Solutions of the Renormalisation Group Equation in Minimal Supersymmetric Standard Model (MSSM).

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Renormalisation Group Equation (RGE) for color and top couplings sector of MSSM has been solved. The mass of the top comes out to be $180.363 \pm 10.876$ GeV and $\beta_{\text{top}} \cong \frac{2}{5}$. It is conjectured that the masses of the other 11 fermions and the CKM phase angle $\phi$ can be theoretically estimated. The results confirm the fact that the quarks and leptons have been created having equal mass $\sim 115$ GeV at the MSSM GUT scale $\sim 2.2 \times 10^{16}$ GeV.

PACS numbers: 12.10.Dm, 12.10.Kt
Keywords: Renormalisation Group Equation, SUSY standard model

The Standard Model for Electroweak and Strong interaction has been proposed with many variations like Standard Model (SM), two Higgs Doublet Model (2HDM), Minimal Supersymmetric Standard Model (MSSM) and such others [1]. Most of them have been phenomenologically successful but this success has brought in relatively large number of free parameters [2]. Besides the gauge couplings of $SU_C(3)$, $SU_L(2)$ and $U_Y(1)$, in the matter sector there are Higgs and Yukawa couplings [2][3]. The aim of a good model building is not only to reduce the number of free parameters but should perturbatively tenable and obtain result in the agreement with experiment. To have an idea of the parameters, the vertex function of the model satisfy the renormalisation group equations [4].

$$\partial^2 \partial t + \beta_{\gamma_i} \frac{\partial}{\partial \beta_{\gamma_i}} + \beta_{C_i} \frac{\partial}{\partial \beta_{C_i}} - \sum_a \gamma_a N_a \Gamma = 0. \tag{1}$$

The notations are usually taken from reference [4]. To give an example of what we imply by reliable calculation is to note that the gauge coupling coefficients $\beta_{\gamma_i}$ satisfy the following equations in MSSM which we shall consider in this letter,

$$16\pi^2 \frac{dg_i(t)}{dt} = c_i g_i^3(t), \quad \text{with} \quad t = \log(\mu/M_Z), \quad i = 1, 2, 3. \tag{2}$$

where $\mu$ is the mass scale of the theory. $c_1=6.6$, and $c_2=1$. This means that the $SU_L(2)$ and $U_Y(1)$ groups are not asymptotically free but $c_3=-3$ means that the color $SU_C(3)$ is asymptotically free. Therefore one can make a perturbative expansion in powers of $g_3$.

It is not certain that the other $\beta$-coefficients obtained from product groups are such that the theory is asymptotically free and one can find reliable perturbative results. There is sufficient evidence to show that the top quark mass coupling decreases as energy increases like $g_3$. So from the entire parameter space of SM, we isolate a small region containing the top coupling and $g_3$ of the color group. The strategy is to see that if the experimentally acceptable results are obtained in this small region; we can then continue this region analytically to gradually encroach and cover the entire parameter space so that the perturbative result will be meaningful everywhere. This was also the basis of reduction of couplings technique of Zimmermann et al [4]. They developed a technique of reduction of coupling with one Higgs parameter space so that the perturbative result will be meaningful everywhere. This was also the basis of reduction of parameter space so that the perturbative result will be meaningful everywhere.

Pendleton and Ross [5] have exclusively extended the Kube, Siebold and Zimmermann’s [6] work to include the other gauge coupling parameters $g_1$, $g_2$ and $g_3$ in the same type of standard model. Their result, taking $g_3$ coupling alone, is also $m_{\text{top}} = \sqrt{\frac{2}{3}} g_3 v = 81$ GeV. Theoretical prediction of the mass of the top, before it was discovered, has been made by several authors [7]. Faraggi has obtained $m_{\text{top}} \sim 175 - 180$ GeV in Superstring derived standard like model, but, in getting this result, he has taken $\frac{m_{\text{top}}}{m_{\mu}} = \frac{1}{8}$ at the unification scale. We attempt here to extend earlier works to include the successful MSS Model into Yukawa coupling parameters calculations.

