TIME-REVERSAL SYMMETRY BREAKING BY AC FIELD: EFFECT OF COMMENSURABILITY IN THE FREQUENCY DOMAIN

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It is shown that the variance of the linear dc conductance fluctuations in an open quantum dot under a high-frequency ac pumping depends significantly on the spectral content of the ac field. For a sufficiently strong ac field the dc conductance fluctuations are much stronger for the periodic pumping than in the case of the noise ac field of the same intensity. The reduction factor $r$ in a static magnetic field takes the universal value of 2 only for the white-noise pumping. In general $r$ may deviate from 2 thus signalling on the time-reversal breaking by the ac field. For the bi-harmonic ac field of the form $A(t) = A_0 [\cos(\omega_1 t) + \cos(\omega_2 t)]$ we predict the enhancement of effects of $T$-symmetry breaking at commensurate frequencies $\omega_2/\omega_1 = P/Q$. In the high-temperature limit there is also the parity effect: the enhancement is only present if either $P$ or $Q$ is even.

1 Introduction

Recently there has been a considerable interest in non-equilibrium mesoscopics. The effect of adiabatic charge pumping has been experimentally observed and analyzed theoretically. Weak localization under ac pumping and the photovoltaic effect in a quantum dot have been theoretically studied. The non-equilibrium noise has been suggested as a cause of both the low temperature dephasing saturation and the anomalously large ensemble averaged persistent current.

Here we study the effect of the high-frequency ac field on the mesoscopic fluctuations of linear dc conductance when both the weak dc voltage and a strong enough high-frequency pump field are applied to the open quantum dot. We will study the dependence of the variance of the dc conductance fluctuations on the ac field intensity for the noiselike and an almost periodic ac field. In particular we focus on the reduction factor $r = 1 + C/D$ for the variance of conductance fluctuations after turning on a strong static magnetic field that kills the cooperon contribution to the variance $\langle \delta G^2 \rangle_C \equiv C$ while leaving the diffuson one $\langle \delta G^2 \rangle_D \equiv D$ unchanged.

This question is of the fundamental importance, since the universality of the reduction factor is related with the time-reversal invariance of the system without the static magnetic field. The external ac field certainly breaks the time-reversal invariance. However, if the characteristic frequency $\omega$ of the ac field is high it is intuitively clear that the effect of time-reversal symmetry breaking is observed only at special conditions.

Below we show that not only the power but also the spectral content of the ac field is important for an effective $T$-symmetry breaking. In particular, the white ac noise is shown to be the most effective way to diminish the conductance fluctuations by dephasing. Yet it does not lead to any deviations from the universal value of $r = 2$. The periodic ac field is least effective in dephasing but it produces the maximum possible (at a given ac power) effect of time-reversal breaking.

An interesting special case arises when the ac field is the superposition of two harmonic fields with different frequencies $\omega_1$ and $\omega_2$. We will show below that both the variance of conductance fluctuations and the inverse reduction factor $1/r$ have very sharp peaks as a function of the ratio $\alpha = \omega_2/\omega_1$ at any rational value of $\alpha = P/Q$ but the height of the peak decreases with increasing the denominator $Q$. Thus we conclude that the statistics of mesoscopic conductance fluctuations reveals the effect of commensurability in the frequency domain on the dephasing and
the time-reversal breaking by the ac field.

2 The Landauer conductance in the time domain

The Landauer conductance \( g = (\gamma V/4) [K(r, r') + K(r', r)] \) of a dot of the volume \( V \) with small contacts at \( r \) and \( r' \) and the electron escape rate \( \gamma \), can be expressed in terms of the exact retarded and advanced electron Green’s functions \( G^{R,A}(r, r', t, t') \) in the time domain\cite{4,10,11}:

\[
K(r, r') = \int dt_1 dt_2 G^R(r, r'; t, t_1) G^A(r', r; t_2, t) F_{t_1 - t_2}
\]  

