The Power of Worldsheets:
Applications and Prospects

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Abstract

We explain how perturbative string theory can be viewed as an exactly renormalizable Weyl invariant quantum mechanics in the worldsheet representation clarifying why string scattering amplitudes are both finite and unambiguously normalized and explaining the origin of UV-IR relations in spacetime. As applications we examine the worldsheet representation of nonperturbative type IB states and of string solitons. We conclude with an analysis of the thermodynamics of a free closed string gas establishing the absence of the Hagedorn phase transition. We show that the 10D heterotic strings share a stable finite temperature ground state with gauge group $SO(16) \times SO(16)$. The free energy at the self-dual Kosterlitz-Thouless phase transition is minimized with finite entropy and positive specific heat. The open and closed string gas transitions to a confining long string phase at a temperature at or below the string scale in the presence of an external electric field.

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1 Introduction

In this talk, I will make the argument that our strongest insights into a fully nonperturbative framework for string/M theory are to be found in the worldsheet formalism for perturbative string theory [2, 3]. On the other hand, since the spacetime low energy effective action is the likely first point of detailed contact with any candidate nonperturbative framework, it is crucial that any wisdom gained from worldsheet calculations is incorporated into formulating precise relations among both the couplings in the spacetime effective action, as well as the quantum amplitudes of the low energy effective theory at the string scale. Such relations may be interpreted as the specification of renormalization conditions on the candidate nonperturbative theory holding at the string mass scale, $\alpha'^{-1/2}$. They will play an important role in the next wave of research into candidate nonperturbative frameworks such as the reduced matrix models of [1].

Perturbative string theory originated in the Dual Model, a phenomenological description of mesons and hadrons that pre-dates the discovery of asymptotic freedom and subsequent development of QCD as the theory of the strong interactions [4, 5]. The spacetime viewpoint and introduction of the worldsheet formalism came with the realization that many properties of dual model amplitudes, such as S–T channel duality, are automatic in a quantum theory of one-dimensional extended objects or strings. The open string endpoints carry charge, and the lowest-lying excitation in the open string spectrum is identified with a vector boson. Nonabelian symmetry is straightforwardly introduced by assigning the endpoint wavefunctions to the fundamental representations of an internal symmetry group. An added bonus is the discovery that the one-loop amplitude of an open string theory always factorizes on a massless spin two closed string excitation, motivating the natural interpretation of string theory as a unified formulation of gravity and gauge theory. The theory is characterized by a dimensionful scale corresponding to the string tension, $\alpha'^{-1/2}$, and a dimensionless coupling, $g_s$, the strength of the cubic string interaction vertex. It is therefore natural to identify the string tension of fundamental strings with the Planck mass scale, at which the gauge and gravitational interactions are of comparable strength. Note that since closed string interactions only result in producing closed strings, it is consistent with perturbative unitarity to have a pure closed string theory with $g_s = g_{\text{closed}}$. In the open and closed string theory, consistency requires in addition the tree level relation between couplings, $g_{\text{closed}} = g_{\text{open}}^2$.

2 A Weyl-invariant Quantum Mechanics

The full beauty of perturbative string theory becomes transparent upon detailed examination of the loop expansion. The world-sheet representation of string loop amplitudes implies the existence of a single graph at each order in loop perturbation theory. The loop amplitudes in string theory can be equivalently interpreted as quantum correlators in a two-dimensional “gauge theory”, where the gauge symmetry is general coordinate invariance [2]. The S-matrix describing the scattering of asymptotic string states is obtained by invoking two-dimensional conformal invariance to represent asymptotic on-shell states as operator insertions that are local in the two-dimensional sense, i.e., on the worldsheet. Off-shell string states are boundaries on the worldsheet—either macroscopic closed loops or macroscopic line segments, localized instead in the embedding spacetime [14]. This correspondence between worldsheet and spacetime pictures has remarkable consequences. Consider
a gauge invariant path integral expression for the generic Greens function at arbitrary order in the string loop expansion. Upon gauge fixing to conformal gauge, the path integral over metrics is restricted to the fiducial representative from each conformally inequivalent class of metrics. This is an ordinary integral over the finite number of moduli parameters of Riemann surfaces of fixed topology. Remarkably, in any critical string theory both the measure of the path integral, the functional determinants, and vertex operator insertions, can be unambiguously computed as functions of the moduli, while preserving the full Diffeomorphism $\times$ Weyl gauge invariance. The result is an unambiguously normalized and ultraviolet finite expression for the Greens function with a well-defined, zero string tension, field theory limit.

