The Chaotic Regime of D-Term Inflation

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Motivation & Result

• Attractive possibility: GUT scale inflation (Cobe normalization), natural in hybrid inflation [Linde, ’94]

• GUT scale inflation can yield large tensor modes (BICEP2?), but then chaotic inflation [Linde, ’83]

• Is the connection between GUT scale and chaotic inflation accidental?

• Standard hybrid inflation ends in `waterfall transition’ [spinodal decomposition,..., tachyonic preheating, Felder et al ’00]

• For small couplings of inflaton to matter, `chaotic inflation’ can emerge after tachyonic preheating, as final stage of inflation
I. D-term hybrid inflation

Appealing framework for inflation with GUT scale energy density [Binetruy, Dvali '94; Halyo '94]; superpotential and Kahler potential for waterfall fields and inflaton (shift symmetry, [Yanagida et al '00]):

\[ W = \lambda \phi S_+ S_-, \]

\[ K = \frac{1}{2} (\phi + \bar{\phi})^2 + S_+ \bar{S}_+ + S_- \bar{S}_- \]

Waterfall transition below critical point

\[ \varphi_c = \frac{g}{\lambda} \sqrt{2\xi} , \]

can be trans-planckian for small Yukawa couplings \( \lambda/g \ll 1 \) (regime used in following discussion, also possible in standard D-term hybrid inflation)
Inflaton potential above critical point (waterfall fields integrated out):

\[ V(\varphi) = V_0 \left( 1 + \frac{g^2}{8\pi^2} \left( \ln x + \ldots \right) \right), \]

with \( x = \lambda^2 \varphi^2 / (2g^2 \xi) \). Below critical point scalar potential for inflaton and waterfall fields (\( M_P = 1 \)):

\[ V(\varphi, s) = \frac{g^2}{8} (s^2 - 2\xi)^2 + \frac{\lambda^2}{4} s^2 \varphi^2 + O(s^4 \varphi^2). \]

Note, potential quadratic in inflaton field (consequence of shift symmetry!) Below critical point waterfall transition (spinodal decomposition; tachyonic preheating [Felder et al '00; ...]), rapid transition to global minimum (completed after `single oscillation'). Following discussion: Hubble parameter takes over role of mass of waterfall field.

Inflation can continue after critical point [Clesse '10; ...., models with small field inflation]; here: large field inflation after critical point, leads to `chaotic inflation' after tachyonic preheating.
Beyond critical point: rapid growth of waterfall field by low momentum ($k < k_\ast \sim H$) quantum fluctuations $\langle s^2(t) \rangle$; mode equation:

$$\ddot s_k + \left( k^2 e^{-2Ht} - \frac{9}{4} H^2 - D^3 t \right) s_k = 0 ,$$

where close to critical point ($t(\varphi_c) = 0$),

$$\varphi(t) \simeq \varphi_c + \dot \varphi_c t ,$$

$$V(s; t) \simeq \frac{1}{2} g^2 \xi^2 - \frac{1}{2} D^3 t \ s^2 + \mathcal{O}(s^4; t^2) ,$$

with $D^3 = \sqrt{2\xi} g \lambda |\dot \varphi_c|$; fluctuations grow faster than exponential, yields quickly classical field; difficult problem: backreaction due to self-interaction of waterfall field.

II. Tachyonic Preheating
Growth of quantum fluctuations depends on initial velocity:

\[
\dot{\varphi}_c = -\frac{\partial \varphi V}{3H} \bigg|_{\varphi_c} = -\frac{g^2 \lambda \ln 2}{4\sqrt{3}\pi^2} \sqrt{\xi}.
\]

For typical parameters \((g^2 = 1/2, \ \lambda = 5 \times 10^{-4}, \ \sqrt{\xi} = 2.8 \times 10^{16} \text{ GeV})\):

\[
H_c \equiv H(\varphi_c) = 9.1 \times 10^{13} \text{ GeV}, \quad \dot{\varphi}_c \simeq -21H_c^2, \quad D \simeq 1.5 H_c.
\]

Estimate of growth by means of Airy functions [Asaka, WB, Covi ’01],

\[
\langle s^2(t) \rangle \simeq \int_0^{H_c} dk \frac{k^2}{2\pi^2} e^{-3H_c t} |s_k(t)|^2 ,
\]

\[
s_k(t) \simeq i\sqrt{\frac{\pi}{2D}} \text{Ai}(Dt) + \sqrt{\frac{\pi}{2D}} \text{Bi}(Dt).
\]

