Abstract. Close encounters and physical collisions between stars in young dense clusters can result in new channels for stellar evolution, and may lead to the formation of very massive stars and black holes via runaway merging. We present some details of this process, using the results of N-body simulations and simple analytical estimates to place limits on the cluster parameters for which it expected to occur. For small clusters, the mass of the runaway is effectively limited by the total number of high-mass stars in the system. For larger clusters, the runaway mass is determined by the fraction of stars that can mass-segregate to the cluster core while still on the main sequence. In typical cases, the result is in the range commonly cited for intermediate-mass black holes. This mechanism may therefore have important implications for the formation of massive black holes and black-hole binaries in dense cluster cores.

1. Introduction: Young Dense Clusters in the Galaxy and Beyond

Among the many massive young clusters now known throughout the local universe, perhaps the most interesting to dynamicists are those in which stellar dynamical time scales are short enough that the cluster can undergo significant structural change during the lifetimes of the most massive stars. In such clusters, dynamical evolution opens up novel avenues for stellar and binary evolution, making possible the creation of entirely new stellar species. One obvious modification to standard stellar evolutionary tracks arises from collisions and
mergers between stars, and we focus on that here. From this perspective, the clusters listed by Portegies Zwart et al. (2004b; Table 1) represent an ideal combination of properties, having ages of less than a few million years and relaxation times of less than a few tens of millions of years. In these clusters, dynamical evolution, traditionally regarded as a “slow” process, actually occurs much more rapidly than the stellar evolution of even the most massive stars. In fact, cluster dynamics controls the early phases of these stars’ lives.

Portegies Zwart et al. (2004b) are primarily concerned with the lifetimes and global structural evolution of young dense clusters in the vicinity of the Galactic center. In this paper we consider mainly the stellar evolutionary aspects of life in such an extreme environment. We start by investigating the circumstances under which collisions are likely to occur, and how a cluster might find its way into such a state. We then present a scenario which may plausibly lead to the formation of very massive stars and (perhaps) intermediate-mass black holes (IMBHs) in sufficiently young, dense systems. Our results are based in large part on detailed N-body simulations of model clusters. Finally, we apply this scenario to recent observations of the starburst galaxy M82. In a companion contribution, Baumgardt et al. (2004) extend these ideas to the subsequent evolution of a cluster containing a massive, compact object.

2. Stellar Collisions and Cluster Structure

We are interested here in the possibility of runaway collisions leading to ultra-massive stars. To appreciate the conditions under which such runaways can occur, consider a massive object moving through a field of background stars of total mass density $\rho$ and velocity dispersion $v$. We assume that the mass $M$ and radius $R$ of the object are large compared to the masses and radii of other stars, and that all velocities are small enough that gravitational focusing dominates the total cross section. In that case, the object’s collision cross section is

$$\sigma \approx 2\pi GMR/v^2,$$  \hspace{1cm} (1)

nearly independent of the properties of the other stars. The rate of increase of the object’s mass due to collisions is therefore

$$\frac{dM}{dt} \approx \rho \sigma v \approx 2\pi GMR\rho/v$$

$$= 6 \times 10^{-11} \left(\frac{M}{M_\odot}\right) \left(\frac{R}{R_\odot}\right)$$

$$\times \left(\frac{\rho}{10^6 M_\odot/pc^3}\right) \left(\frac{v}{10 \text{km/s}}\right)^{-1} M_\odot/\text{yr}. \hspace{1cm} (2)$$

Thus, if the object initially has $M = 100M_\odot$ and $R = 30R_\odot$, we fix $v$ at 10 km/s, and adopt a mass-radius relation $R \propto M^{1/2}$, we find that in order for the object to accrete $10^3 M_\odot$ of material in 5 Myr (to form an IMBH within the typical lifetime of a massive star), the local density must satisfy

$$\rho \gtrsim 5 \times 10^8 M_\odot/pc^3 = \rho_{\text{crit}}, \text{ say}. \hspace{1cm} (3)$$
Such a density is much higher than the mean density of any known star cluster, young or old. For comparison, the average density of the Arches cluster is $\sim 6 \times 10^5 M_\odot/pc^3$, that of a fairly compact globular cluster is $\sim 10^4 M_\odot/pc^3$, while even the most concentrated globular cluster cores have densities $\leq 10^{5-7} M_\odot/pc^3$.

