Featuring the structure functions geometry

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Abstract

We consider geometrical properties of the polarized and unpolarized structure functions and provide definition for the $b$-dependent structure functions. It is shown that unitarity does not allow factorized form of the structure functions over the $x$ and $b$ variables. We conclude that the spin of constituent quark has a significant orbital angular momentum component.

Introduction

The behavior and dependence of the structure functions on the Bjorken $x$ is among the most actively discussed subjects in the unpolarized and polarized deep-inelastic scattering. The particular role here belongs to the small $x$ region where asymptotical properties of the strong interactions can be studied. The characteristic point of low-$x$ region is an essential nonperturbative nature of the underlying dynamics in the whole region of $Q^2$\cite{1,2}. Despite the results of perturbative QCD calculations are in a good agreement with the latest HERA data, the conceptual feasibility of the perturbative QCD methods in this region has not been justified.

Of course, the shortcomings of various model approaches to the study of this nonperturbative region are also evident. However, one can hope to gain from these models an information which cannot be obtained from the perturbative methods (cf. \cite{3}). Among the possible extensions there could...
be considerations of the geometrical features of the structure functions, i.e. dependence of the structure functions on the transverse coordinates or in other words on impact parameter. This dependence would allow one to gain an information on the spatial distribution of the partons inside the parent hadron and the spin properties of the nonperturbative intrinsic hadron structure. The geometrical properties of structure functions should play an important role under analysis of the lepton–nuclei deep–inelastic scattering and in the hard production in the heavy–ion collisions.

1 Definition and interpretation of $b$–dependent structure functions

In this note we study the $b$–dependence of the structure functions along the line used in \[\text{[4]}, \text{i.e. we suppose that the deep–inelastic scattering is determined by the aligned-jet mechanism } \text{[1]. There are serious arguments in favor of its leading role and dominance over the other mechanism known as a color-transparency. The aligned-jet mechanism is essentially nonperturbative one and allows to relate structure functions with the discontinuities of the amplitudes of quark–hadron elastic scattering. These relations have the following form } \text{[5, 6]}

\[
\begin{align*}
q(x) &= \frac{1}{2} \text{Im}[F_1(s, t) + F_3(s, t)]|_{t=0}, \\
\Delta q(x) &= \frac{1}{2} \text{Im}[F_3(s, t) - F_1(s, t)]|_{t=0}, \\
\delta q(x) &= \frac{1}{2} \text{Im}F_2(s, t)|_{t=0}. 
\end{align*}
\]

(1)

The functions $F_i$ are helicity amplitudes for the elastic quark-hadron scattering in the standard notations for the nucleon–nucleon scattering. We consider high energy limit or the region of small $x$.

The structure functions obtained according to the above formulas should be multiplied by the factor $\sim 1/Q^2$ – probability that such aligned–jet configuration occurs \[\text{[1].}

The amplitudes $F_i(s, t)$ are the corresponding Fourier-Bessel transforms of the functions $F_i(s, b)$.
The relations Eqs. (1) will be used as a starting point under definition of the structure functions which depend on impact parameter. According to these relations it is natural to give the following operational definition:

\[ q(x, b) \equiv \frac{1}{2} \text{Im}[F_1(x, b) + F_3(x, b)], \]
\[ \Delta q(x, b) \equiv \frac{1}{2} \text{Im}[F_3(x, b) - F_1(x, b)], \]
\[ \delta q(x, b) \equiv \frac{1}{2} \text{Im}F_2(x, b), \]

and \( q(x), \Delta q(x) \) and \( \delta q(x) \) are the integrals over \( b \) of the corresponding \( b \)-dependent distributions, i.e.

\[ q(x) = \frac{Q^2}{\pi^2 x} \int_0^\infty bdbq(x, b), \quad \Delta q(x) = \frac{Q^2}{\pi^2 x} \int_0^\infty bdb\Delta q(x, b) \]

and

\[ \delta q(x) = \frac{Q^2}{\pi^2 x} \int_0^\infty bdb\delta q(x, b). \]

The functions \( q(x, b), \Delta q(x, b) \) and \( \delta q(x, b) \) depend also on the variable \( Q^2 \) and have simple interpretations, e.g. the function \( q(x, b, Q^2) \) represent probability to find quark \( q \) in the hadron with a fraction of its longitudinal momenta \( x \) at the transverse distance

\[ b \pm \Delta b, \quad \Delta b \sim 1/Q \]

from the hadron geometrical center. Interpretation of the spin distributions directly follows from their definitions: they are the differences of the probabilities to find quarks in the two spin states with longitudinal or transverse directions of the quark and hadron spins.

