GRAVITATIONAL WAVE RECOIL OSCILLATIONS OF BLACK HOLES: IMPLICATIONS FOR UNIFIED MODELS OF ACTIVE GALACTIC NUCLEI

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ABSTRACT

We consider the consequences of gravitational wave recoil for unified models of active galactic nuclei (AGNs). Spatial oscillations of supermassive black holes (SMBHs) around the cores of galaxies following gravitational wave (GW) recoil imply that the SMBHs spend a significant fraction of time off-nucleus, at scales beyond that of the molecular obscuring torus. Assuming reasonable distributions of recoil velocities, we compute the off-core timescale of (intrinsically type 2) quasars. We find that roughly one-half of major mergers result in a SMBH being displaced beyond the torus for a time of \(10^{7.5}\) yr or more, comparable to quasar activity timescales. Since major mergers are most strongly affected by GW recoil, our results imply a deficiency of type 2 quasars in comparison to Seyfert 2 galaxies. Other consequences of the recoil oscillations for the observable properties of AGNs are also discussed.

Subject headings: galaxies: active — galaxies: evolution — quasars: general

1. INTRODUCTION

Gravitational waves emitted anisotropically during gravitational collapse carry away linear momentum. As a result, the center of mass of the collapsing object recoils (Peres 1962). Configurations of coalescing spinning black holes can result in recoil velocities of hundreds to thousands of km s\(^{-1}\) (e.g., Campanelli et al. 2007; González et al. 2007; Herrmann et al. 2007; Pollney et al. 2007; Brügmann et al. 2008; Dain et al. 2008), scaling to a maximum of \(4000\) km s\(^{-1}\) (Campanelli et al. 2007; Baker et al. 2008) for maximally spinning equal-mass binaries with antialigned spins in the orbital plane, and as large as \(10^4\) km s\(^{-1}\) in hyperbolic encounters (Healy et al. 2008). Kicks large enough to remove SMBHs from galaxies have potentially far-reaching consequences for SMBH and galaxy assembly, and predict interstellar and intergalactic quasars (e.g., Madau et al. 2004; Merritt et al. 2004; Madau & Quataert 2004; Haiman 2004; Boylan-Kolchin et al. 2004; Loeb 2007; Volonteri 2007; Gualandris & Merritt 2008).

Komossa et al. (2008a) reported the detection of a recoil candidate with a projected kick velocity of \(2650\) km s\(^{-1}\), based on a kinematically offset broad-line region (BLR), and a system of very narrow emission lines which lack the usual ionization stratification. Apart from spectroscopic signatures (Merritt et al. 2006; Bonning et al. 2007), offset quasars could be detected by their temporarily flaring accretion disks (Shields & Bonning 2008; Lippai et al. 2008; Schmittman & Krolik 2008), by tidal disruption flares from the bound (and unbound) population of stars (Komossa & Merritt 2008), and via the bound compact star cluster itself (Merritt et al. 2008). One key consequence of gravitational wave recoil is long-lasting oscillations of the SMBH around the galaxy core, implying that SMBHs may spend as long as \(10^9\) yr off-nucleus, with an amplitude of parsecs or kiloparsecs (Merritt et al. 2004; Madau & Quataert 2004; Gualandris & Merritt 2008, hereafter GM08). In this Letter, we consider consequences of these “recoil oscillations” for unified models of active galactic nuclei (AGNs).

According to unified models, AGNs are intrinsically similar, but their appearance depends strongly on the line of sight of the observer toward the “central engine” (reviews by Antonucci 1993; Elitzur 2007). Along certain sight lines, a dusty torus consisting of molecular clouds blocks the observer’s view, hiding some core components, especially the BLR, which therefore is only directly visible in “type 1” AGNs. The unified model has been very successful in explaining observed properties of AGNs, including the presence of hidden BLRs detected in polarized light (e.g., Antonucci & Miller 1985; Zakamska et al. 2005), and has been corroborated by recent imaging and spectroscopy of the torus (e.g., Jaffe et al. 2004).

