Charged pion photoproduction is proposed as a new method to study halo nuclei. In particular, it is demonstrated that the reaction \( ^{17}\text{O}(\gamma, \pi^-)^{17}\text{F}(\frac{1}{2}^+, 0.495 \text{ MeV}) \) is well-suited to investigate the well-known proton halo in \(^{17}\text{F}\) which is not amenable to radioactive beam experiments. The cross sections are shown to be very sensitive to the halo structure of the loosely bound proton.
Much information on the momentum distributions of halo nucleons in halo nuclei has been gained from heavy ion collisions. One of the most striking experimental feature of reactions in which a halo system is broken up is the extremely narrow momentum distribution of the fragments, reflecting the spatially extended nature of the halo wave functions with r.m.s radii of $5 - 7$ fm. Among the best-studied cases of neutron-rich nuclei are $^{11}$Be as a single-neutron halo and $^{11}$Li as a loosely bound two-neutron halo. The analogues of these systems near the proton drip line are all unbound. Proton halos are hampered by the additional Coulomb barrier outside the nuclear surface that tends to cut off the tail of the proton wave function.

The best candidates for proton halos are those formed by valence protons in $s$ states. Such an example with a clear signature for a proton halo is the first excited $\frac{1}{2}^+$ state of $^{17}$F; the separation energy of the valence proton is only $105$ keV. This state plays an important role in stellar nucleosynthesis since radiative proton capture on $^{16}$O is responsible for the breakout from the carbon-nitrogen-oxygen (CNO) cycle. This transition proceeds from a continuum $p$-state to this excited state and is strongly enhanced by the tail of the valence proton of the bound state. Such a tail is the typical signature of a halo structure. The halo nature of the proton has also been inferred from the large Thomas-Ehrman Coulomb shift relative to the $\frac{3}{2}^+, T = \frac{1}{2} (0d_{\frac{3}{2}})$ ground state and from the large $1/2^+$ to $5/2^+$ $B(E2)$ value. Furthermore, in a study of the first forbidden $\beta$-decay of $^{17}$Ne to the excited state of $^{17}$F the decay rate was found to be almost a factor two above the rate obtained from the well-known mirror decay of $^{17}$O to the first excited state of $^{17}$O. While this largest mirror asymmetry for first-forbidden decays between bound states recorded to date was considered to be clear evidence of an extended proton structure in $^{17}$F, Millener argued that these differences can be explained in terms of charge-dependent effects. These charge-dependent effects themselves also arise from the modifications of the radial wave functions of the valence nucleons due to the loose binding of the $s$-state valence proton.

The reason for the difficulties in the interpretation of this (excited state) single proton halo is that its reaction cross section and momentum distribution cannot be studied in radioactive beam experiments. Herein, we propose to use the reaction $^{17}$O($\gamma, \pi^-)$ to extract the proton halo structure of this state. In principle, halo states may be formed by single charge exchange reactions on stable nuclei that fall into two broad classes: those mediated by the strong interaction ($[p,n], (n,p)$, or $(\pi^\pm, \pi^0)$) and those mediated by the electromagnetic interaction ($[(\gamma, \pi^\pm)]$). The latter are favored for the study of halo states as they are soft electromagnetic reactions, and can largely preserve the momenta of the valence nucleons. The purpose of this investigation is not only to demonstrate the feasibility of pion photoproduction experiments leading to halo nuclear states, but also to show that such experiments will indeed be sensitive to the details of the wave functions of the valence nucleons. We stress that this investigation is exploratory and meant to stimulate theoretical discussion and experimental efforts. Ideally, one would also require complementary data from nucleon or pion charge exchange reactions leading to the same halo states. Together with the photopion data, they would provide the most sensitive tests available for the halo wave functions.

