Virtual Gravitons at the LHC

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Outline

Large extra dimensions

Real and virtual gravitons at LHC

RG improved virtual gravitons

Outlook
Large extra dimensions

Large extra dimensions (ADD)

– new physics at LHC: cannot always look for supersymmetry
– Einstein–Hilbert action for low fundamental Planck scale

\[ S = -\frac{1}{2} \int d^4x \sqrt{|g|} M_D^2 R \rightarrow -\frac{1}{2} \int d^{4+n}x \sqrt{|g|} M_D^{2+n} R \]

\[ = -\frac{1}{2} (2\pi r)^n \int d^4x \sqrt{|g|} M_D^{2+n} R \]

\[ \equiv -\frac{1}{2} \int d^4x \sqrt{|g|} M_{\text{Planck}}^2 R \]

⇒ express the 4D Planck scale in terms of fundamental Planck scale

\[ M_{\text{Planck}} = M_D (2\pi r M_D)^{n/2} \]

Numbers to make it work

– wanted \(r M_D \gg 1\)
– constraints from gravity tests above \(O(\text{mm})\)
– \(M_D = 1 \text{ TeV} \ll M_{\text{Planck}}\) fine for \(n \gtrsim 2\)

⇒ signatures of strong gravity in extra dimension?

| \(M_D = 1 \text{ TeV}\) |
|---|---|
| \(n\) | \(r\) |
| 1 | \(10^{12} \text{ m}\) |
| 2 | \(10^{-3} \text{ m}\) |
| 3 | \(10^{-8} \text{ m}\) |
| \(\ldots\) | \(\ldots\) |
| 6 | \(10^{-11} \text{ m}\) |
Large extra dimensions at the LHC

Minimal model: only gravitons in extra dimensions

- only the interacting (tensor) graviton  [QCD/QED massless, $M_D = 1$ TeV]

\[(\Box + m^2_k) G^{(k)}_{\mu\nu} = -\frac{T_{\mu\nu}}{M_{\text{Planck}}}\]

\[\Delta m \sim \frac{1}{r} = 2\pi M_D \left( \frac{M_D}{M_{\text{Planck}}} \right)^{2/n} = \begin{cases} 0.003 \text{ eV} & (n = 2) \\ 0.1 \text{ MeV} & (n = 4) \\ 0.05 \text{ GeV} & (n = 6) \end{cases}\]

- KK graviton tower with mass splitting $\Delta m \ll \text{GeV}$  [below LHC resolution]

universal couplings via $-\frac{T_{\mu\nu}}{M_{\text{Planck}}}$

$\Rightarrow$ LHC effective theory: KK gravitons light, weakly coupled continuum

Real emission and virtual gravitons  [Giudice, Rattazzi, Wells; Han, Lykken, Zhang;...]

- integration over continuous KK tower  $[dm/|k| = 1/r; (d\sigma) \propto 1/M^2_{\text{Planck}}]$

\[(d\sigma) \rightarrow \int dm (d\sigma) S_{n-1} m^{n-1} r^n = \int dm (d\sigma) \frac{S_{n-1} m^{n-1}}{(2\pi M_D)^n} \left( \frac{M_{\text{Planck}}}{M_D} \right)^2\]

\[\mathcal{A} = \frac{1}{M^2_{\text{Planck}}} \frac{1}{s - m^2} \rightarrow S_{n-1} \frac{\Lambda^{n-2}_{\text{cutoff}}}{M^n_{D}} \frac{M^n_{D}}{\Lambda^{n+2}_{\text{cutoff}}}\]

$\Rightarrow$ $1/M_D$ interaction after integration over KK tower

$\Rightarrow$ explicit UV cutoff $\Lambda_{\text{cutoff}}$ or RG improvement?
Real gravitons at LHC

**Effective theory of real gravitons**  [Giudice, Rattazzi, Wells; Vacavant, Hichliffe...]

- real graviton emission  \( pp \rightarrow G_{KK} + \text{jets} \)  [coupling \( G \sim 1/M_D^{2+n} \)]
- recoil against hard jet  [with \( E_j \gtrsim M_D/4 \)]
  background: radiation of \( Z \rightarrow \nu\bar{\nu} \)
- towers of ADD gravitons  \( dN \propto S_{n-1} \left(M_{\text{Planck}}/M_D\right)^2 m^{n-1} dm \)
  cutoff \( M_{KK} = 0 \) for \( E_{\text{parton}} > \Lambda_{\text{cutoff}} \sim M_D \)
- observables: total rate or 5\( \sigma \) discovery reach
- little UV sensitivity for \( \Lambda_{\text{cutoff}} \rightarrow \infty \), small RG effects expected?

