Lightest Higgs Boson and Relic Neutralino in the MSSM with CP Violation

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Abstract. In a minimal supersymmetric extension of the standard model (MSSM) with explicit CP violation a mass of the lightest Higgs boson $H_1$ in the range $7 \text{ GeV} \lesssim M_{H_1} \lesssim 10 \text{ GeV}$ is experimentally allowed by present accelerator limits. In the same scenario a lightest neutralino as light as $2.9 \text{ GeV}$ can be a viable dark matter candidate, provided that a departure from the usual GUT relation among gaugino masses is assumed.

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1 Introduction

In SUSY models, CP violating phases in the soft terms can considerably enrich the phenomenology without violating existing constraints. In particular, in such a scenario it is well known that the mass of the lightest Higgs boson $H_1$ can be much lighter compared to the CP conserving case. We wish here to take a fresh look at how light $m_{H_1}$ can be, and discuss the cosmological lower bound to the mass of the relic neutralino $\chi$, when standard assumptions are made for the origin and evolution of its relic density. For more details of this analysis, see [1].

2 MSSM with explicit CP violation: the CPX scenario

In the presence of sizable CP phases in the relevant soft SUSY breaking terms, a significant mixing between the scalar and pseudo–scalar neutral Higgs bosons can be generated in the Higgs potential at one loop [2]. As a consequence of this, the three neutral MSSM Higgs mass eigenstates, labeled in order of increasing mass as $M_{H_1} \leq M_{H_2} \leq M_{H_3}$, have no longer definite CP parities, but become mixtures of CP-even and CP-odd states. Due to the large Yukawa couplings, the CP-violating mixing among the neutral Higgs bosons is dominated by the contribution of third-generation squarks and is proportional to the combination:

$$\frac{3m(A_f \mu)^2}{16\pi^2 m_{f_2}^2 - m_{f_1}^2},$$

with $f = t, b$. Here $\mu$ is the Higgs–mixing parameter in the superpotential and $A_f$ denotes the trilinear soft coupling.

In presence of CP violation, the mixing among neutral Higgs bosons is described by a $3 \times 3$ real orthogonal matrix $O$:

$$(\phi_1, \phi_2, \phi_3)^T = O (H_1, H_2, H_3)^T,$$

where the elements $O_{ij}$ are the CP-even components of the $i$-th Higgs boson, while $O_{ai}$ is the corresponding CP-odd component.

The Higgs-boson couplings to the SM and SUSY particles can be modified significantly due to the CP violating mixing. Among them, one of the most important ones is the Higgs-boson coupling to a pair of vector bosons, $g_{HVV}$, which is responsible for the production of Higgs bosons at $e^+e^-$ colliders:

$$\mathcal{L}_{HVV} = gM_W \left(W^+ W^- \mu + \frac{1}{2c_W} Z \mu Z^\mu\right) \sum_{i=1}^3 g_{H_iV VH_i},$$

where

$$g_{H_iV VH_i} = c_{3\beta} O_{\phi_{1i}} + s_{3\beta} O_{\phi_{2i}},$$

when normalized to the SM value. Here we have used the following abbreviations: $s_{3\beta} \equiv \sin 3\beta$, $c_{3\beta} \equiv \cos 3\beta$, $t_{3\beta} \equiv \tan 3\beta$, etc. We note that the two vector bosons $W$ and $Z$ couple only to the CP-even components $O_{\phi_{1,2i}}$ of the $i$-th Higgs mass eigenstate, and the relevant couplings may be strongly suppressed when the $i$-th Higgs boson is mostly CP-odd, $O_{ai}^2 \sim 1 \gg O_{\phi_{1i}}^2, O_{\phi_{2i}}^2$.

