The fault tree simulation modeling

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Abstract. This article explains differences in mathematical and simulation modeling approaches for solving quantitative analysis of the fault tree. With basic knowledge of reliability of complex systems and priorities in determining the phenomenon of component failure that is critical to system failure. It brings a look at fault tree solution based on statistical view and using simulation modeling with variable time step and graphical representation of events.

1. Introduction
Growing reliability and safety requirements make it necessary to provide efficient methods and tools for modeling and evaluating large technical systems. Such tools should on the one hand have solid mathematical and formal foundations while on the other hand presenting an easily understandable modeling paradigm to the user. Fault tree models are clearly structured, but unable to model some aspects of system behavior, such as multi-state components or failure and repair dependencies between different parts of the system. It seems natural to remove these drawbacks of the fault tree modeling paradigm by combining it with stochastic systems, which are especially well suited for modeling complex stochastic dependencies [1].

Stochastic analysis using mathematical modeling is performed using the probability distributions of failures of individual fault tree components. It is implemented on the principle of a program cycle with uniform time step shift. In each step of the cycle, we determine probability of failure of components and determine resulting failure probability based on deterministic relations. Obtained results give a statistical course but are still deterministic. Simulation modeling is closer to real conditions. Effects are randomly generated, outputs are variable, processed and recorded. Simulation step is subsequently repeated with new set of generated values. Repetition takes place many times (it can be hundreds of thousands or more repetitions). At the end of the process, distribution of these outputs shows the most probable values and estimates, but also their boundaries that are reasonable to expect [2, 3]. A set of mathematical and logical relations was used for presented simulation, which expresses behavior of modeled system components (with respect to the objective of this simulation). Random influences are included in the form of probabilistic characteristics, including time. The model allows repeated calculations and input data changing. Such an approach has many advantages. Most notably [4]:

- substituting real system experimentation
- applicable in cases that cannot be solved analytically
- applicable to a large number of random effects
- ability to model time,
- possibility of verifying solution obtained by another method
- better understanding of real system.
In order to use resulting gate distributions, we have to process obtained failure rate data statistically, find random distributions of variables and determine their parameters. Problems of determination of parameters of distribution of probability of failures are solved either by testing hypotheses for known distribution of probabilities from calculated summary data of experimental \(cdf\) and there is also option of using bootstrapping. For discrete simulation of series-parallel systems failure in FTA, we use principle of solving the probability of failure of components entering gates at the lowest level. The resulting gate probability is included in calculation of higher level probability up to the probability of top event failure [5].

2. **Discrete simulation with variable time step**

Fault tree failures are modeled by generating a probability distribution of time periods between individual components 1, 2 and 3 failures from failure probability distributions [6]:

- Weibull (Beta \(= 1.5\), Eta \(= 1000\)), \(\beta\) – beta is the shape parameter, \(\eta\) – eta is the scale parameter.
- Weibull (Beta \(= 1.1\), Eta \(= 800\)).
- Exponential \((\lambda = 3000)\), \(\lambda\)= constant rate.

Generated time intervals represent beginning of component failure. We keep track of failure times in the event calendar.

In case of critical system failure event from components entering OR gate, failure characteristics of n components are characterized by random vectors \((X_1, X_2 \ldots X_n)\), which represent probability of components failure. It is a series sub-system represented at each simulation step by a minimal event of \(X (s) = \min (X_1, X_2 \ldots X_n)\). This means that we select the minimal failure time from failure times of individual components. This event characterizes the timeline offset when a system failure occurs. Failure times of other components remain in the event calendar and a new failure time is generated for the component that caused system failure to be included in the event calendar. Minimal time and component that causes system failure are determined again [7, 8].

![Figure 1. Simulation of OR gate failures.](image)

Critical event from failure times of components entering AND gate from n components characterized by random vectors \((X_1, X_2 \ldots X_n)\) is represented by maximal event \(X (p) = \max (X_1, X_2 \ldots X_n)\).

This is a parallel sub-system. Which means that, we choose the maximal failure time. We move the simulation time and register system failure. Then, we generate failure times again for all components and repeat the procedure.
These two principles are used in case of analysis of series-parallel systems (figures 1 and 2).

Model of the system formed by OR and AND gates, each made up of three components, grouped in series-parallel system is created by logical rules based on the aforementioned principles. OR gate is formed by components F1, F2, F3. OR gate components are renumbered. Failure probability distributions and distribution parameters were retained F4 = F1, F5 = F2, F6 = F3 [9].

Simulation modeling takes place within a million hours of operation. Only failures that are involved in system failures are displayed. Result of the simulation shows how individual components contribute to the system failure.

As for serial components, main fraction of failures is caused by F1 and F2. Component F3 is hardly involved in failures. Obviously, parallel components F4–F6 are involved in smaller fraction of failures compared to serial components [10].

From occurrence of failures in figure 3 it is clear that such a system would not be real in the serial branch and should be optimized, which can be solved by simulation modelling [11].

3. Determination of resulting CDF of simulation model

By processing times between system failures, we can form and apply hypotheses about the type and parameters of their probability distribution. From three formed and attested hypotheses / Normal, Weibull, Exponential / Weibull probability distribution seems most suitable [12].
Figure 4. Verification of the Weibull distribution hypothesis with a quantile of 0.999.

Cumulative density of series-parallel system failures with Weibull distribution OR and AND gates with parameters $Weibull (Beta=1.4, Eta=1016)$ has similar course of probability of failures in both experimental and parametric expression as we can see in figure 4 [13].

Figure 5. Experimental $cdf$ in comparison with $cdf$ obtained by testing hypotheses up to 4 500 hours.

However, if we compare the results of mathematical and simulation modeling, we can see that the theoretical results of mathematical modeling differ from simulation modeling results (figure 5).

Figure 6. Comparison of mathematical and simulation modeling results to 0.99 percent quantile.

Resulting simulation model $cdf$ (figure 6), is shallower, comparing data after 2 000 hours. This is due, in contrast to the mathematical model, to the fact that some components are involved in failure rates only sporadically and their theoretical failure rate is not exhausted.
4. Conclusion
Discrete simulation with variable time step gives us an idea of individual components and gates failures occurrence in the formation of system failures solved by a fault tree analysis. It allows us to acquire and verify the distribution of probability and distribution parameters of system failure rate. However, \(cdf\) results obtained by time-limited simulation, show that this rate achieves more acceptable results in the initial phase. The course of the model is more closely related to the actual occurrence of failures in operation [14].

Simulation was performed depending on the time of operation. The course of the simulation shows that in unbalanced systems some components are only little involved in failure rate formation. This in practice confirms the theory of the weakest component of the system.

5. References
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