Abstract—A polar-coded transmission (PCT) scheme with joint channel estimation and decoding is proposed for channels with unknown channel state information (CSI). The CSI is estimated via successive cancellation (SC) decoding and the constraints imposed by the frozen bits. SC list decoding with an outer code improves performance, including resolving a phase ambiguity when using quadrature phase-shift keying (QPSK) and Gray labeling. Simulations with 5G polar codes and QPSK show gains of up to 2 dB at a frame error rate (FER) of $10^{-4}$ over pilot-assisted transmission for various non-coherent models. Moreover, PCT performs within a few tenths of a dB to a coherent receiver with perfect CSI. For Rayleigh block-fading channels, PCT outperforms an FER upper bound based on random coding and within one dB of a lower bound.

Index Terms—polar codes, fading channel, blind estimation, non-coherent communication, pilot-assisted transmission

I. INTRODUCTION

THE communication setting where channel state information (CSI) is not available at the transmitter or receiver is known as non-coherent communication [1] Ch. 10.7]. A common approach to address the lack of CSI is to embed pilot symbols in the transmitted symbol string, have the receiver estimate the CSI based on the pilots, and use the estimated CSI to decode. This approach is called pilot-assisted transmission (PAT) [2] with mismatched decoding [3, Ex. 5.22], [4]–[8].

PAT has two disadvantages for short block lengths: mismatched decoding reduces reliability and pilot symbols reduce rate significantly at low to moderate signal-to-noise ratio (SNR) [6], [7], [9]–[11]. Both problems can be partially mitigated with sophisticated signal processing. For instance, one may use iterative channel estimation and decoding [12]–[18], or two-stage algorithms that consider pilot symbols as part of the codebook [19], [20], or even maximum likelihood (ML) decoding. Nevertheless, there is a fundamental performance degradation due to using pilot symbols [9].

We propose a pilot-free two-stage polar-coded transmission (PCT) scheme to jointly estimate the CSI and data with an adjustable complexity that can be made comparable to PAT. In the first stage, successive cancellation list (SCL) decoding and the polar code constraints are used to estimate the CSI. In the second stage, mismatched SCL decoding proceeds with this estimate. Gains of up to 2 dB are shown at a frame error rate (FER) of $10^{-4}$ as compared to classic PAT schemes for several non-coherent settings.

A related method to estimate CSI uses the parity-check constraints of a low-density parity-check (LDPC) code [21], [22]. However, SCL decoding of polar codes naturally provides soft estimates of frozen bits. Moreover, polar codes are usually used with a high-rate outer code [23], [24] that can resolve CSI ambiguities, e.g., the phase ambiguity when using quadrature phase-shift keying (QPSK) and Gray labeling [21]. Of course, one may consider outer codes for LDPC codes as well. Other low-complexity methods for non-coherent channels are described in, e.g., [21], [25]–[28]. We remark that our focus is on QPSK but the ideas extend to higher-order modulations. One may also combine PAT and PCT to optimize performance.

This paper is organized as follows. Sec. II introduces notation, the system model, polar codes, and PAT. Sec. III describes our joint channel estimation and decoding algorithm. Sec. IV demonstrates the effectiveness of the method for short polar codes concatenated with an outer cyclic redundancy check (CRC) code and QPSK. Sec. V concludes the paper.

II. PRELIMINARIES

Uppercase letters, e.g., $X$, denote random variables and lowercase letters, e.g., $x$, denote their realizations. The probability distribution of $X$ evaluated at $x$ is written as $P_X(x)$ or $P(x)$ when the argument is the lower-case version of the random variable. We similarly treat densities $p_X(x)$ or $p(x)$. For $a \leq b$ we write $x_{a:b}$ for the row vector $(x_a, \ldots, x_b)$. Lower case bold letters, e.g., $x$, also denote row vectors. Capital bold letters, e.g., $X$, denote random vectors. All-zeroes and all-ones vectors are denoted as $0$ and $1$, respectively. The notation $x_{0:a}$ refers to the element-wise bit-flipped version of a binary vector $x_{a:0}$. We write $[N] = \{1, \ldots, N\}$ and use calligraphic letters, e.g., $S$, for sets otherwise. A subvector $x_S$ of $x_N$ is formed by appropriately ordered elements with indices in $S$. The cardinality of $S$ is denoted as $|S|$. We write $\|\cdot\|$ for the $l_2$-norm and $\langle\cdot,\cdot\rangle$ for the inner product of two vectors. Finally, $\mathbb{F}^\otimes m$ refers to the $m$-fold Kronecker product of a matrix $\mathbb{F}$ where $\mathbb{F}^\otimes 0 = 1$.

