The torsion cosmology in Kaluza-Klein theory

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Abstract

We have studied the torsion cosmology model in Kaluza-Klein theory. We considered two simple models in which the torsion vectors are $A_{\mu} = (\alpha, 0, 0, 0)$ and $A_{\mu} = a(t)^2(0, \beta, \beta, \beta)$, respectively. For the first model, the accelerating expansion of the Universe can be not explained without dark energy which is similar to that in the standard cosmology. But for the second model, we find that without dark energy the effect of torsion can give rise to the accelerating expansion of the universe and the alleviation of the well-known age problem of the three old objects for appropriated value of the model parameter $\beta$. These outstanding features of the second torsion cosmology model have been supported by the Type Ia supernovae (SNIa) data.

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I. INTRODUCTION

Many observations [1, 2, 3] have been strongly confirmed that the expansion of our present Universe is accelerating rather than slowing down. This late time cosmic acceleration can not be explained by the four known fundamental interactions in the standard models, which is the greatest challenge today in the modern physics. Within the framework of Einstein’s general relativity, an exotic component with negative pressure called dark energy is invoked to explain this observed phenomena. The simple candidate of dark energy which is consistent with current observations is the cosmological constant [4], which is a term that can be added to Einstein’s equations. This term acts like a perfect fluid with an equation of state $\omega = -1$, and the energy density is associated with quantum vacuum. However, it is very difficult to understand in the modern field theory since the vacuum energy density is far below the value predicted by any sensible quantum field theory. Moreover, it has also been plagued by the so-called coincidence problem. Thus, a lot of the dynamical scalar fields, such as quintessence [5], k-essence [6], phantom [7] and quintom field [8, 9], have been put forth as an alternative of dark energy. However, so far, the nature of dark energy is still unclear.

On the other hand, it is argued that in Einstein theories gravity is not well understood and an important ingredient is missing which may account for such observed phenomena. One of such an ingredient is the torsion which is vanished in Einstein theories. It is widely believed that the presence of torsion will change the character of the gravitational interaction because that in this case the gravitational field is described not only by spacetime metric, but also by the torsion field. Recently, a lot of investigations have indicated that the torsion plays the important role in the modern physics [10]. Cosmological models with torsion were pioneered by Kopczyński [11] in the last century. Thereafter, the bouncing cosmological model with torsion has been proposed by Kerlick [12] in which the torsion was imagined as playing role only at high densities in the early universe. The effects of torsion field on the inflation in cosmology has been investigated in [13, 14]. Recently some authors have begun to study torsion as a possible reason of the accelerating universe [15]. The study of dynamics and the statefinder diagnostic in torsion cosmology have been studied in [16, 17].

Recently, the cosmology in Kaluza-Klein theory [18, 19, 20] has attracted a considerable attention [21, 22, 23, 24, 25, 26, 27, 28, 29]. The Kaluza-Klein theory was introduced first by Kaluza [18] to unify Maxwell’s theory of electromagnetic and Einstein’s gravity theory by adding the fifth dimension. In 1926, Klein [19, 20] proposed that the fifth dimension is compactified by being curled up in a circle of very small radius, so that
the extra dimension is not observable except on very high energy scales. Due to its potential function to unify the fundamental interactions, Kaluza-Klein theory has been regarded as a candidate of fundamental theory. Moreover, the fantastic idea of extra dimensions promotes various higher dimensional theories, including the well-known string theory \[30\]. Thus Kaluza-Klein theory has been revived in recent years in the modern physics, such as in supergravity \[31\] and superstring theories \[32\]. Many of the papers currently published in the context of Kaluza-Klein theory deal with cosmology. Cho \[25, 26\] has studied the 4+1 dimensional Kaluza-Klein cosmology and find that it can solve some of the problems in the standard big bang model (including the horizon problem and the flatness problem) in a very natural way. Wesson et al \[27\] have investigated the cosmological constant problem in Kaluza-Klein cosmology. Li \[28\] has considered the inflation in Kaluza-Klein theory and obtain a relation between the fine-structure constant and the cosmological constant. Some authors argued that Kaluza-Klein theory can be effective in accounting for the dark constituent of the universe \[29\]. In Ref. \[33\], Shankar et al incorporates the torsion into the five dimensional Kaluza-Klein theory \[18, 19, 20\] and find that there exists some non-vacuum solutions. Moreover, they discuss furtherly the implications of Kaluza-Klein theory with torsion fields in cosmology and obtain that the presence of torsion fields changes the dynamical equations for a flat Friedmann-Robertson-Walker spacetime. This means that the torsion fields will play the important roles in the evolution of the Universe. The motivation of this paper is to study concretely the properties of the torsion cosmology in the Kaluza-Klein theory and then perform the constraints on this cosmology model by using the Type Ia supernovae (SNIa) data.

