The Advantage Regret-Matching Actor-Critic

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Abstract

Regret minimization has played a key role in online learning, equilibrium computation in games, and reinforcement learning (RL). In this paper, we describe a general model-free RL method for no-regret learning based on repeated reconsideration of past behavior. We propose a model-free RL algorithm, the Advantage Regret-Matching Actor-Critic (ARMAC): rather than saving past state-action data, ARMAC saves a buffer of past policies, replaying through them to reconstruct hindsight assessments of past behavior. These retrospective value estimates are used to predict conditional advantages which, combined with regret matching, produces a new policy. In particular, ARMAC learns from sampled trajectories in a centralized training setting, without requiring the application of importance sampling commonly used in Monte Carlo counterfactual regret (CFR) minimization; hence, it does not suffer from excessive variance in large environments. In the single-agent setting, ARMAC shows an interesting form of exploration by keeping past policies intact. In the multiagent setting, ARMAC in self-play approaches Nash equilibria on some partially-observable zero-sum benchmarks. We provide exploitability estimates in the significantly larger game of betting-abstracted no-limit Texas Hold'em.

1 Introduction

The notion of regret is a key concept in the design of many decision-making algorithms. Regret minimization drives most bandit algorithms, is often used as a metric for performance of reinforcement learning (RL) algorithms, and for learning in games [3]. When used in algorithm design, the common application is to accumulate values and/or regrets and derive new policies based on these accumulated values. One particular approach, counterfactual regret (CFR) minimization [39], has been the core algorithm behind super-human play in Computer Poker research [4, 29, 6, 8].

We investigate the problem of generalizing these regret minimization algorithms over large state spaces in the sequential setting using end-to-end function approximators, such as deep networks. There have been several approaches that try to predict the regret, or otherwise, simulate the regret minimization: Regression CFR (RCFR) [38], advantage regret minimization [20], regret-based policy...
All of these approaches have focused either on the multiagent or single-agent problem exclusively, some have used expert features, while others try to search to scale. Another common approach is based on fictitious play [18, 19, 23, 28], a simple iterative self-play algorithm based on best response. A common technique among several of these algorithms is to use reservoir sampling to maintain a buffer that represents a uniform sample over past data, which is used to train a classifier representing the average policy. In Neural Fictitious Self-Play (NFSP), this produced competitive policies in limit Texas Hold’em [19], and in Deep CFR this method was shown to approach an approximate equilibrium in a large subgame of Hold’em poker. A generalization of fictitious play, policy-space response oracles (PSRO) [23], stores past policies and a meta-distribution over them, replaying policies against other policies, incrementally adding new best responses to the set, which can be seen as a population-based learning approach where the individuals are the policies and the distribution is modified based on fitness. This approach only requires simulation of the policies and aggregating data; as a result, it was able to scale to a very large real-time strategy game [37]. In this paper, we describe an approximate form of CFR in a training regime that we call retrospective policy improvement. Similar to PSRO, our method stores past policies. However, it does not store meta-distributions or reward tables, nor do the policies have to be approximate best responses, which can be costly to compute or learn. Instead, the policies are snapshots of those used in the past, which are retrospectively replayed to predict a conditional advantage, which used in a regret matching algorithm produces the same policy as CFR would do. In the single-agent setting, ARMAC is related to POLITEX [1], except that it is based on regret-matching [17] and it predicts average quantities rather than explicitly summing over all the experts to obtain the policy. In the multiagent setting, it is a sample-based, model-free variant of RCFR with one important property: it uses trajectory samples to estimate quantities without requiring importance sampling as in standard Monte Carlo CFR [24], hence it does not suffer from excessive variance in large environments. This is achieved by using critics (value estimates) of past policies that are trained off-policy using standard policy evaluation techniques. In particular, we introduce a novel training regime that estimates a conditional advantage \( W_i(s, a) \), which is the cumulative counterfactual regret \( R_i(s, a) \), scaled by factor \( B(s) \) that depends on the information state \( s \) only; hence, using regret-matching over this quantity yields the policy that CFR would compute when regret-matching to the same (unscaled) regret values. By doing this entirely from sampled trajectories, the algorithm is model-free and can be done using any black-box simulator of the environment; hence, ARMAC inherits the scaling potential of PSRO without requiring a best-response training regime, driven instead by regret minimization.

2 Background

In this section, we describe the necessary terminology. Since we want to include the (partially-observable) multiagent case and we build on algorithms from regret minimization we use extensive-form games notations [33]. A single-player game represents the single-agent case where histories are aggregated appropriately based on the Markov property.

A game is a tuple \( \langle \mathcal{N}, A, S, \mathcal{H}, Z, u, \tau \rangle \), where \( \mathcal{N} = \{1, 2, \ldots, n\} \) is the set of players. By convention we use \( i \in \mathcal{N} \) to refer to a player, and \(-i\) for the other players \( (\mathcal{N} - \{i\}) \). There is a special player \( c \) called chance (or nature) that plays with a fixed stochastic strategy (chance’s fixed strategy determines the transition function). \( A \) is a finite set of actions. Every game starts in an initial state, and players sequentially take actions leading to histories of actions \( h \in \mathcal{H} \). Terminal histories, \( z \in Z \subset \mathcal{H} \), are those which end the episode. The utility function \( u_i(z) \) denotes the player \( i \)'s return over episode \( z \). The set of states \( S \) is a partition of \( \mathcal{H} \) where histories are grouped into information states \( s = \{ h, h', \ldots \} \) such that the player to play at \( s \), \( \tau(s) \), cannot distinguish among the possible histories (world states) due to private information only known by other players. Let \( \Delta(X) \) represent all distributions over \( X \): each player’s (agent’s) goal is to learn a policy \( \pi_i : S_i \rightarrow \Delta(A) \), where \( S_i = \{ s \mid s \in S, \tau(s) = i \} \). For some state \( s \), we denote \( A(s) \subseteq A \) as the legal actions at state \( s \), and all valid state policies \( \pi(s) \) assign probability 0 to illegal actions \( a \notin A(s) \).

