Questions and Remarks About Clans in Multiparticle Dynamics

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Abstract

The fact that several important effects in multiparticle dynamics, on which QCD has not yet satisfactory predictions, have been interpreted in terms of the validity of negative binominal (Pascal) regularity and related clan properties at the level of simpler substructures, raises intriguing questions on clan properties in all classes of collisions, the main one being whether clans are observable objects or merely a mathematical concept. We approach this problem by studying clan masses and rapidity distributions in each substructure for $e^+e^-$ annihilation and $hh$ collisions, and find that such properties can indeed characterise the different components. These results support the idea that clans could be observable, a challenging problem for future experiments.
1 Introduction

As well known, the concept of clan has been introduced in the eighties in order to interpret the occurrence of the approximate NB (Pascal) regularity in final charged particles multiplicity distributions (MD’s) of the full sample of events both in full phase space and in restricted rapidity intervals in all classes of collisions in the GeV region.

Clans were defined as group of particles of common ancestor with at least one particle per clan; by assumption no correlations exist among clans. Accordingly, the production process was understood as a two-step process: to independently produced clans in the first step (they are Poissonianly distributed), it follows a second step in which each clan decays into final particles with a logarithmic distribution. The average number of clans, \( \bar{N} \), in rapidity intervals at fixed c.m. energy and at various c.m. energies, as well as the average number of particles per clan, \( \bar{n}_c \), characterised fully the multiplicity distribution properties in various classes of high energy collisions. These two parameters are linked to standard NB (Pascal) distribution parameters \( \bar{n} \) (the average charged multiplicity) and \( k \) \((1/k = D^2/\bar{n}^2 - 1/\bar{n} , D \) being the dispersion of the MD) by two non-trivial relations

\[
\bar{N} = k \ln \left(1 + \frac{\bar{n}}{k}\right) \quad \text{and} \quad \bar{n}_c = \frac{\bar{n}}{\bar{N}}. \tag{1}
\]

We learned, within the just recalled elementary interpretation, that \( \bar{N} \) in \( e^+e^- \) annihilation is larger at the same c.m. energy than in hadron-hadron collisions, and that \( \bar{n}_c \) is much smaller in the former than in the latter case. In deep inelastic scattering (DIS), the situation turned out to be intermediate between the previous two: \( \bar{N} \) behaves as in hadron-hadron collisions but \( \bar{n}_c \) as in \( e^+e^- \) annihilation. In addition, in all classes of collisions, clans are larger (they contain more particles) in central rapidity intervals than in peripheral ones.

These remarkable properties of clans were obtained in a quite simple framework and suggested an interesting clan picture at parton level by using generalised local parton-hadron duality (GLPHD) . It was found that partonic clans behave as bremsstrahlung gluon jets originated by the initial quark (the dominant vertex in \( e^+e^- \) annihilation is \( q \to q + g \) ) and are generated very probably at quite low virtualities (this consideration explains the high number of clans and the relatively low population of partons [particles] per clan in this case). This interpretation should be compared with what happens very reasonably in the same picture in hadron-hadron collisions, where bremsstrahlung gluon jets are thought to be generated at quite high virtualities and to have a lot of virtuality space for generating larger partonic cascades (the process is dominated here by the gluon self-interaction vertex and stronger colour exchanges).

More accurate analyses of final charged particles MD’s at higher energies (at LEP , UA5 ) revealed violations of the regularity. A shoulder was seen in the MD’s both in full phase space and in rapidity intervals in \( e^+e^- \) annihilation and \( hh \) collisions, which were not described by a single NB(Pascal)MD. The death of the regularity was celebrated as an expected and sound fact. Experimental complexity was winning over theoretical simplicity.

A different school of thought pointed out that the NB (Pascal) regularity was not dead in multiparticle dynamics, but simply that it was working at a more fundamental level of investigation, i.e., at the level of different substructures characterising various classes
of collisions. The violation of NB (Pascal) regularity in the full sample of events in a high energy collision was considered as the indication of the existence of substructures (or different classes of events), and the suggestion was to explore the validity of the regularity in these substructures. It was shown that the regularity was surviving in the separate 2- and 3-jet samples of events in $e^+e^-$ annihilation and, presumably, in soft and semi-hard components in $hh$ collisions.

