Electromagnetic currents of elementary particles
from a second order formalism

E G Delgado-Acosta\textsuperscript{1}, M Napsuciale\textsuperscript{2} and S Rodriguez\textsuperscript{3}

\textsuperscript{1} Instituto de F"{i}sica, Universidad Autónoma de San Luis Potosí, Av. Manuel Nava 6, San Luis Potosí, SLP 78290, México
\textsuperscript{2} Departamento de Física, Universidad Guanajuato, Loma del Bosque 103, León, GTO 37150, México
\textsuperscript{3} Facultad de Ciencias Físico Matemáticas, Universidad Autónoma de Coahuila, Edificio D, Unidad Campus Camerredondo, Saltillo, COAH 25280, México

E-mail: \textsuperscript{1}german@ifisica.uaslp.mx, \textsuperscript{2}mauro@fisica.ugto.mx, \textsuperscript{3}simonrodriguez@uadec.edu.mx

Abstract. We use a second order formalism based on Poincaré symmetry, and known as Covariant Projector Formalism to calculate the electromagnetic currents of spin 1/2, spin 1 and spin 3/2 particles. We restrict to parity conserving processes and obtain the currents in terms of the gyromagnetic factor $g$ alone. This parameter is then fixed by calculating the forward Compton scattering cross section. In the end we reproduce the Dirac moments for the electron from a more general second order Lagrangian, we also reproduce the Standard Model results for the $W$ boson. In the spin 3/2 case we show that the RS current is incomplete as it does not include the spin 1 piece of the vector-spinor representation. As a consequence, when we calculate the spin 3/2 EM moments, instead of the universal value $g = 2$, we find $g = 2/3$ as the gyromagnetic ratio for a Rarita-Schwinger particle.

1. Introduction

The present study is devoted to the electromagnetic (EM) interaction of particles with spin 1/2, spin 1, and spin 3/2. The interest in this matter is motivated by existing problems of the conventional description. For example, the Proca equation describes spin 1 massive particles \cite{1}

\begin{equation}
\left[\left(-p^2 + m^2\right)g_{\mu\nu} + p_{\mu}p_{\nu}\right]A^\nu = 0,
\end{equation}

but in the presence of an EM field it fails to predict the correct EM moments of a charged spin 1 massive particle such as the $W$ boson. Spin 3/2 particles are usually described by the Rarita-Schwinger equation \cite{2}

\begin{equation}
(\gamma \cdot p - m)\psi^\alpha = 0, \quad p^{\alpha}\psi_\alpha = 0,
\end{equation}

which is plagued by the Velo-Zwanziger problem, allowing for a super-luminal propagation of the wave fronts when coupled to an EM field. We show that the above problems are avoided within a more elaborated formalism suggested in \cite{3}, based on covariant projectors. The advantages of the covariant projector formalism can be summarized as follows:
• It is based on minimal coupling and allows for the construction of an EM interaction written in terms of undetermined parameters that can be fixed by physical requirements, among them, well behaved forward Compton scattering cross sections.
• In the \((1/2,0) \oplus (0,1/2)\) Dirac representation of the homogeneous Lorentz group (HGL) it reproduces, upon an appropriate choice of parameters, the EM properties of a spin 1/2 fermion described by the Dirac Lagrangian.
• It adequately describes the EM interaction of massive spin 1 particles in the \((1/2,1/2)\) representation of the HLG.
• In the spin 3/2 case it provides an equation of motion with causal propagation of the wave fronts by restricting the gyromagnetic factor to the universal value of \(g = 2\).

In the next section we shall highlight the essentials of the covariant projector formalism.

