Possibility of forming a stable Bose-Einstein condensate of $2^3S_1$ positronium atoms

Y. Zhang$^{1,2}$, M.-S. Wu$^1$, J.-Y. Zhang$^{1,3}$, Y. Qian$^4$, X. Gao$^3$, and K. Varga$^5$

$^1$ State Key Laboratory of Magnetic Resonance and Atomic and Molecular Physics, Innovation Academy for Precision Measurement Science and Technology, Chinese Academy of Sciences, Wuhan 430071, China
$^2$ School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 100049, China
$^3$ Beijing Computational Science Research Center, Beijing 100193, China
$^4$ Department of Computer Science and Technology, East China Normal University, Shanghai 200062, China
$^5$ Department of Physics and Astronomy, Vanderbilt University, Nashville, Tennessee 37235, USA

(Dated: June 14, 2022)

The confined variational method in conjunction with the orthogonalizing pseudo-potential method and the stabilization method is used to study the low energy elastic scattering between two spin-polarized metastable positronium Ps($2^3S_1$) atoms. Explicitly correlated Gaussian basis functions are adopted to properly describe the complicated Coulomb interaction among the four charged particles. The calculated $s$-wave scattering length ($\approx 8.5 a_0$) is positive, indicating the possibility of forming a stable Bose-Einstein condensate of fully spin-polarized Ps($2^3S_1$) atoms. Our results will open a new way of experimental realization of Ps condensate and development of $\gamma$-ray and Ps($2^3S_1$) atom lasers.

PACS numbers: 34.80.Bm, 34.80.Uv, 03.65.Nk

The positronium (Ps) atom, a hydrogen-like bound system of an electron and a positron, has two ground states, the singlet $1^3S_0$ state and the triplet $1^3P_1$ state, known respectively as para-Ps (p-Ps) and ortho-Ps (o-Ps). $P$-Ps has a lifetime of 0.125 ns [1] and decays into two gamma photons, and o-Ps has a lifetime of 142 ns [2] and decays into three gamma photons. The longer lived o-Ps atoms were amongst the first candidates [3] for achieving Bose-Einstein condensation (BEC), a phase transition of a boson gas where a macroscopic number of bosons occupy a same quantum state below a critical temperature. BEC is one of the most interesting phenomena in quantum systems of bosons that has important applications such as in testing the weak equivalence principle and in studying gravitational effects on quantum systems [4]. Historically, the first BEC was realized in an ensemble of Rb atoms in 1995 [5], which opened a new era of ultracold physics. For o-Ps, the smallness of its mass allows for much higher BEC temperature of 20-30 K than ordinary atoms around 200 nK [6, 7]. However, a realization of BEC of o-Ps atoms has been hindered by its very short lifetime.

The formation and observation of a Ps BEC has been of extraordinary interest though it is very challenging. Since Platzman and Mills suggestion of a possible way of creating an o-Ps BEC in 1994 [3], some significant progress has been made both theoretically and experimentally. Low energy scattering between two ground-state Ps atoms has been extensively studied for calculating scattering cross sections and the $s$-wave scattering lengths [8–12]. These quantities are critical for determining possibility of forming a stable ground-state Ps BEC and for designing experimental configurations. In order to probe Ps densities, some low-energy scattering properties of the ground- and $2s$-state of Ps have been computed using hyperspherical coordinates [13]. For modeling a BEC process of o-Ps atoms confined in a porous silica material, Morandi et al. [14] showed that the condensation process is compatible with the o-Ps lifetime, which strongly depends on the external electromagnetic field [15]. There are also some theoretical works on $\gamma$-ray laser [16, 17] and spinor dynamics [18, 19] based on BEC of Ps atoms. From experimental side, significant progress has been made in the area of Ps-laser physics due to the breakthrough development of the Surko type buffer gas positron trap [20–22]. Recently, implanting high density bursts of polarized positrons into a porous silica film in a high magnetic field, Cassidy et al. produced a highly spin-polarized (96%) o-Ps gas [23]. Moreover, the suppression of the Zeeman mixing of the $2^1P$ and $2^3P$ states of Ps observed in high magnetic fields [24] makes laser cooling of Ps feasible [25–28]. Typically, Ps atoms produced in most porous materials can approach room temperature at the currently highest achievable density $10^{16}$ cm$^{-3}$ [23]. To form a Ps BEC, one needs not only increase the Ps density but also significantly reduce the temperature of Ps gas. However, the short lifetime of o-Ps seriously limits application of advanced cooling techniques, such as laser cooling, developed for ordinary atoms [29]. So far, o-Ps has been the only focus of all Ps-BEC related studies, although some experimental and theoretical researches have been conducted on the longer-lived metastable and Rydberg states of Ps [30–38].

