Consequences of Gravity-Induced Couplings in Theories with Many Particle Species

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Abstract

In theories with many copies of the Standard Model virtual black hole exchange may produce effective higher dimensional operators that can be treated below the cutoff scale as fundamental vertices of interspecies non-gravitational interaction. We consider the vertex that couples fermions of one species through magnetic moment to photons of other species, and study the quantum corrections it generates. In particular, we find kinetic mixing between photons of different species produced via fermion loops. Diagonalization of gauge kinetic terms then renders the fermions millicharged under other species’ electromagnetism. We explore some phenomenological consequences of such effects by considering possible observable signatures in collider experiments and constraining the interaction strength. The derived bounds are in agreement with non-democratic nature of micro black hole coupling.

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1 Introduction

Theories with a large number $N$ of particle species [1] provide a simple solution to the hierarchy problem by lowering the effective gravitational cutoff to a scale [2]:

$$\Lambda_G = \frac{M_P}{\sqrt{N}}.$$  (1)

Lowering of the gravitational cutoff finds justification from consistency of large distance black hole physics [1, 2], from black hole entropy considerations [3], and from quantum information theory [4]. The hierarchy problem is explained by the large $N$ number of species with cutoff scale $\Lambda$ by assuming that gravity becomes strong at the scale $\Lambda_G \approx \Lambda$. Given this one will start probing quantum gravity effects in particle collisions around the scale $\Lambda \sim \text{TeV}$. In particular, production of microscopic black holes will take place by collisions of Standard Model (SM) particles at that energy scale.

In this scenario virtual black hole exchange can generate interspecies scattering processes. The intermediate black hole states are heavy, with mass $\mathcal{O}(\Lambda)$. They can be integrated out below the scale $\Lambda$. By doing so one ends up having effective higher dimensional operators, which of course will be compatible with the symmetries. In the framework where the other species are exact copies of the SM, or some other copies with photon-like fields, one can have the following gauge invariant operator:

$$\mathcal{L}_{\text{int}} = \sum_{i \neq j} \frac{\lambda_{ij}}{M_p} F_i^{\mu\nu} \bar{\psi}_j \sigma_{\mu\nu} \psi_j.$$  (2)

Here fermions belonging to the $j$-th species couple via magnetic moment to the photon in the $i$-th species. The dimensionless coupling constants $\lambda_{ij}$ are real, non-vanishing and non-diagonal. Below the scale $\Lambda$, the above interaction can be treated as a fundamental vertex of non-gravitational coupling among different particle species. Such non-gravitational couplings may have very interesting consequences, which we investigate in this paper. The point is that from fermion loops there will be quantum corrections to the photon propagators, which will render the photon kinetic terms non-diagonal. One can always diagonalize the kinetic terms by redefining the photon fields. But the latter inevitably endows fermions belonging to our SM with tiny charges under electromagnetic gauge groups of other species.

The organization of this paper is as follows. In Section 2 we compute Feynman diagrams involving fermion loops that renormalize the photon propagators, and give rise to kinetic mixing between photons of different species. The diagram is regularized by using Pauli-Villars regularization technique [5]. We then diagonalize the photon kinetic terms, which results in redefinition of fermion covariant derivatives. In particular, fermions become millicharged under other species’ electromagnetism. In Section 3 we discuss phenomenological consequences of such effects by considering scattering processes where we...
have an incoming fermion-antifermion pair belonging to our SM, and an outgoing pair of other species. Such processes, potentially observable in collider experiments, put bounds on the dipole moment couplings. The draw concluding remarks in Section 4.

2 Photon Kinetic Mixing via Fermion Loop

The effective interaction under consideration is:

$$\mathcal{L}_{\text{int}} = \sum_{i \neq j} \frac{\lambda_{ij}}{M_p} F_i^{\mu \nu} \bar{\psi}_j \sigma_{\mu \nu} \psi_j.$$  \hspace{1cm} (3)

To find the momentum space Feynman rules, we define photon momentum as outgoing, so that $\partial^\mu A^\nu \rightarrow +ip^\mu \bar{A}^\nu$. We have a vertex with an incoming fermion-antifermion pair of the $j$-th species, and an outgoing photon in the $i$-th species. If the photon carries a Lorentz index $\nu$, with momentum $p^\mu$ attached to it, the corresponding vertex-factor is:

$$\text{Vertex} = \frac{2i}{M_p} \lambda_{ij} p^\mu \sigma_{\mu \nu}.$$  \hspace{1cm} (4)

Such a vertex will give rise to diagrams where a photon $A_{k, \mu}$ of the species $k$ goes into a photon $A_{i, \nu}$ of another species $i$ via fermion loop.

