The oscillatory dynamics of nanoelectromechanical systems (NEMS) is at the heart of many emerging applications in nanotechnology. For common NEMS, such as beams and strings, the oscillatory dynamics is formulated using a dissipationless wave equation derived from elasticity. Under a harmonic ansatz, the wave equation gives an undamped free vibration equation; solving this equation with the proper boundary conditions provides the undamped eigenfunctions with the familiar standing wave patterns. Any harmonically driven solution is expressible in terms of these undamped eigenfunctions. Here, we show that this formalism becomes inconvenient as dissipation increases. To this end, we experimentally map out the position- and frequency-dependent oscillatory motion of a NEMS string resonator driven linearly by a non-symmetric force at one end at different dissipation limits. At low dissipation (high $Q$ factor), we observe sharp resonances with standing wave patterns that closely match the eigenfunctions of an undamped string. With a slight increase in dissipation, the standing wave patterns become lost and waves begin to propagate along the nanostructure. At large dissipation (low $Q$ factor), these propagating waves become strongly attenuated and display little, if any, resemblance to the undamped string eigenfunctions. A more efficient and intuitive description of the oscillatory dynamics of a NEMS resonator can be obtained by superposition of waves propagating along the nanostructure.

Nanoelectromechanical systems (NEMS) have enabled a number of nanotechnologies for monitoring the environment\cite{1}, storing and processing information\cite{2,3}, and applying controllable forces to physical\cite{4} and biological nanosystems\cite{5,6}. NEMS-based detection of individual atoms and molecules\cite{7,8}, single charge quanta\cite{9,10}, and vibrations of single microorganisms\cite{11,12} has established the potential of NEMS sensors. NEMS are also at the forefront of fundamental physical science, opening up studies in quantum mechanics\cite{13}, optomechanics\cite{14}, Brownian motion\cite{15,16}, fluid mechanics\cite{17,18,19,20}, and nanoelectronics\cite{21,22,23,24}.

In a typical implementation\cite{25}, one actuates linear oscillations of the NEMS resonator using a force transducer and looks for changes in the phase, frequency, or dissipation due to interactions. For proper operation, the user must know how exactly the nanomechanical structure is moving under the actuation forces. The oscillatory NEMS dynamics is typically determined using a dissipationless wave equation, e.g., the beam equation or the string equation, derived from elasticity. After the harmonic ansatz, one obtains the undamped free vibration equation and solves it subject to boundary conditions\cite{26,27}. This approach provides the undamped eigenfunctions that correspond to standing wave patterns on the structure. These well-known patterns emerge from the interference of undamped waves reflecting back and forth from the boundaries of the structure. The undamped eigenfunctions form a complete set, and the driven harmonic motion of NEMS, even in the presence of dissipation, can be expressed as an expansion in terms of these eigenfunctions\cite{26–30}. The practical aspects of the expansion, however, become cumbersome with increasing dissipation. Since waves get attenuated along the structure and at the boundaries, one needs a large, if not infinite, number of terms in the eigenfunction expansion. Here, we illustrate these complications by examining the position- and frequency-dependent oscillatory dynamics of a NEMS resonator driven by a non-symmetric harmonic force at different dissipation limits. Instead of an expansion including a large number of undamped eigenfunctions, we describe the dynamics efficiently by superposing waves that are attenuated along the beam.

Our experiments are performed on silicon nitride doubly-clamped beams with respective linear dimensions along $x$, $y$, and $z$ axes of $l \times b \times h \approx 50 \ \mu m \times 900 \ \text{nm} \times...
10.364 10.368
-360
-180
0
Phase [Degree]
Position [µm]
Frequency [MHz]
10.364 10.368

