Dynamic susceptibility spectra analysis of ferromagnetic spheres via micromagnetic simulations

D Djuhana, C Kurniawan, and A T Widodo

Department of Physics, Faculty of Mathematics and Natural Sciences (FMIPA) Universitas Indonesia, Depok 16424, Indonesia

Center for Physics Research, The Indonesian Institute of Sciences (LIPI), Kawasan Puspitek No. 441-442 Serpong, Banten 15314, Indonesia

Corresponding author: dede.djuhana@sci.ui.ac.id

Abstract. Herein, we report the dynamic susceptibility spectra of ferromagnetic spheres with diameters varying from 50 to 100 nm determined via micromagnetic simulations. The dynamic susceptibility spectra were calculated based on magnetization and the external field response in the frequency domain using Fourier transforms. The peak of the imaginary part of the dynamic susceptibility spectra, which indicates the resonance frequency of the ferromagnetic material, is interestingly near Kittel’s resonance frequency.

1. Introduction
Understanding the high-frequency response of magnetic systems is important for both fundamental and applied research [1,2]. A previous study explained the relation between high-frequency response and magnetization is characterized using the dynamic susceptibility tensor [3]. Many studies have focused on the microscale dynamic susceptibility spectra in the shaped magnetic systems, such as nanopillars [4,5], rectangular structures [6], nanodots [7], and stripes [8]. However, there has been insufficient research on the microscale spheres of ferromagnetic materials.

This study reports the simulated results of the dynamic susceptibility spectra of ferromagnetic spheres modeled in a public-domain micromagnetics software package. The dynamic susceptibility spectra are calculated based on magnetization and the external field response in the frequency domain. The results show that the peak of imaginary part of the dynamic susceptibility spectra is close to Kittel’s resonance frequency. It means that the micromagnetic dynamic susceptibility of ferromagnetic nanosphere materials can be predicted both numerically and analytically.

2. Micromagnetic simulation
Herein, we used the public-domain micromagnetic simulator OOMMF [9] to numerically solve the Landau–Lifshitz–Gilbert (LLG) equation [10]. The diameter of the ferromagnetic spheres modeled herein was varied from 50 to 100 nm. A dynamic exponential magnetic field, 
\[ H(t) = 1000 \exp(-10^6 t), \ t \geq 0 \] (\( H(t) \) in A/m and \( t \) in s), was applied along the \( x \)-direction of the sphere and perpendicular to the spin configuration (\( y \)-direction) [4], as shown in figure 1. The material and parameters are listed in table 1 [11]. The damping factor was \( \alpha = 0.05 \), and the cell size was \( 2.5 \times 2.5 \times 2.5 \) nm\(^3\). The dynamic susceptibility spectra were calculated based on magnetization and the external field behavior in the frequency domain using Fourier transform.
Further, the resonance frequency originates from the discretization of the demagnetizing factors that act on the nanospheres. For example, Ni was 28.2 GHz, and Py was 27.9 GHz. The dynamic susceptibility spectra predicted from Kittel’s formula and the expected anisotropy field of the nanospheres are plotted in the frequency domain. The imaginary part of the dynamic susceptibility spectra is related to the resonance frequency behavior [3,4]. The real and imaginary parts of the dynamic susceptibility spectra are shown in Figure 2. In this figure, the dynamic susceptibility spectra are around the GHz range, as expected at the nanometer scale.

