Determining Compositeness of Hadronic Resonances: the $\Lambda(1405)$ Radiative Decay and the $a_0(980)$-$f_0(980)$ Mixing

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Recently the concept of compositeness has been developed so as to distinguish whether interested hadrons are hadronic molecules or not. Here, in terms of compositeness, we investigate $\bar{K}N$ molecular structure of the $\Lambda(1405)$ resonance with the $\Lambda(1405)$ radiative decay and $K\bar{K}$ molecular structure of the $a_0(980)$ and $f_0(980)$ resonances with the $a_0(980)$-$f_0(980)$ mixing.

**KEYWORDS:** hadronic molecules, compositeness, $\Lambda(1405)$ radiative decay, $a_0(980)$-$f_0(980)$ mixing

1. Introduction

Although excellent successes of the constituent quark model tell us that ordinary hadrons consist of three quarks ($qqq$) for baryons and a quark-antiquark pair ($q\bar{q}$) for mesons [1], there should exist exotic hadrons, which are not able to be classified as $qqq$ for baryons and $q\bar{q}$ for mesons since the fundamental theory of strong interaction, QCD, does not prohibit such exotic systems as long as they are color singlet. In fact, there are several experimental indications that some hadrons do not fit into the classifications by the constituent quark model. For instance, the hyperon resonance $\Lambda(1405)$ has an anomalously light mass among the negative parity baryons and has been expected to be a $\bar{K}N$ molecular state rather than a three-quark state [2]. Moreover, the lightest scalar meson nonet shows inverted spectrum from the expectation with the $q\bar{q}$ composition and hence various exotic configurations have been proposed for the scalar mesons such as compact tetra-quark states [3] or $K\bar{K}$ molecules for $a_0(980)$ and $f_0(980)$ [4]. In addition, it is encouraging that charged quarkonium-like states were observed in the heavy-quark sector by the Belle collaboration [5].

Among exotic hadrons, hadronic molecules are of special interest since they are unique in that their constituents are not quarks but hadrons themselves, which will give various characteristic properties to hadronic molecules. For instance, a hadronic molecule can be a spatially extended object due to the absence of strong quark confining force. Actually, spatial size of $\Lambda(1405)$ was theoretically studied in Refs. [6–8] and it was found that its spatial size largely exceeds the typical hadronic size $\lesssim 0.8$ fm. Furthermore, the uniqueness of hadronic molecules allows us to construct their two-body wave functions in terms of the hadronic degrees of freedom, and recently compositeness was introduced as the contribution from the two-body wave function to the normalization of the total wave function for hadrons so as to identify hadronic molecules [9–12]. Since the compositeness can be evaluated from experimental observables, there is a possibility to determine experimentally the structure of candidates of hadronic molecules such as $\Lambda(1405)$. 
2. Dynamically generated hadrons and compositeness

When a hadron-hadron two-body interaction $V$ is sufficiently strong, the interaction can dynamically generate a bound state composed of the two hadrons. In this condition, the bound state pole appears in the complex energy plane of the scattering amplitude $T(s)$, which can be obtained from the Lippmann-Schwinger equation as

$$T(s) = V + VGT,$$

(1)

with the Mandelstam variable $s$ and the hadron-hadron two-body loop function $G(s)$. The bound state pole is described by the pole position $s_{\text{pole}}$ and its residue $g^2$ as

$$T(s) = \frac{g^2}{s - s_{\text{pole}}} + \text{(regular at } s = s_{\text{pole}}),$$

(2)

where $g$ can be interpreted as the coupling constant of the bound state to the hadron-hadron two-body state. In Refs. [10,13] it was found that the pole position $s_{\text{pole}}$ and coupling constant $g$ are related to the two-body wave function. In the following we treat a two-body scattering in a relativistic framework, in which the two-body wave function for the bound state generated with the Lippmann-Schwinger equation (1) is expressed as [12]

$$\tilde{\Psi}(q) = \frac{g}{s_{\text{pole}} - [\omega(q) + \omega'(q)]^2}, \quad \omega(q) \equiv \sqrt{m^2 + q^2}, \quad \omega'(q) \equiv \sqrt{m'^2 + q^2},$$

