Conjugate heat transfer in the mode of thermal gravitational-capillary convection in the model of a fuel tank, after a sudden heating of the sidewall

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Abstract. The evolution of unsteady gravitational-capillary convection in a layer of ethyl alcohol with a free surface after sudden electric heating of one of the vertical walls of a rectangular cavity was investigated numerically. The effect of the incoming flow of hot liquid on the time evolution of the temperature field on the opposite thin metal wall of the cavity was investigated. The calculations were carried out by the finite element method in the conjugate two-dimensional formulation with the Prandtl number Pr = 16, and the range of Grashof numbers determined by the heat flux density, \(33 \times 10^3 \leq \text{Gr} \leq 28 \times 10^6\). It is shown that the maximum local temperature gradients occur on the wall near the liquid-gas interface.

1. Introduction

When creating aviation equipment, the tasks of simultaneously reducing the weight and increasing the reliability of thin-walled structures are solved. During take-off, landing, and the initial stages of reaching the cruising speed of the aircraft, the thermal state of the thin-walled structures significantly depends on the processes of non-stationary conjugate convective heat transfer in the fuel tanks and the air layers of the fuselage. Therefore, the problem of carrying out accurate calculations of thermal stresses in thin walls with unsteady gravitational-capillary convection and conjugate heat transfer is relevant [1]. Similar problems are typical for many technical devices in heating or cooling regimes. Thermogravitational convective flows develop in unevenly heated volumes of liquid located in the gravity field, and in the presence of a non-isothermal free liquid-gas interface, thermocapillary convection develops. This affects the poorly studied local characteristics of conjugate heat transfer and the temperature distribution in the thin wall [2, 3]. A series of works have been carried out in the S.S. Kutateladze Institute of Thermophysics SB RAS, aimed at studying the effect of conjugate natural convective heat transfer on the temperature distribution in thin walls [2-6]. The development of unsteady gravitational-capillary convection in a layer of ethyl alcohol with a free surface after sudden electric heating of one of the vertical walls of a rectangular cavity has been investigated experimentally. Using thermal imaging techniques, the development of the flow and temperature fields on the free surface of the liquid layer and the effect of the incoming flow of hot liquid on the evolution over time of the temperature field on the opposite thin wall of the cavity were investigated. Studies of nonstationary conjugate convective heat transfer in the computational domain, corresponding to the conditions of the physical experiment were carried out numerically in this paper. Convective heat
transfer from the outside of the thin wall and the free surface of the liquid was considered. The evolution of convective flows has been studied, considering the thermocapillary effect on the free surface of the liquid, and temperature fields in the liquid, gas, and the thin wall after sudden heat supply to its inner side. Numerical studies allowed supplementing significantly the experimental results with data on non-stationary temperature fields in a liquid and gas.

2. Model

When conducting the numerical simulation, nonstationary conjugate gravitational-capillary convective heat transfer was considered in the computational domain corresponding to the conditions of the physical experiment in a two-dimensional conjugate formulation using the finite element method [7]. A nonstationary dimensionless equation of thermal conductivity was used in the simulation of conductive heat transfer:

\[
\frac{\partial T}{\partial t} - \frac{1}{Pr} \frac{T}{\lambda_f} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) = 0
\]

Here \( t \) is the dimensionless time, \( T \) is the dimensionless temperature, \( \lambda_f \) is the thermal conductivity of the thin wall made of steel 1X13, \( \lambda_l \) is the thermal conductivity of ethanol, \( x \) and \( y \) are horizontal and vertical coordinates, respectively. The Prandtl number \( Pr = \nu/\alpha_f = 16 \), where \( \nu \) is the kinematic viscosity of ethanol, \( \alpha_f \) is the thermal conductivity coefficient of ethanol.

To simulate thermogravitational convection, a dimensionless system of Navier-Stokes equations, energy, and continuity in the Boussinesq approximation, written in the terms vortex, stream function, and temperature, was used:

\[
\begin{align*}
\frac{\partial T}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} &= \frac{1}{Pr} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \\
\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} &= -\omega \\
\frac{\partial \omega}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial \omega}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \omega}{\partial y} &= \left( \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right) + Gr \frac{\partial T}{\partial x}
\end{align*}
\]

Here \( Gr = g \beta H^4 q^2 \alpha_f^{-1} \) is the Grashof number, where \( g \) is the acceleration of gravity, \( \beta \) is the volume expansion coefficient of ethanol, \( \alpha \) is a dimensionless vortex, \( \psi \) is a dimensionless stream function, \( q \) is the density of the heat flux on the heated wall. The height of the liquid layer was chosen as the geometric dimension. The temperature scale is \( \Delta T = \lambda_f H \alpha_f^{-1} \). The velocity scale is \( \nu/H \), where \( \nu \) is the kinematic viscosity of the liquid, the time scale is \( H^2/\nu \).

