Influenced by lateral liquid sloshing in partially filled tanks, tank vehicles are apt to encounter rollover accidents. Due to its strong nonlinearity and loading state uncertainty, it has great challenges in tank vehicle active control. Based on the model-free adaptive control (MFAC) theory, the roll stability control problem of tank trucks with different tank shapes and liquid fill percentages is explored. First, tank trucks equipped with cylinder or elliptical cylinder tanks are modelled, and vehicle dynamics is analyzed. The tank dynamic model is used to provide I/O data in the controlled system. Next, the control objective of tank vehicle rollover stabilization is analyzed and the controlled variable is selected. Subsequently, differential braking and active front steering controller are designed by MFAC algorithm. Finally, the effectiveness of the designed controllers is verified by simulation, and the difference between the controllers is analyzed. The controller designed by MFAC algorithm is proven to be adaptive to vehicle loading and driving states. The controlled system has great robustness.

1. Introduction

Tank vehicles are widely used in road transportation of liquid cargoes. As affected by liquid sloshing in partially filled tank, the tank vehicle usually has poor driving performance, especially rollover stability [1, 2]. The active control on it is of great significance.

Model based and data driven are the two main approaches proposed for the control-system design. In the model-based method, the system dynamic model is first derived, and then, a controller is designed based on that. The tank vehicle is complex fluid-solid coupling multibody system. Its dynamics is greatly influenced by liquid sloshing, which makes it difficult to construct an accurate dynamic model [3, 4]. To simplify the modelling of the tank vehicle, the equivalent mechanical model is popularly used to approximately describe liquid sloshing in partially filled tank; then, the coupling mechanism of liquid sloshing and vehicle driving could be expressed easily and the vehicle dynamic modelling could be achieved. As an essential nonlinear system, the complex dynamic model renders it difficult for many control methods based on system dynamics to design the rollover stabilization controller. Furthermore, as liquid sloshing is greatly influenced by tank shapes, the equivalent mechanical model for liquid sloshing would change with it [5–10]. By far, only tank truck equipped with the horizontal cylinder or elliptical cylinder tank was modelled [6, 9–14]. Quite few studies on the modelling of tank truck with other tank shapes have been reported. Owing to this, the control algorithm developed by model-based design strategy is poorly adaptive. It is difficult for the model-based control method to achieve wide usage in practical application.

Two common data-driven controllers, PID and fuzzy logic controller, had been studied by many researchers to achieve tank vehicle rollover and yaw stabilization. Zhao et al. designed a PID controller to obtain the additional yaw moment and applied differential braking on tires to improve vehicle roll stability. The authors used the difference between the practical and theoretical roll angle to design the PID controller [15]. Hu and Zhao analyzed the yaw stability of tractor tanker semitrailer and then conducted a control on the tractor’s yaw rate. Error between the practical tractor yaw rate and its desired value, as well as its derivation change rate, was used to design the fuzzy logic controller [16]. Li
used the axle lateral load transfer ratio (LTR) as the controlled variable to design active antirollover bar by PID control algorithm [17]. Zeng used active front steering, active suspension, and their combination, respectively, to restrain liquid sloshing in tanks. Liquid oscillation angle was used as the controlled variable [18]. Besides the PID and fuzzy logic control method, Acarman and Özgüner used the sliding mode control method to regulate liquid sloshing frequency, which would greatly reduce the settling time of the vehicle driving state [19].

