Relationships between structure and microwave dielectric properties in cotton fabrics

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Abstract

The microwave dielectric properties of cotton fabrics in the fabric thickness direction are investigated in relation to the fabric construction, thread count and solid volume fraction (SVF) under the five different relative humidity (RH) conditions. The dielectric constants of the fabric samples exhibit increasing trends with RH, and this is primarily due to more abundant free water in the samples. For most of woven and knitted samples, an increase in thread count results in a higher dielectric constant and this is associated with the increase in SVF. By comparing the woven and knitted samples of the same SVFs, it is found that the woven fabrics tend to have higher dielectric constants than the knitted samples at near 2.45 GHz. This observation of the dielectric properties of cotton fabrics clearly indicates that although the SVF is primarily responsible for the resulting dielectric properties of fabrics, other structural parameters must also be considered in dielectric analysis. Based on in-situ analysis of yarn orientations in the given samples, we claim that the yarn orientation plays an important role in the microwave dielectric properties of cotton fabrics.

1. Introduction

With the recent booming of wearable electronics and e-textiles, the dielectric properties of textile materials have been featured in the research and development of textile-based microwave interfaces [1]. Wearable microwave interfaces such as antennas and transmission lines have been inherently integrated into fabrics by weaving [2], knitting [3], embroidering [4] and interfacing [5] electrically conductive yarns and fabrics. These approaches to add electronic functionalities to conventional fabrics not only enable constructions of flexible and lightweight devices in a highly wearable form with the pre-existing textile manufacturing techniques [6], but could also enhance electronic performance. For instance, certain textile fabrics including cotton were reported to have dielectric properties desirable for the fabrication of microwave devices [1, 5]. The highly porous nature of fabric structure results in a permittivity close to that of air which enables development of a microwave systems having low dielectric loss [1, 5].

The real part of the relative permittivity (dielectric constant) of textile substrates influences the impedance matching, and it also dominates the device size and other performance. For example, a higher dielectric constant could reduce the size of patch antennas, while a higher gain and a wider bandwidth are attainable with a substrate of a lower dielectric constant [1, 7]. On the other hand, the imaginary part of the relative permittivity and loss tangent also influences the device performance. While a higher imaginary part of the relative permittivity (and hence a higher loss tangent) could help to increase the impedance bandwidth, a higher efficiency and a higher gain could be achieved with a substrate of a lower imaginary part of the relative permittivity (and hence a lower loss tangent) [8]. Therefore, engineering the dielectric properties of textile materials is crucial for design and production of high-performance textile systems for various microwave applications.
The dielectric constant of conventional fabrics such as cotton, polyester and nylon are reported to be in the low range, typically sitting between 1.0 and 2.1 [1, 9–11]. It needs to be noted that, in the context of microwave engineering, even such small variation could be fatal to system optimization. For instance, the resonant (operating) frequency of many resonant structures including patch antennas exhibits a critical dependence on the dielectric constant of the built-in substrate [7, 12]. Therefore, a fine tuning (impedance matching) must be performed with the utmost care and attention to the dielectric properties of the textile substrate for a successful antenna operation [7, 9].

However, microwave investigations on textiles are arduous [13]. At microwave frequencies, certain EM phenomena are enhanced, such as the radiation loss, conductor loss, dielectric loss, surface wave loss and capacitive coupling, and these phenomena make microwave circuits more complex for data collection and analysis compared to the low-frequency dielectric measurements [14]. Accordingly, despite increasing interests in wearable electronics, there is currently no standardized method of microwave dielectric measurements that are specifically developed for textile substances [13].

In previous reports, the microwave dielectric properties of cotton fabrics were studied by the microstrip resonator [15] and the resonant patch antenna [9, 10] methods at discrete frequencies. These dielectric characterization techniques were successfully adopted from the antenna theory [13, 16]. Based on the patch antenna method, the monotonic relationship between the relative humidity (RH) and the dielectric constant of a cotton fabric was reported and discussed [9]. In another report [17], the microstrip line method was also suggested to measure the dielectric properties of cotton/polyester blend fabrics at broadband frequencies.

