Comparison of sensitivity matrix calculation methods for EMT system based on TMR for permeability detection

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Abstract. This paper proposed a new electromagnetic tomography (EMT) system based on tunnel magnetoResistance (TMR) for the detection of permeability, in which measured sensitivity is not affected by excitation frequency. In addition, we firstly put forward an improved simulation model, which has a similar distribution of magnetic field with real system. Afterwards, the perturbation method and the direct method were used to calculate the sensitivity matrices respectively, some simulation experiments were performed for reconstructing images and correlation coefficient between original images and reconstructed images based on different sensitivity matrices were calculated after that. The calculation shows that the sensitivity matrix calculated by the perturbation method is more accurate for detecting permeability in the new EMT system.

1. Introduction
As a new type of non-contact imaging technique, Electromagnetic tomography (EMT) is applicable to a range of industrial applications. When applied in three-phase fluidized beds, it can provide real-time images of some fluid in a section of pipe. This paper firstly proposed the feasibility of EMT technology in detecting the distribution of fluid with permeability, while all EMT systems can only be applied to obtain the distribution of conductivity [1][2][3][4].

In almost all EMT systems, coil are selected as the sensor to measure voltage, but its measuring sensitivity is related to the excitation frequency according to the principle of electromagnetic induction, the low frequency will lead to the low sensitivity. Instead of coil sensor, we choose tunnel magnetoResistance (TMR) which is a kind of new magnetic sensitive component of high sensitivity and measure magnetic flux density directly, it helps the system get rid of the limit of excitation frequency to sensitivity. Due to its detection principle is different with coil sensor, an appropriate sensitivity matrix calculation method becomes necessary.

Many researchers use straight line to replace real coil in the two-dimensional simulation, but the distribution of simulation magnetic field is close to concentric circles with the straight line as the center, which is different from the reality. Therefore there is a discrepancy between simulation model and reality in sensitivity matrices. We improve the simulation model which has a similar distribution to practice, then bring the perturbation method and the direct method to calculate the sensitivity matrices. In the end, the comparisons of image reconstruction results between the two methods were presented.
2. Theoretical basis
The Maxwell's equations for the harmonic version of the EMT system are defined as,
\[
\begin{align*}
\nabla \times \vec{H} &= \vec{J} + j \omega \varepsilon \vec{E} \\
\nabla \times \vec{E} &= -j \omega \vec{B} \\
\n\nabla \cdot \vec{B} &= 0 \\
\n\nabla \cdot \vec{D} &= \rho 
\end{align*}
\] 
(1)

Where \( \vec{H} \) is the vector of magnetic field intensity, \( \vec{J} \) is current density, \( \omega \) is excitation frequency, \( \varepsilon \) is permittivity, \( \vec{E} \) is electric field intensity, \( \vec{B} \) is magnetic induction intensity \( \vec{D} \) is potential shift vector, \( \rho \) is charge density.

For simplicity, neglect the displacement current and the free charge in the measured field, \( \vec{H} = \vec{J} = \vec{B} = \vec{D} = 0 \). In the static isotropic medium, \( \vec{B} = \varepsilon \vec{E}, \vec{B} = \mu \vec{H}, \vec{J} = \sigma \vec{E} \). Introduced vector magnetic potential \( \vec{A} \), which are defined as \( \vec{B} = \nabla \times \vec{A} \). Then the first equation in equation (1) becomes
\[
\nabla \times (\mu^{-1} (\nabla \times \vec{B})) = \sigma \vec{E} 
\] 
(2)

Through some similar derivation, the second equation in equation (1) becomes
\[
\vec{E} = -j \omega \vec{A} 
\] 
(3)

Substituting equation (3) into (2) gives the diffusion equation about \( \vec{A} \)
\[
\nabla^2 \vec{A} = -j \omega \mu \vec{A} 
\] 
(4)

The vector magnetic potential varies with the change of the conductivity and permeability in medium, so that the distribution of permeability can be obtained by using magnetic field detection.

3. Simulation model and sensitivity matrix
In figure 1, an 8-coils EMT sensor array is designed. The diameter of pipeline is 100mm, the outer diameter, inner diameter and height of the excitation coil are 40mm, 24mm and 5mm respectively, the excitation frequency is 1 kHz. TMR sensors have an output proportional to the magnetic flux density on the direction of its sensitive axis. In this paper, the sensitive axis points the axle centre of the pipeline.

As shown in figure 2(a), straight line is used to replace practical coil in two-dimensional simulation\(^5\), which is called model A in this article. The distribution of magnetic field is approximated as concentric circles which are completely different from reality. This paper puts forward a new model, in which two rectangles corresponding to the central intersecting surface of the actual exciting coil are set up to replace straight line, and we name it model B. Excitation result is showed in figure 2(b), it shows that model B has the closer distribution with practical system compared with model A.