It should be noted that the unitarity plays crucial role in the direct probabilistic interpretation of the function \( q(x, b) \). Indeed due to unitarity

\[ 0 \leq q(x, b) \leq 1. \]

The integral \( q(x) \) is a quark number density which is not limited by unity and can have arbitrary nonnegative value. Thus, the given definition of the \( b \)-dependent structure functions is self-consistent. Of course, spin distributions \( \Delta q(x, b) \) and \( \delta q(x, b) \) are not positively defined.
2 Unitarity and structure function geometrical profiles

The unitarity can be fulfilled through the $U$–matrix representation for the helicity amplitudes of elastic quark–hadron scattering. In the impact parameter representation the expressions for the helicity amplitudes are the following \[4\]

\[
\begin{align*}
F_{1,3}(x, b) &= U_{1,3}(x, b)/[1 - iU_{1,3}(x, b)], \\
F_{2}(x, b) &= U_{2}(x, b)/[1 - iU_{1}(x, b)]^2
\end{align*}
\] (6)

Unitarity requires $\text{Im}U_{1,3}(x, b) \geq 0$. The $U$–matrix form of unitary representation contrary to the eikonal one does not generate itself essential singularity in the complex $x$ plane at $x \to 0$ and implementation of unitarity can be performed easily. Therefore we use this representation and not the method of the eikonalization. The model which provides explicit form of helicity functions $U_{i}(x, b)$ has been described elsewhere \[4\]. A hadron consists of the constituent quarks aligned in the longitudinal direction and embedded into the nonperturbative vacuum (condensate). The constituent quark appears as a quasiparticle, i.e. as current valence quark surrounded by the cloud of quark-antiquark pairs of different flavors. The strong interaction radius of the constituent quark $Q$ is determined by its Compton wavelength.

Spin of constituent quark, e.g. $U$-quark in this approach is given by the sum:

\[
J_U = 1/2 = S_{uv} + S_{\bar{q}q} + L_{\bar{q}q} = 1/2 + S_{\bar{q}q} + L_{\bar{q}q}.
\] (7)

In the model an exact compensation between the total spin of the quark-antiquark cloud and its angular orbital momenta occurs, i.e.

\[
L_{\bar{q}q} = -S_{\bar{q}q}.
\] (8)

In this approach based on effective Lagrangian the gluon degrees of freedom are overintegrated.

On the grounds of the experimental data for polarized DIS we arrive to conclusion that the significant part of the spin of constituent quark is due to the orbital angular momentum of the current quarks inside the constituent one \[3\].
The explicit expressions for the helicity functions $U_i(x, b)$ at small $x$ can be obtained from the corresponding functions $U_i(s, b)$ given in [4] by the substitute $s \simeq Q^2/x$ and at small values of $x$ they get the form:

$$U_{1,3}(x, b) = U_0(x, b)[1 + \beta_{1,3}(Q^2)m_Q \sqrt{x}/Q],$$

$$U_2(x, b) = g_f^2(Q^2)m_Q^2 x \frac{\exp[-2(\alpha - 1)m_Q b/\xi]}{Q^2} U_0(x, b),$$  \hspace{1cm} (9)

where

$$U_0(x, b) = i\tilde{U}_0(x, b) = i\left[\frac{a(Q^2)Q}{m_Q \sqrt{x}}\right]^{n+1} \exp[-Mb/\xi].$$  \hspace{1cm} (10)

$a, \alpha, \beta, g_f$ and $\xi$ are the model parameters, some of them in this particular case of quark-hadron scattering depend on the virtuality $Q^2$. The meaning of these parameters (cf. [4]) is not crucial here; note only that $m_Q$ is the average mass of constituent quarks in the quark-hadron system of $n + 1$ quarks and $M$ is their total mass, i.e. $M = \sum_{i=1}^{n+1} m_i$. We consider here for simplicity a pure imaginary amplitude. Using definition of the $b$-dependent structure functions given in Sec. 1 and Eqs. (6) we obtain at small $x$:

$$q(x, b) = \frac{\tilde{U}_0(x, b)}{1 + \tilde{U}_0(x, b)},$$  \hspace{1cm} (11)

$$\Delta q(x, b) = \frac{\beta_-(Q^2)m_Q \sqrt{x}}{Q} \frac{\tilde{U}_0(x, b)}{[1 + \tilde{U}_0(x, b)]^2},$$  \hspace{1cm} (12)