(1)

where \( F_t = \pi T t \sinh^{-1}(\pi T t) \) is the Fourier-transform of the derivative of the Fermi distribution function for electrons in the leads and \( f(t) = \frac{1}{T} \int_{T/2}^{-T/2} dt f(t) \) denotes time averaging during the observation time \( T \). We consider a chaotic or disordered quantum dot with the number of open channels \( M \gg 1 \) and the electron escape rate \( \Delta \ll \gamma = \Delta M \ll E_c \) where \( \Delta \) is the mean level separation and the Thouless energy \( E_c \) is the inverse ergodic time. We also assume the dephasing rate \( \gamma_{\text{int}}(T) \) caused by electron interaction to be smaller than the escape rate \( \gamma \). In this situation the charging effects are negligible, the perturbative diagrammatic analysis (see Fig.1) is possible and the ergodic zero-dimensional approximation applies. In this approximation the diffuson and the cooperon contribution to the variance of conductance fluctuations is given by\cite{1}:

\[
D \propto \int dt' dt'' D_{\eta}(t, t') D_{-\eta}(t', t'') F_{t_1 - t_2}^2
\] 

(2)

\[
C \propto \int dt' d\eta' C_{t'+t}(-\eta' - 2t', -\eta + 2t') C_t(\eta, \eta') F_{2t'}^2.
\] 

(3)

where averaging over \( t \) and \( \eta \) is assumed.

In Eqs.\( \ref{2}, \ref{3} \) the time-dependent diffuson \( D_{\eta}(t, t') \) and cooperon \( C_t(\eta, \eta') \) are certain products of electron retarded \( (G^R) \) or advanced \( (G^A) \) Green’s functions averaged over disorder:

\[
\langle G^R(t_+, t'_+) G^A(t'_-, t_-) \rangle = \delta(\eta - \eta') D_{\eta}(t, t'), \quad \langle G^R(t_+, t'_+) G^A(t_-, t'_-) \rangle = \frac{1}{2} \delta(t - t') C_t(\eta, \eta'),
\] 

(4)

where \( t_\pm = t \pm \eta/2, t'_\pm = t' \pm \eta'/2 \).
In the ergodic regime they are independent of the coordinates and are given by:

\[ D_\eta(t,t') = \Theta_{\eta-\eta'} \exp \left[ - \int_{t'}^t \Gamma_d(\eta, \xi) \, d\xi \right] \]

(5)

\[ C_t(\eta, \eta') = \Theta_{t-t'} \exp \left[ - \frac{1}{2} \int_{\eta'}^\eta \Gamma_e(t, \xi) \, d\xi \right], \]

(6)

where \( \Theta_t \) is the step-function.

The functions \( \Gamma_d(\eta, \xi) \) and \( \Gamma_e(t, \xi) \) describe the dephasing caused by the ac field and by the electron escape into leads. For a ring with the diffusive electron motion and the circular electric field \( E(t) = -\frac{\partial}{\partial t} A(t) \) cased by the time-dependent flux through it, these functions are given by [4-6]:

\[ \Gamma_d(\eta, \xi) = \gamma + D \left( A(\xi + \eta/2) - A(\xi - \eta/2) \right)^2, \]

(7)

\[ \Gamma_e(t, \xi) = \gamma + D \left( A(t + \xi/2) + A(t - \xi/2) \right)^2. \]

(8)

where \( \gamma \) is an electron escape rate, \( A(t) \) is the vector-potential, and \( D \) is an electron diffusion coefficient.

The same expressions hold for a single connected dot with a homogeneous longitudinal electric field \( E(t) \) (the vector-potential \( A(t) \) is unambiguously defined by the condition \( A(t) = E(t) \) = 0) but only in the high-frequency limit \( \omega \gg E_c \). In the adiabatic limit \( \omega \ll E_c \) one obtains [4-6]:

\[ \Gamma_d(\eta, \xi) = \gamma + C(\eta^2 + \xi^2), \]

(9)

\[ \Gamma_e(t, \xi) = \gamma + (C/4)(\xi^2 + \eta^2), \]

(10)

where \( C \sim L^2/E_c \), \( L \) being the dot size.

The existence of two different forms Eqs. (7), (8) and Eqs. (9), (10) is the manifestation of the fact that there are two different time-dependent random matrix theories for each Dyson universality class.

