Gravitational radiation of generic isolated horizons

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From the similarity between null infinity and horizons, we show how to set up proper frames near generic isolated horizons. The asymptotic expansion and reference spin frame are used to study gravitational radiation near generic isolated horizons and it turns out that the news function appears on non-expanding horizon. We also verify that the surface gravity is constant on (weakly) isolated horizon. The corresponding conserved quantities and relevant asymptotic symmetry groups which allow gravitational radiation of generic isolated horizons are obtained from asymptotic expansion.

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I. INTRODUCTION

The boundary of black hole is defined as a region of no escape. However, the definition of event horizon cannot give a realistic description of how a black hole grows since it is too global. The event horizon can be located only after we know the global structure of space-time. Hence, the purpose of generalization of the event horizon to the 'quasi-local' horizons is to let the observer detect the horizon. The stationary horizon excludes the situation of radiation outside the horizon. Therefore, Ashtekar et al [1] propose the idea of the generic isolated horizon. It is less restrictive than the usual Killing horizon. We do not need to require any symmetry or assume that the space-time is globally stationary. If gravitational collapse occurs, the final stage of the black hole is isolated and in the equilibrium state therefore will not radiate anymore. However, there might be some gravitational and matter fields radiation which is far away from the black hole. The horizon will finally reach an equilibrium state and settle down to an isolated horizon (IH). The purpose of this framework is to probe the properties of black hole which are themselves in equilibrium but allows non-trivial dynamics in the exterior region. It allows one to assign mass and angular momentum to the black hole in terms of values of the fields on the horizon itself without referring to null or space-like infinity. It also leads to a generalization of the zeroth and first laws of black hole mechanics.

It is expected that black holes are rarely in equilibrium in Nature. By using generic isolated horizons as a basis, the ideas can be generalized to a dynamical horizon definition by a space-like hypersurface rather than null hypersurface in the non-expanding horizon definition. The horizon geometry of dynamical horizon is time dependent and it allows a quantitative relation between the growth of the horizon area and the flux of energy and angular momentum across it [2].

We start from Ashtekar’s most general definition of isolated horizon called the non-expanding horizon (NEH). It requires the degenerate metric to be independent of time. If we further require the extrinsic curvature (the rotation one form) to be time independent then it gives the definition of weakly isolated horizon (WIH). Here, the NEH resembles Killing horizon up to the first order and WIH further up to the second order. In WIH, the black hole zeroth law holds. One can further require the full derivative operator to be time independent, and it yields the definition of IH.

Unlike Ashtekar’s three dimensional analysis our work is a fully four dimensional approach. This approach allows one to consider the next order contribution from the neighborhood of the isolated horizon. It allows us to calculate the amount of mass-energy flux cross or near the horizon. Although there is no well-defined mass or energy density (including gravitational field) in general relativity (GR), it does have well-defined mass or energy associated to a two surface, i.e., quasi-locally. Unlike Newtonian theory, it does not have an unique expression of quasi-local mass or energy in GR [2]. We use a quasi-local formula based on spinor fields to define the mass of a black hole. Asymptotic expansions give a way to study the geometry near black holes or null infinity. Using the similarity to the asymptotic expansion for the null infinity we can set up a frame, certain gauge choices near the boundary of the horizon. Therefore, we can find the asymptotic expansion of the Newman-Penrose (NP) coefficients with respect to radius and compare this with the exact solutions we know. The asymptotic expansion for the null infinity and the horizon are quite different geometrically. As we approach null infinity we consider an asymptotically flat space-time, however, the approach near the horizon is not necessarily asymptotically flat. For the null infinity, we take the incoming tetrad $n$ as a generator of null infinity that generates different cuts with respect to different times. The outgoing tetrad $\ell_a$ can be chosen as tangent to null ray that can be parameterized by using affine parameter $r$. On the other hand, $\ell_a$ can be the gradient of the surface of a constant retarded time $u$. On the non-dynamical
horizon, we take the outgoing tetrad $\ell$ as the generator of the horizon that can generate different cross sections with respect to different advanced times, say $v$. The ingoing tetrad $n_a$ can be chosen as tangent to a null ray that can be parameterized by using the affine parameter $r$. This light ray goes into the horizon. The tetrad $n_a$ can be the gradient of the surface of a constant retarded time $v$. In this paper, we use convention $(+,−,−,−)$ and NP equations in p. 46-p. 50 of [4].

