The Analysis and Compensation of Elastic Unbalance Torque of the Three-axis Air-bearing Simulator

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Abstract. The most important interfering torque of a three-axis air-bearing simulator is the displacement of the center of mass in the gravity field caused by structural elasticity. In order to characterize the torque, a mathematical model of the interference moment was established. Based on the model, it is suggested that the vertical stiffness and horizontal stiffness of the structure should be equal as far as possible during the structural design, and the elastic unbalance moment can be compensated by the vertical offset of the center of mass of the air floating platform relative to the rotation center after the initial attitude leveling. ABAQUS was used to build a simulation model of the air floating platform, and the changes of the structure's centroid before and after the gravitational field was applied were extracted by software to simulate the centroid deviation caused by the elastic deformation of the structure, which was used as the characterization to conduct discrete optimization of the structure. The optimal structural parameters were obtained. Then the disturbance torque curve and the corresponding initial centroid offset after initial centroid compensation were calculated by mathematical model. The results are of positive guiding significance to the design of three-axis air-bearing simulator.

Keywords: Three-axis air-bearing simulator; Unbalance torque; Mass-center compensation.

1. Introduction

The three-axis air-bearing simulator is the most advanced physical simulation equipment for the satellite attitude control system, which can simulate the attitude motion of the satellite in the outer space without disturbing moment environment[1-2]. In recent years, a number of air-bearing simulators have been established at home and abroad, and a lot of research work on satellite attitude has been done by using them[3-6].

In the ground environment, during the full physical simulation of satellite motion by three-axis air-bearing simulator, it is inevitably affected by many interference moments. Among all kinds of interfering torques, the most important interfering torques are gravity torques, including static unbalance torques, elastic unbalance torques and gravity gradient unbalance torques[7]. The literature [8-9] mainly analyzes the static unbalance moment. The platform of the air floating platform is regarded as a rigid body, and it is assumed that the rotation center of the air floating platform coincidently with the center of mass of the structure is the best state. The above research is applicable to the small size of the three-axis air floating platform, whose static unbalance moment accounts for a large proportion of the weight moment. In the literature [10-11], the mathematical model of gravity gradient unbalance moment is established and the simulation analysis is carried out. However, the simulation results can judge that the unbalance
moment generated by gravity gradient is much smaller than the other two kinds of unbalance moments, which can be ignored in the actual analysis. Literature [12-14] established an air platform model based on the kinematic elastodynamics method, took the static unbalanced moment and the elastic unbalanced moment into comprehensive consideration, and fitted the relationship between the disturbance moment and the structural deformation through the simulation of the centroid offset of the platform body under multiple attitude groups. A method of compensating the elastic unbalance moment by means of static unbalance moment is put forward by setting the vertical distance between the center of mass of the platform and the rotary center at a small distance. The above research did not analyze the deformation of the platform body, but only carried out numerical fitting.

In view of this, this paper adopts the analytical method to establish the formula of the interference moment of the three-axis air-bearing simulator after the initial centroid compensation, and adopts the simulation software to model and calculate the structure of the platform and optimize the structure of the platform. The analysis results have positive guiding significance for the structural design of the three-axis air-bearing simulator.

2. The Establishment of Disturbance Torque Model of Three-axis Air-bearing Simulator after Centroid Compensation

2.1. Fundamental Assumption

In order to establish the disturbance torque model, the following basic assumptions are introduced:

- The air-bearing simulator adopts tiled layout and umbrella-shaped scheme, and is designed with axisymmetric and periodic symmetric structure. It is considered that the centroids offset in the two horizontal directions are equal, that is \( \Delta xx = \Delta yy \);
- Because the model of the air-bearing simulator is relatively complex, but its stress state is under the action of gravity perpendicular to the table, and its thickness to diameter ratio is greater than 1:15, the air-bearing simulator can be considered as the thin plate bending problem. When the thin plate bending deformation occurs, the deflection is far less than the thickness of the plate, which belongs to the small deformation problem;
- In the physical relationship of stress and strain, the stress component parallel to the surface of the middle plane \( \tau_{xz}, \tau_{zy} \) can be ignored as a secondary factor compared with other normal stresses.

