Initial stress state at installation of a single-layer lattice dome due to errors of its assembly

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Abstract. The paper presents the results of the numerical modeling of the assembly process of single-layer lattice domes with respect to random deviations in lengths of individual bars which was performed with a special purpose software SBORKA developed by the author. Modeling of the installation process of the dome frame was performed by sequential calculation of the coordinates of its nodes through simulation of the connection of individual bars in these nodes. The inaccuracies of the bar lengths used were random variables with a normal distribution law. Due to errors in the length of the bars, the actual geometric scheme of the dome frame is distorted in relation to the design scheme. To obtain objective data on such distortions, numerical computer modeling was performed repeatedly with statistical processing of the results obtained. The sequence of assembly of the dome frame was chosen such that the largest number of different types of rods were used. Distortions in the geometric scheme of a single-layer lattice dome do not allow its frame to be installed simultaneously on all support nodes. First, the dome frame is installed only on three support nodes, and then due to deformation under its own weight, the remaining support nodes are included in the work. This deformation of the dome frame leads to the appearance of initial stress state in the bars. In this research, computer static analysis of a single-layer lattice dome was performed for the action of the self-weight load of its frame. The analysis was made taking into account the sufficiency of the load to exhaust the existing gaps in the support nodes. Based on the results of these calculations, the initial stress state in the bars of the dome frame are obtained, which should be taken into account in the analysis and design of the structure.

1. Introduction
Frames of single-layer lattice domes consist of a large number of individual straight bars, which are connected to each other in the joints during installation. Due to random inaccuracies in their manufacture [1], the actual dimensions of the bars differ from the nominal values. Accumulation of assembly errors yield to the distortion of the geometric scheme of the bar system. Distortions in the geometric scheme of the dome frame make it difficult to connect the elements to each other and create gaps in the support nodes during installation of the fully assembled structure. The study of possible errors of single-layer lattice domes allows estimating the possible initial forces and helps to improve the reliability of structures.

Previously, the author developed a computer program SBORKA, which allows for numerical simulation of the assembly of single-layer lattice domes from individual bars with respect to the errors.
of their lengths. The algorithm of this computer program is based on the Monte Carlo method [2, 3]. The simulation is performed repeatedly with subsequent statistical analysis [4, 5] of possible deviations of the frame nodes from the design position. The actual size of a bar differs from the nominal size by the amount of random deviation [6], which is set within the tolerance. The tolerance defines possible upward or downward deviations of the actual size from the nominal one, and the deviation itself is subject to the normal law [7, 8]. Therefore, the actual bars from which the dome frames are assembled can be longer or shorter than the nominal length. In the process of simulated computer assembly of the dome frame as a bar system, the value and the sign of a deviation are set randomly [9, 10]. In this study, we consider the frame of a single-layer lattice dome, characterized by the fact that the bars directed from the top to the base have the same length (Figure 1).

The dome is spherical with a span of 20.23 m and a height of 5.54 m, a radius of curvature of 12 m, and a length of descending rods of 3 m. The dome frame has triangular grid cells, belongs to the axisymmetric form of the grid structure, and consists of six repeating sectors. In accordance with the construction requirements for linear dimensional tolerances, the allowed deviations of ±2 mm were applied to the rods and ±3 mm to the radius of curvature [1]. The dome frame is assembled from the top to the supports.

The dome frame is supported on a horizontal plane only in six nodes. It is obvious that even with a slight distortion of the geometric scheme of the frame, it is impossible to install the dome on six support nodes at once. At the initial time, only three nodes will be installed on the horizontal plane, and the remaining three will have air gaps. Due to deformation under the self-weight of the dome, these gaps disappear and the remaining nodes are included in the work. This deformation of the dome frame will cause in initial forces in it, which will result a stress state in the individual bars.

The goal of the study was to determine the values of possible gaps in the support nodes based on statistical computer modeling of the assembly and subsequently to study the stress state associated with the deformation of the dome frame up to including all six support nodes in the work.

