Vortex arrays in neutral trapped Fermi gases through the BCS-BEC crossover

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**Vortex arrays in type-II superconductors reflect the translational symmetry of an infinite system.** There are cases, however, such as ultracold trapped Fermi gases and the crust of neutron stars, where finite-size effects make it complex to account for the geometrical arrangement of vortices. Here, we self-consistently generate these arrays of vortices at zero and finite temperature through a microscopic description of the non-homogeneous superfluid based on a differential equation for the local order parameter, obtained by coarse graining the Bogoliubov-de Gennes (BdG) equations. In this way, the strength of the inter-particle interaction is varied along the BCS-BEC crossover, from largely overlapping Cooper pairs in the Bardeen-Cooper-Schrieffer (BCS) limit to dilute composite bosons in the Bose-Einstein condensed (BEC) limit. Detailed comparison with two landmark experiments on ultracold Fermi gases, aimed at revealing the presence of the superfluid phase, brings out several features that make them relevant for other systems in nature as well.

The Meissner effect provides evidence of the superconducting phase below the critical temperature \(T_c\) (refs. 1,2). In particular, in type-II superconductors the flux of an applied magnetic field \(H\) is partially or totally expelled from the material, depending on its strength. Two critical values of \(H\) are distinguished, such that for \(0 < H < H_{c1}\) the magnetic flux is totally expelled, whereas for \(H_{c2} < H < H_{c1}\) the system allows the penetration of an ordered array of vortices. Both \(H_{c1}\) and \(H_{c2}\) depend on the temperature \(T\), making \(T\) depend on \(H\).

These considerations apply to large systems containing an (essentially) infinite number of particles, such that the number of vortices can also be considered infinite for all practical purposes. There exist, however, systems with a finite number of particles \(N\) where superfluid behaviour occurs. They include nuclei\(^3\) with \(N \approx 10^{12}\) and ultracold trapped Fermi gases\(^4\) typically with \(N \approx 10^4-10^9\). In these systems, orbital effects, which in superconducting materials are associated with an applied magnetic field, result from rotating the sample as a whole\(^5\) or through synthetic gauge fields\(^6-8\).

In particular, ultracold Fermi gases have attracted special interest recently\(^8-10\), because their inter-particle interactions can be varied almost at will by means of Fano–Feshbach resonances\(^11-14\). These circumstances make the Meissner-like effect in these systems depend also on the coupling parameter \((k_{F}a_{h})^{-1}\) (see Methods). In addition, surface effects may become prominent in systems with finite \(N\), as a finite fraction of the sample can become normal in the ‘outer’ region as a result of rotation, whereas the ‘inner’ region remains superfluid. The Meissner-like effect in these bounded systems can be connected with related effects occurring in nuclei, where the moment of inertia gets quenched by superfluidity\(^9\), and in the inner crust of neutron stars, where arrays of vortices form close to the surface\(^15-16\).

Two experiments were performed by setting an ultracold trapped Fermi gas into rotation, aimed at revealing the presence of the superfluid phase. In the first experiment\(^17\), arrays of vortices were generated at low temperature throughout the BCS–BEC crossover, providing the first direct evidence for the occurrence of the superfluid phase in these systems. In the second experiment\(^18\), the superfluid behaviour was revealed by the quenching of the moment of inertia, which was measured at unitarity (where \((k_{F}a_{h})^{-1} = 0\) for increasing temperature until its classical value of the normal state was reached. In both experiments, the two lowest hyperfine atomic levels were equally populated for a gas of \(^{6}\)Li Fermi atoms contained in an ellipsoidal trap of radial and axial frequencies \(v_{r}, v_{z}\), respectively (see Methods), and the system was assumed to reach a (quasi)-equilibrium situation at any given rotation frequency \(v\). These experiments complement each other, in that they involve determining the total angular momentum and controlling the appearance of vortices.

