Non-leptonic $B$ decays involving tensor mesons

G. López Castro$^1$ and J. H. Muñoz$^{1,2}$

$^1$ Departamento de Física, Cinvestav del IPN, Apdo. Postal 14-740, 07000 México, D.F., MEXICO.

$^2$ Departamento de Física, Universidad del Tolima, A.A. 546, Ibagué, COLOMBIA.

Abstract

Two-body non-leptonic decays of $B$ mesons into $PT$ and $VT$ modes are calculated using the non-relativistic quark model of Isgur et al.. The predictions obtained for $B \rightarrow \pi D_2^*$, $\rho D_2^*$ are a factor of $3 \sim 5$ below present experimental upper limits. Interesting patterns are obtained for ratios of $B$ decays involving mesons with different spin excitations and their relevance for additional tests of forms factor models are briefly discussed.

PACS numbers: 12.39.Jh, 13.25.Hw, 14.40.Nd
Weak non-leptonic decays of $B$ mesons involving mesons of intrinsic orbital momentum $l \geq 1$ in final states, are expected to be very suppressed \cite{1, 2}. The experimental values for $B$ decay into orbitally excited charmed mesons, which are allowed at lowest order via external $W$-emission diagrams, exhibit the following suppressions \cite{1}:

\begin{align*}
\frac{B(B^+ \rightarrow D_s^0 \rho^+)}{B(B^+ \rightarrow D^0 \rho^+)} & \leq 0.35 \\
\frac{B(B^+ \rightarrow D_{10}^0 \rho^+)}{B(B^+ \rightarrow D^0 \rho^+)} & \leq 0.104 \\
\frac{B(B^+ \rightarrow D_{1+}^0 \pi^+)}{B(B^+ \rightarrow D^0 \pi^+)} & = 0.28 \\
\frac{B(B^0 \rightarrow D_s^+ \rho^+)}{B(B^0 \rightarrow D^- \rho^+)} & \leq 0.63.
\end{align*}

Predictions for these $B$ decays are important because they would provide additional tests for the factorization hypothesis and form factor models used to describe exclusive modes in $B$ decays. On the other hand, analogous ratios of $B$ decays involving lowest lying mesons ($l = 0$), namely $B \rightarrow XV/XP$ ($V$ and $P$ stand for vector and pseudoscalar mesons), such as \cite{1}: $B^+ \rightarrow \overline{D}_0^0 \rho^+/\overline{D}_0^0 \pi^+ = 2.53$, $B^+ \rightarrow \overline{D}_0^0 \rho^+/\overline{D}_0^0 \pi^+ = 2.98$, $B^0 \rightarrow D^- \rho^+/D^- \pi^+ = 2.60$ and $B^0 \rightarrow D^{*-} \rho^+/D^{*-} \pi^+ = 2.81$, show the expected behavior due to the three degrees of freedom of vector particles.

Non-leptonic decays of $B$ and $D$ mesons involving scalar and tensor mesons have been calculated previously in a series of papers by Katoch and Verma \cite{3, 4, 5} using the non-relativistic quark model of Ref. \cite{2}. B decays into final states involving tensor mesons ($J^{PC} = 2^{++}$) are not suppressed by phase-space considerations as in the case of $D$ decays. Actually, upper limits for $B$ decays into $PT$ and $VT$ channels ($T$ denotes the tensor meson) at
the level of $10^{-3} - 10^{-4}$, both for Cabibbo-favored and Cabibbo-suppressed modes, are reported in the literature [1]. Furthermore, according to the Particle Data Group (see p.99 in Ref. [1]), the multiplet of tensor mesons is well established among the ones for the $q\bar{q}$ assignement for the case of four flavors.

In this paper we compute the branching fractions for $B$ decays into $PT$ and $VT$ final states using the non-relativistic quark model of Isgur et al. [2]. While there are not available predictions for the $VT$ decay modes, our results for the $PT$ modes differ quantitatively from those of Ref. [3]. We present possible reasons for this discrepancy.

