Control Charts for the Shape Parameter of Skewed Distribution

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Abstract: The weighted distributions are useful when the sampling is done using an unequal probability of the sampling units. The Weighted Power function distribution (WPFD) has applications in the fields of reliability engineering, management sciences and survival analysis. WPFD is more beneficial in Statistical process control (SPC). SPC is defined as the use of statistical techniques to control a process or production method. SPC tools and procedures can help to monitor process behaviour, discover problems in internal systems, and find solutions for production issues. To identify and remove the variation in different reliability processes and also to monitor the reliability of machines where the number of errors follows WPFD, we develop control charts to keep the process in control. A memory-based control chart like an exponentially weighted moving average (EWMA) control chart and an extended exponentially weighted moving average (EEWMA) control chart are discussed and compared each other. The proposal of these control charts is based on the modified maximum likelihood estimator (MMLE) under the shape parameter of WPFD. We have presented Monte Carlo simulation technique and a real-life application to compare the proposed control charts. This study shows that an EEWMA control chart based on MMLE performs better than EWMA control chart, when the underlying distribution of the errors in process monitoring follows WPFD. These findings can be useful for researchers and practitioners in dealing with production errors and optimizing the output.

Keywords: Weighted Power function distribution; EWMA control chart; EEWMA control chart; modified maximum likelihood estimator; management sciences; quality charts

1 Introduction

The different production processes in industries face commonly two types of variations: common cause variation and special cause variation. Common cause variation always exists even if the process is designed very well and maintained very carefully. This variation should be relatively small in magnitude and is uncontrollable and due to many small unavoidable causes. A process is said to be in statistical control if only common cause variation is present. The variations outside this common cause pattern are called
special cause variations. These variations are subject to some problem in the system, like the poor tuning of
equipment, controller fell asleep or got absent, the computer stopped working, a poor lot of raw material,
machine break down. A process working under both types of variation is said to be out of control.

These variations may affect the production process and ultimately result in a defective output. The
consequences of it are pretty severe. Companies have to face high cost as well as their reputation will be
at stake. Therefore, in order to avoid variations in the process, there is a need to use control charts to
keep the process under control. Zaka et al. [1] introduced a new class of distribution called WPFD in
order to have more application in applied sciences such as biosciences engineering, management
sciences. The weighted distributions are helpful when the sampling is done using an unequal probability
of the sampling units. WPFD is more useful in Statistical process control. Statistical process control
(SPC) is defined as the use of statistical techniques to control a process or production method. SPC tools
and procedures can help to monitor process behavior, discover issues in internal systems, and find
solutions for production problems. Two types of control charts are commonly studied, memory less
control chart i.e., Shewhart control chart and memory based control charts such as EWMA and HEWMA
etc. Many works have been done in this regard, such as the EWMA control charts firstly by Roberts [2],
and recently by Li et al. [3] and Nguyen et al. [4]. The cumulative-sum (CUSUM) control charts firstly
by Page [5], Sanusi et al. [6], Haq et al. [7] and Hossain et al. [8]. The mixed EWMA-CUSUM control
charts by Abbas et al. [9], Ajadi and Riaz [10]. The hybrid exponential weighted moving average
(HEWMA) control charts due to Shamma et al. [11] and Haq [12].

In real life scenario, this is not always possible to fulfil the normality assumption for the distribution of
error during the process. A very few works in literature is about this situation of not normal process
distribution including Noorossana et al. [13], Lin et al. [14], Erto et al. [15], Liang et al. [16] and Ahmed
et al. [17]. The EWMA statistic is widely used for shift detection in the ongoing process either to monitor
qualitative or quantitative physical phenomena. The generalized form of the existing EWMA statistic was
introduced by Naveed et al. [18] and named it as EEWMA.

