A study of the heat parameters during bidirectional solidification

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Abstract

The temperature field and heat parameters are important in controlling metal liquid crystallinity in unidirectional and bidirectional solidification. The temperature field can be divided into three cases: a liquid temperature field; solid temperature field; and a temperature field on the solid–liquid (S–L) interface. Heat parameters can be divided into two cases: technical heat parameters; and solidification heat parameters. The temperature field on the S–L interface and solidification heat parameters are the most important for the structures and properties of materials. The temperature field on the S–L interface is determined by the alloy system, and solidification heat parameters are related to the temperature field of the environment and technical heat parameters. The temperature field on the S–L interface is closely related to the solidification heat parameters.

A theoretical model describing precisely the temperature field on the S–L interface during bidirectional solidification was proposed. A series of heat parameters, including temperature gradients $G$, solidification rate $R$, cooling velocity $V$ and characteristic temperature $T_c$, have been derived from this model. A superalloy has been chosen as the experimental object in order to verify the theoretical model. The theoretical calculations are found to be in agreement with the experimental results. © 2001 Published by Elsevier Science Ltd.

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1. Introduction

There are three cases of variations in the temperature field during bidirectional solidification (BDS): variation of liquid metal; variation of solid metal; and variation of the solid–liquid (S–L) interface.

Being continuous mediums and single phase, liquid and solid states of a metal accord with the law of thermal conduction [1]. On the S–L interface, it is very difficult to establish a thermal conduction equation, since a liquid transforms into a solid and there is release of the latent heat of fusion.

The purpose of this paper is to study the law of the temperature field on the interface during BDS, and to evaluate a chain of the following heat parameters: temperature gradients $G$, solidification rate $R$, cooling velocity $V$ and characteristic temperature $T_c$.

2. Theoretical model

Suppose that the sample of BDS is a disc, and that the direction of solidification is from the perimeter to the centre. The disc is horizontally axisymmetric so that the effect of $\theta$ angle will be slight. Suppose that the solidification time is $\tau$, the distance of a solidified disc at zero solidification time is zero and at $\tau$ s is $r$, and that the temperature of the metal on the interface varies with $\tau$ and $r$. Temperature $T$ is then a function of solidification time $\tau$ and radii $r$, namely $T = T(r, \tau)$.

According to the physical senses of $G$ and $V$

\[
\frac{\partial T(r, \tau)}{\partial r} \bigg|_{\tau = \tau_0} = G,
\]

\[
\frac{\partial T(r, \tau)}{\partial \tau} \bigg|_{r = r_0} = V.
\]

Suppose the partial derivatives of $T(r, \tau)$ with respect to $\tau$ and $r$ are existent and continuous, then:

\[
\frac{\partial^2 T(r, \tau)}{\partial \tau \partial r} = \frac{\partial^2 T(r, \tau)}{\partial r \partial \tau}.
\]

Applying Eq. (1) and Eqs. (2) and (3)

\[
\frac{\partial G}{\partial \tau} = \frac{\partial V}{\partial r}.
\]

Studying the problem on the S–L interface, the relation of
the solidification heat parameters can thus be used:

\[ V = -GR. \]  

(5)

The minus sign indicates the direction of \( V \) as opposed to that of \( R \).

\( R \) remains constant within a small range. Substituting Eq. (5) for Eq. (4), then:

\[ \frac{\partial V}{\partial r} = -R \frac{\partial G}{\partial r} \]  

(6)

thus:

\[ \frac{\partial G}{\partial \tau} = -R \frac{\partial G}{\partial r} \]  

(7)

or:

\[ \frac{\partial^2 T}{\partial \tau \partial r} = -R \frac{\partial^2 T}{\partial r^2}. \]  

(8)

Eq. (8) is the equation of the temperature field of the S–L interface. The solutions of Eq. (8) [2] are:

\[ T = a_1 e^{a_2 r - a_3 \tau} + a_4, \]  

(9)

\[ a_1 = K \frac{R}{\lambda}, \]  

(10)

\[ a_2 = \frac{\lambda}{R}, \]  

(11)

\[ a_3 = -\lambda, \]  

(12)

\[ a_4 = C, \]  

(13)

where \( a_1, a_2, a_3 \) and \( a_4 \) are the coefficients of the temperature field and \( K, \lambda \) and \( C \) are constants.

3. Experiments and verification of the model

Thermocouples are located at different places of the mould. The temperature values of different places within

the metallic liquid within the mould at various times are recorded automatically.

A superalloy based on nickel is used, resembled PWA1422 alloy [3], of which the temperature values of the S–L interface are about 1307–1366°C [4]. Considering a definite cooling velocity and super-cooling, the temperature values of the S–L interface are actually about 1260–1330°C. By means of BDS equipment, a series of experimental data of temperature \( T \), time \( \tau \) and radii \( r \) between liquid and solid phase points are measured. By means of a computer [5], the coefficients of the temperature field \( a_1, a_2, a_3 \) and \( a_4 \) are determined.