The resulting expressions for the field theory Greens functions are free of ultraviolet regulator ambiguity. More importantly, in an infrared finite string theory, they are also free of ambiguity in the choice of renormalization scheme [3, 9]. Both properties are a consequence of having maintained manifest two dimensional general coordinate invariance in computing the full string theory Greens function prior to taking the field theory limit defined as follows: we factorize on massless mode exchange in either open or closed string sectors, projecting also onto the massless on- or off-shell modes in any external states, and integrating out the worldsheet modulus dependence of the resulting expression. The result is a coupling in the field theory in which we can smoothly take the zero string tension limit. Thus, the expression for any renormalized string theory n-point Greens function, including in particular the corresponding field theory limit, is independent of dependence on the string tension, $\alpha'^{-1/2}$, which plays the role of an ultraviolet cutoff in spacetime.

String theory can therefore be more simply understood as a renormalizable scale invariant quantum mechanics with a single Wilsonian renormalization in the two-dimensional sense: the worldsheet cosmological constant renormalizes to zero in the worldsheet infrared regime, simultaneous with setting the string tension to zero. Thus, spacetime ultraviolet corresponds to worldsheet infrared [3]. Corrections to the tree-level relations for the coupling constants and masses— measured in units of string tension, are given at each order in the string loop expansion by an unambiguously normalized, and finite, 2d general coordinate invariant string amplitude [3]. Factorizing on the lowest-lying modes— those that are massless at tree level, gives the loop corrections to the tree-level values of the coupling constants and masses in the field theory. Once again the results are independent of dependence on the spacetime uv cutoff. But, more importantly, due to modular invariance of closed string amplitudes or open-closed world-sheet duality of open and closed string amplitudes, the worldsheet infrared and worldsheet ultraviolet asymptotics of any string amplitude are closely related [5]. A well-known consequence is the existence of spacetime UV-IR relations in any perturbative string theory [3, 13, 8, 24].

The renormalizability of perturbative string theory is obscured from the viewpoint of the spacetime Lagrangian. Since the renormalized Greens functions for massless fields are computed in an $\alpha'$ expansion using the nonrenormalizable Wilsonian effective action with ultraviolet cutoff of order the string scale, it is simply not possible in this framework to infer that the cutoff can in fact be removed. The conclusion that string theory is perturbatively renormalizable relied crucially on the existence of an all orders in $\alpha'$ worldsheet representation of the Greens functions, recasting the computation as a problem in quantum mechanics instead of in a field theory. A remarkable consequence is that, unlike in quantum field theories, the dimensionless vacuum energy density in string theory— where we rescale by the spatial volume and inverse temperature defining, more
precisely, the effective action functional \[3, 24, 25\], is finite: a calculable quantity in any infrared finite background of the string, as was first noted in \[3\].

The discovery of supersymmetry in the mass spectrum of free strings with worldsheet fermionic degrees of freedom results in supersymmetric string theories, where nonrenormalization theorems protect the tree-level relations between masses and couplings. Rapid developments in the eighties culminated in the discovery of anomaly free superstring theories with massless fermions in chiral representations of the gauge group \[4, 5\]. Ultraviolet finiteness was firmly established at the one-loop level for the ten-dimensional SYM-supergravity obtained in the low energy field theory limit of vanishing string tension: the tower of excitations with Planck scale masses decouples, leaving only massless modes but with the key non-renormalizable couplings necessary for establishing the absence of gauge and gravitational anomalies. The nonrenormalizable terms are the chief remnant signal in the ten-dimensional Wilsonian effective Lagrangian of its string theory origin.

Discovery of the anomaly-free and UV finite type IB O(32) theory of open and closed strings, the non-chiral type IIA and chiral type IIB closed strings, and the heterotic O(32) and \(E_8 \times E_8\) closed strings, brings the number of distinct perturbatively consistent ten-dimensional supersymmetric string theories to five. This puzzling multiplicity of consistent perturbation theories was finally removed with the insightful union of the five perturbative string limits by strong-weak coupling and target space dualities. Finally, with the identification of Dbranes as nonperturbative gauge-gravity solitons of zero width, charged also with respect to one or more antisymmetric p-form potential where \(0 \leq p \leq 10\), and playing a crucial role in the conjectured string dualities, we have come full circle. We return to discover a new, and deep, underlying relationship between nonperturbative string theory and gauge theory. The focus on reduced matrix models in \[1\], and citations thereof, should be understood in light of these previous results.

