Exploring effects of magnetic field on the hadron resonance gas

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received 15 July 2016; accepted in final form 17 October 2016
published online 4 November 2016

PACS 25.75.-q – Relativistic heavy-ion collisions
PACS 12.38.Mh – Quark-gluon plasma

Abstract – We present a study of the effects of magnetic fields on fluctuations and correlations in the hadron resonance gas model. We find significant changes in the fluctuations of net baryon number, electric charge and strangeness. This is also reflected in various fluctuation ratios along the freezeout curve.

Introduction. – Heavy-ion collisions (HIC) are investigated both theoretically and experimentally to understand the properties of nuclear matter at extreme conditions. One of the most important issues addressed in HIC is the possibility for nuclear matter to undergo a phase transitions to quark matter. At low baryon density and high temperature nuclear matter is expected to smoothly cross over \([1]\) to a quark gluon plasma (QGP) phase. Whereas, at high baryon density and low temperature the system is expected to have a first-order phase transition \([2–4]\).

The study of the effect of magnetic field on the phase transition has become a subject of intense research in the last few years. The phase transition in Quantum Chromodynamic (QCD) systems is usually expected to occur around the QCD energy scales \(\Lambda \sim 200\,\text{MeV}\). So one should be interested in the magnetic fields with strength \(B \sim (200\,\text{MeV})^2 \sim 2 \times 10^{18}\,\text{G}\). Non-central relativistic HIC may create extremely strong magnetic field \((\sim m_\pi^2 \sim 10^{18}\,\text{G})\) due to the relativistic motion of the charged particles. The magnetic field \((B)\) may reach up to order of 0.1 \(m_\pi^2\), \(m_\rho^2\) and 15 \(m_\pi^2\) for SPS, RHIC and LHC energies, respectively \([5]\).

Magnetic fields can induce many interesting phenomena in QCD matter. For example the chiral magnetic effect, \(i.e.,\) electric charge separation induced by chirality imbalance, along an external magnetic field, which also results in \(P\) and \(CP\) violation \([6,7]\). On the other hand, magnetic catalysis \([8]\) and inverse magnetic catalysis \([9,10]\) can affect the phase diagram of QCD matter.

Lattice QCD studies show that in the presence of magnetic field the critical temperature may increase \([11,12]\). Effects on hadron mass modification have also been reported \([13]\). The presence of magnetic fields may increase the fluctuations and correlations \([14]\), as well as elliptic flow coefficient \([15]\) of hot QCD matter. Anisotropic electric conductivity of hadronic matter is an effect of strong magnetic field which has been found in lattice studies \([16]\). Such anisotropy in conductivity should create an anisotropy in the dilepton emission rate with respect to reaction plane and should be observed \([17,18]\). The final observables in relativistic HIC may also depend on the evolution of the magnetic field. Depending on the electrical conductivity of the medium the magnetic diffusion time \(\tau_{\text{mag}}\) may vary from 0.3 fm to 150 fm \([19,20]\). So for large diffusion time a constant magnetic field may be considered.

In view of the importance of magnetic fields, we plan to study its effect on fluctuations and correlations of strongly interacting matter, using the Hadron Resonance Gas (HRG) model \([21–27]\). Here the confined phase of QCD is modelled as a non-interacting gas of hadrons and resonances. Recently, the hadronic EOS for non-zero magnetic fields has been studied within the HRG model \([25]\). On the other hand, the study of fluctuations and correlations of conserved charges is a reliable way to study the phase transition. These quantities behave quite differently in the hadronic and quark phases \([28–30]\). Moreover, the fluctuations are expected to be enhanced near phase boundary and are related to the critical behaviour of strongly interacting matter \([31,32]\). Furthermore, by studying event-by-event fluctuations \([33]\) as a function of beam energy \([34,35]\) it would be possible to learn more about the QCD phase diagram. The paper is organised as
follows. First we discuss the HRG model in the presence of a magnetic field. Thereafter we present our results and finally we conclude.

**HRG model in the presence of magnetic field.**

The grand canonical partition function of HRG may be written as sum of all the partition functions $Z_i$ of each hadron $i$, where

$$
\ln Z_i = \pm V g_i \int \frac{d^3p}{(2\pi)^3} \ln [1 \pm e^{-\left(E_i - \mu_i\right)/T}].
$$

(1)

Here $V$ is the volume of the system, $g_i$ is the degeneracy factor, $T$ is the temperature, $E_i = \sqrt{p^2 + m_i^2}$ is the single-particle energy, $m_i$ is the mass and $\mu_i = B_i \mu_B + S_i \mu_S + Q_i \mu_Q$ is the chemical potential. In the last expression, $B_i$, $S_i$, $Q_i$ are, respectively, the baryon number, strangeness charge and the particle, $\mu$'s are corresponding chemical potentials. The upper and lower signs correspond to fermions and bosons, respectively. We have incorporated all the hadrons listed in the particle data book [36] up to mass of 3 GeV.