The PID and fuzzy logic control are classic, typically offline data-driven control method. The controller design by those methods has a simple structure. However, tank vehicle dynamics is greatly affected by tank shape, liquid fill percentage, and cargo physical characters, thus rendering it difficult for those controllers to maintain a consistent control performance. Controller parameters need to be readjusted to stabilize the system. Therefore, the model-free adaptive control (MFAC) method, an online data-driven control method with good robustness and adaptability, is proposed to stabilize tank vehicle. The MFAC algorithm was proposed by Hou in 1994 [20]. It is a data-driven control method based on system online data. The algorithm performs dynamic linearization at each working point of the controlled system by the aid of I/O data, thus describing system dynamics by this incremental dynamic linear model. Based on the dynamic linear model, the adaptive controller could be designed [21, 22]. The pseudo-partial derivative in the dynamic linear model is insensitive to the time-varying structure and time-varying phase of the system; thus, the controller exhibits strong robustness and adaptability. Wang Wang studied the fault-tolerant control problem for a class of discrete-time systems subjected to the sensor fault. For the purpose of obtaining better control performance, a model-free adaptive fault-tolerant controller is developed by employing more past control information [23]. Jiang et al. explored the heading control problem of unmanned surface vehicles with uncertainties based on MFAC theory. The conventional MFAC algorithm was developed to solve problems existing in USVs' heading control [24]. Jiang et al. studied the heading tracking problem of the six-wheel independent drive and four-wheel independent steering unmanned ground vehicle with variable wheelbase under the influence of uncertainties [25]. Li et al. addressed a pattern-moving-based modified model-free adaptive control (PMFAC) scheme and illustrated the convergence of its tracking error for a class of nonlinear systems with the unknown model and multithreshold quantized observations [26].

Therefore, the paper aims to investigate adaptive rollover stabilization for tank trucks by the MFAC method, and its outline is as follows. In Section 1, tank trucks equipped with cylinder or elliptical cylinder tanks are modelled, and vehicle dynamics is analyzed. The dynamic model of tank trucks is used to provide I/O data in the controlled system. In Section 2, control objective is analyzed and the controlled variable is selected; then, controllers based on differential braking and active front steering strategies are designed by the MFAC method. In Section 3, the effectiveness of the designed controllers is verified by simulation, and the difference between the two controllers is analyzed. In the end, a major conclusion about this research is given.

2. Dynamic Modelling and Analysis of Tank Truck

2.1. Equivalent Mechanical Model for Lateral Liquid Sloshing in Partially Filled Tanks. Tanks with circular or elliptical cross section are widely used in tank trucks. To simplify the modelling of liquid sloshing, Zheng et al. used an equivalent elliptical pendulum to describe liquid lateral sloshing in these tanks [6, 7, 11], as shown in Figure 1.

The movement of the elliptical pendulum should be equivalent to lateral liquid sloshing in partially filled tanks. Thus, the pendulum should have the same dynamics as liquid sloshing. By the principle of dynamic equivalence, parameters of the elliptical pendulum could be modelled as the function of liquid sloshing dynamics [14]:

\[
\begin{align*}
    b_p &= \frac{g}{(\omega^2 \zeta^2)} \\
    a_p &= \frac{F}{\max(m_a)} \\
    m_p &= \frac{F}{\max(m_a)} \\
    m_f &= m - m_p \\
    b_f &= \frac{m b_{cg} - m_p (b - b_p)}{m_f}
\end{align*}
\]

where \( b_p \) is half length of the minor axis of the elliptical pendulum, \( a_p \) is half length of the major axis of the elliptical pendulum, \( g \) is acceleration of gravity, \( \zeta \) is length ratio of the major and minor axis of tank cross section, \( m_p \) is the pendulum ball mass, \( m_f \) is the static liquid mass, \( m \) is the liquid cargo mass, \( \omega \) is liquid oscillation frequency, \( F \) and \( a_p \) are the peak liquid sloshing force in a sloshing cycle and the corresponding sloshing lateral acceleration at the same time, \( b_{cg} \) is the vertical distance between the liquid center of mass and tank lowest point when liquid-free surface is leveled, \( b \) is half length of the major axis of the tank, and \( b_f \) is the vertical distance between the static liquid and tank lowest point.

After the sloshing frequency and force of lateral liquid sloshing have been obtained, parameters of the elliptical pendulum can be obtained according to equation (1). By curve fitting, they can be modelled as the function of the tank shape and liquid fill percentage, that is,
\[
\frac{b_p}{b} = 1.087 + 0.6999\Delta - 0.1407\zeta - 0.9291\Delta^2 - 1.178\zeta\Delta + 0.05495\zeta^2 - 0.03353\Delta^3 + 0.5404\zeta\Delta^2 + 0.1518\zeta^2\Delta,
\]
\[
\frac{m_p}{m} = 0.7844 - 1.729\Delta + 0.3351\zeta + 1.156\Delta^2 + 0.7256\zeta\Delta - 0.1254\zeta^2 - 0.3219\Delta^3 - 0.9152\Delta^2 + 0.08043\zeta^2\Delta,
\]

where \(\Delta\) is liquid fill percentage, which is the ratio of liquid level height to the tank height.

