Associated production of $J/\psi$-mesons and open charm and double open charm production at the LHC

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Theoretical predictions of cross sections and properties of the $J/\psi$-meson production in association with an open charm hadron and formation of two open charm hadrons from two $c\bar{c}$ pairs in the LHC conditions are presented. Processes in both single and in double parton scattering mechanisms are included into consideration. Special attention is paid to the kinematic limits of the LHCb detector for which comparison with the newest experimental data is carried out.

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I. INTRODUCTION

In recent work of the LHCb collaboration [1] data on the double $J/\psi$-meson production at 7 TeV energy is presented. At first blush value of the double $J/\psi$ production cross section reported $(5.1 \pm 1.1 \, \text{nb})$ is accordant within uncertainty limits with the predictions obtained in the leading order (LO) QCD calculations [2, 3]. These calculations lead to the total cross section value of $10 \div 27 \, \text{nb}$ and to $3 \div 5 \, \text{nb}$ in the kinematic limits of the LHCb detector.
It is well known that such calculations include big uncertainties connected with the hard scale selection, next to the LO (NLO) contributions and allowing for the relative motion of \(c\)-quarks in the \(J/\psi\)-meson. It is known that the last of this factors increases cross section of double quarkonia production in \(e^+e^-\)-annihilation in several times \([4–7]\).

Apart from uncertainties in the partonic cross section of double \(J/\psi\) production a new problem arises in the LHC conditions. The hadronic cross section appears to be three orders of magnitude higher than the cross section of the partonic subprocess. This phenomenon due to the high luminosity of low-\(x\) gluons with fraction of proton momenta of about \(10^{-4} \div 10^{-3}\), which contribute most to the processes in question. Such an enhancement gave rise to discussion of double parton interactions in a single \(pp\)-collision (DPS) \([8–11]\) with independent production of particles considered in each interaction. In works \([12–14]\) mechanism of double \(J/\psi\) production in DPS approach was considered and it was shown that DPS can give significant contribution to the channel in question in the LHCb detector conditions.

Although SPS\(^1\) and DPS models predict somewhat different kinematic distributions for the \(J/\psi\) pairs produced, the question if enhancement of statistics gained allows to distinguish this mechanisms remains open. On the other hand at least in the LO there is a qualitative difference between predictions for the \(J/\psi + \chi_c\) production obtained in the SPS and DPS models.

Moreover, additional DPS contribution should obviously express itself in other channels of the four \(c\)-quark domain: in the associated production of \(J/\psi + D^2\) and in the four \(D\)-meson production\(^3\). In the beginning of 2012 first LHCb results for the channels listed were presented \([15]\). It is interesting to understand the interplay of the SPS and DPS mechanisms in these channels. Currently there are estimations of cross sections of the SPS processes contributing these final states in the LO perturbative QCD formalism \([16–21]\). In the current work we review results obtained in the LO perturbative QCD for the SPS contribution and estimate DPS contributions for the channels mentioned.

\(^1\) We will address formation of considered final states in a single parton interaction as SPS.

\(^2\) In the following we will refer to the \(J/\psi + D\) production for the production of \(J/\psi\) and a \(c\bar{c}\) pair, from which at least one \(c\)-quark hadronize into an observed open charm hadron.

\(^3\) In the following we will refer to the four \(D\)-meson production for the production of a \(c\bar{c}c\bar{c}\) configuration, from which at least two \(c\)-quarks hadronize into observed open charm hadrons.
II. FOUR HEAVY QUARK PRODUCTION IN THE SINGLE GLUON-GLUON INTERACTION

One of the first research, in which the possibility to observe four heavy quark production at colliders was discussed is \[16\]. In this work cross sections of subprocesses $gg \rightarrow Q_1 \bar{Q}_1 Q_2 \bar{Q}_2$ and $q\bar{q} \rightarrow Q_1 \bar{Q}_1 Q_2 \bar{Q}_2$ was estimated within LO perturbative QCD approach for the kinematical conditions of the LHC and SSC.

Slightly later the analogous processes in which quark and antiquark are bind in a doubly-heavy meson were investigated. Production of the $S$-wave $B_c$-meson in the $gg \rightarrow B_c + b + \bar{c}$ and $q\bar{q} \rightarrow B_c + b + \bar{c}$ processes was estimated in the works \[22\]–\[29\]. Calculation of the $P$-wave $B_c$-meson production cross section was done in the studies \[30\]–\[32\]. These researches continued in the works \[33\]–\[35\]. Associated $J/\psi$ and $D$-meson production, as well as $\Upsilon$ and $B$-meson production, was also estimated within the same technique in \[17\]–\[21\].

It is worth to note that doubly heavy baryon production implies production of two heavy quarks. Therefore, assuming that the doubly heavy baryon is created in the heavy diquark hadronization, one can study the doubly heavy baryon production by analogy with the $B_c$-meson production \[17\]–\[36\]–\[39\].