(3)

where $m$ and $m'$ are masses of the constituent hadrons. By using the two-body wave function, we can define compositeness of the bound state, $X$, as the contribution of the two-body wave function to the normalization of the total wave function [9, 11, 12]:

$$X = \int \frac{d^3q}{(2\pi)^3} \frac{\omega(q) + \omega'(q)}{2\omega(q)\omega'(q)} \left[\tilde{\Psi}(q)\right]^2 = -g^2 \frac{dG}{ds}(s = s_{\text{pole}}),$$

(4)

where the normalization factor $[\omega(q) + \omega'(q)]/[2\omega(q)\omega'(q)]$ guarantees the Lorentz invariance of $X$. We note that the compositeness is not an observable and hence is a model dependent quantity. On the other hand, components which cannot be reduced to hadronic two-body configurations, such as compact $q\bar{q}$ and $q\bar{q}q\bar{q}$ states, contribute to the elementariness $Z$, and its expression is [12]

$$Z = -g^2 \left[ G^2 \frac{dV}{ds} \right]_{s = s_{\text{pole}}}. $$

(5)

Then the sum of the compositeness $X$ and elementariness $Z$ gives the normalization of the total wave function as [7, 12]

$$\langle \Psi|\Psi \rangle = X + Z = -g^2 \left[ \frac{dG}{ds} + G^2 \frac{dV}{ds} \right]_{s = s_{\text{pole}}} = 1.$$  

(6)

The above discussions can be straightforwardly extended to the resonance states in coupled-channel approach, but we note that for resonance states both the compositeness $X$ and elementariness $Z$ becomes complex although the sum is exactly unity, $X + Z = 1$.

Studies on hadron-hadron scatterings with the Lippmann-Schwinger equation (1) and hadronic resonances dynamically generated as poles in Eq. (2) have been performed especially in the so-called chiral unitary approach for meson-meson [14] and meson-baryon [15] scatterings. In the chiral unitary approach the interaction $V(s)$ is taken from chiral perturbation theory and the approach successfully reproduces the light scalar and vector mesons in the meson-meson scatterings and $\Lambda(1405)$ in the
meson-baryon scatterings. The compositeness of these hadronic resonances was theoretically evaluated in Refs. [8, 12] and it was found that \( \Lambda(1405) \) and \( f_0(980) \) are dominated by the \( K\bar{N} \) and \( K\bar{K} \) composite states with the compositeness \( X_{K\bar{N}} \) and \( X_{K\bar{K}} \) close to unity, respectively, while compositeness of other scalar and vector mesons is not large compared to unity.

We here emphasize that, although the compositeness is not an observable, we can evaluate it from experimental observables via appropriate models. In this study we employ the expression in Eq. (4). The key is to determine the resonance pole position of the scattering amplitude and the coupling constant obtained on the pole position. Actually the determination can be done by using the \( \Lambda(1405) \) radiative decay for \( \Lambda(1405) \) [16] and the \( a_0(980)-f_0(980) \) mixing for \( a_0(980) \) and \( f_0(980) \) [17], as we will discuss below.

