Dust-acoustic solitary waves and shocks in strongly coupled quantum plasmas

Yunliang Wang\textsuperscript{1} and A. P. Misra\textsuperscript{2}

\textsuperscript{1}Department of Physics, School of Mathematics and Physics, University of Science and Technology Beijing, Beijing 100083, China
\textsuperscript{2}Department of Mathematics, Siksha Bhavana, Visva-Bharati University, Santiniketan-731 235, West Bengal, India

We investigate the propagation characteristics of electrostatic dust-acoustic (DA) solitary waves and shocks in a strongly coupled dusty plasma consisting of inertialless electrons and ions, and strongly coupled inertial charged dust particles. A generalized viscoelastic hydrodynamic model with the effects of electrostatic dust pressure associated with the strong coupling of dust particles, and a quantum hydrodynamic model with the effects of quantum forces associated with the Bohm potential and the exchange-correlation potential for electrons and ions are considered. Both the linear and weakly nonlinear theory of DA waves are studied by the derivation and analysis of dispersion relations as well as Korteweg-de Vries (KdV) and KdV-Burgers (KdVB)-like equations. It is shown that in the kinetic regime ($\omega \tau_m \gg 1$, where $\omega$ is the wave frequency and $\tau_m$ is the viscoelastic relaxation time), the amplitude of the DA solitary waves decays slowly with time with the effect of a small amount of dust viscosity. However, the DA shock-like perturbations can be excited in the hydrodynamic regime with $\omega \tau_m \ll 1$. The analytical and numerical solutions of the KdV and KdVB equations are also presented and analyzed with the system parameters.

PACS numbers: 52.27.Gr, 52.27.Lw, 52.35.Mw, 52.35.Sh, 52.35.Tc

I. INTRODUCTION

Dusty plasmas, which usually consist of electrons, ions, and extremely massive (typically $10^9 - 10^{12}$ times the mass of protons), highly charged ($10^2 - 10^4$ times the elementary charge) nanometer to millimeter-sized dust particles, has been an increasingly important area of research in planetary ring systems (e.g., the spokes in Saturns rings), Earth’s atmosphere (e.g., the regions of mesosphere and ionosphere), astrophysics (e.g., interstellar media, molecular dusty clouds, star forming clouds, and supernovae such as the Eagle Nebula), semiconductor manufacturing, as well as in a variety of low-temperature laboratory devices and tokamak edges [1–4]. The presence of massive, charged dust particles may bring about new collective modes such as dust-acoustic (DA) or dust-ion acoustic (DIA) waves depending on whether the dust particles are mobile or stationary. Since Rao et al. [5] first predicted the existence of DA waves, and their experimental observation by Barkan et al. [6] in dusty plasmas, the linear and nonlinear properties of such low-frequency waves have been extensively studied during the past few years (See, e.g., Refs. [7, 8]). In DA waves, while the restoring force comes from the pressures of the inertialless electrons and ions, the dust particles provide the inertia. In addition to DA waves, there may also be associated coherent nonlinear structures such as DA solitary waves, which arise due to a delicate balance between the nonlinear and dispersive effects, or shocks due to a dominant role of dissipation in the system. The evolution of such DA solitary waves and shocks can be described by a Korteweg-de Vries (KdV) equation and a Korteweg-de Vries Burgers (KdVB) equation respectively.

When a plasma is cooled to a relatively low-temperature or its number density is relatively high, the thermal de Broglie wavelength of the charged particles becomes comparable to the dimension of the system or the distance between the charged particles. Accordingly, the quantum effects, such as the quantum diffraction (tunneling) associated with the Bohm potential and the exchange-correlation of electrons and ions should be considered in the description of dusty plasmas. We mention that in the description of equations of motion, the electron-electron and ion-ion interactions can be separated into a Hartree term due to electrostatic potential and an exchange correlation term which can be described as a functional of the particle number density \[ N \]. In the past, a quantum hydrodynamic (QHD) model for dusty plasmas was used to investigate the linear DA waves, and it was shown that the quantum effects significantly modify the dispersion properties of the modes [11]. The quantum effects have also been shown to greatly influence the nonlinear structures like solitons [12–15], double layers [16], as well as and modulational instability [17] of nonlinear DA waves. Since dust particles are much heavier than the electrons and ions, the quantum effects for them are usually less important than those of electrons and ions. Accordingly, many authors have considered the quantum diffraction effects for inertialless electrons and ions, while the dusty grains were treated as classical as described by a fluid equation [18–21] or to form only as a static background [22–24].

It is well known that the low-frequency DA waves in strongly coupled dusty plasmas is quite different from that in weakly coupled systems [25–29]. A dusty plasma can enter the strongly (weakly) coupled regime when the Coulomb coupling parameter $\Gamma_d = \frac{e^2}{\epsilon_0 k_B T_d}$
\[ Z_d^2 e^2/k_B T_d r_{d0} \equiv \Gamma e^e, \] defined as the ratio of the interaction potential energy to the dust kinetic energy, is larger (smaller) than unity. Here, \( e \) is the elementary charge, and \( Z_d, T_d \) and \( r_{d0} = (3/4\pi n_{d0})^{1/3} \) are, respectively, the charge number, temperature and Wigner-Seitz radius (mean interparticle distance) of the dust particles. Also, \( \kappa = r_{d0}/\lambda_{Dd} \) is a measure of the screening of the dust charge by the plasma over a distance of the dust Debye length \( \lambda_{Dd} = \sqrt{k_B T_d/4\pi n_{d0} Z_d^2 e^2} \). It follows that the high charge number and low-temperatures of dust particles may lead to the strong coupling regime, i.e., \( \Gamma_d > 1 \). However, when \( \Gamma_d \gg 1 \), crystallization can occur, and so dusty plasmas may be important for phase transitions, which is another active topic of research. Such strongly coupled dusty plasmas are ubiquitous in compact astrophysical objects (e.g., the interior of white dwarfs, neutron stars), giant planetary interiors (e.g., Jupiter), and in a laboratory (e.g., ultracold plasmas by laser compression of matter), as well as nonideal plasmas for industrial applications. In strongly coupled dusty plasmas, viscosity often plays an important role on the wave modes as well as on the nonlinear structures, such solitons and shocks \[30, 31\]. For example, we will show that in the kinetic regime (\( \omega \tau_m \gg 1 \), where \( \omega \) is the wave frequency and \( \tau_m \) is the viscoelastic relaxation time) while it may reduce the solitary wave amplitude, the other hydrodynamic regime (\( \omega \tau_m < 1 \)) can give rise to the generation of shock-like perturbations.

Recent theoretical developments have indicated that strong correlations of charged dusts significantly modify the dispersion properties of collective plasma modes as well as the characteristics of nonlinear structures such solitons and shocks \[30, 31\]. In fact, a number of works can be found in the literature dealing with linear and nonlinear properties of electrostatic waves in strongly coupled dusty plasmas. To mention, a theoretical investigation on the linear and nonlinear properties of ion-acoustic waves was made by Ghosh et al. \[32\] in a strongly coupled quantum plasma taking into account the effects of quantum statistical pressure as well as the quantum recoil effects of the degenerate electrons and the strong correlation effects of classic ions. They considered the hydrodynamic regimes to study the DA shock waves due to the effects of viscous damping. Ourabah et al. \[33\] studied the effects of exchange-correlation on the nonlinear quantum ion-acoustic solitary waves. Furthermore, Consens et al. \[34\] investigated the nonlinear propagation of DA shocks in a strongly coupled dusty plasma. In other works, the evolution of wave envelopes associated with the modulational instability in the kinetic regime was investigated in strongly coupled plasmas with relativistically degenerate electrons \[34\]. On the other hand, the stimulated Raman scattering and Brillouin scattering instabilities of coherent circularly polarized electromagnetic waves were also studied in strongly coupled quantum plasmas with degenerate electrons and strongly correlated ions \[35\].

Our aim in this work is to investigate the linear and nonlinear properties of DA solitary waves and shocks in a strongly coupled dusty plasma. We account for strong coupling between the dust grains by using a generalized viscoelastic hydrodynamic model presented by Gozdadinou et al. \[36\], along with the dust pressure as prescribed by Yaroshenko et al. \[37\]. We use a QHD model with the effects of quantum Bohm potential as well as the exchange-correlation potentials for both electrons and ions. We show that in the hydrodynamic regime (\( \omega \tau_m \ll 1 \)), the evolution of DA waves can be described by a KdV equation exhibiting viscous dissipation with a shocklike structure \[38\]. In the kinetic regime (\( \omega \tau_m \gg 1 \)), the DA solitary waves can be excited in a strong-coupling regime, which can be described by Korteweg-de Vries (KdV) equation \[38\] or modified KdV equation \[39\] with a damping term due to dust viscosity. The latter is shown to reduce the wave amplitude with time. The effects of strong coupling of dust particles together with the effective electrostatic pressure as well as the quantum diffraction and the exchange correlations of electrons and ions are studied on the profiles of the DA waves.

II. PLASMA MODEL

A. Basic equations

We consider the nonlinear propagation of low-frequency (\( kV_{T_d} < \omega < kV_{Te,i} < \omega_{pd} \)) electrostatic waves in an unmagnetized dusty plasma consisting of inertial strongly coupled classical dusts and inertialess quantum electrons and ions with weak interparticle interactions. We assume that the dust particles each have constant mass and size, and a fixed charge number \( Z_d \). The latter, in general, varies, and can introduce a new low-frequency wave eigen mode as well as a dissipative effect (wave damping) into the system. However, we neglect such effect on the assumption that the typical DA time scale is much larger than the dust grain charging time (or charging rate of dust grains is very high compared to the dust plasma oscillation frequency). The collisions of all particles are also neglected in the considered interval of time. We also assume that the ratio of electric charge to mass of dust grains remains much smaller than those of both electrons and ions. Furthermore, the size of dust grains is assumed to be small compared to the average interparticle distance. Since the electrons and ions are much lighter than the dust particles, they are assumed to be governed by QHD equations with the quantum forces associated with the Bohm potential as well as the exchange-correlation potential. The latter is based on the local-density approximation in which the number density is restricted to be less than a critical value \[40\]. Thus, the dynamics of electrons and ions can be described by the following QHD equations \[41, 42\]

\[
0 = -q_j \frac{\partial \phi}{\partial x} + \frac{\partial V_{xe,j}}{\partial x} - \frac{\partial V_{Bj}}{\partial x} - \frac{1}{n_j} \frac{\partial P_j}{\partial x},
\]

(1)
where $\phi$ is the electrostatic scalar potential, $q_j$ is the charge (with $q_e = -e$ and $q_i = e$), $n_j$ is the number density (with its equilibrium value $n_{j0}$) of $j$-species particles ($j = e$ for electrons and $j = i$ for singly charged ions). The typical forces on the right-hand side of Eq. (1) can be described as follows:

The **first term** is the electrostatic force caused by the separation of charges (in absence of any externally applied electric field) in the plasma, and is expressed as a gradient of $\phi$.

