Comment on “Dynamics and Directionality in Complex Networks”

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In a recent letter [1], authors proposed a residual degree gradient (RDG) method to enhance networks’ synchronizability only by flipping the direction of the edges without changing the entire topology and the total weight. In each step, they select the node with minimum residual degree, and set all of its residual edges pointing to it (Different from [1], please note that the edge direction here is the direction of information flow). The letter indicates RDG method will finally obtain a network embeds an oriented spanning tree. They also claim that RDG can enhance the network synchronization, contrary to the randomly assigned directional network (RAD).

However, in some cases, the RDG method can not enhance the synchronizability of the original networks and will actually result in a directed network with synchronizability \( R = \lambda_2^\gamma \lambda_N < 0 \) [2]. That means the RDG method may create more than one “root node” (the node without any input). A simple example is given in Fig.1(a). According to the rule of RDG, node 1 (\( k = 2 \)) will be selected first and the two remaining community will be left disconnected. So two “root nodes” (3 and 7) are created respectively. In this case, the RDG network can never reach complete synchronized state.

We use RDG method in Watts-Strogatz small-world networks [3] and random scale-free networks [4]. Specifically, we make sure there is no isolated nodes or communities in those original networks. When RDG network is selected first and the two remaining community will be left disconnected. So two “root nodes” (3 and 7) are created respectively. In this case, the RDG network can never reach complete synchronized state.

To solve the problem of RDG method, we proposed a so-called residual betweenness gradient (RBG) method. As known to all, degree only reflects local information. Instead of the degree, we take the edge betweenness into account, which embeds the global information. Firstly, we define \( s_i \) in each node as \( s_i = \sum_{j=1}^{N} l_{ij}^\theta \), where \( l_{ij} \) is the betweenness of edge between \( i \) and \( j \) and \( 0 \leq \theta \leq 1 \). In fact, when \( \theta = 0 \), \( s_i = k_i \) and RBG is RDG. In each step, we select the node with minimum \( s_i \) and set all of its residual edges pointing to it. If there are multiples of the same rank \( s_i \), we choose the node with smaller initial \( s_i \) first. Fig.1(b) and Fig.2 show that RBG can solve RDG’s problem. And the detail performance of RBG method will be reported elsewhere.

In conclusion, because RDG method consider only local information (degree), it will sometimes cause synchronization failure. Based on the edge betweenness, we claim the RBG method can solve this problem.

**References**

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[2] Compared with \( \lambda_{\min}^\gamma / \lambda_{\max}^\gamma \) in [1], this index can detect disconnected directed networks whose \( \lambda_2^\gamma = 0 \).
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[4] M. E. J. Newman et al., Phys. Rev. E 64, 026118 (2001).