N-particle Bogoliubov vacuum state

Jacek Dziarmaga and Krzysztof Sacha

Intytut Fizyki Uniwersytetu Jagiellońskiego, ul. Reymonta 4, 30-059 Kraków, Poland

We consider the Bogoliubov vacuum state in the number-conserving Bogoliubov theory proposed by Castin and Dum [Phys. Rev. A 57, 3008 (1998)]. We show that in the particle representation the vacuum can be written in a simple diagonal form. The vacuum state can describe the stationary \( N \)-particle ground state of a condensate in a trap, but it can also represent a dynamical state when, for example, a Bose-Einstein condensate initially prepared in the stationary ground state is subject to a time-dependent perturbation. In both cases the diagonal form of the Bogoliubov vacuum can be obtained by basically diagonalizing the reduced single particle density matrix of the vacuum. We compare \( N \)-body states obtained within the Bogoliubov theory with the exact ground states in a 3-site Bose-Hubbard model. In this example, the Bogoliubov theory fails to accurately describe the stationary ground state in the limit when \( N \to \infty \) but a small fraction of depleted particles is kept constant.

I. INTRODUCTION

Bose-Einstein condensate (BEC) is a state of \( N \) bosons where all particles occupy the same single particle state \( \left| 0 \right> \). Such a state can be an eigenstate of the system only in an ideal case when particles do not interact. However, dilute atomic gases offer a possibility for experimental realization of states that are not too far from ideal BEC \cite{1}. In a dilute system the effect of interactions is weak and resulting quantum depletion of a condensate is not greater than 1\% in a typical experiment \cite{2}.

From the point of view of theoretical description of a system weak interactions allow one to employ various perturbation approaches. The approach commonly used in the field is the Bogoliubov theory which basically relies on substitution of a full quantum many-body Hamiltonian by an effective Hamiltonian which is a quadratic approximation of the exact Hamiltonian \cite{1, 3}. In the original version of the Bogoliubov theory a condensate state is identified with a coherent state that for a system of massive particles, in non-relativistic quantum mechanics, can not be acceptable. In the literature one may find several versions of number conserving Bogoliubov theory \cite{3, 5, 6, 7, 8}. In the present publication we consider the theory proposed by Castin and Dum \cite{3, 8}. In this introductory part we will briefly describe the key elements of the theory.

Let us begin with a case where \( N \) particles, interacting via zero range potential \( V(\mathbf{r}) = g_0 \delta(\mathbf{r}) \), are trapped in a time-independent potential \( U(\mathbf{r}) \). We would like to get approximation for the ground state of the system. In the Castin and Dum theory bosonic field operator \( \hat{\psi} \) is decomposed into an operator \( \hat{a}_0 \) that annihilates a particle in a condensate mode \( \phi_0 \) and an annihilation operator \( \delta\hat{\psi} \) defined in the subspace orthogonal to \( \phi_0 \),

\[
\hat{\psi}(\mathbf{r}) = \phi_0(\mathbf{r})\hat{a}_0 + \delta\hat{\psi}(\mathbf{r}). \tag{1}
\]

We are interested in \( N \)-particle states where a contribution coming from the subspace orthogonal to \( \phi_0 \) is small,

\[
\langle \hat{a}_0^\dagger \hat{a}_0 \rangle \approx N, \tag{2}
\]

where \( \frac{dN}{N} \ll 1 \). Substituting (1) into the Hamiltonian of a system and performing expansion in a "small parameter \( \delta\hat{\psi}^\dagger \) up to second order leads to the stationary Gross-Pitaevskii equation for \( \phi_0 \)

\[
H_{GP}\phi_0 = \left[ -\frac{\hbar^2}{2m}\nabla^2 + U(\mathbf{r}) + g_0 N|\phi_0|^2 - \mu \right] \phi_0 = 0, \tag{3}
\]

and to an effective Hamiltonian quadratic in \( \delta\hat{\psi} \). Castin and Dum have collected operators \( \delta\hat{\psi} \) and \( \hat{a}_0 \) in such a way that the effective Hamiltonian,

\[
\hat{H}_{\text{eff}} = E_0(N) + \frac{1}{2} \int d^3r \left( \hat{\Lambda}^\dagger, -\hat{\Lambda} \right) L \left( \hat{\Lambda}/\hat{\Lambda}^\dagger \right), \tag{4}
\]

depends only on operators

\[
\hat{\Lambda}(\mathbf{r}) = \frac{\delta\hat{\psi}(\mathbf{r})}{\sqrt{N}}. \tag{5}
\]

