THE CIRCUMSTELLAR DISK OF THE Be STAR $\upsilon$ AQUARII AS CONSTRAINED BY SIMULTANEOUS SPECTROSCOPY AND OPTICAL INTERFEROMETRY

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ABSTRACT

Omicron Aquarii is a late-type, Be shell star with a stable and nearly symmetric H\textalpha emission line. We combine H\textalpha interferometric observations obtained with the Navy Precision Optical Interferometer covering 2007 through 2014 with H\textalpha spectroscopic observations over the same period and a 2008 observation of the system’s near-infrared spectral energy distribution to constrain the properties of $\upsilon$ Aqr’s circumstellar disk. All observations are consistent with a circumstellar disk seen at an inclination of $75^\circ \pm 3^\circ$ with a position angle on the sky of $110^\circ \pm 8^\circ$ measured East from North. From the best-fit disk density model, we find that 90\% of the H\textalpha emission arises from within 9.5 stellar radii, and the mass associated with this H\textalpha disk is $1.8 \times 10^{-10}$ of the stellar mass, and that the associated angular momentum, assuming Keplerian rotation for the disk, is $1.6 \times 10^{-8}$ of the total stellar angular momentum. The occurrence of a central quasi-emission feature in Mg II $\lambda 4481$ is also predicted by this best-fit disk model and the computed profile compares successfully with observations from 1999. To obtain consistency between the H\textalpha line profile modeling and the other constraints, it was necessary in the profile fitting to weight the line core (emission peaks and central depression) more heavily than the line wings, which were not well reproduced by our models. This may reflect the limitation of assuming a single power law for the disk’s variation in equatorial density. The best-fit disk density model for $\upsilon$ Aqr predicts that H\textalpha is near its maximum strength as a function of disk density, and hence the H\textalpha equivalent width and line profile change only weakly in response to large (factor of $\sim 5$) changes in the disk density. This may in part explain the remarkable observed stability of $\upsilon$ Aqr’s H\textalpha emission line profile.

\textit{Key words:} circumstellar matter – stars: emission-line, Be – stars: individual ($\upsilon$ Aqr) – techniques: interferometric

\textit{Supporting material:} machine-readable table

1. INTRODUCTION

$\upsilon$ Aqr (HR 8402, HD 209409) is a bright, Be shell star of spectral type B7Ie. Rivinius et al. (2006) note that $\upsilon$ Aqr has had stable H\textalpha emission and does not exhibit V/R variations, thus excluding a prominent one-armed spiral density wave in the disk (Okazaki 1991; Hanuschik et al. 1995). However, $\upsilon$ Aqr does possess a central quasi-emission (CQE) feature in Mg II $\lambda 4481$ (Rivinius et al. 2006), consistent with a high viewing angle for the disk (Hanuschik 1996). Stoeckley & Buscombe (1987) used the shape and widths of He i $\lambda 4471$ and Mg ii $\lambda 4481$ and a gravitational darkening model for the central star to estimate the inclination angle for $\upsilon$ Aqr, finding $i > 82^\circ$, consistent with its shell designation. Hubrig et al. (2009) claimed detection of a weak magnetic field in $\upsilon$ Aqr of about 100 G at 3$\sigma$; however, Bagnulo et al. (2012) conclude that the polarization detected with the FORS1 VLT instrument was instrumental in nature. $\upsilon$ Aqr is not known to have a binary companion, a result strengthened by direct adaptive optics imaging observations in the K-band with the VLT (Oudmaijer & Parr 2010). The $v \sin i$ of $\upsilon$ Aqr is 282 km s$^{-1}$, giving a $V/V_{cm}$ ratio of 0.74 (Touhami et al. 2013). This is consistent with the consensus that rapid rotation is a key driver behind the Be phenomena (Howarth 2007; Rivinius 2013; Rivinius et al. 2013).

Being both bright ($m_V = 4.69$) and close ($d = 134$ pc; based on \textit{Hipparcos} parallax), $\upsilon$ Aqr has been a target of recent interferometric studies. Meilland et al. (2012) included it in their survey of Be stars and Touhami et al. (2013) resolved it in the K-band, finding a major axis of 1.525 $\pm$ 0.642 mas as measured by fitting a geometric star-plus-Gaussian disk to the observed visibilities.

Optical interferometry using H\textalpha emission has proven to be very effective in resolving the circumstellar emission of Be stars. The strength of H\textalpha can result in detectable emission extending to many stellar radii (Tycner et al. 2005; Grundstrom 2006). The combination of H\textalpha interferometry and contemporaneous H\textalpha spectroscopy has been shown to be a powerful tool to constrain the physical properties of circumstellar disks of Be stars (Jones et al. 2008; Tycner et al. 2008). In this current work, we attempt to constrain the physical parameters of the H\textalpha-emitting circumstellar disk surrounding $\upsilon$ Aqr using this approach. We attempt to find a unified disk density model that reproduces the observed H\textalpha emission profile, H\textalpha interferometric visibilities, the near-IR spectral energy distribution, and the existence of a CQE feature in Mg II $\lambda 4481$, all using the numerical codes \textsc{bedisk} (Sigut & Jones 2007) and \textsc{beray} (Sigut 2011).

2. OBSERVATIONS

2.1. Spectroscopy

Spectroscopic observations in the H\textalpha region have been obtained using the Solar Stellar Spectrograph on the John S. Hall telescope at Lowell Observatory. Thirty individual spectra
are available for observing seasons from 2005 through 2014. The raw echelle spectral frames have been processed using the standard reduction routines developed by Hall et al. (1994) for the interferometer. The H\textsc{$\alpha$} profile of \textit{o} Aqr is shown as the error bars. The shell parameter, defined as \( F_p/F_e \), is 2.2, and the spectral resolving power is 10\(^5\). The H\textsc{$\alpha$} equivalent width as a function of the Julian date of the observations. The mean equivalent width (19.9 Å) is shown as the dotted line, and the 1\(\sigma\) variation is shown as the error bar.

