On the question of quark confinement in the QED interaction

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If we approximate light quarks as massless and we apply the Schwinger mechanism to light quarks, we will reach the conclusion that a light quark $q$ and its antiquark $\bar{q}$ will be confined as a $(q\bar{q})$ boson in the Abelian U(1) QED gauge interaction in $(1+1)$D, as in an open string. Could such a QED-confined $(q\bar{q})$ one-dimensional open string in $(1+1)$D be the idealization of a flux tube in the physical world in $(3+1)$D? If so, the QED-confined $(q\bar{q})$ bosons could show up as neutral QED mesons in the mass region of many tens of MeV (PRC81(2010)064903 & JHEP08(2020)165). Is it ever possible for a quark and an antiquark to be produced and to interact in QED alone to form a confined QED meson? Is there any experimental evidence for the possible existence of the QED meson (or QED mesons)? Experimental confirmation of the recently observed X17 particle at about 17 MeV and the E38 particle at about 38 MeV will shed light on the question of quark confinement in QED in $(3+1)$D.

I. INTRODUCTION

We are dedicating this special issue to the memory of Jean Cleymans1, a distinguished pioneer in the thermodynamics of confined and deconfined quarks and gluons. Jean has made many important contributions, well described by many of his colleagues in this volume. He was always receptive to new ideas and was willing to enter into discussions with an open mind. The subject matter of quark confinement was central to his main interest. In paying tribute to Jean’s contributions, it is fitting to discuss an interesting question in connection with the confinement of quarks.

As is well-known, quarks2 interact in the QCD (quantum chromodynamical) interaction and the QED (quantum electrodynamical) interaction. The common understanding is that the confinement of quarks arises from the non-Abelian nature of the QCD interaction in which the gluons mediate the QCD interaction between quarks. The gluons also interact among themselves. The self-interactions of gluons build a bridge connecting the quarks and confining the quarks.

The confinement of quarks is a peculiar phenomenon because quarks cannot be isolated. We can get an idea about such a peculiar property by asking whether a quark and its antiquark are confined in the QCD interaction at a certain energy. The answer is that they are confined only at the eigenenergies of the QCD eigenstates, which span a confined spatial region. However, at other energies different from those of the QCD quark-antiquark eigenenergies, bound states of an interacting quark and antiquark do not exist, nor are there any continuum states of an isolated quark and antiquark at these energies. In contradistinction, for isolatable particles such as an electron and a positron interacting in QED, states exist at all energies, either as bound $(e^+e^-)$ states or as as continuum states of an isolated electron and positron. Therefore, a quark and an antiquark can be described as being confined in a certain interaction, if there exist confined $(q\bar{q})$ eigenstates at the

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2 We use the term “quarks” to include implicitly also their antiparticles, the antiquarks, if no ambiguity arises. The term “antiquark(s)” will be explicitly used, if ambiguities may arise.
eigenenergies when a quark and an antiquark interact in that interaction, in conjunction with the absence of continuum states of isolated quarks and antiquarks at other energies.

For the non-Abelian QCD interaction, the question on quark confinement in QCD can be inferred from the QCD potential between a static quark and a static antiquark. For example, the quark and the antiquark appear as static external probes represented by time-like world lines at fixed spatial locations in a Wilson loop, given in terms of the product of link variables. The area law of the Wilson loop gives a linear QCD interaction potential between the quark and the antiquark, which leads to QCD meson eigenstates in the confining quark-antiquark potential and the absence of continuum states of an isolated quark and antiquark at other energies.

However, for quarks interacting in the QED interaction, the question of quark confinement in (3+1)D cannot be answered by just studying the static QED potential between a static quark and antiquark only, because there are important dynamical quark effects associated with light quarks, such as the Schwinger mechanism [1, 2], which may play an important role in the question of confinement as we shall discuss in Section 3. In particular, if we consider quark confinement just from the viewpoint of the static QED potential between a quark and an antiquark, we will reach the conclusion that a static quark and antiquark will be deconfined in QED in (3+1)D because the quark-antiquark QED interaction belongs to the weak-coupling regime [3–16], as we shall discuss in more detail in Section 7. However, a serious question arises because if a static quark and antiquark are deconfined in QED in (3+1)D, an isolated quark and antiquark will appear as fractional charges. In a contradicting manner, no such deconfined quark and antiquark in the form of isolated fractional charges have ever been observed in (3+1)D, even though there exists no physical laws to forbid a quark and an antiquark to interact in QED alone. The absence of fractional charges suggests that the question of quark confinement in QED in (3+1)D needs to be carefully examined to include additional dynamical quark effects. We will need to return to the basic description of quark confinement in terms of the existence of confining eigenstates at quark-antiquark eigenenergies and the absence of continuum states of isolated quarks and antiquarks at other energies.

Out of scientific curiosity with encouraging suggestions from theories and experiments, we venture to explore whether quarks are confined when they interact in QED alone, without the QCD interaction. On this question of the confinement for a quark and an antiquark interacting in QED, we shall limit our attention implicitly to light quarks unless specified otherwise. Light quarks have rest masses of only a few MeV [17] and can be approximated as massless. If we approximate light quarks as massless and we apply the Schwinger mechanism [1, 2] to quarks, we will reach the conclusion that a light quark and a light antiquark will be confined in QED in (1+1)D as in an open string [18–25]. Could such a QED-confined one-dimensional $q\bar{q}$ open string in (1+1)D space-time be the idealization of a flux tube in the physical (3+1)D, with the quark and the antiquark at the two ends of the flux tube? If so, the confined $q\bar{q}$ states in (1+1)D will show up as a neutral QED mesons in (3+1)D. Is it ever possible for a quark and an antiquark to be produced and to interact in QED alone so as to form a confined QED meson? Is there any experimental evidence to indicate the possible existence of the QED $q\bar{q}$ meson or mesons?

Such questions have not been brought up until recently for obvious reasons. It is generally perceived that a quark and an antiquark interact with the QCD and the QED interactions simultaneously, with the QED interaction as a perturbation, and the occurrence of a stable and confined state of the quark and the antiquark interacting in the QED interaction alone, without the QCD interaction, may appear impossible. Furthermore, the common perception is that only the QCD interaction with its non-Abelian properties can confine a quark and an antiquark, whereas the QED interaction is Abelian. It has been argued that even if a quark and an antiquark can interact in the QED interaction alone, the QED interaction by itself cannot confine the quark and the antiquark, as in the case of an electron and a positron, so the quark and the antiquark cannot be confined even if
they can interact with the QED interaction alone.

In the presence of many questions and many common contrary perceptions, it is clear that there is a need to explain the unfamiliar concepts of quark confinement in QED in detail and to discuss the resolutions to the common objections. In Section 2, in response to the question on the possibility of quarks and antiquarks interacting in QED alone, we present examples of reactions in which a \((q\bar{q})\) pair may be produced and may interact in QED alone. In Section 3, we apply the Schwinger mechanism to quarks interacting in QED and show that a quark and an antiquark approximated as massless are confined in QED in \((1+1)D\). In Section 4, we make the quasi-Abelian approximation of the non-Abelian QCD dynamics to search for stable collective excitations in QCD. The quasi-Abelian approximation allows the generalization of the Schwinger mechanism from QED in \((1+1)D\) to \((\text{QCD}+\text{QED})\) in \((1+1)D\). We obtain the open string model of QCD and QED mesons. In Section 5, we use the open string model of QCD and QED mesons in \((1+1)D\) as a phenomenological model. We fit the masses of \(\pi^0\), \(\eta\), and \(\eta'\) to infer on the QCD coupling constant and the flux tube radius. Extrapolation of the open string model to the QED interaction with the QCD fine-structure coupling constant, we predict the masses of the QED mesons. In Section 6, we discuss the decay modes of the QED mesons and examine some recent experimental observations of anomalous particles in the mass region of many tens of MeV produced in low-energy \(pA\), and high-energy \(e^+e^-\), hadron-hadron, and nucleus-nucleus collisions. We compare the masses of the experimental anomalous particles with the predicted QED meson masses. There is a reasonable agreement of the predicted QED meson masses with the observed experimental masses of the anomalous particles, placing the QED mesons as good candidates for these particles. In Section 7, we examine the question of quark confinement in QED from the viewpoint of lattice gauge calculations which indicate that quarks in QED in \((3+1)D\) are not confined. We discuss the lattice gauge calculation results of deconfined quarks in QED in \((3+1)D\), which contradicts the experimental absence of fractional charges. It is therefore suggested that the Schwinger dynamical quark effects may need to be included in future lattice gauge calculations. We present our conclusions and discussions in Section 8.

