It is commonly accepted that credit market frictions are an important source of macroeconomic fluctuations. But what is the link between the two? And what is the driving factor of asset prices volatility? To answer these questions, we have introduced a specific credit friction, limited commitment, in a general equilibrium model with production and investment in productive capital, where agents can trade bonds. The model always displays a stationary equilibrium where bonds are traded. More importantly, limited commitment may generate stochastic endogenous fluctuations driven by self-fulfilling volatile expectations (sunspots), yielding credit and investment cycles and bond price volatility consistent with data.

Keywords: Credit Frictions, Limited Commitment, Indeterminacy, Sunspots

1. INTRODUCTION

The recent financial turmoil has shown that macroeconomic fluctuations are tied to the functioning of credit markets. One possible explanation is that credit market frictions amplify and propagate exogenous shifts in fundamentals. More recently some works have relied on exogenous financial shocks as a source of macroeconomic fluctuations [see Jermann and Quadrini (2012), Shi (2015)]. In this work, we provide an alternative explanation that does not rely on shifts in preferences, technologies, and other payoff relevant fundamentals, nor on exogenous perturbations to the financial sector. Instead, we consider a credit market friction, limited commitment, which by introducing endogenous borrowing constraints, generates stochastic endogenous fluctuations driven by self-fulfilling volatile expectations.
(sunspots), yielding credit and investment cycles and asset price volatility consistent with data.\textsuperscript{2} This means that animal spirits in credit markets are an important source of macroeconomic fluctuations.

A similar conclusion is reached by Gu et al. (2013) that also introduce limited commitment into a very stylized model that can be reinterpreted as a Kehoe and Levine (1993) pure endowment economy. Even though the authors characterize the properties of the one-dimension global dynamics of their model, they do not consider a productive asset such as capital, and therefore they are unable to study the consequences of limited commitment on asset prices. To address this issue, we introduce limited commitment in a general equilibrium model with production and investment in a productive asset, where agents can trade bonds. In this way, we obtain a richer dynamic framework, better suited to assess fluctuations in standard macroeconomic variables as well as credit and investment cycles and asset prices fluctuations.\textsuperscript{3} More precisely, we depart from the standard one good representative agent macroeconomic model by introducing two types of goods, consumption and capital, and two infinitely lived types of agents, entrepreneurs (agents that need to borrow to finance their investment) and workers (agents that want to transfer consumption intertemporally). The consumption good is produced by firms using capital and labor in a linearly homogeneous technology. Entrepreneurs are more impatient and have access to a technology that transforms the consumption good into next period capital. Workers are more patient, work every period, but have no technology to transfer consumption intertemporally. Finally, entrepreneurs have limited commitment and can choose to default.

If entrepreneurs could commit to repay, we would recover the main features of the Ramsey model: Workers buy claims on future capital (bonds) to transfer consumption across time, entrepreneurs operate their technology but do not save, and the price of new capital is pinned down by the properties of the capital-producing technology.\textsuperscript{4} The economy exhibits one unique stationary first best equilibrium with positive capital, which is a saddle, one state variable being predetermined and the other non-predetermined. Therefore, indeterminacy is not possible and endogenous fluctuations (deterministic or stochastic) do not emerge.

The introduction of entrepreneurs’ limited commitment may change significantly the properties of equilibria. Cooperative behavior is sustained by the threat of future exclusion from credit markets, which defines a no-default constraint\textsuperscript{5}: entrepreneurs’ willingness to honor their promises depends on the continuation value of their business, which in turn depends on their expectations about the future. The model always exhibits a stationary equilibrium where bonds are traded.\textsuperscript{6} Depending on the severity of the commitment problem, this can be the first best steady state, or a second best featuring an endogenous borrowing constraint, without or with entrepreneurs investing in their technology. We interpret this investment as collateral.

More importantly, when the no-default constraint binds, and even in the absence of collateral, we now obtain stochastic endogenous fluctuations (sunspots). The
price of new capital is no longer pinned down by the properties of the capital-producing technology, but can now jump in order to accommodate changes in expectations, with investment reacting accordingly. Therefore, the two state variables are now non-predetermined, and, although the steady state is still a saddle, indeterminacy (and therefore sunspots) emerges. This, as shown by our simulation exercise, can result in fluctuations that match credit and investment cycles observed in the data. Moreover, the model generates a volatility of the price of bonds which is consistent with the one observed in the data.

Our work builds on two branches of the literature. From the first one, we borrow the use of limited commitment to generate endogenously debt limits [see Kehoe and Levine (1993), Albuquerque and Hopenhayn (2004), Azariadis and Kaas (2007), Gu et al. (2013)]. However, the setup considered in these studies is different from ours. Either they consider exchange economies without a productive asset, or restrict their analysis to partial equilibrium models. The second branch of the literature investigates, in monetary overlapping generation (OLG) models, the role of credit market distortions on the emergence of local indeterminacy, bifurcations, and endogenous cycles. Examples are Azariadis and Smith (1998) and Boyd and Smith (1997), where private information between borrowers and lenders produces multiple stationary equilibria and changes local dynamics, introducing indeterminacy and endogenous fluctuations. Our paper differs from these works in two important dimensions: First, while they use an OLG framework, we consider an economy populated by infinitely lived agents, so that limited commitment is the only friction we introduce. Second, we do not impose any restriction on endogenous variables that implicitly determines the pattern of trade.

