Periodicity of the giant vortex states in mesoscopic superconducting rings

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Abstract

The giant vortex states of a multiply connected superconductor, with radius comparable to the penetration depth and the coherence length, are theoretically investigated based on the nonlinear Ginzburg-Landau theory, in which the induced magnetic field by the super-currents is accurately taken into account. The solutions of Ginzburg-Landau equations are found to be actually independent of the angular momentum L in a gauge invariant point of view, provided that the hole is in the center. Different cases with the paramagnetic current, the diamagnetic current, and the coexistence of the above two, have been studied numerically. The interpretation of the L-independent solutions of Ginzburg-Landau equations is given based on the same principle of Aharonov-Bohm effect, and could be observed by Little-Parks like oscillations near the phase boundary.

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I. INTRODUCTION

In 1962, Little and Parks\(^1\) demonstrated that in multiply connected system, the superconducting transition temperature \(T_c\) is a periodic function of the applied magnetic field \(H\) in the axial direction of a superconducting thin loop. The oscillations of \(T_c(H)\) with a period of the flux quantum \(\Phi_0 = \hbar c/2e\) are a consequence of the fluxoid quantization constraint which was first predicted by F. London.\(^2\) The free energy of the superconducting state is periodic in the unit of flux quanta, while the free energy of normal state is almost independent of the flux. Therefore the transition temperature should be a periodic function of the enclosed flux.

Recent advancements in micro-fabrication and technique of Hall magnetometry\(^3\) make it possible to detect the properties of superconducting samples with sizes comparable to the coherence length \(\xi\) and the London penetration depth \(\lambda\) at the temperature well below \(T_c\). The superconducting states in such a mesoscopic system reveal unique properties, which depend on the size and geometry of the sample. Such confinement effects have been clearly demonstrated experimentally in nanosized superconducting square loops\(^4\), disks\(^5\) and rings.\(^6\) In the last decade, the properties of mesoscopic superconductors have drawn growing attention, both experimentally and theoretically(see, e.g., Refs. 7-15).

In particular, as a doubly connected topological sample, superconducting ring like structures have also been studied extensively in the past years\(^16,17\). In contrast to bulk superconductivity, the symmetry of the sample play an important role when the sample scales become comparable with \(\xi\) (or \(\lambda\)). The effective Ginzburg-Landau(GL) parameter \(\kappa = \lambda/\xi\) is significantly increased, and the magnetic response of a type I superconductor even presents a shape of a type II superconductor. On the basis of GL theory, Berger and Rubinstein predicted that there is a surface in a nonuniform mesoscopic superconducting loop, where the order parameter vanishes due to the existence of the "singly connected state".\(^18\) Using the linearized GL equation, Bruyndoncx \textit{et al}. studied superconducting loops of finite width and found an dimensional crossover from different behaviors of the phase boundary as the geometry of the loop changes.\(^19\) Taking the induced magnetic fields into account, Baelus \textit{et al}.\(^20\) investigated vortex states in a thin superconducting disk with a hole and noticed a transition from the giant vortex state to the multivortex state arises when the size of the disk becomes large enough. They also showed that the multi vortex state is stable when
circular symmetry is broken.

Recently, Pedersen et al. experimentally investigated the flux quantization of a mesoscopic loop with a \( \mu \) Hall magnetometer. The magnetic field intensity periodicity observed in the magnetization measurements indicates a gradual transition from large flux avalanches to single flux jumps as the external magnetic field intensity is increased. Such a sub-flux quantum shift was interpreted as a consequence of a giant vortex state nucleating towards either the inner or the outer boundary of the loop.\(^{21}\) Their observation leads to a new interest in the phase transition in superconducting loops\(^{22,23,24}\). At low magnetic field, multiple flux jumps and irreversible behavior were also observed by Vodolazov et al.\(^{25}\) in thin Al superconducting rings with sufficiently large radii. The also showed the possibility of the existence of flux avalanches by numerical calculations based on the time dependent GL model. The nucleation of superconductivity in rings has been studied both experimentally and theoretically by Morelle et al.\(^{26}\), A transition from a one-dimensional to a two-dimensional regime is seen when the magnetic field with small holes is increased. The phase transitions due to the superconducting vortices enter into the sample were also reported by Bourgeois et al. in mesoscopic square loops\(^{27}\). The measurements of specific heat as a function of external magnetic field exhibit oscillations with changing periodicity, depending on both the temperature and applied flux.

