Robust Resource Allocation for PD-NOMA-Based MISO Heterogeneous Networks with CoMP Technology

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Abstract—In this paper, we consider a hybrid scheme of coordinated multi-point (CoMP) technology in MISO heterogeneous communication networks based on power domain non-orthogonal multiple access (PD-NOMA). We propose a novel method based on matching game with externalities to realize the hybrid scheme where the number of the cooperative nodes are variable. Moreover, we propose a new matching utility function to manage the interference caused by CoMP and NOMA techniques. We also devise robust beamforming to cope with the channel uncertainty. In this regard, we focus on both no CSI and partial CSI cases to increase the achievable data rate. We provide the complexity analysis of both schemes which shows that the complexity of the partial CSI approach is more than that of the no CSI method. Results evaluate the performance of proposed CoMP scheme and the sensibility of our methods.

Index Terms— CoMP technology, hybrid scheme, matching game with externalities, PD-NOMA, robust beamforming, probabilistic constraint, no CSI, partial CSI.

I. INTRODUCTION

The idea of heterogeneous networks (HetNets) is to bring the network access point closer to the user by which the performance of resource usage and communications is improved. HetNets consist of several types of base stations (BSs), e.g., macrocells and femtocells, with different capabilities, transmit powers, and coverages [1], [2]. In such a network, due to spectrum reuse and dense utilization of BSs, intercell interference is of most concern. Coordinated multipoint (CoMP) is one of promising techniques to alleviate the effect of intercell interference. Joint Processing (JP), which can be fell into Joint Transmission (JT-CoMP) and Dynamic Point Selection (DPS), is one variety of CoMP in which data can be simultaneously transmitted from all of the cooperative nodes. Nevertheless, this scheme has many challenges such as high signaling and latency [3]. Since the cooperation process in high order may not be required, we offer a method which allows cooperative nodes to be variable based on the conditions [1] and [5]. Therefore, devising efficient resource allocation schemes for JT-CoMP is of most importance.

To increase the spectral efficiency, power domain non-orthogonal multiple access (PD-NOMA) scheme is presented in which the same spectrum is shared among several users (called NOMA set) whose signals are superimposed, and the resulting signal is transmitted towards these users. Each user performs successive interference cancellation (SIC) to cancel the interference from other users which depends on the specified (SIC) ordering of that user. Devising an efficient resource allocation schemes for PD-NOMA-based transmission could be very challenging due to the fact that determining the NOMA set, transmit power of users in the NOMA set, and the SIC ordering would be very complicated in real scenarios.

The performance of the resource allocation method used in the network is highly depends on the availability of the channel state information at the transmitter (CSIT). However, in most practical scenarios, such a perfect knowledge is not available due to the limited capacity of the feedback channels, high signaling, and estimation errors [2], [3]. In such cases, the network should adopt robust methods to cope with the imperfectness of CSIT.

Motivated by the above mentioned facts, our goal is to devise an efficient robust resource allocation scheme for MISO networks based on the CoMP and the PD-NOMA. In such scheme, finding the NOMA set, the set of BSs performing CoMP for users, transmit power variables, and robustness against channel certainty is attained by formulating the resource allocation problem into an optimization problem and solving the resulting optimization problem via efficient iterative algorithms.

A. Related Workes

In [21], we consider a homogeneous network which works based on JT scheme when uncertainty of CSI is taken into account. In order to solve this kind of non-convex problems, some approximations or equivalents can be employed [16], [18] and [19]. In [21], we use the Bernstein approximation and the worst-case method.

The authors in [22] consider a single-carrier network and propose the stochastic difference of convex programming (SDC) algorithm. In [23], worst-case optimization constraint is rewritten as a linear matrix inequalities by the S-procedure method. In [2], a single-carrier HetNet without any cooperation process is considered and robustness solutions in no CSI and partial CSI feedback are proposed which are based on Bernstein inequality and SDR method. In [26], a time-division duplex (TDD) based HetNet with hybrid analog design for MBS are proposed. To find digital beamforming vectors, a power minimization problem with outage probability constraints which are approximated by the Bernstein-type inequality is solved.

Matching theory has been applied to solve the optimization problems in 5G networks. Most of the existing methods for multi-dimensional matching problems fall into two categories [6]: 1) transform the multi-dimensional matching problem into the two-dimensional matching problems as to the pairing algorithm in [7]; 2) construct the hypergraph model [8] or k-set packing problem [9]. Although there are some works which employ the matching theory, none of them investigated...
cooperative NOMA based network and they ignored externalities and their framework are traditional. In [10], a matching-theory-based user scheduling and the optimal sensing duration adaptation are proposed in an alternate iteration framework for a cognitive OFDM-NOMA systems where externalities are ignored in the matching algorithm. In [11], a greed bidirection subchannel matching scheme without externalities is provided for NOMA system by selecting the users who have the maximum subchannel energy efficiency. In [12], the matching theory is used to manage the co/cross-tier interferences between D2D and cellular communications caused by resource sharing. Hence, the matching is an effectual tool to manage all interferences in a network and we apply this idea in our proposed framework.

Authors in [13], consider an OFDMA network in the uplink transmissions case. They employ a one-to-many matching game theory algorithm for user association and a one-to-one matching game for channel allocation problems. Moreover, the transmission process is assumed in the non-cooperative mode where the CSI is perfectly known. In this paper, we propose a new algorithm for hybrid cooperative node association via many-to-many matching game and sub-channel allocation in PD-NOMA-based MISO system via many-to-one matching game based on the general framework of [13]. Unlike the methods in [13], there are some extra-interferences due to the NOMA and CoMP in our considered network. Moreover, we assume externalities in our matching game to insure stability of our proposed method.

Authors in [14], consider externalities in their spectrum with a set of femtocell base stations (FBS) as transmission nodes, necessarily are not in a fixed cooperative set (CS). In this regard, we propose a novel method based on matching games. Generally, unlike other existing works in CoMP design, we assume each antenna of a FBS can join to different CSs.

**B. Contributions**

Since the 5G network employs some techniques which lead to some design challenges as extra-interferences and unnecessary cooperations, we aim to model a practical and flexible network based on these techniques and attempt to propose some methods for use of advanced 5G networks. As far as we find out, there is no comprehensive work which consider the joint BS association and channel allocation in cooperative NOMA systems especially in the MIMO case with imperfect CSIT effect.

The main contributions and features of this paper can be summarized as follows:

- **CoMP Scheme**: To eliminate unnecessary cooperations, we consider a hybrid scheme where different antennas, as transmission nodes, necessarily are not in a fixed cooperative set (CS). In this regard, we propose a novel method based on matching games. Generally, unlike other existing works in CoMP design, we assume each antenna of a FBS can join to different CSs.

- **Architecture of Network**: Based on our researches, all of the mentioned papers in the related works section which jointly investigate robustness of the CoMP and MIMO networks discussed a single-carrier network or they consider SISO multicarrier networks with perfect CSI. In this paper, we assume a cooperative network in multi-carrier conditions considering uncertainty of the CSIT. This problem has a three dimensional matching concept. Hence, we propose a new matching algorithm with a low complexity. Since multiplexing of multiple users on the same frequency channel leads to co-channel interference (CCI), SIC must be performed at the receivers. In this regard, we introduce a new probabilistic SIC constraint which is strongly intractable.

- **Advanced Interference Management**: In order to remove the extra-interference due to SIC and CoMP, we propose a novel approach to apply an advanced interference management method based on the matching utility functions. We pair the cooperative nodes in the CoMP set and users which are multiplexed on the same subcarriers in such a way to reduce the interference with harmful effect on other users. Moreover, to achieve a stable solution in practical networks, we consider externalities and employ swap-matching idea.

- **Robustness Method**: We consider robustness in both stochastic and deterministic cases. The considered scenario is more intractable due to probabilistic SIC constraint. To solve the proposed robust optimization problem, a novel alternative sequential algorithm is proposed. Moreover, the convergence of the iterative algorithms are proved and their computational complexities are investigated.

**C. Organization**

The rest of this paper is organized as follows: The considered system model is presented in Section II. The proposed resource allocation problem is formulated in Section III. In Section IV, a new matching game based solution is proposed. We investigate the convergence and the computational complexity of the proposed methods in Section V. Simulation results are in Section VI. and the paper is concluded in Section VII.

Notations: We use \( \odot \) to define Hadamard product of two vectors while \( \ast \) represents the common matrix multiplication. \( (a,b) \) represents inner product of two matrices and \( A \succeq 0 \) indicates that \( A \) is a positive semidefinite (PSD) matrix. In addition, \( ||.||_F \) and \( ||.||_2 \) denote Frobenius norm of the matrix and Euclidean norm of a vector, respectively. Trace of a matrix are defined via trace [A]. The conjugate transpose of a matrix \( A \) is denoted by \( A^H \). The complex space of \( n \)-dimensional vectors is described using \( \mathbb{C}^n \). \( \lambda_{\max}(.) \) denotes the maximum eigenvalue of matrix. Re \( \{.\} \) and \( \mathbb{E}\{.\} \) are the real part and mean of associated argument. The maximum number of linearly independent row vectors in the matrix is shown using \( \text{rank}(.) \). The expression \( A \subset B \) defines \( A \) as a subset of the set \( B \) and \( \cup \) denotes the union of two sets. \( \{A\}(\alpha) \) equals a subset contains all of the elements of set \( A \) except element \( \alpha \).

