Multi objective functions of constraint optimal power flow based on modified ant colony system optimization technique

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Abstract. This article presents the development of Ant Colony System algorithm for solving the constrained Optimal Power Flow (OPF) problem. In this modification of ant colony, a local and global update of pheromone has been used. This optimization technique is used to adjust an optimal control variables of the system while satisfying the constraint of the state variables in their limits. The proposed algorithm tested on the IEEE-30 bus with different cases of single and multiple OPF objectives. The obtained results are analysed and compared with other previous studies. This comparison illustrates the efficiency of the ACS technique for solving different OPF problems of complex objective functions where the propose algorithm of ACS has been reduce the single objective function of total fuel cost, power losses, voltage profile improvement and the total generation emission to minimum value of 800.83 $/h, 3.2723 MW, 0.1194 pu and 0.2066 ton/h respectively with reduction of these objective function of 11.216%, 43.991%, 89.660%, 13.952% respectively. The authors have been used the MATLAB software for programming the propose technique without using any simulators. The Ant Colony System has the facility of simplicity in analysis, easy in program, faster in execution, less iteration with accurate result over the other optimization techniques.

1. Introduction
Optimal Power Flow (OPF) is a non-convex system and this problem is generally difficult to resolve for searching and finding an optimum power system operating point. It has been suggested in 1962 [1] and has since become one of the most important tasks of operation, control, production and observing power in modern energy systems, it has been tested to minimize the single or multi-objective function problem such as fuel cost, power transmission losses, generation emission, voltage deviation, etc. with limits to be met the balance of power flow, load bus voltage and generator capability [2]. Optimal power flow program assists to determine the optimal state of operation of a power system and the conforming settings of the control variables for better economic and safe operation [3]. The OPF problem could be solved in two methods, the first method of optimization is the traditional method which includes the quadratic programming, linear and nonlinear programming, etc. These methods provide appropriate results, but in practice, they suffer from difficulty in finding the global optimum and having complex calculations. Also, these techniques are depending on convex system theory, linearity, differentiability and continuity while OPF is non convex, non-differential system with discreet parameters. The second method of optimization is known the artificial intelligence optimization which has been used to overcome the problems and the complications of the traditional optimization methods. Some of these modern optimization techniques are: Differential Evolution DE [4], Particle Swarm Optimization PSO [5], Genetic Algorithm GA [6], Ant Colony System ACS [7, 8], Artificial Bee Colony ABE [9], … etc. These optimization techniques upon the real animal behaviour and evolutionary algorithms [10]. PSO is difficult in program and analysis but has accurate result, GA
is simple in program and analysis but takes large time in execution and so on for others methods where each technique has specific properties over the others. In this article eight cases of single- and multi-objective functions of Optimal Power Flow OPF problems are considered on IEEE 30 bus system to demonstrate the efficiency of the proposed algorithm of modified ant colony system. The four cases of single objective function are the total fuel cost, power losses, voltage deviation improvement and the total generation emission, while the four case of the multi-objective function are fuel cost with active power losses, fuel cost with voltage deviation improvement, fuel cost with fuel emission and fuel cost with active power losses with fuel emission with voltage deviation improvement. Ant Colony System is simple in analysis and program and succeed to solve the OPF problem with less iteration, less time in execution with minimum and accurate result.

The remainder of this article is organized as follows: In Section 2, the OPF is mathematically formulated, Section 3 presents explanation of ACS. Then, modified ACS in constrained OPF is presented in Section 4. Application (IEEE-30 bus) of the proposed ACS algorithm to the OPF problems explained in Section 5. Next, Section 6 summarizes explanation of results, comparison, and discussion using modified ACS algorithm to solve the OPF problems. Conclusions are mentioned in Section 7.

2. Optimal Power Flow OPF problem mathematical formulation

OPF can be expressed as a nonlinear optimization problem and solving. It achieves a set of selected optimal control variables which can optimize predefined power system objectives while taking into consideration the system operating constraints. Mathematically the OPF can be represented as follows:

\[
\begin{align*}
\text{minimize} & \quad F(x, \mu) \\
g(x, \mu) &= 0 \\
h(x, \mu) &\leq 0
\end{align*}
\]

where \(x\) is the vector of state variables; \(\mu\) is the vector of control variables; \(F(x, \mu)\) is the objective function of OPF; \(g(x, \mu)\) is a function that represents a set of equality constraints; \(h(x, \mu)\) is a function that represents a set of inequality constraints \([2,10]\).

The vector of control variables (\(\mu\)) and state variables (\(x\)) of the optimal power flow problem are represented in equations (4) and (5) respectively as follows:

\[
\begin{align*}
\mu &= [P_{G2} \ldots P_{GNg}, V_{G1} \ldots V_{Gng}, Q_{G1} \ldots Q_{Gnc}, T_1 \ldots T_{NT}]
\end{align*}
\]

\[
\begin{align*}
x &= [P_{Gs}, V_{L1} \ldots V_{NL}, Q_{G1} \ldots Q_{GNG}]
\end{align*}
\]

where the control variables in any power system have 4 type with a constraint of minimum and maximum limits. These control variables are the active power of the generator at PV buses (without slack) \(P_G\); the magnitude voltage of the generator \(V_G\); the tap changer of the transformer \(T\) and yhe VAR compensative of shunt capacitors \(Q_C\). Some articles used one type of variables or two or three or four. It is clear that best result will be at four type of control variables, therefore the authors in this article have been used 4 control variables.

