Demonstration of double EIT using coupled harmonic oscillators and RLC circuits

Joshua Harden¹, Amitabh Joshi¹ and Juan D Serna²

¹ Department of Physics, Eastern Illinois University, Charleston, IL 61920, USA
² School of Mathematical and Natural Sciences, University of Arkansas at Monticello, Monticello, AR 71656, USA

E-mail: ajoshi@eiu.edu and serna@uamont.edu

Received 26 September 2010, in final form 31 December 2010
Published 9 February 2011
Online at stacks.iop.org/EJP/32/541

Abstract
Single and double electromagnetically induced transparencies (EIT) in a medium, consisting of four-level atoms in the inverted-Y configuration, are discussed using mechanical and electrical analogies. A three-coupled spring–mass system subject to damping and driven by an external force is used to represent the four-level atom mechanically. The equations of motion of this system are solved analytically, which revealed single and double EIT. On the other hand, three coupled RLC circuits are used, as the electrical analogue, to explore and experimentally demonstrate single and double EIT. The simplicity of these two models makes this experiment appropriate for undergraduate students and easy to incorporate into a college physics laboratory.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Atomic media have the physical characteristic of absorbing light of certain frequencies. It is possible, for example, that a medium absorbs two slightly different light frequencies simultaneously. However, and perhaps more intriguing, we observe that for certain atomic configurations of the medium, the level of absorption of one of the frequencies can be controlled by the other frequency making the medium virtually transparent to the former frequency. This phenomenon is called electromagnetically induced transparency (EIT) [1, 2]. Usually, EIT occurs in vapours of three-level atomic systems, where laser lights (coherent light sources) drive two different atomic transitions sharing one common level (known as the ‘probe’ and ‘coupling’ field transitions). In the same way that a ‘coupling field’ controls the properties of an EIT medium determining the amount of absorption of a ‘probe field’, the dispersive
properties of the medium also get modified resulting in the reduction of the group velocity of light inside it. Physically, EIT can be understood as a process of quantum interference between two atomic states of a medium involving two indistinguishable quantum paths that lead to a common final state. In addition to the EIT phenomenon, double EIT occurs when a four-level atomic system is exposed to three laser sources driving three different transitions with one common level. The three transitions are described as the ‘probe’, the ‘coupling’ and the ‘pumping’ field transitions. In this case, two strong electromagnetic fields, i.e. the coupling and the pumping fields, control the medium in determining the absorption and propagation of the probe field.

The phenomenon of EIT, first observed two decades ago using high-power lasers in strontium vapour [3], has been extensively investigated during the past years in atomic beams [4, 5], plasma [6, 7], optical cavities [8–11] and Bose–Einstein condensates [12, 13]. It has also been studied theoretically and experimentally for media consisting of three- and four-level atoms [14–22].

Besides absorption of light, there are other substantial changes observed if a medium exhibits EIT, such as the modified index of refraction [23], which can give rise to the reduction of the group velocity of a light pulse [24], or even a complete stop of light in the medium [25]. Important applications of EIT include lasing without population inversion [26, 27], enhanced nonlinear optical processes [28], quantum computation and telecommunications [29, 30], quantum memory [31] and optical switches [32].

During the past two decades, the study of quantum-classical analogies in physics has gained some momentum as they prove to be very useful in helping to understand the fundamental physical concepts and the applicability of different theories [33]. It is important to note that these analogies bring to light the fact that similar mathematical models can be applied to both quantum and classical phenomena, though these theories differ in both formalism and fundamental concepts. Recently, a number of these classical analogies of different quantum optical systems have been reported. For example, stimulated resonance Raman effect [34], rapid adiabatic passage in atomic physics [35], vacuum Rabi oscillation [36], number-phase Wigner function and its relation to the usual Wigner function [37], and EIT in three-level systems [38]. In a recent work, the response of a coupled array of nonlinear oscillators to parametric excitation has been calculated in the weak nonlinear limit using secular perturbation theory and the exact results for small arrays of oscillators have been used to guide the analysis of the numerical integration of the model equations of motion for large arrays. Such results provide qualitative explanations for experiments involving a parametrically excited micromechanical resonator array [39].

The double EIT phenomenon is very important in EIT-based atomic memory systems. Systems displaying multiple EIT could be useful in the bifurcation of quantum information in multiple channels temporarily, which can then be used in multiplexing required in certain quantum information protocols. The release of stored information from multiple channels could be separately controlled by manipulating the group velocity of individual channels (via their control fields) in such systems. Hence double EIT is an important phenomenon for quantum information processing and quantum computing and thus, it needs its introduction and realization in the simplest form to the readers.

The goal of this work is to demonstrate double EIT in four-level systems using two classical analogies: mass–spring systems and RLC circuits. For that purpose, we first describe the atom as a damped, harmonic oscillator driven by an external force [40]. Three different masses connected by springs and subject to frictional forces (damping) are used to represent the four-level atom. The destructive interference of the normal modes of oscillation of the masses is equivalent to the quantum interference that originates EIT. Second, we explore experimentally
Figure 1. Schematic energy level diagram of a four-level system in the inverted-Y configuration. Here $\omega$, $\omega_c$ and $\omega_r$ are the frequencies of the probe, coupling and pumping fields, respectively, whereas $\Delta$, $\Delta_c$ and $\Delta_r$ are their corresponding frequency detunings.

the electrical analogue of double EIT using three coupled RLC circuits. The power delivered to one of these coupled oscillating circuits is measured as a function of the frequency of a driving source of alternating voltage. The electrical equivalence of the power transmitted to the circuit with the power absorbed by an atomic medium allows us to investigate, directly from the circuit, the characteristic patterns of single and double EIT.