Although an increasing number of studies is found in literature related to the microwave dielectric properties of cotton fabrics thanks to the growing recognition of their potential applications and significance [1], fundamental investigations on the effect of textile structure, such as fabric construction (e.g., woven and knitted), thread count and solid volume fraction (SVF), on the microwave dielectric properties are rarely discussed for cotton fabrics. As air-fiber-moisture mixtures, the dielectric properties of cotton fabrics are dependent on the microstructures and moisture content of cotton fibers, in addition to the way of mixing the fibers with air [18]. In order to find a reference point to design a cotton fabric with engineered dielectric properties for development of textile-integrated microwave apparatus, a comprehensive study on relationships between the fabric structural parameters and the microwave dielectric properties of cotton fabrics is necessary. Also, establishing a structure-dielectric property relationship could lead to a novel analytical technique for the cotton fabric structure [19].

This paper, therefore, investigates the effect of the fabric construction, thread count and SVF on the dielectric properties of woven and knitted cotton fabrics at controlled RH levels. The patch antenna method was adopted from the literature [10], and the dielectric constant was characterized near 2.45 GHz by operating a vector network analyzer (VNA) to discuss the role of fabric structure on the microwave dielectric properties of cotton fabrics.

2. Experimental details

2.1. Research questions
Both fabric construction and thread count were investigated as structural parameters in this work since these are the most common design parameters considered by woven and knit designers. Also, because the dielectric mixture theory [19] states that the dielectric properties of a mixture (fabric as an air-fiber system) could depend on the shape of an inclusion (fiber or yarn) and its volume fraction, the key structural parameters were further analyzed in terms of SVF and yarn orientation. Based on this, the research questions (RQs) were formulated as follows:

RQ1: How do the thread count and SVF affect the dielectric properties of plain woven and plain knit cotton fabrics?

RQ2: Do the fabric construction (either plain woven or plain knit) and yarn orientation affect the dielectric properties of cotton fabrics under the controlled SVF?

The flowchart shown in figure 1 summarizes research procedures. Samples of woven and knitted cotton fabrics were produced following the previous research [20]. Then, the physical and hygroscopic properties of these samples were characterized. The dielectric characterization was performed by the resonant (patch antenna) method [10], and finally the dielectric constants were discussed in relation to the thread count, fabric construction, SVF and yarn orientation.

2.2. Yarn preparation
Cotton fibers are a natural product and hence their shapes, sizes, chemical compositions and microstructures vary significantly depending on various genetic and environmental factors [21]. Particularly, the variation in the
cellulose crystallinity was reported to impact the moisture absorbing properties and hence the dielectric properties \([9, 21]\). In order to eliminate the factors associated with fiber and yarn composition and structure, a 100% cotton yarn that is compatible for both weaving and knitting processes was prepared and used in this work. The details of yarn preparation procedure are described elsewhere \([20]\). The resulting yarn was a 5-ply yarn in a linear density of 4.6 Ne (equivalent to 0.128 g m\(^{-1}\)) with 100 Z-twists per meter.

### 2.3. Fabric preparation, laundering, and conditioning

Five plain-woven and five plain-knit (single jersey) samples were produced in various thread counts and SVFs by following the procedures reported in \([20]\). The produced fabrics were cleaned by a home laundry machine with cold water for 30 min. After drying the samples at room temperature, the samples were pre-conditioned in an environmentally controlled laboratory (65 (±5)% RH and 21 (±2)°C) for over 24 hours. The RH and temperature of the room were managed to comply with the ASTM standard D1776. Prior to moisture and dielectric measurements, the fabric samples were conditioned at 80%, 50%, 35%, and 20% RH (21 °C) for over 24 hours. For this additional conditioning, a 30-cubic-foot environmental chamber (Parameter Generation & Control, Inc.) was used. The environmental fluctuations in this chamber were kept more precisely within the ranges of ±2.5% RH and ±0.2 °C.

### 2.4. Physical and hygroscopic properties

The physical properties of the woven and knitted fabrics are given in tables 1 and 2, respectively. As reported in \([20]\), picks per inch (PPI) of the woven samples was ranging from 9 to 22, while ends per inch (EPI) remained constant at 24. The knitted samples had courses per inch (CPI) ranging from 15 to 24, and their wales per inch (WPI) was in the range of 11 to 15 \([20]\). Two pairs of the woven and knit fabric samples (W4 and K2; W5 and K3) had the same SVFs.

