Determining the Mass of Supersymmetric Scalars at the CLIC Multi-TeV $e^+e^-$ Collider

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Abstract

The determination of the smuon mass at the CLIC multi-TeV $e^+e^-$ linear collider has been studied for two CMSSM benchmarks. Results are given for both the analysis of the muon energy spectrum and the threshold scan method. The effects of detector resolution, beam-beam interactions and accelerator-induced backgrounds are discussed. The energy spectrum technique is also applied to the $t \rightarrow t\tilde{g}$ process to determine the scalar top mass, in scenarios with the gluino lighter than the squarks.

1 Introduction

A multi-TeV $e^+e^-$ linear collider (LC) is expected to complete the investigation of the SUSY particle spectrum, in the case Supersymmetry is realised in nature and the particle partners exhibit a mass spectrum extending up to 1 TeV, and beyond. In this case, precise determinations of the masses and widths of these particles will represent a central part of the collider physics programme. Sets of SUSY benchmarks have recently been proposed, which take into account constraints from direct searches at LEP-2 and other colliders and data from $b \rightarrow s\gamma$ and cosmology [2, 3]. Most of these benchmarks have the masses of the heaviest sparticles beyond the reach of the

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LHC and of a TeV-class $e^+e^-$ linear collider. The LHC is expected to reveal the existence of Supersymmetry and perform a study of the sparticle properties in the vast majority of conceivable scenarios and a 0.5-1.0 TeV $e^+e^-$ collider can improve the accuracy in the study of the lightest SUSY states. However, higher energies are likely to be needed to access all the sparticles species and acquire the data necessary to clarify the nature of Supersymmetry breaking.

A first mapping of the potential of a $3-5$ TeV $e^+e^-$ collider, such as CLIC [4], in comparison to the LHC and to a lower energy $e^+e^-$ collider was performed, based on a simple assumption on the minimum observable product of production cross section and decay branching fraction into well reconstructable final states [2]. Beyond detection, it is important to assess the ability of CLIC to perform accurate determinations of sparticle masses, widths and decay branching fractions, taking into account the details of beam-beam interactions and the resulting luminosity spectrum and accelerator-induced backgrounds [1]. We study here the expected CLIC accuracy on sfermion masses, using a realistic simulation. Results are presented for the Supersymmetric partner of the muon, $\tilde{\mu}_L$ and for the scalar top, $\tilde{t}_2$. Section 2 presents the benchmarks considered and the CLIC parameters. There are two different techniques for the determination of the smuon mass: the analysis of the energy distribution of the resulting muon in the two body $\tilde{\mu}_L$ decay and the energy scan close the the production threshold. These are discussed in 3.1 and 3.2 respectively. In Section 4 we discuss the perspectives for extending the energy spectrum analysis to $\tilde{t}_2$ decays and Section 5 has the conclusions.

2 SUSY Benchmark and Simulation

The post-Lep CMSSM benchmark points E and H, proposed in [2], have been chosen for this study. Point E $^1$ is representative of the focus region, characterised by heavy sleptons and squarks. The $\mu_0^L + \mu_0^L$ threshold is just accessible at $\sqrt{s} = 3$ TeV. The gluino ($M_{\tilde{g}} = 697$ GeV) being lighter than the squarks, decays such as $\tilde{g} \rightarrow q\bar{q}$ are open for these parameters. It is interesting to observe that such inverted scenario, with the gluino lighter than the squarks, is also common to benchmarks inspired to effective field theories derived from strings [5]. The mass of the LSP is 120 GeV and those of the heavier neutralinos are in the range 200-300 GeV. Point H $^2$ is located at the end-