Calculations show that gluon-gluon interactions provide the main contribution into the four heavy quark production in the LHC experiments. Quark-antiquark annihilation amounts to about 10%. That is why production in the gluon-gluon interactions is mainly discussed in this paper.

Usually calculations are made under an assumption that initial gluons are real and their transverse momenta are negligible (the collinear approach). To simulate real distribution over the transverse momenta of initial gluons in our studies we use the Pythia 6.4 MC generator \[40\]. In this connection it is worth noting researches \[18\]–\[19\] where transverse momenta and virtualities of gluons are taken into account in the framework of the $k_T$-factorization approach.

III. PAIR PRODUCTION OF $J/\psi$-MESONS IN THE LHCB DETECTOR

Production of two charmonia in SPS can be described within perturbative QCD by the fourth order in $\alpha_S$ Feynman diagrams. For the $J/\psi$-mesons pair formation invariant masses
and quantum numbers \((1-^-)\) of two \(c\bar{c}\) pairs are fixed.

Cross section of the hard subprocess of two \(c\bar{c}\) pair formation in the color-singlet \((1_C)\) state with \(m_{c\bar{c}} \approx m_{J/\psi}\) is proportional to\(^4\)

\[
\dot{\sigma}(gg \rightarrow J/\psi J/\psi) \sim \frac{\alpha_s^4 |\psi(0)|^4}{m_{J/\psi}^8},
\]

where \(|\psi(0)|\) is the value of the \(c\bar{c}\) wave function in the \(J/\psi\)-meson at the origin. Emergence of this factor due to the approximation in which momenta of \(c\) and \(\bar{c}\) quarks are parallel and their relative momentum is neglected in the matrix element of the subprocess in question \((\delta\)-approximation). At large invariant masses of the \(J/\psi\)-meson pair cross section \(^1\) decreases with the rise of the full energy squared \(\hat{s}\) as

\[
\dot{\sigma}(gg \rightarrow J/\psi J/\psi) \sim \frac{\alpha_s^4 |\psi(0)|^4}{\hat{s}^4}.
\]

Numerical result of \(4.1 \pm 1.2 \text{ nb} [2]\) derived in the assumptions listed was obtained using the hard subprocess scale equal to the transverse mass of one of the \(J/\psi\)-mesons produced and using the CTEQ5L proton pdfs [43]. As mentioned above this value agrees within uncertainty limit with the experimental value of \(5.1 \pm 1.1 \text{ nb}\) measured in [1]. Variation of the hard scale from the one half to two transverse masses of the \(J/\psi\)-meson produced changes the cross section value from 5.1 nb to 3 nb. If CTEQ6LL pdfs [44] are used, cross section has maximum at the scale of about one transverse mass of \(J/\psi\) and amounts to 3.2 nb. Cross sections are less at both half and double scales and are 2.8 nb and 2.6 nb respectively. Presence of extremum due to the fact that with rise of the scale strong coupling constant decreases while gluon density grows. As well as in the manuscript [2] we include contribution from the production and decay of the \(\psi(2S)\) state into the \(J/\psi\)-mesons yield.

Fig. 1 shows distributions over the invariant mass of the \(J/\psi\) pairs calculated within the assumptions mentioned for the different hard scale choices and pdf sets in comparison with the experimental data reported by LHCb in [1]. One can see that the shape of the distributions predicted is nearly the same. What concerns experimental distribution, it looks tilted to the bigger invariant mass values. We would like to notice that this fact can be accounted for by the relative \(c\)-quarks motion in the \(J/\psi\)-meson. With this aim we calculated cross section of the process in question averaged by some “duality” region of the

\(^4\) For the precise expression see [2, 41, 42].
Figure 1: Distribution over the invariant mass of the $J/\psi$-meson pairs compared with the LHCb measurement. Solid curves were obtained with $m_{T}^{J/\psi}$ as the hard scale, dashed — $2 \cdot m_{T}^{J/\psi}$ and dotted — $0.5 \cdot m_{T}^{J/\psi}$. For every scale choice upper curve corresponds to the CTEQ5L, lower — to the CTEQ6LL pdf used.

c$\bar{c}$ invariant mass:

$$\hat{\sigma}^{\text{dual}}(gg \rightarrow J/\psi(\psi')J/\psi(\psi')) \approx \int \int_{2m_{c}}^{2m_{D} + \Delta} \frac{d^{2}\sigma(gg \rightarrow (c\bar{c})_{1C}^{S=1} + (c\bar{c})_{1C}^{S=1})}{dm_{c\bar{c}1} dm_{c\bar{c}2}} dm_{c\bar{c}1} dm_{c\bar{c}2}, \quad (3)$$

where $m_{D}$ is the $D$-meson mass. The $c$-quark mass was taken equal to

$$m_{c} = 1.25 \text{ GeV}.$$  

The $\Delta$ parameter can be selected in such a way that the value of the pair production cross section obtained coincides with the total production cross section of the $J/\psi J/\psi$, $J/\psi \psi'$ and $\psi' \psi'$ final states calculated in the $\delta$-approximation. If one takes $\sqrt{s}/2$ for the hard scale of the subprocesses considered the correspondence is reached at $\Delta = 0.3$ GeV:

$$\hat{\sigma}^{\text{dual}}(gg \rightarrow J/\psi(\psi')J/\psi(\psi'), \Delta = 0.3 \text{ GeV}) \approx 4.4 \text{ nb.}$$

