SPECTRA OF MASER RADIATION FROM A TURBULENT, CIRCUMNUCLEAR ACCRETION DISK. II.
HIGH-VELOCITY FEATURES

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ABSTRACT

Calculations for radiative transfer in a turbulent medium are presented that emphasize the high-velocity, spectral line features due to masing at the sides of a Keplerian disk. The focus is on understanding the highly refined observational data about the 22 GHz masers in the thin, subparsec disk around the presumed massive black hole at the nucleus of the galaxy NGC 4258. Only a minimal description of velocity irregularities in the disk—we use turbulent velocities created by statistical sampling with a Kolmogorov-like power spectrum—is necessary to obtain spectra that reflect the basic properties of the observations. A small number of sharp features are produced at the sides of the disk and are dispersed over Doppler velocities of some 200 km s\(^{-1}\) as in the observations. By treating the radiation as emerging at the observed angle of 7\(^\circ\) from the average plane of the disk, we demonstrate that the flux of maser radiation from the high-velocity features and from the center of the disk are compatible with the premise that the physical properties of the front and the sides of the disk are statistically equivalent. Spectra computed with rms velocities of turbulence that are equal to the speed of sound are in satisfactory agreement with the observed spectra when the 7\(^\circ\) angle is included in the calculations. A limited effort is reported to examine the changes in the observed spectra with time that result from the Keplerian rotation.

 Subject headings: accretion, accretion disks — black hole physics — galaxies: individual (NGC 4258) — galaxies: nuclei — masers — turbulence

1. INTRODUCTION

In our initial investigation (Wallin, Watson, & Wyld 1998a, hereafter Paper I), we began to examine how irregularities in the velocities of the gas might contribute to creating the “clumpy” spectra of line radiation that are often observed at radio and other wavelengths. There is reason to believe that irregularities in velocities caused by turbulence, convection, waves, etc., occur widely in diffuse astrophysical environments. In Paper I, velocity fields that represent turbulence were created by standard methods involving statistical sampling with Kolmogorov-like power spectra and Gaussian amplitudes. These velocity fields were combined in Paper I with the Keplerian velocities of rotation to describe certain maser emission from the nearly edge-on, thin disk around the presumed black hole at the nucleus of the galaxy NGC 4258. Because of the uniquely refined observational description of this disk (Watson & Wallin 1994; Miyoshi et al. 1995; Moran et al. 1995), which makes possible a close comparison with calculations, it serves as the focus of our investigations. To compare with these observations, spectra were computed for the 22 GHz maser radiation that is observed near the systemic velocity of that galaxy. This radiation propagates close to the plane and nearly radially from near the center of the plane of this disk as it is projected on the sky. Turbulent velocities of some magnitude are strongly indicated for accretion disks by the need for viscous dissipation to allow the transport of accreting matter through the disk. Despite the simplifying idealizations that were made to calculate this central (or systemic) emission in Paper I, the resulting spectra are similar to what is observed. The calculations demonstrate, for example, that narrow spectral features with breadths comparable to the thermal Doppler velocity are prevalent even though the rms turbulent velocities can be an order of magnitude greater than the thermal velocities. They also relate the radial extent of the masing gas, the magnitude of the turbulent velocity, and to some degree the amplification by the maser to observational properties such as the overall Doppler width of the systemic emission, the deviations of the velocities and locations of the features from that given by the Keplerian relationship, and the appearance of the spectra. Turbulent velocities have also been shown to lead to clumpy spectra and images similar to what is observed for certain masers in molecular clouds (Sobolev, Wallin, & Watson 1998) and, including polarization properties, for certain circumstellar masers (Wiebe & Watson 1998). The early investigation of Deguchi (1982) demonstrated that even simpler assumptions about irregularities in the velocities lead to clumpy appearances for astrophysical masers.

