Nuclear medium effects in structure functions

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Abstract. We discuss the nuclear-medium effects in the weak structure functions $F_2(x, Q^2)$
and $F_3(x, Q^2)$ in the charged current neutrino and antineutrino induced deep inelastic reactions
in some nuclei which are being used in the present or proposed neutrino oscillation experiments.
Our study of the medium effects has been done using relativistic nuclear spectral functions which
incorporates Fermi motion, binding energy and nucleon correlations. We also consider the pion
and rho meson cloud contributions calculated from a microscopic model for meson-nucleus self-
energies. Using these structure functions, the results for the differential cross section have been
obtained and compared with some of the available experimental data.

1. Introduction
At the present time many ongoing and proposed neutrino oscillation experiments are being
performed to determine precisely the parameters of neutrino mixing matrix, emphasis has also
been laid towards better understanding of the low and intermediate energy neutrino interactions
and specifically attention has been brought to study both theoretically and experimentally
the strong dynamics of the nucleon and nucleus that affect these interactions. Statistically
plentiful data are available from T2K, MiniBooNE, MINOS, SciBooNE, etc. and more results
are expected from the Main INjector Experiment $\nu$-A(MINER$\nu$A) being performed at Fermi
Lab. MINER$\nu$A is studying the medium effects in neutrino-nucleus cross-section measurements
using various nuclear targets like helium, carbon, oxygen, iron and lead using muon neutrinos in
the energy region of 1-20 GeV and the aim is to study effect of nuclear medium in the calculation
of the event rates.
In the few GeV energy region the contribution to the cross section comes from the quasielastic, inelastic as well as the deep inelastic processes. Several theoretical calculations have been performed for the quasielastic and inelastic one pion production processes. In the deep inelastic region, both experimentally as well as theoretically, limited efforts have been made to understand the medium effects for the weak interaction induced processes. In the case of experimental measurements performed for $F_{2,3}^{W\text{eak}}$ the error bars are large and need better precision. In general, nuclear modifications in the weak structure functions $F_{2,3}^{W\text{eak}}(x, Q^2)$ may be different from the nuclear modification in $F_2^{EM}$. Particularly neutron data is important in the determination of valence quark distributions in the nucleon. This is due to the fact that the parity-violating ($F_3^{W\text{eak}}$) structure function directly probes into the valence distributions. Precise determination of parton distribution functions (PDFs) is also necessary for the new physics at the colliders. But for the determination of PDFs, nuclear medium effects should be properly accounted for. Nuclear-medium effects may be responsible for the anomaly observed in the NuTeV experiment for the weak mixing angle $\sin^2\theta_W$ [1]. The weak structure functions $F_2^N(x, Q^2)$ and $F_2^\bar{N}(x, Q^2)$ have also been measured using neutrino (antineutrino) beams [2, 3, 4, 5, 6, 7, 8, 9] with some nuclear targets. CERN Dortmund Heidelberg Saclay Warsaw CDHSW [3] and NuTeV [9] collaborations have studied differential scattering cross sections in iron.

CERN Hybrid Oscillation Research apparatUS CHORUS collaboration [10, 11] has performed high-statistics measurement of the differential (anti)neutrino cross-sections using mainly lead target at various neutrino and antineutrino energies as a function of Bjorken scaling variables $x$ and $y$. Analysis is being performed at the Neutrino Oscillation MAgnitic Detector (NOMAD) [12] for the weak structure functions and cross section measurements with carbon target using neutrino beam. The Neutrino Scattering On Glass (NuSOnG) experiment [13] has been proposed at Fermilab to study the structure functions in the deep inelastic region using neutrino scattering on carbon target. The Oscillation Project with Emulsion-tRacking Apparatus (OPERA) experiment [14] is a long baseline experiment using lead emulsion target and the main purpose of the experiment is the observation of $\nu_\mu$ to $\nu_\tau$ oscillations in the direct appearance mode and the charged current deep inelastic scattering process is expected to be dominant. Therefore, several experimental activities are going on to study neutrino as well as antineutrino event rates in the deep inelastic region.

On the theoretical side, for the weak interaction induced processes in the deep inelastic region, the studies can be divided into two parts. One in which nuclear medium effects have been phenomenologically described in terms of a few parameters which are determined by fitting the experimental data of charged leptons and (anti)neutrino deep inelastic scattering from various nuclear targets. The various phenomenological studies differ in the choice of the data sets (Lepton-Nucleus+Drell Yan data, Lepton-Nucleus+Drell Yan+$\nu(\bar{\nu})$-Nucleus data, and $\nu(\bar{\nu})$-Nucleus data), experimental cuts in their analysis, parametrization of the parton distributions, etc. for example see the discussions given in Refs. [15, 16, 17]. Another approach is to understand the dynamics of the nucleon in the nuclear medium. This approach has been used by few authors [18, 19, 20] and more work is needed in this direction.

