Precision Diboson Observables for the LHC

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based on arXiv:1510.08451 (accepted by JHEP)
with Chris Frye, Jakub Scholtz, Matt Strassler
Why precision? Why dibosons?

- entering the era of precision Higgs physics
- …but not all EW physics easy to tie to Higgs measurements
  - BSM might not be easy to connect either
    - whither 750 GeV?
- non-Higgs precision measurements at the LHC are the next frontier
Why precision? Why dibosons?

- entering the era of precision Higgs physics
- ...but not all EW physics easy to tie to Higgs measurements
  - BSM might not be easy to connect either
   - whither 750 GeV?
- non-Higgs precision measurements at the LHC are the next frontier
- large part of this effort hinges on precision calculations
  - many talks here: D. Heymes, M. Worek, A. Huss, F. Dreyer, S. Kallweit ...
- but clever choices of observables can also help
  - cf. hadronic, Higgs decay ratios
- diboson rates related by $SU(2)_L \times U(1)_Y$ relations
  - broken at low energies, but restored above the EW scale
  - is there any way to take advantage of this?
Plan

• **Leading order**
  ▶ Structure of partonic cross sections
  ▶ Ratio observables

• **Next-to-leading order**
  ▶ NLO corrections for $\sigma(\gamma\gamma)$, $\sigma(Z\gamma)$, $\sigma(ZZ)$
  ▶ Photon isolation and gluon fusion

• **Experimental benefits**
(Nearly) massless gauge bosons at high $\sqrt{s}$

expand around unbroken $SU(2)_L \times U(1)_Y$ (gauge bosons: $w^a, b$)
- corrections at $(m_W, Z/E)^2$

\[ bb_1 = bb : \quad |bb\rangle \]

\[ wb_3 = wb : \quad \{|w^+ b\rangle, |w^3 b\rangle, |w^- b\rangle\} \]

\[ ww_1 : \quad |w^- w^+\rangle + |w^- w^+\rangle - |w^3 w^3\rangle \]

\[ ww_3 : \quad \{|w^+ w^3\rangle - |w^3 w^+\rangle, |w^+ w^-\rangle - |w^- w^+\rangle, |w^3 w^-\rangle - |w^- w^3\rangle\} \]
(Nearly) massless gauge bosons at high $\sqrt{s}$

expand around unbroken $SU(2)_L \times U(1)_Y$ (gauge bosons: $w^a$, $b$)

- corrections at $(m_{W,Z}/E)^2$

Schematically define coupling-stripped amplitudes:

\[ a_1 = (2), \quad a_3 = (1) + (2) \]

Matrix elements of interest:

\[ a_1 \sim M(bb) \quad \sim M(ww_1) \propto t-, u- \text{-channel} \]
\[ a_3 \sim M(ww_3) \propto s-, t-, u- \text{-channel} \]
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expand around unbroken $SU(2)_L \times U(1)_Y$ (gauge bosons: $w^a, b$)

- corrections at $(m_W, Z/E)^2$
- need $\phi^a$ goldstones to unitarize

Schematically define coupling-stripped amplitudes:

\[ a_1 = (2), \quad a_3 = (1) + (2), \quad a_L = (3) \]

Matrix elements of interest:

\[ a_1 \sim M(bb) \sim M(wb) \sim M(ww_1) \propto t-, u\text{-channel} \]
\[ a_3 \sim M(ww_3) \propto s-, t-, u\text{-channel} \]
\[ a_L \sim M(\phi\phi) \propto s\text{-channel} \]
Amplitudes at high energies

boson statistics fix amplitudes under $\hat{t} \leftrightarrow \hat{u}$

- $a_1, a_L$ symmetric
- $a_3$ antisymmetric

$\implies a_3$ vanishes at threshold, $\text{Re}(a_1^\dagger a_3)$ vanishes in asymmetries

\[
|a_1|^2 = \frac{\hat{t}}{\hat{u}} + \frac{\hat{u}}{\hat{t}}
\]

\[
2\text{Re}(a_1^\dagger a_3) = \frac{\hat{t} - \hat{u}}{2\hat{s}} + \frac{1}{4} \left( \frac{\hat{t}}{\hat{u}} - \frac{\hat{u}}{\hat{t}} \right)
\]

