Modeling of the turbulent burning velocity for planar and Bunsen flames over a wide range of conditions

Zhen Lu\textsuperscript{1,2}, and Yue Yang\textsuperscript{1,2,3}\textsuperscript{*}

\textsuperscript{1}State Key Laboratory for Turbulent and Complex Systems, College of Engineering, Peking University, Beijing 100871, China; \textsuperscript{2}BIC-ESAT, Peking University, Beijing 100871, China; \textsuperscript{3}HEDPS-CAPT, Peking University, Beijing 100871, China

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We develop and assess a model of the turbulent burning velocity $s_T$ over a wide range of conditions. The aim is to obtain an explicit $s_T$ model for turbulent combustion modeling and flame analysis. The model consists of sub models of the stretch factor and the turbulent flame area. The stretch factor characterizes the flame response of turbulence stretch and incorporates detailed chemistry and transport effects with a lookup table of laminar counterflow flames. The flame area model captures the area growth based on Lagrangian statistics of propagating surfaces and considers the effects of turbulence length scales and fuel characteristics. The present model predicts $s_T$ via an algebraic expression without free parameters. We assess the model using 490 cases of the direct numerical simulation or experiment reported from various research groups on planar and Bunsen flames over a wide range of conditions, covering fuels from hydrogen to $n$-dodecane, pressures from 1 to 30 atm, lean and rich mixtures, turbulence intensity ratios from 0.1 to 177.6, and turbulence length ratios from 0.5 to 66.7. Despite the scattering $s_T$ data in the literature, the comprehensive comparison shows that the proposed $s_T$ model has an overall good agreement over the wide range of conditions, with the averaged modeling error of 28.1%.

Turbulent burning velocity, Turbulent premixed flame, Flame speed

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1. Introduction

The turbulent burning velocity (or the turbulent flame speed) $s_T$ is one of the most important statistics for turbulent premixed combustion\textsuperscript{[1-4]}. It is an indicator of the reactant consumption rate or a measure of the flame propagation speed, closely related to the fuel efficiency, heat release rate, and flame dynamics. Among problems related to $s_T$, a predictive model with a small set of key characteristic parameters is of practical interest for industrial design and combustion modeling. The $s_T$ model is used in various turbulent combustion models\textsuperscript{[1, 2]} to close the nonlinear source term, as a crucial component in the modeling of turbulent premixed combustion. An explicit expression for $s_T$ also helps to build the low-order, analytical models\textsuperscript{[5, 6]} on the turbulent flame dynamics. However, extensive studies have shown that $s_T$ depends on a variety of factors\textsuperscript{[1-4]}, which poses an enormous challenge for developing a simple predictive model of $s_T$.

Various scaling laws and empirical models\textsuperscript{[7-15]} have been proposed for $s_T$ (also see review articles\textsuperscript{[2-4]}) based on data of the experiment and direct numerical simulation (DNS). These algebraic models involve various flow parameters, dimensionless numbers, and model parameters for fitting the data. The major issue is that the model parameters are sensitive to flame configurations, flow conditions, and the definition of $s_T$\textsuperscript{[16]}, requiring \textit{ad hoc} adjustment for different conditions. The lack of a theoretical framework leads to the failure of the fitted correlations in a wide range of conditions, including turbulence intensity, integral length scale,
Theoretical development of the $s_T$ model is generally based on the flamelet concept [1] and the flame area estimation [17]. The fractal theory was applied to estimate the flame area in turbulence and then to develop a variety of $s_T$ models [18, 19]. Gouldin [18] modeled the flame area as a power law function for the ratio of the outer and inner cutoffs of the fractal geometry. Various values for the cutoffs, fractal dimension, and model constants were then reported [1, 19]. Another approach to model the flame area is via the $G$-equation [20]. Yakhot [21] derived a model based on the dynamic renormalization group and the $G$-equation. Peters [1] obtained an algebraic expression through the balance of turbulent production, flame propagation, and scalar dissipation terms. These models mostly ignore the diffusive turbulence effects on flamelets.

In addition to the modeling of the flame area, many studies [22-26] argued that small-scale turbulence could enhance the flame propagation by turbulence transport. For turbulent premixed flames at high Reynolds numbers, Zimont [23] derived a model by combining the small-scale turbulence effect on enhancing turbulent transport and the large-scale turbulence effect on wrinkling the flame surface. Romney and Yakhot [22] extended the flamelet-based model [21] with enhanced local flame speed by small-scale turbulence. Based on experimental observation at strong turbulence conditions, various correction factors [24-26] have been proposed to model the turbulence transport effects. Modeling flame area and small-scale turbulence characterize the turbulence effects on $s_T$, but neglect the flame stretch.

Thermo-diffusive effects modify the burning velocity of stretched flames [1, 27]. The thermo-diffusive effect on flamelet speed is represented by the stretch factor $I_0$ [28], which has been investigated numerically and experimentally [3, 29-33]. Diffusively neutral flames have $I_0 \approx 1$ [3], while the thermo-diffusive effects cause $I_0$ away from unity [31-33]. The thermo-diffusive instability also enhances or suppresses the growth of the flame surface. Trouvé and Poinso [34] showed that the area of unstable flames is larger than that of stable ones. Furthermore, Aspden et al. [35, 36] observed that the thermo-diffusive effects alter the thickening/thinning of the flamelets. These effects on the flamelet speed, flame area, and flamelet thickness imply a strong dependence of $s_T$ on the thermo-diffusive effects and the flame stretch. Several models [12-15] account for the thermo-diffusive effects through the Lewis number ($Le$) or the Markstein number. There are some efforts to incorporate the effects of detailed transport and chemistry in $s_T$ modeling [28, 29, 37]. A library of strained laminar flame was built to calculate $I_0$ [37], and then an empirical model of $I_0$ with fitting experiment data was developed to reduce the use of pre-computed libraries [28]. However, most existing models treat the flame stretch effects on $s_T$ as an isolated factor besides the turbulence field. This limitation hinders a robust performance on the prediction of $s_T$ over a wide range of conditions.

Recently, You and Yang [38] proposed a model of $s_T$ from the Lagrangian perspective. This model predicts $s_T$ for flames of several simple fuels at 1 atm, with universal turbulence-related model constants based on Lagrangian statistics of propagating surfaces [39, 40] in non-reacting homogeneous isotropic turbulence (HIT). Later Lu and Yang [41] investigated the lean hydrogen turbulent premixed flames at a range of pressures. They developed a model of $I_0$ with flamelet libraries to characterize the strong flame stretch effect on $s_T$ at high pressures. They also introduced $I_0$ into the flame area model to incorporate the influence of the thermo-diffusive effects. The $s_T$ model with effects of the flame area and stretch factor was validated with DNS results. However, the model application is restricted to a simple flame geometry, i.e., statistically planar flame propagation in HIT. Additionally, the ratio between the integral turbulence length and the flame thermal thickness is close to unity, and fuels are relatively simple such as hydrogen and methane in previous works [38, 41].

