Unifying Classical and Bayesian Revealed Preference

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Abstract

This paper relates the key results of two lines of work in revealed preference theory, namely, Bayesian revealed preference (Caplin and Dean, 2015) and classical revealed preference with non-linear budget constraints (Forges and Minelli, 2009). Classical revealed preference tests for utility maximization given known budget constraints. Bayesian revealed preference tests for costly information acquisition given a utility function. Our main result shows that the key theorem in Caplin and Dean (2015) on Bayesian revealed preference is a special case of the Afriat-type feasibility inequalities (Afriat, 1967) for general (non-linear) budget sets. Our second result exploits this connection between classical and Bayesian revealed preference to construct a monotone convex information acquisition cost from decision maker’s utilities and decisions in Bayesian revealed preference.

Keywords: Afriat’s Theorem, Revealed preference, Costly Information acquisition, Rational Inattention, Blackwell ordering, Utility Maximization Theory

JEL Classification: D11, D80

1 Introduction

Afriat’s theorem (Samuelson, 1938; Houthakker, 1950; Afriat, 1967; Varian, 1982) in classical revealed preference gives a necessary and sufficient condition so that a finite time series of linear budget constraints and consumption bundles can be rationalized by a concave monotone utility function. Forges and Minelli (2009) generalize...
Afriat’s theorem to general (non-linear) budget sets and construct a utility function that rationalizes the budget constraints and consumption bundles. More recently, in a Bayesian context, Caplin and Dean (2015) give necessary and sufficient conditions so that a sequence of utility functions, action selection policies and a prior can be rationalized by a monotone non-parametric information acquisition cost. In this paper, we refer to the test for costly information acquisition in Caplin and Dean (2015) as Bayesian revealed preference.

Both classical and Bayesian revealed preference test for economics based rationality. So it seems plausible that there exists a one-to-one correspondence between the results of Forges and Minelli (2009) and Caplin and Dean (2015). The main result of this paper is to construct a one-to-one correspondence between Bayesian and classical revealed preference. Our key finding is that the NIAC (No Improving Attention Cycles) condition of (Caplin and Dean, 2015, Theorem 1) in Bayesian revealed preference is a special case of the General Axiom of Revealed Preference (GARP) (Varian, 1982) used widely in classical revealed preference literature. To the best of our knowledge, this result is new. To prove this result, we extend the revealed preference result of Forges and Minelli (2009) to probability vectors in the unit simplex equipped with the Blackwell partial order (Blackwell, 1953).

A useful consequence of Theorem 4 is that the widely used GARP condition (2) in classical revealed preference is equivalent to a generalization (10) of the standard NIAC condition of Caplin and Dean (2015) in the Bayesian case. Specifically, we show that NIAC (Caplin and Dean, 2015) is equivalent to the feasibility of Afriat inequalities when the Lagrange multipliers are set to a constant. Indeed, GARP is an acyclic condition due to unconstrained Lagrange multipliers (marginal utility values) and is a weaker condition than the cyclical monotonicity condition of standard NIAC (Caplin and Dean, 2015).

Extending the revealed preference test of Afriat (1967) to more general partially ordered sets of consumption bundles dates back to Richter (1966), and more recently, to Nishimura et al. (2017) where the consumption bundles are partially ordered via first-order stochastic dominance. Freer and Martinelli (2016, 2021) generalize the classical revealed preference test to the partial order over probability distributions (mixed strategies). Unlike the problem setting in this paper, the decision maker in Freer and Martinelli (2021) does not update its belief via Bayes rule. The subtle distinction between Freer and Martinelli (2021) and our work is that the decision maker’s choice in this paper lies in the Cartesian product of probability simplices and thus requires a different partial order. Chambers et al. (2020) consider a generalized decision model (compared to Caplin and Dean (2015)) for the Bayesian agent, and give necessary and sufficient conditions for Bayesian rationality, namely, NIAS and GACI (Generalized Axiom of Costly Information) that generalize Theorem 1 in Caplin and Dean (2015). In this paper, we focus primarily on the result of Caplin and
and its connection to classical revealed preference. In spite of a unification flavor in the result of Chambers et al. (2020) where GACI and GARP are discussed in a similar vein, the variable map in the unification result of this paper is distinct from that used by Chambers et al. (2020) to formulate GACI. Finally, Freer and Martinelli (2022) unify multiple approaches in classical revealed preference theory under an algebraic axiom of revealed preference. Our result builds on Freer and Martinelli (2022) in that we connect classical revealed preference to Bayesian revealed preference (Caplin and Martin, 2015; Caplin and Dean, 2015) where the consumer’s response takes the form of probabilistic information structures and the aim is to test for costly information acquisition.

Since we will unify Bayesian and classical revealed preference, the reader might wonder: how to abstract Bayes rule into the classical revealed preference formulation? It is here that the Blackwell partial order is used. In the Bayesian framework, the decision maker computes the posterior belief of the state of nature via Bayes rule using a measurement (observation). We will show that the consumption bundle in classical revealed preference translates to the attention strategy (which is an observation likelihood) that links the decision maker’s prior belief to its posterior belief. In the classical case, a higher consumption good yields a larger utility for the decision maker. Thus, the utility function is a monotonically increasing function of the consumption bundle with respect to the natural (element-wise) partial order on Euclidean space. In complete analogy, for the Bayesian case, a more accurate attention strategy (in the Blackwell sense) results in a higher expected utility of the Bayesian decision maker. Hence, the expected utility in the Bayesian sense is monotonically increasing in the decision maker’s chosen attention strategy (response) with respect to the Blackwell order. This analogy is crucial for the main unification result of this paper, Theorem 4. For the reader’s convenience, Theorem 4 is schematically shown in Fig. 1.

Several reasons motivate our paper. Classical revealed preference and Bayesian revealed preference are developed largely independently in the literature; a notable exception being Varian (1983) where a classical revealed preference method is proposed to identify expected utility maximization. However, unlike Caplin and Dean (2015), Varian (1983) assumes the probability distribution over states of nature as an exogenous variable and is not encoded into the agent’s response. With our unification result, the results in these two areas can enrich each other. Moreover, apart from applications in economics, revealed preference methods have been applied in areas like machine learning, specifically for inverse reinforcement learning (Ng et al., 2000; Lopes et al., 2009; Dimitrakakis and Rothkopf, 2011; Hoiles et al., 2016, 2020; Pattanayak and Krishnamurthy, 2020) and interpretable AI (Pattanayak and Krishnamurthy, 2021).

