Wideband spectrum sensing based on modulated wideband converter with nested array

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Abstract
Several spectrum sensing systems based on sub-Nyquist sampling have been extensively studied to deal with difficulties of traditional wideband spectrum sensing in cognitive radio (CR) networks. The modulated wideband converter (MWC) is an effective application and has drawn considerably more attention over the past few years. In this study, MWC with a nested array (NA) system is proposed to improve spectrum sensing performance, which is an array sensing system with sub-Nyquist sampling. The simulations show that the connection of co-array and MWC obtains the lower minimal system sampling rate than MWC. Second, the proposed system enables frequency and power spectrum estimation with sub-Nyquist sampling for more sources than sensors. Finally, our alternative spectrum sensing system outperforms other conventional methods in terms of sensing accuracy and design complexity.

1 INTRODUCTION

Spectrum resources are a kind of non-renewable resource. With the rapid growth of human communication services, limited spectrum resources are scarce. Improving the real-time utilisation of spectrum resources is expected to be an effective way to alleviate the shortage of spectrum resources [1]. Cognitive radio (CR) [2] is an effective solution for solving spectrum scarcity issue by exploiting its sparsity, which improves real-time utilisation of target bands by accurately monitoring the frequency bands already allocated to the primary users (PUs), capturing the spectral holes of the frequency band and finally providing the opportunity for the secondary users (SUs) spectrum access. The technology of spectrum sensing can be divided into narrowband and wideband sensing technologies. In the narrowband technology of spectrum sensing, energy detection [3] is one of the most common methods because of its low implementation cost. One weakness of this technology is that it cannot distinguish the user signals from the noise well, so it has poor performance at low signal-to-noise ratio (SNR) and is unable to identify unoccupied spectral opportunities over a wide frequency band. Therefore, it is necessary to apply CRs to high-frequency fields [4] to find unoccupied spectral bands quickly. To solve the obstacle, wideband spectrum sensing technology has attracted the attention of researchers.

A few studies [5, 6] focus on wideband spectrum sensing. In [5], a multiband joint detection algorithm based on Nyquist sampling is introduced, which can be used to detect spectrum opportunities on multiple frequency bands simultaneously. In [6], wavelet-based spectrum sensing has been proposed. Since it requires an analogue-to-digital converter (ADC) running at the Nyquist sampling rate, it also has high implementation and computational complexity. Therefore, many spectrum sensing applications deal with wideband signals leading to extremely high Nyquist rates. To overcome this obstacle, several sampling architectures and recovery algorithms of signals which are beneficial from the compressed sensing (CS) theory [7] have been proposed [8–11]. The emerged theory of CS is a technology with a sampling rate of less than twice, only a small number of sampled signals data can be perfectly reconstructed to obtain the original information, thereby breaking limitation of sampling. The methods in [8] used the multicoset sub-Nyquist sampling to reconstruct multiband signals at low rates. But it requires synchronisation between time shift elements and sufficient bandwidth of commercial ADCs in case of signal distortions in time-domain. In [9], Tropp et al. proposed the random demodulator that solves the problem of wideband spectrum sensing by generating a random matrix to compress and measure signals to obtain a small amount of important measurement data. The modulated wideband converter (MWC)
sampling scheme [10, 11] is able to implement with off-the-shelf ADCs, which can reconstruct blind and sparse signals at under-sampling condition in frequency-domain. So far, sub-Nyquist-based wideband sensing technology has been widely studied in the academia and industry. In [12], the authors use CS to compress the sample vector into a smaller vector, and reconstruct the spectrum by minimising the $l_1$ norm. However, the multirate asynchronous sub-Nyquist sampling approaches relies on the noise power as prior knowledge. MWC does not have these disadvantages since it can recover signal without prior knowledge of signal or noise position. To address wireless channel fading due to the lack of space diversity exploitation in the process of spectrum sensing, the author proposes a multi-antenna system based on MWC called the multi-antenna generalised modulated converter (MAGMC) [13]. In [14], a blind sub-Nyquist spectrum sensing algorithm based on MWC sub-Nyquist sampling framework is proposed, referred to as the residual energy ratio-based detector (RERD). RERD no longer requires prior knowledge of the monitored spectrum and makes the way of performing spectrum sensing more autonomous. Unfortunately, there are still shortcomings with using MWC for wideband spectrum sensing. First, the main difficulty is choosing different periodic functions $p_0(t)$ that alternates between the levels $\pm 1$ for each channel to let their Fourier coefficients fulfil CS requirements. Next, the sampler of MWC is one sensor composed of analogue processing channels so that all sensing channels are disrupted by the same noise, which causes the whole process of spectrum sensing to be difficult to stably accomplish in practical condition. In [15], Eldar et al. suggest a sub-Nyquist sampling scheme based on the MWC system with known or identical direction of arrivals (DOAs), and they also derived a system to solve joint spectrum sensing and DOA estimation is called Compressed Carrier and DOA Estimation (CaSCADE). Another method in [16] is proposed, which applies the co-prime array (CPA) to the MWC. The CPA-based MWC can detect more targets than traditional ULA-based MWC and reduce the effect of mutual coupling of ULA. Nevertheless, it is difficult to choose the optimal number of samplers in the condition of designing equations for a large number of elements since the number of CPAs must follow the prime principle. This problem also leads to the minimum sampling rate being very high for the sub-Nyquist sampling scheme. Another obstacle is that CPA has holes in the co-array so that the ULA part of the co-array is smaller than those of the nested array (NA).

