Variable Star Classification with a Multiple-input Neural Network

T. Szklenár1,2,3, A. Bódi1,2,3, D. Tarczay-Nehéz1,2,3, K. Vida1,2,3,4, Gy. Mező1,2,3, and R. Szabó1,2,3,4

1 Konkoly Observatory, Research Centre for Astronomy and Earth Sciences (ELKH), H-1121 Budapest, Konkoly Thege Miklós út 15-17, Hungary
2 CSFK, MTA Centre of Excellence, H-1121 Budapest, Konkoly Thege Miklós út 15-17, Hungary
3 MTA CSFK Lendület Near-Field Cosmology Research Group, Konkoly Observatory, Budapest, Hungary
4 ELTE Eötvös Loránd University, Institute of Physics, Budapest, Hungary

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Abstract

In this experiment, we created a Multiple-Input Neural Network, consisting of convolutional and multilayer neural networks. With this setup the selected highest-performing neural network was able to distinguish variable stars based on the visual characteristics of their light curves, while taking also into account additional numerical information (e.g., period, reddening-free brightness) to differentiate visually similar light curves. The network was trained and tested on Optical Gravitational Lensing Experiment-III (OGLE-III) data using all OGLE-III observation fields, phase-folded light curves, and period data. The neural network yielded accuracies of 89%–99% for most of the main classes (Cepheids, δ Scuti, eclipsing binaries, RR Lyrae stars, Type-II Cepheids), only the first-overtone anomalous Cepheids had an accuracy of 45%. To counteract the large confusion between the first-overtone anomalous Cepheids and the RRab stars we added the reddening-free brightness as a new input and only stars from the LMC field were retained to have a fixed distance. With this change we improved the neural network’s result for the first-overtone anomalous Cepheids to almost 80%. Overall, the Multiple-input Neural Network method developed by our team is a promising alternative to existing classification methods.

Unified Astronomy Thesaurus concepts: Astronomy data analysis (1858); Delta Scuti variable stars (370); RR Lyrae variable stars (1410); Eclipsing binary stars (444); Neural networks (1933); Cepheid variable stars (218); Classification (1907)

1. Introduction

Recent sky surveys obtained a vast amount of data that pose previously unseen challenges for astronomers during their analysis. While observations of a few targets can be processed manually, this is not feasible in the case of several or hundreds of thousands of targets. Classification of variable stars is a typical case for utilizing automated data analysis, which can be based on different statistical properties of the light curves like mean, standard deviation, kurtosis, skewness (see, e.g., Nun et al. 2015), Fourier decomposition (Kim & Bailier-Jones 2016), color information (Miller et al. 2015), or applying different machine-learning methods, including random forest (see e.g., Breiman 2001) or deep learning (Zhang & Bloom 2021).

In our previous paper, we introduced an experiment to classify variable stars based on their light curves as images—similarly to what a human astronomer would perform (Szklenár et al. 2020, hereafter referred to as Paper I). For this purpose, we selected five main variable star classes: δ Scuti, eclipsing binaries, RR Lyrae, and anomalous and Type-II Cepheids. This experiment showed that this method is able to classify the different variable types observed by the Optical Gravitational Lensing Experiment (OGLE; Udalski et al. 2015) with 77%–99% accuracy for light curves in the OGLE-III and OGLE-IV databases.

As shown in Paper I, image-based classification of variable stars using a convolutional neural network is a viable method. Although this method can achieve very high accuracy, due the similarity of the phase-folded light curves, to further increase the classification accuracy it requires the usage of additional data. A recurrent neural network (RNN)—as a standard solution for time-series data—could help to distinguish similar features of light curves, like as it was done for identifying stellar flares in Vida et al. (2021). The latter uses Kepler light curves, which, unlike the OGLE light curves, are very well sampled, essentially continuous, and do not contain as many large gaps as the light curves we use. Another main difference is that the cadence is uniform for all Kepler targets, while it is inhomogeneous in case of the OGLE. Therefore, application of the RNN would require extremely meticulous and disproportionately massive preprocessing in our case. In this paper, we extend our experiment by supplying different physical parameters (e.g., period, magnitude) as an auxiliary input to the classifier networks in the hope of improving their performance. The main goal of this work is to investigate the effectiveness of a Multiple-input Neural Network (MINN) in general, where numerical data are expected to help distinguish the main variable stars, and their subtypes as well. To achieve this, we used numerical data of periods and magnitudes attached to images of phase-folded light curves of OGLE-III periodic variable stars (Udalski et al. 2008).

This paper is structured as follows. In Section 2, we present our data and the process of data augmentation. Section 3 describes the MINN, while our results are discussed in Section 4. The paper closes with Section 6, in which we give the concluding remarks on our results, respectively.

2. Data and Methods

The OGLE provides one of the most extensive data sets of variable stars with reliable, human expert verified classifications, which is crucial to train and test a machine-learning algorithm. In Paper I, a convolutional neural network was constructed to...
classify the periodic variable stars based on the observations of the OGLE. Paper I considered only the light curves obtained in the field of the Large Magellanic Cloud, in order to keep the sample as homogeneous as possible. Similar to Paper I, we primarily used the OGLE-III data set, but now we aim to extend our data sample with the Small Magellanic Cloud, the Galactic bulge, and the Galactic disk data sets.

2.1. Observational Data

The OGLE-III catalog lists more than 100,000 variable stars, containing measurements from the Galactic bulge, the Galactic disk, and the Magellanic Clouds. The observations were obtained in the \(I\) and \(V\) bands. As the amount of \(I\)-band observations exceeds many times the \(V\)-band measurements (about 15 times more), we chose to work with the \(I\)-band data only. Along with the photometric observations, the OGLE-III catalog contains some fundamental parameters of the objects (e.g., periods, amplitudes, colors). We collected these values for every variable star presented in our research. From the available information, we utilized the periods and calculated the Wesenheit index (van den Bergh 1975):

\[
W = I - 1.55(V - I),
\]

which served as an additional input parameter for the classification process.

In this work, we focus on six different main variable star types: anomalous Cepheids (ACep; Soszyński et al. 2008), classical Cepheids (Cep; Soszyński et al. 2008; Soszyński et al. 2010c, 2011a), \(\delta\) Scuti (DSct; Poleski et al. 2010), eclipsing binaries (ECL; Graczyk et al. 2011; Pawlak et al. 2013; Pietrukowicz et al. 2013; Soszyński et al. 2016), RR Lyrae stars (RRLyr; Soszyński et al. 2009, 2010a, 2011b), and Type-II Cepheids (T2Cep; Soszyński et al. 2008, 2010b, 2011a). The main variable star classes are divided into several subclasses, excluding those that have only a few members or contain mostly noisy light curves. Table 1 lists the final number of variable stars per classes collected from the OGLE-III database. We note in passing that we used classical Cepheids (Cep) in this work, while we omitted them in Paper I.

