The energy conservation optimization design of the cutting edges of the twist drill based on Dijkstra’s algorithm

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Abstract The energy conservation optimization design about geometrical shape and dimensions of the cutting edges (lips) of the twist drill is meant to determine the main cutting edge curve in the rake face that minimizes the drilling power. Dijkstra’s algorithm is introduced and applied to serve the above purpose. The rake surface of the twist drill is discretized into a set of nodes and arcs between the adjacent nodes, which together compose a digraph. Cutting angles of elementary cutting tools on the lips of a twist drill are determined by the method of displaying planar image; those angles, together with undeformed chip thickness and width, are substituted into the empirical model of cutting force for elementary cutting tool to obtain the weights of arcs in the digraph, that is, the drilling power. Then, a main cutting edge curve in the rake face that minimizes the drilling power is generated using Dijkstra’s algorithm. Meanwhile, an improvement in Dijkstra’s algorithm procedures is put forward to reduce time and space complexities of the process and improve smoothness and machinability of the cutting edge curve. A computer program is developed with Matlab 2011b to determine the main cutting edge curve. The calculation results with 0.50 % carbon steel show that the new curved cutting edge reduces the drilling power by 7~8 %, compared to a conventional straight cutting edge.

Keywords Twist drill · Drilling power · Energy conservation · Cutting lip · Shortest path · Dijkstra’s algorithm

1 Introduction

Conventional twist drills are widely used in manufacturing, of which the flake is usually grinded into a conical surface and the main cutting edge (lip) a straight line. An enterprise in China may sell 1000,000,000 conventional high-speed steel (HSS) twist drills annually. However, their drilling performance is not satisfactory. Related problems are many and various, among which is the undesired distribution of the cutting angles of the main cutting edge due to the limitation of the grinding structure and the grinding method, resulting in poor drilling performance and a higher drilling power [1]. For a long time, people have been making huge effort to reduce the energy consumption in drilling by improving the grinding structure of the drill point [1–4]. Among the numerous kinds of improved drills, the most outstanding is the multifacet drill (MFD), which is called the Ni Zhifu drill in China. It is reported that the MFD reduces the energy consumption by 20 % compared to the standard twist drill when drilling plain carbon steel [1, 2].

Unfortunately, the MFD is difficult to be marketed and has a limited range of application because of its especially complicated grinding method. Under the instruction of the Planar Display Theory [5, 6], which aims at mapping of images on a projection plane, Xiong and Shi et al. [7, 8] put forward a curved cutting edge design that controls the distribution of rake angles of the main cutting edge by changing the orientation of the edge, and thus reduces its energy consumption in drilling. This design discards the requirement of straight-line cutting edge of the standard grinding process of the conventional twist drill (without changing its manufacturing process and the original structure). A curve-edged drill that maximizes the value of the rake angle at each point along the tool cutting edge (hereafter referred to as the “drill with large rake angles”) is developed, the flank face of which is grinded as a simple
ruled surface [9, 10]. In this way, grinding problems that block the marketing of the optimized drill are well solved and a new path for energy conservation optimization of the drill tip grinding structure is opened. The drilling torque is reduced by about 12–20 % compared to conventional drills, with remarkable energy-saving effect. Xiong [11], Xiong and Fang et al. [12] further introduce the specific method to optimize the main cutting edge curve and reduce the energy consumption in cutting based on certain given cutting angle distributions of the main cutting edge, such as rake angle distribution, inclination angle distribution, tool cutting edge angle distribution, etc. As is well-known, the cutting power of cutting tools is not only affected by the cutting angle distribution but also by undeformed chip thickness, undeformed chip width, and cutting parameters. Change in the orientation of the main cutting edge will also cause change in cutting angles and in undeformed chip thickness and width. Nevertheless, changes in the three elements may not be in the same direction, and thus may result in heterogeneous change in cutting energy consumption. Therefore, it is almost impossible for the optimized main cutting edge curve based on any randomly given cutting angle distribution to be the most energy-saving curve. The problem is that the given cutting angle distribution is very unlikely to be the optimal distribution. Then what kind of cutting angle distribution is the optimal one? In other words, what kind of main cutting edge curve is the most energy-saving?

The question is essentially how to determine the main cutting edge curve in the helicoid rake face of the twist drill that minimizes the drilling power, which is a typical functional optimization problem. However, “functional analysis” [13] has not provided a general method to determine the functional extremums of such a curve on a hook surface.

From a mathematical perspective, the abovementioned problem is of the same nature with the determination with a traffic navigation software of the shortest path between any two points in a given traffic network. Based on this insight, this essay makes an attempt to apply Dijkstra’s algorithm [14–17] which settles the shortest path problem of the graph theory to the energy conservation optimization of the main cutting edge curve of the twist drill. A specific method used in determining the main cutting edge curve in the helicoid rake face of the twist drill that minimizes the drilling power is introduced. The idea is as follows: The rake surface of the twist drill is discretized into a set of nodes and arcs between the adjacent nodes, which together compose a digraph. The arc between any adjacent nodes (hereafter referred to as “ECT” standing for “elementary cutting tool”) is a possible part of the main cutting edge curve with the minimum drilling power (hereafter referred to as “MCECMDP”); relational models of ECT cutting forces with cutting angles and cutting parameters are established through cutting experiments; the cutting angles and the cutting parameters of the ECTs that constitute the main cutting edge curve of the twist drill are figured out; the drilling forces are calculated through the above models, and then the drilling power can be obtained; the ECT drilling power being the weight of arc in the rake surface digraph, the shortest path between the source and the destination composed of adjacent arcs, the sum of whose weights is the minimal, is located through Dijkstra’s algorithm. This path is exactly the MCECMDP. Programmed calculation shows that the MCECMDP is not the curve with the largest rake angles; when machining a 0.50 % carbon steel workpiece with a drill with a diameter of φ25 mm, the drilling power of a MCECMDP is reduced by 7.45 % compared to that of the straight main cutting edge of a standard twist drill.