We start by introducing the effective weak hamiltonian for non-leptonic, Cabibbo favored, $b$ decays:

$$H_{eff} = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}V_{ud}^*[a_1(\bar{c}b)(\bar{d}u) + a_2(\bar{d}b)(\bar{c}u)] + V_{tb}V_{ts}^*[a_1(\bar{c}b)(\bar{s}c) + a_2(\bar{s}b)(\bar{c}c)] \right\}$$

where $(\bar{q}q')$ is a short notation for the $V-A$ current, $G_F$ denotes the Fermi constant and $V_{ij}$ are the relevant Cabibbo-Kobayashi-Maskawa mixing factors. The numerical values $a_1 = 1.15$ and $a_2 = 0.26$, which are obtained from a fit to $B \rightarrow PP, VP$ decays [3], will be used throughout this paper.

Since the internal $W$-emission diagrams are color suppressed, the leading contribution to the decays under consideration are given by the $W$-external diagram. Following Refs. [4, 5, 6], we write the decay amplitudes in the

\footnote{Note that the scalar+tensor final states are suppressed respect to $VT$ and $PT$ because the $< S | V_{\mu}^{\text{weak}} | 0 >$ amplitude gives a further suppression factor.}
following general form:

\[ \mathcal{M}(B \to PT) = i \frac{G_F}{\sqrt{2}} (\text{CKM factors})(\text{QCD factor}) f_P \epsilon_{\mu \nu} p_B p_B^\nu F_{B \to T}(m_P^2) \]  

(2)

and

\[ \mathcal{M}(B \to VT) = \frac{G_F}{\sqrt{2}} (\text{CKM factors})(\text{QCD factor}) m_V^2 f_V \epsilon_{\psi \tau} F_{\psi \tau}(m_V^2) \]  

(3)

where

\[ F_{B \to T}(m_P^2) = k + b_+(m_B^2 - m_T^2) + b_- m_P^2 \]  

(4)

\[ F_{\psi \tau}(m_V^2) = \epsilon_{\mu}^*(p_B + p_T)_{\sigma} [ih \epsilon_{\mu \nu \sigma \rho} g_{\psi \tau}(p_V)_\tau (p_V)_\rho + k \delta_{\psi}^\mu \delta_{\tau}^\sigma \]  

\[ + b_+(p_V)_\psi (p_V)_\tau g^{\sigma \mu}] \]  

(5)

In the above expressions, \( \epsilon_{\mu}^* \) denotes the polarization four-vector of \( V \), \( \epsilon_{\mu \nu} \) is the symmetric and traceless tensor describing the polarization of tensor mesons (\( p_T^\mu \epsilon_{\mu \nu} = \epsilon_{\mu \nu} p_T^\nu = 0 \)) and \( p_i \) correspond to the four-momenta of the particles. The argument in the functions \( F_{B \to T} \) means that the form factors \( h, k, b_+ \) and \( b_- \) should be evaluated at those values (namely, \( m_P^2 \) or \( m_V^2 \)). The term (QCD factor) in the amplitudes refers to either \( a_1 \) or \( a_2 \). Note that in the above expressions we have only one contribution to the decay amplitudes because the matrix element \( <T|j_{\mu}^{\text{weak}}|0> \) vanishes identically.

The hadronic matrix elements used in the previous expressions are defined as follows:

\[ <T|j_{\mu}|B> = ih \epsilon_{\mu \nu \rho \sigma} \epsilon_{\nu \beta} p_B (p_B + p_T)_{\phi} (p_B - p_T)_{\rho} + k \epsilon^{* \mu \nu} (p_B)_\nu \]  

\[ + \epsilon^{* \alpha \beta} (p_B)_{\alpha} (p_B)_{\beta} [b_+ (p_B + p_T)_{\mu} + b_- (p_B - p_T)_{\mu}] \]  

(6)

\[ <P|A_{\mu}|0> = if_P p_P^\mu \]  

(7)

\[ <V|V_{\mu}|0> = m_V^2 f_V \epsilon_{\mu}. \]  

(8)
The form factors $k$, $h$, $b_+$ and $b_-$ which describe the $B \to T$ transition, have been calculated in the non-relativistic quark model of Ref. [2]. Note that $b_-$ gives a negligible contribution to $PT$ modes, while it does not appear at all in the decay amplitude for $VT$ modes because $p_T^\mu \varepsilon_\mu = 0$.