For a constant magnetic field $B_z$, the single-particle energy levels for charged and neutral particles are respectively given by [37–39]

$$
E_{i,\pm}^{B_z} = \sqrt{p_z^2 + (m_i^2 + |q_i| B_z (2n + 2s_2 + 1) + 2s_2 k_i B_z)^2}.
$$

(2)

and

$$
E_{i,0}^{B_z} = \sqrt{p_z^2 + (m_i^2 + p_z^2 + p_y^2 + 2s_2 k_i B_z)^2},
$$

(3)

where $q_i$ is the charge of the particle $n$ is any positive integer corresponding to allowed Landau levels, $s_2$ are the components of spin in the direction of magnetic field, and $k_i$ is the anomalous magnetic moment. For a given $s$, there are $2s + 1$ possible values of $s_2$. The gyromagnetic ratios are taken as $g_i = 2|q_i|/c$ for all charged hadrons [25,40]. Therefore the pressure for the $i$-th hadron may be written as

$$
P_i = \frac{g_i T}{2\pi^3} \sum_{s_2} \int \pm d^3p \ln [1 \pm e^{-\left(E_{i,\pm}^{B_z} - \mu_i\right)/T}],
$$

for $Q_i = 0$, (4)

and

$$
P_i = \frac{g_i T}{2\pi^3} |Q_i| c B_z \sum_{s_2} \sum_n \int_0^\infty \pm dp_z
$$

$$
\times \ln [1 \pm e^{-\left(E_{i,0}^{B_z} - \mu_i\right)/T}],
$$

for $Q_i \neq 0$, (5)

where $g_i$ is the degeneracy other than spin. There are several simplifying assumptions that we are using here. Firstly we are considering a non-interacting HRG. Secondly we are considering point-like hadrons with their masses remaining unaffected by the ambient temperature, chemical potentials and magnetic fields. These are common assumptions that are regularly used to study the strongly interacting matter in the context of heavy-ion collisions as may be found in refs. [21–27] and references therein. Similar assumptions are also used by various lattice QCD groups to set the scales of their results for various thermodynamic quantities [41,42]. This is true also in the studies of magnetised neutron stars as may be found in refs. [37,38,43–45].

Also, apart from the above thermal contribution, there is also a vacuum part which is in general divergent and needs regularisation and renormalisation. However, theories with particles having spin > 1 are not renormalizable [46]. This is probably reflected through the fact that the renormalized contributions from particles with spin > 1 in the HRG vacuum are negative [25]. Since many of the thermodynamic quantities including fluctuations and correlations of conserved charges are unaffected by the vacuum part, we have neglected its contribution altogether.

**Results.**

The $n$-th-order fluctuation and correlation are given, respectively, by the diagonal and off-diagonal components of the susceptibility

$$
\chi_{xy}^n = \frac{\partial^{|x+k|}(\sum_i P_i / T^4)}{\partial (\mu_x / T)} \frac{\partial (\mu_y / T)}{\partial ^{|x+k|}},
$$

(6)

where $n = j + k$, $\mu_x/\mu_y$ is the chemical potential for conserved charge $x/y$, with $x, y = B / S / Q$.

Figure 1 shows the variation of second-order ($\chi^2$) and fourth-order ($\chi^4$) susceptibilities of different conserved charges with $T$ at $\mu = 0$. At low $T$, the dominating contribution to $\chi_B$ comes from protons and neutrons. With increase in $T$, other heavier baryons populate and contribute to $\chi_B$ and hence susceptibilities increase with increase in temperature. Since all the baryons have baryon number ±1, there is no difference in magnitude for higher-order susceptibility for $\chi_B$. Both $\chi_B^2$ and $\chi_B^4$ increases with increase in $B$ especially at high temperature. The number density of protons increase with the magnetic field. At high temperature the $\Delta$ particles get excited. The number density of $\Delta^{++}$ (1232) particles increases with magnetic field even faster than that for the proton for two reasons. Firstly, as the spin of $\Delta^{++}$ (1232) is 3/2 the effective mass of $\Delta^{++}$ (1232) increases with magnetic field [25]. Secondly, for a spin-(3/2) and doubly charged particle the effects of degeneracy and magnetic field make the density increase faster. As a result the susceptibilities increase perceptibly at high temperature with the increase in magnetic field.

