Measurement of $CP$ observables in $B^\pm \rightarrow DK^{*\pm}$
decays using two- and four-body $D$ final states

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ABSTRACT: Measurements of $CP$ observables in $B^\pm \rightarrow DK^{*\pm}$ decays are presented, where $D$ denotes a superposition of $D^0$ and $\bar{D}^0$ meson states. Decays of the $D$ meson to $K^-\pi^+$, $K^-K^+$, $\pi^-\pi^+$, $K^-\pi^+\pi^-\pi^+$ and $\pi^-\pi^+\pi^-\pi^+$ are used and the $K^{*\pm}$ meson is reconstructed in the $K^0_S\pi^\pm$ final state. This analysis uses a data sample of $pp$ collisions collected with the LHCb experiment, corresponding to integrated luminosities of 1 fb$^{-1}$, 2 fb$^{-1}$ and 1.8 fb$^{-1}$ at centre-of-mass energies $\sqrt{s} = 7$ TeV, 8 TeV and 13 TeV, respectively. The sensitivity of the results to the CKM angle $\gamma$ is discussed.

KEYWORDS: B physics, CKM angle gamma, CP violation, Hadron-Hadron scattering (experiments)

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1 Introduction

A key characteristic of the Standard Model is that CP violation originates from a single phase in the CKM quark-mixing matrix [1, 2]. In the Standard Model the CKM matrix is unitary, leading to the condition $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$, where $V_{ij}$ are the CKM matrix elements. This relation is represented as a triangle in the complex plane, with angles $\alpha$, $\beta$ and $\gamma$, and an area proportional to the amount of CP violation in the quark sector of the Standard Model [3]. Overconstraining this unitarity triangle may lead to signs of physics beyond the Standard Model. The CKM angle $\gamma \equiv \arg \left( \frac{V_{ub}V_{cd}^*}{V_{cd}V_{ub}^*} \right)$ is the least well-known angle of the CKM unitarity triangle. The latest published LHCb combination from direct measurements with charged and neutral $B$ decays to a $D$ meson (reconstructed in one of a variety of final states) and a kaon is $\gamma = (72.2^{+6.8}_{-7.3})^\circ$ [4]. A global fit to the CKM triangle by the CKMtfitter group [5] obtains a $\gamma$ value of $(66.9^{+0.9}_{-3.4})^\circ$, where this determination of $\gamma$ excludes all direct measurements. The uncertainties on the indirect measurement are expected to decrease as lattice QCD calculations become more accurate. Therefore, precision at the level of $1^\circ$ on a direct measurement of $\gamma$ would test the consistency of the direct and indirect measurements and thereby the Standard Model. This precision can be achieved through a combination of measurements of various $B$ decays that are sensitive to $\gamma$.

Direct measurements of $\gamma$ can be made by exploiting the interference between $b \to c\pi s$ and $b \to u\pi s$ transitions. These transitions are present in $B \to D^{(*)}K^{(*)}$ decays. This analysis measures CP violation in $B^- \to DK^*(892)^-$ decays, with $K^*(892)^- \to K_{S}^{0}(\pi^+\pi^-)\pi^-$, where the inclusion of charge-conjugate processes is implied, except when discussing ratios or asymmetries between $B^+$ and $B^-$ decays.
where $D$ denotes a superposition of $D^0$ and $\bar{D}^0$ meson states. In this paper $K^{*-}$ is used to represent the $K^*(892)^-$ resonance. The effect of the interference is observed by reconstructing the $D$ meson in a final state accessible to both $D^0$ and $\bar{D}^0$ meson states, which gives sensitivity to the weak phase $\gamma$. In this analysis, only $D$ mesons decaying to two or four charged kaons and/or pions are considered. The branching fraction of $B^- \to DK^{*-}$ is of a similar magnitude to $B^- \to D\bar{K}^-$, which has been extensively analysed at LHCb [6–8]. However, the reconstruction efficiencies associated with the $K^{*-} \to K^0_S\pi^-$ decay are lower due to the presence of a long-lived neutral particle.

Two main classes of $D$ decays are used. The first employs $D$ decays into the CP-even eigenstates $K^+K^-$ and $\pi^+\pi^-$; these are referred to here as the “GLW” decay modes [9, 10]. The second class of decay modes involves $D$ decays to $K^\mp\pi^\pm$, which is not a CP eigenstate. In the favoured decay, the pion from the $D$ meson and that from the $K^{*-}$ meson have opposite charge, while in the suppressed decay (referred to here as the “ADS” [11, 12] decay mode) the pion from the $D$ meson that from the $K^{*-}$ meson have the same charge. The favoured mode is used as a control mode for many aspects of the analysis since no CP asymmetry is expected. The ADS decay mode is a combination of a CKM-favoured $B^- \to D^0K^{*-}$ decay, followed by a doubly Cabibbo-suppressed $B^- \to DK^-\pi^+$ decay, and a Cabibbo- and colour-suppressed $B^- \to \bar{D}^0K^*$ decay, followed by a Cabibbo-favoured $\bar{D}^0 \to K^+\pi^-$ decay. Both paths to the same final state have amplitudes of similar size, and interference effects are therefore magnified in comparison to the GLW decay modes, where the decay path via the CKM-favoured $B^- \to D^0K^{*-}$ dominates. Studies of $B^- \to DK^-$ and $B^0 \to DK^{*0}$ decays have been published by the LHCb collaboration [6, 13].

The GLW and ADS methods can be extended to the $D \to K^+K^-\pi^\mp\pi^\pm$ and $D \to \pi^+\pi^-\pi^+\pi^-$ inclusive four-body final states, provided external information is available on the overall behaviour of the intermediate resonances, averaged over phase space [14, 15]. These channels have previously been studied for $B^- \to DK^-$ decays [6], and are included in this paper for the first time in $B^- \to DK^{*-}$ decays. The $B^- \to DK^{*-}$ channel has previously been investigated by the BaBar collaboration using a variety of two-body $D$ decay modes [16]. Also, both the BaBar and Belle collaborations have performed studies on $B^- \to DK^{*-}$ with $D \to K^0_S\pi^+\pi^-$ [17, 18].

Twelve quantities, collectively referred to as CP observables, are measured in this analysis.

