10 = 6 + 4

Frank D. (Tony) Smith, Jr.
e-mail: tsmith@innerx.net
P. O. Box 370, Cartersville, GA 30120 USA
WWW URL:
http://www.innerx.net/personal/tsmith/TShome.html

Abstract

Some physics models have 10 dimensions that are usually decomposed into:
4 spacetime dimensions with local Lorentz Spin(1,3) symmetry
plus
a 6-dimensional compact space related to internal symmetries.

A possibly useful alternative decomposition is into:
6 spacetime dimensions with local Conformal symmetry
of the Conformal Group C(1,3) = Spin(2,4) = SU(2,2)
plus
a 4-dimensional compact Internal Symmetry Space
that can be taken to be complex projective 2-space $\mathbb{CP}^2$
which, since $\mathbb{CP}^2 = SU(3)/U(2)$,
is a natural representation space for $SU(3)$
and on which $U(2) = SU(2) \times U(1)$ can be represented naturally by local action.
# Contents

1 Decomposition of 10 Dimensions ............................................. 2  
   1.1 6-Dimensional Conformal spacetime ................................ 2  
   1.2 4-Dimensional Internal Symmetry Space ........................... 3  

2 Superstrings, Dixon, and D4-D5-E6-E7 .................................. 4  
   2.1 Superstrings ............................................................ 4  
   2.2 Geoffrey Dixon’s Division Algebra model .......................... 4  
   2.3 the D4-D5-E6-E7 model .............................................. 5  

3 Acknowledgements ............................................................. 6
1 Decomposition of 10 Dimensions

Some physics models have 10 dimensions that are usually decomposed into:

4 spacetime dimensions with local Lorentz $Spin(1, 3)$ symmetry

plus

a 6-dimensional compact space related to internal symmetries.

A possibly useful alternative decomposition is into:

6 spacetime dimensions with local $C(1, 3) = Spin(2, 4) = SU(2, 2)$ Conformal symmetry.

plus

a 4-dimensional compact Internal Symmetry Space.

1.1 6-Dimensional Conformal spacetime

Conformal symmetries and some of their physical applications are described in the book of Barut and Raczka [1].

The Conformal group $C(1, 3)$ of Minkowski spacetime is the group $SU(2, 2) = Spin(2, 4)$. As $Spin(2, 4)$, the Conformal group acts on a 6-dimensional $(2,4)$-space that is related to the 6-dimensional $CP^3$ space of Penrose twistors [2].

It is reasonable to consider the 6-dimensional Conformal space as the spacetime in the dimensional decomposition of 10-dimensional models because Conformal symmetry is consistent with such physics structures as:

Maxwell’s equations of electromagnetism;
the quantum theoretical hydrogen atom;

the canonical Dirac Lagrangian for massive fermions, as shown by Liu, Ma, and Hou [3];

gravity derived from the Conformal group using the MacDowell-Mansouri mechanism, as described by Mohapatra [4];

the Lie Sphere geometry of spacetime correlations; and

the Conformal physics model of I. E. Segal [5].

1.2 4-Dimensional Internal Symmetry Space

An example of a possibly useful 4-dimensional compact Internal Symmetry Space is complex projective 2-space $\mathbb{C}P^2$.

Since $\mathbb{C}P^2 = SU(3)/U(2)$, it is a natural representation space for $SU(3)$.

Further, $U(2) = SU(2) \times U(1)$ can be represented naturally on $\mathbb{C}P^2 = SU(3)/U(2)$ as a local action.

Therefore, all three of the gauge groups of the Standard Model $SU(3) \times SU(2) \times U(1)$ can be represented on the 4-dimensional compact Internal Symmetry Space $\mathbb{C}P^2 = SU(3)/U(2)$.

