Rolling of a cylinder on the deformable surface with account of viscous lubricant

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Abstract. A friction unit describing in this article consists of an absolute solid cylinder and a deformable surface. The cylinder rolls on the surface. The viscous liquid is applied to the surface as a lubricant. The surface is getting deformed if the cylinder is rolling under the load. This article demonstrates that if the cylinder is rolling without load, a locally thin layer of lubricant is formed between the surface and the cylinder. When the cylinder is rolling on the surface, the surface is deforming by the load. A thin layer of lubricant between the cylinder and the surface also exists. Moreover, the presence of a layer of the lubricant leads to an increase of an impact area and to a decrease in the load per surface unit. The distribution of pressure in the thin layer of liquid is calculated. The presence of the layer of viscous incompressible liquid in a cylinder-surface pair leads to an increase of a contact area and more even pressure distribution over the surface. As a result, the measures of pressure on the surface is significantly reduced. The rolling cylinder makes less pressure than it is calculated in statics.

1. Research object
The role of viscous lubricant in friction units such as a cylinder and a surface is studying. A friction unit describing in this article consists of an absolute solid cylinder and a deformable surface. The cylinder is rolling on the surface. The viscous liquid is applied to the surface as a lubricant. The surface is getting deformed if the cylinder is rolling under the load.

2. Practical significance of the work
Theoretical questions and practical recommendations on the designing of friction unit are developed.

3. Purposes of the study
To simulate the effect of the lubricant when a cylinder is rolling on the surface:
- rolling of the cylinder without load;
- rolling of the cylinder with load and the surface deformation;
- the role of the viscous lubricant in the rolling of the cylinder.

4. Problem formulation
Rolling of the cylinder on the flat surface is considered (Figure 1). A layer of viscous liquid is applied to the surface as the lubricant.
The lubricant should fall between the surface and the cylinder during its rolling. As a result, the cylinder will not roll directly on the surface, but on the lubricant layer. The lubricant layer significantly affects the process of the rolling during the operation of friction unit. The layer of the liquid prevents a direct contact between the cylinder and the surface, and therefore the cylinder-surface pair is protected from sticking, which reduces the material loss. The layer of the lubricant significantly reduces stress in metal at the point of contact [1]. It is known, that modern friction units can have very high stress in metal at the points of contact. The occurrence of such stress can be explained by the fact that the contact area between the cylinder and the surface is very little and it leads to high concentration of powers. For instance, in the common liquid bearing units the stress can be up to 800 N/mm². Such stress can lie beyond the limits of elastic deformation and should unavoidably lead to rapid destruction of the rolling surface. The layer of lubricant at the contact point increases the area of the contact larger and it leads to more uniform distribution of pressure. As a result, the stress in the rolling surface is significantly decreasing. Thus, in reality when we roll the cylinder on surface, the surface experiences less stress, then when a cylinder is static.

Figure 1. Rolling of the cylinder on the flat surface is considered.

This model of the rolling cylinder on the surface with a presence of the lubricant layer considers the fact that the lubricant is under significant pressure too. In contrast, a standard hydrodynamic lubrication does not account this fact. Moreover, it is possible to find a full solution of the problem with the rolling cylinder.

5. Presence of a thin layer between the surfaces
Considering two surfaces, where pieces of surfaces are replaced with tangential planes at first approximation. It can be assumed that the tangential planes are parallel, and when the layer is thin, it is possible to use the same parametrization with the parallel basic vectors [2]:

\[ \vec{h}_o = h_o \vec{n} = h_o \vec{N}, \]  \hspace{1cm} (1)

where \( h_o \) – thickness of a layer between two surfaces.

\[ \vec{h}_o = \vec{\rho}_o + (\vec{R}_o - \vec{r}_o). \]  \hspace{1cm} (2)

Vector \( \vec{h}_o \) can be formed in two ways:
1) The difference \( (\vec{R}_o - \vec{r}_o) \), by giving the equations of the surfaces (Picture 2).
2) The vector of displacement \( \vec{\rho}_o \), which determines the relative position of the surfaces with the final angle of rotation \( \theta \) (Picture 3).

Layer thickness with Gaussian curvatures taken into account:
$$h = h_0 + \frac{1}{2}(R_1 + \hat{R}_1) \delta x_1^2 + \frac{1}{2}(K_1 + \hat{K}_1) \delta x_2^2.$$  \hspace{1cm} (3)

Figures 2 and 3 show how the local thickness of the layer is formed for both ways. In the case when a cylinder is rolling on the surface without load, a thin layer is formed between the cylinder and the surface locally, along the generatrix of the cylinder.

If the cylinder is under load, then the surface is getting deformed and the layer is formed similarly to the movement “a cylinder in the cylinder”.

6. Main equations in a contact-hydrodynamic theory of lubricant

We first assume that both surfaces are absolutely firm. The thickness of the layer between these surfaces is:

$$h = h_0 + \frac{1}{2}[(\hat{b}_{11} - b_{11})\delta x^2 + (\hat{b}_{22} - b_{22})\delta y^2].$$ \hspace{1cm} (4)

The equations of the surfaces are replaced by approximations:

$$h_1 = \frac{1}{2}(b_{11}\delta x^2 + b_{22}\delta y^2); \quad h_2 = h_0 + \frac{1}{2}(\hat{b}_{11}\delta x^2 + \hat{b}_{22}\delta y^2).$$ \hspace{1cm} (5)