In the past 20 years, pion photoproduction from light nuclei has developed into an important tool in addition to electron and proton scattering to examine the nuclear structure of light nuclei. The four ingredients in the theoretical description of the $(\gamma, \pi^\pm)$ reaction...
are: 1) the nuclear structure transition density matrix elements, 2) the amplitude for the elementary pion photoproduction process off a free nucleon, 3) the single-particle shell-model wave functions, and, 4) the final state interaction of the outgoing pion with the residual nucleus. Using the notation of Ref. [10] the many-body matrix elements may be written as

$$\langle J_f M_f T_f P_f; \pi | T | J_i M_i T_i P_i; \gamma \rangle = \sum_{\alpha, \alpha'} \left( J_f M_f T_f P_f \right| a^\dagger_{\alpha'} a_{\alpha} \left| J_i M_i T_i P_i \right\rangle \langle \alpha'; \pi | t | \alpha; \gamma \rangle ,$$

(1)

where $\alpha$ denotes $\{n, l, j, m, \rho\}$ and the indices $i$ and $f$ refer to the initial and final nuclear states. The nuclear structure matrix element is expanded in terms of the one-body transition density matrix elements (OBDME), reduced in both spin and isospin, i.e.

$$\Psi_{JT}(\alpha', \alpha) = \frac{1}{\sqrt{(2J + 1)(2T + 1)}} \left( J_f T_f \right| \left[ \left[ a^\dagger_{\alpha'} \times \tilde{a}_{\alpha} \right]^{JT} \right| \left| J_i T_i \right\rangle .$$

(2)

The one-body transition operator, $t$, is given by

$$t = \left( L + i \vec{\sigma} \cdot \vec{K} \right) \frac{T-\beta}{\sqrt{2}}$$

(3)

$$= \sum_{S, M_S} i^S (-1)^{M_S} \sigma^S_{-M_S} K^{S^\beta}_{M_S} \frac{T-\beta}{\sqrt{2}},$$

(4)

where $L$ and $\vec{K}$ are the spin 0 and spin 1 transition operators [10], respectively. As the elementary pion production amplitude we choose the simple operator of Blomqvist and Laget (BL) [11,12] which has been designed specifically with nuclear applications in mind. While more sophisticated approaches have been developed in recent years [13] which give a good description of the $(\gamma, \pi)$ multipoles they are less suitable for nuclear applications since their off-shell extrapolation is not straightforward. The BL amplitude incorporates the essential features and gives an adequate description of the $N(\gamma, \pi)N$ cross section data. The BL operator consists of tree-level Born terms that include the model-independent Kroll-Ruderman (KR) term, as well as the contribution from the $\Delta(1232)$. Future studies should use the more advanced production operators that adequately include unitarity dynamically.

The matrix element of this operator between bound nuclear states is

$$\langle \alpha'; \pi | t | \alpha; \gamma \rangle = \int d^3 p d^3 q' \Psi^*_{\alpha'}(p') \varphi^{(-)*}_{\pi}(q', \vec{q}) t_{\gamma, \pi}(p, p', \vec{K}, \vec{q}) \Psi_{\alpha}(p)$$

(5)

where $p$ is the momentum of the initial nucleon and $p' = p + \vec{K} - \vec{q}'$ from momentum conservation. The outgoing (distorted) pion wave function, obtained as a solution of the Schrödinger equation with the optical potential of Stricker, McManus and Carr [14], is denoted by $\varphi^{(-)*}_{\pi}(q', \vec{q})$.

The matrix element may be expressed as [10]

$$\langle \alpha'; \pi | t | \alpha; \gamma \rangle = \sum_{LSJM} i^S (-1)^{j-m+\frac{1}{2}-\rho'+s} \sqrt{6} \sqrt{(2j' + 1)(2L + 1)(2S + 1)}$$

$$\times \left( \begin{array}{ccc} \frac{1}{2} & \frac{1}{2} & 0 \\ -\rho' & 1 & \beta \end{array} \right) \left( \begin{array}{ccc} j' & j & J \\ -m' & m & M \end{array} \right) \left\{ \begin{array}{ccc} l' & \frac{1}{2} & j' \\ l & \frac{1}{2} & j \end{array} \right\} f^{(a')LSJ}_{-M}.$$
In $LS$-coupling the radial integral, $I_{M}^{(a'a)LSJ}$, therein is given by

$$I_{M}^{(a'a)LSJ} = \int d^3p \, d^3q' \, \varphi^*_{n'l'j'}(p') \varphi_{nlj}(p) \varphi_{\pi}^{(+)}(q', \vec{q}) \left[ Y_{l'm'j'}(\hat{p}') \times Y_{lmj}(\hat{p}) \right] L \times K^S_{JM}. \quad (7)$$

This integral is dependent on the single particle wave functions of the outgoing pion, and of the initial and final state nucleons involved in the transition. It is through this integral that the halo wave function enters into the calculation. Hence the pion photoproduction reaction probes the whole wave function of the halo nucleon. This is in contrast to measurements of the momentum distributions of the halo nucleons in heavy ion collisions, where the part of the wave function inside the core is hidden [15].