For the selected fabric samples of the same SVFs (W4, W5, K2 and K3), the yarn orientation was further analyzed based on its fabric construction. Micro-computed tomography (micro-CT) (Bruker SkyScan 1174) was used to measure the average degrees of in-situ yarn angles relative to the z-axis (fabric thickness direction) within

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**Table 1. Physical properties of the woven samples (65% RH at 21 °C) (data adopted from \([20]\)).**

| Sample# | EPI | PPI | Thickness (mm) | Weight (g m\(^{-2}\)) | SVF |
|---------|-----|-----|----------------|------------------------|-----|
| W1      | 24  | 9   | 1.67           | 255.9                  | 0.10|
| W2      | 24  | 12  | 1.39           | 225.7                  | 0.11|
| W3      | 24  | 15  | 1.40           | 230.9                  | 0.11|
| W4      | 24  | 20  | 1.14           | 245.7                  | 0.14|
| W5      | 24  | 22  | 1.11           | 261.4                  | 0.15|
The woven samples showed lower average yarn angles than the knit samples, which indicated that the woven structures had more yarns aligned along the fabric thickness than the knitted structures. The procedural details of the yarn orientation measurements can be found in appendix.

The moisture contents of the conditioned fabric samples were determined under the five different RH conditions based on the ASTM standard D629. As depicted in figure 3, the moisture content increased almost linearly with RH.

| Sample# | WPI | CPI | Thickness (mm) | Weight (gm$^{-2}$) | SVF |
|---------|-----|-----|----------------|---------------------|-----|
| K1      | 11  | 15  | 1.73           | 347.8               | 0.13|
| K2      | 12  | 16  | 1.64           | 353.9               | 0.14|
| K3      | 13  | 17  | 1.61           | 376.5               | 0.15|
| K4      | 14  | 20  | 1.42           | 398.2               | 0.18|
| K5      | 15  | 24  | 1.44           | 460.3               | 0.21|

Figure 2. Average yarn angles relative to the fabric thickness axis.

Figure 3. Moisture contents of the cotton fabric samples at 21 °C (data adopted from [20]).
2.5. Microwave dielectric characterization

In the resonant (patch antenna) method of dielectric characterization [10], the dielectric constant along the substrate’s thickness direction is analytically determined from the dimensions (figure 4(a)) and resonant (operating) frequency of the rectangular patch. According to the transmission line theory [7, 16, 22], the effective dielectric constant ($\varepsilon_{r, eff}$) of the substrate and resonant frequency of the antenna ($f_r$, hertz) are related as follows:

$$L_p = \left[ \frac{c}{2f_r \sqrt{\varepsilon_{r, eff}}} \right] - 2\Delta L_p$$

where $L_p$ is the patch length (meters); $c$ is the speed of light in free space; and $\Delta L_p$ is the additional length of the patch (meters) caused by the fringing fields and is given by:

$$\Delta L_p = \frac{0.412h}{(\varepsilon_{r, eff} + 0.3)\left(\frac{W_p}{h} + 0.264\right)}$$

where $W_p$ and $h$ are the patch width and substrate height, respectively. The effective dielectric constant is related to the dielectric constant ($\varepsilon'$) as follows [7, 16, 22]

$$\varepsilon'_{r, eff} = \left[ \frac{\varepsilon'_r + 1}{2} \right] + \left[ \frac{\varepsilon'_r - 1}{2} \right] \left[ 1 + \frac{12h}{W_p} \right]^{\frac{1}{2}}$$

Thus, the dielectric constant of the substrate can be determined from the resonant frequencies through equations (1)–(3).

In this work, the antennas were designed to have the dimensions given in table 3. The antenna dimensions were optimized by running EM simulations using a 3D full-wave EM simulator (Ansys HFSS®) [10] based on speculative values of dielectric constants (table 3) of the fabric samples. This speculation was decided according to the previous reports that the typical dielectric constants of cotton fabrics are within the range of 1.3–2.0 at

![Figure 4. (a) Antenna model and (b) antenna sample.](image)

Table 3. Antenna dimensions (in millimeters) determined based on speculative values of dielectric constants.

