The disordered Fulde–Ferrel–Larkin–Ovchinnikov state in d-wave superconductors

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Abstract. We study the Fulde–Ferrel–Larkin–Ovchinnikov (FFLO) superconducting state in disordered systems. We analyze the microscopic model, in which the d-wave superconductivity is stabilized near the antiferromagnetic quantum critical point, and investigate two kinds of disorder, namely, box disorder and point disorder, on the basis of the Bogoliubov–de Gennes (BdG) equation. The spatial structure of the modulated superconducting order parameter and the magnetic properties in the disordered FFLO state are investigated. We point out the possibility of the ‘FFLO glass’ state in the presence of strong point disorders, which arises from the configurational degree of freedom of the FFLO nodal plane. The distribution function of the local spin susceptibility is calculated and its relationship with the FFLO nodal plane is clarified. We discuss the nuclear magnetic resonance (NMR) measurements for CeCoIn$_5$.

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1. Introduction

Fulde–Ferrel–Larkin–Ovchinnikov (FFLO) superconductivity was predicted in the 1960s by Fulde and Ferrel [1] and also by Larkin and Ovchinnikov [2]. In addition to the \( U(1) \)-gauge symmetry, spatial symmetry is spontaneously broken in the FFLO state owing to the modulation of the superconducting (SC) order parameter. After nearly 40 years of fruitless experimental searches for FFLO states, recent experiments appear to give the first evidence of such a phase (see [3] and references therein). Moreover, the FFLO phase is attracting growing interest in other related fields such as cold fermion gases [4] and high-density quark matter [5].

Extensive studies of the FFLO state were triggered by the discovery of a novel SC phase at high fields and low temperatures in the heavy fermion superconductor CeCoIn\(_5\) [6, 7]. Possible FFLO states have also been discovered in some organic materials [8]–[12]. All these candidate materials are close to the antiferromagnetic quantum critical point (AFQCP), and thus d-wave superconductivity is expected. Although it has been expected that the AFQCP significantly influences the SC state, almost all the theoretical works on the FFLO state are based on the weak coupling theory and neglect the antiferromagnetism. We have examined the FFLO state near the AFQCP by analyzing the two-dimensional Hubbard model using the fluctuation-exchange (FLEX) approximation, and found that the d-wave FFLO state is stable in the vicinity of the AFQCP owing to some strong coupling effects [13].

Another intriguing relationship between FFLO superconductivity and antiferromagnetism has been indicated in CeCoIn\(_5\). Several experimental results suggest the emergence of an FFLO state in CeCoIn\(_5\) [3], [14]–[17]. On the other hand, nuclear magnetic resonance (NMR) and neutron scattering data rather indicate the presence of antiferromagnetic (AFM) order in the high field phase of CeCoIn\(_5\) [18, 19]. The pressure dependence of the phase diagram [17] seems to be incompatible with the AFM order in the uniform SC state, because the AFM order is suppressed by pressure in the other Ce-based heavy fermions (for a review, see [20]), whereas the high field phase of CeCoIn\(_5\) is stabilized by pressure [17]. Therefore, it is expected that the coexistent state of FFLO superconductivity and AFM order will be realized in CeCoIn\(_5\) at ambient pressure, where the AFM moment is induced by the Andreev bound states around the FFLO nodal plane [21].

Another important issue in FFLO superconductivity is the role of disorders. In this paper, we investigate the d-wave FFLO state near the AFQCP in the presence of randomness on the basis of the mean-field Bogoliubov–de Gennes (BdG) equations. The role of disorder in the FFLO state has been investigated by many authors [22]–[27], and it has been shown that the FFLO state is suppressed by the disorders. However, the disorder average is approximately taken into the account in these studies, and therefore, the regular spatial structure is artificially restored. The spatial inhomogeneity is accurately taken into account using the BdG equations adopted in this paper. We focus on the spatial structure of the disordered FFLO state and clarify its relationship with the magnetic properties.