Using the epochs and periods from the OGLE-III catalog, the light curves have been phase folded and transformed into 1-bit (black and white) images with a size of 512 \times 512 pixels (see Figure 1). To phase fold the light curves of pulsating variables we used the pulsation periods, while for eclipsing binaries we used the orbital periods (i.e., twice of the formal periods). The horizontal range of the phase-folded light curve is set by the range of light variation (e.g., twice of the formal periods). To clean the data set, phase-folded light curves were fitted with a Savitzky–Golay filter (Savitzky & Golay 1964). Afterward, points that were further than three standard deviations away from the mean of the residual light curves have been excluded.

2.2. Constructing the Training and Testing Sample

Table 1

| Main Type | Subtype | LMC | SMC | Galactic Bulge | Galactic Disk | Total |
|-----------|---------|-----|-----|----------------|--------------|-------|
| ACep      | ...     | 83  | ... | ...            | ...          | 83    |
| F         | 62      | ... | ... | ...            | ...          | 62    |
| IO        | 21      | ... | ... | ...            | ...          | 21    |
| Cep       | ...     | 3262| 4485| 28             | ...          | 7775  |
| F         | 1818    | 2626| 21  | ...            | ...          | 4465  |
| IO        | 1238    | 1644| 4   | ...            | ...          | 2886  |
| IOO       | 206     | 215 | 3   | ...            | ...          | 424   |
| DSct      | ...     | 2788| ... | ...            | ...          | 2788  |
| SINGLEMODE| 2696    | ... | ... | ...            | ...          | 2696  |
| MULTIMODE | 92      | ... | ... | ...            | ...          | 92    |
| ECL       | ...     | 23,993| 6138| ...           | 7434         | 37,565|
| EC        | 1048    | 777 | ... | ...            | 7 434        | 9 259 |
| ED        | 16443   | 5361| ... | ...            | ...          | 21 804|
| ESD       | 6502    | ... | ... | ...            | ...          | 6 502 |
| RRLyr     | ...     | 23,637| 2366| 16,835         | ...          | 42,838|
| RRAB      | 17693   | 1933| 11755| ...         | ...          | 31 381|
| RRC       | 4958    | 175 | 4989 | ...        | ...          | 10 122|
| RRD       | 986     | 258 | 91  | ...           | ...          | 1 335 |
| T2Cep     | ...     | 186 | 36  | 357           | ...          | 579   |
| BLHer     | 64      | 17  | 156 | ...            | ...          | 237   |
| RVTau     | 42      | 9   | 73  | ...            | ...          | 124   |
| WVir      | 80      | 10  | 128 | ...            | ...          | 218   |

Note. Subtypes with only a few members or mostly noisy light curves were excluded.
data augmentation approach, which is different from that used in Paper I. To increase the sample of underrepresented classes, we applied Gaussian process regression (see Section 2.3).

First, we focused only on the classification of the main variable star classes. In this case, data augmentation was applied to construct a data set consisting of 10,000 and 10,500 light curves for classes with two and three subtypes, respectively. During the generation of artificial light curves, the ratio of the various subtypes within a class was taken into account. For example, the original data set contains 83 ACePS, of which 62 are fundamental mode pulsators (F subtype), while 21 are first-overtone pulsators (1O subtype). In this case, we created an augmented data set in which the ratio of F and 1O subtypes are 7470–2530, representing the original proportion. Table 2 lists the number of original and augmented data sets for each variable star type. Note that in those cases where the number of samples within a given subtype in the original data set is more than that is required in the final sample (e.g., in case of eclipsing binaries, RR Lyrae stars), data augmentation was not applied. Instead, the desired number of stars was randomly selected from the original data set.

The data set used for training and testing the neural network contained 62,000 images; 9000 light curves were used from each main variable star type for training/validation with a ratio of 70%/30% and 500 lightcurve images were selected from each variable star subtype for testing purposes.

Second, as many of the subtypes in a given main variable star class show explicitly distinguishable lightcurve shapes we decided to perform training using the subtypes separately.

We divided the main classes into subtypes as they are labeled in the OGLE catalog (e.g., Cepheids pulsating in the fundamental mode—F, first radial overtone—1O, etc.). In this case, each subtype was balanced to contain 5000 light curves, either augmenting the data set or randomly selecting light curves from the original sample. For the training and validation of classification of 16 different labels we sampled 4500 images. For testing, 500 lightcurve images were selected from every variable star subtype without any overlapping with the training and validation samples. Altogether we used 80,000 light curves in this article, which was enough to train and test both the main types and the subtypes. For more details, see Table 2.

Here we note that, in order to avoid false predictions we ensured that the teaching and testing samples do not overlap. For a given star the original and synthetic light curves are only present in one of the three steps: teaching, validation, or testing.

2.3. Generating Synthetic Light Curves

To generate synthetic light curves, a physical model is needed for each variable type represented in our sample. As we lack such a model set, we need a method that is flexible enough to model the different lightcurve shapes, and provide reliable uncertainties. To overcome this problem, the Gaussian process (GP) regression is used.

GPs are stochastic, continuous, nonparametric models (Rasmussen & Williams 2006). GP is a distribution over functions, which is fully described by its mean and a covariance matrix or kernel function. The data are represented with a general multivariate Gaussian distribution

\[
p(m|t, \alpha) = \mathcal{N}(\mu(t), K(t, \alpha)) \tag{2}
\]

where \(m(t)\) is the time series of observations, and \(m\) and \(t\) are the vectors of fluxes and time, respectively; \(\mathcal{N}\) depicts a Gaussian distribution with the mean function \(\mu(t)\) and covariance matrix \(K(t, \alpha)\), and \(\alpha\) is a vector of hyperparameters characterizing the covariance matrix. The mean function can be set to any function; however, it is often considered to be zero. The kernel function describes the covariance between any pair of the points drawn from a given GP. Usually a kernel function assumes that the covariance between points is a function of the distance between the points of the independent variable, which is the time (or phase) in case of a light curve.

