Bridging resonant leptogenesis and low-energy CP violation with an RGE-modified seesaw relation

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Abstract

We propose a special type-I seesaw scenario in which the Yukawa coupling matrix $Y_\nu$ can be fully reconstructed by using the light Majorana neutrino masses $m_i$, the heavy Majorana neutrino masses $M_i$ and the PMNS lepton flavor mixing matrix $U$. It is the RGE-induced correction to the seesaw relation that helps interpret the observed baryon-antibaryon asymmetry of the Universe via flavored resonant thermal leptogenesis with $M_1 \approx M_2 \ll M_3$. We show that our idea works well in either the $\tau$-flavored regime with equilibrium temperature $T \approx M_1 \in (10^9, 10^{12})$ GeV or the ($\mu + \tau$)-flavored regime with $T \approx M_1 \in (10^5, 10^9)$ GeV, provided the light neutrinos have a normal mass ordering. We find that the same idea is also viable for a minimal type-I seesaw model with two nearly degenerate heavy Majorana neutrinos.

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1 Introduction

A special bonus of the canonical (type-I) seesaw mechanism \[5\] is the thermal leptogenesis mechanism \[6\], which provides an elegant way to interpret the mysterious matter-antimatter asymmetry of our Universe. The key points of these two correlated mechanisms can be summed up in one sentence: the tiny masses of three known neutrinos $\nu_i$ are ascribed to the existence of three heavy Majorana neutrinos $N_i$ (for $i = 1, 2, 3$), whose lepton-number-violating and CP-violating decays result in a net lepton-antilepton number asymmetry $Y_L$ which is finally converted to a net baryon-antibaryon number asymmetry $Y_B$ as observed today.

In the standard model (SM) extended with three right-handed neutrinos and lepton number violation, it is the following seesaw formula that bridges the gap between the masses of $\nu_i$ (denoted as $m_i$) and those of $N_i$ (denoted as $M_i$):

$$M_\nu = -v^2 \left( Y_\nu M_R^{-1} Y_\nu^T \right),$$

where $M_\nu$ represents the light (left-handed) Majorana neutrino mass matrix, $v \simeq 174$ GeV is the vacuum expectation value of the SM neutral Higgs field, $M_R$ stands for the heavy (right-handed) Majorana neutrino mass matrix, and $Y_\nu$ is a dimensionless coupling matrix describing the strength of Yukawa interactions between the Higgs and neutrino fields. The eigenvalues of $M_\nu$ (i.e., $m_i$) can be strongly suppressed by those of $M_R$ (i.e., $M_i$) as a consequence of $M_i \gg v$ (for $i = 1, 2, 3$), and that is why $m_i \ll v$ naturally holds.

Although such a seesaw picture is qualitatively attractive, it cannot make any quantitative predictions unless the textures of $M_R$ and $Y_\nu$ are fully determined \[7\]. Without loss of generality, one may always take the basis in which both the charged-lepton mass matrix $M_l$ and the heavy Majorana neutrino mass matrix $M_R$ are diagonal (i.e., $M_l = D_l \equiv \text{Diag}\{m_e, m_\mu, m_\tau\}$ and $M_R = D_N \equiv \{M_1, M_2, M_3\}$). In this case the undetermined Yukawa coupling matrix $Y_\nu$ can be parametrized as follows — the so-called Casas-Ibarra (CI) parametrization \[8\]:

$$Y_\nu = \frac{i}{v} U \sqrt{D_\nu} O \sqrt{D_N},$$

where $U$ is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix \[9\] used to diagonalize $M_\nu$ in the chosen basis (i.e., $U^\dagger M_\nu U^* = D_\nu \equiv \text{Diag}\{m_1, m_2, m_3\}$), and $O$ is an arbitrary complex orthogonal matrix. This popular parametrization of $Y_\nu$ is fully compatible with the seesaw formula in Eq. (1), but the arbitrariness of $O$ remains unsolved.