To get information about the absorption and dispersion of light in the four-level atomic medium, we need to solve a large system of the density matrix equations numerically [19]. However, the equations of motion that describe the mechanical and electrical systems can be solved analytically and hence the double EIT phenomenon could be studied with more ease in the two analogue systems using the analytical solutions. The merit of analytic solutions is that it clearly brings out the functional dependence of the double EIT phenomenon on several parameters. On the other hand, the circuits used in this experiment show realistic forced, damped harmonic oscillations that can be easily built and may be incorporated into an undergraduate physics laboratory, and help students and teachers to appreciate the complex quantum phenomena of EIT and double EIT put together in a very simplified manner both theoretically and experimentally.

2. Model and basic equations

We considered a medium consisting of four-level atoms in the so-called inverted-Y configuration as shown in figure 1. The levels $|1\rangle$ and $|2\rangle$ were coupled by a ‘probe’ field of frequency $\omega$, in whose absorption and dispersion we were interested. The level $|2\rangle$ was connected to the lower level $|0\rangle$ by a strong ‘coupling’ field of frequency $\omega_c$, and to the upper level $|3\rangle$ by the strong ‘pumping’ field of frequency $\omega_r$. Only the atomic transitions $|1\rangle \leftrightarrow |2\rangle$, $|0\rangle \leftrightarrow |2\rangle$ and $|2\rangle \leftrightarrow |3\rangle$ were dipole allowed.

In a typical double EIT experiment, quantum interference is introduced by driving the upper two levels with strong coherent fields. Under appropriate conditions, the medium becomes transparent (zero absorption) for the probe field. In the absence of the coupling and pumping fields, we may observe a regular absorption resonance profile. However, under certain conditions, the addition of either the coupling or pumping fields prevents the absorption of energy by the medium, and the transmitted intensity as a function of the probe frequency shows a narrow peak of induced transparency called single EIT (or just EIT). When both coupling and pumping fields are simultaneously present, they together control the absorption
Figure 2. Imaginary and real parts of the susceptibility $\chi$, as a function of probe detuning $\Delta$, for different parametric conditions. The profiles show double EIT for a four-level atom in an inverted-Y configuration with $\gamma_1 = \gamma_2 = \gamma_3 = 1.0$ and $\gamma_0 = 10^{-4}$. The other parameters for plots (a, b), (c, d) and (e, f) are $(\Omega_c = 1.0, \Omega_r = 2.5, \Delta_c = 0, \Delta_r = 0)$, $(\Omega_c = 2.0, \Omega_r = 3.0, \Delta_c = 0, \Delta_r = 0)$ and $(\Omega_c = 1.0, \Omega_r = 2.0, \Delta_c = 0.8, \Delta_r = 1.8)$, respectively. All the parameters have the dimension of frequency.

and propagation of the probe field, and thus double EIT may be observed in the transmitted intensity profile of the probe field [41].

The absorptive and dispersive properties of the atomic system can be studied by calculating the electrical susceptibility of the system. When the atom–field interaction is determined by the density matrix equation and the corresponding off-diagonal (coherence) component of the probe transition is $\rho_{12}$, the complex susceptibility $\chi$ is given by $\chi = \mu_p \rho_{12} / E_p$, in which $\mu_p$ and $E_p$ represent the dipole moment and the field amplitude for the probe transition. The susceptibility $\chi = \chi' + i\chi''$ is a complex quantity such that its real (imaginary) part determines the dispersive (absorptive) property of the atomic medium for the probe field. The intensities of the driving fields determine the effects observed in double EIT, as depicted in figure 2 for the radiative decay constants $\gamma_1 = \gamma_2 = \gamma_3 = 1.0$ and $\gamma_0 = 10^{-4}$. The Rabi frequencies $\Omega_c$ and $\Omega_r$ are directly proportional to the coupling and pumping field strengths, respectively, and must be comparable with all damping rates $\gamma_i$ present in the medium. Figure 2(a) clearly
Figure 3. Absorption coefficient for the probe beam versus probe frequency for the $5S_{1/2}, F = 1 \rightarrow 5P_{1/2}, F' = 2$ transition of $^{87}$Rb. The lower solid curve is for the no pumping field. The upper solid curve is for the pumping field tuned at the $5S_{1/2}, F = 2 \rightarrow 5P_{1/2}, F' = 2$ transition with intensity $I_c = 19.6$ W cm$^{-2}$ at the cell centre. The dotted curve is the theoretical result for $\Gamma_1 + \Gamma_2 = 6.0$ MHz, $\Gamma_3 = 0.1$ MHz, $\delta \omega_1 = \delta \omega_2 = 2.5$ MHz, $\Delta \omega_{dp} = 530$ MHz and an effective pumping Rabi frequency $\Omega_2 = 105$ MHz (adapted from figure 4 of [43] with permission).

shows double EIT (two dips at $\Delta = 0$) at the exact resonance conditions of the coupling and pumping fields, i.e. $\Delta_c = \Delta_r = 0$ and $(\Omega_c = 1.0, \Omega_r = 2.5)$. The corresponding dispersive property is given in figure 2(b). Furthermore, strong coupling and pumping fields may induce ac-Stark splitting of the excited levels [2] and [3] under resonant conditions. When the coupling and pumping fields are strong, the splitting expands and the absorption spectrum displays the Autler–Townes doublets [42]. In figure 2(c), the values of the coupling and pumping fields' Rabi frequencies are stronger $(\Omega_c = 2.0, \Omega_r = 3.0$ with other parameters unchanged) in comparison to figure 2(a) and hence the width of the two EIT dips becomes broader due to a wider splitting of the Autler–Townes doublets. The corresponding dispersive properties under this parametric condition are displayed in figure 2(d). Finally, the effects of off-resonant coupling and pumping fields $(\Omega_c = 1.0, \Omega_r = 2.0, \Delta_c = 0.8, \Delta_r = 1.8)$ are displayed in figures 2(e) and (f).

