The Color Excesses of Type Ia Supernovae from Single-Degenerate Channel Model

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Abstract Single Degenerate model is the most widely accepted progenitor model of type Ia supernovae (SNe Ia), in which a carbon-oxygen white dwarf (CO WD) accretes hydrogen-rich material from a main sequence or a slightly evolved star (WD+MS) to increase its mass, and explodes when its mass approaches the Chandrasekhar mass limit. During the mass transfer phase between the two components, an optically thick wind may occur and the material lost as the wind may exist as circumstellar material (CSM). Searching the CSM around progenitor star is helpful to discriminate different progenitor models of SNe Ia. Meanwhile, the CSM is a source of color excess. The purpose of this paper is to study the color excess produced from the single-degenerate progenitor model with optically thick wind, and reproduce the distribution of color excesses of SNe Ia. Meng et al. (2009) systemically carried out binary evolution calculation of the WD+MS systems for various metallicities and showed the parameters of the systems before Roche lobe overflow and at the moment of supernova explosion in Meng & Yang (2009). With the results of Meng et al. (2009), we calculate the color excesses of SNe Ia at maximum light via a simple analytic method. We reproduce the distribution of color excesses of SNe Ia by our binary population synthesis approach if the velocity of the optically thick wind is taken to be of order of magnitude of 10 km s\textsuperscript{-1}. However, if the wind velocity is larger than 100 km s\textsuperscript{-1}, the reproduction is bad.

Key words: Stars: white dwarfs - stars: supernova: general

1 INTRODUCTION

Although type Ia supernovae (SNe Ia) showed their importance in determining cosmological parameters, e.g. $\Omega_M$ and $\Omega_L$ \cite{Riess1998, Perlmutter1999}, the progenitor systems of SNe Ia have not been confidently identified yet \cite{Hillebrandt2000, Leibundgut2000}. It is widely believed that a SN Ia is from thermonuclear runaway of a carbon-oxygen white dwarf (CO WD) in a binary system. The CO WD accretes material from its companion to increase its mass. When its mass reaches its maximum stable mass, it explodes as a thermonuclear runaway and almost half of the WD mass is converted into radioactive nickel-56 \cite{Branch2004}. Two basic scenarios have been discussed over the last three decades. One is a single degenerate (SD) model, which is widely accepted \cite{Whelan1973}. In this model, a CO WD increases its mass by accreting hydrogen- or helium-rich matter from its companion, and explodes when its mass approaches the Chandrasekhar mass limit. The companion may be a main-sequence star (WD+MS) or a red-giant star (WD+RG) \cite{Yungelson1995, Li1997, Hachisu1999a, Hachisu1999b, Nomoto1999, Nomoto2003, Hachisu2003}.
An alternative is the theoretically less favored double degenerate (DD) model (Iben & Tutukov 1984; Webbink 1984), in which a system consisting of two CO WDs loses orbital angular momentum by gravitational wave radiation and merges finally. The merger may explode if the total mass of the system exceeds the Chandrasekhar mass limit (see the reviews by Hillebrandt & Niemeyer 2000 and Leibundgut 2000). In theory, a large amount of circumstellar materials (CSM) may be produced via an optically thick wind for the SD model (Hachisu et al. 1996), while there is no CSM around DD systems. Then, a basic method to distinguish the two progenitor models is to find the CSM around progenitor systems.

Evidence for CSM was first found in SN2002ic (Hamuy et al. 2003), which has shown extremely pronounced hydrogen emission lines that have been interpreted as a sign of strong interaction between supernova ejecta and CSM. The discovery of SN2002ic may uphold the SD model (Han & Podsiadlowski 2006). Recently, the evidence for CSM was found in a normal SN Ia (SN 2006X) defined by Branch, Fisher & Nugent (1993) and the CSM is proposed to be from a wind from a red-giant companion (Patat et al. 2007a). The CSM may play a key role to solve the problem of the low value of reddening ratio of external galaxy (Wang 2005), which is very important for precision cosmology (Wang et al. 2008).

If a SN Ia is surrounded by a large amount of CSM, its color observed should be redder than its intrinsic color, which results in a color excess, \( E(B-V) \). Reindl et al. (2005) showed the color excesses of more than one hundred SNe Ia at maximum light, which suggests a mission to check which progenitor model of SNe Ia can explain the distribution of the color excesses. Recently, Meng et al. (2009) performed binary stellar evolution calculations for more than 25,000 close WD binary systems with various metallicities, and present all the parameters of the systems for SNe Ia before the Roche lobe overflow (RLOF) and at the moment of supernova explosion in a following paper (Meng & Yang 2009). In their works, the prescription of Hachisu et al. (1999a) for the accretion efficiency of hydrogen-rich material was adopted by assuming an optically thick wind (Hachisu et al. 1996), and then their works provide a possibility to check whether the SD model with optically thick wind can reproduce the distribution of color excesses of SNe Ia obtained from observation or not. The purpose of this paper is to check the possibility, and this work is based on the results from Meng et al. (2009).

In section 2, we describe our model. We show the results in section 3 and give discussions and conclusions in sections 4 and 5.

2 MODEL AND PHYSICS INPUTS

2.1 the mass distribution of lost hydrogen-rich material

As described in section 1, the color of a supernova is reddened by the CSM surrounding the supernova. We first check whether there exists enough CSM resulting from optically thick wind. The following is a simple description about how to obtain the CSM. As described in Meng et al. (2009), in a WD + MS system, the companion fills its Roche lobe at MS or during HG, and transfers material onto the WD. If the mass-transfer rate, \(|\dot{M}_2|\), exceeds a critical value, \(\dot{M}_{cr}\), we assume that the accreted hydrogen steadily burns on the surface of WD and that the hydrogen-rich material is converted into helium at the rate of \(\dot{M}_{cr}\). The unprocessed matter is assumed to be lost from the system as an optically thick wind at a rate of \(\dot{M}_{wind} = |\dot{M}_2| - \dot{M}_{cr}\) (Hachisu et al 1996). Adopting the prescription of Hachisu et al. (1999a) on WDs accreting hydrogen-rich material from their companions, Meng et al. (2009) obtained the initial parameters of WD + MS systems for SNe Ia, and at the same time, the final parameters at the moment of SN Ia explosion, such as \(M_{SN}\). In this paper, incorporating the binary evolution results in Meng et al. (2009) into the rapid binary evolution code developed by Hurley et al. (2000, 2002), we carry out a series of binary population synthesis (BPS) studies for various Z. In each BPS study, \(10^7\) binaries are generated by Monte Carlo simulation and a circular orbit is assumed for all binaries. The basic parameters for the simulations are same to those in Meng et al. (2009). It can be shown that a WD+MS system may originate from three possible evolution channels, namely, the He star channel, the EAGB channel.
and the TPAGB channel (see Meng et al. 2009 for details). We assume that a SN Ia is produced if the initial parameters of a WD + MS system, e.g. initial orbital period $P_{\text{orb}}$ and initial secondary mass $M_{i2}$, locate in the appropriate regions of the parameters for SNe Ia at the onset of RLOF. We can obtain the companion mass at the moment of explosion by interpolation in the three-dimensional grid ($M_{i\text{WD}}$, $M_{i2}$, log $P$) of the more than 25,000 close WD binary system calculated in Meng et al. (2009). In Fig. 1 we show the mass distribution of hydrogen-rich material lost as the optically thick wind for various metallicities, $M_{\text{wind}} = (M_{i2} + M_{i\text{WD}}) - (M_{i\text{SN}} + M_{i\text{WD}})$, where superscript i and SN represent the initial and final values for white dwarf and secondary, respectively, and $M_{i\text{WD}}$ is assumed to be $1.378 M_\odot$. We can see from the figure that the distribution of the lost mass peaks at about $0.3 M_\odot$ and has a high-mass tail. The amount of the lost material may be as large as $2.5 M_\odot$, which should contribute to the color excess of SNe Ia.