The spatial structure of the s-wave FFLO state in the presence of weak box disorder has been investigated in [28]. It is expected that the response to the disorder will be quite different between the s-wave superconductor and the d-wave one, because s-wave superconductivity is robust against disorder in accordance with Anderson’s theorem [29]. The d-wave FFLO state in the presence of moderately weak point disorders has previously been investigated, and the configuration transition from the two-dimensional structure to the one-dimensional one has been pointed out [30].
In this paper, we show that the spatial structure of disordered FFLO states depends on
the character of the disorders to a significant extent. In the case of weak box disorders, the SC
order parameter has distorted nodes, while a more complicated spatial structure indicating the
FFLO glass state is induced by strong point disorders. In the former, the magnetic properties are
governed by the spatial nodes of the SC order parameters, on which the local spin susceptibility
is larger than that in the normal state. On the other hand, the magnetic properties are dominated
by the disorder-induced antiferromagnetism in the latter.

It is expected that most of our results will be generally applicable to the FFLO state with
non-s-wave pairing. For example, the spatial structure of the SC order parameter is independent
of the details of the Hamiltonian. On the other hand, the disorder-induced antiferromagnetism
is a characteristic property of systems near the AFQCP. Therefore, the magnetic properties in
the presence of point disorders are significantly affected by the AFQCP.

The paper is organized as follows. In section 2, we formulate the BdG theory for the
microscopic model that describes the d-wave superconductivity near the AFQCP. The phase
diagram for the magnetic field and temperature in the clean limit is shown in section 3. The
roles of weak box disorders and strong point disorders are investigated in sections 4 and 5,
respectively. The results are summarized and some discussions are given in section 6.

2. Formulation

Our theoretical analysis is based on the following model:

\[ H = H_0 + H_1, \]  

\[ H_0 = t \sum_{\langle \vec{i}, \vec{j} \rangle, \sigma} c_{\vec{i}, \sigma}^\dagger c_{\vec{j}, \sigma} + t' \sum_{\langle \vec{i}, \vec{j} \rangle, \sigma} c_{\vec{i}, \sigma}^\dagger c_{\vec{j}, \sigma} + \sum_{\vec{i}} (W_{\vec{i}} - \mu) n_{\vec{i}} - g H \sum_{\vec{i}} S_{\vec{i}}^z, \]  

\[ H_1 = U \sum_{\vec{i}} n_{\vec{i}, \uparrow} n_{\vec{i}, \downarrow} + V \sum_{\langle \vec{i}, \vec{j} \rangle} n_{\vec{i}} n_{\vec{j}} + J \sum_{\langle \vec{i}, \vec{j} \rangle} \vec{S}_{\vec{i}} \cdot \vec{S}_{\vec{j}}, \]  

where \( \vec{S}_{\vec{i}} \) is the spin operator at the site \( \vec{i} \), \( n_{\vec{i}, \sigma} \) is the number operator at site \( \vec{i} \) with spin \( \sigma \), and \( n_{\vec{i}} = \sum_{\sigma} n_{\vec{i}, \sigma} \). The pair of brackets \( \langle \vec{i}, \vec{j} \rangle \) and \( \langle \langle \vec{i}, \vec{j} \rangle \rangle \) denote the summation over the nearest neighbour sites and the next nearest neighbour sites, respectively. We assume a two-
dimensional square lattice. The candidate materials for the FFLO state, namely, CeCoIn\textsubscript{5} and
organic superconductors, have quasi-two-dimensional Fermi surfaces. We adopt the unit of
energy \( t = 1 \), and we set \( t'/t = 0.25 \).

We study two kinds of disorder, which are taken into account in the third term of equation
(2). One is box disorder in which the site diagonal potential \( W_{\vec{i}} \) is randomly distributed within
\(-\sqrt{3}W : \sqrt{3}W\). We multiply it by \( \sqrt{3} \) so that the root mean square is \( \bar{W} = \sqrt{\langle |W_{\vec{i}}|^2 \rangle} = W \).
The other is point disorder, where \( W_{\vec{i}} = 0 \) or \( W_{\vec{i}} = W \). We assume \( W \ll \varepsilon_F \) in the former and
\( W \gg \varepsilon_F \) in the latter. Then, the box disorder is regarded as a Born scatterer, while the point
disorder gives rise to the unitary scattering. The randomness is represented by \( W \) in the former,
while the concentration of impurity sites, where \( W_{\vec{i}} = W \), determines the randomness in the
latter. The chemical potential enters equation (2) as \( \mu = \mu_0 + \frac{U}{2} n_0 \), where \( n_0 \) is the number
density at \( U = V = J = H = W = 0 \). We set \( \mu_0 = -0.8 \) for which the electron concentration
is 0.8 < \( n < 0.9 \).
The on-site repulsive interaction is given by $U$, whereas $V$ and $J$ stand for the attractive interaction and AFM exchange interaction between nearest neighbour sites, respectively. We take into account the AFM interaction $J$ to describe the FFLO state near the AFQCP. The interaction $V$ stabilizes the d-wave superconductivity, which we focus on. These features, namely the d-wave superconductivity and AFQCP, can be self-consistently described using the FLEX approximation on the basis of the simple Hubbard model [13]. But here, we assume the interactions $V$ and $J$ for simplicity in order to investigate the inhomogeneous system. The last term in equation (2) describes the Zeeman coupling due to the applied magnetic field. We assume the $g$-factor, $g = 2$.

