Iterative deconvolution technique for measurements of diffraction-limited images on optical microscopes

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(Received 10 March 2014; accepted 9 May 2014)

Diffraction limit is usually a thorny problem in an optical inspection system. In this investigation, a model-based deconvolution technique was developed to recover diffraction-limited images, where images with sizes smaller than the diffraction limit could be recognized. Experiments were carried out with a traditional microscope at 200× magnification coupled with a halogen light source for a series of line width samples. The point spread function of the imaging optics was first obtained from an estimated model and then combined with a nonlinear deconvolution algorithm to calculate the full width at half maximum and reconstruct the line widths. Experimental results indicate that a measurement error below one pixel size of the measurement system is achievable. Accordingly, the target of nanoscale line width inspection based on a low cost and real-time image processing technique can be fulfilled, which greatly increases the ability of nanoscaling on optical microscopes.

Keywords: deconvolution; optical microscope; diffraction limit; point spread function; line width inspection

1. Introduction

With the accelerated development of electronic products, the line width and spacing of circuit patterns in wafers are constantly narrowing. Currently, the line width of SRAM chips has narrowed down approaching the 22 nm scale. In order to assure the production quality and examination efficiency, an inspection system with nanometer resolution and high inspection speed becomes an important requirement for electronic manufacturers.

In traditional imaging system, the resolution can be affected by physical factors such as imperfections in the lenses or misalignment of the components. Moreover, there is a fundamental maximum to the resolution of even a perfect optical system because of diffraction. As it is known, the spatial resolution of a given instrument is proportional to the size of its objective and inversely proportional to the wavelength of the illumination. In microscopes, the finest detail that can be resolved is determined by the focal spot size. A tightly focused light with wavelength \( \lambda \), traveling through an objective with numerical aperture (NA), will form a spot size of diameter \( d = \frac{\lambda}{2(NA)} \) at the focus [1]. The use of low NA non-immersion type objectives in such case limits the spatial resolution to roughly \( \lambda/2 \). The Abbe diffraction limit is a serious challenge for observation of sub-wavelength structures using optical microscopes. Several techniques such as structured illumination [2], interferometry [3], near-field optics [4], and STED microscopy [5] have been developed to produce images with higher resolution than that are allowed by the diffraction limit. However, the cost and complexity associated with these techniques are generally impractical for high volume inspection in a production line. Other commercial instruments are capable of line width inspection with nanometer resolution such as scanning electron microscopes [6] and atomic force microscopes. Although the resolution achieved from these measurement systems is fantastic, they are also not practical because of the necessity for offline inspection and demanding maintenance costs.

Recently, through-focus method [7–10] has been developed for line width inspection using a bright-field microscope as the measurement system. This technique requires that images from different focal positions of the specimen were gathered via vertical scanning. A through-focus algorithm converts the light intensity of the focal points into a series of mathematical data to allow reconstruction of an auto-focus curve for comparison of the most obvious points of discrepancy. Numerical comparison is conducted with a known database to implement line width inspection. While this approach provides nanometer measurement precision, it is usually more suited for measurements of a periodic structure with sub-micron line width. Furthermore, as the through-focus method requires vertical scanning, it still struggles to reach the requirement of inline inspection, which is usually needed in an automated optical inspection system.

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In this study, an image recovery system that adopts a traditional optical microscope system was developed for real-time inspection of specimens with line widths beyond the diffraction limit. A point spread function (PSF) model of the imaging optics derived from the Fresnel-Kirchhoff diffraction formula was first estimated before integration with a nonlinear deconvolution method [11] to process blurred diffraction images. This model considers the wavelength configuration, NA of the objective, mounting position of the specimen, and detector spectrum. Therefore, it provides a more realistic approximation of the PSF and reduces deconvolution time as opposed to viable but tedious alternatives [12,13], where a priori information about the object is often necessary. The L–R algorithm proposed by Richardson and Lucy [14,15] was adopted to deal with the noise interference. Experimental results show that a measurement error below one pixel of the imaging system is achievable. The advantages of this method include low-cost image processing, avoid massive and enormous databases, achieve inline inspection, and continued upgrade through improved hardware and optimized estimation model.