In Fig. 1 we can see that there does not seem to be a systemic trend with metallicity. Actually, the influence of metallicity on the $M_{\text{wind}}$ is complicated. $M_{\text{wind}}$ is mainly determined by $M_{i2}$ and $M_{i\text{WD}}$. The two parameters are both systemically affected by metallicity, but the tendency is reversed, i.e. the peak of the distribution of companion move to higher mass with metallicity, while the peak for WD mass to lower mass (see Figs. 9 and 10 in Meng et al. 2009). In addition, the metallicity also affect the mass-transfer rate between WD and its companion, and then $M_{i\text{SN}}$ (Langer et al. 2000). The complicated influence of metallicity on the distribution of $M_{\text{wind}}$ results in a non- systemic trend of the distribution with metallicity. We also noticed that the percentage of high $M_{\text{wind}}$, e.g. $M_{\text{wind}} > 2.0 M_\odot$, increases with metallicity. High $M_{\text{wind}}$ is mainly determined by the upper boundary of the companion mass, which moves to higher mass with metallicity (see Fig. 4 in Meng et al. 2009). So, a binary system producing SN Ia with high metallicity may loss more hydrogen-rich material by optically thick wind, and then the high-metallicity model in Fig. 1 shows a higher percentage of high $M_{\text{wind}}$. 

![Fig. 1](image-url)
Fig. 2 The distribution of color excesses of SNe Ia at maximum light for various metallicities, where wind velocity is assumed to be 10 km s$^{-1}$. The solid histogram is from observation, and the bar represents its maximum error (Reindl et al. 2005). The black dashed line is the sum of those of $Z=0.01$, 0.02 and 0.03, where the weights for the three components are 20%, 40% and 40% respectively. Via K-S test, the percentages indicated in the figure show the confident level that the distributions of color excess from theory and observation are indistinguishable.

2.2 model

Bohlin et al. (1978) found

$$E(B - V)/n_H = 1.72 \times 10^{-22} \text{mag cm}^2,$$

for the Galaxy, where $n_H$ is total hydrogen column density and this relation linearly depends on metallicity (Draine 2003). The linear relation can be fitted by

$$E(B - V)/n_H = (17.4 \times Z/Z_{\odot} - 0.454) \times 10^{-23} \text{mag cm}^2,$$

where the fitting data are from Bohlin et al. (1978), Koorneef (1982), Fitzpatrick (1985) and Martin et al. (1989). In the following, we describe how to obtain the total hydrogen column density, $n_H$.

While the CSM around the progenitor of a SN Ia may be asymmetric, for simplicity, we assume that the mass loss of optically thick wind is spherically symmetric and the mass-loss rate is constant during the whole mass-transfer phase. Then, hydrogen number density is only a function of the distance of wind material from the progenitor star, i.e.

$$n(r) = ar^{-2},$$

where $a$ is a coefficient determined by

$$\int_{r_0}^{r_m} ar^{-2} \cdot 4\pi r^2 dr = N_H,$$
where \( r_0 \) is the radius of progenitor, \( r_m \) is the maximum distance which hydrogen-rich material can arrive as optically thick wind at the moment of SN Ia explosion, and \( N_H \) is the total number of hydrogen atom. The total hydrogen column density at maximum light is calculated from

\[
\int_{r_1}^{r_m} 4\pi r^{-2}dr = n_H,
\]

where \( r_1 \) is the distance of explosion ejecta from explosion center at maximum light. From equations (4) and (5), we can obtain

\[
n_H = \frac{N_H}{4\pi r_m r_1}.
\]

\( N_H \) can be obtained from

\[
N_H = \frac{X M_{\text{wind}}}{m_H},
\]

where \( M_{\text{wind}} \) is the total mass lost as optically thick wind, \( m_H \) is the mass of a hydrogen atom and \( X \) is the mass fraction of hydrogen and is set to be

\[
X = 0.76 - 3.0Z,
\]

(Pols et al. 1998). \( r_m \) can be obtained from

\[
r_m = V_{\text{wind}} t_d,
\]

where \( V_{\text{wind}} \) is the velocity of the optically thick wind, and \( t_d \) is the delayed time from the onset of mass transfer to the moment of SN Ia explosion. From the results of Meng et al. (2009), \( t_d \) can be approximated by

\[
\log(t_d/\text{yr}) = -\frac{2}{3} M_i^2 + 7.8,
\]

where \( M_i^2 \) is the initial mass of the secondary (the mass donor unit in solar mass) in a WD + MS system. \( t_d \) from the equation is a rough estimation of the mean value for a certain \( M_i^2 \) and has an error of about 50%. \( r_1 \), the distance of explosion ejecta from explosion center at maximum light, can be obtained by the product of the velocity of supernova ejecta and the rise time of light curve of SNe Ia. We simply assume that the velocity of ejecta is 10000 km s\(^{-1}\) (Gamezo et al. 2003) and the rise time of light curve of SNe Ia is 20 day (Conley et al. 2006; Strovink 2007). The ejecta velocity adopted here corresponds to a typical photospheric velocity (Wang et al. 2003) and might be lower than terminal ejecta velocity (Wang 2006). However, the uncertainty resulting from the ejecta velocity is moderate and accepted. We will discuss its influence in section 4. The rise time does not significantly affect the final results.

Meng et al. (2009) performed binary stellar evolution calculations for more than 25,000 close WD binary systems with various metallicities and Meng & Yang (2009) presented the distribution of all the parameters for these close systems before the RLOF and at the moment of SN Ia explosion. Incorporating their results into the binary population synthesis code of Hurley et al. (2000, 2002), we obtain the distribution of the wind mass, \( M_{\text{wind}} \) (see Fig. 1) and the color excess via equations 1 and 6. The basic parameters for Monte Carlo simulations are same to that in Meng et al. (2009) when primordial binary samples are generated. Because the code is valid just for \( Z \leq 0.03 \), only five metallicities (i.e. \( Z = 0.03, 0.02, 0.01, 0.004 \) and 0.001) are examined here.

