Modified Stojkovic-Stanimirovic method to find redundant constrains in linear programming problems

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Abstract. Important techniques in linear programming are modeling and solving practical optimization. A redundant constraint is a constraint that can be remove from a system of linear constraints without changing the feasible region. Method for identifying redundant constraints which in the process only uses the objective function and inequality constraints, i.e., Stojkovic-Stanimirovic method. In this paper we give a modified method of Stojkovic-Stanimirovic method to detect redundant constraints in linear programming problems. The advantages of employing the propose method is still to use the different objective function on linear programming associated the same feasible set have the same result. To clarify these method is given an example cannot be solve using detect Stojkovic-Stanimirovic method but can be solve using propose method.

1. Introducing
Optimization linear problems are one of the most useful in several areas of science [1]. Abstractions of real-world situations can be viewed in linear programming problems [2]. Solving the linear programming problems, we tend to include all possible constraints that will increase the number of iterations and computational work [3]. Simplex method and interior method, there are methods for solving the linear programming problem [4]. Simplex method is the mostly method for solving linear programming problems.[5].

Usually effect on large-scale linear programming problem includes redundant constraints [6]. Redundant constraints in linear programming problem do not change feasible region [7]. Redundant constraints waste computational effort [7]. Duplicate rows in a linear programming problem are said also redundant constraints [8]. The presence redundant constraints may cause degeneracy [9]. Degenerate may cause cycling, it may occur that the simplex algorithm returns to a previously constructed tableau, in the other hand the same sequence of tableau or repeated over and over again [8].

Many methods identifying redundant constraints and removed them [9]. Paulraj proposed Heuristic method [10]. Heuristics method can identify redundant constraints, constraints that do not contribute to the solution [8]. Telgen proposed a deterministic method is using minimum ratio criteria as in simplex method to identify redundant constraints [11]. Stojkovic-Stanimirovic method based on theorem by Stojkovic-Stanimirovic[11,12]. Objective function and constraints are used Stojkovic-Stanimirovic method to identify redundant constraints.

In this paper, we discuss a Modification of the Stojkovic-Stanimirovic method while depends on the objective function. Given an example that cannot be solve using detect Stojkovic-Stanimirovic method but can be solve using propose method.
Stanimirovic method but can identify redundant constraints using Modified Stojkovic-Stanimirovic method. Modified Stojkovic-Stanimirovic method removes step 2 in Stojkovic-Stanimirovic method.

2. Method
Consider systems of linear inequalities constraints are as follow,

\[ Ax \leq b \]
\[ x \geq 0, \]

where \( A \in R^{mxn} \) matrixs positive with \( m \geq n, x \in R^m, 0 \in R^m \) and \( b \in R^m \).

Consider the feasible region associated with the system (2.1) is

\[ S = \{ x \in R^n \mid Ax \leq b, x \geq 0 \}, \]

and let of \( S_k \) is the feasible region associated \( Ax \leq b \) without inequalities constraint \( a_kx \leq b_k \) can be written as,

\[ S_k = \{ x \in R^n \mid a_k x \leq b_i, i = 1, ..., m, i \neq k \}, \]

where \( S_k > 0 \) is redundant in the system inequality (1) if and only if \( S = S_k \). Its label is \( R \). Classification of redundant constraints by Jan Telgen among others are weakly redundant and strongly redundant constraints [8]. Weakly redundant constraint is defined as follows,

**Definition 2.1.** [8] Constraint \( a_kx \leq b_k \) is weakly redundant if a constraint is redundanct constraints and \( s_k = \min(s_k(x)|x \in S_k) \).

Let weakly redundant will be referred to by label \( WR \) and label \( SR \) is assigned to strongly redundant. Strongly redundant constraints are defined as follows,

**Definition 2.2.** [8] Constraint \( a_kx \leq b_k \) weakly redundant if a constraint is redundant constraints and \( s_k > 0 \) where \( s_k = \min(s_k(x)|x \in S_k) \).

