Abstract—An underlay cognitive radio network with energy harvesting is considered which operates in slotted fashion. The primary user (PU) transmits with a constant power in each slot, while the secondary user (SU) either harvests energy from primary’s transmission or transmits its data. We propose an optimal offline harvest-or-transmit strategy where in each slot, SU takes a decision whether to harvest energy or transmit its data limiting interference at the primary receiver. We aim to maximize the achievable rate of SU under energy causality and interference constraints. The optimization problem is formulated as a mixed integer non-linear program and the optimal harvest-or-transmit policy is obtained using generalized Benders decomposition algorithm. Through simulations, we analyze the effects of various system parameters and interference constraint at the primary receiver on the optimal policy.

I. INTRODUCTION

In a wireless communication system, two major challenges are to achieve high spectral and energy efficiency. One of the possible solution for these two challenges is energy harvesting cognitive radio network (EH-CRN) [1]. In EH-CRNs, a set of users namely licensed (primary) and unlicensed (secondary) users (PU and SU respectively) share the same spectrum while harvesting energy from the environment. EH-CRNs have been studied operating in interweave mode in [2]–[5], overlay mode in [6]–[9] and underlay mode in [10]–[12].

In underlay CRNs, the PU and SU users coexist and the SU transmits along with PU while limiting the interference at primary receiver (PR). In [10], an underlay EH-CRN is considered where SU harvests energy at the beginning of each slot. The authors used geometric waterfilling with peak power constraint to obtain an optimal offline power allocation policy for SU which maximizes the throughput. In [11], the cooperation between energy harvesting PU and SU is considered at energy level. In each slot, SU may transfer some fraction of its energy to PU and transmits along with it. Authors obtained transmission policies maximizing SU’s throughput and showed that energy cooperation helps secondary improve its performance. In [12] and [13], authors considered a scenario where the first fraction of each slot is used by the SU to harvest energy from the PU’s transmission, and the remaining fraction is used for the data transmission. The authors obtained a suboptimal myopic transmission policy in [12] and an optimal offline transmission policy in [13] maximizing SU’s achievable throughput under outage constraint of PU.

We consider the system model similar to [12] and [13]. However in our model, each slot is dedicated either for energy harvesting or information transfer (harvest-or-transmit policy). This policy makes the switching between the harvesting module and transmission module less complex by allowing less frequent switching ($N-1$ switchings in worst case as compared to $2N-1$ in [12] and [13] for $N$ slots). In addition, unlike the time sharing policy, switching occurs only at the end of the slot which results in less complex switching circuitry. We are interested in finding an optimal offline harvest-or-transmit policy which acts as a benchmark for the online and suboptimal offline policies for the system model under consideration, and gives an upper bound on the system performance. The channel gains can be obtained using any channel prediction technique [14]. Our contributions in this paper are as follows:

- Aiming to maximize the achievable rate of SU over a finite number of slots under PU’s interference constraint and SU’s energy causality constraint, we formulate the optimization problem as a non-convex mixed integer non-linear program (MINLP). Then, we convert the non-convex MINLP into an equivalent convex MINLP and obtained the optimal harvest-or-transmit policy using generalized Bender’s decomposition (GBD) algorithm.
- We then analyze the effects of various system parameters on the optimal harvest-or-transmit strategy through simulations.
- Finally, we compare the optimal policy with the myopic policy proposed in [12], and show that the former outperforms the latter in terms of achievable rate.

II. SYSTEM MODEL

We consider an underlay CRN where the SU harvests energy from the transmission of PU and stores it in its infinite sized battery. In the system model shown in Fig. 1, the primary transmitter (PT) transmits with power $p_p$ in all the slots and remains active for total $M$ slots. In addition, the PU has an interference constraint $P_{int}$ which is needed to be satisfied in each slot. The information of PU’s availability is not needed to be known at the SU and in this case, the SU will follow a policy assuming that the interference constraint of PU is needed to be satisfied in all the transmission slots. However, if PU’s availability is known, the SU can optimize its transmission strategy which will improve its throughput. In either case, as long as the PU is present, in the $i$th slot, the ST
decides either to harvest energy from primary’s transmission or transmit its data with power \( p_s^i \). The PT and ST are assumed to be in close vicinity so that the effects of multipath fading on harvested energy can be neglected. We consider a case where the ST operates for \( N > M \) slots and \( M \) is known, and therefore it can either harvest or transmit in the first \( M \) slots with efficiency \( 0 \leq \alpha \leq 1 \), and transmits without harvesting in the remaining \( N - M \) slots. The proposed policy can also be modified for \( N = M \) and \( N < M \), and the extension to these cases is straightforward. We assume that the battery at the ST has an initial energy of \( E_0 \). We assume quasi static Rayleigh fading channel. Therefore, the power gains of all the channel links are i.i.d. exponentially distributed. We consider the slot length \( \tau \) to be 1 second so that terms power and energy can be used interchangeably. However, the proposed policy can be modified for any value of \( \tau \).