Despite the role played by these experiments, no theoretical account exists for the structure of complex arrays of quantum vortices in a rotating superfluid with a finite number of particles in realistic traps as functions of inter-particle coupling and temperature, nor for the related behaviour of the moment of inertia. This is because accounting theoretically for these complex arrays of vortices across the BCS–BEC crossover becomes computationally prohibitive when relying on a standard tool such as the BdG equations, which extend the BCS mean field to non-uniform situations\(^9\). Here, we tackle this difficult computational problem by resorting to a novel differential equation for the local gap parameter \(\Delta(r)\) at position \(r\), through which it is possible to find solutions in a rotating trap also for large numbers of vortices. This equation was introduced in ref. 20 (see Methods) and rests on the assumption that the phase \(\varphi(r)\) of the complex function \(\Delta(r) = |\Delta(r)|e^{i\varphi(r)}\) varies more rapidly than the magnitude \(|\Delta(r)|\). Accordingly, this equation was referred to as a local phase density approximation (LPDA) to the BdG equations. In the following, we shall determine the behaviour of the gap parameter, density and current for a rotating trapped Fermi gas using the LPDA equation, while varying the thermodynamic variables \((k_{F}a_{h})^{-1}\) and \(T\), as well as the angular frequency \(\Omega = 2\pi v\).

Our theoretical analysis highlights several features which are of interest also to other branches of physics, such as: the deviations from Feynman’s theorem for the vortex spacing in an array; the bending of a vortex filament at the surface of the cloud; the shape...
of the central vortex surrounded by a finite number of vortices; the emergence of a bi-modal distribution for the density profile in the superfluid phase; the orbital analogue of a breached-pair phase; the yrast effect similar to that occurring in nuclei; the temperature versus frequency phase diagram in a finite-size system.

Previous work on vortices in Fermi gases considered a single vortex with cylindrical symmetry at zero temperature, either within a BdG approach in the BCS regime\(^1\) and across the BCS–BEC crossover\(^2,3\) or including energy-density-functional corrections at unitarity\(^4\). Vortex lattices were considered only for a strictly two-dimensional geometry, at zero temperature in the BCS regime within a static\(^5\) and a dynamic\(^6\) BdG approach, and close to \(T_c\) at unitarity within a Ginzburg–Landau approach\(^7\). None of these works could address a comparison with the experimental data of refs. 17, 18, nor bring out the physical features highlighted above.

We first consider the conditions of the experiment of ref. 17 with \(N = 2 \times 10^5\) and an aspect ratio 2.5 (see Methods), with an atomic cloud large enough to support many vortices at low temperature. In this experiment, the trap was set in rotation at an optimal stirring frequency \(\nu_{\text{opt}} = 45\) Hz.

Figure 1 shows false colour images of the array of vortices obtained theoretically at unitarity and zero temperature for the experimental set-up of ref. 17 with \(\Omega = \Omega_{\text{opt}} = 0.8 \Omega_c\), as seen from the top at \(z = 0\) [panel (a)] and from the side at \(x = 0\) [panel (b)] (where \(\Omega_c = 2\pi \nu_{\text{opt}}\)). In Fig. 1a, the array contains 137 vortices, whose locations are not fixed a priori but are self-consistently determined by solving the LPDA equation (see Supplementary Information). The spacing among vortices in the triangular array increases away from the trap centre. Near the trap centre this spacing is found to be \(1/(\pi \sqrt{3} \Omega)\), consistent with Feynman’s theorem for the number of vortices per unit area \(n_v = 2m\Omega^2/\pi \hbar\), which applies to an infinite array of vortices\(^8\) (where \(\hbar\) is the Planck constant \(\hbar\) divided by \(2\pi\)). In Fig. 1b, 11 vortex filaments are distinguished and seem to bend away from the \(z\) (axial) direction upon approaching the boundary of the cloud, so as to meet the surface perpendicularly with no current leaking out of the cloud (this feature was noted previously for a single vortex in an elongated trapped bosonic cloud large enough to support many vortices at low temperature. In this experiment, the trap was set in rotation at an optimal stirring frequency \(\nu_{\text{opt}} = 45\) Hz.

