Study on pore fabric, dilatancy, dissipation function and yield function for sand

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Abstract. The mechanical behavior of sand is affected by the anisotropic fabric. Integrated with the micromechanics theory, and linked with void fabric tensor, the relations among macroscopic volumetric strain, shear strain and void fabric are established, and it is further shown that dilatancy and anisotropy under applied shear force of sand are necessarily induced. By means of thermomechanics theory, the inclined elliptical yield function in dissipation space is built, and considering the effects of anisotropic fabric, there exists three important lines, just as the normal consolidation line, phase transformation line and peak value line, and corresponding with three characteristic lines passing through the origin in true stress space which relatively truly reflect the mechanical characteristics for sand, especially dense sand.

1. Preface
The mechanics of granular materials is widely used in engineering, but this theory is still immature. There are still many challenging problems to be studied. There are generally two methods to establish the constitutive model of granular materials, namely the continuum mechanics method and microscopic mechanical methods. At present, most of the existing constitutive models of granular materials are based on continuum mechanics and are used to describe the mechanical properties of soils, such as the use of soil as an isotropic continuum, and to simulate the behavior of soil. Many valuable theories, such as the framework of critical state soil mechanics, are based on this; the second is based on the method based on micromechanics which considers soil as having regular and irregular arrangements. Aggregates of discrete particles. In models, the contact distribution of particles usually uses probability distribution functions to reflect their anisotropy (Horne, 1965)¹; Oda, 1972a & b²[³].

Since the continuous plastic theory can not directly explain the microscopic properties of the irreversible shape of the granular material, the detailed study of the microscopic properties of the granular material is mathematically complicated, which leads to great limitations in the application of this method. This paper starts from the theory of particle mechanics. The pore fabric of sand is discussed, and the relationship between pore structure anisotropy, dissipation function and real stress space yield function is analyzed in detail with thermodynamic theory. The theory of establishing boundary surface constitutive model based on thermodynamic theory is further discussed. frame.

2. Strain tensor expression based on pore fabric
For granular materials, the most interesting study is not the reaction of individual particle but how the particles are arranged in space under external force and their overall reaction. So the mechanical
behavior of the granular material is closely related to its microstructure. Here, we assign the microscopic and mesoscopic magnitudes of the particles to the microstructure. In the triaxial compression experiment on sand, Oda (1982) observed that the granular material had different strength due to the different compression direction. Moreover, he observed that the long axis direction of the non-spherical particles tended to rotate perpendicular to the maximum compressive stress, so the mechanical behavior of the granular material is affected by its microstructure. Generally, void ratio $e$ or porosity $n$ indicates the degree of compaction of the granular material structure, but these quantities do not indicate the directionality of the granular material structure. The internal structure of the randomly distributed granular material can be described by the geometric statistical properties of the particle and its spatial distribution. The higher-order microstructure variable $\mathbf{d}$ of the "fabric tensor" can usually represent the distribution and orientation of particles and pores (Oda et al., 1982; Mehrabadi, 1982). Using the famous Voronoi-Delaunay checkerboard grid, Bagi (2006) derived the mathematical description of the internal structure of the granular material, and it is pointed out that the spatial distribution of the contact point of the particle is the most basic factor when the dilatancy of the material is considered. Li and Li (2009) proposed to quantify the internal structure of granular materials by contact-based grid system. In three-dimensional case, the continuous pore space is composed of the smallest pore units, and the unit boundaries are statically determined. The microstructure expression of the strain tensor is briefly described below.

