An integrated Smart Grid system has been developed for matching the production and consumption of electric power based on a prediction of changes in the battery capacity. Advanced decisions on the change in power transmission capacity have made it possible to regulate voltage in the distribution system by maintaining the power factor of the photoelectric charging station. Voltages at the input to the hybrid inverter and in the distribution system were measured to assess their ratio. Comprehensive mathematical and logical modeling of the photoelectric charging station was performed based on the mathematical substantiation of architecture and operation maintenance. A dynamic subsystem including such components as mains, a photoelectric module, a hybrid inverter, batteries, a two-way counter Smart Meter and a charger formed the basis of the proposed technological system. Time constants and coefficients of mathematical models of dynamics in terms of estimation of changes in the battery capacity and power factor of the photoelectric charging station were determined. A functional estimate of changes in the battery capacity and power factor of the photoelectric charging station was obtained. Maintenance of voltage in the distribution system was realized based on resulting operation data to estimate a change in the battery capacity. Advanced decision-making has made it possible to raise the power factor of the photoelectric charging station up to 40 % due to matching the electric power production and consumption. Maintenance of operation of the photoelectric charging station using the developed Smart Grid technology has enabled prevention of peak loading of the power system due to a 20 % reduction of power consumption from the network.

Keywords: photoelectric charging station, rechargeable battery, hybrid inverter, two-way counter Smart Meter

1. Introduction

Distributed generation of electric energy using renewable sources requires connection to Smart Grid technologies for integration into the electric network. For example, study [1] based on predicting voltage changes in a storage battery is devoted to connection to Smart Grid technologies. A technology of maintaining change in the capacity of a storage battery during the measurement of electrolyte temperature in a set of accumulators was presented. The use of an integrated system of estimating a change in voltage based on matching electrochemical and diffusion processes of charge and discharge enables making advanced decisions on boosting to prevent impermissible overcharge and discharge. Study [2] tackles forecasting change in the battery capacity for connection to Smart Grid technologies. It presents an integrated system of maintaining the operation of a wind-solar electrical system. Making advanced decisions on changing the power of a thermodlectric accumulator is based on establishing a ratio between the voltage measured at the input to a hybrid charge controller and at an inverter output when measuring frequency. Up to 30 % reduction of thermodlectric accumulator charge time was provided based on a change of the number of turns of a circulating pump motor to change flow rate and temperature of heated water.

There are devices for charging electric vehicles which differ from each other by the type of current used and charging time. For example, Mode 3 of a charging station operation using alternating current makes it possible to charge electric cars of medium power in 4 hours using a 10-kW charger. Fast charge in Mode 4 using direct current restores the capacity of electric car batteries to 80 % in half an hour. A serious complication occurring in the use of AC charging stations consists of a risk of peak loading of mains. In conditions of the growing number of electric vehicles and irregularity of charge, there is a need to build charging stations using renewable energy sources.

Maintaining of power factor of the photoelectric charging station as regards redistribution of produced and consumed electricity is an urgent problem of further development of Smart Grid technologies. To this end, it is necessary to predict changes in battery capacity and power factor of the photoelectric charging station when measuring the input voltage of a hybrid inverter and voltage in the distribution system to assess their ratio. Advanced decisions concerning change in the level of power transmission to the electric network make it possible to adjust the voltage in the distribution system to maintain the balance of active and reactive power without the use of additional devices. The power factor of photoelectric charging stations is maintained by coordination of production and consumption of electric energy. This enables the prevention of peak loads in the electric network in conditions of satisfaction of growing consumer demands.
2. Literature review and problem statement

Optimization of distributed power generation conventionally requires the improvement of intelligent control systems for both production and consumption of electricity. Loading of distributed generation of energy consumes active and reactive powers which form a total power. Active power as a ratio of active power to total power is estimated by the power factor called cosφ where φ is the angle of phase shift between current and voltage. Active power is directed at the provision of efficiency while reactive power is a measure of energy exchange between a generator and an inductive load. Reactive power is directed at creating magnetic fields. Its absence would render impossible the operation of the inductive load. To estimate the reactive power, \( \tan\phi \) is used which is related to the active power as follows: \( \cos\phi = 1/\sqrt{1+\tan^2\phi} \).

There must be a balance between the generation and consumption of active and reactive power in the electric network. While frequency in the power system is the main indicator of active power maintenance, the voltage in the distribution network is the indicator of reactive power maintenance. A method of calculating the electrical load of civilian facilities was proposed in [3]. In contrast to the definition according to present-day regulations, its novelty consists in improving the accuracy of the calculation. Its essence consists in modeling schedules of loading the power receivers, their synthesis at inputs of civilian facilities. Primary information for modeling is based on earlier measurement of parameters of loading schedules and modes of operation of similar power supply systems. The well-known VVO concept makes it possible to change power consumption based on voltage regulation in the distribution system using a change in reactive power. An intelligent converter was proposed in [4] for voltage regulation in distribution networks by absorbing or supplying reactive power (Var) to or from the network using the control function Volt–Var. This paper investigates capacitive (i.e., Var-injection) and inductive (i.e., Var-absorption) effects of using an intelligent inverter and its ability to influence voltage at the distribution level. When the intelligent converter inputs reactive power, this increases the distribution voltage. Conversely, the voltage gets smaller when the intelligent inverter absorbs the reactive power. A VVO optimization model for prioritizing sensitivity to data changes based on accurate measurement was presented in [5] to improve programs of response and power consumption.

The use of special compensating devices at the level of consumption, such as synchronous compensators, intelligent inverters, etc. which are able to both generate and absorb reactive power, is an indispensable component to compensate for voltage changes. The cost of installing and maintaining these devices can be quite high.

