Superluminal pulse transmission through a phase-conjugating mirror

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We theoretically analyze wave packet transmission through a phase-conjugating mirror and show that the transmission of a suitably chosen input pulse is superluminal, i.e. the peak of the pulse emerges from the mirror before the time it takes to travel the same distance in vacuum. This pulse reshaping effect can be attributed directly to the dispersion relation in the nonlinear medium constituting the mirror. Thus, for the first time a connection is laid between optical phase conjugation and superluminal behavior. In view of its additional amplifying ability, a phase-conjugating mirror is a most promising candidate for an experimental observation of tachyonic signatures.

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It is well known that transmission of a wave packet through an absorbing medium gives rise to pulse reshaping by dispersion [1]. The pulse seems to have been transmitted with a speed larger than the speed of light, or "superluminally". Similar behavior has recently been observed in tunneling experiments using nondissipative dielectric mirrors [2] and has been explained in terms of destructive interference between causally propagating consecutive components of the pulse [3]. Superluminal particles, or "tachyons", were first fully studied in the sixties [4]. Theoretically, several models were developed which give rise to tachyonic collective modes in, for example, systems of inverted pendula [5] or inverted two-level atoms [6]. However, experiments carried out to directly observe tachyon-like excitations were so far without success [7]. Here, we consider a phase-conjugating mirror (PCM) consisting of a pumped nonlinear optical material [8]. The dispersion relation in this material is shown to be tachyonic, giving rise to group velocities larger than the speed of light. We analyze the transmission of a gaussian wave packet incident upon the PCM and find that its peak can be transmitted superluminally. This effect is a direct consequence of the dispersion relation and does not violate causality, since the peak of the transmitted pulse is not causally related to the peak of the input, but originates from the forward tail of this incoming pulse. In order to transmit information one could eg. use a discontinuous incident signal. This kind of signal also exhibits the superluminal peak-advancement, but the discontinuity (information) is transmitted with the speed of light, in full agreement with causality. A measurement of pulse transmission through a PCM thus provides an experimental signature of optical tachyonic excitations.

Our PCM consists of an optical medium with a large third-order susceptibility $\chi^{(3)}$. The medium is pumped by two intense counterpropagating laser beams of frequency $\omega_0$. When a weak probe beam of frequency $\omega_0 + \delta$ is incident on the material, a fourth beam will be generated due to the nonlinear polarization of the medium. This so-called conjugate wave propagates with frequency $\omega_0 - \delta$ in the opposite direction as the probe beam [9].

In order to derive the dispersion relation for an electromagnetic excitation in this pumped medium, we consider the one-dimensional wave equation for a nonmagnetic, nondispersive material in the presence of a nonlinear polarization

$$
\left( \frac{\partial^2}{\partial x^2} - \epsilon_r \epsilon_0 \mu_0 \frac{\partial^2}{\partial t^2} \right) E(x, t) = \frac{1}{\epsilon_0 c^2} \frac{\partial^2}{\partial t^2} P_{NL}(x, t)
$$

(1)

with $P_{NL}(x, t) = \chi^{(3)} \mathcal{E}^3(x, t)$. The total field is taken as the sum of four monochromatic plane waves

$$
E(x, t) = \sum_{\alpha=1,2,p,c} E_\alpha(x, t) = \sum_{\alpha=1,2,p,c} \mathcal{E}_\alpha(x) e^{-i\omega_\alpha t} + c.c.,
$$

(2)

where we have labeled the two pump beams as 1 and 2 and the probe and conjugate as p and c. $\mathcal{E}_\alpha$ denotes the complex amplitude of the $\alpha$th field which propagates with frequency $\omega_\alpha$. We substitute (2) in (1) and select the phase-conjugation terms in the polarization. Assuming the pump beams to be non-depleted and applying the appropriate phase-matching conditions [8], we arrive at two coupled equations for the amplitudes of the probe and conjugate waves which are, using $\delta \ll \omega_0$ and taking $\epsilon_r = 1$ [3]

$$
\left( -\frac{\epsilon_r^2}{2\omega_0} \frac{\partial^2}{\partial x^2} - \frac{\omega_0}{2} - \kappa c \right) \begin{pmatrix} \mathcal{E}_p(x) \\ \mathcal{E}_c^*(x) \end{pmatrix} = \delta \begin{pmatrix} \mathcal{E}_p(x) \\ \mathcal{E}_c^*(x) \end{pmatrix}
$$