We examine the model equation (1) using the BdG theory by taking into account the Hartree terms arising from $U$ and $J$ in addition to the mean field of the SC order parameter. The Hartree term due to the attractive interaction $V$ may lead to the charge order if we assume a large attractive $V$. However, we ignore this possibility since the charge ordered state is not stabilized in the systems near the AFQCP, and this is an artificial consequence of the simplified model in equation (1).

The mean-field Hamiltonian is obtained as

$$H = t \sum_{\langle \mathbf{r}_i, \mathbf{r}_j \rangle, \sigma} \bar{c}_{i, \sigma}^\dagger c_{j, \sigma} + t' \sum_{\langle \mathbf{r}_i, \mathbf{r}_j \rangle, \sigma} \bar{c}_{i, \sigma}^\dagger c_{j, \sigma} + \sum_{\mathbf{r}_i, \sigma} W_{i, \sigma} n_{i, \sigma} - \sum_{\langle \mathbf{r}_i, \mathbf{r}_j \rangle, \sigma} \{ \delta \bar{c}_{i, \sigma}^\dagger c_{j, \sigma} + \text{c.c.} \},$$

where $W_{i, \sigma} = W_i + U (n_{i, \sigma} - \frac{1}{2}) - \mu$. The summation of $\delta$ is taken over $\delta = (\pm \frac{\pi}{2}, 0), (0, \pm 1)$. The pair potential is obtained as $\delta \bar{c}_{i, \sigma}^\dagger c_{j, \sigma} + \text{c.c.}$ for $i = j + \delta$, and is otherwise 0. The thermodynamic average $\langle \rangle$ is calculated on the basis of the mean-field Hamiltonian, equation (4). The free energy is obtained as

$$F = - \sum_{\alpha} \log[1 + \exp(-E_{\alpha}/T)] + \sum_{\mathbf{r}_i} W_{i, \downarrow} - \frac{1}{2} \sum_{\mathbf{r}_i, \sigma} \left( U (n_{i, \sigma}) + \frac{1}{2} J \sum_{\delta} \langle S_{i+\delta} \rangle \right) \langle n_{i, \sigma} \rangle + \sum_{\mathbf{r}_i, \mathbf{r}_j} \Delta_{j-i}^\dagger \delta \bar{c}_{i, \alpha}^\dagger c_{j, \alpha} + \text{c.c.},$$

where $E_{\alpha}$ is the energy of the Bogoliubov quasi-particles. We numerically solve the mean-field equations and determine the stable phase by comparing the free energy of self-consistent solutions.

The electron concentration and the magnetization at the site $\mathbf{r}$ are obtained as $n(\mathbf{r}) = \langle n_{\mathbf{r}, \uparrow} + n_{\mathbf{r}, \downarrow} \rangle$ and $M(\mathbf{r}) = \langle n_{\mathbf{r}, \downarrow} - n_{\mathbf{r}, \uparrow} \rangle$, respectively. The order parameter of superconductivity is described by the pair potential $\Delta_{j-i}$. The main component of the pair potential has d-wave symmetry, although a small extended s-wave component is induced in the inhomogeneous system. The d-wave component of the SC order parameter is obtained as

$$\Delta^d(\mathbf{r}) = \Delta_{\mathbf{r}, \mathbf{r}+\mathbf{a}} + \Delta_{\mathbf{r}, \mathbf{r}-\mathbf{a}} - \Delta_{\mathbf{r}, \mathbf{r}+\mathbf{b}} - \Delta_{\mathbf{r}, \mathbf{r}-\mathbf{b}},$$

where $\mathbf{a} = (1, 0)$ and $\mathbf{b} = (0, 1)$.