2. Covariant projector formalism

The second order formalism used here consists in the construction of an equation of motion for a state \(\psi^{(m,s)}\) transforming in a given multi-spin representation, \(\psi^{(m,s_1,s_2,\ldots,s)}\), of the Poincaré group as a projection onto its spin \(s\) sector, and the mass, \(m\). The projector is expressed in terms of the two Casimir operators of the group, the squared momentum operator \(P^2\) and the squared Pauli-Lubanski operator \(W^2\). The equation of motion for a mass \(m\) and spin \(s\) has the initial form

\[
P^{(m,s)} \psi^{(m,s)} = \psi^{(m,s)},
\]

and in general can be rewritten as

\[
(\Gamma_{\mu\nu} p^\mu p^\nu - m^2) \psi^{(m,s)} = 0,
\]

where the \(\Gamma_{\mu\nu}\) tensor is expressed in terms of the \(P^2\) and \(W^2\) operators, its explicit form however has to be worked out in the particular representation of interest. It is important to note that the projection does not restrict the form of the antisymmetric part of the \(\Gamma_{\mu\nu}\) tensor, to see this we write the equation as

\[
(\Gamma_{\mu\nu} \partial^\mu \partial^\nu + m^2) \psi = 0.
\]

It is obvious that the antisymmetric part of \(\Gamma_{\mu\nu}\) is irrelevant in the free case since the derivatives \(\partial^\mu\) and \(\partial^\nu\) commute, however it becomes activated when introducing interactions via the gauge principle, leading to non-commuting covariant derivatives, \(\partial \rightarrow D = \partial + ieA\),

\[
[D^\mu, D^\nu] = ieF^{\mu\nu}.
\]

In effect, in order to preserve generality of the coupled theory, one has to define the \(\Gamma_{\mu\nu}\) tensor with the most general antisymmetric part allowed in a given representation. We will refer to this method either as covariant projector formalism, [3], or, NKR formalism.

2.1. Spin 1/2 in the \((1/2,0) \oplus (0,1/2)\) representation

Following the method described above, we can write an equation of motion for a state \(\psi\) in this representation as

\[
(\Gamma_{\mu\nu}^S p^\mu p^\nu - m^2) \psi = 0.
\]

In the present work we restrict to parity conserving processes so that the most general tensor to be considered is

\[
\Gamma_{\mu\nu}^S = g_{\mu\nu} - ig_S M_{\mu\nu}^S,
\]
where $g_S$ is an arbitrary parameter and $M^S_{\mu\nu}$ are the generators of the representation $(1/2, 0) \oplus (0, 1/2)$ of the HLG

$$M^S_{\mu\nu} = \frac{i}{4} (\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu) = \frac{i}{4} [\gamma_\mu, \gamma_\nu] = \frac{1}{2} \sigma_{\mu\nu},$$

with $\gamma^\mu$ being the Dirac gamma matrices. The associated free Lagrangian is

$$L^S_{\text{free}} = (\partial^\mu \psi)(\Gamma^S_{\mu\nu} \partial^\nu \psi - m^2 \bar{\psi} \psi).$$

The EM interaction is introduced by minimal coupling,

$$\partial^\mu \rightarrow D^\mu = \partial^\mu + ieA^\mu,$$

where $e$ is the charge of the particle, the resulting gauged Lagrangian being

$$L^S = L^S_{\text{free}} - ej^\mu A^\mu + e^2 \bar{\psi}(\Gamma^S_{\mu\nu} + \Gamma^S_{\nu\mu}) \psi.$$ (12)

Then the EM current in momentum space is found to be

$$j^S_\mu(p', \lambda'; p, \lambda) = e \eta^\alpha(p', \lambda') \left[ (p' + p)_\mu G^S_{\mu\nu} + M^S_{\mu\nu}(p' - p)_\nu \right] u(p, \lambda),$$

where where the $u(p, \lambda)$ spinors transform in the $(1/2, 0) \oplus (1/2, 1/2)$ representation of the HLG.

In effect, the method used directly provides a current that parallels the Gordon decomposition of the Dirac current.

