CDM-variant cosmological models – I: Simulations and preliminary comparisons

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ABSTRACT

We present two matched sets of five dissipationless simulations each, including four presently favored minimal modifications to the standard cold dark matter (CDM) scenario. One simulation suite, with a linear box size of $75 h^{-1}$ Mpc, is designed for high resolution and good statistics on the group/poor cluster scale, and the other, with a box size of $300 h^{-1}$ Mpc, is designed for good rich cluster statistics. All runs had 57 million cold particles, and models with massive neutrinos (CHDM-$2\nu$) had an additional 113 million hot particles. We consider separately models with massive neutrinos, tilt, curvature, and a nonzero cosmological constant ($\Lambda = 3H_0^2\Omega_\Lambda$) in addition to the standard CDM model. We find that the dark matter in each of our tilted $\Omega_0 = 1$ model (TCDM), and our open $\Lambda = 0$ (OCDM) model with $\Omega_0 = 0.5$ has too much small-scale power by a factor of $\sim 2$, while CHDM-$2\nu$ and SCDM are acceptable fits. In addition, we take advantage of the large dynamic range in detectable halo masses afforded by the combination of the two sets of simulations to test the Press-Schechter approximation. We find good fits at cluster masses for $\delta_{c,g} = 1.27$–1.35 for a Gaussian filter and $\delta_{c,t} = 1.57$–1.73 for a tophat filter. But, when we adjust $\delta_c$ to obtain a good fit at cluster mass scales, we find that the Press-Schechter model overpredicts the number density of halos compared to the simulations by a weakly cosmology-dependent factor of 1.5–2 at galaxy and group masses. It is impossible to obtain a good fit over the entire range of masses simulated by adjusting $\delta_c$ within reasonable bounds.

Key words: large-scale structure of universe – dark matter – cosmology:theory – cosmic microwave background

1 INTRODUCTION

The COBE DMR detection of anisotropies in the cosmic microwave background (Smoot et al. 1992) made it very clear that the ‘standard’ structure formation scenario of cold dark matter (Blumenthal et al. 1984; Davis et al. 1985) cannot simultaneously account for fluctuations on very large and very small scales. That model made several very restrictive assumptions about cosmological parameters – that space-time is homogeneous, isotropic and globally flat; that there is no cosmological constant; that fluctuations from homogeneity are Gaussian-distributed and nearly scale-independent at horizon crossing; that the Hubble parameter $h \equiv H_0 / (100 \text{ km s}^{-1} \text{ Mpc}^{-1})$ is 0.5; and that the number of free parameters is minimized. The obvious fixes to the problem of excess small-scale power (when normalizing power spectra to the COBE anisotropy) are to make one of the following modifications to the model:

(i) tilt the primordial spectrum,
(ii) allow a nonzero cosmological constant but retain globally flat geometry,
(iii) allow the universe to be open,
(iv) add hot dark matter (i.e., neutrinos with masses of a few eV), or
(v) lower the Hubble parameter much further ($h \sim 0.3$–0.4).

Each of these modifications adds only one free parameter to the cosmology. In this paper, we consider the most viable models from each class above except the last, and simulate them with an $N$-body code in two suites, with equivalent initial conditions across all the models. We do not consider a ‘low-$H_0$' model (Bartlett et al. 1995) because of increasingly solid observational evidence that $h \geq 0.5$.

Deciding on cosmological parameters is to some extent an iterative process. Much can be done using the Press-Schechter approximation, but the assumptions that go into it are not necessarily realistic (for example, spherical symmetry – see Jain & Bertschinger 1994 and Monaco 1993). Therefore, it is useful as a first approximation to calculating the mass functions, and we use it to perform an approximate cluster normalization, using guesses about other cosmological parameters. We run a set of simulations and use them to test the Press-Schechter approximation, and make several preliminary comparisons to observational data. In a companion paper (Gross et al. 1998), we recalibrate the Press-Schechter approximation and use it to derive refined estimates of model normalization and $\Omega_0$ from several different data sets, and make more careful comparisons to cluster abundance. Subsequent papers will use simulations based on the refined normalizations.

In section 2.1, we describe our specific models from each class of CDM-variant models and explain why we chose the parameters as we did. In section 2.2, we briefly describe the implementation of the particle-mesh algorithm we used for this study. We explain our halo finding algorithm and the effect of mass resolution upon it in section 3 and report the simulation results in section 4. Finally, in section 5, we give our conclusions.

2 SIMULATIONS

2.1 Models

Given the long list of modifications to the cold dark matter scenario in the previous section, we could construct a model by adjusting every parameter in order to fit all the available observational data. However, in addition to being aesthetically displeasing, the physical significance of such a model would be unclear. As a result, we have tried to minimize the number of modifications to the relatively simple standard cold dark matter scenario by investigating each of the modifications mentioned above in a separate model. The exceptions to this policy are that in addition to any one of modifications (ii)–(iv), we allow a small tilt, up to $n = 0.9$, in order to simultaneously fit the COBE and cluster data, and we allow the Hubble parameter to be adjusted within reasonable observational bounds according to the requirements of the model. Larger tilts are not allowed because they tend to cause disagreements with high-multipole cosmic microwave background data.

We explore the large parameter space by running a large suite of linear calculations and comparing the output to appropriate observational constraints. Constraints that we consider in choosing model parameters for more detailed nonlinear analysis are:

- (i) the abundance of Abell clusters, as measured by X-ray temperature profiles (White, Efstathiou, & Frenk 1993; Biviano et al. 1993, hereafter WEF93 and BGGM93, respectively). We assume that cluster masses may be underestimated by up to a factor of two, motivated by results from cluster density mapping with gravitational lensing (Squires et al. 1996; Squires et al. 1994; Miralda-Escudé & Babul 1995; Wu & Fang 1996; Wu & Fang 1997; figure 1).
- (ii) microwave background anisotropies for $\ell \leq 800$ (figure 2), as measured by several recent CMB detection experiments (Tegmark 1996; Netterfield et al. 1997; Scott et al. 1997; Platt et al. 1997; figure 2). The linear estimates of these parameters are shown in figure 4 and in figures 6, 8, and 10 for the models we consider.

In most of the previous work with modified CDM models, the most ‘extreme' values of the model parameters have been chosen (i.e. as far from SCDM as was considered observationally plausible). For example, low-$\Omega_0$ models typically have values of $\Omega_0 \sim 0.2$ – 0.3. However, in every case, while solving some of the problems with SCDM, this introduces new problems or conflicts with other observational constraints. Thus our approach will be somewhat different. We use our previous experience with linear and non-linear tests of CDM-variant models, as well as the published results of others, to find models that represent a ‘middle ground' between SCDM and the most extreme version of the particular class of model. In this way, we hope to choose the ‘best' rather than the most extreme case, and to identify models that agree with the widest possible range of observations.

For most models, we presume a baryon abundance of $\Omega_b = 0.025 h^{-2}$, consistent with the Tytler, Fan, & Burles (1994) cosmic deuterium abundance measurement. Normalization is accomplished by calculating low multipoles using an enhanced version of the linear code from Holtzman (1988) and comparing to the four-year COBE DMR anisotropy measurements (Górski et al. 1996; Górski, private communication).

For comparison to other studies, we also simulated the standard cold dark matter (SCDM) model with bias $b = \sigma_8 = 1.5$. That model is intended to approximately match observed cluster abundances at the cost of being inconsistent with the COBE anisotropy measurements. For this model, we presumed there were no baryons in the Universe, and used the BBKS transfer function (Bardeen et al. 1986) used in previous studies, that is,

$$P(k) = Ak \frac{\left[ \ln(1 + 2.34q) \right]^2}{(2.34q)^2} \times$$

$$\left(1 + 3.89q + (16.1q)^2 + (5.46q)^3 + (6.71q)^4\right)^{-1/2}$$

$\Omega_b$ Burles & Tytler (1997a, 1997b) have very recently remeasured the deuterium abundance and found it to be 20 per cent lower, 0.019 ± 0.001. This makes a very small change in the power spectrum, and the most significant effect is to make agreement with high-$\ell$ cosmic microwave background measurements (figure 2) more difficult. The height of the first Doppler peak depends strongly on $\Omega_b$. 

