Invoking the virial theorem to understand the impact of (dry) mergers on the $M_{bh}$-$\sigma$ relation

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ABSTRACT

While dry mergers can produce considerable scatter in the (black hole mass, $M_{bh}$)–(spheroid stellar mass, $M_{*}$,sph) and $M_{bh}$–(spheroid half-light radius, $R_{e}$,sph) diagrams, the virial theorem is used here to explain why the scatter about the $M_{bh}$–(velocity dispersion, $\sigma$) relation remains low in the face of such mergers. Its small scatter has been claimed as evidence of feedback from active galactic nuclei (AGNs). However, it is shown that galaxy mergers also play a significant role. The collision of two lenticular (S0) galaxies is expected to yield three types of merger product (a core-Sérsic S0, an elliptic ES,e or an elliptical E galaxy), depending on the remnant’s orbital angular momentum. It is shown that the major merger of two S0 galaxies with $M_{e}$,sph $\sim$ 10$^{11}$ M$_{\odot}$ advances the system along a slope of $\sim$5 in the $M_{bh}$-$\sigma$ diagram, while a major E+E galaxy merger moves a system slightly along a trajectory with a slope of $\sim$9. Mergers of lower-mass S galaxies with $M_{e}$,sph $\sim$ 10$^{10}$ M$_{\odot}$ move slightly along a trajectory with a slope of $\sim$3, thereby further contributing to the steeper distribution for the E (and Es,e) galaxies in the $M_{bh}$-$\sigma$ diagram, reported here to have a slope of 7.27$\pm$0.91, compared to the S0 galaxies which have a slope of 5.68$\pm$0.60. This result forms an important complement to the AGN feedback models like that from Silk & Rees, providing a more complete picture of galaxy/(black hole) coevolution. It also has important implications for nanohertz gravitational wave research.

Key words: galaxies: bulges – galaxies: elliptical and lenticular, cD – galaxies: structure – galaxies: interactions – galaxies: evolution – (galaxies:) quasars: supermassive black holes

1 INTRODUCTION

Before it was observed, Silk & Rees (1998) predicted a relation between a halo’s central supermassive black hole mass, $M_{bh}$, and the host halo’s velocity dispersion $\sigma$, such that $M_{bh} \propto \sigma^6$. Haehnelt et al. (1998) built on this by predicting $M_{bh} \propto M_{*}$,sph$^{6.7}$. Using rotational velocities to probe the halo mass, Ferrarese (2002) reported $M_{bh} \propto M_{*}$,sph$^{6.5}$, which with $M_{sph}$ is the total (dark matter, stellar, and gas) mass. Although, using rotational velocities for 48 spiral (S) galaxies, Davis et al. (2019b) have since measured the notably steeper relation $M_{bh} \propto M_{*}$,sph$^{4.5 \pm 0.66}$, Curiously, working with the stellar masses of bulges/spheroids, $M_{*}$,sph, Graham & Sahu (2022a) have reported $M_{bh} \propto M_{*}$,sph$^{6.4 \pm 0.17}$ and $M_{bh} \propto M_{*}$,sph$^{5.3 \pm 0.15}$, respectively, for elliptical (E) galaxies and the bulges of lenticular (S0) galaxies.

The non-linear nature of the low- and high-mass end of the $M_{bh}$-$M_{*}$,sph relation(s) was, however, already noted by Graham (2016, and references therein) and Sahu et al. (2019a), respectively.

Many potential pathways exist for massive black holes to grow at the centres of galaxies (e.g., Rees 1984). One of these is engendered through ‘dry’ galaxy mergers rather than active galactic nuclei (AGNs), leading to the generation of long-wavelength gravitational waves when the black holes merge (e.g., Rees 1974; Begelman et al. 1980; Khan et al. 2011; Sesana 2013) and the likely creation of E galaxies. Indeed, Sahu et al. (2019a) have revealed that E galaxies and their massive black holes represent a subsequent generation in the family tree of galaxy-(black hole) coevolution. In the $M_{bh}$-$M_{*}$,sph diagram, E galaxies are offset from the bulges of S0 galaxies by almost an order of magnitude in the $M_{bh}$ direction. This is not simply due to the exclusion of the S0 galaxies’disc mass because they are additionally offset by a factor of $\sim$2 in the $M_{bh}$-$M_{*}$,gal diagram. Graham & Sahu (2022a) explain how this offset reflects the creation of E galaxies from the merger of S0 galaxies, which themselves follow a quadratic $M_{bh}$-$M_{*}$,gal relation. That is, AGN feedback (e.g. Silk & Nusser 2010; Costa et al. 2014; Heckman & Best 2014) has not established the (black hole)-galaxy relation for.

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1 This $\sigma$ is the assumed constant velocity dispersion of a dark matter halo described by an isothermal sphere.

2 Coupled with $M_{bh} \propto M_{*}$,gal$^{0.05}$ (Davis et al. 2018), this implies $M_{halo}/M_{*}$,gal $\propto M_{*}$,sph$^{0.3}$ that is, lower mass spiral galaxies have higher halo-to-stellar mass ratios.
the E galaxies, but rather the presence of a central heat source — dubbed a ‘Benson Burner’ after Benson et al. (2003) — has maintained the merger-established $M_{bh}/M_{sph}$ (and $M_{bh}/M_{gal}$) connection by preventing gas from cooling to form new stars or reignite the central quasar. Within clusters, ram-pressure stripping from the hot X-ray gas likely performs a similar role for the more centrally-located galaxies (e.g. Haines et al. 2006; Grossi et al. 2015).

This order of magnitude offset in the $M_{bol}/M_{sph}$ diagram is mirrored in the $M_{bol}/M_{gal}$ diagram, providing an opportunity to use the spheroids’ effective half-light radii, $R_{e, sph}$, in the virial theorem ($M_{dyn,sph} \propto \sigma^2 R_{e, sph}$; e.g., Ciotti 2021) to give some insight into the distribution in the $M_{bol}/\sigma$ diagram (Merritt 2000; Ferrarese & Merritt 2000; Gebhardt et al. 2000). Here, $\sigma$ is the stellar velocity dispersion of the stars. The low scatter about the $M_{bol}/\sigma$ relation has long been heralded as a sign that AGN feedback regulates the coevolution of galaxies and their central black holes. However, for the E galaxies, it is not AGN feedback but mergers which have established their $M_{bol}/M_{sph}$ ratio. In this paper, it is explained how mergers also maintain a tight $M_{bol}/\sigma$ relation, with small departures expected to cause a steepening of the relation for the E (and ES) galaxies.

Section 2 describes the data sample, shown in Sections 2 and 3. In essence, it is used to reveal why the large, merger-induced jumps in $M_{sph}$ and $R_{e, sph}$ — once the disc stars from S0 galaxies become the outer regions of a merger-built E galaxy — leave little room for increases in $\sigma$. It is shown that such collisions roughly move galaxies along the $M_{bol}/\sigma$ relation or produce small offsets leading to a steepening at mid-to-high masses. A brief summary is provided in Section 4.