To get (4), sometimes it was necessary to introduce by hand the operator \( \hat{a}_0/\sqrt{N} \), i.e. a quantity of the order of one, see (2). Diagonalization of (4) reduces itself to diagonalization of a non-hermitian operator

\[
\mathcal{L} = \left( \begin{array}{cc} H_{GP} + g_0 N \hat{Q} |\phi_0|^2 \hat{Q}^\dagger & g_0 N \hat{Q} \phi_0^2 \hat{Q}^\dagger \\ -g_0 N \hat{Q}^\dagger \phi_0^2 \hat{Q} & -H_{GP} - g_0 N \hat{Q}^\dagger |\phi_0|^2 \hat{Q} \end{array} \right), \tag{6}
\]

where \( \hat{Q} = 1 - |\phi_0\rangle\langle \phi_0 | \). Generally the \( \mathcal{L} \) operator is diagonalizable and, for the ground state of the system we are interested in, its eigenvalues \( E_m \) are real. Owing to the symmetries of \( \mathcal{L} \) (i.e. \( \sigma_x \mathcal{L} \sigma_x = -\mathcal{L}^\dagger \) and \( \sigma_z \mathcal{L} \sigma_z = \mathcal{L} \)), where \( \sigma_x, \sigma_z \) are Pauli matrices) it is not difficult to show that if

\[
|\psi^L_m \rangle = \begin{pmatrix} u_m \\ v_m \end{pmatrix}, \tag{7}
\]

is a right eigenvector of \( \mathcal{L} \) corresponding to an eigenvalue \( E_m \), the left eigenvector is

\[
|\psi^R_m \rangle = \sigma_z |\psi^L_m \rangle, \tag{8}
\]
is a right eigenvector corresponding to \(-E_m\). These properties imply that the eigenvectors can be divided into two classes. The first class (the so-called family "+"\) consists of eigenvectors which can be normalized so that
\[
\langle \psi^R_m | \sigma_z | \psi^R_m \rangle = \langle u_m | u_m \rangle - \langle v_m | v_m \rangle = \delta_{nm} .
\] (10)
Employing transformation (9) we obtain, from eigenvectors belonging to the family "+", eigenvectors that form the family "−" where \( \langle \psi^R_i | \sigma_z | \psi^R_j \rangle = -\delta_{ij} \). There are always two eigenvectors of \( \mathcal{L} \)
\[
\begin{pmatrix}
\phi_0 \\
0
\end{pmatrix}, \quad \begin{pmatrix}
0 \\
\phi_0^*
\end{pmatrix},
\] (11)
corresponding to zero eigenvalue. Collecting all eigenvectors of \( \mathcal{L} \) one can decompose the identity operator in the following form
\[
\mathbf{i} = \left( \langle \phi_0 |, 0 \right) + \left( 0, \langle \phi_0^* | \right)
+ \sum_{m \in ^{+\prime}} \left[ \left( \langle u_m |, \langle v_m | \right) - \langle v_m^* |, \langle u_m^* | \right).
\] (12)
From the analysis performed it is clear that
\[
\langle u_m | \phi_0 \rangle = 0,
\langle v_m | \phi_0^* \rangle = 0,
\] (13)
and in the subspace of functions orthogonal to a condensate wave function \( \phi_0 \) the completeness relation can be written in the following form
\[
\mathbf{i} = \sum_{m \in ^{+\prime}} \left( \langle u_m | \langle u_m \rangle | - | v_m^* \rangle \langle v_m^* \rangle \right).
\] (14)
Having calculated Bogoliubov modes \( (u_m, v_m) \) one can easily find diagonal form of the effective Hamiltonian
\[
\hat{H}_{\text{eff}} = \tilde{E}_0(N) + \sum_{m \in ^{+\prime}} E_m \hat{b}_m \hat{b}_m^\dagger,
\] (15)
where so called quasi-particle annihilation operators
\[
\hat{b}_m = \frac{1}{\sqrt{N}} \left( \hat{a}^\dagger_0 \langle u_m \rangle | \tilde{\psi} \rangle - \hat{a}_0 \langle v_m \rangle | \tilde{\psi}^\dagger \right),
\] (16)
fulfill a bosonic commutation relation
\[
[\hat{b}_m, \hat{b}_m^\dagger] = \delta_{nm} + \mathcal{O}(1/N).
\] (17)
The ground state of the Hamiltonian (15) is a Bogoliubov vacuum state that is annihilated by all quasi-particle annihilation operators
\[
\hat{b}_m | 0 \rangle_B = 0.
\] (18)
Excited states can be generated by means of the quasi-particle creation operators
\[
| m_1, m_2, \ldots \rangle = \prod_{k=1}^{\infty} \left( \frac{\hat{b}^\dagger_k}{\sqrt{m_k}} \right) | 0 \rangle_B .
\] (19)

