Filamentation of laser beams and excitation of ion acoustic wave in non-paraxial region

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Abstract
This paper contains the work on the filamentation of a laser beam and its effect on excitation of ion acoustic waves in a hot collision less plasma in non-paraxial region. To consider the non-paraxial region, the amplitude and the eikonal part of the beams are expended in terms of $r^2$ and $r^4$. On account of the non-uniform intensity distribution along the wave front of the laser beam, the ponderomotive force becomes finite. This leads to modification of the plasma density profile in the plane transverse to the beam axis and hence also the modification in the propagation characteristics of the waves propagating in the plasma. It is shown that the Filamentation of the waves can occur and this considerably affects the ion acoustic wave.

Keywords: Laser Plasma, Filamentation, Ion Acoustic Wave
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1. Introduction
The nonlinear interaction of two co-axial beams of electromagnetic waves in plasma gives rise to a longitudinal magnetic force at the beat frequency. Resonant excitation of electrostatic waves occurs when the beat frequency and the difference of the wave vectors of the input waves satisfy the linear dispersion relation for a natural mode of the plasma. Most previous studies have been concerned with the excitation of electron electrostatic waves (usually Langmuir waves) by use of laser beams. The study of ion acoustic wave (IAW) also becomes important due to its role in stimulated Brillouin scattering (SBS), decay instabilities, particle heating and other non-linear phenomenon in laser plasma interaction as well as in space plasma. Specifically, the nonlinear evolution of IAW may play an important role in nonlinear saturation of SBS process.

A lot of experimental and theoretical studies have been reported about the excitation of the IAW in plasma. The excitation of the IAW in collision-less plasma was studied first in late 70s and early 80s, by using whistler waves ¹-³ and by the beating of two laser beams ⁴. ⁵. Salimullah⁶, ⁷ studied the IAW excitation by two micro wave beams in semiconductor plasma with and without static magnetic field and
Pottelette\textsuperscript{8} also reported the excitation of the IAW by the slow ion beam. In recent generation of the IAW is also reported by the beating of two kinetic Alfv\text{en} waves\textsuperscript{9}. Anomalous effects, such as nonlinear reflection due to nonlinear interaction of radiation with excitations in the plasma medium play an essential role for the problem of plasma heating by lasers\textsuperscript{10}. Tsytovich et al. have studied the effects of IAW, excited by laser radiation, on the nonlinear reflection and absorption in a one-dimensional case considering an inhomogeneous plasma layer\textsuperscript{11}. For laser powers exceeding a certain threshold value an appreciable nonlinear reflection was found even when the plasma layer was linearly transparent. A nonlinear absorption that amounted to a few percent was also caused by the ion-acoustic waves, excited during the process. Experiments on IAW were widely studied in double plasma devices and Q-machines\textsuperscript{12-16}. There are two main methods to excite linear and nonlinear waves in these devices. The first one is the double-plasma (DP) mode which is operated by applying signals between two vacuum chambers biased at different voltages. The second is the grid excitation mode which is operated by applying signals to the grid immersed in the device. As early as the 1970s, a qualitative interpretation of the trapped ion instability was proposed by Taylor\textsuperscript{17} and Ikezi et al.\textsuperscript{18} for the DP excitation. Recently, the effects of a slow ion beam (IB) on the IAW excitation were considered by Nakamura et al. The experiments showed that the IAW can be transformed into the fast ion beam mode when the density of the ion beam was increased to a critical value\textsuperscript{19, 20}. On the other hand, the experiments by Honzawa and Nagasawa suggested that the fast ion beam can limit the amplitude of the ion-acoustic solitons or result in the amplification of the solitons depending on the rise time of the ramp signals\textsuperscript{21, 22}. It was shown that the ion beam plays an important role in the excitation of the linear as well as nonlinear IAWs\textsuperscript{23}. Nonlinear evolution of ion acoustic wave at the beat wave frequency of two co-propagation laser beams, in hot, collision-less, homogeneous plasma is studied in non-paraxial region in the present paper. Only non-linearity considered here is due to the ponderomotive force. In section 2.1 model equations for the mutual interaction of two laser beams and differential equations governing the intensity of laser beams in the plasma are formulated in non-paraxial region. The equation for ion acoustic wave (excited at the beat wave frequency of two laser beams) is presented in section 2.2. The results and the conclusion of the present investigation are discussed in section 3.