2. Methodology

Image restoration techniques can be divided into spatial domain processing and frequency domain processing. Spatial domain restoration focuses on the removal of image noise, while frequency domain restoration focuses on image recovery from a variety of ambiguous situations. Figure 1 illustrates the basic model for the process of image degradation and restoration. Within a spatial domain, the degraded image can be defined as

\[ g(x, y) = h(x, y) * f(x, y) + \eta(x, y). \] (1)

As shown, the imaging process of a specimen is summarized as a degradation function \( h(x, y) \) along with a noise effect \( \eta(x, y) \) on the input image, creating a degraded image \( g(x, y) \), where “\( * \)” represents a convolution process. The focus of this research is to create an estimated recovery image from the degraded image. In principle, the more understanding one has of \( h(x, y) \) and \( \eta(x, y) \), the closer the estimated image \( \hat{f}(x, y) \) will be to the original image \( f(x, y) \). According to Fresnel–Kirchhoff diffraction formula, the light field complex amplitude \( E(P) \) for a single point \( P \) on the image plane is shown as

\[ E(P) = \frac{A}{i\lambda} \int \frac{e^{ikl} e^{ikr}}{r} \left[ \frac{\cos(nl) - \cos(nr)}{2} \right] d\sigma, \] (2)

where \( l \) represents the distance between the light source \( S \) to a certain point \( Q \) on the aperture plane \( \Sigma \), \( r \) is the distance between \( Q \) and \( P \), \( (n, l) \) is the angle between \( SQ \) and the normal to the aperture, \( (n, r) \) is the angle between the normal to the aperture and \( PQ \), \( A \) is the unit amplitude, and \( \lambda \) is the wavelength. The square of the absolute value of the complex amplitude stands for the intensity distribution of the image or the PSF. Since the degraded image is obtained from the integration of PSF from each point light source on the optical system, the degradation function of the optical system can be expressed as

\[ h(x, y) = \left| \frac{A}{i\lambda z_1} \int \frac{e^{ikl} e^{ikr}}{l} \left[ \frac{\cos(nl) - \cos(nr)}{2} \right] d\sigma \right|^2. \] (3)

For a far-field microscope, approximation can be adopted to simplify the expression as \( \frac{\cos(nl) - \cos(nr)}{2} \approx 1 \) and \( r \approx Z_1 \), where \( Z_1 = a \frac{\sqrt{1 - (NA)^2}}{2NA} \) is the vertical distance from the aperture plane to the image plane and \( a \) is the diameter of the aperture. Therefore, the degradation function of the optical system can be simplified to

\[ h(x, y) = \left| \frac{1}{i\lambda z_1} \int \frac{\tilde{E}(Q) e^{ikr}}{r} d\sigma \right|^2, \] (4)

where \( \tilde{E}(Q) = \frac{A}{\lambda^2} \) is the light field complex amplitude on the aperture plane.

The actual image is also accompanied with some noise interference. Since noise is randomly induced and cannot be precisely estimated, the recovery operation of images cannot be directly processed with a filtration
algorithm. As such, the L–R algorithm presented by Richardson and Lucy [14,15] is adopted to recover the image,

$$
\hat{f}_{k+1}(x,y) = \hat{f}_k(x,y) \left[ h(-x, -y) \ast \frac{g(x,y)}{h(x,y) \ast \hat{f}_k(x,y)} \right]
$$

(5)

where the L–R algorithm uses a maximum-likelihood estimation as a basis and the Poisson regression model to modulate the image. The estimation function of the original image, \(\hat{f}(x,y)\), is constructed based on Bayes' theory [16,17]. After an initial estimate of \(\hat{f}(x,y)\), iteration of the image deconvolution process is implemented based on (5). Through successive iterations, the optimal nondegraded image as well as the most viable PSF is calculated to gradually eliminate noise during the image recovery process. The image extraction process of this system utilizes a discrete cosine transform coupled with vertical scanning. Based on the light source spectrum, CCD spectrum, objective parameters, and sample environment, the optimal PSF and \(\hat{f}(x,y)\) can be estimated in advance.

The nonlinear characteristic of (5) occurs in the right side of the equation, where \(\hat{f}(x,y)\) appears in the denominator. Although the iterative process of the L–R algorithm is important in effectively resisting the destructive interference of noise during the image recovery process, it is also usually difficult to estimate a suitable iteration times. In theory, the deconvolution process would make an image clearer. However, when the number of iteration approaches a certain amount, the visibility \(V\) which is defined as the modulation or contrast between bright and dark intensities over the mean intensity in the same restored image gradually decays because the strength of the main image is reduced. Image error is also accumulated with the increase in iterations since the estimation of the PSF is usually not perfect. Figure 2(a) shows the relationships between the visibility of an image and iteration number for a grating with 200 nm line width and 15 lines. As shown, the visibility increases gradually from 60 to 90% after 20 iterations, then drops rapidly to below 30% after 40 iterations. Figure 2(b) shows the iteration result for an ideal image without noise. The visibility of the image increases continuously with iteration times but the incremental increase also becomes slow after 20 iterations. From these results, the default number of iterations for the deconvolution process is 20.

