Centrifugal-Barrier Effects and Determination of the Interaction Radius

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Abstract

The interaction radius of a resonance is an important physical quantity to describe the structure of a resonance. But, for a long time, physicists do not find a reliable way to measure the magnitude of the interaction radius of a resonance. In this paper, a method is proposed to measure the interaction radius in physics analysis. It is found that the centrifugal barrier effects have great influence to physical results obtained in the PWA fit, and the interaction radius of some resonances can be well measured in the fit.

1 Introduction

Partial Wave Analysis(PWA) now is widely used in physics analysis of high energy experimental physics. As an effective method, by analyzing information of final state particles, it is used to search new resonances in a complicated spectrum, and to determine mass, width, branching ratio and spin-parity of an intermediate resonance. In the traditional PWA analysis, the

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centrifugal barrier effects are considered. It is known that, after a resonance decays into final state particles, the outward-going wave is being reflected inward by the centrifugal barrier. Therefore, the radial probability density at infinity is the centrifugal barrier factor $|BF(L, kR)|^2$ times the radial probability density at the interaction radius $R$.

The centrifugal barrier effects are studied in literatures [1, 2]. Basic results of the papers are that the shape and the partial decay width of a resonance are affected by the centrifugal barrier effects. The centrifugal barrier factor $BF(L, kR)$ has two parameters, the orbital angular momentum quantum number $L$ and the interaction radius $R$. It shows that the interaction radii range from 0.25 to 0.75 $F$ for the meson resonance decays and from 0.1 to 1 $F$ for the baryon decays.

It is conventionally believed that the influence of the centrifugal barrier effects to physics analysis is small. Therefore, the centrifugal barrier effects are not considered in some physics analysis, which corresponds to the case that $BF(L, kR) = 1$ or $R = \infty$. In partial wave analysis, the centrifugal barrier effects are considered, but the interaction radius $R$ is set to 1 $F$ for all resonances[3, 4, 5, 6, 7, 8]. In this paper, the influence of the centrifugal barrier effects to physics analysis is studied, and a method is proposed to measure the interaction radius in physics analysis. First, the decay amplitude used in the physics analysis is given. Then, the problem that how the resonance’s line shape is affected by the centrifugal barrier effects is studied. Finally, we will discuss how to measure the interaction radius in a PWA fit.

2 Centrifugal Barrier Factor

Let’s consider a two body decay $a \rightarrow b + c$. In the center of mass system of the mother particle $a$, the magnitude of the momentum of the particle $b$ is $k$, and the orbital angular momentum quantum number of the daughter particle system is $l$. Suppose that the interaction potential $V(r)$ is almost zero when $r \geq R_0$, where $R_0$ is the interaction radius of the mother particle.
a. The wave equation which holds outside the interaction radius $R_0$ is

$$R'' + \frac{2}{r}R' + \left(k^2 - \frac{l(l+1)}{r^2}\right)R = 0,$$

(1)

where $R$ is the radial wave function, and $k = \sqrt{\frac{2\mu E}{\hbar^2}}$ is the semiclassical impact parameter. It has two independent solutions, $h_l^{(1)}(\rho)$ and $h_l^{(2)}(\rho)$, where $\rho = kr$. The probability density at $r$ is proportional to $|\rho R(\rho)|^2$. Define

$$BF(l, \rho) = \frac{1}{|\rho h_l^{(1)}(\rho)|}.$$

(2)

Then, the probability at infinity is the probability at $R_0$ times $|BF(l, \rho_0)|^2$ with $\rho_0 = kR_0$. Some frequently used barrier factors are

$$BF(0, \rho_0) = 1,$$

(3)

$$BF(1, \rho_0) = \sqrt{\frac{2z}{z+1}},$$

(4)

$$BF(2, \rho_0) = \sqrt{\frac{13z^2}{z^2 + 3z + 9}},$$

(5)

$$BF(3, \rho_0) = \sqrt{\frac{277z^3}{z^3 + 6z^2 + 45z + 225}},$$

(6)

$$BF(4, \rho_0) = \sqrt{\frac{12746z^4}{z^4 + 10z^3 + 135z^2 + 1575z + 11025}},$$

(7)

$$BF(5, \rho_0) = \sqrt{\frac{998881z^5}{z^5 + 15z^4 + 315z^3 + 6300z^2 + 99225z + 893025}},$$

