Black Hole Remnant in Massive Gravity

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The possibility of a nonzero graviton mass has been widely pursued in the literature. In this work we investigate a black hole solution in massive gravity with a degenerate fiducial metric often used in the literature. We find that the end state of Hawking evaporation leads to black hole remnant, which could help to ameliorate the information paradox. We prove that these remnants only exist in anti-de Sitter spacetime. Nevertheless, we speculate on their possible relevance to our Universe as dark matter candidate, in view of the possibility that our Universe could be inherently anti-de Sitter-like, with a transient accelerated expansion phase.

I. INTRODUCTION: A BRIEF ACCOUNT OF MASSIVE GRAVITY

Black hole remnants are the stable or meta-stable end state of Hawking evaporation, in the sense that Hawking radiation may stop as the mass of the black hole reaches the Planck scale, due to new physics of quantum gravitational nature. They can arise from different theories, or from various quantum gravity inspired phenomenological models. Properties of black hole remnants have been studied in the literature [1, 2]. See [3] for a recent review of the subject. In this paper, our main objective is to find black hole remnants in the theory of massive gravity, without additional gauge fields, which to our knowledge has not been explored before. Given that massive gravity is nowadays a popular candidate for modified theory of gravity (despite its various short comings), this is a topic worth studying – can black hole remnant arise simply by endowing graviton with a mass?

We begin with some background on massive gravity for completeness. Einstein’s general relativity can be cast as a theory of massless spin-2 gravitons. Generalizations to massive gravity theories have several motivations, including an attempt to explain the observed accelerated expansion of the Universe. One could also investigate massive gravity as an extension of general relativity, to see if such a theory is consistent, or if general relativity is the unique consistent spin-2 theory of gravitation. Recent observations by LIGO has put a tight bound on graviton’s mass [5, 6], but well-known work of Boulware and Deser [7] showed that a generic extension of the Fierz-Pauli (FP) theory [8] to curved backgrounds will give rise to ghost instabilities, now known as the “BD-ghost”. A special generalization to a nonlinear and stable massive gravity has been introduced by de Rham, Gabadadze and Tolley (dRGT) [9–11], which was subsequently shown to be ghost-free by Hassan and Rosen [12–14]. As we shall see below, dRGT massive gravity comes with two metric tensors, one of which is a fixed spacetime background, as realized by Hassan and Rosen. A natural generalization to having both gravitons being dynamical was then sought by Hassan and Rosen, later known as the bimetric or bi-gravity theory [15]. However, in this work, we shall focus on the original massive gravity with a fixed background. In addition to cosmological implications, the nonzero graviton mass allows one to model field theories with momentum dissipation in holography, without the need to employ the more traditional lattice method in the anti-de Sitter bulk [16, 17].

We note that dRGT massive gravity suffers from some problems. Firstly, there is a lack of viable FLRW cosmological solutions [4, 18]. More accurately, massive gravity does not admit FLRW solutions if a flat reference metric is assumed. However such solutions can exist with other choices of the reference metric. There are also fundamental problems related to the well-posedness of the theory, and “micro-acausality” (arbitrarily small closed causal curve) [19–23] (it is likely that these problems are avoided in the bimetric generalization, given the recent understanding of its complicated causal structure [24]). Nevertheless, there is still merit in further understanding the various aspects of the theory. For example, the effects of nonzero graviton mass on the structure of neutron stars [25] and white dwarfs [26] have been studied recently. The results showed that the maximum mass of these stars can be about three times the solar mass, i.e. more massive than in general relativity. In this work, we will focus on demonstrating that massive gravity admits black hole remnants. Interestingly, this type of remnants exist in anti-de Sitter spacetime, but not in de Sitter one. We will discuss its possible implications for information paradox of black holes. We also
speculate on the relevance of these remnants in our actual Universe as possible dark matter candidate; although the Universe is currently undergoing accelerated expansion, it is possible that this is a transient period, and the Universe is actually inherently anti-de Sitter-like. That is to say, current observation does not rule out the possibility that our Universe will be asymptotically anti-de Sitter in the future. The cosmological constant would eventually dominate the evolution of the Universe, slowing its expansion. The current phase of accelerating expansion could be the results of other fields, whose effects might become subdominant compared to the cosmological constant in the far future. A recent study that attempted to address the current tension regarding values of the Hubble constant measured by low z observations and high z Planck measurement from CMB has also proposed such a scenario [27].

