Design and research on optimization of single point mooring system

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Abstract. In this paper, we establish a statics model to analyze the force balance of the buoys, four steel pipes and steel buckets, and we derive the relationship between the inclination angle of steel pipes and steel buckets and the draught depth and wind speed, which is solved by moment in mechanics. Next, by applying the catenary curve's equation to per unit of anchor chain, we can obtain the vertical height of anchor chain, thus the total depth of sea water can be obtained. Finally, the variable step search algorithm is used to search the draught depth, and we can obtain each parameter of the optimization result.

1. Introduction

In the past 40 years, with the development of offshore petroleum exploration and marine transportation, single-point mooring [1] technology has developed rapidly. The biggest advantage of single point mooring is move the wharf from the shore to the sea, which solves the contradiction that most of the ports in the world are narrower, shallower and smaller in scale, unable to match the development of large oil tankers and super large tankers. The reasonable design of the subsea observation network provides a guarantee for the accuracy of the measurement. The reasonable allocation of mooring system is particularly important. This paper mainly focuses on the design and research of positioning system. Domestic scholars have certain studies in this aspect. For example, Sun et al [2] mainly researched the CALM type single point mooring system. They focused on the rigidity of the mooring system, buoy displacement and anchor chain, the force of the mooring cable, length of anchor chain section and Oil tanker's bow angle. However, they didn’t consider the influence of environmental conditions.

In this paper, the factor of environment is considered in detail. According to the different mooring modes of buoys, there are two mooring modes [3]: single point mooring and multipoint mooring. Because the mooring mode of single point is convenient for layout and recovery, the moving area of buoy on the water surface is small, and the positioning of buoy is very good, so the single point mooring mode is studied. The transmission node of near shallow sea observation network consists of buoy system, mooring system and underwater acoustic communication system. According to the NDBC (National data buoy Center) [4], the buoy mooring system can be divided into three types [2] according to the different water depth in the sea area: shallow sea area, middle depth sea area and deep sea area. Since the depth of water studied is 18m, the buoy is moored by a full anchor chain system.

The buoy system of a transmission node can be reduced to a cylinder. The mooring system (as shown in Figure (1)) is make up of steel pipe, steel drum, heavy ball, electric welding anchor chain and special anchor of anti-drag. The anchor chain is composed of ordinary chain no rings. When the angle between the anchor chain and the seabed does not exceed 16 degrees [5], the measurements
good. Otherwise the anchor will be dragged away and the node will be lost. The cylindrical steel drum of sealed is equipped with underwater acoustic communication system. The steel pipe is connected to the bottom of fourth section drum and the top of the electric anchor chain. The steel drum works best in the vertical state. When the inclination angle (the angle between the steel drum and the vertical line) is no more than 5 degrees [5], it works in good condition. Otherwise, the mooring system works to be bad. A heavy ball can be hung at the link between the steel barrel and the electric anchor chain to control the inclination angle of the steel drum.

This paper takes wind, water flow and depth into consideration. The goal is to research how to determine the type of anchor chain, length and the mass of heavy ball, so as to minimize the buoy’s draft depth, the swimming area and the inclination angle of the steel pipe.

2. Nomenclature

In this paper, 0 denotes buoy, 1 to 4 denotes the first to the fourth section of steel pipe respectively, 5 denotes steel drum. Where $i, j = 0, 1, 2, 3, 4, 5$, the detailed indications as shown in Table 1.

| Symbol | Meaning | Symbol | Meaning |
|--------|---------|--------|---------|
| $F_{wind}$ | Offshore wind loads on buoys. | $\Delta l$ | The unit length of anchor chain. |
| $h_i$ | The steel anchor chain corresponding to the vertical height. | $\beta$ | The angle between action line and horizontal direction of anchor chain tension. |
| $T_i$ | The force of the first section of the steel tube to the buoy. | $H_i$ | Vertical height corresponding to the whole anchor chain. |
| $H$ | The height of buoy. | $H_{total}$ | Total seawater depth |
| $h$ | The draught depth of buoys. | $H_i$ | Vertical height corresponding to steel barrel. |
| $V_{steel}$ | The volume of seawater discharged from steel pipes. | $\gamma$ | The inclination angle of the steel barrel is in line with the scope of work. |
| $l$ | Half of the length of the steel pipe. | $m_i$ | Quality of heavy goods |
| $d$ | Buoys’ bottom diameter. | $H_i$ | Vertical height of four section steel pipe. |
| $s_b$ | The base area of buoy. | $G_i$ | The gravity of the object $i$. |
| $T_{ij}$ | The acting force between $i$ and $j$,where $(T_i = T_j)$. | $\rho_{i,j}$ | Buoyancy of object $i$ in the seawater. |
| $\theta_i$ | The angle between the force $T_i$ and the horizontal line. | $\phi_i$ | The angle that between the object $i$ and the horizontal line. |

3. Theoretical introduction

3.1. Variable step search [6] algorithm

Variable step search algorithm is a search method for solving optimization problems. Its search direction is closed to the optimal descent direction, so this method improves the work efficiency and reduces the programming workload greatly. The method of variable step search is also quite simple by MATLAB. As long as we set the step factor and input the search target, we can get the optimal result.