For Bayesian revealed preference, it suffices to ensure weak monotonicity of the expected utility with respect to the partial order defined on the space of attention strategies. One well-known partial order that satisfies this condition is the Blackwell order.
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| Classical revealed preference | Bayesian revealed preference |
|------------------------------|-----------------------------|
| Afriat's test ([Afriat (1957)](Afriat (1957))) | Test for costly information acquisition ([Caplin and Dean (2015)](Caplin and Dean (2015))) |
| (Known linear budget constraint, unknown utility function) | (Known utility function, unknown cost, constant multipliers for expected utility) |
| Generalized Afriat’s test for non-linear budgets ([Forges and Minelli (2009)](Forges and Minelli (2009))) | Generalized test for costly information acquisition ([Theorem 3](Theorem 3)) |
| (Known non-linear budget constraints, unknown utility function) | (Known utility function, unknown cost, unknown multipliers for expected utility) |
| Generalized Afriat’s test for unknown non-linear budgets ([Theorem 2](Theorem 2)) | Equivalent via [Theorem 4](Theorem 4) |
| (Known utility function, unknown non-linear budget constraint) | |

Fig. 1 Schematic illustration of the main result in this paper. In Theorem 2, we devise a revealed preference test for known utility functions but unknown budget constraints. In Theorem 3, we state necessary and sufficient conditions for costly information acquisition of a decision maker, where the decision maker follows a generalized decision model compared to [Caplin and Dean (2015)](Caplin and Dean (2015)). Finally, in our main result, Theorem 4, we construct a one-to-one correspondence map between the revealed preference tests of Theorems 2 and 3.

2 Background

Since our main result relates classical and Bayesian revealed preferences, this section summarizes the main results of [Forges and Minelli (2009)](Forges and Minelli (2009)) (classical revealed preference) and [Caplin and Dean (2015)](Caplin and Dean (2015)) (Bayesian revealed preference).

2.1 Classical revealed preference (nonlinear budget)

**Theorem 1 ([Forges and Minelli (2009)](Forges and Minelli (2009)))** Given budget constraints \( g_k : \mathbb{R}_+^m \to \mathbb{R} \) and consumption bundles \( \beta_k \in \mathbb{R}_+^m \) such that \( g_k \) is a non-decreasing, locally non-satiated and continuous function, \( \{ \beta | g_k(\beta) \leq 0 \} \) is compact and \( g_k(\beta_k) = 0, k = 1, \ldots, K \). Then the following statements are equivalent.

1) There exists a monotone, continuous utility function \( u : \mathbb{R}_+^m \to \mathbb{R} \) that rationalizes the data set \( \{ \beta_k, g_k \}, k = 1, \ldots, K \). That is,

\[
\beta_k \in \arg\max_{\beta} u(\beta), \quad g_k(\beta) \leq 0 \tag{1}
\]

2) The data set \( \{ \beta_k, g_k \}, k = 1, \ldots, K \) satisfies GARP:

\[
\beta_k \geq_H \beta_j \implies g_j(\beta_k) \geq 0, \quad \forall k \neq j, \tag{2}
\]

where the relation \( \beta_k \geq_H \beta_j \) (‘revealed preferred to’) means there exists indices \( i_1, i_2, \ldots, i_L \) such that \( g_k(\beta_{i_1}) \leq 0, g_{i_1}(\beta_{i_2}) \leq 0, \ldots, g_{i_L}(\beta_j) \leq 0 \).

3) There exist positive scalars \( u_k, \lambda_k > 0, k = 1, 2, \ldots, K \) such that the following inequalities hold:

\[
u_s - u_t - \lambda_t g_L(\beta_s) \leq 0 \quad \forall t, s \in \{1, 2, \ldots, K\} \tag{3}\]
The monotone utility function $u$ defined as

$$u(\beta) = \min_{k \in \{1, \ldots, K\}} \{u_k + \lambda_k g_k(\beta)\}$$

constructed using $u_k$ and $\lambda_k$ rationalizes $\{\beta_k, g_k\}, k = 1, \ldots, K$ by satisfying (1).

Theorem 1 says that a sequence of budget constraints and consumption bundles are rationalized by a utility function if and only if a set of linear inequalities (3) has a feasible solution. The GARP condition (2) is equivalent (due to Varian (1982)) to the cyclical consistency condition proposed in Afriat (1967).

### 2.2 A modification of classical revealed preference. Sequence of utility functions and a single budget constraint

To establish a correspondence between classical and Bayesian revealed preference, we will need a modification of the result of Forges and Minelli (2009).

Forges and Minelli (2009) assume known a sequence of budget constraints and consumption bundles $\{g_k, \beta_k\}$, and reconstruct a utility function $u$ that rationalizes the observed variables (1). Consider the scenario where we know a sequence of utility functions and consumption bundles $\{u_k, \beta_k\}$. We assume the decision maker faces budget constraints of the form $\{g(\beta) - \gamma_k \leq 0\}$, where both the cost $g$ and positive scalars $\gamma_k$ are unknown and must be reconstructed from $\{u_k, \beta_k\}$. How to test for the existence and reconstruct a sequence of budget constraints $\{g(\beta) - \gamma_k\}$ that rationalizes the finite time series $\{u_k, \beta_k\}$ of utility functions and consumption bundles?

In complete analogy to Theorem 1, Theorem 2 below states necessary and sufficient conditions for the existence of a budget constraint when the utility functions and consumption bundles are known.

**Theorem 2** (Known utility, unspecified budget constraint) Given a sequence of monotone, continuous, locally non-satiated utility functions $u_k : \mathbb{R}_+^m \rightarrow \mathbb{R}$ and consumption bundles $\beta_k \in \mathbb{R}_+^m$, $k = 1, 2, \ldots, K$. Assume the decision maker faces a budget constraint of the form $\{g(\beta) - \gamma_k \leq 0\}$ for every $k$. Then the following statements are equivalent:

1) There exists a monotone, continuous, locally non-satiated cost $g : \mathbb{R}_+^m \rightarrow \mathbb{R}$ and positive scalars $\gamma_k$ such that the sequence of budget constraints $\{g(\beta) - \gamma_k \leq 0\}$ rationalize the data set $\{\beta_k, u_k\}, k = 1, \ldots, K$. That is,

$$\beta_k \in \arg\max_{\beta} u_k(\beta), \quad g(\beta) - \gamma_k \leq 0.$$  

Since both $u_k(\cdot)$ and $g(\cdot)$ are monotone, $\beta_k$ satisfies the condition $g(\beta_k) = \gamma_k$.

2) The data set $\{\beta_k, u_k(\beta_k)\}, k = 1, \ldots, K$ satisfies GARP (2).