Might we be able to generate conditions more conducive to mergers by assuming that a cluster is born very centrally concentrated (e.g. Portegies Zwart et al. 2004, Merritt et al. 2004)? As a simple limiting model of a very condensed cluster, consider the nearly isothermal system of total mass $M_c$ and half-mass radius $r_h$, described by the density profile

$$\rho(r) = \frac{M_c}{8\pi r_h r^2},$$

$$M(r) = \frac{1}{2} M_c \left( \frac{r}{r_h} \right),$$

for $0 \leq r \leq 2r_h$. Densities exceeding $\rho_{\text{crit}}$ are found for $r < r_{\text{crit}}$, where

$$r_{\text{crit}} = \sqrt{\frac{M_c}{8\pi r_h \rho_{\text{crit}}}},$$

$$= 1.8 \times 10^{-3} \left( \frac{v}{10 \text{ km/s}} \right) \text{ pc},$$

where $v = \sqrt{GM_c/2r_h} \sim 10 \text{ km/s}$ for all the clusters of interest here. However, the total mass contained within this radius is just

$$M_{\text{crit}} = 40 \left( \frac{v}{10 \text{ km/s}} \right)^3 M_\odot.$$

Given the highly optimistic assumptions needed to accrete even a small fraction of this mass onto the original object, it is clear that, for reasonable cluster parameters, there is far too little initial mass in the high-density region to accomplish the task of forming a $1000 M_\odot$ object in the time available.

3. Cluster Dynamics

Thus it seems that collisions in a static cluster core cannot lead to the formation of an ultramassive object. However, it is well known that cluster dynamical evolution can result in conditions much more favorable for a runaway merger to occur. Here we briefly describe the relevant processes and their consequences. (We note in passing that essentially the same result could be achieved if there were significant initial mass segregation in the cluster, but there is currently no firm evidence to support this assumption.)

The evolution of a cluster is governed by its half-mass relaxation time, the time scale on which two-body encounters transport energy around the system:

$$t_{rh} \approx 0.14 \frac{M_c^{1/2} r_h^{3/2}}{G^{1/2} m \ln \Lambda} \approx 5 \left( \frac{v}{10 \text{ km/s}} \right)^3 \left( \frac{\bar{\rho}}{10^5 M_\odot/pc^3} \right)^{-1} \text{ Myr}$$
Here, \( N \) is the number of stars in the system, \( m = M_c/N \) is the mean stellar mass, \( \bar{\rho} = 3M_c/8\pi r_h^3 \) is the mean cluster density, and \( \ln \Lambda \sim \ln(0.1N) \sim 10 \). For an equal-mass system, the time scale for dynamical evolution—the core collapse time—is about \( 15t_{rh} \), too long to cause significant structural change in a few million years as required here. However, the presence of even a modest range in masses greatly accelerates the process of core collapse (Spitzer 1987). The time scale for a star of mass \( M \) to sink to the cluster center as equipartition reduces its velocity is

\[
t_s(M) \sim \frac{m}{M} t_{rh},
\]

where we note that typical mass spectra (e.g. Scalo 1986 or Kroupa 2001) have \( m \sim 0.4–0.6 M_\odot \) (\( 0.5 M_\odot \) is used in Eq. 8) and maximum mass \( \sim 50–100 M_\odot \).

From N-body simulations, Portegies Zwart & McMillan (2002) find that the most massive (\( \gtrsim 20M_\odot \)) stars segregate rapidly to the cluster center, forming a dense stellar subcore on a time scale \( t_{cc} \sim 0.2t_{rh} \). A central density increase of 2–3 orders of magnitude is typical, boosting even a relatively low-density core into the range where collisions become common, and greatly increasing the reserve of raw material available to form a collision runaway. The collisions naturally involve the most massive stars in the cluster, and the low relative velocities typical of these systems ensures that the colliding stars merge with minimal mass loss (J. Lombardi 2004, private communication).

In systems having \( t_{cc} \lesssim 5 \) Myr, corresponding to \( t_{rh} \lesssim 25 \) Myr, essentially all the massive stars in the cluster can reach the center before exploding as supernovae and hence participate in the runaway process. Using the parameters presented by Portegies Zwart et al. (2004b; Table 1), we find that the Arches and Westerlund I fall into this category, while the Quintuplet, NGC 3603, and R 136 all come close. In these cases, the maximum mass of the runaway is limited primarily by the total number of massive stars in the system—a few tenths of 1 percent of the total cluster mass. In less dense or more massive clusters, such as MGG-11, the relaxation time is longer and only a fraction of the massive stars initially present in the system can reach the center in the time available, but, as we will see, their greater number may still ensure that a runaway can occur.