The remainder of this paper is organized as follows: in the next section we review briefly the torsion cosmology in Kaluza-Klein theory. And then we study the cosmic expansion history and check the age problem of the high redshift objects for the models. In Sec.III, we perform the constraints on this cosmology model by using the SNIa data and present our results. Finally in the last section we will include our conclusions.

II. THE TORSION COSMOLOGY IN KALUZA-KLEIN THEORY

In this section, we first review briefly the torsion cosmology in Kaluza-Klein theory which proposed by Shankar et al \[33\], where the torsion tensor \( S_{ab}^c \) is defined as the antisymmetric part of connection in a coordinate basis

\[
S_{jk}^i = \Gamma^i_{jk} - \Gamma^i_{kj},
\]
where $\Gamma^i_{jk}$ is the affine connection with torsion and has a form
\[
\Gamma^i_{jk} = \hat{\Gamma}^i_{jk} + K^i_{jk}. \tag{2}
\]
The $\hat{\Gamma}^i_{jk}$ is the usual Christoffel symbol and the $K^i_{kj}$ is the well-known contorsion tensor
\[
K^i_{jk} = \frac{1}{2} [S^i_{kj} + S^i_{k}j + S^i_{j}k]. \tag{3}
\]
Using some constraints and conditions, Shankar et al. \cite{33} found the five dimensional Einstein equations in the Kaluza-Klein theory
\[
G_{ij} = \tilde{R}_{ij} + \frac{1}{2} g_{ij} \tilde{R} = \tilde{T}_{ij}, \tag{4}
\]
can be split into
\[
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \frac{1}{2} A_\mu A_\nu \Phi^2 R = T_{\mu\nu}, \\
-\frac{1}{2} A_\mu \Phi^2 R = T_{\mu5}, \\
-\frac{1}{2} \Phi^2 R = T_{55}. \tag{5}
\]
Here $g_{ij}, \tilde{R}_{ij}$ and $\tilde{T}_{ij}$ are the five dimensional metric, Ricci curvature tensor and the energy-momentum tensor in Kaluza-Klein theory respectively. The $g_{\mu\nu}, R_{\mu\nu}$ and $T_{\mu\nu}$ are corresponding tensors in the four dimensional hypersurface. $A_\mu$ and $\Phi$ are vector and scalar torsion fields. For the vacuum solution with $\tilde{T}_{ij} = 0$ in 5D spacetime, one can obtain from Eq. (5) that the Ricci scalar $R = 0$ and $R_{\mu\nu} = 0$, which means that the 4D metric $g_{\mu\nu}$ obtained from vacuum solutions in 5D spacetime are identical to the vacuum solutions in the torsion free 4D spacetime \cite{33}. The converse is also true. Therefore, for the vacuum solutions, we can not distinguish the 4D spacetime from the 4D hypersurface within the 5D spacetime with torsion \cite{33}. In other word, the effects of torsion can not be detected in this case. However, for the non-vacuum solution, one can construct new 4D metric $g_{\mu\nu}$ which do not exist in torsion free 4D spacetime \cite{33}. Comparing with the solutions in Einstein’s theory, we can acquire some information about torsion.