In the present work, we study nuclear medium effects on the structure functions $F_2(x, Q^2)$ and $F_3(x, Q^2)$ in some nuclei like carbon, iron and lead. We shall treat iron and lead to be a nonisoscalar nuclear targets. This study has been performed using a relativistic nuclear spectral function [21] to describe the momentum distribution of nucleons in the nucleus within a field-theoretical approach where nucleon propagators are written in terms of this spectral function, and nuclear many-body theory is used to calculate it for an interacting Fermi sea in nuclear matter. A local-density approximation is then applied to translate these results to finite nuclei [19, 20, 22]. We have assumed the Callan-Gross relationship for nuclear structure functions $F_1^A(x)$ and $F_2^A(x)$. The contributions of the pion and rho meson clouds are taken into account in a many-body field-theoretical approach which is based on Refs. [22, 23].
have taken into account target mass correction (TMC) following Ref. [24] which has significant effect at low $Q^2$, moderate, and high Bjorken x. To take into account the shadowing effect, which is important at low $Q^2$ and low x, and modulates the contribution of pion and rho cloud contributions, we have followed the works of Kulagin and Petti [18, 25]. Using structure functions $F_2^A$ and $F_3^A$ as an input we have calculated differential scattering cross section for charged current neutrino and antineutrino induced reactions. The Next-to-leading Order(NLO) evolution has been incorporated using the prescription of Moch et al [26]. In the next section we present the formalism in brief and in Sec. III we present the results and discussions. In Sec. IV we conclude our findings.

2. Formalism

The expression of the cross section, for deep inelastic scattering (DIS) of neutrino with a nucleon target induced by charged current reaction

$$\nu_l(k) + N(p) \rightarrow l^{-}(k') + X(p'), \ l = \mu,$$

is written as

$$\sigma = \frac{1}{e_{\nu_{\mu}} \left(2E_{\nu}(k)\right) \left(2E_{l}(p')\right)} \int d^4 p' \ N \prod \left(\frac{2M_{2}'}{2E_{l}'(p')}\right) \prod \left(\frac{1}{2\omega'}\right) \sum \sum |T|^2 \left(2\pi\right)^4 \times \delta^4 \left(p + k - k' - \sum_{i=1}^{N} p_i'\right)$$

(2)

where $f$ stands for fermions and $b$ for bosons in the final state $X$. The index $i$ is split into $l$ and $j$ for fermions and bosons respectively,

$T$ is the invariant matrix element for the above reaction and is, written as

$$-i T = \left(\frac{iG_F}{\sqrt{2}}\right) \bar{u}_l(k')\gamma^\alpha (1 - \gamma_5) u_\nu(k) \left(\frac{m_W^2}{q^2 - m_W^2}\right) \langle X | J_\alpha | N \rangle .$$

After performing the phase space integration in Eq.(2), the double differential scattering cross section evaluated for a nucleon target in its rest frame is expressed as

$$\frac{d^2 \sigma_N}{dQ^2 dE'} = \frac{G_F^2}{(2\pi)^2} \frac{|k'|}{|k|} \left(\frac{m_W^2}{q^2 - m_W^2}\right)^2 L_{\nu,\bar{\nu}}^{\alpha\beta} W_N^{\alpha\beta},$$

(4)

where $G_F$ is the Fermi coupling constant, $m_W$ is the mass of the W boson, $k$ is the incoming neutrino four momentum and $k'$ is the outgoing four momentum of the lepton, $q = k - k'$ is the four momentum transfer and $\Omega', E'$ refer to the outgoing lepton.

The lepton tensor for antineutrino(neutrino) scattering $L_{\nu,\bar{\nu}}^{\alpha\beta}$ is given by

$$L_{\nu,\bar{\nu}}^{\alpha\beta} = k^\alpha k'^\beta + k'^\alpha k^\beta - k. k' g^{\alpha\beta} \pm i e^{\alpha\beta\gamma\delta} k_\gamma k'^\delta,$$

(5)

and the hadronic tensor $W_N^{\alpha\beta}$ is defined as

$$W_N^{\alpha\beta} = \frac{1}{2\pi} \sum_{X} \sum_{s_{N}} \prod_{i=1}^{n} \left(\frac{2M_i'}{2E_{l}'(p')}\right) \prod_{j=1}^{\tilde{N}} \left(\frac{1}{2\omega_j}\right) \langle X | J_{\alpha} | N \rangle \langle X | J_{\beta} | N \rangle \left(2\pi\right)^4 \delta^4 \left(p + q - \sum_{i=1}^{n} p_i'\right),$$

(6)

where $q$ is the momentum of the virtual $W$, $s_{N}$ the spin of the nucleon and $s_i$ the spin of the fermions in $X$. $N$ is a nucleon, $X$ is a jet of n hadrons consisting of fermions(f) and bosons(b).
in the final state labeled by \( l \) and \( j \) in the following. In the case of antineutrino \( \langle X | J_\alpha | N \rangle \) is replaced by \( \langle X | J_\alpha | N \rangle \). The most general form of the hadronic tensor \( W_{j\beta}^N \) is expressed as

\[
W_{j\beta}^N = \left( \frac{q_\alpha q_\beta}{q^2} - g_{\alpha\beta} \right) W_{1}^{\nu(\beta)} + \frac{1}{M^2} \left( p_\alpha - \frac{p.q}{q^2} q_\alpha \right) \left( p_\beta - \frac{p.q}{q^2} q_\beta \right) W_{2}^{\nu(\beta)} \frac{1}{M^2} q_\alpha q_\beta W_{3}^{\nu(\beta)} + \frac{1}{M^2} q_\alpha q_\beta W_{4}^{\nu(\beta)} + \frac{1}{M^2} (p_\alpha q_\beta + g_{\alpha\beta} q_\beta) W_{5}^{\nu(\beta)} + \frac{1}{M^2} (p_\alpha q_\beta - g_{\alpha\beta} p_\beta) W_{6}^{\nu(\beta)},
\]

where \( M \) is the nucleon mass and \( W_{i}^{\nu(\beta)} \) are the structure functions, which depend on the scalars \( q^2 \) and \( p.q \). The terms depending on \( W_4, W_5 \) and \( W_6 \) in Eq. (7) do not contribute to the cross section in Eq. (4) in the limit of lepton mass \( m_l \to 0 \).

