Hole-Hole Interaction Effect in the Conductance of the Two-Dimensional Hole Gas in the Ballistic Regime

Y.Y. Proskuryakov\textsuperscript{1}, A.K. Savchenko\textsuperscript{1}, S.S. Safonov\textsuperscript{1}, M. Pepper\textsuperscript{2}, M.Y. Simmons\textsuperscript{2}, D.A. Ritchie\textsuperscript{2}

\textsuperscript{1} School of Physics, University of Exeter, Stocker Road, Exeter, EX4 4QL, U.K.
\textsuperscript{2} Cavendish laboratory, University of Cambridge, Madingley Road, Cambridge CB3 0HE, U.K.

On a high mobility two-dimensional hole gas (2DHG) in a GaAs/GaAlAs heterostructure we study the interaction correction to the Drude conductivity in the ballistic regime, \( k_B T \tau/h > 1 \). It is shown that the ‘metallic’ behaviour of the resistivity \((d\rho/dT > 0)\) of the low-density 2DHG is caused by hole-hole interaction effect in this regime. We find that the temperature dependence of the conductivity and the parallel-field magnetoresistance are in agreement with this description, and determine the Fermi-liquid interaction constant \( F_0^\sigma \) which controls the sign of \( d\rho/dT \).

It is well known that electron-electron interaction gives rise to a quantum correction to the classical (Drude) conductivity caused by impurity scattering \([4]\). Its manifestation can be quite different in the two regimes which relate the quasi-particle interaction time, \( h/k_B T \), to momentum relaxation time, \( \tau \): diffusive \((k_B T \tau/h < 1)\), and ballistic \((k_B T \tau/h > 1)\). So far the interaction correction to the conductivity of two-dimensional \(2D\) systems has been studied, both theoretically and experimentally, only in the diffusive regime, which is applicable to low-mobility \(\tau\) \(2D\) systems \([4]\). Experimentally, the interaction correction was seen to be negative and produce a logarithmic decrease of the resistivity with increasing temperature, similar to the interference correction due to weak localisation \(\text{(WL)}\). Theory, however, suggests that the sign of the interaction effect in the diffusive regime can be different, dependent on the value of the interaction constant \( F_0^\sigma \). Thus the correction can become positive and give rise to a ‘metallic’ temperature dependence with \( d\rho/dT > 0 \).

The role of interactions in the conductance of \(2D\) systems has now been intensely discussed, after the observation of the ‘metallic’ behaviour in some low-density, high-mobility \(2D\) systems \([4]\). If one attempts to apply the conventional interaction theory, it can be seen that in high mobility structures the diffusion approximation becomes invalid even at low \( T \). Recently, a theory of the interaction correction in the \textit{ballistic} and intermediate regimes has been developed \([4]\). Stimulated by this theory, in this work we examine the role of the hole-hole interaction effects in a \(2D\) hole gas \(2D\text{HG}\) which shows a ‘metallic’ \( \rho(T) \). We analyse the temperature dependence of the conductivity and positive magnetoresistance in parallel field, and show that these two main features of the ‘metallic’ state can be explained by the interaction effect. It has the same origin as the logarithmic correction studied earlier \([4]\), but now manifests itself in the ballistic regime.

The interaction theory \([4]\) considers elastic (coherent) electron scattering on the modulated density of other electrons \(\text{(Friedel oscillation)}\) caused by an impurity with a short-range potential. The phase of the Friedel oscillation, \(\Delta\rho \propto \exp(i2k_v r)\), is such that the wave scattered from the impurity interferes constructively with the wave scattered from the oscillation, Fig. 1 (a), leading to the quantum correction to the Drude conductivity \(\sigma_0\). The model \([4]\) gives several predictions to be tested experimentally. Firstly, the logarithmic correction in the diffusive regime of multiple impurity scattering, becomes a linear temperature dependence in the case of a single scatterer, at \( k_B T \tau/h > 1 \):

\[
\delta\sigma(T) = \frac{e^2}{\pi \hbar} \frac{k_B T \tau}{h} \left( 1 + \frac{3 F_0^\sigma}{1 + F_0^\sigma} \right)
\]

\[
= \sigma_0 \left( 1 + \frac{3 F_0^\sigma}{1 + F_0^\sigma} \right) \frac{k_B T}{E_F}
\]

where \( F_0^\sigma \) is the Fermi liquid interaction parameter in the triplet channel. The coefficient in the temperature dependence originates from two contributions: one due to exchange processes \(\text{(Fock)}\) and another due to direct interaction \(\text{(Hartree)}\). Similar to the diffusion regime, the sign of \( d\rho/dT \) depends on the constant \( F_0^\sigma \). It is important that for a given \( F_0^\sigma \), dependence \(\rho(T)\) can be ‘metallic’ even when the logarithmic correction at smaller \( k_B T \tau/h \) is ‘insulating’. It is also interesting to note that according to \([4]\) the actual transition to the ballistic regime occurs at \( 0.1 k_B T \tau/h \), so that experiments on high-mobility structures can be easily driven into the ballistic regime.

