One loop scattering on D-branes

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Abstract

We analyze one loop scattering amplitudes of the massless states on a stack of D3-branes. We use the vertex operators that have been obtained in the direct open string analysis developed in arXiv:0708.3452. The method does not have the obstacle of the D9 computation which is associated with the appearance of an $\epsilon$-tensor. The divergence structure is not the same as the D9 brane case. What makes the analysis deviate from the D9 brane case is that the momenta of the states have non-zero components only along the brane directions. We ponder on the possibility that the one-loop divergence may be canceled by adding additional vertex operators at the tree level. We anticipate that they will be “exponentiated” to the free string action, with the resulting action to constitute a non-linear sigma model of the D-brane/AdS geometry.
1 Introduction

An open string is an interesting object for its end points among other things. They are the places where the gauge symmetry is carried through the Chan-Paton factors. Potentially they can stick together thereby converting the original open string into a closed string. More recently it has been discovered that they may be attached on a hyper-plane, a D-brane [1, 2, 3]. With its end points attached, the open string will move on the D brane and may scatter when it comes across another open string. Because of this it has to be a due course of study to analyze various scattering amplitudes on a stack of Dp branes, especially with \( p < 9 \). Certain pieces of information on the amplitudes may be obtained by applying T-duality to the results for the D9 brane. However, the range of the information obtained in that manner is limited: many additional pieces could be obtained by directly considering an open string on lower dimensional D-branes. Furthermore a whole new picture seems to emerge, as we propose below, on how the geometry would arise\(^2\) as a way to cope with the open string loop divergences, which does not have an analogue in the D9 brane physics. The picture has implications to the open-closed string type dualities, the matrix theory conjectures and AdS/CFT conjecture. Eventually it will provide a new scheme for unification of gauge theory and gravity.

The scattering physics on D-branes will obviously be relevant for AdS/CFT correspondence and its generalization, which is in fact the main motivation of the work. Although the methodology of the present paper is general we will consider the D3 brane case to be specific. In the stronger version of the AdS/CFT conjecture it is stated that the \( D = 4 \ N = 4 \) SYM theory is fully equivalent, without taking any limit such as the large N limit, to the closed string theory on AdS\(_5 \times S^5\). A low energy limit of an open string is \( D = 4 \ N = 4 \) SYM: a SYM theory result should be acquired by taking a small \( \alpha' \) limit of the corresponding open string computation. In light of the conjecture what it means is that the massive open string modes do not play a role in producing the same results as those of the dual closed string theory. This should be so in spite of the fact that they are a natural (i.e., stringy) extension of the SYM. During the past few years evidence along this line has been collected. However the conjecture still remains a conjecture. Furthermore there have been attempts to deduce or derive the conjecture which only led toward its weaker form, but not necessarily toward the stronger form [10, 11]. (See [12] also.) Therefore we believe it is of prime importance to understand whether (and if so, how) the full open stringy analysis figures into the picture.

\(^1\)Scattering on D-branes was studied in NSR formulation by several authors [4, 5, 6, 7, 8, 9]. We will use the Green-Schwarz formulation for a reason that will become clear later. Also our approach is different in that we do not introduce independent closed string fields.

\(^2\)We emphasize that the geometry would arise as a result of the flat space computations. More comments can be found in sec 3.

\(^3\)This would really be the strongest version.
For that purpose it may be useful to consider scattering of open strings on a stack of D3-branes. The first task will be construction of vertex operators on D3-branes. In \cite{13} massless vertex operators have been constructed and their tree level scattering has been analyzed. One of the reasons why such an independent analysis is necessary rather than relying on applying T-duality on the D9 physics, is the non-commutativity of the quantum corrections and T-duality (or dimensional reduction) in the current setting. An analogy may be helpful. In a standard quantum field theory it is well-understood that there is no connection between the quantum corrections of a dimensionally reduced theory and those of the original theory. The reason is that the reduced theory loses some degrees of freedom. One has a similar situation here. One can easily see it from the world-sheet perspective. When computing the one loop correction one takes the trace over the momentum, \[ \int d^D p < p | \cdots | p > \]. In the D9 case one takes \[ D = 10 \]. However for the D3 case one should take \[ D = 4 \]. This makes a difference in the divergence behavior of the one loop. In the D9 case renormalization of the string tension is sufficient to absorb the divergence. As we will see below additional counter terms (presumably infinitely many of them for all order cancellation) may be required in the case of D3-branes.