100 nm, all from the same batch. There is a 2-µm gap between the beam and the substrate. The beams are under tension as inferred from their resonance frequencies in vacuum, the two identical u-shaped gold thin-film nanoresistors of thickness of 135 nm and width of 120 nm on the anchors of the NEMS act as electrothermal actuators. The dimensions along x are $\xi_1 \approx 800$ nm and $\xi_2 \approx 600$ nm. (b) Numerical simulations of electrothermal actuation near (but not exactly at) the fundamental resonance frequency in vacuum. Sinusoidal current input to the nanoresistor at $f = 2.5895$ MHz (black curve) results in nanomechanical oscillations of the beam at $f = 5.179$ MHz. The response of the beam at its center is shown. The small phase between the current and displacement is due to the thermal inertia and the mechanical response of the resonator. The color maps are the temperature profiles of the region in inset of (a) at five instants. Typical powers dissipated on the nanoresistor are 1 $\mu$W, 50 $\mu$W and 100 $\mu$W in vacuum, air, and water, respectively. (c) Heterodyne optical interferometer. AOM: acousto-optic modulator; $\lambda/2$: half wave plate; $\lambda/4$: quarter wave plate; PBS: polarizing beam splitter; BS: beam splitter; PD: photodetector; PID: proportional-integral-derivative controller; VCO: voltage controlled oscillator. The signal on PD$_1$ is used for feedback; PD$_2$ is connected to a lock-in amplifier via a mixer for driven measurements. The lock-in amplifier is used in 2f mode. (d) Phase (top) and amplitude for a beam as functions of frequency and position at its first harmonic resonance in vacuum.
FIG. 2. Oscillatory dynamics of a 50-µm-long resonator at the low dissipation limit, i.e., in vacuum. (a-c) Normalized rms amplitudes \( W_{\text{rms}} \) and relative phases \( \phi_{\text{rel}} \) (insets) of the beam oscillations, as a function of \( x \) for its first three modes. Data points are from measurements, and the background shadings are \( |\sin (\frac{n \pi}{Q} x)| \). The phase values change by exactly 180° at the nodes. (d-f) Normalized time-dependent amplitude \( w(x,t) \) for the first three modes, as a function of \( x \) during an oscillation cycle. Relevant parameters of the modes are listed in Table I.

Hence, the actuation force is assumed to be independent of frequency.

The harmonically-driven linear dynamics of the NEMS resonator, i.e., the rms amplitude \( W_{\text{rms}} \) and phase \( \phi \) of its oscillations as functions of position, is measured in a heterodyne optical interferometer\(^{21}\). Fig. 1(c) shows the schematic diagram of the optical setup\(^{21,32}\). An XYZ stage is used to position the laser spot along the \( x \) axis in Fig. 1(a). Fig. 1(d) shows a representative data set: \( W_{\text{rms}} \) and \( \phi \) as a function of \( x \) and drive frequency for the first harmonic mode resonance of the NEMS in vacuum. The measured \( \phi \) can be understood as the phase of the NEMS oscillation with respect to the sinusoidal drive force and includes all the parasitic phases coming from the measurement circuit. We perform the measurements in vacuum, air, and water, corresponding respectively to the low \( (Q \gtrsim 10^3) \), intermediate \( (20 \lesssim Q \lesssim 70) \), and high dissipation \( (Q \sim 1) \) regimes. In vacuum and air, we measure the resonator amplitude and phase around the resonance frequencies [Supporting Information Figures S1 and S2]; in water, we sweep the frequency over our entire frequency range.

We first discuss the oscillatory dynamics of the NEMS at very low dissipation \( (Q \approx 20 \times 10^3) \), as shown in Fig. 2. From measurements shown in Supporting Information Figure S1, we obtain the spatial dependence of the rms resonance amplitude \( W_{\text{rms}} \) and the relative phase \( \phi_{\text{rel}} \) at each resonance frequency \( f_n = \frac{n}{Q} \) [Fig. 2(a-c)]. \( W_{\text{rms}} \) are the peak values in the amplitude vs. frequency curve for each resonance (Figure S1), and \( \phi_{\text{rel}} \) is the phase with respect to the phase of the first data point around \( x = 0 \), i.e., \( \phi_{\text{rel}}(x) = \phi(x) - \phi (x = 2.5 \mu m) \), at \( f_n \). The time-dependent motion of the beam at frequency \( f_n \) can be reconstructed from \( W_{\text{rms}} \) and \( \phi_{\text{rel}} \) as

\[
\begin{align*}
\phi(x, t) &= \sqrt{2} W_{\text{rms}}(x) \sin (\omega_n t + \phi_{\text{rel}}(x)) \quad \text{by advancing the dimensionless time } \omega_n t \text{ over a cycle.}
\end{align*}
\]

Fig. 2(d-f) shows normalized \( w(x,t) \) for the first three modes. Supporting Information includes video files of these modes.