In Appendix A, a range of dynamical-to-stellar mass ratios are presented, along with caveats about inferring the presence of dark matter in E galaxies from these ratios. Appendix B reports on three galaxies flagged for exclusion in Section 2.2.

2 THE DATA SAMPLE

2.1 Data Source

Correcting a false assumption in Shankar et al. (2016) regarding the use of, what turned out to be inconsistent, stellar mass-to-light ratios, the sample of galaxies with directly measured black hole masses is not biased with respect to the galaxy population at large (Sahu et al. 2022).

This research forms an extension of Graham & Sahu (2022a) and Graham & Sahu (2022b), which was based on an initial sample of 104 galaxies with directly measured supermassive black hole masses and Spitzer Space Telescope (SST) imaging at 3.6 $\mu$m. Two of those galaxies were bulgeless spiral galaxies (NGC 4395 and NGC 6926), and an additional nine were excluded from the regression analysis due to the reasons given in Graham & Sahu (2022a, see the end of their Section 2.1). Briefly, five of the nine excluded galaxies are well-recognised merger remnants. Shown here in the figures, they are explained in Graham & Sahu (2022b) in terms of (cold gas)-rich mergers in which the system’s angular momentum is not cancelled and thus a disc is present in the merger remnant. These galaxies are not included in the regression analyses. The four other galaxies are Circinus (another unrelaxed spiral galaxy For et al. 2012), NGC 5055 (a spiral galaxy assigned an incorrect black hole mass)\(^4\), NGC 4342 (a stripped S0 galaxy with a questionable spheroid mass), and the S0 galaxy NGC 404 (Nguyen et al. 2017), which is the only galaxy in the sample with $M_{bh} \leq 10^7 M_\odot$ and therefore potentially carrying too much weight in regression analyses. While such low-mass black holes are much sought after, there are not many because it is challenging to spatially-resolve their sphere of gravitational influence (Merritt 2013) — although future instruments hold much promise (e.g., Graham et al. 2021b, their Section 5.1). Moreover, the velocity dispersion can be influenced, if not dominated, by the disc and, when present, the bar in these low-mass galaxies. This latter issue is left for future work given this paper’s focus on roughly the upper-half of the $M_{bol}/\sigma$ diagram, where E galaxies reside. Here, NGC 7457 is also excluded. As discussed in Sahu et al. (2019b), this peculiar S0 galaxy was already recognised as having an oddly low velocity dispersion relative to its peers, plus other unusual kinematic properties such as cylindrical rotation about its major axis (Sil’chenko et al. 2002; Molaeinezhad et al. 2019).

The above exclusions collectively reduced the working sample to 92, involving 35 E galaxies (including ten ES,e galaxies), 31 S0 galaxies (including two core-Sérsic S0 galaxies and four ES,b galaxies), and 26 S galaxies.\(^5\) Although, as seen in the following subsection, each of these three subsamples is further reduced by one. For the regression, following Graham & Sahu (2022a), the ES,e and ES,b galaxies (Graham & Sahu 2022b, see their Table 1) are treated as though they are E and S0 galaxies, respectively. Galaxies excluded from the regression analyses are still plotted in Section 3.

The spheroid sizes have come from Graham & Sahu (2022a, their Table 1) and are based on the updated distances reported there. These sizes were derived from the multicomponent galaxy decompositions that are shown for every galaxy in Savorgnan & Graham (2016), Davis et al. (2019a), Sahu et al. (2019a), or the Appendix of Graham & Sahu (2022a). The spheroid stellar masses stem from the spheroid magnitudes obtained in the above galaxy decompositions, coupled with a colour-dependent stellar mass-to-light ratio. These ratios are based on the Into & Portinari (2013) stellar population models, as adjusted by Equation 4 in Graham & Sahu (2022a) to a Kroupa (2002) initial mass function.

With one exception (NGC 1300), the central stellar velocity dispersions, $\sigma$ — tabulated in Sahu et al. (2019b) and taken from the HyperLeda database\(^6\) (Paturel et al. 2003) — are used. NGC 1300 was previously flagged by Sahu et al. (2019b) as a notable outlier in the $M_{sph}/\sigma$ diagram. Their velocity dispersion for NGC 1300 appears to be in error,\(^7\)\(^8\) Batcheldor et al. (2005, their Table 1) report values around 82 to 90 km s\(^{-1}\). A value of 90±12 km s\(^{-1}\) is adopted here. (Davis et al. 2019a) might be the inner part of an anti-truncated disc, blurring the definition of what is a bulge.

\(^4\) The often reported mass is the total mass interior to 300 pc (Blais-Ouellette et al. 2004).

\(^5\) Liller (1966) introduced the ES galaxy notation, with the subtypes ES,e and ES,b.

\(^6\) http://leda.univ-lyon1.fr

\(^7\) HyperLeda reports values from 145 to 304 km s\(^{-1}\) taken from two studies.

\(^8\) It is noted that NGC 1300 was also flagged by Graham & Sahu (2022a, their Fig 8) as something of an outlier in the $M_{bol}/M_{sph}$ diagram. This may in part be due to a possibly missed barlens (dominant over $\sim10^{-35}$") in the decomposition (Davis et al. 2019a), but may also reflect an overly large black hole mass measurement (Atkinson et al. 2005).
are excluded from the regression analyses, while the other labelled galaxies are retained. The slopes are $4.24 \pm 0.47$, $3.68 \pm 0.47$, and $2.49 \pm 0.56$ for the E (E, E, and S) and S (S, S, and S) samples, respectively. For reference, when using $M_{\text{bul}}$ rather than $M_{\text{gal}}$, the slopes are $4.14 \pm 0.55$, $2.71 \pm 0.45$, and $1.90 \pm 0.35$.

2.2 Checking for outliers in the $M_{\text{gal}}$-$\sigma$ diagram

Before proceeding, the $M_{\text{gal}}$-$\sigma$ diagram (Fig. 1) was inspected for potentially biasing data points. Three such galaxies were detected (NGC 2787, NGC 4291, and NGC 4945). Failure to identify and remove outlying data points, not representative of the population at large, can bias a regression. This may occur when the outlying data point is located near the end of a trend, causing a fitted regression line to shift towards the outlier. Sometimes the reason for discrepant data is identified and can be corrected, while other times, it remains unknown. These three galaxies identified in Fig. 1 are discussed in Appendix B. They are excluded from the regression analyses that follow and together with NGC 7457, mentioned in the previous subsection, results in a sample that is smaller by four than used in Graham & Sahu (2022a).

