Heavy flavor dibaryons

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Abstract

We study the two-baryon system with two units of charm looking for the possible existence of a loosely bound state resembling the $H$ dibaryon. We make use of a chiral constituent quark model tuned in the description of the baryon and meson spectra as well as the $NN$ interaction. The presence of the heavy quarks makes the interaction simpler than in light baryon systems. We analyze possible quark-Pauli effects that would be present in spin-isospin saturated channels. Our results point to the non-existence of a charmed $H$-like dibaryon, although it may appear as a resonance above the $\Lambda_c\Lambda_c$ threshold.

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I. INTRODUCTION

Hadronic molecules have recently become a hot topic on the study of meson spectroscopy as well-suited candidates for the structure of the so-called XYZ states [1]. This explanation seems compulsory in the case of the charged charmonium-like states first reported by Belle [2] and charged bottomonium-like states reported later on also by Belle [3]. In the two-baryon system there is a well-established molecule composed of two light baryons, the deuteron. Another well-known candidate is the H dibaryon suggested by Jaffe [4], a bound state with quantum numbers of the ΛΛ system, i.e., strangeness $-2$ and $(T)J^P = (0)0^+$.

These two scenarios have recently triggered several studies about the possible existence of molecules made of heavy baryons [5–9]. The main motivation originates from the reduction of the kinetic energy due to large reduced mass as compared to systems made of light baryons. However, such molecular states that have been intriguing objects of investigations and speculations for many years, are usually the concatenation of several effects and not just a fairly attractive interaction. The coupling between close channels or the contribution of non-central forces used to play a key role for their existence. Some of these contributions may be reinforced by the presence of heavy quarks while others will become weaker.

The understanding of the hadron-hadron interaction is an important topic nowadays. To encourage new experiments seeking for evidence of theoretical predictions, it is essential to make a detailed theoretical investigation of the possible existence of bound states, despite some uncertainty in contemporary interaction models [10]. It is the purpose of this work to analyze the interaction of two-baryons with two units of charm and its application to study the possible existence of a hadronic molecule with the quantum numbers of the $\Lambda_c\Lambda_c$ system, $(T)J^P = (0)0^+$. When tackling this problem, one has to manage with an important difficulty, namely the complete lack of experimental data. Thus, the generalization of models describing the two-hadron interaction in the light flavor sector could offer insight about the unknown interaction of hadrons with heavy flavors. Following these ideas, we will make use of a chiral constituent quark model (CCQM) tuned on the description of the $NN$ interaction [11] as well as the meson [12] and baryon [13, 14] spectrum in all flavor sectors, to obtain parameter-free predictions that may be testable in future experiments. Such a project was already undertaken for the interaction between charmed mesons with reasonable predictions [15], what encourages us in the present challenge. Let us note that the study of the interaction between charmed baryons has become an interesting subject in several contexts [16] and it may shed light on the possible existence of exotic nuclei with heavy flavors [17].

The paper is organized as follows. We will use Sec. II for describing all technical details of our calculation. In particular, Sec. II A presents the description of the two-baryon system quark-model wave function, analyzing its short-range behavior looking for quark-Pauli effects. Sec. II B briefly reviews the interacting potential, and section II C deals with the solution of the two-body problem by means of the Fredholm determinant. In Sec. III we present our results. We will discuss the baryon-baryon interactions emphasizing those aspects that may produce different results from purely hadronic theories. We will analyze the character of the interaction in the different isospin-spin channels, looking for the attractive ones that may lodge resonances. We will also compare with existing results in the literature. Finally, in Sec. IV we summarize our main conclusions.
II. THE TWO-BARYON SYSTEM

A. The two-baryon wave function

We describe the baryon-baryon system by means of a constituent quark cluster model, i.e., baryons are described as clusters of three constituent quarks. Assuming a two-center shell model the wave function of an arbitrary baryon-baryon system can be written as:

\[
\Psi_{B_1B_2}(\vec{R}) = A \sqrt{1 + \delta_{B_1B_2}} \left\{ \Phi_{B_1}(123; -\frac{\vec{R}}{2}) \Phi_{B_2}(456; \frac{\vec{R}}{2}) \right\}_{\text{LST}} + (-1)^f \left\{ \Phi_{B_2}(123; -\frac{\vec{R}}{2}) \Phi_{B_1}(456; \frac{\vec{R}}{2}) \right\}_{\text{LST}},
\]

