Vector theory of gravity: Universe without black holes and solution of dark energy problem

Anatoly A Svidzinsky

Department of Physics & Astronomy, Texas A&M University, College Station, TX 77843, United States of America

E-mail: asvid@physics.tamu.edu

Received 23 January 2017, revised 17 August 2017
Accepted for publication 16 October 2017
Published 20 November 2017

Abstract

We propose an alternative theory of gravity which assumes that background geometry of the Universe is fixed four dimensional Euclidean space and gravity is a vector field $A_k$ in this space which breaks the Euclidean symmetry. Direction of $A_k$ gives the time coordinate, while perpendicular directions are spatial coordinates. Vector gravitational field is coupled to matter universally and minimally through the equivalent metric $f_{ik}$ which is a functional of $A_k$. We show that such assumptions yield a unique theory of gravity, it is free of black holes and, to the best of our knowledge, passes all available tests.

For cosmology our theory predicts the same evolution of the Universe as general relativity with cosmological constant and zero spatial curvature. However, the present theory provides explanation of the dark energy as energy of longitudinal gravitational field induced by the Universe expansion and yields, with no free parameters, the value of $\Omega_\Lambda = 2/3 \approx 0.67$ which is consistent with the recent Planck result $\Omega_\Lambda = 0.686 \pm 0.02$. Such close agreement with cosmological data indicates that gravity has a vector, rather than tensor, origin. We demonstrate that gravitational wave signals measured by LIGO are compatible with vector gravity. They are produced by orbital inspiral of massive neutron stars which can exist in the present theory. We also quantize gravitational field and show that quantum vector gravity is equivalent to QED. Vector gravity can be tested by making more accurate measurement of the time delay of radar signal traveling near the Sun; by improving accuracy of the light deflection experiments; or by measuring propagation direction of gravitational waves relative to laser interferometer arms. Resolving the supermassive object at the center of our Galaxy with VLBA could provide another test of gravity and also shed light on the nature of dark matter.

Keywords: alternative theory of gravity, dark energy, dark matter, detection of gravitational waves by LIGO, quantum vector gravity, galactic centers, neutron star mass limit

(Some figures may appear in colour only in the online journal)

Contents

1. Introduction 2
2. Postulates of the vector theory of gravity 4
3. Equivalent metric 5
4. Action for gravitational field 5
5. Equations for classical gravitational field 6
1. Introduction

A century ago, Albert Einstein completed the general theory of relativity [1]. Einstein’s theory then became an accepted theory of gravity. In general relativity the space–time geometry $g_{ik}$ (metric tensor) is the gravitational field described by the action

$$S_{GR} = -\frac{c^3}{16\pi G} \int d^4x \sqrt{-g} g^{ik}R_{ik} - \int \rho \sqrt{g} \frac{dx^i}{dt} \frac{dx^k}{dt} dx^i dx^k,$$

(1)

where $G$ is the gravitational constant, $c$ is the speed of light and $R_{ik}$ is the Ricci tensor. The second term in equation (1) describes interaction between gravitational field and matter with the rest mass density $\rho(t, r)$. Variation of the action (1) with respect to $g_{ik}$ yields Einstein equations

$$R_{ik} = \frac{8\pi G}{c^4} \left( T_{ik} - \frac{1}{2} g_{ik} T \right),$$

(2)

where $T_{ik}$ is the energy–momentum tensor of matter. Einstein equations (2) are a consequence of the postulate that space–time geometry $g_{ik}$ is gravitational field.

One should mention that so far general relativity was accurately tested only at weak gravitational field [2, 3] and thus it is not a theory fully confirmed experimentally. Observations of binary pulsars yet have not provided an accurate test of general relativity at strong gravity. Even though neutron stars in the binary systems are relativistic objects, they are sufficiently well-separated and all aspects of their orbital behavior and gravitational wave generation in general relativity are characterized only by their net masses and angular momentum. As a result, observations of binary pulsars tested Einstein equations for the weak time-dependent field and also the equivalence principle which guarantees effacement of the bodies (relativistic) internal structure. One should note, however, that strong internal gravitational fields of neutron stars can affect orbital dynamics and gravitational wave generation in alternative theories of gravity that violate the strong equivalence principle [2–4]. Only in this sense observations of binary pulsars is a test of strong gravity.

Recent direct detection of gravitational waves from a binary ‘black hole’ merger by the LIGO team [5–7] is also not an accurate test of strong gravity. Such detection was unable to constrain higher-order post-Newtonian parameters with a reasonable accuracy [8, 9]. Obtained bounds on relative deviations in the post-Newtonian parameters are of the order of $O(1)$. We show in section 14 of this paper that the LIGO signal can be interpreted in the framework of vector gravity as being produced by a merger of massive neutron stars, rather than black holes, which yields radiation waveform compatible with the LIGO data. We also demonstrate that stable neutron stars with a simple linear ‘causal’ equation of state can have masses up to about $35M_\odot$ in vector gravity (see section 13). Neutron star mass can be much larger if ‘causality’ constraint on the equation of state is not imposed. Moreover, as we show in section 15, in vector gravity, compact objects composed of dark matter can have masses exceeding billions solar masses.

In 1998, published observations of Type Ia supernovae by the high-Z Supernova Search Team [10] followed in 1999 by the Supernova Cosmology Project [11] suggested that the expansion of the Universe is accelerating. Since then, these observations have been corroborated by several independent sources. Measurements of the cosmic microwave background, gravitational lensing, and the large-scale structure of the cosmos as well as improved measurements of supernovae have been consistent with the lambda-CDM model, which is the current standard model of cosmology. According to this model, a mysterious dark energy gives the main contribution to the Universe composition. Work done in 2013 based on the Planck spacecraft observations of the cosmic microwave background gave the most accurate estimate of about 68% of dark energy in the Universe [12].

There are questions that general relativity is unable to answer without extending the model. E.g., general relativity alone can not explain why Universe is spatially flat; it does not provide a mechanism of matter generation at the Big Bang; and it can not explain the value of the cosmological term (mysterious dark energy). It also predicts existence of
singularities such as black holes when a massive star collapses into a point with zero volume and infinite matter density. One can argue that general relativity becomes invalid in the vicinity of singularities and a quantum theory of gravity will remove them. In contrast, the present theory is free of such singularities at the classical level. Namely, the end point of a gravitational collapse is not a point singularity but rather a stable star with a reduced mass. One should mention that black holes have never been observed directly and ‘evidences’ of their existence are based on the presumption that general relativity describes gravity for strong field. Until signatures of the event horizons are found the existence of black holes will not be proven.

Here we propose an alternative theory of gravity which is a Lagrangian-based vector field theory in fixed four-dimensional Euclidean space. The present vector theory is a metric theory of gravity which means that space–time is endowed with a symmetric equivalent metric $g_{\mu\nu}$ formed out of the vector field and Euclidean metric. Matter and nongravitational fields respond only to the space–time metric $g_{\mu\nu}$. The world lines of test bodies are geodesics of that metric and in local freely falling frames the nongravitational laws of physics are those of special relativity. Our theory is prior-geometric for it contains the nongravitational laws of physics and gives us a hint about composition of dark matter (see section 15 and [13]).

As we show in section 11, quantization of the linearized equations of vector gravity yields quantum theory which is equivalent to QED.

An interesting feature of our theory is that equations for gravitational field can be solved analytically for arbitrary static mass distribution (see section 6). If point masses are located at $r_1$, $r_2$, ... $r_N$ then exact solution of the field equations for the equivalent metric is

$$f_{\mu\nu} = \begin{pmatrix} e^{2\phi} & 0 & 0 & 0 \\ 0 & -e^{-2\phi} & 0 & 0 \\ 0 & 0 & -e^{-2\phi} & 0 \\ 0 & 0 & 0 & -e^{-2\phi} \end{pmatrix}, \quad (3)$$

where

$$\phi(r) = -\frac{m_1}{|r - r_1|} - ... - \frac{m_N}{|r - r_N|} \quad (4)$$

and $m_k$ ($k = 1, ..., N$) are constants determined by the value of masses. Solution (3) is free of black holes: photons with a radial velocity component can always escape from gravitationally compact objects. In recent years, the evidence for the existence of ultra-compact supermassive objects at centers of galaxies has become very strong. It is important to note that present solution (3) not only argues that such objects are not black holes, but also can explain quantitatively their observed properties and give us a hint about composition of dark matter (see section 15 and [13]).

Before we proceed with building the vector theory of gravity we discuss an algorithm that we use to construct the theory. Classical electrodynamics is an example of a successful field theory which is very well tested. It postulates that electromagnetic field is a four-dimensional vector $A_k$ in Minkowski space–time. The conserved 4-current density $j^k$ is the source of the electromagnetic field which is coupled to $A_k$ through the Lorentz invariant term in the action

$$S_{\text{coupl}} = -\frac{1}{c^4} \int d^4x A_k j^k. \quad (5)$$

Such postulate allows us to construct classical electrodynamics in a unique way using symmetries of equation (5). Namely, conservation of current yields that $S_{\text{coupl}}$ is invariant under the gauge transformation $A_k \rightarrow A_k + \partial \phi / \partial x^k$. Action for the electromagnetic field $S_{\text{field}}$ must possess the same symmetries as the coupling term $S_{\text{coupl}}$. Such a requirement, together with the condition that $S_{\text{field}}$ must be quadratic in field derivates yields a unique answer for $S_{\text{field}}$

$$S_{\text{field}} = -\frac{1}{16\pi c} \int d^4x \left( \frac{\partial A_k}{\partial x^i} \frac{\partial A_i}{\partial x^k} - \frac{\partial A_k}{\partial x^i} \frac{\partial A_i}{\partial x^k} \right) \quad (6)$$

Variation of the total action of the system $S = S_{\text{field}} + S_{\text{coupl}} + S_{\text{matter}}$ with respect to $A_k$ yields Maxwell equations and variation with respect to the particle trajectories gives the Lorentz force. Thus, the whole classical electrodynamics is uniquely assembled from the structure of the coupling term (5).

In the next sections we will use the same algorithm to construct the classical vector theory of gravity. Namely, first
we postulate how the gravitational field is coupled to matter. Then, using symmetries of the coupling term, we uniquely assemble the total action and obtain the classical field equations from the variational principle. One should note that symmetries of the coupling term in vector gravity are very different from the symmetries of equation (5). As a consequence, the vector theory of gravity substantially differs from classical electrodynamics.

When we make a transition from the classical to quantum electrodynamics we must make an additional assumption, namely, that the quantum of the vector field \( A_k \), the photon, is an elementary particle. This postulate yields that photon is a boson because fermion field does not transform as a vector. However, a product of two fermion fields can transform as a vector. E.g., a four-vector field \( A_k \) can be created from a fermion–antifermion pair as [14]

\[
A_k = \Psi^\dagger \gamma_0 \gamma_k \Psi,
\]

where \( \gamma_4 \) are \( 4 \times 4 \) gamma matrices and \( \Psi \) is the fermion field. In this regard there is an interesting proposal that the photon is not an elementary particle but rather a composite particle formed of fermion–antifermion pairs (the so called composite theory of photon). The idea that the photon is a composite particle dates back to 1932, when Louis de Broglie [15] suggested that the photon is composed of neutrino–antineutrino pairs. Recall that many composite bosons, such as Cooper pairs, atoms with total integer spin, deuterons, pions, and kaons, are not perfect bosons because of their internal fermion structure, however in the asymptotic limit they are essentially bosons [16]. This suggests that photon could be a particle composed of spin-1/2 fermions as well. Work on the composite theory of photon continues to be of some interest [16–20].

The present paper deals with the vector gravity, rather than photons. However, as we show, the measured value of the energy loss by binary stars due to emission of gravitational waves can be explained in the vector gravity only if the quantum of the vector gravitational field (the graviton) is a composite particle formed of fermion–antifermion pairs. This fact makes a link between vector gravity and the composite theory of photon. We quantize gravitational field in section 11 assuming that graviton is a composite particle and obtain a wonderful result that quantum vector gravity is equivalent to QED.

One of the motivations for the composite photon theory is the lack of a well defined wave function for a single photon. In particular, the authors of the classic book on Quantum Optics [21] say: ‘There is, strictly speaking, no such a thing as a photon wave function’. If this is the case, in order for QED to be compatible with first quantization the photon can not be an elementary particle, but rather a particle composed of fermions. Recall that for fermions, described by the Dirac equation, the wave function is well defined. Hence, the assumption that photon is composed of fermions seems natural. For QED such assumption yields essentially no detectable experimental consequences (see section 11 and [16]). However, as we show in section 11, for quantum vector gravity the consequences are dramatic.

There is also a proposal that a fundamental unified theory of the gravitational, electroweak and strong interactions can be formulated in terms of only fermionic degrees of freedom [22, 23].

2. Postulates of the vector theory of gravity

The present vector theory of gravity is based on four postulates:

1. Background geometry of the Universe is a fixed four dimensional Euclidean space with metric \( \delta_{ik} = \text{diag}(1, 1, 1, 1) \). Such space is completely isotropic and has no preferred directions.

2. In the four dimensional Euclidean space there is a dynamical 4-vector field \( A_k \) (the gravitational field) which breaks the symmetry. Namely, direction of \( A_k \) is now preferred and this direction becomes the time coordinate. Directions perpendicular to \( A_k \) are three spatial coordinates.

3. Vector gravitational field is coupled to matter and all nongravitational fields through the equivalent metric \( f_{ik} \) which is an algebraic function of \( A_k \) and the background Euclidean metric \( \delta_{ik} \). Gravitational field couples universally and minimally to all the fields of the Standard Model by replacing everywhere the Minkowski metric \( \eta_{ik} \) with the equivalent metric \( f_{ik} \) and replacing partial derivatives with covariant derivatives formed from \( f_{ik} \). In particular, the trajectories of freely falling bodies are geodesics of the equivalent metric \( f_{ik} \) - Action for a point particle with mass \( m \) moving in the gravitational field reads

\[
S_{\text{matter}} = -mc \int \sqrt{f_{ik} \, dx^i \, dx^k},
\]

where \( c \) is the speed of light. Action (7) has the same form as in general relativity, however, the tensor gravitational field \( g_{ik} \) of general relativity is now replaced with the equivalent metric \( f_{ik} \). One should note that the Einstein equivalence principle is a consequence of the action (7).

4. The quantum of the vector field \( A_k \) (the graviton) is not an elementary particle, but rather it is a composite particle formed of massless fermion–antifermion pairs. Emission and absorption of a graviton corresponds to creation and annihilation of particle–antiparticle pairs.

For construction of the classical equations for the vector field \( A_k \) it does not matter whether the graviton is an elementary particle or a composite particle. However, processes involving gravitons, e.g. emission of gravitational waves, might depend on the graviton composition. The present vector theory of gravity can quantitatively explain the energy loss by binary stars orbiting each other provided that graviton is a composite particle.

The postulates 1–3 outlined above allow us to construct the classical vector theory of gravity in a unique way using symmetries of the action \( S_{\text{matter}} \) (7). Such symmetries uniquely specify the structure of the total action. We proceed with assembling the classical theory in the next sections.
3. Equivalent metric

In appendix A we show that Einstein equivalence principle yields the following unique expression for the equivalent metric $f_{ik}$ in terms of the vector field $A_k$ and the background Euclidean metric $\delta_{ik}$

$$f_{ik} = -\delta_{ik} + \left( A + \frac{1}{A} \right) A_i A_k A^2,$$

where

$$A = \sqrt{A_i A_k \delta^{ik}}.$$

Throughout the paper we use the usual conventions. Namely, unless otherwise noted, there is summation over repeated indices. Lower case Latin indices (i, k, l, ... ) label four dimensional coordinates (range 0, 1, 2, 3 ), while lower case Greek letters $\alpha, \beta, \gamma$ denote spatial coordinates (range 1, 2, 3).

In Cartesian coordinate system if we chose $x^0$-axis along the direction of $A_k$ the equivalent metric is diagonal and reads

$$f_{ik} = \text{diag}\left( A, -\frac{1}{A}, -\frac{1}{A}, -\frac{1}{A} \right).$$

Since $A_k$ is a dynamical variable one can make a transformation $A_k \rightarrow F(A)A_k$, where $F$ is an arbitrary function of $A$. Such a transformation changes the norm of $A_k$. It also modifies the expression for the metric (8) and field equations. However, the physical answer, e.g., motion of particles in gravitational field is independent of how we normalize the vector field. Thus, we can choose the field normalization in any suitable way.

Instead of $A_k$, it is convenient to introduce new independent functions, a scalar $\phi$ and a unit vector $u_k$, according to the relations

$$A = e^{2\phi}, \quad u_k = \frac{A_k}{A}.$$

The vector $u_k$ has the unit norm

$$u_i u_k \delta^{ik} = 1.$$

In terms of the unit vector $u_k$ and the scalar $\phi$ the equivalent metric (8) reads

$$f_{ik} = -e^{-2\phi} \delta_{ik} + 2 \cosh(2\phi) u_i u_k,$$

while metric $\tilde{f}_{ik}$ inverse to $f_{ik}$, defined as $\tilde{f}^{ik} f_{jm} = \delta^i_m$, is

$$\tilde{f}^{ik} = -e^{2\phi} \delta_{ik} + 2 \cosh(2\phi) u'^i u'^k,$$

where

$$u'^i = \delta^{ik} u_k.$$

In the present paper raising and lowering of the indices is performed using Euclidean metric $\delta_{ik}$, unless otherwise stated. We denote determinant of $f_{ik}$ as $f$. Equation (9) yields

$$\sqrt{-f} = e^{-2\phi}.$$

One should note that in the literature there were attempts to construct a vector theory of gravity in background Minkowski (rather than Euclidean) metric $\eta_{ik} = \text{diag} (1, -1, -1, -1)$ by Rastall [24, 25] and by the author [26]. The equivalent metric obtained in such theories has a form similar to our equation (9) in which Euclidean metric $\delta_{ik}$ is replaced with $-\eta_{ik}$ and $\cosh(2\phi)$ is replaced with $\sinh(2\phi)$. However, the main achievement of the present paper compared to the previous work on vector gravity is the discovery of how to obtain the action for the gravitational field $S_{\text{gravity}}$ based on symmetries of the coupling term $S_{\text{matter}}$. In the previous attempts to construct the theory $S_{\text{gravity}}$ has not possessed symmetries of $S_{\text{matter}}$ which yielded no success.

4. Action for gravitational field

Next we construct action of the system in terms of $\phi$ and $u_k$. The total action for the gravitational field and matter is given by

$$S = S_{\text{gravity}} + S_{\text{matter}},$$

where $S_{\text{matter}}$ is the action of matter written in curvilinear coordinates with the metric $f_{ik}$. We obtain the action for the gravitational field $S_{\text{gravity}}$ using the requirement that symmetries of $S_{\text{matter}}$ and $S_{\text{gravity}}$ must be the same. $S_{\text{matter}}$ possesses the following symmetry: it is invariant under coordinate transformations that leave background Euclidean metric $\delta_{ik}$ intact. This symmetry is exact. In addition there are approximate symmetries. For small deviations of $\phi$ from a constant value $\phi_0$ and small deviations of $u_k$ from (1, 0, 0, 0) in the rescaled coordinates

$$x^0 \rightarrow e^{-\phi_0} x^0, \quad x^\alpha \rightarrow e^{\phi_0} x^\alpha$$

the equivalent metric is given by

$$f_{ik} = \eta_{ik} + \begin{pmatrix} h_{00} & h_{01} & h_{02} & h_{03} \\ h_{01} & h_{00} & 0 & 0 \\ h_{02} & 0 & h_{00} & 0 \\ h_{03} & 0 & 0 & h_{00} \end{pmatrix},$$

where

$$h_{00} = 2(\phi - \phi_0), \quad h_{0\alpha} = 2 \cosh(2\phi_0) u_\alpha,$$

$\alpha = 1, 2, 3$ and $\eta_{ik} = \text{diag} (1, -1, -1, -1)$ is the Minkowski metric. For small deviations of $f_{ik}$ from the Minkowski metric, using

$$\delta S_{\text{matter}} = -\frac{1}{2c} \int d^4 x \sqrt{-f} \tilde{f}^{ik} \delta f_{ik},$$

we have

$$\delta S_{\text{matter}} \approx -\frac{1}{2c} \int d^4 x (T^{00} h_{00} + 2 T^{0\alpha} h_{\alpha 0} + T^{\alpha \beta} h_{\alpha \beta}),$$

where $T^{ik}$ is the energy–momentum tensor of matter. Recall that the energy–momentum tensor $T^{ik}$ is obtained as a functional derivative of $L_{\text{matter}}$ with respect to the metric tensor $g_{ik}$ [27]

$$T^{ik} = -\frac{2}{\sqrt{-g}} \frac{\delta (\sqrt{-g} L_{\text{matter}})}{\delta g_{ik}},$$

where $L_{\text{matter}}$ is the nongravitational part of the Lagrangian density of the action written in curvilinear coordinates with
metric \( g_{\mu \nu} \)

\[
S_{\text{matter}} = \frac{1}{c} \int d^4 x \sqrt{-g} L_{\text{matter}}.
\]

Therefore [27]

\[
\delta S_{\text{matter}} = -\frac{1}{2c} \int d^4 x \sqrt{-g} T^{\mu \nu} \delta g_{\mu \nu}
\]

which yields equation (15).

One can see from equation (16) that in the rescaled coordinates the action \( S_{\text{matter}} \) is independent of the background cosmological field \( \phi_0 \). This is one of the symmetries of \( S_{\text{matter}} \). Another symmetry can be found by taking into account equation (16) and approximate energy conservation in the Minkowski metric

\[
\frac{\partial T^{00}}{\partial x^0} + \frac{\partial T^{0\alpha}}{\partial x^\alpha} = 0
\]

which yields that action \( S_{\text{matter}} \) is approximately invariant under the gauge transformation

\[
h_{00} \to h_{00} + 2 \frac{\partial \psi}{\partial x^0}, \quad h_{0\alpha} \to h_{0\alpha} + \frac{\partial \psi}{\partial x^\alpha}
\]

upto the terms quadratic in the mass velocity \( V^\alpha = dx^\alpha/d\tau \). Here \( \psi \) is an arbitrary scalar function.

In addition, there is approximate Lorentz invariance. Namely, the line element that enters the matter action (7) can be approximately written in the metric (13) as

\[
dx^2 = f_\delta dx^i dx^k \approx \eta_{ik} dx^i dx^k + h_{00}(dx^0)^2 + 2h_{0i}dx^i dx^0 + h_{00} d\tau^2
\]

which is invariant (upto the terms of the order of \( V^3/c^3 \)) under transformation

\[
x^0 \to \left(1 + \frac{V^2}{2c^2}\right)x^0 + \frac{1}{c} \mathbf{V} \cdot \mathbf{r}, \quad \mathbf{r} \to \mathbf{r} + \frac{\mathbf{V}}{c} x^0,
\]

\[
h_{00} \to h_{00} \left[1 + \frac{2V^2}{c^2}\right] - 2\frac{V^0}{c} h_{0i}, \quad h_{0i} \to h_{0i} - 2\frac{V^i}{c} h_{00},
\]

where \( \mathbf{V} \) is a constant (velocity) vector and \( \alpha = 1, 2, 3 \). In equation (18) \( h_{0i} \) and \( d\tau/dx^0 \) are of the order of \( V/c \).

Requirement that \( S_{\text{gravity}} \) must also possess these symmetries, namely, \( S_{\text{gravity}} \) should be invariant under Euclidean transformations; for small deviations from the background field be independent of \( \phi_0 \) after scaling transformation (12); and approximately invariant under the gauge (17) and low-velocity Lorentz (19), (20) transformations, allows us to find \( S_{\text{gravity}} \) uniquely\(^1\). In appendix B we obtain that the gravitational field action in the background four dimensional Euclidean space is

\[
S_{\text{gravity}} = \frac{c^3}{8\pi G} \int d^4x \left\{ \frac{\partial \phi}{\partial x^0} \frac{\partial \phi}{\partial x^0} \left[-\delta^0k + (1 - 3e^{-4\phi}) u^mu^m u^k\right] \frac{\partial^2 \phi}{\partial x^0 \partial x^k} \right. \\
+ \cosh^2(2\phi) \frac{\partial u^\mu}{\partial x^0} \frac{\partial u^\mu}{\partial x^0} \left( \delta^0k - 3e^{-4\phi} \delta^0k - (1 + e^{-4\phi}) \delta^0k\right) \\
\left. + 2(1 + e^{-4\phi}) \frac{\partial \phi}{\partial x^0} \frac{\partial u^\mu}{\partial x^0} \right\}
\]

(21)

where \( G \) is the gravitational constant.

Action (21) is written in Euclidean metric, it has no free parameters and serves as a foundation of the present theory of gravity. Our derivation of the action (21) is unique and, hence, the classical vector theory of gravity is also a unique consequence of the postulates 1–3.

5. Equations for classical gravitational field

Taking variation of the total action (11) with respect to \( \phi \) and unit vector \( u_k \) yields four equations for the classical gravitational field in the background Euclidean space (see appendix C for derivation details)

\[
\delta u^m = -2\delta^m u^k + (1 + 3e^{-4\phi}) u^m u^ku^k \frac{\partial^2 \phi}{\partial x^m \partial x^k} \\
+ 2\delta^m \left(3e^{-4\phi} - 1\right) u^mu^k \frac{\partial \phi}{\partial x^m} \frac{\partial u^k}{\partial x^k} \\
+ 2\left(e^{-4\phi} - 2e^{-4\phi} - 1\right) \delta^m u^mu^k - \left(1 + 3e^{-4\phi}\right) \delta^m u^mu^k \\
- \left(2e^{-4\phi} + 3e^{-4\phi} - 1\right) \delta^m u^mu^k \frac{\partial \phi}{\partial x^m} \frac{\partial u^k}{\partial x^k} \\
+ \cosh(2\phi) \left[2e^{-2\phi} \frac{\partial u^k}{\partial x^0} = \frac{\partial u^k}{\partial x^0} \Delta u^k + \left(2e^{-2\phi} + 2e^{-2\phi}\right) u^m u^mu^k \right] \\
\frac{\partial u^m}{\partial x^m} \frac{\partial u^m}{\partial x^m} \frac{\partial u^k}{\partial x^k} \\
+ 2e^{-2\phi} \left(2\phi \frac{\partial u^k}{\partial x^0} = \frac{\partial u^k}{\partial x^0} \Delta u^k + \left(2e^{-2\phi} + 2e^{-2\phi}\right) u^m u^mu^k \right]
\]

(22)

where \( \Delta u^m = \frac{\partial u^m}{\partial x^m} - \frac{u^m}{c^2} \frac{d\tau}{dx^0} \) is the energy–momentum tensor of matter, \( T = T^m_{\phantom{m}m} f_{mk} \) is the trace of the energy–momentum tensor and \( f_{\mu k} \) is the metric inverse to \( f_{\mu k} \) given by equation (10). The right-hand side of equations (22) is the source of the gravitational field.

Equations (22) for the scalar \( \phi \) and the unit vector \( u_k \) are the main equations of the vector theory of gravity. They are written in Euclidean metric which means that raising and lowering of indexes is carried out using metric \( \delta_{ik} = \text{diag} \)
(1, 1, 1, 1). Equations (22) play the same role in vector gravity as Einstein equations in general relativity.

In our theory the motion of particles in gravitational field is described by the same equation as in general relativity

\[
\frac{d^2x^k}{ds^2} = \frac{1}{2} \left[ \frac{\partial f^k_{ij}}{\partial x^i} - \frac{\partial f^k_{ij}}{\partial x^j} - \frac{\partial f^j_{ik}}{\partial x^j} \right] dx^i dx^j, \tag{23}
\]

where ds = \sqrt{f_{ik} dx^i dx^k}. In equation (23) the metric \(g_{ik}\) of general relativity is replaced with the equivalent metric \(f_{ik}\). We obtain equation of particle motion in appendix D.

Next we explore solutions of the classical gravitational field equations (22) for various cases.

6. Static gravitational field

In this section we consider gravitational field produced by rest matter distributed with density \(\rho(r)\). This situation is idealized since gravitational interaction will lead to mass motion unless there are other forces that keep matter at rest. Here we solve gravitational field equations assuming that masses are static. Self-consistent solution of the problem requires solving the field equations together with the equation of motion for masses. Here we replace the latter with the constraint that position of masses is fixed.

For static field \(u_0 = (1, 0, 0, 0)\), scalar \(\phi\) depends only on spatial coordinates \(r\) and the equivalent metric (9) reads

\[
f_{ik} = \begin{pmatrix}
e^{2\phi(r)} & 0 & 0 & 0 \\
0 & -e^{2\phi(r)} & 0 & 0 \\
0 & 0 & -e^{2\phi(r)} & 0 \\
0 & 0 & 0 & -e^{2\phi(r)}
\end{pmatrix}, \tag{24}
\]

\(\sqrt{-f} = e^{-2\phi}\), while the inverse metric is

\[
f^{ik} = \begin{pmatrix}
e^{-2\phi} & 0 & 0 & 0 \\
0 & e^{-2\phi} & 0 & 0 \\
0 & 0 & e^{-2\phi} & 0 \\
0 & 0 & 0 & e^{-2\phi}
\end{pmatrix}. \tag{25}
\]

For static masses the energy–momentum tensor of matter has only one nonzero component \(T^{(0)}\) which depends on spatial coordinates. This component can be found from the conservation equation

\[
T^{ik}_{;k} = 0, \tag{26}
\]

where ‘;’ stands for the covariant derivative with metric \(f_{ik}\). Equation (26) yields \(T^{00} = \rho c^2 e^\phi\) and \(T = \rho c^2 e^{2\phi}\), where \(\rho\) is the mass density which is independent of \(\phi\).

For static gravitational field, equations (22) reduce to a single equation for \(\phi(r)\)

\[
\Delta \phi = \frac{4\pi G}{c^2} \rho e^\phi. \tag{27}
\]

In the Newtonian limit equation (27) yields \(\Delta \phi = 4\pi G \rho e^\phi / c^2\) and, thus, \(c^2 \phi(r)\) has a meaning of gravitational potential.

Exponential metric solution (24) is free of black holes for any mass distribution and field strength. For a point mass \(M\) located at \(r = 0\) equation (27) leads to \(c^2 \Delta \phi = 4\pi GM\delta(r)\) and has a solution \(\phi = -GM/c^2 r\). For \(N\) point masses located at \(r_1, \ldots, r_N\) equation (27) yields

\[
\Delta \phi = 4\pi \sum m_i \delta(r_i - r) + m_N \delta(r_N), \tag{28}
\]

where \(m_1, \ldots, m_N\) are positive constants. Solution of equation (28) is

\[
\phi(r) = \frac{m_1}{|r - r_1|} + \ldots + \frac{m_N}{|r - r_N|}. \tag{29}
\]

We discuss motion of particles in static gravitational field in appendix E. For a star of mass \(M\) and radius \(R\) equation (29) reduces to \(\phi(r) = -GM/c^2 r (r > R)\) and using equation (E1) for energy conservation we obtain that escape velocity for a particle from the stellar surface is

\[
v = c_s \sqrt{1 - e^{2\phi(R)}}, \tag{30}
\]

where \(c_s = c e^{2\phi(R)}\) is the speed of light at the stellar surface (see equation (E9)). Equation (30) shows that escape velocity is always smaller then \(c_s\) (\(c_s \leq c\)).

One should note that exponential metric (24) was also obtained for static field in some alternative theories of gravity \([25, 26, 28–32]\) and based on simple physical arguments in \([24, 33, 34]\). Stability of compact stars in the exponential metric has been investigated in the literature. It has been shown that solution (24) predicts that stars do not collapse into a point singularity but rather form stable compact objects with no event horizon and finite gravitational redshift \([35]\).

7. Post-Newtonian limit

Post-Newtonian limit applies when the gravitational field is weak, and the motion of the matter is slow. It is sufficiently accurate to encompass all solar-system tests of gravity performed so far. In the post-Newtonian formalism the metric is expanded in a small parameter \(\epsilon\). The ‘order of smallness’ is determined according to the rules that matter velocity is of order \(V \sim \epsilon^{1/2}\) and gravitational constant \(\sim \epsilon\). A consistent post-Newtonian limit requires determination of \(g_{00}\) correction through \(O(\epsilon^2)\), \(g_{0i}\) through \(O(\epsilon^{3/2})\), and \(g_{ij}\) through \(O(\epsilon)\) [2].

We compare the vector theory of gravity with general relativity in the post-Newtonian limit in the cosmological reference frame (the mean rest frame of the Universe in which the Universe appears isotropic) for which background equivalent metric \(f_{ik}\) is diagonal and after rescaling coordinates reduces to Minkowski metric. As it is shown in [2] , comparison of any metric theory of gravity with general relativity can be done in any suitable reference frame. The rest
Boundary conditions for the gravitational field $g_{ik}$ from the Minkowski metric $\eta_{ik}$

$$g_{ik} = \eta_{ik} + h_{ik}.$$  

In the post-Newtonian limit of general relativity in the post-Newtonian gauge [2]

$$h_\beta^\alpha = -h_{00}h_\beta^\alpha$$  \hspace{1cm} (31)

and the tensor gravitational field $g_{ik}$ is described by four independent functions $h_{ik}$, so that metric is given by

$$g_{ik} = \eta_{ik} + \begin{pmatrix} h_{00} & h_{01} & h_{02} & h_{03} \\ h_{01} & h_{00} & 0 & 0 \\ h_{02} & 0 & h_{00} & 0 \\ h_{03} & 0 & 0 & h_{00} \end{pmatrix}.$$  \hspace{1cm} (32)

In the present vector theory of gravity in the post-Newtonian limit after rescaling of coordinates in the cosmological reference frame the equivalent metric $f_{ik}$ also has the form of equation (32) (see equation (13)). Therefore, in both theories in the post-Newtonian limit the gravitational fields are described by an equal number of independent functions which are coupled with matter in the same way. Since, by construction of both theories, such coupling uniquely specifies the total action and, hence, the field equations, the general relativity and the vector theory of gravity are identical in the post-Newtonian limit.

To convince the reader that this is indeed the case, in appendix F we show directly that in the post-Newtonian limit in the cosmological reference frame the equivalent metric

$$\frac{1}{2} \Delta h_{00} + \frac{3}{2} \frac{\partial^2 h_{00}}{\partial x^0 \partial x^0} - \frac{\partial^2 h_{00}}{\partial x^0 \partial x^0} + \frac{1}{2} h_{00} \Delta h_{00} - \frac{1}{2} (\nabla h_{00})^2 \right) = \frac{8 \pi G}{c^4} T_{00} - \frac{1}{2} \frac{8 \pi G}{c^4} T_{00}.$$  \hspace{1cm} (33)

$$\frac{1}{2} \Delta h_{00} - \frac{\partial h_{00}}{\partial x^0} + \frac{\partial^2 h_{00}}{\partial x^0 \partial x^0} = \frac{8 \pi G}{c^4} T_{00}.$$  \hspace{1cm} (34)

Boundary conditions for $h_{ik}$ are also the same and, therefore, both theories are equivalent in the post-Newtonian limit.

We made comparison of the theories in the cosmological reference frame assuming that matter moves relative to this frame with nonrelativistic speed. Since both theories are equivalent in such a frame in the post-Newtonian limit they are also equivalent in a frame moving with nonrelativistic velocity relative to the mean rest frame of the Universe. This is the case because in vector gravity the equivalent metric $f_{ik}$, by its definition, is also a tensor under general coordinate transformations. If equations of vector gravity and general relativity give the same metric in one reference frame then in any other frame the two metrics will coincide because both of them transform in the same way under coordinate transformation from one frame to another.

This result can be also obtained by noting that equations (33) and (34) are invariant under the low-velocity Lorentz transformation

$$\frac{\partial}{\partial x^0} \rightarrow \frac{\partial}{\partial x^0} - \frac{V}{c} \nabla,$$

$$\frac{\partial}{\partial x} \rightarrow \frac{\partial}{\partial x} - \frac{V}{c} \frac{\partial}{\partial x^0} + \frac{V}{c^2} \left( \frac{\partial}{\partial x} \right)$$  \hspace{1cm} (35)

$$h_{00} \rightarrow h_{00} \left( 1 + \frac{2V^2}{c^2} \right) - \frac{2V^2}{c^2} h_{00}, \quad h_{0a} \rightarrow h_{0a} - \frac{V^2}{c^2} h_{00}.$$  \hspace{1cm} (36)

$$T^{00} \rightarrow T^{00} + \frac{2V^2}{c^2} h_{00}, \quad T^{0a} \rightarrow T^{0a} + \frac{V^2}{c^2} T^{00}.$$  \hspace{1cm} (37)

for which $h_{ik}$ and $T^{ik}$ transform as symmetric tensors (keeping in mind that in the post-Newtonian limit $h_{00} = h_{00}$), and $V$ is a constant (velocity) vector. Thus, for any reference frame moving with nonrelativistic speed relative to the mean rest frame of the Universe (e.g., frame of the solar system) equations of vector gravity and general relativity are equivalent in the post-Newtonian limit and have the form of equations (33) and (34).