- The CP asymmetry for the favoured decay mode
  
  $$A_{K^*} = \frac{\Gamma (B^- \to D(K^+\pi^-)K^*) - \Gamma (B^+ \to D(K^+\pi^-)K^*)}{\Gamma (B^- \to D(K^-\pi^+)K^*) + \Gamma (B^+ \to D(K^+\pi^-)K^*)}.$$  

- The CP asymmetry for the $D \to K^+K^-$ decay mode
  
  $$A_{KK} = \frac{\Gamma (B^- \to D(K^+K^-)K^*) - \Gamma (B^+ \to D(K^+K^-)K^*)}{\Gamma (B^- \to D(K^+K^-)K^*) + \Gamma (B^+ \to D(K^+K^-)K^*)}.$$  

- The CP asymmetry for the $D \to \pi^+\pi^-$ decay mode
  
  $$A_{\pi\pi} = \frac{\Gamma (B^- \to D(\pi^+\pi^-)K^*) - \Gamma (B^+ \to D(\pi^+\pi^-)K^*)}{\Gamma (B^- \to D(\pi^+\pi^-)K^*) + \Gamma (B^+ \to D(\pi^+\pi^-)K^*)}.$$
The ratio of the rate for the $D \to K^+K^-$ decay mode to that of the favoured decay mode, scaled by the branching fractions

$$R_{KK} = \frac{\Gamma (B^- \to D(K^+K^-)K^{*-}) + \Gamma (B^+ \to D(K^+K^-)K^{*+})}{\Gamma (B^- \to D(K^+K^-)K^{*-}) + \Gamma (B^+ \to D(K^+K^-)K^{*+})} \times \frac{\mathcal{B}(D^0 \to K^-\pi^+)}{\mathcal{B}(D^0 \to K^+K^-)}. \quad (1.4)$$

The ratio of the rate for the $D \to \pi^+\pi^-$ decay mode to that of the favoured decay mode, scaled by the branching fractions

$$R_{\pi\pi} = \frac{\Gamma (B^- \to D(\pi^+\pi^-)K^{*-}) + \Gamma (B^+ \to D(\pi^+\pi^-)K^{*+})}{\Gamma (B^- \to D(\pi^+\pi^-)K^{*-}) + \Gamma (B^+ \to D(\pi^+\pi^-)K^{*+})} \times \frac{\mathcal{B}(D^0 \to K^-\pi^+)}{\mathcal{B}(D^0 \to \pi^+\pi^-)}. \quad (1.5)$$

The ratio of the rate for the ADS decay mode to that of the favoured decay mode for $B^+$ decays

$$R^+_{K\pi} = \frac{\Gamma (B^+ \to D(K^{-\pi^+})K^{*-})}{\Gamma (B^+ \to D(K^+\pi^-)K^{*-})}. \quad (1.6)$$

The ratio of the rate for the ADS decay mode to that of the favoured decay mode for $B^-$ decays

$$R^-_{K\pi} = \frac{\Gamma (B^- \to D(K^+\pi^-)K^{*-})}{\Gamma (B^- \to D(K^{-\pi^+})K^{*-})}. \quad (1.7)$$

The $CP$ asymmetry for the favoured $D^0 \to K^-\pi^+\pi^-\pi^+$ decay mode

$$A_{K\pi\pi} = \frac{\Gamma (B^- \to D(K^{-\pi^+\pi^-\pi^+})K^{*-}) - \Gamma (B^+ \to D(K^+\pi^-\pi^+\pi^-)K^{*+})}{\Gamma (B^- \to D(K^{-\pi^+\pi^-\pi^+})K^{*-}) + \Gamma (B^+ \to D(K^+\pi^-\pi^+\pi^-)K^{*+})}. \quad (1.8)$$

The $CP$ asymmetry for the $D \to \pi^+\pi^-\pi^+\pi^-$ decay mode

$$A_{\pi\pi\pi} = \frac{\Gamma (B^- \to D(\pi^+\pi^-\pi^+\pi^-)K^{*-}) - \Gamma (B^+ \to D(\pi^+\pi^-\pi^+\pi^-)K^{*+})}{\Gamma (B^- \to D(\pi^+\pi^-\pi^+\pi^-)K^{*-}) + \Gamma (B^+ \to D(\pi^+\pi^-\pi^+\pi^-)K^{*+})}. \quad (1.9)$$

The ratio of the rate for the $D \to \pi^+\pi^-\pi^+\pi^-$ decay mode to that of the favoured decay mode, scaled by the branching fractions

$$R_{\pi\pi\pi} = \frac{\Gamma (B^- \to D(\pi^+\pi^-\pi^+\pi^-)K^{*-}) + \Gamma (B^+ \to D(\pi^+\pi^-\pi^+\pi^-)K^{*+})}{\Gamma (B^- \to D(\pi^+\pi^-\pi^+\pi^-)K^{*-}) + \Gamma (B^+ \to D(\pi^+\pi^-\pi^+\pi^-)K^{*+})} \times \frac{\mathcal{B}(D^0 \to K^-\pi^+\pi^-)}{\mathcal{B}(D^0 \to \pi^+\pi^-\pi^+\pi^-)}. \quad (1.10)$$

The ratio of the rate for the four-body ADS decay mode to that of the four-body favoured decay mode for $B^+$ decays

$$R^+_{K\pi\pi\pi} = \frac{\Gamma (B^+ \to D(K^{-\pi^+\pi^-\pi^-})K^{*-})}{\Gamma (B^+ \to D(K^+\pi^-\pi^+\pi^-)K^{*+})}. \quad (1.11)$$

The ratio of the rate for the four-body ADS decay mode to that of the four-body favoured decay mode for $B^-$ decays

$$R^-_{K\pi\pi\pi} = \frac{\Gamma (B^- \to D(K^+\pi^-\pi^+\pi^-)K^{-*})}{\Gamma (B^- \to D(K^{-\pi^+\pi^-\pi^-})K^{*-})}. \quad (1.12)$$
The asymmetries $A_{K\pi}$ and $A_{K^\mp\pi}$ should be essentially zero due to the very small interference expected in the configuration of $B$ and $D$ decays. Due to negligible direct CP violation in $D$ decays [19], the observables $A_{KK}$ and $A_{\pi\pi}$ should be equal and are often labelled together as $A_{CP+}$; similarly the observables $R_{KK}$ and $R_{\pi\pi}$ should be equal and are labelled $R_{CP+}$. The analogous observables to $R_{CP+}$ and $A_{CP+}$ for the ADS mode are $R_{ADS}$ and $A_{ADS}$. However, $R_{ADS}$ and $A_{ADS}$ are not used for the ADS decay mode, instead the ratios are measured separately for the positive and negative charges. The reason for this choice is that the uncertainty in $A_{ADS}$ depends on the value of $R_{ADS}$, therefore these observables are statistically dependent, raising problems for the low yields expected in the ADS mode. Hence the statistically independent observables $R^+_{K\pi}$ and $R^-_{K\pi}$ are preferred.