### 2.2. Modelling of Tank Truck

As a component of the vehicle, the movement of the tank coincides with that of the vehicle. That is to say, the tank translates along vehicle Y-axis and rotates on vehicle roll axis. Consequently, the reference frame of liquid sloshing is a noninertial coordinate system with both lateral translation and roll rotation, and liquid sloshing is affected by the movement of the vehicle.

While the elliptical pendulum model is used to equivalently describe liquid sloshing in partially filled tanks, its dynamics in the noninertial coordinate system should be derived, that is,

\[
\ddot{\theta}(a_p^2\sin^2\theta + b_p^2\cos^2\theta) + \frac{1}{2}\dot{\theta}^2(a_p^2 - b_p^2)\sin(2\theta) - gb_p\cos\theta + 2\eta\dot{\theta}(a_p^2\sin^2\theta + b_p^2\cos^2\theta) - V(\dot{\beta} + r)e_p\sin\theta - r\dot{e}_p a_p \sin\theta + \phi a_p (H_\beta \sin\theta - b_p) + \frac{1}{2}\dot{\theta}^2 a_p^2 \sin(2\theta) + \frac{1}{2}\phi^2(a_p^2 - b_p^2)\sin(2\theta) - \phi r e_p b_p \cos\theta + \phi^2 H_\beta b_p \cos\theta = 0,
\]

where \(\theta\) is pendulum oscillation angle. When vehicle is stationary, the value is \(\pi/2\), \(\eta\) is dimensionless damping coefficient of lateral liquid sloshing, \(V\) is vehicle driving speed, \(\beta\) is vehicle body slip angle, \(r\) is the vehicle yaw rate, \(e_2\) is the coordinate of the liquid center of mass along vehicle X-axis, \(\phi\) is the vehicle roll angle, and \(H_j\) is the height from the tank center to vehicle roll axis.

Since equation (3) contains nonlinear composite terms such as \(\dot{\theta}^2\), \(\phi^2\), \(r^2\), and \(\phi r\), liquid sloshing is an essential nonlinear subsystem that is difficult to linearize near its equilibrium point.

In lateral direction, vehicle dynamics along Y-axis can be expressed by

\[
(m_v + m_u + m_f + m_p)\dot{V} + (m_c + m_v e + m_f e_2 + m_p e_2)\dot{r} + (m_e h_s - m_f H_1 - m_p H)\dot{\phi} - \dot{\theta} m_p a_p \sin\theta = 2(F_f + F_r) - Vr (m_v + m_u + m_f + m_p) + m_p a_p \cos\theta (r^2 + \phi^2 + \dot{\theta}^2),
\]

where \(m_v\) and \(m_u\) are vehicle sprung and unsprung mass, respectively, \(c\) is the coordinate of the center of gravity of vehicle sprung mass along X-axis, \(e\) is the coordinate of the center of gravity of vehicle unsprung mass along X-axis, \(h_s\) is the vertical distance from the center of gravity of vehicle sprung mass to vehicle roll axis, \(H_1\) is the height from the static liquid to vehicle roll axis, \(H\) is the vertical distance from pendulum ball to vehicle roll axis, which is expressed by \(H = H_f - b_p \sin\theta\), and \(F_f\) and \(F_r\) are lateral tire-road forces of front and rear axles, respectively.

Also, in lateral direction, vehicle dynamics about Z-axis can be expressed by

\[
(m_c + m_v e + m_f e_2 + m_p e_2)\dot{V} + (I_{xz} + I_{zw} + I_{zz} = m_v e^2 + m_f e_2^2 + m_p e_2^2)\dot{r} + (I_{xx} - m_e h_s - m_f e_2 H_1 - m_p e_2 H)\dot{\phi} - \dot{\theta} m_p a_p \sin\theta = 2(F_f l_f - F_r l_i) - Vr (m_c + m_v e + m_f e_2 + m_p e_2) + m_p a_p \cos\theta (r^2 + \phi^2 + \dot{\theta}^2),
\]