The origin of the difference between the D9- and D3- analyses is that the transverse momentum components are zero for the D3-brane case \cite{13}. We extend the study to one loop and in particular consider four point scattering amplitudes as shown in the figure below. The outer boundary of the annulus is attached to the D3 branes as a result of the Dirichlet boundary condition of an open string. The inner boundary lies in the bulk. Since the momentum of the all four open string states are along the longitudinal direction \cite{13} it is natural to believe that inner boundary should represent the closed string “propagating” into the transverse space but with zero momentum. Put another way the closed string should be non-propagating. In this setting, therefore,

\footnote{Some cautionary remarks may be needed here. In the conventional setup where one starts both with an open string and a closed string, T-duality will commute with the quantum corrections. This is because one would account for the momentum and winding states and computes the quantum loops before taking a large/small radius limit: the D9 brane results will translate into the D3 brane results via T-duality. The current setup is as if one takes the limit first and therefore the D3-brane results cannot be recovered from the corresponding D9 results. Once more an analogy with a quantum field theory may be useful. If one compactifies on a circle and keeps all the momentum and winding modes, there will be a connection between the two theories. However, then one is not dealing with the dimensionally reduced theory. The current setup of a pure open string is analogous to a dimensionally reduced theory. For one thing we just saw that some of the zero modes get lost. We believe that the correct approach to obtain the D3 brane quantum effects is as we present in this paper, at least in the purely open string setup. Whether such a purely open string frame-work really exists is debatable. If the conjecture of this paper can be verified to high loop orders, it will be an indication that the answer is affirmative.}

\footnote{After this work was published, it has been verified at the one-loop \cite{14}. The two loop extension has also been initiated more recently \cite{15}.}
the status of a closed string is very different from that of an open string. In terms of
the low energy field theory it would mean that the closed string would appear as an
insertion of certain composite operators, whereas an open string would be propagating,
fundamental degrees of freedom. Since an insertion of a closed string vertex operator
is associated, according to the common lore, with change in the metric it is likely that
the effect of the loop is to deform it. To what metric will it deform? The only natural
candidate is the supergravity metric solution for a stack of D3 branes (or AdS$^5 \times S^5$
when the number of the branes is large).

![Figure 1: One loop four point open string scattering on a stack of D3 branes](image)

The amplitude computations will be performed in an operator formulation. However, it is not the same as the one in [16]: instead of treating the first state and the
last state as a bra-state and a ket-state we put all vertex operators on an equal foot-
ing. We insert the vertex operators constructed in [13]. What we find as a pleasant
surprise is that the present formulation seems free of the well-known limitation of the
light-cone gauge for D9 brane that is associated with setting $k^\pm = 0$ for simplicity.

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6 The motivation for considering such a formulation is to have a convenient set-up for a future-
check of the conjecture that is put forward in sec 3. With the setup one can use the standard Wick
contraction techniques on the fields. The check will require many lengthy amplitude computation
with many vertex operators inserted. In the existing operator formulation one must deal with multiple
oscillator products and it will be hard to maintain a reasonable level of confidence in the accuracy
of the computations. It is absolutely advantageous to employ the standard quantum field theoretic
technique, the Wick contraction. It may not be just a matter of convenience: as will be discussed in
the beginning of sec 2.
With $k^\pm = 0$ one cannot compute $M$-point amplitudes with $M > 6$. The reason is the appearance of the $\epsilon$-tensor with eight indices, which is not clear how to covariantize.\(^7\) As a matter of fact one should prove that such terms are absent as a separate task. What saves the case of D3 brane (or for that matter other cases of D$p$-branes with a low enough $p$) from similar difficulty is that the momenta have only two non-zero components therefore making the $\epsilon$-tensor term vanish. By treating all the operators on an equal footing, the method has a more direct link to the path integral approach.\(^8\)

The rest of the paper is organized as follows: We start in Sec2 with a brief review of necessary ingredients. For a review of the string theory in general see [16, 17, 1]. To check the validity of the operator formulation with all operators on equal footing we compute various three- and four- point amplitudes at the tree level. Throughout the computations we use dimensional regularization. In Sec3 we consider one loop four point amplitudes and work out the divergence structure. The analysis goes differently from that of the D9 brane: it is not connected to the D9 result via T-duality. This is due to the fact that the momenta of the states are only along the D-branes but not along the transverse directions. Motivated by a physical picture we propose a mechanism of divergence cancellation by inserting additional vertex operators. We make a schematic analysis for a quartic vertex operator for an illustration that it produces terms with the correct pole structure. A more complete study with the inclusion of a complete set of the vertex operators will be given elsewhere. We also discuss how the geometry might arise in this setting. We put forward a coherent way to view AdS/CFT and similar type dualities assuming the validity of the proposed picture. We conclude with discussions of a few other issues and future directions.

