Probabilistic power flow based on slice sampling for distribution network containing distributed generations

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Abstract. With the wide application of distributed generations (DG) in power system, some problems appear to the power flow calculation of distribution network. In the methods of probabilistic power flow calculation based on Monte Carlo simulation (MCS), the Gibbs sampling algorithm needs a large number of complex iterative operations to get more accurate results. Aiming at the problem of the algorithm, a Markov Chain Monte Carlo (MCMC) simulation method based on slice sampling algorithm is proposed and applied to probabilistic power flow calculation of the distribution network containing distributed generation. Finally, the IEEE-33 node system is used for simulation. The results show that the slice sampling algorithm can significantly improve the computational accuracy of the traditional MCMC method. In the meantime, the slice sampling is faster and more stable than Gibbs sampling under the same number of sampling iterations.

1. Introduction
Distributed generations is usually operated in the way of access to distribution network, which changes the large-scale centralized power supply for the traditional power grid, and it can play the advantages of simple installation of distributed power and flexible power generation. On the other hand, the power of the distributed generations that has the randomness and the intermittence is usually influenced by renewable energy, which brings great challenge to the planning and optimal control of grid-connected system.

For the power system with distributed generations including wind power and photovoltaic, the randomness of distributed generations make the power flow distribution of the system become a probabilistic problem. The traditional power flow calculation method is difficult to deal with randomness, thus it brings great problems to the analysis and optimization control of the system. However, probabilistic power flow method can solve these problems well. Probabilistic power flow is an important measure of power system operation analysis by analyzing the probability and statistics characteristics of the random disturbance of the power system, and obtaining the state variables of the system, the probability density function of the power flow distribution, the probability distribution function and so on. Probabilistic power flow methods can be commonly used to analytic method, Monte Carlo simulation method and point estimation method [1-3]. Among them, the analytical method can achieve fast calculation speed by simplified convolution calculation, but the accuracy of calculation is not high, and the independent assumption of each input variable is not consistent with...
the actual power system operation. The point estimation method gets the each order moment of the parameters by running two times the number of random parameters. Compared with the analytic method, the calculation time is increased. Although the accuracy of calculation is slightly improved, the sampling value is not universal. The Monte Carlo simulation method (MCS) has high calculation accuracy when the sample size is large enough, but its calculation is too great and the time is too long.

Therefore, the slice sampling algorithm is introduced into probabilistic power flow calculation of distributed generations system based on Monte Carlo method. It not only effectively avoids the problem of too long calculation time in Monte Carlo method, but also shows that the slice sampling algorithm improves the coverage of the sampling value in the random variable distribution and the sampling efficiency by comparing with the Gibbs sampling algorithm. When the IEEE-33 node system is transformed by introducing the distributed generations, the Gibbs sampling algorithm and the slice sampling algorithm are respectively used to calculate the probability power flow. The results show the efficiency and accuracy of the proposed algorithm.

2. Output model of distributed generations

2.1. Output model of wind power DG
Through the relevant literatures, we know that the Gauss mixture model can be used to establish the output probability model or the wind speed model. Among them, the power probability model of the wind farms is mostly used in connecting to the grid of the large-scale wind farms, but the model in this paper is not the large-scale wind farm rather than the small wind power station. For the model of wind power DG, first of all, most of the wind speed model is established. After the wind speed data can be acquired through the wind speed model, the wind power DG of probability model is finally obtained by formula (1). In the meantime, on the basis of Gauss mixed distribution, this paper uses the weighted Gaussian Mixture Distribution to combine with several Gauss distribution functions together for approximately analyzing a large number of complex data distributions, and it can fit the wind speed fluctuation characteristics and improve accuracy of the wind speed probability model through the Gauss distribution of different weights. The model parameters are solved by DAEM algorithm [4].

![Figure 1. Probability density fitting curve of weighted Gaussian mixture distribution.](image-url)

The weighted Gaussian mixture distribution based on the DAEM algorithm is used to fit the wind speed after selecting the wind speed data. As shown in figure 1, second-order and fifth-order Gaussian
mixture wind speed models are respectively used to show the fitting effect. Obviously, the fitting effect of the wind speed model based on the weighted Gaussian mixture distribution is good. With the increase of the order, the fitting effect is better. In this paper, the five order weighted Gaussian mixture wind speed model is adopted.