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It will be helpful to state the gauge sector values for later use. We shall take $t_X = \log(M_X/M_Z) = 33.0$, $4\pi/g_1^2 = 24.6$ and $M_S = M_Z$. The values of $g_1^2$, $g_2^2$, $g_3^2$ at $M_Z$ obtained from the R.G. equation above, are $4\pi/g_1^2 = 59.24$, $4\pi/g_2^2 = 29.85$ and $4\pi/g_3^2 = 8.85$. These are reasonable and consistent with the experimental results.

In this letter, we are primarily concerned in the top mass within the top-color sector. In the renormalisation group equation for the top in MSSM, there is an added complication of having two Higgs. Therefore we write for the top

$$m_{top} = M_{top}(t) v_{top}(t).$$

Conventionally,

$$v_{top}(t) = v \sin(\beta_{top}(t)),$$

where $v = 174$ GeV.

Letting $g_1 = g_2 = 0$, $m_{lepton} = 0$, $m_{quark} = 0$, except the top, the MSSM, RGE is

$$16\pi^2 \frac{d m_{top}}{dt} = \left[ \frac{16}{3} g_3^2 + 3 M_{top}^2 \right] m_{top},$$

$$16\pi^2 \frac{d g_3}{dt} = -3 g_3^3,$$

and

$$16\pi^2 \frac{d v_{top}}{dt} = \left[ \frac{3}{20} g_1^2 + \frac{3}{4} g_2^2 + 0.0 \times g_3^2 - 3 M_{top}^2 \right] v_{top},$$

$$= -3 M_{top}^2 v_{top}.$$ 

The top Yukawa and color sector equations, which have to be solved self consistently, become

$$g_3^3 \frac{d m_{top}}{d g_3} = \left( \frac{16}{9} g_3^2 - \frac{m_{top}^2}{v_{top}^2} \right) m_{top},$$

$$g_3 v_{top} \frac{d m_{top}}{d g_3} = \left( \frac{m_{top}^2}{v_{top}^2} \right) v_{top}.$$ 

Following reference [4], we introduce $\rho_{top}(g_3) = \frac{m_{top}^2}{g_3^2}$. From equation (9), this satisfies the equation

$$g_3^2 \frac{\rho_{top}}{g_3} = 14 \frac{\rho_{top}}{g_3} - \frac{4 \rho_{top}^2}{v_{top}^2}.$$ 

To simplify further, we set $\rho_{top} = \Gamma_{top} v_{top}^2$, and using equation (10) and (11), we get

$$g_3 \frac{d (\Gamma_{top} v_{top})}{d g_3} = g_3 \left[ \frac{d \Gamma_{top} v_{top}}{d g_3} + 2 \Gamma_{top} v_{top} \frac{d v_{top}}{d g_3} \right]$$

$$= g_3 \left[ \frac{d \Gamma_{top} v_{top}}{d g_3} + 2 \Gamma_{top} \frac{m_{top}^2}{v_{top}^2} g_3 \right]$$

$$= g_3 v_{top}^2 \frac{d \Gamma_{top}}{d g_3} v_{top}^2 + 2 \Gamma_{top} v_{top}^2.$$ 

Equating equation (11) to equation (12), we obtain

$$g_3 \frac{d \Gamma_{top}}{d g_3} = \frac{14}{9} \Gamma_{top} - 4 \frac{\Gamma_{top}^2}{g_3^2}.$$ 

The equation, exhibiting the pole, is

$$\frac{d \Gamma_{top}}{\Gamma_{top} \left( \frac{14}{9} - \Gamma_{top} \right)} = 4 \frac{dg_3}{g_3}.$$
On integration,
\[
\Gamma_{\text{top}} = \frac{7}{18} \left[ \frac{\frac{14}{g_3^2}}{C_{\text{top}} + \frac{14}{g_3^2}} \right],
\] (17)