The numerical calculation is carried out on the $N = 100 \times 100$ lattice in the clean limit and on the $N = 40 \times 40$ lattice for disordered systems. We have confirmed that qualitatively the same results are obtained for 100 × 100 and 40 × 40 lattices in the clean limit.
3. Phase diagram in the clean limit

We first determine the phase diagram for the normal, uniform Bardeen–Cooper–Schrieffer (BCS) and FFLO states in the clean limit. We determine the stable state by comparing the free energy of these states. The order of phase transition is numerically determined by analyzing both the order parameter and free energy. The free energy of the two phases crosses at the first-order phase transition. A discontinuous jump of the SC order parameter also shows the first-order transition. We show that both on-site repulsion $U$ and AFM interaction $J$ are necessary to reproduce the phase diagram of CeCoIn$_5$ [3].

Figure 1 shows the phase diagram for (a) $U = 2.2$ and $J = 0$, (b) $U = 0.9$ and $J = 0.54$ and (c) $U = 0$ and $J = 0.6$. For the parameters in (b), the second-order phase transition occurs between the uniform BCS state and the FFLO state (BCS–FFLO transition). The phase transition from the normal state to the uniform BCS state and the FFLO state is of first order at the temperature below the tricritical point, which is slightly higher than the end point of the BCS–FFLO transition. A conventional second-order SC transition occurs above the tricritical point. These features of the phase diagram in figure 1(b) are consistent with the experimental results for CeCoIn$_5$ [3, 6, 7, 31, 32].

Note that the shape of the BCS–FFLO transition line seems to be incompatible with the experimental results for CeCoIn$_5$. A large positive slope $\partial H_{\text{BC}}(T)/\partial T > 0$, where $H_{\text{BC}}(T)$ is the magnetic field at the BCS–FFLO transition, has been reported in the experiments. This feature does not appear in figure 1(b). That has been reproduced by taking into account the self-energy correction arising from the spin fluctuation near the AFQCP [13]. Here we neglect the self-energy correction for simplicity. This means that the mean-field theory adopted in this paper underestimates the stability of the FFLO state. This is not important for the spatial structure of the FFLO state in the presence of randomness, on which we focus in this paper.

A serious discrepancy between the theory and the experiment is seen in the phase diagram for $J = 0$ (figure 1(a)) and $U = 0$ (figure 1(c)). The FFLO state is completely suppressed for $J = 0$, whereas the first-order transition to the SC state is suppressed for $U = 0$. Thus, the phase diagram near the Pauli–Chandrasekhar–Clogston limit is significantly affected by the electron correlation. These results can be understood on the basis of the Fermi liquid theory. It has been
Figure 2. (a, b) Typical spatial dependence of the d-wave SC order parameter \( \Delta_d(r) \) in the presence of the box disorder for \( W = 0.1 \) and 0.3, respectively. (c, d) Spatial dependence of the local spin susceptibility \( M(r)/H \) for \( W = 0.1 \) and 0.3, respectively. We assume \( T = 0.02 \) and \( H = 0.24 \) in (a) and (c), and adopt \( T = 0.02 \) and \( H = 0.225 \) in (b) and (d). We set \( U = 0.9 \), \( J = 0.54 \) and \( V = 0.8 \) in the following results.

shown that the FFLO state is suppressed by the negative Fermi liquid parameter \( F_{0a} \), while the first-order transition to the SC state is suppressed by the positive parameter \( F_{0a} \) [33]. Within the mean-field theory, the on-site repulsion \( U \) and the AFM interaction \( J \) give rise to negative and positive \( F_{0a} \), respectively. The consistency between figure 1(b) and experimental results [3, 6, 7, 31, 32] indicates that the local spin fluctuation, which is essential for the formation of heavy fermions [34], coexists with the AFM spin fluctuation in CeCoIn$_5$. We adopt the parameters in figure 1(b) in the following sections.

4. Box disorder

We investigate in this section the FFLO state in the presence of box disorders, where the site potential \( W_i \) is randomly distributed within \([-\sqrt{3}W : \sqrt{3}W]\). Since we assume that \( W \ll \varepsilon_F \), all of the sites are weakly disordered.