If SMBHs and the BLRs bound to them spend a significant time displaced from the nucleus, this will have profound consequences for obscuration-based unified models of AGNs. This statement holds true whether the obscuration originates in parsec-scale molecular tori (Antonucci 1993) or compact star-forming regions (Levenson et al. 2001), or is due to absorption associated with the host galaxy itself, which may work on scales of \(\sim 100\) pc (Maiolino & Rieke 1995). Here we argue that a significant fraction of the quasar population is expected to be in a regime such that the SMBH and the BLR bound to it is displaced beyond the obscuring region, implying a deficiency of type 2 (obscured) AGNs among quasars.

2. RELEVANT SPATIAL SCALES IN THE AGN CORE

After the kick, matter remains bound to the SMBH within a region whose radius \(r_k\) is given by

\[
 r_k = \frac{GM_{\text{BH}}}{v_k^2} \approx 0.43 \left(\frac{M_{\text{BH}}}{10^8 M_\odot}\right) \left(\frac{v_k}{10^3 \text{ km s}^{-1}}\right)^{-2} \text{ pc},
\]

where \(v_k\) is the kick velocity (e.g., Merritt et al. 2006). The size \(r_{\text{BLR}}\) of the BLR of AGNs has been determined from re-
vibration mapping (Peterson 2007), and scales with AGN luminosity as

\[ r_{BLR} = 0.1 \left( \frac{\lambda L_{\lambda}(5100 \, \text{Å})}{10^{45} \text{ ergs s}^{-1}} \right)^{0.69} \text{ pc} \]  

(Kaspi et al. 2005), where the luminosity at 5100 Å, \( \lambda L_{\lambda}(5100 \, \text{Å}) = 0.1 L \) and \( L \) is the AGN bolometric luminosity. The size of the molecular torus is still poorly constrained from observations. Recent measurements of dusty gas in the Seyfert galaxies NGC 1068 and Circinus suggest an extent of 2–3 pc (Jaffe et al. 2004; Davies et al. 2007; Tristram et al. 2007). It is reasonable to assume that the inner edge of the torus, \( r_{\text{tor, in}} \), is beyond the dust sublimation radius (e.g., Netzer 1990; Nenkova et al. 2008), which is given by

\[ r_{\text{tor, in}} \approx 0.4 \left( \frac{L}{10^{45} \text{ ergs s}^{-1}} \right)^{1/2} \left( \frac{1500 \, \text{K}}{T_{\text{sub}}} \right)^{2.6} \text{ pc}, \]  

with an outer radius likely not much larger than \( \sim 20 \, r_{\text{tor, in}} \) (Elitzur 2007), and where \( T_{\text{sub}} \) is the dust sublimation temperature. Comparison of these three relations shows that a large fraction of the BLR remains bound to the recoiling hole, while structures of the size of the torus or larger will typically be left behind. Oscillation amplitudes of \( \pm 10–20 \) pc will therefore move the SMBH beyond the torus scale, except for the highest SMBH masses, where part of the torus will remain bound.

3. OSCILLATION AMPLITUDES AND OSCILLATION DURATIONS IN QUASARS

We base our discussion on the \( N \)-body simulations of GM08, who computed SMBH trajectories after GW recoil. Key model parameters are the SMBH mass \( M_{\text{BH}} \) and galaxy mass \( M_{\text{gal}} = 10^9 M_{\odot} \), the kick velocity \( v_k \), and the galaxy structural parameters. We concentrate here on the mass range of SMBHs that is typical for the bulk of quasars (\( M_{\text{BH}} \approx 10^4 \) to a few \( \times 10^5 M_{\odot} \)). We adopt the relation between effective radius \( R_e \) and \( M_{\text{gal}} \) defined by the nine luminous (\( \sim 22 \leq M_r \leq \sim 24 \)) quasar host galaxies in the sample of Wolf & Sheinis (2008), i.e., \( \log (R_e/\text{kpc}) \approx -5.61 + 0.55 \log (M_{\text{gal}}/M_{\odot}) \). The models of GM08 on which we base our discussion had postkick core radii \( r_c \), consistent with the range defined by the brightest E galaxies with resolved cores (Ferrarese et al. 2006), i.e., \( 0.01 \leq r_c/R_e \leq 0.03 \). We focus here on the spherical “A1” galaxy models from GM08; scaling of the \( N \)-body results to physical units was done following their equation (4). Dark matter halos were ignored.