The central values for the decay constants of pseudoscalar mesons relevant for our calculations are (in GeV units): $f_\pi = 0.131$, $f_{\eta_c} = 0.384$ \cite{1,3} and $f_D = 0.217$, $f_{D_s} = 0.241$ (we assume the isospin symmetry relation $f_{D^0} = f_{D^+}$). The value for the $D_s$ decay constant \cite{3} includes two recent determinations from the $D_s \to \mu \nu$ decay \cite{4}, and $f_D$ is obtained using the theoretical prediction $f_D/f_{D_s} = 0.90$ \cite{5} based on a lattice calculation\cite{6}.

On the other hand, the dimensionless decay constants of vector mesons are: $f_{\rho^-} = 0.2713$ (from $\tau \to \rho \nu_\tau$), $f_{J/\psi} = 0.087$ (from $J/\psi \to e^+e^-$), while for the decay constants of $D^{*+}$ and $D_s^*$ we have estimated their values using the approximate scaling relation $m_V f_V \sim$ constant (see for example \cite{10}). Indeed, using $m_V f_V = 0.231$ GeV (which is obtained from a simple average of $f_{\rho^-}$, $f_{K^{*-}}$, $f_{J/\psi}$ and $f_{\Upsilon(1S)}$), we obtain $f_{D^*} = 0.1144$ and $f_{D_s^*} = 0.109$.

The expressions for the decay amplitudes in the exclusive modes of $B$ decays are given explicitly in Table 1 for each of the $VT$ modes allowed in the leading approximation; our expressions for the $PT$ amplitudes coincide with those given in Table 1 of Ref. \cite{3}.

From the previous expressions we obtain the following decay rates for the $PT$ modes:

$$\Gamma(B \to PT) = |A(B \to PT)|^2 \left(\frac{m_B}{m_T}\right)^2 \frac{|\vec{p}_T|^5}{12\pi m_T^2} \quad (9)$$

\footnote{Other lattice computations are consistent with this result for $f_D/f_{D_s}$, while QCD sum rules predict a smaller value ($\approx 0.8$) for this ratio (see \cite{3} for a summary of results).}
where

$$\mathcal{A}(B \rightarrow PT) = \frac{G_F}{\sqrt{2}} (\text{CKM factors})(QCD \text{ factor}) f_P F_{B \rightarrow T} (m_P^2), \quad (10)$$

and the expression for the decay into the VT channels is given by:

$$\Gamma(B \rightarrow VT) = \frac{G_F^2}{48\pi m_T^2} (\text{CKM factors})^2 (QCD \text{ factor})^2 \cdot m_V^2 f_V^2 [\alpha |\vec{p}_V|^2 + \beta |\vec{p}_V|^4 + \gamma |\vec{p}_V|^3] \quad (11)$$

where $\alpha$, $\beta$ and $\gamma$ are quadratic functions of the form factors evaluated at $q^2 = m_V^2$, namely:

$$\alpha = 8m_B^4 b_+^2$$
$$\beta = 2m_B^2 [6m_V^2 m_T^2 h^2 + 2(m_B^2 - m_T^2 - m_V^2) k b_+ + k^2]$$
$$\gamma = 5m_T^2 m_V^2 k^2.$$

In the above expressions for the decay rates, $\vec{p}_{V(P)}$ denotes the three-momentum of the $V(P)$ meson in the $B$ rest frame ($\vec{p}_{V(P)}$ depends, of course, on the specific decay considered).

