Modelling non-dust fluids in cosmology

Adam J. Christopherson,1 Juan Carlos Hidalgo,2,3 and Karim A. Malik4

1School of Physics and Astronomy, University of Nottingham, University Park, Nottingham, NG7 2RD, United Kingdom
2Instituto de Astronomía, UNAM, Ciudad Universitaria, 04510, México D.F., México
3Institute of Cosmology and Gravitation, University of Portsmouth, Dennis Sciama Building, Burnaby Road, Portsmouth, PO1 3FX, United Kingdom
4Astronomy Unit, School of Physics and Astronomy, Queen Mary University of London, Mile End Road, London, E1 4NS, United Kingdom

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Currently, most of the numerical simulations of structure formation use Newtonian gravity. When modelling pressureless dark matter, or ‘dust’, this approach gives the correct results for scales much smaller than the cosmological horizon, but for scenarios in which the fluid has pressure this is no longer the case. In this article, we present the correspondence of perturbations in Newtonian and cosmological perturbation theory, showing exact mathematical equivalence for pressureless matter, and giving the relativistic corrections for matter with pressure. As an example, we study the case of scalar field dark matter which features non-zero pressure perturbations. We discuss some problems which may arise when evolving the perturbations in this model with Newtonian numerical simulations and with CMB Boltzmann codes.

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I. INTRODUCTION

Current observations indicate that the universe in which we live is almost homogeneous and isotropic. However, it is also known that small initial departures from homogeneity and isotropy give rise to the structures we observe today. Thus, while the universe is well approximated on large scales by a homogeneous and isotropic Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime, the existence of large scale structure and inhomogeneities in the Cosmic Microwave Background (CMB) tell us that this is not the complete picture [1].

In order to model inhomogeneities we utilise a technique that is well established in many branches of physics and applied mathematics, namely, we take an approximate solution and add small perturbations. In cosmology, this technique is called cosmological perturbation theory, and requires the addition of small inhomogeneous perturbations on top of the FLRW background, such that the system still solves Einstein’s field equations (e.g. Refs. [2–4]). When considering the dynamics on sufficiently small scales, and for fluids which are pressureless, it is enough to use Newtonian physics [5]. Inhomogeneous perturbations of Newtonian cosmology have been studied for many years and these are the equations that are used when performing large numerical simulations of galaxy formation [5, 7]. However, as simulations get more sophisticated and the observations more precise, it is possible to distinguish those components which exhibit pressure perturbations. For example, to fully account for the effects of inhomogeneous scalar fields as dark energy (e.g. [8–11], dark matter [12–15], or a unified field [16–19], in the formation of structures, one should employ more general equations with the input of general relativity.

In this paper we re-visit the question of how to relate Newtonian and cosmological perturbation theories [1]. We show that, for pressureless systems, the equations governing cosmological perturbations of an FLRW spacetime reduce to the equivalent Newtonian equations on using gauge invariant variables – the metric potential in the longitudinal gauge, and the density contrast in the comoving gauge or the total matter gauge. Drawing on this equivalence, we then investigate the situation for fluids with pressure and/or pressure perturbations. We find that one can write the Poisson equation in the usual way, but that the continuity and Euler equations then differ, depending upon the equation of state parameter and the adiabatic sound speed.

We then go on to study how to relate the two perturbation theories in a general scalar field model. We find that, as expected, the Poisson equation is identical to the Newtonian case, but that the Euler and continuity equations differ, depending now on the equation of state parameter and the effective speed of propagation of perturbations through the system. Finally we discuss the Jeans scale in the scalar field dark matter models, where the background equation of state parameter is zero. We find that this scale depends upon $c_{ph}^2$, the phase speed, or speed of propagation of perturbations. We close, in Section IV, with a brief discussion.

