Spectral Madness - Widening the Scope of a Partial Dynamic Symmetry

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Abstract

If one examines two-body matrix elements from experiment, one notices that not only \( J=0 \) \( T=1 \) lies low, but also \( J=1, T=0 \) and \( J=J_{\text{max}}=2 \) \( T=0 \). It is sometimes thought that one needs both \( T=1 \) and \( T=0 \) two-body matrix elements to get equally spaced spectra of even I states, i.e., vibrational spectra. We here attempt to get equally spaced levels with only those that have \( T=1 \) (even \( J \)). As an example, we perform single-j \( f_{7/2}^{\dagger} \) calculations in \(^{44}\)Ti and \(^{46}\)Ti. We then shift gears and decide to play around with the input two particle matrix elements (not worrying about experiment) to generate interesting spectra, e.g., rotational spectra, with and then without \( T=0 \) two-body matrix elements. We also consider simple interactions (e.g. "123","1234" and "12345") and find an expanded partial dynamical symmetry.

1 Introduction

In a Nature article by B. Cederwall et al.[1], they report findings of equally spaced levels \( J=0, 2, 4, 6 \) in the 8 hole nucleus \(^{92}\)Pd. Their calculated B(E2)'s [1,2] are not consistent with a simple vibrational interpretation. In the “supplements material” of the Nature article they feel that they have an isoscalar spin aligned coupling scheme and emphasize configurations in which a neutron and proton couple to \( J=J_{\text{max}}=9 \) in the \( g_{9/2} \) shell. Clearly, they emphasize the importance of odd-J \( T=0 \) two-body matrix elements. Robinson et al.[3] noted that in a large space calculation the static quadrupole moment of the lowest \( 2^+ \) state of \(^{92}\)Pd was very small, consistent with the vibrational picture. On the other hand, \(^{96}\)Cd turned out to be prolate, and \(^{88}\)Ru, oblate.

Previous to this, Robinson et al.[4] made a study of doing full fp calculations of another 8 particle system, \(^{48}\)Cr. They compared the results of including all two-body matrix elements with those in which all \( T=0 \) two-body matrix elements were set to zero. The qualitative discussions in the 2 cases were quite different. In a lanl preprint by Robinson et al.[5], a figure is shown which confirms the implications of refs [1,2] that dire consequences occur when \( T=0 \)
two-body matrix elements are set to zero. With one of the interactions used, the first 6+ and 8+ states are nearly degenerate and the B(E2)'s 8 to 6 and 6 to 4 are very small. On the other hand, for 48Cr the emphasis was that it was hard to tell which of the calculations agreed better with experiment – full interaction of T=1 only. Sure, there were some differences, but both spectra looked similar – sort of vibrational, but with a tendency to rotation. The B(E2)'s were larger in the full calculation, but this could be accommodated to a large extent by changing the effective charges.

Although the above works serve as a stimulus for we are about to do, we would like to separate ourselves from discussions and arguments about the relative merits therein. Rather, we start anew and address the problem of how two-body matrix elements affect the spectra of more complex nuclei.

2 Attempt to generate near equally spaced spectra with only T=1 two-body matrix elements

In 1963 and 1964, calculations were performed to obtain wave functions and energy levels in the f_{7/2} shell by Bayman et al. [6], McCullen et al. [7] and French et al. [8]. At that time, the T=1 two-body matrix elements were well known but not so T=0. In 1985, the T=0 matrix elements were better known and the calculations were repeated by Escuderos et al. [9]. The two-particle matrix elements, obtained mainly from the spectrum of 42Sc, are shown in the table. Note that not only J=0 T=1, but also J=1 T=0 and J=7 T=0, are low lying.

