A novel model for calculating critical droplet entrainment rate of gas well considering droplet deformation and multiple parameters

Heng Xiao\(^1\)\(^2\) | Xiao-Ping Li\(^1\)\(^2\) | Xiao-Hua Tan\(^1\)\(^2\) | Jinhan Li\(^1\)\(^2\) | Chong Han\(^3\) | Hang Xiao\(^3\) | Zongzhi Han\(^4\) | Lina Cao\(^5\)

\(^1\)Petroleum Engineering School, Southwest Petroleum University, Chengdu, Sichuan, China
\(^2\)Science and Technology Cooperation Project of the CNPC-SWPU Innovation Alliance, Chengdu, Sichuan, China
\(^3\)Sinopac East Sichuan Natural Gas Pipeline Co., Ltd, Yichang, Hubei, China
\(^4\)Sichuan ChuanGang Gas Co., Ltd, Chengdu, Sichuan, China
\(^5\)China ZhenHua Oil Co., Ltd, Beijing, China

Abstract
When a spherical droplet moves parallel to the direction of motion of its surrounding gas medium, it will deform into an ellipsoidal droplet. In this paper, an exposition of a novel model for predicting the minimum flow rate for the continuous removal of liquids from gas wells based on two theories is presented. First, droplet breakup principle is utilized to microscopically obtain the maximum size parameters of ellipsoidal droplets, and second, droplet force equilibrium is adopted to macroscopically calculate the critical droplet entrainment rate. Moreover, effect of variations in the windward area (attributed to droplet deformation) on the drag coefficient is determined. Impacts of variations in pressure, temperature, and pipe diameter on the gas-liquid two-phase friction resistance and gas-liquid surface tension are also considered. For model verification and comparison with other droplet entrainment models, field data from 3436 gas wells are utilized. The field data well validates the results with a 92% accuracy, indicating that the new model has a better comprehensive performance in determining whether or not a gas well is loaded. Besides, parameter sensitivity analyses including the effects of drag coefficient, surface tension, and friction coefficient on the minimum flow rate of gas are performed. Finally, the relationship curve of critical droplet entrainment rates corresponding to different pressures and pipe diameters is established.

Key Words
drag coefficient, droplet deformation, droplet entrainment rate, friction coefficient, liquid loading, surface tension

1 | INTRODUCTION

In most gas fields, depletion development has been widely employed, which caused formation pressure to decline, and as a result, the gas well production also decreases because the pressure is insufficient to continuously remove liquids from the gas well. The liquids, accumulating at the bottomhole and imposing an additional back pressure on the formation, further reduce the effective energy to remove the liquids and restricting well productivity; in severe cases, this may even...
kill the well. This phenomenon is called liquid loading. It is evident that the investigation of the mechanism on the continuous removal of liquids from the gas well and the reduction in liquid loading are crucial to gas well production.

As for the continuous entrainment of droplets from the gas well, Turner\(^1\) proposed a continuous film model and an entrained droplet model (Table 1). The film model describes the upward velocity profile of the liquid film along the pipe wall instead of providing a direct exposition of the critical gas flow rate. Accordingly, the minimum gas phase flow rate necessary to initiate the upward movement of the liquid membrane can be determined indirectly. The continuous film model, however, is more complex and impractical; consequently, the entrained droplet model has been widely utilized in the gas fields. The principle of the entrained droplet model theory is that the gas flow rate should be sufficient to carry the largest droplet out of the wellbore, thereby maintaining gas well unloading. Furthermore, the maximum droplet size is determined by the Weber number, and the critical droplet entrainment rate is obtained through the droplet force equilibrium. The entrained droplet model, however, has certain defects. For example, it may indicate that a gas well remains unloaded although the gas flow rate is considerably less than that computed by Turner’s entrained droplet model. This phenomenon is attributed to two reasons: a solid spherical model instead of an oscillating droplet model is adopted, and the Weber number is tested in air rather than in the gas well. In this case, Turner’s model increases the critical droplet entrainment rate by 20% to satisfy all well conditions.

Coleman\(^2\) suggested that most wellhead flowing pressures adopted from those obtained by the Turner model exceed 3.45 MPa (Table 1). Based on Coleman’s validation, it is not necessary to increase the critical droplet entrainment rate by 20% when the wellhead flowing pressures are less than 3.45 MPa.

It is known that droplets deform and break up.\(^3\) Previous studies show that droplet breakups can be classified into the following types: vibrational, bag, bag-and-stamen, chaotic, stripping, and catastrophic.\(^4\) It is also observed that before breaking up, a spherical droplet deforms into an ellipsoidal droplet.

Considering a pressure differential between the two ends of a liquid droplet along the high-speed gas flow direction, Li et al\(^7\) improved Turner’s model by modifying the previous assumption that the spherical liquid droplet transforms into an ellipsoid (Table 1). Such a shape transformation leads to variations in the windward area and drag coefficient of the droplet. The critical gas flow rate obtained by Li’s model is 38% of the rate yielded by Turner’s model. Although the models of Coleman and Turner take into account the maximum droplet diameter, both neglect droplet deformation, which is considered in the model of Li. The three entrained droplet models, however, neglect the effect of liquid-gas ratio on the minimum flow rate.

Tan\(^8\) derived a novel model of continuous liquid removal from the gas well (Table 1). The model is based on the theory that the total surface energy of the liquid phase is equal to the turbulent kinetic energy of the gas phase. Moreover, the maximum droplet diameter (obtained by means of the Weber number) is replaced with the average droplet diameter (calculated by considering the impact of liquid holdup on the maximum droplet diameter). Different critical droplet entrainment rates for various liquid-gas ratios can be obtained using the above approach. Tan,\(^8\) however, ignored the droplet deformation resulting from the pressure differential caused by the high-speed gas turbulence.

In the four typical entrained droplet models above, the gas-liquid surface tension and drag coefficient are all constant. The gas-liquid surface tension, however, varies with temperature and pressure,\(^9\) and the drag coefficient is a function of droplet deformation and Reynolds number.\(^17\)

Wang et al\(^18\) incorporated the influence of droplet deformation caused by the pressure differential on the drag coefficient but did not consider surface tension variation.

Few researchers consider the effects of pipe diameter on the minimum gas rate of the entrained droplet model. For example, Sarkhi and Hanratty\(^19\) analyzed the liquid-gas annular flow and obtained the droplet size distribution of this flow pattern. They found that the droplet size increased with the pipe diameter. In considering various tube diameters, Guanyi Wang\(^20\) established a novel entrainment model wherein the Reynolds number is calculated using a new approach. The Laplace length is utilized as the length scale, and the entrainment rate and superficial liquid flow rate are nondimensionalized as rate factors.

The aim of this research was to effectively describe and consistently integrate the effect of multiple parameters (including drag coefficient, friction coefficient, and gas-liquid interface tension), which are attributed to variations in pressure, temperature, and pipe diameter, on critical droplet entrainment rate and to formulate a model that is applicable to the simulation of the continuous removal of droplets from the gas well. The formulated model is based on two theories: droplet breakup principle and droplet force equilibrium. The former is utilized to microscopically obtain the maximum size parameters of ellipsoidal droplets, and the latter is adopted to macroscopically calculate the critical droplet entrainment rate. Moreover, multiple parameters, such as droplet deformation, surface tension, drag coefficient, and gas-liquid two-phase friction coefficient, which are attributed to variations in pressure, temperature, and pipe diameter, are considered.