3. The \( \bar{K}N \) compositeness of \( \Lambda(1405) \) from its radiative decay width

Although the hyperon resonance \( \Lambda(1405) \) decays to \( \pi\Sigma \) with the branching ratio 100\% [1], the radiative decay of \( \Lambda(1405) \), \( \Lambda(1405) \to \Lambda\gamma \) and \( \Sigma^0\gamma \), is in principle possible. Indeed, in PDG there are “experimental” data on the \( \Lambda(1405) \) radiative decay width evaluated from an isobar model fitting of the decays of the \( K^- p \) atom [18]: \( \Gamma_{\Lambda\gamma} = 27 \pm 8 \) keV and \( \Gamma_{\Sigma^0\gamma} = 10 \pm 4 \) keV or \( 23 \pm 7 \) keV, which imply that the branching ratio of the radiative decay is \( \sim 0.1\% \). The \( \Lambda(1405) \) radiative decay is closely related to the structure of \( \Lambda(1405) \) as an \( E1 \) transition, and here we investigate a relation between the \( \bar{K}N \) compositeness for \( \Lambda(1405) \) and its radiative decay width via the \( \Lambda(1405)-\bar{K}N \) coupling constant as a free parameter.

Our formulation of the \( \Lambda(1405) \) radiative decay is based on that developed in Ref. [19]. The Feynman diagrams relevant to the radiative decay are shown in Fig. 1, which is obtained in the picture that the photon is emitted from meson-baryon components inside \( \Lambda(1405) \) without considering compact \( qqgqgq \) or \( qqqqg \) state for \( \Lambda(1405) \). As one can see from the diagrams, the decay amplitude contains coupling constants of \( \Lambda(1405) \) to meson-baryon states, thus we can relate the compositeness of \( \Lambda(1405) \) with the decay width via the coupling constant [see Eq. (4)]. In this study we fix the \( \Lambda(1405) \) pole position by using the mass and width taken from PDG [1]: \( s_{pole} = (M_{\Lambda(1405)} - i\Gamma_{\Lambda(1405)}/2)^2 \) with \( M_{\Lambda(1405)} = 1405 \) MeV and \( \Gamma_{\Lambda(1405)} = 50 \) MeV. The absolute value of the \( \Lambda(1405)-\bar{K}N \) coupling constant is determined by the absolute value of the \( \bar{K}N \) compositeness with Eq. (4), while the \( \Lambda(1405)-\pi\Sigma \) coupling constant is fixed by the \( \Lambda(1405) \) decay width, \( \Gamma_{\Lambda(1405)} = 50 \) MeV. The relative phase between \( \Lambda(1405)-\bar{K}N \) and \( \pi\Sigma \) coupling constants is not known, hence we calculate both maximally constructive and destructive conditions in order to evaluate the allowed range of the radiative decay width. We finally note that the divergence coming from each diagram in Fig. 1 is cancelled when contributions from three diagrams are summed due to the gauge and Lorentz symmetries.

In this strategy we can calculate the allowed range of the \( \Lambda(1405) \) radiative decay width with respect to each value of the absolute value of the \( \bar{K}N \) compositeness, and the result is shown in Fig. 2. The most prominent property of the radiative decay width can be seen in the \( \Lambda\gamma \) decay mode. Namely, the allowed range of the \( \Lambda\gamma \) decay width increases almost linearly with a small band as the absolute value of the \( \bar{K}N \) compositeness, \( |X_{\bar{K}N}| \), increases. This behavior comes from the fact that the contributions from the \( \pi^+\Sigma^- \) and \( \pi^-\Sigma^+ \) components are largely cancelled with each other in the

![Fig. 1. Feynman diagrams for the \( \Lambda(1405) \) radiative decay employed in this study [16]. In the figure (a) \( M \) and \( B \) denote mesons and baryons, respectively, and \( \gamma^0 \) is \( \Lambda \) or \( \Sigma^0 \).](image)
Λγ decay mode and hence the K−p component dominates the Λγ decay, as discussed in Ref. [19]. Therefore, a large Λγ decay width implies a large Λγ component inside Λ(1405) and thus the radiative decay Λ(1405) → Λγ is suited to investigate the Λγ component. On the other hand, the Σ0γ decay width is nonzero even when the Λγ component is absent, |X_{Λγ}| = 0, since there is no cancellation between contributions from the π±Σ± component in contrast to the Λγ decay mode. Then comparing with the “experimental” value of the decay width, we can estimate the Λγ component as |X_{Λγ}| = 0.5 ± 0.2 from Γ_{Λγ} = 27 ± 8 keV, |X_{Λγ}| > 0.5 from Γ_{Σγ} = 10 ± 4 keV, and |X_{Λγ}| can be arbitrary for Γ_{Σγ} = 23 ± 7 keV. This estimation implies that Λγ seems to be the largest component inside Λ(1405). We note that the Λ(1405) pole position dependence of our result is small as discussed in Ref. [16].