In order to obtain the unpolarized rates given in Eqs. (9,11), we have used the following expression for the sum over polarizations of the tensor meson:

$$P_{\mu\nu\alpha\beta} = \sum_\lambda \varepsilon_{\mu\nu}(p,\lambda) \varepsilon^*_{\alpha\beta}(p,\lambda)$$
$$= \frac{1}{2} (\theta_{\mu\alpha} \theta_{\nu\beta} + \theta_{\mu\beta} \theta_{\nu\alpha}) - \frac{1}{3} \theta_{\mu\nu} \theta_{\alpha\beta}, \quad (12)$$

where $\theta_{\mu\nu} \equiv -g_{\mu\nu} + p_\mu p_\nu / m_T^2$. Observe that $P_{\mu\nu\alpha\beta}$ satisfies the following identities:

$$P_{\mu\alpha\beta}^\mu = P_{\mu\nu\alpha}^\nu = 0$$
$$P_{\mu\nu\alpha\beta} \varepsilon^{\alpha\beta} = \varepsilon_{\mu\nu}$$
$$P_{\mu\nu\rho\sigma} P_{\alpha\beta}^{\rho\sigma} = P_{\mu\nu\alpha\beta}.$$
Now, let us address some comments on the expressions for the decay rates given in Eqs. (9,11):

(1) The decay rate in the $PT$ mode, Eq. (9), contains an additional factor $(m_B/m_T)^2$ with respect to Eq. (6) in Ref. [3]. This would enhance the prediction for the $PT$ modes by a factor of around $4 \sim 17$, depending on the specific decay mode, with respect to the predictions of Ref. [3]. The origin for this discrepancy could be the expression used for the sum over polarizations of tensor mesons. Note that using Eq. (12), we reproduce the results given in Refs. [11] for the strong decay rates of tensor mesons.

(2) The $|\vec{p}|^5$-dependence in Eq. (9) indicates that only the $l = 2$ wave is allowed for the $PT$ system, while in the $VT$ decay modes the $l = 1$, 2 and 3 are simultaneously allowed, as expected.

The numerical values for the corresponding branching ratios are given in Table 2, where we have used the $B$ lifetimes values reported in Ref. [1], and $|V_{bc}| = 0.041$, $|V_{cs}| = 0.975$ and $|V_{ud}| = 0.9736$ [1]. When we computed the rates involving an isoscalar tensor meson ($f_2$ or $f'_2$), we have used an octet-singlet mixing angle $\theta_T = 28^o$ or equivalently the deviation from ideal mixing $\phi_T \equiv \arctan(1/\sqrt{2}) - \theta_T \approx 8.3^0$, as in Ref. [3].

From Table 2, we can raise the following conclusions:

(1) The values for the $PT$ branching fractions are larger by a factor of $2.6 \sim 15$ than those given in Ref. [3]. The reason for this enhancement has been explained above; the smallest enhancement factor corresponds to the $B \to D_s D_2^*$ decay mode, while the largest value corresponds to $B \to \eta_c K_2^*$. Note that part of the difference between our results and those in Ref. [3] arises from the smaller values that we use for the $D$, $D_s$ decay constants.

(2) When we consider the ratio for the decay modes $VT/PT$, $V$ and $P$ having
identical quark content, we obtain

\[
\frac{\Gamma(B \to VT)}{\Gamma(B \to PT)} = \frac{m_v^2 f_V^2}{2 m_B f_P} \frac{1}{F^{B \to T}(m_P^2)^2} \frac{\alpha |\vec{p}_V|^7 + \beta |\vec{p}_V|^5 + \gamma |\vec{p}_V|^3}{|\vec{p}_P|^5}.
\] (13)

This ratio turns out to be independent of \(a_{1,2}, G_F\) and Kobayashi-Maskawa mixing factors and would provide a clean test of the factorization hypothesis and of the ISGW model \([2]\). In the last column of Table 2 we show the ratios corresponding to Eq. (13). The results indicate that the ratios \(VT/PT \sim 3\) for processes which amplitudes are proportional to \(a_1\), which are normally expected due to the three degrees of freedom of vector particles. Note, however, that for the other processes, \((VT)/(PT)\) differs substantially from 3 because these decays are not allowed at tree level through the emission of external \(W\) (they proceed \(\text{via}\) amplitudes proportional to \(a_2\)).