3.2. Moment balance

The distance from the axis of rotation to the action line of force is called the arm of force. The product of force and arm is called the moment of force on the axis of rotation. The case which the resultant force of a couple of forces acting on an object is zero is called the moment balance.
4. Design of single point mooring system

4.1. Model analysis

In order to simplify the study of the system, we present the following assumptions: 1) The system is in a good environment without considering the impact of environmental loads such as currents and waves, and there is no relaxation fracture at the joints of the various parts of the structure. 2) Because of static analysis, the influence of wave on system load is neglected. 3) Sea current is plane current, and there is no component in the vertical direction, so the analysis is simplified as a two-dimensional model problem. 4) Anchor chain is a flexible component, which does not bear shear stress and transfer torque. 5) Anchor does not enter the soil. 6) Anchor chain does not undergo elastic deformation. 7) Cable and instrumentation is untwisted and no rotation occurs. 8) Inertia force and buoys movement at sea level don’t be considered.

4.1.1. Force analysis of buoys. We suppose the direction of the X axis is water direction (unidirectional plane flow). The forces acting on the buoy include \( G_0 \), \( F_{\text{float}}^0 \), and \( T_{10} \), as shown in Figure 2. Thus, we can obtain buoy’s gravity is \( G_0 = m \times g \) and buoyancy in water \( F_{\text{float}}^0 = \rho_{\text{seawater}} g V = \rho_{\text{seawater}} g s h \), where \( g = 9.8 N/kg \). According to the balance state of buoys, there is

\[
\begin{align*}
T_{10} \cos \theta_1 &= F_{\text{wind}}^0, \\
T_{10} \sin \theta_1 + G_0 &= F_{\text{float}}^0.
\end{align*}
\]  

By solving Equation (1) static equilibrium equations, we can obtain \( T_{10} \) and \( \theta_1 \)

\[
T_{10} = \sqrt{F_{\text{wind}}^0 + (F_{\text{float}}^0 - G_0)^2},
\]

\[
\theta_1 = \arctan \frac{F_{\text{float}}^0 - G_0}{F_{\text{wind}}^0}.
\]

4.1.2. Force analysis of steel pipe. We set up the coordinate system as shown in Figure 3. We take the stress analysis of the first section of steel pipe as an example to build the model. The first section of the steel pipe subjected forces have \( G_1, F_{\text{float}}^1, T_{01} \) and \( T_{21} \).

Firstly, we can obtain the gravity of steel pipe, where \( g = 9.8 N/kg \), \( G_1 = m \times g \). In the sea, the steel pipes’ buoyancy is \( F_{\text{float}}^1 = \rho_{\text{seawater}} g V_{\text{steel}} \). We can obtain the following equation:

\[
\begin{align*}
F_{\text{float}}^1 + T_{01} \sin \theta_1 &= G_1 + T_{21} \sin \theta_2, \\
T_{01} \cos \theta_1 &= T_{21} \cos \theta_2.
\end{align*}
\]  

By solving Equation (4), we can obtain \( T_{21} \) and its horizontal angle \( \theta_2 \) as follows:
\[ T_{i2} = \sqrt{\left( F_{\text{float}} + T_{0i} \sin \theta_i - G_i \right)^2 + (T_{ni} \cos \theta_i)^2}, \quad \theta_2 = \arctan \frac{F_{\text{float}} + T_{ni} \sin \theta_i - G_i}{T_{0i} \cos \theta_i}. \] (5)

Secondly, the force arm of \( T_1 \) buoyancy \( F \) and gravity \( G \) at point \( C \) is obtained by decomposing the force \( T_1 \), and the total torque is obtained as follows: (Regulation: counter clockwise in positive direction, clockwise in negative direction.)

\[ F_{\text{float}} \cos \varphi_i - G_i \cos \varphi_i + 2I_i \sin \theta_i \cos \varphi_i - 2I_i \cos \theta_i \sin \varphi_i = 0. \]

Where, \( F_{\text{float}} \) is buoyancy of steel tubes, \( 2I \) is the length of steel pipe. Thus, \( \varphi_i \) can be written as:

\[ \varphi_i = \arctan \frac{\rho_{\text{seawater}} \frac{g T}{\rho_{\text{steel pipe}} - G_i} + 2T_0i \sin \theta_i}{2T_{0i} \cos \theta_i}. \] (6)