2.3 Galaxies built from dry mergers

Sahu et al. (2019a, their Fig. 8) revealed that E galaxies are offset from S0 galaxies in the $M_{\text{gal}}$-$M_{\text{gal}}$ diagram. Fig. 2 helps explain this offset between the current sample of E (E, S0, and S) and S (S, S0, and S) galaxies in the $M_{\text{bul}}$-$M_{\text{gal}}$ diagram. The ES,e elliptical galaxies have previously been referred to as 'disc ellipticals' (Nieto et al. 1988; Scorza & Bender 1995). Graham & Sahu (2022b) discusses why they likely formed from a major merger event making them more akin to elliptical galaxies, while the S, S0 elliptical galaxies, also known as 'compact galaxies' (Zwicky & Kowal 1968; Zwicky & Zwicky 1971) and 'relic red nuggets' or 'relic galaxies' (e.g., Ferré-Mateu et al. 2017), likely never accreted a large-scale disc, making them more akin to classical bulges. Given that Sersic S0 galaxies have grown their discs through (angular momentum)-building (cold gas)-rich accretion and merger events (e.g., Graham et al. 2015; Hon et al. 2022a; Jackson et al. 2022), while core-Sersic S0 galaxies are likely built from significant, relatively dry mergers, the two core-Sersic S0 galaxies are best shifted from the S0 to the S galaxy bin when dealing with galaxy stellar masses. Such a redesignation only affects Fig. 2, which is not used for our upcoming analysis of the $M_{\text{bul}}$-$\sigma$ diagram.

Due to the (steeper than linear) $M_{\text{bul}} \propto M_{\text{gal}}^2$ relation for S galaxies, the (dry merger)-built E galaxies will not move along this relation but effectively shift to the right. Additional major dry mergers will move systems along a path with a slope of 1 in the $M_{\text{bul}}$-$M_{\text{gal}}$ diagram. It appears that (cold gas)-rich mergers involving S galaxies also produce merger remnants shifted rightward of the $M_{\text{bul}} \propto M_{\text{gal}}^2$ relation for S galaxies.

The (Liller 1966) ES galaxies with intermediate-scale discs have previously been recognised, on a morphological basis, as an intermediary between E and S0 galaxies (e.g., Graham 2019b, and references therein). With recourse to their $M_{\text{bul}}/M_{\text{gal}}$ ratios, Fig. 2 now shows their evolutionary connection. The ES,e galaxies appear as something of a bridging population, an intermediary step facilitating the speciation of E galaxies from S0 galaxies. Through the mating of two S0 galaxies to produce an ES,e galaxy, and the mating of two ES,e galaxies (or E+ES,e) to produce an E galaxy, one can start to see the lineage and transformation of these galaxies. Similarly, two S0 galaxies may merge to form an E galaxy, with the subsequent merger of two E galaxies forming a bigger E galaxy, such as a brightest cluster galaxy (BCG). Mergers are a well-known phenomenon, but an exploration of galaxy pairings, as systems evolve to higher masses in the $M_{\text{bul}}$-$M_{\text{gal}}$ diagram, has only recently been observed Graham & Sahu (2022a, their Figures 5–8). In the following section, this is extended to the observed $M_{\text{bul}}$-$M_{\text{gal}}$, $M_{\text{bul}}$-$\sigma$, and $M_{\text{bul}}$-$M_{\text{gal}}$ diagrams.

3 ANALYSIS AND DISCUSSION

3.1 Dynamical masses and black hole scaling relations

Here, the dynamical mass is defined as

$$M_{\text{dyn}} = 5\sigma^2 R_e / G.$$ (1)
with the virial coefficient of 5 taken from Cappellari et al. (2006). As noted earlier, the equivalent-axis\(^9\) effective half-light radii of the spheroids, \(R_*\), are listed in Graham & Sahu (2022a) and the central velocity dispersions, \(\sigma\), are listed in Sahu et al. (2019b).

Fig. 3 shows the spheroid dynamical mass (Equation 1) plotted against the spheroid stellar mass, taken from Graham & Sahu (2022a). On average, the E galaxies roughly have \(M_{\mathrm{dyn,sph}}/M_{\ast,sph} \approx 2.3\), while the bulges of the disc galaxies have a lower ratio of around 1. As discussed in Appendix A, it is premature to conclude that the higher ratio in the merger-built E galaxies is due to dark matter halos of the progenitor disc galaxies contributing within (the now larger) \(R_{\ast,sph}\) of the E galaxies.

Using \(M_{\mathrm{dyn,sph}} \approx 2.3M_{\ast,sph} = 5\sigma^2 R_{\ast,sph}/G\), the E galaxies follow

\[
M_{\ast,sph} \approx (5/2.3)\sigma^2 R_{\ast,sph}/G. \tag{2}
\]

Fig. 4 presents the \(M_{\ast,sph}/M_{\ast,gal}\) and \(M_{\mathrm{dyn,sph}}/R_{\ast,sph}\) diagrams from Graham & Sahu (2022a), showing the offset between the E galaxies and the bulges of the S0 galaxies. An additional third panel showing the dynamical mass of these spheroids, as per Equation 1, has been added. Following Graham & Sahu (2022a), Bayesian model fitting using Stan (Carpenter et al. 2017; Team 2016)\(^{10}\) has been employed. The approach is described in Davis et al. (2019a, their Appendix A) and was used to obtain a symmetrical linear regression between the plotted quantities. The regressions are based on almost the same sample of 35 E and 32 ES/S0 galaxies shown in Graham & Sahu (2022a, their Table 1). As discussed in Section 2, the E sample is reduced by one (NGC 4291) and the ES/S0 sample by two (NGC 2787 and NGC 7457). Another distinction from the figures in Graham & Sahu (2022a) is that Fig. 4 provides a more detailed morphological description using the designations given in Graham & Sahu (2022b). This encompasses labelling the brightest cluster galaxies (BCGs) and differentiating between two potential types of ES galaxy, namely the compact ES,b systems and the larger ES,e systems, likely built by mergers of S0 galaxies.

\(^9\) The ‘equivalent axis’, also known as the geometric-mean axis, is such that the radius \(R_{\mathrm{equi}} = \sqrt{R_{\mathrm{maj}}R_{\mathrm{min}}}\) provides a light profile equivalent to that coming from a circularised version of the projected spheroid light.

\(^{10}\) https://mc-stan.org/
revealed the constraint from the virial theorem. That is, insight into why $M_{\text{bh}}$ varies with $\sigma$ in the way it does would have been absent. The $M_{\text{bh}}$-$\sigma$ relations in Fig. 5 shall be visited in the following subsection. A couple of relevant observations should, however, first be made.

Given the trend in Fig. 3, the separation between the E galaxies and both the S and S0 galaxies is greater in the $M_{\text{bh}}$-$M_{\text{dyn,sph}}$ diagram (Fig. 4, right-hand panel) than it is in the $M_{\text{bh}}$-$M_{\text{sph}}$ diagram (Fig. 4, left-hand panel), with increased separation of roughly log(2.3) = 0.36 dex in the log $M_{\text{dyn,sph}}$ direction.

