Numerical simulation of dimension effect on critical field of rectangular superconductor

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Abstract. Numerical simulation of dimension variations was observed to determine the critical field of rectangular superconductors. The object was superconductor type-II and considered in the vacuum. The external magnetic field is applied to the \( XY \)-plane superconductors perpendicularly. The TDGL equations and the boundary condition were applied to the assumptions and solved using \( \psi U \) method numerically. Afterward, it yielded numerical simulation that can be used to study the lower critical field \( (H_{c1}) \) and the higher critical field \( (H_{c3}) \). The input is dimension variations \( N_x \times N_y \) and the value of Ginzburg-Landau parameter \( \kappa = 2.00 \) for all dimension variations. The result of the numerical simulation showed that when the ratio between circumference and area of the rectangular superconductor decreased, the value of \( H_{c1} \) and \( H_{c3} \) would also decrease.

1. Introduction
The Time Dependent Ginzburg-Landau (TDGL) equations can be used to analyze irreversible phenomena in a superconductor. The phenomena are related to the dynamical properties of superconducting magnetic vortices. In other words, the TDGL equations can be used to study superconducting properties [1-3]. The TDGL equations have high nonlinearity, so numerical solution is a possible tool to do [4]. Numerical analysis of TDGL equations for studying superconducting properties had been widely practiced in earlier studies [5-7]. The \( \psi U \) method is the most commonly used numerical method to solve TDGL equations. The approach using \( \psi U \)-method is to preserves the gauge invariance of TDGL equations after they are discretized and to obtain numerical convergence at high magnetic field [8].

The numerical analysis of the TDGL equations by the \( \psi U \)-method is interesting to study. Dynamical superconductor and its properties have been studied in various geometric shapes and superconducting states by applying TDGL equations which were solved by \( \psi U \) method [5,9].

The TDGL equations can be written as [5,7],
\[
\frac{\partial \psi(\mathbf{r},t)}{\partial t} = \left( \nabla - i \mathbf{A}(\mathbf{r},t) \right)^2 \psi(\mathbf{r},t) + \psi(\mathbf{r},t) - \left| \psi(\mathbf{r},t) \right|^2 \psi(\mathbf{r},t) \quad (1)
\]
\[
\sigma \frac{\partial \mathbf{A}(\mathbf{r},t)}{\partial t} = \frac{1}{2i} \left[ \psi(\mathbf{r},t) \nabla \psi(\mathbf{r},t) - \psi(\mathbf{r},t) \nabla \bar{\psi}(\mathbf{r},t) + 2i \bar{\psi}(\mathbf{r},t) \mathbf{A}(\mathbf{r},t) \right] - \kappa^2(\mathbf{r}) \nabla \times (\nabla \times \mathbf{A}(\mathbf{r},t) - \mathbf{H}_{ext}(\mathbf{r},t)) \quad (2)
\]
If the superconductor in vacuum, the boundary condition for \( \psi \) and \( \mathbf{A} \) as follows
\[ \hat{n} \cdot [\nabla - iA] \psi = 0 \]  
(3)

and the magnetic field boundary condition is
\[ \mathbf{H}_{\text{ext}} = \nabla \times \mathbf{A} \]  
(4)

where \( n \) is the normal vector of the superconducting boundary, \( \psi \) is (the complex-valued) parameter order, \( A \) and \( \mathbf{H}_{\text{ext}} \) are the vector potential and the external magnetic field. In the above equations, \( \mathbf{r} \) as a length is expressed in unit scale of the coherence length \( \xi(T) \), \( \mathbf{t} \) as a time is expressed in unit scale \( \tau(T) = \frac{\xi(T)}{2 \kappa(T)} \), \( \psi \) in \( \phi_{0} = (\alpha(T)/\beta)^{1/2} \), \( \mathbf{A} \) in \( A_{0} = \mu_{0} \mathbf{H}_{c2}(T) \xi(T) \), \( \mathbf{H}_{\text{ext}} \) is scaled in \( \mathbf{H}_{c2}(T) \), \( \sigma \) in \( \sigma_{0} = \frac{1}{\mu_{0} \kappa(T)} \), \( \mathbf{H}_{c2}(T) \) is the upper critical field, \( \sigma \) is the electrical conductivity, \( \alpha(T) \) and \( \beta \) are expansion coefficients, \( \kappa(T) \) is Ginzburg-Landau parameter, \( D \) is phenomenological diffusion constant [5,7].

In this paper, we study the critical field that appeared due to the influence of dimension variations by applying the Time Dependent Ginzburg-Landau equations and the boundary conditions that were solved by \( \psi \mathbf{U} \) method numerically. It would yield numerical simulation that can be used to study the lower critical field \( (H_{c1}) \) and the higher critical field \( (H_{c3}) \).

