Development of a mathematical model of sound insulation with flexible panels

E A Romanenko, G S Russkikh, Z N Sokolovskiy

Omsk State Technical University, Mira ave., 11, Omsk, 644050, Russia
E-mail: elinaromanenko18@gmail.com, russkikh.gs@mail.ru, ninasok@yandex.ru

Abstract. For sound insulation in the presence of restrictions related to transportation and installation conditions, practically the only possible constructive solution is the use of flexible panels. Also, the flexible panel provides reliable protection in the most difficult area - low frequencies, so modernization of the panel design is an actual topic for research. The object of the study is a mesh-plate panel. Flexibility is ensured by the following constructive solution - fixing the main soundproofing plates on a steel mesh bearing with basalt fabric with a gap. The sound insulation of the panel has been investigated experimentally. The goal is to develop analytical techniques evaluation flexible sound insulation panels based on the fundamental positions of industrial and architectural acoustics, sound insulation, and synthesis of published experimental studies of the effect of gaps (acoustic holes and voids). The calculation results are consistent with tests insulation panels manufactured at a sufficient level. As a result, it was described how the sound insulation of the panel is affected by the size, shape and filling of acoustic holes in the gap between the plates. Suggested ways to reduce their negative impact and optimize the design of a flexible soundproof panel.

1. Introduction
Currently, the development of structures for effective sound insulation of low frequencies gives rise to many technical solutions. The usual sandwich panels are replaced by panels with different perforation configurations [1], [2], acoustic membranes [3], [4] and fibrous structures [5]. Acoustic holes are present in all these cases. The using of adequate theoretical models allows adjusting the design parameters to achieve the desired level of sound insulation.

In this paper we consider a flexible sound insulating barrier in the form of a plate-mesh panels (hereinafter - PMP) protected Russian Federation patents [6], [7].

The PMP has a woven mesh, two layers of square (TxT) metal plates with total thickness $h$, arranged symmetrically relative to the mesh. Four basalt fabric layers are used as an elastic gasket. To provide the desired flexibility of the plate are mounted at a distance $t$ from each other, and also have bevels in the region of the side surfaces, thus increasing PMP’s flexibility.

2. Statement of the problem
To develop a complete mathematical model, it is necessary to evaluate the sound insulation of the working panel, taking into account main features. The general view and the element of the PMP are shown in Fig. 1.

![Figure 1. General view and element of the PMP](image)
The ratio of the total area of PMP to size LxL square slots between the plates is:

\[ \frac{S_0}{S} = 0.0301 \text{ for plates} \]

The layers of the basalt fabric liner practically do not resist bending, they are not a continuous obstacle to the sound within the width of the gaps \( t \) and are also characterized by a certain ratio of the area of the gaps and the total area:

\[ \frac{S_0}{S} = 0.0258 \text{ for gasket} \]

Depending on the type of fabric and its weaving method, this parameter may be different. Influence on the permeability of the mutual arrangement of layers of fabric during installation is not controlled and not taken into account.

In the general case, the varied design parameters that affect the sound insulation of the PMP are:

- material, thickness \( h \) and dimensions \( T_1 \) and \( T_2 \) of the main insulating plates,
- the \( t \) value (i.e. \( \frac{S_0}{S} \) plates) and the shape of the gap,
- material and thickness \( h \) of the gasket (i.e. \( \frac{S_0}{S} \) gaskets),
- the sizes of the PMP \( L_1 \) and \( L_2 \) (in the actual PMP \( T_1 = T_2, L_1 = L_2 \)).

The authors do not know purposeful experimental studies of the degree of sound insulation of PMP, depending on the design parameters, now. There is reliable data only on experimental studies of the sound insulation of a working PMP [8], carried out by one of the authors, and the results of the experiments [9] in comparison with sound insulation: a solid plate, a plate having a narrow slit, a circular hole and plates with a 16th round holes uniformly distributed over the area at

\[ \frac{S_0}{S} = 0.001 \]

in all three experiments.

Theoretical relationships for calculating sound insulation obtained in the basic studies on architectural acoustics, including taking into account acoustic holes and slits. In [9] it is partially used for the assessment of flexible insulation panels, but not taken into account:

- the influence of the gaps between the plates and the sound permeability of basalt fabric,
- features of sound insulation during acoustic tests,
- relatively large amplitude of the oscillations of the PMP with a limited free (protected) volume.