It has been shown that the two-dimensional FFLO state rather than the one-dimensional FFLO state can be stable [3, 35]. This is the case in our calculation in the clean limit \( (W = 0) \); however, a weak disorder \( (W = 0.1) \) stabilizes the one-dimensional FFLO state as shown in figure 2. This is qualitatively consistent with the results for moderately weak point disorders [30].

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Figures 2(a) and (b) show a typical spatial dependence of the order parameter of d-wave superconductivity $\Delta^d(\vec{r})$ in the FFLO state for $W = 0.1$ and 0.3, respectively. For $W = 0.1$, the spatial structure of the SC order parameter is almost regular, which is approximated by $\Delta^d(\vec{r}) = \Delta_0 \cos(q_0 r_x)$ (figure 2(a)). On the other hand, we see a spatially modulated structure of the SC order parameter for $W = 0.3$ (figure 2(b)). Figures 2(c) and (d) show the spatial dependence of local spin susceptibility $\chi(\vec{r}) = M(\vec{r})/H$ for $W = 0.1$ and 0.3, respectively. In both cases, the magnetization $M(\vec{r})$ is induced around the spatial line node of the SC order parameter, where $\Delta^d(\vec{r}) = 0$. In particular, for a moderate disorder $W = 0.3$, the spatial distribution of the magnetization $M(\vec{r})$ follows the spatial nodes of the SC order parameter.

In order to illuminate the features of the FFLO state, we show the spatial dependences of $\Delta^d(\vec{r})$ and $M(\vec{r})/H$ in the BCS state. Figure 3(a) shows the SC order parameter at $H = 0.18$, where the uniform BCS state is stable in the clean limit. We see that the SC order parameter is nearly uniform in the presence of moderately strong disorders $W = 0.3$, except for the suppression around $\vec{r} = (35, 28)$. The local spin susceptibility $\chi(\vec{r}) = M(\vec{r})/H$ is increased around $\vec{r} = (35, 28)$ because the superconductivity is suppressed there (figure 3(b)). We see a checkerboard structure of the local spin susceptibility, which is similar to high-$T_c$ cuprates [36, 37]. This checkerboard structure is induced by the quasi-particle interference effect [38, 39]. The quasi-particle interference effect occurs in the FFLO state too; however, the spatial dependence due to the quasi-particle interference effect is much smaller than that arising from the inhomogeneous SC order parameter in the FFLO state.

To show the spatial dependences more clearly, we show the local spin susceptibility, SC order parameter and electron concentration along $\vec{r} = (x, 1)$. We see an enhancement of local spin susceptibility around the spatial nodes of the FFLO state, in addition to spatial fluctuation in the atomic scale (figures 4(b) and (e)). The large spatial dependence in the FFLO state is in contrast to the small oscillation for $x < 23$ in the BCS state (figure 4(a)). The latter arises from the quasi-particle interference effect. The spatial fluctuation around $x = 30$ in the BCS state is induced by the inhomogeneity of the SC order parameter (figure 4(d)). The local spin susceptibility in the normal state is governed by the weak atomic scale oscillation (figure 4(c)), which can be regarded as a weak disorder-induced antiferromagnetism (see section 4). We find no clear relationship between the local spin susceptibility and the electron concentration in the FFLO state. The latter is shown in figure 4(f).
Figure 4. Spatial dependences along $\vec{r} = (x_1)$ for $W = 0.3$. We assume the same disorder potential as that in figures 2(b) and (d). Upper panel: the local spin susceptibility in (a) the BCS state ($T = 0.02$ and $H = 0.18$), (b) the FFLO state ($T = 0.02$ and $H = 0.225$) and (c) the normal state ($T = 0.2$ and $H = 0.25$). Lower panel: the d-wave order parameter in (d) the BCS state and (e) the FFLO state. (f) The electron concentration $n(\vec{r})$ in the FFLO state.

Here we show the distribution function of the local spin susceptibility $P(M/H)$, which is expressed as

$$P(x) = \left\langle \frac{1}{N} \sum_\vec{r} \delta(x - (M(\vec{r})/H)) \right\rangle_{av},$$  \hspace{1cm} (7)

where $\langle \rangle_{av}$ denotes the random average. This distribution function is measured by the spectrum of NMR measurements.