Note that it is the complex phases hidden in $Y_\nu$ that govern the CP-violating asymmetries $\varepsilon_{\alpha \alpha}$ between the lepton-number-violating decays $N_i \rightarrow \ell_\alpha + H$ and $N_i \rightarrow \ell_\alpha + \bar{H}$ (for $i = 1, 2, 3$ and $\alpha = e, \mu, \tau$) \[6\]. In particular, the flavored asymmetries $\varepsilon_{\alpha \alpha}$ depend on both $(Y_\nu^*)^{\alpha i} (Y_\nu)^{\alpha j}$ and $(Y_\nu^*)_{ij} (Y_\nu)_{ij}$ (for $j \neq i = 1, 2, 3$), but the unflavored asymmetries $\varepsilon_i \equiv \varepsilon_{ie} + \varepsilon_{i\mu} + \varepsilon_{i\tau}$ are only dependent upon $(Y_\nu^*)_{ij} (Y_\nu)_{ij}$ \[15\]. Given the CI parametrization of $Y_\nu$ in Eq. (2), one can immediately see that $\varepsilon_i$ have nothing to do with the PMNS matrix $U$ \[23\], while $\varepsilon_{\alpha \alpha}$ will depend directly on $U$ if $O$ is assumed to be real \[26\].

Note also that both $U$ and $D_\nu$ in Eq. (2) are defined at the seesaw scale $\Lambda_{SS} \gg v$, which can be related to their counterparts at the Fermi scale $\Lambda_{EW} \sim v$ via the one-loop renormalization-
In the SM, we find that at the seesaw scale but also in the matrix (i.e., \( O \)).

\[ Y_\nu (\Lambda_{SS}) = \frac{i}{v} I_0 T_1 U (\Lambda_{EW}) \sqrt{D_\nu} (\Lambda_{EW}) O \sqrt{D_N} (\Lambda_{SS}) , \]

(3)

where \( T_1 = \text{Diag}\{ I_e, I_\mu, I_\tau \} \), and the evolution functions \( I_0 \) and \( I_\alpha \) (for \( \alpha = e, \mu, \tau \)) are given by

\[ I_0 = \exp \left[ -\frac{1}{32\pi^2} \int_0^{\ln(\Lambda_{SS}/\Lambda_{EW})} \left[ 3g_2^2(t) - 6y_t^2(t) - \lambda(t) \right] dt \right] , \]

\[ I_\alpha = \exp \left[ -\frac{3}{32\pi^2} \int_0^{\ln(\Lambda_{SS}/\Lambda_{EW})} g_\alpha^2(t) dt \right] \]

(4)

in the SM with \( g_2, \lambda, y_t \) and \( y_\alpha \) standing respectively for the SU(2)\(_L\) gauge coupling, the Higgs self-coupling constant, the top-quark and charged-lepton Yukawa coupling eigenvalues. Eq. (3) tells us that the unflavored CP-violating asymmetries \( \varepsilon_i \) should also have something to do with the PMNS matrix \( U \) at low energies because of a slight departure of \( T_1 \) from the identity matrix. This new observation makes it possible to establish a direct link between unflavored thermal leptogenesis and low-energy CP violation under the assumption that \( O \) is a real matrix, but one may still frown on the uncertainties associated with \( O \).

In this work we simply assume the unconstrained orthogonal matrix \( O \) to be the identity matrix (i.e., \( O = 1 \)), so as to reconstruct the Yukawa coupling matrix \( Y_\nu \) in terms of not only \( M_i \) at the seesaw scale but also \( m_i \) and \( U \) at low energies. Considering the fact of \( y_t^2 \ll y_\mu^2 \ll y_\tau^2 \ll 1 \) in the SM, we find that \( I_e \approx I_\mu \approx 1 \) and \( I_\tau \approx 1 + \Delta_\tau \) are two excellent approximations, where

\[ \Delta_\tau = -\frac{3}{32\pi^2} \int_0^{\ln(\Lambda_{SS}/\Lambda_{EW})} y_\tau^2(t) dt \]

(5)

denotes the small \( \tau \)-flavored effect. Then the expression of \( Y_\nu \) in Eq. (3) can be somewhat simplified and explicitly written as

\[ Y_\nu = \frac{i}{v} I_0 \begin{pmatrix} \sqrt{m_1 M_1 U_{e1}} & \sqrt{m_2 M_2 U_{e2}} & \sqrt{m_3 M_3 U_{e3}} \\ \sqrt{m_1 M_1 U_{\mu1}} & \sqrt{m_2 M_2 U_{\mu2}} & \sqrt{m_3 M_3 U_{\mu3}} \\ \sqrt{m_1 M_1 U_{\tau1}} & \sqrt{m_2 M_2 U_{\tau2}} & \sqrt{m_3 M_3 U_{\tau3}} \end{pmatrix} + \Delta_\tau \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \]