II. COULD A \(q\bar{q}\) PAIR BE PRODUCED AND INTERACT IN QED ALONE?

The proposal of quark confinement in QED involves the hypothesis that a quark and an antiquark could be produced and could interact in QED alone without the QCD interaction. From the static quark and antiquark viewpoint, the common perception is that a quark and an antiquark interact simultaneously in QCD and QED. However, from the dynamical viewpoint of the quantum field theory of quarks interacting in the QED \(U(1)\) interaction and the QCD \(SU(3)\) interaction, we can envisage that quarks can exchange a gluon in a QCD interaction. They can also exchange a photon in a QED interaction. There is no theorem nor basic physical principle that forbids a quark and an antiquark to exchange a single photon or multiple photons and interact in QED only. What is not forbidden is allowed, in accordance with Gell-Mann’s Totalitarian Principle[26]. So, it is permitted to explore the hypothesis that a quark and an antiquark could interact in QED alone.

Experimentally in low-energy \(e^+\) and \(e^-\) collisions, a single \((q\bar{q})\) pair may be produced as shown in Fig. 1(a) and 1(b), and in high-energy \(e^+\) and \(e^-\) collisions, many \((q\bar{q})\) pairs may be produced as shown in Fig. 1(c),

\[
e^+ + e^- \rightarrow \gamma^* \rightarrow q + \bar{q}, \tag{1a}
\]
\[
e^+ + e^- \rightarrow \gamma^* + \gamma^* \rightarrow q + \bar{q}, \tag{1b}
\]
\[
e^+ + e^- \rightarrow \gamma^* \text{ or } Z^0 \rightarrow (q\bar{q})^n. \tag{1c}
\]

In reactions 1(a) and 1(b), the incident \(e^+\) and \(e^-\) pair is in a colorless color-singlet state, and thus the produced \(q + \bar{q}\) pair must combine with their interacting gauge boson \(\gamma\) or \(g\) in a colorless color-
singlet final state. The produced $q$ resides in the color-singlet $3$ representation, and the produced $\bar{q}$ in the $3^*$ representation. They can combine to form the color singlet $1$ and the color octet $8$ configurations,

$$3 \otimes 3^* = 1 \oplus 8.$$  \hfill (2)

The produced $q$ and $\bar{q}$ in their coupled color-singlet configuration can interact in the QED interactions and combine with a photon to form a color-singlet $[(q\bar{q})^8 g^8]^1$ final state, where the superscripts denote color multiplet indices. Similarly, the produced $q$ and $\bar{q}$ in their color-octet configuration can interact in the QCD interactions and combine with a gluon to form a color-singlet $[(q\bar{q})^8 g^8]^1$ final state. A $(q\bar{q})$ pair will be produced at the eigenenergy of a QCD-confined $[(q\bar{q})^8 g^8]^1$ state as a QCD meson.

In a similar way, a $(q\bar{q})$ pair will be produced at the eigenenergy of a QED-confined $[(q\bar{q})^1 \gamma^1]^1$ state as a QED meson, if there is a QED-confined $[(q\bar{q})^1 \gamma^1]^1$ eigenstate at that QED eigenenergy. At all other energies, no $(q\bar{q})$ pair will be produced because continuum states of an isolated $q$ and $\bar{q}$ do not exist. In other words, reactions involving the production of a $q$ and a $\bar{q}$ contains the density of final-states factor, which is a delta-function at the eigenenergies of the confined $(q\bar{q})$ eigenstates of an interacting $q$ and $\bar{q}$ pair.

We can examine the $e^+ e^- \rightarrow q + \bar{q}$ reactions in Figs. 1(a) and 1(b) with a center-of-mass energy $\sqrt{s}(q\bar{q})$ in the range $(m_q + m_{\bar{q}}) < \sqrt{s}(q\bar{q}) < m_\pi$, where the sum of the rest masses of the light quark and light antiquark is of order a few MeV and $m_\pi \sim 135$ MeV \cite{17}. If there is a confined $[(q\bar{q})^1 \gamma^1]^1$ QED eigenstate in this energy range, then a confined $(q\bar{q})$ pair will be produced as a QED meson. In this energy range below $m_\pi$, a confined $(q\bar{q})$ pair, if it can ever be produced at a possible QED meson eigenstate, can only come from the QED interaction but not from the QCD interaction, because the QCD interaction with a gluon exchange would endow the $(q\bar{q})$ pair as a composite $[(q\bar{q})^8 g^8]^1$ QCD meson with a center-of-mass energy $\sqrt{s}(q\bar{q})$ beyond this energy range, in a contradictory manner. It is therefore possible that if a $(q\bar{q})$ QED meson eigenstates exist below the pion mass $m_\pi$, then a quark and an antiquark can be produced and can interact in the QED interaction alone to form a QED meson in this energy range. At energies other than the QED meson eigenenergies in this energy range below $m_\pi$, the $e^+ e^-$ collisions will probe the dynamics of a quark and antiquark interacting in QED alone, without the QCD interaction. In this energy range, the absence of fractional charges in $e^+ e^-$ collisions will indicate the absence of the continuum isolated quark and antiquark states when a quark and an antiquark interact in the QED interaction alone.

As the $e^+ e^-$ collision energy increases, many $(q\bar{q})$ pairs will be produced as shown in Fig. 1(c). For example, at the $Z^0$ resonance energy in the DELPHI experiments \cite{27,30}, many $(q\bar{q})$ pairs will be produced. Most of the produced $(q\bar{q})$ pairs will materialize as $(q\bar{q})$ QCD mesons labeled schematically as $h_i$ in Fig. 1(c). However, there may be a small fraction of the $(q\bar{q})$ pairs which

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Production of a quark $q$ and an antiquark $\bar{q}$ by the collision of $e^-$ and $e^+$ with a single intermediary virtual photon in (a) and two intermediary virtual photons in (b), with the production of many $(q\bar{q})$ pairs in (c).}
\end{figure}
will have invariant masses below the pion mass. If there is a confined \((q\bar{q})\) QED meson state at the appropriate eigenenergy below \(m_\pi\), then the \((q\bar{q})\) pair will be produced as a QED meson, shown schematically as the \(X\) particle in Fig. 1(c).

![Diagram](image)

**FIG. 2:** (a) \((q\bar{q})\) production by the fusion of two virtual gluons in the deexcitation of a highly-excited \(^4\text{He}\)(ABCD) \(\rightarrow\) \(^4\text{He}(\text{ground state})\) \((A'B'CD) + (q\bar{q})(X)\). (b) \((q\bar{q})\) production in hadron-hadron or a nucleus-nucleus collision by \(A + B \rightarrow A' + B' + (q\bar{q})^n \rightarrow A' + B' + \sum_i h_i + (q\bar{q})(X)\).