where \( A \) is the antisymmetrization operator accounting for the possible existence of identical quarks inside the hadrons. The symmetry factor \( f \) satisfies \( L + S_1 + S_2 - S + T_1 + T_2 - T + f = \text{odd} \). For non-identical baryons indicates the symmetry associated to a given set of values LST. The non-possible symmetries correspond to forbidden states. For identical baryons, \( B_1 = B_2 \), \( f \) has to be even in order to have a non-vanishing wave function, recovering the well-known selection rule \( L + S + T = \text{odd} \). In the case we are interested in, two baryons with a charmed quark, the antisymmetrization operator comes given by

\[
A = \left( 1 - \sum_{i=1}^{2} \sum_{j=4}^{5} P^{\text{LST}}_{ij} - P^{\text{LST}}_{36} \right) (1 - \mathcal{P}),
\]

where \( P^{\text{LST}}_{ij} \) exchanges a pair of identical quarks \( i \) and \( j \) and \( \mathcal{P} \) exchanges identical baryons. There appear two different contributions coming either from the exchange of light quarks \( (i = 1, 2 \text{ and } j = 4, 5) \) or the exchange of the charm quarks \( (i = 3 \text{ and } j = 6) \). If we assume gaussian 0s wave functions for the quarks inside the hadrons, the normalization of the two–baryon wave function \( \Psi_{B_1B_2}(\vec{R}) \) of Eq. (1) can be expressed as,

\[
\mathcal{N}^{\text{LST}}_{B_1B_2}(R) = \mathcal{N}^{L}_{0}(R) - C_1(S,T) \mathcal{N}^{L}_{\text{ex1}}(R) - C_2(S,T) \mathcal{N}^{L}_{\text{ex2}}(R).
\]

![Different diagrams contributing to the two-baryon normalization kernel as indicated in Eq. (3). The vertical thin solid lines represent a light quark, u or d, while the vertical thick solid lines represent the charm quarks.](image-url)
\( \mathcal{N}_{\text{di}}^L(R), \mathcal{N}_{\text{ex1}}^L(R), \) and \( \mathcal{N}_{\text{ex2}}^L(R) \) stand for the direct and exchange radial normalizations depicted in Fig. 1 whose explicit expressions are

\[
\begin{align*}
\mathcal{N}_{\text{di}}^L(R) &= 4\pi \exp \left\{ -\frac{R^2}{8} \left( \frac{4}{b^2} + \frac{2}{b_c^2} \right) \right\} i_{L+1/2} \left[ \frac{R^2}{8} \left( \frac{4}{b^2} + \frac{2}{b_c^2} \right) \right], \\
\mathcal{N}_{\text{ex1}}^L(R) &= 4\pi \exp \left\{ -\frac{R^2}{8} \left( \frac{4}{b^2} + \frac{2}{b_c^2} \right) \right\} i_{L+1/2} \left[ \frac{R^2}{8} \left( \frac{2}{b_c^2} \right) \right], \\
\mathcal{N}_{\text{ex2}}^L(R) &= 4\pi \exp \left\{ -\frac{R^2}{8} \left( \frac{4}{b^2} + \frac{2}{b_c^2} \right) \right\} i_{L+1/2} \left[ \frac{R^2}{8} \left( \frac{4}{b^2} - \frac{2}{b_c^2} \right) \right],
\end{align*}
\]

where, for the sake of generality, we have assumed different gaussian parameters for the function of the light quarks \( (b) \) and the heavy quark \( (b_c) \). In the limit where the two hadrons overlap \( (R \to 0) \), the Pauli principle may impose antisymmetry requirements not present in a hadronic description. Such effects, if any, will be prominent for \( L = 0 \). Using the asymptotic form of the Bessel functions, \( i_{L+1/2} \), we obtain,

\[
\begin{align*}
\mathcal{N}_{\text{di}}^{L=0} &\xrightarrow{R \to 0} 4\pi \left\{ 1 - \frac{R^2}{8} \left( \frac{4}{b^2} + \frac{2}{b_c^2} \right) \right\} \left[ 1 + \frac{1}{6} \left( \frac{R^2}{8} \left( \frac{4}{b^2} + \frac{2}{b_c^2} \right) \right)^2 + \ldots \right], \\
\mathcal{N}_{\text{ex1}}^{L=0} &\xrightarrow{R \to 0} 4\pi \left\{ 1 - \frac{R^2}{8} \left( \frac{4}{b^2} + \frac{2}{b_c^2} \right) \right\} \left[ 1 + \frac{1}{6} \left( \frac{R^2}{8} \left( \frac{2}{b_c^2} \right) \right)^2 + \ldots \right], \\
\mathcal{N}_{\text{ex2}}^{L=0} &\xrightarrow{R \to 0} 4\pi \left\{ 1 - \frac{R^2}{8} \left( \frac{4}{b^2} + \frac{2}{b_c^2} \right) \right\} \left[ 1 + \frac{1}{6} \left( \frac{R^2}{8} \left( \frac{4}{b^2} - \frac{2}{b_c^2} \right) \right)^2 + \ldots \right].
\end{align*}
\]