At $\Delta = 0.5$ GeV the cross section estimated in the duality approach is close to the value
Figure 2: Distribution over the invariant mass of the $J/\psi$-meson pairs in the “duality” approach compared with the LHCb measurement. Solid curve was obtained with $\Delta = 0.5$ GeV, dashed — with $\Delta = 0.3$ GeV and dotted — in the $\delta$-approximation.

reported by the LHCb experiment:

$$\sigma^{\text{dual}}(gg \rightarrow J/\psi(\psi')J/\psi(\psi'), \Delta = 0.5 \text{ GeV}) \approx 5.8 \text{ nb.}$$

Increase of $\Delta$ leads to the growth of the total cross section on the one hand, and improves agreement in the $m_{J/\psi J/\psi}$ distribution on the other (Fig. 2).

In the LHC environment huge density of low-$x$ gluons leads to the increase of the multiple gluon-gluon interactions probability within one proton-proton collision. In the DPS approach, which implies production of particles concerned in two independent subprocesses, the cross section is written down as following:

$$\sigma_{AB}^{\text{DPS}} = m \frac{\sigma_{\text{SPS}}^{A} \sigma_{\text{SPS}}^{B}}{\sigma_{\text{eff}}}. \tag{4}$$

where the $\sigma_{\text{eff}} = 14.5$ mb parameter was measured in the four jets and three jets plus photon modes by the CDF and D0 detectors [45, 46]. The $m$ parameter equals 1 for identical subprocesses and 2 for different. For the $J/\psi$ pairs production in the LHCb conditions expression (4) leads to the following cross section value:

$$\sigma_{\text{DPS}}^{pp \rightarrow J/\psi J/\psi + X} = 4 \text{ nb.} \tag{5}$$
Figure 3: Distribution over the difference of the azimuthal angles of the $J/\psi$-mesons produced. Curve designations coincide with Fig. 1.

Known inclusive production cross section of the $J/\psi$ meson in the LHCb kinematic limits, $\sigma_{J/\psi} = 10.5 \, \mu b$, was used. In the work [14] authors note that the DPS contribution can be located at bigger $J/\psi$ pair invariant masses than the SPS one.

One of the proposed methods to distinguish the DPS signal from the SPS one is to study correlations between azimuthal angles of two mesons produced or between their rapidities [13, 14]. However analysis involving modelling in the Pythia generator shows that correlations presenting in collinear approach completely go out when including transverse momenta of the initial gluons into consideration (Fig. 3). To be more precise, depending on the scale choice collinear or anticollinear directions of the $J/\psi$ momenta can dominate. Moreover, at the standard scale of one $J/\psi$ transverse mass the relative angle correlation is absent at all. It is the model implemented in the Pythia generator which is completely responsible for the distribution over the transverse momenta of the initial gluons, so model-independent prediction on the $\Delta \phi$ distribution can not be derived. Investigation of the rapidity correlation appears to be more fruitful. In spite of narrowness of LHCb rapidity window ($2.0 < y < 4.5$) it appears to be sufficient to test QCD predictions which state that the difference in rapidity between the $J/\psi$-mesons produced does not exceed 2 units of rapidity (Fig. 4). At the current stage DPS predicts no correlations between products of
two partonic interactions at all.

Apart from the correlation studies investigation of the $P$-wave state contributions to the total $J/\psi$ production can be fruitful. Indeed, section rules emerging in the CS LO pQCD consideration \[47\] imply significant limitation on the final states which can appear in the gluon fusion process. According to the $C$-parity conservation occurrence of the $C$-odd $J/\psi\chi_C$ final state should be suppressed in the SPS. As in DPS formation of charmonia occurs independently, it does not have any suppression in this channel. That is why DPS should dominate in the $J/\psi\chi_C$ state production (possibly followed by the $\chi_C \to J/\psi\gamma$ decay). It was pointed out in the work \[13\] that similar situation takes place in the $J/\psi\Upsilon$ mode in which SPS and DPS lead to different hierarchy of the $pp \to J/\psi J/\psi$, $pp \to J/\psi\Upsilon$ and $pp \to \Upsilon\Upsilon$ cross sections.

As it was noted, we have taken into account contribution of the $pp \to J/\psi\psi(2S)$ and $pp \to \psi(2S)\psi(2S)$ processes followed by the $\psi(2S) \to J/\psi X$ decays to the total $J/\psi$ pairs yield. $\psi(2S)$-mesons originating from these processes can also be detected by a leptonic decay, just like $J/\psi$. It is interesting to compare ratios of $J/\psi J/\psi$ and $J/\psi\psi(2S)$ yields predicted by SPS and DPS models.

