Hawking radiation as tunneling from Gravity’s rainbow

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Abstract

Planck scale corrections arising from deformed special relativity on Hawking radiation in Parikh and Wilczk’s tunneling framework are studied. We calculate the emission rate of massless particles tunneling though the corrected horizon of modified black holes from gravity’s rainbow. In the tunneling process, when a particle get across the quantum horizon, the metric fluctuation not only due to the energy conservation but also quantum effects of the space-time are taken into account. Our results show that, the emission rate is related to the changes of the black hole’s quantum corrected entropy and consistent with an underlying unitary theory. In the modified black hole, by using black hole thermodynamics, a series of quantum correction terms including a logarithmic term to the Bekenstein-Hawking entropy are obtained. Correspondingly, the Planck scale corrected emission spectrum is obtained and it deviates from the thermal spectrum.

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I. INTRODUCTION

Based on Hawking’s great discovery \[1, 2\] that black holes have emission of thermal radiation, the General Relativity, Quantum Mechanics and Thermodynamics have a link with profound physical significance and the black hole thermodynamics get a solid fundament. However, following this, the information loss paradox emerges \[3, 4\]. It is that, since a thermal spectrum can not bring out any information other than only one parameter of temperature, when a black hole evaporate away completely, all information about the matter making up of the black hole will be lost. Accordingly, Hawking argued that the formation and evaporation of a black hole are not commanded by the quantum mechanics \[3\]. That is to say, the pure states of matter forming the black hole evolve into the mixed states of thermal radiation, thus, the underlying unitary theory of the quantum mechanics is violated. Besides, in the light of Hawking’s description, the black hole evaporation is a quantum tunneling effect \[5\]. That is to say, due to vacuum fluctuations near the horizon, when a pair of particles are spontaneously created just inside the horizon, the positive energy particle can tunnel out the horizon to the infinity. At the same time, the negative energy particle remains behind the horizon and effectively lowers the mass of the black hole because the negative energy orbit can not exist outside the horizon. However, the actual derivation of Hawking radiation did not proceed in this way, most of which based upon quantum field theory on a fixed background space-time without considering the gravity back-reaction of the emitted particles and the quantum fluctuation of the space-time \[6, 7\].

Recently Hawking has put forward \[4\] his new viewpoint on the information loss paradox that the information hidden in a black hole could come out if Hawking radiation was not exactly thermal but had some corrections. And that, Parikh and Wilczek have presented a semi-classical method of calculating the emission rate by implementing the Hawking radiation as a tunneling process from the horizon, in which the non-thermal spectrum i.e. back-reaction corrected radiation spectrum is obtained \[8, 9, 10\]. This simply and availably method presents the quantum tunneling description on Hawking radiation and support information conservation in the radiation process of black holes. In the tunneling approach, the particles get across classically forbidden trajectories, staring just behind the horizon onward to infinity, and the potential barrier is created just by the particle’s self-gravitation. The crucial point of the program is that the energy conservation has been taken into account.
and the background is allowed to fluctuate because of the particle’s back-reaction. Following this method, some recent research has been dedicated to extend this tunneling study to many cases including static or stationary black holes [11, 12, 13, 14, 15], cosmological horizons [16, 17] and different kinds of tunneling particles such as massive and charged particles [12, 13]. The same results, that is, Hawking radiation is not purely thermal spectrum, unitary theory is satisfied and information is conserved, are obtained. However, the quantum effects of space-time have not been considered in Parikh and Wilczek’s original work [8, 9, 10] and much less attention in the literature was paid to the particle’s tunneling from quantum horizon [18, 19].

It is remarkable that, Hawking radiation opens an important window to quantum gravity. Meanwhile, for comprehensively understand Hawking radiation and some closely related problems, such as the information loss puzzle and the origin of black hole entropy, the Planck-scale physics is necessary. In fact, in the study of black hole thermodynamics, quantum gravity has acquired notable achievements but it has still many challenges. For example, which is here of interest, the derivations in string theory support the idea that Hawking radiation can be described within a manifestly unitary theory, but it remains a mystery how information is returned [20]. And that, the black hole emission spectrum obtained from loop quantum gravity is still a Hawking’s thermal spectrum [21, 22]. In addition, both string theory and loop quantum Gravity are successful in statistically explanation of black hole entropy, but on the quantum correction to Bekenstein-Hawking (B-H) entropy, the two leading candidate theories of quantum gravity have a contradiction [23, 24, 25, 26, 27, 28]. In the two Plank-scale physics, the coefficient of the leading-order correction has been presented with different values. And also, even in loop quantum gravity, there is still debate on the coefficient [27, 28].