![Figure 1](image-url)

**Figure 1.** The surface increment $dS$.

Under the condition of uniform strain, an arbitrary representative unit $\text{Rev.}$ is set, and the volume is $V$, and the boundary surface is closed. Then, for the representative unit $\text{Rev.}$, the compatibility condition is necessarily satisfied, namely

$$
\oint_{S} d(u,S) = 0
$$

(1)

Among them $S$ a continuous boundary surface representing a representative unit (Rev.), as shown in Figure 1, $u$ represents the representative unit displacement. Here the direction is kept consistent with the soil mechanics and the pressure is positive. Under the compressive stress, the average displacement gradient tensor of the representative unit $\text{Rev.}$ can be expressed as

$$
\bar{e}_{ij} = \frac{1}{V} \oint_X e_{ij} dV = -\frac{1}{V} \oint_X u_{i,j} dV
$$

(2)

According to the scatter theorem, the formula (2) can be re-expressed as

$$
\bar{e}_{ij} = \frac{1}{V} \oint_X u_{i,j} n_{i}(x) dS = \frac{1}{V} \oint_X u \otimes n dS
$$

(3)

Among them $n(x)$ represents the normal direction of point $x$ on the boundary surface $dS$ that the direction points to the inside. Then, By using the statistical probability method to describe the edge $n$, the displacement gradient of the direction is expressed as (for detailed derivation to see $^{[7]}$)

$$
\bar{e} = \frac{1}{V} \sum_{\omega} \mathbf{u} \otimes \Delta n = \frac{1}{V} \sum_{\omega} \bar{\mu}(n) n \otimes \Delta n_{\omega} + \Omega
$$

$$
= \frac{1}{V} E \Delta u^{\omega} \oint_{\omega} \bar{\mu}(n) n \otimes n d\Omega
$$

(4)
Here $\mu'$ represents pore element polygon vector\(^{[7]}\) for granular materials which is closely related to the direction distribution of the length of the pore vector. The integral range is the entire space $\Omega$, and $E_0$ is the standardization factor, and $E_0=\sqrt{\int_\Omega \Delta \omega} \Delta \omega$ which is respectively equal to $2\pi$ and $4\pi$ in two and three dimensions, $\Delta \omega$ indicates the average relative displacement at the contact point of the microelement which is set to a constant under uniform strain conditions.

For variables $\mu'$ is closely related to the direction distribution of the length of the pore vector, so the tensor can be established based on the directional distribution of the pore vector. Considering the pore length $\mu(n)$ on the direction of the representative pore unit $n$ is the direction of the average length of all pore vectors in the sample $n$. Therefore, the fabric tensor based on the directional distribution of the average pore vector length is defined to describe the shape and orientation of the average pore unit, $\mu(n)$ represents the distance from the center of the micro-porous unit to the boundary of the pore unit in space $\omega \Omega$, and $n$ is a unit vector representing the direction of the aperture, ignoring the high order, along the direction $n$, pore vector $\mu(n)$ can be approximated as

$$\mu(n)=\mu_0(1+d): (n \otimes n) = \mu_0(1+n\cdot d\cdot n)$$  \(5\)

Here $\mu(n) = \mu(n)$, $d$ is a second-order tensor with no trace and represents the anisotropy of the pore structure, $\mu_0 = E_0\int_\Omega \mu(n)d\Omega$ representing the average length of the pores. Substituting formula (5) into (4), we get

$$e = \frac{1}{V} E_0 \Delta \omega \int_\Omega \mu(n)n \otimes nd\Omega = \frac{1}{V} E_0 \Delta \omega \int_\Omega \mu_0(1+n\cdot d\cdot n)n \otimes nd\Omega$$  \(6\)

The above formula is the expression of the displacement gradient, and the rotation tensor is removed, so the expression of the strain tensor is

$$\bar{e}_{ij} = (\bar{e}_j + \bar{e}_j)/2$$  \(7\)

Then under the action of compressive stress, the volumetric strain expression in the directional $n$ is

$$\bar{e}_v(n) = \bar{e}_v(1+n\cdot d\cdot n \otimes n)$$  \(8\)

Let’s say $e_v = E_0 \Delta \omega \mu_0/V$, then formula (8) can be converted into

$$e_v(n) = e_v(1+n\cdot d\cdot n \otimes n)$$  \(9\)

Here $e_v(n)$ indicates pore volume change rate in the unit vector $n$ under the compressive stress, and $e_v$ for the macroscopic volumetric strain of the granular material.

