Denoising the ECG Signal Using Ensemble Empirical Mode Decomposition

Wahiba Mohguen
CCNS Laboratory
Department of Electronics
Faculty of Technology
University of Ferhat Abbas - Setif 1
Setif, Algeria
wasotel@yahoo.fr

Saad Bouguezel
CCNS Laboratory
Department of Electronics
Faculty of Technology
University of Ferhat Abbas - Setif 1
Setif, Algeria
bouguezelsaad@yahoo.com

Abstract—In this paper, a novel electrocardiogram (ECG) denoising method based on the Ensemble Empirical Mode Decomposition (EEMD) is proposed by introducing a modified customized thresholding function. The basic principle of this method is to decompose the noisy ECG signal into a series of Intrinsic Mode Functions (IMFs) using the EEMD algorithm. Moreover, a modified customized thresholding function was adopted for reducing the noise from the ECG signal and preserve the QRS complexes. The denoised signal was reconstructed using all thresholded IMFs. Real ECG signals having different Additive White Gaussian Noise (AWGN) levels were employed from the MIT-BIH database to evaluate the performance of the proposed method. For this purpose, output SNR (SNRout), Mean Square Error (MSE), and Percentage Root Mean square Difference (PRD) parameters were used at different input SNRs (SNRin). The simulation results showed that the proposed method provided significant improvements over existing denoising methods.

Keywords—denoising; ECG; EMD; EEMD; customized thresholding

I. INTRODUCTION

Empirical Mode Decomposition (EMD) is a powerful algorithm for splitting non-stationary signals [1]. The goal of EMD is to represent the signals as sums of zero-mean oscillating components, named Intrinsic Mode Functions (IMFs) via a sifting process [1]. Signal reconstruction is achieved by summing all IMFs and the residual. EMD techniques have been used for signal denoising, and specifically, those based on thresholding were developed in [2-10]. A denoising technique can be based on signal estimation using all the previously thresholded IMFs [3-13]. Since the useful information of the signal is often concentrated on low-frequency IMFs (last IMFs) and the noise is primarily located in high-frequency IMFs (first IMFs), another approach is to perform denoising by partial construction of the signal with the IMFs that contain useful information [2, 14]. Authors in [2] proposed a method for estimating the energy of noisy IMFs from a theoretical model and IMFs’ energies of the test signal, and the signal was reconstructed partially by using only the IMFs that contained useful information, eliminating those that essentially maintained noise. In [14], an EMD consecutive mean square error (EMD-CMSE) method was developed for IMF selection. Since Electrocardiogram (ECG) signals are nonstationary and nonlinear methods, a wavelet thresholding technique was proposed in [15, 16] without preserving ECG components such as QRS complexes [17]. A customized thresholding function was proposed in [18] to overcome the disadvantages of hard and soft thresholding functions [15, 16]. EMD combined with a customized thresholding function (EMD-Custom) can be useful for reducing noise and significantly improve the results of EMD soft and hard thresholding [3, 4, 8]. To overcome the drawbacks of EMD such as mode mixing (presence of oscillations of different amplitudes in one mode) [1], a variant of the EMD algorithm called Ensemble Empirical Mode Decomposition (EEMD) was proposed in [19]. EEMD was based on averaging the modes obtained from EMD applied to several trials of Additive Gaussian White Noise (AWGN) added to the signal. The EEMD decomposition resolved efficiently the mode mixing and has been widely used in noise reduction. Moreover, EEMD achieved better denoising performance than EMD with a reduced number of trials.

The main objective of this paper is to propose a denoising method for ECG signals using EEMD and a modified custom thresholding function. The basic principle of the proposed method is to decompose the noisy signal into a series of IMFs using the EEMD algorithm and then use the modified custom thresholding function. The denoised signal is reconstructed using all the thresholded IMFs. Denoising experiments were used on MIT-BIH ECG signals to assess the performance of the proposed method [20] with different AWGN levels. Three standard parameters were used at different input SNR (SNRin): output SNR (SNRout), Mean Square Error (MSE), and percentage Root Mean square Difference (PRD). The proposed method is compared to EMD-CMSE [14], EMD-Custom [8], and wavelet [15,16] denoising methods.

