to nucleosynthesis (at \(T_0\)) is then
\[
N_{\text{eff}} = 3 + 2 \left( \frac{43}{247} \right)^{4/3} \approx 3.194. \tag{44}
\]

5.2 The Inert Singlino Decoupling Temperature

The light neutrinos are kept in equilibrium via their electroweak interactions. For all the light neutrinos there are the following tree-level diagrams:

For the electron neutrinos there is also the following additional diagram:

We express the cross-section for processes relevant for keeping muon and \(\tau\) neutrinos in equilibrium as
\[
\langle \sigma_{\nu_\mu,\nu_\tau},v \rangle = k_2 \frac{T^2}{m_Z^2} \left( \frac{5/3}{s_W^2} \right)^2 \frac{g_1^4}{s_W^4} X^4, \tag{45}
\]
where \(k_2\) is another constant and
\[
X^4 = ((-1/2 + s_W^2)(1/2))^2 + 1/4s_W^4 \approx 0.031. \tag{46}
\]
Note that using the GUT normalised $U(1)_Y$ gauge coupling $g_1$ we have
\[
\frac{g_2}{c_W} = \sqrt{\frac{5}{3}} \frac{g_1}{s_W}. \tag{47}
\]
The cross-section for electron neutrinos with their extra diagram is then
\[
\langle \sigma_{\nu_e \nu} \rangle = k_2 \frac{T^2}{m_Z^4} \left( \frac{5}{3} \right)^2 g_1^4 \frac{s_W^4}{2} Y^4, \tag{48}
\]
where
\[
Y^4 = \left( \frac{1}{2} + s_W^2 \right)^2 + \frac{1}{4} s_W^4 \approx 0.147. \tag{49}
\]
We express the number densities of all Weyl fermions still in equilibrium with the photon as
\[
n_{\nu_e} = n_{\nu_e} = n_{\nu_\mu} = n_{\nu_\tau} = k_3 T^3 \tag{50}
\]
and the expansion rate is given by
\[
H = k_1 \sqrt{g_\text{eff} T^2}. \tag{51}
\]
The neutrino decoupling temperature $T^\nu$ can then be approximated by
\[
\langle \sigma_{\nu} \rangle n^\nu = H \tag{52}
\]
\[
\Rightarrow \quad (T^\nu)^3 = K \sqrt{g_\text{eff} m_Z^4 \frac{s_W^4}{2}} \frac{1}{X^4}, \tag{53}
\]
\[
(T^\nu_\nu)^3 = K \sqrt{g_\text{eff} m_Z^4 \frac{s_W^4}{2}} \frac{1}{X^4}, \tag{54}
\]
with $K = k_1 / k_2 k_3$. A more detailed calculation finds that in the SM (with only neutrinos, electrons and photons contributing to $g_\text{eff}$) $T^\nu_\mu, \nu_\tau \approx 3.7$ MeV and $T^\nu_\nu \approx 2.4$ MeV, the muon and $\tau$ neutrinos decoupling earlier.

At temperatures above the strange quark mass the processes relevant for keeping the inert singlinos in equilibrium are as follows:

The part of the $Z'$ current illustrating the relevant $U(1)_N$ charges is
\[
J^\mu_{Z'} = ( \bar{L} \ e_R \ \bar{Q} \ \bar{u}_R \ \bar{d}_R \ \bar{\sigma} ) \gamma^\mu \ \frac{1}{\sqrt{40}} \begin{pmatrix} (2) L \\ (1) e_R \\ (1) Q \\ (1) u_R \\ (2) d_R \\ (5) \bar{\sigma} \end{pmatrix} \tag{55}
\]
and the total cross-section taking into account all of these diagrams is then (neglecting the small $Z-Z'$ mixing)

\[
\langle \sigma_{\tilde{\tau}} v \rangle = k_2 \frac{T^2}{m_{Z'}^4} 2g_4^4 \frac{Z'^4}{(40)^2},
\]

where

\[
Z'^4 = (5)^2[2(2)^2 + 2(1)^2 + 3(1)^2 + 3(1)^2 + 6(1)^2 + 6(2)^2 + 3(2)^2] = 1450,
\]

leading to an approximate singlino decoupling temperature of

\[
(T_{\tilde{\tau}})^3 = K g_s g_{\text{eff}}^4 m_{Z'}^4 \frac{1}{g_4^4} \frac{(40)^2}{Z'^4},
\]

\[
\Rightarrow \left( \frac{T_{\tilde{\tau}}}{T_{\nu e}} \right)^3 = \sqrt{g_s^4 g_{\text{eff}}^4} \left( \frac{m_{Z'}}{m_{\tau}} \right)^4 \frac{(40)^2(5/3)^2 Y^4}{g_4^4 Z'^4}.
\]

