Axions without Peccei-Quinn Symmetry

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We argue that the axion arising in the solution of the strong CP problem can be identified with the Majoron, the (pseudo-)Goldstone boson of spontaneously broken lepton number symmetry. At low energies, the associated $U(1)_L$ becomes, via electroweak parity violation and neutrino mediation, indistinguishable from an axial Peccei-Quinn symmetry in relation to the strong interactions. The axionic couplings are then fully computable in terms of known SM parameters and the (as yet unknown) Majorana mass scales, as we illustrate by computing the effective couplings to photons and quarks at two loops. Together with previous results our proposal provides further evidence that the known particle physics phenomena can all be explained without introducing intermediate scales of any kind between the electroweak scale and the Planck scale.
1 Introduction

The solution of the strong CP problem by means of the Peccei-Quinn mechanism [1] is commonly assumed to require the presence of a chiral $U(1)_{PQ}$ symmetry (Peccei-Quinn symmetry) which is not part of the standard model (SM), as well as an independent new scale $\gtrsim O(10^{10})$ GeV beyond the SM. When spontaneously broken, the PQ symmetry gives rise to a (pseudo-)Goldstone boson, the axion [2, 3]. The latter is usually described by a pseudoscalar field transforming by constant shifts under $U(1)_{PQ}$. The absence of CP violation in the strong interactions is then explained by the fact that any contribution to the $\theta$ parameter can be absorbed into such a shift, so the problem is solved if the axion vacuum expectation value dynamically adjusts itself to zero [4]. To accommodate the extra $U(1)_{PQ}$ the available models realizing this idea invariably need to introduce (so far unobserved) new particles and large scales beyond the SM, such as new heavy quarks or non-standard Higgs fields [5, 6].

In [7] a minimal extension of the SM was proposed, based on the hypothesis that quantum mechanically broken conformal symmetry stabilizes the electroweak hierarchy, with only the right-chiral neutrinos $\nu^c_R$ and one complex scalar field

$$\phi(x) = \varphi(x) \exp \left( \frac{ia(x)}{\sqrt{2}\mu} \right)$$

(1)

as new ingredients (for alternative models based on conformal symmetry, see [8]; a similarly minimalistic scenario without exact conformal symmetry had already been developed in great detail in [9]). The field $\phi$ is a singlet under the SM gauge symmetries and couples only to right-chiral neutrinos, see (2) below. If $\phi$ acquires a vacuum expectation value by (possibly radiatively induced) spontaneous symmetry breaking, a Majorana mass term is generated for the right-chiral neutrinos. The phase $a(x)$ then gives rise to a (pseudo-)Goldstone particle (called ‘Majoron’) associated with the spontaneous breaking of global $U(1)_L$ lepton number symmetry [10]. The crucial feature of our proposal is that it requires all mass scales to arise from the quantum mechanical breaking of classical conformal invariance. Therefore in any consistent implementation of this scheme there cannot exist intermediate scales of any kind between the electroweak scale and the Planck scale. This holds in particular true for the masses of the light neutrinos whose smallness is naturally explained here with appropriate neutrino Yukawa couplings.
\( \sim O(10^{-5}) \) \(^1\) and without the need to introduce a large Majorana mass ‘by hand’. In this paper we show that likewise, and contrary to widely held expectations, no extra large scale is required for the solution of the strong CP problem either.

As argued in [7] the Majoron has several features in common with the axion, and the smallness of its couplings can be tied to the smallness of neutrino masses. In this Letter, we go one step further and propose that the Majoron actually \textit{is} the axion, with \textit{computable} effective couplings to SM particles, and the neutrino Yukawa couplings as the only unknown parameters (a possible link between light neutrinos and the invisible axion had already been suggested in [11, 12]). In other words, we claim that lepton number symmetry \( U(1)_L \) is transmuted, via electroweak parity violation and neutrino mixing, into a \( U(1) \) symmetry that, in relation to the strong interactions, is indistinguishable from the standard axial Peccei-Quinn symmetry at low energies. We present exact expressions for the (UV finite) two-loop integrals describing the coupling of the axion to photons and (light) quarks; the main technical novelty here is the consistent use of the off-diagonal neutrino propagators (6) below. From the quark couplings one can estimate the coupling of the axion to gluons, which comes out naturally tiny.