Let \( \pi \) denote a joint policy. Define the state-value \( v_{\pi,i}(s) \) as the expected (undiscounted) return for player \( i \) given that state \( s \) is reached and all players follow \( \pi \). Let \( q_{\pi,i} \) be defined similarly except also conditioned on player \( \tau(s) \) taking action \( a \) at \( s \). Formally, \( v_{\pi,i}(s) = \sum_{a \in A} \pi(a | s) v_{\pi,i}(s) = \sum_{a \in A} \pi(a | s) v_{\pi,i}(s) \).
\[ \sum_{(h,z) \in Z(s)} \eta^\pi(h|s)\eta^\pi(h,z)u_i(z), \] where \( Z(s) \) are all terminal histories paired with their prefixes that pass through \( s \), \( \eta^\pi(h|s) = \frac{\eta^\pi(h)}{\eta^\pi(s)} \), where \( \eta^\pi(s) = \sum_{h' \in s} \eta^\pi(h') \), and \( \eta^\pi(h,z) \) is the product of probabilities of each action taken by the players’ policies along \( h \) to \( z \). The state-action values \( q_{\pi,i}(s,a) \) are defined analogously. Standard value-based RL algorithms estimate these quantities for policy evaluation. Regret minimization in zero-sum games uses a different notion of value, the **counterfactual value**: 
\[ v^\pi_i(s) = \sum_{(h,z) \in Z(s)} \eta^\pi_i(h)\eta^\pi(h,z)u_i(z), \]
where \( \eta^\pi_i(h) \) is the product of opponents’ policy probabilities along \( h \). We also write \( \eta^\pi_i(h) \) the product of player \( i \)’s own probabilities along \( h \). Under the standard assumption of perfect recall, we have that for any \( h, h' \in s \), \( \eta^\pi_i(h) = \eta^\pi_i(h') \). Thus counterfactual values are formally related to the standard values \([34]\):
\[ v^\pi_i(s) = \beta_{-i}(\pi,s) \sum_{h \in s} \eta^\pi_{-i}(h). \]
Also, \( q^\pi_i(s,a) \) is defined similarly except over histories \((ha,z) \in Z(s), \) where \( ha \) is history \( h \) concatenated with action \( a \).

Counterfactual regret minimization (CFR) is a tabular policy iteration algorithm that has been at the core of the advances in Poker AI \([39]\). On each iteration \( t \), CFR computes counterfactual values \( q^\pi_{i,t}(s,a) \) and \( v^\pi_{i,t}(s) \) for each state \( s \) and action \( a \in A(s) \) and the regret of *not* choosing action \( a \) (or equivalently the advantage of choosing action \( a \)) at state \( s \), \( r^t(s,a) = q^\pi_{i,t}(s,a) - v^\pi_{i,t}(s). \)

CFR tracks the cumulative regrets for each state and action, \( R^T_t(s,a) = \sum_{t=1}^T r^t(s,a). \)

Define \((x)^+ = \max(0,x)\); regret-matching then updates the policy of each action \( a \in A(s) \) as follows \([17]\):
\[ \pi^{t+1}(s,a) = \text{NormalizedReLU}(R^T_t(s,a), a) = \begin{cases} \frac{R^{t+1}_T(s,a)}{\sum_{b \in A(s)} R^{t+1}_T(s,b)} & \text{if } \sum_{b \in A(s)} R^{t+1}_T(s,b) > 0, \\ 0 & \text{otherwise}. \end{cases} \]

In two-player zero-sum games, the mixture policy \( \pi^T \) converges to the set of Nash equilibria as \( T \to \infty \).

Traditional (off-policy) Monte Carlo CFR (MCCFR) is a generic family of sampling variants \([24]\). In **outcome sampling** MCCFR, a behavior policy \( \mu_i \) is used by player \( i \), while players \( -i \) use \( \pi_{-i} \), a trajectory \( \rho \sim (\mu_i, \pi_{-i}) \) is sampled, and the **sampled counterfactual value** is computed:
\[ q^\rho_{i,t}(s,a \mid \rho) = \frac{1}{\eta^i_{(\mu_i, \pi_{-i})}(z)} \eta^i_{(\mu_i, \pi_{-i})}(ha,z)u_i(z), \]

if \((s,a) \in \rho \), or 0 otherwise. \( q^\rho_{i,t}(s,a \mid \rho) \) is an unbiased estimator of \( q^\pi_{i,t}(s,a) \) \([24]\) Lemma 1].

However, since these quantities are divided by \( \eta^i_{(\mu_i, \pi_{-i})}(z) \), the product of player \( i \)'s probabilities, (i) there can be significant variance introduced by sampling, especially in problems involving long sequences of decisions, and (ii) the ranges of the \( \tilde{v}^\rho_i \) can vary wildly (and unboundedly if the exploration policy is insufficiently mixed) over iterations and states, which could make approximating the values in a general way particularly challenging \([33]\). Deep CFR and Double Neural CFR are successful large-scale implementations of CFR with function approximation, and they get around this variance issue by using external sampling or a robust sampling technique, both of which require a perfect game model and enumeration of the tree. This is unfeasible in very large environments or in the RL setting where full trajectories are generated from beginning to the end without having access to a generative model which could be used to generate transitions from any state.

### 3 Retrospective Policy Improvement

Retrospective policy improvement focuses on learning from past behavior, by storing complete descriptions of value functions and/or policies, deriving a new policy from them via aggregation, rather than e.g. greedily optimizing a current policy. Specifically, a retrospective agent finds a new policy by playing back through past policies, rather than maintaining a buffer of past data. The general idea is common in learning partially-observable multiagent games, where algorithms are built upon regret minimization and learning from expert advice \([9]\). Regret minimization in self-play can approximate various forms of equilibria \([31]\). In single-agent settings, they can also approximate optimal policies in Markov decision processes \([1]\), even when the reward function changes over time \([14][21]\), and solve a general class of robust optimization problems \([13]\). We also show some appealing properties in the single-agent case via some illustrative examples (Section 4.4)
3.1 The Advantage Regret-Matching Actor-Critic

ARMAC is a model-free RL algorithm motivated by CFR. Like algorithms in the CFR framework, ARMAC uses a centralized training setup and operates in epochs that correspond to CFR iterations. Like RCFR, ARMAC uses function approximation to generate policies. ARMAC was designed so that as the number of samples per epoch increases and the expressiveness of the function approximator approaches a lookup table, the generated sequence of policies approaches that of CFR. Instead of accumulating cumulative regrets— which is problematic for a neural network—the algorithm learns a conditional advantage estimate \( \bar{W}(s,a) \) by regression toward a history-dependent advantage \( A(h,a) \), for \( h \in s \), and uses it to derive the next set of joint policies that CFR would produce. Indeed we show that \( \bar{W}(s,a) \) is an estimate of the cumulative regret \( R(s,a) \) up to a multiplicative factor which is a function of the information state \( s \) only, and thus cancels out during the regret-matching step. ARMAC is a Monte Carlo algorithm in the same sense as MCCFR: value estimates are trained from full episodes. It uses off-policy learning for training the value estimates (i.e. critics), which we show is sufficient to derive \( \bar{W} \). However, contrary to MCCFR, it does not use importance sampling. ARMAC is summarized in Algorithm 1.