1 Note, throughout this paper we work within the confines of linear perturbation theory.
II. MODELLING INHOMOGENEITIES

A. Newtonian perturbations

Let us first study the theory of perturbations in Newtonian physics. We consider inhomogeneous perturbations about a homogeneous background, and so the energy density \( \rho \) is

\[
\rho(x, t) = \bar{\rho}(t) \left( 1 + \delta_N(x, t) \right), \tag{2.1}
\]

where \( \bar{\rho}(t) \) is the homogeneous background energy density and \( \delta_N(x, t) \) is the inhomogeneous density contrast. On introducing the inhomogeneous Newtonian potential, \( \Phi_N(x, t) \) and fluid velocity \( \vec{v}_N(x, t) \), the linearised conservation and Euler equations are, respectively, \([2, 20]\)

\[
\begin{align*}
\delta_N + \frac{1}{a} \cdot \vec{v}_N &= 0, \tag{2.2} \\
\dot{\vec{v}}_N + H \vec{v}_N &= -\frac{1}{a} \vec{\nabla} \Phi_N - \frac{1}{a \bar{\rho}} \vec{\nabla} \delta_P_N, \tag{2.3}
\end{align*}
\]

where \( H = \dot{a}/a \) is the Hubble parameter, a dot denotes a derivative with respect to coordinate time and \( \delta P_N \) denotes the pressure perturbation. The Newtonian potential and the density contrast are then related through the Poisson equation

\[
\nabla^2 \Phi_N = 4 \pi G a^2 \bar{\rho} \delta_N, \tag{2.4}
\]

where the Laplacian is defined as \( \nabla^2 = \vec{\nabla} \cdot \vec{\nabla} \). On utilising the relationship between the energy density perturbation and the pressure perturbation for a barotropic fluid, \( \delta P = c_s^2 \delta \rho \), we can combine the fluid equations into a second order equation

\[
\frac{\partial^2 \delta_N}{\partial t^2} + 2H \frac{\partial \delta_N}{\partial t} = 4 \pi G \bar{\rho} \delta_N + c_s^2 \nabla^2 \delta_N. \tag{2.5}
\]

B. Cosmological perturbations

While the theory of Newtonian perturbations is sufficient for modelling small scale physics involving only pressureless dust particles, the dynamics of the universe are governed by general relativity. Therefore, we must consider relativistic perturbation theory. Since Einstein’s theory relates the geometry of the universe to its matter content, we must consider perturbations of both the matter and the FLRW spacetime metric.

The most general, linear scalar perturbations to the FLRW metric are \([2, 20]\)

\[
ds^2 = a^2(\eta) \left[ -\left( 1 + 2\phi \right) d\eta^2 + 2B_i d\eta^i d\eta^j \right] + \left\{ \left( 1 - 2\psi \right) \delta_{ij} + E_{ij} \right\} d\eta^i d\eta^j, \tag{2.6}
\]

where we now use conformal time \( \eta \), related to coordinate time \( t \) through \( dt = a d\eta \). A unique problem which arises in the relativistic theory is the problem of gauge dependence. Since general relativity is covariant, and splitting the spacetime into a background and a perturbation is not a covariant process, we introduce extra spurious coordinate dependence (see, e.g., Refs. \([1, 22, 23]\)). This can be resolved in a systematic manner, as was first shown by Bardeen \([24]\), by considering gauge-invariant variables – quantities that do not change under a gauge transformation. A popular choice of variables amount to setting \( E \) and \( B \) to zero, resulting in the FLRW metric in the so-called longitudinal or Newtonian gauge, with the gauge invariant variables \( \Phi \) and \( \Psi \)

\[
ds^2 = a^2(\eta) \left[ -(1 + 2\Phi) d\eta^2 + (1 - 2\Psi) \delta_{ij} d\eta^i d\eta^j \right]. \tag{2.7}
\]

The conservation and Euler equations are then reduced from the form without fixing the gauge

\[
\begin{align*}
\delta \rho' + 3H (\delta \rho + \delta P) - 3(\bar{\rho} + \bar{P}) \psi' &= 0, \\
\left[ (\bar{\rho} + \bar{P}) v_v \right]' + 4H(\bar{\rho} + \bar{P}) v_v + (\bar{\rho} + \bar{P}) \Phi + \delta P_v = 0, \tag{2.10}
\end{align*}
\]

