The Dynamics of SIR (Susceptible-Infected-Recovered) Epidemic Model in Greater Noakhali for Pneumonia and Dysentery

Jamal Uddin, Md. Jamal Hossain, Mohammad Raquibul Hossain

Department of Applied Mathematics, Noakhali Science and Technology University, Noakhali-3814, Bangladesh

Email: 1jamal.rubel1206056@gmail.com, 2z_math_du@yahoo.com, 3raquib.math@gmail.com

Corresponding Author: Md. Jamal Hossain
Email: z_math_du@yahoo.com

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Abstract

We study the SIR model for the mathematical modeling of diseases of greater Noakhali. This model describes the spread of infectious diseases in which an individual may move from susceptible to infected and to recover. We discussed the mathematics behind the model and various tools for judging effectiveness in a certain territory. We completed the paper with an example using the infectious diseases, Pneumonia and Dysentery, commonly the children are infected. The current results of this paper are greatly instructive for us to further understand the epidemic spreading and design some fruitful prevention and disposal strategies to fight the epidemics.

Keywords: SIR Model, Effective removal rate, Basic reproductive ratio, Effective reproductive ratio.

I. Introduction

The SIR (susceptible-infected-removed) model developed by Ronald Ross, William Hamer, and others in the early twentieth century consists of a system of three coupled non-linear ordinary differential equations, which does not possess an evident cue solution. The term of epidemiology is used to human diseases for a large time now. Epidemiology is concerned with disease spreading within populations. The word ‘epidemic’ or ‘endemic’ is a word which is currently related to tragic events of monumental proportions such as when the pneumonia or dysentery hit the world. Both the pneumonia and the dysentery cause the death of millions of people every year.

The dynamics of SIR (Susceptible-Infected-Recovered) epidemic model with a limited resource for treatment and immigrants are proposed and analyzed. It is assumed that the population is divided into three classes known as susceptible, infected and recovered. The existence, uniqueness and boundedness of the solution of
the model have been investigated. The local stability analysis of SIR epidemic model without treatment and immigrants is discussed analytically. We have discussed elaborately about SIR epidemic model with assumptions, limiting behavior, basic reproductive ratio, effective reproductive number and found the actual number from the tables of “populations under 0-5 infected by Pneumonia in Noakhali district”, “populations under 0-5 infected by dysentery in Noakhali district”, “populations under 0-5 infected by Pneumonia in Lakhshmipur district”, “populations under 0-5 infected by dysentery in Lakhshmipur district”. We have discussed the future prediction of susceptible, infected and recovered population under the age 0-5 using SIR epidemic models.

II. Preliminaries

Population

In biology, a population is all the organisms of the same group or species, which live in a particular geographical area, and have the capability of interbreeding [VII]. The area that is used to define a sexual population is defined as the area where inter-breeding is potentially possible between any pair within the area, and where the probability of interbreeding is greater than the probability of cross-breeding with individuals from other areas [IV].

Susceptible

Individuals that are susceptible have in the case of the basic SIR model, who are not infected but could become infected that is they are able to catch the disease. Once they have it, they move into the infected axil. That is the state of being easily affected, influenced, or harmed by something.

Susceptible population is represented by $S(t)$. The rate of change of S with respect to time $t$ is given by

$$\frac{dS}{dt} = -rSI$$

where $r > 0$ is infectious rate and I is infected population.

Example of susceptibility in a sentence:

- The virus can infect susceptible individuals.

Infected

Infected individuals can spread the disease to susceptible individuals. The time they spend in the infected compartment is the infectious period, after which they enter the recovered compartment.

Infected population is represented by $I(t)$. The rate of change of I with respect to time $t$ is given by

$$\frac{dI}{dt} = rSI - aI$$
where $a$ is removal rate and $a > 0$.

Example of infected in a sentence:
- If you are sick you should stay home to avoid infecting other people in office or any other place.
- The virus has infected many people.
- They are unable to prevent bacteria from infecting the wound.

**Recovered**

Individuals in the recovered compartment are assumed to be immune for life. Recovered population is represented by the letter $R(t)$. The rate of change $R$ with respect to $t$ is
\[
\frac{dR}{dt} = aI
\]
where $a$ is removal rate and $a > 0$.

Example of recovered in a sentence:
- He had a heart attack but is recovering well.
- She recovered consciousness in the hospital.