The advantage of the number conserving version of the Bogoliubov theory with respect to the original theory is that one always deals with states of exactly \( N \) particles — application of quasi-particle operators does not change number of particles thanks to the presence of the \( \hat{a}_0 \) and \( \hat{a}_0^\dagger \) operators in Eq. (10).

Having a condensate in the ground state of the trapping potential \( U(\vec{r}) \) (i.e. in the Bogoliubov vacuum state \( | 0 \rangle_B \)) we may apply a time-dependent perturbation. Then \( \phi_0 \) starts evolving according to the time-dependent Gross-Pitaevskii equation and the Bogoliubov modes evolve according to the time-dependent Bogoliubov-de Gennes equations
\[
i \hbar \frac{d}{dt} \begin{pmatrix}
\psi_k(\vec{r}, t) \\
v_k(\vec{r}, t)
\end{pmatrix} = \mathcal{L}(t) \begin{pmatrix}
\psi_k(\vec{r}, t) \\
v_k(\vec{r}, t)
\end{pmatrix} .
\] (20)
However, the system remains in a Bogoliubov vacuum state but this Bogoliubov vacuum and the quasi-particle operators \( \hat{b}_m(t) \) depend on time.

Actually one may define any initial Bogoliubov vacuum state (not necessary the solution for the ground state of a time-independent problem). To this end one has to define \( \phi_0 \) and initial Bogoliubov modes \( (u_m, v_m) \) in such a way that (10) and (13) are fulfilled. For example choosing \( \phi_0 = \psi_0 \) and \( (u_m = \psi_m, v_m = 0) \) for the family "+" (where \( \psi_j \) are the eigenstates of the harmonic oscillator) the initial Bogoliubov vacuum corresponds to a perfect condensate where all particles occupy the ground state wavefunction of the harmonic oscillator.

We should mention that in practice the projection operator \( \hat{Q} \) in the \( \mathcal{L} \) operator (9) can be omitted. One can solve the Bogoliubov-de Gennes equations in the standard form and at the end apply the operator \( \hat{Q} \).

**II. RESULTS**

In this section we will restrict ourselves to analysis of the Bogoliubov vacuum state. The Bogoliubov vacuum is a state of \( N \) particle system where there is no quasi-particle, i.e. it is annihilated by all quasi-particle annihilation operators \( \hat{b}_m \) (quasi-particle vacuum), see (18). The particle vacuum (a state with no particles) is also annihilated by all \( \hat{b}_m \) because of the presence of a particle annihilation operator in each part of the \( \hat{b}_m \) operator, see (10). We have shown (18) (19) that the Bogoliubov vacuum can be obtained from the particle vacuum with the help of some particle creation operator \( \hat{d}^\dagger \),
\[
| 0 \rangle_B \sim \left( \hat{d}^\dagger \right)^M | 0 \rangle .
\] (21)
Indeed, if we require that the $\hat{d}^\dagger$ commutes with all quasi-particle operators,
\[
[\hat{b}_m, \hat{d}^\dagger] = 0,
\] (22)
then
\[
\hat{b}_m \left( \hat{d}^\dagger \right)^M |0\rangle = \left( \hat{d}^\dagger \right)^M \hat{b}_m |0\rangle = 0.
\] (23)

It turns out that two-particle creation operator
\[
\hat{d}^\dagger = \hat{a}^\dagger_0 \hat{a}^\dagger_0 + \sum_{\alpha, \beta = 1}^{\infty} Z_{\alpha \beta} \hat{a}^\dagger_{\alpha} \hat{a}^\dagger_{\beta},
\] (24)
(where $\hat{a}^\dagger_\alpha$’s create atoms in modes $\phi_\alpha$’s orthogonal to the condensate wavefunction $\phi_0$) solves the set of equations \ ([22]). Substituting \ ([24]) to \ ([22]) yields the following equation for the $Z_{\alpha \beta}$ matrix
\[
\langle v_m | \phi^*_\alpha \rangle = \sum_{\beta = 1}^{\infty} \langle u_m | \phi^*_\beta \rangle Z_{\alpha \beta}.
\] (25)

