Different Kinds of Rough Graph and Their Representation Forms

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Abstract. Based on definition and basic properties of rough graph, this paper gives different kinds of rough graph, which are weighted rough graph and directed rough graph. By using representation form of classical rough graph, representation forms of these rough graphs are also given, which can help us to research rough problems conveniently.

Introduction

By combining rough set theory [1] with graph theory [2], reference [3] presents rough graph theory. Rough graph theory is generation of graph theory [4, 5]. Moreover, it improves computing ability of rough set theory. And by using algorithms in rough graph, many actual problems have been solved successfully [6-10].

In this paper, we also do further rough graph theory research and construct weighted rough graph and directed rough graph by enduing the edges of rough graph with weight attribute and direction attribute respectively. Moreover, we give representation forms of weighted rough graph and directed rough graph so that we can research rough problems intuitively.

Weighted Rough Graph and its Representation Forms

Definition 1. Given universe graph $U = (V, E) , V = \{v_1, v_2, \ldots, v_r\} , E = \bigcup e_k(v_i, v_j)$.

$\forall e \in E$ , let mapping $\omega : e \rightarrow \omega(e)$ , the real number $\omega(e)$ is called edge weight of $e$ .

$\Omega([e_{uv}],R) = f(\omega(e))$ is called class weight of edge equivalence class $[e_{uv}]_R$. Where $e \in [e_{uv}]_R$,

$[e_{uv}]_R$ is edge equivalence class between vertex $u$ and vertex $v$ respect to attribute $R$, $f$ is some function.

Definition 2. Rough graph $T = (\overline{R(T)}, \overline{\Omega(T)})$ with class weight of their edge equivalence class is called weighted rough graph.

In the following paragraph, the representation forms of weighted rough graph will be given. They are weight matrix and weight list. It is easy to know that weight matrix and weight list is the extension of adjacency matrix and edge list of classical rough graph respectively[4].

Definition 3. For weighted rough graph $T = (W, X, \Omega)$, supposed $W = \{v_1, v_2, \ldots, v_r\}$ is vertex set of $T$, there corresponds a block matrix $W(T) = [f_{ij}]$, called weight matrix of $T$, where $f_{ij} = (s_{ij}^1, s_{ij}^2, \ldots, s_{ij}^r), s_{ij}' = (\alpha_{ij}^1, \alpha_{ij}^2, \ldots, \alpha_{ij}^r) , \alpha_{ij}^r \in \Omega$ indicates the weight of the $l$th edge
in the $t$ th edge equivalence class between $v_i$ and $v_j$ in $T$. $l = 1, \ldots, r'_i , r'_j \in N$ indicates the number of edge in the $t$ th edge equivalence class between $v_i$ and $v_j$ in $T$. $i, j = 1, \ldots, v$, $t = 1, \ldots, n$, $n$ is total number of edge equivalence class $[e]_R$ in rough graph. Especially, if $i = j$, then let $\omega'^t_{ij} = L$. If $e(v_i, v_j) \notin X$ and $i \neq j$, then let $\omega'^t_{ij} = K$. According to real need, $L$ in the above definition can not be given or is taken as $0, -\infty$ or $\infty$, and $K$ can be taken as $-\infty$ or $\infty$.

Given weighted rough graph $T = (W, X, \Omega)$, supposed $W = \{v_1, v_2, \ldots, v_r\}$ is vertex set of $T$. Table 1 gives weight list of weighted rough graph.

| $\omega'^1_{ij}$ | $\omega'^2_{ij}$ | $\omega'^3_{ij}$ | $\omega'^4_{ij}$ | $\omega'^5_{ij}$ | $\omega'^6_{ij}$ | $\omega'^7_{ij}$ |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|                 |                 |                 |                 |                 |                 |                 |

Where, the serial numbers in the first two rows are given according to the lexicographic order of vertices in rough graph. In the third row, $\omega'^t_{ij} \in \Omega$ indicates the weight of the $t$ th edge in the $t$ th edge equivalence class between $v_i$ and $v_j$ in $T$. $l = 1, \ldots, r'_i , r'_j \in N$ indicates the number of edge in the $t$ th edge equivalence class between $v_i$ and $v_j$ in $T$. $i, j, k = 1, \ldots, v$, $t = 1, \ldots, n$, $n$ is total number of edge equivalence class $[e]_R$ in rough graph. Especially, if $i = j$, then let $\omega'^t_{ij} = L$; if $e(v_i, v_j) \notin X$ and $i \neq j$, then let $\omega'^t_{ij} = K$. According to real need, $L$ can not be given or is taken as $0, -\infty$ or $\infty$, and $K$ can be taken as $-\infty$ or $\infty$.