**Net population**

Net population is the total population which contains susceptible, infected and removal individuals. That is the total population $N = S + I + R$ is constant because
\[
\frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = -rSI + (rSI - aI) + aI = 0
\]

**Effective removal rate**

Effective removal rate is the ratio of the rate at which individuals are removed from the infected category to the rate at which they are added to the same category. It is denoted by $\rho$,
\[
\rho = \frac{a}{r}
\]

**Basic reproductive ratio**

The basic reproduction number (sometimes called basic reproductive ratio, or basic reproductive rate and denoted by, $B_r$) of an infection can be thought of as the number of cases, one case generates on average over the course of its infectious period, otherwise uninfected population.

An important part of modeling diseases is the basic reproductive ratio [III]. The basic reproductive ratio is important since it tells us if a population is at risk from a disease. $B_r$ is affected by the infection and removal rates, i.e., $r$ and $a$, and is obtained by
\[
B_r = \frac{r}{a}S_0 = \frac{S_0}{\rho}
\]
When $B_r < 1$, the infection will die out in the long run.
When $B_r = 1$, the disease occurrence will be constant. But if $B_r > 1$, the infection will be able to spread in a population.

**Effective Reproductive Number**

Effective reproductive number, denoted by $E_r$, is the average number of secondary cases generated by an infectious case during the epidemic. To calculate this number, we multiply the basic reproductive ratio by how many people are susceptible at time $t$, that is

$$E_r = B_r \frac{s_t}{N}$$

(3)

The effective reproductive number is important since it helps researchers and health officials determine how effective their policies are on controlling the disease. When $E_r < 1$, the policies concerning the containing of the disease are effective [III].

**Assumptions of SIR models**

The SIR model is used in epidemiology to compute the amount of susceptible, infected, and recovered people in a population. This model is an appropriate one to use under the following assumptions:

- The population is fixed.
- The only way a person can leave the susceptible group is to become infected. The only way a person can leave the infected group is to recover from the disease. Once a person has recovered, the person received immunity.
- Age, sex, social status, and race do not affect the probability of being infected.
- There is no inherited immunity.
- The member of the population mix homogeneously (have the same interactions with one another to the same degree).
- Removal of infective to the removed class is proportional to the number of infective, i.e., $aI$ is constant, where $a > 0$.
- The incubation period is short enough to be negligible.

The model based on the above assumptions is

$$\frac{dS}{dt} = -rSI$$

(4)

$$\frac{dI}{dt} = rSI - aI$$

(5)

$$\frac{dR}{dt} = aI$$

(6)

**III Development of SIR epidemic model**

From (4) and (5), we have
\[
\frac{dI}{dS} = \frac{rSI - aI}{rSI} = -1 + \frac{a}{rS} = -1 + \frac{\rho}{S}
\]

Where \(\rho = \frac{a}{r}\) is called the effective removal rate, i.e., the ratio of the rate at which individuals are removed from the infected category to the rate at which they are added to the same category.

\[
\Rightarrow dI = \left(-1 + \frac{\rho}{S}\right) dS
\]

\[
\Rightarrow \int dI = -\int dS + \rho \int \frac{1}{S} dS
\]

\[
\Rightarrow I = -S + \rho \ln(S) + \text{Constant}
\]

\[
\Rightarrow 1 + S - \rho \ln(S) = \text{Constant} = I_0 + S_0 - \rho \ln(S_0)
\] (7)

Here, \(R(0) = 0\), so \(0 \leq S + I < N\)

\(I(t)\) will be maximum if

\[
\frac{dI}{dt} = 0
\]

\[
\Rightarrow I(rS - a) = 0
\]

\[
\Rightarrow rS - a = 0 \quad [\text{Since } I \neq 0]
\]

\[
\Rightarrow rS = a
\]

\[
\Rightarrow S = \frac{a}{r}
\]

\[
\therefore S = \rho
\] (8)

Putting \(S = \rho\) in (7), we get

\[
I_{\text{max}} + \rho - \rho \ln \rho = I_0 + S_0 - \rho \ln S_0
\]

\[
\Rightarrow I_{\text{max}} = \rho \ln \rho - \rho + I_0 + S_0 - \rho \ln S_0
\]

\[
\Rightarrow I_{\text{max}} = I_0 + S_0 - \rho + \rho \ln \left(\frac{\rho}{S_0}\right)
\]

\[
\therefore I_{\text{max}} = N - \rho + \rho \ln \left(\frac{\rho}{S_0}\right) \quad [\text{Since } N = I_0 + S_0]
\] (9)

If \(I_0 > 0\) and \(S_0 > \rho\), then the phase trajectory starts with \(S > \rho\). Also in this case \(I\) increases from \(I_0\) and hence an epidemic ensures. If \(S_0 < \rho\), then \(I\) decreases from \(I_0\) and as such no epidemic occurs.