In this work, we will explore an alternative possibility of forming a BEC using metastable Ps$^*$($2^3S_1$) atoms. In the following, the notation Ps$^*$($2^3S_1$) is abbreviated as Ps$^*$. The Ps$^*$ has a lifetime of 1136 ns that is eight times as long as the lifetime of o-Ps [2]. By calculating
the s-wave scattering length for the spin-aligned Ps*-Ps* elastic scattering that governs the interaction between Ps* atoms at low temperatures, we will see whether it is a positive value, which is a key factor for forming a stable BEC. Ps*-atom scattering problem is one of the most difficult problems in atomic collision theory because both projectile and target are composite objects with their internal structures. One has to deal with multi-center integrals of interaction matrix elements. A further complication to these calculations lies in the fact that both colliding Ps* atoms are in the excited 2 S 1 state so that one should have basis functions to be able to accurately describe both short- and long-range (van der Waals) interactions, in particular for low energy scattering.

Based on the existing computational techniques [39, 40], a novel method i.e. the confind variation method (CVM) has been proposed recently [41, 42] for studying low-energy elastic scattering between a simple or composite projectile with an atom. The CVM combined with the orthogonalizing pseudo-potential (OPP) method [43–46] will be applied to study the s-wave elastic scattering between two spin-aligned Ps* atoms. The principal result of this work is that, for the first time, we have established a definitive value of the s-wave scattering length for the spin-aligned Ps* atoms. The principal result of this work is that, for the first time, we have established a definitive value of the s-wave scattering length that has positive sign, indicating that a stable BEC of spin-aligned Ps* atoms can be formed.

Theory for the spin-aligned Ps*-Ps* scattering.—The nonrelativistic Hamiltonian for the four-body system of (e+e+e−e−) can be written in the form (in atomic units)

\[
H = -\sum_{i=0}^{3} \frac{\nabla r_i^2}{2} + \frac{1}{|r_0 - r_1|} - \frac{1}{|r_0 - r_2|} - \frac{1}{|r_0 - r_3|} - \frac{1}{|r_1 - r_2|} - \frac{1}{|r_1 - r_3|} + \frac{1}{|r_2 - r_3|},
\]

(1)

where \(r_0\) and \(r_1\) are the two-positron position vectors, and \(r_2\) and \(r_3\) are the two-electron position vectors. For calculating the s-wave elastic scattering of Ps*-Ps*, we use the OPP method [43–46] to prevent any electron-positron pair from forming the ground state Ps(1 S0). The OPP operator is constructed by summing over the Ps(1 S0) projection operators

\[
\lambda \hat{P} = \lambda \sum_{i=0}^{3} \sum_{j=2}^{3} \hat{P}_{ij}
\]

(2)

where \(\lambda\) is a large positive number and \(\phi_{1 S_0}(r_i - r_j)\) is the wave function of Ps(1 S0). Since a wave function with a nonzero overlap with the Ps(1 S0) orbital tends to increase the energy, an eigenfunction of \(H + \lambda \hat{P}\) for a low energy level will have a very small overlap with \(\phi_{1 S_0}\). The OPP method was first introduced by Krasnopolsky and Kukulin [43] in 1974. Mitroy and Ryzhikh performed a comprehensive numerical investigation on the effects due to different \(\lambda\) and different sizes of basis sets [46] and found that the energies calculated with OPP will converge to those of the \(\bar{QHG}\) Hamiltonian in the projection operator method [47], a method that has been widely used in studying atomic and molecular resonant and excited states. Compared to the \(\bar{QHG}\) method, the OPP method is easier to apply for scattering problems.

