How far away is infinity? An electromagnetics exercise to develop intuition regarding models

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Abstract. The estimation of the electric field in simple situations provides an opportunity to develop intuition about the models used in physics. We propose an activity aimed at university students where the electric field of a finite line of charge is compared, analytically or numerically, with the fields of an infinite line and of a point charge. Contrary to intuition, it is not necessary to get very close for the line charge to be considered infinite, nor to move very far away for the finite line field to resemble that of a point charge. We conducted this activity with a group of students and found that many of them have not yet developed an adequate intuition about the approximations used in electromagnetism.

Introduction Modeling in physics is based on subtle approximations. When teaching the subject, however, the limits of its validity are not always explicitly stated or emphasized. Although teaching strategies based on model building have been developed [1, 2] with proven success, modeling is generally not given due emphasis in traditional courses. Point particles, plane waves and rigid bodies are some of the concepts proposed to approximate reality in different situations, but they are often used superficially. In addition to the students’ own difficulties in understanding and using the different models [3], the lack of proper discussion may lead them to become unmotivated and conceive physics as being far from reality [4].

Teaching electromagnetism at introductory levels does not escape the aforementioned difficulties, involving the frequent use of several idealized concepts such as point charges, dipoles, and infinite solenoids. Additionally, the application of some of the laws of electromagnetism, especially Gauss’ law, requires the consideration of abstract objects, Gaussian surfaces, to apply this law and find expressions for the fields with relative ease. It is also common to make different approximations to obtain expressions of the fields at small or large distances from the sources. In order to develop intuition regarding these concepts, we propose the following exercise.
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Figure 1. The situation presented to the students focuses on the estimation of the electric field at a distance \( d \) from a pen charged by rubbing.

The field of a finite line of charge. Imagine an ordinary elongated insulating object, e.g., a ballpoint pen, which is rubbed against the hair and acquires a negative charge that is distributed more or less uniformly, as shown in Fig. 1. If we consider a nearby point, the electric field will be approximately equal to that of an infinite line with the same linear charge density. The question that naturally arises is how far away can we place ourselves so that the two electric fields differ by less than a certain amount. In the opposite scenario, if we consider a distant point, the electric field will be similar to that of a point charge of equal magnitude. Then what is the minimum distance for the two fields to differ by less than a certain amount? As we will see below, the answers are surprising to most of our students.

We consider the electric field of a bar of length \( L \) with a uniformly distributed charge. The magnitude of the field at a distance \( d \) on a perpendicular line passing through its midpoint is

\[
E_{\text{bar}} = \frac{q}{4\pi\varepsilon_0 d^2} \frac{1}{\sqrt{rac{L^2}{4d^2} + 1}},
\]

the quotient of the two fields results in

\[
\frac{E_{\text{bar}}}{E_q} = \frac{1}{\sqrt{(\frac{L}{2d})^2 + 1}}.
\]

As noted above, for distant points tends both fields tend to have the same value. A possible criterion to consider the fields to be approximately equal is that they differ by less than 5\%, i.e., \( E_{\text{bar}}/E_q > 0.95 \). It is easy to see that this condition is met if \( d > 1.52L \), or that the distance must be greater than one and a half times the length of the bar.

Regarding the comparison between the field of the bar with an infinite line, the electric field needs to be expressed as a function of the linear density of charge \( \lambda = Q/L \),

\[
E_{\text{bar}} = \frac{\lambda}{2\pi\varepsilon_0 d} \frac{1}{\sqrt{1 + (\frac{2d}{L})^2}}.
\]
the quotient of the two fields is

\[ \frac{E_{\text{bar}}}{E_{\infty}} = \frac{1}{\sqrt{1 + \left(\frac{2d}{L}\right)^2}}. \]

As noted above, the field of the bar approaches that of an infinite line when \( d \ll L \), i.e. for points that are very close to the bar. Following the aforementioned criterion of admitting a difference of less than 5%, for \( E_{\text{bar}}/E_{\infty} \), in this case it must be verified that \( d < 0.16L \), i.e., the distance must be less than one sixth of the length of the bar.

A comparison of the electric fields considered above is shown in Fig. 2. For small distances, the fields of the finite and infinite lines tend to have the same value, whereas for large distances, the fields of the point charge and the finite line have approximately the same value. It is worth noting that the correction in the electric field in both cases is proportional to the square of the quotient of the relevant lengths \( d \) and \( L/2 \).

**Using spreadsheets.** The questions posed about the line of charge, as well as similar questions about both the electric and magnetic fields, can be answered numerically using spreadsheets. These spreadsheets are powerful tools that allow students to easily obtain answers, compare the validity of different approximations and quickly analyze the dependence with different parameters [5, 6].

In order to use feasible values in everyday situations, we assume that the pen has a length of \( L = 15\text{cm} \) and is charged with \( q = 10^{-9}\text{C} \), a typical value that can be acquired by rubbing. The different parameters are entered into the spreadsheet, as well as one step of distance. Then two columns are drawn for the electric field of the pen at the observation point: one for the field of a charged particle and one for the quotient of both quantities. Based on Fig. 2, we can see that for this distance it results in \( d = 24\text{cm} \). Therefore, the electric field of a typical pen at a distance slightly greater than one and
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A half times its length behaves approximately like that of a charged particle.

**Final comments.** This exercise was proposed to a group of 37 university students in their first year of General Physics studies, in the form of a multiple-choice questionnaire with the instruction to answer both questions without making numerical calculations. The percentage of correct answers for both questions was about 20%. In the vast majority of answers, the maximum distance for the field to resemble that of a line was overestimated, in many cases by up to two orders of magnitude. Likewise, the distance required for the field to resemble that of a point charge was also underestimated by similar magnitudes. This type of analysis can be performed in the classroom in just a few minutes, allowing students to analyze different scenarios and obtain the answer to the validity of different modeling and approximations numerically. It also allows students to quickly become familiar with the order of magnitude of electric field values in different scenarios. The spreadsheet allows them to extend the analysis to problems without analytical solutions or with more complex solutions. Moreover, the analysis can be extended to any expression of electric and magnetic fields, such as a charged disk or a loop through which current is flowing. We believe that this type of exercise helps students to develop physical intuition about the idealizations used in our courses.

**References**

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