So, results of intelligent control of distributed energy generation based on large-scale seasonal heat storage (ATES) are presented in [6]. The disadvantage of this study consists in the need for spatial planning in connection with the use of construction technology to create ATES. Moreover, exchange with information on dynamic control between ATES systems does not link the use of storage to the estimation of change in power factor. Results of the introduction of an algorithm of stochastic optimization of distributed generation of electric energy with the use of fuzzy logic were presented in [7]. A link between loading of the electrical system and operating costs with flexibility of distributed generation control was established. A limit level of electric power generation using a utility network as a virtual storage was proposed to maintain flexibility of management. However, the design and management strategy at which the results presented in this paper were aimed do not enable expanding the level of distributed power generation.

Known methods of optimizing charging stations are based on both economic and environmental optimization criteria and improvement of power production and consumption management. Satisfaction of consumer demands and obtaining profits by the service providers are the main components of expanding the number of charging stations. Optimization of a charging station integrated into a distribution network with the use of renewable energy sources was presented in [8]. A target function has been developed. Its minimization is based on the sum of the costs of charging electric vehicles from an external network and the costs associated with service delays. Optimization of a charging station connected to the net and using renewable energy sources does not include estimation of power production and consumption in a ratio. The need to develop infrastructure in a connection with establishing the optimal location of charging stations was stated in [9]. An agent-oriented approach based on a genetic algorithm was presented. The proposed multi-agent system takes into account the data of activities in social networks and information on mobility to establish optimal configurations but does not determine the energy aspects of coordination of power production and consumption. A strategy of hybrid power production and charging electric cars was developed in [10] on the basis of an integrated stochastic model of planning. Queue optimization and planning volume of power generated from non-renewable and renewable energy sources taking into account the non-stability of solar radiation were used. Namely, instability of solar radiation that requires a dynamic assessment of voltage changes in the distribution system was not taken into account. A model of multipurpose optimization of a charging station based on the theory of fuzzy numbers was proposed in [11]. An algorithm of a swarm of particles was presented to determine the optimal operation of charging stations with a possibility of testing the model in real operating conditions but without coordination of power production and consumption. Optimization of the charging station operation based on mixed integer programming was presented in [12]. It was solved in a form of a diagram for the day ahead. The purpose of the proposed approach implies maximizing the profit of the charging station owner while satisfying power consumers based on data of charge and discharge and battery replacement during the day. This model takes into account the arrival of customers, changes in the price of electricity from the net, restrictions on connection to the net, and self-destruction of batteries. The very restriction on the net connection does not satisfy consumers because coordination of power production and consumption has not been fulfilled. The study [13] addresses the improvement of power consumption management. A technology of fast charge with direct current based on dynamic estimation of a hybrid station was offered. A decrease in a peak load on the power system during periods of electric vehicle charge and an increase in battery life due to more controlled coordination of the discharge/charge have been established. Battery capacity change was not evaluated and controlled charge/charge uses measurements of charge and discharge voltage without providing prediction of battery capacity change. It was proposed in [14] to increase the capacity of batteries and shift.
the time of charge of electric vehicles to avoid peak loads on the net. The problem of determining how it is possible to reconcile the increase in charge power with the peak demand remains unresolved. Thus, even an increase in battery capacity does not make it possible to reconcile the production and consumption of electric power because the change in battery capacity is not estimated.

It was proposed in [15] to ensure the stability of the power net with the use of additional accumulation and wind energy. The search for maximum power with a variable step which is applied to both the photoelectric and wind part of the station was used. Moreover, it was proposed to use an auxiliary power source with control of redistribution of produced and consumed energy. The system produces additional electricity when the output of photoelectric and wind energy is less than that required for charging. The system electrolyzer produces hydrogen by absorbing additional electrical power available in the system when the production of photoelectric and wind energy exceeds the charging requirement. Thus, an additional energy system acts as a storage tank adjusting the charge power according to energy consumption. The use of an additional power source and an additional storage capacity relates to the lack of assessment of change in the battery capacity as an integral part of the charging station. In conditions of distributed generation of electric energy, the operation of the battery capacity as an integral part of the circuit design of charging stations becomes fundamental in terms of voltage regulation in the distribution system. The neural model of predicting changes in parameters of the electrical system based on distributed parameters [16] does not estimate the change in battery capacity in terms of matching power production and consumption. The presented analysis of literature allows us to assess optimization of the charging stations based on economic and environmental principles of connection to renewable energy sources. Control of electricity consumption and production is carried out with the use of additional devices for voltage regulation in the distribution system, increasing the capacity of batteries, or inclusion of additional storage devices which requires additional costs. The rechargeable battery as a mandatory element of the technological scheme of the photoelectric charging station can become the main element of voltage regulation in the distribution system. This is possible if changes in its capacity are predicted. In this case, the battery becomes the basis for redistribution of electric energy between the network and the photoelectric module, i.e. it becomes a voltage regulator in the distribution system. Moreover, the assessment of change in the battery capacity makes it possible to maintain the power factor of the photoelectric charging station. Therefore, it was proposed to measure the voltage at the input to the hybrid inverter and in the distribution system to assess their ratio. Making advanced decisions to change the level of power transmission to the network will enable the regulation of voltage in the distribution system to match the production and consumption of energy. The above substantiates the need for further studies in this area.

3. The aim and objectives of the study

The study objective is to develop the Smart Grid technology to maintain the operation of a photoelectric (AC) station of charging electric vehicles. This will make it possible to maintain voltage in the distribution system based on a prediction of changes in the battery capacity and power factor of the photoelectric charging station.

