Generation of Spin Current from Lattice Distortion Dynamics: Spin-Orbit Routes

Takumi Funato and Hiroshi Kohno

Department of Physics, Nagoya University, Nagoya 464-8602, Japan

Generation of spin current from lattice distortion dynamics in metals is studied with special attention on the effect of spin-orbit coupling. The lattice distortion by local coordinate transformation, we calculate spin current and spin accumulation with the linear response theory. It is found that there are two routes to the spin-current generation: one via the spin Hall effect and the other via the spin accumulation. The present effect due to spin-orbit coupling can be comparable to, or even larger than, the one based on the spin-vorticity coupling in systems with strong spin-orbit coupling.

In the field of spintronics, spin current occupies a central position for the development of new devices or the discovery of novel physical phenomena. To date we know several methods available to generate spin currents, which include spin pumping,\(^1\) spin Hall effect,\(^4\) spin accumulation at the ferromagnet/nonmagnet interface,\(^5\) and spin Seebeck effect.\(^6\) These are classified as magnetic, electrical, magnetoelectric, and thermal means, respectively.

Recently, there has also been interest in generating spin currents by mechanical means, namely, by converting angular momentum associated with mechanical motion, such as the rigid rotation of a solid or vorticity of a fluid, into spin angular momentum of electrons. In the experiments reported so far, two mechanisms have been considered. One is the acoustic spin pumping by Uchida et al.,\(^7\) which is based on the magnon-phonon coupling. They succeeded in generating spin current by injecting acoustic waves into yttrium iron garnet (YIG) from the attached piezoelectric element. Theoretical analyses were given by Adachi and Maekawa,\(^8\)\(^,\)\(^9\) Keshtgar et al.,\(^11\) and Deymier et al.\(^12\) Another mechanism, proposed by Matsuo et al.,\(^13\)\(^,\)\(^14\) is based on the spin-rotation coupling or the spin-vorticity coupling (SVC). This is the coupling of the spin to the effective magnetic field that emerges in a rotating (non-inertial) frame of reference locally fixed on the material that is in motion. The first experiment for the SVC mechanism was conducted on liquid metals.\(^15\)\(^,\)\(^16\) To realize the SVC mechanism in solids, it was proposed to use surface acoustic waves.\(^17\)\(^,\)\(^18\) Nozaki et al. used Py/Cu bilayer and injected surface acoustic waves into Cu from the attached LiNbO\(_3\) (surface acoustic wave filter).\(^19\) The generated AC spin current was detected via the spin-torque ferromagnetic resonance.

One of the reasons that the mechanical generation of spin current has attracted attention is that it does not rely on spin-orbit interaction (SOI). Therefore, previous works did not pay attention to the effects of SOI. However, it is well expected that SOI plays certain roles in the mechanical processes of spin-current generation. For example, the previous experiments\(^7\)\(^,\)\(^8\) were conducted on systems with an interface, which potentially possesses Rashba SOI. Furthermore, the mechanical generation method may be used in combination with other “conventional” mechanisms that utilize SOI, and thereby enhance spin current.

In this paper, we study a mechanical generation of spin current by focusing on the effects of SOI. As a mechanical process, we consider dynamical lattice deformations of a solid with metallic electrons and with SOI. To treat lattice deformations analytically, we use the method of Tsuneto developed in the context of ultrasonic attenuation in superconductors,\(^20\) which employs a local coordinate transformation. By calculating spin current and spin accumulation induced by dynamical lattice deformations, we found two routes to spin-current generation: one via the spin accumulation and the other via the spin Hall effect. As a related work, Wang et al.\(^21\) derived the Hamiltonian that includes SOI in a general coordinate system starting from the general relativistic Dirac equation, but they did not give an explicit analysis of spin-current generation.

**Model:** We consider a free-electron system in the presence of random impurities and the associated SOI. The Hamiltonian is given by

\[ H = -\frac{\nabla^2}{2m} + V_{\text{imp}}(\mathbf{r}') + i\lambda_{\text{so}} \left[ (\nabla V_{\text{imp}}(\mathbf{r}')) \cdot \sigma \right] \cdot \nabla'. \]  \hspace{1cm} (1)