One may wonder how it is possible that a bosonic quasi-particle annihilation operator may annihilate, in fact, an infinite number of states (for each $N$ there is a Bogoliubov vacuum) \ ([12]). It is due to the fact that, strictly speaking, $b_m$ are not bosonic operators. However, for large $N$ corrections to bosonic commutation relations are small [see \ ([17])] and at the present order of approximation we may consider $b_m$ as bosonic operators that allows us to define (within a subspace of a fixed $N$) a Fock space \ ([19]).

Let us now show that solution of \ ([20]) can be significantly simplified thanks to properties of the Bogoliubov modes \ ([11]). To this end let us multiply \ ([25]) by $\langle \phi^*_\gamma | v_m \rangle$ and then sum over $m$ which leads to the following equation
\[
\langle \phi^*_\gamma | \hat{d}^\dagger | \phi^*_\alpha \rangle = \sum_{\beta = 1}^{\infty} \langle \phi^*_\gamma | \hat{\Delta} | \phi^*_\beta \rangle Z_{\alpha \beta},
\] (26)
where $\hat{d}^\dagger$ and $\hat{\Delta}$ operators are defined as
\[
\hat{d}^\dagger = \sum_{m \in \mathbb{Z}^+*} |v_m\rangle \langle v_m|,
\]
\[
\hat{\Delta} = \sum_{m \in \mathbb{Z}^+*} |v_m\rangle \langle u_m|.
\] (27)

Single particle density matrix corresponding to the Bogoliubov vacuum state reads
\[
\rho(\vec{r}, \vec{r}') = N_0 \phi_0^*(\vec{r}) \phi_0(\vec{r}') + \sum_{m \in \mathbb{Z}^+*} v_m(\vec{r}) v^*_m(\vec{r}'),
\] (28)
which indicates that $\hat{d}^\dagger$ operator is a part of the single particle density operator corresponding to the subspace orthogonal to the condensate wavefunction $\phi_0$. Thus eigenstates of the single particle density matrix
\[
\rho(\vec{r}, \vec{r}') = N_0 \phi_0^*(\vec{r}) \phi_0(\vec{r}') + \sum_{\alpha = 1}^{\infty} dN_\alpha \phi^*_\alpha(\vec{r}) \phi_\alpha(\vec{r}'),
\] (29)
(where $dN_\alpha$ are numbers of particles depleted from a condensate) diagonalize also $\hat{d}^\dagger$
\[
d^\dagger = \sum_{\alpha = 1}^{\infty} dN_\alpha |\phi^*_\alpha\rangle \langle \phi^*_\alpha|,
\] (30)

Employing the completeness relation \ ([14]) and the orthogonality relation \ ([10]) it is easy to show that
\[
d^\dagger \hat{\Delta} = \hat{\Delta} d^\dagger,
\] (31)
which in turn implies that eigenstates of the single particle density matrix allows us to find also diagonal form of the $\hat{\Delta}$ operator
\[
\hat{\Delta} = \sum_{\alpha = 1}^{\infty} \Delta_\alpha |\phi^*_\alpha\rangle \langle \phi^*_\alpha|.
\] (32)
There is another equality fulfilled by the $d^\dagger$ and $\hat{\Delta}$ operators
\[
\hat{\Delta}^* \hat{\Delta} = d\rho^* (1 + d\rho^*),
\] (33)
which allows us to get simple relation between absolute value of $\Delta_\alpha$ and a number of particles $dN_\alpha$ depleted from a condensate:
\[
|\Delta_\alpha|^2 = dN_\alpha (1 + dN_\alpha).
\] (34)

Let us turn back to the equation \ ([20]). If we choose as the basis vectors $\phi_\alpha$ the eigenstates $\phi_\alpha$ of the single particle density matrix we will obtain $d^\dagger$ and $\hat{\Delta}$ in diagonal forms and consequently also $Z_{\alpha \beta}$ matrix in a diagonal form,
\[
Z_{\alpha \beta} = \lambda_\alpha \delta_{\alpha \beta},
\] (35)
where
\[
\lambda_\alpha = \frac{dN_\alpha}{\Delta_\alpha}.
\] (36)