where \(I_{xx}, I_{zw}\) and \(I_{zz}\) are the yaw moment of inertia of vehicle sprung mass, vehicle unsprung mass and liquid cargo around their centroid coordinate system, respectively, \(I_{xx}\) is the product of inertia of vehicle sprung mass around its centroid coordinate system, and \(l_f\) and \(l_i\) are the distance from the center of gravity of the vehicle to front and rear axles, respectively.
Vehicle roll moment about X-axis can be expressed by

\[
\left(-m_r h - m_f H_1 - m_p H\right) \ddot{\phi} + \left(-I_{xx} - m_c ch - m_c e H_1 - m_c e H\right) \dot{\phi} + \left(I_{xx} + m_r h^2 + m_f H^2 + m_p^2\right) \ddot{\phi} + \theta m_d a_p H \sin \theta a = \phi g (m_h + m_r H + m_p H) k_\phi - c_\phi \dot{\phi} + (m_h + m_r H + m_p H) V r - m_p g a r \cos \theta - m_r H a_r \cos \theta (r^2 + \phi^2 + \dot{\phi}^2),
\]

where \(I_{xx}\) and \(I_{xc}\) are the roll moment of inertia of vehicle sprung mass and liquid cargo around their centroid coordinate system, respectively, \(k_\phi\) is suspension angular stiffness, and \(c_\phi\) is suspension angular damping.

Equations (4)–(6) describe three degrees of freedom of tank trucks. Along with equation (3) that describes liquid sloshing dynamics, the tank truck is dynamically modelled. Owing to nonlinear terms existing in the model, the tank truck is an essential nonlinear system that is difficult to approximately linearize near its equilibrium point.

In the state space form, the tank truck’s dynamic model is

\[
\dot{X} = \left[ f_i (\theta, \dot{\theta}, \beta, r, \phi, \dot{\phi}) \right], \quad i = 1, 2, \ldots, 6,
\]

where vehicle state vector \(X\) is \([\theta, \dot{\theta}, \beta, r, \phi, \dot{\phi}]^T\). Magic formula is used to describe tire cornering characteristics, that is,

\[
F_y = D \times \sin (C \times \text{atan}(B \alpha)),
\]

where \(D = -0.0004 m_0^2 + 8.9012 m_c + 163.94\), \(m_c\) is tire vertical load, \(B = 8.4; C = 1.59\), and \(\alpha\) is tire slip angle.

Considering lateral load transfer, vertical loads of the four tires are expressed by

\[
\begin{align*}
m_{FL} &= m_f \left(1 - \frac{h_f \sin \phi}{d_f}\right) + m_{fr} \left(\frac{1}{2} \left(\frac{h + b_f \sin \phi}{d_f}\right)\right) + m_{fl} \left(\frac{1}{2} \frac{(h + b_f \sin \phi)}{d_f}\right) + m_{fr} \left(\frac{1}{2} \frac{(h + b_f \sin \phi)}{d_f}\right), \\
m_{FR} &= m_f \left(1 - \frac{h_f \sin \phi}{d_f}\right) + m_{fr} \left(\frac{1}{2} \left(\frac{h + b_f \sin \phi}{d_f}\right)\right) + m_{fl} \left(\frac{1}{2} \frac{(h + b_f \sin \phi)}{d_f}\right) + m_{fr} \left(\frac{1}{2} \frac{(h + b_f \sin \phi)}{d_f}\right), \\
m_{RL} &= m_f \left(1 - \frac{h_f \sin \phi}{d_f}\right) + m_{fr} \left(\frac{1}{2} \left(\frac{h + b_f \sin \phi}{d_f}\right)\right) + m_{fl} \left(\frac{1}{2} \frac{(h + b_f \sin \phi)}{d_f}\right) + m_{fr} \left(\frac{1}{2} \frac{(h + b_f \sin \phi)}{d_f}\right), \\
m_{RR} &= m_f \left(1 - \frac{h_f \sin \phi}{d_f}\right) + m_{fr} \left(\frac{1}{2} \left(\frac{h + b_f \sin \phi}{d_f}\right)\right) + m_{fl} \left(\frac{1}{2} \frac{(h + b_f \sin \phi)}{d_f}\right) + m_{fr} \left(\frac{1}{2} \frac{(h + b_f \sin \phi)}{d_f}\right),
\end{align*}
\]

where \(m_{FL}, m_{FR}, m_{RL},\) and \(m_{RR}\) are the vertical loads of left front tire, right front tire, left rear tire, and right rear tire, respectively, \(m_{fr}, m_{fl}, m_{fr} m_{fl}, m_{fr}, m_{fl},\) and \(m_{fr}\) are the load distribution of vehicle sprung mass, vehicle unsprung mass, static liquid mass, and pendulum mass on the front and rear axles, respectively, \(d_f\) and \(d_r\) are track width of the front and rear axle, respectively, and \(h\) is the distance from tank lowest point to vehicle roll axis.

