Modeling, simulation and experimental research for MEMS cantilevers of complex geometry

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Abstract. The fundamental resonant frequencies for MEMS cantilevers of complex geometry (paddle-shaped rectangular microbeam, homogeneous on a part of length and nonhomogeneous, layered structure to the wider part of the beam) are calculated. A method of analytical calculation using the Mohr-Maxwell theory is proposed for homogeneous microcantilevers, which is then adapted for non-homogeneous structures. The analytical model has been validated by numerical simulation using finite element method (FEM). The experimental validation has been made using laser-Doppler vibrometry (LDV) by scanning with the Polytec MSA-500 system.

1 Introduction

Microcantilevers are miniaturized mechanical structures widely spread in MEMS (Micro-Electro-Mechanical Systems) devices as sensors and actuators [1]. Applications that have enabled nanotechnology such as atomic force microscopy, chemical and biological particles detection, actuation for optical switching or micromanipulators, and, also, measurement of mechanical properties of different materials, even living cells, rely especially on microcantilevers [2-6].

Different geometries were analyzed (rectangle or paddle shape cantilever, cantilevers with full or inner cut rectangle/triangle/trapeze, a.s.o.) and various materials were used (mono- and polycrystalline silicon, silicon dioxide, silicon nitride or polymers like SU-8), and processed through bulk or surface micromachining, based on the crystalline anisotropy property of a silicon wafer or the sacrificial layer technique, specific for MEMS technologies and, also, by photolithography and etching processes.

The application of a cantilever determines its design, the used materials, the operating characteristics and the fabrication method [7, 8]. Increased uses of microcantilevers for mass sensing applications are due mainly to high sensitivity of these structures. There are two types of operating methods: static deflection methods, which are based on the fact that the adsorbed (substance) mass induces a stress gradient into the structure that causes deformations.

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(deflections) of the beam; resonant (dynamic) detection methods relying on the change of the sensing system mass (by attachment of extraneous matter) or on the combined modification of mass and rigidity (as the case is of a layer – like deposition), which produces a variation in the fundamental resonant frequency (usually bending or torsion).

This paper aims at determining the fundamental resonant frequencies for MEMS cantilever beams of complex geometry, with variable width (paddle-shaped rectangular microbeam), homogeneous - made only by silicon on a part of length and non-homogeneous, layered structure - structure of more thin films deposited on silicon substrate to the wider part of the beam having possible applications as gas sensors. More objectives of research have been identified, as follows:

- proposing an analytical calculation method for such microbeams, a less laborious method of calculation than those presented in other works [9, 10]. The calculation formulas proposed by customizing lead to the relationships established in literature for homogeneous microbeams;
- the performed study, useful in pre-sizing of such structures, is a continuation of the authors’ preoccupations regarding the calculation of microbeams layered along their entire length [7, 8];
- the model of calculation is validated through numerical analysis with FEM, and experimentally, by LDV method, using MSA-500 from Polytec. The experimental investigations have been performed on homogeneous structures made from silicon dioxide layer released from the substrate (the silicon wafer) through plasma and wet etching processes;
- the proposed calculation relationships are used to study the influence of structure’s geometry parameters on both the mass and the spring constant of microbeam, in order to choose the optimal configuration for an application of gas sensor.

The proposed analytical model is described in section 2. In section 3, is given the validation of the analytical calculations through numerical simulation with FEM, using SolidWorks/CosmosM software. Section 4 presents and discusses the results regarding the influence of structure’s geometry parameters on the values of resonant frequency. Section 5 presents the experimental results representing the fabrication process of silicon dioxide structures and the the resonant frequency measurement by LDV. In section 6, interesting conclusions regarding the analyzed MEMS structures have been inferred.