II. THE GENERIC ISOLATED HORIZONS

Firstly, we consider 4-D space-time manifold $(M, g)$ with 3-D sub-manifold $(\Delta, q)$.

a. Definition. $\Delta$ is called a non-expanding horizon (NEH) if (1) $\Delta$ is diffeomorphic to the product $S \times \mathbb{R}$ where $S$ is a space-like two surface. (2) The expansion $\Theta(t)$ of any null normal $\ell$ to $\Delta$ vanishes, where the expansion is defined by $\Theta(t) = \frac{1}{2}q^{ab}\nabla_a \ell_b = -R_{\mu\nu}$, and $\Theta(n) := \frac{1}{2}q^{ab}\nabla_a n_b = R_{\mu\nu}$, where $q^{ab} := -m^a m^b - n^a n^b$ on the tangent space of horizon. The twist on horizon is defined as $\omega_{\text{twist}} := \frac{1}{2}q^{ab} c^d q^{eb} \nabla(a \ell_b) \nabla(c \ell^d)$. The shear on horizon is defined as $[\sigma_{\text{shear}}] := \frac{1}{2}q^{ab} c^d q^{eb} \nabla(a \ell_b) \nabla(c \ell^d) - \Theta^{(t)},$ Since $\ell$ is the null normal of the null hypersurface, it implies the twist free. Moreover, the shear vanishes by using Raychaudhuri equation and the dominate energy condition. Therefore, the gauge conditions on NEH are

$$\kappa \equiv 0, \sigma \equiv 0, \rho \equiv 0. \tag{1}$$

From using these, there must exist a natural connection one form $\omega := \omega_a dx^a$ on $\Delta$ which can be obtained by

$$D_a \ell^b = \omega_a \ell^b. \tag{2}$$

The surface gravity $\kappa(t)$ is defined as

$$\kappa(t) := \omega_a \ell^a \tag{3}$$

On NEH $\Delta$ (measured by $\ell$). Note that we do not have an unique normalization for $\ell$. Under the scale transformation $\ell \mapsto f\ell$, we have $\omega \mapsto \omega + f \ln f$ and $\kappa(t) \mapsto f \kappa(t) + f\m L_{\ell} \ln f$ which leaves Eq. (2) and Eq. (3) invariant.

From (2), we get

$$\m L_{\ell} q_{ab} \equiv D_{\ell} q_{ab} \equiv q_{ab} \omega_a \ell^b + q_{ab} \omega_b \ell^a = 0 \tag{4}$$

for any null normal $\ell$ to $\Delta$. In fact, $\ell$ is an asymptotic Killing vector field as we approach the horizon even though the space-time metric $g_{ab}$ may not admit a Killing vector field in the neighborhood of $\Delta$.

The energy condition from the third point of definition then further implies that $R_{ab}\ell^a$ is proportional to $\ell_a [2]$, that is $R_{ab}\ell^a X^b \equiv 0$, for any vector field $X$ tangent to $\Delta$. Then we then have

$$\Phi_{00} \equiv \Phi_{01} = \Phi_{10} = 0. \tag{5}$$

Because $\ell$ is expansion and shear-free, it must lie along one of the principal null directions of the Weyl tensor. From equation (b) and (k) in p. 46 in [4], we have:

$$\Psi_a \equiv \Psi_1 = 0. \tag{6}$$

The $\Psi_2$ is gauge invariant i.e., independent of the choice of the null-tetrad $(n, m, \pi, \overline{\pi})$ on $\Delta$. We have

$$d \omega = 2(\Im[\Psi_2])^2 \ell \tag{7}$$

where $2\epsilon$ is an area two form. The two form $d \omega$ can also be written as

$$2D_a \omega_{[b]} = 2(0)\pi b]m_{[a} m_{b]}.$$

$\Im[\Psi_2]$ plays the roles of gravitational contributions to the angular-momentum at $\Delta$. Ashetkar et al calls $\omega$ the rotational 1-form potential and $\Im[\Psi_2]$ the rotational curvature scalar.

Using Cartan identity $\m L_\ell \omega_a = d\ell \epsilon_v + i_\ell d \ell$ and (7), the Lie derivative of $\omega$ with respect to $\ell$ is given by

$$\m L_\ell \omega_a \equiv 2 \Im[\Psi_2] \epsilon_v \omega_a + D_a (\ell^b \omega_b) \equiv D_\ell \kappa(t). \tag{9}$$

On NEH, the surface gravity may not be constant. To obtain the zeroth law such that the surface gravity is constant, one may need a further condition, i.e., $\m L_\ell \omega_a = 0$, on NEH. It motivates the definition of weakly isolated horizon.

b. Definition. A weakly isolated horizon (WIH) is a NEH with an equivalence class of null normals under constant transformation. The flow of $\ell$ preserves the rotation 1-form $\omega \m L_\ell \omega_a = 0$, i.e., $[\m L_{\ell}, D]|_{\ell} = 0$.

