Model of a protoplanetary disk forming in-situ the major Uranian satellites before the planet is formed

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ABSTRACT

We fit an isothermal oscillatory density model of Uranus’ protoplanetary disk to the present-day major satellites and we determine the radial scale length of the disk, the equation of state and the central density of the primordial gas, and the rotational state of the Uranian nebula. This disk does not at all look like the Jovian disk that we modeled previously. Its rotation parameter that measures centrifugal support against self-gravity is a lot smaller ($\beta_0 = 0.00507$), as is the radial scale length (only 27.6 km) and the size of the disk (only 0.60 Gm). On the other hand, the central density of the compact Uranian core is higher by a factor of 180 and its core’s angular velocity is about 2.3 times that of Jupiter’s core (a rotation period of 3.0 d as opposed to 6.8 d). Yet, the rotation of the disk is sufficiently slow to guarantee its long-term stability against self-gravity induced instabilities for millions of years.

Keywords. planets and satellites: dynamical evolution and stability—planets and satellites: formation—protoplanetary disks

1. Introduction

In previous work (Christodoulou & Kazanas 2019\textsuperscript{a, b}), we presented isothermal models of the solar and the Jovian primordial nebulae capable of forming protoplanets and protosatellites, respectively, long before the central object is actually formed by accretion processes. This entirely new “bottom-up” formation scenario is currently observed in real time by the highest-resolution (~1-5 AU) observations of many protostellar disks by the ALMA telescope (ALMA Partnership 2015; Andrews et al. 2018; Ruane 2017; Lee et al. 2017). In addition to structural differences, the analytic (intrinsic) and numerical (oscillatory) solutions of the isothermal Lane-Emden equation and the resulting model of the gaseous nebula have been described in detail in van der Marel et al. (2019). In this work, we apply the same model to Uranus’ protoplanetary disk that formed its six major satellites. Our goal is to compare our best-fit model of Uranus’ primordial nebula to Jupiter’s nebula and to find similarities and differences between the two disks that hosted gravitational potential minima in which the orbiting moons could form in relative safety over millions of years of evolution.

As was expected, the two model nebulae are very different in their radial scale lengths (27.6 km versus 368 km, for Uranus and Jupiter, respectively) and their sizes (0.60 Gm versus 12 Gm, respectively) and central densities (55.6 g cm\textsuperscript{-3} versus 0.31 g cm\textsuperscript{-3}, respectively). In addition to structural differences, the disks are significantly different in their remaining physical quantities: Uranus’ core is smaller by about a factor of 2 ($R_1 \approx 0.1$ Gm), the radial density profile is shallower ($k \approx -1$), and there is no need for an outer flat-density region; also, Uranus’ disk enjoys a lot lower rotational support against self-gravity than Jupiter’s disk ($\beta_0 \approx 5 \times 10^{-3}$ versus $\beta_0 \approx 3 \times 10^{-2}$, respectively).

The extremely high gas densities and the mild differential rotation speeds in Uranus’ compact disk signify that its major equatorial moons were formed long before the planet was actually fully formed; but not before the protoplanet was knocked over to its current axial tilt of 98 degrees. This is because all of the inner moons orbit at nearly zero inclination to the planet’s equator and this means that the accretion disk was formed around the protoplanetary core after it had been tilted severely by a very early giant impact.

The analytic (intrinsic) and numerical (oscillatory) solutions of the isothermal Lane-Emden equation and the resulting model of the gaseous nebula have been described in detail in Christodoulou & Kazanas (2019\textsuperscript{b}) for Jupiter’s disk, and there is no need of repeating the descriptions here. In what follows, we apply in § 2 our model nebula to the major moons of Uranus and we compare the best-fit results to Jupiter’s extended Model 2. In § 3 we summarize and discuss our results.

2. Physical Model of Uranus’ Protoplanetary Disk

2.1. Best-Fit Uranian disk model

The numerical integrations that produce oscillatory density profiles were performed with the MATLAB ode15s integrator (Shampine & Reichelt 1997; Shampine et al. 1999) and the optimization used the Nelder-Mead simplex algorithm as implemented by Lagarias et al. (1998). This method (MATLAB routine fminsearch) does not use any numerical or analytical gradients in its search procedure which makes it extremely stable numerically, albeit somewhat slow.

In Fig. 1 we show the best optimized fit to the semimajor axes of the moons of Uranus. The innermost small moon Cordelia was used along with the other satellites (Puck, Miranda, Ariel, Umbriel, Titania, and Oberon) because its inclusion improves the best fit substantially. The need for including Cordelia
moon Oberon \((R_{\text{max}} \approx 0.60 \, \text{Gm})\). The next outer density peak lies at a distance of 0.95 Gm around which no moon is known. In fact, the disk of Uranus must have been really small \((< 1 \, \text{Gm} \) in radial extent) because the next outer irregular moon, Francisco, has a semimajor axis of 4.3 Gm.

