Study of 3-flavor QCD Finite Temperature Phase Transition with Staggered Fermions

X. Liao

Physics Department, Columbia University, New York, NY 10027, USA

We have studied the 3-flavor, finite temperature, QCD phase transition with staggered fermions on an $N_t = 4$ lattice. By studying a variety of quark masses we have located the critical point, $m_c$, where the first order 3-flavor transition ends as lying in the region $0.32 < m_c < 0.35$ in lattice units.

1. INTRODUCTION

The flavor and mass dependence of finite temperature QCD phase transition has been studied extensively in the past [1]. Three flavor QCD is especially interesting because of the existence of a second order end point of the first order phase transition line which is conjectured to belong to the 3-d Ising universality class [2]. It is similar to systems such as the liquid-gas [4], the 3-d Ising [5], the 3-d 3-state Potts model [6] and the SU(2) Higgs model [7]. It’s worth mentioning that the field mixing phenomenon [4] is quite important in explaining quantitatively some critical behaviors of these systems, but it does not have an important effect on the behaviors we are discussing.

In this study, we extend our previous effort [3] to locate the end point using the standard Wilson gauge action and unimproved staggered fermion action. There are many approaches to locate the end point, which is characterized by a diverging correlation length. In this paper, we study the discontinuity of the order parameter $\langle \bar{\psi} \psi \rangle$ [2] and the meson screening masses (including the scalar singlet $\sigma$ meson) [8] along the first order phase transition line and extrapolate to the end point. These two approaches work well in the region with relatively strong first order phase transition. Another method called the Binder intersection method [9] is based on the finite size scaling of symmetry sensitive quantities such as $B_4 = \langle (\Delta M)^4 \rangle / \langle (\Delta M)^2 \rangle^2$, in which $M$ is the magnetic-like observable. This method can be used to study the region close to the end point and does not require a knowledge of critical exponents. However, it’s rather expensive to obtain a large amount of statistically independent samples from full QCD simulations using existing evolution algorithms, especially in the critical region where the critical slowdown is problematic. The Binder intersection method is currently under study and is not reported here.

2. EVOLUTION

To determine the first order phase transition line, we evolve the system starting from both cold and hot initial configurations. We use the hybrid Monte Carlo R algorithm with a trajectory length of 0.5 molecular dynamic time and step sizes ranging from 0.01 to 0.0125 depending on quark mass. If a strong first order phase transition exists, we should be able to observe two-state coexistence at large enough volume. This is indeed found at the small quark masses 0.015 and 0.02 (Fig. 1) on an $L = 16$ lattice. At $m = 0.025$, tunneling between the two phases happens in our simulation on an $L = 16$ lattice (Fig. 2). However, this tunneling is not found on a larger, $L = 32$ lattice (Fig. 3), indicating that the phase transition is still first order. For $m = 0.035$, no two-state signal is found on either $L = 16$ (Fig. 4) or $L = 32$ lattices. The system also shows significant critical slowdown behavior and large fluctuations, which indicate a long cor-
relation length and closeness to a second order critical point. Crossover behavior is observed at $m = 0.04$.

Figure 1. The $\langle \bar{\psi}\psi \rangle$ evolution and histogram for $m = 0.02$ and $L = 16$ at $\beta_c = 5.1235$

Figure 2. The $\langle \bar{\psi}\psi \rangle$ evolution and histogram for $m = 0.025$ and $L = 16$ at $\beta_c = 5.132$

3. SCREENING MASS

Theoretical study \cite{9} shows that the scalar singlet meson becomes massless while other mesons stay massive at the end point. To improve statistics, we measure the screening correlators in all three spatial directions and average them. The disconnected diagram needed for the flavor singlet case is measured using a noise estimation technique \cite{10}. We did a careful study of the effect of the type and number of noise sources to ensure that the noise from noise estimator is smaller than the gauge configuration noise. Figure 3 shows the time dependence of a typical correlator we obtained. The meson screening masses are measured at the critical $\beta$ value where a two-state signal is observed. It becomes difficult for large quark masses, near $m_c$ which needs very large volume to reduce frequency of tunneling events.

Figure 3. The $\langle \bar{\psi}\psi \rangle$ evolution and histogram for $m = 0.025$ and $L = 32$ at $\beta_c = 5.132$

Figure 4. The $\langle \bar{\psi}\psi \rangle$ evolution and histogram for $m = 0.035$ and $L = 16$ at $\beta_c = 5.151$

Figure 5. Screening correlators

Figure 6 shows the result of the pseudoscalar and scalar flavor singlet ($\sigma$) screening masses. Extrapolating using the form $M_\sigma \propto (m_c - m)^{1/2}$, we get $m_c = 0.034(3)$ from the confined phase and $m_c = 0.032(2)$ from the deconfined phase.
4. **⟨ψψ⟩ DISCONTINUITY**

The discontinuity in the order parameter $⟨\bar{ψ}ψ⟩$ decreases with increasing quark mass and finally disappears at $m_c$. We fit the data with the mean field exponent $β = 1/2$ (Fig. 6), defined as $\Delta⟨\bar{ψ}ψ⟩ \propto (m_c - m)^β$, and find $m_c = 0.0334(17)$. Fitting to the 3-d Ising exponent of 0.3285 gives a much poorer $χ^2$.

5. **SCALE SETTING**

We have done a scale setting calculation on an $8^3 \times 32$ lattice. The pseudoscalar to vector mass ratio is 0.3134(34) and 0.3666(32) for $m = 0.025$ and $m = 0.035$, respectively. If we use the physical $ρ$ mass for the vector meson, this implies an SU(3) symmetric pseudoscalar mass of 281 Mev at $m = 0.035$.

6. **CONCLUSIONS**

In conclusion, we have obtained the approximate location of the end point by studying the behavior of order parameter discontinuity and meson screening masses as quark mass approaches the end point of first order phase transition line. From our scale setting calculation, we conclude that there is no first order phase transition for three flavor QCD at the strange quark mass. Higher statistics and larger volume studies in the critical region are needed to determine more accurately the critical point, critical exponents and universality class.

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