(6)

in which the scale indices \( \Lambda_{SS} \) and \( \Lambda_{EW} \) have been omitted for the sake of simplicity, but one should keep in mind that the values of \( m_i \) and \( U_{ai} \) (for \( i = 1, 2, 3 \) and \( \alpha = e, \mu, \tau \)) are subject to the Fermi scale \( \Lambda_{EW} \). With much less arbitrariness, we are going to show that such a special RGE-modified seesaw scenario allows us to account for the observed baryon-to-photon ratio

\footnote{A similar RGE-modified CI parametrization of \( Y_\nu \) has been given in the case of the minimal supersymmetric standard model (MSSM) extended with the seesaw mechanism.}
\[ \eta \equiv n_B/n_\gamma \simeq (6.12 \pm 0.03) \times 10^{-10} \simeq 7.04 Y_3 \] in today’s Universe \cite{40} by means of flavored resonant thermal leptogenesis with \( M_1 \simeq M_2 \ll M_3 \) \cite{50, 51}. We find that our idea works in either the \( \tau \)-flavored regime with equilibrium temperature \( T \simeq M_1 \in (10^9, 10^{12}) \) GeV or the \((\mu + \tau)\)-flavored regime with \( T \simeq M_1 \in (10^5, 10^9) \) GeV, if the mass spectrum of three light Majorana neutrinos has a normal ordering. In addition, we show that the same idea is also viable for thermal leptogenesis in a minimal type-I seesaw model \cite{50, 51} with two nearly degenerate heavy Majorana neutrinos.

## 2 Resonant leptogenesis

In the type-I seesaw scenario the lepton-number-violating decays \( N_i \to \ell_\alpha + H \) and \( N_i \to \bar{\ell}_\alpha + \bar{H} \) are also CP-violating, thanks to the interference between their tree and one-loop (self-energy and vertex-correction) amplitudes \cite{6, 12–14}. Given \( M_1 \simeq M_2 \ll M_3 \), however, the near degeneracy of \( M_1 \) and \( M_2 \) can make the one-loop self-energy contribution resonantly enhanced \cite{41, 49}. As a result, the flavor-dependent CP-violating asymmetries \( \varepsilon_{i\alpha} \) between \( N_i \to \ell_\alpha + H \) and \( N_i \to \bar{\ell}_\alpha + \bar{H} \) decays (for \( i = 1, 2 \) and \( \alpha = e, \mu, \tau \)) are dominated by the interference effect associated with the self-energy diagram \cite{42, 43}:

\[
\varepsilon_{i\alpha} \equiv \sum_\alpha \left[ \frac{\Gamma (N_i \to \ell_\alpha + H) - \Gamma (N_i \to \bar{\ell}_\alpha + \bar{H})}{\Gamma (N_i \to \ell_\alpha + H) + \Gamma (N_i \to \bar{\ell}_\alpha + \bar{H})} \right] \frac{\text{Im} \left[ (Y_\nu^*)_{ai} (Y_\nu)_{aj} (Y_\nu^\dagger Y_\nu)_{ij} + \xi_{ij} (Y_\nu^*)_{ai} (Y_\nu^*)_{aj} (Y_\nu^\dagger Y_\nu)_{ij} \right]}{(Y_\nu^\dagger Y_\nu)_{ii} (Y_\nu^\dagger Y_\nu)_{jj}}, \quad \frac{\xi_{ij} \xi_j (\xi_{ij}^2 - 1)}{(\xi_{ij} \xi_j)^2 + (\xi_{ij}^2 - 1)^2}, \quad (7)
\]

where \( \xi_{ij} \equiv M_i/M_j \) and \( \xi_j \equiv (Y_\nu^\dagger Y_\nu)_{jj} / (8\pi) \) with the Latin subscripts \( j \neq i \) running over 1 and 2. Taking account of the expression of \( Y_\nu \) in Eq. \( (6) \), we immediately arrive at

\[
(Y_\nu^*)_{ei} (Y_\nu)_{ej} = \frac{I_0^2}{v^2} \sqrt{m_i m_j M_i M_j U_{ei} U_{ej}}, \quad (Y_\nu^*)_{\mu i} (Y_\nu)_{\mu j} = \frac{I_0^2}{v^2} \sqrt{m_i m_j M_i M_j U_{\mu i} U_{\mu j}}, \quad (Y_\nu^*)_{\tau i} (Y_\nu)_{\tau j} = \frac{I_0^2}{v^2} (1 + 2\Delta_\tau) \sqrt{m_i m_j M_i M_j U_{\tau i} U_{\tau j}} + \mathcal{O} (\Delta_\tau^2) , \quad (8)
\]

together with

\[
(Y_\nu^\dagger Y_\nu)_{ij} = \frac{I_0^2}{v^2} \sqrt{m_i m_j M_i M_j} \left( \delta_{ij} + 2\Delta_\tau U_{\tau i} U_{\tau j} \right) + \mathcal{O} (\Delta_\tau^2) . \quad (9)
\]

