BPS solution for eleven-dimensional supergravity with a conical defect configuration.

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ABSTRACT: In this work, we found the solution for the field equations of eleven-dimensional supergravity with a BPS conical topological defect configuration. We chose the solutions that corresponded to a M2-brane where the space-time presents the co-dimension two. The source of the topological defect is a sigma model where the brane tension is connected with the angular deficit. We analyzed the Killing spinor equations, a 3-form gauge potential and Einstein’s equations, proving that it is possible to find a full solution for the system. We analyzed the near-horizon limit proving that it is possible to obtain an $\text{AdS}_4 \times S^1 \times \mathbb{E}^6$ for certain values of the brane tension. We also discussed some applications in the strong coupling theory for condensed matter systems.

KEYWORDS: Conical Defect, Supergravity.
1. Introduction

The importance of a study on eleven-dimensional supergravity is that this theory presents the maximum dimension admitting supersymmetric extended objects [1]. This theory gives us a low-energy effective description of the M-theory. The full description of the M-theory remains unknown. Therefore, the eleven dimensional supergravity can be considered the main ingredient to study the super-unified theory. An important feature was the discovery of a network of dualities that can map the five superstring theories, I, IIA, IIB, heterotic $E_8 \times E_8$ and SO(32), with the eleven-dimensional supergravity. For this reason it is commonly accepted that the M-theory is the mother of these five superstring theories. The other valid type of duality for the string/M-theory, involves the holographic principle which is crucial for investigating strongly coupled field theories where the most important case is called Anti-de-Sitter/Conformal Field Theory (AdS/CFT) correspondence [2]. The AdS/CFT opened a new branch of applications, for example in QCD [3], condensed matter systems [4, 5, 6, 7, 8] and recently the $AdS_4$ black hole from M-theory [9]. The AdS/CFT conjecture establishes the exact relation between AdS supergravity and Super Conformal Field Theories (SCFT). Maldacena [2] showed that it is possible to establish this exact correspondence if we consider the large limit of the color number, $N \to \infty$, of certain SCFT in the ’tHooft coupling with $\lambda \equiv g_{YM}^2 N$ fixed and $\lambda \gg 1$. Considering the perturbative description of $N$ dynamical Dp-branes coupled to strings one can check that at first order in the $\alpha'$ expansion, with all possible dimensionless parameter in game fixed, decouples into a SCFT living in $(p+1)$ dimension and supergravity in flat space. By taking an analog limit ($g_s = g_{YM}^2$) in
the p-Brane solitonic description coming from the corresponding supergravity analysis, one argues that a pair of decoupled systems survive. The first being fluctuations in the near horizon limit of the given p-Brane solution and the second supergravity in flat space-time. In this way one is encouraged to conjecture the duality among the SCFT in (p+1) dimension coming from the string theory side and supergravity in the near horizon geometry of the p-Brane geometry, which is always $AdS_{p+2} \times S_{D-p}$ with D being the number of space time dimensions where the embedding string theory lives. In the previous sketch some assumptions are implicitly done like for instance the validity of the supergravity and perturbative string descriptions of dynamical branes. But these are guarantied provided one works in the large $N$ with fixed $\lambda >> 1$ limit previously mentioned before with the usual identification among both sides of the duality. The field content of the theory is given by a metric field $g_{MN}$, the 3-form potential $A_{MNP}$ and their fermionic partners. In this theory the AdS space time arises naturally as $AdS_4 \times S^7$ and $AdS_7 \times S^4$. An important ingredient of the string/M theory is the brane formulation. This is an effective formulation that encodes the low energy dynamics of the system of branes and forms. In this framework we can construct the M2-brane following the same prescription used for the Dp-brane. A M2-brane has a three dimensional Poincaré invariant sector and a compact $SO(8)$ invariant group. The important feature of this solution is that it contains a near horizon limit $AdS_4 \times S^7$. By the use of a dual 4-form field strength, we can also construct the M5-brane and their near horizon limit is $AdS_4 \times S_7$. Using this same logic, to understand the M-theory, we used a ABJM theory formulated by Aharony, Bergman, Jafferis, and Maldacena that are Super Conformal Chern-Simons (SCCS) gauge theories with $N = 6$ supersymmetry that are related to the M-theory on $AdS_4 \times S^7 / \mathbb{Z}_k$ with N units of flux. This duality holds when we choose the gauge group $U(N)_k \times U(N)_{-k}$ with SCCS level $k << N^{1/5}$. Otherwise, when $N^{1/5} << k << N$, the most appropriatted theory is the dual description in terms of the IIA string theory in $AdS_4 \times CP^3$. The later being also a very interesting regime of the duality with a lot of applications. Despite only having pointed out some implications of the low energy limit of the M-theory, there is no doubt of the importance of their study. For these and others motivations, this work was dedicated to the development of a new class of solutions for eleven dimensional supergravity with a conical defect. This description required that the space time contains a sector that presents the co-dimension two. Topological defects are expected to be formed during phase transitions in the early universe. The most studied, because of their stability, was the cosmic string. This defect is analog to a superconductor in condensed matter, for this reason, they are some times called flux tubes. These tubes appear in materials and when coupled with the gravitation are called cosmic strings. In the universe these structures were not observed, yet. Some effort has been devoted to their detection. These defects were extensively studied in connection with structure formation but now their interest has been reborn.
with the superstring context. Despite the great interest in the study of topological defects, such as cosmic strings in the superstring theory to be the interface within cosmology [19] they are also important for the study of strongly coupled systems in condensed matter [20, 21, 22] with a lot of contexts. In analogy with the D7-branes, which contains this type of defect [23], the formalism should be a M8 brane, however these types of branes are massive, and are not the aim of this work, for this reason we choose to work with the split (3,2,6). We showed that it is possible to find a BPS-like structure, with this type of construction.