The greatest uncertainty of our model is from \( V_{\text{wind}} \). Here, we assume that \( V_{\text{wind}} = 10 \) km s\(^{-1}\) and we will discuss it in section 4.
3 RESULTS

In Fig. 3 we show the distribution of the color excesses of SNe Ia at maximum light for various metallicities. The distribution from observation is also shown by solid histogram in the figure (Reindl et al. 2005). We see in the figure that the SD model with optically thick wind can reproduce the distribution of color excesses of SNe Ia observed. The K-S test shows that the cases of $Z=0.02$ and $Z=0.03$ have the highest confident level that the distributions from theory and observation are indistinguishable. The sample of Reindl et al. (2005) includes the SNe Ia with various metallicities. However, it is difficult to determine the metallicity of the host galaxy of every SN Ia for a large distance (Hamuy et al. 2000). Wang et al. (2006) collected the properties of several SNe Ia from previous papers and noticed that the mean metallicity of the host galaxies of the SNe Ia is $[12 + \log(O/H)]_{\text{mean}} = 8.85 \pm 0.10$ (private communication), which is consistent with solar metallicity $[12 + \log(O/H)]_{\text{solar}} \sim 8.8$ (Zaritsky et al. 1994). Then, our result is consistent with observation although the sample collected by Wang et al. (2006) is small.

There are some SNe Ia whose color excesses are very large (i.e. larger than 1.0), such as SN1999cl (Jha 2002; Reindl et al. 2005), SN2003cg (Elias-Rosa et al. 2006) and SN2006X (Wang et al. 2008). In the sample of Reindl et al. (2005), only one among 113 SNe Ia has a color excess larger than 1. In our simulation, the possibility of the high-color-excess SNe Ia is 2.0% for $Z=0.03$, 0.4% for $Z=0.02$ and 0 for $Z < 0.02$, consistent with that of the observations. This result might imply that the SNe Ia may have a color excess larger than 1 only when their host galaxies have a metallicity larger than 0.02.

Fig. 3 The distribution of color excesses of SNe Ia at maximum light for $Z = 0.02$ and various wind velocities (in km s$^{-1}$). The solid histogram is from observation, and the bar represents its maximum error (Reindl et al. 2005). Via K-S test, the percentages show the confident level that the distributions of color excess from theory and observation are indistinguishable. The black dashed line is the best fitted line, where the weights for the components of $Z=0.01$, 0.02 and 0.03 are 20%, 10% and 70%, respectively.
Observationally, the host galaxies of SN1999cl and SN2006X, i.e. NGC4501 and NGC4321, are both oversolar galaxy (Caputo et al. 2000; Dors Jr & Copetti 2006). Although there is no information about the metallicity of the host galaxy of SN2003cg (NGC3169), NGC3169 is a Sa galaxy and probable has a oversolar metallicity (Willner et al. 1985).

4 DISCUSSION

Our analytic model is so simple. We discuss various uncertainties about our model in this section.

4.1 wind velocity

The major uncertainty of our model is from the assumption that \( V_{\text{wind}} = 10 \, \text{km s}^{-1} \). Although the consistency between the theoretical distribution of color excess and that from observation upholds this assumption, there is no direct observational evidence to verify it. Many observational efforts were projected to find CSM (Hamuy et al. 2003; Aldering et al. 2006; Panagia et al. 2006; Ofek et al. 2007; Patat et al. 2007a,b), and only one observation obtained the constraint of the wind velocity (Patat et al. 2007a). The wind velocity is constrained to be smaller than 50 km s\(^{-1}\) (see Fig. 2 in Patat et al. 2007a), and then Patat et al. suggested that the progenitor of SN2006X should be a WD+RG system. However, we cannot rule out the possibility that the observed CSM is from the optically thick wind since the companion has not been directly observed (Hachisu et al. 2008). Recently, Badenes et al. (2007) explored the relationship between the SD models with optically thick wind for SNe Ia and the properties of the supernova remnants that evolve after the explosion. They found that the optically thick wind with velocity larger than 200 km s\(^{-1}\) would excavate large low-density cavities around the progenitors. The large cavities are incompatible with the dynamics of the forward shock and the X-ray emission from the shocked ejecta in all the SNe Ia remnants as they examined. However, they also showed that if a wind velocity of 10 km s\(^{-1}\) is adopted, the properties of type Ia supernova remnants are well compatible with the prediction from the SD model with optically thick wind. Generally, the escape velocity from a white dwarf is in an order of magnitude of \( 10^3 \, \text{km s}^{-1} \), which is upheld by observations from recurrent novae (Wood & Lockley 2000). In the theoretically framework laid down by Hachisu et al. (1996), an optically thick wind is formed in CO WD envelope with photospheric velocity of \( \sim 10^3 \, \text{km s}^{-1} \) (Hachisu et al. 1999a,b), where the expansion of the photosphere is driven by helium flash in a helium shell on top of CO WD (Kato & Hachisu 1999). Then, in both theory and observation, it seems not to uphold an assumption of a low wind velocity at present. We check the influence of wind velocity on the distribution of color excess for the case of \( Z = 0.02 \) and the results are shown in Fig. 3. We see in the figure that when \( V \geq 100 \, \text{km s}^{-1} \), K-S test gives a low confident level that the distributions of color excess from theory and observation are indistinguishable, while when \( V \leq 50 \, \text{km s}^{-1} \), K-S test shows an acceptable level. This result is similar to that in Badenes et al. (2007).

4.2 delay time

The secondary uncertainty of our model is from \( t_d \). Since optically thick wind may stop before SN Ia explosion, \( t_d \) used in this paper overestimates the delayed time of the wind for some systems. However, this is not a serious problem since for most cases, SNe Ia occur during the wind phase or after the wind phase for a short time (Han & Podsiajłowski 2004; Meng et al. 2009). In addition, \( t_d \) obtained from equation (10) only approximates the mean value for a given \( M_2 \). This is also not a serious problem since we only check an average distribution of color excess. Generally, the peak of the distribution of color excesses moves to lower color excess with \( t_d \), and the peak value increases while the percentage of high color-excess SNe Ia decreases with \( t_d \).
4.3 metallicity

The combination of different metallicities is checked and shown in Fig. 2 by the black dashed line since not all SNe Ia are from a unique metallicity. The weights of the components for $Z=0.01$, $0.02$ and $0.03$ are simply fitted from the samples of Hamuy et al. (2000) and Wang et al. (2006). We also show the best fitted line in Fig. 3 by the black dashed line. We can see from these lines that the combination of different metallicities does not significantly increase the confident level that the distributions for theory and observation are indistinguishable. Actually, for any combination of $Z = 0.01$, $0.02$ and $0.03$, the results will be acceptable, i.e. yielding similar K-S test probability, which is derived from similar K-S test probability for the distributions of $Z = 0.01$, $0.02$ and $0.03$. So, the basic results here still hold.