To illustrate the sensitivity of the cross section to the halo nature of the valence proton, we contrast the difference between the results obtained using the harmonic oscillator (HO) and Woods-Saxon (WS) single particle wave functions. As the charge radius for $^{17}$O is the same as that for $^{16}$O [16], the HO parameter for the present calculations was 1.7 fm, as obtained from an analysis of the elastic electron scattering form factor for $^{16}$O [17]. The WS wave function for the $1s_{\frac{1}{2}}$ proton was obtained by solving the Schrödinger equation with a Woods-Saxon potential that reproduces the single particle binding energy of 105 keV, the separation energy of that proton [2]. The difference between the single particle densities obtained using the two wave functions is shown in Fig. 1. In contrast to the HO density the tail of the WS wave function exhibits the dramatic enhancement of the tail, characteristic of halo particles, and is similar to single-neutron halos such as the $1s_{\frac{1}{2}}$ state of $^{11}$Be. The r.m.s. radius of our WS wave function is 5.1 fm. The enhancement at large $r$ requires a reduction of the strength at short distances to preserve normalization. Note that the node occurs at around 2 fm for both HO and WS densities.

The $^{17}$O and $^{17}$F mirror nuclei are described by pure single particle states outside a closed $^{16}$O core in the $0\hbar \omega$ shell model. This is a simple model: the transverse magnetic form factor for the elastic scattering of electrons from $^{17}$O is suppressed compared to the predictions of the single particle model [18]. Yet the magnetic moment is $-1.894 \, \mu_N$ [2], which is very close to that of the neutron ($-1.913 \, \mu_N$). Our simple model may then be valid as a first approximation. The 0.495 MeV excited state therefore has a much simpler proton halo configuration compared to $^{8}$B. As such, the $^{17}$O($\gamma, \pi^-)^{17}$F reaction is effected by a pure $0d_{\frac{5}{2}} \rightarrow 0d_{\frac{5}{2}}$ (ground state) or $0d_{\frac{5}{2}} \rightarrow 1s_{\frac{1}{2}}$ (excited state) single particle transition. In each case the isovector OBDME, in $jj'$-coupled form, are unity for all given multiplicities.

The results of our calculations of the $^{17}$O($\gamma, \pi^-)^{17}$F differential cross sections, for $E_\gamma = 200$ MeV, are displayed in Fig. 2(a). The result for the ground state cross section obtained using the HO wave function is displayed by the dashed line while that obtained using the WS wave function is displayed by the dotted line. The binding energy for the WS wave function in this case was set to the separation energy, 600 keV [2], of the proton in the ground state. The result of the ground state cross section at forward angles is pronounced, around $3 \, \mu b/\text{sr}$, decreasing to $\sim 1 \, \mu b/\text{sr}$ at backward angles. This enhancement at forward angles may be explained by the exact overlap of the $0d_{\frac{3}{2}}$ neutron and proton single particle wave functions. There is only a negligible difference between the HO and WS results indicating that the ground state is not a halo, as expected. The WS wave function does not have a pronounced tail in this case.

The results of the cross section for the transition to the excited state obtained using the HO and WS wave functions are displayed in Fig. 2(a) by the solid and dot-dashed lines.
respectively. The cross section is suppressed at forward angles by the orthogonality of the $0d_{5/2}$ neutron in $^{17}$O with the $1s_{1/2}$ proton in $^{17}$F*. However, at backward angles the cross section rises to around $1 \mu b/sr$. Using the halo WS wave function instead of the standard HO density reduces the excited state cross section at large angles by a factor of two. That dramatic reduction at large momentum transfer can be traced to the reduction of the $1s_{1/2}$ radial wave function at short distances, as illustrated in Fig. 1.