The only unknown variable here affecting the inert singlino decoupling temperature is then the $Z'$ mass $m_{Z'}$. Rearranging we find

\[
m_{Z'} \approx m_{\tau} \left( \frac{T_{\tilde{\tau}}}{6.60 \text{ MeV}} \right)^{3/4}.
\]

### 5.3 $N_{\text{eff}}$ in the EZSSM

We now check which values of $m_{Z'}$ are consistent with our assumption that the inert singlinos decouple at a temperature between the strange and charm quark masses. For $T_{\tilde{\tau}} < m_c$ we find that we require $m_{Z'} < 4700$ GeV. For $m_{Z'} \sim 1000$ GeV the situation is slightly more complicated. Firstly the temperature of the QCD phase transition is not accurately known and secondly the effective number of degrees of freedom is decreased by so much after the QCD phase transition that even if the inert singlinos were decoupled beforehand the universe may be expanding slowly enough afterwards that they could come back into equilibrium. After checking a range of scenarios we find that for $1300 \text{ GeV} \lesssim m_{Z'} < 4700$ GeV our value of $N_{\text{eff}} = 3.194$ is valid. For $m_{Z'} \lesssim 950$ GeV the inert singlinos decouple at a temperature above the muon mass, but below the pion mass leading to a larger prediction of $N_{\text{eff}} = 4.373$. The current experimental limit in the EZSSM is $m_{Z'} > 892$ GeV [39], so at the time of writing it is possible that the $Z'$-boson is light enough to predict $N_{\text{eff}} = 4.373$. For $Z'$ masses in between these ranges the value of $N_{\text{eff}}$ depends on the details of the QCD phase transition, but is somewhere between these predictions. For inert singlinos decoupling above the pion mass, but after the QCD phase transition we have $N_{\text{eff}} = 4.065$. All of these values are within the 2-sigma measured range $N_{\text{eff}} = 3.80^{+0.80}_{-0.70}$ and closer to the central value than the SM result $N_{\text{eff}} = 3$.

### 6 Benchmark Points

In the following tables we present three benchmark points in the cEZSSM. For all three points we fix $\lambda_{322} = 0.1$ and $\lambda_{331} = \lambda_{312} = 0.0001$ at the EWSB scale. For the $Z^H_2$-breaking couplings we also fix $\lambda_{332} = \lambda_{323} = 0.012$ and $\lambda_{331} = \lambda_{313} = 0.005$ at the EWSB
| Benchmark | 1        | 2        | 3        |
|-----------|----------|----------|----------|
| tan(β)    | 30       | 10       | 3        |
| s [TeV]   | 5        | 4.4      | 5.5      |
| λ_{333} @ GUT scale | -0.3 | -0.37    | -0.4     |
| λ_{322} @ EWSB scale | 0.1  | 0.1      | 0.1      |
| λ_{311} @ EWSB scale | 0.0293 | 0.0403   | 0.0399   |
| κ_{3ii} @ GUT scale | 0.18 | 0.18     | 0.23     |
| M_{1/2} [GeV] | 590     | 725      | 908      |
| m_0 [GeV] | 1533     | 454      | 1037     |
| A [GeV]   | 1375     | 1002     | 413      |

Table 2: The GUT scale parameters of the three benchmark points.

| Benchmark | 1        | 2        | 3        |
|-----------|----------|----------|----------|
| µ [GeV]   | -1086.7  | -1189.5  | -1405.5  |
| λ_{322}s/√2 [GeV] | 353.55 | 331.13   | 388.91   |
| λ_{311}s/√2 [GeV] | 103.59  | 125.38   | 155.17   |
| N at mass [GeV] | 94.07   | 114.49   | 143.50   |
| N at mass [GeV] | -105.12 | -126.45  | -156.57  |
| N at mass [GeV] | 105.14  | 126.47   | 156.62   |
| N at mass [GeV] | 167.05  | 203.19   | 255.47   |
| N at mass [GeV] | -353.77 | -311.29  | -389.12  |
| N at mass [GeV] | 353.78  | 311.30   | 389.13   |
| N at mass [GeV] | -1092.5 | -1194.5  | 1409.6   |
| N at mass [GeV] | 1093.3  | 1194.8   | -1411.2  |
| N at mass [GeV] | -1803.2 | -1572.3  | -1964.7  |
| N at mass [GeV] | 1899.7  | 1688.7   | 2109.9   |
| C at mass [GeV] | 105.04  | 126.41   | 156.52   |
| C at mass [GeV] | 167.05  | 203.19   | 255.46   |
| C at mass [GeV] | 353.78  | 311.30   | 389.13   |
| C at mass [GeV] | -1094.4 | -1196.1  | -1411.3  |
| m_{Z'} [GeV] | 1850.4  | 1628.4   | 2035.4   |
| N_{eff}    | 3.194   | 3.194    | 3.194    |
| Ω_{CDM}h^2 | 0.112   | 0.107    | 0.102    |
| Υ          | 1.1 × 10^8 | 2.3 × 10^8 | 2.3 × 10^8 |
| σ_{SI} [cm^2] | 4.9 × 10^{-48} | 2.5 × 10^{-48} | 1.2 × 10^{-48} |