2.1. Model Equations for Cross-Focusing of Laser Beams

Consider the propagation of two coaxial Gaussian laser beams of frequencies $\omega_1$ and $\omega_2$ along the $z$ direction. The initial intensity distributions of the beams are given by
Where \( r \) is the radial coordinate of the cylindrical coordinate system and \( r_{10} \) and \( r_{20} \) are their initial beam widths. The expression for the ponderomotive force in the presence of two laser beams can be written as

\[
F_p = -\left( \frac{e^2}{4m} \right) \left[ \left( \frac{1}{\omega_1^2} \right) \nabla E_1 \cdot E_1^* + \left( \frac{1}{\omega_2^2} \right) \nabla E_2 \cdot E_2^* \right]
\]

and the modified electron density due to ponderomotive force is

\[
N_{0e} = N_0 \exp \left[ -\frac{3}{4} \left( \frac{m_e}{m_i} \right) \left( \alpha_{1,2} E_1 \cdot E_1^* + \alpha_{2,2} E_2 \cdot E_2^* \right) \right]
\]

where

\[
\alpha_{1,2} = \frac{e^2 m_i}{18k_B T_0 m_e^2 \omega_{1,2}^2}
\]

e and \( m \) are the electric charge and mass, respectively, \( m_i \) is the mass of ion, \( k_B \) is the Boltzmann’s constant, \( T_0 \) is the equilibrium temperature of the plasma and \( N_0 \) is the electron density in the absence of the laser beams.

The effective dielectric constant of the plasma at the frequencies \( \omega_1 \) and \( \omega_2 \) is given by

\[
\varepsilon_{1,2} = \varepsilon_{01,2} + \phi_{1,2} (E_1 \cdot E_1^*, E_2 \cdot E_2^*)
\]

where

\[
\varepsilon_{01,2} = 1 - \frac{\omega_{2,0}^2}{\omega_{1,2}^2}
\]

and

\[
\phi_{1,2} = \left( \frac{\omega_{2,0}^2}{\omega_{1,2}^2} \right) \left[ 1 - \exp \left[ -\frac{3}{4} \left( \frac{m}{M} \right) \left( \alpha_{1,2} E_1 \cdot E_1^* + \alpha_{2,2} E_2 \cdot E_2^* \right) \right] \right]
\]

where plasma frequency \( \omega_{2,0} \) is given by

\[
\omega_{2,0}^2 = 4\pi N_0 e^2 / m
\]

The wave equation governing the electric vectors of the two laser beams in plasma can be written as

\[
\frac{\partial^2 E_{1,2}}{\partial z^2} + \frac{1}{r} \frac{\partial E_{1,2}}{\partial r} + \frac{\partial^2 E_{1,2}}{\partial r^2} + \frac{\omega_{1,2}^2}{c^2} \varepsilon_{1,2} E_{1,2} = 0
\]
In writing eq. (55), we have neglected the $\nabla(\nabla \cdot E)$ term which is justified as long as

$$\epsilon_{1,2} \leq 1$$

assuming the variations of the electric fields as

$$E_{1,2} = A_{1,2}(x, y, z)e^{-ik_{1,2}z}$$

the wave equation becomes as

$$-k_{1,2}^2 A_{1,2} - 2ik_{1,2} \frac{\partial}{\partial z} A_{1,2} + \left( \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial}{\partial r} \right) A_{1,2} + \frac{\omega_{1,2}^2}{c^2} \epsilon_{1,2} A_{1,2} = 0$$

(6)

$A_{1,2}$ is a complex function of space, further assuming the variation of $A_{1,2}$ as Akhmanove et al.

$$A_{1,2} = A_{01,2}(r, z)e^{-ik_{1,2}S_{1,2}(r, z)}$$

(7)

Where $A_{1,2}$ and $S_{1,2}$ are the real function of space. Substituting the eq. (7) into eq. (6) and separating real and imaginary parts of resulting equation, the following equations are obtained.