\(C_{\text{top}}\) is independent of \(g_3\). This is similar to or the general solution of Kubo et al [4] for \(g_1 = g_2 = 0\). \(C_{\text{top}}\) may depend on the other gauge couplings \(g_1, g_2\) at higher energies without invalidating equation (17). We solve the equation for the \(v_{\text{top}}\) by noting that
\[
\frac{m_{\text{top}}^2}{g_3^2 v_{\text{top}}^2} = \frac{\rho_{\text{top}}}{v_{\text{top}}} = \Gamma_{\text{top}}
\] (18)

On simplification
\[
\frac{dv_{\text{top}}}{v_{\text{top}}} = \frac{7}{18} \frac{\frac{14}{g_3^2}}{C_{\text{top}} + \frac{14}{g_3^2}} \frac{dg_3}{g_3}
\] (19)

This can be deduced to be the same as the corresponding equation for \(m_{\text{top}}\) of Faraggi [7] translated to our notation. Integrating equation (19),
\[
v_{\text{top}}^4 = v_{\text{top}}^4 \left( C_{\text{top}} + \frac{14}{g_3^2} \right)
\] (20)

Using \(v_{\text{top}}^2\) in \(m_{\text{top}}^2 = M_{\text{top}}^2 v_{\text{top}}^2\), one obtains the solutions of (9) and (10)
\[
m_{\text{top}} = \sqrt{\frac{7}{18} g_3^2 g_3 \left( \frac{\frac{14}{g_3^2}}{C_{\text{top}} + \frac{14}{g_3^2}} \right) \frac{1}{v}}
\] (21)

\[
v_{\text{top}} = v \left( C_{\text{top}} + \frac{14}{g_3^2} \right)^{\frac{1}{4}}
\] (22)

This can be confirmed by direct substitution.

Observationally, one intends to know the value of \(m_{\text{top}}(t)\) for a given mass \(m_1\), such that \(t_1 = \log \left( \frac{m_1}{M_Z} \right)\). The function in the square bracket, optimally stable like the equation (17), with respect to the variation of \(g_3\) at this mass \(M_1 (t_1)\) or \(t=t_1\), should be
\[
\frac{\partial}{\partial g_3} \left( C_{\text{top}} + \frac{14}{g_3^2} \right) = 0
\] (23)

or
\[
C_{\text{top}}(t_1) = -\frac{3}{4} \frac{14}{g_3^2} (t_1).
\] (24)

Finally, we get from (21), (22) and (24), the solutions of coupled equations (9) and (10) as
\[
m_{\text{top}}(t) = \sqrt{\frac{7}{9} g_3^2 (t_1) \frac{g_3 (t)}{4g_3^2 (t_1)} \left( \frac{1}{4g_3^2 (t_1) - 3g_3^2 (t_1)} \right) \frac{1}{v}}
\] (25)

\[
v_{\text{top}}(t) = \sqrt{\frac{1}{2} \left( 4g_3^2 (t) - 3g_3^2 (t_1) \right) \frac{1}{v}}
\] (26)

\[
M_{\text{top}}^2(t) = \frac{m_{\text{top}}^2}{v_{\text{top}}^2} = \frac{14}{9} g_3^2 (t) g_3^2 (t_1) \frac{1}{4g_3^2 (t_1) - 3g_3^2 (t_1)}
\] (27)

The 'stable' values of \(m_{\text{top}}\) and \(v_{\text{top}}\), at \(t_1\), based on the equation (24), are
\[
m_{\text{top}}(t_1) = \sqrt{\frac{7}{9} g_3^2 (t_1) g_3 (t_1)} v
\] (28)

\[
\sin(\beta(t_1)) = \sqrt{\frac{1}{2} g_3^2 (t_1)}
\] (29)
for comparison with experimental results at mass for which $t=t_1$.