2 Introduction to the shortest path problem and Dijkstra’s algorithm

2.1 The shortest path problem

The shortest path problem is a classical topic in graph theory researches. The essence of the issue is to find out the shortest-distance path between two nodes vertex on a digraph [13, 14] \( G = (V, E) \) \( V \) is the set of all the vertices (nodes), and \( E \) is the set of weighted edges that connect the nodes), that is, to find out the path composed of adjacent edges, of which the sum of weights is minimized.

When a source node \( v_s \) in \( V \) is given and the arc weights in \( G \) are all nonnegative real numbers, the shortest path problem that aims to find the shortest distances from \( v_s \) to all other nodes is referred to as the single-source shortest path problem.

2.2 Dijkstra’s algorithm

Dijkstra’s algorithm is a classical algorithm to solve the single-source shortest path problem.

The solution pattern of Dijkstra’s algorithm is as follows: to divide \( V \) into two sets \( S \) and \( T \). \( S \) is the set of all the ending nodes of the determined shortest paths to the source node \( v_s \) at the initial stage, \( S \) only contains \( v_s \). \( T \) is the set of nodes of the undetermined shortest paths to the source node \( v_s \). Nodes in \( T \) will be moved to \( S \) in an ascending order of the length of the shortest paths to the source node \( v_s \), until there is no node left in set \( T \). The path composed by all the edges that connect the source node \( v_s \) to any node \( v_t \) in order is the shortest path from \( v_s \) to \( v_t \). The sum of the corresponding weight values is the length of the shortest path.

Many practical problems can be abstracted and then transformed into the shortest path problem, such as location of the optimal route in the road traffic network, the optimal transmission of information flow among routers in the computer network, etc. Therefore, Dijkstra’s algorithm has been successfully applied in such fields [16] as operational research,
3 Adjacency matrix of the rake face network

To determine the main cutting edge curve of the twist drill with the minimal drilling power is to find a curve on the helicoid rake face that starts from any point on the tool minor cutting edge, for instance the outer corner C, to a point on the cylindrical spiral passing the chisel edge corner B that ensures the consumed drilling power reaches its minimum when this curve serves as the main cutting edge. Obviously, this is a typical single-source shortest path problem. In order to solve this problem with Dijkstra’s algorithm, the rake surface of the twist drill should be discretized into a set of nodes and edges between adjacent nodes, which together compose a digraph (network). Also, the edge weights should be evaluated to establish an adjacency matrix of the network.

3.1 Equation set of the rake face of the twist drill

The rake face of the twist drill corresponding to the main cutting edge is a helicoid formed through the helical motion of the straight generating line round the axis of the drill. Assuming the drill diameter is \( D \), in the drill structure coordinate system \( Oxyz \) shown in Fig. 1, the equations of the rake face corresponding to the straight main cutting edge BC are

\[
\begin{align*}
  x &= \sqrt{r^2 - r_0^2} \cos \varepsilon + r_0 \sin \varepsilon \\
  y &= \sqrt{r^2 - r_0^2} \sin \varepsilon - r_0 \cos \varepsilon \\
  z &= \sqrt{r^2 - r_0^2} \tan \varphi_0 + T \varepsilon / (2\pi)
\end{align*}
\]

where \( 2\varphi_0 \) is the drill point angle, \( 2\varphi_0 = 118^\circ \); \( T \) is the pitch of the drill flute helicoid, \( T = 2\pi R \tan \beta \), in which \( R = 0.5D \) is the drill radius, \( \beta \) is the helical angle in the outer corner C, \( \beta = 30^\circ \); \( r \) is the radius of any point on the rake face; and \( \varepsilon \) is the turning angle of the point.

The coordinates of points C and B are, respectively, \( (\sqrt{R^2 - r_0^2}, -r_0, \sqrt{R^2 - r_0^2} \tan \varphi_0) \), and \( (r_0 / \tan \psi, -r_0, 0) \).

3.2 Discretization of the rake face

As is shown in Fig. 2, the rake face is discretized into a directed network through the following procedures:

1. In the \( xOy \) plane, the rake face’s projection ABCD (BC is the projection of the main cutting edge of the standard twist drill in the \( xOy \) plane; AD and BC are parallel and there, distance to \( x \) axis is \( r_{01} \)) and is divided into \( n - 1 \) arcs segments by \( n \) concentric circles, of which the radiiuses are distributed with arithmetic progression and the center is \( O \), the origin of coordinates. The radius of the \( i \) th concentric circle is

\[
r_i = R - (i - 1) \left( \frac{R - r_0}{\sin \psi} \right) / (n-1), \quad i = 1, 2, 3 \ldots n
\]

where \( \psi \) is the chisel edge angle of the conventional twist drill. Correspondingly, the rake face is divided into \( n - 1 \) strips of helicoid by \( n \) co-axial cylindrical surfaces.