Table 1 shows the experimental and the theoretical model values.

The difference between the experimental and the theoretical model values is about 1°C.

A least-squares surface method [6] was used in contrast to the present theoretical model for further testing of the model. The computer-generated equation is:

\[ T(r, \tau) = \sum_{i=0}^{5} \sum_{j=0}^{5} a_{ij}(r - \bar{r})^i(\tau - \bar{\tau})^j, \]  

(14)

where \( r \) and \( \tau \) are mean values of solidification distance and time, respectively, and \( a_{ij} \) are the coefficients of Eq. (14).

The difference between values calculated using Eq. (14) and the experimental values is about 15°C.

Thus it is clear that the theoretical model (9) of the temperature field is more suitable for describing the temperature field on the interface during BDS.

4. Determination of the heat parameters of the temperature field

It had always been very difficult to the determine heat parameters of BDS before a suitable theoretical model was set up. Now, the solidification heat parameters can be
determined accurately according to Eq. (9):

\[
G = \frac{\partial T(r, \tau)}{\partial r} = a_1 a_2 \varepsilon^{a_2 r + a_3 \tau},
\]

\[
V = \frac{\partial T(r, \tau)}{\partial \tau} = a_1 a_3 \varepsilon^{a_2 r + a_3 \tau},
\]

\[
R = \frac{a_3}{a_2}.
\]

Discussing the problem of the S–L interface, then:

\[
r = R \tau,
\]

i.e:

\[
a_2 r + a_3 \tau = 0.
\]

Thus Eqs. (15) and (16) can be simplified to:

\[
G = a_1 a_2,
\]

\[
V = a_1 a_3,
\]

The characteristic temperature on the interface \( T_c \) is:

\[
T_c = T|_{r=R} = a_1 + a_4.
\]

\( T_c \) is closely related to the initial stage temperature \( T_0 \), which can be controlled. Generally, when crystal grows bidirectionally, \( T_c \) is \( T_0 \)

\[
T_0 = T|_{\tau=0} = a_1 + a_4.
\]

In this case \( T_c \) indicates the temperature condition required for a crystal to grow bidirectionally.

If the coefficients of Eq. (9), \( a_1, a_2, a_3, \) and \( a_4 \), are explored, a series of heat parameters on the interface, such as \( G, V, R \) and \( T_c \), can be determined by use of Eqs. (17), (20)–(22).

Table 2 shows the values of the coefficients \( a_1, a_2, a_3 \) and \( a_4 \) and the heat parameters of the alloys.

As seen in Table 2, under the present experimental conditions most of the temperature gradients \( G \) are from 1.4 to 6.1°C/mm, cooling velocities \( V \) are from 0.18 to 0.38°C/s, solidification rates \( R \) are from 0.057 to 0.067 mm/s and when the growth of crystal is bidirectional, the characteristic
temperature $T_c$ values are all within the range between liquid and solid phase points (from 1260 to 1330°C).

It is necessary to point out that heat parameters are different under different conditions, but they are fixed when the conditions are fixed.

5. Form and structure of bidirectional solidification

BDS is an advanced technique [7–9]. Samples of the superalloys have been developed with BDS, possessing the following structure:

1. Macrostructure. A BDS sample of the superalloy is made up of a series of regular and radialized columnar crystals (Fig. 1), within which there is a series of fibres along the direction of columnar crystal.

2. Microstructure. The fibres within the columnar crystal are radialized crystal microaxes (Fig. 2), which are arranged regularly along the radius and together with the columnar crystal can improve the properties of the alloy, such as impact toughness, etc. and, above all, the high temperature properties.

6. Conclusions

1. The temperature field on the S–L interface during BDS can be described with the following partial differential equation:

$$\frac{\partial G}{\partial \tau} = -R \frac{\partial G}{\partial r}$$

2. The solution of the partial differential equation is the equation of temperature field on the S–L interface:

$$T = a_1 e^{\alpha r^2} + a_2,$$

where $a_1$, $a_2$, $a_3$ and $a_4$ are coefficients that are determined by the results of experiment.

3. Solidification heat parameters on the S–L interface can be settled by the temperature field equation. The relationships between heat parameters and temperature field are:

$$G = a_1 a_2,$$

$$V = a_1 a_3,$$

$$T_c = a_1 + a_2,$$

$$R = \frac{a_3}{a_2}.$$

4. The theoretical calculations of the temperature field are in agreement with the experimental results. The difference between the experimental values and the theoretical model values is about 1°C.

5. In the present experiments, most of $G$ values are from 1.4 to 6.1°C/mm, the $R$ values are from 0.057 to 0.67 mm/s, and the $V$ values are from 0.18 to 0.38°C/s, and when a crystal grows bidirectionally, the $T_c$ values are all within the temperature range between the liquid and solid phase points of the superalloy based on nickel.

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