A Biased Random-Key Genetic Algorithm for Bandwidth Reduction

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Abstract. The bandwidth minimization problem is a well-known \textit{NP}-hard problem. This paper describes our experience in implementing a biased random-key genetic algorithm for the bandwidth reduction problem. Specifically, this paper compares the results of the new algorithm with the results yielded by four approaches. The results obtained on a set of standard benchmark matrices taken from the SuiteSparse sparse matrix collection indicated that the novel approach did not compare favorably with the state-of-the-art metaheuristic algorithm for bandwidth reduction. The former seems to be faster than the latter. On the other hand, the design of heuristics for bandwidth reduction is a very consolidated research area. Thus, a paradigm shift seems necessary to design a heuristic with better results than the state-of-the-art metaheuristic algorithm at shorter execution times.

Keywords: Bandwidth reduction · Biased random-key genetic algorithms · Ordering · Reordering algorithms · Renumbering · Sparse matrices

1 Introduction

The bandwidth minimization problem consists of labeling the vertices of a graph $G = (V, E)$ (composed of a set of vertices $V$ and a set of edges $E$) with integer numbers from 1 to $n$, where $n = |V|$ is the number of vertices such that the maximum absolute difference between labels of adjacent vertices is minimum. The bandwidth minimization problem is a well-known \textit{NP}-hard problem \cite{1}. Prevalent algorithms for bandwidth reduction are heuristic methods since they attempt to find a labeling that provides a small bandwidth in a reasonable amount of time. Thus, heuristics for bandwidth reduction place nonzero coefficients of a sparse matrix as close to the main diagonal as possible. Figure 1 illustrates an example of applying a heuristic for bandwidth reduction to a matrix.

Let $A = [a_{ij}]$ be an $n \times n$ symmetric adjacency matrix associated with an undirected graph $G = (V, E)$. The bandwidth of matrix $A$ is defined as $\beta(A)$ =
max \[\beta_i(A)\], where \(\beta_i(A) = i - \min_{1 \leq j \leq i} [j : a_{ij} \neq 0]\). Equivalently, the bandwidth of \(G\) for a vertex labeling \(S = \{s(v_1), s(v_2), \cdots, s(v_|V|)\}\) (i.e. a bijective mapping from \(V\) to the set \(\{1, 2, \cdots, |V|\}\)) is \(\beta(G) = \max_{\{v,u\} \in E} [|s(v) - s(u)|]\).

Since bandwidth reduction is related to a vast range of important scientific and engineering applications, such as computational fluid dynamics and structural problems, researchers have proposed a large number of heuristics to reduce the bandwidth of matrices since the 1960s [2]. The Reverse Cuthill-McKee method [3] is the most classical heuristic for bandwidth reduction [4–7]. For instance, this method is available on MATLAB [8,9] and GNU Octave [10] mathematical softwares as the function \textit{symrcm}\(^1\), and on Boost C++ Library [11]\(^2\).

Metaheuristic-based algorithms for bandwidth reduction began to be proposed mainly in the 1990s [12]. As previously mentioned, since bandwidth reduction is a relevant problem in various scientific and engineering fields, practitioners have applied the best-known metaheuristics in the design of heuristics for bandwidth reduction [12], including tabu search [13,14], GRASP [15], genetic algorithm [16,17], ant colony optimization [6,7,18–21], particle swarm optimization [22], simulated annealing [23,24], variable neighbourhood search [25], genetic programming [26–28] and self-organizing maps [29,30].

The VNS-band heuristic [25] yielded better bandwidth results than a heuristic based on the simulated annealing metaheuristic [23], which was considered the previous state-of-the-art heuristic for bandwidth reduction. Torres-Jimenez et al. [24] proposed a Dual Representation Simulated Annealing (DRSA) heuristic for bandwidth reduction. The DRSA heuristic delivered better bandwidth

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\(^1\) https://www.mathworks.com/help/matlab/ref/symrcm.html?requestedDomain=www.mathworks.com, https://octave.sourceforge.io/octave/function/symrcm.html.

\(^2\) http://www.boost.org/doc/libs/1_38_0/libs/graph/doc/cuthill_mckee_ordering.html.
results than the VNS-band heuristic. Based on the results reported [24], the DRSA heuristic is the state-of-the-art metaheuristic-based algorithm for bandwidth reduction.

Researchers have successfully applied the biased random-key genetic algorithms (BRKGA) to several optimization problems such as routing in OSPF networks [31], cell formation in manufacturing [32], and telecommunications [33]. This paper implements a BRKGA [34] for bandwidth reduction. To the best of our knowledge, this is the first heuristic for bandwidth reduction based on BRKGA. This paper compares the results yielded by the novel algorithm with the results obtained by the DRSA heuristic [24], a GRASP heuristic, the Reverse Cuthill-McKee method [3], and a variant of the breadth-first search procedure.