The dominant contribution to $\chi_S$ at low temperatures comes from kaons which have strange quantum number ±1. As a result magnitudes of $\chi_S^3$ and $\chi_S^4$ are similar in the low-$T$ region up to 0.1 GeV. Since $K^\pm$ are spin-zero particles, their populations get suppressed in the presence of $B$. However, since $T$ is small, the population itself is small and hence this suppression is also small. With the increase in $T$, other strange mesons, like $K^*$ (spin one), start populating the system. In the presence of magnetic field the population of $K^\pm$ increases. As the
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Fig. 1: (Colour online) Variation of $\chi^2$ and $\chi^4$ with $T$ at $\mu = 0$ for baryon number, strangeness and electric charge.

Fig. 2: (Colour online) $\chi_{BS}$, $\chi_{QS}$, $\chi_{BQ}$ as a function of temperature at $\mu = 0$.

Contributions of $K^\pm$ and $K^{*\pm}$ to the susceptibilities are opposite in nature, in the presence of $B$, up to a certain temperature ($T < 0.12$ GeV), susceptibilities for strange quantum number do not get affected much by the magnetic field. At higher temperatures, other charged strange hadrons (like $\Sigma^{\pm}$) start populating the system. Furthermore, at high temperature and high magnetic field, the contribution of $K^{*\pm}$, to the susceptibilities, is much more compared to that for $K^{\pm}$. This also makes the susceptibilities increase with magnetic field. Also, at high $T$, several strange baryons like $\Xi, \Omega$ are excited which have strange quantum number $\pm2$ and $\pm3$, respectively. Therefore, in the high-$T$ region, magnitude of $\chi_S$ increase for higher orders.

In the low-temperature domain, $\chi_{BS}^2$ and $\chi_{BS}^4$ decrease in the presence of magnetic field. Beyond a certain temperature, those increase with increase in magnetic field. The dominant contribution to the susceptibilities of conserved electric charges at low temperatures comes from pions which are spin-zero particles. Next dominating contributions are coming from kaons which are also spin-zero particles. Therefore, the populations of these particles get suppressed in the presence of magnetic field. As a result susceptibilities of conserved electric charges decrease with the increase of magnetic field at low temperature. With the increase of temperature $\rho, K^{*}, p, \Delta$ etc., which are spin–non-zero particles, appear in the system and these particles cause the susceptibilities to rise at high temperature.

In fig. 2 we have plotted the correlations (off-diagonal susceptibilities) of conserved charges. For all the cases the correlations increase with magnetic field. The maximum effect is observed in the baryon charge sector as the baryons with higher spin states and charge contribute in this sector. Here the leading contributors are $p, \Sigma^\pm$ and $\Delta$. For all these particles the number density increases with increase in magnetic field, especially at high temperature, as a result the correlation increases. The leading-order contribution to $BS$ comes from $\Lambda$ and $\Sigma^\pm$. As the number densities of these particles increase in the presence of a magnetic field the correlation also increases, especially at high temperature. For the $QS$ sector, the correlation is suppressed at low temperature, in the presence of a
magnetic field, as the major contributions come from $K^\pm$. The number density of $K^\pm$, being a spin-0 particle, decreases in the presence of a magnetic field. As the temperature increases the spin-1 particles $K^{*\pm}$ appear in the medium. This makes the $QS$ correlation increase at high temperature.

**Beam energy dependence of the products of moments.**

Experimentally measured moments such as mean ($M$), standard deviation ($\sigma$), skewness ($S$) and kurtosis ($\kappa$) of conserved charges are used to characterize the shape of charge distribution. The products of moments can be linked with susceptibilities by the following relations:

$$\frac{\lambda_2^2}{\lambda_2^2} = \frac{\sigma_p^2}{M_x}, \quad \frac{\lambda_3^3}{\lambda_3^3} = S_x\sigma_x, \quad \frac{\lambda_4^4}{\lambda_4^4} = \kappa_x\sigma_x^2. \quad (7)$$

These ratios are independent of the volume of the system and play a crucial role for the search of possible critical point in the QCD phase diagram. To make contact between our model and experimental data we need a parametrization of $T$, $\mu^*$'s and $B$ with $\sqrt{s}$. Such parametrization exist for multiplicities of identified hadrons in the absence of magnetic field [23,24] for central collisions. Here we use this parametrization to check effects of magnetic fields on the fluctuation ratios of net proton and net charge data given in [34,35]. However with this parametrization, a one-to-one comparison with experimental data is not expected to be perfect as the system may not have equilibrated completely, leading to differences of fluctuations from those at thermal and chemical equilibrium. Secondly the parameters for central collisions are not the same as peripheral collisions. Lastly the effects of magnetic field have not been included in the above parametrization. We therefore seek a qualitative comparison of the fluctuations with non-zero magnetic fields vis-a-vis those at $B = 0$. A further example for possible future net-kaon fluctuation ratios is also presented.