**Directed Rough Graph and its Representation Forms**

**Definition 4.** Given universe graph $U = (V, E)$, $V = \{v_1, v_2, \ldots, v_r\}$, $E = \bigcup_{\forall e \in E} \{e\}$, $\forall e \in E$, let mapping $d : e \rightarrow d(e)$, $d(e)$ is called edge direction of $e$ and $d(e) \in \{+,-\}$. The edge with direction is called arc, which is denoted as $a$. If $d(e) = +$, then arc $a$ is called with positive direction, which is denoted as $a^+(v_i, v_j)$, where, $v_i$ is called tail of $a^+$, $v_j$ is called head of $a^+$. If $d(e) = -$, then arc $a$ is called with negative direction, which is denoted as $a^-(v_i, v_j)$, where, $v_j$ is called head of $a^-$, $v_i$ is called tail of $a^-$. 

**Definition 5.** Let $d(e) \in \{+,-\}$ is direction of edge $e$, given $R$-edge equivalence class $[e_{uv}]_R$ between vertex $u$ and vertex $v$, where, $R$ is attribute set on $E$. If $d(e) \subseteq R$, then $d(e)$ is called class direction of edge equivalence class $[e_{uv}]_R$, where, $e \in [e_{uv}]_R$. The edge equivalence class with class direction is called arc equivalence class, which is denoted as $[a_{uv}]_R$. If $d(e) = +$, then arc equivalence class $[a_{uv}]_R$ is called with positive direction, which is denoted as $[a_{uv}]_R^+$, where, $u$ is called tail of $[a_{uv}]_R^+$, $v$ is called head of $[a_{uv}]_R^+$. If $d(e) = -$, then arc equivalence class $[a_{uv}]_R$ is called with negative direction, which is denoted as $[a_{uv}]_R^-$. If $d(e) = -$, then arc equivalence class $[a_{uv}]_R$ is called with negative direction, which is denoted as $[a_{uv}]_R^-$, where, $u$ is called head of $[a_{uv}]_R^-$, $v$ is called tail of $[a_{uv}]_R^-$. 

**Table 1. Weight list of weighted rough graph.**
Definition 6. Rough graph \( T = (\overline{R}(T), \overline{R}(T)) \) with class direction of its arc equivalence class is called directed rough graph, which is denoted as \( D = (\overline{R}(D), \overline{R}(D)) \) or \( D = (V, A) \). Rough graph \( T \) is called basic rough graph of directed rough graph \( D \).

Assumption In the following discussion, if supposed the arc which takes \( v_i \) as tail and \( v_j \) as head has positive direction, then the arc which takes \( v_j \) as head and \( v_i \) as tail has negative direction. Vice versa.

Just like rough graph, directed rough graph can also be simply represented as a figure. This figure representation of directed rough graph can be obtained by adding arrow to edges in figure of its basic rough graph, where, arrow directs to the head of arc. However, the simplest representation method is a matrix or a table which takes smaller storage space. Similar to the definitions of adjacency matrix and edge list of rough graph[4] and weight matrix and weight list of weighted rough graph, in the following paragraph we will give representation forms of directed rough graph, which are adjacency matrix and arc list.

Definition 7. For any directed rough graph \( D = (\overline{R}(D), \overline{R}(D)) \), let \( \{v_1, v_2, \cdots, v_r\} \) is vertex set of \( D \), there corresponds a block matrix \( A(D) = [a_{ij}] \), which is called arc adjacency matrix of \( D \), where, when \( i \leq j = 1, 2, \cdots, \nu \), 
\[
\begin{align*}
  a_{ij} &= (r_{ij}^{-}, r_{ij}^{m+}, \cdots, r_{ij}^{m'}) \\
  r_{ij}^{-}, r_{ij}^{m+} &\in N 
\end{align*}
\]
indicates the number of arc in the \( h \) th arc equivalence class which takes \( v_i \) as tail and \( v_j \) as head in \( D \). \( t = 1, \cdots, l \), \( l \) is the total number of arc equivalence class \([a]_{\nu} \) which takes \( v_i \) (\( i \leq j \)) as tail and \( v_j \) (\( i \leq j \)) as head in \( D \). When \( i \geq j, j = 1, \cdots, \nu \), 
\[
\begin{align*}
  a_{ij} &= (r_{ij}^{l+}, r_{ij}^{-}, \cdots, r_{ij}^{m-}) \\
  r_{ij}^{-}, r_{ij}^{l+} &\in N 
\end{align*}
\]
indicates the number of arc in the \( h \) th arc equivalence class which takes \( v_i \) as tail and \( v_j \) as head in \( D \). \( h = 1, \cdots, m \), \( m \) is the total number of arc equivalence class \([a]_{\nu} \) which takes \( v_i \) (\( i \geq j \)) as tail and \( v_j \) (\( i \geq j \)) as head in \( D \).