2.2. Mathematical Model Establishment

In the gravitational field environment, the centroid offset compensation is considered to be completed initially, that is, the centroid of the platform is initially adjusted to the axis below the rotation center, and the corresponding centroid coordinate is \( C_i (0, z_i) \) (considering the same deformation in both horizontal directions). The gravitational field is removed, and only the initial centroid deviation is taken into account. The centroid position changes and becomes \( C_0 (x_0, z_0) \).

When the vertical (Z-direction) gravitational field is applied:

\[
\Delta zz = Gk_{zz}
\]

\[
\Delta xx = Gk_{zx}
\]

Type: \( G \) —the air-bearing simulator weight; \( k_{zz}, k_{zx} \) —Stiffness coefficient.

Similarly, when a horizontal (X-direction) gravitational field is applied:

\[
\Delta xx = Gk_{xx}
\]

\[
\Delta xz = Gk_{xz}
\]

The following formula can be derived from the change of the position of the center of mass before and after applying the gravitational field:
The following is to find the disturbance torque caused by gravity at any angle. Consider the rotation center coordinate system as a fixed coordinate system and the gravitational field is flipped. The center of mass goes from \( C_0(x_0, z_0) \) to \( C_\alpha(x_\alpha, z_\alpha) \).

The equation of the gravitational field line is:

\[
\cos \alpha \cdot x - \sin \alpha \cdot z = 0
\]  

(9)

Then, according to the distance formula from the point to the straight line, the distance from the position of the center of mass \( C_\alpha(x_\alpha, z_\alpha) \) to the equation of the straight line of gravity, namely, the formula of the moment arm of gravity is:

\[
L = \left| \frac{Ax_0 + By_0 + Cz_0 + D}{\sqrt{A^2 + B^2 + C^2}} \right| = \left| \cos \alpha \cdot x_\alpha - \sin \alpha \cdot z_\alpha \right|  
\]  

(10)

Substituting the coordinate formula of the center of mass position:

\[
L = \cos \alpha (-Gk_{xz} + G \cos \alpha k_{zx} + G \sin \alpha k_{zx} - \sin \alpha (z_1 - Gk_{zz} + G \cos \alpha k_{zx} + G \sin \alpha k_{zx}))  
\]  

\[
= -z_1 \sin \alpha + \Delta xx \sin \alpha \cos \alpha + \Delta zz (\sin \alpha - \sin \alpha \cos \alpha) - \Delta xz \sin^2 \alpha + \Delta zz (\cos^2 \alpha - \cos \alpha)  
\]  

(11)

According to the third hypothesis, ignore \( \Delta xx \) and \( \Delta xx \), and finally get:

\[
L = -z_1 \sin \alpha + \Delta zz \sin \alpha \cos \alpha + \Delta zz (\sin \alpha - \sin \alpha \cos \alpha)  
\]  

\[
= (\Delta z - z_1) \sin \alpha \cos \alpha + (\Delta zz - z_1) \sin \alpha  
\]  

(12)

According to the formula, the following conclusions can be reached:

- If the vertical stiffness and horizontal stiffness can achieve equal stiffness, i.e. \( \Delta xx = \Delta zz \), the magnitude of the moment arm depends on the value of \( \Delta zz - z_1 \). The smaller this value is, the smaller the moment arm is and the smaller the interference moment is;
• $\Delta z$ is the deformation of the stepping body in the vertical gravitational field environment, that is, the centroid offset caused by the deformable structure of the platform is considered without considering the mismatch between the center of mass and the center of rotation. This value is the structural characteristic. $z$, refers to the deviation of the center of the actual platform body in the vertical direction, which is the measured absolute value, considering that the center of mass does not coincide with the center of rotation and the platform body is deformed. In the subsequent platform body adjustment, the actual position of the center of mass should be adjusted with this value as the target;

• $\Delta z - z$ is the initial centroid adjusted to the relative displacement quantity, and is the non-coincidence quantity between the preset initial centroid and the rotation center. This initial non-coincidence quantity is designed to compensate the interfering torque caused by the structural deformation caused by the difference between $\Delta x$ and $\Delta z$.