Fig. 6 reveals that while there is a tight $M_{\text{sph}}$-$R_{\text{eff,sph}}$ relation for spheroids, there is a tendency for the E and ES,e galaxies to have larger sizes than the bulges of S, S0 and ES,b galaxies at a given stellar mass. This offset has previously been shown in Graham & Worley (2008, their Fig. 13) and Gadotti (2009, his Fig. 13). This explains why the offset between these populations in the $M_{\text{bh}}$-$R_{\text{eff,sph}}$ diagram (Fig. 4, left-hand panel) is slightly greater than it is in the $M_{\text{bh}}$-$M_{\text{sph}}$ diagram (Fig. 4, middle panel). Had all systems exactly followed the (steeper than linear) $M_{\text{sph}} \propto R_{\text{eff,sph}}^{1.12}$ relation (see Table 1), one would have expected the size of the offsets to be reversed, i.e., slightly smaller in the $M_{\text{bh}}$-$R_{\text{eff,sph}}$ diagram.

The inclusion of BCGs with elevated sizes, due to the inclusion of intracluster light (ICL), will lead to a reduction of slope in the $M_{\text{sph}}$-$R_{\text{eff,sph}}$ diagram for the E galaxies. As noted in Graham & Sahu (2022b), while the sample used here contains BCGs, these BCGs are not expected to have elevated sizes due to the ICL. For reference, at $M_{\text{sph}} > 3 \times 10^{10} M_\odot$, Méndez-Abreu et al. (2021) report a slope in the $M_{\text{sph}}$-$R_{\text{eff,sph}}$ diagram of 1.15 ± 0.05, agreeing with the trend seen here for all spheroids. Möllenhoff & Heidt (2001, their Fig. 8d) report a slope of 1.19 between the near-IR luminosity and half-light radii of bulges in bright spiral galaxies.

However, in passing it is noted that Laurikainen et al. (2010)
and Balcells et al. (2007), their Eq. 3) report, respectively, slopes of 1.6 and 2.6 for S0 bulges. Furthermore, Gadotti (2009) analysed SDSS data and reported notably different slopes in the $M_{\text{sph}}$-$R_{\text{sph}}$ diagram of 5, 3.33, and 2.63 for what they considered pseudobulges, classical bulges, and elliptical galaxies, respectively. In contrast, the current investigation finds slopes of roughly 1 to 1.5 for the S, S0 and E samples in the mass range shown. Although, there are no spheroids with masses below $\sim 2 \times 10^{10} M_\odot$ in these samples, and they appear to introduce some curvature into, and steepening of, the relation for bulges, as seen in Laurikainen et al. (2010, their Fig. 8a), Graham (2019a, his Fig. 17 and 18), based on data in Graham & Worley (2008, their Fig. 12), Méndez-Abreu et al. (2021, their Fig. 8), and thus Lange et al. (2016). Nonetheless, the analysis of the present sample’s SST images and the analysis of SDSS images by Hon et al. (2022a, their Fig. 16) — obtained from multicomponent decompositions which account for potentially biasing structures such as bars, inner discs, nuclear star clusters or depleted cores — find sizes at $M_{\text{sph}} \sim 2 \times 10^{10} M_\odot$ which are 2 to 3 times smaller than suggested by the above works. This is explored further in Hon et al. (2022b).

3.2 Quantifying Dry Mergers

Returning to the $M_{\text{bh}}$-$\sigma$ diagram, the impact of dry E-building mergers is quantified here.

Considering an equal mass E+E merger in which $M_{\text{bh}}$ doubles and thus log $M_{\text{bh}}$ increases by $-0.3$ dex, the jump in $R_{\text{sph}}$ in the $M_{\text{sph}}$-$R_{\text{sph}}$ diagram (0.30/2.50 = 0.12) and the jump in $M_{\text{dyn,sph}}$ in the $M_{\text{sph}}$-$M_{\text{dyn,sph}}$ diagram (0.3/1.58 = 0.19) is associated with an expected jump of 0.035 dex in log $\sigma$. This stems from Equation 1, from which one has $\delta \log M_{\text{dyn,sph}} = 2 \delta \log \sigma + \delta \log R_{\text{sph}}$. It translates to movement along a slope of $-8.6 = (0.3/0.035)$ in the $M_{\text{sph}}$-$\sigma$ diagram. That is, a steepening is predicted at high masses due to E+E mergers. Mergers between ES galaxies and between ES and E galaxies are expected to yield a similar result. Such mergers will contribute to the steeper $M_{\text{bh}}$-$\sigma$ relation observed by Bogdán et al. (2018) for BCGs and by Sahu et al. (2019b) for core-Sérsic galaxies and those with high velocity dispersions $>270$ km s$^{-1}$. (Graham & Scott 2013). This trend can be compared with idealised simulations and analytic studies of dry and gas-poor mergers between E galaxies on parabolic orbits, in which the velocity dispersion does not increase by much (e.g., Ciotti & van Albada 2001; Nipoti et al. 2003; Volonteri & Ciotti 2013, see their Fig. 3).

Another way to tackle this is to consider an E+E galaxy merger in which the dynamical, spheroid and black hole mass double. From the E galaxy $M_{\text{bh}}$-$R_{\text{sph}}$-$\sigma$ relation (Fig. 6), one has $\log M_{\text{sph}} \propto 1.51 \log R_{\text{sph}}$ (see Table 1, and thus a 0.30 dex increase in stellar mass corresponds to a 0.20 ($=0.30/1.51$) dex increase in spheroid size. The virial relation $\log M_{\text{dyn,sph}} \propto \log R_{\text{sph}} + 2 \log \sigma$ informs us that a 0.30 dex increase in the dynamical mass, coupled with a 0.20 dex increase in $R_{\text{sph}}$, requires a 0.05 dex increase in $\sigma$. Now, given the associated 0.3 dex increase in $M_{\text{bh}}$, this leads to movement along a slope of 6 ($=0.3/0.05$) in the $M_{\text{bh}}$-$\sigma$ diagram. This approach works when $R_{\text{sph}}$ and $\sigma$ are suitable for use in the virial theorem. To illustrate the need for caution, Graham (2019a) and Sahu et al. (2020) reported on how the stellar mass-size relation for ETGs and spheroids changed for sizes defined by radii enclosing different fractions of light than the canonical 50 per cent value associated with the effective half-light radii, $R_e$. These changes affect both typical dynamical mass estimates (based on $\sigma^2 R$) and implications for baryon-to-(dark matter) ratios. This is left for now, but further discussion is provided in Appendix A.