The **second term** represents the exchange-correlation force, which appears due to electron-electron and ion-ion interactions, and is given for $j$-th species particles as

$$V_{xcj} = -0.985e^2 n_j^{1/3} \times \left[ 1 + \frac{0.034}{a_{BJ} n_j^{1/3}} \ln \left( 1 + 18.376 a_{BJ} n_j^{1/3} \right) \right],$$

where $a_{BJ} = \hbar^2/m_j e^2$ is the Bohr radius. In the weakly nonlinear limit, we consider $18.376 a_{BJ} n_j^{1/3} < 1$ which holds if $n_j \lesssim 1.087 \times 10^{21}$ cm$^{-3}$ and $n_i \lesssim 5.34 \times 10^{26}$ cm$^{-3}$. Thus, for $n_j \lesssim n_e \equiv 1.087 \times 10^{21}$ cm$^{-3}$ with $j = e, i$, Eq. (2) gives

$$V_{xcj} \approx -1.6 e^2 n_j^{1/3} + \frac{\hbar^2}{m_j} \left( 5.65 n_j^{2/3} - 69.27 a_{BJ} n_j \right) = C_{j0} + C_{j1} \delta n_j + C_{j2} \delta n_j^2 + \cdots ,$$

where $\delta n_j \equiv n_j/n_{j0} - 1 \ll 1$ and the coefficients are

$$C_{j0} = -1.6 \frac{e^2}{\epsilon} n_j^{1/3} + \frac{\hbar^2}{m_j} \left( 5.65 n_j^{2/3} - 69.27 a_{BJ} n_{j0} \right),$$

$$C_{j1} = -\frac{1}{3} \frac{e^2}{\epsilon} n_j^{1/3} + \frac{\hbar^2}{m_j} \left( 11.3 - 3 a_{BJ} n_j \right),$$

$$C_{j2} = \frac{1}{9} \left( 1.6^2 \frac{e^2}{\epsilon} n_j^{1/3} - 5.65 \frac{\hbar^2}{m_j} n_j^{2/3} \right).$$

The **third term** describes the quantum force associated with the Bohm potential, which appears due to the electrons or ion tunnelling effects through a potential barrier [12]. This is given by $F_{Bj} = \partial V_{Bj}/\partial x$, where

$$V_{Bj} = -\frac{\hbar^2}{2 m_j} \frac{1}{\sqrt{n_j}} \frac{\partial^2 \sqrt{n_j}}{\partial x^2},$$

and the **fourth or last term** is the pressure gradient force for thermal motion of electrons and ions.

We note that $V_{xcj}$ is approximated with the number density satisfying $n_{j0} \lesssim n_j \equiv 1.087 \times 10^{21}$ cm$^{-3}$, and in this regime electrons and ions may not be degenerate. However, other quantum effects, such as tunnelling and exchange-correlation effects may no longer be negligible. Typically, for astrophysical plasmas, degeneracy of particles of species $j$ may occur at $n_{j0} \gtrsim 10^{27}$ cm$^{-3}$ and $T_j \gtrsim 10^7$ K, whereas in metals, particles are degenerate at $n_{j0} \sim 10^{23}$ cm$^{-3}$ and $T_j \lesssim 10^5$ K. Thus, we consider the pressures of electrons and ions $P_j$ as given by the following polytropic equation of state

$$\left( \frac{P_j}{P_{j0}} \right) = \left( \frac{n_j}{n_{j0}} \right)^{\gamma_a},$$

where $P_{j0} = n_{j0} k_B T_j$ and $\gamma_a = 3$ for one-dimensional propagation of waves.

On the other hand, the dynamics of strongly coupled dust particles can be described by a generalized hydrodynamic (GH) model, which reads [26]

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x} (n_d v_d) = 0,$$

$$\left( 1 + \tau_m \frac{d}{dt} \right) \left( m_d n_d \frac{dv_d}{dt} + g_d n_d \frac{\partial \phi}{\partial x} + \frac{\partial P_d}{\partial x} \right) = \eta \frac{\partial^2 v_d}{\partial x^2},$$

where $n_d$, $v_d$ and $m_d$, respectively, denote the number density (with equilibrium value $n_{d0}$), velocity and mass of negatively charged dust grains, $q_d = -Ze$ with $Z_d$ denoting the number of electrons residing on the dust grain surface and $e$ is the elementary charge. In Eq. (8), $\tau_m$ denotes the viscoelastic relaxation time given by [26]

$$\tau_m = \frac{\bar{n}}{n_{d0} k_B T_d} \left( 1 - \frac{\gamma_d \mu_d + 4}{15} (\Gamma_d) \right)^{-1},$$

where $\bar{n} = \zeta + 4\eta/3$ is the coefficient of the effective dust viscosity in which $\eta$ and $\zeta$ are, respectively, the shear and bulk viscosities, $\gamma_d$ is the dust adiabatic index and $u(\Gamma_d)$ is a measure of the excess internal dust energy. The expression for $u(\Gamma_d)$ can be given for $k \rightarrow 0$ as [27, 28, 34]

$$u(\Gamma_d) \approx \begin{cases} -0.90 \Gamma_d + 0.95 \Gamma_d^{1/4} + 0.18 \Gamma_d^{1/4} - 0.80 & (1 \leq \Gamma_d \leq 160), \\ 1.5 - 0.90 \Gamma_d + 2980 \Gamma_d^{-2} & (160 < \Gamma_d \leq 300), \\ 1.5 - 0.90 \Gamma_d + 9.6 \Gamma_d^{-1} + 840 \Gamma_d^{-2} + 1.1 \times 10^4 \Gamma_d^{-3} & (300 < \Gamma_d \leq 2000). \end{cases}$$

We note that the relaxation time $\tau_m$ in Eq. (9) represents two characteristic time scales to describe two classes of wave modes with frequency $\omega$, namely the hydrodynamic modes with $\omega \tau_m \ll 1$ and the modes with $\omega \tau_m \gg 1$, i.e., the kinetic modes. Furthermore, the compressibility parameter $\mu_d$ appearing in Eq. (9) is given by [26]

$$\mu_d = \frac{1}{k_B T_d} \left( \frac{\partial P_d}{\partial n_d} \right)_{T_d} = 1 + \frac{1}{3} \frac{u(\Gamma_d)}{\Gamma_d} + \frac{\Gamma_d}{9} \frac{\partial u(\Gamma_d)}{\partial \Gamma_d}. \quad (11)$$

Since $u(\Gamma_d)$ is negative for increasing values of $\Gamma_d$, $\mu_d$ can change its sign. It has been shown that for values of $\Gamma_d$ in $1 < \Gamma_d < 10$, this change of sign can cause the dispersion curve to turn over with the group velocity going to zero and then to negative values [26].

We further note that in Eq. (8), apart from the electrostatic force (gradient of $\phi$) due to separation of charges, there are other two forces, namely the dust pressure ($P_d$)
and the dissipative force (\( \propto \dot{\eta} \)) which maintain the equilibrium of dust particles. We consider the dust pressure to include the thermodynamic contribution as well as the pressure due to mutual electrostatic repulsion of like charged dust particles \([34, 36, 37]\), i.e.,

\[
P_d = \gamma_d k_B T_d n_d,
\]

where \( T_d = T_s + \mu_d T_d \) is the effective dust temperature in which \( T_s \) appears due to electrostatic interactions between strongly coupled dusts, and is given by \([30, 37]\)

\[
T_s = \frac{N_{nn}}{3} \Gamma_d T_d (1 + \kappa) e^{-\kappa}.
\]

Here, \( N_{nn} \) is determined by the dust structure and corresponds to the number of nearest neighbors (e.g., in the crystalline state, \( N_{nn} = 12 \) for the fcc and hcp lattices, \( N_{nn} = 8 \) for the bcc lattice). Although, the parameter \( \mu_d \) can be negative \([cf. \text{Eqs. (10)} \text{and (11)}]\) for increasing values of \( \Gamma_d \), \( T_s \) may be comparable or even dominate over \( \mu_d T_d \) for \( \Gamma_d >> 1 \), and so the effective temperature \( T_d \) \((> 0)\) in the limit of \( \kappa \to 0 \) is most likely due to the strong coupling of dust grains.

The system of Eqs. (1), (7), and (8) is then closed by the Poisson equation

\[
\frac{\partial^2 \phi}{\partial x^2} = 4\pi e (n_e - n_i + Z dn_d).
\]

In equilibrium, the overall charge neutrality condition reads

\[
n_{io} = n_{i0} + Z dn_d.
\]

So far we have described the basic equations \([1, 7, 8]\) and \([13]\) for plasmas with strongly coupled dusts and weakly coupled nondegenerate electrons and ions. However, there may be a situation in which electron or ion number density is very high, e.g., in astrophysical plasmas (such as those in the cores of white dwarf stars). In this case, not only the quantum mechanical effects (such as tunelling) are important, the effects of quantum statistical (Fermi-Dirac) pressure for degenerate species should be taken into account. However, we mention that since in the weakly nonlinear limit, the exchange-correlation potential can only be expressed as a power series for \( n_j \lesssim n_c \equiv 1.087 \times 10^{21} \text{ cm}^{-3} \), the exchange-correlation effects of electrons and ions can be disregarded in degenerate dense plasmas where \( n_j > 10^{21} \text{ cm}^{-3} \) may be satisfied. Thus, in the case of DA wave propagation in strongly coupled dusty plasmas with degenerate electrons and ions (Hereafter we call this case as degenerate case for brevity), the equation of state \([6]\) may be replaced by the following equation for nonrelativistic degenerate electrons and ions \([14]\)

\[
P_j = \frac{1}{5} \frac{\hbar^2}{m_e} (3\pi^2)^{2/3} n_j^{5/3}.
\]

### B. Normalization

We normalize the physical quantities as well as the space and time coordinates in terms of new variables as: \( V_d = v_{d} / V_T \), \( N_j = n_j / n_{j0} \), \( \Phi = Z_d e \phi / \gamma_d k_B T_d \), \( \nabla x_{ej} = Z_d V_{xej} / \gamma_d k_B T_d \), \( X = x / \lambda_T \) and \( T = \tau_{wd} \), where \( V_T = \sqrt{\gamma_d k_B T_d / m_d} \) is the effective dust thermal speed, \( \lambda_T = \sqrt{\gamma_d k_B T_d / 4\pi n_{d0} Z_d^2 e^2} \) is the effective Debye length and \( \omega_{pd} = V_T / \lambda_T \) is the dust plasma oscillation frequency. Thus, Eqs. (1), (7), (8) and (14) can be recast in the following nondimensional forms:

\[
\pm \frac{\partial \Phi}{\partial x} - \sigma_j \frac{\partial N_j^{2/3}}{\partial x} + H_j^2 \frac{\partial}{\partial x} \left( \frac{1}{\sqrt{N_j}} \frac{\partial^2 \sqrt{N_j}}{\partial x^2} \right) = 0,
\]

\[
\frac{\partial N_d}{\partial T} + \frac{\partial}{\partial x} (N_d V_d) = 0,
\]

\[
\left( 1 + \tau_m \frac{d}{dT} \right) \left( N_d dV_d / dT - N_d \frac{\partial \Phi}{\partial x} + \partial N_d / \partial x \right) = \eta^* \frac{\partial^2 V_d}{\partial x^2},
\]

\[
\frac{\partial^2 \Phi}{\partial x^2} = \mu N_e - (1 + \mu) N_i + N_d,
\]

where the symbol \( \pm \) in Eq. \([17]\) stand, respectively, for electrons and ions, \( \sigma_j = 3 Z_d T_j / 2 \gamma_d T_d \), \( H_j = \sqrt{m_d Z_d / 2 m_j} (\hbar \omega_{pd} / \gamma_d k_B T_d) \), \( \tau_m = \tau_m \omega_{pd} \), \( \eta^* = \eta \omega_{pd} / n_{d0} \gamma_d k_B T_d \) and \( \mu = n_{e0} / Z_d n_{d0} \).

