Analysis of Flow Models for Aerostatic Thrust Bearings with Porous Material

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Abstract. The numerical determination of static characteristics of bearings allows a cost-efficient and fast pre-design. In this study, two flow models for aerostatic thrust bearings with pressurized porous material are presented and analyzed. The models are based on the coupling of the Reynolds equation for lubricants (REL) and the determination of pressure drop through porous material by Darcy’s law. The simplified model is based on the assumption of a one-dimensional axial flow through porous media. The extended model considers the three-dimensional flow in the porous body. The analysis includes pressurized air from 4 to 9 bar(a) with nominal clearance of 5 to 60 µm. Commercial CFD software was used to verify the results. The extended model allows a more accurate prediction about the performance in the critical gap range. In total, the results show good agreement with CFD within a short computation time.

1. Introduction

Aerostatic bearings are increasingly being used in turbomachinery due to their advantages. These bearings allow high positioning and high rotational speed. Furthermore, lubricants like oil could cause contamination of the working fluid, e.g., in compressors for cooling machines or air conditioning systems. The Reynolds equation for lubricants (REL) allows a fast calculation of the pressure in the lubricating film. The detailed derivation of the REL can be found in [1] and [2]. Gargiulo [3] coupled Darcy’s law with REL to calculate gas bearings with porous material. These results were successfully validated with measurements. Important work has been done in this field by Mori [4], Sneck [5] and Sun [6]. Kobayashi introduced a model for a herringbone-grooved journal bearing and thrust bearing system, made of porous material [7]. Yoshimoto [8] investigated numerically and experimentally the static characteristics of porous gas bearings with restricted surface layer. Metal, graphite and ceramic can be applied as porous. These materials are investigated among others by Otsu [9], Yoshimoto [8], and Durazo-Cardenas [10]. This paper relates to the theoretical and numerical modeling of thrust bearings with a porous material. Two models (RELD and RELFD) were presented and validated with commercial software. A ceramic matrix composite (CMC) developed by German Aerospace Center (DLR) was used as porous material [11]. This study builds on the previous work, which deals with the modeling of radial bearings with porous media [12].
2. Aerostatic Thrust Bearing with Porous Material

The lubricating film (1) forms in axial direction between the shaft shoulder and the bearing (see Fig. 1). The radial gap between shaft and bearing is assumed to be significantly larger than the axial gap and is neglected. The contact surface of the bearing is flat and the bearing has a porous (2) and a non-porous (solid) (3) part. The porous material is a restrictor which is pressurized with the supply pressure $p_s$. No misalignment is assumed between shaft and bearing.

![Figure 1: Aerostatic thrust bearing with porous material](image)

A detailed derivation of the REL can be read in [1] and [2]. The Navier-Stokes equations are simplified with the following assumptions. Viscous forces are much larger than inertial and body forces. The lubricating film is considered to be an incompressible Newtonian fluid with constant viscosity and density. The porous material has isotropic porosity and laminar flow. Additionally, the velocity gradients across the $z$-dimension are larger than in $r$- and $\varphi$-direction in porous restrictor. The velocities $\frac{\partial u}{\partial z}$, $\frac{\partial v}{\partial z}$, $\frac{\partial w}{\partial z}$, $\frac{\partial u}{\partial r}$, $\frac{\partial v}{\partial r}$, $\frac{\partial w}{\partial r}$ are neglected. The resulting simplified NS-equations in $r$- and $\varphi$-directions are Eq. (1) and Eq. (2).

$$0 = -\frac{\partial p}{\partial r} + \mu \frac{\partial^2 u}{\partial z^2} \quad (1)$$

$$0 = -\frac{1}{r} \frac{\partial p}{\partial \varphi} + \mu \frac{\partial^2 v}{\partial z^2} \quad (2)$$

$U_a$, $V_a$, $W_a$, $U_b$, $V_b$, and $W_b$ stand for the velocity boundary condition on the shaft and bearing in $r$-, $\varphi$- and $z$-direction and are substituted into the integration of Eq. (1) and Eq. (2).

$$u(z = h) = U_a \quad v(z = h) = V_a \quad w(z = h) = W_a$$

$$u(z = 0) = U_b \quad v(z = 0) = V_b \quad w(z = 0) = W_b$$
The mass conservation equation is introduced to close the simplified momentum equations, yielding the general form of REL in cylindrical coordinates.