2 The Conventional EWMA Control Chart

Let the distribution of the underlying process having the sequence \( \{X_t\} \) is normal. Also, let the
0 ≤ \( \lambda \) ≤ 1, is a known constant. Now EWMA statistics is given by
\[
W_t = \lambda X_t + (1 - \lambda) W_{t-1}.
\]

The smoothing constant \( \lambda \) plays a very important here. As it approaches to zero, it becomes sensitive for
a small and moderate shift in mean and as it becomes close to one. It approaches to Shewart control chart. The
control limits for EWMA are given below
\[
UCL_{W_t} = \mu + L \star \sqrt{V(W_t)}
\]
\[
LCL_{W_t} = \mu - L \star \sqrt{V(W_t)}
\]
\[
E(W_t) = \mu
\]
\[
V(W_t) = \sigma^2 \left(1 - (1 - \lambda)^2t\right) \left(\frac{\lambda}{2 - \lambda}\right)
\]

3 The Traditional Extended Exponentially Weighted Moving Averages Control chart

When the distribution of the process is normal, the EEWMA control chart was introduced by Naveed
et al. [18]. The EEWMA control chart by Naveed et al. [18] is given as
\[ Z_t = \lambda_1 T_t - \lambda_2 T_{t-1} + (1 - \lambda_1 + \lambda_2) Z_{t-1} \]

where \(0 \leq \lambda_1 \leq 1\) and \(0 \leq \lambda_2 \leq \lambda_1\). \(T_{t-1}\) is represents the previous value of the variable and \(Z_{t-1}\) denotes the previous value of a statistic.

The mean and variance are given as

\[ E(Z_t) = \mu \]

And

\[ \text{var}(Z_t) = \sigma^2 \left[ (\lambda_1^2 + \lambda_2^2) \left\{ \frac{1-\left(1-\lambda_1+\lambda_2\right)^2}{2(\lambda_1-\lambda_2)-2(\lambda_1-\lambda_2)^2} \right\} - 2a\lambda_1\lambda_2 \left\{ \frac{1-(1-\lambda_1+\lambda_2)^2}{2(\lambda_1-\lambda_2)-(\lambda_1-\lambda_2)^2} \right\} \right] \]

### 4 Proposed Extended Exponentially Weighted Moving Averages Control Chart Under Non-Normality

Following Zaka et al. [1], it is assumed that the process random variable \(x_1, x_2, x_3, \ldots, x_t\) are independently and identically distributed following WPFD with probability density function (pdf) and cumulative density function (cdf) for the WPFD are given respectively by

\[ f(x) = \frac{2\beta x^{2\gamma-1}}{\beta^2};\quad 0 < x < \beta \]

and

\[ F(x) = \left( \frac{x}{\beta} \right)^{2\gamma}, \quad \text{where} \quad \beta \quad \text{and} \quad \gamma \quad \text{are the scale and shape parameters.} \]

Modified Maximum Likelihood Estimator (MMLE) is used to construct memory less and memory-based control charts to monitor the shape parameter of a process that follows a WPFD. The estimator for the shape parameter of WPFD defined by Zaka et al. [1] is given as,

\[ \hat{\gamma}_{\text{MMLE}} = \frac{-1 + \sqrt{1 + \frac{\beta^2}{\gamma^2}}}{2}. \]  \hspace{1cm} (1)

The variance of the \(\hat{\gamma}_{\text{MMLE}}\) is given by

\[ \text{Var}(\hat{\gamma}_{\text{MMLE}}) = E((\hat{\gamma}_{\text{MMLE}} - \gamma)^2). \]  \hspace{1cm} (2)

Now using Zaka et al. [1] and Naveed et al. [18], the EEWMA statistic is given as

\[ EEW_t = \lambda_1 \hat{\gamma}_{\text{MMLE}(t)} - \lambda_2 \hat{\gamma}_{\text{MMLE}(t-1)} + (1 - \lambda_1 + \lambda_2) EEW_{t-1} \]

For \(t = 1\)

\[ EEW_1 = \lambda_1 \hat{\gamma}_{\text{MMLE}(1)} - \lambda_2 \hat{\gamma}_{\text{MMLE}(0)}(1 - \lambda_1 + \lambda_2) EEW_0 \]