⇒ explicit cutoff irrelevant due to phase space  [and gluon densities]
Virtual gravitons at LHC

Effective theory of virtual gravitons

- virtual graviton in $s$ channel $pp \rightarrow \mu^+ \mu^-$
- reconstructed $m_{\mu\mu}$ for photon, $Z$, graviton
- divergent D8 operator

\[ S = \frac{S_{n-1}}{M^2_{D} + n} \int dm \frac{m^{n-1}}{s + m^2} = \begin{cases} \frac{4\pi}{M^4_{\text{eff}}} & \text{(effective scale)} \\ \frac{S_{n-1}}{M^4_{D}} \frac{1}{2} \left( \frac{\Lambda_{\text{cutoff}}}{M_D} \right)^{n-2} & \text{(NDA)} \\ \frac{S_{n-1}}{M^4_{D}} \frac{1}{n-2} \left( \frac{\Lambda_{\text{cutoff}}}{M_D} \right)^{n-2} & \text{(cutoff $\Theta$)} \end{cases} \]

\[ M_{\text{eff}}[\text{TeV}] \]

5σ: $pp \rightarrow l^+l^-$ (D8)

\[ \Lambda_{\text{eff}}[\text{TeV}] \]

- 100 fb$^{-1}$
- 10 fb$^{-1}$
Virtual gravitons at LHC

Effective theory of virtual gravitons [Giudice & Strumia; Giudice, Strumia, TP; Kachelries & Plümacher,...]

- virtual graviton in s channel $pp \rightarrow \mu^+ \mu^-$
- reconstructed $m_{\mu\mu}$ for photon, $Z$, graviton
- divergent D8 operator [leading constant in $\sqrt{s}/\Lambda_{\text{cutoff}}$]

$$S = \frac{S_{n-1}}{M_D^{2+n}} \int dm \frac{m^{n-1}}{s + m^2} = \begin{cases} \frac{4\pi}{M_4^{4+\epsilon}} S_{n-1} \frac{1}{2} \left( \frac{\Lambda_{\text{cutoff}}}{M_D} \right)^{n-2} & \text{(effective scale)} \\ \frac{S_{n-1}}{M_D^{4+\epsilon}} \frac{1}{n-2} \left( \frac{\Lambda_{\text{cutoff}}}{M_D} \right)^{n-2} & \text{(NDA)} \\ \frac{S_{n-1}}{M_D^{4+\epsilon}} \frac{1}{n-2} \left( \frac{\Lambda_{\text{cutoff}}}{M_D} \right)^{n-2} & \text{(cutoff } \Theta) \end{cases}$$

- scaling of rates $M_D^{\text{max}} \sim \Lambda_{\text{cutoff}}^{(n-2)/(n+2)}$

$\Rightarrow$ explicit cutoff needed for virtual gravitons

![Graph showing the relationship between $M_D$ and $\Lambda$](image)
Virtual gravitons at LHC

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S = \frac{S_{n-1}}{M_D^{2+n}} \int dm \frac{m^{n-1}}{s + m^2} = \begin{cases} 
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\frac{S_{n-1}}{M_4^n} \frac{1}{2} \left( \frac{\Lambda_{\text{cutoff}}}{M_D} \right)^{n-2} \\
\frac{S_{n-1}}{M_4^n} \frac{1}{n-2} \left( \frac{\Lambda_{\text{cutoff}}}{M_D} \right)^{n-2} 
\end{cases}
\]

(leading constant in \( \sqrt{s}/\Lambda_{\text{cutoff}} \))

– scaling of rates \( M_D^{\text{max}} \sim \Lambda_{\text{cutoff}}^{(n-2)/(n+2)} \)
⇒ explicit cutoff needed for virtual gravitons

String theory as UV completion [e.g. Cullen, Perelstein, Peskin; Antoniadis, Benakli, Laugier...]