The (e+e+e−e−) system possesses rich symmetries including the electron interchange symmetry, the positron interchange symmetry, the inversion parity, and the charge parity. These symmetries can be described by a permutation group isomorphic to the molecular point group \(D_{2h}\). A detailed analysis of the symmetries is presented in Ref. [48]. For the fully spin-aligned Ps*-Ps* scattering, the total spin operators \(S^2\) and \(S_z\) have good quantum numbers \(S = S_z = 2\), and this scattering state can be classified according to the irreducible representations of the \(D_{2h}\) group as \(B_1\) symmetry. In the calculation of the s-wave elastic scattering, even parities are used for both inversion and charge conjugation. After taking these symmetries into account, the total symmetry projector applied to the spacial part of the basis function is \((1 + P_{02}P_{13})(1 - P_{01})(1 - P_{23})\), where \(P_{ij}\) is the permutation of the spatial coordinates of particles \(i\) and \(j\).

The Hamiltonian operator which is evaluated in the variational calculation usually commutes with all the permutation operators. Therefore we can perform a convenient implementation where all the permutational operators are applied to the ket

\[
\langle \psi | \hat{H} | \psi \rangle = \langle \psi | \hat{H} | \mathcal{P} | \psi \rangle,
\]

(3)

In the OPP method, the projection operators do not commute with all the permutation operators. The total symmetry projector \(\mathcal{P} = (1 + P_{02}P_{13})(1 - P_{01})(1 - P_{23})\), adapting wave function to the correct \(B_1\) symmetry actually, should be applied on both bra and ket. The matrix elements of Hamiltonian operator and OPP operator are written as

\[
\hat{H}_{ij} + \lambda \hat{P}_{ij} = \langle \psi_i | \hat{H} + \lambda \hat{P} | \psi_j \rangle,
\]

(4)

which makes convergence rather slower. Meanwhile the OPP operator \(\lambda \hat{P}\) could result in linear dependence problems. Variational calculation became numerically unstable with respect to further enlargement of the ECG basis.

Of crucial importance to this work is the use of explicitly correlated Gaussians (ECGs) [39, 49–51] to describe the Coulomb interaction between the charged particles. An ECG basis can not only describe the correlations among the charged particles, but also allows us to evaluate the Hamiltonian matrix elements analytically. After separating out the center-of-mass motion from the
required by the CVM to be weak at the boundary

$$\rho \rightarrow 0 \quad (i \geq 1)$$

where $$x_i = r_i - r_0$$. The independent parameters $$A_{ij}$$ contained in symmetric matrices $$A^n$$ are optimized through the energy minimization using the confined variational method (CVM). The CVM is simple and powerful in the sense that it converts a problem of continuum states to a problem of bound states by adding a confining potential $$\chi_{cp}(\rho)$$ to the Hamiltonian. This method provides a framework for optimizing wave functions in the interaction region using bound-state techniques. The advantages of using the CVM have been demonstrated by solving some long-term intractable problems, including the $$e^+ e^- e^- e^-$$ scattering and Ps-H$_2$ scattering, where the calculated annihilation parameters are, for the first time, in agreement with precise experimental values [52, 53]. In this work, the confining potential is chosen to be

$$\chi_{cp}(\rho) = 0, \quad \rho < R_0,$$

$$\chi_{cp}(\rho) = G(\rho - R_0)^2, \quad \rho \geq R_0,$$

where $$\rho$$ is the distance between the two centers of mass of two electron-positron pairs, and $$G$$ is a small positive number. We set $$R_0 = 50 a_0$$ because the long-range interaction $$V_L = -C_6/\rho^6$$, where $$C_6 = 27320$$ a.u. [54], is required by the CVM to be weak at the boundary $$R_0$$. Due to the exchange symmetries between the identical particles and their indistinguishability, the Schrödinger equation for the confined Ps$^*$-Ps$^*$ system can be written in the form

$$[H + \lambda \hat{P} + \chi_{cp}(\rho_1) + \chi_{cp}(\rho_2)] \Psi(x) = E \Psi(x),$$

where $$\rho_1 = |x_1 + x_2 - x_3|/2$$, $$\rho_2 = |x_1 - x_2 + x_3|/2$$, and $$x$$ stands for $$(x_1, x_2, x_3)$$ collectively. Besides the ECG basis for the short-range interaction region, as a supplement a set of exterior basis functions are designed to describe the long-range interaction between the two Ps$^*$, as listed below