Since \( Q \) is very high, we ignore the dissipation and solve the undamped string equation with fixed-fixed boundary conditions to obtain

\[
\begin{align*}
\phi_n(x) &= \frac{\sin (k_n x) \sin (\omega_n t)}{\sin (k_n L)}
\end{align*}
\]

Here, \( k_n = n \frac{\pi}{L} \) is the wave number and \( f_n = \frac{n}{Q} \) is the corresponding eigen-frequency. The speed of flexural waves in the beam is \( c = \sqrt{\frac{E}{\rho}} = 510 \pm 10 \text{ m/s} \), based on experimental values\(^{21}\) of tension \( \tau = 68 \pm 4 \mu N \) and...
mass per unit length $\mu = \rho_s bh = 26.6 \pm 0.3 \times 10^{-11}$ kg/m with the density being $\rho_s = 2960 \pm 30$ kg/m$^3$. The shadings in Figs. 2(a-c) are based on this solution for $n = 1, 2$ and 3. We note that $w(x,t)$ can also be written as a sum of two undamped propagating waves as $w(x,t) = \Re\{\frac{1}{2}e^{i(k_{x x}-\omega t)} - \frac{1}{2}e^{-i(k_{x x}+\omega t)}\}$, with $\Re$ denoting the real part of the complex expression. We emphasize that the boundary conditions are not trivial: there are undercuts and the gold nanoresistors around $x = 0$ and $x = l$. The rigidity of the beam also becomes appreciable near the clamps. Regardless, the low dissipation makes these complications negligible.

Now, we turn to the oscillatory dynamics of the same resonator in the intermediate dissipation limit by repeating the experiment in air. Since the quality factors, $Q^{(a)}_n$, in air are still relatively high (Table I), the modes are well separated in frequency (Supporting Information Figure S2). At a first glance, $W_{rms}$ and $\phi_{rel}$ data in Fig. 3(a-c) look similar to those in Fig. 2. The resonance frequencies also do not deviate much from their vacuum values (Table I). Upon more careful comparison with Fig. 2, however, we notice that the amplitudes in Fig. 3 become asymmetric with respect to the beam center and decay noticeably away from the actuator. To highlight these features, we show in the background of Fig. 3(a-c) the undamped eigenfunctions $|\sin(\frac{n\pi}{l} x)|$. The step jumps in $\phi_{rel}$ in Fig. 2(b-c) become smooth in Fig. 3(b-c), indicating that the waves are propagating along the $x$. The corresponding $w(x,t)$ constructed from the data are shown in Fig. 3(d-f). Supporting Information includes video files, where the zero crossings of $w(x,t)$ move slightly along the $x$.

We model the dynamics in air using the string equation with uniform viscous damping$^{15,21,33-35}$:

$$\mu \frac{\partial^2 w}{\partial t^2} + \gamma \frac{\partial w}{\partial t} - \tau \frac{\partial^2 w}{\partial x^2} = f(x,t). \tag{1}$$

Here, $\gamma$ is the damping per unit length and $f(x,t) = \Re\{F(x)e^{i\omega t}\}$ is the applied force per unit length with $F(x)$ being the complex force amplitude. Considering only the domain $\xi_1 < x < \xi_2$ in which $F(x) \approx 0$, we write the general solution to Eq. 1 as

$$w(x,t) = \Re\{Ae^{i(k_{x x}-\omega t)} + Be^{-i(k_{x x}+\omega t)}\}. \tag{2}$$