Two EIT dips moved away from the $\Delta = 0$ position because of finite detunings of the coupling and pumping fields. The details of this theoretical work on double EIT are discussed in [19]. The single EIT observed experimentally in a three-level $\Lambda$-type atomic system and double EIT in a four-level tripod-type atomic system are shown in figures 3 and 4, respectively. Clearly in the optical spectrum, a single dip is observed in the absorption spectrum of a single EIT system and two dips are observed in a double EIT system. The experimental conditions are mentioned in the captions of figures 3 and 4 and can be further explored in [43, 44].

2.1. Mechanical spring analogue of single and double EIT-like phenomena

The Lorentz model [40, 47] is recognized as one of the classical models for the atom that works incredibly well for describing the interaction of light with matter. The basic assumption made in this model is that the bounded electrons within the neutral atom oscillate about their
Figure 4. Experimental results for probe absorption as a function of the probe frequency detuning.

\[ \Delta T = 85 \text{ MHz}, \text{ and effective decay rates } \gamma_1 = 1.5 \text{ MHz}, \gamma_2 = 1.5 \text{ MHz}, \gamma_3 = 0.5 \text{ MHz}, \gamma_e = 3.8 \text{ MHz}. \text{ The laser powers are given in terms of the Rabi frequencies } \Omega_C = 40 \text{ MHz}, \Omega_p = 2.5 \text{ MHz}, \Omega_T = 8 \text{ MHz} \text{ (adopted from figure 3(a1) of [44] with permission).} \]

In the classical model of double EIT, we described the atom as a damped harmonic oscillator of mass \( m_1 \) attached to a rigid support by a spring of force constant \( \kappa_1 \) and driven by a harmonic force \( F = F_0 e^{-i(\omega t + \phi)} \). To this mass–spring combination were attached two other masses originally at rest, \( m_2 \) and \( m_3 \) that were connected to mass \( m_1 \) by springs of force constants \( \kappa_{12} \) and \( \kappa_{13} \), respectively. These two masses were also fixed, from the other side, to rigid supports by springs of force constants \( \kappa_2 \) and \( \kappa_3 \), respectively (see figure 5(a)).

It is always a matter of importance and interest to know at what rate energy is transmitted into the driven oscillator, and how this power is absorbed as a function of the frequency \( \omega \) [45]. In the typical situation of a damped harmonic oscillator \( m_1 \) driven by a harmonic force \( F \), a standard absorption resonance profile is observed. However, if either \( m_2 \) or \( m_3 \) is allowed to move due only to the forces from the springs they are attached to (with force constants \( \kappa_{12} \) and \( \kappa_2 \), and \( \kappa_{13} \) and \( \kappa_3 \), respectively), this will avoid absorption in a limited region of the resonance profile, and the transmitted power as a function of the driving force frequency will show a narrow peak of induced transparency (single EIT) [38].

In this physical model of the atom, the spring attaching masses \( m_1 \) and \( m_2 \) (with the force constant \( \kappa_{12} \)) emulated the coupling field between atomic levels \( |0\rangle \) and \( |2\rangle \), whereas the spring connecting masses \( m_1 \) and \( m_3 \) (with the force constant \( \kappa_{13} \)) emulated the pumping field...
Demonstration of double EIT using coupled harmonic oscillators and RLC circuits 547

Figure 5. Coupled damped harmonic-oscillator model showing (a) double EIT and (b) single EIT features.

between levels [2] and [3]. The probe field was then modelled by the harmonic force acting on mass \( m_1 \). These analogues remind us the description of the fields in terms of harmonic oscillators [46]. Now, if we allow both masses \( m_2 \) and \( m_3 \) to move simultaneously under the conditions described above, we will observe double EIT features.

To describe the classical evolution of this system, we used a fixed set of one-dimensional Cartesian coordinates \( x_1, x_2 \) and \( x_3 \), representing the positions of the masses from their equilibrium positions. Thus, the equations of motion could be written as

\[
\ddot{x}_1(t) + \gamma_1 \dot{x}_1(t) + \omega_2^2 x_1(t) - \Omega_{1c}^2 x_2(t) - \Omega_{1r}^2 x_3(t) = \left( \frac{F_0}{m} \right) e^{-i\omega t},
\]

\[
\ddot{x}_2(t) + \gamma_2 \dot{x}_2(t) + \omega_2^2 x_2(t) - \Omega_{1c}^2 x_1(t) = 0
\]

\[
\ddot{x}_3(t) + \gamma_3 \dot{x}_3(t) + \omega_2^3 x_3(t) - \Omega_{1r}^2 x_1(t) = 0,
\]

where we assumed that \( \phi = 0 \) and \( m_1 = m_2 = m_3 = m \). The other parameters were defined as follows: \( \omega_1^2 = (\kappa_1 + \kappa_{12} + \kappa_{13})/m \), \( \omega_2^2 = (\kappa_2 + \kappa_{12})/m \), \( \omega_3^2 = (\kappa_3 + \kappa_{13})/m \), \( \Omega_{1c}^2 = \kappa_{12}/m \) and \( \Omega_{1r}^2 = \kappa_{13}/m \). The damping parameters \( \gamma_i \) (viscous damping) represented the mechanical equivalent to the spontaneous decay rates of the three excited states in the inverted-Y atomic configuration.