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with \( q = kh^{-2} \) and \( A \) adjusted so that the rms fractional variance in mass in spheres of radius 8 \( h^{-1} \) Mpc estimated using linear theory is \( \sigma_8 = 0.667 \).

The simplest way to solve the problem of excess power on small scales is by ‘tilting’ the spectrum, that is, by changing the \( k \) factor in equation (1) to \( k^n \), with \( n < 1 \). However, the price paid is that choosing \( n = 0.9 \) is the largest allowable tilt that is still marginally consistent with the large-multipole cosmic microwave background spectrum. We find that \( n = 0.9 \), when we COBE normalize the model we find that it tends to overproduce clusters at \( M = 6 \times 10^{14} \) \( h^{-1} M_\odot \), according to the Press-Schechter estimate, unless we use a rather low value of the Hubble parameter, \( h = 0.45 \). Although this is not favored by most of the current observational data, we conclude that this choice of parameters constitutes the best compromise amongst the observational constraints that we have imposed.

Another fix is to add a little hot dark matter, usually assumed to be in the form of a massive neutrino. Previously studied versions of this class of model typically postulate a single species of neutrino with significant mass, and a fraction \( \Omega_\nu = 0.3 \) of the critical density in the form of hot dark matter. This model was ruled out, based on its inability to reproduce the observed abundances of Damped Lyman \( \alpha \) systems (DLAS) at \( z \sim 3 \) (Kauffmann & Charlot 1990). Models with lower fractions of hot dark matter (\( \Omega_\nu = 0.2 \)) are more plausibly consistent with constraints from DLAS (Klypin et al. 1995a), but still have much too much small scale power and thus overproduce clusters at \( z = 0 \). However, as pointed out by Primack et al. (1994, see also Pogosyan & Starobinsky 1995) if the hot dark matter is divided into two species of neutrino with equal masses, the power on cluster scales is reduced by 20 per cent without affecting smaller or larger scales. This lowers cluster abundances without worsening potential early structure formation problems (small-scale power) or compromising the COBE normalization. We find reasonable agreement with observed cluster abundances with \( \Omega_\nu = 0.2, \Omega_b = 2, h = 0.50, \) and \( n = 1 \); or alternatively, with a small tilt \( n = 0.9 \) and a higher Hubble parameter \( (h = 0.6) \). We chose the former, based on concerns about the age of the Universe. However, we ran the simulations before the Hubble Key Project.

Figure 3 summarizes the expected mass functions on the group and cluster mass scales, as estimated from the Press-Schechter approximation, with a Gaussian filter. Using the calibration with \( N \)-body simulations from Borgani et al. (1997a), we use \( \delta_{c,g} = 1.5 \) for the model with massive neutrinos and \( \delta_{c,g} = 1.3 \) for all other models, in this figure (but cf. Table 3 for best-fit \( \delta_{c,g} \) and \( \delta_{c} \) to our simulation results). The observational cluster abundance estimates plotted are in reasonable agreement with these mass functions, especially if the mass estimates are low as indicated by some gravitational lensing estimates.

In figure 2 we compare each model to several recent CMB measurements, using the CMBFAST program of Seljak & Zaldarriaga (1996). We also show the four most recently announced CMB results on the figure. Not shown are systematic calibration errors of 14 per cent for Saskatoon and 20 per cent for Python III. Note that the OCDM model is strongly inconsistent with the Saskatoon points, and our choice of \( \Omega_0 = 0.5 \) is at the 95 per cent confidence lower limit for an open model (Linder & Barbosa 1997). Also note that the models with even the relatively mild tilt of \( n = 0.9 \) are at best in marginal agreement with the Saskatoon data around the first Doppler peak.

Figure 4 shows the linear power spectra at the present epoch. As one might expect, all the spectra nearly cross at a wavenumber of a few tenths \( h \) Mpc\(^{-1} \), corresponding to cluster scales. Also, we show some of the window functions used in the normalization procedure described above.

\( \Omega_0 \neq 0 \) class of models has been well studied, typically with \( \Omega_0 = 0.3 \) and \( h \sim 0.65-0.7 \). However, analysis of earlier \( N \)-body simulations has shown that when nonlinear effects are included, this model produces a power spectrum/correlation function with too high an amplitude on small spatial scales compared to observations, unless galaxies are strongly anti-biased with respect to the dark matter (Klypin, Primack & Holtzman 1993; hereafter KP93; Jenkins et al. 1997). Ghigna et al. (1997) have also shown that the void probability function for this model is in disagreement with observations. Therefore we have chosen a model with a slightly higher value of the matter density \( (\Omega_0 = 0.4) \) and a tilt \( (n = 0.9) \) to reduce small-scale power and correlations.

Many observers favor an open cosmology and a high Hubble parameter, consistent with local density estimates and the Hubble Key Project. The lowest reasonable value of \( \Omega_0 \), given initial Gaussian fluctuations as assumed in all CDM-variant models considered here, is constrained to be above 0.3 at \( > 4 \sigma \) confidence (Nusser & Dekel 1993; cf. also Dekel & Rees 1994; Bernardeau et al. 1995). We adopt \( \Omega_0 = 0.5 \) as a ‘reasonable’ value for OCDM, noting that even this relatively high \( \Omega_0 \) leads to a power spectrum lower than that indicated by the POTENT analysis (Kolatt & Dekel 1997), see also figure 4. Our linear code is not capable of determining low multipole cosmic microwave background fluctuations for OCDM, as it uses a plane wave expansion that is only appropriate for flat cosmologies. Instead, we use fitting functions for the normalization \( \delta_I(\Omega_b) \) and the transfer function \( T(k) \) given by Liddle et al. (1996) hereafter LLRV96.

2 After we ran this model, LLRV96 was superseded by Bunn & White (1997) and Hu & White (1997). Those papers’ \( \sigma_8 \) values agree to high precision with LLRV96 if one lowers \( \Omega_b \) from 0.255 to 0.215, which Bunn & White (1997) favor anyway. However, the transfer function shapes are somewhat different, and the LLRV96 normalization is to the COBE 2-year data, so the power on scales of a few hundred \( h^{-1} \) Mpc may be up to 20 per cent low compared to Bunn & White (1997) and Hu & White (1997). Using a BBKS-style fit as all three papers do, rather than integrating the Boltzmann equation directly, introduces an error of similar magnitude, even with the improved shape parameter described in Hu & Sugiyama (1994, equation D-29). We therefore neglect the difference between the Bunn & White (1997) and LLRV96 spectra.
Table 1. Model parameters and linear results for both simulation suites.

| Model      | Age$^a$ | $h^b$ | $\Omega_0$ | $\Omega_c$ | $\Omega_\Lambda$ | $\Omega_\nu$ | $N_{eff}^c$ | $\sigma_8^d$ | $\tilde{\sigma}_8^e$ | $V_{rms}^f$ | $N_{rms}^g$ |
|------------|---------|-------|-------------|-------------|------------------|-------------|------------|-------------|------------------|-------------|------------|
| observations |     |       |             |             |                  |             |            |             |                  |             |            |
|             | 375    | $5 \times 10^{-6}$ |             |             |                  |             |            |             |                  |             |            |
| 1-σ errors |         | $85 \times 2 \times 10^{-6}$ |             |             |                  |             |            |             |                  |             |            |
| CHDM-2Ω     | 13.0   | 0.5   | 1.0         | 0.7         | 0.1              | 0.2         | 0.0        | 0.0         | 1.0              | 2.0         | 0.719       |
| OCDM        | 12.3   | 0.6   | 0.5         | 0.431       | 0.069            | 0.0         | 0.0        | 0.0         | 1.0              | 0.0         | 0.773       |
| SCDM        | 13.0   | 0.5   | 1.0         | 1.0         | 0.0              | 0.0         | 0.0        | 0.0         | 1.0              | 0.0         | 0.667       |
| TCDM        | 14.5   | 0.45  | 1.0         | 0.9         | 0.1              | 0.0         | 0.0        | 0.0         | 0.0              | 0.0         | 0.732       |
| TACDM       | 14.5   | 0.6   | 0.4         | 0.365       | 0.035            | 0.0         | 0.0        | 0.0         | 0.6              | 0.9         | 0.878       |

a Time since the Big Bang in Gyr.

b Presumed Hubble parameter, in units of 100 km s$^{-1}$ Mpc$^{-1}$.

c ‘Tilt’ of the primordial spectrum; $P(k) \propto k^n$.

d Number of massive neutrinos presumed. The equivalent mass of a neutrino is $m_\nu = \frac{\Omega_\nu h^2}{N_{eff}} \cdot 92$ eV.

e rms mass fluctuation in a sphere of radius 8 $h^{-1}$ Mpc.