2. Each arc between AD and BC is divided into \( m - 1 \) equal segments (so the grids in the rake face are different in...
size). Assuming \( j \) is the equipartition point number, for the points on AD, \( j = 1 \), and for the points on BC, \( j = m \). Therefore, \( nm \) (i.e. \( n \times m \)) nodes are obtained. The \( j \) th node on the \( i \) th arc is marked as \( P_{ij} \), and the angle between \( OP_{ij} \) and \( x \) axis is

\[
\theta_{ij} = (j-1) \left( \tan^{-1} \left( \frac{y_{ij}}{x_{ij}} \right) + \tan^{-1} \left( \frac{y_{ij}}{x_{ij}} \right) \right) / (m-1) + \tan^{-1} \left( \frac{y_{ij}}{x_{ij}} \right), \quad j = 1, 2, 3, \ldots m
\]

Assuming the coordinates of \( P_i \) are \((x_{ij}, y_{ij}, z_{ij})\), since \( P_{ij} \) is on the rake face, the coordinates can be obtained on the related geometrical relations and the equation set (1):

\[
x_{ij} = r_i \cos \theta_{ij}, \quad y_{ij} = r_i \sin \theta_{ij}, \quad z_{ij} = t_i / \tan \varphi_0 + T \varepsilon / (2\pi)
\]

where \( t_i = \sqrt{r_i^2 - r_0^2} \) and the parameter \( \varepsilon \) can be determined through the equation set (1) based on the values of the coordinates \((x_{ij}, y_{ij})\):

\[
\begin{bmatrix} \cos \varepsilon \\ \sin \varepsilon \end{bmatrix} = \begin{bmatrix} t_1 & r_0 \\ -r_0 & t_1 \end{bmatrix}^{-1} \begin{bmatrix} x_{ij} \\ -y_{ij} \end{bmatrix}
\]

\[
= \begin{bmatrix} t_1 & r_0 \\ -r_0 & t_1 \end{bmatrix}^{-1} \begin{bmatrix} r_i \cos \theta_{ij} \\ -r_i \sin \theta_{ij} \end{bmatrix}
\]

3. For programming convenience, all the nodes are renumbered from 1 to \( nm \). Assume node \( P_g \) is renumbered \( g \), and node \( P_{ij+1} \) is renumbered \( h \), then

\[
g = (i-1) \quad m + j, \quad h = im + k
\]

Hereafter, \( P_g \) and \( P_h \) are used to represent the same nodes \( P_{ij} \) and \( P_{ij+1} \) on the condition that no misunderstanding would be caused.

4. Connect node \( P_g \) and node \( P_h \). If the projection length \( l_{ijk} \) of the line segment \( P_gP_h \) in the \( xOy \) plane satisfies

\[
l_{ijk} = |P_gP_h|_{xOy} = \sqrt{(x_g-x_h)^2 + (y_g-y_h)^2} \leq \Delta r_i
\]

where

\[
\Delta r_i = \sqrt{r_i^2 - r_{i+1}^2}, \quad i = 1, 2, 3, \ldots, n-1
\]

In other words, if there is one, but only one intersection between the \( i + 1 \)th arc and the line segment \( P_gP_h \) obtained by connecting node \( P_g \) and node \( P_{ij+1} \), then line \( P_gP_h \) is one edge of the directed network (digraph) of the rake face.

### 3.3 Adjacency matrix

The adjacency matrix \( A \) of the directed network of the rake face is a \( nm \times nm \) square matrix. Its element \( w_{gh} \) (\( g = 1, 2, 3 \ldots \)) is the weight value of the edge that connects the \( g \) th and \( h \) th nodes.

Any edge in the directed network of the rake face may be a segment of the MCECMDP that is to be determined. The drill part that corresponds to the edge \( P_gP_h \) viewed as an ECT that can operate cutting independently, its cutting edge is the edge \( P_gP_h \), and its rake face is the tangent plane to the rake face at the midpoint of this edge. If the edge is short enough, it can be substituted by the straight line that connects its ends. Since it is the main cutting edge with the minimal drilling power that is to be determined, the cutting power (indicated as \( \Delta P_{gh} \)) of the ECT is defined as the weight of the edge \( P_gP_h \).

\[
w_{gh} = \Delta P_{gh}, \quad |P_gP_h|_{xOy} \leq \Delta r_i
\]

If the length of the edge that connects node \( P_g \) and node \( P_h \) does not satisfy the Eq. (7), it means there exists no such edge that can connect these two nodes. In this case, the corresponding cutting power \( \Delta P_{gh} \) of the ECT would be infinite. Therefore,

\[
w_{gh} = \begin{cases} \Delta P_{gh}, & |P_gP_h|_{xOy} \leq \Delta r_i \\ \infty, & |P_gP_h|_{xOy} > \Delta r_i \end{cases}
\]

By calculating the drilling powers of all the ECTs, that is, the weights of all edges, the adjacency matrix \( A \) of the rake face discrete network (digraph) can be obtained.

### 4 Calculation of edge weight

The terminologies and definitions employed in this section follow those recommended by ISO 3002 [18, 19].

In order to determine the cutting power of the ECT, its cutting angles and its undeformed chip thickness and width should be calculated first.

#### 4.1 ECT cutting angles

ECT cutting angles are defined as the cutting angles of the main cutting edge of the twist drill at the midpoint of the ECT cutting edge, including the working rake angle \( \gamma_\text{w} \), the working normal rake angle \( \gamma_\text{n} \), the working tool inclination angle \( \lambda_\text{w} \), and the working cutting edge angle \( \kappa_\text{w} \). The method of displaying planar image [5, 6] is applied to determine the ECT cutting angles.