II. BLACK HOLE REMNANTS IN MASSIVE GRAVITY

The action of dRGT massive gravity can be written as Hilbert-Einstein action with suitable nonlinear interaction terms [10]:

$$ I = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left( R + m^2\mathcal{U}(g, \phi^\alpha) \right), $$

where $R$ and $\mathcal{U}$ are, respectively, the Ricci scalar and the effective potential of graviton which modifies the gravitational sector with a nonzero graviton mass $m$. Note that despite appearance, dRGT gravity should not be viewed as a “scalar-tensor” theory – the scalar fields are St"uckelberg scalars, introduced as a mean to restore the general covariance of the theory [28]. Note that we do not include the cosmological constant a priori in the action, though of course this can be done just as well as in general relativity. The reason for omitting the cosmological constant is in view of the original motivation of the massive gravity theory to explain the accelerating expansion of the Universe without resorting to a cosmological constant.

The Newton constant is dimensionful, but we will set its value as unity for simplicity. This means that graviton mass $m$ has dimension inverse length ($c = 1 = h$), but terms like $M/r \equiv GM/c^2r$ are dimensionless. This follows the convention of, e.g., [29–31]. The effective potential $\mathcal{U}$ can be written as

$$ \mathcal{U}(g, \phi^\alpha) = \mathcal{U}_2 + \alpha_3 \mathcal{U}_3 + \alpha_4 \mathcal{U}_4, $$

in which

$$ \mathcal{U}_2 = \mathcal{K}^2 - 3\mathcal{K}^2, $$

$$ \mathcal{U}_3 = \mathcal{K}^3 - 2 \mathcal{K}^2 \mathcal{K} + 2 \mathcal{K}^3, $$

$$ \mathcal{U}_4 = \mathcal{K}^4 - 6 \mathcal{K}^2 \mathcal{K}^2 + 8 \mathcal{K}^3 \mathcal{K} + 3 \mathcal{K}^2 - 6 \mathcal{K}^4, $$

where

$$ \mathcal{K}^\mu = \delta^\mu_\nu - \sqrt{g} g^{\mu\sigma} f_{ab} \partial_\nu \phi^a \partial_\nu \phi^b, $$

in which

$$ f_{ab} \text{ is an appropriate non-dynamical reference metric and the rectangular bracket denotes the traces, namely } [\mathcal{K}] = K^\mu_\nu \text{ and } [\mathcal{K}^n] = (\mathcal{K}^n)^b_a. $$

We now consider a 4-dimensional static, spherically symmetric spacetime with metric ansatz

$$ ds^2 = -g(r)dt^2 + \frac{dr^2}{g(r)} + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2). $$

The reference metric essentially plays the role of a Lagrange multiplier to eliminate the BD ghost, and also different choices of the reference metrics give different theories. Here we shall follow the same choice for non-dynamical reference metric in the following form [16, 29, 31]:

$$ f_{ab} = \text{diag}(0, 0, c^2, c^2 \sin^2 \theta), $$

where $c$ is a positive constant with dimension of length. We should emphasize that the property of massive gravity is such that the choice of reference metric does affect what kind of solutions are allowed, so this black hole solution depends on the choice made above. (For detailed study on black hole solutions in dRGT theory, see [32]) Admittedly, the proof of ghost-freeness of dRGT theory [13, 14] assumes that the reference metric is invertible, so for degenerate metric (i.e. its rank is smaller than its dimension) like Eq. (8) one has to analyze the BD ghost separately[16, 17]. It was shown that in [33], the aforementioned non-dynamical reference metric in Eq.(8), does indeed give rise to ghost-freeness. However, since the existence of BD ghost depends not only on the background but also on the values of free parameters $\alpha$ and $\beta$ (or equivalently $\alpha_3$ and $\alpha_4$), this delicate issue is beyond the scope of the current work. Our aim is less ambitious: taking the theory with the aforementioned reference metric,
which has been considered numerous times in the literature, what can we say about the existence of black hole remnant?