On general grounds the effective couplings of \( a(x) \) can only be of a very restricted type. Because Goldstone bosons interact only via derivatives, the perturbative effective action at low energies contains only terms \( \propto \mathcal{X}^\mu \partial_\mu a \), where \( \mathcal{X}^\mu \) are local expressions in the SM quantum fields. At lowest order there are only three candidates for \( \mathcal{X}^\mu \): (i) a Chern-Simons current, which by partial integration is equivalent to a coupling \( a \text{Tr} \bar{W}_{\mu} \bar{W}^\mu \) (where \( W_{\mu} \) can be any SM gauge connection), (ii) a vector current \( J^\mu_V \) and (iii) an axial current \( J^\mu_A \). Being mediated by the weak interactions the fermionic bilinears contributing to \( \mathcal{X}^\mu \) and involving charged SM fermions all appear in ‘V – A’ form. Therefore, whenever \( \partial_\mu J^\mu_V \approx 0 \) by some approximate\(^2\) conservation law, \( a(x) \) couples like a \textit{pseudoscalar} to photons, gluons, quarks and electrons.

\(^1\)Recall that the appearance of a similar ratio for the charged leptons is an experimental fact: \( m_e/m_\tau < 10^{-5} \).

\(^2\)By this we mean neglecting all terms involving neutrinos or the scalar field \( \phi \) in the relevant currents, as well as baryon or lepton number violating ‘sphaleron-like’ contributions, because these will give negligible contributions to all processes considered in this paper.
2 Neutrino Lagrangian and propagators

We refer to [13, 14] for basic properties of the SM, and here only quote the Yukawa couplings

\[ \mathcal{L}_Y = (\overline{L}_i \Phi Y_{ij}^E E^j + \overline{Q}_i \Phi Y_{ij}^U D^j + \overline{Q}_i \epsilon \Phi^* Y_{ij}^U U^j + \overline{L}_i \epsilon \Phi^* Y_{ij}^\nu \nu^j + \phi N^T C^{-1} Y_{ij}^M N^j + \text{h.c.}) \] (2)

and the neutrino terms in the Lagrangian, see (4) below. Here \( Q^i \) and \( L^i \) are the left-chiral quark and lepton doublets, \( U^i \) and \( D^i \) the right-chiral up- and down-like quarks, while \( E^i \) are the right-chiral electron-like leptons, and \( N^i \equiv \nu^i_R \) the right-chiral neutrinos (we suppress all indices except the family indices \( i, j = 1, 2, 3 \)). \( \Phi \) is the usual Higgs doublet, and \( \phi \) is the new complex scalar field introduced in (1). As is well known, one can use global redefinitions of the fermion fields to transform the Yukawa matrices \( Y_{ij}^E, Y_{ij}^U \) and \( Y_{ij}^M \) to real diagonal matrices. By contrast, the matrices \( Y_{ij}^D \) and \( Y_{ij}^\nu \) may exhibit (strong) mixing. Besides the standard (local) \( SU(3)_c \times SU(2)_w \times U(1)_Y \) symmetries, the Lagrangian (2) admits two global \( U(1) \) symmetries, baryon number symmetry \( U(1)_B \) and lepton number symmetry \( U(1)_L \). The latter is associated with the Noether current

\[ J^\mu_{L} := \overline{\nu} \gamma^\mu L^i + \overline{E} \gamma^\mu E^i + \overline{N} \gamma^\mu N^i - 2i \phi^\dagger \overset{\leftrightarrow}{\partial}^\mu \phi \] (3)

The fact that \( \phi \) carries lepton charge is crucial for the proposed transmutation of \( U(1)_L \) into a PQ-like symmetry.

