Self-consistent quasiparticle model for quark-gluon plasma

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Abstract
Here we present a self-consistent quasi-particle model for quark-gluon plasma and apply it to explain the non-ideal behaviour seen in lattice simulations. The basic idea, borrowed from electrodynamic plasma, is that the gluons acquire mass as it propagates through plasma due to collective effects and is approximately equal to the plasma frequency. The statistical mechanics and thermodynamics of such a system is studied by treating it as an ideal gas of massive gluons. Since mass or plasma frequency depends on density, which itself is a thermodynamic quantity, the whole problem need to be solved self-consistently.

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1 Introduction:

The quasiparticle model of quark-gluon plasma (qQGP) is used to study the thermal properties of quark-gluon plasma (QGP) \[1, 2, 3, 4, 5\]. QGP is a plasma of quarks and gluons which exhibits a collective behaviour because of the interactions, governed by quantum chromodynamics (QCD). At extremely high temperature, the interaction may be very weak because of the asymptotic freedom and hence the plasma is almost ideal gas of quarks and gluons. However, near and above the critical $T_c$, the coupling constant $\alpha_s$ is not weak and hence QGP may be non-ideal gas of quarks and gluons. Here $T_c$ is the critical temperature of phase transition or cross over from hadrons to QGP. The lattice simulation of QCD at finite temperature \[6\] (LGT) and the elliptical flow observed in relativistic heavy ion collisions (RHICs) signal the existence of non-ideal QGP. The problem of confinement-deconfinement of QCD and the search for QGP in RHICs depend on the properties of QGP at temperature close to $T_c$. Since the strong coupling constant $\alpha_s$ is not very small near $T_c$, it may be difficult to solve QCD analytically. Many attempts in this line were not completely successful \[7, 8, 9\]. Therefore, several phenomenological models with two or more fitting parameters, like confinement models \[10\], strongly coupled plasma (SCQGP) \[11\], strongly interacting plasma (sQGP) \[12\], qQGP \[1, 2, 3, 4, 5\], liquid model \[13\] and so on, were proposed. By adjusting parameters of the model one may fit the LGT results. Here we propose a new quasiparticle model which has only one adjustable parameter and is the revised version of the widely studied qQGP \[5, 11, 14\].

2 Self-consistent qQGP:

Let us recollect first various versions of qQGP models. It was first proposed by Peshier et. al. \[1\] and with two adjustable parameters, they were able to fit LGT results of \[15\] on gluon plasma. However, it fails to fit the more recent LGT results as shown in \[11\]. Furthermore, Gorenstein and Yang \[14\] pointed out that there is a thermodynamic (TD) inconsistency in this model and by imposing a stringent constraints, which they called TD consistency relation or the reformulation of statistical mechanics (SM), they tried to solve the problem. Recently, in a series of papers \[5\], we pointed out the reason for the TD inconsistency and revised their model using the standard SM in a TD consistent way. The revised model of qQGP, without TD consistency relation or the reformulation of SM, fits remarkably well many LGT results with a single system dependent parameter \[5\]. We also showed how the TD consistency relation comes out from our formulation.
The basic idea in our model is that we start from the energy density of quasiparticles instead of ideal gas pressure-partition function relation. Further, we assume that quasiparticle acquire an additional temperature dependent mass and this thermal mass is equal to the plasma frequency, $\omega_p$, because of the collective effects of QGP. We used, as an approximation, the expression for $\omega_p$ from perturbative QCD results and assumed the two-loop running coupling constant as function of temperature. So we just have one adjustable parameter related to QCD scale parameter which appears in $\alpha_s(T)$. In this letter, we further revise our qQGP model by considering a density dependent plasma frequency, instead of the perturbative QCD results which is valid at extremely high temperature. It is motivated from a similar work in ultra-relativistic $(e, e^+ \text{ and } \gamma)$ system \[16, 17\] where one uses ultra-relativistic plasma frequency which depends on density and temperature.

Here we consider a simplest and well studied system, gluon plasma (GV). It is a system of gluons interacting via QCD interaction at finite temperature, $T$. As we discussed earlier, it is difficult to solve it analytically since the coupling constant is not weak. Therefore, we consider a phenomenological model where the thermal properties may be obtained by studying the thermal excitations of plasma modes. These thermal excitations are called quasi-gluons with the quantum numbers of gluons and with the thermal mass equal to the plasma frequency. Thus we have a gas of non-interacting or ideal quasi-gluons and following the standard SM \[18\], the logarithm of grand partition function or q-potential is given by

$$q = -\sum_{k=0}^{\infty} \ln(1 - e^{-\beta \epsilon_k}) ,$$

where $\epsilon_k$ is the single particle energy of quasi-gluon, i.e, gluon with temperature dependent mass, given by,

$$\epsilon_k = \sqrt{k^2 + m^2(T)} ,$$

where $k$ is the momentum and $m$ is the thermal mass which is equal to the plasma frequency. $\beta$ is defined as $1/T$. Instead of using the QCD perturbative expression for plasma frequency as done in the earlier qQGP models \[2, 5\], we model it as