To achieve this objective, the following tasks were set:

– offer voltage maintenance in a distribution system based on predicting a change in the battery capacity and power factor of the photoelectric charging station. A change in the ratio of measured voltages at the input to the hybrid inverter and in the distribution system must be estimated;
– construct a block diagram and perform comprehensive mathematical modeling to obtain a reference estimate of the change in the battery capacity and power factor of the photoelectric charging station;
– offer making of advanced decisions on the change in the level of power supply to the network to maintain voltage in the distribution system. To this end, construct a block diagram and perform logical modeling to obtain a functional estimate of the change in the battery capacity and power factor of the photoelectric charging station;
– work out a block diagram and perform logical modeling to obtain an integrated Smart Grid system for maintaining the operation of the photoelectric charging station at the decision-making level;
– ensure matching of power production and consumption on the basis of forecasting changes in the battery capacity and power factor of the photoelectric charging station to maintain voltage in the distribution system.

4. Materials and methods used in the study

Architecture and mathematical substantiation of the photoelectric charging station were offered based on the methodological and mathematical substantiation of the architecture of technological systems [1, 2]. (Fig. 1)

![Fig. 1. Architecture and mathematical substantiation of the architecture of the photoelectric charging station where RB is a rechargeable battery; Smart Meter is a two-way counter of changes in the level of power transmission to the network; 1 — the charging unit; 2 — the discharging unit; 3 — the unit of assessing the functional efficiency](image-url)
A photoelectric charging station is a dynamic system. Its operation involves the reproduction of changes in external and internal effects and initial conditions, such as changes in solar radiation, power consumption for charging electric vehicles, a voltage in the distribution system, etc. Therefore, when designing a photoelectric charging station, an integrated dynamic subsystem is laid down in its base (Fig. 1). The integrated dynamic subsystem includes the following components: mains, photoelectric solar panels, a hybrid inverter, rechargeable batteries, a two-way Smart Meter counter and a charger. When representing the system design as the organization of a complex system, it was expanded by building up the dynamic subsystem blocks that forecast the process components around its base. Other components of the technological system include the units of charge and discharge and functional efficiency estimation in a coordinated interaction with the dynamic subsystem (Fig. 1).

In Fig. 1:
- \( PHCHS(t) \): the photoelectric charging station; \( t \) – time, s;
- \( D(t) \) – integrated dynamic subsystem (mains, a photoelectric module, a hybrid inverter, a rechargeable battery, a two-way Smart Meter counter and a charger);
- \( P(t) \) – properties of the photoelectric charging station elements; \( x \): effects (changes in solar radiation, power consumption for charging electric vehicles, voltage in the distribution system, etc.);
- \( f(t) \) – measured parameters (voltage at the input to the hybrid inverter, voltage in the distribution system);
- \( K(t) \) – coefficients of mathematical description of dynamics of change in the capacity of the storage battery, power factor of the photoelectric charging station;
- \( y(t, z) \) – predicted output parameters (battery capacity, power factor of the photoelectric charging station);
- \( z \) – coordinate of the length of the battery plates, m;
- \( d(t) \) – dynamic parameters (battery capacity, power factor of the photoelectric charging station);
- \( F(t) \) – summary functional data;
- \( LC(t) \) – logical relations for performance control, identification of the photoelectric charging station state, respectively;
- \( R(t) \) – logical relations in \( PHCHS(t) \) concerning confirmation of the correctness of the decisions made by the units of the photoelectric charging station.

Indices: \( i \) – number of blocks in the photoelectric charging station; \( 0, 1, 2 \) – initial stationary mode, external and internal nature of effects.

Proceeding from the system-structural substantiation of the architecture of the technological systems \([1, 2]\), the relation category is considered as organizing interactions within elements of the dynamic subsystem and units of the technological system. This provides an opportunity to monitor performance and identify the state of dynamic subsystems and confirm new operating conditions from the units of the technological system on the basis of the developed method of the cause-effect graph (Fig. 2).

For example, the \( CT_i \) unit evaluates the change in initial operating conditions caused by the appearance of effects, e.g. changes in solar radiation, power consumption relative to the charge of electric vehicles, voltage in the distribution system, etc. Next, using a chain of cause-effect relations when the previous assessment of the event is the cause of the next one, a final information message is obtained from the \( CT_f \) unit that makes it possible to make decisions on the process maintenance, such as change of power transmission to the net and power consumption for charging electric vehicles. After appropriate decision-making, new conditions of operation of the power system are confirmed using the second part of the graph of the cause-effect relations with respect to the parameters assessed according to the first part of the graph. Decision-making and verification of correctness of decisions made based on logical connections within the dynamic subsystem are final if the dynamic subsystem receives confirmatory estimates of the correctness of decision-making from relevant units.

Mathematical substantiation of maintenance of the photoelectric charging station operation (2), (Fig. 3) was offered based on the methodology of mathematical description of dynamics of power systems and the method of the graph of cause-effect relations (Fig. 2). Mathematical description of the architecture of the photoelectric charging station (1) is the basis of the proposed substantiation (Fig. 1). Prediction of changes in the battery capacity and the power factor of the photoelectric charging station enables making advanced decisions to change the level of power transmission to the network in order to maintain voltage in the distribution system. The change in the ratio of voltage at the input to the hybrid inverter and voltage in the distribution system is assessed.