## 2 Computations with operators on equal footing

In this section we discuss the computations of various amplitudes in a setting where all the vertex operators are treated on an equal footing. In other words all of them are inserted between a bra- and ket- states which are the vacuum states. The set-up will provide a convenient stage for a future-check of the conjecture that we put forward in section 3.

\(^7\)Presumably it is this $\epsilon$ problem that the type of the formulation of this work was not used in the past.

\(^8\)The path integral that we are referring to is not that of [16] but will be a hybrid approach of [16] and [1]. The path integral approach of [16] is rather unwieldy in the sense that they rely on the oscillator wave functions. The wave functions, especially the fermionic ones, seems to be complicated. Here we use the “conventional” vertex operators, i.e., the ones constructed [13]. They are the vertex operators in the Green-Schwarz formulation for the states on D3 branes.
An $M$-point amplitude in general is given by

$$A_M = \int d\mu < \prod_{i=1}^{M} V(k_i) >$$

(1)

The measure $d\mu$ is given by

$$d\mu = |(x_1 - x_2)(x_1 - x_M)(x_2 - x_M)| \int dx_3 ... dx_{M-1} \prod_{l=1}^{M-1} \theta(x_r - x_{r+1})$$

(2)

The vertex operators for the massless states have been obtained in [13]: we refer to [13] for them and for our conventions. The bosonic and the fermionic propagators are respectively

$$<X^i X^j> = -2\alpha'\eta^{ij} \ln |x - x'|$$

$$<S^{a_1}_{1} S^{a_2}_{1}> = \frac{\delta^{a_1 a_2}}{x_1 - x_2}$$

(3)

2.1 four point amplitudes

One can apply the formulation to various three point amplitudes. We do not present the result, instead refer the interested readers to [18]. We turn to the various four point amplitudes. Recall the limitation of setting $k^\pm = 0$ in the operator formulation of a D9 brane [16, 17]. It does not allow one to compute $M$-point amplitude with $M > 6$: with $M > 6$ one encounters an $\epsilon$-tensor with 8 indices and it is not clear how to covariantize the result. A pleasant surprise is that the limitation is absent in the current formulation of a D3 brane where the $\epsilon$-tensor appears in the four point amplitude already. Basically the reason is that the present approach treats the first state and the final state on equal footing with all the other states, whereas in the operator method they appear as a bra and ket respectively. Therefore one expects to face the $\epsilon^{(8)}$ issue already at a four point level. What saves the formulation is that with the D3-brane the $\epsilon$-term vanishes due to the fact that the momenta only have two non-zero components in the brane directions. Through several examples below, we again will demonstrate that the present formulation yields the same results as the conventional operator method.

Let’s consider four point amplitudes in the order of increasing complexity. The simplest is the four scalar amplitude.

$$I_{\phi\phi\phi\phi} = < \prod_{i=1}^{4} (\zeta^{m_i} X^{m_i} + \zeta^{m_i} R^{m_i v_i} k^{v_i}) e^{ik_i X} >$$

(4)
There are six different types of terms: $XXXX, XXXR, XXRR, XRRR$ and $RRRR$. In the dimensional regularization the second type of terms vanish. The result is precisely the same as the result of the operator formulation [13]. Omitting the factor $\frac{g^2}{2} \alpha'^2 \text{Tr}(\lambda^a \lambda^b \lambda^c \lambda^d) \frac{\Gamma(-s/2)\Gamma(-t/2)}{\Gamma(-s-t/2)}$ one gets

$$A_{\phi\phi\phi} = \frac{1}{4} (su \xi_1 \cdot \xi_2 \cdot \xi_3 + tu \xi_1 \cdot \xi_2 \cdot \xi_4 + st \xi_2 \cdot \xi_4 \cdot \xi_1 \cdot \xi_3)$$  \hspace{1cm} (5)$$

Many terms that would be otherwise present vanish due to the fact that $\xi \cdot k = 0$. It is more involved to compute the two vector and two scalar scattering amplitude. We illustrate this with

$$I_{\phi AA\phi} = \langle \prod_{i=1}^{4} (\xi_{m_i} X^{m_i} + \xi_{m_i} R^{m_i v_i} k^{v_i}) e^{ik_i \cdot X} \prod_{i=2}^{3} (\zeta_{u_i} \hat{X}^{u_i} - \zeta_{u_i} R^{u_i v_i} k^{v_i}) e^{ik_i \cdot X} >$$  \hspace{1cm} (6)$$