Figure 1 shows the mean profile with the 1\(\sigma\) variation shown as the error bars. The lower panel of this figure also shows the H\textsc{$\alpha$} equivalent width (EW) as a function of Julian date over the nine-year period covered by the observations. The mean EW is 19.9 Å, with a 1\(\sigma\) variation of only 0.9 Å or 4.5\%. The profile has a peak-to-continuum contrast of 3.75, and the H\textsc{$\alpha$} shell parameter, defined as the ratio of the average flux in the emission peaks divided by the flux at line center, is 2.2. This identifies \textit{o} Aqr as a shell star following Hanuschik (1996): shell stars have ratios in excess of 1.5 based on the correlation of the H\textsc{$\alpha$} shell parameter with net, line-center absorption in weak, optically thin Fe\textsc{ii} lines (Hanuschik 1996). Typically for shell stars, the viewing inclination of the system is in excess of 70\(^\circ\).

Finally, Figure 1 shows that the emission in H\textsc{$\alpha$} extends to \( \approx \pm 400 \) km s\(^{-1}\). Using the adopted mass and radius for \textit{o} Aqr from Table 1, the velocity at the inner edge of a Keplerian disk is \( \approx 500 \) km s\(^{-1}\). As the inclination of the system must be large and \( \sin i \approx 1 \), there is no evidence of disk emission beyond the velocities available in the disk. Note that this would remain the case even if \textit{o} Aqr were critically rotating (see discussion below); the disk would then start at 1.5 \( R_e \) due to the geometric distortion caused by rapid rotation, and the velocity of the inner edge of the disk would drop to \( \approx 400 \) km s\(^{-1}\).

\footnote{We have an additional H\textsc{$\alpha$} spectrum from 2015 June 24 (JD = 2457198) which is consistent with the profile and equivalent widths of Figure 1.}

2.2. Interferometry

We have acquired interferometric observations of \textit{o} Aqr using the Navy Precision Optical Interferometer (NPOI) on a total of 58 nights covering five observing seasons: 2007 June, 2011 October, 2012 October through November, 2013 October through December, and 2014 July. The NPOI is a long-baseline interferometer that can measure the fringe contrast between various telescope pairs (i.e., baselines) for up to six telescopes simultaneously (Armstrong et al. 1998). The fringe contrast represents the measure of the degree of coherence between the light beams from separate telescopes, and when expressed as a squared visibility (\( V^2 \)) it represents the normalized Fourier power of the brightness distribution of the source on the sky (Hummel 2008). Therefore, assuming the source is spatially resolved, it allows the angular extent of the source to be constrained.

The processing of NPOI data has been conducted using the OYSTER (Optical Interferometer Script Data Reduction) package developed by Christian Hummel, which follows the procedures outlined in Hummel et al. (1998) with additional bias corrections using off-fringe measurements (Hummel et al. 2003). Typically, for an unresolved point source on the sky, no loss of fringe contrast would be expected; however, atmospheric and instrumental effects will contribute toward loss of coherence between light beams from separate telescopes. These effects are typically removed from the data by interleaving the observations of the target star with observations of a source of a known angular diameter (i.e., a calibrator star), which allows the determination of instrumental and atmospheric response functions, which in turn can be divided out of the data of the target star. However, because the light at the beam combiner of NPOI is dispersed over 16 spectral channels covering the wavelength range 560–870 nm, and the H\textsc{$\alpha$} emission line is contained in a single 15 nm wide spectral channel, it is possible to calibrate the H\textsc{$\alpha$} visibilities with respect to continuum channels. This was accomplished by adopting an angular diameter for the central star of 0.222 mas (based on the distance and radius listed in Table 1) and following the method outlined in Tycner et al. (2003) with the additional step of small-channel-to-channel fixed-pattern removal (Tycner et al. 2006a) that utilized observations of two calibrator stars, \textit{η} Aqr (for 2007, 2012, and 2014) and \textit{t} Aqr (for 2011 and 2013).

The final calibrated interferometric data set for \textit{o} Aqr from the spectral channel containing the H\textsc{$\alpha$} emission line consists of a total of 994 distinct \( V^2 \) measurements and these are shown in

| Parameter             | Value   |
|-----------------------|---------|
| Mass\(^{*}\) (\(M_\odot\)) | 4.2     |
| Radius\(^{*}\) (\(R_\odot\)) | 3.2     |
| Luminosity (\(L_\odot\)) | \(3.6 \times 10^5\) |
| \(T_{\text{eff}}(\text{K})\) | 14,000  |
| log(\(g\))             | 4.0     |
| Distance\(^{\text{a}}\) (pc) | 134     |
| Angular Diameter (mas) | 0.222   |

Notes.

\(^{a}\) Adopted from Townsend et al. (2004).

\(^{b}\) Based on Hipparcos parallax (Perryman et al. 1997).
Figure 2. NPOI squared visibilities from the Hα channel for o Aqr (N = 994) as a function of the magnitude of the spatial frequency. The symbol colors indicate the observing seasons: black (2007), red (2011), blue (2012), green (2013), and yellow (2014). The signature of a central star as represented by a uniform disk with a diameter of 0.222 mas is shown as the dashed-line.

Table 2

| Julian Date (JD = 2450000) | Spatial Frequency ι (10^6 cycles rad^(-1)) | Spatial Frequency ν (10^6 cycles rad^(-1)) | V^2 ± 1σ | Baseline^a |
|---------------------------|-------------------------------------------|-------------------------------------------|----------|------------|
| 4264.946                  | 77.594                                    | 61.891                                    | 0.757 ± 0.047 | AN–W7     |
| 4264.946                  | 115.830                                   | 24.900                                    | 0.699 ± 0.087 | E6–W7     |
| 4264.990                  | 69.858                                    | 61.141                                    | 0.788 ± 0.049 | AN–W7     |
| 4264.990                  | 118.350                                   | 23.709                                    | 0.749 ± 0.056 | E6–W7     |
| 4265.952                  | 76.495                                    | 61.799                                    | 0.829 ± 0.112 | AN–W7     |

Notes.
^a The baseline entries for the NPOI instrument are explained in Armstrong et al. (1998).
(This table is available in its entirety in machine-readable form.)