The flavored CP-violating asymmetries in Eq. \( (7) \) turn out to be

\[
\varepsilon_{i\alpha} = 2\Delta_\tau \left[ \text{Im} \left( U_{\tau i}^* U_{\tau j} U_{\alpha i}^* U_{\alpha j} \right) + \xi_{ij} \text{Im} \left( U_{\tau j}^* U_{\tau i} U_{\alpha i}^* U_{\alpha j} \right) \right] \frac{\xi_{ij} \xi_j (\xi_{ij}^2 - 1)}{(\xi_{ij} \xi_j)^2 + (\xi_{ij}^2 - 1)^2} + \mathcal{O} (\Delta_\tau^2) , \quad (10)
\]

\(^2\)For such a heavy Majorana neutrino mass spectrum, the role of \( N_3 \) in thermal leptogenesis is expected to be negligible because its contribution has essentially been washed out at \( T \simeq M_1 \simeq M_2 \).
where $\alpha = e, \mu, \tau$ and $j \neq i = 1, 2$; and $\zeta_j = I_0^2 \left(1 + 2\Delta_i |U_{ij}|^2\right) m_j M_j / (8\pi v^2) + \mathcal{O}(\Delta_i^2)$. One can see that $\varepsilon_{i\alpha} \propto \Delta_i$ holds, and hence $\varepsilon_{i\alpha}$ will be vanishing or vanishingly small if $O = 1$ is taken but the RGE-induced effect is neglected. Note that the first term in the square brackets of Eq. (10) depends only on a single combination of the two so-called Majorana phases $\rho$ and $\sigma$ of $U\ [7]$, denoted here as $\phi \equiv \rho - \sigma$; and the second term is only dependent on the Dirac phase $\delta$ of $U$. So a direct connection between the effects of leptonic CP violation at high- and low-energy scales has been established in our RGE-assisted seesaw-plus-leptogenesis scenario.

In the flavored resonant thermal leptogenesis scenario under consideration, the CP-violating asymmetries $\varepsilon_{i\alpha}$ are linked to the baryon-to-photon ratio $\eta$ as follows [52, 53]:

$$\eta \simeq -9.6 \times 10^{-3} \sum_{\alpha} \left(\varepsilon_{1\alpha} \kappa_{1\alpha} + \varepsilon_{2\alpha} \kappa_{2\alpha}\right),$$

(11)

where $\kappa_{1\alpha}$ and $\kappa_{2\alpha}$ are the conversion efficiency factors, and the sum over the flavor index $\alpha$ depends on which region the lepton flavor(s) can take effect. To evaluate the sizes of $\kappa_{i\alpha}$, let us first of all figure out the effective light neutrino masses

$$\tilde{m}_{i\alpha} \equiv \frac{v^2 |(Y_\nu)_{i\alpha}|^2}{M_i} = I_0^2 \left(1 + 2\Delta_i \delta_{i\sigma}\right) m_i |U_{i\alpha}|^2 + \mathcal{O}(\Delta_i^2),$$

(12)

Then the so-called decay parameters $K_{i\alpha} \equiv \tilde{m}_{i\alpha} / m_\ast$ can be defined and calculated, where $m_\ast = 8\pi v^2 H(M_1)/M_1^2 \simeq 1.08 \times 10^{-3}$ eV represents the equilibrium neutrino mass and $H(M_1) = \sqrt{8\pi^3 g_*/90M_1^2 / M_{pl}}$ is the Hubble expansion parameter of the Universe at temperature $T \simeq M_1$ with $g_* = 106.75$ being the total number of relativistic degrees of freedom in the SM and $M_{pl} = 1.22 \times 10^{19}$ GeV being the Planck mass.

- For $M_i \gtrsim 10^{12}$ GeV (for $i = 1, 2$), all the leptonic Yukawa interactions are flavor-blind. In this case the unflavored leptogenesis depends on the overall CP-violating asymmetry $\varepsilon_i = \varepsilon_{i\tau} + \varepsilon_{i\mu} + \varepsilon_{i\tau} \simeq 0$ in our scenario, as one can easily see from Eq. (10).