We report the following results

$$m_{\text{top}}(m_Z) = 180.92 \text{ GeV}$$  \(30\)
$$m_{\text{top}}(m_X) = m_U = 110 \text{ GeV}$$  \(31\)
$$\tan\beta(M_Z) = 1.1$$  \(32\)
$$\tan\beta(M_X) = 0.9$$  \(33\)

These are the results from the explicitly known asymptotically free region. Experimentally, the top quark has a mass of $174.3 \pm 5.1 \text{ GeV}$ and is in excellent agreement with our result. Even though the extrapolation is too drastic, the decrease in the value of running $\tan\beta$ as the mass changes from $M_Z$ to $M_X$, confirms the result which was first reported by Parida and Purkayastha [9]. The value of tangent hovers around one. So, to get an overall picture of the general nature of solutions of the MSSM R.G. Equations, it is fairly adequate to solve the equations for the fermion couplings $M_F$ like $m_{\text{top}}$ of equation 3.

We continue to expand the parameter space, retaining the top and bottom Yukawa couplings ($m_{\text{top}}, m_{\text{bottom}}$) and color gauge coupling $g_3$. The equations are [7]

$$16\pi^2 \frac{d}{dt} \log M_{\text{top}} = \left[ -\frac{16}{3} g_3^2 + 6M_{\text{top}}^2 + M_{\text{bottom}}^2 \right] M_{\text{top}}$$  \(34\)
$$16\pi^2 \frac{d}{dt} \log M_{\text{bottom}} = \left[ -\frac{16}{3} g_3^2 + M_{\text{top}}^2 + 6M_{\text{bottom}}^2 \right] M_{\text{bottom}}$$  \(35\)

We eliminate $M_{\text{bottom}}$ from r.h.s, by noting

$$16\pi^2 \frac{d}{dt} \log \left( \frac{M_{\text{top}}^6}{M_{\text{bottom}}} \right) = \left[ -\frac{80}{3} g_3^2 + 35 M_{\text{top}}^2 \right]$$  \(36\)

From equation (21) $M_{\text{top}}^2 = m_{\text{top}}^2 v_t^2 = 7 \frac{18}{14} g_3^2 + \frac{14}{3} C_{\text{top}} + \frac{1}{3}$

This is easily solved for $M_{\text{bottom}}$. After some algebra, we get at $t=t_1=0$ i.e. $M_Z$

$$M_{\text{bottom}}(M_Z) = 2 \frac{1}{18} \left( \frac{7}{18} \right)^3 g_3 - \frac{103}{108}$$  \(38\)

hence

$$m_{\text{bottom}} = M_{\text{bottom}} v_{\text{bottom}} = M_{\text{bottom}} (v^2 - v_{\text{top}}^2)^\frac{1}{2} \approx 5.31 \text{ GeV}$$  \(39\)

This is a very good result in view of the shrunk region of parameter space.

As a step further, we expand the region of validity of perturbation to include SU(2) and U(1) i.e. $g_2 \neq 0$ and $g_1 \neq 0$. Then the equation for top becomes

$$16\pi^2 \frac{dM_{\text{top}}}{dt} = \left[ -\frac{13}{15} g_1^2 - 3g_2^2 - \frac{16}{3} g_3^2 + 6M_{\text{top}}^2 \right] M_{\text{top}}$$  \(40\)

This equation [40] has been exactly solved by Deo and Maharana [8] and the result is

$$1 = \frac{M_{\text{top}}^2}{m_U^2} a_{\text{top}} + b_{\text{top}}$$  \(41\)

where

$$a_{\text{top}} = \left( \frac{g_1^2}{g_6^2} \right)^\frac{\kappa_{\text{U}}}{2} \left( \frac{g_2^2}{g_6^2} \right)^\frac{\kappa_{\text{U}}}{2} \left( \frac{g_3^2}{g_6^2} \right)^\frac{\kappa_{\text{U}}}{2},$$  \(42\)