3 The limit of high frequencies

Eqs. (4), (5) can be significantly simplified in the limit of high frequencies \( \omega \gg \gamma \). At such a high-frequency pumping one can replace \( \Gamma_d(\eta, \xi) \) and \( \Gamma_e(t, \xi) \) in Eqs. (4) by the time averages \( \Gamma_d(\eta) = \Gamma_d(\eta, \xi) \) and \( \Gamma_e(t) = \Gamma_e(t, \xi) \). Then we obtain:

\[ \frac{D}{g_0} = \int_{-T/2}^{T/2} \frac{d\eta}{2T} \left( \frac{\gamma^2}{\Gamma_d(\eta)} \right) \int_0^\infty e^{-t \Gamma_d(\eta)} F_t^2 \, dt. \]

(11)

\[ \frac{C}{g_0} = \int_{-T/2}^{T/2} \frac{dt}{T} \int_{-\infty}^t dt' \frac{\gamma^2 F_c^2 e^{-2(t-t') \Gamma_e(t')}}{2(\Gamma_e(t) + \Gamma_e(t'))}, \]

(12)

where \( g_0 = \pi \gamma/4 \Delta \) is the mean conductance, \( T \) is the observation time, and and \( F_t = \pi T t \sinh^{-1}(\pi T t) \) is the Fourier-transform of the Fermi distribution function.

In Fig.2 we show the result of a direct numerical evaluation of integrals in Eqs. (11), (12) at \( T = 0 \). One can see how the high-frequency limit is reached for the inverse reduction factor in the case of harmonic pumping. Eqs. (11), (12) can be further simplified in the limit of low temperatures \( T \ll \gamma \) where \( F_t \approx 1 \) and in the limit of high temperatures \( T \gg \omega(\gamma T)^{-1/2} \) where \( F_t^2 \approx (\pi/6T)^2 \delta(t) \) and we have:

\[ \left\langle \delta G^2 \right\rangle_{D,C} = \frac{\pi \gamma^2}{12T} \int_{-T/2}^{T/2} \frac{dt}{T} \frac{1}{\Gamma_{d,c}(t)}. \]

(13)

Below we consider only these limiting cases.
4 Conductance fluctuations for the noise and harmonic ac pumping

There is a dramatic difference between the noiselike ac field with the short correlation time \( \tau_0 \sim \omega^{-1} \ll \tau_\varphi \) and the harmonic ac field \( A(t) = A_0 \cos(\omega t) \) with \( \omega \gg \gamma \). In the case of the white-noise pumping the time-average of the cross-terms \( \overline{A_tA_\xi+\eta/2A_\xi-\eta/2} \) and \( \overline{A_t+\xi/2A_t-\xi/2} \) in Eqs.(10) or Eqs.(10),(11) is zero and we obtain the same, time-independent dephasing rates \( \Gamma_d = \Gamma_c = \gamma + 1/2\tau_\varphi \) for the cooperons and the diffusons with \( \eta \neq 0 \), where

\[
\tau_\varphi^{-1} = \begin{cases} 
\frac{4DA^2}{4CE^2} & \omega \gg E_c \\
\frac{4DA^2}{4CE^2} & \omega \ll E_c 
\end{cases}
\]

is proportional to the pumping intensity. For convenience of the further analysis we introduce the dimensionless intensity \( I \):

\[
I = \frac{1}{\tau_\varphi \gamma} \tag{15}
\]

which for the harmonic pumping is equal to the number of absorbed/emitted field quanta \( \hbar \omega \) for the time \( \gamma^{-1} \) electron spends inside the dot.

Then Eqs.(2),(3) give \( C = D \), that is the diffuson and the cooperon contributions to the variance of conductance fluctuations are equal to each other. Hence the time-reversal symmetry is effectively unbroken.