Because we expected the motion to be oscillatory, we attempted solutions of the form \( x_i = B_i e^{-i\omega t} \), where \( B_i \) are constants \( i = 1, 2, 3 \). Substituting these expressions for the displacements into the equations of motion, we found that the displacement of \( m_1 \) (atom displacement) was given by

\[
x_1(t) = \frac{\left( \frac{F_0}{m} \right) e^{-i\omega t}}{\left( \omega_1^2 - \omega^2 - i \gamma_1 \omega \right) - \Omega_{1c}^2 - \omega_2^2 - i \gamma_2 \omega}. \tag{3}
\]

In the Lorentz oscillator model [40, 47], the electrical polarization \( P \) (or the susceptibility \( \chi = P/F \)) induced in the atom by the external force field \( F \) is directly proportional to \( x_1 \), for the polarization is defined as \( P = Ne x_1 \), where \( N \) is the number of atoms per unit volume, and \( e \) is the electronic charge. The real and imaginary parts of \( x_1 \) give the dispersion and absorption properties of the atom, respectively. A graphical analysis of (3) will allow us to explore these two important properties of light propagation. The frequency differences (detuning) of the probe, coupling and pumping fields with respect to the external driving field were defined as
Delta = \omega_1 - \omega, \Delta_c = \omega_2 - \omega \text{ and } \Delta_r = \omega_3 - \omega, \text{ respectively. These definitions are slightly different from what is used in optical double EIT nomenclature [19].}

2.2. Electrical analogue of double EIT: coupled RLC circuits

There is a well-known correspondence between a driven damped harmonic oscillator and an electrical circuit consisting of a resistor \( R \), an inductor \( L \) and a capacitor \( C \) connected in series to an alternating voltage source \( V \) [48]. The importance of this correspondence is that RLC circuits are easy to build in the laboratory and may be used as excellent examples of non-mechanical oscillations. We used these circuits to demonstrate experimentally and study theoretically single and double EIT by analysing the dissipation of electric power in the resistance. The circuit that showed double EIT behaviour is shown in figure 6(a). This circuit was made up of three loops of RLC circuits. The resistance, inductance and capacitance of the loops were represented by \( R_i \), \( L_i \) and \( C_i \), respectively (\( i = 1, 2, 3 \)). The first loop with resistance \( R_1 \), inductance \( L_1 \), and capacitances \( C_1 \) and \( C/2 \) represented the atom. The resistance accounted for the spontaneous radiative decay of the second excited level \( |2\rangle \) to level \( |1\rangle \). The capacitance \( C \), shared by the first and second loops, provided the link between the atom and the coupling field, whereas the other capacitance \( C \), shared by the first and third loops, linked the atom with the pumping field.

In this circuit, the loop that modelled the atom (loop 1) had a resonance frequency that represented the transition energy from the ground state \( |1\rangle \) to the excited state \( |2\rangle \). The probability of populating this excited state was maximum when the alternating voltage source \( V \) was in resonance with the resonance frequency of this loop (or in resonance with the
Demonstration of double EIT using coupled harmonic oscillators and RLC circuits

\[ |1\rangle \rightarrow |2\rangle \) transition. However, with a three-loop configuration, we had two other possible ways to accomplish this excitation since we were using the analogue of a four-level atom in the inverted-Y configuration. For instance, loop 1 (representing the atom) could have also been excited either by the coupling loop 2 (|0\rangle \rightarrow |2\rangle), the pumping loop 3 (|3\rangle \rightarrow |2\rangle) or both.

The EIT was studied by examining the frequency dependence of the transmitted power from the voltage source \( V = V_s e^{-i\omega t} \) to the resonant first loop. If the currents flowing in three different loops of the circuit are written as \( I_1(t) = \dot{q}_1(t), I_2(t) = \dot{q}_2(t) \) and \( I_3(t) = \dot{q}_3(t) \), the following system of coupled differential equations for the charges is found:

\[
\begin{align*}
\dot{q}_1(t) + \gamma_1 \dot{q}_1(t) + \omega_1^2 q_1(t) - \Omega_2^2 q_2(t) - \Omega_3^2 q_3(t) &= (V_s/L_1) e^{-i\omega t} \\
\dot{q}_2(t) + \gamma_2 \dot{q}_2(t) + \omega_2^2 q_2(t) - \Omega_3^2 q_3(t) &= 0 \\
\dot{q}_3(t) + \gamma_3 \dot{q}_3(t) + \omega_3^2 q_3(t) &= 0
\end{align*}
\]

(4)

where \( \gamma_i = R_i/L_i, \omega_i^2 = 1/(L_i C_i) \) (with \( i = 1, 2, 3 \)) and \( \Omega_i^2 = \Omega_2^2 = 1/(L_1 C) \). The equivalent capacitances for these loops were

\[
\begin{align*}
C_{e1} &= \frac{(C/2) C_1}{C/2 + C_1} \\
C_{e2} &= \frac{C C_2}{C + C_2} \\
C_{e3} &= \frac{C C_3}{C + C_3}
\end{align*}
\]

(5)

It was easy to compare (3) with (4) and conclude that both models described the same physical phenomenon.