Figure 5(a) clearly shows the double peak structure of $P(M/H)$ in the FFLO state for a weak disorder ($W = 0.1$). A peak around $M/H = 0.3$ arises from the region where the SC order parameter is large, whereas the other peak around $M/H = 0.55$ comes from the Andreev bound states localized around the spatial nodes of the SC order parameter. It has been shown that this double peak structure also appears in the FFLO state in the presence of a vortex lattice when the Maki parameter is large [40].

As the disorder potential $W$ increases, the double peak structure of $P(M/H)$ in the FFLO state vanishes, as shown in figure 5(b). The width of the peak in $P(M/H)$ is broader in the FFLO state than in the BCS state. These results seem to be consistent with the NMR measurement of CeCoIn$_5$ [16], which shows a single and broad peak whose position moves to the large $M/H$ in the high field SC phase. Note that the peak in $P(M/H)$ in the BCS state moves to the large $M/H$ with an increase in the disorder potential $W$, since the residual DOS is induced by disorders in the d-wave superconductors [41]. This is in contrast to the FFLO state, where the average of local spin susceptibility $M/H$ is slightly affected by the randomness.

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5. Point disorder

We now turn to the point disorder, in which $N = 40 \times 40$ sites are divided into the host sites where $W_i = 0$ and the impurity sites where $W_i = W$. We assume that $W = 40 \gg \varepsilon_F$ so as to give rise to the unitarity scattering. The impurity concentration is set as $N_{\text{imp}}/N = 0.05$, where $N_{\text{imp}}$ is the number of impurity sites. We investigated ten samples for the impurity distribution, and found that the distribution in figure 6(a) gives a typical result. We adopt this sample in the following results.

Figure 6(b) shows the suppression of the SC order parameter around the impurity sites in the BCS state. We see that the local spin susceptibility is significantly enhanced around the
impurity sites (figure 6(c)). The maximum of the local spin susceptibility is much larger than the spin susceptibility in the normal state of clean systems. This is because of the disorder-induced antiferromagnetism, which has been investigated in the nearly AFM Fermi liquid state [42], and in the pseudogap state [43] of high-$T_c$ cuprates. The disorder-induced antiferromagnetism is a ubiquitous phenomenon in the systems near the AFQCP, such as high-$T_c$ cuprates, organic materials and heavy fermion systems. Clear experimental evidence for the disorder-induced antiferromagnetism has been obtained in high-$T_c$ cuprates [44]–[46].

A complicated spatial structure is realized at high fields, where the FFLO state is stable in the clean limit. Then, the free energy shows a multi-valley structure. There are many local minima of free energy with respect to the spatial structure of the SC order parameter. Figures 7(a)–(e) show five examples of the self-consistent solutions of the BdG equation for the impurity distribution shown in figure 6(a). The local spin susceptibility in each solution is shown in figures 7(f)–(j). The difference in condensation energy is small between these states. The condensation energy is maximum in the ‘FFLO2’ state shown in figure 7(b) among the solutions obtained by us. However, we obtain the solution of the ‘FFLO3’ state shown in figure 7(c) when we choose the SC order parameter near $T_c$ as an initial state of the mean-field equation. The ‘FFLO1’ state in figure 7(a) is obtained when the uniform BCS state is chosen to be an initial state. This means that the FFLO3 state can be stabilized as a metastable state by decreasing the temperature through $T_c$, whereas the FFLO1 state may be realized by increasing the magnetic field through the BCS–FFLO transition.

The condensation energy is increased by aligning the spatial nodes of the SC order parameter on the ‘dirty region’ where the local concentration of impurity sites is large. Many spatial configurations of the SC order parameter have a similar condensation energy because there are many configurations of spatial nodes that match the dirty region. The situation is similar to the vortex glass state which is induced by the configurational degree of freedom of the quantum vortices [47, 48]. The analogy with the vortex glass state indicates the possibility of an ‘FFLO glass’ state, which is realized by the paramagnetic de-pairing effect in random systems.