We next make a search for a set of T=1 two-body matrix elements which will give close to equal spaces spectra for even I states in 44Ti, and for which all the T=0 (odd-J) matrix elements are set equal to zero. We find a good choice for J=0, 2, 4, 6 are, respectively, 0.00, 1.00, 1.60, 2.00.

| J  | MBZE | T=0 | MBZE T=1 only | MeV   |
|----|------|-----|---------------|-------|
| 0  | 0.000| 1   | 0.611        | 0.000 |
| 2  | 1.586| 3   | 1.490        | 2.000 |
| 4  | 2.815| 5   | 1.510        | 1.600 |
| 6  | 3.242| 7   | 0.616        | 2.000 |

3 Spectra (MeV) of 44Ti and 46Ti

Table II: Spectra of 44Ti and 46Ti
We have not made an exhaustive search the optimum T=1 matrix elements so as to obtain an equally spaced spectrum of $^{44}\text{Ti}$, but it is more than sufficient to get the point across. With a perfect vibrator the I=12$^+$ state would be at $6 \times 0.8302 = 4.9812$ MeV. In fact, it is at 4.8524 MeV. The 8 → 6 splitting is significantly different for the 2 → 0 splitting of 0.8397 MeV. It is, in fact, 1.3105 MeV.

| I  | MBZE $^{44}\text{Ti}$ | MBZE $^{46}\text{Ti}$ | T=1 only $^{44}\text{Ti}$ | T=1 only $^{46}\text{Ti}$ |
|----|----------------|----------------|----------------|----------------|
| 0  | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2  | 1.1631 | 1.1483 | 0.8302 | 0.8397 |
| 4  | 2.7900 | 2.2225 | 1.5723 | 1.5535 |
| 6  | 4.0618 | 3.1575 | 2.1508 | 1.9492 |
| 8  | 6.0842 | 4.8720 | 3.4643 | 3.1042 |
| 10 | 7.3839 | 6.3334 | 4.2524 | 4.0223 |
| 12 | 7.7022 | 8.0257 | 4.8524 | 4.9490 |

We can also play the game of obtaining rotational spectra. This is easier. We consider 3 cases:

1. Set all two-particle matrix elements to J(J+1). Then it is easy to show that for $^{44}\text{Ti}$, one also gets a perfect I(I+1) spectrum with the I=2 state at 6 MeV and the I=12 at 156 MeV.

### Table III: Perfect rotator spectrum for $^{44}\text{Ti}$

| I  | E   | T=1 only scaled |
|----|-----|----------------|
| 0  | 0   | 0              |
| 2  | 6   | 6              |
| 4  | 20  | 19.58          |
| 6  | 42  | 41.16          |
| 8  | 72  | 70.16          |
| 10 | 110 | 107.80         |
| 12 | 156 | 150.61         |

4 Rotational Spectra.

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3
2. We next refer to Table III. In the first spectral column we have the perfect rotator spectrum. We obtain this from a \( J(J+1) \) 2-body spectrum where both \( T=0 \) and \( T=1 \) 2-body matrix elements are included. In the next column we have a spectrum in which only the \( T=1 \) matrix elements are included but we rescale them so the first \( 2^+ \) state comes at 6 MeV, the same as in the first column. Without rescaling the \( 2^+ \) state would be at 4.625 MeV and the \( 12^+ \) at 116.10 MeV. We do not obtain a perfect rotational spectrum but still a fairly good one.

3. If we set all two-body \( T=1 \) matrix elements to zero and set the \( T=0 \) ones to \( J(J+1) \), we get a most fascinating spectrum. The even \( I \) states with \( I=0, 2, 4, 6 \) and \( 8 \) are at zero energy whilst the \( 10^+ \) and \( 12^+ \) states are at 94 MeV and 156 MeV respectively. While at first glance surprising, there is an easy explanation for the multiple ground state zeros. All states of zero energy have isospin \( T=2 \). They are therefore double analogs of states in \( ^{44}\text{Ca} \), which in our model space consists of 4 valence neutrons. The neutron-neutron interaction occurs only in the \( T=1 \) channel, but in this case all three of the \( T=1 \) matrix elements are set to zero. This explains why the \( T=2 \) states in \( ^{44}\text{Ti} \) lie at zero energy. A closer examination shows that there are two \( I=2^+ \) and two \( I=4^+ \) states at zero energy. But it is well known that in \( ^{44}\text{Ca} \) such states occur twice. One \( I=2^+ \) state has seniority \( v=2 \) and the other \( v=4 \); likewise for \( I=4^+ \). Note that there are no \( I=10^+ \) or \( 12^+ \) states of the \( (f_{7/2})^4 \) configuration in \( ^{44}\text{Ca} \). This explains why these states do not appear at zero energy in \( ^{44}\text{Ti} \).