4.4 interstellar extinction

While we are not sure about the existence of the optically thick wind (it is a prediction by a model, not an observed fact; Hachisu et al. 1990), we are absolutely sure about the existence of the interstellar extinction (IE). In the paper, we did not consider the influence of IE since it is difficult to separate it from CSM dust. However, we know that Type Ia SNe, as any other celestial object, suffer from interstellar reddening, arising in material that has nothing to do with the circumstellar environment of the exploding star. Let’s consider the case of SN 2006X, whose color excess is certainly larger than $E(B-V)=1$, while is most likely arising in a cold molecular cloud which has nothing to do with the explosion site (Wang et al. 2008). So, for the case suffering interstellar reddening, we should remove the influence of IE on color excess. But at present, it is very difficult to do this. However, it is a clear effect that IE will lead to certain directions of where parameters such as $V_{\text{Wind}}$ and $V_{\text{ejec}}$ should go, e.g. moving some very reddened observed SNe Ia to lowering reddened one would yield a higher $V_{\text{Wind}}$ or $V_{\text{ejec}}$. For example, if we assume rather arbitrarily and simply that all observed SNe Ia suffer a extinction of $E(B-V)_{\text{host}} = 0.1$ within its host galaxy, our model suggests $V_{\text{Wind}} = 20$ km s$^{-1}$ or $V_{\text{ejec}} = 20000$ km s$^{-1}$. Similarly, if $E(B-V)_{\text{host}} = 0.2$, our model suggests $V_{\text{Wind}} = 50$ km s$^{-1}$ or $V_{\text{ejec}} = 50000$ km s$^{-1}$. So, the effect of IE could be counteracted by another uncertainties, $V_{\text{ejec}}$, and the basic results in this paper are still valid. (see the discussion about ejecta velocity below).

If the color excess is from CSM as shown in this paper, one may argued that we would have seen the CSM material through radio and X-ray emission, arising in the shock produced by the fast moving SN ejecta crashing into the CSM (Stockdale et al. 2006). However, Panagia et al. (2006) showed a non-detection result for radio for 27 SNe Ia via the Very Large Array (VLA) observations, even including SN 2002ic, which indicated that mass-lose rate should be lower than $\sim 3 \times 10^{-6}$ $M_\odot$ yr$^{-1}$. We take a typical case to check whether our model contradicts with observations. We set companion mass to $2.1 M_\odot$ (see Fig. 10 in Meng et al. 2009), which corresponds to a delay time of $2.5 \times 10^3$ yr. The mass of lost material is set to $0.3 M_\odot$ (see Fig. 1). The mean mass-lose rate is $12 \times 10^{-8} M_\odot$ yr$^{-1}$, which is higher but still comparable to that inferred from observations. Attempts to detect radio emission from SN 2002ic with the VLA were unsuccessful (Berger et al. 2003; Stockdale et al. 2003). SN 2002ic is the first case to show the signal of CSM (Hamuy et al. 2003) and the amount of the CSM is $0.5-6 M_\odot$ (Wang et al. 2004; Chugai & Yungelson 2004; Uenishi et al. 2004; Kotak et al. 2004) and its properties may be explained successful by the WD+MS model used here (Han & Podsiadlowski 2006). In addition, another recent twins of SN 2002ic (SN 2005gj, Aldering et al. 2006) has also not been detected using the VLA (Soderberg & Frail 2005). These non-detection results may indicate that the mechanism that is successful used for SN Ib/c may not work for SNe Ia (Panagia et al. 2006). So, our model is at least not inconsistent with observation at present.

4.5 ionization and ejecta velocity

Generally, when a SN explodes, its radiation field is quite strong, especially at distance $r_H < 10^{16}$ cm (for SN 2006X, $r_H < 4 \times 10^{15}$, Patat et al. 2007a). This radiation can easily ionize hydrogen and
Fig. 4 The distribution of color excesses of SNe Ia at maximum light for different ejecta velocity, where wind velocity is assumed to be 10 km s\(^{-1}\). The ejecta velocity is unit in 10\(^4\) km s\(^{-1}\). The black dashed line is the best fitted line, where the weights for the components of Z=0.01, 0.02 and 0.03 are 20%, 10% and 70%, respectively.

...evaporate dust up to quite large distances, causing the disappearance of reddening. In our simple model, this effect is not considered completely. We check the influence of the effect.

We set \(r_H = 10^{16}\) cm and assume that all hydrogen atoms are ionized within the shell with radius of \(r_H\). The column density of ionized hydrogen is \(n_{H,0} = \int_{r_1}^{r_H} ar^{-2}dr\), where \(r_1 = 1.73 \times 10^{15}\) cm (a ejecta velocity of 10\(^4\) km s\(^{-1}\) and a rise time of 20 day are assumed). Then, the relative uncertainty from the ionization effect is \(\frac{n_{H,0}}{n_{H}} = \frac{(r_H - r_1)r_0}{(r_H - r_1)r_{H,0}} \approx \frac{r_0}{r_{H,0}} \approx 0.83\) (\(r_m \gg r_1\)). The uncertainty seems too large. However, this situation may be improved by taking a higher ejecta velocity and a smaller distance \(r_H\). A distance of \(r_H = 4 \times 10^{15}\) cm like SN 2006X may reduce the uncertainty to 0.57.

The ejecta velocity used here may be smaller than the terminal velocity of SN Ia which can be as high as 3 \(\times 10^4\) km s\(^{-1}\) (Wang 2006). This value may reduce the uncertainty from the ionization effect to 0 \(\sim\) 0.48, which means that no more than a half of CSM are evaporated up to large distances and have no influence on the color excess of SNe Ia. Then, the ionization effect at most decreases the confident level that theory and observation are indistinguishable from 74% to 65%.

In Fig. 4 we show the influence of ejecta velocity on the distribution of the color excess of SNe Ia. The influence is similar to that of wind velocity. From the figure, we can see that the peak of the distribution increases with the ejecta velocity, and when \(V_{ejec} < 5 \times 10^4\) km s\(^{-1}\), the distributions of color excess from theory and observation are indistinguishable on the confident level of higher than 50%. Fortunately, the effect of ionization and a higher ejecta velocity is same to that of interstellar, and their influence on the decrease of the confident level may counteract each other. Then, we can say that the distribution of color excess between theory and observation are indistinguishable on the level of higher than 59%, even when \(V_{ejec} = 3 \times 10^4\) km s\(^{-1}\).
For the discussion above, our model may get a meaning result although our analytic model is so simple.