#### 4.1.1 Formation of image points and image lines on the projection plane

Assume the coordinates of the end points of the edge \( P_gP_h \) to be \((x_g, y_g, z_g)\) and \((x_h, y_h, z_h)\). The coordinates \((x_{ghm}, y_{ghm}, z_{ghm})\)
and the radius of its midpoint \( P_{im} \) are, respectively, as follows:

\[
\begin{align*}
\begin{bmatrix}
 x_{ghm} \\
 y_{ghm} \\
 z_{ghm}
\end{bmatrix} &= \frac{1}{2}\begin{bmatrix}
 x_g + x_h \\
 y_g + y_h \\
 z_g + z_h
\end{bmatrix}, \\
\begin{bmatrix}
 r_{ghm} \\
 \end{bmatrix} &= \frac{1}{2}\begin{bmatrix}
 r_g + r_h \\
 \end{bmatrix}
\end{align*}
\] (11)

Assume the rotation speed of the twist drill to be \( N/\text{r} \cdot \text{s}^{-1} \) and its feed to be \( f/\text{m} \cdot \text{r}^{-1} \). The image points and image lines can be created in the projection plane according to the method of displaying planar image [5, 6], as shown in Fig. 3. \( d \) in the figure is the image line of the tangent line (which is substituted by line \( P_{ghm}P_{ghm} \) of the ECT cutting edge at its midpoint; \( Q, P_e, T_e, R_e, N, F \) are, respectively, the image points of the rake face, the working reference plane, the working cutting edge plane, the working orthogonal plane, the cutting edge normal plane, and the assumed working plane at this midpoint. Based on the definitions of these planes and their relations and the Eq. set (1) for the rake face, the homogeneous coordinates [5, 6, 8] of these images can be obtained.

\[
d = \begin{bmatrix}
 x_h - x_g \\
 y_h - y_g \\
 z_h - z_g
\end{bmatrix}^T = N, \quad (12)
\]

\[
Q = \begin{bmatrix}
 \frac{dy}{dx} \\
 \frac{d\lambda}{dx} \\
\end{bmatrix}_{\lambda=0}
\]

\[
= \begin{bmatrix}
 \frac{\tan \phi - 2\pi r_0 \sin \pi - 2\pi \sqrt{r^2_r - r_0^2} \cos \phi}{-T \cos \tan \phi + 2\pi r_0 \cos \pi - 2\pi \sqrt{r^2_r - r_0^2} \sin \phi}
\end{bmatrix}
\]

\[
= \begin{bmatrix}
 \frac{\tan \phi - 2\pi r_0 \sin \pi - 2\pi \sqrt{r^2_r - r_0^2} \cos \phi}{T + 2\pi \sqrt{r^2_r - r_0^2} \sin \phi - r_0 \cos \phi}
\end{bmatrix}
\]

\[
P_e = v_e = \begin{bmatrix}
 -\pi (z_g + z_h) \\
 -f \pi (z_g + z_h)
\end{bmatrix}^T, \quad (14)
\]

\[
T_e = d \times v_e, \quad (15)
\]

\[
R_e = P_e \times T_e, \quad (16)
\]

and

\[
F = \begin{bmatrix}
 -(x_g + x_h) \\
 0 (z_g + z_h)
\end{bmatrix}^T \div 2, \quad (17)
\]

where \( v_e \) is the image line of the cutting velocity \( v_e \) at the midpoint.

4.1.2 Calculation of the cutting angles

According to the method of displaying planar image [5, 6], the Euclidean angle \( \theta_{ACB} \) between the planes \( A \) and \( B \) which is measured in the plane \( C \) is equal to the non-Euclidean angle \( \theta_{ACB} \) formed by the image points \( A, B, \) and \( C \). Based on the homogeneous coordinates, their value can be obtained [5, 6]:

\[
\theta_{ACB} = \theta_{ACB} = \tan^{-1}\left(\sqrt{C^T C \begin{bmatrix}
 A_1 \\
 B_1 \\
 C_1
\end{bmatrix} \begin{bmatrix}
 A_2 \\
 B_2 \\
 C_2
\end{bmatrix} \begin{bmatrix}
 A_3 \\
 B_3 \\
 C_3
\end{bmatrix} \begin{bmatrix}
 C^T \\
 B^T \\
 A^T
\end{bmatrix}}\right). \quad (18)
\]

Substituting the homogeneous coordinates obtained from Eqs. (12)~(17) in Eq. (18) yields the values of all the cutting angles, that is, the working rake angle \( \gamma_{0e} \), the working normal rake angle \( \gamma_{ne} \), the working tool inclination angle \( \lambda_{te} \), and the working cutting edge angle \( \kappa_{te} \) Fig. 3:

\[
\begin{align*}
\gamma_{0e} &= \theta_{QRP_e}, \\
\gamma_{ne} &= \theta_{QNP_e}, \\
\lambda_{te} &= \theta_{QTP_e}, \\
\kappa_{te} &= \theta_{FPe},
\end{align*} \quad (19)
\]

4.2 Undeformed chip thickness and cut width

The undeformed chip thickness \( a_{gh} \) and the cut width \( a_{wgh} \) of the ECT are, respectively, [1, 8]

\[
a_{gh} = f \sin \kappa_{regh} v_{gh} / (2v_{gh}), \quad (20)
\]

and [1, 8]

\[
a_{wgh} = d \lambda_{regh} \cos \lambda_{wgh}, \quad (21)
\]

where \( \kappa_{regh} \) and \( \lambda_{wgh} \) are, respectively, the working cutting edge angle and the working tool inclination angle of the
ECT; \( v_{gb}/v_{gbh} \) is the ratio of the rotational speed \( v_{gb} \) to cutting velocity \( v_{gbh} \), and

\[
v_{gb}/v_{gbh} = \frac{(x_{gbh})^2 + (y_{gbh})^2 + (f/2\pi)^2 + (y_{gbh})^2}{(x_{gbh})^2 + (y_{gbh})^2},
\]
(22)

\( dl_{gb} \) is the length of the ECT cutting edge, and

\[
dl_{gb} = \sqrt{(x_h-x_g)^2 + (y_h-y_g)^2 + (z_h-z_g)^2}.
\]
(23)

4.3 Empirical models of ECT cutting forces

The ECT cutting forces are calculated with the empirical relational models of the cutting forces with the cutting angles, the undeformed chip thickness, and the cut width that are obtained through cutting experiments.