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( \rho h^3 \frac{\partial p}{\partial r} \right) + \frac{1}{r} \frac{\partial}{\partial \varphi} \left( \rho h \frac{\partial p}{\partial \varphi} \right) = \frac{1}{r} \frac{\partial}{\partial r} \left( h \rho \left( \frac{U_a + U_b}{2} \right) \right) + \frac{1}{r} \frac{\partial}{\partial \varphi} \left( h \rho \left( \frac{V_a + V_b}{2} \right) \right) - \frac{U_a \rho}{r} \frac{\partial h}{\partial r} - \frac{V_a}{r} \rho \frac{\partial h}{\partial \varphi} + \rho \left( W_a - W_b \right) + \frac{\partial \rho}{\partial t} h \quad (3)
\]

3. Reynolds Equation for Lubricants with Darcy’s Law (RELD) for Thrust Bearings

The calculation of the static characteristics is based on the general REL in Eq. (3). No slip boundary condition is assumed at shaft or bearing. The velocities of the shaft in \( r \)-direction \( U_a \) and in \( z \)-direction \( W_a \) are set to zero. The velocity of the shaft in \( \varphi \)-direction \( V_a \) (circumferential velocity) is equal to \( 2\pi nr \). In the simplifications, the inertial terms were neglected and the bearing has a flat inner surface. As a result, the film thickness in \( r \)- and \( \varphi \)-direction is constant. Based on the assumptions, \( V_a \) does not affect the calculation of the pressure.

\[
\frac{\partial h}{\partial r} = \frac{\partial h}{\partial \varphi} = 0 \quad U_a = U_b = V_a = W_a = 0 \quad V_a = 2\pi nr
\]

The velocity in \( z \)-direction \( W_b \) is calculated by Darcy’s law, Eq. (4). The RELD refers to a purely axial flow through the porous material. The velocity components of the bearing in \( r \)- and \( \varphi \)-direction are neglected and set to zero. Integration through the porous body results to Eq. (6). According to the assumption \( w(z = 0) = W_b \) and Eq. (6), the velocity of the bearing in \( z \)-direction equals to Eq. (7).

\[
dp = \frac{\mu}{\alpha} \int \wp \wz \, dz \quad (4) \quad \int \wp \, dp = \frac{\mu}{\alpha} \int \wp \wz \, dz \quad (5)
\]

\[
w(z) = \frac{\alpha}{\mu} \left( \wp \wp - \wp \wp \right) \quad (6) \quad \wp = \frac{\alpha}{\mu} \left( \wp \wp - \wp \wp \right) \quad (7)
\]

The resulting governing Eq. (8) is solved numerically with the finite difference method. The two-dimensional grid is shown in Fig. 2. The adjacent porous and non porous material of the lubricant
are labeled. The partial derivatives are approximated with second order central difference schemes. The successive over-relaxation (SOR) method with a relaxation value of 1.4 was used for the iterative calculation. For $r = r_0$ and $r = r_3$ Dirichlet boundary conditions were used with the static atmospheric pressure $p_a$. At $\varphi = \varphi_1$ and $\varphi = \varphi_2$ periodic was set as a boundary condition.

$$h^3 \frac{\partial p}{\partial r} + h^3 r \frac{\partial^2 p}{\partial r^2} + \frac{h^3 \partial^2 p}{\partial \varphi^2} = -p_a \left( \frac{12\alpha r}{z_p} \right) + p \left( \frac{12\alpha r}{z_p} \right)$$

(8)

4. Reynolds Equation for Lubricants with Full Darcy’s Law (RELFD) for Thrust Bearings

The RELFD model is based on the general REL with a fully discretized porous material. For the velocities of the shaft $U_a$, $V_a$ and $W_a$, the same conditions apply as at the RELD model. Furthermore, the lubricant film thickness in $r$- and $\varphi$- directions are constant. The RELFD assumes a three-dimensional flow within the porous medium. As a result, the velocities $U_b$, $V_b$, and $W_b$ are based on Darcy’s law. The combination of the assumptions and the general REL leads to the governing Eq. (9) for lubricant.