Definition 8. For arc adjacency matrix \( A(D) \) of directed rough graph \( D = (\overline{R}(D), \overline{R}(D)) \), there corresponds a pair of arc adjacency matrix, that is \( A(D) = (\overline{A}(D), \overline{A}(D)) \). \( A(D) = [b_{ij}] \) is called lower approximate arc adjacency matrix of \( D \), where, when \( i \leq j = 1, 2, \cdots, \nu \), 
\[
\begin{align*}
  b_{ij} &= (z_{ij}^{1+}, z_{ij}^{2+}, \cdots, z_{ij}^{l+}) \\
  z_{ij}^{1+}, z_{ij}^{2+} &\in N 
\end{align*}
\]
indicates the number of arc in the \( t \) th arc equivalence class which takes \( v_i \) as tail and \( v_j \) as head in \( \overline{R}(D) \). \( t = 1, \cdots, l \), \( l \) is the total number of arc equivalence class \([a]_{\nu} \) which takes \( v_i \) (\( i \leq j \)) as tail and \( v_j \) (\( i \leq j \)) as head in \( D \). When \( i \geq j, j = 1, \cdots, \nu \), 
\[
\begin{align*}
  b_{ij} &= (z_{ij}^{m-}, z_{ij}^{m-}, \cdots, z_{ij}^{l}) \\
  z_{ij}^{m-}, z_{ij}^{l} &\in N 
\end{align*}
\]
indicates the number of arc in the \( h \) th arc equivalence class which takes \( v_i \) as tail and \( v_j \) as head in \( \overline{R}(D) \). \( h = 1, \cdots, m \), \( m \) is the total number of arc equivalence class \([a]_{\nu} \) which takes \( v_i \) (\( i \geq j \)) as tail and \( v_j \) (\( i \geq j \)) as head in \( D \).

\( \overline{A}(D) = [c_{ij}] \) is called upper approximate arc adjacency matrix of \( D \), where, when \( i \leq j = 1, 2, \cdots, \nu \), 
\[
\begin{align*}
  c_{ij} &= (q_{ij}^{1+}, q_{ij}^{2+}, \cdots, q_{ij}^{l+}) \\
  q_{ij}^{1+}, q_{ij}^{2+} &\in N 
\end{align*}
\]
indicates the number of arc in the \( t \) th arc equivalence class which takes \( v_i \) as tail and \( v_j \) as head in \( \overline{R}(D) \). \( t = 1, \cdots, l \), \( l \) is the total number of arc equivalence class \([a]_{\nu} \) which takes \( v_i \) (\( i \leq j \)) as tail and \( v_j \) (\( i \leq j \)) as head in \( D \). When \( i \geq j, j = 1, \cdots, \nu \), 
\[
\begin{align*}
  c_{ij} &= (q_{ij}^{m-}, q_{ij}^{m-}, \cdots, q_{ij}^{l}) \\
  q_{ij}^{m-}, q_{ij}^{l} &\in N 
\end{align*}
\]
indicates the number of arc in the \( h \) th
arc equivalence class which takes \( v_i \) as tail and \( v_j \) as head in \( \overline{R}(D) \). \( h = 1, \ldots, m \), \( m \) is the total number of arc equivalence class \([a]_R\) which takes \( v_i \) \((i \geq j)\) as tail and \( v_j \) \((i \geq j)\) as head in \( D \).

It is obvious that when all arcs belong to one arc equivalence class with respect to \( R \), and \( R \) does not contain direction attribute, the above arc adjacency matrix of directed rough graph is adjacency matrix of classical directed graph.

**Definition 9.** For the pair of arc adjacency matrix \((A(D), \overline{A}(D))\) of directed rough graph \( D = (\overline{R}(D), \overline{R}(D)) \), there corresponds a pair of class adjacency matrix \((T(D), \overline{T}(D))\). \( \overline{T}(D) = [d_{ij}] \) is called lower approximate class adjacency matrix of \( D \), where, when \( i \leq j = 1, 2, \ldots, \nu \), \( d_{ij} = (x_{ij}^{l+}, x_{ij}^{l-}, \ldots, x_{ij}^{l-}) \),

\[
x_{ij}^{l+} = \begin{cases} 1, & \text{there exist the } t\text{th edge equivalence class which} \\
0, & \text{takes } v_j \text{ as tail and } v_j \text{ as head in } \overline{R}(D) \end{cases}
\]

