Polarization multistability of cavity polaritons

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New effects of polarization multistability and polarization hysteresis in a coherently driven polariton condensate in a semiconductor microcavity are predicted and theoretically analyzed. The multistability arises due to polarization-dependent polariton-polariton interactions and can be revealed in polarization resolved photoluminescence experiments. The pumping power required to observe this effect is of orders of magnitude lower than the characteristic pumping power in conventional bistable optical systems.

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Introduction.—Cavity polaritons are elementary excitations of semiconductor microcavities with extremely small effective mass \( m^{*} \) ranging from \( 10^{-4} \) to \( 10^{-5} \) of the free electron mass \( m \).

In the low density limit they behave as weakly interacting bosons and their Bose-Einstein condensation (BEC) has been recently claimed \[2\]. Under resonant excitation two main nonlinear mechanisms have been identified, the polariton parametric scattering \[3, 4, 5, 6, 7, 8\] and bistability of the polariton system \[9, 10, 11, 12, 13\]. These two nonlinear mechanisms often coexist, being of the same origin \[11, 12\].

Another important peculiarity of the cavity polaritons is related to their polarization or pseudospin degree of freedom. Polaritons have two possible spin projections on the structure growth axis, \( \pm 1 \), corresponding to the right and left circular polarizations of the counterpart photons. In case of non-zero in-plane wave-vector \( k \neq 0 \) these two components are mixed by TE-TM splitting \[14\], but we will be interested in \( k = 0 \) case. The mixing of the \( k = 0 \) states appears due to the polariton-polariton interaction, which depends on the spin orientation. Namely, the interaction of polaritons in triplet configuration (parallel spin projections on the structure growth axis) is different from that of polaritons in singlet configuration (antiparallel spin projections). The spin-dependent polariton coupling strongly affects the predicted superfluid properties of the polariton system \[12, 16\] and leads to remarkable nonlinear effects in polariton spin relaxation, such as self-induced Larmor precession and inversion of the linear polarization during the scattering act \[17\].

In this Letter we show that the interplay between the nonlinearity caused by the polariton-polariton interactions and the polarization dependence of these interactions results in a remarkable multistability of a driven polariton condensate, contrary to the usual optical bistability in the spin-less nonlinear case. We consider the ground state \( (k = 0) \) polariton mode having a finite lifetime which is coherently pumped at normal incidence to the microcavity plane by a \( cw \) laser of variable intensity, frequency, and polarization. We calculate the resulting intensity and polarization of the polariton field and show that, for a given polarization of the pump and depending on the history of the pumping process, the polariton polarization can, in general, take three different values. For instance, a linearly polarized laser can result in a strongly right-circularly, strongly left-circularly, or in a linearly polarized polariton state. The whole system is therefore not simply bistable but tri- or even multistable. It is worth noting that this conversion from linear to circular polarization arises without any TE-TM splitting \[18\] and represents a new nonlinear effect.

Qualitatively, the multistability can be understood as follows. Assume that the cavity is illuminated by a laser light at normal incidence at the frequency above the bottom of the lowest polariton branch (LPB). At low pumping, the pump is not in resonance with the polariton eigenstate, so that the population of the driven mode remains low. At higher pumping, polariton-polariton interactions lead to the blue-shift of the LPB, so that it approaches the pump laser frequency. At resonance, the population jumps up abruptly. If the pumping power is then decreased, the population of the polariton mode jumps down back, but at a lower threshold. As a result the typical S-shape dependence of the intra-cavity field on the pumping intensity appears as Fig. 1 shows, which means the formation of a hysteresis cycle. The additional polarization degree of freedom makes this picture much more complex and rich. At elliptically polarized pumping the hysteresis for right- and left-circularly polarized
In the quasi-stationary case can be written as (we set driven spinor Gross-Pitaevskii equation [15, 19], which leads to the formation of the linearly polarized condensate, where \( \tau \) is the polariton lifetime, \( \alpha \) is the circular polarization intensity.

Coherent polarization evolution.---We consider a semiconductor microcavity in the strong coupling regime pumped by a laser light perpendicular to the cavity plane \((k = 0)\), with spatially uniform and slowly changing with time intensity \( p \) and frequency \( \omega \) near the bottom of the LPB, where \( \sigma = \pm 1 \) is the circular polarization index. We neglect the non-parabolicity of the polariton dispersion, and the wave vector dependence of exciton and photon fractions. These approximations are well justified since the effects we discuss are taking place in a narrow energy and wave vector range close to the polariton ground state at \( k = 0 \) with the eigenfrequency \( \omega_0 \).