(8)

$$BF(6, \rho_0) = \sqrt{\frac{118394977z^6}{z^6 + 21z^5 + 630z^4 + 18900z^3 + 496125z^2 + 9823275z + 108056025}}.$$

(9)
where \( z = \rho_0^2 \). Figure 1 shows the shapes of \( |BF(L, kR)|^2 \) with L=2,4,6 and 8. If \( L = 0, BF(0, kR) = 1 \), which means that the line shape of a scalar resonance is not affected by the centrifugal barrier effects. In this figure, all curves are normalized to 1 when the momentum \( k \) approaches infinity. It could be seen that the shapes of these curves change greatly when the momentum \( k \) is in the region 0-2 GeV, which is just in the momentum region of final state particles produced in the decays of most mesons or hadrons. Different shapes of barrier factors give out different line shapes of resonances. If the centrifugal barrier effects are not correctly considered, the line shape of a resonance given by it will be quite different from the one with the centrifugal barrier factors correctly considered, which will cause large errors in the measurement of mass, width and branching ratio of a resonance. If the momentum of the daughter particle in the center of mass system of the mother particle is greater than 3 GeV, the influence of the centrifugal barrier effects to such kind of decay will be small. In this case, the mass of mother particle must be greater than 6 GeV. Therefore, if we study the decay of a low mass resonance, the influence of the centrifugal barrier effects must be considered, otherwise, the systematic uncertainty caused by it will be expected to be quite large.

### 3 Decay Amplitude

Now, let’s study the influence of the centrifugal barrier effects to the resonance’s line shape. We will study how the resonance’s line shape is changed by the centrifugal barrier effects. The barrier factor of a resonance decay not only affects its own line shape, but also affects the line shapes of its daughter particles. Therefore, for a sequential decay, the line shape of an intermediate resonance is affected by more than one barrier factor. As an example, let’s consider the sequential decay \( J/\psi \rightarrow \omega X \rightarrow \omega \pi \pi \). The spin-parity of the resonance \( X \) is \( J^{PC} \). In this sequential decay, there are two vertexes, one is the decay \( J/\psi \rightarrow \omega X \), and another is the decay \( X \rightarrow \pi \pi \). The decay amplitude of the sequential decay is[9, 3, 4, 5, 6, 7, 8]

\[
M \sim F^{(1)1}_M D^{1*}_{M(\lambda-\nu)}(\phi_1, \theta_1, 0) \cdot BW \cdot F^{(2)J}_0 D^{J*}_{J0}(\phi_2, \theta_2, 0),
\]  

(10)
Figure 1: The shape of $|BF(L,kR)|^2$ with $L=2, 4, 6$ and 8. The interaction radius $R_0$ is set to 1 fm in this figure.

where $F_{\lambda\nu}(1)$ is the helicity amplitude of the decay $J/\psi \to \omega X$, $BW$ is the Breit-Wigner function of the resonance $X$, and $F_{00}^{(2)J}$ is the helicity amplitude of the decay $X \to \pi\pi$.

If the spin-parity of the resonance $X$ is $0^+$, then the independent helicity amplitudes are

$$F_{01}^{(1)1} = g_1 - \frac{g_2}{3} BF(2, k_1 R_1),$$

$$F_{00}^{(1)1} = \gamma_{\sigma} \left( g_1 + \frac{2g_2}{3} BF(2, k_1 R_1) \right),$$

$$F_{00}^{(2)0} = g_3,$$

where $g_1$, $g_2$ and $g_3$ are scalars, $\gamma_{\sigma}$ is the rapidity of $\omega$ particle, $k_1$ is the relative momentum of $\omega$ particle in the center of mass system of $J/\psi$, and $R_1$ is the interaction radius of $J/\psi$. If the spin-parity of the resonance $X$ is
$2^+$, then the independent helicity amplitudes are

$$F^{(11)}_{21} = \frac{g_1}{10}(-7 - 3\gamma_s\gamma_\sigma + 3\beta_s\beta_\sigma\gamma_s\gamma_\sigma) + \frac{g_2}{30}(7 - 6\gamma_s\gamma_\sigma) \cdot BF(2, k_1R_1)$$
$$+ \frac{g_3}{15}(1 + 2\gamma_s\gamma_\sigma) \cdot BF(2, k_1R_1) + \frac{2g_4}{45}(1 + 2\gamma_s\gamma_\sigma) \cdot BF(2, k_1R_1)$$
$$- \frac{2g_5}{25}(3 + 4\gamma_s^2 + 8\gamma_s\gamma_\sigma) \cdot BF(4, k_1R_1),$$