In the present study, we extend the Lagrangian-based modeling approach [38, 41] to develop a $s_T$ model for a wide range of conditions. Several new modeling ingredients are added into the existing model of $s_T$, including a scaling of turbulence length scales and an empirical model of a fuel-dependent coefficient for instabilities and fuel chemistry. This effort makes a crucial step towards a universal $s_T$ model for various turbulent flames without free parameters. We then use 490 DNS/experimental cases to assess the model performance. The datasets from a number of research groups cover fuels from hydrogen to n-dodecane, pressure from 1 to 30 atm, lean and rich mixtures, and a wide range of turbulence parameters. Flame configurations include planar and Bunsen flames, in accordance with the same consumption-based concept for model development. In addition, it is inevitable to include empirical parameters in modeling of $s_T$ due to the many factors influencing $s_T$. The inclusion of flame stretch effects with detailed chemistry and transport also introduces uncertainty through the chemical kinetic model. We quantify the model uncertainty with respect to the parameters and chemical kinetic model to extend the model applicability.

We remark that extensive efforts [1, 2] have been made to develop the model of $s_T$ with regard to the importance of $s_T$, but there is no consensus on whether or not we can obtain a universal predictive model of $s_T$ [27] due to the issues of inconsistent definitions, multiple parameter dependencies, measurement uncertainties, and scattering data points of $s_T$. Reactant species, equivalence ratio, pressure, etc.
The present study aims to develop an explicit $s_T$ model for a wide range of conditions with a systematic modeling approach and a large $s_T$ database with 490 cases. The proposed model cannot ensure the perfect prediction for each case, but it is able to achieve an overall good performance compared with existing models.

The rest of this paper is organized as follows. We present DNS/experimental cases of the turbulent planar and Bunsen flames used for model development and assessment in Sect. 2, and develop the model of $s_T$ in Sect. 3. Comprehensive comparisons of model predictions against DNS/experimental data of $s_T$ are presented in Sect. 4. Conclusions are drawn in Section 5.

2. DNS and experimental cases

We collect a number of DNS and experimental datasets to develop and assess the model of $s_T$ for planar and Bunsen flames over a wide range of conditions. Each dataset consists of a series of DNS/experimental cases for a realistic fuel in one referenced paper. The cases in a dataset are under various conditions, e.g., the pressure $p$, equivalence ratio $\phi$, unburnt temperature $T_u$, turbulence intensity $u'$, and integral length scale $l_i$. In particular, every case has a value of $s_T$ measured via either DNS or experiment.

Table 1 lists the datasets employed in the present study, together with the ranges of $p$, $\phi$, $T_u$. These datasets include

| Dataset | Configuration | Fuel | $p$ (atm) | $\phi$ | $T_u$ (K) | Mean error (%) | rms error (%) |
|---------|---------------|------|-----------|--------|-----------|----------------|---------------|
| 1. Aspden et al., 2011 [42] | planar | H$_2$ | 1 | 0.31,0.4 | 298 | 28.7 | 26.1 |
| 2. Aspden et al., 2015 [43] | planar | H$_2$ | 1 | 0.4 | 298 | 9.0 | 6.6 |
| 3. Lu and Yang, 2020 [41] | planar | H$_2$ | 1-10 | 0.6 | 300 | 9.0 | 5.0 |
| 4. Zhang et al., 2021 [44] | planar | H$_2$ | 1.5,10 | 0.6 | 300 | 9.4 | 8.6 |
| 5. Aspden et al., 2016 [35] | planar | CH$_4$ | 1 | 0.7 | 298 | 31.8 | 10.4 |
| 6. Aspden et al., 2017 [36] | planar | CH$_4$ | 1 | 0.7 | 298 | 15.0 | 3.9 |
| 7. Wang et al., 2017 [45] | planar | CH$_4$ | 20 | 0.5 | 810 | 26.7 | 7.0 |
| 8. Lapointe et al., 2015 [46] | planar | C$_7$H$_{16}$ | 1 | 0.9 | 298, 500, 800 | 17.1 | 9.8 |
| 9. Savard et al., 2017 [32] | planar | C$_8$H$_{18}$ | 1, 20 | 0.9 | 298 | 9.9 | 4.2 |
| 10. Aspden et al., 2017 [36] | planar | C$_{12}$H$_{26}$ | 1 | 0.7 | 298 | 7.4 | 4.8 |
| 11. Kobayashi, 2002 [47] | Bunsen | CH$_4$ | 1-30 | 0.9 | 300 | 32.4 | 18.2 |
| 12. Kobayashi et al., 2005 [8] | Bunsen | CH$_4$ | 1-10 | 0.9 | 300, 573 | 41.7 | 15.0 |
| 13. Fragner et al., 2015 [48] | Bunsen | CH$_4$ | 1-4 | 0.7-1.0 | 300 | 8.3 | 10.7 |
| 14. Muppala et al., 2005 [13] | Bunsen | CH$_4$ | 1,5,10 | 0.9 | 298 | 22.8 | 16.7 |
| 15. Yuen and G¨ulder, 2013 [49] | Bunsen | CH$_4$ | 1 | 0.6-1.0 | 300 | 22.1 | 13.1 |
| 16. Tamadonfar and G¨ulder, 2014 [50] | Bunsen | CH$_4$ | 1 | 0.7-1.0 | 298 | 43.6 | 8.3 |
| 17. Tamadonfar and G¨ulder, 2015 [51] | Bunsen | CH$_4$ | 1 | 0.7-1.35 | 298 | 17.6 | 12.2 |
| 18. Wabel et al., 2017 [26] | Bunsen | CH$_4$ | 1 | 0.75 | 298 | 45.0 | 20.1 |
| 19. Wang et al., 2015 [52] | Bunsen | CH$_4$ | 5, 10 | 1.0 | 298 | 41.8 | 9.8 |
| 20. Zhang et al., 2018 [53] | Bunsen | CH$_4$ | 1 | 0.89 | 298 | 6.5 | 4.3 |
| 21. Cohé et al., 2009 [54] | Bunsen | CH$_4$ | 1,9 | 0.6 | 298 | 47.0 | 13.7 |
| 22. Venkateswaran et al., 2015 [29] | Bunsen | CO/H$_2$ | 1.5,10 | 0.5-0.7 | 298 | 24.4 | 20.8 |
| 23. Zhang et al., 2018 [53] | Bunsen | CO/H$_2$ | 1 | 0.5-0.7 | 298 | 29.9 | 13.0 |
| 24. Cohé et al., 2007 [55] | Bunsen | CH$_2$/H$_2$ | 1-9 | 0.6-0.8 | 298 | 47.7 | 13.5 |
| 25. Zhang et al., 2020 [56] | Bunsen | CH$_2$/H$_2$ | 1 | 0.69-0.91 | 298 | 18.7 | 14.8 |
| 26. Ichikawa et al., 2019 [57] | Bunsen | CH$_2$/NH$_3$ | 5 | 0.9 | 298 | 15.5 | 7.2 |
| 27. Muppala et al., 2005 [13] | Bunsen | C$_2$H$_4$ | 5, 10 | 0.7 | 298 | 24.2 | 10.8 |
| 28. Tamadonfar and G¨ulder, 2015 [51] | Bunsen | C$_2$H$_6$ | 1 | 0.7-1.45 | 298 | 23.7 | 12.4 |
| 29. G¨ulder et al., 2000 [58] | Bunsen | C$_2$H$_8$ | 1 | 0.8,1.0 | 300 | 46.2 | 9.9 |
| 30. Zhang et al., 2018 [53] | Bunsen | C$_3$H$_8$ | 1 | 0.76 | 298 | 3.5 | 3.1 |
| 31. Yuen and G¨ulder, 2013 [49] | Bunsen | C$_3$H$_8$ | 1 | 0.7-1.0 | 300 | 20.0 | 12.9 |
| 32. Tamadonfar and G¨ulder, 2015 [51] | Bunsen | C$_3$H$_8$ | 1 | 0.8-1.35 | 298 | 18.8 | 7.8 |
| 33. Muppala et al., 2005 [13] | Bunsen | C$_3$H$_8$ | 5 | 0.9 | 298 | 32.9 | 17.3 |
490 DNS/experimental cases reported by worldwide research groups in 25 journal papers, and cover a wide range of conditions. The fuel species vary from hydrogen to large hydrocarbon molecules, with the pressure up to 30 atm and the equivalence ratio from very lean to rich. Figure 1 plots the parameters of the 490 cases in the diagram of turbulent premixed combustion. Here, \( Ka = \left( \frac{u'}{s_L^0} \right)^2 \left( \frac{\delta_L^0}{l} \right)^2 \) is the Karlovitz number, where \( s_L^0 \) and \( \delta_L^0 \) denote the laminar flame speed and thermal thickness of the unstrained one-dimensional laminar flame, respectively. The scattered data points indicate a broad distribution of case parameters, with \( u'/s_L^0 \) from 0.1 to 177.6 and \( l/\delta_L^0 \) from 0.5 to 66.7.

