Many Fermi polarons at nonzero temperature

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Abstract

An extremely polarized mixture of an ultracold Fermi gas is expected to reduce to a Fermi polaron system, which consists of a single impurity immersed in the Fermi sea of majority atoms. By developing a many-body T-matrix theory, we investigate spectral properties of the polarized mixture in experimentally relevant regimes in which the system of finite impurity concentration at nonzero temperature is concerned. We explicitly demonstrate presence of polaron physics in the polarized limit and discuss effects of many polarons in an intermediate regime in a selfconsistent manner. By analyzing the spectral function at finite impurity concentration, we extract the attractive and repulsive polaron energies. We find that a renormalization of majority atoms via an interaction with minority atoms and a thermal depletion of the impurity chemical potential are of significance to depict the many-polaron regime.

1. Introduction

Understanding effects of impurities immersed in an environment is one of the key issues in physics. In nuclear physics, heavy hadrons in nuclear matter such as charm hadrons are now discussed in context of impurity problems [1]. In condensed matter physics, a number of impurities problems have been examined for a long time, depending on conditions of impurities such as mobile or immobile and presence or absence of a spin-exchange interaction [2]. A particularly fundamental class of the problems is the polaron in which a mobile impurity interacts with an environment [3, 4]. The concept of the polaron appears in a variety of the materials such as metal, semiconductor, and superconductor systems [5, 6].

Currently, there is a growing interest in an ultracold atomic gas as a quantum simulator of polaron physics [7–17]. The Feshbach resonance available in an ultracold atomic gas allows us to control an interaction between impurity and bath and to investigate the strong coupling regime, which is generally challenging in quantum many-body physics [18]. In addition, by using radio-frequency (rf) spectroscopy, we can address spectral properties of the systems including excited branches [19]. For example, rf spectroscopy experiments confirmed existence of a repulsive polaron, which is a quasiparticle associated with a repulsive interaction and is a metastable excited many-body state [10–16]. The repulsive polaron also receives attention in terms of the realization of repulsive many-body states such as itinerant ferromagnetism [20–24].

Interpretations of polaron experiments in ultracold atomic gases are grounded on the theoretical analyses in which the system with a single impurity at the zero temperature is assumed. In the case of the Fermi polaron whose bath consists of fermions, due to such assumptions, theoretical treatments such as variational methods [25–31], T-matrix approximation [32–35], functional renormalization [36, 37], and diagrammatic Monte Carlo [38–42] are successfully applied. In the case of finite polarization, the polaron–polaron interaction is discussed [43–47]. In reality, however, none of these theoretical assumptions are exactly satisfied in corresponding experiments; the temperature is about from centesimal to few tenths of the Fermi temperature [48] and impurity concentration is of the order of 10%. Thus, it is important to directly analyze such regimes in terms of many-body calculations accessible to the strong coupling regime.
In this paper, we examine spectral properties in the polarized mixture of an ultracold Fermi gas with a many-body \( T \)-matrix theory, which allows us directly to plug in the finite temperature and the impurity concentration effects. We demonstrate that by shifting impurity concentration, the spectral function of impurities shows crossover behaviors from a single polaron to many polarons. By analyzing the spectral function in detail, we extract the polaron energy as a function of impurity concentration. We point out that a renormalization of majority atoms due to minority atoms plays a crucial role in understanding the system at a finite density, which has been overlooked in previous studies. In addition, we show that the impurity chemical potential is largely affected by finite temperature effects compared to other quantities. We also predict a quasiparticle-like peak in a high-energy regime of the spectral function of majority atoms, which cannot be captured with single-impurity theories and may be measured with rf spectroscopy.