As one generally believed viewpoint, the existence of a minimally observable length order of Planck length is a universal feature of quantum gravity [29, 30]. Recently, such character has invoked some research on the fate of Lorentz symmetry at Planck scale. The reason is that, the character of Plank scale physics, which in principle may contract any object to arbitrarily small size by Lorentz boost, seemly leads to a apparent confliction with Lorentz symmetry. At present, when keeping Planck energy as an invariant scale, namely a universal constant for all inertial observers, to preserve the relativity of inertial frames, a deformed formalism of special relativity has been proposed [31, 32]. This deformed special relativity...
(DSR) points to the possibility that the usual energy momentum relation in special relativity may be modified in term of the ratio of particle’s energy to Planck energy. As the main prediction of DSR, modified dispersion relations (MDR) has got both experimental and theoretical supports \[32, 33\]. In the same time, great efforts have been devoted to DSR and its implications \[32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43\]. In \[38, 39\], based on DSR, the quantum corrections to B-H entropy have been investigated and obtained. And, in \[41\], DSR has been extended to general relativity. The main feature of DSR incorporating with the curvature of space-time is that the geometry of space-time depend on the energy of a particle moving in it. That is to say, for the space-time with Planck scale correction effects arising from DSR, there are different geometries of space-time for particles with different energies. The modified geometries of space-time are described by one parameter family of metric as a function of particle energy observed by an inertial observer, namely the rainbow metric. The Schwazschild solution of the rainbow metric has been presented in \[41\]. And that, it’s some thermodynamics quantities and asymptotic flatness have been investigated in \[42, 43\], respectively.

In this paper, we study the particle’s tunneling from the modified Schwazschild solution. Our aim is to incorporate Plank scale physics with Parikh-Wikzek’s semi-classical method of investigating Hawking radiation and then to investigate some Planck scale modification effects on black hole evaporation, entropy and information paradox. In a general way, the black hole radiation spectrum have arbitrarily high frequencies and their energy can go below the Planck energy \[19, 44, 45\]. Therefore, in Parikh and Wilczek’ tunneling framework, the Planck scale modification effects on the Hawking radiation should be taken into account. In the present tunneling investigation, while a particle tunneling though the quantum corrected horizon, the metric fluctuation not only due to the energy conservation but also quantum effects of the space-time is taken into account. Here, the quantum effects of space-time are presented by using the rainbow metric obtained from DSR. The results of the paper show that, when the Plank scale modification of space-time is incorporated with the particle’s self-gravitation in the tunneling program, the emission rate is related to the changes of the modified black holes entropy and consistent with an underlying unitary theory. Here, the black hole entropy include the quantum correction to Bekenstein-Hawking (B-H) entropy. By using the first law of black hole thermodynamics to the modified black holes, a series of quantum correction terms including a logarithmic term are obtained and
the result is consistent with a rigorously statistical calculation in loop quantum gravity [28]. Correspondingly, the emission spectrum with correction arising from energy conservation also quantum gravity effects is obtained. The corrected spectrum is departure from the pure thermal spectrum and it’s statistical correlations is discussed.

The paper is organized as follows. In Sec. II the modified Schwarzschild solution from the gravity’s rainbow in the context of DSR is introduced briefly and its some thermodynamics quantities is investigated. Then in Sec. III by using the Parikh-Wikzek’s framework, the emission rates of massless particles tunneling through the horizon of the modified black holes are calculated. Then in Sec. IV the entropy of the modified black hole is investigated and the quantum correction include a logarithmic item to the B-H entropy is obtained. Accordingly, the deviation of the emission spectrum of the modified black hole to the thermal spectrum is obtained. The last part is the summary and conclusion.