Both in SPS and in DPS approaches ratio of different meson pair yields can be estimated
using values of the $c\bar{c}$ wave functions in the charmonia at the origin:

$$\sigma(pp \to J/\psi J/\psi) : \sigma(pp \to J/\psi(2S)) : \sigma(pp \to \psi(2S)\psi(2S)) \approx \psi_{J/\psi}(0)^4 : 2 \cdot \psi_{J/\psi}(0)^2 \psi_{\psi(2S)}(0)^2 : \psi_{\psi(2S)}(0)^4 \approx 1 : 1 : 0.3.$$ (6)

where $\psi_{J/\psi}(0) = 0.21 \text{ GeV}^{3/2}$, $\psi_{\psi(2S)}(0) = 0.16 \text{ GeV}^{3/2}$. A more accurate estimate which allows for different meson masses leads to the relation

$$\sigma(pp \to J/\psi J/\psi) : \sigma(pp \to J/\psi(2S)) : \sigma(pp \to \psi(2S)\psi(2S)) \approx 1.7 : 1 : 0.15.$$ (7)

Accounting contributions from the $\psi(2S)$ decays in the channels discussed one gets finally

$$\sigma(pp \to J/\psi J/\psi) : \sigma(pp \to J/\psi(2S)) : \sigma(pp \to \psi(2S)\psi(2S)) \approx 2.2 : 1 : 0.13.$$ (8)

What concerns DPS, using inclusive $J/\psi$ and $\psi(2S)$ production cross sections equal to $10.5 \mu b$ [48] and $1.88 \mu b$ [49] respectively, one gets

$$\sigma(pp \to J/\psi J/\psi) : \sigma(pp \to J/\psi(2S)) : \sigma(pp \to \psi(2S)\psi(2S)) = \frac{\sigma_{J/\psi}^2}{\sigma_{J/\psi}^2} \cdot 2 \cdot \frac{\sigma_{\psi(2S)}^2}{\sigma_{\psi(2S)}^2} \cdot \frac{\sigma_{\psi(2S)}^2}{\sigma_{\psi(2S)}^2} = 2.8 : 1 : 0.9.$$ (9)

It can be seen that DPS predicts slightly larger suppression of the $J/\psi\psi(2S)$ production compared to SPS. The main reason of it is that inclusive $J/\psi$ production cross section already includes contribution from the $\chi_C$ decays, which can amount up to $20 \div 30\%$ [50, 51]. If one excludes expected contribution of $\chi_C$ decays by taking $J/\psi$ production cross section equal $0.8 \times 10.5 \mu b = 8.4 \mu b$, then DPS prediction amounts to

$$\sigma(pp \to J/\psi J/\psi) : \sigma(pp \to J/\psi(2S)) : \sigma(pp \to \psi(2S)\psi(2S)) = \frac{\sigma_{J/\psi}^2}{\sigma_{J/\psi}^2} \cdot 2 \cdot \frac{\sigma_{\psi(2S)}^2}{\sigma_{\psi(2S)}^2} \cdot \frac{\sigma_{\psi(2S)}^2}{\sigma_{\psi(2S)}^2} = 2.2 : 1 : 0.11.$$ (10)

Up to the uncertainties this relation coincides with the SPS prediction [8]. Uncertainties in the cross section ratios predicted by DPS can be estimated by the largest relative uncertainty
in the measurement of the cross sections involved. This uncertainty is maximum for the \( \psi(2S) \) measurement and reaches 20\% \[49\]. Unfortunately difference between relation \([9]\), which suspects feed-down from the \( \chi_{C} \) production, and relations \([8]\), \([10]\), which do not, is of the same order. Nonetheless it would be interesting to measure ratio of the \( J/\psi J/\psi \) and \( J/\psi \psi(2S) \) yields experimentally.

### IV. ASSOCIATED PRODUCTION OF \( J/\psi \) AND \( D \) MESON IN THE LHCB DETECTOR

To compare predictions for the \( gg \to J/\psi c\bar{c} \) and \( gg \to c\bar{c} c\bar{c} \) processes with experiment some model of the \( c \)-quark transition into a specific hadron should be used. The most common hardonization model is based on the assumption that charm hadron moves approximately in the same direction that the initial \( c \) quark does and obtains some fraction \( z \) of the quark momentum with the probability \( D_{c \to H}(z) \) (so called fragmentation function). At the scale of about \( c \)-quark mass the mean \( z \) value is about 0.7. Two following parametrizations are used in our calculations: the standard parametrization of Pythia 6.4 and a pQCD motivated parametrization of BCFY \[52\] with the parameter values obtained in \[53\].