The state variables also have a constraints of minimum and maximum limits. These state variables in this article are three type as follow:

The active power of the slack generator \(P_{Gs}\); the magnitude load voltage \(V_L\) and the reactive power for all generators including the slack generator and \(NL\) is the number of load buses.

\(NG\) is the number of generators; \(NC\) is the number of shunt VAR compensators; \(NT\) is the number of transformers; \(NL\) is the number of load buses.

The optimal power flow constraints are classified into two parts: equality and inequality constraints. The equality constraints represent the power flow equations which are represented as follow equations
\[ P_{Gi} - P_{Li} - V_i \sum_{j=1}^{NB} V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) = 0 \]  
\[ Q_{Gi} - Q_{Li} - V_i \sum_{j=1}^{NB} V_j (G_{ij} \sin \theta_{ij} + B_{ij} \cos \theta_{ij}) = 0 \]

where \( P_{Gi} \) and \( Q_{Gi} \) are the generated active and reactive power at \( i \)-th bus respectively; \( P_{Li}, Q_{Li} \) are the active and reactive load demands at \( i \)-th bus; \( G_{ij}, B_{ij} \) are conductance and susceptance between bus \( i \) and \( j \) respectively and \( NB \) is the total number of buses.

The inequality constraints are classified also into two parts as follows:

- **The inequality constraints of the control variables:**
  \[ P_{Gi}^{\text{min}} \leq P_{Gi} \leq P_{Gi}^{\text{max}} \quad i = 1, 2, ..., NG \]  
  \[ V_{Gi}^{\text{min}} \leq V_{Gi} \leq V_{Gi}^{\text{max}} \quad i = 1, 2, ..., NG \]  
  \[ Q_{Gi}^{\text{min}} \leq Q_{Gi} \leq Q_{Gi}^{\text{max}} \quad i = 1, 2, ..., NC \]  
  \[ T_i^{\text{min}} \leq T_i \leq T_i^{\text{max}} \quad i = 1, 2, ..., NT \]

where \( P_{Gi}^{\text{min}}, P_{Gi}^{\text{max}} \) are the minimum and maximum active power of the generator at the \( i \)-th PV bus; \( V_{Gi}^{\text{min}}, V_{Gi}^{\text{max}} \) are the minimum and maximum magnitude voltage limits of the generator at the bus \( i \) respectively; \( Q_{Gi}^{\text{min}}, Q_{Gi}^{\text{max}} \) are the minimum and maximum limits of shunt VAR compensator at the \( i \)-th load bus; \( T_i^{\text{min}}, T_i^{\text{max}} \) are the lower and upper limits of tap settings of transformer \( i \).

- **The inequality constraints of the state variables:**
  \[ Q_{Li}^{\text{min}} \leq Q_{Li} \leq Q_{Li}^{\text{max}} \quad i = 1, 2, ..., NG \]  
  \[ V_{Li}^{\text{min}} \leq V_{Li} \leq V_{Li}^{\text{max}} \quad i = 1, 2, ..., NL \]  
  \[ P_{Gs}^{\text{min}} \leq P_{Gs} \leq P_{Gs}^{\text{max}} \quad s = \text{slack bus} \]

where \( Q_{Li}^{\text{min}}, Q_{Li}^{\text{max}} \) are the minimum and maximum generator reactive power limits at bus \( i \) respectively; \( V_{Li}^{\text{min}}, V_{Li}^{\text{max}} \) are the minimum and maximum magnitude voltage limits at load bus \( i \); \( P_{Gs}^{\text{min}}, P_{Gs}^{\text{max}} \) are the minimum and maximum active power limits of the slack generator \( s \).

### 3. Ant Colony Search Algorithm ACS

ACS was introduced by Marco Dorigo [11] which is based on the behaviour of real ant. A colony of artificial ants is able to find the shortest possible path (optimal path) between their nest and food source by pheromone trail laying on the ground. The pheromone is a chemical substance liberated by the ants that evaporates after a period of time. It is used as the key guide between the ants allowing the ants to select the shortest path when looking for food in the next move. The shortest path that has the highest concentration of pheromone is better than the path with a lower concentration of pheromone [11]. For further explanation and to illustrate this process, suppose the ants are move from the nest to acquire the food as shown in (Fig. 1). Firstly, if no obstacle exists, the ants will go directly for the food (Fig. 1a) subsequently, if there was an obstacle that prevents the direct path, the ants in this case will randomly choose the right or left of the obstacle to move (Fig. 1b). After some time, the path that collects more pheromone will be the shortest path, therefore the ants will focus on it and discard the other path (Fig. 1c). This behaviour forms the essential idea of the ant colony system [12].
4. Modified ant colony system to solve the optimal power flow problem

The power system constraints are accomplished by combining the control variable settings between the maximum and minimum values while maintaining the state variables within their limits. Fig. 1 shows the ACS algorithm from the nest (home) to the food (destination) to find the shortest path for a particular objective function. The number of stages (layers) is equal to the number of control variables \( i = 1, \ldots, N \) and the number of states (nodes) in a particular stage is equal to the number of candidate values between the minimum and maximum limits \( j = 1, \ldots, M \). The ants will start to move from the home colony and they will travel through each stage and stop at the destination in each iteration. Each ant selects the state of each stage to complete one tour or one path. For example, an ant chooses a vector of control variables to finish one path shown in Figure (2), this vector (one path) represents the solution \( \{x_{12}, x_{23}, x_{33}, x_{41}, x_{52}, \ldots, x_{N2}\} \).