**II. SYSTEM MODEL**

We assume that the spectrum of macrocell base stations (MBSs) are different from others and each of them shares their spectrum with a set of femtocell base stations (FBS) as \( F = \{1,...,F\} \) which are operating inside the coverage of a macrocell. Hence, each user of femtocell can just be affected by interference of the macrocell with common spectrum. Consequently, we consider a HetNet presented at Fig. 1 with
employ subcarrier $n$, each user $k$ adopts the SIC technique [25]. User $k$ can decode and cancel the interference from other users with a lower channel gain $\| \mathbf{h}_{F,n} \| < \| \mathbf{h}_{F,k,n} \|$. Thus, the indistinguishable interferences must be taken into account. Consequently, SINR of FUE $k$ on the $n^{th}$ subcarrier is given by

$$\Gamma_{(w_k, n, \rho_{k,n}, \mathbf{h}_{F,k,n}, \mathbf{h}_{F,M,k,n})} = \frac{\| \mathbf{h}_{F,k,n} (w_k, n \circ \rho_{k,n}) \|^2}{I_{F,M,k,n} + I_{F,k,n} + \sigma_{k,n}^2}$$ (2)

where $I_{F,M,k,n} = \| \mathbf{h}_{F,M,k,n} \|^2$ is the interference from MBS and $I_{F,k,n} = \Sigma_{i \in K} \| \mathbf{h}_{F, i,n} \| \| \mathbf{h}_{F,k,n} \|$.

$\| \mathbf{h}_{F,k,n} (w_i, n \circ \rho_{i,n}) \|^2$ expresses the summation of inter-cell interference of other FBSs and intra-cell interference due to the PD-NOMA approach. $w_k, n = [(\mathbf{w}_k, n^T), ..., (\mathbf{w}_k, n^{F})]^T \in \mathbb{C}^{FT}$, $\rho_{k,n} = [(\mathbf{\rho}_k, n^T), ..., (\mathbf{\rho}_k, n^{F})]^T \in \mathbb{C}^{FT}$ and $\sigma_{k,n}^2$ is the noise power. $\mathbf{\rho}_{k,n} \in \mathbb{C}^{FT}$ is the subcarrier indicator vector of the $f^{th}$ femtocell. Accordingly, the achievable data rate at user $k$ on subcarrier $n$ is formulated by

$$r_{k,n} = \log_2 (1 + \Gamma_{(w_k, n, \rho_{k,n}, \mathbf{h}_{F,k,n}, \mathbf{h}_{F,M,k,n})})$$ (3)

A. Channel State Information and Robustness

Practically, receiver has uncertain estimation of channel coefficients. Uncertainty can be caused by various reasons, such as estimation errors, feedback quantization, hardware limitations and delays. To model the uncertainty of CSI, we choose additive error model for a simple communication channel model which is shown as follows:

$$\mathbf{h}_{F,k,n} = \mathbf{h}_{F,k,n} + \mathbf{e}_{F,k,n}$$ (4)

where the notations $\mathbf{h}_{F,k,n}$ and $\mathbf{e}_{F,k,n}$ denote the estimated imperfect channel coefficients and the error vector [32]. We consider two schemes of CSIT imperfection as follows:

- In no CSI case, we assume $\mathbf{e}_{F,k,n}$ is a norm bounded vector for analytical convenience. In this regard, we consider the Euclidean ball-shaped uncertainty set as follows:

$$\mathcal{H} = \{ \mathbf{h}_{F,k,n} : \mathbf{h}_{F,k,n} = \mathbf{h}_{F,k,n} + \mathbf{e}_{F,k,n}, \| \mathbf{e}_{F,k,n} \| \leq \zeta_{k,n} \},$$ (5)

where $\zeta_{k,n}$ defines error bound on the uncertainty region. $\mathbf{h}_{F,k,n}$ is defined as the estimation coefficients of channels which is the center of the assumed ball [24] and [32].

- In partial CSI case, we use the Bernstein approach to approximate the chance constraint. To apply this inequality, we assume that the error vectors follow the complex Gaussian distribution as [27], [29] and [30]. Hence, $\mathbf{e}_{F,k,n}$ is a complex Gaussian random vector with specific fixed mean and covariance matrix as

$$\mathbf{e}_{F,k,n} \sim \mathcal{C}_n \mathbf{F}_{F,k,n} \mathbf{v}_{F,k,n},$$ (6)

where $\mathbf{C}_n \mathbf{F}_{F,k,n} \geq 0$ is the covariance matrix of $\mathbf{e}_{F,k,n}$ and $\mathbf{v}_{F,k,n}$ is the complex Gaussian random vector, i.e., $\mathbf{v}_{F,k,n} \sim \mathcal{C}_n (\mathbf{0}, \mathbf{I})$ and $\mathbf{I}$ is an identity matrix.
III. PROBLEM FORMULATION

The beamformer vector of transmitters must be designed based on the channel models such that guarantees the outage occurs below a small predetermined probability threshold as follows:

1) We define achievable data rate based on (2) with considering uncertainty sets similar to (5) in a worst-case approach as \( r(h_{F,k,n}, w_k, \rho_{k,n}) \geq R_k \) where channel coefficients must be considered in uncertainty set \( H \) and \( R_k \) is the minimum required achievable data rate.

2) In probabilistic case, we define achievable data rate with considering imperfect CSI as

\[
\Pr \left\{ \log_2 (1 + \Gamma (w_k, n, \rho_{k,n})) \geq R_k \right\} \geq 1 - \beta, \tag{7}
\]

where \( R_k \) and \( \beta \) respectively denote the target rate for FUE \( k \) and the maximum tolerable outage probabilities. Moreover, to perform the process of decoding at user \( j \) and removing the CCI from user \( k \) on the same subcarrier \( n \), the following SIC constraint must be satisfied:

\[
\Pr \left\{ \Gamma (w_k, n, \rho_{k,n}, h_{F,j,n}, h_{F,M,j,n}) \geq 1 - \beta \right\}. \tag{8}
\]

In the more general concept, we assume that CS which includes different antennas in various FBSs is variable and not necessarily all of these nodes are in the CS. Therefore, the nodes related to a CS could be determined by solving an optimization problem. This problem has three dimensional matching concept which is hard to solve. To address this problem, we define the antennas as transmission nodes in a new set as \( A = \{1, \ldots, F * T_f\} \). Therefore, we define \( \rho_{k,n}^a \) as a three dimensional variable. To solve this problem, we transform it into two two-dimensional matching problems. Hence, we define some new integer variables as \( \chi_{k,a} \) and \( \nu_{k,n} \) so that \( \rho_{k,n}^a = \chi_{k,a} * \nu_{k,n} \). The variable \( \chi_{k,a} \) denotes that the cooperative transmission node \( a \) is assigned to the FUE \( k \) or not, and \( \nu_{k,n} \) denotes these cooperative nodes can use the subcarrier \( n \) to FUE \( k \). The optimization problem for maximizing the total throughput of the considered network is described as follows:

\[
\begin{align*}
\max_{W_{b}, \nu} & \quad \sum_{n \in N} \sum_{k \in K} r_{k,n}, \tag{9a} \\
\text{s.t.} & \quad (7), \quad (8).
\end{align*}
\]

where \( R_k \) and \( \beta \) respectively denote the target rate for FUE \( k \) and the maximum tolerable outage probabilities. Moreover, to perform the process of decoding at user \( j \) and removing the CCI from user \( k \) on the same subcarrier \( n \), the following SIC constraint must be satisfied:

\[
\begin{align*}
\Pr \left\{ \sum_{k \in K} ||h_{F,k,n}^f (w_k, n, \rho_{k,n})||^2 \leq \epsilon_M \right\} \geq 1 - \alpha, \tag{9b} \\
0 < \sum_{n \in N} \sum_{k \in K} ||h_{F,k,n}^f (w_k, n, \rho_{k,n})||^2 \leq P_{\max}, \forall f \in F \tag{9c} \\
\sum_{a \in A} \chi_{k,a} \leq F_{\max}, \forall k \in K, \tag{9d} \\
\sum_{k \in K} \chi_{k,a} \leq N_a, \forall a \in A, \tag{9e} \\
\sum_{k \in K} \nu_{k,n} \leq q_{\max}, \forall n \in N, \tag{9f} \\
\sum_{n \in N} \nu_{k,n} \leq 1, \forall k \in K, \tag{9g} \\
\chi_{k,a}, \nu_{k,n} \in \{0, 1\}, \forall k \in K, n \in N, a \in A, \tag{9h}
\end{align*}
\]

where all of the beamformer vectors of FBSs are considered in a matrix as \( W \). Matrix \( \rho \) defines a three dimensional matrix for all subcarrier indicators so that \( \rho = \chi * \nu \). Practically, there are some extra-effects of FBSs on the macrocell [2]. In the CoMP network, these effects are not ignorable. It is intelligent to assume an interference power constraint on the MUE. In this regard, [26] is considered to improve the overall performance of the network where \( \epsilon_M \) is the preset target value and \( \alpha \) denotes the maximum tolerable outage
where the maximum order of cooperation defines limiting the total number of coordinated transmission nodes antenna as in [13]. To manage signaling volume and delays, we introduce a complexity constraint as (9g). It is applied for limiting the total number of coordinated transmission nodes where the maximum order of cooperation defines $F_{\text{max}}$. In (9f) and (9g), we assume that spectrum can be shared between MBS and FBSs while each subcarrier can be assigned to a set of users and every user can be served by each subcarrier $n$. $q_{\text{max}}$ denotes the maximum number of interfered users through the PD-NOMA technique.