It is worth mentioning here that as it was shown in \[54–58\], there are models in which hadronization is not described by simple fragmentation. For example, it is reasonable to suppose that \( c \)-quark can pull a light quark from the sea without loosing any momentum. In this case it can be formally assumed that \( D_{c \to H}(z) = \delta(z) \). Moreover it can be supposed that in some cases the final hadron momentum is even larger (by a quantity of about \( \frac{m_{q}}{m_{c}} p_{c} \)) than the initial \( c \)-quark momentum.

All mentioned possibilities have been considered in the present estimations of the cross section values. Nevertheless it should be stressed that these estimations are too rough to give preference to some particular hadronization model.

Recently cross section value of the associated production of \( J/\psi \) together with a \( D \)-meson has been measured by the LHCb collaboration for the following kinematical region \[15\]:

- \( J/\psi \) meson is produced in the rapidity region \( 2.0 < y_{J/\psi} < 4.0 \);

- one charmed hadron is produced in rapidity region \( 2.0 < y_{D} < 4.0 \) and has transverse momenta \( 3 \text{ GeV} < p_{T}^{D} < 12 \text{ GeV} \).
Calculations within the LO of pQCD lead to the cross section value $20 \div 60$ nb depending on the scale choice and the $c$-quark hadronization model [17, 18, 20, 21] (the scale value was varied from $\sqrt{s}/4$ to $\sqrt{s}$). Nevertheless, as it was shown in paper [27], interaction of the sea $c$-quark from one proton with gluon from the other can essentially contribute to the $J/\psi$-meson and $c$-quark associated production, i.e. the subprocess $cg \to J/\psi c$ should also be taken into account, as well as the main subprocess $gg \to J/\psi c\bar{c}$. It is natural for such an approach, that problems connected with double counting and non-zero $c$-quark mass essentially impede an accurate estimation of the calculation uncertainties. It can be assumed that this method is already valid at the transverse momenta of the charmed hadron $p_T^D > 3$ GeV $\approx 2m_c$ and that interference contributions are small. Also one can try to avoid double counting by subtracting the part due to the direct gluon splitting from the total $c$-quark structure function:

$$\tilde{f}_c(x, Q^2) = f_c(x, Q^2) - \frac{\alpha_s(Q^2)}{4\pi} \int_x^1 \frac{dz}{z} \left[ \left( \frac{x}{z} \right)^2 + \left( 1 - \frac{x}{z} \right)^2 + \frac{2m_c^2(x - z)}{z^2 Q^2} \right] f_g(z, Q^2),$$  \hfill (11)  

where splitting function is taken from [59].

The cross section value of the subprocesses $cg \to J/\psi c$ was found to be about $10 \div 40$ nb. Therefore the contribution of such corrections to the $J/\psi + D$ associated production is of the same order as the contribution of the main subprocess $gg \to J/\psi c\bar{c}$. Thus the calculations within pQCD lead to the cross section value of about $30 \div 100$ nb for the $J/\psi + D$ production in the LHCb fiducial region. It should be noticed that in contrast to the charmonia pairs production in the associated charm production there are no $C$-parity selection rules. So one should expect not only feed-down from the $J/\psi + \psi(2S)$ production but also from the $J/\psi + \chi_C$ one. This contributions can increase observed $J/\psi + D$ cross section by up to 50%.

In the Fig. 5 cross section distribution over the $J/\psi$ meson transverse momentum in the $gg \to J/\psi + c\bar{c}$ subprocess is shown in comparison with the LHCb experimental data. The $d\ln \sigma/dp_T$ distributions are plotted, i.e. spectra are normalized to unity. Both $J/\psi$ and associated charmed hadron produced in the events plotted are limited to the LHCb fiducial region. It can be seen that at least in the high $p_T^{J/\psi}$ region the predicted slope is in a good agreement with the experimental data. It should be noticed that $p_T^{J/\psi}$ distribution in the

$^5$ Form now on summation with charge conjugate mode is implied.
Figure 5: Distribution over the transverse momentum of the $J/\psi$-meson in the $J/\psi + D$ production compared with the LHCb measurement (points for $J/\psi$ produced together with $D^0$ or $D^+$-meson are shown). Solid curves were obtained with the hard scale value of $1 \cdot m_{J/\psi}$, dashed — $2 \cdot m_{J/\psi}$ and dotted — $0.5 \cdot m_{J/\psi}$. Dot-dashed curve corresponds to the collinear gluon approximation.

inclusive $J/\psi$ production measured by LHCb exhibits significantly more rapid decrease with the $p_{T}^{J/\psi}$ growth.

Cross section distribution over the $D$-meson transverse momentum for the same $gg \to J/\psi + c\bar{c}$ subprocess is given in the Fig. 6 and demonstrates good agreement with the LHCb measurement. As in the previous figure, both spectra are normalized to unity. In contrast to the $J/\psi$ signal, both predicted and measured spectra are similar to those in the inclusive $D$-meson production at LHCb [48].