II. THE MODIFIED BLACK HOLES FROM GRAVITY’S RAINBOW

To investigate the Hawking radiation as tunneling from a quantum corrected horizon, firstly we briefly introduce the modified Schwarzschild solution from gravity’s rainbow and analyze its some thermodynamics quantities.

The staring point and main result of DSR is MDR [32], namely

\[ E^2 f_1^2 (E; \lambda) - p^2 f_2^2 (E; \lambda) = m_0^2, \]  \hspace{1cm} (1)

where \( f_1 \) and \( f_2 \) are two energy functions from which a specific formulation of boost generator can be defined, in which \( \lambda \) is a parameter of order the Planck length. The equation concretely indicates that, MDR is energy dependent. It is to say, particles with different energy \( E \) have different energy-momentum relations.

From DSR, it has been pointed out that the flat space-time has the invariant [40, 41]

\[ ds^2 = -\frac{dt^2}{f_1^2} + \frac{dr^2}{f_2^2} + \frac{r^2}{f_2^2} d\Omega^2. \]  \hspace{1cm} (2)

Thus, the DSR space-time is endowed with an energy dependent quadratic invariant, that is, an energy dependent metric, namely rainbow metric.

By extending the Eq. (2) to incorporate curvature, in [41], the modified Schwarzschild solution has been demonstrated in terms of energy independent coordinates and the energy
independent mass parameter $M$ as
\[dS^2 = -(\frac{1-\frac{2GM}{r}}{f_1^2})dt^2 + \frac{1}{f_2^2(1-\frac{2GM}{r})}dr^2 + \frac{r^2}{f_2^2}d\Omega^2.\] (3)

Obviously, the metric is also depend on the energy of particle moving in it. That is, if a given observer probes the space-time using the quanta with different energies, he will conclude that space-time geometries have different effective description. Here, the particle’s energy $E$ denotes it’s total energy measured at infinity from the black hole. By this, the present space-time is endowed with a Plank-scale modification and has some quantum effects.

In addition, from (3) we can see the modified Schwarzschild solution is asymptotically DSR. And that, it has been pointed that the asymptotically DSR space-times has equality with the usual asymptotically flat space-times \[43\]. Then, using the Komar integrals, we define the total Arnowitt-Deser-Misner (ADM) mass $M_{ADM}$ of the quantum effected space-time as
\[M_{ADM} = -\frac{1}{8\pi G} \int_s \varepsilon_{abcd} \nabla^c \xi^d = \frac{M}{f_1 f_2}.\] (4)

We find that, for the modified Schwarzschild black holes, the ADM mass is not equal to the mass parameter $M$. It shows that the quantum corrected space-time have topological defects. And that, the total energy of the space-time is depend on the energy of the probe particle. This is because the quantum correction effects of the black hole arise from DSR, which has energy dependence.

Also, From the metric (3), we can see its horizon $r_+ = 2GM$ is universal for all observers and at the usual place as the usual Schwarzschild black hole. However, the horizon area
\[A = \frac{16\pi G^2 M^2}{f_2^2}\] (5)
is different from the usual value and depend on the particle’s energy. This should has some modification effects on the black hole thermodynamics.

Besides, the surface gravity on the horizon is defined by
\[\kappa = -\frac{1}{2r-r_+} \sqrt{-g^{rr}(g^{tt})'} g^{tt}.\] (6)

Thus, from (3), we can obtain the surface gravity of the modified black hole as
\[\kappa = \frac{f_2}{f_1} \frac{1}{4GM}.\] (7)

Obviously, it also depends on the particle’s energy.
Therefore, the temperature of the modified black hole is obtained as

\[ T = \frac{\kappa}{2\pi} = \frac{f_2}{f_1} \frac{1}{8\pi GM}. \] (8)

It show that the temperature of the modified Schwarzschild solution is dependent on the energy of probe particle. That is, using the quanta with different energy, an observer at infinity will probe different effective temperature for the quantum corrected black hole.

Obviously, in the modified black holes, the energy dependence of thermodynamics quantities arises from DSR and is the exhibition of quantum effects of the space-time. And that, analysis on the characters is necessary for the applications of energy conservation and black hole thermodynamical law in the quantum corrected space-time.