3) There exist positive scalars $\hat{g}_k, \lambda_k > 0$, $k = 1, 2, \ldots, K$ such that the following inequalities hold:

$$\hat{g}_s - \hat{g}_t - \lambda t(u_t(\beta_s) - u_t(\beta_t)) \geq 0 \quad \forall t,s.$$  

The sequence of constraints $\{g(\beta) - \hat{g}_k \leq 0\}$, $k = 1, 2, \ldots, K$ satisfies (5) and rationalizes $\{\beta_k, u_k\}$, where $g(\cdot)$ is defined as

$$g(\beta) = \max_{k \in \{1, \ldots, K\}} \{\hat{g}_k + \lambda_k (u_k(\beta) - u_k(\beta_k))\}.$$  

We denote the inequalities (6) in abstract notation as CRP($\{u_k, \beta_k\}$).
The proof of Theorem 2 is in the appendix. At first sight, Theorem 2 appears to be a dual statement to the optimization in Theorem 1. Instead of establishing the existence and recover a utility given a sequence of known budget constraints, Theorem 2 aims to establish the existence of the budget constraint given a sequence of utility functions. However, the proof of Theorem 2 provided in the appendix, does not use duality. Theorem 2 tests if there exists a single cost $g$ and positive threshold values $\gamma_k$ such that $\beta_k$ maximizes utility $u_k$ given budget constraint $g_k(\beta) \leq \delta_k$.

Remarks.

• Why consider the above modification of classical revealed preference?

In the next section, we will map the setting in Theorem 2 to the Bayesian revealed preference problem of Caplin and Dean (2015).

• Why choose the form $g(\beta) \leq \gamma_k$ for the unknown budget?

The revealed preference test (6) in Theorem 2 has a degenerate solution if $\gamma_k$ is the same for all $k$ since the reconstructed cost $g$ will be a constant function.

• Theorem 2 assumes the elements in the sequence of constraints $\{g(\beta) - \gamma_k\}$ differ only by a scalar shift. This assumption can indeed be relaxed to allow any sequence of budget constraints. But the reconstructed constraints (7) are restricted to the space of monotone piece-wise linear convex functions identical up to a constant. As a result, the original constraints are non-identifiable since the reconstruction in (7) in Theorem 2 projects the original constraints to a smaller subspace of constraints.

2.3 Bayesian revealed preference

Since our aim is to relate Theorem 2 to Bayesian revealed preference, we review the key result of Caplin and Dean (2015).

Theorem 3 (Bayesian revealed preference (Caplin and Dean [2015])) Consider a finite set of states $\mathcal{X}$, actions $\mathcal{A}$, observations $\mathcal{Y}$ and prior pmf $\pi_0$ over $\mathcal{X}$. Given utility function $U_k : \mathcal{X} \times \mathcal{A} \rightarrow \mathbb{R}$ and attention strategy $\alpha_k : \mathcal{X} \rightarrow \Delta(\mathcal{Y})$, $k = 1, 2, \ldots, K$. We assume that, given observation $y$, the decision maker’s action maximizes the conditional expected utility, for all $y \in \mathcal{Y}$. Then, the following statements are equivalent.

1) There exists a monotone information acquisition cost $C(\alpha)$ and positive scalars $\lambda_k$ that rationalize the dataset $\{U_k, \alpha_k\}, k = 1, 2, \ldots, K$:

$$\alpha_k \in \arg\max_\alpha \lambda_k J(\alpha, U_k) - C(\alpha).$$

In (8), $J(\cdot)$ is the decision maker’s expected utility given attention strategy $\alpha$ and utility function $U$, defined as:

$$J(\alpha, U) = \sum_{y \in \mathcal{Y}, x \in \mathcal{X}} \pi_0(x)\alpha(y|x)U(x, a^*(y)),$$

$$a^*(y) \in \arg\max_{a \in \mathcal{A}} \sum_x \pi_0(x)\alpha(y|x)U(x, a).$$

2) Given dataset $\{U_k, \alpha_k\}, k = 1, 2, \ldots, K$, there exist positive scalars $c_k$, $\lambda_k$ such that the following inequalities hold:

$$c_j - c_k - \lambda_k \left( J(\alpha_j, U_k) - J(\alpha_k, U_k) \right) \geq 0,$$
where $J(\cdot)$ is defined in (9). We denote the inequalities (10) in abstract notation as \( \text{BRP}(\{ U_k, \alpha_k \}) \) and refer to (10) as the generalized NIAC condition.

3) There exists a monotone, weakly informative, convex cost $C(\alpha)$ of information acquisition and positive scalars $\lambda_k$ that rationalize the dataset $\{ U_k, \alpha_k \}, k = 1, 2, \ldots, K$ and satisfy (8).

**Proof:**

Statement (1) $\implies$ (2): From the definition of $\alpha_k$ in (8), the following inequalities result:

$$
\alpha_k \in \arg\max_\alpha \lambda_k J(\alpha, U_k) - C(\alpha)
\implies \alpha_k \in \arg\max_{\alpha \in \{ \alpha_1, \ldots, \alpha_M \}} \lambda_k J(\alpha, U_k) - C(\alpha)
\implies \lambda_k J(\alpha_k, U_k) - C(\alpha_k) \leq \lambda_j J(\alpha_j, U_k) - C(\alpha_j), \forall j \in \{1, 2, \ldots, M\}, j \neq k.
$$

Setting $C(\alpha_j) = c_j$ in the above inequality results in inequality (10) of statement (2).

Statement (2) $\implies$ (3): Let $\{ c_k, \lambda_k \}_{k=1}^M$ denote a feasible solution of inequality (10). Define cost $C(\alpha)$ as:

$$
C(\alpha) = \max_{k=1,2,\ldots,M} c_k + \lambda_k (J(\alpha, U_k) - J(\alpha_k, U_k))
$$

It can be immediately verified from the above definition that if (10) holds, then $C(\alpha_k) = c_k$ for all $k$. It only remains to show that the above cost $C$ rationalizes the dataset $\{ U_k, \alpha_k \}$, i.e., condition (8) holds. Fix index $k$. Then:

$$
C(\alpha) = \max_{k=1,2,\ldots,M} c_k + \lambda_k (J(\alpha, U_k) - J(\alpha_k, U_k))
\implies C(\alpha) \geq c_k + \lambda_k (J(\alpha, U_k) - J(\alpha_k, U_k)), \forall \alpha
\implies C(\alpha) - \lambda_k J(\alpha, U_k) \geq C(\alpha_k) - \lambda_k J(\alpha_k, U_k), \forall \alpha
\implies \alpha_k \in \arg\min_{\alpha} C(\alpha) - \lambda_k J(\alpha, U_k)
\implies \alpha_k \in \arg\max_{\alpha} \lambda_k J(\alpha, U_k) - C(\alpha) \equiv (8).
$$

Statement (3) $\implies$ (1): The proof is trivial since a monotone weakly informative convex cost is a special case of a monotone cost.