As a consequence, the vector theory of gravity, similarly to general relativity, yields no post-Newtonian preferred frame and preferred location effects. For vector gravity the ten post-Newtonian parameters introduced to compare metric theories of gravity with each other [2, 3] have the same values as in general relativity.

To convince the most sceptical reader, in appendix J we investigate the post-Newtonian limit of vector gravity in the framework of the parametrized post-Newtonian formalism following the 9-step procedure of [2] and explicitly calculate the ten post-Newtonian parameters. As expected, they are equal to those in general relativity.

8. Weak field limit

8.1. Linearized gravitational field equations

In this section we linearize equations for classical gravitational field in the cosmological reference frame assuming that unit vector $u_k$ slightly deviates from $(1, 0, 0, 0)$. For small deviations of $\phi$ from a constant value $\phi_0$ and $|u_k| \ll 1$, keeping linear terms, equations (22) yield

$$\Delta \phi + 3e^{-4\phi_0} \frac{\partial^2 \phi}{\partial x^0 \partial x^0} = 2e^{-2\phi_0} \frac{\partial^2 \mu_0}{\partial x^0 \partial x^0} - \frac{8 \pi G}{c^4} \left( T^{00} - \frac{T^{00}}{2} \right) + \cos(2\phi_0) \left( e^{2\phi_0} \frac{\partial^2 \mu_0}{\partial x^0 \partial x^0} - 2e^{-2\phi_0} \frac{\partial^2 \mu_0}{\partial x^0 \partial x^0} \right)$$

$$-2e^{-2\phi_0} \frac{\partial^2 \phi}{\partial x^0 \partial x^0} = \frac{8 \pi G}{c^4} T^{00}.$$  \hspace{1cm} (37)

In the rescaled coordinates

$$x^0 \rightarrow e^{-\phi_0} x^0, \quad x^\alpha \rightarrow e^{\phi_0} x^\alpha,$$

the equivalent metric is given by
\[ f_{\alpha k} = \eta_{\alpha k} + \left( \frac{h_{00} h_{01} h_{02} h_{03}}{h_{01} h_{00} 0 0} \right) \left( \begin{array} {cccc} h_{02} & 0 & 0 & 0 \\ 0 & h_{00} & 0 & 0 \\ 0 & 0 & h_{03} & 0 \\ 0 & 0 & 0 & h_{00} \end{array} \right), \]

where
\[ h_{00} = 2(\phi - \phi_0), \quad h_{0a} = 2\cosh(2\phi_0)u_a \]
and \( \alpha = 1, 2, 3 \). In terms of \( h_{0a} \) equations for the gravitational field in the weak field limit read
\[ \Delta h_{00} + 3 \frac{\partial^2 h_{00}}{\partial x^0 \partial x^0} - 2 \frac{\partial^2 h_{0a}}{\partial x^0 \partial x^a} = \frac{16\pi G}{c^4} \left(T^{00} - \frac{T}{2} \right). \] \tag{38}

\[ \left( \frac{\partial^2}{\partial x^0 \partial x^0} - \Delta \right) h_{0a} + \frac{\partial^2 h_{0b}}{\partial x^0 \partial x^b} - 2 \frac{\partial^2 h_{00}}{\partial x^a \partial x^0} = \frac{16\pi G}{c^4} T^{0a}. \] \tag{39}

In these equations the energy–momentum tensor of matter \( T^{\alpha \beta} \) is written in Minkowski metric. Equations (38) and (39) are invariant up to the terms of the order of \( V^2/c^4 \) under low-velocity Lorentz transformation \( (35)-(37) \). Therefore, they remain the same in inertial reference frames moving with non-relativistic velocity relative to the rest frame of the Universe. Equation (23) yields that no relativistic motion of particles in weak gravitational field is described by the following equation
\[ \frac{1}{c^2} \frac{dV^\alpha}{dr} = \frac{\partial h_{0\alpha}}{\partial x^0} - \frac{1}{2} \frac{\partial h_{00}}{\partial x^\alpha} \left( \frac{V^0}{c} \right)^2 + \frac{\partial h_{00} V^\alpha}{\partial x^0} c, \] \tag{40}

where \( V^\alpha = dx^\alpha/dr \) is the particle velocity.

In appendix G we explore an analogy between weak gravity and electrodynamics and show that equations for weak classical gravitational field are analogous to Maxwell’s equations in a medium with negative refractive index.

### 8.2. Energy density and energy flux in the classical limit of vector gravity

In appendix H we derive expression for the energy density and energy density flux (Poynting vector) for the weak classical gravitational field interacting with matter that moves with nonrelativistic velocity. We find the following expression for the energy density
\[ w = - \frac{c^4}{32\pi G} \left[ 3 \left( \frac{\partial h_{00}}{\partial x^0} \right)^2 - (\nabla h_{00})^2 + \left( \frac{\partial h}{\partial x^0} \right)^2 + \text{curl}^2 \mathbf{h} \right] + \rho c^2 + \frac{1}{2} \rho c^2 h_{00} + \frac{1}{2} \rho V^2, \] \tag{41}

where \( \rho \) is the mass density, \( \mathbf{V} \) is the matter velocity and three dimensional vector
\[ \mathbf{h} = h^{0\alpha}. \]

The energy density flux is given by
\[ \mathbf{S} = \frac{c^5}{16\pi G} \left[ - \left( \frac{2}{c^2} \frac{\partial \mathbf{h}}{\partial x^0} + \nabla h_{00} \right) \frac{\partial h_{00}}{\partial x^0} + \frac{\partial \mathbf{h}}{\partial x^0} \times \text{curl} \mathbf{h} \right] + \rho c \mathbf{V}. \] \tag{42}

Equation of the energy conservation reads
\[ \frac{\partial \mathbf{S}}{\partial x^0} + \text{div} \mathbf{S} = 0. \]

#### 8.3. Gravitational waves

The weak field limit homogeneous equations for the classical gravitational field
\[ \Delta h_{00} + 3 \frac{\partial^2 h_{00}}{\partial x^0 \partial x^0} - 2 \frac{\partial^2 h_{0a}}{\partial x^0 \partial x^a} = 0, \] \tag{43}

\[ \left( \frac{\partial^2}{\partial x^0 \partial x^0} - \Delta \right) h_{0a} + \frac{\partial^2 h_{0b}}{\partial x^0 \partial x^b} - 2 \frac{\partial^2 h_{00}}{\partial x^a \partial x^0} = 0 \] \tag{44}

have solutions describing waves propagating with the speed of light \( c \). Taking \( \partial/\partial x^a \) from equation (43) and \((1/2)\partial/\partial x^a \) from equation (44) and adding them together we obtain
\[ \frac{\partial^2}{\partial x^a \partial x^0} \left( \frac{\partial h_{00}}{\partial x^0} - \frac{1}{2} \frac{\partial h_{00}}{\partial x^a} \right) = 0. \]

Thus, for the time-dependent solutions describing gravitational waves
\[ \frac{\partial h_{00}}{\partial x^0} - \frac{1}{2} \frac{\partial h_{00}}{\partial x^a} = 0 \]

and equations (43), (44) reduce to separate wave equations for \( h_{00} \) and \( h_{0a} \)
\[ \left( \frac{\partial^2}{\partial x^0 \partial x^0} - \Delta \right) h_{00} = 0, \]
\[ \left( \frac{\partial^2}{\partial x^0 \partial x^a} - \Delta \right) h_{0a} = 0. \]

Field equations have two classes of solutions corresponding to transverse and longitudinal waves. For transverse waves
\[ \frac{\partial h_{0a}}{\partial x^a} = 0, \quad h_{00} = 0 \]

and, e.g., for a transverse wave propagating along the \( x \)-axis the equivalent metric reads
\[ f_{\alpha k} = \eta_{\alpha k} + \left( \begin{array} {cccc} 0 & 0 & h_{0x}(t, x) & h_{0z}(t, x) \\ 0 & 0 & 0 & 0 \\ h_{0z}(t, x) & 0 & 0 & 0 \\ h_{0x}(t, x) & 0 & 0 & 0 \end{array} \right). \] \tag{45}
According to equations (41) and (42), the energy density and the energy density flux of the transverse gravitational wave is

$$ w_t = -\frac{c^4}{32\pi G} \left[ \left( \frac{\partial h}{\partial x^a} \right)^2 + \text{curl}^2 h \right], \quad (46) $$

$$ S_u = \frac{c^5}{16\pi G} \frac{\partial h}{\partial x^a} \times \text{curl}(h). \quad (47) $$

Thus, graviton has negative energy in the classical description of the gravitational field. This result has important implication for cosmology. If graviton energy is negative and the matter energy is positive this suggests that at the Big Bang matter was created at the expense of generation of the negative energy gravitons. We address this issue in section 10. In section 11 we show that in the quantum limit (present epoch) the graviton energy is positive. In this limit the energy density and the energy density flux are given by the same equations (46) and (47) but with the opposite sign.

For a plane transverse wave

$$ h = h_0 \cos(\omega t - kr) $$

Equation (47) yields

$$ S_u = -\frac{c^4h_0^2\omega}{16\pi G} \kappa \sin^2(\omega t - kr), $$

that is Poynting vector which gives the direction of the energy flow is opposite to the wave vector k in the classical limit. This is analogous to propagation of electromagnetic waves in a medium with negative refractive index [36]. Thus, vacuum for the classical vector gravitational field is left-handed.

For a longitudinal wave $h_{00} = 0$ and $\partial h_{00}/\partial x^a = 2\partial h_{00}/\partial x^0$. For such wave propagating along the x-axis the metric oscillates as

$$ f^\text{long}_{ik} = \eta_{ik} + \begin{pmatrix}
    h_{00}(t, x) & h_{00}(t, x) & 0 & 0 \\
    h_{00}(t, x) & h_{00}(t, x) & 0 & 0 \\
    0 & 0 & h_{00}(t, x) & 0 \\
    0 & 0 & 0 & h_{00}(t, x)
\end{pmatrix}. \quad (48) $$

As we show in section 12, binary stars orbiting each other do not emit longitudinal gravitational waves. Quantum mechanical analysis of section 11.4 yields the same answer. However, longitudinal gravitational waves can be generated during star mergers or in early Universe.

One should mention that gravitational waves in the vector gravity substantially differ from those in general relativity. In general relativity the metric for weak plane gravitational waves propagating along the x-axis in a properly chosen coordinate system reads [27]

$$ g_{ik} = \eta_{ik} + \begin{pmatrix}
    0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 \\
    0 & h_{yy}(t, x) & h_{yz}(t, x) & 0 \\
    0 & h_{yz}(t, x) & -h_{yy}(t, x)
\end{pmatrix}. \quad (49) $$

where $h_{yy}$ and $h_{yz}$ are small perturbations obeying the wave equation $\Box h_{yy} = 0, \quad \Box h_{yz} = 0,$

where $\Box$ is the d’Alembertian operator. By making proper change of coordinates one can transform equations (45) and (48) for plane waves in vector gravity to

$$ f^a_{ik} = \eta_{ik} + \begin{pmatrix}
    0 & 0 & 0 & 0 \\
    0 & h_{yy}(t, x) & h_{yz}(t, x) & 0 \\
    0 & h_{yz}(t, x) & 0 & 0 \\
    0 & 0 & 0 & h(t, x)
\end{pmatrix}. \quad (50) $$

$$ f^\text{long}_{ik} = \eta_{ik} + \begin{pmatrix}
    0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 \\
    0 & -2h(t, x) & 0 & 0 \\
    0 & 0 & h(t, x) & 0
\end{pmatrix}. \quad (51) $$

which have different structure than general relativistic equation (49). Thus, measuring polarization of gravitational waves with gravitational wave detectors could provide a test of the vector gravity.

One should note, however, that both in general relativity and vector gravity the polarization of gravitational waves emitted by orbiting binary stars is transverse, that is wave produces motion of test particles in the plane perpendicular to the direction of wave propagation. The reader might get an impression that equation (50) does not describe a transverse wave. This illusion appears because metric (50) is written in coordinate system in which test particles do not move under the influence of gravitational wave. The original metric (45) clearly shows that the wave is transverse. According to equation (40), a rest particle (or mirrors of an interferometer) will move under the influence of the gravitational wave (45) with time-dependent velocity $V^a = h_{00}c.$ Metric (50) is obtained from (45) by making a coordinate transformation into the co-moving frame

$$ x'^a = x^a - \int_0^t V^a dt,$$
which yields the following relation between components of the metrics (45) and (50): \( h_{yy} = h_{0y}, h_{zz} = h_{0z} \).

Signal of the LIGO-like interferometer with arms of length \( L \) along the direction of unit vectors \( \hat{a} \) and \( \hat{b} \) is proportional to the relative phase shift \( \Delta \varphi \) of electromagnetic waves traveling a roundtrip distance \( 2L \) along the two arms (for details see section 16)

\[
\Delta \varphi = \omega \frac{L}{c} [h^{\alpha \beta}(\hat{a}_i \hat{a}_j - \hat{b}_i \hat{b}_j)],
\]

(52)

where \( \omega \) is the frequency of electromagnetic wave and \( h^{\alpha \beta} \) is a perturbation of the metric in the reference frame in which interferometer mirrors do not move (frame of equations (49) and (50)). For the gravitational wave propagating along the \( x \)-axis equation (52) yields for gravitational wave (49) in general relativity

\[
\Delta \varphi = \omega \frac{L}{c} [(h^{yy}(\hat{a}_x^2 - \hat{b}_x^2 + \hat{b}_y^2 - \hat{a}_y^2) + 2h^{yz}(\hat{a}_y \hat{a}_z - \hat{b}_y \hat{b}_z))],
\]

(53)

while for the transverse wave (50) in vector gravity we obtain

\[
\Delta \varphi = \omega \frac{2L}{c} [(h^{yy}(\hat{a}_x \hat{a}_y - \hat{b}_x \hat{b}_y) + h^{zz}(\hat{a}_z \hat{a}_z - \hat{b}_z \hat{b}_z))].
\]

(54)

Equations (53) and (54) show that vector gravity and general relativity predict qualitatively different effect of the gravitational wave on the interferometer signal. Namely, general relativistic gravitational wave of any polarization (arbitrary \( h^{yy} \) and \( h^{yz} \)) produces no signal when gravitational wave propagates parallel to the interferometer plane at \( 45^\circ \) angle relative to one of its perpendicular arms (see figure 1(a)). For these orientations the gravitational wave in vector gravity can produce signal.

On the other hand, transverse gravitational wave in vector gravity (for arbitrary \( h^{yy} \) and \( h^{yz} \)) yields no signal if gravitational wave propagates in the direction perpendicular to the interferometer plane (see figure 1(b)), or along one of the interferometer arms.

This difference can be used to distinguish between general relativity and vector gravity in experiments with several gravitational wave interferometers, e.g., in a joint run of the LIGO–Virgo network. We discuss details of such experiment in section 16.

9. Cosmology

In this section we apply our theory to evolution of the Universe. Cosmological model of the Universe is an effective model which replaces a complicated infinitely large spatially nonuniform system of moving matter with those that has uniform density distribution and zero velocity. In such a model the unit gravitational field vector is \( u_i = (1, 0, 0, 0) \) and the scalar \( \phi \) depends only on time. However, correct description of the Universe evolution requires solution of the full system of spatially nonuniform equations for the gravitational field and then averaging of the result over the large scales. Symmetry arguments yield that such an averaging can give an effective gravitational field action with an additional cosmological term of the form

\[
S_{\text{cosm}} = -c\Lambda \int \mathrm{d}^4x \sqrt{-f},
\]

(55)

where \( \Lambda \) is a constant independent of the gravitational field. The structure of \( S_{\text{cosm}} \) follows from the requirement that after rescaling of coordinates (12) the action should be independent of the background cosmological field. This is one of the symmetries of the full action and the effective action must possess such a symmetry. The cosmological term appears due to replacement of the exact equations with the equations for the averaged metric which is spatially uniform and isotropic. Since the term (55) is an effective, \( \Lambda \) depends on the choice of coordinate system, namely on the reference frame in which we perform the spatial averaging.

For cosmology, instead of \( \phi \), it is convenient to use a scale (expansion) factor

\[
a = e^{-\phi}
\]

as a variable. In terms of \( a \) the equivalent metric (9) reads

\[
f_{ik} = \begin{pmatrix}
\frac{1}{a^2} & 0 & 0 & 0 \\
0 & -a^2 & 0 & 0 \\
0 & 0 & -a^2 & 0 \\
0 & 0 & 0 & -a^2
\end{pmatrix},
\]

(56)

and \( \sqrt{-f} = a^2 \). Components of the energy–momentum tensor of matter can be obtained from the conservation equation (26). For cold Universe (\( P = 0 \)) there is only one nonzero component which is given by \( T^{00} = p c^2 / a^4 \) and, hence, \( T = p c^2 / a^4 \), where \( p \) is a constant that has a meaning of the matter density for \( a = 1 \).

Spatial part of nonlinear gravitational field equations (22) \( (i = \alpha = 1, 2, 3) \) gives a simple linear equation for \( a(t, \mathbf{r}) \)

\[
\frac{\partial^2 a}{\partial x^i \partial t} = 0.
\]

(57)

The solution of this equation with the initial condition \( a(t = 0, \mathbf{r}) = b(\mathbf{r}) \) is

\[
a(t, \mathbf{r}) = a(t) + b(\mathbf{r}),
\]

(58)

where \( a(t) \) is an arbitrary function of time such that \( a(0) = 0 \). If at the Big Bang \( (t = 0) \) the Universe was inhomogeneous then subsequent expansion makes the spatially uniform term \( a(t) \) in equation (58) much larger than \( b(\mathbf{r}) \) which is time independent. Therefore, we can omit \( b(\mathbf{r}) \) and treat metric as spatially uniform.

Thus, present theory predicts that shortly after Big Bang the Universe becomes spatially flat and homogeneous on the large scales regardless of the initial condition. This is not the case for general relativity and known as the problems of large-scale homogeneity and flatness of the Universe. To resolve these problems, cosmological models based on
general relativity require stage of inflation and introduction of additional hypothetical field, the inflaton, responsible for inflation. In contrast, vector gravity does not need additional fields.

Temporal part of equations (22) \((i = 0)\) with the cosmological term yield the following equation for \(a(t)\)

\[
-\frac{d}{dt}(\dot{a}^2) = \frac{8\pi G}{3} \left( \frac{\rho}{a^3} - \Lambda \right)
\]

(59)

which shows that matter decelerates expansion of the Universe, while the \(\Lambda\)-term accelerates it (provided \(\Lambda > 0\)). In equation (59) a dot denotes derivative with respect to time \(t\). Integration of equation (59) gives

\[
\dot{a}^2 = \frac{8\pi G}{3} \left( \frac{\rho}{a^3} + \Lambda \right) + \frac{C}{a^2},
\]

(60)

where \(C\) is an integration constant which is proportional to the total energy density of the Universe. Indeed, in the metric (56) the action (11) with the additional cosmological term (55) reads

\[
S = -\int d^4x \left[ \frac{3c^2}{8\pi G} \dot{a}^2 - c\Lambda a^2 + \frac{\rho c^2}{a} \right].
\]

Hence, the Lagrangian density is

\[
L = -\frac{3c^2}{8\pi G} \dot{a}^2 - c^2 \Lambda a^2 - \frac{\rho c^2}{a}
\]

(61)

which yields the following conserved Hamiltonian density (the total energy density) \(w\)

\[
w = \dot{a} \frac{\partial L}{\partial \dot{a}} - L = -\frac{3c^2}{8\pi G} \dot{a}^2 + c^2 \Lambda a^2 + \frac{\rho c^2}{a}.
\]

(62)

Therefore, integration constant \(C\) in equation (60) is

\[
C = -\frac{8\pi G}{3c^2} w.
\]

(63)

Observations indicate that \(C = 0\), or, at least, that in the present epoch the term \(C/a^2\) in equation (60) is small compared to the other terms. Since the term \(C/a^2\) evolves as \(1/a^2\), while the matter term is proportional to \(1/a^3\) the total energy density in the early Universe must be equal to zero with very high precision. Namely, for small \(a\) the term \(C/a^2\) becomes very small as compared to the matter contribution. Thus, positive energy of matter in the Universe is balanced by the negative energy of the gravitational field giving zero net energy. This result can be considered as an observational evidence that matter in the Universe has been produced at the expense of generation of the gravitational field with negative energy.

On the other hand, in the metric (56) (for zero spatial curvature), Einstein equations (2) with the extra cosmological term \(-\frac{8\pi G}{3c^2} \Lambda a^2\) in the right-hand side are equivalent to the equations of vector gravity. We will show such equivalence for the energy–momentum tensor of matter in more general form

\[
T^\mu_\nu = \begin{pmatrix} \rho(t) c^2 & 0 & 0 & 0 \\ 0 & -P(t) & 0 & 0 \\ 0 & 0 & -P(t) & 0 \\ 0 & 0 & 0 & -\Lambda \end{pmatrix},
\]

(64)

where \(P(t)\) is the matter pressure. In the metric (56), Einstein equation

\[
R_{00} = \frac{8\pi G}{c^4} \left( T_{00} - \frac{1}{2} g_{00} T - \Lambda c^2 g_{00} \right)
\]

and the temporal part of equations (22) \((i = 0)\) with the cosmological term yield the same evolution equation

\[
-\frac{d}{dt}(\dot{a}^2) = \frac{4\pi G}{3} \left( \rho(t) + \frac{P(t)}{c^2} - 2\Lambda \right)
\]

(65)

which has two unknown functions \(a(t)\) and \(\rho(t)\) (equation of state \(P = P(\rho)\) fixes the relation between \(P\) and \(\rho\), and, therefore, \(P\) is not independent). Additional equation can be taken, e.g., in the form of the energy–momentum relation \(T_{00} = 0\) which is the same in both theories. In the metric (56) this relation yields

\[
\dot{\rho}(t) = -3 \left( \rho(t) + \frac{P(t)}{c^2} \right) \dot{a}.
\]

(66)

For cold Universe \(P(t) = 0\) and integration of equation (66) yields \(\rho(t) = \rho/a(t)^3\), where \(\rho\) is independent of \(t\). Thus, for cold Universe, equation (65) gives the previous equation (59).

Since equations (65) and (66) are identical in both theories, general relativity for spatially flat metric and vector gravity predict the same evolution of the Universe which agrees with available cosmological data for a certain value of \(\Lambda\). General relativity, however, does not predict the value of \(\Lambda\).

To find the value of the cosmological constant \(\Lambda\) in vector gravity we must start from the full system of equations (22) for nonuniform matter distribution without the cosmological term and average them over the large scales. We perform this procedure for the linearized equations for which the answer can be found exactly. Then we will match the nonlinear equation (59) with the exact linearized equations (which do not have the \(\Lambda\)-term) and obtain the value of \(\Lambda\) in the effective cosmological model.

We calculate \(\Lambda\) in our coordinate system, that is at the present time. To do so we linearize equation (59) near \(a = a_{\text{now}}\) and rescale time and coordinates as \(t \rightarrow a_{\text{now}} t\) and \(x_0 \rightarrow x_0/a_{\text{now}}\). In the vicinity of the present time the equivalent metric is

\[
f_{ik} = \begin{pmatrix} 1 + h_{00} & 0 & 0 & 0 \\ 0 & -1 + h_{00} & 0 & 0 \\ 0 & 0 & -1 + h_{00} & 0 \\ 0 & 0 & 0 & -1 + h_{00} \end{pmatrix},
\]

where \(h_{00} = -2(a - a_{\text{now}})/a_{\text{now}}\) and linearization of
equation (59) yields the following equation for $h_{00}$

$$3h_{00} + 16\pi G\Lambda = 8\pi G \frac{\rho}{a_{\text{now}}}.$$  \hspace{1cm} (67)

On the other hand, the full system of linearized nonuniform equations without cosmological term is (see equations (38) and (39))

$$\Delta h_{00} + 3\frac{\partial^2 h_{00}}{\partial x^0 \partial x^0} - 2\frac{\partial^2 h_{00}}{\partial x^0 \partial x^3} = \frac{8\pi G}{c^4} T_{00} \text{ now},$$  \hspace{1cm} (68)

$$\left(\frac{\partial^2}{\partial x^0 \partial x^0} - \Delta\right)h_{00} + \frac{\partial^2 h_{03}}{\partial x^0 \partial x^3} - 2\frac{\partial^2 h_{00}}{\partial x^3 \partial x^0} = \frac{16\pi G}{c^4} T_{00} \text{ now},$$  \hspace{1cm} (69)

where we took into account that for nonrelativistic matter $\tilde{T}^{00} = T^{00}$. Taking $2\partial / \partial x^3$ from both sides of equation (68) and divergence from both sides of equation (69), adding them together and using the continuity equation

$$\frac{\partial T_{00} \text{ now}}{\partial x^0} + \frac{\partial T_{00} \text{ now}}{\partial x} = 0$$

we obtain

$$\frac{\partial^2}{\partial x^0 \partial x^3} \left(2 \frac{\partial h_{00}}{\partial x^0} - \frac{\partial h_{03}}{\partial x^3}\right) = 0.$$  \hspace{1cm} (70)

Integration yields

$$\frac{\partial}{\partial x^0} \left(2 \frac{\partial h_{00}}{\partial x^0} - \frac{\partial h_{03}}{\partial x^3}\right) = F(r),$$  \hspace{1cm} (71)

where $F(r)$ is a function of spatial coordinates.

Equation (70) has the following physical meaning. Change in time of the spatial scale (given by $h_{00}$) can be viewed as motion of masses relative to each other. This matter current produces longitudinal vector field $h_{03}$ which, according to equation (70), has nonzero divergence. Relation between $\partial h_{00} / \partial x^0$ and $\partial h_{03} / \partial x^3$ should be independent of what causes $h_{00}$ to change (expansion of the Universe or motion of a nearby star). If time dependence of $h_{00}$ is produced by a moving star then solution is bound and average of the full derivative in the left-hand side of equation (71) over time vanishes. Since $F(r)$ is independent of time we obtain $F(r) = (F(r))_t = 0$ and, therefore, equation (71) reduces to

$$\frac{\partial^2 h_{03}}{\partial x^0 \partial x^3} = 2\frac{\partial^2 h_{00}}{\partial x^0 \partial x^0}. \hspace{1cm} (72)$$

Equation (72) must be also valid for the evolution of the Universe. Physically, expansion of the Universe changes spatial scale which can be treated as an effective matter current directed away from a local observer. Such current produces longitudinal vector field $h_{03}$ (according to equation (72) change of $h_{00}$ with time induces $h_{03}$). This vector field makes the third term in (68) nonzero which acts as the cosmological term in the evolution equation. Since, according to equation (41), the energy density of the longitudinal vector field is negative the cosmological term accelerates expansion of the Universe.

Substituting equation (72) into equation (68) we find

$$\frac{\partial^2 h_{00}}{\partial x^0 \partial x^0} = \Delta h_{00} - \frac{8\pi G}{c^4} T_{00} \text{ now}.$$  \hspace{1cm} (73)

Next we average equation (73) over the large scales so that $T_{00} \text{ now}$ becomes spatially uniform. In the effective cosmological model the averaged metric (56) is diagonal and depends only on time (see discussion after equation (58)). Therefore, spatial averaging of $\Delta h_{00}$ must be equal to zero and equation (73) yields

$$\frac{\partial^2}{\partial x^0 \partial x^0} \langle h_{00} \rangle = \frac{8\pi G}{c^4} \langle T_{00} \rangle \text{ now}.$$  \hspace{1cm} (74)

On the other hand, spatial averaging of equation (72) gives that

$$\left\langle \frac{\partial h_{03}}{\partial x^3} \right\rangle = 2\frac{\partial}{\partial x^0} \langle h_{00} \rangle$$  \hspace{1cm} (75)

is not equal to zero. This is consistent with our assumptions. Indeed, equation (75) can be satisfied by choosing $h_{03}$ as an odd function of coordinates, $h_{03} \propto x_f$. Thus, spatial averaging of $h_{03}$ in the local region yields zero and, hence, the averaged metric $\langle h_{00} \rangle$ is diagonal and spatially homogeneous.

Comparing equation (67) with equation (74) and taking into account that

$$\langle T_{00} \rangle = \frac{\rho c^2}{a_{\text{now}}^2}$$
we find the following value for the cosmological constant
\[ \Lambda = \frac{2 \rho}{a_{\text{now}}^3}. \]

Thus, at the present time the cosmological term contribution (dark energy) is twice as much as the energy of matter. Therefore, the ratio between the energy density due to the cosmological constant and the critical density of the Universe \( \Omega_{\text{critical}} = \frac{3a_{\text{now}}^2}{8 \pi G} = \rho_{\text{matter}} + \Lambda \) is
\[ \frac{\Lambda}{\Omega_{\text{critical}}} = \frac{2}{3} \approx 0.67. \] (76)

This is also the case in any other reference frame. That is an observer who lives billion years before or after would find the same answer for \( \Lambda/\Omega_{\text{critical}} \) in his reference frame. Our prediction has no free parameters and agrees with the recent Planck result which measured \( \Lambda/\Omega_{\text{critical}} \) and obtained 0.686 ± 0.02 [12].

In the present theory the \( \Lambda \)-term appears as a solution of equations. It comes from the gravitational part (left-hand side) of equations (22) as a result of averaging of the terms \( \frac{\partial^2 h_{\alpha\beta}}{\partial^2 \phi \partial^2 \phi} \) over the large scales. Such average does not vanish in equation (68) if we make the transition to the infinitely large size of the Universe properly.

The physical meaning of the dark energy becomes clear if we compare the exact expression for the weak field limit Lagrangian density with those obtained from the effective cosmological action containing the \( \Lambda \)-term. Keeping only the relevant contributions the weak field limit Lagrangian density in the rescaled coordinates reads (see equation (B6))
\[ L = -\frac{3c^2}{32\pi G} (\dot{h}_{00})^2 - \frac{c^2}{32\pi G} h^2 - \rho c^2, \]
where \( h = h^{\alpha\alpha} \) depends on time and spatial coordinates. On the other hand, linearization of the effective cosmological Lagrangian (61) yields
\[ L = -\frac{3c^2}{32\pi G} (\dot{h}_{00})^2 - c^2 \Lambda - \rho c^2. \]
Thus,
\[ \Lambda = \frac{1}{32\pi G} (\dot{h}^2) \]
and dark energy is the average energy of the longitudinal part of the gravitational field. From the perspective of a local observer the change in the spatial scale caused by the Universe expansion is equivalent to a matter current directed away from the observer. Such current generates the time-dependent longitudinal field \( h \) which is analogous to generation of the vector potential \( A \) by a current in classical electrodynamics. The value of the current depends on the matter density and on the expansion rate of the Universe which, in turn, is a function of the matter density. Thus, the value of the cosmological constant \( \Lambda \) is determined by the averaged matter density in the reference frame of the observer (matter density at the moment the observer measures \( \Lambda \)).

One should mention that quantization of the gravitational field involves only radiative part of the field. Since dark energy originates from the non radiative part it is a pure classical effect which can be described by the classical field equations.

10. Contents of the Universe

Vector theory of gravity explains the nature of dark energy as the energy of longitudinal gravitational field induced by the Universe expansion. Such energy is negative and accelerates expansion of the Universe. According to the present theory, the Universe is made of matter (dark matter and ordinary matter) and gravitational field (see figure 2). Observations indicate that the total energy of the Universe is equal to zero (see discussion after equation (63)).

As we show in section 8.3, classical field equations yield that the graviton has negative energy. This suggests that at the Big Bang matter was created at the expense of generation of the negative energy gravitons. Since the pressure is determined by the wave momentum, rather than energy, the gas of gravitons has positive pressure \( P \). For such a gas the equation of state reads
\[ P = -\frac{w_g}{3}, \] (77)
where \( w_g \) is the graviton energy density. Since the energy momentum tensor of an isotropic gas is
\[ T^{ik} = \begin{pmatrix} w_g & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & P \end{pmatrix} \]
we obtain that for the gas of gravitons
\[ T^{00} - \frac{1}{2} T^{00} T = \frac{1}{2} (w_g + 3P) = 0. \] (78)

According to equations (22), the combination \( T^{00} - \frac{1}{3} T^{00} T \) is the source of gravitational field. This combination is also the source of gravitational field in general relativity. Namely, it determines gravitational mass of an object (Komar mass) in terms of the energy–momentum tensor. Since for the primordial gravitational waves \( T^{00} - \frac{1}{3} T^{00} T = 0 \) they do not produce gravitational field and, hence, they do not contribute to the gravitational acceleration measured by an accelerometer. Equation (78) also yields that primordial gravitons do not change the cosmological equation (59) and, thus, they have no effect on expansion rate of the Universe.

According to the postulate 4 of our theory the graviton is a composite particle, namely it is composed of fermion–antifermion pairs. Since no more than one fermion can occupy the same quantum state the matter generation at the Big Bang has continued until fermion states were filled. The following Universe expansion practically did not change the fermion occupation number and states remain filled. These filled states form a vacuum in the present epoch. As we show
in the next section, for the filled vacuum the graviton energy is positive and, thus, vacuum is stable. For the filled vacuum creation of a graviton corresponds to creation of fermion–antifermion hole pairs out of the filled fermion states.

Analogy with the composite photon theory \[20\] suggests that the fermion–antifermion pairs that compose the graviton are coupled to matter with the gravitational constant \( G \), while a single fermion interacts with a much weaker (perhaps zero) coupling constant. As a consequence, the energy scales associated with the graviton and its constituent fermion are very different. The Planck energy is the characteristic quantum energy scale for the graviton and the corresponding Planck frequency is

\[
\omega_{Pl} = \sqrt{\frac{c^5}{hG}} = 1.8 \times 10^{43} \text{s}^{-1}.
\]

Perhaps at the Big Bang the gravitational waves have been generated to the Planck frequency. However, states of the constituent fermions are filled up to a much higher frequency which is determined by their coupling constant. Such frequency cannot be predicted in the framework of the present theory.

Constituent fermions have both positive and negative energy states. The net energy of the graviton gas produced at the Big Bang is negative. Probably shortly after the Big Bang the negative energy of the graviton gas has been transferred to the kinetic energy of the Universe expansion (the first term in the right-hand side of equation \(62\)) and the negative and positive energy states became filled symmetrically. This subject, however, requires detailed analysis of cosmological models and is beyond the scope of our paper.

### 11. Quantization of gravitational field

Here we quantize gravitational field assuming that graviton is composed of fermion–antifermion pairs. Since quantization procedure is similar to quantization of electromagnetic field we start from a brief review of the field quantization in electrodynamics.

#### 11.1. Classical electrodynamics

Classical electrodynamics is a vector field theory in four dimensional Minkowski space-time. Electromagnetic field is a 4-vector \( A^4 = (A_0, A) \) in this space–time, while electric current density is a 4-vector \( j^k = (\rho, j) \), where \( \rho \) and \( j \) are the electric charge and spatial current densities. The conserved 4-current density \( j^k \) is coupled to \( A_k \) through the Lorentz and gauge invariant term in the action

\[
S_{\text{coul}} = -\frac{1}{c^2} \int d^4x A_k j^k.
\]

The total action of the system is \( S = S_{\text{field}} + S_{\text{coul}} + S_{\text{matter}} \), where action for free electromagnetic field is

\[
S_{\text{field}} = -\frac{1}{16\pi c} \int d^4x \left( \frac{\partial A_k}{\partial x^l} - \frac{\partial A_l}{\partial x^k} \right) \left( \frac{\partial A^k}{\partial x^l} - \frac{\partial A^l}{\partial x^k} \right).
\]

and for nonrelativistic motion of particles

\[
S_{\text{matter}} = \int dt \sum_a \frac{m_a r_a^2}{2}.
\]

where the sum is over all particles \( a \) having positions \( r_a \), masses \( m_a \), and electric charges \( q_a \). Electric current density is given by

\[
j = \sum_a q_a \frac{d}{dt}(r - r_a(t)).
\]

Particle momentum conjugate to \( r_a \) is

\[
p_a = \frac{\partial L}{\partial \dot{r}_a} = m_a \dot{r}_a + \frac{q_a}{c} A(r_a),
\]

while momentum \( \pi^k \) conjugate to the field \( A_k \) reads

\[
\pi^k = \frac{\partial L}{\partial \dot{A}_k} = -\frac{1}{4\pi} \int \left\{ E^k = 0, \quad k = 0 \right\} \left( E^k, \quad k = 1, 2, 3, \right. \]

where \( L \) is the Lagrangian density and \( E = -\nabla A^0 - \partial A^j/\partial x^j \) is the electric field.

