Fast forward Adiabatic Quantum Dynamics On Two Dimensional Dirac Equation

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Abstract. We study a scheme of accelerated adiabatic quantum dynamics. This scheme was originally proposed by Masuda-Nakamura. The strategy of combining two opposite idea: infinitely-large timemagnification factor and infinitely-small growth rate of adiabatic parameter was elucidated. We apply the proposed method to two dimensional system with electric field and magnetic field using two dimensional Dirac equation. We settle the quasi-adiabatic dynamics (QAD) by adding the regularization terms to the original vector potential and scalar potential and accelerate it with use of a large time-scaling factor which realizes QAD on shortened time scale. These terms multiplied by the velocity function give the counterdiabatic terms that generate the fast forward potential. The fast forward potential can accelerate the dynamics of the system from initial state to the final state with the same condition.

1. Introduction

Acceleration dynamics of nanostructure can be very important to optimize the electronic properties. The discovery of graphene [1, 2], a two-dimensional layer of graphite, and the massless Dirac character of the low energy electrons moving has attracted much interest in physics due to its important electronic properties. Graphene also an appropriate material to develop electronic devices. It is consisting of an isolated single atomic layer of graphite, is an ideal realization of such a two-dimensional system. If we want to accelerate the system we should accelerate the dynamics of each electron, atom or molecules to reach their desired target states in shorter time.. Masuda and Nakamura [3] proposed a theory of fast forward to accelerate quantum dynamics with use of additional phase and driving potential, and Khujakulov and Nakamura [4] showed that this theory is useful to enhance the quantum tunneling power. The theory of fast-forward guarantees to accelerate any given quantum evolution and to obtain the desired target state on shortened time scale, by fast forwarding the reference quantum dynamics. This theory was further developed to accelerate the quasi-static or adiabatic quantum dynamics [5, 6, 7], and constitutes one of the promising means to the shortcut to adiabaticity (STA) [8, 9, 10, 11, 12, 13, 14]. The relationship between the fast forward and STA is nowadays clear [7, 15, 16]).

While the theory of fast forward has been limited to orbital dynamics of atoms, molecules or Bose-Einstein condensates, we recently proposed a scheme of fast forward of adiabatic spin dynamics
Regularization of adiabatic dynamics of Dirac equation

Before embarking on the fast forward of adiabatic dynamics of Dirac dynamics, we shall find the regularization term of scalar and vector potential. Firstly we consider the standard dynamics of Dirac equation as the following

\[ i\hbar \frac{\partial \Psi}{\partial t} = H_0 \Psi(x, y, t) \]  

(1)

with Hamiltonian is defined as

\[ H_0 = (p_x + A_x)\sigma_x + (p_y + A_y)\sigma_y + mc^2\sigma_z + V_0 \]  

(2)

where \( \sigma_x(y), p_x(y) \) and \( A_x(y) \), \( V_0 \) are Pauli matrices, momenta, vector potentials and scalar potentials respectively.

Time dependent Dirac equation can be rewritten as

\[ i\hbar \frac{\partial \Psi_0}{\partial t} = [(-i\hbar \partial_x + A_x^0)\sigma_x + (-i\hbar \partial_y + A_y^0)\sigma_y + mc^2\sigma_z + V_0]\Psi_0 \]  

(3)

For our notation convenience, we denote prior wave functions \((\Psi_1^0, \Psi_2^0)\) The matrix form of Dirac equation is written as

\[ i\hbar \begin{pmatrix} \frac{\partial \Psi_1}{\partial t} \\ \frac{\partial \Psi_2}{\partial t} \end{pmatrix} = \begin{pmatrix} mc^2 + V_0 & \pi_+^0 \\ \pi_-^0 & -mc^2 + V_0 \end{pmatrix} \begin{pmatrix} \frac{\partial \Psi_1}{\partial t} \\ \frac{\partial \Psi_2}{\partial t} \end{pmatrix} \]  

(4)

where

\[ \pi_\pm^0 = \pi_x^0 \pm i\pi_y^0 \]  

(5)

