Gamma-ray strength function and pygmy resonance in rare earth nuclei

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The γ-ray strength function for γ energies in the 1–7 MeV region has been measured for 161,162Dy and 171,172Yb using the (3He,αγ) reaction. Various models are tested against the observed γ-ray strength functions. The best description is based on the Kadmenski˘ı, Markushev and Furman (KMF) model and the Lorentzian M1 model. A γ-ray bump observed at Eγ ∼ 3 MeV is interpreted as the so-called pygmy resonance, which has also been observed previously in (n,γ) experiments. The parameters for this resonance have been determined and compared to the available systematics.

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1. INTRODUCTION

The concept of γ-ray strength functions was introduced in the fundamental work of Blatt and Weisskopf [1]. They showed that the square of the γ-transition matrix element connecting highly excited states is proportional to the level spacing Dγ of the initial states i with equal spin and parity. Therefore, the ratio of the partial radiative width Γγ of the states i which is connected to γ transitions with transition energy Eγ and populating some low lying levels) and Dγ was suggested to be important for the description of γ transitions in the continuum. The corresponding model-independent definition of the γ-ray strength function1 is given by $f_{KL} = \Gamma_i / (E_\gamma^{2L+1} D_i)$, where $L$ is the multipolarity of the γ transition and X refers to the electric or magnetic character of the γ transition. The γ-ray strength function is now considered as a measure for the average electromagnetic properties of nuclei and is fundamental for the understanding of nuclear structure and reactions involving γ rays.

Experimentally, the main information on the γ-ray strength function has been obtained from the study of photoabsorption cross-sections [3]. It is commonly adopted that the E1 strength function is determined by the properties of the giant electric dipole resonance (GEDR) around its resonance energy, typically $E_\gamma \sim 10$–20 MeV. However, serious lack of information persists at lower γ-ray energies. It was assumed that the tail of the Lorentzian describing the GEDR determines the E1 strength function at these energies. The only experimental data on the E1 γ-ray strength function between compound states with γ-ray energies below 2 MeV have been obtained using the 143Nd(n,γα) reaction [3].

These data show that the extrapolation of the GEDR to low energies fails to describe the experimental values of the E1 strength function and indicates a finite value of $f_{E1}$ in the limit $E_\gamma \to 0$. As a result, a model for the E1 strength function was developed by Kadmenski˘ı, Markushev and Furman (KMF) [4] which takes into account the energy and temperature dependence of the GEDR width. Today, this model and its empirical modifications [5] are frequently used in the description of experimental data, but at the same time, it needs additional experimental verification.

The E1 strength function is not solely governing the γ-ray emission for lower γ-ray energies. Other multipo- larities, and especially the M1 strength function, play important roles as well. The experimental information on the γ-ray strength of M1 transitions is more scarce. It is commonly assumed that the M1 strength is well described by the Weisskopf model [6], where the dipole γ-ray strength function is independent of the γ energy. But some experiments indicate the existence of an M1 giant resonance originating from spin-flip excitations in the nucleus [6]. Also, the analysis of γ-ray spectra from (n,γ) reactions [8] indicates that the use of the M1 giant dipole resonance model gives a better fit to the experimental data than the Weisskopf model.

Special attention [8,9] has been devoted to the anomalous bump found in the γ-ray spectra of the (n,γ) and (d,γγ) reactions at low energies. The same bump has probably also been observed in the (3He,αγ) reaction [3]. A previous work [9] shows that the energy of the γ-ray bump increases with neutron number in the N = 82–126 region. The bump is called the pygmy resonance due to the considerably lower strength compared to the GEDR. The pygmy resonance has first been ex-

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‡In literature, also called the radiative strength function.
plained by the enhancement of the E1 strength function [8]. However, one can not rule out a possible M1 character connected to orbital M1 strength (scissors mode) in nuclei, which was first observed in electron scattering experiments [14].

The γ-ray strength function is difficult to measure in the γ-decay between highly excited states, since the decay rate also depends on the number of accessible levels. The analysis of γ-ray strength functions from the spectra of (n,γ) reactions [6] shows that the results depend crucially on the level density model employed. Therefore, conclusions based on a certain level density formula should be considered as preliminary and thus, need confirmation.

