The Explicit Procedures for Reconstruction of Full Set of Helicity Amplitudes in Elastic Proton-Proton and Proton-Antiproton Collisions

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Abstract. The explicit procedures are described for reconstruction of the full set of helicity amplitudes in proton-proton and proton-antiproton elastic scattering. The procedures are based on the derivative relations for the helicity amplitudes in $s$-channel, on the explicit parametrization of the leading spin non flip amplitudes and crossing - symmetry relations. Asymptotic theorems are used for definition of free parameters in derivative relations. We also study the Odderon influence on the helicity amplitude reconstruction. Reconstruction procedures are valid at extremely wide energy domain and broad range of momentum transfer. These procedures might be useful in studying the spin phenomena in proton-proton and proton-antiproton elastic scattering.

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INTRODUCTION

Elastic scattering of hadrons has always been a crucial tool for study the dynamics of strong interaction. In the absence of the strong interaction theory the predictions or interpretation of the experimental observables are furnished by the phenomenological approaches. One of the reliable approaches is the asymptotical model. Other models (like Regge model) suffer from necessity to introduce many free parameters which should be defined by fitting to the experimental data. In such a situation the direct reconstruction of the scattering matrix from the complete set of the experimental data will be the appropriate method. Unfortunately such set of experiments were never been fulfilled in the high energy region. One may hope that such program will be realized at RHIC, FAIR (PAX project) and other facilities in near future. In order to make the predictions for the measurable observables in those facilities one needs to have a method which should be well justified, contains a small number of free parameters and applicable at wide range of the kinematics variables. Below we propose such a technique. Its applicability may be tested by the joint consideration of proton-proton and proton-antiproton elastic scattering data.

RECONSTRUCTION PROCEDURE

Two methods have been suggested for building the full set of helicity amplitudes for elastic $pp$-collisions in [1, 2]. We describe in details the method for deducing the helic-
ity amplitudes based on crossing-symmetry relation and the derivative relations here. The amplitudes for the binary reaction $A + B \rightarrow C + D$ in $s-$, $t-$, and $u-$ channels all depend upon the Mandelstam variables and are described by just one set of analytic functions evaluated in different regions of variables $s,t,u$. Thus the following preliminary expression for full set of helicity amplitudes of $p\bar{p}$ elastic scattering via set of helicity amplitudes for $pp$ elastic reaction have been derived in \[2\]:

\[
\begin{align*}
\Phi^{p\bar{p}}_1 &= 1/2 \left[ \sin^2 \psi \left( \Phi^{pp}_1 + \Phi^{pp}_2 + \Phi^{pp}_3 \right) + (1 + \cos^2 \psi) \Phi^{pp}_4 \right], \\
\Phi^{p\bar{p}}_2 &= 1/2 \left[ \sin^2 \psi \left( \Phi^{pp}_1 + \Phi^{pp}_3 - \Phi^{pp}_4 \right) - (1 + \cos^2 \psi) \Phi^{pp}_5 \right], \\
\Phi^{p\bar{p}}_3 &= 1/2 \left[ \sin^2 \psi \left( \Phi^{pp}_1 + \Phi^{pp}_2 - \Phi^{pp}_4 \right) - (1 + \cos^2 \psi) \Phi^{pp}_6 \right], \\
\Phi^{p\bar{p}}_4 &= 1/2 \left[ (1 + \cos^2 \psi) \Phi^{pp}_1 - \sin^2 \psi \left( \Phi^{pp}_3 + \Phi^{pp}_2 - \Phi^{pp}_4 \right) \right] + 2 \Phi^{pp}_5 \sin \psi, \\
\Phi^{p\bar{p}}_5 &= 1/2 \cos \psi \left[ \sin \psi \left( \Phi^{pp}_1 + \Phi^{pp}_2 + \Phi^{pp}_3 - \Phi^{pp}_4 \right) + 2 \Phi^{pp}_6 \right]
\end{align*}
\]

where

\[
\cos \psi = \sqrt{\frac{st}{(s-4m^2_p)(t-4m^2_p)}}, \quad \sin \psi = \frac{m_p \sqrt{s-4m^2_p}}{\sqrt{t(t-4m^2_p)}} \sin \theta; \quad \cos \theta = 1 + \frac{2t}{s-4m^2_p};
\]