After reasonably selecting the wind speed model and taking into account the probabilistic characteristics of wind speed, the wind power output is obtained through the probability density function of the wind turbines output power.

\[
f(P_v) = \begin{cases} \int_0^{x_{ci}} f(x)dx + \int_{x_{ci}}^{x_{co}} f(x)dx & P_v = 0 \\ \frac{k}{k_c} \exp \left[ - \left( \frac{P_v - k_2}{k_c} \right)^\frac{1}{k_1 - k_2} \right] & 0 < P_v < P_r \\ \int_{x_r}^{x_\infty} f(x)dx & P_v = P_r \end{cases} \tag{1}
\]

\[k_1 = \frac{P_c}{x_r - x_{ci}} \tag{2}\]

\[k_2 = k_1 x_{ci} \tag{3}\]

Where \(P_c\): Power rating of wind turbine, \(x_{ci}\): Cut-in wind speed, \(x_{co}\): Cut-off wind speed, \(x_r\): Rated wind speed.

2.2. Output model of photovoltaic DG

The output of photovoltaic power plants which is similar to wind power output is closely related to the intensity of light. Therefore, the output of the photovoltaic power plant is also random and fluctuant. In order to minimize the impact of randomness and volatility on the system, it is necessary to establish a photovoltaic power plant output probability model which is in accordance with the actual accident situation.

As shown in figure 2, the light intensity is fitted by the Beta distribution [5].

![Figure 2. Fitting of light intensity on 7:00-9:00 a.m. of a certain area.](image)

The model of illumination mainly includes Weibull distribution, t location-scale distribution,
logistic distribution, lognormal distribution, extreme value distribution and Beta distribution. Among them, the model effect is the most widely used considering with surrounding environmental factors. A period of time illumination is similar to Beta distribution.

\[
f(x) = \frac{1}{\sqrt{2\pi \sigma_x}} \exp\left(-\frac{(x - \mu_x)^2}{2\sigma_x^2}\right)
\]

\[
f(Q) = \frac{1}{\sqrt{2\pi \sigma_Q}} \exp\left(-\frac{(Q - \mu_Q)^2}{2\sigma_Q^2}\right)
\]

Where \(\mu_P\): The mean value of the active power probability distribution of the load, \(\mu_Q\): The mean value of the reactive power probability distribution of the load, \(\sigma_P\): The standard deviation of the active power probability distribution of the load, \(\sigma_Q\): The standard deviation of the reactive power probability distribution of the load.

3. Probabilistic power flow calculation based on slice sampling algorithm for distributed generations

The slice sampling algorithm needs to introduce auxiliary variables in the sampling process. When the single auxiliary variable is introduced, the sampling process can be regarded as the Gibbs sampling algorithm under the probability density function image.

Assuming that a variable \(x\) is extracted from a probability distribution in a set, its probability density function is proportional to a function \(f(x)\). This method can be achieved by introducing an auxiliary variable \(y\) and defining a joint distribution function of \(x\) and \(y\), where \(y\) is located under the function \(f(x)\) curve. The joint distribution of \((x, y)\) will be sampled before the \(y\) can be removed for \(x\) sampling. However, it is difficult to generate mutually independent sample points from \(U\), so it needs to define a Markov chain that converges to this uniform distribution. The feasibility method of generating sample points in this paper is to use the Gibbs sampling method: first, given the conditional distribution of the \(x\) to \(y\) (in the region \((0, f(x))\)), then given the conditional distribution of the \(y\) to \(x\) (in the region \(S=\{x:y<f(x)\}\), where in \(S\) is known as the slice and defined by \(y\)), finally alternatively sampled from two conditional distributions.

In this paper, an improved MCMC method based on slice sampling algorithm is used to solve probabilistic power flow with distributed generations\(^7\). The calculation process is as follows:

- A probabilistic model of wind power DG and photovoltaic DG is established, and the related modeling process has outlined in the first part of this paper.
- Slicing. That is to determine the real number \(y\) in the region \((0, f(x))\) to generate slicing \(S\) (slicing \(S\) should contain the initial sampling value).
- The determination of region. That is to find a region \(I=\{L, R\}\) around \(X\) and to contain most of the slicing \(S\).
- Sample generation. Assuming the sampling scale is \(N\), the slice sampling algorithm is used to sample the output probability model of the wind power DG and the photovoltaic DG \(f_w(x), f_s(x)\) and the load model \(f_l(x)\) from the normal distribution, and getting the Markov chain of the
variables \([P_w, P_s, P_l, Q_l]\). It supposes that wind power DG and photovoltaic DG adopt constant power factor control \((\cos \phi_w = 0.95, \cos \phi_s = 0.9)\). Where

\[
Q = P \cdot \tan \phi
\]  

(7)

The Markov chain \(Q\) of wind power DG and photovoltaic DG can be obtained from the formula (7), and the sample space of random variables is obtained for probabilistic power flow calculation.

