Correlation, Linear Complexity, Maximum order Complexity on Families of binary Sequences

Zhixiong Chen¹, Ana I. Gómez², Domingo Gómez-Pérez∗³, Andrew Tirkel⁴
1. Key Laboratory of Applied Mathematics of Fujian Province University, Putian University, Putian, Fujian 351100, P. R. China
2. Universidad Rey Juan Carlos, Spain
3. Universidad de Cantabria, Spain
4. Scientific Technologies, Australia

July 27, 2021

Abstract

Correlation measure of order $k$ is an important measure of randomness in binary sequences. This measure tries to look for dependence between several shifted version of a sequence. We study the relation between the correlation measure of order $k$ and another two pseudorandom measures: the $N$th linear complexity and the $N$th maximum order complexity. We simplify and improve several state-of-the-art lower bounds for these two measures using the Hamming bound as well as weaker bounds derived from it.

Keywords. Pseudorandom sequences, Binary sequences, Correlation measure of order $k$, $N$th linear complexity, $N$th maximum order complexity

*Corresponding author: domingo.gomez@unican.es.
1 Introduction

For a positive integer $N$, the $N$th linear complexity $L(S, N)$ of a binary sequence $S = (s_i)_{i=0}^\infty$ over the two-element finite field $\mathbb{F}_2 = \{0, 1\}$ is the smallest positive integer $L$ such that there are constants $c_0, c_1, \ldots, c_{L-1} \in \mathbb{F}_2$ with

$$s_{i+L} = c_{L-1}s_{i+L-1} + \ldots + c_0s_i, \quad \text{for } 0 \leq i < N - L.$$  \hspace{1cm} (1)

We use the convention $L(S, N) = 0$ if $s_0 = \ldots = s_{N-1} = 0$ and $L(S, N) = N$ if $s_0 = \ldots = s_{N-2} = 0 \neq s_{N-1}$. The $N$th linear complexity is a measure for the predictability of a sequence and thus its unsuitability in cryptography. If $S$ is $T$-periodic, we have $L(S, N) = L(S, 2T)$ for $N \geq 2T$. This number is the linear complexity of the sequence $S$.

Analogously, the $N$th maximum-order complexity $M(S, N)$ of a binary sequence $S = (s_i)_{i=0}^\infty$ is defined as the smallest positive integer $M$ such that there is a polynomial $f(x_1, \ldots, x_M) \in \mathbb{F}_2[x_1, \ldots, x_M]$ with

$$s_{i+M} = f(s_i, s_{i+1}, \ldots, s_{i+M-1}), \quad \text{for } 0 \leq i < N - M,$$

see [13, 12, 19]. Again, if the sequence is $T$-periodic, $M(S, N) = M(S, 2T)$ for $N \geq 2T$. This is called maximum-order complexity of $S$.

Obviously, we have $M(S, N) \leq L(S, N)$, so maximum-order complexity is a finer measure of pseudorandomness than linear complexity.

Let $k$ be a positive integer. The $(N$th$)$ correlation measure of order $k$ of $S$ is defined as

$$C_k(S, N) = \max_{U,D} \left| \sum_{n=0}^{U-1} (-1)^{s_{n+d_1} + s_{n+d_2} + \ldots + s_{n+d_k}} \right|,$$

where the maximum is taken over all $U \leq N - k + 1$ and $D = (d_1, d_2, \ldots, d_k)$ with integers $0 \leq d_1 < d_2 < \ldots < d_k \leq N - U$. This is an adaptation to the binary case of the definition concerning sequences over $\{ -1, +1 \}$, introduced by Mauduit and Sárközy [13].

Brandstätter and Winterhof [11] proved the following relation between the $N$th linear complexity and the correlation measures of order $k$:

$$L(S, N) \geq N - \max_{1 \leq k \leq L(S, N)+1} C_k(S, N), \quad \text{for } N \geq 1.$$  \hspace{1cm} (2)

Recently, Işık and Winterhof [11] have derived an analogous result concerning the $N$th maximum-order complexity:

$$M(S, N) \geq N - 2^{M(S, N)+1} \cdot \max_{1 \leq k \leq M(S, N)+1} C_k(S, N), \quad \text{for } N \geq 1.$$  \hspace{1cm} (3)
Roughly speaking, any sequence with small correlation measure up to a sufficiently large order $k$ must have a high $N$th maximum-order complexity (and hence $N$th linear complexity) as well. For surveys on linear complexity and related measures of pseudorandomness, see [9, 15, 16, 21, 22, 25].

