Dam-break release of a gravity current in a power-law channel section

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Abstract. The evolution of gravity currents generated in open channels which represent those present in estuaries and many hydraulics structures are studied analytically and experimentally. We are particularly interested in the effects of the cross-section shape on the dam-break flows that satisfy the shallow-water equations by considering an equivalent channel cross-section represented by a power law. A stratified two-layer system is assumed and an analytical model based on the balances of mass and momentum in the frontal region, whose results are successfully compared with those obtained by means of experiments, is proposed. The solutions describe the dynamics of the gravity currents with sharp fronts and can be applied to a wide range of natural situations where the turbulence is not dominant.

1. Introduction
Gravity currents are flows driven by a difference in density between the current and the ambient fluid into which it penetrates. As they are present in numerous natural, man-made and industrial events, the comprehension of its features is important for a variety of practical situations [1]. Gravity currents are affected by the topography over which they flow, the entrainment of the ambient fluid and, in many cases, the particulate matter they carry. Most part of the effort has been directed towards investigating the release of such flows in flat-bottomed rectangular tanks, in open horizontal or inclined channels, and considering lock-exchange or dam-break configurations. Also significant, the studies of gravity currents propagating along channels of different cross-section shapes have been addressed in the geophysics context [2-6] although these flows are considered to be fed by a constant flux rather than by a constant volume.

The release of a volume of fluid from behind a lock into an ambient fluid of a different density is a well known situation in hydraulics. Interest in dam-break problems arises as a result of governmental regulations to prevent dramatic events which may become dangerous to human beings or nature, as occurs in Italy among other mountainous countries. The dam-break flow is that produced by the instantaneous release of a liquid by means of the sudden removal of a barrier. It may be thought as the idealized situation of that occurring when the wall of a reservoir open to the atmosphere collapses and the water initially contained within spreads, for example, along a valley or a gorge. The mass of water then flows horizontally under gravity, representing the non-Boussinesq limit in which the air is the fluid of smaller density and, as a consequence, its resistance to the water motion may be neglected. The one-dimensional problem (i.e. for wide cross-section channels) was solved by St. Venant (see for reference).
example [7]) in the shallow-water equations approximation, assuming that the vertical accelerations are small enough with respect to gravity g. This hypothesis is valid except immediately after the dam collapse, so that the eventual errors derived from it are not significant for long periods of time [8].

A rather simple form of the dam failure problem in a dry channel was first solved by Ritter [9], who used the Saint-Venant equations in the characteristic form, under the hypothesis of instantaneous failure in a horizontal rectangular channel without bed resistance. A description of the subsequent studies about this problem was summarized by Zanuttigh & Lamberti [10]. Geometric effects on sudden releases were first investigated by Shih-Tun Su and Barnes [11], who obtained water surface profiles and mean velocity distributions for rectangular, triangular, and parabolic cross sections, represented by a power-law variation of the area with the depth in the case of a dam break in a dry right channel. Zanuttigh & Lamberti [10] analytically investigated the cross-sectional shape effects on the initial discharge of a dam-break flow assuming an equivalent channel section represented by a power-law variation of the section width. They considered three parameters characterizing the section: the channel reference width (not necessarily the bank-full width), the bed elevation, and the power-law index assumed to be constant along the channel. Velocity and bore speed for some typical sectional shapes (rectangular, triangular, concave, and convex banks) were determined as functions of the relative flow depth difference and of the power-law index. Later, Zanuttigh & Lamberti [12] extended the Weighted Averaged Flux (WAF) method by E. F. Toro [13] from rectangular to generic regular cross sections, in order to represent the geometric effects of the cross-section on wave propagation along channels. Assuming a power-law variation of the channel width, the 1D conservative shallow water equations, their characteristic form and the shock propagation equations were presented. The exact Riemann solver was derived and applied to the dam-break problem in valleys with different shape in order to test its efficiency and to check the accuracy order of solutions obtained by approximating the real cross-section with an equivalent rectangle. More recently Biscarini et al. [14] presented numerical simulations of free surface flows induced by a dam-break comparing the shallow water approach to full three-dimensional simulations based on the solution of the complete set of Reynolds-Averaged Navier-Stokes (RANS) equations coupled to the Volume of Fluid (VOF) method. The methods assessment and comparison were carried out on a dam-break over a flat bed without friction, a dam break over a triangular bottom sill and a dam break flow over a 90° bend. Experimental and numerical literature data were compared to their results.