Figure 1a shows trajectories of kicked SMBHs scaled to a galaxy with SMBH mass \( 5 \times 10^4 M_{\odot} \). Kick velocities were \( v_k/v_{\text{esc}} = 0.3, 0.5, 0.7, 0.9 \), corresponding to \( v_k = (360, 590, 830, 1070) \) \( \text{km s}^{-1} \) for a central escape velocity \( v_{\text{esc}} = 1185 \) \( \text{km s}^{-1} \) of our galaxy model. As discussed by GM08, SMBH oscillations continue well beyond the time (“phase I”) that would be predicted by applying Chandrasekhar’s dynamical friction formula assuming a fixed galaxy core (e.g., Madau & Quataert 2004; Blecha & Loeb 2008). The time of onset of these long-term, or “phase II,” oscillations is indicated by the open circles in Figure 1a. Including the effect of the phase II oscillations, Figure 1a shows that the SMBH’s motion persists for more than \( \sim 10^7 \) yr if \( v_k \approx 450 \) \( \text{km s}^{-1} \).

4. COMPARISON WITH OBSERVATIONS

4.1. Unified Models and Type 1/Type 2 Fractions

About 70% of the nearby Seyfert galaxies are type 2 (e.g., Schmitt et al. 2001); i.e., they lack a BLR in their (unpolarized) optical spectra. The fraction of the high-luminosity equivalents, type 2 quasars, is less well known and still subject to a number of selection effects (e.g., Halpern & Moran 1998; Reyes et al. 2008), even though studies generally indicate a deficiency of type 2 quasars. While in past studies type 2 quasars were ob-
servationally very rare or absent, significant numbers have been found in recent X-ray and optical surveys (e.g., Norman et al. 2002; Zakamska et al. 2005; Brusa et al. 2007). There is a systematic trend such that the fraction of type 2 sources, or equivalently the amount of X-ray absorption, decreases with increasing source luminosity (e.g., Simpson 2005; Barger et al. 2005; Reyes et al. 2008; Hasinger 2008), while the simplest possible version of the unified model would imply a constant type 1/type 2 ratio. Models have been proposed which account for this luminosity dependence, e.g., by invoking changes in the properties of the obscurer as L increases (e.g., Nenkova et al. 2008; Ballantyne 2008). Recoil oscillations inevitably affect the numbers of obscured versus unobscured sources and therefore have potentially profound implications for unified models. How do they affect the ratio of type 1/type 2 quasars in comparison to the number of type 1/type 2 Seyfert galaxies? There is increasing evidence that quasar activity is powered by major mergers while Seyfert activity may have other triggers including bars, minor mergers, or random accretion of molecular clouds (e.g., Sanders et al. 1988; Urrutia et al. 2008; Hopkins et al. 2008; Hasinger 2008 and references therein). Recoil oscillations have highest amplitudes in major mergers, and we may therefore expect that the frequency and properties of (type 2) quasars are strongly affected by recoil oscillations. Can this explain the relative scarcity of type 2 quasars in comparison with type 2 Seyfert galaxies, and the more general trend that type 2–ness (X-ray absorption) decreases with luminosity?

4.2. Rate Estimates

Since a large fraction of quasars are believed to be triggered by major mergers, in order to estimate the fraction of quasars that occur at or above a given kick velocity, we carried out rate estimates relevant for major mergers with random spin distributions (Campanelli et al. 2007; Schnittman & Buano 2007; Baker et al. 2008; note that the actual kick velocities could be smaller in gas-rich systems if the mechanism discussed by Bogdanović et al. 2007 is at work, but see § 3 of Schnittman & Krolik 2008). We first updated these previous estimates, based on the recent kick formula of Baker et al. (2008; essentially identical results were obtained using the Lousto & Zlochower 2008 version of the kick formula), and assuming random orientations of the spin vectors of both SMBHs and a distribution of SMBH mass ratios in the range $0.3 \leq q \leq 1.0$ where $q \equiv m_2/m_1$, $m_2 \leq m_1$ relevant for major mergers. SMBH spins were drawn from a distribution such that $a_1 \leq a \leq a_2$ with $a \equiv S/m^2$ the dimensionless spin, with $a_1 = 0.5$, $a_2 = 0.9$. The distributions of $m$ and $a$ were assumed to be uniform in the logarithm between these limits. For this base model, we find that about 50% of major mergers have kick velocities above 500 km s$^{-1}$. At or above these kick velocities, total oscillation timescales start to be on the order of quasar lifetimes, which are about $10^9$–$10^{9.5}$ yr (e.g., Yu & Tremaine 2002; Hopkins et al. 2005; see review by Martini 2004).

