Cooper pair tunneling in circuits with substantial dissipation: 
the three-junction R-pump for single Cooper pairs

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We propose a circuit (we call it R-pump) comprising a linear array of three small-capacitance 
superconducting tunnel junctions with miniature resistors \( R > R_Q \approx \hbar/4e^2 \approx 6.5 \, \text{k} \Omega \) attached to the 
ends of this array. Owing to the Coulomb blockade effect and the effect of dissipative environment 
on the supercurrent, this circuit enables the gate-controlled transfer of individual Cooper pairs. The 
first experiment on operating the R-pump is described.

INTRODUCTION

The Coulomb blockade effect in circuits with small-
capacitance tunnel junctions provides the means of ma-
nipulating single charge quanta (see, for example, the 
review paper by Averin and Likharev [1]). If periodic 
signals of frequency \( f \) are applied to the gate electrodes 
of the circuit, a train of single charges \( q \) can move across 
an array of junctions so that charge pumping, giving rise 
to the current \( I = qf \), is achieved. It was experimentally 
proven that the normal-state metallic circuits enable sin-
gle electrons \( (q = e) \) to be effectively pumped at frequen-
cies \( f \) of about several MHz [2]. Moreover, the accuracy 
of single-electron pumping can nowadays meet the re-
quirements of fundamental metrology, viz. \( \delta I/I \approx 10^{-8} \).

Unlike pumping of electrons, the pumping of Cooper 
pairs \( (q = 2e) \) in superconducting circuits has not been 
that successful so far. The only experiment had been 
carried out in 1991 by Geerligs et al. [3] with a three-
junction Al sample. Although this experiment did evi-
dence a pumping of the pairs, the pumping was strongly 
disturbed by several factors: the Landau-Zener transi-
tions, Cooper pair co-tunneling, quasiparticle tunneling, 
etc. As a result, the shape of the current plateau at 
\( I = 2ef \) in the I-V curve was far from being perfect.

Recently, in their theoretical paper Pekola et al. [4] 
concluded that pumping of Cooper pairs with reason-
able accuracy in a three-junction array was impossible. 
For practical values of parameters (the ratio of the 
Josephson coupling energy to the charging energy 
\( \lambda \equiv E_J/E_c = 0.01 - 0.1 \)) they evaluated the inaccu-
acy of pumping \( \delta I/I \) to be as much as 9%-63%. This is 
because of intensive co-tunneling of pairs in the short ar-
aves, i.e. the process of tunneling simultaneously across 
several \( (\geq 2) \) junctions. To suppress the co-tunneling 
and, in doing so, to improve the characteristics of the 
pump they proposed to considerably increase the num-
er of junctions \( N \geq 3 \) and, hence, of the gates \( (N-1) \). 
However, such modification would make operation of the 
circuit more complex.

In this paper we propose an alternative way to im-
prove Cooper pair pumping in a three-junction array. 
We modify the bare circuit by attaching to the ends of 
the array the miniature resistors (see the electric dia-
gram of this device, which we call R-pump, in Fig. 1). 
Their total resistance \( R \) exceeds the resistance quantum 
\( R_Q \equiv \hbar/4e^2 \approx 6.5 \, \text{k} \Omega \) so that the dimensionless parameter 
\[ z = \frac{R}{R_Q} \gg 1. \] (1)

The self-capacitance of these resistors should not be much 
larger than the junction capacitance \( C \). In this paper we will 
analyze how such resistors affect tunneling and co-
tunneling of Cooper pairs in the three-junction array and 
present preliminary experimental data.

PECULIARITIES IN OPERATING THE R-PUMP

In general terms, the principle of operation of the R-
pump remains the same as that of the pump without 
resistors (for details of the Cooper pair pump operation 
see Ref. [4]). Two periodic signals \( V_1(t) \) and \( V_2(t) \) are 
used to the gates to form an elliptic trajectory in the 
parameter plane \( V_1 - V_2 \). At zero voltage \( V \) across 
the pump, this trajectory encircles in the clockwise (counter-

![FIG. 1: Schematic of the R-pump for single Cooper pairs. Three superconducting junctions of type Al/AlOx/Al form two small islands. Miniature normal metal (Cr) resistors accomplish the structure. The device is driven by two harmonic voltages \( V_{1,2} = V_{0,1,2} + A \cos(2\pi ft + \frac{\pi}{2}) \), where the dc offset \( V_{0,1,2} \) determines the centre of the cycling trajectory in the \( V_1 - V_2 \) plane; the phase shift \( \theta \) = \( \frac{\pi}{2} \).]
clockwise) direction the triple point of the boundaries between the stability domains for adjacent charge configurations with an excess Cooper pair. A complete cycle results in the charge \( q = 2e \) \((q = -2e)\) being sequentially transferred across all three junctions of the array.