The corresponding operator result in the conventional formulation, apart from $\frac{g^2}{2} \text{tr}(\lambda^a \lambda^b \lambda^c \lambda^d)$, is [13]

$$\langle k^1, \xi^1 | V_g(k^2, \zeta^2) V_g(k^3, \zeta^3) | k^4, \xi^4 \rangle = \xi^1 \xi^4 \left[ \frac{1}{4} su \zeta^2 \cdot \zeta^3 - \frac{1}{2} (u \zeta^2 \cdot k^1 \zeta^3 \cdot k^4 + s \zeta^2 \cdot k^4 \zeta^3 \cdot k^1) \right]$$  \hspace{1cm} (7)$$

where we have omitted the factor $\frac{\Gamma(-s/2)\Gamma(-t/2)}{\Gamma(-s-t/2)}$. We break the computation of (6) into pieces. First we check the coefficient of the $\xi^1 \cdot \xi^4 \zeta^2 \cdot \zeta^3$ term, and subsequently the remaining terms. In all of the following computations we omit the common factor $\frac{\Gamma(-s/2)\Gamma(-t/2)}{\Gamma(-s-t/2)}$. There are three contributions:

$$XXXX \Rightarrow \frac{su/4}{1 + t/2}$$

$$XXRR \Rightarrow \frac{tsu/4}{1 + t/2}$$

$$RRRR \Rightarrow \frac{t^2}{4} \frac{su/4}{1 + t/2} - \frac{1}{8} stu$$  \hspace{1cm} (8)$$

They add up to yield

$$\frac{1}{4} su$$

which is indeed the correct coefficient. The results for the remaining terms can be summarized similarly. Unlike above the $XRRR$-terms contribute. One can work them out explicitly using the identities given in the appendix. The result is

$$XXXX \Rightarrow -\frac{1}{2} \left( u \zeta^2 \cdot k^1 \zeta^3 \cdot k^4 + s \zeta^2 \cdot k^4 \zeta^3 \cdot k^1 + \frac{su/2}{1 + t/2} \zeta^2 \cdot k^3 \zeta^3 \cdot k^2 \right)$$

7
\[ XXRR \Rightarrow \frac{su/4}{1 + t/2} \zeta^2 \cdot k^3 \cdot \zeta^3 \cdot k^2 - \frac{t}{2} \frac{su/4}{1 + t/2} \zeta^2 \cdot k^3 \cdot \zeta^3 \cdot k^2 \]
\[-\frac{1}{4} \left( t^2 \zeta^2 \cdot k^4 \cdot \zeta^3 \cdot k^4 - tu \zeta^2 \cdot k^3 \cdot \zeta^3 \cdot k^4 - ts \zeta^2 \cdot k^4 \cdot \zeta^3 \cdot k^2 \right) \]
\[ XRRR \Rightarrow \left( \frac{1}{4} u^2 \zeta_2 \cdot k_3 \cdot \zeta_3 \cdot k_4 - \frac{1}{4} su \zeta^2 \cdot k^3 \cdot \zeta^3 \cdot k^1 - \frac{1}{4} tu \zeta^2 \cdot k^4 \cdot \zeta^3 \cdot k^4 \right. \\
\quad \left. + \frac{1}{4} ts \zeta^2 \cdot k^4 \cdot \zeta^3 \cdot k^1 + \frac{1}{4} s^2 \zeta^2 \cdot k^4 \cdot \zeta^3 \cdot k^2 \right. \\
\quad \left. - \frac{1}{4} tu \zeta^2 \cdot k^4 \cdot \zeta^3 \cdot k^4 \right) \]
\[ RRRR \Rightarrow \frac{t}{2} \frac{su/4}{1 + t/2} \zeta^2 \cdot k^3 \cdot \zeta^3 \cdot k^2 - \frac{1}{2} su \zeta^2 \cdot k^3 \cdot \zeta^3 \cdot k^2 \\
\quad - \frac{1}{4} \left( st \zeta^2 \cdot k^4 \cdot \zeta^3 \cdot k^1 + tu \zeta^2 \cdot k^1 \cdot \zeta^3 \cdot k^4 \right) \]

(9)

where we have omitted the common factor \( \zeta^1 \cdot \zeta^4 \). Adding all four contributions one reproduces the last two terms of (7).

\[ - \frac{1}{2} \left( u \zeta^2 \cdot k^1 \cdot \zeta^3 \cdot k^4 + s \zeta^2 \cdot k^4 \cdot \zeta^3 \cdot k^1 \right) \]

(10)

Note that \( XXXX, XXRR, XRRR, RRRR \) terms individually produce terms of the type \( \zeta \cdot k \zeta \cdot k \zeta \cdot k \) which equal zero since \( \zeta \cdot k = 0 \). This completes the discussion of the two scalar and two vector amplitude. The computation of the four vector amplitude

\[ I_{4v} = \langle \prod_{i=1}^{4} \left( \zeta^u_i X^u_i - \zeta^u_i R^u_i v^u_i \right) e^{ik \cdot X} \rangle \]

