Modelling the static and dynamic characteristics of pneumatic muscle

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Abstract. Pneumatic muscle is a new type of actuator that has a number of advantages over pneumatic cylinders, such as higher power-to-weight ratio, smooth speed adjustment, and longer operating life. Pneumatic muscle-based industrial devices can be used in combustible, explosive, and contaminated environments in different industry fields. This article is dedicated to the modeling of pneumatic muscles manufactured by FESTO, a global leader in the production of this type of pneumatic actuators. The existing pneumatic muscle models have several significant drawbacks: they can only be applied to specific types of pneumatic muscles and are inappropriate for FESTO pneumatic muscles, their accuracy is low, and they contain too many adjusting factors. The model presented in this article makes it possible to obtain static characteristics with up to 10% accuracy. The aim of this study is to assess the feasibility of application of the developed expression for modeling the dynamic characteristics of pneumatic muscle and to develop a comprehensive mathematical model of pneumatic muscle for describing the processes of raising and lowering a load.

Pneumatic muscle is a linear single acting actuator. It consists of a rubber tube reinforced by braided mesh and with fitting caps on both sides. When filled with compressed gas, the pneumatic muscle expands in the radial direction and contracts in the axial direction, generating a significant force (for FESTO MAS 40, the maximum force is 5,700 N [1]). Compared to a pneumatic cylinder, the pneumatic muscle has a higher power-to-weight ratio, smooth speed adjustment [2], and longer operating life (50 million cycles for FESTO pneumatic muscles [1]). Pneumatic muscles are also explosion-fireproof and can operate in hazardous and dusty environments.

Nowadays, there are many studies dedicated to the development of industrial manipulators using pneumatic muscles [3-6]. At the same time, different ways of pneumatic muscle positioning are being developed [7-9], and some progress has already been made in this regard (for example, study [8] has achieved ± 0.1º accuracy for rotation movement and ± 0.02 mm [9] for linear movement).

By now, many authors have attempted to model the static characteristics of the pneumatic muscle. The basic expressions are presented in studies [10, 11]. In order to improve convergence with experimental data, various authors make allowance for the thickness of the pneumatic muscle bladder [10], stretching properties of mesh threads [12], diameters and conical shape of the ends of pneumatic muscle [13]. Study [3] represents the force of pneumatic muscle as a function of relative contraction of the pneumatic muscle ε and applied pressure p, F (ε, p), and adds an empirical function k(p) to the expression. The expressions in the above-mentioned studies show good convergence with the
experiment (8-15% error). However, they are true only for specific types of pneumatic muscles and cannot be applied to FESTO pneumatic muscles. Expressions [14, 15] have been obtained specifically for the purposes of modeling of FESTO pneumatic muscles, but they are empirical and contain at least five adjusting factors.

The dynamic characteristics of muscle-based manipulators with FESTO pneumatic muscles are presented in studies [16-18]. Their authors use empirical expressions for the force generated by the pneumatic muscle, which require a large number of adjusting factors. The aim of this study is to examine the dynamic characteristics of pneumatic muscle with a new refined model to assess its efficiency with regard to different loading weights and to assess the applicability of the model in the further development of a muscle-based manipulator.

A refined geometric model has been developed for examining the static characteristics of the pneumatic muscle [19, 27-30]. In order to establish a correlation between the force generated by the pneumatic muscle, applied pressure, and geometric parameters of the pneumatic muscle, the proposed model examines a cell of the bladder with length $l$, width $r$, and mesh angle $2\alpha$ (Figure 1, a).

![Figure 1. Pneumatic muscle calculation model: a) fragment with crossing cord threads, b) cell of the pneumatic muscle bladder with cord mesh, c) cell elasticity calculation model.](image)

After pressurization, forces start to affect the cell, changing the muscle's physical dimensions. The cell is affected by pressure force $F_{pr}$, which is a result of the impact of pressurized gas on the bladder of the pneumatic muscle, and by axial force $F_{ax}$, which results from its contraction in the axial direction and impedes this contraction. The cell is also affected by restoring forces $F_1$ and $F_2$ that causes elastic bladder to return to its initial parameters.

In order to determine the correlation between the resulting forces, it is necessary to examine the equilibrium at points 1 and 2 (Figure 1, b). The expression for force $F_{ax}$ generated by the pneumatic muscle per one cell is as follows:

$$F_{ax} = \frac{F_{pr} - F_1 - F_2 \tan \alpha}{\tan \alpha}$$  \hspace{1cm} (1)

where $\alpha$ is the mesh angle after pressurization.

The equation of forces $F_{pr}$ obtained in [19] look as follows:

$$F_{pr} = \frac{(p_s - p_A)DL}{2}$$  \hspace{1cm} (2)

where $p_s$ is the air pressure in the supply line, $p_A$ is the atmospheric pressure, $D$ is diameter of the tube after pressurization, $L$ is the length of the tube after pressurization.