For \(t = 2\)

\[ EEW_2 = \lambda_1 \hat{\gamma}_{\text{MMLE}(2)} - \lambda_2 \hat{\gamma}_{\text{MMLE}(1)}(1 - \lambda_1 + \lambda_2) EEW_1 \]

\[ EEW_2 = \lambda_1 \hat{\gamma}_{\text{MMLE}(2)} - \lambda_2 \hat{\gamma}_{\text{MMLE}(1)} + (1 - \lambda_1 + \lambda_2) \left( \lambda_1 \hat{\gamma}_{\text{MMLE}(1)} - \lambda_2 \hat{\gamma}_{\text{MMLE}(0)} + (1 - \lambda_1 + \lambda_2) EEW_0 \right) \]

Let \(a = (1 - \lambda_1 + \lambda_2)\)

\[ EEW_2 = \lambda_1 \hat{\gamma}_{\text{MMLE}(2)} - \lambda_2 \hat{\gamma}_{\text{MMLE}(1)} + a\lambda_1 \hat{\gamma}_{\text{MMLE}(1)} - a\lambda_2 \hat{\gamma}_{\text{MMLE}(0)} + a^2 EEW_0 \]
\[ EEW_2 = \lambda \hat{\gamma}_{\text{MMLE}(2)} + (a\lambda_1 - \lambda_2) \hat{\gamma}_{\text{MMLE}(1)} - a\lambda_2 \hat{\gamma}_{\text{MMLE}(0)} + a^2 EEW_0 \]

Let \( b = (a\lambda_1 - \lambda_2) \)

\[ EEW_2 = \lambda \hat{\gamma}_{\text{MMLE}(2)} + b\hat{\gamma}_{\text{MMLE}(1)} - a\lambda_2 \hat{\gamma}_{\text{MMLE}(0)} + a^2 EEW_0 \]

\[ EEW_3 = \lambda \hat{\gamma}_{\text{MMLE}(3)} + b^2\hat{\gamma}_{\text{MMLE}(2)} + ab\hat{\gamma}_{\text{MMLE}(1)} - a^2 \lambda_2 \hat{\gamma}_{\text{MMLE}(0)} + a^3 EEW_0 \]

\[ EEW_4 = \lambda \hat{\gamma}_{\text{MMLE}(4)} + b^3\hat{\gamma}_{\text{MMLE}(3)} + ab^2\hat{\gamma}_{\text{MMLE}(2)} - a^2 b \lambda_2 \hat{\gamma}_{\text{MMLE}(1)} + a^3 \lambda_2 \hat{\gamma}_{\text{MMLE}(0)} + a^4 EEW_0 \]

\[ EEW_i = \lambda \hat{\gamma}_{\text{MMLE}(i)} + b^i\hat{\gamma}_{\text{MMLE}(i-1)} + ab^{i-1}\hat{\gamma}_{\text{MMLE}(i-2)} + a^2 b \lambda_2 \hat{\gamma}_{\text{MMLE}(i-3)} + a^3 \lambda_2 \hat{\gamma}_{\text{MMLE}(0)} + \ldots + a^{i-2} b^i \hat{\gamma}_{\text{MMLE}(1)} - a^{i-1} \lambda_2 \hat{\gamma}_{\text{MMLE}(0)} + a^i EEW_0 \]

By taking expectation, we get

\[ E(EEW_i) = \lambda \gamma + b \gamma + ab \gamma + a^2 b \lambda_2 \gamma + a^3 \lambda_2 \gamma + \ldots + a^{i-2} b \gamma - a^{i-1} \lambda_2 \gamma + a^i \gamma \]