3 UV Asymptotics of Open and Closed Strings

Covariant open and closed string amplitudes are expressed as reparameterization invariant sums over open Riemann surfaces with boundaries. Surfaces with croscaps must be included in the Polyakov sum whenever a Dirichlet boundary is localized on an orientifold, an orientation-reversing hyperplane in the embedding spacetime \[4\]. In \[8\] I describe the worldsheet representation of the generic open and closed string amplitude with Dirichlet boundaries in the Fenchel-Nielsen parameterization of the moduli space of Riemann surfaces—distinguished by both the simplicity of the measure for moduli, and knowledge of the domain of modular integration \[7\]. \[8\] examines the infrared and ultraviolet finiteness of the generic orientable bosonic open and closed string amplitude introducing, for the sake of pedagogy, both ultraviolet and infrared regulators on the worldsheet, preserving both T-duality and open-closed worldsheet duality. This prescription constitutes an explicit violation of scale invariance, but it can be verified that the regulators can be removed at the end of the calculation \[8\]. \[8\] includes a self-contained account of a beautiful formalism linking the eigenvalue problem for the full set of invariant differential operators on the worldsheet to the geometry of the integration domain of the moduli of the Riemann surface, following uniformization to the hyperbolic upper half plane. The eigenvalue spectrum of the scalar Laplacian has, in turn, a precise isomorphism to the length spectrum of geodesics, enabling both deduction of the Fricke-Klein moduli, and an explicit characterization of the boundary of the modular domain.
These results enable a rather simple analysis of the spacetime ultraviolet and infrared asymptotics of generic multiloop amplitudes. Using Selberg trace techniques [7], and Huber’s exponential bound on the asymptotic growth in the number of boundary geodesics of fixed length \( L \), in the limit of large \( L \), I show conclusively that the spacetime ultraviolet limit is benign [8], even in open string amplitudes where the potentially divergent contribution from the lowest-lying modes of the open string spectrum is included in the modular integration. Invoking open-closed worldsheet duality, we can always map this regime into the \( \ell \to 0 \) regime dominated by the lowest-lying closed string modes. It can be shown that the potential divergence from the “pinched handle” is given by the shortest length geodesic, corresponding to exchange of a closed string tachyon [7]. Thus, in the absence of both open and closed string tachyons, or in an infrared finite string theory, ultraviolet finiteness is automatic [5].

4 UV Asymptotics of Off-shell String Amplitudes

A manifestly Weyl-invariant Poyakov path integral representation of off-shell string amplitudes was introduced in the ingenious paper [10]. In [11], we formulate the problem of computing from first principles the pair correlation function of Wilson loops with fixed spatial separation in constant external Maxwell (electric) field. This configuration is a probe of sub-string-scale short distance physics. We assume Wilson loops with fixed spatial separation, \( r \), where \( r \) is much smaller than the loop length, \( l \). Note that our gauge theory results will not rely on taking a large \( N \) limit, and are more fruitfully interpreted in the abelian theory on the worldvolume of each Dbrane, the total number of Dbranes, \( N \), being fixed by the overall consistency of the string theory [3]. The Dbrane carries, in addition, a constant external electric field. In the case of the type I and type II strings, it may also support one or more constant higher rank antisymmetric gauge potentials coupling to Dbranes, as in [12, 15].

To obtain a gauge theory correlation function leading to a closed form expression for the short distance sub-string-scale manifestation of the familiar heavy quark potential, we extract the worldsheet infrared “field theory” asymptotics of the off-shell closed string propagator between loops, \( C_i \), \( C_f \), with fixed spatial separation \( r \) in spacetime. The loops represent the proper time evolution of a pair of semiclassical heavy quarks— in slow relative motion in a direction, \( \hat{i} \), orthogonal to the direction of fixed spatial separation, \( \hat{j} \). Thus, the pair of quarks is free to move as a unit in the embedding spacetime, albeit at small velocity, while maintaining a fixed spatial separation. In the open and closed bosonic string theory, the Wilson loops lie in the worldvolume of the spacefilling D25branes. The small “velocity”, \( \tanh u \), is simply the strength of the constant electric field, \( F_0 i \), as in [13].