How much of the total quasar population is ultimately affected by recoil oscillations then depends on (1) the time $t_{\text{vis}}$ the kicked SMBH + BLR spends beyond the obscuring torus (i.e., at distances greater than $r_{\text{torus}} \approx r_{\text{torus, out}}$ from the galaxy center (eq. [3]) and therefore appears as “type 1” rather than “type 2,” in comparison to (2) the total quasar lifetime. We computed $t_{\text{vis}}$ on a grid in $r_{\text{torus, out}}$ for each of the nine N-body trajectories in GM08 ($0.1 \leq v_{\text{esc}}/H_{\text{esc}} \leq 0.9$). Given arbitrary values of $v_{\text{esc}}$, $M_{\text{gal}}$, and $r_{\text{torus}}$, the value of $t_{\text{vis}}$ was then computed via interpolation between the nine discrete kick velocities of GM08.

Figure 1b shows, for our base model, the fraction $f$ of kicked SMBHs that spend more than $3 \times 10^7$ yr at $r > r_{\text{torus}}$, for a range of $(M_{\text{gal}}, r_{\text{torus}})$ values. For $r_{\text{torus}} \approx r_{\text{torus, out}}$, this fraction is $0.5 \pm 0.1$ with a weak dependence on galaxy mass. We conclude that a significant fraction of mergers would result in recoiling SMBHs that spend of order a quasar lifetime above the obscuring torus and would therefore appear as type 1 quasars even if “intrinsically” type 2. We examined the robustness of these results under different assumptions about the prerecoil distributions of spins and masses. Maximally spinning, equal-mass SMBH mergers affect a fraction of up to $f \sim 0.75$ of the population (same assumptions as above), while major mergers with intermediate SMBH spins of $a = 0.3$ imply $f \sim 0.05–0.15$.

If we assume that the type 1/type 2 fraction of quasars is intrinsically (i.e., in the absence of recoils) the same as in Seyfert galaxies ($\sim70\%$ type 2, $\sim30\%$ type 1), we predict that a fraction $\sim50\%$ of these type 2 quasars will appear instead as type 1 at any given time. Several factors affect these estimates: idealizations in the galaxy models used in the N-body simulations, uncertainties in the distribution of masses and spins as discussed above, uncertainties especially in the thickness of the torus in the most massive galaxies, and various selection effects in the measurements of type 1/type 2 ratios in dependence of SMBH mass. We also recall that if equations (1) and (3) strictly hold, in the most massive quasars a fraction of the torus will remain bound to the recoiling SMBH so absorption/extinction would not fall to zero. Finally, we note that if the quasars’ total lifetime is actually composed of several shorter merger episodes on the order of $10^8$ yr each, even recoil oscillations with velocities as small as $\sim200$ km s$^{-1}$ would affect a large fraction of the quasar population.

4.3. Implications

So far, we have distinguished between Seyfert galaxies (low-mass SMBHs) and quasars (high-mass SMBHs). Can we also reproduce the observed trend ($\S$ 4.1) that obscuration fraction decreases with quasar luminosity (i.e., mass)? In our base model, the dependence of mean oscillation timescale on galaxy mass is weak. However, since the likelihood of a major merger (as opposed to other types of fueling) is believed to increase strongly with AGN luminosity (galaxy mass), the observed trend would arise naturally, since the most luminous AGNs are increasingly likely to be triggered by major mergers.