The CP observables measured in this analysis can be related to the physics parameters to be determined, namely $\gamma$, $r_B$ and $\delta_B$. The parameter $r_B$ is the ratio of the magnitudes between the suppressed and favoured amplitudes of the $B$ decay and $\delta_B$ is the strong-phase difference between these amplitudes. The expected value is $r_B \sim 0.1$, similar to that in the $B^- \rightarrow DK^+$ decay. Both $r_B$ and $\delta_B$ are averaged over the region of $DK_0^0\pi^-$ phase space corresponding to the $K^+$ selection window. A coherence factor, $\kappa$, accounts for the contribution of $B^- \rightarrow DK_0^0\pi^-$ decays that are not due to an intermediate $K^*(892)^-$ resonance [20], where $\kappa = 1$ denotes a pure $K^*(892)^-$ contribution. Given there is a negligible effect from both charm mixing [21] and CP violation in $D$ decays [19], the relationships between the CP observables and physics parameters are given in the following equations,

$$A_{CP+} = \frac{2\kappa r_B \sin \delta_B \sin \gamma}{1 + r_B^2 + 2\kappa r_B \cos \delta_B \cos \gamma}, \quad (1.13)$$

$$R_{CP+} = 1 + r_B^2 + 2\kappa r_B \cos \delta_B \cos \gamma, \quad (1.14)$$

$$R^\pm_{K\pi} = \frac{r_B^2 + (r_{K\pi}^D)^2 + 2\kappa r_B r_{K\pi}^D \cos(\delta_B + \delta_{K\pi}^D \pm \gamma)}{1 + r_B^2 + 2\kappa r_B \cos \delta_B \cos \gamma}, \quad (1.15)$$

$$A_{\pi\pi\pi\pi} = \frac{2\kappa (2F_{4\pi} - 1) r_B \sin \delta_B \sin \gamma}{1 + r_B^2 + 2\kappa (2F_{4\pi} - 1) r_B \cos \delta_B \cos \gamma}, \quad (1.16)$$

$$R_{\pi\pi\pi\pi} = 1 + r_B^2 + 2\kappa (2F_{4\pi} - 1) r_B \cos \delta_B \cos \gamma, \quad (1.17)$$

$$R^\pm_{K^\mp\pi\pi} = \frac{r_B^2 + (r_{K^\mp\pi\pi}^D)^2 + 2\kappa r_B r_{K^\mp\pi\pi}^D \cos(\delta_B + \delta_{K^\mp\pi\pi}^D \pm \gamma)}{1 + (r_B r_{K^\mp\pi\pi}^D)^2 + 2\kappa r_B r_{K^\mp\pi\pi}^D \cos(\delta_B - \delta_{K^\mp\pi\pi}^D \pm \gamma)}. \quad (1.18)$$

These relationships depend on several parameters describing the $D$ decays, which are taken from existing measurements. The parameters $r_{D}^{K\pi}$ and $\delta_{D}^{K\pi}$ are the magnitude of the amplitude ratio and the strong-phase difference between the suppressed and favoured amplitudes of the $D$ decay, namely $D^0 \rightarrow K^+\pi^-$ and $D^0 \rightarrow K^-\pi^+$ respectively [22]. Similarly, the parameters $r_{D}^{K^\mp\pi\pi}$ and $\delta_{D}^{K^\mp\pi\pi}$ are the equivalent quantities for the decays $D^0 \rightarrow K^+\pi^-\pi^+$ and $D^0 \rightarrow K^-\pi^+\pi^-$, averaged over phase space [23, 24]. Two-body $D \rightarrow K^\mp\pi\pm$ decays are characterised by a single strong phase, however for multibody $D \rightarrow K^\mp\pi^+\pi^-\pi^\pm$ decays the strong phase varies over the phase space. By averaging the strong phase variation the interference effects are diluted. This effect is accounted for by the parameter $\kappa_{K^3\pi}$ [23, 24]. The parameter $F_{4\pi} \sim 0.75$ [15] accounts for the fact that $D \rightarrow \pi^+\pi^-\pi^+\pi^-$, though predominantly CP even, is not a pure CP eigenstate.
2 Detector, online selection and simulation

The LHCb detector [25, 26] is a single-arm forward spectrometer covering the pseudorapidity range $2 < \eta < 5$, designed for the study of particles containing $b$ or $c$ quarks. The detector includes a high-precision tracking system consisting of a silicon-strip vertex detector (VELO) surrounding the $pp$ interaction region, a large-area silicon-strip detector (TT) located upstream of a dipole magnet with a bending power of about 4 Tm, and three stations of silicon-strip detectors and straw drift tubes placed downstream of the magnet. The tracking system provides a measurement of momentum, $p$, of charged particles with a relative uncertainty that varies from 0.5% at low momentum to 1.0% at 200 GeV/$c$. The minimum distance of a track to a primary vertex (PV), the impact parameter (IP), is measured with a resolution of $(15 + 29/p_T) \mu m$, where $p_T$ is the component of the momentum transverse to the beam, in GeV/$c$. Different types of charged hadrons are distinguished using information from two ring-imaging Cherenkov detectors (RICH). Photons, electrons and hadrons are identified by a calorimeter system consisting of scintillating-pad and preshower detectors, an electromagnetic calorimeter and a hadronic calorimeter. Muons are identified by a system composed of alternating layers of iron and multiwire proportional chambers, and gas electron multiplier detectors.

The online event selection is performed by a trigger [27], which consists of a hardware stage, based on information from the calorimeter and muon systems, followed by a software stage, which applies a full event reconstruction. Signal events considered in the analysis must fulfil hardware and software trigger requirements. At the hardware trigger stage, events are required to have a muon with high $p_T$ or a hadron, photon or electron with high transverse energy in the calorimeters. At the software stage, at least one charged particle should have high $p_T$ and large $\chi^2_{IP}$ with respect to any PV, where $\chi^2_{IP}$ is defined as the difference in the vertex-fit $\chi^2$ of a given PV fitted with and without the considered track. The software trigger designed to select $b$-hadron decays uses a multivariate algorithm [28] to identify a two-, three- or four-track secondary vertex with a large scalar sum of the $p_T$ of the associated charged particles and a significant displacement from the PVs. The PVs are fitted with and without the $B$ candidate, and the PV with the smallest $\chi^2_{IP}$ is associated with the $B$ candidate.