3. Fabric changes due to deformation

The change in particle arrangement and its pore structure due to shear deformation is actually a change in fabric. Since Philofsky and Finn (1967)\(^{[8]}\) established a method for measuring strain using the Principle of stereography, early researchers began to study the relationship between fabric and strain, Kanatani (1984)\(^{[9]}\) deeply studied and developed the relationship between strain and different tensor. Satake (1989)\(^{[10]}\) proposed that in granular materials, the average strain is a function of the relative displacement between the particles and the branch vector connecting the centers of the particles. For the particle material fabric, the relative position between the particles changes, and accordingly the pore structure also changes. Therefore, for the dual structural system of granular material: pore structure and solid structure\(^{[7]}\) are complementary each other, and any structure can describe the internal structure of the granule. Numerical experiments have proved\(^{[7]}\) that the normal anisotropy of particle contact is closely related to the anisotropy of pore structure. The direction distribution of contact normal tends to the direction of pore structure. Therefore, the changes of volumetric strain and pore fabric tensor under compressive stress will be discussed below. We can get the differential expression for formula (9)
can be connected to the triaxial stress model, that is, "Reynolds dilatancy" effects (Boehler, 1987)[11]: $d(\varepsilon_{ij}) = d(\varepsilon_{ij} \cdot \varepsilon_{ij})$, the changes of the fabric with the partial or shear strain $\varepsilon_{ij}$ can be connected, in general, this function is very complicated, for the sake of simplicity, it is assumed here $d\varepsilon_{ij}$ and $d\varepsilon_{ij}$ have the same spindles(Muhunthan et al., 1996[12]), so the relationship can be expressed as

$$\varepsilon_{ij} = \alpha d(\varepsilon_{ij} \cdot \varepsilon_{ij})$$

Here $\alpha$ represents the anisotropic parameters of the fabric reflecting the basic trend of soil fabric development. For partial strain $\varepsilon_{ij}$, under triaxial stress $d\varepsilon_{ij}$ also represents the shear strain increment. Setting $d(\varepsilon_{ij} \cdot \varepsilon_{ij}) = d\varepsilon_{ij}$, and substituting it into formula (11), we can get

$$d\varepsilon_{ij} = d\varepsilon_{ij} + \alpha d\varepsilon_{ij}$$

The above formula shows the rate of change of volumetric strain $d\varepsilon_{ii}$ under compressive stress in granular materials considering anisotropy is coupled to the rate of change with partial strain $d\varepsilon_{ij}$, that is to say, the change of the partial strain will also lead to the change of volumetric strain.

4. Decomposition of plastic strain

In the existing constitutive model of granular materials, it is generally considered that the change in volume of the material is derived from the compression of the pores, and the plastic volumetric strain is only caused by the change of the compressive stress, and has nothing to do with the soil skeleton. In fact, through the above analysis, The relationship between volumetric strain and pore structure indicates that for granular materials such as dense sand or medium dense sand, the change of volume under shear stress includes two processes. First, when the axial strain is small, dense sand or medium dense sand produce shear under compressive stress $\varepsilon_{ij}$, and then with the development of anisotropic particle organization, will also produce additional volume changes $\alpha d\varepsilon_{ij}$, that is, "Reynolds dilatancy" effects (since the sand particles are significantly larger than the clay, as the shear strain increases, the particles at the shear plane will rise, shift, pull, and pull out, accompanied by an increase in volume - dilatancy). In dense or medium-density sand, another source of plastic volumetric strain is shear-induced volumetric strain-dilatation. Therefore, the macroscopic plastic strain of the granular material can be expressed as