II. WAVELET DENOISING

Wavelet denoising is a powerful tool for removing the noisy component of a corrupted data sequence [15, 16]. Its
basic steps are:

- **Decompose**: Choose a wavelet and a level \(N\). Compute the wavelet decomposition of the signal at level \(N\).
- **Threshold detail coefficients**: For each level from 1 to \(N\), select a threshold and apply soft thresholding to the detail coefficients.
- **Reconstruct**: Compute wavelet reconstruction using the original approximation coefficients of level \(N\) and the modified detail coefficients of levels 1 to \(N\).

This work used the Symlet wavelet (sym8), while thresholding can be performed by using the soft or hard thresholding function proposed in [15, 16] was used. Thresholding was performed by using soft thresholding. The universal threshold was divided into a set of approximations, and detail coefficients were thresholded using soft thresholding. The universal threshold estimator proposed in [15, 16] was used.

### III. EMD-CUSTOM THRESHOLDING

EMD-custom suggests the decomposition of a noisy signal to noisy IMFs via the EMD algorithm [1]. After that, the noisy denoted IMFs were thresholded using a customized thresholding function [18]. Finally, the denoised signal was reconstructed using all thresholded IMFs. The outline of the EMD-Custom [8] method is demonstrated in Figure 1.

![Outline of the EMD-Custom](image)

**Fig. 1.** Outline of the EMD-Custom.

### IV. EEMD ALGORITHM

The EEMD method [19] overcomes the "mode mixing" problem of the EMD method and consists of:

- Adding a white noise \(w^j(t)\) to the original signal \(x(t)\):
  \[
x^j(t) = x(t) + w^j(t), \quad 1 \leq j \leq N_e
  \]
  where \(N_e\) is the ensemble number.
- Decomposing the noisy signal \(x^j(t)\) into IMFs by the EMD method to obtain the corresponding IMF of each order denoted \(z^j_i(t)\), where \(i\) is the IMF order, \(j\) is the trial index, \(N\) is the number of IMFs, and \(1 \leq i \leq N\).
- Calculating the mean of the corresponding IMFs as the final signal IMF, by:
  \[
  Z_{EEMD}(t) = \frac{1}{N_e} \sum_{j=1}^{N_e} z^j_i(t), \quad 1 \leq i \leq N
  \]

### V. PROPOSED DENOISING METHOD

The proposed method suggests the decomposition of the noisy signal to noisy IMFs via the EEMD algorithm [19]. Afterward, the noisy IMFs denoted as \(f_i(t)\) are thresholded using a modified custom thresholding function. Let \(s(t)\) be a noisy signal given as:

\[
s(t) = x(t) + w(t)
\]

where \(x(t)\) is the noiseless signal and \(w(t)\) is an independent noise of finite amplitude. The proposed EEMD-Custom method consists of the following steps:

- **Decompose** the noisy signal \(s(t)\) by the EEMD algorithm to extract the noisy IMFs \(f_i(t)\).
- **Apply** a modified custom thresholding function on the noisy IMFs \(f_i(t)\). A modification of the customized thresholding function [18] is introduced to define a new one as:

\[
\hat{z}_i(t) = \begin{cases} 
  f_i(t) - \text{sgn}(f_i(t))[1 - \alpha]r_i, & \text{if } |f_i(t)| \geq \tau_i \\
  0, & \text{if } |f_i(t)| \leq \gamma
\end{cases}
\]

where \(0 < \gamma < \tau_i\), \(0 \leq \alpha \leq 1\) and \(\tau_i\) is the universal threshold reported in [15, 16] defined as:

\[
\tau_i = C \sqrt{E_i} / 2 \ln(n)
\]

where \(C\) is a constant depending on the type of the signal, \(n\) is the length of the signal, and \(E_i\) is given by:

\[
E_i = \frac{\sum_{j \geq i} f_i(t)}{\theta^{-i}}, \quad i = 2, 3, 4, ..., N
\]

where \(E_i^2\) is the energy of the first IMF, obtained as:

\[
E_i^2 = \left( \frac{\text{median}(f_i(t))}{0.6745} \right)^2
\]

where \(\theta = 0.719\) and \(\rho = 2.01\) are empirically calculated constants [2].