The analysis presented is based on $pp$ collision data corresponding to an integrated luminosity of 1 fb$^{-1}$ at a centre-of-mass energy of 7 TeV collected in 2011, 2 fb$^{-1}$ at 8 TeV collected in 2012 (jointly referred to as Run 1), and 1.8 fb$^{-1}$ at 13 TeV collected in 2015 and 2016 (referred to as Run 2). There are several differences between data collected in Run 1 and Run 2. The main difference is the higher $b\bar{b}$ production cross-section in Run 2 [29]. The average number of $pp$ interactions per bunch crossing is reduced to 1.1 in Run 2 compared to 1.7 in Run 1. The net effect is that, despite the higher energy of the collisions, the background levels and signal-to-background ratios in Run 1 and Run 2 for the type of decay analysed here are similar. Before the start of Run 2, the aerogel radiator was removed from the first RICH detector [30], which improves the detector resolution. Hence, for momenta typical of decays in this analysis, the particle identification criteria have resulted in an increased efficiency of signal selection while simultaneously decreasing...
the rate of misidentified backgrounds. For the $B^- \to D(K^-\pi^+)K^{*-}$ decay mode, the combination of higher $b\bar{b}$ production cross-section, improved particle identification and improvements to the online selection in Run 2 have resulted in a factor of three increase in the yield for a given integrated luminosity.

Simulated event samples are used for the study of efficiencies. In the simulation, $pp$ collisions are generated using PYTHIA [31, 32] with a specific LHCb configuration [33]. Decays of hadronic particles are described by EVTGEN [34], in which final-state radiation is generated using PHOTOS [35]. The interaction of the generated particles with the detector, and its response, are implemented using the GEANT4 toolkit [36, 37] as described in ref. [38].

3 Offline selection

The $K^{*-}$ meson is reconstructed in the decay $K^{*-} \to K^0_S\pi^-$ and the $K^0_S$ meson is reconstructed through its decay to two charged pions. If the pions from the $K^0_S$ decay leave sufficient hits in the VELO to be included in the track reconstruction, the reconstructed $K^0_S$ meson is called “long”. Due to the high boost from the $pp$ collision many $K^0_S$ particles decay outside the VELO. If the pions from the $K^0_S$ decay do not leave sufficient hits in the VELO, the reconstructed $K^0_S$ meson is called “downstream”, with the first hits being recorded in the TT, which typically results in poorer mass resolution. These $K^0_S$ reconstruction types are treated as separate data samples and a slightly different selection is applied to each.

Reconstructed $B$ candidates are formed by combining a $K^{*-}$ candidate with a $D$ candidate, which are required to form a good-quality vertex. For each $D$, $K^{*-}$, and $K^0_S$ candidate the reconstructed meson masses are required to lie within $25 \text{MeV}/c^2$ around the $D$ mass, $75 \text{MeV}/c^2$ around the $K^{*-}$ mass, and $15 \text{MeV}/c^2$ around the $K^0_S$ mass for long candidates and $20 \text{MeV}/c^2$ for downstream candidates [39]. A kinematic fit [40] is performed on the full $B$ decay chain constraining the $B$ candidate to point towards the PV, and the $D$ and $K^0_S$ candidates to have their known masses [39]. To suppress charmless backgrounds, the $D$ decay vertex is required to be well-separated from and downstream of the $B^-$ decay vertex. Also, the $K^0_S$ decay vertex is required to be well-separated from and downstream of the $B^-$ decay vertex in order to suppress $B^- \to D\pi^-\pi^+\pi^-$ decays. The selection window of $\pm 75 \text{MeV}/c^2$, 1.5 times the $K^*(892)^-$ natural width, is required to suppress $B^- \to DK^0_S\pi^-$ decays that do not proceed via an intermediate $K^*(892)^-$ resonance. Further suppression of these decays is achieved by requiring the magnitude of the cosine of the $K^0_S$ helicity angle to be greater than 0.3. The $K^0_S$ helicity angle is defined as the angle between the $K^0_S$ and the $B^-$ momentum vectors in the $K^{*-}$ rest frame. This requirement retains 97% of true $K^{*-}$ decays, which are distributed parabolically in this variable, while rejecting 30% of the background.

Requirements, based mainly on the RICH system, are applied to all $D$ decay products to identify them as kaons or pions. These selections are applied such that each $D$ candidate is assigned a unique category. Cross-feed between the $K^-\pi^+$, $K^+K^-$ and $\pi^+\pi^-$ $D$ final states is negligible because after misidentification of a $\pi^-$ meson as a $K^-$ meson (or vice versa) the reconstructed mass of the $D$ meson lies outside the $D$ mass selection window. However, the favoured decay $B^- \to D(K^-\pi^+)K^{*-}$ can appear in the $B^- \to D(\pi^-K^+)K^{*-}$
sample due to misidentification of both $D$ decay products. To suppress this, a veto is applied to the ADS decay mode. The $D$ mass is reconstructed assuming the mass hypotheses of the decay products are swapped. If the resulting value is within 15 MeV/$c^2$ of the nominal $D$ mass, the candidate is removed from the sample, after which any remaining contamination is negligible while retaining 92% of the signal. Similarly a 15 MeV/$c^2$ veto selection is applied to the four-body ADS decay mode to prevent the contamination of $B^- \rightarrow D(K^{-}\pi^+\pi^-\pi^+)K^{*-}$ in the $B^- \rightarrow D(\pi^-K^+\pi^-\pi^+)K^{*-}$ sample. The swapped $D$ mass hypothesis is considered for both $\pi^+$ mesons separately, resulting in a combined signal efficiency for the vetoes of 90%.