**Figure 1.** An example of selecting the shortest path

**Figure 2.** ACS for the OPF problem
In the artificial ACS that used to solve the problem of Optimal Power Flow, the state transition rule is applied to select the next node to be visited by the ant. The state transition rule is given by equation (15).

$$p_{(r,s)}^k = \frac{[\tau_{r,s}]}{\sum [\tau_{r,l}]} \quad l, s \in T_r^K$$

where \((\tau_{r,s})\) shows the intensity of the pheromone deposited between node \(r\) and node \(s\); \(r\) and \(s\) represent the previous and current node respectively; \(T_r^K\) indicates the set of nodes that still to be visited by ant \(k\) positioned on node \(r\); \(K\) represent the ant number. This equation gives the probability of moving an ant \(k\) in node \(r\) to node \(s\) during a tour [13].

An improved local and global update equation was used that differs from previously used (old version ACS) and demonstrated its ability to enhance the exploration of the guidance path and this increases the accuracy to create the optimal path for the control variables. These equations were derived from [13,14]. The pheromone trails are regularly updated at each iteration according to following equation

$$\tau_{(r,s)} = \rho \tau_{(r,s)} + \Delta\tau_{(r,s)}^k$$

where \(\rho\) is the continuation of the pheromone trail while \((1-\rho)\) reflects the evaporation; \(\Delta\tau_{(r,s)}^k\) represent the desirability of the trail \((r, s)\). The best possible performance is related with the shorter distance depending on the application problem. To complete a tour (path implementation), an ant needs to choose a random trail and deposit pheromone into the trail where the pheromone amount is dependent on the pheromone update of equation (16). After all ants finish their tours, global pheromone is updated in the trails \((r, s)\) of the best ant tour completed. After constructing the ant's tour, each ant will be able to use a local updating rule. This local pheromone will be applied after passing the trail \((r, s)\) while making the path of stages as shown in equation (16). The pheromone update \(\Delta\tau_{(r,s)}^k\) is defined as:

$$\Delta\tau_{(r,s)}^k = \frac{1}{D \times \text{Obj}}$$

Where \(D\) is a large number (positive constant); \(\text{Obj}\) is the objective function [13]. When all ants have completed a tour, the global updating rule is implemented to trails \((r, s)\) that belong to the best ant tour (that has minimum objective function) as follows:

$$\tau_{(r,s)} = (1-\rho)\tau_{(r,s)} + \rho \Delta\tau_{(r,s)}^{bs} \quad \forall (r, s) \in L^{bs}$$

$$\Delta\tau_{(r,s)}^{bs} = 1/(\text{Obj})^{bs}$$

where \((\text{Obj})^{bs}\) is the best objective function according to the best tour; \((L^{bs})\) is the best tour achieved during the algorithm. The equations (18, 19) illustrates the update of the pheromone trail, where both the evaporation and the deposition of new pheromone are applied only to the \((L^{bs})\) trail [13],[14].1--4a The main advantage of the modified ACS in comparison with the traditional ACS algorithms is obtaining the best path of the control variables according to the local and global update of pheromone, therefore the proposed approach shows the best performance in choosing the shortest distance and the optimum success rate of paths.
5. Application
The IEEE 30 bus system shown in figure (4) is used to test the effectiveness of the proposed algorithm of the Ant Colony System. This test system comprises 6 generation units installed at buses 1, 2, 5, 8, 11 and 13. The slack generator has been placed at bus one. This system has 41 transmission lines including four transformers at lines (4-12), (6-9), (6-10), (28-27) and 9 shunt VAR compensator capacitors installed at the load buses 10, 12, 15, 17, 20, 21, 23, 24 and 29 [15]. Therefore, this IEEE 30 bus system includes 24 control variables as follows:
6 magnitude generators $[V_{G1}, V_{G2}, V_{G5}, V_{GB}, V_{G11}, V_{G13}]$;
4 tap changer of the transformers $[T_{4-12}, T_{6-9}, T_{6-10}, T_{28-27}]$;
9 shunt injection capacitance \[Q_{C10}, Q_{C12}, Q_{C15}, Q_{C17}, Q_{C20}, Q_{C21}, Q_{C23}, Q_{C24}, Q_{C29} \];
5 active power of the generators without slack generator \[P_{G2}, P_{G5}, P_{G6}, P_{G13}, P_{G13} \];
Table 1 show the generators coefficients for the cost and emission objective function.