**Discussion.** Theorem 3 is stated slightly differently to Theorem 1 in Caplin and Dean (2015) but is useful for our purposes. The key differences are:

- Eq. (10) in Theorem 3 is equivalent to the NIAC condition of Caplin and Dean (2015) Theorem 1 with the added constraint that $\lambda_k = 1$ for all $k$. Eq. (10) can be interpreted as a test for relative optimality: For every experiment $k$, does attention strategy $\alpha_k$ maximize $\lambda_k J(\cdot, U_k) - C(\cdot)$ over the finite set of strategies $\{ \alpha_k \}$. Indeed, for arbitrary (non-constant) $\lambda_k$, NIAC cannot be represented as a cyclical monotonicity condition. We prove this equivalence below:

Eq. (10) $\lambda_k = 1 \implies$ NIAC (Caplin and Dean 2015): Fix a sequence of indices.
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$k_1, k_2, \ldots, k_m$, $m \leq K$, where $k_i \in \{1, 2, \ldots, K\}$, $\forall i = 1, 2, \ldots, m$. Further, suppose there exists a feasible solution $\{c_k\}$ to the inequalities in (10) with the added constraint that $\lambda_k = 1$. Then, the following inequalities result from the feasibility of (10):

$$J(\alpha_{k_1}, U_{k_1}) - c_{k_1} \geq J(\alpha_{k_2}, U_{k_1}) - c_{k_2}$$

$$J(\alpha_{k_2}, U_{k_2}) - c_{k_2} \geq J(\alpha_{k_3}, U_{k_2}) - c_{k_3}$$

$$\vdots$$

$$J(\alpha_{k_{m-1}}, U_{k_{m-1}}) - c_{k_{m-1}} \geq J(\alpha_{k_m}, U_{k_{m-1}}) - c_{k_m}$$

$$J(\alpha_{k_m}, U_{k_m}) - c_{k_m} \geq J(\alpha_{k_1}, U_{k_m}) - c_{k_1}$$

$$\Rightarrow \sum_{i=1}^{m} \left( J(\alpha_{k_i}, U_{k_i}) - J(\alpha_{k_{i+1}}, U_{k_i}) \right) \geq 0,$$ where $k_{m+1} = k_1$

$$= \text{NIAC} \quad \text{(Caplin and Dean (2015), Theorem 1)}.$$ 

The last inequality results from a telescoping sum due to the feasibility of (10) and the constraint $\lambda_k = 1$.

\text{NIAC} \quad \text{(Caplin and Dean (2015))} \Rightarrow \quad \text{Eq. (10)} \quad \lambda_k = 1; \quad \text{Caplin and Dean (2015, Proof of Sufficiency, Sec. 10.2)} \text{ prove that, if NIAC is satisfied, then there exist shadow prices } \{C_k\} \text{ (from duality theory (Boyd and Vandenberghe 2004)) that satisfy the following inequalities:}$

$$J(\alpha_{k}, U_{k}) - C_{k} \geq J(\alpha_{j}, U_{k}) - C_{j}, \quad \forall j, k = 1, 2, \ldots, K,$$

which is precisely the set of feasibility inequalities $\text{BRP}(\{U_k, \alpha_k\})$ (10) when $\lambda_k$ is set to 1.

- \text{Caplin and Dean (2015)} assume the attention strategy $\alpha_k$ is measured in noise as the action selection policy $\hat{\alpha}_k = \{p(a|x)\}$. $\hat{\alpha}_k$ is a noisy version (in the sense of \text{Blackwell (1953)}) of attention strategy $\alpha_k$. It can be shown using Blackwell dominance that the necessary and sufficient conditions (10) for Bayesian rationality imply Theorem 1 of \text{Caplin and Dean (2015), hence (10) is a stronger condition. Our unification result, Theorem 4 still holds if $\alpha_k$ in Theorem 3 is replaced by $\hat{\alpha}_k$.}$

- \text{Caplin and Dean (2015)} assume a single utility function $U$ and associate a decision problem $A_k \subseteq A$, a subset of allowable actions for every $k$. Without loss of generality, in Theorem 3, we assume the same set of allowable actions $A$, but different utility functions $U_k$.

- \text{Theorem 3 assumes that for every given observation $y$, the decision maker’s action $a$ maximizes its conditional expected utility $E_x\{U(x, a)\mid y\}$. This assumption implies the NIAS condition of \text{Caplin and Dean (2015)} holds and helps better highlight the saliency of our main result in the following section. However, for concreteness, Sec. 3 discusses how the NIAS condition can be formulated as a testable condition by a simple reformulation of the Bayesian revealed preference problem.}
Intuition behind Theorems 2 and 3: Theorem 2 assumes known utility functions, consumption bundles, and tests for the existence of budget constraints that rationalizes the known variables (5). In contrast, Theorem 3 assumes known utility functions, attention strategies (analogous to consumption bundles), and tests for the existence of a cost that rationalizes the known variables (8). Given the abstract description of the two revealed preference setups, it is intuitive that there exists a one-to-one correspondence between the two results. This relationship between Theorems 2 and 3 is established in the next section, thereby unifying classical and Bayesian revealed preference.

3 Main Result. Unification of Classical and Bayesian revealed preference

This section unifies classical and Bayesian revealed preference. Specifically, Theorem 4 below says that the test for Bayesian rationality (10) in Theorem 3 is equivalent to the classical revealed preference test (6) of Theorem 2. The key takeaway of Theorem 4 is:

The NIAC condition of Caplin and Dean (2015) is a special case of GARP (2). If NIAC (Caplin and Dean, 2015) holds, then the GARP condition (2) holds. If GARP holds and Afriat inequalities (6) are feasible with the Lagrange multipliers $\lambda k$ set to 1, then NIAC (Caplin and Dean, 2015) holds.