$$d\varepsilon_{ii} = d\varepsilon_{ii} - \alpha d\varepsilon_{ij}$$

As can be seen from the above formula, the macroscopic plastic volumetric strain increment $d\varepsilon_{ii}$ includes two components: one component is the change in pore volume caused by the change of compressive stress of the granular material directly $d\varepsilon_{ii}$ -shearing, the other component is shear-induced volumetric change ($\alpha d\varepsilon_{ij}$) - dilatancy, since the pore structure of the particles is anisotropic during the loading process, this part of the volumetric deformation has been in a dilatant state and is dominant in the volume change. In the correction of the body strain in the Cambridge model, it usually refers to the first volumetric change-shear, which is completely obtained by compressive stress change, and does not take into account the shear-induced part of the dilatancy.
Therefore, the correction of the stress dilatancy relationship reflected by the Cambridge model has certain limitations. Below, we will further analyze this part of the shear-induced volumetric change based on the thermodynamic framework. \((\alpha \epsilon^p)\) - the relationship between the dilatancy and the dissipation function and the yield function of the real stress space.

5. Thermodynamic analysis

Today, there are many mechanical models that describe the mechanical behavior of sand and other particulate materials. In order to make these models more realistically reflect the mechanical phenomena observed in the laboratory of particulate materials, these problems are generally solved by increasing the model parameters. However, all These models are inseparable from some basic mechanical concepts. These concepts have been verified by practice and laid a solid theoretical foundation: critical state, stress dilatancy theory and state parameters. Most of the models are based on rate-independent, elastoplastic solids. The classical theoretical framework, therefore, the yield surface, the flow rule, etc. need to be determined, and many models assume the associated flow rule, but in fact the sand should be unrelated flow.

We use thermodynamic theory for it is a more rigorous method and does not require many artificial assumptions in traditional methods. The constitutive equations based on thermodynamics not only satisfy the laws of thermodynamics, but also have a clear physical explanation based on energy storage and dissipation. It is especially important to establish the close relationship between dissipation, yield function and flow law, and plastic dilatancy and induced anisotropy. The yield function, the law of flow, the form of non-correlated flow laws can be inferred from the dissipative part of the energy balance equation. Naturally to get the nature of frictional materials (such as soil, rock, concrete, etc.) in the unstressed flow in the real stress space. In addition, using thermodynamic methods, the micro-macro behavior of simulated particulate materials can be observed and is linked to macroscopic behavior. Therefore, thermodynamic theory further regulates the constitutive equations of a wider range of material types, including soil and granular materials, providing a unified theoretical framework.

6. Sand anisotropy model

6.1 Pure friction material shearing model

According to the principle of thermodynamics, it can be known that the two key points to establish the dissipation function \(\delta \Phi\) is the choice of stress and strain variables. First, most isotropic hardening comes from compressive strain, and when the pressure is sufficiently large, the particles are rearranged and crushed. Sand or particles are defined herein as pure friction materials (Collins and Muhunthan, 2003[14]), under the action of shearing, this material usually undergoes a large shear deformation before the particle breakage occurs; that is, the normal consolidation stress of the material \(p_c\) is very large when the particle breaks. Because this article does not consider particle breakage, the stress variable of the dissipative function \(\delta \Phi\) is no longer dependent on the consolidation pressure (Collins and Muhunthan, 2003[14]). Secondly, through the analysis of the mechanism of the plastic volumetric deformation of sand, the macroscopic plastic volumetric strain increment of dense sand or medium dense sand \(\epsilon^p\) includes two parts: one is the volumetric compression strain \(\epsilon^c\) caused by the current compressive stress change. Second, another plastic deformation induced by shear deformation - dilatancy \((\alpha \epsilon^p)\), which is closely related to the pore fabric anisotropy. In fact, this part of the volumetric increment associated with the pore structure anisotropy \((\alpha \epsilon^p)\) is the result of particle rigid body motion, which is Reynolds dilatancy, even if the particles are rigid, completely smooth particles will dilate, so Collins (2007)[15], Collins and Muhunthan (2003)[14] assume that Reynolds dilatancy only indicates the internal motion constraint of the material, and the pressure of the associated reaction force and the storage energy generated by the shear force component are balanced against each other.
The dissipative energy associated with this part of the dilatancy is zero. Therefore, the dissipative function $\partial \Phi$ should be directly related to stress-related volumetric strain $d\varepsilon^\rho$ which is not related to this part of the dilatation induced by shear (Kanatani, 1982[9]; Goddard, 1990[17]). For convenience of description, here is defined $\alpha=\tan \theta$, based on the analysis of the above two aspects, set the dissipation function $\partial \Phi$ for