2.2. Spin 1 in the $(1/2, 1/2)$ representation

The equation of motion relevant to this case has the form

$$(\Gamma^V_{\alpha\beta\mu\nu} p^\mu p^\nu - m^2 g_{\alpha\beta}) V^\beta = 0.$$ (15)

For particles in parity conserving processes we define the general $\Gamma^V_{\alpha\beta\mu\nu}$ tensor as

$$\Gamma^V_{\alpha\beta\mu\nu} = g_{\alpha\beta} g_{\mu\nu} - \frac{1}{2} (g_{\alpha\nu} g_{\beta\mu} + g_{\alpha\mu} g_{\beta\nu}) - i \left( g - \frac{1}{2} \right) [M^V_{\mu\nu}]_{\alpha\beta},$$

where $M^V_{\mu\nu}$ are the generators of the representation $(1/2, 1/2)$,

$$[M^V_{\mu\nu}]_{\alpha\beta} = i (g_{\alpha\nu} g_{\beta\mu} - g_{\alpha\mu} g_{\beta\nu}).$$ (17)

Following the same procedure as in the previous section, the EM current is derived as

$$j^V_\mu(p', \lambda'; p, \lambda) = -e \eta^\alpha(p', \lambda') \left[ (p' + p)_\mu g_{\alpha\beta} + ig[M^V_{\mu\nu}]_{\alpha\beta}(p' - p)_\nu \right] \eta^\beta(p, \lambda),$$

where the $\eta^\alpha(p, \lambda)$ transform in the $(1/2, 1/2)$ representation of the HLG.
2.3. Spin 3/2 in the (1/2, 1/2) \( \otimes [(1/2, 0) \oplus (0, 1/2)] \) representation

In this case the equation of motion has the form:

\[
(\Gamma_{\alpha\beta\mu\nu} p^\mu p^\nu - m^2 g_{\alpha\beta}) \psi^{\beta} = 0. \tag{19}
\]

There are various antisymmetric structures allowed in this representation, and for a parity conserving particle the \( \Gamma_{\alpha\beta\mu\nu} \) tensor is written in terms of five undetermined parameters:

\[
\Gamma_{\alpha\beta\mu\nu} = -\frac{1}{3} g_{\alpha\nu} g_{\beta\mu} - \frac{1}{6} i \sigma_{\alpha\mu} g_{\beta\nu} - \frac{1}{3} g_{\alpha\mu} g_{\beta\nu} + \frac{2}{3} g_{\alpha\beta} g_{\mu\nu} + \frac{i}{3} g_{\mu\nu} \sigma_{\alpha\beta} \\
- \frac{i}{6} g_{\beta\nu} \sigma_{\alpha\mu} + \frac{i}{6} g_{\mu\nu} \sigma_{\beta\nu} \\
- g \frac{5}{2} g_{\alpha\beta} \sigma_{\mu\nu} + g \nu \left( g_{\alpha\mu} g_{\beta\nu} - g_{\alpha\nu} g_{\beta\mu} \right) + i d \left( g_{\beta\nu} \sigma_{\alpha\mu} - g_{\beta\mu} \sigma_{\alpha\nu} \right) \\
+ i c \left( g_{\alpha\mu} \sigma_{\beta\nu} - g_{\alpha\nu} \sigma_{\beta\mu} \right) + i f \gamma^5 \epsilon_{\alpha\beta\mu\nu}, \tag{20}
\]

The number of unknown coefficients gets reduced by eliminating the spin 1/2 sector coupling contained in the \( \psi^\mu \) representation and identifying the gyromagnetic factor as \( g = 2 f + g \nu \). In so doing, the EM current for a particle on mass-shell is found as

\[
j_\mu(p', \lambda'; p, \lambda) = -e \bar{\psi}(p', \lambda') \left[ (p' + p) \mu g_{\alpha\beta} + ig[M_{\mu\nu}^{3/2}]_{\alpha\beta}(p' - p)^\nu \right] \psi^\beta(p, \lambda), \tag{21}
\]

where

\[
[M_{\mu\nu}^{3/2}]_{\alpha\beta} = M_{\mu\nu}^S g_{\alpha\beta} + [M_{\mu\nu}^V]_{\alpha\beta}, \tag{22}
\]

are the generators of the \( (1/2, 1/2) \otimes [(1/2, 0) \oplus (0, 1/2)] \) representation of the HLG. This current also decomposes in a similar way to the cases seen before. This is on variance with the Rarita-Schwinger current which on-shell is written in terms of an \( A \)-parameter (see [4] for example) as

\[
j_\mu^{RS}(p', \lambda'; p, \lambda) = -e \bar{\psi}(p', \lambda') [2m \gamma^\mu g_{\alpha\beta}] \psi^\beta(p, \lambda). \tag{23}
\]