- **Reconstruct** the signal using:

\[
\hat{x}(t) = \sum_{i=1}^{N} \hat{z}_i(t) + r(t)
\]

where \(r(t)\) is the residual signal.

### VI. RESULTS AND DISCUSSION

In this section, the results of the proposed denoising method are assessed compared to three denoising methods: wavelet denoising [15, 16], EMD-CMSE [14], and EMD-Custom [8]. The proposed EEMD-Custom algorithm was applied to 8 real biomedical ECG signals using the MIT-BIH database [20], labeled 111m, 112m, 113m, 114m, 115m, 116m, 121m, and 122m. An AWGN was added to each clean ECG signal at different \(SNR_{in}\) levels: -4dB, 0 dB, 4 dB, 8 dB, and 12 dB. The data length was 2048. At first, each noisy ECG signal was decomposed into a series of IMFs via the EEMD algorithm, and subsequently, the modified customized thresholding function (4) was utilized to threshold all IMFs for reducing noise and preserve QRS complexes. Thresholding can be used to detect QRS complexes [21]. So, the combination between EEMD and the modified customized thresholding function can be considered as an R peak preservation technique, as the IMFs containing high-frequency signal information (QRS complex) were thresholded by the modified customized thresholding function to preserve the QRS complexes.

Finally, the denoised signal was reconstructed using all thresholded IMFs. Three standard parameters, \(SNR_{out}\), \(MSE\), and \(PRD\), were used to evaluate the capabilities of the proposed method at different \(SNR_{in}\), which were respectively given as:

\[
SNR_{out} = 10 \log_{10} \frac{\|x(t)\|^2}{\|r(t)\|^2}
\]

\[
MSE = \frac{1}{N} \sum_{t=1}^{N} (x(t) - \hat{x}(t))^2
\]

\[
PRD = \frac{\|x(t) - \hat{x}(t)\|}{\|x(t)\|}
\]
The performance of the proposed EEMD-Custom method was evaluated for different values of ensemble number \( N_e \) (10, 20, 30, 40, 50, 100, 200, 250, 300). Figure 2 depicts the \( \text{SNR}_{out} \) for different values of \( N_e \) at \( \text{SNR}_{in}=4 \text{dB} \) on ECG record 112m. Figure 3 displays the plot of \( \text{SNR}_{out} \) for different values of \( N_e \) at \( \text{SNR}_{in}=4 \text{dB} \) on ECG records 114m, 116m, and 122m. As it can be observed, the \( \text{SNR}_{out} \) increases as \( N_e \) increases. Moreover, the proposed method achieved a significant improvement when \( N_e \) was high. Based on the results, an ensemble number of 200 was selected as the best EEMD parameter. Furthermore, as the results of the proposed EEMD-Custom method were influenced by the \( \alpha \) value in (4), an appropriate value of \( \alpha \) should be determined.

\[
\text{SNR}_{out} = 10 \log_{10} \frac{\sum_{n=1}^{N} x(t)^2}{\sum_{n=1}^{N} (x(t)-\hat{x}(t))^2} \tag{9}
\]

\[
\text{MSE} = \frac{1}{N} \sum_{n=1}^{N} (x(t)-\hat{x}(t))^2 \tag{10}
\]

\[
\text{PRD} = 100 \times \sqrt{\frac{\sum_{n=1}^{N} (x(t)-\hat{x}(t))^2}{\sum_{n=1}^{N} x(t)^2}} \tag{11}
\]