A recent paper that, like us, also generates indeterminacy and self-fulfilling business cycles with imperfect credit enforcement is Liu and Wang (2014). However, their indeterminacy mechanism differs significantly from ours, being equivalent to the more standard increasing returns to scale channel.

The rest of the paper is organized as follows. In the next section, we present the model. In Section 3, we obtain the limited commitment solution. In Section 4, we study local dynamics and discuss our main results. In Section 5, we present a simulation exercise and show that our simulation results match important business-cycle features. Finally, Section 6 concludes. Proofs and computations are relegated to an online appendix.

2. THE MODEL

Time is discrete, lasts forever, and is indexed as \( t = 0, 1, 2, \ldots \). There are two goods, a capital and a consumption good, and we take the consumption good as numeraire. Each period there is a perfectly competitive sector—or a representative firm—that produces the consumption good \( Y_t \) using capital \( K_t \) and labor \( L_t \) in a linearly homogeneous technology, \( Y_t = F(K_t, L_t) = L_t F(K_t/L_t) \). Both capital and labor are necessary for production, meaning \( F(0, L_t) = F(K_t, 0) = 0 \). We
assume that capital used in production fully depreciates, hence consumption goods can only be produced if new capital is provided each period. Furthermore, we assume that consumption is nonstorable across periods.

The economy is populated by a measure 1 of heterogeneous agents: a fraction \( \alpha \) of them being workers and a fraction \((1 - \alpha)\) being entrepreneurs. Workers are endowed with one unit of time every period and can work for the representative firm in exchange for the competitive wage \( w_t = F_L(K_t, L_t) \). Differently, entrepreneurs are endowed with a technology that transforms the consumption good into the capital good. This technology uses as input \( x_t \) units of the consumption good in period \( t \) and returns \( g(x_t) = x_t^\nu \) units of capital at time \( t + 1 \). We assume that \( 0 < \nu < 1 \). Workers cannot operate this technology, but can buy in a competitive credit market claims on next period capital, which, in our stylized economy, we call bonds. We denote by \( q_t \) the price of each bond, which is the amount of consumption goods needed at time \( t \), for a claim on one unit of capital at time \( t + 1 \). Furthermore, we denote by \( b_{W,t+1} \) and \( b_{E,t+1} \), respectively, workers and entrepreneurs net demand for bonds at time \( t \), where we use the convention that a positive value stands for purchases, and a negative value represents sales. Trading over these claims is subject to limited commitment: Entrepreneurs can divert resources from their investment technology and repudiate their promises. They will only choose to repay if it is in their interest. This means that credit contracts must adjust to be self-enforcing.

Workers have no disutility from working and are risk-averse. For the sake of simplicity, we consider a log utility function: \( U_t = \sum_{\tau=t}^{\infty} \beta^{\tau-t} \log(c_{W,\tau}) \), where \( c_{W,\tau} \) denotes a worker’s consumption in time \( \tau \). Differently, entrepreneurs are risk-neutral, and their preferences are defined over consumption according with the utility function \( V_t = \sum_{\tau=t}^{\infty} \gamma^{\tau-t} c_{E,\tau} \), where \( c_{E,\tau} \) denotes the consumption of an entrepreneur in time \( \tau \). The parameters \( \beta \) and \( \gamma \) are the discount factors of workers and entrepreneurs, respectively. We assume that entrepreneurs are more impatient than workers: \( \beta > \gamma \).

The timing is the following: At the beginning of period \( t \), after the investment technology returns \( x_{t-1}^\nu \) units of capital, the entrepreneur repays/redeems his claims \( b_{E,t} \). Therefore, his wealth consists in \( x_{t-1}^\nu + b_{E,t} \) units of capital. This capital is supplied to the representative firm, in exchange for \( r_t(x_{t-1}^\nu + b_{E,t}) \) units of consumption, where \( r_t \) is time \( t \) rental rate of capital. Perfect competition ensures that \( r_t = F_K(K_t, L_t) \). A worker, instead, enters time \( t \) with claims on \( b_{W,t} \) units of capital. After redeeming these claims, he supplies one unit of time and capital \( b_{W,t} \) to the representative firm, earning \( w_t + r_t b_{W,t} \). Capital then fully depreciates, and both workers and entrepreneurs have to choose how much to consume and how much to save. A worker can only use his savings to purchase capital for the next period. Therefore, \( b_{W,t} \geq 0 \) holds for all \( t \). Differently, an entrepreneur can use his savings either to buy or to sell claims on future capital or to invest in his technology, where investment has to be nonnegative. We can then write a worker budget constraint as \( c_{W,t} + q_t b_{W,t+1} = w_t + r_t b_{W,t} \) and an entrepreneur budget constraint as \( c_{E,t} + q_t b_{E,t+1} + x_{t}^E = r_t(b_{E,t} + x_{t-1}^\nu) \). Moreover, since entrepreneurs
can only borrow to invest in their technology (i.e., they cannot borrow to consume in the current period) the following inequality must hold: $x_t + q_t b^E_{t+1} \geq 0$.11