In Fig. 3:
- \( SOPHCHS(t) \) – the Smart Grid maintenance of operation of the photoelectric charging station; \( t \) – time, s;
- \( D(t) \) – the integrated dynamic subsystem (mains, a photoelectric module, a hybrid inverter, a rechargeable battery, a two-way counter Smart Meter, a charger);
- \( P(t) \) – properties of \( SOPHCHS \) elements;
- \( MM(t) \) – mathematical modeling of dynamics as regards estimation of change in the battery capacity and power factor of the photoelectric charging station;
- \( AI(t) \) – reference information;
- \( C(t) \) – workability control;
- \( MD(t) \) – decision making;
- \( S(t) \) – state identification;
Fig. 3. Mathematical substantiation of the Smart Grid to maintain the operation of the photoelectric charging station

- \( \text{LC}(t) \), \( \text{LMD}(t) \), \( \text{LS}(t) \) – logical relations in \( C(t) \), \( \text{MD}(t) \), \( \text{S}(t) \), respectively;
- \( f(t) \) – resulting functional information;
- \( NC(t) \) – new operating conditions;
- \( x(t) \) – effects (change in solar radiation, power consumption relative to the charge of electric vehicles, voltage change in the distribution system, etc.);
- \( y(t) \) – measured parameters (voltage at the output of the photoelectric module, voltage in the distribution system);
- \( K(t) \) – coefficients of mathematical description of dynamics of changes in the battery capacity, power factor of the photoelectric charging station;
- \( y(t \ z) \) – output parameters, such as analytical assessment of changes in the battery capacity and the power factor of the photoelectric charging station;
- \( d(t) \) – dynamic parameters of estimating changes in the battery capacity and power factor of the photoelectric charging station;
- \( z \) – coordinate of the length of the battery plates, m;
- \( T_h \) – time constants that characterize thermal storage capacity of electrolyte and metal, s;

Indices: \( i \) – number of \( \text{SOPCHS}(t) \) units (the units of charge, discharge, functional efficiency evaluation); 0, 1, 2 – initial mode, external and internal nature of effects.

Mathematical description (2) (Fig. 2, 3) and mathematical substantiation of the architecture of the technological system (1) (Fig. 1) enables maintenance of operation of the photoelectric charging station using the following actions:

- operability control (\( \text{CIDS}(t) \)) of the integrated dynamic subsystem based on mathematical (\( \text{MMIDS}(t, z) \)) and logical (\( \text{LCIDS}(t) \)) modeling to obtain a reference (\( \text{MIDS}(t) \)) assessment of changes in the battery capacity and power factor of the photoelectric charging station;
- performance control (\( \text{CIDS}(t) \)) of the integrated dynamic subsystem based on mathematical (\( \text{MMIDS}(t, z) \)) and logical (\( \text{LCIDS}(t) \)) modeling to obtain functional (\( \text{FIIDS}(t) \)) assessment of changes in the battery capacity and power factor of the photoelectric charging station;
- decision making (\( \text{MDIDS}(t) \)) using functional information (\( \text{FIIDS}(t) \)) obtained on the basis of logical modeling (\( \text{LMDIDS}(t) \));
- decision making to change the level of power transmission to the network to maintain the power factor of the photoelectric charging station (\( \text{FIIIDS}(t) \));
- identification (\( \text{SIDS}(t) \)) of new operating conditions of the photoelectric charging station \( \text{NCF}(t) \) based on logical modeling (\( \text{LSIDS}(t) \)) as part of the integrated dynamic subsystem and confirmation of new operating conditions based on logical modeling (\( \text{RIDS}(t) \)) from the system blocks.

Prediction of changes in the storage battery capacity and power factor of the photoelectric charging station was offered to be made according to formulas (1), (2) (Fig. 1–3).

The voltage at the input to the hybrid inverter and voltage in the distribution system are measured to assess their ratio.

Transfer functions by channels: “battery capacity – voltage at the input to the hybrid inverter”, “power factor of the photoelectric charging station – voltage in the distribution system” are presented as follows:

\[
W_{\text{CE} \to \text{LI}} = \frac{K_{\text{CE}} K}{(T + 1)\beta - 1} (1 - e^{-\beta}),
\]

\[
W_{\text{PF} \to \text{LI}} = \frac{K_{\text{PF}} K}{(T + 1)\beta - 1} (1 - e^{-\beta}),
\]

where

\[
K_{\text{CE}} = \frac{I_U}{U_1 - U_2}; \quad K_{\text{PF}} = \frac{I_1 (U_1 - U_2)}{N},
\]

\[
\begin{align*}
K & = \frac{n(\theta_a - \sigma)}{G} ; \quad L = \frac{G C e}{a h_0} \\
T_e & = \frac{G C e}{a h_0} ; \quad \beta = \frac{T_e S + \varepsilon + 1}{1 - L} \\
T_a & = \frac{G C e}{a h_0} ; \quad \varepsilon = (1 - L) \\
\gamma & = \frac{(T + 1)\beta - 1}{\beta} ; \quad \xi = \frac{z}{L},
\end{align*}
\]

where \( CE \) is the battery capacity, Ah; \( PF \) – power factor of the photoelectric charging station; \( I_1, I_2 \) – currents at the input of the hybrid inverter, in the distribution network, respectively, A; \( U_1, U_2 \) – voltages at the input to the hybrid inverter and in the distribution system, respectively, V; \( N \) – the power of the photoelectric charging station, kW; \( G \) – specific heat, kJ/(kg \cdot K); \( \alpha \) – heat exchange coefficient, kW/(m² \cdot K); \( G \) – consumption of substance, kg/s; \( \sigma \) – specific weight of a substance, kg/m³; \( h \) – specific surface area, m²/m; \( \theta_a, \theta \) – the temperature of electrolyte at the battery output and at the distribution wall, respectively, K; \( z \) – the coordinate of the length of the battery plates, m; \( T_e, T_m \) – time constants that characterize thermal storage capacity of electrolyte and metal, s; \( n \) – dependence of the heat exchange coefficient on the flow rate; \( \varepsilon \) – time, s; \( S \) – the Laplace transform parameter; \( S = \omega j \); \( \omega \) – frequency, 1/s.

Indices: \( 0 \) – output stationary mode; \( e \) – electrolyte; \( m \) – metal wall.