2. Formulation

We consider the grand canonical Hamiltonian for the two-component Fermi mixture interacting through the broad Feshbach resonance \( [18] \) (we set \( \hbar = k_B = 1 \)),

\[
H = \sum_{\mathbf{k}, \sigma} \varepsilon_{\mathbf{k}, \sigma} c_{\mathbf{k}, \sigma}^\dagger c_{\mathbf{k}, \sigma} + g \sum_{\mathbf{p}, \mathbf{q}, \mathbf{k}} \varepsilon_{\mathbf{p}, \uparrow}^\dagger c_{\mathbf{q}, \downarrow} c_{\mathbf{q}+\mathbf{r}, \downarrow} c_{\mathbf{p}+\mathbf{r}-\mathbf{k}, \uparrow},
\]

where \( c_{\mathbf{p}, \sigma} \) represents the fermionic annihilation operator with momentum \( \mathbf{p} \) and pseudospin \( \sigma = \uparrow, \downarrow \). \( \varepsilon_{\mathbf{k}, \sigma} = \frac{p^2}{2m} - \mu_\sigma \) is the kinetic energy of atoms with mass \( m \) measured from the chemical potential \( \mu_\sigma \). The interatomic interaction is local and the coupling constant \( g \) can be characterized with the \( s \)-wave scattering length \( a_{s} \). Notice that the system volume is taken to be unity. Below, without loss of generality, we assume that \( \Upsilon(\frac{1}{2}) \) is the majority (minority) spin.

To obtain the polaron energy \( \omega_{\text{pol}} \in \mathbb{R} \), we determine the pole \( \omega_{\text{pol}} \in \mathbb{C} \) of \( G_\sigma(\mathbf{p}, \omega + i\delta) \) by solving the selfconsistent equation

\[
\omega_{\text{pol}} = \Sigma_\sigma(\mathbf{p} = 0, \omega_{\text{pol}} + i\delta) = \mu_\parallel + \mu_\perp - i\Gamma,
\]

(4)

In general, \( \omega_{\text{pol}} \) locates on the complex plane of \( \omega_\parallel \) and especially in the case of repulsive polaron near the unitarity in which the \( s \)-wave scattering length diverges, the imaginary part of the self-energy is non-negligible. Therefore, we rewrite equation (4) as

\[
\omega_{\text{pol}} + \mu_\perp = \omega_{\text{pol}} - i\Gamma,
\]

(5)

with the decay rate \( \Gamma \in \mathbb{R} \). Here, \( \omega_{\text{pol}} \) and \( \Gamma \) are related to the self-energy as

\[
\omega_{\text{pol}} = \text{Re} \Sigma_\sigma(\mathbf{p} = 0, \omega_{\text{pol}} - \mu_\perp - i\Gamma + i\delta),
\]

\[
\Gamma = -\text{Im} \Sigma_\sigma(\mathbf{p} = 0, \omega_{\text{pol}} - \mu_\perp - i\Gamma + i\delta).
\]

(6)

(7)

By solving the above two equations, we can obtain \( \omega_{\text{pol}} \) and \( \Gamma \), respectively. In addition, the chemical potential \( \mu_\sigma \) is obtained from the so-called number equation

\[
n_\sigma(\mu_\sigma) = T \sum_{\mathbf{p}, \omega_\parallel} G_\sigma(\mathbf{p}, \omega_\parallel),
\]

(8)

where \( n_\sigma \) represents the particle density of atoms with the state \( \sigma \). In this work, we define the impurity concentration \( y \) as \( y = n_\downarrow/(n_\perp + n_\downarrow) \).

To obtain a reasonable self-energy, we use many-body \( T \)-matrix theories, which are known to reproduce fundamental properties in spin-balanced \([49–52]\) and polaron limits \([32–35]\). The simplest type of the \( T \)-matrix theories is the non-selfconsistent approximation whose self-energy is composed of the bare Green’s function. However, such an approximation does not contain an interaction between impurities, which is inevitable to
discuss the finite impurity concentration case. To overcome the drawback of the non-selfconsistent approximation, we adopt an extended $T$-matrix approximation (ETMA) \[33-37\], which contains the interaction between impurities (see figure 1) and therefore meets the purpose of the paper. In this formalism, as diagrammatically shown in figure 1(a), the self-energy $\Sigma_{\sigma}(\mathbf{p}, i\omega_n)$ is given by

$$\Sigma_{\sigma}(\mathbf{p}, i\omega_n) = T \sum_{\mathbf{q}, \nu_n} f(\mathbf{q}, i\nu_n) G_{\sigma}^{-1}(\mathbf{q} - \mathbf{p}, i\nu_n - i\omega_n), \tag{9}$$