Recently [15–17], a new experimental technique has been developed, based on a set of primary experiments [14]. However, one can not rule out a possible M1 characteristic connected to orbital M1 strength (scissors mode) [8]. However, one can not rule out a possible M1 characteristic connected to orbital M1 strength (scissors mode) [8].

In light-ion reactions with one charged ejectile. The technique allows to disentangle the γ-ray spectra into a γ-energy dependent function F(Eγ) (which, as will be shown below, can be uniquely connected to the γ-ray strength function) and the level density ρ(E). It makes it possible to study the γ-ray strength function and level density independently from each other, in contrast to what can be done by using radiative neutron capture techniques. In previous works [18–20] the extracted level densities ρ have been utilized to deduce thermodynamical properties for several rare earth nuclei. In this paper, however, we will focus on the F(Eγ) function.

The experimental method is described in Sect. II. In Sect. III we give a short outline of the models used to describe the experimental data, and in Sect. IV we compare various predictions to the experimental data. Our results are also compared to data from the (n,γ) reaction performed by others. Conclusions are given in Sect. V.

II. EXPERIMENTAL METHOD

The experiments were carried out with 45 MeV 3He-projectiles at the Oslo Cyclotron Laboratory (OCL). The particle-γ coincidences are measured with the CACTUS multidetector array [21] using the (3He,αγ) reaction on 162,163Dy and 172,173Yb self-supporting targets. The charged ejectiles were detected with eight particle telescopes placed at an angle of 45° relative to the beam direction. An array of 28 NaI γ-ray detectors with a total efficiency of ~15% of 4π surrounded the target and particle detectors.

The experimental extraction procedure and the assumptions made are described in Refs. [6,15] and references therein. From the α-γ-coincidences, spectra of the total γ-ray cascade can be sorted out according to the initial excitation energy E. These spectra are the basis for making the first generation (or primary) γ-ray matrix P(E, Eγ), which is factorized according to the Brink-Axel hypothesis [23,24]

\[ P(E, Eγ) \propto F(Eγ)ρ(E - Eγ). \]  

Here, F and ρ are the γ-ray energy dependent factor and the level density, respectively. It is now possible to determine F and ρ by an iterative procedure. The first trial function for ρ is simply taken as a straight line and the corresponding F is determined by Eq. (1). Then, a \( χ^2 \) minimum is calculated for each data point of F and ρ, keeping the others fixed. This procedure is repeated about 50 times, until a global least square fit to the ~1400 data points of the P(E, Eγ) matrix is achieved.

It has been shown [17] that if one solution for F and ρ has been found, functions of the form

\[ \tilde{ρ}(E - Eγ) = Aρ(E - Eγ) \exp(α[E - Eγ]) \]  

\[ F(Eγ) = BF(Eγ) \exp(α Eγ) \]  

give exactly the same fit to the P(E, Eγ) matrix. The values of A, B and α can be determined by additional conditions. The A and α parameters are used for absolute normalization of the level density ρ: They are adjusted to reproduce (i) the number of levels observed in the vicinity of the ground state and (ii) the neutron resonance spacing at the neutron binding energy Bn. Further details on the extraction procedure and the simulation of errors are given in Ref. [17]. In the following we will concentrate on the γ-ray energy dependent function F(Eγ) and its normalization.

We assume that the main contributions to the derived F function are from E1 and M1 γ-transitions and that the accessible levels of positive and negative parity are equal in number for any energy and spin i.e.

\[ ρ(E - Eγ, I_f, ±Π_f) = \frac{1}{2}ρ(E - Eγ, I_f). \]  

Thus, the observed F is expressed by a sum of the E1 and M1 γ-ray strength functions only

\[ BF(Eγ) = [f_{E1}(Eγ) + f_{M1}(Eγ)]Eγ^3, \]  

where B is the unknown normalization constant. Our experiment does not provide the possibility to derive the absolute normalization of F(Eγ) (see Eq. (1)), therefore, the normalization constant has to be determined from other experimental data. The experimental, average total radiative width of neutron resonances (Γγ) at the neutron binding energy Bn can e.g. be written in terms of F. To show this, we start with Eq. (3.1) of Ref. [16]