The problem with these bounds is that they seem to be far from tight. Even if the correlation measure is close to the expected value for a random binary sequence, the bounds above are far from expected.

Due to the constraints on the $N$th correlation measure given by Gyarmati and Mauduit [10], which implies that the correlation measure is bigger than $\sqrt{N}$ for many orders, the lower bound in Equation (2) is $2\sqrt{N}$. For $M(S, N)$, the lower bound can not be greater than $(\log N)/2$, see e.g. [11]. Notice that the expected $N$th linear complexity of a random binary sequence is $N/2$ [20]. For the $N$th maximum-order complexity, the expected value is $2\log N$ [13].

In this work, we discuss the higher order correlation of binary sequences, improving the lower bounds shown in Equations (2) and (3). Then, we review the literature and improve the lower bounds on linear complexity and maximum-order complexity of several known sequences.

Our results are based on the Hamming bound on error-correcting codes (see e.g. [18, Theorem 3.4.6]). Additionally, we use the following definition for the periodic correlation measure of order $k$ of a $T$-periodic binary sequence $S$,

$$\theta_k(S) = \max_D \left| \sum_{n=0}^{T-1} (-1)^{s_{n+d_1} + s_{n+d_2} + \ldots + s_{n+d_k}} \right|,$$

where $D = (d_1, \ldots, d_k)$, with $0 \leq d_1 < d_2 < \ldots < d_k < T$.

A binary sequence $S$ is said to have a full peak in the aperiodic correlation measure of order $k$ if $C_k(S, N) = N - k + 1$. It has a half peak if $C_k(S, N) \geq N/2$. The same definitions apply also for $\theta_k(S)$, the periodic correlation measure of order $k$.

We suppress “of order $k$” when referring to the correlation measure when the order $k$ is clear from the context.

## 2 Higher-order correlation measure

We prove below a link between the linear complexity of a sequence and its correlation measure. Before, we state a direct consequence of the Hamming bound [18, Theorem 3.4.6].
Lemma 1. Let $p$ be a prime number and $C \subseteq \mathbb{F}_p^T$ a linear subspace of dimension $d$ (i.e. a linear code over $\mathbb{F}_p$). If, for some integer $t > 0$,

$$\sum_{i=0}^{[\frac{(t-1)/2]}{2}} \binom{T}{i}(p-1)^i > p^{T-d},$$

there exists a nonzero vector $\vec{v} \in C$ with at most $t$ nonzero components.

The strong relation between cyclic codes and periodic sequences allows using the previous lemma to relate the linear complexity of a sequence with the existence of full peaks in the periodic correlation measure.

Theorem 2. Let $S = (s_i)_{i=0}^\infty$ be a $T$-periodic binary sequence with linear complexity $L$. If, for some integer $t > 0$,

$$\sum_{i=0}^{[\frac{(t-1)/2]}{2}} \binom{T}{i} \geq 2^L,$$

the sequence has a full peak in the periodic correlation measure $\theta_k(S)$ for some $k$ with $1 < k \leq t$, i.e., $\theta_k(S) = T$.

Proof. Let $C \subseteq \mathbb{F}_2^T$ be the linear subspace generated by

$$(s_0, s_1, \ldots, s_{T-1}), (s_1, s_2, \ldots, s_0), \ldots, (s_{T-1}, s_0, \ldots, s_{T-2});$$

i.e. a sequence’s period and all its shifted versions. We denote by $C^\perp$ the orthogonal subspace of $C$, i.e.

$$C^\perp = \left\{(c_0, \ldots, c_{T-1}) \in \mathbb{F}_2^T : \sum_{i=0}^{T-1} c_i s_{n+i} = 0, \forall n \geq 0 \right\}.$$ 

It is trivial to check that $\dim(C) = L$, and hence $\dim(C^\perp) = T - L$. By Lemma [Lemma 1](with $p = 2$), there exists a vector in $C^\perp$ with exactly $k \leq t$ nonzero components. Let $d_1, \ldots, d_k$ be their indices, so

$$\sum_{j=1}^{k} s_{n+d_j} = 0, \forall n \geq 0.$$ 

This implies that there is a full peak in periodic correlation measure of order $k$. \qed
For the aperiodic correlation, we have the following result.