Electromagnetic field \( A_k \) has four real components. However, since momentum \( \pi^0 \) conjugate to \( A_0 \) vanishes the time component \( A_0 \) is not a dynamical field. This means that \( A_0 \) is not an independent degree of freedom but rather it is a functional of \( A \) and electric charge density \( \rho \)[37]. In addition, gauge symmetry implies that there are only two independent physical degrees of freedom because one of the degrees of freedom can be eliminated by gauge fixing. These two independent degrees of freedom of the electromagnetic field corresponding to radiation are quantized, their quantum is a photon.

Classical Hamiltonian of the system is a functional of \( r_a, p_a, A_k \) and \( \pi_k \)

\[
\mathcal{H} = \sum_a p_a \cdot \dot{r}_a + \int d^3x (\pi^k A_k - L)
\]

\[
= \int d^3x \left[ \frac{1}{8\pi} (E^2 + B^2) - \frac{1}{4\pi} A_0 \text{div} E \right]
\]

\[
+ \sum_a \left[ \frac{1}{2m_a} \left( p_a - \frac{q_a}{c} A \right)^2 + q_a A_0 \right],
\]

where \( B = \text{curl} A \).

One can decompose \( E \) and \( B \) into longitudinal and transverse parts

\[
E = E_{\text{lon}} + E_{\text{tr}}, \quad B = B_{\text{tr}}
\]

such that \( \text{div} E_{\text{tr}} = 0 \) and \( \text{div} B_{\text{tr}} = 0 \). This decomposition can be done in a straightforward way if we write \( E \) as a Fourier series

\[
E(r) = \sum_p E(p)e^{ip\cdot r}
\]

Then

\[
E_{\text{lon}}(r) = \sum_p \hat{p} (\hat{p} \cdot E(p))e^{ip\cdot r},
\]

where \( \hat{p} \) is a unit vector in the direction of \( p \), and

\[
E_{\text{tr}} = E - E_{\text{lon}}.
\]
Having in mind that photon originates from the transverse electromagnetic field we decompose 4-vectors $A_k$ and $j_k$ into transverse spatial part $A_{t,i}$, $j_{ti}$ and the remaining piece which contains longitudinal and time-like components, e.g.

$$A^k = A^k_{t,i} + A^k_{l},$$

(79)

Terms in the right-hand side of equation (79) do not transform as 4-vectors, only their sum does. In particular, for the spatial part of $A^k$ we have

$$A = A_{t,i} + A_{m}.$$  

Decomposition of the field into the longitudinal and transverse components decouples Hamiltonian into two independent pieces $\mathcal{H} = \mathcal{H}_{t,i} + \mathcal{H}_{m}$, where Hamiltonian of the transverse field is

$$\mathcal{H}_{t,i} = \frac{1}{8\pi} \int d^3x (E^2_{t,i} + B^2) + \sum_\mu \frac{1}{2m_\mu}\left(p_\mu - \frac{q_\mu}{c} A_\mu\right)^2$$

(80)

and

$$E_{t,i} = -\frac{\partial A_{t,i}}{\partial \phi}.$$  

For the longitudinal and transverse fields Maxwell equations also decouple. In particular, for transverse field Maxwell equations read

$$\text{curl}B = \frac{4\pi}{c} j_{t,i} + \frac{1}{c} \frac{\partial E_{t,i}}{\partial t}, \quad \text{curl}E_{t,i} = -\frac{1}{c} \frac{\partial B_{t,i}}{\partial t}. \quad (81)$$

11.2. Quantization of electromagnetic field in elementary photon theory

In conventional quantization of the electromagnetic field the $\mathcal{H}_{t,i}$, part of the Hamiltonian remains classical [38]. Part of the transverse field that corresponds to radiation (field without sources) is quantized and is described by the following Hamiltonian operator [37]

$$\hat{\mathcal{H}}_{t,i} = \sum_{p,\mu=1,2} \hbar c p_\mu \hat{A}_{p,\mu}^+ \hat{A}_{p,\mu} + \sum_\mu \frac{1}{2m_\mu} \left(p_\mu - \frac{q_\mu}{c} \hat{A}_{\mu}(\epsilon_\mu_\mu)\right)^2,$$

(82)

where

$$\hat{A}_{p,\mu}(r) = \sum_{\mu',\mu''=1,2} \sqrt{\frac{2\pi\hbar c}{pV}} (\epsilon_{\mu,\mu'} \hat{A}_{p,\mu'} \epsilon_{p,\mu''} + \text{h.c.}),$$

$\epsilon_{p,\mu}$ ($\mu = 1, 2$) are unit three-dimensional polarization vectors perpendicular to the photon wave vector $\mathbf{p}$. $V$ is the photon volume and operators $\hat{A}_{p,\mu}$ obey Bose–Einstein commutation relations

$$[\hat{A}_{p,\mu}, \hat{A}_{p',\mu'}^+] = \delta_{p,\mu'} \delta_{p',\mu}, \quad \mu, \mu' = 1, 2,$$

and all other commutators are equal to zero. Operators $\hat{A}_{p,\mu}^+$ and $\hat{A}_{p,\mu}$ describe creation and annihilation of a spin 1 photon with wave vector $\mathbf{p}$ and polarization $\mu$. Electric

$$\hat{E}_{t,i}(r) = i \sum_{p,\mu=1,2} \sqrt{\frac{2\pi\hbar c}{pV}} (\epsilon_{p,\mu} \hat{A}_{p,\mu} \epsilon_{p,\mu}^\dagger \mathbf{e} - \text{h.c.}),$$

and magnetic

$$\hat{B}(r) = i \sum_{p,\mu=1,2} \sqrt{\frac{2\pi\hbar c}{pV}} (\mathbf{p} \times \epsilon_{p,\mu} \hat{A}_{p,\mu} \epsilon_{p,\mu}^\dagger \mathbf{e} - \text{h.c.})$$

field operators obey commutation relations

$$[\hat{A}_{\alpha}(r_1), \hat{E}_{\nu}(r_2)] = -4\pi i \hbar \delta^{\nu\beta} \delta(r_1 - r_2), \quad \alpha, \beta = 1, 2, 3,$$

(83)

$$[\hat{A}_{\alpha}(r), \hat{B}_{\beta}(r)] = 0,$$

(84)

$$[\hat{H}_{t,i}, \hat{E}_{t,i}] = -i\hbar \text{curl} \hat{B}, \quad [\hat{H}_{t,i}, \hat{B}] = i\hbar \text{curl} \hat{E}_{t,i}. \quad (85)$$

where

$$\dot{\hat{H}}_{t,i} = \sum_{p,\mu=1,2} \hbar c p_\mu \dot{\hat{A}}_{p,\mu}^+ \hat{A}_{p,\mu}.$$  

(86)

In the Heisenberg picture the Heisenberg equations of motion

$$\frac{d\hat{E}_{t,i}}{dt} = \frac{i}{\hbar}[\hat{H}_{t,i}, \hat{E}_{t,i}], \quad \frac{d\hat{B}}{dt} = \frac{i}{\hbar}[\hat{H}_{t,i}, \hat{B}]$$

yield Maxwell’s equations for the transverse field (81) in the operator form

$$\text{curl} \hat{B} = \frac{4\pi}{c} \hat{j}_{t,i} + \frac{1}{c} \frac{\partial \hat{E}_{t,i}}{\partial t}, \quad \text{curl} \hat{E}_{t,i} = -\frac{1}{c} \frac{\partial \hat{B}}{\partial t}. \quad (87)$$

One should emphasize that only a part of the transverse electromagnetic field is quantized, namely the part that corresponds to radiation. For example, magnetic field produced by stationary currents is transverse but is not quantized and remains classical.

11.3. Photon as composite particle

In the composite photon theory the elementary particle is a massless spin 1/2 fermion and photon is composed of the fermion–antifermion pairs. In free space the Dirac equation for massless spin 1/2 fermion, described by a four-component spinor $\Psi = (\psi_1, \psi_2)$, reads

$$\gamma^\mu \partial_\mu \Psi = 0,$$

(88)

where $\gamma^\mu$ are gamma matrices which can be written in terms of $2 \times 2$ sub-matrices taken from the Pauli matrices and the $2 \times 2$ identity matrix $I$. In the Weyl (chiral) basis the gamma matrices have the form

$$\gamma^0 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & \sigma_1 \\ -\sigma_1 & 0 \end{pmatrix},$$

$$\gamma^2 = \begin{pmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{pmatrix}, \quad \gamma^3 = \begin{pmatrix} 0 & \sigma_3 \\ -\sigma_3 & 0 \end{pmatrix}$$
and Pauli matrices are
\[ \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \]

Recall that \( \gamma^\mu \) are fixed under Lorentz transformations in the forms given above. Lorentz invariance of the Dirac equation is achieved by proper transformation of spinor \( \Psi \) that counterbalances the transformation of \( \partial_\mu \). In the Weyl (or chiral) representation of the Dirac matrices the Weyl spinors \( \psi_\beta \) and \( \bar{\psi}_\beta \) do not mix under Lorentz transformations.

Solutions of the Dirac equation are arbitrary superposition of four plane wave spinors
\[ \Psi_a = u_a(p)e^{-ipct + ip \cdot r}, \quad \Psi_b = u_b(p)e^{ipct + ip \cdot r}, \quad \Psi_c = u_c(p)e^{-ipct - ip \cdot r}, \quad \Psi_d = u_d(p)e^{ipct - ip \cdot r}, \]
where \( p \) is the wave vector, \( p = |p| \) and for \( p \) oriented along the positive direction of the \( z \)-axis
\[ u_a(p) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad u_b(p) = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad u_c(p) = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad u_d(p) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}. \]

Energy of particles described by spinors \( \Psi_a \) and \( \Psi_d \) is positive \( \varepsilon = \hbar cp \), while fermions corresponding to spinors \( \Psi_b \) and \( \Psi_c \) have negative energies \( \varepsilon = -\hbar cp \). The helicity of a particle is right-handed if the direction of its spin is the same as the direction of its motion. It is left-handed if the directions of spin and motion are opposite. \( u_a \) and \( u_c \) are right-handed spinors, while \( u_b \) and \( u_d \) are left-handed spinors. Helicity of a massless particle is Lorentz invariant.

General solution of the free Dirac equation can be written as
\[ \Psi = \sum_p [a_p u_a(p)e^{-ipct + ip \cdot r} + b_p u_b(p)e^{ipct + ip \cdot r} + c_p u_c(p)e^{-ipct - ip \cdot r} + d_p u_d(p)e^{ipct - ip \cdot r}], \]
where \( a_p, b_p, c_p \) and \( d_p \) are arbitrary constants which transform as scalars under Lorentz transformation.

Out of \( u_a(p), u_b(p), u_c(p) \) and \( u_d(p) \) one can construct four linearly independent 4-vectors in 4-dimensional space-time:
\[ u^\dagger_a(p)\gamma^0\gamma^\mu u_b(p) = (0, 1, -i, 0), \quad (89) \]
\[ u^\dagger_d(p)\gamma^0\gamma^\mu u_c(p) = (0, -1, -i, 0), \quad (90) \]
\[ u^\dagger_b(p)\gamma^0\gamma^\mu u_d(p) = u^\dagger_c(p)\gamma^0\gamma^\mu u_a(p) = (1, 0, 0, -1), \quad (91) \]
\[ u^\dagger_c(p)\gamma^0\gamma^\mu u_d(p) = u^\dagger_d(p)\gamma^0\gamma^\mu u_a(p) = (1, 0, 0, 1). \quad (92) \]

Other combinations give zero vectors. Vectors (89) and (90) are transverse, while (91) and (92) are combinations of the longitudinal and time-like vectors. Recall that wave vector \( p \) is chosen to be oriented along the positive direction of the \( z \)-axis.

We map the fermion field into the real transverse field \( A_\mu \)
\[ A_\mu(t, r) = \sum_p [A_{p\mu}(t)e^{ip\cdot r} + A^*_\mu(t)e^{-ip\cdot r}] \quad (93) \]
such that \( A_\mu(t, r) \) obeys Maxwell equations for free transverse field. In terms of the Fourier components \( A_{p\mu}(t) \) this can be done in the following way
\[ A_{p\mu}(t) = A_p e^{-ip\cdot ct} \gamma_\mu \epsilon_{p+1} + A_{p\mu,2} e^{-ip\cdot ct} \gamma_\mu \epsilon_{p+2}, \quad (94) \]
where \( \epsilon_{p+1} \) and \( \epsilon_{p+2} \) are the spatial unit polarization vectors of the left and right circularly polarized photons respectively. Equations (89) and (90) indicate that \( A_{p+1} \) and \( A_{p+2} \) should be chosen as
\[ A_{p+1} = \sum_{k|p} F_1(p, k)\gamma_\mu a_{p+k} b_{-k}, \quad A_{p+2} = \sum_{k|p} F_2(p, k)\gamma_\mu a_{p+k} b_{-k}, \]
where summation is over all fermion states with wave vectors \( k \) parallel to \( p \).

In order for \( A_{p\mu} \) to transform as a transverse field under Lorentz transformations the spectral functions \( F_1(p, k) \) and \( F_2(p, k) \) must be scalars. Therefore they can depend only on the absolute values of the 4-vectors and their dot products. \( F_1(p, k) \) is a factor in front of \( a_{p+k} b_{-k} \) which is a product of two fermion states with 4-wave vectors \( (p + k, p + k) \) and \((-k, -k)\). Absolute values of these 4-vectors and their dot product (with Minkowski metric) are equal to zero. Therefore, spectral function \( F_1(p, k) \) is independent of \( p \) and \( k \). Similar arguments yield that \( F_2(p, k) \) is also a constant which can be chosen arbitrary. We choose \( F_1 = F_2 = 1/\sqrt{4N_f} \), where \( N_f \) is the number of fermion states with wave vectors parallel to \( p \), and assume that \( N_f \) is independent of \( p \).

11.3.1. Field quantization in the composite photon theory.

Next we quantize the fermion field by replacing \( a_p, b_p, \epsilon_p \) and \( d_p \) with operators that obey canonical anticommutation relations
\[ \hat{a}_p \hat{a}^\dagger_p + \hat{a}^\dagger_p \hat{a}_p = \delta_p \delta_{p'}, \quad \hat{b}_p \hat{b}^\dagger_p + \hat{b}^\dagger_p \hat{b}_p = \delta_p \delta_{p'}, \quad (95) \]
\[ \hat{c}_p \hat{c}^\dagger_p + \hat{c}^\dagger_p \hat{c}_p = \delta_p \delta_{p'}, \quad \hat{d}_p \hat{d}^\dagger_p + \hat{d}^\dagger_p \hat{d}_p = \delta_p \delta_{p'}, \quad (96) \]

All other anticommutators are equal to zero. The second quantized Hamiltonian for the free fermion field is
\[ \hat{H}_0 = \sum_p [\varepsilon_p(\hat{a}_p^\dagger \hat{a}_p + \varepsilon_p(\hat{b}_p^\dagger \hat{b}_p + \varepsilon_p(\hat{c}_p^\dagger \hat{c}_p + \varepsilon_p(\hat{d}_p^\dagger \hat{d}_p), \quad (97) \]
where \( \varepsilon_p = \varepsilon_d(p) = \hbar cp \) and \( \varepsilon_p = \varepsilon_s(p) = -\hbar cp \).

The free transverse field (93) (field without sources) now becomes an operator which in the Schrödinger picture reads
\[ \hat{A}_\mu(r) = \sum_p [\hat{A}_{p\mu} e^{ip\cdot r} + \hat{A}^*_\mu e^{-ip\cdot r}], \quad (98) \]
phys. scr. 92 (2017) 125001

a

\( N = + \hat{\mathcal{A}}_{1} \epsilon_{p} + \hat{\mathcal{A}}_{2} \epsilon_{p} \)

and

\[
\hat{\mathcal{A}}_{1} = \frac{1}{\sqrt{|N|}} \sum_{k|p} \hat{a}_{p+k} \hat{b}_{k}, \tag{99}
\]

\[
\hat{\mathcal{A}}_{2} = \frac{1}{\sqrt{|N|}} \sum_{k|p} \hat{a}_{p+k} \hat{c}_{k}. \tag{100}
\]

One should mention that under charge conjugation operation \( \hat{a} \leftrightarrow \hat{\gamma} \), \( \hat{b} \leftrightarrow \hat{\delta} \). Changing index of summation in equations (99) and (100) we then obtain that under charge conjugation \( \hat{\mathcal{A}}_{1} \leftrightarrow \hat{\mathcal{A}}_{2} \). That is in our quantization scheme the photon is identical to its own antiparticle. This agrees with recent antihydrogen experiments at CERN [39].

Since fermion \( a \) is right-handed the operator \( \hat{a}_{p+k} \) creates a spin 1/2 fermion with spin parallel to \( p + k \) and energy \( \epsilon_{a} = \hbar c (p + k) \). On the other hand, \( \hat{b}_{k} \) creates a spin 1/2 antifermion with spin antiparallel to \( k \) (that is parallel to \( k \)) and negative energy \( \epsilon_{b} = -\hbar k \). Thus, the combination \( \hat{b}_{k} \hat{a}_{p+k} \) creates a fermion–antifermion pair with the total energy \( \varepsilon = \hbar c p \) and spin 1 parallel to \( p \). Recall that for left (right) circularly polarized photon the photon spin is parallel (antiparallel) to the wave vector \( p \). Therefore, operator \( \hat{\mathcal{A}}_{+1} (\hat{\mathcal{A}}_{+2}) \) creates a left (right) circularly polarized photon with spin 1. According to equations (99) and (100), emission of a single photon corresponds to creation of \( N_{p} \) fermion–antifermion pairs with equal probability 1/\( N_{p} \).

Equations (95) and (96) yield the following commutation relations for operators \( \hat{\mathcal{A}}_{p,\mu} \) and \( \hat{\mathcal{A}}_{p,\mu} \):

\[
[\hat{\mathcal{A}}_{p,\mu}, \hat{\mathcal{A}}_{p',\mu'}] = \delta_{p,p'} \delta_{\mu,\mu'} + \frac{1}{|N|} \sum_{k|p|p'} \left( \delta_{k,k} \hat{a}_{p+k} \hat{a}_{p'} + \delta_{p,k,p'+k} \hat{b}_{k} \hat{b}_{k} \right) \tag{85}
\]

\[
[\hat{\mathcal{A}}_{p}, \hat{\mathcal{A}}_{p'}] = \delta_{p,p'} \delta_{\mu,\mu'} + \frac{1}{|N|} \sum_{k|p|p'} \left( \delta_{k,k} \hat{a}_{p+k} \hat{a}_{p'} + \delta_{p,k,p'+k} \hat{b}_{k} \hat{b}_{k} \right) \tag{102}
\]

Terms under the sum are written in the normal order, that is annihilation operators are placed to the right of the creation operators. If the total number of fermion states is very large compared to the number of occupied states the commutation relations for the vector operator become exactly the same as in the conventional quantum electrodynamics, namely, in the limit \( N_{p} \to \infty \) we obtain

\[
[\hat{\mathcal{A}}_{p,\mu}, \hat{\mathcal{A}}_{p',\mu'}] = \delta_{p,p'} \delta_{\mu,\mu'}, \quad \mu, \mu' = 1, 2 \tag{103}
\]

and all other commutators are equal to zero. Roughly, the correction term to the Bose–Einstein commutation relations is of the order of the ratio of the number of fermions in the system to the total number of fermion states in the Universe. Such correction is negligible.

One should mention that the number of fermion states can be much larger than the number of photon states. For example, for electromagnetic field in a cavity only certain photon states satisfy boundary conditions. However, there is no such constraint on the fermion states because fermions do not interact directly with the cavity walls. The boundary condition constrains the total sum in equations (99) and (100), that is values of the photon wave vector \( p \). However, there is no constraint on the values of the summation index \( k \).

In the composite theory of the photon the transverse part of the classical electromagnetic Hamiltonian (80) describing radiation field interacting with electric charges is replaced by the operator

\[
\hat{\mathcal{H}}_{0} = \mathcal{H}_{0} + \frac{1}{2m_{e}} \left( \mathbf{p}_{e} - \frac{q_{e}}{c} \hat{\mathcal{A}}_{e}(\mathbf{r}_{e}) \right)^{2},
\]

where \( \mathcal{H}_{0} \) is the Hamiltonian for the free field (97) and \( \hat{\mathcal{A}}_{e}(\mathbf{r}) \) is given by equations (98)–(100).

In the composite photon theory the electric and magnetic field operators

\[
\hat{E}_{e}(\mathbf{r}) = i \sum_{p_{\mu}=1,2} \frac{2\pi \hbar e}{V} \left( \epsilon_{p_{\mu}} \hat{\mathcal{A}}_{p_{\mu}} e^{i \mathbf{p} \cdot \mathbf{r}} - \text{h.c.} \right), \tag{104}
\]

\[
\hat{B}(\mathbf{r}) = i \sum_{p_{\mu}=1,2} \frac{2\pi e}{V} \left( \mathbf{p} \times \epsilon_{p_{\mu}} \hat{\mathcal{A}}_{p_{\mu}} e^{i \mathbf{p} \cdot \mathbf{r}} - \text{h.c.} \right) \tag{105}
\]

obey the same commutation relationships (82)–(85) as in the case of the elementary photon theory. Namely, equations (82) and (83) are a direct consequence of equation (103), while relations (84) and (85) are satisfied because for the free fermion field Hamiltonian (97), as in the case of the free photon Hamiltonian (86), we obtain

\[
[\hat{\mathcal{H}}_{0}, \hat{\mathcal{A}}_{p,\mu}] = -\hbar c p \hat{\mathcal{A}}_{p,\mu}, \quad \mu = 1, 2.
\]

Therefore, in the composite photon theory the Heisenberg equation of motion also yields Maxwell’s equations for the transverse field (87). Thus, in the limit \( N_{p} \to \infty \) the composite photon theory yields the same quantum electrodynamics as the elementary photon theory and, in particular, photons obey Bose–Einstein statistics.

In the composite theory the fermions do not bind together to form photons. It is not the interaction between fermion and antifermion that binds them together into photons, but rather the manner in which fermions interact with charged particles that leads to the simplified description of light in terms of the composite photons [40].

11.4. Quantization of gravitational field

In the vector theory of gravity for weak gravitational field and nonrelativistic motion of masses the Lagrangian of the field interacting with matter reads

\[
\mathcal{L} = \frac{e^{4}}{32\pi G} \int d^{3}x \left( -3 \frac{\partial h_{00}}{\partial x^{0}} \frac{\partial h_{00}}{\partial x^{0}} - \frac{\partial h_{00}}{\partial x^{0}} \frac{\partial h_{00}}{\partial x^{0}} - \frac{\partial h_{00}}{\partial x^{0}} \frac{\partial h_{00}}{\partial x^{0}} - \frac{\partial h_{00}}{\partial x^{0}} \frac{\partial h_{00}}{\partial x^{0}} \right)
\]
where \( h_{0k} \) are components of the equivalent metric, \( h = h_{00} \) and the sum is over all masses \( m_a \) having positions \( r_a \) and velocities \( \dot{r}_a \). Particle momentum conjugate to \( r_a \) is

\[
\mathbf{p}_a = \frac{\partial L}{\partial \dot{r}_a} = m_a \dot{r}_a + m_a c \mathbf{h}(r_a),
\]

while momentum \( \Pi^k \) conjugate to the field \( h_{0k} \) reads

\[
\Pi^k = \frac{\partial L}{\partial \dot{h}_{0k}} = -\frac{c^3}{16\pi G} \left\{ \begin{array}{c}
\pi^0, \\
k = 1, 2, 3
\end{array} \right. 
E^k, 
\]

where \( L \) is the Lagrangian density,

\[
\alpha^0 = \text{div} \mathbf{h} + 3 \frac{\partial h_{00}}{\partial x^4},
\]

and

\[
E = -\nabla h_{00} - \frac{\partial \mathbf{h}}{\partial x^0}.
\]

The classical Hamiltonian of the system is a functional of \( r_a, \mathbf{p}_a, h_{0k} \) and \( \Pi_k \)

\[
\mathcal{H} = \sum_a \mathbf{p}_a \cdot \dot{r}_a + \int d^3x \Pi^k h_{0k} - L
\]

\[
= -\frac{c^4}{32\pi G} \int d^3x \left( \mathbf{B}^2 + 2 \mathbf{E}^2 + \frac{1}{3} (\alpha^0 - \text{div} \mathbf{h})^2 - 2h_{00} \text{div} \mathbf{E} \right) + \frac{1}{2m_a} \left[ \mathbf{p}_a - m_a c \mathbf{h}(r_a) \right]^2,
\]

where \( \mathbf{B} = \text{curl} \mathbf{h} \). Transverse gravitational field \( \mathbf{h}_v \) interacting with matter is described by the Hamiltonian

\[
\mathcal{H}_v = -\frac{c^4}{32\pi G} \int d^3x \left( \mathbf{E}_v^2 + \mathbf{B}^2 \right) + \frac{1}{2m_a} \left( \mathbf{p}_a - m_a c \mathbf{h}(r_a) \right)^2.
\]

defining radiation field interacting with matter is replaced with the operator

\[
\mathcal{H}_v = \mathcal{H}_v + \frac{1}{2m_a} (\mathbf{p}_a - m_a c \mathbf{h}_a)^2,
\]

where \( \mathcal{H}_v \) is the Hamiltonian of the free fermion field \( \phi_1 \) and \( \mathbf{h}_v(r) \) is given by equation \( (109) \) in which

\[
\hat{h}_{v,1} = \frac{1}{\sqrt{|N_1|}} \sum_{k \parallel \mathbf{p}} \hat{a}_{-k} \hat{b}_{p+k},
\]

\[
\hat{h}_{v,2} = \frac{1}{\sqrt{|N_1|}} \sum_{k \parallel \mathbf{p}} \hat{d}_{-k} \hat{c}_{p+k},
\]

respecting the composite states with wave vectors \( \mathbf{p} \parallel \mathbf{p} \) and summation is taken over all such states.

Since fermion \( a \) is right-handed the operator \( \hat{a}_{-k}^+ \) creates a spin \( \frac{1}{2} \) fermion with spin parallel to \( -\mathbf{k} \) and energy \( \varepsilon_a = \hbar c k \). On the other hand, \( \hat{b}_{p+k}^+ \) creates a spin \( \frac{1}{2} \) antifermion with spin antiparallel to \( \mathbf{p} \parallel \mathbf{p} \) and energy \( \varepsilon_a = -\hbar c (p \parallel k) \). The combination \( \hat{b}_{p+k}^+ \hat{a}_{-k} \) with \( k \parallel \mathbf{p} \) creates a fermion–antifermion pair with the total negative energy \( \varepsilon = -\hbar c p \parallel k \) and spin \( 1 \) antiparallel to \( \mathbf{p} \). The combination \( \hat{c}_{p+k}^+ \hat{d}_{-k} \) with \( k \parallel \mathbf{p} \) creates a fermion–antifermion pair with the same energy \( \varepsilon = -\hbar c p \parallel k \) but with spin parallel to \( \mathbf{p} \). If we adopt the convention that for the left (right) circularly polarized graviton the graviton spin is parallel (antiparallel) to the wave vector \( \mathbf{p} \) then operator \( \hat{h}_{v,1}^+ \) \( (\hat{h}_{v,2}^+) \) creates a right (left) circularly polarized graviton with spin 1. Emission of a single graviton corresponds to creation of \( N_1 \) fermion–antifermion pairs.

Operators \( \hat{h}_{v,1} \) and \( \hat{h}_{v,2} \) obey the following commutation relations

\[
[\hat{h}_{v,1}, \hat{h}_{v,1}^+] = \delta_{p,p'}
\]

\[
-\frac{1}{N_1} \sum_{k \parallel \mathbf{p}} \sum_{k' \parallel \mathbf{p}} (\delta_{k,k'} \hat{b}_{p+k}^+ \hat{b}_{p+k} + \delta_{k,k'} \hat{b}_{p+k} \hat{b}_{p+k}''),
\]

\[
[\hat{h}_{v,2}, \hat{h}_{v,2}^+] = \delta_{p,p'}
\]

\[
-\frac{1}{N_1} \sum_{k \parallel \mathbf{p}} \sum_{k' \parallel \mathbf{p}} (\delta_{k,k'} \hat{c}_{p+k}^+ \hat{c}_{p+k} + \delta_{k,k'} \hat{c}_{p+k} \hat{c}_{p+k}''),
\]

which also can be written in the form

\[
[\hat{h}_{v,1}, \hat{h}_{v,1}^+] = -\delta_{p,p'}
\]

\[
+ \frac{1}{N_1} \sum_{k \parallel \mathbf{p}} \sum_{k' \parallel \mathbf{p}} (\delta_{k,k'} \hat{b}_{p+k} \hat{b}_{p+k}' + \delta_{k,k'} \hat{b}_{p+k} \hat{b}_{p+k}''),
\]

\[
[\hat{h}_{v,2}, \hat{h}_{v,2}^+] = -\delta_{p,p'}
\]

\[
+ \frac{1}{N_1} \sum_{k \parallel \mathbf{p}} \sum_{k' \parallel \mathbf{p}} (\delta_{k,k'} \hat{c}_{p+k} \hat{c}_{p+k}' + \delta_{k,k'} \hat{c}_{p+k} \hat{c}_{p+k}'').
\]
In equations (112) and (113) the terms under the sum are written in the normal order, while in equations (114) and (115) the order is the opposite. At the moment of Big Bang the fermion states are mostly empty, that is the total number of fermion states is very large compared to the number of occupied states. In this case in the limit \( N_f \to \infty \) equations (112) and (113) yield Bose–Einstein commutation relations for operators \( \hat{h}_{p,i} \) and \( \hat{h}_{p,2} \):

\[
[\hat{h}_{p,\mu}, \hat{h}^{\dagger}_{p',\mu'}] = \delta_{p,p'} \delta_{\mu,\mu'}, \quad \mu, \mu' = 1, 2.
\] (116)

All other commutators are equal to zero. At this stage of the Universe evolution the graviton has negative energy \( \varepsilon(p) = -\hbar c p \) which causes cosmological inflation.

Shortly after the Big Bang the fermion states become filled and remain filled in the present epoch. Such filled states form a new vacuum. If the total number of fermion states is very large compared to the number of empty states we must use equations (114) and (115) that in the limit \( N_f \to \infty \) give

\[
[\hat{h}_{p,\mu}, \hat{h}^{\dagger}_{p',\mu'}] = -\delta_{p,p'} \delta_{\mu,\mu'}, \quad \mu, \mu' = 1, 2
\] (117)

which differs from the Bose–Einstein commutation relations by the minus sign in the right-hand side. However, for operators

\[
\hat{A}_{\mu} = \hat{h}_{-\mu}, \quad \hat{A}^{\dagger}_{\mu} = \hat{h}^{\dagger}_{-\mu}
\]
equation (117) yields Bose–Einstein commutation relations

\[
[\hat{A}_{p,\mu}, \hat{A}^{\dagger}_{p',\mu'}] = \delta_{p,p'} \delta_{\mu,\mu'}, \quad \mu, \mu' = 1, 2.
\] (118)

while equation (116) gives relations with the minus sign

\[
[\hat{A}_{p,\mu}, \hat{A}^{\dagger}_{p',\mu'}] = -\delta_{p,p'} \delta_{\mu,\mu'}, \quad \mu, \mu' = 1, 2.
\] (119)

Operator \( \hat{A}_{\mu} \) (\( \hat{A}^{\dagger}_{\mu} \)) creates a graviton with positive energy \( \varepsilon(p) = \hbar c p \) and spin 1 parallel (antiparallel) to \( p \). Thus, in the present epoch the graviton energy is positive. Creation of such graviton corresponds to annihilation of fermion–antifermion pairs out of the filled vacuum states (creation of holes). In terms of \( \hat{A}_{\mu} \) and \( \hat{A}^{\dagger}_{\mu} \) the field operator (109) reads

\[
\hat{h}_\nu(r) = \sum_{p,\mu=1,2} \frac{8\pi G\hbar}{pVc^3} (\epsilon_{p\mu} \hat{A}_{-\mu} e^{ip\cdot r} + \text{h.c.}),
\]

where now \( \epsilon_{p1} \) and \( \epsilon_{p2} \) are spatial unit polarization vectors of the left and right circularly polarized gravitons respectively.

Commutators of \( \hat{A}_{\mu} \) gravitoelectric

\[
\hat{E}_\nu(r) = i \sum_{p,\mu=1,2} \frac{8\pi G\hbar}{Vc^3} \left( \epsilon_{p\mu} \hat{A}_{\mu} e^{ip\cdot r} - \text{h.c.} \right),
\]

and gravitomagnetic

\[
\hat{B}(r) = i \sum_{p,\mu=1,2} \frac{8\pi G\hbar}{pVc^3} (p \times \epsilon_{p\mu} \hat{A}_{\mu} e^{ip\cdot r} - \text{h.c.})
\]

field operators with the free field Hamiltonian (97) are the same as in electrodynamics, namely

\[
[\hat{H}_0, \hat{A}_{\mu}] = -\hbar c p \hat{A}_{\mu}, \quad \mu = 1, 2.
\]

This result is independent of the commutation relation between \( \hat{A}_{p,\mu} \) and \( \hat{A}^{\dagger}_{p,\mu'} \). In addition, the commutator

\[
[\hat{h}_\nu(r'), \hat{B}_\alpha(r)] = 0, \quad \alpha, \beta = 1, 2, 3
\]

remains the same. However, sign of the commutation relation between \( \hat{h}_\nu(r) \) and \( \hat{E}_\alpha(r) \) depends on the vacuum state, namely

\[
[\hat{h}_\nu(r'), \hat{E}_\alpha(r)] = \mp i \frac{16\pi G\hbar}{c^3} \delta^{\beta\gamma}\delta(r - r').
\] (120)

The lower sign in equation (120) corresponds to equation (119), that is to the vacuum with empty fermion states. The upper sign follows from equation (118) obtained for the filled vacuum and is the same as in quantum electrodynamics.

In the Heisenberg picture the Heisenberg equations of motion yield Maxwell-like equations for the transverse gravitational field in the operator form

\[
\text{curl} \hat{\mathbf{B}} = \pm i \frac{16\pi G}{c^3} \mathbf{j}_\nu + \frac{1}{c} \frac{\partial \hat{E}_\nu}{\partial t}, \quad \text{curl} \hat{\mathbf{E}}_\nu = -\frac{1}{c} \frac{\partial \hat{B}}{\partial t},
\] (121)

where \( \mathbf{j}_\nu \) is the transverse part of the mass current density

\[
\mathbf{j} = \sum_a m_a \mathbf{r}_a \delta(\mathbf{r} - \mathbf{r}_a(t)).
\]

Hamiltonian (108) gives the following equation of motion of mass \( m \) in transverse gravitational field

\[
m\ddot{\mathbf{r}} = c^2 \left[ m \mathbf{E}_\nu + \frac{m}{c} (\mathbf{r} \times \mathbf{B}) \right].
\] (122)

Comparison of equations (121) and (122) with those of quantum electrodynamics yields that quantum vector gravity, up to irrelevant numerical factor, is equivalent to QED for the upper sign in equation (121), that is for the filled vacuum (present epoch).

The lower sign in equation (121) corresponds to the vacuum with empty fermion states. This is the classical limit of the quantum vector gravity which reproduces the classical weak field equations for the transverse field. In the present epoch the fermion states are filled and we must take the upper sign in equation (121). Thus, quantum mechanical analysis yields that evolution equations describing gravitational radiation in the present epoch are different from those that follow from the classical Lagrangian (106). Namely, in classical equations describing radiation, \( \mathbf{j}_\nu \) must be taken with the opposite sign.

One should mention that the difference in equations appears only when we are dealing with the radiation part of the transverse field which is quantized. Transverse gravitational field produced by stationary mass currents is not quantized and is described by the same classical equations. As a consequence, the post-Newtonian limit of vector gravity is entirely classical and post-Newtonian equations are not modified by the quantum mechanical analysis. This is also the
case for Universe evolution after the end of the inflation stage. In particular, dark energy comes from the classical longitudinal part of the gravitational field.