where

$$f(\mathbf{q}, i\nu_n) = \frac{g}{1 + g\chi(\mathbf{q}, i\nu_n)}, \tag{10}$$

is the many-body $T$-matrix ($\nu_n = 2n\pi T$ is the bosonic Matsubara frequency). In equation (10), the lowest-order-pair-correlation function $\chi(\mathbf{q}, i\nu_n)$ is given by

$$\chi(\mathbf{q}, i\nu_n) = T \sum_{\mathbf{p}, \omega_\nu} G_{\beta}^{\sigma}(\mathbf{p} + \mathbf{q}, i\omega_\nu + i\nu_n) G_{\sigma}^{\beta}(\mathbf{p}, -i\omega_\nu)$$

$$= \sum_{\mathbf{p}} \frac{1 - f(\xi_{\mathbf{p} + \mathbf{q}, \nu_n}) - f(\xi_{\mathbf{p}, -\nu_n})}{\xi_{\mathbf{p} + \mathbf{q}, \nu_n} + \xi_{\mathbf{p}, -\nu_n} - i\nu_n}, \tag{11}$$

where $f(x) = 1/(e^{x/T} + 1)$ is the Fermi distribution function. In equation (11), $G_{\sigma}^{\beta}(\mathbf{p}, i\omega_\nu) = 1/(i\omega_\nu - \xi_{\mathbf{p}, \sigma})$ is the bare Green’s function. Physically, $f(\mathbf{q}, i\nu_n)$ describes superfluid fluctuations in the particle–particle channel [31]. Since the dressed Green’s function $G_{\sigma}$ in equation (9) (or figure 1(a)) involves the self-energy $\Sigma_{\sigma}$, the polaron–polaron interaction process described by figure 1(b) is automatically included in the self-energy of minority atoms $\Sigma_{\sigma}$. We note that $\Sigma_{\sigma}(\mathbf{p}, i\omega_n)$ is numerically obtained by self-consistently solving equation (9) with calculating $\mu_s$ from equation (8), as shown in figure A1.

Recently, it was shown that the ETMA well reproduces thermodynamic properties in spin-balanced systems [38, 39]. In what follows, we demonstrate that the ETMA also provides reasonable results on spectral properties in the polarized system such as the polarons. In this work, we focus on the relevant parameter regimes to the recent experiments. After discussing the comparison between our results and the previous works of experiments as well as theories at the low temperature and impurity density regime, we clarify effects of finite temperature and impurity density.

### 3. Result

We first show how our many-body $T$-matrix theory works well even in the zero impurity density limit at the low temperature through a comparison between our numerical results and the recent experimental measurements [16] as well as previous theoretical studies. In our formalism, the zero impurity density limit is achieved by putting the large chemical potential difference $\mu_s - \mu_g$ such that the impurity concentration $\gamma = n_i/(n_i + n_g) \lesssim 10^{-3}$ is enough small. The left panel of figure 2 shows the attractive or repulsive polaron energy $\omega_{\text{pol}}(\mathbf{q})$ as a function of inverse scattering length $(k_i a_i)^{-1}$ with the Fermi momentum of majority atoms $k_i$. In our calculation, the temperature is fixed at $T = 0.03T_F$ (where $T_F$ is the Fermi temperature of majority atoms). Our results show good agreements with recent experimental results in $^6$Li Fermi gases [16]. We note that while the experiment [16] has been done at a bit higher impurity density and higher temperature compared with our theoretical input, the differences do not lead to significant consequences on the polaron energy as discussed below. In addition, in the zero impurity density limit, the

![Figure 1](image_url)
ETMA reduces to the non-selfconsistent $T$-matrix approximation, which is known to describe polaron properties quantitatively, since the majority one-particle Green's function $G_{ij}^{p}(p, \omega, n)$ in the ETMA reduces to non-interacting one $G_{ij}^{p,0}(p, \omega, n) = 1/(\omega + \xi_{p,\omega})$ in the zero impurity density limit. Thus, our approach based on the ETMA turns out to be a natural extension of the non-selfconsistent $T$-matrix approximation with a single impurity to discuss finite temperature and density in the Fermi polaron system.