On the other hand, Monaghan et al. [15] investigated high-Reynolds numbers gravity currents of constant volume and propagating horizontally along a rectangular tank with a V-shaped bottom filled with a lighter fluid. They showed that a numerical simulation based on a box model with the Froude number of the head defined using the distance from the top of the current to the bottom of the valley predicts the evolution of the position of the head in close agreement with the experiments. The major differences between these gravity currents and those flowing along a flat bottom were indicated. Also Marino and Thomas [16-17] introduced a theoretical treatment describing the dynamics of the counter-flows, which only includes the effect of the cross-section shape. As the front condition for steady (surface and bottom) flows generated in non-rectangular cross-section channels is analytically derived independently for each current, the results can be used to study situations where the attention is focused on a single current as in geophysical and engineering flows. The theoretical findings were compared with those provided by laboratory experiments where lock-exchange flows in open and closed channels of rectangular, triangular and parabolic cross-section were generated.

We experimentally and analytically investigate the gravity currents generated by the removal of a barrier that initially separates two fluids of slightly different densities, and provide an analytical model based on mass and momentum balances in the shallow-water approximation that describes the evolution of the front velocity and height profile of the dam-break flows generated in open channels with cross-sections defined by an index related to geometry. It is found that the current dynamics is affected in different ways. The coefficients of the power-law that describe the evolution of the front position with time are greater in the cases in which the cross-section shape is farther from being
rectangular and the relative density difference is smaller, while the height profile tends to be uniform. These differences between the flows can affect the estimates of erosion and sedimentation effects on river and estuaries beds. The experiments indicate that the mixing does not significantly modify the frontal motions in stratified environments. Thus, the analytical solutions may be applied to a huge range of natural situations in which turbulence and mixing are not dominant.

2. Analytical solution for a dam-break gravity current in a power-law cross-section channel

We consider a horizontal channel with a cross-section shape that is uniform in the \( x \)-direction. As usual, a power-law variation of a channel width in terms of the flow depth is assumed in the form

\[
y = \frac{1}{2} b z^\alpha,
\]

where \( z \) and \( y \) are the vertical and transverse coordinates of the channel cross-section, while \( \alpha \) and \( b \) are constants. In particular, \( b = \frac{2w}{h_0} \) is determined by the width \( w \) and the depth \( h_0 \) of the channel occupied by the fluid. Figure 1 illustrates the cross-section shape associated with different values of \( \alpha \).

![Figure 1. Cross-section shapes of a channel according to the values of \( \alpha \)-parameter.](image)

A fluid of density \( \rho_1 \) occupies the channel from the rear wall \( (x = 0) \) up to the position \( x_0 \) where a vertical barrier is inserted to separate it from another fluid of density \( \rho_2 < \rho_1 \). Both fluids fill their rooms up to a depth \( h_0 \) and are initially at rest. The volume \( V \) of the dense fluid is

\[
V = \int_0^{x_0} \int_0^{h_0} 2y \cdot dz \cdot dx = \int_0^{h_0} b \cdot x_0 \cdot h_0^{\alpha+1} \frac{\alpha}{\alpha + 1}
\]

As soon as the gate is removed, the dense fluid collapses generating a layer that extends up to \( x = x_f(t) \) where the front (or a height discontinuity) moves with velocity \( u_f \). When inertial forces dominate, the equations of mass and momentum conservation in the shallow-water approximation for a channel of cross-section shape defined by (1) take the form [18]:

\[
\frac{\partial h^{\alpha+1}}{\partial t} + \frac{\partial}{\partial x} \left( u h^{\alpha+1} \right) = 0
\]

(3)

\[
\frac{\partial u}{\partial t} + \frac{\partial}{\partial x} \left( g^* h + u^2 / 2 \right) = 0
\]

(4)

where \( h \) is the current height, \( t \) is time, \( u \) is the component of fluid velocity in the \( x \)-direction and \( g^* = g(\rho_2 - \rho_1)/\rho_1 \) is the initial value of the reduced gravity (\( g \): gravity acceleration).

