Behavioral modeling for synthesizing n-scroll attractors

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Abstract: In this letter, the nonlinear behavioral model of a third-order chaotic system based on CFOAs is introduced. The proposed model is more realistic than PWL approaches widely used in the literature, since herein real physical active device performance parameters along with its parasitic elements are taken into account in the modeling process. As a consequence, chaotic attractors at 1-D can not only be better forecasted, but since chaotic waveforms numerically and experimentally generated have a random behavior, statistical tests are used to measure the similitude between them. Experimental results of the chaotic system designed with the AD844AN integrated circuit are gathered, showing good agreement with theoretical simulations.

Keywords: chaos, multi-scroll, saturated nonlinear function series, current-feedback operational amplifier

Classification: Integrated circuits

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1 Introduction

Chaotic circuits find many applications in data encryption, secure communications, bio-medical engineering and random number generation [1], and some them have been implemented by a huge number of electronic devices. Particularly on the generation of continuous time n-scroll attractors by using universal active devices [2, 3], a piece-wise linear (PWL) approach is often used to model the nonlinear part of the third order chaotic system [4, 5, 6, 7, 8]. Despite of that PWL models are relatively easy to build and numerical simulations can be realized, in practice, the predicted performance of the chaotic system differs substantially from reality. These differences are more prominent when the operating frequency of the chaotic attractors are pushed to operate in higher frequencies and this is due to that any information related with the real physical active device performance parameters is included in the numerical simulations. In this context, the works recently reported in [9, 10] address not only the nonlinear modeling of an operational amplifier (Op Amp), including the most influential performance parameters, like the dynamic range (DR), gain bandwidth product (GB), DC gain ($A_{DC}$) and slew rate (SR), but based on that nonlinear model, the behavioral model of the saturated nonlinear function series (SNFS) along with the chaotic system were also derived and validated through numerical and experimental tests. This paper continues along the same line to study and enhance the generation of continuous time n-scroll attractors, but unlike [9, 10], the current feedback operational amplifier (CFOA) is chosen as active device [11]. Because a CFOA presents better performance parameters than Op Amps, chaotic waveforms at 1-D can numerically be forecasted in higher frequencies [9, 10] and with a better accuracy than those reported in [6, 8]. Experimental results are gathered, showing good agreement with numerical tests.

2 SNFS architecture with CFOAs

The SNFS topology with CFOA taken from [6] is shown in Fig. 1(a). In this way, following the analysis in [9, 10], the behavioral model for SNFS can easily be deduced whether each basic block is modeled as shown in Fig. 1(b). Hence
\[
\frac{dV_n(x(t))}{dt} = p_n - \frac{GB}{A_{DC}} V_n(x(t)),
\]

\[
p_n = \begin{cases} 
\frac{A_{v1} R_{an}}{R_{an} + R_y} - x(t) & \text{if } x(t) > \frac{R_{an} A_{v2} SR}{(R_{an} + R_y) GB} \\
\pm A_{v1} Bp_j - x(t) & \text{if } \frac{R_{an} A_{v2} SR}{(R_{an} + R_y) GB} \leq x(t) \leq \frac{R_{an} A_{v2} SR}{(R_{an} + R_y) GB} \\
\frac{A_{v2} R_{an}}{R_{an} + R_y} - x(t) & \text{if } x(t) < -\frac{R_{an} A_{v2} SR}{(R_{an} + R_y) GB}
\end{cases}
\]

which is limited by

\[
x(t) \frac{R_{an}}{R_{an} + R_y} + V_{ns} \frac{R_{an}}{R_{an} + R_y} \leq v_z(t) \leq x(t) \frac{R_{an}}{R_{an} + R_y} + V_{ps} \frac{R_{an}}{R_{an} + R_y}
\]

where \( R_{x,y,z,w} \) and \( C_{y,z} \) are the parasitic resistances and capacitances on the \( y-, x-, z- \) and \( w- \) terminal, respectively [11]. \( A_{v1}, A_{v2} \) and \( A_i \) model the finite open-loop voltage and current gains of the controlled sources, \( V_{ns} \) and \( V_{ps} \) are the negative and positive saturation voltages, and \( \omega_p \) is the dominant pole corner frequency of the open-loop CFOA; \( \omega_p = R_{an} + \sqrt{R_{an}^2 + R_{an} R_y} \) is the dominant pole corner frequency of the open-loop CFOA, \( A_{DC} = \frac{A_{v2} A_{an} R_y}{R_{an} + R_y} \frac{R_{an}}{R_{an} + R_y} \frac{R_{an}}{R_{an} + R_y}, GB = \frac{A_{v1} A_{v2} R_{an}}{(R_{an} + R_y) (R_{an} + R_y) C_y}, Bp_j \) is the \( j \)-th breakpoint [6, 7] and each \( \pm Bp \) is a DC voltage source which is used to compare if \( x(t) \) is larger or smaller than \( \pm Bp \). Once that the behavioral model for the \( n \)-building block has been deduced, the behavioral model for the SNFS can be derived [6, 9, 10] by modeling each basic block by (1) and (2). This means that the SNFS dynamic behavior is approached by a set of \( n \)-differential equations, which results in \( n + 1 \) number of plateaus [9, 10]. Therefore, each output voltage from Fig. 1(a) must numerically be computed and after, the total output current is obtained as