Theorem 4 (Unification of Classical and Bayesian Revealed Preference) Consider the Bayesian revealed preference problem of Theorem 3. Given the sequence of utility functions and attention strategies $\{U_k, \alpha_k\}, k = 1, 2, \ldots, K$. Then,

1. The Bayesian revealed preference test, $BRP(\cdot)$ (10) in Theorem 3, is equivalent to the classical revealed preference test, $CRP(\cdot)$ (6) in Theorem 2:

$$BRP(\{U_k, \alpha_k\}) \equiv CRP(\{u_k, \beta_k\})$$

The classical revealed preference parameters $\{u_k, \beta_k\}$ in (11) are defined in terms of $\{U_k, \alpha_k\}$ as follows:

$$\beta_k = \alpha_k, u_k(\beta) = J(\alpha, U_k),$$

where $J(\cdot)$ is the Bayesian decision maker’s expected utility (9).

2. The generalized NIAC condition (10) in Theorem 3 holds if and only if the sequence $\{\alpha_k, J(\alpha_k, U_k) - J(\cdot, U_k)\}$ satisfies GARP (2).

3. The reconstructed cost $g$ (7) in Theorem 2 is equivalent to the information acquisition cost $C$ reconstructed as:

$$C(\alpha) = \max_{k \in K} \{c_k + \lambda_k(J(\alpha, U_k) - J(\alpha_k, U_k))\},$$

where the positive scalars $c_k, \lambda_k$ are obtained by solving (10). The reconstructed cost $C$ also satisfies the axiomatic properties of weak monotonicity, mixture feasibility and normalization as theorized in Caplin and Dean (2015, Theorem 2).

The proof of Theorem 4 is discussed in detail in the next section. Recall that $BRP(\cdot)$ denotes the Bayesian revealed preference test (10) in Theorem 3 and $CRP(\cdot)$
Table 1 | Variable map that relates classical and Bayesian revealed preference (RP) in Theorem 4. The variable $J(\alpha, U)$ (9) is the expected utility given utility function $U$ and attention strategy $\alpha$. The key result is that the test for Bayesian rationality, BRP($\cdot$) (Theorem 3) can be formulated as the classical revealed preference test, CRP($\cdot$) (Theorem 2) via a careful parameter mapping.

denotes the classical revealed preference test (6) in Theorem 2. For the reader’s convenience, Table 1 presents the results of Theorem 4 in a tabular form to relate the various variables in classical and Bayesian revealed preference.

**Discussion.**

Theorem 4 presents two key results. First, it establishes the one-to-one correspondence between classical and Bayesian revealed preference and relates the variables used in both approaches. In comparison to classical revealed preference, we see from Table 1 that in the Bayesian case, the “effective” utility function is the expected utility $J(\cdot, U)$ that encodes both the prior pmf $\pi_0$ and utility $U$. The cost $g$ in classical revealed preference translates to the information acquisition cost $C$ in the Bayesian case. The scalars $\gamma_k$ are simply the cost $C$ evaluated at the attention strategies $\alpha_k$.

Second, Theorem 4 explicitly reconstructs a monotone convex cost of information acquisition in terms of the solution to the testable conditions (10) in Theorem 3. The construction has an Afriat-type flavor to it. Afriat (1967) ‘stitches’ together a piece-wise linear, concave utility function in terms of the utility function evaluated at the observed consumption bundles and the budget constraints, that rationalizes the data. In complete analogy, the convex cost $C(\cdot)$ (13) is constructed using the information acquisition cost evaluated at observed attention strategies $\alpha_k$ and the expected utility functional $J$. The reconstructed utility function in Afriat (1967) is locally non-satiated, monotone and concave, and rationalizes the observed price and consumption bundles. In complete analogy, the reconstructed cost (13) is weakly monotonic (in information (Blackwell partial order)), mixture feasible (convex) and normalized, and rationalizes the observed utility functions and attention strategies.
Remarks.

- **NIAC** ([Caplin and Dean, 2015]) and testing for quasilinear utility maximization: Brown and Calsamiglia [2007, Theorem 2.2] propose testable conditions for quasilinear utility maximization, namely, the feasibility of a set of linear inequalities. Due to the FOC of the quasilinear utility maximization problem, the linear inequalities are a special case of Afriat’s inequalities with the Lagrange multipliers set to 1. Equivalently, the cyclical monotonicity condition of Brown and Calsamiglia [2007, Theorem 2.2] is a special case (constant Lagrange multipliers) of the acyclic GARP condition (2). Following this analogy, the standard NIAC condition of Caplin and Dean [2015] is equivalent to testing for quasilinear utility maximization via the variable map of Table 1.

- For the sake of completeness, we remark that the point estimate below of the information acquisition cost $C(\cdot)$ in the style of Rockafellar [2015, Theorem 24.8] also rationalizes the dataset $\{\alpha_k, U_k\}$:

$$C(\alpha) = \sup_D \{J(\alpha_m, U_m) - J(\alpha, U_m) + \ldots + J(\alpha_1, U_1) - J(\alpha_2, U_1)\},$$

where the supremum is taken over all finite subsets of the dataset $\{\alpha_k, U_k\}$. The cost $C(\cdot)$ is weakly monotone in information and mixture feasible and can be proved in a way similar to Step 3 in Sec. 4. Statement (3) in Theorem 4 supplements (14) by providing an Afriat-type set-valued reconstruction of $C$.

- The NIAS assumption in Theorem 3 may be relaxed by introducing the decision maker’s action policy $\varepsilon_k : \mathcal{Y} \to \mathcal{A}$ given observation $y$ in the problem formulation and separately testing for the optimality of $\varepsilon_k$ via the following inequality:

$$\mathbb{E}_x \{U_k(x, a) - U_k(x, \varepsilon_k(y)) \mid y\} \leq 0, \forall a \in \mathcal{A}, y \in \mathcal{Y}.$$

The above inequality is precisely the NIAS condition. The NIAS condition can be encoded as an active constraint condition by a simple reformulation of Theorem 2 without affecting the unification result.

- In the parameter mapping of Table 1, the “response” $\alpha_k$ in the Bayesian setup lies in the unit simplex of probability mass functions. More precisely, the response belongs to the space $\Delta(\mathcal{Y})^{\mid \mathcal{X}}$, where $\Delta(\mathcal{Y})$ is the unit simplex of pmfs over the observation set $\mathcal{Y}$. Clearly, with respect to the natural element-wise partial order of Euclidean spaces, the expected utility $J(\alpha, U)$ and information acquisition cost $C(\alpha)$ are not monotonically increasing in $\alpha$. Hence, the unification result of Theorem 4 involves equipping $\Delta(\mathcal{Y})^{\mid \mathcal{X}}$, the compact space of attention strategies with a different partial order, namely, the Blackwell order [Blackwell, 1953] for probability measures. That Theorem 2 holds, and consequently, the unification result Theorem 4 holds with respect to the Blackwell partial order is the key topic of discussion in the outline of Proof of Theorem 4 in Sec. 4.
4 Proof of Theorem 4

The proof of our unification result, namely, Theorem 4 comprises three steps. First, we show that the assertions of Theorem 2 hold with respect to the Blackwell partial order. Second, we replace the variables in the classical setup with the equivalent variables (based on Table 1) from the Bayesian problem and show that the classical revealed preference test (6) of Theorem 2 is identical to (10). This establishes the correspondence between classical and Bayesian revealed preference methods. Third, we show the reconstructed information acquisition $C$ satisfies the three axiomatic properties, namely, weak monotonicity in information, mixture feasibility and normalization, hence concluding our proof.