In summary, with a \( J(J+1) \) two-body interaction one gets a perfect rotation spectrum in more complex nuclei, e.g., \( ^{44}\text{Ti} \). If this interaction is only in the \( T=1 \) channel, one gets an imperfect, but still fairly good rotational spectrum. If this interaction is only in the \( T=0 \) channel one gets a spectrum with multiple \( T=2 \) degeneracies at zero energy and the spectrum does not at all resemble a rotational spectrum.

![Improper Rotation](image1.png)

![Perfect Rotation](image2.png)

*Disclaimer:* Correct value of imperfect rotation is true divided by the constant 1.07206024.
5 Fun Spectra

We here look around for interesting relations between input 2-particle spectra and more complicated systems. As before, the latter will be $^{44}$Ti as 2 protons and 2 neutrons in the $f_{7/2}$ shell. We use trial and error. We have found one interesting case which we here mention. The two-body interaction matrix elements are 0, 0, 1, 0, 2, 0, 3, 0. We call this the 123 interaction. In other words, all the T=0 matrix elements are set to zero and the T=1’s are set to J/2. The resulting spectra are shown in table IV. Of special interest is that the levels from I=6 to I=12 equally spaced with a separation of 1.5 MeV, almost twice the 2 → 0 splitting. Thus, as shown in table IV, we have achieved a “vibrational spectrum” from I=6 to I=12, but not from I=0 to I=6.

Of even more interest are the structures of the I=6 and I=8 wave functions in tables VI and VII. That the (2,4) and (4,2) configurations have the same value is not a surprise. It follows from charge symmetry, and the same sign from the fact that we are dealing with a T=0 state. What is a surprise is that for I=6 we have a multitude of zeros – (2,6), (4,4), (6,2), (6,4) and (6,6). There are corresponding zeros for I=8. We note one common feature – things seem to separate into classes such that configurations with the same value of the sum of the proton-proton and neutron-neutron angular momenta act the same. For I=6 the (6,0) and (4,2) coefficients are non-zero. The $J_p + J_n$ sum is 6. For (2,6) and (4,4) the sum is 8 and for this class the coefficients are all zero. And (6,6) gives us 12 and here we also get a zero coefficient.

A more complete picture is afforded if we include the odd angular momenta. This is shown in Table XXIX. We show all the equally spaced levels and all the angular momenta of degenerate T=0 states. We note that for a given energy, all the states have the same value of $(J_p + J_n)$. That quantity is listed in the first column. Next comes the energy E, a bit rounded off to show more clearly that the spacing is 1.5 MeV. Then we list, for a given energy, the angular momenta of the states with that energy. For example, for 4.65 MeV, all states have $J_p + J_n = 8$. There are 3 degenerate states in this case with I=6, 7, 8.

If we go to the $g_{9/2}$ shell we get a similar behaviour (now with the 1234 interaction), but starting from I=8 and ending at I=14. The spacings are still 1.5 MeV and the I=8 and beyond wave functions are very strange looking.

Some of the results have been known before. In the $f_{7/2}$ case there are several states at the same energy 6.15 MeV. They have angular momenta 3, 7, 9 and 10. But this is not specific to the 123 interaction – it applies to any interaction acting only for T=1. This has been noted and explained [10,11,12,13] as an example of a partial dynamical symmetry. They noted that all these T=0 states have the same dual quantum numbers $(J_p, J_n)$, not just the sum $(J_p + J_n)$. A common feature of these angular momenta is that they cannot occur or a system of 4 identical nucleons in the $f_{7/2}$ shell e.g. $^{44}$Ca. This can be seen in the tables of identical-particle fractional parentage coefficients, B.F. Bayman and A. Lande [14]. The conditions that are imposed by the non-existence of these states can be used to show that indeed the states 3, 7, 9, and 10 with the same $(J_p, J_n)$ are degenerate. What is new in this work is that with the specific 123 (T=1)
interaction, two other angular momenta $I=6$ and $8$ enter the game. Note that we are considering only $T=0$ states. There are no $T=0$ states with $I=1$ or 11 in the $(f_{7/2})^4$ configuration.