Secondly, for a wide range of parameter \( F_0^\sigma \) the model allows the change of the sign of \( d\rho/dT \) with parallel magnetic field - the effect seen in recent experiments. Magnetic field suppresses the correction in the triplet channel in Eq. \([4]\), resulting in a universal, positive correction to the Drude conductivity in magnetic field, \(\sigma_0^B\), and hence the ‘insulating’ behaviour of \(\rho(T)\):

\[
\delta\sigma = \sigma_0^B \frac{T}{E_F} \text{ at } B \geq B_S.
\]
Here $B_S$ is the field corresponding to the full spin polarization of the 2D system, $B_S = 2E_F/g^*\mu_B$, where $g^*$ is the Lande g-factor, $\mu_B$ is the Bohr magneton, and $T_F$ is the Fermi temperature. Note that the same functional dependence as in Eq. \((4)\) was derived in \([6]\), where the authors consider the same physical phenomenon in terms of the temperature effect on screening of the impurity potential. The model \([5]\) has been applied to the analysis of the linear $\rho(T)$ in several experiments on Si MOSFETs and GaAs structures, although no quantitative agreement with the model was achieved. It is important to mention that model \([5]\) considers only the Hartree potential of interacting electrons and ignores the Fock contribution. As a result, it only predicts the positive sign of $d\rho/dT$ and does not allow the change in $d\rho/dT$ with magnetic field.

The Fermi-liquid constant $F_0^\sigma$ has a significant physical meaning. It can be considered as the ratio of exchange and kinetic energies, and it also comes into the magnetic susceptibility \([3]\):

$$
\chi(n) = \frac{\chi_0}{1 + F_0^\sigma}.
$$

Recently, there have been reports that in the 'metallic' state of the 2DEG in Si-MOSFETs the $g$-factor diverges when approaching the 'metal'-'insulator' transition \([2]\), indicating the ferromagnetic (Stoner) instability expected in low-density 2D systems \([3]\). In our 2DHG the parameter $r_s$ is twice as large as in \([3]\); $r_s \sim 20$ near the crossover, so that we can also expect a manifestation of the Stoner instability. Therefore, our analysis of the conductance in terms of Eq. \((1)\) can give the value of the interaction parameter $F_0^\sigma$ and show how close to the Stoner instability the system is - from Eq. \((1)\) the instability is expected to occur at $F_0^\sigma = -1$.

The experiments have been performed on a (311)A GaAs/AlGaAs heterostructure with a peak mobility of $6.5 \times 10^5$ cm$^2$/Vs, which shows the crossover from 'metal' to 'insulator' at $p \sim 1.5 \times 10^{10}$cm$^{-2}$, Fig. 1 (b). A standard four-terminal lock-in technique has been used for resistivity measurements at temperatures down to 50 mK, with currents of 1-10 nA to avoid electron heating. The hole density $p$ in the 'metallic' region is varied by the gate voltage in the range $(2.09 - 9.4) \times 10^{10}$cm$^{-2}$, corresponding to the interaction parameter $r_s = 10 - 17$ (with the effective mass $m^*$ taken as 0.38m$_e$ \([3]\)). In \([3]\) the 'metallic' character of a higher density 2DHG in GaAs was explained in terms of inelastic scattering between two hole subbands, split due to spin-orbit interaction. In our low-density structures the effect of the band splitting is negligible.

