Symmetry Energy of Nuclear Matter
and Properties of Neutron Stars in a Relativistic Approach

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ABSTRACT

Asymmetric nuclear matter is treated in the formalism of Dirac-Brueckner approach with Bonn one-boson-exchange nucleon-nucleon interaction. We extract the symmetry energy coefficient at the saturation to be about 31 MeV, which is in good agreement with empirical value of 30±4 MeV. The symmetry energy is found to increase almost linearly with the density, which differs considerably from the results of non-relativistic approaches. This finding also supports the linear parameterization of Prakash, Ainsworth and Lattimer. We find, furthermore, that the higher-order dependence of the nuclear equation of state on the asymmetry parameter is unimportant up to densities relevant for neutron stars. The resulting equation of state of neutron-rich matter is used to calculate the maximum mass of neutron star, and we find it to be about 2.1M⊙. Possible mechanisms for the softening of the equation of state are also discussed.

1 Introduction

Although the fact that the equation of state of nuclear matter contains a symmetry energy term has been known since the early days of nuclear physics, the experimental and theoretical study of the symmetry energy and its density dependence is becoming an increasingly interesting topic, mainly because of the recent development of radioactive ion beam facilities that allow one to study
the structure and reactions of neutron-rich nuclei [1, 2, 3], in which the symmetry energy plays an important role. The recognition that the symmetry energy, especially its density dependence, has a profound effect on the properties of neutron stars [4, 5, 6, 7, 8] also makes the experimental and theoretical determination of this quantity very relevant and useful.

Empirically, the symmetry energy coefficient \( S_2(\rho_0) \) in nuclear matter at the saturation density \( \rho_0 \) can be extracted from the systematic study of the masses of atomic nuclei, based on, e.g., the liquid droplet model [9, 10] or the macroscopic-microscopic model [11, 12]. This, however, determines the symmetry energy only for small asymmetry parameter \( \alpha \) (\( \alpha = (N - Z)/A \)) and for densities around \( \rho_0 \). From the experience with symmetric nuclear matter we know that the determination of the compression modulus \( K \) at \( \rho_0 \) does not uniquely constrain the equation of state at high densities. Similarly, the determination of \( S_2(\rho_0) \) does not guarantee an unambiguous determination of symmetry energy at high densities which is needed for the study of neutron star properties.

The situation changes with the recent advances in the development of various radioactive ion beam facilities around the world that will produce nuclei with large neutron excess near and beyond the drip-line. The study of the structure of these neutron-rich nuclei allows us to determine the symmetry energy for large asymmetry parameter and extract possible higher-order dependence on \( \alpha \). Furthermore, the collisions of neutron-rich nuclei at relativistic energies, during which nuclear matter with densities up to \((2-3)\rho_0\) is created, make it possible to study experimentally the density dependence of the symmetry energy [13, 14].

On the theoretical side, the symmetry energy has been studied over many years based on various models and approaches, which can roughly be divided into phenomenological and microscopic, each of which can be subdivided into relativistic and non-relativistic. Hartree-Fock [15] and Thomas-Fermi [16] calculations with Skyrme-type effective nucleon-nucleon interactions have been carried out to study various nuclear properties, including symmetry energy. The symmetry energy coefficient \( S_0 \) from these calculations ranges from about 27 to 38 MeV, and is in agreement with the empirical value of \( 30 \pm 4 \) MeV [17]. Since the the parameters in the Skyrme forces are adjusted to fit nuclear matter and finite nuclei properties, this degree of agreement is fully expected.

Another phenomenological approach that has been used extensively in the study of nuclear properties is quantum hadrodynamics (QHD) which is based on the relativistic field theory [18, 19]. The symmetry energy in this approach has two contributions [20, 21]; one comes from the ‘kinetic’ energy difference between symmetric and asymmetric matter, and the other arises from
the exchange of the rho meson which couples to neutron and proton with opposite sign. The symmetry energy in this approach ranges from about 35 to 40 MeV \[21, 22, 23\], somewhat larger than the empirical value of 30 ± 4 MeV.