IV. MATCHING GAME BASED RESOURCE ALLOCATION

Optimization problem described in (9) is a probabilistic mixed-integer nonlinear programming (MINLP) problem. Generally, we tend to perform a joint transmission nodes assignment and subcarrier allocation with beamforming. To solve (9), we choose an iterative based framework which has three independent phases: 1) Given an initialized beamforming vectors, we propose a new algorithm based on many-to-many matching game algorithm where the output of this phase introduces $x^*$. 2) Given $x^*$ and $W^*$, the subcarrier assignment problem can be solved by a many-to-one matching game algorithm where the output of this phase introduces $\nu^*$. 3) At third phase, beamforming design based on the proposed association at previous phases is performed. These phases are sequentially applied until the problem converges to a feasible solution of $\{x^*, \nu^*, W^*\}$.

A. Cooperative Transmission Node Selection (CTNSA) Algorithm

Since the clustering is an important issue in the MIMO-5G networks, we investigate it in a general problem as follows:

$$\mathcal{P}_{\text{CTNSA}} : \max_{x} \sum_{n \in N} \sum_{k \in K} c_{k,n}^a,$$

s.t. (9d), (9e), (9f).

As we mentioned before, we use (9d) and (9f) for this algorithm which works based on many-to-many matching concept. By the definition of the different antennas as transmission nodes, $\chi_{k,a}$ defines node $a$ is joined to the cooperative set of user $k$ or not. In this problem, we describe a CS with $A_k \subset A$ as the set of transmission nodes assigned to the FUE $k$ and also $K_a \subset K$ as the set of users associated to node $a$. Additionally, we define $N_a \subset N$ as the set of available subcarriers at node $a$ while the set of FUEs multiplexing on subcarrier $n$ is denoted by $C_n \subset K$.

1) Definition of a matching function: Optimization problem $\mathcal{P}_{\text{CTNSA}}$ can be defined by a tuple $(A, K, \chi_{A,C}, \chi_{K,C})$. Here, $\chi_{A,C}$ and $\chi_{K,C}$ denote the sets of the preference relations of FUEs and transmission nodes, respectively. We define two disjoint finite sets of players $A$ and $K$ and also a mapping function $\mu_{CS}$ such that: 1) $a \in \mu_{CS}(k) \iff k \in \mu_{CS}(a)$; 2) $|\mu_{CS}(k)| \leq F_{\text{max}}$ and $|\mu_{CS}(a)| \leq N_a$. Instead of (7) and (9b), we introduce some utility functions to each component of players until they construct their preference list in a decreasing order. We use the average received SINR over all subchannels as utility function of the FUE, which is the most common criterion for user association [13, 15] as follows:

$$\varphi_{\text{CS}}^k(a) = \log_2(1 + \sum_{n \in N_a} \gamma_{k,n}^a),$$

In this case, $\gamma_{k,n}^a$ describes independent effect of each node $a$ on the received SINR as

$$\gamma_{k,n}^a = \frac{\chi_{k,a} \nu_{k,n}^a \hat{h}_{F,k,n}^a \hat{w}_{k,n}^a}{I_{F,M,k,n} + I_{F,k,n} + \sigma_{k,n}^2}, \forall k \in K_a, n \in N_a,$$

where the interference $I_{F,k,n} = \sum_{i \in K \setminus \{k\}} \hat{h}_{F,k,n}^i |\hat{h}_{F,k,n}^i|^2 |\hat{h}_{F,i,n}^a|^2$. In [13], a new function is introduced as a utility function for user association at uplink transmission of HetNets, we implement the same function for our problem as

$$\varphi_{CS}^k(k) = \gamma_{CS} \sum_{n \in N} \frac{|\hat{h}_{F,k,n}^a \hat{w}_{k,n}^a|^2}{2\hat{h}_{F,k,n}^a - 1} - \Theta_{k,a},$$

where $\gamma_{CS}$ is a weighting parameter capturing the average direct channel gain from the node $a$ to the FUE. Although the CoMP can improve the received signal of users, it causes significant interference in other receivers, i.e., MUE and FUE $\forall (i \neq k)$ especially in the case of $|\hat{h}_{F,k,n}^i| \geq |\hat{h}_{F,k,n}^a|$ which SIC procedure is unsuccessful. Therefore, we propose an advanced interference management through introducing $\Theta_{k,a}$ which quantifies the aggregated interference that node $a$ causes to the MUE and also the other FUE $i$ on all subchannels which is defined as $\Theta_{k,a} = \sum_{n \in N_a} (\Theta_{M,F,k,n} + \Theta_{F,k,n})$ where

$$\Theta_{M,F,k,n} = c_{M,F}^n \varpi^n |\hat{h}_{M,F,n}^a|^2,$$

$$\Theta_{F,k,n} = \sum_{i \in K \setminus \{k\}} |\hat{h}_{F,i,n}^a|^2 c_{F}^a |\hat{h}_{F,k,n}^a|^2.$$ (14a) and (14b) decrease the interference of node $a$ on MUE and other FUEs, respectively.

$$\varpi^n = \max \left(0, (\sum_{i \in K_a} |\hat{h}_{M,F,n}^i|^2 - \epsilon_M)/\epsilon_M \right)$$

is defined to quantify the degree of violation of the constraint (9b). $c_{M,F}^n$ and $c_{F}^a$ are the costs per unit of the interference power at the MUE and FUE $i$, respectively. The proportions of $c_{M,F}^n$ and $c_{F}^a$ can be set, based on the priority of users. For example, $c_{M,F}^n \gg c_{F}^a$ indicates the priority of MUEs and guarantees that solutions with harmful effect on the MUEs can be blocked.

2) Description of the stopping criterion: We remind that our purpose is the hybrid scheme. Hence, we introduce a new criterion in addition to (9d) which limits the size of CS as required. Accordingly, we define

$$\gamma_{k,n} = \frac{|\sum_{a \in A_k} \chi_{k,a} \nu_{k,n}^a \hat{h}_{F,k,n}^a \hat{w}_{k,n}^a|^2}{I_{F,M,k,n} + I_{F,k,n} + \sigma_{k,n}^2},$$

as the SINR when CS of the FUE $k$ include node $a$. As well as, the average received SINR is defined $\Phi_k = \log_2(1 + \sum_{n \in N} \gamma_{k,n})$. Now, we introduce a new criterion constraint which determines node $a$ can join to the CS or not. Actually, because of the hybrid scheme, the number of the cooperative
Algorithm 1 Matching CTNSA Algorithm.

\section*{Step 1: Initialization:}
1. Preset $L_a^{req} = \emptyset$, $L_a^{rej} = \emptyset$, $F_{max}$, $N_a \forall k, a$.

\section*{Step 2: Utility Computation:}
2. Construct $P_{k,CS}$ using $\phi_{CS}^{k}(a) \forall k$.

\section*{Step 3: Find stable matching ($\mu_{CS}$) without externalities:}
3. while $\Sigma_{a, b} b_{k-a}^c \neq 0$ do
4. for each unassociated FUE $k$ do
5. while $D_{CS}^b \leq \epsilon$ do
6. find $a = \arg\max_{a \in \nu_{k,CS}} \phi_{CS}^{k}(a)$.
7. $b_{k-a}^c = 1$.
8. for each node $a$ do
9. $L_a^{req} \leftarrow \{k : b_{k-a}^c = 1, k \in K\}$.
10. construct $\nu_{a,CS}$ using $\phi_{CS}^{a}(k)$.
11. repeat
12. if $|A_k \cup \{a\}| \leq F_{max}$
13. accept $k = \arg\max_{a \in \nu_{k,CS}} \phi_{CS}^{k}(a)$.
14. $K_a := K_a \cup \{k\}$.
15. $A_k := A_k \cup \{a\}$.
16. end if
17. until $|K_a| = N_a$
18. $L_a^{req} := L_a^{req} \setminus K_a$.
19. remove node $a \in \nu_{k,CS}, \forall k \in L_a^{req}$.
20. end while
21. end while
22. Output: $\mu_{CS}$.