In figure 3 we compare our models to the matter power spectrum recently measured from bulk flows by Kolatt & Dekel (1997). We only use the three data points that Kolatt & Dekel use for their own statistical analysis, because for larger wavenumbers smoothing lowers the power significantly. SCDM disagrees at about the 2.5σ level, also reflected in its low value of $V_{rms}$ in table 1. OCDM and TACDM disagree because the value of $P(\sim 0.1$ h Mpc$^{-1}$) is fixed by comparing the observed density of clusters (WEF93, BGGMM93; Borgani et al. 1997)$^4$ to the Press-Schechter prediction, and they have low values of $f(\Omega_0, \Omega_\Lambda) \equiv D_{a}/D_{a} \approx \Omega_0^{6/4}$, where $D(\Omega_0, \Lambda, t)$ is the linear growth factor and $a(t)$ is the expansion parameter. The combination of cluster abundances and bulk-flow power spectrum measurements favors $f \sim 1$, for the currently favored classes of CDM-variant models.

There is currently significant controversy over the proper normalization of our model parameters, and our OCDM and TACDM normalizations are higher than the recent fits reported in Eke, Cole, & Frenk (1996), based on cluster X-ray temperature distributions (Henry & Arnaud 1991)$^5$ although they are consistent with the older analyses of WEF93 and the newer cluster velocity dispersion measurements of Borgani et al. (1997a). We have reanalyzed the Eke, Cole, & Frenk (1996)$^6$ calculation, and he gets slightly higher low-$\Omega_0$ normalizations of $\sigma_8 = 0.86$ and 0.72 for our TACDM and OCDM models, respectively. These normalizations are close to those we have chosen (table 1).

2.2 Algorithm

A classic problem with gravitational simulations is the ‘overmerging’ problem, where small scale structure in highly overdense regions is not resolved. Part of the problem is physical – real galaxies form much denser cores than dissipationless halos can, because the baryons can dissipate energy (but cf. Klypin, Gottlöber, & Kravtsov 1997). Aside from that, numerical limitations can make the problem vastly worse. There are two numerical effects to consider: force resolution and sampling of initial conditions and bound structures. Improving either of these requires vast amounts of memory and processing time, so there is an inherent tradeoff.

Recently, the more popular approach has been to improve the forces by using hybrid (Hockney & Eastwood 1988; Couchman 1991, Xu 1993, for example) or adaptive-mesh (Kravtsov, Klypin, & Khokhlov 1997, for example) force solvers, at the expense of either poor sampling of initial fluctuations or small box sizes. We choose a complementary approach, where we try to balance the sampling of density in a large box with the force resolution. We still require a large dynamic range in order to sample small scales well and simultaneously simulate a large volume for comparison to redshift surveys. Since the two requirements imply an enormous number of particles, computer time limitations force us to use the fastest code available. We choose a standard particle-mesh (Hockney & Eastwood 1988) algorithm, parallelized...
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to run on a distributed-memory message-passing system. This type of code produces adequate forces at about 1.5 grid cells (KNP97, Appendix A), but we double this distance to be conservative. So, we require that we have 3^3 times as many grid cells as particles, for the high resolution suite. We choose a grid cell size of 65 h^{-1} kpc, with N_g = 1152^3 grid cells and N_p = 384^3 = 57 million ‘cold’ particles. For the large-volume case, we wish to follow the dynamics only of clusters of galaxies, so we can afford to coarsen the density grid slightly. We find that a cell size of 390 h^{-1} kpc is adequate for following the dynamics of \( \gtrsim 10^{14} h^{-1} M_\odot \) objects, and expect information about smaller objects to come from the high resolution simulations. The slight coarsening of the density resulted in a substantial advantage in running time.

Initial conditions were calculated using a parallelized Zel’dovich (1970) approximation. For CHDM models, we started with a uniform grid of cold particles, and two neutrinos at the position of every cold particle. Cold particles and neutrinos were offset from the grid using separate cold+baryon and hot power spectra, and consistent velocities were derived from the offsets using scale-dependent lin-

ear growth rates calculated by a refinement of the Holtzman (1989) code. In addition, equal and opposite random thermal velocities were chosen for each pair of neutrinos from a redshifted relativistic Fermi-Dirac distribution (Klypin et al. 1993).

We adopted the form of the equations of motion used in Kates, Kotok, & Klypin (1991) generalized to arbitrary cosmology:

\[
\nabla^2 \phi = \frac{3}{2a} \delta \equiv \frac{3}{2a} \delta \rho / \Omega_c , \quad (2)
\]

\[
\frac{d\vec{p}_i}{da} = -\dot{a} \nabla \phi (\vec{x}_i) , \quad (3)
\]

and

\[
\frac{d\vec{x}_i}{da} = \frac{\vec{p}_i}{a^2 \dot{a}} \quad (4)
\]

where \( \dot{a} \) is given by the Friedmann equation with time variable \( H_0 \),

\[
\dot{a} = a^{-1/2} \sqrt{\Omega_c + \Omega_{\nu} + \Omega_{\Lambda} a^3 + \Omega_\kappa a} . \quad (5)
\]

Time discretization was a standard ‘leapfrog’ scheme (cf. Hockney & Eastwood 1988), with even steps in the expansion parameter \( a \). To reduce the expense of the simulations, the timestep was chosen only to stabilize bound structures at the final timestep, rather than keep all structures on the scale of the grid spacing stable. This is only a problem for the cores of clusters, which have the highest velocities. For clusters, we assume an upper bound of particle velocities of 1200 km s^{-1} today and a minimum diameter of any given bound structure equal to the linear cell size. Stability for such an object requires that particles take at least one timestep to traverse the object. So, the required condition is

\[
\Delta a \equiv \dot{a} \Delta t \leq \frac{H_0 L}{N_g \sqrt{3} v_{\text{max}}} \quad (6)
\]

independent of cosmology because the condition is evaluated at the present epoch and \( H_0 L \) is chosen to be the same for all models. Plugging in \( v_{\text{max}} = 1200 \) km s^{-1}, \( H_0 L = 7500 \) km s^{-1} and \( N_g = 1152^3 \) gives \( \Delta a \lesssim 0.005 \), or 200 timesteps for the high resolution suite. Such a low \( v_{\text{max}} \) will not model the interiors of clusters well, since they are observed to have velocity dispersions larger than that, but to remain bound to the cluster, particles have the much looser requirement that they not traverse the whole cluster in one timestep. As large cluster radii are up to about 50 grid cells, the effective stability limit is 20 per cent the speed of light inside a large cluster, for the high resolution suite, presuming that the cluster is adequately modeled by an isothermal sphere. We checked that the choice of timestep was adequate by running a 25 h^{-1} Mpc box CHDM-2ν simulation with 384^3 grid cells (which has the same 65 h^{-1} kpc cell size as the high resolution suite) for 200 timesteps and for 300 timesteps. The resulting mass functions were not significantly different. For the large volume suite, clusters do not cover nearly as many cells as in the high resolution suite, and so the velocity limit is much higher. Our choice of 150 timesteps corresponds to a limiting speed of 5000 km s^{-1} if particles are not to cross one cell in a timestep. The suite parameters are summarized in table 1.