4.3.1 Turning experiments on the end face of sleeve parts

A set of turning experiments with HSS end-cut tools on 0.50 % carbon steel were designed, which adopt the L25(5\(^6\)) orthogonal array based on the three-factor experimental results so as to establish the empirical models of the cutting forces. The experiments were conducted on a CM6140-type horizontal lathe. The objects to be machined are turned 0.50 % carbon steel sleeve parts (with a hardness of HBS241, an outer diameter of \( \phi \)115 mm, and an inner diameter of \( \phi \)110 mm, Fig. 4). The cutting forces were measured by a Kistler three-component piezoelectric dynamometer with the computer data acquisition system. Altogether, 25 sets of HSS end-cut tools (with the cutting edge angle \( \kappa_r=90^\circ \)) with different rake angles and inclination angles were grinded, and a second round of cutting experiments were made, in which the cutting tools were resharpened. Due to limited space, Table 1 shows the results so as to establish the empirical models of the cutting forces.

Table 1: Results of face cutting experiments on sleeve parts

| No. | \( \gamma_0^\circ \) | \( \lambda_\circ \) | \( f/\text{mm} \times \text{r}^{-1} \) | \( v_c/\text{mm} \times \text{min}^{-1} \) | \( F_r / \text{N} \) | \( F_a / \text{N} \) | \( F_t / \text{N} \) |
|-----|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 1   | -30             | -30             | 0.04            | 3.5             | 163.2           | 31.8            | 209.5           |
| 2   | -30             | -15             | 0.05            | 7.1             | 140.7           | 34.8            | 245.2           |
| 3   | -30             | 0                | 0.065           | 14.1            | 371.8           | 14.0            | 352.5           |
| 4   | -30             | 0.08            | 22.3            | 457.0           | -52.3           | 326.9           |
| 5   | -30             | 0.1             | 28.3            | 568.0           | -80.4           | 381.5           |
| 6   | -15             | -30             | 0.05            | 14.1            | 127.7           | 62.8            | 218.5           |
| 7   | -15             | 0.065           | 22.3            | 393.4           | 57.5            | 349.4           |
| 8   | -15             | 0.08            | 28.3            | 406.1           | 12.2            | 343.8           |
| 9   | -15             | 15               | 0.1             | 3.5             | 255.2           | -29.5           | 390.2           |
| 10  | -15             | 0.04            | 7.1             | 110.1           | -41.8           | 182.1           |
| 11  | 0                | -30             | 0.065           | 28.3            | 262.4           | 79.2            | 296.7           |
| 12  | 0                | -15             | 0.08            | 3.5             | 175.1           | 40.7            | 316.2           |
| 13  | 0                | 0                | 0.1             | 7.1             | 138.7           | 17.4            | 291.4           |
| 14  | 0                | 15               | 0.04            | 14.1            | 84.0            | -10.9           | 151.4           |
| 15  | 0                | 30               | 0.05            | 22.3            | 126.6           | -50.7           | 196.6           |
| 16  | 15               | -30              | 0.08            | 7.1             | 91.6            | 59.1            | 250.6           |
| 17  | 15               | -15              | 0.1             | 14.1            | 125.6           | 41.4            | 273.3           |
| 18  | 15               | 0                | 0.04            | 22.3            | 71.1            | 4.4             | 132.0           |
| 19  | 15               | 15               | 0.05            | 28.3            | 87.3            | -16.8           | 161.9           |
| 20  | 15               | 30               | 0.065           | 3.5             | 77.8            | -50.2           | 198.5           |
| 21  | 30               | -30              | 0.1             | 22.3            | 84.2            | 84.1            | 271.6           |
| 22  | 30               | -15              | 0.04            | 28.3            | 51.7            | 19.1            | 115.9           |
| 23  | 30               | 0                | 0.05            | 3.5             | 39.2            | 5.0             | 130.1           |
| 24  | 30               | 15               | 0.065           | 7.1             | 55.8            | -13.4           | 155.4           |
| 25  | 30               | 30               | 0.08            | 14.1            | 103.1           | -81.5           | 232.7           |

Fig. 4: Face cutting experiments on sleeve parts

revolution; \( v_c \) is the working cutting speed, which is calculated based on the rotation speed and the workpiece pitch diameter (with the feed speed ignored); \( F_r \) is the axial thrust force; \( F_a \) is the radial thrust force, and \( F_t \) is the main cutting force.