Finally, the normalization kernel of Eq. \( \text{(3)} \) can be written for the S wave in the overlapping region as,

\[
\mathcal{N}_{B_i B_j}^{L=0ST} \xrightarrow{R \to 0} 4\pi \left\{ 1 - \frac{R^2}{8} \left( \frac{4}{b^2} + \frac{2}{b_c^2} \right) \right\} \left\{ [1 - 3C(S,T)] + \ldots \right\},
\]

where \( C(S,T) = \frac{4}{9}C_1(S,T) + \frac{1}{9}C_2(S,T) \) for systems with two charmed baryons. For the particular case of the \( N\Xi_{cc} \) system \( C(S,T) = \frac{1}{3}C_1(S,T) \). The values of \( C(S,T) \) are given in Table 1. Thus, the closer the value of \( C(S,T) \) to 1/3 the larger the suppression of the normalization of the wave function at short distances, generating Pauli repulsion. It is the channel \( \Lambda_c \Lambda_c \) \( (T, J) = (0, 0) \), with \( C(S,T) = 2/9 \), where the norm kernel gets smaller

**TABLE I:** \( C(S,T) \) spin-isospin coefficients, as defined in the text, for \( L = 0 \) partial waves.

| \( B_i B_j \) | \( T = 0 \) | \( T = 1 \) | \( T = 2 \) |
|--------------|--------|--------|--------|
| \( \Lambda_c \Lambda_c \) | \( N\Xi_{cc} \) | \( N\Xi_{cc} \) | \( N\Xi_{cc} \) |
| \( J = 0 \) | \( 2/9 \) | \( -1/9 \) | \( 0 \) | \( 5/27 \) | \( -1/18 \) | \( -1/9 \) |
| \( J = 1 \) | \( -2/27 \) | \( -1/9 \) | \( -1/9 \) | \( 16/81 \) | \( 1/54 \) | \( -5/81 \) |

\(^1\) Note that in the \( N\Xi_{cc} \) case the antisymmetrization operator of Eq. \( \text{(2)} \) becomes much more simpler \( \mathcal{A} = (1 - 3P_{ij}^{LST}) \), and only the first exchange diagram of Fig. \( \text{(1)} \) will contribute with the two charm quarks on the same baryon.
at short distances. One would only find Pauli blocking \([18]\) in excited states like \(\Sigma^*_c\Sigma^*_c\) \((T, J) = (2, 3)\), where \(C(S, T) = 1/3\), due to lacking degrees of freedom to accommodate the light quarks present on this configuration, four \(u\) quarks with spin up. However, this partial wave could only exist for \(L = \text{odd}\), and then the Pauli blocking may get masked by the centrifugal barrier.

We show in Fig. 2 the normalization kernel given by Eq. (3) for \(L = 0\) and four different channels: \(\Lambda_c\Lambda_c\) with \((T, J) = (0, 0)\), \(\Sigma_c\Sigma_c\) with \((T, J) = (0, 0)\) and \((2, 0)\), and \(N\Xi_{cc}\) with \((T, J) = (1, 0)\). In the first case \(C(S, T)\) is positive and close to 1/3, what gives a small normalization kernel. In the last case \(C(S, T)\) is also positive but smaller, giving rise to a slightly larger normalization kernel. In the other two cases \(C(S, T)\) is zero or negative, showing a large norm kernel at short distances and therefore one does not expect any Pauli effect at all.

B. The two-body interactions

The interactions involved in the study of the two-baryon system are obtained from a chiral constituent quark model \([11]\). This model was proposed in the early 90’s in an attempt to obtain a simultaneous description of the nucleon-nucleon interaction and the light baryon spectra. It was later on generalized to all flavor sectors \([12]\). In this model hadrons are described as clusters of three interacting massive (constituent) quarks, the mass coming from the spontaneous breaking of the original \(SU(2)_L \otimes SU(2)_R\) chiral symmetry of the QCD Lagrangian. QCD perturbative effects are taken into account through the one-gluon-
exchange (OGE) potential \[19\]. It reads,

\[
V_{\text{OGE}}(\vec{r}_{ij}) = \frac{\alpha_s}{4} \vec{\lambda}_i \cdot \vec{\lambda}_j \left\{ \frac{1}{r_{ij}} - \frac{1}{4} \left( \frac{1}{2m_i^2} + \frac{1}{2m_j^2} + \frac{2\vec{\sigma}_i \cdot \vec{\sigma}_j}{3m_im_j} \right) \frac{e^{-r_{ij}/r_0}}{r_{ij}^2} - \frac{3S_{ij}}{4m_im_jr_{ij}^3} \right\},
\]