Why is this a worldsheet representation of the short distance Wilson loop correlator, and not that of the short distance potential between bosonic Dbranes discussed in [13]? The two string computations differ in the specification of the boundary value problem on the worldsheet. In [11], we sum over all worldsheets of cylindrical topology connecting two loops with fixed spatial separation, moving as a unit within the worldvolume of a space-filling D25brane. The computation in [13] gives instead the potential between a pair of D0branes in slow relative motion, with a slow monotonic increase in spatial separation, although \( r(\tau) \) is assumed to be a sub-string-scale distance. Wilson loop boundary conditions in the gauge theory require that we impose Dirichlet boundary conditions
in the string theory, and D25branes are, of course, hypersurfaces in the embedding spacetime where bosonic open string endpoints satisfy a Dirichlet boundary condition. It will become apparent that the physics is \textit{abelian}, independent of the total number of D25branes which is determined by the overall consistency of the string theory.

Consider the classic problem of giving an analytic description of the short distance sub-string-scale manifestation of the abelian flux tube linking a pair of semiclassical heavy quarks. We cannot, of course, extend the effective flux tube picture of abelian gauge theory into the short distance regime since the theory is not asymptotically free. However, in the presence of constant external fields, we can probe \textit{arbitrarily} short sub-string-scale distances in an open and closed string theory \cite{13, 11, 12}. For the bosonic string, where we have no control on strong-weak coupling duality, we will simply complete the systematics of this worldsheet calculation ignoring the question of what dynamics stabilizes such a short abelian flux tube. The crucial new element in the worldsheet calculation of the off-shell closed string propagator is boundary reparameterization invariance. We complete the work begun in \cite{10}, giving a simple extension of Polyakov’s treatment of the manifestly two-dimensional Diffeomorphism $\times$ Weyl invariant path integral over bulk worldsheet metrics to the path integral over boundary einbeins, assuming, for simplicity, a 1-to-1 mapping of each worldsheet boundary into a corresponding circular Wilson loop \cite{11}.

In the worldsheet infrared limit, and with $l >> r$, we obtain an expression for the sub-string-scale manifestation of the closed time propagator of a flux tube in a nonsupersymmetric abelian gauge theory. This is, of course, only a formal solution to the boundary value problem since we have not addressed the dynamics responsible for the formation of this classical configuration. Extracting the effective potential, $V_{\text{eff}} = -\Gamma_0/TV$, where $\Gamma_0$ is the gauge theory limit of the off-shell closed string propagator, we find a universal static potential, $(d - 2)/r$, where $d = 26$ is the dimension of the embedding spacetime. The velocity dependent corrections— present also in the analogous result for the type IIB superstring, can be computed in a systematic double expansion in small velocity and short distance. The result probes distance scales down to $r_{\text{min}}^2 \sim 2\pi\alpha'\tanh^{-1}v$, assuming nonrelativistic velocity $v$.

Ref. \cite{12} gives the supersymmetrization of this result, summing over surfaces of cylindrical topology between closely separated circular Wilson loops lying in the worldvolume of a spacefilling Dbrane in the type IIB superstring. The phases in the path integral representation of the off-shell type IIB closed string propagator between Wilson loops are, a priori, unknown, and correspond to the weighted sum over worldsheet spin structures. We will require the absence of both the leading contribution, which is a tachyonic mode, and the next-to-leading term, which gives the static potential. This ensures that upon application of open-closed worldsheet duality— which maps the short distance gauge potential into a long distance supergravity potential \cite{13, 14}, we preserve the full N=2 spacetime supersymmetry for vanishing constant external field. Turning on a constant electric field induces an external field-dependent short distance potential, also breaking half of the spacetime supersymmetries. For Wilson loops, $C_i$, $C_f$, wrapped about some spacelike compact direction the potential takes the form: $V(r) = 2^4\pi^{7/2}\alpha'^4\Gamma(9/2)\left(\left(\frac{\tanh^{-1}\alpha'}{\sqrt{\alpha'}}\right)^4\right)$.

A puzzle addressed in \cite{13} is to explain what accounts for the stability of the short abelian flux tube. Consider a configuration of closely separated parallel IB soliton strings: D1branes, with worldvolume quantized constant $C_0$ and $B_{\mu\nu}$ potential, wrapped about the compact $X^9$ coordinate and with fixed spatial separation in the $X^8$ direction. They move as a unit within the worldvolume...
of a space-filling D9brane under the action of an external electric field. In nine dimensions, the type IB $O(32)$ string is the strong coupling dual of the heterotic $SO(32)$ string, with identical non-abelian gauge group. Thus, this is nothing but the strong coupling limit of a pair of closely separated wrapped heterotic soliton strings, in a nonperturbative background characterized by both Yang-Mills gauge fields and discrete moduli. A nice check is to consider the inverse of the orientation reversal projection, arriving at a configuration of localized parallel wrapped soliton strings, in the absence of an external electric field, in the 9d IIB massive supergravity \[14\]. The Dstrings couple to the Ramond-Ramond $*F_{10}$ scalar background field strength. A $T^9$-duality gives a pair of D0branes in the worldvolume of the D8brane stack with half-integer quantized $B_{NS-NS}$ field strength. The radius $R_{9B}$ corresponds to the mass parameter of the massive IIA supergravity \[14\], which is quantized in integer units of the inverse IIB radius.