- For $10^9$ GeV $\lesssim M_i \lesssim 10^{12}$ GeV, the $\tau$-flavored Yukawa interaction is in thermal equilibrium and thus the $\tau$ flavor can be distinguished from $e$ and $\mu$ flavors in the Boltzmann equations [52, 53]. In this case one has to consider two classes of lepton flavors: the $\tau$ flavor and a combination of the indistinguishable $e$ and $\mu$ flavors. We are then left with the flavored CP-violating asymmetries $\varepsilon_{i\tau}$ and $\varepsilon_{i\mu} + \varepsilon_{i\mu}$ together with the flavored decay parameters $K_{i\tau}$ and $K_{i\tau} + iK_{i\mu}$, and the latter can be used to determine the corresponding conversion efficiency factors.

- For $10^5$ GeV $\lesssim M_i \lesssim 10^9$ GeV, the $\mu$- and $\tau$-flavored Yukawa interactions are both in thermal equilibrium, making the $\mu$ and $\tau$ flavors distinguishable. That is why all the three lepton flavors should be separately treated in this case.

Now that we are dealing with resonant leptogenesis, let us define a dimensionless parameter $d \equiv (M_2 - M_1) / M_1 = \xi_{21} - 1$ to measure the level of degeneracy for two of the three heavy Majorana neutrinos. Allowing for $d \ll 1$, we have $\kappa_{1\alpha} \simeq \kappa_{2\alpha} \equiv \kappa(K_\alpha)$ with $K_\alpha \equiv K_{1\alpha} + K_{2\alpha}$.
Given the initial thermal abundance of heavy Majorana neutrinos, the efficiency factor \( \kappa(K_\alpha) \) can be approximately expressed as \[ \kappa(K_\alpha) \approx \frac{2}{K_\alpha z_B(K_\alpha)} \left[ 1 - \exp \left( -\frac{1}{2} K_\alpha z_B(K_\alpha) \right) \right], \] (13)

where \( z_B(K_\alpha) \approx 2 + 4K_\alpha^{0.13} \exp(-2.5/K_\alpha) \).

We proceed to numerically illustrate that our resonant leptogenesis scenario works well. First of all, the values of \( I_0 \) and \( \Delta_\tau \) at the seesaw scale are illustrated in Fig. 1 with \( \Lambda_{SS} \in [10^5, 10^{12}] \) GeV in the SM. Adopting the standard parametrization of \( U \) [7], we need to input the values of eleven parameters: two heavy neutrino masses \( M_1 \) and \( M_2 \) (or equivalently, \( M_1 \) and \( d \)); three light neutrino masses \( m_i \) (for \( i = 1, 2, 3 \)); three lepton flavor mixing angles \( \theta_{12}, \theta_{13} \) and \( \theta_{23} \); and three CP-violating phases \( \delta, \rho \) and \( \sigma \) (but only \( \delta \) and the combination \( \phi \equiv \rho - \sigma \) contribute). For the sake of simplicity, here we only input the best-fit values of \( \theta_{12}, \theta_{13}, \theta_{23}, \delta, \Delta m^2_{21} \equiv m_2^2 - m_1^2 \) and \( \Delta m^2_{31} \equiv m_3^2 - m_1^2 \) (or \( \Delta m^2_{32} \equiv m_3^2 - m_2^2 \)) extracted from a recent global analysis of current neutrino oscillation data [66,57]: \( \sin^2 \theta_{12} = 0.310, \sin^2 \theta_{13} = 0.02241 \) (or 0.02261), \( \sin^2 \theta_{23} = 0.558 \) (or 0.563), \( \delta = 222^o \) (or 285\(^o\)), \( \Delta m^2_{21} = 7.39 \times 10^{-5} \) eV\(^2\), and \( \Delta m^2_{31} = 2.523 \times 10^{-3} \) eV\(^2\) (or \( \Delta m^2_{32} = -2.509 \times 10^{-3} \) eV\(^2\)) for the normal (or inverted) neutrino mass ordering. Then we are left with only four unknown parameters: \( m_1 \) (or \( m_3 \)), \( M_1 \), \( d \) and \( \phi \).

Given the above inputs, we can estimate the size of \( K_\alpha \) with the help of Eq. (12). It is found that \( K_e \gtrsim 2.4, K_\mu \gtrsim 2.9 \) and \( K_\tau \gtrsim 2.6 \) in the normal neutrino mass ordering case; or \( K_e \gtrsim 44.9, K_\mu \gtrsim 20.8 \) and \( K_\tau \gtrsim 26.4 \) in the inverted mass ordering case. Now that \( K_\alpha > 1 \) holds in either case, any lepton-antilepton asymmetries generated by the lepton-number-violating and CP-violating decays of \( N_3 \) with \( M_3 \gg M_1 \simeq M_2 \) can be efficiently washed out. It is therefore safe to only consider the asymmetries produced by the decays of \( N_1 \) and \( N_2 \).