For the computation of loop diagrams it is convenient to employ \( SL(2, \mathbb{C}) \) spinors [7]. With \( \nu^i_L \equiv \frac{1}{2} (1 - \gamma^5) \nu^i \equiv \bar{\nu}^{i\alpha} \) and \( \nu^i_R \equiv \frac{1}{2} (1 + \gamma^5) \nu^i \equiv N^i_\alpha \), the neutrino part of the free Lagrangian reads (see [15] for conventions)

\[ \mathcal{L} = \frac{1}{2} \left( \nu^{i\alpha} \phi_{\alpha\beta} \bar{\nu}^{j\beta} + N^{i\alpha} \phi_{\alpha\beta} \bar{N}^{j\beta} \right) + m_{ij} \nu^{i\alpha} N^j_\alpha + \frac{1}{2} M_{ij} N^{i\alpha} N^j_\alpha + \text{c.c.} \] (4)

after spontaneous breaking of conformal and electroweak symmetries. Consequently, the (complex) Dirac and Majorana mass matrices are given by \( m_{ij} = Y_{ij}^E \langle H \rangle \) and \( M_{ij} = Y_{ij}^M \langle \varphi \rangle \), respectively (where \( \langle H \rangle^2 \equiv \langle \Phi^\dagger \Phi \rangle \)). Rather than diagonalize the fields w.r.t. these mass terms, we work with non-diagonal propagators and the interaction vertices from (2). Defining

\[ \mathcal{D}(p) := \left[ p^4 - p^2 (M^{-1} m^T m^* M + m^T m + M M^*) + m^T m M^{-1} m^T m^* M \right]^{-1} \] (5)
we obtain the matrix propagators (in momentum space)

\[
\begin{align*}
\langle \nu_i^{\alpha} \nu_j^{\beta} \rangle &= i \left[ m^* M D(p) m^\dagger \right]^{ij} \varepsilon_{\alpha\beta} \\
\langle \nu_i^{\alpha} \bar{\nu}_j^{\beta} \rangle &= i \left[ (M^T)^{-1} \left\{ p^2 - M M^* - (M^*)^{-1} m^\dagger m M^* \right\} D(p) m^T \right]^{ij} \eta_{\alpha\beta} \\
\langle N_i^{\alpha} N_j^{\beta} \rangle &= i \left[ M^* p^2 D(p)^* \right]^{ij} \varepsilon_{\alpha\beta} \\
\langle N_i^{\alpha} \bar{N}_j^{\beta} \rangle &= i \left[ (p^2 - M^{-1} m^T M^* M) D(p) \right]^{ij} \eta_{\alpha\beta} \\
\langle \nu_i^{\alpha} N_j^{\beta} \rangle &= i \left[ m^* \left\{ p^2 - (M^*)^{-1} m^\dagger m M^* \right\} D(p) \right]^{ij} \varepsilon_{\alpha\beta} \\
\langle \nu_i^{\alpha} \bar{N}_j^{\beta} \rangle &= -i \left[ m^* M D(p) \right]^{ij} \eta_{\alpha\beta},
\end{align*}
\]

(6)

together with their complex conjugate components. Evidently, these propagators allow for maximal mixing in the sense that every neutrino component can oscillate into any other (also across families). For the UV finiteness of the diagrams to be computed below it is essential that some of the propagator components fall off like \( \sim p^{-3} \), unlike the standard Dirac propagator. Taking \( M_{ij} \) diagonal it is not difficult to recover the mass eigenvalues as predicted by the standard see-saw formula [16, 17, 18, 19].

With the above propagators and the (extended) SM Lagrangian we can now proceed to compute various effective low energy couplings involving the ‘axion’ \( a \) which are mediated by neutrino mixing via two or three-loop diagrams. Here we present only the results for photon-axion and quark-axion couplings, cf. the diagrams depicted below. Further results and detailed derivations will be given in a forthcoming publication [20].