The force analysis of other steel pipes are the same as above. We can obtain the \( ith \), \( i = 2, 3, 4 \) section of the steel pipes as follows:

\[ T_{(i+1)i} = \sqrt{\left( F_{\text{float}} + T_{(i-1)i} \sin \theta_i - G_i \right)^2 + (T_{(i-1)i} \cos \theta_i)^2}, \]
\[ \theta_{(i+1)i} = \arctan \frac{F_{\text{float}} + T_{(i-1)i} \sin \theta_i - G_i}{T_{(i-1)i} \cos \theta_i}, \]
\[ \varphi_i = \arctan \frac{\rho_{\text{seawater}} \frac{g T}{\rho_{\text{steel pipe}} - G_i} + 2T_{(i-1)i} \sin \theta_i}{2T_{(i-1)i} \cos \theta_i}. \] (7)

Then we can obtain the relation between \( \varphi_i, h \).

**4.1.3. Force analysis of steel drum.** We assume the steel barrel subjected force are \( G_5, T_{45}, T_{45}, T_5 \) and we don’t think about the volume of heavy ball.

Based on the balance condition of moment of force, we can obtain the following equations:

\[ F_{\text{float}}^5 + T_{45} \sin \theta_5 = \left( m_5 + m_6 \right) g + T_{75} \sin \theta_7, \]
\[ T_{45} \cos \theta_5 = T_{75} \cos \theta_7, \]
\[ F_{\text{float}}^5 l_5 \cos \varphi_5 - \left( m_5 + m_6 \right) g l_5 \cos \varphi_5 + 2l_5 T_{45} \sin \theta_5 \cos \varphi_5 - 2l_5 T_{45} \cos \theta_5 \sin \varphi_5 = 0. \]

By solving these equations, we can obtain:

\[ T_{75} = \sqrt{\left( F_{\text{float}}^5 + T_{45} \sin \theta_5 - \left( m_5 + m_6 \right) g \right)^2 + \left( T_{45} \cos \theta_5 \right)^2}, \]
\[ \theta_7 = \arctan \frac{F_{\text{float}}^5 + T_{45} \sin \theta_5 - \left( m_5 + m_6 \right) g}{T_{45} \cos \theta_5}, \]
\[ \varphi_5 = \arctan \frac{F_{\text{float}}^5 - \left( m_5 + m_6 \right) g + 2T_{45} \sin \theta_5}{2T_{45} \cos \theta_5}. \] (8)

**4.1.4. Force analysis of anchor chain.** Firstly, we assume that the anchor chain is located in a vertical plane, without considering the three-dimensional deformation of the anchor chain, and the flow has no vertical component. We establish a coordinate system, as shown in Figure 4.

Then, we select a segment of anchor chain for analysis, as shown in Figure 5. The tension at both ends are \( T, \Delta T \). The anchor chain self weight is \( q \Delta l \) (\( q \) is weight per unit length of anchor chain, \( \Delta l \)
is the unit length of anchor chain, \( T \) is the component of force on anchor chain, \( a = T / q \). The catenary curve's Equation [7-9] developed by Henri O. Berteaux is adopted, we get catenary curve's equation in common condition as follows:

\[
y = a \cosh\left(\frac{x}{a}\right) + \ln(a \tan\beta - \sec\frac{x}{a})a.
\]

We can obtain the relational expression about \( \beta \) by solving Equation (9):

\[
\beta = \arctan y,
\]

Therefore, the vertical height corresponding to each unit anchor chain as follows:

\[
h_i = \Delta l \sin \beta_i (i = 1, 2, L, 210).
\]

Finally, the vertical height corresponding to the whole anchor chain is obtained according to:

\[
H_3 = \sum_{i=1}^{210} h_i = \sum_{i=1}^{210} \Delta l \sin \beta_i.
\]

**Figure 3.** Stress analysis diagram of steel pipe. **Figure 4.** Posture chart of anchor chain.

According to Equation (9) to (12), we can obtain the formula of underwater depth.

4.1.5. **Solving the underwater depth by searching method.** By simplify Equation (7) and (12), we can obtain the vertical height of the four sections of steel pipe:

\[
H_1 = \sum_{i=1}^{L} l_i \sin \phi_i.
\]

The vertical height of the steel drum can be obtained according to:

\[
H_2 = l_5 \sin \phi_5.
\]

Thus the total depth of sea water can be obtained according to:

\[
H_{total} = H_1 + H_2 + H_3.
\]

It is programmed by MATLAB, we search the scope of \( h \) by the given goal of \( H_{total} \) with approaching 18 meters but not exceed 18 meters. We can obtain:

\[
\gamma = \frac{\pi}{2} - \phi_5.
\]

Then, we can get the inclination angle of steel barrel \( \gamma \) is 7.3855, when \( v_{invalid} = 36 \text{m/s} \). In fact, the range of \( \gamma \) is limited to \([0, \pi/2]\). Next, we adjust the quality of the heavy ball to meet the requirement.