Based on the receiver architecture MWC, this study considers a spectrum sensing system that uses MWC scheme based on an NA. First, the signals go through the NA antennas. Then the received signals are multiplied by the same periodic function, then filtered by a low-pass filter (LPF) and sampled at a low rate ADC below the Nyquist rate. NA configuration consists of contiguous virtual components (no holes) that can utilise the complete correlation information when signals are more than practical sampling sensors. One advantage of NA-based MWC is the simplicity of choosing the optimal number of samplers to attain the minimum system sampling rate. Another advantage is that each sampling sensor is corrupted by uncorrelated noises so that this configuration allows for noise averaging that increases signal to noise ratio (SNR).

2 | SIGNAL AND ARRAY MODEL

Consider $K$ uncorrelated far-field narrow-band complex signals $\{x_i(t)\}_{i=1}^{K}$ impinging on the array with unknown disjoint frequencies. Each signal $x_i(t)$ is modulated by a carrier frequency $f_i \in \mathbb{R}$ and bandlimited to $B = [-1/2T, 1/2T]$, namely $\min_i |f_i - f_j| > |B|$. The received sparse wideband signal as $u(t)$ which is assumed to be bandlimited to $F = [-f_Nyq/2, f_Nyq/2]$, such that

$$u(t) = \sum_{i=1}^{K} x_i(t) e^{2\pi f_i t}.$$  (1)

We denote by $f_{Nyq}$ the Nyquist rate of $u(t)$. The Fourier transform of $u(t)$ is given by

$$U(f) = \int_{-\infty}^{\infty} u(t) e^{-2\pi f t} dt = \sum_{i=1}^{K} X_i(f - f_i),$$  (2)

where $X_i(f)$ is the Fourier transform of $x_i(t)$. The Fourier transform of $u(t)$ is zero for every $f \notin F$.

3 | NA BASED ON MWC

Consider a two-level NA consisting of two ULAs, which has $N_1$ and $N_2$ elements. The distance between elements of two ULAs is $d$ and $d_2 = (N_1 + 1)d$, respectively. The sets of sensors locations that can be defined by $S_1 = \{md, m = 1, 2, \ldots, N_1\}$ and $S_2 = \{n(N_1 + 1)d, n = 1, 2, \ldots, N_2\}$, where $d \leq \lambda$. The definition of $\lambda$ is the half-wavelength of the signals whose frequency is the highest. Therefore, there are $N_1 + N_2 = N$ sensors in total. We choose the number of sub-array elements to be $N_1 = N_2 = N/2$ for convenient discussion and assume that $N$ is even in this work. Similar arguments can be applied for the odd case as well. The structure of NA composed of $N_1 = N_2 = 3$ is illustrated in Figure 1.