The covariance matrix, \(K\), is defined as:

\[
K_{ij} = \sigma_i^2 \delta_{ij} + k_{ij}(\tau_{ij}), \tag{3}
\]

where \(\sigma_i^2\) is the measurement error given for the \(i\)th observation, \(\delta_{ij}\) is the Kronecker delta function, and \(k_{ij}(\tau_{ij})\) is the kernel with \(\tau_{ij} = |t_i - t_j|\); the distance between \(i\)th and \(j\)th time points.

The GP regression optimizes the set of parameters \(\alpha\) by minimizing the log-likelihood function

\[
\ln L(\alpha) = -\frac{1}{2} r^T K^{-1} r - \frac{1}{2} \ln |K| - \frac{N}{2} \ln(2\pi), \tag{4}
\]

where \(r\) is the residual after subtracting the mean model from the observations, and \(N\) is the number of data points.
As the computational cost of GP regression scales with $O(n^3)$, we have to choose an implementation that is computationally efficient. In this work we used the GP implemented in the exoplanet\(^5\) python package (Foreman-Mackey et al. 2021).

The first step of GP modeling is to choose an adequate kernel. In the exoplanet implementation the kernel is a mixture of exponential functions:

$$k_\alpha(\tau_{ij}) = \sum_{m=1}^{M} a_m \exp(-c_m \tau_{ij}). \tag{5}$$

If the $a_m$ and $c_m$ parameters are complex numbers, $a_m \rightarrow a_m \pm ib_m$, $c_m \rightarrow c_m \pm id_m$, then Equation (5) can be rewritten as a sum of sine of cosine terms and the result is a mixture of quasiperiodic oscillators:

$$k_\alpha(\tau_{ij}) = \sum_{m=1}^{M} \left[ a_m \exp(-c_m \tau_{ij}) \cos(d_m \tau_{ij}) + b_m \exp(-c_m \tau_{ij}) \sin(d_m \tau_{ij}) \right], \tag{6}$$

where the parameter set $\alpha = \{a_m, b_m, c_m, d_m\}$.

From the available kernels we chose the RotationTerm, which is a mixture of two SHO terms, and can be used to model stochastic variability in a time series. In the Fourier space, the SHO term represents a stochastically-driven, damped harmonic oscillator with power spectral density (Foreman-Mackey et al. 2017):

$$S(\omega) = \frac{2}{\pi} \frac{S_0 \omega_0^4}{(\omega^2 - \omega_0^2)^2 + \omega_0^2 Q^2}, \tag{7}$$

where $\omega$ is an angular frequency, $\omega_0$ is the frequency of the undamped oscillator, $Q$ is the oscillator’s quality factor, and $S_0$ is proportional to the power at $\omega = \omega_0$,

$$S(\omega_0) = \frac{2}{\pi} S_0 Q^2. \tag{8}$$

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\(^5\) exoplanet is a toolkit for probabilistic modeling of astronomical time series, which is built upon theano (Theano Development Team 2016), PyMC3 (Salvatier et al. 2016), and celerite (Foreman-Mackey et al. 2017).

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Table 2

Data Used From OGLE-III

| Main Type | Subtype | Original\(^a\) | Main Type Sample Training/Testing | Subtype Sample Training/Testing | OGLE-III LMC Field Only |
|-----------|---------|---------------|----------------------------------|----------------------------------|------------------------|
| ACep      | ...     | 83            | 9 000/1 000                      | 4500/500                        | 83                     |
|           | IO      | 21            | 2277/500                         | 4500/500                        | 21                     |
|           | F       | 62            | 6723/500                         | 4500/500                        | 62                     |
| Cep       | ...     | 7775          | 9000/1500                        | 4500/500                        | 3262                   |
|           | IO      | 2886          | 3341/500                         | 4500/500                        | 1818                   |
|           | IO2O    | 424           | 491/500                          | 4500/500                        | 1238                   |
|           | F       | 4 465         | 5168/500                         | 4500/500                        | 206                    |
| DSct      | ...     | 2788          | 9000/1000                        | 4500/500                        | 2788                   |
|           | MULTIMODE | 92             | 297/500                          | 4500/500                        | 2 696                  |
|           | SINGLEMODE | 2696         | 8703/500                         | 4500/500                        | 92                     |
| ECL       | ...     | 37,565        | 9000/1500                        | 4500/500                        | 23,993                 |
|           | EC      | 9259          | 2218/500                         | 4500/500                        | 1008                   |
|           | ED      | 21,804        | 5224/500                         | 4500/500                        | 16,443                 |
|           | ESD     | 6502          | 1558/500                         | 4500/500                        | 6502                   |
| RRLyr     | ...     | 42,838        | 9000/1500                        | 4500/500                        | 23,637                 |
|           | RRAB    | 31 381        | 6593/500                         | 4500/500                        | 17,693                 |
|           | RRC     | 10,122        | 2127/500                         | 4500/500                        | 4958                   |
|           | RRD     | 1335          | 280/500                          | 4500/500                        | 986                    |
| T2Cep     | ...     | 579           | 9000/1500                        | 4500/500                        | 186                    |
|           | BLHer   | 237           | 3684/500                         | 4500/500                        | 64                     |
|           | RVTau   | 124           | 1927/500                         | 4500/500                        | 42                     |
|           | WVir    | 218           | 3389/500                         | 4500/500                        | 80                     |
| Total used |        | 54,000        | 8000/8000                        | 72,000                          | 72,000/8000            |

Note.

\(^a\) The original OGLE-III data set from the LMC, SMC, galactic bulge, and disk fields.
Foreman-Mackey et al. (2017) showed that if the parameters are chosen properly, then Equation (7) can be matched to Equation (6), of which the kernel can be rewritten as

\[
\begin{align*}
 k_{\text{SHO}}(\tau; S_0, Q, \omega_0) &= S_0 \omega_0 Q \exp\left(-\frac{\omega_0 \tau}{2Q}\right) \\
 &= \begin{cases} \\
 \cosh(\eta \omega_0 \tau) + \frac{1}{2\eta Q} \sinh(\eta \omega_0 \tau), & 0 < Q < 1/2 \\
 2(1 + \omega_0 \tau), & Q = 1/2, \\
 \cos(\eta \omega_0 \tau) + \frac{1}{2\eta Q} \sin(\eta \omega_0 \tau), & 1/2 < Q \\
\end{cases}
\end{align*}
\]

where \( \eta = |1 - (4Q^2)^{-1}|^{1/2} \).