The complex wave vector $k$ is found as

$$k = k_R + ik_I = \frac{\omega}{c} \left(1 + i \frac{\gamma}{\mu \omega}\right)^{1/2}. \tag{3}$$
FIG. 4. Oscillatory dynamics of the 50-µm resonator at high dissipation. (a) \( W_{rms} \) and phase (inset) of the resonator as a function of drive frequency at different \( x \) positions on the resonator. (b) Color map of \( W_{rms} \). (c) \( W_{rms} \) as a function of frequency at \( x = 10 \mu m \), with peaks corresponding to the first three resonances. (d-f) Normalized \( W_{rms} \) as a function of \( x \) at the peak frequencies in (c) (vertical arrows in (b)): 1.17 MHz, 3.45 MHz, and 6.01 MHz. The insets show \( \phi_{rel} \). (g-i) Normalized \( w(x,t) \) constructed from the data in (d-f). A wave propagates from \( x \approx 0 \) to \( x \approx l \) in all cases. The wave decays at different length scales.

where \( k_R = \Re\{k\}, \ k_I = \Im\{k\}, \) and \( \Re \) and \( \Im \) respectively denote the real and imaginary components. Eq. 2 can thus be rewritten as

\[
 w(x,t) = \Re\{A e^{-k_I x e^{i(k_R x - \omega t)}} + B e^{k_I x e^{-i(k_R x + \omega t)}}\},
\]

with \( A \) and \( B \) being the complex amplitudes of the right- and left-propagating waves, respectively. We can fit the data in Fig. 3(a-c) using Eq. 4, as shown in Fig. S3. The best fits provide the complex \( k \) values listed in Table I. Using \( \gamma \approx \frac{\mu_0 \omega}{Q_n} \), we expand Eq. 3 to find

\[
 k_n^{(a)} \approx \frac{n\pi}{l} \left( 1 + i \frac{1}{2Q_n^a} \right). \]

The values for \( k_n^{(a)} \) are very close to \( \frac{n\pi}{2l} \); \( k_n^{(a)} \) are roughly a factor of two to three larger than \( \frac{n\pi}{2Q_n^{(a)}} \). We note that the fits in Fig. S3 are approximations only and can be improved by modeling...
TABLE I. Parameters for the first three modes of the NEMS resonator in vacuum and air: $\omega_n/2\pi$ and $Q_n$ respectively refer to the mode frequency and quality factor; $k_n$ is the mode wave number. Air values are indicated by $a$; $R$ and $I$ correspond to real and imaginary components.

| Mode | $\omega_n/2\pi$ (MHz) | $Q_n$ (m$^{-1}$) | $k_n$ (m$^{-1}$) | $\omega_n^{(a)}$ (MHz) | $Q_n^{(a)}$ | $k_n^{(a)R}$ (m$^{-1}$) | $k_n^{(a)I}$ (m$^{-1}$) |
|------|----------------------|-----------------|-----------------|----------------------|--------------|-------------------------|-------------------------|
| 1    | 5.179                | 23.6 $\times 10^4$ | 6.28 $\times 10^4$ | 5.163                | 32 $\pm$ 5   | 6.58 $\times 10^4$      | 2.21 $\times 10^4$      |
| 2    | 10.367               | 22.8 $\times 10^4$ | 12.57 $\times 10^4$ | 10.315               | 54 $\pm$ 5   | 13.18 $\times 10^4$     | 3.76 $\times 10^3$      |
| 3    | 15.578               | 20.3 $\times 10^4$ | 18.85 $\times 10^4$ | 15.504               | 73 $\pm$ 5   | 19.76 $\times 10^4$     | 4.04 $\times 10^3$      |