Our system consists of a NA $(N_1, N_2)$ composed of $N$ sensors that have the same sensing pattern as shown in Figure 2. The work pattern of each sensor channel is like a single channel of MWC which collects signals simultaneously.
The received signal from each sensor is mixed to the baseband through a periodic mixing function \( p(t) \) whose period is \( T_p = 1/f_p \) and then filtered by a LPF with cut-off frequency \( f_s/2 \). Finally, it is sampled at a rate \( f_s \geq f_p \) below the Nyquist rate. To derive the relation about sample sequences, we define \( F_s = \{-f_s/2, f_s/2\} \) and \( F_p = \{-f_p/2, f_p/2\} \). Let \( u_n(t) \) be the received signal at the \( n \)th sensor of NA.

\[
u_n(t) = \sum_{i=1}^{K} X_i(t + \tau_n) e^{j2\pi f_p(t + \tau_n)}, \tag{3}
\]

where \( \tau_n(\theta) = \frac{d}{c} \cos(\theta) \) is the relative time advance of the \( n \)th sensor from the direction \( \theta \) with respect to the first one at the origin. \( \theta \) is the known or identical DOA and \( c \) denotes the speed of light.

Due to the narrow-band assumption, the approximation in Equation (3) can be written as

\[
u_n(t) \approx \sum_{i=1}^{K} X_i(t) e^{j2\pi f_i(t + \tau_n)}, \tag{4}
\]

The Fourier transform of the received signal \( u_n(t) \) is given by

\[
U_n(f) = \int_{-\infty}^{\infty} u_n(t) e^{-j2\pi f t} dt = \sum_{i=1}^{K} X_i(f - f_i) e^{j2\pi f_i t \tau_n}, \tag{5}
\]

where \( X_i(f) \) is the spectrum of the \( i \)th baseband signal before modulation. Then the next step is mixing the received signal with a periodic mixing function \( p(t) = \sum_{p=1}^{\infty} a_p e^{j2\pi p't/p} \) with period \( T_p = 1/f_p \) in each sensor, where \( a_p = \frac{1}{T_p} \int_{0}^{T_p} p(t) e^{-j2\pi p't/p} dt \). Therefore, the analogue multiplication \( \tilde{z}_n(t) \) is given by

\[
\tilde{z}_n(t) = u_n(t) p(t) = \sum_{i=1}^{K} X_i(t) e^{j2\pi f_i (t + \tau_n)} \sum_{l=-\infty}^{\infty} a_l e^{j2\pi l f_p t} \]

\[
= \sum_{i=1}^{K} e^{j2\pi f_p t \tau_n} \sum_{l=-\infty}^{\infty} a_l X_i(t) e^{j2\pi (f_i - f_p) t}. \tag{6}
\]

We denote the Fourier transform of \( \tilde{z}_n(t) \) by

\[
\tilde{Z}_n(f) = \int_{-\infty}^{\infty} \tilde{z}_n(t) e^{-j2\pi f t} dt
\]

\[
= \sum_{i=1}^{K} e^{j2\pi f f \tau_n} \sum_{l=-\infty}^{\infty} a_l X_i(f - f_i - l f_p)
\]

\[
= \sum_{l=-\infty}^{\infty} a_l U_n(f - l f_p). \tag{7}
\]

Therefore, the sum in Equation (7) contains at most \( \lfloor \frac{f_{Nyq}}{f_p} \rfloor \) non-zero terms since \( U(f) = 0, \forall f \neq F_p. \) [\( \lfloor \cdot \rfloor \) means to return the nearest integer towards positive infinity. Then the mixed signal \( Z_n(f) \) goes through an ideal LPF \( H(f) \) with cut-off frequency \( f_s/2 \). After filtering, we have

\[
Z_n(f) = Z_n(f) H(f)
\]

\[
= \sum_{i=1}^{K} e^{j2\pi f f \tau_n} \sum_{l=-L_0}^{L_0} a_i X_i(f - f_i - l f_p), f \in F_s \tag{8}
\]

where \( L_0 \) is chosen as the smallest integer such that the sum contains all non-zero contributions. The exact value of \( L_0 \) is calculated by \( L_0 = \lfloor \frac{f_{Nyq}}{f_p} \rfloor - 1. \)

We define \( \tilde{X}_i(f) \) as \( \sum_{l=-\infty}^{\infty} a_l X_i(f - f_i - l f_p) \) to obtain

\[
Z_n(f) = \sum_{i=1}^{K} \tilde{X}_i(f) e^{j2\pi f f \tau_n}, \tag{9}
\]

where \( \tilde{X}_i(f) \) is the output signal at baseband after modulation. \( \tilde{X}_i(f) \) is a cyclic shifted and scaled version of \( X_i(f) \) in the interval \( F_p \) as shown in Figure 3.