Averaging the operator equation (121) over the state vector yields Maxwell-like equations for the average fields $E_{\alpha}$ and $B$. The averaged equations lead to the following expression for the energy of the radiation field interacting with matter

$$W_l = \pm \frac{e^2}{32\pi G} \int d^4x (E_{\alpha}^2 + B^2) + \sum_a \frac{m_a e^2}{2},$$

and the energy flux density (Poynting vector) of the radiation gravitational field

$$S = \pm \frac{e^2}{16\pi G} E_{\alpha} \times B.$$  

(123)

(124)

Energy of the graviton in the classical limit is a unit vector in the direction of $\hat{\epsilon}$, and $\hat{\epsilon}$ is a unit vector in the direction of $\hat{p}$. Classical analysis yields that such waves are not emitted or absorbed by orbiting stars (see next section). Quantum consideration gives the same answer. Indeed, equations (91), (92), (107), (126) and (127) suggest that longitudinal gravitational waves must be quantized by replacing $\hat{h}_l$ and $h_{00}$ with operators

$$\hat{h}_l(r) = \sum_p \frac{32\pi G h}{3pVc^5} (\hat{\rho} \hat{p} e^{ip\cdot r} + \text{h.c.}),$$

$$h_{00}(r) = -\frac{1}{2} \sum_p \frac{32\pi G h}{3pVc^5} (\hat{\sigma} e^{ip\cdot r} + \text{h.c.}),$$

(128)

(129)

describes a composite particle with negative energy $\varepsilon(p) = -\hbar c \hat{p}$. Commutator of $\hat{h}_l$ with the free field Hamiltonian (97) is

$$[\hat{H}_0, \hat{h}_l] = \hbar c \hat{p} \hat{h}_l.$$  

(133)

In the limit $N_l \to \infty$ we obtain commutation relations

$$[\hat{h}_p, \hat{h}_p^+] = [\hat{h}_p, \hat{h}_l] = 0.$$  

for both empty and filled vacuum. $\hat{h}_p$ and $\hat{h}_p^+$ also commute with the transverse field (graviton) operators. Thus, if longitudinal waves are coupled with matter through the operators $\hat{h}_p$ and $\hat{h}_p^+$, the commutator of $\hat{h}_p$ with the interaction Hamiltonian will be equal to zero. As a result, the Heisenberg equations of motion involving the full Hamiltonian will also give the free field equations (131) and (132) without sources. This means that evolution of the longitudinal waves is not affected by matter. Therefore, longitudinal waves are not produced by matter in the classical (empty vacuum) and quantum (filled vacuum) limits. However, they can be generated in the early Universe or at the stage of merger of massive neutron stars.

Commutation relations (133) indicate that longitudinal and time-like components of the gravitational field behave as classical quantities.

12. Radiation of gravitational waves by system of masses

In this section we consider radiation of gravitational waves by a system of stars moving with nonrelativistic velocities. Our analysis is also valid for neutron stars which produce strong gravitational field in their vicinity. We assume that stars have masses $M_i$ ($i = 1, 2, \ldots$) and move with velocities $\mathbf{V}(t) \ll c$. Spacing between starts is large compared to their dimension, however, the total size of the system is much smaller that the radiation wavelength.

Strong gravitational field of neutron stars demands to keep nonlinear terms in the equations for the gravitational field. Such terms are not confined to a compact region, but extend over all space. However, nonlinear terms decay as $1/r^2$ away from the star and can be large only in the vicinity of a neutron star. We average the gravitational field equations (22) over the volume large compared to the stellar size but much smaller than spacing between stars. After such averaging the source of the gravitational field becomes a sum of $\delta$-functions localized at the star positions. Nonlinear terms in the stellar vicinity are subsumed into the $\delta$-function sources. Nonlinear terms far from the star could give a small correction to the solution in the wave zone of the order of $G^2$ which we neglect.

As a result, after averaging we obtain the following linear equations for the gravitational field in the cosmological...
reference frame (see equations (38) and (39))

\[
\Delta h_{00} + 3 \frac{\partial^2 h_{00}}{\partial x^0 \partial x^0} - 2 \frac{\partial^2 h_{03}}{\partial x^0 \partial x^3} = \frac{8 \pi G}{c^2} \sum_i M_i \delta(\mathbf{r} - \mathbf{r}(t)),
\]

\[
\left( \frac{\partial^2}{\partial x^0 \partial x^0} - \Delta \right) h_{00} + \frac{\partial^2 h_{03}}{\partial x^0 \partial x^3} - 2 \frac{\partial^2 h_{00}}{\partial x^0 \partial x^0} = \frac{16 \pi G}{c^3} \sum_i M_i^* V_i^\alpha \delta(\mathbf{r} - \mathbf{r}(t)),
\]

where \(\mathbf{r}(t)\) are the radii vectors of the stars. In equation (134) \(M_i\) are the stellar masses measured by a distant observer. However, for neutron stars, \(M_i^*\) in equation (135) could differ from \(M_i\) due to strong-field effects and depend on the stellar equation of state. Next we show that \(M_i^* = M_i\) at least up to the second order in the stellar velocity.

Taking \(\partial / \partial x^0\) from equation (134) and \((1/2) \partial / \partial x^\alpha\) from equation (135) and adding these equations together we obtain

\[
3 \frac{\partial^2}{\partial x^0 \partial x^0} \left( \frac{\partial h_{00}}{\partial x^0} - \frac{1}{2} \frac{\partial h_{03}}{\partial x^0} \right) = \frac{8 \pi G}{c^2} \sum_i \left( M_i \frac{\partial}{\partial x^0} \delta(\mathbf{r} - \mathbf{r}(t)) + \frac{1}{c} \frac{\partial}{\partial x^0} M_i^* V_i^\alpha \delta(\mathbf{r} - \mathbf{r}(t)) \right),
\]

Using

\[
\frac{\partial}{\partial x^0} \delta(\mathbf{r} - \mathbf{r}(t)) = - \frac{1}{c} \frac{\partial}{\partial x^0} [V^\alpha \delta(\mathbf{r} - \mathbf{r}(t))]
\]

one can write equation (136) as

\[
3 \frac{\partial^2}{\partial x^0 \partial x^0} \left( \frac{\partial h_{00}}{\partial x^0} - \frac{1}{2} \frac{\partial h_{03}}{\partial x^0} \right) = \frac{8 \pi G}{c^2} \sum_i \left( M_i - M_i^* \right) \frac{\partial}{\partial x^0} \delta(\mathbf{r} - \mathbf{r}(t)).
\]

The left-hand side of equation (137) is of the order of \((V/c)^3\). This yields that \(M_i - M_i^*\) in the right-hand side are of the order of \((V/c)^2\).

Thus, with the required accuracy one can take \(M_i^* = M_i\) in equations (134) and (135). Solution of equations (134) and (135) satisfying the proper boundary condition is given by the retarded potentials

\[
h_{00} = -\frac{2G}{c^2} \sum_i \frac{M_i}{|\mathbf{r} - \mathbf{r}(t_i)|},
\]

\[
h_{00} = \frac{4G}{c^3} \sum_i \frac{M_i V_i^\alpha(t_i)}{|\mathbf{r} - \mathbf{r}(t_i)|},
\]

where \(t_i\) is solution of the equation \(t = t_i + |\mathbf{r} - \mathbf{r}(t_i)|/c\). Retarded potentials describe outgoing waves with phase velocity directed away from the source. For solution (138) and (139)

\[
\frac{\partial h_{00}}{\partial x^0} = \frac{1}{2} \frac{\partial h_{03}}{\partial x^0} = 0.
\]

In appendix I we provide an alternative derivation of the gravitational field produced by an orbiting neutron star which is valid in the \(V^2/c^3\) order in the stellar velocity and arbitrary strength of the stellar gravitational field \(\phi\). The answer for the equivalent metric is given by equations (II3)–(II5). The analytical result includes both near and far field regions in a single equation which is valid for arbitrary \(\phi\) but omits retardation effects. In the far field the answer reduces to equations (I16) and (I17) of appendix I which match the retarded potentials (138) and (139) obtained here using linearized equations. Such a match justifies omission of the nonlinear terms in the present equations (134) and (135).

Since \(\sum M_i\) is constant, the time dependent part of \(h_{00}\) in equation (138) vanishes at large \(r\) as \(1/r^2\). Thus, \(h_{00}\) does not contribute to the emission of gravitational waves, as in the case of electromagnetic radiation in classical electrodynamics.

In the present theory, a graviton is not an elementary particle, but rather it is composed of fermion–antifermion pairs. Solution (139) is obtained in the classical limit when vacuum corresponds to empty fermion states. However, fermion states are filled in the present epoch and, as we showed in the previous section, for filled vacuum the equations describing radiation of gravitational waves must be modified by changing the sign of the mass current to the opposite. As a consequence, solution (139) must be replaced with

\[
\mathbf{h} = \frac{4G}{c^3} \sum_i \frac{M_i V_i(t_i)}{|\mathbf{r} - \mathbf{r}(t_i)|},
\]

where \(\mathbf{h} = h^{0\alpha} = -h_{\alpha 0}\). For filled vacuum emission of a graviton corresponds to absorption of fermion–antifermion pairs out of the filled vacuum states or creation of fermion–antifermion holes which propagate away from the source. According to equation (123), for filled vacuum graviton has positive energy and binary stars orbiting each other are losing their energy by emitting gravitational waves. Equation (140) has a very similar form to the vector potential produced by moving charges \(e\)

\[
\mathbf{A} = \frac{1}{c} \sum_i \frac{e_i V_i(t_i)}{|\mathbf{r} - \mathbf{r}(t_i)|},
\]

and, according to equations (124) and (125), expressions for the Poynting vectors of the radiation field in vector gravity and electrodynamics are also similar. As a consequence, to obtain the answer for the power \(P\) of emission of gravitational waves by the system of masses we can apply the formula of classical electrodynamics for the radiation of electromagnetic waves by a system of charges

\[
P = \frac{2}{3} \frac{d^2}{c^3} + \frac{1}{180 c^5} \mathbf{D}_{\alpha \beta} \mathbf{D}^\alpha_{\beta} + \frac{2}{3} \frac{m^2}{c^3},
\]

where

\[
\mathbf{d} = \sum e \mathbf{r}
\]

is the electric dipole moment of the system,

\[
\mathbf{D}_{\alpha \beta} = \sum e (3x_{\alpha} x_{\beta} - r^2 \delta_{\alpha \beta})
\]
is the quadrupole moment,  
\[ \mathbf{m} = \frac{1}{2c^2} \sum e(r \times V) \]
is the magnetic moment and the sum is over all charges in the system.

Comparison of equations (140) and (141) yields that in order to obtain power loss due to emission of gravitons one should in the electrodynamics equations replace the electric charges with \( q_i \rightarrow 4GM_i/c^2 \). In addition, equation (124) gives that gravitational energy density flux for the radiation field is  
\[ S = -\frac{c^5}{16\pi G} \frac{\partial h}{\partial x^0} \times \text{curl}(h), \]
which is in a factor \( c^4/4G \) greater than the corresponding expression for the electromagnetic waves  
\[ S = \frac{c}{4\pi} \mathbf{E}_r \times \mathbf{H} = -\frac{c}{4\pi} \frac{\partial \mathbf{A}}{\partial x^0} \times \text{curl}(\mathbf{A}). \]
Thus, \( P \) for gravity must be also multiplied by \( c^4/4G \).

Combining all factors together we finally obtain the following expression for the power loss by the system of masses due to emission of gravitational waves  
\[ P = \frac{8G}{3c^3} d^2 + \frac{G}{45c^5} D_{\alpha \beta} + 2G/3c^5 L^2, \]
where we introduced the dipole moment of the system  
\[ d = \sum \mathbf{M}_i r, \]
the quadrupole moment of masses  
\[ D_{\alpha \beta} = \sum M(3x_\alpha x_\beta - r^2 d_{\alpha \beta}), \]
and the net angular momentum  
\[ L = \sum \mathbf{M}(r \times V). \]
In these equations the summation is over all masses.

With the required accuracy, the stellar trajectories can be calculated in the Newtonian limit. In such limit  
\[ \mathbf{d} = \sum_i M_i V_i \]
is the total linear momentum of the isolated system which is a conserved quantity. Therefore, \( \mathbf{d} \) vanishes and, hence, there is no dipole radiation. For an isolated system the total angular momentum \( \mathbf{L} \) is also conserved and, thus, the last term in equation (144) also vanishes. The fact that \( M_i^0 = M_i \) in equations (134) and (135) guarantees that inertial mass that determines the dipole moment is the same as mass that generates gravitational waves. One should mention that possible small deviation of \( M_i^0 \) from \( M_i \) in our equation (135) due to strong-field effects (which is of the order of \( (V/c)^2 \)) might yield contribution to the dipole radiation of the order of \( d^2 \propto (V/c)^8 \), while \( L^2 \propto (V/c)^{10} \). These contributions are much smaller than the quadrupole emission which is proportional to \( (V/c)^6 \).

As a result, the quadrupole radiation gives the dominant contribution to the energy loss. The quadrupole term in equation (144) coincides with the Einstein’s formula obtained in general relativity. The rate of loss of angular momentum from a system of bodies emitting gravitational waves is also given by the same equation as in general relativity

\[ \frac{dL_{\alpha\beta}}{dt} = -\frac{2G}{45c^5} e_{\alpha\beta\gamma} D_{\gamma\delta} D_{\delta\gamma}, \]

where \( e_{\alpha\beta\gamma} \) is the Levi-Civita symbol. Equation (145) is obtained in [27] directly from the rate of energy loss by the system which is given by the same quadrupole formula in both theories. As a consequence, for nonrelativistic motion the present theory yields the same orbital decay of binary stars from gravitational radiation as general relativity.

The energy flux at infinity carried out by gravitational radiation is balanced by an equal loss of mechanical or orbital energy by the system \( W \). This loss of energy results in a decrease in the orbital period \( T \) given by Kepler’s third law [2]

\[ \frac{T}{T_0} = \frac{3}{2} \frac{W}{W_0}, \]

Such decrease in the orbital period has been measured for several binary systems and agreed with the predictions of general relativity. Thus, it also agrees with the vector theory of gravity.

In other alternative theories of gravity, while the inertial dipole moment may remain uniform, the ‘gravity wave’ dipole moment need not, because the mass that generates gravitational waves depends differently on the internal gravitational binding energy of each body than does the inertial mass [2, 3]. In such theories, the additional form of gravitational radiation damping (dipole radiation) could be significantly stronger for neutron stars than the usual quadrupole damping. Our vector theory of gravity predicts no dipole gravitational radiation because it satisfies the strong equivalence principle at least to the post-Newtonian order.

### 13. Neutron star mass limit in vector gravity

According to general relativity an object of nuclear density and more than about \( 3M_0 \) would be a black hole [41, 42]. Here we examine the neutron star upper mass limit in the vector theory of gravity. To calculate the maximum mass of a neutron star, one must have an equation of state for matter at high density which is very uncertain at present.

Taking \( T_k^0 \) as the energy momentum tensor of a perfect fluid, namely  
\[ T_k^0 = \varepsilon, \quad T_i^0 = -P, \]
all others are 0, with energy density \( \varepsilon = \varepsilon(r) \) and pressure \( P = P(r) \) the field equations for static gravitational field described by the equivalent metric

\[ f_{ik} = \begin{pmatrix} e^{2\phi} & 0 & 0 & 0 \\ 0 & -e^{-2\phi} & 0 & 0 \\ 0 & 0 & e^{-2\phi} & 0 \\ 0 & 0 & 0 & -e^{-2\phi} \end{pmatrix}, \]
become
\[ \Delta \phi = \frac{4\pi G}{c^2}(\varepsilon + 3P)e^{-2\phi}. \]

The energy–momentum relation
\[ T^i_k = 0, \quad (146) \]
where covariant derivative is taken using equivalent metric \( f_{ik} \)
\[ T^i_k = \frac{1}{\sqrt{-f}} \frac{\partial}{\partial x^k} \left( \sqrt{-f} T^i \right) - \frac{1}{2} \frac{\partial f_{il}}{\partial x^l} T^i_k, \]
yields equation of hydrostatic equilibrium
\[ \frac{\partial P}{\partial r} = -(P + \varepsilon) \frac{\partial \phi}{\partial r}, \]
where \( P \) is related to \( \varepsilon \) by an equation of state
\[ P = P(\varepsilon). \]

The boundary conditions at the stellar center \( r = 0 \) and the surface \( r = R \) read
\[ \phi'(0) = 0, \quad P(R) = 0. \]
Taking into account that outside the star \( \phi(r) = -GM/rc^2 \), continuity of \( \phi \) and \( \phi' \) yields additional boundary condition at the star surface
\[ R\phi'(R) = -\phi(R). \]
Matching solution inside and outside the star yields expression for the stellar mass \( M \) in terms of \( \phi(R) \)
\[ M = \frac{Rc^2}{G}\phi(R). \]

It is now necessary to choose an equation of state. We take it in the form which have been studied previously in general relativity \([43]\)
\[ P = b[\varepsilon - \varepsilon_0(\phi)], \quad (147) \]
where \( b \) is a constant and \( \varepsilon_0(\phi) \) is the energy density above which the equation of state becomes ‘stiff.’

Very often in literature the so called ‘causality’ condition is imposed to the equation of state: \( dP/d\varepsilon \leq 1 \), that is \( b \leq 1 \) in equation \((147)\). The condition requires that the speed of sound in the stellar matter \( c_s \) can not exceed the speed of light \( c \). However, though the last statement is true it does not mean that we must impose the restriction \( dP/d\varepsilon \leq 1 \) to the possible equation of state. The point is that equations of hydrodynamics, which result in \( c_s = c\sqrt{dP/d\varepsilon} \), are derived under the assumption of instantaneous interactions between particles (local approximation). However, real interactions propagate with the speed of light. As a result, if \( dP/d\varepsilon > 1 \) the equations become substantially nonlocal and static compressibility \( dP/d\varepsilon \) no longer describes the speed of sound. In this regime the speed of interaction propagation imposes the restriction on the speed of compression waves in matter. As a consequence, the speed of sound never exceeds the speed of light no matter what is the value of the static compressibility \( dP/d\varepsilon \).

One should mention that some authors also express caution about the ‘causality’ condition constraint on the equation of state. E.g., Kalogera and Baym say \([42]\): ‘The connection between the zero frequency sound velocity being greater than the speed of light and violation of causality, while physically plausible, is a tricky question ... We are not aware of a general proof yet that the ground state of matter must obey \( dP/d\varepsilon \leq 1 \).’

In equation \((147)\) \( \varepsilon_0(\phi) \) depends on the gravitational potential \( \phi \). To find this dependence we consider spatially uniform fluid placed in a spatially uniform gravitational field.
Figure 5. Gravitational redshift $z$ at the surface of a neutron star as a function of central pressure $P(0)$ in vector theory of gravity for equation of state (149) with $b = 1/3, 1$ and $1.3$.

$\phi$ that depends on time. Such consideration is similar to the cosmological model of the Universe. Since $\varepsilon = \varepsilon(\phi)$ and $P = P(\phi)$ the change of $\phi$ causes change of $\varepsilon$ and $P$. Then the energy–momentum relation (146) yields the equation

$$\dot{\varepsilon} = 3(\varepsilon + P)\dot{\phi}.$$  

Assuming that $\varepsilon(\phi)$, $\varepsilon_0(\phi)$ and $P(\phi)$ are proportional to the same function of $\phi$ we obtain

$$\varepsilon(\phi), P(\phi) \propto e^{\alpha \phi},$$

where

$$\alpha = 3\left(1 + \frac{P}{\varepsilon}\right)$$

is independent of $\phi$. For the equation of state (147) we find

$$\varepsilon_0(\phi) = \varepsilon_0 \exp\left[3\left(1 + \frac{b^2 \varepsilon_0(\phi)}{P(\phi) + b \varepsilon_0(\phi)}\right)\phi\right]$$

(148)

which is an algebraic equation for $\varepsilon_0(\phi)$. In equation (148) $\varepsilon_0$ is the value of $\varepsilon_0(\phi)$ at $\phi = 0$. It is convenient to introduce dimensionless coordinate, pressure and energy density as

$$r \rightarrow r_0 r, \quad P \rightarrow \varepsilon_0 P, \quad \varepsilon \rightarrow \varepsilon_0 \varepsilon$$

where

$$r_0 = \frac{c^2}{\sqrt{3\pi G \varepsilon_0}}.$$  

For the dimensionless functions the field equation and equation of hydrostatic equilibrium read

$$\Delta \phi = (\varepsilon + 3P)\varepsilon^{-1}\phi,$$

$$\frac{\partial P}{\partial r} = -(P + \varepsilon)\frac{\partial \phi}{\partial r},$$

while the boundary conditions are

$$\phi'(0) = 0, \quad P(R) = 0, \quad R\phi'(R) = -\phi(R).$$

Dimensionless equation of state (147) is

$$P = b(\varepsilon - \xi),$$

(149)

where $\xi$ is given by a solution of the dimensionless algebraic equation

$$\xi = \exp\left[3\left(1 + b - \frac{b^2 \xi}{P + b \xi}\right)\phi\right].$$

Stellar mass $M$ is obtained from the formula

$$\frac{M}{M_0} = -R\phi(R),$$

where

$$M_0 = \frac{c^2 r_0}{G} = \frac{c^4}{\sqrt{4\pi G \varepsilon_0}}.$$  

For $\varepsilon_0/c^2 = 10^{14}$ g cm$^{-3}$ we have

$$r_0 = 32.8 \text{ km}, \quad M_0 = 22.3 M_\odot.$$  

In figures 3–5 we plot mass of a neutron star, its radius and surface gravitational redshift as a function of the central pressure $P(0) \equiv P(r = 0)$ in the vector theory of gravity for the equation of state (149) for various values of the stiffness parameter $b = 1/3, 1$ and $1.3$. Figure 3 shows that the stiffer the equation of state, the higher the star mass limit. For example, for $b = 1$ the maximum gravitational mass of a stable neutron star is $M_{\text{max}} = 1.52 M_0 = 34 M_\odot$ for $\varepsilon_0/c^2 = 10^{14}$ g cm$^{-3}$. This mass is obtained for $dP/d\varepsilon \leq 1$. Radius of star with maximum mass is $R = 1.98 r_0 = 65$ km. The effects of stellar rotation may increase the maximum mass by about 25% [41].

Star is stable provided $\partial M/\partial P(0) > 0$ [44]. For $b = 1$ there are three intervals of stability for nonrotating stars, namely $M < 0.31 M_\odot$, $0.5 M_\odot < M < 1.00 M_\odot$ and $1.37 M_\odot < M < 1.52 M_\odot$ which for $\varepsilon_0/c^2 = 10^{14}$ g cm$^{-3}$ give $M < 7 M_\odot$, $11 M_\odot < M < 22 M_\odot$ and $30 M_\odot < M < 34 M_\odot$ respectively. It is interesting to note that masses of compact objects in the merging binary systems obtained by the LIGO team based on the gravitational wave detection reported so far ($29^{+12}_{-5} M_\odot$ and $36^{+3}_{-2} M_\odot$ [5]; $14.2^{+3.8}_{-3.7} M_\odot$ and $7.5^{+3}_{-2} M_\odot$ [6]; $31.2^{+8.9}_{-6.5} M_\odot$ and $19.4^{+3.5}_{-2.8} M_\odot$ [7]), within the error bar fit in the mass intervals for which neutron stars are stable.

However, stability intervals are sensitive to the choice of the equation of state which is very uncertain at high matter density. Our choice of the equation of state (149) is only an example. In addition, inclusion of stellar rotation can considerably widen stability regions. Nevertheless, vector gravity predicts existence of gaps in the neutron star mass distribution, although position of the gaps depends on the uncertain equation of state. One should note that stellar-mass compact objects with masses up to about $16 M_\odot$ have been discovered in X-ray binaries [45]. Thus, there is a wide gap between $19 M_\odot$ and $29 M_\odot$ measured by LIGO. Future observations will
fill this interval with more data and test the prediction of vector gravity about existence of gaps. It is interesting to note that a \(3-5 M_\odot\) gap has been found in the low-mass part of the measured compact object mass distribution in the Galaxy [46, 47]. If position of the mass gaps is obtained from observations this information can be used to determine the equation of state of matter in the dense stellar cores by matching stability regions with the astronomical data.

Supermassive compact objects with masses of \(>10^{5}M_\odot\) have been discovered in galactic centers [48]. In section 15 we argue that such supermassive compact objects are made of dark matter and, according to vector gravity, can have masses in a range \(\sim 10^{3}-10^{10}M_\odot\). It is interesting to note that compact objects with masses in the interval \(\sim 10^{2}-10^{5}M_\odot\) have not yet been detected beyond doubt.

We found that, as in general relativity, the neutron star mass limit varies roughly as \(1/\sqrt{\varepsilon_0}\), where \(\varepsilon_0\) is the energy density above which the equation of state becomes ‘stiff.’ Unlike general relativity, the stiffer the equation of state, the higher the mass limit. Our numerical simulations show that \(M_{\text{max}}\) increases with increasing \(b\). E.g., for \(b = 3\) we find \(M_{\text{max}} = 13.4M_\odot \approx 300M_\odot\). This result is somewhat similar to those in the bimetric theory of gravity which for \(dP/d\varepsilon > 1\) yields that the upper mass limit \(M_{\text{max}}\), unlike the general-relativistic case [43], can be arbitrary large [49].

In vector gravity, as in general relativity, if the stellar mass exceeds a certain value \(M_{\text{max}}\), there does not exist any static solution of the field equations, and therefore the star must undergo collapse. However, there is a great difference between the predictions of the two theories as to what will happen during the process of collapse. In the case of gravitational collapse in the framework of general relativity, once the surface of the star has entered the Schwarzschild sphere, one has a black hole. It is believed that the matter of the star and its radiation are then permanently trapped in the black hole [50].

In vector gravity, as well as in other alternative theories of gravity with no event horizons, since there does not appear to be anything corresponding to the Schwarzschild sphere, the inner part of an unstable star will first contract and then expand [51]. The contracting outer part could collide with the expanding inner part. The collision could result in the ejection of the outer envelope and hence in a loss of mass which makes the star stable again [51]. Another possible scenario is that a neutron star with a limiting mass swirling baryons will radiate their mass equivalents for stability yielding substantial additional radiation of internal origin. Such radiation could appear as copious photon or neutrino emissions or gravitational radiation [35].

The end point of a gravitational collapse in vector gravity is not a point singularity but rather a stable object with a reduced mass. Merger of two neutron stars with masses close to the upper limit leads to formation of a stable star with a higher baryon number but not the mass. The net mass of the two merging stars is reduced due to greater gravitational binding energy of the merger. The excess energy is radiated away, e.g., by neutrino emission or by other mechanisms.

In vector gravity there is no gravitational collapse of a star into a point singularity because such an object would have zero mass. Indeed, let us consider two static point masses separated by a distance \(R\) and assume that \(m_1\) and \(m_2\) are the values of masses at infinite separation. In vector gravity, static gravitational field is described by equation (27) which for the case of two masses reduces to

\[
\Delta \phi = \frac{4\pi G}{c^2} \left( m_1 \delta (\mathbf{r} - \mathbf{r}_1) + m_2 \delta (\mathbf{r} - \mathbf{r}_2) \right) e^\phi
\]

and has the following solution

\[
\phi(\mathbf{r}) = -\frac{G}{c^2} \left[ \frac{m_1 e^{\phi(\mathbf{r})}}{|\mathbf{r} - \mathbf{r}_1|} + \frac{m_2 e^{\phi(\mathbf{r})}}{|\mathbf{r} - \mathbf{r}_2|} \right],
\]

(150)

where

\[
\phi_{1,2}(R) = -\frac{Gm_{1,2}}{c^2 R}.
\]

The net mass of the system \(m\) is determined by the asymptotic of equation (150) at large \(r\) which gives

\[
m = m_1 \exp \left(-\frac{Gm_2}{c^2 R}\right) + m_2 \exp \left(-\frac{Gm_1}{c^2 R}\right).
\]

(151)

For \(R \to 0\) the net mass vanishes. If we gradually move masses closer to each other the net mass of the system decreases to zero due to negative contribution of the gravitational potential energy. A star collapsed to a point would also have zero kinetic energy (in the classical consideration) and, hence, the total energy of such collapsed object would be equal to zero. As a consequence, spatial point singularities do not exist in vector gravity. However, for simplicity, in many problems masses can be approximated as point masses similar to the concept of point charges in electrodynamics.

14. Tests of the theory of gravity

In this section we compare predictions of the vector theory of gravity with observations. References [2–4] provide a detail procedure of how to compare metric theories of gravity with experimental tests and show viability of the theory. Here we follow this procedure step by step and show that vector gravity passes all available tests.

Post-Newtonian limit. In vector gravity there is a preferred cosmological reference frame in which background vector gravitational field has only time component. This is a reference frame in which the large-scale distribution of matter is isotropic (presumably the rest frame of the cosmic background radiation). As we show in section 7, in such cosmological frame in the post-Newtonian limit, equations of the vector gravity as well as the boundary conditions are equivalent to those in general relativity. Thus, in any frame moving with a non relativistic speed with respect to the cosmological reference frame (such as our solar system) both theories remain equivalent provided we are not going beyond the post-Newtonian approximation. As a consequence, vector gravity yields the same values for the ten PPN parameters as general relativity [2–4]. In appendix J we
investigate the post-Newtonian limit of vector gravity in the framework of the parametrized post-Newtonian formalism and explicitly calculate the ten post-Newtonian parameters. As expected, they are equal to those in general relativity.

The post-Newtonian limit is sufficient to describe the gravitational physics of the solar system and the experimental tests one can perform there [2–4]. To some degree, it can also describe the gravity of binary-pulsar systems. Since vector gravity and general relativity are equivalent in the post-Newtonian limit, they both pass every high-precision test in the solar system, where gravitational fields are relatively weak. In those familiar precincts, they correctly predict red-shifting, light deflection by a massive body, Shapiro time delay, precession of planetary orbits, the strict equivalence of gravitational and inertial mass, lack of the preferred-frame and preferred-location effects, etc.

Vector gravity, as well as general relativity, is built on the Einstein equivalence principle which states that matter is coupled in a universal manner to a single tensorial metric. Extension of the Einstein equivalence principle to gravitational experiments is known as the strong equivalence principle which states that local gravitational physics is independent of the position and velocity of the local reference frame. Alternative theories of gravity involving additional fields or fixed background geometry tend to violate the strong equivalence principle [2–4]. However, since vector gravity is equivalent to general relativity in the post-Newtonian limit the vector gravity obeys the strong equivalence principle in this limit.

**Gravitational radiation by binary pulsars.** There are several tests of gravity beyond the solar system. Gravitational radiation by binary pulsars provides a tool for testing relativistic gravity. In general relativity, the gravitational waves emitted by a slowly-moving system are dominantly quadrupole and there is no monopole or dipole radiation. This is because the field equations of general relativity insist that monopole and dipole moments are the total mass and the total momentum of the system which are constants if the system is isolated [2–4]. There is no reason to expect that a generic alternative theory will predict the suppression of monopole and dipole emission. However, as we show in section 12, for vector gravity there is no monopole and dipole radiation due to the same reason as for general relativity. Moreover, we show that in our theory the gravitational energy loss by binary stars is described by the same formula as in general relativity. Our analysis of section 12 is also valid for neutron stars which are relativistic objects. Energy loss by binary pulsars due to emission of gravitational radiation was measured for several systems and served as a quantitative test of Einstein equations for weak time-dependent field. The present theory also passes this test.

One should note that studies carried out in the wake of the discovery of the Hulse–Taylor binary pulsar in 1974 [52] revealed that a number of otherwise respectable alternative theories of gravity predicted the emission of the negative energy [2]. Once the orbital period of the binary pulsar was shown to decrease in response to the emission of gravitational waves (that is total energy of the binary system decreases with time), these theories were ruled out. Since in the present theory the graviton energy is negative in the classical limit, the vector theory of gravity would also predict emission of the negative energy by binary pulsars if we would not postulate that graviton is composed of fermion–antifermion pairs by analogy with the composite theory of photon.

One should mention that theories with negative graviton energy are somewhat appealing because they provide a natural mechanism of matter generation at the Big Bang without involving additional cosmological fields. In the present theory, as soon as fermion states are filled shortly after the Big Bang the matter generation stops and Universe subsequently evolves according to the usual hot Universe scenario. For filled vacuum the graviton energy becomes positive (see section 11) and emission of a graviton corresponds to creation of fermion–antifermion hole pairs out of the filled fermion states. Since fermion states are filled in the present epoch the binary pulsars orbiting each other emit positive energy gravitons which, as we show in section 12, yields exactly the same energy loss by the binary systems as predicted by general relativity.

**Structure and motion of compact objects.** Precise orbital data obtained for binary pulsars permits the direct measurement of the mass of a neutron star and the study of relativistic orbital effects (such as periastron shifts) in systems containing compact objects possessing strong gravitational field. In alternative theories of gravity, strong gravitational field involved in the neutron star can make significant differences in relativistic orbital effects. When dealing with a system such as the binary pulsar one must employ a method for deriving equations of motion for compact objects that involves solving the full relativistic equations for the regions inside and near each body, solving the post-Newtonian equations in the interbody region and matching these solutions [2].

Most alternative theories of gravity possess additional gravitational fields (dynamical or fixed), whose values in the matching region can influence the structure of each body, and, as a consequence, affect its motion. Namely, mass of the compact object may depend on the boundary values of the auxiliary fields leading to modification of the body’s motion. Thus, the location and velocity of the body relative to the external gravitational environment can affect its structure and motion. This is known as the preferred location and preferred frame effects. Orbital data obtained for binary pulsars show lack of such effects for compact objects at least in the $V^2/c^2$ order, where $V$ is the neutron star velocity [2]. Namely, observations show that binary pulsars move the same way as if they were weak-field post-Newtonian bodies.

Present vector theory of gravity contains an auxiliary nondynamical field, the flat Euclidean metric $\delta_{\alpha\beta}$, which yields a possibility of the preferred frame and preferred location effects. In appendix 1 we investigate this question and demonstrate lack of such effects for binary pulsars in the $V^2/c^2$ order (this might also be valid in higher orders). Namely, we show that equivalent metric produced by a moving neutron star is independent of the external gravitational background and of the star velocity relative to the background. The metric is characterized only by the object’s
cosmological constant $\Omega_\Lambda = 2/3 \approx 0.67$ which agrees with the recent Planck result $\Omega_\Lambda = 0.686 \pm 0.02$ [12]. Thus, vector gravity also passes the ‘dark energy’ test.

**Direct detection of gravitational waves by laser interferometers.** Possibility of gravitational wave detection by laser interferometers was first suggested in 1962 [53], shortly after invention of a laser. Recently LIGO team reported first observation of a transient gravitational-wave signal from orbital inspiral and merger of two compact objects losing their energy due to gravitational-wave emission [5]. Over 0.2 s, the signal increased in frequency and amplitude in about 8 cycles from 35 to 150 Hz, where the amplitude reached a maximum. Then waveform decayed undergoing damped oscillations (see top part of figure 6). Interpretation of the signal in [5] is based on general relativity which yields that the merging objects are two black holes with masses $29.4_{-4}^{+4} M_\odot$ and $36.2_{-4}^{+5} M_\odot$. Numerical relativity gives that at the maximum of the waveform amplitude the objects are separated by a distance $r_{\text{max}} \approx 350$ km [5].

Here we discuss interpretation of the LIGO signal GW150914 in the framework of vector gravity and calculate the gravitational radiation waveform. For simplicity we suppose that the two orbiting bodies have equal masses $m$ and move with velocity $V$ along circular trajectories of diameter $r$ around their common centre of mass. In Schwarzschild coordinates, the line element for the Schwarzschild metric of a point mass $m$ has the form

$$\text{d}s^2 = \left(1 - \frac{2Gm}{c^2r}\right)c^2\text{d}t^2 - \left(1 - \frac{2Gm}{c^2r}\right)^{-1}\text{d}r^2 - r^2\text{d}\Omega^2.$$  

(152)

In these coordinates equation (152) yields that for $m = 35M_\odot$ the Schwarzschild radius is

$$R_{S1} = \frac{2Gm}{c^2} = 103 \text{ km}$$

which is comparable with the 350 km separation between objects at the peak of the waveform amplitude. Based on this observation, the decay of the radiation waveform in the LIGO signal is commonly interpreted as damped oscillations of two merging black holes relaxing to a final stationary Kerr configuration [5].