– Veneziano form factor
\[
\frac{\Gamma(1 - \alpha's) \Gamma(1 - \alpha't)}{\Gamma(1 - \alpha'(s + t))} = \frac{\Gamma(1 - s/M_S^2) \Gamma(1 - t/M_S^2)}{\Gamma(1 - (s + t)/M_S^2)} = 1 - \frac{\pi^2}{6} \frac{st}{M_S^4} + \mathcal{O} \left( M_S^{-6} \right)
\]

– string resonances above \( \Lambda_{\text{cutoff}}: \sqrt{n} M_S \)
Graviational fixed point

**Matched graviton propagator**  [Reuter; Fischer & Litim]

- effective action: \( \Gamma_k = 1/(16\pi G_k) \int d^{4+n} x \sqrt{g} [-R(g) + \cdots] \)  [Percacci’s talk]
- gravity weak enough at high energies?
- IR — no running; \( M_D \) regime — strong effects; UV — fixed point
- iterative approach: start with anomalous dimension of graviton propagator

\[
P(s, m) = \begin{cases} 
\frac{1}{s + m^2} & \sqrt{s}, m < k_{\text{trans}} \\
\frac{M_D^{n+2}}{(s + m^2)^{n/2+2}} & \sqrt{s}, m > k_{\text{trans}}
\end{cases}
\]

- IR and UV contributions by virtual gravitons  [UV: gluon pdf \( \rightarrow \) leading in \( \sqrt{s}/m \)]

\[
S^{(\text{FP})} = S_{n-1} M_D^4 \left( \frac{k_{\text{trans}}}{M_D} \right)^{n-2} \frac{n-1}{n-2} = (1 + (n-2)) S^{(\Theta)}
\]

- needed and not needed:
  (1) gravitational coupling \( G \sim 1/M_D^{2+n} \)
  (2) transition scale \( k_{\text{trans}} \sim M_D \)
  (3) no artificial UV cutoff \( \Lambda_{\text{cutoff}} \)  [setting \( M_{\text{KK}} = 0 \)]

\( \Rightarrow \) UV fixed point regularizes KK integral
LHC signature

test artificial $\Lambda_{\text{cutoff}}$ setting $M_{\text{KK}} = 0$

- perfect decoupling, as expected [similar to real emission]
- mild effects for $k_{\text{trans}} = M_D \pm 10\%$ [more details to be studied]
- reach largely independent of $n$
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Shape of graviton kernel

- non-trivial structure from interference [shown $n = 3$]
  small $m_{\ell\ell}$: factor $S \propto (n - 1)$
  large $m_{\ell\ell}$: factor $S^2 \propto (n - 1)^2$
- UV contribution not negligible
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predicted LHC rates

- \( S^{(\text{FP})} \): UV regime in addition to \( S^{(\Theta)} \equiv S^{\text{IR}} \)

| \( \sigma \) [fb] | \( n = 3 \) | \( n = 6 \) |
|------------------|------------------|------------------|
| \( M_D \)        | 2 TeV  | 5 TeV  | 8 TeV  | 2 TeV  | 5 TeV  | 8 TeV  |
| \( S^{(NDA)} \)  | 43.6   | 0.18   | 0.0053 | 263    | 1.11   | 0.031  |
| \( S^{(\Theta)} \) | 173    | 0.72   | 0.0204 | 66     | 0.28   | 0.008  |
| \( S^{(FP)} \)   | 408    | 1.24   | 0.0317 | 398    | 1.21   | 0.031  |

⇒ fixed–point graviton effect stable and large
Outlook

Gravitons at LHC

– effective field theory:
  (1) real emission accidentally well defined
  (2) virtual–graviton predictions cutoff dependent
– fixed point picture: gravity weak at large scales  [non-perturbative asymptotic safety]
  leading effect: anomalous dimension  [Hewett & Rizzo: running coupling]
  KK theory well defined without explicit cutoff
⇒ testable at the LHC  [Litim & TP, to appear tomorrow morning]