(11)

goes parallel although it is more involved. There are four different kinds of terms as before. The computation of \( \zeta \cdot \zeta \cdot \zeta \cdot \zeta \)-type of terms is similar to those of above. Let’s consider the terms of the form \( \zeta \cdot k \zeta \cdot k \zeta \cdot \zeta \). To be specific we take the example of \( \zeta_1 \cdot \zeta_2 \). The results can be summarized as

\[ XXXX \Rightarrow -\alpha'(u \zeta_3 \cdot k_2 \zeta_4 \cdot k_1 + t \zeta_3 \cdot k_1 \zeta_4 \cdot k_2) - \frac{\alpha' t \alpha'u}{1 + \alpha's} \zeta_3 \cdot k_4 \zeta_4 \cdot k_3 \]
\[ XXRR \Rightarrow \frac{\alpha' t \alpha'u}{1 + \alpha's} \zeta_3 \cdot k_4 \zeta_4 \cdot k_3 \\
\quad - \left( \frac{1}{4} s^2 \zeta_3 \cdot k_2 \zeta_4 \cdot k_2 - \frac{1}{4} su \zeta_3 \cdot k_2 \zeta_4 \cdot k_3 - \frac{1}{4} st \zeta_3 \cdot k_4 \zeta_4 \cdot k_2 \\
\quad + \frac{s}{2} \frac{\alpha' t \alpha'u}{1 + \alpha's} \zeta_3 \cdot k_4 \zeta_4 \cdot k_3 \right) \]

(12)
\[ XRRR \Rightarrow \left( -\frac{1}{4} st \zeta_3 \cdot k_2 \zeta_4 \cdot k_2 + \frac{1}{4} su \zeta_3 \cdot k_2 \zeta_4 \cdot k_1 + \frac{1}{4} t^2 \zeta_3 \cdot k_4 \zeta_4 \cdot k_2 - \frac{1}{4} tu \zeta_3 \cdot k_4 \zeta_4 \cdot k_1 \right) \]
$-\frac{1}{4} us \zeta_3 \cdot k_2 \zeta_4 \cdot k_2 + \frac{1}{4} ts \zeta_3 \cdot k_1 \zeta_4 \cdot k_2 + \frac{1}{4} u^2 \zeta_3 \cdot k_2 \zeta_4 \cdot k_3 - \frac{1}{4} tu \zeta_3 \cdot k_1 \zeta_4 \cdot k_3$}

$RRRR \Rightarrow \frac{s}{2} \frac{\alpha \prime t\alpha u}{1 + \alpha \prime s} \zeta_3 \cdot k_4 \zeta_4 \cdot k_3 - \frac{1}{2} tu \zeta^3 \cdot k^4 \zeta^4 \cdot k^3$

$- \left( \frac{1}{4} su \zeta_3 \cdot k_2 \zeta_4 \cdot k_1 + \frac{1}{4} st \zeta_3 \cdot k_1 \zeta_4 \cdot k_2 \right)$

(13)

Adding all four contributions one gets as the coefficient of $\zeta^1 \cdot \zeta^2$

$$- \frac{1}{2} (u \zeta_3 \cdot k_2 \zeta_4 \cdot k_1 + t \zeta_3 \cdot k_1 \zeta_4 \cdot k_2) + \frac{1}{8} tu \zeta_3 \cdot k_4 \zeta_4 \cdot k_3$$

(14)

It is precisely the same result as the one obtained in the conventional operator method. Finally one can show that the terms of the type $\zeta \cdot k \zeta \cdot k \zeta \cdot k \zeta \cdot k$ completely cancel among themselves, confirming the conventional operator result that those types of terms are absent.

### 3 Analysis of the one loop divergence

The above method should be applicable to one loop computations. We will not pursue it here. Rather we use the conventional operator method [16, 17] which is simpler, to find various one loop amplitudes. The divergence structure is different from that of the D9. As has been emphasized previously it is due to the fact that the Fock space momentum takes the non-zero values only in the brane directions. After obtaining the one loop amplitudes we ponder on how to remove the divergence. We will carry out some preliminary check to see whether it is possible to remove it by adding counter terms in the action, such as quartic $X$-terms, and evaluating their contributions at the tree level. For that we again will turn to the present operator method. We make a conjecture on how the counter-terms may be connected to the geometry.

#### 3.1 one loop divergence

In the operator method of [16, 17], the one loop divergence can easily be computed in analogy with the D9 brane case. The only difference occurs in the bosonic zero modes, i.e., the momentum, when one takes the trace. The difference, although looking minor, is crucial since it is what makes the factor of $(\ln w)$ below different from the D9 case, which in turn changes the pole structure. The divergence cancellation mechanism through an introduction of geometry (which we propose below) hinges on this difference.