Transfer functions by channels: “battery capacity – voltage at the input of the hybrid inverter”, “power factor of the photoelectric charging station – voltage in the distribution system” were obtained by solving a system of nonlinear differential equations using the Laplace transform. The systems of differential equations include an equation of state as an estimate of the physical model of the photoelectric charging station, an equation of energy of the battery charge and discharge, an equation of heat balance for the wall of the
battery plates. The equation of charge and discharge energies was composed with the representation of a change in electrolyte temperature in pores of the plates and above the plates both in time and along the spatial coordinate of the battery plates. The transfer functions include coefficients \(K_{ce}, K_{pf}\) which estimate change in the battery capacity and the power factor of the photoelectric charging station. When analyzing the obtained mathematical model, internal parameters to be diagnosed as being a part of coefficients of the equations of dynamics \(K_{ce}, K_{pf}\) were established. In real conditions of operation of the power system at a transition from stationary states and under external and internal effects, reorganization of the coefficients of equations of dynamics in time occurs because of a change in the diagnosed internal parameters.

A real part of the transfer functions was selected:

\[
O(\omega) = \frac{(L_A + t_e)^2}{(A_o + \frac{1}{A_e})},
\]

(5)

The coefficient \(K\) includes the temperature of the dividing wall \(\theta\):

\[
\theta = (\alpha_{1} (\sigma_1 + \sigma_2) / 2) + (A (t_1 + t_2) / (\alpha_1 + A)),
\]

(6)

where \(\sigma_1, \sigma_2\) – temperature of electrolyte at the inlet and outlet of the battery, \(K\); respectively; \(t_1, t_2\) – temperature of electrolyte in pores of the plates and above the plates at the battery inlet and outlet, respectively; \(K, \alpha\) – heat exchange coefficient, kW/(m\(^2\)K). Index \(e\) – electrolyte.

\[
A = 1 / (B_n / \lambda_n + 1 / \alpha),
\]

(7)

where \(\delta\) – the battery plate wall thickness; \(\lambda\) – thermal conductivity of metal of the battery plate, kW/(m\(^2\)K). Indi ces: \(m\) – a metal wall of the battery plate.

To use the real part \(O(\omega)\), the following coefficients were obtained:

\[
A_e = e - T_e \omega^2;
\]

(8)

\[
A = e + 1;
\]

(9)

\[
B_e = T_e \omega^2 + T_e \omega + T_e \omega;
\]

(10)

\[
B = T_e \omega;
\]

(11)

\[
C = \frac{B_e - A_e B}{A + B_e^2};
\]

(12)

\[
D = \frac{A_e B - A_e B}{A + B_e^2};
\]

(13)

\[
L = 1 - e^{-L} \cos(-\xi D);
\]

(14)

\[
M = -e^{-L} \sin(-\xi D).
\]

(15)

The transfer functions (3), (4) obtained by applying the method of operators for solving the system of nonlinear differential equations hold the Laplace transform parameter, \(S(S=\omega_0)\), where \(\omega_0\) is frequency, 1/s. For a transition from the frequency domain to the time domain, the real part (5) which was obtained by mathematical processing of transfer functions was marked out. Namely, this part belongs to integrals (16), (17). This makes it possible to obtain dynamic characteristics of change in the battery capacity and power factor of the photoelectric charging station with the use of inverse Fourier transform.

\[
CE(t) = \frac{1}{2\pi} \int_{0}^{\infty} K_{ce} K_{PF} \sin(\omega t) d\omega,
\]

(16)

\[
PF(t) = \frac{1}{2\pi} \int_{0}^{\infty} K_{ce} K_{PF} \sin(\omega t) d\omega,
\]

(17)

where \(CE\) – the battery capacity, Ah; \(PF\) – power factor of the photoelectric charging station.

5. The results obtained in studying maintenance of operation of the photoelectric charging station

5.1. Reference assessment of change in the battery capacity and power factor of the photoelectric charging station

According to the proposed block diagram (Fig. 4), Tables 1–3 present results of comprehensive mathematical modeling of the photoelectric charging station.

| Operation levels | \(N_e\), kW | \(U_1\), V | \(U_2\), V | \(m\) |
|------------------|-------------|-------------|-------------|------|
| Level 1          | 17.3        | 520         | 400         | 0.58 |
| Level 2          | 21.3        | 640         | 400         | 0.71 |
| Level 3          | 25.3        | 760         | 400         | 0.84 |
| Level 4          | 30.0        | 900         | 400         | 1    |

Note: \(N_e\) – the power of photoelectric charging station; \(U_1, U_2\) – voltages at the input to the hybrid inverter and in the distribution system, respectively; \(V, m\) – power level of the photoelectric charging station.

| Operation levels | \(a_{h}\), W/(m\(^2\)K) | \(a_{h}\), W/(m\(^2\)K) | \(k\), W/(m\(^2\)K) |
|------------------|--------------------------|--------------------------|-----------------------|
| Charge           | 15.298                   | 15.232                   | 3.359                 |
| Discharge        | 14.67.56                 | 13.552.5                 | 0.973                 |

Note: \(a_{h}\) – coefficient of heat transfer from the electrolyte to a wall of the battery plate when charged, W/(m\(^2\)K); \(a_{h}\) – coefficient of heat transfer from the wall of the battery plate to the electrolyte when discharged, W/(m\(^2\)K); \(k\) – coefficient of heat exchange, W/(m\(^2\)K).

| Operation levels | \(T_e\), s | \(T_e\), s | \(\epsilon\) | \(\zeta\) | \(L, m\) |
|------------------|------------|------------|-------------|------------|-------|
| Charge           | 1467.56    | 13352.5    | 0.973       | 0.647      | 35.51 |
| Discharge        | 1466.94    | 13346.86   | 0.973       | 0.647      | 35.51 |

Time constants and the coefficients that are components of mathematical models of dynamics of the photoelectric charging station.