It turns out in fact that the weighted superposition of the mentioned substructures, each described by a NB(Pascal) MD, reproduces approximately three observed behaviours in final charged particles MD's in both classes of collisions: the first one is the mentioned shoulder effect in $P_n$ vs $n$ ($P_n$ is the MD); the second one are $n$-oscillations in $H_n = K_n/F_n$ vs $n$ (where $F_n$ are factorial moments, $K_n$ cumulant moments), and the third one is the general behaviour of the forward-backward multiplicity correlation strength.

The fact that three important effects on which QCD has not yet satisfactory predictions (the only claim is the onset of the hard gluon vertex) and which can be interpreted in terms of the same cause, i.e., the validity of the NB (Pascal) regularity at a more elementary level of investigation than initially thought, raises intriguing questions on clan properties in all classes of collisions, the main one being: are clans observable, or is clan concept a purely statistical one like cluster expansion in statistical mechanics? In order to approach the problem, we decided to proceed by asking ourselves the following preliminary questions.

- Are clans massive objects?
- If clans are massive, are clan masses different in different classes of events (or substructures) in a given collision?
- If clans are massive, what about clan masses in different classes of collisions?

2 First question.

A quantitative answer to the first question has been given by A.Bialas and A. Szczerba; it was stimulated by the observed qualitative properties of clan structure analysis when applied to multiplicity distributions in rapidity intervals in hadron hadron collisions for the full sample of events collected by UA5 Collaboration. Here as discussed previously, the charged particle multiplicity distributions in rapidity intervals are of course of NB (Pascal) type, $P_n^{NB}(k, \bar{n})$, with $k$ and $\bar{n}$ increasing with rapidity and with particles generated by each independently produced clan according to a logarithmic distribution, $P_n^L(\beta)$, with $\beta = \bar{n}/(\bar{n} + k)$.

Two assumptions were at the basis of the mentioned generalisation of standard clan structure analysis to rapidity intervals: they concern the distributions of clans in rapidity and the angular distribution in clan decay respectively.

As previously mentioned, clans are Poissonianly distributed and independently emitted in bremsstrahlung-like fashion. Using energy and (longitudinal) momentum conservation, the single-clan (pseudo)-rapidity density has been written as

$$
\frac{dN}{dy} = \lambda (1 - x_+)^\lambda (1 - x_-)^\lambda
$$

(2)
with 
\[ x_\pm \equiv \frac{m}{\sqrt{s}} e^{\pm y}, \]  
(3)
where \( m \) is the (average) transverse mass of the clan (\( m_T = \sqrt{m^2 + p_T^2} \), it will be called \( m \) in following), \( \lambda \) is a parameter closely related to the plateau height, that is, to the average number of clans per unit (pseudo)-rapidity \( y \), and \( \sqrt{s} \) is the c.m. energy. Notice that Eq. (2) limits clan emission to the interval \(|y| < \ln(\sqrt{s}/m)\).

Assuming furthermore that each clan produces particles according to a logarithmic MD, whose generating function is
\[ g_{\log}(z) \equiv \sum_{n=1}^{\infty} z^n P_n^L(\beta) = \frac{\ln(1-z\beta)}{\ln(1-\beta)}, \]  
(4)
with the average multiplicity per clan, \( \bar{n}_c \), given by
\[ \bar{n}_c = \frac{\beta}{(\beta - 1) \ln(1 - \beta)}, \]  
(5)
then the generating function for the MD in the (pseudo)-rapidity interval \( \Delta \eta \) turns out to be [13]
\[ G(z; \Delta \eta) = \exp \left\{ \int dN \ln \left[ \frac{1 - \beta / (1-\beta) p(y; \Delta \eta)(z - 1)}{\ln(1 - \beta)} \right] dy \right\}; \]  
(6)
\( p(y; \Delta \eta) \) is the fraction of particles, produced by a clan of (pseudo)-rapidity \( y \), falling within \( \Delta \eta \); it was also assumed that for a fixed clan multiplicity the MD of particles falling within \( \Delta \eta \) is binomial, i.e., particles emitted by each clan are emitted independently from each other. The integration is over the full range allowed by the kinematical limits of Eq. (2). The Authors of Ref. [13] assumed further, for the probability density function within a clan to produce a particle at \( \eta \), given that the clan is at \( y \), a form based on the hypothesis of isotropic decay:
\[ \phi(\eta; y) = \left[ 2 \omega \cosh^2 \left( \frac{\eta - y}{\omega} \right) \right]^{-1} \]  
(7)
(\( \omega = 1 \) gives isotropic decay if \( \eta \) is pseudo-rapidity; the width of the distribution is proportional to \( \omega \)) and thus computed
\[ p(y; \Delta \eta) = \int_{\Delta \eta} \phi(\eta; y) d\eta. \]  
(8)