2.3. Simulation Model Establishment
The finite element model of the air-bearing simulator is established by ABAQUS. After the initial model is established, the whole structure is analyzed by finite element. In calculation, the whole system can be regarded as composed of small units connected by finite nodes, and $m_i$ is the mass of a single unit, and $u_i$ is the elastic deformation of the center of mass of the unit.

Using the formula

$$u_c = \frac{\sum_{i=1}^{n} m_i u_i}{\sum_{i=1}^{n} m_i}$$

(13)

to calculate the deformed body core. ABAQUS software can realize the extraction of the center of mass before and after the deformation of the structure. The key of the extraction of the center of mass is the discrete solution quality of the finite element.

The product of the discrete element volume and the element density calculated according to the software is the element mass. The deformation vector of the centroid in the body coordinate system can be calculated according to the centroid formula deduced above.

The process of calculating the centroid offset using ABAQUS is shown in the figure 2.

3. Establishment and Optimization of Structure Model of Three-axis Air-bearing Simulator

3.1. Structural Model of Three-axis Air-Bearing Simulator
The three-axis air-bearing simulator comprises an air floating ball bearing and an instrument platform, as shown in the figure 3. The instrument platform is composed of a supporting flange, a load-bearing cylinder, a top plate, a bottom plate, the first layer of partition, the second layer of partition and the vertical partition, etc. The overall structure is made of aluminum alloy riveting structure.

Using Abaqus to complete the establishment of simulation model. In order to simplify the calculation, and to truthfully reflect the main mechanical characteristics of the platform structure, the instrument platform is modeled using plate and shell units. For the air floating ball bearing, the lower part is regarded as a rigid body, only its quality attribute is considered, and the fixed support constraint is made, while the upper part of the area connected with the instrument platform is regarded as a flexible body. Furthermore, small holes and small size structures in unimportant areas are also ignored.

3.2. Structural Optimization of Air Floating Platform
According to the formula of the mathematical model of the interference moment arm, it can be judged that when the stiffness of the structure is equal in all directions, the elastic unbalance moment is 0. However, it is quite difficult to make the stiffness in all directions equal, but it is very necessary to make them close enough. Therefore, in the design stage of air floating platform, it is necessary to optimize the
structure of the platform. In addition, for any Angle, the smaller the magnitude of the centroid deformation is, the smaller the magnitude of the interference arm is, and the higher the overall stiffness is, the smaller the influence of the calculation error in the design stage is. Therefore, the following two aspects should be considered in the optimization analysis of the air floating platform:

- Equal stiffness design, that is, vertical stiffness and horizontal stiffness should be as consistent as possible;
- High stiffness design, that is, to maximize the vertical stiffness and horizontal stiffness.

Figure 2. Diagram of the calculation process of mass-center offset.

According to the requirements, the center of the air float ball is located between the first and second layer of partition. For the top plate, the bottom plate, the first layer of partition and the second layer of partition, the relative to the center of the ball to maintain a symmetrical arrangement. If the top plate and the first layer of partition thickness increases, the vertical stiffness and horizontal stiffness were enhanced, but the weight were also increased at the same time, in order to guarantee to near its center of mass, the second layer of partition and the bottom plate should be a corresponding increase thickness, which will reduce the vertical stiffness and horizontal stiffness. Therefore, the four-layer equipment mounting plate is designed to match in pairs, and the corresponding structure is hollowed out to reduce weight.