Thus, the following expressions, where we neglect anisotropic stresses, \( \Pi \),

\[
\begin{align*}
\delta \rho_v' + (\bar{\rho} + \bar{P})(\nabla^2 v_v - 3 \Psi') &= -3H(\delta \rho_v + \delta P_v), \\
[\bar{\rho} + \bar{P}] v_v' + 4H(\bar{\rho} + \bar{P}) v_v + (\bar{\rho} + \bar{P}) \Phi + \delta P_v &= 0. \tag{2.11}
\end{align*}
\]

Here the subscript \( v \) denotes matter variables in the longitudinal gauge; \( v \) is the velocity potential, i.e. \( v^i = \nabla^i v \) with \( v^i \) the fluid three velocity; and the momentum potential is \( V \equiv v + B \).

We can then specialise to a barotropic fluid with equation of state \( P = w(\rho) \bar{\rho} \), and whose pressure perturbation can be related to the energy density perturbation through

\[
\delta P = c_s^2 \delta \rho, \tag{2.12}
\]

where \( c_s^2 = \bar{P}'/\bar{\rho}' \) is the adiabatic sound speed\(^2\). Thus, for this system, Eqs. \((2.10)\) and \((2.11)\) become

\[
\begin{align*}
\delta \rho_v' + (1 + w)(\nabla^2 v_v - 3 \Psi') &= 3H(w - c_s^2)^2, \\
v_v' + H(1 - 3c_s^2) v_v + \Phi + \frac{c_s^2}{1 + w} \delta \ell &= 0. \tag{2.15}
\end{align*}
\]

\(^2\) The adiabatic sound speed, equation of state parameter and its derivative are related through

\[
\frac{w'}{1 + w} = -3H(c_s^2 - w). \tag{2.13}
\]

For the case where \( w \) is constant, this relationship guarantees that \( w = c_s^2 \).
The Einstein equations then give that $\Phi = \Psi$ (in the case of zero anisotropic stress, as is true for any perfect fluid), and the Poisson equation is
\[
\nabla^2 \Psi = 4\pi G a^2 \rho \left( \delta_c - 3H(1+w)v_\ell \right). \tag{2.16}
\]
In order to draw an equivalence between this and the Newtonian Poisson equation, we consider that the energy density perturbation transforms under the gauge transformation $x^\mu \rightarrow x^\mu + \delta x^\mu$ as
\[
\tilde{\rho} = \rho + \rho' \delta \eta, \tag{2.17}
\]
we get for the comoving density contrast, in terms of the longitudinal density contrast and the velocity perturbation \[21,\]
\[
\delta_\ell = \delta_c - 3H(1+w)v_\ell. \tag{2.18}
\]
Then we obtain the Poisson equation \[21, 25\]
\[
\nabla^2 \Psi = 4\pi G a^2 \bar{\rho} \delta_\ell, \tag{2.19}
\]
which is equivalent to Eq. \[2.4\] upon the identification
\[
\Psi = \Phi_N, \quad \delta_\ell = \delta_N. \tag{2.20}
\]
This equivalence is the one we follow in the rest of the paper. Historically, this is the reason why the longitudinal gauge is at times called the Newtonian gauge.

### C. Correspondences

In this Section we relate the relativistic equations to the Newtonian equations. We take heed from the Poisson equation which, as stated above, relates the density contrast in the comoving gauge to the metric potential in the longitudinal gauge:
\[
\nabla^2 \Psi = 4\pi G a^2 \rho \delta_c. \tag{2.21}
\]
In order to obtain an equivalence for the set of equations, we must first ensure that we are consistent with the density contrast that we use. With the aid of the background Friedmann equations and one of Einstein’s perturbed equations,
\[