The problem was solved under the following boundary conditions. The right-side wall was heated by electric current; the heat flow was set on the outer surface of the wall: \( \partial T/\partial n \big|_{l_2} = q \). The lower wall was thermally insulated: \( \partial T/\partial n \big|_{l_1} = 0 \). The thermocapillary effect was considered at the free boundary:

\[ \omega \big|_{l_3} = -\frac{Ma}{\alpha_f} \frac{\partial T}{\partial x}, \] where is the Marangoni number \( Ma = \frac{\partial \sigma}{\partial T} \frac{H \cdot \Delta T \cdot c_p}{\alpha_f \nu \lambda_f} \), here \( c_p \) is the heat capacity of alcohol. At the interface boundaries, the conditions of ideal thermal contact were set:

\[ -\frac{\lambda_f}{\lambda_x} \frac{\partial T}{\partial n} \big|_{n_{1,3}, n_{1,3,4}} = -\frac{\partial T}{\partial n} \big|_{n_{1,3,4}}, -\frac{\lambda_l}{\lambda_f} \frac{\partial T}{\partial n} \big|_{l_{1,4}, l_{1,4,4}} = -\frac{\partial T}{\partial n} \big|_{l_{1,4,4}}, T \big|_{n_{1,3,4}, l_{1,4,4}} = T \big|_{l_{1,3,4}, l_{1,4,4}}, \] here \( \lambda_f \) is the thermal conductivity of the air. Non-flowing \( \psi \big|_{1,2,3} = 0 \) and sticking \( \omega \big|_{1,2,3} = \frac{\partial V}{\partial z} \big|_{1,2,3} = \frac{\partial V}{\partial r} \big|_{1,2,3} \) conditions are set on all hard
surfaces. On the left, upper, and part of the right border of the area, the open border condition is set: 
\[ \frac{\partial T}{\partial n_{1,6}} = 0, \frac{\partial \omega}{\partial n_{1,6}} = 0, \frac{\partial \psi}{\partial n_{1,6}} = 0. \]

Figure 1. The scheme of the experimental stand:
1 - liquid; 2 - transparent sidewalls of the cavity are made of polycarbonate; 3 - thin wall (cold wall); 4 - heater; 5 - thermal imager - FLIR X650sc; 6 – PC.

Figure 2. The computational domain: I - liquid, II - air, 1 - hot wall, 2 - lower wall, 3 - thin wall, 4 - free surface, 5 - open border on top, 6 - open border on the left.

An uneven grid with triangular elements and 77,949 nodes was used. The thermophysical parameters of ethanol were taken as: \( \rho_f = 807.75 \text{ kg/m}^3 \) - density, \( \beta_f = 0.00105 \text{ 1/K} \) - volume expansion coefficient, \( \nu_f = 14.83 \times 10^{-7} \text{ m}^2/\text{s} \) - kinematic viscosity, \( \lambda_f = 0.179 \text{ W/(m} \cdot \text{K)} \) - thermal conductivity, \( c_{pf} = 2403.22 \text{ J/(kg} \cdot \text{K)} \) - heat capacity. Thermophysical parameters of air: \( \rho_a = 1.2019 \text{ kg/m}^3 \) - density, \( \beta_a = 0.00346 \text{ 1/K} \) - coefficient of volumetric expansion, \( \nu_a = 149.46 \times 10^{-7} \text{ m}^2/\text{c} \) - kinematic viscosity, \( \lambda_a = 0.0254 \text{ W/(m} \cdot \text{K)} \) - thermal conductivity, \( c_{pa} = 1006 \text{ J/(kg} \cdot \text{K)} \) - heat capacity. The thermal conductivity of steel 1X13 \( \lambda_s = 28 \text{ W/(m} \cdot \text{K)} \). \( \partial \sigma / \partial T = 875 \times 10^{-7} \text{ N/(m} \cdot \text{K)} \). The height of the liquid layer \( H = 0.1 \text{ m}. \) The thickness of the thin walls is 0.58 mm.

3. Results and discussion
After turning on the heater, during a short incubation period, the right-side wall and the wall layer of the liquid were heated. Then, the liquid heated on the wall began to float, and cold liquid began to leak to the wall. Due to the dependence of the surface tension of the liquid on the temperature, a thermocapillary effect acted on the non-isothermal free surface, forcing the liquid to move away from the heated wall. Thus, the process of advancing the thermal front was observed on the free surface of the liquid. In a physical experiment, a thermal imager was used to measure the temperature field on a free surface. As a result of the processing of thermal imaging films, the dependence of the time when the heat front reached a thin wall on the density of heat fluxes on the heated wall was constructed (Figure 3).