In the degenerate case, the momentum balance equations given by Eqs. \([17]\) are to be replaced by the following normalized equations

\[
\pm \frac{\partial \Phi}{\partial x} - \sigma_j \frac{\partial N_j^{2/3}}{\partial x} + H_j^2 \frac{\partial}{\partial x} \left( \frac{1}{\sqrt{N_j}} \frac{\partial^2 \sqrt{N_j}}{\partial x^2} \right) = 0,
\]

where \( \sigma_j = Z_d T_{Fj} / \gamma_d T_d \) with \( k_B T_{Fj} = h^2 (3 \pi^2 n_j)^{2/3} / 2 m_j \) denoting the Fermi energy density for \( j \)-th species particles. All other expressions/equations will remain the same as above.

### C. Parameter regimes

We note that the degeneracy parameter for a particle of species \( j \) is given by \( \chi_j \equiv T_{Fj} / T_d = (1/2) (3\pi^2)^{2/3} (n_{j0} \lambda_B^3)^{2/3} \), where \( \lambda_B = h / \sqrt{k_B T_j m_j} \) is the thermal de Broglie wavelength, \( E_{Fj} = k_B T_{Fj} \) is the Fermi energy. Thus, depending on the thermal energy \( k_B T_j \), particles are said to be nondegenerate or degenerate if \( \chi_j \leq 1 \) and the number density \( n_{j0} \) remains below or above the quantum concentration \( n_{qj} \equiv (2 m_j k_B T_j / h^2)^{3/2} / 3 \pi^2 \). Typically, for astrophysical dense plasmas with \( n_{j0} \gtrsim 10^{27} \text{ cm}^{-3} \), the degeneracy condition, i.e., \( \chi_j > 1 \) is satisfied for \( T_j \lesssim 10^7 \text{ K} \), whereas
in metals, particles are degenerate at $n_{j0} \sim 10^{23}$ cm$^{-3}$ and $T_j \lesssim 10^5$ K. Thus, particles with lower densities, i.e., $n_{j0} \lesssim 10^{21}$ cm$^{-3}$ and temperature $T_j \lesssim 10^5$ K are truly nondegenerate.

When the particles are degenerate, their weak or strong interparticle interactions is given by the quantum coupling parameter $\Gamma_{qj} = 4\pi e^2 n_{j0}^{1/3} / E_{Fj} \sim J_{rj}/a_B$. Since $E_{Fj} \propto n_{j0}^{2/3}$, the parameter $\Gamma_{qj}$ decreases below one with increasing values of $n_{j0} (\gtrsim 10^{27}$ cm$^{-3}$). Thus, in dense plasmas, degenerate electrons and ions become even more ideal with thermal compression than charged dust particles. On the other hand, the criterion of ideality/nonideality for nondegenerate particles of species $j$ is given by $\Gamma_{qj} = Z_j^2 e^2 / k_BT_{rj} n_{j0}$. Since electrons and ions are considered as ideal (weakly coupled) and charged dust particles as nonideal (strongly coupled) we must have $\Gamma_{e,i} < 1$ and $\Gamma_d > 1$ for strongly coupled dusty nondegenerate plasmas, and $\Gamma_{q(e,i)} < 1$ and $\Gamma_d > 1$ for strongly coupled dusty plasmas with degenerate electrons and ions.

It has been found that $\nu_{vis} \equiv \nu_{vis,pd} / n_{j0} g_B^d T_d$ (after $\tau_m$ being normalized by $\omega_{pd}^{-1}$) in Eq. (9) assumes a value $\sim 1$ in $1 < \Gamma_d < 10$, and it increases as $\propto \Gamma_d^{3/2}$ for $\Gamma_d \geq 10$. Furthermore, its values may be high for $\Gamma_d < 1$. For example, $\nu_{vis} \sim 45$ for $\Gamma_d = 0.1$, $46.4$ for $\Gamma_d = 100$, and $118.5$ for $\Gamma_d = 200$. Thus, $\tau_m (\propto C_{vis})$ becomes high not only in the weak coupling ($\Gamma_d < 1$) regime, but also in the high coupling ($\Gamma_d > 1$) regimes. Thus, the kinetic modes ($\omega \tau_m \gg 1$) exist either in weakly coupled regimes ($\Gamma_d < 1$) or in strongly coupled plasmas ($\Gamma_d > 1$) where the condition $\omega \tau_m \gg 1$ is satisfied. In the range of $1 < \Gamma_d < 10$, $\tau_m / \omega_{pd}$ is typically of the order of unity, and so the kinetic condition may no longer be valid, and we can have only the hydrodynamic modes with $\omega \tau_m \ll 1$.

Furthermore, in the weakly nonlinear expansion of the exchange-correlation potential for nondegenerate electrons and ions, it has been assumed that $n_{j0} \lesssim 1.087 \times 10^{21}$ cm$^{-3}$. Also, in the degenerate case (i.e., strongly coupled dusty plasmas with weakly relativistic degenerate electrons and ions) the particle density may vary in the regime $n_{j0} \lesssim 1.087 \times 10^{26}$ cm$^{-3}$ and temperature $T_j \lesssim 10^5$ K. Thus, we can classify three separate parameter regimes as three cases for the kinetic and hydrodynamic modes as follows:

Case I: $\chi_j < 1$ for $j = e, i, d$, $\Gamma_{e,i} < 1$, $\Gamma_d \gg 1$ and $\omega \tau_m \gg 1$. This represents the propagation of kinetic wave modes in a plasma composed of weakly coupled nondegenerate quantum electrons and ions (with the effects of Bohm potential and exchange-correlation potential), and strongly coupled charged dust particles. In this case, the ranges of values of the number density, thermodynamic temperature etc. are $n_{j0} \lesssim 10^{21}$ cm$^{-3}$, $n_{i0} \lesssim 10 n_{e0}$, $T_e \approx T_i \approx 10^5$ K, $T_d \approx T_i/2$ and $Z_d \approx 15$.

Case II: $\chi_j < 1$ for $j = e, i, d$, $\Gamma_{e,i} < 1$, $1 < \Gamma_d < 10$ and $\omega \tau_m \ll 1$. That is, hydrodynamic wave modes may exist in a plasma composed of weakly coupled nondegenerate quantum electrons and ions (with the effects of Bohm potential and exchange-correlation potential), and strongly coupled dust particles. The parameter regimes are $n_{j0} \sim 10^{15} - 10^{18}$ cm$^{-3}$, $n_{i0} (> n_{e0}) \lesssim 10 n_{e0}$, $T_e \approx T_i \approx 10^5 - 10^6$ K, $T_d \approx T_i/2$ and $Z_d \approx 15$.

Case III: The degenerate case in which $\chi_j > 1$ for $j = e, i, d$, $\Gamma_{q(e,i)} < 1$, $\Gamma_d \gg 1$ and $\omega \tau_m \gg 1$. This represents the existence of kinetic wave modes in a plasma composed of weakly coupled and degenerate (weakly relativistic) quantum electrons and ions (with the effects of Bohm potential and no exchange-correlation potential), and strongly coupled dust particles. The parameter regimes are $n_{j0} \lesssim 10^{26}$ cm$^{-3}$, $n_{i0} > n_{e0}$, $T_e \approx T_i \lesssim 10^7$ K, $T_d \lesssim T_i$ and $Z_d > 1$.

### III. Linear Dispersion Law

We derive a general linear dispersion relation for strongly coupled nondegenerate plasmas from the normalized system of equations (17)–(20) to identify different collective modes. We split up the physical quantities into unperturbed and perturbed (with subscript $G$) parts as $N_j = 1 + N_{jG}$, $V_d = V_{dG}$, and $\Phi = \Phi_G$, where all the perturbations are assumed to be much lower than unity and vary as plane waves $\sim \exp(ikx - i\omega t)$, with $\omega$ and $k$ denoting, respectively, the wave frequency and the wave number of perturbations. Thus, we obtain the following dispersion relation

$$1 + \frac{\mu}{k^2 \kappa_e} + 1 + \frac{\mu}{k^2 \kappa_i} - \left( \frac{\omega^2 - k^2 \cC}{1 - i\omega \tau_m} \right)^{-1} = 0, \quad (22)$$

where the parameters $\kappa_j (j = e, i)$ for electrons and ions are given by

$$\kappa_j = 2\sigma_j - C_j + \frac{1}{2} k^2 H_j^2 \equiv D_j + \frac{1}{2} k^2 H_j^2, \quad (23)$$

in which $\sigma_j$ is the contribution from dust thermal temperature, $C_j \equiv Z_d C_j / \gamma_d k_B T_d$ is from the exchange-correlation potential, and $H_j$ is due to the quantum Bohm potentials of electrons and ions.

We find that the dispersion relation (22) for low-frequency DA waves is greatly modified by the quantum diffraction effects ($\propto H_j$), the exchange-correlation effects ($\propto C_j$) and the dust fluid viscosity ($\propto \tau_m$ or $\eta^*$) associated with the strong coupling of the dust particles. As mentioned before, the memory function $\tau_m$ (proportional to the dust fluid viscosity) defines two characteristic time scales to distinguish between two classes of wave modes, namely, "hydrodynamic modes" ($\omega \tau_m \ll 1$) and "kinetic modes" ($\omega \tau_m \gg 1$). While in the low-frequency limit, the memory function $\tau_m$ stands for the ordinary dissipative term in hydrodynamic regime, the relatively high-frequency limit gives rise the viscosity to act as a source of damping to the DA waves.