Algorithm 1: Advantage Regret-Matching Actor-Critic

\begin{enumerate}
\item \textbf{input} : initial set of parameters \( \theta_0 \), number of players \( n \)
\item \textbf{for} epoch \( t \in \{0,1,2,\cdots\} \) \textbf{do}
  \item reset \( D \leftarrow \emptyset \)
  \item Let \( \pi'(s) = \text{NORMALIZEDRELU}(\bar{W}_{\theta_t}(s)) \) (i.e. Eq. 1)
  \item Let \( q_{\theta_i}(h) = \sum_{a \in A(h)} \pi'(h,a)q_{\theta_{\pi'}}(h,a) \)
  \item Let \( \mu_t^i \) be a behavior policy for each player \( i \)
  \item \textbf{for} episode \( k \in \{0,1,\cdots,K_{\text{act}}\} \text{ mod } n \) \textbf{do}
    \item Sample \( j \sim \text{UNIF}\{\{0,1,\cdots,t-1\}\} \)
    \item Sample trajectory \( \rho \sim (\mu_i, \pi_{\rho,i}^\prime) \)
    \item let \( d \leftarrow (i,j,\{u_i(\rho)\}_{i \in N}) \)
    \item \textbf{for} history \( h \in p \) where player \( i \) acts \textbf{do}
      \item let \( s \) be the state containing \( h \)
      \item let \( \bar{r} = \{q_{\theta_t'}(h,a') - v_{\theta_t'}(h)\}_{a' \in A(s)} \)
      \item let \( a \) be the action that was taken in \( \rho \)
      \item f append \((h,s,a,\bar{r},\pi'(s))\) to \( d \)
    \item \textbf{end}
    \item add \( d \) to \( D \)
  \item \textbf{end}
  \item \textbf{for} learning step \( k \in \{0,1,\cdots,K_{\text{learn}}\} \text{ do}
    \item Sample a random episode/batch \( d \sim \text{UNIF}(D) \);
    \item for history and corresponding state \((h,s) \in d\) \textbf{do}
      \item Use \text{TreeBackup}(\lambda) \text{ to train the critic } q_{\theta_t'}(h,a)
      \item If \( \pi(s) = i \): train \( \bar{W}_{\theta_t'} \) to predict \( A(h,a) \)
      \item If \( \tau(s) = -i \): train \( \bar{W}_{\theta_t'} \) to predict \( \pi'(s) \)
    \item \textbf{end}
  \item \textbf{end}
  \item Save \( \theta_t \) for future retrospective replays
  \item \( \theta_{t+1} \leftarrow \theta_t \)
\end{enumerate}

In practice, rather than using \( h \) as input to our approximators, we use a concatenation of all players’ observations, i.e. an encoding of the augmented information states or action-observation histories [10][22]. In some games this is sufficient to recover a full history. In others there is hidden state from all players, we can consider any chance event to be delayed until the first observation of its effects by any of the players in the game. Thus, the critics represent an expectation over those hidden outcomes. Since this does not affect the theoretical results, we choose this notation for simplicity. Importantly, ARMAC remains model-free: we never enumerate chance moves explicitly or evaluate their probabilities which may be complex for many practical applications.
(see Lemma 1). Once \( \hat{W}_i^t(s, a) \) is trained, the next joint policy \( \pi^{t+1}(s, a) \) can be produced by normalizing the positive part as in Eq. 1. After each training epoch the joint policy \( \pi^t \) is saved into a past policy reservoir, as it will have to be loaded and played during future epochs. Lastly, an average policy head \( \hat{\pi}^t \) is also trained via a classification loss to predict the policy \( \pi^t \) over all time steps \( t' \leq t \). We explain its use in Section 4.

Using a history-based critic allows ARMAC to avoid using importance weight (IW) based off-policy correction as is the case in MCCFR, but at the cost of higher bias due to inaccuracies that the critic has. Using IW may be especially problematic for long games. For large games the critic will inevitably rely on generalization to produce history-value estimates.

To save memory, reservoir sampling with buffer of size of 1024 was used to prune past policies.

### 3.2 Theoretical Properties

Each epoch \( t \) estimates \( q_{\pi^t,i}(h, a) = \sum_z \eta^{z}(h, z) u_i(z) \) and value \( v_{\pi^t,i}(h) = \sum_a \pi^t(h, a) q_{\pi^t,i}(h, a) \) for the current policies \( (\pi^t) \). Let us write the advantages \( A_{\pi^t,i}(h, a) = q_{\pi^t,i}(h, a) - v_{\pi^t,i}(h) \). Notice that we learn functions of the history \( h \) and not state \( s \).

At epoch \( T \), in order to deduce the next policy, \( \pi^{t+1} \), CFR applies regret-matching using the cumulative counterfactual regret \( R_i^T(s, a) \). As already discussed, directly estimating \( R_i^T \) using sampling suffers from high variance due to the inverse probability \( \eta_i^{(a,\pi_{-i})}(z) \) in (2). Instead, ARMAC trains a network \( \hat{W}_i^T(s, a) \) that estimates a conditional advantage along trajectories generated in the following way: For player \( i \) we select a behavior policy \( \mu_i^T \) providing a good state-space coverage, e.g. a mixture of past policies \( (\pi_j^T)_{t=T} \), with some added exploration (Section 3.3 provides more details). For the other players \(-i\), for every trajectory, we choose one of the previous opponent policies \( \pi_j^T \) played at some epoch \( j \) chosen uniformly at random from \( \{1, 2, \ldots, T\} \). Thus at epoch \( T \), several trajectories \( \rho^j \) are generated by following policy \( (\mu_i^T, \pi_j^T) \), where \( j \sim U(\{1, 2, \ldots, T\}) \).

Then at each step \((h, a)\) along these trajectory \( \rho^j \), the neural network estimate \( \hat{W}_i^T(s, a) \) (where \( s \triangleright h \)) is trained to predict the advantage \( A_{\pi,i}(h, a) \) using the empirical \( \ell_2 \) loss: \( \hat{L} = \left[ \hat{W}_i^T(s, a) - A_{\pi,i}(h, a) \right]^2 \).

Thus if the network has sufficient capacity, it will minimize this average loss, thus our estimate \( \hat{W}_i^T(s, a) \) will converge (when the number of trajectories goes to infinity) in each state-action pair \((s, a)\), such that the reach probability \( \frac{1}{T} \sum_{j=1}^T \eta^{(\mu_i^T, \pi_j^T)}(s)(\mu_i^T(s, a)) > 0 \), to the conditional expectation

\[
\hat{W}_i^T(s, a) = \sum_{h \triangleright s} \frac{1}{T} \sum_{j=1}^T \eta^{(\mu_i^T, \pi_j^T)}(h, \pi_{-i}(h, a)) \frac{1}{T} \sum_{h \triangleright s} \eta^{(\mu_i^T, \pi_j^T)}(s) = \frac{1}{T} \sum_{j=1}^T A_{\pi,i}(h, a) \eta^{(\mu_i^T, \pi_j^T)}(s)
\]

Notice that \( \hat{W}_i^T \) does not depend on the exploratory policy \( \mu_i^T \) for player \( i \) chosen in round \( T \). After several trajectories \( \rho^j \) our network \( \hat{W}_i^T \) provides us with a good approximation of the \( W_i^T \) values and we use it in a regret matching update to define the next policy, \( \pi^{t+1}(s, a) = \text{NORMALIZEDRELU}(\hat{W}_i^T(s, a)) \), i.e. Equation 1. The following lemma (proved in Appendix D) shows that if \( \hat{W}_i^T(s, a) \) is sufficiently close to the \( \hat{W}_i^T(s, a) \) values, then this is equivalent to CFR, i.e., doing regret-matching using the cumulative counterfactual regret \( R_i^T \).

