Inclusive Production of $h_c(h_b)$ States via $e^+e^-$ Annihilation

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We calculate the inclusive production of $h_c(h_b)$ at $e^+e^-$ colliders with the center-of-mass energy from the CLEO-c energy to $Z^0$ boson mass at leading order of nonrelativistic QCD. At $Z^0$ boson mass, the cross sections are $39 \sim 703$fb for $h_c$(1p), $37 \sim 61$fb for $h_c$(1p) and $44 \sim 73$fb for $h_b$(2p). At the B-factory, it is $86 \sim 212$fb for $h_c$(1p). For $h_c$ at the CLEO-c and $h_b$(1p, 2p) at the B-factory, the perturbative QCD expansion is not good and the results are much smaller than the experimental measurements. It is clearly shown in all the results that the color-octet state ($1S^0_c$) contributes dominantly while the color-singlet state ($1P^1$) contributes small or even negative part, in contrast to the case of inclusive $J/\psi$ production where the color-singlet state is found to contribute dominantly.

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In recent years, many experimental measurements for P-wave quarkonia $h_c, h_b(1^{+−})$ have been achieved. The related branch ratios were measured [1][8], the masses of them are measured precisely [3][11], and the cross sections for $h_c(h_b)$ production via $e^+e^-$ annihilation at the CLEO-c (B-factory) are also measured [10][11]. More experimental measurements on P-wave quarkonium states $h_c, h_b$ could be expected in the future. It provides a new place to test or improve our knowledge of quantum chromodynamics (QCD) on heavy quarkonium production and decay.

To study heavy quarkonium decay and production processes, nonrelativistic QCD(NRQCD) [12] is a successful factorization theorem, in which calculation is factorized into process-dependent short-distance coefficients, to be calculated perturbatively in the strong-coupling constant $\alpha_s$ expansions, and universal long-distance matrix elements (LDMEs), to be extracted from experiment. In this scheme, the process $\gamma \gamma \rightarrow h_c(h_b)$ pair can be produced in any Fock state $n = 2S^1+L^0_p$ with definite spin $S$, orbital angular momentum $L$, total angular momentum $J$, and $a=1,8$, where $a=1$ is for color-singlet (CS) and $a=8$ is for color-octet (CO) states which finally evolve into physical quarkonia through nonperturbative processes. The relative importance of all the CO and CS states are estimated by velocity scaling rules, which weigh each of the LDMEs by a definite power of the heavy-quark velocity $v$ in the limit $v \ll 1$. In this way, the theoretical predictions are organized in double expansions in $\alpha_s$ and $v$.

Based on heavy quark spin symmetry of NRQCD, the LDMEs for $h_c(h_b)$ are simply related to that for $\chi_c(\chi_b)$, therefore, studies in the past already supply the information of the LDMEs for $h_c(h_b)$ from the fit of $\chi_c(\chi_b)$ related processes. Experimental data [13][14] shows that $J/\psi$ production from $\chi_c$ feeddown count on a large part in $J/\psi$ hadroproduction. Based on the experimental data [13] and LO calculation on $\chi_c$ hadroproduction, an estimate of the CO LDME for $\chi_c$ and $h_c$ was given in Ref. [15][16]. Employing these matrix elements, the calculations of $h_c$ hadroproduction at the Tevatron [17] and LHC [18][19] predicted a significant yield. Photoproduction of $h_c$ was investigated in Ref. [20] by using a CO LDME extracted from the decay $B \rightarrow \chi_cJ + X$, the results indicated a significant cross section at the DESY HERA. Recently, the $\chi_c$ hadroproduction calculated up to QCD next-to-leading order (NLO) with an estimate of the CO LDME for $\chi_c$ is given in Ref. [21]. In the study of polarization for prompt $J/\psi$ hadroproduction [22], the CO LDME for $\chi_c$ is given by fitting the Tevatron and LHCb data.

In this paper, we study heavy quarkonium P-wave states $h_c(h_b)$ inclusive production via $e^+e^-$ annihilation by calculating the related cross sections at the leading-order(LO) of NRQCD. Detailed results are given at the $Z^0$ boson peak, B-factory and CLEO-c. The discussions on the comparison between our calculation and the experimental measurements are given.