As in double $J/\psi$ production, essential angle and rapidity correlations in the $gg \to J/\psi c\bar{c}$ process are predicted by pQCD. Within collinear approach $J/\psi$ and $D$ mesons move in the opposite directions in most cases. However no concrete prediction can be made when taking into account transverse gluon motion in the framework of the Pythia generator as the distribution is highly sensitive on the scale selection (see Fig. 7).

What concerns distribution over the rapidity difference between $J/\psi$ and $D$-meson produced, from Fig. 8 one can see that contrary to the two $J/\psi$-meson production, LHCb rapidity window appears to be too narrow to observe rapidity correlations predicted in the $gg \to J/\psi c\bar{c}$ subprocess. We omit discussion of correlations between $D$ and $\bar{D}$ mesons in the
Figure 6: Distribution over the transverse momentum of the $D$-meson in the $J/\psi + D$ production compared with the LHCb measurement (points for $D^0$ and $D^+$-mesons are shown). Curve designations coincide with Fig. 5.

$J/\psi + D$ associated production as LHCb analysis focuses on events in which one co-produced $D$ meson is observed.

The cross section value obtained by LHCb collaboration,

$$\sigma^{\exp}(pp \to J/\psi + D^0(D^+, D_s^+, \Lambda_c^+) + X) \approx 300 \text{ nb},$$

is several times larger than the SPS prediction of $30 \div 100 \text{ nb}$.

Let us now address to the simultaneous production of $J/\psi$ and open charm in two gluon-gluon interactions. Within the DPS approach cross section of the associated $J/\psi$ and $D$-meson production can be expressed as follows:

$$\sigma_{J/\psi D}^{\text{DPS}} = \frac{\sigma_{J/\psi} \sigma_D}{\sigma_{\text{eff}}},$$

where $\sigma_{J/\psi}$ and $\sigma_D$ are cross sections of the inclusive $J/\psi$ and $D$-meson production in the LHCb acceptance correspondingly. Recalculated for the fiducial region discussed ($2 < y < 4$, $p_T^D > 3 \text{ GeV}$), these values are $9 \mu\text{b}$ and $380 \mu\text{b}$ respectively [48, 60]. As always summation with the charge conjugate state is assumed. Unpublished cross section of the $\Lambda_c$ inclusive production is not included in consideration. Thus the associated $J/\psi$ and $D$ meson production cross section for the LHCb kinematical region within the DPS model can
Figure 7: Distribution over the difference of the azimuthal angles of the $J/\psi$ and $D$-meson in the $J/\psi + D$ production. Curve designations coincide with Fig. 5.

be estimated as

$$\sigma_{J/\psi D}^{DPS} = 240 \text{ nb}.$$  \hfill (14)

As earlier, numerical value of $\sigma_{\text{eff.}} = 14.5 \text{ mb}$ \hfill [45, 46] was used. One can see that DPS prediction is several times larger than the SPS one and within uncertainty limits agrees with the experimental value \hfill (12).

V. FOUR D-MESON PRODUCTION IN THE LHCB DETECTOR

In the same LHCb studies \hfill [15] production of four $c$-quarks is investigated. Events in which two open charm hadrons both containing $c$-quark (or both containing $\bar{c}$-quark) are produced in the fiducial region $2.0 < y < 4.0, 3 \text{ GeV} < p_T < 12 \text{ GeV}$ were selected.

The calculation within LO of QCD in SPS approach gives for this kinematical region cross section value of

$$\sigma_{\text{pQCD}}^{\text{bQCD}}(gg \to c\bar{c}c\bar{c}) \sim 50 \div 500 \text{ nb}$$

depending on the scale selection and the $c$-quark hadronization model used.

There is an indication that interaction with sea $c$-quarks contribute essentially into this process, as well as into the associated production of $J/\psi$ and $c$. According to our preliminary
Figure 8: Distribution over the difference of rapidities of the $J/\psi$ and $D$-meson in the $J/\psi + D$ production. Solid curve corresponds to the LHCb kinematic limits imposed, dashed — to the absence of kinematic limits.

estimation, cross section value for the process $cg \rightarrow cc\bar{c}$ (plus charge conjugate) is about

$$\sigma^{pQCD}(cg \rightarrow cc\bar{c}) \sim 200 \div 500 \text{ nb}$$

depending on the scale selection and the $c$-quark hadronization model used. The $c$-quark structure function has been taken in the form (11).

The interactions between two sea $c$-quarks can also be considered. Our estimations show that this process can give a contribution comparable to the two processes mentioned above:

$$\sigma^{pQCD}(cc \rightarrow cc) \sim 40 \div 200 \text{ nb}.$$ 

Thus one can conclude that predictions obtained in the LO pQCD within SPS approach underestimate the experimental value of about 3$\mu$b [15]. Also it is worth mentioning that the experimental spectra shapes also can not be exactly reproduced.

Nevertheless some futures of the experimental spectra can be understood from such calculations using different kinematical cuts. For example the local minimum near 6 GeV in the experimental cross section distribution over the invariant mass $m_{cc}$ of two charmed particles is probably connected with the cut on the minimum transverse momenta at the LHCb data
are assumed independent.

