Supercurrent through Graphene: Effects of Vanishing Density of States

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1. Introduction
Graphene has been stimulating researches on physics induced by novel band structures [1, 2, 3]. The superconducting properties related to graphene physics have also been studied both theoretically and experimentally. One of the fundamental questions on this topic is whether or not the supercurrent flows through a conductor with a vanishing density of state (DOS) at Fermi level originating from “Dirac cones”. Existence of such supercurrent is shown both theoretically and experimentally [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16], however the role of vanishing DOS has not been clarified yet especially at finite temperatures.

The present authors have calculated the critical current through superconductor-graphene-superconductor Josephson junction using tunnel Hamiltonian approximation, and analyzed its temperature and junction-length dependences [17]. It was found that these dependences show peculiar behavior originating from band structure of graphene, including oscillating behavior in bilayer case [18].

In this paper we focus on the role of vanishing DOS of monolayer graphene on the critical current through S-G-S junction. It is shown that the vanishing DOS does not prohibit the supercurrent even at the absolute zero temperature, however suppression actually occurs at low temperature leading to a re-entrant behavior.

2. Model
The low-energy effective Hamiltonian of graphene is given in the following form,

$$H^{\text{eff}} \simeq \sum_{k\sigma} E_k \left( \alpha_k^{\dagger} \alpha_k^{\sigma} - \beta_k^{\dagger} \beta_k^{\sigma} \right).$$

Here, we limit the electron hopping to the nearest neighbors. The quasi-particle annihilation operators with positive ($E_k$) and negative ($-E_k$) energies are denoted by $\alpha_k^{\sigma}$ and $\beta_k^{\sigma}$,
respectively, where $\vec{k}$ is the wave vector and $\sigma$ the spin index. In case of monolayer graphene, the dispersion is given by $E_K = |\gamma_k| = |te^{i\vec{k} \cdot \vec{\delta}}(1 + e^{i\vec{a}_1 \cdot \vec{\delta}} + e^{i\vec{a}_2 \cdot \vec{\delta}})|$ where $\vec{\delta}$, $\vec{a}_1$ and $\vec{a}_2$ are shown in fig. 1 (a) and $t$ is the in-plane hopping matrix element. Near the $K$ point, $|\gamma_K|$ is approximated by $\hbar v_F |\vec{k} - \vec{K}|$ with $\hbar v_F = 3\alpha a/2$ and the same holds for $K'$, where $a$ is the distance between neighboring carbon atoms. ($\vec{K}$ is the wave vector of $K$-point.) These structures are usually described as Dirac cones and the density of state behaves like $D(E) \propto |E|$. Thus, without doping the DOS is vanishing at Fermi level.

![Figure 1](image_url)

**Figure 1.** (a) The lattice structure of graphene and the definition of $\vec{a}_1$, $\vec{a}_2$, and $\vec{\delta}$. (b) A point-contact type and (c) a realistic S-G-S junction. Here $S_L$ and $S_R$ indicate superconducting leads and $d$ is their separation.

Let us consider the tunneling of electrons between graphene and a superconducting lead. We assume a point-contact type junction such as one depicted in fig. 1 (b). The tunneling Hamiltonian has two contributions, namely tunneling through $A$- and $B$-sublattice sites of graphene, which we denote by $H_T^{(A)}$ and $H_T^{(B)}$, respectively. In terms of quasi-particles $\alpha_{\vec{k}\sigma}$ and $\beta_{\vec{k}\sigma}$, $H_T^{(A)}$ and $H_T^{(B)}$ are given as

$$H_T^{(A)} = \frac{t'}{\sqrt{n_G n_L}} \sum_{\vec{k}, \vec{l}, \sigma} \left\{ e^{-i\theta_{j}} e^{-i(\vec{k} - \vec{l}) \cdot \vec{\delta}} \alpha_{\vec{k}\sigma}^+ \left[ \alpha_{\vec{l}\sigma} - \beta_{\vec{l}\sigma} \right] + H.c. \right\}$$

$$H_T^{(B)} = \frac{t'}{\sqrt{n_G n_L}} \sum_{\vec{k}, \vec{l}, \sigma} \left\{ e^{-i(\vec{k} - \vec{l}) \cdot \vec{\delta}} \alpha_{\vec{k}\sigma}^+ \left[ \alpha_{\vec{l}\sigma} + \beta_{\vec{l}\sigma} \right] + H.c. \right\}$$

where $e^{i\theta_{j}} = \gamma_j/|\gamma_j|$, and $c_{\vec{k}\sigma}$ is the electron operator in the leads. $\vec{r}_j^{(A)}$ is the $A$-(B-)sublattice point of the $j$-th cell, where tunneling of electrons takes place. $t'$ is the tunneling matrix element and $n_G$ and $n_L$ are number of lattice sites in graphene and lead, respectively. The critical current through S-G-S junction is obtained by the perturbative expansion with respect to $t'$. Here we introduce $\Theta$, the phase difference between two superconducting leads. When the tunneling sites are in the same sublattice, we obtain