$$\Phi_{ext} = \exp(-\frac{1}{2} \alpha_i \rho_1) \phi_{Ps^*}(x_3) \overline{\phi_{Ps^*}}(x_1 - x_2),$$

where $$\phi_{Ps^*}$$ is the Ps$^*$ wave function written as a linear combination of 20 ECGs that give rise to an energy eigenvalue of $$-0.06249999999$$ a.u. very close to the exact value of $$-0.0625$$ a.u. for $$2^3S_1$$. A total of 10 even-tempered exponents $$\alpha_i$$ are generated using $$\alpha_i = \alpha_1/T^{i-1}$$ with $$\alpha_1 = 0.0001$$ and $$T = 1.78$$.

The phase shifts were extracted from wave functions using the stabilization method [40]. After omitting the confining potential in Eq (8), the Schrödinger equation was solved in the full interior and exterior regions to generate a set of positive energy pseudostates. The phase shift was derived by fitting the density distribution $$C(\rho)$$ to the asymptotical density distribution in the range $$\rho \in [40a_0, 45a_0]$$, where $$C(\rho)$$ is defined as

$$C(\rho) = \int dx_1 \int dx_2 \int dx_3 \delta((x_1 + x_2 - x_3)/2 - \rho) \times |\Psi(x_1, x_2, x_3)|^2.$$
TABLE II. Wave numbers \( k_i \), corresponding phase shifts \( \delta_{ki} \), and determined \( s \)-wave scattering length \( A_0 \) for the fully spin-aligned \( \text{Ps}^-\text{Ps}^- \) elastic scattering, with the OPP parameter \( \lambda = 1000 \). In the first column, the first entry is the dimension of the interior region basis and the second entry is for the exterior basis.

| \( N \)       | \( k_1(a_0^{-1}) \) | \( k_2(a_0^{-1}) \) | \( k_3(a_0^{-1}) \) | \( k_4(a_0^{-1}) \) | \( k_5(a_0^{-1}) \) | \( \delta_{k_1} \) (rad) | \( \delta_{k_2} \) (rad) | \( \delta_{k_3} \) (rad) | \( \delta_{k_4} \) (rad) | \( \delta_{k_5} \) (rad) | \( A_0(a_0) \) | \( r_0(a_0) \) |
|-------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|-------------|-------------|
| 3000+10     | 0.0402           | 0.0855           | 0.1327           | 0.1608           | 0.1842           | -0.3408          | -0.7942          | -1.385           | -1.781           | -2.042           | 8.0         | 6.1         |
| 4000+10     | 0.0361           | 0.0768           | 0.1185           | 0.1312           | 0.1665           | -0.3043          | -0.7651          | -1.204           | -1.365           | -1.757           | 8.2         | 5.6         |
| 5000+10     | 0.0330           | 0.0699           | 0.1085           | 0.1187           | 0.1520           | -0.2751          | -0.6823          | -1.094           | -1.235           | -1.469           | 8.3         | 5.3         |

Ps\(^{-}\)-BEC but also the dynamics of phase transition can be investigated using the mean-field theory \([63]\). The necessary conditions for a realization of Ps\(^{-}\)-BEC are as follows. Firstly, one should be able to produce a sufficiently high number of polarized Ps\(^+\) atoms in a confined void. Secondly, the Ps\(^+\) gas has to be cooled down to sufficiently low temperature. For cooling it can be achieved by thermalization through collisions of Ps\(^+\) with the walls of the void, Ps\(^+\)-Ps\(^+\) scattering, laser cooling, and other cooling methods. There has been a long interest in producing Ps\(^+\) \([64–66]\) due to their potential applications \([37]\) in testing quantum electrodynamics (QED), in atom interferometry, and in gravitational interaction of antimatter. The available techniques of Ps\(^+\) production include radio frequency transition from laser-excited Ps\((2\,^3\text{P})\) in a weak magnetic field \([64]\), two-photon Doppler-free Ps\((1\,^3\text{S})\)-Ps\((2\,^3\text{S})\) laser excitation \([65, 66]\), single-photon excitation of Ps\((1\,^2\text{S})\) to Ps\((2\,^3\text{P})\) in an electric field \([34]\), and radiative decay of Ps\((3\,^3\text{P})\) generated by single-photon excitation of Ps\((1\,^3\text{S})\) \([35, 36, 38]\). In particular, the efficiency of Ps\(^+\) production has recently been increased to 30\% by stimulating the Ps\((3\,^3\text{P})\)-Ps\(^+\) transition using a laser pulse, and further improvement in efficiency is still possible \([38]\).