Now let us use the observed value of \( \eta \) to constrain the parameter space of \( \phi \) and \( d \) by allowing \( m_1 \) (or \( m_3 \)) and \( M_1 \) to vary in some specific ranges; or to constrain the parameter
Figure 2: A viable RGE-assisted resonant leptogenesis scenario for the $\tau$-flavored regime with temperature $T \simeq M_1 \in (10^9, 10^{12}]$ GeV in the normal neutrino mass ordering case: the parameter space of $d$ and $\phi$ (upper panels) with some given values of $m_1$ and $M_1$; and the parameter space of $m_1$ and $M_1$ (lower panels) with some given values of $d$ and $\phi$.

space of $m_1$ (or $m_3$) and $M_1$ by allowing $\phi$ and $d$ to vary in some specific ranges, and by taking account of both the $\tau$-flavored regime with $T \simeq M_1 \in (10^9, 10^{12}]$ GeV and the ($\mu + \tau$)-flavored regime with $T \simeq M_1 \in (10^5, 10^9]$ GeV. We find no parameter space in the inverted neutrino mass ordering case, in which the conversion efficiency factors are strongly suppressed. Our RGE-assisted resonant leptogenesis scenario is viable in the normal neutrino mass ordering case, and the numerical results for the $\tau$- and ($\mu + \tau$)-flavored regimes are shown in Figs. 2 and 3 respectively. Some brief discussions are in order.

- The $\tau$-flavored regime (i.e., $T \simeq M_1 \in (10^9, 10^{12}]$ GeV). As can be seen in the upper panels
Figure 3: A viable RGE-assisted resonant leptogenesis scenario for the \((\mu + \tau)\)-flavored regime with temperature \(T \simeq M_1 \in (10^5, 10^9)\) GeV in the \textit{normal} neutrino mass ordering case: the parameter space of \(d\) and \(\phi\) (upper panels) with some given values of \(m_1\) and \(M_1\); and the parameter space of \(m_1\) and \(M_1\) (lower panels) with some given values of \(d\) and \(\phi\).

Of Fig. 2, \(\phi\) is mainly allowed to lie in two possible ranges: \([0, 2\pi/5]\) and \([\pi, 7\pi/5]\); and the dimensionless parameter \(d\) satisfies \(d \lesssim 4 \times 10^{-5}\). These two ranges of \(\phi\) differ from each other just by a shift or reflection; and they are symmetric about \(\phi = \pi/5\) and \(\phi = 6\pi/5\), respectively. Such a feature can easily be understood. Considering \(\varepsilon_{1e} + \varepsilon_{1\mu} + \varepsilon_{1\tau} = 0\) and \(M_1 \simeq M_2\), we have \(\eta \propto \varepsilon_{1\tau} + \varepsilon_{2\tau} \propto \sin 2(\phi - \varphi_\tau)\) with \(\varphi_\tau \equiv \arg (U_{\tau 1} ^* U_{\tau 2} e^{i\delta})\) being dominated by the CP-violating phase \(\delta\) whose value is around \(19\pi/20\). And thus if \(\phi\) is replaced by \(\pi + \phi\) (or \(7\pi/5 - \phi\)) and \(2\pi/5 - \phi\) (or \(12\pi/5 - \phi\)), the value of \(\eta\) will keep unchanged. Note that even if \(\phi = 0\) holds, there can still exist some parameter space for
the four free parameters. In this special case the Dirac CP phase $\delta$, which is sensitive to leptonic CP violation in neutrino oscillations, is the only source of CP violation in our flavored resonant leptogenesis scenario. As shown in the lower panels of Fig. 2, $M_1$ varies in the range $(10^9, 10^{12})$ GeV and $m_1 \lesssim 0.01$ eV holds. But for a given value of $d$, the parameter space of $M_1$ is generally constrained to a specific range; and when $d$ decreases, the allowed range of $M_1$ increases correspondingly. For the most part of the allowed range of $\phi$, the smallest neutrino mass $m_1$ can approach zero with a given value of $d$, and the parameter space of $M_1$ is mainly described by $m_1 \in [2 \times 10^{-3}, 10^{-2}]$ eV and $M_1 \in [4 \times 10^6, 10^9]$ GeV.

