Model for the Optimal Control of the Cross-Lapper Drive

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Received 7 July 1998; received in revised form 18 December 1998

Abstract

This paper presents the mathematical model of a cross-lapper elaborated to satisfy the needs of computer simulation of its motion. An algorithm for calculating the optimal driving moment profile to provide motion according to the given functions of velocity and acceleration has been described. Some results of the calculations are shown on graphs.

1. Introduction

A cross-lapper is a textile machine for producing uniform layers of fibers having suitable width and thickness. Suitable thickness is obtained by folding the card web many times, one on top of the other, by moving the collecting conveyor belt crosswise to the folding (see Fig. 1). A cross-lapper cooperates with a carding set cleaning, opening and blending fibers to produce a card web. A layer of fibers (web) made by a cross-lapper is next stitched and this way non-woven material is manufactured.

The cross-lapper takes off a card web on its conveyor belt. Two carriages making to-and-fro movements in opposite direction fold the card web on the collecting conveyor belt. The machine has two drives:
• reversal drive, which moves the carriages,
• belt conveyor drive, which moves the belts of the conveyors forward.

Pneumatic cylinders provide adequate tension in the conveyor belts as also do in toothed driving belts (see Fig. 1 and Fig. 2).

Securing proper control of the reversal drive is a major problem in construction of the machine. The carriages moved by a servo-motor must stop themselves very precisely at their dead points because the edges of the web must lie in a straight line. On the other hand the reverse phase of motion should be as quick as possible. The velocity of a card web supply is constant in time while the carriages stop themselves for a very short moment at their dead points, which may cause unwanted concentration of fibers along the edges of the web. In the construction the dynamic properties of the machine should also be taken into consideration.

The cross-lapper drives frequently use microprocessors. This makes it possible to find a drive control strategy that both satisfies requirements of the textile process engineering and limits vibration of the machine as well as decreasing its dynamic overloads.

2. Calculation model

The calculation model will be used further in solving the problem of the drive control, so it should be as simple as possible. However it must take into account the most important properties of the machine. According to the needs of the analysis the model shown in Fig. 2 is proposed.

The model has four degrees of freedom: $x_1$ is the displacement of the carriage, $x_2$ is the displacement of the piston in the cylinder, $\varphi_1$ is the angle of rotation of the reversal drive wheel, $\varphi_2$ is the angle of rotation of the upper carriage. The conveyor belts are treated as non-flexible in the model because it has been proved in static stretching tests that the driving belts are about 20 times more flexible than the conveyor belt.

Equations of motion may be obtained from the second form of Lagrange’s equations:

$$\frac{d}{dt} \left( \frac{\partial E}{\partial q} \right) + \frac{\partial D}{\partial q} + \frac{\partial V}{\partial q} = P,$$

components of potential energy $V$, dissipation function $D$ and kinetic energy $E$ may be calculated from the following expressions:
• components of potential energy $V$:
1° potential energy of elastic deformation of the segment of driving belt which is modeled by the spring having elasticity $k_1$: \[ V_1 = \frac{1}{2} \delta_1 k_1 (\varphi_1 r_0 - x_1)^2, \quad (2.1) \]
\[ \delta_1 = \begin{cases} 1 & \text{when } x_1 < \varphi_1 r_0, \\ 0 & \text{in opposite case} \end{cases} \quad k_1 = \frac{2EF}{l_1^0 - x_1}, \]

2° potential energy of elastic deformation of the segment of driving belt which is modeled by the spring having elasticity $k_2$: \[ V_2 = \frac{1}{2} \delta_2 k_2 \left( \varphi_2 r_0 + \frac{1}{2} x_1 - \varphi_1 r_0 \right)^2, \quad (2.2) \]
\[ \delta_2 = \begin{cases} 1 & \text{when } \varphi_2 r_0 + \frac{1}{2} x_1 > \varphi_1 r_0, \\ 0 & \text{in opposite case} \end{cases} \quad k_2 = \frac{2EF}{l_2^0 + \frac{1}{2} x_1}, \]

3° potential energy of elastic deformation of the segment of driving belt which is modeled by the spring having elasticity $k_3$: \[ V_3 = \frac{1}{2} \delta_3 k_3 \left( x_1 - \varphi_3 r_0 + \frac{1}{2} x_1 \right)^2, \quad (2.3) \]
\[ \delta_3 = \begin{cases} 1 & \text{when } \frac{1}{2} x_1 + x_3 > \varphi_3 r_0, \\ 0 & \text{in opposite case} \end{cases} \quad k_3 = \frac{2EF}{l_3^0 + \frac{1}{2} x_1 + x_3}, \]

where $EF$ is the rigidity of stretching of belt, $r_0$ is the radius of wheel having inertia moments $J_0^z$ and $J_1^z$, $l_1^0$ and $l_3^0$ are initial length of proper segments of belt.