The so called CPX scenario is defined as a showcase benchmark point for studying CP-violating Higgs-mixing phenomena [3]. Its parameters are all defined at the electro–weak scale, and are chosen in order to
enhance the combination in Eq. (1). In this scenario, SUSY soft parameters are fixed as follows:

\begin{align*}
M_Q^3 &= M_D^3 = M_{D^c} = \\
M_{3} &= M_{E_3} = M_{SUSY}; |\mu| = 4M_{SUSY}, \|A_{t,b,\tau}\| = 2M_{SUSY}, |M_3| = 1 \text{ TeV,}
\end{align*}

where, with a usual notation, Q, L, U, D and E indicate chiral supermultiplets corresponding to left– and right–handed quarks and leptons. In this scenario \(\tan \beta\), \(M_{H^\pm}\), and \(M_{SUSY}\) are free parameters. As far as CP phases are concerned, we adopt, without loss of generality, the convention \(\arg(\mu) = 0\), while we assume a common phase for all the \(A_i\) terms, \(\Phi_A \equiv \arg(A_i) = \arg(A_b) = \arg(A_\tau)\). As a consequence of this, we end–up with two free physical phases: \(\Phi_A\) and \(\Phi_3 = \arg(M_3)\).

In addition to the parameters fixed by the CPX scenario, we need to fix the gaugino masses \(M_1, M_2\) for our study. We take them as free parameters independently of \(M_3\) since, for \(M_3\) once chosen, to relax the usual relations at the electro–weak scale: \(M_1/M_2 = g_1^2/g_2^2\) with \(g_1, g_2\) =gauge coupling constants, which originate from the assumption of gaugino–mass unification at the GUT scale. The neutralino \(\chi\) is defined as usual as the lowest–mass linear superposition of \(B\)-ino \(\tilde{B}\), \(W\)-ino \(\tilde{W}^{(3)}\), and of the two Higgsino states \(\tilde{H}_1^0, \tilde{H}_2^0\):

\begin{equation}
\chi \equiv a_1 \tilde{B} + a_2 \tilde{W}^{(3)} + a_3 \tilde{H}_1^0 + a_4 \tilde{H}_2^0.
\end{equation}

In Ref. 4 it was proved that in a CP–conserving effective MSSM with \(M_1 << M_2\) light neutralinos of a mass as low as 7 GeV are allowed. Indeed, for \(M_1 << M_2\) the LEP constraints do not apply, and the lower bound on the neutralino mass is set by the cosmological bound. In the following we will assume vanishing phases for \(M_1\) and \(M_2\), and we will fix for definiteness \(M_2 = 200\) GeV (the phenomenology we are interested in is not sensitive to these parameters in a significant way). On the other hand, we will vary \(M_1\), which is directly correlated to the lightest neutralino mass \(m_{\chi}\).

In this work, we rely on CPsuperH 5 for the computation of mass spectra and couplings in the MSSM Higgs sector.

### 3 Experimental constraints on the CPX scenario

In the CPX scenario the lightest Higgs boson is mostly CP odd and its production at LEP is highly suppressed since \(|g_{H,VV}| << 1\) though it is kinematically accessible. As a consequence of this, taking \(\Phi_A = \Phi_3 = 90^\circ\) and \(M_{SUSY} = 0.5\) TeV, the combined searches of the four LEP collaborations at \(\sqrt{s} = 91 - 209\) GeV reported the following allowed interval for a very light \(H_1\) 6, which we will focus on in our analysis:

\begin{equation}
M_{H_1} \lesssim 10 \text{ GeV} \quad \text{for} \quad 3 \lesssim \tan \beta \lesssim 10.
\end{equation}

![Fig. 1. The Thallium EDM $\hat{d}_{Tl} \equiv d_{Tl} \times 10^{24} \tan^2 \beta$ in the CPX scenario with $M_{SUSY} = 0.5$ TeV in the region $M_{H_1} \lesssim 15$ GeV and $3 < \tan \beta < 10$. The shaded regions correspond to different ranges of $|\hat{d}_{Tl}|$, as shown: specifically, the narrow region consistent with the current thallium EDM constraint, $|\hat{d}_{Tl}| < 1$, is denoted by black squares. In the blank shaded region we have $|\hat{d}_{Tl}| > 100$. The region below the thick solid line is excluded by data on $\tau'(1S)$ decay. For comparison, the thin line shows an estimation of the same boundary obtained using the tree–level coupling taking $O_{111} = 1$, i.e. $|\hat{d}_{Tl}| = \tan \beta$. Also shown are the three contour lines of the rescaled $\hat{B}(B_s \rightarrow \mu \mu) = (B_{s} \rightarrow \mu \mu) \times 10^7$: $\hat{B}(B_s \rightarrow \mu \mu) = 2$ (solid), 20 (dotted), and 200 (dashed).](image)