In NRQCD framework, at LO of $\alpha_s$ and $v^2$, cross sections for $H$ state production can be expressed as

$$\sigma(H) = f_n P_{[1]} (\alpha H (1P^1_{[1]})) + f_{n[8]} P_{[8]} (\alpha H (1S^0_{[8]})), \quad (1)$$

where $f_n$ denotes the short-distance coefficient corresponding to the NRQCD operator $\alpha H (n)$ and the LDMEs of the operators are

$$\langle \alpha H (1S^0_{[8]}) \rangle = \langle 0 | \psi^{+} T a H a H^+ T a \psi | 0 \rangle,$$

$$\langle \alpha H (1P^1_{[1]} ) \rangle = \langle 0 | \psi^{+} (\sqrt{D}/2) a H^+ a H \chi^{+} (\sqrt{D}/2) \psi | 0 \rangle,$$

where $H$ represents the hadron state $h_c$, or $cc(1P^1_{[1]}, 1S^0_{[8]})$ states. The CS operator $\alpha H (1P^1_{[1]})$ is of order $v^2$ while the CO operator $\alpha H (1S^0_{[8]})$ is of order $v^0$. However, to hadronize into $h_c$, the CO state have to emit at least one soft gluon, which rises the order of the LDME $\langle \alpha H (1S^0_{[8]}) \rangle$ to $\alpha_s v^2$, the same order of $v$ as the CS LDME $\langle \alpha H (1P^1_{[1]} ) \rangle$.

The leading processes for inclusive $h_c$ production are...
listed as
\[ e^+e^- \rightarrow c\bar{c} (1 P_1^{[1]} ) + g + g, \]
\[ e^+e^- \rightarrow c\bar{c} (1 P_1^{[1]} ) + c + \bar{c}, \tag{3} \]
\[ e^+e^- \rightarrow c\bar{c} (1 S_0^{[8]} ) + g. \]

It is easy to see that the three processes are of the same order of \( v^2 \) at LO. The cross sections for the second and third processes are infrared (IR) divergence free and can be calculated directly. The cross section for the first process, although at LO, is IR divergent when one of the gluons is soft. To deal with the divergence, the NRQCD factorization formulas involved are as follows,

\[ \sigma(h_c) = f_{1 P_1^{[1]}} \langle O^{h_c} (P_1^{[1]} ) \rangle + f_{1 S_0^{[8]}} \langle O^{h_c} (1 S_0^{[8]} ) \rangle \tag{4} \]
and
\[ \sigma(1 P_1^{[1]}) = f_{1 P_1^{[1]}} \langle O^{1 P_1^{[1]}} (P_1^{[1]} ) \rangle + f_{1 S_0^{[8]}} \langle O^{1 P_1^{[1]}} (1 S_0^{[8]} ) \rangle, \]
\[ \sigma(1 S_0^{[8]}) = f_{1 P_1^{[1]}} \langle O^{1 S_0^{[8]}} (1 P_1^{[1]} ) \rangle + f_{1 S_0^{[8]}} \langle O^{1 S_0^{[8]}} (1 S_0^{[8]} ) \rangle. \tag{5} \]