### 2.2. Physical parameters from the best-fit model

Using the scale length of the disk \(R_0\) and the definition \(R_0^2 = c_0^2/(4\pi G \rho_0)\), we write the equation of state for the Uranian circumplanetary gas as

\[
\frac{c_0^2}{\rho_0} = 4\pi G R_0^2 = 6.39 \times 10^6 \, \text{cm}^2 \, \text{g}^{-1} \, \text{s}^{-2},
\]

where \(c_0\) and \(\rho_0\) are the local sound speed and the local density in the inner disk, respectively, and \(G\) is the gravitational constant. For an isothermal gas at temperature \(T\), \(c_0^2 = RT/\mu\), where \(\mu\) is the mean molecular weight and \(R\) is the universal gas constant. Hence, eq. (1) can be rewritten as

\[
\rho_0 = 13.0 \left(\frac{T}{\mu}\right) \, \text{g} \, \text{cm}^{-3},
\]

where \(T\) and \(\mu\) are measured in degrees Kelvin and \(\text{g} \, \text{mol}^{-1}\), respectively.

For the coldest gas with \(T \geq 10 \, \text{K}\) and \(\mu = 2.34 \, \text{g} \, \text{mol}^{-1}\) (molecular hydrogen and neutral helium with fractional abundances \(X = 0.70\) and \(Y = 0.28\) by mass, respectively), we find that

\[
\rho_0 \geq 55.6 \, \text{g} \, \text{cm}^{-3}.
\]

This extremely high value implies that the conditions for protosatellite formation were already in place during the early isothermal phase (Tohline 2002) of the Uranian nebula.

Using the above characteristic density \(\rho_0\) of the inner disk in the definition of \(\Omega_j \equiv \sqrt{2\pi G \rho_0}\), we determine the Jeans frequency of the disk:

\[
\Omega_j = 4.8 \times 10^{-3} \, \text{rad} \, \text{s}^{-1}.
\]

Then, using the model’s value \(\beta_0 = 0.00507\) in the definition of \(\beta_0 \equiv \Omega_j/\Omega_j\), we determine the angular velocity of the uniformly-rotating core \((R_1 \leq 0.0967 \, \text{Gm})\), viz.

\[
\Omega_0 = 2.5 \times 10^{-5} \, \text{rad} \, \text{s}^{-1}.
\]

For reference, this value of \(\Omega_0\) for the core of the Uranian nebula corresponds to an orbital period of \(P_j = 3.0\) d. This value is close to the present-day orbital period of Ariel (2.5 d), but it is not near the orbital period of the largest moon Titania (8.7 d). This is a deviation from what we found for the solar system and for Jupiter. Nevertheless, the large outer moons of Uranus are all comparable in mass and size, so our previous finding remains valid: the angular velocity of the core of the primordial nebula is comparable to the present-day angular velocities of the largest regular satellites.

### 2.3. Comparison between the models of Uranus and Jupiter

We show a comparison between the physical parameters of the best-fit models of Uranus and Jupiter in Table 1. Obviously, these two protoplanetary disks are very different in most of their physical properties. The disk of Uranus is a lot smaller \((R_{\text{max}})\), more
compact ($R_0$), and denser ($\rho_0$) by a factor of 180. In addition, the disk of Uranus is just as cold (assuming that $T = 10$ K), a lot heavier ($\Omega_1$), and its core is rotating ($\Omega_2$) more than twice as fast (still, this is a slow rotation with a period $P_0$ of only 3.0 d).

The power-law index of the Uranian nebular model is $k \approx -1$ (surface density $\Sigma \propto R^{-1}$), unlike the Jovian nebula and the solar nebula ($k \approx -1.5$, $\Sigma \propto R^{-1.5}$). This range of values of $k$ has been observed in studies of young circumstellar disks in the pre-ALMA era (Andrews & Williams 2007; Hung et al. 2010; Lee et al. 2018, and references within).

Although the disk of Uranus is small, it is still very heavy and hosts high densities of gas, thus also of ices and planetesimals. As we found for Jupiter’s disk, these conditions support a “bottom-up” hierarchical formation in which protosatellites are seeded early inside such nebular disks and long before their protoplanets are fully formed; these compact moon systems complete their formation in $< 0.1$ Myr (Harsono et al. 2018) and long before the central stars become fully formed (Greaves & Rice 2010).

### Table 1. Comparison of the protoplanetary disks of Jupiter and Uranus

| Property | Symbol (Unit) | Jupiter’s Model 2 | Uranus’ Best-Fit Model |
|----------|--------------|------------------|------------------------|
| Density power-law index | $k$ | -1.4 | -0.96 |
| Rotational parameter | $\beta_0$ | 0.0295 | 0.00507 |
| Inner flat-density radius | $R_1$ (Gm) | 0.220 | 0.0967 |
| Outer flat-density radius | $R_2$ (Gm) | 5.37 | - |
| Scale length | $R_0$ (km) | 368 | 27.6 |
| Equation of state | $c_s^2/\rho_0$ (cm$^2$ g$^{-1}$ s$^{-2}$) | $1.14 \times 10^9$ | $6.39 \times 10^6$ |
| Minimum core density for $T = 10$ K, $\pi = 2.34$ | $\rho_0$ (g cm$^{-3}$) | 0.31 | 55.6 |
| Isolational sound speed for $T = 10$ K, $\pi = 2.34$ | $c_0$ (m s$^{-1}$) | 188 | 188 |
| Jeans gravitational frequency | $\Omega_j$ (rad s$^{-1}$) | $3.6 \times 10^{-4}$ | $4.8 \times 10^{-3}$ |
| Core angular velocity | $\Omega_0$ (rad s$^{-1}$) | $1.1 \times 10^{-5}$ | $2.5 \times 10^{-5}$ |
| Core rotation period | $P_1$ (d) | 6.8 | 3.0 |
| Maximum disk size | $R_{\text{max}}$ (Gm) | 12 | 0.60 |

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