Possibility of a Femtosecond Pulse Propagation in a Nonlinear Chirped Soliton Mode in a Medium with Induced Photoluminescence

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Abstract. The soliton mode of a femtosecond pulse propagation in a medium with induced photoluminescence is theoretically investigated. Our study is based on a pair of nonlinear Schrödinger equations, derived for the slowly varying amplitudes of the basic wave and the luminescence wave. Under assumption of the mutual trapping of the waves, we discuss a bistable mode of the waves propagation and derived two families of nonlinear chirped elliptic soliton solutions corresponding to this bistable mode. We confirmed our analytical results by numerical simulation results, obtained using conservative finite-difference scheme.

1. Introduction
Photoluminescence, induced from the noble metals, is being widely investigated nowadays because of the use of metal nanoparticles in numerous applications, for example [1-4]. Today, the investigations deal with the various aspects of this phenomenon, including the significant enhancement of the photoluminescence from metal nanoparticles in comparison with metal films [5-7]. A number of mathematical models are proposed up to now to describe the peculiarities of the photoluminescence from noble metals and metal nanoparticles [8-10].

Among the important problems of laser radiation propagation under the condition of induced photoluminescence of a medium, an achievement of the self-similar mode at such propagation is of great interest. This mode is very important for data retrieving from multilayered data storage devices based on noble metal nanoparticles [1,3,4]. Below we investigate a special kind of the self-similar mode of laser pulse propagation in a medium with induced photoluminescence – a mode of the nonlinear chirped soliton. Opposite to the case of classical soliton, this laser pulse propagation mode is characterized by nonlinear frequency chirp of a pulse. It should be noted that the chirped solitons play an important role in optical communication links and optical signal processing systems [11,12]. Mathematically, chirped solitons can be obtained, for example, as solutions of the generalized Schrödinger equation with various nonlinear terms [12-14]. To our knowledge, for the first time, an analytical and numerical investigation of the nonlinear chirped solitons at a femtosecond pulse propagation in a cubic nonlinear waveguide was carried out in [15,16].

To write the equations describing a process under consideration we use the semi-classical approach: we use the approximation of a slowly varying envelope to describe the electric field propagation in the framework of a set of the nonlinear Schrödinger equations, and to describe a response of a medium, possessing of the three energy levels, we use a density matrix formalism. Then
we believe that the medium response is quasi steady-state and therefore we neglect the time-dependence of the matrix density elements [17,18]. Under these assumptions we demonstrate an existence of two families of nonlinear chirped elliptic solitons for a bistable mode of the pulse propagation – a family of solitons on a “pedestal” [19] and a family of singular periodic travelling wave or stationary solutions [20]. We also demonstrate their existence using a computer simulation.

It should be noted that various families of the Schrödinger equation solitons described by elliptic functions were obtained in [19,20], for example, using various ansätze. Similar families of periodic travelling wave solutions - cnoidal waves and singular periodic travelling wave solutions - can also be found in Korteweg-de Vries equation (see, for example, [21]).

2. Problem statement
We consider a femtosecond laser pulse propagation in a medium with induced photoluminescence in the framework of the semi-classical approach [17,18] and the slowly varying envelope for the electric field wave. We consider one-photon absorption of laser energy and there energy levels of atoms (figure 1). We suppose that the populations of the higher energy levels 2 and 3 are equal to zero before the action of the laser pulse while the population of the lowest energy level 1 is equal to unity. We believe that the basic wave at the frequency \( \omega_{21} \), which is equal to the transition frequency between the energy levels 3 and 1, falls on the medium causing the population growth of the energy level 3. The energy transition between the close energy levels 2 and 3 occurs without emission while the energy transition between the energy levels 2 and 1 corresponds to the luminescence wave at the frequency \( \omega_{21} \). We also suppose that the medium response can be described as stationary one in the framework of the density matrix formalism.

Figure 1. Schemes of the atom energy levels and frequencies of waves under consideration.