| Units Number | $a_i$ ($$/h)$ | $b_i \times 10^{-2}$ ($/$/MWh) | $c_i \times 10^{-4}$ ($$/$/MW$^2$/h) | $\alpha_i \times 10^{-2}$ (Ton/h) | $\beta_i \times 10^{-4}$ (Ton/MWh) | $\psi_i \times 10^{-6}$ (Ton/MW$^2$/h) | $\zeta_i \times 10^{-4}$ (Ton/MWh) | $\lambda_i \times 10^{-2}$ (Ton/MWh) |
|--------------|---------------|-------------------------------|-------------------------|-----------------|-------------------|---------------------------|------------------|------------------|
| 1            | 0             | 200                           | 37.5                    | 4.091           | -5.554            | 6.49                      | 2.0              | 2.857            |
| 2            | 0             | 175                           | 175.0                   | 2.543           | -6.047            | 5.638                     | 5.0              | 3.333            |
| 5            | 0             | 100                           | 625.0                   | 4.258           | -5.094            | 4.586                     | 0.01             | 8.0              |
| 8            | 0             | 325                           | 83.0                    | 5.326           | -3.55             | 3.38                      | 20.0             | 2.0              |
| 11           | 0             | 0                             | 250.0                   | 4.258           | -5.094            | 4.586                     | 0.01             | 8.0              |
| 13           | 0             | 0                             | 250.0                   | 6.131           | -5.555            | 5.151                     | 10.00            | 6.667            |

**Table 1. Generator cost and emission coefficients**

6. Results
In this article, eight cases have been considered for optimal power flow based on the ant colony system, four cases of single objective function and four cases of multi objective function as illustrate as follows:

6.1 Single objective function cases
Fuel cost, active power losses, the voltage profile improvement and the emission of the generation are the four cases of single objective function that have been considered.

6.1.1. Case 1: fuel cost (Cost ($$/h))
This is the most fundamental objective function of the OPF which has been studied almost in all literatures. The correlation between fuel cost ($$/h) and generated power (MW) is roughly determined by the quadratic relationship, the objective function that must be minimized is modelled as:
\[ F = \text{Cost} = \sum_{i=1}^{NG} a_i + b_i P_{Gi} + c_i P_{Gi}^2 \]  

(20)

where \(a_i, b_i, c_i\) are fuel cost coefficients of the \(i\)-th generator illustrated in table 1.

6.1.2. Case 2: active power losses (Loss(MW))

In this case, the total active power transmission losses can be expressed as follows:

\[ F = \text{Loss} = \sum_{k=1}^{NTL} G_{ij} \left( V_i^2 + V_j^2 - 2V_i V_j \cos \theta_{ij} \right) \]  

(21)

where \(\theta_{ij} = \theta_i - \theta_j\) is the voltage angle difference between bus \(i\) and bus \(j\); \(V_i, V_j\) represent the voltage magnitude at bus \(i\) and \(j\) respectively; \(G_{ij}\) is the conductance between bus \(i\) and \(j\) and \(NTL\) is the number of transmission lines.

6.1.3. Case 3: The voltage profile improvement (Voltage Deviation \(VD(pu)\))

The voltage profile is a measure of the quality of the voltage in the system. The improvement of the voltage profile includes minimizing the deviation of load bus (PQ bus) from the unity as shown below:

\[ VD = \sum_{i=1}^{NPQ} |V_i - 1| \]  

(22)

Where \(VD\) is the total voltage deviation at the load buses; \(NPQ\) is the number of load buses and \(V_i\) is the magnitude voltage at bus \(i\).

6.1.4. Case 4: The generation emission of gases (Em(ton/h))

Global warming is one of the most important concerns of the modern world, for which the power industries have a great responsibility, so the emission of gases in the environment is to be minimized. These emission gases can be expressed as:

\[ Em = \sum_{i=1}^{NG} \alpha_i + \beta_i P_{Gi} + \psi_i P_{Gi}^2 + \xi_i \exp(\lambda_i P_{Gi}) \]  

(23)

where \(Em\) is the emission function (ton/h); \(\alpha_i, \beta_i, \psi_i, \xi_i, \lambda_i\) are the emission coefficients of the \(i\)-th generator which illustrates in Table 1.

6.2. Multi objective function cases

Four cases of fuel cost with active power losses, fuel cost with voltage deviation, fuel cost with emission and finally fuel cost with emission, power losses and voltage deviation have been used for multi objective function.

6.2.1. Case 5: fuel cost with active power losses

The aim of this section (multi-objective) is to minimize both of the fuel cost and the active power losses at the same time. This can be satisfied by multiplying a weight factor to one of the objective function, which is expressed as follow:
where, $P_{\text{loss}}$ is the active power loss calculated in equation (21) and $\lambda_p$ is a weighting factor randomly chosen for minimum multi objective function. In this article $\lambda_p$ is chosen as 40 [16].

6.2.2. Case 6: fuel cost with voltage deviation
The collective objective function of fuel cost and voltage profile is expressed as:

$$F = \text{Cost} + \lambda_{VD} VD$$  (25)

where $VD$ is the total voltage deviation calculated in equation (22) and $\lambda_{VD}$ is a weighting factor which is chosen [16].

6.2.3. Case 7: fuel cost with emission control
The objective function of minimum fuel cost and emission is considered as:

$$F = \text{Cost} + \lambda_{em} E_m$$  (26)

where $E_m$ is the emission function determined in equation (23) and $\lambda_{em}$ is a weighting factor which is equal to 1000 for minimum multi objective function of cost and emission.