### 3. Theory

Investigation of the effect on the sound insulation holes slots devoted a number of works by foreign authors. A mathematical model and a series of experiments to determine the sound transmission through the holes and cracks were studied in [10]. Shows three options of the depth of circular openings - 0.036 m, 0.072 m and 0.144 m; three options slit depth - 0.051 m, 0.076 m and 0.152 m.

Diffraction phenomenon in acoustics discussed in [11]. Three different cases are given: the width and depth of the slit are equal to the wavelength, the width of the slit is equal to the wavelength (depth is 10 times smaller), the width of the slit is equal to the wavelength (depth is 5 times larger).

An analytical model of the acoustic resistance of round holes of finite depth is given in [12]. Application of these models to the studied case we are difficult because of differences in the key geometric dimensions and, consequently, the physical processes occurring in the system.

In this paper attempted to closest approach, and corrections of the theoretical curves and noise calculation algorithm applied to the design features the PMP and its operating conditions, including on the basis of mathematical processing of results known experiments. Program and setting of further clarifying the test was determined.
When calculating the sound insulation known PMPs are sequentially used for sound insulation according to the individual layers of the structure with and without taking into account the acoustic holes.

The sound insulation of the continuous layer without taking into account the dimensions and vibrations of the PMP was calculated using the impedance method:

\[
R_c = 10 \lg \left[ \cos^2(hk_2) + \frac{1}{4} \left( \frac{\rho_1 c_1}{\rho_2 c_2} + \frac{\rho_2 c_2}{\rho_1 c_1} \right) \sin^2(hk_2) \right]
\]  

(1)

where:
- \( k \) – wave number,
- \( \rho, c \) – acoustic resistance of media, at the interface of which there is an effect of sound insulation,
- \( h \) – layer thickness.

The sound insulation of a layer having cracks - acoustic holes, is calculated by the formula [9]:

\[
R = R_1 - 10 \lg \left[ \frac{1 + \varphi}{1 + \frac{S_0}{S}} \cdot \frac{R_1 - R_0}{10} \right] 
\]  

(2)

where:
- \( R_0 \) – sound insulation of acoustic hole,
- \( \frac{S_0}{S} \) – calculated (geometric) ratio of the area of the acoustic hole and the area of the layer,
- \( \varphi \) – dimensionless coefficient taking into account an increase in sound transmission diffuse field conditions depends on the hole size and frequency and, in the author's opinion, can be \( \varphi = 1..10 \). The value of the coefficient is refined on the basis of mathematical processing of the experiments [9].

The dependence of \( \varphi \) on the ratio of the effective length of the hole and the wavelength of sound represented as:

\[
\varphi(l_w, \varphi_w) = \begin{cases} 
1, & \text{if } l_w \geq \lambda \\
\varphi_w \left( \frac{\lambda - l_w}{\lambda} \right), & \text{if } l_w < \lambda \\
\varphi \geq 1 
\end{cases}
\]  

(3)

where \( l_w, \varphi_w \) are the effective hole size and the effective maximum value \( \varphi \), respectively.

In the literature the concept of a "large" and "small" acoustic aperture is introduced, determined by the ratio of its dimensions to the wavelength, i.e. the same hole may have the properties of a large acoustic hole in the high-frequency range and a small one in the low-frequency region. It is also obvious the effect on the soundproof ratio of the wavelength of sound and the size of the barrier. In this paper, these positions are specified by introducing a correction function for the equivalent relative size of the acoustic hole

\[
\frac{S_0}{S}(l_w, \varphi_w) = \frac{S_0}{S} \begin{cases} 
1, & \text{if } l_w \geq \lambda \\
\frac{l_w}{\lambda}, & \text{if } l_w < \lambda 
\end{cases}
\]  

(4)

Formulas (3), (4) in the calculation of sound insulation with allowance for acoustic holes are substituted in (2), which takes the form:
\[
R_c^2 = R_1 - 10 \log \left[ \frac{1 + \varphi \left( l_w \cdot \varphi_w \right) S_0}{S_1} \frac{R_1 - R_0}{10} \right] \quad (5)
\]

The effective values of the parameters \( l_w, \varphi_w \) were determined from the condition of the best agreement (the maximum value of the reliability of the approximation \( R^2 \to 1 \) due to minimization of the sum of the squares of the experimental differences [9] and the values of sound insulation calculated by (5)) and amounted to:

**Table 1. Treatment of experiments [9]**

| Experiment        | Size, m | Effective Size \( l_w, m \) | Maximum effective coefficient, \( \varphi_w \) | Reliability of approximation \( R^2 \) | Average error of approximation \( \overline{A}, \% \) |
|-------------------|---------|------------------------------|-----------------------------------------------|-------------------------------------|----------------------------------------|
| Narrow gap        | 0.001*1 | 0.055                        | 10                                            | 0.996                               | 2.69                                   |
| Round hole        | d=0.036 | 0.087                        | 8.69                                          | 0.998                               | 3.80                                   |
| 16 round holes    | d=0.009 | 0.173                        | 12.78                                         | 0.924                               | 8.79                                   |

The reliability of the approximation calculates by the formula:

\[
R^2 = 1 - \frac{\sum_i^n (y - Y)^2}{\sum_i^n y^2 - \sum_i^n Y^2} \quad (6)
\]

where \( y \) and \( Y \) are calculated and experimental values of sound insulation, \( n \) - the number of experimental values.

The average error of approximation calculates from the formula:

\[
\overline{A} = \frac{1}{n} \sum_i^n \left( \frac{y_i - Y_i}{y_i} \right) \cdot 100\% \quad (7)
\]

Taken into account design features and applications PMP:
- reduced stiffness and dependence of the natural oscillation frequency both on the cylindrical rigidity, and on the dimensions of the plates, gaps and stiffness of the carrier grid;
- relatively large amplitude oscillations PMP \( U_{\text{max}} \) under sound pressure as a consequence of the flexibility of the PMP;
- the commensurability of the volume of air displaced by vibrations with the volumes of the protection zone \( V_2 \) and the source zone \( V_1 \)

\[
\Delta V = S \cdot U_{\text{max}} \cdot K_u \quad (8)
\]

where \( K_u \) is coefficient that takes into account the shape and amplitude of the oscillations. For a working PMP having a square shape, when pivoted \( K_u \approx 4/\pi^2 \)

- the additional pressure fluctuations \( \Delta p \) in these zones were calculated according to the Boyle-Mariotte law:

\[
(V_i - (-1)^i \cdot \Delta V) \left( p + (-1)^i \cdot \Delta p_i \right) \cdot V_i \Delta p \rightarrow \Delta p_i = \frac{\Delta V}{V_i}
\]
the change in the phase of the oscillations $K_{ud}$ upon passing through resonance taking into account the loss coefficient $\eta$ under oscillations.

To take into account these features, the derivation of the formula for the calculation of sound insulation at low frequencies was modernized and an equation obtained for calculating the sound insulation at frequencies of sound effect $\omega$ close to the frequency of the first harmonic of natural oscillations $\omega_r$.

$$ R_\omega = 10 \log \left[ \left( 1 + \frac{\eta \cdot m \cdot \omega}{2 \rho_0 \cdot c_0} \left( \frac{\omega_r}{\omega} \right)^2 \right)^2 + \left( \frac{m \cdot \omega}{2 \rho_0 \cdot c_0} \left( 1 - \left( \frac{\omega_r}{\omega} \right)^2 - \frac{K_{slit} \cdot K_v \cdot p \cdot S \cdot (V_1 + V_2)}{V_1 \cdot V_2 \cdot \omega^2 \cdot 2 \rho_0 \cdot c_0} \right) \right)^2 \right] $$

where $\omega_r = 2 \pi (5 \ldots 27)$ Hz (when the position of the PMP is changed from vertical to horizontal); the loss factor $\eta = 0.6$, corresponding to the normal damping coefficient of mechanical oscillations $\beta = 0.3$.

For working PMP obtained $S_0 / S = 0.0301$ for plates, $S_0 / S = 0.0258$ for gasket. To take into account the uncertainty of the overlapping of holes in the layers of basalt fabric, the coefficient $K_{hole} = (1/3 \ldots 1/2)$ was introduced.

The algorithm for calculating the sound insulation of the PMP includes the following sequence of calculations:

1. Calculation $R_\omega$ according to (9) for a solid plate without taking into account the holes and gaps.
2. Calculation of $R_c$ from (1) for laying layers without taking into account permeability.
3. Calculation $R_c^s$ according to (5) for layers of the gasket taking into account the slits at $R_c$ from point 2 and $R_0$ (for air).
4. Calculation of $R_c^s$ PMPs according to (5) with $R_c = R_\omega$ from item 1 and $R_0 = R_c^s$ from item 3.

The algorithm provides a lower bound, since it ignores small (due to the small differences impedances) soundproofing gasket between the base plate and the support mesh.

4. Experimental results

The experiments [9] were carried out on an aluminum plate $L = 1 \text{ m}$ and $h = 0.003 \text{ m}$ at a constant size of the holes in the thickness, in contrast to the variable cross section and a larger slit depth in the working PMP.