Again from (4) & (6), we get

\[
\frac{dS}{dr} = -\frac{r}{a} S = -\frac{s}{\rho} \quad (\text{where } \rho = \frac{a}{r})
\] (10)

Integrating (10), we obtain

\[
\int \frac{dS}{S} = -\frac{1}{\rho} \int dR
\]
\[ \ln S = -\frac{R}{p} \]
\[ \ln S - \ln S_0 = -\frac{R}{p} \]
\[ \frac{S}{S_0} = e^{-\frac{R}{p}} \]
\[ S = S_0 e^{-\frac{R}{p}} \]  

(11)

By applying Euler’s method of systems, we can solve the differential equations (4), (5) & (6). The solutions to the differential equations are:

\[ S_{n+1} = S_n - rS_n I_n (t_{n+1} - t_n) \]  
\[ I_{n+1} = I_n \{1 + (rS_n - a)(t_{n+1} - t_n)\} \]  
\[ R_{n+1} = R_n + aI_n (t_{n+1} - t_n) \]  

(12) to (14)

Where \( S_{n+1}, I_{n+1} \) and \( R_{n+1} \) are the number of susceptible, infected and recovered people at time \( t_{n+1} \). Time difference \( (t_{n+1} - t_n) \) is a small change of time and will be equal for all. It is important to note that researchers and health officials first collect data at a given period of time. The equations (12) to (14) are primarily used to calculate \( r \) and \( a \).

**Populations under 0-5 year infected by pneumonia in Noakhali district**

Table 1: Children under 0-5 years old threatened by pneumonia last 5 years in Noakhali district

| Year | Net Population N(t) | Susceptible person S(t) | Infectious person I(t) | Recovered person R(t) |
|------|----------------------|-------------------------|------------------------|-----------------------|
| 2012 | 471440               | 464373                  | 3624                   | 3443                  |
| 2013 | 479737               | 469230                  | 5388                   | 5119                  |
| 2014 | 487797               | 472811                  | 7685                   | 7301                  |
| 2015 | 496160               | 484840                  | 5805                   | 5515                  |
| 2016 | 504667               | 485961                  | 9593                   | 9113                  |
| 2017 | 513320               | 488114                  | 12926                  | 12280                 |

Source: “Civil Surgeon Office Noakhali and Population & Housing Census 2011, Zila Report: Noakhali”, Bangladesh Statistical Bureau, Bangladesh [I].

From equation (12) and (14), we can write

\[ r = \frac{S_n - S_{n+1}}{S_n I_n (t_{n+1} - t_n)} \] and \( a = \frac{R_{n+1} - R_n}{I_n (t_{n+1} - t_n)} \)  

(15)
Using this equation (15), we can get the values of \( r \) and \( a \) for each period. These \( r \) and \( a \) are shown in the following table:

### Table 2: Infection and Removal rate of pneumonia during last 5 years

| Year  | Susceptible person \( S(t) \) | Infection rate \( r \) | Removal rate \( a \) | Effective removal rate \( \rho \) | If \((S > \rho)\) No epidemic occurs | If \((S < \rho)\) Epidemic occurs |
|-------|-------------------------------|------------------------|----------------------|----------------------------------|-----------------------------------|-----------------------------------|
| 2012  | 464373                        | -                      | -                    | -                                | -                                 | -                                 |
| 2013  | 469230                        | 2.86 \( \times 10^{-6} \) | 0.46                 | 161703                           | No epidemic occurred              | -                                 |
| 2014  | 472811                        | 1.42 \( \times 10^{-6} \) | 0.41                 | 288732                           | No epidemic occurred              | -                                 |
| 2015  | 484840                        | 3.31 \( \times 10^{-6} \) | 0.23                 | 69486                            | No epidemic occurred              | -                                 |
| 2016  | 485961                        | 3.98 \( \times 10^{-7} \) | 0.62                 | 1557788                          | Epidemic occurred                 | -                                 |
| 2017  | 488114                        | 4.62 \( \times 10^{-7} \) | 0.33                 | 714285                           | Epidemic occurred                 | -                                 |

Here the negative sign of \( r \) and \( a \) should be neglected since \( r \) and \( a \) are always greater than zero.