6.2.4. Case 8: fuel cost with voltage deviation, active power losses and emission control
The aim of this objective function is to minimize a four objective of generation fuel cost, active power losses in the transmission line, voltage deviation and the generation emission simultaneously. The objective function is defined by:

$$F = \text{Cost} + \lambda_{VD} VD + \lambda_p P_{\text{loss}} + \lambda_{em} E_m$$  (27)

where $\lambda_{VD} = 21$, $\lambda_p = 22$ and $\lambda_{em} = 19$ are the weighting factors, which are selected to balance between the objectives[16]. Table 2 and table 3 show the 24 optimal control variables, the four objective function and the first state variable of the active power of the slack generator for both the single and multi-objective function respectively based on the proposed algorithm of Ant Colony System (ACS) optimization technique. Also table 2 and table 3 show that the first state variables of the slack active power are between their minimum and maximum limits, and therefore satisfied the condition of constraint optimal power flow based on Ant Colony System.

| Control Variable | Min. | Max. | initial | $P_{G2}$ | $P_{G5}$ | $P_{G6}$ | $P_{G11}$ | $P_{G13}$ | $V_{G1}$ | $V_{G2}$ | $V_{G5}$ | $V_{G8}$ | $V_{G11}$ | $V_{G13}$ | $T_{4-12}$ |
|------------------|------|------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| $P_{G2}$         | 20   | 80   | 80      | 48.685  | 74.702  | 61.448  | 69.083  |
| $P_{G5}$         | 15   | 50   | 50      | 21.055  | 49.766  | 26.717  | 49.607  |
| $P_{G6}$         | 10   | 35   | 20      | 24.802  | 34.189  | 15.474  | 34.832  |
| $P_{G11}$        | 10   | 30   | 20      | 10.429  | 29.753  | 26.966  | 29.069  |
| $P_{G13}$        | 12   | 40   | 20      | 13.831  | 39.467  | 36.378  | 39.951  |
| $V_{G1}$         | 0.95 | 1.1  | 1.05    | 1.0931  | 1.0924  | 1.0159  | 1.0760  |
| $V_{G2}$         | 0.95 | 1.1  | 1.04    | 1.0724  | 1.0887  | 1.0062  | 1.0668  |
| $V_{G5}$         | 0.95 | 1.1  | 1.01    | 1.0491  | 1.0652  | 1.0162  | 1.0568  |
| $V_{G8}$         | 0.95 | 1.1  | 1.01    | 1.0465  | 1.0706  | 1.0095  | 1.0677  |
| $V_{G11}$        | 0.95 | 1.1  | 1.05    | 1.0816  | 1.0155  | 1.0839  | 1.0925  |
| $V_{G13}$        | 0.95 | 1.1  | 1.05    | 1.0840  | 1.0504  | 0.9956  | 1.0571  |
| $T_{4-12}$       | 0.9  | 1.1  | 1.078   | 1.0510  | 1.0996  | 0.9452  | 1.100   |
| Case | Cost+ | Cost+ | Cost+ |
|------|-------|-------|-------|
|      | End   | End   | End   |
|      | VD    | Em    | VD+Em |
| Case 5 |      |       |       |
| Case 6 |      |       |       |
| Case 7 |      |       |       |
| Case 8 |      |       |       |
| P_{62} | 20 | 80 | 80 |
| P_{65} | 15 | 50 | 50 |
| P_{68} | 10 | 35 | 20 |
| P_{61} | 10 | 30 | 20 |
| V_{61} | 0.95 | 1.1 | 1.05 |
| V_{62} | 0.95 | 1.1 | 1.04 |
| V_{65} | 0.95 | 1.1 | 1.01 |
| V_{68} | 0.95 | 1.1 | 1.01 |
| V_{61} | 0.95 | 1.1 | 1.05 |
| V_{63} | 0.95 | 1.1 | 1.05 |
| T_{4} | 0.9 | 1.1 | 1.078 |
| T_{6} | 0.9 | 1.1 | 1.069 |
| T_{6} | 0.9 | 1.1 | 1.032 |
| T_{28} | 0.9 | 1.1 | 1.068 |
| Q_{10} | 0 | 5 | 0 |
| Q_{12} | 0 | 5 | 0 |
| Q_{15} | 0 | 5 | 0 |
| Q_{17} | 0 | 5 | 0 |
| Q_{20} | 0 | 5 | 0 |
| Q_{24} | 0 | 5 | 0 |
| Objective Function | 943.63 | 843.63 | 953.10 |
| Cost ($/h) | 3.5417 | 7.3763 | 3.2723 |
| Loss (MW) | 0.9997 | 0.5613 | 1.1548 |
| Em (ton/h) | 0.2066 | 0.3569 | 0.2401 |
| P_{slack} | 200 | 50 | 50 | 0 | 5 | 0 | 1.2267 | 0.9257 | 4.0185 |
| P_{slack} | 50 | 200 | 99.242 | 173.84 |

Table 3. Optimal settings of control variables of multi objective function.
Table 4 shows that the second state variables of the reactive power for the generators are within their minimum and maximum limits. Therefore, the propose algorithm satisfied the condition of the constraint optimal power flow. This condition is very important because most of the optimization technique fail to get the reactive power within their limits for constraint optimal flow. Table 5 shows that the third state variables of the magnitude load voltage are also within their minimum limit of 0.9 pu and maximum limit of 1.1 pu and also satisfied the condition of constraint optimal power flow OPF.

