The Effect of a Rubber Sheet on the Dynamic Response of a Machine Foundation Located over a Small Thickness of Soil Layer

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Abstract. Placing a machine footing over a small thickness of soil layer, which is located over a bedrock, could encounter many challenges due to the bed’s notable stiffness in comparison to the soil. The advantages of using rubbers to protect facilities (structures, machine foundations, nearby footings and equipment, etc.) from vibration and control its consequences are well known nowadays. In this study, the benefits of employing a small thickness of rubber sheet (12 mm) on the dynamic response of a machine foundation which is located over four thicknesses of soil (210, 420, 630, and 840 mm) has been investigated. The soil layer is located over an artificial bedrock that is consisted of a thick concrete layer. The tests have been conducted in a vast test pit of size 2500×2500 mm and a depth of 840 mm by using a semi large-scale machine foundation model with a square concrete foundation of width 400×400×100 mm. It was observed that, by increasing the soil layer thickness, the resonant frequency and amplitude of the vibrating system decreases. Moreover, by employing a rubber sheet beneath the machine footing, the resonant frequency of the vibrating system significantly decreases especially for a small thickness of the soil layer. Although, using a rubber sheet could slightly increase the resonant amplitude, but the benefit of the resonant frequency-changing capability of the rubber sheet is too impressive by taking the resonant frequency of the system far enough from the unchangeable working frequency of the machine and preventing the resonant phenomenon to happen.

1. Introduction
Machines have been used for centuries to ease the life of humans. As these machineries become more advanced, the generated vibration turned into an engineering problem, and protecting the machine’s bed and any facilities nearby became vital. Previous researches [1-20] have been investigated analytically, experimentally and numerically many methods to estimate the dynamic response of the machine foundations and well-known methods such as Mass-Spring-Dashpot and Modified Elastic Half-Space had introduced. However, the dynamic response of a machine foundation that is located over a small thickness of the soil is not well-known. As the thickness of the soil over a bedrock (artificial one, like concrete slab beneath a small thickness of soil or the real bedrock consist of significantly stiff soil) is not sufficient to almost dissipate the vibration which is generated by the machine foundation, it is expected the dynamic response of the machine foundation being affected by the bedrock. Some researchers have been investigated the effect of the soil thickness over an artificial bedrock on the
dynamic response of a machine foundation. Baidya and Krishna [21] found that the dynamic response of a machine foundation over a limited thickness of soil over a bedrock becomes independent from the bedrock as the soil thickness gets larger than 2.5 times the foundation width. A similar observation has been made by Ramesh and Prathap Kumar [22]. They found the thickness about 2.5 times the foundation width.

The rubber has been used as a part of many dampers and energy dissipater equipment. Remarkable energy dissipation properties of the rubber and its application in geotechnical and earthquake engineering has been assessed by many researchers in many forms of sheets, grains, shredded etc. [13, 23-30]. As the rubber has much smaller stiffness in comparison to the soil and considering its notable damping properties, it could be an ideal material to be used as a method to improve the dynamic response of machine foundations.

In this study, the benefits of employing a rubber sheet (as it is simple to be used in comparison to other methods such as sand rubber particles mixture) to improve the dynamic response of a machine foundation which is located over a limited thickness of soil over an artificial bedrock, have been investigated.

2. Analytical Relations
In this study, the Mass-Spring-Dashpot theory has been used and the motion equation (Equation 1) of the foundation back-calculated to reach the equivalent dynamic properties of the bed (G and ζ for equivalent shear modulus and equivalent total damping properties as Equations 3 and 4) corresponding to its dynamic response (Ż, Z̈ and Z are vertical acceleration, velocity, and displacement, Zres is resonant amplitude and fres resonant frequency). The “m_e” is the total eccentricity mass and ω is the angular frequency.

\[ m\ddot{z} + c\dot{z} + Kz = m_eω^2 sin(ωt) \]  
\[ z = \frac{m_eω^2}{\sqrt{(1 - (\frac{ω}{ω_s})^2)^2 + (2ζ\frac{ω}{ω_s})^2}} \]  
\[ z_{res} = \frac{m_e}{2ζ\sqrt{1 - ζ^2}} \]  
\[ G = \frac{(1-ν)(2πf_{res})^2 m}{4τ_b} \]  

3. Test Materials and Methods
The soil has been used in this study was a Well Graded Sand (SW) according to the Unified Soil Classification System, ASTM-D2487-17 [31]. The maximum dry unit of the soil, the angle of internal friction (φ), the unit weight and moisture content of the test fill material was about 20.42 kN/m³, 40.5 degrees, 19.5 kN/m² and 2% respectively. The rubber has been used in this study was a rubber sheet of size 500×500 mm in plan and a thickness of 12mm with a density of 16.7kN/m³.