We look for solutions of (3) and (4) in the form

\[
h(x,t) = h_f(t) H(\eta)
\]

(5)
where \( \eta = x / x_f \) and the sub-index \( f \) denotes the values of the variables in the front. These are self-similar solutions since the dimensionless distributions of the current height and velocity have the same form at any time, that is \( H(\eta) \) and \( U(\eta) \) do not depend on \( t \). Thus the time evolution is established only by the variation of the current length \( x_f(t) \), that in turn determines \( u_f(t) \) and \( h_f(t) \).

The dense fluid volume is constant if mixing is negligible and, as the current profile is self-similar, it results that

\[
V = \int_0^{x_f} \int_0^h 2y \cdot dz \cdot dx = \frac{b \cdot I \cdot x_f \cdot h_f^{\alpha+1}}{\alpha+1}
\]

with the constant shape factor

\[
I = \int_0^1 H^{\alpha+1} d\eta.
\]

Differentiating (7) with respect to time, it is found that

\[
\frac{dh_f}{dt} = -\frac{h_f u_f}{(\alpha+1)x_f}.
\]

In addition, a local balance of mass and momentum in the front gives place to a constant densimetric Froude number

\[
Fr = \frac{u_f}{\sqrt{g h_f}},
\]

of the order of unity (\( Fr \sim 1 \)). Integrating (10) in time with \( u_f = dx/\sqrt{dt} \) and using (7), the front position evolves according to

\[
\frac{x_f}{x_0} = \xi \left( \frac{t}{t_c} \right)^\beta,
\]

where

\[
\beta = \frac{2\alpha+2}{2\alpha+3},
\]

\[
\xi = \left( \frac{Fr}{\beta} \right)^\beta \left( \frac{1}{Fr} \right)^{1/(2\alpha+3)}
\]

and

\[
t_c = \frac{x_0}{\sqrt{g h_0}}
\]

is the characteristic time determined by the characteristic length \( x_0 \) and the velocity \( \sqrt{g h_0} \) of an internal wave propagating in shallow waters.

In this way the self-similar solutions are found. For \( \alpha = 0 \) the result agrees with that for gravity currents running in rectangular cross-section channels where \( \beta = 2/3 \). For \( \alpha \neq 0 \) the exponent \( \beta \) depends only on the channel cross-section shape, while the coefficient (13) depends also on the Froude number at the front. The released volume and the reduced gravity merely influence the characteristic distance \( x_0 \) and time \( t_c \). In addition, the relationship between the width \( w \) and depth \( h_0 \) of the channel can be neglected under the assumed approximations since only the shape is of interest.
The current height profiles are determined by solving the equations (3) and (4). Differentiating (10), and employing (9), we obtain

$$\frac{d u_f}{d t} = -\frac{1}{2(\alpha + 1)} u_f^2.$$  \hspace{1cm} (15)

On the other hand, by replacing (5) and (6) in (3) and (4), and using (9) and (15), it is found that

$$H^{\alpha+1} \left( \frac{dU}{d\eta} - 1 \right) + (U - \eta) \frac{dH^{\alpha+1}}{d\eta} = 0$$  \hspace{1cm} (16)

$$- \frac{U}{2(\alpha + 1)} + \frac{dU}{d\eta} (U - \eta) + \frac{1}{Fr^2} \frac{dH}{d\eta} = 0.$$  \hspace{1cm} (17)

The solution for any value of $\alpha$ and $Fr$ results:

$$U = \eta$$  \hspace{1cm} (18)

$$H = 1 - \frac{Fr^2}{4(\alpha + 1)} \left( 1 - \eta^2 \right).$$  \hspace{1cm} (19)

Then, velocity increases linearly from $x = 0$ to $x = x_f$ independently of the cross-section shape. Figure 2 shows the distributions $H(\eta)$ given by (19). Again, for $\alpha = 0$ the result coincides with that of gravity currents in rectangular channels. For increasing $\alpha$ the distribution tends to be uniform but $H \approx 1$ only for large values of $\alpha$.