Figure 3. The (u, v) plane coverage for all NPOI interferometric observations. Different colors represent individual observing seasons: 2007 (black), 2011 (red), 2012 (blue), 2013 (green), and 2014 (yellow).
within 10% of the values in the range 2.3.

Several thousand Hα line profiles were computed covering a range in \( \rho_0 \), \( n \), \( i \), and various disk truncation radii \( R_d \). Hα profiles for the 50 combinations of \( (\rho_0, n) \) used in the BEDISK calculations were interpolated down to grid spacings of \( \Delta \log \rho_0 = 0.1 \) and \( \Delta n = 0.1 \). Five disk radii were considered \( (R_d = 5, 12, 25, 50, \) and 100 \( R_*) \), along with 11 viewing inclinations, covering \( 0°\)–\( 90° \), for each model. Figure 4 shows the distribution of disk density parameters \( (n, \log \rho_0) \) that match the observed mean Hα EW within 2\( \sigma \) (19.9 ± 1.8 \( \AA \)). Note that there are many values of \( R_d \) and \( i \) corresponding to each \( (n, \log \rho_0) \) pair; Figure 4 indicates a match if one or more combinations of \( R_d \) and \( i \) match the observed EW range.

To further refine the model, a match to the Hα line profile was sought. A figure of merit, \( \mathcal{F} \), for each model profile was found by taking the average absolute fractional deviation between each model profile (convolved to a resolving power of \( 10^5 \)) and an observed profile:

\[
\mathcal{F} = \frac{1}{N} \sum_i \left| \frac{F_i^\text{mod} - F_i^\text{obs}}{F_i^\text{obs}} \right|.
\]

Here, \( F_i^\text{mod} \) is the model flux computed with BERAY, \( F_i^\text{obs} \) is the observed flux, and the sum over \( i \) is for all \( N \) wavelengths in the range \( 6550 \lesssim \lambda_i \lesssim 6570 \). The minimum in \( \mathcal{F} \) then defined the best-fit model.\(^6\)

Figure 5 shows the five best-fitting profiles to the first available Hα spectrum in our series (2005 June 26). A good match to the peak height and central depth is obtained for the model with \( \rho_0 = 6.0 \times 10^{-12} \text{ g cm}^{-3}, n = 2.0, R_d = 25 R_*, \) and \( i = 75° \). However, the computed models are all slightly narrower at the base of the line, and the EW of the best-fitting Hα line profile is 16 \( \AA \), somewhat less than that of the observed profile. Figure 4 also shows the top 17 fitting profiles in the \( (n, \log \rho_0) \) plane that have \( \mathcal{F} \) within 10% of the best-fitting model. A much narrower region is now permitted, \(-11.5 < \log \rho_0 < -11 \) and \( 2 < n < 2.3 \). Among the top 10% of line profile fitting models, the mode of \( i \) is 75° and the mode of \( R_d \) is 25 \( R_* \). The result \( i = 75° \) is consistent with the classification of \( \alpha \) Aqr as a shell star based on the Hα shell parameter defined by Hanuschik (1996).

Fitting all available Hα line profiles (2005–2014) results in disk parameters that vary only slightly relative to those found for the 2005 June 26 profile. All best-fit parameters are identical with the exception of \( \rho_0 \), which assumes values

\(^6\) We note that other choices for the fit figure of merit, such as \( (F_i^\text{mod} - F_i^\text{obs})^2/\sigma \), where \( \sigma \) is a wavelength-independent uncertainty, or \( (F_i^\text{mod} - F_i^\text{obs})^2/F_i^\text{obs} \), select the same best-fitting models, with small permutations of the order. We prefer the form given in Equation (3) because equal terms indicate the same percentage deviation at each wavelength.
of $5.0 \times 10^{-12} \, \text{g cm}^{-3}$ (12 spectra), $6.0 \times 10^{-12} \, \text{g cm}^{-3}$ (10 spectra), and $7.0 \times 10^{-12} \, \text{g cm}^{-3}$ (2 spectra). The average $\rho_0$, weighted by the number of spectra, is $(5.6 \pm 0.7) \times 10^{-12} \, \text{g cm}^{-3}$ where the quoted uncertainty is the 1σ variation.

The influence of the model parameters $\rho_0$, $n$, and $R_d$ on the line profile figure of merit $F$ is shown in Figure 6, which plots each parameter versus $F$ in a separate panel. This results in a series of horizontal lines for each parameter value (reflecting the discrete values of that parameter considered) with the leftmost value giving the smallest $F$ achievable with that choice. For example, for $i = 20\degree$, no combination of the remaining model parameters $\rho_0$, $n$, and $R_d$ can result in $F < 0.6$. However, models near $i = 75\degree$ produce the overall minimum in $F$ of around 0.18. As illustrated in the figure, the observed line profiles discriminate most strongly in inclination, followed by $\rho_0$ and $n$. The variation of $F$ with $R_d$ rules out small disks with $R_d \lesssim 5 \, R_\odot$, but the discrimination for larger disks is poor as these disks typically encompass the complete Hα formation region, and larger disks have little impact on the relative Hα flux profile.

The inability of our models to fit the Hα line wings and core simultaneously suggests that we attempt to minimize the influence of the line wings on the fitting procedure in order to gauge the effect on the models selected. To this end, we considered a revised, core-weighted, figure of merit, $F_{cw}$, of the form

$$F_{cw} = \frac{1}{N} \sum_i w_i \frac{\left| F_i^\text{mod} - F_i^\text{obs} \right|}{F_i^\text{obs}},$$

where the weights were chosen to be small in line wings and large in the core. The function

$$w_i = \frac{F_i^\text{obs}}{F_c^\text{obs}} - 1,$$

where $F_c^\text{obs}$ is the observed continuum flux (i.e., equal to unity as the spectra are continuum-normalized) achieves this effect. Minimizing $F_{cw}$ results in best-fit profiles that better fit the line core and the top five such profiles are shown in Figure 7. Interestingly, the top model parameters now favor a higher $\rho_0$ and larger $n$ compared with the $w_i = 1$ minimization. The best fit to the 2005 June 26 profile is $\rho_0 = 1.0 \times 10^{-10} \, \text{g cm}^{-3}$ and $n = 2.7$ for $R_d = 25 \, R_\odot$ and $i = 75\degree$. Fitting all available spectra from 2005 through 2014, and choosing the top-fitting profile in each case, gives essentially identical parameters except that $\rho_0$ varies from $5.0 \times 10^{-11}$ through $1.0 \times 10^{-10} \, \text{g cm}^{-3}$. The average, weighted by the number of fitting spectra, is $\rho_0 = (6.8 \pm 0.2) \times 10^{-11} \, \text{g cm}^{-3}$, nearly a factor of ten larger than the previous best-fit profiles based on the $w_i = 1$ figure of merit.