If we assume that both surfaces are deformable, then the components of the vector of displacement for both surfaces are:

$$h_1 = \frac{1}{2}(b_{11}\delta x^2 + b_{22}\delta y^2) - \Delta u_z^1; \quad h_2 = h_0 + \frac{1}{2}(\hat{b}_{11}\delta x^2 + \hat{b}_{22}\delta y^2) + \Delta u_z^2.$$ \hspace{1cm} (6)
The velocity components for the surfaces $S_1$ and $S_2$ are determined. We assume the hypothesis, that a coefficient of viscosity is exponentially depends on hydrodynamic stress at the point of contact between a solid and a liquid

$$\mu = \mu_H e^{\alpha p}. \quad (7)$$

Stress distribution equation looks like

$$
\frac{\partial}{\partial x} \left[ (h_2 - h_1)^3 \frac{\partial \Pi}{\partial x} \right] + \frac{\partial}{\partial y} \left[ (h_2 - h_1)^3 \frac{\partial \Pi}{\partial y} \right] = 6\mu_n \frac{\partial}{\partial x} \left[ (U_2 + U_1)(h_2 - h_1) \right] + \\
+ 6\mu_n \frac{\partial}{\partial y} \left[ (W_2 + W_1)(h_2 - h_1) \right] + 12\mu_n \left[ (V_2 - V_1) - (U_2 \frac{\partial h_2}{\partial x} - U_1 \frac{\partial h_1}{\partial x}) \right] - \left( W_2 \frac{\partial h_2}{\partial y} - W_1 \frac{\partial h_1}{\partial y} \right), \quad (8)
$$

and the function

$$\Pi = -\frac{1}{\alpha} e^{-\alpha p} \quad (9)$$

satisfies the boundary conditions, $p$ – hydrodynamic stress. It is possible to determine hydrodynamic stress in the thin layer using the formula (9)

$$p = -\frac{1}{\alpha} \ln(-\alpha \Pi). \quad (10)$$

The deviator part of stress tensor is determined at the contact point. And then, the tangential stresses $P_{\tau}$ are found. Depending on the magnitude of the force $P_{\tau}$, the condition of rolling or rolling with sliding can be fulfilled.

Equations of the cylinder movement lay in the area of nonholonomic mechanics.

The task of contact-hydrodynamic theory of lubricant is fully formulated. This task plays an important role in solving the problem when switching from contact-hydrodynamic theory of lubricant, to purely hydrodynamic. Observing the thickness of the layer and a decrease in dependency of viscosity from the pressure on the way of solving a contact-hydrodynamic problem, we can replace the transition from the function $\Pi$ to hydrodynamic stress, when

$$\alpha \approx 0. \quad (11)$$

In this case, we can neglect the deformation of the medium $S_1$ and $S_2$, which form the friction unit.

Contact-hydrodynamic problem in this formulation can have two special cases.
1. Common contact problems, when there is no lubricant layer.
2. The Kapitsa problem, when only hydrodynamic equations and no deformation of mediums $S_1$ and $S_2$, which form the cylinder-surface pair, are considered. In this case, the equation for stress distribution can be replaced by the usual Raymonds Equation, but the solution is laying in the area of nonholonomic mechanics.
The solving results of the hydrodynamic problem. The cylinder rolls on the surface. A viscous fluid is applied between the cylinder and the surface as a lubricant. The calculations results of pressure distribution for cylinder rolling with various workloads $F$ are presented in the Figure 4.

![Figure 4. The solving results of the hydrodynamic problem](image)

Workloads $F = 0.204; F = 0.154; F = 0.121; F = 0.042; F = 0.006$

7. A numerical example
A steel cylinder with a diameter of 1 cm rolls along a liquid layer with a viscosity of 0.04 Pa, workload $= 5.3$ kg/sm$^2$. The pressure will be less than 330 kg/sm$^2$. The thickness of the lubricant layer is 1 mkm. In a case when the cylinder rolls without taking into account the lubricating layer at a workload $= 5.3$ kg/sm$^2$, then pressure will be equal to 1800 kg/sm$^2$.

8. Conclusions
Solving the surface geometry problem based on tensor analysis gives an answer to the question of the presence of a thin layer between the cylinder surface and the rolling surface. In a case when the cylinder rolls on the surface, a thin layer is formed locally along the cylinder. Upon deformation of the rolling surface, a thin layer is formed similarly to the movement of the cylinder in the cylinder [3].

The viscous fluid on the rolling surface flows into the layer between the cylinder and the surface. The presence of a viscous liquid in the current layer between the cylinder and the surface leads to an increase in the contact area and a more uniform pressure distribution.

With increasing cylinder loads, surface deformations and nonholonomicity of the model should be taken into account [4].

The contact-hydrodynamic problem is solved in the partial case when only the equations of hydrodynamics are taken into account and there are no deformations of the surfaces forming a cylinder-rolling surface pair.
References

[1] Zavyalov O.G. The effect of a gear part to the deformable surface with consideration of the lubricant layer // The eighth Polyakhov reading. Thesis of reports at the International scientific mechanic conference. 30 January – 2 February 2018 / Saint Petersburg: Saint Petersburg State University, 2018, p. 98

[2] Izmaylov V.V., Novoselova M.V., Contact solidness of machine parts and effect of micro geometrics of contacting surfaces on it / Friction loss, vol.39, № 1, 2018, p.41 – 49.

[3] M.V. Korovchinskiy Teoreticheskiye osnovy raboty podshipnikov skol'zheniya. M .: Mashgiz, (1959)

[4] I.A. Solov'yov Vychislitel'naya matematika na smartfonakh, kommunikatorakh i noutbukakh s ispol'zovaniem programmnykh sred Python: uchebnoye posobiye / I. A. Solov'yov, A. V. Chervyakov, A. YU. Repin. SPb .: Lan' (2011)