We also show more briefly results in table XXX for the 135 interaction $0, 0, 1, 0, 3, 0, 5, 0$. The spacings are now 3 MeV, double those for the 123 interaction. We note that the results for $(J_p+J_n) = 10$ and 12 are the same as in table XXIX. This is to be expected, since they involve angular momenta not present for 4 identical nucleons in the $f_{7/2}$ shell and are therefore true for any $T=1$ interaction. The main difference is that with 135 there is only one special $I=6^+$ state, whereas with 123 there were 2 special $6^+$ states.

If we go to the $g_{9/2}$ shell we get a similar behaviour, (now with the 1234 interaction), but starting from $I=7$ and ending at $I=14$. The results are shown in table XXXI. The spacings are still 1.5 MeV and the $I=7$ and beyond wave functions have fixed $(J_p+J_n)$. As an example there are degerate states at 7.29 MeV with $I=10, 11,$ and 12, as well as states at 8.79 MeV with $I=11, 13,$ and 14. In the $(g_{9/2})^4$ configuration of identical particles, e.g. neutrons, the following angular momenta cannot occur: 1, 11, 13, 14, 15, 16. There are no $T=0$ states with $I=1$, so this angular momentum is not under consideration.

In the next section we will be showing wave function as column vectors with ammplitudes $D^I(J_p,J_n)$, such that for a i state of total angular momentum $I$ $|D^I(J_p,J_n)|^2$ is the probability that the protons couple to $J_p$ and the neutrons to $J_n$. Note that for an $N=Z$ nucleus e.g. $^{44}$Ti one has the following relation for a state of isospin $T$:

$$D(J_p,J_n) = (-1)^{I+T} D(J_n,J_p).$$

5.1 123 Tables in $f_{7/2}^{44}$Ti

**Table IV: Fun Spectra and Differences**

| $I$ | E   | Diff. |
|-----|-----|-------|
| 0   | 0.0000 |       |
| 2   | 0.7552 | 0.7552 |
| 4   | 1.8338 | 1.0786 |
| 6   | 3.1498 | 1.3160 |
| 8   | 4.6498 | 1.5000 |
| 10  | 6.1498 | 1.5000 |
| 12  | 7.6498 | 1.5000 |

**Table V: Wave functions for I=3**
Table VI: Wave functions for $I=6$

| $J_p$ | $J_n$ | $D(J_p,J_n)$ | $D(J_p,J_n)$ |
|------|------|-------------|-------------|
| 2    | 2    | 0.0000      | 0.0000      |
| 2    | 4    | 0.0000      | 0.0000      |
| 4    | 2    | 0.0000      | 0.0000      |
| 4    | 4    | 0.0000      | 0.0000      |
| 4    | 6    | -0.7071     | 0.7071      |
| 6    | 4    | 0.7071      | 0.7071      |
| 6    | 6    | 0.0000      | 0.0000      |

Table VII: Wave functions for $I=8$

| $J_p$ | $J_n$ | $D(J_p,J_n)$ |
|------|------|-------------|
| 0    | 6    | 0.3953      |
| 2    | 4    | 0.5863      |
| 2    | 6    | 0.0000      |
| 4    | 2    | 0.5863      |
| 4    | 4    | 0.0000      |
| 4    | 6    | 0.0000      |
| 6    | 0    | 0.3953      |
| 6    | 2    | 0.0000      |
| 6    | 4    | 0.0000      |
| 6    | 6    | 0.0000      |

Table VIII: Wave functions for $I=9$

| $J_p$ | $J_n$ | $D(J_p,J_n)$ |
|------|------|-------------|
| 4    | 6    | -0.7071     |
| 6    | 4    | 0.7071      |
| 6    | 6    | 0.0000      |