5 SUMMARY AND CONCLUSIONS

In summary, if a wind velocity of $\sim 10 \text{ km s}^{-1}$ is adopted, the SD model with optically thick wind may reproduce the distribution of color excess of SNe Ia obtained from observation, which might support the SD model for SNe Ia. However, if a wind velocity larger than 100 km s$^{-1}$ is adopted, the reproduction is bad. Our results are similar to those of Badenes et al. (2007). Then, it should be encouraged to perform more detailed observations about the velocity of CSM around progenitors of SNe Ia.

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Appendix A: THIS SHOWS THE USE OF APPENDIX

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The Color Excesses of Type Ia Supernovae from Single-Degenerate Channel Model

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Abstract Single Degenerate model is the most widely accepted progenitor model of type Ia supernovae (SNe Ia), in which a carbon-oxygen white dwarf (CO WD) accretes hydrogen-rich material from a main sequence or a slightly evolved star (WD +MS) to increase its mass, and explodes when its mass approaches the Chandrasekhar mass limit. During the mass transfer phase between the two components, an optically thick wind may occur and the material lost as the wind may exist as circumstellar material (CSM). Searching the CSM around progenitor star is helpful to discriminate different progenitor models of SNe Ia. Meanwhile, the CSM is a source of color excess. The purpose of this paper is to study the color excess produced from the single-degenerate progenitor model with optically thick wind, and reproduce the distribution of color excesses of SNe Ia. Meng et al. (2009) systemically carried out binary evolution calculation of the WD +MS systems for various metallicities and showed the parameters of the systems before Roche lobe overflow and at the moment of supernova explosion in Meng & Yang (2009). With the results of Meng et al. (2009), we calculate the color excesses of SNe Ia at maximum light via a simple analytic method. We reproduces the distribution of color excesses of SNe Ia by our binary population synthesis approach if the velocity of the optically thick wind is taken to be of order of magnitude of 10 km s⁻¹. However, if the wind velocity is larger than 100 km s⁻¹, the reproduction is bad.

Key words: Stars: white dwarfs - stars: supernova: general

1 INTRODUCTION

Although type Ia supernovae (SNe Ia) showed their importance in determining cosmological parameters, e.g. Ω_M and Ω_Λ (Riess et al. 1998, Perlmutter et al. 1999), the progenitor systems of SNe Ia have not been confidently identified yet (Hillebrandt & Niemeyer 2000; Leibundgut 2000). It is widely believed that a SN Ia is from thermonuclear runaway of a carbon-oxygen white dwarf (CO WD) in a binary system. The CO WD accretes material from its companion to increase its mass. When its mass reaches its maximum stable mass, it explodes as a thermonuclear runaway and almost half of the WD mass is converted into radioactive nickel-56 (Branch 2004). Two basic scenarios have been discussed over the last three decades. One is a single degenerate (SD) model, which is widely accepted (Whelan & Iben 1973). In this model, a CO WD increases its mass by accreting hydrogen- or helium-rich matter from its companion, and explodes when its mass approaches the Chandrasekhar mass limit. The companion may be a main-sequence star (WD+MS) or a red-giant star (WD+RG) (Yungelson et al. 1995, Li & van den Heuvel 1997, Hachisu et al. 1999a, Hachisu et al. 1999b, Nomoto et al. 1999, 2003).
Evidence for CSM was first found in SN2002ic (Hamuy et al. 2003), which has shown extremely pronounced hydrogen emission lines that have been interpreted as a sign of strong interaction between supernova ejecta and CSM. The discovery of SN2002ic may uphold the SD model (Han & Podsiadlowski 2006). Recently, the evidence for CSM was found in a normal SN Ia (SN 2006X) defined by Branch, Fisher & Nugent (1993) and the CSM is proposed to be from a wind from a red-giant companion (Patat et al. 2007a). The CSM may play a key role to solve the problem of the low value of reddening ratio of external galaxy (Wang 2005), which is very important for precision cosmology (Wang et al. 2008).

If a SN Ia is surrounded by a large amount of CSM, its color observed should be redder than its intrinsic color, which results in a color excess, \( E(B - V) \). Reindl et al. (2005) showed the color excesses of more than one hundred SNe Ia at maximum light, which suggests a mission to check which progenitor model of SNe Ia can explain the distribution of the color excesses. Recently, Meng et al. (2009) performed binary stellar evolution calculations for more than 25,000 close WD binary systems with various metallicities, and present all the parameters of the systems for SNe Ia before the Roche lobe overflow (RLOF) and at the moment of supernova explosion in a following paper (Meng & Yang 2009). In their works, the prescription of Hachisu et al. (1999a) for the accretion efficiency of hydrogen-rich material was adopted by assuming an optically thick wind (Hachisu et al. 1996). Adopting the prescription of Hachisu et al. (1999a) on WDs accreting hydrogen-rich material from their companions, Meng et al. (2009) obtained the initial parameters of WD + MS systems for SNe Ia, and at the same time, the final parameters at the moment of SN Ia explosion, such as \( M_{SN} \). In this paper, incorporating the binary evolution results in Meng et al. (2009) into the rapid binary evolution code developed by Hurley et al. (2000, 2002), we carry out a series of binary population synthesis (BPS) studies for various Z. In each BPS study, \( 10^7 \) binaries are generated by a Monte Carlo simulation and a circular orbit is assumed for all binaries. The basic parameters for the simulations are same to those in Meng et al. (2009).

2 MODEL AND PHYSICS INPUTS

2.1 the mass distribution of lost hydrogen-rich material

As described in section 1, the color of a supernova is reddened by the CSM surrounding the supernova. We first check whether there exists enough CSM resulting from optically thick wind. The following is a simple description about how to obtain the CSM. As described in Meng et al. (2009), in a WD + MS system, the companion fills its Roche lobe at MS or during HG, and transfers material onto the WD. If the mass-transfer rate, \( |\dot{M}_2| \), exceeds a critical value, \( \dot{M}_{cr} \), we assume that the accreted hydrogen steadily burns on the surface of WD and that the hydrogen-rich material is converted into helium at the rate of \( \dot{M}_{cr} \). The unprocessed matter is assumed to be lost from the system as an optically thick wind at a rate of \( \dot{M}_{wind} = |\dot{M}_2| - \dot{M}_{cr} \) (Hachisu et al. 1996). Adopting the prescription of Hachisu et al. (1999a) on WDs accreting hydrogen-rich material from their companions, Meng et al. (2009) obtained the initial parameters of WD + MS systems for SNe Ia, and at the same time, the final parameters at the moment of SN Ia explosion, such as \( M_{SN} \). In this paper, incorporating the binary evolution results in Meng et al. (2009) into the rapid binary evolution code developed by Hurley et al. (2000, 2002), we carry out a series of binary population synthesis (BPS) studies for various Z. In each BPS study, \( 10^7 \) binaries are generated by Monte Carlo simulation and a circular orbit is assumed for all binaries. The basic parameters for the simulations are same to those in Meng et al. (2009).
Fig. 1 The mass distribution of hydrogen-rich material lost as optically thick wind for different metallicities.

and the TPAGB channel (see Meng et al. 2009 for details). We assume that a SN Ia is produced if the initial parameters of a WD + MS system, e.g. initial orbital period $P_{\text{orb}}^i$ and initial secondary mass $M_2^i$, locate in the appropriate regions of the parameters for SNe Ia at the onset of RLOF. We can obtain the companion mass at the moment of explosion by interpolation in the three-dimensional grid $(M_{\text{WD}}^i, M_2^i, \log P_i)$ of the more than 25,000 close WD binary system calculated in Meng et al. (2009).