In the hydrodynamic regime ($\omega \tau_m \ll 1$), the dispersion relation (22) reduces to

$$\omega^2 - k^2 + i\omega k^2 \eta^* - k^2 / \kappa = 0, \quad (24)$$

where $\kappa = \kappa_e + \kappa_i$.
where $\mathcal{K} = k^2 + \mu/\kappa_e + (1 + \mu)/\kappa_i$. Assuming that the wave frequency has real and imaginary parts, i.e., $\omega = \omega_r + i\omega_i$ and the wave number $k$ is real, and separating the real and imaginary parts, we obtain from Eq. (24) as

$$\omega_r = k \left( 1 - \frac{1}{4} k^2 \eta^2 + \frac{1}{\mathcal{K}} \right)^{1/2},$$

$$\omega_i = -\frac{1}{2} k^2 \eta^*.$$  

(25)

(26)

It follows that the low-frequency DA waves suffers a viscous damping in the hydrodynamic regime, which was also observed in strongly coupled dusty plasmas with Boltzmann distributed electrons and ions [20]. In the long-wavelength limit, the phase velocity of the DA wave (obtained from the real part of $\omega$) assumes a constant value, i.e., the wave becomes dispersionless. From Eq. (25), we also find that the DA wave frequency increases with increasing values of $k$. However, the wave has cut-off frequencies at $k = 0$ and $k = k_c$ (> 0) which satisfies $\mathcal{K} + 1/(1 - \eta^2)/4 = 0$. Clearly, this zero-frequency mode occurs due to the effect of dust viscosity in the plasma. We note that the values of the parameters $H_j$ and $C_j$, which are due to quantum diffraction and exchange-correlation effects, increase with increasing values of the number density $n_j$. As $H_j$ increases, the values of $\mathcal{K}$ get enhanced and so is $\omega_r$. Furthermore, as the contribution from $C_j$ increases but still smaller than that from $\sigma_j$, the parameter $D_{jl}$ decreases (or $\mathcal{K}$ increases), resulting into a decrement of $\omega$. The upper panel of Fig. 1 exhibits a plot of $\omega_r$ with respect to $k$. It is found that as the coupling parameter $\Gamma_d$ increases with increasing values of $Z_d$ (i.e., reducing the magnitudes of $H_j$ and terms $\propto C_j$), the wave frequency increases with a cut-off at lower wave number (see the solid and dashed lines). However, a small enhancement of the ion number density, keeping all others unchanged, leads to a significant increase of $\omega_r$ with cut-off at higher $k$ (See the dotted line). Here, though the parameters $\Gamma_d$ and $H_j$ are slightly increased, the contributions from exchange-correlation ($C_j$) remain lower (compared to those for the solid curve).

On the other hand, for relatively high-frequency electrostatic waves, we obtain the following dispersion relation for the kinetic wave modes ($\omega \tau_m \gg 1$) as

$$\omega = k \left( 1 + \frac{\eta^2}{\tau_m} + \frac{1}{\mathcal{K}} \right)^{1/2},$$

$$k \left[ \frac{T_s}{T_{df}} + \frac{T_d}{\gamma_d T_{df}} \left( 1 + \frac{4}{15} u(\Gamma_d) \right) + \frac{1}{\mathcal{K}} \right]^{1/2}. \quad (27)$$

Clearly, the viscosity is no longer a dissipative effect, however, it modifies the dispersion curve via the parameters $T_s$, $T_d$ and $\Gamma_d$. In Eq. (27), though the term $\propto T_s$ may be negative for typical plasma parameters as in Cases I-III, the term $\propto T_d$ is always larger than the other terms, and hence $\omega$ is always real for all $k$ [34]. However, in absence of $T_s$, there must exist a critical wave number above which the kinetic wave modes do not exist. Furthermore, Eq. (27) shows that the phase speed of the wave remains greater than the effective dust thermal speed and assumes a constant value in the long-wavelength limit $k \to 0$. Again, since the term in the square root is typically $\lesssim 1$ for $\Gamma_d \gg 1$, the low-frequency ($< \omega_{pd}$) waves exist for $k < 1$. Typical behaviors of the kinetic mode is shown in the lower panel of Fig. [1] It is seen that the wave frequency decreases with increasing values of the electron number density (and so of $\Gamma_d$, $H_j$ and terms $\propto C_j$).

The dotted line (which has the same qualitative behaviors as the solid or dashed lines) in the upper panel of Fig. [1] represents the kinetic mode that may be excited in strongly coupled degenerate dense plasmas (without the exchange-correlation effects) in the regime as in Case III with the following dispersion relation (for $k \ll 1$)

$$\omega = k \left( 1 + \frac{\eta^2}{\tau_m} + \frac{1}{D} \right)^{1/2},$$

where $D = \mu/D_\zeta + (1 + \mu)/D_\zeta$ with $D_{\zeta} = 2\sigma_{\zeta}/3$. Note that the hydrodynamic modes do not exist for strongly coupled degenerate dense plasmas.

IV. NONLINEAR EVOLUTION OF DA WAVES

We consider the propagation of slowly varying, weakly nonlinear and weakly dispersive DA solitary waves in strongly coupled dusty plasmas. In the previous section, we have seen that both the hydrodynamic and kinetic wave modes become dispersionless (with constant phase velocity) in the long-wavelength limit (i.e., the length much larger than the effective Debye length which ensures the collective behaviors of the plasma not to disappear). This implies that in a frame moving with the phase velocity, say $V_p$, the time derivatives of all physical quantities should vanish. So, for a finite $\epsilon$ with $0 < \epsilon \lesssim 1$, we can expect slow variations of the wave amplitude in the moving frame of reference. Thus, we introduce the stretched coordinates as $\xi = \epsilon^{1/2}(X - V_p T)$, $\tau = \epsilon^{3/2} T$, where $V_p$, normalized by the effective dust thermal speed $V_T$, will be shown later as equal to the expressions for the phase velocities (in the limit of $k \to 0$) in both hydrodynamic and kinetic regimes from Eqs. (24), (27) and (28). The dependent variables, namely $N_d$, $V_d$ and $\Phi$ are expanded in powers of $\epsilon$ as

$$f = f^{(0)} + \sum_{n=1}^{\infty} \epsilon^n f^{(n)},$$

(29)

where $f^{(0)} = 1$ for $N_d$, and $f^{(0)} = 0$ for $V_d$ and $\Phi$. In what follows, we substitute the stretched coordinates and the expansion into Eqs. (17), (20) and equate different powers of $\epsilon$ to obtain a set of reduced equations as given
FIG. 1. (Color online) The low-frequency hydrodynamic (upper panel) and kinetic (lower panel) wave modes in different plasma regimes. The parameters used for the hydrodynamic modes [Eq. (25)] are (i) for the solid line: \( n_{\text{e0}} \sim 10^{18} \text{ cm}^{-3}, n_{\text{i0}} = 1.5n_{\text{e0}}, Z_{\text{d}} = 5, T_{\text{e}} \sim T_{\text{i}} \sim 10^5 \text{ K} \) and \( T_{\text{d}} = 0.5T_{\text{e}} \) for which \( \Gamma_{\text{d}} = 3, H_{\text{c}} = 0.0043, H_{\text{i}} = 10^{-4}, C_{\text{c}} = -0.017 \) and \( C_{\text{i}} = -0.02 \), where \( C_{\text{c}} = Z_{\text{d}}C_{\text{jC}}/\gamma_{\text{i}d}k_{\text{B}T_{\text{d}}} \), (ii) for the dashed line: \( Z_{\text{a}} = 10 \) and all other parameters remain the same as for the solid line for which \( \Gamma_{\text{a}} = 10, H_{\text{c}} = 0.003, H_{\text{i}} = 7 \times 10^{-5}, C_{\text{c}} = -0.012 \) and \( C_{\text{i}} = -0.014 \), and (iii) for the dotted line: \( n_{\text{e0}} = 2n_{\text{e0}} \) and all other parameters remain the same as for the solid line for which \( \Gamma_{\text{d}} = 4, H_{\text{c}} = 0.005, H_{\text{i}} = 1.6 \times 10^{-4}, C_{\text{c}} = -0.014 \) and \( C_{\text{i}} = -0.018 \). The waves corresponding to these lines have cut-offs at \( k = 0.85, 0.83 \) and 1.36 respectively.

The parameters used for the kinetic modes [Eq. (27)] are (i) for the solid line: \( n_{\text{e0}} \sim 10^{19} \text{ cm}^{-3}, n_{\text{i0}} \sim 10n_{\text{e0}}, Z_{\text{d}} = 20, T_{\text{e}} \sim T_{\text{i}} \sim 10^5 \text{ K} \) and \( T_{\text{d}} = 0.5T_{\text{e}} \) for which \( \Gamma_{\text{d}} = 35, H_{\text{c}} = 0.008, H_{\text{i}} = 2 \times 10^{-4}, C_{\text{c}} = -0.005 \) and \( C_{\text{i}} = -0.01 \), (ii) for the dashed line: \( n_{\text{e0}} \sim 5 \times 10^{24} \text{ cm}^{-3} \) and all other parameters remain the same as for the solid line for which \( \Gamma_{\text{d}} = 165, H_{\text{c}} = 0.02, H_{\text{i}} = 6 \times 10^{-4}, C_{\text{c}} = -0.005 \) and \( C_{\text{i}} = -0.01 \), and (iii) for the dotted line: \( n_{\text{e0}} \sim 10^{24} \text{ cm}^{-3}, n_{\text{i0}} \sim 200n_{\text{e0}}, Z_{\text{d}} = 2, T_{\text{e}} \sim T_{\text{i}} \sim 10^8 \text{ K} \) and \( T_{\text{d}} = 0.5T_{\text{e}} \) for which \( \Gamma_{\text{d}} = 215, H_{\text{c}} = 19, H_{\text{i}} = 0.6 \). The kinetic mode [Eq. (28)] corresponding to the dotted line is generated in strongly coupled degenerate (nonrelativistic) dense plasmas without the exchange-correlation effects of electrons and ions.

in the following two subsections corresponding to Cases I and II, i.e., for kinetic and hydrodynamic modes in the propagation of DA solitary waves and shocks in strongly coupled dusty plasmas. Case III will be discussed along with Case II.