In close analogy with the D9 brane computation one can show that a one-loop four
point amplitude of the massless states in general is given by

$$A(1, 2, 3, 4) = g^4 G K \int \frac{dw}{w} \int \left( \prod_{r=1}^{3} \frac{d\rho_r}{\rho_r} \right) \left( \frac{-2\pi}{\ln w} \right)^2 \prod_{r<s} (\psi_{rs})^{k_r \cdot k_s}$$

(15)

where $G$ is the group theory factor

$$\text{Tr}(\lambda^a \lambda^b \lambda^c \lambda^d) + \text{Tr}(\lambda^a \lambda^d \lambda^c \lambda^b) + \text{Tr}(\lambda^a \lambda^c \lambda^b \lambda^d)$$

$$+ \text{Tr}(\lambda^a \lambda^d \lambda^b \lambda^c) + \text{Tr}(\lambda^a \lambda^b \lambda^d \lambda^c) + \text{Tr}(\lambda^a \lambda^c \lambda^d \lambda^b)$$

(16)

The factor $K$ is a kinematic factor and depends on the states under consideration. For the four vector amplitude for example it is given by

$$K = t_{i_1 j_1 i_2 j_2 i_3 j_3 i_4 j_4}$$

(17)

with

$$t_{i_1 j_1 i_2 j_2 i_3 j_3 i_4 j_4} = \text{Tr}(R_{i_1 j_1}^0 R_{i_2 j_2}^0 R_{i_3 j_3}^0 R_{i_4 j_4}^0)$$

(18)

For the four scalar scattering it is

$$-\frac{1}{4} (su \xi_1 \cdot \xi_4 \cdot \xi_2 \cdot \xi_3 + tu \xi_1 \cdot \xi_2 \cdot \xi_3 \cdot \xi_4 + st \xi_2 \cdot \xi_4 \cdot \xi_1 \cdot \xi_3)$$

(19)

For other amplitudes the $K$ should be replaced appropriately. Note that the power of the $\left( \frac{-2\pi}{\ln w} \right)^2$ is different from the D9: instead of five it is two for the D3 case.

Note that the divergence of (15) comes from $w \sim 1$ and has the pole structure of

$$\int_0^1 dw \frac{1}{w(\ln w)^2}$$

(24)

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9 To compare with the D9, the divergence structure of (15) can be put in the following variables

$$q = e^{\frac{2\pi}{\ln w}}, \quad \nu_r = \frac{\ln \rho_r}{\ln w}$$

(20)

Eq (15) now takes the form of

$$A(1, 2, 3, 4) = g^4 G K (-8\pi^4) \int \frac{dq}{q(\ln q)^3} F(q^2)$$

(21)

where

$$F(q^2) = \int \prod_{r=1}^{3} \frac{d\nu_r}{\nu_r} \prod_{1 \leq r < s \leq 4} \left[ \sin \pi (\nu_s - \nu_r) \prod_{n=1}^{\infty} \left( 1 - 2q^{2n} \cos 2\pi (\nu_s - \nu_r) + q^{4n} \right) \right]^{k_r \cdot k_s}$$

(22)

Compared with the D9 where the logarithmic factor disappears, the degree of divergence is more serious as

$$\int \frac{dq}{q(\ln q)^3}$$

(23)
In the next section we discuss a possible mechanism to cancel the divergence after discussion of physical motivation. Before we get to that we express, for comparison later, the pole structure of (24) in a new coordinate, \(\ln w \equiv -y\): the pole structure takes the form of

\[
\int_0^\infty dy \frac{1}{y^2}
\]  

(25)

### 3.2 anticipated mechanism for divergence cancelation

The presence of the \((\ln q)^3\)-factor in (21), which does not have an analog in the D9 case, seems to suggest a more radical measure for the divergence cancellation. We discuss the possibility that the one-loop divergence may be cancelled with additional vertex operators at the tree level. We expect that they originate from the curved geometry in the sense explained below.

Let’s go back to the figure in the introduction where we have pointed out that the status of a closed string is different from that of an open string. Whereas the open strings are the fundamental degrees of freedom, the closed strings are not in a several regards: first of all they have been *generated* by open string quantum effects. Secondly, they seem to be non-propagating due to the momentum restriction on the open string states. Since it is not fundamental degrees of freedom a natural way to realize them should be via composite\(^{10}\) operators. The crucial question is then, how should they be introduced? Here we propose that they be introduced in such a way to cancel (at least potentially) the open string divergences, in other words, as counter terms.