Note that
\[
|\lambda_\alpha| = \sqrt{\frac{dN_\alpha}{dN_\alpha + 1}}.
\] (37)

Finally the Bogoliubov vacuum state in the particle representation can be written in a nice diagonal form
\[
|0\rangle_B \sim \left( \hat{a}^\dagger_0 \hat{a}^\dagger_0 + \sum_{\alpha = 1}^{\infty} \lambda_\alpha \hat{a}^\dagger_\alpha \hat{a}^\dagger_\alpha \right)^{N/2} |0\rangle,
\] (38)
where $\hat{a}^\dagger_\alpha$ operators create particles in modes that are eigenstates of the single particle density matrix.

The analysis performed shows beautiful properties of the Bogoliubov modes. Almost all we need in order to obtain the Bogoliubov vacuum is given by eigenstates and eigenvalues of the single particle density matrix. The only things that can not be determined from this matrix are the phases of $\Delta_\alpha$. 
FIG. 1: Panel (a) shows number of atoms depleted from a condensate wavefunction, i.e. dN, versus N for Ω = 0.01N, calculated in the exact state (full circles), calculated directly in the Bogoliubov vacuum state in the particle representation (open circles), and calculated with the help of Eq. (46) (open squares). In panel (b) the squared overlap between the exact ground state of the system |ψ₀⟩ and the Bogoliubov vacuum |0⟩ₜ in the particle representation is presented.

FIG. 2: Panel (a): the solid line corresponds to the exact ground state |ψ₀⟩, the dashed line to the Bogoliubov prediction |0⟩ₜ and the dotted line to the perfect condensate state |N : φ₀⟩ where all atoms occupy the condensate mode. Panel (b): the solid line corresponds to the exact ground state |ψ₀⟩ and the dashed line to the Bogoliubov prediction |0⟩ₜ for N = 20 and Ω = 0.1 (dN = 2.5, |ψ₀|₀⟩ = 0.76). The states are projected on |n₁, n₂, n₃⟩ where nᵢ is a number of particles in an i-well.

III. COMPARISON OF THE BOGOLIUBOV THEORY WITH EXACT SOLUTIONS FOR A MODEL SYSTEM

Knowledge of the Bogoliubov vacuum state in the particle representation allows for precise comparison of the Bogoliubov theory with exact solutions for model systems. In Ref. [9] we have analyzed N particles in a double well potential within the tight binding approximation where we have found that, quite surprisingly, the Bogoliubov theory gives extremely good predictions for the ground state and low lying excited states in the entire range of the system parameters. This is rather an exception than a rule and therefore in order to test the theory we will focus now on a slightly more complex system, i.e. N particles in a triple well potential. Restricting the single particle Hilbert space to the ground states in each well only, the Hamiltonian of the system reads

\[ H = -\Omega \sum_{(i,j)} cᵢ^† cⱼ + \frac{1}{2} \sum_{j=1}^{3} cᵢ^† cᵢ (cᵢ^† cᵢ - 1), \tag{39} \]

where \( cᵢ \) denotes bosonic operator that annihilates a particle in an i-well, \( \Omega \) stands for tunneling rate between neighboring wells. This is the only parameter because the particle interaction coefficient was chosen as energy scale. For \( \Omega \gg N \) the ground state of the system is a BEC where all atoms occupy the same condensate wavefunction \( |0⟩ₜ \). On the other hand \( \Omega \ll 1/N \) defines the Mott insulator regime where the Bogoliubov theory should definitely stop working.

The ground state of the Gross-Pitaevskii equation corresponding to the triple-well system reads

\[ |ψ₀⟩ = \frac{1}{\sqrt{3}} (1,1,1) \tag{40} \]

and the solutions of the Bogoliubov-de Gennes equations are \( \bar{u} \)

\[ u_± = \frac{X φ_±}{\sqrt{X^2 - 1}}, \quad v_± = \frac{-φ_±}{\sqrt{X^2 - 1}}, \tag{41} \]

where

\[ φ_+ = \frac{1}{\sqrt{3}} \left( 1, e^{2\pi/3}, e^{-2\pi/3} \right), \quad φ_- = \frac{1}{\sqrt{3}} \left( 1, e^{-2\pi/3}, e^{2\pi/3} \right) \tag{42} \]

and

\[ X = \left( 1 + \frac{9\Omega}{N} \right) \sqrt{\left( 1 + \frac{9\Omega}{N} \right)^2 - 1}. \tag{43} \]

