Modeling the thermal response of geothermal boreholes during peak heating and cooling demands

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Abstract. Designing a geothermal heat exchanger requires special care with peak heating and cooling demands of the building as material damages in the installation can occur due to the extreme temperatures reached by the heat carrying liquid. These peak demands last a few days at most and require the inclusion of the thermal inertia of the heat carrying liquid, the grout, and the ground located close to the boreholes in the theoretical model used to predict the thermal response of the geothermal heat exchanger. The present work develops such a model by expanding the grout and ground temperatures in terms of conveniently chosen multipoles.

1. Introduction
Nowadays it is well clear that the massive burning of fossil fuels has negative consequences for the global environment. In that line, heating and cooling of buildings represents more than 25% of the world energy consumption [1], and because of it, the use of energy-efficient HVAC (heating, ventilation, and air conditioning) systems is a highly beneficial and necessary action. One example are geothermal HVAC systems that consist in a water-to-water heat pump connected to a geothermal heat exchanger composed of geothermal boreholes. Each borehole, typically drilled between 50 and 300 meters into the ground, is equipped with one or more U-shaped pipes through which a liquid circulates to exchange heat with the surrounding ground. During the hottest and coldest moments of the year the employed heat carrying liquids reach their maximum and minimum operating temperatures. These need to be in a certain prescribed temperature range, typically between 0 and 35 degrees Celsius, determined by safety requirements, structural integrity considerations, and environmental concerns. To account for this when designing a geothermal HVAC system, theoretical models for the thermal response of geothermal heat exchangers are required to incorporate the thermal inertia of the heat carrying liquid, of the grout, and of the ground located close to the boreholes. Several of such models have already been proposed. Some of them approximate the geometry of the borehole by an axisymmetric configuration with an equivalent pipe centered at the borehole [2, 3, 4, 5, 6, 7, 8, 9, 10]. Others concentrate all thermal inertia of the grout into a few spatial locations and formulate models for the heat exchange between those locations and the rest of the borehole [11, 12, 13, 14, 15, 16, 17, 18, 19]. A third group of models perform a detailed numerical simulation of the complete borehole [20, 21, 22, 23, 24].
Despite their usefulness, these models present their own limitations: Analytical models require
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significant geometrical or physical approximations and exact numerical simulations require excessive computational resources. With that in mind, the present work provides a model that takes into account the aforementioned thermal inertia without any simplifications in the geometrical or physical configuration of the borehole. For this, the unsteady heat conduction equation in the grout and the ground is solved exactly by expanding the grout and ground temperatures in terms of infinite conveniently chosen multipoles centered at the different pipes. In the following sections the essence of the developed model is explained while a full detailed description can be found in [25].

2. Transient thermal response of slender geothermal boreholes

A geothermal borehole consists in a borehole of depth $H$ and radius $r_b$ drilled vertically into the ground, where straight pipes are placed inside through which a heat carrying liquid flows and exchanges heat with the surrounding ground. Due to its slenderness, $H/r_b \gg 1$, vertical heat conduction in grout and ground is negligible [26]. So, for most of the borehole, the problem to solve is two imensional and perpendicular to the borehole. With this in mind, the dependence with depth of the bulk temperatures of the liquid in the different pipes is irrelevant for the problem to solve here. This dependence only needs to be taken into account when solving the energy conservation equations along the pipes for which the results presented in this work and in [25] are required. Figure 1 depicts a typical borehole configuration with two pipes, although configurations with more pipes also exist. Each pipe $j$ inside the borehole is characterized by its outer radius $r_{pj}$, its wall thickness $d_j$, its thermal conductivity $k_j$, its time-dependent bulk temperature of the liquid $T_j(t)$, and its convective heat transfer coefficient $h_j$. The system is subject to the heat injection rate demanded by the building, which is set as a response to the weather and the operational circumstances of the building. As such, the heat injection rate has a characteristic time $t_q$ with a wide spectrum of values. This characteristics time leads to a characteristic penetration length $\sqrt{\alpha t_q}$ that determines the region affected by thermal inertias [26]. The present work focuses on characteristic times $t_q$ of the order of hours or days which lead to characteristic penetration lengths that are comparable to the borehole radius, $\sqrt{\alpha t_q} \sim r_b$. As such, the thermal inertia of the heat carrying liquid, the grout, and the ground close to the borehole, at radial distances of order $r_b$, need to be taken into account.

![Figure 1. Scheme of a typical geothermal borehole with a single U-shaped pipe inside.](image-url)
3. Formulation of the problem
The two-dimensional unsteady heat conduction problem in a plane perpendicular to the borehole is described by

\[
\frac{1}{\alpha} \frac{\partial T}{\partial t} = \frac{1}{r_0} \frac{\partial}{\partial r_0} \left( r_0 \frac{\partial T}{\partial r_0} \right) + \frac{1}{r_0^2} \frac{\partial^2 T}{\partial \theta_0^2},
\]

with radial and azimuthal coordinates \((r_0, \theta_0)\) parting from the borehole center and with the thermal diffusivity \(\alpha\) being equal to the one in the grout, \(\alpha_b\), when \(r_0 < r_b\) and equal to the one in the ground, \(\alpha_g\), when \(r_0 > r_b\).

At the borehole wall, continuity conditions in temperature and normal heat flux are imposed:

\[
T \bigg|_{r_0=r_b^-} = T \bigg|_{r_0=r_b^+}, \quad -k_b \frac{\partial T}{\partial r_0} \bigg|_{r_0=r_b^-} = -k_g \frac{\partial T}{\partial r_0} \bigg|_{r_0=r_b^+},
\]

being \(k_b\) and \(k_g\) the thermal conductivities of the grout and the ground, respectively.