$^1$ $m_{1/2} = 300$, $m_0 = 1500$, $\tan\beta = 10$, $sign(\mu) = +$, $A = 0$

$^2$ $m_{1/2} = 1500$, $m_0 = 419$, $\tan\beta = 20$, $sign(\mu) = +$, $A = 0$
Table 1: Main properties of the sparticles considered in this analysis

| Point | Mass (GeV) | Width (GeV) | $\sigma(e^+e^-\to \bar{\mu}\bar{\mu})$ (fb) | BR($\bar{\mu}\to \mu\chi_1^0$) |
|-------|------------|-------------|--------------------------------|-----------------|
| E $\tilde{\mu}_R$ | 1433 | 21. | 0.03 | 0.35 |
| E $\tilde{\mu}_L$ | 1427 | 7.9 | 0.02 | 0.95 |
| H $\tilde{\mu}_R$ | 710 | 0.06 | 0.92 | 1.00 |
| H $\tilde{\mu}_L$ | 1150 | 0.6 | 0.59 | 1.00 |
| E $\tilde{t}_1$ | 994 | 42. | 0.65 | 0.20 |
| E $\tilde{t}_2$ | 1303 | 76. | 0.12 | 0.48 |

point of the coannihilation tail at large $m_{1/2}$ and moderate $\tan \beta$. The heavy sparticle spectrum makes this a highly problematic point at the LHC. However, at $\sqrt{s}=3$ TeV, smuons are accessible in $e^+e^-$ collisions and their masses can be determined through the muon energy distribution. The LSP mass, $M_{\chi_1^0}$, is 665 GeV. Events have been simulated using the PYTHIA 6.2 Monte Carlo generator. Parameters have been tuned to reproduce sparticles masses and widths to a good accuracy. Initial state radiation (ISR) has been included. Muon energy distributions have been obtained using full GEANT-3 simulation of a discrete silicon tracker consisting of twelve layers located from 3.5 cm up to 180 cm in radius and a solenoidal magnetic field $B$ of either 4 or 6 T. Backgrounds and $\tilde{t}$ decays have been processed using the SIMDET parametric simulation [6], describing the detector response determined using GEANT. The CLIC luminosity spectrum has been folded to the production cross sections using the CALYPSO interface. $\gamma\gamma \to$ hadrons and parallel muon backgrounds have been overlayed to the generated signal and background events for the smuon study.

### 3 Muon Mass Determination

The study has been performed for $e^+e^-\to \tilde{\mu}_L\tilde{\mu}_L \to \mu^+\chi_1^0\mu^-\chi_1^0$. The three main sources of background, also leading to two muons plus missing energy, are i) $e^+e^-\to W^+W^-\to \mu^+\mu^-\nu\bar{\nu}$, ii) $e^+e^-\to W^+W^-\bar{\nu}\nu \to \mu^+\mu^-\nu\bar{\nu}_\mu\nu_e\bar{\nu}_e$ and iii) $e^+e^-\to \chi_1\chi_2, \chi_2\chi_2 \to \mu^+\mu^-\nu\bar{\nu}\chi_0\chi_0$. These backgrounds can be suppressed by requiring central production and decay kinematics compatible with those characteristic of smuon pair production. A multi-dimensional discriminant based on $M_{\mu\mu}$, $M_{\text{recoil}}$, $M_{\text{det}}$.
\[ E_{\text{missing}}, \mu\mu \text{ Acollinearity, } |\cos \theta_{\text{Thrust}}|, E_t \text{ and } E_{\text{hem}} \text{ has been applied. The signal efficiency is flat with the muon energy.} \]