The direct method was proposed by Dyck D N t al in 1994\(^6\), which is obtained by dot product of vector \( \vec{H} \) from two fields.
\[ S_{\rho} = -j\omega H_{A} \cdot H_{B} = -j\omega (H_{AX} \cdot H_{BX} + H_{AY} \cdot H_{BY}) \quad (5) \]

Where \( H_{A} \) and \( H_{B} \) are magnetic field intensity produced by exciting the exciting coil and receiving coil separately, then \( H_{AX}, H_{AY} \) represent the components of \( H_{A} \) in the direction of X and Y.

In perturbation method, it is necessary to divide the sensitive field into small enough units according to the size of the disturbed sample, then move the sample gradually to obtain signal at the boundary. The magnetic induction intensity is obtained as measured signal. The formula is as follows,

\[ S = \Delta B \cdot (\Delta \mu)^{-1} \quad (6) \]

Where \( \Delta B = B - B_0 \). \( B \) is the measured value respect to the permeability disturbance. \( B_0 \) is the value without the permeability disturbance. Due to the sensitive axis characteristic of TMR, it is necessary to extract the magnetic flux density on the sensitive axis of each TMR sensor before calculation.

| Method | Perturbation method | Direct method |
|--------|---------------------|---------------|
| Model A | ![Image](image1.png) | ![Image](image2.png) |
| Model B | ![Image](image3.png) | ![Image](image4.png) |

4. Image reconstruction results

To evaluate the sensitivity calculation methods described by equation (5) and equation (6), simulation experiments were carried out using a magnetic cylinder 20mm in diameter and 100 s/m in permeability. The Landweber algorithm is used for image reconstruction and has the form,

\[ G_{k+1} = G_k + \alpha S^T (C - SG_k) \quad (7) \]

Where the constant \( \alpha \) is known as the factor and is used to control the convergence rate, we selected \( \alpha = 0.04 \). \( C \) is the vector of measured changes, \( G \) is the vector of normalized image of the magnetic distribution, and \( S \) is the sensitivity matrix.

| Method | Perturbation method | Direct method |
|--------|---------------------|---------------|
| Real distribution | ![Image](image5.png) | ![Image](image6.png) |
| Model A | ![Image](image7.png) | ![Image](image8.png) |
| Model B | ![Image](image9.png) | ![Image](image10.png) |

Table 2 indicates that the reconstructed images based on model B have a higher imaging precision than have those based on model A, which explains the need for improving the simulation model. Then
the correlation coefficients are introduced to evaluate the spatial similarity between the real images and reconstructed images made by the two sensitivity calculation methods for comparison. The correlation coefficients are defined as equation (8)\(^7\).

\[
R_{xy} = \frac{\sum_{i=1}^{N} (g_{i}^{\text{rec}} - \bar{g}^{\text{rec}})(g_{i}^{\text{ref}} - \bar{g}^{\text{ref}})}{\sqrt{\sum_{i=1}^{N} (g_{i}^{\text{rec}} - \bar{g}^{\text{rec}})^2 \sum_{i=1}^{N} (g_{i}^{\text{ref}} - \bar{g}^{\text{ref}})^2}}
\]

Where, \(g_{i}^{\text{rec}}\) and \(g_{i}^{\text{ref}}\) are the gray value of original image and reconstructed image in pixel \(i\). \(\bar{g}^{\text{rec}}\) and \(\bar{g}^{\text{ref}}\) are the average value of \(g_{i}^{\text{rec}}\) and \(g_{i}^{\text{ref}}\) respectively. Evaluation results are shown in Table 3, from the comparison we can get a conclusion that the EMT system can applied to detect distribution of permeability and the perturbation method does better than that of direct method in image reconstruction.

| Flow pattern | Perturbation method/Direct method |
|--------------|----------------------------------|
| flow pattern 1 | 0.870/0.840 |
| flow pattern 2 | -0.012/-0.059 |
| flow pattern 3 | 0.626/0.534 |

5. Conclusions
A new EMT system based on TMR is described to help EMT system get rid of the limit of excitation frequency to system sensitivity. An improved simulation model is given in the solution of forward problem, which has a closer magnetic field distribution with real EMT system than has the initial simulation model. The perturbation method and the direct method are introduced to calculate the sensitivity matrix respectively. After that, the correlation coefficients between real images and reconstructed images were calculated, which explained that the perturbation method can produce better images than can the direct method. In conclusion, the EMT system based on TMR has the ability to detect magnetic material in the field, therefore it can applied to parameter detection in some three-phase fluidized beds including magnetic catalyst which takes ferromagnetic materials as core.

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