It is finally worth mentioning that the normal neutrino mass ordering is currently favored over the inverted one at the $3\sigma$ level, as indicated by a global analysis of today’s available experimental data on various neutrino oscillation phenomena \[56\, 58\]. This indication is certainly consistent with our RGE-assisted resonant leptogenesis scenario.

3 On the minimal seesaw

Since we have focused on resonant leptogenesis with $M_1 \simeq M_2 \ll M_3$ based on the type-I seesaw mechanism, it is natural to consider a minimized version of this scenario by switching off the heaviest Majorana neutrino $N_3$. That is, we can simply invoke the minimal type-I seesaw model \[50\, 51\] with two nearly degenerate heavy Majorana neutrinos to realize resonant leptogenesis. In this case the Yukawa coupling matrix is a $3 \times 2$ matrix, and thus the arbitrary orthogonal matrix $O$ in the CI parametrization of $Y_\nu$ is also a $3 \times 2$ matrix. To remove the uncertainties associated with $O$, we may take

$$O = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \text{or} \quad O = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix},$$

(14)
corresponding to the normal \((m_1 = 0)\) or inverted \((m_3 = 0)\) neutrino mass ordering. Then the expression of \(Y_\nu\) in Eq. (6) can be simplified to

\[
Y_\nu = \frac{i}{v} I_0 \begin{pmatrix}
\sqrt{m_2^1 M_1 U_{e2}} & \sqrt{m_3^2 M_2 U_{e3}} \\
\sqrt{m_2^1 M_1 U_{\mu_2}} & \sqrt{m_3^2 M_2 U_{\mu_3}} \\
\sqrt{m_2^1 M_1 U_{\tau_2}} & \sqrt{m_3^2 M_2 U_{\tau_3}}
\end{pmatrix} + \Delta_\tau \begin{pmatrix}
0 & 0 \\
0 & 0 \\
\sqrt{m_2^1 M_1 U_{\tau_2}} & \sqrt{m_3^2 M_2 U_{\tau_3}}
\end{pmatrix}
\]

(15)

with \(m_1 = 0\), \(m_2 = \sqrt{\Delta m^2_{21}}\) and \(m_3 = \sqrt{\Delta m^2_{31}}\); or

\[
Y_\nu = \frac{i}{v} I_0 \begin{pmatrix}
\sqrt{m_1^1 M_1 U_{e1}} & \sqrt{m_2^2 M_2 U_{e2}} \\
\sqrt{m_1^1 M_1 U_{\mu_1}} & \sqrt{m_2^2 M_2 U_{\mu_2}} \\
\sqrt{m_1^1 M_1 U_{\tau_1}} & \sqrt{m_2^2 M_2 U_{\tau_2}}
\end{pmatrix} + \Delta_\tau \begin{pmatrix}
0 & 0 \\
0 & 0 \\
\sqrt{m_1^1 M_1 U_{\tau_1}} & \sqrt{m_2^2 M_2 U_{\tau_2}}
\end{pmatrix}
\]

(16)

with \(m_3 = 0\), \(m_2 = \sqrt{-\Delta m^2_{32}}\) and \(m_1 = \sqrt{-\Delta m^2_{32} - \Delta m^2_{21}}\). In other words, the mass spectrum of three light neutrinos is fully fixed by current neutrino oscillation data in the minimal seesaw model, so the uncertainty associated with the absolute light neutrino mass scale disappears. Another bonus is that one of the Majorana phases of \(U\) (i.e., \(\rho\)) can always be removed thanks to the vanishing of \(m_1\) or \(m_3\), and therefore we are left with only two low-energy CP-violating phases (i.e., \(\delta\) and \(\sigma\)) which affect the flavored CP-violating asymmetries \(\varepsilon_{i\alpha}\). In our numerical calculations we simply input the best-fit values of \(\theta_{12}, \theta_{13}, \theta_{23}, \delta, \Delta m^2_{21}\) and \(\Delta m^2_{31}\) (or \(\Delta m^2_{32}\)) as given below Eq. (13). Then the observed value of \(\eta\) can be used to constrain the parameter space of \(\sigma\) and \(d\) by allowing \(M_1\) to vary in some specific ranges; or to constrain the parameter space of \(M_1\) and \(d\) by allowing \(\sigma\) to vary in \((0, 2\pi]\). We find that in this minimal type-I seesaw model our RGE-assisted resonant leptogenesis scenario is viable only for the normal neutrino mass ordering with \(m_1 = 0\) and only in the \((\mu + \tau)\)-flavored regime. The numerical results are briefly illustrated in Fig. 4.