\Psi' = -H \Psi - 4\pi G a^2 (1+w)\bar{\rho}v_\ell, \tag{2.22}
\]
we can rewrite Eq. \[2.10\] as
\[
\delta_\ell ' + 3H(c_s^2 - w)\delta_\ell + (1+w) \left( 3H \Psi + \nabla^2 v_\ell \right)
+ \frac{9}{2} H^2 (1+w)^2 v_\ell = 0. \tag{2.23}
\]
Using now the relationship between $\delta_\ell$ and $\delta_c$, Eq. \[2.18\], and its derivatives, the continuity equation can be written as
\[
\delta_c ' - 3Hw\delta_c + (1+w)\nabla^2 v_\ell = 0. \tag{2.24}
\]
Furthermore, the Euler equation, \[2.11\], is
\[
v'_\ell + Hv_\ell = -\Psi - \frac{c_s^2}{1+w} \delta_c. \tag{2.25}
\]
From this we can see that, in the case of a pressureless dust for which $w = c_s^2 = 0$, the evolution equations reduce to
\[
\delta_\ell ' + \nabla^2 v_\ell = 0, \tag{2.26}
\]
\[
v'_\ell + H v_\ell + \Psi = 0. \tag{2.27}
\]
These equations are finally identical to the Newtonian \[2.2\] and \[2.3\]. Thus we can establish a mathematical equivalence between Newtonian and relativistic velocity for the case of dust,
\[
v_\ell = uN. \tag{2.28}
\]
The above equations can be recast as a second order differential equation for the density contrast as
\[
\delta_c '' + H \delta_\ell ' = 4\pi G a^2 \bar{\rho} \delta_c, \tag{2.29}
\]
which governs the evolution of matter density perturbations. Here the second term is a suppression of the perturbations with the expansion of the universe, and the term on the right hand side sources the growth of perturbations due to the gravitational instability. This is the equivalent form to the Newtonian perturbation theory (as expected for a non-relativistic species), as obtained in Eq. \[2.5\].

The exact mathematical equivalence between the set of variables presented and their Newtonian counterpart, is already known and has been presented for the Poisson and the evolution equations in the context of other problems \[25-27\].

However, should one wish to consider fluids other than dust, with non-zero pressure either in the background or in the perturbations, then the relativistic equations must be used instead. In such cases, the exact mathematical equivalence with the Newtonian counterpart does not hold and the form of the equations in numerical studies must change.

This is especially important for, say, hot dark matter, or for a system containing dark energy perturbations. In the following, we will consider the case of scalar field models of dark matter.

### III. SCALAR FIELDS AS PERFECT FLUIDS

In this section we aim to present the minimal additions to the ordinary hydrodynamic equations to treat cosmological structure formation when the matter components support pressure. Specifically, models of scalar fields treated as fluids with zero effective pressure, yet with non-negligible pressure perturbations, have been considered to account for the dark matter component \[13, 28, 29\].
The basic model features a canonical scalar field oscillating at the bottom of its potential with a period much smaller than the Hubble time and any other dynamical times. A model of dark matter which has received increased attention is the scalar field dark matter model (SFDM), specifically in the form of a Bose-Einstein condensate [30, 31], the theory is robust enough to study the numerical problem of structure formation (recent attempts are [32–34]). In contrast to cold dark matter (CDM), this component exhibits pressure perturbations.

It is well known that a scalar field system cannot be modelled as a barotropic fluid (except on super-horizon scales [37]), and in fact a consistent fluid equivalence must consider a more general perfect fluid [38]. Indeed, as elucidated in Ref. [39], if one is to interpret a scalar field as a fluid, a distinction must be made between $w$ and $c_s^2$. Furthermore, the speed with which pressure perturbations propagate is described by the effective sound speed, or phase speed, $c_{ph}^2$ defined, in the fluid rest frame, as

$$c_{ph}^2 = \frac{\delta P}{\delta \rho} \bigg|_{\nu f}.$$  \hspace{2cm} (3.1)

For a scalar field with a canonical kinetic term, this is equal to unity. However, for a non-canonical scalar field with pressure and energy density depending upon both the field, $\varphi$, and its kinetic term, $X \equiv -\frac{1}{2}g^{\mu\nu}\partial_\mu \varphi \partial_\nu \varphi$, this can differ from one and is given by [10] [37]

$$c_{ph}^2 = \frac{P}{\rho, X}.$$  \hspace{2cm} (3.2)