The $\mathcal{N} = 1$ supergravity model for eleven dimension space-time involves a set of massless fields which carry a representation of supersymmetry. Since supersymmetry assigns to each bosonic degree of freedom a corresponding fermionic one, we can obtain a relation between bosons and fermions by supersymmetric transformations. The action of the model, invariant by the SUSY transformation, contains only the metric field $g_{MN}$ and the three form potential $A_{MNO}$ [24]. The bosonic part of the low-energy action is given by

$$S = \frac{1}{4} \int d^Dx \sqrt{-g} R - \frac{1}{48} \int d^Dx \sqrt{-g} F_{MNOP} F^{MNOP}$$
$$+ \frac{1}{(12)^4} \int d^Dx \epsilon^{MNOQRSTU VW} F_{MNOP} F_{QRSTUVW} A_{UVW}$$

(1.1)

where the capital letters index all space from $M = 0...10$. This action is invariant in $\mathcal{N} = 1$ supersymmetric transformations given by the following equations

$$\delta e^N_M = -i \bar{\epsilon} \Gamma^N_M \psi_M$$
$$\delta \psi_M = D_M \epsilon - \frac{1}{288} (\Gamma_M^{LOPQ} + 8 \Gamma^{OPQ} \delta_M^L \Gamma^PQ) F_{LOPQ} \epsilon = \bar{D}_M \epsilon$$
$$\delta A_{MNO} = \frac{3}{2} \bar{\epsilon} \gamma^{[MN} \psi_{O]}.$$ 

(1.2)

(1.3)

(1.4)

We organized this work as follows. Section 2 is dedicated to the construction of the Killing spinor equations as well as the formulation of BPS conditions. In Section 3, we discuss the brane action, which is a sigma model. We also analyze the complete solution using the field equations for the 3-form gauge, we discuss the equations of Einstein and the topological conical configuration of the defect. The analysis of the near horizon limit AdS were presented in Section 4. Finally, in Section 5, we present our main results with a discussion about a dimensional reduction of eleven dimensions to ten. We also make some analyses about the applications for future works.

2. Killing spinors for the M2-brane with a (3,2,6) split.

In this section, we consider the construction of the M2-brane BPS solution with a
\((3,2,6)\) split. The disposition of this brane, according to the pattern, is

\[
M2 : 1 \ 2 \ \odot \ \odot \ \odot \ \odot \ \odot \ \odot \ \odot \ (2.1)
\]

The fields here only depend on the sector of the co-dimension two that was represented by \(\odot\). The other transverse sector is represented by \(\otimes\) that presents six coordinates. We labeled this pattern as \((3,2,6)\), where the underlined number refers to the dependence of the fields. In this split, the D=11 coordinates can be written as

\[
X^M = (X^\mu, Y^m, Z^{\tilde{m}}) \ (2.2)
\]

where we labeled \(\mu = 0, 1\) and 2 as the Minkowiski space-time, \(m = 1, 2\) is the conical defect transverse coordinates and \(\tilde{m} = 1, \ldots, 6\) refers to the other transverse coordinates. The metric is as follows

\[
ds^2 = e^{2A(y_1,y_2)}(-dt^2 + dx_1^2 + dx_2^2) + e^{2B(y_1,y_2)}(dy_1^2 + dy_2^2) + e^{2C(y_1,y_2)} \sum_{n=1}^{6} dz_n dz_n \ (2.3)
\]

we considered the ansatz of the three-form gauge field as follows

\[
A_{\mu\nu\rho}(y_1,y_2) = \pm \frac{c}{3} g^{\mu\nu\rho} e^{E(y_1,y_2)} \ (2.4)
\]

where \(c\) is a constant that in this moment is considered arbitrary and \(3g\) is the metric determinant that gives us Levi Civita’s tensor definition that in a gravitational context is \(\epsilon_{\mu\nu\rho} = g_{\mu\alpha}g_{\nu\beta}g_{\rho\lambda}\epsilon^{\alpha\beta\lambda}\). We considered all other components such as \(A_{MNO}\), the graviton and the gravitino \(\psi_M\) as zero. We can see that \(A, B, C\) and \(E\) depend only on \(y^m\). We chose this theory due to the fact that it is the simplest case where there is a conical defect in 11 dimensions. In this split there are four arbitrary functions \((A, B, C\) and \(E\)) which are reduced to one due to the requirement of the field configuration, \((2.3)\) and \((2.4)\), preserve some unbroken supersymmetry. For this reason, there are Killing spinors that satisfy the following equation

\[
\bar{D}_M \epsilon = 0 \ (2.5)
\]

where \(\bar{D}_M\) is the super covariant derivative appearing in gravitino’s supersymmetric transformation. We can write this \((1.3)\) in the following manner

\[
\delta \psi_M = (D_M + A_M^{(1)} + A_M^{(2)}) \epsilon = \bar{D}_M \epsilon \ (2.6)
\]