Research of influence of variable cross-section and the depth of the effect on the sound insulation gap requires further experimentation and refinement. In the present work, when calculating the sound insulation of the working memory bandwidth, the following are first taken as the closest results of determining the effective parameters for the gap: $t = 0.001 \text{ m}$ and for basalt tissue: an experiment with 16 uniformly distributed holes.
Figure 2. Comparison of the experimental values of sound insulation [6] and calculated by (5)
If we take as a calculated curve approximating a curve, then $R^2 = 0.97$, and the average
approximation error does not exceed 15%.
Figure 3 shows the result of calculating the insulation operating PMP developed algorithm at $\omega_r = 2 \cdot \pi \cdot 13.5$ and the set of experimental values $f = (15 \ldots 200)$ Hz and $f = (200 \ldots 10000)$ Hz averaged values.

![Figure 3](image.png)

Figure 3. Comparison of the experimental values of soundproofing of the memory bandwidth and
calculated according to the algorithm with different parameters $V$ and $K_{hole}$
The correspondence between experiments and calculations was estimated by the value of the indicators $R^2$ and $\bar{A}$. If $V_1 = V_2 = (2 \ldots 10)$ m$^3$, $K_{hole} = (1/3 \ldots 1)$, then $R^2 = 0.965 \ldots 0.976$, $\bar{A} = (13.467 \ldots 14.377)$ %.

5. Discussion of results
Comparison of the graphs shows that after $f \geq 200$Hz, the calculation results are almost identical, i.e.
the effect of the dynamics of the memory bandwidth on vibration isolation is manifested only at low
frequencies.
For the working PMP, the results of the sound insulation calculations are in satisfactory agreement
with the experimental data (taking into account the lower estimate and spread of the experimental data
for the 6 tests for the determination of sound insulation - the line shows a linear filter line at 5 points).

6. Conclusions
The technique of analytical assessment soundproofing flexible panels having structural features as
cracks and holes.
To refine the algorithm, the following additional experiments are necessary:
1. To definition of sound insulation for different combinations of $T$, $t$ and $h$ in a conventional
plate and PMP, as well as for different orientations and the number of layers of basalt fabric. The
purpose of the tests: refinement of the quantities $l_c$, $\varphi_c$ and their adequate geometric calculation.
2. To develop the design of the memory bandwidth with a significantly smaller $t$, for
example, with one-sided flexibility.
3. Development of the methodology for calculating the first form of natural oscillations of the
memory bandwidth for a different from the horizontal position in space.

References
[1] Meng H, Galland M A, Ichchou M, Bareille O, Xin F X and Lu T J 2017 J. Composite struct. 182 pp 1-11
[2] Simon F 2018 J. of Sound and Vibr. 421 pp 1-16
[3] Langfeldt F, Riecken J, Gleine W and Estorff O 2016 J. of Sound and Vibr. 373 pp 1-18
[4] Li S, Mao D, Huang S and Wang X 2018 *J. Appl. Acoustics* **130** pp 92-98
[5] Yang Y, Li B, Chen Z, Sui N, Chen Z, Saeed M, Li Y, Fu R, Wu C and Jing Y 2016 *J. Composites B:Engin.* **96** pp 281-286
[6] Zubarev A V, Tribelskiy I A, Adonin V A and Malyutin V I 2007 *Sound Insulation Panel* [http://www1.fips.ru/wps/portal/IPS_Ru#1518716254883] Patent 2340478 Russian Federation
[7] Tribelskiy I A, Adonin V A, Bobrov S P, Denisov V D, Bokhan V V and Gidion V A 2012 *The sound-insulating panel and the method of its manufacturing* [http://www1.fips.ru/wps/portal/IPS_Ru#1518716254883] Patent 2457123 Russian Federation
[8] Tribelskiy I A 2011 *The results of tests, design estimates and justification of the choice of design options for ZP panels and passage nodes through ZP, schemes for fixing ZP panels to the supporting contour* Omsk FSUE "Progress" p 145
[9] Bogolepov I I 2009 *J. Engineering and construction* **1** pp 17-20
[10] Oldham D J and Zhao X 1993 *J. of Sound and Vibr.* **161** pp 119-35
[11] Berezovski A and Engelbrecht J 2016 *J. Wave Motion* **60** pp 35-45
[12] Yang D and Morgans A S 2016 *J. of Sound and Vibr.* **384** pp 294-311