3) Structure of the CTNSA Algorithm: According to our descriptions on the network, we propose Algorithm 1 to perform cooperative node selection problem in an advanced manner. In this algorithm, after initialization, each user $k$ constructs its preference list $P_{k,CS}$ based on the utility function $\phi_{CS}^{k}(a)$ in (11). Afterward, user $k$ sends an attachment request $b_{k-a}^c$ to the most preferred transmission node $a$. This node adds user $k$ to its request list. Next, node $a$ constructs its preference list based on the new utility function $\phi_{CS}^{a}(k)$ which we have proposed it at (13), previously and check out the limitations on the size of CS and number of users which are connected to it. If these limitations can be satisfied, accepts request of the user $k$. In the same way, the user $k$ send request to the next preferred node as the other cooperative node until the quality criteria proposed at (16) be satisfied or the size of CS be overflowed.

4) Matching with externalities: As the performance of each FUE and transmission nodes is strongly affected by the dynamic formation of other FUE-FBSs links due to the dependence of the utility functions on externalities, the proposed game can be classified as a many-to-many matching game with externalities. Anomalous to many other papers which work in small cell domain and deals with conventional matching games, we assume that the individual players utilities practically are affected by the other players preferences. In fact, due to externalities, a player may prefer to change its preference order in response to the formation of other UE-SBS links. Therefore, we employ a new stability concept, based on the idea of swap-matching [14]. For the given matching $\mu_{CS}$, a swap-matching for any possible pair of FUEs $k , m \in K$ and FBSs $a, i \in A$ where $(a, k), (i, m) \in \mu_{CS}$, $a \in A_k$ and $i \in A_m$ is defined as $\mu_{CS,i,a} = \{\mu_{CS}(a, k) \cup \{m, k\} \}$. The given matching is stable if there exist no swap-matchings $\mu_{CS,i,a}$ such that $\phi_{CS}(\mu_{CS,i,a}) > \phi_{CS}(\mu_{CS})$ or in the other words, $\mu$ is stable if there is not any transmission node which FUE $k$ prefer to replaced in its accepted set ($A_k$) and there is not any FUE which node $a$ prefer to serve in its accepted set ($K_a$). In order to find a stable matching ($\mu_{CS}$), we propose Algorithm 2 which can update matching $\mu_{CS}$ based on the externalities. Line (5) of the algorithm indicate that FUE $k$ may prefer node $i$ based on the updated utility function. the Algorithm 2 monitors any preferred requests based on the given network and matching.

Algorithm 2 Swap-matching Algorithm.

\section*{Step 1: Perform initial matching:}
1. Import $\mu_{CS}$ through results in Algorithm 1

\section*{Step 2: Swap-matching Evaluation:}
2. repeat
3. the utility $\phi_{CS}$ is updated based on the current $\mu_{CS}$.
4. construct $\nu_{a,CS}$ and $\nu_{a,CS}$ based on the new $\phi_{CS}$.
5. if $(i, \mu_{CS,i,a}) \succ k, CS (a, \mu_{CS})$.
6. $b_{k-a}^c = 1$.
7. node $i$ computes $\phi_{CS,i,k}(\mu_{CS,i,a})$.
8. if $(k, \mu_{CS,i,a}) \succ i, CS (k, \mu_{CS})$.
9. $\mu_{CS} \leftarrow \mu_{CS,i,a}$.
10. end if
11. until $\forall(i, \mu_{CS,i,a}) \succ k, CS (a, \mu_{CS})$ and $(k, \mu_{CS,i,a}) \succ i, CS (k, \mu_{CS})$.
12. Output: $\mu_{CS}$.

B. enhanced Subcarrier Allocation (eCA)

The subcarrier allocation in PD-NOMA systems is investigated in many researches using the SCA approach or matching theory but none of them has directorship on the resource allocation design such that the interference of NOMA approach can be decreased. To solve this resource allocation problem, we propose a new method based on many-to-one matching structure.

$$\mu_{CA} \max_{\nu} \sum_{n} \sum_{k} r_{k,n} \nu_{k,n}, \quad (17a)$$
$$\text{s.t. } (9f), (9g), (9h).$$

In this problem $\nu_{k,n}$ indicate the allocation of subcarrier $n$ to user $k$. Optimization problem $\mu_{CA}$ can be defined by a tuple $(N, K, \nu_{CA}, \nu_{CA})$. Here, $\nu_{CA}$ and $\nu_{CA}$ denote the sets of the preference relations of FUEs and subcarriers, respectively.
Similar to the CTNSA algorithm, mapping function $\mu_{CA}$ is defined such that: 1) $n \in \mu_{CA}(k) \leftrightarrow k \in \mu_{CA}(n)$: $|\mu_{CA}(k)| \leq 1$ and $|\mu_{CA}(n)| \leq q_{\text{max}}$. As we mentioned at the previous algorithm, we propose some utility functions in order to decrease the impact of the extra-interference of PD-NOMA systems which can not be removed using SIC procedure. The utility function of the FUE $k$ and each subcarrier are described as follows:

$$\varphi_{CA}^k(n) = r_{k,n} = \log_2(1 + \gamma_{k,n}),$$ (18)

$$\varphi_{CA}^n(k) = T_{CA} \frac{r_{k,n} - R_k}{r_{k,n}} - \Theta_{k,n}.$$ (19)

To minimize the total interference caused by user $i$, $\forall i \in K$, we employ a new parameter as $\Theta_{k,n} = \sum_{a \in A}(\Theta_{a}^{CA} + \Theta_{a})$ where $T_{CA}$ is a weighting parameter.

We use Algorithm 3 to assign subcarriers in an advanced manner. For simplicity, we define the set of users with the same subcarrier as $C_n$ where each user constructs its preference list based on achievable data rate and sends an attachment request to all of the nodes in the CS which are determined in CTNSA algorithm. The called subcarrier accepts or rejects this proposal based on its preference list and utility function. If the subcarrier satisfies and the maximum number of users with the same subcarrier does not overflow, it can be assigned to user $k$. After the eCA algorithm we swap a matching algorithm similar to Algorithm 2.

Algorithm 3 Matching eCA Algorithm.

**Step 1: Initialization:**
1. Preset $L_{\text{req}} = \emptyset$, $L_{\text{req}} = \emptyset$, $q_{\text{max}}$, $\forall k,n$.

**Step 2: Utility Computation:**
2. construct $T_{k,CA}$ using $\varphi_{CA}^k(n)$ $\forall k$.

**Step 3: Find stable matching:**
3. while $\sum_{n \neq k} b_{k,n}^CA(t) \neq 0$ do
4. for each FUE $k$ without subcarrier do
5. $n = \arg\max_{n \in \mu_{CA}} \varphi_{CA}^n(k)$.
6. $b_{k,n}^CA = 1$.
7. for each node $n$ do
8. $L_{\text{req}} \leftarrow \{k: b_{k,n}^CA = 1, k \in \sum_{a \in A} K_a\}$.
9. construct $>_{\text{CA}}$ using $\varphi_{CA}^n(k)$.
10. repeat
11. $k = \arg\max_{k \in >_{\text{CA}}} \varphi_{CA}^n(k)$.
12. assign $n$ to the FUE $k$.
13. $C_n := C_n \cup \{k\}$.
14. until $|C_n| = q_{\text{max}}$.
15. $L_{\text{req}} \leftarrow L_{\text{req}} \setminus C_n$.
16. remove subchannel $n \in >_{\text{CA}}$, $\forall k \in L_{\text{req}}$.
17. end while
18. Output: $\mu_{CA}$.

1. **Worst-Case in No-CSI Situation:** Based on the additive error model and (5), we define uncertainty of channels in the Euclidean ball-shaped uncertainty sets as $H_{F,k,n}$, $H_{M,k,n}$ and $H_{M,F,n}$. In this case, we define $\zeta_{k,n}$, $\eta_{k,n}$ as the error bounds on the uncertainty region of the channel coefficients $h_{F,k,n}$, $h_{M,k,n}$ and $h_{M,F,n}$, respectively.

