Ultracold atoms in optical lattices are optimum candidates for quantum computation \[1\] and quantum simulation \[2, 3\] thanks to the possibility to control and access the atomic state. They have demonstrated great capabilities in reproducing ideal solid state systems, a remarkable example being the realization of the Mott Insulator (MI) phase \[4, 5\]. Furthermore, experiments are now beginning to access a rather unexplored class of quantum systems involving correlations and disorder \[6\]. The need for effective diagnostic techniques being able to observe these quantum phases and compellingly establish their nature has become notable. Few years ago Altman \textit{et al.} suggested in ref. \[7\] the use of Hanbury Brown and Twiss (HBT) spatial interferometry to probe hidden order in strongly correlated phases of ultracold atoms. This proposal has been recently employed to experimentally investigate quantum statistically correlated lattice systems \[8, 9, 10\] and momentum pair-correlated fermions after molecule dissociation \[11\]. Meanwhile, theoretical works have extended the use of noise interferometry to study disordered systems \[12, 13\] and to observe a predicted supersolid phase \[14\].

In this work we break the ordered domains of a one-dimensional MI in a controlled way, by means of a second periodic potential superimposed on the main lattice. The MI is a state of matter in which strong interactions force bosonic particles to localize at the lattice sites, in the homogeneous case with exactly the same number of particles per site. This energetic stiffness of the phase remains even if, as it happens in usual experiments with cold atoms in optical lattices, a ”mildly” varying trapping potential is superimposed. This variation of site energy produces the well known ”wedding cake” structure \[15\], in which the system rearranges regularly alternating MI domains with different occupation number and superfluid islands. Locally the main properties of the homogeneous system are still preserved, and, as confirmed by experiments \[8, 10\] the global system has mostly an ordered insulating character. A more intimate alteration of the MI structure can be instead obtained by a non-monotone scrambling of the energy offsets of the sites, employing a two-color lattice.

As earlier shown in ref. \[6, 17\], the superposition of an additional weak lattice to a MI state is enough to profoundly affect the excitation spectrum of the system. When this second lattice is strong enough, we expect the atoms to redistribute over the lattice breaking the ordered structure of the MI. In this Letter we describe an experiment where we have detected this redistribution using the noise interferometry technique in a yet unexploited way. In the experiments performed until now, the noise correlation has only given information on the strongly correlated character of the MI and on the spatial ordering of the in-trap density distribution, namely the fact that atoms are located at equidistant positions in space. In this work we further demonstrate the powerfulness of this diagnostic technique by showing that it can be used to detect modifications in the ordered site filling typical of a MI state, whereas the density distribution after time-of-flight (TOF) does not show qualitative differences.

In the experiment we load a $^{87}\text{Rb}$ Bose-Einstein condensate (BEC) containing typically $3 \times 10^5$ atoms in a 2D optical lattice (confining lattice) with height $s_l = 40$, in units of the recoil energy $E_{R1} = \hbar^2 k_1^2/2m \approx h \times 3.33$.
kHz, where $2\pi/k_1 = 830.3(1)$ nm is the wavelength of the lattice light we derive from a Titanium-Sapphire laser and $m$ is the atomic mass. In this way we create a bidimensional array of 1D atomic tubes with maximum 200 atoms, among which the tunneling is highly suppressed on the timescale of the experiment. Simultaneously, we apply along the axis of each atomic tube a third optical lattice (main lattice) with the same spacing and height $s_1 = 16$.

The system can be described by the Bose-Hubbard Hamiltonian [1]

$$\hat{H} = -J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \frac{1}{2} U \sum_j \hat{n}_j(\hat{n}_j - 1) + \sum_j \epsilon_j \hat{n}_j \quad (1)$$

where $\hat{a}_j$ and $\hat{a}_j^\dagger$ are respectively the annihilation and creation operators of one boson in the $j$-th site of the lattice and $\hat{n}_j$ is the number operator. The relevant energy scales in the process are the tunneling energy $J$, the on-site repulsive atomic interaction $U$ and the site energies $\epsilon_j$ determined by the external trapping potential. In our realization, the system is deeply in the MI phase: the interaction energy is $U = h \times 2.7$ kHz and the tunneling along the tubes direction (that we will call $x$ direction in the following) is $J = h \times 20$ Hz. The lattice beams have a gaussian shape with a typical waist of $\sim 200 \mu$m and cause an additional external harmonic confinement. The resulting trapping frequencies are $\omega_x = 2\pi \times 77$ Hz along the direction of the tubes and $\omega_\perp = 2\pi \times 110$ Hz orthogonally.