### 3.1 The Energy Distribution method

If the centre-of-mass energy, \( \sqrt{s} \), is significantly larger than twice the sparticle mass, \( M_{\tilde{\mu}} \), this can be determined by an analysis of the energy spectrum of the muon, emitted in the two-body \( \tilde{\mu} \rightarrow \chi^0_1 \mu \) decay (see Figure 3.1). The two end-points, \( E_{\text{min}} \) and \( E_{\text{max}} \), of the spectrum are related to the \( \tilde{\mu} \) and \( \chi^0_1 \) masses and to the \( \tilde{\mu} \) boost by:

\[
E_{\text{max/min}} = \frac{M_{\tilde{\mu}}}{2} \left(1 - \frac{M_{\chi^0_1}^2}{M_{\tilde{\mu}}^2}\right) \times \left(1 \pm \sqrt{1 - \frac{M_{\tilde{\mu}}^2}{E_{\text{beam}}^2}}\right) \tag{1}
\]

from which either the smuon mass \( M_{\tilde{\mu}} \) can be extracted, if \( M_{\chi^0_1} \) is already known, or both masses can be simultaneously fitted. This technique, already considered

![Figure 1: Left: Muon energy spectra in the decay \( \tilde{\mu} \rightarrow \mu\chi^0_1 \) for \( M_{\tilde{\mu}} = 1150 \text{ GeV} \) and \( M_{\chi^0_1} = 660 \text{ GeV} \) obtained at \( \sqrt{s} = 3 \text{ TeV} \), assuming the baseline CLIC luminosity spectrum. Right: Accuracy on the determination of the \( \tilde{\mu} \mu 
and \( \chi^0_1 \) masses by a two-parameter fit to the muon energy distribution. The lines give the contours at 1 \( \sigma \), 68\% and 95\% C.L. for 1 \( ab^{-1} \) of data at \( \sqrt{s}=3 \text{ TeV} \).](image)
Table 2: Results of the 1-parameter $\chi^2$ fit to the muon energy distribution, obtained for different assumption on the $\delta p/p^2$ momentum resolution and the beamstrahlung spectrum. Accuracies are given for an integrated luminosity of 1 ab$^{-1}$.

| $\delta p/p^2$ | Beamstrahlung | Fit Result (GeV) |
|---------------|--------------|-----------------|
| 0.            | none         | 1150 ± 10       |
| $3.0 \times 10^{-5}$ | none     | 1150 ± 12       |
| $4.5 \times 10^{-5}$ | none     | 1151 ± 12       |
| $4.5 \times 10^{-5}$ | Std.      | 1143 ± 18       |

for the determination of squark masses [7], has been extended also to sleptons for the LHC and a TeV-class LC [8]. It is interesting to consider it here also for its implications on the required momentum resolution in the detector. Two values of the solenoidal magnetic field $B = 4$ and 6 T have been tested, corresponding to momentum resolutions $\delta p/p^2$ of $4.5 \times 10^{-5}$ and $3.0 \times 10^{-5}$ GeV$^{-1}$ respectively. No appreciable difference on the resulting mass accuracy has been observed from these two momentum resolution, since the dominant smearing is due to beamstrahlung. In fact, at Clic, the main issue is the significant beamstrahlung smearing of the luminosity spectrum, and thus of the effective $E_{\text{beam}}$ value. The corresponding effect has been estimated by using both a perfectly well known and constant beam energy and that corresponding to the baseline Clic parameters at a nominal $\sqrt{s} = 3$ TeV. Results are summarised in Table 3.1. The smuon mass has been extracted by a $\chi^2$ fit to the muon energy spectrum by fixing $M_{\chi^0_1}$ to its nominal value (see Table 3.1. The fit has been also repeated leaving both masses free and performing a simultaneous two-parameter fit. The results are $M_{\tilde{\mu}} = (1145 \pm 25)$ GeV and $M_{\chi^0_1} = (652 \pm 22)$ GeV (see Figure 3.1).