Since $|H_{F,k,n} h_{F,k,n}|^2$ is a nonlinear expression, we can apply the SDR method where $\nu^{H}A\nu = \text{trace}[A\nu^{H}]$. In this solution, we have confidence that $V_k,n = (w_k,n \circ p_k,n)^*(w_k,n \circ p_k,n) = W_k,n \circ g_k,n$ and $p_k,n = |p_k,n|^2$. The expression $\Delta_{F}$ is defined as a norm-bounded matrix, i.e., $\|\Delta_{F}\| \leq \epsilon_{F}$ and $H_{F,k,n} = h_{F,k,n}^H h_{F,k,n}$. Hence, $\epsilon_{F,k}$, $\epsilon_{M,k}$ and $\epsilon_{M,F,k}$ can be found as follows:

$$\|\Delta_{F,k,n}\| \leq \epsilon_{F,k} = \zeta_{k,n}^2 + 2\zeta_{k,n}^2 |h_{F,k,n}|,$$

$$\|\Delta_{M,k,n}\| \leq \epsilon_{M,k} = \eta_{k,n}^2 + 2\eta_{k,n}^2 |h_{F,k,n}|,$$

$$\|\Delta_{M,F,k,n}\| \leq \epsilon_{M,F,k} = \eta_{k,n}^2 + 2\eta_{k,n}^2 |h_{M,F,k,n}|.$$ (20)

In order to define the critical situations, we apply (20) with minimizing the numerator and maximizing the denominator of the SINR of users. Hence, (7) can be written as

$$\text{trace}(H_{F,k,n} - \epsilon_{F,k} I_{FT} F_{W,k,n} \circ g_k,n) - (2R_{c} - 1)(I_{W} + I_{F} + I_{M}) \geq (2R_{c} - 1)\sigma_{k,n}^2.$$ (21)

where $I_{W} = \sum_{n \neq k} \|h_{W,k,n}\| = \text{trace}(H_{F} + \epsilon_{F} I_{FT} F_{W,k,n} \circ g_k,n)$ and $I_{M} = \text{trace}(H_{M,F,k,n} + \epsilon_{M,F} I_{FT} F_{W,k,n} \circ g_k,n)$. As the same way, the critical equivalent of (9b) can be expressed as

$$\sum_{k \in K} \text{trace}(H_{M,F,k,n} + \epsilon_{M,F} I_{FT} F_{W,k,n} \circ g_k,n) \leq \epsilon_{M}.$$ (22)

The final problem in the worst-case scenario is rewritten as follows:

$$\max_{W_{k,n}, g_k,n} \left\{ \sum_{k \in K} \sum_{n \in N} \text{trace}(W_{k,n} \circ g_k,n) \right\}
\text{s.t.} \quad (8), (21) \text{ and } (22)
\quad 0 < \sum_{k \in K} \sum_{n \in N} \text{trace}(W_{k,n} \circ g_k,n) \leq P_{\text{max}},
\quad \text{if}\ W_{k,n} \geq 0, \text{ rank}(W_{k,n}) = 1.$$ (23a)

Due to the non-convex rate function, the optimization problem (23) is non-convex. To tackle this issue, the SCA approach with difference of two concave functions (D.C.) approximation method is used. In order to apply this method, at first the rate function in the objective is written as

$$r_{k,n} = f_{k,n} - g_{k,n},$$ (24)
where
\[
\begin{align*}
  f_{k,n} &= \log_2 \left( \text{trace}[ (H_{F_{k,n}} - \varepsilon_{F_{k,n}} I_{p_{F}}) (W_{k,n} \circ g_{k,n}) ] + I_{F_{M_{k,n}}}^W + I_{F_{k,n}}^W + \sigma^2_{k,n} \right), \\
  g_{k,n} &= \log_2 (I_{F_{M_{k,n}}}^W + I_{F_{k,n}}^W + \sigma^2_{k,n}).
\end{align*}
\]
By applying the D.C. approximation, \( g_{k,n} \) is approximated as follows
\[
\begin{align*}
  g_{k,n}(W_{k,n}) &\approx g_{k,n}(W_{k,n}^{[t-1]} + \langle \nabla g_{k,n}(W_{k,n}^{[t-1]}), (W_{k,n} - W_{k,n}^{[t-1]}) \rangle),
\end{align*}
\]
where
\[
\nabla g_{k,n}(W_{k,n}) = \begin{cases} 
0, \\
(H_{F_{k,n}} + \varepsilon_{F_{k,n}} I_{p_{F}})^T \circ g_{k,n}, \\
\ln(2) (I_{F_{M_{k,n}}}^W + I_{F_{k,n}}^W + \sigma^2_{k,n})^{-1/2}, 
\end{cases} 
\quad \forall i = k,
\]
\[
\nabla g_{k,n}(W_{k,n}) = \begin{cases} 
0, \\
\ln(2) (I_{F_{M_{k,n}}}^W + I_{F_{k,n}}^W + \sigma^2_{k,n})^{-1/2}, 
\end{cases} 
\quad \forall i \in K, \| h_{F_{k,n}} \| > \| h_{F_{k,n}} \|.
\]

To approximate (8) as the SIC constraint, we employ a combinatorial method. At the first stage, we apply the D.C. approximation on the SINR functions which are at two sides of the SIC inequality as follows:
\[
\begin{align*}
  \Gamma(w_{k,n}, p_{k,n}, h_{F_{k,n}}, h_{F_{M_{k,n}}}) &= f'_{k,n} - g'_{k,n}, \\
  \Gamma'(w_{k,n}, p_{k,n}, h_{F_{k,n}}, h_{F_{M_{k,n}}}) &= f''_{k,n} - g''_{k,n},
\end{align*}
\]
where \( f'_{k,n} \) and \( g'_{k,n} \) are the numerator and denominator of \( \Gamma(w_{k,n}, p_{k,n}, h_{F_{k,n}}, h_{F_{M_{k,n}}}) \), respectively. Gradient of \( g'_{k,n} \) are described as follow
\[
\nabla g'_{k,n}(W_{k,n}, h_{F_{k,n}}) = \alpha_{i,j}(h_{F_{j,n}} h_{F_{j,n}}^T) \circ g_{i,n},
\]
where \( \alpha_{i,j} \) equals one \( \forall i \in K, \| h_{F_{i,n}} \| > \| h_{F_{i,n}} \| \) and equals zero for others. As (29) and (30) are approximated by the D.C. solution where \( \nabla g''_{k,n}(W_{k,n}, h_{F_{k,n}}) \) can be calculated like (31). Next, we apply (32) and Euclidean ball-shaped uncertainty set on the approximated SIC constraint. According inequality (32) changes as follows
\[
\begin{align*}
  \text{trace}[ (H_{F_{j,n}} - H_{F_{k,n}} - (\varepsilon_{F_{j,n}} + \varepsilon_{F_{k,n}}) I_{p_{F}}) (W_{k,n} \circ g_{k,n}) ] + \text{trace}[ (H_{F_{k,n}} + \varepsilon_{F_{k,n}} I_{p_{F}}) T(W_{k,n}^{[t]} h_{F_{j,n}}) ] - \text{trace}[ (H_{F_{k,n}} - \varepsilon_{F_{k,n}} I_{p_{F}}) T(W_{k,n}^{[t]} h_{F_{j,n}}) ] + \sigma_{k,n} - \sigma_{j,n} \geq 0, \forall j, k \in K, n \in N.
\end{align*}
\]
where
\[
T(W_{k,n}^{[t]}, h_{F_{j,n}}, e_{i,n}) = \left( -\sum_{i \in K, \| h_{F_{i,n}} \| > \| h_{F_{i,n}} \|} [W_{k,n}^{[t-1]} + \alpha_{i,j}(W_{k,n}^{[t-1]} - \alpha_{i,j} W_{k,n}^{[t-1]} + \alpha_{i,j} W_{k,n}^{[t-1]} \circ g_{i,n}) ] - \sum_{i \in K, \| h_{F_{i,n}} \| > \| h_{F_{i,n}} \|} [W_{k,n}^{[t-1]} + \alpha_{i,j} W_{k,n}^{[t-1]} \circ g_{i,n}) ] \right)
\]
Since rank(.) is a non-convex constraint, it can be guaranteed with Gaussian randomization method. By applying the D.C. approximation, the optimization problem (23) is approximated by a convex function which can be solved by CVX toolbox.