(3)

Here $\kappa \equiv \kappa_0 e^{i\phi} = \frac{\omega_0}{4\epsilon_0 c^2} \chi^{(3)} \mathcal{E}_1 \mathcal{E}_2$ is the pumping induced coupling strength (per unit length) between the probe and conjugate wave. Because the above matrix is anti-hermitian, the system is said to be dissipatively coupled [10]. Trying a harmonic wave solution in (3) yields the dispersion relation

$$
k^2 = \frac{\omega_0^2}{c^2} \pm \frac{2\omega_0}{c^2} \sqrt{\delta^2 + (\kappa_0 c)^2},
$$

(4)
which is plotted in Fig. 2(c). In vacuum \( \kappa_0 = 0 \), and (3) reduces to the four solutions \( k = \pm (\omega_0 \pm \delta)/c \). Note that the group velocity in the nonlinear medium is always larger than \( c \), the speed of light in vacuum.

In view of this dispersion relation, the question arises how wave packets will be transmitted through a PCM. Because of the superluminal group velocity, tachyonic effects are expected. We begin by considering the transmission amplitude for a monochromatic probe beam incident on a PCM, see Fig. 1.

![FIG. 1. Reflection and transmission in one dimension of a probe beam (solid line) incident from vacuum on a phase-conjugating mirror. Dotted (dashed) arrows denote probe (conjugate) reflected and transmitted beams.](image)

Matching the plane wave solutions of (4) in the PCM at \( x = 0 \) and \( x = L \) to the probe and conjugate waves outside the cell yields for the probe transmission amplitude at \( x = L \) the well-known result (choosing \( \phi = 0 \))

\[
t_{pp}(\delta) = \frac{\beta}{\beta \cos(\beta L) - i \delta \sin(\beta L)}
\]

where

\[
\beta = \frac{1}{c} \sqrt{\delta^2 + (\kappa_0 c)^2}.
\]

Now consider a probe pulse \( \mathcal{E}_p(0, t) = \int_{-\infty}^{\infty} d\delta \, \mathcal{E}_p(0, \delta) \, e^{-i\delta t} \) incident at \( x = 0 \). Using an analogous two-sided Laplace Transform technique as Fisher et al. [11] employed for phase-conjugate reflected pulses, we obtain for the probe pulse at \( x = L \)

\[
\mathcal{E}_p(L, t) = \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} ds \, t_{pp}(is) \tilde{\mathcal{E}}_p(0, s) e^{st}
\]

with

\[
\tilde{\mathcal{E}}_p(0, s) = \int_{-\infty}^{\infty} dt' \mathcal{E}_p(0, t') e^{-st'}.
\]

\( \mathcal{E}_p(L, t) \) is thus expressed as an integral in the complex \( s \)-plane of the input pulse times the transmission amplitude \( t_{pp} \). The integration contour in (7) defined by \( \gamma \) must be chosen to the right of all singularities of \( t_{pp} \).

A direct calculation of these singularities leads to two possible operating regimes of the PCM, in which the pulse-reshaping upon transmission is entirely different. For \( \kappa_0 L < \pi/2 \), all singularities of \( t_{pp} \) lie in the left half \( s \)-plane [11] and \( \mathcal{E}_p(L, t) \) is always finite. On the other hand, if \( \kappa_0 L > \pi/2 \) there will be at least one singularity in the right half \( s \)-plane. We then find exponential growth of the transmitted probe field and hence the mirror is said to be in an unstable (or "active") operating regime [8]. Here we restrict ourselves to the case of stable operation. The effects of the instability will be discussed elsewhere [13]. In view of the above, for \( \kappa_0 L < \pi/2 \), we may take \( \gamma = 0 \) in (7).

The substitution \( s \to -i\delta \) transforms (7) into a Fourier integral, which can then be easily evaluated numerically [11][13].

Fig. 2 shows the result for a gaussian input pulse \( \mathcal{E}_p(0, t) = e^{-\alpha t^2} e^{i\beta t} \) which is centered around frequency \( \delta_0 \) in the spectral domain. In (a) \( |\mathcal{E}_p(L, t)| \) is plotted as a function of \( t \) (in units of \( L/c \)) for various values of \( \delta_0 \), corresponding to different positions on the dispersion curve (Fig. 2(c)).

