Chases and Bag-Set Certain Answers

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Abstract. In this paper we show that the chase technique is powerful enough to capture the bag-set semantics of conjunctive queries over IDBs and IDs and TGDs. In addition, we argue that in such cases it provides efficient (LOGSPACE) query evaluation algorithms and that, moreover, it can serve as a basis for evaluating some restricted classes of aggregate queries under incomplete information.

1 Introduction

One of the main query answering techniques for unions of conjunctive queries over incomplete databases is the chase [8][2][7]. Incomplete databases (IDBs) can be seen as a set I of first order logic (FO) ground facts (the database instance) and a set Σ of FO axioms known as integrity constraints or dependencies. Unions of conjunctive queries (UCQs), on the other hand, are logical specifications of SQL SELECT-PROJECT-JOIN-UNION queries belonging to the positive-existential fragment of FO (with no function symbols). Answering a query over an incomplete database boils down to returning the values of all the closed substitutions (or groundings) under which the query is entailed by the incomplete database. Such set of values is known as certain answers [2][10]. When Σ is a set of inclusion and tuple-generating dependencies (IDs and TGDs), then the chase is known to provide efficient (LOGSPACE in the size of the data) algorithms for computing the set of certain answers.

As we all know, queries in relational database systems return, in practice, bags or multisets of answers. This is because it can be quite costly to delete repeated answers [5][1]. Arguably, this property still holds for incomplete databases that integrate relational datasources. The main contributions of this paper are thus the following: (i) extending the notion of certain answers to cover bags and (ii) showing that chases are powerful enough to capture these bags of certain answers when evaluating a UCQ over an IDB with IDs and TGDs. These results outline algorithms that, on the one hand, efficiently compute these bags of certain answers and, on the other hand, can be extended to more expressive classes of queries.

2 Preliminaries

Let R := \{R_1, ..., R_n\} be a set of relation symbols each with an associated arity (a positive integer), called database schema. Let Dom be a countably infinite set of constants called domain. A tuple over Dom is any finite sequence c of domain constants. A fact over R and Dom is a relational atom over R and Dom. It is said to be ground when it contains no variables and positive when, in addition, it is not negated. We will denote by Fact(R) the set of all positive facts. Furthermore, we will denote by Sent(R) the set of all FO sentences (closed formulas) over R and Dom. Notice that Fact(R) ⊊ Sent(R).

A union of conjunctive queries q (UCQ) of arity n is a rule over R of the form q(x) ← φ_0(x, y_0, c_0) ⋁ ... ⋁ φ_k(x, y_k, c_k) where q(x) is called the query’s head, x is a sequence of n ≥ 0 distinguished variables and φ_0(x, y_0, c_0) ⋁ ... ⋁ φ_k(x, y_k, c_k) is the the query’s body, where the φ_i, s, for i ∈ [0, k], are conjunctions of atoms. A UCQ is said to be boolean if x is an empty sequence.

A database instance or, simply, database (DB), of R is a finite set I ⊊ Fact(R) of ground facts. The active domain of I, adom(I), denotes the (finite) set of constants that occur among the facts in I. We will denote by Inst(R) the class of all such DBs. An incomplete database (IDB) of R is a pair (Σ, I) where Σ ⊊ Sent(R) is a finite set of FO axioms called dependencies or integrity constraints and I is a DB of R.
A grounding over UCQ $q$ is a function $\gamma: \text{Var}(q) \to \text{Dom}$, where $\text{Var}(q)$ denotes the set of variables of $q$. Groundings can be extended to sequences of variables, FO atoms and FO formulas in the usual way. We will denote by $q\gamma$ the grounding of the body of a UCQ $q$ by $\gamma$. The set of all such groundings will be denoted $\text{Var}(q)\text{Dom}$.

A grounding $\gamma$ is said to satisfy a UCQ $q$ w.r.t. a DB $I$ iff $I \models q\gamma$, i.e., if $I$ is a model (in FO terms) of $q\gamma$, the grounded body of $q$. It is said to satisfy $q$ w.r.t. to an IDB $(\Sigma, I)$ iff $\Sigma \cup I \models q\gamma$, i.e., if $\Sigma \cup I$ entails (in FO terms) $q\gamma$ [12][10].

3 Bag-Set Semantics and Chases

Let $X$ be a set. A bag or multiset $B$ (over $X$) is a function $B: X \to \mathbb{N} \cup \{\infty\}$. We say that an $x \in X$ belongs to $B$ whenever $B(x) \geq 1$, where the integer $B(x)$ is called the multiplicity of $x$ in $B$. As with sets, bags will be denoted by extension or intension using the special brackets $\{\cdot\}$.