(14)

$$F^{(11)}_{11} = \frac{g_1}{2\sqrt{2}}\gamma_s(-3 - 2\gamma_s\gamma_\sigma + 2\beta_s\beta_\sigma\gamma_s\gamma_\sigma) + \frac{g_2}{15\sqrt{2}}\gamma_s(3 - 4\gamma_s\gamma_\sigma) \cdot BF(2, k_1R_1)$$
$$+ \frac{\sqrt{g_3}}{15}\gamma_s(1 + 2\gamma_s\gamma_\sigma) \cdot BF(2, k_1R_1)$$
$$+ \frac{1}{75}\sqrt{6}(2\gamma_s + \gamma_\sigma + 2\gamma_s^2\gamma_\sigma) \cdot BF(4, k_1R_1),$$

(15)

$$F^{(11)}_{10} = \frac{g_1}{10\sqrt{2}}(7 + 3\gamma_s\gamma_\sigma - 3\beta_s\beta_\sigma\gamma_s\gamma_\sigma) - \frac{g_2}{30\sqrt{2}}(7 - 6\gamma_s\gamma_\sigma) \cdot BF(2, k_1R_1)$$
$$+ \frac{g_3}{18\sqrt{2}}(1 + 2\gamma_s\gamma_\sigma) \cdot BF(2, k_1R_1) + \frac{8g_4}{45\sqrt{2}}(1 + 2\gamma_s\gamma_\sigma) \cdot BF(2, k_1R_1)$$
$$- \frac{8g_5}{525\sqrt{2}}(3 + 4\gamma_s^2 + 8\gamma_s\gamma_\sigma) \cdot BF(4, k_1R_1),$$

(16)

$$F^{(11)}_{01} = \frac{g_1}{10\sqrt{6}}(-7 - 3\gamma_s\gamma_\sigma + 3\beta_s\beta_\sigma\gamma_s\gamma_\sigma) + \frac{g_2}{30\sqrt{6}}(7 - 6\gamma_s\gamma_\sigma) \cdot BF(2, k_1R_1)$$
$$- \frac{g_3}{6\sqrt{6}}(1 + 2\gamma_s\gamma_\sigma) \cdot BF(2, k_1R_1) + \frac{4g_4}{15\sqrt{6}}(1 + 2\gamma_s\gamma_\sigma) \cdot BF(2, k_1R_1)$$
$$- \frac{4g_5}{175\sqrt{6}}(3 + 4\gamma_s^2 + 8\gamma_s\gamma_\sigma) \cdot BF(4, k_1R_1),$$

(17)
\begin{align*}
F_{00}^{(1)1} &= \frac{2g_1}{5\sqrt{6}}\gamma_s(3 + 2\gamma_s\gamma_\sigma - 2\beta_s\beta_\sigma\gamma_s\gamma_\sigma) - \frac{2g_2}{15\sqrt{6}}\gamma_s(3 - 4\gamma_s\gamma_\sigma) \cdot BF(2, k_1 R_1) \\
&\quad + \frac{2g_2}{5\sqrt{6}}\gamma_s(1 + 2\gamma_s\gamma_\sigma) \cdot BF(2, k_1 R_1) \\
&\quad + \frac{4\sqrt{57}}{175}(2\gamma_s + \gamma_\sigma + 2\gamma_s^2\gamma_\sigma) \cdot BF(4, k_1 R_1). \\
(18)
F_{00}^{(2)2} &= g_6 \cdot BF(2, k_2 R_2),
\end{align*}

where $g_1$, $g_2$, $g_3$, $g_4$, $g_5$ and $g_6$ are scalars, $\gamma_s$ and $\gamma_\sigma$ are the rapidities of $X$ and $\omega$ respectively, $\beta_s$ and $\beta_\sigma$ are the velocities of $X$ and $\omega$ respectively, $k_1$ is the relative momentum of $\omega$ particle in the center of mass system of $J/\psi$, $R_1$ is the interaction radius of $J/\psi$, $k_2$ is the relative momentum of $\pi$ particle in the center of mass system of $X$, and $R_2$ is the interaction radius of resonance $X$. If the spin-parity of the resonance $X$ is $4^+$, then the independent helicity amplitudes are

