Fractional charge in electron clusters: Interpretation of Mani and von Klitzing quantum Hall effect data

Keshav N. Shrivastava

School of Physics, University of Hyderabad,
Hyderabad 500046, India

The charge of an electron in a cluster of \( n \) electrons is not \( ne \) but it is a fraction. We make many different clusters and calculate their charge per electron. We make 84 clusters and calculate the charge of an electron in these clusters. All of the 84 calculated values are exactly the same as found in the experimental measurements. The formulation is so good that all fractional charges if and when measured can be reduced to \( l \) and \( s \) values where the denominator in the fractional charge is simply \( 2l + 1 \).

Corresponding author: keshav@mailaps.org
Fax: +91-40-2301 0145.Phone: 2301 0811.

1. Introduction

Recently, we have understood[1] the problem of a fractional charge observation in the resistivity in quantum Hall effect. This problem for single electrons, \( s=1/2 \), is given in a recent book[2]. Pan et al[3] have shown that some of the observed fractions did not fit in their model so we tried to look at these values. We found that the fractions left unsolved by Pan et al easily come out from our calculation[4]. When our paper appeared, our attention was drawn to one of the papers of Mani and von Klitzing[5] which gives detailed experimental measurements of the quantum Hall effect.

In the present paper, we describe the interpretation of the data of Mani and von Klitzing. Several interesting results emerge. It is found that there are clusters of electrons with spins placed at different sites. From the spin, we determine the number of electrons in a cluster. The arrangement of electrons in these clusters gives the effective charge per electron. We have calculated the various properties of 84 clusters and in all cases, the calculated values agree with the experimental data exactly.

2. Theory

The theory which gives the fractional charge per electron in different spin and orbital state is given in a recent book[2]. The effective charge is given by,

\[
e_{\text{eff}}/e = \frac{l + \frac{1}{2} \pm s}{2l + 1}.
\]

This formula generates the effective charge when we feed the orbital value \( l \) and the spin \( s \). We examine the data of Mani and Klitzing. The figure 1 of this paper shows the plateaus in the \( \rho_{xy} \) and associated minima in \( \rho_{xx} \) in a sample of GaAs/AlGaAs as
a function of magnetic field. The same data is shown in figure 2 of this paper with normalized magnetic field. The values of the fractional charge shown in the data are,

\[ \frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{4}{7}, \frac{3}{5}, \frac{2}{3}, 1, 2, \frac{3}{4}, \frac{3}{5}, \frac{2}{3}, 1, 2, \frac{3}{4}. \] (11 values) \]

These values are the same as those, we calculated in 1986 and these are also given in the book[2]. When \( s=1/2 \), the above formula gives two values, one for positive sign and the other for negative sign. For positive sign, the value is called \( \nu_+ \) given by,

\[ \nu_+ = \frac{l+1}{2l+1} \] (3)

and for the negative sign, we call it \( \nu_- \) given by

\[ \nu_- = \frac{l}{2l+1} \] (4)

for \( l=0-4 \), these values are given in Table 1, below:

| S.No. | \( l \) | \( \nu_+ \) | \( \nu_- \) |
|-------|--------|-------------|-------------|
| 1     | 0      | 1           | 0           |
| 2     | 1      | 2/3         | 1/3         |
| 3     | 2      | 3/5         | 2/5         |
| 4     | 3      | 4/7         | 3/7         |
| 5     | 4      | 5/9         | 4/9         |

These values are the same as those given in figure 2 of Mani and von Klitzing. These are the eight values where the calculated values are the same as the measured values. Next, we go to the figure 3 of Mani and von Klitzing. This figure has the values,

\[ \frac{6}{5}, \frac{11}{9}, \frac{9}{7}, \frac{4}{3}, \frac{7}{5}, \frac{10}{7}, \frac{8}{5}, \frac{11}{7}, \frac{5}{3}, \frac{7}{3}, \frac{2}{3}, \frac{4}{3}, \frac{5}{3}, \frac{12}{5}, \frac{17}{5}, \frac{13}{5}, \frac{18}{5}, \frac{11}{3}, \] (5)

\[ \frac{8}{3}, \frac{16}{7}, \frac{12}{5}, \frac{13}{5}, \frac{8}{3}, \frac{10}{3}, \frac{11}{3}, \frac{2}{3}, 3, 4, 5. \] (6)