In terms of the Bjorken variables \( x \) and \( y \) defined as

\[
x = \frac{Q^2}{2M\nu}, \quad y = \frac{\nu}{E_\nu}, \quad Q^2 = -q^2, \quad \nu = \frac{p.q}{M}
\]

we can write the expression for the differential scattering cross section (in the limit of lepton mass \( m_l \to 0 \)) as

\[
\frac{d^2\sigma^{\nu(p)}}{dxdy} = \frac{G_F^2 M E_\nu}{\pi} \left\{ xy^2 F_1^{\nu(p)}(x,Q^2) + \left( 1 - y - \frac{xyM}{2E_\nu} \right) F_2^{\nu(p)}(x,Q^2) + xy(1-y/2) F_3^{\nu(p)}(x,Q^2) \right\},
\]

where the + (−) sign stands for the neutrino (antineutrino) cross section, and the \( F_i^{\nu(p)}(x,Q^2) \) are dimensionless structure functions defined as

\[
F_1^{\nu(p)}(x,Q^2) = M W_1^{\nu(p)}(\nu,Q^2), \quad F_2^{\nu(p)}(x,Q^2) = \nu W_2^{\nu(p)}(\nu,Q^2), \quad F_3^{\nu(p)}(x,Q^2) = \nu W_3^{\nu(p)}(\nu,Q^2)
\]

\( F_1 \) and \( F_2 \) are related by the Callan-Gross relation [27] leaving only two unknown structure functions \( F_2^{\nu(p)}(x,Q^2) \) and \( F_3^{\nu(p)}(x,Q^2) \). The nucleon structure functions are determined in terms of parton distribution functions for quarks and anti-quarks. The parton distribution functions for the nucleons have been taken from CTEQ6.6 [28] collaboration for our numerical calculations.

**Figure 1.** Self-energy diagram of the neutrino in the nuclear medium associated with the process of deep inelastic neutrino-nucleon scattering.

**Figure 2.** Self-energy diagram of the W-boson in the nuclear medium.
3. Neutrino nucleus scattering

In a nuclear medium the expression for the cross section given in Eq.(4) is modified as:

\[
\frac{d^2\sigma^A_{\nu,\bar{\nu}}}{d\Omega dE} = \frac{G_F^2}{(2\pi)^2} \left| \frac{m_W^2}{q^2 - m_W^2} \right|^2 L_{\alpha\beta} W^A_{\alpha\beta},
\]

where \(W^A_{\alpha\beta}\) is the nuclear hadronic tensor defined in terms of nuclear hadronic structure functions \(W_{iA}(x, Q^2)\) through Eq.(7).

In our formalism the neutrino nuclear cross sections are obtained in terms of neutrino self energy \(\Sigma(k)\) in the nuclear medium which also defines the dimensionless nuclear structure functions \(F_i^A(x, Q^2)\). This is when compared with the equation for the differential scattering cross section for neutrino scattering with a nucleon with momentum \(p = (E, p)\) in the rest frame of the nucleus

\[
\frac{d^2\sigma^N_{\nu,\bar{\nu}}}{d\Omega dE} = \frac{G_F^2}{(2\pi)^2} \left| \frac{m_W^2}{q^2 - m_W^2} \right|^2 L_{\alpha\beta} W^N_{\alpha\beta},
\]

gives the nuclear hadronic tensor \(W^A_{\alpha\beta}\). With proper choice of tensor components \(\alpha, \beta\), the dimensionless nuclear structure functions \(F_i^A(x, Q^2)\) are obtained[22].

The probability per unit time for the neutrino to collide with nucleons when traveling through nuclear matter may be expressed as:

\[
\Gamma(k) = -\frac{2m_\nu}{E_\nu(k)} \text{Im} \Sigma(k),
\]

and the cross section for neutrino scattering from an element of volume \(d^3r\) and surface \(dS\) in the nucleus is given by

\[
d\sigma = \Gamma dtdS = \Gamma \frac{dt}{dt} dtdS = \frac{\Gamma}{E_\nu(k)} \frac{E_\nu(k)}{|k|} d^3r = -\frac{2m_\nu}{|k|} \text{Im} \Sigma d^3r.
\]

The neutrino self-energy in nuclear matter corresponding to Fig.1 is given by,

\[
\Sigma(k) = \frac{-iG_F}{\sqrt{2}m_\nu} \int \frac{d^4k'}{(2\pi)^4} \frac{1}{k'k - m_\nu^2 + i\epsilon} \left( \frac{m_W^2}{q^2 - m_W^2} \right)^2 L_{\alpha\beta} \Pi^{\alpha\beta}(q),
\]

where \(L_{\alpha\beta}\) is given by Eq.(5) and \(\Pi^{\alpha\beta}(q)\) is the \(W\) self-energy in the nuclear medium and is written with the help of Fig.2 as:

\[
-\Pi^{\alpha\beta}(q) = \int \frac{d^4p}{(2\pi)^4} G(p) \sum_{X} \sum_{p_{i1}} \prod_{j=1}^{n} \int \frac{d^4p'_j}{(2\pi)^4} \prod_{j'} iG_i(p'_j) iD_j(p'_j) \frac{-G_F m_W^2}{\sqrt{2}} X X^* J^*_f |N| J^f|N| |p - p - \sum_{i=1}^n p'_i|.
\]