The short tube of abelian flux runs between D0brane sources with fixed spatial separation. It is amusing that this configuration fits both the geometry, and quantized constant background fields, of a well-known massive string soliton of the nine-dimensional IIA supergravity \[14\]. In a constant external electric field, $F^{09}$, a pair of D8branes, carrying a D0brane and its image brane, will be separated by a sub-string-scale distance in a spatial direction orthogonal to the direction of nearest separation, which is held fixed, and which lies within the worldvolume of the D8brane. The difference between computing the potential between a pair of D0brane sources moving as a unit within the worldvolume of the embedding D8brane, and that between a pair of D0branes in slow relative motion with respect to each other is simple: the $r^{-7}$ falloff computed in \[13\] is replaced by the more rapid $r^{-9}$ falloff, characterizing the mass of the bound state of D0branes moving as a unit under the influence of the external electric field. Note that the Wilson loops, $C_i$, $C_f$, are a fixed distance $r$ apart, but there is no constraint on their position. Thus, we have a zero mode corresponding to the center of mass motion of the D0brane pair, parallel to their direction of nearest separation.

What if we take the strong coupling limit of the type I' theory assuming the nonabelian gauge group $E_8 \times E_8$? The result can be interpreted as a membrane of finite sub-string-scale width in M theory stretched between 10d walls at the endpoints of the finite interval $X^{10}$, the strong coupling limit of the heterotic $E_8 \times E_8$ string.

## 5 String Theory in Noncommutative Spacetime

The manifestly Weyl invariant path integral computation of the closed string tachyon scattering amplitude at one-loop given in \[3\], is extended to an analogous result for the scattering of open string tachyons on the boundaries of both the planar, and nonplanar, cylinder amplitude in bosonic string theory, and in the presence of a constant antisymmetric tensor two-form potential, in \[9\]. The worldvolume of the Dbrane coupled to a background two-form potential is a noncommutative spacetime \[11\]. The string amplitudes are nevertheless found to be both finite and unambiguously normalized, including in the limit of zero string tension. We infer, by similar arguments as given for vanishing external field, the exact Wilsonian renormalizability of open and closed string theory on a noncommutative space. The ultraviolet cutoff can be taken to infinity. The ordinary integral over the cylinder modulus can, in fact, be carried out in closed form upon taking the worldsheet infrared limit of the expression for the planar amplitude. We demonstrate that the momentum dependent
phase factor in the one-loop noncommutative field theory amplitude can indeed be interpreted as a wave-function renormalization, and we obtain its explicit form in terms of the star product. Coupling constant renormalization is exactly reminiscent of that in commutative spacetime, with identical short distance singularity but for a finite renormalization of the open string mass scale: 

\[(2\pi\alpha')^{-(p+1)/2}\text{det}(1 + B),\]

where for simplicity, we assume a Dpbrane with \(p\) odd, and \(\leq 25\). The higher effective string tension implies that open string modes probe shorter distances than the closed string scale.

The opposite regime of zero momentum transfer is dominated by the lightest closed string modes, or the worldsheet ultraviolet asymptotics. We will find that the zero momentum transfer limit of the nonplanar amplitude is ordinary gravity with closed string masses scaling in units of the bare string tension, \((2\pi\alpha')^{-1/2}\). Thus, the infrared regime is completely benign and the puzzling inconsistencies of the infrared regime present in noncommutative field theory are circumvented.

6 Type I Duals of the CHL Models

In the appendix of [15], I gave an analysis of enhanced gauge symmetry and nonperturbative type I’ states using an extended IIB-IB-I’ chain in conjunction with S- and T- dualities. The argument is as follows. The states in the spinor representation of \(O(16)\) necessary to obtain the 9D IB dual of the \(E_8\times E_8\) enhanced symmetry point in the moduli space of the 9D heterotic string requires nonperturbative IB states. Consider the nonperturbative states associated with a pair of D1branes wrapped in the \(X^9\) direction and lying in the worldvolume of D9branes carrying \(O(16)\times O(16)\) Chan-Paton factors. A \(T_9\) duality maps this into a type I’ background, with a D0brane lying in the worldvolume of the stack of 8 D8branes on either of two orientifold planes. The spinor states are associated with configurations of eight D1-D9 strings, or their \(T_9\)-dual D0-D8 strings, at each of two orientifold planes. The counting of extra massless modes is given by the \((SU(2))^8\) decomposition of the vector and spinor representations of \(SO(16)\). The D1-D9 strings live in doublets of the eight \(SU(2)\)'s, and the projection to the spinor and conjugate spinor is isomorphic to the heterotic weight lattice of \(E_8\). It will be of great interest to give a more detailed description of such nonperturbative excitations of the D1-D9 IB configuration.