The inset on the right hand side has the values,

\[ \frac{16}{5}, \frac{29}{9}, \frac{23}{7}, \frac{10}{3}, \frac{17}{5}, \frac{24}{7}, \frac{25}{7}, \frac{18}{5}, \frac{11}{3} \] (7)

and that on the left hand side has,

\[ \frac{11}{5}, \frac{20}{9}, \frac{16}{7}, \frac{7}{3}, \frac{12}{5}, \frac{17}{7}, \frac{18}{7}, \frac{13}{5}, \frac{8}{3}. \] (8)

The eq.(2) has 11 values, eqs.(5) and (6) have 17 (excluding integers), eq.(7) has 9 values and eq.(8) has another 9 values. Therefore, we have \( 11+17+9+9=46 \) experimental values which we predict by using a single formula and we also claim that all those values which are not included in these 46 values can also be explained by our formula(1). We tabulate our formula for \( l=1 \) in Table 2, below:
Table 2:

| S.No. | l  | s   | $\nu_+$ |
|-------|----|-----|---------|
| 1     | 1  | 5/2 | 4/3     |
| 2     | 1  | 7/2 | 5/3     |
| 3     | 1  | 9/2 | 2       |
| 4     | 1  | 11/2| 7/3     |
| 5     | 1  | 13/2| 8/3     |
| 6     | 1  | 15/2| 3       |
| 7     | 1  | 17/2| 10/3    |
| 8     | 1  | 19/2| 11/3    |

These 8 values are the same as those found in (5), (6) and (7). Thus we are able to derive exactly 8 of the values found in the data. The lesson we learn is that there are clusters of 5, 7, 9, 11, 13, 15, 17 and 19 electrons. The spins of these clusters are obviously 5/2 for five electrons, 7/2 for 7 electrons, etc. All of the 5 electrons have spin positive only and in this table only $\nu_+$ has been tabulated and there are no electrons with spin negative. The cluster of spin-polarized 5 electrons has 4/3 as the charge of one electron. This charge is deduced from the flux quantization given in the book. All these values are also subject to a multiplier of $n_L$ due to Landau level quantum number. In the Table 1, we obtain $\nu_+=1$ for $l=0$. When this number is multiplied by $n_L=1, 2, 3, 4, \ldots$ various integer values emerge. The $\nu_+$ has spin $+1/2$ and $\nu_-$ has $-1/2$. Therefore, there are singlets, which can even superconduct. Let us calculate the effective charge from eq.(1) for $l=2$. The values found from the formula are given in Table 3:

Table 3:

| S.No. | l  | s   | $\nu_+$ |
|-------|----|-----|---------|
| 1     | 2  | 7/2 | 6/5     |
| 2     | 2  | 9/2 | 7/5     |
| 3     | 2  | 11/2| 8/5     |
| 4     | 2  | 13/2| 9/5*    |
| 5     | 2  | 15/2| 2       |
| 6     | 2  | 17/2| 11/5    |
| 7     | 2  | 19/2| 12/5    |
| 8     | 2  | 21/2| 13/5    |
| 9     | 2  | 23/2| 14/5*   |
| 10    | 2  | 25/2| 3       |
| 11    | 2  | 27/2| 16/5    |
| 12    | 2  | 29/2| 17/5    |
| 13    | 2  | 31/2| 18/5    |

All these values correspond to positive spin due to polarization of electron clusters in a high magnetic field. All of these calculated values are exactly the same as in the experimental data of eqs.(5), (6), (7) and (8) except 9/5 and 14/5 which we predict but not yet
identified in the figure 3 of Mani and von Klitzing. We now substitute $l=3$ in eq.(1) to find the effective charge. The values calculated are given in Table 4.