Our result of the effective mass $m^*$ subtracted from $G_{ij}^{p}(p, \omega, i\delta)$ near the single impurity limit is shown in the right panel of figure 2 and is consistent with the previous work [11]. The small difference between the previous and our works comes from the finite temperature effects as shown in the inset of the right panel of figure 2. It is quite natural that $m^*$ decreases with increasing the temperature since the temperature effects gradually suppress the interaction effects. This is the reason why our calculated $m^*$ at $T = 0.03T_F$ is smaller than that of the previous work obtained at $T = 0$. On the other hand, the experimental results [16] show heavier effective masses than our evaluation in spite of the fact that the experimental temperature $T = 0.1T_F$ is higher than our case. We also numerically checked that the effect of a finite impurity density does not lead such significant difference. The large mass renormalization in the recent experiment [16] cannot be explained by finite temperature or impurity density effect by means of the ETMA.

The left panel of figure 3 shows the residue $Z$ of minority Green's function at $\omega = \omega_{pole}$, which is calculated as

$$Z^{-1} = -\frac{\partial}{\partial \omega} G_{ij}^{-1}(p = 0, \omega, i\delta) \bigg|_{\omega = \omega_{pole}}. \tag{12}$$

Our results of $Z$ for attractive and repulsive polarons show good agreement with the theoretical study based on the functional renormalization group (FRG) at $T = 0$ [36], which non-perturbatively involves higher order corrections such as three-body process. From this comparison, one can find that the residue $Z$ is essentially described well by the ladder–approximation scheme at the single–impurity limit.

However, the decay rate of repulsive polarons $\Gamma$ obtained from equations (6) and (7) is generally smaller compared to FRG results [36] since our calculation does not incorporate the effect of three-body decay associated with atom–dimer scatterings [60] as well as the decay to attractive polarons, which can be considered by replacing $G_{ij}^{0}$ in $\chi(q, i\omega_n)$ with dressed one $G_{ij}^{0}$ [11]. Since $G_{ij}^{0}$ is concerned, the ETMA may reproduce the decay rate of polaron-to-bare-atom transition $\Gamma_{PP}$ rather than that of polaron-to-polaron transition $\Gamma_{PP}$ calculated in the previous work at the single-impurity limit with exactly $T = 0$ [11]. Although the correct physical process may be the latter, the former is closer to the experimental result. In addition, our result involves finite temperature effects which enhance the decay of the quasi-particles [47], which is visible in the weak repulsive interacting regime where the collisional effects are relatively small.

We next look at how impurity concentration $y$ affects the chemical potential $\mu_y$. We note that $\mu_y = \xi_F$ in the single impurity case at $T = 0$. However, as shown in figure 4, $\mu_y$ deviates from the Fermi energy and decreases with increasing $y$ due to the self-energy shift $\text{Re} \Sigma_{ij}(p, \omega + i\delta)$ associated with the strong pairing interaction. This renormalization effect on majority atoms becomes more remarkable when the pairing interaction gets
stronger. Furthermore, the shifts of \( \mu_{\sigma} \) are not explained by the simple mean-field shift \( S = s_{p} - n_{m} M_{F} \), since the scattering length \( a_{s} \) diverges near the unitarity limit. However, the shift of \( m \uparrow \) is proportional to \( n \downarrow \) even at \( y = 0 \). By using the linear fitting with respect to \( n \downarrow \) in the small impurity density regime \((y < 0.2)\), we obtain

\[
\mu_{\uparrow} = \frac{\varepsilon_{F}}{E_{F}} \left[ 1 - 0.526 \frac{n_{\downarrow}}{n_{\uparrow}} \right] \equiv \varepsilon_{F} \left[ 1 - 0.526 \frac{y}{1 - y} \right] \quad \quad (13)
\]