In the above expression \(G_i(p'_j)\) and \(D_j(p'_j)\) are respectively the nucleon and meson relativistic propagators in the final state which are taken as the standard free relativistic propagators. \(G(p)\) is the nucleon propagator with mass \(M\) and energy \(E(p)\) in the initial state, which is calculated for a relativistic nucleon in the interacting Fermi sea.

The relativistic Dirac propagator \(G(p)\) for a free nucleon, is written in terms of the contribution from the positive and negative energy components of the nucleon described by the Dirac spinors \(u(p)\) and \(\bar{u}(p)\) using their appropriate normalizations is written as

\[
G^0(p) = \frac{\not{p} + M}{p^2 - M^2 + i\epsilon} = \frac{M}{E(p)} \left\{ \frac{\sum_r u_r(p)\bar{u}_r(p)}{p^0 - E(p) + i\epsilon} + \frac{\sum_r \bar{u}_r(-p)\bar{u}_r(-p)}{p^0 + E(p) - i\epsilon} \right\}
\]
The nucleon propagator \( G(p) \) is then calculated by making a perturbative expansion of \( G(p) \) in terms of \( G^0(p) \) given in Eq.(17) by retaining the positive energy contributions only (the negative energy components are suppressed). This perturbative expansion is summed in ladder approximation to give [22]:

\[
G(p) = \frac{M}{E(p)} \sum_r u_r(p) \bar{u}_r(p) \left[ \int_{-\infty}^{\mu} d\omega \frac{S_h(\omega, p)}{p^0 - \omega - i\eta} + \int_{-\infty}^{\mu} d\omega \frac{S_p(\omega, p)}{p^0 - \omega + i\eta} \right],
\]

(18)

where \( \Sigma^N(p^0, p) \) is the nucleon self energy in nuclear matter taken from Ref. [21]. The relativistic nucleon propagator \( G(p) \) in a nuclear medium is then expressed as [22]:

\[
G(p) = \frac{M}{E(p)} \sum_r u_r(p) \bar{u}_r(p) \left[ \int_{-\infty}^{\mu} d\omega \frac{S_h(\omega, p)}{p^0 - \omega - i\eta} + \int_{-\infty}^{\mu} d\omega \frac{S_p(\omega, p)}{p^0 - \omega + i\eta} \right],
\]

(19)

where \( S_h(\omega, p) \) and \( S_p(\omega, p) \) being the hole and particle spectral functions respectively, which are given by [22, 21]:

\[
S_h(p^0, p) = \frac{1}{\pi} \frac{M}{E(p)} \frac{Im\Sigma^N(p^0, p)}{(Re\Sigma^N(p^0, p))^2 + (Im\Sigma^N(p^0, p))^2}
\]

(20)

for \( p^0 \leq \mu \)

\[
S_p(p^0, p) = \frac{1}{\pi} \frac{M}{E(p)} \frac{Im\Sigma^N(p^0, p)}{(Re\Sigma^N(p^0, p))^2 + (Im\Sigma^N(p^0, p))^2}
\]

(21)

for \( p^0 > \mu \).

The normalization of this spectral function is obtained by imposing the baryon number conservation which is written as:

\[
2V \int \frac{d^3p}{(2\pi)^3} \int_{-\infty}^{\mu} S_h(\omega, p) d\omega = B = 1.
\]

(22)

In the local density approximation, we do not have a box of constant density, and the reaction takes place at a point \( r \), lying inside a volume element \( d^3r \) with local density \( \rho_p(r) \) and \( \rho_n(r) \) corresponding to the proton and neutron. Therefore, the upper limit in the integration over nucleon momentum in Eq. (22) is the local Fermi momentum \( k_{F_{p,n}}(r) \) of the nucleon given by:

\[
k_{F_p}(r) = \left[ 3\pi^2 \rho_p(r) \right]^{1/3}, k_{F_n}(r) = \left[ 3\pi^2 \rho_n(r) \right]^{1/3}.
\]

(23)

This makes the spectral function \( S_h(\omega, p) \) density dependent i.e. \( S_h(\omega, p, k_F(r)) \) and the normalization condition given in Eq. (22) is modified to

\[
2 \int \frac{d^3p}{(2\pi)^3} \int_{-\infty}^{\mu} S_h(\omega, p, k_{F_{p,n}}(r)) d\omega = \rho_{p,n}(r)
\]

(24)

For a symmetric nuclear matter of density \( \rho(r) \), there is a unique Fermi momentum given by \( k_F(r) = \left[ \frac{3}{2} \pi^2 \rho(r) \right]^{1/3} \) for which we obtain

\[
4 \int \frac{d^3p}{(2\pi)^3} \int_{-\infty}^{\mu} S_h(\omega, p, k_F(r)) d\omega = \rho(r),
\]

(25)

where \( \rho(r) \) is the baryon density for the nucleus.
This leads to the normalization condition given by
\[
4 \int d^3r \int \frac{d^3p}{(2\pi)^3} \int_{-\infty}^{\mu} S_h(\omega, p, \rho(r)) \, d\omega = A \quad (26)
\]

In the antineutrino case the expressions obtained are very similar. \( L^{\alpha\beta} \) appears, as in Eq. (5), with a minus sign in front and in the \( W \) self-energy, Eq. (16), we have \( \langle X|J_{\alpha}^{\dagger}|N \rangle \), instead of \( \langle X|J_{\alpha}|N \rangle \).