In another example as shown in Fig. 2(a), we show an excited state of \(^4\text{He}\) nucleus, that has been prepared in a low-energy \(p + ^3\text{H}\) collision. For example, the \(0^-\) excited state at 20.02 MeV of \(^4\text{He}\) can be formed by placing a proton in the stretched-out \(p\) state interacting with the \(^3\text{H}\) core in Fig. 2(a) \([30, 38]\). The excited \(^4\text{He}^*\) state can be depicted equivalently as pulling a proton out of a strongly bound alpha particle in a stretched-out configuration. The deexcitation of the excited system \(^4\text{He}^*\) down to the alpha particle ground state \(^4\text{He}_{g.s.}\) can occur by the proton emitting a virtual gluon which fuses with the virtual gluon from the \(^3\text{H}\) core, to lead to the production of a quark and an antiquark pair as shown in Fig. 2(a).

Gluons reside in the color-octet \(8\) representation. In the fusion of gluons in the reaction \(g + g \rightarrow q + \bar{q}\), the fusion gives rise to color multiplets as

\[
8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus 10 \oplus 10 \oplus 27, \tag{3}
\]

which contains the color-singlet component, \(1\), among other color multiplets. There is thus a finite probability in which a color-singlet \((q\bar{q})\) pair can be produced by gluon fusion. At very low energies in the deexcitation of the \(^4\text{He}^*\) nucleus, this gluon fusion process may occur if a QED meson eigenstate \(X\) exist at the appropriate energy, as shown schematically in Fig. 2(a).

In high-energy hadron-hadron collisions \([31, 35]\) and nucleus-nucleus collisions \([39, 40]\), many \((q\bar{q})\) pairs may be produced as depicted schematically in Fig. 2(b),

\[
A + B \rightarrow A' + B' + (q\bar{q})^n. \tag{4}
\]

The invariant masses of most of the produced \((q\bar{q})\) pairs will exceed or equal to the pion mass, and they will materialize as QCD mesons and labeled as \(h_i\) in Fig. 2(b). However, there may remain a small fraction of the color-singlet \((q\bar{q})\) pair with an invariant masses below \(m_\pi\), which allows the quark and the antiquark to interact in QED alone, without the QCD interaction, to lead to a possible QED meson eigenstate labeled schematically as \(X\) in Fig. 2(b).

In other circumstances in the deconfinement-to-confinement phase transition of the quark-gluon plasma in high-energy heavy-ion collisions, a deconfined quarks and a deconfined antiquark in close spatial proximity can coalesce to become a \((q\bar{q})\) pair with a pair energy below the pion mass, and they can interact in QED alone to lead to a possible QED meson.
### III. LIGHT QUARKS ARE CONFINED IN QED IN (1+1)D

Having settled on the possibility for the production of a quark and an antiquark pair interacting in QED alone, we proceed to explore whether there can be confined \((q\bar{q})\) eigenstates for quarks interacting in QED. Previously Schwinger showed that a massless fermion and antifermion interacting in an Abelian QED \(U(1)\) gauge interaction with a coupling constant \(g_{2D}\) in \((1+1)D\) are confined as a neutral boson with a mass

\[
m = \frac{g_{2D}}{\sqrt{\pi}}.
\]

The coupling constant \(g_{2D}\) has the dimension of a mass. Such a confining mechanism, involving a massless fermion and a massless antifermion to form a neutral massive boson in a gauge interaction, can be called the Schwinger mechanism\(^3\). The confinement in \((1+1)D\) occurs for all strengths of the coupling constant, including the weak as well as the strong interactions.

The QED interaction between a quark and its antiquark is a \(U(1)\) Abelian gauge interaction. Light quarks have rest masses of order a few MeV \([17]\), and they can be approximated as massless fermions. We can therefore apply the Schwinger mechanism to light quarks interacting in the QED interaction to conclude that light quarks are confined in QED in \((1+1)D\).

We can review here the mechanism how (light) quarks are confined in QED in \((1+1)D\) \([1, 2, 41]\). The quark vacuum is the lowest-energy state of the system with quarks filling up the (hidden) negative-energy Dirac sea and interacting with the QED interaction. It is defined as the state with no valence quarks as particles above the Dirac sea and no valence antiquarks as holes below the Dirac sea. A local QED disturbance \(A^\mu\) will generate stable collective particle-hole excitations of the quark-QED system as in a quantum fluid. Subject to the disturbing gauge field \(A^\mu\) with a coupling constant \(g_{2D}\) in \((1+1)D\), the massless quark field \(\psi(x)\) satisfies the Dirac equation,

\[
\gamma_\mu (p^\mu - g_{2D} A^\mu) \psi = 0.
\]

The gauge field \(A^\mu\) instructs the quark field \(\psi\) how to move. The quark field \(\psi\) in turn generates the quark current \(j^\mu\). The quark current \(j^\mu\) in turn generates the gauge field \(A^\mu\). By imposing the Schwinger modification factor to ensure the gauge invariance of the quark Green’s function, the quark current \(j^\mu(x)\) at the space-time point \(x\) induced by \(A^\mu\) can be evaluated. After the singularities from the left and from the right cancel, the gauge-invariant quark current \(j^\mu\) is found to relate explicitly to the perturbing QED gauge field \(A^\mu\) by

\[
j^\mu = -\frac{g_{2D}}{\pi} \left( A^\mu - \partial^\mu \frac{1}{\partial_\mu \partial_\eta} \partial_\eta A^\nu \right).
\]

It is easy to see that the above relationship between \(j^\mu\) and \(A^\mu\) satisfies gauge invariance upon a change of the the gauge in \((A^\mu)^' = A^\mu - \partial^\mu \Lambda\), for any local function of \(\Lambda\). The quark current \(j^\mu\) in turn generates the gauge fields \(A^\mu\) through the Maxwell equation,

\[
\partial_\mu F^{\mu\nu} = \partial_\mu (\partial^\mu A^\nu - \partial^\nu A^\mu) = g_{2D} j^\nu = g_{2D} \bar{\psi} \gamma^\nu \psi.
\]

A stable collective particle-hole excitation of the quark system occurs, when the disturbance \(A^\mu\) gives rise to the quark current \(j^\mu\) which in turn leads to the gauge field \(A^\mu\) self-consistently. We

\(^3\) For a review of the Schwinger mechanism, see Chapter 6 of \([41]\). For recent works and a generalization of the Schwinger model and the relation to the unparticle physics, see \([42–46]\).
impose this self-consistency condition of the gauge field and current by substituting the relation (7) to the Maxwell equation (8). We get both $j^\mu$ and $A^\mu$ to satisfy the Klein-Gordon equation

$$\partial^\nu \partial_\nu A^\mu + \frac{g_\text{D}^2}{\pi} A^\mu = 0,$$

$$\partial^\nu \partial_\nu j^\mu + \frac{g_\text{D}^2}{\pi} j^\mu = 0,$$

for a bound and confined boson with a mass given by $m = g_\text{D}^2/\sqrt{\pi}$ as in Eq. (5).

Consequently, the application of the Schwinger mechanism to quarks approximated as massless leads to a confined $(q\bar{q})$ boson state in QED in (1+1)D. The confined boson state can be viewed in two equivalent ways [18, 21]. It can be depicted effectively as a QED-confined one-dimensional open string, with a quark and an antiquark confined at the two ends of the open string subject to an effective linear two-body confining interaction. A more basic and physically correct description considers the boson as the manifestation of a collective particle-hole excitation involving the coupled self-consistent responses of quark current $j^\mu$ and the gauge field $A^\mu$. The quark confinement arises because the quark current $j^\mu$ and the gauge field $A^\mu$ depend on each other in a self-consistent manner and such self-consistency leads to a stable and self-sustainable space-time variations of both the current $j^\mu$ and the gauge field $A^\mu$. Through the Dirac equation for quarks, a space-time variation of the gauge field $A^\mu$ leads to a space-time variation in the quark current $j^\mu$, which in turn determines the space-time variation of the gauge field $A^\mu$ through the Maxwell equation [1, 2, 41]. As a consequence of such self-consistent dependencies, a quantized local space-time collective variations of the quark current $j^\mu$ and the QED gauge field $A^\mu$ can sustain themselves indefinitely at the lowest eigenenergy of the QED-confined $(q\bar{q})$ state in a collective motion as a one-dimensional open string with a mass, when the decay channels for the confined collective state are turned off for such an examination [18, 21].