$$\partial \Phi = \rho \sqrt{(d\varepsilon^\rho + \tan \theta d\varepsilon^\rho)^2 + M^2 d\varepsilon^\rho^2}$$

(15)

Thus, the complete plastic work expression is

$$\delta W = \rho d\varepsilon^\rho + q d\varepsilon^\rho$$

$$= (\rho d\varepsilon^\rho + \xi d\varepsilon^\rho) + (\tau d\varepsilon^\rho + \tau d\varepsilon^\rho)$$

$$= (\rho d\varepsilon^\rho + \xi d\varepsilon^\rho) +$$

$$p\sqrt{(d\varepsilon^\rho + \tan \theta d\varepsilon^\rho)^2 + M^2 d\varepsilon^\rho^2}$$

(16)

Dividing both sides of the above equation by $d\varepsilon^\rho$, the general dilatancy relationship of pure friction particle materials expressed by angle can be obtained as

$$\tan \phi = \frac{\alpha}{\beta}$$

(17)

$$= \frac{\alpha}{\beta} = \frac{(\tan \theta - \tan \phi)}{\tan \theta - \tan \phi} + \tan \phi$$

Among them: $\tan \phi = q/\rho$, $\beta = p/\rho$, $\tan \theta = \xi/\rho$, $\phi$, $\psi$ represents sliding friction angle, internal friction angle and dilatancy angle, respectively, and satisfy $d\varepsilon^\rho/d\varepsilon^\rho = -\tan \psi$.

Based on above formula, corresponding dissipative stress components are

$$\pi = \frac{\partial \delta \Phi}{\partial (d\varepsilon^\rho)} = \rho \frac{(d\varepsilon^\rho + \tan \theta d\varepsilon^\rho)}{\partial \Phi}$$

$$\tau = \frac{\partial \delta \Phi}{\partial (d\varepsilon^\rho)} = \rho \frac{(d\varepsilon^\rho + \tan \theta d\varepsilon^\rho) \tan \theta + M^2 d\varepsilon^\rho)}{\partial \Phi}$$

(18)

(19)

Through the above two formulas can be derived as

$$\tau - \tan \theta \pi = M^2 p^2 d\varepsilon^\rho / \partial \Phi$$

(20)

Then it can get a tilted elliptical yield surface equation in the dissipating space $(\pi, \tau)$

$$\frac{\pi^2}{\rho^2} + \frac{(\tau - \tan \theta \pi)^2}{M^2 p^2} = 1$$

(21)

As shown in Figure 2, the pure friction material dissipates the space yield function as a slanted ellipse with a tilt angle of $\theta$. 
Conversion formula (19) and (20), Using formula (21) available, it can get
\[ \text{d} \varepsilon^p = \Phi \left( \frac{\pi}{p^2} \frac{(\sigma - \tan \theta \tau) \tan \theta}{M^2 p^2} \right) \]
\[ \text{d} \varepsilon^q = \Phi \left( \frac{\sigma - \tan \theta \tau}{M^2 p^2} \right) \]
(22)

Then dilatation equation \( D \) can be derived as
\[ D = \frac{\text{d} \varepsilon^p}{\text{d} \varepsilon^q} = -\tan \phi = M^2 \frac{\pi}{\tau - \tan \theta \phi} - \tan \theta \]
(23)