The \( k = 0 \) polariton wave function \( \psi_\sigma \) satisfies the driven spinor Gross-Pitaevskii equation [15, 19], which in the quasi-stationary case can be written as (we set \( \hbar = 1 \))

\[
\left[ \frac{\omega - \omega_0}{\tau} - \frac{i}{\tau} + \alpha_1 |\psi_\sigma|^2 + \alpha_2 |\psi_{-\sigma}|^2 \right] \psi_\sigma + \frac{p_\sigma}{\sqrt{4\tau}} = 0, \tag{1}
\]

where \( \tau \) is the polariton lifetime, \( \alpha_1(2) \) is the matrix element of polariton-polariton interaction in the triplet(singlet) configuration, respectively. The exchange interaction is suppressed for polaritons in the singlet configuration since it involves energetically split-off dark excitons as intermediate states. As a result, there is an attraction between polaritons of opposite sign, \( \alpha_2 < 0 \), and polariton-polariton interaction is strongly anisotropic \( |\alpha_2| \ll |\alpha_1| \) [23].

At thermodynamic equilibrium the polariton BEC leads to the formation of the linearly polarized condensate, which minimizes the free energy of the system [13]. In the case of \( cw \) driven system the situation is qualitatively different. The polariton system is intrinsically out of equilibrium and its state is now driven by the pump. The parameters of the driven mode, however, are not uniquely defined. E.g., with a linearly polarized pump the polariton condensate can change its polarization either to the left- or to the right-circular, increasing this way the blue-shift and entering into resonance with the pump. The left and right circularly polarized condensates can appear in this case with equal probabilities.

Equation (1) can be used to relate the occupation of the condensate \( N = |\psi_{+1}|^2 + |\psi_{-1}|^2 \) and the degree of its circular polarization \( \rho_c = (|\psi_{+1}|^2 - |\psi_{-1}|^2)/N \) with the pump intensity \( I = |p_{+1}|^2 + |p_{-1}|^2 \) and the pump polarization degree \( \rho_p = (|p_{+1}|^2 - |p_{-1}|^2)/I \). Namely,

\[
\frac{I}{4\tau} = |\Omega|^2 + \tau^{-2} + (\alpha_1 - \alpha_2)(1 - \rho_c^2)\Omega N \\
+ (1/4)(\alpha_1 - \alpha_2)^2(1 - \rho_c^2)N^2]N, \tag{2}
\]

\[
\rho_p I/4\tau = |\Omega|^2 + \tau^{-2} - (1/4)(\alpha_1 - \alpha_2)^2(1 - \rho_c^2)N^2]\rho_c N, \tag{3}
\]

where \( \Omega = \omega - \omega_0 - \alpha_1 N \)

It follows from Eqs. (2) and (3) that only for the pump with a pure circular polarization, \( \rho_p = \pm 1 \), the polarizations of the polariton system and the pump coincide. In this case the present spinor model can be reduced to the scalar model [11, 12]. For elliptical pumping the polarizations of the polariton system and of the laser are different due to the different blue shifts for right and left circular polarized components. Due to the effect of the self-induced Larmor precession [17] the polarization ellipse of the driven mode is rotated with respect to the polarization ellipse of the pump by some angle which depends on the pump intensity. Moreover, in this regime even the signs of circular polarization degrees of the polaritons and the pump can be different, as we show in the next subsection.

Multistability and hysteresis.---The polarization and occupation of the driven mode, \( \rho_c \) and \( N \), can be determined self-consistently from Eqs. (2) and (3), which can have several solutions depending on the value and polarization of the pump.

The condensate population \( |\psi_{\pm1}|^2 \) as a function the pump intensity \( |p_{\pm1}|^2 \) for the case of fully circular polarized excitation (when the polarization of the driven mode coincides with that of the pump) is shown in Fig. 4. The energy of the laser \( \omega \) is chosen above the energy of the bare polariton state, \( \omega - \omega_0 = 3 \) meV, so that the curve shows the classical S-shape. The matrix element \( \alpha_1 = 6\pi E_b a_B^2/S \), where \( a_B = 100 \) Å is the two dimensional exciton Bohr radius, \( E_b = 8 \) meV is the exciton binding energy, \( x = 1/4 \) is the squared exciton fraction, and \( S = 100 \) \( \mu m^2 \) is the laser spot area. The polariton life-time is \( \tau = 2 \) ps. These parameters are typical for

![FIG. 1: The condensate population \(|\psi_\sigma|^2\) versus pumping intensity \(|p_\sigma|^2\) for the circularly polarized pump (\( \sigma = \pm 1 \)). Dashed line shows the unstable region. \( I_1 \) and \( I_2 \) mark the intensities of two jumps corresponding to the increase and decrease of the pump, respectively. The numbers shown (scaled to the excitation area \( S \approx 100 \) \( \mu m^2 \)) are typical for cavity polariton experiments.](image-url)
a GaAlAs microcavity. We have denoted by $I_1$ the laser intensity corresponding to the bending point of the S-shape taking place with the increase of intensity, and $I_2$ is the laser intensity corresponding to the bending point in the case of the intensity decrease.

