Fermi Constant from Muon Decay Versus Electroweak Fits and Cabibbo-Kobayashi-Maskawa Unitarity

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Abstract: The Fermi constant GF is extremely well measured through the muon lifetime, defining one of the key fundamental parameters in the standard model (SM). Therefore, to search for physics beyond the SM (BSM) via GF, the constraining power is determined by the precision of the second-best independent determination of GF. The best alternative extractions of GF proceed either via the global electroweak (EW) fit or from superallowed decays in combination with the Cabibbo angle measured in kaon, or D decays. Both variants display some tension with GF from muon decay, albeit in opposite directions, reflecting the known tensions within the EW fit and hints for the apparent violation of Cabibbo-Kobayashi-Maskawa unitarity, respectively. We investigate how BSM physics could bring the three determinations of GF into agreement using SM effective field theory and comment on future perspectives.

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The Fermi constant \( G_F \) is extremely well measured through the muon lifetime, defining one of the key fundamental parameters in the standard model (SM). Therefore, to search for physics beyond the SM (BSM) via \( G_F \), the constraining power is determined by the precision of the second-best independent determination of \( G_F \). The best alternative extractions of \( G_F \) proceed either via the global electroweak (EW) fit or from superallowed \( \beta \) decays in combination with the Cabibbo angle measured in kaon, \( \tau \), or \( D \) decays. Both variants display some tension with \( G_F \) from muon decay, albeit in opposite directions, reflecting the known tensions within the EW fit and hints for the apparent violation of Cabibbo-Kobayashi-Maskawa unitarity, respectively. We investigate how BSM physics could bring the three determinations of \( G_F \) into agreement using SM effective field theory and comment on future perspectives.

**Introduction.**—The numerical value of the Fermi constant \( G_F \) is conventionally defined via the muon lifetime within the standard model (SM). Even though this measurement is extremely precise [1–3] at

\[
G_F = 1.1663787(6) \times 10^{-5}\ \text{GeV}^{-2},
\]

at the level of 0.5 ppm, its determination of the Fermi constant is not necessarily free of beyond the SM (BSM) contributions. In fact, one can only conclude the presence or absence of BSM effects by comparing \( G_F^\mu \) to another independent determination. This idea was first introduced by Marciano in Ref. [4], concentrating on Z-pole observables and the fine-structure constant \( \alpha \). In addition to a lot of new data that have become available since 1999, another option already mentioned in Refs. [4,5]—the determination of \( G_F \) from \( \beta \) and kaon decays using Cabibbo-Kobayashi-Maskawa (CKM) unitarity—has become of particular interest due to recent hints for the (apparent) violation of first-row CKM unitarity. These developments motivate a fresh look at the Fermi constant: in particular, on its extraction from a global electroweak (EW) fit and via CKM unitarity, as will be discussed in the first part of this Letter.

The comparison of the resulting values for \( G_F \) then serves as a model-independent measure of possible BSM effects. It shows that with modern input, the two independent extractions are close in precision yet still lagging behind muon decay by almost three orders of magnitude. Therefore, the BSM sensitivity is governed by the uncertainty of these two indirect determinations. Since the different \( G_F \) measurements turn out to display some disagreement beyond their quoted uncertainties, the second part of this Letter is devoted to a systematic analysis of possible BSM contributions in SM effective field theory (SMEFT) [6,7] to see which scenarios could account for these tensions without being excluded by other constraints. This is important to identify BSM scenarios that could be responsible for the tensions, which will be scrutinized with forthcoming data in the next years.

**Determinations of \( G_F \).**—Within the SM, the Fermi constant \( G_F \) is defined by, and is most precisely determined from, the muon lifetime [2]

\[
\frac{1}{\tau_{\mu}} = \frac{(G_F^\mu)^2 m_\mu^5}{192\pi^3} (1 + \Delta q),
\]

where \( \Delta q \) includes the phase space, QED, and hadronic radiative corrections. The resulting numerical value in Eq. (1) is so precise that its error can be ignored in the following. To address the question whether \( G_F^\mu \) subsumes BSM contributions, however, alternative independent determinations of \( G_F \) are indispensable; and their precision limits the extent to which BSM contamination in \( G_F^\mu \) can be detected.
In Ref. [4], the two best independent determinations were found as

\[ G_F^{\ell\ell'}(\mu) = 1.1650(14)_{-6}^{+11} \times 10^{-5} \text{ GeV}^{-2}, \]
\[ G_F^{(3)} = 1.1672(8)_{-7}^{+18} \times 10^{-5} \text{ GeV}^{-2}, \]