III. TUNNELING PROBABILITY IN MODIFIED BLACK HOLES

DSR and gravity’s rainbow are low energy effect of quantum gravity. It is that, the modified black holes (3) is a coarse grained model of space-times at semi-classical level. Here, we assume that Parikh and Wilczk's quantum tunneling program of investigating Hawking radiation is still holds for the large modified black holes. Therefore, in this section, following Parikh and Wilczk's tunneling framework, we calculate the tunneling probability of massless particles in the quantum modified black hole. The novel point of the present tunneling investigation is that the quantum effect of geometry are considered in the tunneling process.

In Parikh and Wilczk’s tunneling scheme, the particle behind the horizon can tunnel out along a classically forbidden trajectory and the tunneling probability is given by means of WKB approximation. That is, the emission rate can be expressed as the imaginary part of the action for the trajectory 8, 9, 10

\[ \Gamma \sim \exp(-2 \text{Im} I). \] (9)

For calculating the action \( I \) in the modified black holes showing as (3), the coordinate singularity at the horizon must be removed. Here, we introduce a new time coordinate \( t \) and follow a Painleve type coordinate transformation. Letting

\[ dt_{s} = dt - F(r) dr, \] (10)
and
\[ \frac{1}{f_2^2 \left(1 - \frac{2GM}{r}\right)} - \frac{1}{f_1^2} F^2 (r) = 1, \]  
(11)

then we have
\[ ds^2 = - \frac{1}{f_1^2} dt^2 + \frac{2}{f_1 f_2} \sqrt{1 - f_2^2 \left(1 - \frac{2GM}{r}\right)} dt dr + \frac{f_2^2}{f_2^2} \left( d\theta^2 + \sin^2 \theta d\varphi^2 \right). \]  
(12)

It is easy to find that the Painleve-like metric of the modified black holes has some advantages for us to implement the calculation on the emission rate of particle tunneling through a quantum corrected horizon. Firstly, none of the components of either the metric or the inverse metric diverges at the horizon. Secondly, the coordinate system has Killing vector \( \partial/\partial t \). In addition, as expected, the metric has Planck scale correction effects showing as the energy dependence. It denotes that, even if the black hole has a fixed mass parameter \( M \), the emitted particles with different energy will be affected by different metric.

It is assumed that a massless particle with energy \( E = \frac{1}{f_1} \omega \) measured at infinity tunnels out the horizon of the modified black hole. For the massless particle, its motion equation can be given by the radial null geodesics on the geometry (12). Let \( ds^2 = 0 \), in the presence of Planck scale effects, we have the radial null geodesic as
\[ \dot{r} = \frac{dr}{dt} = \frac{1}{f_1 f_2} \left[ \pm 1 - \sqrt{1 - f_2^2 \left(1 - \frac{2GM}{r}\right)} \right], \]  
(13)

where "+" corresponding outgoing particles, "-" corresponding ingoing particles.

However, if we enforce the energy conservation of the space-time, when the particle tunnels out the horizon, the mass of the black hole should vary. That is, the back-reaction of emitted particles should affect the background geometry. In spherical symmetry space-time, the back-reaction effects of emitted shell have been investigated in detail [47]. Here, we treat the particle as a s-wave i.e. a shell. Thus, when the particle radiate outside the horizon of the modified black hole, since the particle’s self-gravitation, the mass parameter \( M \) in the metric (12) should be replaced with \( M - \omega \) [8, 9, 10, 47]. This is consistent with Birkhoff’s theorem. The theorem tell us that, in the spherical symmetry space-time, the only effect on the geometry due to the s-wave is to provide a junction condition for matching the total mass inside and outside the shell. Then we get the geometry between the horizon
and the spherical shell as

$$\text{ds}^2 = -\left(\frac{1-\frac{2G(M-\omega)}{r}}{f_1^2}\right)dt^2 + \frac{2}{f_1 f_2} \sqrt{1 - \frac{f_2^2}{f_1} \left(1-\frac{2G(M-\omega)}{r}\right)} \text{d}t \text{d}r + \text{d}r^2 + \frac{r^2}{f_2^2} \left(\text{d}\theta^2 + \sin^2 \theta \text{d}\varphi^2\right).$$

(14)

And that, we can see the locations of the horizon before and after the particle’s emission are $r_i = r_+(M) = 2GM$ and $r_f = r_+(M-\omega) = 2G(M-\omega)$, respectively. Thus, by the shrinking of the black hole, the tunneling barrier is created by the emitted particle itself due to the energy conservation of the space-time. Furthermore, the set of the barrier is not affected by the Planck scale modification effects of the black hole-emitted particle system.