In the calculations of this paper, the focus is on the masing features that arise from the sides of the Keplerian disk in NGC 4258 at “high” Doppler velocities (\(\pm 1000\) km s\(^{-1}\)) relative to the systemic velocity of that galaxy. Unlike the features close to the systemic velocity that are so numerous that they are nearly continuous in space and in Doppler velocity, the maser emission from the sides of the disk consists of only a small number of narrow features spread over some 200 km s\(^{-1}\) in Doppler velocities and emerges from a region comparable in linear extent with the radius of the disk. To compute the spectra of the high-velocity features, care with the numerics is needed that is somewhat different from what is required for the nearly continuous spectra near the systemic velocity. The results of our computations presented in this paper demonstrate that modest irregularities in the velocities lead naturally to high-velocity masing features that are similar to the observed features for plausible values of the relevant parameters. They also show that the emission features at both the center and the sides of the disk can be obtained for the same values of the parameters that describe the masing medium—mainly, the rate for the
pumping, the rms turbulent velocity and the radial extent of the masing gas. With its small number of well-spaced features, the emission at the sides emphasizes that clumpiness is important for the maser radiation. At the same time, the location and Doppler velocities of this emission indicates that it arises mainly along the radius of the disk that is perpendicular to the line of sight, as would be expected when the optical depths calculated for a uniform Keplerian disk determine the emission (e.g., Watson & Wallin 1994). The observed “triplet” nature of the spectrum for the disk as a whole also follows in the latter interpretation. Our calculations demonstrate how these seemingly conflicting considerations of clumpiness and smoothness combine to create the observed pattern of the emission. Examining the various appearances that can occur for disk spectra as a result of nothing more than statistical variations offers guidance for recognizing disks from observational data. Preliminary reports of certain aspects of our investigation have been presented previously (Wallin, Watson, & Wyld 1998b; Watson, Wallin, & Wyld 1998).

In §2 the rays are treated as entirely in the plane of the disk when computing spectra for the high-velocity maser emission from the side of the disk. For the masing disk in NGC 4258, the viewing angle measured from the plane of the disk is about 7°. This is reflected in §3 where computations are performed for the emission that is due to rays that are inclined to the plane of the disk by this angle of 7°. Here, we demonstrate that the emission from the center and from the sides of the disk can be created with the same values for the relevant parameters. However, the appearance of spectra computed for rays that are in the plane of the disk seems to agree better with the observations for the side of the disk in NGC 4258. The observational data can be interpreted as evidence that warps are present in the sides of this disk. These may create a plane for the emission in which the line of sight is effectively “in the plane” for the region from which the radiation is observed. We retain the idealization of Paper I that the pumping is a smooth function of radius. Although this is primarily a simplification, reasoning has been advanced in Paper I that it is plausible. The pumping may not be uniform because, for example, the optimal conditions for pumping may be a result of the localized dissipation of turbulent energy. To some degree the velocities of these regions will reflect the turbulence of the medium. Our calculations thus are still likely to provide an indication of how chance coincidences in the Doppler velocities of these regions can occur and create the bright masing features. Finally, all radiation is calculated in the unsaturated limit. Evidence was provided in Paper I that this approximation should be satisfactory for the central emission from NGC 4258. In §5 we will discuss why it may also be satisfactory for the emission from the sides of the disk. The methods for creating the turbulent velocity fields and for computing the emission of the maser radiation have been described in detail in Paper I and that description will only be summarized here. Efforts have been made to develop approximate ways to treat the effects of saturation in an irregular medium, but so far the results have not been satisfactory. Spectra are presented at a sequence of times in §4 to examine the changes that result in the emission from both the center and the sides of the disk due to the variations in the pattern of the turbulent velocities along the lines of sight. Keplerian rotation is the main cause for these variations.

2. EMISSION IN THE PLANE OF THE DISK

As in Paper I, the turbulent velocities are created within a rectangular volume that encompasses the portion of the disk that is the source of the maser radiation of interest. The location of this volume that is used in calculating the emission from a side is indicated in Figure 1. The observed emission from the sides of the disk in NGC 4258 is created in a region with a radial extent from about \( R_0 \) to \( 2R_0 \), where \( R_0 \) is the radial location from which the systemic emission arises (4 \( \times 10^{17} \) cm). The size of the grid in the rectangular volume is \( 2048 \times 2048 \times 16 \). The ratio 16/2048 is chosen to reflect approximately the ratio of the thickness \( H \) of the disk to this radius \( R_0 \). The velocities are obtained by first making statistical choices from a Gaussian distribution for the amplitudes of the components of the vector potential for the velocities where, for example,