2 Analytical Computation Model

The under study structure, paddle-shaped rectangular microbeam and the geometrical parameters are represented in figure 1. In [9], it is used the method of integrating the differential equation of the deformed elastic line with the imposition of boundary and the continuity conditions for calculating the static displacement in the relation of the fundamental frequency determination of the homogeneous microbeam with step-variable widths (as in figure 1); in [10], the same result is achieved using the Castigliano method [11, 12]. The static displacement is:

\[ \delta_{st} = \frac{\partial U}{\partial P} \]  

where: \( U \) - the strain energy stored by the microbeam; \( P \) - the applied load.

Both methods are laborious.
Fig. 1. Sketch of the under study microbeam.

Fig. 2. Schemes for the analytical computation of the static deflection (displacement).

In this paper, it is proposed to use the Mohr-Maxwell (unit load) method [12], according to which the static displacement is calculated with the relationship:

$$\delta_{st} = \sum_{i=1}^{2} \int \frac{M_i(x) m_i(x) dx}{E_i I_{yi}(x)}$$

(2)

where: $M(x)$ is the bending moment in a certain section $x$, produced by the force $F$ (figure 2a); $m(x)$ is the bending moment in the same section, produced by an unit load that is applied in the section whose displacement is determined, on the direction of this displacement (figure 2b); $E_i I_{yi}(x)$ are the bending rigidities corresponding to the two microbeam sections, of lengths and widths $l_i, w_i$ and $l_2, w_2$, respectively (figures 2c and 2d).

$$E_1 I_{y1} = E_1 \frac{w_1 h^3}{12}; \quad E_2 I_{y2} = E_2 \frac{w_2 h^3}{12}.$$  

(3)

For homogeneous microbeams made from silicon, $E_1 = E_2 = E$ ($E$ – Young’s modulus of silicon); $I_{yi}(x)$ - the individual moments of inertia for the two sections.

According to the schemes from figures 2a and 2b, it results:

$$M(x) = -Fx; \quad m(x) = -x.$$  

(4)
Substituting (4) in (2), it results:

\[
\delta_{st} = \int_0^{l_2} \frac{(-Fx)(-x)\,dx}{EI_{y_2}} + \int_{l_2}^{l_2+l_1} \frac{(-Fx)(-x)\,dx}{EI_{y_1}}
\]

\[
\delta_{st} = \frac{F}{3} \left( \frac{l_1^3 + 3l_1^2l_2 + 3l_1l_2^2}{EI_{y_1}} + \frac{l_2^3}{EI_{y_2}} \right).
\]  

(5)

The spring constant of the microbeam is:

\[
k = \frac{F}{\delta_{st}}
\]

(7)

The first eigenfrequency (the fundamental one) results then with the relationship [13]:

\[
f = \frac{1}{2\pi} \sqrt{\frac{k}{0.24m}}
\]

(8)

where: 0.24 m is the effective mass of the microbeam, and

\[
m = \rho_{Si}h(w_1l_1 + w_2l_2)
\]

(9)

where: \(\rho_{Si}\) – silicon density.

Considering the expressions of the bending rigidities (3), it will result that, in case of a homogeneous microbeam, its spring constant is:

\[
k = \frac{Eh^3}{4} \left[ \frac{w_1w_2}{w_1l_2^3 + w_2l_1^3 + 3l_1l_2(l_1 + l_2)} \right].
\]

(10)

Thus it reaches the expression of the spring constant established in [9, 10], but using a simpler algorithm. In addition, the method can be extended also to non-homogeneous microbeams on certain parts, as it is proposed in this work. For example, on the part of length \(l_2\), the cross-section can be a non-homogeneous structure (as shown in figure 3), composed of silicon (the substrate), silicon dioxide layer and polymer SU-8 layer for adsorbing the gases. On this length, it will be determined an equivalent bending rigidity \(EI_2\), as in the case of the bars of non-homogeneous sections [11]. This equivalence was used by authors in [8] for a non-homogeneous microbeam on its entire length.