Surprisingly, as pointed out in [61], this shift is the same-order of the mean-field shift with \( a = 1/k_{F} \) given by

\[
\Sigma_{MF}(a = 1/k_{F}) = \frac{4\pi}{mk_{F}} n_{\uparrow} \simeq 0.424 \frac{n_{\downarrow}}{n_{\uparrow}} \varepsilon_{F}. \quad (14)
\]

Since the chemical potential plays a crucial role in the thermodynamics of a unitary Fermi gas in which \( \mu_{\sigma} / \varepsilon_{F} \) in the unpolarized case takes a universal constant called Bertsch parameter [62], we expect that the origin of prefactor 0.526 in the second term of the right hand side of equation (13) would be important in terms of the thermodynamics of the many polarons. We emphasize that these renormalization effects cannot be captured with single-impurity theories. The renormalization is of the order of a tenth of the Fermi energy in the typical
cold-atom experiments whose impurity concentration is 0.1 to 0.3. We expect that such a significant shift can be measured with the state-of-the-art precision thermodynamic measurement [59].

The inset of figure 4 shows the impurity chemical potential $\mu_\downarrow$, which monotonically increases with increasing $y$ and decreases with increasing the interaction strength. At the zero temperature, $\mu_\downarrow$ is equivalent to the attractive polaron energy $\omega_{qp}$ at $y \to 0$, since $\mu_\downarrow = E_{N_\downarrow=1} - E_{N_\downarrow=0}$ is defined as the energy needed to add an impurity with zero momentum to the system where $E_{N_\downarrow} (N_\downarrow \in \mathbb{Z})$ is the energy in the presence of $N_\downarrow$ impurities.

Indeed, this definition is equivalent to $\mu_\downarrow = \left( \frac{\partial E}{\partial n_\downarrow} \right)_S$ at the thermodynamic limit, where $E$ and $S$ are the internal energy and entropy, respectively. At a finite temperature, however, we have to carefully notice the difference between $\mu_\downarrow$ and $\omega_{qp}$. An important point is that at a finite temperature there is the contribution from thermal excited states with nonzero momenta in addition to one from the ground state with the zero momentum. Figure 5(a) shows the impurity chemical potential $\mu_\downarrow$ and $\omega_{qp}$ of the unitarity limit as a function of $y$ at several temperatures. In general, $\mu_\downarrow$ is smaller than $\omega_{qp}$ in the small impurity density region ($y \approx 0$). In addition, $\mu_\downarrow$ decreases with increasing the temperature, whereas $\omega_{qp}$ slightly shifts due to the temperature effects. Except for the strong coupling regime beyond polaron-molecule (or polaron-BEC) transition, the number equation of impurities for $\mu_\downarrow$ can approximately be given by

$$n_\downarrow \approx \sum_p Z_p f \left( \frac{p^2}{2m_\downarrow} - \mu_\downarrow + \omega_{qp}^2 \right),$$  

where $Z_p$ and $m_\downarrow^e$ are the residue and effective mass of an attractive polaron, respectively. For simplicity, we neglect the decay rate of an attractive polaron as well as the repulsive branch. At $T = 0$, the solution of equation (15) for the low impurity density limit ($n_\downarrow \to 0$) is apparently $\mu_\downarrow = \omega_{qp}$ since the Fermi distribution function $f(x)$ becomes a step function $\theta(-x)$. On the other hand, at finite temperature, such solution have to be $\mu_\downarrow \to -\infty$ because the summation over momenta in equation (15) involves the contribution from high momentum region associated with the finite temperature. This large negative $\mu_\downarrow$ reflects the fact that a few
polarons at finite temperature behave as a classical Boltzmann ensemble. Indeed, if one measures the temperature by using the Fermi temperature of impurities $T_{Fi}$, one can obtain

$$\frac{T}{T_{Fi}} = \left(\frac{n_l}{n_i}\right)^2 \frac{T}{T_F},$$

(16)

which diverges in the limit of $n_l \to 0$ with fixed $T/T_F$. In contrast, the region where $\mu_1 > \omega_{qp}$ at the large impurity density can be regarded as the Fermi degenerate regime of attractive polarons. In this case, they make a soft Fermi surface with the effective Fermi energy $\varepsilon_{Fi} = \mu_1 - \omega_{qp}$. To access such a regime, the temperature must be much smaller than $T_{Fi} = (n_l/n_i)^2 T_F$. In figure 6, we summarize the different regimes in the Fermi polaron system. We also note that the curves shown in figure 6 are shifted below if the effective mass is considered, since $T_{Fi}$ is generally in inverse proportion to the effective mass.

