MIMO channel estimation with arbitrary angle of arrival incident power spectrum for wireless communications

Jie Zhou1 | Xueying Wang1 | Hao Jiang1 | Shigenobu Sasaki2 | Genfu Shao3

1 Department of Electronic and Electrical Engineering, Nanjing University of Information Science and Technology, Nanjing, People’s Republic of China
2 Department of Electronic and Electrical Engineering, Niigata University, Niigata, Japan
3 Department of Communication Engineering, Hangzhou Dianzi University, Hangzhou, People’s Republic of China

Correspondence
Hao Jiang, Department of Electronic and Electrical Engineering, Nanjing University of Information Science and Technology, 210044, Nanjing, People’s Republic of China.
Email: jianghao@nuist.edu.cn

Funding information
National Nature Science Foundation of China, Grant/Award Numbers: 61771248, 61971167

Abstract
This study proposes an approximate algorithm for the arbitrary angle of arrival (AoA) incident power spectrum in wireless environments. The approach expands approximate algorithms for small angles to the large-angle AoA PDF. To achieve this, the fading correlation of wireless channels is calculated and derived for various theoretical fittings and measured data. Then, the multiple-input multiple-output (MIMO) channel model is reconstructed for special environments using spatial fading correlation (SFC). Following this, the approximate algorithm and its complexity are investigated in depth in the SFC of multi-antenna arrays with small AoA angles under uniform, Gaussian, and Laplacian distributions, which can function in macrocell and microcell environments. Further, the closed form formulas for SFC are derived for the small angle spread of incident signals. The formulas are extended to simulate these cases with large angle spreads, which result in an increasing calculation complexity, particularly for the actual measured results. The selection of the number of samples and the weighting coefficient in the large angle spreads and its fitting accuracy are considered in detail. Based on this relationship, the properties of the temporal correlation and channel of the MIMO uniform linear array are studied and discussed.

1 INTRODUCTION

Massive multiple-input multiple-output (MIMO) antenna arrays (AAs) are the most promising candidates for system capacity and signal quality enhancement in 5G wireless communications. [1–3] Owing to the significant gains achieved by exploiting the multi-path richness of the channel, such arrays have been of great interest in recent years. Generally, MIMO systems have been considered for providing an effective solution means of achieving maximal spectral efficiency and high channel capacity. The authors of [4] proposed a visual scattering channel model for multibounced propagation rays in dense urban communication scenarios. In [5], the authors proposed a three-dimensional (3D) wideband geometry-based channel model for MIMO vehicle-to-vehicle (V2V) communications, in which a two-cylinder model was introduced to describe moving vehicles and multiple confocal semi-ellipsoid models were utilised to depict the internal surfaces of tunnel walls. Furthermore, in [6], the authors proposed a geometry-based stochastic model for capturing small-scale fading channel characteristics in key 5G communication scenarios, such as massive MIMO, high-speed train, V2V, and millimeter wave communication situations. In a MIMO system, the temporal and spatial statistical properties [7–9] of the received multi-path signals in the channel, such as the distributions of the angle of arrival (AoA) and angle of departure (AoD), play important roles in performance analysis. Rigorous analysis and performance investigation are not possible without knowledge of these statistical properties of the channel. A massive MIMO system is an enhanced version of a conventional MIMO system that uses an enormous number of antenna units, utilising a base station (BS) and mobile station (MS) equipped with dozens or hundreds of antenna units. Notably, the calculations for these systems are quite complex, and a large amount of central processing unit (CPU) time is required to investigate and simulate the system performance. Additionally, it is well known that the correlation in fading across multiple diversity channels results in degradation of the gain obtained by a MIMO system. On the other hand, incorporating MIMO technology into a small wireless device implies that the antenna units are limited by the spacing of the device,
which produces the spatial fading correlation (SFC). Therefore, the properties of the propagation channel and the characteristics of the antenna units are key factors in designing these systems. [10]

Prior studies, such as [11] and [12], have shown the SFCs of linear antenna arrays (LAAs) with AoAs following uniform, Gaussian, Laplacian, and Von Mises (VM) power distributions. More recently, [13] and [14] proposed a method of investigating a circular antenna array (CAA) as well as approximate algorithms to improve the computing speed when the angle spread is small. However, the performance of these approaches has not been studied, particularly when there are numerous antenna units. In several special environments, such as industrial, microcell urban, and dense urban street scenarios, the AoA distribution may be arbitrarily asymmetric, with a $2\pi$ angle spread. Some reports [15–17] have proposed sampling methods to solve this problem; in each of the corresponding studies, an arbitrary AoA distribution was simulated using the approximate SFC with different sampling pulses and various weight coefficients. However, the sampling pulse distributions considered were limited and unrelated to the measurement data. [18–23] It can be intuited that utilising new sampling pulses to simulate an arbitrary distribution and measured data is likely to provide superior performance to that of a channel in the estimation of a massive MIMO system. On the other hand, recent research on channels has revealed that the received waves approach from finite distinct directions with different delays. This characteristic is evident because, in the Internet of Things (IoT) era, the scatterers cannot be distributed uniformly throughout the coverage area, which makes the wireless coverage area small (e.g. in the case of a picocell). Then, owing to the large size of the massive antenna structure, it is necessary to consider spherical waves. Therefore, it is essential to study approximate algorithms and simulation methods for large angle spreads. [24–27]

In this work, we propose an approximate algorithm for an arbitrary AoA incident power spectrum, with the objective of expanding small-angle approximate algorithms to PDFs with large AoAs. The main contributions of this article can be summarised as follows:

- The fading correlation of wireless channels was calculated and derived for various theoretical fittings and measured data. According to the SFC properties, the MIMO channel model was reconstructed for special random environments, such as IoT applications. Then, the approximate algorithm and its SFC complexity were investigated in depth, with small AoA spreads following uniform, Gaussian, and Laplacian distributions, which can function in macrocell and microcell environments.
- The closed-form formulas for the SFC with a small angle spread were derived. The formulas were extended to simulate cases with large angle spreads, which involve complex calculations, particularly when processing actual measured results.
- The selections of the number of samples and weighting coefficient in cases with large angle spreads as well as the fitting accuracy were analysed in detail, and the properties of the temporal correlation and channel capacity of an MIMO uniform linear array (ULA) were studied.