In order to determine \(EI_2\), it will be established first the position of the centroidal axis (\(z_G\) coordinate), figure 3:

\[
z_G = \frac{\sum_{i=1}^{N} E_i A_i z_i}{\sum_{i=1}^{N} E_i A_i}
\]

(11)
where: \( N \) – the number of layers in the composite cantilever, in this case \( N = 3 \); \( E_i, A_i \) – Young’s modulus and the cross-sectional area of each individual layer of the composite beam.

The bending rigidity of the microbeam on the non-homogeneous length is determined then with the relationship:

\[
EI_2 = \sum_{i=1}^{N} E_i I_i = \sum_{i=1}^{N} E_i \left[ \frac{w_2 \cdot h_i^3}{12} + A_i (z_i - z_G)^2 \right].
\]  

(12)

The spring constant of the microbeam, homogeneous – silicon (Si) structure on the length \( l_1 \) and non-homogeneous – layered structure (Si, SiO\(_2\) and polymer (SU-8)) on the length \( l_2 \) is:

\[
k = \frac{3}{l_1^3 + 3l_1^2 l_2 + 3l_1 l_2^2 + l_2^3} \cdot \frac{E_1 I_1}{EI_2}.
\]  

(13)

The first eigenfrequency will be determined with the equation (8), where:

\[
m = \rho_{Si} h (w_1 l_1 + w_2 l_2) + w_2 l_2 \left( \rho_{SiO_2} h_2 + \rho_{polymer} h_3 \right),
\]  

(14)

\( \rho_i \) – the density of the individual layers - silicon (the substrate), silicon dioxide layer and polymer SU-8 layer.

### 3 Finite Element Analysis and Validation of Analytical Model

In order to validate the analytical results, a finite element analysis has been made. The program used for the analysis was SolidWorks/CosmosM. For the homogeneous silicon microbeam two models were used: one of them, a 2D model, realized by meshing in thin shell elements (with 4 nodes per element - SHELL4) and the second one, a 3D model, using solid elements (with 8 nodes per element - SOLID8). The computing effort for the 3D model is significantly higher. Thus, for the homogeneous microbeam, 16,000 SHELL4 elements and 60,000 SOLID8 elements have resulted.
For the microbeam partly non-homogeneous (layered over the wider \( l_2 \) length) is useful 3D model, which allows a more elements meshing on the thickness of each layer (4 elements on the \( Si \) substrate thickness, 2 elements on the \( SiO_2 \) layer thickness and 3 elements on the \( SU-8 \) polymer layer thickness).

In figure 4, it is represented the variation of fundamental frequencies determined analytically and numerically with 2D and 3D FEM models, as a function of \( l_2/l_1 \) ratio. Multiple values of the geometrical parameters \( w_1, w_2 \) (beam widths) and \( l, l_1, l_2 \) (beam lengths) have been considered: \( w_1/w_2=0.5; \ w_2=100 \ \mu m; \ l=400 \ \mu m \) (figure 4a) and \( l=200 \ \mu m \) (figure 4b).

The results are compared both in case of homogeneous microbeams, from silicon (for which the analytical relationship is established in literature) and in case of non-homogeneous structures layered on the wider part, of length \( l_2 \) (for which a calculation method is proposed in this paper): thickness of silicon substrate, \( h_1=15 \ \mu m \); thickness of \( SiO_2 \) layer, \( h_2=1.7 \ \mu m \); thickness of \( SU-8 \) layer, \( h_3=4 \ \mu m \).

It was also studied the influence of the geometrical parameters on the frequency response of the complex microstructure, in order to use it as a gas sensor.

The physical (density, \( \rho \)) and elastic (Young’s modulus, \( E \) and Poisson’s ratio, \( \nu \)) properties of materials are given in table 1, chosen from literature, also studied by authors in previous works [7-8].

| Table 1. Material properties. |
|-------------------------------|
| Material | \( E \) [GPa] | \( \nu \) | \( \rho \) [kg/m\(^3\)] |
|-----------------|-----|-----|-------------|
| Silicon         | 165 | 0.22| 2330        |
| \( SiO_2 \)     | 70  | 0.17| 2200        |
| Polymer (SU-8)  | 4.4 | 0.22| 1190        |

The following matters have been observed:

1) The homogenous microbeams have been meshed using two models: 2D – SHELL4 elements and 3D – SOLID8 elements. Differences between the fundamental frequencies obtained with the two models are insignificant. Consequently, it was considered that the 3D model, necessary for microbeams partially non-homogeneous to allow multiple elements in the thickness of a layer, is satisfactory in terms of precision of the results.