The matrix elements of operators are calculated in dimensional regularization scheme at LO with an ultraviolet cutoff (NRQCD scale) \( \mu_A = m_c \) and are given as:
\[ \langle O^{1 S_0^{[8]}} (1 P_1^{[1]} ) \rangle = 0, \tag{6} \]
\[ \langle O^{1 P_1^{[1]}} (1 S_0^{[8]} ) \rangle = - \frac{\alpha_s}{\pi m_c^2} u_c^* N_c^2 \langle O^{1 P_1^{[1]}} (1 P_1^{[1]} ) \rangle, \]
where \( N_c = 3 \) for SU(3) gauge field and \( u_c \) is defined as
\[ u_c^* = \frac{1}{\epsilon_I R} - \frac{5}{3} + \ln \left( \frac{\pi \mu_R^2}{\mu_A^2} \right) \tag{7} \]
with \( \mu_R \) being the renormalization scale. The divergence in this LDME will cancel that in \( \sigma(1 P_1^{[1]}) \), which can be isolated by using the two-cutoff phase space slicing method \cite{23} as
\[ \sigma(1 P_1^{[1]}) = \sigma^S (1 P_1^{[1]}) + \sigma^S (1 P_1^{[1]}) \tag{8} \]
where \( \sigma^S \) and \( \sigma^S \) are from the gluon-soft and hard region, respectively. The boundary of the two regions is \( E_g = \frac{N_c^2}{\alpha_s} \delta_s \), where \( E_g \) is the energy of the soft gluon, \( \sqrt{s} \) is the center-of-mass (CM) energy of \( e^+e^- \) colliders and \( \delta_s \) is an arbitrary positive number small enough to make the soft approximation good enough. The soft part can be expressed as
\[ \sigma^S (1 P_1^{[1]}) = - \frac{\alpha_s}{3\pi m_c^2} u_c^* N_c^2 \langle O^{1 S_0^{[8]}} \rangle \tag{9} \]
where
\[ u_c^* = \frac{1}{\epsilon_I R} + \frac{s + 4m_c^2}{s - 4m_c^2} \ln\left( \frac{s}{4m_c^2} \right) + \frac{4\pi \mu_R^2}{s \delta_s^2} - \gamma_E - \frac{1}{3}. \tag{10} \]

Using Eq.\( 5 \) and Eq.\( 6 \), we obtain the expressions of short-distance coefficients in Eq.\( 4 \) as
\[ f_{1 S_0^{[8]}} = \frac{\sigma(1 S_0^{[8]})}{\langle O^{1 S_0^{[8]}} (1 S_0^{[8]} ) \rangle}, \tag{11} \]
\[ f_{1 P_1^{[1]}} = - \frac{N_c^2}{\alpha_s} u_c^* f_{1 S_0^{[8]}} + \frac{\sigma^S (1 P_1^{[1]})}{\langle O^{1 P_1^{[1]}} (1 P_1^{[1]} ) \rangle}. \tag{12} \]

It is clearly shown that all the short-distance coefficients are IR divergence free and the cross section for the first process is well defined. To calculate \( \sigma(1 S_0^{[8]}) \) and \( \sigma^S (1 P_1^{[1]}) \) for the first process and the cross sections for the second and third processes, we apply our Feynman Diagram Calculation package (FDC) \cite{24} to generate all the needed FORTRAN source.

From the heavy quark spin symmetry of the NRQCD Lagrangian, it is obvious that the LDME \( \langle O^{h_c} (1 S_0^{[8]} ) \rangle \) for the intermediate state \( c\bar{c} (1 S_0^{[8]} ) \) evolving into \( h_c \) is exactly the same as that for the intermediate state \( c\bar{c} (3 S_1^{[8]} ) \) evolving into \( \chi_c \) at LO of \( v^2 \). It gives
\[ \langle O^{h_c} (1 S_0^{[8]} ) \rangle \approx \langle O^{\chi_c} (3 S_1^{[8]} ) \rangle. \tag{14} \]

We will employ the LDME value obtained in Ref.\( 16 \) as \( \langle O^{\chi_c} (3 S_1^{[8]} ) \rangle = 0.0098 \text{GeV}^3 \), which is from a fit of the Tevatron experimental data with the LO calculation for transverse momentum \( p_t \) distribution of \( \chi_c \) hadroproduction and is independent on the NRQCD scale \( \mu_A \) since the range of \( p_t \) in the fit does not include \( p_t = 0 \) where \( \mu_A \) is involved in the calculation to factorize IR divergence. However, the result in our calculation is \( \mu_A \)-dependent, therefore we have to fix a value of \( \mu_A \).

