**PρLog: a system for rule-based programming**

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1 Brief overview

PρLog [11] is a rule-based system that supports programming with individual, hedge, function and context variables. It extends Prolog with rule-based programming capabilities to manipulate sequences of terms, also known as *hedges*. The four kinds of variables help to traverse tree forms of expressions both in horizontal and vertical directions, in one or more steps. It facilitates to have expressive pattern matching that often helps to write short and intuitive code.

Another important feature of PρLog is the use of strategies. They provide a mechanism to control complex rule-based computations in a highly declarative way. With the help of strategies, the user can combine simpler transformation rules into more complex ones. In this way, PρLog conveniently combines the whole Prolog power with rule-based strategic programming features.

PρLog is based on ρLog calculus [17], where the inference system basically is the SLDNF-resolution with normal logic program semantics [14]. It has been successfully used in the extraction of frequent patterns from data mining workflows [20], XML transformation and web reasoning [7], modeling of rewriting strategies [9] and access control policies [18], etc.

The ρLog calculus has been influenced by the ρ-calculus [4, 5], which, in itself, is a foundation for the rule-based programming system ELAN [2]. There are some other languages for programming by rules, such as, e.g., ASF-SDF [3], CHR [12], Maude [6], Stratego [21], Tom [1]. The ρLog calculus and, consequently, PρLog differs from them, first of all, by its pattern matching capabilities. Besides, it adopts logic programming semantics (clauses are first class concepts, rules/strategies are expressed as clauses) and makes a heavy use of strategies to control transformations. Earlier works about ρLog and its implementation in Mathematica include [16, 19, 15].

PρLog is available at [https://www.risc.jku.at/people/tkutsia/software/prholog](https://www.risc.jku.at/people/tkutsia/software/prholog).

2 The PρLog language

We write PρLog constructs in typewriter font. They are *terms* (including a special kind of terms, called *contexts*) and hedges. These objects are constructed from function symbols without a fixed arity (so called unranked or variadic function symbols), a special constant *hole* (called the *hole*), and individual, functional, context and hedge variables. These variables are denoted by the identifiers whose names start respectively with $i_-$, $f_-$, $c_-$, and $h_-$ (e.g., $i_X$, $f_X$, $c_X$, $h_X$). Terms $t$ and hedges $h$ are constructed in a standard way:

- **Terms**:
  
  $$t ::= \text{hole} \mid i_X \mid f(h) \mid f_X(h) \mid c_X(t)$$

- **Hedges**:
  
  $$h ::= \text{eps} \mid t \mid h_X \mid (h.h)$$
where $f$ is a function symbol and $\text{eps}$ stands for the empty hedge and is omitted whenever it appears as a subhedge of another hedge. A context is a term with a single occurrence of the hole constant. Application of a context $C$ to a term $t$ is a term derived by replacing the hole in $C$ with $t$. For instance, applying $f(i_X,g(i_Y,\text{hole}),a)$ to $g(b,\text{hole})$ gives another context $f(i_X,g(i_Y,g(b,\text{hole})),a)$, while applying it to $g(b,c)$ gives a non-context term $f(i_X,g(i_Y,g(b,c)),a)$.

A substitution is a mapping from individual variables to hole-free terms, from hedge variables to hole-free hedges, from function variables to function variables and symbols, and from context variables to contexts, such that all but finitely many individual, sequence, and function variables are mapped to themselves, and all but finitely many context variables are mapped to themselves applied to the hole. This mapping can be extended to terms and hedges in the standard way. For instance, for a substitution $\sigma = \{c_{\text{Ctx}} \mapsto f(\text{hole}), i_{\text{Term}} \mapsto g(h_X), f_{\text{Funct}} \mapsto g, h_{\text{Hedge1}} \mapsto \text{eps}, h_{\text{Hedge2}} \mapsto (b,c)\}$ and a hedge $h = (c_{\text{Ctx}}(i_{\text{Term}}), f_{\text{Funct}}(h_{\text{Hedge1}}, a, h_{\text{Hedge2}}))$, by applying $\sigma$ to $h$ we get the hedge $\sigma(h) = (f(g(h_X)), g(a,b,c))$.

Matching problems are pairs of hedges, one of which is ground (i.e., does not contain variables). Such matching problems may have zero, one, or more (finitely many) solutions, called matching substitutions or matchers. For instance, the hedge $(h_1, f(i_X), h_2)$ matches $(f(a), f(b), c)$ in two different ways: one by the matcher $\{(h_1 \mapsto \{\}, i_X \mapsto a, h_2 \mapsto (f(b), c)\}$ and another one by the matcher $\{(h_1 \mapsto f(a), i_X \mapsto b, h_2 \mapsto c)\}$. Similarly, the term $c_X(f_Y(a))$ matches the term $f(a, g(a))$ with the matchers $\{c_X \mapsto f(hole, g(a)), f_Y \mapsto f\}$ and $\{c_X \mapsto f(a, g(hole)), f_Y \mapsto g\}$. Matching is the main computational mechanism in PpLog.