where \(\lambda\) are the SU(3) color matrices, \(r_0 = \hat{r}_0/\mu\) is a flavor-dependent regularization scaling with the reduced mass of the interacting pair, and \(\alpha_s\) is the scale-dependent strong coupling constant given by \[12\],

\[
\alpha_s(\mu) = \frac{\alpha_0}{\ln \left((\mu^2 + \mu_0^2)/\gamma_0^2\right)},
\]

where \(\alpha_0 = 2.118\), \(\mu_0 = 36.976\) MeV and \(\gamma_0 = 0.113\) fm\(^{-1}\). This equation gives rise to \(\alpha_s \sim 0.54\) for the light-quark sector, \(\alpha_s \sim 0.43\) for \(\bar{c}c\) pairs, and \(\alpha_s \sim 0.29\) for \(\bar{u}c\) pairs.

Non-perturbative effects are due to the spontaneous breaking of the original chiral symmetry at some momentum scale. In this domain of momenta, light quarks interact through Goldstone boson exchange potentials,

\[
V_\chi(\vec{r}_{ij}) = V_{\text{OSE}}(\vec{r}_{ij}) + V_{\text{OPE}}(\vec{r}_{ij}),
\]

where

\[
V_{\text{OSE}}(\vec{r}_{ij}) = -\frac{g_{\chi}^2}{4\pi} \frac{\Lambda^2}{\Lambda^2 - m_\sigma^2} m_\sigma \left[ Y(m_\sigma r_{ij}) - \frac{\Lambda}{m_\sigma} Y(\Lambda r_{ij}) \right],
\]

\[
V_{\text{OPE}}(\vec{r}_{ij}) = \frac{g_{\chi}^2}{4\pi} \frac{m_\pi^2}{12m_i m_j} \frac{\Lambda^2}{\Lambda^2 - m_\pi^2} m_\pi \left\{ \left[ Y(m_\pi r_{ij}) - \frac{\Lambda^3}{m_\pi^3} Y(\Lambda r_{ij}) \right] \vec{\sigma}_i \cdot \vec{\sigma}_j + \left[ H(m_\pi r_{ij}) - \frac{\Lambda^3}{m_\pi^3} H(\Lambda r_{ij}) \right] S_{ij} \right\} (\vec{r}_i \cdot \vec{r}_j).
\]

\(g_{\chi}^2/4\pi\) is the chiral coupling constant, \(Y(x)\) is the standard Yukawa function defined by \(Y(x) = e^{-x}/x\), \(S_{ij} = 3 (\vec{\sigma}_i \cdot \vec{r}_{ij})(\vec{\sigma}_j \cdot \vec{r}_{ij}) - \vec{\sigma}_i \cdot \vec{\sigma}_j\) is the quark tensor operator, and \(H(x) = (1 + 3/x + 3/x^2)Y(x)\).

Finally, any model imitating QCD should incorporate confinement. Being a basic term from the spectroscopic point of view it is negligible for the hadron-hadron interaction. Lattice calculations suggest a screening effect on the potential when increasing the interquark distance \[20\],

\[
V_{\text{CON}}(\vec{r}_{ij}) = \{-a_c(1 - e^{-\mu_r r_{ij}})\} (\vec{\lambda}_i \cdot \vec{\lambda}_j).
\]

Once perturbative (one-gluon exchange) and nonperturbative (confinement and chiral symmetry breaking) aspects of QCD have been considered, one ends up with a quark-quark interaction of the form

\[
V_{q_i q_j}(\vec{r}_{ij}) = \begin{cases} \{q_i q_j = nn\} \Rightarrow V_{\text{CON}}(\vec{r}_{ij}) + V_{\text{OGE}}(\vec{r}_{ij}) + V_\chi(\vec{r}_{ij}) & , \\ \{q_i q_j = cn/cc\} \Rightarrow V_{\text{CON}}(\vec{r}_{ij}) + V_{\text{OGE}}(\vec{r}_{ij}) & , \end{cases}
\]

where \(n\) stands for the light quarks \(u\) and \(d\). Notice that for the particular case of heavy quarks \((c\ or\ b)\) chiral symmetry is explicitly broken and therefore boson exchanges do not contribute. The parameters of the model are the same that have been used for the study of the \(ND\) system \[21\] and for completeness are quoted in Table \[11\]. The model guarantees a nice description of the light \[13\] and charmed \[14\] baryon spectra.
In order to derive the $B_n B_m \rightarrow B_k B_l$ interaction from the basic $qq$ interaction defined above, we use a Born-Oppenheimer approximation. Explicitly, the potential is calculated as follows,

$$V_{B_n B_m (LST) \rightarrow B_k B_l (L' S'T')}(R) = \xi_{LST}^{L' S'T'}(R) - \xi_{LST}^{L' S'T'}(\infty),$$  