This interpretation, however, should be taken with caution due to general covariance of Einstein’s theory. Recall that suitable nonlinear change of coordinates can make the value of the gravitational radius much smaller than the position between two objects. For comparison with vector gravity that yields a spatially isotropic line element

$$\text{d}s^2 = \exp\left(-\frac{2Gm}{c^2r}\right)c^2\text{d}t^2 - \exp\left(\frac{2Gm}{c^2r}\right)(\text{d}x^2 + \text{d}y^2 + \text{d}z^2)$$

(153)

we must write Schwarzschild metric (152) in isotropic coordinates by making nonlinear coordinate transformation $r \rightarrow (1 + Gm/2c^2 r)^{1/2} r$. This transformation reduces the value of the Schwarzschild radius 4 times but changes $r_{\text{max}}$ only a little from 350 km to $\approx 300$ km. In isotropic

---

**Figure 6.** Top: estimated gravitational-wave strain amplitude from GW150914 event as a function of time obtained by numerical relativity. Reproduced from [5]. CC BY 3.0. Bottom: metric component $g_{00}$ as a function of interstellar separation $r$ in general relativity (solid line) and vector gravity (dashed line). The unit of distance is Schwarzschild radius in isotropic coordinates $R_{S2}$. The cross denotes the location of the maximum of the radiation waveform amplitude. Vertical lines separate regions of orbital inspiral, ringdown inspiral and merger of two neutron stars of 60 km radii and masses $35M_\odot$. **

Kepler-measured mass $M$, and is independent of its internal structure. In the region far from the neutron star in the post-Newtonian limit the metric in our theory is given by the same formula as in general relativity. Thus, the matching procedure described above must yield the same result, whether the body is a neutron star of mass $M$ or a post-Newtonian body of mass $M$. Therefore, in vector gravity motion of compact objects is described by the same equations as motion of weak-field stars and coincides with predictions of general relativity in the $V^2/c^2$ order. Hence, vector gravity passes the binary pulsar test.

**Cosmological test.** Cosmology provides another important test of gravitational theories. As we show in section 9, for cosmology with a general equation of state of matter the present theory gives the same evolution of the Universe as general relativity with cosmological constant and zero spatial curvature. Thus, vector theory of gravity passes the cosmological test and, in particular, provides the same explanation for the cosmic microwave background radiation and the helium abundance as general relativity. Moreover, vector gravity yields, with no free parameters, the value of the

---

*Phys. Scr.* *92* (2017) 125001

A A Svidzinsky
coordinates the Schwarzschild line element reads
\[
ds^2 = \left(1 - \frac{Gm}{2c^2r}\right) c^2dr^2 - \left(1 + \frac{Gm}{2c^2r}\right) \left(dx^2 + dy^2 + dz^2\right).
\]
Equation (154) gives that for \( m = 35M_\odot \) the radius of Schwarzschild sphere in isotropic coordinates is
\[
R_{S2} = \frac{Gm}{2c^2} = 25.7 \text{ km}
\]
which is much smaller than separation between objects at the onset of the waveform ringdown stage \( r_{\text{max}} \approx 300 \text{ km} \). In the post-Newtonian formalism the metric (154) is expanded in the small parameter
\[
\epsilon = \frac{V^2}{c^2} = \frac{Gm}{2c^2r} = \frac{R_{S2}}{r},
\]
where \( V \) is the object velocity in the binary system which in the Newtonian gravity is given by
\[
V^2 = \frac{Gm}{2r}.
\]
For \( r = r_{\text{max}} \) equation (156) yields
\[
\epsilon = \frac{R_{S2}}{r_{\text{max}}} = 0.08 \ll 1
\]
and, therefore, decay of the radiation waveform in the LIGO signal actually begins at a relatively weak gravity. One should note that estimate (158) is independent of the value of mass \( m \) which factors out from equations.

As a consequence, interpretation of the decaying part of the radiation waveform in isotropic coordinates is qualitatively different. Namely, the decay occurs at the stage of orbital inspiral when two objects are yet considerably far from their merger (see top part of figure 6). The LIGO signal becomes smaller than noise before the two objects actually start to merge.

To compare vector gravity with general relativity we plot \( g_{00} \) component of the metric, given by equations (153) and (154), as a function of separation between stars \( r \) for both theories. The result is shown in figure 6 (bottom). General relativity yields solid line, while \( g_{00} \) for vector gravity is shown as dashed line. Vertical lines in the plot separate three regions of the orbital inspiral, ringdown inspiral and merger of two neutron stars of 60 km radii and masses 35\( M_\odot \). The figure demonstrates that \( g_{00} \) in both theories is practically indistinguishable up to the point of merger.

Next we calculate the radiation waveform in vector gravity and show that it is compatible with the LIGO data. We assume that two compact stars with masses \( m \) move in the \( x-y \) plane along circular orbits of diameter \( r \) with tangential velocity \( V = r\dot{\theta}/2 \), where \( \theta \) is the azimuthal angle in the \( x-y \) plane. Since the effects of gravity are expected to be relatively weak even during the ringdown stage the loss of the system’s angular momentum \( L = mrV \) can be accurately described by the quadrupole formula [27]
\[
\frac{dL_{00}}{dt} = -\frac{2G}{45c^3} \epsilon^{(\alpha)(\beta)} D_{\alpha\beta} D_{\gamma\gamma},
\]
where components of the quadrupole moment tensor are
\[
D_{xx} = \frac{m}{2} r^2 (3 \cos^2 \theta - 1), \quad D_{xy} = \frac{m}{2} r^2 (3 \sin^2 \theta - 1),
\]
\[
D_{yy} = D_{yx} = \frac{3m}{4} r^2 \sin(2\theta), \quad D_{zz} = -\frac{m}{2} r^2.
\]
Keeping the leading order term we obtain
\[
\frac{m}{2} \frac{d}{dt}(rV)^2 = -\frac{256Gm^2V^2}{5c^3 r}.
\]
For Newtonian gravity \( V \propto 1/\sqrt{r} \) and the left-hand side of equation (159) can be written as \( d(rV)/dt = V\dot{r}/2 \) which yields the following equation of the orbit decay
\[
\dot{r} = -\frac{512GmV^4}{5c^3 r}.
\]
In the wave zone perturbation of the metric due to gravitational wave propagating along the \( x \)-axis is given by [27]
\[
h_{00} \propto \dot{D}_{xx} \propto V^2 \sin(2\theta), \quad h_{00} = 0.
\]
The signal of the LIGO-like interferometer with perpendicular arms laying in the \( xy \) plane is proportional to
\[
h = h_{00} \propto V^2 \sin(2\theta),
\]
where for the orbital motion
\[
\dot{\theta} = \frac{2V}{r}
\]
Equation of motion of a particle in metric yields the following relation

\[ \frac{V^2}{c^2} = \frac{\partial g_{00}}{\partial r} + \frac{1}{r} + \frac{\partial^2 F}{\partial r^2} \]

Introducing dimensionless velocity, distance and time

\[ V \rightarrow V_c, \quad r \rightarrow R_{SS} r, \quad t \rightarrow \frac{R_{SS} t}{c} \]

we find for the case of vector gravity

\[ V^2 = \frac{e^{-8/r}}{r - 1} \]

and

\[ V^2 = \frac{r^4 (r - 1)}{(r + 1)^6} \]

for general relativity. Substituting this into equations (160)–(162) we obtain equations for the orbit decay \( r(t) \) and generated radiation waveform \( h(t) \). In the dimensionless coordinates the equations read

\[
\dot{r} = \frac{-1024}{5} \frac{e^{-16/r}}{r(r - 1)^2}, \\
\dot{\theta} = \frac{2e^{-4/r}}{r^4 - 1}, \\
h = \frac{A e^{-8/r}}{r - 1} \sin(2\theta + \varphi_0)
\]

for vector gravity, and

\[
\dot{r} = -\frac{1024}{5} \frac{r^4 (r - 1)^2}{(r + 1)^{12}}, \\
\dot{\theta} = \frac{2r^2}{r^4 - 1}, \\
h = \frac{A r^4 (r - 1)}{(r + 1)^6} \sin(2\theta + \varphi_0)
\]

for general relativity. In these equations \( A \) and \( \varphi_0 \) are free (fitting) parameters that depend, in particular, on the unknown distance to the binary system and its initial phase of motion. Mass \( m \) is another free parameter that determines the scale of dimensional coordinates.

It turns out that equations (166)–(168) and (169)–(171) are sufficiently accurate to describe the observed LIGO signal. In figure 7 we plot radiation wave strain \( h(t) \) (in arbitrary units) as a function of time obtained by numerical solution of equations (166)–(168) in vector gravity (solid line) and equations (169)–(171) for the case of general relativity (dashed line). The free parameters are chosen to get the best fit of the GW150914 event signal. Figure shows that radiation waveforms obtained in both theories are practically indistinguishable. One should note that the decaying part of the waveform corresponds to the orbital inspiral rather than damped oscillations of the merged system. Radiation waveform decay occurs because, according to equations (164) and (165), deviation from the Newtonian gravity results in slowing down the orbital motion which, according to equation (161), reduces the wave amplitude.
To show that our result is compatible with the LIGO data we process the vector gravity radiation waveform of figure 7 using bandpass and spectral whitening algorithm described in the LIGO tutorial on signal processing [54] and compare the obtained waveform with the respectively filtered LIGO signal reported in [5]. The results are summarized in figure 8. Top row shows strain $h(t)$ as a function of time for the gravitational-wave event GW150914 observed by the LIGO Hanford detector (dotted line) [5] and the best fit waveform obtained in vector gravity (red solid line). All time series are filtered with a bandpass and band-reject filters in the same way as in [5]. Second row compares the LIGO Hanford signal with the best fit waveform of numerical relativity taken from [5]. Bottom row shows residuals after subtracting the filtered vector gravity and numerical relativity waveforms from the filtered detector time series. One can see that within the limits of detector noise both theories yield radiation waveforms which are compatible with the LIGO signal.

In section 13 we show that in vector gravity the masses of compact objects in the merging binary systems obtained by the LIGO team based on the gravitational wave detection reported so far $(29.4^{+4}_{-6} M_{\odot} \text{ and } 36.4^{+5}_{-9} M_{\odot})$ [5], $14.2^{+3.5}_{-1.9} M_{\odot}$ and $7.5^{+2.3}_{-1.3} M_{\odot}$ [6], $31.2^{+8.6}_{-6.0} M_{\odot}$ and $19.4^{+5.3}_{-4.0} M_{\odot}$ [7]) fit in the mass intervals for which neutron stars are stable. Thus, interpretation of the LIGO signals as orbital inspiral and merger of two massive neutron stars, rather than black holes, is plausible in vector gravity.

Polarization of gravitational waves in vector gravity differs from those in general relativity (see section 8.3). Since only two interferometers were involved in the recent detection of gravitational waves by LIGO, the wave polarization was not measured. Simultaneous detection of gravitational waves by multiple instruments is able to distinguish between vector gravity and general relativity. We discuss details of such experiment in section 16.

15. Galactic centers and dark matter problem

In the present theory, static gravitational field is described by the equivalent exponential metric (24). Metric (24) was also obtained in [24–26, 28–33]. Exponential metric (24) predicts no black holes, but rather compact objects with no event horizon and finite gravitational redshift.

In recent years, the evidence for the existence of an ultracompact concentration of dark mass at centers of galaxies has become very strong. However, a proof that such objects are black holes rather than compact objects without event horizon is lacking. If the present theory of gravity is correct then the compact supermassive objects at galactic centers are unlikely composed of baryonic matter. Indeed, as we show in section 13, mass of a compact (neutron star like) baryonic object with ‘causal’ equation of state in vector gravity does not exceed a few dozen solar masses, but the objects at galactic centers possess masses up to a few $10^9 M_{\odot}$. Even though there is no general proof that state of matter must obey the ‘causality’ constraint $dP/d\varepsilon \leq 1$ it is unlikely that equation of state can be so stiff to make neutron star-like objects of billion solar masses stable. Hence, likely those objects are made of dark matter of non baryonic origin. This fact gives us an opportunity to determine composition of dark matter based on observations of supermassive objects at galactic centers.

In the previous paper [13] we found that properties of compact objects at galactic centers can be explained quantitatively assuming they are made of dark matter axions and the axion mass is about 0.6 meV. Analysis of [13] is based on the assumption that static gravitational field is described by the exponential metric (24) rather then by general relativity. A full time-dependent theory of gravity was unnecessary for calculations made in [13]. The present paper provides such a theory and justifies our previous choice of the exponential metric.

Axions are one of the leading particle candidates for the cold dark matter in the Universe [55]. Interaction of axions with QCD instantons generates the axion mass $m$ and periodic interaction potential [56]

$$V(\varphi) = m^2 F^2 [1 - \cos(\varphi/F)],$$

where $\varphi$ is a real scalar axion field and $F$ is the Peccei–Quinn symmetry breaking scale. The interaction potential (172) has degenerate minima $V = 0$ at $\varphi = 2\pi n F$, where $n$ is an integer number. As a consequence, axions can form bubbles. Bubble mass is concentrated in a thin surface (interface between two degenerate vacuum states). In the exponential metric the potential energy of a spherical bubble with radius $R$ is given by [13]

$$U(R) = 4\pi \sigma R^2 \exp \left(\frac{GM}{c^2 R}\right),$$

where $\sigma$ is the surface energy density and $M$ is the fixed bubble mass. $U(R)$ has a shape of a well. At $R \gg GM/c^2$ one can omit gravity and $U(R) \approx 4\pi \sigma R^2$ is just a surface energy (tension) which tends to contract the bubble. At $R \ll GM/c^2$ gravity effectively produces large repulsive potential which forces the bubble to expand. As a result, the bubble radius $R(t)$ oscillates between two turning points.

In [13], based on quantitative analysis of available data, we argued that such oscillating axion bubbles, rather than supermassive black holes, could be present at galactic centers. Recent observations of near-infrared and x-ray flares from Sagittarius A*, which is believed to be a $4 \times 10^6 M_{\odot}$ black hole at the Galactic center, show that the source exhibits about 20 min periodic variability [57–59]. An oscillating axion bubble can explain such variability. Known value of the bubble mass at the center of our Galaxy and its oscillation period yields the axion mass of about 0.6 meV. Size of the axion bubble at the center of the Milky Way oscillates between $R_{\text{min}} \approx 1 R_\odot$ and $R_{\text{max}} \approx 1 \text{AU} \approx 210 R_\odot$.

Further, as shown in [13], the axion bubbles with no free parameters (if we fix $m = 0.6$ meV based on Sagittarius A* flare variability) quantitatively explain the upper limit (a few $10^9 M_{\odot}$) on the supermassive ‘black hole’ mass found in analysis of the measured mass distribution [60]. Also, with no free parameters the bubble scenario explains observed lack of
supermassive ‘black holes’ with mass \( M \lesssim 10^6 M_\odot \). For such low-mass bubbles the decay time \( t \propto M^{9/2} \) becomes much shorter then the age of the Universe and, as a result, such objects are very rare.

One should note that results of [13] describe bubbles which are already formed and relatively isolated. Thus, Active Galactic Nuclei whose bubbles are currently under formation or strongly interact with the galactic environment should be excluded. A sample of predominantly inactive galaxies for which direct supermassive ‘black hole’ mass measurements have been catalogued shows lack of such objects with \( M \lesssim 10^6 M_\odot \) (see figure 1 in [61]). On the other hand, both limits on the bubble mass in Active Galactic Nuclei can be somewhat wider. For instance, in such galaxies, fast vaporization of a low-mass bubble could be reduced by the back flow of galactic axions into the bubble which extends its lifetime. Such a scenario leads to observable consequences. Namely, it predicts that at the low-mass end of the ‘black hole’ versus host galaxy bulge mass diagram the ‘black hole’ masses must be lower than predicted by the relation established using galaxies having predominantly higher-mass ‘black holes’. In addition, at the low-mass end it should not be correlation between the ‘black hole’ mass and bulge luminosity because bubbles loose mass fast on a time scale of the bulge evolution. Thus, a wide range of bubble masses can exist at almost the same bulge luminosity at the low-mass end. Observations support both these predictions [62]. ‘Black holes’ with estimated mass in the range \( 10^2\sim 10^6 M_\odot \) have been found in Active Galactic Nuclei and their masses lie substantially below the scaling relation defined by the massive systems [62–64]. In addition, at the lower end the measured ‘black hole’ masses span a much wider range at fixed bulge luminosity [62].

Observation of the Galactic center with a very long baseline interferometry (VLBI) within the next few years will be capable to test theories of gravitation in the strong field limit. Such an observation will allow us to distinguish between the black hole (predicted by general relativity) and the oscillating axion bubble scenario. A defining characteristic of a black hole is the event horizon. To a distant observer, the event horizon casts a relatively large shadow. For a black hole of mass \( M \) the intrinsic diameter of Sagittarius A* at a wavelength of 1.3 mm was determined using VLBA [71]. The intrinsic diameter of Sagittarius A* was found to be \( < 0.3 \) AU \( \approx 65 R_\odot \) which is less than the expected apparent size of the event horizon of the presumed black hole. Such observation might indicate lack of black holes, in agreement with the present theory.

Existence of dark matter axions with the predicted mass of about 0.6 meV can be experimentally tested in future ARIADNE [72] and Orpheus [73] experiments.

16. Testing vector gravity with gravitational wave interferometers

In spite of fundamental differences, vector gravity and general relativity yield for the experimentally tested regimes quantitatively very close predictions which allowed both theories to pass available tests. However, prediction of the matter behavior at strong gravity and interpretation of the Universe evolution on large scales depends on the theory of gravity we are using. Thus, there is a need for a feasible test which can distinguish between vector gravity and general relativity and rule out one of the two theories. Here we propose such an experiment that can be done in the nearest years using gravitational wave interferometers.

Both in general relativity and vector gravity the polarization of gravitational waves emitted by orbiting binary objects is transverse, that is wave yields motion of test particles in the plane perpendicular to the direction of wave propagation. However, as we show here, dependence of the laser interferometer signal on the orientation of the interferometer arms relative to the propagation direction of the gravitational wave is different in the two theories.

Gravitational wave produces motion of the interferometer mirrors and changes phase velocity of light. Both of these effects contribute to the relative phase shift of light traveling in the perpendicular arms of the Michelson interferometer. For certain propagation directions of the gravitational wave relative to the arms the two contributions cancel each other yielding zero net phase shift. Those are the directions of zero response of the interferometer for which gravitational wave can not be detected for any transverse polarization. As we show, directions of the zero response are different for gravitational waves in vector gravity and general relativity. Detection of a wave in the direction of the zero response predicted by a theory of gravity will rule out such theory.

In vector gravity for a weak transverse plane gravitational wave propagating along the \( x \)-axis the equivalent metric is given by equation (45)

\[
g_{ik} = \eta_{ik} + \begin{pmatrix} 0 & 0 & h_{0x}(t, x) & h_{0c}(t, x) \\ 0 & 0 & 0 & 0 \\ h_{0x}(t, x) & 0 & 0 & 0 \\ h_{0c}(t, x) & 0 & 0 & 0 \end{pmatrix}, \quad (173)
\]

32
where \( g_{\mu \nu} \) is Minkowski metric and \( h_{00}, h_{0i} \) are small perturbations obeying the wave equation. A rest particle (or mirrors of an interferometer) will move under the influence of the gravitational wave (173) with a time-dependent velocity \( V^\alpha = h_{0\alpha}c \) \((\alpha = x, y, z)\) perpendicular to the direction of the wave propagation. By making a coordinate transformation into the co-moving frame of the test particle \( x'^\alpha = x^\alpha - \int V^\alpha dt \), the metric (173) reduces to

\[
g_{ab} = \eta_{ab} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_{xy}(t, x) & h_{xz}(t, x) & 0 \\ 0 & h_{yx}(t, x) & 0 & 0 \\ 0 & h_{zx}(t, x) & 0 & 0 \end{pmatrix},
\]

(174)

where \( h_{xy} = h_{0y}, h_{xz} = h_{0z} \). Metric (174) is written in the coordinate system in which test particles do not move under the influence of the gravitational wave.

On the other hand, in general relativity for a weak gravitational wave propagating along the \( x \)-axis the metric in the co-moving frame evolves as [27]

\[
g_{ab} = \eta_{ab} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_{xy}(t, x) & h_{xz}(t, x) & 0 \\ 0 & h_{yx}(t, x) & 0 & 0 \\ 0 & h_{zx}(t, x) & 0 & 0 \end{pmatrix} - \epsilon \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_{xy}(t, x) & h_{xz}(t, x) & 0 \\ 0 & h_{yx}(t, x) & 0 & 0 \\ 0 & h_{zx}(t, x) & 0 & 0 \end{pmatrix},
\]

(175)

Here we investigate a response of a laser interferometer with perpendicular arms on a gravitational wave in the two theories of gravity. In gravitational field with a metric \( g_{ab} \) the Maxwell’s equations for the electromagnetic field vector \( A^k \) in the absence of charges read

\[
\frac{\partial}{\partial x^k} \left[ \sqrt{-g} g^{kl} \frac{\partial A^l}{\partial x^j} - \frac{\partial A^j}{\partial x^l} \right] = 0,
\]

(166)

while the Lorenz gauge equation is

\[
\frac{\partial}{\partial x^k} (\sqrt{-g} A^k) = 0,
\]

(177)

where \( A^k = g^{kl} A_l \) and \( g = \det (g_{\mu \nu}) \). Gravitational wave causes oscillation of \( g_{ab} \) in space and time. However, since frequency of the gravitational waves is much smaller than frequency of the electromagnetic waves traveling in the interferometer one can disregard derivatives of the metric in equations (166) and (177). Then equations (166) and (177) reduce to

\[
g^{kl} \frac{\partial^2 A^l}{\partial x^k \partial t^j} - g^{lm} \frac{\partial A^j}{\partial x^m} = 0,
\]

(178)

\[
\frac{\partial A^k}{\partial x^k} = 0.
\]

(179)

Combining them together we obtain

\[
\frac{\partial^2 A^k}{\partial x^k \partial t^j} = 0.
\]

(180)

In the co-moving frame there is no motion of the interferometer mirrors and the phase shift of light traveling along the two arms appears due to difference in the light phase velocity. Calculation of the gravitational wave signal can be done in any reference frame because the phase shift

\[
\Delta \varphi = \int k_i dx^i
\]

(181)

is invariant under general coordinate transformations and, thus, the interferometer signal is independent of the frame. In equation (181) \( k_i = (\omega/c, \mathbf{k}) \) is the photon four dimensional wave vector.

Substitute \( g^{kl} = \eta^{kl} + h^{kl} \) in equation (180), where \( h^{kl} \) is a small perturbation that has only spatial components \( h^{\alpha \beta} \) \((\alpha, \beta = x, y, z)\), yields the following propagation equation for the electromagnetic wave

\[
\left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 + h^{\alpha \beta} \frac{\partial^2}{\partial x^\alpha \partial x^\beta} \right) A^i = 0.
\]

(182)

In this equation, \( h^{\alpha \beta} \) can be approximately treated as constants since they vary slowly as compared to the fast variation of \( A^i \). Looking for solution of equation (182) in the form \( A^i \propto e^{-i \omega t + i k \mathbf{r}} \), where \( k \) is the wave vector of the electromagnetic wave, we obtain the following dispersion relation for light

\[
\omega^2 = c^2 k^2 - h^{\alpha \beta} k_\alpha k_\beta,
\]

and, hence, the phase velocity of the electromagnetic wave is (see also [53])

\[
V_{ph} = \frac{\omega}{k} \approx c \left( 1 - \frac{1}{2} h^{\alpha \beta} \hat{k}_\alpha \hat{k}_\beta \right),
\]

(183)

where \( \hat{k} = k/k \).

Equation (183) shows that presence of the gravitational wave leads to the change of the phase velocity of light which depends on the direction of the light propagation \( \hat{k} \). If arms of the interferometer are oriented along unit vectors \( \hat{a} \) and \( \hat{b} \) then difference in the phase velocities of the laser light propagating along the two arms is

\[
\Delta V_{ph} = \frac{c}{2} h^{\alpha \beta} (\hat{a}_\alpha \hat{a}_\beta - \hat{b}_\alpha \hat{b}_\beta).
\]

(184)

Signal of the LIGO-like (Michelson) interferometer with arms of length \( L \) oriented along the directions \( \hat{a} \) and \( \hat{b} \) is proportional to the relative phase shift \( \Delta \varphi \) of electromagnetic waves traveling a roundtrip distance \( 2L \) along the two arms

\[
\Delta \varphi = k \frac{2L}{c} \Delta V_{ph} = \frac{\omega L}{c} h^{\alpha \beta} (\hat{a}_\alpha \hat{a}_\beta - \hat{b}_\alpha \hat{b}_\beta),
\]

(184)

where \( \omega \) is the frequency of electromagnetic wave and \( h^{\alpha \beta} \) is the spatial perturbation of the metric in the reference frame in which interferometer mirrors do not move (frame of equations (174) and (175)).

For the gravitational wave propagating along the \( x \)-axis, equation (184) yields for the gravitational wave (175) in
general relativity

$$\Delta \varphi = \frac{\omega L}{c} \left[ h^{yy}(\hat{a}_y^2 - \hat{b}_y^2 + \hat{b}_z^2 - \hat{a}_z^2) + 2h^{yz}(\hat{a}_y\hat{a}_z - \hat{b}_y\hat{b}_z) \right],$$

(185)

while for the transverse wave (174) in vector gravity we obtain

$$\Delta \varphi = \frac{2\omega L}{c} \left[ h^{yy}(\hat{a}_y\hat{a}_y - \hat{b}_y\hat{b}_y) + h^{yz}(\hat{a}_y\hat{a}_z - \hat{b}_y\hat{b}_z) \right].$$

(186)

Equations (185) and (186) show that vector gravity and general relativity predict qualitatively different effects of the gravitational wave on the interferometer signal. Namely, general relativistic gravitational wave of any polarization (arbitrary $h^{yy}$ and $h^{yz}$) produces no signal when gravitational wave propagates parallel to the interferometer plane at $45^\circ$ angle relative to one of its perpendicular arms (see figure 1(a)). E.g., this is the case for $\hat{a} = (1, 1, 0)/\sqrt{2}$ and $\hat{b} = \pm (1, -1, 0)/\sqrt{2}$. For these orientations the gravitational wave in vector gravity can produce signal, namely,

$$\Delta \varphi = \frac{2\omega L}{c} h^{yy}. \quad (187)$$

On the other hand, gravitational wave in vector gravity (for arbitrary $h^{yy}$ and $h^{yz}$) yields no signal if gravitational wave propagates in the direction perpendicular to the interferometer plane, e.g. $\hat{a} = (0, 1, 0)$ and $\hat{b} = (0, 0, 1)$ (see figure 1(b)), or along one of the interferometer arms, e.g. $\hat{a} = (1, 0, 0)$, $\hat{b} = (0, 0, 1)$ or $\hat{a} = (0, 1, 0)$, $\hat{b} = (1, 0, 0)$. For these orientations the gravitational wave in general relativity can produce signal.

This difference can be used to test theories of gravity in polarization experiments with several LIGO-like interferometers. The experiment can be conducted with three interferometers. Simultaneous detection of the gravitational wave by all three instruments allows us to determine the direction of the wave propagation by measuring the wave arrival times at the interferometer locations and using information about the waveform [74–76]. In vector gravity the gravitational waveform produced by inspiral of two objects can be specified to the same extent as in general relativity and, thus, the wave source can be localized on the sky with a similar accuracy in both theories.

The issue of localization of gravitational wave signals with a detector network has been discussed previously in many publications (see, e.g., [74–88]). The accuracy with which the source can be localized on the sky depends upon the timing accuracy in each of the detectors, the network geometry and the angle between the plane of the detectors and the signal location. A detector network with widely separated detectors yields the best localization ability. Depending on the source orientation the LIGO–Virgo network can determine source location with accuracy $20 \div 100$ deg$^2$ [79]. Prospects for observing and localizing gravitational-wave transients to areas of 5 deg$^2$–20 deg$^2$ with Advanced LIGO and Advanced Virgo gravitational-wave detectors over the next decade are discussed in [89].

By detecting many events one can accumulate statistics and find a distribution of the direction of the detected gravitational waves relative to the interferometer arms. Namely, one can measure the distribution function $N(\theta, \phi)$ defined as $dN = N(\theta, \phi) d\Omega$, where $dN$ is the number of events for which detected gravitational waves propagate inside the solid
angle $d\Omega = \sin(\theta)d\theta d\phi$. Here $\theta$ and $\phi$ are the polar and azimuth angles in the spherical coordinate system in the frame of the interferometer arms (see figure 9).

Taking square of equations (185), (186) and averaging over polarization of the gravitational waves we obtain that distribution function in vector gravity is

$$N(\theta, \phi) = \sin^2(\theta)[1 - \sin^2(\theta)\cos^2(2\phi)],$$

while for general relativity we find

$$N(\theta, \phi) = \cos^2(\theta) + \frac{1}{4} \sin^4(\theta)\cos^2(2\phi).$$

Functions (188) and (189) are normalized such that $N_{\text{max}} = 1$. We plot $N(\theta, \phi)$ given by equations (188) and (189) in figure 10. The two distributions look very different and, hence, the experiment we are proposing should be able to distinguish between them with available localization accuracy.

Vector gravity predicts that distribution $N(\theta, \phi)$ will have dips in the directions perpendicular to the interferometer plane $- \theta = 0, \pi$ and along the interferometer arms $- \theta = \pi/2, 0, \pi/2, 3\pi/2$ (see figure 10(a)). The dips appear because for these propagation directions the interferometer cannot detect the transverse gravitational wave. In the case of general relativity the dips will appear in the directions for which wave propagates in the interferometer plane at $45^\circ$ angle relative to one of the interferometer arms $- \theta = \pi/2$ and $\phi = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$ (figure 10(b)).

One should mention that vector gravity also predicts existence of longitudinal gravitational waves which are not emitted by orbiting binary stars. However, they might be generated during the star mergers or in early universe. For such a wave propagating along the $x$-axis the equivalent metric in the co-moving frame reads

$$g_{ab} = \eta_{ab} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -2h(t, x) & 0 & 0 \\ 0 & 0 & h(t, x) & 0 \\ 0 & 0 & 0 & h(t, x) \end{pmatrix}.$$  (190)

For the longitudinal wave (190) the interferometer signal is

$$\Delta \varphi = \frac{3\omega L h}{c}(\hat{a}_t^2 - \hat{a}_c^2),$$

which yields the following distribution function $N$

$$N(\theta, \phi) = \sin^4(\theta)\cos^2(2\phi).$$

We plot this function in figure 11.

Longitudinal gravitational waves produce no interferometer signal if they propagate at equal angles relative to the interferometer arms (more exactly when $\hat{k} \cdot \hat{a} = \pm \hat{k} \cdot \hat{b}$). Thus, propagation direction perpendicular to the interferometer plane shown in figure 1(b) is the direction of zero response for all kind of gravitational waves in vector gravity. However, waves propagating along the arms can produce signal if they are longitudinal.

The experiment we are proposing is crucial for our understanding of the nature of gravity and can test whether gravity has a tensor or a vector origin. Simultaneous detection of gravitational waves in at least three instruments is necessary for the experiment. A joint scientific run of the two LIGO interferometers in the US and the Virgo interferometer in Italy is capable of distinguishing between tensor and vector origin of gravity. Such joint runs can begin in 2017 when the Virgo instrument will reach the required sensitivity.

### 17. Summary

Einstein’s general relativity is an elegant theory of gravity which is based on the assumption that space–time geometry is a tensor gravitational field. However, beauty of the theory does not guarantee that theory describes the nature. So far general relativity has passed all available tests of gravity. To the best of our knowledge, the vector theory of gravity we are proposing in this paper also passes all available tests as we discuss in section 14. General relativity, however, cannot explain the nature of dark energy. In contrast, vector gravity is free of such drawback.

Our alternative theory of gravity is based on the assumption that gravity is a vector field in a fixed background four dimensional Euclidean space which is coupled to matter universally and minimally through the equivalent metric $f_a$ which is a functional of the vector field. We show that present theory is the only possibility that can be obtained from this assumption.

There are several motivations for the vector theory of gravity. It provides an appealing explanation of how the difference between space and time appeared in the originally totally symmetric Euclidean Universe. Namely, the vector gravitational field breaks the symmetry of the four dimensional Euclidean space. Direction of the vector field gives the time coordinate, while perpendicular directions are spatial coordinates.

Vector gravity also suggests a natural mechanism of matter generation at the Big Bang. Namely, vector theory of gravity yields that at the moment of Big Bang the energy of gravitational waves is negative and, thus, matter can be created at the expense of generation of the negative energy gravitons. This mechanism has an analogy with emission of electromagnetic waves by an electric dipole (or a quadrupole) placed in a dispersionless medium with negative refractive index. In a dispersionless medium with negative dielectric constant $\varepsilon$ and negative magnetic permeability $\mu$ the energy density of the electromagnetic field

$$w = \frac{1}{8\pi}(\varepsilon E^2 + \mu H^2)$$

is negative. As a result, photons emitted by an oscillating dipole placed in such medium carry away negative energy yielding exponential growth of the dipole oscillations [90]. Thus, system is unstable with respect to generation of electromagnetic waves and acceleration of electric charges placed in such medium.

For vector gravity the vacuum of empty fermion states acts as a dispersionless medium with negative refraction (see appendix G). Such vacuum is unstable with respect to
generation of gravitational waves and heating up the Universe. Vector gravity suggests that at the Big Bang the Universe was heated up by this mechanism. The vacuum instability leads to exponential growth of the gravitational field and matter generation. This is the era of cosmological inflation. Thus, vector gravity predicts the inflation stage. In vector gravity there is no need for an additional cosmological field that would supply energy for matter generation. However, such additional field, the inflaton, is a necessary ingredient of cosmological models based on general relativity for which graviton energy is always positive [91].

At some point the heating of the Universe came to an end. Thus, it must be a mechanism which stopped the heating process. A need for it motivated us to postulate, by the analogy with the composite theory of photon, that in vector gravity the graviton is a composite particle formed of fermion–antifermion pairs and graviton emission corresponds to creation of such pairs. The constituent fermion is an elementary spin 1/2 massless particle which has positive and negative energy states. This assumption explains why heating of the Universe stopped. Since no more than one fermion can occupy the same quantum state the matter generation at the Big Bang has continued until fermion states were filled and the Universe became extremely hot. Subsequent evolution of the Universe is described by the usual hot Universe theory. The following expansion of the Universe practically did not change the fermion occupation number and fermion states remain filled in the present epoch.

Transition from the originally four dimensional Euclidean geometry to the equivalent metric of Minkowski character occurred at the point of Big Bang. Recall that in terms of the unit vector \( u_k \) and the scalar \( \phi \) the equivalent metric in vector gravity reads

\[
\hat{f}_{ik} = -e^{-2\phi}\delta_{ik} + 2\cosh(2\phi)u_iu_k.
\]

Before the Big Bang the Universe should be described quantum mechanically so that \( f_{ik} \) and \( u_k \) are replaced with operators

\[
\hat{f}_{ik} = -e^{-2\phi}\delta_{ik} + 2\cosh(2\phi)\hat{u}_i\hat{u}_k.
\]

Before the Big Bang the vector gravitational field had no preferred direction and was undergoing quantum fluctuations. For this state of the Universe the quantum mechanical average of the field operator is equal to zero

\[
\langle \hat{u}_k \rangle = 0,
\]

while

\[
\langle \hat{u}_i\hat{u}_k \rangle = \frac{1}{4}\delta_{ik}.
\]

For such state the expectation value of the metric operator (194) is

\[
\langle \hat{f}_{ik} \rangle = \frac{1}{4}(e^{2\phi} - 3e^{-2\phi})\delta_{ik}.
\]

That is before the Big Bang the equivalent metric has Euclidean character.

![Figure 12](https://example.com/image.png)

**Figure 12.** Explanation of dark energy in vector gravity. Expansion of the Universe generates matter current \( \mathbf{j}_m \) directed away from an observer \( O \). Such current induces longitudinal gravitational field \( \mathbf{h} = k\mathbf{v} \) in a similar way as electric current creates vector potential in classical electrodynamics. Spatial averaging of \( \mathbf{h} \) over the local (shaded) region yields zero and, therefore, averaged metric is spatially isotropic. However, average energy density associated with \( \mathbf{h} \), \( w_h = -e^2k^2/32\pi G \), does not vanish after spatial averaging. This energy is the mysterious dark energy. Contrary to matter, it has negative energy density and accelerates expansion of the Universe.