The covariant derivative part involving the spin connection is given by \(D_M \epsilon = \partial_M \epsilon - \frac{1}{4} \omega_{M}^{AB} \Gamma_{AB} \), where \(M\) is the Lorentz index and \(A\) is the flat index. The convention is \(e_A^M e_{BM} = \eta_{AB}\) and \(e_A^M e_{NA} = g_{MN}\). The covariant derivative involving the flux field contribution is as follows

\[
\omega_M^{AB} = e_N^B e_A^O \Omega_{M}^{NO} - e_A^O \partial_M e_{O}^{B} \ (2.7)
\]

\[
A_M^{(1)} = -\frac{1}{288} \Gamma_{M}^{NOPQ} F_{NOPQ} \ (2.8)
\]

\[
A_M^{(2)} = \frac{1}{36} \Gamma^{OPQ} \delta_L^{M} F_{LOPQ} \ (2.9)
\]
where $F_{MNOP} = 4 \partial_{(M} A_{NOP)}$. The first term in (2.7) is the Christoffel symbol that we labeled as $\Omega_{MNO}$ to differ from the Dirac Matrix where we originally use $\Gamma$. The Dirac Matrices, $\Gamma_A$, in $D = 11$ satisfy $[\Gamma_A, \Gamma_B]_+ = 2 \eta_{AB}$ and the metric signature is $\eta_{AB} = diag(-,+,...+)$. We consider the decomposition of the $\Gamma$ matrix, that respecting the $(3,2,6)$ split, is shown as follows

$$\Gamma_A = (\gamma_\alpha \otimes \Sigma_3 \otimes \Gamma_7, \mathbb{I} \otimes \Sigma_\alpha \otimes \mathbb{1}, \mathbb{I} \otimes \Sigma_3 \otimes \Theta_\bar{a}), \quad (2.10)$$

where $\gamma_\alpha$, $\Sigma_\alpha$ and $\Theta_\bar{a}$ Dirac matrices correspond to $D = 3$, $D = 2$ and $D = 6$ dimensions respectively, with

$$\Sigma_3 \equiv \Sigma_1 \Sigma_2 \quad (2.11)$$

$$\Gamma_7 \equiv \Gamma_1 \Gamma_2 \ldots \Gamma_6. \quad (2.12)$$

The consistent form to write the spinor is as follows

$$\epsilon = \epsilon_1 \otimes \epsilon_2 (y_1, y_2) \otimes \epsilon_3 \quad (2.13)$$

where $\epsilon_1$ is a constant spinor of SO(1,2), $\epsilon_2$ is a spinor of co-dimension 2 sector and $\epsilon_3$ is a constant spinor of $D=6$ transverse sector. The super covariant derivatives with a three split are as follows

$$\bar{D}_\mu = \partial_\mu - \frac{1}{2} \gamma_\mu e^{-A} \Sigma^a \partial_a e^A \Sigma_3 \Gamma_7 \mp \frac{1}{6} \gamma_\mu e^{-A} \Sigma^m \partial_m e^E \quad (2.14)$$

$$\bar{D}_a = \partial_a + \frac{1}{4} e^{-B} (\Sigma_a \Sigma^m - \Sigma^m \Sigma_a) \partial_m e^B \mp \frac{1}{24} e^{-A} (\Sigma_a \Sigma^m - \Sigma^m \Sigma_a) \partial_m e^E \Sigma_3 \Gamma_7 \mp \frac{1}{6} e^{-A} \partial_a e^E \Sigma_3 \Gamma_7 \quad (2.15)$$

$$\bar{D}_{\bar{m}} = \partial_\bar{m} + \frac{1}{4} e^{-C} (\Theta_{\bar{m}} \Sigma^a - \Sigma^a \Theta_{\bar{m}}) \partial_a e^C \mp \frac{1}{24} e^{-A} (\Theta_{\bar{m}} \Sigma^a - \Sigma^a \Theta_{\bar{m}}) \partial_a e^E \Sigma_3 \Gamma_7 \quad (2.16)$$

The constraint to preserve unbroken supersymmetry is given by (2.5) considering (2.14), (2.15) and (2.16) in (2.10) it leads to, with $c=1$, a constraint for the metric function $A$,

$$-\frac{1}{6} \gamma_\mu \Sigma^a \partial_a E (1 \pm \Sigma_3 \Gamma_7) \epsilon = 0 \rightarrow A = \frac{1}{3} E. \quad (2.17)$$