We observe that in the scenario analyzed by the LEP collaborations one has $|\mu| = 2$ TeV. For this large value of $|\mu|$, the neutralino is a very pure $B$-ino configuration, with a Higgsino contamination $a_3 \approx 0.02$. As will be shown in Section 5 this has important consequences for the phenomenology of relic neutralinos, in particular suppressing their annihilation cross section, and restricting the possibility of having a relic abundance in the allowed range only to the case of resonant annihilation. So, the exploration of different possibilities with lower values of $|\mu|$ could in principle be very relevant for relic neutralinos. However, this would require a re–analysis of LEP data which is beyond the scope of this paper.

CP phases in the MSSM are significantly constrained by the EDM measurements. In particular, the EDM of the Thallium atom provides currently the most stringent constraint on the MSSM scenario of our interest. In Fig. 1 we show the rescaled Thallium EDM $\hat{d}_{Tl} \equiv d_{Tl} \times 10^{24}$ in units of e cm in the $M_{H_1}$-$\tan \beta$ plane. Here, we consider only the contributions from the Higgs-mediated two–loop diagrams 7. Different ranges of $|\hat{d}_{Tl}|$ are shown explicitly by different shadings. In the blank unshaded region we have obtained $|\hat{d}_{Tl}| > 100$. We also note that the Thallium EDM constraint can be evaded by assuming cancellations between the two–loop contributions and other contributions, such as those from first– and second–generation sfermions.

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**Cosmology and Astrophysics**

**Contributed Talk**

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**3 Experimental constraints on the CPX scenario**

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$M_{H_1} \lesssim 10 \text{ GeV} \quad \text{for} \quad 3 \lesssim \tan \beta \lesssim 10$. (7)
In this way the allowed region shown in Fig. 1 can be enlarged. The amount of cancellation can be directly read–off from Fig. 1 For instance, in the region \( |\tilde{t}_{\chi}| < 10 \) it would be less severe than 1 part in 10.

In the region of Eq. (7) the bottomonium decay channel \( \Upsilon(1S) \to \gamma H_1 \) is kinematically accessible \([8]\), and the experimental upper bounds on this process can be directly converted to a constraint in the plane \( \tan \beta \cdot M_{H_1} \). The result is shown in Fig. 1, where the thin (red) line corresponds to the limit obtained by neglecting finite–threshold corrections induced by gluino and chargino exchanges, and by setting \( O_{\text{al}} = 1 \), while the thick solid line shows the same constraint when threshold corrections and the true value of \( O_{\text{al}} \) are used. From Fig. 1 one can see that, when the following constraints are combined: (i) the LEP constraint; (ii) Thallium EDM; (iii) the limit from bottomonium decay, the allowed parameter space is reduced to:

\[
7 \text{ GeV} \lesssim M_{H_1} \lesssim 7.5 \text{ GeV} \quad \text{and} \quad \tan \beta \simeq 3. \tag{8}
\]

This region may be enlarged to

\[
7 \text{ GeV} \lesssim M_{H_1} \lesssim 10 \text{ GeV} \quad \text{and} \quad 3 \lesssim \tan \beta \lesssim 5, \tag{9}
\]

if we assume 10 %-level cancellation in the Thallium EDM.

In light of the above discussion and for definiteness, from now on we will fix \( M_{H_1} = 7.5 \text{ GeV} \) and \( \tan \beta = 3 \) in our analysis. Taking into account the CPX parameter choice of Eq.(5) with \( \Phi_\lambda = \Phi_3 = 90^\circ \) and \( M_{\text{SUSY}} = 0.5 \text{ TeV} \), this implies, in particular: \( M_{H^+} \simeq 147 \text{ GeV} \), \( M_{H_2} \simeq 108 \text{ GeV} \), \( M_{H_3} \simeq 157 \text{ GeV} \).

As will be discussed in the following sections, if the pseudoscalar Higgs boson mass is in the range \([8]\), a CPX light neutralino with \( m_\chi \lesssim M_{H_1}/2 \) can be a viable DM candidate. Due to their very pure \( B \)–ino composition, and to the quite low value of \( \tan \beta \), neutralinos in this mass range evade constraints coming from accelerators. For instance, in the CPX light neutralino mass range the present upper bound to the invisible width of the \( Z \)–boson implies \( |a_1^2 - a_2^2| \lesssim 5 \) a few percent, a constraint easily evaded in this case.