Big Bang is the point of phase transition at which the gravitational field vector acquires nonzero expectation value \( \langle \hat{u}_k \rangle = u_k = 0 \). This expectation value can serve as a transition order parameter. Now the four dimensional space has a preferred direction \( u_k \). If fluctuations of the field around \( u_k \) are small one can use a mean-field description and replace operators with their mean values: \( \hat{u}_k \to u_k \), etc. Choosing direction of the vector gravitational field as a time coordinate we have \( u_k = (1, 0, 0, 0) \) and the equivalent metric (193) now reads

\[
\hat{f}_{ik} = \begin{pmatrix}
e^{2\phi} & 0 & 0 & 0 \\
0 & -e^{-2\phi} & 0 & 0 \\
0 & 0 & -e^{-2\phi} & 0 \\
0 & 0 & 0 & -e^{-2\phi}
\end{pmatrix}.
\]

That is geometry has the character of the Minkowski space–time in the ordered phase of the Universe. In Minkowski geometry the initial vacuum of empty fermion states is unstable towards generation of matter in the expense of production of the negative energy gravitons. Universe enters the stage of inflation which ends when fermion states become filled.

These filled states act as a new vacuum for the evolution of the Universe after the heating stage. As we show in section 11, for the filled vacuum the graviton energy is positive and, thus, vacuum is stable. For such vacuum, emission of a graviton corresponds to creation of fermion–antifermion hole pairs out of the filled fermion states. Binary stars orbiting each other are loosing their energy by emitting positive energy gravitons. As we show in section 12, the rate of the energy loss by the binary system is given by the same quadrupole formula as obtained in general relativity. Thus, vector gravity also passes the binary pulsars tests.
For the Universe expansion the present theory gives the same answer as general relativity with cosmological constant and zero spatial curvature. However, zero spatial curvature of the Universe is a solution of our equations, while in general relativity the spatial curvature is a free parameter. Thus, vector theory of gravity does not have the Euclidicity problem, that is why space is almost perfectly Euclidean on large scales.

Moreover, the vector theory of gravity solves the dark energy problem. Namely, the theory yields, with no free parameters, the value of the cosmological constant \( \Omega_0 = 2/3 \approx 0.67 \) which agrees with the recent Planck result \( \Omega_0 = 0.686 \pm 0.02 \) [12]. General relativity failed to predict the value of \( \Omega_0 \), but the present vector theory of gravity passes this cosmological test. This result is crucial since it points to the vector nature of gravity rather than a tensor field.

The present theory provides an explanation of the dark energy as the energy of longitudinal (div \( \mathbf{h} = 0 \)) gravitational field induced by the Universe expansion (see figure 12). Namely, time variation of the spatial scale caused by the Universe expansion produces matter current directed away from an observer. Such current generates longitudinal part of the vector gravitational field (similarly to generation of the vector potential by the electric current in classical electrodynamics) which possesses negative energy and accelerates expansion of the Universe.\(^2\)

Since value of the current depends on a reference frame the value of the cosmological constant \( \Lambda \) depends on the time \( t \) at which the observer measures \( \Lambda \). As we show in section 9, the value of \( \Lambda \) is given by \( \Lambda = 2\rho/a^3(t_0) \) and the contribution from the cosmological term to the expansion rate of the Universe at time \( t_0 \) is twice larger than those of matter in any reference frame. Therefore, according to our theory, Universe will expand forever at a continually decelerating rate, with expansion asymptotically approaching zero. This is what is expected for the flat Universe in absence of exotic forms of energy.

Mathematically, the cosmological term appears in vector gravity as a result of spatial averaging of the gravitational field equations. Namely, exactly spatially inhomogeneous equations yield that in the vicinity of time \( t_0 \) and \( \mathbf{r} = 0 \) the solution for the equivalent metric is

\[
f_{ik} = \begin{pmatrix}
1 + h_{00} & h_{0x} & h_{0y} & h_{0z} \\
h_{0x} & -1 + h_{00} & 0 & 0 \\
h_{0y} & 0 & -1 + h_{00} & 0 \\
h_{0z} & 0 & 0 & -1 + h_{00}
\end{pmatrix},
\]

where, according to equation (72),

\[
h_{0i}(t, \mathbf{r}) = \frac{2}{c} h_{00}(t_0)(t - t_0) x^i.
\]

That is \( h_{0i} \) is induced by the Universe expansion (more exactly by the acceleration of expansion \( h_{00} \)). Spatial averaging of \( h_{0i} \) in the local region yields zero because \( h_{00} \) is an odd function of spatial coordinates \( x^i \). Therefore, the averaged metric \( f_{ik} \) is spatially isotropic. However, since \( h_{00} \) enters the evolution equation (68) as divergence its contribution does not vanish in the equation after spatial averaging and equation (68) yields

\[
3 \frac{\partial^2}{\partial x^i \partial x^j} (h_{00}) - 2 \frac{\partial}{\partial x^0} \left( \frac{\partial h_{00}}{\partial x^0} \right) = \frac{8\pi G}{c^4} (T_{00}^{\text{now}}).
\]

The second term in equation (196) is the cosmological (dark energy) term which, according to equation (195), is equal to \(-4h_{00}(t_0)/c^2\). The dark energy term appears because Universe expansion induces \( h_{00} \), which itself affects Universe evolution. The value of the cosmological constant \( \Lambda \) in the effective nonlinear evolution equation (59) is determined by matching this equation with the local evolution of the Universe in the vicinity of the observer’s time \( t_0 \). As a consequence, the value of \( \Lambda \) depends on the average matter density at time \( t_0 \), that is it depends on the observer’s reference frame.

According to the vector gravity, the contents of the Universe are somewhat different from those predicted by general relativity. The total energy density \( w \) of the Universe in the effective cosmological model is given by equation (62)

\[
w = -\frac{3c^2}{8\pi G} a^2 \dot{a}^2 + c^2 \dot{a}^2 + \frac{\kappa c^2}{a},
\]

which is the energy density in the fixed background Euclidean space. The net energy of the Universe is equal to zero (\( w = 0 \)), and positive energy of matter is compensated by the negative energy of the gravitational field (see figure 2).

Introducing \( X = a^2 \) one can rewrite equation (197) as an equation of energy conservation for a particle in an external potential \( U(X) \)

\[
\frac{3}{32\pi G} X^2 + U(X) = \text{const},
\]

where

\[
U(X) = -\Delta X = \frac{\rho}{\sqrt{X}}.
\]

The term with \( \Delta \) in \( U(X) \) decreases with increasing \( X \), while the matter term increases. Thus, the cosmological \( \Lambda \)-term accelerates expansion of the Universe while matter causes deceleration.

In the post-Newtonian limit the vector gravity gives the same answer as general relativity. As we explain in section 7, the reason for such a coincidence is symmetry of the action \( S_{\text{matter}} \). Namely, in the post-Newtonian limit the symmetries of \( S_{\text{matter}} \) coincide in both theories. By construction of both theories, the symmetries of \( S_{\text{matter}} \) uniquely determine the whole classical theory of gravity. Thus in the post-Newtonian

\[^2\] This value of \( \Omega_0 \) was obtained assuming that matter in the Universe is nonrelativistic. However, small fraction of matter (e.g., neutrinos) is relativistic which can slightly modify \( \Omega_0 \). Measuring deviation of \( \Omega_0 \) from 2/3 in future more accurate observations could give us information about the amount of relativistic matter.

\[^3\] Universe expansion induces non radiative longitudinal gravitational field which is not quantized. This is different from graviton which is a quantized transverse field. Since graviton is a composite particle there are constraints imposed by the Pauli exclusion principle on its generation. However, for the classical longitudinal field there are no such constraints.
limit when symmetries of $S_{\text{matter}}$ coincide the two theories become equivalent.

For strong field, vector gravity gives a very different result and yields no singularities such as black holes. A defining characteristic of a black hole is the event horizon. So far there were no observations of the event horizon and, thus, a proof of black holes existence is lacking.

The current vector theory is not equivalent to general relativity even in the weak field limit. For example, it predicts different polarization of weak gravitational waves. As we show in section 11, quantization of the vector gravitational field can be performed in a way similar to the quantization of electromagnetic field in the composite photon theory. Namely, gravitational field is decomposed into free field (corresponding to radiation) which is quantized assuming that graviton is composed of fermion–antifermion pairs and the residual non radiative part of the field which remains classical. In particular, the post-Newtonian limit as well as cosmological evolution of the Universe are described by the part of the gravitational field which is not quantized. As a result, classical field equations (22) are applicable for these problems.

We show in section 11 that quantization of the free transverse gravitational field for the filled vacuum yields quantum theory which is equivalent to QED. At the moment of Big Bang the vacuum fermion states are empty. This is the classical limit of quantum vector gravity which yields classical evolution equations for the free gravitational field with negative energy of gravitational waves. As we show in appendix G, classical equations for the weak gravitational field are analogous to Maxwell’s equations in a medium with $\varepsilon = \mu = -1$. For the filled vacuum (quantum limit) the classical equations (22) no longer describe radiation field. In this case the quantum mechanical treatment must be used to describe evolution of the radiation (quantized) part of the gravitational field. As we show in section 11, quantum mechanical analysis yields that for the filled vacuum the equations for the radiation field are analogous to Maxwell’s equations with $\varepsilon = \mu = 1$ and the graviton energy is positive.

The present theory, if confirmed, can also lead to a break through in the problem of dark matter. Namely, the theory predicts that likely the supermassive compact objects at galactic centers have non baryonic origin and, thus, yet undiscovered dark matter particle is a likely ingredient of their composition. As a result, observations of such objects can allow us to predict the nature of dark matter. In the previous paper [13] we showed that properties of compact objects at galactic centers can be explained quantitatively assuming they are made of dark matter axions and the axion mass is about 0.6 meV. Analysis of [13] was based on the exponential metric (24) for the static gravitational field rather than general relativity. The present theory of gravity justifies our previous use of the exponential metric.

The vector theory of gravity can be tested in several ways. For example, one can examine gravity beyond the post-Newtonian limit in the solar system by improving the accuracy of Shapiro time delay experiment (time delay of a radar signal traveling near the Sun), or improving precision of the light deflection measurements by placing an optical interferometer with microarcsecond resolution into Earth orbit [92]. Vector gravity differs from general relativity in the post–post-Newtonian regime which has not been accurately tested to date.

Future detection of gravitational waves from binary mergers with improved sensitivity or detection of merger events with louder signals might be able to constrain the higher-order post-Newtonian parameters with a reasonable accuracy [9]. One can also test the vector theory of gravity by measuring propagation direction of gravitational waves relative to the interferometer arms (see section 8.3). Such measurement can be performed by detecting a signal from the same event by several LIGO-like interferometers [8] and is able to distinguish between vector gravity and general relativity (see figure 1 and section 16).

Another possibility is to resolve the supermassive object at the center of our Galaxy with the EHT. If general relativity is correct we must see a steady shadow from a black hole. If the present theory is right then shadow might appear and disappear periodically with a period of about 20 min as we predicted in [13]. Observation of such oscillations will also provide evidence for dark matter axion with mass in meV range.

Finally, we want to emphasize that vector gravity and general relativity are constructed in a unique way and have no adjustable parameters. Thus, both theories make fixed predictions and every new test of the theory is potentially a deadly test. A verified discrepancy between observation and prediction would kill the theory.

Despite fundamental differences, vector gravity and general relativity yield for the experimentally tested regimes quantitatively very close predictions which allowed both theories to hold up under extensive experimental scrutiny. In order to determine whether gravitational field has a vector or a tensor origin additional tests are required. Such tests are crucial for understanding of our Universe.

In section 16 we propose an experiment which can rule out one of the two theories in the next few years. The test is based on measurement of the gravitational wave propagation direction relative to the interferometer arms using several instruments and can be performed in a joint run of the LIGO–Virgo interferometer network. Such joint runs can start in 2017 when the Virgo instrument will reach the required sensitivity.

Although it is remarkable that general relativity (GR), born 100 years ago, has managed to pass many unambiguous observational and experimental tests, it actually has unwanted issues. For example,

- GR is not compatible with quantum mechanics.
- GR can not explain why Universe is spatially flat. Models involving cosmic inflation are needed to fix the problem. In contrast, in vector gravity the spatially flat Universe comes out as a solution of equations.
- GR does not provide a mechanism of matter generation at the Big Bang. An additional field with negative energy, the inflaton, is required to resolve the issue. In vector
gravity the mechanism of matter generation is part of the theory. No extra fields are necessary.

- GR can not explain the value of the cosmological term. In contrast, vector gravity predicts, with no free parameters, the value of the cosmological constant that agrees with observations.

- Existence of space–time singularities for which geometry is ill-defined is a generic feature of GR. Schwarzschild solution describing a static black hole is an example of a curvature singularity, where geometrical quantities characterizing space–time curvature, such as the Ricci scalar, take on infinite values. In GR such a singularity is unavoidable once the gravitational collapse of an object with realistic matter properties has proceeded beyond a certain stage. In contrast, stars in vector gravity do not collapse into a singularity. In vector gravity black holes do not exist and the end point of the gravitational collapse is a stable star with a reduced mass.

- Energy and momentum of the gravitational field in GR do not form a tensor quantity under an arbitrary coordinate transformation. Recall that conservation laws reflect a symmetry of the background space–time, namely its homogeneity and isotropy. In GR, which identifies gravitational field with the metric tensor $g_{\mu\nu}$, the real space is a space with Riemannian geometry, and this does not admit symmetries corresponding to displacements and rotations [93]. As a consequence, conservation laws do not hold in GR. If we want to make a theory compatible with the conservation laws we must postulate existence of a fixed symmetric background geometry [93]. We do so in the present vector theory of gravity.

The mentioned arguments point on the vector, rather than a tensor, nature of the gravitational field.

**Acknowledgments**

I am very grateful to the Institute for Quantum Science and Engineering of Texas A&M University for providing opportunity and resources for conducting research. This work was supported by the Office of Naval Research (Award No. N00014-16-1-3054) and the Robert A Welch Foundation (Award A-1261).

**Appendix A. Derivation of the equivalent metric**

Metric tensor determines the line element for infinitesimal coordinate displacement $dx$

$$ds^2 = f_{ik} dx^i dx^k.$$ 

Let us consider vector field $A_k$. The most general form of the equivalent metric $f_{ik}$ which can be constructed from the background Euclidean metric $\delta_{ik} = \text{diag} (1, 1, 1, 1)$ and $A_k$ is

$$f_{ik} = - F \delta_{ik} + (F + G) \frac{A_i A_k}{A^2},$$ (A1)

where $F$ and $G$ are scalar functions of

$$A = \sqrt{A_i A_k \delta^{ik}}.$$ 

If we chose $x^0$ axis along the direction of $A_k$ then $A_k = (A, 0, 0, 0)$ and the equivalent metric is diagonal

$$f_{ik} = \text{diag}(G, -F, -F, -F).$$ (A2)

To find a relation between $F$ and $G$ one can consider a particular case of gravitational field which has two nonzero components $A_0$ and $A_1$ and apply Einstein equivalence principle. Let us assume that a test particle moves along the $x$-axis under the influence of such field. Particle velocity is a function of time $V = V(t)$ and the line element for the particle in the gravitational field reads

$$ds^2 = f_{00} c^2 dt^2 + 2 f_{0i} c dt dx^i + f_{ij} dx^i dx^j.$$ (A3)

The same motion is obtained if the particle is at rest in Minkowski space–time, but the reference frame moves with velocity $V(t)$. Making a change of coordinate $x \rightarrow x + \int^t V(t') dt'$ in the Minkowski line element

$$ds^2 = c^2 dt^2 - dx^2$$

we obtain that the interval in the moving frame is

$$ds^2 = dt^2 (c^2 - V^2) - 2 V dt dx^i dx_i.$$ (A4)

According to the Einstein equivalence principal, the interval (A4) must be equal to (A3) which yields three equations

$$f_{00} = 1 - V^2/c^2, \quad f_{0i} = -V/c, \quad f_{xx} = -1.$$ (A5)

Taking into account equation (A1) and $A_0^2 + A_1^2 = A^2$, equations (A5) give

$$F = \frac{V^4}{4 c^4} + 1 + \frac{V^2}{2 c^2}, \quad G = \frac{V^4}{4 c^4} + 1 - \frac{V^2}{2 c^2},$$

that is

$$GF = 1.$$ (A6)

Equation (A6) fixes the relation between $G$ and $F$ for arbitrary $A$.

One should note that normalization of $A_k$ is not unique. Namely, $A_k$ can be multiplied by an arbitrary scalar function of $A$ which yields another vector field. Our theory is independent of the field normalization and equation (A6) is the only constraint on the equivalent metric we have. For example, one can choose norm of $A_k$ such that

$$G = 1.$$ (A7)

Then equivalent metric $f_{ik}$ is given by

$$f_{ik} = - \frac{\delta_{ik}}{A} + \left( A + \frac{1}{A} \right) \frac{A_i A_k}{A^2}.$$ (A8)
Metric $f^{ik}$ inverse to $f_{ik}$, defined as $f^{ik}f_{im} = \delta^i_m$, reads

$$f^{ik} = -A\delta^{ik} + \left( A - \frac{1}{A} \right) A^{ik} A^j.$$

(A9)

where $A' = \delta^{ik}A_k$ and the following relationships are satisfied

$$A' A_m f^{im} = A, \quad A_k f^{ik} = \frac{A'}{A} \sqrt{-f} = \frac{1}{A}. \quad \text{(A10)}$$

where $f = \det(f_{ik})$. In Cartesian coordinate system if we chose $x^0$ axis along the direction of $A_k$ the equivalent metric reads

$$f_{ik} = \text{diag} \left( A, -\frac{1}{A} - \frac{1}{A} \right).$$

(A11)

### Appendix B. Derivation of gravitational field action

#### B.1. Weak field limit

First we find gravitational field action for small deviation of the gravitational field from a constant value $\phi = \phi_0$ and $u_k = (1, 0, 0, 0)$. For small deviation the equivalent metric is

$$f_{ik} = \eta_{ik} + \begin{pmatrix}
    h_{00} & h_{01} & h_{02} & h_{03} \\
    h_{01} & h_{00} & 0 & 0 \\
    h_{02} & 0 & h_{00} & 0 \\
    h_{03} & 0 & 0 & h_{00}
\end{pmatrix}, \quad \text{(B1)}$$

where $|h_{01}| \ll 1$. One can obtain the weak field action for the gravitational field from the requirement that the action must be invariant under the gauge transformation (17)

$$h_{00} \rightarrow h_{00} + 2 \frac{\partial \psi}{\partial x}, \quad h_{03} \rightarrow h_{03} + \frac{\partial \psi}{\partial x^6}$$

upto the $V^2/c^2$ order. One can look for the weak-field action as a combination of gauge invariant terms. Introducing

$$B_0 = \frac{h_{00}}{2}, \quad B_0 = h_{03}$$

and a gauge-invariant combination

$$F_{ik} = \frac{\partial B_k}{\partial x^i} - \frac{\partial B_i}{\partial x^k}$$

the general form of the action which is gauge invariant upto the $V^2/c^2$ order is

$$S_{\text{gravity}} = C \int d^4x \left[ F^{ik}F_{ik} + C_1 \frac{\partial h_{00}}{\partial x^0} \frac{\partial h_{00}}{\partial x^0} + C_2 \left( \frac{\partial h_{00}}{\partial x^0} \frac{\partial h_{00}}{\partial x^0} - \frac{1}{4} \frac{\partial h_{00}}{\partial x^0} \right) \right]. \quad \text{(B2)}$$

where terms with $C_1$ and $C_2$ break gauge invariance in the $V^3/c^3$ order. One can find coefficients $C_1$ and $C_2$ from the requirement of the low-velocity Lorentz invariance. Straightforward calculations yield that action (B2)

$$S_{\text{gravity}} = C \int d^4x \left[ -\frac{1}{2} \left( \frac{C_2}{2} \right) \frac{\partial h_{00}}{\partial x^0} \frac{\partial h_{00}}{\partial x^0} \right]$$

+ $2\frac{\partial h_{00}}{\partial x^0} \frac{\partial h_{00}}{\partial x^0} - 2\frac{\partial h_{00}}{\partial x^0} \frac{\partial h_{00}}{\partial x^0}$

$$+ (2C_2) \frac{\partial h_{00}}{\partial x^0} \frac{\partial h_{00}}{\partial x^0} + C_1 \frac{\partial h_{00}}{\partial x^0} \frac{\partial h_{00}}{\partial x^0} \right] \right) \quad \text{(B3)}$$

is invariant under transformations (19) and (20) provided

$$C_1 = -6, \quad C_2 = 6.$$  

Indeed, under transformation

$$\frac{\partial}{\partial x^0} = \frac{\partial}{\partial x^0} - \frac{V}{c} \frac{\partial}{\partial \tau}, \quad \frac{\partial}{\partial x^0} = \frac{\partial}{\partial x^0} + \frac{V}{c} \frac{\partial}{\partial \tau} + \frac{V}{2c^2} \left( \frac{\partial}{\partial \tau} \right) \right) \quad \text{(B4)}$$

$$h_{00} \rightarrow h_{00} \left[ 1 + \frac{2V}{c^2} \right] = \frac{\partial h_{00}}{\partial x^0} - \frac{\partial h_{00}}{\partial x^0} - \frac{\partial h_{00}}{\partial x^0} - \frac{\partial h_{00}}{\partial x^0} \right] \quad \text{(B5)}$$

the action (B3) transforms as (we keep terms upto $V^3/c^3$ order)

$$S_{\text{gravity}} \rightarrow S_{\text{gravity}} + C \int d^4x \left[ -\frac{C_2}{6} \left( \frac{\partial h_{00}}{\partial x^0} \right)^2 \right.$$

$$+ \left( 2C_1 + 3 \right) \frac{\partial h_{00}}{\partial x^0} \frac{\partial h_{00}}{\partial x^0} + \left( C_1 - 6 \right) \frac{\partial h_{00}}{\partial x^0} \frac{\partial h_{00}}{\partial x^0} + \left( C_1 - 6 \right) \frac{\partial h_{00}}{\partial x^0} \frac{\partial h_{00}}{\partial x^0}$$

+ $\left( 6 - C_2 \right) \frac{\partial h_{00}}{\partial x^0} \frac{\partial h_{00}}{\partial x^0} + C_1 \left( \frac{7}{4} \right)$

Thus if $C_1 = -6$ and $C_2 = 6$ the action is invariant.

The overall constant factor $C$ in equation (B3) is obtained by matching the action (B3) with the Newtonian limit. The factor $C$ must be independent of the background field $\phi_0$ because such independence is one of the symmetries of $S_{\text{matter}}$. The final expression for the action of weak gravitational field reads

$$S_{\text{gravity}} = \frac{c^3}{32\pi G} \int d^4x \left[ \frac{\partial \phi}{\partial x^0} \frac{\partial \phi}{\partial x^0} - \frac{\partial h_{00}}{\partial x^0} \frac{\partial h_{00}}{\partial x^0} \right.$$

$$+ \frac{\partial h_{00}}{\partial x^0} \frac{\partial h_{00}}{\partial x^0} + \frac{\partial h_{00}}{\partial x^0} \frac{\partial h_{00}}{\partial x^0} + \frac{3\partial h_{00}}{\partial x^0} \frac{\partial h_{00}}{\partial x^0} \right] \quad \text{(B6)}$$

where $G$ is the gravitational constant.

### B.2. Arbitrary gravitational field

The most general form of the gravitational field action that can be constructed from the scalar $\phi$ and the unit vector $u_k$ in four dimensional Euclidean space $\theta_k$ is

$$S_{\text{gravity}} = \frac{c^3}{64\pi G} \int d^4x \left[ \frac{\partial \phi}{\partial x^0} \frac{\partial \phi}{\partial x^0} \right. \left( F_i^j(\phi) \delta^{ij} + F_4(\phi) u^i u^j \right. \right.$$

$$+ \left. \frac{\partial u_i}{\partial x^0} (F_i^j(\phi) \delta^{ij} u^j + F_k(\phi) \delta^{ik} \delta^{jk}) + F_4(\phi) \delta^{ij} \delta^{jk} u^i u^j \right. \left. + F_k(\phi) \frac{\partial \phi}{\partial x^0} \frac{\partial \phi}{\partial x^0} \delta^{ij} u^i u^j \right] \quad \text{(B7)}$$

The unknown functions $F_1,...,F_5$ can be obtained by matching the action with the weak field limit. For small deviations of $\phi$ from a constant $\phi_0$ and $u_k$ from $(1, 0, 0, 0)$ the equivalent
Thus in equation (F) the action \( S \) reads

\[
S = \frac{c^3}{64\pi G} \int dx^4 \left\{ (F_1 + F_2) \frac{\partial \phi}{\partial x^0} \frac{\partial \phi}{\partial x^0} + F_3 \frac{\partial u_0}{\partial x^0} \frac{\partial u_0}{\partial x^0} + \left( F_4 + F_2 \right) \frac{\partial u_3}{\partial x^0} \frac{\partial u_3}{\partial x^0} \right\} + 2 \cos(\phi_0) u_0 \left( u_0 \right) u_0 \left( u_0 \right)
\]

while action \( S_{\text{grav}} \) reduces to

\[
S_{\text{grav}} = \frac{c^3}{64\pi G} \int dx^4 \left\{ (F_1 + F_2) \frac{\partial \phi}{\partial x^0} \frac{\partial \phi}{\partial x^0} + F_3 \frac{\partial u_0}{\partial x^0} \frac{\partial u_0}{\partial x^0} + F_4 \frac{\partial u_3}{\partial x^0} \frac{\partial u_3}{\partial x^0} \right\} \]

where \( F_1, ..., F_4 \) are taken at \( \phi = \phi_0 \). In the rescaled coordinates

\[
x^0 \to e^{-\phi_0} x^0, \quad x^\alpha \to e^{\phi_0} x^\alpha
\]

the action \( S_{\text{grav}} \) reads

\[
S_{\text{grav}} = \frac{c^3}{64\pi G} \int dx^4 \left\{ (F_1 + F_2) e^{\phi_0} \frac{\partial \phi}{\partial x^0} \frac{\partial \phi}{\partial x^0} + F_3 e^{\phi_0} \frac{\partial u_0}{\partial x^0} \frac{\partial u_0}{\partial x^0} + F_4 e^{\phi_0} \frac{\partial u_3}{\partial x^0} \frac{\partial u_3}{\partial x^0} \right\}
\]

and the equivalent metric is

\[
f_{\alpha\beta} = g_{\alpha\beta} + 2\phi \delta_{\alpha\beta} + 2 \cos(\phi_0) \left( \begin{array}{ccc} 0 & u_1 & u_2 & u_3 \\ u_1 & 0 & 0 & 0 \\ u_2 & 0 & 0 & 0 \\ u_3 & 0 & 0 & 0 \end{array} \right)
\]

Thus in equation (B1)

\[
h_{00} = 2\phi_0, \quad h_{0\alpha} = 2 \cos(\phi_0) u_\alpha.
\]

In terms of \( h_{00} \) and \( h_{0\alpha} \), the action \( S_{\text{grav}} \) reads

\[
S_{\text{grav}} = \frac{c^3}{64\pi G} \int dx^4 \left\{ F_1 \frac{\partial h_{00}}{\partial x^0} \frac{\partial h_{00}}{\partial x^0} + F_4 \frac{\partial h_{00}}{\partial x^0} \frac{\partial h_{00}}{\partial x^0} + \left( F_3 + F_2 \right) \frac{\partial h_{00}}{\partial x^0} \frac{\partial h_{00}}{\partial x^0} \right\} + 2 \cos(\phi_0) h_{00}
\]

Matching this with the weak field limit action \( S_0 \) we obtain

\[
F_1 = 8, \quad F_2 = 8 - 24e^{-4\phi}, \quad F_3 = -16e^{-2\phi} \cos^2(2\phi),
\]

\[
F_4 = 8 \cosh^2(2\phi), \quad F_5 = -8 \cosh^2(2\phi),
\]

\[
F_6 + F_7 = 32e^{-2\phi} \cosh(2\phi).
\]

The functions \( F_6(\phi) \) and \( F_7(\phi) \) yet remain undetermined, only their sum. In order to find these functions we need to investigate symmetries of the action in the higher order in the post-Newtonian expansion parameter \( \epsilon \). The ‘order of smallness’ is determined according to the rules that matter velocity is of order \( V \sim \epsilon^{1/2} \) and gravitational constant \( G \sim \epsilon \).

Making change of functions

\[
h_{00} = 2 \cos(\phi_0) u_\alpha,
\]

\[
e^{2\phi} = e^{2\phi} - 2 \cos(\phi_0) u_\alpha^2,
\]

or

\[
\Phi \approx \phi - \frac{h_{00}^2}{2(1 + e^{4\phi})}
\]

and taking into account that \( h_{00} \sim \epsilon^{1/2} \) and \( dx^\alpha / dx^0 \sim \epsilon^{1/2} \) we obtain the following expression for the square of the interval up to the terms of the \( \epsilon^3 \) order

\[
dx^2 = e^{2\phi}(dx^0)^2 + 2h_{00}(dx^0 dx^\alpha - e^{-2\phi}(dx^\alpha)^2).
\]

Interval \( B(15) \), and hence \( S_{\text{matter}} \), is invariant under transformation

\[
x^0 \to e^{-a} x^0, \quad x^\alpha \to e^{a} x^\alpha,
\]

\[
\Phi \approx \phi + a, \quad h_{00} \to h_{00},
\]

where \( a \) is an arbitrary constant. Therefore, action for the gravitational field \( S_{\text{grav}} \) must also possess such symmetry in the \( \epsilon^3 \) order. Taking into account equations \( B(10)-(B12) \) and making change of functions \( B(13) \) and \( B(14) \) we obtain that \( S_{\text{grav}} \) up to the terms of the \( \epsilon^3 \) order reads

\[
S_{\text{grav}} = \frac{c^3}{8\pi G} \int dx^4 \left\{ - \frac{\partial \Phi}{\partial x^0} \frac{\partial \Phi}{\partial x^0} - 3e^{-4\phi} \frac{\partial \Phi}{\partial x^0} \frac{\partial \Phi}{\partial x^0} \right\}
\]

\[
+ \frac{1}{4} \left( \frac{\partial h_{00}}{\partial x^0} \right)^2 - \frac{1}{4} \frac{\partial h_{00}}{\partial x^0} \frac{\partial h_{00}}{\partial x^0} - 2e^{-2\phi} \frac{\partial h_{00}}{\partial x^0} \frac{\partial h_{00}}{\partial x^0} \right.
\]

\[
+ h_{00} \frac{\partial \Phi}{\partial x^0} \frac{\partial h_{00}}{\partial x^0} - h_{00} \frac{\partial h_{00}}{\partial x^0} \frac{\partial h_{00}}{\partial x^0} - 2e^{-2\phi} h_{00} \frac{\partial \Phi}{\partial x^0} \frac{\partial \Phi}{\partial x^0} \right.
\]

\[
- e^{-4\phi} \frac{\partial h_{00}}{\partial x^0} \frac{\partial h_{00}}{\partial x^0} + F_3(\Phi) h_{00} \left( \frac{\partial h_{00}}{\partial x^0} \frac{\partial h_{00}}{\partial x^0} \right)
\]

Please note that there is a factor \( 1/G \) in front of the integral and, hence, expression under the integral must be calculated up to the \( \epsilon^4 \) order. Action \( B(18) \) is invariant under transformation \( B(16) \) and \( B(17) \) provided

\[
F_7(\Phi) = 2F \cosh^2(2\Phi),
\]

where \( F \) is a constant independent of \( \Phi \). To find \( F \) we ought to dig symmetries deeper.
Let us consider stationary gravitational field for which equivalent metric is independent of time and make the following gauge transformation

\[ h_{0a} \rightarrow h_{0a} + e^{2\phi} \frac{\partial \psi}{\partial \alpha}. \]  

(B19)

where \( \psi \sim e^{3/2} \) is a function of spatial coordinates. Taking into account that

\[ \delta S_{\text{matter}} = -\frac{1}{2c} \int d^4x \sqrt{-f} T^a_{ik} \delta f_{ik}, \]

where \( T^{ik} \) is the energy–momentum tensor of matter, we obtain that under the gauge transformation (B19) the action \( S_{\text{matter}} \) transforms as

\[ S_{\text{matter}} \rightarrow S_{\text{matter}} - \frac{1}{c} \int d^4x \sqrt{-f} T^{00} e^{2\phi} \frac{\partial \psi}{\partial \alpha}. \]

Using \( \sqrt{-f} = e^{-2\phi} \) and integrating by parts we find

\[ S_{\text{matter}} \rightarrow S_{\text{matter}} + \frac{1}{c} \int d^4x \partial T^{00}_{\alpha} \frac{\partial \psi}{\partial \alpha}. \]

(20)

Conservation equation \( T^k_{ik} = 0 \) yields

\[ \frac{\partial}{\partial x^k} (\sqrt{-f} T^{00}) = \frac{\sqrt{-f}}{2} T^{00}_{ijkl} \delta f_{jk}, \]

which for stationary field reduces to

\[ \frac{\partial}{\partial x^0} (\sqrt{-f} T^0_0) = 0. \]

Keeping in mind that \( T^0_0 = f_{00} T^{00} + f_{03} T^{03} \) and \( f_{00} = e^{2\phi} \), we find

\[ \frac{\partial T^{00}_{\alpha}}{\partial x^0} = -\frac{\partial}{\partial x^0} [e^{-2\phi} h_{00} T^{30}] \sim e^{3/2}. \]  

(B21)

Therefore, the last term in equation (B20) is of the order of \( e^4 \) and, thus, action \( S_{\text{matter}} \) is invariant under gauge transformation (B19) in the \( e^3 \) order for stationary field. Now we apply gauge transformation (B19) to the action (B18). Keeping terms upto the \( e^3 \) order we obtain that for stationary field \( S_{\text{gravity}} \) transforms as

\[ S_{\text{gravity}} \rightarrow S_{\text{gravity}} + \frac{c^3}{8\pi G} \int d^4x e^{2\phi} \frac{\partial \Phi}{\partial \alpha} \left( h_{00} \frac{\partial^2 \Phi}{\partial x^0 \partial \alpha} - \frac{\partial \psi}{\partial \alpha} \frac{\partial h_{00}}{\partial \alpha} + e^{2\phi} \frac{\partial \psi}{\partial \alpha} \frac{\partial \Phi}{\partial \alpha} \right). \]

Hence, \( S_{\text{gravity}} \) is gauge invariant provided \( F = 0 \), that is \( F_0(\phi) = 0 \) and, according to equation (B12),

\[ F_0 = 32e^{-2\phi} \cosh(2\phi). \]

(B22)

Now all functions in the action (B7) are uniquely determined.