2. Methodology

Numerical computer modeling of the actual geometric shape of the frame of the single-grid dome under study is carried out repeatedly using the SBORKA program. The construction of the model starts from the top of the dome at node 1, that is previously placed in the design position. The position of node 2 in the direction of bar 1-2 is determined. Then the coordinates of the nodes are calculated sequentially as points of intersection of the actual lengths of the bars [11, 12] on the surface of the sphere with a real radius that constantly changes within the allowed deviation. First, the coordinates of the nodes 12, 22, 32, 42, and 52 are calculated. Next, the coordinates of the nodes 56, 3, 6, 13, 16, 23, 26, 33, 36, 43, 46, and 53 are calculated around the top of the dome, and then similarly, the coordinates of the other nodes of the frame. Figure 2 shows the numbers of nodes of the dome, as well
as the numbers of cells of its lattice frame in the order of their up building from the center to the support contour.

Upon completion of the assembly, the simulation is performed for the installation process of the lattice frame of the single-layer dome on a horizontal plane (Figure 3). Since the geometric shape of the frame is distorted in the process of assembly, support nodes cannot lie in the same plane, and the installation was performed only on nodes 5, 25 and 45. Since it is not known in advance at which computer assembly these nodes are lower than others, then after installing the dome on the plane, the procedure included in the SBORKA program was performed for a comparative assessment of the position of all support nodes. If at least one of the nodes 15, 35, or 55 was below the horizontal reference plane, then such a computer simulation of the installation was discarded.

Statistical analysis of errors in the positions of the nodes of the dome frame, provided in the SBORKA program, allows determining possible distortions of its geometric scheme. The study of distortions of the actual geometric scheme of the dome frame as a bar system was performed in accordance with the generally accepted methods of stochastic problems [13-15] by the coordinates of nodes.

Figure 2. Numbering of the nodes of the dome frame and the sequence of connection of the bars in the nodes.

Figure 3. Installation of the assembled frame on a plane.
As expected, it turned out that the total number of computer simulations of the assembled frame of the actual geometric scheme far exceeded the number of cases set in this study. Thus, with the total number of numerical computer simulations of the dome frame equal to 400,000, only in 56559 cases the support nodes 5, 25 and 45 were the lowest. Therefore, they served as supports that first transferred the load from the frame’s self-weight to the horizontal plane. The results of statistical processing of possible errors in the position of the support nodes of the single-layer dome under study for these cases are presented in Table 1.

Table 1. Statistical data on errors \( \delta_j \) in reference nodes.

| № of node | \( s(\delta_x) \) mm | \( m(\delta_x) \) mm | \( s(\delta_y) \) mm | \( m(\delta_y) \) mm | \( s(\delta_z) \) mm | \( m(\delta_z) \) mm |
|-----------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| 5         | 1,24                  | 0,14                  | 2,42                  | 0,02                  | 0,00                  | 0,00                  |
| 15        | 2,95                  | -1,04                 | 1,79                  | 0,41                  | 1,15                  | 1,45                  |
| 25        | 2,04                  | -0,07                 | 1,54                  | 0,14                  | 0,00                  | 0,00                  |
| 35        | 1,22                  | 0,16                  | 3,83                  | -1,09                 | 1,16                  | 1,46                  |
| 45        | 3,25                  | -0,11                 | 1,92                  | -0,13                 | 0,00                  | 0,00                  |
| 55        | 3,25                  | 0,84                  | 2,35                  | 0,68                  | 1,16                  | 1,45                  |

The proposed method of calculating the coordinates of nodes will not fully correspond to the actual installation of the dome frame. One reason is that for each newly mounted node, the actual distance to the center of the dome sphere is used. The other reason is that part of the bars were not used in the computer simulation of the assembly, although when installing a real frame, they have to be installed immediately on each tier. However, numerical simulation on a computer of the assembly process of a single-layer frame of the dome with its subsequent installation on a horizontal plane leads to the formation of its actual geometric shape adequately close to the real one.

In addition to errors in the position of the support nodes, errors in the distances between the support nodes were also determined. Statistical data on possible errors in the distances between the support nodes of the single-layer dome frame are shown in Table 2. Errors in the distances between the support nodes of the frame are much higher than the allowed deviations provided for them. However, when installed on a horizontal plane, they do not affect the stress state of the bars of a single-layer lattice dome. Although they are of significant importance for performance of the installation work related to fixing support nodes in the design position. Problems associated with these errors can be solved by using flexible design solutions or measures that compensate for errors in the distances between the support nodes on the mounting plane.