In all the selected datasets, \( s_T \) is computable based on the definition of the global consumption speed, which is an important criterion for our dataset selection. It is noted that there are several definitions for \( s_T \), i.e., the global consumption speed, local consumption speed, and local displacement speed [3, 27], and which definition to be employed depends on the flame configuration and measurement method. Since the calculated value of \( s_T \) may vary with its definition [3], the comparison of \( s_T \) from the data and the model must be based on the same definition.

In general, there are two flame configurations for the datasets in Table 1. The configuration of DNS datasets is the statistically planar turbulent premixed flame propagating in HIT, which has been extensively studied for turbulence-flame interactions [3, 4]. The DNS considers the turbulent flames in a box with inflow/outflow boundaries at the streamwise direction and periodic boundary conditions at the spanwise directions. The statistically one-dimensional propagating flames are free of mean strain and inhomogeneity, while a forcing term is added in the momentum equation to maintain the turbulence intensity. The consumption speed in the DNS is calculated by the integration of the fuel consumption rate over the entire computational domain as [27]:

\[
s_T = \frac{1}{\rho_u A_L (Y_{F,b} - Y_{F,a})} \int_{\Omega} \rho \dot{m}_F dV,
\]

where \( \rho_u \) is the density of unburnt mixture, \( A_L \) is the flame surface area of the laminar flame, \( Y_{F,b} \) and \( Y_{F,a} \) are the mass fractions of fuel species in unburnt and burnt mixtures, respectively, \( \Omega \) denotes the computational domain, and \( \dot{m}_F \) is the reaction rate of the fuel species. In Table 1, all the DNS datasets are labeled by “planar”.

The other configuration of experimental datasets is the turbulent premixed Bunsen flame, where the flame is anchored to the burner rim. For the Bunsen-type burner, the turbulence is typically generated by a perforated plate installed upstream of the burner outlet. A pilot is used to stabilize the flame over a range of turbulence intensities. Although the turbulence parameters \( u' \) and \( l \) may be nearly uniform, the wrinkling of the flame surface varies along the flame height, causing the measured \( s_T \) “geometry-dependent” [3]. For the Bunsen flames, the global consumption speed can be calculated by the ratio of the total mass flow rate \( \dot{m} \) of reactants and the averaged flame area \( A \) as [3]:

\[
s_T = \frac{\dot{m}}{\rho_u A},
\]

where \( A \) is calculated by an averaged progress variable \( \langle c \rangle \) isocontour obtained with time-averaging of instantaneous flame images of radical signals. It is necessary that all of the reactants pass through the flame brush in the experiment to calculate \( s_T \) via Eq. (2). The burner exit is enveloped by the flame brush for the Bunsen flames, satisfying the requirement. Another widely adopted approach to measure \( s_T \) is the angle method [47]. It utilizes the tip angle of the \( \langle c \rangle \) profile and the mean burner exit velocity to determine \( s_T \). As reported by Kobayashi et al. [8], \( s_T \) obtained via these two methods are consistent. A number of groups reported \( s_T \) over a wide range of conditions in turbulent Bunsen flames, and these datasets are labeled by “Bunsen” in Table 1. Although various values of \( \langle c \rangle \) from 0.05 to 0.5 were used [8,26,51], \( s_T \) data calculated with small \( \langle c \rangle \) from 0.05 to 0.2 is employed in the present study, in accordance with the turbulence parameters measured near the burner exit.

Moreover, there are several other flame geometries conventionally adopted for experimental measurements on \( s_T \), such as V-shaped flames [59], counterflow flames [33], swirl flames [9], and spherical flames [60,61], but different definitions of \( s_T \) were employed in these experiments. In principle, the \( s_T \) model should be validated against data obtained with the same definition [3], so the model assessment with different \( s_T \) definitions is only briefly discussed in Sect. 4.4.

3. Model development

In the present modeling approach, we first loosely decouple the contributions of different processes to \( s_T \), and model each
process explicitly. Then, all the sub models are combined into a model of $s_T$ without free parameters.

Utilizing the consumption-based definition of $s_T$ and the flamelet concept [28], the Damköhler hypothesis [17] suggests

$$\frac{s_T}{\delta} = I_0 \frac{A_T}{A_L}, \quad (3)$$

where $A_T$ denotes the turbulent flame area. The form of Eq. (3) implies that the turbulence influence on $s_T$ is decomposed into two parts, the stretch factor $I_0$ due to the flame response under flow variations [27,28], and the flame area ratio $A_T/A_L$ due to the surface stretching in turbulence. For modeling $A_T/A_L$, we assume the reaction layer is thin and the flame surface can be identified. Most cases listed in Table 1 satisfy this condition.

It is noted that several studies [4,22-26] suggest that small-scale eddies can penetrate into the preheat zone of turbulent flames, broadening the flame thickness and enhancing the local burning velocity. Consequently, a correction factor for small-scale turbulence is added into Eq. (3), and various corrections factors [23-25] based on the turbulence diffusivity have been proposed. Meanwhile, Aspden et al. [35,36] reported that the broadening of the preheat zone is fuel-dependent. They observed thinner flames for the lean hydrogen mixture in turbulent combustion. Due to the lack of a universal description of the flame thickening for different fuels, the present model has not included the small-scale turbulence effects.

In the present model, $I_0$ and $A_T/A_L$ in Eq. (3) are modeled separately, and the thermo-diffusive effects on the flame area are considered via the local flame speed in the modeling of $A_T/A_L$. Next, we introduce the sub models involved in Eq. (3) for predicting $s_T$.

### 3.1 Stretch factor

The flame stretch factor

$$I_0 = \frac{\langle s_L \rangle_A}{\delta_L}, \quad (4)$$

characterizes the effect of chemical kinetics and molecular transport on $s_T$, and it links the mean laminar flamelet consumption speed $\langle s_L \rangle_A$ to the unstretched laminar flame speed $\delta_L$, where $\langle \cdot \rangle_A$ denotes the average over the flame surface. The model of $I_0$ was proposed by Lu and Yang [41] and is improved in the present study on the modeling of turbulence stretch.