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4 Specifically, the Cornell Theory Center SP2, but the code is portable to any system supporting MPI, including heterogeneous workstation clusters and most modern supercomputers.

5 Though the implementation, and especially parallelization, of...
Comparison of cosmological models to selected CMB observations

Figure 2. Model Comparison to Cosmic Microwave Background. All models except SCDM are consistent with COBE four year data (Górski et al. 1996; Górski, private communication). Circles, solid squares, open squares and asterisks are the COBE four year power spectrum (Tegmark 1996), Saskatoon 1995 results (Netterfield et al. 1997), CAT detection (Scott et al. 1996) and Python III results (Platt et al. 1997), respectively. Not shown are systematic normalization errors of 14 and 20 per cent, for Saskatoon and Python III, respectively. The curves are all calculated using the cmbfast program of Seljak & Zaldarriaga (1996). Cosmological parameters correspond to models considered in this paper, except for SCDM. The normalization is adjusted so that the low harmonics match the output of our linear code. CMBFAST is capable of calculating larger multipoles than our linear code. SCDM is shown here with $\Omega_b = 0.1$, since all the high-$\ell$ features in the CMB spectrum are dependent upon baryon interactions, but was actually simulated with no baryons.

The Zel’dovich (1970) approximation is only valid when the two-species particle mesh code described above is much less trivial than one might suppose, discussion of the code has been omitted for space considerations. The interested reader may find a detailed description of the code, its implementation on the Cornell SP2, and several code tests in Gross (1997).

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Table 2. Simulation parameters for both simulation suites

| Suite    | Box size $h^{-1}$ Mpc | $N_{\text{res}}$ | Cell size $h^{-1}$ kpc | $N^a_{\text{cold}}$ | $M^b_{\text{cold}}$ $(\Omega_c + \Omega_b) h^{-1} M_{\odot}$ | $N_{\text{steps}}$ | $M^c_{\text{min}}$ $\Omega_0 h^{-1} M_{\odot}$ | $f^d$ |
|----------|------------------------|------------------|------------------------|---------------------|-----------------------------------------------|------------------|-----------------------------------------------|-------|
| high res | 75                     | 1152$^3$         | 65                     | 384$^3$             | 2.09 x 10$^9$                                | 200              | 3.4 x 10$^{11}$                               | 0.078 |
| low res  | 300                    | 768$^3$          | 390                    | 384$^3$             | 1.34 x 10$^{11}$                              | 150              | 7.3 x 10$^{13}$                               | 0.043 |

$^a$ Number of cold particles; for models with massive neutrinos, $N_{\text{hot}} = 2N_{\text{cold}}$.

$^b$ Mass of cold particles; for models with massive neutrinos, $M_{\text{hot}} = M_{\text{cold}}\Omega_c/2(\Omega_c + \Omega_b)$.

$^c$ Halo detection cutoff, from the restriction that halos must be larger than the grid size.

$^d$ Fractional error in mass for the smallest halos identifiable.

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Figure 3. Linear power spectra used in our simulation suites. Also shown are two of the window functions used in normalizing the models: $k^2 W^2(rk)$ with $r = 8 h^{-1}$ Mpc for $\sigma_8^2$ and $W^2(rk)$ with $r = 50 h^{-1}$ Mpc for $V_3^2$. Here $W(x) = 3(\sin(x) - x \cos(x)) x^{-3}$. Also shown (for illustrative purposes only) is the equivalent window function for approximate COBE normalization using the pure Sachs-Wolfe effect, $j_{10}(d_h k)/(2\pi d_h^2 k^2)$ where $j_{10}(x)$ is the 10th order spherical Bessel function and $d_h$ is the horizon distance. The version plotted has the amplitude raised by a factor of 100 for visibility and uses $d_h = 2c/H_0 = 6000 h^{-1}$ Mpc, which is appropriate for $\Omega_0 = 1$. For OCDM, the horizon distance is 7470 $h^{-1}$ Mpc and for TACDM, it is 8810 $h^{-1}$ Mpc, so the window function moves a small distance to smaller $k$ in those cases. A similar window function for cluster abundance doesn’t exist because it doesn’t have the form of a convolution. In an extremely rough sense, the scales are comparable to those sampled by $\sigma_8$.

Figure 4. Linear power spectrum comparison to bulk flow measurements. The curves are all a magnification of figure 3 multiplied by $f^2(\Omega_c, \Omega_b) = (aD_0/D)$. The three data points are from Kolatt & Dekel (1997), $f(\Omega_c, \Omega_b)$ was calculated exactly, using equation (C.3.14) of Gross (1997) and its analytic derivative.

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Brings the rms fluctuations well below 1. The initial time was chosen so that the rms overdensity on the grid scale was $\delta_{\text{rms}} \lesssim 0.2$. This was $z = 30$–60, depending on the model. Particle data and halo catalogs were stored at four equally spaced intervals in $a$ during execution. The large volume simulation suite used the same starting times as the high resolution suite even though they could have been started somewhat later due to the poorer resolution. The extra computation involved is about one timestep and is therefore negligible.

Random numbers are necessary to model inflation-generated Gaussian fluctuations and random phases in the density field. Such randomness introduces highly significant variation from simulation to simulation, commonly referred to as ‘cosmic variance.’ Because we can only observe one universe, quantifying the effect of cosmic variance is very important and is a separate issue from variations between models due to different physics. In these suites, we have separated the effects by picking a single random number seed for each suite, checking that the largest 26 waves do not have any fluctuations larger than a factor of 2, and rerunning one model with a different seed. That is, within each suite, the random numbers for each model within a suite are all the same, and large wavelength fluctuations are restricted to a smaller range than Gaussian statistics would permit, in an attempt to prevent rare statistical flukes from compromising expensive simulations (as happened in Klypin et al. 1993). This means the structures are approximately in the same
place, and when one also considers the cluster abundance criterion discussed in section 2.1 there is roughly the same number, distribution and positions of 5 × 10^{14} h^{-1} M_\odot clusters in all the models in a given suite. Note that the models do have different power spectra and fluctuation growth rates, so distributions can differ for objects with different masses.

3 DARK MATTER HALOS

3.1 Halo finding algorithm

We identify dark matter halos using a spherical overdensity algorithm similar to that of KNP97, with some of the limitations removed:

(i) We define candidate halos as the centers of all density maxima containing an overdensity greater than \( \delta \equiv \delta_p / \Omega_0 \rho_c = 50 \). A density maximum is defined as a cell whose density is greater than its six Cartesian neighbors. Just in case there are other halos hiding in those six neighbors, we also consider each of them to be candidates. Note that the finite grid size (as in all other grid-based halo finders, such as DENMAX, [Eke & Bertschinger 1994]) will introduce a minimum separation between halos, which may cause small halos to be missed, in which turn will require a mass cut.

(ii) Each candidate halo then has the location of its center set iteratively to the center of mass of all the particles inside a sphere of diameter equal to the cell size (65 h\(^{-1}\) kpc in our high resolution case). Halos are expected to have a minimum size of the order of the grid size, so this procedure moves the candidate halo to the peak of the density maximum. Of course, since we have defined more than one candidate for each detected maximum, some candidates will converge on the same halo. The smaller mass object in a given pair is to be missed, in which turn will require a mass cut.

(iii) We perform a central overdensity cut. All halos that don’t enclose a mean overdensity sufficient for virialization according to the spherical collapse model (see Gross 1997, appendix C, and references therein) at the end of the center-of-mass detection phase are presumed not to be virialized objects and are discarded. Typically, this reduces the number of halos by a factor of 2–3, though the number is model dependent.

(iv) We now estimate at what radius the mean enclosed overdensity \( \delta \equiv \delta_p / \Omega_0 \rho_c \) falls to \( \delta_{\text{vir}} \), the virial radius of the halo in spherical infall models. For each halo, we count the number of particles within five radii up to five grid cells (325 h\(^{-1}\) kpc in this case) away from the center, convert that to density, and interpolate the radius at which \( \delta = \delta_{\text{vir}} (r_{\text{vir}}) \) using power-law cubic splines. If five radii is not large enough to enclose \( r_{\text{vir}} \), we search five more radii, each twice as long as the original radii. This is repeated until we enclose \( r_{\text{vir}} \).