Figure 4 also shows the selected models in the $(n, \log \rho_0)$ plane that fit within 10% of the best-fit model; the core-weighted fits include a much wider range of models. In conclusion, while this new weighting is arbitrary, it will become instructive when we discuss the fit to the observed near-IR SED of o Aqr in Section 4.3.

### 4.2. NPOI Hα Interferometry

#### 4.2.1. Geometric Models

We first fit two very simple geometric models to the entire set of 994 NPOI visibilities: a nearly unresolved star (represented by a uniform disk with an angular diameter of 0.222 mas) plus either a uniform elliptical or a Gaussian elliptical disk representing the circumstellar contribution. If $V_4$ is the visibility of the star and $V_D$ that of the disk, then the

\[ \text{In contrast to Equation (3), including models within 20% or 30% of the best model does not significantly increase the range of models in the $(n, \log \rho_0)$ plane.} \]
model visibilities can be represented as

\[
V^2 = \left[ c_b \, V_e(0.222 \text{ mas}) + (1 - c_b) \, V_D(a, b, \phi) \right]^2.
\] (6)

Here, the major and minor axes of the ellipse \((a, b)\), the position angle of the major axis \((\phi)\), and the fractional contribution of the star to the visibilities, \(0 < c_b < 1\), are free parameters. Detailed forms for \(V_e\) and \(V_D\) for both uniform and Gaussian elliptical disks are given in Berger (2003) and Tycner et al. (2006).

Table 3 gives the results of these fits. The star-plus-elliptical Gaussian disk fits the observations with a reduced chi-squared of \(\chi^2/\nu = 1.193\), yielding a major axis (given by the FWHM of the Gaussian) of 2.65 \(\pm\) 0.09 mas (or equivalently a radial extent given by half-width at half-maximum of 1.19 \(R_*\)). However, the fit is unconstrained along the minor axis, consistent with \(o\) Aqr being unresolved in this dimension, and no estimate of the axial ratio is possible with this model. The position angle on the sky (measured East from North) is \(\phi = 110^\circ \pm 6^\circ\).

The uniform elliptical disk model produces a slightly poorer fit \((\chi^2/\nu = 1.139)\) and a significantly larger major axis, 4.15 \(\pm\) 0.15 mas, as expected based on the geometrically different description of the extent of the emitting region (diameter of uniform disk versus FWHM of a Gaussian). The axial ratio is found to be \(r = 0.20 \pm 0.21\), again consistent with the minor axis not being sufficiently resolved. The position angle on the sky for this model was found to be \(\phi = 111^\circ \pm 5^\circ\).

Finally, we note that assuming a geometrically thin disk, an axial ratio of \(r = 0.20 \pm 0.21\) implies a viewing inclination of \(i = 78^\circ \pm 12^\circ\), consistent with \(i = 75^\circ\) found from the \(H\alpha\) line profile modeling of Section 4.1.

A star-plus-elliptical Gaussian disk model with a fixed axial ratio of \(r = 0.2\) was also tried, and this gave a reduced chi-squared of \(\chi^2/\nu = 1.097\), just marginally better than the unconstrained fit. With this model, the major axis was 2.58 \(\pm\) 0.09 mas and the position angle on the sky, \(\phi = 110^\circ \pm 2^\circ\). Figure 8 compares the visibilities of this star-plus-Gaussian disk with the fixed axial ratio of \(r = 0.2\) to the NPOI observations.

In the fitting procedure, \(c_b\) was treated as a free parameter, with all models finding \(c_b = 0.87\). However, as noted by Tycner et al. (2006), \(c_b\) is essentially

\[
c_b = \frac{\Delta}{\Delta + \text{EW}_{H\alpha}}
\] (7)

where \(\Delta\) is the width of the NPOI spectral channel containing the \(H\alpha\) emission, 150 \(\AA\), and \(\text{EW}_{H\alpha}\) is the EW of the \(H\alpha\) emission. Using \(\text{EW}_{H\alpha} = 19.9\ \AA\), we find \(c_b = 0.88\), in excellent agreement with the value recovered by the fits.

\begin{table}[h]
\centering
\begin{tabular}{|l|c|c|c|c|c|}
\hline
Model & Major Axis & Axial Ratio & Position Angle & \(c_b\) & \(\chi^2/\nu\) \\
Star+ & (mas) & & (deg) & & \\
\hline
Uniform Disk & 4.15 \(\pm\) 0.15 & 0.20 \(\pm\) 0.21 & 111 \(\pm\) 5 & 0.873 \(\pm\) 0.002 & 1.139 \\
Gaussian Disk & 2.65 \(\pm\) 0.10 & 0 & 107 \(\pm\) 6 & 0.870 \(\pm\) 0.003 & 1.101 \\
Gaussian Disk & 2.58 \(\pm\) 0.09 & \(\equiv\) 0.2 & 110 \(\pm\) 2 & 0.864 \(\pm\) 0.003 & 1.097 \\
\hline
\end{tabular}
\caption{Geometric Model Fits to the NPOI Visibilities}
\end{table}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure8.png}
\caption{Best-fitting star-plus-elliptical Gaussian disk geometric model. The axial ratio is fixed at \(r = 0.2\), as discussed in the text. The red symbols give the predicted model visibilities at the spatial frequencies of the observations, which are shown in light gray with \(1\sigma\) error bars. The visibilities of the major and minor axes of the model are shown as the solid lines.}
\end{figure}

Touhami et al. (2013) found a major axis of 1.525 \(\pm\) 0.642 mas, an axial ratio of \(r = 0.249 \pm 0.059\), and a position angle of \(\phi = 107.5 \pm 2.2\) for \(o\) Aqr based on K-band continuum interferometry and Gaussian elliptical fits that included the star. Their best-fit model had \(\chi^2/\nu = 1.80\). This K-band continuum major axis found by Touhami et al. (2013) is about 50% smaller than the \(H\alpha\) major axis given in Table 3.