In Table 3 were obtained based on the parameters of heat exchange for charge and discharge of the battery presented in Table 2.
5.2. Functional assessment of changes in the battery capacity and power factor of the photoelectric charging station

A block diagram of maintaining the operability of the technological system was developed (Fig. 5) for maintenance of operation of the photoelectric charging station based on the proposed mathematical substantiation of the Smart Grid (1)–(4).

Control of operability of the photoelectric charging station (Fig. 5) provides an opportunity to obtain summarized data on making advanced decisions to maintain voltage in the distribution system.

Fig. 4. Block diagram of comprehensive mathematical modeling of the photoelectric charging station: \( N_e \), \( N_{ec} \) — the power of photoelectric module and charge of electric cars, respectively, kW; \( CE \) — battery capacity, Ah; \( U_1 \), \( U_2 \) — voltage at the input to the hybrid inverter and in the distribution system, respectively, V; \( n \) — the number of photoelectric panels; \( m \) — the power level of the photoelectric charging station

Fig. 5. Block diagram of the photoelectric charging station functioning control: \( U_1 \), \( U_2 \) — voltages at the input to the hybrid inverter and in the distribution system, respectively, V; \( CE \) — battery capacity, Ah; \( KF \) — power factor of the photoelectric charging station; \( CT \) — event control; \( Z \) — logical relations; \( d \) — dynamic parameters; \( x \) — effects; \( f \) — parameters measured; \( y \) — parameters predicted; \( K \) — coefficients of mathematical description; \( t \) — time. Indices: \( c \) — control of operability; \( ceup \), \( ccll \) — constant calculated value of the parameter of upper, lower levels of operation, respectively; \( cccl \) — constant calculated value of the level of operation parameter; \( 0 \), \( 1 \), \( 2 \) — initial stationary mode, external, internal influences; \( 3 \) — coefficients of dynamics equations; \( 4 \) — significant predicted parameters; \( 5 \) — dynamic parameters
5.3. Maintaining of voltage in the distribution system based on a prediction of changes in the battery capacity

Based on the proposed mathematical substantiation of the Smart Grid (1)–(4), a block diagram (Fig. 6) of maintenance of the photoelectric charging station has been developed based on maintaining the distribution system voltage.

Voltage maintenance in the distribution system (Fig. 6) makes it possible to ensure the operation of the photoelectric charging station.

5.4. The Smart Grid system of maintaining the operation of the photoelectric charging station at the decision-making level

A comprehensive integrated system has been developed (Table 4) for maintaining the operation of the photoelectric charging station based on a prediction of changes in the battery capacity and power factor of the photoelectric charging station. Advanced decisions on change in the level of transmission of electrical energy to the network make it possible to maintain voltage in the distribution system through maintaining the power factor of the photoelectric charging station. Continuous measurement of the voltage at the input to the hybrid inverter and in the distribution system takes place to assess their ratio.

The integrated Smart Grid system of maintenance of operation of the photoelectric charging station (Table 4) provides an opportunity to coordinate electric power production and consumption.

### Table 4

| Time, $10^3$ s | Change in parameters | $\Delta CE(t)/\Delta E(t)$ | $CE(t), \text{Ah}$ | $\Delta PF(t)/\Delta PF(t)_1$ | $PF(t)$ |
|---------------|---------------------|---------------------------|-------------------|-------------------------------|--------|
| 0             | Charge – discharge $U_1=520 \text{ V}; U_2=400 \text{ V}; N_e=17.3 \text{ kW}; m=0.58$ | 1 | 600 | 0.2400 | 0.6760 |
| 3             | Charge – discharge $U_1=544 \text{ V}; U_2=395 \text{ V}; N_e=18.1 \text{ kW}; m=0.6$ | 0.8425 | 576.38 | 0.30 | 0.7000 |
| 6             | Charge – discharge $U_1=568 \text{ V}; U_2=390 \text{ V}; N_e=18.9 \text{ kW}; m=0.63$ | 0.7364 | 560.47 | 0.3560 | 0.7224 |
| 9             | Charge – discharge $U_1=592 \text{ V}; U_2=383 \text{ V}; N_e=19.7 \text{ kW}; m=0.66$ | 0.66 | 549.01 | 0.4140 | 0.7456 |
| 12            | Charge – discharge $U_1=616 \text{ V}; U_2=380 \text{ V}; N_e=20.5 \text{ kW}; m=0.68$ | 0.6023 | 540.35 | 0.4720 | 0.7688 |
| 15            | Decision making $m=0.71; U_1=640 \text{ V}; U_2=400 \text{ V}; N_e=21.3 \text{ kW}$ | 0.6154 | 542.31 | 0.48 | 0.7720 |
| 18            | Charge – discharge $U_1=664 \text{ V}; U_2=395 \text{ V}; N_e=22.1 \text{ kW}; m=0.74$ | 0.5969 | 535.45 | 0.5380 | 0.7952 |
| 21            | Charge – discharge $U_1=688 \text{ V}; U_2=390 \text{ V}; N_e=22.9 \text{ kW}; m=0.76$ | 0.5328 | 529.93 | 0.5960 | 0.8184 |
| 24            | Charge – discharge $U_1=712 \text{ V}; U_2=385 \text{ V}; N_e=23.7 \text{ kW}; m=0.79$ | 0.5025 | 525.39 | 0.6540 | 0.8416 |
| 27            | Charge – discharge $U_1=736 \text{ V}; U_2=380 \text{ V}; N_e=24.5 \text{ kW}; m=0.82$ | 0.4771 | 518.58 | 0.7120 | 0.8648 |
| 30            | Decision making $m=0.84; U_1=760 \text{ V}; U_2=400 \text{ V}; N_e=25.3 \text{ kW}$ | 0.4872 | 520.085 | 0.72 | 0.8860 |
| 33            | Charge – discharge $U_1=784 \text{ V}; U_2=395 \text{ V}; N_e=26.1 \text{ kW}; m=0.87$ | 0.4450 | 518.25 | 0.7779 | 0.8912 |
| 36            | Charge – discharge $U_1=810 \text{ V}; U_2=390 \text{ V}; N_e=27 \text{ kW}; m=0.9$ | 0.4286 | 516 | 0.84 | 0.9160 |
| 39            | Charge – discharge $U_1=834 \text{ V}; U_2=385 \text{ V}; N_e=27.8 \text{ kW}; m=0.93$ | 0.4142 | 513.84 | 0.8980 | 0.9392 |
| 42            | Charge – discharge $U_1=858 \text{ V}; U_2=380 \text{ V}; N_e=28.6 \text{ kW}; m=0.95$ | 0.4154 | 514.02 | 0.9559 | 0.9624 |
| 45            | Decision making $m=1; U_1=900 \text{ V}; U_2=400 \text{ V}; N_e=30 \text{ kW}$ | 0.4341 | 511.22 | 1 | 0.98 |