The Ps\(^+\)-BEC can be applied to study fundamental physics and create new technologies once it is formed. Ideally, it is possible to realize the transformation from Ps\(^-\)-BEC to o-Ps-BEC through the stimulated transition from Ps\(^+\) to o-Ps. It is also possible to produce gamma-ray laser through the stimulated transition from Ps\(^+\) to p-Ps followed by the corresponding two-photon annihilation. Moreover, a coherent beam of Ps\(^+\) atoms, the so-called Ps\(^+\) atom laser, can be generated from a Ps\(^+\) BEC. Employing a Ps\(^+\) atom laser as the Ps\(^+\) source will significantly improve the accuracy of measurements on matter-antimatter gravitational interaction and on Ps precision spectroscopy \([67, 68]\) which have been proposed to test QED and physics beyond the Standard Model \([69, 70]\) such as the dark matter, and it will be of great benefit to producing cold antihydrogen atoms \([71, 72]\). Furthermore, a coherent beam of Ps\(^+\) atoms as a tool will enrich Ps chemistry to study various interactions with other atoms and molecules.

Summary.— In this work, the near-zero-energy \( s \)-wave elastic scattering between two fully-spin-aligned Ps\(^+\) has

FIG. 1. \( s \)-wave phase shift Cot(\( \delta_k \)) for the fully spin-aligned Ps\(^+\)-Ps\(^+\) scattering as a function of wave number \( k \) computed with three different values of the OPP parameter \( \lambda \). The lines represent effective-range fits to the phase shifts using Eq. (11).

TABLE II. \( s \)-wave scattering length for the fully spin-aligned Ps\(^+\)-Ps\(^+\) elastic scattering obtained with different values of the OPP parameter \( \lambda \).

| \( \lambda \) | 1000 | 3000 | 5000 |
|--------------|------|------|------|
| \( A_0(a_0) \) | 8.3  | 8.4  | 8.5  |
| \( r_0(a_0) \) | 5.3  | 5.4  | 5.7  |
been studied using a combined approach of OPP method, CVM, and stabilization method. The calculated s-wave scattering length represents the first determination of this quantity. The positive value of the scattering length (≈ 8.5a₀) is particularly significant since it demonstrates the feasibility of forming a stable BEC of fully-spin-aligned Ps⁺ atoms and hence it becomes possible for developing γ-ray and Ps⁺ lasers based on Ps⁺-BEC.

Acknowledgments—J. Y. Z acknowledges S. Yi for valuable discussion and hospitality during his visit at the Institute of Theoretical Physics, Chinese Academy of Sciences. We would also like to thank Z.-C. Yan and W.-M. Liu for their helpful discussion. J. Y. Z. was supported by the Hundred Talents Program of the Chinese Academy of Sciences. X. G. is supported by the National Natural Science Foundation of China (Grant Nos. 11774023 and 11861121001). The research is supported by the Key Research and Development Program of China (Grant No. 2016YFA0302104).