From the above review, it is clear that the Schwinger mechanism is a many-body phenomenon containing dynamical quark effects beyond the potential interaction between a static quark and a static antiquark alone.

IV. GENERALIZATION OF THE SCHWINGER MECHANISM FROM QUARKS IN QED IN (1+1)D TO QUARKS IN (QCD+QED) IN (1+1)D

The Schwinger mechanism applies to all Abelian gauge theories with massless quarks in (1+1)D. Even though QCD is a non-Abelian gauge theory, many features of the QCD mesons (such as quark confinement, meson states, and meson production), mimick those of the Schwinger model for the Abelian gauge theory in (1+1)D, as pointed out by Casher, Kogut, and Susskind [17], Belvedere et al. [48], t’Hooft [49], Sekeido et al. [50], and Suzuki et al. [51] This suggests that for questions of quark confinement in QCD, QCD meson states, and the QCD meson production, the non-Abelian QCD interaction can be appropriately approximated as a quasi-Abelian interaction in (1+1)D [18, 21]. In such a quasi-Abelian interaction in (1+1)D dynamics, the Schwinger mechanism is applicable not only to quarks interacting in QED but also to quarks interacting in QCD [17]. We would like to review here how the Schwinger mechanism of quark confinement can be generalized from QED in (1+1)D to (QED+QCD) in (1+1)D, for stable collective excitations [18, 21].

Because of the three-color nature of the quarks, the quark current $j^\mu$ and the gauge field $A^\mu$ are $3\times3$ color matrices with 9 matrix elements. The 9 matrix elements in the color space can be separated naturally into color-singlet and color-octet subgroups of generators. Specifically, quarks reside in the $3$ representation and antiquarks reside in the $3^*$ representation, and they form a direct product of $3 \otimes 3^* = 1 \oplus 8$, with a color-singlet $1$ subgroup and a color-octet $8$ subgroup. The quark current $j^\mu$ and the QED and QCD gauge field $A^\mu$ are $3\times3$ color matrices which can be expanded in
terms of the nine generators of the U(3) group,

\[ j^\mu = \sum_{i=0}^{8} j^\mu_i t^i, \quad A^\mu = \sum_{i=0}^{8} A^\mu_i t^i, \quad t^0 = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \]

where \( t^0 \) is the generator of the U(1) color-singlet subgroup and \( t^1, t^2, ..., t^8 \) are the eight generators of the SU(3) color-octet subgroup.

In the general case of quark and QCD gauge field dynamics, the QCD gauge field \( A^\mu(X) = \sum_{i=1}^{8} A^\mu_i(X) t^a \) has eight non-Abelian degrees of freedom with \( A^\mu_i(X), a = 1, 2, ..., 8 \) in the color SU(3) generator space. The solution of the quark dynamics in QCD will correspondingly couple these eight degrees of freedom in the eight \( t^a \) directions. Such couplings in the non-Abelian degrees of freedom will lead to color excitations, the majority of which will not lead to stable collective excitations.

There is a known and simple way to get stable collective excitations of the QCD gauge field and quark current as carried out in [18, 21] by introducing a unit vector \( \tau^1 \) randomly in the eight-dimensional SU(3) generator space,

\[ \tau^1 = \sum_{a=1}^{8} n_a t^a, \quad \text{with} \quad \sqrt{n_1^2 + n_2^2 + ... + n_8^2} = 1, \]

where \( n_a \) are the components of generator vectors \( \tau^1 \) in the eight-dimensional \( t^a \) space, with \( n_a = \text{tr}(\tau^1 t^a)/2 \) oriented randomly. We restrict our considerations in the color-octet subspace with a fixed orientation of \( \tau^1 \) but allow the current \( j^\mu \) of the gauge field \( A^\mu \) projected along \( \tau^1 \) to vary. We represent the dynamics of the current \( j^\mu \) and the gauge field \( A^\mu \) of the quark-QCD-QED system in terms of the amplitudes \( j^\mu_\lambda \) and \( A^\mu_\lambda \) with \( \lambda = 0, 1 \) [18, 21]

\[ j^\mu = j^\mu_0 \tau^0 + j^\mu_1 \tau^1, \quad \text{and} \quad A^\mu = A^\mu_0 \tau^0 + A^\mu_1 \tau^1, \quad \lambda = \begin{cases} 0 \quad \text{QED} \\ 1 \quad \text{QCD} \end{cases}, \]

where \( \tau^0 = t^0 \) and the generators \( \tau^0 \) and \( \tau^1 \) satisfy \( 2\text{tr}(\tau^\lambda \tau^{\lambda'}) = \delta^{\lambda\lambda'} \), with \( \lambda, \lambda' = 0, 1 \). We vary the amplitudes \( j^\mu_0 \) and \( A^\mu_0 \) without varying the orientation \( \tau^1 \) in the 8-dimensional color-octet subspace. The randomness of the orientation of such a unit vector \( \tau^1 \) in the generator space ensures that it can cover a significant part of the available color space. Furthermore, because \( \tau^0 \) and \( \tau^1 \) commute, the dynamics of the current and gauge field in the generator subspace is Abelian.

Fixing the orientation of the unit vector \( \tau^1 \) is a special truncation of the non-Abelian QCD dynamics in a subspace which can lead to stable collective excitations. For lack of a proper name, we shall call such a special truncation the quasi-Abelian approximation of the non-Abelian dynamics. In such a quasi-Abelian approximation, the QED interaction and the QCD interaction are nonetheless retaining their individual characters with different generators \( t^0 \) and \( t^1 \).

Because of the self-consistency condition for stable collective excitations, the gauge field and the current field of each subgroup depend on each other within the same color subgroup. As a consequence, the two different currents and the gauge fields in their respective subgroups possess independent and self-consistent stable collective QED and QCD excitations at different energies, as shown in [18, 21]. The collective QCD and QED excitations can lead to the known QCD mesons and new, unknown QED mesons.

Through the solution of the Dirac equation for the quarks, the gauge-invariant quark current \( j^\mu_\lambda \)
can be evaluated as a function of the applied gauge field $A^\mu_\lambda$ to be

$$j^\mu_\lambda = -\frac{g_{2D\lambda}}{\pi} \left( A^\mu_\lambda - \partial^\mu \frac{1}{\partial \eta \partial \eta} \partial_\nu A^\nu_\lambda \right), \quad \lambda = \begin{cases} 0 & \text{QED} \\ 1 & \text{QCD} \end{cases}. \quad (13)$$

The Maxwell equations for the QED and QCD gauge fields in such a quasi-Abelian approximation of non-Abelian dynamics becomes

$$\partial_\mu (\partial^\mu A^\nu_\lambda - \partial^\nu A^\mu_\lambda) = g_{2D\lambda} j^\nu_\lambda, \quad \lambda = \begin{cases} 0 & \text{QED} \\ 1 & \text{QCD} \end{cases}. \quad (14)$$

Upon substituting such a relation $(13)$ to the Maxwell equation $(14)$, we get $j^\mu_\lambda$ and $A^\mu_\lambda$ both satisfy the Klein-Gordon equation

$$\partial_\nu \partial^\nu A^\mu_\lambda = 0, \quad \text{and} \quad \partial_\nu \partial^\nu j^\mu_\lambda = 0, \quad \lambda = \begin{cases} 0 & \text{QED} \\ 1 & \text{QCD} \end{cases}, \quad (15)$$

which gives a confined boson with a mass given by

$$m_\lambda = \frac{g_{2D\lambda}}{\sqrt{\pi}}, \quad \lambda = \begin{cases} 0 & \text{QED} \\ 1 & \text{QCD} \end{cases}. \quad (16)$$

Thus, the QED and QCD currents and gauge fields satisfy their corresponding Klein-Gordon equations with masses $m_\lambda$, depending on their respective coupling constants $g_{2D\lambda}$ [18, 21]. There are independent collective QED and QCD excitations of the quark-QCD-QED system in $(1+1)$D where these excitations can be described as open-string states of $(q\bar{q})$ pairs.