Numerical experiments were carried out at the height of the liquid layer \( H = 100 \text{ mm} \) and the set of heater capacities \( q = 2, 27, 274, \) and \( 1710 \text{ W/m}^2 \). Conjugate temperature fields in liquid, gas, and solids, as well as conjugate velocity fields in liquid and gas, were obtained. The obtained data significantly supplement the data of physical experiments, providing data on the temperature distribution in liquids and solids, detailed data on hydrodynamic processes in liquids, and data on heat transfer from the free surface of the liquid. The dependences of the time of the advance of the heat front to the thin wall depending on the heater power were compared. The results of numerical simulation and physical experiments have demonstrated a good coincidence.
Figure 3. The dependence of the time of the advance of the heat front from the heated to the cold wall on the density of the heat flow on the heater with $H = 100$ mm.

The results obtained numerically make it possible to specify the time when the heat front touches a thin wall. Figure 4 shows isotherms and temperature fields at the moment when the heat front touches a thin wall, depending on the heater power. It is noticeable that the maximum point of the thermal front is located under the free surface. With all the heater's capacities, the heat front falls apart and the head part separates from it, which is the first to reach a thin wall at a level slightly lower than the liquid level. This coincides with the experimental data.

Figure 4. Isotherms (the dotted line shows additional isotherms) and the temperature field at the moment when the thermal front reaches the thin wall at power and at time points: a - $q = 2$ W/m$^2$, $t = 22$ min 28 s; b - 27, 6 min 38 s; c - 274, 152 s; d = 1710, 108 s.

During the numerical simulation, data on the temperature field and the velocity field in the air were obtained. Figure 5 shows that as the thin wall warms up, ascending streams of air and descending streams in the liquid form on its outer surfaces. Ascending streams of liquid and air are formed on the surface of the hot wall. As a result, a form of advective flow is formed in the air with ascending flows along the sidewalls and a descending flow in the middle of the layer. At some point, a rising stream begins to form from the air heated on the free surface of the liquid. However, under the influence of the central descending air flow, it is pushed back to one of the vertical walls, which can lead to a local temperature surge.
Figure 5. Combined temperature field (color) and velocity field (arrows) at heater power $q = 274$ W/m$^2$ at time $t = 90$ min.

Figure 6a shows the longitudinal temperature gradient on the free surface of the liquid, which is the driving force of the thermocapillary effect. It is noticeable that the greatest effect will be achieved in the wall areas, especially near the hot wall. According to the evolution of the distribution of the vertical temperature gradient on the free surface, a feature at time $t = 12,000$ seconds in the region $2 \leq x \leq 2.2$ is clearly visible, associated with the formation of an ascending air flow from the free surface. A similar feature, although slightly noticeable, exists for the longitudinal temperature gradient, in this area at this moment it changes sign.

Figure 6c shows the evolution of the horizontal temperature gradient on the outer surface of the thin wall over time. It is noticeable that under the influence of the incoming flow of cold air from the open border, the wall is unevenly cooled to the level $y = 85$ mm. After that, the wall begins to actively
warm up under the influence of hot liquid flowing on it, spreading near the free surface, and the flow of air heated at the free surface, which makes a comparable contribution to the heating of the wall.

4. Conclusion
The nonstationary conjugate convective heat transfer is studied numerically in the computational domain corresponding to the conditions of the physical experiment. Convective heat transfer from the outside of the thin wall and the free surface of the liquid was considered. The evolution of convective flows has been studied, considering the thermocapillary effect on the free surface of the liquid, and temperature fields in the liquid, gas, and the thin wall after sudden heat supply to its inner side.

It is shown that with an increase in the heater power, the velocity of the front of the heated liquid along the free surface increases under the action of surface tension forces. After the heated front reaches the cold wall, it begins to monotonically heat up in the area of the liquid-gas interface under the impact of an incoming flow of hot liquid and above this area under the impact of an incoming flow of heated air at the free surface.

It is shown that the conjugate heat transfer between liquid and air can have a noticeable effect on the process of heat transfer from the free surface of the liquid and affect the process of heating a thin wall.

This research was conducted under project No. 0257-2021-0003, state registration No. 121031800213-0, under RFBR project No. 19-08-00707 A, and under RFBR-NSO project No. 19-48-540003 _а._

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