Figures 4 and 5 show the \( \text{SNR}_{out} \) of the proposed EEMD-Custom method as a function of \( \alpha \) for different ECG signals. The \( \text{SNR}_{out} \) of five ECG signals at various \( \text{SNR}_{in} \) values is presented in Table I to provide quantitative analysis. The proposed EEMD-Custom denoising method on the ECG record 112m reached an improvement of SNR equal to 6.95dB at \( \text{SNR}_{in}=4 \text{dB} \) compared to the wavelet denoising method. An improvement of 3.89dB was obtained at \( \text{SNR}_{in}=0 \text{dB} \) compared to EMD-CMSE, while an improvement of 4.46dB was obtained at \( \text{SNR}_{in}=4 \text{dB} \) compared to the EMD-Custom. The proposed EEMD-Custom denoising method improved SNR by 6.36dB, 4.34dB, and 3.87dB on the ECG 121m at \( \text{SNR}_{in}=4 \text{dB} \) compared to wavelet denoising [2], EMD-CMSE [5,6] and EMD-Custom [8], respectively. As it can be noted, the proposed method worked better for high and low values of \( \text{SNR}_{in} \). Moreover, the values of \( \text{MSE} \) and \( \text{PRD} \) of ECG signals for different \( \text{SNR}_{in} \) values are presented in Tables II and III, respectively. The \( \text{MSE} \) and \( \text{PRD} \) values should be small for better denoising and preserving ECG signal details. Lower \( \text{PRD} \) and \( \text{MSE} \) values indicate better preservation of physiological information in ECG signal processing [17]. As it can be noted, the proposed EEMD-Custom method provided less \( \text{MSE} \) and \( \text{PRD} \) than the other methods.
Mohguen & Bouguezel: Denoising the ECG Signal Using Ensemble Empirical Mode Decomposition

### TABLE I. SNRout Obtained by Different Methods

| Signals | Methods | SNRin (dB) | Wavelet (Sym 8) | EMD-CMSE | EMD-Custom | Proposed EEMD-Custom |
|---------|---------|------------|----------------|----------|------------|---------------------|
| ECG 111. M | -4 | 1.44 | 2.33 | 2.83 | 4.48 |
| | 0 | 5.38 | 5.87 | 6.51 | 7.79 |
| | 4 | 9.24 | 9.75 | 10.41 | 11.34 |
| | 8 | 12.90 | 12.79 | 14.26 | 14.69 |
| | 12 | 16.16 | 14.82 | 17.56 | 17.69 |
| ECG 112. M | -4 | 4.61 | 7.89 | 7.10 | 11.56 |
| | 0 | 8.58 | 10.27 | 10.94 | 14.16 |
| | 4 | 12.44 | 13.78 | 14.31 | 15.95 |
| | 8 | 16.10 | 16.36 | 17.54 | 18.03 |
| | 12 | 19.36 | 18.71 | 20.77 | 20.99 |
| ECG 113. M | -4 | 4.21 | 6.37 | 6.13 | 8.44 |
| | 0 | 7.57 | 7.38 | 7.65 | 9.20 |
| | 4 | 10.30 | 10.54 | 12.00 | 12.48 |
| | 8 | 12.16 | 12.36 | 15.56 | 15.75 |
| | 12 | 15.85 | 16.04 | 18.48 | 18.84 |
| ECG 114. M | -4 | 4.13 | 2.96 | 3.95 | 5.86 |
| | 0 | 5.35 | 6.12 | 7.55 | 9.20 |
| | 4 | 9.17 | 9.41 | 11.12 | 12.54 |
| | 8 | 12.75 | 12.75 | 14.89 | 15.77 |
| | 12 | 16.10 | 16.36 | 18.48 | 18.84 |
| ECG 115. M | -4 | 4.26 | 5.64 | 5.96 | 7.54 |
| | 0 | 8.06 | 7.94 | 9.28 | 9.98 |
| | 4 | 11.23 | 10.30 | 12.10 | 12.60 |
| | 8 | 13.68 | 13.81 | 15.71 | 16.27 |
| | 12 | 15.24 | 16.88 | 19.02 | 19.32 |
| ECG 116. M | -4 | 4.66 | 6.68 | 7.15 | 11.02 |
| | 0 | 8.64 | 16.52 | 10.36 | 12.98 |
| | 4 | 12.58 | 12.49 | 13.57 | 15.44 |
| | 8 | 16.42 | 16.66 | 17.56 | 18.37 |
| | 12 | 20.07 | 18.75 | 20.93 | 21.14 |
| ECG 121. M | -4 | 4.57 | 6.95 | 6.45 | 8.60 |
| | 0 | 8.72 | 8.14 | 8.85 | 10.62 |
| | 4 | 12.14 | 11.81 | 13.02 | 12.99 |
| | 8 | 15.42 | 15.03 | 16.05 | 16.47 |
| | 12 | 18.03 | 18.32 | 19.17 | 19.68 |
| ECG 122. M | -4 | 4.61 | 7.89 | 7.10 | 11.56 |
| | 0 | 8.58 | 10.27 | 10.94 | 14.16 |
| | 4 | 12.44 | 13.78 | 14.31 | 15.95 |
| | 8 | 16.10 | 16.36 | 17.54 | 18.03 |