In this case, the slowly varying envelopes for the basic and the luminescence waves are governed by the pair of Schrödinger equations:

\[
\begin{align*}
\frac{\partial A_{21}}{\partial z} + iD_{2,1} \frac{\partial^2 A_{21}}{\partial t^2} + \xi_{21} \left( 1 + \bar{\sigma}_{21} |A_{21}|^2 + \sigma_{31} |A_{31}|^2 \right) A_{21} &= 0, \\
\frac{\partial A_{31}}{\partial z} + iD_{2,2} \frac{\partial^2 A_{31}}{\partial t^2} + \xi_{31} \left( 1 + \bar{\sigma}_{21} |A_{21}|^2 + \sigma_{31} |A_{31}|^2 \right) A_{31} &= 0.
\end{align*}
\]

Above \( A_{21} \) and \( A_{31} \) are dimensionless slowly varying envelopes of the luminescence wave packet and the incident basic wave packet normalized on the square roots from the maximum incident pulse intensity \( I_{31}^{\text{max}} \) on the basic frequency. Parameters \( D_{2,1} \) and \( D_{2,2} \) characterize the second order dispersion (SOD) of the luminescence wave and the basic wave, respectively. Coefficients \( \xi_{21} \), \( \xi_{31} \), \( \bar{\sigma}_{21} \), \( \sigma_{31} \), and \( \sigma_{21} \) describe the self-action and cross-modulation of the interacting waves and are related to the physical variables in the following manner:
In formulas (3), the parameter $\tau_p$ is the duration of incident basic pulse. The parameter $\gamma_{mn}$ is the damping rate of the off-diagonal element and it characterizes the relaxation time of the density matrix off-diagonal element, $k_{21}$ is the wave vector of the basic wave; $d_{mn}$ is a dipole moment associated with the transition between energy levels $m \rightarrow n$, $w_{mn}$ is the rate per atom, at which the population decays due to spontaneous transitions between the energy level $m$ and the energy level $n$. $\hbar$ is the Planck's constant. Deriving equations (1)-(3), we suppose that the energy level transition between the close energy levels 2 and 3 occurs without emission and we neglected the decay rates for the transitions from the lower energy level 1 to the higher energy levels 2 and 3.

Note that parameters $\xi_{21}$, $\xi_{31}$ have opposite signs depending on the sign of the difference between the energy levels 2 and 3. In particular, if the energy of the level 3 is larger than the energy of the level 2 (figure 1a), the parameter $\xi_{21}$ is negative while the parameter $\xi_{31}$ is positive, and vice versa (figure 1b).

Equations (1) and (2) are supplemented with initial and boundary conditions

$$A_{21}(z=0,t) = A_{01}(t), \quad A_{31}(z=0,t) = A_{01}(t), \quad 0 < t < L_z, \quad (3)$$

$$A_{m1}(z,t=0) = A_{m1}(z,t=L_z) = A_{01}(z), \quad 0 \leq z \leq L_z, \quad m = 2,3, \quad (4)$$

here $L_z$ characterizes the time interval under consideration, $L_z$ characterizes the pulse propagation distance. Below we consider that both waves - the basic wave and the luminescence wave – fall on a medium and their characteristics (pulse duration, pulse shape) are the same. It means that we discuss the conditions of the soliton mode appearance for the wave propagation under their mutual-trapping.

3. Self-similar solution

In order to formulate the eigenvalue problem to develop a self-similar solutions of the problem under consideration, we use the following representation for the slowly varying amplitudes:

$$A_{21}(z,t) = B_1(\zeta)e^{-i\lambda_1\zeta}, \quad A_{31}(z,t) = B_1(\zeta)e^{-i\lambda_2\zeta}, \quad \zeta = t + \nu z, \quad (5)$$

where $\lambda_1$ and $\lambda_2$ are real parameters which represent the eigenvalues, parameter $\nu$ characterises the travelling wave velocity. Substituting representations (5) into equations (1) and (2), we obtain the following equations

$$\nu \frac{dB_1}{d\zeta} + iD_{21} \frac{d^2B_1}{d\zeta^2} + i\xi_{21}(1 + \bar{x}_{21} |B_1|^2 + \bar{x}_{31} |B_2|^2)B_1 = i\lambda_2 B_1, \quad (6)$$
\begin{equation}
\nu \frac{dB_2}{d\zeta} + iD_{2,2} \frac{d^2 B_2}{dt^2} + i \xi_{31} \left( 1 + \bar{\omega}_{21} |B_1|^2 + \bar{\omega}_{21} |B_2|^2 \right) B_2 = i \lambda_2 B_2 .
\end{equation}