The real part from eq. (6) is

$$2\frac{\partial^2 S_{1,2}}{\partial z^2} + \left( \frac{\partial S_{1,2}}{\partial z} \right)^2 = \alpha_{1,2}^2 \epsilon_{1,2} + 1 - 1 - k_{1,2}^2 \frac{\partial^2 A_{01,2}}{\partial r^2} + 1 - 1 - \frac{\partial A_{01,2}}{\partial r}$$

(8)

Where

$$A_{01,2}^2 = \left( 1 + \frac{\alpha_{01,2} r^2}{r_{1,2}^2 f_{1,2}} + \frac{\alpha_{21,2} r^4}{r_{1,2}^2 f_{1,2}^2} \right) \left( \frac{E_{1,2}^2}{f_{1,2}} \right) e^{\left( -\frac{r^2}{r_{1,2}^2 f_{1,2}^2} \right)}$$

(9)

are the laser beam intensities, $f_{1,2}$ are the dimensionless beam width parameters for beam 1 and 2 respectively, and

$$S_{1,2} = \frac{r^2}{2f_{1,2}} \frac{df_{1,2}}{dz} + \frac{r^4}{r_{1,2}^4} S_{2,1,2}$$

(10)

By substituting eqs. (4), (9) and (10) in eq. (8) and equating the coefficient of $r^2$ on both sides of the resulting equation, the governing equation of beam width parameters $f_{1,2}$ is:

$$\frac{d^2 f_{1,2}}{dz^2} = \frac{1}{k_{1,2}^2 f_{1,2}^2} \frac{8\alpha_{21,2} - 2\alpha_{01,2} - 3\alpha_{1,2}^2}{f_{1,2}^2 + \frac{a_{1,2}^2 \epsilon_{1,2}}{2c^2 k_{1,2}^2}} \frac{3m}{4M} \left( \frac{\alpha_{1,2} E_{1,2}^2}{r_1^2 f_1^4} + \frac{\alpha_{2,2} E_{2,2}^2}{r_2^2 f_2^4} \right)$$

(11)

In a similar way, by equating the coefficient of $r^4$ on both sides of the resulting equation, we obtained the following equations
The imaginary part of equation (6) is given by

\[
\frac{\partial S_{1,2}}{\partial z} = \frac{3m\alpha_p^2}{16Mc^2k_{1,2}^2} \left\{ (1 - 2\alpha_{01} + 2\alpha_{21}) \frac{\alpha_{01}E_{01}^2}{r_1^4f_1^6} + (1 - 2\alpha_{02} + 2\alpha_{22}) \frac{\alpha_{02}E_{02}^2}{r_2^4f_2^6} \right\} e^{\frac{3m}{4M} \left( \frac{\alpha_{E1}}{f_1} + \frac{\alpha_{E2}}{f_2} \right)} \]

\[
- \left( \frac{7\alpha_{01,2}\alpha_{21,2} + 2\alpha_{21,2} + \alpha_{01,2}^2 - \alpha_{01,2}^3}{k_{1,2}^2r_{1,2}^6f_{1,2}^6} \right) - \frac{4S_{21,2}}{f_{1,2}} \frac{df_{1,2}}{dz} \]

(12)

The imaginary part of equation (6) is given by

\[
\frac{\partial A_{01,2}^2}{\partial z} + \frac{\partial S_{1,2}}{\partial r} \frac{\partial A_{01,2}}{\partial r^2} + A_{01,2}^2 \left( \frac{\partial^2 S_{1,2}}{\partial r^2} + \frac{1}{r} \frac{\partial S_{1,2}}{\partial r} \right) = 0 \]

(13)

By substituting Eqs. (9) and (10) into eq. (13) and equating the coefficient of \( r^2 \) on both sides of the resulting equation, we obtained the equations for the coefficient \( \alpha_{01,2} \)

\[
\frac{\partial \alpha_{01,2}}{\partial z} = -\frac{16S_{21,2}f_{1,2}^2}{r_{1,2}^2} \]