In general, for a scalar field, the pressure perturbation is no longer proportional to the energy density perturbation. Instead the relationship between pressure perturbations and energy density perturbations in an unspecified gauge is

$$\delta P = c_s^2 \delta \rho + (c_{ph}^2 - c_s^2) \left[ \delta \rho + \rho' (v + B) \right].$$  \hspace{2cm} (3.3)

The second term on the right hand side is often referred to as the non-adiabatic pressure perturbation $\delta P_{\text{nad}}$. In this case, Eqs. (2.10) and (2.11) become

$$\delta \ell'_f + (1 + w)(\nabla^2 \delta \ell_f - 3 \Psi') = 3\mathcal{H}(w - c_{ph}^2) \delta \ell_f + 9\mathcal{H}^2 (1 + w)(c_{ph}^2 - c_s^2) \nu_f,$$  \hspace{2cm} (3.4)

$$\nu'_f + \mathcal{H}(1 - 3w)\nu_f + \Phi + \frac{w}{1 + w} \delta \ell_f + \frac{w'}{1 + w} \nu_f = 3\mathcal{H}(c_{ph}^2 - c_s^2) \nu_f.$$  \hspace{2cm} (3.5)

As shown in the previous section, in order to write the equations in the variables employed in simulations, we present the equations in longitudinal gauge and then introduce the density contrast in the comoving gauge. Following a similar procedure, using the background and perturbed Einstein equations, we have

$$\delta \ell' + 3\mathcal{H}(c_{ph}^2 - w) \delta \ell + 3\mathcal{H}^2 (1 + w) \Psi + (1 + w) \nabla^2 \nu_f + 9\mathcal{H}^2 (1 + w)(c_s^2 - c_{ph}^2 + \frac{1 + w}{2}) \nu_f = 0,$$  \hspace{2cm} (3.6)

which in light of Eq. (2.18) can then be written as

$$\delta \ell' - 3\mathcal{H} w \delta \ell + (1 + w) \nabla^2 \nu_f = 0.$$  \hspace{2cm} (3.7)

Interestingly, this is identical to the equation for the barotropic fluid presented earlier. However, the Euler equation does not take the same form as Eq. (2.25). Instead, in terms of the comoving density contrast, it is

$$\nu'_f + \mathcal{H} \nu_f = -\Psi - \frac{c_{ph}^2}{1 + w} \delta \ell.$$  \hspace{2cm} (3.8)

This form shows the advantage of using the same variables that are identified with the Newtonian counterpart in the dust case. These last two equations present explicitly the minimal relativistic modification with respect to the dust case in equations (2.2) and (2.3).

The combination of these two equations yields a Klein-Gordon equation – a generalisation of Eq. (2.29) for $\delta$ with an extra Laplacian term,

$$\delta \nu'' c_{ph}^2 \nabla^2 \delta \ell + \mathcal{H} (6w - 3c_s^2 - 1) \delta \ell' - 3\mathcal{H}^2 \left( \frac{3}{2} + 4w - \frac{3}{2} w^2 - 3c_s^2 \right) \delta \ell = 0.$$  \hspace{2cm} (3.9)

The above equation is reduced to Eq. (2.29) in the limit when $w = c_s^2 = c_{ph}^2 = 0$.

As a characteristic feature of the problem one can determine, directly from Eq. (3.9) the instability scale for density perturbations. For the most general case we obtain the Jeans wavenumber

$$k_j^2 = \frac{3}{2} \mathcal{H}^2 \left( 1 + 8w - 3w^2 - 6c_s^2 \right).$$  \hspace{2cm} (3.10)

This scale is associated to a power spectrum cutoff, and is a characteristic feature of non-dust components. In the

\[3\] There are conversely, models which unify dark energy and dark matter under a single degree of freedom [e.g. 35, 36]. The set of equations in the previous section are sufficient to work with in these cases, with the equation of state as the only degree of freedom.

\[4\] Note that the equivalence between Eqs. (3.1) and (3.2) is only true for a single scalar field, which is the case we study in this paper.
case of the scalar field acting as a dark matter component, for which \( w_{\text{eff}} = 0 \), the Jeans’ wavelength reduces to

\[
\lambda_J = \frac{c_{\text{ph}} \sqrt{\pi}}{G \rho (1 - 6c_s^2)}.
\]  

(3.11)

This is consistent with the fact that the speed of propagation of perturbations is given by \( c_s^2 \).