The variance of conductance fluctuations in this case is given by:

\[
\frac{D}{g_0^2} = \frac{C}{g_0^2} = \begin{cases} 
\frac{1}{2} \left(1 + \frac{1}{2}\right)^{-2}, & T \ll \gamma + \frac{1}{2\tau_\varphi} \\
\frac{\pi \gamma}{12T} \left(1 + \frac{1}{2}\right)^{-1}, & T \gg \gamma + \frac{1}{2\tau_\varphi} 
\end{cases} \tag{16}
\]

For the harmonic pumping the dephasing rates are periodic functions of time:

\[
\Gamma_d(\eta) \equiv \Gamma_d(\eta, \xi) = \gamma + \tau_\varphi^{-1} \sin^2(\omega\eta/2), \quad \Gamma_c(t) \equiv \Gamma_c(t, \xi) = \gamma + \tau_\varphi^{-1} \cos^2(\omega t), \tag{17}
\]

and Eqs.(11,12) no longer lead to the same result.

For large pumping intensities \( I = (\tau_\varphi \gamma)^{-1} \gg 1 \) the dephasing functions Eq.(13) are small in the ‘no-dephasing’ windows near zeros \( \eta_n \) and \( t_n \) of \( \sin^2(\omega\eta/2) \) and \( \cos^2(\omega t) \). It is these no-dephasing windows of the width \( \omega^{-1} \Gamma^{-1/2} \) that make the dominant contribution to the magnitude of all phase-coherent effects at high pumping intensities \( I \gg 1 \).

In order to compute the diffuson and the cooperon contribution to the conductance fluctuations one can expand \( \Gamma_d(\eta) \) and \( \Gamma_c(t) \) in Eqs.(11),(12) near \( \eta_n \), and \( t_n \), perform integrations
from $-\infty$ to $+\infty$ over $\delta \eta = \eta - \eta_n$ and $\delta t = t - t_n$ and sum over all $n$. The result depends on the relation between the temperature of leads $T$ and $\gamma$. For $T \ll \gamma \ll \tau^{-1}$ we obtain:

$$\frac{D}{g_0^2} \approx \frac{1}{4\sqrt{T}}, \quad \frac{C}{g_0^2} \approx \frac{1}{21^2} \quad (I \gg 1).$$  \hspace{1cm} (18)

One can see that the diffuson contribution $D$ to the conductance fluctuations is parametrically larger than the cooperon one $C$ even in the absence of a static magnetic field. This signals on the extremely strong $T$-breaking effect in this regime with the reduction factor $r = 1 + 2I^{-1/2}$ close to 1.

However, it is enough to raise the temperature of leads to make the $T$-breaking effect negligible. At high temperatures $T \gg \omega \sqrt{I}$ and strong pumping $I \gg 1$ Eq. (13) gives the same value

$$\frac{D}{g_0^2} = \frac{C}{g_0^2} = \frac{\pi \gamma}{12I} \frac{1}{\sqrt{T}},$$  \hspace{1cm} (19)

for the diffuson and the cooperon contribution to conductance fluctuations, and the reduction factor $r = 2$.

Comparing Eq. (16) with Eqs. (18), (19) we also conclude that at high intensities $I \gg 1$ the suppression of mesoscopic fluctuations by the noiselike ac field is always stronger than the suppression by a harmonic field of the same power. This is because the ‘no dephasing’ windows are absent in the case of noise.

5 Conductance fluctuations for an almost periodic ac field and statistics of zeros of the dephasing functions.

The idea of the ‘no-dephasing windows’ in the vicinity of zeros of the dephasing functions $\Gamma_d(\eta) - \gamma$ and $\Gamma_c(t) - \gamma$ can be applied to a general case of an almost periodic ac field. For such an ac filed the dephasing functions have a certain density of complex zeros $z_n = x_n + iy_n$ with a small complex part $y_n$:

$$\rho(y) = \sum_n \langle \delta(x - x_n)\delta(y - y_n) \rangle_x,$$

where $\langle ... \rangle_x = (T)^{-1} \int_{|x|<T/2} dx ...$ stands for the averaging over $x$.

Since only the time variables in the ‘no dephasing windows’ contribute to the variance of mesoscopic fluctuations at large pumping intensities, one can express the dependence of the variance on the dimensionless intensity $I$ in terms of the density of zeros $\rho(y)$.