**Lemma 1.** The policy defined by \( \text{NORMALIZEDRELU}(\hat{W}_i^T(s, a)) \) is the same as the one produced by CFR when regret matching is employed as the information-state learner:

\[
\pi^{t+1}(s, a) = \frac{R_i^{t+1}(s, a)}{\sum_b R_i^{t+1}(s, b)} = \frac{W_i^{t+1}(s, a)}{\sum_b W_i^{t+1}(s, b)}.
\]

The \( W^T(s, a) \) estimate the expected advantages \( \frac{1}{T} \sum_{j=1}^T A_{\pi,i}(h, a) \) conditioned on \( h \in s \). Thus ARMAC does not suffer from the variance of estimating the cumulative regret \( R_i^T(s, a) \), and in the
case of infinite capacity, we can prove that, from any \((s, a)\), the estimate \(\hat{W}^T(s, a)\) is unbiased as soon as the \((s, a)\) has been sampled at least once (see Lemma 2 in the Appendix).

### 3.3 Adaptive Policy Selection

Figure 1: An average reward a given policy modulation scores against opponent \(\check{\pi}^t\) as a function of time (measured in acting steps). The brown curve is a random uniform policy (i). Cyan, orange and blue is (ii) with \(\epsilon \in \{0.0, 0.01, 0.05\}\) respectively. Pink, green and yellow is (iii) with \(\epsilon \in \{0.0, 0.01, 0.05\}\) respectively.

ARMAC dynamically switches between what policy to use based on estimated returns. For every \(t\) there is a pool of candidate policies, all based on the following four policies: (i) random uniform policy. (ii) several policies defined by applying Eq. 1 over the current epoch’s regret only \((q_\theta(h, a) - v_\theta(h))\), with different levels of random uniform exploration: \(\epsilon \in \{0.0, 0.01, 0.05\}\). (iii) several policies defined by the mean regret, \(\bar{\pi}^t\) as stated in Algorithm 1, also with the same level of exploration. (iv) the average policy \(\bar{\pi}^t\) trained via classification. The purpose of generating experiences using those policies is to facilitate the problem of exploration and to help produce meaningful data at initial stages of learning before average regrets are learnt. Each epoch, the candidate policies are ranked by cumulative return against an opponent playing \(\bar{\pi}^t\). The one producing highest rewards is used half of the times. When sub-optimal policies are run for players \(-i\), they are not used to train mean regrets for player \(i\), but can be used to train the critic. Typically (but not always), (ii) produces the best policy initially and allows to bootstrap the learning process with the best data (Fig. 1). In later stages of learning, (iii) with the lowest level of \(\epsilon\) starts providing better policies and gets consistently picked over other policies. The more complex the game is, the longer it takes for (iii) to take over (ii).

Exploratory policy \(\mu^T_i\) is constructed by taking the most recent neural network with 50% probability or otherwise sampling one of the past neural networks from replay memory uniformly and modulating it by the above described method. Such sampling method provides both on-policy trajectories for learning \(q_\theta(h, a)\) while also making sure that previously visited states get revisited.

### 3.4 Single-Agent Environments

Despite ARMAC being based on commonly-used multiagent algorithms, it has properties that may be desirable in the single-agent setting. First, similar to policy gradient algorithms in the common “short corridor example” [36, Example 13.1], stochastic policies are representable by definition, since they are normalized positive mean regrets over the actions. This could have a practical effect that entropy bonuses typically have in policy gradient methods, but rather than simply adding arbitrary entropy, the relative regret over the set of past policies is taken into account.

Second, a retrospective agent uses a form of directed exploration of different exploration policies [2]. Here, this is achieved by the simulation \((\mu^T_i, \pi_{-i}^t)\), which could be desirable whenever there is

Figure 2: The (a) Multi-headed network architecture, and (b) Exploration example.
overlapping structure in successive tasks. \( \mu^T \) here is an exploratory policy, which consists of a mixture of all past policies (plus random uniform) played further modulated with different amounts of random uniform exploration (more details are given in Section 3.3). Consider a gridworld illustrated in Fig. 2(b). Green squares illustrate positions where the agent gets a reward and the game terminates. Most of RL algorithms would find the reward of +1 first as it is the closest to the origin \( S \). Once this reward is found, a policy would quickly learn to approach it, and finding reward +2 would be problematic. ARMAC, in the meantime, would keep re-running old policies, some of which would pre-date finding reward +1, and thus would have a reasonable chance of finding +2 by random exploration. This behaviour may also be useful if instead of terminating the game, reaching one of those two rewards would start next levels, both of which would have to be explored.

These properties are not necessarily specific to ARMAC. For example, POLITEX (another retrospective policy improvement algorithm [1]) has similar properties by keeping its past approximators intact. Like POLITEX, we show an initial investigation of ARMAC in Atari in Appendix B. Average strategy sampling MCCFR [16] also uses exploration policies that are a mixture of previous policies and uniform random to improve performance over external and outcome sampling variants. However, this exact sampling method cannot be used directly in ARMAC as it requires a model of the game.

3.5 Network architecture

ARMAC can be used with both feed-forward (FF) and recurrent neural networks (RNN) (Fig. 2(a)). For small games where information states can be easily represented, FF networks were used. For larger games, where consuming observations rather than information states is more natural, RNNs were used. The critic was evaluated by feeding both player observations, but to the respective sides of the network. Policies only get their own observations to the respective side of the network. Both sides of the network in Fig. 2(a) share weights. More details can be found in Appendix in Section F.

4 Empirical Evaluation

For partially-observable multiagent environments, we investigate Imperfect Information (II-) Goofspiel, Liar’s Dice, and Leduc Poker and betting-abstracted no-limit Texas Hold’em poker (in Section 4.1). Goofspiel is a bidding card game where players spend bid cards collect points from a deck of point cards. Liar’s dice is a 1-die versus 1-die variant of the popular game where players alternate bidding on the dice values. Leduc poker is a two-round poker game with a 6-card deck, fixed bet amounts, and a limit on betting. Longer descriptions of each games can be found in [28]. We use OpenSpiel [23] implementations with default parameters for Liar’s Dice and Leduc poker, and a 5-card deck and descending points order for II-Goofspiel. To show empirical convergence, we use NashConv, the sum over each player’s incentive to deviate to their best response unilaterally [25], which can be interpreted as an empirical distance from Nash equilibrium (reaching Nash at 0).