Applying Kirchhoff’s second law to the three loops of the circuit [49], with loop currents \( I_1, I_2 \) and \( I_3 \), we obtained

\[
\begin{align*}
[R_1 - i(2X_C + X_C_1 - X_{L_1})]I_1 + iX_C I_2 + iX_C I_3 &= V \\
iX_C I_1 + [R_2 - i(X_C + X_C_2 - X_{L_2})]I_2 &= 0 \\
iX_C I_1 + [R_3 - i(X_C + X_C_3 - X_{L_3})]I_3 &= 0
\end{align*}
\]

(6)

where \( X_C = 1/(\omega C) \) and \( X_C_i = 1/(\omega C_i) \) (\( i = 1, 2, 3 \)) were the capacitive reactances and \( X_{L_i} = \omega L_i (i = 1, 2, 3) \) were the inductive reactances. From the above system of equations, it was found that

\[
I_1 = \left( \frac{A + iB}{A^2 + B^2} \right) V
\]

(7)

where, for convenience, we defined

\[
\begin{align*}
A &= R_1 + \frac{R_2 X_C^2}{R_2^2 + [X_{L_2} - (X_C + X_C_2)]^2} + \frac{R_3 X_C^2}{R_3^2 + [X_{L_3} - (X_C + X_C_3)]^2} \\
B &= X_{L_1} - (2X_C + X_C_1) - \frac{X_C^2 [X_{L_2} - (X_C + X_C_2)]}{R_2^2 + [X_{L_2} - (X_C + X_C_2)]^2} - \frac{X_C^2 [X_{L_3} - (X_C + X_C_3)]}{R_3^2 + [X_{L_3} - (X_C + X_C_3)]^2}
\end{align*}
\]

(8a)

(8b)

The electrical power in the \( R_1 L_1 C_{e1} \) loop was obtained by multiplying (7) by the voltage. The in-phase and out-of-phase components of the power were associated with the energy dissipated by the resistive (\( P_R \)) and the energy stored by the reactance (\( P_X \)) parts of the circuit, giving the following expressions:

\[
P_R = \frac{A |V_s|^2}{A^2 + B^2} \quad \text{and} \quad P_X = \frac{B |V_s|^2}{A^2 + B^2}
\]

(9)

where \( A \) and \( B \) were given by (8a) and (8b).
3. Results and discussion

We first studied the absorption and dispersion properties of the spring–mass system at exact resonance conditions $\Delta_c = \Delta_r = 0$ of the coupling and pumping fields. Figures 7(a) and (b) display the curves for the absorption and dispersion of the probe field, respectively. The Rabi frequencies and radiative decays (damping) used were $\Omega_c = 3.0$, $\Omega_r = 2.3$, $\gamma_1 = 1.0$, $\gamma_2 = 0.1$ and $\gamma_3 = 10^{-4}$ (all these quantities given in units of the atomic decay $\gamma_1$). Double EIT was observed in the absorption curve at $\Delta = 0$, where two dips of different widths, one inside the other, clearly became visible (figure 7(a)). When the coupling and pumping frequencies were changed to $\Omega_c = 2.7$ and $\Omega_r = 3.0$, we noticed from the absorption curve that when the pumping field increased reducing its relative difference with the coupling field, the second dip became wider (figure 7(c)). On the other hand, figure 7(d) shows how, in the vicinity of $\Delta = 0$, the peaks of dispersion flipped in a smoother way. The change of frequency detunings brought in further interesting changes as depicted in figures 7(e) and (f), where we set $\Delta_c = \Delta_r = 0.1$. Because of these detuning changes, the two EIT peaks separated from each other and moved away relative to the $\Delta = 0$ position.

The double EIT features changed to those of single EIT (for the spring–mass system in figure 5(b)) when the zero-limit condition for either the coupling or pumping fields
Demonstration of double EIT using coupled harmonic oscillators and RLC circuits

Figure 8. Imaginary and real parts of the susceptibility $\chi$ for the single EIT-like system as a function of the detuning $\Delta$. The radiative decays are $\gamma_1 = 1.0$, $\gamma_2 = 10^{-4}$ and $\gamma_3 = 0.0$. For plots (a) and (b), $\Delta_c = 0.0$ and $\Omega_c = 2.3$. For plots (c) and (d), $\Delta_c = 0.0$ and $\Omega_c = 3.0$. For plots (e) and (f), $\Delta_c = 0.1$ and $\Omega_c = 2.3$. All the parameters have the dimension of frequency.

was considered (i.e. $\Omega_c = 0$ or $\Omega_c = 0$). The absorption and dispersion curves showed characteristics of standard EIT, as observed in figures 8(a) and (b), respectively. The parameters used for this case were $\Omega_c = 2.3$, $\Omega_r = 0.0$, $\gamma_1 = 1.0$, $\gamma_2 = 10^{-4}$ and $\gamma_3 = 0.0$. The effects of the coupling field strength on this system are now shown in figures 8(c) and (d). The broadening in the EIT peak was apparent and caused by the coupling field increase. In figures 8(e) and (f), the only parameter changed was $\Delta_c = 0.1$, leaving the other parameters as before. Clearly, the EIT moved away from the centre of the graph.

We next looked at the behaviour of $P_R$ and $P_X$ as a function of the frequency detuning $\delta = \omega - \omega_R$ for different initial conditions of the parameters $R$, $L$ and $C$. In figure 9, the effects of the coupling and pumping frequency detunings, in the double EIT scenario, are shown when parameters $L_2$ and $L_3$ took on different values. The solid and dashed lines represent the absorption and dispersion of light, respectively. Figure 9(a) shows that, at exact resonance conditions for both the coupling and pumping fields with the probe field ($\Delta_c = \Delta_r = 0$), there was only a single dip in the curve (like single EIT). This happened because both EIT dips occurred at the same location. The corresponding dispersion curve also shows this particular characteristic. A separation of the two EIT dips in the absorption line occurred when $L_3$ was increased, as shown in figure 9(b). The dispersion line also moved apart, showing the typical dispersion characteristics of double EIT. This showed how the second dip moved towards the
Figure 9. Power transferred to the $R_1L_1C_1$ circuit (figure 6(a)) as a function of the detuning $\delta = \omega - \omega_R$. This detuning was defined as the difference between the driving field frequency $\omega$ and the resonance frequency of the circuit $\omega_R$. The parameters used were $R_2 = R_3 = 5.0$ $\Omega$, $R_1 = 50$ $\Omega$, $C_1 = C_2 = C_3 = 0.1$ $\mu$F, $C = 0.2$ $\mu$F and $L_1 = 0.0010$ H. For plots (a) $L_2 = 0.00010$ H and $L_3 = 0.0010$ H; (b) $L_2 = 0.0010$ H and $L_3 = 0.0015$ H; (c) $L_2 = 0.0020$ H and $L_3 = 0.0003$ H and (d) $L_2 = 0.0005$ H and $L_3 = 0.0003$ H. The solid line represents $P_R$, whereas the dashed line represents $P_X$. $P$ is given in arbitrary units.