The graph is given by

$$\Pi^{\mu \nu} = \left( \frac{2i}{M_p} \right)^2 (-1) \int \frac{d^4k}{(2\pi)^4} \sum_j \text{Tr} \left[ \lambda_{ij} \lambda_{kj} \Pi^{\mu \nu}_{j}(p, m_j) \right]$$

where $\Pi^{\mu \nu}_{j}(p, m_j)$ is defined as

$$\Pi^{\mu \nu}_{j}(p, m_j) \equiv \frac{4}{M_p^2} p^\mu p^\nu \int \frac{d^4k}{(2\pi)^4} \left( \frac{k^\alpha(k - p)^\beta}{k^2 - m_j^2} \right) \text{Tr}(\gamma^{\rho \sigma} \gamma^{\lambda \mu} \gamma^{\lambda \nu} + m_j^2 \text{Tr}(\gamma^{\rho \sigma} \gamma^{\lambda \nu}))$$.

The above integral seems quadratically divergent for large internal momentum $k$. To give it a meaning we employ the technique of Pauli-Villars regularization \[5\]. This amounts to minimally coupling the photons to additional spinor fields with a very large mass $M_s$. These fields might have ghost couplings. This prescription implies the replacement \[5\]:

$$\Pi^{\mu \nu}_{j}(p, m_j) \rightarrow \bar{\Pi}^{\mu \nu}_{j}(p) = \Pi^{\mu \nu}_{j}(p, m_j) + \sum_{s=1}^{S} C_s \Pi^{\mu \nu}_{j}(p, M_s).$$  \hspace{1cm} (7)
where the constants $C_s$ will be chosen such that the integrals converge. The minimal coupling of the additional fields implies that gauge invariance is preserved by the regularization procedure.

A convenient way to evaluate (7) is to introduce auxiliary variables to elevate the propagator denominators into exponential factors by the identity:

$$\frac{i}{k^2 - m^2 + i\epsilon} = \int_0^\infty d\alpha e^{i\alpha(k^2-m^2+i\epsilon)}.$$  \hspace{1cm} (8)

After some shift of variable one can perform the momentum integrals to obtain

$$\Pi_{\mu\nu}^j (p, m_j) = -i\frac{\pi^2 M_p^2}{p^{\mu\nu} - \eta^{\mu\nu} p^2} \int_0^\infty \int_0^\infty \frac{d\alpha_1 d\alpha_2}{(\alpha_1 + \alpha_2)^2} e^{-i(\alpha_1 + \alpha_2)(m_j^2 - \frac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2} p^2)} \times \left[ \frac{\alpha_1 \alpha_2}{(\alpha_1 + \alpha_2)^2} p^2 + m_j^2 \right].$$ \hspace{1cm} (9)

Notice that $\Pi_{\mu\nu}^j (p, m_j)$ is proportional to $(p^{\mu\nu} - \eta^{\mu\nu} p^2)$, as required by gauge invariance. To perform the integrations in (9), we use the identity:

$$1 = \int_0^\infty \frac{d\lambda}{\lambda} \delta \left( 1 - \frac{\alpha_1 + \alpha_2}{\lambda} \right).$$ \hspace{1cm} (10)

Inserting this into (9), and then making the rescaling $\alpha_i \rightarrow \lambda \alpha_i$, we obtain

$$\Pi_{\mu\nu}^j (p, m_j) = -\frac{i}{\pi^2 M_p^2} \left( p^{\mu\nu} - \eta^{\mu\nu} p^2 \right) \int_0^\infty \int_0^\infty d\alpha_1 d\alpha_2 \delta(1 - \alpha_1 - \alpha_2)(\alpha_1 \alpha_2 p^2 + m_j^2) \times \int_0^\infty \frac{d\lambda}{\lambda} e^{-i\lambda(m_j^2 - \alpha_1 \alpha_2 p^2)}. $$ \hspace{1cm} (11)

The integral over $\lambda$ looks logarithmically divergent at the point $\lambda = 0$. As we will see, if we choose the coefficients $C_s$ in (7) such that

$$C_s = -2\delta_s^1 + \delta_s^2,$$ \hspace{1cm} (12)

$$\sum_{s=1}^S C_s M_s^2 = -2M_1^2 + M_2^2 = -m_j^2,$$ \hspace{1cm} (13)

we can render (11) convergent. Furthermore, let us pick $p$ such that $p^2 < 4m_j^2$, which corresponds to the threshold of pair creation. Because $\alpha_1, \alpha_2$ are positive definite and satisfy $\alpha_1 + \alpha_2 = 1$, we have $\alpha_1 \alpha_2 \leq 1/4$. Therefore the quantity $(m_j^2 - \alpha_1 \alpha_2 p^2)$ is positive and the integration contour in the complex $\lambda$ plane can be rotated by $-\pi/2$, so that

$$\int_0^\infty \frac{d\lambda}{\lambda} e^{-i\lambda(m_j^2 - \alpha_1 \alpha_2 p^2)} \rightarrow \ln \left( \frac{\Lambda^2}{m_j^2} \right) - \ln \left( 1 - \frac{\alpha_1 \alpha_2 p^2}{m_j^2} \right),$$ \hspace{1cm} (14)

where we have defined

$$\Lambda \equiv \frac{M_1^2}{M_2}.$$ \hspace{1cm} (15)
The above combined with condition (13) gives

\[
a_1 \equiv \frac{M_i^2}{\Lambda^2} = 2 \left( 1 - \frac{m^2_j}{4\Lambda^2} \right) + O(m_j^4/\Lambda^4),
\]