![Graphs showing empirical convergence for different games](image)

Figure 3: NFSP and MC-RCFR on the Leduc Poker, II-GoofSpiel with 5 cards and Liars Dice

We compare empirical convergence to approximate Nash equilibria using a model-free sampled form of regression CFR [38] (MC-RCFR). Trajectories are obtained using outcome sampling MCCFR [24], which uses off-policy importance sampling to obtain unbiased estimates of immediate regrets \( \hat{r} \), and average strategy updates \( \hat{s} \), and individual (learned) state-action baselines [31] to reduce variance. A regressor then predicts \( \hat{R} \) and a policy is obtained via Eq. 1 and similarly for the average strategy. Each episode, the learning player \( i \) plays with an \( \epsilon \)-on-policy behavior policy (while opponent(s) play
on-policy) and adds every datum \((s, \hat{r}, \pi(\hat{R}))\) to a data set, \(D\), with a retention rule based on reservoir sampling so it approximates a uniform sample of all the data ever seen. MC-RCFR is related, but not equivalent to, a variant of DeepCFR [5] based on outcome sampling (OS-DeepCFR) [35]. Our results differ significantly from the OS-DeepCFR results reported in [35], and we discuss differences in assumptions and experimental setup from previous work in Appendix A. As with ARMAC, the input is raw bits with no expert features. We use networks with roughly the same number of parameters as the ARMAC experiments: feed-forward with 4 hidden layers of 128 units with concatenated ReLU [32] activations, and train using the Adam optimizer. We provide details of the sweep over hyper-parameters in Appendix A.

Next we compare ARMAC to NFSP [19], which combines fictitious play with deep neural network function approximators. Two data sets, \(D^{RL}\) and \(D^{SL}\), store transitions of sampled experience for reinforcement learning and supervised learning, respectively. \(D^{RL}\) is a sliding window used to train a best response policy to \(\bar{\pi}^{-i}\) via DQN. \(D^{SL}\) uses reservoir sampling to train \(\bar{\pi}^{i}\), an average over all past best response policies. During play, each agent mixes between its best response policy and average policy. This stabilizes learning and enables the average policies to converge to an approximate Nash equilibrium. Like ARMAC and MC-RCFR, NFSP does not use any expert features.

Convergence plots for MC-RCFR and NFSP are shown in Figure 3, and for ARMAC in Figure 4. NashConv values of ARMAC are lower (Liar’s Dice) and higher (Goofspiel) than NFSP, but significantly lower than MC-RCFR in all cases. MC-RCFR results are consistent with the outcome sampling results in DNCFR [26]. Both DNCFR and Deep CFR compensate for this problem by instead using external and robust sampling, which require a forward model. As with NFSP, and RL more generally, introducing bias to reduce variance may be worthwhile when scaling to large environments. So, next we investigate the performance of ARMAC in a much larger game.

### 4.1 No-Limit Texas Hold’em

![Figure 5: ARMAC results in No-Limit Texas Hold’em trained with FCPA action abstraction evaluated using LBR-FC metric. The y-axis represents the amount LBR-FC wins against the ARMAC-trained policy. The x-axis indicates days of training. The left graph shows the learning curve in a linear scale, while the right one shows the same curve in a log-log scale.](image)

Figure 5: ARMAC results in No-Limit Texas Hold’em trained with FCPA action abstraction evaluated using LBR-FC metric. The y-axis represents the amount LBR-FC wins against the ARMAC-trained policy. The x-axis indicates days of training. The left graph shows the learning curve in a linear scale, while the right one shows the same curve in a log-log scale.
We ran ARMAC on the game of no-limit Texas Hold’em poker, using the common { Fold, Call, Pot, All-in } (FCPA) action/betting abstraction. This game is orders of magnitude larger than benchmark games used above (≈ 4.42 · 10^{13} information states). Action abstraction techniques were used by all of the state-of-the-art Poker AI bots up to 2017. Modern search-based techniques of DeepStack [29] and Libratus [6] still include action abstraction, albeit in the search tree.

Computing the NashConv requires traversing the whole game and querying the network at each information state, which becomes computationally infeasible as the game grows. So, to assess the quality of the resulting policy, we use local best-response (LBR) [27]. LBR is an exploiter agent that produces a lower-bound on the exploitability: given some policy π−i, it does a shallow search using the policy at opponent nodes, and a poker-specific heuristic evaluation at the frontier of the search. LBR found that previous competition-winning abstraction-based Poker bots were far more exploitable than first expected. In our experiments, LBR was limited to the betting abstractions: FCPA, and FC. We used three versions of LBR: LBR-FCPA, which uses all 4 actions within the abstraction, LBR-FC, which uses a more limited action set of { Fold, Call } and LBR-FC12-FCPA34 which has a limited action set of { Fold, Call } for the first two rounds and FCPA for the rest.

We first computed the average return that an ARMAC-trained policy achieves against uniform random. Over 200000 episodes, the mean value was 516 (chips) ± 25 (95% c.i.). Similarly, we evaluated the policy against LBR-FCPA; it won 519 ± 81 (95% c.i.) per episode. Hence, LBR-FCPA was unable to exploit the policy. ARMAC also beat LBR-FC12-FCPA34 by 867 ± 87 (95% c.i.) . Interestingly, ARMAC learned to beat those two versions of LBR surprisingly quickly. A randomly initialized ARMAC network lost against LBR-FCPA by -704 ± 191 (95% c.i.) and against LBR-FC12-FCPA34 by -230 ± 222 (95% c.i.), but was beating both after a mere 1 hour of training by 561 ± 163 (95% c.i.) and 427 ± 140 (95% c.i.) respectively (3 million acting steps, 11 thousand learning steps). [1]

However, counter-intuitively, ARMAC was exploited by LBR-FC which uses a more limited action set. ARMAC scored -46 ± 26 (95% c.i.) per episode after 18 days of training on a single GPU, 1.3 billion acting steps (rounds), 5 million learning steps, 50000 CFR epochs. The trend over training time is shown in Figure 5. To the best of our knowledge, this is the first time a bound on exploitability has been reported for any form of no-limit Texas Hold’em among this class of algorithms.

5 Conclusion and Future Work

ARMAC was demonstrated to work on both single agent and multi-agent benchmarks. It is brings back ideas from computational game theory to address exploration issues while at the same time being able to handle learning in non-stationary environments. As future work, we intend to apply it to more general classes of multiagent games; ARMAC has the appealing property that it already stores the joint policies and history-based critics, which may be sufficient for convergence one of the classes of extensive-form correlated equilibria [11, 15, 12].