The effective sound speed for each canonical SFDM model is equivalent and equal to 1, though for non-canonical models this differs (see e.g. [10]). The specific scales for each model, as well as the growth of perturbations will be treated elsewhere. For our purposes it suffices to note that the \( c_s^2 \) contribution to the last equation would not be manifest in the Newtonian context. This is eventually important in determining the scale of a spectrum cutoff [28, 29].

These simple results show the importance of considering the system with no approximations and argue for the use of the system of equations (3.7) and (3.8) in the forthcoming simulations of structure formation in the general SFDM models.

Another important comment is that care must be taken when studying the SFDM model using CMB codes. Many of the popular Boltzmann codes are written in the synchronous gauge, a gauge specified by demanding that \( \phi = 0 = B \). However, as has been known for some time, these conditions alone do not fix the gauge. That is, the synchronous conditions alone are not a complete gauge choice, and one needs an additional condition. The condition usually employed, e.g. by CAMB [11], is to set the velocity perturbation of the cold dark matter to zero.

This can be done since the dark matter is assumed to have zero pressure perturbations [12]. But \( u_{\text{DM}} = 0 \) is a gauge choice that does not admit pressure perturbations in the dark matter component, since imposing this condition will result in inconsistencies in the theory (c.f. equation (2.9)). Consequently, in order to use CMB codes to study the SFDM model, one must either include an extra CDM component to allow for this condition – which arguably reduces the value of the theory – or use a code that is not written in the synchronous gauge [e.g. 43].

IV. DISCUSSION

In this paper we have revisited the issue of relating Newtonian perturbation theory to relativistic perturbation theory in cosmology. After reviewing both perturbation theories, we explicitly showed how one relates them for the case of a pressureless fluid, such as dark matter. As is well documented in the literature on cosmological perturbation theory, in order to relate the two approaches one must use the Newtonian gauge with the comoving energy density perturbation. With this gauge choice, we have shown that the hydrodynamical and Poisson equations remain identical to the Newtonian prescription.

We then considered the extension of this for fluids with pressure, both for the barotropic and for the perfect fluids taking our lead from the dark matter case. We show that the mathematical equivalence is lost and, instead, the continuity equation differs, depending upon the equation of state parameter, and the Euler equation depends upon \( c_s^2 \).

The major application that we explored in Section III regards scalar field dark matter. In this case, the dark matter species is no longer pressureless, but is instead a scalar field with canonical kinetic term. In addition to the adiabatic sound speed and equation of state parameter, a scalar field also has a speed of propagation of perturbations, which we dubbed the phase, or effective sound speed \( c_{\text{ph}} \). The evolution equations for this scalar field fluid in the longitudinal gauge depend on all the parameters. However, on writing the equations in terms of the comoving energy density perturbation, the Poisson and continuity equations reduce to those of perfect fluid form. The Euler equation, on the other hand, still depends upon all the parameters.

Thus we have shown that, when studying the scalar field dark matter model and treating it as a fluid, one cannot simply use the pressureless cold dark matter or the barotropic fluid equations without potentially finding erroneous results. Instead, one must use the equations given in Section III. These conclusions can be extended to quintessence models as well as those modified gravity theories conformally equivalent to a scalar field.

The inequivalence between both theories is also important in systems containing more than one fluid. For example, as shown in Ref. [44], in a system containing normal CDM and a dark energy component, Eq. (2.29) is no longer satisfied, since one must take into account the dark energy perturbations.

Finally we would like to stress again that the results presented here have been limited to linear or first order, both for Newtonian theory as well as for cosmological perturbation theory. Linear theory is only an approximation, as General Relativity is non-linear. We will extend the results presented here to second order in perturbation theory in future work [45].

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