**Theorem 3.** Let \( S = (s_i)_{i=0}^{\infty} \) be a \( T \)-periodic binary sequence with \( N \)th linear complexity \( L(S, N) \). If, for some integer \( t > 0 \),

\[
\left( \left\lfloor \frac{N}{2} \right\rfloor \right)_t \geq 2^{L(S, N)},
\]

the sequence has a half peak in the aperiodic correlation measure \( C_k(S, N) \) for some \( k \) with \( 1 < k \leq 2t \), i.e., \( C_k(S, N) \geq N/2 \).

**Proof.** Suppose that the sequence satisfies Equation (1), which means that the first \( L(S, N) \) elements and the recurrence generates the next \( N - L(S, N) \). There are at most \( 2^{L(S, N)} \) different sequences of length \( N \) that can be generated by the same linear recursion.

One the other hand, any sequence \( (y_n)_{n=0}^{\infty} \) defined as

\[
y_n = \sum_{j=1}^{t} s_{n+d_j}, \quad \text{with } 0 \leq d_1 < \ldots < d_t < \left\lfloor \frac{N}{2} \right\rfloor,
\]

(4)

can also be generated by that linear recursion. There are at least \( \left( \left\lfloor \frac{N}{2} \right\rfloor \right)_t \) ways of choosing that shift set \( \{d_j\} \).

Therefore, by hypothesis, there exist two different ordered list of shifts: \( \{d_1, \ldots, d_t\} \) and \( \{e_1, \ldots, e_t\} \) such that

\[
\sum_{j=1}^{t} s_{n+d_j} = \sum_{j=1}^{k} s_{n+e_j} \implies \sum_{j=1}^{t} (s_{n+d_j} - s_{n+e_j}) = 0,
\]

(5)

for \( 0 \leq n \leq \left\lfloor \frac{N}{2} \right\rfloor \leq N - \max\{d_t, e_t\} \). Then, there is a half peak in the \( N \)th correlation measure of order at most \( 2t \). \( \square \)

The following result, which we state for its applications, is a direct consequence of Theorem 3.

**Corollary 4.** Given any positive integers \( K \) and \( N \) with \( K^2 < N \). If a binary sequence \( S \) satisfies \( C_k(S, N) < N/2 \) for every \( k < K \), we have

\[
L(S, N) > \frac{1}{2} K (\log N + 1 - \log K) - \frac{1}{2} \log K + \delta,
\]

where \( \delta \) is an absolute constant.
Proof. Because the result of Theorem 3 does not hold, it must be the case that:

$$2^{L(S,N)} \geq \left(\frac{\lfloor N/2 \rfloor}{K/2}\right),$$

(6)

where substitute the combinatorial number by the Stirling approximation

$$\left(\frac{\lfloor N/2 \rfloor}{K/2}\right) \approx \left(\frac{Ne^K}{K}\right)^{K/2} \frac{1}{(2\pi K)^{1/2} \varepsilon},$$

where $\varepsilon$ is some positive constant. Taking logarithms at both sides of Equation (6), we get the result.

We compare Equation (2) and this new bound. First, whenever we can apply the former, Corollary 4 applies as well and the lower bound is improved by a factor of $\log N$. Also, it is enough to obtain a non-trivial bound for $C_k(S,N)$, a strong bound being no longer necessary.

These results have immediate application to the families of binary sequences summarized in Table 1. For those sequences’ definition, as well as parameters and properties, see the book of Golomb and Gong [8].

| Family         | Period | Linear complexity | Bound on $k$ for the existence of a peak |
|----------------|--------|-------------------|------------------------------------------|
| $m$-sequences  | $2^\ell - 1$ | $\ell$            | 3                                        |
| Small Kasami   | $2^\ell - 1$ | $3\ell/2$        | 5                                        |
| Gold codes     | $2^\ell - 1$ | $2\ell$          | 7                                        |
| Large Kasami   | $2^\ell - 1$ | $5\ell/2$        | 9                                        |
| 3-term trace   | $2^\ell - 1$ | $3\ell$          | 9                                        |
| 5-term trace   | $2^\ell - 1$ | $5\ell$          | 11                                       |
| Welch-Gong     | $2^\ell - 1$ | $2^{\ell/3} + 1$ | $(2^{\ell/3} + 1)/\ell$                  |

Table 1: Different families of binary sequences, together with the upper bound on $k$ such that there exists a peak in the periodic correlation measure of order $k$ according to Theorem 2.

Results on Small Kasami and $m$-sequences have already been discovered by Warner [24, 25]. In the case of the Gold codes, Adams [1] presented some results regarding partial peaks and conjectured on full peaks for order 9. Boztas and Parampalli [3] studied the third-order correlation in order to assure the probability of intercept of Gold codes.
We now enunciate a simple theorem of the same flavour for the $N$th maximum order complexity, improving the bound in Equation (3).