\[
\langle |A^y_\nu|^2 \rangle = C \left( k_x^2 + 11.45 k_y^2 + k_z^2 + k_{\min}^2 \right)^{-17/6 - b_x} \tag{1}
\]

is the power spectrum as a function of wavenumber \( k \) for the real part of the component of \( A_k \) in the \( y \) direction of the Cartesian coordinate system. The other required components, including imaginary parts, are obtained from exactly the same power spectrum. Variations in density are thus being ignored. In equation (1), the factor 11.45 is introduced as in Paper I so that the velocities reflect at least

![Figure 1. Schematic diagram for the masing disk at the nucleus of the galaxy NGC 4258. The rectangles indicate the locations of the volumes within which the turbulent velocities are created for computing the maser emission as discussed in the text. Rectangle c represents the location of the volume that was used in Paper I.](image-url)
approximately the anisotropy due to the rotation (e.g., Brandenberg et al. 1995; Stone et al. 1996). We note that a somewhat similar approach has been taken independently by others to account approximately for the anisotropy due to rotation (Heyvaerts, Bardou, & Priest 1996). We have also performed computations in which the factor 11.45 in equation (1) is replaced by unity so that the turbulence is isotropic. The differences in the appearances of the maser spectra computed with and without the factor of 11.45 are negligible in the context here. Where some difference does arise is in the computations for § 3. In our limited computations, we were unable to obtain acceptable amplification factors for the maser spectra from the center and from the sides of the disk with a single set of disk parameters ($\kappa_0$ and $\sigma_i$) when the turbulence is isotropic. As described in § 3, this was achieved for the anisotropic power spectrum given by equation (1).

In calculating the spectra at the side, as well as the spectra at the center in § 3, the rectangular volume is always positioned with the “y-axis” in the azimuthal direction at the center of the volume and thus parallel to the Keplerian velocities. The thickness of the disk is measured along the z-axis. The constant $C$ in equation (1) is determined by the requirement that the resulting turbulent velocities have the

FIG. 2.—Representative spectra computed for maser emission from the side of the disk when the rays are parallel to the plane of the disk. The amplification factor $A$ is shown as a function of the Doppler velocity (measured relative to the 480 km s$^{-1}$ systemic velocity of NGC 4258). These four spectra differ only because they are computed for different statistical realizations of the Kolmogorov turbulent velocity field, that is, as a result of different random numbers chosen by the number generator. The insert in (a) provides an enlarged view of selected spectral features as indicated. In the insert, the spacing in velocity of the fiducial marks is 2 km s$^{-1}$ and the amplifications are arbitrary.
and the actual Doppler velocity of the features. The actual Doppler velocity, and is the difference between the best-fit curve and the actual Doppler velocity of the features.

Here in equation (1), the power spectrum is exactly that of Kolmogorov. For unsaturated masing, the emission at the sides due to rays in the plane of a rapidly rotating Keplerian disk with no turbulent velocities is strongly peaked at the outer edge of the disk when the amplifying maser opacity $\kappa_0$ is constant per unit volume as adopted in Paper I (this is also the case for the central emission). The introduction of turbulent velocities of the magnitude being considered here does not significantly alter that result. Plausible alterations of our basic description that will tend to spread the masing features over the range of Doppler velocities and of spatial locations that are observed at the sides of the disk in NGC 4258 are (1) a radial variation of the pumping (or more precisely of the amplifying opacity $\kappa_0$) or (2) performing the calculations for rays of radiation that are inclined by the angle of $7^\circ$ to the plane of the disk. The latter may seem to be the more natural modification. However, as has been discussed elsewhere, warping of the disk may be such that the observed radiation arises in a region of the disk that is locally tangent to the line of sight so that the observed emission is effectively “in the plane.”