**Step 1.** Lemma 1 below states that the results of Theorem 2 hold identically for the space of attention strategies over a finite set of states and observations, equipped with the Blackwell partial order. Definition 1 defines the Blackwell relation between a pair of attention strategies $\alpha$, $\bar{\alpha}$ by viewing the attention strategies as row-stochastic matrices.

**Definition 1 (Blackwell order [Blackwell, 1953])** Consider two attention strategies $\alpha$, $\bar{\alpha} \in \Delta(\mathcal{Y})^{\mathcal{X}}$, where $\mathcal{X}$ and $\mathcal{Y}$ denote the finite set of states and observations, respectively, in Theorem 3. Then, $\alpha$ Blackwell dominates $\bar{\alpha}$ (denoted as $\alpha \geq_B \bar{\alpha}$) if there exists a row-stochastic matrix $Q$ such that $\bar{\alpha} = \alpha Q$.

The Blackwell order introduces the notion of monotonicity in the space of attention strategies. Intuitively, attention strategy $\alpha$ Blackwell dominates $\bar{\alpha}$ if $\bar{\alpha}$ is a noisy (garbled) version of $\alpha$. The Blackwell order is a partial order, since there exist attention strategy pairs that cannot be ordered via the Blackwell relation (Definition 1).

**Lemma 1** Consider the setup in Theorem 2. Suppose the consumption bundles $\beta$ lies in the space of attention strategies $\Delta(\mathcal{Y})^{\mathcal{X}}$, equipped with the Blackwell partial order $B$ (Definition 1). The utility $u_k : \Delta(\mathcal{Y})^{\mathcal{X}} \rightarrow \mathbb{R}$ and cost $g : \Delta(\mathcal{Y})^{\mathcal{X}} \rightarrow \mathbb{R}$ are monotone with respect to the Blackwell order $B$ for all $k$. Then,

1. Statements (1)-(4) in Theorem 2 are equivalent.
2. The reconstructed cost $g(\beta) = \max_k \{ \bar{g}_k + \lambda_k (u_k(\beta) - u_k(\beta_k)) \}$ is continuous and monotone with respect to the Blackwell partial order over consumption bundles $\beta$. If the utility function $u_k$ is convex for all $k$, then the reconstructed cost is also convex in $\beta$.

Lemma 1 can be proved by repeating the steps as in the proof of Theorem 2 (Appendix A). The only difference is that the natural element-wise partial order over Euclidean spaces is replaced by the Blackwell order when proving statement (1) $\implies$ (2) in Theorem 2, i.e., GARP, holds. The rest of the proof remains unchanged. Lemma 1 says that the results of Theorem 2 which assumes the element-wise partial order over consumption bundles hold identically for the Blackwell partial order.

**Step 2.** Consider the classical revealed preference test (6) of Theorem 2. Since Lemma 1 validates Theorem 2 for the Blackwell partial order, we map the variables...
in (6) to the equivalent variables from the Bayesian revealed preference problem via the mapping in Table 1. That is, let $\beta \equiv \alpha$, $u_k(\beta) \equiv J(\alpha, U_k)$.\footnote{This variable mapping is justified since the continuity of the utility function is preserved. That is, the expected utility functional $J(\alpha, U)$ is continuous in $\alpha$ just like $u_k(\beta)$ is assumed to be continuous in $\beta$.}

Then, we have the following equivalence:

\begin{equation}
\begin{aligned}
\forall t, s : & \exists \bar{g}_k, \lambda_k \geq 0, \\
& \bar{g}_s - \bar{g}_t - \lambda_t(u_t(\beta_s) - u_t(\beta_t)) \geq 0, \\
& \equiv \bar{g}_s - \bar{g}_t - \lambda_t(J_t(\alpha_s, U_t) - J_t(\alpha_t, U_t)) \geq 0, \quad \forall t, s \equiv (10),
\end{aligned}
\end{equation}

where the last line follows from the variable map $\beta \equiv \alpha$, $u_k(\beta) \equiv J(\alpha, U_k)$. The above result shows that the classical revealed preference test (6) is equivalent to the Bayesian revealed preference test (10), when the consumption bundles in the classical case lie in a Blackwell ordered set. Consequently, we observe that the generalized NIAC condition (10) is a Blackwell order version of the GARP condition proposed by Varian (1982).

Finally, note the positive scalars $\bar{g}_k$ in the last inequality above translate to the decision variables $c_k$ in (10). As a consequence, the information acquisition cost $C$ constructed using $c_k$ (13) is equivalent to the cost $g(\cdot)$ that specifies the budget constraints in the classical case, reconstructed using $\bar{g}_k$ in (4).

**Step 3.** Consider the reconstructed information acquisition cost $C(\cdot)$ (13) using the solution $\{c_k, \lambda_k\}$ of the Bayesian revealed preference test (10). The cost $C$ is ordinal\footnote{For the special case when $\lambda_k$ is set to $1$ \textit{a priori} \cite{Caplin2015}, the reconstructed cost $C(\cdot)$ is no more ordinal, but only shift invariant, which suffices for the normalized definition of the information acquisition cost (15).}, i.e., any monotone transformation of $C(\cdot)$ rationalizes the data equally well. Hence, without loss of generality, we modify the definition of $C$ (13) as

\begin{equation}
C(\alpha) = \max_k \{c_k + \lambda_k(J(\alpha, U_k) - J(\alpha_k, U_k))\} - C^*, \\
C^* = \max_k \{c_k + \lambda_k(J_k(\alpha_0, u) - J(\alpha_k, U_k))\}. \tag{15}
\end{equation}

In (15), $J(\cdot)$ is the expected utility function defined in (9) and $\alpha_0$ is the non-informative (uniform conditional probability) attention strategy, i.e., $\alpha_0(y|x) = 1/|\mathcal{Y}|$ for all $y, x$. Combining (10) and (15) above gives $C(\alpha_k) = c_k - C^*$. To show $C$ (15) rationalizes the sequence $\{U_k, \alpha_k\}$ in Theorem 3, fix index $k$ and consider an attention strategy $\alpha$ such that $C(\alpha) \leq C(\alpha_k)$. By definition (15), $0 \geq C(\alpha) - C(\alpha_k) \geq \lambda_k(J(\alpha, U_k) - J(\alpha_k, U_k))$, which implies $J(\alpha, U_k) \leq J(\alpha_k, U_k)$. This inequality holds for all $k$. Hence, the information acquisition cost $C$ (15) rationalizes the sequence $\{U_k, \alpha_k\}$.