**Theorem 5.** If a binary sequence $S$ satisfies $M(S, N) \leq \log N - 2$, it has a half peak in the aperiodic correlation measure of order 2, i.e. $C_2(S, N) \geq N/2$.

**Proof.** In order to simplify the notation, $M = M(S, N) < \log N - 2$. Under the hypothesis and since the first $N$ elements of the sequence can be generated by a polynomial with $M$ variables, i.e.

$$s_{i+M} = f(s_i, s_{i+1}, \ldots, s_{i+M-1}), \text{ for } 0 \leq i < N - M.$$ 

By [13, Proposition 2], the period of the sequence $(s_i)_{i=0}^{\infty}$ is less than $2^M$, see the explanation in the footnote 1.

This means that there exists $0 \leq d_1 < d_2 < 2^M$ such that $s_{i+d_1} = s_{i+d_2}$ for $0 \leq i < N - M - d_2$. The final step is $N - M - d_2 > N - \log N + 2 - N/4 > N/2$ and this finishes the proof. 

\[\Box\]

## 3 Some applications

**Hall’s sextic residue sequence.** The recent work of Aly and Winterhof [2] studied Hall’s sextic sequence, which is a binary sequence with prime period $T = 1 \mod 6$. For such a period and a primitive root modulo $T$, say $g$, Hall’s sextic residue sequence $\mathcal{H} = (h_n)_{n=0}^{\infty}$ is defined as follows: let

$$C_{\ell} = \{g^{6i+\ell} \mid 0 \leq i < (T - 1)/6\}, \quad \ell = 0, 1, \ldots, 5,$$ 

be the cyclotomic cosets modulo $T$ of order 6. Then, for $n \geq 0$,

$$h_n = \begin{cases} 1, & \text{if } n \mod T \in C_0 \cup C_1 \cup C_3; \\ 0, & \text{otherwise}. \end{cases}$$ 

Hall’s sextic sequence has several desirable features of pseudorandomness, one of them being low correlation measure:

$$C_k(\mathcal{H}, N) = O\left(\left(\frac{14}{3}\right)^k k\sqrt{T \log T}\right).$$ 

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1The idea is that the different possibilities for the tuples $(s_i, s_{i+1}, \ldots, s_{i+M-1})$ is, at most, $2^M$. The tuple defines the next element, so this bounds the period of the sequence.
Using this bound and the lower bound proved by Brandstätter and Winterhof \[4\], it is shown in the reference article \[2\] that the $N$th linear complexity is $\Omega(\log T)$. This is improved in the following result.

**Corollary 6.** For any $\varepsilon > 0$, a sufficiently large $T$ and $N > 2T^{1/2+\varepsilon}(\log T)^2$, the $N$th linear complexity of Hall’s sextic sequence $\mathcal{H}$ satisfies

$$L(\mathcal{H}, N) \gg (\log N)^2,$$

where the implied constant depends on $\varepsilon$.

**Proof.** The correlation measure of order $k$ of Hall’s sextic sequence is less than $N/2$ for $k \leq \varepsilon \log T/8$, if $N \gg T^{1/2+\varepsilon}$. This is simple to see substituting in Equation (9),

$$\left(\frac{14}{3}\right)^k k\sqrt{T} \log T \leq \left(\frac{14}{3}\right)^{\log T/8} \log T \sqrt{T} \log T \leq T^{1/2+\varepsilon/2}(\log T)^2 < N/2.$$

By Theorem 3 we have

$$L(\mathcal{H}, N) \geq \varepsilon \log T/16(\log N - \log \varepsilon - \log \log N - 3) - \log \log N + \delta \gg (\log N)^2.$$

This finishes the proof. \[\square\]

**Fermat quotient threshold sequence.** For prime $p$ and an integer $u$ with $\gcd(u, p) = 1$, the **Fermat quotient** $q_p(u)$ modulo $p$ is defined as the unique integer with

$$q_p(u) = \frac{u^{p-1} - 1}{p} \pmod{p}, \quad 0 \leq q_p(u) < p.$$

We also define

$$q_p(kp) = 0, \quad \text{for } k \in \mathbb{Z}.$$

Note that $(q_p(u))$ is a $p^2$-periodic sequence modulo $p$, so $T = p^2$. Then the **binary threshold sequence** $\mathcal{E} = (e_n)_{n=0}^{\infty}$ is defined by

$$e_u = \begin{cases} 
0, & \text{if } 0 \leq q_p(u)/p < \frac{1}{2}; \\
1, & \text{if } \frac{1}{2} \leq q_p(u)/p < 1.
\end{cases}$$