The remainder of this paper is organized as follows. In Sect. 2, a BRKGA for bandwidth reduction is introduced. In Sect. 3, we describe how the experiments were conducted. In Sect. 4, we discuss the results. Finally, in Sect. 5, the main conclusions are presented.

2 A BRKGA for Bandwidth Reduction

A BRKGA is similar to a genetic algorithm, but the former employs different strategies aiming at surpassing some disadvantages related to the latter. Bean [35] introduced the random-key genetic algorithms (RKGA) for combinatorial optimization problems. In RKGA, a permutation vector represents a solution to the problem in context. Vectors of random real numbers, called keys, represent the solutions. A deterministic algorithm, called a decoder, associates a feasible solution of the combinatorial optimization problem with a solution vector. Thus, the algorithm can compute an objective value or fitness function to the solution of the combinatorial optimization problem.

RKGA randomly selects two parents from the entire population to implement the crossover operation. Parents are allowed to be selected for mating more than once in a given generation. The principal objective of an RKGA is to mitigate the difficulty of genetic algorithm operators in dealing with feasible solutions.

A BRKGA selects parents for crossover differently from RKGA (see [34] for a review). BRKGA generates each element combining one individual selected at random from the elite solutions in the current population, whereas the other is a non-elite solution. The selection is said biased because one parent is always an elite individual. Therefore, elite solutions have a high probability of passing their genes to individuals of the new generation.

We followed the mains steps of the metaheuristic approach to design the BRKGA for bandwidth reduction. The BRKGA for bandwidth reduction evolves a population of chromosomes that consists of vectors of real numbers (called keys). Thus, a vector represents each solution. Each component in this vector is a real number in the range [0, 1]. Each solution represented by a chromosome is decoded by a decoding heuristic that receives the vector of keys and builds a solution.

We used a BRKGA framework provided by Toso and Resende [36]. In this framework, the user implements only problem-dependent procedures, such as
the decoding method, which is responsible for decoding a vector of random keys to a solution of the problem in context. The decoding method is also responsible for evaluating the solution and treating penalties if the algorithm generates an infeasible solution. Thus, the decoding method interprets a vector of random keys of the BRKGA.

BRKGA obtains each new individual by combining a randomly selected individual from the elite group \((p_e)\) of the current population \((p)\), and another one from the non-elite set \((p \backslash p_e)\) of individuals. Thus, a single individual can be selected multiple times, and therefore can produce more than one offspring. Since BRKGA requires that \(|p_e| < |p \backslash p_e|\), the probability of BRKGA to select for reproduction \((1/|p_e|)\) an individual from the elite group is greater than the probability of the algorithm to select a non-elite individual \((1/|p \backslash p_e|)\).

BRKGA does not use the standard mutation operator. This operator changes parts of chromosomes with a small probability. Instead, BRKGA employs the concept of mutants. Specifically, BRKGA introduces a fixed number of mutant solutions in the population in each generation. The algorithm randomly generates mutants solutions, equivalently as performed to construct the initial population. Mutants are equivalent to the mutation operator in traditional genetic algorithms, diversifying the search and helping the procedure to escape from local optimal solutions.

A constructive heuristic generates the initial population in the BRKGA for bandwidth reduction. The method is a random breadth-first search procedure. The procedure randomly labels vertices belonging to an adjacency list.

We used the parametrized uniform crossover scheme proposed by Spears and De Jong [37] to combine two parent solutions and produce an offspring. The offspring inherits each key from the best fit of the two parents with probability \(\phi_a > 0.5\) and from the other parent with probability \(1 - \phi_a\).

To accelerate convergence, each time the approach calls the decoder, if the solution is feasible, the BRKGA applies a built-in local search procedure. Specifically, the BRKGA applies a decoding method that takes a vector of random keys as input and returns the difference in bandwidth reduction between the new solution and the original bandwidth of the matrix. After executing the decoding method, the BRKGA performs a local search procedure proposed by Limpg et al. [16]. After applying the local search, the BRKGA converts the newly found solution to a chromosome representation and returns its cost.

3 Description of the Tests

We used the C++ programming language to implement the BRKGA for bandwidth reduction. We employed the same programming language to code the Reverse Cuthill-McKee method, GRASP approach, and a variant of the breadth-first search procedure (i.e., the constructive approach used to generate the initial population in the BRKGA). The workstation used in the execution of the simulations featured an Intel® Core™ i7 (CPU 4.2 GHz, 32 GB of main memory) (Intel; Santa Clara, CA, United States).
The BRKGA for bandwidth reduction uses a specific seed for random number generation at each run. After an exploratory investigation, we set the parameters of the serial BRKGA as follows:

- size of chromosomes: \( n \);
- population size: 1,000;
- elite set fraction: 0.5;
- fraction of population to be replaced by mutants: 0.5;
- probability that offspring inherits an allele from elite parent: 0.5;
- number of independent populations: 1;
- current generation: 0 (so that the BRKGA restarts from the beginning of the approach);
- the best two individuals are combined every 100 generations;
- stopping criterion: 1,000 generations.