**Basic reproductive ratio for pneumonia in Noakhali district**

The basic reproductive number is 2.87 for 2013, since \( S_0 = 464373 \), \( \rho = 161703 \)

\[
B_r = \frac{S_0}{\rho} = \frac{464373}{161703} = 2.87
\]

Similarly, we will find the remaining basic reproductive number for 2014, 2015, 2016 and 2017 from the following table:

### Table 3: Basic reproductive ratio for pneumonia in Noakhali district

| Year  | Susceptible Person \( S(t) \) | Effective removal rate \( \rho \) | Basic reproductive ratio \( B_r \) | Comment                                      |
|-------|-------------------------------|----------------------------------|----------------------------------|-----------------------------------------------|
| 2012  | 464373                        | -                                | -                                | -                                             |
| 2013  | 469230                        | 161703                           | 2.87                             | Infections had spread in the population.       |
| 2014  | 472811                        | 288732                           | 1.63                             | Infections had spread in the population.       |
| 2015  | 484840                        | 69486                            | 6.80                             | Infections had spread in the population.       |
| 2016  | 485961                        | 155778                           | 3.11                             | Infections had spread in the population.       |
| 2017  | 488114                        | 714285                           | 0.68                             | Infection has died out in the long run.         |
Effective reproductive number for pneumonia in Noakhali district

Table 4: Effective reproductive number for pneumonia in Noakhali district

| Year | Net Population N(t) | Susceptible person S(t) | Basic reproductive ratio(B_r) | Effective Reproductive Number (E_r) | Comment |
|------|---------------------|-------------------------|-------------------------------|-----------------------------------|---------|
| 2013 | 479737              | 469230                  | 2.87                          | 2.81                              | Infection spread through the population and infected large number of people |
| 2014 | 487797              | 472811                  | 1.63                          | 1.58                              | Infection spread through the population and infected large number of people |
| 2015 | 496160              | 484840                  | 6.80                          | 6.64                              | Infection spread through the population and infected large number of people |
| 2016 | 504667              | 485961                  | 3.11                          | 2.99                              | Infection spread through the population and infected large number of people |
| 2017 | 513320              | 488114                  | 0.68                          | 0.65                              | Infection would be decreased in case of eliminating the disease |

Populations under 0-5 year infected by dysentery in Noakhali district

Table 5: Children under 0-5 years old threatened by dysentery last 5 years in Noakhali district

| Year | Net Population N(t) | Susceptible person S(t) | Infectious person I(t) | Recovered person R(t) |
|------|---------------------|-------------------------|------------------------|-----------------------|
| 2012 | 471440              | 462313                  | 4804                   | 4323                  |
| 2013 | 479737              | 455524                  | 12744                  | 11469                 |
| 2014 | 487797              | 452201                  | 18735                  | 16861                 |
| 2015 | 496160              | 465753                  | 16004                  | 14403                 |
| 2016 | 504667              | 464794                  | 20986                  | 18887                 |
| 2017 | 513320              | 474135                  | 20624                  | 18561                 |
Using equation (15) we can get the values of $r$ and $a$ for each period. These $r$ and $a$ are shown in the following table:

Table 6: Infection and Removal rate of dysentery during last 5 years

| Year | Susceptible person $S(t)$ | Infection rate $(r)$ | Removal rate $(a)$ | Effective removal rate $(\rho)$ | If $(S > \rho)$ | If $(S < \rho)$ |
|------|--------------------------|----------------------|-------------------|-----------------------------|----------------|----------------|
| 2012 | 462313                   | -                    | -                 | -                           | -              | -              |
| 2013 | 455524                   | $3.06 \times 10^{-6}$ | 1.49              | 486928                      | -              | Epidemic occurred |
| 2014 | 452201                   | $0.57 \times 10^{-6}$ | 0.42              | 736842                      | -              | Epidemic occurred |
| 2015 | 465753                   | $1.60 \times 10^{-6}$ | 0.13              | 81250                       | No epidemic occurred | - |
| 2016 | 464794                   | $0.14 \times 10^{-6}$ | 0.28              | 2153846                     | -              | Epidemic occurred |
| 2017 | 474135                   | $0.96 \times 10^{-6}$ | 0.02              | 20833                       | No epidemic occurred | - |

Basic reproductive ratio for dysentery in Noakhali district

The basic reproductive number for 2013 is 0.95, since $S_0 = 462313$, $\rho = 486928$