The vertical line indicates the time \( t_{tr} = L/c \) needed to traverse the mirror in vacuum. We see that an incident pulse which is centered in the middle of the gap in the dispersion relation is strongly reshaped upon transmission and that its peak appears delayed with respect to \( t_{tr} \) (curve 1). But if the incoming pulse is centered around a frequency further away from \( \delta = 0 \), its peak emerges from the PCM before \( t_{tr} \) (curve 2, enlargement in Fig. 2(b)). The overall reshaping has become less, but the peak traversal time is superluminal. Moving still further up the dispersion curve yields a transmitted pulse with shape almost identical to the incident one and a peak traversal time approaching \( t_{tr} \) from below (curve 3). This is not surprising, since the frequency components of the incident pulse now lie in that part of Fig. 2(c) where the dispersion relation becomes asymptotically linear. The transmitted intensity \( T_{pp} \) then approaches unity,

\[
T_{pp}(\delta) \equiv |t_{pp}(\delta)|^2 = \frac{1 + \left( \frac{\gamma}{\kappa_0 c} \right)^2}{\cos^2(\beta L) + \left( \frac{\delta}{\kappa_0 c} \right)^2} \approx 1 \quad \text{for} \quad \delta \gg \kappa_0 c,
\]

and the pulse propagates almost as in vacuum.

Closer to \( \delta = 0 \), the amplification of the transmitted pulse is larger and the superluminal peak advancement becomes clearly visible, \( t_{peak} \simeq 0.75 t_{tr} \). This advancement is a consequence of the increasingly superluminal group velocity as \( \delta \to 0 \). For a pulse centered around \( \delta_0 \) very close to 0, however, the superluminal effect disappears. Such a pulse contains positive as well as negative frequency components, and the presence of these components with their largely compensating positive and negative group velocities leads to strong reshaping and a delayed peak. The advancement is thus maximized for pulses centered around a frequency \( \delta_0 \) which is small, but still sufficiently far away from 0 to avoid any influence from the lower half of the dispersion curve. On the one hand, the pulse should thus be spectrally narrow, in
order to be as close as possible to the gap region. But on the other hand it should not be too narrow, since on a temporarily very broad pulse the advancement, even if substantial, would not be easily detectable. More quantitatively, we have optimized the ratio peak advancement:

\[
r \equiv r(\Delta t, \delta_0, \kappa_0) = \frac{t_{tr} - t_{pp}}{\Delta t},
\]

where \( \Delta t \) is the temporal width of the incoming pulse. For the gaussian considered in Fig. 2 one finds, by varying simultaneously \( \Delta t \) and \( \delta_0 \) and keeping \( \kappa_0 \) fixed, that the optimal value of \( r \approx 0.073 \). For fixed \( \delta_0 \) (and \( \kappa_0 \) \( r \) decreases fast with increasing temporal width of the incoming pulse.  

We now demonstrate that the superluminal peak advancement does not disagree with the principle of causality, in the sense that the peak of the pulse does not transmit any real information.

![Graph showing transmitted probe pulse intensity vs. time](image)

**FIG. 2.** (a) Transmitted probe pulse \( |E_p(L, t)| \) at \( x = L \) for an incoming gaussian \( E_p(0, t) = e^{-\alpha t^2}e^{i\delta_0 t} \) at \( x = 0 \) (thick solid line). The temporal width (FWHM) of the incoming pulse \( \Delta t \equiv \frac{\ln 2}{\alpha} = 3.3 L/c \), the spectral width \( \Delta \delta = 1.7 c/L \) and \( \kappa_0 L = 1.4 \). The vertical line indicates the time \( t_{tr} = L/c \) needed to traverse the cell in vacuum. (b) Enlargement of \( \delta_0 = 2.3 c/L \), clearly showing advancement of the peak upon transmission. (c) Dispersion relation (solid line) in a \( \chi^{(3)} \)-material. The dashed (dotted) lines correspond to the dispersion relation for the probe (conjugate) waves in vacuum. \( k_0 \equiv \omega_0/c \). The marked positions 1-3 indicate the frequency components in the incident pulses which give rise to the transmitted curves 1-3 in (a).