A potentially dangerous constraint can come from the decay \( B_s \to \mu \mu \), since its dominant SUSY contribution scales as \( \tan^6 \beta |\mu|^2/M^2_{H_1} \), and may have a resonance enhancement when \( H_1 \) is so light that \( M_{H_1} \sim M_{B_s} \). In Fig. 1 we show three contour lines of the rescaled \( \tilde{B}(B_s \to \mu \mu) \equiv (B_s \to \mu \mu) \times 10^7 \cdot B(B_s \to \mu \mu) = 2 \) (solid), 20 (dotted), and 200 (dashed). For the parameters chosen by combining the results from LEP2 searches, Thallium EDM, and Bottomonium decay, Eq. (8), we get: \( B(B_s \to \mu \mu)_{\text{CPX}} \simeq 6 \times 10^{-7} \) taking \( f_{\text{fs}} = 0.23 \text{ GeV} \). This is three times larger than the present 95 % C.L. limit: \( B(B_s \to \mu \mu) < 2 \times 10^{-7} \). This can be easily made consistent with the present experimental constraint if some mild cancellation takes place. The “GIM operative point” mechanism discussed in Ref. [9] may be an example of such cancellation, where the squark mass matrices are flavour diagonal. In particular, we find that \( B(B_s \to \mu \mu)_{\text{CPX}} \) is consistent to the experimental upper bound by choosing \( 0.8 \lesssim \rho \lesssim 0.9 \), where \( \rho \equiv m_q/M_{\text{SUSY}} \) is the hierarchy factor introduced in Ref. [9], with \( m_q \) the soft mass for squarks of the first two generations.

For the discussion of other constraints, see [1].

### 4 The relic density

Taking into account the latest data from the cosmic microwave data (CMB) combined with other observations \([10]\) the 2–\( \sigma \) interval for the DM density of the Universe (normalized to the critical density) is:

\[
0.096 < \Omega_{m} h^2 < 0.122, \tag{10}
\]

where \( h \) is the Hubble parameter expressed in units of 100 km s\(^{-1}\) Mpc\(^{-1}\). In Eq.(10) the upper bound on \( \Omega_{m} h^2 \) establishes a strict upper limit for the abundance of any relic particle. In absence of some resonant effect, the natural scale of the annihilation cross section times velocity \( \sigma_{\text{ann}} v \) of CPX light neutralinos is far too small to keep the relic abundance below the upper bound of Eq.(10) (in particular they are very pure \( B \)–inos and their mass is below the threshold for annihilation to bottom quarks, which is usually the dominant channel of \( \sigma_{\text{ann}} v \) for light neutralinos \([8]\)). However, when \( m_\chi \simeq M_{H_1}/2 \) neutralinos annihilate through the resonant channel \( \chi \chi \to H_1 \to \text{standard particles} \), bringing the relic abundance down to acceptable values. In the Boltzmann approximation the thermal average of the resonant \( \sigma_{\text{ann}} v \) to the final state \( f \) can be obtained in a straightforward way from the following relation among interaction rates:

\[
\frac{n_\chi^2}{2} < \sigma_{\text{ann}} v >_{\text{res}, f} = < \Gamma(\chi\chi \to f) > = < \Gamma(\chi\chi \to H_1)B(H_1 \to f) > = n_{H_1} \Gamma_{\chi}(x_{H_1})K_1(x_{H_1})/K_2(x_{H_1})B_f, \tag{11}
\]

where brackets indicate thermal average, \( \Gamma_{\chi} \) is the zero–temperature \( H_1 \) annihilation amplitude to neutralinos and the thermal average of this quantity is accounted for by the ratio of modified Bessel functions of the first kind \( K_1 \) and \( K_2 \), \( B_f \) is the \( H_1 \) branching ratio to final state \( f \), \( n_\chi = g_\chi m_\chi^2 K_2(x_i)/(2\pi^2 x_i) \) are the equilibrium densities with \( x_i = m_i/T \), \( T \) the temperature, and \( g_\chi \) the corresponding internal degrees of freedom, \( g_\chi = 2 \), \( g_{H_1} = 1 \). The factor of 1/2 in front of Eq.(11) accounts for the identical initial states in the annihilation.