The goal of the GP regression is to represent the different lightcurve shapes with models whose confidence intervals can be used to sample new data sets from the original measurements. As, instead of the time series itself, we use the well-sampled phase-folded light curves for the classification; we used the latter for GP regression too. The optimization of the kernel parameters were based on Bayesian parameter estimation with normally distributed priors. As we only want to represent the different lightcurve shapes and are not interested in the actual parameter values and its uncertainties, to minimize the log-likelihood, we used the \texttt{minimize} method from \texttt{scipy.optimize} (Virtanen et al. 2020).

For each underrepresented variable star, where we needed to augment the data set, we randomly sampled new points from the GP posteriors at random phase values. The number of new points were the same as the original ones. Some example phase-folded light curves, along with their GP fits, and the synthetic data sets can be seen in Figure 2. Figure 3 shows a gallery of artificially generated lightcurve images for subtypes, where data augmentation was needed.

### 2.4. Numerical Data

Due to the similarities of the lightcurve shapes, the classification using only the lightcurve images led to false predictions. We needed additional data to be able to distinguish the different variable stars with even higher accuracy. We created a data file containing every numerical data of the objects used in our project downloaded from the OGLE-III database.\(^6\) These numerical parameters were the star’s pulsation

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\(^6\) https://ogledb.astrouw.edu.pl/~ogle/CVS/
or orbital period, apparent magnitude, and amplitude. This sample was extended with the Wesenheit index.

We worked with 80,000 images and the same amount of numerical data in the final data set. For the augmented light curves, the same physical parameters were given as for the original data set.

3. Multiple-input Neural Network

This work relies on Paper I, which gives us a solid base to try to improve the performance of the convolutional neural network (CNN) and integrate it with new features. The neural network in Paper I used phase-folded lightcurve images as inputs. Although this CNN worked reliably, the similarity of light curves belonging to different variable star classes led to false predictions. As in this paper the light curves are augmented with a different method; we had to make changes to the CNN to be able to extract the fine features. After testing the performance of the CNN we aimed to create a neural network in which—besides the CNN—additional numerical inputs can be used for the classification process. We called this latter architecture a Multiple-input Neural Network.

3.1. Architecture of the Neural Network

The network was developed using the Keras API built over TensorFlow (Abadi et al. 2015), an open source platform for machine learning. For detailed description of the different layers used in the network, see Paper I.

First we designed a multi-input neural network to classify the different variable stars observed in all OGLE-III fields into six main and 16 subclasses, separately. The architecture of this network, which uses two different inputs (images and numerical data), can be seen in Figure 4.

After some experimentation we retained only the period as the most informative parameter as an additional input in this neural network.

In the case of the LMC, the neural network was extended to handle the Wesenheit index as an additional input in order to classify a variable star with similar distances (i.e., absolute magnitudes) into subtypes. This architecture can be seen in Figure 5.

3.2. Image Classification

The architecture of the image classification section of the network can be separated into four different parts. The first part consists of two convolutional layers with $7 \times 7$ and $5 \times 5$ convolutional windows, followed by a dropout and a pooling layer. The purpose of this first section is the extraction of low-level features and the resizing of input images.

The second part contains two blocks that are exactly the same, having three convolutional layers using $3 \times 3$ convolutional windows, ending with one dropout and one pooling layer. In these layers the high-level features are extracted.

The third section has only two convolutional layers with the usual dropout and pooling layers. The output matrix reaches its minimum size after the last pooling layer, where we apply a flattened layer to create a usable input for the last section.

In the fourth section we use only fully connected (FC) layers with a decreasing number of units. The last FC layer applies a softmax activation to classify the images into separate variable star classes or subclasses.

The original resolution of the generated lightcurve images was $512 \times 512$ pixels, which was reduced to $128 \times 128$ pixels during the read-in of the files, and the pixel values were converted to integer numbers. These greatly improved the running time of the algorithm while preserving the information content and yielding the same performance. Figure 8 shows features learned by the convolution neural network (part of the MINN), visualized by images of the latent spaces of the convolutional layers.

Figure 4. Schematic view of the architecture of the Multiple-input Neural Network built in this study. In this case two inputs are present, the first one is the data set of phase-folded lightcurve images, the second one is the numerical data of the periods associated with the given stars. This figure shows the neural network used to classify stars into main variable star classes. When we classified stars into subclasses, the number of the neurons in the last dense layer and those dense layers that are right before the concat layer are changed to 16.

Figure 5. The architecture of our extended Multiple-input Neural Network, where the inputs are the following: phase-folded images, the period, and Wesenheit index of the variable stars. We used this architecture to classify stars observed in the LMC into 16 different variable star subclasses.
### Table 3

| Parameter                        | Tested Values | Chosen Value |
|----------------------------------|---------------|--------------|
| **Architecture**                 |               |              |
| Starting convolution window      | [3 × 3]       | [7 × 7]      |
| Convolution stride               | [5 × 5]       | [7 × 7]      |
| Convolution padding              | [1, 2, 3]     | 1            |
| Convolution activation           | 0             | 0            |
| **Optimization**                 |               |              |
| Dropout probability              | [0.1–0.5]     | 0.3          |
| Pooling type                     | MaxPooling    | ReLU         |
| Pooling size                     | [2 × 2, 3 × 3]| [2 × 2]      |
| Number of convolution layers     | 10            | 10           |
| Number of pooling layers         | 4             | 4            |
| Number of dense layers           | 6             | 6            |
| Dense activation function        | ReLU          | ReLU         |

3.3. Handling Numerical Data

As discussed in Section 2.4, in order to improve the effectiveness of the classification process, we aimed to include additional data for each variable star. OGLE-III database contains basic physical parameters for each target. These are basically floating point numbers, which can be used as input parameters without any preprocessing in the neural network. Thus, a simple dense layer with softmax activation was applied after the read-in of the numerical input, which classifies the stars based on their periods/brightness into separate variable star classes or subclasses. The size of the output changed according to the number of examined types.

3.4. Concatenating the Outputs

As both the image and the numerical data have the same number of output classes, we can concatenate them and use the result as a new input for further classification. The two (or more) concatenated outputs will be handled as a single input into a fully connected layer with 64 units and this will be sent to a softmax dense layer, which will make the final classification.

3.5. Optimizer, Learning Rate, and Batch Size

We tested in detail the performance of the Adam (Adaptive Moment estimation) optimizer with various setups, changing the learning rate between $10^{-1}$ and $10^{-5}$ and the batch size in the range of 32 to 2048. We found that our model performed the best with a learning rate of 0.00025 and a batch value of 256; thus we chose this parameter set in our final model. Table 3 lists the tested and best hyperparameters of the neural network.