References

[1] Yasin Abbasi-Yadkori, Peter Bartlett, Kush Bhatia, Nevena Lazic, Csaba Szepesvari, and Gellert Weisz. Politex: Regret bounds for policy iteration using expert prediction. In Proceedings of the 36th International Conference on Machine Learning, volume 97 of PMLR, pages 3692–3702, 2019.

[2] Adriá Puigdoménech Badia, Pablo Sprechmann, and et al. Never give up: Learning directed exploration strategies. In International Conference on Learning Representations (ICLR), 2020.

[3] A. Blum and Y. Mansour. Learning, regret minimization, and equilibria. In Algorithmic Game Theory, chapter 4. Carnegie Mellon University, 2007.

[4] Michael Bowling, Neil Burch, Michael Johanson, and Oskari Tammelin. Heads-up Limit Hold’em Poker is solved. Science, 347(6218):145–149, January 2015.

[5] Noam Brown, Adam Lerer, Sam Gross, and Tuomas Sandholm. Deep counterfactual regret minimization. CoRR, abs/1811.00164, 2018.
[6] Noam Brown and Tuomas Sandholm. Superhuman AI for heads-up no-limit poker: Libratus beats top professionals. *Science*, 360(6385), December 2017.

[7] Noam Brown and Tuomas Sandholm. Solving imperfect-information games via discounted regret minimization. *CoRR*, abs/1809.04040, 2018.

[8] Noam Brown and Tuomas Sandholm. Superhuman AI for multiplayer poker. *Science*, 365(6456):885–890, 2019.

[9] S. Bubeck and N. Cesa-Bianchi. Regret analysis of stochastic and nonstochastic multi-armed bandit problems. In *Foundations and Trends in Machine Learning*, 5(1):1–122, 2012.

[10] Neil Burch, Michael Johanson, and Michael Bowling. Solving imperfect information games using decomposition. In *Proceedings of the Twenty-Eighth AAAI Conference on Artificial Intelligence (AAAI)*, 2014.

[11] Andrea Celli, Alberto Marchesi, Tommaso Bianchi, and Nicola Gatti. Learning to correlate in multi-player general-sum sequential games, 2019.

[12] Andrea Celli, Alberto Marchesi, Gabriele Farina, and Nicola Gatti. No-regret learning dynamics for extensive-form correlated and coarse correlated equilibria, 2020.

[13] Katherine Chen and Michael Bowling. Tractable objectives for robust policy optimization. In *Advances in Neural Information Processing Systems*, pages 2069–2077, 2012.

[14] Eyal Even-Dar, Sham M Kakade, and Yishay Mansour. Experts in a markov decision process. In *Advances in neural information processing systems*, pages 401–408, 2005.

[15] Gabriele Farina, Tommaso Bianchi, and Tuomas Sandholm. Coarse correlation in extensive-form games, 2019.

[16] Richard Gibson, Neil Burch, Marc Lanctot, and Duane Szafron. Efficient monte carlo counterfactual regret minimization in games with many player actions. In *Advances in Neural Information Processing Systems*, pages 1880–1888, 2012.

[17] S. Hart and A. Mas-Colell. A simple adaptive procedure leading to correlated equilibrium. *Econometrica*, 68(5):1127–1150, 2000.

[18] Johannes Heinrich, Marc Lanctot, and David Silver. Fictitious self-play in extensive-form games. In *Proceedings of the 32nd International Conference on Machine Learning (ICML 2015)*, 2015.

[19] Johannes Heinrich and David Silver. Deep reinforcement learning from self-play in imperfect-information games. *CoRR*, abs/1603.01121, 2016.

[20] Peter H. Jin, Sergey Levine, and Kurt Keutzer. Regret minimization for partially observable deep reinforcement learning. In *ICML*, 2018.

[21] Ian A Kash, Michael Sullins, and Katja Hofmann. Combining no-regret and q-learning. In *Proceedings of The Nineteenth International Conference on Autonomous Agents and Multi-Agent Systems*. International Foundation for Autonomous Agents and Multiagent Systems, May 2020.

[22] Vojtech Kovarík, Martin Schmid, Neil Burch, Michael Bowling, and Viliam Lisý. Rethinking formal models of partially observable multiagent decision making. *CoRR*, abs/1906.11110, 2019.

[23] Marc Lanctot, Edward Lockhart, Jean-Baptiste Lespiau, Vinicius Zambaldi, Satyaki Upadhyay, Julien Pérolat, Sriram Srinivasan, and Finbarr Timbers et al. *OpenSpiel: A framework for reinforcement learning in games*. *CoRR*, abs/1908.09453, 2019. [http://arxiv.org/abs/1908.09453](http://arxiv.org/abs/1908.09453)

[24] Marc Lanctot, Kevin Waugh, Martin Zinkevich, and Michael Bowling. Monte Carlo sampling for regret minimization in extensive games. In Y. Bengio, D. Schuurmans, J. Lafferty, C. K. I. Williams, and A. Culotta, editors, *Advances in Neural Information Processing Systems* 22, pages 1078–1086, 2009.

[25] Marc Lanctot, Vinicius Zambaldi, Audrunas Gruslys, Angeliki Lazaridou, Karl Tuyls, Julien Perolat, David Silver, and Thore Graepel. A unified game-theoretic approach to multiagent reinforcement learning. In *NIPS*, 2017.

[26] Hui Li, Kaiiliang Hu, Zhibang Ge, Tao Jiang, Yuan Qi, and Le Song. Double neural counterfactual regret minimization. *CoRR*, abs/1812.10607, 2018.
[27] Viliam Lisý and Michael H. Bowling. Equilibrium approximation quality of current no-limit poker bots. *CoRR*, abs/1612.07547, 2016.

[28] Edward Lockhart, Marc Lanctot, Julien Pérolat, Jean-Baptiste Lespiau, Dustin Morrill, Finbarr Timbers, and Karl Tuyls. Computing approximate equilibria in sequential adversarial games by exploitability descent. *CoRR*, abs/1903.05614, 2019.

[29] Matej Moravčík and Martin Schmid et al. DeepStack: Expert-level artificial intelligence in heads-up no-limit poker. *Science*, 358(6362), 2017.

[30] D. Precup, R.S. Sutton, and S. Singh. Eligibility traces for off-policy policy evaluation. In *ICML*, 2000.

[31] Martin Schmid, Neil Burch, Marc Lanctot, Matej Moravčík, Rudolf Kadlec, and Michael Bowling. Variance reduction in monte carlo counterfactual regret minimization (VR-MCCFR) for extensive form games using baselines. In *Proceedings of the The Thirty-Third AAAI Conference on Artificial Intelligence*, 2019.