In turbulent flames, $I_0$ depends on the distribution of the curvature and strain rate over the local flamelet. The linear model of the laminar flame speed [27] and symmetric distribution of flame curvature [41,46] suggest that effects of positive and negative flame curvatures tend to cancel out in strong turbulence, and the influence of the strain rate can be approximated with a presumed probability density function. For the axisymmetric Bunsen flames, the round nozzle introduces a negative mean curvature of the flame and makes the distribution of curvature negatively skewed. The DNS of Bunsen flames [62] showed that the skewness decreases with the increase of the turbulence intensity. This study also suggested that the curvature contribution is relatively small as the nozzle diameter, inversely proportional to the mean curvature, is much larger than the flame thickness. Therefore, the mean curvature effects are neglected in the present model. Assuming the Dirac distribution for simplicity, the averaged consumption speed over the flame surface can be approximated as $\langle s_L \rangle_A = s_L$ [27].

The thermo-diffusive effects alter $s_L$ with respect to the stretch on flames. Asymptotic analysis [27] showed a simple relation $s_L/\delta_L = 1 - MaKa$ for weak or moderate stretch, where $Ma$ is the Markstein number. In order to investigate the stretch effects with detailed chemistry and molecular transport, the response of $s_L$ to stretch in one-dimensional stretched flames, such as counterflow and cylindrical flames, can be employed as reference solutions [30,37].

We model $I_0$ using a lookup table $F$ formed by laminar flame data [41] to capture the effects of detailed fuel chemistry and transport. Laminar counterflow flames with two streams of the cold mixture and the corresponding equilibrium product are simulated to build the table $F$. This configuration is similar to the one-dimensional turbulent flame condition, with a pool of hot product mixing with the unburnt mixture. For each counterflow flame solution, the consumption speed is calculated to obtain the ratio $s_L/\delta_L$, and the strain rate $\alpha$ is estimated by the velocity gradient at the location with the maximum fuel consumption rate. Using a series of counterflow simulations from weak to strong stretching, a table of $s_L/\delta_L$ versus $KaLe$ is obtained, where the Karlovitz number of the laminar counterflow flame is calculated as:

$$Ka = \alpha \frac{\delta_L}{s_L}, \quad (5)$$

For a certain condition, the stretch factor of the turbulent premixed flame is retrieved from the table as:

$$I_0(K) = \frac{s_L(K)}{\delta_L} = F \left( K \sqrt{\frac{\nu}{\nu_0}} \right). \quad (6)$$

Here,

$$K = 0.25 \left( \frac{\nu}{\delta_L} \right)^2 Re^{-\frac{1}{2}}$$

(7)
is the model proposed by Bradley et al. [63] for the turbulence stretch effect on flames, and \( p_0 = 1 \text{ atm} \) is a reference value for normalization, where \( Re = u' \ell_1 / \nu \) denotes the Reynolds number. We find that this model is more generalized than that we used in Eq. (7) in Ref. [41].

By incorporating the effects of detailed chemistry and transport, the \( I_0 \) model in Eq. (6) can capture the response of \( s_L \) to stretch for various reactants. Note that this counterflow configuration has the consumption speed decreasing towards zero with the increasing stretch. At the same time, the extinction does not happen as there is support from the equilibrium stream. Therefore, this \( I_0 \) model implies the decrease of \( s_T \) at very large turbulence intensities. It cannot predict global quenching. Our preliminary application of the modeled \( I_0 \) on lean hydrogen flames demonstrated that this type of model significantly improves the prediction of \( s_T \) from the simple model with \( I_0 = 1 \) at a broad range of pressure conditions [41].

### 3.2 Turbulent flame area

To estimate the flame surface ratio \( A_T/A_L \), we apply the modeling approach developed by You and Yang [38] based on theoretical analysis on Lagrangian statistics of propagating surfaces [39, 40]. The essence of this modeling framework is summarized below.

As an important component in Eq. (3), \( A_T/A_L \) is approximated by the area ratio \( A(t^*)/A_L \) of global propagating surfaces at a truncation time \( t^* \), which signals that the characteristic curvature of initially planar propagating surfaces has reached the statistically stationary state in non-reacting HIT. This state resembles the statistical equilibrium state in combustion between the flame area growth due to turbulent stretching and the area reduction due to flame self-propagation.

In the modeling of flame wrinkling in turbulence, the temporal growth of

\[
A(t^*)/A_L = \exp(\xi t^*)
\]  
(8)

is approximated by an exponential function. As the growth of the propagating surface area in HIT is self-similar [38, 40], the constant growth rate

\[
\xi = \mathcal{A} + B L_0^2 f_0^2,
\]

(9)

is approximated by a linear model in terms of \( I_0 \) and the dimensionless laminar flame speed \( s_L^0 = s_L^0 / s_L^0_{\text{ref}} \) normalized by a reference value \( s_L^0_{\text{ref}} = 1 \text{ m/s} \). Here, constants \( \mathcal{A} = 0.317 \) and \( B = 0.033 \) are fitted from Lagrangian statistics of propagating surfaces in non-reacting HIT in the DNS data, and \( I_0^2 \) accounts for the effects of flame stretch on the local burning velocity and the effects of thermo-diffusive instability on the flame area growth.

Considering \( t^* \) at limiting conditions of very weak \( u' / s_L^0 \to 0 \) and very strong \( u' / s_L^0 \to \infty \) turbulence, theoretical analysis and data fit of Lagrangian statistics of propagating surfaces yield

\[
t^* = T_{\infty}^* \left[ 1 - \exp \left( -\frac{CR e^{-1/4} u'}{\xi T_{\infty}^* s_L^0 I_0} \right) \right],
\]

(10)

where \( T_{\infty}^* = 5.5 \) is a universal truncation time determined as when the characteristic curvature of material surfaces reaches the stationary state in HIT. Note that the dependence on the turbulence intensity can be transformed as:

\[
Re^{-1/4} \frac{u'}{s_L^0} = \frac{I_0}{\eta},
\]

(11)

where \( I_0 = v / s_L^0 \) is the diffusive flame thickness and \( \eta \) is the Kolmogorov length scale. Equations (10) and (11) indicate that as \( u' / s_L^0 \) or \( I_0 / \eta \) grows, \( t^* \) increases and approaches \( T_{\infty}^* \).

This implies the increase of the characteristic curvature with \( u' / s_L^0 \) and \( I_0 / \eta \), and an upper limit for the maximum wrinkling (see Fig. 11 in Ref. [38]).

This asymptotic behavior is consistent with the decrease of the inner cutoff reported in a fractal analysis of turbulent premixed flames [18, 64, 65]. Substituting Eqs. (9) and (10) into Eq. (8) yields

\[
\frac{A_T}{A_L} = \exp \left( T_{\infty}^* \left( \mathcal{A} + B L_0^2 f_0^2 \right) \left[ 1 - \exp \left( -\frac{CRe^{-1/4} u'}{\xi T_{\infty}^* s_L^0 I_0} \right) \right] \right),
\]

(12)

where turbulence related constants \( \mathcal{A} \), \( B \), and \( T_{\infty}^* \) are universal, and \( C \) is a fuel-dependent coefficient which will be discussed later.