3. EQUILIBRIUM WITH LIMITED COMMITMENT

3.1. The Workers’ Problem

A worker that enters time $t$ with $b^W_t$ bonds solves the following problem:

$$U_t(b^W_t) = \max_{(c^W_t, b^W_{t+1}) \in \mathbb{R}_+^2} \log(c^W_t) + \beta E_t \left[U_{t+1}(b^W_{t+1})\right]$$

s.t. $c^W_t + q_t b^W_{t+1} = w_t + r_t b^W_t$. (1b)

Due to log preferences $c^W_t > 0$. Moreover, $b^W_{t+1} \geq 0$, so that optimality requires

$$\frac{E_t(c^W_{t+1})}{c^W_t} \geq \frac{\beta E_t(r_{t+1})}{q_t}$$

with equality if $b^W_{t+1} > 0$. The right-hand side of equation (2) gives us the expected discounted real return of a bond, and can also be interpreted as the expected Tobin’s $q$: $r_{t+1}$ is time $t + 1$ value of a unit of capital; by multiplying by $\beta$, we discount it by one period, whereas $q_t$ is today’s cost of replacing capital.

3.2. The Entrepreneurs’ Problem

Entrepreneurs cannot commit to honor their debts and may choose to divert resources from the capital-producing technology, consuming them and enjoying in this way an immediate private benefit. Specifically, an entrepreneur saves $r_t(x^v_{t-1} + b^E_t) - c^E_t$ and receives $-q_t b^E_{t+1}$ units of consumption goods from workers. Afterwards, entrepreneurs can choose to consume an extra amount, which we denote by $\tilde{C}^E_t$, and repudiate the credit contract in the beginning of the following period. This behavior gives entrepreneurs utility $(1 - \theta)$ for each unit of consumption, where $\theta \in (0, 1)$ is a parameter that measures the cost of hiding resources. Upon repudiation, workers can seize entrepreneurs capital, given by $[r_t(x^v_{t-1} + b^E_t) - c^E_t - q_t b^E_{t+1} - \tilde{C}^E_t]$. Easily, entrepreneurs best deviation is to consume $\tilde{C}^E_t = r_t(x^v_{t-1} + b^E_t) - c^E_t - q_t b^E_{t+1}$ investing nothing in their technology. Assuming that the punishment for deviating is (perpetual) exclusion from credit markets, entrepreneurs deviation payoff is $\tilde{V}_{t+1} = (1 - \theta)[r_t(x^v_{t-1} + b^E_t) - c^E_t - q_t b^E_{t+1}]$. To avoid this problem, credit contracts must satisfy the no-default constraint $\gamma E_t[V_{t+1}(b^E_{t+1}, x_t)] \geq \tilde{V}_{t+1}$, i.e., expected future discounted utility (the continuation payoff) must exceed the deviation payoff. Therefore, an entrepreneur that
enters period $t$ with $x_{t-1}^w + b_{t+1}^E$ units of capital solves the following problem:

$$V_t(b_t^E, x_{t-1}) = \max_{(c_t^E, x_t, b_{t+1}^E) \in \mathbb{R}_+^2 \times \mathbb{R}} c_t^E + \nu E_t \left[ V_{t+1}(b_{t+1}^E, x_t) \right]$$

s.t.

$$c_t^E + q_t b_{t+1}^E + x_t = r_t(b_t^E + x_{t-1})$$

$$x_t + q_t b_{t+1}^E \geq 0$$

$$\nu E_t \left[ V_{t+1}(b_{t+1}^E, x_t) \right] \geq (1 - \theta) \left[ r_t(x_{t-1}^w + b_t^E) - c_t^E - q_t b_{t+1}^E \right].$$

Let $\lambda_t$ and $\eta_t$ be the multipliers associated with (3b) and (3c), respectively, whereas $\mu_t$ is the multiplier associated with the no-default constraint (3d). Using (3b), we can rewrite constraint (3d) as

$$\nu E_t \left[ V_{t+1}(b_{t+1}^E, x_t) \right] \geq (1 - \theta)x_t. \tag{4}$$

From the first-order, complementary slackness, and envelope conditions, we obtain

$$1 - \lambda_t \leq 0, \quad \text{with equality if } c_t^E > 0 \tag{5a}$$

$$\eta_t \left( x_t + q_t b_{t+1}^E \right) = 0, \tag{5b}$$

$$\mu_t \{ \nu E_t \left[ V_{t+1}(b_{t+1}^E, x_t) \right] - x_t(1 - \theta) \} = 0, \tag{5c}$$

$$q_t = E_t \left[ \frac{\nu(1 + \mu_t) r_{t+1} \lambda_{t+1}}{\nu(1 + \mu_t) \lambda_{t+1} r_{t+1} v x_{t+1}^{(v-1)} - (1 - \theta) \mu_t} \right], \tag{5d}$$

$$\eta_t = E_t \left\{ \lambda_t - \nu \lambda_{t+1} r_{t+1} v x_{t+1}^{(v-1)} - \mu_t [\nu \lambda_{t+1} r_{t+1} v x_{t+1}^{(v-1)} - 1 + \theta] \right\}. \tag{5e}$$

DEFINITION 3.1. An equilibrium with limited commitment consists in sequences $(c_t^w, b_t^w)_{t=1}^\infty$, $(c_t^E, x_t, b_{t+1}^E)_{t=1}^\infty$, $(q_t, r_t, w_t)_{t=1}^\infty$, $(\lambda_t, \mu_t, \eta_t)_{t=1}^\infty$, and aggregate capital $K_t$ such that

(1) Taking prices as given, workers maximize (1a) and (1b) so that (1b) and (2) hold; entrepreneurs maximize (3a)–(3d) so that conditions (3b), (5a)–(5e) hold.