\(o\) Aqr was also observed with VLT/AMBER by Meillard et al. (2012) in both the K-band continuum and Br\(\gamma\). While \(o\) Aqr was unresolved in the K-band, the Br\(\gamma\) observations were consistent with a kinematic model with \(i = 70^\circ \pm 20^\circ\), a position angle of \(\phi = 120^\circ \pm 20^\circ\), and a disk FWHM of 14 \(\pm\) 1 stellar diameters. Meillard et al. (2012) note sparse \((u, v)\) plane coverage and low signal-to-noise ratio due to poor weather conditions. Nevertheless, the system inclination and position angle of the major axis agree with geometric models of Table 3. One interesting comparison with the current work is that the Br\(\alpha\) disk FWHM found by Meillard et al. (2012) is comparable to the \(H\alpha\) disk FWHM found in this work, something unusual for Be stars where the size of the \(H\alpha\) region is usually 1.5–2 times the Br\(\gamma\) region. Similar sizes for these two regions are further reflected by the very similar peak separations in the emission profiles: 177 km s\(^{-1}\) for Br\(\gamma\) (Meillard et al. 2012) and 162 km s\(^{-1}\) for \(H\alpha\) (Figure 1).
Finally, we note that Yudin (2001) finds an intrinsic polarization in the V-band of 0.6% for α Aqr with a position angle on the sky of +6°. As the polarization vector is expected to be perpendicular to the major axis of the disk, this is consistent with the position angles found in Table 3.

4.2.2. Physical Models

While the geometric fits of the previous section provided very good representations of the observed visibilities, they cannot constrain physical conditions in the disk, such as its density structure. To analyze the NPOI visibilities with physically based models, the beray code (Sigut 2011) was used to produce images of the Be star+disk models on the sky given the observer’s viewing inclination. Each model is specified by a choice of four parameters (μ₀, n, i, and Rₐ), as with the Hα spectroscopic calculations. The Hα image computed by beray was integrated over a 150 Å wavelength interval centered on Hα to match the NPOI observations.

Given the Hα fit results of the previous section, beray images were computed for disk models with μ₀ ranging from 2.5 × 10⁻¹² to 1.0 × 10⁻¹⁰ g cm⁻³ and power-law indices n = 2.0, 2.25, 2.5, 2.75, and 3.0. Inclinations between 68° and 80° (in steps of 2°) and Rₐ = 25 and 50 Rₘ were used. This subset of models includes the entire range of the best-fit Hα profiles.

Given a computed image specified by sky intensities Iₓ, where i = 1...Nsky and j = 1...Nsky, the predicted visibilities were determined by computing the discrete Fourier transform (DFT) of the image following the Zernike–van Cittert theorem (e.g., see Labeyrie et al. 2006). In practice, the beray image was calculated with constant grid spacing on the sky within a linear region spanning R < 20 Rₐ and a logarithmically spaced grid beyond that to reduce the computation time. To prepare for the DFT of the image, the outer, nonlinearly spaced region was interpolated down to the constant spacing of the inner region. As this interpolation is done far away from the star and the image at these locations is smooth, linear interpolation is sufficient. All images used a final constant spacing of 0.05 Rₐ or 0.16 Rₐ. Before the DFT was computed, the image was zero-padded out to R = 62.5 Rₐ or 200 Rₐ. The final images had Nsky = Nsky = 2504. The small grid spacing of the models gives a Nyquist frequency of ∼1.8 × 10¹⁰ cycles per radian, far larger than largest spatial frequency sampled by NPOI, and large enough that visibility values can be expected to be negligible even for the nearly unresolved central star.

To compare with the observed visibilities, two-dimensional interpolation was performed in the DFT images at each of the observed spatial frequencies (u, v) of Figure 3. To fit the position angle of the disk, it proved more computationally efficient to rotate the (u, v) coordinates of the observed spatial frequencies, as opposed to the image itself. The minimum in the reduced χ² defined the best position angle for a given image, and the minimum in reduced χ² over all trial images at their best-fit position angle was used to define the best model.

Over all trial images, the minimum reduced χ² was found to be χ²/ν = 1.081 (N = 994), corresponding to the model with μ₀ = 5.0 × 10⁻¹¹ g cm⁻³, n = 2.0, and Rₐ = 25 Rₘ. Both the i = 78° and i = 80° images corresponding to this model fit the data equally well, and the position angles of the major axis of these best-fit models were 106° and 110°. Thus the physical

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Figure 9. Predicted visibilities based on the Fourier transform of the best-fit model image corresponding to the disk model μ₀ = 5.0 × 10⁻¹¹ g cm⁻³, n = 2.0, Rₐ = 25 Rₘ, and i = 78° (grayscale, scale at top), compared to the observed visibilities (symbols). The position angle of the model is 110°. The symbols are: green circle (model fits to within error bars), red triangle (model below the observations), and blue plus sign (model above the observations). The overall reduced χ² of the fit is 1.081 for N = 994.

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deliberately chosen as a plausible sub-sample of models based on the previous fits to the \( \text{H}\alpha \) emission line profile, and many of these models differ only in their viewing inclination angle over the range \( 68^\circ \)–\( 80^\circ \). The wide range of models consistent with the visibilities includes the smaller ranges consistent with the \( \text{H}\alpha \) profile at the same level.

Note that the diagonal trend to the best-fitting models in the \((n, \log \rho_0)\) diagram is expected and has been noted before (Tycner et al. 2008). For an optically thick disk, the flux is, to first order, just the Planck function at the average disk temperature times the projected surface area of the disk out to \( t = 1 \). Therefore, the various combinations of \( \rho_0 \) and \( n \) that produce similar effective emitting areas will result to first order in observational signatures that match the observations equally well.