The decomposition of the latter current,

\[
j_\mu^{RS}(p', \lambda'; p, \lambda) = -e \bar{\psi}(p', \lambda') \left[ (p' + p) \mu g_{\alpha\beta} + ig S_{\mu\nu} g_{\alpha\beta}(p' - p)^\nu \right] \psi^\beta(p, \lambda), \tag{24}
\]

makes it visible that it does not include any term due to underlying vector representation. As a consequence, this current fails to describe a magnetic coupling proportional to \( g_{3/2} \mathbf{S} \cdot \mathbf{B} \).

3. Multipole expansions

The electromagnetic moments of a particle are defined by means of a multipole expansion of a corresponding current density. We here design such a current density in transforming our electromagnetic currents from above to the Breit frame:

\[
J_\mu^B(s, \lambda, q) = \frac{1}{\omega^2} j_\mu(p', \lambda', p, \lambda), \quad p' = (\omega/2, q/2), \quad p = (\omega/2, -q/2). \tag{25}
\]

The Cartesian electromagnetic moments for a particle of spin \( s \) and polarization \( \lambda \) are then defined as [5]:

\[
Q_E^I(s, \lambda) = b^{I0}( -i \partial_q ) q_E(s, \lambda, q) \bigg|_{q=0}, \tag{26}
\]

\[
Q_M^I(s, \lambda) = \frac{1}{l+1} b^{I0}( -i \partial_q ) q_M(s, \lambda, q) \bigg|_{q=0}, \tag{27}
\]
where the electric density $\varrho_E(s, \lambda)$ and the so-called magnetic density $\varrho_M(s, \lambda)$ read:

$$\varrho_E(s, \lambda, q) = j_0^B(s, \lambda, q), \quad \varrho_M(s, \lambda, q) = \partial_q \cdot [j_B(s, \lambda, q) \times q],$$  \hspace{1cm} (28)

the $b^{00}$ coefficients are obtained from the spherical harmonics as

$$b^{00}(r) = l!\sqrt{4\pi/(2l + 1)}r^l Y_{l0}(\Omega),$$  \hspace{1cm} (29)

so that

$$b^{00}(-i\partial_q) = 1,$$  \hspace{1cm} (30)

$$b^{10}(-i\partial_q) = -i \frac{\partial}{\partial q^3},$$  \hspace{1cm} (31)

$$b^{20}(-i\partial_q) = \frac{\partial^2}{\partial q^1} + \frac{\partial^2}{\partial q^2} - 2 \frac{\partial^2}{\partial q^3},$$  \hspace{1cm} (32)

$$b^{30}(-i\partial_q) = -3i \frac{\partial}{\partial q^3} \left( 3\frac{\partial^2}{\partial q^1} + 3\frac{\partial^2}{\partial q^2} - 2\frac{\partial^2}{\partial q^3} \right),$$  \hspace{1cm} (33)

and so on.

### 3.1. Electromagnetic moments of spin $1/2$ and spin $1$ particles in the covariant projector formalism

We begin with the spin $1/2$ case. In order to obtain explicit expressions for the moments, we need knowledge of the $u(p, \lambda)$, we can use the textbook basis:

$$u\left(p, \frac{1}{2}\right) = N_S \begin{pmatrix} m + p_0 \\ 0 \\ p_1 + ip_2 \\ 0 \end{pmatrix},$$ \hspace{1cm} (34)

$$u\left(p, -\frac{1}{2}\right) = N_S \begin{pmatrix} m + p_0 \\ 0 \\ p_1 - ip_2 \\ -p_3 \end{pmatrix},$$ \hspace{1cm} (35)

with $N_S = [2m(m + p_0)]^{-1/2}$. Inserting this spinors into (14), we find the current needed to obtain the Breit densities (28) and the resulting set of moments as

$$Q_E^{0\left(\frac{1}{2}, \lambda\right)} = e,$$ \hspace{1cm} (36a)

$$Q_M^{1\left(\frac{1}{2}, \lambda\right)} = \frac{egS}{2m} \left\langle \frac{1}{2}, \lambda \right| S_z \left| \frac{1}{2}, \lambda \right\rangle = \frac{egS}{2m} \langle S_z \rangle.$$ \hspace{1cm} (36b)