The Cooper pair transitions in a circuit without dissipation (quasiparticle tunneling is neglected) occur due to Josephson tunneling (supercurrent). The strength of the Josephson coupling \( E_J \equiv \frac{2\pi}{h}I_c \) (here \( I_c \) is the critical current) is assumed to be \( \ll E_c \equiv e^2/2C \), the charging energy, i.e. parameter \( \lambda \ll 1 \). When the domain boundary is crossed, the resonance condition is obeyed and Josephson coupling mixes up the two states, each corresponding to the position of the pair on either side of the junction. (Note that, at the same point, the resonance condition for co-tunneling in the opposite direction across two other junctions is met.) Slow passage across the boundary results in transition of the pair in the desired direction. This process is mapped on the adiabatic level-crossing dynamics (see, for example, Ref. [1]), and the probability of missing the transition is given by the Landau-Zener expression:

\[
\rho_{LZ} = \exp \left( -\frac{\pi E^2}{2\hbar E} \right),
\]

where \( \dot{E} \propto f \) is the velocity of variation of the energy difference \( E \) between the two states.

In contrast to the elastic (and, hence, reversible) tunneling of pairs in the pump without resistors, tunneling in the R-pump is accompanied by dissipation of energy. The strength of the dissipation effect depends on the active part \( \text{Re}Z(\omega) \) of an electromagnetic impedance seen by the tunneling pair, namely by the dimensionless parameter \( z' = \text{Re}Z(0)/RQ \). If the condition of weak Josephson coupling \( E_J \ll E_c/\sqrt{\lambda} \) is satisfied, i.e. \( \lambda \sqrt{\lambda} \ll 1 \), the rate of Cooper pair tunneling is expressed as

\[
\Gamma(E) = \frac{\pi}{2\hbar}E^2_j P(E),
\]

where \( E \) is the energy gain associated with this transition and \( P(E) \) is the function describing the property of the environment to absorb the energy released in a tunneling event. (The discussion of properties as well as numerous examples of \( P(E) \) for different types of electromagnetic environment are given in the review paper by Ingold and Nazarov [2].) The probability of a missed transition is then given by the expression:

\[
\rho_m = \exp \left\{ -\int_{-\infty}^{\infty} \Gamma[E(t)]dt \right\} = \exp \left( -\frac{\pi E^2}{2\hbar E} \right) = \rho_{LZ}.
\]

Here we used the normalization condition \( \int_{-\infty}^{\infty} P(E)dE = 1 \), and the assumption of a value \( \dot{E} \) constant in time was exploited. Thus, in the case of weak coupling, dissipation does not affect the total probability of tunneling \( p = 1 - p_m \). This conclusion is in accordance with the general result obtained by Ao and Rammer [9] for quantum dynamics of a two-level system in the presence of substantial dissipation.

Using the expansion of \( P \) in the region of small \( E \ll E_c \), one arrives at the expression

\[
\Gamma(E) \propto \lambda^2 E^{2z'-1}.
\]

For \( z' < \frac{1}{2} \) the rate \( \Gamma \) is peaked at \( E = 0 \), while in the case \( (z' > \frac{1}{2}) \) the rate is zero at \( E = 0 \) and the maximum is reached at finite energy \( \tilde{E} \leq 4E_C \), or, in other terms, at finite positive voltage \( V \) across the junction. This property of \( \Gamma(E) \) in circuits with substantial dissipation plays the crucial role in the operation of the three-junction R-pump.

For one junction in the three-junction array with gate capacitances \( C_g \ll C \) the effective impedance \( \text{Re}Z(0) \equiv R' = \frac{1}{2}R \), i.e. the parameter \( z' = \frac{1}{2}z \). The factor \( \frac{1}{2} \), which considerably attenuates the effect of resistors, stems from the square of the ratio of the total capacitance of the series array \( (= \frac{1}{2}C) \) to the junction capacitance \( C \) (see a similar network analysis in Ref. [1]).

Accordingly, the equivalent damping for two junctions of the three-junction array is larger and determined by \( R' = \frac{4}{9}R \) and, hence, \( z' = \frac{1}{9}z \). Therefore, the rate of Cooper pair transition simultaneously across two junctions, i.e. the rate of the co-tunneling, at small \( E \) is

\[
\Gamma_{co-t}(E) \propto \lambda^2 E^{\frac{4}{9}z'-3}.
\]

This expression has been obtained by considering the environmental effect in a similar fashion as was done by Odintsov et al. [4] for the co-tunneling of normal electrons in a single-electron transistor. Accordingly, co-tunneling across all three junctions decays even stronger \((\propto \lambda^6 E^{2z-5})\), because the whole array experiences the full resistance \( R \).