There are three kinds of vortex states in mesoscopic systems, the giant vortex state, the multi vortex state and the ring-like vortex state. In terms of thermodynamical stability, the multi-vortex states and giant vortex states can be stable, while ring-like vortices in superconductors seem to be unstable\(^{28,29}\). In present work, we confined our study in the giant vortex states since the mesoscopic superconducting ring has more surfaces compared to the disk with same size. The confinement effects from the boundaries are dominating and it will impose a circular symmetry on superconducting order parameter. Moreover, recent experimental observations also indicate giant vortex states nucleating in mesoscopic superconductors with cylindrical symmetry\(^{9,21}\). For a giant vortex state, the free energy of a type I superconducting ring has been studied theoretically\(^{20}\) in terms of external magnetic field and the winding number \( L \). The vorticity of the ground state changes from \( L \) to \( L + 1 \) at some values as the external magnetic field increases. However, their numerical results seem to show that the properties of giant vortex states are related to the winding number \( L \). In the present work, we theoretically investigate the giant vortex states of a multiply
connected superconductor in terms of the nonlinear GL theory. We show that the winding number \( L \) of a giant vortex state is a tuned function of the flux inside the hole of a ring, and that all the information of giant vortex states can be obtained with a defined winding number \( L \). We also show that the vector potential-tuned giant vortex states have periodicity in the units of \( \Phi_0 \), as demonstrated in Little-Parks experiment.

This paper is organized as follows: In section II, we describe the model of the mesoscopic superconducting ring based on the two-dimensional GL equations. This approach is suitable in two cases: (i) for a ring made from a thin film and (ii) for a high cylinder considered in its cross-section far from the bases. We present a simple proof to show that the Gibbs free energy of a giant vortex state is independent of the vorticity \( L \). In section III, we investigate the giant vortex states numerically and obtain the local magnetic field, the Cooper pair density and the current density. We consider different cases with diamagnetic response, paramagnetic response and the combination of the above two. We also show how the quantum phase tuned by the flux inside the hole. In section IV, we give further discussion and conclusions.

II. THE GL EQUATIONS AND THE L-INDEPENDENCE OF GIBBS FREE ENERGY

In the present paper, we consider a two-dimensional superconducting ring, with internal radius \( r \) and external radius \( R \). The applied magnetic field is along the axial direction and has a uniform value \( H_0 \). When the size of the superconducting ring falls in a mesoscopic regime, where the surface effects are of the same order of magnitude as the bulk effects, the confinement effects from the boundaries are dominating and impose a circular symmetry on the superconducting order parameter and the local magnetic field. Hence the order parameter is expected to be given by

\[
\psi(\rho, \theta) = f(\rho)e^{i\varphi(\rho, \theta)},
\]

and the magnetic field has the form \( \vec{H} = \vec{\nabla} \times \vec{A} = H(\rho)\vec{e}_z \).

We can always choose appropriate gauge so that the vector potential can be written as,

\[
\vec{A} = A(\rho)\vec{e}_\theta
\]
In order to simplify the numerical calculation, we will make the formulas dimensionless by measuring the order parameter $\Psi$ in units of $\frac{\sqrt{-\alpha}}{\beta}$, lengths in units of $\sqrt{2\lambda}$, the magnetic field in units of $\frac{\phi_0}{4\pi \lambda^2}$, the Gibbs free energy in units of $\frac{H_c^2}{4\pi}$, and the vector potential in units of $\frac{\phi_0}{2\sqrt{2\pi \lambda}}$, where $\phi_0 = \frac{hc}{2e}$ and $H_c = \frac{\phi_0}{4\pi \lambda^2} \sqrt{2\kappa}$. Then the Gibbs free energy $g = g_s - g_n$ is given by

$$g = \int \left[ \frac{1}{2}(H - H_0)^2 + \kappa^2(1 - |\Psi|^2)^2 - \kappa^2 + |(\nabla - i\vec{A})\Psi|^2 \right] dV. \quad (3)$$

By minimizing the Gibbs free energy, GL equations can be obtained in the following form:

$$2\kappa^2 \Psi(1 - |\Psi|^2) = (\nabla - i\vec{A})^2 \Psi \quad (4)$$

$$\vec{\nabla} \times \vec{\nabla} \times \vec{A} = (\Psi^* \vec{\nabla} \Psi - \Psi \vec{\nabla} \Psi^*) - 2|\Psi|^2 \vec{A}. \quad (5)$$