Indeed, considering the ansatz (7), the reference metric (8), and the field equation (6), we can obtain the following exact solution [30]

\[ g(r) = 1 - \frac{m_0}{r} + \frac{\Lambda r^2}{3} + \gamma r + \varepsilon, \]  

(9)

where \( m_0 \) is an integration constant related to the mass of the black hole, while \( \Lambda, \gamma \) and \( \varepsilon \) are, respectively [30],

\[ \Lambda = 3m^2(1 + \alpha + \beta), \]
\[ \gamma = -cm^2(1 + 2\alpha + 3\beta), \]
\[ \varepsilon = c^2m^2(\alpha + 3\beta). \]  

(10)

It is notable that by considering the term \( 3m^2(1 + \alpha + \beta) \) equals to \( \Lambda \), one can see that there is a similarity between the obtained black hole solutions in Eq. (9), and the AdS Schwarzschild black holes. So we can consider the term \( 3m^2(1 + \alpha + \beta) \) as the effective cosmological constant. Here we see that the cosmological constant is “emergent” – it comes from the nonzero graviton mass \( m \). It follows directly from Eq. (9) that the Schwarzschild solution is recovered for vanishing massive terms \( (m^2 = 0). \)

Asymptotically locally anti-de Sitter (AdS)-like and de Sitter (dS)-like solutions are possible (depending on the sign of \( 1 + \alpha + \beta \)); for nonzero \( \gamma \) and \( \varepsilon \) the asymptotic geometries are not strictly AdS or dS. The constant term \( \varepsilon \) corresponds to global monopole [30].

Now, we briefly discuss the geometrical structure of this solution. For this purpose, we first look for the obvious singularity (if any) by studying two scalar curvatures: Ricci and Kretschmann scalars. Considering the metric (7), with the solution (9), the Ricci scalar is given by

\[ R = -4\Lambda - \frac{6\gamma}{r} - \frac{2(1 + \gamma)}{r^2}. \]  

(11)

Evidently, we encounter a divergence of the Ricci scalar at the origin \( (\lim_{r \to 0} R = \infty) \). Also, the Kretschmann scalar \( R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} \) is given by a rather lengthy expression:

\[ R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} = \frac{8\Lambda^2}{3} + \frac{8\Lambda\gamma}{r} + 8 \left( \frac{\gamma^2 + \frac{\Lambda}{3} (1 + \varepsilon)}{r^2} \right) + \frac{8\gamma(1 + \varepsilon)}{r^3} + \frac{4(1 + \varepsilon)^2}{r^4} - \frac{8m_0(1 + \varepsilon)}{r^5} + \frac{12m_0^2}{r^6}. \]  

(12)

One can show that this scalar has the following behavior

\[ \lim_{r \to 0} R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} = \infty; \]  

(13)

\[ \lim_{r \to \infty} R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} = \frac{8\Lambda^2}{3}, \]  

(14)

which can confirm that there is a curvature singularity at \( r = 0 \), and also the asymptotic behavior of this solution is AdS\(^1\), since the Kretschmann scalar is \( \frac{8\Lambda^2}{3} \) at \( r \to \infty \).

For clarity, we plot the metric function (9) versus \( r \) in Fig. (1) for specific choice of the parameter values. As one can see, there is a zero to Eq.(9) which corresponds to the event horizon.

In this work we will show that for black hole remnants to exist, it must be the case that \( \Lambda > 0 \), which, as we have emphasized, from the metric in Eq. (9), actually corresponds to anti-de Sitter case.

The physical mass of the black hole is \( M = m_0/2 [29] \), which can be obtained from the Hamiltonian method. For \( m = 0 \), \( M \) reduces to the standard ADM mass of an asymptotically flat Schwarzschild black hole. In order to find the event horizon of black hole, we should solve \( g(r) = 0 \). This gives rise to a cubic equation with at most three real roots. For more details see Fig.(1). In fact, the largest real positive root of the function \( g(r) \) is the event horizon of the black hole, given by

\[ r_+ = \frac{\alpha^2/3 - 2\gamma\alpha^{1/3} - 4(1 + \varepsilon)\Lambda + 4\gamma^2}{2\Lambda\alpha^{1/3}}, \]  

(15)

\[ \text{FIG. 1: The plot of } g(r) \text{ versus } r, \text{ for definiteness here we set } \Lambda = 1, \gamma = 1, m_0 = 0.6, \gamma = -1.45 \text{ (dashed line), } \gamma = -1.51 \text{ (continuous line) and } \gamma = -1.60 \text{ (dashed line).} \]

