Interference effects in f-deformed fields

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Abstract

We show how the introduction of an algebraic field deformation affects the interference phenomena. We also give a physical interpretation of the developed theory.

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Nonlinear systems have always attracted a lot of attention both in classical and in quantum fields. On the other hand, after their introduction [1], also the quantum q-oscillators were interpreted [2] as a nonlinear oscillators with a very specific type of nonlinearity, in which the frequency of vibration depends on the energy of these vibrations, through an hyperbolic cosine function containing the parameter of nonlinearity. But there might exist other types of nonlinearity for which the frequency of oscillations varies with the amplitude by means of a generic function, say $f$. For
this reason, recently, it has been introduced the concept of f-deformed oscillators [3]. The particular case of f-coherent states, called also nonlinear coherent states for the function $f$, expressed in terms of Laguerre polynomials has been shown reachable in trapped ions [4].

The specific and important property of any linear process is the existence of the superposition principle, due to which two solutions of the linear equation may superpose and give rise to another solution of the linear equation. Physically it means the possibility of the interference phenomenon when two different solutions, with appropriate phases both in time and in space domains, produce a stable pattern corresponding to increasing and cancelling amplitudes of both solutions in concrete points of space (time). If the equation has some nonlinearity there is no superposition principle anymore, but if the nonlinearity is small it is clearly intuitive that the interference pattern, characteristics for purely linear vibrations, will be only slightly changed (deformed) according to the influence of the nonlinearity. The nonlinearity may produce generation of other harmonics, which may each other interact, implying the beating phenomenon. In the time domain it could mean a collapse and revival of the interference pattern, as well as in the space domain it could mean the existence of a spoty structure, with sharp enough pattern picture in one spot and with a different pattern in another spot. The influence of the nonlinearity into interference patterns may be traced out by using the influence of the nonlinearity onto the visibility, which is a characteristic of coherence properties of the waves under study.

Here we would discuss the coherence properties of f-deformed fields showing how their deformation could affect the visibility of the interference pattern. We also provide a physical realization of the developed theory within the context of the Bose-Einstein condensate.

Let us start by considering two fields described by the annihilation (creation) operators $\hat{a}$ ($\hat{a}^\dagger$) and $\hat{b}$ ($\hat{b}^\dagger$). The free fields Hamiltonian, in the case of unit frequencies (setting $\hbar = c = 1$), will be

$$\hat{H} = \frac{1}{2} (\hat{a}\hat{a}^\dagger + \hat{a}^\dagger\hat{a}) + \frac{1}{2} (\hat{b}\hat{b}^\dagger + \hat{b}^\dagger\hat{b}) ,$$

which yields an operators evolution in the Heisemberg picture of the form

$$\hat{a}(t) = \hat{a}e^{-it} ; \quad \hat{b}(t) = \hat{b}e^{-it} .$$
The interference effects arise as a consequence of fields superposition, which leads to a total field of the form

$$\hat{\Psi}(t) = [\hat{a}(t)e^{-ik_1x} + \hat{b}(t)e^{-ik_2x}] + \text{h.c.},$$

(3)

where $k_1$ ($k_2$) is the momentum characterizing the field $a$ ($b$). Then, the total field intensity will be

$$I(x, t) = \langle \hat{\Psi}(-)(t)\hat{\Psi}(+)(t) \rangle = \langle [\hat{a}^\dagger(t)e^{ik_1x} + \hat{b}^\dagger(t)e^{ik_2x}] [\hat{a}(t)e^{-ik_1x} + \hat{b}(t)e^{-ik_2x}] \rangle.$$  

(4)

Due to the fact that the two initial fields are independent, the interference fringes appear only if we take the expectation value over particular states of the two modes, like the coherent states

$$|\alpha\rangle_a \otimes |\beta\rangle_b,$$

(5)

obtaining

$$I(x, t) = 2|\alpha|^2\{1 + \cos[\phi(x) - \phi]\},$$

(6)

where for simplicity we have chosen $\beta = \alpha e^{-i\phi}$ and we have set $\phi(x) = (k_1 - k_2)x$. The visibility of the interference fringes is given by [5]

$$V = \frac{I(x, t)_{\text{max}} - I(x, t)_{\text{min}}}{I(x, t)_{\text{max}} + I(x, t)_{\text{min}}},$$

(7)

hence, in Eq. (6) it results as the factor multiplying the cosine term inside the brackets, i.e. $V = 1$.