and

$$b_{\text{top}} = \frac{6}{8\pi^2} \int_{0}^{t_X} dt \left( 1 - g_1^2 e_1 \frac{e_1}{8\pi^2} \right)^\frac{\kappa_{\text{U}}}{2} \left( 1 - g_2^2 e_2 \frac{e_2}{8\pi^2} \right)^\frac{\kappa_{\text{U}}}{2} \left( 1 - g_3^2 e_3 \frac{e_3}{8\pi^2} \right)^\frac{\kappa_{\text{U}}}{2}.$$  \(43\)
\( K_t^V = 13/15, \ K_s^V = 3 \) and \( K_H^V = 16/3 \) are the coefficients of the coupling constant as given in references \(^8\) \(^{11}\). We found that the top mass originated from the unification mass \( m_{U} \sim 114 \) GeV.

This also confirms the perturbative ‘stability’ of this approach. Taking \( g_1 \neq 0 \) and \( g_2 \neq 0 \), and extending to large \( t \)-values, the value of \( m_{U} \) of equation \(^8\), changes by a few GeV only.

In reference \(^8\), it has been shown that all the 12 fermions (quarks and leptons) at the GUT scale had the same mass of about 115 GeV. Encouraged by this, we now enlarge the region of applicability of the analysis for the whole parameter space. Essentially, there are 13 parameters, 12 fermion masses and precisely one CKM phase angle \( \phi \) for the three generations. We now write the full renormalization equation for all the fermions,

\[
16\pi^2 \frac{dM_F(t)}{dt} = A_F M_F^3(t) + [Y_F(t) - G_F(t)] M_F(t)
\]

\[
= A_F M_F^3(t) + Z_F(t) M_F(t). \tag{45}
\]

\( A_F \) is a group theoretic factor whose value is ‘6’ for quarks, i.e. for \( F=1,2,\cdots,6 \) and ‘4’ for the leptons i.e. for \( F=7,8,\cdots,12 \). The positive values indicate that the field theory containing Yukawa couplings only, may not be asymptotically free.

\( Y_F \) is the mixing term which can be put in matrix form

\[
Y_F = \sum_H A_{FH} M_H^4(t) M_H(t), \quad H = 1,2,\cdots,12. \tag{46}
\]

In MSSM, the matrix \( A_{FH} \) is specified by the 144 elements given below \(^11\) \(^{12}\),

\[
A_{FH} = \begin{pmatrix}
0 & 3 & 3 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\
3 & 0 & 3 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\
3 & 3 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\
1 & 0 & 0 & 0 & 3 & 3 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 3 & 0 & 3 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 3 & 3 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 3 & 3 & 3 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 3 & 3 & 3 & 1 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 3 & 3 & 3 & 1 & 1 & 0 & 0 & 0 & 1 \\
3 & 3 & 3 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\
3 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0
\end{pmatrix}, \tag{47}
\]

Then one can write an exact solution for \( M_F(M_Z) \), as given in reference \(^{12}\)

\[
\frac{M_{\text{top}}^2(M_Z)}{M_{\text{top}}^2(M_Z)} = \frac{M_{\text{top}}^2(M_Z)}{M_{\text{top}}^2(M_X)} \exp \left( \frac{1}{8\pi^2} \int_0^{t_X} Z_F(\tau) d\tau \right) + \frac{A_F}{8\pi^2} \int_0^{t_X} dt \exp \left( \frac{1}{8\pi^2} \int_0^t Z_F(\tau) d\tau \right) \tag{48}\]

The first term of equation \(^{15}\) is \( A_F M_F^2 \) which is perturbatively very small except for the top. Therefore we neglect the 2nd term in \(^{15}\). For fermions other than top, i.e. \( F \) varying from 2 to 12,

\[
M_F(M_Z) \simeq m_U \ e^{-\frac{1}{16\pi^2} \int_0^{t_X} Z_F(\tau) \ d\tau }, \tag{49}
\]

\[
\simeq m_U \ e^{-\frac{I_F}{16\pi^2}}, \tag{50}
\]

where

\[
I_F = \int_0^{t_X} Z_F(t) \ dt. \tag{51}
\]