(16)

\[
a_2 \equiv \frac{M_i^2}{\Lambda^2} = 4 \left( 1 - \frac{m^2_j}{2\Lambda^2} \right) + O(m_j^4/\Lambda^4).
\]

(17)

The choice (12,13) also gives us

\[
m^2_j \times \int_0^\infty \frac{d\lambda}{\lambda} e^{-i\lambda(m^2_j-\alpha_1\alpha_2 p^2)} \rightarrow m^2_j \left[ \ln \left( \frac{\Lambda^2}{m^2_j} \right) - \ln \left( 1 - \frac{\alpha_1\alpha_2 p^2}{m^2_j} \right) \right] - 4 \ln 2 \Lambda^2 + (3 \ln 2 + 1) m^2_j,
\]

(18)

where we have used the definitions of \(a_1, a_2\), and some rescalings. Plugging the regularized integrals (14,18) into (11), we finally obtain

\[
\bar{\Pi}^{\mu\nu}_j(p) = \frac{i}{\pi^2 M_p^2} \left( p^\mu p^\nu - \eta^{\mu\nu} p^2 \right) \int_0^1 d\alpha \left[ \alpha (1 - \alpha) p^2 + m^2_j \right] \ln \left[ 1 - \frac{\alpha (1 - \alpha) p^2}{m^2_j} \right]
\]

\[
- \frac{i}{6\pi^2 M_p^2} \ln \left( \frac{\Lambda^2}{m^2_j} \right) (p^2 + 6m^2_j) (p^\mu p^\nu - \eta^{\mu\nu} p^2)
\]

\[
+ \frac{i}{\pi^2 M_p^2} \left[ 4 \ln 2 \Lambda^2 - (3 \ln 2 + 1) m^2_j \right] (p^\mu p^\nu - \eta^{\mu\nu} p^2).
\]

(19)

In the limit \(\Lambda \rightarrow \infty\), then the graph (5) reduces to

\[
\Pi^{\mu\nu} = \left( \frac{4i \ln 2}{\pi^2} \right) \left( \frac{\Lambda^2}{M_p^2} \right) \sum_j \lambda_{ij} \lambda_{kj} (p^\mu p^\nu - \eta^{\mu\nu} p^2).
\]

(20)

Note that higher derivatives do not appear in the leading divergent terms. The above radiative correction renormalizes the photon propagators as:

\[
- \frac{i\eta^{\mu\nu}}{p^2} \delta_{ij} \rightarrow - \frac{i\eta^{\mu\nu}}{p^2} \left[ \delta_{ij} - \left( \frac{4 \ln 2}{\pi^2} \right) \frac{\Lambda^2}{M_p^2} \sum_k \lambda_{ik} \lambda_{jk} \right].
\]

(21)

We need to redefine the photon fields in order to remove their kinetic mixing, i.e. to obtain canonical kinetic terms. Let us assume that all the species are strictly identical, i.e. exact copies of the SM, related by certain permutation symmetry. Then all the non-diagonal elements of the coupling matrix \(\lambda_{ij}\) are the same: \(\lambda_{ij} = \lambda\), for \(i \neq j\), while \(\lambda_{ii} = 0\). This gives

\[
\Delta_{ij} \equiv \left( \frac{4 \ln 2}{\pi^2} \right) \frac{\Lambda^2}{M_p^2} \sum_k \lambda_{ik} \lambda_{jk} \approx \delta \begin{pmatrix}
1 & 1 & \cdots & 1 \\
1 & 1 & \cdots & 1 \\
\vdots & \vdots & \ddots & \vdots \\
1 & 1 & \cdots & 1
\end{pmatrix}.
\]

(22)
where
\[
\delta \equiv \left( \frac{4 \ln 2}{\pi^2} \right) \frac{\Lambda^2}{M_p^2} (N - 1) \lambda^2 \approx \left( \frac{4 \ln 2}{\pi^2} \right) \lambda^2 \ll 1.
\] (23)

Because the matrix $\Delta_{ij}$ is symmetric, the photon kinetic terms can be diagonalized by the field redefinitions:
\[
A^\mu_i \to \sum_j \left( \delta_{ij} - \frac{1}{2} \Delta_{ij} \right) A^\mu_j.
\] (24)

This will result in redefinition of fermion covariant derivatives, so that fermions will acquire millicharge under other species’ electromagnetism:
\[
L_{\text{Dirac}} = -i \sum_{i=1}^{N} \bar{\psi}_i \left[ \partial - m_i - ie \mathcal{A}_i + ie(\delta/2) \sum_{j=1}^{N} \epsilon_{ij} \mathcal{A}_j \right] \psi_i,
\] (25)

where $\epsilon_{ij} = 1$, for $i \neq j$, while $\epsilon_{ii} = 0$.