A. System Model

Consider a scalar block-fading channel, i.e., the fading coefficient $H$ is constant for $n_c$ channel uses and changes independently across $B$ coherence blocks, resulting in a frame size of $n = Bn_c$ symbols. The channel output of the $i$th coherence block is

$$y_i = h_i x_i + z_i, \quad i = 1, \ldots, B$$

where $x_i \in \mathbb{A}^{n_c}$ and $y_i \in \mathbb{C}^{n_c}$ are the transmitted and received vectors, $h_i \in \mathbb{C}$ is a realization of $H$, and $z_i$ is an
additive white Gaussian noise (AWGN) term whose entries are independent and identically distributed (i.i.d.) as $CN(0, 2\sigma^2)$. Neither the transmitter nor the receiver knows $h_i$ or even the probability distribution of $H$. We assume that the noise variance $2\sigma^2$ is known to the receiver; this may be justified by the slow time scale of receiver device variations as compared to fading due to mobility. A vector without subscripts denotes a concatenation of vectors or scalars, e.g., $y = (y_1, \ldots, y_B)$ and $h = (h_1, \ldots, h_B)$.

Consider QPSK with Gray labeling. The input alphabet is $X = \{\pm \Delta \pm j\Delta\}$, $\Delta > 0$, and we map the binary vector $c^2m$ to $x^m_1 \in X^{2m}$ via $\chi : \{0, 1\}^{2m} \rightarrow X^{2m}$ as

$$\chi(c^2m) = (\chi_g(c_1, c_2), \chi_g(c_3, c_4), \ldots, \chi_g(c_{2m-1}, c_{2m}))$$

where $\chi(c^m) = (-1)^{c_1}\Delta + j(-1)^{c_2}\Delta$. The mapping (2) is symmetric, i.e., if $\chi(c^2m) = x$ then $\chi(c'^2m) = -x$.

### B. Polar Codes

A binary polar code of block length $n$ and dimension $K$ is defined by a set $A \subseteq \{0, 1\}$ of indices with $|A| = K$ and the matrix $\mathbb{F}^{\otimes \log_2 n}$, where $n$ is a positive integer power of 2 and $\mathbb{F}$ is the binary Hadamard matrix $[29]$. Encoding is performed as $c_1^N = u_1^N \mathbb{F}^{\otimes \log_2 n}$, where the input vector $u_1^N$ has $K$ uniform information bits $u_A$ and $N - K$ frozen bits $u_{\bar{A}} = 0$ with $F = \{\mathbb{N}\} \setminus A$. A polar code is designed by storing the indices of the most reliable bits under SC decoding in the set $A$ $[29]$, [30]. In this work, we use the channel quality independent beta-expansion construction $[31]$.

An successive cancellation (SC) decoder estimates the bit $u_i$ at decoding stage $i$ as $\hat{u}_i = 0$ if $i \in F$, and otherwise

$$\hat{u}_i = \arg \max_{u_i \in \{0, 1\}} p_{Y_i|U_i} (y, u_i^{-1}|u_i)$$

where the probabilities are approximated recursively by assuming that $U_j$, $i < j \leq N$, are i.i.d. uniform random bits $[29]$. Both encoding and SC decoding can be implemented with complexity $O(N \log_2 N)$ $[29]$.

SC decoding with list size $L$ runs $L$ instances of an SC decoder in parallel $[23]$. Each instance has a different hypothesis on the decoded information bits $\hat{u}_i^{-1}$ at decoding stage $i$, called a decoding path. After decoding stage $N$, the decoder outputs the hypothesis of the most likely path as the estimate $\hat{u}_i^N$. An SCL decoder can be implemented with complexity $O(LN \log_2 N)$ $[23]$.