Here in § 2 we focus on alternative (1). To achieve the desired spreading of the spectrum, the exact variation of $\kappa_0$ is unimportant as long as $\kappa_0$ decreases by about a factor of 2 as the radius increases from $R_0$ to $2R_0$. We adopt the specific form for the computations in § 2 that $\kappa_0$ is proportional to the inverse of the radius. A decrease in the pumping with radius is certainly reasonable. Likely causes for the heating of the gas to the temperatures required for masing include the absorption of X-rays from a central source (Neufeld, Maloney, & Conger 1994) and viscous dissipation resulting from accretion of matter through the disk (Desch, Wallin, & Watson 1998). It is evident that the effects of both of these mechanisms decrease with increasing radius. Especially when augmented by additional considerations (Collison & Watson 1995), these mechanisms also can lead to a relatively smooth opacity $\kappa_0$.

In Figure 2, we present the spectra computed with four representative statistical samplings (or “realizations”) of the turbulent velocity field for the Kolmogorov ($b_\circ = 0$) power spectrum. As in Paper I, the spectra are presented in terms of the amplification factor $A$ that is defined by the expression for the observed flux density

$$S(v) = (2v^2 k T_c/c^2 D^2) \int db dz \{ \exp [\pi (v, b, z)] - 1 \}$$

$$\equiv (2v^2 k T_c/c^2 D^2) HR_0 A,$$  

where $v$ is the Doppler velocity, $v$ is frequency of the maser transition, $T_c$ is the brightness temperature of the continuum radiation that is being amplified by the sides of the disk, $D$ is the distance to the source, $H$ is the thickness of the masing region of the disk, and $R_0$ is the inner radius of the masing region. In equation (2), the seed radiation for the maser is taken to be the background continuum radiation. The gas temperature is probably about 400 K, and the populations of the 616 and 523 states are subthermal in relation to the populations of the lower molecular states of the water molecule. Since the actual $T_c$ are quite uncertain and our discussion encompasses values up to 100 K, including spontaneous emission in the seed radiation would have a negligible effect. That seed radiation due to spontaneous and to background continuum radiation leads to spectra with similar appearances is indicated in detailed calculations where both are included (e.g., Emmering & Watson 1994). Since the flux densities that are observed for the maser features at the sides of the disk are approximately 1 Jy, the amplification factor in equation (2) must be given approximately by

$$(H/4 \times 10^{15} \text{ cm} )(R_0/4 \times 10^{17} \text{ cm}) T_c A \approx 10^{11} \text{ K}.$$  

The sides of the disk are not believed to be amplifying a bright continuum source, but only ordinary extended continuum radiation with a brightness temperature of perhaps 10–100 K. Thus, $A$ in the range $10^5$ to $10^{10}$ seems most

![Schematic diagram showing the location of the presumed central source of continuum radiation in relation to the disk and to the observer. Note that only along the inner rays is the maser amplifying the central continuum source.](image-url)
likely. In practice, we find that spectra with \( A \) from about \( 10^8 \) to \( 10^{12} \) are not qualitatively different in appearance. When the Keplerian velocities are prescribed as they are here since we are attempting to describe the disk in NGC 4258, the quantities that must be specified to compute the spectra in Figure 2 are (1) the thermal dispersion velocity, (2) the ratio of the rms turbulent velocity \( \sigma_i \) to this thermal velocity, and (3) the amplifying opacity \( \kappa_0 \). When (1) and (2) are specified, the choice for \( \kappa_0 \) is effectively determined since it must lead to \( A \) in the proper range. The 22 GHz transition of water consists of three, closely spaced hyperfine components that give it a breadth that is larger than that due to the thermal dispersion of the molecular velocities. Calculations (Nedoluha & Watson 1991) have demonstrated that the 22 GHz transition behaves as if its thermal breadth is about 1.5 km s\(^{-1}\) (FWHM) when the temperature of the gas is about 400 K, as is usually adopted. The thermal dispersion velocity for our computations is chosen to be 1.5 km s\(^{-1}\) (FWHM). Computations have been performed with rms turbulent velocities \( \sigma_i \) as large as 9 km s\(^{-1}\). The spectra shown in Figure 2 are computed with \( \sigma_i = 1.3 \) km s\(^{-1}\)—the speed of sound in the masing gas at the likely temperature (400 K). The essential features of the observed spectrum—a small number of narrow, separated maser features with comparable flux densities that are spread over the relevant interval of Doppler velocities—are reproduced for all of the \( \sigma_i \) within this range.