$$I_c^{AA} = \beta C \sum_{n=-\infty}^{\infty} \frac{2\Delta_0^2 \sin \Theta}{(\hbar \omega_n)^2 + \Delta_0^2} \Omega_{\vec{r}_{jj'}} |\hat{\omega}_n|^2 \left| K_0 \left( |\vec{r}_{jj'}| \frac{\hat{\omega}_n}{\hbar v_F} \text{sgn}(\omega_n) \right) \right|^2$$

where $\omega_n = \omega_n - \delta \mu_G / \hbar$, $\beta = 1/k_B T$ ($k_B$: Boltzmann constant), $T$: temperature) and $\Omega_{\vec{r}_{jj'}} = 2 \left\{ 1 + \cos(\vec{K} - \vec{K'}) \cdot \vec{r}_{jj'} \right\}$, with $\vec{r}_{jj'}$ being $\vec{r}_j^{(A)} - \vec{r}_j^{(B)}$. The chemical potentials of leads are set to zero and that of graphene to $\mu_G$. When the tunneling sites belong to different
sublattices, we obtain

\[ I_c^{AB} = \beta C \sum_{n=-\infty}^{\infty} \frac{\Delta_0^2 \sin \Theta}{(\hbar \omega_n)^2 + \Delta_0^2} \Omega_{r_0} |\tilde{\omega}_n|^2 \left| K_1 \left( |\tilde{r}_{j,j'} + \delta| \tilde{\omega}_n \operatorname{sgn}(\omega_n) \right) \right|^2 \]  

(5)

where \( \Omega_{r_0} = 2 \{ 1 + \cos \left( (K - K') \cdot (\tilde{r}_{j,j'} + \delta) - 2\theta_{j,j'} \right) \} \). Here \( \theta_{j,j'} \) is the angle between the vector, \( \tilde{r}_{j,j'} + \delta \) and the \( x \)-axis. The constant \( C \) is \( \left( \frac{r_{ms,s0}}{4\pi^2 \hbar v_F} \right)^2 \) where \( m \) is the electron mass in the leads, and \( s_0 \) and \( s_1 \) are areas of unit cell in graphene and leads, respectively.

The factor \( \Omega_{r_0} \) and \( \Omega_{r_0} \) give rise to a short range modulation of the critical current as a function of junction site. In fig.2 (a), we have shown the magnitude of critical current at each lattice point (The magnitude is shown by the diameter of circles). As one can see the magnitude oscillates with a period of a few lattice spacing.

3. Critical Current of S-G-S junction

The critical current through a realistic S-G-S junction depicted in fig. 1 (c) can be estimated by adding all the contributions of tunneling sites constituting the electrodes. In this paper we skip this process and give only rough estimations obtained by averaging Eqs. (4) and (5). In fig. 2, we have plotted \( I_c \) as a function of the powers of temperature \( T \). In our calculation temperature (energy) is normalized by transition temperature of superconductors \( T_c (k_B T_c) \) and the unit of length by \( l_0 = \hbar v_F / (k_B T_c) \). As we can see from fig. 2 (b), the critical current is best fitted by \( I_c \propto \exp(-aT^\alpha) \) with \( \alpha = 1 \sim 2 \). In fig. 2 (c), we have directly estimated \( \alpha \) from temperature range \( 0.8T_c < T < 0.95T_c \) by fitting for various \( \mu^G \)'s. The power is largest at \( \mu^G = 0 \) and decreases gradually by doping. It is expected that \( \alpha = 1 \) (normal metal case) at large enough doping, although we could not reach that region in this calculation.

Next we investigate the low temperature region. In fig. 3 the \( \mu^G \) dependence of \( I_c \) is shown. The characteristic feature is that \( I_c \) is suppressed at no doping as compared to finite doping especially at low temperatures \( T \lesssim 0.1T_c \). In some cases, even re-entrant behavior can be observed (see fig. 3 (b) and (c)). A similar behavior can also be seen in the results obtained by González and Perfetto using the Dirac Fermion propagators [10].

4. Discussion

The behavior of S-G-S junction at \( T = 0 \) has been studied by several authors and suppression of critical current at \( \mu^G = 0 \) has been pointed out theoretically[4]. Experimental observation has
Figure 3. (a) Plot of $I_c$ as a function of $T/T_c$ for $\mu^G = 0$, 0.05 and 0.1 with $d = 8l_0$, and (b) with $d = 2l_0$. Here $d = \langle |\vec{r}_{j'-j}| \rangle$ is the junction length. (c) $I_c$ as a function of $\mu^G$ for $T = 0.01$, 0.05 and 0.09. The re-entrant behavior can be clearly seen.

also been done[6, 16]. The theoretical studies until now are mostly based on in-plane S-G contacts and somewhat different from our tunneling model. However we may expect from our results that such “Dirac cone"induced suppression of critical current may survive up to $10\sim20\%$ of $T_c$. We have also predicted the re-entrant behavior of the critical current as a function of temperature. However, this behavior is subtle and easily fades out by a small change of parameters, such as $\mu^G$. In addition, there is some controversy regarding the theoretical treatment of graphene-superconductor interfaces in S-G-S junctions (for example, see Ref. [11]). Therefore, in order to clarify the condition for observing the phenomena predicted in this paper, an extensive study including the effects specific to the wide electrode, such interference and averaging between the contributions of different tunneling sites, is required.

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