Since \( \mu_A \)-dependent term is proportional to \( \ln(\mu_A) \) and it is found that different choices of \( \mu_A \) in a reasonable range around the heavy quark mass \( m_Q \) do not cause larger uncertainty in final results when the CM energy is far above \( 2m_Q \), the default value \( \mu_A = m_Q \) is chosen for \( h_c \) production at both B-factories and Z-factory, and for \( h_b \) production at the Z-factory. For \( h_c \) production at the CLEO-c and \( h_b \) production at B-factories, we give results for different \( \mu_A \) choices to show the uncertainties caused by this parameter. In the numerical calculation, we have the following common choices as
\[ \langle O^{h_c} (P_1^{[1]} ) \rangle = \left( \frac{s}{s_{W}} R_{h_c} (0) \right)^2 = 0.322 \text{GeV}^5, \]
\[ \alpha = 1/128, \]
\[ \sin^2 \theta_W = 0.226, m_z = 91.2 \text{GeV and } t_\gamma = 2.49 \text{GeV}. \]
And other parameters are explicitly given in each individual case. The soft cutoff \( \delta_s \) independence is checked in the calculation and \( \delta_s = 0.0001 \) is used.

For the CLEO-c experiment at the CM energy \( \sqrt{s} = 4.17 \text{GeV} \), only the first and last processes in Eq.\( 4 \) contribute and the gluons in the first process are too soft that the convergence of perturbative calculation in QCD is not good. Therefore, it is not surprising to see that...
the theoretical results can not describe the experimental measurement. To show the uncertainty from the c-quark mass $m_c$, the QCD coupling constant $\alpha_s$ and NRQCD scale $\mu_A$, our results are listed in Table I by choosing $m_c$ as 1.3 GeV, 1.5 GeV and 1.76 GeV, $\alpha_s$ as $\alpha_s(3.0 \text{ GeV}) = 0.26$ and $\alpha_s(\sqrt{s}/2) = 0.30$, and $\mu_A$ as $m_c$ and $m_c/2$.

| $\alpha_s(2m_c)$ = 0.26 | $m_c(\text{GeV})$ | $P_{1}^{1, gg}$ | $S_{1}^{0, g}$ | $\text{Total}$ |
|--------------------------|-----------------|-----------------|-----------------|-----------------|
| 1.3                      | -4.449(-1.534)  | 4.406           | -0.04(2.87)     |
| 1.5                      | -3.057(-1.560)  | 3.014           | -0.04(1.45)     |
| 1.76                     | -1.656(-1.104)  | 1.531           | -0.12(0.43)     |

| $\alpha_s(\sqrt{s}/2)$ = 0.30 | $m_c(\text{GeV})$ | $P_{1}^{1, gg}$ | $S_{1}^{0, g}$ | $\text{Total}$ |
|-----------------------------|-----------------|-----------------|-----------------|-----------------|
| 1.3                         | -5.923(-2.042)  | 5.084           | -0.84(3.04)     |
| 1.5                         | -4.071(-2.077)  | 3.478           | -0.59(1.40)     |
| 1.76                        | -2.204(-1.470)  | 1.766           | -0.44(0.30)     |

Table I: Total Cross sections(pb) for $h_c$ production from $1 P_{1}^{1, gg}$, $S_{1}^{0, g}$ and the sum of them with different $m_c$, $\alpha_s$ and $\mu_A$ at $\sqrt{s} = 4.170$ GeV. Values in the brackets correspond to $\mu_A = m_c$.

The results in the table are all negative for $\mu_A = m_c$ and become positive for $\mu_A = m_c/2$ while the experimental measurements at the CLEO-c $[10]$ give the cross section for $e^+ e^- \to h_c(1P) + \pi^+ \pi^-$ as $15.6 \pm 2.3 \pm 1.9 \pm 0.3$pb. On the results, there are three points to be addressed.