$\therefore B_r = \frac{S_0}{\rho} = \frac{462313}{486928} = 0.95$

Similarly we will find the remaining basic reproductive number for 2014, 2015, 2016 and 2017 from the following table:

Table 7: Basic reproductive ratio for dysentery in Noakhali district

| Year | Susceptible Person $S(t)$ | Effective removal rate $(\rho)$ | Basic reproductive ratio $(B_r)$ | Comment |
|------|--------------------------|--------------------------------|-------------------------------|---------|
| 2012 | 462313                   | -                              | -                             | -       |
| 2013 | 455524                   | 486928                         | 0.95                          | Infection died out in the long run |
| 2014 | 452201                   | 736842                         | 0.62                          | Infection died out in the long run |
| 2015 | 465753                   | 81250                          | 5.57                          | Infections had spread in the population |
| 2016 | 464794                   | 2153846                        | 0.22                          | Infection died out in the long run |
| 2017 | 474135                   | 20833                          | 22.31                         | Infections had spread in the population |
We see that in 2015 and 2017, infection has spread in the population since $R_{0} > 1$. But in 2013, 2014 and 2016, infection has died out in the long run.

**Effective reproductive number for dysentery in Noakhali district**

Table 8: Effective reproductive number for dysentery in Noakhali district

| Year | Net population N(t) | Susceptible person S(t) | Basic reproductive ratio ($R_{0}$) | Effective reproductive number ($E_{r}$) | Comment |
|------|---------------------|------------------------|-----------------------------------|----------------------------------------|---------|
| 2013 | 479737              | 455524                 | 0.95                              | 0.90                                   | Infection would decrease in case of eliminating the disease |
| 2014 | 487797              | 452201                 | 0.62                              | 0.57                                   | Infection would decrease in case of eliminating the disease |
| 2015 | 496160              | 465753                 | 5.57                              | 5.23                                   | Infection spread through the population and infected larger number of people |
| 2016 | 504667              | 464794                 | 0.22                              | 0.20                                   | Infection would decrease in case of eliminating the disease |
| 2017 | 513320              | 474135                 | 22.31                             | 20.61                                  | Infection spread through the population and infected larger number of people |

**Populations under 0-5 year infected by pneumonia in Lakhshmipur district**

Table 9: Children under 0-5 years old threatened by pneumonia last 5 years in Lakhshmipur

| Year | Net Population N(t) | Susceptible person S(t) | Infected person I(t) | Recovered person R(t) |
|------|---------------------|------------------------|----------------------|-----------------------|
| 2012 | 260138              | 236133                 | 12124                | 11881                 |
| 2013 | 264070              | 243259                 | 10511                | 10300                 |
| 2014 | 268061              | 245309                 | 11491                | 11261                 |
| 2015 | 272112              | 252009                 | 10153                | 9950                  |
| 2016 | 276225              | 252457                 | 12004                | 11764                 |
| 2017 | 280399              | 261268                 | 9662                 | 9469                  |
Using equation (15), we can get the values of \( r \) and \( a \) for each period. These \( r \) and \( a \) are shown in the following table:

**Table 10: Infection and Removal rate of pneumonia during last 5 years**

| Year | Susceptible person \( S(t) \) | Infection rate \( r \) | Removal rate \( a \) | Effective removal rate \( \rho \) | If \( (S > \rho) \) No epidemic occurs | If \( (S < \rho) \) Epidemic occurs |
|------|-------------------------------|------------------------|---------------------|-------------------------------|---------------------------------|--------------------------------|
| 2013 | 243259                        | 2.49 \( \times 10^{-6} \) | 0.13                | 52208                         | No epidemic occurred            | -                              |
| 2014 | 245309                        | 0.80 \( \times 10^{-6} \) | 0.09                | 112219                        | No epidemic occurred            | -                              |
| 2015 | 252009                        | 2.38 \( \times 10^{-6} \) | 0.11                | 46218                         | No epidemic occurred            | -                              |
| 2016 | 252457                        | 0.18 \( \times 10^{-6} \) | 0.18                | 1000000                       | -                               | Epidemic occurred              |
| 2017 | 261268                        | 2.91 \( \times 10^{-6} \) | 0.19                | 65292                         | No epidemic occurred            | -                              |