To decrease the magnetic energy, the spatial node is also induced in the ‘clean region’ at high fields. We found that the local spin susceptibility is enhanced around the spatial nodes in

Figure 7. Five self-consistent solutions of the BdG equation at $T = 0.02$ and $H = 0.225$. (a–e) the SC order parameter; (f–j) the local spin susceptibility.
Figure 8. The local spin susceptibility along $\vec{r} = (x, 1)$ in the BCS state (green), the FFLO2 state shown in figures 7(b) and (g) (blue) and the normal state (red). We assume that $T = 0.02$ and $H = 0.18$ in the BCS state, $T = 0.02$ and $H = 0.225$ in the FFLO2 state and $T = 0.06$ and $H = 0.25$ in the normal state, respectively.

To clarify the local spin susceptibility arising from the disorder-induced antiferromagnetism and that from the spatial nodes of the SC order parameter, we show the $M(\vec{r})/H$ along $\vec{r} = (x, 1)$ in figure 8. The normal state, the FFLO2 state and the BCS state are shown for comparison. As shown in figure 6(a), the system is clean at $x < 20$, while it is dirty at $x > 20$. Figure 8 shows that the disorder-induced antiferromagnetism occurs and gives rise to the significant oscillation of the magnetization in the dirty region ($x > 20$). This is a ubiquitous phenomenon near the AFQCP in the absence of translational symmetry. We see that the disorder-induced antiferromagnetism is enhanced in the FFLO state as well as the BCS state. The local spin susceptibility is more significantly affected by the superconductivity in the clean region ($x < 20$). It is shown that the spin susceptibility is larger in the FFLO2 state than in the BCS state because of the presence of a spatial node around $x = 15$ (figure 7(b)).

Figure 9 shows the distribution function of the local spin susceptibility $P(M/H)$, which is defined as

$$P(x) = \left\langle \frac{1}{N} \sum_{\vec{r}} \delta(x - M(\vec{r})/H) \right\rangle_{av}.$$  

The summation $\sum_{\vec{r}}$ is taken over the host sites and therefore the contribution from the impurity sites is eliminated in equation (8). The double peak structure appears in the BCS state as well as in the FFLO2 state because of the disorder-induced antiferromagnetism. This structure vanishes in the FFLO3 state in which many spatial nodes exist in the SC order parameter. These results are incompatible with the NMR measurements for CeCoIn$_5$ [16, 18]. The width of the peak hardly changes through the BCS–FFLO transition in contrast to [16]. This means that the model based on the point disorder is not relevant for CeCoIn$_5$. However, the point disorders can be systematically induced by substituting Ce ions by La ions, or In ions by Cd ions.
Figure 9. Distribution function of the local spin susceptibility $P(M/H)$ in the presence of 5% point disorders. The BCS state (green dash-dotted line), the FFLO2 state in figures 7(b) and (g) (black dashed line), the FFLO3 state in figures 7(c) and (h) (blue solid line) and the normal state (red dashed line) are shown for comparison. The parameters $H$ and $T$ in each state are the same as those in figures 7 and 8.

Figure 10. (a) Typical spatial dependence of the SC order parameter and (b) the distribution function of the local spin susceptibility in the presence of 2% point disorders. We assume that $T=0.02$ and $H=0.235$.

Therefore, it is interesting to investigate the SC state in Ce$_{1-x}$La$_x$CoIn$_5$ and CeCoIn$_5$-$x$Cd$_x$ [49] at high fields.

Before closing this section, we briefly discuss the case of dilute impurities. Figure 10(b) shows the distribution function of the local spin susceptibility in the presence of 2% point disorders. We show the result for a typical distribution of impurities that leads to the SC order parameter shown in figure 10(a). We see the structures around $M/H=0.5$ and 0.8 in addition to the pronounced peak at $M/H=0.25$. The plateau around $M/H=0.5$ is because of the spatial nodes of the SC order parameter, whereas the structure around $M/H=0.8$ arises from the disorder-induced antiferromagnetism.
6. Summary and discussion

We investigated the disordered FFLO state with a focus on the spatial structure of SC order parameter and the magnetic properties. In particular, the d-wave superconductivity near the AFQCP has been investigated in detail. It has been shown that the spatial dependence of the SC order parameter is relatively simple in the case of the box disorder; the FFLO nodal plane is modulated. On the other hand, the spatial structure is complicated in the presence of point disorders. Then, the spatial nodes are strongly pinned to the locally dirty region. There are many configurations in which the nodes are pinned to the point disorders, and therefore a glassy behaviour appears. We called this state ‘FFLO glass’ in analogy with the vortex glass state.