**Table 4.** Reactive power generator unit limits for different cases.

| Unit No. | Min. reactive power (Mvar) | Max. reactive power (Mvar) | Single objective function | Multi objective function |
|----------|---------------------------|---------------------------|--------------------------|-------------------------|
|          | Case1 | Cost | Case2 | Loss | Case3 | EM | Case4 | VD | Case5 | Cost+Loss | Case6 | Cost+EM | Case7 | Cost+VD | Case8 | Cost+Loss+EM+VD |
| 1        | -20   | 200  | 10.12 | -2.19 | 2.65  | -8.00 | 7.34  | 3.90  | -2.5930 | 6.1282 |
| 2        | -20   | 100  | 18.36 | 25.75 | -6.12 | -1.98 | 25.94 | 1.83  | 14.8826 | 27.3087 |
| 5        | -15   | 80   | 38.09 | 24.00 | 31.42 | 60.15 | 12.91 | 28.17 | 69.9029 | 29.4353 |
| 8        | -15   | 60   | 29.33 | 28.97 | 49.53 | 54.89 | 39.56 | 42.84 | 37.2412 | 36.3913 |
| 11       | -10   | 50   | 12.91 | 4.77  | 25.26 | 42.29 | 9.83  | 26.08 | 19.0823 | 14.9980 |
| 13       | -15   | 60   | 30.38 | 26.80 | 21.43 | -10.15 | 20.16 | 26.19 | 5.7714  | 0.9621  |

**Table 5.** Magnitude load voltage (in per unit) for different cases.

| Bus No. | Case 1 | Cost | Case 2 | Loss | Case 3 | Em | Case 4 | VD | Case 5 | Cost+Loss | Case 6 | Cost+EM | Case 7 | Cost+VD | Case 8 | Cost+Loss+EM+VD |
|---------|--------|------|--------|------|--------|----|--------|----|--------|-----------|--------|---------|--------|---------|--------|----------------|
| 1       | 1.0931 | 1.0924 | 1.076  | 1.0159 | 1.0987 | 1.0485 | 1.0266 | 1.0723 |
| 2       | 1.0724 | 1.0887 | 1.0668 | 1.0062 | 1.0870 | 1.0327 | 1.0044 | 1.0589 |
| 3       | 1.0630 | 1.0799 | 1.0690 | 1.0000 | 1.0780 | 1.0331 | 1.0085 | 1.0460 |
| 4       | 1.0565 | 1.0774 | 1.0676 | 0.9966 | 1.0736 | 1.0297 | 1.0047 | 1.0403 |
| 5       | 1.0491 | 1.0652 | 1.0568 | 1.0162 | 1.0498 | 1.0109 | 0.9925 | 1.0317 |
| 6       | 1.0473 | 1.0702 | 1.0608 | 1.0033 | 1.0685 | 1.0253 | 0.9996 | 1.0370 |
| 7       | 1.0397 | 1.0599 | 1.0508 | 0.9999 | 1.0526 | 1.0108 | 0.9879 | 1.0263 |
| 8       | 1.0465 | 1.0706 | 1.0677 | 1.0095 | 1.0729 | 1.0305 | 1.0023 | 1.0403 |
| 9       | 1.0570 | 1.0076 | 1.0463 | 1.0071 | 1.0324 | 1.0105 | 1.0079 | 1.0017 |
| 10      | 1.0314 | 0.9993 | 1.0439 | 1.0072 | 1.0371 | 0.9799 | 1.0107 | 1.0091 |
| 11      | 1.0816 | 1.0155 | 1.0925 | 1.0839 | 1.0504 | 1.0600 | 1.0840 | 1.0305 |
| 12      | 1.0449 | 1.0160 | 1.0295 | 1.0112 | 1.0498 | 0.9817 | 1.0099 | 1.0115 |
| 13      | 1.0840 | 1.0504 | 1.0571 | 0.9956 | 1.0754 | 1.0172 | 1.0474 | 1.0124 |
| 14      | 1.0329 | 1.0038 | 1.0215 | 1.0112 | 1.0371 | 0.9685 | 0.9998 | 1.0015 |
| 15      | 1.0309 | 1.0010 | 1.0227 | 1.0004 | 1.0341 | 0.9659 | 0.9997 | 1.0015 |
| 16      | 1.0326 | 1.0022 | 1.0294 | 1.0032 | 1.0383 | 0.9749 | 1.0030 | 1.0047 |
| 17      | 1.0275 | 0.9962 | 1.0365 | 1.0030 | 1.0341 | 0.9756 | 1.0036 | 1.0051 |
| 18      | 1.0207 | 0.9906 | 1.0206 | 0.9937 | 1.0265 | 0.9614 | 0.9948 | 0.9953 |
| 19      | 1.0177 | 0.9877 | 1.0225 | 0.9930 | 1.0251 | 0.9620 | 0.9951 | 0.9948 |
| 20      | 1.0215 | 0.9918 | 1.0290 | 0.9982 | 1.0298 | 0.9681 | 1.0008 | 1.0001 |
| 21      | 1.0234 | 0.9901 | 1.0345 | 0.9972 | 1.0286 | 0.9669 | 1.0014 | 0.9999 |
| 22      | 1.0242 | 0.9909 | 1.0351 | 0.9981 | 1.0293 | 0.9672 | 1.0020 | 1.0005 |
| 23      | 1.0271 | 0.9939 | 1.0248 | 0.9986 | 1.0289 | 0.9597 | 0.9987 | 0.9989 |
| 24      | 1.0198 | 0.9844 | 1.0271 | 0.9923 | 1.0236 | 0.9511 | 0.9947 | 0.9935 |
| 25      | 1.0347 | 0.9840 | 1.0477 | 1.0028 | 1.0296 | 0.9357 | 1.0026 | 1.0001 |
| 26      | 1.0173 | 0.9657 | 1.0306 | 0.9849 | 1.0121 | 0.9164 | 0.9847 | 0.9821 |
| 27      | 1.0523 | 0.9928 | 1.0691 | 1.0183 | 1.0419 | 0.9355 | 1.0162 | 1.0130 |
| 28      | 1.0396 | 1.0658 | 1.0510 | 0.9979 | 1.0634 | 1.0226 | 0.9919 | 1.0317 |
| 29      | 1.0415 | 0.9865 | 1.0517 | 1.0000 | 1.0356 | 0.9242 | 0.9997 | 1.0075 |
| 30      | 1.0269 | 0.9687 | 1.0402 | 0.9878 | 1.0188 | 0.9071 | 0.9867 | 0.9898 |
Table 6 shows a comparison between the propose algorithm of Ant Colony System and other optimization techniques for the four cases of the single objective function of Fuel Cost ((Cost($/h) (case1)), active power losses ((Loss(MW) (case 2)), Voltage deviation ((VD(pu) (case 3)) and the generation emission ((Em(ton/h) (cases 4)). Table 7, 8, 9 and 10 show the comparison between the proposed algorithm of Ant Colony System and other optimization techniques for the four cases of multi objective function of fuel cost with active power losses (case 5), fuel cost with voltage deviation (case 6), fuel cost with generation emission case (7), and finally the fuel cost with active power losses, voltage deviation and generation emission case (8) respectively. Figures 5, 6, 7 and 8 show the response of single objective function of fuel cost, active power losses, voltage deviation and generation emission with respect the number of iteration based on the propose algorithm of Ant Colony System.