Using Eq.(14) in Eq.(15), we get the expression for the total scattering cross section in the local density approximation as
\[
\sigma = \frac{4\sqrt{2}G_F}{|k|} \text{Im} \int d^3r \int \frac{d^3k'}{(2\pi)^4} \frac{1}{k'^2 - m_{\mu}^2 + i\epsilon} \left( \frac{m_W}{q^2 - m_W^2} \right)^2 L_{\alpha\beta} \Pi^{\alpha\beta}(q) \quad (27)
\]
The imaginary part of the neutrino self energy is evaluated by means of the Cutkosky rules by cutting the Feynman diagram shown in Fig. 2 along the dotted line which puts the particles corresponding to the cut propagators on the mass shell by replacing the fermion and meson propagators by their imaginary parts as
\[
\Sigma(k) \rightarrow 2i\text{Im}\Sigma(k), \quad D(p_j') \rightarrow 2i\theta(p_{0j}) \text{Im}D(p_j') \quad (28)
\]
\[
G(p_j') \rightarrow 2i\theta(p_{0j}) \text{Im}G(p_j'), \quad \frac{1}{k'^2 - m_{\mu}^2 + i\epsilon} \rightarrow 2\pi\delta(k'^2 - m_{\mu}^2) .
\]

After performing the \( k_0' \), \( p_{0j}' \) and \( p_{0j} \) integrations for all momenta in Eq.(27) and using Eqs.(19) and (28), we get the differential scattering cross section which is written in the local density approximation as:
\[
\frac{d^2\sigma_{\nu,\bar{\nu}}}{dQ'dE'} = \frac{G_F^2}{(2\pi)^2} \frac{|k'|}{|k|} \left( \frac{m_W^2}{q^2 - m_W^2} \right)^2 L_{\alpha\beta}^{\nu,\bar{\nu}} W^{A}_{\alpha\beta}, \quad (29)
\]
where
\[
W^{A}_{\alpha\beta} = 4 \int d^3r \int \frac{d^3p}{(2\pi)^3} \int_{-\infty}^{\mu} dp^0 \frac{M}{E(p)} S_h(p^0, p, \rho(r)) W^{N}_{\alpha\beta}(p, q) . \quad (30)
\]

Finally \( W^A_{\alpha\beta}(x, Q^2) \) (i=1-3) are redefined in terms of the dimensionless structure functions \( F^A_i(x, Q^2) \) through
\[
M_A W^A_i(x, Q^2) = F^A_i(x, Q^2), \quad \nu W^A_2(x, Q^2) = F^A_2(x, Q^2), \quad \nu W^A_3(x, Q^2) = F^A_3(x, Q^2) . \quad (31)
\]

We have also assumed the Callan-Gross relationship for nuclear structure functions \( F^A_i(x) \) and \( F^A_j(x) \), therefore, we are left with only two independent structure functions viz. \( F^A_2(x) \) and \( F^A_3(x) \). Taking xy and xx component on both sides of Eq. (30) and \( q \) along the \( z \) axis we can evaluate structure functions \( F^A_2(x) \) and \( F^A_3(x) \).

\[
F^A_2(x, Q^2) = 4 \int d^3r \int \frac{d^3p}{(2\pi)^3} \frac{M}{E(p)} \int_{-\infty}^{\mu} dp^0 S_h(p^0, p, \rho(r)) \frac{x}{x_N} \left( 1 + \frac{2x_N p^0}{M_{NN}} \right) F^N_2(x_N, Q^2) . \quad (32)
\]
\[
F^A_3(x, Q^2) = 4 \int d^3r \int \frac{d^3p}{(2\pi)^3} \frac{M}{E(p)} \int_{-\infty}^{\mu} dp^0 S_h(p^0, p, \rho(r)) \gamma \frac{p^0 - p_z}{p^0 - p_e} F^N_3(x_N, Q^2) . \quad (33)
\]
where
\[
\gamma = \frac{q_z}{q^2} = \left( 1 + \frac{4M^2 x_N^2}{Q^2} \right)^{1/2}, x_N = \frac{Q^2}{2(p^0 q^2 - p_e q_z)} . \quad (34)
\]
Eqs.32 and 33 are used for our base calculations where Pauli blocking and Fermi motion has been taken into account. Thereafter, we have added other medium effects namely shadowing, anti-shadowing, pion, and rho cloud contributions. The expression for the target mass correction (TMC) is taken from Ref. [24] and the shadowing effect has been incorporated using the model of Kulagin and Petti [18, 25]. We shall give the expressions for the pion and rho cloud contributions in the next subsection.