Figure 1 | Arrays of vortices at unitarity for \(\Omega = 0.8 \Omega_c\). a, b, Magnitude of the gap parameter obtained from the self-consistent solution of the LPDA equation at \(T = 0\), seen from the top (a) and from the side (b). c, Radial profile of the (magnitude of the) gap parameter (in units of its value \(\Delta_0\) in between two adjacent vortices) inside the central vortex in the trap for various angular frequencies (in units of \(\Omega_c\)) at \(T = 0\). The inset compares the vortex count over an increasing superfluid section of the cloud with the prediction of Feynman’s theorem (where \(R_c = \sqrt{2E_F/(\hbar^2 \Omega_c^2)}\)) is the Thomas–Fermi radius, with the Fermi energy \(E_F\) and \(\Omega_0\) given in the Methods). d, Profile of the (magnitude of the) gap parameter (broken lines) and density (full lines) for different temperatures (in units of the critical temperature \(T_c\), for \(\Omega = 0\)).

Figure 2 | Comparing the number of vortices with the experimental data. a. Number of vortices \(N_v\) obtained from the solution of the LPDA equation over an extended coupling range across unitarity, for different values of the angular frequency \(\Omega\) (in units of \(\Omega_c\)) and temperature (in units of the Fermi temperature \(T_F = E_F/k_B\), \(k_B\) being the Boltzmann constant). b. Experimental values for \(N_v\) from ref. 17 multiplied by a factor of four (stars) are compared with the results of the present calculation for \(\Omega = 0.8 \Omega_c\) and two different temperatures (in units of \(T_c\)).
cloud\(^n\)). Figure 1c shows the radial profile of the (magnitude of) the gap parameter inside the central vortex for several values of the angular frequency \(\Omega\), whose shape seems to be unmodified once normalized to the ‘asymptotic’ value \(\Delta_0\) in between two adjacent vortices and the radial distance \(\rho = \sqrt{x^2 + y^2}\) is expressed in terms of the local Fermi wavenumber \(k_F(0) = [3\pi^2 n(0)]^{\frac{1}{3}}\), where \(n(0)\) is the density at the trap centre. [Specifically, \(\Delta_0 = (0.87, 0.83, 0.77, 0.68) E_r\) for \(\Omega = (0.0, 0.4, 0.6, 0.8) \Omega_c\).] This is because the healing length of the central vortex (which is about 2\(k_F(0)^{-1}\)) is smaller than the distance between two adjacent vortices (which is about 2\(k_F(0)^{-1}\)) in Fig. 1a where the vortex density is highest. The inset of Fig. 1c shows the \(\rho\)-dependence of the ratio between the number of vortices \(N_v(\rho)\) obtained numerically within a circle \((\rho = \alpha)\) with centre at \(x = y = 0\) and radius \(\rho\), and the corresponding number of vortices \(N_v(\rho) = n_\phi \pi \rho^2\) expected from Feynman’s theorem, for the frequencies \(\Omega = (0.4, 0.6, 0.8) \Omega_c\). Each plot terminates at the boundary \(R_c\) of the superfluid portion of the cloud (see Fig. 1d), which is always smaller than \(R_c\). One concludes that Feynman’s theorem is satisfied, in practice, up to about \(R_c/2\). Finally, Fig. 1d shows the profiles of the (magnitude of the) gap parameter and density at \(z = 0\) along \(x\) for \(\Omega = 0.8 \Omega_c\), and various temperatures in the superfluid phase. A non-negligible portion of

The outer normal component of the cloud past \(R_{\text{cl}}\) becomes normal as a result of rotation, with the result that a bi-modal distribution for the density emerges below a certain temperature. This feature bears analogies with what occurs for a bosonic system, for which the superfluid component emerges near the trap centre as a narrow peak out of a broad thermal distribution\(^n\). Our results suggest that the emergence of a superfluid component below \(T_c\) could be detected also for a rotating fermion system directly from the \textit{in situ} density profiles, avoiding the need to expand the cloud when looking for vortex arrays.