## 2 | NEW ENTRAINED DROPLET MODEL

### 2.1 | Droplet deformation and fracture principle

Based on the droplet deformation, Hinze et al.\(^3\) Z. Liu,\(^5\) and T. Kekesi\(^6\) indicated that the gas stream velocity and pressure...
| Droplet entrainment model | Droplet shape | Droplet size type | Droplet size parameter | Critical droplet entrainment velocity |
|--------------------------|--------------|------------------|------------------------|--------------------------------------|
| Turner model             | Sphere       | Sphere diameter  | $d = \frac{3\pi}{\sqrt[3]{8}}$ | $v_c = 6.6 \left[ \frac{\sigma}{\rho g} \right]^{1/2}$ |
| Coleman model            | Sphere       |                  |                        | $v_c = 4.45 \left[ \frac{\sigma}{\rho g} \right]^{1/2}$ |
| Li Min model             | Ellipsoid    | Ellipsoid z-axis diameter | $h = \frac{2\pi}{\sqrt{3}}$ | $v_c = 2.5 \left[ \frac{\sigma}{\rho g} \right]^{1/2}$ |
|                          |              | Real projection area of ellipsoid | $S = \frac{\pi^2}{2}$ | |
| Tan Xiaohua Model        | Sphere       | Sphere diameter  | $d = \frac{4\pi}{f_d \rho g}$ | $v_c = 3 \left[ \frac{\pi^2}{12f_d^2 \rho g} \right]^{1/2}$ |
|                          |              |                  |                        | $\frac{1}{\sqrt{f_d}} = 1.14 - 2\beta \left( \frac{\rho g}{\beta} + \frac{21}{10^{0.675}} \right)$ |
| This model               | Ellipsoid    | Ellipsoid z-axis radius | $h = \frac{a}{\sqrt[3]{8}}$ | $v_g = \left[ \frac{8\rho g}{f_d (\rho - \rho g)} \right]^{0.25}$ |
|                          |              | Real projection area of ellipsoid | $S = \frac{1}{2} \left[ \frac{\pi}{2} \right]^2 \rho g^2 f_d$ | |
|                          |              |                  |                        | $f_d = 0.0072 + 0.612\text{Re}^{-0.35}$ |
|                          |              |                  |                        | $\sigma_{\text{ge}} = \left[ \frac{9.126}{\text{Re}^{0.25}} + 9.716 \text{Re}^{-0.35} \right]^{1/4}$ |
|                          |              |                  |                        | $C_d = 4.97 \left( \frac{21}{10^{0.675}} + 3.409\text{Re}^{-0.3083} + \frac{3.68 \times 10^{-4} \text{Re}^{-1}}{1 + 4.5 \times 10^{-10} \text{Re}^{2}} \right)$ |
distribution on droplets are unbalanced, whereas a stable gas stream flows through the droplets. The gas flow velocity near the droplet’s equator is high, whereas those at the extremities of the windward and leeward faces of the droplet are low. According to Bernoulli’s law, the pressure imposed on the droplet equator by the airflow is larger than that on the extreme areas of the top and bottom faces of the droplet. The spherical droplet deforms and transforms into an ellipsoidal droplet before breaking up because of the pressure differential. Meanwhile, surface tension prevents the droplet from further deformation and sustains droplet’s integrity (Figure 1). The droplet deformation would reach a plateau when the sum of deformation effect of surface tension and Bernoulli’s pressure differential expended in deformation equals 0. If the pressure differential continues to increase, the droplet deformation will intensify, and the droplet will finally break up. Key equations that depict the key forces governing droplet deformation have been listed in Section 2.2 as Equations (7) and (8).

2.2 | Model construction

2.2.1 | Model assumptions

1. Microscopically, the droplets, which do not break up in high-speed airflow, are exposed to two forces. Turbulent force that tends to break them up; but surface tension sustains their integrity.

2. The turbulent force in the gas phase results in a pressure differential between the windward and leeward faces of the droplet along its direction of motion, thus converting the spherical droplet into an ellipsoid. As a result of the competing effects of these microcosmic forces, a certain steady shape related to the droplet velocity \( v_p \), cylinder, is acquired to predigest the shape of deformed ellipsoid droplet in calculation. As depicted in Figure 2, the volumes of the sphere droplet, the ellipsoid droplet, and the cylinder are equivalent. The ellipsoid droplet windward area, equal to that of the cylinder, is denoted by \( S \). The diameter of the cylinder in the z-axis direction is denoted by \( h \).

3. Macroscopically, equilibrium is attained by droplets that sustain self-weight, buoyancy, and drag force, whereas the maximum droplet velocity is reached through free settling. In order for the gas stream to carry the droplets out of the gas well, the minimum gas stream flow velocity

---

**FIGURE 1** Sketch of the main forces exerted on the droplet microscopically

**FIGURE 2** Droplet deformation and force analysis
By substituting Equation (5) into Equation (4), the total turbulent kinetic energy sustained by the ellipsoidal droplet is as follows:

$$E_T = \frac{3}{4} \rho_g v_{fg}^2 f_{sg} \cdot S \delta h$$

(6)

(3) Conditions for maintaining droplet integrity.

Microscopically, to maintain the integrity and state of the ellipsoid droplet under the influence of surface tension and gas phase turbulence, key equations describing the key forces governing droplet deformation are as follows:

$$\frac{3}{4} \rho_g v_{fg}^2 f_{sg} \cdot S \delta h + \sigma \cdot \delta S = 0$$

(7)

Equation (7) consists of two parts: $\frac{3}{4} \rho_g v_{fg}^2 f_{sg} \cdot S \delta h$ represents the turbulent kinetic energy expended in the deformation $S \delta h$, and $\sigma \cdot \delta S$ indicates the deformation effect of the surface tension forces. When the droplet reaches stable, the total variation of energy will be 0.

Divide both sides of Equation (7) by $\delta h$:

$$\frac{3}{4} \rho_g v_{fg}^2 f_{sg} \cdot S + \sigma \cdot \frac{\delta S}{\delta h} = 0$$

(8)

When the spherical droplet deforms, $S = V/h$ because the volume of the droplet remains unchanged. Taking the derivative of $h$ on both sides of the equation yields the following:

$$\frac{\delta S}{\delta h} = -\frac{V}{h^2} = -\frac{S}{h}$$

(9)

Substitute Equation (9) it into Equation (8).

$$h = \frac{\sigma}{\frac{3}{4} \rho_g v_{fg}^2 f_{sg}}$$

(10)

$$S = \frac{V}{h} = \frac{V}{\sigma} \left( \frac{3}{4} \rho_g v_{fg}^2 f_{sg} \right)$$

(11)

2.2.3 | Determination of critical carryover velocity

When the droplets of the dispersed phase are in a state of force equilibrium in a continuous updraft airflow and remain static relative to wellbore, the velocity of droplets relative to the airflow can be considered as $v_i$, whose direction is opposite that of airflow and equals the updraft velocity, $v_g$. While the updraft velocity is greater than droplet velocity relative to airflow, the droplets are removed from the gas well. This means that the minimum updraft velocity ($v_g$) that can continuously remove the droplets is equal to $v_i$. 

(1) Determination of surface free energy.

According to Adamson,\textsuperscript{21} the surface free energy per unit area is equal to the surface tension between gas and liquid phases:

$$e_s = \sigma$$

(1)

where $e_s$ is the surface free energy per unit area (J/m$^2$); $\sigma$ is the gas-liquid surface tension (N/m).