By adding a weaker second lattice with incommensurate wavelength $2\pi/k_2 = 1076.9(1)$ nm with respect to the main one, along the same direction, we manipulate the third term in the Hamiltonian [1] according to:

$$\epsilon_j = s_2 E_{R2} \sin^2 \left( \frac{j\pi k_2}{k_1} \right) + \frac{m}{2} \omega_x^2 \left( \frac{j\pi}{k_1} \right)^2 \quad (2)$$

with $s_2$ the height of the second lattice expressed in units of the recoil energy $E_{R2} = (h k_2)^2/(2m) \approx h \times 1.98$ kHz. This additional lattice has the main effect of producing a non-periodic modulation of the energy minima over a length scale of $1.8 \mu$m, thus breaking the translational invariance of the main lattice. As far as $s_2 \ll s_1$, the parameters $J$ and $U$ and the spatial position of the energy minima $x_j$ are only slightly affected by the additional lattice [17]. Increasing $s_2$, the ordered atomic distribution in the Mott domains is gradually affected and atoms are expected to rearrange in a non uniform pattern as shown in Fig. [1].

We probe the new density distribution using the noise correlation analysis at fixed height of the main lattice $s_1 = 16$ and various second lattice intensities in the range $0 \leq s_2 \leq 9$. We load the BEC into the final optical lattice configuration, switching on all the lattices at the same time by means of an exponential ramp with time constant 30 ms and duration 140 ms. After holding the atoms for 30 ms in the lattices, we let the atomic system expand by suddenly switching off all the confining potentials. After 20 ms of ballistic expansion, we image the cloud by standard destructive absorption technique. The probe beam has an intensity $I = 0.5$ mW/cm$^2 \approx 0.3 I_{sat}$ where $I_{sat}$ is the saturation intensity of the $D_2$ transition. We illuminate the atomic cloud for 50 $\mu$s and then we image the transmitted light on an interline CCD camera.

From the obtained images we calculate the normalized density-density correlation function analogously to what is done in ref. [8], by:

$$G(d) = \frac{\int d\mathbf{r} \langle \hat{n}(\mathbf{r}) \hat{n}(\mathbf{r} + d) \rangle_t}{\int d\mathbf{r} \langle \hat{n}(\mathbf{r}) \rangle_t \langle \hat{n}(\mathbf{r} + d) \rangle_t} \quad (3)$$

where $\hat{n}(\mathbf{r})$ is the column density detected at the position $\mathbf{r}$ reached by the particles in a time $t$ after releasing from the trap, $d$ is the separation between two detected positions in the expanded atomic cloud and $\langle \rangle_t$ refers to
a statistical averaging over a set of several images taken in the same conditions [18]. A Butterworth high-pass filter is applied on the correlation signal to eliminate the gaussian background appearing as a consequence of shot-to-shot ±25% fluctuations in total atom number. A significative increase in the visibility of the correlation peaks has been obtained by averaging the final image with itself after flipping around the central vertical axis. This procedure is justified by the symmetry of the lattice system under inversion $x \rightarrow -x$.