First, we focus on the $\alpha_s$ expansion. The value of $\alpha_s$ depends on the renormalization scale $\mu_R$, and in the total cross section, there are logarithm terms $\ln(E_s/\mu_R)$ which must be small to make sure that they do not ruin the expansion. $E_s$ can be $m_c$, $s$ and $p_{\text{max}}^g = (s - 4m_c^2)/(2\sqrt{s})$. The smallest scale $p_{\text{max}}^g$ is $1.0(0.6)$ GeV when $m_c$ is 1.5 GeV($m_{h_c}/2$), as a result, there is not a proper choice of $\mu_R$ to lead to good convergence in perturbative QCD expansion. The perturbative expansion becomes better when the CM energy is larger than $\sqrt{s} = 6$ GeV where $p_{\text{max}}^g$ is 2 GeV. For inclusive $h_b$ production at the B-factory, it becomes better when the CM energy is larger than 12 GeV where $p_{\text{max}}^g$ is 1.8 GeV.

Second, we investigate the $v^2$ expansion in shott-distance coefficients. The worst propagators for the $v^2$ expansion in the first process in Eq.(15) is $1/(\mu^2 + q^2)$ where $\mu^2$ and $q^2$ denote the momentum of $h_c$, gluon and the relative momentum of c-quarks in $h_c$, respectively. It is easy to obtain $a$, a Lorentz invariant, as $a = |q|\cos\theta_{q,p_s}/m_c$ in $h_c$ rest frame where $q/m_c$ is the velocity $v$ of c-quark in the quarkonium. Therefore the $a^2$ expansion in the propagator is the nonrelativistic expansion. It is easy to see that validity of the expansion only depends on the value of $a^2$ and has no relation with the momentum of the gluons. In our case of $h_c$, $v^2$ is thought to be larger than that of $J/\psi$, so the expansion is better for $J/\psi$ than that for $h_c$.

Third, we have to check more detail of NRQCD factorization scheme. The default NRQCD scale $\mu_A = m_c$ sets up a boundary between perturbative and non-perturbative parts. The maximum energy of the soft gluons in the non-perturbative part is $\mu_A = m_c$ which even exceeds the maximum gluon energy $p_{\text{max}}^g = 1$ GeV in the phase space in the $e^+ e^- \to h_c(1P) + gg$ at CLEO-c energy. So $\mu_A = m_c$ is not a suitable choice, and $\mu_A = m_c/2$, which is smaller than $p_{\text{max}}^g$ is chosen to present our results. Finally it is clear that the uncertainty from the choice of NRQCD scale $\mu_A$ is larger near production threshold and with choice of $\mu_A = m_c/2$, results for $h_c$ inclusive production at the CLEO-c is 0.30 $\sim$ 3.04pb in our calculation which is just 1/50 $\sim$ 1/5 of the measurement of the exclusive process $e^+ e^- \to h_c(1P) + \pi^+ \pi^-$. In Fig.1, we present the dependence of the total cross section for the inclusive $h_c$ production on the CM energy with $m_c = 1.5$ GeV and $\alpha_s = 0.26$. When the CM energy is smaller than 6 GeV, the perturbative QCD expansion is not good and the total cross section turns out to be sensitive to the choice of NRQCD scale $\mu_A$ and is of large uncertainty. The uncertainty in our result becomes smaller quickly as the CM energy goes larger. We find that there are two peaks in the plot, one is at $Z^0$ boson mass and the other is at about 5 GeV. And the ratio of the cross sections at the two peaks is about 1.1:2, which means that they are in the same order of magnitude. It can also be seen that the $e^+ e^- \to h_c(1P) + gg$ and $e^+ e^- \to h_c(1S) + g$ dominate the inclusive production rate when the CM energy is smaller than 20 GeV, while $e^+ e^- \to h_c(1P) + cc$ dominate that when the CM energy is larger than 20 GeV.