A. Solitary waves in the kinetic regime \( \omega \tau_{\text{m}} \gg 1 \)

We consider the propagation of kinetic wave modes with \( \omega \tau_{\text{m}} \gg 1 \) and the parameters that are discussed in Case I of the previous section. In the lowest order of \( \varepsilon \), we obtain from Eqs. (17)-(20) the following first-order quantities

\[
N_{\text{e}}^{(1)} = D_{\text{c}}^{-1}\Phi^{(1)}, \quad N_{\text{i}}^{(1)} = -D_{\text{i}}^{-1}\Phi^{(1)},
\]

\[
N_{\text{d}}^{(1)} = D_{\text{d}}\Phi^{(1)}, \quad V_{\text{d}}^{(1)} = V_{\text{p}}D_{\text{d}}\Phi^{(1)},
\]

\[
\mu N_{\text{e}}^{(1)} - (1 + \mu)N_{\text{i}}^{(1)} + N_{\text{d}}^{(1)} = 0,
\]

where \( D_{\text{d}} = 1/(1 - V_{\text{p}}^2 + \eta^*/\tau_{\text{m}}) \). Substituting \( N_{\text{e}}^{(1)} \) (for \( j = e, i, d \)) from Eqs. (30) and (31) into Eq. (32), we obtain, for any nonzero perturbation, the following dispersion relation for the DA speed

\[
V_{\text{p}}^2 = 1 + \frac{\eta^*}{\tau_{\text{m}}} + \frac{1 + \mu}{\mu/D_{\text{c}} + 1 + \mu/D_{\text{i}}}
\]

\[
= \frac{T_{\text{e}}}{T_{\text{d}}} + \frac{T_{\text{d}}}{\gamma_{\text{d}}T_{\text{df}}} \left[ 1 + \frac{4}{15}u \left( \frac{\Gamma_{\text{d}}}{\Gamma_{\text{d}}} \right) \right] + \frac{1}{D},
\]

where \( D = \mu/D_{\text{c}} + 1 + \mu/D_{\text{i}} - D_{\text{d}} \). Note that this \( V_{\text{p}} \) has the same expression as \( \omega/k \) of the dispersion relation (27) in the limit of \( k \to 0 \).

In the next higher-order of \( \varepsilon \), we obtain, for the second-order quantities \( N_{\text{e}}^{(2)}, N_{\text{i}}^{(2)}, N_{\text{d}}^{(2)} \), and \( V_{\text{d}}^{(2)} \), the following equations:

\[
D_{\text{jij}}\frac{\partial N_{\text{j}}^{(2)}}{\partial \xi} = \zeta_{\text{j}}D_{\text{jij}}\frac{\partial \Phi^{(2)}}{\partial \xi} - D_{\text{j2ij}}\frac{\partial \Phi^{(1)}}{\partial \xi}
\]

\[
+ \frac{1}{2}H_{\text{jij}}^2 D_{\text{j1l}}\frac{\partial \Phi^{(1)}}{\partial \xi^2},
\]

where \( \zeta_{\text{j}} = \pm 1 \) for electrons \( (j = e) \) and ions \( (j = i) \),

\[
\frac{\partial^2 N_{\text{d}}^{(2)}}{\partial \xi^2} = -D_{\text{d}}\frac{\partial^2 \Phi^{(2)}}{\partial \xi^2} - 2V_{\text{p}}D_{\text{d}}\frac{\partial^2 \Phi^{(1)}}{\partial \xi \partial \tau}
\]

\[
+ D^2 \frac{\partial}{\partial \xi} \left( \frac{\Phi^{(1)}}{\partial \xi} \right)
\]

\[
+ D^2 (1 + D - V_{\text{p}}^2 D) \left( \frac{\partial \Phi^{(1)}}{\partial \xi} \right)^2,
\]

and

\[
\frac{\partial V_{\text{d}}^{(2)}}{\partial \xi} = V_{\text{p}}\frac{\partial N_{\text{d}}^{(2)}}{\partial \xi} + D\frac{\partial \Phi^{(1)}}{\partial \tau} - 2V_{\text{p}}D_{\text{d}}^2\frac{\partial \Phi^{(1)}}{\partial \xi},
\]

where \( D_{\text{jij}} = 2\sigma_{\text{j}} - 2Z_{\text{d}}C_{\text{jC}}/\gamma_{\text{i}d}k_{\text{B}T_{\text{d}}} \) for \( j = e, i, \) and \( \text{D} = 1 + V_{\text{p}}^2 + 2/D \). Eliminating \( N_{\text{e}}^{(2)} \) from Eqs. (34)-(37), we obtain the following modified KdV (mKdV) equation as

\[
\frac{\partial}{\partial \xi} \left[ \frac{\partial \Phi^{(1)}}{\partial \tau} + A\frac{\partial \Phi^{(1)}}{\partial \xi} + B\frac{\partial^2 \Phi^{(1)}}{\partial \xi^2} \right] + \gamma \left( \frac{\partial \Phi^{(1)}}{\partial \xi} \right)^2 = 0,
\]
where the coefficients of nonlinearity, dispersion and damping, i.e., $A$, $B$ and $\gamma$ are
\[
A = \frac{1}{2V_p} \left[ \frac{1}{D_2^2} \left( \mu D_{e2} - \frac{(1 + \mu)D_{e2}}{D_{i1}^2} \right) - D \right],
\]
\[
B = \frac{1}{2V_p D^2} \left[ 1 - \frac{1}{2} \left( \frac{\mu H^2}{D_{e1}} + \frac{(1 + \mu)H^2_i}{D_{i1}^2} \right) \right],
\]
\[
\gamma = \frac{D}{2V_p \tau_m^*}.
\]

Equation (38) is the required KdV equation, which describes the weakly nonlinear and weakly dispersive DA waves in a strongly coupled quantum dusty plasma with the effects of quantum diffraction associated with the Bohm potential, the exchange-correlation of electrons and ions as well as the damping due to dust viscosity. If we set $\eta^* = 0$, i.e., if we neglect the dust viscosity effect, the coefficient $\gamma$ vanishes, and Eq. (38) reduces to the usual KdV equation
\[
\frac{\partial \Phi^{(1)}}{\partial \tau} + A \Phi^{(1)} \frac{\partial \Phi^{(1)}}{\partial \xi} + B \frac{\partial^3 \Phi^{(1)}}{\partial \xi^3} = 0.
\]

In what follows, we examine the range of values of the plasma parameters as in Case I for which the KdV equation (without the damping effect) is applicable to DA waves, and also see the competition between the nonlinear and damping terms in determining whether or not an initial wave steepens. Inspecting on the coefficients $A$, $B$, and $\gamma$, we find that for typical plasma parameters as in Case I, $A > \gamma \gg B$. For example, for $n_e = 10^{21}$ cm$^{-3}$, $n_i = 10 n_e$, $Z_d = 20$, $m_d = 10^{12} m_i$, $m_i = 2000 m_e$, $T_e \approx T_i = 10^5$ K and $T_d = 0.5 T_i$, the coefficients of Eq. (38) are $A = -7.9$, $B = 0.01$ and $\gamma = 2.0$. This implies that the nonlinear and the viscous damping effects become larger than the finite Debye length (dispersive) effects for which the KdV soliton theory may not be applicable to DA waves. In another regime of Case I, e.g., for $n_e = 10^{17}$ cm$^{-3}$, $n_i = 30 n_e$, $Z_d = 20$, $m_d = 10^{12} m_i$, $m_i = 2000 m_e$, $T_e \approx T_i = 10^5$ K and $T_d = 0.5 T_i$, we have the coefficients of Eq. (38) as $A = -2.3$, $B = 0.25$ and $\gamma = 0.05$, i.e., the damping due to dust viscosity can no longer be negligible, but may play crucial roles in reducing the wave amplitude.

We mention that an exact analytic solution of Eq. (38) is much complicated, however, we can seek for an approximate time-dependent soliton solution of Eq. (38) with a small effect of $\gamma$, i.e., when $A \sim B \gg \gamma$. To this end, we rewrite Eq. (38) after integrating with respect to $\xi$ as
\[
\frac{\partial \Phi^{(1)}}{\partial \tau} + A \Phi^{(1)} \frac{\partial \Phi^{(1)}}{\partial \xi} + B \frac{\partial^3 \Phi^{(1)}}{\partial \xi^3} + \gamma \int_{-\infty}^{\infty} \left( \frac{\partial \Phi^{(1)}}{\partial \xi} \right)^2 d\xi = 0,
\]
where we have used the boundary conditions: $\Phi^{(1)} \rightarrow 0$, $\partial \Phi^{(1)}/\partial \xi \rightarrow 0$ as $\xi \rightarrow \pm \infty$. Now, in absence of $\gamma$, a travelling wave (stationary soliton) solution of Eq. (40) can be obtained as
\[
\varphi \equiv \Phi^{(1)} = \Psi \sech^2 \left( \frac{\xi - U_0 \tau}{W} \right),
\]
where $\Psi = 3U_0/A$ is the amplitude, $W = (12B/\Psi A)^{1/2} = \sqrt{4B/\Psi}$ is the width and $U_0 = \Psi A/3$ is the constant phase speed (normalized by $V_p$) of the solitary wave. Figure 2(a) exhibits the profiles of the DA soliton [Eq. (42)] for different parameter values as in Case I in absence of $\gamma$. It is found that the effects of $H_j$ is not so significantly large, however, both the amplitude and width of the soliton get enhanced by the exchange-correlation effects of electrons and ions (see the solid and dashed lines). Also, higher the values of $U_d$, the lower are both the amplitude and width of the soliton (see the solid and dotted lines).