In order to determine restoring forces $F_1$ and $F_2$, the cell is represented as two springs positioned in the radial and axial direction (Figure 1, c). Forces $F_1$ and $F_2$ will then be equal to:

$$F_1 = c_r \Delta y$$  \hspace{1cm} (3)

$$F_2 = c_l \Delta x$$  \hspace{1cm} (4)
where \( \Delta x \) is the change of cell size in the axial direction, \( \Delta y \) is the change of cell size in the radial direction, \( c_r \) is the cell rigidity along the perimeter, \( c_l \) is the cell rigidity along the length. The rigidities indicated below are expressed through the specific rigidity of the material \( c_0 \) and aspect ratio of the cell:

\[
\begin{align*}
    c_r &= \frac{c_0 r_0}{r_0} \\
    c_l &= \frac{c_0 l_0}{l_0}
\end{align*}
\]

where \( r_0 \) is the initial altitude of cell, \( l_0 \) is the initial length of cell.

In order to find the applied forces, it is first necessary to determine the geometric relationship between the change of cell size in the radial and axial direction. Figure 2 shows the parameters of a triangle – a quarter of a rhombus of the cord inside the cell before and after pressurization: \( r_0/2, l_0/2, b, \alpha_0 \) are the initial altitude, length, hypotenuse, and angle of the triangle; \( \Delta x/2 \) and \( \Delta y/2 \) are changes in the length and altitude after pressurization, \( r/2, l/2, b, \alpha \) are the altitude, length, hypotenuse, and angle after pressurization.

\[\text{Figure 2. Calculation model for pneumatic muscle mesh rhombus deformation: a) mesh size before pressurization, b) mesh size after pressurization.}\]

According to Figure 2, the following ratios for length \( l \), angle \( \alpha \), altitude \( r_0/2 \), and altitude change \( \Delta y/2 \) are found [19]:

\[
\begin{align*}
    \frac{l}{2} &= \frac{l_0}{2} - \frac{\Delta x}{2} \\
    \frac{\Delta y}{2} &= \frac{(l_0 - \Delta x) \tan \alpha}{2} - \frac{r_0}{2} \\
    \frac{r_0}{2} &= \frac{l_0}{2} \tan \alpha
\end{align*}
\]

By substituting in (1) the above ratios (2)-(9) an expression for the force generated by one cell (1) is obtained:

\[
F_{ax} = \frac{(p_S - p_A)D_0(1 - \frac{x}{l_0})}{2} \frac{c_0}{\tan \alpha} \frac{1}{l_0} \frac{r_0 - x}{l_0} - \frac{\pi D_0^2 (p_S - p_A)}{4}
\]

Thus, the expression for the force generated by the pneumatic muscle with allowance for the pressure on the fitting caps looks as follows [19]:

\[
T = \pi D_0 \frac{(p_S - p_A)D_0(1 - \frac{x}{l_0})}{2} \frac{c_0}{\tan \alpha} \frac{1}{l_0} \frac{r_0 - x}{l_0} - \frac{\pi D_0^2 (p_S - p_A)}{4}
\]

where \( x \) is the position coordinate of pneumatic muscle.

The following expression for the change of mesh angle is obtained [19]:

\[
\alpha = \arccos \left( 1 - \frac{x}{l_0} \cos \alpha_0 \right)
\]
Next, it is necessary to consider the change of the pneumatic muscle diameter as the bladder is filled with compressed air [21]:

$$D = \frac{D_0 \left(1 - \frac{p}{\rho_0} \right)^{\tan \alpha}}{\tan \alpha}$$  \hspace{1cm} (13)

For the purpose of modeling the dynamic characteristics, a mathematical model of the pneumatic muscle has been developed, which comprises a system of differential equations, including: equation of pneumatic muscle motion; equation of pressure change in the bladder of the pneumatic muscle; equation of changes in the pneumatic muscle geometry during movement.

Figure 3 shows the principle of operation of the pneumatic muscle when lifting a load with mass $m$.

![Figure 3. Principle of operation of the pneumatic muscle.](image)

According to Figure 3, in the initial state, pressure in the bladder of the pneumatic muscle is equal to atmospheric pressure $p_\text{A}$, force $T$ is zero, diameter equals $D_0$, mesh angle is $\alpha_0$. When compressed air is delivered from the supply line with pressure $p_\text{S}$, pressure in the bladder becomes equal to $p$, the pneumatic muscle generates force $T$, thereby lifting the load to coordinate $x$. At the same time, the diameter of the pneumatic muscle becomes equal to $D$ ($D > D_0$), length becomes equal to $L$ ($L < L_0$), volume becomes equal to $V$ ($V > V_0$), mesh angle becomes equal to $\alpha$.

