(Anti)evaporation of Dyonic Black Holes in string-inspired dilaton $f(R)$-gravity

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We discuss dyonic black hole solutions in the case of $f(R)$-gravity coupled with a dilaton and two gauge bosons. The study of such a model is highly motivated from string theory. Our Black Hole solutions are extensions of the one firstly studied by Kallosh, Linde, Ortín, Peet and Van Proeyen (KLOPV) in [arXiv:hep-th/9205027]. We will show that extreme solutions are unstable. In particular, these solutions have Bousso-Hawking-Nojiri-Odintsov (anti)evaporation instabilities.

I. INTRODUCTION

As is known, the low energy limit of a dimensionally reduced superstring theory to $d = 4$ is $\mathcal{N} = 4$ supergravity. There are two versions: $SO(4)$ and $SU(4)$. The first one is invariant under a (rigid) $SU(4) \times SU(1,1)$ symmetry. Black hole solutions of the reduced sector $U(1)^2$ were studied by Kallosh, Linde, Ortín, Peet and Van Proeyen (KLOPV) in Ref. [1]. In particular, they consider $U(1)^2$ charged dilaton black holes. These solutions are Reissner-Nordström-like black holes, or more precisely of dyonic black holes. In particular, the dilaton field is the real part of an initial complex scalar, while the imaginary part is an axion pseudoscalar field. They assumed the axion stabilized to a constant VEV. The effective bosonic action corresponds to the Einstein-Hilbert sector coupled to the dilaton and two $U(1)$ fields. Extreme limits of dyonic solutions are shown to saturate $\mathcal{N} = 4$ supersymmetry in $d = 4$. On the other hand, the presence of non-perturbative stringy effects could modify the effective action in the low energy limit. For instance, higher derivative terms may be generated by Euclidean D-brane or worldsheets instantons. In particular, the Einstein-Hilbert sector coupled to the dilaton and $U(1)$-fields can be extended from $R$ to an analytic function $f(R)$ (See Ref.[1] for a review on this subject).

KLOPP solutions are particularly important in string theory. For instance, the famous derivation of the Hawking BH entropy from BPS microstates shown by Strominger and Vafa is based on five dimensional KLOPP solutions [2]. The Vafa-Strominger result has inspired the so called fuzzball proposal, which has the ambition to solve the BH information paradox [3].

In this paper, we will study black hole solutions in string inspired $f(R)$-gravity, coupled with a dilaton field and two gauge bosons. We assume that the asymptotic space-time is Minkowski’s one. Let us clarify that we will not consider a $f(R)$-supergravity coupled to gauge bosons and dilatons. In fact, it was recently shown that the only $f(R)$-supergravity which is not plagued by ghosts and tachyons is Starobinsky’s supergravity [9,10]. Nevertheless, one can consider the case in which higher derivative terms are generated by exotic instantons or fluxes after a spontaneous supersymmetry breaking mechanism. In this sense, our model, which has a stable vacua and it is not plagued by ghosts and tachyons, is inspired by string theory. Clearly, to calculate instantonic corrections from a realistic stringy model is, at the moment, impossible. We believe that this highly motivates our effective field theory analysis, in which coefficients inside the $f(R)$-functional parametrize our ignorance about the string theory vacua. We will show that extreme dyonic solutions have Bousso-Hawking-Nojiri-Odintsov (BHNO) (anti)evaporation instabilities. In particular, Nojiri and Odintsov have discovered (anti)evaporation instabilities in Reissner-Nordström black holes in $f(R)$-gravity [14]. At posteriori, our result is understood as a generalization of Nojiri-Odintsov calculations in Ref. [14]. On the other hand, the peculiar thermodynamical properties of antievaporating solutions were discussed in our recent paper [15].

FIG. 1. The evolution in time of the extremal dyonic BH horizon is displayed imposing initial condition $\phi_0 = \pm 1$ and $\beta = 1, 1.5, 2, 2.5, 3, 3.5$ (Log-Log-plot). This numerical solution is obtained from EoM at the second order of the perturbation theory. The growing and the decreasing solutions represent antievaporation and evaporation respectively.
II. DILATON-\(f(R)\)-GRAVITY

Let us consider the case of a \(f(R)\)-gravity with two \(U(1)\)-gauge bosons and a dilaton. In particular, we will consider the action

\[
S = \int d^4x \sqrt{-g} \left[ -f(R) + 2\phi^2 \partial\phi \partial\phi \right] + 2\nabla\phi \nabla\phi - e^{-2\phi} \left( 2F_{\mu\lambda} F_{\nu\sigma} g^{\lambda\sigma} - \frac{1}{2} g_{\mu\nu} F^2 \right)
\]

where

\[
F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}
\]

\[
\tilde{B}_{\mu\nu} = \partial_{\mu} \tilde{B}_{\nu} - \partial_{\nu} \tilde{B}_{\mu}
\]

and \(A_\mu, B_\mu\) are gauge bosons of \(U(1) \times U(1)\), we conveniently use unit \(2\kappa(4) = 1\), where \(\kappa(4)\) is the four dimensional gravitational coupling (coming from the Kaluza-Klein reduction of the ten-dimensional gravitational coupling). The action Eq.(1) comes from the \(SO(4), d = 4, N = 4\) supergravity and it is formulated in the Einstein-frame, with an opportune and understood redefinition of the dilaton field.