III. PROBLEM FORMULATION

In the system model considered, in each slot ST decides whether to harvest energy from primary’s transmission or communicate with secondary receiver (SR) with an optimal power. We aim to maximize the achievable rate of ST over all slots under energy availability constraint and interference constraint \( (P_{int}) \) of ST and PR respectively.

Let us take an indicator function \( I_H^i \) such that it takes value 1 if ST harvests energy in the \( i \)th slot and takes value 0 otherwise. Whenever \( I_H^i = 0 \), ST transmits with power \( p_s^i \) in \( i \)th slot and its instantaneous achievable rate is given by Shannon’s capacity formula \( R_i = \log_2 \left( 1 + \frac{p_s^i h_{sp}^i}{\sigma^2 + h_{pp}^i p_p^i} \right) \) bps/Hz for the first \( M \) slots. And in remaining slots, since PU is absent, the instantaneous achievable rate of ST is given as \( R_i = \log_2 \left( 1 + \frac{h_{sp}^i}{\sigma^2} \right) \) bps/Hz, \( i = M + 1, \ldots, N \), where \( p_p^i \) and \( h_{pp}^i \) are the transmit powers of PT and ST in \( i \)th slot respectively, \( h_{sp}^i \) is the i.i.d. exponentially distributed power gains of PT-PR, PT-SR, ST-PR and ST-SR channel link respectively, and \( \sigma^2 \) is the variance of the additive noise at both the receivers, which is assumed to be zero mean Gaussian (AWGN).

The optimization problem \( (P_1) \) of maximizing the achievable rate of ST under energy causality constraints and interference constraint of ST and PR respectively, is written as:

\[
\begin{align*}
\text{max} & \quad f(\bar{p}_s, \bar{I}_H) \\
\text{s.t.} & \quad (1 - I_H^i)p_s^i \leq E_0, \quad i = 2, \ldots, M, \\
& \quad \sum_{i=1}^M (1 - I_H^i)p_s^i + \sum_{j=M+1}^i p_s^i \leq E_0 + \alpha \sum_{j=1}^M I_H^j p_p^j, \\
& \quad (1 - I_H^i)h_{sp}^i p_s^i \leq P_{int}, \quad i = M + 1, \ldots, N, \\
& \quad (1 - I_H^i)p_s^i \geq 0, \quad i = 1, \ldots, M, \\
\end{align*}
\]

(Interference constraint at PR)

where \( f(\bar{p}_s, \bar{I}_H) = \sum_{i=1}^M (1 - I_H^i) \log_2 \left( 1 + \frac{h_{sp}^i}{\sigma^2 + h_{pp}^i p_p^i} \right) + \sum_{i=M+1}^N \log_2 \left( 1 + \frac{h_{sp}^i}{\sigma^2} \right) \), \( 0 \leq \alpha \leq 1 \) is the energy harvesting efficiency, \( E_0 \) is the initial energy available at ST, and \( P_{int} \) is the acceptable interference threshold of primary receiver. Vectors \( \bar{p}_s \) and \( \bar{I}_H \) are such that \( [\bar{p}_s]^i = p_s^i \) and \( [\bar{I}_H]^i = I_H^i \), and \( \bar{p}_s \geq 0 \) and \( \bar{I}_H \in \{0, 1\}^M \).

Convex MINLP

After some manipulations in the constraints, the equivalent convex MINLP \( P_2 \) of optimization problem \( P_1 \) is given as:

\[
\begin{align*}
\text{max} & \quad f(\bar{p}_s) \\
\text{s.t.} & \quad p_s^i \leq (1 - I_H^i)E_0, \\
& \quad \sum_{j=1}^i p_s^j \leq E_0 + \alpha \sum_{j=1}^M I_H^j p_p^j, \\
& \quad \sum_{j=1}^{M+i} p_s^j \leq E_0 + \sum_{j=1}^M \alpha I_H^j p_p^j, \\
& \quad h_{sp}^i p_s^i \leq P_{int}, \\
\end{align*}
\]