Institutions of sequence and context variables can be restricted by regular hedge and regular context languages, respectively. We do not go into the details of this feature of PpLog matching here.

A PpLog atom (\rho-atom) is a quadruple consisting of a hole-free term $st$ (a strategy), two hole-free hedges $h_1$ and $h_2$, and a set of regular constraints $R$ where each variable is constrained only once, written as $st :: h_1 \mapsto h_2$ where $R$. Intuitively, it means that the strategy $st$ transforms $h_1$ to $h_2$ when the variables satisfy the constraint $R$. We call $h_1$ the left hand side and $h_2$ the right hand side of this atom. When $R$ is empty, we omit it and write $st :: h_1 \mapsto h_2$. The negated atom is written as $st :: h_1 = \not\mapsto h_2$ where $R$. A PpLog literal (\rho-literal) is a \rho-atom or its negation. A PpLog clause is either a Prolog clause, or a clause of the form $st :: h_1 \mapsto h_2$ where $R :: body$ (in the sequel called a \rho-clause) where body is a (possibly empty) conjunction of \rho- and Prolog literals.

A PpLog program is a sequence of PpLog clauses and a query is a conjunction of \rho- and Prolog literals. There is a restriction on variable occurrence imposed on clauses: \rho-clauses and queries can contain only PpLog variables, and Prolog clauses and queries can contain only Prolog variables. If a Prolog literal occurs in a \rho-clause or query, it may contain only PpLog individual variables that internally get translated into Prolog variables.

### 3 Inference and strategies

PpLog execution principle is based on depth-first inference with leftmost literal selection in the goal. If the selected literal is a Prolog literal, then it is evaluated in the standard way. If it is a PpLog atom of the form $st :: h_1 \mapsto h_2$, due to the syntactic restriction called well-modedness (formally defined in [9]), $st$ and $h_1$ do not contain variables. Then a (renamed copy of a) program clause $st' :: h_1' \mapsto h_2' :: body$ is selected, such that the strategy $st'$ matches $st$ and the hedge $h_1'$ matches $h_1$ with a substitution $\sigma$. Next, the selected literal in the query is replaced with the conjunction $\sigma(body), id :: \sigma(h_2') \mapsto h_2$, where $id$ is the built-in strategy for identity: it succeeds iff its
right-hand side matches the left-hand side. Evaluation continues further with this new query. Success and failure are defined in the standard way. Backtracking explores other alternatives that may come from matching the selected query literal to the head of the same program clause in a different way (since context/sequence matching is finitary, see, e.g., [8,13]), or to the head of another program clause. Negative literals are processed by negation-as-failure.

When instead of the exact equality one uses proximity as in [10], then in place of \( id \), P\( \rho \)Log introduces another built-in strategy in the new query: \( prox(\lambda) \), which succeeds if iff its right-hand side matches the left-hand side approximately, at least with the degree \( \lambda \). Proximity relations indicate by which degree two expressions are close to each other, where the degree is a real number in \([0,1]\). Proximity with degree 1 means that the terms are equal, while degree 0 means that they are distinct. Hence, \( id \) can be seen as an abbreviation of \( prox(1) \). Proximity is a fuzzy reflexive, symmetric, non-transitive relation, suitable for modeling imprecise, incomplete information.

Some of the other predefined strategies of P\( \rho \)Log and their intuitive meanings are the following:

- **compose\((st_1, st_2, \ldots, st_n)\),** where \( n \geq 2 \), first transforms the input hedge by \( st_1 \) and then transforms the result by \( compose(st_2, \ldots, st_n) \) (or by \( st_2 \), if \( n = 2 \)). Via backtracking, all possible results can be obtained. The strategy fails if either \( st_1 \) or \( compose(st_2, \ldots, st_n) \) fails.

- **choice\((st_1, \ldots, st_n)\),** where \( n \geq 1 \), returns a result of a successful application of some strategy \( st_i \) to the input hedge. It fails if all \( st_i \)'s fail. By backtracking it can return all outputs of the applications of each of the strategies \( st_1, \ldots, st_n \).

- **first_one\((st_1, \ldots, st_n)\),** where \( n \geq 1 \), selects the first \( st_i \) that does not fail on the input hedge and returns only one result of its application. \( first_one \) fails if all \( st_i \)'s fail. Its variation, \( first_all \), returns via backtracking all the results of the application to the input hedge of the first strategy \( st_i \) that does not fail.

- **map\((st)\)** maps the strategy \( st \) to each term in the input hedge and returns the result hedge. Backtracking generates all possible output hedges. \( st \) should operate on a single term and not on an arbitrary hedge. \( map(st) \) fails if \( st \) fails for at least one term from the input hedge.