\[ \langle Γγ(E, I, Π) \rangle = \frac{1}{ρ(E, I, Π)} \sum_{X_L} \sum_{I_f, Π_f} \int_{E_{γ=0}}^{E} dEγ \ Eγ^{E_{γ}^{L+1}} f_{XL}(Eγ)ρ(E - Eγ, I_f, Π_f) \]  

where \( \langle Γγ(E, I, Π) \rangle \) is the average total radiative width of levels with energy E, spin I and parity Π. The summations and integration are going over all final levels with spin I_f and parity Π_f which are accessible by γ radiation with energy Eγ, multipolarity L and electromagnetic
character X. If we, again, assume that only dipole radiation contributes significantly to the sum and that the number of accessible levels with positive and negative parity are equal, we obtain, by combining Eqs. (1) and (2), the average total radiative width of neutron s-wave capture resonances with spins $I_t \pm 1/2$ expressed in terms of the $F(E_\gamma)$ function

$$
\langle \Gamma_\gamma(B_n, I_t \pm 1/2, \Pi_t) \rangle = \frac{1}{2\rho(B_n, I_t \pm 1/2, \Pi_t)} \int_{E_\gamma=0}^{E_n} dE_\gamma BF(E_\gamma) \rho(B_n - E_\gamma) \sum_{j=-1}^{1} g(B_n - E_\gamma, I_t \pm 1/2 + J), \quad (7)
$$

where $I_t$ and $\Pi_t$ are the spin and parity of the target nucleus in the $(n,\gamma)$ reaction and $\rho$ is the experimental level density. Furthermore, we have expressed $\rho$ as the product of the total level density, summed over all spins and the spin distribution $g$. The spin distribution of the level density is given by [2]

$$
g(E, I) = \frac{2I + 1}{2\sigma^2} \exp \left[ -(I + 1/2)^2 / 2\sigma^2 \right], \quad (8)
$$

where $\sigma$ is the excitation-energy dependent spin cut-off parameter. The spin distribution is normalized to $\sum_I g \approx 1$. The experimental value of the average total radiative width of neutron resonances $\langle \Gamma_\gamma \rangle$ is then the weighted sum of contributions with $I_t \pm 1/2$ according to Eq. (7).

Because of methodical difficulties, the functions $F(E_\gamma)$ and $\rho(E)$ cannot be determined experimentally in the interval $E_\gamma < 1$ MeV and $E > B_n - 1$ MeV, respectively. In addition, the data at the highest $\gamma$-ray energies, $E_\gamma > B_n - 1$ MeV, suffer from poor statistics. Therefore, extrapolations of $F$ and $\rho$ were necessary in order to calculate the integral in Eq. (3). The contribution from the extrapolation to the total radiative width in Eq. (3) does not exceed 15%, thus the errors due to a possibly poor extrapolation are expected to be of minor importance.

### III. MODELS FOR E1 AND M1 RADIATION

There have been developed several models for the $\gamma$-ray strength functions $f_{\gamma L}$. The theories behind the models are complicated, and will not be outlined here. However, the resulting strength functions can be written in simple analytical forms. In this work, we have tested various E1 and M1 models. For E1 $\gamma$-transitions these are:

- The standard giant electric dipole resonance (GEDR) model based on the Brink-Axel approach [2][3]

$$
f_{E1}(E_\gamma) = \frac{1}{3\pi^2 \hbar^2 c^2} \frac{\sigma_{E1} E_\gamma \Gamma_{E1}^2}{(E_\gamma^2 - E_{E1}^2)^2 + E_{E1}^2 \Gamma_{E1}^2}, \quad (9)
$$

where $\sigma_{E1}$, $\Gamma_{E1}$ and $E_{E1}$ are the giant electric dipole resonance parameters derived from photodisappearance experiments. ²

- The model of Kadmsenki˘ı, Markushev and Furman (KMF) [3] \n
$$
f_{E1}(E_\gamma) = \frac{1}{3\pi^2 \hbar^2 c^2} \frac{0.7 \sigma_{E1} \Gamma_{E1}^2 (E_\gamma^2 + 4\pi^2 T^2)}{E_{E1} (E_\gamma^2 - E_{E1}^2)^2}, \quad (10)
$$

where $T$ is the temperature of the nucleus which is usually determined as $T = \sqrt{U/a}$ with $U$ being the shifted excitation energy and $a$ the level density parameter. The energy and temperature dependent width of the GEDR in this model is expressed by

$$
\Gamma_{E1}(E_\gamma, T) \approx \frac{\Gamma_{E1}}{E_{E1}^2} (E_\gamma^2 + 4\pi^2 T^2). \quad (11)
$$

These expressions are developed in the framework of a collisional damping model for $E_\gamma < E_{E1}$ and although they should hold for $T \ll 2$ MeV, the absence of thermal shape fluctuations in the model limits their validity to $T < 1$ MeV.