In order to develop our cosmological considerations, let us take into account a flat Friedmann-Robertson-Walker metric of the type
\[
ds^2 = -dt^2 + a(t)^2 (dr^2 + r^2 d\theta^2 + r^2 \sin(\theta)^2 d\phi^2). \tag{6}
\]
Inserting the metric (6) into Eq. (5), we find that due to the presence of the torsion field $A_\mu$ and $\Phi$ the Friedmann equation needs to make some modifications. Thus comparing the case without torsion, we find that the evolution of the Universe will present the different properties. In general, the torsion scalar $\Phi$ and
vector fields $A_\mu$ vary with the time $t$. The dynamical torsion scalar fields in cosmology have been studied extensively in other theories \[15, 16, 17\]. For simplicity, in this paper we adopt $\Phi^2 = 1$ and consider the two simplest models $A_\mu = (\alpha, 0, 0, 0)$ and $A_\mu = a(t)(0, \beta, \beta, \beta)$ and then study their cosmic expansion histories, where $\alpha$ and $\beta$ are constants. These torsion fields ansatze yield that the modified Friedmann equation possesses a more simple form, which is very convenient for us to study the dynamical properties of the model in the following calculations. In order to probe of the properties of the torsion cosmology in Kaluza-Klein theory, here we assume that the Universe contains only the dark matter.

Let us now consider the first model $A_\mu = (\alpha, 0, 0, 0)$ and $\Phi^2 = 1$. Combining Eqs. (6) and (5), we find that the modified Friedmann equation and Raychaudhuri field equation can be expressed simply as

$$H^2 = \frac{2}{3(2 - \alpha^2)} \rho,$$

and

$$\dot{H} = -\frac{1}{2 - \alpha^2} \rho,$$

respectively, where $\rho$ is the energy density of the fluid. The conservation law of the total energy reads

$$\dot{\rho} + 3H \rho = 0,$$

For the dark matter $\omega = 0$, the Eq. (9) are independent of the constant $\alpha$, which are consistent with those in the standard Einstein cosmology. The factor $2/(2 - \alpha^2)$ in Eqs. (7) and (8) is equivalent to the constant of gravity. The deceleration parameter

$$q \equiv -\frac{\ddot{a}}{\dot{a}^2} = -\frac{\dot{H} + H^2}{H^2} = \frac{1}{2}. $$

This means that in the first model the expansion is not accelerating if the Universe contains only the dark matter. Thus, in order to explain the accelerating expansion, we must introduce to the exotic components as in the Einstein cosmology, such as dark energy. Therefore, in the following section we do not consider further the first model.

For the second model $A_\mu = a(t)(0, \beta, \beta, \beta)$ and $\Phi^2 = 1$, repeating the operations above, we obtain that the Friedmann equation, Raychaudhuri field equation and the conservation law of the total energy can be expressed as

$$H^2 = \frac{1}{3} \rho, \quad \dot{H} = -\frac{1 - 2\beta^2}{2 - 3\beta^2} \rho,$$

and

$$\dot{\rho} + 3H \left[\frac{2 - 4\beta^2}{2 - 3\beta^2}\right] \rho = 0.$$
In this case, the Friedmann equation is the same as in the torsion free Einstein cosmology. But both the Raychaudhuri field and the continue equations depend on the torsion parameter $\beta$, which is different from that in the model we discuss previously. The deceleration parameter is

$$q \equiv -\frac{\ddot{a}a}{\dot{a}^2} = -\frac{\dot{H} + H^2}{H^2} = 1 - \frac{3\beta^2}{2 - 3\beta^2}. \quad (13)$$

Obviously, $q$ is a constant which is determined only by the torsion parameter $\beta$. When $\beta^2$ lies in the region $(\frac{1}{3}, \frac{2}{3})$, we find that $q < 0$. This means that the expansion of the Universe can be accelerated without dark energy, which is different from that in the Einstein cosmology.