Next, \( \tilde{z}_n(t) \) is sampled at rate \( f_s \) to obtain the sample sequences, which is the inverse Fourier transform of \( Z_n(f) \). Equation (11) can be written in a matrix form as

\[
\mathbf{Z}(f) = \mathbf{A} \tilde{\mathbf{X}}(f), f \in F_s \tag{10}
\]

where \( \mathbf{Z}(f) = [Z_1(e^{j2\pi f/T}), ..., Z_K(e^{j2\pi f/T})]^T \), the unknown vector \( \tilde{\mathbf{X}}(f) = \{\tilde{X}_1(f), ..., \tilde{X}_K(f)\}^T \) and the matrix \( \mathbf{A} \)
is expressed as

\[ A = \begin{pmatrix} e^{i2\pi f_1 \tau_1} & \cdots & e^{i2\pi f_1 \tau_N} \\ \vdots & \ddots & \vdots \\ e^{i2\pi f_N \tau_1} & \cdots & e^{i2\pi f_N \tau_N} \end{pmatrix}. \quad (11) \]

In the time domain, Equation (12) can be written as

\[ z[\tau] = Ax[\tau], \quad \tau \in \mathbb{Z} \]

By taking noise into account, Equation (14) is modified as

\[ z[\tau] = Ax[\tau] + n[\tau], \quad \tau \in \mathbb{Z} \]

where \( A \) represents the array manifold matrix and the noise \( n[\tau] \) is assumed to be temporally and spatially white, and uncorrelated from each sensor. In this study, we choose Additive White Gaussian Noise (AWGN) which follows a normal distribution with a mean of 0 and a variance of \( \sigma_n^2 \).

4 | RECONSTRUCTION METHOD

4.1 | Carrier frequency recovery with known DOA

The covariance matrix of \( z[\tau] \) is given by

\[
R_{zz} = E\{z[\tau]z^H[\tau]\} \\
= AR_{xx}A^H + \sigma_n^2 I,
\]

where \( R_{xx} = E\{\bar{x}[\tau]\bar{x}^H[\tau]\} \) is also a covariance matrix. The meaning of \((\cdot)^H\) is conjugate transpose. Next, the equivalent covariance matrix \( \hat{R}_{xx} \) of ULA (\( N \)) matching the property of NA (\( N_1, N_2 \)) will be derived. We start by vectoring \( R_{zz} \) to get the following vector:

\[
r_{zz} = \text{vec}(R_{zz}) \\
= (A^* \circ A)r_{xx} + \sigma_n^2 \tilde{I}_n.
\]

Here, \( r_{xx} \) is vector form of \( R_{xx} \), \( \tilde{I}_n = [e_1^T, \ldots, e_N^T]^T \) and \( \circ \) denotes Khatri–Rao product. Comparing Equations (15) with (13), the vector of \( r_{zz} \) behaves like \( z[\tau] \) and the distinct rows of \( A^* \circ A \) are the manifold of NA (\( N_1, N_2 \)). Following Equation [17], we can work with the difference co-array of NA whose sensor locations are given by the distinct values \( S_N = \{nd, n = -M, \ldots, M, M = N_2(N_1 + 1) - 1\} \) instead of from the original NA.

Since the property from NA is a virtual and filled ULA, \( A^* \circ A \) is like a Vandermonde matrix with \( N^2/2 + N - 1 \) distinct rows. Hence, its rank is \( K \leq N^2/2 + N - 1 \).

To obtain the matrix of signals corresponding to this property, first, we need to construct a new matrix \( \hat{A} \) of size \( (N^2/2 + N - 1) \times K \) where \( A^* \circ A \) and \( \hat{A} \) are like a Vandermonde matrix with \( N^2/2 + N - 1 \) distinct rows and \( K \) from \( A^* \circ A \) where we have removed the repeated rows and sorted them so that the row corresponds to the sensor location \( \tilde{R}_{(N^2/2+N−1)\times1} \). This is equivalent to removing the corresponding rows from the vector \( r_{zz} \) and sorting them to get a new vector \( \tilde{r} \)

\[
\tilde{r} = \hat{A}r_{xx} + \sigma_n^2 \bar{c}',
\]