Non-trivial correlations have shown to be present in the second-order correlation function $G_2(d)$ of the density distribution of a bosonic atomic cloud freely expanding from a MI state, with peaks emerging spaced by the distance $2\hbar k t/m$ being $k$ the wavevector of the lattice laser light. Figure 2 shows images of the density distribution after TOF (left column) and the correlation function (right column) both averaged on 5 ± 6 sets of 40 ± 80 images for different values of $s_2$. The TOF images exhibit an unstructured gaussian profile and no significative differences increasing the height of the second lattice (apart from an increase in size along the direction of the two-color lattice). When $s_2 \lesssim 3$ the measured correlation signal presents peaks clearly visible in correspondence of the periodicity of the main lattice ($k_1$ peaks) and no significative alteration with respect to the pure MI case. At $s_2 = 5$ additional peaks are detectable at the periodicity of the second lattice ($k_2$ peaks), in the central row of the noise correlation signal. Similar correlation peaks have been predicted to appear first for hard-core bosons in a quasiperiodic potential [12] and then also for soft-core bosons [13]. The additional peaks strengthen, becoming clearly visible at $s_2 = 9$, where also the beating between the two standing waves ($k_1 - k_2$ peaks) is visible in the upper and lower rows. Note, that the additional peaks only appear along the horizontal direction where the second lattice modifies the site occupation. The $k_1 - k_2$ peaks should be visible also in the central row, but they are obscured by a central black ring artifact deriving from the application of the high-pass filter. Because the second lattice does not appreciably shift the position of the potential wells, in the range of parameters we used, the appearance of the additional peaks arises from a different effect, namely the change in the local atom site occupation induced by the second lattice. Atoms rearrange in the main lattice wells searching for the lowest energy configuration driven by the interplay between the second-color modified site energies and the interactions.

In order to obtain a deeper insight, we have developed a theoretical model working in the zero-tunneling limit which confirms this interpretation. In this limit the Hamiltonian in equation (1) is diagonal on the tensor product of number Fock states in each lattice site, describing a MI state. The ground state is given by the distribution of atoms minimizing the total energy of the system. The lowest energy configuration is computed, fixing the number of atoms, and subsequently noise correlations are calculated according to ref. [7]. In Fig. 3 we report the result of this calculation for different values of the second lattice intensity $s_2 = 0, 2, 5, 9$. The picture shows in column: (a) the density distribution for the central atomic tube, (b) the 2D correlation signal and (c) its cross section along the horizontal line. At $s_2 = 0$ the system shows the typical wedding cake structure, due to the presence of the harmonic trapping potential, with MI domains of $N = 1, 2, 3, 4$ atoms per site. The corresponding noise signal exhibits the peaks at periodicity $k_1$ as expected. When $s_2 = 2$ the MI order begins to be destroyed starting from the edge of each domain, where the site energies scrambling is more efficient due to the different atom occupation of next-neighboring sites with almost the same energy. The noise signal shows weak peaks emerging at the periodicity $k_2$ and $k_1 - k_2$, as can be seen in the cross section picture. Increasing the height of the second lattice to $s_2 = 5$, the density profile gets more and more inhomogeneous, affecting also the bulk of the domains. The extra peaks are now clearly visible in the noise signal. At the maximum value $s_2 = 9,
In Fig. 4 the theoretical and the experimental results for the ratio between the height of the $k_2$ peak and the main $k_1$ peak for different values of $s_2$. Each point in the graph results from the weighted average of 5–9 experimental values. The dotted curve shows the theoretical prediction for $N = 3 \times 10^5$ atoms. The grey zone takes into account possible systematic errors in the measurement of the absolute number of atoms, showing the results of the calculations for a number of atoms ranging from $2 \times 10^5$ to $4 \times 10^5$. We find good agreement between the experimental data and the results of this zero-tunneling model with no free parameters. Results compatible with our experimental findings have also been obtained by T. Roscilde [13] with a Quantum Monte Carlo calculation performed for a one dimensional system with $N = 100$ atoms, corresponding to one atomic tube of our realization.

In conclusion, we have destroyed in a controlled way the Mott Insulator ordered structure by using an additional lattice at a different incommensurate wavelength and we have monitored its progressive degradation via quantum noise interferometry. We have demonstrated that the noise correlation technique can be employed to detect modifications in the occupation of the lattice sites. Furthermore, a zero-tunneling model shows good agreement with the experimental data. Future perspectives of extending our theoretical model to keep into account finite tunneling appear interesting, especially for the search of possible signals that could give information on characterizing complex quantum phases like the Bose Glass [6, 19, 20].

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[18] The direct calculation of the correlation function over several tens of images requires a heavy numerical effort. One can considerably simplify the computation by applying the Wiener-Kinchin theorem, that reduces the problem to the simpler calculation of Fourier transforms ($FT$) and antitransforms ($FT^{-1}$):

$$G(d) = \frac{(FT^{-1}(|FT(n(d))|^2))}{FT^{-1}(|FT(n(d))|^2)}$$  \hspace{1cm} (4)

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