At a special radius of \(X^9\) there is an enhancement of the gauge symmetry to an \(SU(2)\). As noted in [3], the IB string coupling diverges as the heterotic string approaches its self-dual radius. Nevertheless, the multiplicity of additional massless IB states can be inferred by mapping IB to IIB by an inverse orientifold transformation, and then using S-duality to map the wrapped Dstrings to wrapped F-strings: at its self-dual radius, the F-string winding states give the well-known enhancement of \(U(1)\) to a full \(SU(2)\). This argument relies on special properties of theories with 16 supercharges: S-duality is well-established, and orientation reversal is a freely acting \(Z_2\) symmetry. With these two extensions to the moduli space of IB backgrounds with 16 supercharges, we can identify IB strong coupling duals for all of the heterotic CHL orbifolds [16]. This is remarkable and striking confirmation of the validity of type I-heterotic duality [3].
7 The Free Closed String Gas

In [23] I gave a world-sheet representation of the pair correlation function of closely separated time-like Wilson loops, an order parameter for a phase transition to the long string phase in the type I open and closed string gas described more completely in [25]. In formulating the correct set-up for this calculation I became aware of several puzzles and loopholes in the standard treatment of the simpler case of closed string thermodynamics. Let us begin with a gas of free closed bosonic strings, computing the effective action functional at one-loop at finite temperature $\beta=1/T$. In the sequence of papers [24, 25, 23], we establish several points of interest that correct misconceptions in the standard treatment. Our starting point for the free energy of the closed string gas is the modular invariant expression obtained directly from the effective action functional: $F = -W/\beta$, where $W$ is the sum over connected vacuum string graphs at finite temperature. $W$ is an intensive thermodynamic variable. The world-sheet representation of $W$ is the Polyakov path integral without vertex operator insertions.

An important point of interest is the absence of a Hagedorn phase transition in the free energy of a closed string gas in the absence of tachyonic modes. This cannot be illustrated in the pedagogical, but also unphysical, case of the bosonic string gas since the string spectrum contains a zero temperature tachyon. In the pedagogically similar, but physically meaningful, heterotic string gas, it is easy to establish that the free energy is a finite and normalizable function at all temperatures starting from zero [25]. The reason is modular invariance. In particular, notice that quite apart from the occurrence of tachyonic winding modes at high temperature the fermionic closed string gas also has potential tachyonic momentum modes at low temperature. Since there is no tachyonic mode in a supersymmetric fermionic string gas, whether type IIA, IIB, or heterotic, at zero temperature, it would be unphysical to have a tachyon at infinitesimal temperature. In [25], we show that in the absence of Ramond-Ramond backgrounds there are no tachyon-free solutions for a modular invariant one-loop vacuum amplitude at finite temperature in the type II case, which also recover the correct zero temperature limit. On the other hand, in the heterotic string gas, in the presence of a temperature-dependent Wilson line background we are able to successfully suppress the appearance of tachyons at all temperatures starting from zero. The heterotic string gas has no exponential divergences in the free energy and no Hagedorn phase transition. Instead, it displays a self-dual phase transition at $T_C=1/\pi\alpha'^{1/2}$ belonging to the universality class of the Kosterlitz-Thouless phase transition. The transition is in every respect similar to that exhibited by the free closed bosonic string gas, except that the unphysical fixed point entropy that appears for the bosonic string gas [24] is absent in the heterotic case. The free energy, and all of its partial derivatives with respect to temperature, are continuous through the transition. The type II string gases exhibit a similar phase transition: type IIA is mapped to type IIB. The IIA winding modes are interchanged with IIB momentum modes, and vice versa [25].