| Sample # | Speculative dielectric constant | Patch length ($L_p$) | Patch width ($W_p$) | Inset length ($L_i$) | Feedline width ($W_f$) | Gap (g) | Substrate height ($h$) |
|----------|--------------------------------|----------------------|--------------------|---------------------|------------------------|---------|-----------------------|
| W1       | 1.43                           | 49.0                 | 58.0               | 9.0                 | 6.3                    | 2       | 1.67                  |
| W2       | 1.64                           | 46.3                 | 52.0               | 11.0                | 5.2                    | 2       | 1.39                  |
| W3       | 1.62                           | 46.5                 | 53.0               | 9.5                 | 5.0                    | 2       | 1.40                  |
| W4       | 1.72                           | 45.3                 | 52.5               | 9.5                 | 4.2                    | 2       | 1.14                  |
| W5       | 1.76                           | 44.8                 | 52.0               | 8.5                 | 4.3                    | 2       | 1.11                  |
| K1       | 1.62                           | 46.5                 | 53.5               | 10.0                | 6.7                    | 2       | 1.73                  |
| K2       | 1.66                           | 53.0                 | 46.0               | 10.0                | 6.2                    | 2       | 1.64                  |
| K3       | 1.72                           | 44.8                 | 52.0               | 9.0                 | 6.0                    | 2       | 1.61                  |
| K4       | 1.88                           | 43.3                 | 51.0               | 8.5                 | 5.1                    | 2       | 1.42                  |
| K5       | 1.95                           | 42.5                 | 50.5               | 8.0                 | 5.1                    | 2       | 1.44                  |
microwave frequencies \[9, 23\]. The inset feed length \((L_i)\) and gap width \((g)\) were numerically optimized to minimize the impedance mismatch.

The antenna samples (figure 4(b)) were fabricated by mounting an adhesive-backed copper foil as the radiating patch and ground plane on the cotton fabrics. In order to achieve accurate cuts of the copper foil, an electronic cutting machine (Cricut Explore Air™, Provo Craft & Novelty, Inc) was used. Edge-mount SMA connectors (manufacturer part number 142–0701–881, Cinch Connectivity Solutions) were soldered to the feedline and ground plane, and the antenna samples were conditioned at the five different RH levels in the manner described in section 2.3.

The resonant (operating) frequency is the frequency at which the magnitude of the one-port scattering parameter \(S_{11}\) is minimized \[10\]. Thus, the resonant frequencies of the conditioned antenna samples were characterized through \(S_{11}\) measurements. The patch antenna samples were fed with a 50-Ω coaxial cable connected to a VNA (E5071C ENA Series Network Analyzer, Agilent Technologies), and the magnitude of \(S_{11}\) was measured in the frequency range of 1 to 6 GHz.

### 3. Results and discussion

The \(S_{11}\) of the patch antenna samples were measured and the data at 80% RH are plotted in figure 5. From these \(S_{11}\) data, the resonant frequencies of the antenna samples were determined and shown in table 4. Every antenna sample operated successfully as intended within a small frequency variation between 2.59 and 2.87 GHz. Based on these, the dielectric constants of the fabric samples were computed and reported in table 5. Depending on RHs and sample types, the dielectric constants ranged between 1.18 and 1.62. This range coincided with the values previously reported \[1, 9–11\].

As plotted in figure 6, it was observed that the dielectric constant of every fabric sample increased nearly linearly with RH. The Pearson correlation coefficient suggested a strong positive correlation \((1.00 \geq R \geq 0.96)\), except for W2 \((R = 0.69)\). The average increase of permittivity was 0.13 when the RH increased from 20% to
As discussed by [9], these increases in dielectric constants of both woven and knitted cotton fabric samples are most likely due to the more abundant water (particularly, free water) at a higher RH (figure 3).

As the thread count (PPI) of the woven samples increased from 9 to 22 (figure 7(a)), the resulting dielectric constants increased almost linearly by 0.19 on average. Pearson correlation coefficient indicates a strong/moderate positive correlation ($0.92 \geq R \geq 0.82$). From the dielectric mixture theory [18, 19], it is known that a higher SVF could lead to a higher dielectric constant. Also, the dielectric constant of the woven samples increased with SVF showing a strong positive correlation ($0.98 \geq R \geq 0.91$, figure 8). Therefore, it is reasonably interpreted that the higher dielectric constant was measured in the sample with a higher thread count due to the increase in SVF.