For deformed nuclei, the giant dipole resonance is split into two components, hence the sum of two strength functions with different GEDR parameters has been employed.

- The adjusted single-particle model of Weisskopf [1]

where $f_{M1}(E_\gamma)$ is independent of $E_\gamma$ and the absolute value of $f_{M1}$ has been taken from $f_{M1}/f_{E1}$ systematics close to the neutron binding energy [4].

- A Lorentzian based on the existence of a giant magnetic dipole resonance (GMDR) which is assumed to be related to the spin-flip transition between single-particle states [5]. The $\gamma$-ray strength function in this case is determined by

$$
f_{M1}(E_\gamma) = \frac{1}{3\pi^2 \hbar^2 c^2} \frac{\sigma_{M1} E_\gamma \Gamma_{M1}^2}{(E_\gamma^2 - E_{M1}^2)^2 + E_{M1}^2 \Gamma_{M1}^2}. \quad (12)
$$

In the following, we will compare these models to the experimental findings.

²The constant $1/(3\pi^2 \hbar^2 c^2)$ equals $8.6 \times 10^{-8}$ mb$^{-1}$MeV$^{-2}$. 

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IV. RESULTS AND DISCUSSION

Figure 1 shows the experimentally extracted $\gamma$-ray energy dependent factor $F$ and the level density $\rho$ for $^{161,162}$Dy and $^{171,172}$Yb. These data are the very same as in Ref. [1], except that the data of $^{171}$Yb have been retuned by adjusting the parameters $A$ and $\alpha$ to fit the level density based on known discrete levels at low excitation energy. We recognize that the shape of the unnormalized $F$ functions are rather equal for neighboring isotopes indicating that $F$ is a slowly varying function of mass number.

In the extraction procedure, we have used $\gamma$-ray spectra from excitation energy bins between 4 and 8 MeV. This span in excitation energy corresponds to a temperature region of 0.5 to 0.7 MeV for the initial states, and 0.4 to 0.6 MeV for the final states. The spin window in the pick-up reaction is $2 - 6h$, which is assumed to be approximately equal for initial and final states. Hence, the discussion below concerns average properties of nuclei for $T \sim 0.5$ MeV and $I \sim 4h$ with the assumption of equal density of positive and negative parity states.

The normalized experimental $\gamma$-ray strength functions $f = BF/E^3$ for $^{161,162}$Dy and $^{171,172}$Yb are presented in Fig. 3. The experimental values of the average total radiative width ($\Gamma_{\gamma}$) [3] used to determine the normalization constant $B$ are listed in Table I. The figure shows that each experimental curve consists of two components. The first one is a smooth function of the $\gamma$-ray energy and the second one is connected to a local enhancement of the $\gamma$-ray strength function at low $\gamma$-ray energies ($\sim 3$ MeV). The latter component is due to the pygmy resonance, which was first observed in $(n, \gamma)$ reactions [8].

Theoretical curves calculated with the models of Sect. III are shown as dashed curves in Fig. 2. The parameters adopted in the description of the GEDR and GMDR are presented in Table I. The GEDR parameters have been determined from the interpolation of systematics over neighboring isotopes [27]. For the GMDR parameters, there are no rich experimental systematics available. Our parameters have been taken from Ref. [3], namely $E_{M1} = 41A^{-1/3}$ MeV and $\Gamma_{M1} = 4$ MeV. The value of $\sigma_{M1}$ has been derived from $f_{E1}/f_{M1}$ systematics at $\gamma$-ray energies close to the neutron binding energy [29].