### Table 4

| Mean Accuracy | Std Deviation | Best Accuracy |
|---------------|---------------|---------------|
| Training      | 93.90%        | ±0.90%        | 95.38%        |
| Validation    | 93.23%        | ±0.78%        | 94.35%        |

Note. The individual deviation values can be seen in Figure 9, embedded into the confusion matrix.

3.6. Early Stopping

To avoid overfitting, we applied an EarlyStopping callback during training, which monitors the change of the validation loss value. We set the min_delta and patience values to $10^{-4}$ and 19, respectively. These parameters control the minimum change in the monitored quantity to qualify as an improvement during training, and the number of epochs’ with no improvement after which training will be stopped, respectively.

With these parameters, the callback monitors the validation loss change and if it does not decrease by at least $10^{-4}$, the callback runs for another 19 additional epochs, stops the training process, and saves the best weights for further testing.

3.7. Performance

For the training and testing process, we used a GPU-accelerated computer containing NVidia GeForce RTX 2080 Ti GPU cards. The training phase usually took about 300 epochs, where one epoch lasted for 6 s. The whole training and validation phase took about 1.5 hr. The classification of the complete test data set (8000 variable stars) took approximately 3 s.

3.8. K-fold Cross-validation

As the performance of a neural network is highly dependent on the training set, we decided to carry out a K-fold cross-validation test to quantify the reliability of our CNN classification results. Here, the first step is to separate a subset from the whole data set, which is used for the testing. The remaining data set is separated into $k$ distinct parts with equal sizes. From these a single subset is used for testing, and the last $k-1$ sets are used for training the model. The process is repeated $k$ times; each time a different subset is used for testing purposes.

To receive a statistically meaningful result, we performed a tenfold cross-validation on the variable star data set with subtype labels. The images and numerical parameters within each subtype were split into 10 nonoverlapping parts, each containing 500 light curves and periods. From these 10 data packages, 4500 inputs were used for training and 500 were used for testing purposes. After performing the cross-validation, we calculated the mean and standard deviation of the accuracies to characterize how well our neural network works. The results are listed in Table 4 and visualized in Figure 9.

4. Results and Discussion

In the following, we evaluate the training performance, present our classification results on the test sample of the main and subtypes of variable stars, separately. Moreover, we...
examine the performance of the network for stars with known distances in the LMC.

4.1. Training Performance

Figure 6 shows the evolution of accuracy and loss during the training and validation of the network as a function of epoch for both the CNN (left-hand side panel), and the multiple-input neural network (right panel), and for the six main (top row) and sixteen subtype variable star classes (bottom row). Running on the main groups, the convolutional neural network achieved about 90% accuracy, and 140 epochs before the early stopping completed the teaching. If the same neural network was taught with 16 different subclasses, although it was able to continue to run much longer, the accuracy did not change significantly from the 100th epoch, and in terms of the end result it reached significantly lower accuracy than the previous one. The neural network produced the worst loss values in this run. If additional input data were used, the curves smoothed out and the results improved. This is true for neural networks run on both main groups and subgroups. Beside some subtle anomalies, the accuracy increased and the loss decreased continuously. After 400 epochs, where the accuracy reached about 95%, the loss started to flatten and shortly after this point the early stopping terminated the learning. As the training and validation metrics evolved in the same pace, the network did not overfit. The figure also contains a comparison on how well the neural network performs using the variable star main types or the subclasses. Although the performance is very similar, distinguishing between the six main type labels is more accurate than in the 16 subclass case.

4.2. Classification of the Six Main Variable Star Classes

Figure 7 shows the confusion matrix of the classification results of the six main variable star types. Compared to the early test results from the plain convolutional neural network without additional input parameters, like the period (see, e.g., Szklenár et al. 2020), the first tests with additional numerical data showed greatly improved results regarding the identification of the six main variable star types. One can see in Figure 7 that the accuracy of the well-represented variable stars, e.g., RR Lyrae stars, is high (~93%), while in case of ACeps, the classification result is ~50%. The accuracy for most main types is around or over 90%. About 9% of the Type-II Cepheid test sample mix with classical Cepheids, a small amount (six RR Lyrae stars) mix with δ Scutis. Here false classification happened due to short period RR Lyrae stars and long period δ Scutis. The two most accurate classes are the eclipsing binaries and the δ Scutis, where almost every light curve was classified correctly. The classical Cepheid test light curves slightly mix with the RRLyr and Type-II Cepheids.

4.3. Classification of 16 Variable Star Subclasses

In order to test the efficacy and performance of our method we decided to apply our MINN to subclasses as well. Most of the variable star subclasses show explicitly distinguishable lightcurve shapes. The data set was split so that we could perform training for 16 different subtypes, which were selected...
from all the OGLE-III fields. Just like in the previous section, light curves and the period values were used as inputs. We used the same network architecture as for the main variable stars, but the number of predicted classes were changed to 16. The test results for this network are shown in Figure 9. We added the estimated scatter from the K-fold cross-validation to the confusion matrix.

Our main goal was to distinguish 16 different subtypes, we examined how these subclasses perform within their main group. The different main variable star groups were marked with orange boxes in Figure 9 and this shows that although there is scatter between the related subclasses, if we summarize the main types, the precision of the classification is over 90%.

Now we can see that the ACep 1O subtype performs the worst and the test data’s classification is mixing with the RRab group. This is due to the similar shape of the light curves and the periods.

The scatter of the accuracy for the ACep 1O variables is 22%-28%, meanwhile the scatter is lower (around 1%-6%) in case of the other classes.

There is a slight, 5.6% mix between the Cep 1O2O and the RRd subgroups. Both subtypes pulsate with two modes, but we phase fold every light curve only in the dominant mode, so these show significant distortion. Another significant mixing is between the short period BLHer subtype and the classical Cepheid 1O group. The classification of the δ Scuti and ECL types is near perfect, if we look only at the whole main groups. But if we look at the subclasses, distinguishing the two δ Scuti subclasses is difficult; the neural network confused about 10% of the test data with the other subgroup. According to the eclipsing binaries, identifying the detached or semidetached eclipsing binary stars is also a difficult task; the transition between these two groups is continuous (Bódi & Hajdu 2021).