In \( F_3^A(x) \) only valence quarks contribute due to which there is no pion and rho cloud contributions. Target mass correction has also been considered in our base calculations. Numerical calculations in our paper with all medium effects are called as full model calculations. All of the above calculations have been done for isoscalar targets. For non-isoscalar targets like iron and lead where proton and neutron numbers are different few modifications are needed to be done in Eqs.30, 32, and 33.

\[
W_{\alpha\beta}^{(\nu)p} = 2 \int d^3r \int \frac{d^3p}{(2\pi)^3} \frac{M}{E(p)} \int_{-\infty}^{\mu_p} dp^0 S_{h}^{proton}(p^0, \mathbf{p}, k_{F,p}) W_{\alpha\beta}^{(\nu)p} \\
+ 2 \int d^3r \int \frac{d^3p}{(2\pi)^3} \frac{M}{E(p)} \int_{-\infty}^{\mu_n} dp^0 S_{h}^{neutron}(p^0, \mathbf{p}, k_{F,n}) W_{\alpha\beta}^{(\nu)n} 
\]

(35)

where the factor 2 in front of the integral accounts for the two degrees of freedom of the spin of the proton/neutron. There are two different spectral functions, each one of them normalized to the number of protons or neutrons in the nuclear target and are functions of Fermi momentum of protons and neutrons respectively which are given by \( k_{F,p} = (3\pi^2 r_p)^{1/3} \) and \( k_{F,n} = (3\pi^2 r_n)^{1/3} \). For the proton and neutron densities in iron and lead, we have used two-parameter Fermi density distribution and harmonic oscillator density for carbon. The density parameters are taken from Refs. [29, 30].

The expressions for \( F_2^A(x) \) and \( F_3^A(x) \) are obtained as:

\[
F_2^A(x, Q^2) = 2 \int d^3r \int \frac{d^3p}{(2\pi)^3} \frac{M}{E(p)} \left[ \int_{-\infty}^{\mu_p} dp^0 S_{h}^{proton}(p^0, \mathbf{p}, k_{F,p}) F_2^{proton}(x_N, Q^2) \\
+ \int_{-\infty}^{\mu_n} dp^0 S_{h}^{neutron}(p^0, \mathbf{p}, k_{F,n}) F_2^{neutron}(x_N, Q^2) \right] \frac{x}{x_N} \left( 1 + \frac{2x_N p_z^2}{M\nu_N} \right) 
\]

(36)

\[
F_3^A(x, Q^2) = 2 \int d^3r \int \frac{d^3p}{(2\pi)^3} \frac{M}{E(p)} \left[ \int_{-\infty}^{\mu_p} dp^0 S_{h}^{proton}(p^0, \mathbf{p}, k_{F,p}) F_3^{proton}(x_N, Q^2) \\
+ \int_{-\infty}^{\mu_n} dp^0 S_{h}^{neutron}(p^0, \mathbf{p}, k_{F,n}) F_3^{neutron}(x_N, Q^2) \right] \frac{p^0 - p_z}{(p^0 - p_z)^2} 
\]

(37)

Using expressions for structure functions in nucleus for isoscalar as well as non-isoscalar we can calculate differential scattering cross section for charge current in nucleus which is written as:

\[
\frac{d^2\sigma^{\nu(\bar{\nu})A}_{CC}}{dx_A dy_A} = \frac{G_F^2 M_A E_\nu}{\pi} \left( \frac{m_l^2}{Q^2 + m_l^2} \right)^2 \left( \frac{y_A x_A F_1^{\nu(\bar{\nu})A}}{2E_\nu} \right) \\
+ \left\{ 1 - y_A - \frac{M_A x_A y_A}{2E_\nu} \right\} F_2^{\nu(\bar{\nu})A} \pm x_A y_A \left( 1 - \frac{y_A}{2} \right) F_3^{\nu(\bar{\nu})A} 
\]

(38)

\( m_l \) is the mass of lepton, \( E_\nu \) is the incident \( \nu/\bar{\nu} \) energy, \( M_A \) is the mass of nucleus, in \( F_3^A \), +(-)sign is for \( \nu/\bar{\nu} \), \( x_A = \frac{Q^2}{2M_A E_\nu} \) is the Bjorken variable, \( y_A = \frac{E_\nu}{E_\nu} \), \( \nu \) and \( Q^2 = -q^2 \).
3.1. π and ρ mesons contribution to the nuclear structure function

The pion and rho meson cloud contributions to the $F_2$ structure function have been implemented following the many body field theoretical approach of Refs. [22, 23].

The pion structure function $F_{2A,\pi}(x)$ is written as [22];

$$F_{2A,\pi}^A(x) = -6 \int d^3r \int \frac{d^4p}{(2\pi)^4} \frac{\theta(p^0)}{\pi} \frac{\delta Im D(p)}{x_{\pi}} \frac{2M F_{2\pi}(x_{\pi})}{x_{\pi}} \frac{\theta(x_{\pi} - x)}{\theta(1 - x_{\pi})} \theta(1 - x_{\pi}) \theta(1 - x) \theta(1 - x_{\pi})$$  \hspace{1cm} (39)

where $D(p)$ is the pion propagator in the nuclear medium which is given in terms of the pion self energy $\Pi_{\pi}$:

$$D(p) = [p_0^2 - \vec{p}^2 - m_{\pi}^2 - \Pi_{\pi}(p^0, p)]^{-1},$$  \hspace{1cm} (40)

where

$$\Pi_{\pi} = \frac{f_{\pi}^2/m_{\pi}^2} {1 - f_{\pi}^2/m_{\pi}^2 \Pi_{\pi}} .$$  \hspace{1cm} (41)
4. Results and Discussions