2.3. Tank Truck Dynamic Analysis. A widely used 6 × 4 tank truck is selected to analyze its driving performance. The tank equipped on the vehicle has length of 5.8 m and cross-sectional area of 2.5 m². Liquid cargo density is assumed to be 1000 kg/m³. Other basic parameters of the vehicle are listed in Table 1.
trucks, and the roll angle and LTR decrease about 40%–200%. A simple conclusion can be obtained that tankers are prone to nontripping rollover accidents.

3. Controller Design

3.1. Control Objectives and Controlled Variables. LTR and roll angle are variables that can directly illustrate vehicle roll motion. Compared with vehicle roll angle, LTR has a threshold limit. It reaches or almost reaches 1.0, which means a rollover accident would happen. Thus, LTR could be used as a warning signal. The rollover controller is inactive when vehicle LTR is below the set threshold. Otherwise, the rollover controller is activated. Given the consideration that rollover accident will happen as long as LTR reaches 1.0 and the active control is virtually useless, 0.8 is used as the set threshold.

LTR is a comprehensive variable that could not be measured directly, which makes it not a good choice as the controlled variable in the controlled system. It is known that lateral acceleration is the main reason which lead to vehicle roll motion. Under the action of lateral acceleration, the vehicle turns around roll axis. According to Figure 3, the roll dynamics under vehicle steady state can be written as

\[
\Delta F \times d = m_s a_s + m_f H_1 a_f + m_p H a_p + m_s g h_s \phi_s + m_f g h_1 \phi_f + m_p g Y_{ps},
\]

(10)

where \(\Delta F\) is axle lateral load transfer, \(\phi_s\) is vehicle roll angle under steady state, and \(Y_{ps}\) is the coordinate of pendulum ball along vehicle \(Y\)-axis under vehicle steady state.

Under vehicle steady state, vehicle sideslip angular velocity approximates to 0, lateral acceleration is mainly determined by the vehicle yaw rate. Thus, in equation (11), lateral acceleration of vehicle sprung mass, the static liquid, and the pendulum ball could be expressed by

\[
a_s = a_f = a_p = Vr.
\]

(11)

When LTR reaches the set threshold, equation (11) can be rewritten as

\[
0.8(m_s + m_f + m_p) \times d = (m_s h_s + m_f H_1 + m_p H)Vr + m_s g h_s \phi_s + m_f g h_1 \phi_f + m_p g Y_{ps}.
\]

(12)

Lateral acceleration obtained by equation (12) will be the threshold to keep vehicle roll stability. While the practical lateral acceleration exceeds the limitation, a rollover accident would happen.

In equation (12), the static liquid mass \(m_f\) and its vertical position described by \(H_1\), the mass of pendulum ball \(m_p\) and its position described by \(Y_{ps}\), and \(H\) change along with tank shape and cargo fill percentage. Therefore, lateral acceleration threshold for the tank truck with different tank shapes and cargo fill percentages differs. By equation (12), lateral acceleration threshold for LTAB10, LTAB15, and LTAB20 under different liquid fill percentages can be obtained, and the results are shown in Figure 4.
3.2. Control Strategy. Analysis in Section 2.1 tells that tank vehicle rollover control can be achieved by limiting the vehicle yaw rate. Differential braking and active front steering are strategies widely used to control vehicle yaw motion. While differential braking is used to control vehicle yaw motion, the closed-loop system can be expressed by

\[
\dot{X} = \begin{bmatrix} f_1(\theta, \dot{\theta}, \beta, r, \phi, \phi) + [0 \ 0 \ 0 \ \Delta M_z \ 0 \ 0] \end{bmatrix}^T, \quad i = 1, 2, \ldots, 6,
\]

where \(\Delta M_z\) is the additional yaw moment achieved by differential braking on left or/and right tire.