Note that for which applications a discrepancy bound with arbitrary shifts is needed. Most discrepancy bounds on nonlinear pseudorandom numbers found in the literature consider only equidistant shifts.
Using the same techniques, Chen et al. [7] proved a bound on the correlation measure. In Theorem 3 of that paper, they showed that

\[ C_2(\mathcal{E}, N) \ll p(\log p)^3. \] (10)

The following corollary gives a new lower bound on the \(N\)th linear complexity.

**Corollary 7.** For any \(\varepsilon > 0\), a sufficiently large \(p\) and \(N > 2p^{1+\varepsilon}(\log p)^3\), the \(N\)th linear complexity of the binary threshold sequence \(\mathcal{E}\) satisfies

\[ L(\mathcal{E}, N) \gg \log N, \]

where the implied constant depends on \(\varepsilon\).

**Proof.** Again, it is easy to see that if \(N > 2p^{1+\varepsilon}(\log p)^3\), then the correlation of the sequence of order 2 is less than \(N/2\). By Corollary 4 taking \(K = 3\) and using the bound in Equation (10), we get the result. \(\square\)

This improves the bound of order \((\log N - \log p)/\log \log p\), given by Chen et al. [7, Theorem 4]. As shown by this result, even weak bounds lead to improvements on the correlation measure provides information about the linear complexity.

**Error linear complexity profile of sequences.** Another application is to lower bound the \(K\)-error linear complexity profile, i.e. the minimum linear complexity profile among sequences differing from the studied one in at most \(K\) entries. In particular, let us bound the \(K\)-linear complexity of \(\mathcal{E}\) and \(\mathcal{H}\).

**Corollary 8.** For \(N < T(= p^2)\), the \(N\)th linear complexity of the binary threshold sequence \(\mathcal{E}\), allowing at most \(N/6\) entry switches, is greater than \(\log N\).

**Proof.** Notice that a change in \(N/6\) or fewer sequence elements increases the value of the correlation measure of order 2 in \(N/3\). This is trivial to see from the definition, because it modifies at most \(N/3\) terms, so the correlation goes up by \(N/3\).

Together with the bound in Equation (10), we obtain the result. \(\square\)

The proof of the next result follows the same path as the previous one.

**Corollary 9.** For \(N < T\), the \(N\)th linear complexity of Hall’s sextic sequence \(\mathcal{H}\), changing at most \(N/6\), is greater than \((\log N)^2\).

In Table 2 we compare with previous results the obtained bounds for the \(N\)th linear complexity of several sequences. The resulting bound by Theorem 5 on the \(N\)th maximum order complexity for all of the sequences listed in the table is \(\log N - 2\).
| Sequence                          | Previous lower bound | Corollary |
|----------------------------------|----------------------|-----------|
| Logarithm threshold sequence     | \( \log N / \log \log T \)   | \( \log N \)   |
| Two-prime generator sequence     | \( N / \sqrt{T} \)       | \( \sqrt{N} \log N \)   |
| Modified inverse threshold sequence | \( \log N / \log \log T \) | \( (\log N) \log \log T \)   |
| Binary cyclotomic sequence       | \( \log N / \log \log T \) | \( (\log N) \log \log T \)   |
| Inversive threshold sequence     | \( \log N / \log \log T \) | \( (\log N) \log \log T \)   |

Table 2: Bound comparison. The previous results are stated using simplified notation, where \( T \) stands for the period.
4 Conclusions and Acknowledgments

This paper presents generalizations of the results appearing in the articles [4] and [11]. Thanks to these results, it is possible to use these results mount correlation attacks in systems using standard families of binary sequences like Gold codes and Kasami families (see Table 1). The results regarding the aperiodic form of the correlation measure of order $k$ improve the lower bound on the $N$th linear complexity given several papers, as stated in Table 2. For those sequences, we provide new non-trivial lower bounds on the maximum order complexity.

Domingo Gómez-Pérez and Ana I. Gómez are supported by the Spanish Agencia Estatal de Investigación project Secuencias y curvas en criptografía (PID2019-110633GB-I00/AEI/10.13039/501100011033).

Z. Chen was partially supported by the National Natural Science Foundation of China under grant No. 61772292, and by the Provincial Natural Science Foundation of Fujian, China under grant No. 2020J01905.

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