The remainder of this work is organised as follows. In Section 2, the basic MIMO channel model and basic channel properties of massive MIMO are discussed. In Section 3, some explicit closed-form expressions to describe the SFC characteristics of AAs and their approximate algorithms are presented. Section 4 provides the estimation of the SFC among the antenna elements. A method of simulating an arbitrary incident power spectrum with a small angle spread is also shown, and a processing method for the measured channel is proposed. Section 5 discusses the channel characteristics of MIMO AAs used in those environments. Section 6 presents the numerical results and a corresponding discussion, as well as the conclusions.

## 2 | CHANNEL MODEL AND MASSIVE MIMO TECHNOLOGY

In recent times, massive MIMO technologies have received considerable attention in 4G/5G wireless systems because they provide superior spectral efficiency. Equipped with hundreds or even thousands of antenna elements, massive MIMO systems have emerged as enhanced MIMO models to meet increasing demands. For an MIMO system with $N_t$ transmit antennas and $N_r$ receive antennas, the well-known identically distributed (i.i.d) assumption (for which the fades between the antennas in transmit–receive pairs are independent and identical) does not hold in many practical situations. For the spatial correlation channel, the entries of the channel transfer matrix $H$ between transmit–receive stations are generated using the correlation characteristics of the transmitting and receiving fading signals.

$$H = R_t^{1/2} \cdot H \cdot R_r^{1/2}. \tag{1}$$

The ergodic capacity of a MIMO channel is defined as the statistical means values of the instantaneous capacity over time. Here, the channel capacity is

$$E[C] = E[\log_2 |\det(I_{N_t} + \frac{\text{SNR}}{N_r} HH^H})]|. \tag{2}$$

In Equation (1), $R_t$ is the $N_t \times N_t$ receiving correlation matrix, $R_r$ is the $N_r \times N_r$ transmitting correlation matrix, and $H$ is the $N_r \times N_t$ stochastic matrix with complex Gaussian i.i.d entries, which is described by the channel response from the transmitting antenna to the receivers.

In the existing research on massive MIMO channel modelling, typical power spectra, such as uniform, Gaussian, and Laplacian distributions have been considered. As shown in Figure 1, for massive MIMO communication systems, the temporal and spatial cross-correlation functions corresponding to the scattering propagation paths should be deduced. In particular, the SFCs corresponding to different power spectra and channel capacities for massive MIMO systems should
be included. In massive MIMO channel modelling, the traditional planar AA models cannot be applied directly because the variable shapes of mobile receivers and special communication environments make the incident power spectrum arbitrary. Thus, it is very important for channel parametric approximation algorithms to be employed for estimating the performance of the proposed AA model.

In MIMO techniques, multiple antennas are utilised at both the BS and MS levels to exploit the multi-path richness of the channel. One means of achieving high performance involves separating the antenna elements among BSs and MSs sufficiently to obtain diversity. Employing multiple antennas at a BS does not present a significant problem; however, accommodating more antennas in an MS introduces several constraints for practical implementation. Therefore, multiple compact AA configuration candidates should be considered. Promising configurations, such as ULAs, uniform circular arrays (UCAs), and uniform rectangular arrays (URAs), have emerged. As shown for a ULA in Figure 1, the relation between any two antenna elements is linear.

Most channel models developed for single-input single-output transmission systems inaccurately characterise systems with multiple antennas. Thus, it is necessary to extend these models to include the AA geometry description and incident signal AoA. Consider a 3D scattering model that incorporates the spatial AoA power spectra $p_\theta(\theta, \phi)$ and $p_\phi(\theta, \phi)$ in the azimuthal and elevation planes, as well as the vertical and horizontal polarisations. Then, the following normalised equation must be satisfied:

$$\int_0^{2\pi} \int_0^\pi \{p_\theta(\theta, \phi) + p_\phi(\theta, \phi)\} \sin \theta d\theta d\phi = 1.$$  (3)

As in Figure 1, for the MIMO ULA, UCA, and URA, the SFC \([R_{mm}(d, \lambda)]\) between antenna elements at positions \(n\) and \(m\) is defined as in [20, 21].

$$[R_{mm}(d, \lambda)] = \int_0^{2\pi} \int_0^\pi \{e_{\theta_n}(\theta, \phi)e_{\theta_m}^*(\theta, \phi)p_\theta(\theta, \phi)$$

$$+ e_\phi(\theta, \phi)e_\phi^*(\theta, \phi)p_\phi(\theta, \phi)\} \sin \theta d\theta d\phi.$$  (4)

In Equation (4), the superscript $\star$ denotes the complex conjugate. The horizontal polarisation scalar $p_\phi(\theta, \phi)$, that is, the joint PDF of the power spectrum of the AoAs, is considered. Additionally, assuming that the AoA and EoA are independent of each other, the function $p_{\phi}(\theta, \phi)$ can be decomposed into $p_{\phi}(\theta)p_{\phi}(\phi)$. Generally, the analysis of AoA distributions for a macrocell where $p_{\phi}(\theta) = 1$ shows that the key parameter determining the system performance is the spread of the power spectrum, rather than the type of PDF being considered. In this work, uniform, Gaussian, and Laplacian distributions were considered. According to [8] and [9], field measurement data show that the AoA distribution generally has a shape similar to a Laplacian distribution for microcells and a Gaussian distribution for macrocells. Therefore, the three distributions can be expressed as follows [8–10]:

$$p_{\phi}^U(\phi) = \frac{1}{2\Delta}, \quad \text{for} \quad \phi \in [\phi_0 - \Delta, \phi_0 + \Delta],$$  (5)

$$p_{\phi}^C(\phi) = \frac{1}{\sqrt{2\pi\sigma_G}} e^{-(\phi - \phi_0)^2/2\sigma_G^2},$$

$$\text{for} \quad \phi \in [-\Delta - \phi_0, \Delta + \phi_0]$$  (6)

$$p_{\phi}^L(\phi) = \frac{1}{\sqrt{2\pi\sigma_L}} e^{-\frac{1}{2}(\phi - \phi_0)^2/\sigma_L^2}, \quad \text{for} \quad |\phi - \phi_0| \leq \Delta,$$  (7)

where $2\Delta$ is the range of angles about a central AoA $\phi_0$ of the incident signal, and $\sigma_G$ and $\sigma_L$ represent the standard deviations of the Gaussian and Laplacian distributions, respectively, which determine the spread of the distributions.