(13)

where

$$\xi_{LST}^{L' S'T'}(R) = \frac{\langle \Psi_{B_k B_l}^{L' S'T'}(\vec{R}) | \sum_{i<j=1}^{6} V_{i<j}(\vec{r}_{ij}) | \Psi_{B_n B_m}^{L' S'T'}(\vec{R}) \rangle}{\sqrt{\langle \Psi_{B_k B_l}^{L' S'T'}(\vec{R}) | \Psi_{B_k B_l}^{L' S'T'}(\vec{R}) \rangle \sqrt{\langle \Psi_{B_n B_m}^{L' S'T'}(\vec{R}) | \Psi_{B_n B_m}^{L' S'T'}(\vec{R}) \rangle}}. \tag{14}$$

In the last expression the quark coordinates are integrated out keeping $R$ fixed, the resulting interaction being a function of the two-baryon relative distance. The wave function $\Psi_{B_n B_m}^{LST}(\vec{R})$ for the two-baryon system has been discussed in Sec. II.A.

### C. Integral equations for the two-body systems

To study the possible existence of two-baryon molecular states with two units of charm: $\Lambda_c \Lambda_c$, $N \Xi_c$, $\Lambda_c \Sigma_c$, or $\Sigma_c \Sigma_c$, we have solved the Lippmann-Schwinger equation for negative energies looking at the Fredholm determinant $D_F(E)$ at zero energy [22]. If there are no interactions then $D_F(0) = 1$, if the system is attractive then $D_F(0) < 1$, and if a bound state exists then $D_F(0) < 0$. This method permits us to obtain robust predictions even for zero-energy bound states, and gives information about attractive channels that may lodge a resonance in similar systems [15]. We consider a two-baryon system $B_i B_j$ in a relative $S$ state interacting through a potential $V$ that contains a tensor force. Then, in general, there is a coupling to the $B_i B_j D$ wave. Moreover, the two-baryon system can couple to other two-baryon states. We show in Table III the two-baryon coupled channels in the isospin-spin ($T, J$) basis.

Thus, if we denote the different two-baryon systems as channel $A_i$, the Lippmann-Schwinger equation for the baryon-baryon scattering becomes

$$t_{\alpha;\beta;T;J}^{\ell_{\alpha};s_{\alpha};\ell_{\beta};s_{\beta}}(p_{\alpha}, p_{\beta}; E) = V_{\alpha;\beta;T;J}^{\ell_{\alpha};s_{\alpha};\ell_{\beta};s_{\beta}}(p_{\alpha}, p_{\beta}) + \sum_{\gamma=A_1, A_2, \cdots} \sum_{t_s=0,2} \int_0^\infty p_{\gamma}^2 dp_{\gamma} V_{\alpha;\beta;T;J}^{\ell_{\alpha};s_{\alpha};\ell_{\beta};s_{\beta}}(p_{\alpha}, p_{\gamma})$$

$$\times G_{\gamma}(E; p_{\gamma}) t_{\gamma;\beta;T;J}^{\ell_{\gamma};s_{\gamma}}(p_{\gamma}, p_{\beta}; E), \quad \alpha, \beta = A_1, A_2, \cdots, \tag{15}$$

where $t$ is the two-body scattering amplitude, $T$, $J$, and $E$ are the isospin, total angular momentum and energy of the system, $\ell_{\alpha}s_{\alpha}$, $\ell_{\beta}s_{\beta}$, and $\ell_{\gamma}s_{\gamma}$ are the initial, intermediate, and

| TABLE II: Quark-model parameters. |
|-----------------------------------|
| $m_{u,d}$(MeV)  | 313  | $g_{ch}^2/(4\pi)$  | 0.54  |
| $m_c$(MeV)      | 1752 | $m_{a}$(fm$^{-1}$) | 3.42  |
| $b$(fm)         | 0.518| $m_{c}$(fm$^{-1}$) | 0.70  |
| $b_{c}$(fm)     | 0.6  | $\Lambda$(fm$^{-1}$) | 4.2   |
| $\hat{r}_0$ (MeV fm) | 28.170 | $a_{c}$(MeV) | 230   |
| $\mu_c$(fm$^{-1}$) | 0.70  |                           |       |