(2) Capital and labor earn their marginal product,

$$r_t = F_k(K_t, L_t), \tag{6a}$$

$$w_t = F_k(K_t, L_t). \tag{6b}$$

(3) Credit market clears

$$\alpha b_{t+1}^w + (1 - \alpha) b_{t+1}^E = 0. \tag{7}$$
(4) Capital evolves according to

\[ K_{t+1} = (1 - \alpha)x_t^\alpha. \]  

\[ \text{(8)} \]

### 3.3. Stationary Equilibria

In this subsection, we characterize stationary equilibria with positive capital, \( K > 0 \), where bonds are traded.\(^{13}\) Since workers must hold bonds, \( b^W > 0 \), from equation (2) the price of bonds is

\[ q = \beta r. \]  

\[ \text{(9)} \]

If we let \( z = \frac{K}{\alpha} \) denote steady-state capital per worker, from (6a) and (6b) we have \( r = f'(z) \) and \( w = f(z) - zf'(z) \). Using the law of motion of capital (8), we obtain

\[ r = f' \left[ \frac{(1 - \alpha)x^\nu}{\alpha} \right]. \]  

\[ \text{(10)} \]

As all other variables can also be expressed as a function of \( x \), to close the model we only have to determine its level, which depends on whether the no-default constraint binds or not.

**Stationary equilibria with a slack no-default constraint.** In this case, entrepreneurs consume all their income, i.e., \( x + qb^E = 0 \).\(^{14}\) Moreover, as in equation (5d) \( \mu = 0 \), we have \( q = \frac{1}{\nu x^{\nu-1}} \), so that the price of bonds is determined by the capital-producing technology. Combining now the expression for \( q \) with (9) and (10), we obtain

\[ f' \left[ \frac{(1 - \alpha)x^\nu}{\alpha} \right] = \frac{x(1-v)}{\beta^\nu}. \]  

\[ \text{(11)} \]

Equation (11) uniquely determines entrepreneurs’ investment in a stationary equilibrium with a slack no-default constraint. This level of investment is the same as if entrepreneurs had access to a commitment technology. Accordingly, we denote the \( x \) that solves (11) by \( x_{FB} \), where FB stands for the first best.

**Stationary equilibria with bonds and abiding no-default constraint.** From (5c), we have \( \mu \geq 0 \) and \( \gamma V(b^E, x) = (1 - \theta)x \). Using stationarity, from (3a) we obtain \( V(b^E, x) = \frac{x^E}{1-\gamma} \). Combining the two last expressions, we obtain

\[ c^E = \frac{(1 - \theta)(1 - \gamma)x}{\gamma} > 0, \text{ and } \lambda = 1. \]  

\[ \text{(12)} \]

In this case, entrepreneurs may or may not save to invest in their technology. Note that this is not possible when the no-default constraint (4) is slack, where bonds are
always traded and entrepreneurs do not save.\textsuperscript{15} As explained below, we interpret entrepreneurs own investment as collateral.

If entrepreneurs collateral equals zero, from (3b), as (9) still holds, we have \(c^E = rx^v(1 - \nu)\). Equating this expression for \(c^E\) with (12), we obtain

\[
\frac{(1 - \alpha)x^v}{\alpha} = x^{1 - \nu} \left[ \frac{\gamma + \beta(1 - \theta)(1 - \gamma)}{\beta \gamma} \right].
\]

Equation (13) uniquely determines steady-state investment \(x\), when (4) binds, bonds are traded, and entrepreneurs do not save. Let \(x_{BNC}\) denote the level of \(x\) that solves (13), where BNC stands for “bonds and no collateral.” We can prove that \(x_{BNC} < x_{FB}\), and \(q_{BNC} > q_{FB}\).

When entrepreneurs’ savings are positive, solving for \(\mu\) in (5e), where \(\eta = 0\), and replacing its expression and (9) into (5d), we obtain

\[
\frac{(1 - \alpha)x^v}{\alpha} = x^{1 - \nu} \left[ \frac{\gamma + (1 - \theta)(\beta - \gamma)}{\gamma \beta \nu} \right].
\]

Equation (14) uniquely determines \(x\), when (4) binds, bonds are traded, and loans are collateralized.