$$\frac{\partial h}{\partial t} = \frac{\partial h}{\partial r} = 0 \quad U_a = W_a = 0 \quad V_a = 2\pi nr$$

$$U_b = \frac{a}{\mu} \frac{\partial p}{\partial r} \quad V_b = \frac{a}{\mu r} \frac{\partial p}{\partial \varphi} \quad W_b = \frac{a}{\mu} \frac{\partial p}{\partial z}$$

$$\frac{\partial p}{\partial r} \left( h^3 - 6h\alpha \right) + \frac{\partial^2 p}{\partial r^2} \left( h^3 r - 6r h\alpha \right) + \frac{\partial^2 p}{\partial \varphi^2} \left( \frac{h^3}{r} - \frac{6h\alpha}{r} \right) = -12\alpha r \left( \frac{\partial p}{\partial z} \right) = 0$$

(9)

The porous material is discretized in this model with an equidistant numerical grid, as shown in Fig. 3. The combination of the continuity equation and Darcy’s law leads to Eq. (10). Details about the derivation can be read in [3] or in [12]. This equation is used to calculate the flow inside the porous medium.

$$\frac{1}{r} \frac{\partial p}{\partial r} + \frac{\partial^2 p}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 p}{\partial \varphi^2} + \frac{\partial^2 p}{\partial z^2} = 0$$

(10)
Eq. (9) and Eq. (10) were calculated simultaneously by finite difference method. Second-order central differences schemes were used for the calculation of the differential quotients. At \( r = r_1 \) and \( r = r_2 \) of the porous material, second-order Neumann boundary conditions are imposed.

\[
\frac{\partial p}{\partial r} \bigg|_{r=r_1} = 0 = \frac{-3p_i=0 + 4p_i=1 - p_i=2}{2\Delta r}
\]

\[
\frac{\partial p}{\partial r} \bigg|_{r=r_2} = 0 = \frac{-3p_i=NR-1 + 4p_i=NR-2 - p_i=NR-3}{2\Delta r}
\]

At \( z = z_p \) a Dirichlet boundary condition was set with the static pressure \( p_s \). Periodic was set at \( \varphi = \varphi_1 \) and \( \varphi = \varphi_2 \). The non-porous part is considered solid and contains no throughflow. The pressure gradient at \( z = 0 \) was calculated with the following scheme (for details see [13]).

\[
\frac{\partial p}{\partial z} \bigg|_{z=0} \approx \frac{-p_{k=2} + 4p_{k=1} - 3p_{k=0}}{2\Delta z}
\]

5. Computational Fluid Dynamics (CFD)

The full flow in the porous bearing is solved with the commercial solver CFX 18.2. A 60 degree segment of the bearing was used to reduce computation time. Hexahedral meshes were set up for the numerical calculations. Within a grid independence study, an appropriate solution was achieved with 10 grid points in \( z \)-direction and 76 in \( \varphi \)- and \( r \)-direction. Starting from the lubricating film, the computational grid of the porous medium has a growth rate of 1.2. The boundary conditions of the setup can be seen in Fig. 4. Inlet and Outlet are set up with static pressure. Atmospheric pressure was imposed at (1), and supply pressure at (2). No-slip wall is set up in (3) and a rotating no-slip wall at (4). Periodic is imposed on (5) due to the rotational symmetry. The simulations were calculated steady, incompressible and laminar.
6. Results

The results are used to verify the RELD and RELFD models for thrust bearings with commercial software. The parameters of the thrust bearing are:

\[ r_0 = 8 \text{ mm} \quad r_1 = 10 \text{ mm} \quad r_2 = 20 \text{ mm} \quad r_3 = 22 \text{ mm} \quad z_p = 20 \text{ mm} \]
\[ \mu = 1.7 \times 10^{-5} \text{ Pa s} \quad \alpha = 1 \times 10^{-13} \text{ m}^2 \quad n = 10,000 \text{ rpm} \quad p_a = 1 \text{ bar(a)} \]