2) Bernstein Approximation in Partial-CSI Case: In partial-case, we consider that distribution and covariance of the error vectors are clear and because of the independence between antennas in different FBSs, error vectors from all of the FBSs and MBS to the \( k^{th} \) user at the \( i^{th} \) subcarrier can be expressed as \( \epsilon \). Further, we can use the definition of cumulative distribution function (CDF) of exponential random variable (i.e., \( f(x) = \lambda e^{-\lambda x} \) for \( 0 \leq x \) and \( F(x, \lambda) = 1 - e^{-\lambda x} \) for \( x \geq 0 \) where in the paper, \( \lambda = 1 \) and \( x = \epsilon \) can be defined as follows:
\[
\Sigma_{k \in K} \text{trace}[ C_{e_{i,F_{M}} k} (W_{k,n} \circ g_{k,n}) ] \leq \frac{\epsilon}{\ln(1/n)}. \]
Let \( \mathbf{h}_{k,n}, v_{k,n} \) and \( C_{i,k,n}^{1/2} \) be defined as follows:
\[
\begin{align*}
  \mathbf{h}_{k,n} &= (\mathbf{h}_{F_{k,n}}, \mathbf{h}_{F_{M_{k,n}}}), \quad v_{k,n} = (v_{F_{k,n}}, v_{F_{M_{k,n}}}), \\
  C_{i,k,n}^{1/2} &= \begin{pmatrix} C_{i,F_{k,n}}^{1/2} & 0 \\ 0 & C_{i,F_{M_{k,n}}}^{1/2} \end{pmatrix},
\end{align*}
\]
where \( v_{k,n} \sim N(0, I_{p_{F}} + T_{F}) \). If we apply (34) and the SDR method, (9) can be expressed as follows:
\[
\begin{align*}
\max_{W, \rho} & \quad \sum_{k \in K} \sum_{n \in N, i \in K_{k,n}} \\
\text{s.t.} & \quad (5), (23b), (25c), (34) \\
& \quad \Pr \left\{ v_{k,n}^H A_{i,F_{k,n}} (v_{k,n} - \varrho_{k,n}) v_{k,n} + 2 \text{Re} \{ v_{k,n}^H b_{(w_{k,n} \circ \varrho_{k,n})} \} \right\} \geq c_{i,F_{k,n}} \geq 1 - \beta,
\end{align*}
\]
where \( A_{i,F_{k,n}} \), \( b_{(w_{k,n} \circ \varrho_{k,n})} \) and \( c_{i,F_{k,n}} \) are defined as follows:
\[
\begin{align*}
  A_{i,F_{k,n}} &= C_{i,F_{k,n}}^{1/2} W_{k,n} + C_{i,F_{M_{k,n}}}^{1/2}, \\
  b_{(w_{k,n} \circ \varrho_{k,n})} &= C_{i,F_{k,n}}^{1/2} W_{k,n} \mathbf{h}_{k,n}, \\
  c_{i,F_{k,n}} &= W_{k,n} \mathbf{h}_{k,n} + \varrho_{k,n}^2,
\end{align*}
\]
and
\[
\begin{align*}
  W_{k,n} &= \begin{pmatrix} D_{F_{k,n}} & 0 \\ 0 & -m_{m,n} H \end{pmatrix},
\end{align*}
\]
where
\[
\begin{align*}
  D_{F_{k,n}} &= \frac{1}{2} \sum_{i=1}^{n} W_{k,n} \circ \varrho_{k,i} - \sum_{i \in K, \| h_{F_{i,n}} \| > \| h_{F_{i,n}} \|} \| h_{F_{i,n}} \| W_{k,n} \circ \varrho_{i,n}. 
\end{align*}
\]
Since (37b) is a probabilistic inequality, we can use the Bernstein-Type Inequality for stochastic processes of quadratic forms of Gaussian variables with a conservative form lemma. Therefore, (37b) can replaced as follows:
\[
\begin{align*}
\text{trace}(A_{i,F_{k,n}}) - \sqrt{2\xi x - \xi y} &\geq c_{i,F_{k,n}}, \\
\| A_{i,F_{k,n}} \|_F^2 + 2 \| b_{(w_{k,n} \circ \varrho_{k,n})} \| &\leq x k_{n}, \\
y_k, n I_{p_{F}} T_{M} + A_{i,F_{k,n}} &\geq 0, y_k, n \geq 0,
\end{align*}
\]
where \( y = \max \{ \lambda_{\text{max}}(A), 0 \} \), i.e., \( y \) is the maximum eigenvalue of the matrix (\( A \)) and \( y \) and \( x \) are slack variables and \( \xi = -\ln(\beta) \). To minimize the transmit power, \( y \) must be the principal eigenvalue of \( C_{i,F_{M}} m_{m,n} H C_{i,F_{M}}^{1/2} \), i.e.,
where

\[
\mathcal{F}_{F,k,n} = \left\| \mathbf{C}_{e,F,k,n} \mathbf{m}_{k,n} \right\|_F^2 + 2\left\| \mathbf{C}_{i,F,k,n} \mathbf{m}_{k,n} \mathbf{h}_{F,k,n} \right\|^2.
\]

By applying (41a) and (41b), problem (37) can be rewritten as follows:

\[
\max_{W_{k,n}, r \in \mathcal{K}} \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N} T_{k,n}} \mathbf{m}_{k,n}^H \mathbf{C}_{e,F,k,n} \mathbf{m}_{k,n} + \mathbf{m}_{k,n}^H \mathbf{h}_{F,k,n},
\]

s.t. (38), (39a), (39b), (41a) and (41b).

Non-convex rate function can be approximated by (24) where

\[
f_{k,n} = \log_2 \left( \mathbf{h}_{F,k,n}^T (W_{k,n} \circ \mathbf{q}_{k,n}) \mathbf{h}_{F,k,n} + I_{F,k,n} + I_{F,M,k,n} + \sigma_n^2 \right),
\]

and

\[
g_{k,n} = \log_2 \left( I_{F,k,n} + I_{F,M,k,n} + \sigma_n^2 \right),
\]

where

\[
I_{F,M,k,n} = \mathbf{h}_{F,M,k,n}^T \mathbf{m}_{k,n} \mathbf{h}_{F,M,k,n}^H \text{ and } I_{F,k,n} = \sum_{i \neq k} \left( \| \mathbf{h}_{F,k,n} \|_2 \| \mathbf{h}_{F,i,n} \|_2 \| (W_{i,n} \circ \mathbf{q}_{i,n}) \mathbf{h}_{F,i,n} \|_2 + \| \mathbf{h}_{F,k,n} \|_2 \right).
\]

Approximation of \( g_{k,n} \) is calculated as (27) where

\[
\nabla g_{k,n}(W_{k,n}) = \begin{cases} 
0, & \forall i = k, \\
\left( \mathbf{h}_{F,k,n}^T \mathbf{m}_{k,n} \mathbf{h}_{F,k,n}^+ \right) \mathbf{q}_{i,n}, & \forall i \neq k, \| \mathbf{h}_{F,i,n} \|_2 \geq \| \mathbf{h}_{F,k,n} \|_2.
\end{cases}
\]

For the non-convex SIC constraint, we apply Bernstein solution on (29), (29) and (31) as follows

\[
\Pr \left\{ v_{j,n}^H \mathbf{A}' \mathbf{W}_{T_{k,n}} \mathbf{c}_{e,j,n} v_{j,n} + 2 \Re \{ v_{j,n}^H \mathbf{b} \mathbf{W}_{T_{k,n}} \mathbf{c}_{e,j,n} \} \right\} - \frac{2}{\sqrt{m_{j,n} + \sigma_k^2}} v_{j,n}^H = \left( \mathbf{v}_{j,n}^H, \mathbf{b} \mathbf{W}_{T_{k,n}} \mathbf{c}_{e,j,n} \right) \right) \geq 1 - \beta, \forall j, k, n,
\]

where \( \mathbf{A}', \mathbf{A}'', \mathbf{b}', \mathbf{b}'' , c' \) and \( c'' \) are acquired similar to (38). \( \mathbf{W}_{T_{k,n}} \) is defined with the same structure of (39) where instead of \( D_{F,k,n} \) we define \( D_{F,k,n} = W_{T_{k,n}} \circ \mathbf{q}_{k,n} - T_{\mathbf{W}_{T_{k,n}} \mathbf{h}_{F,k,n} \mathbf{a}_{e,j,n}} \). Hence, \( W_{T_{k,n}}'' \) is alike \( W_{T_{k,n}} \) based on

\[
D_{F,k,n} \text{ and } T_{\mathbf{W}_{T_{k,n}} \mathbf{h}_{F,k,n} \mathbf{a}_{e,j,n}}.
\]

After applying the Bernstein inequality with a new variable as \( x' \), we have

\[
\Pr \left\{ v_{j,n}^H \mathbf{A}' \mathbf{W}_{T_{k,n}} \mathbf{c}_{e,j,n} v_{j,n} + 2 \Re \{ v_{j,n}^H \mathbf{b} \mathbf{W}_{T_{k,n}} \mathbf{c}_{e,j,n} \} \right\} - \frac{2}{\sqrt{m_{j,n} + \sigma_k^2}} v_{j,n}^H = \left( \mathbf{v}_{j,n}^H, \mathbf{b} \mathbf{W}_{T_{k,n}} \mathbf{c}_{e,j,n} \right) \right) \geq 1 - \beta, \forall j, k, n.
\]

As the set of constraints that must be employed instead of the probabilistic SIC constraint where \( \theta = \| A \|_2^2 + 2 \| b \|_2 + \| A' \|_2^2 + 2 \| b' \|_2^2 \). If we employ replacement constraints, problem (43) can be efficiently solved with the CVX software package.

**D. The Joint Resource Allocation Algorithm**

We employ Algorithm 4 to solve problem (9). In this algorithm, some parameters are initialized, and after that communications between users and CSs are classified through the CTNSA algorithm. Then the subcarrier indicator can be obtained via the eCA algorithm as variable \( \nu^* \). The beamforming vector for each FBS is calculated using SCA and Gaussian randomization methods, respectively. This sequential iterative procedure continues until converged.

**Algorithm 4 Alternative Sequential Algorithm.**

**Step 1: Initialization:**

1. Choose a feasible \( W, \nu, \epsilon_c, \) and \( \epsilon_M \).
2. Find Stable ASM Solution:

**Step 2: Cooperative Set Up and Clustering:**

3. To find \( \chi^*(l) \), solve (10) for preset \( W \) and \( \nu \).

**Step 3: Subcarrier Allocation Update:**

4. To find \( \nu^*(l) \), solve (17) for preset \( W \) and \( \chi^*(l) \).

**Step 4: Beamforming Design Update:**

5. To find \( W^*(l) \), solve (23) and (43) for fix \( \chi^*(l) \) and \( \nu^*(l) \).

6. end while

**Output** \( W^*, \chi^* \) and \( \nu^* \).

**E. Convergence and stability of the iterative algorithm**

In this section, we prove convergence of the proposed solution. A property of stable algorithm in matching theory is defined as: there is any node that can join to or remove from the stable group which the algorithm calculated before. To prove this statement, we consider a stable group \( G_{n} \) for \( (k, a) \) pairs which is introduced at the end of the iteration \( l \). Then, we try to check that FUE \( E_k, E_k \) can not join to the \( G_{n} \). Because of the preference relation introduced in algorithms, CTNSA and eCA, FUE \( k, E_k \) is the most preferred one which overcomes to \( k \) and \( \Sigma_{k \in K} \Sigma_{a \in N} T_{k,n}(l-1) \leq \epsilon_c \) can not be realized. Similarly, we can derive that the stable pair \( (k, \alpha) \) can not remove from \( G_{n} \) and there is no matching \( \mu \) which
is preferred to stable $\mu^*$. More details of convergence prove of matching game can see Appendix A in [13]. Furthermore, convergence of the swap-matching method follows from some considerations as: the number of possible swaps is finite due to the fact that each FUEs can reach a limited number of FBs in its vicinity and also the subset of these swaps may be preferred.