4.3.2 Empirical models of cutting forces

The experimental data were imported to the statistical analysis software EView [20] so that the following empirical relational formulas of the main cutting force \( F_t \), the axial thrust force \( F_a \), and the radial thrust force \( F_r \) with the cutting angles, the undeformed chip thickness, and the cut width were fitted:

\[
\begin{align*}
F_t &= 2.567 \times 10^8 a_w a_c^{0.8233} \exp(-0.6204 \gamma_n - 0.4271 \lambda^2) \text{ N} \\
F_a &= 1.323 \times 10^8 a_w a_c^{0.455} (\cos \lambda)^{1.35} \sqrt{v_c^{2121} \exp(1.720(1-\sin \gamma_n))} \text{ N} \\
F_r &= 5.000 \times 10^8 a_w (-3.461 \times 10^8 a_c + 1.739 \times 10^8 a_c^2 - 2.509 \times 10^4 a_c^3 \\
&+ 1.208 \times 10^3 a_c^4 - 4.817 a_c^5 + 3.730 a_c^6 - 0.824 a_c^7) \text{ N}
\end{align*}
\]
(24)

where the unit of the cutting angles (including \( \gamma_n \) and \( \lambda \)) is radian, and \( \gamma_n = \tan^{-1}(\tan \gamma_c \cos \lambda) \); \( a_w \) is the cut width, and for the given experimental condition \( a_w = 2.5 \times 10^{-3} \cos \lambda \) (in meters); \( a_c \) is the undeformed chip thickness in meters, and \( a_c = f \); \( v_c \) is in meters per second (m\( \times \)s\(^{-1} \)).
In end-face turning, the differences between the cutting angles ($\gamma_n$, $\lambda_e$, and $\kappa_e$) and the working cutting angles ($\gamma_n\mathrm{wc}$, $\lambda_e\mathrm{wc}$, and $\kappa_e\mathrm{wc}$) are negligible. Therefore, the latter are substituted for the former in calculation in the following sections.

### 4.4 ECT cutting forces

The above empirical formulas are established on the condition that the cutting edge angle $\kappa_e$ is 90°. Since no consideration is given to the influence of the change in the angle on the cutting force while in practice the angle may change along the main cutting edge, Eq. (24) cannot be directly used to calculate the drilling forces of the ECTs which constitute the main cutting edge. Therefore, in reference to [21], it is assumed that

\[
\mathbf{R} = \mathbf{R}_1 \mathbf{R}_2 \mathbf{R}_3 \quad \text{and} \quad \mathbf{R}_n = \mathbf{R}_4 \mathbf{R}_2 \mathbf{R}_3
\]

where

\[
\mathbf{R}_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \mathbf{R}_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \lambda_e & -\sin \lambda_e \\ 0 & \sin \lambda_e & \cos \lambda_e \end{pmatrix} \quad \mathbf{R}_3 = \begin{pmatrix} \cos \gamma_n & 0 & -\sin \gamma_n \\ 0 & 1 & 0 \\ \sin \gamma_n & 0 & \cos \gamma_n \end{pmatrix} \quad \mathbf{R}_4 = \begin{pmatrix} \sin \kappa_e & -\cos \kappa_e & 0 \\ \cos \kappa_e & \sin \kappa_e & 0 \\ 0 & 0 & 1 \end{pmatrix}
\]

(25)

(26)

Then, plug into the following equation the cutting forces $\Delta F_s$, $\Delta F_o$, $\Delta F_t$ predicted by Eq. (24) when the cutting edge angle is 90°, the normal rake angle is $\gamma_n\mathrm{wc}$, and the tool inclination angle is $\lambda_e\mathrm{wc}$.

\[
(\Delta F_{\text{new}}) = \mathbf{R}_n \mathbf{R}^{-1} (\Delta F_s, \Delta F_o, \Delta F_t)^T.
\]

(27)

and the values of $\Delta F_{\text{new}}$, $\Delta F_{\text{new}}$, and $\Delta F_{\text{new}}$ can be obtained, which are the cutting forces when the cutting edge angle is $\kappa_e\mathrm{wc}$, the normal rake angle is $\gamma_n\mathrm{wc}$, and the tool inclination angle is $\lambda_e\mathrm{wc}$.

### 4.5 ECT drilling power

In order to obtain the ECT drilling power, the three cutting forces ($\Delta F_{\text{new}}$, $\Delta F_{\text{new}}$, $\Delta F_{\text{new}}$) calculated with Eq. (27) should be transformed into tangential, radical, and axial components at the position of the ECT on the main cutting edge. Assume the three components to be $\Delta F_{\text{tg}}, \Delta F_{\text{th}}, \Delta F_{\text{th}}$, respectively, with reference to [1], the torque $\Delta T_{gh}$ and axial force $\Delta F_{gh}$ of the ECT between node $g$ and node $h$ are, respectively,

\[
\Delta T_{gh} = \Delta F_{\text{tg}} r_{gh}, \Delta F_{gh} = \Delta F_{\text{th}}.
\]

(28)

The drilling power $\Delta P_{gh}$ (in watts) of the $i$ th ECT, that is, the weight $w_{gh}$ of edge $P_{gh}$ is

\[
w_{gh} = \Delta P_{gh} = N (2\pi \Delta F_{\text{tg}} r_{gh} + |\Delta F_{\text{th}}| f).
\]

(29)

Apparently, the torque $\Delta T_{gh}$ of the ECT can be deemed as the weight $w_{gh}$ of edge $P_{gh}$. However, what is obtained through this procedure is the main cutting edge curve with the minimal torque.