\[
i_s(x(t)) \approx \frac{V_1(x(t))}{R_{an}} + \frac{V_2(x(t))}{R_{an} + R_y} + \cdots + \frac{V_n(x(t))}{R_{an}}
\]

It is important to point out that (1), (2) and (3) can be used to generate any number of plateaus of the SNFS, contrary to PWL approaches where not only two functions are required to generate an even and odd number of
plateaus, but also active device performance parameters to be employed are not considered. Since (1) has been deduced by taking into account the main performance parameters of CFOAs, they can be obtained either from manufactured data sheet or experimental measurements. Table I gives the measured performance parameters and component list of Fig. 1(b).

### Table I. Performance parameters from Fig. 1(b).

| Element | Value       | Tolerance |
|---------|-------------|-----------|
| $R_{an}$ | $1 \, k\Omega$ ($\pm 5\%$) | $R_x$ | $75 \, \Omega$ | $A_i$ | $0.98$ | $SR$ | $2000 \, V/\mu s$ |
| $R_b$ | $1 \, M\Omega$ ($\pm 5\%$) | $R_z$ | $25 \, k\Omega$ ($\pm 5\%$) | $A_{i2}$ | $0.98$ | $GB$ | $8.8575 \, MHz$ |
| $R_{cn}$ | $5 \, k\Omega$ (Pot.) | $C_x$ | $16 \, pF$ | $A_{DC}$ | $56.49 \, dB$ | $V_{ps}$ | $+4 \, V$ |
| $R_y$ | $10 \, M\Omega$ | $C_y$ | $2 \, pF$ | $\omega_{dp}$ | $83.33 \, \text{krad} / \text{s}$ | $V_{ns}$ | $-4 \, V$ |

Part of the component list from Fig. 2.

| Element | Value       | Tolerance |
|---------|-------------|-----------|
| CFOAs | AD844AN | $R_{X,Y,Z}$ | $10 \, k\Omega$ | $\pm 5\%$ |
| $C_{X,Y}$ | $10 \, pF$ | $\pm 20\%$ |
| $C_Z$ | $820 \, pF$ | $\pm 20\%$ |
| $\pm B_{p1}$ | $\pm 2.1 \, V$ |

3 Behavioral modeling of the chaotic system

To enhance the generation of multi-scroll chaotic attractors at 1-D, Fig. 1(a) was embedded in the chaotic circuit from [6] and shown in Fig. 2. Note that the use of $R_s$ does not drastically modify the behavior of the chaotic circuit, since its value is low, and it will be used to indirectly measure $i_s(t)$ in relation with the chaotic signal $x(t)$ [10]. Table I shows the numerical values of the passive elements used in the numerical and experimental tests. Analyzing Fig. 2 and considering (1), (2) and (3), the new nonlinear system of equations to be solved is given as

![Fig. 2. Chaotic circuit taken from [6] with the SNFS shown in Fig. 1(a).](image-url)
As one can observe, the main advantage of this kind of modeling is that real physical performance parameters along with parasitic elements of the CFOA are already included in (4) and as a consequence, n-scroll attractors can numerically be forecasted with a better accuracy compared with PWL models. It is worthwhile noting that this kind of modeling is appealing because it can be used as an alternative and effective way to evaluate not only the complexity of nonlinear systems [12, 13, 14, 15], but also suggest a viable and effective way for improving chaos-based synchronization schemes [1].