In order to cleanly extract the proton halo wave function one requires as little uncertainties from the pion photoproduction process as possible. Hence, we compare plane wave impulse approximation (PWIA) and distorted wave impulse approximation (DWIA) results for the excited state cross section, as obtained using the HO wave function, in Fig. 2(b). The pion final state interaction at these energies affects the cross section by at most 10%; furthermore, the uncertainty due to pion distortion would be only a fraction of this. Thus the size of this effect is much smaller than the sensitivity to the halo wave function.

Much of the excited state cross section comes from the KR term. The other Born terms provide most of the remaining cross section, with at most 10% coming from the $\Delta(1232)$. It has been the latter term which has lead to strong model dependencies in reactions like $^{14}$N$(\gamma, \pi^+)^{14}$C(g.s.) at higher photon energies [13,20]. With a photon energy of 200 MeV one remains well below the kinematic region in which the $\Delta$ resonance is excited. The dominance of the KR term can be traced to that of the $M3$ multipole in the transition, providing about 70% of the cross section (as a $\frac{5}{2}^+ \rightarrow \frac{1}{2}^+$ transition, the $E2$ and $M3$ nuclear multipoles can contribute), and it is this multipole which is known to be almost totally dominated by the KR term [10,21]. Therefore, we may conclude that uncertainties in the elementary amplitude are minimal and will not obscure the extraction of the halo state. Clearly, any remaining small uncertainties from nuclear structure or the elementary operator would affect both calculations equally and therefore would not alter our findings.

The dramatic sensitivity to the halo structure in $^{17}$F* and the size of the differential cross section makes the reaction $^{17}$O$(\gamma, \pi^-)^{17}$F reaction an excellent candidate with which to study the halo experimentally at low-energy monoenergetic photon facilities, such as that at Duke, or using the $(e, e'\pi^+)$ reaction as is possible at Mainz. The comparable magnitude of the ground and excited state cross sections for backward angles will help ease the separation between the two transitions with the appropriate high resolution detectors. The simplifications employed in this investigation may influence the overall magnitude of the cross section and further work is required in order to establish that magnitude. However, as those approximations were applied equally to both the halo and non-halo calculations, the effect of the halo is real. We point out that both the sensitivity and the cross sections are much larger in $^{17}$O$(\gamma, \pi^-)^{17}$F than in comparable processes that lead to neutron-halo nuclei, such as $^{11}$B$(\gamma, \pi^+)^{11}$Be or $^{15}$N$(\gamma, \pi^+)^{15}$C [22].

In conclusion, we have demonstrated that the reaction $^{17}$O$(\gamma, \pi^-)^{17}$F(0.495 MeV) displays a very clear signature of the proton halo in $^{17}$F which cannot be studied in radioactive beam experiments. Thus, the charged pion photoproduction reaction at low energy provides an alternative means by which to study the momentum distributions of the halo nucleons in halo nuclei. In particular, these reactions can probe the effect of the wave function of the halo particle inside the core, a region not accessible to heavy ion collisions. The study of halo nuclei through pion photoproduction therefore has significant potential to expose unique signatures of these exotic structures.
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FIGURES

FIG. 1. Radial wave functions, $R_{nlj}(r)$, for the $1s_{1/2}$ orbit in $^{17}$O. The HO ($b = 1.7$ fm) and WS ($E_B = 105$ keV) wave functions are displayed by the solid and dashed lines respectively.

FIG. 2. Differential cross section for $^{17}$O($\gamma, \pi^{-}$)$^{17}$F at $E_\gamma = 200$ MeV. The cross sections leading to the ground and first excited state of $^{17}$F are displayed in (a) wherein the HO calculation for the excited state is displayed by the solid line, while the WS result is displayed by the dot-dashed line. The ground state cross section obtained using the HO and WS wave functions are displayed by the dashed and dotted lines respectively. The PWIA and DWIA results for the excited state cross section obtained using the HO wave functions are displayed in (b) by the solid and dashed lines respectively.
\[ \rho_{1s1/2} \text{ (fm}^{-3}) \]

Fig. 1
Fig. 2

\[ \frac{d\sigma}{d\Omega} (\mu b/sr) \]

vs

\[ \theta_\pi (\text{deg}) \]