$$\omega_p^2 \propto \alpha_s \frac{n}{T} = a_0 \alpha_s \frac{n}{T} ,$$

which is motivated from a similar work on electrodynamic plasma (EDP) \[16, 17\]. The constant $a_0 = \frac{8\pi}{3}$ in electron-positron EDP plasma and here we fix it by demanding that $\omega_p^2 \rightarrow \frac{g^2 T^2}{4\pi}$ as $T \rightarrow \infty$, the QCD perturbative result. $g^2$ is related to $\alpha_s$ through the relation $\alpha_s = g^2/4\pi$. Further, we use temperature dependent running coupling constant, $\alpha_s(T)$, which is motivated from lattice simulation of QCD \[6\]. $n$ is
the density of gluons which is taken to be the same as the number of quasi-gluons in quasiparticle models. Again from standard SM,

\[ n = \frac{g_f}{2\pi^2} \int_{0}^{\infty} dk k^2 \frac{1}{e^{\beta k^2 + a_0 \alpha_s} - 1}, \tag{3} \]

which may be rewritten as

\[ n = \frac{g_f}{2\pi^2} T^3 \int_{0}^{\infty} dx x^2 \frac{1}{e^{x^2 + a_0 \alpha_s} - 1}, \tag{4} \]

where \( g_f = 16 \), the degeneracy associated with the internal degrees of freedom. These equation need to be solved self-consistently because \( n \) which is to be determined is inside the integral through \( \omega_p \). Note that we don’t use the perturbative QCD expression for plasma frequency, which is appropriate at \( T \to \infty \), instead we calculate it self-consistently. Redefining the variables, the final equation to be solved self-consistently is,

\[ f_g^2 = \int_{0}^{\infty} dx x^2 \frac{1}{e^{x^2 + \tilde{a}^2 f_g^2} - 1} = \tilde{a}^2 f_g^2 \sum_{l=1}^{\infty} \frac{1}{l} K_2(\tilde{a} l f_g), \tag{5} \]

where

\[ \tilde{a}^2 \equiv \frac{g_f}{2\pi^2} a_0 \alpha_s, \]

and

\[ f_g^2 = \frac{2\pi^2}{g_f} \frac{n}{T^3}, \]

where \( K_2 \) is the modified Bessel function. Above equation, Eq. \( (5) \), may be solved numerically to get \( f_g^2 \) and then other TD quantities like energy density, pressure etc. may be calculated.

The energy density is given by,

\[ \varepsilon = \frac{g_f}{2\pi^2} T^4 \int_{0}^{\infty} dx x^2 \frac{\sqrt{x^2 + \tilde{a}^2 f_g^2}}{e^{\sqrt{x^2 + \tilde{a}^2 f_g^2}} - 1}, \tag{6} \]

or

\[ \varepsilon = \frac{g_f}{2\pi^2} T^4 \sum_{l=1}^{\infty} \left[ (\tilde{a} f_g l)^3 K_1(\tilde{a} f_g l) + 3 (\tilde{a} f_g l)^2 K_2(\tilde{a} f_g l) \right], \tag{7} \]

in terms of modified Bessel functions \( K \). Next, the pressure may be obtained from the TD relation,

\[ \varepsilon = T \frac{\partial P}{\partial T} - P, \tag{8} \]

on integration, which is the procedure we follow here. In this analysis, we have neglected vacuum energy or zero point energy with the assumption that the whole thermal properties of gluon plasma may described by the quasi-gluon excitations. A new TD consistent qQGP model with the vacuum energy is presented in Ref. \[5\].
3 Results:

In Fig. 1, we plotted energy density and pressure of GP from our model along with the LGT results [6]. First we solve, self-consistently, the integral equation for the density, Eq. (5), for a given temperature and obtain \( f_g(T) \). We have used 2-loop order running coupling constant, \( \alpha_s(T) \), which is similar to one used in LGT calculations, and is given by,

\[
\alpha_s(T) = \frac{6\pi}{(33 - 2n_f) \ln(T/\Lambda_T)} \left( 1 - \frac{3(153 - 19n_f) \ln(2 \ln(T/\Lambda_T))}{(33 - 2n_f)^2 \ln(T/\Lambda_T)} \right),
\]

where \( \Lambda_T \) is a parameter related to QCD scale parameter and \( n_f \) is the number of flavors which is zero in our case. Once we know \( f_g(T) \), energy density and pressure are calculated using Eq. (7) and Eq. (8) respectively. We adjust only one parameter of our model, \( t_0 = \Lambda_T/T_c \), so that we get the best fit to the energy density of LGT results [6]. We found that \( t_0 = 0.83 \). We need one more integration constant, which is not a parameter, to evaluate pressure which we fix to LGT data at \( T = T_c \). We may as well fix it at \( T = \infty \) such that \( P/P_{SB} = 1 \), where \( P_{SB} \) is the ideal gas pressure, but it is difficult to solve numerically.

In Fig. 2, we plotted \( f_g^2(T) \), \( \alpha_s(T) \) and \( \omega_p^2/(g^2 T^2) \) as a function of \( T/T_c \). The running coupling constant \( \alpha_s(T) \) increases rapidly as \( T \to T_c \) and \( \omega_p^2/(g^2 T^2) \) is small near \( T = T_c \) and asymptotically approaches \( 1/3 \), the QCD perturbative result.
4 Conclusions:

We presented a new quasiparticle model for gluon plasma where gluons acquire mass, approximately equal to the plasma frequency, due to collective effects. Since $\omega_p$ depends on density, which is one of the thermodynamic quantity to be determined from statistical mechanics, we solved the problem self-consistently.

Using this result, energy density and pressure were evaluated and by adjusting a single parameter, related to QCD scale parameter, we got a remarkable good fits to LGT results. Further extension of the model to QGP with quarks may be interesting, but not as simple as GP.

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