(3) Our predictions for the decays \(B^- \to (\pi^-, \rho^-)D_2^0\) and \(\bar{B}^0 \to (\pi^-, \rho^-)D_2^+\) are below the present experimental upper limits reported in \([1]\) by a factor of 3~5. This offers optimistic prospects for their measurements and for additional tests of the model of Ref. \([2]\).

(4) If we use the experimental measurements of \(B \to J/\psi K^*(892)\) and \(B \to D_s D^*\) \([1]\), we can calculate the following ratios from Table 2: \(B^- \to (J/\psi K_{2}^{+-})/(J/\psi K_{2}^{--}(892)) \approx \bar{B}^0 \to (J/\psi K_{2}^{+0})/(J/\psi K_{2}^{*0}\text{K}^*(892)) \approx B^- \to (D_{s}^- D_2^0)/(D_{s}^- D_2^{*0}) \approx B^0 \to (D_{s}^+ D_2^{*-})/(D_{s}^+ D_2^*) \approx 0.05\). Thus, the ratios \((B \to XT)/(B \to XV)\), which measure the effects of the dynamics due the orbital excitation \(V \to T\), turns out to be very suppressed in the \(\Delta s = -1\) channels.

In conclusion, in this paper we have computed the Cabibbo-favored decays of \(B\) mesons into \(VT\) and \(PT\) final states, using the non-relativistic quark model of Isgur \textit{et al.} \([2]\). Our results for the \(\pi D_2^*\) and \(\rho D_2^*\) decay channels are a factor of 3~5 below present experimental upper limits \([1]\). Our
results exhibit interesting patterns for final states mesons of higher orbital momentum excitations, which would offer additional tests for the factorization hypothesis and the form factor models for exclusive $B$ decays.

Acknowledgements

The authors would like to acknowledge financial support from Conacyt (GLC) and Colciencias (JHM).
References

[1] R. M. Barnett et al., The Particle Data Group, Phys. Rev. D54, Part I, (1996).

[2] N. Isgur, D. Scora, B. Grinstein and M. B. Wise, Phys. Rev. D39, 799 (1989).

[3] A. C. Katoch and R. C. Verma, Phys. Rev. D52, 1717 (1995).

[4] A. C. Katoch and R. C. Verma, Phys. Rev. D49, 1645 (1994).

[5] A. C. Katoch and R. C. Verma, Int. Journal of Mod. Phys. A11, 129 (1996).

[6] J. D. Richman, Progress in Understanding Heavy Flavor Physics, plenary talk at ICHEP96, Warsaw (1996).

[7] K. Kodama et al., Fermilab E653 Collaboration, Phys. Lett. B382, 299 (1996); D. Gibaut et al., CLEO Collaboration, CLEO CONF 95-22 (1995).

[8] C. W. Bernard, J. N. Labrenz and A. Soni, Phys. Rev. D49, 2536 (1994).

[9] J. D. Richman and P. R. Burchat, Rev. of Mod. Phys. 67, 893 (1995).

[10] M. Bauer, B. Stech and M. Wirbel, Z. Phys. C34, 103 (1987).

[11] G. Burdman, E. Golowich, J. L. Hewett and S. Pakvasa, Phys. Rev. D52, 6383 (1995); M. Suzuki, Phys. Rev. D47, 1043 (1993); J. J. Godina Nava and G. López Castro, Phys. Rev. D52, 2850 (1995).
\[ \Delta s = 0 \]