Combinatorial background is suppressed using a Boosted Decision Tree (BDT) multivariate discriminant [41]. To train the BDT for two-body decays, simulated $B^- \rightarrow D(K^{-}\pi^+)K^{*-}$ candidates are used as a signal sample and events from the high-mass sideband region of the $B^-$ mass, above 5600 MeV/$c^2$, in the favoured $B^- \rightarrow D(K^{-}\pi^+)K^{*-}$ decay mode are used as a sample of combinatorial background. An analogous strategy is employed in the BDT for four-body decays. Various input quantities are used to exploit the topology of the decay; of particular importance are the $B^{-}$ vertex-fit $\chi^2$ and the $p_T$ asymmetry between the $B^-$ candidate and other tracks from the same PV, defined as

$$A_{p_T} = \frac{p_T^B - p_T^{cone}}{p_T^B + p_T^{cone}} \quad (3.1)$$

where $p_T^B$ is the $p_T$ of the reconstructed $B^-$ signal candidate and $p_T^{cone}$ is the scalar sum of the $p_T$ of all other tracks in a cone surrounding the $B^-$ candidate. This asymmetry is a quantitative measure of the isolation of the $B^-$ candidate. Other input quantities used include the logarithm of the $\chi^2_{DF}$ for various particles and the $p_T$ of the $K_S^0$ candidate (for downstream candidates only). The selection requirement on the BDT output was chosen to minimise the uncertainty on the $CP$ observables. The optimisation is performed separately for the GLW and ADS decay modes. Averaged across the whole dataset used for the analysis, the BDT selection applied to the favoured $B^- \rightarrow D(K^{-}\pi^+)K^{*-}$ channel gives a signal efficiency of 95% (90%) and a background rejection of 94% (95%) for long (downstream) candidates. Similarly, the four-body favoured $B^- \rightarrow D(K^{-}\pi^+\pi^-\pi^+)K^{*-}$ channel gives a signal efficiency of 95% (93%) and a background rejection of 96% (97%) for long (downstream) candidates.

4 Fit to the invariant mass distribution

Extended unbinned maximum likelihood fits are applied to the $B$ candidate mass spectra, in the mass range 4900–5600 MeV/$c^2$, for candidates reconstructed in the favoured decay modes $B^- \rightarrow D(K^{-}\pi^+)K^{*-}$ and $B^- \rightarrow D(K^{-}\pi^+\pi^-\pi^+)K^{*-}$. The same fit model is applied to both spectra. The model consists of a signal component, backgrounds from partially reconstructed decays and a combinatorial background shape. The charmless background has been suppressed to negligible levels, therefore no component is included in the fit. The signal component is described by the sum of two Crystal Ball (CB) functions [42] with the same peak position, which contain small radiative tails that extend towards lower invariant
mass. The signal shape parameters are determined from simulation, except for the common peak position and one of the widths, which are allowed to vary in the fit. The combinatorial background is described by an exponential function. The results of these fits are shown in figure 1.

Backgrounds from partially reconstructed decays include $B \to D^* K^*$ decays where a pion or photon is not reconstructed, namely $B^- \to D^{*0}(D^{0}\pi^0)K^{*-}$, $B^- \to D^{*0}(D^{0}\pi^+)K^{*-}$ and $\overline{B}^0 \to D^{*-}(D^0\pi^+)K^{*-}$. These are decays of $B$ mesons into two vector particles, which are described by three independent helicity amplitudes, corresponding to the helicity states of the $D^*$ meson, denoted by $-1$, 0 and $+1$. The reconstructed $B$-candidate mass distributions for $-1$ and $+1$ helicity states are indistinguishable so these states are collectively named $\pm 1$. Therefore, for each $D^* K^{*-}$ channel, two different components are considered, 0 and $\pm 1$. The shape of these components are determined from simulations and parameterised as Gaussian functions convolved with a second-order polynomial, described in detail in refs. [43, 44], with all parameters fixed in the fit. The ratio between the yields of the three $D^* K^{*-}$ decay modes are fixed according to their branching fractions and selection efficiencies, assuming no CP violation. This procedure assumes that the longitudinal polarisation fraction for $D^0 K^+$ decays is the same for $B^0$ and $B^-$ mesons. The total partially reconstructed yield is allowed to vary as well as the yield ratio between the sum of the 0 shapes and the sum of the $\pm 1$ shapes.

As seen from the fit projections in figure 1, these background contributions are sufficient to describe the overall invariant mass distribution of the favoured decay mode. A number of other backgrounds which could appear close to the signal peak are studied in simulation and found to be negligible, for example $B^- \to D K^{*-} \pi^0$ and $B^- \to D(K^0_S \pi \pi)K^-$. Figure 1 shows that the main background contribution near the signal peak is combinatorial background, while only a small amount of partially reconstructed background enters the signal region. A significant fraction of the combinatorial background is expected to come from $B^- \to D \pi^- X$ decays combined with a real but unrelated $K^0_S$ meson, which is consistent with the observed difference in background level between the $B^- \to D(K^- \pi^+)K^{*-}$ and $B^- \to D(K^+ \pi^-)K^{*-}$ decay modes. In the case of the $B^- \to D(K^+ K^-)K^{*-}$ decay mode, an additional background coming from the decay $A^0_b \to A^+_c(pK^- \pi^+)K^{*-}$ needs to be considered, where the $\pi^+$ meson is not reconstructed and the proton is misidentified as a kaon. The shape of this background is obtained by parameterising the mass distribution from simulated background events; the shape parameters are fixed in the fits described below. The yield of $A^0_b \to A^+_c(pK^- \pi^+)K^{*-}$ compared to signal in the $B^- \to D(K^- \pi^+)K^{*-}$ favoured decay mode is allowed to vary.

Restricting the lower limit of the mass range to 5230 MeV/$c^2$ removes 0.4% of signal and avoids the need to fit the backgrounds from partially reconstructed decays in each of the decay modes. This strategy improves fit stability in the decay modes with lower yields. The shape and yield of the small amount of background from partially reconstructed decays present in all $D$ decay categories above 5230 MeV/$c^2$ is determined and fixed from the fit to data with the favoured decay, adjusted for the smaller branching fractions of the rarer $D$ decays. The yield is estimated to be less than one candidate for all CP-violating decay modes, and therefore uncertainties due to the assumptions present in the initial fit have a
very small effect. These uncertainties in the yield, shape and possible asymmetries in the distribution between $B^+$ and $B^−$ are sources of systematic uncertainty.