$F_2^A(x, Q^2)$ and $F_3^A(x, Q^2)$ structure functions have been calculated using Eqs.(32) and (33) for the carbon target and Eqs.(36) and (37) have been used for the nonscalar nuclear targets like iron and lead, with target mass correction (TMC) and CTEQ6.6 parton distribution functions (PDFs) at LO [28]. We call this as our base (Base) result. Thereafter, we include pion and rho cloud contributions in $F_2^A$ and the shadowing corrections in $F_2^A$ and $F_3^A$, which we call as our full

\[ \frac{d^2\sigma}{dxdy} = \left( \frac{\Lambda^2}{\Lambda^2 + \vec{p}^2} \right) F(p) - \rho \frac{\partial ImD(p)}{\partial \rho} |_{\rho=0} \]

and

\[ x = x_\rho \left( \frac{\rho^0 + \vec{p}^2}{M} \right) \]

Assuming SU(3) symmetry, the pion structure function at LO can be written in terms of pionic PDFs [31, 32] as

\[ F_{2\pi}(x_\pi) = \pi N N \text{ form factor and } \Lambda = 1 \text{ GeV}, f = 1.01, \]

\[ F_{\rho}(x_\rho) = \text{ longitudinal part of the spin-isospin nucleon-nucleon interaction and } \Pi'' \text{ is the irreducible pion self energy that contains the contribution of particle-hole and delta-hole excitations. In Eq.(39), } \]

\[ \delta ImD(p) \text{ is given by } \]

\[ \delta ImD(p) \equiv ImD(p) - \frac{\partial ImD(p)}{\partial \rho} |_{\rho=0} \]

where $\rho_{\pi}(x_\pi)$ is the valence distribution and $\bar{q}_{\pi}(x_\pi)$ is the light SU(3)-symmetric sea distribution.

Similarly, the contribution of the $\rho$-meson cloud to the structure function is written as [22]

\[ F_{2\rho}^A(x) = -12 \int d^2r \int \frac{d^4p}{(2\pi)^4} \theta(p^0) \delta ImD_\rho(p) x_\rho \frac{2MF_{2\rho}(x_\rho)\theta(x_\rho - x)(1 - x_\rho)}{x_\rho} \]

where $D_\rho(p)$ is the $\rho$-meson propagator and $F_{2\rho}(x_\rho)$ is the $\rho$-meson structure function, which we have taken equal to the pion structure function $F_{2\pi}$ using the valence and sea pionic PDFs from Ref. [31]. $\Lambda_\rho$ in $\rho NN$ form factor $F(p) = (\Lambda^2 - m^2_\rho)/(\Lambda^2 + \vec{p}^2)$ has also been taken as 1 GeV.
calculation (Total). The effect of shadowing is about 5-7% at $x=0.1$, $Q^2=1-5$ GeV$^2$ and 1-2% at $x=0.2$, $Q^2=1-5$ GeV$^2$ which dies out with the increase in $x$ and $Q^2$. In the case of $F_2^A$ there are pion and rho cloud contributions. The pion contribution is very dominant in comparison to the rho contribution. Pion contribution is significant in the region of $0.1 < x < 0.4$. Thus, we find that the shadowing correction seem to be negligible as compared to the other nuclear effects. It is the meson cloud contribution which is dominant at low and intermediate $x$ for $F_2$. In Fig.3 we have presented the results for lead nuclear target and also compared our numerical results with the CHORUS data [11]. We find that numerical results at NLO are in good agreement with the CHORUS data [11]. For the results of other nuclear targets like carbon and iron see Ref.[20].

We have shown the results for $\frac{2F_{Pb}}{208F_{i}}$, $\frac{12F_{Pb}}{208F_{i}}$, and $\frac{56F_{Pb}}{208F_{i}}$ ($i=2, 3$) at $Q^2 = 5$ GeV$^2$ in Fig.4 using CTEQ6.6 PDFs at NLO [28]. These results are presented for our base as well as full calculations. The deuteron structure functions have been calculated using the same formulae as for $^{12}$C in Ref. [20], but performing the convolution with the deuteron wave function squared instead of the spectral function. We have used the parametrization given in Ref. [33] for the deuteron wave function of the Paris N-N potential. We observe that the EMC effect is more prominent for ratios of structure functions between a heavy nucleus such as $^{208}$Pb and a much lighter one like $^{12}$C than for ratios between a heavy nucleus ($^{208}$Pb) and a medium-size one like $^{56}$Fe.

The differential scattering cross section for charged current neutrino and antineutrino induced reactions has been presented in Figs.5 and 6 in iron(isoscalar) and lead(nonisoscalar) respectively. In these figures the results for iron have been presented for isoscalar nuclear target as the experimental results are corrected for the isoscalar target [3, 9]. However, if we do nonisoscalar calculations then there would be 2-3% change in the results(not shown here). The medium effects in carbon and iron are small and for details see the discussion given in Ref. [20]. In the case of lead nucleus, for neutrino induced process when the differential scattering cross section is obtained using the full calculation, the change from the results obtained by using the base calculation is around 12-18% for $0.1 < x < 0.2$, that decreases with increase in $x$ and becomes 2-5% for $0.4 < x < 0.5$. For antineutrino induced reaction this difference is around 22-28% for $0.1 < x < 0.2$ which becomes 8-15% for $0.4 < x < 0.5$.