Then we represent the complex amplitudes $B_1(\zeta)$ and $B_2(\zeta)$ in a trigonometric form

\begin{equation}
B_j(\zeta) = f_j(\zeta) e^{\xi_j(\zeta)} , \quad j = 1, 2
\end{equation}

and obtain the following equations for the amplitudes $f_j(\zeta)$ and phases $S_j(\zeta)$:

\begin{equation}
\nu \frac{df_j}{d\zeta} - D_{2,j} f_j \frac{d^2 S_j}{d\zeta^2} - 2D_{2,j} \frac{df_j}{d\zeta} \frac{dS_j}{d\zeta} = 0 , \quad j = 1, 2 ,
\end{equation}

\begin{equation}
\nu f_j \frac{dS_j}{d\zeta} + D_{2,j} \left( \frac{d^2 f_j}{d\zeta^2} - f_j \left( \frac{dS_j}{d\zeta} \right)^2 \right) + \xi_j \left( 1 + \bar{\omega}_{j+1,j} f_j + \bar{\omega}_{j+1,j} f_{j+1} \right) f_j = \lambda_j f_j , \quad j = 1, 2 .
\end{equation}

The order of equations (9) can be reduced:

\begin{equation}
\frac{dS_j}{d\zeta} = \frac{\nu}{2D_{2,j}} + \frac{\alpha_j}{f_j} , \quad j = 1, 2 ,
\end{equation}

here the constant value $\alpha_j$ is determined by the instantaneous frequency $\frac{df_j}{d\zeta}$ and the amplitude $f_j$ at a certain time moment.

Substituting equations (11) into equations (10), we obtain

\begin{equation}
\frac{d^2 f_j}{d\zeta^2} - \frac{\alpha_j^2}{f_j^3} + \frac{\xi_j}{D_{2,j}} \left( \bar{\omega}_{j+1,j} f_j^2 + \bar{\omega}_{j+1,j} f_{j+1}^2 \right) f_j - \left( \frac{\lambda_j - \xi_j}{D_{2,j}} - \frac{\nu^2}{4D_{2,j}^2} \right) f_j = 0 , \quad j = 1, 2 .
\end{equation}

Below we consider a mutual-trapping mode of laser pulses propagation. In this case, the following relation between the amplitudes of the luminescence and the basic waves should be valid

\begin{equation}
f_1(\zeta) = \tilde{c} f_2(\zeta) \equiv f(\zeta)
\end{equation}

with the similarity constant $\tilde{c}$, and function $f(t)$ satisfies the equation

\begin{equation}
\frac{1}{f} \frac{d^2 f}{dt^2} - \alpha_f^2 f^3 + Cf^2 - Q = 0 ,
\end{equation}

here

\begin{equation}
\alpha_f^2 = (\alpha \tilde{c})^2 = \alpha^2 , \quad C = \frac{\tilde{c}_{21}}{D_{2,1}} \left( \frac{\omega_{21} + \bar{\omega}_{21}}{c^2} \right) = \frac{\tilde{c}_{31}}{D_{2,2}} \left( \frac{\omega_{21} + \bar{\omega}_{21}}{c^2} \right) , \quad Q = \frac{\lambda_2 - \tilde{c}_{21}}{D_{2,1}} - \frac{\nu^2}{4D_{2,1}^2} = \frac{\lambda_2 - \tilde{c}_{31}}{D_{2,2}} - \frac{\nu^2}{4D_{2,2}^2} .
\end{equation}