When approaching the BCS limit or for increasing \(T\), the outer normal component increases in size at the expense of the superfluid component in the inner part of the cloud. In particular, at \(T = 0\) the mechanism for activating the outer normal component stems from the presence of the ‘classical’ velocity field \(v_{\text{c}}(\rho) = \mathbf{\Omega} \times \mathbf{\rho}\) in the argument of the Fermi functions entering the coefficients of the LPDA equation (see Methods). This outer normal component can accordingly be referred to as an orbital breached-pair (OBP) phase, in analogy with the breached-pair phase for imbalanced spin populations\(^n\). At a given temperature, the spatial extent of the OBP phase increases upon approaching the BCS limit as the magnitude \(|\Delta(\mathbf{\rho})|\) of the gap parameter becomes progressively smaller, and such a way that the total number of vortices drops considerably when approaching this limit. (A related study of pair-breaking effects in a rotating Fermi gas along the BCS–BEC crossover was made in ref. 32, albeit in the absence of vortices.)

We have performed our calculations over an extended range of \((k_Fa_F)^{-1}\) across unitarity for several values of the angular frequency \(\Omega\), and obtained the number of vortices \(N_v\) shown in Fig. 2a at \(T = 0\) and \(T = 0.1 T_c\). We have obtained \(N_v\) from the total circulation \(\oint \mathbf{d}l \cdot \nabla \varphi = 2\pi N_v\), where the line integral is over a circle which encompasses the whole superfluid portion of the cloud at \(z = 0\). For increasing \(\Omega\), the maximum of \(N_v\) is found to shift towards the BEC side of unitarity. Past this maximum, the temperature has only a minor effect on \(N_v\). In particular, at zero temperature, we find the maximum to occur at \((k_Fa_F)^{-1} = (0.13, 0.27, 0.47)\) for \(\Omega = (0.4, 0.6, 0.8) \Omega_c\). In all cases, a rapid decrease of \(N_v\) occurs when approaching the BCS side of unitarity, in accordance with the above argument for the presence of an OBP phase in the outer portion of the cloud. Figure 2b compares our results with the experimental values of \(N_v\) from ref. 17 at the optimal stirring
frequency $\Omega = 0.8 \Omega_c$. A remarkable agreement is found, provided one multiplies all the experimental data by the same factor of four, irrespective of coupling. (We attribute the discrepancy on the number of vortices at coupling 3.8 to the experimental problems that arise deep in the BEC regime with the relaxation of the highest vibrational state of the molecular potential involved in the Fano–Feshbach resonance of $^7$Li (ref. 17.) The need for this rescaling by a factor of four can be related to our finding (see the inset of Fig. 1c) that Feynman’s theorem is satisfied only in about a quarter of the area of the cloud, leading us to speculate that only those vortices residing in this central portion of the cloud could experimentally be detected after the cloud was expanded. This problem might be overcome by taking in situ images of the two-dimensional vortex distribution, as was recently done for a Bose gas31. Note in this context also the excellent agreement with the experimental data for the position of the maximum, as it results from Fig. 2a for the angular frequency $\Omega = 0.8 \Omega_c$, of the experiment.

We now go on to consider the experiment of ref. 18, which measured the moment of inertia $\Theta$ of the atomic cloud at unitarity as a function of temperature and showed its progressive quenching as the temperature was lowered below $T_c$. The quenching of $\Theta$ (with respect to its classical value $\Theta_c$—see Methods) entails the absence of quantum vortices in the cloud, as these would act to bring the moment of inertia back to $\Theta_c$ according to Feynman’s theorem. To get an estimate for the value of the frequency $\Omega_c$, at which the first vortex enters the cloud of radial size $R_c$, we use the expression

$$h\Omega_c / E_F = 2(k_c R_c)^{-1} \ln(R_c / \xi)$$

where $\xi$ is the healing length of the vortex. This expression is obtained by setting $E - L\Omega = 0$, where $E$ is the energy of an isolated vortex in the laboratory frame and $L$ its total angular momentum33. The detrimental contribution to $\Theta$ originates from the angular kinetic energy outside the vortex core of extent $\xi$.

With the values $k_c \xi = 0.79$ (from ref. 34) and $k_c R_c = 72$ (from the present calculation) at unitarity and zero temperature, we get $\Omega_c = 0.069 \Omega_c$, where $h\Omega_c / E_F = 39.9$ is the trap radial angular frequency from ref. 18. This value of $\Omega_c$ lies within the boundaries of the full numerical calculation (see Supplementary Information), which draws the phase diagram for the temperature dependence of the lower critical frequency $\Omega_c$, about which the first vortex stably appears in the trap and of the upper critical frequency $\Omega_u$, about which the superfluid region disappears from the trap, in analogy to a type-II superconductor.