The connection to the geometry may occur when they get “exponentiated” to the action where they would appear as vertex terms\(^{11}\). Together with the quadratic part of the action they will constitute a non-linear sigma model action. We emphasize that the geometry would arise as a result of the flat space computations. The counter vertex operators will be introduced in attempt to remove the divergence that has resulted from the flat space computation. (For example we are not quantizing the open string in a curved space. As a matter of fact, if the conjecture is true it will actually circumvent the necessity of quantizing the string on a curved space, at least for the open string.) It is similar, in spirit, to what was shown in \([19, 20]\). There it was shown that the effective action of N=4 SYM that contains quantum (and the non-perturbative ) corrections can be interpreted as a non-linear sigma type action with a curved target space. If this anticipation turns out true to this point then we believe that the resulting action should be an action for an open string in a curved geometry. To what geometry would it deform? The only natural candidate is the supergravity metric solution for a stack

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\(^{10}\)The open string vertex operators themselves take the forms of the composite operators. Therefore what is meant by composite here is that they are even more composite and/or of different composite-ness than the open string ones.

\(^{11}\)It will be more clear to see in the path-integral approach where those composite operators will be brought down by the standard procedure.
of D3 branes or AdS$^5 \times S^5$ in an appropriate limit. Stated the other way around the precise set of the vertex operators will be dictated by the non-linear sigma model action and the divergence cancellation. Again we emphasize that the curved geometry should be used in order to guide us in finding the forms of the vertex operators, but not as a space to quantize an open string in. Finding the forms of the counter vertex operators without such guidance would be a hopeless task.

Although the counter terms may reveal aspects of the geometry which part of the geometry is revealed may depend on the dynamics considered. Another possible question is on the realization of the closed string. In the current stage they reveal their presence through the deformed metric. Could they appear on a more fundamental level? We postpone these issues until the conclusion. In the remainder of this section we carry out a preliminary check to set the ideas above on a computational ground.

For an illustration we take the example of the four point scalar scattering. The one loop divergence structure has been presented in the previous section. One needs to find the complete list of the vertex operators that would cancel the divergence. The task involves lengthy computations which involve many vertex operators. Here we only focus on one of them to illustrate the strategy. Recall that a curved metric has the following curvature expansion in a Riemann normal coordinate,

$$g_{MN} = \eta_{MN} - \frac{1}{3} R_{MPNQ}(X_0) X^P X^Q + \cdots$$

(26)

The supergravity metric solution for a stack of D3 branes is given by

$$ds^2 = H^{-1/2} (-dt^2 + (dx^\mu)^2) + H^{1/2} (dx^m)^2$$

$$H = 1 + \frac{4\pi gNl^4}{r^4}$$

(27)

Consider a $r$-expansion of the curvature tensor, $R_{mn pq}$, that results from the D3-brane metric where we treat $r$ to be a large but fixed constant. In the leading order the term that has the least number of fields is

$$R_{mn pq} \sim (\delta_{mq} \delta_{np} - \delta_{mp} \delta_{nq})$$

(28)

Therefore one of the vertex operators that is necessary to cancel the one loop divergence from $< V^m_1 V^m_2 V^m_3 V^m_4 >$ is expected to have the following form

$$\int dx \ C_{mpnq} \dot{X}^m \dot{X}^n X^p X^q$$

(29)

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12The corresponding calculation with the vector vertex operators would be more involved. For this reason and others the advantage of having the explicit forms of the vertex operators is obvious.

13To cancel the divergence using the conventional operator formulation along the line of the conjecture of the present work, the only natural thing to do is to change the Virasora operator $L_0$ to take the ”curved space” effects into account. It is not entirely clear, apart from the technical complexities, whether there will be a valid series expansion in this approach.
When inserted together with the other vertex operators that represent the scattering states, the tree level amplitude

\[ \int d\mu < V_{m_1}^s(x_1) V_{m_2}^s(x_2) V_{m_3}^s(x_3) V_{m_4}^s(x_4) \int dx \ C_{mpnq} \dot{X}_m(x) \dot{X}_n(x) X^p(x) X^q(x) > \]  

produce terms that have the same pole structure as that of the one loop. The measure in front \( d\mu \) is given in (2) and

\[ C_{mpnq} = \text{const} \cdot (\delta_{mq} \delta_{np} - \delta_{mn} \delta_{pq}) \]  

The constant will be a function of the t’ Hooft coupling and will be determined by the requirement of the divergence cancellation.\footnote{It will also contain a certain (positive or negative) power of \( r \). Presumably one should treat \( r \), but not the individual coordinates, as a constant as in the case of a point on a sphere of radius \( r \). This point of view already appeared in the context of AdS/CFT \cite{21}. The detailed mechanism to treat \( r \) should be a part of how open string quantum effects reveal geometry. (The detailed mechanism has now been given in \cite{14}.)}  