3 | SPATIAL CORRELATION FUNCTION AND ITS APPROXIMATION

Termed as the diversity, which includes MIMO, spatiotemporal equalisation, adaptive array, and space–time code technologies, it is generally assumed that each receiving antenna element receives an uncorrelated signal. Conversely, this characteristic
may lead to a strong correlation because of the compact spatial structure. Then, the developed SFC can be used to determine the covariance matrices at both the transmitter and receiver for system capacity investigation in normal and massive MIMO models. Several approaches [11–13] have been used in theoretical SFC analyses. Based on Equations (4)–(7) and some derivations, the SFC between any two antenna elements at a distance \( d \) and positions \( n \) and \( m \) (shown in Figure 1) can be expressed as described in the following sections. Without polarisation of the antenna element, the performance analysis is performed in a directional frequency non-selective Rayleigh fading channel. The channel impulse response \( b(t) \) is then given by \[ b(t) = \sum_{j=1}^{J} \alpha_j(t) e^{j(\phi_j, \theta)} , \] (8)

where \( \alpha_j(t) \) is one of a set of zero-mean complex independent i.i.d random variables, \( e^{j(\phi_j, \theta)} \) is the steering vector of the AA in three dimensions, and \( J \) is the number of multi-path components. The spatial vector parameter is given as azimuth and elevation angles \( \phi \) and \( \theta \), with \( 0 \leq \phi \leq 2\pi \) and \( 0 \leq \theta \leq \pi \). Considering only a wide macrocell and the ULA shown in Figure 2 in a two-dimensional environment with vertically polarised antenna elements, the steering vector for the ULA is given by

\[ e^{j(\phi, \theta)} = [1, e^{jk_y \sin \phi}, \ldots, e^{jk_y(N-1)\sin \phi}]^T, \] (9)

where \( N \) is the number of antenna elements, \( d \) is the antenna spacing, and \( k_y = 2\pi/\lambda \), where \( \lambda \) is the wavelength. \([.]^T\) denotes the transpose. In previous research, such as [9–13], some SFC computational algorithms for ULA cases with various typical AoA power spectra have been investigated and proposed. These are highly convenient for estimating arbitrary power spectra and are summarised in the following sections.

### 3.1 Uniform power distribution model

According to the ULA geometry shown in Figure 1, \( D = 2\pi d/\lambda \). On substituting Equations (9) and (5) into Equation (4) and integrating the result, the real and imaginary parts of SFC \( R_{nm}(d, \lambda) \) between the \( n \)th and \( m \)th antenna elements for a uniform power distribution can be expressed as [11–13]

\[ \Re[R_{nm}(d, \lambda)] = f_0(D(n - m)) + 2 \sum_{i=1}^{\infty} f_{2i}(D(n - m)) \times \cos(2i\phi_0) \sin(2i\Delta), \] (10)

\[ \Im[R_{nm}(d, \lambda)] = 2 \sum_{i=1}^{\infty} f_{2i+1}(D(n - m)) \times \cos((2i + 1)\phi_0) \sin((2i + 1)\Delta), \] (11)

where \( f_i(\cdot) \) denotes the \( i \)th Bessel function. In general, to analyse a macrocell, the scatterers should be distributed at distances such that the AoA spread is small, that is, reduced by \( \Delta \). Making use of the approximations \( \sin \Delta = 0 \) and \( \cos \Delta = 1 \), the following approximation of \( R_{nm}(d, \lambda) \) can be derived:

\[ |R_{nm}(d, \lambda)| \approx \sin(D(n - m)\cos \phi \Delta). \] (12)

### 3.2 Gaussian power distribution model

If the incident signal follows a Gaussian power distribution according to Equation (6), the SFC of the ULA can be derived. Using the same definition of \( D \) and Equation (4) substituted with Equations (6) and (9), the SFC \( R_{nm}(d, \lambda) \) can be determined to be

\[ \Re[R_{nm}(d, \lambda)] = f_0(D(n - m)) + \sum_{i=1}^{\infty} f_{2i}(D(n - m)) \times e^{-2\sigma^2 i^2} \cos(2i\phi_0) \times \Re \left[ \frac{\Delta}{\sqrt{2\sigma}} - \sqrt{2\sigma i m} \right] - \Re \left[ \frac{\Delta}{\sqrt{2\sigma}} + \sqrt{2\sigma i m} \right], \] (13)

\[ \Im[R_{nm}(d, \lambda)] = \sum_{i=1}^{\infty} f_{2i+1}(D(n - m)) e^{-2\sigma^2 (i + 1/2)^2} \times \sin((2i + 1)\phi_0) \times \Re \left[ \frac{\Delta}{\sqrt{2\sigma}} - \sqrt{2\sigma (i + 1/2)m} \right] - \Re \left[ \frac{\Delta}{\sqrt{2\sigma}} + \sqrt{2\sigma (i + 1/2)m} \right], \] (14)

where \( \Re[R_{nm}(d, \lambda)] \) and \( \Im[R_{nm}(d, \lambda)] \) denote the real and imaginary parts of the \( R_{nm}(d, \lambda) \), respectively. Existing research has shown that Gaussian power distributions are suitable for
an analyzing mobile cellular systems. The scatterers are generally considered to be non-uniformly distributed and at a significant distance from the station, which results in a smaller AoA spread, that is, reduction by \( \sigma_C \leq \Delta \). Using the approximations \( \sin \sigma_C = \sin \Delta = 0 \) and \( \cos \sigma_C = \cos \Delta = 1 \), the following approximation of \( R_{\text{inc}}(d, \lambda) \) can be derived:

\[
| R_{\text{inc}}(d, \lambda) | \approx e^{-\frac{(D(n-m) + \phi_0)^2}{2}}.
\]

### 3.3 Laplacian power distribution model

Through Equation (7) in [11], a Laplacian power distribution was proposed to be a realistic model of the incident signal power distribution function in certain circumstances, particularly in microcell environments. Here, using the same definition \( D \) and Equation (4) substituted with Equations (7) and (9), the SFC \( R_{\text{inc}}(d, \lambda) \) of ULA can be determined to be [11–13] through derivation and exact analysis.