Figure 3a shows the moment of inertia $\Theta = L / \Omega$ obtained from the calculation of the total angular momentum $L$ (see Methods) as a function of $\Omega$ at $T = 0$ for three couplings across unitarity for the geometry of ref. 18. The rapid rise of $\Theta$ past the coupling-dependent threshold $\Omega_u$ (which decreases from the BEC to the BCS side of unitarity) is due to the progressive presence of vortices in the trap for increasing $\Omega$. In particular, the plot also reports the number of vortices at unitarity for a few values of $\Omega$, demonstrating that not too many vortices are needed to stabilize $\Theta$ to its classical value. The inset of Fig. 3a provides a closer view of the behaviour of $\Theta$ at unitarity in a narrow range frequency range about $\Omega_u$, where a smooth increase is found to occur before the sharp rise at $\Omega_u$, when the first vortex nucleates. Whereas this effect is essentially suppressed on the BCS side, the increase of $\Theta$ for $\Omega < \Omega_u$ becomes more evident towards the BCS side owing to the progressive presence in the outer part of the cloud of the OBP phase discussed above. A similar effect occurs also in nuclei (where it is referred to as the yrast effect) owing to the finite particle number35–37. With the total particle number $N = 6 \times 10^5$ from ref. 18, this effect is expected to be of the order of $1 / N \approx 10^{-5}$, which indeed corresponds to the values reported in the inset of Fig. 3a. In addition, the linear increase of $\Theta$ before $\Omega_u$ obtained here is in line with a general argument provided in ref. 36.

In ref. 18, the (nominal) angular frequency was estimated at unitarity to be $0.3 \Omega_u$, which is one order of magnitude larger than the threshold $\Omega_u$ for the nucleation of vortices obtained by our calculation. For $\Omega = 0.3 \Omega_u$, our calculation predicts, in fact, that about 20 vortices enter the cloud at unitarity and zero temperature, as shown in Fig. 3b. In ref. 18, on the other hand, the nucleation of vortices was not considered to occur on the basis of a mechanism of resonant quadrupole mode excitation. This absence of vortices might be connected with a transient configuration captured by the experiment, whereas a longer timescale would be required to reach the situation of full thermodynamic equilibrium which is assumed by our calculation, where vortices nucleate just past $\Omega_u$.

Accordingly, we compare the data of ref. 18 to our calculations for the moment of inertia at unitarity for $\Omega = 0.003 \Omega_u < \Omega_u$, in such a way that no vortex appears in the trap at all temperatures below $T_c$. This comparison is reported in Fig. 4, where $\Theta / \Theta_c$ is shown versus $T / T_c$. Even though our calculated value of $T_c (=0.37 T_F)$ differs from that estimated in ref. 18 ($\approx 0.21 T_F$), rescaling the temperature by the respective values of $T_c$, leads to fairly close agreement between theory and experiment. This kind of rescaling is a common practice in condensed matter and was recently considered also for ultracold Fermi gases, when comparing theoretical predictions with experimental data for the temperature dependence of the superfluid fraction38. Also reported in Fig. 4 are the partial moments of inertia contributed by the superfluid (inner) and normal (outer) regions of the atomic cloud, such that $\Theta = \Theta_{\text{inner}} + \Theta_{\text{outer}}$. In the inner region (with $|r| < R_c$), the superfluid and normal components coexist with each other at a given temperature, whereas in the outer region (with $|r| > R_c$) only the normal component survives. The corresponding temperature dependence of $R_c$ is shown in the inset of Fig. 4. We have verified that the moment of inertia in the inner region where $\Delta(r) = 0$ is due only to the normal component which is present at non-zero temperature (see equation (3) in Methods). For $\Omega \rightarrow 0$, our approach reduces to the two-fluid model of ref. 39, which was used recently in ref. 40 to compare with the experimental data of ref. 18 for $\Theta$. In ref. 40, however, a number of numerical approximations were considered to simplify the calculation, which are avoided in the present approach.