For comparison, we also calculated the resonance frequency of a ferromagnetic sphere model using the Kittel’s formula:

$$\omega_r = \gamma \sqrt{(H_0 + H_z + (N_x - N_y)M_s)(H_0 + H_z + (N_x - N_y)M_s)/(2\pi)}$$

where $\gamma$ is the gyromagnetic ratio $2.21 \times 10^8$ mT$^{-1}$s$^{-1}$ [13], $H_0$ is the static magnetic field, and $H_z = 2K/(\mu_0M_s)$ is the anisotropy field. In the sphere model, the demagnetizing factors $N_x = N_y = N_z$ and $N_x + N_y + N_z = 1$, so Kittel’s resonance frequency is:

$$\omega_r = \gamma (H_0 + H_z)/(2\pi)$$

Figure 3 plots the resonance frequency predicted from Kittel’s formula and the frequency peak of the imaginary part of the simulated dynamic susceptibility spectra for Co, Fe, Ni, and Py. Interestingly, the frequency peaks of the imaginary dynamic susceptibility spectra are close to Kittel’s resonance frequency: Co was 42 GHz, Fe was 29.5 GHz, Ni was 28.2 GHz, and Py was 27.9 GHz. The resonance frequency varied with the diameter of the nanospheres, as predicted by Kittel’s formula, which is expected from the geometry of the demagnetizing factors. The small differences between the micromagnetic simulations and Kittel’s formula originated from the discretization of the cubic model in micromagnetic calculations [14]. Further, the resonance frequency originated from dipolar interactions [4,15].

### Table 1. Material and parameters used in micromagnetic simulation [11].

| Materials | Magnetization saturation $M_s$ (Am$^{-1}$) | Exchange stiffness $A$ (Jm$^{-1}$) | Anisotropy constant $K$ (Jm$^{-1}$) |
|-----------|------------------------------------------|-----------------------------------|----------------------------------|
| Co        | $1400 \times 10^3$                      | $14 \times 10^3$                  | $53 \times 10^3$                 |
| Fe        | $1700 \times 10^3$                      | $21 \times 10^3$                  | $48 \times 10^3$                 |
| Ni        | $490 \times 10^3$                       | $9 \times 10^3$                   | $-5.7 \times 10^3$               |
| Py        | $860 \times 10^3$                       | $13 \times 10^3$                  | 0                                |

3. Results and discussion

According to the resonance principle, the dynamic susceptibility of a ferromagnetic sphere model can be calculated from the relation $\chi(\omega) = M(\omega)/H(\omega) = \chi(\omega') - j\chi(\omega'')$, where $M(\omega)$ and $H(\omega)$ represent the magnetization and the external field, respectively, in the frequency domain. We used fast Fourier transforms (FFTs) to convert the time series of the magnetization response and external field into the frequency domain. The imaginary part of the dynamic susceptibility spectra $\chi(\omega'')$ is related to resonance frequency behavior [3,4]. The real and imaginary parts of the dynamic susceptibility spectra are plotted in figure 2. In this figure, the dynamic susceptibility spectra are around the GHz range, as expected at the nanometer scale.

For comparison, we also calculated the resonance frequency of a ferromagnetic sphere model using the Kittel’s formula:

$$\omega_r = \gamma \sqrt{(H_0 + H_z + (N_x - N_y)M_s)(H_0 + H_z + (N_x - N_y)M_s)/(2\pi)}$$

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Figure 2. The dynamic susceptibility of ferromagnetic sphere model profiles for (a) Co, (b) Fe, (c) Ni, and (d) Py for diameter ranging from 50 to 100 nm. Blue lines indicate the real part, and red lines indicate the imaginary part.

Figure 3. Frequency peak in the imaginary part of the dynamic susceptibility spectra (blue line) and Kittel’s equation predictions (red line) for (a) Co, (b) Fe, (c) Ni, and (d) Py spheres with diameters from 50 to 100 nm.

4. Conclusions
Our findings help in understanding the resonance frequency behavior of nanoscale ferromagnetic spheres via micromagnetic simulations, which yielded the dynamic susceptibility spectra. We found that the imaginary part of the frequency peak in the dynamic susceptibility agrees well with analytical solutions for the resonance frequency of the ferromagnetic nanospheres. Engineers can therefore use Kittel’s equation or magnetic simulation to effectively predict resonance responses when designing magnetic devices with microscale ferromagnetic spheres.
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