There are, in summary, 4 free parameters to be used to fit the experimental data: \( \beta, \lambda, \omega \) and \( m \).

Experimental data on \( p\bar{p} \) collisions at 546 GeV are approximately reproduced with the following choice of the parameters in the generalised model of Ref. [13]
\[ \lambda = 0.855, \quad m = 3.15, \quad \omega = 1.45, \quad \beta = 0.90. \]  
(9)

The obtained multiplicity distribution is not indeed of NB type except in full phase space (deviations are significant for \( n < 3 \), and in \( k \) parameter rapidity dependence at the
Table 1: Parameters $\lambda$, $m$, $\omega$ and $\beta$, obtained from fitting Eq. (1) to data for each component at various c.m. energies and in different classes of collisions, are shown in the top part; other quantities derived from the fit parameters are shown in the bottom part.

|        | (a) $pp$ collisions 63 GeV | (b) $p\bar{p}$ collisions 900 GeV | (c) $e^+e^-$ annihilation 91 GeV |
|--------|-----------------------------|------------------------------------|----------------------------------|
| $\lambda$ | 1.14                        | 0.92                               | 1.60                             |
| $m$ (GeV/c$^2$) | 1.80                        | 1.47                               | 0.62                             |
| $\omega$ | 0.84                        | 1.95                               | 1.34                             |
| $\beta$  | 0.79                        | 0.83                               | 0.62                             |
| $dN/dy|_{y=0}$ | 1.15                        | 0.92                               | 1.57                             |
| $\bar{N}$ | 5.59                        | 9.83                               | 11.0                             |
| $\bar{n}_c$ | 2.41                        | 2.69                               | 1.64                             |

The answer to the first question is therefore positive.

### 3 Second question.

This result led us to the next question. We studied first $pp$ collisions at 63 GeV in rapidity intervals: according to our knowledge, at such c.m. energy only one component (the soft one) is usually assumed to control the dynamics of the collision and the shoulder effect is, to a good approximation, negligible.

The average number of particles per clan, the average number of clans, the dispersion, the parameter $k$ and charged particle multiplicities in pseudo-rapidity intervals $\eta_c < 2.5$ are quite well reproduced by the set of parameters shown in Table 1: they are obtained by fitting, with the least square method, the average multiplicity $\langle n \rangle$ and the quantity $D^2/\langle n \rangle^2 - 1/\langle n \rangle$ of the distribution obtained from the generating function (1), in terms of the four parameters $\lambda$, $m$, $\omega$ and $\beta$, to the corresponding moments of the NBMD (namely $\bar{n}$ and $k^{-1}$, respectively) for pseudo-rapidity intervals $\Delta \eta = [-\eta_c, \eta_c]$ with $\eta_c \leq 2.5$ (see Fig. 1).

We remark that

1. soft events only are considered in $pp$ at 63 GeV c.m. energy (in fact, one component only determines reasonably well the $n$-charged particle multiplicity distribution in rapidity...
Figure 1: Average multiplicity, dispersion, average number of clans and average number of particles per clan in different pseudo-rapidity intervals $[-y_c, y_c]$ for the soft component of the MD in $pp$ collisions at 63 GeV: open circles are data [15] and the solid line our fit.

b. the average number of particles per clan, $\bar{n}_c$, is not correctly reproduced for rapidity intervals larger than $\eta_c = 2.5$; a clear bending of $\bar{n}_c$ is visible in the data and not its increase with the rapidity interval considered as shown in the generalised model (clans in central rapidity regions are larger than in more peripheral regions at the border of phase space [1]).