Of the 17 spectra that we have computed with exactly these four statistical realizations as representative of the variations in appearance that can occur due solely to statistical sampling. Figures 2a and 2b are most similar to the observations of the side of the disk in NGC 4258 on which the emission is the stronger. Figures 2c and 2d demonstrate that the emission can be strongest at either the inside edge or the outside edge of the disk, and that there can be fewer features. Figure 2b indicates that the same physical description can also lead to a spectrum with lower amplification. The variation in the amplification factors among the different spectra in Figure 2 suggests that the difference between the flux densities from the two sides of the disk in NGC 4258 may simply be a statistical fluctuation. As in Paper I, we verify the validity of all our computational results by performing computations in which alternate grid points are omitted. In Figure 3 the position-velocity diagram is given for maser features for one of the spectra in Figure 2. Differences in this diagram among the various statistical realizations are obtained by finding, for each Doppler velocity, the impact parameter at which the flux is a maximum (cf. Paper I). The rms deviation in Figure 3 is 1.8 km s\(^{-1}\), whereas that reported from the observations of the spectra for the side of the disk in NGC 4258 is 3.4 km s\(^{-1}\). To obtain an rms deviation of 3.4 km s\(^{-1}\) in our computations, an rms turbulent velocity of 6 km s\(^{-1}\) would be necessary instead of the 1.3 km s\(^{-1}\) which was used for obtaining Figure 3. Accepting the presence of a warp in the disk apparently reduces (or even eliminates) the observed deviations.

### 3. EMISSION AT AN ANGLE TO THE PLANE OF THE DISK

Since the line of sight does make an angle of 7° with the average plane of the masing disk in NGC 4258, consideration should be given to ways in which this angle of inclination might alter the appearance of the maser emission. Because the disk is quite thin, we have so far assumed that these alterations will not be great at the sides of the disk. The inclination does, however, cause the rays at different impact parameters between \( R_0 \) and \( 2R_0 \) to have sufficiently similar path lengths within the masing gas that it is not necessary to introduce a variation of \( \kappa_0 \) with radius in order that the maser emission be spread over the observed range of Doppler velocities and impact parameters. Thus, \( \kappa_0 \) is a constant independent of radius in the computations for this section. It is believed that the emission that arises near the center of the disk and close to the systemic velocity is a result of the amplification of a bright, compact continuum source slightly below the center of rotation of the disk (and of the galaxy NGC 4258). The location and size of this source then determines the radial extent of the disk from which the maser radiation is observed as indicated in Figure 4. It is, of course, necessary that the maser emission due to the “outer central rays” in Figure 4 be weak enough that they are not observed. Even though \( A \) is greater for the outer than for the inner central rays, the outer central rays are weak because they are amplifying only the diffuse background radiation. It is then possible to inquire as to whether there is a unified description of the disk with only a single value between \( R_0 \) and \( 2R_0 \) for the amplifying opacity \( \kappa_0 \), for the rms turbulent velocity \( \sigma_i \), and for the thermal dispersion that leads to the emission at the center and at the sides that is compatible with the observations. That inquiry is the main goal of this section.

When the emission is inclined to the plane of the disk, it is somewhat more convenient to express the spectra in terms of a slightly different amplification factor \( A_i \) from that used.
when the emission is in the plane

\[ S(v) = \left(2\nu^2kT_e/c^2D^2\right) \int db \, dq \left( \exp \left[ \tau(v, b, q) \right] - 1 \right) \]

\[ \equiv \left(2\nu^2kT_e/c^2D^2\right)R^2A_i, \quad (4) \]

where \( q \) is the location on the surface of the disk from which the ray emerges. It is measured from the radius that is orthogonal to the line of sight. A flux density of 1 Jy that is typical of the stronger features from the side of the disk in NGC 4258 then leads to

\[ (R_0/4 \times 10^{17} \text{ cm})^2T_eA_i \approx 10^9 \text{ K}. \quad (5) \]