\(\theta\) - CM scattering angle in the $s$-channel, $m_p$ - proton mass. The system (1) shows apparent analytical forms for full set of amplitudes of elastic $p\bar{p}$ scattering \(\{\Phi^{pp}_i\}_{i=1-6}\) via helicity amplitudes \(\{\Phi^{pp}_j\}_{j=1-5}\) for crossing-symmetrical $pp$ channel. Lets to stress that the relation $\Phi^{pp}_5 = -\Phi^{pp}_6$ has been taken into account in the (1) already. The relation $|\Phi^{pp}_5| = |\Phi^{pp}_6|$ corrects according to G-parity conservation in elastic $p\bar{p}$ collisions \[3\].

But on the other hand the exact correlation between $|\Phi^{p\bar{p}}_5|$ and $|\Phi^{p\bar{p}}_6|$ is open question in general case because of Odderon pole contribution is still a contentious topic.

**Derivative relations**

The $pp$ elastic scattering under study in order to obtain some additional correlations for set of helicity amplitudes \(\{\Phi^{pp}_j\}_{j=1-5}\). Usually the following formula is suggested for spin non-flip amplitudes $\Phi^{pp}_1 = \Phi^{pp}_3$. The derivative relations allow to express spin-flip amplitude $\Phi^{pp}_5$ and spin double-flip amplitude $\Phi^{pp}_4$ via $\Phi^{pp}_1$ \[4\]:

\[
\Phi^{pp}_5(s,t) = C^{pp}_1(s) \frac{\partial}{\partial(\sqrt{-t})} \Phi^{pp}_1(s,t); \quad \Phi^{pp}_4(s,t) = C^{pp}_2(s) \frac{\partial^2}{\partial(\sqrt{-t})^2} \Phi^{pp}_1(s,t),
\]

where $C^{pp}_k(s) = C^{pp}_{k1}(s) + iC^{pp}_{k2}(s)$, $k = 1,2$ - complex parameters in general. The some versions of additional correlation for spin double-flip helicity amplitude $\Phi^{p\bar{p}}_5$ were discussed in \[1,2,3\]. We would like to emphasize that complex parameters $C_k(s) (k = 1,2)$
must be defined for exact knowledge of spin-flip and double-spin amplitudes for \( pp \) elastic scattering.

**Determination of the free parameters**

It seems that the combination of sets of helicity amplitudes for \( pp \) and \( p\bar{p} \) elastic reactions namely in the framework of the method described above and some additional equations result in several ways for analytic determination of parameters \( C_k^{pp} (k = 1, 2) \). The Odderon hypothesis is crucial important for definition of unknown parameters \( C_k^{pp} (k = 1, 2) \) in the derivative relations (2). We suggest to use the asymptotic behavior of total cross section, differential cross section, \( \rho \) and \( B \) parameters in order to obtain the complex unknown parameters in high-energy domain.

The most general case corresponds to possibility for Odderon exchange as well as for Pomeron one. The general Pomeranchuk theorem is satisfied in framework Odderon hypothesis, but the original Pomeranchuk theorem is violated. Thus one can get the following equation system for definition of \( C_k^{pp} (k = 1, 2) \) parameters:

\[
\begin{align*}
\Delta \sigma_{\text{tot}} &\equiv \sigma_{\text{tot}}^{\bar{p}p} - \sigma_{\text{tot}}^{pp} \propto \left[ \text{Im} \Phi_1^{\bar{p}p} (s, t = 0) - \text{Im} \Phi_1^{pp} (s, t = 0) \right], \\
\Delta \rho & = \rho^{\bar{p}p} - \rho^{pp}, \\
\Delta \left( d\sigma_{\text{el}} / dt \right) & = (d\sigma_{\text{el}} / dt)^{\bar{p}p} - (d\sigma_{\text{el}} / dt)^{pp}, \\
\Delta B & = B^{\bar{p}p} - B^{pp}.
\end{align*}
\]  

The definition of the parameters \( C_k^{pp} (k = 1, 2) \) becomes model dependent and non trivial task because of model dependent values are on the left parts of equation in the system (3).