(3)

where the first variant uses the width for \( Z \to \ell^+\ell^- (\gamma) \), while the second employs \( \alpha \) and \( \sin^2 \theta_W \) together with the appropriate radiative corrections. Since the present uncertainty in \( \Gamma[Z \to \ell^+\ell^- (\gamma)] = 83.984(86) \text{ MeV} \) [8] is only marginally improved compared to the one available in 1999, the update

\[ G_F^{\ell\ell'} = 1.1661(16) \times 10^{-5} \text{ GeV}^{-2} \]  

(4)

does not lead to a gain in precision, but the shift in the central value improves agreement with \( G_F^{\mu} \). The second variant, \( G_F^{(3)} \), is more interesting because, here, the main limitation arose from the radiative corrections, which have seen significant improvements regarding the input values for the masses of the top quark \( M_t \), the Higgs boson \( M_H \), the strong coupling \( \alpha_s \), and the hadronic running of \( \alpha \). In fact, with all EW parameters determined, it now makes sense to use the global EW fit (for which \( G_F^{\mu} \) is usually a key input quantity) instead as a tool to determine \( G_F \) in a completely independent way.

The EW observables included in our fit (\( W \) mass, \( \sin^2 \theta_W \), and \( Z \)-pole observables [9,10]) are given in Table I, with the other input parameters summarized in Table II. Here, the hadronic running \( \Delta \alpha_{\text{had}} \) is taken from \( e^+e^- \) data, which are insensitive to the changes in \( e^+e^- \to \text{hadrons} \) cross sections \([11–18]\) recently suggested by lattice QCD [19], as long as these changes are concentrated at low energies \([20–23]\). We perform the global EW fit in a Bayesian framework using the publicly available HEPfit package [24], whose Markov chain Monte Carlo determination of posteriors is powered by the Bayesian Analysis Toolkit [25]. As a result, we find

\[ G_F^{\text{EW}} |_{\text{full}} = 1.16716(39) \times 10^{-5} \text{ GeV}^{-2}, \]

(5)

which is a gain in precision over \( G_F^{(3)} \) in Eq. (3) by a factor of 5. As depicted in Fig. 1, this value lies above \( G_F^{\mu} \) by 2\( \sigma \), reflecting the known tensions within the EW fit [26,27]. For comparison, we also considered a closer analog of \( G_F^{(3)} \) by only keeping \( \sin^2 \theta_W \) from Table I in the fit, which gives

\[ G_F^{\text{EW}} |_{\text{minimal}} = 1.16728(83) \times 10^{-5} \text{ GeV}^{-2}, \]

(6)

which is consistent with Eq. (5) but with a larger uncertainty. The pull of \( G_F^{\text{EW}} \) away from \( G_F^{\mu} \) is mainly driven by \( M_W, \sin^2 \theta_W \) from hadron colliders, \( A_\ell \), and \( A_{\text{FB}}^{0,\ell} \).

As a third possibility, one can determine the Fermi constant from superallowed \( \beta \) decays, taking \( V_{us} \) from kaon or \( \tau \) decays and assuming CKM unitarity (\( |V_{ub}| \) is also needed, but the impact of its uncertainty is negligible). This is particularly interesting, given recent hints for the apparent violation of first-row CKM unitarity known as the Cabibbo angle anomaly (CAA). The significance of the tension crucially depends on the radiative corrections applied to \( \beta \) decays \([45–52]\) but also on the treatment of tensions between \( K_{\ell2} \) and \( K_{\ell3} \) decays \([53]\) and the constraints from \( \tau \) decays \([54]\); see Ref. [55] for more details. In the end, quoting a significance around 3\( \sigma \) should give a realistic representation of the current situation; for definitiveness, we will thus use the estimate of first-row CKM unitarity from Ref. [8]:

\[ |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985(5). \]

(7)

In addition, we remark that there is also a deficit in the first-column CKM unitarity relation [8]:

\[ |V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 0.9970(18). \]

(8)

| Parameter | Prior |
|-----------|-------|
| \( \alpha \times 10^3 \) [8] | 7.2973525664(17) |
| \( \Delta \alpha_{\text{had}} \times 10^4 \) [16,17] | 276.1(1.1) |
| \( \alpha_s (M_Z) \) [8,33] | 0.1179(10) |
| \( M_Z \) (GeV) [8,34–37] | 91.1876(21) |
| \( M_H \) (GeV)[8,38–40] | 125.10(14) |
| \( m_t \) (GeV)[8,41–44] | 172.76(30) |