Growth faster than exponential in time; classical regime quickly reached; approximation only valid as long as backreaction can be neglected.
Estimate of global and local spinodal times:

\[ \langle s^2(t_{sp}) \rangle \simeq 2\xi, \quad \langle s^2(t_{sp}^{loc}) \rangle \simeq s_{\text{min}}^2(\varphi), \]

\[ s_{\text{min}}^2(\varphi) = 2g^2\xi - \frac{\lambda^2}{g^2} \varphi^2. \]
Beyond decoherence time \(|s_k(t_{\text{dec}})\pi_{sk}(t_{\text{dec}})| \equiv R_{\text{dec}} \gg \hbar \equiv 1\),

\[ t_{\text{dec}} \sim \frac{1}{D} \left( \frac{3}{4} \ln(2R_{\text{dec}}) \right)^{2/3} , \]

soft modes \((k < H)\) generate classical waterfall field. Backreaction due to self-interaction of waterfall field can then be taken into account by means of classical field equations:

\[ \ddot{\varphi} + 3H \dot{\varphi} + \frac{1}{2} \lambda^2 s^2 \varphi = 0 , \]

\[ \ddot{s} + 3H \dot{s} - \left( g^2 \xi - \frac{\lambda^2}{2} \varphi^2 \right) s + \frac{g^2}{2} s^3 = 0 . \]

Classical waterfall field, with matching

\[ s(t_{\text{dec}}) = \langle s^2(t_{\text{dec}}) \rangle^{1/2} , \]

quickly reaches local minimum; approach of global minimum much later, in final stage generation of effective mass term for inflaton.
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$100 < s^2(t) >^{1/2}$

```
without backreaction
```

```
with backreaction
```

$t_{\text{dec}}$

matching

$s(t)$

Time $t \ [1 / H_c]$
at the end of inflation standard picture: inflaton oscillations, reheating, etc...
III. Emerging regime of Chaotic Inflation

Beyond local spinodal time waterfall field tracks local minimum; asymmetric trajectory in field space:

\[ V(\varphi, s) \]

\[ s \sim M_{Pl}/10^3 \]

\[ \text{Inflaton field} \quad \varphi \sim M_{Pl} \]

\[ \text{Time} \quad t \sim 1/H_c \]

\[ \text{Number of } e\text{-folds} \quad N_e \]

22.714 22.715 22.716

0 100 200 500

0 0.5 1.0 1.5 2.0

0 5 10 15 20
During coupled slow-roll motion,

\[
\frac{\partial^2 V}{\partial s^2} \gg \frac{\partial^2 V}{\partial \varphi \partial s} \gg \frac{\partial^2 V}{\partial \varphi^2},
\]

\[
\frac{\partial^2 V}{\partial s^2} \gg H_c^2 \gg \frac{\partial^2 V}{\partial \varphi^2}.
\]

Hence, essentially one-field model of inflation, curvature perturbations dominated by quantum fluctuations of inflaton field; effective inflaton potential:

\[
V(\varphi, s_{\text{min}}(\varphi)) = \frac{1}{2} \lambda^2 \xi \varphi^2 \left( 1 - \frac{1}{2} \frac{\varphi^2}{c^2} \right).
\]

Slow-roll parameter:

\[
\epsilon = \frac{1}{2} \left( \frac{V'}{V} \right)^2 \approx \frac{2}{\varphi^2} - \frac{2}{\varphi_c^2}, \quad \eta = \frac{V''}{V} \approx \frac{2}{\varphi^2} - \frac{5}{\varphi_c^2},
\]

\[
\varphi_\epsilon \approx 2 - \frac{4}{\varphi_c^2}, \quad \varphi_\eta \approx 2 - \frac{10}{\varphi_c^2}.
\]
effective inflation potential, becomes approximately quadratic for small inflaton field values
Field value for $N$ e-folds,

$$
\varphi^2(N) \simeq (4N + 2) - \frac{4}{\varphi_c^2} \left( N^2 + N + 1 \right),
$$

yields predictions ($N_* = 60$)

$$
\varphi_* = 14.5, \quad n_s = 0.963, \quad r = 0.083.
$$

Bounds on parameters: (i) requirement of $N_* \geq 60$ beyond critical point,

$$
\lambda \lesssim 1 \times 10^{-3}, \quad \sqrt{\xi} \gtrsim 2 \times 10^{16} \text{ GeV};
$$

(ii) reliability of calculation ($s_{\text{min}}(t_{\text{dec}}) > H/(2\pi)$), yields

$$
\lambda > 1 \times 10^{-4}, \quad \sqrt{\xi} < 1 \times 10^{17} \text{ GeV}.
$$

Cosmic strings: no problem!
Conclusions

D-term inflation attractive framework for describing very early universe (GUT scale, reheating,...)

Realization of inflation depends on coupling strength of inflaton to matter (waterfall) fields; `large’ couplings lead to standard hybrid inflation

For `small’ couplings, regime of `chaotic inflation’ emerges after tachyonic preheating, as final stage of inflation

Prediction: relations between spectral index, tensor-to-scalar ratio, running of spectral index, amplitude of gravitational waves as functions of inflaton coupling strength