Let us now to examine the second model with some old high redshift objects including two old galaxies LBDS 53W091 ($z = 1.55, t = 3.5\text{Gyr}$) and LBDS 53W069 ($z = 1.43, t = 4.0\text{Gyr}$) and the old quasar APM 08279+5255 ($z = 3.91, t = 2.1\text{Gyr}$). The absolute ages of these oldest galactic globular clusters and quasars provide a fundamental constraint on cosmological models because that the universe cannot be younger than its constituents at any redshift. The age of the Universe is

$$t = \frac{1}{H_0} \int_{-\infty}^{-\ln(1+z)} \frac{dx}{h},$$

where $x = \ln a$ and $h = H/H_0$. From the observations of the Hubble Space Telescope Key project, the present

![FIG. 1: The change of the age of Universe with the redshift $z$ for different values of $\beta$. The solid line, dotted, dashed and dot-dashed denote the cases $\beta = 0.61, 0.62, 0.63$ and 0.64, respectively. The large points denote the ages of the three old objects LBDS 53W091 ($z = 1.55, t = 3.5\text{Gyr}$), LBDS 53W069 ($z = 1.43, t = 4.0\text{Gyr}$) and APM 08279+5255 ($z = 3.91, t = 2.1\text{Gyr}$). Here we set $h_0 = 0.72$.](image)

Hubble parameter is constrained to be $H_0^{-1} = 9.776 h_0^{-1}\text{Gyr}$, where $0.64 < h_0 < 0.80$. With Hubble parameter $h_0 = 0.72$ and fractional matter density $\Omega_{m0} = 0.27$, the $\Lambda$CDM model would give an age $t = 1.6\text{Gyr}$ at
$z = 3.91$, which is smaller than the ages 2.1 Gyr inferred from old quasar APM 08279+5255. This is so-called “high-$z$ cosmic age problem” \cite{37}. Recently, it has attracted a lot of attention \cite{37, 38, 39, 40, 41, 42, 43, 44, 45} and various DE models are examined against these old galaxies and quasars. So far, there is no DE model that can pass the examine, and most of these models perform even poorer than $\Lambda$CDM model in solving this problem. Some authors argued that the age problem can be alleviated if one consider the interaction between dark energy and dark matter \cite{45, 46}. However, most of the interacted forms are phenomenological because the natures of dark energy and dark matter are still unclear at present. In this paper, we will examine the second model by these old high redshift galaxies and quasars. We set $h_0 = 0.72$ and draw the curves of age of the universe. From Fig. (1) we find that that for the chosen values of $\beta$ the age problem can be solved in this case. These results imply that the effects of the torsion could help us to understand more about our present Universe.

### III. OBSERVATIONAL CONSTRAINT

We are now in position to use the observational data to fit the second models in the torsion cosmology in the Klaauza-Klein theory. Both the torsion parameters $\alpha$ and $\beta$ are determined by minimizing $\chi^2 = \chi^2_{SN}$. For the Type Ia SNe data, we use the latest 307 Union SNIa data \cite{47} and define

$$
\chi^2_{SN} = \sum_{i=1}^{307} \frac{[\mu_{obs}(z_i) - \mu(z_i)]^2}{\sigma_i^2},
$$

where $\mu_{obs}(z_i)$ and $\sigma_i$ are the observed value and the total error for the supernova dataset, respectively. $\theta$ is the parameter in the model. $\mu(z)$ is the theoretical distance modulus which is given by

$$
\mu(z) = 5 \log_{10} D(z) + \mu_0,
$$

where $\mu_0$ is a nuisance parameter. The luminosity of distance $D(z)$ is

$$
D(z) = \frac{1 + z}{H_0} \int_0^z \frac{dz'}{E(z'; \theta)},
$$

and $E(z; \theta)$ is

$$
E(z; \theta) = \frac{H(z; \theta)}{H_0}.
$$

As in \cite{48}, expanding the $\chi^2_{SN}$ of Eq. (15) with respect to $\mu_0$, we have

$$
\chi^2_{SN} = A(\theta) - 2\mu_0 B(\theta) + \mu_0^2 C,
$$

where

$$
A(\theta) = \sum_{i=1}^{307} [\mu_{obs}(z_i) - \mu(z_i)]^2 / \sigma_i^2
$$

and

$$
B(\theta) = \sum_{i=1}^{307} \mu_{obs}(z_i) \mu(z_i) / \sigma_i^2.
$$

The solution of $\mu_0$ is

$$
\mu_0 = \frac{B(\theta)}{C},
$$

and

$$
A(\theta) - 2\mu_0 B(\theta) + \mu_0^2 C = 0.
$$
where

\[ A(\theta) = \sum_{i=1}^{307} \frac{[\mu_{\text{obs}}(z_i) - \mu(z_i, \mu_0 = 0)]^2}{\sigma_i^2}, \]

\[ B(\theta) = \sum_{i=1}^{307} \frac{\mu_{\text{obs}}(z_i) - \mu(z_i, \mu_0 = 0)}{\sigma_i^2}, \]

\[ C = \sum_{i=1}^{307} \frac{1}{\sigma_i}. \]  