3 Photon-axion vertex

For the low energy effective action we need only retain contributions where all particles circulating inside the loops are much heavier than the external particles. As our first example we determine the effective coupling of the axion to photons via the two-loop diagram in Fig. 1. For small axion momentum \( q^\mu = k_1^\mu - k_2^\mu \) it is possible to derive a closed form expression for the two-loop integral and for arbitrary mixing matrices [20]. Setting \( \mu = \langle \phi \rangle \) in (1) and denoting by \( M_j \) the eigenvalues of the (diagonal) matrix \( M_{ij} \), a lengthy calculation gives the expected kinematical factor \( \epsilon^{\mu\nu\lambda\rho} F_{\mu\nu} F_{\lambda\rho} \propto \epsilon^{\mu\nu\lambda\rho} k_1^{\lambda} k_2^{\rho} \)

5
with coefficient function

$$\frac{i e^2 g_w^2}{64 \sqrt{2} \pi^4} \sum_{i,j} \frac{|m_{ij}|^2}{\langle \varphi \rangle} M_j^2 \int_0^1 dx \int_0^1 dy \int_0^1 dz \int_0^1 dt \, x(1-x)y^2z(1-z)t^3 \times$$

$$\times \left\{ -yt - 6(1-y)zt \right\}^{\frac{1}{M_{ij}^2(x,y,z,t)}} + \frac{y(1-y)[(-2y - 3(1-y)zt]((2-t)m_{ei} - t(1-t)^2k_j^2)}{M_{ij}^2(x,y,z,t)} \right\}$$

(7)

where $m_{ei} \equiv (m_e, m_\mu, m_\tau)$ and

$$M_{ij}(x,y,z,t) := xyztM_j^2 + (1-y)\left[ ztM_W^2 - yt(1-t)k_2^2 + y(1-zt)m_{ej}^2 \right]$$

(9)

The above integral is cumbersome to evaluate in general form, but for small photon momenta $k_1^\mu \approx k_2^\mu$ we get

$$\approx \frac{i \alpha_e \alpha_w}{72 \sqrt{2} \pi^2} \sum_{i,j} |m_{ij}|^2 \left( \frac{\log \frac{M_j^2}{m_e^2}}{m_e^2} \right)^2$$

(10)

Of course, the precise value of the effective low energy coupling depends on the (unknown) values of the Yukawa mass matrices $m_{ij}$ and $M_{ij} = M_j \delta_{ij}$. For $M_j \sim M \sim \langle \varphi \rangle$ the axion-photon vertex is well approximated by

$$L_{a\gamma\gamma}^{\text{eff}} = \frac{1}{4} f_a \alpha F_{\mu\nu} \tilde{F}_{\mu\nu}, \quad f_a^{-1} = \frac{\alpha_e \alpha_w}{72 \sqrt{2} \pi^2 M^2} \left( \frac{\log \frac{M^2}{m_e^2}}{m_e^2} \right)^2$$

(11)

with the standard see-saw relation $\sum m_\nu \sim \sum |m_{ij}|^2 / M$. Substituting numbers we find $f_a = O(10^{16} \, \text{GeV})$ which is outside the range of existing or planned experiments [21]. Thus the smallness of axion couplings gets directly tied to the smallness of the light neutrino masses via (11).

### 4 Quarks and gluons

The effective low energy couplings to light quarks can be analyzed in a similar way. With $P_L \equiv \frac{1}{2}(1 - \gamma^5)$ we parametrize these couplings as

$$L_{aqq}^{\text{eff}} = i \partial_\mu a \left( c_\mu^{aUU} \bar{u}^i \gamma^\mu P_L u^j + c_\mu^{aPP} \bar{d}^i \gamma^\mu P_L d^j \right)$$

(12)

While we use capital letters $U, D, ...$ in (2) to designate chiral spinors, we use small letters $u, d, ...$ for the full (non-chiral) spinors here and below.
Again one can obtain an exact formula for the (UV finite) two-loop integrals; e.g. for the up-like quarks we get

\[ c_{ij}^{UU} = \sum_{k,r,s} g_4^4 |m_{rs}|^2 M_r^2 V_{ik}^r (V_\Gamma^j)^{kj} \times \]

\[ \frac{128 \sqrt{2} \pi^4 \langle \phi \rangle}{\int_0^1 dx \int_0^1 dy \int_0^1 dt \int_0^1 dz \ x(1-x)y^3(1-z)t^3 \times \]