V. PHENOMENOLOGICAL OPEN STRING MODEL FOR QCD MESONS AND QED MESONS IN $(3+1)$D

Under the quasi-Abelian approximation for the non-Abelian dynamics in QCD, the Schwinger mechanism gives the generalized open-string solutions of confined and bound bosons for both QCD and QED, with masses that depend on the magnitudes of the gauge field coupling constants in $(1+1)$D, as given by Eq. $(16)$. For the QCD interaction, such a one-dimensional open-string solution of the lowest energy states for a quark-antiquark system in $(1+1)$D was suggested early on by the dual-resonance model [52], the Nambu and Goto meson string model [53, 55], the ‘tHooft’s two-dimensional meson model [56, 57], the classical yo-yo string model [59], Polyakov’s quantum bosonic string [60], and the Lund model [62]. The open-string description was subsequently supported theoretically for QCD by lattice gauge calculations in $(3+1)$D in which the structure of a flux tube shows up explicitly [63–68]. The open-string description for a quark-antiquark system in high-energy hadron-hadron reactions and $e^+e^-$ annihilations provided the foundation for the $(1+1)$D inside-outside cascade model of Bjorken, Casher, Kogut, and Susskind [47, 69], the yo-yo string model [59], the generalized Abelian Model [48], the projected Abelian model [49], the Abelian dominance model [50, 51], and the Lund model in high-energy collisions [62]. The flux tube description receives experimental support from the limiting average transverse momentum and the rapidity plateau of produced hadrons [41, 47, 69, 71, 73], in high-energy $e^+e^-$ annihilations [76, 80] and $pp$ collisions [81].
While a confined open string in (1+1)D as the idealization of a stable quark-antiquark system in (3+1)D is well known in QCD, not so well-known is the analogous confined open string quark-antiquark system in (1+1)D with a lower boson mass in QED, when we apply the Schwinger mechanism for massless fermions to light quarks in QED in (1+1)D, as discussed in Section 3 [11, 21, 22]. Could the confined quark-antiquark one-dimensional open string in QED in (1+1)D be the idealization of a stable QED $(q\bar{q})$ meson in (3+1)D, just as the confined quark-antiquark Nambu-Goto open string in QCD in (1+1)D can be the idealization of a stable QCD $(q\bar{q})$ meson in (3+1)D?

From the viewpoint of phenomenology, we note that no fractionally-charged particles have ever been observed. Thus, quarks cannot be isolated. Because quarks cannot be isolated, so a quark and an antiquark must be connected by a flux tube or its idealization as a string. The non-isolation of quarks is consistent with the hypothesis that the stable open string $q\bar{q}$ boson solution in (1+1)D is an idealization of a stable $q\bar{q}$ boson system of a confining flux tube between the quark and the antiquark in (3+1)D. On such a hypothesis, the open string description of a quark and an antiquark confined as a boson in QED is a reasonable concept. It can be the basis of a phenomenological description of QED mesons whose validity needs to be constantly confronted with experiments. We can therefore study the question of quark confinement in QED from the phenomenological point of view. It is also necessary to examine separately the question of quark confinement in QED from the lattice gauge calculations viewpoints in (3+1)D, to be taken up in Section 7.

In the phenomenological confrontation of the open string model for QCD and QED mesons in (1+1)D with experimental meson data in (3+1)D, we need an important relationship to ensure that the boson masses calculated in the lower (1+1)D can properly represent the physical boson masses in (3+1)D. The one-dimensional $(q\bar{q})$ open string in (1+1)D is an idealization of a flux tube with a transverse radius $R_T$ in the physical world of (3+1)D. In (3+1)D, the flux tube has a structure with a transverse radius $R_T$, but the coupling constant $g_{4D}$ is dimensionless. In contrast in (1+1)D, the open string does not have a structure, but the coupling constant $g_{2D}$ has the dimension of a mass. The (1+1)D open string can be considered an idealization of the physical meson in (3+1)D with a flux tube with a transverse radius $R_T$, if the coupling constants $g_{2D}$ in (1+1)D and $g_{4D}$ in (3+1)D are related by $\alpha_{4D} = g_{4D}^2/4\pi$. The qualitative consistency of the above equation can be checked by dimensional analysis. Thus, when the dynamics in the higher dimensional 3+1 space-time is approximated as dynamics in the lower (1+1)D, information on the flux tube structure is stored in the multiplicative conversion factor $1/\pi R_T^2$ in the above equation that relates the physical coupling constant square $(g_{4D})^2$ in (3+1)D to the new coupling constant square $(g_{2D})^2$ in (1+1)D. As a consequence, there is no loss of the relevant physical information. The boson mass $m$ determined in (1+1)D is the physical mass in (3+1)D, related to the physical coupling constant $\alpha_{4D}=(g_{4D})^2/4\pi$ and the flux tube radius $R_T$ by

$$m^2 = \frac{4\alpha_{4D}}{\pi R_T^2}. \quad (18)$$

Consequently, the masses of the QED and QCD mesons in (3+1)D in the open-string description are approximately

$$m^2_{QCD} = \frac{4\alpha_{QCD}}{\pi R_T^2}, \quad m^2_{QED} = \frac{4\alpha_{QED}}{\pi R_T^2}. \quad (19)$$
With \( \alpha_{\text{QED}} = \alpha_{\text{QED}} = 1/137 \), \( \alpha_{\text{QCD}} = \alpha_{s} \sim 0.68 \) from hadron spectroscopy \([18, 21]\) and \( R_{T} \sim 0.4 \) fm from lattice QCD calculations \([25]\) and \( \langle p_{T}^{2} \rangle \) of produced hadrons in high-energy \( e^{+}e^{-} \) annihilations \([18]\), we estimate the masses of the open string QCD and QCD mesons to be

\[
m_{\text{QCD}} \sim 458 \text{ MeV}, \quad \text{and} \quad m_{\text{QED}} \sim 48 \text{ MeV}.
\]

The above mass scales provide an encouraging guide for the present task of a quantitative description of the QCD and QED mesons as open strings, using QCD and QED gauge field theories in \((1 + 1)D\).