The Equations of Motion are

\[
0 = \nabla_\mu (e^{-2\phi} F^{\mu\nu})
\]

\[
0 = \nabla_\mu (e^{2\phi} \tilde{G}^{\mu\nu})
\]

\[
0 = \nabla^2 \phi - \frac{1}{2} e^{-2\phi} F^2 + \frac{1}{2} e^{2\phi} \tilde{G}^2
\]

\[
0 = f_R(R) R_{\mu\nu} + \frac{1}{2} (R f_R - f(R)) g_{\mu\nu} - \nabla_\mu \nabla_\nu f_R(R) + g_{\mu\nu} \Box f_R(R)
\]

\[
0 = 2\nabla_\mu \nabla_\nu \phi - e^{-2\phi} (2F_{\mu\lambda} F_{\nu\sigma} g^{\lambda\sigma} - \frac{1}{2} g_{\mu\nu} F^2)
\]

\[
- e^{-2\phi} (2\tilde{G}_{\mu\lambda} \tilde{G}_{\nu\sigma} g^{\lambda\sigma} - \frac{1}{2} g_{\mu\nu} \tilde{G}^2)
\]

A solution of these equations is

\[
ds^2 = e^{2U} dt^2 - e^{-2U} dr^2 - R^2 d\Omega^2
\]

\[
e^{2\phi} = e^{2\phi_0} \frac{r + \Sigma}{r - \Sigma}
\]

\[
F = \frac{Q e^{\phi_0}}{(r - \Sigma)^2} dt \wedge dr
\]

\[
\tilde{G} = \frac{P e^{\phi_0}}{(r + \Sigma)^2} dt \wedge dr
\]

\[
e^{2U} = \frac{(r - r_+)(r - r_-)}{R^2}
\]

\[
R^2 = r^2 - \Sigma^2, \quad \Sigma = \frac{P^2 - Q^2}{2M}, \quad r_\pm = M \pm r_0
\]

\[
r_0^2 = M^2 + \Sigma^2 - P^2 - Q^2 = M^2 + \Sigma^2 - e^{-2\phi_0} P^2 - e^{-2\phi_0} Q^2
\]

The solutions depend on independent parameters \(M, Q, P, \phi_0\). \(M\) is the BH mass, \(\phi_0\) is the asymptotic value of the dilaton field. \(Q_{cl} = e^{\phi_0} Q\) is the F-field electric charge, while \(P_m = e^{\phi_0} P\) is the G-field magnetic charge (electric charge of \(\tilde{G}\)).

These equations imply the relation

\[
C f_R(R_0) = q^2 = \sqrt{Q^2 + P^2} = e^{-\phi_0} \sqrt{Q^2_{cl} + P^2_m}
\]

where \(C\) is an integration constant.

In the case of an extremal dyonic black hole, the metric can be conveniently rewritten as [14]

\[
ds^2 = \frac{M^2}{\cosh^2 x} (dt^2 - dx^2) + M^2 d\Omega^2
\]

This suggests the ansatz

\[
ds^2 = M^2 e^{2\rho(x,t)} (dt^2 - dx^2) + M^2 e^{-2\phi(x,t)} (dt^2 - dx^2) d\Omega^2
\]

and the gravitational EoM can be rewritten as

\[
0 = -(-\tilde{\rho} + 2\phi + \rho'' - 2\phi^2 - 2\rho' \phi' - 2\tilde{\rho} \phi) f_R + \frac{M^2}{2} e^{2\phi} + \frac{\partial}{\partial r} f_R
\]

\[
\tilde{B}_{\mu\nu} = \partial_{\mu} \tilde{B}_{\nu} - \partial_{\nu} \tilde{B}_{\mu}
\]

\[
0 = \frac{M^2}{2} e^{2\phi} - (\tilde{\rho} + 2\phi'' - \rho'' - 2\rho'^2 - 2\rho' \phi' - 2\tilde{\rho} \phi) f_R - \frac{q^2 M^2 e^{2\phi}}{2} + \frac{\partial^2}{\partial x^2} f_R - \frac{\partial f_R}{\partial r} - \frac{\partial f_R}{\partial t}
\]

\[
- \frac{\partial^2}{\partial x^2} f_R + \frac{\partial f_R}{\partial t}
\]

\[
0 = (2\phi' - 2\phi' \phi - 2\rho' \phi - 2\tilde{\rho} \phi') f_R + \frac{\partial^2}{\partial r^2} f_R - \frac{\partial f_R}{\partial r} - \rho \frac{\partial f_R}{\partial t}
\]

\[
0 = -2 M^2 e^{-2\phi} f - e^{-2(\rho + \phi)} (-\tilde{\phi} + \phi'' + 2\phi'^2 + 2\rho'^2) f_R + + f_R + e^{-2(\rho + \phi)} \left( \phi' \frac{\partial f_R}{\partial t} - \phi' \frac{\partial f_R}{\partial r} \right) + \frac{q^2 M^2 e^{2\phi}}{2}
\]