Replacing \( b = (a\lambda_1 - \lambda_2) \)

\[ E(EEW_i) = \gamma \left( \lambda_1 + (a\lambda_1 - \lambda_2) + a(a\lambda_1 - \lambda_2) + a^2 (a\lambda_1 - \lambda_2) + \ldots + a^{i-2} a^2 (a\lambda_1 - \lambda_2) - a^{i-1} \lambda_2 + a^i \right) \]

\[ E(EEW_i) = \gamma \left\{ \lambda_1 (1 + a + a^2 + a^3 + \ldots + a^{i-1}) - \lambda_2 (1 + a + a^2 + a^3 + \ldots + a^{i-1}) + a^i \right\} \]

\[ E(EEW_i) = \gamma \left\{ (\lambda_1 - \lambda_2)(1 + a + a^2 + a^3 + \ldots + a^{i-1}) + a^i \right\} \]

By using geometric series, we get

\[ E(EEW_i) = \gamma \left\{ (\lambda_1 - \lambda_2) \left( \frac{1 - a^i}{1 - a} \right) + a^i \right\} \]

So,

\[ E(EEW_i) = \gamma \left\{ 1 - a^i + a^i \right\} \]

\[ E(EEW_i) = \gamma \]

Let \( \text{Var}(\hat{\gamma}_{\text{MMLE}(0)}) = v = E(\hat{\gamma}_{\text{MMLE}} - \gamma)^2 \), we get

\[ \text{Var}(EEW_i) = \lambda_1^2 v + b^2 v + a^2 b^2 v + a^4 b^2 \lambda_2^2 v + \ldots + a^{2(t-2)} b^2 v - a^{2(t-1)} \lambda_2^2 v \]

Let \( b = (a\lambda_1 - \lambda_2) \)

\[ \text{Var}(EEW_i) = v \left\{ \lambda_1^2 + (a\lambda_1 - \lambda_2)^2 + a^2 (a\lambda_1 - \lambda_2)^2 + a^4 (a\lambda_1 - \lambda_2)^2 + \ldots + a^{2(t-2)} (a\lambda_1 - \lambda_2)^2 - a^{2(t-1)} \lambda_2^2 \right\} \]

\[ \text{Var}(EEW_i) = v \left\{ \lambda_1^2 + a^2 \lambda_1^2 - 2a\lambda_1 + \lambda_2^2 + a^4 \lambda_1^2 - 2a^3 \lambda_1 + a^2 \lambda_2^2 + \ldots + a^6 \lambda_1^2 - 2a^5 \lambda_1^2 + a^4 \lambda_2^2 \right\} + \ldots + a^{2(t-2)} \lambda_1^2 - 2a^{2t-3} \lambda_1 + a^{2(t-2)} \lambda_2^2 + a^{2(t-1)} \lambda_2^2 \}

\[ \text{Var}(EEW_i) = v \left\{ \lambda_1^2 + a^2 \lambda_1^2 - 2a\lambda_1 + \lambda_2^2 + a^4 \lambda_1^2 - 2a^3 \lambda_1 + a^2 \lambda_2^2 + (a^6 \lambda_1^4 - 2a^5 \lambda_1^3 + a^4 \lambda_2^4) + \ldots + a^{2(t-2)} \lambda_1^2 - 2a^{2t-3} \lambda_1 + a^{2(t-2)} \lambda_2^2 + a^{2(t-1)} \lambda_2^2 \right\} \]
\[ Var(EEW_t) = \nu \left\{ \lambda_1^2 + a^2 \lambda_1^2 - 2a\lambda_1 + \lambda_2^2 + a^4 \lambda_1^2 - 2a^3 \lambda_1 + a^2 \lambda_2^2 + (a^5 \lambda_1^4 - 2a^5 \lambda_1^3 + a^4 \lambda_2^4) + \ldots + a^{2(t-2)} \lambda_1^2 - 2a^{2t-3} \lambda_1 + a^{2(t-2)} \lambda_2^2 + a^{2(t-1)} \lambda_2^2 \right\} \]