Consider of maximum linear programming with positive constraints matrix,

Objective function : \[ \max \; x = cx \]
Subject to: \[ Ax \leq b \]
\[ x \geq 0. \]

Where matrix \( A \in R^{mxn} \), vector \( b \in R^m \), vector \( x \in R^m \), vector \( 0 \in R^m \), and vector \( c \in R^n \).

In this paper we give Theorem 2.3 for construction Stojkovic-Stanimirovic algorithm.

**Theorem 2.3** [12] Consider linear programming problem in the system (4). Let \( d_{ij} = \begin{cases} \alpha_{ij} & i = 1, 2, ..., m \text{ and } j = 1, 2, ..., n \\ \beta_{ij} & \end{cases} \) then

1. \[ \text{if } \max \min d_{ij} = \min \max d_{ij} = d_{kl} \text{ then system (2.4) has optimal solution problem.} \]
2. \[ \text{if } \max \min d_{ij} = \min \max d_{ij} = \text{ and if there exist } k, l \text{ such that } d_{kl} \leq d_{ij}, \text{ for all } j = 1, 2, ..., \]

then \( k \)-th constraint is a redundant constraint.

Based on Theorema 2.3 we introduce two algorithms are Stojkovic-Stanimirovic algorithm and Modified Stojkovic-Stanimirovic algorithm. Label \( A1 \) is assigned to Stojkovic-Stanimirovic algorithm and label \( A2 \) is assigned to Modified Stojkovic-Stanimirovic algorithm as follow,

**Algorithm 1** [12] (A1)
Input : \( c \in R^m, A \in R^{mxn}, b \in R^m, x \in R^n \).
Output : Linear constraints without redundant constraints.
Step :  
1. Compute \( d_{ij} = \begin{cases} \alpha_{ij} & i = 1, 2, ..., m \text{ and } j = 1, 2, ..., n \\ \beta_{ij} & \end{cases} \)
2. If \( \max \min d_{ij} = \min \max d_{ij} \), then there is not has redundant constraints, and stop. Otherwise, go to 3.
3. There are found $k$-th and $l$-th constraint so that if $d_{kj} \leq d_{lj}$, for all $j = 1, 2, ..., k$-th constraint is a redundant constraint.

Different between A1 and A2 is in A2 without step 2 in A1. Based on Theorem 2.3 by Stojkovic-Stanimirovic [16] if step 2 in A1 is satisfied then problem has optimal solution so that we remove step 2 in A1 for A2. Modified Stojkovic-Stanimirovic algorithm as follow,

**Algorithm 2 (A2)**

Input : $c \in R^n$, $A \in R^{mxn}$, $b \in R^m$, $x \in R^n$.
Output : Linear constraints without redundant constraints.

Step:
1. Compute $d_{ij} = \left\{ \frac{a_{ij}}{b_{ij}} \right\}$ for all $i = 1, 2, ..., m$ and $j = 1, 2, ..., n$.
2. There are found $k$-th and $l$-th constraint so that if $d_{kj} \leq d_{lj}$, for all $j = 1, 2, ..., k$-th constraint is redundant constraint.

3. Result and Discussion

In the section result and discussion, some examples are given that cannot be solved using Stojkovic & Stanimirovic method but can be completed using Modified Stojkovic-Stanimirovic method. Given 6 examples of linear program problems with the maximum objective function presented in Table 1 and Table 2.

**Table 1. Example 1 - Example 3**

| Objective Function | Example 1 | Example 2 | Example 3 |
|--------------------|-----------|-----------|-----------|
| Subject to         | 1. $7x_1 + 2x_2 \leq 14$ | 2. $3x_1 + 5x_2 \leq 15$ | 3. $x_1 + x_2 \leq 7$ |
|                    | $4x_1 + 3x_2$ | $3x_1 + 2x_2$ | $x_1 + x_2 \leq 7$ |
|                    | $2x_1 + 3x_2 \leq 6$ | $5x_1 + 4x_2 \leq 12$ | $7x_1 + 5x_2 \leq 35$ |
|                    | $x_1 + x_2 \leq 8$ | $4x_1 + 7x_2 \leq 28$ | $3x_1 + 7x_2 \leq 21$ |
|                    | $x_1 + x_2 \leq 4$ | $6x_1 + 7x_2 \leq 42$ | $x_1, x_2 \geq 0$ |
|                    | $x_1, x_2 \geq 0$ | $3x_1 + 5x_2 \leq 15$ | $x_1, x_2 \geq 0$ |

Example 4 has same inequality constraints and different objective function with Example 1. Meanwhile Example 5 has same inequality constraints and different objective function with Example 2, and Example 6 has same inequality constraints and different objective function with Example 3.