Note: $U_1, U_2$ – voltage at the input to the hybrid inverter and in the distribution system, respectively; $V$; $CE$ – battery capacity, Ah; $KF$ – power factor of the photoelectric charging station; $N_e$ – power of the photoelectric module, kW; $m$ – level of power of the photoelectric charging station; $\tau$ – time, s. Indices: $i$ – number of operation levels; $e$ – reference value of the parameter; $1, 2$ – constant calculated value of the parameter of lower and upper levels of operation, respectively.
5.5. Coordination of electric power production and consumption based on voltage maintenance in the distribution system

The battery capacity at a specified time point was determined as follows:

\[
CE_{i+1}(\tau) = CE_i - \left( \frac{\Delta CE_i(\tau)}{\Delta CE_{cc}(\tau)} - \frac{\Delta CE_{i-2}(\tau)}{\Delta CE_{cc}(\tau)} \right) (CE_2 - CE_1),
\]

where \( CE \) – the battery capacity, Ah; \( CE_i, CE_2 \) – initial and final values of the battery capacity, Ah; \( \tau \) – time, s. Indices: cc – constant calculated value of the parameter of lower level of operation; \( i \) – the number of levels of operation of the photoelectric charging station.

The power factor of the photoelectric charging station at the set time is determined as follows:

\[
PF_{i+1}(\tau) = PF_i + \left( \frac{\Delta PF_i(\tau)}{\Delta PF_{cc}(\tau)} - \frac{\Delta PF_{i-2}(\tau)}{\Delta PF_{cc}(\tau)} \right) (PF_2 - PF_1),
\]

where \( PF \) – power factor of the photoelectric charging station; \( PF_1, PF_2 \) – initial and final values of the power factor; \( \tau \) – time, s. Indices – cc – constant calculated value of the parameter of the upper level of operation; \( i \) – the number of levels of operation of the photoelectric charging station.

For example, in a period of \( 15 \times 10^3 \) s (4.17 hrs), the battery capacity was predicted to increase to the level of 542.31 Ah with voltage growth at the input to the hybrid inverter at the level of 540 V. Value of the battery capacity was determined using the formula (18) as follows (Table 4, Fig. 7):

\[
542.31\text{ Ah} = 540.35 + (0.6154 - 0.6023) (600 - 450).
\]

Performing such actions will enable maintenance of voltage in the distribution system to coordinate the production and consumption of electric power.

6. Discussion of the results obtained in studying the Smart Grid technology for maintenance of the photoelectric charging station operation

Under conditions of the growing number of electric cars and uneven consumption of electric energy, there is a necessity of building charging stations with the use of renewable energy sources. Redistribution of produced and consumed electric power takes place based on additional storage devices, large capacities of batteries, etc. Such measures require additional investments leading to higher costs of charging the electric vehicles. The battery, as a mandatory element of the photoelectric charging stations, acquires an additional status of voltage regulator in the distribution system. Therefore, it was proposed to predict changes in the battery capacity and the power factor of the photoelectric charging station. The voltage at the input to the hybrid inverter and in the distribution system was measured to assess their ratio. The established voltage ratio is a part of coefficients \( K_{ce}, K_{pf} \) of the transfer functions (3), (4) having a relation to comprehensive mathematical modeling of the technological system. The comprehensive modeling results in obtaining a reference estimate of the change in the battery capacity and the power factor of the photoelectric charging station (Fig. 4, Tables 1–3). During the operation of the photoelectric charging station, a change in the set ratio of measured voltage occurs at the input to the hybrid inverter and in the distribution system. Therefore, control of the photoelectric charging station operability was performed using the mathematical substantiation of architecture (Fig. 1), maintenance of operation of the photoelectric charging station (Fig. 2, 3) and transfer functions (3), (4). The result of logical modeling consists of the acquisition of the summary data (Fig. 5) on making advanced decisions to change the level of power transmission to the network using a logical decision making process.

Fig. 7. Maintenance of change in the battery capacity:

1–3 — the points where decisions were made to change the level of power transmission to the network

During this period, it was necessary to make an advanced decision to raise the level of power transmission to the network from 0.68 to 0.71. The voltage level in the distribution system was set at 400 V and the power factor of the photoelectric charging station was at 0.7720.