The solutions found indicate that the height and velocity are analytical continuous functions in $0 \leq x < x_f$. At $x = x_f$, a height discontinuity exists as a step from $h_f$ to $h = 0$ while the velocity changes from $u_f$ up to $u = 0$.

![Figure 2](image_url) Figure 2. Height distributions between the origin $x = \eta = 0$ and the front $\eta = 1$ $(x = x_f)$.

3. Experimental validation

3.1. Experimental description

Three sets of laboratory experiments were performed in Perspex channels with non-rectangular cross-sections which are considered as half the shapes defined by equation (2) with $\alpha = 0.44$ (concave), 1.00 (triangular) and 1.79 (convex). This experimental setup therefore includes a vertical wall at the
centerline of the flow to reduce the optical distortion when it is observed laterally. Additional experiments confirm that the results can be extrapolated to the complete section by symmetry without losing validity. The channels are 2.00 m long, 0.20 m wide and 0.29 m high. In addition, one set of experiments was carried out in a rectangular cross-section channel where known results were reproduced. A gate located initially at 0.10 m from the rear of the channel separated two parts: the smaller one contained salt water while the bigger one was filled with fresh water, up to the same level. The experiment started when the barrier is suddenly removed giving place to a gravity current above the bottom. Each set includes four or five experiments with relative density difference \( \Delta \rho / \rho_1 = 1- \gamma \) between 0.2% and 8%. Densities were measured using an Anton Paar 4500 densimeter with a precision of \( 5 \times 10^{-5} \) g/cm\(^3\). The non-rectangular channels were placed inside a greater rectangular cross-section tank filled with fresh water to reduce the optical distortion due to the non parallel or curved lateral walls.

The flows were observed laterally with a CCD video-camera located 6 m away from the tank. Experimental set-up and image processing are similar to those previously reported by Marino et al. [19]. An anamorphic lens was used in order to magnify the vertical scale and reduce the horizontal scale to improve the resolution of the images. Fluorescent strip lights and a light-diffusing screen located behind the tank provided nearly uniform back-lighting. A reference image of the initial intensity distribution of the tank containing only water was taken by averaging a sequence of images registered during 10 s. A calibrated quantity of dye was added to the salt water to provide visualization. After finishing the experiments, the images were processed considering the acquisition system response, converting the captured light intensity to absolute intensity. Then a correction compensating the small spatial variations of the back-lighting was performed and finally the decreasing of the light intensity with respect to the initial reference image was estimated for each pixel. The light attenuation due to the dyed water allows the cross-current averaged concentration to be measured from which the cross-current average density may be inferred. Thus, the intensity in the processed images is proportional to the cross-current average density.

For a rectangular channel the estimate of the density in excess with respect to the non-colored light fluid is direct [19]. For a non-rectangular cross-section, the intensity depends on two variables: mixing and variation of the channel width with depth. This complication is solved by applying an additional operation that uses another reference image of the channel filled with water and a known concentration of dye [16].

From the processed images, the front velocity and the height of the flow are estimated. With these values, Reynolds numbers ranging between 7000 and 35000 are found for the currents generated.