left in comparison to the one displayed in figure 9(a). The separation of the two EIT dips is shown more clearly in figure 9(c) for a different set of parameters $L_2$ and $L_3$. The two dips moved in opposite directions, and double EIT was visible again. The dispersion curve also showed double EIT, and the peaks moved in opposite directions. Figure 9(d) shows that the dips shifted to the right for yet another different set of parameters $L_2$ and $L_3$.

Different values of the radiative decay parameters (damping) also changed the absorption and dispersion curves in double EIT. In the electrical analogue of the atom, the resistance in the circuit loops represented the damping. A comparison of figures 10(a) and (b) shows how the first EIT dip became less pronounced, and its width expanded when resistance $R_2$ increased. When we increased $R_2$ and $R_3$ even more, both EIT dips became even less pronounced, and their widths increased with the resistance increase (compare figures 10(a) and (c)). A large value of $R_2$ caused the first dip to spread out increasing its width and decreasing its depth.

By removing one of the loops, i.e. either $R_2L_2C_2$ or $R_3L_3C_3$, we recovered the two RLC coupled circuits showing single EIT (see figure 6(b)). Figures 11(a)–(d) show the behaviour of $P_R$ and $P_X$ after disconnecting the pumping loop $R_3L_3C_3$. Plot 11(a) clearly shows a
Demonstration of double EIT using coupled harmonic oscillators and RLC circuits

-0.010 0 0.010 0.020
-0.1 0 0.1 0.2

Figure 10. Power transferred to the $R_1L_1C_{e1}$ circuit (figure 4(a)) as a function of the detuning $\delta$. The parameters used were $R_1 = 50 \Omega$, $L_2 = 0.0020 \text{H}$, $L_1 = 0.0010 \text{H}$, $L_3 = 0.0003 \text{H}$, $C_1 = C_2 = C_3 = 0.1 \mu\text{F}$ and $C = 0.2 \mu\text{F}$. For plots (a) $R_2 = 5.0 \Omega$ and $R_3 = 2.0 \Omega$; (b) $R_2 = 15 \Omega$ and $R_3 = 2.0 \Omega$; (c) $R_2 = 30 \Omega$ and $R_3 = 5.0 \Omega$; and (d) $R_2 = 50 \Omega$ and $R_3 = 5.0 \Omega$, respectively. The solid line represents $P_R$, whereas the dashed line represents $P_X$. $P$ is given in arbitrary units.

On the other hand, the experimental results obtained for the coupled RLC circuit shown in figure 6(b) displayed single EIT behaviour. We measured the current flowing through the circuit and calculated the power delivered to the $R_1L_1C_{e1}$ loop. Figure 10 shows the power transmitted $P_R$ as a function of the driving field frequency $\omega$.

In figures 12(a) and (b), the curves A and B illustrate the situation when we opened the switch SW (driven single RLC circuit) and when we closed it (driven RLC circuit coupled to a second RLC circuit). With the open switch, no power was transferred from the circuit loop $R_2L_2C_{e2}$, and the circuit loop $R_1L_1C_{e1}$ behaved like a simple, driven RLC circuit as shown in the figures. However, with the closed switch, we clearly observed a dip (curve B...
Figure 11. Power transferred to the $R_1L_1C_1$ circuit (figure 4(b)) as a function of the detuning $\delta$. The parameters used were $R_2 = 5.0 \Omega$, $R_1 = 50 \Omega$, $C_1 = C_2 = 0.1 \mu F$, $C = 0.2 \mu F$ and $L_1 = 0.0010 H$. For plots (a) $L_2 = 0.0010 H$, (b) $L_2 = 0.0015 H$, (c) $L_2 = 0.0020 H$ and (d) $L_2 = 0.0005 H$. The solid line represents $P_R$, whereas the dashed line represents $P_X$. $P$ is given in arbitrary units.

of these two plots). This dip resembled the single EIT-like dip shown in figures 11(a)–(d). The two B curves observed in figures 12(a) and (b) essentially represented different resonance frequencies for the observed single EIT in the RLC circuits.