Finally, we show our results on the oscillatory dynamics of the resonator at the high dissipation limit ($Q \approx 1$) in Fig. 4. This experiment is performed with the NEMS immersed in water. In Fig. 4(a) shows the rms oscillation amplitude and the phase (inset) of the NEMS as a function of frequency and position, obtained by scanning the drive frequency in the 0.6 – 9 MHz range at each $x$. The colormap in Fig. 4(b) is the top view of the amplitude from Fig. 4(a), showing how the amplitude decays with frequency and position. We observe that the amplitude shows peaks at some frequencies reminiscent of resonances. For instance, $W_{rms}$ as a function of frequency at $x = 10$ µm shown in Fig. 4(c) has three peaks. Taking the values of $W_{rms}$ and phase at frequencies marked by the vertical arrows in Fig. 4(b), i.e., at 1.17 MHz, 3.45 MHz, and 6.01 MHz, we obtain the position dependent data for $W_{rms}$ and $\phi_{rms}$ shown in Fig. 4(d-f) at the peak frequencies in Fig. 4(c). For more insight into the motion of the beam, we construct $w(x,t)$ as above for full cycles of oscillation, as shown in Fig. 4(g-i) and Supporting Information videos. Immediately evident is the fact that $w(x,t)$ are traveling waves that are generated at the actuator at $x \approx 0$ and move toward $x = l$. The waves decay significantly over the length of the beam. Consequently, we can neglect the wave propagating to the left in Eq. 2 and have a simpler mathematical description,

$$w(x,t) \approx \Re\{Ae^{i(kx-\omega t)}\},$$

$$= A'e^{-k_I x} \cos(k_R x - \omega t + \varphi),$$  \hspace{1cm} (5)

where the phase $\varphi$ is adjusted such that $A'$ is real. To estimate $k_R$ and $k_I$ as a function of frequency, we advance the waveform in time (or adjust $\varphi$) until we obtain a peak near the $x = 0$ anchor. This results in data sets such as those shown in Fig. 5(a). In these semilogarithmic graphs, $|w(x,t)|$ is plotted at four frequencies with the x’ axis starting at the peak position. The distance between successive peaks is half the wavelength and provides an estimate for $k_R$ of a given data set. The decaying exponential envelope of each data set, shown by the line in each plot, provides an estimate for $k_I$. In Fig. 5(b) the extracted $k_R$ and $k_I$ are plotted as functions of frequency (lower x-axis) and the viscous boundary layer thickness (upper x-axis) generated by the oscillations, $\delta = \sqrt{\frac{2\eta_f}{\rho_f \omega}}$, where $\rho_f$ and $\eta_f$ are the density and dynamic viscosity of water.

FIG. 5. (a) Semilogarithmic plots showing normalized $|w(x,t)|$ vs. x’ at different drive frequencies, where x’ = 0 marks the peak position of the waveform. Fitting the envelope with a decaying exponential (lines) provides $k_I$; the distance between successive peaks provides $k_R$. (b) $k_I$ (left y) and $k_R$ (right y) as a function of frequency. The upper x axis shows $\delta$. The shading indicates the regions $\delta \lesssim$ gap (dark) and $\delta \gtrsim$ gap (light). The dashed curves show theoretical predictions. Error bars are the uncertainties in the linear fits in (a).
the water, respectively. The $k_R$ data start from 3 MHz since it is impractical to measure the wavelength in the absence of two or more peaks.

We now show how the frequency-dependent spatial profile of driven NEMS oscillations can be related to the physical properties of the fluid by using Stokes’ theory of the oscillating cylinder in a viscous fluid$^{20,29,30}$. We Fourier transform the undamped string equation ($\gamma = 0$ and $f(x,t) = 0$ in Eq. 1) in both space and time with the fluid providing the only force (per unit length), $\tilde{F}_f(k,\omega)$, on the beam:

$$(-\omega^2\mu + \tau k^2)\tilde{W}(k,\omega) = \tilde{F}_f(k,\omega). \quad (6)$$

The fluid force is$^{29}$

$$\tilde{F}_f(k,\omega) \approx \frac{\pi}{4} \rho_f \omega^2 b^2 \Gamma_b(\omega) \tilde{W}(k,\omega),$$

$$\approx \frac{\mu}{\omega^2} T_0 \Gamma_b(\omega) \tilde{W}(k,\omega), \quad (7)$$

where $\Gamma_b(\omega) = \Gamma_b'(\omega) + i\Gamma_b''(\omega)$ is the hydrodynamic function of a blade (found from the cylinder solution$^{30}$); $\mu = \rho b$ is the mass per unit length of the string; and $\rho_f$ and $\eta_f$ are respectively subsumed into $T_0 = \frac{\pi}{4} \frac{\rho b}{\rho c}$ and $\Gamma_b(\omega)$. Substituting $\tilde{F}_f(k,\omega)$ into Eq. 6, we obtain