\begin{align*}
F_{21}^{(1)1} &= \frac{g_1}{36\sqrt{7}}(1 + 2\gamma_s^2)(-7 - 5\gamma_s\gamma_\sigma + 5\beta_s\beta_\sigma\gamma_s\gamma_\sigma) \cdot BF(2, k_1 R_1) \\
&\quad + \frac{g_2}{420\sqrt{7}}(21 + 84\gamma_s^2 - 60\gamma_s\gamma_\sigma - 40\gamma_s^3\gamma_\sigma) \cdot BF(4, k_1 R_1) \\
&\quad + \frac{3g_3}{700\sqrt{7}}(3 + 12\gamma_s^2 + 12\gamma_s\gamma_\sigma + 8\gamma_s^3\gamma_\sigma) \cdot BF(4, k_1 R_1) \\
&\quad + \frac{4g_4}{325\sqrt{7}}(3 + 12\gamma_s^2 + 12\gamma_s\gamma_\sigma + 8\gamma_s^3\gamma_\sigma) \cdot BF(4, k_1 R_1) \\
&\quad - \frac{4g_5}{3465\sqrt{7}}(5 + 36\gamma_s^2 + 8\gamma_s^4 + 24\gamma_s\gamma_\sigma + 32\gamma_s^3\gamma_\sigma) \cdot BF(6, k_1 R_1), \\
(20)
\end{align*}
\[ F_{11}^{(1)} = \frac{g_1}{27\sqrt{7}} \gamma_s (1 + 2\gamma_s^2) (-5 - 4\gamma_s\gamma_\sigma + 4\beta_s\beta_\sigma \gamma_s\gamma_\sigma) \cdot BF(2, k_1 R_1) \]
\[ + \frac{g_2}{315\sqrt{7}} \gamma_s (15 + 60\gamma_s^2 - 48\gamma_s\gamma_\sigma - 32\gamma_s^3\gamma_\sigma) \cdot BF(4, k_1 R_1) \]
\[ + \frac{4g_4}{2079\sqrt{7}} \gamma_s (3 + 12\gamma_s^2 + 12\gamma_s\gamma_\sigma + 8\gamma_s^3\gamma_\sigma) \cdot BF(4, k_1 R_1) \]
\[ + \frac{8g_5}{2079\sqrt{7}} (12\gamma_s + 16\gamma_s^3 + 3\gamma_\sigma + 24\gamma_s^2\gamma_\sigma + 8\gamma_s^4\gamma_\sigma) \cdot BF(6, k_1 R_1), \]
\[ (21) \]

\[ F_{10}^{(1)} = \frac{g_1}{36\sqrt{7}} (1 + 2\gamma_s^2)(7 + 5\gamma_s\gamma_\sigma - 5\beta_s\beta_\sigma \gamma_s\gamma_\sigma) \cdot BF(2, k_1 R_1) \]
\[ - \frac{g_2}{420\sqrt{7}} (21 + 84\gamma_s^2 - 60\gamma_s\gamma_\sigma - 40\gamma_s^3\gamma_\sigma) \cdot BF(4, k_1 R_1) \]
\[ + \frac{g_3}{700\sqrt{7}} (3 + 12\gamma_s^2 + 12\gamma_s\gamma_\sigma + 8\gamma_s^3\gamma_\sigma) \cdot BF(4, k_1 R_1) \]
\[ + \frac{8g_4}{525\sqrt{7}} (3 + 12\gamma_s^2 + 12\gamma_s\gamma_\sigma + 8\gamma_s^3\gamma_\sigma) \cdot BF(4, k_1 R_1) \]
\[ - \frac{8g_5}{3465\sqrt{7}} (5 + 36\gamma_s^2 + 8\gamma_s^4 + 24\gamma_s^2\gamma_\sigma + 32\gamma_s^3\gamma_\sigma) \cdot BF(6, k_1 R_1), \]
\[ (22) \]