The second term represents the impurity potential, \(V_{\text{imp}}(\mathbf{r}') = u_i \sum_j \delta(\mathbf{r}' - \mathbf{R}_j)\), with strength \(u_i\) and at position \(\mathbf{R}_j\) (for \(j\)th impurity), and the third term is the SOI associated with \(V_{\text{imp}}\) with strength \(\lambda_{\text{so}}\) and the Pauli matrices \(\sigma = \sigma^x, \sigma^y, \sigma^z\). When the lattice is deformed, e.g., by sound waves, the Hamiltonian becomes

\[ H_{\text{lab}} = -\frac{\nabla^2}{2m} + V_{\text{imp}}(\mathbf{r}' - \delta \mathbf{R}(\mathbf{r}', t)) + i\lambda_{\text{so}} \left[ (\nabla V_{\text{imp}}(\mathbf{r}' - \delta \mathbf{R}(\mathbf{r}', t)) \right] \times \sigma \cdot \nabla', \]  \hspace{1cm} (2)

where \(\delta \mathbf{R}(\mathbf{r}', t)\) is the displacement vector of the lattice from their equilibrium position \(\mathbf{r}'\).

Following Tsuneto,\(^20\) we make a local coordinate transformation, \(\mathbf{r} = \mathbf{r}' - \delta \mathbf{R}(\mathbf{r}', t)\), from the laboratory (Lab) frame (with coordinate \(\mathbf{r}\)) to a “material frame” (with coordinate \(\mathbf{r}\)) which is fixed to the ‘atoms’ in a deformable lattice. At the same time, the wave function needs to be redefined to keep the normalization condition,

\[ \psi(\mathbf{r}, t) = [1 + \nabla \cdot \delta \mathbf{R}]^{1/2} \psi'(\mathbf{r}', t) + \mathcal{O}(\delta \mathbf{R}^2), \]  \hspace{1cm} (3)

where \(\psi'(\mathbf{r}', t)\) is the wave function in the Lab frame and \(\psi(\mathbf{r}, t)\) is the one in the material frame. Up to the first order
in $\delta \mathbf{R}$, the Hamiltonian for $\psi(r, t)$ is given by
\[ H_{\text{mat}} = H + H'_K + H'_\text{so}, \] (4)
where $H = H_K + H_{\text{imp}} + H_{\text{so}}$ is the unperturbed Hamiltonian defined by $H_{\text{lab}}$ with $\delta \mathbf{R} \neq 0$. Here, $H_K = \sum_k (\varepsilon_f - \varepsilon_k) \psi_k^\dagger \psi_k$ is the kinetic energy, with $\psi_k (\psi_k^\dagger)$ being the electron annihilation (creation) operator. $H_{\text{imp}}$ and $H_{\text{so}}$ describe the impurity potential and impurity SOI, respectively. $H_{\text{imp}} = \sum_{k,k'} V_{k-k'} \psi_k^\dagger \psi_{k'}$, $H_{\text{so}} = \lambda_0 \sum_k V_{k-k'} (k' \times k) \cdot \sigma \psi_k^\dagger \sigma \psi_{k'}$, where $V_{k-k'}$ is the Fourier component of $V_{\text{imp}}(r)$.

Assuming a uniformly random distribution, we average over the impurity positions as $\langle V_k V_{k'} \rangle = n_i \delta_{k-k'} \delta_{k+q, 0}$, and $\langle V_k V_{k'} V_{k''} \rangle = n_i \delta_{k-k'} \delta_{k+q, 0}$. Here, $n_i$ is the impurity concentration. The impurity-averaged retarded/advanced Green function is given by $G^R/\Lambda (\varepsilon) = (\varepsilon + \mu - k^2/2m + i\tau)^{-1}$, with $\gamma = \pi n_i \varepsilon F(N(\mu))(1 + \frac{1}{2} \lambda_0^2 \varepsilon F^2)$ is the damping rate. Here, $N(\mu)$ is the Fermi-level density of states (per spin), and $k_F$ is the Fermi wave number. In this work, we consider the effects of SOI up to the second order.

The effects of lattice distortion are contained in $H'_K$ and $H'_\text{so}$, which come from $H_K$ and $H_{\text{so}}$, respectively. In the first order in $\delta \mathbf{R}$, they are given by
\[ H'_K = \sum_k W^K_n(k) u_{q, \alpha} \psi_{k+q, \alpha}^\dagger, \] (5)
\[ H'_\text{so} = \sum_{k,k'} V_{k-k'} W^{\text{so}}_{ln}(k, k') u_{q, \alpha}^\dagger \psi_{k+q, \alpha}^\dagger \sigma \psi_{k-\alpha}^\dagger, \] (6)

Here, $u_{q, \alpha}$ is the Fourier component of the lattice velocity field, $u(r, t) = \partial_t \mathbf{R}(r, t)$, and we defined (see Fig. 1),
\[ W^K_n(k) = \frac{q \cdot k}{m_0} - 1 \big| k_n, \] (7)
\[ W^{\text{so}}_{ln}(k, k') = \frac{\lambda_0}{\varepsilon F} \big( k \times q \big), k'_{\alpha} - (k' \times q), k_{\alpha} \big], \] (8)