The evolution of the condensate polarization can be conveniently illustrated and understood considering the case $\alpha_2 = 0$, when Eqs. (1) for two values of $\sigma$ are simply decoupled and the two circular components evolve independently [21]. The change of the $\rho_\sigma$ with the total pump intensity is shown in Fig. 2(a) for the elliptically polarized pump with $\rho_\sigma = 0.2$, so that $\sigma^+$ component slightly exceeds $\sigma^-$ one. As one increases the intensity of the pump, the solution moves along the lower branch of the S-curve for both $\sigma^+$ and $\sigma^-$ components. However, the $\sigma^+$ component, since it dominates, reaches the threshold intensity $I_1$ first. The corresponding intensity of the polariton field jumps abruptly to the upper branch. At the same time, the intensity of the $\sigma^-$ pump has not yet reached the $I_1$ bending point. So, the jump of the total polariton density is accompanied by a jump of the circular polarization degree. If we increase further the intensity of the laser, the intensity of the $\sigma^-$ mode also reaches $I_1$. The polariton population increases again, but this now results in an abrupt decrease of the circular polarization degree of the driven mode. If we now reduce the intensity, the reversed process takes place at the pumping intensity $I_2$ so that hysteresis in both the occupation and polarization power dependencies appear.

Fig. 2(b) shows another interesting configuration, where the laser intensity $I$ is kept constant in the domain $I > I_1$ and $I_1 > I/2 > I_2$. The cyan line shows the change of the circular polarization degree of the driven mode $\rho_\sigma$ induced by the laser initially polarized $\sigma^+$ and whose polarization is progressively rotated towards the $\sigma^-$ polarization. One can observe a weak decrease of $\rho_\sigma$, which however remains quite high even when the pumping is linearly polarized. This is due to the fact that the $\sigma^+$ component remains on the upper branch of the S-curve whereas the $\sigma^-$ drops on the lower branch. Then there are two jumps of polarization corresponding to the jumps of $\sigma^+$ and $\sigma^-$ components of the polariton population, and finally the polarization becomes fully $\sigma^-$. The black line shows the evolution of $\rho_\sigma$ with the inverse change of the pump polarization. The stable points corresponding to full linear polarization of the laser are marked with crosses in this Figure. One can see that the internal polarization can be either nearly $\sigma^+$, either nearly $\sigma^-$, or fully linear. The latter case is in fact degenerate. There can be two stable driven mode occupations $N$ for the same value of the external laser intensity $I$.

Fig. 3 shows the functional dependence between $\sigma^+$ and $\sigma^-$ components of the polariton population and the intensity and polarisation of pump calculated accounting for the coupling between polaritons with opposite spins described by $\alpha_2 = -0.1\alpha_1$. This value of $\alpha_2$ corresponds to recent estimations of Ref. [20]. The circular polarization degree of the pump laser is represented by the color of the surface of solution and the intensity of the pump is on the vertical z-axis. The linear part of the polarization of the laser is kept aligned along x-direction.

The green bright areas correspond to nearly linearly polarized pumping. If one increases the intensity of the pumping laser, keeping it linearly polarized, the system follows the black solid line and black arrows shown in the figure. One can see that from the critical point at the end of the solid black line the system can jump into three possible stable points (shown by crosses). One of them corresponds to linearly polarized state and two others to nearly right- and left-circularly polarized states. The choice of the final state by the system is random and is triggered by fluctuations.
Note also that the jump of the system to the circularly polarized final state induces a red shift of the cross-circular component because of the negative sign of $\alpha_2$. The red shift drives this component out of resonance with the pump, which leads to stronger polarization of the final state. The positive feedback in polarization of the condensate would not take place for the case $\alpha_2 = 0$ where only a linearly polarized driven mode would be formed. Experimentally, we expect this effect to have a key impact on the polarization measurements performed with resonantly excited microcavities. It will result in a random sign of the observed polariton polarization changing from one experiment to another.

It should be noted that polarization multistability and chaos in nonlinear optical systems have been studied for more than 30 years (see, e.g., 22). A number of intriguing nonlinear effects have been predicted and observed in different systems, including four-wave mixing in anisotropic crystals 23, 24, magnetic cavities 25 and VCSELs 26. However, in the systems investigated previously extremely high powers (10 MW/cm$^2$ 22) are required for observation of polarization multistability effects linked to optical nonlinearities. The advantage of the microcavities is that in the strong coupling regime the nonlinear threshold corresponds to much lower pumping powers (e.g. 650 W/cm$^2$ 27) which really opens the way to practical realization of new applications like data encryption and communication 28.

Conclusions.—We have analyzed the polarization of the spinor polariton system pumped by a cw laser of variable polarization at $k = 0$ by solving the polarization dependent Gross-Pitaevskii equation. We have shown that for a given polarization of the pumping laser the polariton polarization can take three different values. For instance at linear pumping the resulting polariton state can be right circularly, left circularly or linearly polarized. In realistic cases, the driven mode polarization is random and is triggered by the fluctuations of the external laser. The multistability arises from the interplay between the anisotropy of the polariton-polariton interaction and the bistable behavior of cavity polaritons.

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