Using the same expressions as (2.1) + (2.3) components of dissipation function can be calculated. If the linear model of damping is assumed, coefficients of damping may be estimated using the expression:
\[ b = \frac{Q^{-1}}{\omega} k = qk, \quad (3) \]

where $Q^{-1}$ is the non-dimensional coefficient of losses, $\omega$ is the angular frequency of excitation, $k$ is the coefficient of elasticity, $q$ is damping factor.

• components of kinetic energy $E$:

1° kinetic energy of the reversal drive:
\[ E_0 = \frac{1}{2} J_0 \dot{\varphi}_1^2, \quad (4.1) \]

2° kinetic energy of the upper carriage wheel:
\[ E_1 = \frac{1}{2} J_1 \dot{\varphi}_2^2, \quad (4.2) \]

3° kinetic energy of the carriages and rollers guiding the conveyor belts:
\[ E_2 = \frac{1}{2} m x_1^2, \quad (4.3) \]

4° kinetic energy of the piston in pneumatic cylinder:
\[ E_m = \frac{1}{2} m_p x_3^2, \quad (4.4) \]

After differentiation of equations (2) and (4), and putting the results into equation (1), the differential equations of motion (5) can be obtained.
In equations (5) the terms taking into account the influence of damping have been omitted to obtain a shortened form of notation. Including (3) they have the identical form as the terms taking into account influence of elasticity, but the coefficients $k$ should be replaced by adequate coefficients $b$; and in places of $y$, their derivatives versus time should be inserted. The remainder of the symbols have the following meaning:

$$
\begin{align*}
\dot{y}_1 &= \varphi_1 r_0 - x_1, \\
\dot{y}_2 &= \frac{1}{2} x_1 + x_2 - r_0 \varphi_2, \\
M(t) &= M_0 - c \varphi_1 - M_{\text{reb}} \text{ is the driving moment of the servo-motor diminished by resistant moment of the conveyor belt movement (it can be seen from characteristic of servo-motor Lenze -Germany used in the cross-lapper, that the driving moment } M_{\text{dev}} \text{ is a linear function of the angular velocity } \varphi_1: M_{\text{dev}} = M_0 - c \varphi_1, \text{ hence the constants } M_0 \text{ and } c \text{ may be calculated easily for a specified size of the servo-motor, } M_{\text{reb}} \text{ is the resistant moment of the conveyor belt movement),} \\
F_{\text{xc}} &= \text{is the resistant force of the carriage movement,} \\
S(t) &= 2pF_{3} \left( \ell_{3}^{0} + x_3 \right) \text{is the force in belt tensioned by pneumatic cylinder (pressure } p, \text{ area of piston } F_{3}, \ell_{3}^{0} \text{ is the initial position of piston in pneumatic cylinder; it is assumed that temperature in pneumatic cylinder is constant during motion.}
\end{align*}
$$

3. Solution

Before solving equations (5) the initial values of the variables $\varphi_1, \varphi_2, x_1$, and $x_3$ have to be calculated. This means that the static elongation of the segments of the belt after filling the pneumatic cylinder with compressed air must be found. The proper set of equations may be obtained from (5) when time-dependent terms have been neglected and assuming that $\delta_x = \delta_x^2 = \delta_x^3 = 1$. This resolves to a system of nonlinear equations which can be solved numerically. The procedure based on Brown's modification of the classic Newton's method [1] has been adopted in this paper. The results give initial conditions to solve the exact system of differential equations (5). The system (5) consists of ordinary nonlinear differential equations that can be solved using the Runge-Kutta's fourth order procedure [1]. Therefore it can be said that for given data of the model, variables (coordinates) $\varphi_1$, $\varphi_2$, $x_1$, and $x_3$ can be calculated at every moment of time if the function $M(t)$ is known.

4. Simulation of reversal motion

There are three phases during reversal motion of the carriages: I. start-up phase, i.e. from the moment the drive is switched on to the moment when steady motion is reached, II. steady motion phase, when the carriages move with constant velocities, III. reverse phase (left and right), when direction of the drive changes over.