(v) We define the mass of the halo as the mass enclosed in \( r_{\text{vir}} \). The velocity is the mean velocity of all the particles within \( r_{\text{vir}} \).

(vi) In general, the largest halos in a high resolution run contain much resolved but bound substructure. Because we search for the \( \delta = \delta_{\text{vir}} \) radius, we detect the same regions of space dozens of times for the largest halos. To remove ‘double-counted’ halos, the halos are searched in reverse order by mass to see if they enclose the centers of any smaller halos. If so, the smaller halo is thrown away. Note that the ordering is important because three-body interactions would be non-deterministic otherwise, and throwing away halos that only intersect is too stringent.

One limitation of this algorithm is that it assumes all halos are spherically symmetric, which is demonstrably untrue. But the effect on the mass function is random, rather than systematic, and finding the halos with an algorithm generalized to ellipsoidal distributions does not change the mass function significantly, even though it changes the parameters of individual halos. Because the halos have finite size, one cannot perform mass-weighted correlation function analyses, for distances less than the largest halo radius (about 2–3 h\(^{-1}\) Mpc in radius, typically).

The other limitation is the use of the density grid to identify halo candidates. If one considers a worst-case identification where a large number of particles all collect in one corner of a grid cell, in order to guarantee that all nearby halos are identified, one must draw a sphere which encloses the entire cell, of radius

\[
 r_{\text{min}} = \sqrt[3]{3L/N_h^{1/3}},
\]

where \( L \) is the length of one side of the computational volume and \( N_h \) is the number of grid cells. If halos happen to be bigger than that, then the last step of the halo catalog generator makes it unimportant that we couldn’t see nearby structure. Fortunately, halo extent is trivially related to halo mass because we have defined both where the mean overdensity is \( \delta = \delta_{\text{vir}} \).

3.2 The effect of mass resolution

To what extent should you, the reader, trust the mass functions presented in this paper? To answer that, one must consider several effects. A typical feature in a mass function is that the large-mass end becomes ‘wiggly,’ usually blamed on the scarcity of high mass halos combined with cosmic variance. There is a related effect at somewhat smaller masses, since very large halos tend to have somewhat massive companions. For example, in most models in our high resolution suite, 5 × 10^{13} h^{-1} M_\odot objects are fairly rare, but it is common to see them as companions for 10^{15} h^{-1} M_\odot objects. So, the wiggles may propagate down the mass function, and cosmic variance may have a significant effect on more than just the largest mass scales.

Cosmic variance fortunately leaves a signature, in that the mass function is not smooth at high masses. But, it is quite important to figure out the limiting factors at low mass, where typical mass functions are quite smooth. What limits accuracy here are the effects of finite sized grids and finite numbers of particles.

The effect of the finite sized grid in identifying maxima in the final particle distribution was discussed above, and one must merely translate the minimum radius of a halo \( r_{\text{min}} \) to a minimum mass. Since the halo radius and mass

\[ \text{Note that our definition of } \delta_{\text{vir}} \text{ is related to } Eke, Cole, \& \text{Frenk (1996) by } 1 + \delta_{\text{vir}} = \Delta_{\text{EFP}} / 1. \text{ Our choice is appropriate for the density field calculations in an } N\text{-body code.} \]
are defined as enclosing a mean overdensity of $\delta_{\rm{vir}}$, the mass $M_{\rm{vir}}$ of a halo of radius $r_{\rm{vir}}$ is

$$M_{\rm{vir}} = (1 + \delta_{\rm{vir}}) \frac{4\pi}{3} \Omega_0 \rho_c r_{\rm{vir}}^3.$$  \hspace{1cm} (8)

So, a very conservative mass cut is

$$M_{\rm{min}} = (1 + \delta_{\rm{vir}}) \frac{4\pi}{3} \Omega_0 \rho_c (L/\sqrt{3})^3 N_{\rm{c}}.$$  \hspace{1cm} (9)

Plugging in values for the high resolution suite, the mass cut is $3.4 \times 10^{13} \Omega_0 \ h^{-1} \ M_\odot$. For simplicity, we make the same mass cut on all models, corresponding to $\Omega_0 = 1$.

One might worry that the central density cut described in the previous section could cut too many small halos, because the fairly long timesteps used cause the density within the ‘half-mass’ radius to go down by about a factor of two if the timestep equals the stability limit (Quinn et al. 1997). We perform the central overdensity cut at $r = L/2N_{\rm{c}}$, but the proximity restriction used in deriving equation (9) requires that halo radii in the final catalogs be at least $\sqrt{3}L/N_{\rm{c}}$. If halo profiles fall at least as fast as $r^{-1}$ (whereas the Navarro, Frenk, & White 1994 profile says it should be much steeper than that near the virial radius), then the density fed into the central overdensity cut should be at least a factor of $2\sqrt{3} \approx 3.4$ greater than the virial density. This more than offsets the density smoothing due to timestepping at the stability limit, so we neglect the effect of time steps in our mass resolution analysis. Note that our timesteps are only near the stability limit for virial radii near the detection limit – otherwise, a particle takes many timesteps to cross a halo. Therefore, lowered densities due to long timesteps are only a concern for the smallest detectable halos.

One might also worry that the quality of the force law at scales approaching the grid scale would also result in reduced central density. At the 1.7 grid cells proximity cutoff, the point-mass potential in our simulations is about 90 per cent of the correct $GM/r$ value. With such a force law, the virial theorem requires that the density be also 10 per cent low, to maintain the same velocity dispersion. Thus, some of the smallest halos around the mass cut will not make it into the catalog. In practice, the density profiles for the smallest halos are considerably noisier than 10 per cent due to asphericity and background particles, so we neglect the effect of an oversoftened force law.

Particle discreteness may also affect the halo mass function, because random fluctuations may affect the detection of some of the smallest halos. We consider here how much significance we need to make the expected number of halos of some of the smallest halos. We consider here how much the number of particles inside a halo of mass $M_{\rm{vir}}$ in a simulation box of size $L$ is

$$N_{\rm{vir}} = \frac{M_{\rm{vir}} N_{\rm{p}}}{\Omega_0 \rho_c L^3}.$$  \hspace{1cm} (11)

For counting $N$ particles within $r$, the random variation in number is $\sigma = \sqrt{N}$. Let us suppose there are $N_h$ halos above a given mass, and we wish to detect them all. We presume that counting halos is a Gaussian process and state that the $n$-sigma uncertainty in the detection of the halos corresponds to incorrectly detecting or missing a fraction $\text{erfc}(n/\sqrt{2})$ of the halos. We require detection of all halos, so the fraction missed should be less than $1/\rho L^3$, where $\rho$ is the number density of halos above the mass cutoff, and $L^3$ is the volume of the simulation box where halos are identified. Inverting, we need detections of $\sqrt{2}\text{erfc}^{-1}(1/\rho L^3)$ sigma. The density $\rho$ should really come from the Press-Schechter approximation, given a desired mass cutoff, but the inverse complementary error function is extremely insensitive to the value of its argument, once it becomes much less than one. As an example, for $\rho_{\rm{h}} = 1 h^3 \ Mpc^{-3}$ (appropriate for a mass cutoff a little below $10^{11} h^{-1} \ M_\odot$ for most models), we need to have at least 4.7$\sigma$ detections of all halos. Less significant detections mean it is likely some of them have been missed by random fluctuations. This means every halo must contain at least 23 particles. More generally,

$$N_{\rm{min}} = 2 \left[\text{erfc}^{-1} \left(\frac{1}{\rho_{\rm{h}} L^3}\right)\right]^2$$  \hspace{1cm} (12)

for the rather liberal restriction that we only require detection of the halo.