The constraint for the metric function $B$ is given by

$$-\frac{1}{24} (\Sigma_a \Sigma^m - \Sigma^m \Sigma_a) \partial_m E (1 \pm \Sigma_3 \Gamma_7) = 0 \rightarrow B = -\frac{1}{6} E \quad (2.18)$$

$$\partial_\bar{m} \mp \frac{1}{6} \partial_\bar{m} E \Sigma_3 \Gamma_7 = 0 \rightarrow \epsilon = e^{-E/6} \epsilon_0, \quad (2.19)$$
and finally the constraint for the function \( C \) is
\[
-\frac{1}{24}(\Theta \tilde{m} \Sigma^a - \Sigma^a \Theta \tilde{m}) \partial_a E (1 \pm \Sigma_3 \Gamma_7) = 0 \rightarrow C = -\frac{1}{6} E. \tag{2.20}
\]

then, we can write the metric in the form
\[
ds^2 = e^{\frac{4}{3}E} \left[ -dt^2 + dx_1^2 + dx_2^2 + e^{-E}(dy_1^2 + dy_2^2) \right] + e^{\frac{4}{3}E} \sum_{m=1}^{6} dz_m dz_{\tilde{m}}. \tag{2.21}
\]

In this calculation we omit the tensorial product and the \( \pm \) signs are correlated with our gauge potential ansatz (2.4). The complete solution to the spinor (2.2) is given by
\[
\epsilon = e^{-E/6} \epsilon_0 \tag{2.22}
\]
\[
\mathbb{I} \otimes \Sigma_3 \otimes \Gamma_7 \epsilon_0 = \mp \epsilon_0 = \begin{cases} 
\Sigma_3 \epsilon_{02} = \mp \epsilon_{02} \\
\Gamma_7 \epsilon_{03} = \mp \epsilon_{03}
\end{cases} \tag{2.23}
\]
where \( \epsilon_{02} \) is a constant co-dimension two spinor and \( \epsilon_{03} \) is a co-dimension six spinor.

We showed in this section that our propose satisfies these Killing spinor equations. These equations are responsible for the BPS bound that garanties the stability of our solution. In the next Section, we introduced the brane action. We studied the solution for the function \( E \) by analyzing the gauge field and Einstein equations.

### 3. The brane’s configuration analysis

In the last section we showed that it is possible to find a stable co-dimension two solution analyzing the Killing Spinor equations. In this section we discussed the source of the fields of our system considering the brane action. The brane action is compatible with eleven dimensional supergravity [25] and can be written as
\[
S_M = \mu \int d^3 \xi \left( -\frac{1}{2} \sqrt{-\gamma} \gamma^{ij} \partial_i X^M \partial_j X^N g_{MN} + \frac{1}{2} \sqrt{-\gamma} 
\right.
\]
\[
\pm \frac{1}{3!} \epsilon^{ijk} \partial_i X^M \partial_j X^N \partial_k X^O A_{MNO} \right) \tag{3.1}
\]
where \( \mu \) is the tension of the supermembrane. This action is invariant to \( N = 1 \) supersymmetry transformations given by the following equations
\[
\delta \Psi_M = \bar{D}_M \epsilon(X) \tag{3.2}
\]
\[
\delta \theta = (1 \pm \Gamma) \kappa(\xi) + \epsilon(X) \tag{3.3}
\]
where \( \Psi_M \) is the gravitino and \( \theta \) is the fermionic coordinate, \( \kappa(\xi) \) is the Siegel symmetry parameter and the covariant derivative \( \bar{D}_M \) is the same as (2.6) with a \( \Gamma \) given the following
\[
\Gamma \equiv \frac{1}{3! \sqrt{-\gamma}} \epsilon^{ijk} \partial_i X^M \partial_j X^N \partial_k X^P \Gamma_{MNP} \tag{3.4}
\]
with the BPS bound constraints as

$$\bar{D}_M \epsilon = 0 \quad \Gamma \epsilon = \mp \epsilon \tag{3.5}$$

The antisymmetric tensor field equation is given by the variation of the action (1.1) in relation with the potential $A_{MNO}$ and results in

$$\partial_M (\sqrt{-g} F^{MNOP}) + \frac{1}{1152} \epsilon^{NOPM_1 \ldots M_8} F_{M_1 \ldots M_4} F_{M_5 \ldots M_8} = \mp \kappa \mu \int \epsilon^{ijk} \partial_i X^N \partial_j X^O \partial_k X^P \delta^{11}(x - X). \tag{3.6}$$

The equation of the membrane field is

$$\partial_i (\sqrt{-g} \gamma^{ij} \partial_j X^N g_{MN}) + \frac{1}{2} \sqrt{-g} \gamma^{ij} \partial_i X^N \partial_j X^P \partial_M g_{NP} \pm \frac{1}{3!} \epsilon^{ijk} \partial_i X^N \partial_j X^O \partial_k X^P F_{MNOP} = 0 \tag{3.7}$$

where $\gamma_{ij} = \partial_i X^M \partial_j X^N g_{MN}$. We considered the static gauge choice as

$$X^\mu = \xi^\mu, \quad \mu = 0, 12, \tag{3.8}$$

and the transverse sector as

$$Y^m = \text{constant}, \tag{3.9}$$

$$Z^\tilde{m} = \text{constant} \tag{3.10}$$

It is easy to see that (3.8), (3.9) and (3.10) are compatible with our solution, where $\Gamma_{MNO}$ is reduced to $\Gamma_{\mu \nu \gamma}$ by using the duality relation, $\Gamma_{\mu \nu \kappa} = \epsilon_{\mu \nu \kappa a_1 \ldots a_6} \Sigma^a \Sigma^b \Gamma_{\tilde{a}_1} \ldots \Gamma_{\tilde{a}_6}$, that with the split (2.10) giving us