Below, we state necessary and sufficient conditions for the existence of these equilibria.\textsuperscript{16}

**Proposition 3.2.** Define \(\nu(\theta) \equiv \frac{\gamma}{\gamma + \beta(1 - \theta)(1 - \gamma)}\) and \(\overline{\nu}(\theta) \equiv \frac{\gamma + (1 - \theta)(\beta - \gamma)}{\gamma + \beta(1 - \theta)(1 - \gamma)}\). Then,

\begin{enumerate}
\item An equilibrium with a slack no-default constraint exists if and only if \(\nu \leq \nu(\theta)\).
\item A stationary equilibrium with a binding no-default constraint, bonds, and no collateral exists if and only if \(\nu(\theta) \leq \nu \leq \overline{\nu}(\theta)\).
\item A stationary equilibrium with a binding no-default constraint, bonds, and collateral, exists if and only if \(\overline{\nu}(\theta) \leq \nu < 1\).
\end{enumerate}

This is illustrated in Figure 1, where we partition the parameter space according to the properties of the stationary equilibria with bonds. When the elasticity \(\nu\) is sufficiently small, i.e., \(\nu \leq \nu(\theta)\), entrepreneurs earn significant rents by operating their technology. Permanent exclusion from credit markets is a severe punishment that dissuades them from defaulting, so that the first best levels of credit and investment are sustained. Workers purchase bonds to transfer consumption intertemporally, and the rate of return on capital is too low to convince the more impatient entrepreneurs to invest. For intermediate values of \(\nu\), i.e., \(\nu(\theta) \leq \nu \leq \overline{\nu}(\theta)\), entrepreneurs rents are relatively smaller, and the first best level of credit and investment would violate the no-default constraint. Entrepreneurs are still not investing in their technology because it is costly for them to postpone consumption. Credit and investment adjust (reduce) to avoid that entrepreneurs choose to default. Finally, when \(\nu\) is relatively large (above the red line), entrepreneurs rent are almost
negligible. For these values of $\nu$, entrepreneurs invest a positive fraction of their income in the capital-producing technology. This investment works as collateral: By providing entrepreneurs with a stake in the project, collateral relaxes credit constraints. Easily, given a value of $\nu$, a larger $\theta$ increases the cost of deviation, and helps sustaining larger levels of credit and investment. This explains the positive slope of both lines, $\nu(\theta)$ and $\nu(\theta)$.

4. LOCAL DYNAMICS

We restrict our analysis to the cases in which entrepreneurs do not save, i.e., we assume that $\nu \leq \nu(\theta)$. In this way, we guarantee that our results are not driven by the existence of collateral. Section 4.1 refers to parameter configurations that satisfy $\nu \leq \nu(\theta)$ and Section 4.2 refers to configurations that satisfy $\nu(\theta) \leq \nu \leq \nu(\theta)$.

4.1. Local Dynamics with a Slack No-Default Constraint

Define $z_t = \frac{K_t}{\alpha}$ to be capital per worker. From (8), we can write time $t$ capital per worker as

$$z_t = \frac{1 - \alpha}{\alpha} x_{t-1}^\nu \equiv z(x_{t-1}),$$

so that, using (6a) and (6b), the interest rate and the wage can be written as

$$r_t = f' [z(x_{t-1})] \equiv r(x_{t-1}),$$

$$w_t = f [z(x_{t-1})] - z(x_{t-1}) f' [z(x_{t-1})] \equiv w(x_{t-1}).$$

As the no-default constraint is slack, $\mu_t = 0$, and from (5d) we can write the price of bonds as a function of $x_t$, $q_t = \frac{1}{\nu x_{t-1}^\nu} \equiv q(x_t)$, i.e., the price of bonds is totally
pinned down by the capital-producing technology. Also, since entrepreneurs do not save, from (5b), we have \( b_{E}^{t+1} = -\frac{x_{t}}{q(x_{t})} \equiv b_{E}(x_{t}) \) and using (7) and (3b), we obtain
\[
b_{W}^{t+1} = \frac{1-a}{q(x_{t})} b_{W}(x_{t}) \equiv b_{W}(x_{t}) \quad \text{and} \quad c_{E}^{t} = f'[z(x_{t-1})][x_{t-1} + b_{E}[z(x_{t-1})]] \equiv c_{E}(x_{t-1}).
\]
Substituting the previous expressions in the workers first-order condition (2) and budget constraint (1b), we obtain our two dynamic equations in \( x_{t}, x_{t-1}, c_{W}^{t} \), and \( c_{W}^{t+1} \).\(^{17}\)

\[
\frac{E_{t}(c_{W}^{t+1})}{c_{W}^{t}} = \frac{\beta f'[z(x_{t})]}{q(x_{t})} \quad \text{(18a)}
\]
\[
c_{t}^{W} + q(x_{t})b_{W}(x_{t}) = f[z(x_{t-1})] - z(x_{t-1}) f'[z(x_{t-1})] + f'[z(x_{t-1})]b_{W}(x_{t-1}). \quad \text{(18b)}
\]

It is clear that investment is a predetermined variable, whose behavior is determined by past decisions. However, consumption is a non-predicted variable, whose level is influenced by expectations. Therefore, endogenous fluctuations driven by self-fulfilling volatile expectations (sunspots) may emerge if the steady state is indeterminate. We have indeterminacy when the number of eigenvalues, of the Jacobian matrix of the linearized dynamic system, with modulus less than one is higher than the number of predetermined variables. Since we have one predetermined variable, we will only have indeterminacy if the two eigenvalues are smaller than one, i.e., the steady state is a sink. However, in this environment this will never happen. Indeed, we can prove the following result.\(^{18}\)

**PROPOSITION 4.1.** *If the no default constraint is slack, the stationary equilibrium with bonds is a saddle and never is indeterminate.*

In this case, there is a unique trajectory, \( (c_{W}^{t}, x_{t})_{t=1, \ldots, \infty} \) with a given \( x_{0} \) sufficiently close to the steady state, which remains close to the steady state. This means that forecasting errors are necessarily zero and, in the absence of exogenous shocks on fundamentals, there is a unique convergent path to the steady state, so that endogenous fluctuations do not emerge.