2) The values of the fundamental frequencies analytically determined are higher than those calculated with FEM, both in the case of the homogeneous microbeams and the non-homogeneous ones. The differences can be explained by:
   a) the simplifying assumptions underlying the analytical modeling of microstructure as a bar, when it would be more realistic assumption of thin plate;
   b) the relation for the calculation of the fundamental frequency, Rayleigh’s method, which is approximate;
   c) FEM is an approximate method of calculation of structures, because it replaces a continuous medium with a discreet one. The discretization results in obtaining a structure less rigid (more flexible) than the actual structure, which leads to eigenfrequencies lower than the real ones.

3) The differences between analytical and numerical values of resonant frequency are of the same order of magnitude for the non-homogeneous microbeams as in case of the homogeneous ones, so the proposed calculation method is validated. The deviations of the numerical (FEM) models relative to the analytical ones are listed in table 2. For the same numerical model (3D SOLID), the deviations of resonant frequencies for non-homogeneous microbeams are bigger with 1.7 % to 3.8 % than for the homogeneous microbeams.
| l₂/l | Homogeneous microbeams | Non-homogeneous microbeams |
|------|------------------------|---------------------------|
|      | Dev. 2D SHELL model [%] | Dev. 3D SOLID model [%]   |
|      | Dev. 3D SOLID model [%] |
| 0.2  | 13.75                  | 13.80                     |
|      | 13.91                  | 14.18                     |
| 0.4  | 14.63                  | 15.05                     |
|      | 15.18                  | 15.47                     |
| 0.6  | 13.02                  | 13.08                     |
|      | 13.34                  | 13.72                     |
| 0.8  | 9.75                   | 9.82                      |
|      | 10.43                  | 9.91                      |

**4 Influence of structure’s geometry parameters on the values of resonant frequency**

The curves representing the frequency variation $f_1$ as a function of $l_2/l$ ratio (figure 4) have the same allure for both types of microbeams, non-homogeneous and, respectively, homogeneous, both the short microbeams ($l=200 \mu m$) and the long microbeams ($l=400 \mu m$): a downward curve in the range (0.2 – 0.6) and an upward curve in the range (0.6 – 0.8), so a minimum can be observed between 0.55 and 0.65. The observation is especially valid for the curves plotted based on the analytical calculation.

It should be noted that the influence of geometrical parameter $l_2/l$ (length of the layered part/ the total length of microbeam) on the fundamental frequency is not very high. In figure 5, the variations with $l_2/l$ ratio of the fundamental eigenfrequency $f_1$ [kHz], the spring constant $k$ [N/m] and the mass $m$ [μg] for a non-homogeneous microbeam ($l=400 \mu m; w_1/w_2=0.5; w_2=100 \mu m$), analytical calculation, have been shown. In order to represent sizes of the same order of magnitude, it was considered 100*$m$.