An immediate comparison between Fig. 3 and Fig. 4 which are both associated with the \((\mu + \tau)\)-flavored regime for resonant leptogenesis, tells us that the parameter space in the minimal seesaw case is slightly larger. This observation is attributed to the smaller cancellation among the contributions of three flavors, since the efficiency factor for the \(e\) flavor [i.e., \(\kappa(K_e)\)] is much larger than those for \(\mu\) and \(\tau\) flavors [i.e., \(\kappa(K_\mu)\) and \(\kappa(K_\tau)\)] in the minimal seesaw scenario. Note that if \(\kappa(K_e) = \kappa(K_\mu) = \kappa(K_\tau)\) held, \(\eta\) would vanish due to \(\varepsilon_{i\alpha} + \varepsilon_{i\mu} + \varepsilon_{i\tau} = 0\). As shown in Fig. 4, \(\sigma\) is mainly located in two disconnected intervals \([0, 3\pi/10]\) and \([\pi, 13\pi/10]\). But these two intervals are different from each other only by a shift \((\sigma \to \sigma + \pi)\) or a reflection (about \(\sigma = 13\pi/20\)); and each of them has a symmetry axis \((\sigma = 3\pi/20\) or \(\sigma = 23\pi/20\)). We see that \(M_1 \gtrsim 6.3 \times 10^5\) GeV holds, and \(d\) is allowed to vary in a wide range between \(10^{-11}\) and \(10^{-7}\). When the value of \(M_1\) decreases, the lower and upper bounds of \(d\) are both reduced; meanwhile, the allowed range of \(\sigma\) becomes smaller. That is why when \(M_1\) is smaller than \(10^7\) GeV and \(\sigma\) is switched off (i.e., \(\delta\) is the only source of CP violation), it will be very difficult (and even impossible) to make our RGE-assisted resonant leptogenesis scenario viable.

One may certainly extend the above ideas and discussions from the SM to the MSSM, in which the magnitude of \(\Delta_\tau\) is expected to be enhanced by taking a large value of tan\(\beta\). In
Figure 4: A viable RGE-assisted resonant leptogenesis scenario for the \((\mu + \tau)\)-flavored regime with temperature \(T \simeq M_1 \in (10^5, 10^9) \text{ GeV}\) in the normal neutrino mass ordering case based on the minimal type-I seesaw: the parameter space of \(d\) and \(\sigma\) (left panel) with some given values of \(M_1\); and the parameter space of \(d\) and \(M_1\) (right panel) with some given values of \(\sigma\).

In this case it should be easier to obtain more appreciable CP-violating asymmetries \(\varepsilon_{i\alpha}\), simply because they are proportional to \(\Delta_\tau\). So a successful RGE-assisted resonant leptogenesis can similarly be achieved in the MSSM case. In this connection the main concern is how to avoid the gravitino-overproduction problem \([59–63]\), and a simple way out might just be to require \(M_1 \lesssim 10^9 \text{ GeV}\) and focus on thermal leptogenesis in the \((\mu + \tau)\)-flavored regime.

4 Summary

Based on the type-I seesaw mechanism, we have reconstructed the Yukawa coupling matrix \(Y_\nu\) in terms of the light Majorana neutrino masses \(m_i\), the heavy Majorana neutrino masses \(M_i\) and the PMNS matrix \(U\) by assuming the arbitrary orthogonal matrix \(O\) in the CI parametrization of \(Y_\nu\) to be the identity matrix. To bridge the gap between \(m_i\) and \(U\) at the seesaw scale \(\Lambda_{\text{SS}}\) and their counterparts at the Fermi scale \(\Lambda_{\text{EW}}\), we have taken into account the RGE-induced correction to the light Majorana neutrino mass matrix. This RGE-modified seesaw formula allows us to establish a direct link between low-energy CP violation and flavored resonant leptogenesis with \(M_1 \simeq M_2 \ll M_3\), so as to successfully interpret the observed baryon-antibaryon asymmetry of the Universe. We have shown that our idea does work in either the \(\tau\)-flavored regime with equilibrium temperature \(T \simeq M_1 \in (10^9, 10^{12}) \text{ GeV}\) or the \((\mu + \tau)\)-flavored regime with \(T \simeq M_1 \in (10^5, 10^9) \text{ GeV}\), provided the mass spectrum of three light Majorana neutrinos is normal rather than inverted. We have also shown that the same idea is viable for a minimal type-I seesaw model with two nearly degenerate heavy Majorana neutrinos.
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