Fig. 1 (b) represents the temperature dependence of the resistivity, with the 'metallic' region under study marked by a box. The increase of the resistivity with $T$ can be simply due to phonon scattering, which cannot be ignored in GaAs structures with piezo-electric coupling even at temperatures below 1K. In Fig. 1 (c,d), curves $\rho(T)$ for different densities are plotted together with the theoretical dependence presented as $\rho(T) = \rho_0 + \rho_{ph}$, where $\rho_0 = \sigma_0^{-1} = \rho(T = 0)$ is the residual resistivity due to impurity scattering, obtained by extrapolation to $T = 0$, and $\rho_{ph}$ is the result of the calculations for the phonon scattering in GaAs heterostructures \([2]\). The latter is represented as $\rho_{ph}(T) = \frac{a(T/T_0)^3}{1 + c(T/T_0)^2}$, where parameters $a$ and $c$ depend on the carrier density, effective mass and crystal properties, and $T_0 = k_B^{-1}\sqrt{2m^*S_0^2E_F}$, where $S_0$ is the transverse sound velocity. This relation corresponds to the intermediate temperature range between the Bloch-Gruneisen, $T < T_0$, and the linear, $T > T_0$, regimes, for the case of non-screened phonon scattering. (The criterion $T < T_0/\pi$ \([4]\) for the screened phonon scattering is not satisfied for the majority of our data.) One can see that at the highest $p$, phonon scattering can fully explain the experimental dependence $\rho(T)$. However, with decreasing density another contribution develops, which dominates at low $T$ and low densities. Fig. 2 (c) shows this contribution obtained by subtracting that of phonon scattering. The peak-like shape of $\rho(T)$, with the maximum at $T_{\text{max}} \approx 0.3T_F$, is in qualitative agreement with numerical calculations in \([4]\) where it is explained by the transition to the non-degenerate regime at $T > 0.3T_F$. (Similar $\rho(T)$ dependence with a peak has also been seen in \([3]\)).

In order to compare the results in the low-temperature range of $\rho(T)$ with Eq. \((4)\), we replot in Fig. 2 (b) the data in conductivity form: $\Delta\sigma(T) = (\rho(T) - \rho_{ph}(T))^{-1} - \rho_0^{-1}$. The condition for the ballistic regime $k_B T/h \geq 1$ is satisfied in our structure at $T > 50 - 100$ mK, and a linear fit of $\Delta\sigma(T)$ gives the value of parameter $F_0^\sigma$, which is presented in Fig. 2c for different $p$. The following comments can be made on this result. Firstly, the interaction constant is negative and this provides the 'metallic' slope in $\rho(T)$. Secondly, its absolute value decreases with increasing density, which is in agreement with the expectation that the ratio of the exchange to kinetic energy of quasi-particles decreases to zero at large densities. Thirdly, one can see that the measured value does not exceed 0.42, and when extrapolated to the density of the crossover from 'metal' to 'insulator' ($p \sim 1.5 \times 10^{10}$cm$^{-2}$), is much smaller than the value of $|F_0^\sigma| = 1$ expected for the Stoner instability. This implies that our description of the 'metallic' system as a weakly interacting Fermi-liquid is self-consistent.

Let us now turn to the increase of resistance with parallel field shown in Fig. 3 (a), which is similar to that observed earlier on the 2DHG \([3]\). There is a characteristic feature in the data - a bend, which shifts towards smaller fields as the density is decreased. It was recently shown that this hump corresponds to the magnetic field $B_S$ of full spin polarisation of the 2DHG \([4]\). However, there has been no quantitative analysis of the magnetoresistance in the 2DHG, which we now attempt. We start with the analysis which is similar to that performed on 2D electrons in a Si MOSFET \([3]\) and GaAs heterostructure \([2]\). It is based on the model \([4]\) of positive mag-
netoresistance at $T = 0$, which considers the effect of parallel field on the screening of the impurity potential, affected by the presence of two spin subbands with different Fermi momenta $k_F$. The fact that this model does not consider the Fock component in the interacting potential should not now affect the magnetoconductance result, as the magnetic field only acts on the Hartree term. Fig. 3 (b) shows $\rho(B_{||})/\rho(B_0 = 0)$ as a function of dimensionless magnetic field $\bar{B}/B_S$, with $B_S$ found as a fitting parameter. Its value is shown by the dashed line in Fig. 3 (a) and corresponds to the position of the hump. In accordance with [13], all the data in the density range $p = 1.43 \times 10^{10}$ cm$^{-2}$ collapse on one curve. There is satisfactory agreement with the experiment, apart from a factor, using the relation

$$\sigma = \frac{\rho}{\rho(B_0)} = \frac{\rho(B_{||})}{\rho(B_0)} = \frac{\rho(B_{||})}{\rho(B_0)}$$

In Fig. 3 (b), where a similar analysis on a 2DEG in Si structures showed a rapid increase of the $g$-factor with decreasing density near the 'metallic'-to-'insulator' transition, the $g$-factor in our case decreases with decreasing density. Similar behaviour was recently observed for 2D electrons in GaAs [21].