Moreover, we prove that each of the proposed robust approaches with the DC approximation converges to its local optimum. Objective function of (23) and (43) are approximated by the DC approach where $g_{k,n}$ is concave and $\nabla g_{k,n}(W_{k,n})$ as its gradient is also its super-gradient [33]. Hence, in iteration $l$ we have

$$g_{k,n}(W_{k,n}^{[l]}) \leq g_{k,n}(W_{k,n}^{[l-1]}) +$$

$$\langle \nabla g_{k,n}(W_{k,n}^{[l-1]}), (W_{k,n}^{[l]} - W_{k,n}^{[l-1]}) \rangle.$$ \hspace{1cm} (48)

As the objective function equals (24), we have

$$f_{k,n}(W_{k,n}^{[l]}) - g_{k,n}(W_{k,n}^{[l]}) \geq$$

$$f_{k,n}(W_{k,n}^{[l]}) - [g_{k,n}(W_{k,n}^{[l-1]}) +$$

$$\langle \nabla g_{k,n}(W_{k,n}^{[l-1]}), (W_{k,n}^{[l]} - W_{k,n}^{[l-1]}) \rangle] \geq$$

$$f_{k,n}(W_{k,n}^{[l-1]}) - g_{k,n}(W_{k,n}^{[l-1]}).$$ \hspace{1cm} (49)

Due to (49), after iteration $l$, the objective of (23) and (43) improves against previous solution or is almost equal. As mentioned in [34], the SCA approach with DC approximation is guaranteed to converge to a local optimum. Consequently, we have

$$r_{k,n}(\varrho_{k,n}^{[l-1]}, W_{k,n}^{[l-1]}) \leq r_{k,n}(\varrho_{k,n}^{[l-1]}, W_{k,n}^{[l]}) \leq r_{k,n}(\varrho_{k,n}^{[l]}, W_{k,n}^{[l]}).$$ \hspace{1cm} (50)

Based on (49), solution at the end of each iteration is better than the previous iteration and for a finite set of transmit powers and channel gains, the optimal achievable sum rate is bounded above, procedure of improving the solutions always converges.

V. COMPUTATIONAL COMPLEXITY

In this section, we discuss the computational complexity of the proposed resource allocation approaches. The applied algorithm to solve (20) includes three steps: Cooperative node selection through CTNSA, determining the subcarrier allocation and beamforming. In CTNSA algorithm, we assume that the maximum allowable number of cooperative nodes are used. Therefore, the worst-case complexity of transmission node selection can be determined by $\Sigma_{n=1}^{N_{CS}} K_{CS}(l)(FT_f - a)$ where $K_{CS}(l)$ defines the number of FUEs join into the CS phase at iteration $l$. The complexity of eCA phase is determined by $K_{CA}(l)N_F T_f$ where $K_{CA}(l)$ is the number of FUEs join into the eCACS phase at iteration $l$. It is significant that complexity of both the no-CSI and partial CSI approaches is same at CS and eCA phases while in the beamforming phase they are different. In partial case, CVX selects the SDP method for power allocation. This method can be handled by the intermitter point method with a worst-case complexity of $O(\max\{m,n\}^t n^\frac{3}{2} \times \log(\varrho))$ with the solution accuracy $\varrho$ where $m$ is the number of constraints and $n$ is the problem size [28] and [31]. In power allocation $m = N_c \times (2K^2 + 2K + F)$ and $n$ equals $F \times T_f$. In the worst-case, all of the subcarrier and power allocation stages are similar to the partial method, but the number of constraints in power allocations is $m = N_c \times (2K^2 + K + F)$ and $n$ equals $F \times T_f$.  

VI. SIMULATION RESULTS

In this section, we evaluate the performance and complexity of the proposed algorithms. Moreover, the impact of the CoMP technology in different orders of cooperation is investigated.

A. Simulation setup

We consider a HetNet with 5 FBs where the $f^{th}$ FB is equipped by 2 antennas and $F_{\max}$ as descriptive parameter for the maximum allowable number of cooperative femtocells and receivers on the same subcarrier is preset 5 where the number of subcarriers is $N_c = 10$. The order of MISO is specified, i.e., $T_f = 2$ and $T_m = 8$. We consider that $\sigma_{\theta, n}^2 = 0.01$ for all results and $\epsilon_{\theta, s} = 0.15$. For simplicity, We set the variance of channel coefficients and errors as $\delta_{s}^2 = 1$, $\delta_{b}^2 = 0.05$ and $\delta_{e}^2 = 0.01$ which are equals for all of the users. Robustness parameters
assumed $\zeta = \kappa = 0.05$ and $\beta = 0.1$. Moreover, the target of achievable data rate is assumed $R_k = 0.1$ (bps/Hz) and the maximum power budget of the FBSs sets 40 dBm. For matching algorithms, we select $\gamma_{CS} = \gamma_{CA} = 100$, $c^{\beta}_{FM} = 5$ and $c^{\alpha}_{1} = 0.2$.

B. Simulation results

Fig. 2 shows the achievable sum rate versus $\epsilon_M$. As expected, applying robustness methods decreases achievable data rate as a robustness cost which is rational against perfect CSI conditions. We employ the loosely bounded robust solution as a worst-case method to ensure the performance of the network specially in critical conditions. We consider two different values for $\eta$. Moreover, the described Bernstein approach is employed as probabilistic partial method. We consider that $N_c = 10$, $K = 4$ and $P_{\text{max}} = 40$dBm. It is clear that when CSI is perfect more data rate can be achieved and by increasing the $\epsilon_M$, the sum rate is increased in all of the employed methods. Further, the achievable sum rate in the Bernstein approach is more than the other one and the performance of devised Bernstein method is near the ideal case. In addition to these, with $\eta = 0.15$, more sum rate is achieved compared with $\eta = 0.2$. 

Fig. 3 demonstrates the total achievable data rate versus the different $P_{\text{max}}$. By increasing the power budget of each FBS, the more sum rate can be achieved, but the incremental process is limited in higher values of $P_{\text{max}}$. Specially for worst-case approach, this limitation is strict. The performance of the stochastic approaches in partial CSI case is better than the deterministic approaches per different values of power. Hence, the Bernstein method can achieve to high sum rate with low robustness cost as power budget. Fig. 4 shows that employing CoMP in PD-NOMA systems, increases the achievable data rate and capacity of the network. Moreover, this incremental process is reduced in more number of the cooperative nodes. Therefore, we investigate hybrid scheme of CoMP where the number of nodes in the CS is variable based on the condition of the network.

To compare sensitivity of the proposed methods, we compute achievable sum rate versus variable robustness parameters. We consider that the robustness parameters of deterministic and probabilistic approach, i.e., $\eta$ and $\alpha$ are variable, i.e., each point of horizontal axis in Fig. 6 represents $\alpha$ and $\eta$ so that $\alpha = \eta$. The figure shows that although increasing $\alpha$ increases the achievable sum rate, this increment is not too much against another one. Actually, variation of $\eta$ has a great impact on results and the more $\eta$ increases, the more sum rate are attained in No-CSI. Fig. 6 illustrates that the performance of the worst-case method is related to the allowable error bounds. Therefore, worst-case is an unreliable and sensitive method. The shape of the ellipsoid that we choose in the worst-case method has a deep influence on results. Little $\eta$ gives more achievable data rate though the model of CSI uncertainty set may be inaccurate and unreliable. Furthermore, Fig. 7 shows the number of required iterations to achieve convergence. Based on the comparison performance of proposed methods, each of them converges to its throughput and convergence of the partial CSI method is sharp. Moreover, comparison between the partial and worst-case approaches shows that the computational complexity of the stochastic Bernstein approach is more than worst-case method.

VII. Conclusion

In this paper, a 5G network which works based on hybrid category of CoMP technology was considered. We proposed a new advanced method for the cooperative nodes association and subcarrier allocation problem based on matching game with externalities in this network. Further, due to the uncertainty of CSIT, two robustness schemes were considered, and based on them, two methods were proposed to ensure user’s satisfaction in both no-CSI and partial CSI cases. Because of the high signaling, we devised the worst-case method which is known as a low complexity with low performance method. Moreover, we proposed a probabilistic methods based on Bernstein inequality. As expected, by increasing the demand of users, more resources must be allocated to achieve the high quality performance of the network. To achieve high sum rate
Therefore, we can choose either the worst-case or Bernstein approach for transmit power compared with the no CSI methods. We also took into consideration the trade-off between performance and capacity, the Bernstein approach needs less robustness cost in terms of the computational complexity of the worst-case method is lower than that of the probabilistic approach. Therefore, we can choose either the worst-case or Bernstein approach based on our preference and facilities.