Active front steering applies an additional front-wheel steering angle on the basis of driver steering wheel input to achieve yaw rate control. The closed-loop system under this control strategy can be expressed by

\[
\dot{X} = \begin{bmatrix} f_1(\theta, \dot{\theta}, \beta, r, \phi, \phi) + [0 \ 0 \ 2\Delta F_f(\delta) \ 2\Delta F_y(\delta)l_f \ 0 \ 0] \end{bmatrix}^T, \quad i = 1, 2, \ldots, 6,
\]

where \(\Delta F_f(\delta)\) is tire lateral force of the front axle caused by the additional front-wheel steering angle.

While the vehicle yaw rate is selected as the controlled variable, output of the controlled system is

\[
y = [0 \ 0 \ 0 \ 1 \ 0 \ 0]X.
\]

3.3. Controller Design. The basic principle of the MFAC method is establishing a dynamic linear model that is equivalent to a nonlinear system at every operating point and using the I/O data of the controlled system to estimate the pseudopartial derivative in the incremental dynamic linear model online. Subsequently, the weighted one-step forward controller is designed to realize the data-driven MFAC of nonlinear systems.

The dynamic linearization techniques include the compact form dynamic linearization (CFDL), the partial form dynamic linearization (PFDL), and the full form dynamic linearization (FFDL). The CFDL and PFDL are used for the SISO system. They assume that the partial derivative of the controlled system output is only related to its input. The FFDL are suitable both for the SISO and MIMO system. It assumes that the partial derivative of the controlled system output is related to its input and its previous output.

In this research, control input of the tank truck is the additional yaw moment or the additional front-wheel steering angle, and control output of the tank truck is the vehicle yaw rate. Different from the model-driven controller design, the controller designed by the MFAC method only needs to describe the dynamic relationship between control input and output; other vehicle driving state variables do not need to be measured or estimated. This would get rid of the online estimation of pendulum swing state variables which are extremely difficult to estimate and greatly alleviate controller design difficulty.

Equations (13) and (14) show that both input and output of the controlled system have affection on the vehicle yaw rate. Thus, the controller is designed based on FFDL. It is assumed that the control input in time of \([k - n_u, k]\) and control output in time of \([k - n_y, k]\) have great influence on the vehicle yaw rate at \(k + 1\). On this hypothesis, the linear dynamic model of the controlled system will be

\[
\Delta r(k + 1) = \phi_{n_r,n_u}(k)\Delta H_{n_r,n_u}(k),
\]

where \(\phi_{n_r,n_u}(k) = [\phi_1(k), \phi_2(k), \ldots, \phi_{n_r}(k), \phi_{n_r+1}(k), \ldots, \phi_{n_r+n_u}(k)]^T\) is the pseudopartial derivative matrix of the controlled system, \(\Delta H_{n_r,n_u}(k) = [\Delta r(k), \ldots, \Delta r(k - n_y + 1), \Delta u(k), \ldots, \Delta u(k - n_u + 1)]^T\) is the column vector composed by input and output of the controlled system that has influence on system output at the next moment, \(\Delta r(k), \ldots, \Delta r(k - n_y + 1)\) is the column vector composed by input and output of the controlled system that has influence on system output at the next moment, \(\Delta u(k), \ldots, \Delta u(k - n_u + 1)\) is the column vector composed by input and output of the controlled system that has influence on system output at the next moment. Without the aid of this time-varying linear dynamic model, the relationship between input and output of the controlled system can be obtained.
The estimation result of pseudopartial derivative matrix obtained by system I/O data is

\[
\hat{\phi}_{n_y,n_u}(k) = \hat{\phi}_{n_y,n_u}(k-1) + \frac{\eta_1 \Delta H_{n_y,n_u}(k-1)}{\mu + \|\Delta H_{n_y,n_u}(k-1)\|^2} \left[ r(k) - r(k-1) - \hat{\phi}^T_{n_y,n_u}(k-1) \Delta H_{n_y,n_u}(k-1) \right], \tag{17}
\]

where \(\eta_1\) is the step size factor, \(\eta_1 \in (0, 1)\), and \(\mu\) is the penalty factor, \(\mu > 0\).