On a more microscopic level, it has been the ultimate goal of traditional nuclear physics to describe in a consistent way the properties of nuclear matter, finite nuclei, and nuclear reactions from realistic nucleon-nucleon interactions that are fitted to the nucleon-nucleon scattering and deuteron data. There are basically two approaches to this; namely, variational-type and Brueckner-type. In variational calculations, it is well-known that the realistic two-nucleon potential alone, such as Argonne \(v_{14}\) (AV14) or Urbana \(v_{14}\) (UV14), does not provide a satisfactory description of nuclear matter properties \[24, 25\]. A phenomenological three-nucleon interaction has to be introduced, whose parameters are adjusted so that the variational calculations with the two- and three-body interactions give correct nuclear matter saturation density, binding energy and compression modulus. The symmetry energy coefficient obtained in the variational calculations is about 30 MeV \[24, 25\].

The Brueckner-type approach has both non-relativistic and relativistic versions, known as Brueckner-Hartree-Fock (BHF) \[26, 27\] and Dirac-Brueckner-Hartree-Fock (DBHF) \[28, 29, 30, 31, 32\], respectively. As in the variational calculations, the BHF calculation with realistic two-nucleon interactions such as Bonn and Paris potentials does not provide a good description of nuclear matter properties \[26\]. On the other hand, the DBHF approach, owning to the additional density dependence introduced through the medium modified Dirac spinors, does provide a very good description of nuclear matter properties based entirely on realistic nucleon-nucleon interactions that are constrained by two-nucleon data (nucleon-nucleon scattering and deuteron properties).

In this work, we study systematically the properties of asymmetric nuclear matter in the formalism of the DBHF approach using the Bonn one-boson-exchange (OBE) potential. We will calculate in particular the density dependence of the symmetry energy that is very important for neutron star properties and heavy-ion collisions. Our results will be compared to those of the variational and BHF calculations, and to various phenomenological parameterizations. We will also apply the nuclear equation of state obtained in this study to calculate neutron star properties. In Section 2, we review briefly the DBHF approach and its extension to asymmetric nuclear matter. The results for symmetric energy will be presented in Section 3, while those for neutron stars in Section 4. A short summary is given in Section 5.
2 Dirac Brueckner approach for asymmetric nuclear matter

The essential point of the DBHF approach is the use of the Dirac equation for the description of the single-particle motion in the nuclear medium. The Dirac spinor, which enters the evaluation of in-medium nucleon-nucleon potential, becomes density dependent. This additional density dependence is instructive in reproducing correctly the nuclear matter saturation density and binding energy [30].

The basic quantity in the DBHF calculation is the $\tilde{G}$ matrix which satisfies the in-medium Thompson equation,

$$
\tilde{G}(q', q|P, \tilde{z}) = \tilde{V}(q', q) + \int \frac{d^3k}{(2\pi)^3} \tilde{V}(q', k) \left( \frac{\tilde{m}(k)}{\tilde{E}(k)} \right)^2 \frac{\tilde{Q}(k, P)}{2\tilde{E}(q) - 2\tilde{E}(k)} \tilde{G}(k, q|P, \tilde{z})
$$

where $\tilde{E} = \sqrt{\tilde{m}^2 + (P/2 + k)^2}$, and $\tilde{m} = m + U_S$, with $m$ being nucleon mass in free space. For asymmetric nuclear matter, the angle-averaged Pauli-blocking operator has to be modified and is given by

$$
\tilde{Q}(k, K) = \begin{cases} 
1 & \text{if } \beta_n > 1 \\
(1 + \beta_n)/2 & \text{if } -1 < \beta_n < 1 \text{ and } \beta_p > 1 \\
(\beta_n + \beta_p)/2 & \text{if } \beta_p < 1 \text{ and } 0 < (\beta_n + \beta_p)/2 \\
0 & \text{if } (\beta_n + \beta_p)/2 < 0 \text{ or } \beta_n < -1 
\end{cases}
$$

where

$$
\beta_{n,p} = \frac{K^2/4 + k^2 - k_{F_{n,p}}^2}{Kk}
$$

where $k_{F_n}$ and $k_{F_p}$ are neutron and proton Fermi momenta, respectively, with $k_{F_n} \geq k_{F_p}$.