As the variation in real projection area when the droplet changes from sphere to ellipsoid is $\delta S$, the corresponding deformation effect of surface tension forces is as follows:

$$E_s = \sigma \cdot \delta S$$

(2)

where $E_s$ is the variation in the surface free energy (J).

(2) Determination of turbulent kinetic energy.

Based on the work of White\textsuperscript{22} and Zhang,\textsuperscript{23} the gas turbulent kinetic energy per unit volume can be expressed as:

$$e_r = \frac{3}{2}\rho_g \cdot \frac{v_r^2}{2}$$

(3)

where $e_r$ is the gas turbulent kinetic energy of gas stream per unit volume (J/m$^3$); $\rho_g$ is the gas density (kg/m$^3$); $v_r^2$ is the radial velocity (m/s).

$(S + S\delta h)$ approximately equals $S\delta h$ for $\delta S\delta h$ is little enough to be negligible, when the droplet changes from sphere to ellipsoid, as a result of the interaction of forces, the turbulent kinetic energy expended in the deformation $S\delta h$ will be:

$$E_T = \frac{3}{2}\rho_g \cdot \frac{v_r^2}{2} \cdot S \delta h$$

(4)

where $E_T$ is the turbulent kinetic energy (J).

The square root of the radial velocity is approximately equal to the frictional velocity:\textsuperscript{24,25}

$$\sqrt[4]{\frac{v_r^2}{2}} = v_{sg} \left( \frac{f_{sg}}{2} \right)^{1/2}$$

(5)

where $f_{sg}$ is the friction coefficient with the apparent gas phase velocity.

By substituting Equation (5) into Equation (4), the total turbulent kinetic energy sustained by the ellipsoidal droplet is as follows:

2.2.2 | Maximum droplet size parameters

(1) Determination of surface free energy.

(2) Determination of turbulent kinetic energy.

2.2.3 | Determination of critical carryover velocity

When the droplets of the dispersed phase are in a state of force equilibrium in a continuous updraft airflow and remain static relative to wellbore, the velocity of droplets relative to the airflow can be considered as $v_i$, whose direction is opposite that of airflow and equals the updraft velocity, $v_g$. While the updraft velocity is greater than droplet velocity relative to airflow, the droplets are removed from the gas well. This means that the minimum updraft velocity ($v_g$) that can continuously remove the droplets is equal to $v_i$. 

(1) Determination of surface free energy.

According to Adamson,\textsuperscript{21} the surface free energy per unit area is equal to the surface tension between gas and liquid phases:

$$e_s = \sigma$$

(1)

where $e_s$ is the surface free energy per unit area (J/m$^2$); $\sigma$ is the gas-liquid surface tension (N/m).

As the variation in real projection area when the droplet changes from sphere to ellipsoid is $\delta S$, the corresponding deformation effect of surface tension forces is as follows:

$$E_s = \sigma \cdot \delta S$$

(2)

where $E_s$ is the variation in the surface free energy (J).

(2) Determination of turbulent kinetic energy.

Based on the work of White\textsuperscript{22} and Zhang,\textsuperscript{23} the gas turbulent kinetic energy per unit volume can be expressed as:

$$e_r = \frac{3}{2}\rho_g \cdot \frac{v_r^2}{2}$$

(3)

where $e_r$ is the gas turbulent kinetic energy of gas stream per unit volume (J/m$^3$); $\rho_g$ is the gas density (kg/m$^3$); $v_r^2$ is the radial velocity (m/s).

$(S + S\delta h)$ approximately equals $S\delta h$ for $\delta S\delta h$ is little enough to be negligible, when the droplet changes from sphere to ellipsoid, as a result of the interaction of forces, the turbulent kinetic energy expended in the deformation $S\delta h$ will be:

$$E_T = \frac{3}{2}\rho_g \cdot \frac{v_r^2}{2} \cdot S \delta h$$

(4)

where $E_T$ is the turbulent kinetic energy (J).

The square root of the radial velocity is approximately equal to the frictional velocity:\textsuperscript{24,25}

$$\sqrt[4]{\frac{v_r^2}{2}} = v_{sg} \left( \frac{f_{sg}}{2} \right)^{1/2}$$

(5)

where $f_{sg}$ is the friction coefficient with the apparent gas phase velocity.
As is depicted in Figure 2, the droplet will be in a state of equilibrium when the droplet's weight is equal to the sum of buoyant force and drag force:\(^{26}\)

\[
(\rho_l - \rho_g) g V = \rho_g \frac{V^2}{2} SC_d \quad (12)
\]

Substitute Equations (10) and (11) into (12).

\[
v_g^4 = \frac{8 \sigma g (\rho_l - \rho_g)}{3 \rho_g^2 S g C_d} \quad (13)
\]

The critical droplet entrainment rate of the gas well is:

\[
q_c = 2.5 \times 10^8 \frac{Apv_g}{ZT} \quad (14)
\]

where \(q_c\) is critical droplet entrainment rate (m\(^3\)/d); \(A\) is the pipeline cross-sectional area (m\(^2\)); \(p\) is pressure (MPa); \(Z\) is the gas deviation factor; \(T\) is temperature (K).

### 2.2.4 Determination of drag coefficient

The force exerted by the fluid on the sphere acts in a direction opposite that of the sphere's movement. When relative motion between the sphere and fluid occurs, the fluid force is called drag.\(^{17,27}\) The drag coefficient, also called fluid resistance coefficient, is equal to the ratio of the drag force exerted on the particle by the fluid to the product of the projection area of the particle in its direction of motion and the fluid dynamic pressure. The drag coefficient is a function of Reynolds number.

In the calculations using the models of Turner, Coleman, and Xiao-hua Tan, the drag coefficient has a fixed value of 0.44. Min Li,\(^7\) however, suggested that the drag coefficient should be set to 1 for the windward area, which varies as a result of the droplet deformation in high-speed airflow. Investigations indicate that the drag coefficient is considerably related to droplet shape and Reynolds number.\(^{28-30}\)

The calculation formula for the drag coefficient is usually empirical and derived by fitting a standard resistance curve. This curve demonstrates the relationship between drag coefficient \((C_d)\) and Reynolds number \((Re)\) obtained by testing a single rigid sphere moving at a constant velocity in a stationary, isothermal, incompressible, and infinite flow field. Two methods are invariably utilized to fit the standard resistance curve for the calculation formula of the drag coefficient: global fitting and piecewise fitting.\(^{31}\)

Global fitting refers to the fitting of all points in the standard resistance curve using a single formula that is simple and easy to substitute for subsequent calculations. The current global fitting formulae of the drag coefficient are mainly the equations of Brauer,\(^{32}\) Clift,\(^{33}\) Martin,\(^{34}\) Shao Mingwang,\(^{35}\) Cheng,\(^{36}\) and Mikhailov and Silva Freire.\(^{37}\)

The drag coefficients are calculated by adopting different drag coefficient models and comparing these with the experimental values.\(^{31}\) The results are as follows.

Figure 3 shows that the correlation results of Shao Mingwang, Martin, Cheng, and Mikhailov and Silva Freire approximate the experimental values in the global range where the Reynolds number is between 0.1 and 100,000. In performing a regression analysis of these fitting equations and comparing the average absolute deviation, residual sum of squares, correlation distance, cosine distance, and standardized Euclidean distance of results, it is found that Cheng's nonlinear fitting relationship has the highest accuracy. Meanwhile, Reynolds number is usually between 2000 and 1,000,000 during the regular gas well production; hence, the accuracy of these correlations in this range is verified. The lists in Tables 2 and 3 compare the accuracies of correlations in the entire domain and when the Reynolds number is between 2000 and 1,000,000, respectively.