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final orbital angular momentum and spin, respectively, and \( p_\gamma \) is the relative momentum of the two-body system \( \gamma \). The propagators \( G_\gamma(E; p_\gamma) \) are given by

\[
G_\gamma(E; p_\gamma) = \frac{2\mu_\gamma}{k_\gamma^2 - p_\gamma^2 + i\epsilon},
\]

with

\[
E = \frac{k_\gamma^2}{2\mu_\gamma},
\]

where \( \mu_\gamma \) is the reduced mass of the two-body system \( \gamma \). For bound-state problems \( E < 0 \) so that the singularity of the propagator is never touched and we can forget the \( i\epsilon \) in the denominator. If we make the change of variables

\[
p_\gamma = d \frac{1 + x_\gamma}{1 - x_\gamma},
\]

where \( d \) is a scale parameter, and the same for \( p_\alpha \) and \( p_\beta \), we can write Eq. (15) as

\[
t_{\alpha\beta;IJ}^{\ell_\alpha s_\alpha,\ell_\beta s_\beta}(x_\alpha, x_\beta; E) = V_{\alpha\beta;IJ}^{\ell_\alpha s_\alpha,\ell_\beta s_\beta}(x_\alpha, x_\beta) + \sum_{\gamma=A_1, A_2, \cdots} \sum_{\ell_\gamma = 0, 2} \int_{-1}^{1} d^2\left( \frac{1 + x_\gamma}{1 - x_\gamma} \right)^2 \frac{2d}{(1 - x_\gamma)^2} dx_\gamma \times V_{\alpha\beta;IJ}^{\ell_\alpha s_\alpha,\ell_\beta s_\beta}(x_\alpha, x_\gamma) G_\gamma(E; p_\gamma) t_{\gamma\beta;IJ}^{\ell_\gamma s_\gamma,\ell_\beta s_\beta}(x_\gamma, x_\beta; E).
\]

We solve this equation by replacing the integral from \(-1\) to \(1\) by a Gauss-Legendre quadrature which results in the set of linear equations

\[
\sum_{\gamma=A_1, A_2, \cdots} \sum_{\ell_\gamma = 0, 2} \sum_{m=1}^{N} M_{\alpha\gamma;IJ}^{\ell_\alpha s_\alpha,\ell_\beta s_\beta}(E) t_{\gamma\beta;IJ}^{\ell_\gamma s_\gamma,\ell_\beta s_\beta}(x_m, x_k; E) = V_{\alpha\beta;IJ}^{\ell_\alpha s_\alpha,\ell_\beta s_\beta}(x_n, x_k),
\]

with

\[
M_{\alpha\gamma;IJ}^{\ell_\alpha s_\alpha,\ell_\beta s_\beta}(E) = \delta_{nm}\delta_{s_\alpha s_\gamma}\delta_{s_\beta s_\gamma} - w_m d^2\left( \frac{1 + x_m}{1 - x_m} \right)^2 \frac{2d}{(1 - x_m)^2} \times V_{\alpha\gamma;IJ}^{\ell_\alpha s_\alpha,\ell_\beta s_\beta}(x_n, x_m) G_\gamma(E; p_{\gamma m}),
\]

and where \( w_m \) and \( x_m \) are the weights and abscissas of the Gauss-Legendre quadrature while \( p_{\gamma m} \) is obtained by putting \( x_\gamma = x_m \) in Eq. (18). If a bound state exists at an energy \( E_B \), the determinant of the matrix \( M_{\alpha\gamma;IJ}^{\ell_\alpha s_\alpha,\ell_\beta s_\beta}(E_B) \) vanishes, i.e., \( |M_{\alpha\gamma;IJ}(E_B)| = 0 \).

TABLE III: \( S \) and \( D \) wave two-baryon channels contributing to the different isospin-spin (\( T, J \)) states. See text for details.

| \( J \) | \( T = 0 \) | \( T = 1 \) | \( T = 2 \) |
|-------|-----|-----|-----|
| \( J = 0 \) | \( \Lambda_c \Lambda_c / N\Xi_{cc} \) / \( \Sigma_c \Sigma_c \) | \( N\Xi_{cc} / \Lambda_c \Sigma_c \) | \( \Sigma_c \Sigma_c \) |
| \( J = 1 \) | \( N\Xi_{cc} \) | \( N\Xi_{cc} / \Lambda_c \Sigma_c / \Sigma_c \Sigma_c \) | \( - \) |
III. RESULTS AND DISCUSSION

We will first discuss the interactions derived with the CCQM, centering our attention in the most interesting channel, the flavor singlet channel \((T)J^P = (0)0^+\) with the quantum numbers of the \(\Lambda_c\Lambda_c\) state. This channel might lodge a charmed \(H\)-like dibaryon. We show in Fig. 3 the diagonal and transition central potentials contributing to the \((T)J^P = (0)0^+\) state. It is important to note that the \(\Lambda_c\Lambda_c\) system is decoupled from the closest two-baryon threshold, the \(N\Xi_{cc}\) state, that in the case of the strange \(H\) dibaryon becomes relevant for its possible bound or resonant character [23]. The binding of the \((T)J^P = (0)0^+\) state would then require a stronger attraction in the diagonal channels or a stronger coupling to the heavier \(\Sigma_c\Sigma_c\) state, that as we will discuss below is not fulfilled.

