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Impulsive Pyramid-Vertex and Double-Wedge Diffraction Coefficients
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Abstract
A new time domain version of the uniform description of doubly diffraction at a pair of coplanar skew wedges and vertex diffraction at the tip of a pyramid is here presented. The diffracted fields are uniformly evaluated in closed form via wavefront approximations.

INTRODUCTION
Doubly diffracted (DD) fields at a pair of skew wedges illuminated by an impulsive point source are obtained directly in the TD using a TD spectral synthesis, while the vertex diffracted (VD) field at the tip of a pyramid is constructed via a TD version of the Incremental Theory of Diffraction (ITD) [3] representation of the field scattered at a wedge [2]. The two diffraction mechanisms here considered complement, in a TD ray field framework, the field predicted by the TD-Uniform Theory of Diffraction (UTD) [5], leading to new uniform field expressions for complex scattering problems involving vertices and double edges. In both cases, closed form solutions are determined using wavefront approximations for the evaluation of the TD radiation integrals. TD-DD and TD-VD field responses to an impulsive spherical delta excitation \( \delta(t) \) are obtained directly in the TD using a TD spectral synthesis, while the vertex diffracted (VD) fields are valid only for early times, on and close to (behind) the wavefronts. The field responses \( \hat{\psi}(t) \) to a more general Band Limited (BL) pulsed excitation with large but finite bandwidth are found via convolution between the diffracted field due to an impulsive excitation \( \delta(t) \) and the illuminating waveform \( G(t) \). If the exciting signal \( G(t) \) has no low-frequency components and is thus dominated by high frequencies, the range of validity of the resulting pulsed response is enlarged to later observation times behind the wavefronts. The present TD-DD and TD-VD fields are limited to real time, and matches and compensates the spatial discontinuity of the TD singly diffracted field developed in [3].

TIME DOMAIN DOUBLY DIFFRACTED FIELDS
Let us consider a pair of wedges with soft/hard boundary conditions (BCs) and coplanar edges, illuminated by a spherical source. It is useful to define a spherical coordinate system at each edge with origin at the diffraction point \( Q_i \) (see Fig.1). Our description of the DD mechanism is here constructed entirely in TD, following the procedure in [5] for the FD. In the following, only the TD field diffracted at edge 2 when it is illuminated by the field diffracted at edge 1 (12) will be considered. The DD mechanism (25) can be analogously obtained. The ray geometry for the DD field is depicted in Fig.1 with \( \ell \) the distance between the two diffraction points \( Q_1 \) and \( Q_2 \), and \( \phi_{21} \) the angular coordinate of \( Q_2 \) with respect to \( Q_1 \) measured in the system at edge 1 (2). The singly diffracted field at the first wedge illuminated by an impulsive spherical source at \( P^{(s)}(\phi') \equiv (r', \phi', \theta') \), is expressed as an superposition of impulsive \( \alpha_s \)-spectral spherical sources at \( P^{(s)}(\phi', \theta', \alpha_s) \) weighted by \( G(\phi', \theta', \alpha_s) \). Each \( \alpha_s \)-spectral source provides a diffracted field contribution from edge 2 at the observation point \( P(\phi) \equiv (r, \phi, \theta) \), that is conveniently calculated using reciprocity, i.e., the diffracted field from edge 2 at \( P^{(s)}(\phi', \theta', \alpha_s) \) due to a point source at \( P(\phi) \) is represented as a summation of \( \alpha_s \)-spectral sources at \( P(\phi_1') \) weighted by the spectral \( \alpha_s \)-dependent term \( C^{(s)}(\phi', \theta', \alpha_s, \phi_1') \). The field response \( \psi_{dd}^{s}(t) \) to a more general Band Limited (BL) pulsed excitation with large but finite bandwidth is expressed as an integral over the \( \alpha_s \)-spectral sources representing the radiation by wedge 1, leads to the double integral representation

\[
\psi_{dd}^{s}(t) = \frac{1}{\sqrt{2\pi}} \int_{\phi_{11}}^{\phi_{12}} \int_{\alpha_1}^{\alpha_2} \frac{\delta(t - R(\phi_1', \alpha_s)/c)}{4\pi R(\phi, \phi_1', \alpha_s)} G(\phi, \theta; \alpha_1, \alpha_s) \psi_{dd}^{s}(t) \, d\phi_1' \, d\alpha_s,
\tag{2}
\]
in which $R(\alpha_1, \alpha_2) = |P(\alpha_1 + \phi_1 + \pi) - P(\alpha_2 + \phi_2 + \pi)|$ is the distance between the spectral source and observation whose explicit expression is given in [5], and $c$ is the ambient wavespeed.