[32] Wenling Shang, Kihyuk Sohn, Diogo Almeida, and Honglak Lee. Understanding and improving convolutional neural networks via concatenated rectified linear units. *CoRR*, abs/1603.05201, 2016.

[33] Y. Shoham and K. Leyton-Brown. *Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations*. Cambridge University Press, 2009.

[34] Sriram Srinivasan, Marc Lanctot, Vinicius Zambaldi, Julien Pérolat, Karl Tuyls, Rémi Munos, and Michael Bowling. Actor-critic policy optimization in partially observable multiagent environments. In *Advances in Neural Information Processing Systems (NeurIPS)*, 2018.

[35] Eric Steinberger, Adam Lerer, and Noam Brown. Dream: Deep regret minimization with advantage baselines and model-free learning, 2020.

[36] R. Sutton and A. Barto. *Reinforcement Learning: An Introduction*. MIT Press, 2nd edition, 2018.

[37] Oriol Vinyals, Igor Babuschkin, Wojciech M. Czarnecki, Michaël Mathieu, Andrew Dudzik, Junyoung Chung, David H. Choi, Richard Powell, and et al. Grandmaster level in StarCraft II using multi-agent reinforcement learning. *Nature*, 2019.

[38] Kevin Waugh, Dustin Morrill, J. Andrew Bagnell, and Michael Bowling. Solving games with functional regret estimation. In *Proceedings of the AAAI Conference on Artificial Intelligence*, 2015.

[39] M. Zinkevich, M. Johanson, M. Bowling, and C. Piccione. Regret minimization in games with incomplete information. In *Advances in Neural Information Processing Systems* 20, 2008.
Appendices

A Baseline Details and Hyperparameters

For MC-RCFR, we sweep over all combinations of the exploration parameter, using a (learned) state-action baseline \( [31] \), and learning rate \((\epsilon, b, \alpha) \in \{0.25, 0.5, 0.6, 0.7, 0.9, 0.95, 1.0\} \times \{\text{TRUE, FALSE}\} \times \{0.0001, 0.00005, 0.00001\} \), where each combination is averaged over five seeds. We found that higher exploration values worked consistently better, which matches the motivation of the robust sampling technique (corresponding to \( \epsilon = 1 \)) presented in [26] as it leads to reduced variance since part of the correction term is constant for all histories in an information state. The baseline helped significantly in the larger game with more variable-length episodes.

For NFSP, we keep a set of hyperparameters fixed, in line with [25] and [19]: anticipatory parameter \( \eta = 0.1 \), \( \epsilon \)-greedy decay duration \( 20M \) steps, reservoir buffer capacity \( 2M \) entries, replay buffer capacity \( 200k \) entries, while sweeping over a combination of the following hyperparameters: \( \epsilon \)-greedy starting value \( \{0.06, 0.24\} \), RL learning rate \( 0.1, 0.01, 0.001 \), SL learning rate \( 0.01, 0.001, 0.005 \), DQN target network update period of \( \{1000, 19200\} \) steps (the later is equivalent to 300 network-parameter updates). Each combination was averaged over three seeds. Agents were trained with the ADAM optimizer, using MSE loss for DQN and one gradient update step using mini-batch size 128, every 64 steps in the game.

Finally, note that there are at least four difference in the results, experimental setup, and assumptions between MC-RCFR and OS-DeepCFR reported in [35]:

1. [35] uses domain expert input features which do not generalize outside of poker. The neural network architecture we use is a basic MLP with raw input representations, whereas [35] uses a far larger network. Our empirical results on benchmark games compare the convergence properties of knowledge-free algorithms across domains.

2. The amount of training per iteration is an order of magnitude larger in OS-DeepCFR than our training. In [35], every 346 iterations, the Q-network is trained using 1000 minibatches of 512 samples (512000 examples), whereas every 346 iterations we train 346 batches of 128 samples, 44288 examples.

3. MC-RCFR uses standard outcome sampling rather than Linear CFR [7].

4. MC-RCFR’s strategy is approximated by predicting the OS’s average strategy increment rather than sampling from a buffer of previous models.

Our NFSP also does not use any extra enhancements.

B Initial Investigation of ARMAC in the Atari Learning Environment

While performance on Atari is not the main contribution, it should be treated as a health check of the algorithm. Unlike previously tested multiplayer games, many Atari games have a long term credit assignment problem. Some of them, like Montezuma’s Revenge, are well-known hard exploration problems. It is interesting to see that ARMAC was able to consistently score 2500 points on Montezuma’s Revenge despite not using any auxiliary rewards, demonstrations, or distributional RL as critic. We hypothesize that regret matching may be advantageous for exploration, as it provides naturally stochastic policies which stay stochastic until regrets for other actions becomes negative. We also tested the algorithm on Breakout, as it is a fine control problem. We are not claiming that out results on Atari are state of art - they should be interpreted as a basic sanity check showing that ARMAC could in principle work in this domain.

C Training

Training is done by processing a batch of 64 of trajectories of length 32 at a time. In order to implement a full recall, all unfinished episodes will be continued on the next training iteration by propagating recurrent network states forward. Each time when one episode finishes at a particular batch entry, a new one is sampled and started to be unrolled from the beginning.
Adam optimized with $\beta_1 = 0.0$ and $\beta_2 = 0.999$ was used for optimization. Hyperparameter selection was done by trying only two learning rates: $5 \cdot 10^{-5}$ and $2 \cdot 10^{-4}$. The results reported use $5 \cdot 10^{-5}$ in all games, including Atari.

D Proof of Lemma 1

Proof. First, let us notice that

$$W_T^i(s, a) = \sum_{h \in s} \frac{\sum_{t=1}^T \eta^{\pi_i}(h) A_{\pi_i,i}(h, a)}{\sum_{t=1}^T \eta^{\pi_i}(s)}$$

(5)

$$= \sum_{h \in s} \frac{\sum_{t=1}^T \eta^{\pi_i}(h) A_{\pi_i,i}(h, a)}{\sum_{t=1}^T \eta^{\pi_i}(s)}$$

(6)

$$= \frac{1}{w^T(s)} \sum_{t=1}^T \sum_{h \in s} \eta^{\pi_i}(h) A_{\pi_i,i}(h, a)$$

(7)

where we used the perfect recall assumption in the first derivation, and we define $w^T(s) = \sum_i \eta^{\pi_i}(s)$.