Table 2. Statistical data on errors in the distances \( \delta_l \) between support nodes.

| № of node | \( s(l) \) mm | \( m(l) \) mm | № of node | \( s(l) \) mm | \( m(l) \) mm |
|-----------|----------------|----------------|-----------|----------------|----------------|
| 5 – 15    | 3,50           | 0,01           | 35 – 45   | 3,59           | -0,00          |
| 15 – 25   | 3,50           | -0,02          | 45 – 55   | 3,61           | -0,01          |
| 25 – 35   | 3,46           | 0,00           | 55 – 5    | 3,43           | 0,00           |

Earlier, the author in the process of studying the nature of the distribution of gaps in the support nodes of the dome frames found that the distribution of the gap values between the nodes of the support contour of a single-grid dome and its support plane can be represented as normal with a one-way restriction of the interval (Figure 4) [16, 17].
The limiter for the gaps is the support plane, so vertically calculated statistics $m(\delta_z)$ and $s(\delta_z)$ of the support nodes will be equivalent to the statistics $m_o$ and $s_o$ of the limited normal distribution.

The largest values of gaps on each of the three segments of the support contour can be defined as the sum of the values of the mathematical expectation $m_o$ and the limit deviation $[\delta_c]$. Deviations of the support nodes in different segments of the support triangle can be considered independent of each other. Therefore, the gaps in each segment may differ from each other in value. For each segment of the support triangle, the probability that deviations from the mathematical expectation in the direction of increasing the gap will not exceed the limit values must be at least $P=0.99865$. This is equivalent to $m_n + 3 s_n$ for the normal distribution.

The probability that the sizes of gaps will reach their maximum values at all three segments of the support triangle at the same time is negligible and, therefore, such a case cannot be used for probabilistic analysis. It is known from the probability theory that the probability of simultaneous occurrence of several independent events is equal to the product of the probabilities of occurrence of each event [18, 19], i.e. $P(A\cap B\cap C)=P(A) P(B) P(C)$. Since the probability of occurrence of a limit deviation in one segment is $P=0.00135$, then combinations of deviations in three segments should be selected in such a way that the product of their probabilities is close to this value.

For the convenience of combining different values of gaps in different segments of the support triangle, we will link them to the normal standard deviation $s_n$. For simplicity, let's consider two variants of combinations of gap values in segments (A), (B) and (C) of the support contour – with equal gaps and with one large gap:

1) $3 (m_n + 1.22 s_n)$, where $P(A)=P(B)=P(C)=0.1112$;
2) $2 (m_n + 0)+(m_n + 2.55 s_n)$, where $P(A)=P(B)=0.5, P(C)=0.0054$.

In accordance with these variants, let us determine the values of the limit gaps in the support nodes of the dome frame and present them in Table 3.

| Вариант | № узла | $[\delta_z]$ мм | Вариант | № узла | $[\delta_z]$ мм |
|----------|--------|-----------------|----------|--------|-----------------|
| 1        | 15     | 2.85            | 2        | 15     | 4.38            |
|          | 35     | 2.88            |          | 35     | 1.46            |
|          | 55     | 2.87            |          | 55     | 1.45            |

### 3. Example
In order to investigate the nature of the stress state of the frame of a single-layer dome, resulting from the self-weight of its elements, special computer studies were performed. They were performed on computer models of the dome frame as a spatial bar system [20, 21] using the SCAD software system.

The bar elements of the dome frame are assigned from rectangular pipes with dimensions $160 \times 80 \times 4$ mm (Figure 5), cross-sectional area $A=17.9 \text{ cm}^2$, and elastic section moduli with respect...
to the primary axes of symmetry $W_x = 72.5 \text{ cm}^3$, $W_y = 49.8 \text{ cm}^3$. The choice of the cross section is arbitrary, without strength analysis for acting loads. Connections of the bars at nodes are assigned as rigid, and connections of the support nodes 5, 25, and 45 with the support plane are assigned as hinged.