This flame area model captures the dependence of \( A_T \) on \( u' \) in very weak and strong turbulence [38, 41]. As \( u' \to 0 \), the Taylor expansion of Eq. (12) predicts a linear growth of \( A_T \) with \( u' \). As \( u' \to \infty \), the modeled \( A_T \) reaches an asymptotic state, showing the bending phenomenon of \( s_T \) with \( I_0 \approx 1 \). At \( p = 1 \text{ atm} \), the validation with several DNS datasets showed that the model in Eq. (12) captures the variation of \( s_T \) with \( u' \), outperforming previous models [38].

Moreover, the stretch factor \( I_0 \) in Eq. (12) introduces the thermo-diffusive effects on the local flame speed of the flame surface. The growth of the local flame speed enhances the growth of \( A_T \) [38], and vice versa. Figure 2 shows four typical sets of \( A_T/A_L \) modeling results, with \( Le \) varying from 0.4 for the hydrogen flame to 2.6 for the n-heptane flame. All the cases shown here are calculated with \( p = 1 \text{ atm} \) and \( \ell_1 / s_L^0 = 1 \). The results for methane flames with \( Le \approx 1 \) is similar to the original flame surface model with \( I_0 = 1 \) in
Ref [38]. For thermo-diffusive unstable flames, $I_0 > 1$ leads to the increase of the local flame speed, so the prediction of $A_T/A_L$ with Eq. (12) rises. For stable flames, $I_0 < 1$ mitigates the growth of $A_T/A_L$.

### 3.3 Turbulence length scales

It is well recognized that $s_T$ has a dependence on length scales for turbulent premixed flames. With the balance of turbulent production and scalar dissipation terms, Peters [1] showed that the turbulent flame area scales with the turbulence diffusivity in the thin reaction zone as:

$$
\frac{A_T}{A_L} \sim \sqrt{\frac{D_T}{D}} \sim \sqrt{\frac{u' l}{\delta^3_{t_0} L}}
$$

(13)

The spectral closure of the G-equation [11] yields that the scaling in Eq. (13) applies in both the corrugated flamelet and thin reaction zone regimes. The DNS of planar flames [35,45] also confirms the increase of $A_T/A_L$ with $l_i/\delta^0_L$.

From the perspective of the $A_T/A_L$ modeling with Eq. (8), the effects of the length scale ratio $l_i/\delta^0_L$ come from two aspects. First, the flame surface becomes more wrinkled with the increase of $l_i/\delta^0_L$. Consequently, it takes a longer time to reach the equilibrium state of the flame area, and the truncation time $t^*$ increases with $l_i/\delta^0_L$. Second, $l_i/\delta^0_L$ modifies the growth rate $\xi$ of flame area. Although the growth of propagating surfaces in non-reacting HIT is self-similar [38,40], the flame thickness introduces dependence of the flame surface stretch on the length scale. Meneveau and Poinot [66] showed that $l_i/\delta^0_L$ has significant influence on the flame stretch. An efficiency function of flame stretch was then proposed as a function of $l_i/\delta^0_L$. Therefore, the turbulence length scale should have influences on $t^*$ and $\xi$ in the flame area model Eq. (12).

As the dependence of $A_T/A_L$ on $u'$ has been modeled in Eq. (12) for both weak and strong turbulence conditions, we introduce a scaling of the length scales for the turbulence transport effects and keep the merit of Eq. (12) at the two turbulence limits.

To have a dependence on $l_i/\delta^0_L$ for $t^*$, Eq. (10) is modified as:

$$
t^* = T^*_1 \left\{ 1 - \exp \left[ -C R e^{-1/4} \frac{u'}{\xi T^*_1} \left( \frac{l_i}{\delta^0_L} \right)^{(1/2)} \right] \right\}
$$

(14)

Then, we approximate $A_T/A_L$ by the first-order Taylor expansion of Eq. (8) for $u' \rightarrow 0$ as:

$$
\lim_{u'/\delta^0_L \rightarrow 0} \frac{A_T}{A_L} = 1 + C R e^{-1/4} \left( \frac{l_i}{\delta^0_L} \right)^{(1/2)} \frac{u'}{\delta^0_L T^*_1}
$$

$$
= 1 + C R e^{-1/4} \frac{D u^{1/4}}{\delta^0_L T^*_1}
$$

(15)

where $R e_F = \delta^0_L / \nu$ is the flame Reynolds number with the kinematic viscosity $\nu$. $D = (l_i/\delta^0_L) (\delta^0_L / u')$ is the Damköhler number. As $u'/\delta^0_L \rightarrow \infty$ in very strong turbulence, the flame area modeled by Eq. (12) goes to an asymptotic value. The existence of the upper limit of $A_T/A_L$ is confirmed by experimental [26,49] and numerical [38,41,67] observations. To match the scaling with $l_i/\delta^0_L$, we propose:

$$
\lim_{u'/\delta^0_L \rightarrow \infty} \frac{A_T}{A_L} = \exp \left( \xi T^*_1 \left( \frac{l_i}{\delta^0_L} \right)^{(1/2)} \right)
$$

$$
= \exp \left\{ 1 + \frac{\ln \left( l_i/\delta^0_L \right)}{2 \xi T^*_1} \right\}
$$

(16)

The introduction of $l_i/\delta^0_L$ in Eq. (16) can be taken as a factor on the flame area growth rate, and it is consistent with the observation that the large-scale turbulent motion is efficient to distort the flame surface.

Combining the modifications on the truncation time and flame area growth rate, we obtain a model:

$$
\frac{A_T}{A_L} = \exp \left[ T^*_1 \left( A + B S_{\delta^0_L} u' \right) \frac{1}{2} \ln \left( \frac{l_i}{\delta^0_L} \right) \right]
$$

$$
\times \left\{ 1 - \exp \left[ -C R e^{-1/4} \left( \frac{l_i/\delta^0_L}{\delta^0_L T^*_1} \right)^{(1/2)} \frac{u'}{\delta^0_L T^*_1} \right] \right\}
$$

(17)

by incorporating the turbulence length scale effects into Eq. (12). We remark that the Taylor expansion of Eq. (17) in very weak turbulence is not exactly the same as Eq. (15) due to the factor introduced in Eq. (16), whereas the scaling $s_T \sim D u^{1/4}$ in Eq. (15) is kept. In addition, Eq. (17) leads to a questionable prediction $A_T/A_L < 1$ at the

---

**Figure 2** Comparison of $A_T/A_L$ calculated by model Eq. (12) for different fuels, with $Le$ from 0.4 to 2.6. All cases have $p = 1$ atm and $l_i/\delta^0_L = 1$, and $u'/\delta^0_L$ from 0 to 25.
limit \( l_i/\delta_{L0} \to 0 \), but this modeling artifact only exists for \( l_i/\delta_{L0} < \exp(-2\xi T_{\infty}) = 0.03 \). Regarding to the length scale \( l_i/\delta_{L0} \geq O(1) \) in practical cases in Fig. 1, this shortcoming can be neglected for most applications.

We illustrate the importance of the length scale modeling in \( s_T \) using three sets of DNS of planar flames [68]. It is noted that these DNS cases based on the progress variable and artificial chemistry parameters are not included in Table 1 and Fig. 1. We approximate and artificial chemistry parameters are not included in Table 2 and then Eq. (3) is simplified to \( s_T/\delta_{L0} = A_T/A_L \). The cases are set to have the gas expansion \( \rho_b/\rho_a = 6 \), where \( \rho_b \) is the density of the burnt gas. By adjusting the one-step chemistry coefficient, \( \delta_{L0} \) and \( \delta_{L1} \) in these cases are varied, as listed in Table 2.