4 Numerical and experimental results

Having generated the nonlinear system of equations of the chaotic circuit, numerical simulations can be done by using the fourth-order Runge-Kutta algorithm. In a first step, chaotic waveforms with center frequency of 60 kHz have been experimentally generated and stored into a file, in order to be used as excitation signal in (1), (2) and (3). According to Fig. 3(a), the SNFS behavior is almost equal to the experimental data and the numerical results have a better accuracy compared with PWL models [6, 8]. Additionally, comparisons between experimental data and the proposed nonlinear macro-model for SNFS in the time domain are also shown in Fig. 3(b), where \( V_i(x(t)) \approx i_s(x(t))R_s \). In both figures, a good agreement is observed. In a second step, (4) has been numerically solved in order to generate 4-scrolls. Fig. 4 shows numerical and experimental results, respectively. In these figures, the differences are attributed to the initial conditions used in the numerical
simulations given as \( x(0) = 0, y(0) = 0, z(0) = 0.01 \), however, their statistical distributions are invariant and independent on the initial conditions and on a long time, the chaotic waveforms become relatively similar [13]. In this context, histograms are generated for representing the data distribution, which give an estimation of the probability density function (PDF) of the underlying random variables \( x(t) = x_m(t) \) (for the nonlinear model given by (4)) and \( x(t) = x_e(t) \) (for the experimental data) as shown in Fig. 5, but no information can be obtained about whether the unknown PDF of \( x_m(t) \) is similar to the unknown PDF of \( x_e(t) \). In this case, a nonparametric test can be applied such as Komogorov-Smirnov test (KS-test) [12]. This test quantifies a distance between empirical (or cumulative) PDFs of two samples by assuming that they come from the same distribution (null hypothesis) or from different distributions (alternative hypothesis) [12, 13]. Given that experimentally one can not obtain a data vector sufficiently large whereas numerically it is possible, we will apply the KS-test to these two data vectors, whose histograms are shown in Fig. 5.

**Fig. 3.** Comparison between (3) (blue line) and experimental data (red line) of Fig. 1(a) operating to 60 kHz; (a) SNFS, (b) time-domain.

**Fig. 4.** Comparison between: (a) proposed behavioral model given by (4) and (b) experimental data of Fig. 2 operating to 60 kHz.
Applying KS-test to the two random variables $x_m(t)$ and $x_e(t)$, one can determine if they differ significantly. Test results are shown in Fig. 6(a) and one can see that the maximum vertical deviation between the two curves is given by $D = 0.0858$ and it occurs near $x = 2.1$. Therefore, the maximum difference in cumulative fraction is the supremum $D$ and one can conclude that the cumulative PDFs of the two random variables, $x_m(t)$ and $x_e(t)$, come from the same distribution. Furthermore, since KS-test is a robust test that cares only about the data cumulative distribution, one can also conclude that the random variables $x_m(t)$ and $x_e(t)$ are statistically similar and as a consequence, the n-scrolls at 1-D to be generated on the phase plane must also be similar in a long time. As a result, it is expected that the generation of chaotic waveforms in more directions (e.g., 2-D, 3-D and 4-D) can be better predicted compared with PWL models. We remark that to the best knowledge of the authors, the nonlinear macromodel for SNFS based on CFOAs along with the nonlinear system of equations of the chaotic system given by (4) have not been reported in the literature, until today. We also remark that although

![Fig. 5. Histograms of (a) $x_m(t)$ and (b) $x_e(t)$.](image1)

![Fig. 6. (a) KS-test for $x_m(t)$ and $x_e(t)$ (b) Chaotic spectrum of $x_e(t)$.](image2)
(1) and (2) can be used to study and analyze the complexity of nonlinear circuits and systems based on CFOAs [12, 13, 14, 15], these topics are beyond the framework of this paper. Finally, to verify the operating frequency of the chaotic system, its chaotic spectrum is experimentally obtained and shown in Fig. 6(b). Note that chaotic waveforms are nearly periodic when the output power is concentrated in a single-frequency component and centered near the operating frequency of the chaotic circuit. However, they become chaotic whether the circuit is tuned at its chaotic region of operation. As a consequence, the power is extended to more frequency components into a bandwidth small, with lower and higher frequencies than the center operating frequency, as shown in Fig. 6(b).

5 Conclusions

We have demonstrated numerically that chaotic waveforms of the CFOA-based chaotic circuit can be better forecasted whether real physical active device performance parameters are included in the behavioral model, as given by (4). As a consequence, the proposed model is more realistic and accurate than PWL approaches which are widely used in the synthesis of chaotic attractors. Statistical tests were done in order to validate the deduced behavioral model. In this way, KS-test provides that the random variables $x_m(t)$ and $x_n(t)$, come from the same distribution and as a consequence, the time-series shown in Fig. 3(b) along with the 4-scrolls from Fig. 4 are statistically similar on a long time, independently of the initial conditions. Leveraging the behavioral model given by (1) and (2), not only the behavior of CFOA-based nonlinear electronic circuits can be better analyzed [15], but the common metrics used to evaluate the complexity of nonlinear systems can also be better predicted in future works [12, 13, 14]. Experimental results in the time domain and in the state space using commercially available CFOAs have been shown for illustrating the capability of the proposed behavioral model.

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