Based on the accuracy analysis and the comparison of calculated results of different drag coefficient models with laboratory data when Reynolds number exceeds 2000 (Figure 4), it is found that Shao Mingwang’s formula has the highest accuracy. This formula is thus used in this paper to calculate the drag coefficient.

\[
C_d = \frac{24}{Re} + 3.409 Re^{-0.3083} + \frac{3.68 \times 10^{-5} Re^{1.054}}{1 + 4.5 \times 10^{-7} Re^{1.054}} \quad (15)
\]

The foregoing equations are based on rigid spheres. In the gas well, when the gas-liquid relative velocity is low, the droplets are approximately spherical, and the error is within the allowable range when the rigid sphere
correlation is employed to calculate the drag force exerted by the gas stream on droplets. When the droplet deformation and the deviation of the gas-liquid relative velocity are considerable, however, the adoption of a rigid sphere correlation calculation to determine the drag force will result in a significant error. Considering deformation, the drag coefficient of an ellipsoidal droplet can be obtained by multiplying the drag coefficient of a rigid sphere of the same volume by 4.97.

### TABLE 2  Comparison of calculation accuracies of various widely used drag coefficient models

| Error Analysis Methods | Brauer | Clift | Mingwang Shao | Martin | Cheng | Mikhailov and Silva |
|------------------------|--------|-------|---------------|--------|-------|---------------------|
| Average Absolute Deviation | 44 658.44 | 12 695.71 | 2.56 | 13.46 | 3.60 | 10.11 |
| Residual sum of squares | 4 798 890.61 | 319 317.40 | 113.90 | 701.04 | 24.22 | 1273.60 |
| Correlation distance | 1.11 | 0.70 | 6.40E–5 | 2.1E–4 | 2.88E–5 | 2.75E–5 |
| Cosine distance | 0.86 | 0.54 | 5.52E–5 | 1.9E–4 | 2.50E–5 | 2.58E–5 |
| Standardized Euclidean distance | 8.40 | 2.51 | 2.9E–4 | 1.3E–3 | 3.7E–4 | 1.2E–3 |

### TABLE 3  Comparison of calculation accuracies of different drag coefficient models when Reynolds number range is 2000-100 000

| Error Analysis Methods | Brauer | Clift | Mingwang Shao | Martin | Cheng | Mikhailov and Silva |
|------------------------|--------|-------|---------------|--------|-------|---------------------|
| Average Absolute Deviation | 44 658.44 | 12 695.71 | 2.56 | 13.46 | 3.60 | 10.11 |
| Residual sum of squares | 4 798 890.61 | 319 317.40 | 113.90 | 701.04 | 24.22 | 1273.60 |
| Correlation distance | 1.11 | 0.70 | 6.40E–5 | 2.1E–4 | 2.88E–5 | 2.75E–5 |
| Cosine distance | 0.86 | 0.54 | 5.52E–5 | 1.9E–4 | 2.50E–5 | 2.58E–5 |
| Standardized Euclidean distance | 8.40 | 2.51 | 2.9E–4 | 1.3E–3 | 3.7E–4 | 1.2E–3 |

![Figure 4](image)  
Comparison of calculated results of different drag coefficient models with laboratory data when Reynolds number exceeds 2000

### 2.2.5  Determination of friction coefficient

Equation (13) indicates that the friction coefficient of the gas-liquid two-phase pipe flow is required in the calculation of the critical droplet entrainment rate. For the multi-phase pipe flow, the friction coefficient calculation formulae vary with the flow pattern. When the gas and water phases are in a critical liquid removal state, the flow pattern in the wellbore is an annular mist flow. Yu Xichong suggested that the calculation method for the friction coefficient of monophasic flow could be applied to obtain the friction coefficient between the fluid and pipe wall in a multi-phase pipe flow, including stratified, annular, and slug flows. The friction coefficient of a monophasic fluid can therefore be used to calculate the friction coefficient in the wellbore’s critical liquid removal state.

During regular production, the Reynolds number of fluid flow in the wellbore is in the range 2300-2 × 10^6. The friction coefficient can be calculated using the formulae of Prandtl, Pelagius, Revised Pelagius, Nicholas, Miller, Lees, Drew, IGT, and Towler. The friction coefficients calculated by the foregoing models are compared with laboratory values obtained by Nicholas and other authors. The results are as follows.

Figure 5 shows that the best results are yielded by Lees’ formula because these agree considerably well with experimental values. The formula results are thus validated by laboratory data. Further regression analysis is performed on
the fitted equations, and the average absolute error, residual sum of squares, correlation distance, cosine distance, and standardized Euclidean distance are compared (Table 4). It is found that the nonlinear fitting formula of Lees has the highest accuracy.

Accordingly, this study utilizes Lees’ model\footnote{f_{se} = 0.0072 + 0.612Re^{-0.35}} to calculate the friction coefficient.

\begin{align}
\frac{f}{Re} = 0.25 \frac{\sigma_{bw}}{\rho_w - \rho_h} \left( \frac{T}{T_c} \right)^{0.3125}
\end{align}

\begin{table}
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|}
\hline
Error Analysis Methods & Prandtl & Blasius & Modified & Blasius & Nikuradse & Miller & Lees & Drew & IGT & Towler \\
\hline
Average Absolute Deviation & 0.016 & 0.041 & 0.034 & 0.050 & 0.014 & 0.014 & 0.15 & 0.035 & 0.037 \\
Residual sum of squares & 2.19E−5 & 7.28E−5 & 2.27E−4 & 4.74E−4 & 1.81E−05 & 1.99E−5 & 1.60E−3 & 2.09E−4 & 2.48E−4 \\
Correlation distance & 1.33 E−3 & 2.50E−3 & 4.94E−3 & 3.01E−3 & 1.18E−3 & 1.17E−3 & 1.09E−3 & 4.94E−3 & 5.11E−3 \\
Cosine distance & 1.61 E−4 & 6.62E−4 & 1.52E−3 & 1.33E−3 & 1.65E−4 & 1.48E−4 & 4.16E−4 & 1.52E−3 & 1.71E−3 \\
Standardized Euclidean distance & 3.20 & 10.98 & 6.12 & 8.86 & 3.08 & 2.92 & 25.37 & 6.45 & 6.77 \\
\hline
\end{tabular}
\caption{Comparison of calculation result accuracies of friction coefficients}
\end{table}

2.2.6 | Determination of interfacial tension

In using the models of Turner, Coleman, Min Li, and Xiaohua Tan, the surface tension, \( \sigma \), is fixed at 60 mN/m. Surface tension is related to variations in gas-water density and temperature.\footnote{ where \( \sigma_{bw} \) is the interfacial tension in hydrocarbon-water; \( \rho_h \) is the density of hydrocarbon; \( T_c \) is critical temperature, \( R \). Sutton indicated that the results calculated by this formula are in good agreement with the experimental data of methane-water interfacial tension; however, these do not validate the properties of other gaseous alkanes (other than methane), such as ethane, propane, and butane. In Firoozabadi’s method, the relationship is modified by replacing the critical temperature with 302.881 to calculate the gas-liquid surface tension. Li Yuansheng\footnote{ To select the optimal interfacial tension model, different models (Sutton, Yuansheng Li, Kalantari-Meybodi\textsuperscript{46}) are employed to calculate the interfacial tension between water and gas phases whose density differential is greater than 0.4 g/cm\textsuperscript{3} to match the interfacial tension between gas and water. The following equation is derived:} suggested that based on the experimental data obtained by Firoozabadi to validate the formula, the surface tension values between liquid hydrocarbons and water and between gaseous hydrocarbons and water considerably differ. In view of this, Firoozabadi exclusively utilized data whose density differential is greater than 0.4 g/cm\textsuperscript{3} to match the interfacial tension between gas and water. The following equation is derived:

\begin{align}
\sigma_{gw} = \left[ -6.462(\rho_w - \rho_g) + 9.711(\rho_w - \rho_g)^{0.7192} T^{0.3125}_r \right]^4
\end{align}
This study therefore adopts Yuansheng Li’s model to calculate the surface tension between gas and water. Table 5 summarizes the comparison of accuracies of computed results yielded by different models.