Substituting the expressions of the order parameter and the vector potential into GL equations, we have

$$2\frac{\partial f}{\partial \rho} \frac{\partial \phi}{\partial \rho} + f \frac{\partial \phi}{\partial \rho} + f \frac{\partial^2 \phi}{\partial \rho^2} + f \frac{\partial^2 \phi}{\partial \rho^2 \partial \theta^2} = 0 \quad (6)$$

$$\left( \frac{\partial A}{\rho^2} - \frac{1}{\rho} \frac{\partial A}{\partial \rho} - \frac{\partial^2 A}{\partial \rho^2} \right) \vec{e}_\theta = 2f^2 \left( \frac{1}{\rho} \frac{\partial \phi}{\partial \theta} - A \right) \vec{e}_\theta + 2f^2 \frac{\partial \phi}{\partial \rho} \frac{\partial e_\rho}{\partial \rho} \quad (7)$$

The $\vec{e}_\rho$ term in Equation (7) has to be vanished, then we have $\frac{\partial \phi}{\partial \rho} = 0$. Substituting this equation back into Equation (5), we find the phase of order parameter satisfies

$$\frac{\partial^2 \phi}{\partial \theta^2} = 0 \quad (8)$$

Thus it is convenient to assume $\psi(r) = f(r)e^{iL\theta}$, where $L$ is called as the winding number of the vortex. When the superconductor is described by such a circular symmetric order parameter, it is said to be in giant vortex state. GL equations turn into a set of second-order ordinary differential equations,
\[
\frac{d^2 f}{d\rho^2} + \frac{1}{\rho} \frac{df}{d\rho} - \left( \frac{L}{\rho} - A \right)^2 f + 2\kappa^2 f (1 - f^2) = 0 ,
\]

(9)

\[
\frac{1}{d\rho} \left( \frac{1}{\rho} \frac{d(\rho A)}{d\rho} \right) + 2 \left( \frac{L}{\rho} - A \right) f^2 = 0.
\]

(10)

The boundary conditions assure that no current passes through the surface, which is

\[
\left. \frac{df}{d\rho} \right|_{\rho=r} = \left. \frac{df}{d\rho} \right|_{\rho=R} = 0 ,
\]

(11)

The Gibbs free energy can be written as

\[
g = \int 2\pi \rho G d\rho,
\]

which depends only on the value of the kernel \( \rho G \) in the expression

\[
\rho G = \frac{\rho}{2} (A + \rho \frac{dA}{d\rho} - H_0)^2 + \rho \kappa^2 (1 - f^2)^2 - \rho \kappa^2
\]

\[
+ \left( \frac{df}{d\rho} \right)^2 \rho + \frac{f^2}{\rho} \left( \frac{L}{\rho} - A \right)^2 .
\]

(12)

Notice that in Equations(9), (10) and (12), the winding number choice \( L \) can fix the gauge choice for the vector potential so that

\[
A(L + 1, \rho) = A(L, \rho) + 1/\rho.
\]

(13)

It is straightforward to find that both Gibbs free energy and GL equations are gauge invariant as long as we choose the winding number \( L \) and the vector potential \( \vec{A} \) satisfying Equation(13). Consequently, with different winding number \( L \), GL equations have same solution for the local magnetic field \( \vec{H}(\rho) \), Copper pair density \( f(\rho) \) and current density \( J(\rho) \). It shall be noted that when the winding number changes with gauge transformation Equation(13), the vector potential will change respectively, so that although all the observable physical properties of the superconductor won’t change with winding number, the flux around the superconductor do change. From the gauge transformation Equation(13), it can be predicted that the mesoscopic superconducting ring has the possibility to hold rather strong magnetic field inside its hole. Actually, this gauge-invariant solution has similar interpretation as the well-known Little-Parks experiment. If the winding number and the flux change together through a gauge invariant transformation, the physical properties of the thin-walled superconductor does not change.