The followings were found:
- the mass $m$ increases linearly with $l_2/l$ ratio;
- the spring constant $k$ has a polynomial variation (the best approximation is a 4th degree polynomial) without extreme in the range (0.2 – 0.8);
- the fundamental frequency $f_1$ variation can be approximated best by a 3rd degree polynomial with an extreme easily determined by canceling the derivative, for example, in the studied case the eigenfrequency is minimum (99.69 kHz) for $l_2/l = 0.607$;
- the maximum value of frequency is 112.779 kHz for $l_2/l=0.2$, which means that indeed the geometrical parameter $l_2/l$ does not influence significantly the frequency. This is because for $l_2/l$ ratio with values ranging from 0.2 to 0.6, the mass increasing, which has as effect the frequency decreasing, is high while the spring constant increasing is slower. The spring constant increases then rapidly in the range (0.6 – 0.8), which leads to a slight increase in the frequency;
- it can be said that for the studied microbeam having $l=400 \mu m, w_1/w_2=0.5$ and $w_2=100 \mu m$, the configuration with $l_2/l = 0.607$, for which the eigenfrequency is minimum, is optimal in terms of the ratio between the spring constant and the mass, meaning that it will be the most sensible configuration with the mass variation.
Fig. 4. Variation of the fundamental eigenfrequencies computed analytically and numerically, as a function of $l_2/l$ ratio – (a) $l=400 \, \mu m$; $w_1/w_2=0.5$; $w_2=100 \, \mu m$, (b) $l=200 \, \mu m$; $w_1/w_2=0.5$; $w_2=100 \, \mu m$. 
Fig. 5. Influence of $l_2/l$ ratio on the fundamental eigenfrequency $f_1$, the spring constant $k$ and the mass $m$ for a non-homogeneous microbeam ($l=400 \, \mu m; \, w_1/w_2=0.5; \, w_2=100 \, \mu m$), analytical model.

An addition of mass $M$, through gas adsorption, will change the structure mass, so the eigenfrequency becomes:

$$f = \frac{1}{2\pi} \sqrt[2]{\frac{k}{0.24m + M}}$$

For an utilization as a gas sensor, it means it is better that $l_2/l$ ratio be in the range in which the frequency is minimum, because the additional mass resulted from the gas adsorption has the greatest influence on the fundamental eigenfrequency. It is an observation that appears also to Narducci M et al, [9], but only for homogeneous microbeams. We think that figure 5 from this paper better illustrates the observation, which remains valid for the non-homogeneous microbeams, too.

The geometrical parameter significantly influencing the fundamental frequency is the total length $l$ of microbeam, as it is shown in figure 4. The same thing is illustrated also in figure 6, where the influence of $l/w_2$ ratio is studied. It is found that the difference between the analytical and the numerical values decreases with increasing of $l/w_2$ ratio, which was expected, because for $l/w_2>5$ the bar hypothesis is acceptable. Decrease in the fundamental frequency $f_1$ with increase of $l/w_2$ is due mainly to a lower spring constant (microbeam is more flexible and the static deflection is higher).

In figure 7, the fundamental frequency $f_1$ variation depending on $l_2/l$, for different values of $w_1/w_2$, is represented. It is found that for different values $w_1/w_2$ at $l_2/l=0.6$ approximatively, a minimum appears. The fundamental frequency $f_1$ decreases with $w_1/w_2$ ratio.
Fig. 6. Influence of \( l/w_2 \) ratio on the fundamental frequency \( f_1 \) (\( w_1/w_2=0.5; l_2/l=0.6 \)).

Fig. 7. Influence of the geometrical parameters, \( l_2/l \) and \( w_1/w_2 \) ratios, on the fundamental frequency \( f_1 \) (\( l=400 \mu m; w_2=100 \mu m \)).

5 Experimental results

The experimental investigations have been performed on homogeneous structures made from silicon dioxide layer released from the substrate (the silicon wafer) through MEMS
technologies, meaning combined etching processes, in plasma and wet processes, respectively.

n–(111) silicon wafers of 3 inch diameter and 375 μm thickness were thermally oxidized in wet oxygen atmosphere to obtain a silicon dioxide (SiO₂) layer of about 1.7 μm thickness, used both as protective layer (mask) during etching process, and as constructional material for cantilevers after releasing from substrate (the silicon wafer).