To be satisfy the adiabatic dynamics, we define the regularized spinor as

\[ \psi^{reg} = \begin{pmatrix} \phi_1^{(n)}(x, y, R(t)) \\ \phi_2^{(n)}(x, y, R(t)) \end{pmatrix} e^{i\delta_n(t)}, \]  

(6)

Regularized scalar and vector potential are

\[ V^{reg} = V_0 + \epsilon(\bar{V} + i\bar{U}), \]  

(7)

\[ A_x^0 \pm iA_y^0 \rightarrow A_x^{reg} \pm iA_y^{reg} + \epsilon(\bar{A}_x \pm i\bar{A}_y), \]  

(8)

Where \( \bar{V} \) and \( \bar{U} \) are regularized scalar potential, \( \bar{A}_x \) and \( i\bar{A}_y \) are regularized vector potential. Here \( R \equiv R(t) = R_0 + \epsilon t \) is the adiabatically-changing parameter with \( \epsilon \ll 1 \) and \( \delta_n = -1/ \hbar \int_0^s E_n(R(s)) \). The equation for the regularized spinor in natural unit is

\[ i\hbar \frac{\partial \psi^{reg}}{\partial t} = \begin{pmatrix} m + V_0 + \epsilon(\bar{V} + i\bar{U}) & \pi_-^0 \\ \pi_+^0 & -m + V_0 + \epsilon(\bar{V} + i\bar{U}) \end{pmatrix} \psi^{reg} \]  

(9)
\[
\frac{\partial \psi^{\text{reg}}}{\partial t} = \left( m + V_0 + \varepsilon(\vec{V} + i \vec{U}) \right) \frac{\pi_0^0}{\pi_+^0} - m + V_0 + \varepsilon(\vec{V} + i \vec{U}) \right) \psi^{\text{reg}} \tag{12}
\]

where
\[
\pi_0^0 = -i x + y + A_x^0 + i A_y^0 + \varepsilon(\vec{A}_x + i \vec{A}_y), \tag{13}
\]
\[
\pi_+^0 = -i x - y + A_x^0 - i A_y^0 + \varepsilon(\vec{A}_x - i \vec{A}_y). \tag{14}
\]

On the left hand side of equation 11, we have
\[
i \frac{\partial \psi^{\text{reg}}}{\partial t} = \left[ i \varepsilon \left( \frac{\partial \phi_1^{(n)}}{\partial \rho} + \phi_2^{(n)} \right) + \phi_2^{(n)} \right] E_n e^{i \delta_n} \tag{17}
\]

While the right hand side of equation 11 is
\[
\left( m + V_0 + \varepsilon(\vec{V} + i \vec{U}) \right) \phi_1^{(n)} + (A_x^0 - i A_y^0) + \varepsilon(\vec{A}_x - i \vec{A}_y) - i \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \phi_1^{(n)} + (-m + V_0 + \varepsilon(\vec{V} + i \vec{U}) \phi_2^{(n)}) \times e^{i \delta_n} \tag{18}
\]

Thus, from equation 11 we have
\[
i \varepsilon \left( \frac{\partial \phi_1^{(n)}}{\partial \rho} + \phi_2^{(n)} \right) E_n = \left( m + V_0 + \varepsilon(\vec{V} + i \vec{U}) \phi_1^{(n)} + (A_x^0 - i A_y^0) + \varepsilon(\vec{A}_x - i \vec{A}_y) - i \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \phi_1^{(n)} + (-m + V_0 + \varepsilon(\vec{V} + i \vec{U}) \phi_2^{(n)}) \right) \tag{19}
\]

The zeroth order of \( \varepsilon \) in the above equation is:
\[
\left( \frac{\phi_1^{(n)}}{\phi_2^{(n)}} \right) E_n = \left( \frac{m + V_0 + (A_x^0 - i A_y^0) + \varepsilon(\vec{A}_x - i \vec{A}_y) - i \frac{\partial}{\partial x} + \frac{\partial}{\partial y}}{m + V_0 + (A_x^0 - i A_y^0) + \varepsilon(\vec{A}_x - i \vec{A}_y) - i \frac{\partial}{\partial x} + \frac{\partial}{\partial y}} \right) \phi_1^{(n)} + (-m + V_0) \phi_2^{(n)} \tag{20}
\]

which is nothing but eigenvalue equation.