A simultaneous fit is performed to 56 $B$-meson mass distributions, corresponding to each of the seven $D$ decay modes ($K^−\pi^+$, $K^+\bar{K}^−$, $\pi^+\pi^−$, $K^+\pi^−$, $K^−\pi^+\pi^−$, $\pi^+\pi^−\pi^+\pi^−$ and $K^+\pi^−\pi^+\pi^−$), two $B$-meson charges ($B^+$ and $B^−$), two $K^0_S$ reconstruction types (long and downstream) and two periods of data taking (Run 1 and Run 2). Based on fits to the data and simulation samples, the same signal peak position and width are used for the two periods of data taking, $B$-meson charges and $K^0_S$ reconstruction types, but they are allowed to differ between two- and four-body decay modes. The combinatorial background slope is required to have the same value for all two- and four-body decay modes separately, but can differ between long and downstream categories.
The parameters determined from the simultaneous fit are the yields in the favoured signal decay modes and the CP observables $A_K$, $A_K$, $A_{\pi}$, $R_K$, $R_{\pi}$, $R_K^*$, $R_{\pi}^*$, $A_{K\pi}$, $A_{K\pi}$, $R_{K\pi}$, and $R_{K\pi}^-$. The observables are related to the ratios between the yields through various efficiency corrections, given by

$$R_{hh} = \frac{N(B^- \rightarrow D(h^+h^-)K^{*+})}{N(B^- \rightarrow D(K^-\pi^+)K^{*-})} \times \frac{B(D^0 \rightarrow K^-\pi^+)}{B(D^0 \rightarrow hh)} \times \frac{\epsilon_{sel}(K\pi)}{\epsilon_{sel}(hh)} \times \frac{\epsilon_{PID}(K\pi)}{\epsilon_{PID}(hh)},$$  (4.1)

$$R_{K^*}^+ = \frac{N(B^\pm \rightarrow D(K^+\pi^\pm)K^{*+})}{N(B^\pm \rightarrow D(K^\mp\pi^\pm)K^{*+})} \times \frac{\epsilon_{sel}(K\pi)}{\epsilon_{sel}(K)} \times \frac{1}{\epsilon_{veto}(\pi K)},$$  (4.2)

$$R_{\pi\pi} = \frac{N(B^- \rightarrow D(\pi^+\pi^-\pi^+\pi^-)K^{*-})}{N(B^- \rightarrow D(K^-\pi^+\pi^-\pi^+)K^{*-})} \times \frac{B(D^0 \rightarrow K^-\pi^+\pi^-\pi^+)}{B(D^0 \rightarrow \pi\pi\pi\pi)} \times \frac{\epsilon_{sel}(K\pi\pi\pi)}{\epsilon_{sel}(\pi K\pi\pi)} \times \frac{\epsilon_{PID}(K\pi\pi\pi)}{\epsilon_{PID}(\pi\pi\pi)} \times \frac{1}{\epsilon_{veto}(\pi K K\pi)},$$  (4.3)

$$R_{K\pi}^\pm = \frac{N(B^\pm \rightarrow D(K^+\pi^\pm\pi^\pm\pi^\pm)K^{*\pm})}{N(B^\pm \rightarrow D(K^\mp\pi^\pm\pi^\pm\pi^\pm)K^{*\pm})} \times \frac{\epsilon_{sel}(K\pi\pi\pi)}{\epsilon_{sel}(K\pi\pi)} \times \frac{\epsilon_{PID}(K\pi\pi\pi)}{\epsilon_{PID}(\pi\pi\pi)} \times \frac{1}{\epsilon_{veto}(\pi K K\pi)},$$  (4.4)

where $\epsilon_{sel}$, $\epsilon_{PID}$ and $\epsilon_{veto}$ are the selection, particle-identification and veto efficiencies, respectively, $N$ is the yield of the specified decay and $h$ represents a $\pi$ or $\bar{K}$ meson. The veto is only applied to the ADS decay mode to reduce cross-feed from the favoured decay. These efficiencies are determined from simulation. The selection efficiency for various $D$ decay modes accounts for any differences in kinematics between these modes as well as a tighter BDT cut in the ADS decay mode, which is applied in order to optimise the uncertainty in the CP observables. Any further correction to the four-body observables due to nonuniform acceptance was found to be negligible. The efficiencies cancel for the determination of the CP asymmetries, while corrections are applied for the $B^+$, $B^-$ production asymmetry, $A_{\text{prod}}$, and decay mode dependent detection asymmetries, $A_{\text{det}}$, which are taken from previous LHCb measurements for production asymmetry [45], kaon detection asymmetry [46] and pion detection asymmetry [47]. The value $A_{\text{prod}}$ is assumed to be the same for 7 TeV, 8 TeV and 13 TeV data. A possible difference in $A_{\text{prod}}$ for Run 2 data compared to Run 1 is accounted for as a systematic uncertainty. As the asymmetries are small, $O(1\%)$ or less, the observed uncorrected asymmetry $A_{\text{raw}}$ can be expressed as the sum $A_{\text{raw}} = A_{\text{phys}} + A_{\text{prod}} + A_{\text{det}}$, where $A_{\text{phys}}$ is the CP asymmetry to be extracted. Hence, $A_{\text{prod}}$ and $A_{\text{det}}$ provide additive corrections to the measured asymmetry.

5 Results

The invariant mass spectra and resulting fits to data, combining Run 1, Run 2, long and downstream categories, are shown in figures 2 and 3. The yields determined from the fitted parameters are given in table 1. The Wilks’ theorem statistical significance [48] for the two-body ADS decay mode is 4.2$\sigma$, while for the four-body ADS decay mode it is 2.8$\sigma$. This represents the first evidence of the two-body suppressed decay.

Branching fractions [39], various efficiencies and asymmetries are used as inputs to the simultaneous fit in order to relate the measured yields to the CP observables. Each of these inputs has an associated uncertainty which needs to be propagated to the CP
Figure 2. Result of fits to data for the two-body decay modes with Run 1, Run 2, long and downstream categories summed for presentation. The signal is represented by the red shaded area, the combinatorial background by the dotted blue line and the partially reconstructed background by the solid green line. In the $D^{0} \rightarrow K^{+} K^{-}$ fits the $A_{L_{r}}^{0} \rightarrow A_{L_{r}}^{0} K^{+} K^{-}$ background is represented by the dashed purple line. The total fit is given by the black line. The residuals, shown below each plot, are defined as the difference between the data and the fit value in each bin, normalised by the uncertainty.
Figure 3. Result of fits to data for the four-body decay modes with Run 1, Run 2, long and downstream categories summed for presentation. The signal is represented by the red shaded area, the combinatorial background by the dotted blue line and the partially reconstructed background by the solid green line. The total fit is given by the black line. The residuals are shown below each plot.