Fig. 6. SNRout versus SNRin for different denoising methods.

Fig. 7. MSE versus SNRin for different denoising methods.

Fig. 8. PRD versus SNRin for different denoising methods.

The clean ECG records 112m and 122m, their noisy versions, and denoised ECG records using the proposed EEMD-Custom at SNRin=8dB with Ne=200 are depicted in Figures 9 and 10 respectively. It can be noted that the proposed method removes noise successfully. Figure 11 depicts the denoised ECG record 122m using the wavelet method at SNRin=8dB. A careful comparison of the denoised signals in Figures 10 and 11 shows that the proposed method preserves morphological information of ECG better than the wavelet denoising method. The results also indicate that the proposed method can remove noise from real ECG signals and provide significant improvements in denoising performance. The computational complexity of EEMD can be expressed as:

\[
T_{EEMD} = N_e \cdot T_{EMD}
\]

(12)
demonstrating that EEMD takes more time than EMD.

Fig. 9. ECG 112m signal denoised by the proposed EEMD-Custom method.
TABLE II. MSE OBTAINED BY DIFFERENT METHODS.

| Signals   | Methods        | SNRin (dB) | Wavelet (Sym 8) | EMD-Custom | Proposed EEMD-Custom |
|-----------|----------------|------------|-----------------|------------|---------------------|
| ECG 111.m | EMD-Standard   | -4         | 0.0045          | 0.0330     | 0.0293              |
|           |                | 0          | 0.0163          | 0.0146     | 0.0125              |
|           |                | 4          | 0.0557          | 0.0059     | 0.0051              |
|           |                | 8          | 0.0028          | 0.0029     | 0.0021              |
|           |                | 12         | 0.0013          | 0.0018     | 0.0009              |
| ECG 112.m | EMD-Standard   | -4         | 0.3053          | 0.1342     | 0.1179              |
|           |                | 0          | 0.1221          | 0.0828     | 0.0710              |
|           |                | 4          | 0.0502          | 0.0668     | 0.0527              |
|           |                | 8          | 0.0216          | 0.0263     | 0.0155              |
|           |                | 12         | 0.0102          | 0.0118     | 0.0073              |
| ECG 113.m | EMD-Standard   | -4         | 0.1819          | 0.1278     | 0.1017              |
|           |                | 0          | 0.0736          | 0.0618     | 0.0443              |
|           |                | 4          | 0.0305          | 0.0289     | 0.0195              |
|           |                | 8          | 0.0134          | 0.0134     | 0.0081              |
|           |                | 12         | 0.0085          | 0.0062     | 0.0035              |
| ECG 114.m | EMD-Standard   | -4         | 0.0201          | 0.0122     | 0.0129              |
|           |                | 0          | 0.0092          | 0.0097     | 0.0057              |
|           |                | 4          | 0.0049          | 0.0046     | 0.0030              |
|           |                | 8          | 0.0032          | 0.0029     | 0.0014              |
| ECG 115.m | EMD-Standard   | -4         | 0.2492          | 0.1506     | 0.1342              |
|           |                | 0          | 0.1008          | 0.0896     | 0.0585              |
|           |                | 4          | 0.0418          | 0.0376     | 0.0244              |
|           |                | 8          | 0.0183          | 0.0178     | 0.0109              |
| ECG 116.m | EMD-Standard   | -4         | 0.0089          | 0.0114     | 0.0047              |
|           |                | 0          | 0.2085          | 0.1698     | 0.1340              |
|           |                | 4          | 0.1019          | 0.1263     | 0.0835              |
|           |                | 8          | 0.0580          | 0.0564     | 0.0364              |
| ECG 121.m | EMD-Standard   | -4         | 0.2197          | 0.1381     | 0.1238              |
|           |                | 0          | 0.0878          | 0.0909     | 0.0591              |
|           |                | 4          | 0.0154          | 0.0361     | 0.0282              |
|           |                | 8          | 0.0146          | 0.0138     | 0.0112              |
|           |                | 12         | 0.0063          | 0.0085     | 0.0051              |
| ECG 122.m | EMD-Standard   | -4         | 0.2767          | 0.3997     | 0.1793              |
|           |                | 0          | 0.1131          | 0.1214     | 0.0819              |
|           |                | 4          | 0.0484          | 0.0522     | 0.0395              |
|           |                | 8          | 0.0227          | 0.0248     | 0.0196              |
|           |                | 12         | 0.0124          | 0.0116     | 0.0096              |