In summary, the gas well critical droplet entrainment model in this work is expressed as follows:

\[
\begin{align*}
\sigma_{gw} &= \left[ -6.462 \left( \rho_w - \rho_g \right) + 9.711 \left( \rho_w - \rho_g \right)^{0.7192} \right]^{1/2} \\
C_d &= 4.97 - 24\Re + 3.409\Re^{-0.3083} + \frac{3.68 \times 10^{-5}\Re^{0.3125}}{1 + 4.5 \times 10^{-5}\Re^{1.054}} \\
v_g &= \left[ \frac{8\sigma_g \left( \rho_l - \rho_g \right)}{3\rho_g^2 f_{sg} C_d} \right]^{0.25} \\
f_{sg} &= 0.0072 + 0.612\Re^{0.35}
\end{align*}
\]

(19)

3 | RESULTS AND DISCUSSION

3.1 | Comparison and verification of models

The model proposed in this paper, solved by a multiple parameter iterative algorithm, is verified using the field data of temperatures, pressures, and pipe diameters from 3638 gas field wells in China. The models of Turner, Coleman, Min Li, and Xiao-hua Tan are also compared with these data. Table 1 is a comparison of major droplet entrainment models.

The water loading of gas wells has been estimated in advance based on the manometry data of the bottomhole and production condition: 805 and 2833 gas wells are found unloaded and loaded, respectively. The comparison of liquid loading prediction accuracies of models is summarized in Table 6. Figures 7-11 illustrate the relationship between gas well critical droplet entrainment rate and actual gas production.

As summarized in Table 6, the accuracy rate of the predictions of Turner’s model reaches 92.73% for loaded gas wells; however, the error rate of inaccurate forecasts is 68.82% for unloaded gas wells. Figure 7 and data of number 22-26 in Table 7, however, show that the calculated critical droplet

---

**TABLE 5** Error analysis of different surface tension models

| Error analysis methods | Sutton  | Yuansheng Li | Kalantari-Meybodi45 |
|------------------------|---------|--------------|---------------------|
| Average absolute deviation | 0.14    | 0.034        | 0.045               |
| Residual sum of squares | 62.41   | 3.96         | 6.08                |
| Correlation distance   | 0.067   | 0.016        | 0.026               |
| Cosine distance        | 5.31E−3 | 6.21E−4      | 1.20E−3             |
| Standardized Euclidean distance | 18.50   | 8.68         | 10.59               |

**TABLE 6** Liquid loading prediction accuracies of different droplet entrainment models

| Droplet entrainment models | Turner | Coleman | Min Li | Xiaohua Tan | Heng Xiao |
|----------------------------|--------|---------|--------|-------------|-----------|
| Number of correct predictions of loaded wells | 2627   | 2510    | 1878   | 2157        | 2632      |
| Number of false predictions of loaded wells   | 206    | 323     | 955    | 676         | 201       |
| Number of correct predictions of unloaded wells | 251    | 542     | 735    | 770         | 711       |
| Number of false predictions of unloaded wells  | 554    | 263     | 70     | 35          | 94        |
| Total number of correct predictions            | 2878   | 3052    | 2613   | 2927        | 3343      |
| Accuracy rate of correct predictions of loaded wells | 92.73  | 88.59   | 66.29  | 76.14       | 92.05     |
| Accuracy rate of correct predictions of unloaded wells | 31.18  | 67.33   | 91.3   | 95.65       | 88.32     |
| Error rate of predictions of loaded gas well   | 72.71  | 11.4    | 33.71  | 23.86       | 70.95     |
| Error rate of predictions of unloaded gas well | 68.82  | 32.67   | 86.96  | 43.48       | 11.68     |
| Total accuracy rate of correct predictions     | 79.11  | 83.89   | 71.83  | 80.46       | 91.89     |
entrainment rate, $Q_e$, of this model is greater than the actual gas well production rate, $Q$; this is inconsistent with the condition that the gas well normally produces gas and water. The foregoing indicates that the droplet entrainment rate predicted by Turner’s model has a larger value than the actual rate.

The prediction accuracy of Coleman’s model for loaded gas wells is also significantly greater than that for unloaded gas wells. Figure 8 and the list in Table 6, 7 indicate that the prediction of 2510 gas is 88.60% accuracy, whereas the correct prediction that 263 wells are unloaded has an accuracy of 32.67%. The critical droplet entrainment rate calculated by this model is also generally larger than the pragmatic value.

As for the models of Li Min and Tan Xiaohua, their prediction for loaded gas wells is less accurate than their forecasts for unloaded gas wells with low error rates of 8.87% for 70 wells and 4.348% for 35 wells, respectively. In the prediction of loaded gas wells, inaccurate prediction dominates with error rates of 33.71% and 23.80% for 955 and 676 wells, respectively. Figures 9 and 10, and Data of No. 7, 8, 18, and 19 in Table 7 show that the critical droplet entrainment rates calculated by these two models are less than the gas well production rates, because loaded gas wells have been unreliably predicted. The critical droplet entrainment rate, however, is less than the actual value.

For the model proposed in this paper, the accuracy rates for loaded and unloaded gas wells reach 92.91% and 88.32%, respectively; the total accuracy rate is 91.891%.

Figure 11 and Table 7 depict the relationship between the critical droplet entrainment rate calculated by the proposed model and gas well production rate. It can be observed that
a distinct demarcation exists between loaded and unloaded wells. Only 201 loaded gas wells are found in the area of unloaded gas wells, whereas 94 unloaded gas wells are found in the region of loaded gas wells. This indicates that the proposed model can be adopted to make an accurate evaluation of whether or not a gas well is loaded.

3.2 | Sensitivity analyses of parameters

In summary, the critical droplet entrainment rate is affected by three main factors: pressure, temperature, and pipe diameter. The changes in these three factors will result in changes in Reynolds number, thereby leading to variations in drag coefficient, friction coefficient, and surface tension, and finally influence the droplet entrainment rates. The effects of different drag coefficients, friction coefficients, and surface tensions on the critical droplet entrainment rate are analyzed. The critical droplet entrainment rate that varies with different combinations of pipe diameters and pressures at different wellhead temperatures are plotted.

3.2.1 | Influence of various drag coefficients on critical droplet entrainment rate

Figure 12 describes the relationship between the critical droplet entrainment rate and pipe diameter at different drag coefficients. With drag coefficients of 0.4, 0.8, 1.2, and 1.6, and other parameters fixed, the relationship curve between critical droplet entrainment rate and pipe diameter is plotted. As shown in Figure 11, with the same pipe diameter, the critical droplet entrainment rate decreases as the drag coefficient gradually increases, and at the same drag coefficient, the critical droplet entrainment rate increases with the pipe diameter. According to the definition of drag force, the higher the drag coefficient under a unit pressure differential, the stronger the force exerted by the gas stream on a droplet per unit windward area. In this case, the critical droplet entrainment rate required to lift the same quantity of droplets is the lower entrainment rate corresponding to the gas well flow rate. Similarly, at the same pressure, the critical droplet entrainment rate decreases with the increase in drag coefficient, as shown in Figure 13. Moreover, with the same drag coefficient, the critical droplet entrainment rate increases with pressure. The effect of pressure on the critical droplet entrainment rate is discussed in Section 3.2.4.