\[ Var(EEW_t) = \nu \left\{ \lambda_2^2 + a^2 \lambda_2^2 - 2a\lambda_2 + \lambda_2^2 + a^4 \lambda_2^2 - 2a^3 \lambda_2 + a^2 \lambda_2^2 + (a^5 \lambda_1^4 - 2a^5 \lambda_1^3 + a^4 \lambda_2^4) + \ldots + a^{2(t-2)} \lambda_1^2 - 2a^{2t-3} \lambda_1 + a^{2(t-2)} \lambda_2^2 + a^{2(t-1)} \lambda_2^2 \right\} \]

\[ Var(EEW_t) = \nu \left\{ \lambda_1^2 \left(1 + a^2 + a^4 + \ldots + a^{2(t-1)}\right) + \lambda_2^2 \left(1 + a^2 + a^4 + \ldots + a^{2(t-1)}\right) \right\} \]

\[ -2a \lambda_1 \lambda_2 \left(1 + a^2 + a^4 + \ldots + a^{2(t-2)}\right) \]

The control limits are given as

\[ UCL = \gamma_0 + L\nu \sqrt{\lambda_1^2 + \lambda_2^2} \left(1 - \left(1 - \lambda_1 + \lambda_2\right)^{2t} \right) - 2a \lambda_1 \lambda_2 \left(1 - \left(1 - \lambda_1 + \lambda_2\right)^{2t-2} \right) \]

\[ CL = \gamma_0 \]

\[ LCL = \gamma_0 - L\nu \sqrt{\lambda_1^2 + \lambda_2^2} \left(1 - \left(1 - \lambda_1 + \lambda_2\right)^{2t} \right) - 2a \lambda_1 \lambda_2 \left(1 - \left(1 - \lambda_1 + \lambda_2\right)^{2t-2} \right) \]

### 4.1 Algorithmic Steps

a) Generate a random sample of size \( n = 150 \) on “\( \chi_i \)” from the WPFD are \( x = \beta R^t \) with parameters \( (\beta, \gamma) = (1, 2) \). Where \( R \): random numbers.

b) Compute \( \hat{\gamma}_{\text{MMLE}(t)} \).

c) Repeat steps (a) and (b), for 5000 times and compute \( \hat{\gamma}_{\text{MMLE}(t)}, E\left(\hat{\gamma}_{\text{MMLE}(t)}\right) \) and \( V\left(\hat{\gamma}_{\text{MMLE}(t)}\right) \).

d) Repeat step (c) for 5000 times and compute \( \hat{\gamma}_{\text{MMLE}(t)} \).

e) Compute control limits to construct EEWMA control charts based on \( \hat{\gamma}_{\text{MMLE}(t)} \) (Computed in step (d)).

f) Compute ARL value for each EEWMA control chart that based on \( \hat{\gamma}_{\text{MMLE}(t)} \) given that process is in-control state.

g) Now fix \( ARL_0 = 500 \) for in-control state of the process and search the suitable value of \( L \), so that \( ARL_0 \) for in-control state of process is achieved.
h) Now assume if the process parameter “\(\gamma\)” is shifted by from its true value and compute ARL\(_1\). This step is repeated for different shift values 0, 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.13 and 0.88. Also compute ARL\(_1\) in each case of shift values.

i) Plot ARL\(_s\) values against the values of shift that used in step (g) and (h).

j) It is to note that the procedure of EWMA control chart based on \(^\gamma_{\text{MMLE}}(t)\) observe whether the process following the WPFD is in-control or out of control. If the process is in-control, go to Step (a). Otherwise, record the Run Length, i.e. the process remained in control before it is declared to be out-of-control.

k) Repeat this process 5000 times to obtain the ARLs, SDRLs and percentiles.

4.2 Results and Discussions

We consider the average run length to compare the performance of the proposed estimators. It is taken as the average number of samples that are used before as the process is out of control. ARL\(_0\) indicates the in control ARL, while the ARL\(_1\) describes the out of control ARL. A chart having a larger ARL\(_0\) and smaller ARL\(_1\) is considered better to be used for the process monitoring.