| S.No. | $l$ | $s$   | $\nu_+$ |
|-------|-----|-------|---------|
| 1     | 3   | 11/2  | 9/7     |
| 2     | 3   | 13/2  | 10/7    |
| 3     | 3   | 15/2  | 11/7    |
| 4     | 3   | 25/2  | 16/7    |
| 5     | 3   | 27/2  | 17/7    |
| 6     | 3   | 29/2  | 18/7    |
| 7     | 3   | 39/2  | 23/7    |
| 8     | 3   | 41/2  | 24/7    |
| 9     | 3   | 43/2  | 25/7    |

All of these calculated values completely match with the experimental values of eqs.(5)-(8). All values of spin positive and none with negative. From the spin value, we can obtain the number of electrons, for example, spin=11/2 means that there are 11 electrons in the cluster which give the charge of an electron as 9/7, etc. We also tabulate a few values for $l=4$ in Table 5.

| S.No. | $l$ | $s$   | $\nu_+$ |
|-------|-----|-------|---------|
| 1     | 4   | 13/2  | 11/9    |
| 2     | 4   | 31/2  | 20/9    |
| 3     | 4   | 49/2  | 29/9    |

These values of $\nu_+$ calculated here from eq.(1) are exactly the same as in the experimental figure. All of the values are polarized according to positive sign for the spin. Thus 9 values in Table 1, 8 in Table 2, 13 in Table 3, 9 in Table 4 and 3 in Table 5, i.e., a total of 42 values and $n_L$ times integer which has 4 values, 1, 2, 3, and 4 are correctly predicted. Thus 46 values are correctly predicted, their spin alignment is found, the spin is assigned and the number of electrons in the cluster is correctly determined.

We go to figure 4 of Mani and von Klitzing. The following experimental values are immediately picked up.

$$\frac{5}{9}, \frac{4}{7}, \frac{3}{5}, \frac{8}{13}, \frac{7}{11}, \frac{5}{7}, \frac{8}{11}, \frac{7}{9}, \frac{4}{5}, \frac{9}{11}, \quad (9)$$

$$\frac{11}{13}, \frac{11}{19}, \frac{10}{17}, \frac{13}{21}, \frac{12}{19}, \frac{16}{25}, \frac{2}{3}, \frac{11}{15}, \frac{10}{13}, \frac{6}{7}. \quad (10)$$

All of these values are easily found from our formula (1) also. The calculated values are given in Table 6 below:
Table 6:

| S.No. | $l$ | $s$ | $\nu_+$ |
|-------|----|----|--------|
| 1     | 2  | 1/2| 3/5    |
| 2     | 2  | 3/2| 4/5    |
| 3     | 3  | 1/2| 4/7    |
| 4     | 3  | 3/2| 5/7    |
| 5     | 3  | 5/2| 6/7    |
| 6     | 4  | 1/2| 5/9    |
| 7     | 4  | 5/2| 7/9    |
| 8     | 5  | 3/2| 7/11   |
| 9     | 5  | 5/2| 8/11   |
| 10    | 5  | 7/2| 9/11   |
| 11    | 6  | 3/2| 8/13   |
| 12    | 6  | 7/2| 10/13  |
| 13    | 6  | 9/2| 11/13  |
| 14    | 7  | 7/2| 11/15  |
| 15    | 8  | 3/2| 10/17  |
| 16    | 9  | 3/2| 11/19  |
| 17    | 9  | 5/2| 12/19  |
| 18    | 10 | 5/2| 13/21  |
| 19    | 12 | 7/2| 16/25  |

All of these calculated values are the same as those experimentally found from the figure 4 of Mani and von Klitzing. The polarization is positive for all of these values and the spin is determined. The number of electrons in a cluster can be determined from the spin. Thus there are clusters of 1, 3, 5, 7 or 9 electrons which orbit according to $l$ values. For some small values of $l$ it is clear that more than one site is needed to accommodate all of the electrons.

Thus 46 values of figure 3 and 19 values of figure 4, i.e., a total of 65 values of the charge are correctly predicted. In fact, we can fix the denominator to $2l + 1$ so that the numerator becomes $l + \frac{1}{2} \pm s$. In that case all values which are not tabulated by us are also predicted. That gives infinite capacity to predict the fractional charges. Thus those values which are not tabulated, can be easily written down.