TABLE I. EW observables included in our global fit together with their current experimental values.
which is less significant than Eq. (7) but suggests that if the deficits were due to BSM effects, they would likely be related to $\beta$ decays. For the numerical analysis, we will continue to use Eq. (7), given the higher precision. The deficit in Eq. (7) translates to

$$G_F^{\text{CKM}} = 0.99925(25) \times G_F^p = 1.16550(29) \times 10^{-5} \text{ GeV}^{-2}.$$  

Comparing the three independent determinations of $G_F$ in Fig. 1, one finds the situation in which $G_F^\text{EW}$ lies above $G_F^p$ by $2\sigma$, $G_F^{\text{CKM}}$ lies below $G_F^p$ by $3\sigma$, and the tension between $G_F^\text{EW}$ and $G_F^{\text{CKM}}$ amounts to $3.4\sigma$. To bring all three determinations into agreement within $1\sigma$, an effect in at least two of the underlying processes is thus necessary. This leads us to study BSM contributions to (1) $\mu \rightarrow e\nu\nu$ transitions; (2) $Z \rightarrow \ell^+\ell^-, \nu\nu$, $\alpha_2/\alpha$, and $M_Z/M_W$; and (3) superallowed $\beta$ decays, where the second point gives the main observables in the EW fit, with $\alpha_2/\alpha$ as a proxy for the ratio of $SU(2)_L$ and $U(1)_Y$ couplings. We do not consider the possibility of BSM effects in kaon, $\tau$, or $D$ decays because this would require a correlated effect with a relating symmetry. Furthermore, as shown in Ref. [55], the sensitivity to a BSM effect in superallowed $\beta$ decays is enhanced by a factor of $|V_{ud}|^2/|V_{us}|^2$ compared to kaon, $\tau$, or $D$ decays. This can also be seen from Eq. (7) because $|V_{ud}|$ gives the dominant contribution.

BSM explanations of the discrepancies between these determinations of $G_F$ have been studied in the literature in the context of the CAA [55–56]. In this Letter, we will analyze possible BSM effects in all three $G_F$ determinations using an effective-field-theory approach with gauge-invariant dimension-6 operators [6,7].

SMEFT analysis.—Dimension-6 operators that can explain the differences among the determinations of $G_F$ can be grouped into the following classes: (A) four-fermion operators in $\mu \rightarrow e\nu\nu$, (B) four-fermion operators in $u \rightarrow d\ell\nu$, (C) modified $W_{\mu \nu}$ couplings, (D) modified $W_{\nu\ell\nu}$ couplings, and (E) other operators affecting the EW fit.
\( \mathcal{P}(M - M) = \frac{8(a_{\mu e})^2}{\pi^2} G_F^2 \left| C_{1112}(e)/G_F \right|^2 \)

\[
= 3.21 \times 10^{-6} \left| C_{1112}(e)/G_F \right|^2, \tag{14}
\]

with reduced mass \( \mu_{\mu} = m_{\mu} m_e/(m_{\mu} + m_e) \), the limits become \( |C_{1112}(e)| < 8.6(5.8) \times 10^{-3} G_F \). (4) For \( Q_{\ell\ell}(e) \), again, numerical values of \( |C_{1112}(e)| \approx 0.09 G_F \) are preferred (as for all the remaining Wilson coefficients in this list). Both operators give tree-level effects in \( \mu \rightarrow 3e \), e.g.,

\[
\text{Br}[\mu \rightarrow 3e] = \frac{m_{\mu}^5 G_F}{768\pi} \left| C_{1112}(e) \right|^2 = 0.25 \frac{\left| C_{1112}(e)/G_F \right|^2}{G_F}, \tag{15}
\]

which exceeds the experimental limit on the branching ratio of \( 1.0 \times 10^{-12} [85] \) by many orders of magnitude (the result for \( C_{1112} \) is smaller by a factor of 1/2). (5) The operators \( Q_{\ell\ell}(e) \) contribute at the one-loop level to \( \mu \rightarrow e \) conversion and \( \mu \rightarrow 3e \), as well as at the two-loop level to \( \mu \rightarrow e\gamma \) [86]. Here, the current best bounds come from \( \mu \rightarrow e \) conversion. Using table 3 in Ref. [86], we have

\[
|C_{1112}(e)| < 6.4 \times 10^{-5} G_F, \tag{16}
\]

\[
|C_{1112}(e)| < 2.8 \times 10^{-5} G_F,
\]

excluding again a sizable BSM effect, and this is similar for \( Q_{\ell\ell}(e) = Q_{\ell\ell}(e) \). (6) \( Q_{\ell\ell}(e) \) contribute to \( \tau \rightarrow \mu\mu e \) and \( \tau \rightarrow \mu e e \), respectively, which excludes a sizable effect in analogy to \( \mu \rightarrow 3e \) above [54,87,88].