4.4. Utilizing Brightness for Stars with Known Distances

As we demonstrated above, some subtypes are still hardly distinguishable based on their lightcurve shape and period, e.g., RRab and ACep stars. Here, we carry out an experiment, where additional parameters are fixed, and might be able to break this degeneracy. LMC comes to the rescue, where most of the variable (sub)types are relatively well represented. Restricting ourselves to LMC only is beneficial, since it means that we can fix the distance, and the intrinsic luminosity difference will betray the various variable classes. Here we neglect the depth
of the LMC, since the resulting error in luminosity introduced by this simplification is smaller than the intrinsic luminosity.

The OGLE-III database contains auxiliary information about each star, e.g., its magnitude, both in $I$ and $V$ bands, and the amplitude of variation, which can feed into our Multi-Input Neural Network as well. We calculated the reddening-free Wesenheit index for each selected LMC variable star, and extended our neural network with this new input. The architecture of this network with an additional input can be seen in Figure 5.

As the mixing of the ACep 1O variable star subclass with the RRab stars is very high, we chose to discard every—possibly—foreground RR Lyrae star that is brighter than 18 magnitude (Soszyński et al. 2009). Using this limit, 495 RR Lyrae stars were removed; this way we could reduce the degeneracy between RRab and ACep 1O stars, which have similar periods.

As there is less data in the LMC field only compared to the whole OGLE-III database, we had to generate additional artificial light curves for this training. The used variable star subtypes remained the same, 5000 light curves were used from each subtype. Altogether we used 72,000 light curves for the training and validation process and another 8000 light curves for testing purposes, without any overlapping between the two data sets.

Figure 9. Confusion matrix of the test sample that summarizes the prediction results of 16 variable star subtypes. For this task, the Multiple-input Neural Network was used with lightcurve images and periods as inputs. The individual deviation values from the K-fold cross-validation are shown with red color below the prediction values. For better readability the main variable star classes are marked with orange boxes.

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The confusion matrix in Figure 10 contains our classification result of the test data from the OGLE-III LMC field. The classification result for each variable star was carefully checked; the misclassified stars were further investigated using the OGLE-III and OGLE-IV collection of variable stars.

In Figure 12 the classification result of the 8000 test light curves is shown and to have better understanding about the test results, we illustrated these with three distinct groups: correctly classified stars, mixed stars within their own main group, and incorrectly identified stars. Figure 11 contains typical and interesting misclassified light curves; we will further investigate these stars below.

The removal of the foreground RRab stars greatly improved the neural networks accuracy by the distinction between of the ACep and RRLyr groups. The overall classification accuracy of the ACep variable star subclasses improved greatly at least with 30%. Still 20.4% of the ACep 1O stars are classified as RRab by our MINN, but these misclassifications belong to the OGLE-LMC-ACEP-083, which was reclassified as an RRab star in the OGLE-IV database (see Soszyński et al. 2015a). Indeed, most of the artificial light curves generated from the ACep-083 observations were classified as RRLyr stars.

Regarding the misclassifications of classical Cepheids, two groups can be mentioned. In the first group we can find stars

![Confusion Matrix](image)

*Figure 10. Confusion matrix of the testing phase, which shows the performance of our Multiple-input Neural Network using OGLE-III data only from the LMC field. The inputs were phase-folded light curves and the given star’s period and Wesenheit index. For better readability the main variable star groups are marked with orange boxes.*
with very short periods, especially the 1O2O mode Cepheids, which mix with the δ Scutis. The stars in the other misclassified group are identified as ACep stars; these have a period of about 1 day long and a characteristic, large amplitude light curve. According to the main group of classical Cepheids there is great mixing between the stars with different pulsation modes. About 13% of the 1O mode Cepheids mix with the 1O2O pulsation mode stars.

The OGLE-LMC-CEP-3063 star was classified as a 1O classical Cepheid in the OGLE-III catalog, but has been reclassified as an eclipsing binary with a new ID in the OGLE-IV catalog (OGLE-LMC-ECL-37568, see Soszyński et al. 2015b). Using the period that is given in the OGLE-III catalog, the phase-folded light curve indeed shows a continuous variation that resembles a pulsating star. However, OGLE-IV lists a new period, which is twice the previous one. Using the latter, the phase curve immediately reveals alternating minima, which is characteristic of eclipsing binaries. As we used the OGLE-III data set, our classification has been misled by the wrong period, which is crucial in our case.

Another interesting case is the OGLE-LMC-CEP-3356, which has an uncertain classification in the OGLE-III and OGLE-IV catalogs as a possible RR Lyrae star (Cep; Soszynski et al. 2008). Our classification result shows 82.1% probability that this star is an RRLyr.

The δ Scuti stars performed very well, just 1% of the stars were not identified as δ Scuti. There is a significant mixing between the Singlemode and Multimode stars. We found two interesting cases. The first is OGLE-LMC-DSCT-2662, which is cataloged as an uncertain δ Scuti star in Poleski et al. (2010). Our classification shows 51.4% probability for 1O Cepheid and 45.2% for 1O2O Cepheid. It has an apparent proximity to a bright star, very short period, and a barely noticeable 0.01 magnitude amplitude, therefore it is most probably a blend. OGLE-LMC-DSCT-2788 is much brighter (15.13 mag in I band) than the other data from the LMC field, which will probably makes it a galactic δ Scuti star. Our classification shows 64.9% probability for a 1O Cepheid and 30.6% for 1O2O Cepheid.

The Wesenheit index as additional input did not change much in the classification result of eclipsing binary stars. Although there is significant mixing between the subgroups, only 2% of all test eclipsing stars were identified as a different variable star type.

Examining the results of the RRLyr stars we can conclude that almost all classifications belong to the main RRLyr type; thus mixing occurs mainly between the subgroups only. The RRab and RRc subclasses performed particularly well, but almost 13% of the RRd subclass is mixing between the RRc subgroup. This is understandable, since most of the RRd stars are overtone dominated—the amplitude of the radial overtone mode is higher than that of the fundamental mode.

Regarding the misclassification of Type-II Cepheids the following cases can be mentioned. The WVir star, OGLE-LMC-T2CEP-185, was classified as a classical Cepheid with F-pulsation mode (57.6%). It is a relatively bright—14.5 mag (I-band)—star, with with a 12.7 day long period. This star was labeled as an outlier in the period–radius relation of Type-II Cepheids and was identified as a possible binary star by Groenewegen & Jurkovic (2017a, 2017b).