3.2.2 | Influence of different friction coefficients on critical droplet entrainment rate

Figure 13 illustrates the relationship between the critical droplet entrainment rate and pipe diameter at different friction coefficients. With friction coefficients of 0.01, 0.015, 0.02, and 0.025, and other parameters fixed, the relationship curve between critical droplet entrainment rate and pipe diameter is delineated.

Figure 14 shows that with the same pipe diameter, the critical droplet entrainment rate decreases as the friction coefficient gradually increases. The relationship curve between the critical droplet entrainment rate and pressure gradually shifts downward. Based on the definition of friction coefficient, the larger this coefficient, the greater the friction between gas and water phases and the stronger the gas stream force exerted on the droplets per unit windward area under a unit pressure differential. In this case, the required critical droplet entrainment velocity and corresponding gas well flow rate to lift the same quantity of droplets are lower. Similarly, under the same pressure, the critical droplet entrainment rate decreases with the increase in friction coefficient, as shown in Figure 15. The effect of pressure on the critical droplet entrainment rate at the same friction coefficient is discussed in Section 3.2.4.

3.2.3 | Influence of different surface tensions on critical droplet entrainment rate

Figure 16 depicts the relationship between the critical droplet entrainment rate and pipe diameter at different surface tensions. With surface tensions of 0.05, 0.06, 0.07, and 0.08 N/m, respectively, and other parameters fixed, the relationship curve between the critical droplet entrainment rate and pipe diameter can be constructed. Figure 17 shows that with the same pipe diameter, the critical droplet entrainment rate decreases as the surface tension gradually
TABLE 7  Data and prediction of critical rate for gas wells of different droplet entrainment models

| Data number | wellhead pressure (MPa) | Temperature (°C) | Wellbore diameter (m) | Gas production rate (m³/d) | Water production rate (m³/d) | Production | Turner (m³/d) | Coleman (m³/d) | Min Li (m³/d) | Xiaohua Tan (m³/d) | Heng Xiao (m³/d) |
|-------------|-------------------------|------------------|-----------------------|---------------------------|-----------------------------|------------|---------------|----------------|---------------|--------------------|------------------|
| 1           | 3.1                     | 20               | 0.062                 | 5472                      | 6.02                        | loaded     | 32 493        | 27 078         | 12 308        | 18 187             | 27 259           |
| 2           | 3.3                     | 20               | 0.062                 | 3062                      | 1.98                        | loaded     | 33 520        | 27 933         | 12 697        | 18 031             | 28 214           |
| 3           | 3.3                     | 20               | 0.062                 | 5461                      | 6.28                        | loaded     | 33 520        | 27 933         | 12 697        | 19 157             | 28 214           |
| 4           | 3.6                     | 20               | 0.062                 | 8601                      | 2.04                        | loaded     | 35 009        | 29 174         | 13 261        | 16 463             | 29 601           |
| 5           | 4                       | 20               | 0.062                 | 4055                      | 0                           | loaded     | 36 913        | 30 760         | 13 982        | 46 848             | 20 671           |
| 6           | 4.1                     | 20               | 0.05064               | 8036                      | 24.16                       | loaded     | 24 934        | 20 778         | 9445          | 15 670             | 20 956           |
| 7           | 4.2                     | 20               | 0.05064               | 22 624                    | 6.13                        | loaded     | 25 240        | 19 033         | 9500          | 12 562             | 21 203           |
| 8           | 4.4                     | 20               | 0.0503                | 18 523                    | 0.21                        | loaded     | 25 496        | 21 247         | 9658          | 6167               | 21 794           |
| 9           | 4.51                    | 20               | 0.062                 | 7330                      | 2.59                        | loaded     | 39 225        | 32 688         | 14 858        | 21 090             | 33 539           |
| 10          | 4.54                    | 20               | 0.062                 | 9832                      | 11.2                        | loaded     | 39 358        | 32 798         | 14 908        | 24 509             | 22 177           |
| 11          | 4.78                    | 20               | 0.04094               | 3954                      | 19.61                       | loaded     | 17 617        | 14 681         | 6673          | 11 340             | 22 822           |
| 12          | 4.96                    | 20               | 0.062                 | 12 177                    | 2.3                         | loaded     | 41 175        | 34 312         | 15 597        | 20 294             | 35 362           |
| 13          | 5.3                     | 20               | 0.0506                | 10 504                    | 0.65                        | loaded     | 28 373        | 23 644         | 10 747        | 11 034             | 36 693           |
| 14          | 5.3                     | 20               | 0.05064               | 14 983                    | 10.19                       | loaded     | 28 418        | 23 682         | 10 764        | 17 343             | 36 693           |
| 15          | 5.43                    | 20               | 0.062                 | 15 466                    | 0.53                        | loaded     | 43 132        | 35 943         | 16 338        | 14 822             | 37 192           |
| 16          | 5.6                     | 20               | 0.0409                | 2054                      | 9.92                        | loaded     | 19 070        | 15 892         | 7223          | 12 730             | 24 585           |
| 17          | 7                       | 20               | 0.05064               | 17 719                    | 8.64                        | loaded     | 32 802        | 27 335         | 12 425        | 20 726             | 42 857           |
| 18          | 8.3                     | 20               | 0.062                 | 25 025                    | 0.21                        | loaded     | 53 659        | 44 716         | 20 325        | 14 657             | 47 111           |
| 19          | 9.6                     | 20               | 0.062                 | 44 076                    | 0                           | loaded     | 57 723        | 48 102         | 21 865        | 9 403              | 51 019           |
| 20          | 10.7                    | 20               | 0.062                 | 27 753                    | 4                           | loaded     | 60 842        | 50 702         | 23 046        | 35 366             | 54 070           |
| 21          | 3.2                     | 20               | 0.062                 | 35 649                    | 3.36                        | unloaded   | 33 010        | 27 508         | 12 504        | 12 381             | 27 740           |
| 22          | 3.2                     | 20               | 0.05064               | 15 432                    | 1.2                         | unloaded   | 22 022        | 18 351         | 8 342         | 7813               | 18 272           |
| 23          | 5.5                     | 20               | 0.062                 | 25 955                    | 3.2                         | unloaded   | 43 417        | 36 181         | 16 446        | 20 177             | 24 339           |
| 24          | 8                       | 20               | 0.0503                | 33 227                    | 0                           | unloaded   | 34 661        | 28 884         | 13 129        | 5336               | 29 997           |
| 25          | 9                       | 20               | 0.10605               | 51 222                    | 0.43                        | unloaded   | 163 546       | 136 288        | 61 949        | 47 123             | 49 256           |
| 26          | 10.8                    | 20               | 0.062                 | 55 746                    | 0                           | unloaded   | 61 112        | 50 927         | 23 148        | 10 396             | 23 097           |
| 27          | 13.6                    | 20               | 0.05064               | 58 398                    | 0.05                        | unloaded   | 45 220        | 37 683         | 17 129        | 77 73              | 40 210           |

*The red and bold values are indicate the predicting results of these models is inconsistent with actual condition.*
increases, whereas the critical droplet entrainment rate increases with the pipe diameter at the same surface tension. Based on the definition of surface tension, the greater this parameter, the stronger the droplet capability to maintain integrity is. This means that larger gas phase turbulence is necessary to break up the droplet. In this case, the required critical droplet entrainment velocity and gas well flow rate to lift the same quantity of droplets will be the greater. Similarly, at the same pressure, the critical droplet entrainment rate decreases with the increase in surface tension, as illustrated in Figure 17. The effect of pressure on the critical droplet entrainment rate at the same surface tension is discussed in Section 3.2.4.