All the above discussions are applicable to $h_b$ production at the B-factory with better convergence for non-relativistic expansion since $v^2$ becomes smaller. In the
numerical calculation, we employ the LDMEs, of which the CO ones are estimated based on the NRQCD scaling rule and the CS ones are obtained based on potential model, as \( \langle O_h (1P)(P_1^1) \rangle = 6.1 \text{ GeV}^5 \), \( \langle O_h (2P)(P_1^1) \rangle = 7.1 \text{ GeV}^5 \), \( \langle O_{h(1P)}(1S_0^8) \rangle = 0.43 \text{ GeV}^3 \) and \( \langle O_{h(1P)}(1S_0^8) \rangle = 0.52 \text{ GeV}^3 \). In Table. II and III, the results for \( h_b(1P) \) and \( h_b(2P) \) production at the B-factory are listed, where \( \mu_A = m_b/4 \) is chosen with the same consideration as for \( h_c \) case. We can see that the uncertainty from the choice of NRQCD scale \( \mu_A \) becomes smaller than that in \( h_c \) production at the CLEO-c. In comparison with the experimental measurement in Ref. [11], where exclusive production rate of \( e^+ e^- \to h_b \pi^+ \pi^- \) are 0.416pb and 0.695pb for \( h_b(1P) \) and \( h_b(2P) \), respectively, our results are only 1/13 ~ 1/5 and 1/19 ~ 1/7 for \( h_b(1P) \) and \( h_b(2P) \), respectively.

| \( \alpha_s (2 m_b) = 0.18 \) | | | | |
|---|---|---|---|
| \( m_b \) (GeV) | \( P_1^1 \) gg | \( S_0^8 \) g | Total |
| 4.75 | -3.77(-1.50) | 76.7 | 73.75 |
| 4.95 | -2.78(-1.34) | 53.1 | 50.52 |
| 5.13 | -1.88(-1.05) | 32.7 | 31.32 |

| \( \alpha_s (\sqrt{s}/2) = 0.21 \) | | | | |
|---|---|---|---|
| \( m_b \) (GeV) | \( P_1^1 \) gg | \( S_0^8 \) g | Total |
| 4.75 | -5.13(-2.04) | 89.5 | 84.87 |
| 4.95 | -3.78(-1.82) | 62.0 | 58.60 |
| 5.13 | -2.56(-1.43) | 38.2 | 36.37 |

Table II: Total Cross sections(fb) for \( h_b(1P) \) production from \( P_1^1 \), \( S_0^8 \) and the sum of them with different \( m_b \), \( \alpha_s \) and \( \mu_A \), at \( \sqrt{s} = 10.865 \text{ GeV} \). Values in the brackets correspond to \( \mu_A = m_b/4 \).

| \( \alpha_s (2 m_c) = 0.26 \) | | | | |
|---|---|---|---|
| \( m_c \) (GeV) | \( P_1^1 \) gg | \( P_1^1 \) ee | \( S_0^8 \) g | Total |
| 1.3 | 8.4 | 46.2 | 102.6 | 212 |
| 1.5 | -6.5 | 15.5 | 131.7 | 147 |
| 1.76 | -12.2 | 37.1 | 113.6 | 106 |

| \( \alpha_s (\sqrt{s}/2) = 0.21 \) | | | | |
|---|---|---|---|
| \( m_c \) (GeV) | \( P_1^1 \) gg | \( P_1^1 \) ee | \( S_0^8 \) g | Total |
| 1.3 | 5.5 | 30.2 | 131.1 | 167 |
| 1.5 | -4.2 | 10.1 | 111.2 | 117 |
| 1.76 | -7.9 | 2.9 | 91.7 | 86 |

Table III: Total Cross sections(fb) for \( h_b(2P) \) production from \( P_1^1 \), \( S_0^8 \) and the sum of them with different \( m_c \), \( \alpha_s \) and \( \mu_A \), at \( \sqrt{s} = 10.865 \text{ GeV} \). Values in the brackets correspond to \( \mu_A = m_b/4 \).

| \( \alpha_s (2 m_c) = 0.26 \) | | | | |
|---|---|---|---|
| \( m_c \) (GeV) | \( P_1^1 \) gg | \( P_1^1 \) ee | \( S_0^8 \) g | Total |
| 1.3 | 8.4 | 46.2 | 102.6 | 212 |
| 1.5 | -6.5 | 15.5 | 131.7 | 147 |
| 1.76 | -12.2 | 37.1 | 113.6 | 106 |

Table IV: Total Cross sections(fb) for \( h_c \) production from \( P_1^1 \), \( S_0^8 \) and the sum of them with different \( m_c \) and \( \alpha_s \), at \( \sqrt{s} = 10.6 \text{ GeV} \).