There are three important lines in Figure 2, all of which rotate with the development of shear deformation. The first one is the "normal consolidation line of motion" (KNCL), which satisfies \( \text{d} \varepsilon^p = 0 \), the plastic shear strain increment on the line is 0, but the stress state is not isotropic, according to the formula, in the dissipative stress space to meet: \( \tau \phi = \tan \theta \). The second is the "Phase transformation line" (PTL), the total volumetric strain increment on this line is 0, i.e., \( \text{d} \varepsilon^v = 0 \). At this time, the dilatation \( D = 0 \), shear-induced dilatancy is offset by stress-induced shearing, in the dissipative stress space: \( \tau / \pi = M^2 / \tan \theta + \tan \theta \). The third line is the "Reynolds-Taylor Line" (RTL) which has a dissipative pressure of zero, that is \( \pi = 0 \), \( p = \rho \). According to formula (18), \( \text{d} \varepsilon^p + \tan \phi \text{d} \varepsilon^p = 0 \), the only volume change is caused by the Reynolds shear effect and has
\[ D = \frac{\text{d} \varepsilon^p}{\text{d} \varepsilon^p} = \text{D}_{\text{max}} = -\tan \phi = -\tan \theta \]
(24)

6.2 Constrained shear model of pure friction particle material
The above analysis is mainly for the general deformation of pure friction materials. In order to study the dilatancy relationship Taylor (1948)\[^{[18]}\] proposed, Collins and Muhunthan (2003)\[^{[14]}\] defined a constrained shear deformation of a purely frictional particulate material, which is thermodynamically defined as the stored plastic work not considering the pre-consolidation pressure \( p_c \) and the pure shear flow, the volume fraction of the dissipation increment is 0, and the plastic free energy is not stored, that is \( d \Psi = 0 \). However, the pressure and shear component of the reaction force are not equal to zero. The above viewpoint is consistent with the analysis Arthur et al. (1991)\[^{[10]}\] and Dunstan (1998)\[^{[11]}\] proposed. They believed that for the shear deformation of granular materials (sand), the energy stored in the force chain is quickly dissipated due to friction shear sliding, so no plastic work is
stored and all energy is dissipated. Collins (2003)\textsuperscript{[14]} also stated in the text that for purely frictional particulate materials, as long as the anisotropy is considered in the model, the "quasi-steady-state" shear deformation of the dilatancy must occur, and no plastic properties are stored, but the reaction force component is not equal to zero.

Based on the above analysis, we can get the correlation of constrained shear deformation of pure friction materials:

$$\pi d\varepsilon_p^\tau = 0, \quad r d\varepsilon_q^\tau = 0, \quad d\Phi = p\sqrt{(d\varepsilon_p^\tau + \tan \theta d\varepsilon_q^\tau)^2 + M^2 d\varepsilon_q^\tau}$$

(25)

When the material is dilatant, the volumetric strain increment is not zero, so under such conditions, it must be satisfied $\pi = 0$. Therefore, according to the formula (21), it can be known that when $\pi = 0$, the dissipative stress is at the two vertices of the yield trajectory $A$, $B$, as shown in Figure 2, and satisfied $\rho = p$, that is, "Reynolds-Taylor line", according to And, it can get

$$\tau = M\rho, \quad \tan \psi = \tan \theta$$

(26)

And formula (17) converted into

$$\tan \phi_m = \tan \Theta + \tan \phi$$

(27)

Since the shear deformation of the pure friction material is a pure shear flow, the pre-consolidation pressure $p_c$ is not considered. Storage of plastic free energy, thus plastic free energy increase $d\Psi^p$ only related to the Reynolds effect. According to the above description, the plastic part of free energy can be seen as the function for $d\varepsilon_p^\tau$

$$d\Psi^p (\varepsilon_p^\tau) = \frac{d\Psi^p (\varepsilon_p^\tau)}{d\varepsilon_p^\tau} d\varepsilon_p^\tau = (d\varepsilon_p^\tau + \alpha d\varepsilon_q^\tau) \frac{d\Psi^p (\varepsilon_p^\tau)}{d\varepsilon_p^\tau} = (d\varepsilon_p^\tau + \alpha d\varepsilon_q^\tau)\Psi^p$$