Table IX: Wave functions for $I=10$

| $J_p$ | $J_n$ | $D(J_p,J_n)$ |
|------|------|-------------|
| 4    | 6    | 0.7071      |
| 6    | 4    | 0.7071      |
| 6    | 6    | 0.0000      |
Table X: Wave functions for $I=12$

| $J_p$ | $J_n$ | $D(J_p,J_n)$ |
|-------|-------|-------------|
| 6     | 6     | 1.0000      |

5.2 135 Tables in $f_{7/2}$

Table XI: Wave functions for $I=3$

| $J_p$ | $J_n$ | $D(J_p,J_n)$ | $D(J_p,J_n)$ |
|-------|-------|-------------|-------------|
| 2     | 2     | 0.0000      | 0.0000      |
| 2     | 4     | -0.7071     | 0.0000      |
| 4     | 2     | 0.7071      | 0.0000      |
| 4     | 4     | 0.0000      | 0.0000      |
| 4     | 6     | 0.0000      | -0.7071     |
| 6     | 4     | 0.0000      | 0.7071      |
| 6     | 6     | 0.0000      | 0.0000      |

Table XII: Wave functions for $I=6$

| $J_p$ | $J_n$ | $D(J_p,J_n)$ |
|-------|-------|-------------|
| 0     | 6     | 0.0000      |
| 2     | 4     | 0.0000      |
| 2     | 6     | -0.4743     |
| 4     | 2     | 0.0000      |
| 4     | 4     | 0.7416      |
| 4     | 6     | 0.0000      |
| 6     | 0     | 0.0000      |
| 6     | 2     | -0.4743     |
| 6     | 4     | 0.0000      |
| 6     | 6     | 0.0000      |

Table XIII: Wave functions for $I=7$

| $J_p$ | $J_n$ | $D(J_p,J_n)$ |
|-------|-------|-------------|
| 2     | 6     | 0.0000      |
| 4     | 4     | 0.0000      |
| 4     | 6     | -0.7071     |
| 6     | 2     | 0.0000      |
| 6     | 4     | 0.7071      |
| 6     | 6     | 0.0000      |

Table XIV: Wave functions for $I=8$

| $J_p$ | $J_n$ | $D(J_p,J_n)$ |
|-------|-------|-------------|
Table XV: Wave functions for $I=9$

| $J_p$ | $J_n$ | $D(J_p, J_n)$ |
|-------|-------|---------------|
| 2     | 6     | 0.4882        |
| 4     | 4     | 0.7234        |
| 4     | 6     | 0.0000        |
| 6     | 2     | 0.4882        |
| 6     | 4     | 0.0000        |
| 6     | 6     | 0.0000        |

Table XVI: Wave functions for $I=10$

| $J_p$ | $J_n$ | $D(J_p, J_n)$ |
|-------|-------|---------------|
| 4     | 6     | -0.7071       |
| 6     | 4     | 0.7071        |
| 6     | 6     | 0.0000        |

Table XVII: Wave functions for $I=12$

| $J_p$ | $J_n$ | $D(J_p, J_n)$ |
|-------|-------|---------------|
| 6     | 6     | 1.0000        |

5.3 1234 Tables in $g_{9/2}^{66}$Cd

Table XVIII: Wave functions for $I=3$

| $J_p$ | $J_n$ | $D(J_p, J_n)$ |
|-------|-------|---------------|
| 2     | 2     | 0.0000        |
| 2     | 4     | -0.6598       |
| 4     | 2     | 0.6598        |
| 4     | 4     | 0.0000        |
| 4     | 6     | -0.2449       |
| 6     | 4     | 0.2449        |
| 6     | 6     | 0.0000        |
| 6     | 8     | 0.0090        |
| 8     | 6     | -0.0690       |
| 8     | 8     | 0.0000        |

Table XIX: Wave functions for $I=7$
| $J_p$ | $J_n$ | $D(J_p,J_n)$ |
|------|------|-------------|
| 2    | 6    | 0.0000      |
| 2    | 8    | -0.3459     |
| 4    | 4    | 0.0000      |
| 4    | 6    | 0.6167      |
| 4    | 8    | 0.0000      |
| 6    | 2    | 0.0000      |
| 6    | 4    | -0.6167     |
| 6    | 6    | 0.0000      |
| 6    | 8    | 0.0000      |
| 8    | 2    | 0.3459      |
| 8    | 4    | 0.0000      |
| 8    | 6    | 0.0000      |
| 8    | 8    | 0.0000      |