Here $Q_E^0, Q_M^1$ stand for the charge and dipole magnetic moment of the particle respectively, higher moments are found to be zero. The Dirac results [6, 7] are recovered for $g = 2$, the Compton scattering process providing an additional argument in favor of this value. We now turn to the spin $1$ case. In repeating the procedure for the current in (18), again the explicit
form of $\eta^\alpha(p, \lambda)$ is required,

$$\eta(p, +1) := \frac{N_V}{\sqrt{2}} \begin{pmatrix} - (m + p_0) (p_1 + ip_2) \\ -m^2 - p_0m - p_1^2 - ip_1p_2 \\ -i (p_2^2 - ip_1p_2 + m(m + p_0)) \\ -(p_1 + ip_2) p_3 \end{pmatrix},$$

$$\eta(p, 0) := N_V \begin{pmatrix} (m + p_0) p_3 \\ p_1p_3 \\ p_2p_3 \\ p_3^2 + m(m + p_0) \end{pmatrix},$$

$$\eta(p, -1) := \frac{N_V}{\sqrt{2}} \begin{pmatrix} (m + p_0) (p_1 - ip_2) \\ m^2 + p_0m + p_1^2 - ip_1p_2 \\ -i (p_2^2 + ip_1p_2 + m(m + p_0)) \\ (p_1 - ip_2) p_3 \end{pmatrix},$$

with $N_V = [m(m + p_0)]^{-1}$. In terms of expectation values between vectors of polarization $\lambda$, for a spin 1 particle we obtain:

$$Q_E^2(1, \lambda) = e,$$  \hspace{1cm} (38a)

$$Q_M^1(1, \lambda) = \frac{eg_V}{2m} (S_z),$$  \hspace{1cm} (38b)

$$Q_E^2(1, \lambda) = \frac{e (1 - g_V)}{m^2} (3S_z^2 - S^2),$$  \hspace{1cm} (38c)

where $Q_E^2$ denotes the electric quadrupole moment. These expressions match the known results for the electroweak $W$ gauge boson for $g = 2$ as described by the standard model Lagrangian [8, 9]. We conclude from this that the covariant projector formalism gives an adequate description of spin 1/2 and spin 1 particles. However the result is not a surprising one because the currents in (14) and (18) parallel their counterparts from Dirac and SM Lagrangians [7, 10], in the sense that they decompose into a spinless part proportional to $(p' + p)^2$ and a spin part associated with the term $igM_{\mu\nu}(p' - p)^\nu$.

3.2. Spin 3/2 particles in the Rarita-Schwinger formalism

To make calculations in the $(1/2, 1/2) \otimes [(1/2, 0) \oplus (0, 1/2)]$ representation we need the spin 3/2 basis previously employed in [4],

$$u^\alpha(p, +3/2) = \eta^\alpha(p, +1) u(p, +1/2),$$  \hspace{1cm} (39a)

$$u^\alpha(p, +1/2) = \frac{1}{\sqrt{3}} \eta^\alpha(p, +1) u(p, -1/2) + \sqrt{2} \eta^\alpha(p, 0) u(p, +1/2),$$  \hspace{1cm} (39b)

$$u^\alpha(p, -1/2) = \frac{1}{\sqrt{3}} \eta^\alpha(p, -1) u(p, +1/2) + \sqrt{2} \eta^\alpha(p, 0) u(p, -1/2),$$  \hspace{1cm} (39c)

$$u^\alpha(p, -3/2) = \eta^\alpha(p, -1) u(p, -1/2),$$  \hspace{1cm} (39d)

where $u(p, \lambda)$ and $\eta^\alpha(p, \lambda)$ have been previously introduced in (35) and (37). The calculation of the multipole decompositions of the Rarita-Schwinger spin-3/2 current amounts to the following
set of multipole moments,