3. Experimental results

The measurement system utilizes a bright-field microscope as the base structure. The optical system includes a white light halogen light source, an objective with NA = 0.8 and 100× magnification, and an adjustable tube lens with magnifications of 0.5× to 4×. A CCD camera with 8-bit grey level resolution and pixel dimensions of 1000 × 1000 is used to capture images. The effective pixel resolution at 200× magnification is 37 nm.

Experiments were carried out with several line width samples. The first specimen is a grating structure with 200 nm line width. Figure 3(a) is the original picture of the grating captured by the microscope imaging system. Figure 3(b) is the recovery image after 20 iterations of the deconvolution operation, which was completed in less than two seconds. Figure 3(c) is the SEM picture of the grating to confirm the line width of the grating. Table 1 shows the comparison of the line width inspection before and after deconvolution process. The line width is determined based on the full width at half maximum principle. As shown, the original line width before deconvolution and measured line width after deconvolution are roughly 11 pixels and 6 pixels for each line, respectively. Since the size of each pixel is 37 nm, the measurement error after 20 iterations is 22 nm.
The second specimen to be inspected is a double-line structure. Figure 4(a) shows the original picture of the specimen captured by the imaging system. Figure 4(b) is the recovery image after 20 iteration times of the deconvolution operation. Table 2 shows the comparison of the line width inspection before and after deconvolution. As shown, the measured line width is 7 pixels or equivalent to 259 nm. Figure 4(c) is the SEM image of the structure at 5000× magnification that shows a structure line width of 250 nm. Compared to the SEM image, the difference in the measured width after deconvolution is only 9 nm. Figure 4(d) is the SEM image of the structure at 30,000× magnification to highlight the double-line pattern on the specimen. The extremely narrow spacing between the two lines was not recognized in the deconvolution process as it is also poorly discriminated in the original image. This indicates that the image restoration effect is still limited to the quality and resolution of the original picture.

The third experiment was executed with a simple sub-pixel resolution technique. The specimen is a grating structure with 200 nm line width. Figure 5(a) and (b) show the original picture of the grating and its intensity distribution captured by the microscope imaging system, respectively. The original pixel dimension of the image is 1000 × 1000. It was enlarged to 4000 × 4000 before the deconvolution process such that the measured result from the enlarged image is four times the actual value. Figure 5(c) and (d) are the recovery images and intensity distributions, respectively, after 20 iterations. Table 3 shows the comparison of the line width inspection before and after deconvolution. As shown, the measured line width range after deconvolution is from 5.5 pixels to 6 pixels and the smallest measurement error is only 3.5 nm. The variation may be caused by the different noise conditions in the original image. However, a measurement result with the sub-pixel resolution is obtained and better measurement accuracy could be expected with this proposed sub-pixel method.

The last experiment was carried out for inspection of individual lines. The line width obtained from the SEM image is 240 nm, as shown in Figure 6(a). The diffraction image of the line captured with the microscope is shown in Figure 6(b). The blurred image occupies a width of 10 pixels on the CCD. The recovery image after the deconvolution process is shown in Figure 6(c), where the width of the line is reduced to 7 pixels or 259 nm. The measurement error compared to the SEM result is 19 nm. By enlarging the image four times before deconvolution, sub-pixel resolution is also obtained. The recovery image after deconvolution process is shown in Figure 6(d), where the width of the line is 27 pixels, corresponding to width reduction of 37 (nm/pixel) × 27/4 (pixel) = 249.75 nm. From the above experiments, we can conclude that the measurement error below one pixel of the measurement system can be achieved with the proposed approach. That means the measurement accuracy can be upgraded with the use of a better hardware and sub-pixel technique. When the imaging resolution of a CCD is high enough, the inspection of a nanoscale line width with the proposed deconvolution method can be expected.
Table 1. Line width measurement results for a periodic structure with 200 nm line width.

| Lines | Original image line width (pixels) | Image line width post-deconvolution (pixels) | Measurement error (nm) |
|-------|-----------------------------------|-----------------------------------------------|------------------------|
| 1     | 11                                | 6                                             | 22                     |
| 2     | 11                                | 6                                             | 22                     |
| 3     | 11                                | 6                                             | 22                     |
| 4     | 10                                | 6                                             | 22                     |
| 5     | 10                                | 6                                             | 22                     |
| 6     | 11                                | 6                                             | 22                     |
| 7     | 10                                | 6                                             | 22                     |
| 8     | 11                                | 6                                             | 22                     |
| 9     | 11                                | 6                                             | 22                     |
| 10    | 10                                | 6                                             | 22                     |
| 11    | 10                                | 6                                             | 22                     |
| 12    | 11                                | 6                                             | 22                     |
| 13    | 10                                | 6                                             | 22                     |
| 14    | 11                                | 6                                             | 22                     |
| 15    | 10                                | 6                                             | 22                     |