The controller designed on the basis of the linear dynamic model can be expressed as
\[ u(k) = u(k-1) + \phi_{n_y+1}(k) \frac{\rho n_y+1 \left[ r^* (k + 1) - r(k) \right] - \sum_{i=1}^{n_y} \rho_i \phi_i(k) \Delta r (k - i + 1) - \sum_{i=n_y+2}^{n_y+nu} \rho_i \phi_i(k) \Delta u (k - i + 1)}{\lambda + \left| \phi_{n_y+1}(k) \right|^2} \]  

where \( r^* \) is the limit threshold of vehicle yaw rate, \( \phi_{n_y+1} \) is the estimated value of the \((n_y+1)\)th submatrix of the pseudopartial derivative matrix \( \phi_{n_y, n_y+1} \), \( \phi_i \) is the estimated value of the \(i\)th submatrix of the pseudopartial derivative matrix \( \phi_{n_y, n_y} \), \( \rho \) is the step size factor, \( \rho \subseteq (0, 1] \), and \( \lambda \) is the weighting factor, \( \lambda > 0 \).

4. Simulation Results and Analysis

4.1. The Performance of Differential Braking Control Strategy.

The actual additional yaw moment input under differential braking strategy is
Figure 5: The architecture of tank trucks’ rollover control.

Figure 6: Dynamic response of LTA810 with and without differential braking control. (a) The rear axle LTR, (b) vehicle body roll angle, (c) pendulum oscillation angle, and (d) the additional yaw moment.
where $U$ is the actual additional yaw moment input, $u$ is the theoretical control input calculated by equations (16)–(18), and $M$ is the amplification coefficient.

It has been studied in [6, 7] that tankers are the most prone to encounter with rollover accident when liquid fill percentage is $0.6 \sim 0.8$. $k_{+\text{hus}}$, liquid fill percentage of 0.6 is chosen as the cargo loading state for LTAB10. With vehicle driving speed of 15 m/s, the rear axle LTR of LTAB10 quickly reaches 1.0 while the tanker is given a step input of the front-wheel steering angle of 0.07 rad (Figure 6), and the rollover accident happens. After applying the differential braking that contributes to an additional yaw moment $U$, tanker’s rear axle LTR shall not reach to the rollover threshold of 1.0. Furthermore, by the aid of differential braking, LTAB10 can still keep roll stability while the front-wheel steering angle increased to 0.4 rad or vehicle driving speed increased to 25 m/s.

Under the three vehicle driving conditions, the differential braking controller helps LTAB10 keep roll stability. Under active control, the peak and steady-state value of rear axle LTR are smaller than 0.89 and 0.75, respectively. Lateral liquid sloshing is also restrained by the controller. Figure 6(c) tells that the pendulum oscillation angle does not increase unlimited but keeps around 1.92 rad. It should be
mentioned that the required additional yaw moment increases as vehicle instability aggravates.

Keep the initial pseudopartial derivative matrix, the step size factor, the weight factor, and the penalty factor of the controller constant, and only adjust the limit threshold of tankeryawrate according to Figure 4. k+_heliquidfillpercentage of 0.7 and 0.8 are chosen as the cargo loading state for LTAB15 and LTAB20, respectively. Dynamic response of LTAB15 and LTAB20 with and without control is shown in Figures 7 and 8. In Figure 7, LTAB15 encounters with rollover accident under angular step test with vehicle driving speed of 15 m/s and the front-wheel steering angle of 0.05 rad. In Figure 8, LTAB20 encounters with rollover accident under the angular step test with vehicle driving speed of 15 m/s and the front-wheel steering angle of 0.04 rad.

Figures 6–8 tell that the differential braking controller based on the MFAC method could work in a wide range of factors that have great influence on tanker roll stability, such as tank shape, liquid fill percentage, vehicle driving speed, and front-wheel steering angle. After applying differential braking control, the peak and steady-state value of rear axle LTR is limited below 0.96 and 0.75, respectively. Furthermore, lateral liquid sloshing is also restrained.
4.2. The Performance of Active Front Steering Control Strategy. The actual additional front steering angle input under active front steering strategy can be obtained by equations (16)–(18). Dynamic response of tank vehicles LATB10, LTAB15, and LTAB20 with and without the active front steering controller is shown in Figures 9–11. Liquid fill percentage for these three tankers is 0.6, 0.7, and 0.8, respectively.