$$\Gamma = \mathbb{1} \otimes \Sigma_3 \otimes \Gamma_7 \tag{3.11}$$

where $\Sigma_3$ and $\Gamma_7$ are defined in the eqs. (2.11) and (2.12). We can compute the solution of the gauge field equation (3.6) considering the brane action (3.1), the metric (2.21) and the action (1.1). These, together with the ansatz (2.4) give us the only contribution for the equation of motion as

$$\delta^{mn} \partial_m \partial_n e^{-E(y_1, y_2)} = -16 G \mu \delta^2(y_1, y_2) \tag{3.12}$$

In conclusion we obtained the solution as the following

$$e^{-E(r)} = 1 - 8G \mu \ln \left( \frac{r}{r_0} \right) \tag{3.13}$$

where $r_0$ is the minimum radius of circle and $r = \sqrt{y_1^2 + y_2^2}$. 
Now, we will analyze the Einstein equation from the conical defect in eleven dimensional supergravity. We considered the fermion field as zero and we worked on the bosonic part of the \(D = 11\) theory. The Einstein equation can be written as

\[
R_{MN} - \frac{1}{2} g_{MN} R - \frac{1}{12} (F_{OPQ}^M F^{NOPQ} - \frac{1}{8} g_{MN} F_{OPQR} F^{OPQR} ) = 8 \pi G T_{MN},
\]

where the energy momentum tensor \(T_{MN}\) is given by

\[
T_{MN} = -\mu \int d^3 \xi e^{ijk} \partial_i X^M \partial_j X^N \delta^{11}(x - X) \sqrt{-g}
\]

we redefined the Einstein tensor as \(\tilde{G}_{MN}\) only for commodity where we have identified three different parts. One of these is given by the Minkowski sector,

\[
G_{\mu\nu} = -\frac{1}{4} \delta^{ab} \left[ \partial_a E(y_1, y_2) \partial_b E(y_1, y_2) - 2 \partial_a \partial_b E(x_1, x_2) \right] e^E \eta_{\mu\nu}
\]

the other part corresponds to the co-dimension two sector,

\[
G_{ab} = \frac{1}{4} \left( \left( \partial_a E(y_1, y_2) \right)^2 - \left( \partial_b E(x_1, x_2) \right)^2 \right) / \left( \partial_a E(y_1, y_2) \partial_b E(y_1, y_2) \right) = \frac{1}{4} \left( \left( \partial_a E(x_1, x_2) \right)^2 - \left( \partial_a E(y_1, y_2) \right)^2 \right)
\]

and the last one is given by the co-dimension six sector,

\[
G_{\tilde{a} \tilde{b}} = -\frac{1}{4} \delta^{ab} \partial_a E(y_1, y_2) \partial_b E(y_1, y_2) \eta_{\tilde{a} \tilde{b}}
\]

Using the definition of flux we can write the Einstein equation for the defect where the non zero components for the \(\tilde{G}_{MN}\) are

\[
\tilde{G}_{\mu\nu} = -\frac{1}{2} e^{2E(y_1, y_2)} \delta^{mn} \partial_m \partial_n e^{-E(y_1, y_2)} \eta_{\mu\nu}
\]

the above equation respects the energy momentum configuration

\[
\begin{align*}
T^t_t &= T^x_1 = T^x_2 = 8 G \mu \delta^2(y_1, y_2) e^{\frac{4}{3}E} \\
T^y_{y_1} &= T^y_{y_2} = 0 \\
T^z_{z_1} &= \ldots = T^z_{z_6} = 0
\end{align*}
\]

The energy momentum configuration (3.21) is given by the brane action (3.1) with the brane conditions (3.8-3.10). Analysing only the brane’s contribution of the energy momentum tensor we found an essential aspect of this theory. Despite the eleven dimensional space time, the energy momentum tensor is zero in a transverse plane.
This is a generalization of a four dimensional conical defect for eleven dimensional supergravity. We can consider the redefinition as

$$\rho = \frac{(r/r_0)^{1-4G\mu}}{(1-4G\mu)}$$  \hspace{1cm} (3.22)$$

with the approximation $1 - 8G\mu \ln(r/r_0) \sim (r/r_0)^{-8G\mu}$ we have the metric below as

$$ds^2 = f(\rho)(ds^2_{CD} + dx^2_2) + g(\rho)ds^2_6$$  \hspace{1cm} (3.23)$$

where for convenience we defined the metric with a dimensional conical defect sector given by $ds^2_{CD}$ and the Euclidian sector $ds^2_6$ is given by

$$ds^2_{CD} = -dt^2 + dx_1^2 + dp^2 + \rho^2 d\bar{\theta}^2$$  \hspace{1cm} (3.24)$$