\[
\begin{align*}
\text{RS:} \quad Q_E^0 \left( \frac{3}{2}, \lambda \right) &= e, \quad (40a) \\
Q_M^1 \left( \frac{3}{2}, \lambda \right) &= \frac{eg_s}{2m} \frac{1}{3} \langle S_z \rangle, \quad (40b) \\
Q_E^2 \left( \frac{3}{2}, \lambda \right) &= \frac{e}{m^2} \frac{1}{3} \langle A \rangle, \quad (40c) \\
Q_M^3 \left( \frac{3}{2}, \lambda \right) &= \frac{eg_s}{2m^3} \langle B \rangle, \quad (40d)
\end{align*}
\]

with \( g_S = 2 \). The latter equations show that only the gyromagnetic factor, \( g_S \), of the Dirac-spinor piece of the four-vector-spinor contributes to the magnetic dipole and octupole moments, with the result that one has to ascribe \( g = 2/3 \) to the spin-\( 3/2 \) field. The electric quadrupole moment turns out to be indifferent to both the \( g_S \) and \( g_V \) values. In a forthcoming paper, we shall argue that this shortcoming of the Rarita-Schwinger framework can be avoided within the covariant projector formalism, where the \( g_V \) and \( g_S \) factors will be shown to participate the expressions for the spin-\( 3/2 \) multipoles on equal footings.

4. Compton scattering

4.1. Spin 1/2 particles in the \((1/2, 0) \oplus (0, 1/2)\) representation

We begin with the spin 1/2 case. From the Lagrangian (12) we can extract Feynman rules of first and second order on the charge of the particle, while the propagator can be read form the inverse of equation (7). The cross section of the Compton scattering process is found in terms of the \( g_S \) parameter, the incident photon energy \( \eta = \omega/m \) and the scattering angle in the Lab system \( x = \cos \theta \). In the low energy limit we get:

\[
\left[ \frac{d\sigma_{1/2}(g_S, x, \eta)}{d\Omega} \right]_{\eta \to 0} = \frac{1}{2} (x^2 + 1)r_0^2,
\]

where \( r_0^2 = e^2/(4\pi m) = \alpha/m \) is the so called classical radius of the particle. The latter expression is in accordance with the Thompson limit of the scattering. In forward direction \((x = 1)\) the result is energy-independent,

\[
\left[ \frac{d\sigma_{1/2}(g_S, x, \eta)}{d\Omega} \right]_{\theta = 0} = r_0^2.
\]

Therefore, at this stage, we cannot say much about the gyromagnetic factor since the forward cross section is a well behaved quantity at all energies independently of \( g \). However, the cross section under investigation depends on \( g \) in other directions. Indeed, for the total cross section at high energies one finds

\[
\sigma_{1/2}(g_S, \eta)\big|_{\eta \to \infty} = \frac{3}{128} (g_S - 2)^2 g_S^2 \sigma_T,
\]

where \( \sigma_T = (8/3)\pi r_0^2 \) denotes the Thompson cross section. Then, for the formalism to describe properly the EM interaction of the electron, one has to choose \( g_S = 2 \) with the aim to guarantee a vanishing cross section at high energies and in accordance with the experimental observations. In this way, the EM moments from the second order formalism (36) reproduce the Dirac results when we consider the Compton scattering process. In addition it has been shown [11] that the general result for an arbitrary \( g \) factor can be used to approximately describe a fermion with \( g \neq 2 \), namely the proton with \( g_P \approx 5.58 \), in which case the cross section at a fixed angle will look as presented in Figure 1.
4.2. Spin 1 particles in the (1/2, 1/2) representation
The Compton scattering process can be handled in parallel to the case of spin 1/2 fermions. Also here we are going to search for additional information about the gyromagnetic factor from the behavior of the Compton cross section in the ultraviolet. Also in this case we recover the Thompson result in the low energy limit,

$$\left[ \frac{d\sigma_1(g_V, x, \eta)}{d\Omega} \right]_{\eta \to 0} = \frac{1}{2}(x^2 + 1)r_0^2.$$