The value of the power factor in this period was determined as follows using formula (19) (Table 4, Fig. 8):

\[
0.7720 = 0.7688 + (0.48 - 0.4720) (0.98 - 0.58).
\]
model (Fig. 6). Voltage was maintained in the distribution system. For this purpose, the integrated Smart Grid system of coordination of electric power production in conditions of changing power consumption was obtained based on logical modeling (Fig. 6). For example, in the period from the beginning of the battery discharge to its connection for charging, the battery capacity has dropped by 25% (Table 4, Fig. 7) when the charge voltage at the input to the hybrid inverter has changed from 520 V to 900 V. Advanced decisions were made during this period to raise the level of power transmission to the network (Table 4, Fig. 7, 8) to maintain voltage in the distribution system and consume the battery capacity. Making advanced decisions enables an increase in the power factor of the photoelectric charging station to 40% (Table 4). Maintenance of operation of the photoelectric charging station using the developed Smart Grid technology has made it possible to prevent peak loads on the power system while reducing up to 20% consumption of electricity from the network to charge electric vehicles. Cash profit of 3485 €/year was obtained with the “green tariff” for additional electricity supply to the net. Making a cash profit for charging electric cars up to 30% has made it possible to cover the costs of the photoelectric charging station to 40% (Table 4). Maintenance of operation of the photoelectric charging station using the developed Smart Grid technology has made it possible to prevent peak loads on the power system while reducing up to 20% consumption of electricity from the network to charge electric vehicles. Cash profit of 3485 €/year was obtained with the “green tariff” for additional electricity supply to the net. Making a cash profit for charging electric cars up to 30% has made it possible to cover the costs of the photoelectric charging station to 40% (Table 4). Maintenance of operation of the photoelectric charging station was worked out and logical modeling was performed to control the operability of the photoelectric charging station according to the principle of cause-effect relations. The logic unit has components that estimate the change in voltage in the range of 520–900 V at the input of the hybrid inverter and 400–380 V in the distribution system. Changes in the battery plate wall temperature, coefficients of mathematical models of dynamics, $K_{co}$, $K_{pf}$, the battery capacity, and power factor of the photoelectric charging station were estimated according to the block diagram. Changes in dynamic parameters of the battery capacity from 600 Ah to 511.22 Ah and changes in power factor of the photoelectric charging station from 0.6760 to 0.98 were estimated. Functional estimation of changes in the storage battery capacity and power factor of the photoelectric charging station was obtained in the resultant unit of operability control.

3. It was proposed to make advanced decisions on the change in the level of power transmission to the network to maintain voltage in the distribution system. A block diagram was worked out and logical modeling was performed to control the operability of the photoelectric charging station using the developed Smart Grid technology has made it possible to prevent peak loads on the power system while reducing up to 20% consumption of electricity from the network to charge electric vehicles. Cash profit of 3485 €/year was obtained with the “green tariff” for additional electricity supply to the net. Making a cash profit for charging electric cars up to 30% has made it possible to cover the costs of the photoelectric charging station to 40% (Table 4). Maintenance of operation of the photoelectric charging station was worked out and logical modeling was performed to control the operability of the photoelectric charging station according to the principle of cause-effect relations. The logic unit has components that estimate the change in voltage in the range of 520–900 V at the input of the hybrid inverter and 400–380 V in the distribution system. Changes in the battery plate wall temperature, coefficients of mathematical models of dynamics, $K_{co}$, $K_{pf}$, the battery capacity, and power factor of the photoelectric charging station were estimated according to the block diagram. Changes in dynamic parameters of the battery capacity from 600 Ah to 511.22 Ah and changes in power factor of the photoelectric charging station from 0.6760 to 0.98 were estimated. Functional estimation of changes in the storage battery capacity and power factor of the photoelectric charging station was obtained in the resultant unit of operability control.

4. An integrated Smart Grid system of maintenance of operation of the photoelectric charging station was developed based on the elaborated block diagram of logical modeling. Maintenance for the power factor from 0.58 to 0.98 of the cogeneration system was based on a comparison of voltages at the hybrid inverter input and in the distribution system with reference values. According to the established levels of operation, reference values of voltage at the input to the hybrid inverter were 500 V, 640 V, 760 V and 900 V. Reference value of voltage in the distribution system was 400 V with a functional change in the range of 400–380 V. Coordination of electric power production and consumption was provided based on predicting the changes in the battery capacity and power factor of the photoelectric charging station. Advanced decisions to change the level of power transmission to the network have enabled regulation of voltage in the distribution system. With the charging system power of 30 kW, generation of electrical energy by the photoelectric module mounted about 32351 kWh/year. Nine medium-power electric vehicles which are charged from an 11-kW charger in 3.84 hours require 138758 kWh/year. Maintenance of operation of the photoelectric charging station using the developed Smart Grid technology has made it possible to prevent peak loading of the power system by reducing power consumption for charging electric vehicles up to 20% from 138758 kWh/year to 106349 kWh/year.

7. Conclusions

1. It was proposed to maintain voltage in the distribution system based on a prediction of changes in the battery capacity and power factor of the photoelectric charging station. The change in the ratio of voltage measured at the input to the hybrid inverter and in the distribution system was estimated. For example, when the charge voltage at the input of the hybrid inverter changed from 520 V to 900 V and the battery discharge from 600 Ah to 511.22 Ah (25%), advanced decisions were made to increase the level of power transmission to the network from 0.58 to 1. This has made it possible to maintain voltage in the distribution system by increasing the power factor of the photoelectric charging station from 0.58 to 0.98 (up to 40%).

2. A structural diagram was developed and comprehensive mathematical modeling was applied to obtain reference estimation of change in the storage battery capacity and power factor of the photoelectric charging station. Estimation of measured voltage ratio at the input to the hybrid inverter and in the distribution system is a unifying element of mathematical modeling of dynamics. Parameters of heat exchange in the battery, time constants, and coefficients of mathematical models of dynamics were determined for the established levels of operation. Reference dynamic estimates of change in the battery capacity were obtained (600 Ah, 542.31 Ah, 520.085 Ah, 511.22 Ah) and power factor of the photoelectric charging station (0.6760, 0.7720, 0.8680, 0.98) according to the set levels of operation.

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