### 4 Examples

In this section we bring some examples to illustrate features and the expressive power of P\( \rho \)Log.

**Example 1** (Sorting). The following program illustrates how bubble sort can be implemented in P\( \rho \)Log.

\[
\text{swap}(f\_Ordering) :: (h\_X, i\_I, i\_J, h\_Y) \Rightarrow (h\_X, i\_J, i\_I, h\_Y) :-
\text{not}(f\_Ordering(i\_I, i\_J)).
\]

\[
\text{bubble\_sort}(f\_Ordering) := \text{first\_one}(nf(swap(f\_Ordering))).
\]

In the first clause, the user-defined strategy \( \text{swap} \) swaps two neighboring elements \( i\_I, i\_J \) in the given hedge if they violate the given ordering \( f\_Ordering \). The use of sequence variables \( h\_I, h\_J \) helps to identify the violating place in the given hedge by pattern matching, without the need to explicitly define the corresponding recursive procedure. The sorting strategy \( \text{bubble\_sort} \) is then defined as an exhaustive application of \( \text{swap} \) (via the built-in strategy \( nf \)), which will lead to a sorted hedge. The strategy \( \text{first\_one} \) guarantees that only the first answer computed by \( nf(swap(f\_Ordering)) \) is returned: it does not make sense to sort a hedge in different ways to get the same answer over and over again via backtracking.

The way how the \( \text{bubble\_sort} \) strategy is defined above is just an abbreviation of the clause

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1 This is an experimental feature, not yet included in the official distribution.
bubble_sort(f_Ordering) :: h_X ==> h_Y :-
    first_one(nf(swap(f_Ordering))) :: h_X ==> h_Y.

PpLog allows the user to write such abbreviations. To use this strategy for sorting a hedge, we could call, e.g.,

(?(bubble_sort(=<) :: (1,3,4,3,2) ==> h_X, Result).

and PpLog would return a single result in the form of a substitution:

Result = [h_X --> (1,2,3,3,4)].

Example 2 (Rewriting). One step of rewriting a term by some rule/strategy can be straightforwardly defined in PpLog:

    rewrite_step(i_Str) :: c_Ctx(i_X) ==> c_Ctx(i_Y) :- i_Str :: i_X ==> i_Y.

It finds a subterm i_X in c_Ctx(i_X) to which the strategy i_Str applies and rewrites it. Due to the built-in matching algorithm that finds the relevant instantiation of the context variable c_Ctx, this step corresponds to a leftmost-outermost rewriting step. In [9] we have illustrated how easily other rewriting strategies can be modeled in PpLog.

Example 3 (Using proximity). We assume that a proximity relation between function symbols is given. (Based on it, we can compute proximity between terms as well.) The task is to remove from a given hedge approximate duplicates, i.e., if the hedge contains two elements that are proximal to each other (by a predefined degree), we should get a hedge where only one of the proximal elements is retained. The strategy merge_dubles(i_D) below does it. It checks, whether the hedge contains somewhere two elements i_X and i_Y that are close to each other at least by the degree i_D and removes i_Y. merge_all_dubles(i_D) removes all duplicates and returns one answer:

merge_dubles(i_D) :: (h_X, i_X, h_Y, i_Y, h_Z) ==> (h_X, i_X, h_Y, h_Z) :-
    prox(i_D) :: i_X ==> i_Y.

merge_all_dubles(i_D) := first_one(nf(merge_dubles(i_D))).

Assume our proximity relation is such that a and b are proximal with the degree 0.6 and b is close to c with the degree 0.8. Then we have:

?(merge_dubles(0.5) :: (a,b,d,b,c) ==> h_Ans, Result).
Result = [h_Ans --> (d,c)] ; false.

?(merge_dubles(0.7) :: (a,b,d,b,c) ==> h_Ans, Result).
Result = [h_Ans --> (a,d,c)] ; false.

Due to nontransitivity of proximity relations and the fact that matching finds the first proximal pair (from the left), the order of hedge elements affects the answer. For instance, if we put a at the end, we get

?(merge_dubles(0.5) :: (b,d,b,c,a) ==> h_Ans, Result).
Result = [h_Ans --> (d,c,a)] ; false.

?(merge_dubles(0.7) :: (b,d,b,c,a) ==> h_Ans, Result).
Result = [h_Ans --> (d,c,a)] ; false.

It happens because a and c are not close to each (although a and b as well as b and c are).
5 Summary

The main advantages of $\rho$Log are: compact and declarative code; capabilities of expression traversal without explicitly programming it; the ability to use clauses in a flexible order with the help of strategies. Besides, $\rho$Log has access to the whole infrastructure of its underline Prolog system. These features make $\rho$Log suitable for nondeterministic computations, manipulating XML documents, implementing rule-based algorithms and their control, etc.

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