(14)

In a similar way, by equating the coefficient of \( r^4 \) gives the equation for the coefficient \( \alpha_{21,2} \)

\[
\frac{\partial \alpha_{21,2}}{\partial z} = 8 \left( 1 - 3\alpha_{02} \right) \frac{S_{21,2}f_{1,2}^2}{r_{1,2}^2} \]

(15)

Equation (9) gives the intensity profile of the laser beams in the plasma along with the radial direction. The intensity profile of both laser beams depends on the beam width parameters \( f_{1,2} \) and the coefficients \( (\alpha_{01,2} \text{ and } \alpha_{21,2}) \) of \( r^2 \) and \( r^4 \) in non-paraxial region. Equation (11) determines the focusing/defocusing of the laser beams, along with the distance of propagation in the plasma. In order to have a numerical appreciation of the cross-focusing in non-paraxial region, author has performed numerical computation of the coupled equations (11), (12), (14) and (15) with typical plasma and laser beam parameters. The following set of the parameters has been used in the numerical calculation: \( r_1 = 15\mu, r_2 = 20\mu, \omega_1 = 1.963 \times 10^{14}\text{rad/S}, \omega_2 = 1.778 \times 10^{14}\text{rad/S} \text{ and } \omega_{01}^2 = 0.5\omega_1^2 \). For an initial plane wave front of the laser beams, the initial condition used here are \( f_{1,2} = 1, df_{1,2}/dz = 0 \), and \( \alpha_{01,2} = \alpha_{21,2} = 0 \) at \( z = 0 \).

### 2.2. Excitation of the Ion Acoustic Wave

To study the excitation of ion acoustic wave by beat wave process, we start with the following equations:

The continuity equation

\[
\frac{\partial}{\partial t} N_i + \nabla \cdot (N_i V_i) = 0
\]

(16)

The momentum equation
\[ m_{e,i} \left[ \frac{\partial}{\partial t} V_{e,i} + (V_{e,i} \cdot \nabla)V_{e,i} \right] = qE + \frac{e}{c} V_{e,i} \times B - 2\Gamma_{e,i} MV_{e,i} - \frac{3K_B}{N_{e,i}} T_{e,i} \nabla N_{e,i} \]  

(17)

Poisson’s equation

\[ \nabla E = 4\pi e \left( N_i - N_e \right) \]  

(18)

Where the subscripts e and i stand for electron and ion particles respectively and q = -e for electron and q = e for the ion particle case. In this equation \( N_{e,i} \) are the particle densities, E is the sum of electric field vectors of the electromagnetic waves and the self consistent field, \( V_{e,i} \) are the drift velocities of the particles in the EM field and self consistent field, other symbols have their usual meanings.

Using the equation (16), (17) and (18), and adding the inertia-less electron and inertial ion version of equation (17), we obtain the following equation governing the ion density in the plasma:

\[ \frac{\partial^2 N_i}{\partial t^2} + 2\Gamma_i \frac{\partial N_i}{\partial t} - V_{th}^2 \nabla^2 N_i = \frac{N_{i0}}{4} \nabla^2 (V_{i1} V_{i2}^*) \]  

(19)

where \( 2\Gamma_i \) is the Landau damping factor given by Krall, \( V_{th}^2 \) is the thermal velocity. Therefore the equation for the ion acoustic wave at the difference frequency (\( \Delta \omega = \omega_1 - \omega_2 \)) reduces to

\[ - (\omega_1 - \omega_2)^2 N_i - 2i(\omega_1 - \omega_2) N_i - V_{th}^2 \frac{\partial^2 N_i}{\partial \omega^2} \cong \frac{1}{4} N_{i0} \nabla^2 (V_{i1} V_{i2}^*) \]  

(20)

where \( N_i \) is the component of ion density oscillating at frequency \( \Delta \omega \). The drift velocities of ions in the pump field at the frequency \( \omega_1 \) and \( \omega_2 \), \( V_{i1,2} \) are

\[ V_{i1,2} = - \frac{eE_{1,2}}{m_i \omega_{1,2}} \]