3 Phenomenological Consequences

As long as our Standard Model is concerned, there are two kinds of vertices given by the Lagrangian (25): the one of usual QED, and another QED-like vertex where the photon belongs to another SM. The latter vertex carries a tiny fermion charge of $(-\delta/2)e$, instead of $e$. However, because of the enormous multiplicity of species, the fermion millicharge may produce observable signals in particle colliders.

Let us consider ultra-relativistic electron-positron collision\(^2\). At tree level the $e^+e^-$ pair may produce a fermion-antifermion pair of other species either through the SM photon, or through other species’ photons. The corresponding Feynman diagrams are:

![Figure 2](image1.png)

![Figure 3](image2.png)

The amplitude for the process, considering exact replicas of the SM, is given by:
\[
\mathcal{A}(e^+e^- \to \psi^+_j \psi^-_j) = -\frac{1}{2} \delta \mathcal{M} - \frac{1}{2} \delta \left[ 1 - \frac{1}{2} \delta(N - 2) \right] \mathcal{M},
\] (26)

\(^2\)The center of mass energy however must be taken below the threshold of $t\bar{t}$ production. This is necessary for the regularization of the loop integral, because otherwise the condition $p^2 < 4m^2$ is never satisfied.
where $\mathcal{M}$ is the would-be amplitude if the outgoing pair belonged to our SM instead. The process has a multiplicity of $(N - 1)$, and looks like $e^+e^- \rightarrow \text{invisible}$, when viewed from our SM. The branching ratio is given by

$$
\text{Br} (e^+e^- \rightarrow \text{invisible}) = (N - 1) \frac{|A|^2}{|\mathcal{M}|^2} \approx \delta^2 N \left( \frac{1}{4} \delta N - 1 \right)^2.
$$

Thus such processes are potentially observable, because a small value of $\delta$ may be compensated by a large value of $N$.

Alternatively, one can put bounds on the dipole moment coupling $\lambda$, by using (27). A stringent bound comes from invisible decays of orthopositronium [6]:

$$
\text{Br} (e^+e^- \rightarrow \text{invisible}) \leq 2.1 \times 10^{-8} \ (90\% \ C.L.)
$$

This gives, from (23), with $N \sim 10^{32}$ copies:

$$
\lambda \lesssim 10^{-13} \approx 10^3 (\Lambda/M_P).
$$

We see that $\lambda$ has a suppression factor of $\Lambda/M_P$. This is in fact compatible with the non-democratic nature of black hole couplings with different species [2, 7], which can be derived from unitarity considerations. Indeed, as shown in [2], the off-diagonal couplings of the black holes must be suppressed at least by $1/\sqrt{N}$. After integrating them out one finds that effective interspecies coupling is suppressed by the scale $\Lambda/M_P^2$.

4 Conclusion

In this paper we argued how in a theory of gravitationally coupled many particle species effective non-gravitational interspecies interactions arise because of virtual black hole exchange. We have shown that through radiative corrections magnetic moment-type interspecies coupling renders the SM fermions millicharged under hidden sectors’ electromagnetism. The effect may manifest itself in particle colliders. By considering ultra-relativistic $e^+e^-$ collisions, we put bounds on such couplings, which are in agreement with non-democratic nature of micro black hole coupling.

It is worth pointing out that many extensions of the SM, in particular those coming from string theory, predict hidden $U(1)$ gauge groups, and naturally give rise to the kinetic mixing phenomenon [8, 9]. According to the common lore, gauge kinetic mixing is generated by irrelevant operators that do require the existence of cross-charged fundamental states (e.g. [9, 10]). However, such states are not indispensable. Indeed, the magnetic moment interaction operator (2) considered in this paper produces kinetic mixing without appealing to cross-charged particles.

One would like to see what could be the implications of effective non-gravitational interspecies couplings in the early universe cosmology. Such considerations will probably put more severe bounds on the coupling strength.

Acknowledgments

We would like to thank G. Dvali and N. Weiner for useful discussions and comments. Special thanks to A.E.C. Hernández for providing help with the Feynman diagrams.
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