The following section lists some examples of physics models that have such 10-dimensional spaces: Superstring theory; the Division Algebra model of Geoffrey Dixon; and the $D_4 - D_5 - E_6 - E_7$ physics model.
2 Superstrings, Dixon, and D4-D5-E6-E7

2.1 Superstrings

The 10-dimensional space of Superstring theory is well known, and described in many references, so I will not try to summarize it here. One particularly current and thorough reference is the 2-volume work of Polchinski [6].

2.2 Geoffrey Dixon’s Division Algebra model

Geoffrey Dixon, in his publications and website [7], considers the real division algebras:
the real numbers \(\mathbb{R}\);
the complex numbers \(\mathbb{C}\);
the quaternions \(\mathbb{Q}\); and
the octonions \(\mathbb{O}\).

Dixon then forms the tensor product \(T = \mathbb{R} \otimes \mathbb{C} \otimes \mathbb{Q} \otimes \mathbb{O}\) and considers the 64-real-dimensional space \(T\).

Then Dixon takes the left-adjoint actions \(T_L = \mathbb{C}_L \otimes \mathbb{Q}_L \otimes \mathbb{O}_L\), and notes that \(T_L\) is isomorphic to \(\mathbb{C}(16) = Cl(0, 9) = \mathbb{C} \otimes Cl(0, 8)\).

Then Dixon considers the algebra \(T\) to be the spinor space of \(T_L\).

Then Dixon forms a matrix algebra \(T_L(2)\) as the \(2 \times 2\) matrices whose elements are in the left-action adjoint matrix algebra \(T_L\) and notes that \(T_L(2)\) is isomorphic to \(\mathbb{C}(32) = \mathbb{C} \otimes Cl(1, 9)\).

Dixon describes the matrices \(T_L(2)\) as having spinor space \(T \oplus T\) and \(\mathbb{C} \otimes Cl(1, 9)\) as the Dirac algebra of 10-dimensional \((1,9)\)-space.

Dixon then describes leptons and quarks in terms of reduction of the Dirac spinors of the 10-dimensional \((1,9)\)-space to the Dirac spinors of a
4-dimensional (1,3)-spacetime.

The right-action adjoint matrix algebra $T_R$ is not the same as the left-action adjoint $T_L$, because, although $C_R = C_L$ and $O_R = O_L$, it is a fact that $Q_R \neq Q_L$ (they are isomorphic but not identical).

Since $Q_R = Q$, the part of the matrix algebra $T_R$ that differs from $T_L$ is just $Q$, and the different part of the $2 \times 2$ matrix algebra $T_R(2)$ is just the $2 \times 2$ matrix algebra with quaternion entries $Q(2)$.

In section 6.7 of his book [7], Dixon shows that commutator closure of the set of traceless $2 \times 2$ matrices over the quaternions $Q$, which he denotes by $sl(2, Q)$, is the Lie algebra of $Spin(1, 5)$.

Since the Lie algebra $Spin(1, 5)$ is just the Lie algebra of the Conformal group $C(1, 3) = Spin(2, 4) = SU(2, 2)$ with a different signature,

I conjecture that it might be useful to consider the spacetime part of Dixon’s 10-dimensional (1,9)-space to be the 6-dimensional (1,5)-spacetime of $Spin(1, 5)$.

That would leave a 4-dimensional (0,4)-space to be used as an Internal Symmetry Space.

2.3 the D4-D5-E6-E7 model

The $D_5$ Lie algebra of the $D_4 - D_5 - E_6 - E_7$ physics model corresponds (with Conformal signature) to the Lie algebra $Spin(2, 8)$ of the Clifford algebra $Cl(2, 8)$ whose vector space is 10-dimensional.

As the $D_4 - D_5 - E_6 - E_7$ physics model is described on the web [8], I will not try to summarize it here.
3 Acknowledgements

The idea of 6-dimensional spacetime with Conformal symmetry was motivated by the works of I. E. Segal [5] and by e-mail conversations with Robert Neil Boyd.

The idea of 4-dimensional Internal Symmetry Space was motivated by Cayley calibrations of octonions [9] and by e-mail conversations with Matti Pitkanen.
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