We note that in contrast to $\mu_1$, the spectral property of the attractive polaron at the single-impurity limit is relatively robust against the finite temperature effects, since it is related to the thermodynamic property of majority atoms. At $n_l \to 0$ where $G_1(p, \omega + i\delta) \approx \delta(\omega - \xi_{p,1})$, the self-energy of impurities after the analytic continuation is given by

$$\Sigma_i(p, \omega + i\delta) = \sum_q \int_{-\infty}^{\infty} \text{d}v A_i(q, v) \frac{b(v) + f(\xi_{q-p,1})}{\omega + i\delta + \xi_{q-p,1} - \nu},$$

(17)

where $A_i(q, v) = -\frac{1}{\pi} \text{Im} \tau(q, v_n \to \nu + i\delta)$ is the spectral function of a diatomic pair and $b(v) = 1/(e^{\varepsilon/v} - 1)$ is the Bose distribution function. The finite temperature effects in equation (17) originate from mainly $f(\xi_{q-p,1})$ and $\mu_1 \approx \varepsilon_{Fi} \left[1 - \frac{\pi^2}{12} \left(\frac{T}{T_{Fi}}\right)^2\right]$ [4] (see figure 5(b)) far away from the BEC critical point of molecules. In this way, one can find that spectral polaron properties such as $\omega_{qp}$ determined by equation (4) is deeply related to how majority fermions are affected by the temperature. We also note that the large negative $\mu_1$ does not notably affect $\Sigma_i(p = 0, \omega + i\delta)$ since $\mu_1$ in equation (17) is included in only the molecular branch $A_i(q, v)$.

A renormalization of majority atoms is also visible in the spectral function $A_1(p = 0, \omega)$. In figure 7, we show the spectral function at $v = 4 \times 10^{-4}, 0.18$ and $0.26$ at $(k_0a)^{-1} = 0.2$. It turns out that the stable pole position shifts toward the lower energy with increasing $v$ due to the shift of $\mu_1$. From equation (3), the shift of the peak in figure 7 is directly related to the change of the self-energy of majority atoms as given by Re $\Sigma_i(p = 0, \omega + i\delta)$. This is nothing but the renormalization effect of majority atoms. In addition, we find that a metastable peak associated with the upper branch appears at finite impurity concentration even in the spectral function of majority atoms. The presence of such a peak originates from the upper peak of the minority Green’s function that is explicitly contained in the self-energy of majority atoms. We also confirm that the metastable-peak structure is enhanced in the vicinity of the strong coupling limit. By considering that the intensity of such an upper peak in the majority spectral function is comparable to that in minority spectral function, its experimental validation with rf spectroscopy is promising.
On the other hand, in contrast to majority atoms, the shift of the spectral function $A_i(p = 0, \omega)$ of minority atoms by the finite density is small as shown in the inset of figure 7. In figure 8(a), we show impurity concentration dependence of the attractive polaron energy $\omega_{qp}^a$ obtained from equation (4) at several interaction strengths. We find that $\omega_{qp}^a$ is almost independent of $\gamma$ from the weak coupling region to unitary region. However, in the strong coupling region $[(k_F a_i)^{-1} = 0.4$ in figure 8(a)], the polaron energy turns to slightly increase with increasing $\gamma$. We argue that this indicates the presence of the polaron–polaron interaction, which is indeed known to be positive by means of the Fermi liquid theory [43, 44, 47]. One can interpret that the polaron–polaron interaction effect is visible due to the increase of pairing interaction that overcomes the finite temperature effect. Indeed, the increase of $\omega_{qp}^a$ at $(k_F a_i)^{-1} = 0.4$ starts around $\gamma \simeq 0.2$, where $T/T_F \lesssim 0.1$ estimated by equation (16) and the attractive polarons are in the deep quantum degenerate regime. In this sense, the precise determination of $\mu_i$ is very important even from such viewpoint for the polaron–polaron interaction.