2. Numerical Method
The object studied was superconductor type-II and considered in the vacuum. The superconductor is represented as the rectangular shape in the \( xy \)-direction with dimension \( N_{x} \times N_{y} \). External magnetic field \( (\mathbf{H}_{\text{ext}}) \) uniformly was applied to the \( XY \)-plane superconductor perpendicularly as shown in Figure 1. The Time Dependent Ginzburg-Landau equations and the boundary condition were applied to the assumptions and solved using the \( \psi \mathbf{U} \) method numerically [1,4-7]. The \( \psi \mathbf{U} \) method involves the magnitude of \( \psi \) and \( U \) where the superconductor consists of size cells \( \Delta_{x} \times \Delta_{y} \). \( \mathbf{H}_{\text{ext}} \) was applied to the superconductor as time increases slowly, so that we could obtain the output \( \psi \) as a function of \( \mathbf{H}_{\text{ext}} \).

Because of external magnetic field \( (\mathbf{H}_{\text{ext}}) \) was applied to the superconductor, we could obtain the output \( \psi(r,H_{\text{ext}}) \) where \( r \) is a function of space \( (x,y) \).

![Figure 1. The rectangular superconductor is applied external magnetic field perpendicularly](image)

3. Result and Discussion
The input-dimensional for simulation program is given in Table 1 with a value of \( \kappa = 2.00 \) for all dimension variations, where each grid are composed of cells with size \( \Delta_{x} \times \Delta_{y} = 0.5\xi(T) \times 0.5\xi(T) \). Based on the input, we could obtain \( \psi(r,H_{\text{ext}}) \). Afterward, we could acquire figure \( |\psi|^{2} \) of \( H_{\text{ext}} \) function. \( H_{c1} \) and \( H_{c3} \) can be determined from the first local maximum point and zero points of the \( |\psi|^{2} \) graph.
[10] as shown in Fig. 2, Fig. 3 and Fig. 4. From the $|\psi|^2-H_{ext}$ figure, it appeared that the dimension influences the graph pattern. Since $Hc_1$ and $Hc_3$ were determined by the $|\psi|^2-H_{ext}$ graph, the values of $Hc_1$ and $Hc_3$ were affected by the dimension as well.

Figure 2. $|\psi|^2-H_{ext}$ graph for basis $N_xN_y=12x12$

Figure 3. $|\psi|^2-H_{ext}$ graph for basis $N_xN_y=32x32$
The values of $H_{c1}$ and $H_{c3}$ for each dimension variations were shown in Fig. 5 and Fig. 6. Based on the figures, it can be seen that the input of the dimensions affected the values of $H_{c1}$ and $H_{c3}$ due to simulation calculation $\psi(r,H_{ext})$ used the TDGL equations and boundary condition. The TDGL equations can be used to determine the states of $\psi$ in the middle superconductor. Meanwhile, the boundary condition can be used to determine the states of $\psi$ in the edge superconductor. If the dimension of superconductor is greater, the effect of the TDGL equations on the numerical calculation is more dominant than the boundary conditions. If the dimension of the superconductor is smaller, the effect of boundary condition on the numerical calculation becomes more dominant.

Based on the numerical simulations, we observed the values of $H_{c1}$ and $H_{c3}$ on each basis $N_xN_y$. The circumference of the rectangular superconductor was compared with the area of the rectangular superconductor on each basis $N_xN_y$. On the basis $N_xN_y$, the values of $H_{c1}$ and $H_{c3}$ decreased for each dimension variations. It showed that when the ratio between circumference and area of the rectangular superconductor decreased, the value of $H_{c1}$ and $H_{c3}$ would also decrease. When the result of the ratio differ considerably, the difference in $H_{c1}$ and $H_{c3}$ values would be significant.

![Figure 4. $|\psi|^2-H_{ext}$ graph for basis $N_xN_y=64x64$](image)

![Figure 5. Value of the lower critical field ($H_{c1}$) for each dimension of the superconductor](image)
Figure 6. Value of the higher critical field ($H_{c3}$) for each dimension of the superconductor

Table 1. The input of superconductor dimension

| Basis  | Input Name | $N_x \times N_y$ | Circumference | Area | Circumference/Area |
|--------|------------|-------------------|---------------|------|--------------------|
| 12 x 12| A1         | 6 x 24            | 60            | 144  | 0.42               |
|        | A2         | 8 x 18            | 52            | 144  | 0.36               |
|        | A3         | 9 x 16            | 50            | 144  | 0.35               |
|        | A4         | 12 x 12           | 48            | 144  | 0.33               |
| 32 x 32| B1         | 8 x 128           | 272           | 1024 | 0.26               |
|        | B2         | 16 x 64           | 160           | 1024 | 0.16               |
|        | B3         | 32 x 32           | 128           | 1024 | 0.12               |
| 64 x 64| C1         | 8 x 512           | 1040          | 4096 | 0.25               |
|        | C2         | 16 x 256          | 544           | 4096 | 0.26               |
|        | C3         | 32 x 128          | 320           | 4096 | 0.08               |
|        | C4         | 64 x 64           | 256           | 4096 | 0.06               |

4. Conclusion
We have made the numerical simulation to determine the critical field ($H_{c1}$ and $H_{c3}$) of the rectangular superconductor. We applied the TDGL equations and the boundary condition to the studied superconductor and then it solved by using the $\psi U$ method. In this simulation, we varied the dimension of the superconductor to observe its influence on the value of $H_{c1}$ and $H_{c3}$. The result of the numerical simulation showed that when the ratio between circumference and area of the rectangular superconductor decreased, the value of $H_{c1}$ and $H_{c3}$ would also decrease.

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