Remark that these local dynamic properties are identical to those of the standard Ramsey model: The steady state is locally determinate, it is a saddle, and endogenous fluctuations do not emerge. We can therefore conclude that, if commitment problems are not relevant, the Ramsey environment, where one single good is used both as a consumption and capital good constitutes a good approximation, even if the technology that transforms one into the other is not linear.

### 4.2. Local Dynamics with a Binding No-Default Constraint, Bonds, and No Collateral

From the workers problem, we obtain as before (2) and (1b). Also expressions (15)–(17) still apply. Moreover, since entrepreneurs savings are again zero, using
(3b) and (5b), from the entrepreneurs problem we obtain as before
\[ b_{t+1}^{E} = -\frac{x_{t}}{q_{t}}, \quad (19) \]
\[ c_{t}^{E} = r_{t} \left( x_{t-1}^{\nu} + b_{t}^{E} \right). \quad (20) \]

However, now the no-default constraint is binding, i.e,
\[ (1 - \theta) x_{t} = E_{t} \left[ \gamma c_{t+1}^{E} + \gamma (1 - \theta) x_{t+1} \right]. \quad (21) \]

Combining the credit market clearing condition (7) with (19), we obtain as before that
\[ b_{t+1}^{W} = -\frac{1 - \alpha}{\alpha} b_{t+1}^{E} = \frac{1 - \alpha}{\alpha} x_{t}. \quad (22) \]

Replacing the last expression in the worker’s budget constraint (1b), and using also (15)–(17), we can rewrite it as \( c_{t+1}^{W} + \frac{1 - \alpha}{\alpha} x_{t} = w(x_{t-1}) + r_{t-1} b_{t}^{W} \). Now, if we forward this expression one period, and we rearrange its terms, we obtain \( b_{t+1}^{W} = \frac{c_{t+1}^{W} + \frac{1 - \alpha}{\alpha} x_{t+1} - w(x_{t})}{r(x_{t})} \). Replacing back this expression into (22), we can write \( q_{t} \) as a function of \( x_{t}, x_{t+1} \) and \( c_{t+1}^{W}, q_{t} = \frac{1 - \alpha}{\alpha} x_{t} \frac{c_{t+1}^{W} + \frac{1 - \alpha}{\alpha} x_{t+1} - w(x_{t})}{(1 - \alpha) x_{t}} \equiv q_{t}(c_{t+1}^{W}, x_{t}, x_{t+1}) \).

Clearly, the price is no longer pinned down by technology, but depends on future economic conditions. Finally, replacing this last expression into (2), we obtain
\[ E_{t} \left\{ \frac{c_{t+1}^{W}}{c_{t}^{W}} - \frac{\beta \alpha}{(1 - \alpha) x_{t}} \left[ c_{t+1}^{W} + \frac{1 - \alpha}{\alpha} x_{t+1} - w(x_{t}) \right] \right\} = 0, \quad (23) \]

In the entrepreneur’s problem, if we forward (20) one period, and we replace it into (21) together with (19) and the expression for \( q_{t} \) obtained above, we obtain
\[ (1 - \theta) x_{t} = E_{t} \left\{ \gamma r_{t} [x_{t}^{\nu} - \frac{\alpha}{1 - \alpha} \frac{c_{t+1}^{W} + \frac{1 - \alpha}{\alpha} x_{t+1} - w(x_{t})}{r(x_{t})}] + \gamma (1 - \theta) x_{t+1} \right\}. \quad (24) \]

Equations (23) and (24) are our two dynamic equations, in \( x_{t}, x_{t+1}, c_{t}^{W}, c_{t+1}^{W} \).

It is clear that now, both consumption and investment are non-predetermined variables, whose values are influenced by expectations about the future. Hence, depending on the local stability properties of the steady state, there is potentially room for the existence of equilibria trajectories that exhibit bounded fluctuations, sufficiently close to the steady state, driven by self-fulfilling changes in expectations unrelated to economic fundamentals. This will happen when the steady state is locally indeterminate. With two non-predetermined variables, we will have indeterminacy when the steady state is a sink or a saddle. It is easy to prove the following result.19
PROPOSITION 4.2. Assume that \( \gamma > \min\{s, (1 - \theta)\beta\} \). The steady state with a binding no-default constraint, bonds, and no collateral is a saddle and always locally indeterminate.

Indeed, since we have two non-predetermined variables, when the steady state is a saddle there are an infinite number of combinations of initial values for both variables, that keep the system in the stable arm, so that it will always converge to the steady state, i.e., there are now infinitely many bounded deterministic equilibrium trajectories \((c_t, x_t)_{t=-1,\ldots}\infty\) converging to the steady state. Also, as proved by Grandmont et al. (1998), there are also infinitely many nondegenerate stochastic equilibria driven by self-fulfilling volatile expectations (stochastic endogenous fluctuations or sunspots equilibria), which stay arbitrarily close to the steady state.