If a fraction of all quasars is recoiling at any given time, we should see the corresponding BLR emission-line velocity shifts $v_{\text{esc}}$ in a fraction of the type 1 quasars. Bonning et al. (2007, hereafter B07) set limits on the fraction of emission line velocity shifts observed in a sample of SDSS quasars. In our model, the majority ($\geq75\%$) of kicked SMBHs remain bound to the galaxy and so their velocities quickly drop below the initial value of $v_{\text{esc}}$ most of the time they would therefore be observed with a velocity that is much smaller than $v_{\text{esc}}$.

We carried out Monte Carlo simulations to check the consistency of our recoil model with the B07 limits. Our simulations were designed to crudely mimic the properties of a sample of quasars selected to exhibit both broad and narrow emission lines, as in the B07 sample. We adopted a uniform logarithmic distribution of galaxy masses, $11 \leq$
lead to measurable variability in the absorption and extinction of AGN spectra once the recoiling SMBH passes the individual clouds making up the torus. A number of interesting effects are related to the torus itself: (1) A recoiling SMBH with a bound gas disk that passes through the dense torus (rather than moving perpendicular to it) might cause local shocks, heating, and temporary X-ray emission. (2) During the long-lived “phase II” oscillations, when the SMBH oscillation amplitude is on the torus scale, the SMBH might efficiently accrete from the dense molecular gas at each turning point, causing repeated flares of radiation. Such flares would locally destroy the dust, while photoionization of the dense surrounding gas would produce a strong emission-line response. (3) Torus radii are roughly equal to SMBH gravitational influence radii, so ejection of the SMBH might lead to temporary expansion of the torus since the mass holding it in place is suddenly removed. Isotropic expansion would not affect the column density along the line of sight, seen from the very center. These effects will be addressed in more detail in forthcoming work.

In summary, we have shown that timescales of recoil oscillations are in an interesting regime where they can potentially affect a significant fraction of the quasar population. Knowledge of oscillation timescales and amplitudes is also critical for modeling AGN evolution, delays between starburst and AGN activity after merger, and the cosmic X-ray background.

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2 While strong emission-line variability in response to an X-ray flare has recently been observed, this event is more likely interpreted in terms of stellar tidal disruption (Komossa et al. 2008b).

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$\log_{10}(M_{\text{gal}}/M_{\odot}) \leq 12.5$, and a fixed quasar lifetime of $t_{\text{QSO}} = 3 \times 10^7$ yr; kicks were assumed to have occurred at times that were distributed uniformly and randomly between the epoch of observation and a time $t_{\text{QSO}}$ earlier. The position and velocity of the SMBH at the time of observation was extracted from the appropriate $N$-body model after scaling to physical units using the galaxy mass. If the distance of the SMBH from the galaxy center exceeded the outer torus radius, its radial velocity $v_{\text{gas}}$ was added to the cumulative distribution (assuming a random direction for the recoil); this velocity was identified with the measured velocity offset of the BLR gas in the B07 galaxies. A fraction 30% of SMBHs (the “true” type 1 population) with $r < r_{\text{tor, out}}$ were assumed to have visible BLRs and so were included in the accounting. Figure 1c shows that the predicted fraction of objects with large ($\geq 100$ km s$^{-1}$) velocity shifts is about an order of magnitude smaller than would be inferred from the unmodified distribution of kick velocities; this is due to the deceleration that occurs as the SMBH moves through the galaxy.

B07 found a maximal fraction of $f_{\text{QSO}} = 0.04$ quasars with velocity shifts above $v_{\text{obs}} = 500$ km s$^{-1}$, and a fraction $f_{\text{QSO}} = 0.0035$ with shifts above $v_{\text{obs}} = 1000$ km s$^{-1}$. These limits are consistent with our baseline model within a factor of a few, particularly if we impose an upper limit to the galaxy/ SMBH separation at which a recoiling SMBH would be spectroscopically identified with its host galaxy (shown as the dotted lines in Fig. 1c, assuming $v_{\text{max}} = 10$ kpc).

Finally, we note that recoil oscillations will also have a number of other observable consequences. They will affect the X-ray background and its modeling since a fraction of AGN will be unobscured at any given time. In particular, small-ray background and its modeling since a fraction of sources would be inferred from the unmodified distribution of kick velocities; this is due to the deceleration that occurs as the SMBH moves through the galaxy.

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