In small systems (containing less than a few tens of thousands of solar masses), collision rates are significantly enhanced by the fact that the massive object tends to form binaries, which are then perturbed into eccentric orbits by encounters with other stars (Portegies Zwart & McMillan 2002). Binary-induced mergers increase the collision cross section, but they still require high central densities before the (three-body) binary formation rates become significant. In larger systems, unbound collisions appear to be the norm, and these have been observed in both direct N-body (NBODY4/Starlab/treecode) and Monte-Carlo (Gürkan et al. 2003) cluster simulations.

4. X-ray Sources and Dense Clusters in M82

Recently a bright X-ray point source (M82 X-1) has been observed some 200 pc from the center of the starburst galaxy M82 (Matsumoto et al. 2001; Kaaret et al. 2001). With a luminosity exceeding \( 10^{41} \) erg/s, it is too bright to be an ordinary X-ray binary, while its off-center location in the galaxy argues against its
being a supermassive black hole. The luminosity is consistent with an accreting compact object of at least 350 solar masses, raising the intriguing possibility that it might be an IMBH. The discovery of $54.4 \pm 0.9$ mHz quasi-periodic oscillations (Strohmayer et al. 2003) supports this assertion. Follow-up observations indicate that M82 X-1 is apparently located in the star cluster MGG-11, one of several massive young clusters in the central region of M82. This prompts the obvious question: Could the observed X-ray source be the result of the runaway collision process just outlined?

Figure 1. Region of interest in M82. The background X-ray image is from Matsumoto et al. (2001); M82 X-1 is near the center. Star clusters from Table 3 of McCrady et al. (2003) are indicated by circles and squares, the latter marking the positions of the two star clusters MGG-9 and MGG-11. Magnified near-infrared images of these clusters from McCrady et al. are presented at the upper right (MGG-11) and lower left (MGG-9).

McCrady et al. (2003) have made accurate measurements of the bulk parameters of several clusters in the central regions of M82. Figure 1 shows a composite X-ray (Matsumoto et al.) and near-infrared (McCrady et al.) map of the few hundred parsecs around MGG-11. The relative positional accuracies of both the X-ray and the infrared observations are better than 1 arcsecond. However, the absolute pointing accuracy is much poorer, for both telescopes. Although apparently off-center, the positions of the bright X-ray source and the star cluster MGG-11 are in fact consistent with one another (D. Pooley 2004, private communication). Curiously, the brighter, more massive, and apparently
coeval cluster MGG-9, lying just a few arc seconds from MGG-11 in projection, shows no X-ray emission.

The clusters MGG-9 and MGG-11 have quite similar ages, in the range 7–12 Myr (McCrady et al. 2003). The line-of-sight velocity dispersion of MGG-11 ($\sigma_r = 11.4 \pm 0.8$ km/s) is somewhat smaller than that of MGG-9 ($\sigma_r = 15.9 \pm 0.8$ km/s). Combining these numbers with the projected half-light radii, 1.2 pc for MGG-11 and 2.6 pc for MGG-9, McCrady et al. estimate total cluster masses of $\sim (3.5 \pm 0.7) \times 10^5 M_{\odot}$ for MGG-11, and about four times higher for MGG-9. The mean density of MGG-9 is just under half that of MGG-11, raising a second question: Are such seemingly small differences in cluster parameters sufficient to explain the presence of a $> 350 M_{\odot}$ IMBH in MGG-11 and the absence of a similar object in MGG-9?

Portegies Zwart et al. (2004a; PZBHMM) have addressed this issue using detailed N-body simulations. Starting with MGG-11, they first demonstrate that IMBH formation is a natural outcome of that cluster’s dynamical evolution, and then go on to show that the same processes would have failed to create a runaway in MGG-9. Their calculations were carried out using two independently developed N-body codes, Starlab (see Portegies Zwart et al. 2001) and NBODY4 (Aarseth 1999, Baumgardt 2003). Initial conditions for the model clusters were chosen so that at the present time they have mass functions, luminosities, half-mass radii and velocity dispersions in agreement with the McCrady et al. observations.