The amplitude (30) have poles at

\[ x = x_i, \quad i = 1, \ldots, 4 \]

The pole terms at \( x = x_1 \) vanishes as \( x_1 \to \infty \). The highest order pole terms at \( x = x_i, i \neq 1 \) have the structure of

\[ \int_0^\infty dy \frac{1}{y^2} \]  

where we have not recorded the precise form of the coefficient. Note that it is the same order pole as the one loop divergence given in (25). Eq.(30) also produces terms of a different pole structure, \( \int_0^\infty dy \frac{1}{y^3} \), and/or terms that become divergent as \( x_1 \to \infty \). It is crucial to check that all these unwanted terms must cancel among themselves when the complete list of the vertex operators are once considered together. At the same time the momenta structure must turn out to match that of the one loop for the correct pole terms. We have elaborated on this in \cite{14}.

4 Conclusion

In this letter we have analyzed the one loop divergence structure of the four point scattering amplitudes. We have conjectured the existence of a complete set of additional vertex operators whose origin should be linked to the D-brane/AdS geometry. In weak coupling the relevance of the closed strings seems to be recognized rather indirectly, i.e., through the non-linear sigma model. But is there a circumstance where they become propagating degrees of freedom? We believe that it is when one goes to a large coupling limit where closed string degrees of freedom become fundamental.
It should occur through an open string conversion into a closed string \[22\]. Then it will suggest that AdS/CFT type dualities will be a two-step process. First the open string quantum corrections will “engineer” the curved geometry, which can be viewed as an open string generalization of the gauge theory result of \[23, 24, 19\]. The curved geometry will be introduced as a way to absorb the divergence. At the same time it will serve as a route for the closed strings to exhibit their relevance. At this stage the closed strings are not fundamental degrees of freedom but they become so when one reaches a strong coupling region through an S-duality, as an open string converts to a closed string \[15\].

A few comments on the future directions are in order: One obvious direction is to find those set of the vertex operators, at least order by order. For that purpose the works of \[25, 26\] will provide a useful guide. Another direction is to fully develop the path integral formulation. The setting of the present work is such that one can readily switch to the path integral method. There the vertex “operators” will appear in the standard procedure from the exponentiated action. If our picture is indeed correct one can say that the open string dynamics reveals aspects of the geometry. Which particular part of the geometry becomes revealed depends on the dynamics considered. For example, considering scalar multiple scattering will reveal different pieces of information about the geometry that the vector scattering. So the counter-terms necessary to cancel the divergences will be the geometry information relevant for the dynamics. Also there is a question concerning the radius of the sphere that originates in the large N-limit of the open string engineered geometry. Since there is an S-duality involved to go to the strong coupling it may be the inverse (in terms of the t’Hooft coupling) of the radius of the regular sphere that results from the D3 brane supergravity solution.

Connection between the string divergence cancellation and the field theory cancellation may be an interesting issue as well. The field theory task has been initiated in \[29\] where the starting point is the four dimensional action with \(\alpha’\)-corrections. The action is obtained by dimensional reduction of the ten dimensional action \[30\]. Since the massive modes have been integrated out the “renormalization” process will go differently from the full fledged string analysis. However we expect that they might be properly taken into account by an energy scale. We hope to report on this with better understanding in the future.

Finally a possible connection to the Fischler-Susskind mechanism: A few ingredients (such as the role of the zero momentum states) of the conjectured divergence cancellation are reminiscent of Fischler-Susskind mechanism \[31\, 32\]. One of the differences is the setting: here we start out only with the open string degrees of freedom. Existence of such a setup is not fully established although it is one of the issues that is pursued.

\[15\] One may wonder about the reverse mechanism where a closed string converts into an open string. We suspect that it should be a process that involves a spontaneous symmetry breaking. This issue will be pursued elsewhere.
by the string field theory. According to the string theory common lore, an open string always needs a closed string. However what we are conjecturing is independent of the frameworks: we are conjecturing that the open string divergence should be removed by purely open string vertex operators. If it does not work this way, then closed string vertex operators may be considered. In other words, in the conventional set-up where one includes the closed string degrees of freedom as well, one would try to cancel the divergence by some closed string tadpole divergence. However, it does not seem natural to attempt to remove the divergence of a single integral in (25) with a \( dy d\bar{y} \)-type double integral that the closed string analysis will produce. Furthermore there is another string theory common lore that states that an oriented open string cannot be coupled to an oriented closed string, although the statement is not being brought up too often with the advent of D-brane physics.

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