Definition 1 (Bag-set answers). Let $I$ be a DB and $q$ a CQ of arity $n$ over a schema $R$. We define the bag-set answers of $q$ as the bag:

$$q[I] := \{c \in \text{adom}(I)^n \mid c = \gamma(x) \text{ for some } \gamma \text{ s.t. } I \models q\gamma\}.$$  

Definition 2 (Bag-set certain answers). Let $(\Sigma, I)$ be an IDB and $q$ a UCQ of arity $n$ over a schema $R$. We define the bag-set certain answers of $q$ as the bag:

$$q[\Sigma, I] := \{c \in \text{adom}(I)^n \mid c = \gamma(x) \text{ for some } \gamma \text{ s.t. } \Sigma \cup I \models q\gamma\}.$$  

Notice that by answers we understand the tuples returned by the satisfying groundings. Notice, too, that in the first case we check for groundings that are satisfying over one single DB and in the second for groundings that are satisfying for a whole space of DBs, namely, all the models of the IDB.

Chases generate DBs (a.k.a. canonical or universal DBs) that minimally complete the information held by an IDB [7]. Chases are linked to dependencies. In what follows we will consider only two general kinds of such dependencies, namely, inclusion dependencies (IDs), which are axioms of the form $\forall x(R(x) \rightarrow R'(x))$, and tuple-generating dependencies (TGDs), which are axioms of the form $\forall x(R(x) \rightarrow \exists yR'(x, y))$, as is common in IDB literature [12][8].

Definition 3. (Chase rules) Let $\Sigma$ be a (finite) set of IDs and TGDs. The set $R$ of chase rules generate new facts by applying the dependencies $\rho \in \Sigma$ to previously computed facts:

$$\begin{align*}
\text{ID} & \quad f := R(c) & \rho := \forall x(R(x) \rightarrow R'(x)) & \quad f_{\text{new}} := R'(c) \\
\text{TGD} & \quad f := R(c) & \rho := \forall x(R(x) \rightarrow \exists yR'(x, y)) & \quad f_{\text{new}} := R'(c, c_{\text{new}})
\end{align*}$$

Where $c_{\text{new}}$ is a sequence of fresh constants from $\text{Dom}$. Note that there will be as many rules as dependencies in $\Sigma$. We will denote by $R$ the set of all chase rules (for a given set $\Sigma$ of integrity constraints). Notice that rules in $R$ are in one-to-one correspondence with dependencies in $\Sigma$.

We say that a fact $f$ is derivable from $\Sigma$ and $I$ modulo $R$ and write $\Sigma \cup I \vdash_R f$ whenever (i) $f \in I$ or (ii) there exists a finite sequence $f_0, \ldots, f_n$ such that $f_n = f$ and for each $i \in [0, n]$, either $f_i \in I$ or there exists $j \leq i$ s.t. $f_j$ follows from $\Sigma$ and $f_j$ by some rule in $R$. The chase rules are sound and complete w.r.t. (standard FO) entailment, in the sense that, for all facts $f \in \text{Fact}(R)$, $\Sigma \cup I \vdash_R f$ iff $\Sigma \cup I \models f$.

Definition 4. (Chase) Let $(\Sigma, I)$ be an IDB over $R$ with $\Sigma$ a set of IDs and TGDs. The chase of $(\Sigma, I)$, denoted $\text{chase}(\Sigma, I)$, is defined inductively as follows:

- $\text{chase}_0(\Sigma, I) := I$.
- $\text{chase}_{i+1}(\Sigma, I) := \text{chase}_i(\Sigma, I) \cup \{f_{\text{new}}\}$.
Chases can also be seen (in FO), as the deductive closure \((\Sigma \cup I)^{+R}\) of the IDB (seen as a FO axiomatisation) w.r.t. the set \(Fact(R)\) of atomic facts over \(R\). Formally, \((\Sigma \cup I)^{+R} := \{ f \in Fact(R) \mid \Sigma \cup I \vdash_R f \}\).

**Proposition 1.** Let \((\Sigma, I)\) be an IDB over \(R\), where \(\Sigma\) is a set of IDs and TGDs. Then, \((\Sigma \cup I)^{+R} = chase(\Sigma, I)\).

**Theorem 1.** Let \((\Sigma, I)\) be an IDB over \(R\), with \(\Sigma\) a set of IDs and TGDs. Let \(q\) be a UCQ and \(\gamma\) an arbitrary grounding. Then, \(\text{chase}(\Sigma, I) \models q\gamma\) iff \(\Sigma \cup I \models q\gamma\).

**Proof.** The sense \((\iff)\) is immediate, since \(\Sigma \cup I \subseteq (\Sigma \cup I)^{+R} = chase(\Sigma, I)\). For \((\Rightarrow)\), observe that \((\Sigma \cup I)^{+R} = chase(\Sigma, I)\). Therefore, there exists a finite sequence of applications of chase rules starting from \((\Sigma, I)\) and ending, at some level \(k\) of the chase, with all the facts in \(q\gamma\). But then, since chase rules are sound w.r.t. entailment, \(\Sigma \cup I \models q\gamma\).

**Lemma 1.** Let \((\Sigma, I)\) be an IDB and \(q\) a UCQ Then:
\[
\{ \gamma \in \text{Var}(q) \mid \text{Dom} \mid \Sigma \cup I \models q\gamma \} = \{ \gamma \in \text{Var}(q) \mid \text{Dom} \mid \text{chase}(\Sigma, I) \models q\gamma \}
\]

**Proof.** Let \(\gamma \in \text{Var}(q) \text{Dom}\). By Theorem 1 \(\Sigma \cup I \models \exists \gamma\text{ iff }chase(\Sigma, I) \models q\gamma\), whence the result.