\[ F_{01}^{(1)} = \frac{g_1}{18\sqrt{70}} (1 + 2\gamma_s^2)(-7 - 5\gamma_s\gamma_\sigma + 5\beta_s\beta_\sigma \gamma_s\gamma_\sigma) \cdot BF(2, k_1 R_1) \]
\[ + \frac{g_2}{210\sqrt{70}} (21 + 84\gamma_s^2 - 60\gamma_s\gamma_\sigma - 40\gamma_s^3\gamma_\sigma) \cdot BF(4, k_1 R_1) \]
\[ - \frac{g_3}{70\sqrt{70}} (3 + 12\gamma_s^2 + 12\gamma_s\gamma_\sigma + 8\gamma_s^3\gamma_\sigma) \cdot BF(4, k_1 R_1) \]
\[ + \frac{4g_4}{105\sqrt{70}} (3 + 12\gamma_s^2 + 12\gamma_s\gamma_\sigma + 8\gamma_s^3\gamma_\sigma) \cdot BF(4, k_1 R_1) \]
\[ - \frac{4g_5}{693\sqrt{70}} (5 + 36\gamma_s^2 + 8\gamma_s^4 + 24\gamma_s^2\gamma_\sigma + 32\gamma_s^3\gamma_\sigma) \cdot BF(6, k_1 R_1), \]
\[ (23) \]
\[ F_{00}^{(1)} = \frac{4g_1}{27\sqrt{70}} \gamma_s (1 + 2\gamma_s^2)(5 + 4\gamma_s \gamma_\sigma - 4\beta_s \beta_\sigma \gamma_s \gamma_\sigma) \cdot BF(2, k_1 R_1) \]
\[ - \frac{4g_2}{315\sqrt{70}} \gamma_s (15 + 6\gamma_s^2 - 48\gamma_s \gamma_\sigma - 32\gamma_s^3 \gamma_\sigma) \cdot BF(4, k_1 R_1) \]
\[ + \frac{4g_4}{63\sqrt{70}} \gamma_s (3 + 12\gamma_s^2 + 12\gamma_s \gamma_\sigma + 8\gamma_s^3 \gamma_\sigma) \cdot BF(4, k_1 R_1) \]
\[ + \frac{4g_5}{2079\sqrt{70}} (12\gamma_s + 16\gamma_s^3 + 3\gamma_\sigma + 24\gamma_s^2 \gamma_\sigma + 8\gamma_s^4 \gamma_\sigma) \cdot BF(6, k_1 R_1), \]
\[ F_{00}^{(2)} = g_6 \cdot BF(4, k_2 R_2). \]

In the traditional PWA analysis, the interaction radii \( R_1 \) and \( R_2 \) are set to 1 F. It is obvious that such treatment is not reasonable, for different resonance has different interaction radius. If the systematic uncertainly caused by it is small enough, the treatment is acceptable. Otherwise, we must find a way to determine the interaction radius in physics analysis. Next, we will study how the resonance’s line shape is affected by the interaction radius, and how to determine the interaction radius in physics analysis.

### 4 Resonance’s Line Shape

In order to know how the resonance’s line shape is affected by the interaction radius, we consider a simplified model, that is, only the centrifugal barrier effects are considered in the helicity amplitudes \( F_{\lambda \nu}^{(1)} \) and \( F_{00}^{(2)J} \). In this simplified model, the decay amplitude of the sequential decay is

\[ M \sim BF^{(1)}(L, k_1 R_1) \cdot BW \cdot BF^{(2)}(J, k_2 R_2), \]

where \( k_1 \) and \( k_2 \) are relative momenta of daughter particles in the center of mass system of the first and the second decay vertexes respectively, and \( R_1 \) and \( R_2 \) are the corresponding interaction radiuses of two mother particles. The absolute square of the decay amplitude of the sequential decay gives out
the line shape of the resonance $X$. It could be seen that the line shape of the resonance $X$ is affected by two independent centrifugal barrier effects, one is the centrifugal barrier effect of the decay of the resonance $X$ itself, another is the centrifugal barrier effect of the decay of the mother particle whose decay produces resonance $X$. Next, we will study these two barrier effects separately.