In Fig. 1 we show the mass distribution of hydrogen-rich material lost as the optically thick wind for various metallicities, $M_{\text{wind}} = (M_2^i + M_{\text{WD}}^i) - (M_2^{\text{SN}} + M_{\text{WD}}^{\text{SN}})$, where superscript $i$ and SN represent the initial and final values for white dwarf and secondary, respectively, and $M_{\text{WD}}^{\text{SN}}$ is assumed to be $1.378M_\odot$. We can see from the figure that the distribution of the lost mass peaks at about $0.3M_\odot$ and has a high-mass tail. The amount of the lost material may be as large as $2.5M_\odot$, which should contribute to the color excess of SNe Ia.

In Fig. 1 we can see that there does not seem to be a systemic trend with metallicity. Actually, the influence of metallicity on the $M_{\text{wind}}$ is complicated. $M_{\text{wind}}$ is mainly determined by $M_2^i$ and $M_{\text{WD}}^i$. The two parameters are both systemically affected by metallicity, but the tendency is reversed, i.e. the peak of the distribution of companion move to higher mass with metallicity, while the peak for WD mass to lower mass (see Figs. 9 and 10 in Meng et al. 2009). In addition, the metallicity also affect the mass-transfer rate between WD and its companion, and then $M_{\text{WD}}^{\text{SN}}$ (Langer et al. 2000). The complicated influence of metallicity on the distribution of $M_{\text{wind}}$ results in a non-systemic trend of the distribution with metallicity. We also noticed that the percentage of high $M_{\text{wind}}$, e.g. $M_{\text{wind}} > 2.0M_\odot$, increases with metallicity. High $M_{\text{wind}}$ is mainly determined by the upper boundary of the companion mass, which moves to higher mass with metallicity (see Fig. 4 in Meng et al. 2009). So, a binary system producing SN Ia with high metallicity may lose more hydrogen-rich material by optically thick wind, and then the high-metallicity model in Fig. 1 shows a higher percentage of high $M_{\text{wind}}$. 
Fig. 2 The distribution of color excesses of SNe Ia at maximum light for various metallicities, where wind velocity is assumed to be 10 km s$^{-1}$. The solid histogram is from observation, and the bar represents its maximum error (Reindl et al. 2005). The black dashed line is the sum of those of $Z=0.01, 0.02$ and $0.03$, where the weights for the three components are 20%, 40% and 40% respectively. Via K-S test, the percentages indicated in the figure show the confident level that the distributions of color excess from theory and observation are indistinguishable.

2.2 model
Bohlin et al. (1978) found

$$E(B - V)/n_H = 1.72 \times 10^{-22} \text{mag cm}^2,$$

for the Galaxy, where $n_H$ is total hydrogen column density and this relation linearly depends on metallicity (Draine 2003). The linear relation can be fitted by

$$E(B - V)/n_H = (17.4 \times Z/Z_\odot - 0.454) \times 10^{-23} \text{mag cm}^2,$$

where the fitting data are from Bohlin et al. (1978), Koornneef (1982), Fitzpatrick (1985) and Martin et al. (1989). In the following, we describe how to obtain the total hydrogen column density, $n_H$.

While the CSM around the progenitor of a SN Ia may be asymmetric, for simplicity, we assume that the mass loss of optically thick wind is spherically symmetric and the mass-loss rate is constant during the whole mass-transfer phase. Then, hydrogen number density is only a function of the distance of wind material from the progenitor star, i.e.

$$n(r) = ar^{-2},$$

where $a$ is a coefficient determined by

$$\int_{R_0}^{R_m} ar^{-2} \cdot 4\pi r^2 dr = N_H,$$
where \( r_0 \) is the radius of progenitor, \( r_m \) is the maximum distance which hydrogen-rich material can arrive as optically thick wind at the moment of SN Ia explosion, and \( N_H \) is the total number of hydrogen atom. The total hydrogen column density at maximum light is calculated from

\[
\int_{r_1}^{r_m} ar^{-2} \, dr = n_H, \tag{5}
\]

where \( r_1 \) is the distance of explosion ejecta from explosion center at maximum light. From equations (4) and (5), we can obtain

\[
n_H = \frac{N_H}{4\pi r_m r_1}. \tag{6}
\]

\( N_H \) can be obtained from

\[
N_H = \frac{X M_{\text{wind}}}{n_H}, \tag{7}
\]

where \( M_{\text{wind}} \) is the total mass lost as optically thick wind, \( m_H \) is the mass of a hydrogen atom and \( X \) is the mass fraction of hydrogen and is set to be

\[
X = 0.76 - 3.0Z, \tag{8}
\]

(Pols et al. 1998). \( r_m \) can be obtained from

\[
r_m = V_{\text{wind}} t_d, \tag{9}
\]

where \( V_{\text{wind}} \) is the velocity of the optically thick wind, and \( t_d \) is the delayed time from the onset of mass transfer to the moment of SN Ia explosion. From the results of Meng et al. (2009), \( t_d \) can be approximated by

\[
\log(t_d/\text{yr}) = -\frac{2}{3} M_i^2 + 7.8, \tag{10}
\]

where \( M_i^2 \) is the initial mass of the secondary (the mass donor unit in solar mass) in a WD + MS system. \( t_d \) from the equation is a rough estimation of the mean value for a certain \( M_i^2 \) and has an error of about 50%. \( r_1 \), the distance of explosion ejecta from explosion center at maximum light, can be obtained by the product of the velocity of supernova ejecta and the rise time of light curve of SNe Ia. We simply assume that the velocity of ejecta is 10000 km s\(^{-1}\) (Gamezo et al. 2003) and the rise time of light curve of SNe Ia is 20 day (Conley et al. 2006; Strovink 2007). The ejecta velocity adopted here corresponds to a typical photospheric velocity (Wang et al. 2003) and might be lower than terminal ejecta velocity (Wang 2006). However, the uncertainty resulting from the ejecta velocity is moderate and accepted. We will discuss its influence in section 4. The rise time does not significantly affect the final results.