The results at \( Z^0 \) boson mass are listed in Table. V, VI and Table. VII for \( h_c(1P) \), \( h_b(1P) \) and \( h_b(2P) \) respectively.

Fig. 2 shows the dependence of the total cross section for inclusive \( h_b(1P) \) production on the CM energy. We can see that the \( P_1^1 \) process dominates throughout the \( \sqrt{s} \) range, and the first peak where the cross section reaches its maximum value is at the CM energy 12 GeV, while \( Z^0 \) boson mass is the second peak. And the ratio of the cross sections at the two peaks is about 4:3, which means they are almost of the same value.

We calculate the production of \( h_c \) at the B-factory by changing quark mass and QCD coupling constant to show the uncertainties caused by different choices of the two parameters. The results are listed in Table. IV. In this case, theoretical predictions should be better than those at the CLEO-c since the uncertainty from the choice of \( \mu_A \) becomes much smaller.

Fig. 3 shows the distribution of invariant mass \( M_X \) of the final states with \( h_c \) excluded for \( h_c \) production in the CS processes at B-factory and Z-factory CM energy. The parameter choice is \( m_c = 1.76 \text{ GeV} \) and \( \alpha_s = 0.26 \). Fig. 3 gives that for \( h_b(1P) \) and \( h_b(2P) \) production. The CM energy is 91.2 GeV and b-quark mass for \( h_b(1P) \) and \( h_b(2P) \) are 4.95 GeV and 5.13 GeV, respectively. QCD coupling constant is \( \alpha_s = 0.18 \). The \( S_0^8 \) process contribute in the phase space point where \( M_X \) is zero. However, \( S_0^8 \) state evolves into a P-wave quarkonium requires it emitting soft gluons which will combine with other gluon and hadronize. Therefore, its contribu-
TABLE V: Total Cross sections(fb) for $h_c$ production from $1P_{1}^{[1]}$ $1S_{0}^{[8]}$ and the sum of them with different $m_c$ and $\alpha_s$, at $\sqrt{s} = 91.2$ GeV.

| $\alpha_s(2m_c)$ = 0.26 | $m_c$( GeV) | $1P_{1}^{[1]} gg$ | $1P_{1}^{[1]} c\bar{c}$ | $1S_{0}^{[8]} g$ | Total |
|-------------------------|------------|-----------------|-----------------|-------------|------|
| 1.3                     | 2.51       | 698             | 2.01            | 703         |
| 1.5                     | 1.53       | 339             | 1.74            | 342         |
| 1.76                    | 0.89       | 151             | 1.48            | 153         |
| 1.76                    | 0.22       | 38              | 0.74            | 39          |

| $\alpha_s(\sqrt{s}/2) = 0.13$ | $m_c$( GeV) | $1P_{1}^{[1]} gg$ | $1P_{1}^{[1]} c\bar{c}$ | $1S_{0}^{[8]} g$ | Total |
|-------------------------------|------------|-----------------|-----------------|-------------|------|
| 1.3                          | 0.63       | 175             | 1.01            | 177         |
| 1.5                          | 0.38       | 85              | 0.87            | 86          |
| 1.76                         | 0.22       | 38              | 0.74            | 39          |

TABLE VI: Total Cross sections(fb) for $h_b(1P)$ production from $1P_{1}^{[1]}$ $1S_{0}^{[8]}$ and the sum of them with different $m_b$ and $\alpha_s$, at $\sqrt{s} = 91.2$ GeV.

| $\alpha_s(2m_b)$ = 0.18 | $m_b$( GeV) | $1P_{1}^{[1]} gg$ | $1P_{1}^{[1]} bb$ | $1S_{0}^{[8]} g$ | Total |
|-------------------------|------------|-----------------|-----------------|-------------|------|
| 4.75                    | 0.73       | 10.3            | 49.6            | 61          |
| 4.95                    | 0.63       | 8.24            | 47.5            | 56          |
| 5.13                    | 0.54       | 6.79            | 46.8            | 53          |
| 4.75                    | 0.38       | 5.37            | 35.8            | 42          |
| 4.95                    | 0.33       | 4.30            | 34.3            | 39          |
| 5.13                    | 0.28       | 3.54            | 33.1            | 37          |

