On Free Field Realizations of Strings in BTZ

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\textbf{Abstract}
We discuss realizations of the $SL(2, R)$ current algebra in the hyperbolic basis using free scalar fields. It has been previously shown by Satoh how such a realization can be used to describe the principal continuous representations of $SL(2, R)$. We extend this work by introducing another realization that corresponds to the principal discrete representations of $SL(2, R)$. We show that in these realizations spectral flow can be interpreted as twisting of a free scalar field. Finally, we discuss how these realizations can be obtained from the BTZ Lagrangian.

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1 Introduction

$2 + 1$ dimensional anti-de Sitter space ($\text{AdS}_3$) is one of the recently studied examples of string theory in non-trivial backgrounds. Since $\text{AdS}_3$ is the group manifold of special linear transformations, it can be studied using a $SL(2, R)$ Wess–Zumino–Witten model. However, the physical spectrum of the $SL(2, R)$ WZW model seemed to contain states of negative norm. A resolution to this problem is to utilize an additional symmetry, known as spectral flow, in the theory. In the context of the $SL(2, R)$ WZW model, this symmetry was first discussed in [1, 2], where it was noted that spectral flow was necessary for the modular invariance of the theory. In [3], it was shown how the states with negative norm could be removed from the physical spectrum using spectral flow.

One of the possible extensions of [3] is to study orbifolds of AdS$_3$ space-time. These include AdS$_3/Z_N$ orbifolds, which have been examined in [4, 5]. Another interesting case is the Bañados–Teitelboim–Zanelli (BTZ) black hole [6], which can be obtained by quotienting AdS$_3$ by a boost. As opposed to pure AdS$_3$, specific features of the BTZ black hole include the appearance of space-time horizons and a region that is analogous to a space-like singularity. String theory on BTZ black holes based on the WZW model has been described in [7, 8, 9, 10]. In the context of spectral flow, the model has been analyzed by [11, 12]. Spectral flow in the fermionic sector has recently been discussed in [13].

In this letter, we consider a realization of the $SL(2, R)$ current algebra using free scalar fields. The realization is given in the hyperbolic basis which is the appropriate choice for a BTZ black hole. Similar investigation has been made by Satoh [14] (see also [15]), but the discussion was limited to the principal continuous representations only. We will also clarify the role of the spectral flow in this setup. Spectral flow can be formulated as twisting of a free field in a basis where the generator of the Cartan subalgebra is diagonal. We will also discuss how the scalar field realization can be derived starting from the classical BTZ Lagrangian. We show that the values of the spectral flow parameters obtained in this way coincide with the results of a previous study [11].

2 $SL(2, R)$ Current Algebra

2.1 Realizations in the Hyperbolic Basis

Most of this subsection is a review of [14]. The $SL(2, R)$ current algebra in the hyperbolic basis is expressed as

$$J^2(z) J^\pm (z') \sim \frac{\pm i J^\pm(z')}{z - z'}$$
\[ J^+(z) J^-(z') \sim \frac{-k}{(z - z')^2} - \frac{2iJ^2(z')}{z - z'} \]  
(1)

\[ J^2(z) J^2(z') \sim \frac{k/2}{(z - z')^2} \]

We wish to construct a realization of (1) using free fields. For the description of spectral flow, it is useful to work in a basis where the Cartan current \( J^2 \) is diagonal. A suitable realization is obtained when we introduce three bosonic scalar fields \( X_a \) \( (a = 0, 1, 2) \) obeying the following operator product expansion:

\[ X_a(z) X_b(z') \sim -\eta_{ab} \ln(z - z') \]  
(2)

The signature of the metric \( \eta \) is chosen to be \( \eta = \text{diag}(-1, 1, 1) \). Then the currents satisfying (1) can be constructed as in [14, 16]:

\[ iJ^\pm = e^{\pm \sqrt{2/k} X} \partial \left( \sqrt{k/2} X_0 \mp \sqrt{k'/2} X_2 \right) \]
\[ iJ^2 = \sqrt{k/2} \partial X_1 \]  
(3)

where the notations \( X_\pm = X_0 \pm X_1, k' = k - 2 \) have been introduced.