\[
|a_3|^2 = \frac{\hat{u}t}{4\hat{s}^2} - \frac{1}{8} + \frac{1}{32} \left( \frac{\hat{t}}{\hat{u}} + \frac{\hat{u}}{\hat{t}} \right)
\]

\[
|a_L|^2 = \frac{\hat{u}t}{4\hat{s}^2}
\]
Amplitudes
with mass corrections

boson statistics fix amplitudes under $\hat{t} \leftrightarrow \hat{u}$

- $a_1, a_L$ symmetric
- $a_3$ antisymmetric

$\implies a_3$ vanishes at threshold, $\text{Re}(a_1^\dagger a_3)$ vanishes in asymmetries

$$|A_1|^2 = (\hat{t}\hat{u} - m_1^2 m_2^2) \left( \frac{1}{\hat{t}^2} + \frac{1}{\hat{u}^2} \right) + \frac{2\hat{s}(m_1^2 + m_2^2)}{\hat{t}\hat{u}}$$

$$\text{Re}(A_1^\dagger A_3) = P_s \left( \hat{t}\hat{u} - m_1^2 m_2^2 - \hat{s}(m_1^2 + m_2^2) \right) \left( \frac{1}{\hat{u}} - \frac{1}{\hat{t}} \right)$$

$$+ \frac{1}{4} (\hat{t}\hat{u} - m_1^2 m_2^2) \left( \frac{1}{\hat{u}^2} - \frac{1}{\hat{t}^2} \right)$$

$$|A_3|^2 = \ldots$$

$$|A_L|^2 = P_s^2 \left( \hat{t}\hat{u} - m_1^2 m_2^2 + 2\hat{s}(m_1^2 + m_2^2) \right)$$

finite $m$ effects have uniform structure
\gamma \gamma, Z \gamma, ZZ at LO

\[ \frac{d\hat{\sigma}_{q\bar{q} \to V_1^0 V_2^0}}{d\hat{t}} = \frac{C_{q\bar{q} \to V_1^0 V_2^0}}{\hat{s}^2} |A_1|^2 \]

\[ \gamma = c_W b + s_W w^3, \]
\[ Z = c_W w^3 - s_W b \]

\[ \gamma \gamma, Z\gamma, ZZ \propto bb, wb, ww_1 \]

where

\[ C_{q\bar{q} \to \gamma \gamma} = \frac{1}{2} \frac{\pi \alpha_2^2 s_W^4}{N_c} 2Q^4 \]
\[ C_{q\bar{q} \to Z\gamma} = \frac{\pi \alpha_2^2 s_W^2 c_W^2}{N_c} (L^2 Q^2 + R^2 Q^2) \]
\[ C_{q\bar{q} \to ZZ} = \frac{1}{2} \frac{\pi \alpha_2^2 c_W^4}{N_c} (L^4 + R^4) \]
\[ \gamma, Z\gamma, ZZ \text{ at LO} \]

\[
\frac{d\hat{\sigma}_{q\bar{q}\to V_0^1 V_0^2}}{d\hat{t}} = \frac{C_{q\bar{q}\to V_0^1 V_0^2}}{\hat{s}^2} |A_1|^2
\]

\[
\gamma = c_W b + s_W w^3,
\]

\[
Z = c_W w^3 - s_W b
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\[
C_{q\bar{q}\to ZZ} = \frac{1}{2} \frac{\pi \alpha_2^2 c_W^4}{N_c} (L^4 + R^4)
\]

\[L = T_3 - Y_L t_W^2, \quad R = -Y_R t_W^2\]
\(\gamma\gamma, Z\gamma, ZZ\) at LO

\[
\frac{d\hat{\sigma}_{\bar{q}q \to V_1^0 V_2^0}}{d\hat{t}} = \frac{C_{\bar{q}q \to V_1^0 V_2^0}}{\hat{s}^2} |A_1|^2
\]

\[
\frac{d\sigma_{pp \to V_1^0 V_2^0}}{d\tilde{m}_T} = \frac{1}{s} \int \frac{d\hat{s}}{\hat{s}^2} \left| \frac{d\hat{t}}{d\tilde{m}_T} \right| |A_1|^2 \sum_q C_{\bar{q}q \to V_1^0 V_2^0} \int dy f_q f_{\bar{q}} \quad \mathcal{L}_{12}(\hat{s})
\]