Meng et al. (2009) performed binary stellar evolution calculations for more than 25,000 close WD binary systems with various metallicities and Meng & Yang (2009) presented the distribution of all the parameters for these close systems before the RLOF and at the moment of SN Ia explosion. Incorporating their results into the binary population synthesis code of Hurley et al. (2000, 2002), we obtain the distribution of the wind mass, \( M_{\text{wind}} \) (see Fig. 1) and the color excess via equations 1 and 6. The basic parameters for Monte Carlo simulations are same to that in Meng et al. (2009) when primordial binary samples are generated. Because the code is valid just for \( Z \leq 0.03 \), only five metallicities (i.e. \( Z = 0.03, 0.02, 0.01, 0.004 \) and 0.001) are examined here.

The greatest uncertainty of our model is from \( V_{\text{wind}} \). Here, we assume that \( V_{\text{wind}} = 10 \) km s\(^{-1}\) and we will discuss it in section 4.
3 RESULTS

In Fig. 2 we show the distribution of the color excesses of SNe Ia at maximum light for various metallicities. The distribution from observation is also shown by solid histogram in the figure (Reindl et al. 2005). We see in the figure that the SD model with optically thick wind can reproduce the distribution of color excesses of SNe Ia observed. The K-S test shows that the cases of $Z = 0.02$ and $Z = 0.03$ have the highest confident level that the distributions from theory and observation are indistinguishable. The sample of Reindl et al. (2005) includes the SNe Ia with various metallicities. However, it is difficult to determine the metallicity of the host galaxy of every SN Ia for a large distance (Hamuy et al. 2000). Wang et al. (2006) collected the properties of several SNe Ia from previous papers and noticed that the mean metallicity of the host galaxies of the SNe Ia is $[12 + \log(O/H)]_{\text{mean}} = 8.85 \pm 0.10$ (private communication), which is consistent with solar metallicity $([12 + \log(O/H)]_{\text{solar}} \sim 8.8$ (Zaritsky et al. 1994). Then, our result is consistent with observation although the sample collected by Wang et al. (2006) is small.

There are some SNe Ia whose color excesses are very large (i.e. larger than 1.0), such as SN1999cl (Jha 2002; Reindl et al. 2005), SN2003cg (Elias-Rosa et al. 2006) and SN2006X (Wang et al. 2008). In the sample of Reindl et al. (2005), only one among 113 SNe Ia has a color excess larger than 1. In our simulation, the possibility of the high-color-excess SNe Ia is 2.0% for $Z = 0.03$, 0.4% for $Z = 0.02$ and 0 for $Z < 0.02$, consistent with that of the observations. This result might imply that the SNe Ia may have a color excess larger than 1 only when their host galaxies have a metallicity larger than 0.02.
Observationally, the host galaxies of SN1999cl and SN2006X, i.e. NGC4501 and NGC4321, are both oversolar galaxy (Caputo et al. 2000; Dors Jr & Copetti 2006). Although there is no information about the metallicity of the host galaxy of SN2003cg (NGC3169), NGC3169 is a Sa galaxy and probable has a oversolar metallicity (Willner et al. 1985).

4 DISCUSSION

Our analytic model is so simple. We discuss various uncertainties about our model in this section.

4.1 wind velocity

The major uncertainty of our model is from the assumption that \( V_{\text{wind}} = 10 \text{ km s}^{-1} \). Although the consistency between the theoretical distribution of color excess and that from observation upholds this assumption, there is no direct observational evidence to verify it. Many observational efforts were projected to find CSM (Hamuy et al. 2003; Aldering et al. 2006; Panagia et al. 2006; Ofek et al. 2007; Patat et al. 2007a,b), and only one observation obtained the constraint of the wind velocity (Patat et al. 2007a). The wind velocity is constrained to be smaller than 50 km s\(^{-1}\) (see Fig. 2 in Patat et al. 2007a), and then Patat et al. suggested that the progenitor of SN2006X should be a WD+RG system. However, we can not rule out the possibility that the observed CSM is from the optically thick wind since the companion has not been directly observed (Hachisu et al. 2008). Recently, Badenes et al. (2007) explored the relationship between the SD models with optically thick wind for SNe Ia and the properties of the supernova remnants that evolve after the explosion. They found that the optically thick wind with velocity larger than 200 km s\(^{-1}\) would excavate large low-density cavities around the progenitors. The large cavities are incompatible with the dynamics of the forward shock and the X-ray emission from the shocked ejecta in all the SNe Ia remnants as they examined. However, they also showed that if a wind velocity of 10 km s\(^{-1}\) is adopted, the properties of type Ia supernova remnants are well compatible with the prediction from the SD model with optically thick wind. Generally, the escape velocity from a white dwarf is in an order of magnitude of \( 10^3 \text{ km s}^{-1} \), which is upheld by observations from recurrent novae (Wood & Lockley 2000). In the theoretically framework laid down by Hachisu et al. (1996), an optically thick wind is formed in CO WD envelope with photospheric velocity of \( \sim 10^3 \text{ km s}^{-1} \) (Hachisu et al. 1999a,b), where the expansion of the photosphere is driven by helium flash in a helium shell on top of CO WD (Kato & Hachisu 1999). Then, in both theory and observation, it seems not to uphold an assumption of a low wind velocity at present. We check the influence of wind velocity on the distribution of color excess for the case of \( Z = 0.02 \) and the results are shown in Fig. 3. We see in the figure that when \( V \geq 100 \text{ km s}^{-1} \), K-S test gives a low confident level that the distributions of color excess from theory and observation are indistinguishable, while when \( V \leq 50 \text{ km s}^{-1} \), K-S test shows an acceptable level. This result is similar to that in Badenes et al. (2007).

4.2 delay time

The secondary uncertainty of our model is from \( t_d \). Since optically thick wind may stop before SN Ia explosion, \( t_d \) used in this paper overestimates the delayed time of the wind for some systems. However, this is not a serious problem since for most cases, SNe Ia occur during the wind phase or after the wind phase for a short time (Han & Podsiadlowski 2004; Meng et al. 2009). In addition, \( t_d \) obtained from equation (10) only approximates the mean value for a given \( M_i^2 \). This is also not a serious problem since we only check an average distribution of color excess. Generally, the peak of the distribution of color excesses moves to lower color excess with \( t_d \), and the peak value increases while the percentage of high color-excess SNe Ia decreases with \( t_d \).
4.3 metallicity

The combination of different metallicities is checked and shown in Fig. 2 by the black dashed line since not all SNe Ia are from a unique metallicity. The weights of the components for $Z=0.01$, $0.02$ and $0.03$ are simply fitted from the samples of Hachisu et al. (2000) and Wang et al. (2006). We also show the best fitted line in Fig. 2 by the black dashed line. We can see from these lines that the combination of different metallicities does not significantly increase the confident level that the distributions for theory and observation are indistinguishable. Actually, for any combination of $Z = 0.01$, $0.02$ and $0.03$, the results will be acceptable, i.e. yielding similar K-S test probability, which is derived from similar K-S test probability for the distributions of $Z = 0.01$, $0.02$ and $0.03$. So, the basic results here still hold.