Diagonal form of the Bogoliubov vacuum in the particle representation is defined by the \( \bar{d} \) operator

\[ \bar{d} = \bar{a}^† \bar{a} \left( 1 - \frac{1}{X} \right) \bar{a}^† \bar{a} + \frac{1}{X} \bar{a}^† \bar{a}^† \bar{a} \bar{a}, \tag{44} \]
where $\hat{a}_0^\dagger$ creates a particle in the condensate mode $\phi_0$ while $\hat{a}_1^\dagger$ and $\hat{a}_2^\dagger$ create particles in $\phi_1 = (\phi_+ + \phi_-)/\sqrt{2}$ and $\phi_2 = (\phi_+ - \phi_-)/\sqrt{2}$ modes, respectively. Number of atoms depleted from the condensate mode can be directly calculated from the Bogoliubov vacuum in the particle representation but it is given approximately by

$$
\text{d}N = \int dx \langle \delta \hat{\psi}^\dagger \delta \hat{\psi} \rangle \approx \int dx \langle \delta \hat{\psi}^\dagger \hat{a}_0 \frac{1}{N} \hat{a}_0^\dagger \delta \hat{\psi} \rangle = \int dx \langle \hat{\Lambda}^\dagger \hat{\Lambda} \rangle = \langle v_+|v_+\rangle + \langle v_-|v_-\rangle = \frac{2}{X^2 - 1}.
$$

Increasing $N$ but keeping fixed number of atoms depleted from the condensate mode one approaches the limit where the Bogoliubov theory should become more and more accurate. We have done calculations for different $N$ but for fixed $\Omega/N$ chosen so that $\text{d}N \approx 2/(X^2 - 1) = 1.5$. The squared overlap between the Bogoliubov vacuum state and the exact ground state of the Hamiltonian is depicted in Fig. 1 where one can indeed observe that with increasing $N$ the Bogoliubov solution approaches the exact one. With increasing $N$ also number of depleted atoms calculated directly from the Bogoliubov vacuum state and with the help of the estimation coincide with the exact results. Example of the Bogoliubov and exact states for $N = 20$ is for two values of $\Omega$ are shown in Fig. 2. When $\Omega$ is sufficiently large we get perfect agreement between the Bogoliubov and exact results.

It is generally believed that when the fraction of atoms depleted from a condensate is small, say a few percent, the Bogoliubov theory is reliable. Let us now investigate the limit when $N$ increases but $\text{d}N/N$ remains constant. In Fig. 3 we present the results for different $N$ but for $\text{d}N/N \approx 2/[N(X^2 - 1)] = 0.05$. One can see that the greater $N$ the worse the Bogoliubov predictions. Actually increasing $N$ and keeping $\text{d}N/N$ fixed we go far and far away from the BEC regime where $\Omega \gg N$. Indeed,

$$
\frac{\text{d}N}{N} \to \frac{\sqrt{2}}{6 \sqrt{\Omega N}} + \mathcal{O}(1/N),
$$

and in order to keep $\text{d}N/N$ constant the $\Omega$ must behave like $1/N$. This example shows that even if the fraction of depleted atoms is very small and single particle density matrix can be estimated quite satisfactorily, for very large number of atoms the Bogoliubov theory fails to predict structure of the $N$-body ground state. Basing on the triple-well problem only it is difficult to judge how general is such a behaviour.

**IV. CONCLUSIONS**

We have considered the Bogoliubov vacuum states in the number-conserving version of the Bogoliubov theory. It is shown that the vacuum can be obtained in a simple diagonal form in the particle representation. To this end one has to basically diagonalize single particle density matrix of a system only. The Bogoliubov vacuum can describe eigenstates of a time independent system and also a dynamical case where starting with a system in the ground state particles are perturbed by time dependent force.

$N$-body states obtained with the help of the Bogoliubov theory have been confronted with exact solutions for a triple-well system. In the limit of $N \to \infty$ and $\text{d}N = \text{const.}$, the Bogoliubov vacuum states approach the exact solutions as expected. However, it turns out that in the limit of $N \to \infty$ and for arbitrary small but fixed $\text{d}N/N$, the Bogoliubov theory is not able to predict $N$-body states of the system.

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