**Table 6.** Comparison of the proposed algorithm of Ant Colony System with other optimization

| Optimization techniques | Ref. | Cost ($/h) | Loss (MW) | VD (pu) | Em Ton/h |
|--------------------------|------|------------|-----------|---------|----------|
| Differential Evolution   | [17] | 803.05     |           |         |          |
| Imperialistic Competitive Algorithm | [17] | 802.88     |           |         |          |
| Flower Pollination Algorithm | [18] | 802.79     | 3.5661    |         |          |
| Improved PSO             | [19] | 802.63     |           |         |          |
| Evolutionary Programming | [19] | 802.62     |           |         |          |
| Modified differential evolution algorithm | [20] | 802.37     |           |         |          |
| modified shuffle frog leaping algorithm | [21] | 802.28     |           |         |          |
| Adaptive group search optimization | [22] | 801.75     |           |         |          |
| Particle Swarm Optimization(PSO) | [23] | 801.66     |           |         |          |
| Differential Evolution   | [24] |           | 4.720     |         |          |
| Differential Evolution   | [25] |           | 4.760     |         |          |
| Real coded Genetic Algorithm | [26] |           | 4.501     |         |          |
| black-hole-Based Optimization | [27] |           | 3.5661    |         |          |
| PSO          | [33] | 800.96     | 0.147     |         | 0.2672   |
| GA           | [34] | 800.96     | 0.135     |         |          |
| PSO          | [34] | 800.96     | 0.147     |         | 0.2117   |
| PSO          | [21] | 0.2117     |          |         |          |
| SLFA         | [21] | 0.2096     |          |         |          |
| MSLFA        | [21] | 0.2063     |          |         |          |
| FA           | [21] | 0.2056     |          |         |          |
| ABC          | [30] | 0.217      |          |         |          |
| ABC          | [31] | 0.217      |          |         |          |
| EBBO         | [31] | 0.226      |          |         |          |
| ACO          | [31] | 0.220      |          |         |          |
| EACO         | [31] | 0.231      |          |         |          |
| GSO          | [32] | 0.206      |          |         |          |
| MFO          | [35] | 0.11       |          |         |          |
| PSO-SSO      | [36] | 1.24       |          |         |          |
| HFPSO        | [37] | 0.11       |          |         |          |
| PSO          | [37] | 0.117      |          |         |          |

Ant Colony system (ACS) | 800.83 | 3.2723 | 0.1194 | 0.2066 |
Table 7. Comparison of Ant Colony System with other optimization techniques for the multi objective function (case 5: fuel cost with active power losses) of IEEE 30-bus

| Optimization techniques | Fuel Cost ($/h) | Active power losses(MW) | Reference |
|-------------------------|----------------|-------------------------|-----------|
| MSA                     | 859.1915       | 4.5404                  | [16]      |
| MDE                     | 868.7138       | 4.3891                  | [16]      |
| FPA                     | 855.2706       | 4.7981                  | [16]      |
| MOALO                   | 826.4556       | 5.7727                  | [28]      |
| NKEA                    | 829.4911       | 5.8603                  | [28]      |
| GA                      | 832.22         | 5.81                    | [34]      |
| PSO                     | 839.95         | 5.73                    | [34]      |
| ACS                     | 867.282        | 4.3589                  | Proposed algorithm |