(20)

It is easy to see that when \( \mu_0 = B(\theta)/C \) the formula (19) has a minimum

\[ \hat{\chi}^2_{\text{SN}}(\theta) = A(\theta) - \frac{B(\theta)^2}{C}. \]  

(21)

This means that \( \chi^2_{\text{SN,min}}(\theta) = \hat{\chi}^2_{\text{SN,min}}(\theta) \). Since \( \hat{\chi}^2_{\text{SN,min}}(\theta) \) is independent of \( \mu_0 \), we will minimize \( \hat{\chi}^2_{\text{SN,min}}(\theta) \) rather than \( \chi^2_{\text{SN}}(\theta) \) in the our analysis.

![Graph](image)

**FIG. 2:** The \( \beta-\Delta \chi^2 \) plane for the second model by using the SNIa data. Here \( \Delta \chi^2 = \chi^2 - \chi^2_{\text{min}} \) and \( \chi^2_{\text{min}} = 318.84 \). The dashed line and dash-dotted line in the right figure denote \( \Delta \chi^2 = 1 \) and \( \Delta \chi^2 = 4 \), corresponding to 68.3\% and 95.4\% C.L., respectively.

Now we present the numerical results of fitting the torsion cosmology in Kaluza-Klein theory to the Type Ia SNe data. For the second model \( A_\mu = a(t)(0, \beta, \beta, \beta) \) and \( \Phi^2 = 1 \), we obtain that \( \chi^2_{\text{min}} = 318.84 \) and best-fit value of \( \beta = 0.64689^{+0.00577}_{-0.00694} \) in the range of 1\( \sigma \), which ensures that the deceleration parameter \( q < 0 \) and our present Universe is accelerating. Moreover, we define the quantity \( \Delta \chi^2 = \chi^2 - \chi^2_{\text{min}} \) and then plot the \( \beta-\Delta \chi^2 \) plane in Fig. (2), which supports the best-fit value of \( \beta \) we obtained above. In the range of 2\( \sigma \), we find the model parameter \( \beta \) is \( \beta \in (0.63232, 0.65864) \). Comparing with the chosen value of \( \beta \) in Fig.(1), we find that the age problem of three old objects can be solved in the torsion cosmology in the Kaluza-Klein theory. This implies that the second model here we considered is able to accommodate both the ages of the high redshift
objects and the SNIa measurements. However, it seems impossible for the usual dark energy models. For example, although the ΛCDM model is favored by the observational data, as our discussion previously it is not free from the age problem of the oldest quasar APM 08279+5255. The similar results are obtained in other dark energy models. From these discussion, it is easy to conclude that Kaluza-Klein type theories with torsion within proper constraints will play an important role in cosmology.

IV. SUMMARY

In this paper we have studied the torsion cosmology model in the Kaluza-Klein theory. We considered two simple models in which the torsion vectors are $A_\mu = (\alpha, 0, 0, 0)$ and $A_\mu = a(t)^2(0, \beta, \beta, \beta)$, respectively. For the first model, the accelerating expansion of the Universe can be not explained without dark energy which is similar to that in the standard cosmology. However, for the second model, we find that the expansion of the universe can be accelerated without dark energy and it is free of the age problem of the three old objects for appropriated value of the model parameter $\beta$, which is different from that in the Einstein cosmology. The constraints on the second torsion cosmology models in Kaluza-Klein theory have been studied by the analysis of SNIa data. The best-fit value of the parameter in the model is $\beta = 0.6468$. It is worth noting that in the $2\sigma$ uncertainty the second model is able to accommodate both the ages of the high redshift objects and the SNIa measurements. These means that modified gravity theories obtained by Kaluza-Klein theories with torsion will play an important role in cosmological models.

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