In figure 8(b), we show the calculated repulsive polaron energy $\omega_{qp}^r$, as a function of $\gamma$ in the strong coupling region $[(k_F a_i)^{-1} = 0.4, 0.8$ and 1.2]. In addition, the inset of figure 8(b) is the comparison between $\gamma$-dependence of attractive and repulsive polaron energies at $(k_F a_i)^{-1} = 0.8$, where we set an offset $(=2.5\varepsilon_F)$ on the attractive polaron energy. These results indicate that the repulsive polaron energy does not represent any noteworthy behavior related to the polaron–polaron interaction, which is consistent with the recent experiment [16]. While solely from our numerical data it is difficult to pinpoint the reason of the difference from the prediction of the Fermi liquid theory, the followings could be conceivable: (i) smallness of the polaron–polaron interaction due to the Pauli blocking, (ii) short lifetime of the repulsive polaron (typically of the order of the Fermi time), (iii) finite temperature effect as is the case with attractive polaron.

We note that we stop the calculations of $\omega_{qp}^r$ at the superfluid instability point, which can be identified by the so-called Thouless criterion [63],

$$\tau(q = 0, \mathbf{v}_\text{F} = 0)^{-1} = 0. \quad (18)$$

At the fixed temperature, the Thouless criterion is more likely to be satisfied in the regime $(k_F a_i)^{-1} \gtrsim 0$, where the transition temperature of the superfluid is higher and increases with increasing $\gamma$. To correctly describe the superfluid phase transition in a strongly interacting spin-imbalanced Fermi gas, we have to consider the existence of the first order phase transition and the phase separation [64, 65]. In this paper, we avoid such a regime by focusing on lower impurity concentration. We also note that although the realization of the Fulde–Ferrell–Larkin–Ovchinnikov state [66, 67] has been predicted in a uniform polarized Fermi gas [64], such an exotic superfluid state is known to be unstable against superfluid fluctuations [68, 69] (note however [70]).

Furthermore, to address the more detailed experimental situation, we consider the effect of initial state momentum of impurities in the rf spectrum measurement [16]. We first estimate the averaged momentum of impurities $\bar{p}$ by assuming that the initial state is a non-interacting uniform Fermi gas. The thermal average of the impurity energy $\bar{\varepsilon}$ is defined as
where $\mu_i$ is the chemical potential of the initial state impurities, obtained by the solving

$$\bar{\varepsilon} = \frac{1}{n_i} \sum_p \frac{p^2}{2m} \left( \frac{p^2}{2m} - \mu_i \right).$$

From the above equations, we can obtain $\bar{\rho} = \sqrt{2m\bar{\varepsilon}}$. Figure 9 shows the impurity concentration dependence of $\bar{\rho}$. In the relevant region of the experimental impurity density ($0.1 \leq y \leq 0.3$) and temperature ($T \approx 0.1 T_F$), it is quite small compared to the trapped case reported in the supplemental material of [16]. In the presence of $\bar{\rho}$, the repulsive polaron energy is obtained from

$$\omega_{\rho q}^{\rho}(\bar{\rho}) - i\gamma = \Sigma_{\rho}(\bar{\rho}, \omega_{\rho q}(\bar{\rho}) - \mu_i - i\gamma + i\delta).$$

The dashed line in figure 8 shows calculated $\omega_{\rho q}^{\rho}(\bar{\rho})$ at $(k_Fa_s)^{-1} = 0.4$. As expected, the finite $\bar{\rho}$ leads to the negative shift of $\omega_{\rho q}^{\rho}(\bar{\rho})$ compared to $\omega_{\rho q}(\bar{\rho} = 0)$. In the experimental paper, it is estimated that this negative shift is given by $-\left(1 - \frac{m}{M} \right)\varepsilon$ with $\varepsilon = O(10^{-1}e_F)$ [16]. However, in our case, $\varepsilon$ is smaller than $10^{-2}e_F$ in the relevant region, and the estimated shift is also smaller than $O(10^{-2}e_F)$. This result indicates the importance of effects of a harmonic trap potential to see the mass renormalization effects from the $y$-dependence of polaron energies. Since the harmonic trap enhance the finite temperature effects due to the inhomogeneous density profile [57], it may also be related to the suppression of effects of polaron–polaron interaction in the experiment.
4. Conclusion