where \( \bar{c}' \in \mathbb{R}^{(N^2/2+N−1)\times1} \) is an equivalent vector which is also sorted and replaced of repeated rows. The new steering matrix

\[
\hat{A} = \begin{pmatrix} e^{i2\pi f_1 \tau_{(N^2/4+N/2−1)}} & \cdots & e^{i2\pi f_k \tau_{(N^2/4+N/2−1)}} \\ \vdots & \ddots & \vdots \\ e^{i2\pi f_N \tau_{(N^2/4+N/2−1)}} & \cdots & e^{i2\pi f_K \tau_{(N^2/4+N/2−1)}} \end{pmatrix},
\]

where \( \tau_n = \frac{dn}{\epsilon} \cos(\theta) \), \( (N^2/4+N/2−1) \leq n \leq N^2/4+N/2−1 \).

Each array sensor separation of our sensing system is defined from \( -(N^2/4+N/2−1)d \) to \( (N^2/4+N/2−1)d \). Then this filled ULA is divided into \( N^2/4+N/4+N/2 \) overlapping sub-arrays, each with \( N^2/4+N/4+N/2 \) elements. Consequently, the sensor position of \( \ell \)th sub-array is expressed as

\[
\left\{ (-\ell+1+n)d, n = 0, 1, \ldots, \frac{N^2}{4} + \frac{N}{2} - 1 \right\}.
\]

We can denote the \( \tilde{r} \) in \( \ell \)th sub-array

\[
\tilde{r}_{\ell} = \hat{A}_{\ell}r_{xx} + \sigma_n^2 \bar{c}_{\ell}',
\]

where \( \hat{A}_{\ell} \) consists of the row from \( (N^2/4+N/2−i+1)d \) to \( (N^2/4+N−\ell)d \) of \( \hat{A} \).

Consequently, \( \hat{A}_{\ell} \) can be written as

\[
\hat{A}_{\ell} = \begin{pmatrix} e^{i2\pi f_1(-\ell+i+1)d \cos(\theta)} & \cdots & e^{i2\pi f_K(-\ell+i+1)d \cos(\theta)} \\ \vdots & \ddots & \vdots \\ e^{i2\pi f_1(N^2/4+N-\ell)d \cos(\theta)} & \cdots & e^{i2\pi f_K(N^2/4+N-\ell)d \cos(\theta)} \end{pmatrix}.
\]

Equation (19) also can write as this form

\[
\tilde{r}_{\ell} = \hat{A}_{\ell}\Phi^{-1}r_{xx} + \sigma_n^2 \bar{c}'_{\ell},
\]
where diagonal matrix
\[
\Phi = \begin{pmatrix}
\frac{e^{j2\pi f_1 d \cos(\theta)}}{r} & \cdots & \frac{e^{j2\pi f_K d \cos(\theta)}}{r}
\end{pmatrix},
\]

Then we define
\[
\tilde{R}_f = \tilde{r}_f \tilde{r}_f^H.
\]

Taking the average of \(\tilde{R}_f\), we can get a spatially smoothed matrix
\[
\tilde{R}_z = \frac{1}{N^2/4 + N/2} \sum_{i=1}^{N^2/4 + N/2} \tilde{A}_i \tilde{A}_i^H + \sigma_z^2 I.
\]

In addition, \(\tilde{R}_{zz}\) can be expressed as
\[
\tilde{R}_{zz} = \tilde{R}_z^2.
\]

Therefore,
\[
\tilde{R}_{zz} = \frac{1}{\sqrt{N^2/4 + N/2}} (\tilde{A}_1 \tilde{A}_1^H + \sigma_z^2 I),
\]

where the form of \(\tilde{R}_{zz}\) is same as the signal received by a longer virtual and filled ULA of the structure NA (\(N_1, N_2\)). The definition of \(\tilde{A}\) is the diagonal matrix with covariance elements from \(K\) received signals. The proof and theorem about Equation (24) follow in Equation [18]. Finally, we can apply \(\tilde{R}_{zz}\) to estimate carrier frequencies.

Decomposing \(\tilde{R}_{zz}\) using the singular value decomposition (SVD), we write
\[
\tilde{R}_{zz} = U \sum V^H,
\]

where \(U\) is a left singular vector with size of \(\tilde{R}_{zz}\), \(\Sigma\) is a diagonal matrix with the values of elements which are permuted from large to small and \(V\) is a right singular vector.