| $\alpha_s(\sqrt{s}/2) = 0.13$ | $m_c$( GeV) | $1P_{1}^{[1]} gg$ | $1P_{1}^{[1]} bb$ | $1S_{0}^{[8]} g$ | Total |
|-------------------------------|------------|-----------------|-----------------|-------------|------|
| 4.75                          | 0.44       | 6.26            | 43.3            | 50          |
| 4.95                          | 0.38       | 5.02            | 41.5            | 47          |
| 5.13                          | 0.33       | 4.13            | 40.0            | 44          |

TABLE VII: Total Cross sections(fb) for $h_b(2P)$ production from $1P_{1}^{[1]}$ $1S_{0}^{[8]}$ and the sum of them with different $m_b$ and $\alpha_s$, at $\sqrt{s} = 91.2$ GeV.

In summary, we study the inclusive production of $h_c(1P)$, $h_b(1P)$ and $h_b(2P)$ via $e^+e^-$ annihilation with the center-of-mass energy from the CLEO-c energy to Z$^0$ boson mass at leading order of nonrelativistic QCD. A detailed discussion on the validity of perturbative calculation for the total cross section of $h_c(h_b)$ production at the CLEO-c(B-factory) is given. It is easy to see that the perturbative QCD expansion is not good in these cases. It is also found that validity of the nonrelativistic expansion only depends on the value of $v^2$ and has no relation with the momentum of the related gluons, so the nonrelativistic expansion is available. Besides, $\mu_A = m_c$ is not a suitable choice, and $\mu_A = m_c/2$, which is smaller than the maximum gluon energy 1 GeV in the phase space, is chosen to present the results. In our numerical calculation, the LDMEs $\langle O^{h_c(1S_{0}^{[8]})} \rangle$ and $\langle O^{h_b(1S_{0}^{[8]})} \rangle$ from a
fit of the Tevatron experimental data with the LO calculation for transverse momentum $p_t$ distribution of $\chi_cJ$ hadroproduction are independent on the NRQCD scale $\mu_A$, since the range of $p_t$ in the fit does not include $p_t = 0$ where $\mu_A$ is involved in the calculation to factorize IR divergence. Therefore, in our results, $\mu_A$ independence can not be achieved and it can bring large uncertainty when the CM energy approach the production threshold. At $Z^0$ boson, the cross sections are $39 \sim 703$fb for $h_c(1p)$, $37 \sim 61$fb for $h_0(1p)$ and $44 \sim 73$fb for $h_0(2p)$. At the B-factory, they are $86 \sim 212$fb for $h_c(1p)$, $32 \sim 87$fb for $h_0(1p)$ and $38 \sim 106$fb for $h_0(2p)$ where the production rate for $h_0(1p, 2p)$ are about $5 \sim 19$ times smaller than the experimental measurements. And at the CLEO-c, it is $0.3 \sim 3.1$pb, which is of large uncertainty from the choice of $\mu_A$, far away from the experimental measurement. In another aspect, the results clearly show that the color-octet state ($1S_0^c$) contributes dominantly while the color-singlet ($1P_1^c$) state contributes a small part, in contrast to the case of inclusive $J/\psi$ production at the B-factory, where the color-singlet state is found to contribute dominantly. From previous works at NLO of QCD for inclusive $J/\psi$ production at $e^+e^-$ colliders, it is clear that the QCD NLO correction can enhance the production rate 1.7 times for $e^+e^- \rightarrow J/\psi + cc$ [27] and 1.2 times for $e^+e^- \rightarrow J/\psi + gg$ [28]. It is quite interesting to see the effect from QCD NLO correction on the inclusive $h_c(h_0)$ production in future work.

Note added When we were preparing our draft, a related work appeared at arXiv recently [29], which study on $h_c$ production at B-factories. Once take their renormalization scheme in the calculation of NRQCD corrections to CO operator and their parameters, our calculation on $h_c$ production agrees with their numerical results.

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