\[ -1 + 3y + 3(1-y)zt \]

\[ \left[ xyz M_s + (1-y)\{yt(1-z)M_W^2 + ztm_e + y(1-t)m_{D_k}\} \right]^2 \]

with the CKM matrix \( V_{ij} \). A similar (but not the same) formula is obtained for \( c_{ij}^{DD} \) [20]. In principle, there are also contributions from diagrams with Z-boson exchange, but these can be disregarded for the effective low energy Lagrangian because they involve a purely neutrino triangle with one light neutrino (which is lighter than any external quark). To estimate the integral, we set \( m_{e_i} = m_{d_i} = 0 \) in (13) (which still leaves a convergent integral that can be calculated exactly [20]). Because the CKM matrix is unitary, both \( c_{ij}^{UU} \) and \( c_{ij}^{DD} \) then become proportional to \( \delta_{ij} \) to leading order. Keeping only one lepton flavor in (13) we here quote the result only for two limiting cases: for \( M_j \sim M \gg M_W \) we get

\[ c_{ij}^{UU} = \sum_{k,l} \frac{\alpha_w^2 |m_{kl}|^2}{128 \sqrt{2} \pi^2 \langle \phi \rangle M_j^2} \left[ \left( \log \frac{M_j^2}{M_W^2} - 2 \right)^2 + \frac{2\pi^2}{3} \right] \delta_{ij} \]  

(14)

If instead \( M \sim M_W \) the exact result replaces the square bracket by 0.71. Note that the Majorana mass \( M \) is much closer to the weak scale in [9, 7] than in the usual see-saw scenario, favoring the second value.

By the approximate conservation of the up and down quark vector currents, we can now drop the vectorlike contribution in the effective Lagrangian which thus becomes purely axial to leading order, viz.

\[ \mathcal{L}_{\text{eff}} \rightarrow i\partial_a \left( g_{aUU}^1 \bar{u}^j \gamma^5 \gamma^\mu u^j + g_{aDD}^1 \bar{d}^j \gamma^5 \gamma^\mu d^j \right) \]  

(15)

At subleading order off-diagonal contributions to \( c_{ij}^{UU} \) and \( c_{ij}^{DD} \) will appear with both vector and axial vector interactions. The numerical values of the effective coupling constants can be read off from the above results. Their
precise values are subject to the same caveats as mentioned before (11). With the same assumptions on the Yukawa mass matrices as for (11) we get

\[ g_a^{-1} \equiv g_{aUU}^{-1} \sim g_{aDD}^{-1} \sim \mathcal{O}(10^{-3}) \alpha^2 \sum m_{\nu} / M^2 \tag{16} \]

If \( M \) is not very much larger than the weak scale \( M_W \), we get \( g_{aUU} \sim 10^{18} \) GeV for \( \sum m_{\nu} \approx 1 \) eV.

The axion-gluon coupling involves various three-loop diagrams, now with all six quarks in the loop [7]. For a rough estimate we can shortcut this calculation by integrating the effective vertex (15) by parts, using the anomalous conservation of the axial (color singlet) quark current \(^4\) (see e.g. [22])

\[ \partial_{\mu} (i \bar{q} \gamma^5 \gamma^{\mu} q) = \frac{\alpha_s}{4\pi} \mathrm{Tr} G^{\mu\nu} \tilde{G}_{\mu\nu} \equiv Q \tag{17} \]

with the gluonic topological density \( Q(x) \) (in principle there could appear extra terms \( \propto m_q \bar{q} \gamma^5 q \) on the r.h.s., but Goldstone's Theorem assures us that such non-derivative terms must drop out in the final result (18)). Summing over the six quark flavors and now also the three leptons we thus obtain

\[ \mathcal{L}^{agg}_{\text{eff}} = \frac{18\alpha_s}{4\pi g_a} \mathrm{Tr} G^{\mu\nu} \tilde{G}_{\mu\nu} \equiv 18 g_a^{-1} a Q \tag{18} \]

When the quark mass matrix \( m_q \) is complex there is an extra contribution to this term from the anomalous chiral rotation required to render the quark mass matrix real, resulting in a shift

\[ 18 g_a^{-1} a \quad \rightarrow \quad 18 g_a^{-1} \tilde{a} \equiv 18 g_a^{-1} a + \arg \det m_q \tag{19} \]

Because \( a \) is a Goldstone boson this shift does not affect any other terms in the effective Lagrangian, but merely replaces \( a \) by \( \tilde{a} \) in (18).