The BLHer type star, OGLE-LMC-T2CEP-187, was classified as an anomalous Cepheid with 1O pulsation mode. Another BLHer type star, OGLE-LMC-T2CEP-188 was classified as an anomalous Cepheid by our MINN. The OGLE-IV catalog contains only four measurements in V band. Because of this, the calculated V brightness shows 0.5 magnitude change compared to the OGLE-III data. If the Wesenheit index would be

Figure 11. The light curves of the highlighted misclassified stars from Figure 12.
calculated from this V-band measurement, the star would lie in the ACep region in the period–Wesenheit relation.

There are three RV Tauri stars in the LMC, which show, besides pulsation, long-term mean brightness variation. To phase fold the light curves of these stars, we used the pulsation period, as this is given as the primary variability in the OGLE catalog. In case of the RVTau type star, OGLE-LMC-T2CEP-200, the variation of the mean amplitude is so large that it makes the pulsation pattern unrecognizable after phase folding the light curve, leaving us with a confident classification as an eclipsing binary, whose subgroup contains noisy light curves with very long periods.

OGLE-LMC-T2CEP-203 belongs to the RVTau subtype, the original data were correctly classified as an RVTau star, yet misidentified most of the artificial light curves generated from this star as a WVir. Probably because the light curve’s shape is not stable in time and the generation of artificial light curves removed this information.

5. Trained Weights and Code Availability

We decided to publish the codes and weight files obtained by the neural network on the website of our institute, so that other research groups can use them as well. The files are available from the following link: (https://konkoly.hu/KIK/data_en.html#ML).

6. Conclusions

In this paper we trained, validated, and tested a Multi-Input Neural Network (MINN), which consists of an image classifier convolutional neural network and simple dense layers that are used to handle additional numerical input data. The light curves, from which the input images were generated, were downloaded from the OGLE-III database with the corresponding physical parameters (periods, brightness). To have as much data as possible, we collected light curves from the LMC, SMC, Galactic bulge, and Galactic disk fields. Because of the highly unbalanced number of stars in different classes we generated artificial light curves to have equal amounts of data in each variable star type. The augmented light curves were sampled from the posterior distributions of Gaussian process regression of real observations.

For the classification we tested two kinds of setups: a six-class input where the data sets of the main variable classes were merged, and a 16-class input where all the subclasses were handled separately. The test showed that utilizing the periods beside the images of phase-folded light curves significantly improves the classification results. From the previous 77%—99% (Szkenlár et al. 2020) we were able to improve the accuracy to 89%—99%. Nonetheless, in case of underrepresented variable star subtypes, the classification results are significantly worse than for the other types, even using the additional numerical data. The low number of known anomalous Cepheids (ACeps) prevents us compiling a diverse training sample for the network—even with augmented training data. This explains the poor performance of the network in the case of first-overtone ACep variables (see Figures 7 and 9).

As an experiment, we restricted the training sample only to the LMC field, where the distance of the variable stars was fixed. The extended neural network received three different inputs: the phase-folded light curves, the periods, and the reddening-free brightness (Wesenheit index). The intrinsic luminosity difference helped to distinguish the first-overtone ACep and the RRab variable stars with higher precision.

To be able to handle such an extensive amount of data as the OGLE catalog we used a high-performance GPU-accelerated computer. The generation of artificial lightcurve images took the most of time in the project due to the low scalability of Gaussian process regression. The training and testing of the neural network took about 1.5 hr, after which we saved the weight file that could be used for further prediction and classification of thousands of light curves within just a couple of seconds.

As the various sky survey programs generate such vast amount of data each night, astronomers need to develop methods to be able to identify the different celestial objects in a reliable, accurate, and efficient way. These methods can be used not only by current sky surveys, e.g., the Zwicky Transient Facility (ZTF; Masci et al. 2018), which has a continuously growing data set of ∼1 billion light curves, but also by such future projects, like the Vera C. Rubin Observatory Legacy Survey of Space and Time (LSST; Ivezic et al. 2019). We note that these surveys will have to accumulate enough data to build their own training samples in order to use our method. Depending on the strategy of the survey, some of these will reach that point pretty soon, e.g., quasi-continuous space-based photometric surveys, like TESS or PLATO, while others, like LSST will have to wait for months–years to have sufficient number of data points for a given object. Also, in order to use our method, we need to know whether a given object is a (periodic) variable star or not, we need to know its period and even its phase for more accurate classification. Such auxiliary information will not always be delivered by the official pipeline of a given survey, but for example in the case of LSST, in-kind contributions and brokers might deliver such data.

Training samples tailored to the characteristics of specific surveys and well-designed neural networks can greatly accelerate data analysis with high reliability. We believe that the method we have developed and the ones based on it will be capable of this task in the future.

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Software: Python (van der Walt et al. 2011), Numpy (van der Walt et al. 2011), Pandas (McKinney 2010), Scikit-learn (Pedregosa et al. 2011), Tensorflow (Abadi et al. 2015), Keras (Chollet & others 2018).

Appendix

Period–Luminosity Relation
Figure 12. Test result of the pulsating variable stars from the LMC field, based on their period–Wesenheit index relation. The training and the test was based on phase-folded light curves and the given star’s period and Wesenheit index. The neural network was trained on 16 variable star subtypes. For better readability we grouped them with one color for each main type and only the pulsating variable stars are shown in this figure. The crosses without a border are from the training and validation; the correct classifications are shown with black borders. Those misclassified stars that are plotted with red borders remained in their main group. The triangles with red edges are “true” misclassifications, which are found mostly on the borders of different pulsating variable star types.