To get a better determination of the QCD and QED meson masses, it is necessary to take into account the flavor mixtures \( D_{ij} \), the quark rest masses \( m_{f} \), and the quark electric charges \( Q_{[u,d]}^{QED} \). Using the method of bosonization \([21]\) and including the quark rest mass and the chiral condensate as discussed in \([21]\), we obtain the semi-empirical mass formula for the neutral QCD mesons with \( N_{f} = 3 \). We can use the pion mass to calibrate the chiral condensate \( \langle \bar{\psi}\psi \rangle \). Therefore the masses of neutral QCD mesons is given by

\[
m_{\Lambda}^{2} = \left[ \sum_{f} D_{ij}^{\Lambda} Q_{f}^{j} \right]^{2} \frac{4\alpha_{\Lambda}}{\pi R_{T}^{2}} + m_{\pi}^{2} \frac{\sum_{f} m_{f}(D_{ij}^{\Lambda})^{2}}{m_{ud}}, \quad \lambda = \begin{cases} 0 & \text{QED} \\ 1 & \text{QCD} \end{cases},
\]

where \( m_{ud} = (m_{u} + m_{d})/2 \), \( \lambda = 0 \) for QED and \( \lambda = 1 \) for QCD. \( Q_{f}^{j} \) is the charge number of the quark with the flavor \( f \) with \( Q_{u}^{QED} = 2/3 \), \( Q_{d}^{QED} = -1/3 \), \( Q_{u}^{QCD} = Q_{d}^{QCD} = 1 \). The chiral condensate depends on the interaction type \( \lambda \), specifically on the coupling constant. We note that the chiral current anomaly in the chiral current depends on the coupling constant as \( e^{2} = g^{2} \) as gives Eq. (19.108) of \([33]\)

\[
\partial_{\mu}j_{\mu53}^{i} = -\frac{e^{2}}{32\pi} e^{\alpha_{QED}} F_{\alpha\beta} F_{\gamma\delta},
\]

which shows that the degree of non-conservation of the chiral current is proportional to \( e^{2} \). It is therefore reasonable to infer that the chiral condensate term depends on the coupling constant as \( g^{2} \) or \( \alpha \). Hence, we have \([21]\)

\[
m_{\Lambda}^{2} = \left[ \sum_{f=1}^{N_{f}} D_{ij}^{\Lambda} Q_{f}^{j} \right]^{2} \frac{4\alpha_{\Lambda}}{\pi R_{T}^{2}} + m_{\pi}^{2} \frac{\sum_{f=1}^{N_{f}} m_{f}(D_{ij}^{\Lambda})^{2}}{m_{ud}}, \quad \lambda = \begin{cases} 0 & \text{QED} \\ 1 & \text{QCD} \end{cases}.
\]

For the \( \pi^{0}, \eta, \) and \( \eta' \) the degree of admixture is known as stated in the Particle Data Book. Explicitly, it is given for \( \Phi_{i}^{\text{QCD}} = \sum_{f=1}^{N_{f}} D_{ij}^{\Lambda} |f \rangle \), and \( \Phi_{1} = \pi^{0}, \Phi_{2} = \eta, \) and \( \Phi_{3} = \eta' \), by

\[
\begin{pmatrix}
\Phi_{1} \\
\Phi_{2} \\
\Phi_{3}
\end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix}
\cos \theta_{P} - \sqrt{2} \sin \theta_{P} \\
\cos \theta_{P} - \sqrt{2} \sin \theta_{P} \\
\cos \theta_{P} - \sqrt{2} \sin \theta_{P}
\end{pmatrix} \begin{pmatrix}
0 \\
\bar{u}d \\
\bar{s}s
\end{pmatrix}.
\]

From the tabulation in PDG \([17]\), we find \( \theta_{P} = -24.5^{0} \) and \( m_{s}/m_{u} = 27.3_{-1.3}^{+0.7} \). Using the \( \pi^{0} \) mass as a calibration of the chiral condensate, we search for the flux tube radius \( R_{T} \) and the QCD coupling constant \( \alpha_{\text{QCD}} \) that can describe well the masses of \( \eta \) and \( \eta' \). We find \( \alpha_{\text{QCD}} = 0.68 \pm 0.08 \), and \( R_{T} = 0.40 \pm 0.04 \) fm. By extrapolating to the QED mesons with \( \alpha_{\text{QED}} = 1/137 \), we find an open string isoscalar \( I(J^{\pi}) = 0(0^{-}) \) QED meson state at \( 17.9 \pm 1.6 \) MeV and an isovector \( (I(J^{\pi}) = 1(0^{-}), I_{3} = 0) \) QED meson state at \( 36.4 \pm 4.8 \) MeV. We shall compare these predicted masses with the masses of
the anomalous X17 and E38 observed recently in the next Section.

TABLE I: The experimental and theoretical masses of neutral, $I_3=0$, QCD and QED mesons, obtained with the semi-empirical mass formula \((23)\) for QCD and QED mesons.

| $[I(J^P)]$ | Experimental mass (MeV) | Mass formula Eq. \((23)\) (MeV) |
|---|---|---|
| QCD meson $\pi^0$ | $1[0^+]$ | 134.9768±0.0005 | 134.9$^\ddagger$ |
| | $0[0^+]$ | 547.862±0.017 | 498.4±39.8 |
| | $0[0^-]$ | 957.78±0.06 | 948.2±99.6 |
| QED X17 meson $\eta'$ | $0[0^-]$ | 16.94±0.24$^\ddagger$ | 17.9±1.5 |
| | $1[0^-]$ | 37.38±0.71$^\ddagger$ | 36.4±3.8 |

$\ddagger$ Calibration mass
# A. Krasznahorkay et al., PRC104,044003(2021)
$\ddagger$ K. Abraamyan et al., EPJ Web Conf 204,08004(2019)

VI. POSSIBLE EXPERIMENTAL EVIDENCE FOR THE EXISTENCE OF THE QED MESONS

In a QED meson, the quark and the antiquark can annihilate to lead to the emission of two real photons ($\gamma_1\gamma_2$) as in Fig. 3(a), two virtual photons ($\gamma^*_1\gamma^*_2$) or two dilepton ($e^+e^-$) pairs as in Fig. 3(b), or a single ($e^+e^-$) pair as in Fig. 3(c). A QED meson can be detected by the invariant mass of its decay products.

FIG. 3: Decay of a QED meson $X$: (a) by decaying into two real photons $X \rightarrow \gamma_1 + \gamma_2$, (b) by decaying into two virtual photons which subsequently decay into two ($e^+e^-$) pairs, $X \rightarrow \gamma^*_1 + \gamma^*_2 \rightarrow (e^+e^-) + (e^+e^-)$, and (c) by decaying into a single ($e^+e^-$) pair, $X \rightarrow \gamma^*_1 + \gamma^*_2 \rightarrow e^+e^-$. We have discussed in Section 2 the production of ($q\bar{q}$) pairs in many low- and high-energy $e^+e^-$, hadron-hadron, and nucleus-nucleus collisions. A ($q\bar{q}$) pair will be produced and materialize as a QCD meson final state, when the center-of-mass energy $\sqrt{s}$ of the pair coincides with the eigenenergy of a QCD meson. The ($q\bar{q}$) will be produced and materialize as a QED meson in the energy range $(m_q + m_{\bar{q}}) < \sqrt{s(q\bar{q})} < m_{\pi}$ when $\sqrt{s(q\bar{q})}$ coincides with the eigenenergy of a QED meson. At energies different from the eigenenergies of QCD and QED mesons, no ($q\bar{q}$) pair will be produced, because quarks cannot be isolated.