From the $\tilde{G}$ matrix we can calculate the single-particle potential

$$
\Sigma_{DBHF}(k) = Re \int_0^{k_F} d^3q \left( \frac{\tilde{m}(q)}{\tilde{E}(q)} \right) \left( \frac{\tilde{m}(k)}{\tilde{E}(k)} \right) \langle kq | \tilde{G}(\tilde{z}) | kq - qk \rangle,
$$

which is usually parameterized in terms of the scalar and vector potentials

$$
\Sigma(k) = \frac{\tilde{m}(k)}{\tilde{E}(k)} U_S(k) + U_V(k).
$$

In principle, the scalar and vector potentials are both density and momentum dependent. The momentum dependence for momenta below the Fermi momentum is rather weak and smooth [29].
In a recent work [33], we found that the saturation properties and nuclear equation of state of symmetric matter with explicit momentum dependence are almost the same as those obtained in Ref. [30] under the assumption of momentum independent mean fields. In this work we thus follow Ref. [30] and neglect the momentum dependence of the scalar and vector potentials.

We will use the Bonn A potential from Ref. [30]. The parameters in this potential are fitted to neutron-proton (np) scattering and deuteron properties. It is well known that at low energies, there are both charge independence breaking and charge symmetry breaking [30], namely, np, pp, and nn potentials are not completely identical. The effects of those differences on the bulk nuclear matter properties are, however, found to be extremely small and can well be neglected [34].

In the case of asymmetric nuclear matter, the potential energy of a single particle is

$$E_{pot} = \frac{1}{\int_{0}^{k_{F_n}} d^3k + \int_{0}^{k_{F_p}} d^3k} \left( \int_{0}^{k_{F_n}} d^3k \frac{1}{2} \Sigma_n(k) + \int_{0}^{k_{F_p}} d^3k \frac{1}{2} \Sigma_p(k) \right),$$

(6)

while the kinetic energy is given by

$$E_{kin} = \frac{1}{\int_{0}^{k_{F_n}} d^3k + \int_{0}^{k_{F_p}} d^3k} \left( \int_{0}^{k_{F_n}} d^3k \frac{m^*(k) + k^2}{E^*(k)} + \int_{0}^{k_{F_p}} d^3k \frac{m^*(k) + k^2}{E^*(k)} \right),$$

(7)

The energy per nucleon, or nuclear equation of state, is then given by

$$E = E_{pot} + E_{kin} - m.$$  

(8)

The asymmetric nuclear matter is characterized by the asymmetry parameter

$$\alpha = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}.$$  

(9)

In the left panel of Fig. [4] the neutron and proton scalar and vector potentials are shown as a function of the asymmetry parameter, for three different densities. It is seen that the variation of the proton and neutron potentials is not symmetric with respect to their common value at $\alpha = 0$. For a nucleon at rest in nuclear matter, we can define its single-particle potential as $U_S + U_V$. We find that as the asymmetry parameter increases, the neutron potential becomes less attractive and that of proton becomes slightly more attractive. These differences in proton and neutron potentials may be observed in heavy-ion collisions with radioactive ion beams [13, 14].

In the right panel of Fig. [4] we show the nuclear equation of state for a number of asymmetry parameters. We compare our results with those of Ref. [29] obtained in the BHF approach. As is well-known, the BHF approach saturates nuclear matter at a much too high density. In the DBHF calculation, the nuclear matter saturation properties are better reproduced. The binding energy and the saturation density become progressively smaller as $\alpha$ increases.
3 Symmetry Energy of nuclear matter

In this section, we discuss our results for symmetry energy and its density dependence. We introduce \( \Delta E \) as the energy difference between symmetric and asymmetric nuclear matter,

\[
\Delta E = E(\rho, \alpha) - E(\rho, 0),
\]

with \( E(\rho, \alpha) \) defined in Eq. (7). The results are shown in the left panel of Fig. 2 as a function of \( \alpha^2 \) for six different densities. It is seen that at all densities considered here, \( \Delta E \) increases almost linearly with \( \alpha^2 \), indicating that the \( \alpha^4 \) and higher-order terms are not important. To a good extent we can express the equation of state of asymmetric nuclear matter as

\[
E(\rho, \alpha) = E(\rho, 0) + S_2(\rho)\alpha^2 + S_4(\rho)\alpha^4.
\]

(11)

The usual symmetry energy \( S_2 \) is thus defined as

\[
S_2(\rho) = \frac{1}{2} \left. \frac{\partial^2 E(\rho, \alpha)}{\partial \alpha^2} \right|_{\alpha=0},
\]

(12)

similarly,

\[
S_4(\rho) = \frac{1}{24} \left. \frac{\partial^4 E(\rho, \alpha)}{\partial \alpha^4} \right|_{\alpha=0}.
\]