5. Conclusions

In this paper we have studied structure functions $F_2^A(x)$ and $F_3^A(x)$ in nuclei. For parton distribution functions(PDFs) in nucleon we have used CTEQ6.6 parametrization. Many-body theory is used to describe the spectral function of the nucleon in the nuclear-medium for all $Q^2$. The local density approximation has been used to apply the results for the finite nuclei. The use of the spectral function is to incorporate Fermi motion and binding effects. Target mass correction has been considered. We have taken the effects of mesonic degrees of freedom, shadowing, anti-shadowing in the calculation of $F_2^A$ and shadowing and anti-shadowing effects in the calculation of $F_3^A$. We have found that the mesonic cloud (mainly pion) gives an important contribution to the cross section. Using these structure functions we obtained differential scattering cross section in iron and lead. These numerical results have been compared with the available experimental results. The ratios $\frac{2F_{Pb}}{208F_{i}}$, $\frac{12F_{Pb}}{208F_{i}}$, and $\frac{56F_{Pb}}{208F_{i}}$ ($i=2, 3$) have been calculated using the results from Ref. [20] in the case of carbon and iron, where for deuteron we have used the same formulae as for $^{12}$C in Ref.[20], but performing the convolution with the deuteron wave function squared instead of the spectral function. We find that the nuclear-medium effects are also quite important in the case of deep inelastic scattering and the effect of nuclear medium in $F_2^A(x)$ and $F_3^A(x)$ structure functions is of different nature. Furthermore, we observe that the ratio of the structure functions for the different nuclei are not the same.
Figure 6. (Color online) $\frac{1}{E} \frac{d^2\sigma}{dxdy}$ vs $y$ (left panel is for neutrino and right panel is for antineutrino) at different $x$ for $\nu_\mu (E_{\nu_\mu} = 25 \text{ GeV})$ induced reaction in $^{208}\text{Pb}$. Dotted line is the results using Eq.38 with TMC. Solid (Dashed line) line is full calculation at NLO (LO). The experimental points are from CHORUS [11].

6. References
[1] G. P. Zeller et al., Phys. Rev. Lett. 88, 091802 (2002).
[2] D. Allasia et al. Zeit. Phys.C 28, 321 (1991).
[3] J. P. Berge et al. Zeit. Phys. C 49, 187 (1991).
[4] K. Varvell et al. Zeit. Phys. C 36, 1 (1991).
[5] E. Oltman et al. Zeit. Phys. C 53, 51 (1992).
[6] W. G. Seligman et al. Phys. Rev. Lett. 79, 1213 (1997).
[7] A. V. Sidorov et al. Eur. Phys. J. C 10, 405 (1999).
[8] Bonnie T. Fleming et al. Phys. Rev. Lett. 86, 5430 (2001).
[9] M. Tzanov et al., Phys. Rev. D 74, 012008 (2006).
[10] G. Onengut et al., Phys. Lett. B 632, 65 (2006).
[11] http://choruswww.cern.ch/Publications/DIS-data.
[12] R. Petti, Private Communication.
[13] T. Adams et al. [NuSOnG collaboration], Int. J. Mod. Phys. A 24, 671 (2009).
[14] N. Agafonova et al. (The OPERA collaboration) New J. Phys. 13, 053051 (2011).
[15] M. Hirai, S. Kumano, and T. H. Nagai, Phys. Rev. C 76, 065207 (2007).
[16] K. J. Eskola H. Maukkunen and C. A. Salgado, JHEP 0807, 102 (2008).
[17] I. Schienbein et al. Phys. Rev. D 80, 094004 (2009).
[18] S. A. Kulagin and R. Petti, Phys. Rev. D 76, 094023 (2007).
[19] M. Saaid Athar, S. K. Singh and M. J. Vicente Vacas, Phys. Lett. B 668, 133 (2008).
[20] H. Haider, T. Ruiz Simo, M. Saaid Athar and M.J. Vicente Vacas, Phys. Rev. C 84, 054610 (2011).
[21] P. Fernandez de Cordoba and E. Oset, Phys. Rev. C 46, 1697 (1992).
[22] E. Marco, E. Oset and P. Fernandez de Cordoba, Nucl. Phys. A 611, 484 (1996).
[23] C. Garcia-Recio, J. Nieves and E. Oset, Phys. Rev. C 51, 237 (1995).
[24] I. Schienbein et al., J. Phys. G 35, 053101 (2008).
[25] S. A. Kulagin and R. Petti, Nucl. Phys. A 765, 126 (2006).
[26] S. Moh., J. A. M. Vermaseren and A. Vogt, Nucl. Phys. B 813, 220 (2009).
[27] C. G. Callan, Jr. and D. J. Gross, Phys. Rev. Lett. 22, 156 (1969).
[28] Pavel M. Nadolsky et al., Phys. Rev. D 78, 013004 (2008); http://hep.pa.msu.edu/cteq/public
[29] J. Nieves, M. Valverde and M.J. Vicente Vacas, Phys. Rev.C 73, 025504 (2006).
[30] H. de Vries, C.W. de Jager and C. de Vries, At. Data Nucl. Data Tables 36, 583 (1971).