Table 1. Black hole mass scaling relations

| Type   | slope (A) | midpoint (C) | intercept (B) | $\Delta_{\text{rms}}$ |
|--------|-----------|--------------|---------------|----------------------|
| E (34) | 2.50±0.35 | 0.90         | 9.00±0.12     | 0.58                 |
| S0 (30)| 1.98±0.25 | 0.23         | 8.05±0.15     | 0.52                 |
| S (25) | 2.40±0.42 | 0.25         | 7.41±0.16     | 0.67                 |

| Type   | slope (A) | midpoint (C) | intercept (B) | $\Delta_{\text{rms}}$ |
|--------|-----------|--------------|---------------|----------------------|
| E (34) | 1.68±0.17 | 11.45        | 9.00±0.12     | 0.38                 |
| S0 (30)| 1.65±0.19 | 10.30        | 8.00±0.14     | 0.43                 |
| S (25) | 2.66±0.67 | 10.14        | 7.39±0.14     | 0.57                 |

| Type   | slope (A) | midpoint (C) | intercept (B) | $\Delta_{\text{rms}}$ |
|--------|-----------|--------------|---------------|----------------------|
| E (34) | 1.58±0.18 | 11.78        | 8.99±0.12     | 0.43                 |
| S0 (30)| 1.25±0.13 | 10.30        | 8.02±0.15     | 0.37                 |
| S (25) | 1.55±0.24 | 10.05        | 7.39±0.16     | 0.56                 |

| Type   | slope (A) | midpoint (C) | intercept (B) | $\Delta_{\text{rms}}$ |
|--------|-----------|--------------|---------------|----------------------|
| E (34) | 7.27±0.91 | 2.41         | 8.99±0.12     | 0.43                 |
| S0 (30)| 5.68±0.60 | 2.24         | 8.03±0.15     | 0.45                 |
| S (25) | 6.19±1.11 | 2.13         | 7.39±0.16     | 0.68                 |
| All (89)| 5.90±0.33 | 2.27         | 8.22±0.10     | 0.52                 |

For the Bayesian model fitting, a fractional error of 13.20 and 25 per cent was assigned to the velocity dispersions, radii and dynamical mass, while the individual errors on the spheroid stellar mass and black hole mass, tabulated in Graham & Sahu (2022a), were used. Note: the Bayesian analysis was not designed to minimise the scatter in the vertical direction, reported here as $\Delta_{\text{rms}}$. The lower-case ‘t’ term, $t$, allows for conversions between adopted stellar mass-to-light ratios. For this study, it is equal to 1.0. Within the statistical analyses, the Monte Carlo method results in slopes and intercepts that differ by a few hundreds from run to run. This can be seen in some of the slightly different intercepts above. This variation is, however, well contained within the tabulated parameter uncertainties.

Figure 6. Spheroid (stellar mass)-size diagram similar to Fig. 7 in Graham & Sahu (2022a) but showing more detail regarding the host galaxy’s morphological type. The symbols, colours, lines, and shading have the same meaning as in Figure 5. The relations for the three different morphological types, and the combined sample, are reported in Table 1.
Next, consider equal mass $S0$-$S0$ galaxy mergers taking systems from $\log(M_{bh}/M_*)$ equals 9.0 to 9.5 dex and creating an E galaxy; that is, moving from the $S0$ galaxy to the E galaxy $M_{bh}$-$M_{bh}$ and $M_{bh}$-$R_{sph}$ relations. Following the above procedure, one can calculate changes of 0.0611 dex in log $\sigma$ and a shift along a line with a slope of $\sim$4.9. Such mergers, therefore, largely move systems along the $M_{bh}$-$\sigma$ relation. This movement explains why the $M_{bh}$-$\sigma$ relation has less horizontal scatter than the $M_{bh}$-$M_{sph}$ and $M_{bh}$-$R_{sph}$ relations in which the E galaxies are clearly offset from the $S0$ galaxies due to their lower $M_{bh}/M_{sph}$ ratios which arose when mergers contributed disc stars, as detailed in Graham & Sahu (2022a).

Another observation leading credence to the merger-built E galaxies is made in passing. The stellar density, $\rho_{sph}$, within a sphere of radius $R$ is proportional to $M_{sph}/R_{sph}^4$ using $M_{sph} \propto R_{sph}^{10}$ (as an approximate trend for all spheroids, see Table 1), one has that $\rho_{sph} \propto R_{sph}^{-4}$. This relation is why the density of smaller bulges in disc galaxies is greater than that of larger elliptical galaxies, as shown in Graham (2013). In terms of a dry $S0$-$S0$ merger building an E galaxy, one can think of the pre-merger (initially disc) stars at large radii dispersing to create an elliptical galaxy with its light profile having a shallow quasi-exponential tail. This likely also explains the red-to-blue colour gradient observed in some E galaxies (e.g., Strom et al. 1976; Kim & Ann 1990) if comprised of former disc stars at large radii and former bulge stars at small radii. It also helps explain why such mergers produce the large jump in $R_{sph}$—along with the jump in $M_{sph}$—because $R_{sph}$ is notably larger than $R_{sph}$ for most $S0$ galaxies.

Finally, consider a dry, equal mass $S0$-$S0$ galaxy merger taking two lower-mass $S0$ galaxies with $\log(M_{bh}/M_*)$ equal to 7.7 to one E galaxy with $\log(M_{bh}/M_*)$ = 8.0. This is accompanied by a shift in log $\sigma$ of 0.10 dex, corresponding to movement along a line with a slope of 3.0 in the $M_{bh}$-$\sigma$ diagram. This current $S0$-$S0$ example merger corresponds to a shift in spheroid mass of 0.74 dex (see Fig. 1), which could be achieved by the dry merger of two similar $S0$ galaxies with bulge-to-total ratios of 0.37 prior to merging. This merger is associated with a slight shift away (to higher velocity dispersions) from the mean $M_{bh}$-$\sigma$ relation at mid-(black hole) masses. This can be seen through the red line for E galaxies residing on the right-hand side of the cyan line for $S0$ galaxies at $M_{bh} \sim 10^7 M_\odot$ in the $M_{bh}$-$\sigma$ diagram (Fig. 5). Coupled with the smaller shift (in log $\sigma$) at higher black hole masses — due to E+E mergers — this will contribute to, if not explain, the observed steepening in the $M_{bh}$-$\sigma$ diagram, seen in Fig. 5 and reported by Bogdán et al. (2018) and Sahu et al. (2019b). Curiously, it is noted that at black hole masses below $\sim 10^5 M_\odot$ and $\log(\sigma / km s^{-1}) \leq 2.2$ (or $\sigma \leq 150 km s^{-1}$), the $M_{bh}$-$\sigma$ relation appears less well defined, although this may be a consequence of the small data range and roughly $\pm 0.5$ dex (± 1 sigma) scatter in the $\log M_{bh}$ direction.

3.2.1 Wet mergers

The notion that E galaxies might be built from the merger of two (bulgeless or near bulgeless) disc galaxies (e.g., van Albada & van Gorkom 1977; Toomre 1977) is problematic. The early simulations which gave rise to this idea contained no gas. Such mergers would yield a large jump in spheroid mass which may not be matched by the even greater jump in black hole mass required to maintain the steeper-than-linear $M_{bh}$-$M_{sph}$ trend. For example, consider the hypothetical creation of an E galaxy from the merger of two equal spiral galaxies having $B/T = 0.1$, $M_{sph} = 5 \times 10^{10} M_\odot$, and $M_{bh} = 5 \times 10^5 M_\odot$. Prior to any black hole fueling or star formation, this hypothetical system will have $M_{sph} = 10^{11} M_\odot$, and $M_{bh} = 10^5 M_\odot$. However, E galaxies with these properties are not known (see the middle panel of Figure 4). If there was, however, sufficient quasar activity to drive up the black hole mass to meet with the spheroid stellar mass on the observed $M_{bh}$-$M_{sph}$ relation, then this scenario does not represent the much-acclaimed symbioses in which the black hole regulates the spheroid’s star formation to establish the black hole scaling relations.