Figure 3. Diagram of the air-bearing simulator.
The supporting flange is connected with the air float ball, and the bearing cylinder is the main bearing structure. Like the equipment installation plate, increase the main load-carrying structure of wall thickness will enhance the stiffness of structure itself. But for integral components, in order to ensure that the overall center of mass is located near the center of the air float, the mass of the bottom counterweight increases accordingly, which in turn reduces the stiffness of the instrument platform. Therefore, it is necessary to carry out the structure discrete optimization.

The support flange is optimized in the following positions:
- a) Thickness of the reinforcement plate;
- b) Thickness of central cylinder;
- c) Thickness of the connecting surface of the bearing cylinder;
- d) Thickness of top surface;
- e) Thickness of outer ring.

Figure 4. The optimization position of support flanges.

The optimization results of the five dimensions are shown in Table 1 to Table 5. According to the calculation results, it can be concluded that the thickness of the reinforcement plate has a great influence on the stiffness of the support flange, and the increase of the thickness has an obvious effect on improving the stiffness in both directions of the air floating platform. Therefore, the design value of the maximum thickness of the reinforcement plate should be considered. The thickness of the other several dimensions is not sensitive to the influence of the structural stiffness, so choose the thickness value that makes the vertical stiffness and the horizontal stiffness the closest, and at the same time can reduce the weight of the whole structure.

**Table 1.** The optimization result of reinforcement plate thickness.

| The optimization results (The centroid offset /mm) | Dimension /mm |
|---------------------------------------------|----------------|
|                                             | 30     | 35     | 40     |
| Δxz                                        | 0.05801 | 0.05527 | 0.05338 |
| Δzz                                        | 0.03313 | 0.03126 | 0.02981 |
| Δxz/Δzz                                    | 1.751   | 1.768   | 1.791   |

**Table 2.** The optimization result of central cylinder thickness.

| The optimization results (The centroid offset /mm) | Dimension /mm |
|---------------------------------------------|----------------|
|                                             | 30     | 35     | 40     |
| Δxz                                        | 0.05578 | 0.05551 | 0.05527 |
| Δzz                                        | 0.03157 | 0.03170 | 0.03126 |
| Δxz/Δzz                                    | 1.767   | 1.751   | 1.768   |

**Table 3.** The optimization result of the connecting surface of the bearing cylinder thickness.

| The optimization results (The centroid offset /mm) | Dimension /mm |
|---------------------------------------------|----------------|
|                                             | 25     | 35     | 45     | 55     | 65     |
| Δxz                                        | 0.05209 | 0.05284 | 0.05364 | 0.05438 | 0.05527 |
| Δzz                                        | 0.03022 | 0.03036 | 0.03056 | 0.03092 | 0.03126 |
| Δxz/Δzz                                    | 1.724   | 1.740   | 1.755   | 1.759   | 1.768   |
Table 4. The optimization result of top surface thickness.

| The optimization results (The centroid offset /mm) | Dimension /mm | 20  | 25  | 30  | 40  |
|-----------------------------------------------|----------------|-----|-----|-----|-----|
| Δxx                                           | 0.05490        | 0.05495 | 0.05507 | 0.05527 |
| Δzz                                           | 0.03280        | 0.03232 | 0.03192 | 0.03126 |
| Δxx/Δzz                                       | 1.674          | 1.700  | 1.725  | 1.768 |

Table 5. The optimization result of outer ring thickness.

| The optimization results (The centroid offset /mm) | Dimension /mm | 30 | 35 | 40  | 55  |
|-----------------------------------------------|----------------|----|----|-----|-----|
| Δxx                                           | 0.05430        | 0.05459 | 0.05471 | 0.05527 |
| Δzz                                           | 0.03086        | 0.03097 | 0.03103 | 0.03126 |
| Δxx/Δzz                                       | 1.760          | 1.763  | 1.763  | 1.768 |

The wall thickness of the load-bearing cylinder is mainly considered to be optimized. The calculation results are shown in Table 6. According to the calculation results in the table, it can be interpreted that increasing the thickness of the bearing cylinder can simultaneously improve the two-direction stiffness of the air floating platform and the matching of the two-direction stiffness. Therefore, the maximum design value should be selected for the thickness.