Also the rapid decrease of the cross section at \( m_{cc} > 20 \) GeV can be explained by cut on the maximum transverse momenta:

\[
m^{\text{cut}}_{cc} \approx 2p_{T}^{\text{max}}.
\]

Let us now turn to the DPS contribution to the different \( D \)-meson pairs production. Expression (4) has to be modified as experimentally observed quantities are inclusive production cross sections of particular types of \( D \)-mesons summed together with anti-mesons of the same type. These cross sections can be written down as follows:

\[
s^{\text{incl.}}_{i} = \sigma_{1}p^{c\bar{c}} + \sigma_{2}(2p^{c\bar{c}} - (p^{c\bar{c}})^{2}),
\]

where \( \sigma_{1} \) and \( \sigma_{2} \) are cross sections of one and two \( c\bar{c} \) pair production in a single proton-proton collision respectively and \( p^{c\bar{c}} \) is probability that \( c \) or \( \bar{c} \) quark transits into detected hadron of type \( i \).

In the following we will be interested in events in which both \( c \) and \( \bar{c} \) quarks form two \( D \)-mesons of particular type in the detector acceptance, or it is done by pairs of identical quarks — \( cc \) or \( \bar{c}\bar{c} \). In the first case cross section of the \( i \) type meson pair production can be written down as

\[
s^{\text{diff.}}_{i,i} = \sigma_{1}p^{c\bar{c}} + \sigma_{2}(2p^{c\bar{c}} - (p^{c\bar{c}})^{2} + (p^{c\bar{c}} - p^{c\bar{c}})^{2}/2),
\]

and in the second — as

\[
s^{\text{same}}_{i,i} = \sigma_{2}(p^{c\bar{c}})^{2} + 2(p^{c\bar{c}})(p^{c\bar{c}} - p^{c\bar{c}}) + (p^{c\bar{c}} - p^{c\bar{c}})^{2}/2).
\]

Here \( p^{c\bar{c}} \) stands for the probability for \( c \) and \( \bar{c} \) quarks from one pair to transit into mesons of type \( i \) and \( j \) observed in the detector and probabilities for quarks from the different pairs are assumed independent.

For the different \( i \) and \( j \) types of mesons analogous quantities are written down as

\[
s^{\text{diff.}}_{i,j} = \sigma_{1}p^{c\bar{c}} + \sigma_{2}(2p^{c\bar{c}} - (p^{c\bar{c}})^{2} + 2p^{c\bar{c}}(p^{c\bar{c}} - p^{c\bar{c}}) + 2p^{c\bar{c}}(p^{c\bar{c}} - p^{c\bar{c}}) + (p^{c\bar{c}} - p^{c\bar{c}})(p^{c\bar{c}} - p^{c\bar{c}} - p^{c\bar{c}} - p^{c\bar{c}}))(18),
\]

\[
s^{\text{same}}_{i,j} = \sigma_{2}(0.5(p^{c\bar{c}})^{2} + 2p^{c\bar{c}}(p^{c\bar{c}} - p^{c\bar{c}}) + 2p^{c\bar{c}}(p^{c\bar{c}} - p^{c\bar{c}}) + (p^{c\bar{c}} - p^{c\bar{c}})(p^{c\bar{c}} - p^{c\bar{c}}))(19).
\]
Table I: Cross sections of different $D$-meson pairs production compared with the LHCb results.

| Mode     | $\sigma_{\text{th.}}^{\text{diff.}}, \mu b$ | $\sigma_{\text{exp.}}^{\text{diff.}}, \mu b$ | $\sigma_{\text{th.}}^{\text{same}}, \mu b$ | $\sigma_{\text{exp.}}^{\text{same}}, \mu b$ |
|----------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|
| $D^0D^0$| $7.2 \pm 1.1$                          | $6.2 \pm 0.6$                          | $0.53 \pm 0.2$                         | $0.69 \pm 0.07$                         |
| $D^0D^+$| $6.0 \pm 0.9$                          | $4.0 \pm 0.4$                          | $0.4 \pm 0.1$                         | $0.52 \pm 0.08$                         |
| $D^0D^+_s$| $2.3 \pm 0.4$                      | $1.7 \pm 0.2$                          | $0.16 \pm 0.05$                        | $0.27 \pm 0.05$                         |
| $D^+D^+$| $1.2 \pm 0.2$                          | $0.78 \pm 0.11$                        | $0.087 \pm 0.029$                     | $0.08 \pm 0.02$                         |
| $D^+_sD^+_s$| $0.97 \pm 0.15$                     | $0.55 \pm 0.08$                        | $0.066 \pm 0.022$                     | $0.07 \pm 0.02$                         |
| $D^+_sD^+_s$| $0.19 \pm 0.03$                   | $—$                            | $0.013 \pm 0.005$                     | $—$                                     |