For over several decades in many exclusive measurements in high-energy hadron-hadron collisions [31,35] and in high-energy $e^+e^-$ annihilations [27,30], it has been consistently observed that whenever hadrons are produced, anomalous soft photons in the form of excess $e^+e^-$ pairs, about 4 to 8 times of the bremsstrahlung expectations, are proportionally produced, and when hadrons are not produced, these anomalous soft photons are also not produced [27,35]. The transverse momenta of the excess $e^+e^-$ pairs lie in the range of a few MeV/c to many tens of MeV/c, corresponding to a mass scale of the ($e^+e^-$) pair from a few MeV to many tens of MeV.
In another experiment in the \( p^+7\text{Li} \rightarrow 8\text{Be}^* + e^+ + e^- \) reaction at ATOMKI at a proton beam energy of a few MeV, the \( I(J^\pi)=0(1^+) \) excited state of \( 8\text{Be}^* \) at 18.15 MeV was observed to decay to the \( 8\text{Be} \) ground state with the emission of a neutral “X17” boson with a mass of 16.70±0.35(stat)±0.5(syst) MeV [36]. Supporting evidence for this hypothetical X17 particle has been reported in [84–89]. There occurs in addition the observation of the E38 boson \( 16.84\pm0.16 \) MeV [37]. Earlier observations of similar \( e^+e^- \) pairs with invariant masses between 3 to 20 MeV in the collision of nuclei with emulsion detectors have been reported [33,38]. There occurs in addition the observation of the E38 boson with a mass of 37.38 ± 0.71 MeV from the \( \gamma\gamma \) invariant mass spectrum in high-energy \( pC \), dC, dCu collisions at momenta of 2.75, 3.83 and 5.5 GeV/c per nucleon, respectively, at Dubna [39,40]. There are furthermore possible \( \gamma\gamma \) invariant mass structures at 10-15 MeV and 38 MeV in \( pp \), and \( \pi^-p \) reactions in COMPASS experiments [101–107].

The anomalous soft photon, the X17 particle, and the E38 particles are anomalous particles because they lie outside the domain of known Standard Model particle families. Many different models have been presented to describe these anomalous soft photons as arising from quantized bosons [18,21] or from a continuous spectrum [90–99]. For the X17 and E38 particles, many speculations have also been proposed, including the cold quark-gluon plasma, QED mesons, the fifth force of Nature, the extension of the Standard Model, the QCD axion, dark matter and many others [112–123]. Among the different proposed mechanisms, we shall focus our attention on the proposed QED meson description of the anomalous particles as composite \((q\bar{q})\) systems in which the quark and the antiquark are confined and bound by their mutual QED interactions [18,25]. Such a description has the desirable prospect of linking the three separate phenomena of the anomalous soft photons, the X17 particle, and the E38 particle together in a consistent framework.

Owing to the simultaneous and proportional production of the anomalous soft photons alongside with hadrons in high-energy collisions, a parent particle of an anomalous soft photon is likely to contain elements of the hadron sector, such as a light quark and a light antiquark. Light quarks can be approximated as massless. Massless quarks interacting in QED in (1+1)D give rise to confined open string boson states, in accordance with the Schwinger mechanism, as discussed in Section 3 [11,2]. Schwinger’s \( m = g_{\text{QED}} \sqrt{\pi} \) relation in Eq. (5) will bring the quantized mass \( m \) of a confined \( q\bar{q} \) pair to the lower mass range of the anomalous soft photons, as shown in Eqs. (19) and (20). It is therefore proposed in [18,21] that the Schwinger mechanism of quark confinement in QED in (1+1)D may lead to confined and bound open string \((q\bar{q})\) states which may be the idealization of a flux tube in (3+1)D, showing up as QED-meson states with a mass of many tens of MeV. These QED mesons may be produced simultaneously with the QCD mesons in high-energy collisions in the mechanism as shown in Fig. 2(b) for hadron-hadron collisions and Fig. 1(c) for \( e^+e^- \) annihilations [27,33], and the excess \( e^+e^- \) pairs in the anomalous soft photons may arise from the decays of these QED mesons. Measurements of the invariant masses of excess \( e^+e^- \) and \( \gamma\gamma \) pairs in \( e^+e^- \) collisions will provide tests for the existence of the open string \((q\bar{q})\) QED mesons.

The phenomenological open string description of the the QCD and QED mesons in the last Section indicates that \( \pi^0, \eta, \) and \( \eta' \) particles can be adequately described as open string \((q\bar{q})\) QCD mesons. By extrapolating into the \((q\bar{q})\) QED sector in which a quark and an antiquark interact with the QED interaction, we find an open string isoscalar \( I(J^\pi) = 0(0^-) \) QED meson state at 17.9±1.5 MeV and an isovector \( (I(J^\pi) = 1(0^-), I_3 = 0) \) QED meson state at 36.4±3.8 MeV as listed in Table I. The predicted masses agree with the experimental masses of the X17 [36,38] and the E38 particles [33,39], lending support to the proposal that a quark and an antiquark may be confined in QED. Ongoing experiments to confirm the X17 and the E38 particles are continuing.

In addition to the search for a theoretical understanding on the QED meson masses, it is also necessary to understand the mechanism how these anomalous particles may be be produced. For
the production of the X17 particle in the decays of the excited states of \( ^4\text{He}^* \) and \( ^8\text{Be}^* \) at ATOMKI \[36, 38\], we envisage the scenario that the excited states of \( ^8\text{Be}(1^+\, 18.15 \text{ MeV}) \) and \( ^4\text{He}(0^-\, 20.02 \text{ MeV}) \) are formed by pulling a proton out of one of the alpha-particles of the (\( \alpha \))\(_n\)-nucleus core and by placing the proton on an orbital that is considerably outside the corresponding tritium core as shown in Fig. 2(a). The stretched string-like interaction between the proton and the tritium core polarizes the vacuum so much that the proton may emit a virtual gluon which fuses with the virtual gluon from the \( ^4\text{H} \) core as shown in Fig. 2(a) to lead to the production of a \((q\bar{q})\) pair by the reaction \( g + q \rightarrow q + \bar{q} \). At the appropriate \( \sqrt{s}(q\bar{q}) \) eigenenergy, the QED interaction between the \( q \) and the \( \bar{q} \) may result in the formation of the \((q\bar{q})\) bound state X17 \[18, 21\], which subsequently decays into \( e^+e^- \).

In other high-energy reaction processes as illustrated in Figs. 1 and 2(b), in \( e^-e^- \) annihilations in DELPHI experiments \[29, 30\], in hadron-hadron collisions \[29, 31, 32, 35\], and in nucleus-nucleus collisions at Dubna \[39, 40\], many \((q\bar{q})\) pairs may be produced. While most produced \((q\bar{q})\) pairs will lead to hadron production, there may however be \((q\bar{q})\) pairs with \((m_q + m_{\bar{q}}) < \sqrt{s}(q\bar{q}) < m_\pi \) for which the QED interaction between the quark and the antiquark may lead to the production of the X17 and E38 particles at the appropriate energies.

VII. QUESTIONS ON QUARK CONFINEMENT IN COMPACT QED IN (3+1)D FROM LATTICE GAUGE CALCULATIONS

It has been known for a long time since the advent of Wilson’s lattice gauge theory that a static fermion and a static antifermion in (3+1)D in compact QED interaction has a strong-coupling confined phase and a weak-coupling deconfined phase \[3\]. The same conclusion was reached subsequently by Kogut, Susskind, Mandelstam, Polyakov, Banks, Jaffe, Drell, Peskin, Guth, Kondo and many others \[4–14\]. The transition from the confined phase to the deconfined phases occurs at the coupling constant, \( \alpha_c = g_{\text{crit}}^2/4\pi = 0.988989481 \) \[15, 16\]. The magnitude of the QED coupling constant, \( \alpha_c = 1/137 \), places the QED interaction between a quark and an antiquark as belonging to the weak-coupling deconfined regime. Therefore, a static quark and a static antiquark are deconfined in lattice gauge calculations in compact QED in (3+1)D. Are a static quark and a static antiquark really deconfined in QED in the physical world of (3+1)D?