### 4.6 Validation of the drilling power (force) calculation model

In order to examine the accurateness of the drilling force and the drilling power obtained through the above procedure, the predicted results for the drilling power, the torque, and the axial force of the standard twist drill are compared to the calculation results obtained from the formulas recommended in reference [2], as shown in Table 2. The structural parameters used in the calculation are as follows: diameter: $D$ = 25 mm.; drill core thickness 2 $r_0$ = 3.6 mm.; and half drill point angle: $\varphi_0$ = 59°; chisel edge inclination angle: $\psi$ = 55°; and helical angle at the outer corner C: $\beta$ = 30°. The workpiece material is 0.50% carbon steel. Since the tool inclination angle along the chisel edge is 0, the cutting process is deemed as an orthogonal cutting process, and therefore $F_o$ = 0. The figures in Table 2 reveal that the relative errors ($=|\text{predicted value - value in reference}| \times 100\%$) between the predicted values of the drilling power and the torque and the calculation results from formulas in reference [2] are within ±5.5%, which is acceptable. This indicates that the previously proposed models of the drilling force and the drilling power are accurate.

### 5 MCECMDF based on Dijkstra’s algorithm

#### 5.1 Program design

The steps below describe the procedure based on Dijkstra’s algorithm for determining the MCECMDF on the given rake face and calculating its drilling power (Fig. 5):

1. Input the structural parameters ($D$, $r_0$, $\varphi_0$, $\beta$, $\psi$) of the twist drill, its rotation speed ($N$) and feed ($f$), the search region parameter ($r_0$), and the meshing parameters ($n$, $m$).
2. Do the meshing. Let $i = 1, 2, 3, \ldots$ $n - 1$ and $j = 1, 2, 3, \ldots$ $m$, then calculate the polar coordinates ($r_i$, $\theta_j$) and the spatial coordinates ($x_{gh}$, $y_{gh}$, $z_{gh}$), etc. with Eqs. (1)–(5), (11).
3. Let $i = 1, j = 1, k = 1$. 

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5.2 Preliminary result

In calculation, the structural parameters of the twist drill are identical to those of the standard twist drill mentioned in Section 4.6; the drilling parameters are as follows: rotation speed: \( N = 450 \text{ r} \cdot \text{min}^{-1} = 7.5 \text{ r} \cdot \text{s}^{-1} \); feed: \( f = 0.11 \text{ mm} \cdot \text{r}^{-1} = 1.1 \times 10^{-3} \text{ m} \cdot \text{r}^{-1} \); discrete parameters of the rake face helicoid: \( n = 80, m = 80 \); parameters of search region: distance from straight line AD to \( x: r_0 = 0.35 r_0 \) and the radius of arc AB: \( r_0/\sin \psi \).

5.2.1 Minimal drilling power

The calculated minimal drilling power of the main cutting edge \( P_{\text{min}} \) is 894.888 W (accurate to three decimal places, the same hereafter). The corresponding drilling torque \( T \) is 19.228 N\( \cdot \)m.

Move straight line AD in Fig. 2 till it is infinitely close to straight line BC, and the drilling power and the drilling force of the two main cutting edges of the standard twist drill can be obtained with the abovementioned method. The drilling power \( P_{\text{standard}} \) is 962.409 W and the torque \( T_{\text{standard}} \) is 20.753 N\( \cdot \)m.

The calculated minimal drilling power of the main cutting edge is reduced by 7.016 \% compared to the drilling power of the main cutting edge of the standard twist drill.

### Table 2

Comparison between the drilling power, the torque, and the axial force of the standard twist drill as predicted by the proposed models and the values calculated with the formulas recommended in reference [2].

| Rotating speed \( N/\text{r} \cdot \text{min}^{-1} \) | Feed \( f/\text{mm} \cdot \text{r}^{-1} \) | Drilling power \( P/\text{W} \) | Torque \( T/\text{N} \cdot \text{m} \) | Axial force \( R/\text{N} \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|
|                 |                 | Predicted       | Reference       | Predicted       | Reference       |
| 300             | 0.02            | 169.978         | 165.122         | 5.409           | 5.254           | -560.205        | -608.728        |
|                 | 0.05            | 361.191         | 364.813         | 11.488          | 11.603          | -1117.106       | -1193.738       |
|                 | 0.08            | 531.563         | 548.031         | 16.902          | 17.423          | -1394.141       | -1686.309       |
|                 | 0.11            | 690.701         | 722.152         | 21.956          | 22.950          | -1720.144       | -2131.03        |
|                 | 0.14            | 842.502         | 890.060         | 26.768          | 28.275          | -2238.489       | -2544.311       |
| 450             | 0.02            | 254.980         | 244.171         | 5.409           | 5.180           | -600.799        | -591.933        |
|                 | 0.05            | 541.859         | 539.572         | 11.489          | 11.441          | -1213.265       | -1160.803       |
|                 | 0.08            | 797.524         | 810.643         | 16.904          | 17.181          | -1543.487       | -1639.785       |
|                 | 0.11            | 1036.383        | 1068.273        | 21.959          | 22.633          | -1921.106       | -2072.237       |
|                 | 0.14            | 1264.281        | 1316.723        | 26.773          | 27.887          | -2489.826       | -2474.115       |
| 600             | 0.02            | 339.985         | 322.302         | 5.409           | 5.128           | -631.793        | -580.299        |
|                 | 0.05            | 722.552         | 712.330         | 11.490          | 11.328          | -1286.684       | -1137.989       |
|                 | 0.08            | 1063.548        | 1070.268        | 16.906          | 17.013          | -1657.514       | -1607.556       |
|                 | 0.11            | 1382.182        | 1410.477        | 21.962          | 22.413          | -2074.542       | -2031.508       |
|                 | 0.14            | 1686.245        | 1738.576        | 26.778          | 27.616          | -2681.725       | -2425.488       |

Relative errors (%) 

\( -5.343 \text{~} 5.486 \text{~} -5.330 \text{~} 5.485 \text{~} -19.281 \text{~} 13.066 \)
5.2.2 MCECMDP

The calculated MCECMDP and the distribution curves of the cutting angles and the drilling power are shown in Fig. 6. The following conclusions can be drawn from the figure:

1. For most points on the main cutting edge curve with the minimal drilling power, the corresponding tool inclination angle \( \lambda_{se} \) is not 0. As is known, only when \( \lambda_{se} = 0 \) does the rake angle take the maximum value [5–8]. Therefore, the MCECMDP is not the main cutting edge curve with the largest rake angles.