In summary, for the infrared-finite heterotic string gas with monotonically increasing internal energy and positive specific heat at criticality, we obtain a thermodynamically stable ensemble at all temperatures starting from zero. The nonabelian gauge group is $SO(16)\times SO(16)$. We should note that the analogous Lorentzian time nonsupersymmetric heterotic string ground state was discovered in [22]. Recall that the finite temperature string gas is defined in the presence of a temperature-dependent self-dual background, $A_0(\beta)=1/\beta$. In terms of the low energy gauge theory limit, this corresponds to a modification of the usual axial gauge quantization. Here, $A_0$ has been set to a
temperature dependent constant rather than zero.

We emphasize that gauge fields appear to be essential in order that a given supersymmetric ground state of string theory have a sensible finite temperature description. We have shown that the type IIA and IIB ten-dimensional string gas contains tachyonic modes at any infinitesimal temperature different from zero. A plausible resolution is discussed in [25]. Of necessity, it requires a nontrivial Ramond-Ramond sector and nonperturbative gauge fields.

8 The Transition to the Long String Phase

The open and closed string theory can be mapped into its thermal duality transform, the type \( \tilde{I} \) string containing thermal Dpbranes: extended objects with a \( p \)-dimensional spatial worldvolume [23]. The low energy field theory on the worldvolume of a thermal Dpbrane is a \( p \)-dimensional finite temperature Higgs-gauge-gravity theory. As shown in [23], the ten-dimensional type IB theory also has a stable finite temperature ground state characterized by the nonabelian gauge group \( \text{SO}(16) \times \text{SO}(16) \), and a tachyon-free spectrum at all temperatures different from zero. The amplitude is tadpole-free, and the one-loop free energy is found to vanish identically [25]. We comment that this result is consistent with the weak-strong coupling, heterotic-type I, duality conjectures. This is not too surprising since, at least for infinitesimal temperatures different from zero, the strong-weak duality relations would be expected to hold with small \( \beta \)-dependent modifications. It is not implausible that the heterotic ground state at finite temperature with finite one-loop vacuum cosmological constant and, consequently, a dilaton tadpole, \textit{flows} to strong coupling. This strongly coupled heterotic ground state has a weakly-coupled type I description as given in [25]. While satisfactory from a physical standpoint, these inferences must remain conjectural in the absence of a nonperturbative formalism.

It is natural to look for an analog of the deconfinement phase transition of gauge theories in our string theory calculations. Accordingly, we compute the pair correlator of parallel and closely separated timelike Wilson loops lying in a thermal Dpbrane. Isolating the low temperature behavior of the gas of short open strings, we find a \( T^{10} \) growth characteristic of the ten-dimensional finite temperature gauge theory characterizing this limit. At a temperature at, or below, \( T_C \) we observe a transition to a long string phase characterized by a pair potential approaching a constant. On the other hand, below \( T_C \), we can expand in a Taylor expansion to obtain a \( 1/r^3 \) correction to the leading \( 1/r \) dependence. Precisely at the transition, we observe an inverse power law fall-off with a coefficient independent of the dimensionality of the Dpbrane in question [23, 25]. In the presence of an external electric field the transition temperature is modified with the simple replacement, \( T_C \rightarrow uT_C \), where \( u=\tanh^{-1}F_{09} \) [25], where \( F \) is the electric field strength [23, 25].

9 Conclusions

I have emphasized at the outset the importance of refining our understanding of the details of the worldsheet prescription for string scattering amplitudes because of the insight it gives into the fully nonperturbative string/M theory. In particular, the world-sheet representation of off-shell string amplitudes is largely unexplored territory, with the potential of suggesting many new worldsheet
calculations with field theory limit results that are of significance in both gauge and gravitational physics. Extension of the worldsheet representation to off-shell amplitudes with finite-sized boundary segments is of interest. The inclusion of non-trivial wavefunctions on the boundary representing physical semiclassical D0branes could be of far-reaching importance, raising the possibility of a first-principles computation of both the phase and normalization of the “meson” and “hadron” D0brane bound state wavefunctions. Extended exploration of both strong-weak coupling and target spacetime dualities in the context of off-shell string amplitudes, along the lines of the finite-width bound states described in section IV, remains a goal for future work. In particular, it could facilitate understanding the precise role of the NS spacetime solitons in nonperturbative string/M theory. Finally, the subject of string thermodynamics has far to go. We have exhaustively studied the free string limit. If feasible, an extension of the worldsheet representation that describes non-equilibrium string thermodynamics would be of great interest. A fully nonperturbative formalism that goes beyond the limitations of the worldsheet remains a goal for future work.

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References

[1] T. Banks, W. Fischler, S. H. Shenker and L. Susskind, Phys. Rev. D55 (1997) 5112. N. Ishibashi, H. Kawai, Y. Kitazawa, and A. Tsuchiya, Nucl. Phys. B510 (1998) 158. S. Chaudhuri, hep-th/0202138.