To this end, it is convenient to rewrite (3) as

\[
E_p(L, t) = \int dt' E_p(0, t')H(t, t'),
\]

with

\[
H(t, t') = \frac{1}{2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} ds t_{pp}(is)e^{s(t-t')}
\]

and \( t_{pp} \) given by (3). The integral (11) can be evaluated analytically, following an analogous method as used in [11]. The result is

\[
E_p(L, t) = \sum_{n=0}^{\infty} \int_{-\infty}^{t-(n+\frac{1}{2})\tau} dt' E_p(0, t') L_n(t, t'),
\]

with

\[
L_n(t, t') = \frac{\kappa_0 c}{2} A_n^{\frac{1}{2}} \left( A_n^{n-1} I_{2n-1} \left[ \kappa_0 c \sqrt{(t-t')^2 - (n+\frac{1}{2})^2 \tau^2} \right] - 2 A_n^{n+1} I_{2n+1} \left[ \kappa_0 c \sqrt{(t-t')^2 - (n+\frac{3}{2})^2 \tau^2} \right] + A_n^{n+2} I_{2n+3} \left[ \kappa_0 c \sqrt{(t-t')^2 - (n+\frac{5}{2})^2 \tau^2} \right] \right) A_n = \frac{t-t'-(n+\frac{1}{2})\tau}{t-t'-(n+\frac{3}{2})\tau} \]

\[
\tau \equiv 2L/c = 2t_{tr}, \quad \text{the PCM roundtrip time.}
\]

The expression (12) for \( E_p(L, t) \) in terms of modified Bessel functions \( I_n \) is equivalent to the Fourier integral and serves as a double check for the results in Fig. 2.  

(13) also gives a good illustration of what happens for an incoming non-analytic signal, which is suddenly switched on at \( t = 0 \), so \( E_p(0, t) = E_p(0, t) \Theta(t) \) with \( \Theta \) the Heaviside step function. The transmitted pulse is then given by (14)

\[
E_p(L, t) = \sum_{n=0}^{\infty} \Theta(t - (n+\frac{1}{2})\tau) \int_0^{t-(n+\frac{1}{2})\tau} dt' E_p(0, t') L_n(t, t').
\]

There is no signal emerging at \( x = L \) before time \( t = \frac{1}{2}r = t_{tr} \), so the chopped edge of the pulse travels with the speed of light. The “information” contained
in this abrupt disturbance is thus transmitted causally. Subsequent contributions to the sum in (14) appear after each following roundtrip time.

Similar causal transmission is also seen in the response at $x = L$ for an incident gaussian pulse which is suddenly switched off at its maximum, see Fig. 3.

![FIG. 3. The transmitted probe pulse through a PCM at $x = L$ for a chopped gaussian pulse incident at $x = 0$ (thick solid line). $\kappa_0 L = 1.4$.](image)

For all values of $\delta_0$ the step is again transmitted causally, traveling with the speed of light. One clearly sees that for pulses centered around intermediate frequencies in the dispersion diagram ($\delta_0 \approx 2.4 c/L$) the previously observed advancement of the pulse maximum remains, because it is formed before an observer at $x = L$ learns about the chopped edge of the pulse. After transmission of the step, the pulse decays, since there is no further input signal to the mirror reinforcing and triggering new multiple reflections.

Finally, consider a realistic phase-conjugating mirror, consisting of a cell of length $L \sim 10^{-2}$ m. Reflectivities on the order of 100%, so $\tan^2(\kappa_0 L) \sim 1$, have been reported for PCM’s [2] and hence coupling strengths $\kappa_0 c \sim c/L \sim 10^{10} s^{-1}$ can be reached. As seen from the above, a pulse of temporal width $\sim 0.1$ ns incident on this PCM gives rise to peak advance and delay times $\sim 0.03$ ns, well within range of observation.

Summarizing, we have studied the transmission of wave packets through a phase-conjugating mirror. Based on the tachyonic dispersion relation in the nonlinear PCM medium, superluminal peak traversal times are predicted. Thus, a so far unnoticed link is established between optical phase conjugation and superluminal behavior, which, especially in view of the amplifying properties of a PCM, provides an excellent framework for an experimental observation of a signature of optical tachyons.

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