To understand the mechanism through which equilibrium indeterminacy arises in our model consider that, departing from the steady state, entrepreneurs expect an increase in future investment, \(x_{t+1}\). Such expectation, relaxing the borrowing constraint, implies an increase in current investment \(x_t\) [see (21)]. Since entrepreneurs do not save, \(x_t\) can only increase if workers buy more bonds. For this to happen, the price of bonds \(q_t\), no longer pinned down by technology, adjusts downward in such a way that both \(b_{t+1}^W\) and \(q_b x_t\) increase. This rise in \(x_t\) increases wages and decreases the interest rate at \(t+1\) [see (16) and (17)]. Furthermore from (1b), we can see that the increase in \(q_b b_{t+1}^W\) must be compensated by an identical decrease in \(c_{t+1}^W\) since the left-hand side does not change. From the workers FOC \(E_t(q_t c_{t+1}^W) = E_t(r_{t+1} c_{t+1}^W)\), since its left-hand side significantly decreases, and \(q_t\) decreases, but not dramatically, we have that \(c_{t+1}^W\) must also decrease. Using now (1b) at \(t+1\), as its left-hand side increases and \(c_{t+1}^W\) decreases, we conclude that \(x_{t+1}\) rises, rendering the initial expectation self-fulfilling.

Sunspot equilibria. The stable branch of the saddle is given by the following first-order difference equation: \(\tilde{x}_{t+1} = \lambda_1 \tilde{x}_t\), where a \(\tilde{}\) over a variable denotes percentage deviations from the steady state, e.g., \(\tilde{x}_t = \frac{d_{x_t}}{x_t}\), and \(\lambda_1 < 1\) is the stable eigenvalue of \(J_{BNC}\), the Jacobian Matrix of the linearized version of our dynamic system (23) and (24). Moreover, along the stable arm, we have \(\tilde{c}_{t+1}^W = \frac{\lambda_1 - \beta_{ij}^M}{j_{ij}} \tilde{x}_t\), where \(j_{ij}\) denotes the element \(ij\) of matrix \(J_{BNC}\). However, in this economy sunspots matter. To capture the effects of these nonfundamental shocks, we add forecasting errors \(e_{t+1} = \tilde{x}_{t+1} - E_t(\tilde{x}_{t+1})\), with \(E_t(e_{t+1}) = 0\), obtaining

\[
\tilde{x}_{t+1} = \lambda_1 \tilde{x}_t + e_{t+1}. \tag{25}
\]

Local sunspot equilibria can be obtained by considering that \(e_{t+1}\) follows an i.i.d. stochastic process of bounded support with sufficiently small variance [see Benhabib and Farmer (1999) for further discussion]. Suppose that, starting from the steady state, a forecasting error hits this economy. Since the values of \(x\) and \(c\) are tied by the relationship \(\tilde{c}_{t+1}^W = \frac{\lambda_1 - \beta_{ij}^M}{j_{ij}} \tilde{x}_t\), the economy will come back to the stable arm, which guarantees convergence to the steady state. Forecasting errors now act as independent sources of the business cycles, even in the absence of
intrinsic uncertainty affecting fundamentals, and stochastic bounded equilibrium trajectories, driven by expectations shocks, that stay arbitrarily close to the steady state are possible when the steady state is a saddle. In this case, there is a potential role for stabilization policies. Our theoretical analysis suggests that any policy that aims at stabilizing the economy has to be successful in relaxing the commitment problem up to the point where the no-default constraint does not bind.

5. SIMULATION EXERCISE

We present below a simple simulation exercise, where we introduce sunspot shocks near the saddle steady state with a binding no-default constraint, bonds, and no collateral.

We considered a discount factor for workers $\beta = 0.99$, in line with most calibrations, and to motivate trade, we assumed $\gamma = 0.92$. In the online appendix, we show that $\gamma = 0.92$ gives a plausible excess return $R^e \approx 1.4\%$. We set the capital share at $s = 0.33$, a value consistent with observed data. In the absence of a reliable estimate for the cost $\theta$ of hiding resources, we experimented with several values, and we choose a value of $\theta = 0.6$. We considered a CES production function with $\sigma = 2.5$, consistent with the estimates obtained by Duffy and Papageorgiou (2000), contained in the $(1.24, 3.24)$ interval. We choose $\nu = 0.98$, which guarantees that $0.97 = \nu < \nu < \bar{\nu} = 0.99$. Moreover, this value is sufficiently close to 1, to ensure that the results are not mainly driven by an excessive concavity of the capital-production technology. By construction, the sunspot shocks are i.i.d. shocks. We assume a normal distribution $N(0, \sigma_e)$, where $\sigma_e = 0.0185$ was chosen to match the standard deviation of consumption growth in postwar US data.