5 Axion potential

Being a Goldstone boson, the axion cannot acquire a mass in perturbation theory; likewise its vacuum expectation value remains undetermined in perturbation theory. However, non-perturbative effects can generate a potential

\(^4\)The actual result for the effective coupling (18) follows from a UV finite, hence non-anomalous 3-loop diagram [7]. Within the present scheme, it is ultimately the conformal anomaly which accounts for the non-vanishing coupling in (18).
for the axion and thereby lift the vacuum degeneracy. To compute it we use the formula

$$\langle \exp F \rangle = \exp \left[ \langle F \rangle + \frac{1}{2} (\langle F^2 \rangle - \langle F \rangle^2) + \ldots \right]$$

with $F \equiv 18g_a^{-1} \tilde{a} Q$. Except for possible contributions from the weak interactions which we ignore, there is no $G\tilde{G}$ condensate and we have $\langle Q(x) \rangle = 0$ (likewise $\langle Q^n \rangle$ vanishes for all odd $n$). Hence the axion potential is

$$V_{\text{axion}}(a) = \frac{1}{2} m_{\text{axion}} a^2 + O(\tilde{a}^4)$$

It is important that this potential is written as a function of the shifted axion field $\tilde{a}$ introduced in (19). The axion mass is therefore

$$m_{\text{axion}} = 18g_a^{-1} \left[ \int d^4x \langle Q(x)Q(0) \rangle \right]^{\frac{1}{2}}$$

We conclude that (1) indeed $\theta \equiv \langle \tilde{a} \rangle = 0$ as required for the solution of the strong CP problem, and (2) an axion mass term is generated by non-perturbative effects. Although the value of the $(G\tilde{G})^2$ condensate is apparently not known, we can estimate $m_{\text{axion}} \sim 18g_a^{-1} \Lambda_{QCD}^2 \sim 10^{-8} \text{ eV}$, which may be still compatible with the axion being a (cold) dark matter candidate, at least according to standard reasoning [23, 24], and bearing in mind the considerable uncertainties in these numbers. From (16) it is evident that the viability of this dark matter scenario requires the Majorana scale $M$ to be not much larger than $M_W$, in contrast to the standard see-saw proposal [16, 17, 18]. This is a main new feature of the present proposal: if true, it could be interpreted as additional evidence for a hidden conformal symmetry of the SM [25, 7, 8], such that the observed diversity of scales in particle physics could be explained via quantum mechanically (or even quantum gravitationally) induced logarithmic effects [26].

The main virtue of the present proposal is that it provides a single source of explanation for axion couplings and neutrino masses, tying together in a most economical manner features of the SM previously thought to be unrelated. Given the known SM parameters, and parametrizing the unknown physics in terms of just the Yukawa mass matrices, all relevant couplings are entirely calculable in terms of UV finite diagrams, and naturally come out to be very small without the need for any fine tuning.
Finally, we note that all results in this Letter can be equivalently obtained if we take the scalar field $\phi(x)$ in (1) to be real, absorbing the phase $a(x)$ into a redefinition of the lepton fields. This point will be discussed in much more detail in [20]. The redefinition also shows that the apparent periodicity of $a(x)$ in (1) is spurious because the redefined Lagrangian involves the field $a(x)$ only through its derivatives. Rather, the periodicity parameter for $a$ is set by the effective action (18) and the fact that the gluon term is a topological density (see e.g. [27]).

Acknowledgments: AL and KAM thank the AEI for hospitality and support during this work. We are also grateful to Pierre Fayet for his incisive comments on a first version of this work.

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Fig.1. Axion-photon-photon and axion-quark-quark effective couplings