In this study we have neglected compact qqq and qqqq̄ configurations for Λ(1405), by assuming that Λ(1405) is dominated by the meson-baryon components. However, in general we have to take into account them when we calculate the radiative decay width with |X_{Λγ}| < 1. Thus, here we roughly estimate the behavior of the Λγ decay width taking into account the qqq configuration for Λ(1405) as well. On the one hand, as we have seen above, hadronic molecules can have a radiative decay width of ≤ 100 keV. On the other hand, excited baryons which are expected to have a qqq configuration, such as N(1535), have radiative decay widths of ~ 100 keV−1 MeV, which implies that the radiative decay width accompanied by a quark transition inside the hadron would be relatively large. As a consequence, when we describe Λ(1405) as a mixture of the compact qqq state and the Λγ component, the correct normalization of the total wave function (6), the slope of Γ_{Λγ} would be negative with respect to |X_{Λγ}| and the decay width becomes, for instance, Γ_{Λγ} ~ 200 keV for |X_{Λγ}| = 0 and Γ_{Λγ} ~ 50 keV for |X_{Λγ}| = 1. In this sense, the small “experimental” decay width Γ_{Λγ} = 27 ± 8 keV for Λ(1405) might be a signal that Λ(1405) is a hadronic molecule.

4. The K̄K compositeness of a0(980) and f0(980) from their mixing intensity

Since two scalar mesons a0(980) and f0(980) have almost degenerate masses, there is a possibility that a0(980) and f0(980) are mixed with each other in processes which break isospin symmetry. Especially it was pointed out in Ref. [20] that the mixing effect should be unusually enhanced around the K̄K threshold due to the difference of the phase spaces of K+K− and K0̄K0 [see Fig. 3(a) and (b)]. The a0(980)-f0(980) mixing effect was recently observed in Ref. [21], in which decay of J/ψ to φη was used to confirm the mixing effect. In Ref. [21] the mixing intensity ξ_{fa} was introduced as

\[ ξ_{fa} \equiv \frac{Br(J/ψ → φf_{0}(980) → φa_{0}^{0}(980) → φπ^{0}η)}{Br(J/ψ → φf_{0}(980) → φπ^{0}π)} \]  

and its value was obtained as ξ_{fa} = 0.60 ± 0.20(stat) ± 0.12(sys) ± 0.26(par)% and ξ_{fa|upper limit} = 1.1% (90% C.L.) [21].

The a0(980)-f0(980) mixing should be closely related to the K̄K structure of the scalar mesons a0(980) and f0(980) since the mixing amplitude contains the a0(980)-K̄K and f0(980)-K̄K coupling
constants. Therefore, through the $a_0(980)$- and $f_0(980)$-$K\bar{K}$ coupling constants we can investigate the $K\bar{K}$ component inside the scalar mesons $a_0(980)$ and $f_0(980)$ from their mixing intensity $\xi_{fa}$. Here we give a constraint on the $K\bar{K}$ compositeness of $a_0(980)$ and $f_0(980)$, which we denote as $X_\alpha$ and $X_f$, respectively, from the $a_0(980)$-$f_0(980)$ mixing intensity.