\[
\gamma = c_w b + s_w w^3,
\]

\[
Z = c_w w^3 - s_w b
\]

where

\[
C_{\bar{q}q \to \gamma\gamma} = \frac{1}{2} \frac{\pi\alpha_2^2 s_W^4}{N_c} 2Q^4
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C_{\bar{q}q \to Z\gamma} = \frac{\pi\alpha_2^2 s_W^2 c_W^2}{N_c} (L^2 Q^2 + R^2 Q^2)
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\[
L = T_3 - Y_L t_W^2, \quad R = -Y_R t_W^2
\]
$\gamma\gamma, Z\gamma, ZZ$ at LO

\[
\frac{d\hat{\sigma}_{qq\rightarrow V_0^1 V_0^2}}{d\hat{t}} = \frac{C_{qq\rightarrow V_0^1 V_0^2}}{\hat{s}^2} |A_1|^2
\]

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\]

where

| $V_1^0 V_2^0$ | $\mathcal{L}_{12}^u \cdot 10^5$ | $\mathcal{L}_{12}^d \cdot 10^5$ |
|-----------------|-----------------|-----------------|
| $\gamma\gamma$  | 1.2             | 0.07            |
| $Z\gamma$       | 2.2             | 0.7             |
| $ZZ$            | 1.6             | 3.3             |

$\gamma = c_W b + s_W w^3$,  
$Z = c_W w^3 - s_W b$

$\gamma\gamma, Z\gamma, ZZ \propto bb, wb, wW_1$

\[R_1 \text{ ratios @ LO}\]
\( \gamma\gamma, Z\gamma, ZZ \) at LO

\[
\frac{d\hat{\sigma}_{qq\rightarrow V_0^1V_0^2}}{d\hat{t}} = \frac{C_{qq\rightarrow V_0^1V_0^2}}{\hat{s}^2} |A_1|^2
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\]

\[
\mathcal{L}_{12} (s) \quad \gamma\gamma, Z\gamma, ZZ \propto bb, wb, ww_1
\]

where

| \( V_1^0V_2^0 \) | \( \mathcal{L}_{12}^u \cdot 10^5 \) | \( \mathcal{L}_{12}^d \cdot 10^5 \) |
|---|---|---|
| \( \gamma\gamma \) | 1.2 | 0.07 |
| \( Z\gamma \) | 2.2 | 0.7 |
| \( ZZ \) | 1.6 | 3.3 |

PDFs mostly cancel – \( uu \) dominates
$W^{\pm \gamma}, W^{\pm Z}$ at LO

$W^{\pm \gamma}$ and $W^{\pm Z}$ built from $wb$ and $ww_3 \implies$ Need $A_1$ and $A_3$.

Need $A_L$ for $\phi^\pm \phi^3$ component of $W^{\pm Z}$.

$$
\frac{d\hat{\sigma}_{ud \rightarrow W^{\pm \gamma}}}{d\hat{t}} = \frac{\pi |V_{ud}|^2 \alpha_2^2 s_W^2}{N_c \hat{s}^2} \left( \frac{Y_L^2}{2} |A_1|^2 \pm 2Y_L \text{Re}(A_1^\dagger A_3) + |A_3|^2 \right)
$$

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$$
$W^{\pm\gamma}, W^{\pm Z}$ at LO

$W^{\pm\gamma}$ and $W^{\pm Z}$ built from $w b$ and $w w_3 \implies$ Need $A_1$ and $A_3$.

Need $A_L$ for $\phi^\pm \phi^3$ component of $W^{\pm Z}$.

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\]

Numerically small

$\implies W\gamma/WZ \sim \tan^2 \theta_W$
\( W^{\pm\gamma}, W^{\pm Z} \) at LO

\( W^{\pm\gamma} \) and \( W^{\pm Z} \) built from \( wb \) and \( ww_3 \) \( \implies \) Need \( A_1 \) and \( A_3 \).