Equation (14) is supplemented with boundary conditions

\begin{equation}
f(\zeta = 0) = f(\zeta = L_4) = |A_4^0(0)| .
\end{equation}

Equation (14) can be reduced to following equation with respect to the pulse intensity $I = f^2$:
\[ \frac{dI}{d\varsigma} = \pm \sqrt{-2C\alpha^3 + 4QI^2 + RI - 4\alpha^2} = \pm \sqrt{\psi(I)}, \quad (17) \]

\[ \frac{R}{4} = \left( \frac{df}{d\varsigma} \right)_{\varsigma_o}^2 + \left( \frac{\alpha^2 + \frac{1}{2}Cf'^2 - Qf'^2}{f^2} \right)_{\varsigma_o} \quad (18) \]

The constant \( R \), the sign and the initial condition for equation (17) are determined by the function \( f(\varsigma) \) and its first derivative \( \frac{df}{d\varsigma} \) at coordinate \( \varsigma = \varsigma_o \), which can be chosen in a manner convenient for the further analysis.

Depending on the roots of the function \( \psi(I) \), equation (17) admits either solitary wave solution or periodic wave solution – cnoidal wave solution which is expressed through elliptic functions (elliptic integrals). Note that the solutions of the Schrödinger equation expressed through elliptic functions were also obtained in [19] using the different approach - the ansatz involving linear dependence between the real and imaginary parts of the slowly varying amplitude.

Equations (11), (17) determine nonlinear chirped self-similar solution only if \( \alpha \neq 0 \). If \( C < 0 \), equation (17) admit an unbounded solution which corresponds to a singular periodic solution [20] of the problem (1)-(4). Indeed, in this case, the polynomial \( \psi(I) \) has a positive root \( a \) \( (a > 0) \), so that it possesses positive values \( (\psi(I) > 0) \) if the pulse intensity is larger than \( a : I > a \). If in addition to this root, the polynomial has two other different positive roots \( 0 < c < b < a \), then the problem (1)-(4) admits two solutions: the first one is an unbounded periodic solution, which corresponds to the nonlinear chirped elliptic periodic solution, and the second one corresponds to the nonlinear chirped bright elliptic soliton solution - the soliton solution at the CW background [22] or a soliton at a “pedestal” [19]. This means the bistability of the considered system. Note that the singular periodic solutions correspond to dark solitons (the solitons of limited intensity which have the intensity minimum at the soliton center) on a finite time interval (or on an interval along a spatial coordinate).

In the case of bistability, the nonlinear chirped singular elliptic periodic solution is described by the following pulse shape

\[ I(\varsigma) = b + (a - b)nc^2(\eta\varsigma|m), \quad m = \frac{b - c}{a - c}, \quad \eta = \sqrt{-C\frac{a - c}{2}} \quad (19) \]

and phase distributions

\[ S_2(\varsigma) - \frac{\nu}{2D_{2,2}} = S_1(\varsigma) - \frac{\nu}{2D_{2,1}} = S(\varsigma), \quad (21) \]

\[ S(\varsigma) = \frac{\alpha}{b} \varsigma + \frac{\alpha(b - a)}{ab\eta} \Pi\left( \frac{b}{a}, am(\eta\varsigma|m), m \right). \quad (22) \]

Dependence (19) follows from equation (17) for the initial condition \( I(0) = a \). Phase distributions, determined by equations (21) and (22), are the solutions of equations (11) if \( S_1(0) = S_2(0) = 0 \). In equations (19) and (22), function \( nc(u|m) = cn^{-1}(u|m) \) is the reciprocal of an elliptic cosine; \( \Pi(c, \varphi, m) \) is an incomplete elliptic integral of the third kind and \( am(u|m) \) is a Jacobi elliptic amplitude [23].

For the nonlinear chirped bright elliptic soliton solution, equation (17) yields

\[ I(\varsigma) = c + (b - c)sn^2(\eta\varsigma|m) \quad (23) \]
if $I(0) = c$, and the phases are determined by equations (21), where

$$S(\zeta) = \frac{\alpha}{cn} \Pi \left( \frac{b-c}{c}, am(\eta \zeta \| m), m \right)$$

(24)

if $S_1(0) = S_2(0) = 0$. In equations (23) and (24), the parameters $\eta$ and $m$ are determined by equations (19) and $sn(u \| m)$ is a Jacobi elliptic sine [23].