Therefore

\[ V_{i1} V_{i2}^* = \frac{C_1}{f_1 f_2} B_1^{1/2} B_2^{1/2} e \left( \frac{1}{2 \nu_1 f_1^2} - \frac{1}{2 \nu_2 f_2^2} \right) e^{-i[(k_1 - k_2)z+(k_1 - k_2)z]} \]  

(21)

where

\[ C_1 = \frac{e^2}{m_i^2 \omega_1 \omega_2} \frac{E_{10} E_{20}}{f_1 f_2} \]  

\[ B_1 = 1 + \frac{a_{10} r^2}{r_1^2 f_1^2} + \frac{a_{12} r^4}{r_1^4 f_1^4} \]  

and

\[ B_2 = 1 + \frac{a_{20} r^2}{r_2^2 f_2^2} + \frac{a_{22} r^4}{r_2^4 f_2^4} \]
Equation (20) contains two plasma waves (both at different frequency), the first one is supported by hot plasma and the second by the source term at the difference frequency. The solution of equation (20), with in the WKB approximation can be expressed as\textsuperscript{25,26}:

\[ N_1 = N_{10}(r, z) e^{-i[(k_1 - k_2)z + (k_1 S_1 - k_2 S_2)]} \]  \hspace{1cm} (22)

\( N_{10} \) is a slowly varying real function of the space coordinate. Using equation (20), (21) and (22), the governing equation of the ion density oscillating at the difference frequency (\( \Delta \omega = \omega_1 - \omega_2 \)) can be written as:

\[ V_{\theta b} \frac{\partial^2 N_{10}}{\partial z^2} + 2i \Delta k V_{\theta b} \frac{\partial N_{10}}{\partial z} + \left( \Delta \omega^2 + 2i \Gamma_\Delta \omega - V_{\theta b}^2 \Delta k^2 \right) = -\frac{N_{10} C_1}{4} A \]  \hspace{1cm} (23)

where

\[
A = \left[ \left\{ \left( \frac{r^2}{\sigma_r^2} \right) \frac{1}{f_1^2} \frac{\partial f_1}{\partial z} + \left( \frac{r^2}{\sigma_r^2} \right) \frac{1}{f_2^2} \frac{\partial f_2}{\partial z} \right\} + \left( \frac{r^2}{\sigma_r^2} \right) \frac{1}{f_1^2} \frac{\partial^2 f_1}{\partial z^2} + \left( \frac{r^2}{\sigma_r^2} \right) \frac{1}{f_2^2} \frac{\partial^2 f_2}{\partial z^2} \right) + \left( \frac{r^2}{\sigma_r^2} \right) \frac{1}{f_1^2} \left( \frac{\partial f_1}{\partial z} \right)^2 + \left( \frac{r^2}{\sigma_r^2} \right) \frac{1}{f_2^2} \left( \frac{\partial f_2}{\partial z} \right)^2 \right] \left( \frac{B_1 B_2}{f_1 f_2} \right)^{1/2} e^{-\frac{1}{2f_1^2 f_2^2} \left( \frac{1}{f_1^2} + \frac{1}{f_2^2} \right)}
\]

Equation (23) has been solved numerically by using a finite difference method in the presence of filamentary structure of tow laser beams in paraxial as well as non-paraxial region for the typical set of parameters as used in previous portion of this chapter. Results are presented in the form of graphs in figures 1 to 3.