REFERENCES

[1] U. A, “Coordinated multi-point operation for LTE physical layer aspects (Release 11),” 3GPP TR, 2011-12.
[2] K. Y. Wang, N. Jacklin, Z. Ding, and C. Y. Chi, “Robust MISO Transmitting optimization under outage-based qos constraints in two-tier heterogeneous networks,” IEEE Transactions on Wireless Communications, vol.12, no.4, pp.1883-1897, 2013.
[3] N. Mokari, P. Azmi, S. Parsaeefard and H. Saeedi, “Robust ergodic up-link resource allocation in underlay OFDMA cognitive radio networks,” IEEE Transactions on Mobile Computing, vol. 15, pp. 419-431, March 2015.
[4] D. H. N. Nguyen, Long B. Le, Tho Le-Ngoc, “Optimal dynamic point selection for power minimization in multiuser downlink CoMP,” IEEE Signal Processing Magazine, vol. 16, pp. 619–633, 2016.
[5] R. Irmer, H. Droste, P. Marsch, M. Griebel, G. Fettweis, S. Brueck, H. P. Mayer, L. Thiele, and V. Jungnickel, “Coordinated multipoint: Concepts, performance, and field trials results,” IEEE Communications Magazine, vol.49, pp.102-111, February 2011.
[6] Qing Wei, Wanhu Sun, Bo Bai, Li Wang, Erik G. Strom, and Mei Song, “Resource Allocation for V2X Communications: A Local Search Based 3D Matching Approach,” IEEE International Conference on Communications (ICC), May 2017.
[7] L. Wang and H. Wu, “Fast pairing of device-to-device link underlay for spectrum sharing with cellular users,” IEEE Commun. Lett., vol. 18, no.10, pp. 1803–1806, Oct 2014.
[8] H. Zhang, L. Song, and Z. Han, “Radio resource allocation for device-todevice underlay communication using hypergraph theory,” IEEE Transactions on Wireless Communications, vol. 15, pp. 4852–4861, July 2016.
[9] T. Oosterwijk, “On local search and LP and SDP relaxations for k-set packing,” arXiv preprint arXiv:1507.07459 2015.
[10] Wenjun Xu, Xue Li, Chia-Han Lee, “Joint Sensing Duration Adaptation, User Matching, and Power Allocation for Cognitive OFDM-NOMA Systems,” IEEE Transactions on Wireless Communications, vol.17, pp.1269 – 1282, Feb 2018.
[11] Gongliang Liu, Ruisong Wang, Haijun Zhang, Wenjing Kang, Theodoros A. Victor C. M. Leung. Tsiftsis, “Super-Modular Game-Based User Scheduling and Power Allocation for Energy-Efficient NOMA Network,” IEEE Transactions on Wireless Communications, vol.17, pp.3877 – 3888, June 2018.
[12] Shiva Shamaei, Siavash Bayat ,Ali Mohammad Afshin Henmatyar, “Interference Management in D2D-Enabled Heterogeneous Cellular Networks Using Matching Theory,” IEEE Transactions on Mobile Computing, September 2018.
[13] Tuan LeAnh, Nguyen H. Tran, Walid Saad, Long Bao Le, Dusit Niyato, Tai Manh Ho, and Choong Seon Hong, “Matching Theory for Distributed User Association and Resource Allocation in Cognitive Femtocell Networks,” IEEE Tran. Veh. Technol., vol.66, Issue: 9, pp.8413 – 8428, Sept 2017.
[14] Francesco Pantisano, Mehdi Bennis, Walid Saad, Stefan Valentin, Mrouane Debbah, “Matching with externalities for context-aware user-cell association in small cell networks,” in Proc. IEEE Glob. Commun. Conf. (GLOBECOM), Atlanta, GA, USA, pp.4483 – 4488, Des 2013.
[15] D. Liu, L. Wang, Y. Chen, M. Elkashlan, K.-K. Wong, R. Schober, and L. Hanzo, “User association in 5G networks: A survey and an outlook,” IEEE Communications Surveys Tutorials, vol.18, no. 2, pp.1018 – 1044, 2016.
[16] A. Nemirovski and A. Shapiro, “Convex approximations of chance constrained programs,” SIAM Journal on Optimization, vol. 40, pp. 325-8, Nov 2006.
[17] V. Chandrasekhar, J. G. Andrews, and A. Gatherer, “Femtocell networks: A survey,” IEEE, Communications Magazine, vol. 46, no. 9, pp. 59-67, Sep. 2008.
[18] Z. Hu, L. J. Hong and L. Zhang, “A smooth Monte Carlo approach to joint chance-constrained programs,” IIE Transactions, vol. 45, no. 7, pp. 716–35, Jul 2013.
[19] L. A. Cheng, “A second-order cone programming approach for linear programs with joint probabilistic constraints,” Operations Research Letters, Sep 2012.
[20] L. J. Hong, Y. Yang, and L. Zhang, “Sequential convex approximations to joint chance constrained programs: A Monte Carlo approach,” INFORMS Operations Research, vol.59, no.3, pp.617-630, Jun 2011.
[21] A. Rezaei, P. Azmi, N. Mokari, “Robust Resource Allocation in Cloud-RAN Network Based on CoMP Technology,” in Proc. International Symposium on Telecommunications (IST2016), March 2017, pp.293 – 298.
[22] Y. Shi, J. Zhang, and K. B. Letaief, “Optimal stochastic coordinated beamforming for wireless cooperative networks with csi uncertainty,” IEEE Transactions on Signal Processing, vol.63, no.4, pp.960-973, 2015.
[23] C. Shen, K. Y. Wang, T. H. Chang, Z. Qiu, and C. Y. Chi, “Worst-case sinn constrained robust coordinated beamforming for multicell wireless systems,” in Proc. IEEE International Conference on Communications (ICC), pp.1-5, 2011.
[24] E. A. Gharavol, Y. C. Liang, “Robust downlink beamforming in multi-user MISO cognitive radio networks with imperfect channel state information,” IEEE Transactions on Vehicular Technology, vol. 59, no. 6, pp.2852-2860, Jul 2010.
[25] J. Liberti, S. Moshavi, and P. Zablocky, “Successive interference cancellation,” U.S. Patent 8670418 B2, Mar. 11th, 2014.
[26] Guixian Xu, Chia-Hsiang Lin, Weiguo Ma, Shanzhi Chen and Chong-Yung Chi, “Outage Constrained Robust Hybrid Coordinated Beamforming for Massive MIMO Enabled Heterogeneous Cellular Networks,” IEEE Access, vol.5, no.6, pp.13601 – 13616, March 2017.
[27] S. Parsaefard and A. R. Sharafat, “Robust distributed power control in cognitive radio networks,” IEEE Transactions on Mobile Computing, vol.12, no.4, pp.609-620, 2013.
[28] Z. Q. Luo, W. K. Ma, A. Man-cho So , Y. Ye and S. Zhang, “Semidefinite relaxation of quadratic optimization problems,” IEEE Signal Processing Magazine, vol. 27, no. 3, May 2010.
[29] K.Y. Wang, T.H. Chang and Wing-Kin Ma, “Probabilistic SINR constrained robust transmit beamforming: A Bernstein-type inequality based conservative approach,” in Proc. IEEE International Conference on Speech and Signal Processing (ICASSP), May 2011, pp. 3080-3083.

[30] Vincent K. N. Lau and Y. Cui, “Low complexity delay-constrained beamforming for multi-user MIMO systems with imperfect CSIT,” IEEE Transactions on Signal Processing, vol. 61, no. 16, pp. 4090-9, Aug 2013.

[31] N. Mokari, F. Alavi, S. Parsaeefard, Tho Le-Ngoc, “Limited-Feedback Resource Allocation in Heterogeneous Cellular Networks,” IEEE Transactions on Vehicular Technology, vol.65, Issue: 4, pp.2509 – 2521, April 2016.

[32] Emil Bjornson, Gan Zhengi, Bjorn Ottersten, “Robust Monotonic Optimization Framework for Multicell MISO Systems,” IEEE Transactions on Signal Processing, vol.60, May 2012.

[33] H.H. Kha, H.D.Tuan, Ha H. Nguyen, “Fast Global Optimal Power Allocation in Wireless Networks by Local D.C. Programming,” IEEE Transactions on Wireless Communications, vol.11, no.2, pp.510 – 515, February 2012.

[34] Duy Trong Ngo, Suman Khakurel, Tho Le-Ngoc, “Joint Subchannel Assignment and Power Allocation for OFDMA Femtocell Networks,” IEEE Transactions on Wireless Communications, vol.13, no.1, pp.342 – 355, January 2014.