- The probability power flow calculation is carried out. Assuming that the wind power DG and the PV DG node are PQ nodes, the sample space of each sample is successively replaced by the Newton Ralph Xun power flow calculation formula to find power flow samples of each node and branch.
- System probabilistic evaluation. The stochastic characteristics and the probabilistic statistics index of the output variables are obtained by probability statistics.

The power flow calculation process is shown in figure 3.

**Figure 3.** Flow calculation process based on slice sampling algorithm.

### 4. Example analysis

In this paper, the IEEE-33 node system is used for test. Figure 4 shows the IEEE-33 node system. The line parameters and injection power of the system can be seen in references [8].

The 500 kW wind power DG is connected to the bus 31, and the bus 20 is connected to the 100 kW photovoltaic DG. The wind speed data and the illuminated data are used in the first section of the model, and the constant power factor is adopted. The power factor of the wind power DG is 0.95, and the power factor of the photovoltaic DG is 0.9. The results of the power flow calculation are shown in table 1.
Table 1. Power flow calculation of distributed generations based on slice sampling algorithm.

| bus | Voltage (pu) | bus | Voltage (pu) | bus | Voltage (pu) |
|-----|--------------|-----|--------------|-----|--------------|
| 0   | 1.0000       | 11  | 0.9316       | 22  | 0.9836       |
| 1   | 0.9985       | 12  | 0.9224       | 23  | 0.9821       |
| 2   | 0.9907       | 13  | 0.9247       | 24  | 0.9804       |
| 3   | 0.9849       | 14  | 0.9235       | 25  | 0.9731       |
| 4   | 0.9776       | 15  | 0.9202       | 26  | 0.9705       |
| 5   | 0.9685       | 16  | 0.9323       | 27  | 0.9617       |
| 6   | 0.9687       | 17  | 0.9314       | 28  | 0.9523       |
| 7   | 0.9618       | 18  | 0.9978       | 29  | 0.9465       |
| 8   | 0.9507       | 19  | 0.9936       | 30  | 0.9368       |
| 9   | 0.9424       | 20  | 0.9914       | 31  | 0.9357       |
| 10  | 0.9373       | 21  | 0.9874       | 32  | 0.9356       |

In the meantime, the comparison between the two sampling algorithms and the Monte Carlo simulation method is given in table 2. As far as computation time is concerned, the Monte Carlo simulation method which is commonly used for data contrast is too long because of the huge data space. The time of the slice sampling algorithm and Gibbs sampling is similar. Therefore, the accuracy and stability of the sampling algorithm will be compared.

Table 2. Three methods of power flow calculation time.

| Method     | MCMC | SS-MCMC | G-MCMC |
|------------|------|---------|--------|
| Computation time/s | 1689.50 | 80.68   | 89.23  |

In order to verify the stability of slice sampling algorithm, slice sampling algorithm and Gibbs sampling method are compared for the voltage amplitude of bus 28. From figures 5 and 6, we can see that the slice sampling algorithm has a significant improvement in convergence rate or convergence stability compared with the Gibbs sampling.

Meanwhile, figure 7 got the voltage probability density curves by different data sources. It can be seen that the slice sampling algorithm can be better fitting effect than the Gibbs sampling through the actual voltage data. It keeps the accuracy of the sampling space and the distribution of the target, and almost coincides with the actual voltage. Thus, it reflects that the slice sampling algorithm has a better accuracy than the Gibbs sampling algorithm in the whole sampling process.
Figure 5. Voltage expectation convergence curve of bus 28 based on Gibbs algorithm.

Figure 6. Voltage expectation convergence curve of bus 28 based on slice sampling algorithm.

Figure 7. Comparison of voltage probability density curve of 28 bus.
5. Conclusion
Based on the establishment of DG output probability model, this paper studies the power flow calculation problem of distribution network with DG. Monte Carlo simulation can be commonly used to deal with the randomness of the DG output, but it will greatly reduce the computational efficiency because of its need for large-scale sampling. Therefore, based on the Monte Carlo method to deal with the randomness problem, this paper combines the slice sampling algorithm to improve the computational efficiency and ensure the sampling accuracy, and compares it with the Gibbs sampling method. Finally, the effectiveness of the proposed method is verified through the IEEE-33 node system.

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