The results obtained here demonstrate how vortex arrays evolve from the BCS to the BEC regime when a Fermi superfluid is constrained in a confined geometry, for increasing temperature and angular velocity or a combination of both. Our theoretical approach has relied on a novel differential equation for the spatially varying gap parameter, which considerably reduces the storage space and computational time compared to the BdG equations, thereby advancing the current state-of-the-art standard for the self-consistent generation of complex arrays of quantum vortices under a wide range of physical conditions.

Methods

Methods and any associated references are available in the online version of the paper.

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Author contributions
S.S. carried out the calculations, G.C.S. directed the work and wrote the manuscript, and P.P. contributed to the interpretation of the results and the writing of the manuscript.

Additional information
Supplementary information is available in the online version of the paper. Reprints and permissions information is available online at www.nature.com/reprints. Correspondence and requests for materials should be addressed to G.C.S.

Competing financial interests
The authors declare no competing financial interests.
Methods

The coupling parameter for the BCS–BEC crossover. The BCS–BEC crossover is spanned by the coupling parameter \( (k_{\text{F}} a_{\text{s}})^{-1} \), where \( a_{\text{s}} \) is the scattering length of the two-fermion problem and \( k_{\text{F}} \) the Fermi wavevector related to the (trap) Fermi energy \( E_{\text{F}} \) by \( k_{\text{F}}/2m = E_{\text{F}}/2\hbar^2 \), with \( \hbar = E_{\text{F}}/\Omega_{\text{L}} \). Here, \( m \) is the fermion mass, \( N \) the total particle number, and \( \Omega_{\text{L}} \) and \( \Omega_{\text{R}} \) the radial and axial angular frequencies, respectively. (Throughout the Methods, we set \( \hbar \) equal to unity.) The coupling parameter ranges from \( (k_{\text{F}} a_{\text{s}})^{-1} \leq -1 \), characteristic of the weak-coupling (BCS) regime when \( a_{\text{s}} < 0 \), to \( (k_{\text{F}} a_{\text{s}})^{-1} \geq +1 \), characteristic of the strong-coupling (BEC) regime when \( a_{\text{s}} > 0 \), across the value \( (k_{\text{F}} a_{\text{s}})^{-1} = 0 \) at unitarity when \( a_{\text{s}} \) diverges.

In ref. 17, the ratio aspect between the radial \( (v_{r} = 57 \text{ Hz}) \) and axial \( (v_{z} = 23 \text{ Hz}) \) trap frequencies was chosen to be about 2.5, to maximize the number of vortices produced at the nominal rotation frequency \( v = 45 \text{ Hz} \approx 0.8 v_{z} \). In ref. 18, a larger aspect ratio of about 28 between the radial \( (v = 680 \text{ Hz}) \) and axial \( (v = 24 \text{ Hz}) \) trap frequencies was instead adopted, to minimize the number of vortices produced at the (nominal) rotation frequency \( v = 200 \text{ Hz} \approx 0.3 v_{z} \).

LPDA equation, density and current. The LPDA equation for the gap parameter \( \Delta(r) \) was introduced in ref. 20. For the present problem of a rotating trap with angular velocity \( \Omega \) it reads:

\[
-\frac{m}{4\pi a_{\text{s}}} \Delta(r) = \int_{V} \frac{\nabla \nu}{4m} \Delta(r) - i \int_{V} \nu_{s}(r) \cdot \nabla \Delta(r)
\]

where

\[
\int_{V} \nu_{s}(r) = \int \frac{dk}{(2\pi)^{3}} \left\{ \begin{array}{l}
\frac{ \xi(k) |k| }{ 2E(k) |k|^2 + m |k|^2 } - \frac{E_{s}}{E(k) |k|^2 + m |k|^2 } \end{array} \right.
\]

and

\[
\int_{V} \nu_{s}(r) \cdot \nabla \Delta(r) = \int \frac{dk}{(2\pi)^{3}} \left\{ \begin{array}{l}
\frac{ \xi(k) |k| }{ 2E(k) |k|^2 + m |k|^2 } \end{array} \right.
\]

In these expressions, \( \xi(k) = k^2/2m - \mu + V(r) \), where \( \mu \) is the chemical potential and \( V(r) \) the external (trapping) potential, \( E_{s}(k) = \sqrt{\xi(k)^2 + |\Delta(r)|^2} \), \( E_{s}(k) = \sqrt{\xi(k)^2 + |\Delta(r)|^2} \), and \( f_{s}(r) = (e^{\beta E_{s}(r)} + 1)^{-1} \) the Fermi function.