In Figures 5a and 5b, we present representative spectra for \( A_i \) that are analogous to those of Figure 2. If the extended continuum emission in the neighborhood of the disk has a brightness temperature \( T_e \) of about 10 K, \( A_i \approx 10^8 \) are most appropriate for comparison with the observations. Again, the appearance of these spectra is relatively insensitive to adjustments in the amplifying opacity \( \kappa_0 \) that would lead to \( A_i \) that differ by a factor of 10 or so from those in Figures 5a and 5b. We present spectra computed for a true Kolmogorov variation of the power spectrum, as well as for a modified form in which \( b_p = 0.5 \) in equation (1). The essential features of the observations are present in these spectra, as well—a number of sharp features with breadths down to about 1 km s\(^{-1}\) that are spread over a few hundred km s\(^{-1}\) in Doppler velocities. In their general appearance these spectra do not agree quite as well with the observations as do some of those computed for emission in the plane (Fig. 2). The features seem to be somewhat more numerous than in the observed spectra (but see Herrnstein et al. 1998). However, this should not yet be taken as significant since we have devoted less effort to the computation of emission that is inclined to the plane of the disk. For the computations involving inclined rays at both the side and the center of the disk, a grid size of 1024 \( \times \) 1024 \( \times \) 64 was found to be optimal (within the limitations of our workstation). To provide needed spatial resolution in the direction perpendicular to the plane of the disk, the spacing between grid points in this direction is only 1/8 of that in the other two directions. Thus, the effective \( H/R \) is 8/1024 and is still comparable with what is suggested by the observations.

Using exactly the same geometry and values for the parameters of the disk (i.e., the same constant \( \kappa_0 \) and \( \sigma_1 \) in the gas between \( R_0 \) and 2\( R_0 \) as was used in the computations for Figs. 5a and 5b), spectra also are computed for the central (systemic) emission due to rays that are inclined at the angle of 7° to the plane of the disk. This emission is divided into two components in Figures 6a and 6b—that which emerges from the portion of the disk at radii between \( R_0 \) and 1.2\( R_0 \) and that which emerges from radii between 1.2\( R_0 \) and 2\( R_0 \). The inner portion of the disk is assumed to
be amplifying the strong continuum source near the center, whereas the outer portion is assumed to amplify only diffuse background emission as indicated in Figure 4. Since the assumed compact central source can have a brightness temperature $T_c$ of $10^8$ to $10^7$ K (Herrnstein et al. 1997), whereas the diffuse background is likely to have $T_c$ of only 10–100 K, the contribution to the observed flux from the region between $1.2R_o$ and $2R_o$ can easily be several orders of magnitude smaller than that from the inner portion. It would presumably not have been detected at the sensitivity of the observations that have appeared in the literature. As was the case for the emission from the side of the disk, we have devoted considerably less effort to the computation of central spectra for rays that are inclined to the plane of the disk than we did in Paper I for rays that are in the plane. We have computed only a few realizations and may not have encountered those that agree best in general appearance with the observations. The primary goal was to demonstrate that a disk with statistically identical properties at the sides and along the line of sight from the center leads to maser radiation that is generally consistent with the observed fluxes (and upper limits) of the various components of the disk.

Position-velocity diagrams analogous to that in Figure 3 have been computed for the emission at an angle to the disk shown in Figures 5a and 5b. The deviations from the best-fit curve are similar (about 1.8 km s$^{-1}$) to those in Figure 3 for emission in the plane of the disk. Position-velocity diagrams have also been computed for the central emission in Figures 6a and 6b. An example is given in Figure 7. Although $\sigma_v$ is only 1.3 km s$^{-1}$, the deviations are similar to those of the observational data. In Paper I we concluded that supersonic (but sub-Alfvénic) turbulence with $\sigma_v$ of 6 km s$^{-1}$ or greater would be required if the emission is in the plane of the disk based largely on the position-velocity diagrams. It is now possible to conclude that plausible calculations in which $\sigma_v$ for the turbulent velocities is comparable with the speed of sound lead to spectra that are compatible with the observational data.