**Basic reproductive ratio for pneumonia in Lakhshmipur district**

The basic reproductive number for 2013 is 4.52, since \( S_0 = 236133, \rho = 52208 \)

\[
\therefore B_r = \frac{S_0}{\rho} = \frac{236133}{52208} = 4.52
\]

Similarly we will find the remaining basic reproductive number for 2014, 2015, 2016 and 2017 from the following table:

**Table 11: Basic reproductive ratio for pneumonia in Lakhshmipur district**

| Year | Susceptible person \( S(t) \) | Effective removal rate \( \rho \) | Basic reproductive ratio, \( B_r \) | Comment |
|------|-------------------------------|-------------------------------|-------------------------------|---------|
| 2012 | 236133                        | -                             | -                             | -       |
| 2013 | 243259                        | 52208                         | 4.52                          | Infections had spread in the population. |
| 2014 | 245309                        | 112219                        | 2.17                          | Infections had spread in the population. |
| 2015 | 252009                        | 46218                         | 5.31                          | Infections had spread in the population. |
| 2016 | 252457                        | 1000000                       | 0.25                          | Infection died out in the long run.     |
| 2017 | 261268                        | 65292                         | 3.87                          | Infections had spread in the population. |
In this case from 2013-15 and 2017, infection has spread in the population since $B_r > 1$. But in 2016, infection has died out in the long run since $B_r < 1$.

**Effective reproductive number for pneumonia in Lakhshmipur district**

Table 12: Effective reproductive number for pneumonia in Lakhshmipur district

| Year | Net Population N(t) | Susceptible person S(t) | Basic reproductive ratio ($B_r$) | Effective reproductive number ($E_r$) | Comment |
|------|----------------------|------------------------|----------------------------------|--------------------------------------|---------|
| 2012 | 260138               | 236133                 | -                                | -                                    | -       |
| 2013 | 264070               | 243259                 | 4.52                             | 4.16                                 | Infection spread through the population and infected large number of people |
| 2014 | 268061               | 245309                 | 2.17                             | 1.99                                 | Infection spread through the population and infected large number of people |
| 2015 | 272112               | 252009                 | 5.31                             | 4.92                                 | Infection spread through the population and infected large number of people |
| 2016 | 276225               | 252457                 | 0.25                             | 0.23                                 | Infection would decrease in case of eliminating the disease |
| 2017 | 280399               | 261268                 | 3.87                             | 3.61                                 | Infection spread through the population and infected large number of people |

**Populations under 0-5 year infected by dysentery in Lakhshmipur district**

Table 13: Children under 0-5 years old threatened by dysentery last 5 years in Lakhshmipur

| Year | Net Population N(t) | Susceptible person S(t) | Infected person I(t) | Recovered person R(t) | Infected person (%) | Recovered person (%) |
|------|----------------------|------------------------|----------------------|-----------------------|---------------------|---------------------|
| 2012 | 260138               | 246968                 | 6754                 | 6416                  | 2.60                | 2.45                |
| 2013 | 264070               | 252622                 | 5871                 | 5577                  | 2.22                | 2.11                |
| 2014 | 268061               | 252299                 | 8083                 | 7679                  | 3.02                | 2.86                |
| 2015 | 272112               | 256206                 | 8157                 | 7749                  | 2.99                | 2.85                |
| 2016 | 276225               | 270806                 | 2779                 | 2640                  | 1.01                | 0.96                |
| 2017 | 280399               | 274602                 | 2973                 | 2824                  | 1.06                | 1.01                |
Using equation (15), we can get the values of $r$ and $a$ for each period. These $r$ and $a$ are shown in the following table:

Table 14: Infection and Removal rate of dysentery during last 5 years

| Year | Susceptible person $S(t)$ | Infection rate ($r$) | Removal rate ($a$) | Effective removal rate ($\rho$) | If $(S > \rho)$ No epidemic occurs | If $(S < \rho)$ Epidemic occurs |
|------|---------------------------|----------------------|-------------------|---------------------------------|---------------------------------|--------------------------------|
| 2013 | 252622                    | $3.39 \times 10^{-6}$| 0.12              | 35398                           | No epidemic occurred            | -                              |
| 2014 | 252299                    | $0.22 \times 10^{-6}$| 0.36              | 1636363                         | -                               | Epidemic occurred              |
| 2015 | 256206                    | $1.92 \times 10^{-6}$| 0.01              | 5208                            | No epidemic occurred            | -                              |
| 2016 | 270806                    | $6.99 \times 10^{-6}$| 0.63              | 90128                           | No epidemic occurred            | -                              |
| 2017 | 274602                    | $5.04 \times 10^{-6}$| 0.07              | 13888                           | No epidemic occurred            | -                              |