\(^1\) For de-Sitter case, since \( r \) is bounded, we cannot take \( r \to \infty \) limit. Note that with the metric Eq. (9), \( \Lambda > 0 \) corresponds to anti-de Sitter instead of de-Sitter. This can be a source of confusion, however we follow the form of Eq. (9) which is widely used in the literature.
The square root actually comes with a ± sign. That is to say, the black hole reduces to asymptotically flat Schwarzschild black hole. Dashed and continuous lines are related to black holes in massive gravity and general relativity, respectively. Up left panel: Temperature \((T)\) versus mass \((M)\). Up right panel: Radius \((R)\) versus mass \((M)\). Down left panel: Entropy \((S)\) versus mass \((M)\). Down right panel: Heat capacity \((C)\) versus mass \((M)\).

By substituting \(R_r\) in the physical mass \(M_r(r=R_r)\), one finds that indeed there exists a remnant with mass below which there is no black hole solution:

\[
M_r = \frac{2\Lambda(1+\varepsilon) + \gamma \left( \sqrt{\gamma^2 - (1+\varepsilon)\Lambda - \gamma} \right)}{6\Lambda^2} \times \left( \sqrt{\gamma^2 - (1+\varepsilon)\Lambda - \gamma} \right).
\]

We remark that in the absence of the cosmological constant the remnant mass of black hole reduces to

\[
M_r(\Lambda = 0) = -\frac{(1+\varepsilon)^2}{8\gamma}.
\]

In order to have a positive remnant mass of the black hole in the absence of the cosmological constant (Eq. (21)), we always impose \(\gamma < 0\).

The remnant of entropy is given by

\[
S_r = \frac{\pi}{\Lambda^2} \left( \sqrt{\gamma^2 - (1+\varepsilon)\Lambda - \gamma} \right)^2.
\]

We present our results in Fig. (2), in which the temperature, radius, entropy and heat capacity of a massive gravity black hole, with parameters chosen to be \(\Lambda = 1, \varepsilon = 1, \gamma = -1.5\), are depicted and compared against its GR counter-part. We note in particular that the heat capacity and temperature are both zero when the remnant...
mass is reached. Recall that for $\Lambda > 0$, our black hole remnant is asymptotically anti-de Sitter.

One might wonder if $\Lambda < 0$ solution also exists, which might be more relevant to cosmology? Unfortunately this seems not to be the case. To see this, let us consider Eq. (19) and Eq. (20). We will show that for $\Lambda < 0$ (which would correspond to the de Sitter case), it is impossible to obtain remnants for which the mass and the radius are positive simultaneously. With $\Lambda < 0$, in Eqs. (19) and (20) can be written as

$$R_r = \frac{\sqrt{\gamma^2 + \sigma - \gamma}}{\Lambda},$$

$$M_r = \frac{-2\sigma + \gamma \left( \sqrt{\gamma^2 + \sigma} - \gamma \right)}{6\Lambda^2}$$

in the above equation $\sigma = -(1 + \varepsilon)\Lambda$.

In order to have a real positive remnant radius ($R_r > 0$) for $\Lambda < 0$, the following constraints must be satisfied:

$$\sqrt{\gamma^2 + \sigma} - \gamma < 0,$$  \tag{25}

$$\gamma^2 + \sigma > 0,$$  \tag{26}

(in which due to Eq. (25), we find the requirement that $\sigma < 0$ (or $\varepsilon < -1$). Considering the condition (26), we obtain two possible ranges for $\gamma$: either $\gamma \geq \sqrt{-\sigma}$ or $\gamma \leq -\sqrt{-\sigma}$. However, note that Eq. (25) implies that $\gamma > 0$, so we must have $\gamma \geq \sqrt{-\sigma}$.

(Now, we focus on the remnant mass of black holes given by Eq. (24). In order to have positive value of remnant mass, the numerator of Eq. (24) has to be positive, which contains two factors:

(I) $-2\sigma + \gamma \left( \sqrt{\gamma^2 + \sigma} - \gamma \right),$

(II) $\sqrt{\gamma^2 + \sigma} - \gamma.$

They must either be both positive or both negative. However, factor (II) is nothing but $-\Lambda R_r$, which we want to be positive (for $\Lambda < 0$). According to Eq. (25), the factor (II) is negative. It thus follows that we need factor (I) to also be negative. Since previously $\sigma < 0$, the term $-2\sigma$ in factor (I) is positive. The question is whether the factor (I) can be negative enough. The answer is no. In order to have the negative value for the factor (I), we need $\gamma \left( \sqrt{\gamma^2 + \sigma} - \gamma \right) < 2\sigma$. Since $\gamma \geq \sqrt{-\sigma}$, we define $\gamma := s\sqrt{-\sigma}$ in which $s \geq 1$. Replacing $\gamma = s\sqrt{-\sigma}$ in $\gamma \left( \sqrt{\gamma^2 + \sigma} - \gamma \right) < 2\sigma$, we obtain

$$s\sigma \left( s - \sqrt{s^2 - 1} \right) < 2\sigma.$$  \tag{27}

Since $\sigma < 0$, this means we need

$$s \left( s - \sqrt{s^2 - 1} \right) > 2,$$  \tag{28}

which yields $s < -2\sqrt{3}/3$, contradicting $s \geq 1$. This shows that there is no black hole remnant for $\Lambda < 0$ (or black hole remnant which is asymptotically de Sitter).

III. BLACK HOLE REMNANT AND INFORMATION PARADOX IN ANTI-DE SITTER-LIKE UNIVERSE

There are a few motivations for black hole remnants, one of which is that remnants prevent black holes from becoming arbitrarily hot during the end stage of the evaporation. Probably no one expects Hawking temperature to be truly divergent in the $M \to 0$ limit, but exactly what prevents just such a divergence is not agreed upon. One possibility is simply that new physics comes in at sufficiently high energy, thereby stopping black holes from evaporating further. Just such a possibility was investigated in [36] by appealing to the generalized uncertainty principle (GUP), which modifies quantum mechanics taking into account correction due to strong gravitational field. The remnant solution therein exhibits a rather peculiar property that its temperature is positive – how could a black hole be a remnant (not losing mass) yet continue to have Hawking radiation? One possible way out is to interpret this temperature as the internal energy of the remnant (since $E \sim kT$). The specific heat of the remnant is zero, and therefore it does not interact with the thermal environment [37, 38]. This means the remnant is stable, a pre-requisite for it to serve as dark matter candidate. Indeed, such a black hole remnant derived from GUP has been proposed as possible dark matter candidate [39].

Another virtue of black hole remnant is that it might be able to ameliorate the information paradox of black hole. The usual proposal to preserve quantum information is by having it scrambled and entangled in the Hawking radiation. Consider a black hole formed by a pure state. By unitarity one should recover pure state at the end of the black hole evaporation. The attempt to purify the Hawking radiation has given rise to issues like firewall [40]. The remnant picture, first proposed in [41], avoided this problem by proposing that Hawking radiation is never purified – states behind the horizon and states in the Hawking radiation remains mixed separately, but taken as a whole it is a pure state. Such a proposal is not without problems. For example, in order to hide plenty of quantum states behind the ever shrinking horizon, the Bekenstein-Hawking entropy does not reflect all the interior degrees of freedom. There is also the infinite production problem. Both of these problems are discussed in details in [3]. The bottom line is that despite these issues, remnants should not be dismissed outright, and could well help to resolve the information paradox, especially if they have huge interiors due to non-trivial geometries. All these
comments apply also to our massive gravity remnants, with the caveats that our remnants exist in anti-de Sitter spacetime.

There are two ways in which our remnants can be relevant to the information paradox. The first possibility is more straightforward: as we have explained in the introduction, our Universe could actually be asymptotically anti-de Sitter in the far future, with the current phase of acceleration caused by other fields [42] (This would also avoid the recently raised “swampland” issue of de Sitter space [43]). In [42], a quintessence was used. It is now appreciated that a simple quintessence model is difficult to be realized in string theory without fine tuning, so appealing to one to avoid the Swampland is swapping one difficulty with another [44–47]. However, other more complicated fields could still do the job [48].

Of course, there is the subtlety that the theory still needs to be coupled with these other fields and then strictly speaking the remnant solution would be different (if it still exists). If our Universe is asymptotically de Sitter, as most cosmologists believe, then our massive gravity remnants cannot be straightforwardly applied to understand actual black holes. Nevertheless, it is hoped that black hole remnants in the anti-de Sitter bulk may – eventually – help us to understand how information is preserved via holographic correspondence to a field theory on the conformal boundary.