Table XX: Wave functions for $I=8$

| $J_p$ | $J_n$ | $D(J_p,J_n)$ |
|------|------|-------------|
| 0    | 8    | 0.2792      |
| 2    | 6    | 0.4949      |
| 2    | 8    | 0.0000      |
| 4    | 4    | 0.5951      |
| 4    | 6    | 0.0000      |
| 4    | 8    | 0.0000      |
| 6    | 2    | 0.4949      |
| 6    | 4    | 0.0000      |
| 6    | 6    | 0.0000      |
| 6    | 8    | 0.0000      |
| 8    | 0    | 0.2792      |
| 8    | 2    | 0.0000      |
| 8    | 4    | 0.0000      |
| 8    | 6    | 0.0000      |
| 8    | 8    | 0.0000      |

Table XXI: Wave functions for $I=9$

| $J_p$ | $J_n$ | $D(J_p,J_n)$ |
|------|------|-------------|
| 2    | 8    | 0.6281      |
| 4    | 6    | 0.3248      |
| 4    | 8    | 0.0000      |
| 6    | 4    | -0.3248     |
| 6    | 6    | 0.0000      |
| 6    | 8    | 0.0000      |
| 8    | 2    | -0.6281     |
| 8    | 4    | 0.0000      |
| 8    | 6    | 0.0000      |
| 8    | 8    | 0.0000      |
Table XXII: Wave functions for I=10

| J_p | J_n | D(J_p,J_n) | D(J_p,J_n) |
|-----|-----|-----------|-----------|
| 2   | 8   | 0.3293    | 0.0000    |
| 4   | 6   | 0.6258    | 0.0000    |
| 4   | 8   | 0.0000    | -0.5126   |
| 6   | 4   | 0.6258    | 0.0000    |
| 6   | 6   | 0.0000    | 0.6888    |
| 6   | 8   | 0.0000    | 0.0000    |
| 8   | 2   | 0.3293    | 0.0000    |
| 8   | 4   | 0.0000    | -0.5126   |
| 8   | 6   | 0.0000    | 0.0000    |
| 8   | 8   | 0.0000    | 0.0000    |

Table XXIII: Wave functions for I=11

| J_p | J_n | D(J_p,J_n) | D(J_p,J_n) |
|-----|-----|-----------|-----------|
| 4   | 8   | 0.7071    | 0.0000    |
| 6   | 6   | 0.0000    | 0.0000    |
| 6   | 8   | 0.0000    | 0.7071    |
| 8   | 4   | -0.7071   | 0.0000    |
| 8   | 6   | 0.0000    | -0.7071   |
| 8   | 8   | 0.0000    | 0.0000    |

Table XXIV: Wave functions for I=12

| J_p | J_n | D(J_p,J_n) |
|-----|-----|-----------|
| 4   | 8   | 0.4715    |
| 6   | 6   | 0.7452    |
| 6   | 8   | 0.0000    |
| 8   | 4   | 0.4715    |
| 8   | 6   | 0.0000    |
| 8   | 8   | 0.0000    |

Table XXV: Wave functions for I=13

| J_p | J_n | D(J_p,J_n) |
|-----|-----|-----------|
| 6   | 8   | 0.7071    |
| 8   | 6   | -0.7071   |
| 8   | 8   | 0.0000    |

Table XXVI: Wave functions for I=14

| J_p | J_n | D(J_p,J_n) |
|-----|-----|-----------|
| 6   | 8   | 0.7071    |
| 8   | 6   | 0.7071    |
| 8   | 8   | 0.0000    |
Table XXVII: Wave functions for I=15

| J_p | J_n | D(J_p, J_n) |
|-----|-----|------------|
| 8   | 8   | 1.0000     |

Table XXVIII: Wave functions for I=16

| J_p | J_n | D(J_p, J_n) |
|-----|-----|------------|
| 8   | 8   | 1.0000     |

5.4 Two protons and two neutrons in the h_{11/2} with the 12345 interaction

In this section we will be more brief. We will just show in table XXXII the special states with (J_p + J_n) constant for 2 protons and 2 neutrons in the h_{11/2} shell. We use the 12345 interaction 0, 0, 1, 0, 2, 0, 3, 0, 4, 0, 5, 0. The angular momenta that cannot occur in this case are 1, 15, 17, 18, 19, and 20. Since we are considering only T=0 states, we do not consider I=1 or I=19.