The comparison of Eq. (6) with Eq. (5) taken for the value \( z' = \frac{1}{9}z \) shows that the rate of co-tunneling is drastically depressed if \( z \) is sufficiently large. For example, for \( z = 9 \) (or \( R \approx 58.5 \text{k}\Omega \)) the rate of Cooper pair co-tunneling across two junctions \( \Gamma_{co-t} \propto E^5 \) (i.e. it is similarly small as single-electron co-tunneling across a three-junction normal array [3]). On the other hand, the direct tunneling rate \( \Gamma \propto E \), is similar to that of ordinary electron tunneling. The effective junction resistance in the latter case is \( R_{eff} \approx 0.03\lambda^{-2}RQ \). For the practical value \( \lambda \sim 0.03 \) this formula yields \( R_{eff} \approx 200 \text{k}\Omega \), i.e. the typical value of a normal single electron junction. Since the junction resistance determines the characteristic time constant \( RC \), it can be expected that the frequency characteristics of the Cooper pair R-pump are at least not worse than those of the normal electron counterpart without resistors.
EXPERIMENT

The Al/AlOₓ/Al tunnel junctions and Cr resistors were fabricated in situ by the three-angle shadow evaporation technique through a trilayer mask patterned by e-beam lithography and reactive-ion etching. The tunnel junction parameters of the best suited available sample were found to be: capacitance $C \approx 250 \text{ aF}$ and normal-state resistance $R_j \approx 160 \text{ kΩ}$. The former value yields $E_c \approx 320 \mu \text{eV}$. On the assumption that the Ambegaokar-Baratoff relation between the critical current $I_c$ and the junction resistance is valid, the aforementioned value of $R_j$ yields the value $E_J = \Delta_{\text{Al}} R_Q/2 R_j \approx 4.1 \mu \text{eV}$, where $\Delta_{\text{Al}} \approx 200 \mu \text{eV}$ is the superconducting energy gap of aluminum at low temperature. These parameters of the sample give the ratio of characteristic energies $\lambda \approx 0.013$.

The thin-film Cr resistors (thickness $d \approx 7 \text{ nm}$) had lateral dimensions of about 80 nm by 10 µm. For the purpose of an independent characterization of the resistors and junctions, we attached four resistors in twos to either end of the junction array, resulting in the equivalent resistance $\frac{1}{2} R$ on each side as depicted in Fig. 1. (For the measurement of the I-V curves we used, however, the two-wire configuration.) Either of four resistors had $R \approx 60 \text{ kΩ}$ and reasonably low self-capacitance per unit length, $\approx 60 \text{ aF}/\mu \text{m}$. The resistors exhibited negligibly small ($< 2 \%$) non-linearity of the I-V curve in the region of small voltages at the millikelvin temperature.

The samples were measured in the dilution refrigerator at the temperature $T$ of about 10 mK, i.e. well below the critical temperature of aluminum, $T_c \approx 1.15 \text{ K}$. The bias and gate lines were equipped with pieces of the thermocoax cable which was thermally anchored at the sample holder plate and served as a filter for frequencies above 1 GHz [11]. The typical I-V curves with and without ac drive are presented in Fig. 2. The curves exhibit considerably smeared steps whose position ($I \approx -2ef$) clearly shows the linear current-versus-frequency dependence (although the quality of the steps is noticeably degraded with increasing frequency) [12]. Another feature of the steps is their position on the voltage axes: they appear at finite voltage $V$ applied in the current direction, while at $V = 0$ neither co-tunneling nor pumping effect were observed. We attribute this effect to the damping effect of the resistors, which was too strong in this sample. Thus, the behavior of the Cooper pair R-pump differs significantly from that of the Cooper pair pump without resistors [4].

DISCUSSION

Although our first experiment evidenced the desirable effect of Cooper pair pumping, a further improvement is inevitably needed. First, the sample parameters should be optimized: Josephson coupling should be increased (possibly up to $\lambda = 0.1$) while the resistance value $R$ should be somewhat reduced. These parameter values lead to an increase of the tunneling rate and, as a consequence, to Cooper pair pumping at zero voltage bias $V$. Secondly, the quasiparticle tunneling, resulting in sporadic translations of the cycle trajectory and, hence, to the pumping errors, should be reliably depressed: As the measurements showed, all our Al-Cr samples (as well as those with $E_c < \Delta_{\text{Al}}$) so far suffered from quasiparticle tunneling [13] and did not show the so-called parity effect [14], [15]. It is particularly remarkable that our Cr resistors do not prevent an uncontrolled poisoning of Josephson tunneling by non-equilibrium quasiparticles arriving at the islands from the external circuit. Such "buffer" effect of the normal electrodes was proposed and demonstrated in Al-Cu devices by Joyez et al. [16]. That is why the problem of the poisoning quasiparticle tunneling requires a more radical solution. Probably manufacture of small-capacitance niobium junctions ($\Delta_{\text{Nb}} \approx 1.4 \text{ meV} \gg E_c$) could improve the situation by making the parity effect in these structures stable.

In conclusion, we proposed a simple superconductor-normal metal circuit enabling, in principle, the efficient pumping of Cooper pairs. Due to the energy dissipation in the resistors, the effect of Cooper pair co-tunneling is heavily damped. A possibility of pair pumping was demonstrated and improvements of the experiment are proposed.

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