In addition, it has been proved that the motion of the emitted particle is affected by the geometry between the event horizon and the spherical shell [47]. Thus, considering the background metric’s dynamical effects due to energy conservation and the quantum effects of the space-time, the tunneling particle’s radial motion equation should be modified as

$$\dot{r} = \frac{dr}{dt} = \frac{1}{f_1 f_2} \left(1 - \sqrt{1 - f_2^2 \left(1 - \frac{2G(M-\omega)}{r}\right)}\right).$$

(15)

Also, for the tunneling process, a canonical Hamiltonian treatment gives a simple result for the total action of the black hole-particle system [47],

$$I = \int dt \left( p_t + \frac{dt}{dr} p_r \right),$$

(16)

where $p_t$ and $p_r$ are the conjugate momentum corresponding to Painleve’s coordinates $t$ and $r$, respectively. Here, only the second term in (16) contributes to the imaginary part of the action. Therefore, we can obtain the tunneling probability for an outgoing massless particle by computing the imaginary part of the action, which is

$$\text{Im} I = \text{Im} \int_{t_i}^{t_f} dt \frac{dr}{dt} p_r = \text{Im} \int_{r_i}^{r_f} p_r dr = \text{Im} \int_{r_i}^{r_f} dp_r dr,$$

(17)

where $t_i$ and $t_f$ are the Painleve coordinate times corresponding to $r_i$ and $r_f$, respectively.

To proceed with an explicit computation, we now apply the Hamilton’s equation

$$\dot{r} = \frac{dH}{dp_r} = \frac{dM'_{\text{ADM}}}{dp_r},$$

(18)

there $M'_{\text{ADM}} = \frac{1}{f_1 f_2} M'$is the ADM mass of the modified black hole after emitting a particle with energy $E' = \frac{1}{f_1 f_2} \omega'$. Substituting Eq.(18) into Eq.(17), and switching the order of
integral, we have

\[ \text{Im} I = \text{Im} \int_{r_i}^{r_f} \int_0^{M'_{ADM}} \frac{dM'_{ADM}}{r} dr = \text{Im} \int_M^{M'_{ADM}} \int_r^{r_f} \frac{dM'}{r} \]

\[ = \text{Im} \int_M^{M'_{ADM}} \int_r^{r_f} \frac{f_1}{f_2} \left( 1 + \sqrt{1 - f_2^2 \left( 1 - \frac{r_+}{r} \right)} \right) \frac{dr}{r} \frac{M'}{f_1 f_2}, \quad (19) \]

where \( r'_+ = 2GM' \) is the horizon location after emitting the particle. Considering the particle tunneling through the horizon, we can see that \( r'_+ \) is a single pole in Eq. (19). Then the integral can be evaluated by deforming the contour around the pole. In this way, we finished the integral over \( r \) and get

\[ \text{Im} I = -4\pi G \int_M^{M'_{ADM}} \frac{f_1}{f_2} M' \frac{M'}{f_1 f_2}, \quad (20) \]

Now, for the modified black hole in the tunneling process, we apply the first law of black hole thermodynamics

\[ dM'_{ADM} = T'dS'. \quad (21) \]

In fact, many previous works [13, 14, 15] in Parikh and Wilczk’s tunneling framework have confirmed that the tunneling process is consistent with the first law of black hole thermodynamics. Then, inserting the temperature expression (8) into (21), we have

\[ 4\pi G \frac{f_1}{f_2} M' \frac{M'}{f_1 f_2} = \frac{1}{2} dS', \quad (22) \]

and

\[ \text{Im} I = -\frac{1}{2} \int_S^{S + \Delta S} dS' = -\frac{1}{2} \Delta S, \quad (23) \]

where \( \Delta S = S (M - \omega) - S (M) \) is the difference of the black hole entropies of the modified black hole before and after the emission.