ORCID iDs

T. Szklenár https://orcid.org/0000-0002-5610-7697
A. Bódi https://orcid.org/0000-0002-8585-4544
D. Tarczay-Nehéz https://orcid.org/0000-0003-3759-7616
K. Vida https://orcid.org/0000-0002-6471-8607
Gy. Mező https://orcid.org/0000-0002-0686-7479
R. Szabó https://orcid.org/0000-0002-3258-1909

References

Abadi, M., Agarwal, A., Barham, P., et al. 2015, TensorFlow: Large-Scale Machine Learning on Heterogeneous Systems, https://www.tensorflow.org/
Bódi, A., & Hajdu, T. 2021, ApJS, 255, 1
Breiman, L. 2001, Mach. Learn., 45, 5
Chollet, F. & others 2018, Keras: The Python Deep Learning library, Astrophysics Source Code Library, ascl:1806.022
Foreman-Mackey, D., Agol, E., Ambikasaran, S., & Angus, R. 2017, AJ, 154, 220
Foreman-Mackey, D., Savel, A., Lugger, R., et al. 2021, exoplanet-dev/exoplanet v0.5.0, zenodo, doi:10.5281/zenodo.1998447
Graczyk, D., Sośmyński, I., Poleski, R., et al. 2011, AcA, 61, 103
Groenewegen, M. A. T., & Jurkovic, M. I. 2017a, A&A, 603, A70
Groenewegen, M. A. T., & Jurkovic, M. I. 2017b, A&A, 604, A29
Ivezić, Ž., Kahn, S. M., Tyson, J. A., et al. 2019, ApJ, 873, 111
Kim, D.-W., & Bailer-Jones, C. A. L. 2016, A&A, 587, A18
Maschi, F. J., Laher, R. R., Rulshom, B., et al. 2018, PASP, 131, 018003
McKinney, W. 2010, in Proc. of the 9th Python in Science Conf., ed. S. van der Walt & J. Millman (Austin, TX: SciPy), 51

Miller, A. A., Bloom, J. S., Richards, J. W., et al. 2015, ApJ, 798, 122
Nun, I., Protopapas, P., Sim, B., et al. 2015, arXiv:1506.00010
Pawlak, M., Graczyk, D., Sośmyński, I., et al. 2013, AcA, 63, 323
Pedregosa, F., Varoquaux, G., Gramfort, A., et al. 2011, JMLR, 12, 2825
Pietrukowicz, P., Mróz, P., Sośmyński, I., et al. 2013, AcA, 63, 115
Poleski, R., Sośmyński, I., Udalski, A., et al. 2010, A&A, 60, 1
Rasmussen, C. E., Williams, C. K. I. 2006, Gaussian Processes for Machine Learning (Cambridge, MA: MIT Press)
Salvatier, J., Wiecki, T. V., & Fonnesbeck, C. 2016, PeerJ Comp. Sci., 2, e55
Savitzky, A., & Golay, M. J. E. 1964, AnaCh, 36, 1627
Soszyński, I., Udalski, A., Szymański, M. K., et al. 2010a, AcA, 60, 165
Soszyński, I., Udalski, A., Szymański, M. K., et al. 2010b, AcA, 60, 91
Soszyński, I., Udalski, A., Szymański, M. K., et al. 2008, AcA, 58, 293
Soszyński, I., Udalski, A., Szymański, M. K., et al. 2009, AcA, 59, 1
Soszyński, I., Poleski, R., Udalski, A., et al. 2010c, AcA, 60, 17
Soszyński, I., Udalski, A., Szymański, M. K., et al. 2011a, AcA, 61, 285
Soszyński, I., Dziembowski, W. A., Udalski, A., et al. 2011b, AcA, 61, 1
Soszyński, I., Udalski, A., Szymański, M. K., et al. 2015a, AcA, 65, 233
Soszyński, I., Udalski, A., Szymański, M. K., et al. 2015b, AcA, 65, 297
Soszyński, I., Pawlak, M., Pietrukowicz, P., et al. 2016, AcA, 66, 405
Szklenár, T., Bódi, A., Tarczay-Nehéz, D., et al. 2020, ApJL, 897, L12
van den Bergh, S. 1975, Stars and Stellar Systems (Chicago, IL: Univ. Chicago Press)
Virtanen, P., Gommers, R., Oliphant, T. E., et al. 2020, NatMe, 17, 261
Zhang, K., & Bloom, J. S. 2021, MNRAS, 505, 515

Miller, A. A., Bloom, J. S., Richards, J. W., et al. 2015, ApJ, 798, 122
Nun, I., Protopapas, P., Sim, B., et al. 2015, arXiv:1506.00010
Pawlak, M., Graczyk, D., Sośmyński, I., et al. 2013, AcA, 63, 323
Pedregosa, F., Varoquaux, G., Gramfort, A., et al. 2011, JMLR, 12, 2825
Pietrukowicz, P., Mróz, P., Sośmyński, I., et al. 2013, AcA, 63, 115
Poleski, R., Sośmyński, I., Udalski, A., et al. 2010, A&A, 60, 1
Rasmussen, C. E., Williams, C. K. I. 2006, Gaussian Processes for Machine Learning (Cambridge, MA: MIT Press)
Salvatier, J., Wiecki, T. V., & Fonnesbeck, C. 2016, PeerJ Comp. Sci., 2, e55
Savitzky, A., & Golay, M. J. E. 1964, AnaCh, 36, 1627
Soszyński, I., Udalski, A., Szymański, M. K., et al. 2010a, AcA, 60, 165
Soszyński, I., Udalski, A., Szymański, M. K., et al. 2010b, A&A, 60, 91
Soszyński, I., Udalski, A., Szymański, M. K., et al. 2008, AcA, 58, 293
Soszyński, I., Udalski, A., Szymański, M. K., et al. 2009, A&A, 59, 1
Soszyński, I., Poleski, R., Udalski, A., et al. 2010c, A&A, 60, 17
Soszyński, I., Udalski, A., Pietrukowicz, P., et al. 2011a, A&A, 61, 285
Soszyński, I., Dziembowski, W. A., Udalski, A., et al. 2011b, A&A, 61, 1
Soszyński, I., Udalski, A., Szymański, M. K., et al. 2015a, A&A, 65, 233
Soszyński, I., Udalski, A., Szymański, M. K., et al. 2015b, A&A, 65, 297
Soszyński, I., Pawlak, M., Pietrukowicz, P., et al. 2016, A&A, 66, 405
Szklenár, T., Bódi, A., Tarczay-Nehéz, D., et al. 2020, ApJL, 897, L12
Theano Development Team 2016, arXiv:1605.02688
Udalski, A., Szymański, M. K., Sośmyński, I., & Poleski, R. 2008, A&A, 50, 293
Udalski, A., Szymański, M. K., & Szymański, G. 2015, A&A, 65, 1
van den Bergh, S. 1975, Stars and Stellar Systems (Chicago, IL: Univ. Chicago Press)
van der Walt, S., Colbert, S. C., & Varoquaux, G. 2011, CSE, 13, 22
Vida, K., Bódi, A., Szklenár, T., & Seli, B. 2021, A&A, 652, A107
Virtanen, P., Gommers, R., Oliphant, T. E., et al. 2020, NatMe, 17, 261
Zhang, K., & Bloom, J. S. 2021, MNRAS, 505, 515