Simulation of plastic forming process by variation of geometric parameters

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Abstract. In this paper, the complex research method was used which includes theoretical analysis and verification of the results using numerical simulation in the Pam-Stamp 2G solution. During the determination of the stress-strain condition of the workpiece the functional was solved which allows minimal thickness fluctuations of the thin-walled parts during the forming. The radius of the punch was chosen as a variable parameter. Its calculated values served as input parameters for the simulation.

1. Introduction
Development of the sheet metal deformation theory, considering the minimization of the function between the set uniform thickness value and the technologically possible thickness, allows more possibilities and different approaches to the analysis of such cases. The methods of obtaining accurate geometric dimensions of the complex shaped parts with minimal thickness fluctuations can raise productivity and reduce defects [1, 2]. The stable operation flow is achieved by maximization of the usage of the internal deformational reserves of the metal and by deep analysis of the stress-strain condition of the workpiece. The availability of the developed mathematical apparatus allows the creation of the model, but usually, the difficulties are in the application of the simplifications and assumptions which are required by that apparatus. Adequate mathematical model, however, allows predicting and evaluating the performance of the process in the set conditions and testing the hypothesis about the reasons for the observed effects and undesirable changes [3, 4, 5].

2. The object of the study
Fulfillment of requirements to aviation parts, including thin-walled shells, is possible by analytically representing the condition as the functional and solving it [6]:
\[ \int_F (s_T - s_{set})^2 \, dF \rightarrow \text{min} \]  \hspace{1cm} (1)

where \( s_{set} \) is the set thickness of the part; \( s_T \) is the technologically possible thickness which emerges after the forming of the blank; \( F \) is the area of the median surface of the part.

Using the dependence (1), the condition for the axisymmetric part can be presented as:
\[ \int_{l} (s_f - s_m)^2 \, dl \rightarrow \min, \quad (2) \]

where \( l \) is the length of generatrix of the part.

The function requires the approach of the technologically possible thickness \( s_f \) to the set thickness of the part \( S_{set} \) with the condition of the minimal deviation to the positive or the negative difference between the two functions of the thicknesses. For the thin-walled shells \( \frac{S_{set}}{2R} \leq 0.008 \) to prevent the corrugations on the free portion of the workpiece two processes are used – flanging and forming, since tensile stresses are in action in meridional and tangential directions during the deformation of the flat blank. Forming is more preferable than flanging in terms of obtaining parts with a minimum thickness fluctuations. It can be separated into two stages: the first one takes place until the moment when the free portion of the workpiece touches the tapering part of the punch, the second one – then the cylindrical portion is formed. The second stage is characterized by the presence of the active friction forces, leading to the reduction of the thickness fluctuations of the part. During the forming, stresses and strains have more uniform values across the deformation area.

![Figure 1](image)

Figure 1. Scheme of the forming using the conical punch.

The molding process takes place in two stages. The first stage proceeds until the fit of the workpiece to the conical surface of the punch; the second stage proceeds with the formation of a cylindrical part.

3. Methods and theoretical foundations

Considering the mechanism of the first stage forming process, it should be noted, that on the free portion the meridional stresses are larger of the two kinds. So, from the Laplace equation [7] we get:

\[ \frac{\sigma_\rho}{\sigma_\varphi} = -\frac{R_\varphi}{R_\rho}, \quad (3) \]

where \( |R_\varphi| \geq |R_\rho| \), \( \sigma_\varphi > \sigma_\rho \); \( \sigma_\rho \) is the meridional stresses; \( \sigma_\varphi \) is the tangential stresses; \( R_\rho \) is the radius of the part in the meridional direction; \( R_\varphi \) is the radius of the part in the tangential direction.

Forming of the part takes place due to the thinning of the blank. The forces on the radial surfaces of the punch and the die are directed in the opposite sides, thus compensating each other. Friction forces in the bottom portion of the workpiece are almost neglectable due to the action of the bending momentum in the radial portion of the workpiece [7]. We consider that the stresses picture, due to the relatively small extent of the free portion, is influenced by the additional stresses from the bending and the friction \( \Delta \sigma_\rho \) on the radiuses of the punch and the die (figure 2).

\[ \Delta \sigma_\rho = (1 + \frac{1}{2 \frac{r_u}{s_{ideal} + 1}}) e^{r_u}, \quad (4) \]
where $f_1$ is the friction coefficient on the radii of the punch and the die; $a_i$ is the bending angle of the workpiece; $\frac{r_n}{s_{\text{blank}}} = \bar{r}_s$ is the relative radius of the punch; $s_{\text{blank}}$ is the thickness of the blank.

Figure 2. Scheme of the influence of the bending radius on the stresses.