Using the elaborated model, behavior of elements during specified phases of motion or during complete cycle of motion can be observed. Some results of numerical tests are shown bellow. Numerical data to the tests are from the 5W700 cross-lapper produced by BEFAMA-Poland.

4.1 Optimization of driving moment

The velocity profiles of the carriages should be smooth to ensure uniformity of the produced web and to prolong service life of the machine. In driving systems with microprocessors encoders are used to allow discrete measurement of the carriage position. High frequency measurements enable carriage velocity and acceleration to be obtained by numerical differentiation of the measured signals. Therefore we may require carriage motion to follow given functions of velocity and acceleration.

In this way we can answer the question: what value should the driving moment be at a given moment of time – but ensuring that the maximum drive capability is not exceeded – to warrant that the velocity and acceleration of the carriage are the same as the required value? The task of optimization of one controlling variable (driving moment) with one limitation (characteristic of drive) has been formulated considering the criterion: minimum square of deviation between the real and the required values of velocity or acceleration.

The assumed functions of velocity for start-up and steady motion phases should fulfill practical needs, i.e. a smooth passage of the machine through the phases: start-up – steady
motion and steady motion – reverse – steady motion, should be warranted. This is very advantageous from the operational and textile process engineering points of view. The functions suitable for start-up and steady motion phases can be sine or polynomial.

Table 1 Functions of velocity of start-up and steady motion used in numerical tests

| function  | start-up                  | steady motion                |
|-----------|---------------------------|------------------------------|
| sine      | \(v_i = \frac{v_0}{2} \left[1 + \sin(\pi t' - \frac{1}{2})\right]\) | \(v_i = -v_0 \sin(\frac{\pi t'}{2})\) |
| polynomial| \(v_i = v_0 (3t'^7 - 3t'^5 + t'^3)\) | \(v_i = \frac{v_0}{8} (-15t'^5 + 10t'^3 - 3t')\) |

where

\[0 \leq t' \leq 1 \quad \text{for start-up phase}\]
\[-1 \leq t' \leq 1 \quad \text{for steady motion phase}\]

The problem of optimization of the driving moment has been solved at every step of integrating of equations (5). The classic gold division method has been adopted. Based on the results of numerical tests it is asserted that the minimum square of deviation of acceleration is a better optimization criterion than the minimum square deviation of velocity (the final velocity profiles are more smooth). It is also asserted that the polynomial form of acceleration of the carriage is much more advantageous than the sine (see Fig.3). This can be explained by the condition:

\[\frac{d^2 v_i}{dt'^2} \neq 0 \quad \text{for sine}\]
\[\frac{d^2 v_i}{dt'^2} = 0 \quad \text{for polynomial}\]

Some results of computer simulation of motion for the start-up phase and for the complete motion cycle of the machine are presented further on.

Figure 3 shows comparison of profiles of the driving moments during the start-up phase for the sine and polynomial functions assuming the velocity of the carriage.

Figure 4 shows that the profiles of the driving moment depend on the angle of rotation of the reversal drive wheel and the velocity of the carriage versus its path (position of the carriage measured by encoder) during complete motion cycle of the machine.

As can be seen in Fig.4, the optimization procedure allows the profile of the driving moment to be calculated, which can provide an active damping of vibrations in the machine. Finally motion of the carriage is fluent both during the start-up phase and during the complete motion cycle of the machine.

5. Final remarks

The results of this analysis show that it is possible to elaborate an effective algorithm of the cross-lapper drive control. The algorithm provides the motion of the carriage needed from the textile process engineering point of view. In practice there are two ways in which the task can be realized:

I. a control system with a microprocessor calculates the proper driving moment using the elaborated algorithm in real time and in this manner realizes the assumed motion,

II. based on the results of many computer simulations the mean profile of the driving moment described by a set of parameters is found and this profile is applied in the control system.

The computer simulation applying the algorithm allows verification of the influence of constructional data (masses, inertia moments, elasticity of belts and conveyor belts and the like) and settings (pressure in pneumatic cylinder, duration of start-up phase, velocity of steady motion) in the motion of the machine. In this manner some of the constructional data of the 5W700 (BEFAMA-Poland) cross-lapper have been selected.

The problem described in the paper was undertaken within the grant PB 871/T07/95/09 sponsored by the Committee of Scientific Researches of Polish government.
Fig. 4 Driving moment and velocity of carriage during complete motion cycle of the machine

References

[1] A. Marciniak, D. Gregulec, J. Kaczmarek: Numerical procedures, Nakom, Poznań 1992.