* Email address: jzhang@apm.ac.cn

[1] P. A. M. Dirac, Proc. Camb. Philos. Soc. 26, 361 (1930).
[2] A. Ore and J. L. Powell, Phys. Rev. 75, 1696 (1949).
[3] P. M. Platzman and A. P. Mills, Phys. Rev. B 49, 454 (1994).
[4] D. E. Bruschi, C. Sabini, A. White, V. Baccetti, D. K. L. Oi, and I. Fuentes, New J. Phys. 16, 053041 (2014).
[5] M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell, Science 269, 198 (1995).
[6] M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell, 269, 198 (1995).
[7] C. C. Bradley, C. A. Sackett, J. J. Tollett, and R. G. Hulet, Phys. Rev. Lett. 75, 1687 (1995).
[8] I. A. Ivanov, J. Mitroy, and K. Varga, Phys. Rev. Lett. 87, 063201 (2001).
[9] J. Shumway and D. M. Ceperley, Phys. Rev. B 63, 165200 (2001).
[10] K. Oda, T. Miyakawa, H. Yabu, and T. Suzuki, J. Phys. Soc. Jpn 70, 1549 (2001).
[11] I. A. Ivanov, J. Mitroy, and K. Varga, Phys. Rev. A 65, 022704 (2002).
[12] K. M. Daily, J. von Stecher, and C. H. Greene, Phys. Rev. A 91, 012512 (2015).
[13] M. D. Higgins, K. M. Daily, and C. H. Greene, arXiv:1904.04295 (2019).
[14] O. Morandi, P.-A. Hervieux, and G. Manfredi, Phys. Rev. A 89, 033609 (2014).
[15] N. Cui, M. Macovei, K. Z. Hatsagortsyan, and C. H. Keitel, Phys. Rev. Lett. 108, 243401 (2012).
[16] H. K. Avetissian, A. K. Avetissian, and G. F. Mkrtchian, Phys. Rev. Lett. 113, 023904 (2014).
[17] H. K. Avetissian, A. K. Avetissian, and G. F. Mkrtchian, Phys. Rev. A 92, 023820 (2015).
[18] Y.-H. Wang, B. M. Anderson, and C. W. Clark, Phys. Rev. A 89, 043624 (2014).
[19] C. Zheng, P. Zhang, and S. Yi, Commun. Theor. Phys. 68, 236 (2017).
[20] C. M. Surko, M. Leventhal, and A. Passner, Phys. Rev. Lett. 62, 901 (1989).
[21] T. J. Murphy and C. M. Surko, Phys. Rev. A 46, 5696 (1992).
[22] J. R. Danielson, D. H. E. Dubin, R. G. Greaves, and C. M. Surko, Rev. Mod. Phys. 87, 247 (2015).
[23] D. B. Cassidy, V. E. Meligne, and A. P. Mills, Phys. Rev. Lett. 104, 173401 (2010).
[24] D. B. Cassidy, T. H. Hisakado, H. W. K. Tom, and A. P. Mills, Phys. Rev. Lett. 106, 173401 (2011).
[25] E. P. Liang and C. D. Dermer, Opt. Commun. 65, 418 (1988).
[26] H. Iijima, T. Hirose, M. Irako, M. Kajita, T. Kunitsa, H. Yabu, and K. Wada, J. Phys. Soc. Jpn 70, 3255 (2001).
[27] T. Hirose, T. Asonuma, H. Iijima, M. Irako, K. Kadoya, T. Kunitsa, B. Matsumoto, N. N. Mondal, K. Wada, and H. Yabu, Laser Cooling of Ortho-Positronium: Toward Realization of Bose-Einstein Condensation (2014).
[28] K. Shu, X. Fan, T. Yamazaki, T. Namba, S. Asai, K. Yoshioka, and M. Kuwata-Gonokami, J. Phys. B 49, 104001 (2016).
[29] D. B. Cassidy, Eur. Phys. J. D 72, 53 (2018).
[30] K. P. Ziock, R. H. Howell, F. Magnotta, R. A. Failor, and K. M. Jones, Phys. Rev. Lett. 64, 2366 (1990).
[31] J. Estrada, T. Roach, J. N. Tan, P. Yesley, and G. Gabrielse, Phys. Rev. Lett. 84, 859 (2000).
[32] F. Castelli, I. Boscolo, S. Cialdi, M. G. Giannamarchi, and D. Comparat, Phys. Rev. A 78, 052512 (2008).
[33] D. B. Cassidy, T. H. Hisakado, H. W. K. Tom, and A. P. Mills, Phys. Rev. Lett. 108, 043401 (2012).