Polar codes perform significantly better when combined with an outer CRC code $[23]$. Decoding proceeds as follows: An SCL decoder for the inner polar code produces a list of codewords. The outer decoder discards those not fulfilling the constraints of the outer code. The decoder puts out the most likely of the remaining codewords if there is at least one, and it declares a frame error otherwise. For classic AWGN channels, these modified polar codes are competitive under SCL decoding for short block lengths $[32]$.

### C. Pilot-Assisted Transmission

Consider PAT as shown in Fig. 1 where the first $n_p$ symbols in each coherence block are pilot symbols $x_p^i$ and the remaining $n_d = n_c - n_p$ symbols $x_d^i$ are coded. To keep the overall rate fixed, the $(N, K)$ code is punctured by using quasi-uniform puncturing (QUP) $[33]$ so that the code length after puncturing is $N_{\text{punc}} = N - 2Bn_p = 2Bn_d$ with QPSK. The pilot and coded symbols have the same energy. Upon observing $y$, an ML estimate of the CSI is $\hat{h}_i = (y^t_i, x^t_i)/\|x^t_i\|^2$.

A mismatched decoder uses $\hat{h} = (\hat{h}_1, \ldots, \hat{h}_B)$ to compute the bit-wise log-likelihood ratios (LLRs) that are fed to the SCL decoder, leading to a codeword estimate.

### III. Joint Channel Estimation and Decoding

This section presents a low-complexity joint channel estimation and decoding scheme for polar codes. We do not use pilot symbols, i.e., we have $n_p = 0$ and $x_i = x_d^i$. A random interleaver $\Pi$ permutes the encoded bits $c^i_1$ and is followed by the mapping (2). The channel model is (7).

Let $h_i = r_i e^{j\theta_i}$, where $r_i \in [0, \infty)$ and $\theta_i \in [0, 2\pi)$, $i \in [B]$. We begin by estimating the amplitudes $r_i = |h_i|$ as

$$\hat{r}_i = \left(\sqrt{2\Delta}\right)^{-1} \frac{1}{n_c} \|y_i\|^2 - 2\sigma^2, \quad i = 1, \ldots, B. \tag{3}$$

Let $\beta$ be a number of input bits, and let $A^{(\beta)} = A \cap [\beta]$ and $F^{(\beta)} = F \cap [\beta]$ be sets of information and frozen indices among the first $\beta$ input bits $u_1^\beta$. We use the polar code constraints to estimate the phase as

$$\text{arg max}_{\theta_1, \ldots, \theta_B} \prod_{i=1}^{B} \frac{p_Y|U_i|H}(y, u, h) \tag{4}$$

where $h_i = \hat{r}_i e^{j\theta_i}$, $i \in [B]$. The sum in (4) can be computed by SCL decoding up to decoding stage $[F^{(\beta)}]$ with a list size $L_c = 2^{|A^{(\beta)}|}$. To reduce complexity at the expense of accuracy, one can approximate the calculation with SCL decoding and $L_c$ satisfying $1 \leq L_c < 2^{|A^{(\beta)}|}$. In fact, simulations in Sec. 1V show that small list sizes such as $L_c = 8$ give FER curves close to those of the coherent receiver.

**Remark 1.** The search space in (4) grows exponentially in the number of diversity branches $B$. There are several approaches to reduce complexity and we consider only the symmetry of the likelihood function due to the channel (1) and mapping (2) that halves the search space. We further adopt a coarse-fine search $[21], [34]$ as an efficient optimizer.

**Lemma 1.** Polar-coded modulations with the mapping (2) and the channel (1) have a sign ambiguity for the channel coefficients, i.e., for all $y$, $u$ and $u_1^{N-1}$, we have

$$p_{\hat{y}|U_1^{N-1}, H}(y, 0, h) = p_{\hat{y}|U_1^{N-1}, H}(y, 1, -h) \tag{5}$$