Figure 2 also shows that the Lorentzian E1 model [label a), Eq. (9)] gives an acceptable description for $\gamma$-ray energies near $B_n$, in accordance with the systematics of Kopecky and Uhl for deformed nuclei [23]. But for lower energies this GEDR model overestimates the experimental data. The combination of the KMF E1 model and the Weisskopf M1 model [label b)] fails to describe the data due to the strong M1 component especially in the region of low $\gamma$-ray energies. The KMF model plus the Lorentzian M1 model [label c), Eqs. (10, 12)] is seen to give the best description of the general slope of the $\gamma$-ray strength function. However, since the M1 strength-function model is generally only $\sim 20\%$ of the GEDR model for the investigated nuclei [23], no further conclusion concerning M1 models could be drawn in this work. In detail, the agreement of the last curve with the data for $^{161,162}$Dy is satisfactory, excluding the low energy region where the pygmy resonance is observed. For $^{171,172}$Yb, the slopes of the calculated curves differ somewhat from the experimental ones.

In order to obtain a good parameterization of the $\gamma$-ray strength function which can fit the experimental data, the sum of the KMF E1 and the Lorentzian M1 models has been selected for further modification. In contrast to the common use, where the nuclear temperature is defined as $T = \sqrt{U/a}$, we will keep the temperature fixed (as first proposed by Grudzhevich [28,29]) according to a constant temperature model of the nuclear level density, which is supported by recent findings [40]. The constant temperature model may also be regarded as to mimic the generalized superfluid model of the nuclear level density [41,42] in this excitation energy region. The mean value of the temperature in the excitation energy region under study is $T \sim 0.5$ MeV for the final levels as has been mentioned above. We should point out that the temperature dependence of the GEDR width used in the E1 model is a much disputed topic. Experimental data on damping of the GEDR at low temperatures ($T < 1$ MeV) are absent. At higher temperatures, the damping of the GEDR is intensively studied with inelastic scattering of light particles (e.g. $\alpha$ particles [33]) but different theoretical approaches give ambiguous results. For example, in the adiabatic coupling model [34,35], the increasing width is explained in terms of thermally induced shape fluctuations, yielding in general a $\Gamma \propto \sqrt{t}$ dependence. These shape fluctuations become important for $T > 1 - 2$ MeV.

In the collisional damping model [34] the width of the GEDR is due to collisional damping of nucleons, giving a $\Gamma \propto T^2$ law. Experiments on $^{208}$Pb show that the data set can be fitted by both of these parameterizations [53,57], while new calculations on the collisional damping model using realistic in-medium cross-sections [58] show that the width is in general underestimated within this model. Also, a recent calculation [57] on $^{128}$Sn within the phonon damping model shows good agreement with experiment.

At temperatures appropriate for the present study ($T \sim 0.5$ MeV), pairing correlations [15] and shell effects [34,55] have to be taken into account. Most experimental data on the strength function at the low-energetic tail of the GEDR are obtained from $(n, \gamma)$ reactions, where the quadratic temperature-dependence of the GEDR width [4] is a popular parameterization [4,65]. We therefore use this parameterization, knowing that the model behind can not account properly for the damping mechanism of the GEDR [59].

In order to obtain a good fit of the chosen $\gamma$-ray strength-function models to the data we use the temperature as a free parameter because of the uncertain temperature dependence of the GEDR width in our temperature region. Also, a common normalization constant
The γ-ray energy dependent factor and the level density for $^{161,162}$Dy and $^{171,172}$Yb have been measured using the $(^3\text{He},\alpha\gamma)$ reaction. For the first time, the normalized γ-ray strength function $f(E_\gamma)$ could be extracted from such data.

Various models are tested against the observed γ-ray strength function and the best description is found for the E1 model of KMF with a fixed temperature plus Lorentzian models for the GMDR and the pygmy resonance. The pygmy resonance parameters for $^{161,162}$Dy and $^{171,172}$Yb fit into the available systematics obtained from (n,γ) experiments. Hence, the adopted approach gives consistent γ-ray strength functions for the investigated nuclei.

A few tentative explanations exist for the pygmy resonance. Still, the question remains open whether the pygmy resonance is of E1 or M1 character. Measurements of the electromagnetic character of the pygmy resonance is therefore important in order to pin down the true nature of this peculiar phenomenon.