In Fig. 4 we have separated the contributions of the different terms in Eq. (12) to the two diagonal interactions. As can be seen, the \(\Lambda_c\Lambda_c\) potential is the most repulsive one. It becomes repulsive at short-range partially due to the reduction of the normalization kernel (see Fig. 2). The OGE and OPE can only give contributions through quark-exchange diagrams due to the color-spin-isospin structure of the antisymmetry operator [11, 24]. They generate short-range repulsion that it is compensated at intermediate distances by the attraction coming from the scalar exchange, with a longer range. Thus, the total potential becomes slightly attractive at intermediate distances but repulsive at short range. In the \(\Sigma_c\Sigma_c\) interaction, the presence of a direct (without simultaneous quark-exchange) contribution of the OPE and the opposite sign of most part of the exchange diagrams, generates an overall attractive potential. This is rather similar to the situation in the strange sector but with the absence of the one-kaon exchange potential, what gives rise to a less attractive interaction.

Regarding the character of the interaction, similar results were obtained in Ref. [9] within the quark delocalization color screening model (QDCSM). It is important to note at this
point the difference with hadronic potential models as those of Refs. [5, 6]. As can be seen in Fig. 3(a) of Ref. [5], the \((T)J^P = (0)0^+ \Lambda_c\Lambda_c\) potential is attractive due to the absence of quark-exchange contributions and the dominance of the attraction of the scalar exchange potential. In spite of being attractive, the central potential alone is not enough to generate a bound state. In Ref. [6] they only consider the hadronic one-pion exchange and then, the \(\Lambda_c\Lambda_c\) interaction is zero. Thus all possible attraction comes generated by the coupling to larger mass channels.

As mentioned above, when comparing with the similar problem in the strange sector an important difference arises, the absence of the \(\Lambda_c\Lambda_c\leftrightarrow N\Xi_{cc}\) coupling. As a consequence the mass difference between the two coupled channels in the \((T)J^P = (0)0^+\) partial wave, \(\Lambda_c\Lambda_c\) and \(\Sigma_{cc}\Sigma_{cc}\), is much larger than in the strange sector, making the coupled channel effect less important. Let us note that in the strange sector \(M(N\Xi) - M(\Lambda\Lambda) = 25 \text{ MeV}\) and \(M(\Sigma\Sigma) - M(\Lambda\Lambda) = 154 \text{ MeV}\), this is why the \(N\Xi\) channel plays a relevant role for the \(\Lambda\Lambda\) system [23], as well as why the \(N\Sigma\) state is relevant for the \(N\Lambda\) system [25]. In the charmed sector the closest channel coupled to \(\Lambda_c\Lambda_c\) in the \((T)J^P = (0)0^+\) state is \(\Sigma_c\Sigma_c\), 338 MeV above. This energy difference is similar to the \(N\Delta - NN\) mass difference, the coupled channel effect being still important although it may not proceed through the central terms due to angular momentum selection rules [26]. Heavier channels play a minor role, as it occurs with the \(\Delta\Delta\) channel, 584 MeV above the \(NN\) threshold [24]. In the present case the coupling to the closest channel \(\Sigma_{cc}\Sigma_{cc}\) proceeds through the central potential and one does not expect higher channels, as \(\Sigma_{cc}\Sigma_{cc}^*\) 468 MeV above the \(\Lambda_c\Lambda_c\) threshold, to play a relevant role in quark-model descriptions as shown in the QDCSM model of Ref. [9].

[FIG. 4: Left panel: Contribution of the different terms of the interaction to the \((T)J^P = (0)0^+ \Lambda_c\Lambda_c\) potential. 'OGE' stands for the one-gluon exchange, 'OPE' for the one-pion exchange, 'OSE' denotes the one-sigma exchange and 'Tot' represents the total potential. Right panel: Same as the left panel for the \((T)J^P = (0)0^+ \Sigma_{cc}\Sigma_{cc}\) potential.]
situation seems to be a bit different in hadronic models where the non-central potentials are not regularized by the quark-model wave function [6, 7]. Let us finally note that the coupling between the $\Lambda_c\bar{\Lambda}_c$ and $\Sigma_c\bar{\Sigma}_c$ channels comes mainly given by quark-exchange effects and the direct one-pion exchange potential. Thus, it becomes a little bit stronger than in hadronic theories based on the one-pion exchange potential [6]. The resulting interaction is rather similar to that in the strange sector, as can be seen by comparing with Fig. 1(b) of Ref. [27], the main difference coming from the behavior of the normalization kernel at short distances.