| Decay mode                | $B^-$ yield | $B^+$ yield |
|---------------------------|-------------|-------------|
| $B^\pm \to D(K^\mp \pi^\pm)K^{*\pm}$ | 996 ± 34    | 1035 ± 35   |
| $B^\pm \to D(K^+K^-)K^{*\pm}$          | 134 ± 14    | 121 ± 13    |
| $B^\pm \to D(\pi^+\pi^-)K^{*\pm}$      | 45 ± 10     | 33 ± 9      |
| $B^\pm \to D(K^+\pi^\mp)K^{*\pm}$      | 1.6 ± 1.9   | 19 ± 7      |
| $B^\pm \to D(K^\mp\pi^+\pi^-)K^{*\pm}$ | 556 ± 26    | 588 ± 27    |
| $B^\pm \to D(\pi^+\pi^-\pi^+\pi^-)K^{*\pm}$ | 59 ± 10   | 56 ± 10     |
| $B^\pm \to D(K^+\pi^+\pi^-\pi^+)K^{*\pm}$ | 3 ± 5      | 10 ± 6      |

Table 1. Fitted yields in each of the $B$ decay modes. The uncertainties are statistical only.
observables giving rise to the systematic uncertainties. In the case of the efficiencies, uncertainties arise from a limited sample size of simulated events. Uncertainties on $A_{\text{prod}}$ and $A_{\text{det}}$ are taken from previous LHCb measurements in Run 1 [45–47]. The changes to the detector between the data-taking periods are not expected to significantly affect the $A_{\text{det}}$ measurement. For $A_{\text{prod}}$, a conservative estimate, double the Run 1 uncertainty, is assigned to accommodate a possible dependence of the production asymmetry on the centre-of-mass energy. The systematic uncertainties due to the use of fixed inputs from branching ratios, simulation efficiencies, asymmetry corrections and shape parameters are estimated by performing multiple fits to data where each relevant parameter is varied according to a Gaussian distribution with the width as the assigned uncertainty. The standard deviation of the fitted parameter distribution is assigned as the systematic uncertainty. Correlations between the shape parameters are small, typically less than 10%, and are ignored. Tests, where the most relevant correlations have been included, show a negligible impact on the systematic uncertainty arising from the fixed shape parameters.

Other systematic uncertainties arise from the modelling of the signal and partially reconstructed backgrounds and the effect of any residual charmless $B$ decays. The systematic uncertainties from these sources are computed by generating pseudoexperiments. In each case the generated model is varied according to the systematic effects being estimated. The systematic uncertainty on each observable is taken to be the difference between the mean of the fitted parameter distribution and the generated value. The systematic uncertainty on the partially reconstructed background takes into account uncertainties in the yield and shape parameters, as well as possible asymmetries due to CP violation. The contamination from charmless $B$ decays is consistent with zero, although it has a large uncertainty. Pseudoexperiments are generated with charmless decays according to the fit model, with the number of events fluctuating according to the uncertainty in the fit. The assumption that the slope of the function describing the combinatorial background is the same for all $D$ decay modes has an associated uncertainty. Pseudoexperiments are generated fixing the slope parameters to a different value for each decay mode, where the value used is obtained from fits in the mass region above the $B$ mass. For the ADS mode, a potential background from $B_s^0 \rightarrow D(K^*(1410)^0 \rightarrow K^*(892)^-\pi^+)$, where the $\pi^+$ meson is not reconstructed, is considered. An estimate of the contribution using simulated events and the branching fraction [49] is found to be $2.6 \pm 2.6$ events, which is consistent with observations from data in the region of $B$ mass below the lower limit of the simultaneous fit. The shape of this background is obtained by parameterising the mass distribution from simulated events. A systematic is assigned by performing many fits to data varying the yield according to a Gaussian distribution with the width as the assigned uncertainty. The standard deviation of the fitted parameter distribution is assigned as the systematic uncertainty. A summary of the components of the systematic uncertainties for the CP observables is given in table 2.
| Source | $A_{K\pi}$ | $A_{KK}$ | $A_{\pi\pi}$ | $R_{KK}$ | $R_{\pi\pi}$ | $R_{K\pi}^+$ | $R_{K\pi}^-$ | $A_{KK\pi}$ | $A_{\pi\pi\pi}$ | $R_{\pi\pi\pi}$ | $R_{K\pi\pi}^+$ | $R_{K\pi\pi}^-$ |
|--------|----------|--------|-------------|--------|-------------|--------------|------------|-------------|----------------|----------------|--------------|--------------|
| Statistical | 0.023 | 0.07 | 0.13 | 0.09 | 0.15 | 0.06 | 0.004 | 0.031 | 0.11 | 0.13 | 0.008 | 0.007 |
| Branching fractions | - | - | 0.013 | 0.001 | 0.012 | - | - | - | 0.0008 | 0.027 | - | - |
| Selection efficiencies | - | - | 0.007 | - | 0.006 | 0.0002 | - | - | 0.0008 | 0.014 | - | - |
| PID efficiencies | - | - | 0.002 | - | 0.002 | - | - | - | - | 0.002 | - | - |
| Veto efficiencies | - | - | - | - | - | 0.0001 | - | - | - | - | - | - |
| $A_{\text{prod}}$ | 0.0073 | 0.007 | - | 0.008 | - | - | - | - | 0.0079 | 0.0077 | - | - |
| $A_{\text{det}}$ | 0.0034 | 0.003 | - | 0.003 | - | 0.0001 | - | 0.0034 | 0.0030 | - | 0.0001 | - |
| Signal shape | 0.0011 | 0.003 | 0.011 | 0.003 | 0.027 | 0.0011 | 0.0013 | 0.0017 | 0.0022 | 0.010 | 0.0030 | 0.0038 |
| Combinatorial shape | 0.0012 | 0.003 | 0.004 | 0.005 | 0.009 | 0.0002 | 0.0003 | 0.0001 | 0.0018 | - | 0.0012 | 0.0004 |
| Partially reconstructed shape | 0.0007 | 0.001 | 0.001 | 0.003 | 0.005 | - | 0.0003 | 0.0003 | 0.0005 | 0.002 | 0.0008 | 0.0001 |
| Charmless | 0.0008 | - | 0.002 | 0.003 | 0.007 | - | 0.0003 | 0.0009 | 0.0030 | 0.002 | 0.0008 | 0.0001 |
| $B_s^0 \to \Lambda_c^+ K^-$ | 0.0002 | - | 0.011 | - | 0.001 | 0.0001 | - | - | - | - | - | - |
| $B_s^0 \to DK^*(1410)^0$ | - | - | - | - | - | 0.0005 | 0.0001 | - | - | - | - | - |
| Total systematic | 0.0083 | 0.009 | 0.022 | 0.012 | 0.032 | 0.0012 | 0.0014 | 0.0088 | 0.0093 | 0.032 | 0.0034 | 0.0038 |