The other four-quark operators can only contribute via loop effects, which leads us to conclude that the only viable mechanism proceeds via a modification of the SM operator \( Q_{\ell\ell}(e) \).

**Four-fermion operators in \( d \rightarrow u e v \).**—First of all, the operators \( Q_{\ell\ell}(e) \) give rise to \( d \rightarrow u e v \) scalar amplitudes. Such amplitudes lead to enhanced effects in \( \pi \rightarrow \mu\pi / \tau \rightarrow ev \) with respect to \( \beta \) decays, and therefore can only have a negligible impact on the latter once the stringent experimental bounds [8,89] are taken into account. Furthermore, the tensor amplitude generated by \( Q_{\ell\ell}(e) \) has a suppressed matrix element in \( \beta \) decays.

Therefore, we are left with \( Q_{\ell\ell}(e) \), for which we only consider the flavor combination that leads to interference with the SM. The CAA prefers \( C_{\ell\ell}(e) \approx 0.7 \times 10^{-3} G_F \). Via \( SU(2)_L \) invariance, this operator generates effects in neutral-current (NC) interactions

\[
L_{NC} = C_{\ell\ell}(e) \left( \bar{d}^i \gamma^\mu P_L d - \bar{u}^i \gamma^\mu P_L u \right) \epsilon^\mu P_L e. \tag{17}
\]

Note that since the SM amplitude for \( \bar{u}u(\bar{d}d) \rightarrow e^+ e^- \), at high energies, has a negative (positive) sign, we have constructive interference in both amplitudes. Therefore, the latest nonresonant dilepton searches by ATLAS [90] naively lead to

\[
C_{\ell\ell}(e) \approx 1.6 \times 10^{-3} G_F, \tag{18}
\]

which implies that bringing \( C_{\ell\ell}(e) \) into \( 1 \sigma \) agreement with \( G_F \) via four-fermion operators affecting \( d \rightarrow u e v \) transitions is still possible. However, ATLAS derived the bound for the \( SU(2)_L \) singlet operator, which means that the actual constraint for triplet operators is stronger because it leads to constructive interference in both the up- and the down-quark channels. In consequence, the required value of \( C_{\ell\ell}(e) \) lies at the border of the ATLAS constraint.

**Modified \( W-\nu-d \) couplings.**—There are only two operators modifying the \( W \) couplings to quarks:

\[
Q_{\phi d}^{(3)ij} = \phi_i^\dagger i D^\mu \phi_j \gamma^\mu \gamma^0 j, \tag{19}
\]

\[
Q_{\phi d}^{(3)ij} = \phi_i^\dagger i D^\mu \phi_j \gamma^\mu d_j.
\]

First of all, \( Q_{\phi d}^{(3)ij} \) generates right-handed \( W \)–quark couplings [91–95], which can solve the CAA. In addition, a right-handed \( W-\nu-s \) coupling could also account for the difference between \( K_{e2} \) and \( K_{e3} \) decays [96]. \( Q_{\phi d}^{(3)ij} \) generates modifications of the left-handed \( W \)–quark couplings, and data prefer

\[
C_{\phi d}^{(3)ij} \approx 0.7 \times 10^{-3} G_F. \tag{20}
\]

Due to \( SU(2)_L \) invariance, in general, effects in \( D^0-\bar{D}^0 \) and \( K^0-\bar{K}^0 \) mixing (as well as in \( \Gamma(Z \rightarrow \text{hadrons})/\Gamma(Z \rightarrow \text{leptons}) \)) are generated. However, the former bounds can be avoided by a \( U(2) \) flavor symmetry, and the latter by simultaneous contributions to \( Q_{\phi d}^{(3)ij} \). For a detailed analysis, we refer the reader to Ref. [97].