\[
e^{-2\phi} \left[ \frac{\partial}{\partial r} \left( e^{-2\phi} \frac{\partial f_R}{\partial r} \right) + \frac{\partial}{\partial x} \left( e^{-2\phi} \frac{\partial f_R}{\partial x} \right) \right]
\]

\[
e^{-2\phi} \left[ \frac{\partial}{\partial t} \left( e^{-2\phi} \frac{\partial f_R}{\partial t} \right) + \frac{\partial}{\partial x} \left( e^{-2\phi} \frac{\partial f_R}{\partial x} \right) \right]
\]
Now, let us consider perturbations around the background extremal solution as

\[ \rho = -\ln(\cosh x) + \delta \rho, \quad \varphi = \delta \varphi \]  \hspace{1cm} (12)

The perturbed EoM are

\[ 0 = -\frac{f_R(R_0) + 2M^{-2}f_{RR}(R_0)}{2} \delta R - f_R(R_0)M^{-2}\cosh^2 x (-\delta \ddot{\rho} + 2\delta \dot{\varphi} + \delta \rho'' + 2\tanh x \delta \varphi') \]

\[ -2f_R(R_0)M^{-2}\delta \rho + f_{RR}(R_0)M^{-2}\cosh^2 x (\tanh x \delta R' + \delta R'') \]  \hspace{1cm} (13)

\[ 0 = -\frac{f_R(R_0) + 2M^{-2}f_{RR}(R_0)}{2} \delta R + 2f_R(R_0)M^{-2}\delta \rho \]

\[ -f_R(R_0)M^{-2}\cosh^2 x (\delta \ddot{\rho} + 2\delta \dot{\varphi} - \delta \rho'' + 2\tanh x \delta \varphi') \]

\[ + f_{RR}(R_0)M^{-2}\cosh^2 x (\tanh x \delta R' + \delta R'') \]  \hspace{1cm} (14)

\[ 0 = -\frac{f_R(R_0) + 2M^{-2}f_{RR}(R_0)}{2} \delta R - 2M^{-2}f_R(R_0)\delta \varphi \]

\[ -f_R(R_0)M^{-2}\cosh^2 x (-\delta \ddot{\varphi} + \delta \varphi'') \]

\[ -f_{RR}(R_0)M^{-2}\cosh^2 x (-\delta \dot{R} + \delta R'') \]  \hspace{1cm} (15)

A convenient parametrization of perturbations is

\[ \delta \rho = \rho_0 \cosh \omega \tau \cosh^3 x, \quad \delta \varphi = \varphi_0 \cosh \omega \tau \cosh^3 x \]  \hspace{1cm} (17)

where \( \rho_0, \varphi_0, \beta \) are arbitrary constants.

Solving EoM, we find conditions

\[ \omega^2 = \beta^2 \]  \hspace{1cm} (18)

and

\[ \beta = \beta_\pm = \frac{1}{2} \left[ 1 \pm \sqrt{1 - \frac{4}{3} M^2 \left( \frac{f_R(R_0)}{f_{RR}(R_0)} \right)} \right] \]  \hspace{1cm} (19)

from

\[ \Box \delta \varphi = [\beta^2 + \beta(\beta - 1) \cosh^{-2} x - \omega^2] \delta \varphi \]  \hspace{1cm} (20)

Let us note that \( \beta \) has always a Real part which is positive, implying exponential instabilities. In particular, for \( \phi_0 < 0 \) antievaporation \( \phi_0 > 0 \) evaporation. Clearly, this is not enough to demonstrate that the extremal solution is unstable. So that, we show the numerical solution of the horizon radius obtained by EoM perturbed up to the second order in \( \delta \rho, \delta \varphi \). Finally, we claim that a similar analysis in the case of the \( SU(4) \)-inspired model (despite of \( SO(4) \) gauge group) leads to the same kind of instabilities, as can be easily checked.

### III. CONCLUSIONS

In this paper, we have discussed dyonic BH solutions in \( f(R) \)-gravity coupled with a dilaton and two gauge bosons. We have shown that in the extremal limit of the internal radius saturating the external one, these solutions cannot be stable. In particular, their horizon radius explodes or rapidly decays in time, depending from the initial conditions. Such instabilities are interpreted as classical BH antievaporation and evaporation. Our result can be viewed as an analogous of the Nojiri-Odintsov calculation in Ref. [14].

The implications of our result are not fully understood. For instance, it is conceivable that (anti)evaporation can be related to deep theoretical issues like the holographic conjecture [17], the no-remnant conjecture [18] and, as a cascade, to many important related concepts of string theory and black holes. Such a result also represents an explicit violation of the generalized Birkhoff’s theorem, as an example of unstable spherically symmetric solution.

To conclude, in future companion papers, we hope to generalize our analysis in higher dimensions and in the case of extremal BH lying in a de Sitter or an Anti-de Sitter space-time.

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\[ \text{We mention that some solutions in other extended theories of gravity have also geodetic instabilities [16].} \]
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