Thus the tunneling probability of the massless particle from the quantum corrected horizon is obtained, namely

\[ \Gamma = \exp \left( -2 \text{Im} I \right) = \exp \left( \Delta S \right). \quad (24) \]

We find that the tunneling rate is related to the change of the modified black hole entropy and is consist with an underlying unitary theory. It is the same result obtained from the
usual Schwarzschild black hole\cite{8,9,10}. However, in the quantum corrected space-time, the present black hole entropy should have quantum correction to B-H entropy. This is a radical difference with Parikh and Wilczek’s original results, in which, black hole entropy is obtained and applied as B-H entropy. Accordingly, the emission spectrum of the modified black hole should has corresponding Planck scale corrections to the usual spectrum from the usual black hole. In the next section, by calculating the \ref{22}, we obtain the quantum corrected entropy and the corrected emission spectrum of the modified black hole is given and discussed.

IV. ENTROPY AND RADIATION SPECTRUM OF THE MODIFIED BLACK HOLES

In the present tunneling investigation, to obtain the entropy of the modified black hole by calculating the \ref{22} and based on the entropy to analyze the radiation spectrum, we need the explicit DSR i.e. specific correction functions $f_1$ and $f_2$. Some research has been devoted to the investigation on the explicit MDR models and different functions $f_1$ and $f_2$ have been proposed\cite{32,34}. However, as so far, the standard form of $f_1$ and $f_2$ has not been given and the further investigation are necessary. In the low energy realm i.e. $E/E_p<<1$, where $E_p \equiv 1/\sqrt{8\pi G}$ is the Planck energy, the correspondence principle requires that $f_1$ and $f_2$ approach to unit. Here, for convenience, we take

$$f_1 = f = e^{-\frac{1}{2}E^2/E_p^2}, f_2 = 1. \quad (25)$$

Then, based on the specific MDR, from the Eq.\ref{5} and Eq.\ref{8}, the horizon area and the temperature of the modified black holes are, respectively,

$$A = 16\pi G^2 M^2, \quad (26)$$

$$T^2 = \frac{1}{f^2} \frac{1}{(8\pi GM)^2}. \quad (27)$$

And that, from Eq.\ref{22}, we have the differential equation of the black hole entropy as

$$dS = 8\pi GM f d\left(\frac{M}{f}\right) = 4\pi GdM^2 - 8\pi GM^2 \frac{df}{f}. \quad (28)$$

We find that, for the modified black hole, the entropy equation is depend on the energy of particle. It is to say, the effective black hole entropy has dependence on the energy of
probe particle. Now, for large modified black holes, we use the characteristic temperature by identifying the energy of particles emitted from the black holes with the hole’s temperature \[39, 42, 48, 49\], namely

\[ E = T. \] (29)

This can be understood as a statistical treatment for explicitly obtaining the black hole entropy. It is that, supposing all the emitted particles form an ensemble outside the black hole, then the average energy of the particles is equal to the temperature of the black hole. In other words, we use the particle with energy \( T \) to probe the entropy and ascertain it as the intrinsic entropy i.e. the black hole entropy. Then we have

\[ f = e^{-\frac{1}{2}T^2/E_p^2}, \quad df = -\frac{1}{2E_p^2}dT^2. \] (30)

For the large modified black hole with \( M >> 1/\sqrt{8\pi G} \), we have \( T^2/E_p^2 \sim E_p^2/M^2 << 1 \), and

\[ T^2 = \frac{1}{f^2 (8\pi GM)^2} = \left(1 + \frac{T^2}{E_p^2} + \cdots \right) \frac{1}{(8\pi G)^2 M^2} \simeq \frac{1}{(8\pi G)^2 M^2} + \frac{1}{8\pi G M^2} T^2. \] (31)

Substituting Eq. (26) into Eq. (31) and solving it, we have

\[ T^2 = \frac{1}{4\pi A} \frac{1}{1 - \frac{2G}{A}} = \frac{1}{4\pi A} \left(1 + \frac{2G}{A} + \left(\frac{2G}{A}\right)^2 + \cdots \right) = \frac{1}{4\pi A} \frac{1}{1 - \frac{2G}{A}} \sum_{n=1}^{\infty} \left(\frac{2G}{A}\right)^n. \] (32)