As an alternative cutoff criterion, requiring a 10 per cent or less 1-$\sigma$ error in mass is a more stringent requirement, and every halo must have at least 100 particles, since mass is determined by counting particles within several radii. If one requires a fractional error of $f$ for a minimum halo mass of $M_{\rm{min}}$, one needs at least

$$N_p = \frac{\Omega_0 \rho_c L^3}{f^2 M_{\rm{min}}}$$  \hspace{1cm} (13)

particles in the simulation. The parameters used, and the effective $f$ they allow, are shown in table 3. Figure 6 shows that, with grid sizes and mean interparticle spacings of the order of those used in our suite, the effect of lowering either the grid size or the mean interparticle spacing by a factor of two does not significantly affect the mass function. For this test, we raised the threshold for halo candidate identification from $\delta = 50$ in one cell to $\delta = 70$ because one isolated cold particle in the high $N_\delta$ case gives $1 + \delta = 51.2$.

To explicitly test the effect of grid sizes on our mass functions, we ran five small simulations of the CHDM-2$\nu$ model with $N_\delta = 192^3$ grid cells and $N_p = 3 \times 64^3$ particles, with various-sized boxes. Though these simulations are too small to generate meaningful mass functions on their own, collectively their upper envelope does match the Press-Schechter formula reasonably well, for $\delta_{\rm{c},g} = 1.2$ with a Gaussian filter. Figure 6 shows the five different mass functions. Also shown are lower mass cuts, determined for every model using equation (13). Above the mass cutoffs, every mass function agrees with the one for the next smaller box.

Well below, the mass function slopes are not steep enough, but they agree with the neighboring curves for significant distances below the mass cuts, so it may be reasonable to
extrapolate the mass function further. Every halo detected by the halo finder is represented in the figure, and the locations of the lower mass cuts are indicated by vertical lines. This test could conceivably overproduce clusters because of the extremely poor force and mass resolutions in the largest volume run – a cell width is about the size of an Abell radius. This result does persist for much larger simulations, as discussed below.

4 RESULTS

The connection between simulations and observations is still fairly uncertain, and the least well determined portion of it is the galaxy identification procedure. It is therefore helpful to do as much analysis as one can using quantities that are insensitive to the details of galaxy formation. Currently, only bulk flow motions \cite{KolattDekel97} provide a meaningful matter power spectrum, but the large smoothing required means that the comparison is best made to the linear power spectrum (see figure \ref{fig:massfunctions}). When investigating quantities derived from observations of galaxies (as the vast majority of astronomical observations are) one is forced to make assumptions based on expectations about the nature of galaxy bias, for example the usual expectation that galaxies are more clustered than the dark matter. Figure \ref{fig:massfunctions} shows nonlinear real-space dark matter power spectra for all our models, compared to the APM real-space galaxy power spectrum \cite{BaughElst94}. The OCDM model requires significant antibiasing and the ΛCDM model requires even more. There is no evidence for such strong antibiasing, and it is very difficult to explain physically, especially on such large scales \cite{YEP97, Gauf99}. Additional arguments against strongly scale-dependent antibiasing are given in KPH96.

The process of galaxy formation is not well understood, so one could argue that perhaps there is some mechanism that would give us strong antibiasing. We have created an extreme model for galaxy formation designed to produce as much antibias as possible \cite{KPH96}. Everywhere in the density grid, if there is more than $2.1 \times 10^9 \, h^{-1} M_\odot$ in a grid cell, we presume one galaxy forms there. That mass corresponds to slightly more than the mass due to one isolated particle in the high-resolution SCDM and TCDM simulations (which have the most massive particles in the suite). Such a limit is necessary to prevent placing excess power in the voids due to vestiges of the initial grid there. This is a highly unreasonable model for galaxy formation, as it says that the density of $\gtrsim 2 \times 10^{11} \, h^{-1} M_\odot$ galaxies in the core of the Coma cluster should be the same as in the local group, and this is clearly ruled out observationally. However, even though there is significant antibias on small scales, it is only visible at scales smaller than about $k = 1 \, h \, \text{Mpc}^{-1}$ (see figure \ref{fig:ps}), whereas antibiasing is needed on scales larger than that in order for OCDM or TΛCDM to be consistent with the APM power spectrum. Note that a possible way

\begin{figure}
\centering
\includegraphics[width=0.45\textwidth]{massfunction_dependence.png}
\caption{Effect of raising the number of particles or the number of grid cells by a factor of 8 in a very small CHDM-2$\nu$ 10 $h^{-1}$ Mpc simulation with $N_p = 128^3$ and $N_p = 3 \times 64^3$. The mass functions are not significantly different. A somewhat low mass cutoff of $10^{11} \, h^{-1} M_\odot$ has been applied. The high resolution suite has a linear volume run – a cell width is about the size of an Abell radius. This result does persist for much larger simulations, as discussed below.}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=0.45\textwidth]{massfunction_dependence.png}
\caption{Effect of grid size on mass functions. The curves represent very small simulations of various sizes. The Press-Schechter model was tuned to match the cluster-scale part of the mass function in the largest box. Note that the envelope mirrors the Press-Schechter curve reasonably well, but each individual mass function has a power-law index that is too shallow. Mass functions are limited at the large-mass end by statistics – one simply runs out of enough space to create objects in – and on the small-mass end by some fraction of the halos becoming as small as two grid cells, which means it is not guaranteed that the halo can be resolved from its neighbors, particularly if they are also small halos. The vertical lines represent the lower limit in mass for each run, above which all halos can be detected.}
\end{figure}
Two different high resolution runs of the SCDM case are shown as a guide to how large cosmic variance is. The power in the second SCDM realization is 20–30 per cent lower than that in the first realization for 0.3 ≤ k ≤ 1 h Mpc⁻¹. The APM data are presumably biased with respect to the matter power spectrum, and yet the OCDM, TCDM, and TΛCDM cases require the APM data to be significantly antibiased with respect to the dark matter, with b² ~ 0.6 for OCDM and TCDM and b² ~ 0.5 for TΛCDM at k ~ 1 h Mpc⁻¹. If APM misses galaxies in clustered regions, that would give a low power spectrum on scales of k ≥ 1 h Mpc⁻¹ (see text).

Nonlinear power spectra

Figure 7. Nonlinear real-space dark matter power spectra compared to the APM real-space galaxy power spectrum (Baugh & Efstathiou 1994) of galaxy number-count fluctuations. The simulation power spectra shown here are a composite of the high and low resolution suites, where data from a model’s high resolution run is used at large k and low resolution data is used at small k. Two different high resolution runs of the SCDM case are shown as a guide to how large cosmic variance is. The power in the second SCDM realization is 20–30 per cent lower than that in the first realization for 0.3 ≤ k ≤ 1 h Mpc⁻¹. The APM data are presumably biased with respect to the matter power spectrum, and yet the OCDM, TCDM, and TΛCDM cases require the APM data to be significantly antibiased with respect to the dark matter, with b² ~ 0.6 for OCDM and TCDM and b² ~ 0.5 for TΛCDM at k ~ 1 h Mpc⁻¹. If APM misses galaxies in clustered regions, that would give a low power spectrum on scales of k ≥ 1 h Mpc⁻¹ (see text).

Nonlinear ‘biased’ power spectra

Figure 8. Nonlinear power spectra, assuming an extreme scale-independent biasing scheme. The density field has been set to ‘on’ at any cell containing mass exceeding the largest particle mass in the 75 h⁻¹ Mpc suite, 2.1 × 10⁸ h⁻¹ M₅₀, and ‘off’ everywhere else. That mass cut is most likely lower than anything that could make it into the CfA2 or APM catalogs, except if one assumes an impossibly small mass-to-light ratio. The result of such a bizarre galaxy identification scheme is a bias on large scales, due to clearing out the void regions, and an antibias on small scales, due to removing the high peaks in density. We do comparisons with the high resolution suite because the low resolution suite particle mass is too high.