(13)

The density dependences of \( S_2 \) and \( S_4 \) obtained in our calculation are shown in the right panel of Fig. 2 by the solid curve. \( S_2 \) increases almost linearly with density. Actually a parameterization in terms of \((\rho/\rho_0)^{0.9}\) fits the theoretical curve reasonably well, as shown in the figure by dotted curve. At nuclear matter saturation density, our calculation gives a symmetry energy coefficient \( S_2(\rho_0) \) of about 31 MeV, which is in good agreement with the empirical value of about 30 ± 4 MeV [17]. The BHF and the variational calculations also reproduce the empirical symmetry energy coefficient [25, 26]. The coefficient of \( \alpha^4 \) term is very small in the density region considered here. This means that the approximation of neglecting this term as adopted in Ref. [24, 25] is quite reasonable.

In the left panel of Fig. 3 we compare our results for the density dependence of the symmetry energy with those of Ref. [20] based on the BHF calculations, and of Ref. [25] based on the variational calculation. There are significant differences between the results of these three calculations. In relativistic approaches [34, 35], the symmetry energy increases almost linearly with density, and is considerably larger than those in non-relativistic and variational calculations [25, 26]. The difference between the DBHF and BHF is mainly due to the relativistic effects. In the simple mean-field
approximation to the Walecka-type model, the symmetry energy has a contribution from the 'kinetic energy' difference, which is inversely proportional to \( E_F^* = \sqrt{k_F^2 + m^*^2} \). This contribution is thus larger in relativistic approaches because of the dropping nucleon mass. This also accounts for part of the difference between our results and that of Ref. [25] which is non-relativistic in nature. The remaining difference can be explained by the differences in the nucleon-nucleon potentials used in the two calculations. In the variational calculations [25], the major contribution to the 'potential' part of the symmetry energy comes chiefly from the second-order tensor interaction, which is progressively blocked with increasing density. With the strong \( \rho \)-coupling of the Bonn potential, the second-order tensor force is relatively weak, compared with that of Ref. [25], so that this is not a large effect in our calculation, where the main contribution to the symmetry energy comes from \( \rho \)-meson exchange. The differences in the symmetry energy in these three calculations will have a profound impact on the properties of neutron stars. We hope that future experiments with radioactive ion beams will help to shed light on this problem.

Phenomenologically, Prakash, Ainsworth and Lattimer [5] proposed the following parameterization for the density dependence of the symmetry energy,

\[
S(u) = \left(2^{2/3} - 1\right) \frac{3}{5} E_F^0 \left(u^{2/3} - F(u)\right) + S_0 F(u),
\]  

(14)

with \( F(u) = 2u^2/(1 + u) \), \( u \), or \( \sqrt{u} \), where \( u = \rho/\rho_0 \) and \( E_F^0 \) is Fermi energy at saturation density \( \rho_0 \). In the right panel of Fig. 3, the density dependences of three forms of \( F(u) \) are compared with our results, and it is seen that the \( F(u) = u \) case is very close to our results.

4 Nucleon star properties

In this section we apply the nuclear matter equation of state obtained in the DBHF calculation to the study of neutron star properties. In a real neutron star, there exists a large fraction of protons with electrons and muons maintaining the charge neutrality. We call this star a nucleon star instead of neutron star [37]. We assume that the star is in chemical equilibrium at zero temperature after the neutrinos leave the system. The equilibrium condition requires that the chemical potentials should satisfy

\[
\hat{\mu} = \mu_n - \mu_p = \mu_e = \mu_\mu,
\]  

(15)
where muons start to contribute when $\mu_e > m_\mu$. The nucleon chemical potential difference $\bar{\mu}$ can be calculated once we have the coefficients of symmetry energy from DBHF \[38\],

$$\bar{\mu} = 4(1 - 2x) \left[ S_2(\rho) + 2S_4(\rho)(1 - 2x)^2 \right],$$

(16)

where $x$ being the proton fraction. From the charge neutrality $\rho_e + \rho_\mu = \rho_p$ and Eq. (15), we have

$$\rho x = \frac{1}{3\pi^2 \mu_e^3} + \theta(\mu_e - m_\mu) \frac{1}{3\pi^2} \left( \frac{\mu_e^2 - m_\mu^2}{\rho} \right)^{3/2}.$$  

(17)

By solving Eqs. (16) and (17) for a given density, we can get the proton fraction in nucleon star matter. In Fig. 4, the proton fraction and the energy difference (Eq. (10)) are plotted in the left panel. Muons start to contribute almost at the nuclear saturation density, $\rho_0 = 0.16$ fm$^{-3}$, which leads to a slight bump in the proton fraction.