The first term in $W^K_n$ describes the coupling of the strain $\partial_t \mathbf{R}_n$ to the stress tensor $\sum_k k_n k'_n c_k c_k^\dagger$ of electrons, and modifies the effective mass tensor. Throughout this report, $\mathbf{q}$ represents the wave vector of the lattice deformation, and $\omega$ is its frequency. We assume that the spatial and temporal variations of $\delta \mathbf{R}$ are slow and satisfy the conditions $q \ll \ell$ and $\omega \ll \gamma$, where $\ell$ is the mean free path.

Spin and current density operators are given by
\[ j_{\alpha, \beta}^{\text{po}}(q) = \sigma^{\alpha \beta} (q) = \sum_k \psi_{k+q, \alpha}^\dagger v_0 \psi_{k+q, \beta}, \] (9)
\[ j_{\alpha, \beta}^{\text{so}}(q) = \sum_k \psi_{k+q, \alpha}^\dagger v_0 \psi_{k+q, \beta}^\dagger + j_{\alpha, \beta, i}^{\text{so}}(q), \] (10)

where $\alpha = x, y, z$ specifies the spin direction, $i = x, y, z$ the current direction, and $v_0 = 1$. Here, $j_{\alpha, \beta, i}^{\text{so}}(q)$ is the ‘anomalous’ part of the spin-current density, with $\epsilon_{\alpha ij}$ being the Levi-Civita symbol. We calculate $j_{\alpha, \beta}^{\text{po}}$ in (linear) response $^{21,22}$ to $\mathbf{u}$,
\[ j_{\alpha, \beta}^{\text{po}}(q) = \langle j_{\alpha, \beta}^{\text{po}}(q) \rangle_{\text{so}} = \left[ K_{\alpha \mu}^{\text{po}} + K_{\mu \alpha}^{\text{po}} + K_{\alpha \mu}^{\text{so}} \right] u_{\alpha} \] (12)
where $K_{\alpha \mu}^{\text{po}} (K_{\mu \alpha}^{\text{po}})$ is the skew-scattering (side-jump) type contribution in response to $H'_K$, and $K_{\alpha \mu}^{\text{so}}$ describes the response to $H'_\text{so}$.
The diagrams in (b) can be nonvanishing only when the lattice deformation is anomalous velocity, Eq. (11); hence they contribute only to the spin current. The diagrams in (a) come from the anomalous velocity field via SOI. This contribution is also written with the charge current (Eq. (20)) since the left vertices come from the anomalous velocity field. W e note that these contributions, coming from the divergence part of the charge current [the fusion spin current (previous work)] and the SVC (present work), are derived from the spin connection in the general relativistic Dirac equation. The above result does not include the effects of lattice distortion on the spinorial character of the electron wave function. Such effects are derived from the spin connection in the general relativistic Dirac equation. The total spin current and spin accumulation are given by the sum of the contributions from the SVC (previous work) and SOI (present work). Next, we study the contribution from SVC, an effect originating from the spin connection.

Spin-rotation coupling: For comparing the present result with the previous one that is based on the spin-vorticity coupling (SVC), we also calculate the spin accumulation and spin-current density arising from the dynamical lattice distortion and SOI contributions to Eq. (23) as the first term. In addition, dynamical lattice distortion generates a charge current as well (known as the acousto-electric effect\(25\)), which is then converted to a spin Hall current (in the transverse direction) via SOI. As seen from Eq. (22), spin accumulation is induced by the vorticity of the lattice velocity field via SOI. The resulting diffusion spin current contributes to Eq. (23) as the second term. In addition, dynamical lattice distortion generates a charge current as well (known as the acousto-electric effect\(25\)), which is then converted to a spin Hall current (in the transverse direction) via SOI. As expressed by the second and third terms in Eq. (23). Therefore, there are two routes to the spin-current generation in the present mechanism; one is the “diffusion route” caused by the spin accumulation and the other is the “spin Hall route” that follows the acousto-electric effect.\(26\) This is illustrated in Fig. 5. In the latter (spin Hall) route, the longitudinal component of \(\mathbf{u}\) also induces spin current via the generation of charge current. Finally, we note that the induced spin accumulation (22) and the spin-current density (23) satisfy the spin continuity equation.

\[
\partial_t \langle \sigma^\alpha \rangle_{\text{SOI}} + \nabla \cdot \langle \mathbf{j}^\alpha \rangle_{\text{SOI}} = -\frac{\langle \sigma^\alpha \rangle_{\text{SOI}}}{\tau_{sf}},
\]

(24)

The term on the right-hand side represents spin relaxation due to SOI.