Then, substituting Eq. (30) and Eq. (32) into the Eq. (28), we obtain

\[ dS = d \left(\frac{A}{4G}\right) - \frac{1}{2} d\ln \left(\frac{A}{4G}\right) + \sum_{n=1}^{\infty} \frac{n+1}{n} \frac{1}{2n+1} d \left(\frac{4G}{A}\right)^n. \] (33)

Next, integrating the Eq. (33), up to a constant term, the entropy expression of the modified black holes can be obtained as

\[ S = \frac{A}{4G} + c_0 \ln \left(\frac{A}{4G}\right) + \sum_{n=1}^{\infty} c_n \left(\frac{4G}{A}\right)^n + \text{const}, \] (34)

where \( c_0 = -1/2, \ c_n = (n+1)/n2^{n+1}. \)

It is worth to point out that, the present leading order correction to the B-H entropy goes as the logarithm of the black hole area. It is consistent with many other research’s results (for a review of the correspondence see [50]). In particle, the factor \( c_0 = -1/2 \) is the same as the loop quantum theory prediction by the direct counting the micro-states of black holes.
Of course, this happen on the condition that a special form of MDR shown as Eq. (25) has been proposed.

Now, substituting Eq. (34) into the Eq. (24) and thinking of the Eq. (26), we can obtain the radiation spectrum of the modified black holes, namely,

\[ \Gamma \sim \exp (\Delta S) = \exp (S(M - \omega) - S(M)) \]

\[ = \prod_{n=1}^{\infty} \exp \left( \frac{c_n}{(4\pi G)^n} \frac{1 - (1 - \frac{\omega}{M})^{2n}}{M^{2n} (1 - \frac{\omega}{M})^{2n}} \right) \left( 1 - \frac{\omega}{M} \right)^{-1} \exp \left( -8\pi GM\omega \left( 1 - \frac{\omega}{2M} \right) \right). \] (35)

Compared with the usual self-gravitation correction radiation spectrum from the usual black holes derivation found in \[8, 9, 10\]

\[ \Gamma \sim \exp \left( -8\pi GM\omega \left( 1 - \frac{\omega}{2M} \right) \right), \] (36)

we find the present radiation spectrum has a series of quantum modification factors and it further derived from the pure thermal spectrum. But, if we do not consider the corrections from quantum effects of gravity, i.e. neglecting the logarithmic correction term and the inverse area items in the black hole entropy, the factor of the final exponential in Eq. (35) equals to unit and the radiation spectrum has the same type of non-thermal form shown as (36). And that, if we further overlook the effect of emitted particle’s back-reaction with neglecting \( \omega/M \) in the expression (35), the tunneling rate of the large modified black holes takes the form of Boltzmann factor \( e^{-\beta \omega} \) with \( \beta \equiv 1/T = 8\pi GM \) and the Hawking’s thermal formula is obtained.

Next, for analyzing how to get information from the radiation spectrum, we discuss the statistical correlations for the probabilities of different emission models in the present emission spectrum [10, 18, 19]. The statistical correlation between two emission probabilities of two emitted quanta is measured by the function

\[ \chi (\omega_1 + \omega_2; \omega_1, \omega_2) = \ln(\Gamma(\omega_1 + \omega_2)) - \ln(\Gamma(\omega_1)\Gamma(\omega_2)). \] (37)

Where we assumes that the two quanta with energies \( \omega_1 \) and \( \omega_2 \) are emitted out successively and \( \Gamma(\omega_1 + \omega_2) \) presents the tunneling probability of particle with energy \( \omega = \omega_1 + \omega_2 \). For the Hawking’s thermal emission spectrum, the correlation function is zero showing that the probabilities of different models are independent. And that, for the back-reaction corrected non-thermal spectrum in the usual black hole, the function also vanishes [10]. It is that,
in Parikh-Wikzek’s tunneling program, the back-reaction effects alone do not provide a statistical correlations between different quanta in Hawking radiation.