When the pneumatic muscle is emptied and the load with mass $m$ is lowered, all parameters of the pneumatic muscle become equal to their initial values ($D = D_0$, $L = L_0$, $V = V_0$, $\alpha = \alpha_0$, $T = 0$).

The equation of pneumatic muscle motion will then look as follows:

$$m \ddot{x} = T - P - hx'$$  \hspace{1cm} (14)

where $m$ is the load mass, $T$ is the force generated by the bladder of the pneumatic muscle, $P$ is the weight of the moving parts, $h$ is the damping coefficient.

The equation of pressure change in the bladder of the pneumatic muscle when it is filled/emptied can be written as [20]:

$$p' = a \left[ \frac{k f_S \sqrt{R T_S}}{V_S} \left( \frac{\rho P_S}{P} \right)^{\frac{1}{k}} \sqrt{p^2 - p'^2} \right] + (1 - a) \left[ \frac{k f_E \sqrt{R T_E}}{V_E} \left( \frac{\rho P_E}{P} \right)^{\frac{1}{k}} \sqrt{p^2 - p'^2} - \frac{k p V}{V} \right]$$  \hspace{1cm} (15)

where $T_S$ is the air temperature in the supply line; $k$ is the adiabatic coefficient for air ($k = 1.4$); $R$ is the gas constant ($R = 287$ J/(kg·K)); $f_S$, $\zeta_S$ is the area and resistance factor of the supply line; $f_E$, $\zeta_E$ is the area and resistance factor of the exhaust line; $V$ is the volume of the pneumatic muscle bladder; $a$ is the logical factor that determines the nature of the dynamic pneumatic process in the pneumatic muscle: when $a = 1$, the pneumatic muscle is filled, when $a = 0$ it is emptied.

The equations that describe the changes in pneumatic muscle geometry during movement look as follows [21-26]:
\[ V = \frac{\pi D^2}{4} (L - x) \]  
(16)

\[ V' = \frac{\pi D}{2} (L - x) D' - \frac{\pi D^2}{4} x' \]  
(17)

\[ D' = \left(1 - \frac{x}{L_0}\right) \frac{D_0}{\sin \alpha \cos \alpha} x' \]  
(18)

\[ \alpha' = \frac{\cos \alpha}{\sqrt{1 - \left(\frac{x}{L_0}\right)^2}} x' \]  
(19)

The numerical modelling of Eq. (11)-(13), (14)-(19) allows to obtain the static and dynamic characteristics for the pneumatic muscle. Figure 4 illustrates the relationship between the static characteristics of the pneumatic muscle MAS-20-600 \((D_0 = 20 \text{ mm}, L_0 = 600 \text{ mm})\) obtained during modelling with experimental curves taken from catalog [1]. It can be stated that the calculated results are close to the experimental ones (divergence in modes \(p = 0.7; 0.6; 0.5; 0.4; 0.3 \text{ MPa}\) lies within 10%).

Figure 4. Diagram of correlation between the force generated by pneumatic muscle MAS 20 and relative contraction: solid lines represent experimental curves from the FESTO catalog, dashed lines stand for the results of calculation according to model; T1 with \(p_S = 0.2 \text{ MPa}\); T2 with \(p_S = 0.3 \text{ MPa}\); T3 with \(p_S = 0.4 \text{ MPa}\); T4 with \(p_S = 0.5 \text{ MPa}\); T5 with \(p_S = 0.6 \text{ MPa}\); T6 with \(p_S = 0.7 \text{ MPa}\).

Figure 5 shows the dynamic characteristics during the lifting and lowering of a 40 kg and 60 kg load by the pneumatic muscle with pressure \(p_S = 0.4 \text{ MPa}\). The diagrams prove that the developed expression for the force generated by the pneumatic muscle adequately describes the behavior of the pneumatic muscle when lifting/lowering a load as well as its ability to position the load.

Figure 5. Dynamic characteristics of the MAS-20-600 pneumatic muscle when lifting/lowering a load with the following mass: a) 40 kg, b) 60 kg.

This article is dedicated to the modeling of the static and dynamic characteristics of the pneumatic muscle. A refined equation for the force generated by the pneumatic muscle with a minimum number of parameters and good convergence with the experiment was proposed in this study. Further the applicability of this expression for the evaluation of the dynamic characteristics was assessed. For this purpose, a comprehensive mathematical model of the pneumatic muscle comprising differential
equations of motion and equations of pressure change in the bladder of the pneumatic muscle were developed. It shows good results, which correspond to experimental data and can be used in subsequent development of pneumatic muscle manipulators.

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