The first order of \( \varepsilon \) in equation 19 is
\[
i \left( \begin{array}{c} \phi_1^{(n)} R \\ \phi_2^{(n)} R \end{array} \right) = \left( \begin{array}{c} \vec{V} + i \vec{U} \phi_1^{(n)} + (\vec{A}_x - i \vec{A}_y) \phi_1^{(n)} \\ (\vec{A}_x + i \vec{A}_y) \phi_1^{(n)} + (\vec{V} + i \vec{U}) \phi_2^{(n)} \end{array} \right) \tag{21}
\]

To be explicit Eq.(21) is rewritten as
\[
\phi_1^{(n)} (\vec{V} + i \vec{U}) + \phi_2^{(n)} (\vec{A}_x - i \vec{A}_y) = i \frac{\partial \phi_1^{(n)}}{\partial \rho} \tag{22}
\]
\[
\phi_1^{(n)} (\vec{A}_x + i \vec{A}_y) + \phi_2^{(n)} (\vec{V} + i \vec{U}) = i \frac{\partial \phi_2^{(n)}}{\partial \rho} \tag{22}
\]

Equation (22) is the core equation to obtain regularization term for scalar and vector potential.
Assuming both $\phi_1^{(n)}$ and $\phi_2^{(n)}$ in Eq.(22) to be complex. Using the decomposition $\phi_j^{(n)} = Re[\phi_j^{(n)}] + Im[\phi_j^{(n)}]$ with $j = 1, 2$ and 

$$Re[\phi_1^{(n)}] = \phi_a$$
$$Im[\phi_1^{(n)}] = \phi_b$$
$$Re[\phi_2^{(n)}] = \phi_c$$
$$Im[\phi_2^{(n)}] = \phi_d$$

Two complex equation in Eq.(22) can be decomposed into four real equations as

$$\phi_a^{(n)} \vec{V} - \phi_b^{(n)} \vec{U} + \phi_c^{(n)} \vec{A}_x + \phi_d^{(n)} \vec{A}_y = -\frac{\partial \phi_b^{(n)}}{\partial R}$$
$$\phi_a^{(n)} \vec{A}_x - \phi_b^{(n)} \vec{A}_y + \phi_c^{(n)} \vec{V} - \phi_d^{(n)} \vec{U} = -\frac{\partial \phi_d^{(n)}}{\partial R}$$
$$\phi_b^{(n)} \vec{V} + \phi_a^{(n)} \vec{U} + \phi_d^{(n)} \vec{A}_x - \phi_c^{(n)} \vec{A}_y = \frac{\partial \phi_a^{(n)}}{\partial R}$$
$$\phi_b^{(n)} \vec{A}_x + \phi_a^{(n)} \vec{A}_y + \phi_d^{(n)} \vec{V} + \phi_c^{(n)} \vec{U} = \frac{\partial \phi_d^{(n)}}{\partial R}$$

Eq.(23) is rewritten as

$$\begin{pmatrix} \phi_a & -\phi_b & \phi_c & \phi_d \\ \phi_c & -\phi_d & \phi_a & -\phi_b \\ \phi_b & \phi_a & -\phi_d & \phi_c \\ \phi_d & \phi_c & \phi_b & \phi_a \end{pmatrix} \begin{pmatrix} \vec{V} \\ \vec{U} \\ \vec{A}_x \\ \vec{A}_y \end{pmatrix} = \begin{pmatrix} -\frac{\partial \phi_b^{(n)}}{\partial R} \\ -\frac{\partial \phi_d^{(n)}}{\partial R} \\ \frac{\partial \phi_a^{(n)}}{\partial R} \\ \frac{\partial \phi_c^{(n)}}{\partial R} \end{pmatrix}$$