Figure 3 compares the predictions of \( s_T \) from the present model Eq. (17) of \( A_T/A_L \) with the length scale effects and from the previous model Eq. (12) of \( A_T/A_L \), where \( C = 0.83 \) is further modeled by Eq. (18) with \( Le = 1 \) and \( I_0 = 1 \). The laminar flame parameters in groups R and L of this DNS series are the same, while \( l_i/\delta_{L0} = 2.955 \) and 1.545 are different. We find that the model prediction from Eq. (17) (solid lines) agrees well with the DNS results (symbols), showing the growth of \( s_T/\delta_{L0} \) with \( l_i/\delta_{L0} \). By contrast, the model in Eq. (12) (dash-dotted lines) fails to predict different \( s_T/\delta_{L0} \) with the length scale effect in these groups. In groups T and L, the laminar flame parameters are different and length scale ratios are close, but the contribution from small \( \delta_{L0} \) to \( s_T \) via \( \xi \) in Eq. (9) is negligible. Thus, the model predictions from Eq. (17) are close for these two groups.

### 3.4 Fuel-dependent coefficient

In Eq. (17), the model coefficient \( C \) characterizes the fuel effect on the growth of \( s_T \) in weak turbulence with \( u'/\delta_{L0} = O(1) \). Since hydrodynamic and thermo-diffusive instabilities drive the flame area growth in weak turbulence [69], unstable and stable flames have large and small \( C \), respectively. One way to determine \( C \) is from one or a few available DNS or experimental data points of \( s_T \) in weak turbulence. Alternatively, the constant value \( C = 2.0 \) is suggested for hydrogen mixtures, and \( C = 1.0 \) is recommended for other fuels such as methane [38].

Towards a predictive model of \( s_T \), we reduce the degree of freedom on the determination of \( C \) in the present work. First, we decompose \( C = C_0 I_0(K = 1) \), where \( I_0(K = 1) \) is obtained from stretch factor table \( T \), and \( C_0 \) only depends on the mixture composition. Based on the influence of hydrodynamic and thermo-diffusive instabilities on the flame area growth, we propose an empirical model:

\[
C_0 = \frac{1 - \rho_b/\rho_a}{Le},
\]

for various mixtures. This model implies stronger hydrodynamic and thermo-diffusive instabilities at smaller \( \rho_b/\rho_a \) and \( Le \), respectively, and predicts large \( C_0 \) for unstable cases.

Figure 4 shows the fit of \( C_0 \) from the DNS and experimental cases listed in Table 1 against Eq. (18), where the fitted \( C_0 \) is calculated using the nonlinear least square method for each dataset. The symbol size in Fig. 4 is proportional to the number of cases in each dataset. Note there is an apparent trend that \( C_0 \) decreases with the increase of \( Le \), though the data points are very scattered.

We remark that the severe scattering points of \( C_0 \) in Fig. 4 suggest that \( C_0 \) may depend on multiple parameters rather than a simple function in Eq. (18), so the data-driven methods can be used to fit \( C_0 \) in high-dimensional parameter space in the future work.

### 3.5 Explicit model of \( s_T \)

Substituting Eqs. (6) and (17) with all the model constants into Eq. (3), we have an explicit model of the turbulent burning velocity as:

\[
\frac{s_T}{\delta_{L0}} = I_0 \exp \left\{ \left( 1.742 + 0.182 \delta_{L0}^2 \right) + \frac{1}{2} \ln \left( \frac{L_0}{\delta_{L0}} \right) \right\}
\]
In Eq. (19), the model constants related to turbulence are universal, and the fuel-dependent parameters are obtained by the lookup table from a separate laminar flame calculation or the fit of DNS and experimental data in the literature. Therefore, the model Eq. (19) has no free parameters.

From the algebraic model in Eq. (19), $s_T$ is obtained from a given set of reactant and flow parameters. Specifically, the required inputs for $s_T$ predictions are the reactant species, equivalence ratio $\phi$, unburnt temperature $T_u$, pressure $p$, turbulence intensity $u'$, and integral length $l_i$. Moreover, the free propagation of laminar premixed flames and a series of counterflow flames are calculated to obtain the laminar flame parameters, including laminar flame speed $s_{0L}$, flame thermal thickness $\delta_{0L}$, stretch factor table $F$, and the Lewis number $Le$. Finally, $s_T/s_{0L}$ is calculated by substituting $u'$ and $l_i$ into Eq. (19). Additionally, the model prediction involves the uncertainty quantification for model parameters and chemical kinetics to extend the model applicability, which is detailed in Sect. S1 in the Supplementary Material. The above procedure has been implemented by a modularized code available online [70] with a brief manual in Sect. S2.

4. Model assessment

We validate the model of $s_T$ in Eq. (19) using the DNS and experimental datasets listed in Table 1. The model predictions and DNS/experimental results are first compared for several typical datasets, and the comparisons are categorized into different fuel types. Then, overall model performance is assessed with all datasets. Additionally, more model assessments and the performance comparison of different $s_T$ models are presented in Sects. S3 and S4, respectively.

In the model implementation, separate simulations of the laminar counterflow flames are carried out for each case with the corresponding case conditions to obtain the stretch factor table $F$. One-dimensional free flame simulations are conducted to obtain $\delta_{0L}$ and $\delta_{0T}$. For each DNS case, the same chemical mechanism in the original DNS is applied for the calculation. For experimental cases, the FFCM-1 mechanism [71] is applied for methane related cases, the Davis mechanism [72] is used for syngas cases, the UCSD mechanism [73] is employed for ethane, ethylene, and propane cases. All laminar flame information and thermal transport properties in the present model are calculated using Cantera [74]. For consistency, $u'$, $l_i$, and $s_T$ are taken from references as the raw data and normalized by the calculated $s_{0L}$ and $\delta_{0L}$ in the present study.

When the uncertainty range is presented in the following figures, grey shades denote the uncertainty ranges due to the model parameter uncertainties. Dark and light grey shades represent one and two standard deviations, corresponding to 68% and 95% of confidence intervals, respectively. This uncertainty quantification for model parameters and an additional one for chemical kinetics are discussed in detail in Sect. S1.

![Figure 4](image-url)

**Figure 4** Fit of $C_0$ in Eq. (18) using the DNS/experimental cases listed in Table 1. Each marker represents one set of cases, and the marker size is proportional to the number of cases in the dataset.
case with $\phi = 0.31$ (blue squares), even at a very large $u'/\delta_L^0$ up to 100. By contrast, the variation of $s_T$ with $u'$ shows a typical bending phenomenon for $\phi = 0.4$ (red circles), where $s_T$ stops growing and then decays with $u'$ in strong turbulence. We see that $I_0$ plays a dominant role in the prediction of $s_T/\delta_L^0$, while the flame area $A_T/A_L$ reaches a maximum at large $u'/\delta_L^0$. The present model (solid lines) well captures the different trends for the two equivalence ratios, including the decrease of $s_T$ in strong turbulence with $\phi = 0.4$, which is contributed by the variation of $I_0$ in Eq. (19) with laminar flame parameters. In addition, hydrogen flames at high pressures also have large $I_0$ due to the thermo-diffusive effects [41].