For an important example of the bi-harmonic ac field

$$A(t) = A_0 [\cos(\omega t) + \cos(\alpha \omega t)]$$  \hspace{1cm} (21)

one obtains in the high-temperature limit $T \gg \omega \sqrt{I}$:

$$\frac{\langle \delta G^2 \rangle_{D,C}}{g_0^2} = \frac{\pi^2 \gamma}{3T^2(1 + \alpha^2)} I^{-1} \int_{-\infty}^{+\infty} \frac{dy \rho_{d,c}(y)}{I^{-1} + y^2 (1 + \alpha^2)}^{1/2},$$  \hspace{1cm} (22)

where $\rho_{d,c}$ is the density of zeros that corresponds to the equation (the sign $\pm$ stands for the cooperon(diffuson) part):

$$\cos z + \cos(\alpha z) \pm 2 = 0.$$  \hspace{1cm} (23)
Figure 3: Reduction factor $r$ vs the frequency ratio $\alpha$ in simple commensurate points $\alpha = P/Q$ for low temperatures $T \ll \gamma$ and pumping intensity $I = (\gamma \tau)^{-1} = 20$. Also plotted is $r$ for the golden mean $\alpha = \tau = \sqrt{5} - 1/2$.

6 Effect of commensurability in the frequency domain

There is a drastic difference in the density of zeros $\rho_{d,c}(y)$ for the case of commensurate and incommensurate frequencies $\omega_1 = \omega$ and $\omega_2 = \alpha \omega$ in Eq. (21). For the case of commensurate frequencies $\alpha = P/Q < 1$ the density $\rho(y)$ is the set of $\delta$-functions $(2\pi Q)^{-1} \sum \delta(y - y_n)$ separated by gaps $\Delta y_n \sim 1/Q$. In the limit of incommensurate frequencies $Q \to \infty$ the function $\rho(y)$ is continuous with no singularity at $y = 0$. This difference shows up in the dependence of the diffuson and the cooperon contribution to the variance of conductance fluctuations on the pumping intensity $I$.

Let us consider the case of commensurate frequencies $\alpha = P/Q$ with not very large denominator $Q$. At large dimensionless intensities $I \gg Q^2$ the gap $y_1 \sim 1/Q$ is large compared to $I^{-1/2}$, and one can neglect in Eq. (22) all complex roots with $y_n \neq 0$. Equation (23) with the sign minus has real roots at any $\alpha$. Then one immediately concludes from Eq. (23) that for commensurate frequencies with $Q \ll \sqrt{I}$ the diffusion part of the variance of conductance fluctuations is proportional to $I^{-1/2}$. However, Eq. (23) with the sign plus (relevant for the cooperon contribution) has real solutions only if $P$ and $Q$ are both odd. In this case in the high-temperature limit we have $D = C \propto I^{-1/2}$ as for a strictly harmonic pumping. Thus the symmetry between the diffusion and the cooperon contribution is unbroken. One can see that the condition that both $P$ and $Q$ in $\alpha = P/Q$ in Eq. (21) are odd, is a particular case of a more general condition:

$$A(t + \tau^*) = -A(-t + \tau^*),$$

(24)

that is, by a certain shift $\tau^*$ (in our case $\tau^*$ is a quarter of the period) the vector-potential $A(t)$ can be made an odd function of time. This is the natural definition for the time-reversal symmetry in the absence of a preferential time origin.

If either $Q$ or $P$ is even this condition is violated and the cooperon contribution is anomalously suppressed $C \propto I^{-1}$. In this case the effective $T$-symmetry breaking is strong even in the high-temperature limit (see Fig. 4. and Fig. 5.). Such a parity effect is also present in the low-temperature limit $T \ll \gamma$ as it is seen from Fig. 3 obtained by numerical integration of Eqs. (11-12). However, in this case it is much weaker and the symmetry Eq. (24) no longer implies $C = D$.