To extract signal subspace, we get \(U_\Sigma\) as
\[
U_\Sigma = [U_1, \ldots, U_K].
\]

We define that \(U_1\) consists of first \(N^2/4 + N/2 - 2\) rows and \(U_2\) consists of last \(N^2/4 + N/2 - 2\) rows.

Finally, we perform least squares recovery [19]
\[
\Psi = U_1^* U_2.
\]

Then, the carrier frequencies can be estimated as
\[
\hat{f} = \frac{c}{2\pi d \cos(\theta)} \angle (\text{eig} (\Psi)),
\]

where \(\angle (\cdot)\) denotes the corresponding angle of its argument and \(\text{eig} (\cdot)\) denotes that eigen-decompose argument to get eigenvectors composed of eigenvalues.

### 4.2 Signal power spectrum recovery

In order to enable perfect reconstruction of the power spectrum of transmissions \(\hat{X}(f)\), we choose Equations (12) not (13) to be the recovery setting. Therefore, from Equation (9) we hold that
\[
R_z(f) = E\{\hat{X}(f)\hat{X}^H(f)\} = \begin{cases} 
0, f \in \mathcal{P}_s 
\end{cases}
\]

where \(R_z(f) = E\{\hat{X}(f)\hat{X}^H(f)\}\) and \(A\) is defined in Equation (11).

Then, the next processing steps are similar to the set of equations such as Equations (15), (16) and (19). We vectorise \(R_z(f)\) and then remove the redundancies and obtain the corresponding virtual filled ULA model
\[
\tilde{r}(f) = \tilde{A} \tilde{X}(f), \ f \in \mathcal{P}_s
\]

where \(\tilde{r}(f)\) is the vector form of \(R_z(f)\). The steering matrix \(\tilde{A}\) defined in Equation (17) can be constructed only if the carrier frequency \(f_i\) is recovered. Since \(\tilde{A}\) is constructed, the power spectrum of \(\tilde{X}(f)\) then gets by inverting the steering matrix,
\[
\tilde{\tilde{X}}(f) = \tilde{A}^\dagger \tilde{r}(f),
\]

where \([\tilde{X}(f)]_i = E\{\tilde{X}_i(f)\tilde{X}_i^H(f)\}\) and \([\tilde{\tilde{X}}(f)]_i\) is the power spectrum of \(i\)th transmission \(\tilde{X}_i(f)\). Note that the steering matrix \(\tilde{A}\) is a full rank matrix with size of \((N^2/2 + N - 1) \times K\) since the number of source signals should be less than the number of physical and virtual sensors.

Next, we need to derive the relationship between \(\tilde{X}_i(f)\) and \(X_i(f)\) so that the relationship between \([\tilde{X}(f)]_i\) and \([\tilde{\tilde{X}}(f)]_i\) can be found and then finish signal power spectrum recovery.

In the interval \(\mathcal{P}_p\), the sampled output \(\tilde{X}_i(f)\) is given by
\[
\tilde{X}_i(f) = \sum_{j=-L_0}^{L_0} a_jX_i(f - jf_p - l f_p), \ f \in \mathcal{P}_p
\]

If \(a_j \neq 0\), \(-L_0 \leq l \leq L_0\) and \(f' \in \mathcal{B} \subseteq \mathcal{P}_p\), we can write
\[
\tilde{X}_i(f') = a_{f'}X_i(f' - f_i - l' f_p),
\]

where \(a_{f'} = \lfloor rac{f_f + f_f + f_f^2}{f_p} \rfloor\) and \(l'\) is the index of one of the two \(f_p\)-bins. \(\lfloor \cdot \rfloor\) means to return the nearest integer towards negative infinity.
From Equation (34), $[\tilde{r}_X(f')]$, can be expressed as

$$[\tilde{r}_X(f')]_i \overset{\Delta}{=} E \{ X_i(f') X_i^H(f') \}$$

$$= E \{ a_p X_i(f' - f_i - l' f_p) (a_p X_i(f' - f_i - l' f_p))^H \}$$

$$= a_p^2 E \{ X_i(f' - f_i - l' f_p) X_i^H(f' - f_i - l' f_p) \}$$

$$= a_p^2 [r_X(f' - f_i - l' f_p)],$$  \(i \in \mathbb{T}_p\) \(35\)

where $[r_X(f)]_i \overset{\Delta}{=} E \{ X_i(f) X_i^H(f) \}$ and $[r_X(f' - f_i - l' f_p)]$, denotes $i$th element of the vector $r_X(f' - f_i - l' f_p)$ with size of $K \times 1$.