For the calculation of the $a_0(980)$-$f_0(980)$ mixing we employ three Feynman diagrams shown in Fig. 3. The diagrams (a) plus (b) in Fig. 3 contribute to the mixing at the leading order, while the diagram (c) in Fig. 3 gives a subleading order contribution. From these diagrams we nonperturbatively calculate the full propagators of $a_0(980)$ and $f_0(980)$ and the transition between $a_0(980)$ and $f_0(980)$ by summing up all the contributions $a_0(980) \to f_0(980) \to a_0(980) \to \cdots$. The mixing intensity $\xi_{fa}$ is calculated as the ratio of the two partial decay widths according to the experimental analysis in Eq. (7). In this formulation the model parameters for the mixing intensity $\xi_{fa}$ are the masses and widths of the scalar mesons, $[M_\alpha$ and $\Gamma_\alpha$ for $a_0(980)$ and $M_f$ and $\Gamma_f$ for $f_0(980)$, respectively], and the $a_0(980)$- and $f_0(980)$-$K\bar{K}$ coupling constants. In our strategy, we appropriately fix the masses and widths of the scalar mesons, while the values of the $a_0(980)$- and $f_0(980)$-$K\bar{K}$ coupling constants move independently within appropriate ranges so as to evaluate simultaneously the mixing intensity $\xi_{fa}$ and the absolute value of the $K\bar{K}$ compositeness of $a_0(980)$ and $f_0(980)$, $|X_\alpha|$ and $|X_f|$, respectively, with Eq. (4).

Now we fix the parameters $M_\alpha = M_f = 980$ MeV, $\Gamma_\alpha = 100$ MeV, and $\Gamma_f = 50$ MeV, calculate both the mixing intensity and the $K\bar{K}$ compositeness, and investigate the values of $|X_\alpha|$ and $|X_f|$ favored by the experimental mixing intensity. The result of the allowed region for $|X_\alpha|$ and $|X_f|$ in the $|X_\alpha|$-$|X_f|$ plane is plotted in Fig. 4. In the figure the shaded area corresponds to the allowed region by the experimental mixing intensity $\xi_{fa} = 0.60 \pm 0.20 \pm 0.12 \pm 0.26\%$, and the solid line to the border line for the mixing intensity $\xi_{fa}|_{\text{upper limit}} = 1.1\%$. As one can see from the figure, the region $|X_\alpha| \approx |X_f| \approx 1$ is not favored, which excludes the possibility that both $a_0(980)$ and $f_0(980)$ are simultaneously $K\bar{K}$ molecules. We have also examined several parameter sets and found that in most
parameter sets $|X_a|$ and $|X_f|$ cannot be simultaneously take values close to unity with the constraint of the experimental mixing intensity [17]. This implies that the statement that both $a_0(980)$ and $f_0(980)$ are simultaneously $K\bar{K}$ molecules is questionable.

5. Summary

In this study we have investigated hadronic molecular structure of $\Lambda(1405)$, $a_0(980)$, and $f_0(980)$ in terms of compositeness, which is defined as the contribution of the two-body wave function to the normalization of the total wave function and measures amount of the two-body component inside the hadron. We have seen that the compositeness is not only a theoretical concept but also a quantity which can be evaluated from experimental observables via appropriate models. For instance, from the $\Lambda(1405)$ radiative decay width we have obtained an implication that the $\bar{K}N$ compositeness of $\Lambda(1405)$ is $|X_{\bar{K}N}| \gtrsim 0.5$, and from the $a_0(980)$-$f_0(980)$ mixing intensity we have found that the statement that both $a_0(980)$ and $f_0(980)$ are simultaneously $K\bar{K}$ molecules is questionable.

At last we emphasize again that we can investigate hadronic molecular structure without relying directly upon the underlying theory, QCD, since the constituents of hadronic molecules are hadrons themselves, which are color-singlet states and hence observable. Due to this fact, one can in principle construct hadronic two-body wave functions with hadronic degrees of freedom and use the wave functions to evaluate the two-body component inside the hadronic molecules. In this sense, the $\Lambda(1405)$ radiative decay and the $a_0(980)$-$f_0(980)$ mixing are exactly the experimental phenomena with which one can discuss the structure of hadronic resonances without considering QCD directly.

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