Need \( A_L \) for \( \phi^{\pm} \phi^3 \) component of \( W^{\pm Z} \).

\[
\frac{d\hat{\sigma}_{ud\rightarrow W^{\pm\gamma}}}{dt} = \frac{\pi |V_{ud}|^2 \alpha^2 s^2_W}{N_c s^2} \left( \frac{Y_L^2}{2} |A_1|^2 \pm 2 Y_L \text{Re}(A_1^\dagger A_3) + |A_3|^2 \right)
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\frac{d\hat{\sigma}_{ud\rightarrow W^{\pm Z}}}{dt} = \frac{\pi |V_{ud}|^2 \alpha^2 s^2_W}{N_c s^2} \left( \frac{s_W^4 Y_L^2}{2c_W^2} |A_1|^2 \pm 2s_W^2 Y_L \text{Re}(A_1^\dagger A_3) + 4 c_W^2 |A_3|^2 + \frac{1}{2} |A_L|^2 \right)
\]

Numerically small
\( \implies W\gamma/WZ \sim \tan^2 \theta_W \)
Radiation zero at threshold
\( \implies W\gamma/WZ \sim 0.19 \)
$W^{\pm \gamma}, W^{\pm Z}$

$CP$ symmetry controls $W^{-}V^{0}$ rates:

$$d\hat{\sigma}_S(\bar{u}\bar{d} \rightarrow W^+V^0) = d\hat{\sigma}_S(\bar{d}\bar{u} \rightarrow W^-V^0)$$

$$d\hat{\sigma}_A(\bar{u}\bar{d} \rightarrow W^+V^0) = -d\hat{\sigma}_A(\bar{d}\bar{u} \rightarrow W^-V^0)$$

![Graphs showing C2 and D2 ratios at LO as a function of m_{VV} [GeV]](image-url)
\( W^+ W^- \) built from \( \omega \omega_1, \omega \omega_3, \phi^+ \phi^- \)

Very similar to \( W^\pm \gamma \)

\[
\frac{\sigma_A(W^+ W^-)}{a \sigma_A(W^+ \gamma) + b \sigma_A(W^- \gamma)} \sim \frac{\mathcal{L}_{u\bar{u}}^A - \mathcal{L}_{d\bar{d}}^A}{4 |V_{ud}|^2 s_w^2 Y_L (a L_{ud}^A - b L_{d\bar{u}}^A)}
\]

\[m_{\text{vv}} \text{[GeV]}\]

\( R_3 \) ratios @ LO

\( A_3 \) ratios @ LO
Beyond leading order

- QCD cancellations?
  - how large are the shifts?
  - $SU(2)_L \times U(1)_Y$ relations help – where do they fail?
  - residual uncertainties?

- EW corrections?

- Big issue: the radiation zero
  - LO relations may receive large corrections where present

- Start with $V_1^0 V_2^0$
  - No radiation zero
  - Fully reconstructed (only $Z \rightarrow \ell^+ \ell^-$ here)
  - Good statistics ($ZZ$ tougher)
NLO corrections to diboson production

EW processes at LHC typically have large $O(\alpha_s)$ corrections

$qg \rightarrow V_1 V_2 q$ appears at $O(\alpha_s)$, and $f_g \gg f_{\bar{q}}$

Large uncertainties if distributions are “effectively LO” somewhere in PS

$$\bar{m}_T = \frac{1}{2} (m_{T1} + m_{T2}) = \text{min. } E \text{ at } \theta_{CM} = \pi/2$$

Radiation can never reduce this variable

![Graph showing the ratio of NLO to LO cross-sections versus $m_T$ for different processes: ZZ, Z\gamma, and \gamma\gamma. The graph illustrates the theoretical predictions and their uncertainties.]
NLO corrections to diboson production

EW processes at LHC typically have large $O(\alpha_s)$ corrections

$qg \to V_1 V_2 q$ appears at $O(\alpha_s)$, and $f_g \gg f_{\bar{q}}$

Choose cuts to avoid generating large logs anywhere in phase space

$$H_T < \frac{1}{2} p_{T,\text{min}}^V, \quad p_{T,\text{min}}^V > \frac{1}{2} p_{T,\text{max}}^V$$

Fixed order calculation reliable
Complications in QCD corrects divergence in $qg$ process when $q\gamma$ are collinear cut region out of phase space, or absorb into fragmentation function large logarithm after regulation $\implies$ need tight $\gamma$ isolation
Complications in QCD corrects divergence in $qg$ process when $q\gamma$ are collinear.