Table XXIX: Special states in the f_{7/2}shell (123 interaction)

| J_p + J_n | E (MeV) | I    |
|-----------|---------|------|
| 6         | 3.15    | 3, 6 |
| 8         | 4.65    | 6, 7, 8 |
| 10        | 6.15    | 3, 7, 9, 10 |
| 12        | 7.65    | 10, 12 |

Table XXX: Special states in the f_{7/2}shell (135 interaction)

| J_p + J_n | E(MeV) | I    |
|-----------|--------|------|
| 6         | 4.42   | 3    |
| 8         | 7.42   | 6, 7, 8 |
| 10        | 10.42  | 3, 7, 9, 10 |
| 12        | 13.42  | 10, 12 |
Table XXXI: Special states in the $g_{9/2}$ shell (1234 interaction)

| $J_p+J_n$ | E (MeV) | I   |
|-----------|---------|-----|
| 8         | 4.29    | 8   |
| 10        | 5.79    | 7, 9, 10 |
| 12        | 7.29    | 10, 11, 12 |
| 14        | 8.79    | 11, 13, 14 |
| 16        | 10.29   | 14, 16 |

Table XXXII: Special states in the $h_{11/2}$ shell (12345 interaction)

| $J_p+J_n$ | E (MeV) | I   |
|-----------|---------|-----|
| 10        | 5.46    | 10  |
| 12        | 6.96    | 11, 12 |
| 14        | 8.46    | 11, 13, 14 |
| 16        | 9.96    | 14, 15, 16 |
| 18        | 11.46   | 15, 17, 18 |
| 20        | 12.96   | 18, 20 |

5.5 Explanation

For a detailed explanation of the results we refer the reader to a final version of this work W. Pereira, R. Garcia, L. Zamick, A. Escuderos and K. Neergaard, Int. J. Mod. Phys. E26, 1740021 (2017)

5.6 Final Comment

Previously we had found a partial dynamical symmetry when we set all $T=0$ two-body matrix elements to zero in a single $j$ shell calculation for 2 protons and 2 neutrons [10, 11, 12, 13]. The angular momenta involved were those that could not occur for 4 identical particles. For the states in question one had $(J_p, J_n)$ as good dual quantum numbers. When we consider more restrictive interactions, still with $T=0$ interactions set to zero e.g. “123” in $f_{7/2}$, “1234” in $g_{9/2}$, “12345”
Table: XXXIII: Special states not shown in previous tables

| $j$  | $J_p + J_n$ | $I$       |
|------|-------------|-----------|
| 1/2  | 0           | 0         |
| 3/2  | 2           | 2         |
|      | 4           | 2, 4      |
| 5/2  | 4           | 4         |
|      | 6           | 3, 5, 6   |
|      | 8           | 6, 8      |
| 13/2 | 12          | 12        |
|      | 14          | 13, 14    |
|      | 16          | 15, 16    |
|      | 18          | 15, 17, 18|
|      | 20          | 18, 19, 20|
|      | 22          | 19, 21, 22|
|      | 24          | 22, 24    |
| 15/2 | 14          | 14        |
|      | 16          | 15, 16    |
|      | 18          | 17, 18    |
|      | 20          | 19, 20    |
|      | 22          | 19, 21, 22|
|      | 24          | 22, 23, 24|
|      | 26          | 23, 25, 26|
|      | 28          | 26, 28    |
in $h_\frac{11}{2}$ we get some selected equally spaced levels which are usually multi-degenerate. As well as the old we get new angular momenta as part of the equally spaced spectra. These can occur for systems of identical particles. The wave functions have the constraint that $(J_p + J_n)$ is a constant. This is less constrictive than the previous condition.

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