To solve the equations adduced we will use known inclusive production cross sections of particular $D$-meson types \[48\]. As LHCb collaboration presents these cross sections in bins of rapidity and transverse momenta, they can be recalculated into the fiducial region discussed ($2 < y < 4, 3\text{GeV} < p_T^D < 12\text{GeV}$). We will also assume that the total $c\bar{c}$ production cross section in the $7\text{ GeV}$ proton-proton collisions is known. It was obtained using the Pythia generator calibrated by known inclusive open charm production cross sections in the LHCb acceptance and is equal to $6.1 \pm 0.9\text{mb}$ \[48\]. According to the DPS approach, cross section of two $c\bar{c}$ pairs production in a single proton-proton scattering is given by expression \[4\]:

$$
\sigma_2 = \frac{\sigma_1^2}{2\sigma_{\text{eff.}}} = 1.3 \pm 0.4 \text{mb.}
$$

(20)

However until $\sigma_{i,j}^{\text{diff.}}$ or $\sigma_{i,j}^{\text{same}}$ cross sections are measured there is no sufficient information to derive the $p_{i,j}^{c\bar{c}}$ probabilities. So we will assume that rather rigid kinematic cuts imposed result in the smallness of probability to observe both particles produced from a $c\bar{c}$ pair in the detector. Then neglecting double counting one can write down

$$
p_{i,j}^{c\bar{c}} \approx (p_{i}^{c\bar{c}})^2, \quad p_{i,j}^{c\bar{c}} \approx 2p_{i}^{c\bar{c}}p_{j}^{c\bar{c}}. \tag{21}
$$

Under the assumptions listed equations \[16\] — \[19\] can be solved. Obtained cross sections of pair production of $D^0$, $D^+$ and $D^+_s$ mesons are given in Table I together with the values measured by the LHCb. We would like to stress one more time here that summation with the charge conjugate states is everywhere assumed. Generally speaking, good agreement between the DPS predictions and the experimental results is observed. Nonetheless, it is mentioned in \[15\] that $p_T$-spectra of $D$-mesons in pair production significantly differ from
those in inclusive open charm production, while similar $p_T$-behaviour could be expected in the DPS model.

VI. CONCLUSION

It is well known that the particle production multiplicity increases with the energy of hadronic interactions. Therefore phenomenon of multiple production should be observed for charmed and beauty particles as well, but at the higher energies due to the larger masses. At the LHC energy yield of charm particles (6.1 mb \cite{48}) is comparable to the common light particle yields, so production of two, three and so on pairs should be expected as well as single $c\bar{c}$ pair production. Recently the first data on the four $c$-quark production in the proton-proton interactions have been obtained by the LHCb Collaboration \cite{15}.

From the theoretical point of view processes in single gluon-gluon interactions (such as $gg \rightarrow c\bar{c}c\bar{c}$) are the natural source of multiple charm production. The calculations within LO of pQCD in SPS approach had been done earlier for the process of $J/\psi$ pair production \cite{2,41,42}, $J/\psi + c\bar{c}$ associated production \cite{16,21}, and for the four $c$-quarks production.

The main conclusion to be drawn from these theoretical studies and from the recent LHCb results is that SPS model used together with the LO pQCD can not describe all the data on multiple charm production. The presented analysis shows that only data on $J/\psi$ pair production is in satisfactory agreement with SPS LO pQCD predictions. The predictions obtained for the $J/\psi + D$ associated production, as well as for the four $D$-meson production underestimate the experimental data in several times. As alternative model we consider the simplest model of double parton scattering (DPS). In the frame work of this approach it is assumed that two $c\bar{c}$ pairs are produced independently in two different partonic collisions. DPS predictions on the cross section values fairly agree with the experimental data. As it was shown in \cite{61}, cross section of pair charm production becomes equal to the ordinary $c\bar{c}$ cross section at the energy of about 20 TeV.

It is interesting to note, that for the double $J/\psi$ production predictions of SPS and DPS models are fairly close, for the $J/\psi + D$ associated production the DPS prediction exceeds the SPS one in several times and for the four $D$-meson production excess is even higher. At first glance it seems amazing as an attempt to explain advantage of the DPS model by combinatorial factor only does not lead to distinction in the channels discussed.
Infinitesimality connected with the $\alpha_S$ constant is same for both SPS and DPS: in SPS the factor is $\alpha_S^4$ and in DPS $-\alpha_S^2 \times \alpha_S^2 = \alpha_S^4$. From our point of view the reasonable explanation lies in the different phase volumes for the SPS and DPS production: in SPS final state contains three particles for the $J/\psi+c\bar{c}$ production and four for the $c\bar{c}cc\bar{c}$ production, so differential cross sections of these processes peak at the larger $\sqrt{s}$ values at the expense of phase volume factors. By-turn this leads to the smaller gluon luminosity as compared to the $2 \rightarrow 2$ processes which take place in the DPS model.

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