The deconfined static quark and static antiquark in the lattice gauge results in (3+1)D poses a serious question. There are experimental circumstances in which a quark and an antiquark can be produced and they can interact in QED alone, without the QCD interaction, as we discussed in Section 2. For example, we can study the reactions \( e^+e^- \rightarrow \gamma^* \rightarrow q + \bar{q} \) and \( e^-e^- \rightarrow \gamma^* \rightarrow q + \bar{q} \) with a center-of-mass energy range \((m_q + m_{\bar{q}}) < \sqrt{s}(q\bar{q}) < m_\pi \), where the sum of the rest masses of the quark and the antiquark is of order a few MeV and \( m_\pi \sim 135 \text{ MeV} \) \[17\]. The incident \( e^+ + e^- \) pair is in a colorless color-singlet state, and thus the produced \( q \) and \( \bar{q} \) pair and the quanta mediating their interactions must also combine together into a color-singlet final state. In this energy range, the produced \( q \) and \( \bar{q} \) in their coupled color-singlet \((q\bar{q})^1\) configuration can interact with the colorless Abelian \( U(1) \) QED interaction to form a color-singlet \([(q\bar{q})^1\gamma^1]\) final state, the \([(q\bar{q})^1\gamma^1]\) state will be produced as a QED meson, if there is a QED-confined \([(q\bar{q})^1\gamma^1]\) eigenstate at this eigenenergy. At energies other than the QED meson eigenenergies in this energy range below \( m_\pi \), the \( e^+e^- \) collision will probe the dynamics of a quark and antiquark interacting in QED alone, without the QCD interaction. The absence of fractional charges in collisions in this energy range in \( e^+e^- \) collisions will indicate the absence of the continuum isolated quark and antiquark states in the interaction of a quark and an antiquark in the QED interaction.

The solution of deconfined static quark and static antiquark in the lattice gauge calculations in QED in (3+1)D predicts that the \( q \) and \( \bar{q} \) produced in \( e^+e^- \) collisions in the range of energy below
$m_\pi$ will not be confined and would appear as fraction charges, when the quark interact with the antiquark in QED alone in (3+1)D. However, no such fractional charges have ever been observed. Furthermore, the phenomenological open-string QCD and QED meson model with the hypothesis of a confined $(q\bar{q})$ pair in QED in (3+1)D leads to QED meson and QCD meson spectra in agreement with experimental data, as discussed in Section 6. They indicate that the present-day lattice gauge calculations for compact QED in (3+1)D may not be complete and definitive, because the important Schwinger dynamical quark effects associated with light quarks has not been included. Future lattice gauge calculations with dynamical quarks in compact QED interactions in (3+1)D will be of great interest in clarifying the question of quark confinement in QED.

VIII. CONCLUSION AND DISCUSSIONS

Light quarks have rest masses of only a few MeV. They can be approximated as massless. In accordance with the Schwinger mechanism, massless quarks interacting in the Abelian gauge interactions are confined for all strengths of the gauge interaction in (1+1)D as in an open string, forming a neutral boson with a mass proportional to the magnitude of the coupling constant.

The QED interaction between the quark and the antiquark is an Abelian gauge interaction. The non-Abelian QCD interaction can also be approximated as a quasi-Abelian interaction, on questions of quark confinement and QCD meson states. Therefore, the Schwinger mechanism can be applied to quarks interacting in both the QED interaction and the QCD interaction, leading to confined $(q\bar{q})$ pairs in QED and QCD open string states in (1+1)D, with boson masses depending on the magnitudes of the QCD and QED coupling constants. Such a viewpoint is consistent with the QCD string description of hadrons in the Nambu\cite{53} and Goto\cite{53} string model, the string fragmentation models of particle production \[47, 59, 62\], the Abelian projection model \[49\], and the Abelian dominance model \[50, 51\]. Just as an open string in (1+1)D for a QCD $(q\bar{q})$ system is an adequate idealization of a flux tube for the QCD $(q\bar{q})$ system in (3+1)D, we inquire here whether an open string in (1+1)D for a QED $(q\bar{q})$ system is similarly an adequate idealization of a flux tube for the QED $(q\bar{q})$ system in (3+1)D.

In the phenomenological open string model in (1+1)D for both QCD and QED mesons, we need an important relationship to ensure that the boson mass calculated in the lower (1+1)D can properly represent the mass of a physical boson in (3+1)D. The open string (1+1)D can describe a physical meson if the structure of the flux tube is properly taken into account. This can be achieved by relating the coupling constant in (1+1)D with the coupling constant in (3+1)D and the flux tube radius $R_T$ \[16, 25, 72, 82\]. Using such a relationship, we find that that $\pi^0, \eta, \text{ and } \eta'$ can be adequately described as open string $(q\bar{q})$ QCD mesons. By extrapolating into the $(q\bar{q})$ QED sector in which a quark and an antiquark interact with the QED interaction, we find an open string isoscalar $I,(J^\pi)=0(0^-)$ QED meson state at $17.9\pm1.5$ MeV and an isovector $(I,(J^\pi)=1(0^-), I_3=0)$ QED meson state at $36.4\pm3.8$ MeV. The predicted masses of the isoscalar and isovector QED mesons are close to the masses of the reported X17 and E38 particles observed recently, making them good candidates for these particles. Experimental confirmation of the reported X17 and the E38 particles will shed light on the question of quark confinement for quarks interacting in in Abelian U(1) QED interaction.

On the theoretical front, there is a need for theoretical clarification on the question of confinement with regard to lattice gauge calculations. The results of the present-day lattice gauge calculations would indicate that a static quark and a static antiquark interacting in the compact QED interaction will not be confined in (3+1)D. However, the deconfined solution for static quark and static antiquark in compact QED in (3+1)D in lattice gauge calculations contradicts the experimental absence of fractional charges. This indicates that the present-day lattice gauge calculations for compact QED
in (3+1)D may not be complete and definitive, because the important Schwinger dynamical quark effects associated with light quarks has not been included.

We have constructed a "stretch (2+1)D" flux tube model to investigate the importance of the Schwinger mechanism on quark confinement in QED in (3+1)D [24] by utilizing the Polyakov's transverse confinement in conjunction with Schwinger's longitudinal confinement. We find that the stretch (2+1)D flux tube model leads to quark confinement in compact QED in (3+1)D [25]. Such a quark confinement result of the stretch (2+1)D flux tube model is consistent with the experimental absence of fractional charges. Furthermore, it gives predictions in agreement with experimental QCD and QED meson spectra. It is therefore worthy of further considerations. It is important to find out whether future lattice gauge calculations with dynamical light quarks, in a configuration such as the stretch (2+1)D flux tube configuration, will lead to confined quarks in compact QED in (3+1)D.

The success of the open-string description of the QCD and QED mesons leads to the search for other neutral quark systems stabilized by the QED interaction between the constituents in the color-singlet subgroup, with the color-octet QCD gauge interaction as a spectator field. Of particular interest is the QED neutron with the color-singlet current and the color-singlet QED gauge field reside. In the color-singlet d-u-d system with three different colors, The attractive QED interaction between the u quark and the two d quarks overwhelms the repulsion between the two d quarks to stabilize the QED neutron at an estimated mass of 44.5 MeV. The analogous QED proton has been found to be unstable, and it does not provide a bound state nor a continuum state for the QED neutron to decay onto by way of the weak interaction. Hence the QED neutron may be stable against the weak interaction. It may have a very long lifetime and may be a good candidate for the dark matter. Because QED mesons and QED neutrons may arise from the coalescence of deconfined quarks during the deconfinement-to-confinement phrase transition in different environments such as in high-energy heavy-ion collisions, neutron-star mergers [124,126], and neutron star cores [127], the search of the QED bound states in various environments will be of great interest.

In concluding this paper, we wish to pay tribute to Jean Cleymans for his pioneering contributions to the physics of confined and deconfined quarks. We also wish to pay tribute to Jean's life-long journey in bridging the physics communities from the West, the North, the East, and the inclusion of the South. Born and educated in the West, Jean dedicated his long career in the education of physicists in the South, worked hard to promote the free flow and exchanges among physicists from the East and the North. Jean held it important that contacts and collaborations among physicists of all nations would bring a better understanding among peoples of different nations and would promote the development of science and technology. He will be remebered by all those who know him.

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