(2a)-(2f)

where \( f(\bar{p}_s) = \sum_{i=1}^M \log_2 \left( 1 + \frac{h_{sp}^i}{\sigma^2 + h_{pp}^i p_p^i} \right) + \sum_{i=M+1}^N \log_2 \left( 1 + \frac{h_{sp}^i}{\sigma^2} \right) \). The equivalence between (1) and (2) can be understood as follows. When \( I_H^i = 1 \) for some \( i \leq M \), the constraints (2b) or (2c) results in \( p_s^i \leq 0 \), which along with \( p_s^i \geq 0 \) yields \( p_s^i = 0 \). In this case, the problem reduces to:

\[
\begin{align*}
\text{max} & \quad \sum_{i=1}^M \log_2 \left( 1 + \frac{h_{sp}^i}{\sigma^2 + h_{pp}^i p_p^i} \right) + \sum_{i=M+1}^N \log_2 \left( 1 + \frac{h_{sp}^i}{\sigma^2} \right) \\
\text{s.t.} & \quad p_s^i \leq E_0 + \alpha \sum_{j=1}^M I_H^j p_p^j, \\
& \quad \sum_{j=1}^i p_s^j \leq E_0 + \sum_{j=1}^M \alpha I_H^j p_p^j, \\
& \quad h_{sp}^i p_s^i \leq P_{int}. \\
\end{align*}
\]

(2a’)-(2f’)

The optimization problem \( (P_2) \) is a convex MINLP, which can be solved optimally using the GBD algorithm [15].
case, constraint (2d) would consider only those $p_i^*$’s which are positive. On the other hand when $I_H^* = 1$ for some $i \leq M$, the constraint (2c) outer bounds the $p_i^*$ and hence, has no effect. In this case, the constraints (2d) and (2e) will dominate and represent the energy causality constraints in (1d) and (1e).

The problem (2) is a convex MINLP problem since the objective function is concave in $\mathbf{p}$, and the constraints contain affine inequalities. Since the continuous variable $\mathbf{p}$ and the integer variable $I_H$ are now linearly separable, this problem can be solved efficiently using GBD algorithm [15].

**IV. OPTIMAL HARVEST-OR-TRANSMIT STRATEGY USING GBD ALGORITHM**

The GBD algorithm solves the MINLP in (2) iteratively by decomposing it into two subproblems: a primal and a master problem. The primal problem is obtained by fixing the integer variable $I_H^*$. On the other hand, the master problem is obtained using the Lagrangian of the primal problem [15].

The algorithm starts by solving the primal problem for an initial value of $I_H^* = 0$. Then, the solution $\mathbf{p}^{(1)}$ and optimal Lagrange multipliers of the primal problem are used to update the $I_H^*$ by solving the first iteration of the master problem. This $I_H^{(1)}$ is again used to solve the second iterate of the primal problem, which yields the solution $\mathbf{p}^{(2)}$, and the process continues until convergence is achieved. Following subsections discuss the primal and the master problems.

**A. Primal Problem**

For the $l$th iteration, the primal problem is obtained by substituting the solution of the master problem obtained in the $(l-1)$th iteration, $I_H^{(l-1)*}$ into (2). The primal problem is given as

$$\max_{\mathbf{p} \geq 0} f(\mathbf{p}), \text{ s.t. } (2b)-(2f).$$

Note that the primal problem (4) is a convex optimization problem in $\mathbf{p}$, [16] and therefore can be solved using CVX [17]. As stated earlier, the solution of the primal problem $\mathbf{p}^{(l)*}$ along with the dual variables $\theta^{(l)*}, \lambda^{(l)*}, \delta^{(l)*}$ and $\bar{\mu}^{(l)*}$ is then used to formulate and solve the $l$th iteration of the master problem. The dual variables $\theta, \lambda, \gamma, \delta$ and $\bar{\mu}$ are associated with constraints (2b), (2c), (2d), (2e) and (2f), respectively.