Basic reproductive ratio for dysentery in Lakhsmipur district

The basic reproductive number for 2013 is 6.98, since $S_0 = 246968$, $\rho = 35398$

\[ B_r = \frac{S_0}{\rho} = \frac{246968}{35398} = 6.98 \]

Similarly, we will find the remaining basic reproductive number for 2014, 2015, 2016 and 2017 from the following table:

Table 15: Basic reproductive ratio for dysentery in Lakhsmipur district

| Year | Susceptible $S(t)$ | Effective removal rate ($\rho$) | Basic reproductive ratio, $(B_r)$ | Comment                      |
|------|-------------------|-------------------------------|----------------------------------|-----------------------------|
| 2012 | 246968            | -                             | -                                | -                           |
| 2013 | 252622            | 35398                         | 6.98                             | Infections had spread in the population |
| 2014 | 252299            | 1636363                       | 0.15                             | Infection died out in the long run |
| 2015 | 256206            | 5208                          | 48.44                            | Infections had spread in the population |
| 2016 | 270806            | 90128                         | 2.84                             | Infections had spread in the population |
| 2017 | 274602            | 13888                         | 19.50                            | Infections had spread in the population |
We see that in 2013 and 2015-17, infection has spread in the population since $B_r > 1$. But in 2014, infection has died out in the long run $B_r < 1$.

**Effective reproductive number for dysentery in Lakhshmipur district**

Table 16: Effective reproductive number for dysentery in Lakhshmipur district

| Year | Net Population N(t) | Susceptible person S(t) | Basic reproductive ratio ($B_r$) | Effective reproductive number ($E_r$) | Comment |
|------|----------------------|-------------------------|----------------------------------|--------------------------------------|---------|
| 2012 | 260138               | 246968                  | -                                | -                                    | -       |
| 2013 | 264070               | 252622                  | 6.98                             | 6.68                                 | Infection spread through the population and infected large number of people |
| 2014 | 268061               | 252299                  | 0.15                             | 0.14                                 | Infection would decrease in case of eliminating the disease |
| 2015 | 272112               | 256206                  | 48.44                            | 45.61                                | Infection spread through the population and infected large number of people |
| 2016 | 276225               | 270806                  | 2.84                             | 2.78                                 | Infection spread through the population and infected large number of people |
| 2017 | 280399               | 274602                  | 19.50                            | 19.10                                | Infection spread through the population and infected large number of people |

IV. Future Prediction of SIR Epidemic Model

**Populations under 0-5 year infected by pneumonia in Noakhali district**

Using equation (7) & (12) we will construct the following table for the future prediction of the pneumonia in Noakhali district.
Table 17: Future prediction of population under 0-5 which are threatened by pneumonia in Noakhali

| Year | Net Population N(t) | Susceptible person S(t) | Infectious person I(t) | Recovered person R(t) |
|------|---------------------|-------------------------|------------------------|-----------------------|
| 2018 | 521626              | 499079                  | 11563                  | 10984                 |
| 2019 | 530570              | 490720                  | 20436                  | 19414                 |
| 2020 | 539667              | 500991                  | 19834                  | 18842                 |
| 2021 | 548919              | 491652                  | 29368                  | 27899                 |
| 2022 | 558331              | 501312                  | 29241                  | 27778                 |

Using equation (15) we get the values of r and a for each period. These r and a are shown in the following table:

Table 18: Infection and Removal rate of pneumonia for next 5 years

| Year | Susceptible person S(t) | Infection rate (r) | Removal rate (a) | Effective removal rate (ρ) | If (S > ρ) No epidemic occur | If (S < ρ) Epidemic occur |
|------|-------------------------|--------------------|------------------|---------------------------|----------------------------|----------------------------|
| 2018 | 499079                  | 1.74 × 10^{-6}     | 0.10             | 57471                     | No epidemic will occur     | -                          |
| 2019 | 490720                  | 1.45 × 10^{-6}     | 0.73             | 503448                    | -                          | Epidemic will occur       |
| 2020 | 500991                  | 1.02 × 10^{-6}     | 0.03             | 29411                     | No epidemic will occur     | -                          |
| 2021 | 491652                  | 9.40 × 10^{-7}     | 0.46             | 489361                    | No epidemic will occur     | -                          |
| 2022 | 501312                  | 6.69 × 10^{-7}     | 0.01             | 14947                     | No epidemic will occur     | -                          |

Comment: We have seen that in 2019 epidemic will occur and in 2021 epidemic may occur since ρ is near about S.