Table 3: The low energy neutralino and chargino masses, and associated parameters. The dark matter candidate is the lightest neutralino \( \tilde{N}_1 \), which is predominantly bino. There is a nearby pair of inert neutral Higgsinos \( \tilde{N}_2, \tilde{N}_3 \) and a chargino \( \tilde{C}_1 \) into which \( \tilde{N}_1 \) inelastically scatters during freeze-out, resulting in the correct relic density \( \Omega_{CDM} h^2 \) shown. The predicted values of \( m_{Z'} \) and \( N_{eff} \) are also shown, as is the spin-independent \( N_1 \) direct detection cross-section \( σ_{SI} \).
| Benchmark                      | 1     | 2     | 3     |
|-------------------------------|-------|-------|-------|
| $h_1$ mass [GeV]              | 122.2 | 114.6 | 115.3 |
| $h_2$ mass [GeV]              | 1145  | 987.1 | 1522  |
| $h_3$ mass [GeV]              | 1890  | 1664  | 2080  |
| $H^\pm$ mass [GeV]            | 2106  | 1396  | 1675  |
| $A^0$ mass [GeV]              | 2103  | 1393  | 1673  |
| $m_{s_2}, m_{s_1}$ [GeV]      | 1547  | 518   | 1084  |
| $m_{H_d^2}, m_{H_d^1}$ [GeV]  | 1567  | 611   | 1156  |
| $m_{H_u^2}, m_{H_u^1}$ [GeV]  | 1561  | 599   | 1146  |
| $m_{\tilde{D}_3}$ [GeV]      | 1483  | 503   | 1794  |
| $m_{\tilde{D}_2}, m_{\tilde{D}_1}$ [GeV] | 1443 | 493 | 1775 |
| $m_{\tilde{D}_3}, m_{\tilde{D}_1}$ [GeV] | 2864 | 2321 | 3065 |
| $m_{\tilde{D}_2}, m_{\tilde{D}_1}$ [GeV] | 2840 | 2318 | 3052 |
| $m_{\tilde{t}_1}$ [GeV]      | 1122  | 625.3 | 1110  |
| $m_{\tilde{c}_1}, m_{\tilde{u}_1}$ [GeV] | 1817 | 1774 | 1707 |
| $m_{\tilde{t}_2}$ [GeV]      | 1470  | 1069  | 1546  |
| $m_{\tilde{c}_2}, m_{\tilde{u}_2}$ [GeV] | 1838 | 1224 | 1761 |
| $m_{\tilde{b}_1}$ [GeV]      | 1434  | 1009  | 1512  |
| $m_{\tilde{s}_1}, m_{\tilde{d}_1}$ [GeV] | 1840 | 1226 | 1763 |
| $m_{\tilde{b}_2}$ [GeV]      | 1748  | 1265  | 1818  |
| $m_{\tilde{s}_2}, m_{\tilde{d}_2}$ [GeV] | 1907 | 1278 | 1820 |
| $m_{\tilde{\tau}_1}$ [GeV]  | 1500  | 718.8 | 1259  |
| $m_{\tilde{\tau}_2}$ [GeV]  | 1655  | 731.3 | 1261  |
| $m_{\tilde{\mu}_1}, m_{\tilde{\tau}_1}$ [GeV] | 1708 | 949.2 | 1473 |
| $m_{\tilde{\mu}_2}$ [GeV]   | 1775  | 952.8 | 1474  |
| $m_{\tilde{\tau}_2}$ [GeV]  | 1705  | 945.6 | 1472  |
| $m_{\tilde{\mu}_1}, m_{\tilde{\tau}_1}$ [GeV] | 1774 | 949.5 | 1472 |
| $m_{\tilde{g}}$ [GeV]        | 541.3 | 626.9 | 787.7 |

Table 4: The remaining particle spectrum in the standard notation.
scale. At the GUT scale we fix $\kappa_{333} = \kappa_{322} = \kappa_{311}$ and $\kappa_{3ij} = 0$ for $i \neq j$. The lightest (SM-like) Higgs mass is calculated to second loop order.