### 3.2 Results

The evolution of the front position with time for a rectangular cross-section channel is shown in figure 3a, while figures 3b, 3c and 3d show the cases corresponding to a concave, triangular and convex cross-section shape, respectively. In each case, the theoretical lines describing the slumping phase [19], characterized by a constant front velocity, and the self-similar solution given by equation (11) for long periods of time \( t/t_c > 20 \) are included. It is important to note that the classical slumping phase of gravity currents evolving over flat bottoms is also present for \( \alpha \neq 0 \) cases. For a given value of \( \alpha \), the experimental points show the same trend, that is, for different initial relative density differences they collapse onto common curves with the slope depending only on the cross-section shape, thus indicating that the characteristic parameters \( x_0 \) and \( t_c \) are representative of the spatial and time scales, respectively, of the studied flows.

Figure 4a shows a typical lateral image of a gravity current evolving towards the right in a rectangular cross-section channel, processed as described in Section 2. The presence of instabilities generates mixing between the fluids and makes difficult to determine the height profile with precision. However, it is clear that the front region is higher than the following fluid and that the height decreases progressively up to the left vertical wall, consistently with the theoretical profile presented in figure 2.
Figures 4b, 4c and 4d show lateral views of the currents developed in parabolic, triangular and convex cross-section channels, respectively. Here, the intensity of each pixel is also proportional to the mean density, independently of the particular shape of the cross-section of the channel in which the flows evolve. Although it is observed that the general shape of the flows is the same, for increasing $\alpha$ ($\neq 0$) the frontal zone of breaking waves and intense mixing becomes larger. The current height decreases slightly behind the head in agreement with the theoretical model. The thickness of the gravity current increases with $\alpha$ since the fluid needs more space to flow in narrower channels.

Figure 3. Time evolution of the front position of a gravity current evolving in (a) rectangular ($\alpha = 0$), (b) parabolic ($\alpha = 0.44$), (c) triangular ($\alpha = 1$) and (d) convex ($\alpha = 1.79$) channels. Solid lines represent the solution for the *slumping* stage and dashed-lines are the solution given by (11).
4. Concluding remarks

We analytically and experimentally study the dam-break flows generated in open straight channels with cross-sections shape defined by a geometrical parameter that facilitates the analysis in the shallow water approximation. It is found a self-similar solution that describes the evolution of the front height and velocity for extended currents, that is, when the initial conditions are not important ($x_f >> x_0$). In this case the main characteristic magnitudes are the flow initial length $x_0$ and the velocity $\sqrt{g'h_0}$ of the internal waves propagating in shallow waters. Thus the space and time scales are determined by $x_0$ and $t_c$, independently of the cross-section.

After the classical slumping phase, the evolution of the front follows the power law given by equation (11) in which the similarity exponent only depends on the cross-section shape. The power law coefficient, however, also depends on the Froude number at the front. These facts are based on the total mass conservation, the self-similar height profiles shape [equations (5) and (6)], and the mass and momentum balance in the current front. In addition, the mass and momentum conservation equations in the shallow water approximation allow the determination of the height profiles of the flows and, with them, the calculation of the shape factor $I$ and the coefficient $\xi$ of the self-similar law. Thus, a
The relationship between the Froude number in the front and the respective height profile is obtained, which is different from that corresponding to a flow produced in a rectangular channel.

The experimental results obtained using channels with cross-section shape given by (1) agree with the theoretical treatment provided. The space and time scales found analytically are adequate to align the points on a single curve for different initial relative density difference between the fluids. The front evolution tends to be described by a self-similar solution for $x_f > 20x_0$ and the height profiles are consistent with the theoretical ones. In laboratory flows a non negligible mixing between the fluids exists, which is not considered by theory. Nevertheless, this does not notoriously modify the dynamics of the front since the proposed balances are equally verified. This is because mixing increases the volume and height of the current, but the dilution reduces the difference of density in an analogous proportion. As a result the front velocity, and consequently the Froude number, are not modified substantially (equation 10).

The self-similar solution represents an interesting achievement from theoretical-conceptual and practical point of views in the study of gravity currents evolving in channels with non-rectangular cross-section shape. Such solutions describe the dynamics of the flows with sharp fronts and are applied to a wide range of natural situations in which a well-defined stratification exists. It may also be applied to validate complex numerical simulations that consider more difficult analytical aspects.

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