Let us write the equilibrium equation for the conical portion of the shell:

$$\rho \frac{d \sigma_{\rho}}{d \rho} + \sigma_{\rho} - \sigma_{\theta} - \sigma_{\theta} \tan \alpha = 0,$$

where $\alpha$ is the conical angle at $\rho = \rho_c$, $\rho$ is the independent variable representing the current radius.

The ratio of the stresses $\frac{\sigma_{\rho}}{\sigma_{\theta}}$ we estimate by the limits of the forming, accepting the assumption about the linear nature of changes of the ratio, due to the small extend of the conical portion at $r = r_n = r_m$:

$$a_0 + a_1 (1 + \frac{1}{2r_n + 1}) \frac{\bar{r}}{r_n} \sigma_{\rho} = \sigma_{\theta},$$

where $f$ is the coefficient of friction, on the radii of rounding of the punch and die; $a_i$ is the bend angle of the workpiece along the radius $a_i = 90^\circ - \alpha$.

$$\frac{r_s}{s_{\text{vac}}} = \bar{r}_s.$$  \hspace{1cm} (7)

Let us find the coefficients $a_0$ and $a_1$. If $r_n = r_m = 0$, then the movement of the workpiece in the tangential direction is absent, and from the linking equation:

$$\frac{\sigma_{\theta}}{\sigma_{\rho}} = \mu; \ e_{\theta} \approx e_{\rho},$$

where $\mu$ is the coefficient of the anisotropy of the transversely isotropic body.

If $r_n \rightarrow \infty$, then we consider the element of the part, corresponding to the element on the axis of the symmetry where $\frac{\sigma_{\theta}}{\sigma_{\rho}} = 1; \ e_{\theta} \approx e_{\rho}$.

Considering the above mentioned, we get:

$$\frac{\sigma_{\theta}}{\sigma_{\rho}} = 1 + \frac{\mu - 1}{2r_n}.$$  \hspace{1cm} (9)

Expression (9) demonstrates which ratio of the stresses will take place on the free portion of the workpiece when the radiuses of the punch and the die are equal. Stresses themselves are changing proportional to one parameter, but their ratio remains constant.

Consider the first stage of forming until the workpiece fits the conical surface of the punch and we write the equilibrium force in the free area:

$$\rho \frac{d \sigma_{\rho}}{d \rho} + \sigma_{\rho} - \sigma_{\theta} = 0.$$  \hspace{1cm} (10)

We write the equation (10) taking into account (9):
\[ \rho \frac{d \sigma_\rho}{d \rho} + \sigma_\rho (1 - b_0) = 0. \]  

(11)

We assume that during molding, the specified thickness of the part corresponds to the thickness of the part obtained with its uniform thinning. Determine the stress and thickness in the free area of the workpiece where \( \sigma_\rho > \sigma_{\rho 0} \). We take into account the hardening in the form of a linear function, taking

\[ \beta = 1, \quad k = \frac{2}{1 + \mu} , \text{ and we get:} \]

\[ \sigma_\rho = (\sigma_{\rho 0} + H \cdot e_S k). \]  

(12)

or

\[ \overline{\sigma}_\rho = \sigma_{\rho 0} + 1 - \overline{S}. \]  

(13)

where \( \overline{\sigma}_\rho = \frac{\sigma_{\rho 0}}{H \cdot k}, \quad \overline{\sigma}_{\rho 0} = \frac{\sigma_{\rho 0}}{k - H} \); \( \overline{S} = \frac{S}{S_{\text{max}}}; \sigma_{\rho 0} \) and \( H \) are the constants of mechanical properties.

Substitute the relations (13) into (11):

\[ \overline{\rho} \frac{d \overline{\sigma}_\rho}{d \overline{\rho}} + (\overline{\sigma}_{\rho 0} + 1 - \overline{S})(1 - b_0) = 0, \]  

(14)

where \( \overline{\rho} = \frac{\rho}{R}. \)

Equation (14) we bring to the form:

\[ \frac{d \overline{S}}{d \overline{\rho}} + \overline{S} \frac{(1 - b_0)}{\overline{\rho}} - (\overline{\sigma}_{\rho 0} + 1 - \overline{S})(1 - b_0) = 0. \]  

(15)

Equation (15) is a linear differential equation with respect to \( \overline{S} \). Its solution has the form under boundary conditions: \( \overline{S} = \overline{S}^*, \overline{\rho} = 1. \)

\[ \overline{S} = K + (\overline{S}^* - K) \overline{\rho}^b. \]  

(16)

where \( b = b_0 - 1; \quad K = 1 + \overline{\sigma}_{\rho 0} \).