We have theoretically investigated Fermi polarons at finite impurity concentration and finite temperature within the framework of the many-body $T$-matrix theory, which can also describe polaron properties in the zero impurity density and zero temperature limits. Our results show quantitative or semi-quantitative agreement with current experiments as well as previous works based on single polaron theories at zero temperature.

We have pointed out that majority atoms are affected by the strong pairing interaction with impurities. In particular, we have showed the renormalization effects on the chemical potential as well as quasiparticle spectral function of majority atoms. In the case of minority atoms, the finite temperature effects play a crucial role in their thermodynamic properties such as chemical potential. It is also related to the quantum degeneracy of attractive polarons, which leads to the competition between finite temperature effects and the polaron–polaron interaction. The renormalization of the majority chemical potential and the thermal depletion of minority chemical potential can be observed by recent precise thermodynamic measurements. In addition, we have predicted the appearance of the metastable peak in the high-energy region of majority spectral function. A detailed study on such a metastable many-body state is an interesting future work. Also, metastable peak structure in the spectral function of majority atoms can be detected by rf spectrum measurements.

We have also extracted the polaron energy as a function of impurity concentration to discuss the polaron–polaron interaction. We have found that in the strong coupling region at a low temperature, although the polaron–polaron interaction is visible in the lower branch, this effect is much weaker in the upper branch. In addition, we also have clarified that the mass-renormalization effect on the polaron energy in the uniform case is smaller compared to the case of trapped gas clouds, by considering the initial state momentum of impurities.

In this paper, we have emphasized that these many-body effects in the polaron problem at finite temperature and finite impurity density are beyond previous single impurity theories. While our result successfully reproduces experimental results in several regimes and predict the polaron properties which no one has reported, we found that there are still differences between theories and experiments with respect to the effective mass as well as the decay rate somehow beyond finite temperature and impurity density effects, which remain as our important future problem. In particular, an effect of a harmonic trap is important to compare our results with the observed rf-spectra [16] in detail, and our present work can include such effects by employing the local density approximation [37]. It is also interesting to extend our analyses to mass-imbalanced [10] and two-dimensional systems [12] already realized in ultracold Fermi gases.

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Appendix. Analytic continuation

In general, the analytic continuation is sensitive to noises, and theoretical approaches with statistical errors such as Monte Carlo methods suffers from this procedure from the imaginary time \( \tau \) to the real frequency \( \omega \) \cite{42, 71}. On the other hand, the ETMA used in this work is free from statistical errors, and therefore we can implement the conventional numerical continuation methods. In this work, we adopt the Padé approximation with enough small \( \delta \) in equation (2) \cite{72} to examine the spectral structure in the Fermi polaron system.

In our case, the self-energy has been already calculated in the complex energy plane in terms of the Matsubara frequency located at imaginary energy axis. It is known that the Padé approximation is applicable to the conventional numerical continuation methods. In this work, we adopt the Padé approximation with enough small \( \delta \) in equation (2) \cite{72} to examine the spectral structure in the Fermi polaron system.

In the complex energy plane in terms of the Matsubara frequency located at imaginary energy axis. It is known that the Padé approximation is applicable to the conventional numerical continuation methods. In this work, we adopt the Padé approximation with enough small \( \delta \) in equation (2) \cite{72} to examine the spectral structure in the Fermi polaron system.

Figure A1. Calculated impurity self-energy \( \Sigma (p=0, i\omega_n)/e_F \) at \( T = 0.05T_F, y = 0.12 \), and \( (k_i a)^{-1} = 0 \) and the comparison with the interpolated result obtained by the Padé approximation, where original is the self-energy from equation (9) without the interpolation.

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