(28)

Therefore, the reaction force components are

$$\rho = d\Psi^p / d\varepsilon_p^\tau = \Psi^p, \quad \xi = d\Psi^p / d\varepsilon_q^\tau = \alpha \Psi^p$$

(29)

Then the storage plasticity associated with the Reynolds effect is

$$\rho (-\alpha d\varepsilon_q^\tau) + \xi d\varepsilon_q^\tau = \rho (-\tan \theta d\varepsilon_q^\tau) + (\rho \tan \theta) d\varepsilon_q^\tau = 0$$

(30)

Formula (30) shows that the storage plastic work related to the Reynolds effect is zero, which satisfies $\xi / \rho = \tan \theta = \tan \Theta$ From the analysis in the previous section, the normal consolidation line is also satisfied in the dissipation space: $\tau / \pi = \tan \theta$, so the normal consolidation line (kncl) of the dissipative stress space is in the true stress space. $p-q$ a ray of origin further is converted to

$$\tan \phi_m = \tan \theta + \tan \phi$$

(31)

The above formula is the classic "Taylor stress dilatancy" relationship\textsuperscript{[13]}, that is, the Reynolds-Taylor line, which is also commonly called the line of destruction or peak strength, passing through the real stress space. $p-q$ Origin, which describes the peak state of sand under drainage (the undrained state is the state asymptote).
Figure 3. Yield locus in true stress space for a constrained shear deformation of a purely frictional material

As shown in Figure 3, point C corresponds to the position of reactive component \((\rho, \xi)\) in the real stress space. Comparing Fig. 2 and Fig. 3, for the shear deformation of the pure friction material, the three characteristic lines in the dissipative stress space respectively correspond to the three characteristic lines of the real space starting from the origin, that is, normal consolidation line (KNCL), phase change line (PTL) and peak line (RTL).

For the sake of simplicity, but highlighting the force characteristics of the granular material, the yield function of the real stress space is set as

\[ f = q - \eta p = 0 \]  

(32)

Here \(\eta\) is the stress ratio, although it does not appear in this yield function \(\alpha(\tan \theta)\). But it implied \(\eta\) and anisotropic parameters \(\alpha(\tan \theta)\) is closely related, \(\alpha\) changes as the stress ratio changes, that is, under shear stress, it will inevitably cause dilatancy, and the corresponding stress-induced anisotropy change of \(-\alpha\), and \(\alpha\) also known as the reaction force ratio (in the general stress space, also known as the reaction force than the tensor).

Based on the above analysis, it can be known that the thermodynamic method can properly describe the three characteristic lines of sand as pure friction material: normal consolidation line (KNCL), phase change line PTL and peak line RTL, which better describe the sand. Strain strengthening and softening characteristics, and this is precisely the boundary surface plasticity model of sand (Manzari and Dafalias, 1997)\(^{21}\). The mechanical characteristics of the sand reflected are well matched.

7. Summary

Firstly, from the perspective of micromechanics, the averaging theory is used to investigate the pore structure of granular materials and the increase of extra volume due to structural changes-dilatancy, which should be reflected in the plastic model. Through thermodynamic analysis, the anisotropic parameters of pore structure are introduced into the dissipative function, and the yield surface function of sand as a pure friction particle material in the dissipative space is established. Due to the influence of anisotropic fabric, it exists in the dissipative space. The three important lines, namely the normal consolidation line, the phase change line and the peak line, are correspondingly extended to the three characteristic lines of the original stress space: the normal consolidation line, the phase change line and the peak line. The ground reflects the mechanical characteristics of sand, especially dense sand. Therefore, the constitutive model of sand based on thermodynamic theory is in good agreement with the plastic model of sand boundary.
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