The above result does not include the effects of lattice distortion on the spinorial character of the electron wave function. Such effects are derived from the spin connection in the general relativistic Dirac equation.\(^{13}\) The total spin current and spin accumulation are given by the sum of the contributions from the SVC (previous work\(^{15}\)) and SOI (present work). Next, we study the contribution from SVC, an effect originating from the spin connection.

Spin-rotation coupling: For comparing the present result with the previous one that is based on the spin-vorticity coupling (SVC),\(^{16}\) we also calculate the spin accumulation and spin-current density in response to the vorticity of the lattice velocity field, \(\omega = \nabla \times \mathbf{u}\).\(^{27}\) By treating the SVC Hamiltonian \(H_{SV} = -\frac{1}{2}(\mathbf{\sigma} \cdot \omega)(\mathbf{q}, \omega)\) as a perturbation, one has

\(\langle \mathbf{j}^\alpha_{\text{SV}} \rangle_{\omega} = \chi^\alpha_{\text{SV}}(\mathbf{q}, \omega) \omega(\mathbf{q}, \omega),\)

where \(\chi^\alpha_{\text{SV}}(\mathbf{q}, \omega)\) is the response function. The response function (without vertex corrections) is given as

\(\chi^\alpha(\mathbf{q}, \omega) = \frac{N}{2} \langle \mathbf{j}^\alpha_{\text{SV}}(\mathbf{q}, \omega) \rangle_{\omega}.\)

With ladder vertex corrections, spin accumulation and spin-current density arising from the dynamical lattice distortion via SOI have been obtained as

\[
\left\langle \sigma^\alpha \right\rangle_{\text{SOI}} = \alpha_{\text{SOI}} \tau_{\text{SOI}} \left( \frac{3}{5} D q^2 - i \omega \right) \frac{(i q \times \mathbf{u})^a}{D q^2 - i \omega + \tau_{sf}},
\]

(22)

\[
\left\langle \mathbf{j}^\alpha \right\rangle_{\text{SOI}} = -D q^2 (\alpha^\alpha)_{\text{SOI}} \sigma_t + \alpha_{\text{SOI}} \epsilon_{\text{SOI}} (\mathbf{q}, \omega) q^a u_a + D q^2 \tau_{sf} \mathbf{j}^a,
\]

(23)

where \(\alpha_{\text{SOI}} = \alpha_{\text{SS}} + 2 \alpha_{\text{ij}}^k\) is the ‘total’ spin Hall angle. As seen from Eq. (22), spin accumulation is induced by the vorticity of the lattice velocity field and SOI. The resulting diffusion spin current contributes to Eq. (23) as the first term. In addition, dynamical lattice distortion generates a charge current as well (known as the acousto-electric effect\(25\)), which is then converted to a spin Hall current (in the transverse direction) via SOI, as expressed by the second and third terms in Eq. (23). Therefore, there are two routes to the spin-current generation in the present mechanism; one is the “diffusion route” caused by the spin accumulation and the other is the “spin Hall route” that follows the acousto-electric effect.\(26\) This is illustrated in Fig. 5. In the latter (spin Hall) route, the longitudinal component of \(\mathbf{u}\) also induces spin current via the generation of charge current. Finally, we note that the induced spin accumulation (22) and the spin-current density (23) satisfy the spin continuity equation.

\[
\partial_t \langle \sigma^\alpha \rangle_{\text{SOI}} + \nabla \cdot \langle \mathbf{j}^\alpha \rangle_{\text{SOI}} = -\frac{\langle \sigma^\alpha \rangle_{\text{SOI}}}{\tau_{sf}},
\]

(24)