But, the main problem of finding general solutions, is to calculate \( I_F \). This integral is

\[
I_F = \int_0^{t_X} Z_F(t) dt = \frac{1}{2} \int_0^{t_X} [Z_F(t) + Z_F(-t)] dt = \frac{1}{4} \int_{-t_X}^{t_X} dt[Z_F(t) + Z_F(-t)] \tag{52}
\]

Next, we consider the equality,

\[
dt = \frac{dt}{dM_F} \frac{dM_F(t)}{16\pi^2} = \frac{dM_F(t)}{M_F(A_F M_F^4 M_F + \sum_H A_H M_H^4 M_H - G_F(t))} \tag{53}
\]
The integrand should have the poles like the ones in equation \(10\). The most solutions have \(M_{\text{lepton}} = 0\) as the gauge factor \(G_{\text{lepton}}\) does not contain the color gauge coupling \(g_3\). So, we shall let \(H\) to be summed over from 1 to 6 in \(\sum_H A_{FH}\). Choosing

\[
M_F(t) = m_{UE} e^{in_F \theta_F(t)},
\]

we have

\[
I_F = \frac{i}{2} \frac{1}{12} 16\pi^2 n_F \int_{-\frac{m_U}{m_U}}^{m_U} d\theta_F(t) \left( \frac{\sum_H A_{FH} - \frac{1}{m_U} G_{F}(\theta(t))}{A_F + \sum_H A_{FH} - \frac{1}{m_U} G_{F}(\theta(t))} \right)
\]

\[
\cong \frac{i}{2} \frac{1}{12} 16\pi^2 n_F \sum_G \sum_H A_{GH} \int_{-t_X}^{t_X} d\theta_G(t) \left( \frac{1}{A_F + \sum_H A_{FH} - \frac{1}{m_U} G_{F}(\theta(t))} \right)
\]

where \(n_F\) in an integer. We have neglected \(\frac{1}{m_U} G_{F}(t)\) in the numerator and want to make the factor multiplying \(n_F\), independent of \(F\) by taking the average, using \(\frac{1}{12} \sum_G = 1\). Retracing back, we replace \(M_F(t) = m_{UE} e^{i\theta_F(t)m_U^2}\) and find that

\[
16\pi^2 \frac{dM_G(t)}{M_G(t)} = i d\theta_G(t)m_U^2 = (A_G + \sum_{H=1}^{6} A_{GH} - \frac{1}{m_U} G_G(\theta(t)))m_U^2
\]

so that

\[
I_F = n_F t_X \frac{1}{12} \sum_{G=2}^{12} \sum_{H=1}^{6} A_{GH} = n_F t_X \frac{89}{12}
\]

So the masses of the fermions other than the top is

\[
M_F(M_Z) \simeq m_{UE} \lambda^{n_F}
\]

The Wolfenstein parameter \(\lambda\) turns out to be

\[
\lambda = \exp \left( -\frac{t_X}{16\pi^2} \frac{89}{12} \right) = 0.219
\]

This is an excellent result in spite of the approximate estimates.

Starting from the unification mass \(m_{UE} = 115\) GeV and using the equation \(59\), we now calculate masses of all 11 fermions for different values of \(n_F\) and identify them in the Table I.

| \(n_F\) | Mass (GeV) | Fermions (Quarks, leptons) | Expt. Values (GeV) |
|--------|------------|---------------------------|-------------------|
| 2      | 5.5        | b                         | 5                 |
| 3      | 1.2        | c, \(\tau\)               | 1.4, 1.7          |
| 4      | 0.264      | s, \(\mu\)                | 0.1 - 0.23, 0.01  |
| 5      | 0.06       | ?                         | ?                 |
| 6      | 0.012      | d, \(\nu_e\)              | 0.055 - 0.115, 0.018 |
| 7      | 0.0028     | u                         | 0.003             |
| 8      | 6\times10^{-4} | e                   | 5\times10^{-4}    |
| 9      | 1.33\times10^{-4} | \(\nu_\mu\)           | 1.9\times10^{-4}  |
| 16     | 3\times10^{-9}    | \(\nu_e\)               | 3\times10^{-9}    |