3. Discussion and Conclusion

The intensity profile of both laser beams depends on the beam width parameters f\(_{1,2}\) and the coefficients (\( \alpha_{01,2} \) and \( \alpha_{21,2} \)) of \( r^2 \) and \( r^4 \) in non-paraxial region. Equation (11) determines the focusing/defocusing of the laser beams, along with the distance of propagation in the plasma. In equation (11) first term is responsible for the diffraction, while second and third term (non-linear terms) in the right hand side of the equation are responsible for the converging behavior of the beams during propagation in plasmas. The resultant of these three terms governs the filamentation formation and laser beam propagation in the plasmas. Figure 1 shows the variation of the normalized intensities of laser beams along with the normalized distance of propagation and radial distance. Figure 1 (a & b) are intensity profile for beam 1
and 2 in paraxial region ($\alpha_{01,2} = \alpha_{21,2} = 0$) while Figure 1 (c & d) are the intensity profile in non-paraxial region ($\alpha_{01,2} \neq \alpha_{21,2} \neq 0$). It is obvious that in paraxial region the intensity of laser beams is maximum at $r = 0$ along the distance of propagation as $\alpha_{01,2} = \alpha_{21,2} = 0$. While in non-paraxial region the laser intensity becomes minimum at $r = 0$, assumes a ring structure or a spitted beam profile (Fig. 1 c & d). Figure 1 (c & d) also illustrates the effect of the coefficients of non paraxial rays on the focusing/defocusing of the laser beams. Focusing becomes faster in non-paraxial case in comparison to paraxial case due to the participation of off-axis parts ($\alpha_{01,2} \neq \alpha_{21,2} \neq 0$).

The mutual interaction between two co-propagation laser beams can generate filaments and ion acoustic wave at difference frequency. Behavior of the ion acoustic wave at the difference frequency of two laser beams has been studied by analyzing numerically equation (23). Figure (2) displays the variation of the density profile of ions, at the beat wave frequency. The amplitude of ion acoustic wave directly depends on the density profile of ions (Poisson's equation). As shown in figure (2), the density dips are observed at the location of the filaments of the main laser beams as expected from equation (23).

![Fig. 1. Variation of the intensity of laser beams with normalized distance $\xi (= zc/\omega_1 r_1^2)$ and the radial distance ($r/r_1$). (a & b) are for beam 1 and beam 2 in paraxial region while (c & d) are in non-paraxial region respectively.](image-url)
The governing equation (23) of ion density at beat wave frequency depends not only on the laser beams parameters, like radius, intensity, beam width parameters but also depends on the coefficient of \( r^2 \) and \( r^4 \) \((\alpha_{01,2} \text{ and } \alpha_{21,2})\). The ion density (ion acoustic wave power) is minimum at those positions, where the intensity of laser beams is maximum \((f_1, f_2 \text{ are minimum})\). The focusing of the laser beams tends to increase the power of ion acoustic wave, but the factor \( N_1/N_{10} \) decreases exponentially, as the laser beams focus and hence the resultant of these two factors is responsible for the density dips in the ion density profile. On the axis \((r = 0, \alpha_{01,2} = \alpha_{21,2} = 0)\) the ion density is minimum hence the power of ion acoustic wave is also expected to be minimum for this case. In non-paraxial region as the \((\alpha_{01,2} \neq \alpha_{21,2} \neq 0)\), as radial distance increases, the amplitude of the density dip decreases as shown in figure (2) for various values of radial distance \((r)\). The decrease in the density dip with the increase of \(r\), is attributed by decrease of the intensity of the main laser beams in non-paraxial region.

![Variation of the ion density profile of ion acoustic wave with normalized distance](image)

**Fig. 2.** Variation of the ion density profile of ion acoustic wave with normalized distance \( \xi = zc/\omega_1r_1^2 \) at different normalized radial distance \((r/r_1)\). Solid line, (...) line, (----) line and (-.-.-) lines are for \((r/r_1 = 0, \text{ paraxial case})\), \((r/r_1 = 0.2, 0.4, 0.6 \text{ non-paraxial case})\) respectively.
As the dynamics of ion acoustic wave is becoming complex, author did further diagnostics by constructing the phase portrait of this dynamical system. Figure (3) presents the phase-space plot \( \{dN_{10}/dz, N_{10}\} \) for the ions beating at the difference frequency of two laser beams. It is observed that the chaotic nature of phase pattern increases in non-paraxial region in comparison to paraxial region \((r = 0)\).

In the presence of this type of structure one can study the particle heating. Mainly two mechanisms, Landau damping and stochastic heating are responsible for particle heating in plasma. Due to the chaotic nature of the phase plot, other mechanism, (stochastic heating) can also be applicable here for particle heating.

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