2. The distribution curves of the cutting angles of the MCECMDP are broken lines, among which the one for the cutting edge angle \( \kappa_{re} \) is a completely zigzagged broken line. The main cutting edge curve is not smooth, either, the enlarged version of which shows that some cusps exist on the curve. Since the main cutting edge is grinded by the grinding wheel, and a single feed is unlikely to result in a cusp, the machinability of the main cutting edge resulted from the previous procedure is not satisfactory.

5.3 Densification of the discretized grids on the rake face

The reason for the poor machinability of the above MCECMDP is that the discretized grids are too large.

The smoothness and the machinability of the main cutting edge can be guaranteed by eliminating the cusps on the MCECMDP and on the distribution curves of the cutting angles through grid densification.

If the grid is densified directly, for example, if the values of \( n \) and \( m \) are increased on the condition that the discretized region ABCD remains unchanged, the complexity of the calculation process will increase dramatically as the number of the nodes increases. As is known to all, when Dijkstra’s algorithm is used to solve the single-source shortest path problem, its time complexity will be \( O(p^2) \) [14] if the node number is \( p \).

In other words, the time complexity of the calculation process is proportional to the square of the node number. To avoid excessive time consumption, successive densification by means of gradual diminishment of the discretized region ABCD (Fig. 2) is adopted to solve the problem. The procedure is as follows:

1. Assign a moderately bigger value to \( n \), and a moderately smaller value to \( m \), for example, \( n=100 \) and \( m=50 \), then determine the MCECMDP. Of course, neither the main cutting edge curve or the distribution curves of the cutting angles is smooth.
2. Assume the central angle of the arc between x axis and the node on the newly determined main cutting edge curve where the radius is \( r_i \) to be \( \theta_{\alpha i} \). Construct two curves by connecting nodes \((r_i, \theta_{\alpha i}^\text{bound})\) \((i=1,2,3,...n)\) and \((r_i, \theta_{\alpha i}^\text{+ bound})\) \((i=1,2,3,...n)\), respectively, in successive order. Constrict the discretized region ABCD to the area formed by the two new curves and the two arcs (AD, BC), with \( n \) and \( m \) unchanged. Determine the corresponding MCEC MDP.

3. If the main cutting edge curve and the distribution curves of the cutting angles remain broken, multiply bound by \( k \) \((0<k<1)\) to form a new bound. Then, return to (2). Otherwise, the calculation process comes to the end.

5.4 Calculation results after grid densification

Figure 7 illustrates the calculation results after the above densification procedure when \( n=400,m=50,\text{bound}=5^\circ,k=0.8\), and the other parameters are the same as stipulated in Section 5.2. Figure 7a shows the situation before densification, where some cusps exist apparently on the main cutting edge curve and the distribution curves of the cutting angles; Fig. 7b, c, respectively, show the results after the first and the second rounds of densification, where the curves become more smooth successively but the cusps still exist, though less apparently; Fig.7d is the result after enough rounds of densification, where all the curves have become smooth. It can be seen from the above figures that every round of densification results in moderate improvement in smoothness of the curves. Correspondingly, the calculated value of the minimal drilling power is diminished gradually until it becomes a constant as the grids are successively densified. The values of the minimal drilling power corresponding to Fig. 7a–d are, respectively, 916.383, 912.724, 907.450, and 892.132 W. Compared to the drilling power of the straight edged standard twist drill, which is 963.941 W, the drilling power of the curved main cutting edge is reduced by 7.450 %.

The following two points are worth mentioning:

1. Any point on the secondary cutting edge of the twist drill being the source point, a MCECMMDP whose ending point is somewhere on the cylindrical surface of the drill core can be determined through the abovementioned method, and all these main cutting edge curves have the same spatial shape and the same drilling power. Any one of them \((l)\) can be deemed as the result of the helical motion of another \((l')\) round the drill spindle until it reaches the present position (of \( l \)).

2. The optimization of the main cutting edge curve can only result in a decrease of 7–8 % in drilling power compared to the main cutting edge of the standard twist drill. It can be inferred that the fact that the Ni Zhifu drill and the large-rake-angle drill reduce the energy consumption in drilling by about 20 % is the synthetic effect of multiple improvements in structure such as the curved main cutting edge, the chip breaker, and the shortened chisel edge, etc.
6 Conclusion

1. The first attempt is made to use Dijkstra’s algorithm to determine the MCECMDP by discretizing the helicoid rake surface into a digraph composed of a set of nodes and edges between the nodes.

2. For a given helicoid rake face, the MCECMDP is not the one with the largest rake angles.

3. By grinding the main cutting edges of the standard twist drill as curves, the drilling power of the main cutting edge can be reduced by 7.45% at most compared to the straight edge when cutting the 0.50% carbon steel under the
The circumstance of the given cutting parameters. The fact that the MFD and the large-rake-angle drill reduce the energy consumption in drilling by about 30% is the synthetic effect of multiple improvements in structure such as the curved main cutting edge, the chip breaker, and the shortened chisel edge, etc.

4. The shortest path algorithms can be effectively used to determine the functional extremums of any curve defined in a spatial hook surface through discretization.

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