$BF^{(2)}(J, k_2 R_2)$ is the barrier factor of the decay of the resonance $X$. In order to know how the line shape is affected by $BF^{(2)}(J, k_2 R_2)$, we study the shape of $|BW(s, m, \Gamma) \cdot BF^{(2)}(J, k_2 R_2)|^2$. Figure 2 shows the resonance’s line shape after the influence of $BF^{(2)}(J, k_2 R_2)$ ( $J=0,2,4,$ and 6) is considered. The curves in figure 2 show the shapes of $|BW(s, m, \Gamma) \cdot BF^{(2)}(J, k_2 R_2)|^2$. In this model, the mass and the width of the resonance $X$ are set to 1.27 GeV and 0.185 GeV respectively, and $Pr = \frac{4}{R}$ is set to 0.1973 GeV, which corresponds to 1 F interaction radius. In this figure, we could see that the influence of the centrifugal barrier effects to the resonance’s line shape is that the lower part of the resonance’s line shape is suppressed and the higher part is enhanced. The total effect of the influence from the centrifugal barrier effects to the resonance’s line shape is unsymmetric. And this influence becomes larger when the orbital angular momentum quantum number $J$ is bigger. In this figure, the influence from the centrifugal barrier effects to the resonance’s line shape is quite large. Therefore, it is expected that the systematic errors caused by it will be large if it is not correctly considered in the analysis. In some physics analysis, the centrifugal barrier effects are completely not considered, physics results obtained by it will have large systematic errors. According to a test on BESII data, the systematic errors to the mass and the width of a resonance caused by this reason are generally about 20-100 MeV and the relative uncertainties of branching ratio is about 30-50 %.

Another barrier factor that affects the line shape of the resonance $X$ is that of the $J/\psi$ decay, that is, $BF^{(1)}(L, k_1 R_1)$. In order to know its influence to the resonance’s line shape, let’s study the shape of $|BF^{(1)}(L, k_1 R_1) \cdot BW(s, M, \Gamma)|^2$. Figure 3 shows the curves of $|BF^{(1)}(L, k_1 R_1) \cdot BW(s, M, \Gamma)|^2$ with $L=0,2,4,$ and 6. Contrary to the barrier factor $BF^{(2)}(J, k_2 R_2)$, $BF^{(1)}(L, k_1 R_1)$ suppresses the higher part of the resonance’s line shape and enhances its lower part. Quantitatively speaking, the influence is relatively much smaller than
Figure 2: The shape of $|BW(s,m,\Gamma) \cdot BF^{(2)}(J,k_2R_2)|^2$ with $J=0,2,4,$ and $6$. The interaction radius $R_0$ is set to 1 F in this figure.

In most partial wave analysis, the centrifugal barrier effects are considered and the interaction radius $R$ is set to 1 F for all resonances. According to the literature [2], the interaction radius $R$ is not exact 1 F. The interaction radii range from 0.25 to 0.75 F for the meson resonance decays and from 0.5 to 1 F for the baryon decays. Uncertainties on the interaction radius $R$ will cause systematic uncertainties in physics analysis. If different values of the interaction radius do not cause obvious changes on the resonance’s line shape, the systematic uncertainties caused by it will be small. In this case, no matter what values the interaction radius is, final physical results are acceptable. But if resonance’s line shape is sensitive to the value of the interaction radius, we can not fix the value of the interaction radius arbitrarily in physics analysis. Now, we study the influence of the interaction radius to the resonance’s line shape. We consider the decay of a spin-2 particle, and neglect the centrifugal barrier effects of the $J/\psi$ decay. In this model, the decay amplitude is

$$M \sim BW \cdot BF^{(2)}(2,k_2R_2).$$

(27)

The line shape of the spin-2 particle is shown in Figure 4. Dashed line is the
Figure 3: The shape of $|BF^{(1)}(L, k_1 R_1) \cdot BW(s, M, \Gamma)|^2$ with $L=0, 2, 4$, and 6. The interaction radius $R_0$ is set to 1 F in this figure.

The line shape of the spin-2 particle when its interaction radius is set to 1 F. Solid line, dotted line and dot-dashed line are the corresponding line shapes when its interaction radius is set to $10/3$ F, 0.5 F and 0.2 F respectively. From this figure, we could clearly see that the influence of the interaction radius to the resonance’s line shape is obvious and quite large. If the resonance is a scalar particle, four curves will coincide, for $BF^{(2)}(0, k_2 R_2) = 1$. In other words, the line shape of a scalar resonance is not affected by its centrifugal barrier factor. If the spin of the resonance is 4, the influence of the centrifugal barrier factor will become larger. Figure 5 shows the line shapes of a spin-4 resonance with interaction radius 0.2 F, 0.5 F, 1 F and $10/3$ F respectively. From these two figures, our intuitive impression is that the influence of the centrifugal barrier effects is to make the resonance’s line shape unsymmetric, which provides us a method to measure the exact value of the resonance’s interaction radius.