**Theorem 2.** Let \((\Sigma, I)\) be an IDB with \(\Sigma\) a set of IDs and TGDs. Let \(q\) be a CQ of arity \(n\) and distinguished variables \(x\). Then it holds that:
\[
\{\{\gamma(x) \in \text{adom}(I)^n \mid \gamma \text{ s.t. } \Sigma \cup I \models q\gamma\}\} = \{\{\gamma(x) \in \text{adom}(I)^n \mid \gamma \text{ s.t. } \text{chase}(\Sigma, I) \models q\gamma\}\}
\]

**Proof.** Put, for brevity, \(B := \{\{\gamma(x) \in \text{adom}(I)^n \mid \gamma \text{ s.t. } \Sigma \cup I \models q\gamma\}\} \text{ and } B' := \{\{\gamma(x) \in \text{adom}(I)^n \mid \gamma \text{ s.t. } \text{chase}(\Sigma, I) \models q\gamma\}\}. \) Assume now for contradiction that there exists a tuple \(c\) s.t., w.l.o.g., \(B(c) > B'(c)\) (the other case is analogous). This implies that for some pair of groundings \(\gamma, \gamma'\) from \(\text{Var}(q)\) to \(\text{Dom}\):
- Either \(\gamma = \gamma'\). Then, since, by Lemma 1 the set of groundings \(\gamma: \text{Var}(q) \to \text{Dom}\) s.t. \(\text{chase}(\Sigma, I) \models q\gamma\) or \(\Sigma \cup I \models q\gamma\) are exactly the same, it should be the case that \(c\) occurs in \(B\) and \(B'\) with the same multiplicities, which is impossible.
- Or \(\gamma \neq \gamma'\). In which case it holds that:
  - (i) \(\gamma(x) = \gamma'(x) := c\) and \(\gamma\) and \(\gamma'\) are satisfying assignments for \(q\) and \(\text{chase}(\Sigma, I)\).
  - (ii) either \(\gamma\) or \(\gamma'\) is satisfying for \(q\) and \(\Sigma \cup I\), but not both.
Assume that the non satisfying grounding is \(\gamma\) and suppose, moreover, that no such other grounding \(\gamma''\) behaves like \(\gamma\) or \(\gamma'\). Following, again, Lemma 1 if it is the case that \(\text{chase}(\Sigma, I) \models q\gamma\), then \(\Sigma \cup I \models q\gamma\), i.e., \(\gamma\) is a satisfying grounding for this entailment. But this is impossible.

**Corollary 1.** Let \((\Sigma, I)\) be an IDB with \(\Sigma\) a set of IDs and TGDs. Let \(q\) be a UCQ. Then, \(q[\Sigma, I] = q[\text{chase}(\Sigma, I)]\).

Corollary 1 provides an efficient way of computing \(q[\Sigma, I]\). We can apply results from description logics expressive enough to capture IDs and TGDs [3]. We can use a so-called perfect rewriting algorithm, \(\text{PERFECTREF}(\cdot, \cdot, \cdot)\), such that \(\text{PERFECTREF}(q, \Sigma, I) = q[\text{chase}(\Sigma, I)]\). What a perfect rewriting
does is to, so to speak, "compile" the dependencies in $\Sigma$ into the UCQ $q$ by rewriting $q$’s body accordingly (UCQs are closed under such rewritings). Next, we can evaluate this (expanded) UCQ over $I$, thus reducing query evaluation over IDBs to query evaluation over DBs. Since this does not affect the data complexity of query evaluation, i.e., the complexity of evaluating $q$ over IDB $(\Sigma, I)$ when measured only w.r.t. the number of tuples occurring in $I$, we get LOGSPACE data complexity – the complexity of query evaluation over plain DBs with SQL queries \[11\].

4 Conclusions and Related Work

We have showed that chases can be used to compute not only sets of certain answers, but also their bags. This is because when we apply chase rules, we reason basically over the satisfying groundings, and collect later, in a set, the bindings $c$ of an UCQ $q$’s distinguished variables $x$. Chases are also sound and complete w.r.t. bag-set certain answers when IDBs make use only of IDs and TGDs.

A particularly interesting field of application for these bag chase techniques are SQL aggregate queries (AQs), e.g., count, sum, or avg-queries with no nested conditions \[6\], in incomplete information settings. The semantics of those queries in the relational DB setting typically involves aggregates (bags) as arguments of the aggregation functions. In \[4\] we show that a semantics for AQs over ontologies and IDBs can be achieved by returning first the certain answers over $(\Sigma, I)$ of an auxiliary UCQ $q_{\text{aux}}$ associated to the AQ $q$ and aggregating later over those certain answers, yielding a unique value. This contrasts with \[9\], where chases applied to AQs yield intervals of (dense) values. The results of this paper imply that this can be done efficiently and without deleting duplicates using, say, PERFECTREF$(\cdot, \cdot, \cdot)$, when $\Sigma$ is a set of IDs and TGDs.

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