Simplifying expression (16) by expanding the power function in a series:

\[ \overline{S} = K + (\overline{S}^* - K)(1 + b(\overline{\rho} - 1)]. \]  

(17)

We write the minimization condition taking into account the thickness (17):

\[ \int_{\overline{r}_{\text{nom}}}^{\overline{r}} [\overline{S}_{\text{nom}} - K - (\overline{S}^* - K)(1 + b(\overline{\rho} - 1))]. \rho d \rho \rightarrow \min. \]  

(18)

where \( 1 \leq \overline{\rho} \leq \overline{R}; \overline{R} = \frac{K}{r_{\text{nom}}}; \overline{\rho} = \frac{\rho}{r_{\text{nom}}}. \)

Minimization of the expression (2) as the function can be performed using the technological parameters which remains constant during forming. There are the initial thickness of the blank, forming coefficients, geometrical parameters of the tooling, friction coefficient, some mechanical properties, including the transversely isotropic body parameters among them which is determined due to the nature of the material. For the simplification of the solution sake, in the boundaries of the considered error margin, it is enough to identify the conditions and parameters which are most relevant for the alteration of thickness and can be regulated. For further use in the engineering solution method as the variable parameter, we choose the relative radius of the punch [8]. The expression (18) is represented in the form:

\[ \int_{\overline{r}_{\text{nom}}}^{\overline{r}} [B - 1 - b(\overline{\rho} - 1)]. \rho d \rho \rightarrow \min. \]  

(19)

where \( B = \frac{\overline{S}_{\text{nom}} - K}{\overline{S} - K}; \)

The variation of the functional (18) is carried out by two parameters \( b \) and \( B \).
\[
\frac{\partial}{\partial b} \int_{0}^{\infty} [B - 1 - b (\rho - 1)] d\rho = 0. \tag{20}
\]

From equation (20) we have:
\[
b = \frac{3 (B - 1)}{2 (R - 1)}. \tag{22}
\]

From equation (21) we get:
\[
(B - 1) (R - 1) + b \left( \frac{(R - 1)^2}{2}\right) = 0. \tag{23}
\]

We put (22) in (23) and find:
\[
B = 1; \quad b = 0. \tag{24}
\]

Considering the relation (19) the equality \( B = 1 \) is possible with \( S_{\text{mom}} = S \). We have: \( b_0 = 1 \).

The left side of (9) is performed at a given ratio \( b_0 = 1 \). Value of the relative radius \( r_n \) should tend to infinity \( r_n \to \infty \).

Consider the graph of the change in the relative meridional stress from the bend about the relative radius of rounding of the punch (figure 3).

![Figure 3. The change in the relative meridional stress due to bending.](image)

The graph shows that since \( \frac{r_n}{S_{\text{mom}}} \geq 5 \) the increase in relative stress from bending, it is in hundredths, and the ratio \( \frac{1}{2r_n + 1} \) (6) can be ignored, equating it to zero. Then \( b_0 = 1; \quad b = 0 \) and the expression (17) takes the form: \( S = \overline{S} = S_{\text{mom}} \).

After some transformations we get the following expression:
\[
\tau_n = \frac{1 - \mu}{1 - \frac{\mu}{1 + \text{fctg} \alpha}} - \frac{1}{2}. \tag{25}
\]

4. Simulation results

For the labor-intensive tasks, the application of the modern means of automatization allows us to avoid the significant volume of the routine work and represent the results of the simulation in the graphical
The simulation of the investigated process was implemented using theoretical dependencies, assumptions, and limitations. The adequate mathematical allows to predict the development and degradation of the system, as well as to estimate its performance and survivability under specified conditions and to evaluate a hypothesis about the reasons to the effects observed and possible unwanted changes in the state of the system. So as the result, the technological parameters (tools geometry, shape, and size of the blank, friction coefficient, transversely isotropic body coefficient) which influence the distribution of the thickness in the part obtained was established. Special attention was paid to the geometry of the punch, in particular, its relative radius, since by varying this parameter we aim to get the desired minimal thickness fluctuations (figure 4).

5. Conclusions

It was discovered, that using the punch with the radius of the curvature of 0.9 mm the thickness fluctuations on the conical portion is 10.7%, but using the punch with a curvature radius of 1.6 mm – the thickness unevenness is 8.5% (figure 5). So we get the decrease of the thickness fluctuations at the increase of the punch curvature radius.

Modern technologies of obtaining thin-walled shells are based on the methods which are labor-intensive, since one of the mandatory conditions of the manufacturing is the presence of heat setting and mechanical treatment steps, in order to obtain the set geometrical dimensions (including thickness) of the part. Manufacturing of the thin-walled shells is economical (lower labor-intensity, a higher coefficient of useful usage of metal) to perform, basing on the forming process of the flat blank which allows minifying thickness fluctuations.
6. References

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