Vehicle Path Tracking Maneuver Based on Model Predictive Control Theory

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Abstract. Vehicle path tracking problem has been a hot research topic in the field of automotive safety. Therefore, combined with a 3-DOF vehicle model is established. The basic idea behind the work was to identify the optimal steering angle input during a vehicle travels along a prescribed path. Based on Model Predictive Control (MPC), the steering angle input was determined as the control variable, tracking the desired path was determined as control object. By using the Model Predictive Control, the optimal control problem was converted into a secondary plan problem which was then solved by the active set method. The results show that the minimum error of lateral position for the generated path-tracking trajectory can be good indicators of successful solving of the path-tracking problem in vehicle handling inverse dynamics for MPC. The study can help drivers identify safe lane-keeping trajectories and area easily as well as evaluating performance of emergency collision-avoidance for a vehicle.

1. Introduction
Vehicle driving safety is the urgent key problem to be solved of automobile independent development while encountering emergency collision avoidance with high speed. The growing mobility of people and goods has a very high societal cost in order to protect people from vehicle accidents. Several studies show that the driver is responsible for most accidents, which occur mainly due to distraction and wrong perception and judgment of the traffic and environmental situations around the vehicle. At present, people are working on obstacle avoidance through braking and lane change maneuver solving the handling dynamics problem of vehicle path tracking operation [1, 2].

Model Predictive Control (MPC) is a control algorithm based on predictive model. This predictive model can represent the main control performance parameters of the controlled object. It can judge the future input and output of the controlled object according to the process information. The parallel advantages of theory and computing systems can extend the range of applications for real-time of MPC [3, 4]. MPC can solve the constraint problem of control variable well. Literature [5, 6], and applied MPC to vehicle path tracking and achieved good results [7]. In the paper model of vehicle path tracking maneuver is established and analyzed based on the MPC algorithm.
2. Model of Vehicle Path Tracking Problem

It is assumed that the longitudinal force acting on the front wheels is small, and the influence on the tyre cornering characteristics affected by ground tangential force is ignored with tyre cornering characteristics in linear range. The vehicle movement can be simplified as a 3-DOF vehicle model which is depicted in Fig. 1.

In state space form it is:

\[
\begin{align*}
\dot{x} &= m\ddot{\phi} - 2F_yF_x + 2F_xF_y \\
\dot{y} &= -m\ddot{\phi} + 2F_yF_x + 2F_xF_y \\
\dot{\phi} &= I_z\ddot{\phi} - 2l_yF_y - 2l_yF_y
\end{align*}
\]

(1)

Where \( m \) is the vehicle mass; \( I_z \) is the moment of inertia around the z axis; \( l_y, l_y \) are the distances of front and rear axles from the center of gravity; \( \phi \) is the yaw rate of the vehicle; \( F_x, F_y \) are the traction or brake forces of front and rear wheels.

To calculate the vehicle positions in the Earth coordinate defined by \( X \) and \( Y \), the vehicle velocity in the body coordinate is projected in the Earth coordinate as:

\[
\begin{align*}
X &= x\cos\phi - y\sin\phi \\
Y &= x\sin\phi + y\cos\phi
\end{align*}
\]

(2)

The longitudinal and lateral speed of the tires are \( v_x \) and \( v_y \).

\[
\begin{align*}
v_x &= v_y\sin\delta + v_y\cos\delta \\
v_y &= v_x\cos\delta - v_x\sin\delta
\end{align*}
\]

(3)

Where \( \delta \) is the front steering angle.

It is assumed that the vehicle speed does not change much, and the load transfer of the front and rear axles is ignored, then the vertical loads of the front and rear wheels are:

\[
F_{yf} = \frac{l_ymg}{2(l_y + l_y)}, \quad F_{yr} = \frac{l_ymg}{2(l_y + l_y)}
\]

(4)

The vehicle nonlinear dynamics model can be described as the following state space expression:

\[
\frac{dx(t)}{dt} = f(x(t), u(t)), \quad \eta(t) = h(x(t))
\]

(5)

According to the research content, the paper adopts the semi-empirical model ie Magic Formula (MF). The general expression of the Magic formula is:

\[
F(\tau) = D\sin(C\arctan(B\phi(\tau))) + S
\]

(6)
Where $F$ is the longitudinal or lateral force; $\tau$ represents the longitudinal slip ratio or side angle of the tire; $B$ is the stiffness factor; $C$ is the shape factor of the curve; $D$ is the peak factor of the curve representing the maximum value of the curve; $E$ is the curvature factor of the curve.