Figure 1. A PAT frame structure with $B = 2$ coherence blocks. Dark and white boxes represent pilot and coded symbols, respectively.
Proof. For all $x, y, h$ and $s \in \{-1, +1\}^B$, we have
\[
p(y|x, h) = \prod_{i=1}^{B} p_{Y_i|X_i, H_i}(y_i|s_i x_i, s_i h_i)
\]
as $s^2 = 1$. Recall that $c^N_1 = \Pi^{-1}(\chi^{-1}(x))$ so that $c^N_1 = \Pi^{-1}(\chi^{-1}(x))$. By choosing $s = -1$, we have
\[
p_{Y|C, H}(y|c^N_1, h) = p_{Y|C, H}(y|c^N_1, -h).
\]
(5)

Let $u^N_1$ be the vector such that $c^N_1 = u^N_1 x^\otimes_1$. We have $c^N_1 = (u^N_1, u^N_2 x^\otimes_2)$ because the last row of $X_{\otimes_2}$ is $1$.

Lemma 4 implies that if a polar code is considered for (1), then the decoder cannot resolve the ambiguity on bit $u_N$. This ambiguity occurs for any binary linear block code that has a generator matrix with an all-ones row, which is reflected in the bit $u_N$ for polar codes.

**Theorem 1.** Polar-coded modulations with the mapping $u_i \rightarrow \pi(u_i)$ satisfy
\[
p(y|u_1^N, h) = p_{Y|U_1^N, H}(y|u_1^N, -h)
\]
for all $y, h$ and $u_1^N, i \in [N - 1]$.

Proof. For $i \in [N - 1]$, we have
\[
p(y|u_1^N, h) \overset{(a)}{=} \sum_{u_{i+1}^N} P(u_{i+1}^N) p(y|u_1^N, h)
\]
\[
\overset{(b)}{=} \sum_{u_{i+1}^N} P(u_{i+1}^N) \left[ \sum_{u_N} \frac{1}{2} P(y|u_1^N, h) \right]
\]
\[
\overset{(c)}{=} \sum_{u_{i+1}^N} P(u_{i+1}^N) \sum_{u_N} \frac{1}{2} p_{Y|U_1^N, H}(y|u_1^N, -h)
\]
\[
\overset{(d)}{=} \sum_{u_{i+1}^N} P(u_{i+1}^N) p_{Y|U_1^N, H}(y|u_1^N, -h)
\]
where step (a) follows by the law of total probability and the mutual independence of $U_1^N, U_{i+1}^1$ and $H$; steps (b) and (d) follow by rearranging the sums and noting that $U_N$ is uniform; step (c) follows by Lemma 4.

**Corollary 1.** Polar-coded modulations with the mapping $u_i \rightarrow \pi(u_i)$ and the channel (1) satisfy
\[
p_{Y|U_1^N, H}(y|0, h) = p_{Y|U_1^N, H}(y|0, -h)
\]
for all $y$ and $h$.

Proof. We expand
\[
p_{Y|U_1^N, H}(y|0, h) \overset{(a)}{=} \sum_{u_A} P(u_A) p(y|u_1^N, h)
\]
\[
\overset{(b)}{=} \sum_{u_A} P(u_A) p_{Y|U_1^N, H}(y|u_1^N, -h)
\]
where step (a) follows by the law of total probability and mutually independent $U_A^N, U_{j\otimes 1}$ and $H$; step (b) follows by Theorem 4.

**Remark 2.** An outer code with a minimum distance of at least two can resolve the phase ambiguity.

**IV. NUMERICAL RESULTS**

This section provides Monte Carlo simulation results to compare the performance of PAT and PCT. The SNR is expressed as $E_s/N_0$, where $E_s$ is the energy per symbol and $N_0$ is the single-sided noise power spectral density. The inner code is a $(128, 38)$ polar code and the outer code is a $(64, 14)$ CRC code with generator polynomial $x^6 + x^4 + 1$, resulting in a $(128, 32)$ code. For the QPSK modulator (2) we have $n = 8Bn_c = 64$ channel uses and an overall rate of $R = 0.5$ bits per channel use (bpcu). For PAT, the $(128, 32)$ code is punctured to obtain $Bn_p$ pilot bits in total, resulting in a $(128 - 2Bn_p, 32)$ code. All curves shown in the figures below are for SCL decoding with a list size of $L = 8$ after estimating the CSI. The optimization (4) uses a coarse-fine search with 8 levels in both the coarse and fine search parts (34). The performance is compared for various estimator parameters $\beta$ and $L_e$ and to the coherent receiver with perfect CSI. No puncturing is required for the coherent receiver. As discussed below, the gains of our scheme are similar for $B \in \{1, 2\}$ and with or without fading.