4. VARIATIONS OF THE SPECTRA WITH TIME

Qualitatively, the spectra from the sides of the disk in NGC 4258 vary little over the few years in which they have been observed (see also Nakai et al. 1995). In contrast, the flux from the center changes completely over such periods of time. More rapid changes have also been reported in the flux from the center and identifiable spectral features near the systemic velocity drift in velocity with time. The evolution of the turbulent velocity fields in the disk will cause changes in the pattern of the irregularities and hence in the spectra. Utilizing the techniques described in Chen & Shan (1992) and starting from statistically chosen realizations of the type presented in §§ 2 and 3, we have simulated the evolution of the turbulent velocities over a sufficient period of time to determine that these changes have negligible effect on the spectra within a few years.

What is more important are the changes in the pattern of the velocity field as seen by the observer due to the Keplerian rotation. Not only is the viewing perspective altered as a result of the rotation, but the pattern also is changed because of the differential rotation. We thus create the pattern of turbulent velocities at a sequence of times simply by differentially rotating the locations of the points in the initial grid of turbulent velocities for a realization that is created by the statistical sampling (see Wallin 1997 for additional details). These points are assumed to retain their initial turbulent velocities since changes in the pattern of the velocities due to the evolution of the turbulence was demonstrated to be negligible over the relevant periods of time. Computations involving these rotations have been performed for several realizations to obtain the maser emission from both the sides and the center of the disk. A representative comparison of features from the side of the disk in spectra separated by 1 yr and by 3 yr is shown in Figure 8. For an interval of 3 yr, these features mostly disappeared in the plane of the disk. We are wary that our inability to treat the side and central spectra adequately the effects of partial saturation may be an especially serious limitation in attempting to simulate the time evolution of the emission. The variation with time in the appearance of the emission depends upon changes that

![Fig. 7. —Position-velocity diagram for the maser features in the inner central (or systemic) emission in Fig. 6a.](image-url)
occur over small angles in the angular distribution of the emergent maser radiation. This distribution may be influenced significantly by the competition among partially saturated maser beams to deexcite the inverted population. It is well known from the example of spherical masers that the maser beam tends to become tighter with increasing saturation. Our limited description of the turbulence obtained from only the power spectrum and Gaussian amplitudes may also be a severe limitation in computing the time variations. Especially for the side features, the patterns may be more extended than we calculate in the direction of the rotational velocities. This will tend to reduce the rate at which these features change.

5. DISCUSSION

Numerous considerations that can be expected to influence somewhat the maser emission have been neglected in the simplifying idealizations of our calculations. These include a more realistic description of the irregular velocity fields based on the structure of an actual Keplerian disk including MHD and other aspects of the fluid dynamics, about which indications are only beginning to appear in the literature (e.g., Brandenberg et al. 1995; Stone et al. 1996). Creating velocity fields from the power spectrum for the velocities leads to an incomplete description since it ignores higher order correlations that are reflected, for example, in “intermittencies” (e.g., Frisch 1995). The amplifying maser opacity $\kappa_0$ depends upon the physical state of the medium that in turn depends upon the result of incorporating the detailed heating and cooling mechanisms, and chemical reactions, into the fluid dynamical description of the structure of the gaseous disk. Evidence was presented in Paper I that the approximation of unsaturated masing can be compatible with the information about the central emission in NGC 4258. For the emission from the sides of the disk, the spectral line breadths of certain of the strong features (e.g., 1.1 km s$^{-1}$ for the 1306 km s$^{-1}$ feature Herrnstein et al. 1998) are significantly less than the 1.5 km s$^{-1}$ (FWHM) that would be expected because of hyperfine splittings if these masers were fully saturated (Nedoluha & Watson 1991). Velocity gradients, the merging of different components, etc., can enhance the apparent breadths of features, but only unsaturated (or partially saturated) masing has been recognized as capable of reducing the line breadth. Observational evidence tends to confirm the expectation that hyperfine splittings contribute to the spectral line breadths of galactic 22 GHz water masers (Gwinn 1994). There is thus compelling evidence that the masing features from the sides of the disk also are not fully saturated. We have attempted to incorporate the effects of saturation in at least an approximate way by treating each feature as an independent “linear maser.” The results did not seem to represent an improvement and we have not chosen to present spectra computed in this way. Upon reflection, we believe that such an approximation may miss the key aspect of saturation that intersecting rays of radiation compete for amplification to decrease the apparent size of the maser as recognized, for example, for spherical masers (e.g., Emmering & Watson 1994 for a recent reexamination). This competition may also tend to reduce the flux from the weaker
FIG. 9.—Representative spectrum for the central (or systemic) emission after 0, 0.1, 1, and 3 yr of Keplerian rotation. Features and thus to aid the creation of spectra with a small number of prominent features.