**Modified \( W-\ell-\nu \) couplings.**—Only the operator

\[
Q_{\phi d}^{(3)ij} = \phi_i^\dagger i D^\mu \phi_j \gamma^\mu \gamma^0 \ell_j \tag{21}
\]

generates modified \( W-\ell-\nu \) couplings at tree level. In order to avoid the stringent bounds from charged lepton flavor violation, the off-diagonal Wilson coefficients (in particular, \( C_{\phi d}^{(3)ij} \)) must be very small. Since they also do not generate amplitudes interfering with the SM ones, their effect can be neglected. While \( C_{\phi d}^{(3)ij} \) affects \( G_F \) and \( G_F^{\text{KM}} \) in the same way, \( C_{\phi d}^{(3)ij} \) only enters in muon decay. Therefore, agreement between \( G_F \) and \( G_F^{\text{KM}} \) can be achieved by \( C_{\phi d}^{(3)ij} < 0 \), \( C_{\phi d}^{(3)ij} > 0 \), and \( |C_{\phi d}^{(3)ij}| = \left| C_{\phi d}^{(3)ij} \right| \) without violating lepton flavor universality tests such as \( \pi(\pi) \rightarrow \mu\nu / \pi(\pi) \rightarrow ev \) or \( \tau \rightarrow \mu\nu / \tau(\mu) \rightarrow ev \) [55,58,98]. However, \( C_{\phi d}^{(3)ij} \) also affects \( Z \) couplings to leptons and neutrinos, which enter the global EW fit.

**Electroweak fit.**—The impact of modified gauge-boson–lepton couplings on the global EW fit, generated by \( Q_{\phi d}^{(3)ij} \) and
In a second step, we investigated how these hints of BSM physics can be explained within SMEFT. For BSM physics in $G_F$, we were able to rule out all four-fermion operators (except for $Q_{\ell\ell}^{1111}$, which generates a SM-like amplitude) and modified $W-\ell-\nu$ couplings (from $Q_{\phi\phi}^{(3)ij}$). Therefore, both constructive and destructive interferences are possible, which would bring $G_F$ into agreement with $G_F^{\text{CKM}}$ or $G_F^{\text{EW}}$, respectively, at the expense of increasing the tension with the other determination. To achieve a better agreement among the three different values of $G_F$, BSM effects in $G_F^{\text{CKM}}$ and/or $G_F^{\text{EW}}$ are also necessary. In the case of $G_F^{\text{CKM}}$ only a single four-fermion operator, $Q_{\ell\ell}^{(3)1111}$, and $Q_{\phi\phi}^{(3)ij}$, $Q_{\phi\phi}^{(3)ij}$ remain. Finally, modified gauge-boson-lepton couplings, via $Q_{\phi\phi}^{(3)ij}$ and $Q_{\phi\phi}^{(3)}$, can not only change $G_F^{\text{CKM}}$ and $G_F^{\text{EW}}$ but also affect the EW fit via the $Z$-pole observables, which can further improve the global agreement; see Fig. 2. This figure also demonstrates the advantage of interpreting the tensions in terms of $G_F$, defining a transparent benchmark for comparison both in SMEFT and concrete BSM scenarios; and it allows one to constrain the amount of BSM contributions to muon decay.

Our study highlights the importance of improving the precision of the alternative independent determinations of $G_F^{\text{CKM}}$ and $G_F^{\text{EW}}$ in order to confirm or refute BSM contributions to the Fermi constant. Concerning $G_F^{\text{CKM}}$, improvements in the determination of $|V_{ud}|$ should arise from advances in nuclear-structure [99,100] and EW radiative corrections [101], while experimental developments [102–108] could make the determination from neutron decay [109–111] competitive and, in combination with $K_{\ell3}$ decays, add another complementary constraint on $|V_{ud}|/|V_{us}|$ via pion $\beta$ decay [112,113]. Furthermore, improved measurements of $|V_{ud}|$ from $D$ decays [114] could bring the precision of the first-column CKM unitarity relation close to the first-row one, which, in turn, could be corroborated via improved $|V_{us}|$ determinations from $K_{\ell3}$ decays [115–117]. The precision of $G_F^{\text{EW}}$ will profit in the near future from LHC measurements of $m_t$ and $M_W$, in the midterm from the Belle-II EW precision program [118], as well as in the long term from future $e^+e^-$ colliders such as the FCC-ee [119], ILC [120], CEPC [121], or CLIC [122], which could achieve a precision at the level of $10^{-5}$.

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