TABLE III. PSROB OBTAINED BY DIFFERENT METHODS.

| Signals   | Methods       | SNRin (dB) | Wavelet (Sym 8) | EMD-Custom | Proposed EEMD-Custom |
|-----------|---------------|------------|-----------------|------------|---------------------|
| ECG 111.m | EMD-Standard  | -4         | 84.60           | 76.43      | 72.13               |
|           |                | 0          | 53.79           | 50.86      | 47.21               |
|           |                | 4          | 34.49           | 32.53      | 30.16               |
|           |                | 8          | 22.62           | 22.91      | 19.34               |
|           |                | 12         | 15.55           | 18.13      | 13.23               |
| ECG 112.m | EMD-Standard  | -4         | 58.81           | 40.28      | 44.13               |
|           |                | 0          | 37.20           | 30.64      | 28.36               |
|           |                | 4          | 23.86           | 20.44      | 19.24               |
|           |                | 8          | 15.65           | 15.20      | 13.27               |
| ECG 113.m | EMD-Standard  | -4         | 84.79           | 71.08      | 63.41               |
|           |                | 0          | 53.95           | 66.91      | 41.88               |
|           |                | 4          | 34.75           | 33.82      | 27.77               |
|           |                | 8          | 23.02           | 23.02      | 17.99               |
| ECG 114.m | EMD-Standard  | -4         | 61.58           | 47.99      | 49.35               |
|           |                | 0          | 41.78           | 42.74      | 32.92               |
|           |                | 4          | 30.52           | 29.69      | 23.74               |
|           |                | 8          | 24.64           | 23.53      | 16.67               |
| ECG 115.m | EMD-Standard  | -4         | 84.78           | 65.92      | 62.22               |
|           |                | 0          | 53.94           | 50.85      | 41.07               |
|           |                | 4          | 34.73           | 32.94      | 28.56               |
|           |                | 8          | 22.98           | 22.70      | 17.73               |
| ECG 116.m | EMD-Standard  | -4         | 16.06           | 16.20      | 11.73               |
|           |                | 0          | 39.50           | 40.05      | 34.35               |
|           |                | 4          | 27.41           | 30.52      | 24.81               |
|           |                | 8          | 20.67           | 20.39      | 16.38               |
| ECG 121.m | EMD-Standard  | -4         | 59.45           | 46.34      | 43.88               |
|           |                | 0          | 36.96           | 37.49      | 30.32               |
|           |                | 4          | 24.39           | 23.72      | 20.96               |
|           |                | 8          | 15.08           | 14.68      | 13.23               |
| ECG 122.m | EMD-Standard  | -4         | 58.35           | 44.87      | 47.54               |
|           |                | 0          | 37.75           | 39.12      | 32.14               |
|           |                | 4          | 24.71           | 25.66      | 22.32               |
|           |                | 8          | 16.92           | 22.60      | 15.75               |
|           |                | 12         | 12.54           | 12.12      | 11.00               |