The final expression for the gravitational field action in Euclidean space is

\[ S_{\text{gravity}} = \frac{c^3}{8\pi G} \int d^4x \left[ \frac{\partial \phi}{\partial \alpha} \frac{\partial \phi}{\partial \alpha} (-\delta^{ik} + (1 - 3e^{-4\phi}) u^i u^k) + \cosh^2(2\phi) \frac{\partial u_i}{\partial \alpha} \frac{\partial u_m}{\partial \alpha} (\delta^{im} \delta^{kl} - \delta^{il} \delta^{km}) \right. \]

\[ \left. - (1 + e^{-4\phi}) \delta^{im} u^i u^k) + 2(1 + e^{-4\phi}) \frac{\partial \phi}{\partial \alpha} \frac{\partial u_m}{\partial \alpha} \delta^{im} u^k \right]. \]

Appendix C. Equations for gravitational field

Here we sketch how to derive equations for the gravitational field. The Lagrangian density has two parts

\[ L = L_g + L_{\text{matter}}, \]

where \( L_{\text{matter}} \) depends on the gravitational field via the equivalent metric \( f_{ik} \), while \( L_g \) depends on the field explicitly. We treat \( \phi \) and \( u_0 \) (\( \alpha = 1, 2, 3 \)) as independent functions. Then \( u_0 = 1 - u_1^2 - u_2^2 - u_3^2 \). Equations for the gravitational field are obtained by taking variation of the action with respect to \( \phi \) and \( u_0 \)

\[ \frac{\partial L_g}{\partial \phi} + W_k \frac{\partial f_{ik}}{\partial \phi} = 0, \]

(C1)

\[ \frac{\partial L_g}{\partial u_0} + W_k \left( \frac{\partial f_{ik}}{\partial u_0} - \frac{\partial f_{ik}}{\partial u_0} u_0 \right) = 0, \]

(C2)

where

\[ W_k = \frac{\partial L_{\text{matter}}}{\partial f_{ik}} \]

and we used

\[ \frac{\partial u_0}{\partial u_0} = -u_0. \]

Variational derivatives of \( L_g \) in equations (C1) and (C2) deal with derivatives of functions in \( L_g \) in a usual way. Taking into account that [27]

\[ \delta S_{\text{matter}} = -\frac{1}{2c} \int d^4x \sqrt{-f} T^a_{ik} \delta f_{ik}, \]

we obtain

\[ W_k = -\frac{1}{2} \sqrt{-f} T^k. \]

Using

\[ f_{ik} = -e^{-2\phi} \delta_{ik} + 2 \cosh(2\phi) u_i u_k, \]

we find

\[ \frac{\partial f_{ik}}{\partial \phi} = 2e^{-2\phi} \delta_{ik} + 4 \sinh(2\phi) u_i u_k = -2f_{ik} + 4e^{2\phi} u_i u_k, \]

\[ \frac{\partial f_{ik}}{\partial u_m} = 2 \cosh(2\phi)(\delta_{im} u_k + \delta_{ik} u_m). \]

Plugging this in equations (C1) and (C2) yields

\[ \frac{\partial L_g}{\partial \phi} = 2W + 4e^{2\phi} W_k u_i u_k = 0, \]

(C3)
where
\[ W = W^{ik} f_{ik}. \]
Equations (C3) and (C4) can be written in the form
\[ [W^{jk} - F f^{ik}] u_k - B^i = 0, \tag{C5} \]
where \( F \) is a scalar and \( B^i \) is a vector which we find next. Equation (C5) gives
\[ W^{jk} u_k = W^{i} F e^{-2\phi} u^i + B^i, \tag{C6} \]
\[ W^{ik} u_k = W^{i} F e^{-2\phi} + B^i u_i, \tag{C7} \]
where we used
\[ f^{ik} u_k = e^{-2\phi} u^i, \]
\[ f^{ik} u_k u_k = e^{-2\phi}. \]
Substitution of equations (C6) and (C7) in equations (C3) and (C4) yields
\[ \frac{\partial L_g}{\partial \phi} - 2W + 4e^{2\phi} (F e^{-2\phi} + B^i u_i) = 0, \tag{C8} \]
\[ \frac{\partial L_g}{\partial u_i} + 4 \text{cosh}(2\phi) B^i - \left( \frac{\partial L_g}{\partial u_0} + 4 \text{cosh}(2\phi) B^i \right) u^0 u_0 = 0. \]
Equation (C9) gives
\[ B^i = - \frac{1}{4 \text{cosh}(2\phi)} \frac{\partial L_g}{\partial u_i}. \tag{C10} \]
Substituting this into equation (C8) we obtain
\[ F = \frac{W}{2} + \frac{e^{2\phi} \frac{\partial L_g}{\partial u_0} u_0 - 1}{4 \frac{\partial L_g}{\partial \phi}}. \tag{C11} \]
Equations (C10) and (C11) determine \( B^i \) and \( F \) in equation (C5). Substituting them in equation (C5) we can write equations for the gravitational field as
\[ 2e^{2\phi} (W^{ik} - \frac{W^{jk}}{2}) u_k + \frac{1}{1 + e^{-4\phi}} \left[ \frac{\partial L_g}{\partial u_i} - \frac{\partial L_g}{\partial u_0} u_0 u^0 \right] + \frac{1}{2} \frac{\partial L_g}{\partial \phi} u^i = 0, \]
or
\[ \frac{1}{1 + e^{-4\phi}} \left[ \frac{\partial L_g}{\partial u_0} u_0 u^0 - \frac{\partial L_g}{\partial u_i} \right] = \frac{1}{2} \frac{\partial L_g}{\partial \phi} u^i = - \left( T^{ik} - \frac{T^{jk}}{2} \right) u_k, \tag{C12} \]
where \( T^{ik} \) is the energy–momentum tensor of matter and \( T = T^{ik} f_{ik} \).

What is left is to calculate variational derivatives of \( L_g \) and substitute them into equations (C12). Straightforward but lengthy algebra yields equations (22).

**Appendix D. Motion of particles in gravitational field**

Here we obtain how a test particle with rest mass \( m \) moves in an external gravitational field \( f_{ik} \). Interaction of the particle with the field is described by the action
\[ S_{\text{matter}} = -mc \int \sqrt{f_{ik} d^4x}, \tag{D1} \]
where the integral is taken along the particle trajectory. One can find equation of particle motion varying the action (D1) at fixed \( f_{ik} \) [27]
\[ \frac{\delta S_{\text{matter}}}{\delta x^i} = -\frac{mc}{2} \int \left[ \frac{dx^i}{ds} d^4x \delta f_{ik} + f_{ik} \left( \frac{d^2x^i}{ds^2} + \frac{dx^i}{ds} \frac{dx^k}{ds} \right) \right]. \]
\[ \frac{ds}{d\xi} = \sqrt{f_{ik} d^4x}. \tag{D2} \]
Next we take into account \( \delta f_{ik} = (\partial f_{ik} / \partial x^j) \delta x^i \) and integrate the second term by parts
\[ \frac{\delta S_{\text{matter}}}{\delta x^i} = -\frac{mc}{2} \int \left( \frac{\partial f_{ik}}{\partial x^j} - \frac{\partial f_{ik}}{\partial x^j} \frac{\partial f_{ik}}{\partial x^j} \right) \frac{dx^i}{ds} \frac{dx^k}{ds} - 2f_{ik} \frac{d^2x^i}{ds^2} \frac{dx^k}{ds} \right] ds \delta x^i. \tag{D3} \]
Principle of least action \( S_{\text{matter}} = 0 \) yields the following equation
\[ f_{ik} \frac{d^2x^i}{ds^2} = \frac{1}{2} \left( \frac{\partial f_{ik}}{\partial x^j} - \frac{\partial f_{ik}}{\partial x^j} \frac{\partial f_{ik}}{\partial x^j} \right) \frac{dx^i}{ds} \frac{dx^k}{ds} \tag{D4} \]
Multiplying both sides of equation (D3) by tensor inverse to \( f_{ik} \) we find
\[ \frac{d^2x^i}{ds^2} = \frac{1}{2} \frac{\partial f_{ik}}{\partial x^j} - \frac{\partial f_{ik}}{\partial x^j} \frac{\partial f_{ik}}{\partial x^j} \frac{dx^i}{ds} \frac{dx^k}{ds}. \]
This is equation of motion of a particle in gravitational field \( f_{ik} \).

From equation (D1) we obtain the following Lagrangian of the particle
\[ L = -mc \sqrt{f_{ik} d^4x} \frac{dx^i}{ds} \frac{dx^k}{ds}. \tag{D5} \]

Action (D1) and equation (D4) are invalid for massless particles. Let us consider a massless scalar field \( \chi \). In the gravitational field \( f_{ik} \) the action for \( \chi \) reads
\[ S = \frac{1}{8\pi} \int d^4x \sqrt{-g} \frac{\partial \chi^i}{\partial x^j} \frac{\partial \chi}{\partial x^j}. \tag{D6} \]
Variation of equation (D6) yields the following equation of
motion for the field $\chi$

$$\frac{\partial}{\partial x^\nu} \left( \sqrt{-g} f_{\mu \nu} \frac{\partial \chi}{\partial x^\nu} \right) = 0. \quad (D7)$$

For geometrical optics one can write $\chi$ as $\chi = |\chi| e^{i\psi}$, where $\psi$ (eikonal) has a large value. Substituting this into equation (D7) and keeping only the leading term we obtain eikonal equation in gravitational field

$$\tilde{f}^\mu_{\nu} \frac{\partial \psi}{\partial x^\mu} \frac{\partial \psi}{\partial x^\nu} = 0. \quad (D8)$$

**Appendix E. Motion of particles in static gravitational field**

Here we consider motion of a particle with rest mass $m$ in static gravitational field $\phi(r)$. Equation of particle motion in general case is obtained in appendix D. Equation (D4) for static field (24) reduces to

$$\frac{d(e^{2\phi} \gamma)}{dt} = 0, \quad (E1)$$

$$\frac{d(\gamma e^{-2\phi} V)}{dt} = -\gamma e^2 \left[ e^{2\phi} + \frac{V^2}{c^2} e^{-2\phi} \right] \nabla \phi, \quad (E2)$$

where $\nabla \phi = \partial \phi / \partial r$, $r = x^\alpha$, $V = \partial r / \partial t$ is the particle velocity and

$$\gamma = \frac{e^{-\phi} \sqrt{1 - \frac{V^2}{c^2} e^{-4\phi}}}. \quad (E3)$$

One can also find equation of particle motion (E2) directly from Lagrange’s equation $\frac{\partial L}{\partial \dot{x}^\alpha} = \frac{\partial}{\partial x^\alpha} \left( \frac{\partial L}{\partial \dot{x}^\alpha} \right)$, where the Lagrangian (D5) for static gravitational field reads

$$L = -mc^2 \sqrt{e^{2\phi} - \frac{V^2}{c^2} e^{-2\phi}}. \quad (E4)$$

Equation (E1) follows from equation (E2) if we multiply both sides of equation (E2) by $\gamma e^{-2\phi} V$ and make simple algebraic transformations.

Lagrangian (E4) gives the following expression for the particle generalized momentum $p = \frac{\partial L}{\partial \dot{\gamma}}$

$$p = \gamma e^{-2\phi} m V. \quad (E5)$$

and particle Hamiltonian $H = \frac{\partial L}{\partial \dot{\gamma}} - L$

$$H = e^{2\phi} mc^2 = \sqrt{m^2 c^4 e^{2\phi} + p^2 c^2 e^{4\phi}}. \quad (E6)$$

Thus, equation (E1) is the equation of energy conservation $W = \text{const}$, where

$$W = e^{2\phi} mc^2 = \frac{\gamma e^{-\phi} mc^2}{\sqrt{1 - \frac{V^2}{c^2} e^{-4\phi}}}. \quad (E7)$$

is the particle energy and equation (E2) is the equation for momentum.

For a massless particle one should use equation (D7) which for a static field reads

$$e^{-4\phi} \frac{\partial^2 \chi}{\partial t^2} - c^2 \Delta \chi = 0. \quad (E8)$$

equation (E8) describes propagation of a massless particle with speed

$$v = ce^{2\phi}. \quad (E9)$$

One can see that speed of light depends on the gravitational field $\phi$ and $v \leq c$ if $\phi$ is given by equation (29) with positive masses. By proper rescaling of coordinates in equation (E8) one can remove the factor $e^{-4\phi}$ at any given point. Let us fix $\phi = 0$ at infinite distance from masses. If an observer at infinity sends a light signal towards the Sun then near the solar surface $\phi < 0$ and light will propagate with a smaller speed. This is the explanation of Shapiro time delay in the present theory of gravity. Light signal traveling the same distance arrives with a delay if the light trajectory passes near the Sun. The delay occurs because the speed of light is smaller near the solar surface.

Since equation (E8) does not contain $t$ explicitly the photon frequency $\omega_0$ (measured in time $t$) remains the same during light propagation. However, physical processes occur with different rates at different $\phi$. Gravitational field (24) can be removed at a given point by rescaling time in the factor $\sqrt{f_{00}} = \phi \tau / c e^\phi$ and spatial coordinates by $\sqrt{-f_{\alpha\alpha}} = e^{-\phi}$. In such rescaled coordinates identical atoms emit light with equal frequencies $\omega \times \partial \chi / \partial \tau = e^{-\phi} \partial \chi / \partial t$. Thus we obtain

$$\omega = \omega_0 e^{-\phi}, \quad (E10)$$

where $\omega_0$ is the photon frequency measured in time $t$. Equation (E10) shows that if light emitted by an atom propagates into a region with larger gravitational potential then the detected light frequency is smaller than the identical atom emits at the detection point. This phenomenon is known as gravitational redshift of light. Equation (E10) also shows that in our theory there are no black holes. Indeed for the gravitational field created by a point mass $M$, $\phi = -GM/c^2 r$. Therefore if a photon is emitted at a distance $r$ from the mass $M$ with frequency $\omega$ then an observer at infinity will detect the photon with the energy

$$h \omega_0 = h \omega e^{-GM/c^2 r}. \quad (E11)$$

According to equation (E11) no matter how close the photon is emitted to the mass $M$ the photon’s energy at infinity never becomes zero. This means that photon can escape from the mass $M$ from anywhere. Such a conclusion is dramatically different from prediction of general relativity. In Einstein’s theory photons become trapped by the mass $M$ if they are emitted from a distance smaller then the event horizon (that is point mass $M$ behaves as a black hole).
Appendix F. Equations for metric in post-Newtonian limit

Here we show that Einstein equations

$$R_\alpha^\beta = \frac{8\pi G}{c^4} \left( T_\alpha^\beta - \frac{1}{2} R_\alpha T \right)$$  \hspace{1cm} (F1)$$

and equations of the vector theory of gravity (22) are the same in the post-Newtonian limit. In such limit, components of the Ricci tensor are

$$R_{\alpha\beta} = \frac{\partial}{\partial x^\alpha} \left( \frac{\partial h_0^0}{\partial x^\beta} - \frac{1}{2} \frac{\partial h_0^\beta}{\partial x^0} \frac{\partial h_0^\beta}{\partial x^0} \right) + \frac{1}{2} \frac{\partial h_0^0}{\partial x^0} \frac{\partial h_0^0}{\partial x^0} - \frac{1}{2} \frac{\partial^2 h_0^0}{\partial x^0 \partial x^0} + \frac{1}{4} (\nabla h_{00})^2 - \frac{1}{4} \frac{\partial h_0^0}{\partial x^0} \left( 2 \frac{\partial h_0^\alpha}{\partial x^\alpha} - \frac{\partial h_0^0}{\partial x^0} \right),$$  \hspace{1cm} (F2)$$

$$R_{0\alpha} = \frac{1}{2} \frac{\partial h_0^{\alpha\beta}}{\partial x^0} + \frac{1}{2} \frac{\partial h_0^0}{\partial x^0} \frac{\partial h_0^\beta}{\partial x^0} + \frac{1}{2} \frac{\partial h_0^\beta}{\partial x^0} \frac{\partial h_0^0}{\partial x^0} + \frac{1}{2} \Delta h_{0\alpha},$$  \hspace{1cm} (F3)$$

where $h_0^\alpha = \eta^{\alpha\beta} h_{\beta\alpha}$. Taking into account equation (31) we obtain

$$R_{00} = \frac{1}{2} \Delta h_{00} + \frac{3}{2} \frac{\partial^2 h_0^0}{\partial x^0 \partial x^0} - \frac{\partial^2 h_{0\beta}}{\partial x^0 \partial x^\beta} + \frac{1}{2} h_{00} \Delta h_{00} - \frac{1}{2} (\nabla h_{00})^2, \hspace{1cm} \text{and} \hspace{1cm} R_{0\alpha} = \frac{1}{2} \Delta h_{0\alpha} + \frac{1}{2} \frac{\partial h_{0\beta}}{\partial x^0} \frac{\partial h_0^\beta}{\partial x^0} + \frac{1}{2} \frac{\partial h_0^\beta}{\partial x^0} \frac{\partial h_{0\beta}}{\partial x^0}.$$

As a result, Einstein equations in the post-Newtonian limit read

$$\frac{1}{2} \Delta h_{00} + \frac{3}{2} \frac{\partial^2 h_0^0}{\partial x^0 \partial x^0} - \frac{\partial^2 h_{0\beta}}{\partial x^0 \partial x^\beta} + \frac{1}{2} h_{00} \Delta h_{00} - \frac{1}{2} (\nabla h_{00})^2 = \frac{8\pi G}{c^4} \left( T_{00} - \frac{1}{2} f_{00} T \right),$$  \hspace{1cm} (F4)$$

$$\frac{1}{2} \Delta h_{0\alpha} - \frac{1}{2} \frac{\partial h_{0\beta}}{\partial x^0} \frac{\partial h_0^\beta}{\partial x^0} + \frac{1}{2} \frac{\partial h_0^\beta}{\partial x^0} \frac{\partial h_{0\beta}}{\partial x^0} = \frac{8\pi G}{c^4} T_{0\alpha}. \hspace{1cm} (F5)$$

On the other hand in the cosmological reference frame for small deviations of $\phi$ from a constant value $\phi_0$ ($\delta \phi = \phi - \phi_0$) and $|u_i| \ll 1$, keeping post-Newtonian terms, and taking into account that

$$T_{00} = \int_0^1 T_{00} \, j_{00} \, t_{00} = e^{-4\phi} T_{00}, \hspace{1cm} j_{00} = e^{-4\phi} f_{00}, \hspace{1cm} T_{0\alpha} = -T_{\alpha 0},$$

equations (22) of the vector theory of gravity yield

$$\Delta \phi + 3 e^{-4\phi} \frac{\partial^2 \phi}{\partial x^0 \partial x^0} - 2 e^{-2\phi} \cosh(2\phi_0) \frac{\partial^2 u^0}{\partial x^0 \partial x^0} = \frac{8\pi G}{c^4} e^{-4\phi} \left( T_{00} - T_{f00} \right),$$  \hspace{1cm} (F6)$$

$$e^{2\phi_0} \cosh(2\phi_0) \left( \frac{\partial^2 u^0}{\partial x^0 \partial x^0} - \Delta u^0 \right) - 2 e^{2\phi} \frac{\partial^2 \phi}{\partial x^0 \partial x^0} = - \frac{8\pi G}{c^4} T_{00}. \hspace{1cm} (F7)$$

Next we rescale coordinates as

$$x^0 \rightarrow e^{-\phi_0} x^0, \hspace{1cm} x^\alpha \rightarrow e^{\phi_0} x^\alpha.$$ 

In new coordinates the equivalent metric $f_{\alpha\beta}$ has the form of equation (32) with

$$h_{00} = 2\delta \phi + 2(\delta \phi)^2, \hspace{1cm} h_{0\alpha} = 2 \cosh(2\phi_0) u_\alpha,$$

$$\delta \phi = \frac{h_{00}}{2} - \frac{h_0^0}{4}.$$ 

In the rescaled coordinates equations (F6) and (F7) reduce to

$$\frac{1}{2} \Delta h_{00} - \frac{1}{4} h_{00}^2 + \frac{3}{2} \frac{\partial^2 h_{00}}{\partial x^0 \partial x^0} - \frac{\partial^2 h_{0\beta}}{\partial x^0 \partial x^\beta} = \frac{8\pi G}{c^4} e^{4\phi_0} \left( T_{00} - T_{f00} \right). \hspace{1cm} (F8)$$

$$\frac{1}{2} \Delta h_{0\alpha} - \frac{1}{4} \frac{\partial^2 h_{0\beta}}{\partial x^0 \partial x^\beta} + \frac{\partial^2 h_{00}}{\partial x^0 \partial x^0} = \frac{8\pi G}{c^4} T_{0\alpha}. \hspace{1cm} (F9)$$

Multiplying both sides of equation (F8) by $e^{-4(\phi_0 - \phi)} = e^{2\phi_0}$ and expanding the exponential factor we obtain

$$\frac{1}{2} \Delta h_{00} + h_{00} \Delta h_{00} - \frac{1}{2} (\nabla h_{00})^2 = \frac{8\pi G}{c^4} \left( T_{00} - \frac{T_{f00}}{2} \right). \hspace{1cm} (F10)$$

Using

$$\Delta h_{00}^2 = 2(\nabla h_{00})^2 + 2h_{00} \Delta h_{00}$$

we finally find

$$\frac{1}{2} \Delta h_{00} + \frac{3}{2} \frac{\partial^2 h_{00}}{\partial x^0 \partial x^0} - \frac{\partial^2 h_{0\beta}}{\partial x^0 \partial x^\beta} + \frac{1}{2} h_{00} \Delta h_{00} - \frac{1}{2} (\nabla h_{00})^2 = \frac{8\pi G}{c^4} \left( T_{00} - \frac{T_{f00}}{2} \right). \hspace{1cm} (F10)$$

Equations (F9) and (F10) of the vector theory of gravity are identical to the Einstein equations (F4) and (F5). Boundary conditions are also the same. Thus, in the post-Newtonian limit both theories are equivalent.

Appendix G. Analogy of weak gravity in the classical limit with electromagnetism in medium with negative refractive index

In a medium with dielectric constant $\varepsilon$ and magnetic permeability $\mu$ Maxwell equations describing electromagnetic field read

$$\text{curl} \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \hspace{1cm} (G1)$$

$$\text{div}(\varepsilon \mathbf{E}) = 4\pi \rho, \hspace{1cm} (G2)$$

$$\text{curl} \left( \frac{\mathbf{B}}{\mu} \right) = \frac{1}{c} \frac{\partial (\varepsilon \mathbf{E})}{\partial t} + \frac{4\pi}{c} \rho \mathbf{V}, \hspace{1cm} (G3)$$

where $\rho$ is the electric charge density. In terms of the vector $\mathbf{A}$ and scalar $\varphi$ potentials

$$\mathbf{E} = -\nabla \varphi - \frac{1}{\varepsilon} \frac{\partial \mathbf{A}}{\partial t}, \hspace{1cm} \mathbf{B} = \text{curl} \mathbf{A},$$

the Maxwell equations (G1)–(G3) are
\[ \Delta \varphi + \frac{\partial}{\partial x^0} \text{div} A = -\frac{4\pi}{\varepsilon} \rho_v, \]

where \( A = 4\pi \rho_v V. \)

In the Lorenz gauge

\[ \frac{\partial \varphi}{\partial x^0} + \text{div} A = 0 \]  

(\text{G4})

and for \( \varepsilon = \mu = -1 \) equations reduce to

\[ \left( \Delta - \frac{\partial^2}{\partial x^0 \partial x^0} \right) \varphi = 4\pi \rho_v, \]  

(\text{G5})

\[ \left( \Delta - \frac{\partial^2}{\partial x^0 \partial x^0} \right) A = 4\pi c \rho_v V. \]  

(\text{G6})

On the other hand, equations for the weak classical gravitational field and non relativistic motion of matter with density \( \rho_m \) and velocity \( V \) are (see equations (38) and (39))

\[ \left( \Delta + 3 \frac{\partial^2}{\partial x^0 \partial x^0} \right) h_{00} - 2 \frac{\partial^2 h_{00}}{\partial x^0 \partial x^0} = \frac{8\pi G}{c^3} \rho_v, \]  

\[ \left( \frac{\partial^2}{\partial x^0 \partial x^0} - \Delta \right) h + 2 \nabla \left( \frac{\partial h_{00}}{\partial x^0} - \frac{1}{2} \frac{\partial h_{00}}{\partial x^0} \right) = -\frac{16\pi G}{c^3} \rho_m V, \]  

(\text{G7})

where \( h = h^{0\alpha}, V = V^\alpha \) and \( \nabla = \partial / \partial x^0 \). Introducing

\[ \varphi = \frac{c^2}{2} h_{00}, \quad \tilde{A} = \frac{c^2}{4} h \]

we obtain

\[ \left( \Delta + 3 \frac{\partial^2}{\partial x^0 \partial x^0} \right) \varphi + 4 \frac{\partial}{\partial x^0} \text{div} \tilde{A} = 4\pi G \rho_v, \]  

(\text{G8})

\[ \left( \Delta - \frac{\partial^2}{\partial x^0 \partial x^0} \right) \tilde{A} - \nabla \left( \frac{\partial \varphi}{\partial x^0} + \text{div} \tilde{A} \right) = \frac{4\pi G}{c} \rho_m V. \]  

(\text{G9})

Taking \( \partial / \partial x^0 \) from equation (\text{G9}) and \( (1/2) \text{div} \) from equation (\text{G10}), adding these equations together and using the continuity equation

\[ \frac{\partial \rho_v}{\partial t} + \text{div}(\rho_m V) = 0 \]

we find

\[ \frac{\partial^2}{\partial x^0 \partial x^0} \left( \frac{\partial \varphi}{\partial x^0} + \text{div} \tilde{A} \right) = 0. \]

Therefore one can take

\[ \frac{\partial \varphi}{\partial x^0} + \text{div} \tilde{A} = 0 \]

which is the same equation as the Lorenz gauge condition (\text{G4}) in electromagnetism. Then equations (\text{G9}) and (\text{G10}) reduce to

\[ \left( \Delta - \frac{\partial^2}{\partial x^0 \partial x^0} \right) \varphi = 4\pi G \rho_v, \]  

(\text{G11})

\[ \left( \Delta - \frac{\partial^2}{\partial x^0 \partial x^0} \right) \tilde{A} = \frac{4\pi G}{c} \rho_m V, \]  

(\text{G12})

which have the same form as Maxwell equations (\text{G5}) and (\text{G6}) in the left-handed medium with \( \varepsilon = \mu = -1 \).

The analogy becomes more transparent if we compare expressions for the energy density and energy flux. According to equations (46) and (47), the energy density and the energy density flux of the transverse gravitational wave in terms of \( \varphi \) and \( \tilde{A} \) are

\[ w_r = -\frac{1}{2\pi G} \left[ \frac{\partial \tilde{A}}{\partial x^0} \right]^2 + \text{curl}^2 \tilde{A}, \]

\[ S_r = \frac{c}{4\pi G} \frac{\partial \tilde{A}}{\partial x^0} \times \text{curl}(\tilde{A}), \]

which are similar to those for a transverse (\text{div} \( A = 0 \)) electromagnetic wave in the left-handed medium with \( \varepsilon = \mu = -1 \)

\[ w_{\text{em}} = -\frac{1}{8\pi} \left[ \frac{\partial A}{\partial x^0} \right]^2 + \text{curl}^2 A, \]

\[ S_{\text{em}} = \frac{c}{4\pi} \frac{\partial A}{\partial x^0} \times \text{curl}(A). \]

In such left-handed dispersionless medium the energy density of the electromagnetic field is also negative. However, equation of mass motion in weak gravitational field (40) is somewhat different from the equation of motion of a charge in electromagnetic field.

**Appendix H. Energy density and energy flux for weak classical gravitational field**

One can obtain the energy density and energy flux for classical gravitational field using general formula for the energy–momentum tensor. Namely, if action of the system has the form

\[ S = \frac{1}{c} \int d^4x L \left( A_i, \frac{\partial A_i}{\partial x^k} \right), \]

where the Lagrangian density \( L \) is some function of the quantities \( A_i \), describing the state of the system, and of their first derivatives, then the energy–momentum tensor \( T^{ik} \) of the system can be calculated using equation [27]

\[ T^{ik} = \sum_i \frac{\partial A_i}{\partial x^j} \frac{\partial L}{\partial \frac{\partial A_i}{\partial x^k}} - \delta^i_k L. \]  

(\text{H1})

\( T^{ik} \) obeys the conservation law

\[ \frac{\partial T^{ik}}{\partial x^k} = 0 \]

and, therefore, \( T^{00} \) can be interpreted as the energy density of the system, while vector \( S^i = ct T^{0i} \) (the Poynting vector) is the flux density (the amount of energy passing through unit surface per unit time).

46
For weak gravitational field and nonrelativistic motion of masses the Lagrangian density reads

\[ L = \frac{c^4}{32\pi G} \left( \frac{\partial h_{00} \partial h_{00}}{\partial x^0 \partial x^0} - \frac{\partial h_{0a} \partial h_{0a}}{\partial x^a \partial x^a} + \frac{\partial h_{00} \partial h_{00}}{\partial x^a \partial x^0} - \frac{\partial h_{0a} \partial h_{0a}}{\partial x^a \partial x^0} \right) \]

\[ - \rho c^2 \left( \frac{1}{2} \rho c^2 h_{00} - \rho c^2 V^2 h_{00} + \frac{1}{2} \rho V^2 \right) \]  

(H2)
and components of the equivalent metric \( h_{00} \) can be treated as function describing the state of the gravitational field. Applying equation (H1) we find

\begin{align*}
T^{00} &= \frac{c^4}{32\pi G} \left( 3 \frac{\partial h_{00} \partial h_{00}}{\partial x^0 \partial x^0} - \frac{\partial h_{0a} \partial h_{0a}}{\partial x^a \partial x^0} + \frac{\partial h_{00} \partial h_{00}}{\partial x^a \partial x^0} - \frac{\partial h_{0a} \partial h_{0a}}{\partial x^a \partial x^0} \right) \\
&- \rho c^2 + \frac{1}{2} \rho c^2 h_{00} + \frac{1}{2} \rho V^2, \\
T^{0a} &= \frac{c^4}{16\pi G} \left( \frac{\partial h_{0a} \partial h_{0a}}{\partial x^0 \partial x^0} + \frac{\partial h_{0a} \partial h_{0b}}{\partial x^a \partial x^0} - \frac{\partial h_{0a} \partial h_{0b}}{\partial x^a \partial x^0} - \frac{\partial h_{0a} \partial h_{0b}}{\partial x^a \partial x^0} \right) \\
&+ 2 \frac{\partial \rho}{\partial x^0} h_{00} + \rho c V^0, \hspace{1cm} (H3)
\end{align*}

where \( \rho \) is the mass density, \( V \) is the velocity of nonrelativistic motion and \( \alpha = 1, 2, 3 \). Introducing vector

\[ h = h_{00}, \]
we obtain the following expression for the energy density of the weak classical gravitational field

\[ T^{00} = - \frac{c^4}{32\pi G} \left[ 3 \left( \frac{\partial h_{00} \partial h_{00}}{\partial x^0 \partial x^0} \right)^2 - \left( \nabla h_{00} \right)^2 + \left( \frac{\partial h}{\partial x^0} \right)^2 + \text{curl} \ h \right] \\
+ \rho c^2 + \frac{1}{2} \rho c^2 h_{00} + \frac{1}{2} \rho V^2. \]  

(H5)

The energy density flux is given by

\[ S = \frac{c^5}{16\pi G} \left[ - \left( \frac{\partial h}{\partial x^0} \right) \nabla h_{00} \right] \\
+ \frac{\partial h}{\partial x^0} \times \text{curl} \ h + \rho c V. \]  

(H6)

In the Newtonian limit the energy density of field and matter reads

\[ w = \rho c^2 + \frac{\rho V^2}{2} + \rho c^2 \phi + \frac{c^4}{8\pi G} \left( \nabla \phi \right)^2, \]  

(H7)

where \( c^2 \phi = c^2 h_{00}/2 \) is the Newtonian gravitational potential.

**Appendix I. Cosmological suppression of preferred frame and preferred location effects**

Here we show lack of the preferred frame and preferred location effects for neutron stars orbiting each other. Since gravitational field of a neutron star is not weak we must find symmetries of the action valid when spatial change of \( \phi \) is of the order of unity. Spatial variation of \( \phi \) produced by a neutron star yet substantially smaller than cosmological value \( \phi_{\text{cosm}} \). Indeed, due to expansion of the Universe the spatial scale has been magnified in a factor \( e^{-\phi_{\text{cosm}}} \sim 10^{40} \). Thus, at the present epoch \( e^{-\phi} \gg 1 \) and, therefore, we can disregard exponentially small number \( e^\phi \) compared to the exponentially large value of \( e^{-\phi} \).

In terms of components the equivalent metric (9) reads

\[ f_{00} = e^{2\phi} - 2 \cosh(2\phi) u_1^2, \]

\[ f_{0a} = 2 \cosh(2\phi) u_2 u_a, \]

\[ f_{0b} = -e^{-2\phi} \delta_{0b} + 2 \cosh(2\phi) u_a u_b. \]

Taking into account that \( e^{-\phi} \gg 1 \) and introducing new function

\[ h_{00} = e^{-2\phi} u_0, \]

one can write the equivalent metric as

\[ f_{00} = e^{2\phi}(1 - h_{00}^2), \]

\[ f_{0a} = h_{0a}, \]

\[ f_{0b} = -e^{-2\phi} \delta_{0b}. \]  

(11)

Thus, the square of the interval is

\[ ds^2 = e^{2\phi}(1 - h_{00}^2)(dx^0)^2 + 2h_{0a} dx^0 dx^a - e^{-2\phi} dr^2. \]  

(12)

Motion of stars orbiting each other is not relativistic and, therefore, \( V/c \) is another small parameter in our problem. Keeping terms up to \( V^2/c^3 \) and taking into account that \( e^{-\phi} \gg 1 \) the gravitational field action (21) reduces to

\[ S_{\text{gravity}} = \frac{c^3}{8\pi G} \int d^4 x \left[ - \frac{\partial \phi}{\partial x^0} \frac{\partial \phi}{\partial x^0} + 3e^{-\phi} \frac{\partial \phi}{\partial x^0} \frac{\partial \phi}{\partial x^0} \right] \\
+ 2e^{-2\phi} \frac{\partial \phi}{\partial x^0} \frac{\partial h_{00}}{\partial x^0} + 1 \frac{\partial h_{00}}{\partial x^0} \frac{\partial h_{03}}{\partial x^3} - 1 \frac{\partial h_{03}}{\partial x^3} \frac{\partial h_{03}}{\partial x^3} \\
+ h_{00} \frac{\partial \phi}{\partial x^0} \frac{\partial h_{00}}{\partial x^0} + h_{00} \frac{\partial \phi}{\partial x^0} \frac{\partial h_{00}}{\partial x^0} \]

(13)

The interval (12) and the gravitational field action (13) are invariant under scaling transformation

\[ x^0 \rightarrow e^{-\alpha} x^0, \]

\[ x^0 \rightarrow e^{\alpha} x^0, \]

\[ \phi \rightarrow \phi + a, \]

\[ h_{0a} \rightarrow h_{0a}, \]

(14)

where \( a \) is an arbitrary constant parameter, not necessarily small.

There is also an additional Lorentz-like symmetry of the action valid for the strong gravitational field of a neutron star. Straightforward but lengthy calculation yields that up to the terms of the order of \( V^2/c^3 \) the total action \( S_{\text{gravity}} + S_{\text{matter}} \) is invariant under a coordinate transformation for which derivatives transform as

\[ \frac{\partial}{\partial x^0} \rightarrow \left( 1 + \frac{V^2}{2c^2} \right) \frac{\partial}{\partial x^0} - \frac{V}{c} \frac{\partial}{\partial r} - \frac{V}{c} \frac{\partial}{\partial r} - \frac{V}{c} \frac{\partial}{\partial x^0} + \left( 1 + 8e^{-\phi} - 9e^{-\phi} \right) \frac{V^2}{2c^2} - \frac{e^{2\phi}}{c^2} \frac{\partial}{\partial x^0} \]

(15)

where \( c^2 \phi = c^2 h_{00}/2 \) is the Newtonian gravitational potential.
where $V = V^a$ is a constant (velocity) vector and $h = h^{0a}$. Under this transformation the equivalent metric $f_{ab}$ given by equation (11), transforms as a covariant tensor. Namely, $f_{00}$ transforms as $\partial^a_0 \partial^b_0$, $f_{0\alpha}$ transforms as $\partial^\alpha_0 \partial^b_0$, and so on. Since $f_{ab}$ transforms as a tensor the interval $ds$, and hence $S_{\text{matter}}$, are invariant.

Keeping terms of the proper order the metric transformation reads

$$h_{00} \rightarrow h_{00} - 2 \frac{V^0}{c} \sinh(2\phi),$$

$$e^{2\phi} \rightarrow e^{2\phi} + 2 \frac{V^2}{c^2} \sinh(2\phi) - 2 e^{4\phi} \frac{V^0}{c} h_{00},$$

or

$$\phi \rightarrow \phi + \frac{V^2}{c^2} e^{-2\phi} \sinh(2\phi) - \frac{V^0}{c} e^{2\phi} h_{00}.$$  

Under transformation (16) the volume element $d^4x$ in the action $S_{\text{gravity}}$ transforms as $d^4x \rightarrow d^4x / J$, where $J$ is the Jacobian of the transformation

$$J = 1 + (1 + 12 e^{-4\phi} - 13 e^{4\phi}) \frac{V^2}{c^2} + 2 e^{2\phi} \frac{V^0}{c} h_{00}.$$  

In the post-Newtonian limit (far away from the neutron star) the transformation (16)–(18) reduces to the low-velocity Lorentz transformation

$$x^0 \rightarrow \left(1 + \frac{V^2}{2c^2}\right) x^0 + \frac{1}{c} \mathbf{V} \cdot \mathbf{r}, \quad \mathbf{r} \rightarrow \mathbf{r} + \frac{V^0}{c} x^0,$$

$$\phi \rightarrow \left(1 + \frac{2V^2}{c^2}\right) \phi - \frac{V^0}{c} h_{00},$$

$$h_{00} \rightarrow h_{00} - 4 \frac{V^0}{c} \phi.$$  

Scaling transformation (14) and (15) combined with the Lorentz-like transformation (16)–(18) allow us to eliminate the preferred frame and preferred location from the equations describing motion and gravitational field of a neutron star. Indeed, let us consider a reference frame in which background gravitational field is $\phi^{\text{back}} = \text{const}$ and $h_{00}^{\text{back}} = \text{const}$. The background field is produced by the companion star and the cosmological part. Since the total action is invariant under rotations in the four dimensional Euclidean space $\delta_k$ one can eliminate $h_{00}^{\text{back}}$ by making such a rotation. After this transformation the background field becomes $\phi^{\text{back}} = \phi_0 = \text{const}$ and $h_{00}^{\text{back}} = 0$. Next we perform scaling transformation (14) and (15) with $a = -\phi_0$ which makes $\phi^{\text{back}} = 0$, that is now the background metric is Minkowski metric $\eta_{ik}$. In the new frame, however, the neutron star moves with some velocity $V$. Finally, the Lorentz-like transformation (16)–(18) eliminates $V$, that is in the new reference frame the neutron star is at rest. At the same time, the Lorentz-like transformation does not change the background Minkowski metric $\eta_{ik}$. Indeed, far from the star the field transformation reduces to equations (110) and (111) for which the asymptotic values $\phi = 0$ and $h_{00} = 0$ remain invariant.