Figure 4. (a) Original microscopic image of a double-line structure, (b) restoration image of the structure, (c) SEM picture of the structure at 5000× magnification, and (d) SEM picture at 30000× magnification.
Table 2. Line width measurement results for a double-line structure with 250 nm width.

| Lines | Original image line width (pixels) | Image line width post-deconvolution (pixels) |
|-------|-----------------------------------|---------------------------------------------|
| 1     | 12                                | 7                                          |
| 2     | 12                                | 7                                          |
| 3     | 12                                | 7                                          |
| 4     | 12                                | 7                                          |
| 5     | 12                                | 7                                          |
| 6     | 12                                | 7                                          |
| 7     | 12                                | 7                                          |
| 8     | 12                                | 7                                          |
| 9     | 12                                | 7                                          |
| 10    | 12                                | 7                                          |
| 11    | 13                                | 8                                          |

Figure 5. (a) Original microscopic image of a grating with a line width of 200 nm, (b) the intensity distribution of (a), (c) restoration image of the structure, and (d) the intensity distribution of (c). (The colour version of this figure is included in the online version of the journal.)
4. Conclusion

A beyond diffraction limit line width inspection technique has been developed using iterative-modulated deconvolution algorithm. The algorithm benefits from a realistic PSF model of the microscope system without using prior information about the object. This approach

| Lines | Original image line width (pixels) | Image line width post-deconvolution (pixels) | Measurement error (nm) |
|-------|-----------------------------------|-----------------------------------------------|------------------------|
| 1     | 10                                | 24/4 = 6                                      | 22                     |
| 2     | 10                                | 22/4 = 5.5                                    | 3.5                    |
| 3     | 11                                | 23/4 = 5.75                                   | 12.75                  |
| 4     | 10                                | 24/4 = 6                                      | 22                     |
| 5     | 10                                | 24/4 = 6                                      | 22                     |
| 6     | 11                                | 24/4 = 6                                      | 22                     |
| 7     | 10                                | 24/4 = 6                                      | 22                     |
| 8     | 11                                | 23/4 = 5.75                                   | 12.75                  |
| 9     | 11                                | 24/4 = 6                                      | 22                     |
| 10    | 11                                | 24/4 = 6                                      | 22                     |
| 11    | 10                                | 23/4 = 5.75                                   | 12.75                  |
| 12    | 11                                | 23/4 = 5.75                                   | 12.75                  |
| 13    | 12                                | 24/4 = 6                                      | 22                     |
| 14    | 12                                | 22/4 = 5.5                                    | 3.5                    |
| 15    | 11                                | 24/4 = 6                                      | 22                     |
| 16    | 12                                | 24/4 = 6                                      | 22                     |
| 17    | 10                                | 23/4 = 5.75                                   | 12.75                  |
| 18    | 11                                | 23/4 = 5.75                                   | 12.75                  |
| 19    | 10                                | 22/4 = 5.5                                    | 3.5                    |

Figure 6. (a) SEM image of a single line with 240 nm line width, (b) the diffraction image captured with the microscope, (c) the recovery image after the deconvolution process, and (d) the restoration image via the sub-pixel resolution technique.
reduces the optimal number of iterations to restore high visibility image of line gratings. Successful detection of line grating of up to 200 nm line width is possible using a bright-field microscope equipped with a 37 nm pixel resolution CCD imaging system. The deconvolution process is completed within few seconds. Moreover, image restoration of single and multiple grating lines reveals a measurement error below one pixel. The iterative deconvolution process is currently limited by two major elements. One is the difficulty in focusing on three-dimensional structures since diffraction from multiple layers creates interference with the defocused image. Secondly, the deconvolution process relies on the quality of the original picture, where the measured error is easily produced along with the image loss and nonperfect estimate of the degradation function. However, these limitations could be improved with the use of better hardware and a more accurate PSF model. Since the investigation is mainly based on the image processing technique, a low cost and real-time optical nanoscaling inspection system could be constructed with the proposed approach.

**Acknowledgments**
The authors gratefully acknowledge the support of the Ministry of Science and Technology, Taiwan under project number 102-2218-E-033-002-MY2 and Chung Yuan Christian University.

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