We have shown that the magnetic properties in the SC state significantly depend on the character of disorders. In the FFLO state with box disorders, the magnetic properties are governed by the nodal plane of the SC order parameters, on which the local spin susceptibility is larger than that in the normal state. On the other hand, the magnetic properties in the BCS state are mainly determined by the quasi-particle interference effect, which gives rise to an oscillation with a small amplitude. A weak disorder-induced antiferromagnetism is induced by the inhomogeneity of the SC order parameters in the BCS state as well as the FFLO state.

The magnetic properties are dominated by the disorder-induced antiferromagnetism in the presence of point disorders. We found that the disorder-induced antiferromagnetism is enhanced by the superconductivity in both BCS and FFLO states. The local spin susceptibility in the locally clean region is suppressed in the BCS state, whereas it is increased in the FFLO state owing to the spatial nodes of the SC order parameter.

Finally, we examine the distribution function of the local spin susceptibility $P(M/H)$ and discuss the NMR measurements for CeCoIn$_5$. It is known that the distribution function shows a double peak structure in the clean limit [40]. We have shown that the two peaks merge into a single peak in the presence of box disorders and/or point disorders. However, the distribution function is affected in a different way by box disorder and point disorder. The single peak structure is induced by the box disorders because of the displacement of FFLO nodes, while the disorder-induced antiferromagnetism is the main cause of the broad single peak in the presence of point disorders. We can distinguish these two cases by analyzing the line width of $P(M/H)$. The line shape of $P(M/H)$ is very broad and its width does not change through the BCS–FFLO transition in the case of point disorder (figure 9). This is because the local antiferromagnetism occurs near the point disorders in both BCS and FFLO states (figure 8). On the other hand, the line shape is significantly broadened in the FFLO state through the BCS–FFLO transition in the case of box disorders, as shown in figure 5. The latter seems to be consistent with NMR measurements for CeCoIn$_5$ at the In(1) site [16].

Another NMR spectrum at the In(2) site has shown the double peak structure with large splitting and indicated the AFM order in the high field phase of CeCoIn$_5$ [18]. Clear experimental evidence for the AFM order has been obtained by recent neutron scattering measurements [19]. We have shown that the AFM order can occur in the FFLO state near the AFQCP, and the phase diagram suggested in [18, 19] is consistent with the coexistence of antiferromagnetism and FFLO superconductivity [21]. Although the double peak structure of the NMR line shape is also induced by the disorder-induced antiferromagnetism (figure 9), this is not the case with CeCoIn$_5$. The direction of the AFM moment is parallel to the applied magnetic field in the case of the disorder-induced antiferromagnetism. However, the neutron scattering measurement shows the magnetic moment perpendicular to the magnetic field.
field [19], which indicates spontaneous symmetry breaking. Therefore, true long-range order of antiferromagnetism seems to occur in the experiments of [18, 19].

The results of this paper suggest that the single peak structure of the NMR spectrum at the In(1) site, which is less sensitive to the AFM order than the In(2) site, can be caused by the disorder. But the AFM order could be another cause. We are planning to investigate the magnetic properties in the coexistent state of AFM order and FFLO superconductivity. From the experimental point of view, it would be interesting to investigate the pressure effect in CeCoIn$_5$. It is expected that the AFM order is suppressed by pressure, whereas the FFLO superconductivity is enhanced [13]. The latter has been observed in [17]. Therefore, the pure FFLO state may be realized at high pressures.

According to these considerations, the point disorder is not the main source of the randomness in CeCoIn$_5$. However, point disorder can be induced by substituting Ce ions with La ions or In ions with Cd ions. Thus, Ce$_{1-x}$La$_x$CoIn$_5$ and CeCoIn$_{5-x}$Cd$_x$ [49] will be a playground for FFLO superconductivity with point disorders.

In summary, we have investigated the FFLO SC state in the presence of randomness. The spatial structure of the SC order parameter and magnetic properties are clarified in detail. It has been proposed that the experimental results on CeCoIn$_5$ can be understood by assuming the FFLO state realized in the high field phase and by taking into account both the AFM spin correlation and a weak disorder.

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