We have chosen three points with quite different values of $\tan(\beta)$—30, 10 and 3. This illustrates the fact that $\tan(\beta)$ can be quite low in this model since the SM-like Higgs mass is not constrained to be less than $m_Z|\cos(2\beta)|$ at tree-level as it is in the MSSM.

The mass of the bino DMC $\tilde{N}_1$ is not directly constrained to be above above 100 GeV. However, the lightest pseudo-Dirac inert neutralinos $\tilde{N}_2$ and $\tilde{N}_3$ are almost degenerate with the lightest inert Higgsino chargino $\tilde{C}_1$ and therefore these are constrained to heavier than 100 GeV in order to be consistent with LEP constraints. Furthermore the thermal relic DM scenario outlined in Section 4 requires $\tilde{N}_2$ and $\tilde{N}_3$ not to be too much more massive than $\tilde{N}_1$. In practice the $\tilde{N}_1$ is predominantly bino and its mass cannot be much less than 100 GeV. In Benchmark 1, for example, it is 94 GeV.

Requiring such values for the low energy bino mass $M_1$ and requiring consistent electroweak symmetry breaking in practice means that the SM-singlet VEV $\langle v \rangle$ cannot be too low. This in turn means that the $Z'$ mass is always quite a bit above the experimental limits, more than about 1.5 TeV. In these benchmarks from the constrained scenario the effective number of neutrinos contributing to the expansion rate of the universe prior to BBN $N_{\text{eff}}$ therefore takes on the lower value calculated in Section 5—around 3.2. This is more consistent with data than the SM prediction.

In all benchmark points $\tilde{N}_4$ and $\tilde{C}_2$ are predominantly wino. $\tilde{N}_5$, $\tilde{N}_6$ and $\tilde{C}_3$ are predominantly made up of the rest of the inert Higgsinos states, with masses around $\lambda_{322}s/\sqrt{2}$, whereas $\tilde{N}_7$, $\tilde{N}_8$ and $\tilde{C}_4$ are predominantly made up of the active Higgsinos states, with masses around $\mu$. $\tilde{N}_9$ and $\tilde{N}_{10}$ are mostly superpositions of the active singlino and bino'.

The fact that $\Upsilon \gg 1$ indicates that the inert Higgsino components in the predominantly bino state $\tilde{N}_1$, though small, are large enough such that processes involving $\tilde{N}_1$ up-scattering off of a SM particle into $\tilde{N}_2$ happen overwhelmingly more often than neutralino (co-)annihilation processes. In this way the ratios of the number densities of these particles are able to maintain their equilibrium values.

The spin-independent DMC-nucleon cross-section $\sigma_{\text{SI}}$, as estimated using the results of Choi et al. [33], is quite small for these benchmarks, and is not currently detectable by direct detection experiments. This is due to the predominantly bino nature of the DMC, and the large squark masses.

7 Conclusions

The question of dark matter in the $E_6$SSM illustrates the interesting diversity of possibilities that can arise once one goes beyond the MSSM. The difficulty in making the inert singlinos predicted by this model much heavier than 50 GeV makes them natural dark matter candidates, but also led to a very tightly constrained scenario in which the inert LSP (essentially a mixture of the inert singlinos and inert Higgsinos) is now severely challenged by the most recent XENON100 analysis of 100.9 days of data. Moreover the tightly constrained parameter space makes it practically impossible for such a scenario to be consistent with having universal soft mass parameters.

In this paper we have discussed a new variant of the $E_6$SSM, called the EZSSM,
which involves a novel scenario for Dark Matter in which the dark matter candidate is predominantly bino with a mass close to or above 100 GeV which is fully consistent with XENON100. A successful relic density is achieved via its inelastic scattering into nearby heavier inert Higgsinos during the time of thermal freeze-out. The model also predicts two massless inert singlinos which contribute to the number of effective neutrino species at the time of Big Bang Nucleosynthesis, depending on the mass of the $Z'$-boson which keeps them in equilibrium. For example for $m_{Z'} > 1300$ GeV we find $N_{\text{eff}} \approx 3.2$.

We have studied a few benchmark points in the constrained EZSSM with massless inert singlinos to illustrate this new scenario. The benchmark points show that it is easy to find consistent points which satisfy the correct relic abundance as well as all other phenomenological constraints. The points also show that the typical $Z'$ mass is expected to be around 2 TeV, with the gluino having a mass around 500–800 GeV and squarks and sleptons typically having masses around 1–2 TeV. The direct detection spin-independent cross-sections $\sigma_{\text{SI}} \sim \text{few} \times 10^{-48}$ cm$^2$ are well below current sensitivities.

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