The oxide layer was photo-etched by means of a standard photolithographic technique, followed by etching in an HF – solution. In a first stage, in the opened windows, the silicon was plasma etched through a Bosch process using a DRIE (Deep Reactive Ion Etching) – Plasmalab System 100-ICP (Inductively Coupled Plasma) to a depth of about 40 μm. Two steps were performed: (1) a passivation process to 100 sccm flow rate of C4F3, and (2) an etching process to 100 sccm flow rate of SF6; working pressure 10 mTorr at 15°C, power in RF 5 W, ICP power 700 W and etch rate of 2 – 3 μm/min. So, on the silicon surface from the vertical walls of the etched cavities, the access to the faster etching (110) crystal planes was allowed in order to perform a second etching stage (wet etching), based on the anisotropy of silicon. Potassium hydroxide (KOH) 40% at 80°C (lateral etch rate of about 100 μm/h along the <110> direction) was employed to release the SiO₂ cantilevers from the wafer surface.

More microbeams/chip, as a comb structure, have been performed (figure 8), in order to test different sizes and etching times. The measurement results are shown in figure 9 and table 3. Paddle width \( w_2 = 100 \) μm and length \( l_2 = 100 \) μm, as well as beam total length \( l = 200 \) μm were the same, while the beam width \( w_1 \) had the following values: 80 μm (I), 70 μm (II), 65 μm (III) and 50 μm (IV).

![Fig. 8. Photo of the SiO₂ microbeams, on the Leica DM2700 M optical microscope.](image)

![Fig. 9. Photo of the SiO₂ microbeams, on the Dino-Lite digital microscope (AM-311ST).](image)
Table 3. Measured microbeams data.

| The measured feature | Beam no. | Chip no. |
|----------------------|----------|----------|
| Paddle width, $w_2$ [µm] | 1 99 103 100 | 102 104 101 |
| | 2 99 103 96 | 103 102 100 99 |
| | 3 101 100 97 | 102 103 102 100 |
| | 4 101 100 97 | 102 103 102 100 |
| | 5 101 100 97 | 102 103 102 100 |
| | 6 101 100 97 | 102 103 102 100 |
| | 7 101 100 97 | 102 103 102 100 |
| | 8 101 100 97 | 102 103 102 100 |
| | 9 101 100 97 | 102 103 102 100 |
| | 10 101 100 97 | 102 103 102 100 |
| | 11 101 100 97 | 102 103 102 100 |
| | 12 101 100 97 | 102 103 102 100 |
| | 13 101 100 97 | 102 103 102 100 |
| | 14 101 100 97 | 102 103 102 100 |
| Paddle length, $l_2$ [µm] | 1 99 99 99 | 102 102 101 |
| | 2 104 106 100 97 | 102 103 103 103 |
| | 3 104 102 99 100 | 102 103 103 103 |
| Beam width, $w_1$ [µm] | 1 99 99 99 | 102 102 101 |
| | 2 71 74 67 68 72 70 | 72 70 72 72 71 72 70 68 |
| | 3 58 60 58 65 58 | 63 63 64 63 61 60 63 64 |
| Beam total length, $l$ [µm] | 1 207 200 203 | 204 203 206 |
| | 2 206 204 203 202 | 203 203 204 203 203 206 |
| | 3 206 204 203 204 204 | 206 204 204 204 207 203 203 203 |
| Anchor protuberance length [µm] | 1 50 50 45.9 | 56 47.3 50 |
| | 2 45 47.3 43 44.6 39 | 48.7 43.1 37.6 51 40.3 43.1 39 43.2 43.1 |
| | 3 14 15.4 8.5 12.6 11.2 | 9.8 12.6 9.8 9.1 7.7 7.1 7.7 9.7 |

A non-uniformity of micromachining, of about 3.4%, at level of the wafer, was observed. Beam anchor protuberances have appeared because of the incomplete etching time for the wider microbeams. So, the beam total lengths had to be corrected with this protuberance length, in the frequencies calculations.