According to accelerator and cosmic ray experimental data [6] and phenomenological estimates [7] also the values of \( \sigma_{\text{tot}}^{\bar{p}p} \) are (very) close to corresponding values of \( \sigma_{\text{tot}}^{pp} \) up to energy \( \sqrt{s} = 100 \text{ TeV} \). The usual interpretation of the experimental data for \( \rho \)-parameter is that \( \Delta \rho \) is very small. It should be emphasized that even \( \Delta \rho = 0 \) would not exclude an Odderon, but would only rule out specific models for the soft Odderon. Thus one can suggests \( \Delta \sigma_{\text{tot}} = 0 \) and \( \Delta \rho = 0 \) without full excluding of Odderon pole contribution and without significant violation of general description consequently. One can use only two first equation in the system (3) in the framework of simple suggestions that there is one complex parameter or two clear real / imagine parameters. It should be emphasized that the unknown parameters would be define model independently in these specific cases even taking into account possible presence of the (soft) Odderon contribution.

As shown above the new experimental date and phenomenological investigations will be very important at ultra-high energy \( \sqrt{s} \sim 100 \text{ TeV} \) in particulary for decision some fundamental problems for hadron interactions and distinguishing different theoretical models.

The original Pomeranck theorem, namely, for total cross section and for differential cross section in binary reaction, and Cornille-Martin theorem for the forward slope parameter \( B \) [8] can be used in the framework of hypothesis for presence only Pomeron
exchange. The system for definition of free parameters $C_k^{pp}(k = 1, 2)$ is given by

\[
\begin{align*}
\text{Im}\Phi_{1\bar{p}}^p(s, t = 0) &= \text{Im}\Phi_{1}^{pp}(s, t = 0), \\
\text{Re}\Phi_{1\bar{p}}^p(s, t = 0) &= \text{Re}\Phi_{1}^{pp}(s, t = 0), \\
(d\sigma_{\text{el}}/dt)^{p\bar{p}} &= (d\sigma_{\text{el}}/dt)^{pp}, \\
B^{p\bar{p}} &= B^{pp}.
\end{align*}
\]

This system allows to determine all components of complex free parameters by model independent way. Moreover for Pomeron exchange only there is additional correlation between helicity amplitudes for $p\bar{p}$ elastic reaction, namely, $|\Phi_5^{p\bar{p}}| = |\Phi_6^{p\bar{p}}|$ just as well as for $pp$ scattering. The asymptotic relations and fit of experimental dependences at low and intermediate energies should be used for determination of free parameters for the reconstruction method understudy.

**SUMMARY**

The main results of this paper are following. We suggest the explicit procedure for the reconstruction of the the full set of the helicity amplitudes for the elastic $p\bar{p}$ scattering. This method is based on fundamental crossing-symmetry property, derivative relations for helicity amplitudes and selection the anatlytical expression for the spin non-flip amplitude describing well the $pp$ experimental data. We apply to this spin non-flip amplitude the derivative procedure for finding all $pp$ helicity amplitudes. After proving that we get the good description of $pp$-data we apply the crossing relations in order to find the helicity amplitudes for the $p\bar{p}$ elastic scattering. We introduce the minimum of the free parameters which are fixed through the asymptotic relations taking into account the Pomeron and Odderon contributions. The unified analysis of the $pp$ and $p\bar{p}$ data allows to check in details the proposed method. It seems this analytical method might be useful for direct reconstruction of elements of the scattering matrix at extremely wide initial energy domain and broad range of momentum transfer.

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