Note.  

\( ^a \) The reported parameters are for the 2005 June profile.

Figure 10. NPOI observations with errors compared with the model visibilities based on the Fourier transform of the best-fit model image with disk parameters \( \rho_0 = 5.0 \times 10^{-12} \text{ g cm}^{-3} \) and \( n = 2.0 \) as a function of the magnitude of the spatial frequency (top panel). The fit residuals, \( z = (V_{\text{obs}}^2 - V_{\text{mod}}^2)/\sigma \), are shown in the lower panel.

Figure 11. Histogram of the visibility residuals, \( z = (V_{\text{obs}}^2 - V_{\text{mod}}^2)/\sigma \), for the best-fit physical model. The solid black line shows a Gaussian fit to the residuals, giving \( \mu = -0.1065 \) and \( \sigma = 1.0372 \). The dotted blue line gives the reference Gaussian with \( \mu = 0.0 \), \( \sigma = 1.0 \).

Table 4
| Feature Used to Constrain Fit | Best Fitting Parameter | \( \rho_0 \) (g cm\(^{-3}\)) | \( n \) | \( R_d \) (\( R_\odot \)) | \( i \) | Notes |
|-------------------------------|------------------------|-----------------|-----|-----------------|-----|-------|
| \text{H}\( \alpha \) \( ^a \) | \( \chi^2/\nu = 1.85 \times 10^{-1} \) | 5.0 \times 10^{-12} | 2.0 | 25 | 75\(^\circ\) | 17 models within 10\% |
| \text{H}\( \alpha \) core-weighted | \( \chi^2/\nu = 3.5 \times 10^{-4} \) | 1.0 \times 10^{-10} | 2.7 | 25 | 75\(^\circ\) | 10 models within 10\% |
| V\( ^2 \) Visibility | \( \chi^2/\nu = 1.081 \) | 5.0 \times 10^{-12} | 2.0 | 25 | \( 78^\circ \sim 80^\circ \) | 104 models within 10\% |
| Near-IR SED | \( \chi^2/\nu = 0.49 \) | 1.0 \times 10^{-10} | 3.0 | 25 | 72\(^\circ\) | 17 models with \( \chi^2/\nu < 1 \) |

Adopted Region in Figure 12

6.6 \times 10^{-11} | 2.7 | 25 | 75\(^\circ\) |

Note.  

\( ^a \) The reported parameters are for the 2005 June profile.

Figure 12. Best-fit regions in the \((n, \log \rho_0)\) plane based on the top 10\% of fits to the \text{H}\( \alpha \) emission profile (red, Equation (3); green, Equation (4)), NPOI visibilities (blue), and Touhami et al. (2010) SED (black). The models enclosed for each feature correspond to the “Notes” column in Table 4. The location of the adopted, best-fit model for \( \alpha \) Aqr is shown as the purple circle.
visibility data almost as well as the physical model discussed here.

Figure 13 also shows that as worse-fit models are included in the average, the mean position angle rises steadily, and the $1\sigma$ variation increases dramatically. Finally, we note that for individual images, the discrimination in position angle is good, as the reduced $\chi^2$ varies by more than factor of two over the range in $\phi$ from $0^\circ$ to $180^\circ$.

Given the time span of the NPOI observations, we have also fit subsets of the visibility data. As expected, the more limited 2007 data are consistent with a significantly larger range of physical parameters. We did not analyze the 2011 data separately as they are confined to smaller spatial frequencies (see Figure 2), while analyzing the 2012–2014 data alone gives results that are indistinguishable from the full data set. Given this, it is not possible to detect variability between the 2007 and 2012–2014 data sets.

4.3. The Near-IR SED

Touhami et al. (2010) give optical and near-infrared fluxes (from 2008) for $\alpha$ Aqr at four wavelengths: $\lambda\lambda0.440, 0.680, 1.654$, and $2.179\mu$m. The apparent stability of $\alpha$ Aqr’s disk suggests that it is useful to consider these near-IR fluxes as an additional consistency check on our modeling. B-ex ray was used to compute optical and near-IR spectral energy distributions for the same subset of disk models used to analyze the NPOI visibilities.

To compare to the model fluxes, the Touhami et al. (2010) observations were normalized to the model SED at 0.440 $\mu$m, and the reduced $\chi^2$ of the fit computed for the three remaining observed wavelengths. The uncertainties in the observed fluxes given by Touhami et al. (2010) are typically $<10\%$, and are quoted as the quadratic sum of uncertainties due to instrumental error, errors due to repeatability of the individual observations, and errors associated with the calibration and air-mass corrections.

Figure 14 shows the best fit to the observed near-IR SED. This model has $\rho_0 = 1.0 \times 10^{-10}$ g cm$^{-3}$, $n = 3.0$, $R_d = 25 R_\odot$, and $i = 72^\circ$, and fits the observations well with a $\chi^2/\nu = 0.49$. Also shown in the figure is the worst-fitting model that has the same $R_d$ and $i$. This is a lower-density model, $\rho_0 = 5.0 \times 10^{-12}$ g cm$^{-3}$ and $n = 3.0$, which gives fluxes close to the pure photospheric SED of the star alone. As the observed fluxes have been separately normalized to each model prediction at 0.440 $\mu$m, the model fluxes themselves can be directly compared to each other. This illustrates that the best-fit model predicts an IR excess and optical and UV deficits relative to the photospheric spectrum. The small deficits of $\lesssim 0.1$ mag are a consequence of the obscuration of the photosphere by the disk for the large viewing inclination.

In addition to this best-fit model, 17 of the 560 models considered had a reduced $\chi^2$ of less than unity. All models consistent with the SED of Touhami et al. (2010) are represented by the black ellipse in the $(n, \log \rho_0)$ plane shown in Figure 12.