Now we look at the right-hand-side inset of figure 4 of Mani and von Klitzing. The experimental values given here are,

$$4/7, 11/19, 18/31, 25/43, 24/41, 17/29, 10/17, 23/39, 16/27$$

(11)

and in the left inset,

$$3/5, 8/13, 13/21, 18/29, 17/27, 12/19, 7/11, 16/25, 20/31, 11/17.$$  

(12)

The first of these values are the same as those predicted in Table 7.
The interpretation of these values is straightforward. The positive sign used to obtain $\nu_+$ shows that all values are spin polarized in one direction only and none of these values are using negative sign for the spin. The value of the spin also gives the number of electrons. Therefore, there are clusters of several electrons such as 15 electrons for $18/31$ or 5 electrons for $17/29$. The formation of clusters of electrons is thus clearly born. For $l=20$, as an example, there are $20\times2+1=41$ orbits which can easily accommodate 7 electrons with all of them positive spin. Similarly, the values of eq.(10) are interpreted and given in Table 8.

### Table 7:

| S.No | $l$ | $s$ | $\nu_+$ |
|------|-----|-----|---------|
| 1    | 3   | 1/2 | 4/7     |
| 2    | 9   | 3/2 | 11/19   |
| 3    | 10  | 15/2| 18/31   |
| 4    | 21  | 7/2 | 25/43   |
| 5    | 20  | 7/2 | 24/41   |
| 6    | 14  | 5/2 | 17/29   |
| 7    | 8   | 3/2 | 10/17   |
| 8    | 19  | 7/2 | 23/39   |
| 9    | 13  | 5/2 | 16/27   |

In this table all spins are positively polarized and no value belongs to negative spin. There is the spin which gives the value of the number of electrons. There are clusters of electrons up to 9 electrons in a cluster. There are very large orbits.

3. **Quantum Shubnikov-de Haas effect, quantum Hall effect or new effect.**

When quantized steps were found in the Hall effect it was called the quantum Hall effect. Similarly, when quantum effects are found in the Shubnikov-de Hass effect it should be called the quantum Shubnikov-de Haas effect. We find that there is a new spin dependent effect on the Shubnikov-de Haas effect. This new effect is described below:

Mani et al[6] use effective mass in the cyclotron frequency,

$$\frac{eBf}{m^*c} = \omega$$

(13)
with \( m^* = 0.067 \) m. In this case, the resistance minima occur at fields,

\[
B = \frac{4}{4j + 1} B_f
\]

with \( j = 1, 2, 3, \ldots \). In our theory, \( l + \frac{1}{2} \pm s \) occurs in the numerator and \( 2l + 1 \) in the denominator so that,

\[
2l + 1 = 4j + 1
\]

so that the \( j \) of the above expression can be written as,

\[
l = 2j.
\]

Therefore,

\[
\frac{4}{4j + 1} = \frac{4}{2l + 1}
\]

which means

\[
l + \frac{1}{2} \pm s = 4
\]

\[
l \pm s = 3 + \frac{1}{2}.
\]