Given the symmetry energy and EOS of symmetric nuclear matter, we can get the energy density and pressure and solve the Tolman-Oppenheimer-Volkov (TOV) equations for a spherically symmetric nucleon star,

$$\frac{dM}{dr} = 4\pi r^2 \epsilon,$$

$$\frac{dP}{dr} = -\frac{G}{r(r - 2GM)} \left[ \epsilon + P \right] \left[ 1 + 4\pi r^3 P \right].$$

(18)

In the above equations, the pressure $P(\epsilon)$ determines the stiffness of the TOV equation, On the right panel of Fig. 4, the nucleon star properties are summarized, where the dash-dotted curves give the results without inclusion of muons. The maximum mass nucleon star corresponds to $M_{\text{max}} = 2.1 M_\odot$, $R = 10.8$ km, and the central density $\rho_c = 1.08$ fm$^{-3}$. This mass, we believe, is well determined in our conventional scenario, beginning from the microscopic Bonn two-body interactions, and using the many-body theory as described above. The EOS can be substantially softened, however, by introduction of kaon condensation \[39, 40\] and this brings the maximum mass down to $M_{\text{max}} \sim 1.5 M_\odot$ \[41\]. Introduction of $K^-$-mesons implies an $\sim 50\%$ nucleus components of protons at the higher densities, so that ”nucleon” star, rather than neutron star, then becomes the most appropriate name. A recent compilation by Steve Thorsett quoted by Brown \[37\] shows that well-measured neutron star masses are all less than $1.5 M_\odot$.

On the other hand, within the nuclear many-body framework, Jiang et al. \[42\] have shown that the inclusion of ring diagrams in the DBHF calculations softens the equation of state of symmetric nuclear matter. It would be of interest to extend this to asymmetric nuclear matter, and to see to what extent this improvement in the nuclear many-body approach can improve the agreement with the astrophysics observations.
5 Summary

In summary, we studied the properties of asymmetric nuclear matter in the formalism of the Dirac-Brueckner approach with the Bonn one-boson-exchange nucleon-nucleon interaction. The symmetry energy coefficient at the saturation density obtained in this work is about 30 MeV. This is in good agreement with the empirical value of about 34±4 MeV, and in agreement with other approaches such as the BHF and the variational calculations. The symmetry energy in our study was found to increase almost linearly with the density and agrees with the linear parameterization of Prakash, Ainsworth and Lattimer. At higher densities, the symmetry energy in our calculation is considerably larger than those in the BHF and variational calculations. The difference can be understood as coming from the both the relativistic effects in the ‘kinetic energy’ contribution, and a strong ρ-meson coupling in the Bonn potential that increases the ‘potential energy’ contribution, to the symmetry energy.

We have also applied the resulting equation of state of neutron-rich matter to calculate the maximum mass of nucleon star, and we find it to be about 2.1\(M_\odot\). The corresponding radius and central density are \(R = 10.8\) km and \(\rho_c = 1.08\) fm\(^{-3}\), respectively. This maximum mass is substantially greater than that measured in any neutron stars. It would be brought down considerably if kaon condensation were included in the EOS. We believe our maximum mass of 2.1 \(M_\odot\), without inclusion of kaon condensation, to be the most direct determination of this important parameter, since we begin from a microscopic two-body interaction and use the best presently available technology to calculate the mass.

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Figure 1: Left panel: neutron and proton scalar and vector potentials as a function of asymmetry parameter, for three densities. Right panel: equation of state of nuclear matter for a number of asymmetry parameters.
Figure 2: Left panel: energy difference $\Delta E$ as a function of $\alpha^2$ for several densities with $u = \rho/\rho_0$ and $\rho_0 = 0.166$ fm$^{-3}$. Right panel: density dependence of symmetry parameters.
Figure 3: Left panel: comparison of our results with those of Refs. [25] and [26]. Right panel: comparisons of our results with phenomenological parameterizations of Prakash, Ainsworth and Lattimer [3].
Figure 4: Left panel: proton fraction and $\Delta E$. Right panel: nucleon star properties.