Correspondingly, the expression for the current density within the LPDA approach in the non-rotating frame is:

\[
j(r) = \nu_{s}(r) n(r) + 2 \int \frac{dk}{(2\pi)^{3}} \frac{k}{m} ( E(k) |k|^2 )
\]

Here, \( \nu_{s}(r) = \nabla \psi(r)/(2m) \) is the velocity field of the superfluid component and \( n(r) \) the number density within LPDA:

\[
n(r) = \left\{ \begin{array}{l}
\frac{1}{1 - 2F_s^{*}(k|r|)} \left[ 1 - \frac{1}{1 - 2F_s^{*}(k|r|)} \right] \\
\frac{1}{1 - 2F_s^{*}(k|r|)} \left[ 1 - \frac{1}{1 - 2F_s^{*}(k|r|)} \right] \\
\frac{1}{1 - 2F_s^{*}(k|r|)} \left[ 1 - \frac{1}{1 - 2F_s^{*}(k|r|)} \right]
\end{array} \right.
\]

In the above expressions, \( \xi(k) |k| = \xi(k) |k| + 1/2m v_{s}^2 - mv_{s} \cdot \nu_{s} \), \( E_{s}(k) = \sqrt{\xi(k)^2 + |\Delta(r)|^2} \), \( E_{s}(k) = \sqrt{\xi(k)^2 + |\Delta(r)|^2} \), and \( f_{s}(r) = (e^{\beta E_{s}(r)} + 1)^{-1} \) the Fermi function.

Note that the expression (1) for the current is consistent with that of a two-fluid model (see, for example, ref. 41). This can be seen by expanding the argument of the Fermi function in equation (1) for small values of \( |\nu_{s}(r) - \nu_{s}(r)| \), thereby identifying the normal density through the expression:

\[
n_{n}(r) = -2 \int \frac{dk}{(2\pi)^{3}} \frac{k^2}{3m} \frac{\partial f_{s}(E(k)|r|)}{\partial E(k)|r|}
\]

which depends on temperature through the Fermi function.

The total angular momentum of the system in the rotating trap is obtained in terms of the current density (1) via the expression:

\[
L = \int d\mathbf{r} \times \mathbf{j}(r)
\]

In the text, we have indicated \( L \) the magnitude of \( L \).

In addition, the moment of inertia \( \Theta \) of the system is obtained from \( L = \Theta \Omega \) and calculated from the expression (4) for any value of \( \Omega \) (until the trapping potential can no longer sustain the atomic cloud when set into rotation). In this respect, the present approach is not limited to small values of \( \Omega \) where the formalism of linear response can be employed. Under these circumstances, the moment of inertia \( \Theta \) can be compared with its ‘classical’ counterpart \( \Theta_{d} \) defined as follows:

\[
\Theta_{d} = m \int d\mathbf{r} (\mathbf{\Omega} \times \mathbf{r})^2
\]

where \( n(r) \) is the total number density given by equation (2). In particular, in the limit of vanishing angular velocity, \( \Theta_{d} \) coincides with the rigid-body value \( \Theta_{0} \) as calculated from the density distribution, under the assumption that the whole cloud (including the superfluid part) performs an extremely slow rigid rotation whereby \( \nu_{s} = \nu_{n} \rightarrow 0 \) in equation (2).

References

41. Pethick, C. J. & Smith, H. Bose–Einstein Condensation in Dilute Gases Ch. 10 (Cambridge Univ. Press, 2008).

42. Pitaevskii, L. & Stringari, S. Bose–Einstein Condensation Ch. 14 (Clarendon Press, 2003).