(44)

The more interesting part is the forward cross section since it depends on the $g$-value:

$$\left[ \frac{d\sigma_1(g_V, x, \eta)}{d\Omega} \right]_{\theta = 0} = \left( \frac{1}{24}(g_V - 2)^4 \eta^2 + 1 \right)r_0^2$$

(45)

with $\eta = \omega/m$ as before. We notice that, for an arbitrary $g_V$ parameter, the differential cross section will increase with the energy, as a consequence the total cross section will be undetermined in the high energy limit unless we have $g_V = 2$, in which case we obtain an energy independent forward cross section and a total cross section looks as displayed in Figure 2. Then

the forward Compton scattering process restricts the value of the gyromagnetic factor, and the related EM moments match the moments the $W$ boson. In this way we relate the requirement on finite forward Compton scattering cross sections with the natural multipole moments of the particle.
4.3. Spin 3/2 particles in the \((1/2, 1/2) \otimes [(1/2, 0) \oplus (0, 1/2)]\) representation

The Compton scattering process at low energies is insensitive to the \(g\) parameter where the classical limit is again recovered

\[
\left[ \frac{d\sigma_{3/2}(g, x, \eta)}{d\Omega} \right]_{\eta \to 0} = \frac{1}{2}(x^2 + 1)r_0^2,
\] (46)

the dependence on \(g\) is again present in the forward direction

\[
\left[ \frac{d\sigma_{3/2}(g, x, \eta)}{d\Omega} \right]_{\theta = 0} = r_0^2 + \frac{r_0^2}{81}(g - 2)(5(g - 2)^3 - 18(g - 2) - 36)\eta^2 + \frac{8r_0^2}{81}g^2(g - 2)\eta^4.
\] (47)

Similarly to spin 1 case, the scattering process requires \(g = 2\) for an energy independent forward cross section. The result for the cross section however is more complex than that of the vector case, for a spin 3/2 particle it is found that the choice of the \(g\) parameter can prevent the differential cross section to grow with the energy in the forward direction, but will not do so in other directions. At any rate, the forward scattering constraint still provides a tool to restrict the \(g\) value to \(g = 2\). Same parametrization has been independently found as the only choice capable to ensure causality of the theory [3]. Notice that within the RS formalism the cross section steadily increases with the energy in all directions, including at \(\theta = 0\), [13, 4]

\[
\left[ \frac{d\sigma_{ \text{RS}}(g, x, \eta)}{d\Omega} \right]_{\theta = 0} = \frac{1}{81}(8\eta^4 + 56\eta^2 + 81)r_0^2,
\] (48)

and in disadvantage with respect to the Covariant projector result shown in Figure 3. Inspection of the latter Figure shows that the dashed curve (RS prediction) increases in the forward direction, while the thick one (Covariant projection prediction with \(g = 2\)) approaches the constant value of \(r_0^2\). Choosing \(g = 2\) furthermore would help fixing the EM moments.

5. Conclusions

- The covariant projector formalism allows for a complete description of the electromagnetic interactions of a particle of spin \(s\) embedded in a given representation of the Poincaré group.
- We have shown that the Dirac equation is not the only way to properly describe spin 1/2 fermions.
- By considering a general antisymmetric part in the equation of motion, the covariant projector formalism in the vector case is free of the problems exhibited by the Proca equation in an EM environment.
• Rarita-Schwinger equation fails to properly describe the complete structure of a spin 3/2 particle in the $(1/2, 1/2) \otimes [(1/2, 0) \oplus (0, 1/2)]$ representation of the HLG, this can be seen in the Gordon-like decomposition of the electromagnetic current, as a consequence the EM moments of a RS particle include a dipole magnetic moment with $g = 2/3$ instead of the expected universal value $g = 2$.

• We have found that Compton scattering off a spin 3/2 particle from a second order formalism exhibits a cross section that grows with the energy except in the forward direction when $g = 2$, a condition also required for the causality of the theory, for this reason we regard the covariant projector formalism as a good candidate to describe the natural EM moments of a spin 3/2 particle.

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