This fact has important consequences on the determination of the leakage parameter introduced [11] in the study of particles generated from clans lying in one hemisphere to the opposite one in forward-backward multiplicity correlations and suggests that the leakage parameter should be larger in broader clans than in smaller ones (and not the same throughout all the allowed rapidity range as done in [11]). The proposed value of the leakage parameter in the just mentioned reference should therefore be considered rather an average value between leakage parameters of large and small clans than a rapidity independent value.

We decided then to study $p\bar{p}$ collisions at 900 GeV c.m. energy. According to our experience here not only one but two components are controlling the dynamics of the pro-
cess, i.e., the soft and the semi-hard component \([14]\). We fit therefore the four parameters \(\lambda, \omega, m, \text{ and } \beta\), with the method previously explained, for each component separately using available UA5 data \([9]\), i.e., the NB fits in pseudo-rapidity intervals \([-\eta_c, \eta_c]\) with \(\eta_c = 1 \ldots 5\); because, as discussed in \([1]\), clans emitted close to their kinematical limit \(\ln(\sqrt{s}/m)\) appear to be smaller, we do not make use of full phase space values. The fits turn out to be good, as shown in Fig. 2, except that they do not reproduce well the decrease in the average number of particles per clan close to full phase space. The resulting values of the fit parameters are shown in Table 1b. Figure 3 shows the densities \(dN/dy\), Eq. (2), and \(\phi(\eta; 0)\), Eq. (7), for the two components at 900 GeV separately, using the results of the fit.

In comparing parameters behaviour for the soft component at 63 GeV and 900 GeV we notice that clan masses and distribution widths vary with c.m. energy and the plateau height is slowly decreasing. The decrease of the clan mass in the soft component with increasing energy could be due to the intentionally overlooked contamination of semi-hard events at 63 GeV, as will be discussed in the last section.

**Figure 2:** Fit to the average multiplicity and dispersion in different pseudo-rapidity intervals \([-y_c, y_c]\) for the two components of the MD in \(p\bar{p}\) collisions at 900 GeV: soft component: open circles (data) and solid line (fit); semi-hard component: filled circles (data) and dashed line (fit). Data points are from Ref. \([9]\).
Figure 3: Clan density $dN/dy$, Eq. (2), and single particle pseudo-rapidity probability density in a clan $\phi(y; 0)$, Eq. (7), for the soft (solid line) and semi-hard (dashed line) component at 900 GeV c.m. energy; parameters from Table 1.b.

In comparing, next, parameters behaviour for the soft and semi-hard components at the same c.m. energy (900 GeV) we remark that clan masses and plateau heights are much higher in the semi-hard than in the soft component, whereas the distribution width is much higher in the soft than in the semi-hard component. This fact shows that heavier particles are produced more in semi-hard than in the soft component.

Interestingly the average number of particles per clan is bending in larger rapidity intervals both in the soft and in the semi-hard component suggesting also here that clans are larger in central rapidity intervals than in the peripheral ones.

Accordingly leakage parameters in forward backward multiplicity correlations should be larger when clans have larger masses and their particle content is distributed in more central rapidity intervals.

In conclusion clan masses on the average are different in different classes of events or substructures of a given class of collision.

4 Third question.

It is interesting to check clan properties also in $e^+e^-$ annihilation at LEP c.m. energy within the generalised model of Bialas and Szczerba.

We apply it to the 2-jet and 3-jet samples of events and we find that data are approximately reproduced with the choice of parameters shown in Table 1.

We conclude in comparing the average masses of 2- and 3-jet samples of events in $e^+e^-$ annihilation at LEP energy with the average mass of the soft component at 63 GeV in $hh$ collisions that clan masses in 2-jet and 3-jet events are much lower than in the soft component at ISR energies.