$$\delta q(x, b) = \frac{g_f^2(Q^2)m_Q^2 x}{Q^2} \frac{\exp[-2(\alpha - 1)m_Q b/\xi]}{\xi} \frac{\tilde{U}_0(x, b)}{[1 + \tilde{U}_0(x, b)]^2},$$  \hspace{1cm} (13)

where $\beta_-(Q^2) = \beta_3(Q^2) - \beta_1(Q^2)$. From the above expressions it follows that $q(x, b)$ has a central $b$-dependence, while $\Delta q(x, b)$ and $\delta q(x, b)$ have peripheral profiles. Their qualitative dependence on the impact parameter $b$ is depicted in Fig. 1. The peripheral dependence on impact parameter according to the relation Eq. (13) is the manifestation of a significant presence of the angular orbital momenta in the spin balance of a nucleon.

From Eqs.(11–13) it follows that factorization of $x$ and $b$ dependencies is not allowed by unitarity. However, this result is valid for the small $x$ region only and approximate factorization is possible in the region of not too small $x$ where account of unitarity reduces to the factorization breaking corrections.
The following relation between the structure functions $\Delta q(x, b)$ and $\delta q(x, b)$ can also be inferred from the above formulas

$$\delta q(x, b) = c(Q^2) \frac{\sqrt{x}}{Q} \exp(-\gamma b) \Delta q(x, b).$$  \hspace{1cm} (14)

Thus, the function $\delta q(x, b)$ which describes transverse spin distribution is suppressed by the factors $\sqrt{x}$ and $\exp(-\gamma b)$, i.e. it has a more central profile. This suppression also reduces double-spin transverse asymmetries in the central region in the Drell-Yan production compared to the corresponding longitudinal asymmetries.

The strange quark structure functions have also a more central $b$–dependence than in the case of $u$ and $d$ quarks. The radius of the corresponding quark matter distribution is

$$R_q(x) \simeq \frac{1}{M} \ln Q^2/x$$  \hspace{1cm} (15)

and the ratio of the strange quark distributions to the light quark distributions radii is given by the corresponding constituent quark masses, i.e. for the nucleon this ratio would be

$$R_s(x)/R_q(x) \simeq (1 + \frac{\Delta m}{4m_Q})^{-1},$$  \hspace{1cm} (16)

where $\Delta m = m_s - m_Q$.

Time reversal invariance of strong interactions allows one to write down relations similar to Eqs.\(\text{[1]})\) for the fragmentation functions also and obtain
expressions for the fragmentation functions $D_q^h(z, b)$, $\Delta D_q^h(z, b)$, $\delta D_q^h(z, b)$ which have just the same dependence on the impact parameter $b$ as the corresponding structure functions. The fragmentation function $D_q^h(z, b, Q^2)$ is the probability for fragmentation of quark $q$ at transverse distance $b \pm \Delta b$ ($\Delta b \sim 1/Q$) into a hadron $h$ which carry the fraction $z$ of the quark momentum. In this case $b$ is a transverse distance between quark $q$ and the center of the hadron $h$. It is positively defined and due to unitarity obey to the inequality

$$0 \leq D_q^h(z, b) \leq 1$$  \hspace{1cm} (17)

The physical interpretations of spin–dependent fragmentation functions $\Delta D_q^h(x, b)$ and $\delta D_q^h(x, b)$ is similar to that of corresponding spin structure function. Peripherality of the spin fragmentation functions can also be considered as a manifestation of the important role of angular orbital momenta.

**Conclusion**

It is interesting to note that the spin structure functions have a peripheral dependence on the impact parameter contrary to central profile of the unpolarized structure function. In the considered model where the hadron has aligned structure the peripherality of the spin structure functions implies that the main contribution to the spin of constituent quark is due to the orbital angular momentum. This orbital angular momentum has a nonperturbative origin and does not result from the perturbative QCD evolution. This conclusion provides clue for the possible solution of the problem of the nucleon spin structure. It is interesting to find out possible experimental signatures of the peripheral geometrical profiles of the spin structure functions and the significant role of the orbital angular momentum. One of such indications could be an observation of the different spatial distributions of charge and magnetization at Jefferson Lab [7]. It would also be important to have a precise data for the strange formfactor. It could also be done analyzing both the spin structure and angular distribution in exclusive electroproduction and it worth considering in a separate study.
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