We found that transformations which keep the total action invariant eliminate the background field from the boundary conditions. Thus, field equations and the equation of the star motion do not yield preferred frame and preferred location effects for they are obtained by taking variation of the total action. As a consequence, one can choose a reference frame in which star is static and metric is asymptotically Minkowski. In this frame, in the outer region of a nonrotating static star the gravitational field is described by the equation $\Delta \phi = 0$ with asymptotic boundary conditions $\phi^{\text{back}} = 0$ and $h_{00}^{\text{back}} = 0$ which yields

$$\phi(\mathbf{r}) = -\frac{GM}{c^2 r},$$

where $M$ is a Kepler-measured mass of the star. Thus, solution for the relativistic structure and gravitational field of the star is independent of the background gravitational field.

To obtain gravitational field in a frame in which star moves with velocity $V < c$ one can make Lorentz-like transformation (16)–(18) which yields analytical solution for arbitrary values of $\phi$. Using equations (11) we find the following expression for the equivalent metric produced by the moving star

$$f_{00} = e^{2\phi} + 2 \frac{V^2}{c^2} \sinh(2\phi)(2 - e^{4\phi}),$$

$$f_{0\alpha} = -2 \frac{V^0}{c} \sinh(2\phi),$$

$$f_{\alpha\beta} = \left(-e^{-2\phi} + 2 e^{4\phi} \frac{V^2}{c^2} \sinh(2\phi)\right) \delta_{\alpha\beta},$$

where $\phi$ is the field of the static star written in terms of coordinates in the moving frame. In the far field in the post-Newtonian limit equations (113)–(115) reduce to

$$h_{00} = -\frac{2 GM}{c^2 |\mathbf{r} - \mathbf{V}^0|} \left(1 + \frac{2V^2}{c^2}\right) + \frac{2G M^2}{c^4 |\mathbf{r} - \mathbf{V}^0|^2},$$

$$h_{0\alpha} = -\frac{4G M \mathbf{V}^0}{c^4} \frac{|\mathbf{r} - \mathbf{V}^0|}{|\mathbf{r} - \mathbf{V}^0|^2},$$

$$h_{\alpha\beta} = -\frac{2 GM}{c^4 |\mathbf{r} - \mathbf{V}^0|} \delta_{\alpha\beta}. $$

Our result coincides with those obtained in general relativity (in the gauge $2\partial h_{00}/\partial x^0 - \partial h_{0\alpha}/\partial x^\alpha = 0$) in the post-Newtonian limit far from the star. It is independent of the original background metric as well as motion of the reference frame relative to the background.

### Appendix J. Post-Newtonian limit of vector gravity in the parametrized post-Newtonian formalism

Clifford Will in his book on ‘Theory and experiment in gravitational physics’ [2] presented a method of calculation of the post-Newtonian (PN) limits of any metric theory of gravity. Vector gravity is a metric theory and, thus, approach of [2] can be applied here as well.

The method outlined in the Will’s book consists of 9 steps. Namely, one should start from the basic field equations
of a metric theory of gravity, solve them in the PN limit for the equivalent metric and compare the answer with the parametrized post-Newtonian (PPN) expansion. In appendix F we carried out the first four steps and found that equations of vector gravity in the PN limit are identical to those of general relativity, so do the boundary conditions. If this is the case then equivalence of vector gravity and general relativity in the PN limit is proven. Calculations of steps 5–9 dealing with the actual solution of the equations are simply identical to those in general relativity.

However, one of the referees of our paper believes that calculations of appendix F are not sufficient and all 9 steps must be included. We present them here closely following prescription of [2]. For completeness of the presentation we repeat the first four steps as well.

In the PN formalism the metric is expanded in a small parameter $\epsilon$. The ‘order of smallness’ is determined according to the rules that matter velocity is of order $V \sim \epsilon^{1/2}$ and gravitational constant $G \sim \epsilon$. A consistent PN limit requires determination of $g_{00}$ correction through $O(\epsilon^3)$, $g_{0\alpha}$ through $O(\epsilon^{3/2})$, and $g_{\alpha\beta}$ through $O(\epsilon)$.

Recall, that we use the following convention. Lower case Latin indices $(i, k, m, \ldots)$ label four dimensional coordinates (range 0, 1, 2, 3 ), while lower case Greek letters $\alpha, \beta, \gamma$ denote spatial coordinates (range 1, 2, 3).

**Step 1. Identify the variables.**

In vector gravity the scalar $\phi$ and the unit vector $u_k$ are dynamical gravitational variables and flat background metric $\tilde{\delta}_{ik}$ is the prior-geometrical variable.

**Step 2. Set the cosmological boundary conditions.** Assume a homogeneous isotropic cosmology, and at a chosen moment of time define the values of the variables far from the PN system. Rest frame of the Universe is a convenient choice of the coordinate system.

Please note that PN expansion of a metric theory of gravity and comparison with general relativity can be made in any convenient reference frame. This is true for any metric theory of gravity, including vector gravity. This question is explained well in [2].

For vector gravity, the cosmological boundary conditions are:

$$\phi \to \phi_0, \quad u_k \to (1, 0, 0, 0)$$

far from the PN system.

**Step 3. Expand in a post-Newtonian series about the asymptotic values.**

Expansion of the dynamical gravitational variables in vector gravity is

$$\phi = \phi_0 + \hat{\phi}, \quad u_k = (1, 0, 0, 0) + \hat{u}_k,$$

where $|\hat{\phi}|, |\hat{u}_k| \ll 1$.

**Step 4. Substitute these forms into the field equations, keeping only such terms as are necessary to obtain a final, consistent PN solution for $f_{ik}$.**

Keeping the post-Newtonian terms, and taking into account that

$$T^{00} = f^{00} J^{00} T_{00} = e^{-4\phi} T_{00},$$

$$T^{00} = -T_{00},$$

the basic field equations of vector gravity (22) reduce to the following equations for $\hat{\phi}$ and $\hat{u}_k$

$$\Delta \hat{\phi} + 3e^{-4\phi_0} \frac{\partial^2 \hat{\phi}}{\partial x^0 \partial x^0} - 2e^{-2\phi_0} \cosh(2\phi_0) \frac{\partial^2 \hat{\beta}}{\partial x^0 \partial x^3} = \frac{8\pi G}{c^2} e^{-4(\phi_0 + \hat{\phi})} \left( T_{00} - \frac{T}{2f_{00}} \right),$$  \hspace{1cm} (J1)

$$e^{2\phi_0} \cosh(2\phi_0) \left( \frac{\partial^2 \hat{\beta}}{\partial x^0 \partial x^3} - \Delta \hat{\beta} \right) - 2e^{-2\phi_0} \frac{\partial^2 \hat{\phi}}{\partial x^0 \partial x^0} = -\frac{8\pi G}{c^4} T_{00},$$  \hspace{1cm} (J2)

Next we rescale coordinates as

$$x^0 \to e^{-\phi_0} x^0, \quad x^n \to e^{\phi_0} x^n.$$

In the new coordinates the equivalent metric $f_{ik}$ has the form

$$f_{ik} = \eta_{ik} + \begin{vmatrix} h_{00} & h_{01} & h_{02} & h_{03} \\ h_{10} & h_{00} & 0 & 0 \\ h_{20} & 0 & h_{00} & 0 \\ h_{30} & 0 & 0 & h_{00} \end{vmatrix},$$  \hspace{1cm} (J3)

where $\eta_{ik} = \text{diag} (1, -1, -1, -1)$ is Minkowski metric and $h_{00} = 2\hat{\phi} + 2\hat{\beta}^2$, $h_{0\alpha} = 2\cosh(2\phi_0) \hat{u}_\alpha$,

$$\frac{\gamma}{\phi} = \frac{h_{00}}{2} - \frac{h_{00}^2}{4}.$$  

In the rescaled coordinates in terms of $h_{00}$ and $h_{0\alpha}$ equations (J1) and (J2) read

$$\frac{1}{2}\Delta h_{00} - \frac{1}{4}\Delta h_{\alpha\beta} + \frac{3}{2} \frac{\partial^2 h_{00}}{\partial x^\alpha \partial x^\beta} - \frac{\partial^2 h_{0\beta}}{\partial x^0 \partial x^3} = \frac{8\pi G}{c^4} e^{-4\phi} \left( T_{00} - \frac{T}{2} \right),$$  \hspace{1cm} (J4)

$$\frac{1}{2}\Delta h_{0\alpha} + \frac{1}{4} \frac{\partial^2 h_{0\alpha}}{\partial x^\beta \partial x^\beta} + \frac{\partial^2 h_{00}}{\partial x^0 \partial x^\alpha} = \frac{8\pi G}{c^4} T_{00}. $$  \hspace{1cm} (J5)

Multiplying both sides of equation (J4) by $e^{4\phi_0} \approx e^{2h_{00}}$ and expanding the exponential factor we obtain

$$\frac{1}{2}\Delta h_{00} + h_{00} \Delta h_{00} - \frac{1}{4}\Delta h_{00}^2 + \frac{3}{2} \frac{\partial^2 h_{00}}{\partial x^0 \partial x^0} - \frac{\partial^2 h_{0\beta}}{\partial x^0 \partial x^3} = \frac{8\pi G}{c^4} \left( T_{00} - \frac{T}{2} \right).$$

Using

$$\Delta h_{00}^2 = 2(\nabla h_{00})^2 + 2h_{00} \Delta h_{00}$$

we find

$$\frac{1}{2}\Delta h_{00} + \frac{3}{2} \frac{\partial^2 h_{00}}{\partial x^0 \partial x^0} - \frac{\partial^2 h_{0\beta}}{\partial x^0 \partial x^3} + \frac{1}{2} h_{00} \Delta h_{00} - \frac{1}{2} (\nabla h_{00})^2.$$
In the PN limit, components of the Ricci tensor are

\[ R_{\alpha\beta} = \frac{1}{2} \frac{\partial^2 h_{\alpha}^\mu}{\partial x^\mu \partial x^\beta} + \frac{1}{2} \frac{\partial^2 h_{\beta}^\mu}{\partial x^\alpha \partial x^\mu} - \frac{1}{2} \frac{\partial^2 h_{\beta}^\mu}{\partial x^\alpha \partial x^\mu} + \frac{1}{2} \Delta h_{\alpha\beta}, \]

where \( h_{\beta}^\mu = \eta_{\gamma\beta} h_{\gamma\alpha} \). If we impose the three gauge conditions \( \gamma = 1, 2, 3 \)

\[ \frac{\partial h_{m}^m}{\partial x^m} - \frac{1}{2} \frac{\partial h_{n}^m}{\partial x^n} = 0 \]

equation (J10) becomes

\[ R_{\alpha\beta} = \frac{1}{2} \Delta h_{\alpha\beta} \]

and, in the order \( O(\epsilon) \), Einstein equations (J7) with \( i k = \alpha\beta \) reduce to

\[ \Delta h_{\alpha\beta} = -\frac{8\pi G}{c^4} T_{\alpha\beta}. \]  

On the other hand, in this order, Einstein equations with \( i k = 00 \) yield

\[ \Delta h_{00} = \frac{8\pi G}{c^4} (2T_{00} - T) = \frac{8\pi G}{c^4} T. \]

Comparing equations (J12) and (J13) we find that in the PN limit of general relativity

\[ h_{\alpha}^\mu = -h_{00} \delta_{\alpha}^\mu \]

and, hence, the metric is given by

\[ g_{\alpha\beta} = \eta_{\alpha\beta} + \begin{pmatrix} h_{00} & h_{01} & h_{02} & h_{03} \\ h_{01} & h_{00} & 0 & 0 \\ h_{02} & 0 & h_{00} & 0 \\ h_{03} & 0 & 0 & h_{00} \end{pmatrix}. \]

Metric (J15) has the same form as the equivalent metric (J3) in vector gravity. Plugging equation (J14) into equations (J8) and (J9) we obtain

\[ R_{00} = \frac{1}{2} \Delta h_{00} + \frac{3}{2} \frac{\partial^2 h_{00}}{\partial x^0 \partial x^0} - \frac{\partial^2 h_{03}}{\partial x^0 \partial x^3} + \frac{1}{2} h_{00} \Delta h_{00} - \frac{1}{2} (\nabla h_{00})^2, \]

\[ R_{03} = \frac{1}{2} \Delta h_{03} + \frac{1}{2} \frac{\partial^2 h_{03}}{\partial x^0 \partial x^3} + \frac{\partial^2 h_{00}}{\partial x^0 \partial x^0}. \]

As a result, Einstein equations (J7) with \( i = 0 \) and \( k = 0, 1, 2, 3 \) in the PN limit read

\[ \frac{1}{2} \Delta h_{00} + \frac{3}{2} \frac{\partial^2 h_{00}}{\partial x^0 \partial x^0} - \frac{\partial^2 h_{03}}{\partial x^0 \partial x^3} + \frac{1}{2} h_{00} \Delta h_{00} - \frac{1}{2} (\nabla h_{00})^2 = \frac{8\pi G}{c^4} \left( T_{00} - \frac{T}{2} \right). \]

\[ \frac{1}{2} \Delta h_{03} + \frac{1}{2} \frac{\partial^2 h_{03}}{\partial x^0 \partial x^3} + \frac{\partial^2 h_{00}}{\partial x^0 \partial x^0} = \frac{8\pi G}{c^4} T_{03}. \]

Equations (J16) and (J17) for the four unknown functions \( h_{00} \) and \( h_{03} \) coincide with equations (J6) and (J5) of vector gravity. Boundary conditions are also the same in both theories, namely, far from the PN system

\[ h_{00} \rightarrow 0, \quad h_{03} \rightarrow 0. \]

This is sufficient to conclude that vector gravity and general relativity are equivalent in the PN limit.

**Step 5.** Solve for \( h_{00} \) up to \( O(\epsilon) \).

Only the lowest PN order equation is needed. In this order equation (J6) of vector gravity reduces to

\[ \Delta h_{00} = \frac{16\pi G}{c^4} \left( T_{00} - \frac{T}{2} \right) = \frac{8\pi G}{c^4} \rho, \]

where \( \rho(t, r) \) is the matter density (measured in a frame momentarily comoving with the matter). Using

\[ \Delta \frac{1}{|r - r'|} = -4\pi \delta(r - r'), \]

we obtain that solution of equation (J18) is

\[ h_{00} = -\frac{2}{c^2} U, \]

where

\[ U(t, r) = G \int \frac{\rho(t, r')}{|r - r'|} \, dr'. \]

is the Newtonian gravitational potential with minus sign.

**Step 6.** Solve for \( h_{03} \) up to \( O(\epsilon) \) and \( h_{00} \) to \( O(\epsilon^3/2) \).

According to equations (J3) and (J19), in vector gravity

\[ h_{03} = h_{00} \delta_{03} = -\frac{2}{c^2} U \delta_{03}. \]

Next we note that with the PN accuracy equations (J5), (J6) are invariant under the gauge transformation

\[ h_{00} \rightarrow h_{00} + 2 \frac{\partial \psi}{\partial x^0}, \quad h_{03} \rightarrow h_{03} + \frac{\partial \psi}{\partial x^3}. \]

where \( \psi \) is an arbitrary function of the order of \( O(\epsilon) \). Thus, we can impose one gauge fixing condition which we choose
as in [2, 27]
\[
\frac{\partial h_{0}^{a}}{\partial \alpha^{a}} - \frac{1}{2} \frac{\partial h_{0}^{a}}{\partial \alpha^{a}} = 0
\]

or
\[
\frac{\partial h_{0a}}{\partial \alpha^{a}} = \frac{3}{2} \frac{\partial h_{0}}{\partial \alpha^{a}}.
\]

Then, using equation (J19), equation (J5) reduces to
\[
\Delta h_{0a} = \frac{1}{c^2} \frac{\partial^2 U}{\partial \alpha^{a} \partial t} = \frac{16\pi G}{c^4} T_{0a},
\]

where with the required accuracy
\[
T_{0a} = \rho c V_{a},
\]

and \( V_{a} = dx_{a}/dr \) is the velocity of matter.

Solution of equation (J21) satisfying the proper boundary condition is
\[
h_{0a} = -\frac{4G}{c^3} \int \rho(t, r') V_{a}(t, r') dr' + \frac{1}{c^3} \frac{\partial F}{\partial \alpha^{a} \partial t},
\]

where \( F \) is the solution of the auxiliary equation
\[
\Delta F = U = G \int \rho(t, r') \frac{dr'}{|r - r'|}.
\]

Using the relation \( \Delta r = 2r \), we find
\[
F(t, r) = G \frac{2}{c^3} \int \rho(t, r') |r - r'| dr'.
\]

Applying the continuity equation
\[
\frac{\partial}{\partial t} \rho(t, r) + \frac{\partial}{\partial r}(\rho V) = 0,
\]

where \( V \equiv V^{a} \), we obtain
\[
\frac{\partial}{\partial t} F(t, r) = \frac{G}{c} \int \frac{dr'}{|r - r'|} \frac{\partial}{\partial r} \rho(t, r')
\]
\[
= -\frac{G}{c} \int \frac{dr'}{|r - r'|} \frac{\partial}{\partial r} [\rho(t, r') \nabla V(t, r')]
\]
\[
= \frac{G}{c} \int \frac{dr'}{|r - r'|} \nabla \cdot \rho(t, r') V(t, r') \frac{\partial}{\partial r'} |r - r'|
\]
\[
= \frac{G}{c} \int \frac{dr'}{|r - r'|} \frac{V(t, r') \cdot (r - r')}{|r - r'|}.
\]

Taking derivative with respect to \( \alpha^{a} \) we have
\[
\frac{\partial^2 F(t, r)}{\partial \alpha^{a} \partial t} = -\frac{G}{c} \int \frac{dr'}{|r - r'|} \frac{\partial^2 \rho(t, r') V^{a}(t, r')}{|r - r'|}
\]
\[
+ \frac{G}{c} \int \frac{dr'}{|r - r'|} \frac{\nabla \cdot \rho(t, r') V(t, r') \cdot (r - r') (\alpha^{a} - \alpha^{a})}{|r - r'|^3}.
\]

Substituting this into equation (J22) and taking into account that \( V^{a} = -V_{a}, \alpha^{a} = -\alpha_{a} \), we finally find
\[
h_{0a} = -\frac{7G}{c^3} \int \frac{\rho(t, r') V_{a}(t, r') dr'}{|r - r'|}
\]
\[
- \frac{G}{c^3} \int \rho(t, r') \frac{[V(t, r') \cdot (r - r')] (\alpha_{a} - \alpha_{a})}{|r - r'|^3} dr'.
\]

**Step 7. Solve for \( h_{00} \) to \( O(\epsilon^2) \).**

In the gauge (J20), keeping terms of the \( O(\epsilon^2) \) order, equation (J6) reduces to
\[
\Delta h_{00} + h_{00} \Delta h_{00} - (\nabla h_{00})^2 = \frac{16\pi G}{c^4} \left( T_{00} - \frac{T}{2} f_{00} \right).
\]

Using
\[
(\nabla h_{00})^2 = \frac{1}{2} \Delta h_{00}^2 - h_{00} \Delta h_{00}
\]

we obtain
\[
\Delta \left( h_{00} - \frac{2}{c^2} h_{00}^2 \right) + \frac{2}{c^2} h_{00} \Delta h_{00} = \frac{16\pi G}{c^4} \left( T_{00} - \frac{T}{2} f_{00} \right).
\]

Substituting in the higher-order terms the known lower-order solution \( h_{00} = \frac{-2U}{c^2} \), \( \Delta h_{00} = \frac{8\pi G}{c^4} \rho \) and taking into account that \( f_{00} \approx 1 + h_{00} \), we get
\[
\Delta \left( h_{00} - \frac{2}{c^2} U^2 \right) = \frac{16\pi G}{c^4} \left( T_{00} - \frac{T}{2} \left( 1 - \frac{2U}{c^2} \right) + 2pU \right).
\]

We will use a perfect fluid as a model of matter. Then in curved space–time with the equivalent metric \( f_{ik} \) the energy–momentum tensor of matter reads
\[
T_{ik} = (\varepsilon + P) u_{ik} - P f_{ik},
\]

where \( \varepsilon = \rho c^2 (1 + \Pi) \) is the rest energy density of the fluid, \( \Pi \) is the specific density of thermal energy, \( P \) is the isotropic pressure and \( v^k = dx^k/ds \) is the four-velocity of the fluid element. Here \( ds = \sqrt{f_{ik} dx^i dx^k} \) and \( v_i = f_{ik} v^k \). Trace of \( T_{ik} \) is \( T = \varepsilon - 3P \).

With the required accuracy \( ds \approx c^2 \sqrt{1 + h_{00} - V^2/c^2} \) and, therefore,
\[
\frac{v_i^2}{f_{00} c^2 dr^2} \approx \frac{(1 + h_{00})^2}{1 + h_{00} - V^2/c^2} \approx 1 - \frac{2U}{c^2} + \frac{V^2}{c^2},
\]

which yields
\[
T_{00} \approx \varepsilon \left( 1 - \frac{2U}{c^2} + \frac{V^2}{c^2} \right).
\]

Substituting this into equation (J23) we obtain to the required accuracy
\[
\Delta \left( h_{00} - \frac{2}{c^2} U^2 \right) = \frac{8\pi G}{c^4} \left[ \rho c^2 (1 + \Pi) + 2\rho (V^2 + U) + 3P \right]. \tag{J24}
\]

Solution of equation (J24) is
\[
h_{00} = -\frac{2}{c^2} U + \frac{2}{c^4} U^2 - \frac{G}{c^4} \int [4 \rho(t, r') (V^2(t, r') + U(t, r'))
\]
\[
+ 2c^2 \rho(t, r') \Pi(t, r') + 6P(t, r')] \frac{dr'}{|r - r'|}.
\]
Steps 8 and 9. Equivalent metric and PPN parameters.

The final form for the equivalent metric is

\[ f_{00} = 1 - \frac{2}{c^2} U + \frac{2}{c^4} U^2 - 4\Phi_1 - 4\Phi_2 - 2\Phi_3 - 6\Phi_4, \]  
(J25)

\[ f_{0\alpha} = - \frac{\sigma}{c} \Phi_\alpha, \]  
(J26)

\[ f_{\alpha\beta} = \left(1 + \frac{2U}{c^2}\right)\delta_{\alpha\beta}, \]  
(J27)

where

\[ U = G \int \frac{\rho(t, r')}{|r - r'|} \, dr', \quad \sigma = \frac{G}{c^3} \int \frac{\rho(t, r') V_c(t, r')}{|r - r'|} \, dr', \]
\[ W_0 = \frac{G}{c^3} \int \frac{\rho(t, r') [(V(t, r') \cdot (r - r'))(x_0 - x_0')]}{|r - r'|} \, dr', \]
\[ \Phi_1 = \frac{G}{c^2} \int \frac{\rho(t, r') V_c^2(t, r')}{|r - r'|} \, dr', \]
\[ \Phi_2 = \frac{G}{c^2} \int \frac{\rho(t, r') U(t, r')}{|r - r'|} \, dr', \]
\[ \Phi_3 = \frac{G}{c^2} \int \frac{\rho(t, r') (V_c(t, r'))^2}{|r - r'|} \, dr', \quad \Phi_4 = \frac{G}{c^4} \int \frac{P(t, r')}{|r - r'|} \, dr'. \]

are metric potentials defined in the same way as in [2]. One should note that in [2] the Minkowski metric is chosen as \( \eta_{\alpha\beta} = \text{diag} (-1, 1, 1, 1) \) and, as a result, formulas for \( f_{00} \) and \( f_{0\alpha} \) have the opposite sign.

The metric (J25)-(J27) is written in the standard PPN gauge and, hence, the PPN parameters can be read off immediately

\[ \gamma = \beta = 1, \quad \zeta = 0, \]
\[ \alpha_1 = \alpha_2 = \alpha_3 = \zeta_1 = \zeta_2 = \zeta_3 = \zeta_4 = 0. \]

They are the same as in general relativity. Thus, vector gravity is a fully conservative theory of gravity and predicts no preferred-frame effects in the PN limit. Moreover, in appendix I we show that in vector gravity there are no preferred-frame effects in the V^2/c^2 order in the matter velocity for arbitrary large values of the gravitational potential.

References

[1] Einstein A 1915 Die Feldgleichungen der Gravitation Sitzungsber. Preuss. Akad. Wiss. 1915 844
[2] Will C M 1993 Theory and Experiment in Gravitational Physics (Cambridge: Cambridge University Press)
[3] Will C M 2006 The confrontation between general relativity and experiment Living Rev. Relativ. 9 3
[4] Poisson E and Will C M 2014 Gravity, Newtonian, Post-Newtonian, Relativistic (Cambridge: Cambridge University Press)
[5] Abbott B P et al 2016 Observation of gravitational waves from a binary black hole merger Phys. Rev. Lett. 116 061102
[6] Abbott B P et al 2016 GW151226: observation of gravitational waves from a 22-solar-mass binary black hole coalescence Phys. Rev. Lett. 116 241103
[7] Abbott B P et al 2017 GW170104: observation of a 50-solar-mass binary black hole coalescence at redshift 0.2 Phys. Rev. Lett. 118 221101
[8] Abbott B P et al 2016 Tests of general relativity with GW150914 Phys. Rev. Lett. 116 221101
[9] Abbott B P et al 2016 Binary black hole mergers in the first advanced LIGO observing run Phys. Rev. X 6 041015
[10] Riess A G et al 1998 Observational Evidence from supernovae for an accelerating universe and a cosmological constant Astron. J. 116 1009
[11] Perlmutter S et al 1999 Measurements of \( \Omega \) and \( \Lambda \) from 42 high-redshift supernovae ApJ 517 565
[12] Ade P A R et al (Planck Collaboration) 2014 Planck 2013 results: XVI. Cosmological parameters Astron. Astrophys. 571 A16
[13] Svidzinsky A A 2007 Oscillating axion bubbles as an alternative to supermassive black holes at galactic centers J. Cosmol. Astropart. Phys. 10 018
[14] Bjorken J D and Drell S D 1965 Relativistic Quantum Fields (New York: McGraw-Hill)
[15] de Broglie L 1932 Sur une analogie entre l’électron de Dirac et l’onde électromagnétique C. R. 195 536
[16] Perkins W A 2002 Quasibosons Int. J. Theor. Phys. 41 823
[17] Dvoeglazov V V 2001 Another spinor field approach to the photon Phys. Scr. 64 119
[18] Sen D K 2007 Left and right-handed neutrinos and baryon-lepton masses J. Math. Phys. 48 022304
[19] Perkins W A 2013 The antiparticles of neutral bosons J. Mod. Phys. 4 32
[20] Perkins W A 2014 Composite photon theory versus elementary photon theory J. Mod. Phys. 5 2089
[21] Scully M O and Zubairy M S 1997 Quantum Optics (Cambridge: Cambridge University Press) section 1.5.4.
[22] Hebecker A and Wetterich C 2003 Spiner gravity Phys. Lett. B 574 269
[23] Wetterich C 2004 Gravity from spinors Phys. Rev. D 70 105004
[24] Rastall P 1975 An approach to a theory of gravitation Am. J. Phys. 43 591
[25] Rastall P 1977 Conservation laws and gravitational radiation Can. J. Phys. 55 1342
[26] Svidzinsky A A 2009 Vector theory of gravity in Minkowski space-time: flat Universe without black holes arXiv:0904.3155 [gr-qc]
[27] Landau L D and Lifshitz E M 1995 The Classical Theory of Fields (Course of Theoretical Physics vol 2) (Oxford: Elsevier)
[28] Yilmaz H 1958 New approach to general relativity Phys. Scr. 111 1417
[29] Yilmaz H 1971 New theory of gravitation Phys. Rev. Lett. 27 1399
[30] Rosen N 1974 A theory of gravitation Ann. Phys. 84 455
[31] Chang D B and Johnson H H 1980 Theory of gravitation with an alternative to black holes Phys. Rev. D 21 874
[32] Chang D B and Johnson H H 1980 No black holes: a gravitational gauge theory possibility Phys. Lett. A 77 411
[33] Lindhard J 1981 Static gravitation on the basis of equivalence Phys. Scr. 23 60
[34] Martinis M and Perković N 2010 Is exponential metric a natural space–time metric of Newtonian gravity? arXiv:1009.6017 [gr-qc]
[35] Robertson S L 1999 X-ray novae, event horizons, and the exponential metric Astrophys. J. 515 365
[36] Veselago V G 1968 Electrodynamics of materials with negative index of refraction Sov. Phys.—Usp. 10 509
[37] Cohen-Tannoudji C, Dupont-Roc J and Grynberg G 1987 Photons and Atoms: Introduction to Quantum Electrodynamics (New York: Wiley)
[38] Ginzburg V L 1987 *Theoretical Physics and Astrophysics* 3rd edn (Moscow: Nauka)

[39] Ahmadi M et al 2017 Observation of the 1S–2S transition in trapped antihydrogen *Nature* **541** 506

[40] Jordan P 1935 Zur Neutrinotheorie des Lichtes *Z. Phys.* **93** 464

[41] Friedman J L and Ipser J R 1987 On the maximum mass of a neutron star rotating at high speeds *Astrophys. J.* **314** 594

[42] Kalogera V and Baym G 1996 The maximum mass of a neutron star *Astrophys. J.* **470** L61

[43] Brecher K and Caporaso G 1976 Obese neutron stars *Nature* **259** 377

[44] Glendenning N K 1997 *Compact Stars: Nuclear Physics, Particle Physics, and General Relativity* (New York: Springer)

[45] Casares J and Jonker P G 2014 Mass measurements of stellar and intermediate-mass black holes *Space Sci. Rev.* **183** 223–52

[46] Ozel F, Psaltis D, Narayan R and McClintock J E 2010 The black hole mass distribution in the Galaxy *Astrophys. J.* **725** 1918

[47] Farr W M, Sravan N, Cantrell A, Kreidberg L, Bailyn C D, Mandel I and Kalogera V 2011 The mass distribution of stellar-mass black holes in active galaxies *Astrophys. J.* **740** 101

[48] Greene J E and Ho L C 2007 A new sample of low-mass black holes in active galaxies *Astrophys. J.* **670** 92

[49] Caporaso G and Brecher K 1977 Neutron-star mass limit in the highest angular resolutions *Proc. IAU Symp. #205 (United Kingdom)* ed R T Schlizzi p 28 (2001IAUS..205...28F)

[50] Shen Z Q, Lo K Y, Liang M C, Ho P T P and Zhao J H 2005 A size of ~1 AU for the radio source Sgr A' at the centre of the Milky Way *Nature* **438** 62

[51] Huang L, Cai M, Shen Z-Q and Yuan F 2007 Black hole shadow image and variability analysis of Sgr A' *Mon. Not. R. Astron. Soc.* **379** 833

[52] Doelman S S, Fish V L, Broderick A E, Loeb A and Rogers A E 2009 Detecting flaring structures in Sagittarius A' with high-frequency VLBI *Astrophys. J.* **695** 59

[53] Akiyama K et al 2015 230 GHz VLBI observations of M87: event-horizon-scale structure during an enhanced very-high-energy γ-ray state in 2012 *Astrophys. J.* **807** 150

[54] Doelman S S et al 2008 Event-scale structure in the supermassive black hole candidate at the Galactic Centre *Nature* **455** 78

[55] Arvanitaki A and Geraci A 2014 Resonantly detecting axion-mediated forces with nuclear magnetic resonance *Phys. Rev. Lett.* **113** 161801

[56] Rybka G, Wagner A, Patel K, Percival R and Ramos K 2015 Search for dark matter axions with the Orpheus experiment *Phys. Rev. D* **91** 011701(R)

[57] Günel Y and Tinto M 1989 Near optimal solution to the inverse problem for gravitational-wave bursts *Phys. Rev. D* **40** 3884

[58] Cavalier F, Barsuglia M, Bizouard M-A, Brisson V, Clapson A-C, Davier M, Hello P, Kreckelbergh S, Leroy N and Varvella M 2006 Reconstruction of source location in a network of gravitational wave interferometric detectors *Phys. Rev. D* **74** 082004

[59] Markowitz J, Zanolin M, Cadonati L and Katsavounidis E 2008 Gravitational-wave burst source direction estimation using time and amplitude information *Phys. Rev. D* **78** 122003

[60] Searle A, Sutton P J, Tinto M and Woan G 2008 Robust Bayesian detection of unmodelled bursts *Class. Quantum Grav.* **25** 114038

[61] Searle A, Sutton P and Tinto M 2009 Bayesian detection of unmodelled bursts of gravitational waves *Class. Quantum Grav.* **26** 155017

[62] Fairhurst S 2009 Triangulation of gravitational-wave source waves with a network of detectors *New J. Phys.* **11** 123006

[63] Wen L and Chen Y 2010 Geometrical expression for the angular resolution of a network of gravitational-wave detectors *Phys. Rev. D* **81** 082001

[64] Klimenko S, Vedovato G, Drago M, Mazzolo G, Mitselmakher G, Pankow C, Prodi G, Re V, Salemi F and Yakushin I 2011 Localization of gravitational wave sources with networks of advanced detectors *Phys. Rev. D* **83** 102001

[65] Nissanka S, Sievers J, Dalal N and Holz D 2011 Localizing Compact Binary Inspiralns on the Sky Using Ground-based Gravitational Wave Interferometers *Astrophys. J.* **739** 99

[66] Croce R P, Pierro V, Postiglione F, Principe M and Pinto I M 2102 Robust gravitational wave burst detection and source localization in a network of interferometers using cross-Wigner spectra *Class. Quantum Grav.* **29** 045001

[67] Veitch J, Mandel I, Aylott B, Farr B, Raymond V, Rodriguez C, van der Sluys M, Kalogera V and Vecchio A 2012 Estimating parameters of coalescing compact binaries with proposed advanced detector networks *Phys. Rev. D* **85** 104045
[85] Grover K, Fairhurst S, Farr B F, Mandel I, Rodriguez C, Sidery T and Vecchio A 2014 Comparison of gravitational wave detector network sky localization approximations Phys. Rev. D 89 042004

[86] Kyutoku K and Seto N 2014 Pre-merger localization of eccentric compact binary coalescences with second-generation gravitational-wave detector networks Mon. Not. R. Astron. Soc. 441 1934

[87] Singer L P and Price L R 2016 Rapid Bayesian position reconstruction for gravitational-wave transients Phys. Rev. D 93 024015

[88] Abbott B P et al (LIGO Scientific Collaboration and Virgo Collaboration) 2016 Properties of the binary black hole merger GW150914 Phys. Rev. Lett. 116 241102

[89] Abbott B P et al (LIGO Scientific Collaboration and Virgo Collaboration) 2016 Prospects for observing and localizing gravitational-wave transients with Advanced LIGO and Advanced Virgo Living Rev. Relativ. 19 1

[90] Smith D R and Kroll N 2000 Negative refractive index in left-handed materials Phys. Rev. Lett. 85 2933

[91] Linde A 1990 Particle Physics and Inflationary Cosmology (New York: Harwood Academic)

[92] Reasenberg R D et al 1988 Microarcsecond optical astrometry—an instrument and its astrophysical applications Astron. J. 96 1731

[93] Logunov A A, Loskutov Y M and Mestvirishvili M A 1987 Relativistic theory of gravitation and criticism of general relativity Theor. Math. Phys. 73 1131