Figure 3 shows the of products of moments for net-proton, net-charge and net-kaon as functions of centre-of-mass energy. The number density of kaons is almost always close to 1 as obtained in HRG. This defines the general behavior of these ratios as functions of $\sqrt{s}$.

With non-zero magnetic field. We find that $(\sigma^2/M)_p$ is very sensitive with increasing $\sqrt{s}$. For $(S\sigma)_p$ the magnetic field effect is non-monotonic with $\sqrt{s}$. Both these behaviours are prominently effects from the anomalous magnetic moment of the proton. We checked that without this anomalous part these ratios have insignificant dependence on the magnetic field. For $(\kappa\sigma^2)_p$ there is a small magnetic field dependence only at very low $\sqrt{s}$. For net-charge we find $(\sigma^2/M)_Q$ to have similar dependence.
on magnetic field as that of \((\sigma^2/M)_p\), but the magnitude is now larger due to contributions of the low-mass charged mesons. \((\delta\sigma)_Q\) is found to be most sensitive to the magnetic field for lower \(\sqrt{s}\). In these cases the effect of anomalous magnetic moment is subdominant. The dependence of \((\kappa\sigma^2)_Q\) on magnetic field is small and of the same order as corresponding ratio of net charge.

By construction, the 0–5% centrality data is close to the HRG results for \(B = 0\). However the sensitivity of \(B\) underlines its importance to be considered as a freeze-out parameter in future. On the one hand it seems that most magnetic field effects would be observed for peripheral heavy-ion collisions because a large number of spectator particles would give rise to a large magnetic field. On the other hand a larger system created in the central collisions may sustain even somewhat smaller magnetic field effects. This study basically sets an upper limit for the magnetic field effects.

Finally we present a case for net-kaons. As we saw that the strangeness fluctuations are weakly dependent on the magnetic field, so is the case for all the ratios \((\sigma^2/M)_k\), \((\delta\sigma)_k\) and \((\kappa\sigma^2)_k\). So strangeness fluctuations are not much useful for the detection of effects from magnetic field. We note here that no anomalous magnetic moments were introduced for the mesons as no conclusive experimental data exist for them.

**Conclusion.** We have studied the fluctuations of conserved charges, namely baryon, strangeness and electric charge, using the HRG model in the presence of a magnetic field. We have chosen three values of magnetic fields, the highest of which is close to the predicted magnetic field for non-central HIC at LHC energy. The study was done with several assumptions like neglecting interactions among the hadrons, ignoring the mass and size modifications in the presence of temperature, chemical potential and magnetic field, as well as assuming that the effect of magnetic fields survives long enough for the hadrons to get affected by it. This study basically sets an upper limit for the magnetic field effects.

The baryon number and electric charge number susceptibilities are more sensitive to the magnetic field as compared to the strangeness number susceptibilities. Among the correlators, the baryon-charge correlation is found to be the most sensitive. The ratios of fluctuations that are measurable experimentally also manifest some effects of the magnetic field. We loosely chose thermodynamic parameters fitted for multiplicity at central collisions for \(B = 0\) to see the deviation in an external magnetic field. We found that the fluctuation ratios for the net kaon have a soft dependence on the magnetic field whereas for net proton and net electric charge the dependence is quite strong. A reparametrization of freeze-out conditions with \(B\) as a parameter may be in order and will be studied elsewhere.

We thank CSIR, DST and AvH foundation for support. SS thanks S. Das, S. Prasad, M. Younus and S. Matty for discussion.