cut region out of phase space, or absorb into fragmentation function

large logarithm after regulation $\implies$ need tight $\gamma$ isolation

$gg \rightarrow V_1^0 V_2^0$

$\implies$ formally NNLO, numerically large

Both effects decrease in importance at high energy
Uncertainties: $\sigma$ vs. $R_{1i}$

PDFs and scale choices

both PDF and scale uncertainties at the 1–2% level in ratios
Predicted ratios
$Z\gamma/\gamma\gamma$ at 300 fb$^{-1}$

\[
\sigma_S(Z\gamma)/\sigma_S(\gamma\gamma)
\]

$\Delta R/R$

$m_T$ [GeV]

$\sigma_S(Z\gamma)/\sigma_S(\gamma\gamma)$
Predicted ratios

The other ratios at 3000 fb$^{-1}$

\[ \frac{\sigma_{S(ZZ)}}{\sigma_{S(\gamma\gamma)}} \]

\[ \frac{\sigma_{S(ZZ)}}{\sigma_{S(Z\gamma)}} \]
$SU(2)_L \times U(1)_Y$ structure of diboson rates at LO suggests points to studying certain observables.

Ratio observables based on this structure:
- low uncertainties, small QCD corrections
- candidates for (even) high(er)-precision calculation

Certainly useful for SM studies.

Need to see if sensitivity to new physics improved
- reliable characterization of new resonances?
- increased sensitivity to non-resonant effects?
Ratios for precision! Ratios for understanding!

\[ \frac{\sigma_{S(Z\gamma)}}{\sigma_{S(\gamma\gamma)}} \]

- Stat. Uncertainty (300 fb\(^{-1}\))
- NLO theory
- LO theory
- \(\alpha_{\text{QED}}\)
- NLL EW
- Lepton cuts

\(\Delta R/R\) vs. \(m_T\) [GeV]
Backup slides
Variable bin widths

Problem: very few $Z$ decays to leptons

| $V_1 V_2$ | $N_f + N_b$ | $N_f - N_b$ |
|-----------|-------------|-------------|
| $\gamma \gamma$ | 12 000 | 0 |
| $Z \gamma$ | 2000 | 0 |
| $ZZ$ | 220 | 0 |
| $W^+ \gamma$ | 3300 | $-500$ |
| $W^- \gamma$ | 2100 | 220 |
| $W^+ Z$ | 790 | 33 |
| $W^- Z$ | 520 | $-16$ |
| $W^- W^+$ | 9500 | $-430$ |

Choose bin widths for 5% statistical uncertainty.

$R_{1a} = \sigma_S(Z\gamma)/\sigma_S(\gamma\gamma)$ not statistics limited with 300 fb$^{-1}$

Others will have to wait until 3000 fb$^{-1}$
Staircase isolation

“Practical” smooth-come isolation:

- stairs of width $\Delta R = 0.1$
- heights match smooth cone at midpoint
- never take heights below 25 GeV (pileup/resolution)

Minimize sensitivity to fragmentation functions, protect against large log at small cutoff. Experimentally viable.
## Uncertainty Budget

| Effect                  | $R_{1a}$  | $R_{1b}$  | $R_{1c}$  | Comments                                      |
|-------------------------|-----------|-----------|-----------|-----------------------------------------------|
| $qq \rightarrow VVqq$  | 2–3%      | 3–3.5%    | 1.5–2.5%  | extrapolating $p_{T,\text{min}}' \rightarrow 0$ |
| $\mu_R, \mu_F$ (gg)    | 0.5–1%    | 1%        | 1–2%      | uses NLO $gg \rightarrow \gamma\gamma$        |
| $\mu_R, \mu_F$ (NLO)   | 0.5–1%    | 1.5–2.5%  | 1–1.5%    | varied independently                           |
| PDF                     | 0.5%      | 1–1.5%    | 0.5–1%    | MSTW2008 using MCFM                           |
| $\gamma$ isolation     | < 0.1%    | < 0.1%    | < 0.1%    | Uncertainty in frag. fun.                     |
| $\alpha_{\text{QED}}$  | 7%        | 14%       | 7%        | Fully correlated                              |
| EW (LL)                | +2%       | +3%       | +2%       | EFT scale uncertainty                         |
|                         | −1%       | −1%       | −1%       |                                                |