The TD-DD field $\delta_{ab}$ is obtained through a uniform wavefront approximation of (2) that is dominated by the spectrum around the saddle point $(\alpha_1, \alpha_2) = (0, 0)$ that satisfy $(\partial/\partial \alpha_1)R(\alpha_1, \alpha_2)|_{(\alpha_1, \alpha_2) = (0, 0)} = 0$, for $i = 1, 2$. Furthermore, it exhibits poles in each variable given by the $G$ functions at $\alpha_1 = \phi_1 = (\pm p, \pm (-1)^{i-1} p) + \pi$ ($p = 1, 2$), and $\alpha_2 = \phi_2 = (\pm q, \pm (-1)^{i-1} q) + \pi$ ($q = 1, 2$). As shown in [5] for the FD case, the integrand is here decomposed in its even and odd part with respect to $(\alpha_1, \alpha_2) = (0, 0)$. Then, $R(\alpha_1, \alpha_2)/c$ is approximated with its Taylor expansion up to the second order in a neighborhood of its saddle point as $R(\alpha_1, \alpha_2)/c = t_{dd} + (A^{ef} + Hu + Hv + T)^2 + ...$, with $t^{ef} = (0, 0)/c = (r^1 + \ell + \pi) + T$ (see Fig. 1), and $v = \sin(\sigma_1/2)$, and $u = \sin(\sigma_2/2)$. Furthermore, in each variable, a canonical quadratic polynomial with the same $a_i$-pole of $G$ is used to regularize each integrand in (2), so that it is regular at the wavefront and can thus directly evaluated at $\alpha_1 = a_2 = 0$. This canonical mapping for the wavefront approximation, permits to reduce (2) to a sum of integrals representing our TD-DD transition functions that can be evaluated in closed form. Summarizing, the TD-DD field is given by $\delta_{ab} = \delta_{ef} - \delta_{ef^{+}} + \delta_{ef^{+}}$, where $\delta^{+} = 1/(4\pi c^2)$ is the incident spreading factor at $Q_1$, $\delta_{ef} = d', d''$ is the DD spreading factor, and $\delta_{ef^{+}}$ is the TD double diffraction coefficient evaluated at the retarded time $t - t_{dd}$. Only the first order 

$$\delta_{ef}^{+}(t) = \frac{d'(t)}{8\pi r_{1} r_{2} \sin \beta_{1} \sin \beta_{2}} \sum_{p, q = 1}^{\infty} \frac{\cos(2\pi p \sin \beta_{1} \sin \beta_{2}) \cos(2\pi q \sin \beta_{1} \sin \beta_{2})}{2\pi} \text{U}(t)$$

is here discussed because of space limitation. The TD transition function $P'(\theta, \ell, \ell, \omega)$ is evaluated in closed form closing one integral with the delta Dirac and the other via the Cauchy residue theorem, leading to

$$P'(\theta, \ell, \ell, \omega) = \frac{1}{\nu^2 - 4\pi^2 \nu} \left[ \frac{\delta(\nu - 2\omega \nu)}{\sqrt{1 + 1/\nu}} + \frac{\delta(\nu + 2\omega \nu)}{\sqrt{1 + 1/\nu}} \right]$$

with $\nu = 1 - \omega^2 + \ell + \ell$, and $\text{U}(t) = 0$ or 1, when $\omega > 0$ or $\omega < 0$, respectively. The parameters of $P'$ in (3) are $\delta_{\nu} = 2/((\nu)^3 \sin(2\nu \sin \beta_1 \sin \beta_2) + \sin(2\nu \sin \beta_1 \sin \beta_2))$, $\nu_1 = 2((\nu) \sin \beta_1 \sin \beta_2)$, $r_{1} \ell / (\ell + \ell) \sin(\Phi^{\nu}_{\nu} - 2\pi n_{1} N_{\nu}^{*}) / 2$, in which $N_{\nu}^{*}$ and $N_{\nu}^{*}$ are the integers that most nearly satisfy $\Phi^{\nu}_{\nu} = 2\pi n_{1} N_{\nu}^{*}$ and $\Phi^{\nu}_{\nu} = 2\pi n_{1} N_{\nu}^{*}$, respectively, and $\omega = (r_{1} \ell / (\ell + \ell) / 2)^{1/2}$. Similar expressions apply for the higher order contribution $\delta_{ef^{+}}$. Far from the shadow boundaries (SBs), where $q$ and $p$ -> 0, it can be easily verified that $P'(\theta, \ell, \ell, \omega) -> 1$, whereas the DD wave phenomenology acts like a time integrator. The wavefront approximation here obtained via a direct TD method, exactly coincides with the direct Fourier inversion applied to the high-frequency solutions in [5].