Under different test conditions, the initial pseudopartial derivative matrix, the step size factor, the weight factor, and the penalty factor of the controller keep constant, only the limit threshold of the vehicle yaw rate is adjusted. The simulation shows that the active front steering controller based on the MAFC method plays a positive role on tanker roll stability, and it is adaptive to tank shape, liquid cargo fill percentage, vehicle driving speed, and front-wheel steering angle. By the aid of the active controller, the peak and steady-state value of rear axle LTR are limited below 0.91 and 0.75, respectively. Pendulum oscillation is also restrained.

4.3. Comparison of the Two Control Strategies. Both differential braking and active front steering controllers based on the MFAC method work well for tanker roll stabilization. A comparison between them is carried out to figure out the
Transient characters of vehicle rear axle LTR, such as the settling time, overshoot, steady-state value, and damped oscillation frequency, are calculated for this work. The results are plotted in Figure 12.

The damped oscillation frequency and the steady-state value of LTR under these two control strategies have quite slightly difference, but settling time and overshoot differ a lot. LTR under active front steering control has a much smaller settling time and overshoot than that under differential braking control, and the differences are about 54% and 37%, respectively. The comparison shows that active front steering controller is much more suitable for tanker roll stability.

Figure 10: Dynamic response of LTAB15 with and without active front steering control. (a) The rear axle LTR, (b) vehicle body roll angle, (c) pendulum oscillation angle, and (d) the additional yaw moment.
Figure 11: Dynamic response of LTAB20 with and without active front steering control. (a) The rear axle LTR, (b) vehicle body roll angle, (c) pendulum oscillation angle, and (d) the additional yaw moment.

Figure 12: Continued.
5. Discussion and Conclusions

Tank trucks are apt to encounter with rollover accidents due to lateral liquid sloshing in partially filled tanks, and active rollover control would bring positive influence on it. Given the consideration of the essential nonlinear characters of the tank truck, the model-driven controller design would not be a good choice. Not only is there a giant difficulty in the controller design, but the designed controller will be complex and fragile due to the nonlinear items. Literature review shows two common data-driven controllers, PID and fuzzy logic controllers, have been studied in tank vehicle rollover stabilization. These two data-driven controllers are designed offline. Controller parameters keep constant and cannot be tuned adaptively. Thus, they are generally used in the linear controlled system. Tank vehicle dynamics reveal nonlinear characters, and it is greatly influenced by vehicle loading state (tank shape, liquid fill percentage, and physical characters of liquid cargo are included) and vehicle driving state (vehicle driving speed and the front-wheel steering angle are included). Once vehicle loading or driving state changes, the offline tuned PID or fuzzy logic controller will not be suitable. The changeable vehicle loading and driving states calls for an adaptive data-driven controller.

Therefore, MFAC, one of data-driven methods, is used to design the desired controller in this research. The MFAC method has the advantage of adaptability under system working condition, which meets the needs of the tanker controller design as tank trucks usually have different tank shapes, liquid cargo fill percentage, vehicle driving speed, and front-wheel steering angle. The differential braking and active front steering controller based on the MFAC method are considered in this research. In the controlled system, the vehicle yaw rate is used as the output of the controlled system, and the additional yaw moment or the additional front-wheel steering angle is used as the control input of the controlled system.

Simulation on the controlled system shows that both the differential braking and active front steering controller plays a positive role on tanker roll stability. LTR of vehicle rear axle is used as the variable to reflect vehicle roll stability. With the aid of controller, the peak and steady-state values of LTR are limited below 0.96 and 0.75, respectively. Pendulum oscillation is effectively restrained, too. The simulation also shows the controller’s adaptability to tank shape, liquid fill percentage, vehicle driving speed, and front-wheel steering angle.

A comparison between the two controllers is carried out to figure out the differences. The result shows that the damped oscillation frequency and the steady-state value of LTR under the two control strategies have quite slight difference, but settling time and overshoot differ a lot. The active front steering strategy controller is much more suitable for tank roll stability, as settling time and overshoot of LTR under active front steering control is about 54% and 37% smaller than that under differential braking control.

Data Availability

The tank vehicle information and modelling and controller design data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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