\[
\begin{align*}
\text{Re}[R_{\text{inc}}(d, \lambda)] &= J_0(D(n-m)) + 4 \sum_{i=1}^{\infty} J_2(D(n-m)) \\
& \times \cos(2i\phi_0) \left[ \sqrt{\frac{2}{\sigma_L}} \left( \frac{\sqrt{2}}{\sigma_L} \right)^2 + 4i^2 \right] \\
& + \frac{\epsilon}{\sqrt{2\sigma_L}} \{ 2i\sin(2i\Delta) - \sqrt{2} \cos(2i\Delta) / \sigma_L \} \\
& \left[ \sqrt{2\sigma_L} \left( \frac{\sqrt{2}}{\sigma_L} \right)^2 + 4i^2 \right] \\
\text{Im}[R_{\text{inc}}(d, \lambda)] &= 4 \sum_{i=1}^{\infty} J_{2i+1}(D(n-m)) \\
& \times \sin((2i+1)\phi_0) \left[ \sqrt{\frac{2}{\sigma_L}} \left( \frac{\sqrt{2}}{\sigma_L} \right)^2 + (2i+1)^2 \right] \\
& + \frac{\epsilon}{\sqrt{2\sigma_L}} \{ (2i+1) \sin((2i+1)\Delta) \} \\
& \left[ \sqrt{2\sigma_L} \left( \frac{\sqrt{2}}{\sigma_L} \right)^2 + (2i+1)^2 \right],
\end{align*}
\]

In the closed-form solutions presented in Equations (16) and (17), \( J_n(\cdot) \) is also the \( n \)th Bessel function. An additional objective here is to show that the generalised SFC for the Laplacian power distribution can be approximated. A power distribution is suitable for analysing open microcell wireless environments, in which the scatterers are clustered and are close to the stations. Using the same methods as in the previous sections, the following approximation of \( R_{\text{inc}}(d, \lambda) \) can be derived from a simple equation for a small AoA spread with \( \sigma_L \leq \Delta \):

\[
| R_{\text{inc}}(d, \lambda) | \approx \frac{1}{1 + \frac{\sigma_L^2}{2} [D(n-m) \cos \phi_0]^2}.
\]

### 4 SIMULATION OF ARBITRARY AoA INCIDENT POWER SPECTRUM

As shown in Figure 3, a massive MIMO system is equipped with more antennas (typically tens or hundreds more) than a conventional MIMO system. With such a large number of antennas, it has been demonstrated that a massive MIMO system has several benefits, such as high capacity, simple scheduling design in the frequency domain, and interference averaging based on the large number theorem. In general, a massive MIMO system can be considered as an enhanced version of a conventional MIMO system that utilises an extremely high number of antennas. According to measurements, the conventional MIMO channel models are not sufficient to capture certain characteristics of massive MIMO channels accurately. First, the far-field and plane wavefront assumptions have generally been applied to simplify the channel models. However, owing to the large number of antennas, the plane wavefront assumption is not fulfilled for massive MIMO channels. Instead, a spherical wavefront channel model should be considered. Second, dynamic clusters properties (such as cluster appearance and disappearance, AoA shifts and the non-stationary characteristics) were observed on the AA axis. Channel characterisation was performed, and the
appearance and disappearance of clusters formed arbitrary time variations in geometrical properties such as the AoA and AoD. For convenient simulation of and calculation in situations with arbitrary AoA power distributions, a simplified evaluation model based on the basic function sampling principle in the geometric angle domain was developed for MIMO systems. Based on the aforementioned analysis, the approximate computation formulas for different basic sampling functions were derived. Further, the number of basic sampling functions and weight coefficients were explored in detail. These assessments were performed in an attempt to reduce the complexity of massive MIMO computations significantly. The simulations for arbitrary power distributions can be classified into two types: (1) arbitrary AoA distribution matching, and (2) measurement observation adaptation.

4.1 Arbitrary AoA distribution matching

The spatial channel model differs from the traditional propagation model, which does not consider the spatial angle distribution resulting from the multi-path effect. As shown in Equations (5)–(7), different $\phi$-plane AoA power distributions have been proposed in the existing literature. In a given AoA scenario, these distributions may be substituted into the spatial correlation calculations shown in the previous sections. Further, researchers have only presented approximate spatial correlations suitable for AoA distributions with small spreads. Thus, one objective of the present work was to develop an effective approximate AoA distribution formula to simulate the case of an arbitrary AoA power spectrum with a large angle spread. Here, with an arbitrary distribution, the SFC does not have a closed-form solution, and computationally complex discretised summation is necessary to evaluate the SFC precisely. As shown in Figure 4, this simulation formulation combines multiple basic functions (such as Equations (5)–(7)) to fit a given arbitrary AoA power distribution. By adopting the $2\Delta$ interval in the angle domain for one basic function, the simulated arbitrary power spectrum can be expressed as

$$ R_{nm}(d, \lambda) = \sum_{i=1}^{N} C_i(\phi) R_{nm}(d, \lambda). \quad (20) $$

If the normalised parameter criterion is adopted, the power spectrum is formed post-sampling, using the basic function as the same probability area. The normalised weight coefficients in Equation (20) can be expressed as

$$ C_i(\phi) = \int_{\phi, -\Delta}^{\phi, +\Delta} \rho_{\beta}(\phi) d\phi. \quad (21) $$

To obtain a highly accurate simulation, a small value of $2\Delta$ is generally selected. Thus, Equation (21) can be derived as

$$ C_i(\phi) = \frac{\rho_{\beta}(\phi_i)}{\sum_{i=1}^{N} \rho_{\beta}(\phi_i)}, \quad (22) $$

where $\rho_{\beta}(\phi_i)$ is the $i$th basic function. Selecting the number of functions $N$ and any arbitrary AoA power distribution for a specified wireless transmitting scenario (such as indoor or outdoor, macrocell or microcell), the channel model may finally be represented and simulated using the proposed exact and approximate formulas. Thus, in the approximation, the computation time required for the correlation calculation can be reduced by using the proposed formulation. This approach may be highly applicable to massive MIMO systems, where the AoA power spectrum may correspond to the continuous, whole $2\pi$-angle domain or dispersed multiple clusters, requiring a large number of basis function samples and resulting in a reduction in computational efficiency. Therefore, the disperse probabilities can be sorted based on their values in the analysis, with number $M$ taken as the accumulated probability set following the sorting process. Here, computational precision $\eta$ is defined, and the satisfied inequity is

$$ \sum_{i=1}^{M} \rho_{\beta}(\phi_i) \geq \eta. \quad (23) $$

Therefore, according to Equations (4), (19) and (20), SFC of MIMO systems in arbitrary AoA power spectra can be expressed as

$$ [R_{ij}(d, \lambda)] = \sum_{i=1}^{M} \frac{\rho_{\beta}(\theta_j)}{\sum_{i=1}^{N} \rho_{\beta}(\theta_j)} [R_{ij}(d, \lambda)]_{ij}, \quad (24) $$

where $\rho_{\beta}(\theta_1) \geq \rho_{\beta}(\theta_2) \geq \cdots \geq \rho_{\beta}(\theta_M)$. 