The Tabs. 1–3 is constructed for ARL values when the parameters of underlying process following WPFD using EWMA and EEWMA. Figs. 1–3 presents the ARL values for EWMA and EEWMA. We observed that EEWMA control chart detects earlier on small shifts as compare to EWMA control chart for different choices of \(\lambda_1\) and \(\lambda_2\). The same behaviour is observed from Figs. 1–3.

| Shifts | Proposed EEWMA \(\lambda_1 = 0.10, \lambda_2 = 0.03\) | EWMA \(\lambda = 0.10\) |
|--------|-------------------------------------------------|---------------------|
|        | L = 16.65                                       | L = 16.3            |
| ARL    | SDRL   | P25   | P75   | ARL    | SDRL  | P25   | P75   |
| 0      | 500.509 | 497.0645 | 163.75 | 647.25 | 500.415 | 512.487 | 145.75 | 658.25 |
| 0.01   | 297.865 | 285.0804 | 91.75  | 413.25 | 314.384 | 313.7296 | 92.00  | 438.25 |
| 0.02   | 189.79  | 179.0556 | 65.00  | 250.25 | 200.772 | 191.563  | 70.00  | 263.25 |
| 0.03   | 129.029 | 114.8576 | 46.75  | 174.00 | 136.626 | 129.6943 | 47.00  | 183.0  |
| 0.04   | 88.135  | 73.72819 | 36.0   | 117.0  | 93.874  | 81.6371  | 37.00  | 126.25 |
| 0.05   | 66.053  | 49.85326 | 30.0   | 89.0   | 69.781  | 59.8337  | 28.00  | 91.0   |
| 0.06   | 52.268  | 38.99031 | 24     | 70     | 54.712  | 44.54965 | 23.0   | 73.0   |
| 0.07   | 42.435  | 29.88861 | 20     | 58     | 43.483  | 33.66992 | 19.00  | 59.25  |
| 0.08   | 34.402  | 24.03956 | 18     | 47     | 35.905  | 26.73666 | 18.00  | 46.25  |
| 0.13   | 15.62   | 10.37117 | 10     | 23     | 16.98   | 11.03464 | 9.0    | 22.0   |
| 0.88   | 1       | 0.5663495 | 1     | 2     | 1.35    | 0.5193937 | 1     | 2     |

Table 1: Average run length (ARL), Standard deviation Run length and Percentiles for WPFD under the proposed EEWMA \((\lambda_1 = 0.10, \lambda_2 = 0.03)\) and EWMA \((\lambda = 0.10)\)
Table 2: ARL, Standard deviation Run length and Percentiles for WPFD under the proposed EEWMA ($\lambda_1 = 0.20$, $\lambda_2 = 0.05$) and EWMA ($\lambda = 0.20$)

| Shifts | Proposed EEWMA $\lambda_1 = 0.20$, $\lambda_2 = 0.05$ | EWMA $\lambda = 0.20$ |
|--------|-----------------------------------------------------|----------------------|
|        | ARL        | SDRL       | P25 | P75 | ARL        | SDRL       | P25 | P75 |
| 0      | 500.136    | 493.1804   | 152.0 | 667.5 | 500.606    | 497.0729   | 145.75 | 685.00 |
| 0.01   | 331.041    | 339.1675   | 95.75 | 456.00 | 351.294    | 359.168    | 100.0 | 480.0 |
| 0.02   | 225.598    | 222.0587   | 74.00 | 296.25 | 235.775    | 238.3603   | 71.75 | 318.00 |
| 0.03   | 158.066    | 147.8465   | 51.0 | 217.0 | 170.249    | 169.09     | 51.75 | 228.00 |
| 0.04   | 118.562    | 109.8235   | 38.00 | 163.25 | 127.716    | 118.176    | 40.75 | 178.50 |
| 0.05   | 86.46      | 78.3840    | 30.0 | 119.0 | 92.09      | 84.99995   | 32 | 123 |
| 0.06   | 67.112     | 57.45761   | 25 | 90 | 71.569     | 64.56178   | 25 | 95 |
| 0.07   | 54.088     | 46.67229   | 20 | 74 | 58.698     | 53.25019   | 19.75 | 79.00 |
| 0.08   | 42.084     | 35.33439   | 17 | 57 | 46.133     | 39.63216   | 18 | 63 |
| 0.13   | 17.98      | 14.13157   | 9 | 25 | 19.802     | 16.26079   | 9 | 26 |
| 0.88   | 1         | 0.5033929   | 1 | 2 | 1.237     | 0.4461278   | 1 | 1 |