The energy-momentum tensor is obtained from the currents as follows:

\[ T = \frac{1}{k'} \eta_{ab} (J^a J^b) = -\frac{1}{2} \eta^{ab} \partial X_a \partial X_b + Q \partial^2 X_2 \]  
(4)

The resulting central charge of the system is \( c = 3k/k' \). We see that the energy-momentum tensor represents a system of three free fields, where the space-like field \( X_2 \) is coupled to the background charge \( Q = \frac{-1}{\sqrt{2k}} \). It turns out that this realization corresponds to the principal continuous representations of \( SL(2, R) \).

Next, one would like to know what are the vertex operators in this realization. One finds [14, 16]

\[ V_{j,J}(z) = \exp \left( iJ \sqrt{2/k} X_-(z) + j \sqrt{2/k'} X_2(z) \right) \]  
(5)

The relevant OPEs are then

\[ J^2(z) V_{j,J}(z') \sim \frac{J}{z - z'} V_{j,J}(z') \quad , \quad J^\pm(z) V_{j,J}(z') \sim \frac{J \mp ij}{z - z'} V_{j,J\pm i}(z') \]  
(6)

The conformal weight of the vertex operator \( V_{j,J} \) is related to the second Casimir of \( SL(2, R) \) as

\[ h_{V_{j,J}} = \frac{c_2}{k'} = \frac{-j(j + 1)}{k - 2} \]  
(7)
In the $SL(2, R)$ current algebra, there exist operators that contain only regular terms in their operator products with the currents. These so-called screening operators are given by [14]

$$\eta^\pm = \exp \left( \pm \sqrt{\frac{k}{2}} X_0 - \sqrt{\frac{k'}{2}} X_2 \right)$$
$$S = \partial X_0 \exp \left( -\sqrt{\frac{2}{k'}} X_2 \right)$$  \hspace{1cm} (8)

The expressions for the screening operators are determined up to constant factors and total derivatives.

This construction of a realization of the $SL(2, R)$ current algebra using three scalar fields is not unique. We need another realization that corresponds to the principal discrete representations in order to fully describe the physical spectrum of the $SL(2, R)$ current algebra. In this letter, we point out that this realization is obtained by making the substitutions $X_2 \rightarrow -iX_0$, $X_0 \rightarrow iX_2$ in the previous formulae. Then the currents and the stress tensor become

$$J^\pm = e^{\pm \sqrt{2/k} (X_1 - iX_2)} \partial \left( \sqrt{\frac{k}{2}} X_2 \pm \sqrt{\frac{k'}{2}} X_0 \right)$$
$$ij^2 = \sqrt{\frac{k}{2}} \partial X_1$$
$$T = -\frac{1}{2} \eta^{ab} \partial X_a \partial X_b + Q \partial^2 X_0$$  \hspace{1cm} (9)

with $Q = \frac{i}{\sqrt{2k'}}$. It should be noted that the field coupling to the background charge is now the time-like field $X_0$. The connection between this realization and the discrete representations will be established in the next section.

The vertex operator corresponding to this realization is

$$V_{j,J}(z) = \exp \left( -ij \sqrt{\frac{2}{k'}} X_0(z) - iJ \sqrt{2/k} (X_1(z) - iX_2(z)) \right)$$  \hspace{1cm} (10)

and it has the same dimension and satisfies the same OPEs as in the previous realization [6], (7).

### 2.2 Representations of $SL(2, R)$

If a holomorphic field $X(z)$ is coupled to a background charge $Q$, the mode expansion for $X(z)$ can be written as

$$X(z) = \hat{q} - i (\hat{p} - iQ) \ln z + i \sum_{n \neq 0} \frac{\alpha_n}{n} z^{-n}$$  \hspace{1cm} (11)