**REFERENCES**

1. Aoki Y., Endrodi G., Fodor Z., Katz S. D. and Szabo K. K., Nature, 443 (2006) 675.
2. Asakawa M. and Yazaki K., Nucl. Phys. A, 504 (1989) 668.
3. Ejiri S., Phys. Rev. D, 78 (2008) 074507.
4. Bowman E. S. and Kapusta J. I., Phys. Rev. C, 79 (2009) 015202.
5. Skokov V., Illarionov A. Y. and Toneev V., Int. J. Mod. Phys. A, 24 (2009) 5925.
6. Schäfer T. and Shuryak E. V., Rev. Mod. Phys., 70 (1998) 323.
7. Fukushima K., Kharzeev D. E. and Warringa H. J., Phys. Rev. D, 78 (2008) 074033; Phys. Rev. C, 83 (2011) 011901(R).
8. Shovkovy O. A. and Schmitt A., JHEP, 03 (2011) 033.
9. Brückmann F., Endrodi G. and Kovacs T. G., JHEP, 04 (2013) 112.
10. D’Elia M., Mukherjee S. and Sanfilippo F., Phys. Rev. D, 82 (2010) 051501(R).
11. Bali G. S., Brückmann F., Endrodi G., Fodor Z., Katz S. D., Krieg S., Schäfer A. and Szabo K. K., JHEP, 02 (2012) 044.
12. Luschevskaya E. V., Solovieva O. E., Kochetkova O. A. and Teryaev O. V., Nucl. Phys. B, 896 (2015) 627.
13. Fu W., Phys. Rev. D, 88 (2013) 044009.
14. Mohapatra R. K., Saumia P. S. and Srivastava A. M., Mod. Phys. Lett. A, 26 (2011) 2477.
15. Buividovich P. V., Chernodub M. N., Kharzeev D. E., Kalaydzhyan T., Luschevskaya E. V. and Polikarpov M. I., Phys. Rev. Lett., 105 (2010) 132001.
16. Bratkovskaya E. L., Teryaev O. V. and Toneev V. D., Phys. Lett. B, 348 (1995) 283.
17. Gupta S., Phys. Lett. B, 597 (2004) 57.
18. Tuchin K., Phys. Rev. C, 82 (2010) 034904.
19. Deng W. and Huang X., Phys. Rev. C, 85 (2012) 044907.
20. Braun-Munzinger P., Stachel J., Wessels J. P. and Xu N., Phys. Lett. B, 344 (1995) 43.
21. Cleymans J., Elliott D., Satz H. and Thews R. L., Z. Phys. C, 74 (1997) 319.
22. Cleymans J., Oeschler H., Redlich K. and Wheaton S., Phys. Rev. C, 73 (2006) 034905.
23. Karšch F. and Redlich K., Phys. Lett. B, 695 (2011) 136.
24. Endrodi G., JHEP, 04 (2013) 023.
25. Bhattacharyya A., Das S., Ghosh S. K., Ray R. and Samanta S., Phys. Rev. C, 90 (2014) 034909.
26. Bhattacharyya A., Ray R., Samanta S. and Sur S., Phys. Rev. C, 91 (2015) 041901(R).
27. Jeon S. and Koch V., Phys. Rev. Lett., 85 (2000) 2076.
[29] Asakawa M., Heinz U. W. and Müller B., Phys. Rev. Lett., 85 (2000) 2072.
[30] Ejiri S., Karsch F. and Redlich K., Phys. Lett. B, 633 (2006) 275.
[31] Stephanov M. A., Rajagopal K. and Shuryak E. V., Phys. Rev. Lett., 81 (1998) 4816.
[32] Hatta Y. and Ikeda T., Phys. Rev. D, 67 (2003) 014028.
[33] Stephanov M. A., Rajagopal K. and Shuryak E. V., Phys. Rev. D, 60 (1999) 114028.
[34] Adamczyk L. et al., Phys. Rev. Lett., 112 (2014) 032302.
[35] Adamczyk L. et al., Phys. Rev. Lett., 113 (2014) 092301.
[36] Olive K. A. et al., Rev. Part. Phys., C, 38 (2014) 090001.
[37] Broderick A., Prakash M. and Lattimer J. M., Astrophys. J., 537 (2000) 351.
[38] Broderick A. E., Prakash M. and Lattimer J. M., Phys. Lett. B, 531 (2002) 167.
[39] Bjorken J. D. and Drell S. D., Relativistic Quantum Mechanics (McGraw-Hill, New York) 1964.
[40] Ferrara S., Porrati M. and Telegdi V. L., Phys. Rev. D, 46 (1992) 3529.
[41] Bazavov A. et al., Phys. Rev. D, 86 (2012) 034509.
[42] Borsanyi S. et al., JHEP, 11 (2010) 077.
[43] Mallick R., Schramm S., Dexheimer V. and Bhattacharyya A., Mon. Not. R. Astron. Soc., 449 (2015) 1347.
[44] Chakrabarty S., Bandyopadhyay D. and Pal S., Phys. Rev. Lett., 78 (1997) 2898.
[45] Bandyopadhyay D., Chakrabarty S. and Pal S., Phys. Rev. Lett., 79 (1997) 2176.
[46] Peskin M. E. and Schroeder D. V., An Introduction to Quantum Field Theory (Addison-Wesley, Reading) 1995, p. 842.