Note that such a simulation of single EIT along with experimental demonstrations have also been presented in an earlier work by Garrido Alzar et al [38]. They present a classical analogue of EIT using two coupled harmonic oscillators subject to a harmonic driving force (similar to figure 5(b)) and reproduce the phenomenology observed in EIT by changing the strength of the coherent coupling field. Moreover, these authors also recreate EIT behaviour experimentally using two linearly coupled RLC circuits (similar to figure 6(b)). In their work, the simulations are for the degenerate probe and coupling transitions, showing excellent agreement of the theoretical modelling of EIT (using coupled RLC circuits) with experimental results under similar conditions of parameters. In this paper, we have presented not only an extensive simulation of EIT, including the effects of different strengths of coupling fields and frequency detunings associated with the coupled harmonic oscillators and RLC circuits, but also simulations for a richer phenomenon of double EIT. We have selected different sets of parameters in the experimental simulations to show that single EIT (see figures 12(a) and (b)) and double EIT (see figures 12(c) and (d) to be discussed in the following paragraph) are exhibited for a wider range of parameters, and hence they are quite versatile.
We observed double EIT in the three coupled RLC circuits as shown in figure 6(a). When we experimentally measured the power transferred to the $R_1 L_1 C_{e1}$ loop from the loops $R_2 L_2 C_{e2}$ and $R_3 L_3 C_{e3}$, two dips were visible (see figures 12(c) and (d)). We also noted that the position of the second EIT peak changed for different values of the inductance $L_2$. These dips were the analogues of quantum interference observed in double EIT atomic systems. In this case, the interference occurred because of the power delivered to the resonant $R_1 L_1 C_{e1}$ circuit from the voltage source $V$ and the other two coupled circuits $R_2 L_2 C_{e2}$ and $R_3 L_3 C_{e3}$. Classically, we looked at this phenomenon as the interference between three excitation paths corresponding to the normal modes of oscillation of the coupled harmonic oscillators.

4. Summary

We have presented mechanical and electrical analogies for single and double EIT observed in three- and four-level atomic systems using coupled harmonic oscillator models and RLC
circuits. The mechanical analogy, consisting of a coupled spring–mass system, may be helpful in understanding the observed zero power absorption in single and double EIT phenomena, as a result of destructive interference between the normal modes of oscillation of the system. The dissipation rates of the coupling and pumping oscillators ($\gamma_2$ and $\gamma_3$, respectively) should be small compared with that of the atomic oscillator ($\gamma_1$) for EIT to be observable. The symmetry of the equation of motion of the atom for EIT allows us to study easily the absorption and dispersion of a multilevel system in the inverted-Y (four-level) and $\Lambda$ (three-level) configurations.

The electrical analogy, associated with a coupled RLC circuit, may be helpful to realize the single and double EIT phenomena experimentally. This type of circuit corresponds to the electrical analogue of the mass–spring system. This fact allows us to establish a direct correspondence between an atomic system (based on Lorentz’s approximations) and the RLC circuit. In fact, by changing some circuit parameters such as the inductances and capacitances, it is possible to produce different control fields acting on different atomic transitions. The resistances of the circuit represent the radiative decays of these atomic levels. The Rabi frequencies of these control fields should be large enough from the radiative decays for EIT to be observable.

The interest on these experiments and the final purpose of this work is to help undergraduate students to develop a better understanding of single and double EIT, as well as to improve their experimental skills. These experiments are easy to adopt in any undergraduate physics laboratory and can be used to approach other compelling topics such as quantum coherence and quantum interference, which occur in atomic systems, and are particularly important in observing phenomena such as group velocity reduction of light, superconductivity and superfluidity, and quantum information processing.

Acknowledgments

The authors gratefully acknowledge the Research Corporation, the College of Sciences and the Department of Physics at Eastern Illinois University, and the School of Mathematical and Natural Sciences at the University of Arkansas—Monticello for providing funding and support for this work.

