Bifurcation Analysis of a Relative Short Spherical Aerodynamic Journal Bearing System

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Abstract. This paper studies the nonlinear dynamic behavior and bifurcation of a rigid rotor supported by relative short spherical aerodynamic journal bearings. The modified Reynolds equation is solved by a hybrid numerical method combined with the differential transformation method and the finite difference method. The analytical results reveal a complex dynamic behavior including periodic, sub-harmonic, and quasi-periodic responses of the rotor center. Furthermore, the results reveal the changes which take place in the dynamic behavior of the bearing system as the rotor mass and bearing number increase. The current analytical results are found to be in good agreement with those of other numerical methods. Therefore, the proposed method provides an effective means of gaining insights into the nonlinear dynamics of relative short spherical aerodynamic rotor-bearing systems.

1. Introduction

Aerodynamic journal bearing systems are ideally suited for use in precision instrumentation due to their low noise during rotation and their zero friction when the instruments are used as null devices[1]. In 1994, Zhao et al. [2] investigated the sub-harmonic and quasi-periodic motions of an eccentric squeeze film damper-mounted rigid rotor system. The authors noted that for large values of unbalance and static misalignment, the sub-harmonic and quasi-periodic motions generated at speeds of more than twice the system critical speed were bifurcated from the unstable harmonic solution. Sundararajan & Noah [3] proposed a simple shooting scheme integrated with an arc-length continuation algorithm for the investigation of periodically forced rotor systems. Using this model, the authors predicted the occurrence of periodic, quasi-periodic and chaotic motion for various ranges of the rotor speed. In 2007, Wang [4-6] provides a further understanding of aerodynamic journal bearing systems and shows the dynamic behavior of system with respect to rotor mass and bearing number.

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The remainder of this study is organized as follows. Section 2 develops a mathematical model describing the time-dependent motions of the rotor center of a rigid rotor supported by two relative short spherical aerodynamic journal bearings. Accordingly, Section 3 develops a hybrid method combining the finite difference method (FDM) and the differential transformation method (DTM) to obtain the required solutions. Section 4 presents the simulation results obtained using the proposed hybrid method for the vibrations of the rotor center for various rotor masses and bearing numbers, respectively. Finally, Section 5 draws some brief conclusions.

2. Mathematical Modelling

The spherical aerodynamic journal bearing model developed is based on the assumptions that the gas flow is isothermal and the gas viscosity is constant. The pressure distribution in the gas film between the shaft and the bushing is modeled by the Reynolds’ equation, i.e.

\[
\csc \phi \frac{\partial}{\partial \phi} \left( \frac{1}{\csc \phi} \frac{\partial p}{\partial \phi} \right) + \csc^2 \phi \frac{\partial}{\partial \theta} \left( \frac{p h^3}{\partial \theta} \frac{\partial p}{\partial \theta} \right) = \Lambda_s \frac{\partial}{\partial \phi} (p h) + \sigma \frac{\partial}{\partial \tau} (p h)
\]

where \( p \) is the dimensionless pressure corresponding to the atmospheric pressure, \( P_0 \); \( h \) is the dimensionless gap between the rotating shaft and the bushing, corresponding to the radial clearance, \( C_r \); \( \mu \) is the gas viscosity; \( \phi \) and \( \theta \) are the coordinates, \( \Lambda_s \) is the bearing number, and \( \sigma \) is the squeeze number.

The analysis presented in this paper considers a rotor-bearing system comprised of a perfectly balanced rigid rotor of mass \( m_r \) supported symmetrically on two identical short journal bearings, which in turn are mounted on rigid pedestals. It is further assumed that during the entire operation, the rotor and bearing axes remain aligned. With these idealizations, it is possible to consider one bearing in isolation in which a rotating rotor of mass \( m_r \) has two degrees of translatory oscillations in the transverse plane.

3. Mathematical Formulation of Hybrid Method Combined differential transformation method and finite difference method

Differential transformation is one of the most widely used techniques for solving differential equations due to its rapid convergence rate and minimal calculation error. A further advantage of this method over the integral transformation approach is its ability to solve nonlinear differential equations. [7]