Populations under 0-5 year infected by pneumonia in Lakhshmipur district

Using equation (7) & (12) we will construct the following table for the future prediction of the pneumonia in Lakhshmipur district.
Table 19: Future prediction of population under 0-5 which are threatened by pneumonia in Lakhshmipur

| Year | Net Population N(t) | Susceptible person S(t) | Infectious person I(t) | Recovered person R(t) |
|------|---------------------|-------------------------|------------------------|----------------------|
| 2018 | 284636              | 255104                 | 15145                 | 14387               |
| 2019 | 288938              | 256147                 | 16816                 | 15975               |
| 2020 | 293305              | 261864                 | 16124                 | 15317               |
| 2021 | 297738              | 256619                 | 21087                 | 20032               |
| 2022 | 302237              | 260115                 | 21601                 | 20521               |

Using equation (15) we get the values of \( r \) and \( a \) for each period. These \( r \) and \( a \) are shown in the following table:

Table 20: Infection and Removal rate of pneumonia during next 5 years

| Year | Susceptible person S(t) | Infection rate \( r \) | Removal rate \( a \) | Effective removal rate \( \rho \) | If \( S > \rho \) | If \( S < \rho \) |
|------|-------------------------|------------------------|---------------------|------------------------|----------------|----------------|
| 2018 | 255104                  | 2.44 \( \times \) 10^{-6} | 0.51                | 209016                 | No epidemic will occur | - |
| 2019 | 256147                  | 2.70 \( \times \) 10^{-7} | 0.11                | 407469                 | -               | Epidemic will occur |
| 2020 | 261864                  | 1.33 \( \times \) 10^{-6} | 0.04                | 30075                  | No epidemic will occur | - |
| 2021 | 256619                  | 1.24 \( \times \) 10^{-6} | 0.29                | 233870                 | No epidemic will occur | - |
| 2022 | 260115                  | 6.46 \( \times \) 10^{-7} | 0.02                | 30959                  | No epidemic will occur | - |

**Comment:** We have seen that in 2019 epidemic will occur and in 2021 epidemic may occur since \( \rho \) is near about \( S \).
V. Discussion

In tables 1 & 9 for pneumonia and 5 & 13 for dysentery represent the net populations in Noakhali and Lakhshmipur which contain three categories of individuals such as susceptible, infected and removal individuals for analysis of SIR epidemic model in last five years. From these tables we have got the infection and removal rate $r$ and $a$ by equation (15) using SIR epidemic model, which are represented in table 2. In that table we have found effective removal rate ($\rho$). With the help of $\rho$ and $S(t)$, we conclude that whether epidemic occurred or not. In that case we have seen that no epidemic occurred in 2013, 2014 and 2015 while epidemic occurred in 2016 & 2017. Now in table 3, by using equation (2) we have found the value of basic reproductive ratio which is used for whether infection would spread or not. In that case we have seen from table 3 that during 2013-2016 infection has spread in the population whereas in 2017 infection has died out in the population. In table 4, we have discussed about effective reproductive ratio ($E_r$) for pneumonia in noakhali district which is very important to test how many people have been infected in a single period of time. We estimate the value of $E_r$ from equation (3). In this table we have seen that from 2013-2016, infection spread through the population and infected a large number of people and in 2017, if the disease had been eliminated then infection would decrease and the similar process would occur for Lakhshmipur district for pneumonia and dysentery.

VI. Conclusion

Pneumonia and dysentery are the most common diseases in our country. Every year a number of people die by these diseases, especially children who are under five years. In this paper we have discussed about the dynamics of SIR model developed with the ordinary differential equation. We have tried to test whether epidemic diseases of pneumonia and dysentery occurred or not during last five years (2013-2017) in Noakhali and Lakhshmipur district and whether epidemic will occur or not during next five years (2018-2022). We hope that this research work will come handy for ministry of health. Studying this paper government can be aware of future effect of pneumonia and dysentery. Conscious citizens, officers of civil surgeon office and any other health NGOs in Noakhali, and Lakhshmipur district can study our prediction of dynamics of SIR model for pneumonia and dysentery and they will be able to take necessary steps to prevent the epidemic against these diseases. We hope it will help to take effective measure for those who are going to be infected in future.
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