Table 3: ARL, Standard deviation Run length and Percentiles for WPFD under the proposed EEWMA ($\lambda_1 = 0.30$, $\lambda_2 = 0.15$) and EWMA ($\lambda = 0.30$)

| Shifts | Proposed EEWMA $\lambda_1 = 0.30$, $\lambda_2 = 0.15$ | EWMA $\lambda = 0.30$ |
|--------|-----------------------------------------------------|----------------------|
|        | ARL        | SDRL       | P25 | P75 | ARL        | SDRL       | P25 | P75 |
| 0      | 500.966    | 490.7444   | 143.0 | 685.0 | 500.966    | 497.4742   | 137.00 | 684.25 |
| 0.01   | 355.016    | 360.5669   | 104.0 | 477.5 | 369.121    | 368.5989   | 101.00 | 502.25 |
| 0.02   | 260.056    | 251.002    | 83.75 | 355.00 | 270.565    | 270.2252   | 85.75 | 371.25 |
| 0.03   | 192.215    | 190.9364   | 59.00 | 258.25 | 203.551    | 206.272    | 66.75 | 276.50 |
| 0.04   | 144.88     | 135.8518   | 47.00 | 199.25 | 155.95     | 152.1694   | 47.00 | 213.25 |
| 0.05   | 110.483    | 103.1633   | 37.00 | 152.25 | 121.669    | 114.7251   | 38.00 | 166.0 |
| 0.06   | 84.995     | 79.4971    | 29.00 | 116.25 | 90.49      | 85.77742   | 30.00 | 122.25 |
| 0.07   | 67.581     | 59.47043   | 25.00 | 89.25 | 73.268     | 68.67861   | 23.00 | 102.25 |
| 0.08   | 54.099     | 48.83753   | 21.00 | 72.25 | 59.065     | 54.23303   | 20.0 | 81.0 |
| 0.13   | 22.414     | 18.08457   | 11 | 31 | 24.415     | 21.55605   | 9 | 34 |
| 0.88   | 1         | 0.583489   | 1 | 2 | 1.222     | 0.430001   | 1 | 1 |
Figure 1: ARL for the shape parameter of WPFD using EWMA and EEWMA

Figure 2: ARL for the shape parameter of WPFD using EWMA and EEWMA

Figure 3: ARL for the shape parameter of WPFD using EWMA and EEWMA
5 Simulation Study

In order to see the working procedure of the proposed control charts, a simulation study was carried out. For this purpose, we generated 25 observations from a WPFD for in-control process, and the next 25 observations were generated from the shifted process with 0.01. The estimated values of the proposed EWMA statistic under MMLE were computed for the selected levels of the proposed control charts parameters with $\lambda = 0.20$ and $L = 16.975$. The data and values of the proposed and existing statistic are listed in Tab. 4, and plotted values of these statistics are shown in Fig. 4. Further, the estimated values of the proposed EEWMA statistic under MMLE were computed for the selected levels of the proposed control charts parameters with $\lambda_1 = 0.20$, $\lambda_2 = 0.05$ and $L = 17.245$. The data and values of the proposed and existing statistic are listed in Tab. 4, and plotted values of these statistics are shown in Fig. 5.