Table 8. Comparisons of Ant Colony System technique with other optimization techniques for multi-objective function (case 6: fuel cost with voltage deviation) of IEEE 30-bus system

| Optimization techniques | Fuel Cost ($/h) | Voltage deviation (p.u) | Reference |
|-------------------------|----------------|-------------------------|-----------|
| MOALO                   | 803.0611       | 0.3787                  | [28]      |
| ISA                     | 807.6408       | 0.1273                  | [29]      |
| GA                      | 803.17         | 0.198                   | [34]      |
| PSO                     | 803.41         | 0.248                   | [34]      |
| ACS                     | 805.72         | 0.1380                  | Proposed algorithm |

Table 9. Comparisons of Ant Colony System technique with other optimization techniques for multi-objective function (case 7: fuel cost with generation emission) of IEEE 30-bus system

| Optimization techniques | Fuel Cost ($/h) | Emission ton/h | Reference |
|-------------------------|----------------|---------------|-----------|
| BSA                     | 835.0199       | 0.2425        | [2]       |
| EWA                     | 834.9863       | 0.2423        | [10]      |
| MOALO                   | 831.6764       | 0.2576        | [28]      |
| MOMICA                  | 865.0660       | 0.2221        | [28]      |
| BB-MPSO                 | 865.0985       | 0.2227        | [28]      |
| ACS                     | 833.496        | 0.2526        | Proposed algorithm |

Table 10. Comparisons of Ant Colony System with other optimization techniques for multi-objective function (case 8: fuel cost, voltage deviation, active power losses, emission) of IEEE 30-bus system

| Optimization techniques | Fuel Cost ($/h) | Voltage deviation (p.u) | Active power losses(MW) | Emission (ton/h) | Reference |
|-------------------------|----------------|-------------------------|-------------------------|-----------------|-----------|
| MSA                     | 830.639        | 0.2938                  | 5.6219                  | 0.2525          | [16]      |
| MDE                     | 829.0942       | 0.3034                  | 6.0569                  | 0.2575          | [16]      |
| FPA                     | 835.3699       | 0.3316                  | 5.5153                  | 0.2478          | [16]      |
| MOALO                   | 826.2676       | 0.7160                  | 7.2073                  | 0.2730          | [28]      |
| MOMICA                  | 830.1884       | 0.2978                  | 5.8581                  | 0.2978          | [28]      |
| BB-MPSO                 | 833.0345       | 0.3945                  | 5.6504                  | 0.4448          | [28]      |
| ACS                     | 829.4921       | 0.2834                  | 5.7585                  | 0.2540          | Proposed algorithm |

For the IEEE 30 bus, the fuel cost reduced in case1 to 800.83 $/hr, the active power losses reduced in case2 to 3.2723 MW, the voltage deviation reduced in case3 to 0.1194 pu, the total emission reduced in case 4 to 0.2066 ton/h, the fuel cost with active power losses in case-5 are (867.282 $/h and 4.3589MW), fuel cost with voltage deviation in case-6 are (805.72$/h and 0.1380p.u), fuel cost with generation emission in case-7 are (833.496MW and 0.2526 ton/h), fuel cost with voltage deviation with active power loss with generation emission are (829.4921$/h, 0.2834 p.u, 5.7585MW and 0.2540ton/h) respectively. Figure 5-8 shows the fast convergence of ACS algorithm for Cases 1-4, respectively.
**Figure 5.** Fuel cost ($/h) with number of iteration

**Figure 6.** Active power losses (MW) with number of iteration number

**Figure 7.** Voltage deviation (pu) with number iteration
7. Conclusion
The proposed algorithm of the Ant Colony System ACS is inspired from the ant colony variant including a modification of a local and global pheromone update rules for enhancing routing path exploration. This modified ant colony system algorithm has been implemented to solve the constraint of Optimal Power Flow OPF problems, it provides an optimal control variable settings of active power of the generator at PV buses; magnitude voltage of the generator; tap changer of the transformer and VAR compensative of shunt capacitors in different cases of single objective function of total fuel cost, active power losses, voltage profile improvement and the total generation emission while satisfying the state variables constraints of active power of the slack generator; magnitude load voltage and reactive power for all generators. The system IEEE-30 bus is used to test the effectiveness of the proposed algorithm. The Ant Colony System reduce objective function of fuel cost by 11.216%; active power losses by 43.991%; voltage deviation by 89.660% and fuel emission by 13.952%. Also four cases of fuel cost with active power losses; fuel cost with voltage deviation; fuel cost with fuel emission and finally fuel cost with active power losses with voltage deviation with fuel emission are considered to solve the multi-objective function of OPF problems. In multi-objective OPF, the weighted sum method is used to integrate the multiple objective functions. The proposed algorithm in this study has been achieved good solutions of the OPF problem compared with other optimization techniques reported in the literature. It has proven to be effective, superior, successful and most importantly, has less implementation time to solve the OPF problems.

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