3.2 The Threshold Scan method

An alternative method to determine the $\tilde{\mu}$ mass is an energy scan of the rise of the $e^+e^- \rightarrow \tilde{\mu}^+\tilde{\mu}^-$ cross section, close to its kinematical threshold. It has been shown that an optimal scan consists of just two energy points, sharing the total integrated luminosity in equal fractions and chosen at locations optimising the sensitivities to the $\tilde{\mu}$ width and mass respectively [9]. Including the beamstrahlung effect, induces a shift of the positions of the maxima in mass sensitivity towards higher nominal $\sqrt{s}$
Table 3: Accuracies on the determinations of the smuon masses for the two benchmark scenarios considered in this study using threshold scans with different experimental conditions.

| Point | Beam- strahlung | Pol. | $\sqrt{s}$ (TeV) | $\int \mathcal{L}$ (ab$^{-1}$) | $\delta M$ (GeV) |
|-------|-----------------|------|------------------|-----------------------------|------------------|
| H     | $\tilde{\mu}_L$ none | 0/0  | 3.0-3.5          | 1                           | $\pm$ 11         |
| H     | $\tilde{\mu}_L$ Std. | 0/0  | 3.0-3.5          | 1                           | $\pm$ 15         |
| E     | $\tilde{\mu}_L$ none | 0/0  | 3.8-4.2          | 1                           | $\pm$ 29         |
| E     | $\tilde{\mu}_L$ Std. | 0/0  | 3.8-4.2          | 1                           | $\pm$ 36         |
| E     | $\tilde{\mu}_L$ none | 80/60| 3.8-4.2          | 1                           | $\pm$ 17         |
| E     | $\tilde{\mu}_L$ Std. | 80/60| 3.8-4.2          | 1                           | $\pm$ 22         |

energies. For benchmark point E, the cross section at $\sqrt{s}=3$ TeV is too small for an accurate measurement. Higher centre-of-mass energy, 4 TeV, and polarised beams need to be considered. By properly choosing the beam polarisation, not only the pair production cross sections are increased but also their sensitivity to the smuon masses. Results are summarised in Table 3.

4 Scalar Top Analysis

In the context of the benchmark point E, the process $e^+e^- \rightarrow \tilde{t}_1\tilde{t}_1, \tilde{t}_2\tilde{t}_2$ followed by the decay $\tilde{t}_{1,2} \rightarrow t\tilde{g}$ offers a possibility to determine the scalar top mass based on the reconstructed top energy only, following a procedure similar to that adopted for the smuon mass. The same relation as Eq. 1 between the top energy spectrum endpoints and the masses and $\sqrt{s}$ energy holds in this case, with $M_{\tilde{t}}$ and $M_{\tilde{g}}$ replacing $M_{\tilde{\mu}}$ and $M_{\chi_1^0}$ respectively. By determining the endpoints of the top quark energy spectrum produced in $\tilde{t} \rightarrow t\tilde{g}$, the masses of the stop and the gluino can be extracted. However, here the situation is complicated by the fact that top quarks may also be produced in $\tilde{g}$ decays or other $\tilde{t}$ decays, such as $\tilde{t} \rightarrow t\chi^0$. Figure 4 shows the distribution of the energy of top quarks produced in $e^+e^- \rightarrow \tilde{t}_2\tilde{t}_2$ events. Although the expected endpoints are diluted, it is still possible to extract $M_{\tilde{t}}$ from the shape of the top energy spectrum. A $\chi^2$ fit, including the contributions from other top sources in signal events, indicates that a relative accuracy $\delta M_{\tilde{t}_2}/M_{\tilde{t}_2} = \pm 7.5\%$ can be achieved with 3 ab$^{-1}$, without including beamstrahlung effects.
5 Conclusions

A preliminary study of sfermion reconstruction at the CLIC multi-TeV collider has shown that smuon masses can be measured with typical accuracies of 1.3-3.0 % for $\mathcal{L} = 1 \text{ ab}^{-1}$ for two benchmark points, using either the muon energy technique, when $\sqrt{s} \gg 2 \times M_\tilde{\mu}$, or a threshold scan. The accuracy of the threshold scans is significantly enhanced by the availability of polarised beams. The energy spectrum technique has also been extended to the scalar top in the case when the decay $\tilde{t}_2 \rightarrow t \tilde{g}$ is kinematically allowed.

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