4.4 interstellar extinction

While we are not sure about the existence of the optically thick wind (it is a prediction by a model, not an observed fact, Hachisu et al. 1990), we are absolutely sure about the existence of the interstellar extinction (IE). In the paper, we did not consider the influence of IE since it is difficult to separate it from CSM dust. However, we know that Type Ia SNe, as any other celestial object, suffer from interstellar reddening, arising in material that has nothing to do with the circumstellar environment of the exploding star. Let’s consider the case of SN 2006X, whose color excess is certainly larger than $E(B-V)=1$, while is most likely arising in a cold molecular cloud which has nothing to do with the explosion site (Wang et al. 2008). So, for the case suffering interstellar reddening, we should remove the influence of IE on color excess. But at present, it is very difficult to do this. However, it is a clear effect that IE will lead to certain directions of where parameters such as $V_{\text{Wind}}$ and $V_{\text{ej}}$ should go, e.g. moving some very reddened observed SNe Ia to lowering reddened one would yield a higher $V_{\text{Wind}}$ or $V_{\text{ej}}$. For example, if we assume rather arbitrarily and simply that all observed SNe Ia suffer a extinction of $E(B-V)=0.1$ within its host galaxy, our model suggests $V_{\text{Wind}} = 20$ km s$^{-1}$ or $V_{\text{ej}} = 20000$ km s$^{-1}$. Similarly, if $E(B-V)_{\text{host}} = 0.2$, our model suggests $V_{\text{Wind}} = 50$ km s$^{-1}$ or $V_{\text{ej}} = 50000$ km s$^{-1}$. So, the effect of IE could be counteracted by another uncertainties, $V_{\text{ej}}$, and the basic results in this paper are still valid. (see the discussion about ejecta velocity below).

If the color excess is from CSM as shown in this paper, one may argued that we would have seen the CSM material through radio and X-ray emission, arising in the shock produced by the fast moving SN ejecta crashing into the CSM (Stockdale et al. 2006). However, Panagia et al. (2006) showed a non-detection result for radio for 27 SNe Ia via the Very Large Array (VLA) observations, even including SN 2002ic, which indicated that mass-lose rate should be lower than $\sim 3 \times 10^{-8}$ $M_\odot$ yr$^{-1}$. We take a typical case to check whether our model contradicts with observations. We set companion mass to $2.1 M_\odot$ (see Fig. 10 in Meng et al. 2009), which corresponds to a delay time of $2.5 \times 10^6$ yr. The mass of lost material is set to $0.3 M_\odot$ (see Fig. 1). The mean mass-lose rate is $12 \times 10^{-8}$ $M_\odot$ yr$^{-1}$, which is higher but still comparable to that inferred from observations. Attempts to detect radio emission from SN 2002ic with the VLA were unsuccessful (Berger et al. 2003; Stockdale et al. 2003). SN 2002ic is the first case to show the signal of CSM (Hamuy et al. 2003) and the amount of the CSM is $0.5-6 M_\odot$ (Wang et al. 2004; Chugai & Yungelson 2004; Uenishi et al. 2004; Kotak et al. 2004) and its properties may be explained successful by the WD+MS model used here (Han & Podsiadlowski 2006). In addition, another recent twins of SN 2002ic (SN 2005gj, Aldering et al. 2006) has also not been detected using the VLA (Soderberg & Frail 2005). These non-detection results may indicate that the mechanism that is successful used for SN Ib/c may not work for SNe Ia (Panagia et al. 2006). So, our model is at least not inconsistent with observation at present.

4.5 ionization and ejecta velocity

Generally, when a SN explodes, its radiation field is quite strong, especially at distance $r_H < 10^{16}$ cm (for SN 2006X, $r_H < 4 \times 10^{15}$, Patat et al. 2007a). This radiation can easily ionize hydrogen and
evaporate dust up to quite large distances, causing the disappearance of reddening. In our simple model, this effect is not considered completely. We check the influence of the effect.

We set $r_H = 10^{16}$ cm and assume that all hydrogen atoms are ionized within the shell with radius of $r_H$. The column density of ionized hydrogen is $n_{H,0} = \int_{r_1}^{r_H} n(r) r \, dr$, where $r_1 = 1.73 \times 10^{15}$ cm (a ejecta velocity of $10^4$ km s$^{-1}$ and a rise time of 20 day are assumed). Then, the relative uncertainty from the ionization effect is $\frac{n_{H,0}}{n_{H}} = \frac{(r_{H,0} - r_1) r_{H,0}}{(r_{m} - r_1) r_{H,0}} \approx \frac{r_{H,0} - r_1}{r_{H,0}} \approx 0.83$ ($r_m \gg r_1$). The uncertainty seems too large. However, this situation may be improved by taking a higher ejecta velocity and a smaller distance $r_H$. A distance of $r_H = 4 \times 10^{15}$ like SN 2006X may reduce the uncertainty to 0.57.

The ejecta velocity used here may be smaller than the terminal velocity of SN Ia which can be as high as $3 \times 10^4$ km s$^{-1}$ (Wang 2006). This value may reduce the uncertainty from the ionization effect to 0 $\sim$ 0.48, which means that no more than a half of CSM are evaporated up to large distances and have no influence on the color excess of SNe Ia. Then, the ionization effect at most decreases the confident level that theory and observation are indistinguishable from 74% to 65%.

In Fig. 4 we show the influence of ejecta velocity on the distribution of the color excess of SNe Ia. The influence is similar to that of wind velocity. From the figure, we can see that the peak of the distribution increases with the ejecta velocity, and when $V_{\text{ejec}} < 5 \times 10^4$ km s$^{-1}$, the distributions of color excess from theory and observation are indistinguishable on the confident level of higher than 50%. Fortunately, the effect of ionization and a higher ejecta velocity is same to that of interstellar, and their influence on the decrease of the confident level may counteract each other. Then, we can say that the distribution of color excess between theory and observation are indistinguishable on the level of higher than 59%, even when $V_{\text{ejec}} = 3 \times 10^4$ km s$^{-1}$.
For the discussion above, our model may get a meaning result although our analytic model is so simple.

5 SUMMARY AND CONCLUSIONS

In summary, if a wind velocity of \( \sim 10 \text{ km s}^{-1} \) is adopted, the SD model with optically thick wind may reproduce the distribution of color excess of SNe Ia obtained from observation, which might support the SD model for SNe Ia. However, if a wind velocity larger than 100 km s\(^{-1}\) is adopted, the reproduction is bad. Our results are similar to those of Badenes et al. (2007). Then, it should be encouraged to perform more detailed observations about the velocity of CSM around progenitors of SNe Ia.

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Appendix A: THIS SHOWS THE USE OF APPENDIX

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