The corresponding Virasoro generators are then

$$L_n = \frac{1}{2} \sum_{\ell} \alpha_{\ell} \alpha_{n-\ell} + iQn\alpha_n + \frac{1}{2} Q^2 \delta_{n,0}$$  \hspace{1cm} (12)
The $SL(2,R)$ invariant vacuum state $|0\rangle$ is invariant under the global conformal transformations, $L_{0,\pm 1} |0\rangle = 0$. Using the realization (4), the vacuum state is found to be labeled by the background charge,

$$|0\rangle = p^0 = p^1 = 0, \quad p^2 = iQ = -i/\sqrt{2k'}$$

States corresponding to the vertex operators (5), i.e. the primary states, are then

$$|j; J\rangle = \lim_{z \to 0} V_{j,J}(z) |0\rangle = p^0 = p^1 = \sqrt{2/k} J, \quad p^2 = i(Q - \sqrt{2/k'} j)$$

From this expression, we see that the relation between $j$ and the momentum component $p^2$ is

$$j = -\frac{1}{2} + i\sqrt{k'/2} \, p^2$$

Let us compare this with the $j$-values of the principal continuous representation $\hat{C}$ of $SL(2,R)$. The unitary condition of $\hat{C}$ is the following:

$$j = -\frac{1}{2} + is, \quad s \in R$$

Hence we conclude that the realization (5) actually belongs to $\hat{C}$, since the momentum component $p^2$ is real.

However, we introduced another realization (10) where the background charge is coupled to a time-like field. This realization leads to a different set of vacuum and primary states:

$$|0\rangle = p^1 = p^2 = 0, \quad p^0 = iQ = -1/\sqrt{2k'}$$

$$|j; J\rangle = p^1 = -ip^2 = \sqrt{2/k} J, \quad p^0 = iQ - \sqrt{2/k'} j$$

The corresponding $j$-value is now

$$j = -\frac{1}{2} - \sqrt{k'/2} \, p^0$$

Now, we compare this with the $j$-values of the discrete representations $\hat{D}^\pm$ of $SL(2,R)$. The unitarity condition

$$j < -\frac{1}{2}$$

is satisfied if time is flowing forward, $p^0 > 0$.

There exists a problem with the Hilbert space of the discrete representations. Namely, their spectrum is afflicted by ghosts (states with negative norm). The ghosts appear in
the discrete representations when \( j < -k/2 \). The spectrum could be truncated by hand, but doing so would create a lower bound on the second Casimir \( c_2 = -j(j+1) \), which is related to the mass of a state. In string theory, such a bound would be considered artificial. A way out of this conundrum was given in [3] using the spectral flow symmetry of the theory.

The fact that \( J^\pm \) shifts \( J \) by \( \mp ij \) as in equation (6) might first appear puzzling, but this is actually a feature of the hyperbolic basis [17, 10, 11]. The operator \( J^2 \) now corresponds to a non-compact direction of the target space, and has a continuous spectrum. Therefore the states in the Hilbert space must be defined as

\[
|\Phi\rangle = \int_{-\infty}^{\infty} dJ \Phi(J) |j; J\rangle
\]

The action of the currents \( J^2, J^\pm \) on the states are

\[
J_0^2 |\Phi\rangle = \int_{-\infty}^{\infty} dJ J \Phi(J) |j; J\rangle
\]

\[
J_0^+ |\Phi\rangle = \int_{-\infty}^{\infty} dJ f(J) \Phi(J - i) |j; J\rangle
\]

\[
J_0^- |\Phi\rangle = \int_{-\infty}^{\infty} dJ g(J + i) \Phi(J + i) |j; J\rangle
\]

The functions \( f(J), g(J) \) correspond to matrix elements of \( J_0^+, J_0^- \).

### 2.3 Twisting and Spectral Flow

We now turn to discuss spectral flow in the free field realization. Our goal is to show how spectral flow can be interpreted as twisting of the field \( X_1 \) in the realizations introduced in section 2.1. The BTZ space-time has the topology of a solid cylinder with one compact space-like direction. Twisted sectors of the theory are generated by winding the string around the compact dimension. It will be shown in section 3 that the winding of the string can be accomplished by twisting the field \( X_1 \).