![FIG. 2.](Color online) Profiles of the soliton solution (at $\tau = 0$, the upper panel) without any damping effect and the time dependent amplitude of the soliton (48) (with the effect of damping due to dust viscosity, the lower panel) of the KdV equation (19) in strongly coupled quantum plasmas with non-degenerate electrons and ions. In the upper panel, the solid (dashed) curve corresponds to the case with (without) the effects of exchange-correlation of electrons and ions. The parameter values corresponding to the solid and dashed curves are $n_e \sim 10^{21}$ cm$^{-3}$, $n_i = 10 n_e$, $Z_d = 20, T_e \sim T_i = 10^5$ K and $T_d = 0.5 T_i$, for which $\Gamma_d = 165$, $A = -7.9$, $B = 0.008$ for the solid line, and $\Gamma_d = 165$, $A = -8.3$, $B = 0.007$ for the dashed line. The dotted line is for $n_i = 50 n_e$, and all other parameters are the same as for the solid line. In this curve, $\Gamma_d = 290$, $A = -11$, $B = 0.004$. In the lower panel, the solid and dashed curves are corresponding to the parameters $n_e \sim 5 \times 10^{27}$ cm$^{-3}$, $n_i = 30 n_e$ (and all other parameters are the same as for the solid curve in the upper panel), for which $\Gamma_d = 19$, $A = -2.3$, $B = 0.25$ and $\gamma = 0.05$, and $n_e = 10^{18}$ cm$^{-3}$, $n_i = 20 n_e$ (and all other parameters are the same as for the solid curve in the upper panel), for which $\Gamma_d = 24$, $A = -2.6$, $B = 0.16$ and $\gamma = 0.07$. Next, to find the solitary wave solution of Eq. (41)
with the effect of a small amount of $\gamma$, we first integrate Eq. (41) with respect to $\xi$ to obtain
\[ \frac{\partial}{\partial \tau} \int_{-\infty}^{t+\infty} \varphi \, d\xi = 0. \] (43)
This shows that Eq. (41) conserves the total number of particles. Furthermore, multiplying Eq. (41) by $\varphi$ and integrating over $\xi$ yields
\[ \frac{\partial}{\partial \tau} \int_{-\infty}^{\infty} \varphi^2(\xi, \tau) d\xi \leq 0, \] (44)
in which the equality sign holds only when $\varphi = 0$ for all $\xi$. Equation (44) states that an initial perturbation of the form (42) for which
\[ \int_{-\infty}^{t+\infty} \varphi^2 \, d\xi < \infty, \] (45)
will decay to zero. Thus, the wave amplitude $\Psi$ is not a constant, but decreases slowly with time. Next, we perform a perturbation analysis of Eq. (41) assuming $\gamma (\gg \epsilon)$ is a small parameter with $1 - A \sim B \gg \gamma$ (in magnitudes). In the previous discussion [after Eq. (40)], we have seen that this condition may be satisfied for plasma parameters corresponding to Case I. So, we introduce a new space coordinate $z$ in a frame moving with the solitary wave and normalized to its width as
\[ z = \left( \xi - \frac{A}{3} \int_{0}^{\tau} \Psi d\tau \right) / W, \] (46)
where $\Psi$ is assumed to vary slowly with time and $\Psi = \Psi(\gamma, \tau)$. We also assume that $\varphi \equiv \varphi(z, \tau)$. Following Refs. 47, 48, and generalizing the multiple time scale analysis with respect to $\gamma$, we obtain, after few steps, the following soliton solution of Eq. (41) with the effect of dust viscosity
\[ \varphi = \tilde{\varphi} \sech^2 z + O(\gamma), \] (47)
where $\tilde{\varphi} = \Psi_0 (1 + \tau/\tau_0)^{-2/3}$, $\Psi = \Psi_0$ at $\tau = 0$ and $\tau_0$ is given by
\[ \tau_0^{-1} = 3\gamma \sqrt{\frac{A\Psi_0^3}{3B}} \int_{-\infty}^{+\infty} \sech^2 z \tanh^2 z \, dz \approx 4 \frac{\gamma}{5} \sqrt{\frac{A\Psi_0^3}{3B}}. \] (48)
This shows that the amplitude of the DA solitary waves decreases slowly with time with the effect of a small amount of the dust viscosity ($\gamma$). From Eq. (48), it is also seen that as the wave amplitude decreases, the propagation speed also slows down in widening the pulse width.

We numerically investigate the properties of the wave amplitude $\tilde{\varphi}$ of the solution (48) for different plasma parameters as in Case I such that $A \gtrsim B \gg \gamma$ holds. The results are shown in Fig. 2(b). We find that the wave amplitude slows down with time. A small increase of the amplitude with the effects of exchange-correlation is seen to occur. Furthermore, an increase in the electron number density, which increases the quantum effects and hence the dispersive coefficient $A$ and $\Gamma_d$, leads to a significant decrease of the wave amplitude.

On the other hand, the evolution equation for the propagation of DA solitary waves in strongly coupled degenerate dense plasmas without the exchange-correlation effects of electrons and ions (Case III) can be obtained as
\[ \frac{\partial}{\partial \xi} \left[ \frac{\partial \Phi^{(1)}}{\partial \tau} + \tilde{A} \frac{\partial \Phi^{(1)}}{\partial \xi} + \tilde{B} \frac{\partial^3 \Phi^{(1)}}{\partial \xi^3} \right] + \tilde{\gamma} \left( \frac{\partial \Phi^{(1)}}{\partial \xi} \right)^2 = 0, \] (49)
where the corresponding coefficients of nonlinearity, dispersion and damping, i.e., $\tilde{A}$, $\tilde{B}$ and $\tilde{\gamma}$ are
\[ \tilde{A} = \frac{1}{2V_p} \left[ \frac{1}{3D^2} \left( \frac{\mu}{D_e} - \frac{1 + \mu}{D_i} \right) - \tilde{D} \right], \]
\[ \tilde{B} = \frac{1}{2V_p D^2} \left[ 1 - \frac{1}{2} \left( \frac{\mu H_e^2}{D_e^2} + \frac{1 + \mu}{D_i} \frac{H_e^2}{D_i} \right) \right], \]
\[ \tilde{\gamma} = \frac{D}{2V_p \tau_m}. \]
where $V_p = (1 + \eta^*/\tau_m + 1/D)^{1/2}$, $\tilde{D} = 1 + V_p^2 + 2/D$.

The properties of the solitary waves and wave amplitude with the effects of viscous dissipation are shown in Fig. 3. We find that the properties of the solitary waves in degenerate dense plasmas remain similar to those in non-degenerate plasmas [Fig. 2]. However, a reduction in the electron concentration (hence an increase in both $|A|$ and $\gamma$, and decrease in $B$, $\Gamma_d$, $H_e$ and $H_i$) leads to a decrease (in magnitude) in both the amplitude and width of the DA wave. It is also found that in relatively dense plasmas, the amplitude of the DA soliton takes longer time to vanish by the effects of dust viscosity.

### B. Shock-like waves in the hydrodynamic regime $\omega \tau_m \ll 1$

We study the nonlinear propagation of DA waves in strongly coupled nondegenerate quantum plasmas in the hydrodynamic regime $\tau_m = \omega_{pd} \tau_m \ll 1$. For this study, we note that the dissipative effect associated with the dust viscosity will play an important role in the formation of DA shock-like waves. We also assume that this dissipative effect is small for which $\eta^* \sim o(\epsilon^{1/2})$. We repeat here that the hydrodynamic modes are not so relevant for the parameters in strongly coupled dense plasmas with degenerate electrons and ions, and we skip the relevant details here. Following the same procedure as in the previous subsection for the nonlinear kinetic waves, we obtain the same set of equations in the lowest and next lowest orders of $\epsilon$ from Eqs. (17), (18), and (20).
obtains the following two equations

\[ N = \lim_{\varepsilon \to 0} \left( A K \right) \]

Finally, eliminating the second-order quantities, namely 

\[ N_j, V_d, \text{and } \Phi \] from Eqs. (34), (35), (37) and (52), we obtain the KdV-Burgers (KdVB) equation as

\[ \frac{\partial \Phi}{\partial \tau} + A \Phi \frac{\partial \Phi}{\partial \xi} + B \frac{\partial^2 \Phi}{\partial \xi^2} = \frac{1}{2} \eta^* \frac{\partial^2 \Phi}{\partial \xi^2}, \]

except Eq. (19). From the latter, on equating the co-efficients of the lowest and next lowest order of \( \varepsilon \), one obtains the following two equations

\[ -V_p \frac{\partial V_d^{(1)}}{\partial \xi} + \frac{\partial N_d^{(1)}}{\partial \xi} - \frac{\partial \Phi^{(1)}}{\partial \xi} = 0, \]

\[ -V_p \frac{\partial V_d^{(2)}}{\partial \xi} + V_d^{(1)} \frac{\partial V_d^{(1)}}{\partial \xi} - V_p N_d^{(1)} \frac{\partial V_d^{(1)}}{\partial \tau} + \frac{\partial V_d^{(1)}}{\partial \tau} + \frac{\partial N_d^{(2)}}{\partial \xi} - \frac{\partial \Phi^{(2)}}{\partial \xi} - N_d^{(1)} \frac{\partial \Phi^{(1)}}{\partial \xi} = \eta^* \frac{\partial^2 V_d^{(1)}}{\partial \xi^2}. \]

FIG. 3. (Color online) Profiles of the soliton solution (upper panel) without any damping effect and the time dependent amplitude of the soliton (lower panel) of the KdV equation (49) in strongly coupled dense plasmas with degenerate electrons and ions (with no exchange-correlation effect) for different set of parameters. The parameter values corresponding to solid and dashed lines are (a) \( n_{e0} \sim 10^{14} \text{ cm}^{-3}, n_{i0} = 200 n_{e0}, Z_d = 2, T_e \sim T_i \sim 10^5 \text{ K} \) and \( T_d = 0.5 T_i \) for which \( \Gamma_d = 215, H_e = 19, H_i = 0.6, \gamma = 1.2, B = 0.98 \) and \( \gamma = 0.1 \), and (b) \( n_{e0} \sim 10^{18} \text{ cm}^{-3}, \) and all others are the same as for the solid line, for which \( \Gamma_d = 170, H_e = 17, H_i = 0.55, \gamma = 1.4, B = 0.64 \)

FIG. 4. (Color online) Analytic shock solution (56) of the KdV-Burgers equation (53) for different sets of parameters (c) \( n_{e0} \sim 10^{18} \text{ cm}^{-3}, \) \( n_{i0} = 5 n_{e0} \), \( Z_d = 10 T_e \sim T_i \sim 10^5 \text{ K} \) and \( T_d = 0.4 T_i \), for which \( \Gamma_d = 5, A = -1.6, B = 0.58 \) and \( \gamma = 0.7 \), and (d) \( Z_d = 15 \) and all others the same as for plot (c) for which \( \Gamma_d = 9.8, A = -1.8, B = 0.4 \) and \( \gamma = 0.68 \). Also, in each plot \( C_1 = -0.1 \) and \( C_2 = 5 \). The shock profile is seen to be monotonic in nature.

except Eq. (19). From the latter, on equating the co-efficients of the lowest and next lowest order of \( \varepsilon \), one obtains the following two equations

\[ -V_p \frac{\partial V_d^{(1)}}{\partial \xi} + \frac{\partial N_d^{(1)}}{\partial \xi} - \frac{\partial \Phi^{(1)}}{\partial \xi} = 0, \]

\[ -V_p \frac{\partial V_d^{(2)}}{\partial \xi} + V_d^{(1)} \frac{\partial V_d^{(1)}}{\partial \xi} - V_p N_d^{(1)} \frac{\partial V_d^{(1)}}{\partial \tau} + \frac{\partial V_d^{(1)}}{\partial \tau} + \frac{\partial N_d^{(2)}}{\partial \xi} - \frac{\partial \Phi^{(2)}}{\partial \xi} - N_d^{(1)} \frac{\partial \Phi^{(1)}}{\partial \xi} = \eta^* \frac{\partial^2 V_d^{(1)}}{\partial \xi^2}. \]