We now show using convexity and Blackwell dominance that the cost $C$ (15) possesses three axiomatic properties of weak monotonicity in information (K1), mixture feasibility (K2) and normalization (K3) as theorized in Caplin and Dean (2015, Theorem 2).

**K1.** The cost $C$ is weakly monotonic in information if for any two attention strategies $\alpha, \hat{\alpha}, C(\hat{\alpha}) \leq C(\alpha)$, when $\alpha \geq_B \hat{\alpha}$. Here $Q$, $\alpha$ and $\hat{\alpha}$ are row stochastic matrices.
Condition K1 can be viewed as a monotonicity condition with respect to the Blackwell order.

**Proof.** Due to Blackwell dominance (Blackwell 1953), if $\alpha$ Blackwell dominates $\tilde{\alpha}$, then because $J(\cdot, U)$ is convex, it follows that $J(\alpha, U) \geq J(\tilde{\alpha}, U)$ for all $k$. Hence, the following inequalities hold:

\[
C(\tilde{\alpha}) = \max_k \{ c_k + \lambda_k (J(\tilde{\alpha}, U_k) - J(\alpha_k, U_k)) \} - C^* \\
\leq \max_k \{ c_k + \lambda_k (J(\alpha, U_k) - J(\alpha_k, U_k)) \} - C^* \\
\implies C(\tilde{\alpha}) \leq C(\alpha)
\]

**K2.** The cost $C$ is mixture feasible if for attention strategies $\alpha, \eta, \psi$ related as $\alpha = \theta \eta + (1 - \theta) \psi, \theta > 0$, cost $C$ satisfies $C(\alpha) \leq \theta C(\eta) + (1 - \theta) C(\psi)$.

**Proof.**

\[
C(\alpha) + C^* = \max_k \{ c_k + \lambda_k (J(\alpha, U_k) - J(\alpha_k, U_k)) \} \\
= \max_k \{ c_k + \lambda_k (J(\theta \eta + (1 - \theta) \psi, U_k) - J(\alpha_k, U_k)) \} \\
\leq \max_k \{ c_k + \lambda_k (\theta J(\eta, U_k) + (1 - \theta) J_k(\psi, U_k) - J(\alpha_k, U_k)) \} \\
\text{(since $J(\cdot, U_k)$ is convex)} \\
\leq \theta \max_k \{ c_k + \lambda_k (J(\eta, U_k) - J(\alpha_k, U_k)) \} \\
+ (1 - \theta) \max_k \{ c_k + \lambda_k (J(\psi, U_k) - J(\alpha_k, U_k)) \} \\
\text{(since the max operation is convex)} \\
\implies C(\alpha) \leq \theta (C(\eta) + C^*) + (1 - \theta) (C(\psi) + C^*) - C^* \\
\implies C(\alpha) \leq \theta C(\eta) + (1 - \theta) C(\psi)
\]

**K3.** The cost $C$ is normalized if $C(\alpha^*) = 0$, for $\alpha^*(y|x) = 1/|Y|$ (zero information gained).

**Proof.** This holds true from the definition of $C$ in (15).

## 5 Conclusion and Future Work.

In this paper, we established the connection between classical and Bayesian revealed preference. Our main finding is that the NIAC condition (Caplin and Dean 2015) in Bayesian revealed preference is a special case of GARP (Varian 1982) in classical revealed preference under a different partial order and a different state space (probability simplex). We exploit this result to construct a monotone convex information acquisition cost in Bayesian revealed preference. The construction procedure resembles that of the utility function reconstructed from consumer data in Afriat (1967).
In future work it is worthwhile exploiting this unification to study revealed preference in Bayesian versions of potential games building on Deb [2008, 2009], market games building on Forges and Minelli [2009], inverse reinforcement learning building on Ng et al. [2000], Pattanayak and Krishnamurthy [2020], and dynamic revealed preference building on Crawford [2010].

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Appendix A Proof of Theorem 2

Statement (1) $\implies$ (2). Fix indices $j, k$. Suppose there exist indices $i_1, i_2, \ldots, i_L$ such that $u_k(\beta_k) - u_k(\beta_{i_1}) \leq 0, u_i(\beta_{i_1}) - u_i(\beta_{i_2}) \leq 0, \ldots, u_i(\beta_{i_L}) - u_i(\beta_j) \leq 0$. If [5] holds, then we must have $g(\beta_{i_1}) \geq g(\beta_k), g(\beta_{i_2}) \geq g(\beta_{i_1}), \ldots, g(\beta_j) \geq g(\beta_{i_L})$, which implies $g(\beta_k) \leq g(\beta_j)$. Now, assume $u_j(\beta_j) - u_j(\beta_k) < 0$. By local non-satiation of $g$ and continuity of $u_j$, there exists a consumption bundle $\beta$ such that $u_j(\beta) > u_j(\beta_j), g(\beta) < g(\beta_k) \leq g(\beta_j) \implies g(\beta) < g(\beta_j)$, which contradicts our assumption. Hence, $u_j(\beta_j) - u_j(\beta_k) \leq 0$, and the sequence $\{\beta_k, u_k(\beta_k) - u_k(\beta_j)\}, k = 1, 2, \ldots, K$ satisfies GARP [2].