From (9), the condition of WIH basically preserves the black hole zeroth law. Because $\ell$ is tangent to $\Delta$, the evolution equation is in fact a constraint. See (B21) and (B22) of [2]. Therefore, given a NEH, we can select a canonical $[\ell]$ by requiring $(\Delta, [\ell])$ to be a WIH satisfying

$$\m L_\ell \mu = 0 \text{ or } \dot{\mu} = 0. \tag{10}$$

$\Delta$ generically admits an unique $[\ell]$ such that the incoming expansion is time independent. This result will establish that a generic NEH admits an unique $[\ell]$ such that $(\Delta, [\ell])$ is a WIH on which the incoming expansion $\mu$ is time independent.
c. Definition. A weakly isolated horizon (Δ, [ℓ]) is said to be isolated horizon (IH) if \([|\mathcal{L}_\ell, D]| V = 0\), for all vector fields \(V\) tangential to \(\Delta\) and all \(\ell \in [\ell]\).

From this definition, we have \([|\mathcal{L}_\ell, D]| V \equiv 0\) and \([|\mathcal{L}_\ell, D]| n = 0\). The first one gives the surface gravity is constant by previous argument. So \(\epsilon \equiv 0\). The second one gives \(\pi \equiv \mu \equiv \lambda \equiv 0\).

III. ASYMPTOTIC STRUCTURE AND COORDINATE TRANSFORMATIONS NEAR GENERIC ISOLATED HORIZONS

Frame setting, gauge choice and gauge conditions

We choose the incoming null tetrad \(n_a = \nabla_a v\) to be gradient of the null hypersurface \(v = \text{const.}\), it gives \(g^{ab}v_av_a = 0\). We further choose \(m, \overline{m}\) tangent to the two surface. These gauge choices lead to

\[
\nu = \mu - n = \rho - \overline{n} = \gamma + \overline{\gamma} = \pi - \alpha - \overline{\beta} = 0,
\]

(11)

From the definition of NEH, the gauge conditions are

\[
\kappa = \kappa_0 \rho' + O(r^2), \quad \rho = \rho_0 \rho' + O(r^2), \quad \sigma = \sigma_0 r' + O(r^2), \quad \epsilon - \tau = O(r^2),
\]

where \(\rho_0 := [\Psi_0^0 - \partial_0 \Psi_0 + \pi_0 \pi_0], \quad \sigma_0 := [-\overline{\partial}_0 \overline{\pi}_0 + \pi_0 \pi_0]\). The rest of NP coefficients are \(O(1)\). The Weyl tensor has the fall off (refer to equation (6))

\[
\Psi_0 = O(r'), \quad \Psi_1 = O(r')
\]

(12)

where \(r' = r - r_\Delta\). In order to preserve orthogonal relation \(\ell^a n_a = 1, \quad m^a m_a = 0\), we can choose the tetrad as

\[
\ell^a = (1, U, X^3, X^4), \quad n^a = (0, -1, 0, 0), \quad m^a = (0, 0, \xi^3, \xi^4).
\]

We first expand NP spin coefficients, tetrad components \(U, X^k, \xi^k\) and Weyl spinors \(\Psi_k\) with respect to \(r'\) and substitute them into NP equations to get following equations:

\[
\begin{align*}
\dot{\rho}_0 &= \partial_0 \rho_0 - \overline{\partial}_0 \overline{\pi}_0 - \kappa_0 \kappa_0, \quad \dot{\sigma}_0 = \partial_0 \sigma_0, \\
\pi_0 &= \kappa_0 = 0, \quad \lambda_0 = \overline{\partial}_0 \overline{\pi}_0 - \pi_0 \pi_0 - 2\lambda_0 \epsilon_0, \\
\dot{\rho}_0 - \partial_0 \dot{\kappa_0} = 0, \quad \dot{\beta}_0 - P \hat{\nabla} \epsilon_0 = 0, \\
\mu_0 &= \partial_0 \pi_0 + \pi_0 \pi_0 + \partial_0 \sigma_0 \pi_0, \\
2\lambda_0 \epsilon_0 + \partial_0 \sigma_0 &= -\Psi_1^0, \\
\dot{\delta}_0 \sigma_0 &= -\Psi_1^0, \quad \dot{\delta}_0 \lambda_0 - \dot{\overline{\delta}_0} \mu_0 = -\Psi_3^0, \\
\Psi_2^0 &= \overline{\nabla} \beta_0 - P \hat{\nabla} \epsilon_0 + \alpha_0 \sigma_0 - \beta_0 \overline{\pi}_0 - 2\lambda_0 \lambda_0, \\
2\overline{\lambda}_0 \epsilon_0 &= -\Psi_2^0, \\
\kappa_0 &= -2 \overline{\nabla} \hat{\nabla} \epsilon_0, \quad \hat{P} = 0, \\
\overline{\nabla} \hat{\nabla} \ln P &= \beta_0 - \overline{\pi}_0, \\
\dot{\psi}_1^0 &= -3\lambda_0 \Psi_2^0, \quad \dot{\psi}_2^0 = 0, \\
\dot{\psi}_3^0 &= \overline{\psi}_2^0 - 3\lambda_0 \psi_2^0 - 2\lambda_0 \psi_3^0, \\
\dot{\psi}_4^0 &= -\overline{\psi}_3^0 - 3\lambda_0 \overline{\psi}_3^0 - 4\lambda_0 \psi_4^0.
\end{align*}
\]

where the complex derivative is defined as \(\hat{\epsilon} := \frac{\partial}{\partial z} + i \frac{\partial}{\partial \overline{z}}\).

Surface gravity: from NEH to WIH

Here we prove that the surface gravity for a rotating WIH is also constant. From (d) in p. 46 and complex conjugate of (e) in p. 46, we have

\[
\dot{\pi}_0 = \frac{d}{dv} (\alpha_0 + \overline{\beta}_0) = 2 \overline{\nabla} \hat{\nabla} \epsilon_0 = -\pi_0.
\]

(13)

Using (c) in p. 46, \(\dot{\pi}_0 = -\kappa_0\). It implies \(\kappa_0 - \pi_0 = 0\). Therefore \(\kappa_0\) is real. Using (b) in p. 46, we get \(\kappa_0 = 0\), i.e., \(\dot{\delta}_0 \epsilon_0 = 0\) (i.e., \(P \hat{\nabla} \epsilon_0 = 0\)) on NEH.

We make a coordinate choice \(r_0 = -\frac{1}{\mu_0}\) on the NEH and it becomes a WIH. This gives \(\mu_0 = 0\). Applying time derivative on (b) in p. 46 and using \(\kappa_0 = 0\) from (b) in p. 49, we then get \(\epsilon_0 = 0\).

It then gives us that \(\epsilon_0\) is constant on WIH. So the surface gravity \(\kappa_0 \equiv \text{Re} \epsilon_0\) is constant on WIH. For NEH, the surface gravity is not necessary constant.

Coordinate transformations near generic isolated horizons

We look at the coordinate transformations on the horizon which are similar to those of the Newman-Unti or BMS group. Under such coordinate transformations, the metric form is preserved. The metric components can be expanded in terms of \(r'\):

\[
\begin{align*}
g_{r' r'} &= -1, \quad g_{\overline{r'} \overline{r'}} = g_{r' r'} = 0, \\
g_{r' \overline{r'}} &= -2U = -2\epsilon_0 r' - \epsilon_1 r'^2 + O(r^3) \\
g_{r' \overline{r'}} &= -X^k = -X_{\overline{r'} r'} + O(r^2) \\
g_{m n} &= -(-\xi^m \overline{\xi}^n + \overline{\xi}^m \xi^n) = -2 P \overline{\delta} m n + O(r'),
\end{align*}
\]

(14-17)

3 Here the number of NP equations refer to p. 46-p. 50 in [4].
where \( k, m, n = 3, 4 \). We expand the new coordinates \((\tilde{v}, r', x^m)\), in terms of \( r' \) to obtain
\[
\begin{align*}
\tilde{v} &:= V_0 + V_1 r' + V_2 r'^2 + O(r'^3), \\
r' &:= R_1 r' + R_2 r'^2 + O(r'^3), \\
x^m &:= K_0^m + K_1^m r' + K_2^m r'^2 + O(r'^3).
\end{align*}
\]