Where $x_{\text{dyn}}, u_{\text{dyn}}$ and $\eta_{\text{dyn}}$ are the state, control and the output which are denoted respectively as $x_{\text{dyn}} = [\phi, \omega, y, x]^T$, $u_{\text{dyn}} = [\delta]$ and $\eta_{\text{dyn}} = [\phi, y]^T$.

3. Design of MPC Controller

3.1. Linear steering model

The lateral distance $y_c$ can be described as Eq. 7.

$$y_c = \int v(\beta + \varphi) dt = \int (\beta + \int \omega, dt) dt$$

(7)

Where $\beta$ is the side slip angle; $\varphi$ is the yaw angle.

The state equation of the vehicle motion can be expressed as:

$$\frac{d}{dt} \begin{bmatrix} \beta \\ \varphi \\ y_c \\ \delta \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ v & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \beta \\ \varphi \\ y_c \\ \delta \end{bmatrix} + \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix}$$

(8)

Where

$$a_{11} = \frac{k_1 + k_2}{mu}; \quad a_{12} = \frac{k_1a - k_2b}{mu^2}; \quad a_{21} = \frac{k_1a - k_2b}{l_z};$$

$$a_{22} = \frac{(k_1a^2 + k_2b^2)}{l_zu}; \quad b_{21} = -\frac{k_1a}{l_z}; \quad b_{11} = -\frac{k_1}{mu}.$$  

Where $k_1$ and $k_2$ are the synthesized stiffness of front and rear tires.

3.2. MPC controller

Combined with Eq.(8), the output equation can be expressed as:

$$y(t) = \begin{bmatrix} y_c \\ F_c \\ \delta \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -C & C & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \beta \\ \varphi \\ y_c \\ \delta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

(9)

The vehicle dynamics model should be discretized firstly. The cost function designed by the MPC controller should ensure that the vehicle tracks the desired trajectory quickly and accurately. So it is necessary to control the deviation of the state variable of the system and optimize the control variable. It is assumed that the given expected trajectory and yaw angle functions are $y_{\text{ref}}, \varphi_{\text{ref}}$ respectively. Then the expected output of each predicted time domain is $\eta_{\text{dyn,ref}} = [\varphi_{\text{ref}}, y_{\text{ref}}]$. So the cost function can be expressed as:

$$J(k) = \sum_{k=1}^{N_p} ||\eta_{\text{dyn}}(t+k | t) - \eta_{\text{dyn,ref}}(t+k | t)||^2_Q + \sum_{k=1}^{N_p} ||AU_{\text{dyn}}(t+k | t)||^2_R + \rho \varepsilon^2$$

(10)

The first item on the right side of the equal sign indicates the tracking ability of the system to the reference trajectory. The second item indicates the requirement for the change of the control variable. And the third item can ensure that the cost function has a feasible solution in each prediction range. $N_p$ is the prediction horizon; $U_{\text{dyn}}=[\delta]$; $Q, R$ are the control horizon; $\rho$ is the weight coefficient; $\varepsilon$ is the relaxation factor ($\varepsilon > 0$).
Affected by external factors such as the vehicle performance and road adhesion coefficient, only a certain control variable limit constraint and increment constraint can be satisfied in the control process to ensure the vehicle completes the path tracking process quickly and smoothly. Therefore, the front wheel angle needs to be constrained, and the expression form of the constraint for control variable is:

\[ U_{\text{min}}(t+k) \leq U(t+k) \leq U_{\text{max}}(t+k), k = 0, 1, \ldots, N_c - 1 \]  

(11)

The increment constraint of the control variable is:

\[ \Delta U_{\text{min}}(t+k) \leq \Delta U(t+k) \leq \Delta U_{\text{max}}(t+k), k = 0, 1, \ldots, N_c - 1 \]  

(12)

When the vehicle is at low speed, the kinematic constraints have a greater impact on control effect. And as the vehicle speed increases, the influence of dynamic characteristics on motion planning and control effect becomes more obvious.