The results yielded by the DRSA method were taken from the original publication [24]. The DRSA heuristic was executed in a processor AMD Opteron™ 6274 running at 2.2 GHz [24]. The authors of the DRSA algorithm [24] presented the best bandwidth achieved by their method and the running times in seconds required to yield the best bandwidth. However, we were not able to find the timeout set for the heuristic.

4 Results

In this section, we evaluate the results obtained by the BRKGA for bandwidth reductions against the DRSA algorithm, GRASP approach, the Reverse Cuthill-McKee method, and a variant of the breadth-first search (Rand-BFS) procedure when applied to 43 matrices taken from the SuiteSparse matrix collection [38]. Table 1 shows the name, size, and original bandwidth (\( \beta_0 \)) of the matrices used in this computational experiment. Additionally, the table shows the bandwidth results and CPU times obtained by the DRSA (taken from the original publication [24]), BRKGA (using the evolutionary algorithm in one execution carried out in each matrix), and GRASP approaches. The Reverse Cuthill-McKee method (RCM) took less than 0.01 seconds to compute each of the matrices used in this study. The variant of the breadth-first search procedure is even faster than the Reverse Cuthill-McKee method.

The last line of Table 1 shows \( \sum \beta \), i.e., the sum of bandwidth results yielded by the algorithms. The same line shows the average time of the heuristics evaluated.

As expected, the other algorithms analyzed in this study dominated the Rand-BFS procedure in terms of bandwidth reduction. The metaheuristic algorithms returned better bandwidth results than did the Rand-BFS procedure in 42 matrices. The Rand-BFS procedure yielded the same bandwidth result as the BRKGA and GRASP algorithms when applied to the matrix gr_30_30. In particular, the Rand-BFS procedure delivered a better bandwidth result than the RCM method in this matrix. On the other hand, the RCM method yielded
Table 1. Bandwidth results yielded by several methods when applied to 43 matrices.

| Matrix    | n   | $\beta_0$ | DRSA($\beta$) t(s) | BRKGA($\beta$) t(s) | GRASP($\beta$) t(s) | RCM | Rnd-BFS |
|-----------|-----|-----------|-------------------|-------------------|-------------------|-----|---------|
| dwt_209   | 209 | 184       | 2                 | 33                | 16                | 33  | 1       |
| gre_216a  | 216 | 36        | 2                 | 33                | 16                | 33  | 1       |
| dwt_221   | 221 | 187       | 3                 | 15                | 14                | 15  | 5       |
| impcol_e  | 225 | 92        | 9                 | 63                | 36                | 67  | 2       |
| dwt_245   | 245 | 115       | 2                 | 33                | 13                | 34  | 1       |
| bcspw04   | 274 | 265       | 3                 | 37                | 15                | 37  | 1       |
| ash292    | 292 | 24        | 4                 | 24                | 18                | 24  | 1       |
| can_292   | 292 | 282       | 6                 | 57                | 27                | 60  | 1       |
| dwt_310   | 310 | 28        | 11                | 13                | 20                | 13  | 1       |
| gre_343   | 343 | 49        | 4                 | 28                | 22                | 28  | 1       |
| dwt_361   | 361 | 50        | 23                | 15                | 24                | 15  | 1       |
| dwt_419   | 419 | 356       | 45                | 32                | 31                | 32  | 1       |
| bcsttk06  | 420 | 47        | 15                | 48                | 116               | 49  | 3       |
| bcsttk07  | 420 | 47        | 14                | 49                | 102               | 49  | 3       |
| impcol_d  | 425 | 406       | 39                | 844               | 56                | 55  | 1       |
| hor_131   | 434 | 421       | 4                 | 7                 | 49                | 17   | 1       |
| bcspr05   | 433 | 435       | 7                 | 49                | 17                | 49  | 1       |
| can_445   | 445 | 403       | 11                | 74                | 32                | 74  | 1       |
| 494_bus   | 494 | 428       | 18                | 55                | 17                | 55  | 1       |
| dwt_503   | 503 | 452       | 50                | 51                | 66                | 52  | 2       |
| gre_512   | 512 | 64        | 9                 | 56                | 25                | 36  | 1       |
| fs_541_l  | 541 | 540       | 1                 | 465               | 181               | 433  | 117 |
| dwt_592   | 592 | 259       | 53                | 40                | 45                | 42  | 1       |
| 662_bus   | 662 | 335       | 14                | 77                | 26                | 78  | 1       |
| fs_680_l  | 680 | 600       | 27                | 17                | 50                | 20  | 1       |
| 685_bus   | 685 | 550       | 32                | 145               | 57                | 31   | 1       |
| can_715   | 715 | 611       | 182               | 116               | 70                | 116  | 4    |
| fs_760_l  | 760 | 740       | 39                | 39                | 146               | 41   | 4    |
| bcsttk19  | 817 | 567       | 9                 | 7                 | 9                 | 59   | 2    |
| bp_0      | 822 | 820       | 1219              | 386               | 92                | 417  | 2    |
| bp_1000   | 822 | 820       | 4988              | 457               | 140               | 496  | 5    |
| bp_1200   | 822 | 820       | 1165              | 467               | 142               | 488  | 5    |
| bp_1400   | 822 | 820       | 2140              | 475               | 141               | 484  | 5    |
| bp_1600   | 822 | 820       | 2261              | 469               | 144               | 488  | 5    |
| bp_200    | 822 | 820       | 1639              | 471               | 108               | 459  | 3    |
| bp_400    | 822 | 820       | 2341              | 480               | 115               | 479  | 5    |
| bp_600    | 822 | 820       | 1315              | 488               | 121               | 485  | 4    |
| bp_800    | 822 | 820       | 1065              | 454               | 131               | 492  | 5    |
| can_838   | 838 | 837       | 70                | 111               | 118               | 119  | 4    |
| dwt_878   | 878 | 519       | 85                | 33                | 64                | 35   | 3    |
| gr_30_30  | 900 | 31        | 57                | 48                | 65                | 48   | 4    |
| dwt_918   | 918 | 839       | 62                | 46                | 65                | 46   | 2    |
| dwt_992   | 992 | 513       | 107               | 52                | 194               | 52   | 5    |
| $\sum \beta$ and average time | 3828 | 470 | 6143 | 71 | 6258 | 5 | 7036 | 8386 |
better bandwidth results than did the Rand-BFS procedure in 37 matrices. The latter returned better bandwidth results than did the former in five matrices.

