ON THE FREE SET NUMBER OF TOPOLOGICAL SPACES
AND THEIR $G_\delta$-MODIFICATIONS

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A transfinite sequence of points of a topological space $X$ is a free sequence in $X$ if the closure of any initial segment of it is disjoint from the closure of the corresponding final segment. A subset $S \subset X$ is free in $X$ if it admits a well-ordering that turns it into a free sequence in $X$. We let $F(X) = \sup \{|S| : S \text{ is free in } X\}$, and call it the free set number of $X$.

We present several new inequalities involving $F(X)$ and $F(X_\delta)$, where $X_\delta$ is the $G_\delta$-modification of $X$:

- $L(X) \leq 2^{2^{F(X)}}$ if $X$ is $T_2$ and $L(X) \leq 2^{F(X)}$ if $X$ is $T_3$;
- $|X| \leq 2^{2^{F(X)} \cdot \omega(X)} \leq 2^{2^{F(X)} \cdot \chi(X)}$ for any $T_2$-space $X$;
- $F(X_\delta) \leq 2^{2^{F(X)}}$ if $X$ is $T_2$ and $F(X_\delta) \leq 2^{F(X)}$ if $X$ is $T_3$.

We also present several forcing constructions of spaces that shed some light on the sharpness of these inequalities.

All the results are joint with L. Soukup and Z. Szentmiklóssy.

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