In the presence of rotation, the mode expansion for a scalar field compactified on a circle of radius \( R = \text{Re} R = \frac{1}{2}(R + \bar{R}) \) becomes

\[
X_1(z, \bar{z}) = q^1 - \frac{i}{2} (\bar{p}^1 + mR) \ln z - \frac{i}{2} (\bar{p}^1 - m\bar{R}) \ln \bar{z} + i \sum_{n \neq 0} \frac{1}{n} \left( \alpha_n^1 z^{-n} + \bar{\alpha}_n^1 \bar{z}^{-n} \right)
\]

The imaginary part of \( R \) is related to the intrinsic angular momentum of the rotating target space. If the system is non-rotating, \( \bar{R} = R \). The integer \( m \) measures the winding of the field \( X_1 \) around the compact dimension. Also, the momentum becomes quantized, \( p^1 = n/R \).
The mode expansion (23) tells that twisting acts as shifting of the momentum $p^1$. Effectively, twisting is

$$
\alpha_0^1 \rightarrow \alpha_0^1 - \sqrt{2k} w, \quad \bar{\alpha}_0^1 \rightarrow \bar{\alpha}_0^1 + \sqrt{2k} \bar{w}
$$

where we have introduced $w = -mR/\sqrt{2k}$, $\bar{w} = -\bar{m}\bar{R}/\sqrt{2k}$ for convenience. Consequently, the currents transform under (24) as

$$
J^\pm(n) \rightarrow \tilde{J}^\pm(n) = J^\pm(n) e^{\pm iw}, \quad \bar{J}^\pm(n) \rightarrow \tilde{\bar{J}}^\pm(n) = \bar{J}^\pm(n) \bar{z}^{\pm i\bar{w}}
$$

and the transformation of the energy-momentum tensor is the following:

$$
T(z) \rightarrow \tilde{T}(z) = T(z) + \frac{w}{z} J^2(z) + \frac{kw^2}{4z^2}
$$

Notice that this transformation is independent of the representations $\hat{C}$ and $\hat{D}^\pm$.

The transformed currents $\tilde{J}^a$ satisfy the $SL(2, R)$ algebra (1) and the Virasoro algebra. Thus the transformation (24) generates a new representation of the current algebra, which should be taken into account when considering the Hilbert space of the theory.

In terms of modes, the transformation reads

$$
J^\pm_n \rightarrow J^\pm_n \pm iw, \quad \bar{J}^\pm_n \rightarrow \bar{J}^\pm_n \pm i\bar{w}
$$

$$
J^2_n \rightarrow J^2_n + \frac{k}{2} w \delta_{n,0}, \quad \bar{J}^2_n \rightarrow \bar{J}^2_n - \frac{k}{2} \bar{w} \delta_{n,0}
$$

$$
L_n \rightarrow L_n + w J^2_n + \frac{k}{4} w^2 \delta_{n,0}, \quad \bar{L}_n \rightarrow \bar{L}_n - \bar{w} \bar{J}^2_n + \frac{k}{4} \bar{w}^2 \delta_{n,0}
$$

This result is in exact agreement with [11]. Hence we identify the transformation (24) as spectral flow in the free field realization. Because the transformation (24) is nothing but twisting, we conclude that spectral flow in the free field realization is twisting of the field $X_1$. A remaining task is to find out what is the compactification radius $R$. We will discuss this in section 3.

The importance of spectral flow comes from the fact that it can be used to eliminate the appearance of ghosts in the discrete representations of $SL(2, R)$. In general, the spectral flowed discrete representations are related by

$$
\hat{D}^\pm_{j,w} = \hat{D}^\pm_{-k/2-j}
$$

It was shown by Maldacena and Ooguri [3] that spectral flow naturally imposes the limit $-1/2 > j > -(k - 1)/2$ on the values of $j$. Within this range of $j$, ghosts do not appear in the spectrum of the discrete representations. If spectral flow is taken to be a symmetry of the theory, the theory will be free of ghosts.

\footnote{Note that also the screening charges remain invariant.}
3 Lagrangian Approach

3.1 BTZ Coordinates

The aim here is to show explicitly how the free fields $X_a$ introduced in the previous section are related with the fields appearing in the sigma model form of the BTZ Lagrangian. For a correct description of the $SL(2, R)$ current algebra at the quantum level, one needs to resolve some subtleties arising from the transformation of the classical Lagrangian. For the AdS$_3$ case, similar discussion has been given in [18, 19, 20].