The thermal stability of the massive gravity remnant is demonstrated by the fact that the heat capacity is zero, much like the remnant obtained from GUP mentioned above (this is not always the case for all GUP models, see e.g., [38]). This means that we have a thermodynamically inert and stable remnant. However, unlike the GUP remnant, its temperature is also zero. This is in fact much more natural – no mass is lost via Hawking emission and thus the remnant is stable. While we could argue that the GUP remnant temperature is really its internal energy, this feels somewhat contrived in comparison. Since our black hole has no electrical charge, the remnant is not like an extremal charged black hole, which could continue to radiate (despite having zero temperature) via non-thermal processes such as Schwinger pair production [3]; the remnant is arguably more stable and long-lived.

IV. DISCUSSIONS

In this work we investigated whether dRGT massive gravity can admit remnant scenario, and found that it is indeed possible. To our knowledge, this is the first black hole remnant found in dRGT massive gravity. The black hole tends to zero temperature remnant with vanishing specific heat, at which point it stops evaporating and becomes stable. The remnant only has positive mass for $\gamma < 0$. Massive gravity remnant could help to ameliorate the information paradox, modulo the usual challenges [3]. Here we discuss several issues and outlook for future works. Note that in [49], a solution in dyonic massive gravity was discussed in which there is a “remnant temperature”, i.e. in the limit of vanishing radius, the temperature is nonzero – it is not a black hole remnant in the sense studied in this work.

In this work, we chose the reference metric

$$f_{ab} = \text{diag}(0, 0, c^2, c^2 \sin^2 \theta).$$

(29)

Since different reference metric might give different results, a more detailed analysis is required to find out how our results may change if another reference metric is chosen. In particular, although our result shows that remnants can only exist in asymptotically anti-de Sitter spacetime, other reference metric may allow remnants to exist in asymptotically de Sitter spacetime as well. This will require further investigations.

It is worth commenting on the cutoff scale of massive gravity theory. The cutoff scale of any theory is the scale beyond which the theory breaks down (in the sense of effective field theory), and one would expect new physics to come in at higher energy scale. The cutoff scale of general relativity (with massless graviton) is the Planck scale. As mentioned in [11], the cutoff scale for massive gravity is not the same as the strong-coupling scale of the theory. Furthermore, the latter need not necessarily mean that there is new physics, but only that perturbation theory breaks down. In fact strong gravity tends to raise the strong coupling scale [50]. Except from very near the singularities, black hole solutions we discussed here are valid in massive gravity theory.

Demonstrating that dRGT massive gravity admits black hole remnant solutions is only the first step in the analysis. One needs to consider the actual evolution of the black hole under Hawking evaporation. That is to say, one has to study the mass loss rate $dM/dt$. The importance of doing so is to check if the remnant state is attainable, i.e. if it can be reached in a finite time, such an analysis would be important to study the Page time [51–53] of the black hole. (Conversely, even if there is no remnant, one could have an “effective remnant” if the evaporation rate is infinite [38].) Presumably if the third law of black hole thermodynamics is valid for such black hole, it would take infinite amount of time to reach zero temperature state. In addition to the mass loss rate $dM/dt$, one should also study the sparsity of the Hawking radiation [38, 54, 55], which affects the lifetime of the black hole. This is beyond the scope of the current paper, and is left for future works.

As mentioned in Sec.(I), dRGT massive gravity suffers from a variety of problems, most notably the causality issue which plagues the theory with superluminal propagation and arbitrarily small closed causal curves, thus rendering the theory rather unpredictable. In addition, a “god-given” reference metric is somewhat unsatisfactory. These has led to the considerations of bimetric (Hassan-Rosen) theory [15], in which the reference metric $f_{ab}$ is dynamical. Such a theory has some advantages over the original massive gravity [56], and its causal structures and constraints are gradually being understood [24, 57], though more research is clearly needed.
Finally, let us speculate on the possibility that massive gravity remnants may be dark matter candidate. Black hole remnants as dark matter is of course not a new idea, see, e.g. [58–60] for some early examples. If our Universe is fundamentally anti-de Sitter, which the current phase of accelerated expansion caused by another field, say a quintessence, then it is possible that massive gravity remnants may play a role as dark matter. In addition, the idea that massive gravitons might be dark matter themselves had been proposed quite a few years back [61]. Massive gravitons remain possible as dark matter candidate in the context of bimetric gravity [62–65]. If remnants exist in that theory they could serve as an additional dark matter candidate.

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