The simplest halo-related quantity to investigate is the density of bound objects as a function of mass. Such ‘mass functions’ and close relatives such as the X-ray temperature function (as in Eke, Cole, & Frenk 1996, for example) are often estimated from the Press-Schechter approximation instead of from simulations. Though it has been checked against scale-free simulations (Efstathiou et al. 1988; Bond et al. 1991; Lacey & Cole 1993), and against specific SCDM, ΛCDM and CHDM models (Carlberg & Couchman 1983; Jain & Bertschinger 1994; Klypin et al. 1995), meaning the power perhaps shouldn’t be suppressed quite as much, and galaxy formation will further raise the power. Figure 8 shows the models’ redshift space power spectra, compared to the combined CfA2 and SSRS2 redshift space power spectrum (da Costa et al. 1993). Given our choices of model normalization and cosmological parameters, the TΛCDM matter power spectrum is nicely consistent with the observed galaxy power spectrum, but that leaves no room for galaxy formation or velocity bias effects. As for the real-space nonlinear power spectrum comparison (figure 7), this requires significant antibiasing for TΛCDM on scales of 0.3–1 h Mpc⁻¹. Note that undersampling the velocity field will miss the large velocities by making the halos physically larger, so it does not make sense to perform redshift space comparisons on the large volume suite.
estimates for all models at the intermediate mass of
are a significant factor of 1.5–2 below the Press-Schechter
functions are consistent, and that
Press-Schechter parameters used in that figure. Note that
mated from both suites of simulations, and table 3 shows the
for a Gaussian window function.

Figure 9. Redshift space power spectrum, compared to the
combined CfA2 and SSRS2 redshift space power spectrum (da Costa
et al. 1996). Notice that, while ACDM is a good match to this
power spectrum, there is no room for galaxy formation or velocity
bias.

Walter & Klypin 1996, Bond & Myers 1996), previous stud-
ies have focused only on a narrow range of masses, typically
at the cluster scale. With our large simulations, we can check
the approximation over four orders of magnitude in mass. The
Press-Schechter formula we use is Klypin et al. (1995b),
equations (1–2), evaluated at z = 0:

\[ N(M) = \sqrt{\frac{2}{\pi}} \frac{\delta_c}{\sigma_m} \int r \epsilon(r') \exp \left[ -\frac{\delta_c^2}{2\sigma^2(r')} \right] \frac{dr'}{r'^3}, \]

(14)

where

\[ \epsilon(r) = \frac{1}{2\pi} \int_0^\infty k^3 P(k) W(kr) \frac{dW(kr)}{dk} \frac{dk}{ck}, \]

(15)

\[ \sigma^2(r) = \frac{1}{2\pi^2} \int_0^\infty k^2 P(k) W^2(kr) \frac{dk}{ck}, \]

(16)

\[ W(x) = \begin{cases} \frac{1}{\sqrt{2\pi}} \sin(x) - x \cos(x) & \text{tophat} \\ \frac{1}{\sqrt{2\pi}} e^{-x^2/2} & \text{Gaussian} \end{cases} \]

(17)

\[ r = \left( \frac{1}{\alpha_m P_c(\Omega_m)} \right)^{1/3} \]

(18)

and \( \alpha_m = 4\pi/3 \) for a tophat window function, and \( (2\pi)^{3/2} \)
for a Gaussian window function.

Figure 10 shows the cumulative mass functions esti-
mated from both suites of simulations, and table 3 shows the
Press-Schechter parameters used in that figure. Note that
in the overlapping region, the two sets of simulation mass
functions are consistent, and that the high resolution results
are a significant factor of 1.5–2 below the Press-Schechter
estimates for all models at the intermediate mass of \( 10^{13} \)
\( h^{-1} M_\odot \) and below.

This result has been verified recently by other groups. Bryan &
Norman (1998) see a somewhat stronger discrepancy at
\( 10^{13} h^{-1} M_\odot \), using a spherical overdensity method,
for cosmological parameters very close to our CHDM-2ν,
OCDM and SCDM choices (though with substantially larger
grid cell sizes for ODM and SCDM). Somerville et al.
(1998) also see an equivalent discrepancy in the differential
multiplicity function \( n(M) \) at \( z = 0 \) in the \( \tau \)CDM
cosmology, for which the power spectrum is very similar to our
CHDM-2ν. This result depends upon a completely independent
simulation Jenkins et al. (1997) modeled using adaptive
P3M, with halos identified using the Friends-of-Friends
method. The halo mass function for the SCDM model from
the Jenkins et al. (1997) simulations is virtually identical
to ours (G. Lemson, private communication). Given these
confirmations, we do not believe that the medium-mass discrep-
ancy we see is an artifact of our simulation method or
halo finding algorithm.

As figure 11 shows, the intermediate and low mass discrep-
cy cannot be fixed by adjusting the value of \( \delta_c \), particu-
larly at a mass of \( \sim 5 \times 10^{12} h^{-1} M_\odot \), where the curves
cross. Our values of \( \delta_{c,t} \) and \( \delta_{c,g} \) are consistent with
Borgani et al. (1997b), except we find that the CHDM-2ν \( \delta_c \)’s
are not significantly different from the other models. The \( \delta_{c,t} \)
values we find for the tophat case are consistent with the
spherical collapse model.

The simulation mass functions in figure 11 fall below the
Press-Schechter predictions for most of their range. For ex-
ample, in the SCDM high-resolution run, only 60 per cent of
the particles are within halos with \( M > 3.4 \times 10^{12} h^{-1} M_\odot \)
at \( z = 0 \). The Press-Schechter prediction is only very slightly
larger, about 62 per cent for \( \delta_{c,t} = 1.672 \) and 65 per cent for
the spherical collapse value \( \delta_{c,t} = 1.686 \). The mass deficit
due to smaller abundance of low-mass halos halos in the
simulations is almost completely compensated for by a small
excess of very large clusters. The Press-Schechter approxi-
mation assumes that all the mass must be in halos of some
size, and this analysis indicates that a significant fraction of
the mass of the universe should be in small halos. The
Press-Schechter approximation indicates that for SCDM, as
much as 20 per cent of the mass is in halos as small as
\( 10^{7} h^{-1} M_\odot \), which is not identifiable in any present-day
cosmological simulation. The simulations show a significant
amount of matter that is not in collapsed objects. Most of
the mass lies in filaments connecting the clusters, many of
which have only a few identified halos on them. It is con-
ceivable that much of this mass may be unresolved halos,
since any \( N \)-body simulation must have a resolution and/or
timestep limit below which forces are ‘soft,’ resulting in dis-

| Model     | \( \delta_{c,t} \) | \( \delta_{c,g} \) |
|-----------|------------------|-----------------|
| CHDM-2ν   | 1.571            | 1.273           |
| ODM       | 1.693            | 1.293           |
| SCDM \( b = 1.5 \) | 1.672 | 1.236 |
| TCDM      | 1.630            | 1.252           |
| TACDM     | 1.732            | 1.355           |

\( \delta_{c,t} \) and \( \delta_{c,g} \) have been chosen to get the same number density
of clusters with \( M > 5.5 \times 10^{14} h^{-1} M_\odot \) as the large-volume simulation, for each model.
CDM-variant models – I

Figure 10. Cumulative halo mass functions, with Press-Schechter fits. In each panel, the relevant mass functions estimated from the two simulation suites is shown by the full curves. Small mass cuts have been applied at $M = 3.4 \times 10^{11} \ h^{-1} M_\odot$ and $2.2 \times 10^{13} \ h^{-1} M_\odot$, for the large and small volume simulations, respectively. Each panel also shows Gaussian (dashed curves) and tophat (dotted curves) Press-Schechter mass functions, with $\delta_{c,t}$ and $\delta_{c,g}$ adjusted to agree with the large-volume simulations at $5.5 \times 10^{14} \ h^{-1} M_\odot$. The values of $\delta_{c,t}$ and $\delta_{c,g}$ used are given in table 3. The data points correspond to the observations of BGGMM93 and WEF93, as in figure 1.

rupture of structure smaller than the limit. Such a mechanism must be present, since filament halos are necessarily not very big, but it is not clear how much of the mass that can account for.