The metaheuristic algorithms dominated the RCM method. The DRSA algorithm yielded better bandwidth results than did the RCM method in 41 matrices. Moreover, the latter delivered the same bandwidth results as the former in six test problems. The RCM method delivered a better bandwidth result than did the BRKGA when applied to the matrix gre.512. The GRASP algorithm yielded better bandwidth results than did the RCM method in 35 matrices. The latter provided the same bandwidth results as the former when applied to seven matrices. The RCM method delivered better bandwidth results than did the GRASP algorithm when applied to the matrix bp.1000.

In general, the BRKGA returned better bandwidth results than did the GRASP algorithm at longer running times. The BRKGA yielded better bandwidth results than did the GRASP algorithm in 17 matrices. Furthermore, the latter provided better bandwidth results than did the former in seven test problems. Both algorithms returned the same bandwidth in 19 matrices.

The DRSA algorithm yielded better bandwidth results than did the four other heuristics evaluated in 40 test problems. The DRSA and BRKGA approaches provided the same results in three cases (matrices gre.216a, gre.343, and fs.680.1). We could not compare the running times of the algorithms because the studies used different machines.

We performed statistical tests for pairwise comparison. Specifically, we used the Wilcoxon matched-pairs signed-rank test to analyze the bandwidth reduction yielded by the DRSA and BRKGA approaches. The null hypothesis for the test was that the algorithms yielded identical results, i.e., the median difference was zero. The alternative hypothesis was that the median difference is positive. Thus, we used a two-tailed test because we wanted to determine if there was any difference between the groups we were comparing (i.e., the possibility of positive or negative differences). In this case, $R_+ = 820$ and $R_- = 0$. The test statistic is, therefore, $T = 0$. Thus, $T \leq T_{crit(\alpha=0.01,40)} = 247$. We have statistically significant evidence at $\alpha = 0.01$ (i.e., the level of significance 0.01 is related to the 99% confidence level), to show that the median difference is positive, i.e., that the DRSA algorithm dominated the BRKGA. In this case, $p$-value equals $3.56056e^{-8}$. The use of a corrected significance level of $1 - (1 - \alpha)^{1/2} = 0.005$ (Dunn-Šidák correction) or $\alpha/2 \approx 0.005$ (Bonferroni correction) confirms the previous statement.

5 Conclusions

This paper describes our experience in implementing a BRKGA for bandwidth reduction. The algorithm did not compare favorably with the state-of-the-art metaheuristic-based heuristic for bandwidth reduction. Although the novel approach yielded better bandwidth results than a GRASP approach and classical
methods for bandwidth reduction, namely the Reverse Cuthill-McKee and a variant of the breadth-first search procedure, the DRSA algorithm dominated the BRKGA approach. The design of a parallel BRKGA for bandwidth reduction is a future step in this investigation.

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