![Computer 3D model of the dome frame.](image)

To determine the internal forces, computer models of the frame of the single-layer lattice dome were analyzed for action of its self-weight for three different static schemes. The first scheme corresponds to the design concept of the dome frame, when its frame is supported in six nodes (Figure 6). The second scheme corresponds to the situation when the weight of the frame is transferred to the support plane through three support nodes (Figure 7). It reflects the behavior of the lattice frame system for the first variant of the gaps in the support nodes. In addition, it also reflects the intermediate stage of work for the second variant of the gaps, when the deflections did not yet compensate for small gaps. The third scheme corresponds to the situation when the weight of the frame is transferred to a support plane through five support nodes (Figure 8). It reflects the work of the lattice frame system for the second variant of the gaps in the support nodes, but at the stage when two support nodes with small gaps have already started working, and the node with a large gap has not yet reached the support plane.

The study of changes in the stress state of the single-layer lattice dome was performed for the bars near the support nodes as experiencing the greatest force effects. As a result of the deformation of the frame, axial forces and bending moments occurred in these bars, which caused normal stresses in their sections. To quantify the state of the elements, these stresses were calculated using the formula

$$\sigma_i = \frac{N_i}{A_i} \pm \frac{M_{x,i}}{W_{x,i}} \pm \frac{M_{y,i}}{W_{y,i}}.$$

Table 4 shows the results of calculations of normal stresses in the most loaded bars of the dome frame for the design scheme of work and for both variants of possible errors in the support nodes.

Data in Table 4 shows that due to the inaccuracy of the assembly, gaps may appear in the support nodes, and stresses in the contour bars can be significant. This is especially important in view of the fact that initial stresses may superimpose with the stresses caused by design loads applied to undeformed scheme. For the sake of objectivity, it should be noted that the greatest stresses appear at the extreme points of the sections of contour bars that are in a combined state of bending and
compression. It should also be remembered that the considered values of the limit gaps in the support nodes of the frame of a single-layer dome are characterized as events of low probability.

**Figure 6.** Distribution of internal forces $N$ and $M$ in the bars of the dome for the design scheme.

**Figure 7.** Distribution of internal forces $N$ and $M$ in the bars of the dome for three support nodes.

**Figure 8.** Distribution of internal forces $N$ and $M$ in the bars of the dome for five support nodes.
Table 4. Stresses in the bars of the dome due to errors in the support nodes.

| № of node | Design scheme | First variant of gaps | Second variant of gaps |
|-----------|---------------|-----------------------|------------------------|
| 4 – 5     | -0.724        | -0.872                | -1.081                 |
| 4 – 8     | 0.493         | 0.356                 | 0.576                  |
| 5 – 8     | -0.549        | -1.460                | -2.082                 |
| 7 – 8     | -0.301        | -0.543                | -0.698                 |
| 7 – 9     | 0.253         | 0.460                 | 0.694                  |
| 8 – 9     | -0.415        | -1.103                | -1.601                 |

4. Conclusions

Statistical analysis of errors in the assembly and installation of the frame of a single-layer dome, provided in the SBORKA program, allow obtaining important data for the study of its initial stress state. The analysis made with this data leads to the conclusion that the self-weight of the dome frame is a sufficient load for elimination of possible gaps in the support nodes.

The results of the conducted research show that deformation of the dome frame under its self-weight in the presence of gaps in the support nodes leads to the initial stress state in its bars. The level of stress depends on the geometrical scheme of the dome frame; from the sequence of the installation of its bars and the type of installation in the design position, from the type and stiffness of the individual frame elements, and from the assigned tolerances on the lengths of the bars.

It can be stated with certainty that if there are more than three support nodes in a real dome frame that is assembled from individual bars with some tolerance and subsequently mounted as a whole, then gaps in the support nodes will inevitably appear. These gaps will lead to deformation of the frame under its self-weight resulting in the initial stress state, that is different from the stress state of the design geometric scheme.

To ensure that the single-layer lattice dome can be assembled, and to avoid force assembly, it is necessary to provide design solutions or technological measures that compensate for the resulting errors in the geometric scheme of its frame during the assembly. Otherwise, it is necessary to perform the analysis of the dome frame for the effect of forced installation of the the bars-inserts.

To reduce the level of the initial stress state in a dome frame that is installed as a fully assembled structure it is essential to either provide compensators of the gaps in the support nodes, or to perform the analysis of the frame with respect to the initial stress state in the elements of the frame resulting from its deformation due to gaps in support nodes.

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