3.2.4 | Influence of different pressures on critical droplet entrainment rate

Investigations suggest that when the pressure and pipe diameter are fixed, the influence of different temperatures on the critical droplet entrainment rate is not significant. The change in critical droplet entrainment rate is 0.24% for every 5°C change in temperature. Figure 18 shows the relationship between the critical droplet entrainment rate and pressure at different temperatures when the pipe diameter is 0.0403 m. The correlation curves plotted at various temperatures are practically coincident. Besides, this section presents the delineation and influence of different pressures on critical droplet...
entrainment rate when the pipe diameters are 0.0403, 0.0506, 0.0620, and 0.10305 m, respectively.

Figure 19 illustrates the relationship between the critical droplet entrainment rate and wellhead pressure corresponding to different pipe diameters. When the pressure is fixed, the critical droplet entrainment rate increases with the pipe diameter. This phenomenon can be attributed to three reasons. First, macroscopically, the increase in pipe diameter will decrease the drag coefficient according to its expression, and a greater critical droplet entrainment rate will be required to lift isometric droplets based on Equation (12). Second, microscopically, the friction coefficient decreases with the gas phase turbulence. To lift the droplet out of the pipe, however, the gas phase turbulence should be greater than friction; hence, a large critical droplet entrainment rate is required. Third, the increase in pipe diameter leads to the increase in pipe cross-sectional area, thus also requiring a large critical droplet entrainment rate. Apart from the foregoing, it is found that the influence of pressure on the critical droplet entrainment rate is complicated. First, it affects the densities of both gas and liquid phases. This impacts the corresponding Reynolds number, which influences the drag coefficient, friction coefficient, and surface tension. According to Equation (14), when the pipe diameter is constant, the critical droplet entrainment rate increases with pressure, which is consistent with the trend shown in Figure 19.
### 4 CONCLUSIONS

1. The spherical droplet transforms into an ellipsoidal droplet because of the effect of pressure differential. In this study, the maximum ellipsoidal droplet size parameters are calculated microscopically based on the balance between gas phase turbulence and droplet surface tension. Meanwhile, the critical droplet entrainment rate is obtained macroscopically through the force equilibrium between gravity, drag, and buoyancy. A novel model of the critical droplet entrainment rate for gas wells is established, which integrally considers the effect of changes in temperature, pressure, and pipe diameter on drag coefficient, surface tension, and friction coefficient.

2. Field data from 3638 gas wells of a certain gas field are utilized to verify the proposed model and other typical models. The results suggest that the proposed novel model affords the best predictions of actual gas well conditions and may be employed to determine whether or not a gas well is loaded.

3. The sensitivity analysis of the effects of drag coefficient, surface tension, and friction coefficient on the critical droplet entrainment rate of gas wells is performed, and the relationship curve of critical droplet entrainment rates corresponding to different pressures and pipe diameters is established. The results are beneficial in engineering reference.

### ACKNOWLEDGMENTS

This article has been supported by the National Natural Science Foundation of China (51704246) and Sichuan Science and Technology Project (Grant No. 2020YFSY0031).

### NOMENCLATURE

- $A$: cross-sectional area of pipeline, $m^2$
- $h$: diameter in z-axis direction, $m$
- $e_s$: surface free energy per unit area, $W/m^2$
- $E_s$: surface free energy, $W/m$
- $e_T$: turbulent kinetic energy of gas stream per unit volume, $W/m^3$
- $E_T$: Turbulent kinetic energy, $W/m$
- $f_g$: friction coefficient at apparent velocity of gas phase
- $g$: gravity
- $g_w$: gas-water interfacial tension, $mN/m$; $Z$: gas deviation factor
- $g_{sw}$: gas-water interfacial tension, $mN/m$; $Z$: gas deviation factor
- $l$: droplet velocity, $m/s$
- $N$: number of gas well
- $P$: pressure, MPa
- $Q_c$: critical droplet entrainment rate of gas well, $m^3/d$
- $R$: gas deviation factor
- $S$: windward area of droplet, $m^2$
- $T$: pseudoreduced temperature
- $T_c$: critical temperature
- $T$: temperature, K
- $v_g$: minimum flow velocity of gas stream, $m/s$
- $v_l$: droplet velocity, $m/s$
- $v_{rms}$: radial velocity, $m/s$
- $\sigma$: gas-liquid surface tension, $N/m$
- $\sigma_{gw}$: gas-water interfacial tension, $mN/m$; $Z$: gas deviation factor

### REFERENCES

1. Turner RG, Hubbard MG, Dukler AE. Analysis and prediction of minimum flow rate for the continuous removal of liquids from gas wells. *J Petrol Technol*. 1969;21(11):1475-1482. https://doi.org/10.2118/2198-PA

2. Coleman SB, Clay HB, McCurdy DG, Norris III LH. A new look at predicting gas-well load-up. *J Petrol Technol*. 1991;43:329-333. https://doi.org/10.2118/20280-PA

3. Hinze JO. Fundamentals of the hydrodynamic mechanism of splitting in dispersion processes. *AIChE J*. 1955;1:289-295. https://doi.org/10.1002/aic.690010303

4. Wierzba A. Deformation and breakup of liquid drops in a gas stream at nearly critical Weber numbers. *Exp Fluids*. 1990;9:59-64. https://doi.org/10.1007/bf00575336

5. Liu Z, Reitz RD. An analysis of the distortion and breakup mechanisms of high speed liquid drops. *Int J Multiph Flow*. 1997;23:631-650. https://doi.org/10.1016/S0301-9322(96)00086-9

6. Kékesi T, Amberg G, Prahl Wittberg L. Drop deformation and breakup. *Int J Multiph Flow*. 2014;66:1-10. https://doi.org/10.1016/j.ijmultiphaseflow.2014.06.006

7. Li M, Lei S, Li S. New View on Continuous-removal Liquids from Gas Wells. In *Proceedings of SPE Permian Basin Oil and Gas Recovery Conference, Midland, Texas*, 2001; p. 6.

8. Tan XF, Li XP, Liu JY. Model of continuous removal Liquids from gas wells by droplet diameter estimation. *J Nat Gas Sci Eng*. 2013;15:8-13.

9. Li YS, Teng SN, Yang ZX, Liao HJ, Ma L, Li N. Critical liquid carrying flow rate model with consideration of interfacial tension and droplet deformation effect. *Oil Drill Prod Technol*. 2017;39:218-223.

10. Sutton RP. Fundamental PVT Calculations for Associated and Gas/Condensate Natural-Gas Systems. *SPE Reservoir Eval Eng*. 2007;10:270-284.

11. Sutton R. An improved model for water-hydrocarbon surface tension at reservoir conditions. *SPE Annual Technical Conference and Exhibition*, 2009. https://doi.org/10.2118/124968-MS

12. Rushing JA, Newsham KE, Van Fraassen KC, Mehta SA, Moore GR. Laboratory Measurements of Gas-Water Interfacial Tension at HP/HT Reservoir Conditions. In *Proceedings of CIPC/SPE Gas Technology Symposium 2008 Joint Conference, Calgary, Alberta, Canada*, 2008; p. 17.