5 Remarks and questions for future experiments

Assuming the validity of NB (Pascal) regularity and its interpretation in terms of clan structure for each substructure (or component or subsample of events) characterising dif-
ferent classes of high energy collisions it was shown in previous works that the weighted superposition of NB (Pascal) MD’s (one for each component) describes quite well experimental final charged particle MD properties which a single NB (Pascal) MD is unable to reproduce. It should be pointed out that QCD has (up to now) quite poor prediction on all just mentioned experimental facts: experts claim that they are all consequences of the onset of hard gluon radiation and rely fully on purely complicated higher order perturbative calculations to be eventually performed in the future. From a stricter, and complementary, phenomenological point of view, the above mentioned successes in describing data in terms of the weighted superposition mechanism of two (or eventually more) NB (Pascal) MD’s and the related interpretation in terms of clans of the different substructures of a collision raise intriguing problems, which we summarised in the following question: are clans observable objects?

This paper is an attempt to answer this interesting question. It concerns mainly energy and rapidity dependence of average clan masses of the substructures characterising $e^+e^−$ annihilation and $hh$ collisions. Results are not inconsistent with general expectations and support the idea that clans could be indeed observable.

Some warnings are needed. Our work is based on informations and extrapolations not coming from dedicated experiments on the subject, as these are not available at present. We are therefore aware of the intrinsic limitations of our approach. For instance the separation between soft and semi-hard events at 900 GeV in $hh$ collisions comes (as already pointed out) from a fit [1] proposed in order to reproduce observed experimental data on MD’s. The question still on the carpet is indeed what is a soft and what is a semi-hard event in $hh$ collisions. Some progress has been made by CDF Collaboration [17], but a definite generally accepted answer is lacking. Similarly at 63 GeV c.m. energy (ISR) in view of the lack of a clean separation between soft and semi-hard events, all events were taken to be soft (and consequently a single NB (Pascal) MD was used for describing the full sample of events). It cannot be excluded that the relatively high average clan mass found in the soft component at 63 GeV c.m. energy ($m \approx 1.80$ GeV/$c^2$) (no semi-hard events were assumed to contribute to the full sample of events) with respect to that of the average clan mass of the soft component at 900 GeV ($m \approx 1.47$ GeV/$c^2$) (here both soft and semi-hard events [according to Fuglesang’s fit] were assumed to contribute to the full sample of events) might be the consequence of the contamination of a certain percentage of semi-hard events at 63 GeV which at present we are unable to disentangle and which will modify the calculated average clan mass of the soft component.

In addition, the separation between 2-jet and 3-jet events in $e^+e^−$ annihilation is jet-finder algorithm dependent and it might be that one should consider the 3-jet sample events separated into two extra subsamples of events (e.g., Mercedes-like events and hard-gluon events). All these remarks make of course our conclusions questionable. These consideration notwithstanding, it seems to us interesting to explore the possibility that clans do have mass. This search should be done in our opinion in future experiments in two steps.

Firstly, it is important to check with dedicated experimental work existing substructures in the full sample of events of each class of collisions and verify then that they agree with NB (Pascal) behaviour or at least with infinitely divisible distribution (IDD) behaviour (which maintains the Poissonian nature of the first step of the production process). Secondly, one should apply the analysis of the present approach to each component
in order to determine clan masses properties. In this respect it is relevant to remember
that the average masses we are referring to are average transverse masses and therefore
one should pay attention to the transverse momentum component fraction contributing
to the real average clan mass which might be disturbing in the separation between soft
and semi-hard events.

It is clear that if clans are massive and their average masses vary with energy and
rapidity the next question is: do clans have other other quantum numbers? If clans
are real objects, do they continue to be independently produced and to decay with no
correlations among particles generated by different ancestors at all c.m. energies, or there
exists a threshold beyond which clans loose their original independence, start to overlap
and then to interact among themselves, modifying the initial simplicity of clan structure
interpretation of NB (Pascal), or IDD, regularities?

Other interesting remarks and subtle questions can be added to the previous ones.
They are too premature in our opinion. The point we want to make, following the results
of this approach, is that if clans are massive objects (with eventually other quantum num-
bers) and their masses vary with c.m. energy and rapidity, and clan masses are different in
different substructures of high energy collisions, the hadronization process itself should be
consistent with the new scenario which emphasises the role of clans with respect to that of
final charged particles. The hadronization process in this perspective will be dominated
by massive clans (primaries?) whose search and characteristic properties study will be a
possible new frontier in multiparticle dynamics in all high energy collisions, a challenging
problem to experimentalists of the next generation accelerators.

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