We use \( \tilde{g}^{\hat{ab}} = \partial x^a / \partial \tilde{x}^\hat{a} \partial x^b / \partial \tilde{x}^\hat{b} \) to transform the metric into the new coordinates, and then obtain the conditions for the metric components. From \( g^{\hat{kk}} \), we get the condition \( \partial K_0^k / \partial x^i + \partial K_0^k / \partial x^j X_{ij} + 2K_0^l \partial K^k_0 / \partial x^l = 0 \) to make \( X^k_0 \) be zero in the new coordinates. \( R_1 \) can be solved from the condition of preserving lowest order of \( g^{11} \) and \( V_0 \) can be integrated from condition of \( g^{01} \) (see [8]). Therefore, when \( r \) approaches \( r_\Delta \) we have the infinitesimal coordinate transformation on horizon which is
\[
\begin{align*}
\tilde{v} &:= V_0 = \frac{1}{\epsilon_0} \ln(G(x^k) + e^{\sigma_0}) \\
r' &:= R_1 r' = (G(x^k) e^{-\sigma_0} + 1) r' \\
x^k &:= K^k_0(v, x^k).
\end{align*}
\]

From the coordinate transformation (21) near horizon, it gives the asymptotic symmetric group transformation near a generic isolated horizon. It is similar with the asymptotic symmetric group (the so called BMS group or Newman-Unti group) near null infinity. When \( \tilde{v} = v + H(x^k), x^k = x^k \), it then defines the analogues of supertranslations which generate different cuts on generic isolated horizons.

IV. CONSTANT SPINORS FOR THE GENERIC ISOLATED HORIZONS: FRAME ALIGNMENT

In this section, we adopt a similar idea of Bramson’s asymptotic frame alignment [3] to set up spinor frames on horizon. Firstly, we demand the conditions on spinor frames to be parallelly transported along the horizon generators \( \ell^a \) direction on \( \Delta \), so
\[
\lim_{r' \to 0} DZ_{\Delta} = 0,
\]
and also the conditions of the frames on different generators on \( \Delta \) are:
\[
\lim_{r' \to 0} \delta Z_{\Delta} = \lim_{r' \to 0} \tilde{\delta} Z_{\Delta} = 0.
\]

It leads to the six conditions for the constant spinor \( \lambda_\Delta \) are 4
\[
\begin{align*}
\lambda_0^0 - \epsilon_0 \lambda_0^0 &= 0, \quad \text{i.e.,} \quad \rho_0 \lambda_0^0 = 0, \\
\lambda_1^0 + \epsilon_0 \lambda_1^0 &= \pi_0 \lambda_0^0, \quad \text{i.e.,} \quad \rho_0 \lambda_1^0 = \pi_0 \lambda_0^0, \\
\delta_0 \lambda_0^0 &= 0
\end{align*}
\]

(24) (25) (26)

To avoid confusion with the spinor \( \lambda_A \), we use another symbol \( -\sigma_0' \) to represent \( \lambda_0 \), i.e., the NP shear of \( n \).

We use the condition (24) and the fact that \( \rho_0 \delta_0 = \delta_0 \rho_0 \) on the horizon. Apply \( \rho_0 \) on (26), we find
\[
0 = \rho_0 \delta_0 \lambda_0^0 = \delta_0 \rho_0 \lambda_0^0 = 0.
\]

(30)

So condition (24) and (26) are compatible.

Apply \( \rho \) on (27) and use condition (24) and (25), we have
\[
0 = \Psi_2 \lambda_0^0.
\]

(31)

Hence the condition (24), (25) and (27) are not compatible unless \( \Psi_2 = 0 \).

From the previous analysis, we conclude that the compatible frame alignment conditions for the generic isolated horizon are (24), (26) and (27). Equation (26) and (27) are Dougan-Mason’s holomorphic conditions [5]. Here we see that the conditions of the spinor field to be asymptotically constant on NEH implies the Dougan-Mason holomorphic conditions on the cuts of the NEH. These equations will be used together with the Nester-Witten two form to define the quasi-local energy-momentum. The time related condition (24) will tell us how the energy momentum changes with time along NEH and will be useful to calculate the energy flux across the horizon.