13. Firoozabadi A, Ramey Jr HJ. Surface tension of water-hydrocarbon systems at reservoir conditions. *J Can Pet Technol*. 1988;27:9. https://doi.org/10.2118/88-03-03

14. Duan R, Sun C, Jiang SJM. A new surface tension formulation for particle methods. *Int J Multiph Flow*. 2020;124:103187.

15. Najafi-Marghmaleki A, Tatar A, Barati-Harooni A, Mohebbi A, Kalantari-Meybodi M, Mohammadi AH. On the prediction of interfacial tension (IFT) for water-hydrocarbon gas system. *J Mol Liq*. 2016;224:976-990. https://doi.org/10.1016/j.molliq.2016.10.083

### ORCID

Heng Xiao https://orcid.org/0000-0002-5645-3629
16. Zhu J, Yang Z, Li X, Hou L, Xie S. Experimental study on the micro-characteristics of foams stabilized by viscoelastic surfactant and nanoparticles. Colloids Surf, A. 2019;572:88-96. https://doi.org/10.1016/j.colsurfa.2019.03.087

17. Bagheri G, Bonadonna C. On the drag of freely falling non-spherical particles. Powder Technol. 2016;301:526-544. https://doi.org/10.1016/j.powtec.2016.06.015

18. Wang Z, Bai H, Zhu S, Zhong H, Li Y. An entrained-droplet model for prediction of minimum flow rate for the continuous removal of liquids from gas wells. SPE Journal. 2015;20(05):1135-1144. https://doi.org/10.2118/174077-PA

19. Al-Sarkhi A, Hanratty TJ. Effect of pipe diameter on the drop size in a horizontal annular gas-liquid flow. Int J Multiph Flow. 2002;28:1617-1629. https://doi.org/10.1016/s0301-9322(02)00048-4

20. Wang G, Sawant P, Ishii M. A new entrainment rate model for annular two-phase flow. Int J Multiph Flow. 2020;124:103185. https://doi.org/10.1016/j.ijmultiphaseflow.2019.103185

21. Adamson AW. The Physical Chemistry of Surfaces. 5th edn. New York: John Wiley & Sons. 1990.

22. White BR. Particle dynamics in two-phase flows. Encyclopedia of Fluid Mech. 1986;4:240-282.

23. Zhang H-Q, Wang Q, Sarica C, Brill JP. A unified mechanistic model for slug liquid holdup and transition between slug and dispersed bubble flows. Int J Multiph Flow. 2003;29:97-107. https://doi.org/10.1016/s0301-9322(02)00111-8

24. Dukler AE, Hubbard MG. A model for gas-liquid slug flow in horizontal and near horizontal tubes. Ind Eng Chem Fundam. 1975;14(4):337-347. https://doi.org/10.1021/i160056a011

25. Chen XT, Cai XD, Brill JP. A general model for transition to dispersed bubble flow. Chem Eng Sci. 1997;52:4373-4380. https://doi.org/10.1016/s0009-2509(97)80857-3

26. Zhang H, Liu Q, Qin B, Bo H. Modeling droplet-laden flows in moisture separators using k-d trees. Ann Nucl Energy. 2015;75:452-461. https://doi.org/10.1016/j.anucene.2014.08.036

27. Zhong HG, Zhou X, Jiang YY, Guo FJ, Zhang Q, Li ZM. Discussion on variation law of drop drag Coefficient under different production conditions in gas wells. Liaoning Chem Ind. 2017;46:1190-1192.

28. Haider A, Levenspiel O. Drag coefficient and terminal velocity of spherical and nonspherical particles. Powder Technol. 1989;58:63-70.

29. Helenbrook BT, Edwards CF. Quasi-steady deformation and drag of uncontaminated liquid drops. Int J Multiph Flow. 2002;28:1631-1657.

30. Zhu B, Tang H, Zhao F, Tang H. Numerical simulation of particulate suspension transport and permeability impairment in an actual rough fracture under normal stresses. Energy Sci Eng. 2020;8(4):1165-1180. https://doi.org/10.1002/es3.576

31. Wei N, Meng YF, Li YQ, et al. Research on the liquid droplet’s drag coefficient for continuous removal of liquid from gas well. Nat Gas Technol. 2007;50-52+54+95.

32. Brauer H, Mewes D. Strkouml;mungswiderstand sowie station&uuml;rer und instation&uuml;rer Stoff- und W&auml;rmeverlust an Kugeln. Chem Ing Tec. 1972;44:865-868.

33. Clift R, Gauvin WH. Motion of entrained particles in gas streams. Can J Chem Eng. 2010;49:439-448.

34. Martin DIH. Wärme- und Stoffübertragung in der Wirbelschicht. Chem Ing Tec. 1980;52:199-209.

35. Shao MW, Xi CD. Parallel formula of drag coefficient of spherical particle sedimentation. Chem Eng Des. 1994;1(1):16–16.

36. Cheng NS. Comparison of formulas for drag coefficient and settling velocity of spherical particles. Powder Technol. 2009;189:395-398.

37. Mikhailov MD, Freire APS. The drag coefficient of a sphere: An approximation using Shanks transform. Powder Technol. 2013;237:432-435. https://doi.org/10.1016/j.powtec.2012.12.033

38. Yan J. Numerical Simulation of Gas-Liquid Two-Phase Flow in Wellbore. Master. Southwest Petroleum University; 2005.

39. Fu J, Su Y, Jiang W, Xing X, Li B. Multiphase flow behavior in deep water drilling: The influence of gas hydrate. Energy Sci Eng. 2020;8(4):1386-1403. https://doi.org/10.1002/es3.600

40. Mu L, Wang LL, Wen YJ. Dynamic analysis on liquid loading on gas well. J Oil Gas Technol. 2005;144-146+110.

41. Yu XC, Feng SC. Analysis of oneway friction coefficient in multiphase tube flow. Oil Gas Field Surf Eng. 2001;3:44-13.

42. Wang MK, Lin RY, Zhang Y. Analysis on frictionial resistance coefficient of single-phase fluid inside pipeline. OGST. 1998;17:22-26.

43. Adamson AW. Physical chemistry of surfaces[J]. Journal of the Electrochemical Society. 1960;124(5):582.

44. Ouyang LB, Aziz K. Steady-state gas flow in pipes. J Petrol Sci Eng. 1996;14:137-158.

45. Towler BF, Pope TL. New equation for friction-factor approximation developed. Oil Gas J. 1994:92:14.

46. Kalantari Meybodi M, Daryasafar A, Karimi M. Determination of hydrocarbon-water interfacial tension using a new empirical correlation. Fluid Phase Equilib. 2016;415:42-50. https://doi.org/10.1016/j.fluid.2016.01.037

47. Hassan M, Nielsen R, Calhoun JJJ. Effect of pressure and temperature on oil-water interfacial tensions for a series of hydrocarbons. J Oil Gas Technol 1953;5:299-306.

48. Jennings Jr HY, Newman GHJS. The effect of temperature and pressure on the interfacial tension of water against methane-normal decane mixtures. Soc Petrol Eng J 1971;11:171-175.

How to cite this article: Xiao H, Li X-P, Tan X-H, et al. A novel model for calculating critical droplet entrainment rate of gas well considering droplet deformation and multiple parameters. Energy Sci Eng. 2021;9:812–827. https://doi.org/10.1002/ese3.836