The knitted samples also exhibited a strong positive correlation ($0.93 \geq R \geq 0.90$) between the thread count and their dielectric properties. With the increase in CPI from 15 to 24 and the associated increase in WPI from 11 to 15, the dielectric constant increased by 0.12 on average (figure 7(b)). Moreover, same to the woven

![Figure 6. Dielectric constants plotted as a function of the RH.](image)

Table 4. Operating frequency of each antenna sample (GHz).

| Sample # | 80% | 65% | 50% | 35% | 20% |
|----------|-----|-----|-----|-----|-----|
| W1       | 2.59| 2.60| 2.60| 2.61| 2.63|
| W2       | 2.79| 2.79| 2.77| 2.82| 2.85|
| W3       | 2.73| 2.73| 2.76| 2.78| 2.80|
| W4       | 2.70| 2.71| 2.75| 2.78| 2.81|
| W5       | 2.69| 2.72| 2.76| 2.78| 2.82|
| K1       | 2.63| 2.64| 2.69| 2.71| 2.75|
| K2       | 2.68| 2.69| 2.74| 2.76| 2.80|
| K3       | 2.71| 2.73| 2.78| 2.80| 2.84|
| K4       | 2.69| 2.76| 2.80| 2.81| 2.87|
| K5       | 2.69| 2.71| 2.78| 2.81| 2.87|

Table 5. Dielectric constants of each fabric sample.

| Sample # | 80% | 65% | 50% | 35% | 20% |
|----------|-----|-----|-----|-----|-----|
| W1       | 1.28| 1.27| 1.27| 1.26| 1.23|
| W2       | 1.24| 1.24| 1.27| 1.21| 1.18|
| W3       | 1.29| 1.29| 1.26| 1.24| 1.21|
| W4       | 1.42| 1.40| 1.36| 1.32| 1.29|
| W5       | 1.46| 1.43| 1.38| 1.35| 1.31|
| K1       | 1.38| 1.37| 1.32| 1.30| 1.25|
| K2       | 1.37| 1.35| 1.30| 1.28| 1.23|
| K3       | 1.42| 1.39| 1.34| 1.32| 1.27|
| K4       | 1.55| 1.47| 1.43| 1.41| 1.35|
| K5       | 1.62| 1.59| 1.50| 1.47| 1.40|
samples, these increases in the dielectric constants with thread counts are associated with the increase in SVF $0.98 \geq R \geq 0.97$, figure 8.

It was noted consistently through figures 6–8 that the woven and knitted samples with the low SVFs (W1, W2, and K1) did not sit well within the general trends of dielectric changes. A possible reason is an insufficient thread count to present a decent fabric structure. There were excessively loose yarns that were not interlaced tightly each other within W1, W2, and K1. The arrangement of fibers and air within those structures might not have been comparable to other tighter samples.

By comparing the woven and knitted fabrics of the same SVFs, it was found that the woven fabrics showed higher dielectric constants than the knitted samples under the controlled RH (figure 9). The calculated $p$-values from the paired t-tests for the fabric construction were well below 0.01 for both sample pairs. These differences in the dielectric constants of the woven and knitted cotton fabric samples clearly indicate that although the SVF is primarily responsible for the resulting dielectric properties of fabrics, an additional structural parameter must also be considered in dielectric analysis of textiles.

One possible influence on the difference in dielectric constants of the woven and knitted samples would be the orientation of fibers. As discussed by the dielectric mixture theory (such as the extension of the Maxwell–Garnet rule), the orientation of high aspect ratio materials affects the permittivity of the mixture, and the permittivity of the fabric in its normal (fabric thickness) direction could be higher as more fibers are oriented in the normal direction (table 6) [19]. In addition, cotton fibers are well known to exhibit a higher local permittivity along the fiber axis than the radial direction due to their highly oriented crystal structures. This microstructural dielectric anisotropy would also contribute to a higher out-of-plane permittivity when more fibers are oriented in the normal direction (table 6) [24].
As demonstrated in figure 2, the yarn orientations of the woven samples were higher than those of the knitted samples of the same SVFs. By assuming that average fiber orientation is almost parallel to yarn orientation in a fabric sample, the observation that the dielectric constant of W4 was larger than that of K2 can be explained by the evidence that the fibers in W4 had a more orientation in the fabric thickness (electric field) direction than K2. Similarly, the larger dielectric constant of W5 than K3 can be explained by the evidence that the fibers in W5 had a more orientation in the fabric thickness direction. Therefore, the fiber orientation could effectively elucidate the different dielectric properties between the woven and knitted samples under the controlled SVF and RH.