**TABLE I**: Identification of fermions
The values given in the table I are estimates only, based on the relation (59). The remaining free parameter is the CKM phase angle $\phi$ defined through Wolfenstein parametrization [13] and is given as [14]

$$\lambda = \left( \frac{M_d}{M_s} + \frac{M_u}{M_c} + 2 \sqrt{\frac{M_d M_u}{M_s M_c}} \cos \phi \right)^{\frac{1}{2}}. \quad (61)$$

Using the Table I for the values of $n_F$, we get

$$\cos \phi \simeq -\frac{\lambda}{2} \simeq -0.1 \quad (62)$$

and this gives $\phi \simeq 95^\circ$.

The mixing angles $c_i, s_i, i=1,2,3,4$ of the CKM matrix as given for quarks is

$$V_{CKM} = \begin{pmatrix}
    c_1 c_2 - s_1 s_2 e^{-i\phi} & s_1 + c_1 s_2 e^{-i\phi} & s_2 (s_3 - s_4) & s_3 - s_4 \\
    -c_1 s_2 - s_1 e^{-i\phi} & -s_1 s_2 + (c_1 c_2 c_3 c_4 + s_1 s_4) e^{-i\phi} & s_3 (s_3 - s_4) & (s_3 c_4 + s_1 s_4) e^{i\phi} \\
    s_1 (s_3 - s_4) & -c_1 (s_3 - s_4) & s_3 - s_4 & \lambda \\
    0 & 0 & 0 & 0
\end{pmatrix} \quad (63)$$

From Table I,

$$s_1 = \left( \frac{M_d}{M_s} \right)^{\frac{1}{2}} = \lambda, \quad s_2 = \left( \frac{M_u}{M_c} \right)^{\frac{1}{2}} = \lambda^2; \quad s_3 = \left( \frac{M_d M_s}{M_b^2} \right)^{\frac{1}{2}} = \lambda^3 \quad \text{and} \quad s_3 - s_4 = \lambda^2 A(t) \quad (64)$$

The SUSY RG equations are [15],

$$\frac{d\lambda}{dt} = 0, \quad (65)$$

and

$$16\pi^2 \frac{d}{dt} \log A(t) = -(M_{top}^2 + M_b^2). \quad (66)$$

So $s_1, s_2, s_4$, and $\phi$ do not change with energy. Further more, to satisfy the equation (66), we must equate $A(t)$ as

$$A(t) = \left[ \frac{M_b(M_{top}(t))}{M_{top}(M_Z)} \right]^{-1/7}, \quad A(M_X) = 1.12 \quad \text{and} \quad A(M_Z) = 1.5. \quad (67)$$

The entire CKM matrix elements are calculable. Choosing $A(M_X)=1.1$ in the CKM matrix for all $t$, we can determine $m_U$ from $M_{top}$ and then the rest of the masses from the equation (59) and, as in reference [12],

$$M_F(t) = m_U \lambda^{n_F(1 - \frac{1}{7})} \quad (68)$$

Thus we are able to find all the free parameters in the matter sector of MSSM, subject to the accuracy of the above relation. In the gauge sector, the two parameters $M_X$ and $\alpha_{GUT}$ are the only inputs in the model.

In this letter, we have found the top mass from the MSSM RGE, using the group theoretic coefficients, as 180 GeV in close agreement with experiment. Using group constants, we have calculated a unification mass and have been able to find an approximate equation for the eleven other fermions, the CKM phase angle $\phi$ in terms of the Wolfenstein parameter. The approach made above to solve RGE should be studied further with greater detail for the eleven fermions separately and much more accurately.

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