**FIGURE 4** Sampling and approximation principle for an arbitrary spectrum using basic functions
4.2 Measurement observation adaptation

The emerging 5G wireless communication system is a ubiquitous information network with a multiband and ultradense heterogeneous system. The network is expected to function in almost imaginable environments; it also needs to work in different frequency bands. Stanford University, Rice University, Tokyo University of Technology, and NTT are actively conducting channel and measurement studies for specific scenarios, and they have obtained large amounts of measured data. Some research, such as that conducted by Tokyo University of Technology and NTT, [19, 20] is focused to modelling and measuring the channels of urban streets in Tokyo. In [23], the researchers focused on measurements on the campus of North-western Polytechnical University (NPU); these observations led to the need for more sophisticated models of the angle characteristics in urban radio channels. It was then concluded that the traditional spatial domain geometry-based single-bounce elliptical model must be modified to perform estimations in these special scenarios. The method of obtaining accurate fitting for these measured data is based on different practical scenarios. The exponential fitting regression method may be used to process the measured data. The large angle expansion is fitted step by step to obtain more accurate weighting coefficients on finishing sampling via the basis function. Based on its characteristics, the adaptation of measurement observations is suitable for combining double-ray models with the least squares method to obtain the optimal fitting of measured AoA data.

As shown in Figure 6, the asymmetric and urban street channel results of [20–23] demonstrate that both measured data and asymmetric theoretical analysis channel data have exponential characteristics. Thus, the exponential non-linear fitting scheme can be used to simulate them. In this study, the distribution characteristics of the AoA power spectrum were investigated, and piecewise exponential non-linear fitting was used to simulate the measured AoA data within the angle domain [0, 2π]. Generally, the exponential function regression equation is assumed to be as follows [24]:

\[ y = a \cdot b^x. \] (25)

First, the curve equation of Equation (25) is linearised, that is, the logarithms are taken on both sides of the formula: \( \log(y) = \log(a) + \log(b) \). By transformation, this formula can be expressed as \( Y = A + Bx \), where \( Y = \log(y), A = \log(a), \) and \( B = \log(b) \). Since Equation (1) is a regression relationship, the sum obtained directly after linearisation is often highly unsatisfactory, which may lead to a significant deviation between the fitting curve and actual measured data. In this study, exponential functions were directly adopted for the non-linear fitting approach. First, it was assumed that the measured data sequence was \( (x_i, y_i), i = 1, 2,... n \) in the region \( [0, 2\pi] \), and all those data points were assumed to follow an exponential distribution. Based on the principle of the least squares method, the sum of the squares of the errors was taken to be

\[ Q = \sum_{i=1}^{n} \delta_i^2 = \sum_{i=1}^{n} (ab^x - y_i)^2. \] (26)

To minimise \( Q \), the partial derivatives of \( a \) and \( b \), respectively, were obtained in Equation (26). They were brought to 0 as follows:

\[ \frac{\partial Q}{\partial a} = 2 \sum_{i=1}^{n} (ab^x - y_i)ab^x = 0, \]

\[ \frac{\partial Q}{\partial b} = 2 \sum_{i=1}^{n} (ab^x - y_i)ab^x - 1x_i = 0. \] (27)

Equation (27) then becomes

\[ a \sum_{i=1}^{n} b^{x_i} - \sum_{i=1}^{n} y_i b^{x_i} = 0, \]

\[ a \sum_{i=1}^{n} x_i b^{x_i} - \sum_{i=1}^{n} y_i b^{x_i} = 0. \] (28)

The solution of Equation (28) can be obtained as follows:

\[ \sum_{i=1}^{n} x_i y_i b^{x_i} - \sum_{i=1}^{n} y_i b^{x_i} = 0, \] (29)

Parameters \( b \) and \( a \) can be obtained from Equation (29) and substitution of \( b \) in Equation (28), respectively. When analysing large measured datasets, \( a \) and \( b \) can be quickly calculated and
simulated by a supercomputer. The approximate value of \( b \) can be obtained by dichotomy. Equation (29) can be written as a functional formula, in which the values of \( b_1 \) and \( b_2 \) may estimated to make \( f(b_1)\leq 0 \) (assuming \( f(b_1) < 0 \) and \( f(b_2) > 0 \)). Then, there exists a solution of \( f(b) = 0, b = b' \), in the interval \([b_1 : b_2]\). If the midpoint of the interval is taken and satisfies \( f(b_k) = 0 \), then the solution of Equation (24) is \( b = b_k \). If \( f(b_k) > 0 \), the interval \([b_1 : b_k]\) is retrieved, and the above process is repeated until the accuracy requirement is satisfied. The exponential function is used as a direct non-linear fitting approach to obtain the fitting formulas for the asymmetric channel model [18] or the measured data. [19, 20] Additionally, the theoretical fitting of the measured data can be completed using the algorithm from Section 3.1. In the future, analysis based on the measured data can be deepened to make the massive MIMO simulation describe the actual channels more closely and to improve the reliability of the obtained results.

5 RESULTS AND DISCUSSION

In the previous section, numerical and computer simulation results based on the ULA antenna elements shown in Figures 2 and 3 were presented for illustration and comparison. This can be used for any arbitrary choice of antenna patterns and incident signal distributions. However, it is important to validate the proposed model using the specified standard angle distributions and antenna patterns because this study was primarily focused on the guidelines provided by approximate computations and the matching of arbitrary AoA distributions and measurement results used globally in certain literature. Depending on the antenna elements of interest, the channel SFC can differ significantly from one element pair to another, even under the same channel conditions. The conditions were numerically computed using computer simulation programs written in MATLAB and run on a computer with an Intel Pentium IV 3 GHz CPU, 256 GB of random access memory, and a 1 TB hard disk. Hence, the general characteristics of the sensitivity of the array to various incident signal distributions were fully represented. A computation method for the mixtures of arbitrary AoA distributions was introduced in this paper. This method may be used to estimate the parameters of the individual components of a physical channel model constituting the mixture parameters shown in Table 1. The exact and approximate computation algorithms were described in the previous sections.