Establish a variable step search model [10]. The force analysis of steel pipe is shown in Figure 6.

The relation formula between the inclination angle \( \gamma \) and \( \phi_5 \) as follows:

\[
\tan \phi_5 = \tan \left(\frac{\pi}{2} - \gamma\right) = \cot \gamma, \tan \phi_5 = \frac{F_{float} - (m_a + m_h)g + 2T_{15} \sin \theta_5}{2T_{15} \cos \theta_5}.
\]

we can obtain the following equation:

\[
\gamma = \frac{\pi}{2} - \phi_5.
\]
\[
\gamma = \arccot \frac{F_{\text{flow}}^5 - (m_s + m_e)g + 2T_{45} \sin \theta_5}{2T_{45} \cos \theta_5},
\]

Therefore, we can obtain the relation between anchor chain and seabed:

\[
\alpha = \arctan \frac{H_5}{\sum_{i=1}^{10} \frac{h_i}{\tan \beta_i}}.
\]

It is programmed by MATLAB, we can determine the step factor \( \mu \) given the search scope \( \gamma \in [0, 5] \) and \( \alpha \in [0, 16] \). The searching target is the depth of water approaching 18 meters but not exceeding 18 meters, then we can obtain the range of heavy ball's weight.

\[ \text{Figure 5. Force analysis of anchor chain.} \]

\[ \text{Figure 6. Force analysis of steel pipe.} \]

5. Numerical results

It is assumed that the buoy system can be simplified a cylinder, its bottom diameter and height are 2 meters. The buoy’s mass is 1000 kg, and the anchor’s weight is 600 kg. There are four sections of steel pipes with a length of 1 meters and its diameter is 50 millimeters. The mass of each section is 10 kg. The length of cylindrical steel barrel is 1 meter and its diameter is 30 cm. The total mass of equipment and steel barrel are 100 kg. There are common type and parameters of near shallow sea observation network in the Table 2.

| Model | Length (mm) | Quality (kg/m) |
|-------|-------------|----------------|
| I     | 78          | 3.2            |
| II    | 105         | 7              |
| III   | 120         | 12.5           |
| IV    | 150         | 19.5           |
| V     | 180         | 28.12          |

(1) We assume that the type of transmission node is II type welding anchor chain with 22.05 meters long, weight ball is 1200 kg. The sea bed is flat, the water depth is 18 meters. The sea water density is \( 1.025 \times 10^3 \text{ kg/m}^3 \). The transmission node is laid out in it. If the sea water is static and sea surface wind speed is 12 m/s and 24 m/s respectively. It is programmed by MATLAB, we search the scope of \( h \) by the given goal of \( H_{\text{total}} \) with approaching 18 meters but not exceed 18 meters. We can calculate the inclination angle, draft depth and swimming area of the steel barrel and each steel pipe by solving Equation (2) to (15), which is shown in Table 3.

(2) Under the assumption of (1), when the sea wind speed is 36 m/s, the angle between anchor point seabed is not more than 16 degrees, and the inclination angle of steel drum is not more than 5
degrees, by changing the mass of heavy ball, we can calculate the corresponding parameters of mooring system, which is shown in the Table 4.

Table 3. Search result.

| Wind speed (meters per second) | Depth of draught (meter) | Inclination angle of steel barrel (degree) | Inclination angle of steel tube (degree) |
|-------------------------------|--------------------------|------------------------------------------|----------------------------------------|
| 12                            | 0.726                    | 88.9625                                  | 88.9979                                 |
| 24                            | 0.763                    | 86.3153                                  | 86.4209                                 |

Table 4. Results of each parameter.

| Wind speed (meters per second) | Depth of draught (meter) | Inclination angle of steel tube (degree) | Inclination angle of steel barrel (degree) |
|-------------------------------|--------------------------|------------------------------------------|----------------------------------------|
| 36                            | 1.05                     | 82.8685                                  | 82.6145                                 |

6. Conclusions

In this work, we combine the variable step-search method to the research, and we use the knowledge of physics and plane geometry. Finally, we can obtain various parameters of mooring system. However, the distortion and rotation of the cable and the inertia of the object itself are not considered in this paper, so the parameters will produce some errors in the process of modeling and analysis.

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