From Eq. (35), we have the emission rates of the two particles of $\omega_1$ and $\omega_2$ as, respectively,

$$\ln[\Gamma(\omega_1)] = -8\pi GM\omega_1 \left(1 - \frac{\omega_1}{2M}\right) - \ln \left(1 - \frac{\omega_1}{M}\right) + \sum_{n=1}^{\infty} \frac{c_n}{(4\pi G)^n} \frac{1 - \left(1 - \frac{\omega}{M}\right)^{2n}}{M^{2n} \left(1 - \frac{\omega_1}{M}\right)^{2n}},$$

(38)

$$\ln[\Gamma(\omega_2)] = -8\pi G(M - \omega_1)\omega_2 \left(1 - \frac{\omega_2}{2(M - \omega_1)}\right) - \ln \left(1 - \frac{\omega_2}{M - \omega_1}\right) + \sum_{n=1}^{\infty} \frac{c_n}{(4\pi G)^n} \frac{1 - \left(1 - \frac{\omega_2}{M - \omega_1}\right)^{2n}}{(M - \omega_1)^{2n} \left(1 - \frac{\omega_2}{M - \omega_1}\right)^{2n}}.$$

(39)

Alternatively, a single emission of the particle with energy $\omega$ is

$$\ln[\Gamma(\omega_1 + \omega_2)] = -8\pi GM(\omega_1 + \omega_2) \left(1 - \frac{\omega_1 + \omega_2}{2M}\right) - \ln \left(1 - \frac{\omega_1 + \omega_2}{M}\right) + \sum_{n=1}^{\infty} \frac{c_n}{(4\pi G)^n} \frac{1 - \left(1 - \frac{\omega_1 + \omega_2}{M}\right)^{2n}}{M^{2n} \left(1 - \frac{\omega_1 + \omega_2}{M}\right)^{2n}}.$$

(40)

Thus, we obtain the correlation function of the two tunneling models as

$$\chi(\omega_1 + \omega_2; \omega_1, \omega_2) = 0.$$

(41)

We find that, for the quantum corrected black hole, the correlation function between two sequential emission rate still vanishes as in the usual black hole. That is to say, the Planck scale corrected non-thermal spectrum obtained in the modified black hole, up to high order correction factors, has not provided the clear mechanism for recovery of information from the tunneling process. It implies that, how to straightway obtain information from the emission rate of black hole is still an open problem.

V. SUMMARY AND DISCUSSIONS

In the present work, incorporating Planck scale physics with the Parikh-Wikzek’s tunneling program \[8, 9, 10\], Hawking radiation as tunneling of particles in the modified black hole from the rainbow metric is investigated. While the particles tunnel across the horizon, by applying DSR and it’s quantum space-time effects, the quantum gravity effects of the black hole-particles system are taken into account. Thus, both the geometry of space-time
and the energy momentum relation of particle depend on the particle’s energy. Therefore, in the tunneling process from the quantum corrected black hole, the background metric is dynamical, due to not only energy conservation but also the quantum effects of geometry. We find that, in the context of DSR and gravity’s rainbow, the tunneling probabilities of massless particle from the quantum corrected horizon are related to the changes of the modified black holes entropy, and the derived emission spectrum depart from the pure thermal spectrum, but it is consistent with an underlying unitary theory.

For the modified black hole, by analyzing it’s some thermodynamics quantities and using the first law of black hole thermodynamics, the entropy with a series of quantum correction terms is obtained. Here, the leading order correction item is the logarithm of the black hole area and the expression of black hole entropy is consistent with a statistical calculation in loop quantum gravity \[28\]. Accordingly, the Planck scale corrected emission spectrum in the modified black hole is obtained and it deviates from the thermal spectrum. However, though the emission spectrum present a series of quantum correction factors, it has not provide a desired correlation between different emission models. Then, in the quantum corrected black hole, how to encode information and obtain it by the tunneling process is not as clear as in the usual black hole. Meanwhile, from the entropy calculation in the modified black hole, a specific MDR of Eq\[25\] is selected and the obtained result of entropy formula Eq\[34\] support the choice. Black hole entropy is an important landmark to Planck scale physics. As a low-energy quantum gravity effect, different models of MDR with some underlying meaning in the quantum gravity should be tested with black hole entropy.

The research here not only provides further evidence to support the Parikh-Wikzek’s tunneling program, which gives an explicit calculation to investigate Hawking radiation and related problem, but also gives an extension for the tunneling analysis from classical black hole to a quantum corrected black hole. And that, the work can be extended to other modified black holes from gravity’s rainbow. On this issue, further work is in progress.
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