Interestingly, the near-IR SED is consistent only with the $H_\alpha$ line profile fitting when the core-weighted figure of merit is used, Equation (4). This situation is summarized in Figure 12. All three contemporaneous observational constraints—the $H_\alpha$ line profile (fit using core weighting), the NPOI visibilities, and the near-IR SED of Touhami et al. (2010)—imply a best-fit model of $\rho_0 = 6.6 \times 10^{-11}$ g cm$^{-3}$, $n = 2.7$ with $R_d = 25 R_\odot$ seen at an inclination of $i = 75^\circ$. These will be adopted as the disk parameters for $\alpha$ Aqr over the time period considered.

Figure 12 also shows that the disk parameters of the best-fit, uniformly weighted $H_\alpha$ profile (i.e., fit with Equation (3)) are not consistent with those based on the near-IR SED of Touhami et al. (2010). Nevertheless, the spatial extent of the $H_\alpha$ disk is much larger than that contributing to the near-IR flux. The extended wings of $H_\alpha$ are not fully reproduced by any of our model $H_\alpha$ profiles, and this may reflect additional material close to the star not accounted for in our assumption of a single power-law description of radial density fall-off.
4.4. Disk Density Variations

The consistency of the Hα EW and line profile seen in Figure 1 suggests that the density in \( o \) Aqr’s disk is very stable over this time period. However, it is important to understand the possible limitations of using Hα as a proxy for disk density stability. This is illustrated in Figure 15, which shows the EW (left panel) and line profile (right panel) of Hα as a function of the disk base density \( \rho_0 \) for the model with \( n = 2.7, R_d = 25 R_\odot \) and \( i = 75^\circ \). The maximum predicted EW is 14.1 Å at \( \log \rho_0 = -10.0 \). Near this maximum, both the Hα profile and EW are very insensitive to changes in the disk density: for example, as the density increases from \( 5.0 \times 10^{-11} \text{ g cm}^{-3} \), the maximum increase in the EW is about 10% before returning to the starting value by \( 2.5 \times 10^{-10} \text{ g cm}^{-3} \). The line profiles for \( 5.0 \times 10^{-11} \) and \( 2.5 \times 10^{-10} \text{ g cm}^{-3} \) (black and blue profiles in Figure 15, respectively) are virtually identical. Thus, in this range of disk parameters, small variations in either the EW or profile of Hα can mask large changes in the disk density.

In the current case of \( o \) Aqr, it is significant that a disk density model consistent with all considered observational constraints (Hα profile fit with core weighting, the visibilities, and the near-IR SED) for parameters \( \rho_0 = 6.7 \times 10^{-11} \text{ g cm}^{-3} \) and \( n = 2.7 \) is very near the maximum predicted model strength. This provides a natural explanation for the observed stability of the Hα line profile and EW. As noted above, even large changes in the overall disk density, up to a factor of approximately five, will lead to only small changes in the observed Hα profile.

4.5. The CQE Feature in Mg\( \equiv \)\( \lambda 4481 \)

Rivinius et al. (2006) note that \( o \) Aqr exhibits a CQE feature in the core of Mg\( \equiv \)\( \lambda 4481 \). CQE features are apparent emission “bumps” in the cores of some lines, particularly those of Mg\( \equiv \), He\( \equiv \), and Fe\( \equiv \). Despite their appearance as relative emission, CQE features are a pure absorption effect caused by the velocity shift (in the observer’s frame) of the local atomic line profile in a Keplerian-rotating disk viewed nearly edge-on (Hanuschik 1996). Because of this geometrical requirement, and the somewhat special circumstances of their formation, CQE features can be a useful test of a particular disk model. In this section, we show that the appearance of a CQE feature in Mg\( \equiv \)\( \lambda 4481 \) is consistent with the disk density parameters found for \( o \) Aqr in this work.

Rivinius et al. (2006) present a Mg\( \equiv \)\( \lambda 4481 \) profile from 1999 (somewhat outside the time-frame of the present work) that shows a CQE feature with a central amplitude of just less than 1% \( (F_c/F_m = 1.008) \) where \( F_c \) is the line-center flux and \( F_m \) is the flux minimum just outside the core; see Figure 16. To see if this is consistent with the disk model proposed for \( o \) Aqr, we have used BEARAY to compute Mg\( \equiv \)\( \lambda 4481 \) line profiles for circumstellar disks with \( \rho_0 \) values of \( 5 \times 10^{-12}, 10^{-11}, 5 \times 10^{-11} \) and \( 10^{-10} \text{ g cm}^{-3} \), all with \( n = 2.5, R_d = 25 R_\odot \), for viewing inclinations of \( i = 65^\circ, 75^\circ, 80^\circ, \) and \( 85^\circ \). We have assumed an equatorial velocity of 290 km s\(^{-1}\) for \( o \) Aqr, corresponding to \( V_{\text{ISCO}} = 0.74 \) (Touhami et al. 2013). Photospheric profiles were computed assuming LTE as this is a reasonable approximation at the \( T_{\text{eff}} \) of \( o \) Aqr (Sigut & Lester 1995).

Figure 16 shows that for \( \rho_0 = 5 \times 10^{-11} \text{ g cm}^{-3} \), a CQE feature of the correct amplitude is predicted for \( i = 75^\circ \) and \( 80^\circ \). Interestingly, a CQE feature is not predicted for densities less than \( \rho_0 = 10^{-11} \text{ g cm}^{-3} \). Thus the appearance of a CQE feature in Mg\( \equiv \)\( \lambda 4481 \) is consistent with the proposed disk density model found in this work and shown in Figure 12. Note that as these observations are outside the time-frame considered in this work, we have not attempted to find a best-fit profile to Mg\( \equiv \)\( \lambda 4481 \). Nevertheless, the general agreement we find is a non-trivial test of our proposed disk density model for \( o \) Aqr.

4.6. The Mass and Angular Momentum Content of the Hα Disk

Combining Hα spectroscopy, NPOI interferometric visibilities, and the near-IR SED, we determined a best-fit disk model of \( \rho_0 = 6.6 \times 10^{-11} \text{ g cm}^{-3}, n = 2.7, R_d = 25 R_\odot \) seen at an inclination of \( i = 75^\circ \). The position angle of the major axis on the sky was found to be \( 110^\circ \pm 8^\circ \).
To determine the mass in the H\(\alpha\) disk implied by this model, we have computed an additional best fit for the best-fit parameters but for \(i = 0^\circ\). Plotting \(2\pi R_\|\) versus \(R\), where \(I\) is the model intensity at distance \(R\), we find that 90% of the H\(\alpha\) disk light comes from \(R < 9.5\ R_\star\). Using the best-fit disk parameters for \((n_\alpha, r_\alpha)\) above and the disk density model given by Equation (1), we find an enclosed disk mass of \(1.5 \times 10^{23}\ g\) or \(~1.8 \times 10^{-10}\ M_\star\).