### 4. Conclusions

The microwave dielectric properties of cotton fabrics in a variety of structures were successfully characterized in the fabric thickness direction by the patch antenna method. As expected from [9], it was shown that as the RH decreased, the dielectric constant almost linearly decreased because of the reduced amount of free water in the cotton fabric samples. The investigations on the effect of the thread count in both woven and knitted samples suggested that structural parameters could also be playing an additional critical role. In order to test this hypothesis, the effect of the fabric construction was examined by comparing the woven and knitted fabrics of the same SVFs. It was revealed that the woven fabrics exhibited larger dielectric constants than the knitted fabrics of the same SVFs under the controlled RHH.

These permittivity differences between the woven and knitted samples were successfully elucidated by the evidence that the woven samples had more yarn orientation in the fabric thickness direction than the knitted samples. These findings and analyses suggested that both SVF and yarn (fiber) orientation critically impacted the resulting dielectric properties of cotton fabrics. Therefore, microwave dielectric characterization, if performed accurately, could become a novel structural analysis tool for fabrics. Furthermore, the structure-dielectric property relationships developed in this research could be a reference point to engineer a fabric structure for microwave systems established on a cotton fabric.

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Appendix

The average orientations of the yarns within the fabric samples (W4, W5, K2 and K3) were determined based on CT image analysis. The cotton fabric samples were immersed in a Lugol’s solution (4% potassium iodide and 2% iodine dissolved in 94% distilled water supplied from J Crow Company) for 24 hours and dried in a fume hood at room temperature for another 24 hours.

A micro-CT system (Bruker SkyScan 1174), which provides the spatial (theoretical) resolution of 6.41 μm, was operated at 40 kV and 0.67 mA to capture cone-beam x-ray projections of the stained cotton fabric samples. The acquired projection images were then reconstructed into cross-sectional images by using a reconstruction software (Bruker NRecon).

Now, the angle between two arbitrary vectors, \( \mathbf{a} = (a_1, a_2, a_3) \) and \( \mathbf{b} = (b_1, b_2, b_3) \), are given in the definition of their inner product [25]:

\[
\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| \cdot |\mathbf{b}| \cos \theta
\]

or

\[
\theta = \arccos \left( \frac{a_1 b_1 + a_2 b_2 + a_3 b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2} \cdot \sqrt{b_1^2 + b_2^2 + b_3^2}} \right)
\]

in the Cartesian coordinate system. Accordingly, the angle between the axis normal to the fabric plane (\( \tilde{e} = (0, 0, 1) \) and the discretized orientation vector of the yarn (\( \vec{l}_i = (e_1, e_2, e_3) \), figure A1) is given by:

\[
\theta = \arccos \left( \frac{e_3 - s_3}{\sqrt{(e_1 - s_1)^2 + (e_2 - s_2)^2 + (e_3 - s_3)^2}} \right)
\]

The average yarn orientation (\( \bar{\theta} \)) can therefore be obtained by taking an average of angles (\( \theta_i \)) over the discretized yarn length (\( l_i \)) as:

\[
\bar{\theta} = \frac{\theta_1 l_1 + \theta_2 l_2 + \cdots + \theta_n l_n}{l_1 + l_2 + \cdots + l_n} = \frac{\sum_{i=1}^{n} \theta_i l_i}{\sum_{i=1}^{n} l_i}
\]

where \( i \) and \( n \) are the discretization index and total number of discretization, respectively. For the plain-woven samples, the average yarn orientation was calculated for the weft and warp yarns respectively using Equation (9), and then the total average was obtained by using the averaging formula:

\[
\bar{\theta}_{woven} = \frac{\bar{\theta}_{weft} \cdot PPI + \bar{\theta}_{warp} \cdot EPI}{PPI + EPI}
\]

In order to find the discretized vectors in the fabric samples, a 3D visualization software (Bruker DataView) was operated, and cross-sectional (tomographic) images (figure A2(a)) were generated. The plied yarns were then plotted in the Cartesian coordinate as shown in figure A2(b). The examined lengths of the yarns for each fabric samples are given in table A1 along with the corresponding average discretized lengths. These yarn orientation measurements included at least 4 repeat units in both woven and knitted samples to ensure that the population was properly represented.
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