Table 4: Simulated data

| $EW_t$ | LCL   | UCL   | $EEW_t$ | LCL   | UCL   |
|--------|-------|-------|---------|-------|-------|
| 1.999742 | 1.958025 | 2.081975 | 1.999922 | 2.001297 | 2.038704 |
| 2.032648 | 1.936621 | 2.103379 | 2.009740 | 1.984710 | 2.055290 |
| 2.022034 | 1.922676 | 2.117324 | 2.013428 | 1.969518 | 2.070482 |
| 2.001997 | 1.912700 | 2.127300 | 2.009999 | 1.956267 | 2.083733 |
| 2.000829 | 1.905254 | 2.134746 | 2.007248 | 1.945017 | 2.094983 |
| 2.026739 | 1.899560 | 2.140440 | 2.013095 | 1.935621 | 2.104379 |
| 2.038168 | 1.895137 | 2.144863 | 2.020617 | 1.927854 | 2.112146 |
| 2.027747 | 1.891667 | 2.148333 | 2.022756 | 1.921479 | 2.118521 |
| 2.034679 | 1.889923 | 2.151077 | 2.026333 | 1.916270 | 2.123730 |
| 2.021543 | 1.886742 | 2.153258 | 2.024896 | 1.912028 | 2.127972 |
| 2.007367 | 1.885002 | 2.154998 | 2.019637 | 1.908582 | 2.131418 |
| 1.991564 | 1.883608 | 2.156392 | 2.011215 | 1.905786 | 2.134214 |
| 1.958908 | 1.882489 | 2.157511 | 1.995523 | 1.903520 | 2.136480 |
| 1.987081 | 1.881590 | 2.158410 | 1.992991 | 1.901684 | 2.138316 |
| 1.974873 | 1.880866 | 2.159134 | 1.987555 | 1.900198 | 2.139802 |
| 1.990290 | 1.880282 | 2.159718 | 1.988376 | 1.898995 | 2.141005 |
| 1.965196 | 1.879811 | 2.160189 | 1.981422 | 1.898022 | 2.141978 |
| 1.987566 | 1.879430 | 2.160570 | 1.983265 | 1.897234 | 2.142766 |
| 1.984756 | 1.879123 | 2.160877 | 1.983712 | 1.896597 | 2.143403 |
| 2.001607 | 1.878874 | 2.161126 | 1.989081 | 1.896081 | 2.143919 |
| 1.993854 | 1.878673 | 2.161327 | 1.990513 | 1.895664 | 2.144336 |
| 2.050537 | 1.878511 | 2.161489 | 2.008520 | 1.895326 | 2.144674 |
| 2.002203 | 1.878379 | 2.161621 | 2.006625 | 1.895052 | 2.144948 |

(Continued)
In Fig. 5, we noted that the proposed EEWMA control chart under MMLE detected a shift at the 32th sample, while in Fig. 4; the EWMA control chart under MMLE could not detect the shift. Hence, this shows that the proposed EEWMA control chart under MMLE has a greater ability to detect smaller shifts earlier, as compared to the EWMA control chart.
6 Conclusions

We have discussed the process monitoring for WPFD. In real life, we may face the situation that any specific process does not follow the normal distribution. But the distribution of the errors in the process becomes WPFD. In the current work, we first estimate the parameters of the distribution of the errors during any process by using MMLE, which was claimed better to estimate the parameters of WPFD. By using the MMLE, we then modified the quality control charts used in literature, such as EWMA and EEWMA control charts. We see that EEWMA control chart under MMLE can be used to monitor the process when the underlying distribution of the errors in process monitoring follows WPFD. It is therefore hoped that the findings of this study will be useful for researchers in different fields of applied sciences. It will be helpful to identify the error in time and making strategies to deal with it. Companies can control their cost and improve product quality by applying the proposed quality chart processes.

Figure 4: Graph of simulated data of the proposed EWMA control chart under MMLE

Figure 5: Graph of simulated data of the proposed EEWMA control chart under MMLE
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