To demonstrate the usefulness of these derivations and to obtain an algorithm to verify the coincidence of the exact and approximate analysis, an MIMO ULA was considered, as shown in Figure 2. It is worth mentioning that the spatial fading functions are Bessel, which indeed fluctuates when increasing the antenna spacing but the general trend is to decrease with the increase of the antenna spacing. Here, the antenna elements were designed to be non-directional and arranged in a linear architecture. The formula presented herein could also be used in specific applications in which the directional antenna element is introduced or the elements are arranged in any geometry. Then, from Figure 2, assuming that \( \phi_0 = \pi / 3 \), \( m = 2 \), \( n = 4 \), the angle spread \( \Delta \) or \( \sigma \) ranges from zero (representing an infinitesimal-width sector) to 2\( \pi \), which was set to 0°, 2°, 5°, 10°, and 20° to obtain the results in Figure 7 for uniform, truncated Gaussian, and Laplacian angle distributions. In addition, there is no directional bias for the AA, and a strong correlation exists between the two antenna elements if the angle spreads are zero. As expected, Figure 7(b) demonstrates that the correlation SFC decreases as \( d/\lambda \) increases. Further, as \( \Delta \) increases, the correlation decreases, in the manner of sinc vibrations with larger angle spread.

The numerical results were calculated using Equations (10) and (11) for the exact analysis, and Equation (12) for the approximate analysis. It can be observed that the approximate equation provides a very good fit to the exact results up to 5° and up to 10° as \( d/\lambda \leq 1 \). Comparing these results, as \( \phi_0 \) increases, the fit is improved for a moderate value of \( \Delta \). When the angle spread \( \Delta \) is below about 5° to 10°, the approximate results are in good agreement with the exact results that are dependent on the incident angle \( \phi_0 \). Figure 7(b) and (c) shows the exact and approximate correlation results for Gaussian and Laplacian angle distributions with various standard deviations \( \sigma_G \) and \( \sigma_L \) for the incident angle \( \phi_0 = 60° \). For \( \sigma \) up to 10°, the approximate results well fit the exact results with an abstract error within 0.10. Comparing Figure 7(a)-(c), the SFC results for the uniform distribution decrease more quickly in the main lobe, but the secondary correlation peaks are lacking for the other two distributions. Furthermore, the output signals between the

### Table 1 Detailed calculation parameters

| Name         | Values | Meaning                                  | Notice          |
|--------------|--------|------------------------------------------|-----------------|
| \( n \)      | 4      | n-th antenna element                     | Refs. [20] and [21] |
| \( m \)      | 2      | m-th antenna element                     | Refs. [20] and [21] |
| \( k \)      | 2      | Angular spread factor                    | Refs. [10]      |
| \( \phi \)   | 0–\( \pi \) | Angle of signal arrival                | Refs. [16] and [17] |
| \( N \)      | [18, 24, 36, 720] | Number of sampling basic function    | Refs. [13–15]  |
| \( \Delta/\sigma \ (\text{deg}) \) | [0, 2, 5, 10, 20] | Angular spread                        | Refs. [11–13]  |
| \( d/\lambda \) | 0.5 or 0.15 | Parameter of uniform antenna            | Refs. [9]       |
| \( \Phi(\phi) \) | Uniform, Gaussian, Laplacian | PDF of angle of arrival                | Refs. [8–10]   |
| Matching methods | Non-linear fitting scheme               | Theoretical and measurement results   | Refs. [24]      |
array elements are closely related if the angle spread is very small. Clearly, the increase in $d/\lambda$ between the antenna elements on the receiver side results in a low receiver antenna correlations. Additionally, the SFCs of the three angular distributions were compared to each other when the angle spread was set to 20°. The results indicate that the Laplacian distribution simulation model was able to capture the channel characteristics with high computation accuracy, as shown in Figure 7(d), when the approximate computing algorithm was adopted.

The correlations obtained using the three distributions in Figure 7 share trends similar to $Sinc$ and exponential variations. Exact numerical integration requires more time than approximate derivation with a small angle spreads, as shown in Table 2. Based on the results, the main efficiency comparison was made between the exact calculations and approximation formulation. It was found that for the three distributions, the approximate computation times were reduced by 98.3%, 98.2% and 98.2% compared to those of exact and direct integration. Notably, the simulated results are in good agreement with the accurate and approximated ones, particularly in the case of a small angle spread. This finding confirms the accuracy of the derivation and simulation. In addition, because the correlation of the signals received by different antennas will affect the performance of the communication systems, approximate results are helpful for optimising the layout of massive AAs. As in the above work, the approximate SFC results only fit well when calculated with either of the closed-form formulas under the condition of small angle spread. As the angle spread increases beyond 10°, less desirable approximate results may be produced. Therefore, each of the multiple AoA distributions should also be chosen to have a small angle spread, and in this case, a spread of less than 10° provides confidence in the accuracy. As described in Section 4, the aforementioned approximation can be extended to arbitrary AoA scenarios, as shown in Figure 4. For an arbitrary AoA distribution that cannot easily be described by a mathematical formula, a discretised summation may be made for its MIMO SFC evaluation. Thus, using the proposed approximation formulation, the computation time required for the correlation calculation can be reduced or used to perform the analysis with a larger angle spread. It is important that to validate the proposed model is validated using the specified standard angle distribution. In this context, the characteristics of azimuth angles are generally well captured by various special density spectra. In recent years, the VM distribution has received significant attention in modelling non-isotropic propagation due to its close association with radio environments. This distribution is given by

$$p_{\phi}(\phi) = \frac{\exp(\kappa \cos(\phi - \phi_0))}{2\pi I_0(\kappa)}, \quad \phi \in [-\pi, +\pi],$$

where $I_0(\kappa)$ is the modified Bessel function of order $n$, $\phi_0$ is the mean AoD/AoA (also called the central AoA signal angle in Section 2), and $1/\kappa$ is a measure of dispersion. In Figures 8–10, the estimated correlation values from the direct numerical integration model provide a far more accurate fit to the simplified calculation curves as the sample number $N$ increases. This
TABLE 2 Efficiency comparison of three distributions in Figure 7 (ms)

| $\Delta/\sigma$ | Uniformly | Gaussian | Laplacian |
|-----------------|-----------|----------|-----------|
|                 | Accurate  | Approximate | Accurate  | Approximate | Accurate  | Approximate |
| $0^\circ$       | 1.969     | 0.033    | 2.720     | 0.048    | 1.937     | 0.035    |
| $2^\circ$       | 2.012     | 0.016    | 3.427     | 0.060    | 1.916     | 0.033    |
| $5^\circ$       | 1.867     | 0.015    | 3.903     | 0.040    | 1.921     | 0.033    |
| $10^\circ$      | 1.872     | 0.015    | 2.897     | 0.034    | 1.915     | 0.033    |
| $20^\circ$      | 1.874     | 0.015    | 3.351     | 0.033    | 1.914     | 0.033    |