Statement (2) $\implies$ (3). From the proof of Forges and Minelli [2009] Proposition 3 (see also Foster et al. [2004] Sections 2 and 3), if the sequence $\{\beta_k, u_k(\beta_k) - u_k(\beta_j)\}$ satisfies GARP, then there exist positive scalars $\gamma_k, \lambda_k$ that satisfy the following inequality.

$$\gamma_j - \gamma_k + \lambda_k(u_k(\beta_k) - u_k(\beta_j)) \leq 0 \quad \forall j, k. \quad (A1)$$

Define $\hat{\gamma}_k = M - \gamma_k$, where $M$ is an arbitrary positive scalar that upper bounds $\gamma_k$ for all $k$. By construction, $\hat{\gamma}_k > 0$. Eq. [A1] can be further simplified in terms of the variable $\gamma_k$ as follows.

$$\gamma_j - \gamma_k + \lambda_k(u_k(\beta_k) - u_k(\beta_j)) \leq 0$$

$$\implies -\gamma_j - (-\gamma_k) - \lambda_k(u_k(\beta_k) - u_k(\beta_j)) \geq 0$$

$$\implies (M - \gamma_j) - (M - \gamma_k) + \lambda_k(u_k(\beta_j) - u_k(\beta_k)) \geq 0$$

$$\implies \hat{\gamma}_j - \hat{\gamma}_k + \lambda_k(u_k(\beta_j) - u_k(\beta_k)) \geq 0 \equiv [6].$$

Consider the reconstructed cost $g(\beta) = \max_k \{\gamma_k + \lambda_k(u_k(\beta) - u_k(\beta_k))\}$. The cost $g$ is monotone and continuous since it is a point-wise maximum of monotone continuous functions. Using the fact that inequality [6] holds, we have $g(\beta_k) = \hat{\gamma}_k$. Hence, the decision maker’s budget constraints are given by $\{g(\cdot) - \hat{\gamma}_k \leq 0\}$. To see that the above budget constraint sequence rationalizes the sequence $\{u_k, \beta_k\}$, fix index $k$ and consider consumption bundle $\beta$ such that $g(\beta) \leq \hat{\gamma}_k$. By definition, $0 \geq g(\beta) - \hat{\gamma}_k \geq \lambda_k(u_k(\beta) - u_k(\beta_k))$, which implies $u_k(\beta) \leq u_k(\beta_k)$ since $\lambda_k > 0$. Hence, the budget sequence $\{g(\cdot) - \hat{\gamma}_k \leq 0\}$ rationalizes the data.
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Statement (3) $\iff$ (1). The utility function $u_k$ is assumed to be locally non-satiated for all $k$. Hence, every function in the set $\{\bar{g}_k + \lambda_k(u_k(\beta) - u_k(\beta_k)), k = 1, 2, \ldots, K\}$ is locally non-satiated. Since $g(\beta) = \max_k \{\bar{g}_k + \lambda_k(u_k(\beta) - u_k(\beta_k))\}$ (7) is a point-wise maximum of finitely many locally non-satiated functions, $g(\cdot)$ is monotone, continuous and locally non-satiated by construction.

References

Afriat, S.N. 1967. The construction of utility functions from expenditure data. *International economic review* 8(1): 67–77.

Blackwell, D. 1953. Equivalent comparisons of experiments. *The annals of mathematical statistics*: 265–272.

Boyd, S.P. and L. Vandenberghe. 2004. *Convex optimization*. Cambridge university press.

Brown, D.J. and C. Calsamiglia. 2007. The nonparametric approach to applied welfare analysis. *Economic Theory* 31(1): 183–188.

Caplin, A. and M. Dean. 2015. Revealed preference, rational inattention, and costly information acquisition. *The American Economic Review* 105(7): 2183–2203.

Caplin, A. and D. Martin. 2015. A testable theory of imperfect perception. *The Economic Journal* 125(582): 184–202.

Chambers, C.P., C. Liu, and J. Rehbeck. 2020. Costly information acquisition. *Journal of Economic Theory* 186: 104979.

Crawford, I. 2010. Habits revealed. *The Review of Economic Studies* 77(4): 1382–1402.

Deb, R. 2008, Jan. Interdependent preferences, potential games and household consumption. MPRA Paper 6818, University Library of Munich, Germany.

Deb, R. 2009. A testable model of consumption with externalities. *Journal of Economic Theory* 144(4): 1804–1816.

Dimitrakakis, C. and C.A. Rothkopf 2011. Bayesian multitask inverse reinforcement learning. In *European workshop on reinforcement learning*, pp. 273–284. Springer.

Forges, F. and E. Minelli. 2009. Afriat’s theorem for general budget sets. *Journal of Economic Theory* 144(1): 135–145.

Fostel, A., H.E. Scarf, and M.J. Todd. 2004. Two new proofs of afriat’s theorem. *Economic Theory* 24(1): 211–219.
Freer, M. and C. Martinelli. 2016. A representation theorem for general revealed preference.

Freer, M. and C. Martinelli. 2021. A utility representation theorem for general revealed preference. *Mathematical Social Sciences* 111: 68–76.

Freer, M. and C. Martinelli. 2022. An algebraic approach to revealed preference. *Economic Theory* 1–26.

Hoiles, W., V. Krishnamurthy, and A. Aprem. 2016. Pac algorithms for detecting nash equilibrium play in social networks: From twitter to energy markets. *IEEE Access* 4: 8147–8161.

Hoiles, W., V. Krishnamurthy, and K. Pattanayak. 2020. Rationally Inattentive Inverse Reinforcement Learning Explains YouTube commenting behavior. *The Journal of Machine Learning Research* 21(170): 1–39.

Houthakker, H.S. 1950. Revealed preference and the utility function. *Economica* 17(66): 159–174.

Lopes, M., F. Melo, and L. Montesano. 2009. Active learning for reward estimation in inverse reinforcement learning. In *Joint European Conference on Machine Learning and Knowledge Discovery in Databases*, pp. 31–46. Springer.

Ng, A.Y., S.J. Russell, et al. 2000. Algorithms for inverse reinforcement learning. In *Icml*, Volume 1, pp. 2.

Nishimura, H., E.A. Ok, and J.K.H. Quah. 2017. A comprehensive approach to revealed preference theory. *American Economic Review* 107(4): 1239–63.

Pattanayak, K. and V. Krishnamurthy. 2020. Necessary and sufficient conditions for inverse reinforcement learning of bayesian stopping time problems. *arXiv preprint arXiv:2007.03481*.

Pattanayak, K. and V. Krishnamurthy. 2021. Behavioral economics approach to interpretable deep image classification. rationally inattentive utility maximization explains deep image classification. *arXiv preprint arXiv:2102.04594*.

Richter, M.K. 1966. Revealed preference theory. *Econometrica: Journal of the Econometric Society*: 635–645.

Rockafellar, R.T. 2015. *Convex analysis*. Princeton university press.

Samuelson, P.A. 1938. A note on the pure theory of consumer’s behaviour. *Economica* 5(17): 61–71.

Varian, H.R. 1982. The nonparametric approach to demand analysis. *Econometrica: Journal of the Econometric Society*: 945–973.
Varian, H.R. 1983. Nonparametric tests of models of investor behavior. *Journal of Financial and Quantitative Analysis*: 269–278.