With these ideas in mind and following the procedure described in Sec. II C we have performed a full coupled channel calculation of the $(T) J^P = (0)0^+$ state. The results are shown in Fig. 5. In the left panel we show by the dashed line the Fredholm determinant of the $\Lambda_c\bar{\Lambda}_c$ channel alone. The Fredholm determinant is large, indicating a barely small attractive interaction. When the coupling to the heavier $\Sigma_c\bar{\Sigma}_c$ channel is included, solid line in Fig. 5, the system gains attraction, but it is not enough as to get a bound state. The main uncertainty when determining the baryon-baryon interaction in quark models with charmed baryons would be the harmonic oscillator parameter of the charm quark. We have explored the results for different values of $b_c$. The results are shown in the right panel of Fig. 5 and as can be seen in no case the $(T) J^P = (0)0^+$ state would become bound. Note that in Ref. [28] it was argued that the smaller values of $b_c$ are preferred to get consistency with calculations based on infinite expansions, as hyperspherical harmonic expansions [29], where the quark wave function is not postulated. This also agrees with simple harmonic oscillator relations $b_c = b_n\sqrt{m_c/m_n}$. The smaller values of $b_c$ give rise to the less attractive results. For the
larger values of $b_C$, if a loosely bound state could be generated, the electromagnetic repulsion arising in the $(T)J^P = (0)0^+$ channel due to the electric charge of the $\Lambda_c^+$ might dismantle the bound state.

Thus, without the strong transition potentials reported in the QDCSM model of Ref. [9] or the strong tensor couplings occurring in the hadronic one-pion exchange models of Refs. [6,7], it seems difficult to get a bound state in this system. We have recently illustrated within the quark model [30] how the coupled channel effect between channels with an almost negligible interaction in the lower mass channel works for generating bound states. Although for the four-quark problem, in this reference it is demonstrated (see Fig. 2 of Ref. [30]) how when the thresholds mass difference increases, the effect of the coupled channel diminishes, which is an unavoidable consequence of having the same hamiltonian to describe the hadron masses and the hadron-hadron interactions.

We have also analyzed the other $(T)J^P$ channels shown in Table III with similar conclusions, the weak interaction in the charm sector and the absence of channel coupling between close mass channels works against the possibility of having dibaryons with two units of charm. For the sake of completeness we have calculated the Fredholm determinant for all cases and it is shown in Fig. 6.

One should finally note that the problem of double heavy dibaryons has also been approached in the literature by means of six-quark calculations. The group of Grenoble [31] addressed this problem within a pure chromomagnetic interaction obtaining several candidates to be bound. There also interesting results based on relativistic six-quark equations constructed in the framework of the dispersion relation technique [32] with a rich spectroscopy of double charmed and beauty heavy dibaryons. Future experimental results will

FIG. 6: Left panel: Fredholm determinant of the $(T)J^P$ channels where the $N\Xi_{cc}$ state is the lowest threshold. Right panel: Fredholm determinant of the $(T)J^P$ channel where the $\Sigma_c^+\Sigma_c^+$ state is the lowest threshold.
help to scrutinize among the different models, and in this way to improve our phenomenological understanding of QCD in the highly non-perturbative low-energy regime. This challenge could only be achieved by means of a cooperative experimental and theoretical effort.

IV. SUMMARY

In short summary, we have studied the baryon-baryon interaction with two units of charm making use of a chiral constituent quark model tuned in the description of the baryon and meson spectra as well as the $NN$ interaction. Several effects conspire against the existence of a loosely bound state resembling the $H$ dibaryon. First, the interaction is weaker than in the strange sector. Second, there is no coupling between close mass thresholds, like $\Lambda_c\Lambda_c \leftrightarrow N\Xi_{cc}$, the closest threshold being more than 300 MeV above. Finally, the existence of a weak attraction may be killed by the electromagnetic repulsion absent in the strange sector. Thus, our results point to the nonexistence of low-energy dibaryons with two units of charm and in particular, to the nonexistence of a stable charmed $H$-like dibaryon. Given that the interaction in the $(T)J^P = (0)0^+$ is attractive, this state may appear as a resonance above the $\Lambda_c\Lambda_c$ threshold.

Weakly bound states are usually very sensitive to potential details and therefore theoretical investigations with different phenomenological models are highly desirable. The existence of these states could be scrutinized in the future at the LHC, J-PARC and RHIC providing a great opportunity for extending our knowledge to some unreached part in our matter world.

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