$$k_R + ik_I = \frac{\omega}{c} \sqrt{(1 + T_0 \Gamma_b'(\omega)) + iT_0 \Gamma_b''(\omega)}. \quad (8)$$

indicating that one can determine $\rho_f$ and $\eta_f$ from measured $k_R$ and $k_I$. This approach could be complementary to that based on fitting the frequency response of the NEMS at a single point. In fact, this theory can be extended to obtain properties of viscoelastic fluids as well$^{36,37}$.

The dashed line in Fig. 5(b) show $k_R$ and $k_I$ predicted from Eq. 8 using experimental values of $c$ and $T_0$ and calculated $\Gamma_b(\omega)$. These predictions match well with our experimental data for frequencies $\gtrsim 2.5$ MHz. The measured $k_I$ deviates from theory at low frequency because of the added squeeze damping$^{20}$.

As more emphasis is put on precision measurements in fluids$^{38,39}$, the spatial decay of the amplitude of driven NEMS resonators could have significant implications. For NEMS-based mass sensing and mass spectrometry, deconvoluting the mass and position of the adsorbed analyte molecule on the NEMS from frequency shifts requires a detailed knowledge of the oscillatory amplitude of the resonator in multiple modes$^{8,40}$. Of particular importance is the behavior of the nodes in the intermediate dissipation regime: since there is a travelling wave along the structure with a small amplitude, there are no true nodes. Similarly, in dynamic AFM in air and liquids$^8$, where the microcantilever is driven should affect the tip amplitude and tip-sample interactions. Another relevant area is fundamental studies in fluid dynamics using NEMS and microcantilever resonators$^{15,29}$. The accuracy of an eigenfunction expansion containing a few eigenmodes should be assessed carefully in liquids$^{15,29}$.

In summary, our results here will be of relevance to research and technology involving NEMS, AFM, and even macroscopic mechanical resonators.

**SUPPLEMENTARY MATERIAL**

See the supplementary material for three figures and nine videos.

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**AUTHOR DECLARATIONS**

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

K.L.E., C.T., and M.G. devised the project. C.T. performed the experiments and analyzed the data. C.Y., I.I.K. and M.S.H. fabricated the samples. G.M., C.T., and A.L. contributed to the wave modal analysis and the data. M.M. and S.K. performed the FEM analysis. H.G. characterized the samples. O.S. contributed to the experimental set up. C.T. and K.L.E. wrote the manuscript with comments from all authors.

**DATA AVAILABILITY**

The data that support the findings of this study are available from the corresponding authors upon reasonable request.

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Supplementary Figures for
“Dynamics of NEMS Resonators across Dissipation Limits”

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FIG. S1: (a-c) Vacuum data showing the rms amplitude and phase of the 50-\(\mu\)m resonator as a function of frequency around its first three resonances at different positions.

FIG. S2: (a-c) Air data showing the rms amplitude and phase of the 50-\(\mu\)m resonator as a function of frequency around its first three resonances at different positions.
FIG. S3: (a-c) Air data and wave fits (continuous curves). Here, the wave is of the form $w(x, t) = \Re\{A e^{-k_I x} e^{i(k_R x - \omega t)} + B e^{k_I x} e^{-i(k_R x + \omega t)}\}$. To make the fits, the $W_{rma}$ and $\phi_{rel}$ data was used to construct the wave at an instant in time ($\omega t = \frac{\pi}{2}$). Since the dissipation is small, the real component of the wave was determined using the resonance frequency $\omega_n$ as $k_R \approx \frac{\omega_n}{c}$.

After fixing $A$ to $A = 0.5$, a search was performed to find the set of parameters $k_I$ and $B$ that provided the best fits: the error between the normalized experimental wave data and the wave $w(x, t) = \Re\{A e^{-k_I x} e^{i(k_R x - \omega t)} + B e^{k_I x} e^{-i(k_R x + \omega t)}\}$, for each mode, was minimized. The found parameters are noted on the plots.