Denoising the ECG Signal Using Ensemble Empirical Mode Decomposition
VII. CONCLUSION

This paper presented a novel denoising method based on the EEMD algorithm for noise removal from ECG signals by introducing a modified custom thresholding function. Three standard parameters SNR\textsubscript{out}, MSE, and PRD were used for evaluating the capabilities of the proposed method at different values of SNR\textsubscript{in}. The simulation results on MIT-BIH ECG signals showed clearly that the proposed method provided better SNR\textsubscript{out} and lesser MSE and PRD compared to other well-known denoising methods. Therefore, the proposed method is characterized as highly suitable for denoising ECG signals.

REFERENCES

[1] N. E. Huang et al., "The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis," Proceedings of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences, vol. 454, no. 1971, pp. 903–995, Mar. 1998, https://doi.org/10.1098/rspa.1998.0193.

[2] P. Flandrin, P. Gonçalvès, and G. Rilling, "Empirical Mode Decomposition and Its Applications," vol. 5, Singapore: World Scientific, 2005, pp. 57–74.

[3] A. O. Boudraa, J. C. Cexus, and Z. Saaidi, "EMD-Based Signal Noise Reduction," International Journal of Signal Processing, vol. 1, no. 1, pp. 33–37, 2004.

[4] A.-O. Boudraa and Jean-Christophe Cexus, "Denoising via Empirical Mode Decomposition," presented at the International Symposium on Communications, Control and Signal Processing (ISCCSP '06), Marrakech, Morocco, Mar. 2006.

[5] Y. Kopsinis and S. McLaughlin, "Empirical mode decomposition based soft-thresholding," in 2008 16th European Signal Processing Conference, Aug. 2008, pp. 1–5.

[6] Y. Kopsinis and S. McLaughlin, "Development of EMD-Based Denoising Methods Inspired by Wavelet Thresholding," IEEE Transactions on Signal Processing, vol. 57, no. 4, pp. 1351–1362, Apr. 2009, https://doi.org/10.1109/TSP.2009.2013885.

[7] G. Yang, Y. Liu, Y. Wang, and Z. Zhu, "EMD interval thresholding denoising based on similarity measure to select relevant modes," Signal Processing, vol. 109, pp. 95–109, Apr. 2015, https://doi.org/10.1016/j.sigpro.2014.10.038.

[8] W. Mohguen and R. E. Bekka, "Empirical Mode Decomposition Based Denoising by Customized Thresholding," International Journal of Electronics and Communication Engineering, vol. 11, no. 5, pp. 519–524, Mar. 2017.

[9] W. Mohguen and R. E. Bekka, "New Denoising Method Based on Empirical Mode Decomposition and Improved Thresholding Function," Journal of Physics: Conference Series, vol. 787, Jan. 2017, Art. no. 012014, https://doi.org/10.1088/1742-6596/787/1/012014.

[10] W. Mohguen and R. E. Bekka, "An Empirical Mode Decomposition Signal Denoising Method Based on Novel Thresholding," presented at the 5th International Conference on Control & Signal Processing (CSP-2017), Kairouan, Tunisia, Oct. 2017.

[11] M. V. Sarode and P. R. Deshmukh, "Image Sequence Denoising with Motion Estimation in Color Image Sequences," Engineering, Technology & Applied Science Research, vol. 1, no. 6, pp. 139–143, Dec. 2011, https://doi.org/10.48084/etasr.54.

[12] W. Helali, Z. Hajaiej, and A. Cherif, "Real Time Speech Recognition based on PWP Thresholding and MFCC using SVM," Engineering, Technology & Applied Science Research, vol. 10, no. 5, pp. 6204–6208, Oct. 2020, https://doi.org/10.48084/etasr.3759.

[13] M. Atif, Z. H. Khan, S. Khan, F. Akhtar, and A. Rajput, "Storage Optimization using Adaptive Thresholding Motion Detection," Engineering, Technology & Applied Science Research, vol. 11, no. 2, pp. 6869–6872, Apr. 2021, https://doi.org/10.48084/etasr.3951.