The test has been conducted in a test pit of size 2500×2500 mm and a depth of 1340 mm in natural ground. The 500 mm concrete was cast at bottom of the test pit to form an artificial bedrock. Therefore the total available depth of the test pit which is ready to be filled with the backfill was about 840 mm.
A machine foundation model with a concrete foundation of size 400×400 mm and thickness of 100 mm has been used. The apparatus has been subjected to the patent privilege [32, 33] by the name of FMRT apparatus and can estimate the dynamic response of the machine foundation and the equivalent dynamic properties of its bed by conducting Steady-State vibration test and Free Vibration test. The measurement system includes an electromotor that runs an oscillator, an inverter to control the electromotor frequency and a digital datalogger that logs the acceleration sensors and geophones data. All the electronic circuits and control systems including signal transfers are developed by the authors. In this study, a series of Steady-State vibration tests according to the Indian Standard [34] has been conducted by using four thicknesses of the soil over the bedrock (210, 420, 630 and 840 mm) and a 12 mm thick rubber sheet at the frequency range of the 10-70 Hz by using the dynamic angular force of 0.09744 N.m (total of 8 test) and total static weight of 2.1kN prepared by using concrete weights. Every layer was compacted by the FMRT apparatus to reach a unit weight of 19.5 kN/m$^3$. The test program is illustrated in Table 1.

| Soil Thickness Ratio B/H | Rubber Sheet Thickness (mm) | Static Weight (kN) | Dynamic Angular Force (N.m) | Number of Tests |
|--------------------------|-----------------------------|--------------------|----------------------------|-----------------|
| 0.525, 1.05, 1.575, 2.10 | 0                           | 2.1                | 0.9744                     | 4               |
| 0.525, 1.05, 1.575, 2.10 | 12                          | 2.1                | 0.9744                     | 4               |

4. Results and Discussion
The dynamic response of the FMRT apparatus located over four thicknesses of soil (210, 420, 630 and 840mm) by employing or without employing a 12 mm thick rubber sheet is shown in Figure 1. It shows that the resonant frequency of the FMRT over a rubber sheet is less than a case of no rubber sheet. Moreover, it seems that by using a rubber sheet, the resonant amplitude as well as the vertical amplitude of the machine foundation model increases. However, by increasing the soil thickness, the variation of the resonant frequency and amplitude gets insignificant.

Figure 2 shows the soil thickness ratio (H/B) variation versus resonant frequency, resonant amplitude, equivalent total damping ratio and equivalent shear modulus in the case of using a rubber sheet or without using a rubber sheet. It shows that in the case of using no rubber sheet, by increasing the H/B ratio the resonant frequency, resonant amplitude and equivalent shear modulus decreased and the equivalent total damping ratio increases up to the H/B = 1.575. By further increasing the H/B, the above parameters variation becomes negligible. Moreover, in the case of using a rubber sheet, it was observed that the resonant frequency, equivalent total damping ratio and equivalent dynamic shear modulus increases and resonant amplitude decreased up to the H/B = 1.575. Similar to the case of no rubber sheet, by further increasing the H/B, the variation of the above parameters becomes negligible. Therefore, the effective thickness of the soil over an artificial bedrock in this study was observed about 630 mm or the effective depth of the soil was found 1.575 times of the foundation width. Moreover, it was observed that the resonant frequency and equivalent dynamic shear modulus of the system with a rubber sheet are smaller than the system without a rubber sheet and the vertical resonant amplitude and equivalent total damping ratio of the system with a rubber sheet is slightly larger than the system without a rubber sheet (note that this observation was due to small surcharge over the rubber sheet as the rubber damping properties is depends on the confining pressure and dynamic shear strain, this conclusion was provided by a previous study [13]).
Figure 1. The dynamic response of the FMRT apparatus over four thicknesses of soil, (a) 210, (b) 420, (c) 630 and (d) 840mm by employing or without employing a 12 mm thick rubber sheet.

The stiffness of the artificial bedrock is much smaller than the soil. Therefore, in the case of a small thickness of the soil, the stiffness of the bed has a much larger share of the equivalent stiffness/equivalent shear modulus of the foundation models bed. By increasing the thickness of the soil up to the effective depth ($H/B=1.575$ in this study) the bedrock’s stiffness share becomes negligible. In this study, it was observed that the effective depth of the soil over the bedrock was independent of the rubber sheet. However, the rubber sheet has a remarkable effect to reduce the resonant frequency especially for a small thickness of the soil (47% reduction). Reduction of the resonant frequency could be an improvement as it could take the resonant frequency far enough from an unchangeable working frequency of the machine foundation if it was needed.
Figure 2. The soil thickness ratio (H/B) variation versus (a) resonant frequency, (b) resonant amplitude, (c) equivalent total damping ratio and (d) equivalent shear modulus for the case of using a rubber sheet or without using a rubber sheet

5. Conclusions
In this study, the effect of using a rubber sheet to improve the dynamic response of a machine foundation located over a limited thickness of soil over an artificial bedrock has been experimentally investigated. The following conclusion has been made:

1) The effective depth of the soil over the artificial bedrock was found about 1.575 times the foundation width.

2) Using a rubber sheet could decrease the resonant frequency of the machine foundation system especially in a case of a small thickness of soil over the bedrock. The reduction of the resonant frequency is an improvement as it can take the resonant frequency far enough from the unchangeable working frequency.

3) The resonant amplitude of the machine foundation could slightly increase by using a rubber sheet.
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