Although the turbulence intensities are large for the cases in Fig. 5, Aspden et al. [35,36] observed that the hydrogen flames have the thinner flame thickness with increasing turbulence intensity. They explained this phenomenon with the increasing reaction rate due to thermo-diffusive effects. It is argued that the thinner flame thickness makes small eddies harder to penetrate the preheat zone, so Eq. (3) can still apply for the lean hydrogen flames at strong turbulence conditions.

### 4.2 Methane

Methane is the largest component of natural gas, one of the major energy sources. The methane/air mixture has $L_e$ close to 1, resulting in $I_0 \approx 1$. As implied in Eq. (3), the turbulent burning velocity is mainly controlled by the flame area ratio.

In Fig. 6, the model predictions of $s_T$ (solid lines) are assessed by the DNS results (symbols) of lean methane premixed flames with $\phi = 0.7$, $p = 1$ atm, and different length ratios $l_i/\delta_L^0 = 4$ and $l_i/\delta_L^0 = 1$ from Aspden et al. [35,36]. We confirm that $I_0 \approx 1$ (dashed lines) for the methane flames, so the prediction of $s_T/\delta_L^0$ is governed by $A_T/A_L$ in the present model. The DNS results also indicate that the turbulent burning velocity increases with $l_i/\delta_L^0$. In general, $s_T/\delta_L^0$ with $l_i/\delta_L^0 = 4$ is about two times of that with $l_i/\delta_L^0 = 1$, consistent with the scaling in Eq. (13). With the length-scale effects incorporated in the modeling of $A_T$ in Eq. (17), our model predicts the rise of $s_T$ due to increased turbulence length scales.

Gülder and co-workers [24,49,58] investigated the effects of strong turbulence and small scales on the flame area and burning velocity. They argued that small-scale turbulence is important in enhancing turbulent flame propagation for $u'/\delta_L^0 > 7$. Figure 7 compares the experimental measurements and the model Eq. (19) for the methane flames with different equivalence ratios. The model predicts well for most cases with $u'/\delta_L^0 < 20$. Meanwhile, the model underestimates $s_T$ in the $\phi = 0.6$ cases, especially at the strong turbulence condition. A similar modeling discrepancy is found for the extreme turbulence cases of Wabel et al. [26]. The model fails to predict the persistent growth of $s_T/\delta_L^0$ at $u'/\delta_L^0$ up to 200, which may be due to the lack of the small-scale turbulence effects in the present model. As mentioned in Sect. 3, there have been some efforts [4,23-25] to model the small-scale turbulence effects on the local burning velocity based on turbulence field. However, the flame thickening was found to be fuel-dependent [35,36], so we still need further investigations to develop a general model for the small-scale turbulence effect.
For most cases employed for the model validation, the model estimation of \( s_T \) from Eq. (19) generally agrees with the DNS/experimental data. Although the predictions are not perfect for some cases, they correctly capture the general variation trends of \( s_T \), and basically cover the data points with reasonable uncertainty ranges. More model validations with other experiment data at various conditions are reported in Supplementary material.

4.3 Large hydrocarbon fuels

Large hydrocarbon molecules can be found in many practical fuels such as gasoline, diesel, and jet fuels. They are also employed to construct surrogate models for engineering applications. For the large hydrocarbon fuels, the large \( Le \) reduces the stretched laminar burning velocity \( s_L \), and the response of \( s_L \) to flame stretch is generally not sensitive to pressure [32]. The small stretch factor for the large hydrocarbon fuel leads to a lower value of \( s_T \), and the model prediction agrees well with the DNS result. Figure 8 compares model predictions (solid lines) and DNS results (symbols) of iso-octane premixed flames [32] at 1 and 20 atm. The 143-species skeletal mechanism [76] was used in our calculation of \( I_0 \) for this case. The model prediction is insensitive to pressure, consistent with the DNS observation. Compared with the methane mixture, the large \( Le \) of iso-octane leads to faster decay of \( I_0 \) with \( u'/s_L^0 \) (dashed line). With the modeled \( I_0 \) pre-computed from laminar flames, the model correctly predicts that \( s_T \) for heavy fuels is smaller than that for light fuels. We observe similar results in the comparison against the n-dodecane cases [36]. Note that the length scale ratio \( l_i/\delta_L^0 = 1 \) for the DNS dataset in Fig. 8. For practical conditions with \( l_i/\delta_L^0 \sim O(100) \), the scaling of Eq. (13) in the model Eq. (19) suggests reasonable \( s_T/s_L^0 \sim O(10) \).

4.4 Overall performance of model prediction

In the present study, we assess the model Eq. (19) using 490 DNS and experimental results of \( s_T \) over a wide range of fuels, equivalence ratios, pressures, turbulence intensities, and turbulence length scales. The validations above and additional ones in Sect. S3 in Supplementary Material have demonstrated that the proposed model gives overall good predictions of \( s_T \). To quantify the model accuracy, the modeling error for each case is defined by

\[
\epsilon = \frac{|s_T,\text{model} - s_T,\text{data}|}{s_T,\text{data}} \times 100\%,
\]

where \( s_T,\text{model} \) is the \( s_T \) predicted by the model in Eq. (19), and \( s_T,\text{data} \) is the \( s_T \) data obtained from DNS or experiment. The mean and root-mean-square (rms) errors for each dataset are summarized in Table 1.

Besides the model assessment through the representative cases for each type of fuel, Fig. 9 presents comprehensive comparisons between the model and DNS/experimental results for all the datasets listed in Table 1. Figure 9a is obtained from the model in Eq. (19) without free parameters. The corresponding averaged modeling error \( \bar{\epsilon} \) over all the cases is 28.1%.

Considering the scattering of the DNS and experimental data across different groups and the intrinsic measurement and statistical uncertainties involved in the datasets, the averaged modeling error 28.1% appears to be acceptable. This performance is the best among existing \( s_T \) models. In addition, existing \( s_T \) models are also assessed using the 490 cases...
in Table 1. We find that the combination of sub models of the turbulence intensity, pressure, Lewis number, and fuel chemistry is important for the accurate prediction of $s_T$ over a wide range of conditions. For example, the model [13] considering turbulence, pressure, and Lewis-number effects has relatively small $\bar{e} = 39.4\%$, and the models [1, 7, 23] considering turbulence effects have $\bar{e}$ larger than 44%. The detailed model assessment for the existing models is presented in Sect. S4 in Supplementary material.

On the other hand, the empirical model of the fuel-dependent coefficient in Eq. (18) can be improved regarding the severe scatter of $C_0$ in Fig. 4. If we apply $C_0$ presented in Fig. 4 for the corresponding dataset, i.e., the model of Eq. (19) has one free parameter $C_0$, the model prediction is significantly improved in Fig. 9b, and $\bar{e}$ is reduced from $28.1\%$ to $15.5\%$. This large reduction demonstrates that the complexity of the fuel property and the hydrodynamic instability in weak turbulence is a major source of the uncertainties in the modeling of $s_T$.

We remark that the model prediction is not satisfactory in some cases. For example, the model underestimates $s_T$ for the cases with the high turbulence intensities in Refs. [26,49]. The improvement may need proper modeling of the small-scale turbulence effects on the local burning rate. Furthermore, the model is not able to predict flame quenching.