Table 2. Summary of systematic uncertainties. Uncertainties are not shown if they are more than two orders of magnitude smaller than the statistical uncertainty.
The CP observables determined from the fit shown in figures 2 and 3 are

\[
A_{K\pi} = -0.004 \pm 0.023 \pm 0.008 \\
A_{KK} = 0.06 \pm 0.07 \pm 0.01 \\
A_{\pi\pi} = 0.15 \pm 0.13 \pm 0.02 \\
R_{KK} = 1.22 \pm 0.09 \pm 0.01 \\
R_{\pi\pi} = 1.08 \pm 0.14 \pm 0.03 \\
R_{K\pi}^+ = 0.020 \pm 0.006 \pm 0.001 \\
R_{K\pi}^- = 0.002 \pm 0.004 \pm 0.001 \\
A_{K\pi\pi\pi} = -0.013 \pm 0.031 \pm 0.009 \\
A_{\pi\pi\pi\pi} = 0.02 \pm 0.11 \pm 0.01 \\
R_{\pi\pi\pi\pi} = 1.08 \pm 0.13 \pm 0.03 \\
R_{K\pi\pi\pi}^+ = 0.016 \pm 0.007 \pm 0.003 \\
R_{K\pi\pi\pi}^- = 0.006 \pm 0.006 \pm 0.004
\]

where the first uncertainty is statistical and the second is systematic. The correlation matrices for the statistical and systematic uncertainties are given in tables 3 and 4, respectively. The large correlations of the systematic uncertainties are mainly due to contributions from production and detection asymmetries. Combined results from the $K^+K^-$ and $\pi^+\pi^-$ decay modes, taking correlations into account, are

\[
R_{CP^+} = 1.18 \pm 0.08 \pm 0.01 \\
A_{CP^+} = 0.08 \pm 0.06 \pm 0.01
\]

where the first uncertainty is statistical and the second is systematic. In addition, $R^+$ and $R^-$ for the $K^+\pi^-$ and $K^+\pi^-\pi^+\pi^-$ decay modes can be transformed into the more commonly used $R_{ADS} = (R^+ + R^-)/2$ and $A_{ADS} = (R^- - R^+)/ (R^- + R^+).$ These results, taking correlations into account, are

\[
R_{K\pi}^{ADS} = 0.011 \pm 0.004 \pm 0.001 \\
A_{K\pi}^{ADS} = -0.81 \pm 0.17 \pm 0.04 \\
R_{K\pi\pi\pi}^{ADS} = 0.011 \pm 0.005 \pm 0.003 \\
A_{K\pi\pi\pi}^{ADS} = -0.45 \pm 0.21 \pm 0.14
\]

where the first uncertainty is statistical and the second is systematic. The measured asymmetries and ratios for the two-body $D$ meson decay modes are consistent with, and more precise than, the previous measurements from BaBar [16].

6 Interpretation

The CP observables measured in this analysis can be used to determine the physics parameters $r_B$, $\delta_B$ and $\gamma$, via eqs. (1.13)–(1.18). The parameter $\kappa$ is estimated by generating many amplitude models for $B \rightarrow DK_{S}^{0}\pi$ decays [50] consisting of various resonant
components whose relative amplitudes and phases are varied within limits according to the existing branching fraction measurements. The components used in the model are $B^- \rightarrow D^0 K^+(892)^-$ and the LASS lineshape [51]. The LASS lineshape is used to describe the $K\pi$ S-wave, which includes a nonresonant term and the $K_S^0(1430)^-$ resonance. Contributions from other resonances e.g. $K^*(1680)^- \rightarrow K_S^0 \pi^-$ and $D_S^+(2460)^- \rightarrow D\pi^-$, are considered to be negligible in the selected $K^{*-}$ region and are not included in the model. For each model, the value of $\kappa$ is determined in the region of phase space defined by the $K^{*-}$ mass window and $K_S^0$ helicity angle requirements. The mean of the resulting distribution gives an estimate for $\kappa$ of 0.95 ± 0.06. The parameters $r_{D}\pi$, $\delta_{D}\pi$, $r_{D}K_{3}\pi$, $\delta_{D}K_{3}\pi$, $\kappa K_{3}\pi$ and $F_{4}\pi$ are also required as external inputs and are taken from refs. [15, 22–24].
Figure 4. Contour plots showing 2D scans of physics parameters $\gamma$ versus $r_B$ (left) and $\gamma$ versus $\delta_B$ (right). The dashed lines represent the $\Delta \chi^2 = 2.30, 6.18, 11.8$ contours, corresponding to 68.3%, 95.5%, 99.7% confidence levels (CL), respectively. The colour scale represents $1 - \chi^2$.

Using the measured values of the $CP$ observables, their uncertainties and the covariance matrices, a global $\chi^2$ minimisation is performed, resulting in a minimum $\chi^2$ of 3.0 with 9 degrees of freedom. A scan of physics parameters is performed for a range of values and the difference in $\chi^2$ between the parameter scan values and the global minimum, $\Delta \chi^2$, is evaluated. The confidence level for any pair of parameters is calculated assuming that these are normally distributed, which enables the $\Delta \chi^2 = 2.30, 6.18, 11.8$ contours to be drawn, corresponding to 68.3%, 95.5%, 99.7% confidence levels, respectively. These are shown in figure 4. The data are consistent with the value of $\gamma$ indicated by previous measurements $[4, 5]$, $\sim 70^\circ$, and result in a value of $r_B = 0.11 \pm 0.02$. This value of $r_B$ is determined at the point where the global $\chi^2$ of the fit is minimised.

7 Conclusions

A study of the $B^- \rightarrow DK^{*-}$ decay mode is presented where the $D$ meson decays to two- and four-body final states consisting of charged kaons and/or pions. The $CP$ observables $R_{CP}^{+}$, $A_{CP}^{+}$, $R_{K\pi}^{\pi}$, $R_{K\pi\pi}$, $A_{K\pi\pi}$, $R_{K\pi\pi}^{K\pi\pi}$ and $R_{K\pi\pi}^{K\pi\pi}$ are measured from the high purity sample obtained from $pp$ collision data recorded with the LHCb detector in Run 1 and Run 2. The measurement of the $CP$ asymmetries in the two-body decay modes and their ratio to the favoured decay mode is consistent with and more precise than the previous determination $[16]$. While no bounds on $\gamma$ are quoted due to the limited sensitivity of this decay mode in isolation, $B^- \rightarrow DK^{*-}$ decays will become valuable in constraining $\gamma$ in the future, as more data are collected and more $D$ decay modes are analysed.
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