Finally, eliminating the second-order quantities, namely 

\[ N_j, V_d, \text{and } \Phi \] from Eqs. (34), (35), (37) and (52), we obtain the KdV-Burgers (KdVB) equation as

\[ \frac{\partial \Phi}{\partial \tau} + A \Phi \frac{\partial \Phi}{\partial \xi} + B \frac{\partial^2 \Phi}{\partial \xi^2} = \frac{1}{2} \eta^* \frac{\partial^2 \Phi}{\partial \xi^2}, \]

where

\[ A = \frac{1}{2 V_p^7} \left[ \frac{1}{D^2} \left( \mu D_{c2} - \frac{(1 + \mu) D_{d2}}{D_{r1}^3} \right) - D D' \right], \]

with \( D' = 2 + 3/D \) and the phase velocity of the DA shock waves is given as

\[ V_p' = \left( 1 + \frac{1}{D} \right)^{1/2}, \]

which has the same expression as \( \omega_{r}/k \) obtained from the dispersion relation [Eq. (25)] of the hydrodynamic mode in the limit \( k \to 0 \). We note that the dispersion coefficient \( B \) remains the same, while the nonlinear and the damping terms get modified. The KdV-Burgers equation (53) has a shock wave solution of the form \( [49, 50] \)

\[ \Phi^{(1)} = C_1 - \frac{3 \eta^*}{25 A B (1 + C_2 \varepsilon^2)^2}, \]

with the parameter

\[ \theta = -\frac{\eta^*}{10 B} \left[ \xi - \left( AC_1 - \frac{3 \eta^2}{100 A B^2} \right) \tau \right], \]

where \( C_1 \) and \( C_2 \) are arbitrary constants. The profiles of this shock solution, which are monotonic in nature, are shown in Fig. 4 for different plasma parameters relevant for Case I. It is found that an increase in the dust charge number \( Z_d \), which, in turn, increases the coupling parameter \( \Gamma_d \) and the nonlinear coefficient \( |A| \), and decreases the coefficients of dispersion and dissipation, leads to an increase in the shock height (amplitude), but a reduction in the shock width. The shock exhibited are indeed monotonic in nature.
We also numerically investigate the travelling wave solution of the KdV-Burgers equation \((53)\) with the same plasma parameters as in Fig. 4 for Figs. 5 (c) and (d) and also with different values of the electron number density [for Figs. 6 (a) and (b)]. In a moving frame of reference \(\zeta = \xi - u_0 t\), where \(u_0\) is the constant velocity of the stationary shock waves, the KdVB equation \((53)\) reduces to a third-order ordinary differential equation. This equation is then integrated once with respect to \(\zeta\) subject to the boundary conditions \(\psi(0) = \beta\) and also with different values of the electron number density \([\text{for Figs. 6 (a) and (b)}]\). In a moving plasma environments with lower particle number density \([\text{Subplots 6 (a) and (b)}]\) and also with different values of the electron number density \([\text{for Figs. 6 (c) and (d)}]\) and also with different values of the electron number density \([\text{for Figs. 6 (c) and (d)}]\).

By introducing \(d\Phi(1)/d\zeta = \psi\), Eq. \((58)\) can be rewritten as

\[
B \frac{\partial^2 \Phi(1)}{\partial \zeta^2} - \frac{\eta^*}{2} \frac{\partial \Phi(1)}{\partial \zeta} - u_0 \Phi(1) + \frac{A}{2} \Phi(1)^2 = 0. \tag{58}
\]

By introducing \(d\Phi(1)/d\zeta = \psi\), Eq. \((58)\) can be rewritten as

\[
\frac{d\Phi(1)}{d\zeta} = \psi, \quad \frac{d\psi}{d\zeta} = \frac{1}{2B} \left(2u_0 \Phi(1) - A\Phi(1)^2 + \eta^* \psi\right). \tag{59}
\]

In the \((\Phi(1), \Psi)\) plane, Eq. \((59)\) has two singular points, namely \((0, 0)\) and \((2u_0/A, 0)\). While the former is a saddle point corresponding to the equilibrium downstream state, the nature of the latter (corresponding to the upstream state) can be determined from the asymptotic behaviors of the solution of the form \(\sim \exp(\beta \zeta)\) of the linearized Eq. \((59)\). This gives

\[
\beta = \frac{1}{4B} \left(\eta^* \pm \sqrt{\eta^* + 16B u_0}\right). \tag{60}
\]

Thus, the singular point is a stable node or stable focus according to when \(\eta^* + 16Bu_0 \geq 0\). While the stable node corresponds to monotonic shock front, the stable focus always corresponds to the oscillatory nature. We note that for the parameters relevant for Case I, \(B\) is mostly positive, and so the condition \(\eta^* + 16Bu_0 > 0\) holds good.

We numerically solve Eq. \((59)\) by the standard 4th-order Runge-Kutta scheme with a step size \(\Delta \zeta = 0.001\) considering the parameters in Case I. The results are shown in Fig. 5 in which only the monotonic nature of the profile is seen to occur. Plots (a) and (b) are for different electron number densities, whereas plots (c) and (d) are for the same parameters as in left and right panels of Fig. 4.

The time developments of the shock profiles, given by Eq. \((58)\), are also shown in Fig. 6 for different shock strengths \([\text{Subplots 6 (a) to 6 (c)}]\) as well as for different plasma environments with lower particle number density and low dust temperature \([\text{Subplot 6 (d)}]\). From Figs. 6 (a) to 6 (c) it is found that the shock width increases with time for increasing values of the viscosity coefficient \(\eta^*\). However, from Fig. 6 (d) the shock is found to be stable.

FIG. 5. (Color online) Numerical travelling wave solution of the KdV-Burgers equation \((53)\) for different sets of parameters \((a) n_{e0} = 2 \times 10^{17} \text{ cm}^{-3}, n_{i0} = 5n_{e0}, Z_d = 5, T_d \sim 10^5 \text{ K}\) and \(T_d = 0.1T_i\) for which \(\Gamma_d = 3.7, A = -0.94, B = 2.9\) and \(\eta^* = 0.2\), \((b) n_{e0} = 3 \times 10^{17} \text{ cm}^{-3}\) and all others the same as for plot (a) for which \(\Gamma_d = 4.2, A = -1, B = 2.3\) and \(\eta^* = 0.3\), \((c) n_{e0} = 10^{18} \text{ cm}^{-3}, n_{i0} = 5n_{e0}, Z_d = 10, T_d \sim T_i \sim 10^5 \text{ K}\) and \(T_d = 0.4T_i\) for which \(\Gamma_d = 5, A = -1.6, B = 0.58\) and \(\eta^* = 0.7\), and \((d) Z_d = 15\) and all others the same as for plot (c) for which \(\Gamma_d = 9.8, A = -1.8, B = 0.4\) and \(\eta^* = 0.68\). The shock profile is seen to be monotonic in nature.

V. SUMMARY AND CONCLUSION

To summarize, we have investigated the propagation characteristics of dust-acoustic solitary waves and shocks in a strongly coupled dusty quantum plasma. A generalized viscoelastic model with the effects of electrostatic dust pressure due to strong coupling of dust particles as prescribed by Gozadinos et al. \[39\], as well as a quantum hydrodynamic model with the effects of quantum force associated with the Bohm potential and the quantum exchange-correlation of both electrons and ions are considered. In the linear theory, a general dispersion relation is derived from which we have obtained two classes of modes, namely kinetic \((\omega \tau_m \gg 1)\) and hydrodynamic \((\omega \tau_m \ll 1)\). In the nonlinear regime these modes are shown to propagate as solitary and shock waves respectively. However, the amplitude of these solitary waves is seen to decay slowly with time by the effect of small amount of dust viscosity. The results can be summarized as follows:

**Linear dust-acoustic waves:**

(i) Propagation of low-frequency \((\omega < \omega_{pd})\) dust-acoustic waves in strongly coupled dusty plasmas is possible both in the hydrodynamic \((\omega \tau_m \ll 1)\) and kinetic \((\omega \tau_m \gg 1)\) regimes.

(ii) In the hydrodynamic regime, strong coupling of dust particles \((\Gamma_d)\) together with weak effects from the Bohm potential \((H_b)\) and the exchange-correlation \((\propto C_j)\) by an increase in the dust charge number \((Z_d)\) reduces the real part \((\omega_r)\) of the wave frequency having
FIG. 6. (Color online) Contour plots of shock solutions of the KdV-Burgers equation (53) for different shock strengths (a) \( \eta^* = 0.08 \), (b) \( \eta^* = 0.2 \), and (c) \( \eta^* = 0.7 \). Plots show that the shock width increases with time for increasing values of the viscosity coefficient \( \eta^* \). The other parameter values are \( \Gamma_d = 5 \), \( A = -1.6 \) and \( B = 0.6 \) corresponding to \( n_{e0} \sim 10^{18} \text{ cm}^{-3} \), \( n_{i0} = 5n_{e0} \), \( Z_d = 10, T_e \sim T_i \sim 10^5 \text{ K} \) and \( T_d = 0.4T_i \). Subplot (d) is for other plasma environments where \( n_{e0} = 3 \times 10^{17} \text{ cm}^{-3} \), \( n_{i0} = 5n_{e0} \), \( Z_d = 10, T_e \sim T_i \sim 10^5 \text{ K} \) and \( T_d = 0.1T_i \) for which \( \Gamma_d = 4 \), \( A = -1 \), \( B = 2.4 \) and \( \eta^* = 0.3 \). It is found that shock may be stable in a regime with relatively lower particle number density and low dust temperature.

Cut-offs at lower wave numbers. The value of \( \omega_r \) increases for wave numbers less than its critical value, i.e., \( k < k_{cr} \). For \( k > k_{cr} \), \( \omega_r \) decreases to zero. These zero-frequency modes appear due to the imaginary term in the dispersion relation by the effects of dust viscosity, and thus may also occur in other dissipation mechanism such as dust-neutral collisions [50]. A significant increase in \( \omega_r \) by a small enhancement of ion concentration, and hence with higher values of \( \Gamma_d, H_e \) and \( H_i \), but lower values of \( C_j \) is seen to occur with a cut-off at higher \( k \).

(iii) In the kinetic regime, the viscosity is no longer a dissipative effect, however, it increases the wave frequency via the temperature \( T_e \) and the strong coupling of dust particles (\( \Gamma_d \)).