The model prediction is also dependent on the flame geometry. Figures 10a, b compare the model predictions against the planar and Bunsen flame data, respectively. The model prediction for planar flames with $\bar{e} = 18.1\%$ is much better than that for Bunsen flames with $\bar{e} = 29.9\%$. Here are two possible reasons for the difference. First, the model in Eq. (19) is developed based on the flame propagation in HIT. This condition is the same for the DNS of planar flames. On the other hand, the Bunsen flames have geometric effects on the turbulence condition. For a given set of $u'$ and $l_s$ measured near the burner exit in the Bunsen dataset, the actual turbulence parameters are not constant in the flow field, and the flame wrinkling changes along with the height. In addition, the Bunsen flame geometry poses a negative averaged curvature on the flame, and a hot pilot stream is typically provided in an experiment to maintain a stable flame. These factors can alter the laminar and turbulent flame speeds. Thus, accurate modeling of the geometric effects on $s_T$ requires further investigations on the influences of the turbulence parameter distribution and flame shape. Second, the experimental dataset itself has very scattering data points. Different experimental setups and methods introduce large discrepancies between experimental datasets [2,3]. This is also implied as the large scatter of $C_0$ near Le $\approx 1$ in Fig. 4.

Although the present model has been only validated for turbulent planar and Bunsen flames due to the requirement of the same consumption-based $s_T$ definition, it can be applied to flames with other geometries. Our preliminary test shows that the present $s_T$ model prediction qualitatively agrees with the experimental data of V-flames of the methane/air mixture [59] and counterflow flames of the blended fuel mixture of CH$_4$/H$_2$ and C$_3$H$_6$/H$_2$ [33]. However, different $s_T$ definitions were used in these flame data. For spherical flames, most experiments observed the accelerating outwardly propagating flame front [60,61] which has not reached a statistically stationary state, and $s_T$ was reported as an average over a range of flame radii [61]. Since the present model does not include this transient effect in turbulent flame development, it tends to overpredict $s_T$ for spherical flames. Thus, the modeling of $s_T$ for the flames with complex geometries remains an open problem.

Figure 9  Comparison between DNS/experiment results and model predictions of $s_T/s_L^0$ for all the 490 data cases in Table 1. a The proposed model Eq. (19); b the model with ad hoc $C_0$. The symbol shape represents the fuel species, and the color denotes the pressure, which is the same as those in Fig. 1.
5. Conclusions

We develop a model of the turbulent burning velocity for a wide range of conditions covering various fuels, equivalence ratios, pressures, and turbulence intensities and length scales. Starting from the definition of the consumption speed, the model of $s_T$ involves the turbulence effects on the flame stretch and the flame area growth in Eq. (3).

The present model of $s_T$ in Eq. (19) has two major submodels. First, the flame response under turbulence stretch is characterized by the stretch factor $I_0$ in Eq. (6), and $I_0$ is retrieved from a lookup table calculated from laminar counterflow flames in the implementation. This model of $I_0$ incorporates the Lewis number and pressure effects for a variety of fuels. Second, the flame area model [38] based on Lagrangian statistics of propagating surfaces is improved to consider the effects of turbulence length scales and fuel characteristics. The modeling of the truncation time and area growth rate includes $l_t/\delta_L^0$ in Eqs. (14) and (16) to recover the scaling of the flame area ratio with the turbulence length scale. An empirical model in Eq. (18) for the fuel-dependent coefficient $C_0$ is proposed to quantify the effects of instabilities and fuel chemistry in weak turbulence. In the implementation [70], $s_T$ is explicitly calculated from the algebraic model in Eq. (19) with several given reactant and flow parameters. Therefore, this model has no free parameter.

We perform a comprehensive assessment for the $s_T$ model using 490 DNS/experimental cases reported from various research groups (see Table 1). The datasets for validation cover fuels from hydrogen to $n$-dodecane, pressures from 1 to 30 atm, and lean and rich mixtures. The model predictions and DNS/experimental results have an overall good agreement over the wide range of conditions, with the averaged modeling error 28.1%.

The features of the present model are summarized as follows. (1) The present model keeps the merit of previous ones [38, 41] on predicting the bending phenomenon of $s_T$ via modeling competing mechanisms of growth and reduction of the turbulent flame area. (2) The incorporation of the scaling for turbulence length scales extends the existing model to a wide range of turbulence parameters with $u'/s_L^0$ from 0.1 to 106.8 and $l_t/\delta_L^0$ from 0.5 to 51.5, so that the present model correctly predicts the growth of $s_T$ with $l_t/\delta_L^0$.

(3) Effects of detailed chemistry and transport are considered via the look-up table of $I_0$. This submodel characterizes the thermo-diffusive effects of reactants on $s_T$, which is important to obtain correct bending curves of $s_T$ for thermo-diffusive unstable mixtures with $Le < 1$. (4) The modeled fuel-dependent coefficient $C_0$ describes the influence of instabilities in weak turbulence, which makes the present model applicable for various fuel mixtures.

On the other hand, the present model shows notable discrepancies in a few cases due to the reasons below, which need further improvement. First, a major source for the model discrepancy is from the fit of $C_0$ in Eq. (18) from very scattering data points in Fig. 4. With an ideal model of $C_0$, the averaged modeling error can be reduced from 28.1% to 15.5%. Second, the uncertainties in the experiment measurement with varying unsteadiness, inhomogeneity, and mean strain also lead to notable model discrepancies. The averaged modeling error 29.9% for the Bunsen flames from experimental datasets is much larger than 18.1% for the planar flame from DNS datasets. Third, small-scale turbulence effects on thickening flamelets and enhancing $s_T$ at extreme turbulence conditions and curvature effects have not been considered.
We remark that the validation for the present $\tau_T$ model is restricted to planar and Bunsen flames. The model application for other flame geometries and different $\tau_T$ definitions requires further investigation. Moreover, this model still needs to be extended for practical combustion problems such as flame kernel development, swirling flame stabilization, and spray combustion.

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宽工况范围下自由传播平面火焰与本生灯火焰的湍流燃烧速度建模

卢臻，杨越

摘要 本文提出并验证了一种适用于宽工况范围的湍流燃烧速度模型。该模型具有显式表达式，可用于湍流燃烧理论分析和模型构建。模型主要由拉伸因子与湍流火焰面积两部分构成。拉伸因子描述了湍流拉伸导致的局部火焰速度变化。基于层流火焰的建表方法使拉伸因子模型能够考虑实际化学和输运特性的影响。火焰面积模型根据自传播面的拉格朗日统计信息刻画了湍流火焰面的增长规律，并考虑了湍流尺度和燃料特性的影响。模型参数主要由流动及火焰参数、通用常数和层流火焰结果查表得到。最后应用统一公式预测宽工况条件下的湍流燃烧速度变化。本文应用宽工况范围下的自由传播平面火焰与本生灯火焰直接数值模拟和实验数据对模型开展评估与验证。数据涵盖氢气至正十二烷等不同燃料、1–30个大气压、贫燃至富燃等不同当量比、湍流脉动速度与层流火焰速度比0.1–177.6、湍流积分长度尺度与层流火焰厚度比0.5–66.7等条件下的共490个湍流燃烧工况。对比不同工况的实验和模拟数据，模型预测的平均误差为28.1%。相较其他现有模型，该模型能正确描述不同燃料在宽工况范围下湍流燃烧速度变化趋势的差异。