[2] A. M. Polyakov, Phys. Lett. B103 207 (1980).

[3] J. Polchinski, Comm. Math. Phys. 104 (1986) 37.

[4] M. Green, J. Schwarz, and E. Witten, String Theory, Volumes I & II (Cambridge) 1987.

[5] J. Polchinski, String Theory, Volumes I & II (Cambridge) 1998. See, Ch. 9.5, 13.5, 8.2, and 9.8.

[6] J. Polchinski and E. Witten, Nucl. Phys. B460 (1996) 525.

[7] A. Selberg, J. Indian Math. Soc. Vol. 20, 47 (1956). H. P. McKean, Comm. Pure & Applied Math., Vol. XXV, 225 (1972). S. Wolpert, Ann. Math. 169 (1969) 323; Comm. Math. Phys. 112 (1987) 283. E. D’Hoker and D.-H. Phong, Nucl. Phys. B269 (1986) 205.

[8] L. Bers, Bull. London Math. Soc. 4 (1972) 257. L. Keen, Annals Math. Studies, Vol. 66 (1971) 205. H. Huber, Math. Annals 138 (1959) 1. S. Chaudhuri, JHEP 003 (1999) 008.

[9] S. Chaudhuri and E. Novak, JHEP 008 (2000) 027.

[10] A. Cohen, G. Moore, P. Nelson, and J. Polchinski, Nucl. Phys. B267 143 (1986).
[11] S. Chaudhuri, Y. Chen, and E. Novak, Phys. Rev. D62 (2000) 026004.

[12] S. Chaudhuri and E. Novak, Phys. Rev. D62 (2000) 046002.

[13] M. Douglas, D. Kabat, P. Pouliot, and S. Shenker, Nucl. Phys. B485 (1997) 85. C. P. Bachas, Phys. Lett. B374 (1996) 37.

[14] E. Bergshoeff, M. de Roo, M. Green, G. Papadopoulos, P. Townsend, Nucl. Phys. B470 113. C. Hull, JHEP 9811 (1998) 027. B. Janssen, P. Meessen, and T. Ortin, Phys. Lett. B453 (1999) 229. M. Massar and J. Troost, Phys. Lett. B458 (1999) 283.

[15] S. Chaudhuri, Nucl. Phys. B591 (2000) 243.

[16] S. Chaudhuri, G. Hockney, and J. Lykken, Phys. Rev. Lett. 75 (1995) 2264. S. Chaudhuri and J. Polchinski, Phys. Rev. D52 (1995) 7168. A. Mikhailov, Nucl. Phys. B534 (1998) 612.

[17] R. Hagedorn, Nuovo Cim. Suppl. 3 (1965) 147. G. Hardy and S. Ramanujan, Proc. Lond. Math. Soc. 17 75 (1918). K. Huang and S. Weinberg, Phys. Rev. Lett. 25 (1970) 895. P. Salomonson and B. Skagerstam, Nucl. Phys. B268 (1986) 349. J. Atick and E. Witten, Nucl. Phys. B310 (1988) 291.

[18] B. Maclain and B. Roth, Comm. Math. Phys. 111 (1987) 1184. K. O’Brien and Chung-I Tan, Phys. Rev. D36 (1987) 1184.

[19] S. Chaudhuri, H. Kawai, and S.-H. H. Tye, Phys. Rev. D36 1148 (1987). H. Kawai, D. Lewellen, and S.-H.H. Tye, Nucl. Phys. B288 1 (1987).

[20] P. Ginsparg, Nucl. Phys. B295 (1988) 153.

[21] J. M. Kosterlitz, J. Phys. C7 (1974) 1046.

[22] N. Seiberg and E. Witten, Nucl. Phys. B276 (1986) 272. L. Alvarez-Gaume, P. Ginsparg, G. Moore, and C. Vafa, Phys. Lett. B171 (1986) 155. L. Dixon and J. Harvey, Nucl. Phys. B274 93 (1986). H. Kawai, D. Lewellen, and S.-H. H. Tye, Phys. Rev. D34 (1986) 3794.

[23] S. Chaudhuri, Phys. Rev. Lett. 86, 1943 (2001), hep-th/0008131.

[24] S. Chaudhuri, Phys. Rev. D65 (2002) 066008, hep-th/0105110.

[25] S. Chaudhuri, hep-th/0208112. See, also, the related discussion in hep-th/0203058.