$$ds^2_6 = dr'^2 + r'^2 d\Omega_5^2$$  \hspace{1cm} (3.25)$$

where $r' = \sqrt{z_1^2 + ... + dz_6^2}$ with $f(\rho) = \left[1 - 2\Delta \ln \frac{\rho}{\rho_0}\right]^{-\frac{2}{3}}$ and $g(\rho) = \left[1 - 2\Delta \ln \frac{\rho}{\rho_0}\right]^{1/3}$ with $\rho_0 = r_0/\kappa$, $\kappa = (1 - 4G\mu)$ and $\Delta = \frac{4G\mu}{1-4G\mu}$. This metric corresponds to the conical defect with warp factors. The sector $ds^2_{CD}$ of this metric is locally known as Minkowiski with $\bar{\theta} = \kappa \theta$, like a cosmic string in four dimensions. If $\kappa \leq 1$ or $\kappa \geq 1$ we can write the range of the angular sector as $0 \leq \bar{\theta} \leq 2\pi \kappa$. This conical geometry has an angular deficit given by $\delta \theta = 8\pi G\mu$. The charge can be calculated using the same framework developed in [26, 27] and by using the Dirac quantization we can prove that this charge is quantized. We also can see that the metric (3.23) presents a warp factor [28] similar to the ones in a scalar tensor theory in a weak field approximation [29].

4. The $AdS_4 \times S^1 \times E^6$ near the horizon limit

In this section let us consider the near horizon limit AdS for SCFT. We can write the metric (3.23) as

$$ds^2 = \left(\frac{r}{r_0}\right)^{4\alpha} (-dt^2 + dx_1^2 + dx_2^2) + \left(\frac{r}{r_0}\right)^{-2\alpha} d^2r + \left(\frac{r}{r_0}\right)^{-2\alpha} (r^2 d\bar{\theta}^2 + ds_6^2)$$  \hspace{1cm} (4.1)$$

where $\alpha$ depends of brane tension $\mu$ as $\alpha = \frac{4}{3}G\mu$. We saw in last sections that the $r_0$ is related with the minimum radial length. The Euclidean metric $ds^2_6$ is given by (3.25). Now let us relate this radial length with the low energy limit. The decoupled limit is obtained by taking the 11 dimensional Planck length to zero $l_p \rightarrow 0$, keeping the world volume energies fixed and taking the separation $U^a \equiv r/l_p^a$ and $V^a' \equiv r'/l_p'$ fixed. With this limit we put the parameter $\alpha$, ”$a$” and ”$b$” as arbitrary and we
analyzed here the conditions for AdS. With this transformation the metric \((4.1)\) becomes

\[
ds^2 = \frac{U^{4\alpha a}}{(l_p^{-b} r_0)^{4\alpha}} (-dt^2 + dx_1^2 + dx_2^2) + \frac{a^2 U^{2\alpha(1-\alpha) - 2}}{l_p^{-2b(1-\alpha)} r_0^{-2\alpha}} dU^2 + \frac{U^{2\alpha(1-\alpha)}}{l_p^{-2b(1-\alpha)} r_0^{-2\alpha}} d^2 \theta
\]

\[
+ \frac{a^2 U^{-2\alpha} V^{2\alpha}}{l_p^{2\alpha} r_0^{-2\alpha}} dV^2 + \frac{U^{-2\alpha} V^{2\alpha}}{l_p^{2\alpha} r_0^{-2\alpha}} V^{2a' - 2\alpha} d^2 \Omega_6
\]

Let us consider \((l_p^{-b} r_0)^{4\alpha} = \frac{a^2 r_0^{2\alpha}}{l_p^{2\alpha(1-\alpha)}} = R_{AdS}^2\) then we get \(a = 1/2\). It is easy to see the low energy \(AdS_4\) near the horizon limit when \(\alpha \to 1\). There is a non zero flux of the dual four-form field strength on the on \(S^1\). In this limit \(a' = 1/2\), and \(V/U \sim 1\). The metric is as follows

\[
ds^2 = \frac{U^2}{R_{AdS}^2} (-dt^2 + dx_1^2 + dx_2^2) + \frac{R_{AdS}^2}{U^2} dU^2 + (2 R_{AdS})^2 d^2 \theta
\]

\[+ dz^2 + R^2 d\Omega_5^2\] (4.3)

where \(z = R^2 \ln V\). We can see that this near horizon limit’s metric is the product of an Anti-de-Sitter space-time in the form of \(AdS_4 \times S^1 \times \mathbb{R}^6\). We can analyze the metric \((4.3)\) and we can see that this limit corresponds to a fix \(G\mu\) responsible for the AdS near the horizon limit. In this theory we have \(a = 1/2\) and \(b = 3/2\), both compatible with the Maldacena analysis. The AdS radius is given by \(R_0 = 2 R_{AdS} = l_p (2^5 \pi^2 N)^{1/6} = r_0\) so we can relate the minimum radius of the circle with the scale of M2-brane.