Finally, after a change of variables, we have

$$[r_X(f')]_i = \frac{1}{a_p^2} [\tilde{r}_X(f' - f_i - l' f_p)], \quad f' \in \mathbb{T}_p$$  \(36\)

The power spectrum $r_X(f')$ can be reconstructed perfectly from sub-Nyquist samples if $f_i \geq f_p \geq B$.

5 | SIMULATION AND DISCUSSION

5.1 | Comparison with other schemes related to MWC

Although NA-, CPA-[16], ULA-based system [15] and MWC [10, 11] all can sample the multiband signals below the Nyquist rate, they have different properties. First, the sampler of MWC is one sensor that consists of $N_1 + N_2$ channels to finish analogue processing. However, the NA- and ULA-based system use $N_1 + N_2$ sensors and each sensor works like one channel. Accordingly, all the MWC channels are impacted by the same antenna noise but array-based sampling system has uncorrelated antenna noise. Then, a practical obstacle of the MWC is choosing realisable periodic functions $\{\rho_n(t)\}$ in hardware condition. The NA-, CPA- and ULA-based system allow all sensors to use the same mixing function $\rho(t)$, making the task easier.

The NA-based MWC composed of $N = N_1 + N_2$ physical sensors can provide $O(N^2)$ degrees of freedom while CPA only can provide $O(N_1 N_2)$ in same physical sensors condition. Therefore, the maximum detectable number of sources of our proposed system is more than CPA-based MWC. In other words, NA-based system is better at detecting more signal sources than physical sensors. Since NA-based system has more detection elements by constructing a longer-filled ULA through sensing in the condition of same physical sensors, it has better robustness to noise than CPA- and ULA-based system.

5.2 | Simulation results

In the NA-based system, an NA with $N_1 = 3$ and $N_2 = 3$ is deployed. Therefore, the number of physical sensors is $N_1 + N_2 = 6$ in total and the equivalent-filled ULA with $2N_2(N_1 + 1) = 23$ elements whose positions are given by the set $S_N = \{-11d, -10d, \ldots, 0d, \ldots, 10d, 11d\}$. The received signals from each physical sensor are corrupted by uncorrelated AWGN. The AWGN of MWC is from one sensor and distributed for each channel. To set similar conditions for comparison, the number of physical sensors in ULA-based MWC is same as that of NA-based system. In all simulations, we choose $f_p = 1.3B$ to satisfy $f_i \geq f_p \geq B$ which is the reconstruction condition. The number of snapshots $Q = 400$ is also the same parameter in each simulation.

In the first scenario, we compare the performance of the proposed NA-based system with the ULAs-, CPA-based system and MWC. The code of MWC can be found in [20]. Consider a multiband signal model containing three uncorrelated signals with $K = 3$, $\theta = 0^\circ$ and $B = 50$MHz. Since the carriers $f_i$ can be chosen uniformly at random in $[-\frac{f_{Nyq}}{2}, \frac{f_{Nyq}}{2}]$ with $f_{Nyq} = 10$GHz, we assume carrier frequency $\tilde{f} = [-1, 1, 2]$ GHz in this case. To evaluate the sensing accuracy of the proposed system, we define the root mean square error (RMSE) by

$$RMSE = \frac{1}{f_{Nyq}} \sqrt{\frac{1}{K} \sum_{k=1}^{K} (\hat{f}_k - f_k)^2}$$  \(37\)

where $\hat{f}_k$ and $f_k$ are the estimated frequency and actual frequency of $k$th transmission, respectively. A total of 500 Monte Carlo simulations are used in each scenario. The SNR is from $-10$ to $10$ dB with step size 2 dB. The simulation result is depicted in Figure 4. Since the number of sources is less than that of physical sensors, all methods can detect all the sources and successfully finish spectrum sensing. It can be observed that the proposed system outperforms the ULA-based system because the NA can produce an equivalent virtual and filled ULA that has much larger aperture than the ULA-based scheme and has reduced mutual coupling compared to ULAs. In addition, the
sensing accuracy of MWC is constrained by the mixing functions $p_n(t)$ and worse than the remaining methods. NA-based MWC system does not have such limitation and is better than other methods under different SNR conditions except non-compressed method. With increasing SNR, the RMSE of our proposed system is gradually becoming more closely related to the non-compressed method, especially in high SNR regimes.