**A. Single Coherence Block ($B = 1$)**

Consider the channel (1) with $B = 1$, $r_1 = 1$, and uniformly distributed phase $\Theta_1 \sim U(0, 2\pi)$. Fig. 2 compares PAT and PCT. The best PAT performance for the FERs of interest was achieved with $n_p = 14$, i.e., 14 pilot symbols gave the lowest SNR for FERs ranging from $10^{-2}$ to $10^{-4}$ in Fig. 2. For smaller $n_p$ the quality of the channel estimate limits performance, and for larger $n_p$ the puncturing weakens the polar code and limits performance.
PCT performs within 0.3 dB of the receiver with perfect CSI if the estimator is run with $L_e = 8$ and up to the last frozen bit with $\beta = 113$. It thereby outperforms PAT by about 1.5 dB at a FER of $10^{-4}$. Observe that if the estimator is run up to the last frozen bit before the first information bit, i.e., $\beta = 47$, then the performance is worse than for PAT. The parameters $\beta = 113$ and $L_e = 1$ provide a good trade-off between complexity and performance when combined with a second-stage SCL decoding with a list size $L = 8$.

Table I compares the number of visited nodes per frame in the polar decoding tree along with the FER at $E_s/N_0 = 1$ dB. Each visited node corresponds to an input bit (including the frozen bits) visited by the algorithm [35, Remark 4]. For PCT, we state the sum of the number of nodes visited by the estimator and the number of nodes visited by the decoder. The number of visited nodes with PAT and perfect CSI is thus the same. Observe that PCT with $\beta = 113$ and $L_e = 1$ visits a similar number of nodes as PAT with a list size $L = 32$ (the difference is less than 10%) and it reduces the error probability by one order of magnitude. We remark that measuring the complexity by the number of visited nodes is pessimistic for PCT since most of the visited nodes are frozen bits. Hence, simplified SC decoders [36, 37] can significantly reduce complexity.

B. Two Coherence Blocks ($B = 2$)

We next consider $B = 2$ coherence blocks. Fig. 3 shows the FER for $r_i = 1$ and $\Theta_i \sim U(0, 2\pi), i \in \{1, 2\}$. Fig. 4 shows the FER for a Rayleigh block-fading channel with $H_i \sim CN(0, 1), i \in \{1, 2\}$. The best performance for PAT was achieved with $n_p = 7$ pilot symbols per coherence block for both cases. Observe that, in both cases, PCT outperforms PAT by about 2 dB at a FER $\approx 10^{-4}$. Moreover, PCT approaches the performance of a coherent receiver with perfect CSI.

Table I

| Method | FER | Visited Nodes |
|---|---|---|
| PAT ($n_p = 14, L = 8$) | $8.43 \times 10^{-4}$ | 631 |
| PAT ($n_p = 14, L = 32$) | $3.16 \times 10^{-3}$ | 2223 |
| PCT ($\beta = 47, L_e = 1, L = 8$) | $3.36 \times 10^{-2}$ | 1383 |
| PCT ($\beta = 61, L_e = 8, L = 8$) | $3.20 \times 10^{-3}$ | 2151 |
| PCT ($\beta = 113, L_e = 1, L = 8$) | $3.50 \times 10^{-4}$ | 2439 |
| PCT ($\beta = 113, L_e = 8, L = 8$) | $1.00 \times 10^{-4}$ | 8807 |
| Perfect CSI ($L = 8$) | $2.40 \times 10^{-5}$ | 631 |

V. CONCLUSIONS

A PCT scheme was proposed that estimates CSI via SCL decoding and the constraints imposed by the frozen bits. An outer code improves reliability and resolves phase ambiguities. Simulation results show that PCT significantly outperforms PAT schemes with a similar complexity and approaches the performance of a coherent receiver.

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Figure 4. Performance of PAT and PCT for a Rayleigh block-fading channel and $\beta = 2$. A (128, 32) polar code was used with QPSK and the overall rate is $R = 0.5$ bpcu. SCL decoding uses a list size of $L = 8$ for all cases.

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