FIGURE 8 SFC and computational error versus $d/\lambda$ between two antenna elements with various incident AoA power spectra, where $\phi_0 = 0^\circ$, $\kappa = 2$ and $\Delta_S/\sigma = 20^\circ$. (a) SFC results, (b) Absolute error

FIGURE 9 SFC and computational error versus $d/\lambda$ between two antenna elements with various incident AoA power spectra, where $\phi_0 = 0^\circ$, $\kappa = 2$ and $\Delta_S/\sigma = 10^\circ$. (a) SFC results, (b) Absolute error

finding validates the proposed novel SFC function under an arbitrary AoA power distribution. It can be seen that $\Delta$ or $\sigma$ is set to $20^\circ$, the VM distribution has $\kappa = 2$ and $\phi_0 = 0$, and the number of sampling basic functions $N = 18$ in Figure 8(a); $\Delta$ or $\sigma$ is set to $15^\circ$ and $N = 24$ in Figure 9(a); and $\Delta$ or $\sigma$ is set to $10^\circ$ and $N = 36$ in Figure 10(a). Figures 8(b)–10(b) depict the absolute error in three simplified models with different sample numbers $N$. For comparison, the absolute error of the uniform distribution model is significantly greater than those of the other two simplified models. It is also evident that when $d/\lambda \leq 1.5$, the error is relatively large, but the fitting degree gradually improves with increasing $d/\lambda$. In addition, the error increases with increasing sampling angle $\Delta_S$ as shown in Figure 4. Therefore, to achieve the optimal results, the
antenna spacing should be slightly larger and the sampling interval should be smaller. From Table 3, it is apparent that the simplified calculation model provides a higher computational efficiency than the numerical integration scheme. In Figures 8–10, the results show that the performance of the Laplacian pulse is similar to that of the Gaussian pulse but better than that of the uniform pulse. Moreover, high computational efficiency is observable with the proposed approach based on three types of pulse sampling in the angle domain and numerical integration. Evidently, the proposed method can reduce the complexity and computing time by 99.37%, 99.28% and 99.3% at $N = 72$ (i.e. $\Delta = 5^\circ$ and $\sigma = 5^\circ$). The proposed method is suitable for any arbitrary AoA scenario, enabling high computational accuracy and significantly short computing time.

Based on the aforementioned results, the approximate computation formulas for different sampling basic functions were derived. Further, the SFC for any arbitrary AoA scenario was efficiently investigated as a function of the number of basic sampling functions. The proposed approach can also simulate the arbitrary asymmetry distributions and measurement observations. An asymmetric geometry-based statistical channel model for cellular mobile systems, proposed in [18], was used, as shown in Figure 5. When the mobile station was located at the edge of the macrocell, the incident signal PDF based on the geometrical model (Figure 5) was derived as shown in Figure 11(a) with various radio parameters (see [18]). Because of the asymmetric geometry, we selected results (marked ‘used’) that showed that the curve of the AoA PDF was asymmetric and had two ‘corners’ and also with the width of the incident
signal spread and that could be expressed using a theoretical formula. Using the approach proposed herein, the theoretical results were sampled using the number of sampling basic functions \( N \); the estimated SFCs are shown in Figure 11(b) with sampling parameters of 4° and 16°. From the results, it is clear that for the truncated Laplacian distribution, the fitting improves with increasing \( d/\lambda \) and the angle spread decreases. For \( d/\lambda \leq 1.5 \), the numerical results indicate that the absolute error is large. Additionally, it is evident that the proposed model ensures a good fit, particularly for the sampling basic functions with PDFs with small AoA spreads.

As described in Section 4.2, accurate modelling of the spatial radio channel is crucial. A field measurement campaign was conducted to characterise the spatial propagation accurately. The incident signal distributions and their angle spreads in both the azimuth and elevation domains were measured in [23]. The measurements exhibited a highly complex relationship between the angle spread and surrounding environment. Figure 12(a) presents these observations, which indicate that more sophisticated models of the angular characteristics in urban radio channels (on the NPU campus) are required. Based on the collected data and using the non-linear fitting scheme, the distributions were tested against truncated Gaussian and Laplacian densities; the latter emerged as a best fit Laplacian PDF. As an actual example, as shown in Figure 12(a), a best fit Laplacian PDF can be expressed with the angular spread \( \sigma_L = 7° \). Using the approach proposed herein and sampling the theoretical results using the number of sampling basic functions \( N \), the SFCs were estimated to be as shown in Figure 12(b), with slow exponentially decreasing attenuation. From the results, considering sampling parameters of 4° and 16°, it is apparent that even for the truncated Laplacian distribution, the fit improves with increasing \( d/\lambda \) and the angle spread decreases. The numerical results indicate that the absolute error is large and decreases with increasing \( N \). Thus, the proposed algorithm model provides a good fit, particularly for basic sampling functions with small angle spreads. Thus, the effectiveness of the proposed fitting algorithm was verified, and the measured AoA distribution is plotted in Figure 12(a). Figure 12(b) compares the theoretical results with those of the proposed simplified model. In future work, we will focus on further reducing the computation time and improving the computation accuracy, factors that are becoming increasingly crucial for the incorporation of massive MIMO technology.

6 CONCLUSIONS

This work proposed an approximate algorithm for an arbitrary AoA incident power spectrum that can be expanded for PDFs with large AoA. Then, the approximate algorithm and its complexity in terms of the SFCs of MIMO multi-antenna arrays, which are implemented in mobile cellular applications, were investigated in depth. The derived analytical formulas were provided to demonstrate the accuracy of the proposed approximation algorithm. Moreover, a novel simplified model for the power spectrum of an arbitrary AoA, PDFs with symmetrical/asymmetrical AoAs, and actual measured results were proposed. The numerical results show that the SFC decreases with increasing antenna spacing, while the amplitude fluctuation of the absolute error decreases with increasing \( d/\lambda \). It was demonstrated that the approximations are effective for standard deviations of approximately 10° or less based on the incident signal angle \( \phi_0 \). It can also be seen that the Gaussian distribution decreases more slowly in the main lobe, but lacks the secondary correlation peak contraction obtained in a uniform distribution. The proposed model fits well with numerical integration and reduces the SFC computation time by at least 100 times. Further, by modelling the calculation formulas as the weighted superpositions of limited numbers of basic sampling functions, the complexity of the spatial correlation evaluation of the MIMO fading signal can be reduced significantly. It can be concluded that the proposed algorithms enable good approximation and have low computational complexity. It is believed that this research work provides a foundation for further development of 5G massive MIMO systems.
ACKNOWLEDGEMENTS
The authors would like to thank the anonymous reviewers for their constructive comments, which greatly helped improve this paper. The authors also acknowledge Prof. Fumiyuki Adachi for his help in finishing this paper, Department of Electrical and Electronic Engineering, Tohoku University, Japan. This research was supported by the National Nature Science Foundation of China (No.61771248 and 61971167), and a project funded by the priority academic program development of the Jiangsu higher education institutions.