**TIME DOMAIN VERTEX DIFFRACTED FIELDS**

Let us consider a pyramid (see Fig.1a) with $M$ edges and $M$ faces on which either hard or soft boundary conditions (b.c.) are imposed on each face. The edges are counterclockwise counted when observing the pyramid from the vertex, and the $m$-th face is that delimited by the $m$-th and $(m+1)$-th edges. In our formulation the pyramid is thought as the superposition of...
of \(m\) wedges sharing common faces and all intersecting at the pyramid vertex. On each \(m\)-th edge both a spherical and a cylindrical reference systems are defined with the origin at the vertex, the \(r_m\) axis along the edge and the \(\phi_m\) axis lying on the plane containing the \(m\)-th face. The exterior angle of the \(m\)-th wedge is denoted by \(\alpha_m\). The pyramid is illuminated by a spherical impulsive source \(\psi(0) = 1 / (1 + r/c)^2/4\pi r^2\). In [2] a useful representation for the field diffracted by a wedge is given. Furthermore, as shown in [1], the field diffracted by a curved or a truncated wedge can be expressed as a superposition of these incremental field contributions arising from the edge itself. Thus, the field diffracted by each truncated wedge of the pyramid can be estimated integrating the incremental field contributions along each \(m\)-th semi-infinite straight edge as

\[
\psi = \sum_{m=1}^{M} \int_{0}^{\infty} G(r_m, \theta_m, \phi_m; r_m(C), \eta_m) \frac{F(t) - F(t')}{r_m} dr_m
\]

where \(G(r_m, \theta_m, \phi_m; r_m(C), \eta_m)\) is defined in (1), \(r_m(C)\) is the incremental source at the \(m\)-th edge, \(\eta_m\) is the distance between the source \(P\) and the \(m\)-th edge, and \(F(t)\) is the spherical Hankel function of the first kind of order zero, with \(t = (t' - r/c) / (c / r_m)\) and \(t' = (t' - r/c) / (c / r_m)\) (see Fig.9b). Furthermore, a canonical quadratic polynomial with the same zero of \(G(r_m, \theta_m, \phi_m; r_m(C), \eta_m)\) is used to regularize each integrand in (5), so that it is regular at the wavefront and can thus directly evaluated at \(\zeta = 0\). This canonical mapping for the wavefront approximation, permits to reduce (5) to a sum of integrals with simple poles and delta Dirac with argument a quadratic polynomial, which can be evaluated by a spherical impulsive source \(\psi(0) = 1 / (1 + r/c)^2/4\pi r^2\).

VECTORIAL CASE AND ILLUSTRATIVE EXAMPLES

The solution for the problem of an Electromagnetic spherical wave, arbitrarily polarized, illuminating either a couple of wedges or a pyramid can be solved in a ray framework, 202
The total field preserves its continuity thanks to the use of the hard and soft respective diffraction coefficients to build dyadic diffraction coefficients in the ray fixed reference systems [5], [3], $D_f(t)$, $D_g(t)$. This emphasizes the calculation (Fig.2) for the DD field due to an electric dipole illuminating a couple of wedges.

At this aspect, the TD-DD singly diffracted field is itself in its transition with a direct or reflected wavefront. At this double transition regime, the DD field time dependence is as shown in Fig.2b. When the source and the observer ye both close to the SB limit, the $D_f(t)$ behavior, not shown here for space limitation.

When the source and the observer both are at a distance far from their transition, i.e. they lie sufficiently below the edges plane. In these deep shadow region (Fig.2a), the only contribution arriving at the observer is the DD ray that is shaped as the primitive of the Rayleigh pulse. This is consistent with the $1/\omega$-frequency dependence of the FD-DD field.

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