Even if the tidally-engaged but not yet wed galaxies experienced bar formation, which led to cold gas inflow that fuelled a bulge-building central starburst (e.g., Barnes & Hernquist 1996), many of the stars in the proposed E galaxy merger remnant would still have come from the discs of the two single galaxies. Once again, any quasar-fuelled growth of the merging black holes has not regulated the mass of these pre-existing disc stars, which will likely dominate the final stellar mass budget. This tension is alleviated if spiral galaxy collisions do not build E galaxies but instead build disc galaxies with less massive spheroids. This gels with binary disc simulations in which the system’s net orbital angular momentum is not nulled by their merger (e.g., Naab et al. 2006), and with the need for dissipation in the formation of some ETGs, that is, some $S0$ galaxies (see Ciotti & van Albada 2001). As discussed in Graham & Sahu (2022b), this scenario also meshes well with the disc structures observed in the remnants of (cold gas)-rich mergers, and it can readily accommodate the $M_{bh}/M_{sph}$ ratios in these remnants.

While the current sample size of five such (cold gas)-rich merger products is small, their alignment with (i) the E galaxy’s $M_{bh}$-$M_{sph}$ relation but also (ii) the spiral galaxy’s $M_{bh}$-$R_{sph}$ relation, requires consideration beyond the present analysis of how dry mergers influence the $M_{bh}$-$\sigma$ diagram. Potential evolutionary pathways from the S to the S0 sequence are presented in Silk & Rees (1998) for the $M_{bh}$-$\sigma$ diagram, and in Graham & Sahu (2022a) for the $M_{bh}$-$M_{sph}$ diagram, while Graham (2022, in preparation) presents a more complete picture in which $S0$ galaxies can also evolve into S galaxies.

While discussing mergers involving spiral galaxies, it is noted that some spiral galaxies may acquire (their initial and/or grow their central massive black hole via the accretion of lower-mass dwarf galaxies. For example, Nikhuli, the suspected remains of a dwarf galaxy’s nuclear star cluster, may be delivering an intermediate-mass black hole (IMBH) to NGC 4424 (Graham et al. 2021a). Chandra X-ray Observatory imaging of 35 spiral galaxies expected to harbour an IMBH has recently revealed central X-ray point sources in 14 of them (Graham et al. 2021b), possibly signalling the suspected abundance of IMBHs (Silk 2017). Meanwhile, a similar campaign which looked at dwarf ETGs found just three candidates from a sample of 30 (Gallo et al. 2008; Graham & Soria 2019), perhaps due to the reduced fuel supply, and thus quiter black holes, in these relatively (cold gas)-poor systems.

Finally, it is noted that increases in black hole mass due to the collision of massive black holes does not void the argument of Soltan (1982), which makes a case for quasar-fuelled black hole growth explaining the bulk of the mass locked up in black holes today (e.g., Shankar et al. 2004). What is presented here is a case for an addendum to the Soltan argument, or at least additional knowl-
edge enabling a fuller understanding, in which the black hole mass function undergoes evolution not captured by the Soltan scenario. Dry mergers neither increase the total amount of mass locked up in black holes nor do they negate past quasar activity, but they do require it be scaled back in the most massive, merger-built, black holes. As discussed in Graham & Sahu (2022a), quasars and Seyferts tend to occur in disc galaxies rather than massive E galaxies, which have hot/radio mode activity rather than cold/quasar mode activity.

3.3 A consistency check

Following Graham (2012), a consistency check is performed to test whether the \( M_{bh} - M_{sph} \) and \( M_{bh} - \sigma \) relations combine to give \( M_{sph} - \sigma \) relations consistent with expectations. Sahu et al. (2019b, their Table 5) have reported slopes of 2.10±0.41, 2.97±0.43, and 5.16±0.53 for the \( M_{sph} - \sigma \) relations for late-type galaxies (LTGs), Sérésic ETGs, and core-Sérésic galaxies, respectively. These slopes are broadly in accord with samples of low-to-intermediate ETGs (Tonry 1981) which used galaxy rather than spheroid magnitudes, and samples of massive ETGs (Schechter 1980) likely dominated by elliptical galaxies. The slopes for the S, S0 and E galaxies in the \( M_{sph} - \sigma \) diagram (Fig. 1) are 2.49±0.56, 3.48±0.47, and 4.24±0.54. Combining these slopes with those from the \( M_{bh} - M_{sph} \) relations in Table 1 yields expected slopes for the \( M_{sph} - \sigma \) relation of 6.62 (≈ 2.49 × 2.66), 5.74 (≈ 3.48 × 1.65), and 7.12 (≈ 4.24 × 1.68). This is in accord with the above explanation surrounding the merger-induced transformation of galaxies.

4 SUMMARY AND CONCLUDING REMARKS

It has been explored how the largest black holes came to be, with vital input from published galaxy morphology analyses which separated bulges from discs (and other galaxy components), yielding spheroid sizes and masses for use in Clausius’ far-reaching mechanical theorem applicable to heat (Clausius 1870). Better known as the ‘virial theorem’, it is routinely applied to pressure-supported stellar systems. Comparing the dynamical-to-stellar masses of the spheroids has enabled one to understand the changes in the velocity dispersion arising from S0 mergers and the emergence of the E galaxies, with their lower \( M_{bh}/M_{sph} \) ratios at a given spheroid mass. While such dry galaxy mergers can yield a substantial jump in the \( M_{bh} - M_{sph} \) diagram, the virial theorem was used to explain why there is not a substantial jump in the \( M_{bh} - \sigma \) diagram, thereby maintaining, if not creating, a low level of scatter, at least in the upper half of this diagram where E/S,e and E galaxies reside. This work additionally explains and quantifies the steepening of the \( M_{bh} - \sigma \) relation due to the production of E/S,e and E galaxies.

The revelation, and refinement, of (galaxy morphology)-dependent black hole scaling relations over the past decade has consequences for many astrophysical phenomena related to massive black holes. One of these is the generation of long-wavelength gravitational radiation associated with massive black holes (Hobbs et al. 2010; Burke-Spolaor et al. 2019). Although it has long been recognised that E galaxies form from mergers, there has been some latency in establishing the E galaxy black hole scaling relations. There are, of course, additional merger remnants than just E/S,e galaxies in which black holes are expected to coalesce, such as the core-Sérésic S0 galaxies and the gas-rich mergers like the Centaurus galaxy, Graham & Sahu (their Eq. 1 2022b) provide a general, and hopefully useful, single \( M_{bh} - M_{sph} \) relation for all of these merger-built systems. The logarithmic slope of the relation is ~2, dramatically different from the slope of ~1 which has often been used in the past. To give just one example, Mapelli et al. (2012) revealed how a steeper slope of ~2, rather than ~1, for the \( M_{bh} - M_{sph} \) relation yields an order of magnitude reduction to the expected detection rate of extreme mass ratio inspiral (EMRI) events by the Laser Interferometer Space Antenna (LISA: Danzmann & LISA Study Team 1997).