Our two low-$\Omega_0$ models produce fewer clusters in simulations than the other models (figure 10). If X-ray temperature cluster masses are correct, this presents no problem for those models. However, if the indications of larger cluster masses from gravitational lensing are correct, the low-$\Omega_0$ models require revision by using less tilt (in the case of TACDM) or a larger value of $H_0$. The former would help lessen the disagreement with high-multipole cosmic microwave background measurements (figure 2), as a weaker tilt would raise the first Doppler peak, but will lead to
the need for even stronger anti-bias to reconcile small-scale power with the APM observations. If X-ray temperature masses are correct, our parameters for TCDM and CHDM-2ν produce too many clusters. For CHDM-2ν, the normalization used here was actually about 10 per cent higher than the preferred four-year COBE normalization, so reducing the normalization by this factor would probably be enough, though this will exacerbate early structure formation problems. For TCDM, the only options are to either increase the tilt, which is highly disfavored by the small-angle cosmic microwave background data as already noted, or further reduce the Hubble parameter, which is also strongly disfavored by observations.

The statements above all take the COBE normalization as a fixed constraint. Alternatively, we could turn the problem around and use the clusters to determine normalizations and tilts, with $H_0$ (and $\Omega_0$ and $\Omega_\Lambda$) as a given. This is explored further in Gross et al. (1998).

Figure 11. Press-Schechter mass functions for TΛCDM. The high and low density TΛCDM mass functions from simulations are shown in the solid curves. From top to bottom at $M = 10^{15} \, h^{-1} M_\odot$, the dashed curves show Press-Schechter mass functions with Gaussian filters for $\delta_c = 1.0$, 1.2, and 1.4, and the dotted curves show tophat filters for $\delta_c = 1.4$, 1.6, and 1.8. Press-Schechter mass functions can be made to agree with our simulations for masses above about $5 \times 10^{13} \, h^{-1} M_\odot$, but not for masses smaller than that.
One would now like to investigate statistics such as correlation functions, void probability functions (Ghigna et al. 1997; shape statistics (Dave et al. 1997) and other sophisticated statistics. However, to compare to observations, we need to know how many galaxies form in each halo. Previous studies (KNP97; Nothenius, Klypin, & Primack 1999; Ghigna et al. 1997, for example) have used ad hoc ‘breakup’ prescriptions to assign galaxies to halos. We intend to populate our halos with galaxies using a more physically motivated approach (as in Kauffmann, Nusser, & Steinmetz 1999) based on semi-analytic models including simplified treatments of gas processes, star formation, supernova feedback, and galaxy-galaxy merging (Somerville 1999; Somerville & Primack 1998). As a result, we do not attempt to include any complicated galaxy identification algorithms here.

For certain statistics, one can partially compensate for the effect of overmerging by mass weighting. This approach is less than ideal because it does not restore the small-scale spatial information lost in the overmerging process. Mass weighting is equivalent to presuming a halo contains a number of galaxies proportional to its mass, and putting all the weight on the largest halo radius (typically 2–3 Mpc). Since very massive halos are rare objects for physically interesting cosmological models, all mass weighted statistics must be unduly influenced by small-number statistical noise.

We calculate the mass-weighted autocorrelation function for the high resolution runs, and the results are shown in figure 12. In this figure, a halo mass cut of $M = 3 \times 10^{11} \, M_\odot$ was used, although the mass weighting makes it insensitive to the mass cut. The mass weighting creates a spread in the correlation values large enough to prevent the test from discriminating among models. To within the spread visible in figure 12, all models are roughly consistent with the Stromlo-APM autocorrelation function (Loveday et al. 1995). However, there are a few trends visible in the figure. SCDM and TCDM are systematically lower in amplitude than the other models, but the effect is not very significant given the spread.

5 CONCLUSIONS

We have run two suites of simulations with 57 million cold particles in boxes of 75 and 300 h$^{-1}$ Mpc, with the goal of studying interesting variants of the CDM family of cosmological models. In this paper, we have made preliminary comparisons of the $z = 0$ simulation outputs to data for all models. In Smith et al. (1998), we used the lower resolution suite, plus some additional simulations, to generalize the Peacock & Dodds (1994, 1996) procedure for recovery of the linear power spectrum corresponding to a given cosmological model from observational data. In Wechsler et al. (1998), we showed that the most massive halos at redshifts $z \sim 3$, or objects that trace their distribution, can account for the observed clustering of Lyman-break objects (Steidel et al. 1998) for all cosmologies except SCDM. More detailed comparisons with observations require assumptions about galaxy formation and will be treated in subsequent work.

Subject to the usual caveats about the uncertainty of galaxy formation, we reach the following conclusions in the present paper:

(i) Based on the results of KPH96, who found that CDM models with $\Omega_m \sim 0.3$ would require strong scale dependent anti-bias in order to be consistent with the APM power spectrum (Baugh & Efstathiou 1994), we investigated a variant of the CDM model with $\Omega_m = 0.4$ and a tilt of $n = 0.9$. We find that this model still requires large antibias of $b^2 \equiv P_{\text{APMgal}}/P_{\text{lin}} \sim 0.5$ at $k = 1 \, h \, \text{Mpc}^{-1}$. Even in a simple model in which galaxies are extremely anti-biased with respect to dark matter halos, the problem persists on scales of $r \sim 6 \, h^{-1} \, \text{Mpc}$, because this scale is larger than the size of individual halos. To get anti-bias on these scales, there would have to be many ‘barren’ halos containing no galaxies. OCDM and TCDM are only slightly better, still requiring a strong antibias of $b^2 \sim 0.6$. Other models considered require a weaker antibias at that scale.

(ii) The TCDM dark matter redshift space power spectrum agrees very well with the redshift space galaxy power spectrum from CfA2+SSRS2 (da Costa et al. 1994). This leaves no room for the ‘positive’ bias expected in normal galaxy formation, or for velocity biases. For comparison, OCDM and TCDM each have room for a modest bias of $b^2 \sim 1.2$ at $k = 0.5 \, h \, \text{Mpc}^{-1}$, and CHDM-2v and SCDM each need $b^2 \sim 1.5$.

(iii) All models considered here are consistent with the Stromlo-APM real-space correlation function (Loveday et al. 1993) on scales of 2–20 h$^{-1}$ Mpc, largely due to a large spread in the model estimates of the correlation function.
because of mass weighting and small-number statistics for large mass objects.

(iv) The Press-Schechter approximation fits the abundance of cluster-mass halos very well, with top-hat $\delta_{c,1}=1.57\pm1.73$ and Gaussian $\delta_{c,G}=-1.27\pm1.35$. However, it overpredicts the number density of galaxy and small group mass objects by a factor of $\sim 2$, only weakly dependent on cosmology, and very weakly dependent on $\delta$. On mass scales of $\sim 5\times 10^{12}\ M_{\odot}$, it is not possible to compensate for the discrepancy by adjusting $\delta$ within reasonable bounds.

In summary, we conclude that none of the models we have investigated can be strongly ruled out by the kind of analysis performed here. The CHDM-2 model gives the best overall agreement with the linear and non-linear tests we have considered here, assuming that galaxies are positively biased with respect to the dark matter. Gawiser & Silk (1998) have shown that a similar CHDM model with $\Omega_m = 0.2$ in $\nu = 1$ neutrino species is a much better fit to microwave background and galaxy distribution data than any other popular cosmological model. Preliminary analysis based on the dark matter alone has shown that the related CHDM-2r model considered in this paper is plausibly consistent with high redshift observations of Lyman-break galaxies (Wechsler et al. 1998) and damped Lyman-α systems (Klypin et al. 1995a), but it remains to be seen whether this model will produce enough early galaxy formation once a more realistic treatment of gas processes and star formation is included. More detailed modelling of galaxy formation will also be necessary to determine whether the small-scale clustering properties of the low-$\Omega_0$ models are indeed inconsistent with the observations. In any case we conclude that models with $\Omega_0 \sim 0.5$ are in better overall agreement with the observations than the lower values ($\Omega_0 \sim 0.2-0.3$) usually considered (e.g. Jenkins et al. 1997). A powerful constraint on $\Omega_0$, the evolution of cluster abundance with redshift, will be considered in a companion paper (Gross et al. 1998).

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