Let us now introduce a deformation of the fields by means of the following Hamiltonian [3]

$$\hat{H} = \frac{1}{2} (\hat{A} \hat{A}^\dagger + \hat{A}^\dagger \hat{A}) + \frac{1}{2} (\hat{B} \hat{B}^\dagger + \hat{B}^\dagger \hat{B}),$$

(8)

where

$$\hat{A} = \hat{a}f(\hat{n}_a, \hat{n}_b); \quad \hat{B} = \hat{b}f(\hat{n}_a, \hat{n}_b),$$

(9)

with $f$ a generic function of the operators $\hat{n}_a = \hat{a}^\dagger \hat{a}$ and $\hat{n}_b = \hat{b}^\dagger \hat{b}$. It should be remarked that in this form one introduces a coupling between the two modes $a$ and $b$, other than a self interaction of the fields.
By using the commutation properties of the fields operators, the Hamiltonian (8) can be rewritten as
\[ H(\hat{n}_a, \hat{n}_b) = \frac{1}{2} \left[ (\hat{n}_a + \hat{n}_b) f^2(\hat{n}_a, \hat{n}_b) + (\hat{n}_a + 1) f^2(\hat{n}_a + 1, \hat{n}_b) + (\hat{n}_b + 1) f^2(\hat{n}_a, \hat{n}_b + 1) \right], \] (10)
which determines the following time evolution of the operators
\[ \hat{a}(t) = \hat{a} \exp \left\{ -i \left[ H(\hat{n}_a, \hat{n}_b) - H(\hat{n}_a - 1, \hat{n}_b) \right] t \right\}; \] (11)
\[ \hat{b}(t) = \hat{b} \exp \left\{ -i \left[ H(\hat{n}_a, \hat{n}_b) - H(\hat{n}_a, \hat{n}_b - 1) \right] t \right\}. \] (12)
Thus, inserting Eqs. (11) and (12) into Eq. (4) and using the state s of Eq. (5), we get
\[ I(x, t) = 2|\alpha|^2 \left[ 1 + \Re \left\{ e^{-2|\alpha|^2} \sum_{n_a,n_b=0}^{\infty} \frac{|\alpha|^{2(n_a+n_b)}}{n_a!n_b!} e^{i[H(n_a,n_b+1) - H(n_a+1,n_b)]t} e^{i(\phi - \phi(x))} \right\} \right]. \] (13)
If we further suppose to have the function f symmetric under the exchange \( \hat{n}_a \leftrightarrow \hat{n}_b \), then the visibility becomes
\[ V = e^{-2|\alpha|^2} \sum_{n_a,n_b=0}^{\infty} \frac{|\alpha|^{2(n_a+n_b)}}{n_a!n_b!} e^{i[H(n_a,n_b+1) - H(n_a+1,n_b)]t}, \] (14)
which clearly shows the time dependence through the specific function f.

Alternatively, one can consider the two mode deformed fields not entangled, i.e.
\[ \hat{A} = \hat{a} f_a(\hat{n}_a); \quad \hat{B} = \hat{b} f_b(\hat{n}_b), \] (15)
in this case the modified Hamiltonian is given by
\[ H(\hat{n}_a, \hat{n}_b) = \frac{1}{2} \left[ \hat{n}_a f_a^2(\hat{n}_a) + \hat{n}_b f_b^2(\hat{n}_b) + (\hat{n}_a + 1) f_a^2(\hat{n}_a + 1) + (\hat{n}_b + 1) f_b^2(\hat{n}_b + 1) \right], \] (16)
and the visibility takes the same form of Eq. (14) provided to have \( f_a = f_b \).

We now apply the above arguments to atom optics, where an increasing interest has been devoted to the Bose-Einstein condensates after their observation [3]. First, we take
\[ f_a^2(\hat{n}_a) = \kappa \hat{n}_a + (1 - \kappa); \quad f_b^2(\hat{n}_b) = \kappa \hat{n}_b + (1 - \kappa), \] (17)
which gives an Hamiltonian of the type
\[ H(\hat{n}_a, \hat{n}_b) = \hat{n}_a + \hat{n}_b + \kappa \left( \hat{n}_a^2 + \hat{n}_b^2 \right). \] (18)
It could describe two trapped condensates each with self collisional effects, with $\kappa$ representing the collisional rate between the atoms within each condensate \[7\]. In this case the argument of the exponential in Eq. (14) takes the form $2i\kappa(n_b - n_a)t$ leading to collapses and revivals of the visibility \[7\].

Second, we take

$$f^2(\hat{n}_a, \hat{n}_b) = \kappa (\hat{n}_a + \hat{n}_b) + (1 - \kappa),$$

(19)

which gives an Hamiltonian of the type

$$H(\hat{n}_a, \hat{n}_b) = \hat{n}_a + \hat{n}_b + \kappa (\hat{n}_a + \hat{n}_b)^2,$$

(20)

which could describe two trapped condensates, including cross collisional effects as well \[8\]. Of course one could consider a cross collisional rate different from $\kappa$, but it will result a function $f$ no longer symmetric in the exchange $\hat{n}_a \leftrightarrow \hat{n}_b$, and for semplicity we will not take into account this eventuality here. It is easy to see that in the particular case of Eq. (20), the visibility does not show time dependence. It means that the correlation between the two modes, induced by the Hamiltonian (20), provides to maintain the initial visibility.

In conclusion, we have investigated the superposition mechanism of f-deformed fields showing that the formalism of deformed oscillators could result a powerfull tool in atom optics.

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