The control input of the actuator at high speed and the slip caused by friction between the tire and the ground as well as the dynamic nonlinear constraints such as the roll caused by lateral acceleration are more severe than those at low speed. In this case, constraining the dynamic model can further ensure the safety and driving stability of the vehicle during the path tracking process at high speed. The dynamic constraints include three aspects, the side slip angle, the adhesion coefficient condition and the angular velocity of front wheel, which can be expressed as:

\[ -12^\circ < \beta < 12^\circ \text{ (Good road surface)} \]

\[ -2^\circ < \beta < 2^\circ \text{ (Ice-snow road surface)} \]

\[ a_{y,\text{min}} \leq a_y \leq a_{y,\text{max}} \quad |\dot{\delta}| \leq 0.436 \text{ rad/s} \]

Where \( a_{y,\text{min}}, a_{y,\text{max}} \) are the maximum and minimum values of the acceleration.

Based on the above objective function and constraints, the optimization problem based on model predictive control in each control cycle can be expressed as:

\[ \min J(k) \]  

(13)

After completing the solution of Eq. 13 and the limitation of constraints in each control cycle, a series of control input increments are obtained. And the first input in each cycle control sequence is taken as an actual control input increment. After the obtained input enters the next control cycle, the above control process is repeated cyclically, and tracking control of the reference trajectory can be realized. Then the path tracking problem at high speed is transformed into a quadratic programming problem. The effective quadratic solution is used to solve the quadratic programming problem after transformation [8].

4. Numerical Simulation

In order to verify the effectiveness of the proposed method the paper uses Carsim and Simulink joint simulation to solve the problem. And in order to verify the control effect of the controller, this paper selects the vehicle model that comes with Car-Sim software as the reference model.

The double lane change road which is shown in Fig. 2 is treated as the given path for the path tracking problem.
Figure 2. Double lane change road.

It is set that control step is 0.01s and the control horizon is 5. The weight matrix is set as:

\[
Q = \begin{bmatrix}
200 & 0 & 0 \\
0 & 100 & 0 \\
0 & 0 & 100
\end{bmatrix}, \quad R = 5 \times 10^4, \quad \rho = 1000.
\]

For the simulation, the calculation parameters are shown in Table 1.

Table 1. Simulation parameters.

| Parameter | Value |
|-----------|-------|
| m/ kg     | 1818.2|
| I_z/ kg.m^2 | 3885  |
| l_a/ m    | 1.463 |
| l_b/ m    | 1.585 |
| k_1/N.rad^{-1} | -62618 |
| k_2/N.rad^{-1} | -110185 |
| \phi      | 0.8   |

Figs. 3-7 are simulation results of lateral displacement, absolute error of the lateral displacement, steering rate of the steering wheel, yaw rate and lateral acceleration of the vehicle respectively when \(N_p=12\).
It can be seen from Fig. 3 that during the entire path tracking process, the driving trajectory of the vehicle is in good agreement with the given double lane change trajectory, which indicates that the vehicle can travel according to the given path well.

Fig. 4 shows the absolute error of the vehicle travel trajectory deviating from the given path. It can be seen from the figure that there are two large errors occur at longitudinal displacements of 60m and 140m, that is, the absolute errors of the vehicle deviating from the given path are bigger when the vehicle enters and drive out of the double lane change road.

Fig. 5 is the simulation result of the steering rate of the steering wheel. From the figure it can be seen that the steering rate is larger at the longitudinal displacements of 70m and 120m respectively, which indicates that the driver is busy, and the driver is prone to fatigue and tension, so the driver should control the steering wheel well.

Figs. 6-7 show the simulation results of yaw rate and lateral acceleration, respectively. It can be seen from Fig.6 that the vehicle yaw rate has peak values at longitudinal displacements of 50m, 80m, 120m and 150m respectively. At this time, the driver should control the speed of the vehicle and the steering angle to track the given path. At the same time, in the longitudinal displacements of 50m, 80m, 120m and 150m, the lateral acceleration has peak values. At this time, the driver should control the steering wheel to prevent dangerous situations such as tail and even rollover in the path track process.

5. Conclusion

The path tracking scenario is analyzed while the MPC method is utilized to identify steering rate input for a vehicle tracking the desired path. Accordingly, a 3-DOF simplified vehicle model has been used to describe the motion of the vehicle. Then the problem of the path tracking maneuver is formulated as an optimal control problem. Finally, the optimal control problem is solved by active set method. The simulation results show that MPC method can be applied to solve the path tracking problem and achieve better control results.

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