Nevertheless, our calculations here and in Paper I demonstrate that incorporating the effects of velocity irregularities at only a minimal level of detail leads to maser spectra that are compatible with key features of the observations. In particular, narrow spectral features result. For the emission from the sides of the disk, these are sharp, well separated and similar in number to those of the observed spectra. The “rare” (in space and frequency) masing features at the sides are to be contrasted with the spectrum at the center of the disk where the features are close together and nearly merge. The presumed large difference between the amplification factors at the sides and at the center plays an important role in this difference. Large, exponential amplification at the sides tends to lead to only a small number of features that are the brightest. Because of the presumed central source, the amplification at the center is not so large and more features can be of comparable brightnesses.

The calculations also demonstrate that the seemingly conflicting characteristics of the observed maser emission can be reconciled. That is, they demonstrate how the “clumpiness” in the spectra and images can occur together with the tendency for the emission to appear only near the center and at the sides of the disk as is predicted from the locations of the largest optical depths based on the velocity gradients in a Keplerian disk. The longer paths with velocity coherence at these locations increase the likelihood that separated components of the gas can amplify at the same Doppler velocity, and thus create bright “aligned masers” as has been imagined in other contexts (Deguchi & Watson 1989). Variations in the maser amplification among the various realizations allows the possibility that the differences between the fluxes from the two sides of the disk in NGC 4258 may be due solely to statistical fluctuations. Our main effort in recreating the appearance of the emission from the side of the disk has been devoted to emission in the limit in which the small (7°) angle of inclination is ignored. To examine whether the calculated spectra can be compatible with the idealization that the disk is, at least in a statistical sense, the same along the line of sight to the center as it is at the sides, it is necessary to treat the inclination of the line of sight to the average plane of the disk. When this is done and likely values are adopted for the brightness temperatures of the presumed compact continuum source and of the diffuse background radiation, the flux densities that follow from the computed amplification factors for the emission from the center and from the sides are compatible with what is observed when the statistical characteristics of the velocity fields and of the amplifying opacity $\kappa_0$ are the same in the two segments of the disk.

For the emission from the sides of the disk that is in the plane, as well as that which is inclined at the 7° angle, satisfactory spectra are computed with rms turbulent
velocities ($v_t = 1.3 \text{ km s}^{-1}$) that are equal to the sound velocity of the mainly H$_2$ gas at the temperature (400 K) customarily adopted for strong masing at 22 GHz by the water molecule. Satisfactory spectra also result when these turbulent velocities are used to compute the emission from the center of the disk when the rays are inclined to the plane of the disk as we calculate in § 3. This allows us to revise our previous conclusion that the turbulent velocities must be supersonic based only on the emission in the plane computed in Paper I. In addition to the various considerations mentioned in the foregoing that are omitted in our idealization of the disk, we also do not attempt to treat explicitly the possible warping at the sides of the disk. Locally, this warping may cause the emission to be effectively “in the plane” for the region of the disk where it is being created. We thus reason that our spectra calculated for emission in the plane can be relevant for interpreting the observations. Finally, we note that sharp spikes appear occasionally in the spectra when there is especially good alignment of separated velocity components (Figs. 6a and 6b; also Paper I) and that these may be related to the maser flares that have been observed.

Although our computations for the time variations of the spectra were limited, they do demonstrate that the characteristic timescales for changes are comparable with those that are observed and that the central spectrum varies much more rapidly than does the spectrum from the sides. Motions of individual masering features in the central emission can be followed for several years to demonstrate their drift in Doppler velocity and changes in flux. The evolution of the turbulent velocities is negligible over the relevant periods of time. Changes in the pattern of the turbulence along the line of sight to the observer occur as a result of the Keplerian rotation.

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