References

[1] Harris S E 1997 Electromagnetically induced transparency Phys. Today 50 36–42
[2] Fleischhauer M, Imamoğlu A and Marangos J P 2005 Electromagnetically induced transparency: optics in coherent media Rev. Mod. Phys. 77 633–73
[3] Boller K J, Imamoğlu A and Harris S E 1991 Observation of electromagnetically induced transparency Phys. Rev. Lett. 66 2593–6
[4] Firstenberg O, Shuker M, Pugatch R, Fredkin D R, Davidson N and Ron A 2008 Theory of thermal motion in electromagnetically induced transparency: effects of diffusion, Doppler broadening, and Dicke and Ramsey narrowing Phys. Rev. A 77 043830
[5] Zhang Y, Nie Z, Zheng H, Li C, Song J and Xiao M 2009 Electromagnetically induced spatial nonlinear dispersion of four-wave mixing Phys. Rev. A 80 013835
[6] Litvak A G and Tokman M D 2002 Electromagnetically induced transparency in ensembles of classical oscillators Phys. Rev. Lett. 88 095003
[7] Shvets G and Wurtele J S 2002 Transparency of magnetized plasma at the cyclotron frequency Phys. Rev. Lett. 89 115003
[8] Werner M J and Imamoğlu A 1999 Photon–photon interactions in cavity electromagnetically induced transparency Phys. Rev. A 61 011801
[9] Bentley C L, Liu J and Liao Y 2000 Cavity electromagnetically induced transparency of driven-three-level atoms: a transparent window narrowing below a natural width Phys. Rev. A 61 023811
[10] Dantan A and Pinard M 2004 Quantum-state transfer between fields and atoms in electromagnetically induced transparency Phys. Rev. A. 69 043810
[11] Yang W, Joshi A and Xiao M 2005 Chaos in an electromagnetically induced transparent medium inside an optical cavity Phys. Rev. Lett. 95 093902
[12] Kuang L M, Chen Z B and Pan J W 2007 Generation of entangled coherent states for distant Bose–Einstein condensates via electromagnetically induced transparency Phys. Rev. A 76 052324
[13] Weatherall J Q, Search C P and Jääskeläinen M 2008 Quantum control of electromagnetically induced transparency dispersion via atomic tunneling in a double-well Bose–Einstein condensate Phys. Rev. A 78 013830
[14] Yamamoto K, Ichimura K and Gemma N 1998 Enhanced and reduced absorptions via quantum interference: a classical model driven by a RF field Phys. Rev. A 58 2460–6
[15] Joshi A, Brown A, Wang H and Xiao M 2003 Controlling optical bistability in a three-level atomic system Phys. Rev. A 67 041801
[16] Brown A W and Xiao M 2004 Modulation transfer in an electromagnetically induced transparency system Phys. Rev. A 70 053830
[17] Yang L, Zhang L, Li X, Han L, Fu G, Manson N B, Suter D and Wei C 2005 Autler–Townes effect in a strongly driven electromagnetically induced transparency resonance Phys. Rev. A 72 053801
[18] Olson A J and Mayer S K 2009 Electromagnetically induced transparency in rubidium Am. J. Phys. 77 116–21
[19] Joshi A and Xiao M 2003 Electromagnetically induced transparency and its dispersion properties in a four-level inverted-Y atomic system Phys. Lett. A 317 370
[20] Wang H, Brown A W and Xiao M 2007 Opening four-wave mixing and six-wave mixing channels via dual electromagnetically induced transparency windows Phys. Rev. Lett. 99 123603
[21] Li S, Yang X, Cao X, Zhang C, Xie C and Wang H 2008 Enhanced cross-phase modulation based on a double electromagnetically induced transparency in a four-level tripod atomic system Phys. Rev. Lett. 101 073602
[22] Joshi A 2009 Phase-dependent electromagnetically induced transparency and its dispersion properties in a four-level quantum well system Phys. Rev. B 79 115315
[23] Xiao M, Li Y Q, Jin S Z and Gea-Banacloche J 1995 Measurement of dispersive properties of electromagnetically induced transparency in rubidium atoms Phys. Rev. Lett. 74 666–9
[24] Hau L V, Harris S E, Dutton Z and Behroozi C H 1999 Light speed reduction to 17 m/s in an ultracold atomic gas Nature 397 594–8
[25] Liu C, Dutton Z, Behroozi C H and Hau L V 2001 Observation of coherent optical information storage in an atomic medium using halted light pulses Nature 409 490–3
[26] Moppart J and Corbín-R 2000 Lasing without inversion J. Opt. B: Quantum Semiclass. Opt. 2 R7
[27] Wu H, Xiao M and Gea-Banacloche J 2008 Evidence of lasing without inversion in a hot rubidium vapor under electromagnetically-induced-transparency conditions Phys. Rev. A 78 041802
[28] Wang H, Goosksey D and Xiao M 2001 Enhanced Kerr nonlinearity via atomic coherence in a three-level atomic system Phys. Rev. Lett. 87 073601
[29] Schmidt H and Ram R J 2000 All-optical wavelength converter and switch based on electromagnetically induced transparency Appl. Phys. Lett. 76 3173–5
[30] Ottaviani C, Vitali D, Artoni M, Cataliotti F and Tombesi P 2003 Polarization qubit phase gate in driven atomic media Phys. Rev. Lett. 90 197902
[31] Héotet G, Peng A, Johnsson M T, Hope J J and Lam P K 2008 Characterization of electromagnetically-induced-transparency-based continuous-variable quantum memories Phys. Rev. A 77 012323
[32] Bermel P, Rodríguez A, Johnson S G, Ioannopoulos J D and Soljačić M 2006 Single-photon all-optical switching using waveguide cavity quantum electrodynamics Phys. Rev. A 74 043818
[33] Dragoman D and Dragoman M 2004 Quantum-Classical Analogies (Berlin: Springer)
[34] Hemmer P R and Prentiss M G 1988 Coupled-pendulum model of the stimulated resonance Raman effect J. Opt. Soc. Am. B 5 1613–23
[35] Shore B W, Gromovyy M V, Yatsenko L P and Romanenko V I 2009 Simple mechanical analogs of rapid electromagnetically-induced-transparency conditions Phys. Rev. A 79 041802
[36] Schmidt H and Ram R J 2000 All-optical wavelength converter and switch based on electromagnetically induced transparency Appl. Phys. Lett. 76 3173–5
[37] Ottaviani C, Vitali D, Artoni M, Cataliotti F and Tombesi P 2003 Polarization qubit phase gate in driven atomic media Phys. Rev. Lett. 90 197902
[38] Garrido Alzar C L, Martinez M A G and Nussenzveig P 2002 Classical analog of electromagnetically induced transparency Am. J. Phys. 70 37–41
[39] Lifshitz R and Cross M C 2003 Response of parametrically driven nonlinear coupled oscillators with application to micromechanical and nanomechanical resonator arrays Phys. Rev. B 67 134302
[40] Allen L and Eberly J H 1987 Optical Resonance and Two-Level Atoms (New York: Dover)
[41] Petrosyans D and Kurizki G 2002 Symmetric photon–photon coupling by atoms with Zeeman-split sublevels Phys. Rev. A 65 053833
[42] Autler S H and Townes C H 1955 Stark effect in rapidly varying fields Phys. Rev. 100 703–22
[43] Li Y Q and Xiao M 1995 Electromagnetically induced transparency in a three-level Λ-type system in rubidium atoms Phys. Rev. A 51 R2703–6
[44] Li S, Yang X, Cao X, Xie C and Wang H 2007 Two electromagnetically induced transparency windows and an enhanced electromagnetically induced transparency signal in a four-level tripod atomic system J. Phys. B: At. Mol. Opt. Phys. 40 3211–9
[45] French A P 1971 Vibrations and Waves (New York: Norton)
[46] Scully M O and Zubairy M S 1997 Quantum Optics (Cambridge: Cambridge University Press)
[47] Lorentz H A 1952 The Theory of Electrons (New York: Dover)
[48] Marion J B and Thornton S T 1995 Classical Dynamics of Particles and Systems 4th edn (Philadelphia, PA: Saunders)
[49] Symon K R 1971 Mechanics 3rd edn (Reading, MA: Addison-Wesley)