[34] A. M. Alonso, S. D. Hogan, and D. B. Cassidy, Phys. Rev. A 95, 033408 (2017).
[35] S. Aghion et al. (AEgIS Collaboration), Phys. Rev. A 98, 013402 (2018).
[36] C. Amsler et al. (AEgIS Collaboration), Phys. Rev. A 99, 033405 (2019).
[37] D. B. Cassidy, H. W. K. Tom, and A. P. Mills, AIP Conf. Proc. 1037, 66 (2008).
[38] M. Antonello et al. (AEgIS Collaboration), arXiv:1904.09004 (2019).
[39] Y. Suzuki and K. Varga, Stochastic Variational Approach to Quantum-Mechanical Few-Body Problems (Springer, New York, 1998).
[40] J. Y. Zhang and J. Mitroy, Phys. Rev. A 78, 012703 (2008).
[41] J. Mitroy, J. Y. Zhang, and K. Varga, Phys. Rev. Lett. 101, 123201 (2008).
[42] J. Y. Zhang, J. Mitroy, and K. Varga, Phys. Rev. A 78, 042705 (2008).
[43] V. M. Krasnopol’skij and V. I. Kulikin, Yad. Fiz. 20, 883 (1974).
[44] A. K. Bhatia, A. Temkin, and J. F. Perkins, Phys. Rev. 153, 177 (1967).
[45] A. K. Bhatia and A. Temkin, Phys. Rev. A 11, 2018 (1975).
[46] J. Mitroy and G. Ryzhikh, Comput. Phys. Comm. 123, 103 (1999).
[47] G. G. Ryzhikh, J. Mitroy, and K. Varga, J. Phys. B 31, 3965 (1998).
[48] D. M. Schrader, Phys. Rev. Lett. 92, 043401 (2004).
[49] S. F. Boys, Proc. R. Soc. London A 258, 402 (1960).
[50] K. Singer, Proc. R. Soc. London A 258, 412 (1960).
[51] V. Cenek and J. Rychlewski, J. Chem. Phys. 98, 1252 (1993).
[52] J.-Y. Zhang, J. Mitroy, and K. Varga, Phys. Rev. Lett. 108, 043401 (2012).
103, 223202 (2009).
[53] J.-Y. Zhang, M.-S. Wu, Y. Qian, X. Gao, Y. Y.-J., K. Varga, Z.-C. Yan, and U. Schwingenschlögl, arXiv:1803.03026 (2018).
[54] Y. Zhang, M.-S. Wu, and J.-Y. Zhang, (unpublished).
[55] G. W. F. Drake, Springer Handbook of Atomic, Molecular and Optical Physics (Springer, New York, 2006) p. 668.
[56] B. Gao, J. Phys. B 37, 4273 (2004).
[57] X. Gao and J.-M. Li, Phys. Rev. A 89, 022710 (2014).
[58] X. Gao, X.-Y. Han, and J.-M. Li, J. Phys. B 49, 214005 (2016).
[59] Y. Zhang, M.-S. Wu, J.-Y. Zhang, and K. Varga, (unpublished).
[60] Y. K. Ho, Phys. Rep. 99, 1 (1983).
[61] G. F. Gribakin and V. V. Flambaum, Phys. Rev. A 48, 546 (1993).
[62] M. J. Jamieson, A. Dalgarno, and M. Kimura, Phys. Rev. A 51, 2626 (1995).
[63] F. Dalfovo, S. Giorgini, L. P. Pitaevskii, and S. Stringari, Rev. Mod. Phys. 71, 463 (1999).
[64] A. P. Mills, S. Berko, and K. F. Canter, Phys. Rev. Lett. 34, 1541 (1975).
[65] S. Chu, A. P. Mills, and J. L. Hall, Phys. Rev. Lett. 52, 1689 (1984).
[66] M. S. Fee, A. P. Mills, S. Chu, E. D. Shaw, K. Danzmann, R. J. Chichester, and D. M. Zuckerman, Phys. Rev. Lett. 70, 1397 (1993).
[67] A. P. Mills and M. Leventhal, Nucl. Instrum. Methods Phys. Res. B 192, 102 (2002).
[68] M. Oberthaler, Nucl. Instrum. Methods Phys. Res. B 192, 129 (2002).
[69] S. G. Karshenboim, Phys. Rev. Lett. 104, 220406 (2010).
[70] S. Kotler, R. Ozeri, and D. F. J. Kimball, Phys. Rev. Lett. 115, 081801 (2015).
[71] A. S. Kadyrov, C. M. Rawlins, A. T. Stelbovics, I. Bray, and M. Charlton, Phys. Rev. Lett. 114, 183201 (2015).
[72] B. Mansoulié and on behalf of the GBAR Collaboration, Hyperfine Interact. 240, 11 (2019).