\( t = 1, \ldots, l \), \( l \) is the total number of arc equivalence class \([a]_R\) which takes \( v_i \) \((i \leq j)\) as tail and \( v_j \) \((i \leq j)\) as head in \( D \). When \( i \geq j = 1, 2, \ldots, \nu \), \( d_{ij} = (x_{ij}^{l-}, x_{ij}^{l+}, \ldots, x_{ij}^{l-}) \),

\[
x_{ij}^{l-} = \begin{cases} 1, & \text{there exist the } t\text{th edge equivalence class which} \\
0, & \text{takes } v_j \text{ as tail and } v_j \text{ as head in } \overline{R}(D) \end{cases}
\]

\( t = 1, \ldots, l \), \( l \) is the total number of arc equivalence class \([a]_R\) which takes \( v_j \) \((i \geq j)\) as tail and \( v_j \) \((i \geq j)\) as head in \( D \). When \( i \geq j = 1, 2, \ldots, \nu \), \( d_{ij} = (x_{ij}^{l-}, x_{ij}^{l+}, \ldots, x_{ij}^{l-}) \),

\[
y_{ij}^{h-} = \begin{cases} 1, & \text{there exist the } h\text{th edge equivalence class which} \\
0, & \text{takes } v_j \text{ as tail and } v_j \text{ as head in } \overline{R}(D) \end{cases}
\]

\( h = 1, \ldots, m \), \( m \) is the total number of arc equivalence class \([a]_R\) which takes \( v_i \) \((i \geq j)\) as tail and \( v_j \) \((i \geq j)\) as head in \( D \). \( \overline{T}(D) = [e_{ij}] \) is called upper approximate class adjacency matrix of \( D \), where, when \( i \leq j = 1, 2, \ldots, \nu \), \( e_{ij} = (y_{ij}^{l+}, y_{ij}^{l-}, \ldots, y_{ij}^{l-}) \),

\[
y_{ij}^{l-} = \begin{cases} 1, & \text{there exist the } t\text{th edge equivalence class which} \\
0, & \text{takes } v_j \text{ as tail and } v_j \text{ as head in } \overline{R}(D) \end{cases}
\]

\( t = 1, \ldots, l \), \( l \) is the total number of arc equivalence class \([a]_R\) which takes \( v_i \) \((i \leq j)\) as tail and \( v_j \) \((i \leq j)\) as head in \( D \). When \( i \geq j = 1, 2, \ldots, \nu \), \( e_{ij} = (y_{ij}^{l-}, y_{ij}^{l+}, \ldots, y_{ij}^{l-}) \),

\[
y_{ij}^{l+} = \begin{cases} 1, & \text{there exist the } h\text{th edge equivalence class which} \\
0, & \text{takes } v_j \text{ as tail and } v_j \text{ as head in } \overline{R}(D) \end{cases}
\]
\( h = 1, \ldots, m, m \) is the total number of arc equivalence class \([a]_R\) which takes \( v_i \) (\( i \geq j \)) as tail and \( v_j \) (\( i \geq j \)) as head in \( D \).

Given directed rough graph \( D = (V, A), V = \{v_1, v_2, \ldots, v_v\} \) is vertex set of \( D \). Table 2 shows arc list of directed rough graph.

| \( \ldots \) | \( i \) | \( \ldots \) | \( j \) | \( \ldots \) | \( \ldots \) | \( j \) | \( \ldots \) |
| \( \ldots \) | \( j \) | \( \ldots \) | \( i \) | \( \ldots \) | \( \ldots \) | \( i \) | \( \ldots \) |
| \( \ldots \) | \( r_{ij}^{1+} \) | \( \ldots \) | \( r_{ij}^{l+} \) | \( \ldots \) | \( r_{ji}^{l-} \) | \( \ldots \) | \( r_{ji}^{m-} \) | \( \ldots \) |

Where, the first row shows serial number of tail vertices, the second shows serial number of head vertices. In the third row, \( r_{ij}^{l+} \in N \) indicates the number of arc in the \( t \) th arc equivalence class which takes \( v_i \) as tail and \( v_j \) as head in \( D \). \( t = 1, \ldots, l \), \( l \) is the total number of arc equivalence class \([a]_R\) which takes \( v_i \) as tail and \( v_j \) as head in \( D \). \( r_{ij}^{h+} \in N \) indicates the number of arc in the \( h \) th arc equivalence class which takes \( v_j \) as tail and \( v_i \) as head in \( D \). \( h = 1, \ldots, m \), \( m \) is the total number of arc equivalence class \([a]_R\) which takes \( v_j \) as tail and \( v_i \) as head in \( D \).

**Conclusion**

To research rough problems by using rough graph theory conveniently, this paper constructs weighted rough graph and directed rough graph by enduing the edges of rough graph with weight attribute and direction attribute respectively. Moreover, representation forms of weighted rough graph and directed rough graph are presented which makes foundation for real computation. In the future, we will do further algorithm research in different kinds of rough graph so that more and more rough problems can be solved by applying rough graph theory.

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