At the outer surface of pipe \(j\), a Robin-type boundary condition is imposed whose thermal resistance \(R_{pj}\) accounts for the convective transport of heat inside the pipe and for the steady-state heat conduction through the thin pipe wall [27]:

\[
-k_b r_j \frac{\partial T}{\partial r_j} \bigg|_{r_j=r_{pj}} = \frac{T_j(t) - T \big|_{r_j=r_{pj}}}{R_{pj}}, \quad \text{with} \quad R_{pj} = \frac{1}{(r_{pj} - d_j) h_j} + \frac{1}{k_j} \ln \left( \frac{r_{pj}}{r_{pj} - d_j} \right).
\]

This condition is implemented \(N_p\) times, being \(N_p\) the number of pipes inside the borehole. Finally, the unperturbed ground temperature \(T_\infty\) is imposed as initial condition and as boundary condition far from the borehole:

\[
t = 0 : \quad T = T_\infty, \quad r_0 \to \infty : \quad T \to T_\infty.
\]

Once the temperature distribution in grout and ground is known, the heat injection rates per unit pipe length \(q_j(t)\) of the \(N_p\) pipes are obtained [27]. These are of interest for the analysis of the convective transport of heat along the pipes [26, 28]:

\[
q_j(t) = - \int_{-\pi}^{\pi} k_b r_{pj} \frac{\partial T}{\partial r_j} \bigg|_{r_j=r_{pj}} d\theta_j.
\]

4. Solution to the problem
The time dependency of the problem is addressed through the Laplace transform, which applied to (1) leads to the following equation for the Laplace-transformed temperature \(\tilde{T}\):

\[
\frac{s}{\alpha} \left[ \tilde{T} - \frac{T_\infty}{s} \right] = \frac{1}{r_0} \frac{\partial}{\partial r_0} \left( r_0 \frac{\partial \tilde{T}}{\partial r_0} \right) + \frac{1}{r_0^2} \frac{\partial^2 \tilde{T}}{\partial \theta_0^2},
\]

where \(s\) is the complex-valued position in the Laplace plane, and to the following expression for the boundary condition (4) far from the borehole:

\[
r_0 \to \infty : \quad \tilde{T} \to \frac{T_\infty}{s}.
\]

All other equations in Section 2 remain the same, except that a tilde (\(\sim\)) is used as superscript to denote the Laplace-transformed character of the variables.

To solve the Laplace-transformed problem, the temperature in the grout and the ground is written as
\[ \tilde{T} = \frac{T_\infty}{s} + \sum_{i=1}^{N_\mu} \sum_{n=-\infty}^{\infty} C_{in} F_{in}, \]  

being \( F_{in} \) the so-called multipoles [29], with subindex \( i \) determining the pipe around which they are centered. The multipoles are composed by modified Bessel functions of the first and second kind, which arise by solving (6) through separation of variables, and by coefficients that multiply them. These coefficients are found by expanding the multipoles \( F_{in} \) in Fourier series and imposing condition (2) on them. Once that is settled, in order to find the multipole intensities \( C_{in} \), condition (3) is imposed with a Fourier expansion of \( \tilde{T} \) over the wall of each pipe \( j \). A detailed description of the outlined derivation can be found in [25].

5. Results
A test case with a single U-shaped pipe inside a borehole is presented next. Its geometrical and physical properties, which are the same as the ones in [28], represent a real-world borehole in which the U-shaped pipe is close to the borehole wall as a consequence of the typical deviations from verticality of the borehole. The scenario of the present test case depicts a cooling demand of the building in which the temperature of the downward-flowing liquid in pipe 1 is higher than the temperature of the upward-flowing liquid in pipe 2. Their values are \( T_1 = 35^\circC \) and \( T_2 = 30^\circC \), respectively. In Figure 2 the real and imaginary parts of the Laplace-transformed temperature \( \tilde{T} \) are shown for \( s = \sqrt{\iota(\alpha_g/r_b^2)/100} \), where \( \iota = \sqrt{-1} \). It can be seen that the thermal perturbation induced by the two pipes reaches up to distances comparable to the borehole radius.

![Figure 2](image_url)

**Figure 2.** (Left) real and (right) imaginary parts of the Laplace-transformed temperature \( \tilde{T} \) of the transient thermal response of a borehole with a single U-shaped pipe inside.

6. Potential and application of the model
To achieve an optimal design for a geothermal HVAC system, all its regimes of operation have to be considered. As mentioned in the introduction of the present paper, a prescribed temperature range needs to be abided during peak heating and cooling demands of the building to avoid structural damages and safety issues. For it, the time evolution of the bulk fluid temperature in the pipes must be predicted. This time evolution is significantly affected by the thermal inertias of the heat carrying liquid, the grout, and the ground close to the borehole [4, 5, 7, 19, 25] so that not considering them leads to erroneous sizing of the borehole and failure to comply with
the prescribed temperature range of the HVAC system. In fact, if a model that does not consider thermal inertias is used, the resulting calculations lead in general to an oversized borehole with a depth larger than what is required, incurring in unnecessary costs. Consequently, the model discussed in the present work and in [25] serves to better design geothermal HVAC systems.

7. Conclusions
In the present work an exact model for the thermal response of geothermal boreholes to peak heating and cooling demands has been presented. Using a carefully chosen multipole expansion, this model, as opposed to the state of the art, takes into account the thermal inertia inside the borehole and its surrounding ground without the need of any artificial simplifications in the borehole's configuration.

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