### 5. Discussions and Remarks

In this work we analyzed the topological defect with a conical deficit in eleven dimensions. In our prescription we found the BPS solution analyzing the Killing spinor, gauge field and Einstein equations. We considered the space time with a \((3,2,6)\) split where the co-dimension two represents the defect and generates the flux. We showed that the split is consistent with the M2-Brane configuration and that it is possible to describe a topological defect energy density with the sigma models. This split gives us the BPS solutions for Killing spinors without intersection\([30]\) or rotation branes \([34]\). We analyzed the Einstein equations and showed that the configuration is a general form for conical defect like a cosmic string with warp factors in eleven dimensions. Despite similarities with the results of theories with lower dimensions, it is important to observe that the eleven dimensional supergravity theory gives us a different solution for the usual conical defect in gravitation. This solution presents the mass for length unit \(\mu\), that is the tension of the brane, in the Planck scale. In a solution for this defect in ten dimensions, \(G\mu \geq 10^{-3}\), but in the cosmic string the quantity is \(G\mu \leq 10^{-5}\). The other difference is that our solution presents a
natural superconductivity given by the presence of the BPS SUSY bound that admit fermionic superconductivity \([31, 32]\). In our work we analyzed the possibility of the existence of this solution directly given by the M2-brane, but this was only the first step to understand our solution. Our prescription is general, consisting in a vast variety of applications. Here we will discuss some of these applications. Nowadays there are many ways to get low dimension prescriptions. Our framework contains the AdS near horizon limit important for the holographic principle. We analyzed this limit and concluded that the metric is \(AdS_4 \times S^3 \times \mathbb{R}^6\). The interesting feature here is that this limit, considering the \((3,2,6)\) split, is only possible because of the existence of a topological defect. This limit occurs to the value \(G\mu = 7.5 \times 10^{-1}\). This value is compatible with the fact of the ten dimensions to be \(G\mu \geq 10^{-3}\). There are a few studies about intersecting and rotationing branes \([33]\) that obtain a configuration \(AdS_3 \times S^3 \times \mathbb{R}^5\). In our framework we can obtain the \(AdS_3\) near the horizon limit in the resulting theory after the compactification into ten dimensions. We have a dimensional reduction of space-time and world volume in \(D = 11\) that reduces the combined type IIA supergravity-superstring field equations into \(D = 10\). The split \((10,1)\) gives us \(x^M = (x^\mu, x^2)\), where \(\mu = (0,1,3,...9)\). We used the same definition standard \([10]\) and consider that the dilaton is related with the \(g_{22}\) component. After the reduction we got \(g_{22} = e^{4\phi/3}\) with \(g_{MN} = e^{-\phi/6}g_{\mu\nu}\) and the gauge potential became, \(A_{\mu\nu} = B_{\mu\nu}\). Considering the solution \([11]\), with our split we had after the reduction the dilaton and the 2-form of gauge given by \(B_{01} = \pm e^{-E}\) and \(e^\phi = e^{4E}\) with the brane condition \(X^\mu = \xi^\mu, \mu = 0,1, X^a = \text{constant}\) and \(Z^{\tilde{m}} = \text{constant}\). We used the same procedure for \(AdS_4\) near the horizon limit of the last section and we found \(AdS_3 \times S_1 \times \mathbb{R}^6\). We found the \(AdS_3\) near the horizon limit without intersection brane in ten dimensions. We can obtain more complicated solutions by intersection branes \([30]\) or adding angular momentum \([34]\), this is a subject to future works. The idea is to analyze the possibility of intersecting branes with the defect appearing in the Minkowiski sector \([8]\). Another important application is the study of strongly coupled condensed matter systems. The topological defects type cosmic strings can be viewed as vortex configuration where the only difference is the coupling with gravitation. We had seen that the conical defect configuration in the \(AdS_4\) can, via holography, give us the \((2 + 1)\) models where the defect is preserved on the boundary via metric\([35]\). Therefore we believe that the vortices in eleven dimensions supergravity theory can give us an interesting prescription to the study of the holographic principle mainly for graphene-like materials. Another important method to understand these systems is the formulation developed by \([36, 37]\) with the study of conserved charges that are preserved on the boundary. In materials graphene like has been shown that a vortex configuration can give us a mass gap. But if the material is put into a cone shape there appear polarized currents that resembles a gravitational deformation caused by the angular deficit like cosmic string\([20, 21, 22]\). The idea now is to develop our framework to understand these effects.
among others.

ACKNOWLEDGEMENT: The author would like to thank the ICTP in Trieste (Italy). The work was partially supported by a CNPq/Brazil fellowship.

References

[1] W. Nahm, Nucl. Phys. B 135, 149 (1978).

[2] J. M. Maldacena, Adv. Theor. Math. Phys. 2, 231 (1998) [hep-th/9711200];
   N. Itzhaki, J. M. Maldacena, J. Sonnenschein and S. Yankielowicz, Phys. Rev. D 58, 046004 (1998) [hep-th/9802042].

[3] R. C. Brower and C. -ITan, Nucl. Phys. B 662, 393 (2003) [hep-th/0207144].

[4] D. Cassani and A. F. Faedo, JHEP 1105, 013 (2011) [arXiv:1102.5344 [hep-th]].

[5] Y. Bu, Phys. Rev. D 86, 046007 (2012) [arXiv:1211.0037 [hep-th]].

[6] A. Donos and J. P. Gauntlett, JHEP 1012, 002 (2010) [arXiv:1008.2062 [hep-th]].

[7] S. A. Hartnoll, J. Polchinski, E. Silverstein and D. Tong, JHEP 1004, 120 (2010)
   [arXiv:0912.1061 [hep-th]].