V. THE QUASI-LOCAL ENERGY-MOMENTUM OF AN ISOLATED HORIZON

By using Nester-Witten two form together with the compatible constant spinor conditions which are Dougan-Mason’s holomorphic conditions (26) and (27) for the NEH, the quasi-local momentum integral near a NEH is
\[
I(r') = -\frac{1}{4\pi} \oint S \left[ \lambda_0^0 \partial \lambda_1 - \lambda_1 \partial \lambda_0^0 + \lambda_0 \partial' \lambda_1 - \lambda_1 \partial' \lambda_0 - \lambda_0 \lambda_0' (\mu + \rho) - \lambda_1 \lambda_1' (\rho + \mu) \right] dS.
\]

(32)

Moreover, the horizon momentum \( P_{\Delta A'} \) can be written as
\[
P_{\Delta A'}(S_\Delta) = I(\Delta) \lambda_{\Delta} \lambda_{\Delta'}
\]

(33)

where \( \lambda_{\Delta} \) is constant spinor on two surface of NEH. From the result of the asymptotic expansion for the generic isolated horizons, we can re-interpret the quasi-local energy-momentum integral of the generic isolated horizons (NEH) as
\[
I(\Delta) = -\frac{1}{4\pi} \oint S \left[ \psi_2 - \mu_0 \right. dS.
\]

(34)

where \( \psi_2 = \psi_2' = M + i L \) and \( \delta_0 \pi_0 = A - i L \) with \( M, L, A \) are function of \( (v, \theta, \phi) \).
VI. NEWS FUNCTION AND CONSERVED QUANTITIES OF GENERIC ISOLATED HORIZONS

In order to match the Kerr solution that its flux vanishes, we rescale the spinor field. Firstly, the constant spinors $\lambda_0^0$ and $\lambda_1^0$ are rescaled by using the following relation

$$\tilde{\lambda}_0^0 = \lambda_0^0 e^{-\epsilon_0 dv}, \quad \tilde{\lambda}_1^0 = \lambda_1^0 e^{-\epsilon_0 dv},$$

(35)

and it yields the new rescaled momentum integral

$$\tilde{I}(r_\Delta) = e^{-2\epsilon_0 dv} I(r_\Delta) = -\frac{1}{4\pi} \int \mu_\lambda \tilde{\lambda}_0^0 \tilde{\lambda}_0^0 dS_\Delta.$$

(36)

The three compatible conditions (24), (26) and (27) then become

$$\dot{\tilde{\lambda}}_0^0 = 0, \quad \delta_0 \tilde{\lambda}_0^0 = 0, \quad \delta_0 \tilde{\lambda}_1^0 - \mu_0 \tilde{\lambda}_0^0 = 0$$

(37)

where we use $\delta_0 \epsilon_0 = 0$ from asymptotic expansion and they are still compatible under rescaling.

By using this new rescaling constant spinor frame, we apply the time derivative on the quasi-local energy-momentum of NEH (34) and thus we get

$$\dot{I}(r_\Delta) = -\frac{1}{4\pi} \int \mu_\lambda \tilde{\lambda}_0^0 \tilde{\lambda}_0^0 dS_\Delta$$

(38)

We claim that (38) is quasi-local energy flux near NEH. The area will not change for the generic isolated horizons. From (b) in p. 49, we have $\Psi_2^0 = 0$, therefore, we can find ten conserve quantities which are

$$G_m = \int Y_{2,m} \Psi_2^0 dS. \quad (m = -2, -1, ..., 2)$$

(39)

Here these conserved quantities corresponds to three different type of generic isolated horizons. Firstly, the most general NEH does not need to require any stationary. Therefore, we have mass loss or mass gain from the outgoing radiation along NEH. Secondly, we have $\dot{\mu}_0 = 0$ on WIH. Therefore, there is no gravitational radiation on WIH From our asymptotic expansion, it further implies two conditions $\epsilon_0 = 0, \tilde{\epsilon}_0 = 0$. This part is different from Ashtekar’s construction. The most restrict definition is IH. It further needs the condition $\lambda_0 = -\sigma_0^0 = 0$. This is a fully stationary case in our construction.

VII. CONCLUSIONS

We work out the coordinate transformations which carry out the asymptotic symmetric group near the generic isolated horizon. Asymptotically constant spinors can be used to define the quasi-local energy-momentum of the horizon. Searching for the compatible conditions of constant spinors of horizons offers us a way to chose for the reference frame when measuring these quasi-local quantities. We find that the news function exists only for NEH. It indicates the radiation outside the equilibrium black hole. The radiation will not cross the horizon, therefore the area will not increase. The news function of NEH will vanish while we make a special choice of affine parameter $\sigma_\Delta = -\frac{\Delta}{\mu_\ell}$. This result refers to that a generic NEH admits an unique $[\ell]$ such that $(\Delta, [\ell])$ is a WIH on which the incoming expansion $\mu_0$ is time independent [1]. The conserved quantities of the generic isolated horizon is easily shown from the equations of the asymptotic expansion. For a stationary case, it corresponds to mass and angular momentum.

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