Notice that $w^T(s)$ depends on the state only (and not on $h$). Now the cumulative regret is:

$$R_T^i(s, a) = \sum_{t=1}^K q^{\pi_i,i}(s, a) - v^{\pi_i,i}(s)$$

$$= \sum_{t=1}^T \eta^{\pi_i}(s) (q^{\pi_i,i}(s, a) - v^{\pi_i,i}(s))$$

$$= \sum_{t=1}^T \eta^{\pi_i}(s) \sum_{h \in s} \frac{\eta^{\pi_i}(h) (q^{\pi_i,i}(h, a) - v^{\pi_i,i}(h))}{\eta^{\pi_i}(s)}$$

$$= \sum_{t=1}^T \sum_{h \in s} \eta^{\pi_i}(h) A_{\pi_i,i}(h, a)$$

$$= w^T(s) W_T^i(s, a).$$

Finally, noticing that regret matching is not impacted by multiplying the cumulative regret by a positive function of the state, we deduce

$$\frac{R_T^{i,+}(s, a)}{\sum_b R_T^{i,+}(s, b)} = \frac{(w^T(s) W_T^i(s, a))^+}{\sum_b (w^T(s) W_T^i(s, b))^+} = \frac{W_T^{i,+}(s, a)}{\sum_b W_T^{i,+}(s, b)}.$$
Lemma 2. Consider the case of a tabular representation and define the estimate $\hat{W}^T_i(s, a)$ as the minimizer (over $W$) of the empirical loss defined over $N$ trajectories

$$\hat{L}_{(s,a)}(W) = \frac{1}{N} \sum_{n=1}^{N} [W - A_{\pi_n,i}(h, a)]^2 \mathbb{I}\{ (h, a) \in \rho^n \text{ and } h \in s \},$$

where $\rho^n$ is the $n$-th trajectory generated by the policy $(\mu^T_i, \pi^T_{i-1})$ where $j_n \sim \mathcal{U}\{1, \ldots, T\}$. Define $N(s, a) = \sum_{n=1}^{N} \mathbb{I}\{(h, a) \in \rho^n \text{ and } h \in s \}$ to be the number of trajectories going through $(s, a)$. Then $\hat{W}^T_i(s, a)$ is an unbiased estimate of $W^T_i(s, a)$ conditioned on $(s, a)$ being traversed at least once:

$$\mathbb{E}[\hat{W}^T_i(s, a)|N(s, a) > 0] = W^T_i(s, a).$$

Proof. The empirical loss being quadratic, under the event $\{N(s, a) > 0\}$, its minimum is well defined and reached for

$$\hat{W}^T_i(s, a) = \frac{1}{N(s, a)} \sum_{n=1}^{N(s, a)} A_{\pi_n,i}(h_n, a),$$

where $h_n \in s$ is the history of the $n$-th trajectory traversing $s$. Let us use simplified notations and write $A_n = A_{\pi_n,i}(h, a)\mathbb{I}\{(h, a) \in \rho^n \text{ and } h \in s \}$ and $b_n = \mathbb{I}\{(h, a) \in \rho^n \text{ and } h \in s \}$. Thus

$$\mathbb{E}[\hat{W}^T_i(s, a)\mathbb{I}\{ \sum_{m=1}^{N} b_m > 0 \}] = \mathbb{E} \left[ \sum_{n=1}^{N} A_n \mathbb{I}\{ \sum_{m=1}^{N} b_m > 0 \} \right] = \sum_{n=1}^{N} \mathbb{E} \left[ A_n \mathbb{I}\{ \sum_{m=1}^{N} b_m > 0 \} \right].$$

Now, $\mathbb{E}[A_n\sum_{m=1}^{N} b_m] = \mathbb{E}[A_n b_n] \mathbb{E}[b_n\sum_{m=1}^{N} b_m]$ since given $b_n$, $A_n$ is independent of $\sum_{m=1}^{N} b_m$. Thus

$$\mathbb{E}[\hat{W}^T_i(s, a)\mathbb{I}\{ \sum_{m=1}^{N} b_m > 0 \}] = \sum_{n=1}^{N} \mathbb{E}[A_n b_n] \mathbb{E}\left[ \frac{\mathbb{E}[b_n\sum_{m=1}^{N} b_m] \mathbb{I}\{ \sum_{m=1}^{N} b_m > 0 \}}{\sum_{m=1}^{N} b_m} \right]$$

Since $\sum_{n=1}^{N} b_n \mathbb{I}\{ \sum_{m=1}^{N} b_m > 0 \} = \mathbb{E}\left[ \frac{\sum_{n=1}^{N} b_n \mathbb{I}\{ \sum_{m=1}^{N} b_m > 0 \}}{\sum_{m=1}^{N} b_m} \right] = \mathbb{P}(\sum_{m=1}^{N} b_m > 0)$, by a symmetry argument we deduce $\mathbb{E}\left[ \frac{b_n \mathbb{I}\{ \sum_{m=1}^{N} b_m > 0 \}}{\sum_{m=1}^{N} b_m} \right] = \frac{1}{N} \mathbb{P}(\sum_{m=1}^{N} b_m > 0)$ for each $n$. Thus

$$\mathbb{E}[\hat{W}^T_i(s, a)|N(s, a) > 0] = \frac{1}{N} \sum_{n=1}^{N} \mathbb{E}[A_n b_n] = \mathbb{E}[A_1 b_1]$$

which is the expectation of the advantage $A_{\pi_n,i}(h, a)$ conditioned on the trajectory $\rho^j$ going through $h \in s$, i.e. $W^T_i(s, a)$ as defined in (3).
F Neural Network Architecture

The following recurrent neural network was used for no-limit Texas Hold’em experiments. Two separate recurrent networks with shared parameters were used, consuming observations of each player respectively. Each of those networks consisted of a single linear layer mapping input representation to a vector of size 256. This was followed by a double rectified linear unit, producing a representation of size 512 then followed by LSTM with 256 hidden units. This produced an information state representation for each player $a_0$ and $a_1$.

Define architecture $B(x)$, which will be reused several times. It consumes one of the information state representations produced by the previously mentioned RNN: $h_1 = \text{Linear}(128)(x)$, $h_2 = \text{DoubleReLU}(h_1)$, $h_3 = h_1 + \text{Linear}(128)(h_2)$, $B(a) = \text{DoubleReLU}(h_3)$.

The immediate regret head is formed by applying $B(s)$ on the information state representation followed by a single linear layer of the size of the number of actions in the game. The same is done for an average regret head and mean policy head. All those $B(s)$ do not share weights between themselves, but share weights with respective heads for another player.

The global critic $q(h)$ is defined in the following way. $n_A = \text{Linear}(128)$, $n_B = \text{Linear}(128)$, $a_0 = n_A(s_0) + n_B(s_1)$, $a_0 = n_B(s_0) + n_A(s_1)$, $h_1 = \text{Concat}(a_0, a_1)$, $h_2 = B(h_1)$ and finally $q_0(s_1, s_2)$ and $q_1(s_1, s_2)$ are evaluated by a two linear layers on top of $h_2$. $B(x)$ shares architecture but does not share parameters with the ones used previously.