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3Here, the phrase "close to" should be appropriate, since the normalization in other works is often a factor of 2 uncertain.
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### TABLE I. The parameters used for calculation of $\gamma$-ray strength functions.

| Nucleus | $E_1^{(1)}$ (MeV) | $\sigma_1^{(1)}$ (mb) | $\Gamma_1^{(1)}$ (MeV) | $E_1^{(2)}$ (MeV) | $\sigma_1^{(2)}$ (mb) | $\Gamma_1^{(2)}$ (MeV) | $E_M$ (MeV) | $\sigma_M$ (mb) | $\Gamma_M$ (MeV) | $\langle \Gamma_\gamma \rangle$ (meV) |
|---------|-------------------|------------------------|------------------------|-------------------|------------------------|------------------------|-------------|----------------|----------------|----------------|
| $^{161}\text{ Dy}$ | 12.13 | 210 | 2.6 | 15.8 | 250 | 5.05 | 7.66 | 1.60 | 4.0 | 108 |
| $^{162}\text{ Dy}$ | 12.13 | 210 | 2.6 | 15.8 | 250 | 5.05 | 7.65 | 1.49 | 4.0 | 113 |
| $^{171}\text{ Yb}$ | 12.25 | 239 | 2.6 | 15.5 | 302 | 4.80 | 7.50 | 1.50 | 4.0 | 63 |
| $^{172}\text{ Yb}$ | 12.25 | 239 | 2.6 | 15.5 | 302 | 4.80 | 7.50 | 1.76 | 4.0 | 75 |

### TABLE II. The parameters obtained from the fit.

| Nucleus | $E_{py}$ (MeV) | $\sigma_{py}$ (mb) | $\Gamma_{py}$ (MeV) | $T$ (MeV) | $K$ |
|---------|----------------|--------------------|---------------------|----------|-----|
| $^{161}\text{ Dy}$ | 2.69(4) | 0.49(5) | 1.37(22) | 0.29(11) | 1.34(11) |
| $^{162}\text{ Dy}$ | 2.73(5) | 0.42(4) | 1.35(25) | 0.34(10) | 1.08(8) |
| $^{171}\text{ Yb}$ | 3.35(6) | 0.65(7) | 0.97(16) | 0.34(3) | 1.22(10) |
| $^{172}\text{ Yb}$ | 3.48(7) | 0.45(5) | 1.30(23) | 0.32(2) | 1.24(6) |
FIG. 1. The observed level density $\rho$ and the $\gamma$-ray energy dependent factor $F$ for $^{161,162}$Dy and $^{171,172}$Yb.
FIG. 2. The observed γ-ray strength functions (data points with error bars) for $^{161,162}$Dy and $^{171,172}$Yb. The dashed curves are calculations where a) denotes the Lorentzian GEDR model [Eq. (9)], b) the KMF model [Eq. (10)] plus a Weisskopf estimate for M1 transitions, and c) the KMF model [Eq. (10)] plus a Lorentzian GMDR model [Eq. (12)]. For b) and c), the temperature is given by $T = \sqrt{U/a}$. The solid curves are the KMF model with constant temperature [Eq. (10)] plus a Lorentzian GMDR model [Eq. (12)] plus a Lorentzian pygmy resonance model [Eq. (13)] (see text).
FIG. 3. Pygmy resonance parameters from the present (\(^3\)He,αγ) reaction (filled circles) compared to those from the (n,γ) reaction [8] (open circles) as function of neutron number \(N\). In the upper panel, the resonance energy \(E_{py}\) is displayed as data points and the width \(Γ_{py}\) is given by the length of the lines through the data points. The cross-sections with error bars are shown in the lower panel. For the (\(^3\)He,αγ) reaction, the quantity \(σ_{py}\) is plotted and assigned an additional systematic error of 20% from the normalization in Eq. [5]. For the (n,γ) reaction, the quantities \(σ_{py}\) (open circles) and \(kσ_{py}\) (open squares) [8] are plotted.
FIG. 4. The total $\gamma$-ray spectrum for $^{162}$Dy. The data points with error bars are taken from the $^{161}$Dy(n,\gamma)$^{162}$Dy reaction \cite{39}$\gamma$-ray strength function and the level density extracted from the present $^{162}$Dy($^{3}$He,\alpha\gamma)$^{162}$Dy data. The calculation is performed by averaging over 100 keV intervals.