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III. NUMERICAL RESULTS OF NON-LINEAR GL EQUATIONS

The non-linear GL equations (9-10) can be represented as four first-order ordinary differential equations, thus four boundary conditions are required. Apparently the boundary conditions equation (11) is not sufficient. We set the value of the vector potential at the external boundary of the ring since the real magnetic field can not be obtained experimentally although the applied magnetic field is known. We apply the “shooting method”\textsuperscript{30}, in which trial integrations are “launched” that satisfy the boundary condition at one endpoint. The discrepancies from the desired boundary condition at the other endpoint are used to adjust the starting conditions, until the boundary conditions at both endpoints are ultimately satisfied. The relative accuracy is set to be $10^{-6}$ for the numerical calculation.

As it has been discussed in the previous section, the full GL equations can be obtained with a fixed winding number such as $L = 0$. Three kinds of magnetic responses can be obtained numerically as we change the boundary condition of vector potential. In Figure.1 and Figure.2, we show that in the case of the diamagnetic response, Cooper pair density decreases and magnetic field increases when the radius becomes larger. Since the magnetic field at the external boundary are much larger than that at the internal boundary, the super-current induces an negative magnetic field to keep the applied magnetic field out of the superconducting ring from the external boundary so that the part closer to the inner side of the ring always have larger Cooper pair density. In the case of paramagnetic response, the situation will be opposite. The magnetic field at the internal boundary are much larger. The Cooper pair density increases and the magnetic field decreases when the radius becomes larger. The super-current induces a positive magnetic filed in order to keep the applied field out of the internal boundary. In the case of combination of both effects, the magnetic field first decreases until it reaches a minimum and then it increases with the increase of the radius, the super-current keeps the magnetic field out of the ring from both boundaries. The total magnetic field reaches its minimum in the middle of the ring while the Cooper pair density show different behaviors for type II and type I. The type I ring has a positive surface energy, since the magnetic field has much larger value at the internal boundary, the positive surface energy keeps the Cooper pairs away. In Fig.3. the calculated super-current as a function of radius is displayed. The current at the sample boundary is not equal to zero which implies that the flux is not quantized. Diamagnetic response has a negative super-
current while paramagnetic response has a positive one. Moreover, the superconducting state can consist of a combination of the diamagnetic and paramagnetic Meissner state, and the super-current density goes to zero at the circle where the magnetic field reaches a minimum. This also indicates a certain effective radius inside the ring through which circular area the flux is exactly quantized, which has been studied both experimentally\cite{21} and theoretically\cite{31}. It shall be noted that this “combination” of superconducting state can only have diamagnetic response in the outer part of ring and paramagnetic response in the inner part. There is no possibility of having the response vice versa because the super-current has to keep magnetic field from ”invading” from both the internal and the external boundary. We have studied the case for different $\kappa$ as 0.5 and 1.0, which represents type I and type II superconductor respectively. There is not much difference in the behaviors of a giant vortex state when only $\kappa$ changes, except for the value of Gibbs free energy. We have calculated the free energy for all the cases.