[8] C. A. B. Bayona, C. N. Ferreira and V. J. V. Otoya, Class. Quant. Grav. 28, 015011
   (2011) [arXiv:1003.5396 [hep-th]];

[9] N. Halmagyi, M. Petrini and A. Zaffaroni, JHEP 1308, 124 (2013) [arXiv:1305.0730
   [hep-th]].

[10] K. Pilch, P. van Nieuwenhuizen and P. K. Townsend, Nucl. Phys. B 242, 377 (1984);
    M. J. Duff and K. S. Stelle, Phys. Lett. B 253, 113 (1991); M. J. Duff and
    C. N. Pope, In *Trieste 1982, Proceedings, Supersymmetry and Supergravity '82*,
    183-228 and London Imp. Coll. - ICTP-82-83-07 (83,REC.MAR.) 49p;
    P. G. O. Freund and M. A. Rubin, Phys. Lett. B 97, 233 (1980); M. J. Duff, ” The
    World in Eleven Dimension: Supergravity, Supermembranes and M-Theory”, Studies
    in High Energy Physics Cosmology and Gravitation, IOP, Publishing Ltd 1999.

[11] M. J. Duff, R. R. Khuri and J. X. Lu, Phys. Rept. 259, 213 (1995) [hep-th/9412184].

[12] O. Aharony, O. Bergman, D. L. Jafferis and J. Maldacena, JHEP 0810, 091 (2008)
    [arXiv:0806.1218 [hep-th]].

[13] B. Chandrasekhar and B. Panda, Int. J. Mod. Phys. A 26, 2377 (2011)
    [arXiv:0909.3061 [hep-th]].

[14] S. Giardino and H. L. Carrion, JHEP 1108, 057 (2011) [arXiv:1106.5684 [hep-th]]
    (and ref therein)
[15] C. Kalousios, M. Spradlin and A. Volovich, JHEP 0907, 006 (2009) [arXiv:0902.3179 [hep-th]] (and references therein).

[16] M. Naghdi, Phys. Rev. D 88, 026013 (2013) [arXiv:1302.5294 [hep-th]].

[17] M. Sadegh Movahed and S. Khosravi, JCAP 1103, 012 (2011) [arXiv:1011.2640 [astro-ph.CO]].

[18] M. S. Movahed, B. Javanmardi and R. K. Sheth, arXiv:1212.0964 [astro-ph.CO].

[19] E. J. Copeland and T. W. B. Kibble, Proc. Roy. Soc. Lond. A 466, 623 (2010) [arXiv:0911.1345 [hep-th]].

[20] M. A. H. Vozmediano, M. I. Katsnelson and F. Guinea, Phys. Rept. 496, 109 (2010).

[21] A. Cortijo and M. A. H. Vozmediano, Nucl. Phys. B 763, 293 (2007) [Nucl. Phys. B 807, 659 (2009)] [cond-mat/0612374].

[22] A. Cortijo and M. A. H. Vozmediano, Europhys. Lett. 77, 47002 (2007) [cond-mat/0603717 [cond-mat.str-el]].

[23] E. A. Bergshoeff, J. Hartong, T. Ortin and D. Roest, Fortsch. Phys. 55, 661 (2007); A. Dabholkar, G. W. Gibbons, J. A. Harvey and F. Ruiz Ruiz, Nucl. Phys. B 340, 33 (1990);

[24] E. Cremmer, B. Julia and J. Scherk, Phys. Lett. B 76, 409 (1978); E. Cremmer and B. Julia, Nucl. Phys. B 159, 141 (1979); E. Cremmer and S. Ferrara, Phys. Lett. B 91, 61 (1980); L. Brink and P. S. Howe, Phys. Lett. B 91, 384 (1980).

[25] E. Bergshoeff, M. J. Duff, C. N. Pope and E. Sezgin, Phys. Lett. B 199, 69 (1987).

[26] R. C. Myers and M. J. Perry, Annals Phys. 172, 304 (1986).

[27] R. Bousso and J. Polchinski, JHEP 0006, 006 (2000) [hep-th/0004134].

[28] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999)

[29] M. E. X. Guimaraes, Class. Quant. Grav. 14, 435 (1997) [gr-qc/9610007].

[30] J. P. Gauntlett, In *Seoul/Sokcho 1997, Dualities in gauge and string theories* 146-193 [hep-th/9705011].

[31] S. C. Davis, A. -C. Davis and M. Trodden, Phys. Lett. B 405, 257 (1997) [hep-ph/9702360].

[32] P. .Brax, C. van de Bruck, A. C. Davis and S. C. Davis, JHEP 0606, 030 (2006) [hep-th/0604198].

[33] H. J. Boonstra, B. Peeters and K. Skenderis, Phys. Lett. B 411 (1997) 59 [hep-th/9706192].
[34] J. P. Gauntlett, R. C. Myers and P. K. Townsend, Phys. Rev. D 59, 025001 (1998) [hep-th/9809065].

[35] A. Ballon-Bayona, C. N. Ferreira and V. J. V. Otoya, Phys. Rev. D 87, 106007 (2013) [arXiv:1302.0802 [hep-th]].

[36] A. O'Bannon, Phys. Rev. D 76, 086007 (2007) [arXiv:0708.1994 [hep-th]].