In order to validate the theoretical and technological results, tests of determining the fundamental frequency have been made using laser-Doppler vibrometry by scanning with the MSA-500 Micro System Analyzer - Polytec. The Laser-Doppler Vibrometer (LDV) is a very precise optical transducer for determining the vibration velocity and displacement in a sample point. It works by sensing the frequency shift of back scattered light from a moving surface. By moving the measurement point to predefined positions, a Scanning LDV provides the full picture of a device’s out-of-plane vibrational behavior. There are no discrete frequencies at which measurements must be performed. Frequency data over the instrument’s bandwidth are available within milliseconds per sample point [14, 15]. The measurements were made on individual chips bonded onto a metal rigid plate of $12 \times 12 \times 5$ mm$^3$. The plate was fixed through an elastic double adhesive layer on a piezoelectric excitor consisting of a multilayer ceramics piezoelectric actuator mounted in an elastic pre-tensioned housing. As it was expected, high resonant frequencies have resulted. Figure 10 demonstrates geometry and 3D motion of the microcantilevers at the first (fundamental) vibration mode viewed on MSA-500.

In table 4, a comparison between the obtained results is presented. The experimental study has been made on SiO$_2$ homogeneous structures having a thickness comparable to that of thin films (1.7 µm). This fact leads to changes of magnitude order of the structures’ spring constant ($k$) and, consequently, of the ratio between the analytical and numerical values of resonant frequencies, $f_i$: the analytical values are lower than the numerical ones. The
Experimental values of resonant frequencies are situated between the analytical and numerical values, which validates them. Shortening of total length of microbeams with the anchor protuberance length diminishes the resonant frequencies and increase the difference between the numerical and analytical values \((l/w_2)\) ratio decreases). The geometrical parameters \(w_1/w_2\) and \(l_2/l\) influence the fundamental frequency in a similar way previously presented: the fundamental frequency \(f_1\) decreases with \(w_1/w_2\) ratio and the \(l_2/l\) ratio does not influence significantly the frequency, but it is important in finding the most sensible structure configuration with the mass variation. In case of beam no. IV, for which the anchor protuberance length is equal to zero, the values of these parameters, are most close to the theoretical ones.

![3D motion of SiO2 paddle microcantilever structure at the first (fundamental) vibration mode viewed on MSA–500.](image)

**Table 4.** Comparison of results.

| Beam no. /chip 14 | \(w_1/w_2\) | \(l_2/l\) | \(l/w_2\) | \(k\), [N/m] | \(f_1\), [Hz] \(f_1\), [Hz] \(f_1\), [Hz] |
|------------------|-------------|-------------|-------------|---------------|-----------------|-----------------|
|                  | analytical  | FEM         | LDV         |               |                 |                 |
| I                | 0.782       | 0.660       | 1.544       | 1.908         | 60761           | 86643           | 87970           |
| II               | 0.686       | 0.632       | 1.645       | 1.471         | 53960           | 80059           | 79910           |
| III              | 0.640       | 0.532       | 1.933       | 0.807         | 37643           | 50870           | 50700           |
| IV               | 0.504       | 0.490       | 2.000       | 0.564         | 32329           | 41506           | 41020           |

**5 Conclusions**

The analytical calculation model, proposed in this paper, has been validated through customization for homogeneous microbeams, by comparison to the relationships established in the literature. It means that the deducted relationships are more general and can be used for microbeams with different configurations.

The comparison between the results obtained by analytical calculation and numerical simulation with finite element method has also led to satisfactory values in terms of precision. The experimental values of resonant frequencies have validated the theoretical results and they have demonstrated the technological possibility of achieving.

For homogeneous microbeams, if the thickness is very small, there are relatively large differences between the values calculated analytically and the numerical ones, validated experimentally. In this case, the adopted bar model, consecrated in literature, is not
appropriate/realistic. Using thin plate theory could lead to more accurate results, but finding the analytic relationships becomes a difficult problem, even for homogeneous microbeams.

Based on the analytical relationships, various geometric and material parameters can change, in order to identify the optimal configuration of the structure for a particular application. Numerical simulation can be made further only for this configuration. The results validation enables then the production of such microstructures, thus saving time and materials.

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