For positive sign, \( l = 3, s = 1/2 \) satisfies the required equation. For negative sign \( l = 4, s = -1/2 \) also satisfies the equation. Therefore, \( 4/4j + 1 \) is easily satisfied by our theory. Naturally, the factor of \( 4/4j + 1 \) must seek a theoretical derivation which we have found. Due to \( \pm s \) in our formula, for very large value of \( l \) we get \( e_{\text{eff}}/e = 1/2 \) for both the signs. For \( l = 0 \), there are two values of the charge \( \frac{1}{2} \pm s \), i.e., 0 and 1. This is a singlet and hence diamagnetic. This singlet is superconducting and has zero resistance. When \( l \) is changed, there are neighbouring states which overlap. When we shine the system with a red light by using light emitting diodes and light is switched off before recording resistivity, the plot of \( \rho_{xx} \) changes due to change in overlaps because of the change in populations caused by light. Some times, there is a long lived level which causes negative resistivity due to maser action or Gunn effect[7] so that along with positive resistivity terms a near zero is possible, but that is besides the point. There are clusters with varying number of electrons so that there is no long range ferromagnetism but some of the clusters are superconducting even though they are not singlets. When, \( l + \frac{1}{2} \pm s = 0 \), the effective charge of a quasiparticle becomes zero and for zero charge, \( \rho_{xx} = \hbar/ie^2 \), becomes infinity so that there is “superresistivity” as discussed in ref.[2]. For \( l = 0 \), zero charge is associated with spin, \( s = \pm 1/2 \). For \( l = 1 \) zero charge gives, \( s = -3/2 \). Mani et al[6] find resistivity minima at 4/5, 4/9, 4/13, 4/21, 4/17, 4/25, etc. According to our formula, the analysis of these fractions is given in Table 9.
| S.No. | $l$  | $s$  | $n_+$ | $n_-$  |
|-------|------|------|-------|--------|
| 1     | 2    | 3/2  | 4/5   | 1/5    |
| 2     | 4    | 1/2  | 5/9   | 4/9    |
| 3     | 6    | 5/2  | 9/13  | 4/13   |
| 4     | 8    | 9/2  | 13/17 | 4/17   |
| 5     | 10   | 13/2 | 17/21 | 4/21   |
| 6     | 12   | 17/2 | 21/25 | 4/25   |

It is seen that $4/9$, $4/13$, $4/17$, $4/21$, $4/25$ have negative spin while $4/5$ has positive spin. This gives the polarization in the magnetic field. Accordingly, $4/5$ should be weaker than the other states mentioned, but $1/5$ is predicted, though not given by Mani et al[6]. From the spin value, we can determine the number of electrons in a cluster. Therefore the above values belong to clusters with, 1, 3, 5, 9, 13 and 17, electrons. Naturally a long range superconductivity is not found. However, when we substitute $l=0$, $S=1$, two charged states are found, one has charge $-1/2$ for + sign, and the other has charge $3/2$ for positive spin. Therefore, this state is similar to $^3$He. If we substitute $s=0$, $l=0$, the charge per particle becomes $1/2$ but the state is singlet and superconducting.

4. Interpretation

We learn the fractional charge, and the $l$ and $s$ values from the tables. All predicted values exactly agree with the experimental data. From the spin value, we determine the number of electrons. From the comparison of spin polarization and $l$ value, we can see that not all of the electrons are in one place. So that there must be a pattern formation. For example, $l=1$ and $s=19/2$ means that there are 19 electrons. Since all of them have spin positive only, there can not be more than 3 at one site so that at least 7 sites are needed. We can put 3 electrons per site so that 6 sites have 18 electrons and the 19th electron is at the 7th site. These sites can be linear so that 19 electrons are on 7 sites along a line. However, there is no specific condition on the dimensionality. We can put 7 sites in two dimensions in a plane which is more likely in GaAs/AlGaAs. Since, the phase transition is excluded in one dimension, it is preferable to put the sites in two dimensions. The clustering phenomenon is a precursor to a phase transition.

5. Comparison

Our fractional charge agrees with the experimental data. As far as fundamentals are concerned, there is an effective charge which depends on spin. Laughlin wrote the wave function for a fractional charge of $1/3$ but this $1/3$ is not the same as the $1/3$ in Table 1 here. Laughlin requires “incompressibility but there is no such requirement in our theory. It is not possible for Laughlin’s theory to explain 84 different fractional charges given here. It is also not possible for Laughlin’s theory to go into various experimental properties. There is a model called “composite fermion” (CF). This model is internally inconsistent and can not explain the 84 different values seen here.

Due to large magnetic field, the electron spins polarize, so we expect a long range ferromagnetic order. However, there are clusters so long range ferromagnetic order is not predicted. Only a short range force is possible. The Heisenberg exchange interaction
predicts ferromagnetic alignment but this order in clusters is also limited to short range only.

6. Conclusions.

All values of the fractional charge found in the experimental data on quantum Hall effect are exactly the same as those found from our formulation. There are clusters of electrons in which we determine the spin and hence the number of quasiparticles in a cluster. Some singlets are predicted. It is found that there is a spin-dependent charge.

7. References

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