**Table 2. Example 4 - Example 6**

| Objective Function | Example 4 | Example 5 | Example 6 |
|--------------------|-----------|-----------|-----------|
| Subject to         | 1. $7x_1 + 2x_2 \leq 14$ | 2. $3x_1 + 5x_2 \leq 15$ | 3. $x_1 + x_2 \leq 7$ |
|                    | $5x_1 + 7x_2$ | $3x_1 + 2x_2$ | $x_1 + x_2 \leq 7$ |
|                    | $2x_1 + 3x_2 \leq 6$ | $5x_1 + 4x_2 \leq 12$ | $7x_1 + 5x_2 \leq 35$ |
|                    | $x_1 + x_2 \leq 8$ | $4x_1 + 7x_2 \leq 28$ | $3x_1 + 7x_2 \leq 21$ |
|                    | $x_1 + x_2 \leq 4$ | $6x_1 + 7x_2 \leq 42$ | $x_1, x_2 \geq 0$ |
|                    | $x_1, x_2 \geq 0$ | $3x_1 + 5x_2 \leq 15$ | $x_1, x_2 \geq 0$ |

Examples in Table 1 and Table 2 are solved using Stojkovic-Stanimirovic method (A1) and Modified Stojkovic-Stanimirovic (A2) method is presented in Table 3.
Example 1 in Table 3 shows that there are respectively two feasible constraints (FR) are 1st and 5th constraint. Three redundant constraints are 2nd, 3rd, and 4th constraint. There are weakly redundant constraints. Example 1 without strongly redundant constraint. Example 1 can be solve by Stojkovic-Stanimirovic method and Modified Stojkovic-Stanimirovic method. Stojkovic-Stanimirovic and Modified Stojkovic-Stanimirovic can be detect three redundant constraints. Example 4 has same inequality constraints and different objective function with Example 1. Example 4 cannot be solved by Stojkovic-Stanimirovic method. Example 4 can be solved by Stojkovic-Stanimirovic method. Modified Stojkovic-Stanimirovic method can be detect three redundant constraint in Example 4. The other example is analogous to Example 1 and Example 4.

### Table 3. Results identification of redundant constraints

| No | Example | FR  | WR  | SR  | R   | A1  | A2   |
|----|---------|-----|-----|-----|-----|-----|------|
| 1  | Example 1 | 2 {1,5} | 3 {2,3,4} | -   | 3 {2,3,4} | 3 {2,3,4} | 3 {2,3,4} |
| 2  | Example 2 | 2 {1,2} | -   | 3 {3,4,5} | 3 {3,4,5} | -   | 3 {3,4,5} |
| 3  | Example 3 | 2 {2,4} | 2 {3,5} | 1 {1} | 3 {1,3,5} | 3 {1,3,5} | 3 {1,3,5} |
| 4  | Example 4 | 2 {1,5} | 3 {2,3,4} | -   | 3 {2,3,4} | -   | 3 {2,3,4} |
| 5  | Example 5 | 2 {1,2} | -   | 3 {3,4,5} | 3 {3,4,5} | -   | 3 {3,4,5} |
| 6  | Example 6 | 2 {2,4} | 2 {3,5} | 1 {1} | 3 {1,3,5} | -   | 3 {1,3,5} |

4. Conclusion

Based on Example 1 – Example 6 disadvantage of Stojkovic-Stanimirovic methods is if there have the same constraints and different coefficient objective function possible different results. Meanwhile, advantage of Modified Stojkovic-Stanimirovic method is if there have the same constraints and different coefficient objective function possible same results.

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