The investigated parameters on the load capacity are the supply pressure \( p_s \) and the gap thickness \( h \). The load capacities at \( p_s = 4 - 9 \text{ bar(a)} \) and \( h = 5 - 60 \mu\text{m} \) are shown in Fig. 5. With increasing supply pressure and reduction of the gap width, an increase of the load capacity is recognizable. In the range of 10 to 60 \( \mu\text{m} \), all models are in good agreement and the maximum difference in load capacity is 20 N.
The highest discrepancy is in the critical range of $h = 5 \mu m$. The RELD model has a maximum deviation to the CFD of 24 N at a supply pressure of 4 bar(a) and a deviation of 62 N at 9 bar(a). On the other hand, the load capacities of the RELFD model result in deviations of 4 N and 7 N at the same operating points. Fig. 6 shows the rotationally symmetric pressure distribution of the lubricating film at $p_s = 9$ bar(a) and $h = 5$ and 30 $\mu m$. Depending on the radius, the adjacent porous or non-porous area is marked. As already shown in the load capacity diagram, the calculated pressure profiles of all three models are in good agreement with a gap thickness of 30 $\mu m$. In the range of 5 $\mu m$, the pressure curve of the RELFD is qualitatively and quantitatively closer to the CFD result. Fig. 7 to Fig. 10 show the pressure distribution of the RELFD and CFD models within the porous material. The reduction of the lubricating film thickness imposes on the porous material a non-uniform pressure distribution near the lubricant. The consistency between CFD and 3D model is good at a lubricating film thickness of 30 $\mu m$. A more significant deviation between CFD and RELFD can be seen in the range of 5 $\mu m$. The pressure isolines of the 3D model reflect higher pressure in the area of the lubricating film, which results in a higher load capacity.
7. Summary and Outlook
This paper complements the work of [12] by analyzing two flow models for thrust bearings with porous material. The models RELD and RELFD were presented and verified with commercial software. The RELD needs less computational resources based on the simplifications. The RELFD model has higher accuracy in the determination of the static characteristics. Three supply pressures and lubrication film thicknesses from 5 to 60 µm were investigated. CMC, with a permeability of \( \alpha = 1 \times 10^{-13} \text{m}^2 \) was used as porous material. The RELD and the RELFD allow results comparable to commercial solvers. Depending on the film thickness, the RELD model is sufficient. The RELD models a one-dimensional axial flow through the porous medium. With lubricant film thicknesses between 10 and 60 µm, this assumption is sufficient to derive a good statement regarding the load capacity of the bearing. This model enables a good pre-design even with low computational resources. For low lubricant film thicknesses like 5 µm, the simplified RELD is less accurate in determining the load capacity. The RELFD model includes the components in \( r \)- and \( \varphi \)- direction in addition to the axial components. The RELFD is in good agreement with CFD in the critical film gap of 5 µm. We propose to expand the model in future to include the compressibility of gases. In further steps, a validation with measurements will take place at the test rig of the Chair of Fluid Mechanics and Fluid Machinery of Technical University in Kaiserslautern (TUK).

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Nomenclature

- \( L \) Load capacity / N
- \( N \) Number of nodes
- \( \mu \) Viscosity / Pa s
- \( \rho \) Density / kg m\(^{-3}\)
- \( U_a \) Velocity on shaft in \( r \)- direction / m s\(^{-1}\)
- \( U_b \) Velocity on bearing in \( r \)- direction / m s\(^{-1}\)
- \( V_a \) Circumferential velocity of shaft / m s\(^{-1}\)
- \( V_b \) Circumferential velocity of bearing / m s\(^{-1}\)
- \( W_a \) Velocity on shaft in \( z \)- direction / m s\(^{-1}\)
- \( W_b \) Velocity on bearing in \( z \)- direction / m s\(^{-1}\)
- \( \alpha \) Permeability / m\(^2\)
- \( \varphi \) Coordinate in circumferential direction / °
- \( h \) Width of gap / m
- \( n \) Number of revolutions of shaft / rpm
- \( p_a \) Atmospheric pressure (absolute) / bar(a)
- \( p_s \) Supply pressure (absolute) / bar(a)
- \( r \) Coordinate of radius / m
- \( t \) Time / s
- \( z \) Coordinate in axial direction / m

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