The term on the right-hand side represents spin relaxation due to SOI.
current density are obtained as
\[
\langle \sigma^0 \rangle_{SV} = \frac{N(\mu)}{2} \frac{Dq^2 + r_{sf}^2}{Dq^2 - i\omega + r_{sf}^2} \omega^2, 
\]
\[
\langle \sigma^0 \rangle_{SV} = -i\omega N(\mu) \frac{Dq_i}{Dq^2 - i\omega + r_{sf}^2} \omega^2. 
\]
They satisfy the spin continuity equation,
\[
(\partial_t + r_{sf}^-)(\langle \sigma^\alpha \rangle_{SV} + \nabla \cdot \langle J^\alpha \rangle_{SV}) = \frac{N(\mu)}{2} r_{sf}^\alpha \omega^2, 
\]
with a source term \((\sim \omega)\) on the right-hand side. Alternatively, one may define the “spin accumulation” \(\delta\mu = \mu^\uparrow - \mu^\downarrow\) by \(\langle \sigma^\alpha \rangle_{SV} = n_1 - n_1 = N(\mu)(\delta\mu^\uparrow + i\omega\delta\mu^\downarrow)/2\), where the spin quantization axis has been taken along the \(\hat{a}\) direction. Then, Eq. (25) leads to
\[
(\partial_t - D\nabla^2 + r_{sf}^-)\delta\mu^\alpha = -\frac{\hbar}{2} \omega \delta\mu^\alpha. 
\]
This is the basic equation used in Ref. 16 to study spin-current generation. Therefore, in the SVC mechanism, only the transverse acoustic waves generate spin current, and the generated spin current is purely of diffusion origin. These are in stark contrast with the SOI-induced mechanism.

Comparison: To see the magnitude of the present effect, we estimate the diffusion spin current generated via SVC, Eq. (22), relative to the one due to SVC, Eq. (25),
\[
R_{\text{diff}}(f) \equiv \left| \langle J_{\text{diff}} \rangle_{SV} \right| / |\langle J_{\text{diff}} \rangle_{SOI}| = \frac{8}{3\pi} \epsilon_F \tau \frac{v_s}{h} \left[ 1 + \frac{6\pi Df}{5v_a} \right], 
\]
where \(f = \omega/2\pi\) is the frequency and \(v_s = \omega/q\) is the (phase) velocity of acoustic waves, \(\epsilon_F = \hbar^2 k_F^2/2m\) is the Fermi energy, and \(h\) has been recovered. This ratio is larger for higher frequency \(f\), and for materials with stronger SOI.

For CuIr, the spin Hall angle is \(2\alpha_{SH} = 2.1\pm0.6\%\), independent of impurity concentration, which is dominated by the extrinsic, skew-scattering process. In the nearly free electron approximation with the Fermi wave number \(k_F = 1.36 \times 10^{10}\) m\(^{-1}\), Fermi velocity \(v_F = 1.57 \times 10^5\) m/s, effective mass \(m^* = 8.66 \times 10^{-31}\) kg, and resistivity \(\rho_{res} = 7.5 \mu\Omega\)m (for 3% Ir), we estimate the scattering time as \(\tau_{\text{imp}} = 5.3 \times 10^{-15}\) s, and the diffusion constant as \(D_{\text{imp}} = 4.35 \times 10^{-3}\) m\(^2\)/s, due to impurities. With the speed of the Rayleigh type surface acoustic wave, \(v_s = 3.80 \times 10^3\) m/s, on a single crystal of LiNbO\(_3\), we obtain
\[
R_{\text{diff}}(f) = 1.51 \sqrt{1 + (1.14 \times f)^2}, 
\]
where \(f\) is expressed in GHz. The diffusion spin current \(\langle J_{\text{diff}} \rangle_{SOI} \rangle\) via SOI is comparable to, or even larger than, that from SVC in metals with strong SOI. It is thus expected that the total contribution \(\langle J_{\text{diff}} \rangle_{SOI}\) to both the diffusion spin current and the spin Hall current, can be larger than \(\langle J_{\text{SOI}} \rangle_{SV}\). The magnitude itself is, however, small; \(\langle J_{\text{SOI}} \rangle_{SV} = 10^{20} \sim 10^{24}\) m\(^2\) s\(^{-1}\) = \(10 \sim 10^4\) A m\(^{-2}\) for \(r_{sf}^- = 0 \sim 5 \times 10^{13}\) s\(^{-1}\), \(\delta\tau = 1\) Å, and \(f = 3.8\) GHz, as in the case of the SVC mechanism.

To summarize, we studied the generation of spin current and spin accumulation by dynamical lattice distortion in metals with SOI at the impurities. We identified two routes to the spin-current generation, namely, the “spin Hall route” and the “spin diffusion route.” In the former route, a charge current is first induced by dynamical lattice distortion, which is then converted into a spin Hall current. In the latter route, a spin accumulation is first induced from the vorticity of the lattice velocity field, which then induces a diffusion spin current. The result suggests that the spin accumulation (hence the associated diffusion spin current) generated via SOI is larger than that due to SVC for systems with strong SOI. Similar effects are expected in systems with other types of SOI, such as Rashba, Weyl, etc., and such studies will be reported elsewhere. In this connection, we note that Xu et al. recently reported an experiment on the mechanical spin-current generation (due to magnon-phonon coupling) in a system with Rashba SOI.

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