which is abbreviated as

$$Ax = y$$

where $det(A) \neq 0$. The solution of Eq.(24) is written as

$$x = A^{-1}y$$

which gives the solution for regularized scalar and vector potential as followed

$$\vec{V} = \frac{a\phi_a + c\phi_b - b\phi_c - d\phi_d}{\phi_a^2 + \phi_b^2 + \phi_c^2 + \phi_d^2}$$

$$\vec{U} = \frac{-a\phi_a + c\phi_b + b\phi_c + d\phi_d}{\phi_a^2 + \phi_b^2 + \phi_c^2 + \phi_d^2}$$

$$\vec{A}_x = \frac{1}{(\phi_a^2 + \phi_d^2) - (\phi_b^2 + \phi_d^2)^2} \times [\phi_a^2(-a\phi_c + d\phi_b + c\phi_d) + \phi_c^2(a\phi_c - c\phi_d)]

+ b\phi_a^3 + d\phi_b^3 + \phi_b(2b\phi_c\phi_d - d\phi_c^2 + d\phi_d^2) - (\phi_a^2 + \phi_b^2)(a\phi_c + c\phi_d)$$
\[ +\phi_a(-2\phi_b(\alpha \phi_d + c \phi_c) + b \phi_d^2 - b \phi_d^2 + b \phi_b^2 + 2d \phi, \phi_d)) \] 

\[ \bar{\Lambda}_y = \frac{1}{(\phi_d^2 + \phi_b^2) - (\phi_d^2 + \phi_b^2)} \times \left[ -\phi_d^2(\alpha \phi_d + b \phi_b + c \phi_c) + \phi_b^2(\alpha \phi_d + c \phi_c) \right. \] 

\[ +d \phi^3 - b \phi^2 + (\phi_d^2 + \phi_b^2)(c \phi_c - a \phi_d) + \phi_b(-b \phi_d^2 + b \phi_d^2 - 2d \phi, \phi_d) \] 

\[ +\phi_a(2\phi_b(\alpha \phi_c - c \phi_d) + 2b \phi_c \phi_d + d \phi_b^2 - d \phi_b^2 + d \phi_b^2)) \] 

where \( a = -\frac{\partial \phi_b^{(n)}}{\partial R}, b = -\frac{\partial \phi_b^{(n)}}{\partial R}, c = -\frac{\partial \phi_c^{(n)}}{\partial R}, \) and \( d = -\frac{\partial \phi_d^{(n)}}{\partial R} \). Equation (27), (28), (29) and (30) are the solution for regularization terms as written in Eq.(22).

3. **Fast-Forward of Adiabatic Dynamics of Dirac Equation**

For fast forward scheme, we define the fast-forwarded spinor as

\[ \Psi_{FF}(x, y, t) = \Psi_{regy}(x, y, R(\Lambda(t))) \] 

\[ = \left( \begin{array}{c} \phi_1^{(n)}(x, y, R(\Lambda(t))) \\ \phi_2^{(n)}(x, y, R(\Lambda(t))) \end{array} \right) e^{i\delta_n(\Lambda(t))}, \] 

while in this time \( \delta_n(\Lambda(t)) = -\int_0^t s E_n(R(\Lambda(s))) \) in natural unit. Here \( \Lambda(t) \) is an advanced time defined by

\[ \Lambda(t) = \int_0^t a(t') dt', \]

with the standard time \( t. \) \( a(t) \) is a magnification time-scale factor given by \( a(0) = 1, a(t) > 1(0 < t < T_{FF}) \) and \( a(t) = 1(t \geq T_{FF}) \). We consider the fast forward dynamics with a new time variable which reproduces the target state \( \Psi_0(R(T)) \) in a shorter final time \( T_{FF} \) defined by

\[ T = \int_0^{T_{FF}} a(t) dt. \]

The explicit expression for \( a(t) \) in the fast-forward range \( (0 \leq t \leq T_{FF}) \) is typically given by [5] as:

\[ a(t) = \bar{a} - (\bar{a} - 1) \cos \left( \frac{2\pi}{T_{FF}} \right), \]

where \( \bar{a} \) is the mean value of \( a(t) \) and is given by \( \bar{a} = T/T_{FF} \).