Table 6. The optimization result of load-bearing cylinder thickness.

| The optimization results (The centroid offset /mm) | Dimension /mm | 9.5 | 10  | 11  |
|-----------------------------------------------|----------------|-----|-----|-----|
| Δxx                                           | 0.02443        | 0.02422 | 0.02301 |
| Δzz                                           | 0.02321        | 0.02311 | 0.02272 |
| Δxx/Δzz                                       | 1.053          | 1.048  | 1.013  |

In addition to the main load-bearing structure, the thickness of the vertical partition also has an effect on the stiffness of the air floating platform. If the thickness is too large, the offset of the center of mass of the air floating platform will increase. However, if the thickness is too thin, it will be deformed when the spaceborne equipment and instruments are installed, which will make the center of mass of the air floating platform shift, thus increasing the unbalance moment. Therefore, the thickness of the vertical partition component is also optimized. The calculation results are shown in Table 7. According to the calculation results in the table, it can be interpreted that increasing the thickness of the vertical partition can improve the vertical stiffness of the air floating platform, but at the same time, the horizontal stiffness will decrease, and the stiffness gap between the two directions will become larger and larger. According to the design of equal stiffness, the minimum value of thickness should be considered.

Table 7. The optimization result of vertical partition thickness.

| The optimization results (The centroid offset /mm) | Dimension /mm | 8  | 10  | 12  |
|-----------------------------------------------|----------------|----|-----|-----|
| Δxx                                           | 0.02301        | 0.02457 | 0.02595 |
| Δzz                                           | 0.02272        | 0.02164 | 0.02103 |
| Δxx/Δzz                                       | 1.013          | 1.135  | 1.234  |
According to the above optimization analysis and comprehensive consideration of vertical stiffness, transverse stiffness, structural weight and other factors, the final selected structural parameters are shown in the table below.

### Table 8. The optimization result of the structure.

| Item                          | thickness /mm |
|-------------------------------|---------------|
| reinforcement plate           | 40            |
| central cylinder              | 35            |
| support flanges               |               |
| connecting surface of the bearing cylinder | 25 |
| top surface                   | 20            |
| outer ring                    | 30            |
| load-bearing cylinder         | 11            |
| vertical partition            | 8             |

4. **Calculation Results of Moment Compensation for Air Floating Platform**

According to the requirements of the air floating platform body, the platform body needs to achieve rotation within the range of 30°. The air floating platform body is designed according to the optimization results. The deformation cloud images of the horizontal and vertical forces of the optimized air floating platform are shown in Figure 5. The maximum deformation is 0.138mm and 0.085mm, and the corresponding centroid offsets are $\Delta x = 0.02301mm$, $\Delta z = 0.02272mm$. Then, the optimal value $z_1$ calculated according to formula (12) is 0.02297mm, and the initial centroid offset distance is 0.25μm. Combined with the overall weight of the floating platform is about 6t, the interference torque of the reference platform varies with the Angle within the rotation range of 30°, and the maximum interference torque within the movement range of 30° is 0.00048N•m, as shown in Figure 6.

![Figure 5. The deformation cloud map of the air floating platform under horizontal force and vertical force](image)

![Figure 6. The curve of the disturbance torque with angle](image)
5. Conclusion
In this paper, the relationship between the interference moment of the three-axis air-bearing simulator and the design parameters of the platform is established by means of analytical and simulation analysis. Taking the equal stiffness and high stiffness of the three-axis of the air platform as the objective function, the size of the supporting flange, load-bearing cylinder and vertical partition of the main structure of the air platform is discretely optimized, and the optimal structure is obtained. The maximum disturbance moment is 0.00048N•m in the range of 30° when the initial center of mass is offset by 0.25μm from the center of rotation. The research results are of guiding significance to the design and centroid leveling of the three-axis air-bearing simulator and the engineering realization of the interference moment compensation should be demonstrated afterwards.

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