Using these values for the parameters, we simulated 500 draws of series of length $T = 100$ each. In Figure 2 below, we present the series generated by simulating a single draw. In the top-left panel, we show the steady-state deviations of worker’s consumption and investment. The two variables move together because $\tilde{c}_t^W = \lambda_1 - \lambda_2 \tilde{x}_t$ and, as in the data, investment is more volatile than consumption. In the top-right panel, we show the simulated series for investment and output. The model performs quite well in generating an excess volatility in investment: The standard deviation of investment (computed averaging over the 500 draws) is indeed 3.1 times larger than the standard deviation of output. The corresponding number in the data is 4.26, as reported in Meh and Moran (2010). In the left-middle panel, we observe that the interest rate is countercyclical, whereas real wages are procyclical, as documented in King and Watson (1996). In the right-middle panel, we plot the evolution of credit contracts (bonds) purchased by workers and investment. As we can see, credit is more volatile than investment: The average ratio of the respective standard deviations is 2.65. Even though this number is larger than the one observed in the data, the model does a good job in capturing the direction of the inequality.
Our model is able to generate price volatility in the credit market consistent with the one observed in the data. The model generates an average ratio of the standard deviation of the price of bonds $q_t$ and output equal to 8.05. This number compares with a value of 5.26, if we use the Moody’s BAA Corporate Bond index, which only includes higher rated corporate bonds. As price variability is higher for lower rated bonds, the standard deviation of the price of all kind of bonds will be higher than Moody’s index. This suggests that our results reflect the overall volatility in the prices of bonds, which has been difficult to match by previous works. It is also interesting to further discuss credit price volatility within the model. In the bottom left-hand panel, we plot deviations of the interest rate $r_t$ and of the price of bonds $q_t$ from steady-state values. We found that $q_t$ is almost 10 times more volatile than $r_t$. While $r_t$ is pinned down by technology, $q_t$ is not, being very responsive to shocks in expectations. Finally, in the bottom right-hand panel, we plot deviations of $q_t$ and of $q_t^{FB} = 1/\nu(x_t)^{\nu-1}$, that represents what would have been the price of one bond, if price was determined as in the first best environment. We can see that $q_t$ is dramatically more volatile than $q_t^{FB}$, which does not fluctuate at all. Since all this excess volatility is due to the existence of limited commitment, this suggests that this friction qualifies as an important mechanism in explaining the price volatility observed in credit markets.

6. CONCLUDING REMARKS

We have introduced a specific credit friction, limited commitment, in a macroeconomic general equilibrium model with production and investment in productive capital, where agents can trade bonds. We show that this economy always displays
one equilibrium where bonds are traded. Depending on the severity of the commit-
ment problem, this can be the first best steady state, or a second best with or
without collateral. Furthermore, we show that with limited commitment, indeter-
mindacy and stochastic fluctuations driven by self-fulfilling volatile expectations
may emerge. This, as shown by our simulation results, yields credit and investment
cycles that match those observed in the data. More importantly, the model gener-
ates a volatility of the price of bonds consistent with real data. This result holds
because the credit market friction considered, limited commitment, delinks asset
prices from economic fundamentals, making them determined by expectations
about future economic conditions. All this suggests that limited commitment may
play an important role in explaining volatility in credit markets. Moreover, our
conclusions can be relevant for the design of macroeconomic policies aimed at
stabilizing economic fluctuations. Finally, it would be interesting to complement
this work by studying the dynamic properties of the stationary equilibrium with
collateral and by analyzing the global dynamics of our model. We leave this and
other questions to further research.

NOTES

1. See Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Gertler and Kiyotaki (2011).
2. In our model expectation shocks, by relaxing and tightening endogenous borrowing constraints,
function in a similar way to the exogenous financial shocks of Jermann and Quadrini (2012) and Shi
(2015).
3. In the absence of limited commitment, Gu et al. (2013) obtain a sequence of static problems, i.e.,
they have no dynamics, besides the one linked to the emergence of borrowing constraints. In contrast,
in our case without limited commitment, we replicate the two-dimensional dynamics of the Ramsey
model.
4. With a linear technology this price equals 1, so that capital and consumption are perfect substi-
tutes, as in the one good Ramsey model.
5. The no-default constraint imposes borrowing limits that prevent default.
6. However, in contrast with Gu et al. (2013), where the no-credit equilibrium always exists, a
stationary equilibrium with positive capital, where bonds are not traded, may or may not exist. This
shows that the distinction between productive capital and credit is important, since in our model the
no credit equilibrium does not coincide with the no capital equilibrium. Note that this last equilibrium
always exists in our case.
7. The indeterminacy mechanism is explained in detail in Section 4.2.
8. Indeed, a fixed cost that firms need to pay to stay in operation makes their model isomorphic to
a representative agent model with increasing returns to scale. See Benhabib and Farmer (1999), for an
excellent survey.
9. This is a standard assumption to motivate trade, see Quadrini (2011).
10. Note that the representative firm profits are zero.
11. This constraint, where, without any loss in generality, we set the right-hand side to zero, is
needed to prevent a Ponzi scheme.
12. Since we want to focus on equilibria with capital accumulation, we assume \( x_t > 0 \).
13. \( K = 0 \) is a trivial stationary equilibrium, where all variables are zero. A stationary equilibrium
with \( K > 0 \) and no bonds also exists if and only if \( v \geq \frac{1}{\phi + \gamma} \) (proof available in the online appendix).
14. The proof is provided in the online appendix.
15. The reader can find the proof in the online appendix.
16. The proof is provided in the online appendix.
17. Note that the dynamic system (18a) and (18b) coincides with the dynamic system of the first best equilibrium, where the no-default constraint (3d) does not enter the entrepreneurs’ problem.
18. The proof is available online.
19. The proof is available online.
20. Note that these are sufficient but not necessary conditions.
21. With an almost linear technology ($\nu$ close to 1), in the first best capital and consumption would be almost perfect substitutes, so that the price of bonds $q_{FB}$ would be always close to 1, not changing along the cycle.

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