We postulate that the fast-forward spinor satisfies the Dirac equation as follows

\[ i \frac{\partial \Psi_{FF}}{\partial t} = \left( \begin{array}{cc} m + V_{FF} & \pi_{FF}^t \\ \pi_{FF}^t & -m + V_{FF} \end{array} \right) \Psi_{FF} \]

where

\[ \pi_{FF}^t = i \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + A_{FF}^x + iA_{FF}^y, \]

\[ \pi_{FF}^t = i \frac{\partial}{\partial x} - \frac{\partial}{\partial y} + A_{FF}^x - iA_{FF}^y. \]

The LHS of equation 33 up to order \( \varepsilon \) is

\[ i \frac{\partial \Psi_{FF}}{\partial t} = \left[ i \alpha \varepsilon \left( \begin{array}{c} \frac{\partial \phi_1^{(n)}}{\partial R} \\ \frac{\partial \phi_2^{(n)}}{\partial R} \end{array} \right) + \left( \begin{array}{c} \phi_1^{(n)} \\ \phi_2^{(n)} \end{array} \right) E_n \right] e^{i\delta_n} \]

It is understood the time parameter in the RHS of the above equation is \( \Lambda \) except for \( \alpha \) which is depend on \( t. \) Using equation 29 and 32 we have
\[
\frac{\partial \Psi}{\partial t} = \begin{pmatrix}
m + V_0 + \alpha \epsilon F \\
A_0^x + iA_0^y - i \frac{\partial}{\partial x} + \alpha \epsilon (A_x + iA_y)
\end{pmatrix}
\begin{pmatrix}
A_0^x - iA_0^y - i \frac{\partial}{\partial x} + \alpha \epsilon (\hat{A}_x - i\hat{A}_y)
\end{pmatrix}
\begin{pmatrix}
\phi_1^{(n)} \\
\phi_2^{(n)}
\end{pmatrix}
\times e^{i\delta_n}
\]

(37)

Thus, from equation 37, we have

\[
V_{FF} = V_0 + \alpha \epsilon F,
\]

\[
A_{x}^{FF} - iA_{y}^{FF} = A_{x}^0 - iA_{y}^0 + \alpha \epsilon (\hat{A}_x - i\hat{A}_y),
\]

\[
A_{x}^{FF} + iA_{y}^{FF} = A_{x}^0 + iA_{y}^0 + \alpha \epsilon (\hat{A}_x + i\hat{A}_y).
\]

(38)

Therefore, we have

\[
\begin{align*}
V_{FF} &= V_0 + v(t) \bar{V}, \\
A_{x}^{FF} &= A_{x}^0 + v(t) (\hat{A}_x), \\
A_{y}^{FF} &= A_{y}^0 + v(t) (\hat{A}_y).
\end{align*}
\]

(39) \hspace{2cm} (40) \hspace{2cm} (41)

Where \( v(t) \) is velocity function which is defined as \( v(t) = \alpha \epsilon \). Here \( V_{FF} \) is fast forward scalar potential and \( A_{x}^{FF} \) and \( A_{y}^{FF} \) are fast forward vector potential. Driving scalar and vector potential as obtained in Eq.(39), (40) and (41) are the driving energy that can enhance the dynamics of Dirac particle in two dimension under electromagnetic field (EMF).

4. Conclusions

We presented a scheme of the fast forward of adiabatic dynamics to accelerate the 2+1 dimension of Dirac particle. The two dimensional system under electromagnetic field is regularized by adding regularization term to scalar and vector potential. By solving four algebraic equation which is include the adiabatic parameter, we obtain the regularization term of scalar and vector potential. Fast forward dynamics of Dirac particle is obtained by multiplying the adiabatic parameter and time scaling parameter with the regularization term. The fast forward driving scalar and vector potential can accelerate the system of Dirac particle from an initial state to the desired final state in a short time with the same wave function.

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