Using the value of $B_S$ one can obtain the effective $g$-factor, using the relation $g^* = 2E_F/\mu_B B_S$, whose dependence on the density is shown in the inset to Fig 3 (b). Note that contrary to [7], where a similar analysis resulted with the prediction given by Eq. (2) we analyse the dependence on the density is shown in the inset to Fig 3 (b). We are grateful to I.L. Aleiner, B.L. Altshuler and B.N. Narozhny for stimulating discussions, and EPSRC and ORS award funds for financial support.

$$\rho(B_{||}) = g^* \mu_B B_{||}$$

in parallel field of a low-density (large $r_s$) 2D hole gas in the 'metallic' phase, near the crossover in the sign of $\rho(T)$. We have demonstrated that the 'metallic' character of $\rho(T)$ and the positive magnetoconductance are caused by the hole-hole interaction in the ballistic limit $k_B T \tau/h > 1$. We have found the Fermi liquid constant $F_0^*$, which determines the sign of $\rho(T)$. Its value near the crossover appears to be significantly smaller than expected for the ferromagnetic instability.

$$\rho(B_{||}) = \frac{\rho(B_{||})}{\rho(B_0)} = \frac{\rho(B_{||})}{\rho(B_0)}$$

We are grateful to I.L. Aleiner, B.L. Altshuler and B.N. Narozhny for stimulating discussions, and EPSRC and ORS award funds for financial support.

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Figure captions:

Fig. 1. (a) Diagram of electron scattering by an impurity and the Friedel oscillation it produced. 
(b) Temperature dependence of the resistivity at different hole densities near the crossover in the sign of $d\rho/dT$. 
(c,d) Resistivity on the ‘metallic’ side of the crossover (symbols), and contribution to $\rho(T)$ due to phonon scattering (solid lines).

Fig. 2 (a) Impurity scattering contribution $\Delta\rho$ against dimensionless temperature at different $p$. (For clarity, curves in (a) and (b) are offset vertically from zero value at $T = 0$.) (b) The same data as in (a) but in conductivity form, with linear fitting. (c) Fermi liquid parameter versus hole density. Open symbols show the result obtained from the analysis of $\rho(T)$ at zero magnetic field; closed symbols show the result from the analysis of the parallel-field magnetoresistance, Fig. 4 (b).

Fig. 3 (a) Dependence of the resistivity on parallel magnetic field at $T = 50$ mK and $p = 1.43, 1.57; 1.75; 2.03; 2.26; 2.49; 2.83; 3.36 \times 10^{10}$ cm$^{-2}$, from bottom to top. 
(b) Scaled data, with an added curve $\rho(B_{||})$ for $p = 8.34 \times 10^{10}$ cm$^{-2}$; solid line is the result of the model \[19\]. Inset: dependence of the effective $g$-factor on the hole density, obtained from the value of $B_S$.

Fig. 4. (a) Temperature dependence of the conductivity at $B_{||} = B_S$ for different hole densities. Coefficient $\alpha = 0.53$ is obtained for $p = 2.49, 2.83 \times 10^{10}$ cm$^{-2}$; $\alpha = 0.62$ for $p = 2.26 \times 10^{10}$ cm$^{-2}$; $\alpha = 0.74$ for $p = 2.03 \times 10^{10}$ cm$^{-2}$; and $\alpha = 0.92$ for $p = 1.43, 1.57, 1.75 \times 10^{10}$ cm$^{-2}$. Inset: $\rho(B_{||})$ for $p = 2.26 \times 10^{10}$ cm$^{-2}$, at different temperatures: $T = 0.1, 0.2, 0.3, 0.45, 0.6, 0.8$ K. 
(b) Magnetococonductivity against $B_{||}^2$, at $T = 0.6$ K for densities $p = 2.03, 2.26, 2.49, 2.83, 3.36 \times 10^{10}$ cm$^{-2}$. 

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\[ \Delta p(r) \propto \frac{1}{r^2} \exp(i2k_{\text{F}}r) \]
\[ \Delta \rho (k\Omega m) \]

\[ T / T_F \]

(a) 

\[ \Delta \sigma_{\text{int}} (\text{mS}) = \frac{1}{(\rho - \rho_{\text{ph}})} - \frac{1}{\rho_0} \]

(b) 

\[ F_0^\sigma (p) \]

\[ p = 4.1 \times 10^{10} \text{ cm}^{-2} \]

\[ p = 4.7 \times 10^{10} \text{ cm}^{-2} \]
(a) \( \alpha = 0.53 \)

(b) \( p = 2.03 \times 10^{10} \text{ cm}^{-2} \)

\[ \sigma(T)/\sigma(T=0) \]

\[ [\sigma(B) - \sigma(0)] \text{ (mS)} \]

\[ B_{\parallel}^2 (T^2) \]