To formulate the master problem, we need the Lagrangian of the primal problem which is given in (3). The Karush-Kuhn-Tucker (KKT) stationarity conditions are:

$$\Psi_1 - \mu_1 h_{sp}^i = \sum_{j=1}^{M-1} \gamma_j^i - \sum_{j=1}^{N-M} \delta_j^i = 0,$$

$$\Psi_i - \mu_i h_{sp}^i - \lambda_j^i = \sum_{j=1}^{M-1} \gamma_j^i - \sum_{j=1}^{N-M} \delta_j^i = 0, \forall i \in \{1, \ldots, M\},$$

$$\frac{h_{sp}^i}{\sigma^2 + h_{sp}^i p_s} \sum_{j=1}^{N-M} \delta_j^i = 0, \text{ for } i = M + 1, \ldots, N,$$

where $\Psi_i = \frac{h_{sp}^i}{\sigma^2 + h_{sp}^i p_s}, \forall i$. The complementary slackness conditions are

$$\theta^* \left[p_s^1 - (1 - I_H^*) E_0\right] = 0,$$

$$\mu_i^* \left[h_{sp}^i p_s^i - P_{int}\right] = 0, \quad i = 1, \ldots, M,$$

$$\lambda_i^* \left[p_i^i + 1 - (1 - I_H^{1+i}) E_0\right] = 0, \quad i = 1, \ldots, M - 1,$$

$$\gamma_i^* \left[\sum_{j=1}^{M} p_j^s - E_0 - \sum_{j=1}^{M} \alpha I_H^* p_j^p\right] = 0, \quad i = 1, \ldots, M - 1,$$

$$\delta_i^* \left[\sum_{j=1}^{M+i} p_j^s - E_0 - \sum_{j=1}^{M} \alpha I_H^* p_j^p\right] = 0, \quad i = 1, \ldots, N - M,$$

where $\zeta = \left\{ \sum_{j=1}^{M} p_j^p + E_0 \right\}$. The dual variables associated with non-negativity constraints can be neglected for mathematical ease. The optimal transmit power of the ST in the $l$th iteration is given under KKT conditions as:

$$p_s^{i(l)} = \begin{cases} \frac{1}{\gamma_j^i} - \frac{\alpha^2}{h_{sp}^i} \sum_{j=1}^{M-1} \gamma_j^i, & i = 1, \\ \frac{1}{\gamma_j^i} - \frac{\alpha^2}{h_{sp}^i} \sum_{j=1}^{N-M} \delta_j^i, & i = 2, \ldots, M, \end{cases}$$

$$p_s^{i(l)} = \begin{cases} \frac{1}{\gamma_j^i} - \frac{\alpha^2}{h_{sp}^i} \sum_{j=1}^{N-M} \delta_j^i, & i = M + 1, \ldots, N, \end{cases}$$

where $\gamma_j^i = \theta^* h_{sp}^i + \sum_{j=1}^{M-1} \gamma_j^i + \sum_{j=1}^{N-M} \delta_j^i, \delta_j^i = \mu_i^* h_{sp}^i + \lambda_j^i - \sum_{j=1}^{M-1} \gamma_j^i + \sum_{j=1}^{N-M} \delta_j^i, \gamma_j^i = 2, \ldots, M, \text{ and } \delta_j^i = 2, \ldots, N - M.$ Note that the $[x]^+$ takes care of the non-negativity constraint. The optimal primal and dual variables in $l$th iterations are obtained using CVX [17]. The master problem for $l$th iteration is explained in next subsection.

**B. Master Problem**

The solution of the primal problem up to the $l$th iteration, $\mathbf{p}^{(1), \ldots, l)*}$ and the dual variables $\theta^{(1), \ldots, l)*}, \lambda^{(1), \ldots, l)*}, \gamma^{(1), \ldots, l)*}, \delta^{(1), \ldots, l)*}$ and $\mu^{(1), \ldots, l)*}$ are used to formulate the master problem in the $l$th iteration as

$$\max_{t \geq 0} t \left[ \begin{array}{c} \sum_{I_H \in \{0, 1\}^M} \mathbf{C}(\mathbf{p}^{(j)*}, \theta^{(j)*}, \lambda^{(j)*}, \gamma^{(j)*}, \delta^{(j)*}, \mu^{(j)*}) \\ \text{s.t. } \mathbf{t} \leq \sum_{j=1}^{l} \mathbf{t} \\ \mathbf{t} \in \{1, 2, \ldots, l\} \end{array} \right], \quad j \in \{1, 2, \ldots, l\}. $$

The optimization problem in (6) is a mixed integer linear program (MILP) in $I_H$ and $t$. Thus, (6) can be solved efficiently using MOSEK [18]. The solution of the primal and master problems lower and upper bound the solution of the original problem (2). We represent these lower and upper bounds with $LB(j)$ and $UB(j)$, respectively, where $j$ is the iteration number. The $LB(j)$ and $UB(j)$ are non-decreasing and non-increasing, respectively, with the iteration number $j$ which guarantees the convergence of the GBD algorithm [15]. The Algorithm 1 summarizes the GBD algorithm with $C$ representing a constraint set in which (6b) is added in each iteration.
The primal problem can be solved in polynomial time as it is a convex optimization problem. However, the master problem is NP-hard as it is an integer programming problem. However, GBD can be solved efficiently using any commercial optimization software such as MOSEK [18].

