Two-Particle Correlations in the Wave Function and Covariant Current Approaches

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Received March 26, 2008

Abstract—We consider two-particle correlations which appear in relativistic nuclear collisions owing to the quantum statistics of identical particles in the frame of two formalisms: wave-function and current parametrizations. The first one is based on solution of the Cauchy problem, whereas the second one is a so-called current parametrization of the source of secondary particles. We argue that these two parametrizations of the source coincide when the wave-function at freeze-out times is put into a specific correspondence with a current. Then, the single-particle Wigner density evaluated in both approaches gives the same result.

PACS numbers: 25.75.-q, 25.75.Gz, 12.38.Mh

DOI: 10.1134/S1063778808090202

1. INTRODUCTION

The models and approaches which are used to describe the processes occurring in the reaction region in relativistic heavy-ion collisions are examined by comparison of provided predictions with experimental data on single-, two-, and many-particle momentum spectra, which contain information about the source at the early stage (photons, dileptons) and at the stage of so-called freeze-out (hadron spectra). Two-particle correlations or the Hanbury–Brown–Twiss interferometry (HBT) encapsulates information about the space–time structure and dynamics of the emitting source [1–7]. Usually, consideration of the correlations which occur in relativistic heavy-ion collisions assumes that (i) the particles are emitted independently (or the source is completely chaotic), and (ii) finite-multiplicity corrections can be neglected. Both approximations are expected to be good for high-energy nuclear collisions with large multiplicities. Then, correlations reflect (a) the effects from symmetrization (antisymmetrization) of the amplitude to detect identical particles with certain momenta and (b) the effects which are generated by the final-state interactions (FSI) of the detected particles between themselves and with the source. At first sight, one can regard the FSI as a contamination of “pure” particle correlations. But it should be noted that the FSI depend on the structure of the emitting source and thus provide as well information about source dynamics [8].

Several surprising questions motivated by new experimental data appeared recently in the HBT. For instance, the experimental measurements on two-pion correlations [9–12] give the ratio of $R_{\text{out}}/R_{\text{side}} \approx 1$, which is much smaller than that predicted theoretically (the so-called RHIC HBT puzzle). This raises the question to what extent some of the model predictions are consistent with experimental measurements [13, 14], or maybe the observed discrepancies are due to such an “apples-with-oranges” comparison. All this drew attention and inspired a more detailed discussion of the theoretical background of the HBT. In the present paper, we are going along this line, and we would like to clarify a question concerning different kinds of parametrization exploited in the HBT.

The nominal quantity expressing the correlation function in terms of experimental distributions [2] is

$$C(k_a, k_b) = \frac{P_2(k_a, k_b)}{P_1(k_a) P_1(k_b)},$$

where $P_1(k) = E d^3 N / d^3 k$ and $P_2(k_a, k_b) = E_a E_b d^3 N / (d^3 k_a d^3 k_b)$ are single- and two-particle cross sections.
In the absence of final-state interactions, the theoretical expression for the two-particle correlator reads

\begin{equation}
C(q, K) = 1 + \frac{\int d^4X e^{iq^x} S(X, K)}{\int d^4XS(X, K + q/2) \int d^4YS(Y, K - q/2)}
\end{equation}

where \( K = (k_a + k_b)/2 \), and \( q = k_a - k_b \). This expression was obtained in the different approaches. In the so-called wave-function approach \[8\], source function \( S(X, K) \) is defined in the following way:

\begin{equation}
S_{w}(X, K) = \int d^4xe^{iK^x} \times \sum_{\gamma,\gamma'} \rho_{\gamma\gamma'} \psi_{\gamma}(X + x/2) \psi_{\gamma'}^{*}(X - x/2),
\end{equation}

where \( \rho_{\gamma\gamma'} \) is the density matrix which in thermal equilibrium has the form \( \rho_{\gamma\gamma'} = \delta_{\gamma\gamma'} \exp(-E_{\gamma}/T) \). The wave-function \( \psi_{\gamma}(t, x) \) is taken at freeze-out times, i.e., \( t \in t_\Sigma \). Freeze-out hypersurface \( \Sigma \) is a spatial surface which moves in space in the same way as, for instance, the surface of a balloon during pumping. It represents an imaginary border between two domains: inside the surface, a strong dynamics takes place, whereas outside the surface, the particles propagate outward freely. The wave-function at freeze-out times can be regarded as the initial one for its further history, and because its further evolution is free (we do not discuss FSI thus far), it can easily be taken into account. As is intuitively understood, the free evolution can reverse back and the resulting cross section and other measurable quantities are determined in the same way as, for instance, the surface of a balloon during pumping.

Correlation function (2) was derived first in the model, where the essential point is a parametrization of the source by use of the currents \( J_{\gamma}(x) \) \[1\] (see also \[15\]), which then become the constituent elements of the source function:

\begin{equation}
S_{\gamma}(X, K) = \int d^4xe^{iK^x} \times \sum_{\gamma,\gamma'} \rho_{\gamma\gamma'} J_{\gamma}(X + x/2) J_{\gamma'}^{*}(X - x/2).
\end{equation}

As a matter of fact, both approaches should give the same result in the region where they are valid. The goal of this paper is to find the relation between source functions (3) and (4) obtained in the wave-function approach and covariant-current approach, respectively.

2. SINGLE- AND TWO-PARTICLE CROSS SECTIONS WITHOUT FSI

In this section, we consider the two-particle quantum statistical correlations when one neglects the FSI of the detected particles. This phenomenon is visualized most transparently on the basis of standard quantum mechanics. First, we briefly consider the so-called wave-function parametrization of the source in a nonrelativistic approach. This approach allows one to take also into consideration the FSI \[8\]. A relativistic picture is considered on the basis of the current parametrization and then on the basis of the wave-function parametrization of the source. We compare these two approaches in a nonrelativistic sector and put into correspondence the source functions (3) and (4). After that, the same comparison is carried out for the relativistic sector.

2.1. Wave Function Parametrization of the Source: Nonrelativistic Approach

The probability to detect two particles which are created in relativistic heavy-ion collisions and have definite asymptotic momenta \( k_a \) and \( k_b \) is compared usually with the probability to detect independently two particles with the same momenta. That is why we first turn to consideration of the single-particle spectrum.

Let us consider a single-particle state \( \psi_{\gamma} \) emitted by the source. Its propagation to the detector is governed by the Schrödinger equation

\begin{equation}
\frac{\partial \psi_{\gamma}(x, t)}{\partial t} = \hat{h}(x)\psi_{\gamma}(x, t),
\end{equation}

where \( \hat{h}(x) = -\frac{1}{2m}\nabla^2 \). The index \( \gamma \) denotes a complete set of single-particle quantum numbers. Equation (5) is solved by \( \psi_{\gamma}(x, t, t_0) = \exp[-i\hat{h}(x)(t - t_0)]\psi_{\gamma}(x, t_0) \) in terms of the single-particle wave-function at some initial time \( t_0 \) (see Fig. 1). For a spherically symmetric fireball, the