The total flux through the area of the superconducting ring is directly obtained from local magnetic field. The flux inside the superconducting ring is found to be not necessarily quantized. For example, the calculated flux for the giant vortex states considered in Fig.1.(a) is around 8.7, 9.8 and 18.0 respectively. The size of the ring is carefully chosen since it was found experimentally that the physical properties of a mesoscopic superconducting system depend on its size. The size of the ring we considered here is large enough to be superconducting when more than one flux enter. Meanwhile it is also narrow enough to maintain the cylindrical symmetry of giant vortex states by the boundary condition. As it has been generalized\cite{20}, the flux inside the hole of the ring is not quantized. Our numerical results also show non-quantized solutions for the flux inside the hole. However, the concept of winding number $L$ in the present work is different from that reviewed in their work. The Gibbs free energy of the giant vortex state will not change with $L$, while the flux inside the hole changes correspondingly. For example, the flux in the hole of the combination magnetic response in Fig.1(b). is 0.8 when $L = 7$. When the winding number change to $L = 8$, the solution of GL equations is the same. The only change is that the flux in the hole there is 1.8, which means one more flux enters inside the hole. A transition from a state with vorticity $L$ to one with vorticity $L + 1$ happens when a vortex, carrying unit vorticity, enters at the outer radius, crosses the ring, and then annihilates at the inner radius. The numerical results are consistent with the theoretical analysis obtained in Section II.
It is well known that the phase shift of an electron wave function resulting in an alteration in the interference pattern of a double slit electron diffraction experiment in the presence of a potential magnetic field, even if the magnetic field is shielded so that diffracted electrons do not pass through it. What is going to happen if only the flux inside the hole of the superconducting ring is changed while the magnetic field is not. If the additional flux is quantized, according to the gauge invariant transformation Equation(13), all the parameters of the giant vortex state will not be influenced except that the winding number \( L \) changes to \( L + 1 \). It’s also interesting to study the case when additional flux is not quantized. Experimentally, this additional flux could be obtained by setting a very small but long solenoid inside the hole. The solenoid will hold in the magnetic field inside its coils, but the uncurled vector potential will also appear outside the coil and it will affect the superconducting states. In Fig.4, we provide the numerical calculation of the Gibbs free energy displayed as a function of the additional flux. It is clearly shown that the Gibbs free energy is a periodic function of the "field free" flux with a period of the flux quantum \( \Phi_0 = \frac{hc}{2e} \). The periodicity of Gibbs free energy with the additional flux quantum(or winding number \( L \)) is reminiscent of Aharonov-Bohm effect, and such kind of quantum periodicity in the free energy of superconducting pairs has been directly observed via Little-Park experiment. If the additional flux is not quantized, the free energy of the vortex states will change. As a result, the magnetic field inside and outside the superconducting ring will also change, assuming that the additional flux will not change the magnetic field in the hole of the ring except for the area occupied by the solenoid. In such a way, the "invisible" vector potential \( \vec{A} \) becomes "visible". If the applied magnetic field is fixed, the additional flux in the solenoid can even cause the free energy to become lower. Based on our numerical results, it can be predicted that when the superconducting state is close enough to the phase boundary, Little-Parks like oscillations could be observed experimentally in a rather "fat" ring for both the critical temperature \( T_c \) and critical applied field \( H_c \), as the additional flux changing continuously.

IV. SUMMARY

In conclusion, we have theoretically studied the giant vortex states in terms of the non-linear GL equations for a superconducting ring. We provide a gauge invariant transformation and show that the solutions can be independent of the winding number \( L \). We also calculate
the numerical results for different magnetic responses of the superconductor, and discuss the concept of the winding number in details. For a superconducting ring, there are giant vortex states that hold more than one flux quantum inside both the superconductor and the hole. The flux is found to be not quantized, while the winding number \( L \) is still an integer, but not necessarily to be the number of vortices anymore. The GL equations can be fully solved without changing the value of the winding number \( L \). We also indicate that the periodicity of Gibbs free energy \( G \) with additional flux in mesoscopic superconducting ring is based on the same principle of Aharonov-Bohm effect, and can be shown with a Little-Parks-like oscillation.

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Figures

FIG. 1: (Color online). The amplitude of the order parameter as a function of radius for different type of superconductors for (a) $\kappa=1.0$ and (b) $\kappa=0.5$. The internal boundary of the ring is at $r = 10.0$, and the external boundary at $R = 12.0$. Three different magnetic responses are shown respectively. Solid curve for the diamagnetic response, dash-dotted curve for the combination magnetic response, and dotted curve for the paramagnetic response.

FIG. 2: (Color online). Magnetic field as a function of radius for different type of superconductors for (a) $\kappa=1.0$ and (b) $\kappa=0.5$. The internal boundary of the ring is at $r = 10.0$, and the external boundary at $R = 12.0$. Three different magnetic responses are shown respectively. Solid curve for the diamagnetic response, dash-dotted curve for the combination magnetic response, and dotted curve for the paramagnetic response.

FIG. 3: (Color online). Super-current density as a function of radius for different type of superconductors for (a) $\kappa=1.0$ and (b) $\kappa=0.5$. The internal boundary of the ring is at $r = 10.0$, and the external boundary at $R = 12.0$. Three different magnetic responses are shown respectively. Solid curve for the diamagnetic response, dash-dotted curve for the combination magnetic response, and dotted curve for the paramagnetic response. Notice there appears a zero current circle at the effective radius for the combination magnetic response.

FIG. 4: Gibbs free energy as a function of additional flux in the solenoid. The internal boundary of the ring is at $r = 10.0$, and the external boundary at $R = 12.0$. GL parameter $\kappa$ is chosen as $1.0$, and $G_0$ is the Gibbs free energy when external magnetic field vanishes.