Assuming Keplerian rotation for the disk, well established for Be stars (Rivinius et al. 2013), we find a total angular momentum associated with the H\(\alpha\)-emitting disk of \(3.5 \times 10^{43}\ g\ cm^2\ s^{-1}\). The stellar angular momentum is \(J_\star = \beta^2 M_\star R_\star V_{eq}\) where \(\beta\) is the radius of gyration, \(~0.2\) (Claret & Giménez 1898), and \(V_{eq}\) is \(o\ Aqr\’s\) equatorial velocity, \(290\ km\ s^{-1}\). We find \(J_\alpha = 2.2 \times 10^{31}\ g\ cm^2\ s^{-1}\), making the angular momentum associated with the H\(\alpha\)-emitting disk \(~1.6 \times 10^{-8}\ J_\star\).

To give an indication of the robustness of these values for the total mass and angular momentum content of the H\(\alpha\)-emitting disk, we list in Table 5 the mass and angular momentum values for the best-fit disk models for each of the considered observational constraints separately, as given in Table 4. The disk radius that encloses 90% of the H\(\alpha\) light comes from \(o\ Aqr\), covering the period 2007 through 2014. Using predicted visibility obtained from the NPOI for the Be shell star \(o\ Aqr\), we find that 90% of the H\(\alpha\) emission is computed for each model and is given in Table 5. The range is about a factor of three in disk mass and a factor of five in angular momentum content. This is significantly smaller than the range in the disk base density \(\rho_\alpha\) alone as higher densities are associated with larger values for the power-law index \(n\).

5. CONCLUSIONS

We have analyzed a large set \((N = 994)\) of H\(\alpha\) interferometric visibilities obtained from the NPOI for the Be shell star \(o\ Aqr\), covering the period 2007 through 2014. Using predicted visibilities based on physical disk models computed by the BENDISK and BEARAY codes, we find best-fit disk parameters that are consistent with an analysis of the H\(\alpha\) emission line profile and the near-IR SED of Touhami et al. (2010) from the same time period. We note that these physically based BEARAY images can fit the observed visibilities down to the level associated with observational uncertainties (i.e., down to the level of \(\chi^2/\nu \sim 1\)).

The best-fit disk model with \(\rho_\alpha = 6.6 \times 10^{-11}\ g\ cm^{-3}, n = 2.7, R_\alpha = 25\ R_\star\), implies a disk mass associated with the H\(\alpha\)-emitting region of \(~1.8 \times 10^{-10}\ M_\star\) and an angular momentum content of the disk of \(~1.6 \times 10^{-8}\ J_\star\), where \(M_\alpha\) and \(J_\alpha\) are the mass and angular momentum of the central B star in \(o\ Aqr\).

Over the nine years of H\(\alpha\) spectroscopic observations, from 2005 until 2014, we find variations in its EW of typically less than 5%. However, our best-fit disk density model is at the maximum strength of H\(\alpha\) that can be produced for any value of \(\rho_\alpha\) given a power-law index of \(n = 2.7\) seen at \(i = 75^\circ\). For this model, variations in the disk density, \(\rho_\alpha\), of up to a factor of \(~5\) would not lead to noticeable changes in the H\(\alpha\) EW or its profile, and variations of this magnitude cannot be excluded over the time period considered.

We further test our model by comparing to the 1999 CQE feature in Mg \(\text{II}\ \lambda 4481\) observed by Rivinius et al. (2006) and find that a very similar feature can be produced by our best-fit disk density model, representing an additional and highly non-trivial success of our modeling.

Finally, we note that in order to produce fits to the H\(\alpha\) line profile consistent with the other constraints, the fits needed to be weighted more heavily in the line core (emission peaks and central depression) at the expense of the line wings. This may reflect the limitation of the assumption of a single power law for the disk’s equatorial density.

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| Diagnostic | \((R_\alpha/R_\|)\)^a | \(M_\alpha\) | \(M_\alpha/M_\star\) | \(J_\alpha\) \(\text{g cm}^2\ s^{-1}\) | \(J_\alpha/J_\star\) |
|-----------|----------------|--------|----------------|----------------|----------------|
| H\(\alpha\) \((\mathcal{F})\) | 19.5 | \(1.0 \times 10^{24}\) | \(1.2 \times 10^{-10}\) | \(3.7 \times 10^{43}\) | \(1.7 \times 10^{-8}\) |
| H\(\alpha\) \((\mathcal{F}_{\text{CW}})\) | 8.3 | \(2.0 \times 10^{24}\) | \(2.3 \times 10^{-10}\) | \(4.4 \times 10^{43}\) | \(2.0 \times 10^{-8}\) |
| \(V^2\) | 19.5 | \(1.0 \times 10^{24}\) | \(1.2 \times 10^{-10}\) | \(3.7 \times 10^{43}\) | \(1.7 \times 10^{-8}\) |
| near-IR SED | 3.3 | \(5.8 \times 10^{23}\) | \(6.9 \times 10^{-11}\) | \(9.0 \times 10^{42}\) | \(4.2 \times 10^{-8}\) |
| Adopted\h | 9.5 | \(1.5 \times 10^{24}\) | \(1.8 \times 10^{-10}\) | \(3.5 \times 10^{43}\) | \(1.6 \times 10^{-8}\) |

Notes.

^\(R_\alpha\) is the disk radius that encloses 90% of the integrated H\(\alpha\) light.

\(R_\alpha\) This is the best-fit model to all three constraints (H\(\alpha\) \((\mathcal{F}_{\text{CW}}), V^2,\) and near-IR SED) shown in Figure 12.
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