Since the initial and the current central densities of both clusters are unknown, the concentration parameter $c$ (the logarithm of the ratio of the tidal radius to the core radius) is treated as a free parameter controlling the initial central density of the models. PZBHMM find that, for $c > 2$ (which for “King” 1966 models is equivalent to a dimensionless central potential $W_0 \geq 9$) the MGG-11 models show runaway growth via repeated collisions. The mass-segregation time scale of a $50 M_{\odot}$ star in MGG-11 is $t_s \sim 4$ Myr (Eq. 9). Thus, massive stars in MGG-11 can easily reach the center of the cluster before leaving the main sequence. Given the high central density of MGG-11, once those stars have accumulated in the center, a runaway collision is inevitable, leading to IMBHs with masses in the range 800–3000 $M_{\odot}$. No episode of runaway growth occurs in the MGG-11 models with $c < 2$, nor in any of the MGG-9 simulations, regardless of initial concentration. In MGG-9, $t_s \gtrsim 15$ Myr even for 100 $M_{\odot}$ stars, so mass segregation cannot occur in the time available and no runaway is seen.

Figure 2 presents a representative sample of results from a number of simulations performed for a broad range of cluster parameters. It shows the growth in mass of the star that will ultimately become the most massive object in the cluster. Following detailed supernova calculations by Heger et al. (2003), stars having masses greater than 260 $M_{\odot}$ are assumed to collapse to black holes without significant mass loss. The stellar evolution models for stars with masses between 50 and 1000 $M_{\odot}$ are based on work by Stothers & Chin (1997) and Ishii et al. (1999). The quantitative differences between the simulations performed with Starlab and those using NBODY4 are due mainly to the different radii (and hence cross sections) assumed for very massive stars in those two packages.

The solid and dashed curves in the figure show the runaway mass as a function of time for a Salpeter (1955) IMF with a lower limit of $1 M_{\odot}$ and $c \approx 2.1$
5. Summary and Discussion

Rapid mass segregation in a dense star cluster leads to an effective core collapse on a time scale \( \sim 0.2 r_{\text{rh}} \) for typical initial mass functions. This in turn can lead to a runaway series of collisions in the cluster core and the possible formation of a \( \sim 1000 M_{\odot} \) IMBH there. We therefore expect an association be-

\((W_0 = 9)\) and \( c \approx 2.7 \) \((W_0 = 12)\). The dash-dotted curves are for two models with \( W_0 = 9 \) with an upper limit to the IMF of 50\( M_{\odot} \), instead of the standard 100\( M_{\odot} \) used in the other calculations; these runs were terminated at the moment the runaway star exploded as a supernova. The dash-3-dotted curve shows the result for \( W_0 = 12 \) with a Salpeter IMF, with 10% of the stars in primordial binaries—any tendency of these systems to arrest core collapse is effectively offset by the larger collision cross sections of the binary components. Finally, the dotted curve shows results for \( W_0 = 9 \) and a Kroupa (2001) IMF with a minimum mass of 0.1\( M_{\odot} \), in a simulation of 585,000 stars.
tween ultraluminous X-ray sources and the cores of dense young star clusters. A leading candidate for such an association is M82 X-1, which appears to lie in the massive young cluster MGG-11. On the basis of N-body simulations and elementary considerations of the time scale on which massive stars sink to the cluster center, we can readily explain why MGG-11 might host an IMBH while its more luminous neighbor MGG-9 does not. High initial central concentrations are required in order for this process to operate even in MGG-11, but we note that all of the “local” clusters listed in Table 1 of Portegies Zwart et al. (2004b) are in fact very centrally condensed.

Of course, it must be conceded that next to nothing is known about the detailed evolution and ultimate fate of stars hundreds or thousands of times more massive than the Sun, so we should perhaps not take too seriously the predictions of a \(2000M_\odot\) “star” in some of our simulations. Nevertheless, the simulations described here do make it clear that the hearts of these dense stellar systems can easily produce conditions suitable for repeated stellar collisions. The collision runaway at the center of such a system should be extremely luminous and eminently observable during its short lifetime. Observations of the cores of dense young star clusters in our Galaxy and beyond may thus shed light on the structure and lifetimes of such ultramassive stellar objects.

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