In solving the Reynolds equation for the current aerodynamic journal bearing system, the differential transformation method is used for taking transformation with respect to the time domain \( \tau \), and hence Eq. (2) becomes

\[
\csc \phi \otimes M \otimes \frac{\partial H}{\partial \theta} + \csc \phi \otimes J \otimes \frac{\partial Q}{\partial \theta} = 2 \Lambda_s \frac{\partial P}{\partial \phi} \otimes H + 2 \Lambda_s \frac{\partial P}{\partial \phi} \otimes P + 2 \sigma \frac{\partial P}{\partial \tau} \otimes H + 2 \sigma \frac{\partial P}{\partial \tau} \otimes P
\]

where

\[
Q(k) = P^2 = P \otimes P = \sum_{l=0}^{k} P_{i,j}(k-l)P_{i,j}(l)
\]

\[
I(k) = H^2 = H \otimes H = \sum_{l=0}^{k} H_{i,j}(k-l)H_{i,j}(l)
\]

\[
J(k) = H^2 = H \otimes H = \sum_{l=0}^{k} H_{i,j}(k-l) \sum_{m=0}^{l} H_{i,j}(l-m)H_{i,j}(m)
\]
The finite difference method is then used to discretize Eq. (2) with respect to the θ and φ directions. Note that Eq. (2) is discretized using the second-order-accurate central-difference scheme for both the first and the second derivatives.

4. Results and Discussions

4.1. Case I

The gas bearing is loaded with a rotational velocity of \( \omega = 1288 \text{ rad/s} \) and the rotor mass \( m_r \) is chosen as the bifurcation parameter.

4.1.1. Dynamic orbits and phase portraits

Figures 1.1(a)…1.4(a) show that the dynamic orbit of the rotor center is regular at a low value of the rotor mass (\( m_r = 2.1 \text{ kg} \)), but becomes irregular at \( m_r = 3.48 \text{ kg} \). At a rotor mass of \( m_r = 5.19 \text{ kg} \), the rotor center behaves irregular quasi-periodic motion. At rotor mass values of \( m_r = 10.79 \text{ kg} \), the rotor center performs subharmonic-2T-periodic motion.

4.1.2. Power spectra

Figures 1.1(b)…1.4(b) show the dynamic response of the rotor center as a function of the rotor mass in the horizontal direction. At a low rotor mass of \( m_r = 2.1 \text{ kg} \), the power spectra indicate that the rotor center performs T-periodic motion. As the rotor mass is increased to \( m_r = 3.48 \text{ kg} \), it can be seen that the rotor center motion transits to quasi-periodic motion. When the rotor mass is further increased to \( m_r = 5.19 \text{ kg} \), the rotor center performs quasi-periodic motion in the horizontal direction. Finally, at rotor mass values of \( m_r = 10.79 \text{ kg} \), the power spectra indicate that the rotor center performs, 2T sub-harmonic motion.

4.1.3. Bifurcation diagrams and Poincaré maps

Figure 1 The trajectory of rotor center at \( m_r = 2.1, 3.48, 5.19 \text{ and } 10.79 \text{ kg} \) (figures 1.1a-1.4a) and (figures 1.1b-1.4b) power spectrum of rotor displacement in horizontal direction (at \( \omega = 1288 \text{ rad/s} \)).

The bifurcation diagrams in figure 2 plot the rotor center displacement against the rotor mass \( m_r \) for values of the rotor mass in the range 1.0 to 12.0 kg. Observing Figures 2, it is seen that the rotor center performs T-periodic motion in both the horizontal and the vertical directions at low values of the rotor mass, i.e. \( m_r < 3.48 \text{ kg} \). This is confirmed by the Poincaré map shown in figure 3(a) for a rotor mass of 2.1 kg. However, at \( m_r = 3.48 \text{ kg} \), the T-periodic motion transits to quasi-periodic motion (see figure...
3(b)). From figure 2, it is seen that the quasi-periodic motion and T-periodic motion are both appeared over the rotor mass range $3.48 \leq m_r < 3.59$ kg. As the rotor mass is increased from 3.59 kg to 8.09 kg, the quasi-periodic motion persists over the rotor mass range $3.59 \leq m_r \leq 8.09$ kg. Figure 2 shows that this quasi-periodic motion reverts to T-periodic motion at $m_r = 8.1$ kg. However, at $m_r = 10.79$ kg, the T-periodic motion is replaced by 2T-periodic motion in the horizontal and vertical directions. Figure 2 shows that this 2T-periodic motion is maintained over the rotor mass interval $10.79 \leq m_r < 11.73$ kg, but reverts to T-periodic motion over the rotor mass range of $11.73 \leq m_r < 12$ kg.

![Figure 2 Bifurcation diagrams: (a) X(nT) and (b) Y(nT) versus rotor mass $m_r$ at $\omega = 1288$ rad/s.](image)

Figure 2 Bifurcation diagrams: (a) X(nT) and (b) Y(nT) versus rotor mass $m_r$ at $\omega = 1288$ rad/s.