Application of updated (galaxy morphology)-dependent black hole scaling relations may prove beneficial for expectations from, and design of, some gravitational wave detectors, such as the Einstein Telescope (ET: Punturo et al. 2010; Gair et al. 2011; Huerta & Gair 2011), the Cosmic Explorer (CE: Reitze et al. 2019; Evans et al. 2021), the Deci-Hertz Interferometer Gravitational wave Observatory (DECIGO: Kawamura et al. 2011; Ishikawa et al. 2021), and TianQin (Luo et al. 2016; Mei et al. 2021). It should also aid

13 These slopes are based on the use of \( M_{sph}/L_{sph} \) = 0.6.
with expectations array for, and assist with implications of, data from pulsar timing array (PTA) projects such as that at the Parkes radio telescope (PTTA: Manchester et al. 2013; Goncharov et al. 2021), the European PTA (EPTA: Stappers et al. 2006; Chen et al. 2021), and other promising ventures (e.g., Hobbs et al. 2010; Joshi et al. 2018; Bailes et al. 2020). Excitingly, the North American Nanohertz Observatory for Gravitational Waves (NANOGrav) recently reported a detection (Arzoumanian et al. 2020) which might be consistent with primordial solar-mass and planet-mass black holes (Kohri & Terada 2021; Domènech & Pi 2022). PTAs such as this one are not sensitive to the higher frequency gravitational ra-

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5 DATA AVAILABILITY

The imaging data underlying this article is available in the NASA/IPAC Infrared Science Archive, while the kinematic data was sourced from HyperLeda. The derived spheroid masses and sizes are tabulated in Graham & Sahu (2022a), where the velocity dispersions shown in Fig. 5 are conveniently listed in Sahu et al. (2019b).

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APPENDIX A: VIRIAL COEFFICIENTS

In the 1990s, researchers started to explore the implications of broken structural homology, i.e., the observation that bulges and elliptical galaxies are not described by de Vaucouleurs (1948)’ R1/4 law’ but instead by Sérsic (1963)’ R1/6 model’. The variety of stellar density profiles, quantified by the Sérsic index, n, implied a variety of velocity dispersion profiles (Ciotti 1991) and thus aperture velocity dispersion profiles (Ciotti & Lansoni 1997, their Fig. 7); Graham & Colless (1997, their Figures 7–8). These are required in order to uphold these pressure-supported systems. The range of profiles means that the measured central velocity dispersion, σc, relative to the desired virial velocity dispersion depends not only on the size of the central aperture but also on the spheroid’s Sérsic index. To avoid the practical difficulties with reading from a figure, Prugniel & Simien (1997, see their Figures 9–11) also provided a useful table accounting for the different profile shapes when an aperture velocity dispersion is measured within 0.1 R_e. 

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Their tabulated $S_p(n)$ term is the virial coefficient $^{14}$, when using $\sigma = \sigma_{ap}(R < 0.1R_e)$, for calculating the virial mass via

$$M_{\text{virial}} = S_p(n)\sigma^2 R_e/G.$$  \hfill (A1)

Bertin et al. (2002, their Fig. D.1) refer to $S_p(n)$ as $K_v(n)$, and for velocity dispersions measured in central apertures of $R_e/8$ or $R_e/10$, their Equation 11 parameterised these tabulated virial coefficient as

$$K_v(n) = \frac{73.32}{10.465 + (n - 0.94)^2} + 0.954.$$  \hfill (A2)

In Fig. A1b, Equation A2 was used to derive a virial mass for the spheroids. This mass differs from panel a), in which a constant virial coefficient of 5 was used (see Equation 1). The difference between the two panels in Fig. A1 is similar to that seen in Forbes et al. (2008, their Figures 12–13). Pursuing this further is beyond the scope of the present investigation into the $M_{bh}$-$\sigma$ relations, for which a constant virial coefficient suffices. However, the use of the (Sérsic $n$)-dependent $K_v(n)$ term in Fig. A1 serves as a reminder of why one cannot yet infer trends regarding potential fractions of dark matter. This point is also illustrated through the use of a different scale radius. As detailed in Graham (2019a), the use of the half-light radius, $R_e$, has always been arbitrary, without any physical significance beyond containing 50 per cent of the light.

Moreover, its use has led to the misunderstanding of galaxy connections. Here, the use of scale radii containing 10 per cent of the spheroid light is briefly explored. To help offset this reduction from the 50 per cent half-light radius, the coefficient in Equation 1 is increased by a factor of 5.

In Fig. A1c, a variant of Equation 1 has been used, such that $M_{\text{dyn}} = 5 \times (\bar{S}\sigma^2 R_{0.1})/G$. Here, $R_{0.1}$ is the projected radius on the sky enclosing the central 10 per cent of the spheroid light. The equation for the transformation between $R_e \equiv R_{0.5}$ and $R_{0.1}$ can be found in Graham (2019a, his Equation 22). This size transformation equation is a function of the Sérsic index, $n$.

There are several additional factors regarding the virial coefficient. For example, while the E galaxies may have an average $R_e$ of around 6 kpc, and thus the 0.595 kpc aperture velocity dispersions (see Golev & Prugniel 1998) from HyperLeda are, on average, measured at 0.1 $R_e$, there is a range of $R_e$ from ~2 to 20 kpc for the E galaxies used here. Moreover, the bulges of the disc galaxies have sizes down to ~100 pc.

In the 1990s, it also became apparent that pressure-supported elliptical galaxies were not as common as previously thought and that discs in ETGs were much more abundant than had been realised (Capaccioli 1990; Rix & White 1990; Graham et al. 1998, 2003b). Unless kinematic decompositions are performed on the galaxy spectra, the dynamics of the disc will contribute to the measured velocity dispersion when the bulge is small. The use of larger apertures, such as ‘effective apertures’, will increasingly bias the slope of the $M_{bh}$-$\sigma$ relation at the low-mass end where the galaxies’ discs dominate over their bulges. That is to say, the ‘effective velocity dispersion’, $\sigma_e$, within the galaxy effective half-light radius of S0 galaxies, also known as ‘fast rotators’, is not applicable for use in the current form of the virial theorem given $R_{\text{ap}}$ is many times greater than $R_{\text{sph}}$. Basically, $\sigma_e$ is partly the disc velocity dispersion for the disc galaxies, which typically have $R_{\text{ap}}/R_{\text{gal}} \approx 0.2$–0.4, or $R_{\text{ap}}/R_{\text{disc}} \approx 0.12$–0.24 (e.g., Graham & Worley 2008).