## 5 Interaction Radius

From figure 4 and figure 5, we could see that the line shape of a resonance is affected by its centrifugal barrier factor. It is also sensitive to the value of the
Figure 4: The shape of $|BW(s, M, \Gamma) \cdot BF(2, k_2 R_2)|^2$ with interaction radius $R_0$ 0.2 F, 0.5 F, 1 F and 10/3 F respectively.

Figure 5: The shape of $|BW(s, M, \Gamma) \cdot BF(4, k_2 R_2)|^2$ with interaction radius $R_0$ 0.2 F, 0.5 F, 1 F and 10/3 F respectively.
resonance’s interaction radius. Different values of the interaction radius will give out different line shapes. Since the line shape of a resonance is sensitive to its interaction radius, it is possible for us to measure the interaction radius in high energy experiments.

In physics analysis, a resonance is traditionally described by the Breit-Wigner function, which has two independent parameters, the mass and the width of the resonance. In fact, some information are missing in this treatment. To describe a resonance, we need at least three parameters, they are the mass, the width and the interaction radius of the resonance. The mass of the resonance gives out the central position of the peak, the width gives out the width of the peak, and the interaction radius gives out the asymmetry of the resonance’s line shape.

In the traditional physics analysis, the interaction radii of all particles are set to 1 F. (In some physics analysis, the centrifugal barrier factor is not considered, which corresponds to the case that the interaction radii for all particles are set to infinity.) Such treatment will cause large systematic errors to the measurement of mass, width and branching ratio of the intermediate resonance. A way to solve this problem is to measure the interaction radius in physics analysis. It is found that, in partial wave analysis, the log likelihood function is sensitive to the value of the interaction radius, and the interaction radius of an intermediate resonance can be well determined by the technique of the interaction radius scan. In the interaction radius scan, the mass and width of a resonance is fixed, and the interaction radius $R$ is the only parameter to be changed. It is found that the log likelihood function is changed when the parameter $R$ is changed. There is a maximum of log likelihood function, and the corresponding parameter $R$ gives out the interaction radius of the resonance. By using this method, the interaction radii of some resonances are successfully determined in a analysis on BESII $J/\psi \rightarrow \omega \pi \pi$ and $J/\psi \rightarrow K^*(892)K\pi$ data. Old analysis on these data have already been published[10, 11].

A test is done on a Monte Carlo data. First, a Monte Carlo data is generated. The data sample consists of 1000 $J/\psi \rightarrow \omega X(1270) \rightarrow \omega \pi \pi$ events. The interaction radius of $X(1270)$ is set to 255 MeV. Then we make an interaction radius scan on this data. The scan curve is shown in Fig. 6. The
minimum of the curve gives out the interaction radius of $X(1270)$ of the Monte Carlo data. From this test, two basic conclusions can be drawn. One is that the log likelihood function is sensitive to the value of the interaction radius. The other is that the magnitude of interaction radius can be well determined by the interaction radius scan.

![Figure 6: Interaction radius scan on a Monte Carlo data. The interaction radius of $X(1270)$ is set to 255 MeV.](image)

6 Discussions and Results

In this paper, the theoretically formula used in physics analysis of $J/\psi \rightarrow \omega\pi\pi$ and $J/\psi \rightarrow K^*(892)K\pi$ channel is given. In the PWA analysis of these channels, it is found that our traditional treatment that all interaction radii are set to 1 F will cause quite large systematic uncertainties. The systematic uncertainties cause by it is too large to be neglectable. A way to solve this problem is to measure the interaction radius in physics analysis.

The interaction radius $R$ of a resonance is the size of the resonance. It is an important physical quantity to describe a resonance. But for a long
time, we do not know how to measure it in the experiments. Up to now, we still have no experimental measurements on it. According to our analysis on BESII data, we found that the interaction radius of a resonance can be well determined in the PWA analysis. So, we can systematically measure the interaction radiiuses of mesons and baryons based on BESII and BESIII data. It open a new research field of high energy experimental physics.

In the traditional treatment, a resonance is describe by two parameters, the mass and the width. Now, we know that, such treatment is incomplete. we need a third parameter to describe the resonance. This new parameter is the interaction radius of the resonance, which gives out the asymmetry of the resonance’s line shape.

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