REFERENCES
1. Lee, W.C.Y.: Effects on correlation between two mobile radio base-station antennas. IEEE Trans. Commun. COM-21(11), 1214–1224 (1973)
2. Liu, Y., et al.: 3D non-stationary wideband tunnel channel models for 5G high-speed train wireless communications. IEEE Trans. Intell. Transp. Syst. 21(1), 259–272 (2020)
3. DOCOMO 5G White Paper: 5G Radio Access: Requirements, Concept and Technologies. NTT DOCOMO, INC (July 2014)
4. Jiang, H., et al.: Analysis of geometric multi-bounced virtual scattering channel model for dense urban street environments. IEEE Trans. Veh. Technol. 66(3), 1903–1912 (2017)
5. Jiang, H., et al.: A 3-D non-stationary wideband geometry-based channel model for MIMO vehicle-to-vehicle communications in tunnel environments. IEEE Trans. Veh. Technol. 68(7), 6257–6271 (2019)
6. Wu, S., et al.: A general 3D non-stationary 5G wireless channel model. IEEE Trans. Commun. 66(7), 3065–C3078 (2018)
7. Tayade, PP, Rohokale, VM.: Enhancement of spectral efficiency, coverage and channel capacity for wireless communication towards 5G. In: Proceedings of IEEE ICPC, Pune, India, pp. 1–5, 8–10 Jan 2015
8. Damnjanovic, A., et al.: A survey on 3GPP heterogeneous networks: Wireless communications. IEEE Wireless Commun. 18(3), 10–21 (2011)
9. Avazov, N., Patzold, M.: A geometric street scattering channel model for car-to-car communication systems. In: IEEE International Conference on Advanced Technology for Communication, Da Nang, Vietnam, pp. 224–230, 2–4 Aug 2011
10. Ma, Y.Y., Patzold, M.: Modeling and statistical characterization of wideband indoor radio propagation channels. In: Proceedings of the IEEE ICUMT’10, pp. 777–783 (2010)
11. Avazov, N., Patzold, M.: A novel wideband MIMO car-to-car channel model based on a geometrical semicircular tunnel scattering model. IEEE Trans. Veh. Technol. 65(8), 2631–2648 (2015)
12. Teal, P.D., et al.: Spatial correlation for general distributions of scatterers. IEEE Signal Processing Lett. 9(10), 305–309 (2002)
13. Zhou, J., et al.: Analysis of MIMO antenna array based on 3D Von Mises Fisher distribution. J. China Univ. Posts Telecomun. 22(2), 1–12 (2015)
14. Tsai, J.A., et al.: Spatial fading correlation function of circular antenna arrays with Laplacian energy distribution. IEEE Commun. Lett. 6(5), 178–180 (2002)
15. Sieskul, B.T., et al.: Spatial fading correlation for semicircular scattering: Angular spread and spatial frequency approximations. IEEE International Conf. on Commun. and Electronics, Nha Trang, pp. 216–221, 11–13 Aug 2010
16. Zhou, J., et al.: Generalized spatial correlation equations for antenna arrays in wireless diversity reception: Exact and approximate analyses. IEEE Trans. Commun. E84-B(5), 1–5 (2004)
17. Hsieh, P.C., Chen, F.C.: A new MIMO spatial correlation approximation of large angular spread. IEEE Antennas and Propagation Society International Symposium, pp. 1909–1912, 9–15 Jun 2007
18. Chuang, J.T., Chen, F.C.: Comparison of MIMO spatial correlation approximations under large angular spread. IEEE Antennas and Propagation Society International Symposium, San Diego, CA, pp. 1–4, 5–11 Jul 2008
19. Yong, S.K., Honpsson, J.S.: Three dimensional spatial fading correlation model for compact MIMO receivers. IEEE Trans. Wireless Commun. 4(6), 2856–2869 (2005)
20. Gutierrez, C.A., Patzold, M.: Sum of sinusoids based simulation of flat fading wireless propagation channels under non-isotropic scattering conditions. In: Proceedings of IEEE Globalcom, Washington, pp. 3842–3846, 4–12 Dec 2007
21. Zhou, J., et al.: Asymmetric geometrical based statistical channel model between directional and omni-directional transceivers and their MIMO receiving performance. IET Commun. 8(1), 1–10 (2014)
22. Ghorashi, M., et al.: Microcell urban propagation channel analysis using measurement dData. Proc. IEEE VTC 3(1), 1728–1731 (2005)
23. Ghorashi, M., et al.: A single bounce channel model for dense urban street microcell. URSI-Japan MEETING, Japan, 502 (2006)
24. Riaz, M., et al.: A generalized 3-D scattering channel model for spatiotemporal statistics in mobile-to-mobile communication environment. IEEE Trans. Veh. Technol. 64(10), 4399–4410 (2015)
25. Cheng, X., et al.: Envelope level crossing rate and average fade duration of nonisotropic vehicle-to-vehicle Ricean fading channels. IEEE Trans. Intell. Transportation Syst. 15(1), 62–72 (2013)
26. Wang, J., et al.: Angular spread measurement and modeling for 3D MIMO in urban macrocellular radio channels. Proc. IEEE ICC W8, 20–25 (2014)
27. Gradshteyn, I.S., Ryzhik, I.M.: Table of Integrals, Series and Products, 5th edn. Academic, San Diego (1994)

How to cite this article: Zhou J, Wang X, Jiang H, Sasaki S, Shao G. MIMO channel estimation with arbitrary angle of arrival incident power spectrum for wireless communications. IET Commun. 2021;15:232–244. https://doi.org/10.1049/cmu2.12049