In the second scenario, we test the RMSE performance of our sensing system and ULA-based system under different number of signals conditions. The formula for RMSE is defined in Equation (37). The proposed system can identify signals up to $N^2/4 + N/2 - 1 = 11$ when there are six physical sensors. The practical sampling channels of the CPA-based MWC are eight in same physical sensors condition since the degrees of freedom of NA are more than CPA in some conditions [21]. In the ULA-based system, the minimal number of sampling channels is $K + 1$ when there are $K$ uncorrelated signal sources [15]. The SNR is 0 dB. The number of signals $K$ is between 1 and 11. Figure 5 shows that the proposed system is superior to other systems and also has a good performance of RMSE when there are more signals than the physical sensors. This is because the NA-based system has larger apertures than other sensing systems when they have the same number of physical sensors, as shown in Figure 6. In the NA-based MWC system, the optimal number of virtual apertures is $N^2/4 + N/2 - 1$. Figure 6, the number of virtual apertures of our method is larger than that of others methods with the augment of the number of physical sensors $N$. Accordingly, this property allows our system to accommodate more signal sources and has better robustness to noise.

In the third scenario, the success probability of NA-based system is shown in Figure 7. The success probability is defined as the number of Monte Carlo simulation with RMSE < 5%, divided by the number of simulations. In this example, we test the probability of success under different numbers of signals. The setup of simulation is same as most parts of the second scenario. Consider a multiband signal model containing $K$ uncorrelated signal sources, the number of which is between 1 and 11. The number of physical sensors in this test is 6. The number of Monte Carlo simulation is 500 as before. In Figure 7, the success probability decreased significantly when $K > 9$, indicating that virtual units from co-array are not as stable as physical sensors in this condition. Therefore, the number of sources no longer needs to highly approximate maximal detectable number of signal sources if signals are needed perfect reconstruction.

In the last scenario, minimal sampling rate of different sensing systems including NA-, CPA- and ULA-based system is taken into consideration. We define the minimum sampling rate $f_{ms}$ by $f_{ms} = N^* \times f_s$, where $N^*$ is the number of sampling channels and $f_s$ is sampling rate of each channel. As mentioned above, the only limitation of $f_s$ is $f_s \geq f_p \geq B$. In the ULA-based system, the minimum sampling rate is $f_{ms} = (K - 1) \times f_s$, referencing to the second scenario. In our derived system, the minimum sampling rate can be obtained by finding the
optimal solution $N_{op}$ from the limitation $N^2/4 + N/2 - 1 > K$. Therefore, the minimum sampling rate of NA-based MWC is $f_{ms} = N_{op} \times f_s$. Next, we can obtain the minimal sampling rate of CPA-based MWC is $f_{ms} = (2N_{op}^1 + N_{op}^2 - 1) \times f_s$, where $N_{op}^1$ and $N_{op}^2$ is the optimal solution of $\gcd(N_1, N_2) = 1$ and $N_1 \times N_2 + N_1 > K$ referring to [21, 22]. We choose the number of signals $K \in [10, 100]$ to complete the experiment. The optimal number $N_{op}$ can be calculated through the mathematic formulas shown above. The simulation result is depicted in Figure 8. It is evident that our proposed system has a lower minimum sampling rate, especially $K$, which is a large number.

6 CONCLUSION

In this study, a NA-based MWC array system is considered for wideband spectrum sensing using sub-Nyquist sampling to enhance the sensing performance. The basic procedure of the proposed system involves array sensors sampling of signals, followed by carrier frequency estimation and power spectrum detection. The key point of the proposed wideband spectrum sensing system is the NA processing. The sampler of our proposed system is an NA that can construct a longer virtual array and acquire a larger virtual aperture to detect more signal sources than sensors and achieve better sensing performance. Comparing with other methods, our proposed configuration can reduce the minimal sampling rate of system and complexity to design and implement. Simulation results confirm that the proposed system can achieve better sensing performance at a lower sampling rate with fewer sensors.

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