The result of our calculation is shown in Fig. 2 where the neutralino relic abundance \( \Omega_{\chi} h^2 \) is shown as a function of the mass \( m_\chi \). The asymmetric shape of the curve in Fig. 2 is due to the fact that thermal motion allows neutralinos with \( m_\chi < M_{H_1}/2 \) to reach the center–of–mass energy needed to create the resonance, while this is not possible for \( m_\chi > M_{H_1}/2 \). In the same figure, the two horizontal lines indicate the range of Eq.(10).
that one can have $<\sigma_{\text{ann}}v> \approx 0 < <\sigma_{\text{ann}}v>$. In fact, as already shown in Section 2, the thermal motion in the early Universe ($x_{\chi} < x_f \approx 20$) allows neutralino resonant annihilation when $m_{\chi} < M_{H_1}/2$. However, for the same neutralinos the contribution of the resonance to $<\sigma_{\text{ann}}v>\approx 0$ can be negligible at present times, since their temperature in the halo of our Galaxy is of order $x_{\chi,0} \approx 10^{-6} \ll x_f$. This implies that the annihilation cross section can be large enough in the early Universe in order to provide the correct relic abundance, but not so large at present times as to drive indirect signals beyond observational limits. As a consequence of this, in [1] we have discussed in detail signals for indirect Dark Matter searches and shown that they are compatible with the present experimental constraints, as long as $m_{\chi} \approx M_{H_1}/2$. On the other hand, part of the range $m_{\chi} \approx M_{H_1}/2$ allowed by cosmology is excluded by antiproton fluxes. Moreover, in [1] prospects of detection in future DM searches have been discussed, showing that CPX neutralinos might indeed produce a detectable signal. Finally, as far as a very light Higgs boson $H_1$ is concerned, we observe that the LHC might not be able to detect it, and a Super $B$ factory could thus be needed for its observation [12].

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**Fig. 2.** Relic abundance as a function of the neutralino mass $m_{\chi}$ for the CPX scenario with $M_{H_1}=7.5$ GeV, $\tan\beta=3$, $M_{\text{SUSY}}=0.5$ TeV and $\Phi_A = \Phi_3 = 90^\circ$. The two horizontal lines indicate the interval of Eq. (10).

In this scenario the neutralino relic abundance can fall in the range of Eq. (12) only with some level of tuning at the boundaries of the allowed mass range. For intermediate values of $m_{\chi}$ either the neutralino is a sub-dominant component of the DM, or some non-thermal mechanism for its cosmological density needs to be introduced. Of course all our considerations are valid if standard assumptions are made about the evolution of the early Universe (e.g. about the reheating temperature at the end of inflation, the energy budget driving Hubble expansion, entropy production, etc).

### 5. Dark matter searches

Neutralinos in the halo of our Galaxy can be searched for through direct and indirect methods. In particular, due to their very low mass and cross section, CPX light neutralinos are quite hard to detect through direct detection, although one proposal exists for such a hard task [11]. As far as indirect searches of CPX light neutralinos are concerned, in our scenario the neutralino relic density $\Omega_{\chi} h^2$ is driven below the observational limit by the resonant enhancement of the annihilation cross section $<\sigma_{\text{ann}}v>$. The same cross section calculated at present times, $<\sigma_{\text{ann}}v>_0$, enters into the calculation of the annihilation rate of neutralinos in our galaxy. This could produce observable signals, like $\gamma’s$, $\nu’s$ or exotic components in Cosmic Rays (CR), like antiprotons, positrons, antideuterons. Note, however,

\[ 2.93 \text{ GeV} \lesssim m_{\chi} \lesssim 5 \text{ GeV}. \] (12)

In this scenario the neutralino relic abundance can fall in the range of Eq. (12) only with some level of tuning at the boundaries of the allowed mass range. For intermediate values of $m_{\chi}$ either the neutralino is a sub-dominant component of the DM, or some non-thermal mechanism for its cosmological density needs to be introduced. Of course all our considerations are valid if standard assumptions are made about the evolution of the early Universe (e.g. about the reheating temperature at the end of inflation, the energy budget driving Hubble expansion, entropy production, etc).