The BTZ metric [6] for a 2 + 1 dimensional black hole is

$$ds^2 = -\frac{(r^2 - r_+^2)(r^2 - r_-^2)}{r^2}dt^2 + \frac{r^2 dr^2}{(r^2 - r_+^2)(r^2 - r_-^2)} + r^2 \left( d\theta - \frac{r_+ - r_-}{r^2} dt^2 \right)^2 \quad (29)$$

Here $r_-$ and $r_+$ are the inner and outer horizons, respectively. The angular coordinate has periodicity $\theta \sim \theta + 2\pi$. The time coordinate is usually taken to be non-compact, $-\infty < t < \infty$.

The mass and the angular momentum of the black hole are given in terms of the inner and outer horizon, $M_{BH} = r_+^2 + r_-^2$ and $J_{BH} = 2r_+r_-$. For a non-rotating black hole, the inner horizon goes to zero: $r_- = 0$.

To proceed with the analysis, we transform the metric (29) into a more convenient form. We make an analytic continuation to Euclidean time $t_E = it$ and make a coordinate transformation

$$\gamma(z) = \sqrt{\frac{r^2 - r_+^2}{r^2 - r_-^2}} e^{(r_+ - r_-)(t - it_E)}$$
$$\bar{\gamma}(\bar{z}) = \sqrt{\frac{r^2 - r_+^2}{r^2 - r_-^2}} e^{(r_+ + r_-)(t + it_E)}$$
$$\phi = -\frac{1}{2} \ln \left( \frac{r^2 - r_+^2}{r^2 - r_-^2} \right) - (r_+\theta + ir_-t_E) \quad (30)$$

The resulting metric is the Poincare patch for Euclidean AdS$_3$. The Lagrangian describing string propagation on this manifold can be written in the sigma model form as

$$L = k(\partial \phi \bar{\partial} \phi + e^{2\phi} \partial \gamma \bar{\partial} \gamma) \quad (31)$$

The level number $k$ has been included for the correct normalization of the path integral. The periodicity of the BTZ angular coordinate $\theta$ translates into the following periodic identifications for $\gamma, \bar{\gamma}, \phi$:

$$\gamma(z) \sim \gamma(z)e^{2\pi \Delta_-}, \quad \bar{\gamma}(\bar{z}) \sim \bar{\gamma}(\bar{z})e^{2\pi \Delta_+}, \quad \phi(z, \bar{z}) \sim \phi(z, \bar{z}) - 2\pi r_+ \quad (32)$$
where $\Delta_\pm$ is given in terms of the inner and outer horizon as $\Delta_\pm = r_+ \pm r_-$. A common trick is to rewrite the Lagrangian by introducing new fields $\beta, \bar{\beta}$. The original Lagrangian (31) can then be recovered from

$$L = k \partial \phi \bar{\partial} \phi - \beta \bar{\partial} \gamma - \bar{\beta} \partial \bar{\gamma} - \frac{1}{k} \beta \bar{\beta} e^{-2\phi}$$

(33)

by integrating over the fields $\beta, \bar{\beta}$. At quantum level, however, the above transformation leads to an anomalous factor in the functional measure. After evaluation of this factor (for details, see [19]), one gains the effective Lagrangian:

$$L = k' \partial \phi \bar{\partial} \phi - \frac{\phi}{4} \sqrt{\hbar} R^{(2)} - \beta \bar{\partial} \gamma - \bar{\beta} \partial \bar{\gamma} - \frac{1}{k} \beta \bar{\beta} e^{-2\phi}$$

(34)

Here $R^{(2)}$ is the scalar curvature in two dimensions. It is useful to think of the interaction term $L_{int} = -\frac{1}{k} \beta \bar{\beta} e^{-2\phi}$ as a screening current. In the limit $\phi \to \infty$, it can be treated perturbatively.