\[ \mathcal{L}(\hat{p}_s, \theta, \lambda, \gamma, \delta, \mu) = f(\hat{p}_s) + \theta \left[ (1 - I^H_i) E_0 - p^{\star}_s \right] + \sum_{i=1}^{M} \mu_i \left[ \int_{i} - h^{sp}_i p^i \right] + \sum_{j=1}^{M-1} \lambda_j \left[ (1 - I^{j+1}_H) \left\{ \sum_{i=1}^{M} p^i_p + E_0 \right\} - p^{j+1}_s \right] \\
+ \sum_{j=1}^{M-1} \gamma_j \left[ E_0 + \sum_{i=1}^{j} \alpha I^H_i p^i_p - \sum_{i=1}^{j} p^i_s \right] + \sum_{j=1}^{N-M} \delta_j \left[ E_0 + \sum_{i=1}^{M} \alpha I^H_i p^i_p - \sum_{i=1}^{M+j} p^i_s \right]. \tag{3} \]

The GBD algorithm is as follows:

**Algorithm 1 GBD algorithm**

**Initialization:** Choose \( I^{(0)}_H \), convergence parameter \( \epsilon \) arbitrarily. Set \( C \leftarrow \emptyset \) and \( j \leftarrow 1 \).

Set \( \text{STOP} \leftarrow 0 \)

while \( \text{STOP} \neq 1 \) do

Obtain \( p^*_s, \theta^*, \lambda^*, \gamma^*, \delta^*, \mu^* \) and \( LB(j) \) by solving (4).

\( C \leftarrow C \cup \{ j \} \)

Obtain \( \bar{I}^{(j)}_H \) and \( UB(j) \) by solving (6).

if \( |UB(j)-LB(j)| \leq \epsilon \) then

\( \text{STOP} \leftarrow 1 \)

end if

Set \( j \leftarrow j + 1 \)

end while

return \( (\hat{p}_s, \bar{I}^H) \)

V. RESULTS

We study the performance of the optimal harvest-or-transmit strategy in this section. We assume quasi static i.i.d. Rayleigh distributed channel links with variances \( \sigma^2_{pp} = \sigma^2_{ps} = \sigma^2_{sp} = \sigma^2_{ss} = 0.1 \) and \( \sigma^2 = 0.1 \).

A. Effect of \( P_{int} \)

In Fig. 2, the effect of \( P_{int} \) on the average EH and average Tx time for the first \( M \) slots is shown. After \( M \) slots, the PU becomes silent and the SU can not harvest RF energy from it. We assume that the PT transmits with power \( p_p = 1 \) W in all the \( M \) slots and initial energy in the battery \( E_0 = 2 \) J. It is evident that as \( P_{int} \) decreases, the average EH time increases and average Tx time decreases (the average Tx time is the duration as long as PT remains active). When \( P_{int} \) approaches zero, average EH-time approaches \( M \) and average Tx-time approaches 0, i.e., the ST harvests energy as long as PT is active.

Fig. 3 shows the average achievable rate of ST under the optimal policy for different energy harvesting efficiency and different interference constraints at PR. The average is obtained over different channel realizations. For the simulation purpose, the number of primary slots, \( M \) is assumed to be
N − 2. From the Fig. 3, it is evident that as the interference constraint at the primary receiver loosens, i.e., $P_{\text{int}}$ increases, and thus secondary transmitter is able to transmit with higher power, which results in higher achievable rate. When the interference constraint becomes too stringent, the secondary power, which results in higher achievable rate. When the primary receiver receives more interference from PT and ST causes more interference to PR due to which ST can not transmit with higher power. Also, when ST-PR and PT-SR links are weak, performance degrades due to similar reasons.

VI. Conclusions

We obtained the optimal harvest-or-transmit policy of an underlay EH-CRN using GBD algorithm and studied the effects of different system parameters. We observed that the optimal EH (Tx) time increases (decreases) as $P_{\text{int}}$ decreases. Also, we analyzed the effects of $P_{\text{int}}$ on average achievable rate and observed that it reduces as $P_{\text{int}}$ decreases. The effect of various channel conditions on average achievable rate has also been studied. In addition, we showed that the proposed policy achieves a higher rate than that of the myopic policy proposed in the literature.

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