| Process | Amplitude \( \times (V_{cb}V_{ud}^*) \) |
|---------|----------------------------------|
| \( B^- \rightarrow \rho^- D^0 \) | \( a_1 f_{\rho^-} m_{\rho^-}^2 \mathcal{F}^{B \rightarrow D^*}_{\mu \nu} (m_{\rho^-}^2) \) |
| \( B^- \rightarrow D^{*0} a_2^- \) | \( a_2 f_{D^{*0}a_2^-} m_{D^{*0}}^2 \mathcal{F}^{B \rightarrow a_2} (m_{D^{*0}}^2) \) |
| \( B^0 \rightarrow \rho^- D^*^{++} \) | \( a_1 f_{\rho^-} m_{\rho^-}^2 \mathcal{F}^{B \rightarrow D^*}_{\mu \nu} (m_{\rho^-}^2) \) |
| \( B^0 \rightarrow D^{*0} a_2^0 \) | \(- a_2 f_{D^{*0}a_2^0} m_{D^{*0}}^2 \mathcal{F}^{B \rightarrow a_2} (m_{D^{*0}}^2)/\sqrt{2} \) |
| \( B^0 \rightarrow D^{*0} f_2 \) | \( a_2 f_{D^{*0}f_2} m_{D^{*0}}^2 \cos \phi_T \mathcal{F}^{B \rightarrow f_2} (m_{D^{*0}}^2)/\sqrt{2} \) |
| \( B^0 \rightarrow D^{*0} f_2' \) | \( a_2 f_{D^{*0}f_2'} m_{D^{*0}}^2 \sin \phi_T \mathcal{F}^{B \rightarrow f_2'} (m_{D^{*0}}^2)/\sqrt{2} \) |

Table 1. Decay amplitudes for the CKM-favored \( B \rightarrow VT \) channels with \( \Delta s = 0, -1 \). The tabulated amplitudes must be multiplied by \((G_F/\sqrt{2})\epsilon^{\mu\nu}\).
| $\Delta s = 0$ | $B \to PT$ | $\text{BR}(B \to PT)$ | $B \to VT$ | $\text{BR}(B \to VT)$ | $VT/PT$ |
|---|---|---|---|---|---|
| $B^- \to \pi^- D^0_2$ | 4.07 $\cdot 10^{-4}$ | $B^- \to \rho^- D^0_2$ | 1.13 $\cdot 10^{-3}$ | 2.79 |
| $B^- \to D^0_2 \pi^-$ | 1.35 $\cdot 10^{-5}$ | $B^- \to D^*_2 a_2^-$ | 2.23 $\cdot 10^{-5}$ | 1.65 |
| $B^0 \to \pi^- D^+_2$ | 4.06 $\cdot 10^{-4}$ | $B^0 \to \rho^- D^+_2$ | 1.13 $\cdot 10^{-3}$ | 2.80 |
| $B^0 \to D^0_2 \pi^-$ | 6.79 $\cdot 10^{-6}$ | $B^0 \to D^*_2 a_2^0$ | 1.13 $\cdot 10^{-5}$ | 1.66 |
| $B^0 \to D^0 f_2$ | 7.34 $\cdot 10^{-6}$ | $B^0 \to D^* f_2$ | 1.17 $\cdot 10^{-5}$ | 1.60 |
| $B^0 \to D^0 f'_2$ | 8.73 $\cdot 10^{-8}$ | $B^0 \to D^* f'_2$ | 2.01 $\cdot 10^{-7}$ | 2.30 |

| $\Delta s = -1$ | $B \to PT$ | $\text{BR}(B \to PT)$ | $B \to VT$ | $\text{BR}(B \to VT)$ | $VT/PT$ |
|---|---|---|---|---|---|
| $B^- \to D^-_s D^0_2$ | 2.69 $\cdot 10^{-4}$ | $B^- \to D^*_s D^0_2$ | 1.05 $\cdot 10^{-3}$ | 3.91 |
| $B^- \to \eta_c K^{*+}_2$ | 8.08 $\cdot 10^{-6}$ | $B^- \to J/\psi K^{*+}_2$ | 7.62 $\cdot 10^{-5}$ | 9.43 |
| $B^0 \to D^-_s D^*_2$ | 2.69 $\cdot 10^{-4}$ | $B^0 \to D^*_s D^*_2$ | 1.05 $\cdot 10^{-3}$ | 3.91 |
| $B^0 \to \eta_c K^{*0}_2$ | 7.53 $\cdot 10^{-6}$ | $B^0 \to J/\psi K^{*0}_2$ | 7.55 $\cdot 10^{-5}$ | 10.03 |

Table 2. Branching ratios for Cabibbo-favored $B \to PT$, $VT$ decays with $\Delta s = 0, -1$. Last column shows the ratio for $VT/PT$ branching ratios.