Furthermore, Graham (2001) presented the ‘iceberg’ model for spiral galaxies, in which the bulge-to-disc size ratios of spiral galaxies are somewhat constant, but the surface brightness of the bulge becomes increasingly faint relative to the inner disc. This reduction in the prominence of the bulge light in late-type spiral galaxies results in a greater contribution of disc light to the measured velocity dispersion. This contribution will contribute to an observed flattening of the $M_{bh}$-$\sigma$ distribution. With ongoing efforts by many teams to probe $M_{bh} \lesssim 10^6 M_\odot$ at $\sigma \lesssim 100$ km s$^{-1}$ comes

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\footnote{This should not be confused with the virial factor, $f$, used to convert AGN virial masses into black masses (e.g., Peterson 1993; Laor 1998). The value of $f$ is often derived using the $M_{bh}$-$\sigma$ relation coupled with reverberation mapping data (e.g., Onken et al. 2004; Graham et al. 2011; Yu et al. 2019).}
a growing need to be mindful of whether $\sigma$ remains representative of the spheroid. If the disc dominates the signal, it may reduce the measured value of $\sigma$. This would represent a break from probing the coevolution of spheroids and their black holes, as the disc velocity dispersion seems unlikely to be regulated by the mass of the black hole.

While the spheroidal component of galaxy light is separated during the multicomponent decompositions of the galaxy images, there is an implicit assumption that the central velocity dispersion reflects the spheroid light and is not contaminated by other components, such as the disc. This assumption is increasingly incorrect in lower mass galaxies where the bulges are smaller and fainter. This implicit assumption can be addressed with two-component kinematic models akin to bulge-disc luminosity models. Following Scorza & Bender (1995) and others, Oh et al. (2020) have shown how the bulge and disc velocity dispersions can be separated. This separation shall be explored in future work, mindful that some of the E galaxies used here (e.g., NGC: 821; 3377, 3607; 4291, and 4621) also display major-axis rotation, sometimes reaching 50-100 km s$^{-1}$. These galaxies are likely mergers in which not all of the orbital angular momentum of the stars has been cancelled, as discussed in Graham & Sahu (2022b).

Moreover, the calculated $K_n(\pi)$ term (Equation A2) for $\sigma_{0,1}$ is based on the Sérsic $R_{1/e}$ radial distribution of stellar matter, treating the spheroids as non-rotating spheres with isotropic dynamical structure. As such, this term does not account for the influence of dark matter. For those wishing to explore this further, it is noted that modifications for varying levels of orbital anisotropy, i.e., radial-to-tangential orbits and dark matter (or radially varying stellar mass-to-light ratios), were introduced in the pioneering work by Ciotti et al. (1996) and Ciotti & Lanzoni (1997) for spheroidal Sérsic $R_{1/e}$ profiles of E galaxies. Density-potential-velocity dispersion profiles for triaxial core-Sérsic models can be found in Terzić & Sprague (2007).

Graham et al. (2011) discuss additional caveats regarding measurements of the central velocity dispersions. One of these pertains to dynamically-hot, (dry merger)-built spheroids in which the coalescence of the binary black hole has scoured out the central stellar phase space to create spheroids with core-Sérsic light profiles (Begelman et al. 1980; Graham et al. 2003a; Merritt & Milosavljević 2005; Merritt et al. 2007).15 Removing a galaxy’s inner stars results in a new dynamical structure with a reduced central velocity dispersion (Terzić & Graham 2005). This modification will act in a sense to further steepen the high-mass end of the $M_{sph}$-$\sigma$ relation, where core-Sérsic galaxies are found (Sahu et al. 2019b). This will be explored in future work, from a theoretical/mathematical perspective and using N-body simulations.

**APPENDIX B: OUTLIERS IN THE $M_{sph}$-$\sigma$ DIAGRAM**

NGC 4291 was previously flagged by Sahu et al. (2019b) as a notable outlier in the $M_{sph}$-$\sigma$ diagram. In Savorgnan & Graham (2016), NGC 4291 was modelled as an E galaxy. It is one of the lowest mass E galaxies in the sample used here and has the smallest size of the E galaxies in this sample. The residual (galaxy minus model) light profile in Savorgnan & Graham (2016, their Fig. 61) reveals a ‘snake-like’ pattern suggestive that the single Sérsic function fit beyond the depleted core may be inadequate.16 The galaxy might, therefore, have a disc component, and, depending on its (unknown) bulge-to-total stellar mass ratio, it may better mesh with the $M_{sph}$-$\sigma$ relation for S0 galaxies.

Due to the torquing effect that it may have on the linear regressions, Sahu et al. (2019a) flagged and excluded NGC 2787. It is the S0 galaxy with the lowest spheroid mass and the second smallest spheroid size in the S0 galaxy sample. The galaxy was brought to attention by (Erwin et al. 2003) for containing both a classical bulge and a ‘pseudobulge’, and Sil’chenko & Afanasiev (2004) note that it has a polar disc plus two nuclear rings which do not reside in the galaxy’s main disc plane. The galaxy has most recently been modelled in Graham & Sahu (2022b). Its prominent nuclear disc within the inner couple of arcseconds (Ledo et al. 2010) may have elevated the measured velocity dispersion relative to the small $R_{sph} = 140$ pc classical bulge.17 However, it may be that the spheroid, i.e., the classical bulge, mass is low, partly explaining its offset in the colour-magnitude diagram Graham & Sahu (2022a, their Fig. A1). Although, including the ‘barlens’ (12.26 mag at 3.6 $\mu$m, AB) with the bulge (11.66 mag at 3.6 $\mu$m, AB) would boost the current spheroid luminosity by just 58 per cent, or 0.20 dex.

Sahu et al. (2020) excluded the S galaxy NGC 4945 because it is a more than a 2-sigma outlier in their $M_{bh}$–(Sérsic $n$) diagram, possibly indicating the Sérsic index and thus spheroid mass is not correct. NGC 4945 has the lowest spheroid stellar mass of the S galaxies. As with NGC 2787, this galaxy is excluded due to its ability to skew the regressions. Sahu et al. (2020) flagged and excluded a further three outlying galaxies, which are retained here. One of these is NGC 3998, seen as something18 of an outlier in Fig. 1 but its exclusion/inclusion has no significant impact here. Another is NGC 5419, which has since been remodelled in Graham & Sahu (2022b). The third galaxy is NGC 3377, also flagged in Graham & Sahu (2022a, their Fig. 8) and mentioned in Fig. 2 but otherwise retained here. These systems are labelled in Fig. 1.

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15 Prior to the core-Sérsic model, galaxies with depleted cores were studied by King & Minkowski (1966), King & Minkowski (1972), Faber et al. (1997, and references therein).

16 If NGC 4291 is reclassified from E to S0, then the partially depleted core in NGC 4291 (Trujillo et al. 2004; Dullo & Graham 2012) would make it one of three S0 galaxies in the sample thought to have been built by a dry merger.

17 Based on the image analysis, the ‘barlens’, bar and disc do not dominate the light until beyond ~8$''$.

18 NGC 3998 resides 2.5-sigma (2.5 $\Delta m_{0}$) from the $M_{sph}$-$\sigma$ relation for 30 S0 galaxies (constructed without NGC 2787).