(iv) In the weakly nonlinear limit, the exchange-correlation potential is expressed as a function of the electron/ion number density satisfying \( n_{j0} \lesssim 10^{21} \text{ cm}^{-3} \). In this parameter regime, the electrons and ions are assumed to be nondegenerate, and hence the quantum effects, such as tunnelling and exchange-correlation do not greatly influence the linear wave modes. However, such a parameter restriction for the number density may be irrelevant in different plasma environments, e.g., in strongly coupled dusty quantum plasmas with degenerate (nonrelativistic) electrons and ions without any exchange correlation of them. In this situation, low-frequency dust-acoustic modes exist only in the kinetic regime, and quantum effects are to enhance the wave frequency with \( k \).

Weak nonlinear dust-acoustic solitary waves in the kinetic regime:

(a) Low-frequency dust-acoustic kinetic wave modes may propagate as solitary waves, however, with the effect of a small amount of the dust viscosity which slows down the wave amplitude with time. This viscosity can be comparable to or even dominate over the dispersion due to separation of charges and quantum tunneling effects, and hence can no longer be negligible in strongly coupled dusty degenerate or nondegenerate quantum plasmas. In the case of plasmas with nondegenerate electrons and ions (Case I), both the amplitude and width (in magnitude) of the solitary waves get enhanced by the effects of the exchange-correlation of the nondegenerate species. In this case, an increase in the ion number density, which increases the dust coupling parameter \( \Gamma_d \), the nonlinear coefficient \( A \) and the damping coefficient \( \gamma \), but decreases the dispersion coefficient \( B \), leads to a significant decrease (in magnitude) in both the amplitude and width of the soliton. The properties of the soliton remain almost similar in dense plasmas with degenerate species (Case III). In this case, an increase in the particle number density (hence increasing values of \( \Gamma_d, H_j \) and \( B \), and decreasing values of \( A \) and \( \gamma \)) leads to an increase (in magnitude) in both the amplitude and width of the soliton.
(b) An approximate soliton solution of the KdV equation \([40]\) with the effect of damping is obtained by a perturbation approach following Refs. \([47, 48]\) on the assumption that the coefficients of Eq. \([40]\) satisfy \(1 \sim A \gtrsim B \gg \gamma\). This holds for relatively low-density plasmas. It is shown that the amplitude of the soliton gets reduced with time with a small effect of the dust viscosity. The qualitative behaviors of these solitons remain similar both for nondegenerate and degenerate plasmas. However, in the latter it is noticed that the higher the number density the longer is the time for the amplitude to assume its zero value.

**Weakly nonlinear dust-acoustic shocks in the hydrodynamic regime:**

(I) Low-frequency dust-acoustic hydrodynamic wave mode propagates as shocks in strongly coupled nondegenerate quantum plasmas in which particle number density is relatively lower than plasmas which support the existence of solitary (kinetic) waves. Both analytical and numerical (stationary) solutions of the KdB-Burgers equation are obtained, and are found in good agreement with the same set of parameters. The shock profile is found to be monotonic in nature in the regime \(1 < \Gamma_{d} < 10\) and with \(|A| > B > \eta^{3}/2\). By increasing the coupling parameter \(\Gamma_{d}\) as well as the nonlinearity \(|A|\), and so decreasing the dispersion and dissipation effects, it is found that the shock height (or amplitude in magnitude) increases with a reduction of its width.

(II) Numerical solutions of the KdVB equation \([53]\) reveal that the shock width decreases with time as the strength of dissipation increases. That is, the shock width is directly proportional to the dust viscosity coefficient \(\eta\). This suggests that one can obtain more accurate values of the viscosity from the shock thickness together with the simulation results. It is also found that shock may be stable in a regime with relatively lower particle number density and dust temperature.

To conclude, the theoretical results should be useful for the identification of dust-acoustic collective plasma modes and the formation of nonlinear structures like solitons and shocks in strongly coupled dusty quantum plasmas such as those in laboratory and space plasmas as well as in dense astrophysical environments where electrons and ions are degenerate or partially degenerate.

**ACKNOWLEDGMENTS**

This work was supported by the National Natural Science Foundation of China (No. 11104012) and the Fundamental Research Funds for the Central Universities (No. FRF-TP-09-019A, No. FRF-BR-11-031B), and partially supported by the SAP-DRS (Phase-II), UGC, New Delhi, through sanction letter No. F.510/4/DRS/2009 (SAP-I) dated 13 Oct., 2009, and by the Visva-Bharati University, Santiniketan-731 235, through Memo No. REG/Notice/156 dated January 7, 2014.

[1] P. K. Shukla and A. A. Mamun, *Introduction to Dusty Plasma Physics* (Institute of Physics, Bristol, 2002).
[2] S. V. Vladimirov, K. Ostrikov, and A. A. Samarian, *Physics and Applications of Complex Plasmas* (Imperial College Press, London, 2005).
[3] V. E. Fortov, A. G. Khrapak, S. A. Khrapak, V. I. Molotkov, and O. F. Petrov, Phys. Usp. **47**, 447 (2004).
[4] A. V. Ivlev, S. A. Khrapak, A. G. Khrapak, and G. E. Morfill, Phys. Rep. **421**, 1 (2005).
[5] N. N. Rao, P. K. Shukla, and M. Y. Yu, Planet. Space Sci. **38**, 543 (1990).
[6] A. Barkan, R. L. Merlino, and N. D’Angelo, Phys. Plasmas **2**, 3563 (1995).
[7] P. K. Shukla, Phys. Rev. E **61**, 7249 (2000).
[8] P. K. Shukla, Phys. Plasmas **10**, 1619 (2003).
[9] P. Hohenberg and W. Kohn, Phys. Rev. **136**, B864 (1964).
[10] W. Kohn and L. J. Sham, Phys. Rev. **140**, A1133 (1965).
[11] P. K. Shukla and S. Ali, Phys. Plasmas **12**, 114502 (2005).
[12] S. Ali and P. K. Shukula, Phys. Plasmas **13**, 022313 (2006).
[13] W. F. El-Taibany and M. Wadati, Phys. Plasmas **14**, 042302 (2007).
[14] S. K. El-Labany, N. M. El-Siragy, W. F. El-Taibany, and E. E. Behery, Phys. Plasmas **16**, 093701 (2009).
[15] A. Mushtaq, Phys. Plasmas **14**, 113701 (2007).
[16] W. M. Moslem, P. K. Shukla, S. Ali, and R. Schlickeiser, Phys. Plasmas **14**, 042107 (2007).
[17] A. P. Misra and A. R. Chowdhury, Phys. Plasmas **13**, 072305 (2006).
[18] W. Masood, A. M. Mirza, and S. Nargis, Phys. Plasmas **15**, 103703 (2008).
[19] S. A. Khan, W. Masood, and M. Siddiq, Phys. Plasmas **16**, 013701 (2009).
[20] M. Sadiq, S. Ali, and R. Sabry, Phys. Plasmas **16**, 013706 (2009).
[21] Y. Wang, Z. Zhou, H. Qiu, F. Wang, and Y. Lu, Phys. Plasmas **19**, 013704 (2012).
[22] H. Ur-Rehman, S. A. Khan, W. Masood, and M. Siddiq, Phys. Plasmas **15**, 124501 (2008).
[23] M. Akbari-Moghanjoughi, Phys. Plasmas **17**, 052302 (2010).
[24] M. Akbari-Moghanjoughi, Phys. Plasmas **17**, 123709 (2010).
[25] M. Rosenberg and G. Kalman, Phys. Rev. E **56**, 7166 (1997).
[26] P. K. Kaw and A. Sen, Phys. Plasmas **5**, 3552 (1998).
[27] R. T. Farouki and S. Hamaguchi, Phys. Rev. E **47**, 4330 (1993).
[28] W. L. Slattery, G. D. Doolen, and H. E. DeWitt, Phys. Rev. A **21**, 2087 (1980).
[29] V. E. Fortov, I. T. Lakubov, and A. G. Khrapak, *Physics of strongly coupled plasma* (Clarendon Press, Oxford,
[30] S. E. Cousens, S. Sultana, I. Kourakis, V. V. Yaroshenko, F. Verheest, and M. A. Hellberg, Phys. Rev. E 86, 066404 (2012).

[31] S. E. Cousens, V. V. Yaroshenko, S. Sultana, M. A. Hellberg, F. Verheest, and I. Kourakis, Phys. Rev. E 89, 043103 (2014).

[32] S. Ghosh, N. Chakrabarti, and P. K. Shukla, Phys. Plasmas 19, 072123 (2012).

[33] K. Ourabah and M. Tribeche, Phys. Rev. E 88, 045101 (2013).

[34] A. P. Misra and P. K. Shukla, Phys. Rev. E 85, 026409 (2012).

[35] P. K. Shukla, B. Eliasson, and L. Stenflo, Phys. Rev. E 86, 016403 (2012).

[36] G. Gozadinos, A. V. Ivlev, and J. P. Boeuf, New J. Phys. 5, 32 (2003).

[37] V. V. Yaroshenko, F. Verheest, H. M. Thomas, and G. E. Morfill, New J. Phys. 11, 073013 (2009).

[38] B. M. Veeresha, S. K. Tiwari, A. Sen, P. K. Kaw, and A. Das, Phys. Rev. E 81, 036407 (2010).

[39] S. Ghosh, M. R. Gupta, N. Chakrabarti, and M. Chaudhuri, Phys. Rev. E 83, 066406 (2011).

[40] S.-H. Mao and J.-K. Xue, Phys. Scr. 84, 055501 (2011).

[41] G. Manfredi and F. Hass, Phys. Rev. B. 64, 075316 (2001).

[42] G. Manfredi, Fields Inst. Commun. 46, 263 (2005).

[43] L. Brey, J. Dempsey, N. F. Johnson, and B. I. Halperin, Phys. Rev. B 42, 1240 (1990).

[44] S. Chandrasekhar, Mon. Not. R. Astron. Soc. 95, 207 (1935).

[45] S. Ichimaru, H. Iyetomi, and S. Tanaka, Phys. Rep. 149, 91 (1987).

[46] V. M. Atrazhev and T. Lakubov, Phys. Plasmas 2, 2624 (1995).

[47] M. S. Janaki, B. Dasgupta, M. R. Gupta, and B. K. Som, Phys. Scripta 45, 368 (1992).

[48] A. Barman and A. P. Misra, Phys. Plasmas 21, 073708 (2014).

[49] A. D. Polyanin and V. F. Zaitsev, Handbook of Nonlinear Partial Differential Equations, 2nd ed. (CRC Press, Boca Raton, FL, 2012), p. 885.

[50] A. P. Misra, N. C. Adhikary, and P. K. Shukla, Phys. Rev. E 86, 056406 (2012).