![Figure 3 Poincaré maps of rotor center trajectory at: (a) $m_r = 2.1$, and (b) 3.48 kg.](image)

Figure 3 Poincaré maps of rotor center trajectory at: (a) $m_r = 2.1$, and (b) 3.48 kg.

4.2. Case II

In the second dynamic analysis case, the rotor mass is specified as $m_r = 3.2$ kg and the bearing number $\Lambda$ is chosen as the bifurcation parameter.

4.2.1. Dynamic orbits and phase portraits

Figures 4.1(a),..,4.4(a) show that the orbits of the rotor center are regular at low bearing numbers (i.e. $\Lambda = 1.2$), but become irregular at $\Lambda = 1.82$. At a bearing number of $\Lambda = 2.93$ and 6.43, the rotor center motion reverts to irregular, sub-harmonic and quasi-periodic motion, respectively.

4.2.2. Power spectra

Figures 4.1(b),..,4.4(b) show the dynamic response of the rotor center as a function of the bearing number in the horizontal direction. It is seen that the rotor center performs harmonic motion in both the horizontal direction at a bearing number of $\Lambda = 1.2$. However, when the bearing number is increased to $\Lambda = 1.82$, the power spectra show that the rotor center exhibits quasi-periodic motion. And the quasi-periodic motion becomes sub-harmonic motion with a period of 2T at $\Lambda = 2.93$. Finally, at $\Lambda = 6.43$, the rotor motion reverts to quasi-periodic motion in the horizontal direction.
4.2.3. Bifurcation diagrams and Poincaré maps
The bifurcation diagrams in figure 5 plot the rotor center displacement against the bearing number $A$ over the range $0.5 \leq A < 7.0$. At a low value of the bearing number ($A=1.2$), the rotor center performs T-periodic motion in both the horizontal and the vertical directions (see figure 6(a)). However, at $A=1.82$, the T-periodic motion becomes unstable and is replaced by quasi-periodic motion. Figure 5 shows that the rotor center performs quasi-periodic motion over the bearing number interval $1.82 \leq A < 2.49$ except $A=2.26$ (T-periodic motion). As shown in figure 5, this quasi-periodic motion is replaced by T-periodic motion in both the horizontal and the vertical directions for bearing numbers in the range $A=2.49$ to 2.92. At a bearing number of $A=2.93$, the T-periodic motion abruptly transits to 2T-periodic motion as shown in figure 6(b). Then, as the bearing number is increased over the range 2.93 $\leq A < 3.03$, the rotor center behaves 2T-periodic motion. The evolution of the rotor center motion behavior over the bearing number range $2.49 \leq A < 6.43$ can be summarized as follows: T-2T-T-2T-T-2T. Finally, as $A$ is increased from $A=6.43$ to $A=7$, the rotor center motion evolves through the following behaviors: quasi-2T-quasi.

![Image](image_url)

Figure 4 The trajectory of rotor center at $A=1.2, 1.82, 2.93$ and 6.43 (Figures 4.1a-4.4a) and (Figures 4.1b-4.4b) power spectrum of rotor displacement in horizontal direction (at $m_r = 3.2$ kg).

5. Conclusions
Regarding the behavior of the bearing system with respect to variations in the rotor mass, the results indicate that the rotor center exhibits a stable behavior when it performs T or 2T-periodic motion. At rotor mass of $m_r = 2.1$ kg and 3.48kg, the Poincaré maps present the form of a single point and a closed curve indicating that the rotor center performs T-periodic and quasi-periodic motion, respectively. However, under quasi-periodic motion, the rotor becomes unstable; particularly in the two intervals. Regarding the effect of the bearing number on the behavior of the bearing system, the rotor center exhibits an unstable behavior when it performs quasi-periodic motion in the three intervals.

The results presented in this study provide general guidelines for the design of a relative short spherical aerodynamic journal bearing system and indicate suitable operating conditions which suppress non-periodic motion and therefore prevent unstable behavior.
Figure 5 Bifurcation diagrams: (a) $X(nT)$ and (b) $Y(nT)$ versus bearing number $\Lambda$ over interval $0.5 \leq \Lambda < 5$ at $m_r = 3.2$ kg.

Figure 6 Poincaré maps of rotor center trajectory at (a) $\Lambda = 1.2$, and (b) 2.93

Acknowledgement
The financial support of this research by National Science Council of R.O.C., under the project No. NSC-95-2221-E269-016 is greatly appreciated.

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