### 3.2 Currents

The free part of the Lagrangian (34) leads to the Wakimoto free field realization [21] of the $SL(2, R)$ current algebra. In this realization, coordinates $\beta, \gamma (\bar{\beta}, \bar{\gamma})$ constitute a holomorphic (antiholomorphic) system of bosonic ghosts. Their OPE is given by

$$\beta(z) \gamma(z') \sim \frac{1}{z - z'}$$

(35)

and a similar relation holds for the antiholomorphic fields. In the hyperbolic basis, the holomorphic currents of the $SL(2, R)$ current algebra are:

$$i J^+(z) = \beta(z)$$

$$i J^-(z) = ((\gamma \gamma) \beta)(z) + \sqrt{2k'} (\gamma \partial \phi)(z) + k' \partial \gamma(z)$$

$$i J_2(z) = (\gamma \beta)(z) + \sqrt{k'/2} \partial \phi(z)$$

(36)

Now, we introduce three scalar fields $X_a$ and (re-)bosonize the $(\beta, \gamma)$ system. The OPEs for the fields $X_a$ have been defined in [2]. The transformation between $\beta, \gamma, \phi$ and $X_a$ is

$$\beta(z) = e^{-\sqrt{2/k} X_-(z)} \partial \left( \sqrt{k'/2} X_0(z) - \sqrt{k'/2} X_2(z) \right)$$

$$\gamma(z) = e^{\sqrt{2/k} X_+(z)}$$

$$\phi(z) = 1/\sqrt{2k'} X_2(z) - 1/\sqrt{2k} X_+(z)$$

(37)
Not surprisingly, the resulting currents and the energy-momentum tensor coincide with the ones given in the previous section \cite{1,2}.

Next we consider the periodicity of the coordinates. From \eqref{32} we find that
\begin{equation}
X_-(z) \sim X_-(z) + \pi \sqrt{2k} \Delta_-, \quad \tilde{X}_-(\tilde{z}) \sim \tilde{X}_-(\tilde{z}) + \pi \sqrt{2k} \Delta_+ \tag{38}
\end{equation}

In the spirit of the previous section, we require that the twisted sectors of the system are generated by twisting of the field \(X_1\). The respective periodicities for holomorphic and antiholomorphic part of the coordinate \(X_1\) are then
\begin{equation}
X_1(z) \sim X_1(z) - \pi \sqrt{2k} \Delta_- \quad \tilde{X}_1(\tilde{z}) \sim \tilde{X}_1(\tilde{z}) - \pi \sqrt{2k} \Delta_+ \tag{39}
\end{equation}

In this setup, \(X_0\) and \(X_2\) are not periodic.\footnote{If we considered a single cover of the \(SL(2, R)\) group manifold, the time-like coordinate \(X_0\) would be periodic.} The same periodicities are obtained for the discrete representations.

Comparing the mode expansion \eqref{23} and the spectral flow transformation \eqref{24}, we find
\begin{equation}
w = m \Delta_- \quad \tilde{w} = m \Delta_+ \tag{40}
\end{equation}

A similar relation was found in \cite{11} using classical considerations.

\section{Summary}

We have examined a realization of the \(SL(2, R)\) current algebra in the hyperbolic basis using free scalar fields. The energy-momentum tensor of the model has a simple form, where one of the scalar fields couples to a gravitational background charge. We have shown that the realizations belong to the principal continuous representations \(\hat{C}\) if the field coupling to the background charge is space-like, and to the principal discrete representations \(\hat{D}^{\pm}\) if the coupled field is time-like. This has a nice analogy in \cite{3}, where it was found that long string states generated by spectral flow of space-like geodesics correspond to principal continuous representations, and short string states generated by spectral flow of time-like geodesics correspond to principal discrete representations.

We demonstrated how spectral flow could be interpreted as twisting of a free scalar field in this model. Using the BTZ Lagrangian as a starting point, we reviewed the construction of the free field realization of the \(SL(2, R)\) current algebra. The periodic conditions of the BTZ coordinates imply that only a discrete set of values is allowed for the spectral flow parameters. These values also agree with \cite{11}.

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