Now consider the case $Q \gg \sqrt{I}$. In this case the function $\rho_{d,c}(y)$ in Eq. (23) is effectively averaged over the interval $\delta y \sim I^{1/2}$ and can be replaced by a constant. As the result both the diffusion and the cooperon part of the variance is proportional to $I^{-1} \ln I$ and the effect
Figure 4: The reduction factor $r$ as a function of the frequency ratio $\alpha$ in the high-temperature limit $T \gg \omega \sqrt{I}$ for $I = 4$ and $1/T\omega = 0.05$ and 0.02. The dips at $\alpha = \frac{1}{3}, \frac{2}{3}, \frac{1}{2}$ and $\frac{1}{4}$ are seen while there is no dip for $\alpha = \frac{1}{5}$ where both $P$ and $Q$ are odd. The width of peaks is of the order of $1/\omega T$.

Figure 5: The reduction factor $r$ as a function of the frequency ratio $\alpha$ in the high-temperature limit $T \gg \omega \sqrt{I}$ for $I = 8$ and $1/T\omega = 0.02$. Small dips at $\alpha = P/Q$ with larger denominator $Q$ ($PQ$ is even) become visible.

of $T$-breaking drastically decreases. That is why the depth of dips in the reduction factor $r$ at $\alpha = P/Q$ rapidly decrease with increasing the denominator $Q$ at a given intensity $I$ (see Fig.4 and Fig.5.). The same trend holds for the low-temperature regime too (Fig.3) but the reduction factor $r$ does not reach its universal value even for the ‘most irrational’ frequency ratio $\alpha = (\sqrt{5} - 1)/2$.

It is possible to obtain an explicit analytical expression for the cooperon and the diffuson contribution to the variance of conductance fluctuations in the high-temperature limit $T \gg \omega \sqrt{I}$:

$$D_{g_0} \approx \frac{\gamma}{6T} \sum_{n,m=-\infty}^{+\infty} \tilde{\delta}(n - \alpha m) \left\{ I^{-1}K_0 \left( \frac{2\sqrt{n^2+m^2}}{\sqrt{I}} \right), \quad n^2 + m^2 > 0 \right\} \frac{1}{I^{-1}\ln(2\sqrt{I})}, \quad m = n = 0,$$

$$C_{g_0} \approx \frac{\gamma}{6T} \sum_{n,m=-\infty}^{+\infty} (-1)^{n+m} \tilde{\delta}(n - \alpha m) \left\{ I^{-1}K_0 \left( \frac{2\sqrt{n^2+m^2}}{\sqrt{I}} \right), \quad n^2 + m^2 > 0 \right\} \frac{1}{I^{-1}\ln(2\sqrt{I})}, \quad m = n = 0,$$ (25)

where $K_0(x)$ is the Bessel function and $\tilde{\delta}(\omega x)$ is the spectral lineshape function of the harmonic component normalized so that $\tilde{\delta}(0) = 1$. Fig.4 and Fig.5 are obtained from Eqs.(25),(26) for
Figure 6: The total variance of conductance fluctuations (in units of the ensemble-average dc conductance) at $T \ll \gamma$ as a function of the frequency ratio $\alpha$ for $I = 20$ and $1/T \omega = 0.01$. The width of peaks is of the order of $1/T \omega$.

the Gaussian lineshape $\tilde{\delta}(x) = e^{-x^2 \omega^2 T^2}$. Eqs.(25),(26) reveal an important and a bit counter-intuitive feature of the $\alpha$ dependences. The electron escape rate $\gamma$ enters only in the dimensionless intensity $I$ (see Eq.(13)) that controls the magnitude of peaks/dips in the $\alpha$-dependences. The width of peaks/dips is determined by the spectral linewidth $\delta = \mathcal{T}^{-1}$ of the harmonic components and can be much smaller than $\gamma/\omega$.

The commensurability effect is present not only for the reduction factor $r$ but also for the total variance of conductance fluctuations. In Fig.6 the total variance is plotted as a function of the frequency ratio $\alpha$ for the case of low temperatures $T \ll \gamma$.

In conclusion, the effect of ac pumping on the statistics of conductance fluctuations possesses an interesting dualism: the periodic ac fields that are least effective in suppressing of conductance fluctuations by dephasing, turn out to be the most effective in breaking the time-reversal symmetry. An important exception is the periodic ac field $A(t)$ that obeys the symmetry Eq.(24). At high temperatures its effect on the reduction factor $r$ is negligible while the conductance fluctuations are much larger than for the white-noise of the same power.

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