According to equations (19) and (23), the pulses amplitudes for both modes – the bright and the singular elliptic soliton mode - are periodic functions with the same period

$$L_{n,\text{per}} = \frac{2}{\eta} K(m),$$

(25)

where $K(m)$ is a complete elliptic integral of the first kind [23], parameters $\eta$ and $m$ are determined by equation (19).

4. Computer simulation results

To verify our analytical formulas (19)-(25), we used them as the incident distributions for the basic wave and the luminescence wave complex amplitudes for the following parameter set:

$$\xi_{21} = -4, \, \xi_{31} = 2, \, D_{21} = D_{22} = 1, \, \omega_{21} = -1, \, \omega_{31} = 0.25, \, \bar{\omega}_{21} = 1/12, \, \bar{\omega}_{31} = 2.$$  

(26)

This parameter set yields the similarity constant $\tilde{\epsilon} = \sqrt{3}$ and parameter $C = -2/3$ of the cubic equation $\psi(I) = 0$ (equation (17)). We have also chosen the other parameters of the cubic equation as $Q = -8/15, \, R = 13/65$ and $\alpha = \sqrt{160}, \,$ so that all the three roots of the cubic equation are positive and different: $a = 1, \, b = 0.5, \, c = 0.1$; the solutions period is $L_{1,\text{per}} = 6.607971$ and the eigenvalues are $\lambda_1 = -4.53333, \, \lambda_2 = 1.466667$ for $\nu = 0, \, \lambda_3 = -4.283333, \, \lambda_2 = 1.716667$ for $\nu = 1$ and $\lambda_4 = -4.523333, \, \lambda_2 = 1.476667$ for $\nu = 0.2$ (equation (15)).

In this case, according to the results in the previous section 3, the bistable mode of waves propagation takes place. So, the luminescence and the basic waves can propagate in two different modes: the nonlinear chirped singular elliptic periodic solution mode and the nonlinear chirped bright elliptic soliton mode.

Note that in our numerical simulations, we consider the singular periodic solution over its half period. As it was already mentioned, in this case, the singular solution corresponds to a dark soliton mode (the soliton of limited intensity with the minimum intensity in the soliton center). So below we will refer to this mode of pulse propagation as a dark mode. We also considered one period of solution for the bright soliton mode.

So, to obtain the mentioned modes of laser pulse propagation, we set the following initial conditions

$$A_{21}(z = 0, t) = \sqrt{I(t-\tau)} \exp(iS_1(t-\tau)), \quad 0 < t < L_t,$$

(27)

$$A_{31}(z = 0, t) = \tilde{\epsilon}^{-1} \sqrt{I(t-\tau)} \exp(iS_3(t-\tau)), \quad 0 < t < L_t,$$

(28)

and boundary conditions

$$A_{22}(z, t = t_b) = \sqrt{I(t_b + \nu z)} \cdot \exp \left( i \left( S_1(t_b + \nu z) - \lambda_1 z \right) \right), \quad 0 \leq z \leq L_z,$$

(29)

$$A_{32}(z, t = t_b) = \tilde{\epsilon}^{-1} \sqrt{I(t_b + \nu z)} \cdot \exp \left( i \left( S_2(t_b + \nu z) - \lambda_2 z \right) \right), \quad 0 \leq z \leq L_z,$$

(30)
where \( \tau = L_t/2 \), \( t_b = \pm L_t/2 \) for the dark (singular) elliptic soliton mode and \( \tau = 0 \), \( t_b = \{0, L_t\} \) for the bright elliptic soliton mode. The functions \( I(\zeta), S_1(\zeta) \) and \( S_2(\zeta) \) are determined by formulas (19), (21) and (22) if we consider the dark (singular) elliptic soliton mode, or by formulas (21), (23) and (24) if we consider the bright elliptic soliton mode. The time interval \( L_t \) is chosen as the solution period \( (L_t = L_{t\text{per}}) \) for the bright elliptic soliton mode and as a half of this period \( (L_t = 0.5L_{t\text{per}}) \) for the dark (singular) elliptic soliton mode.

Note that for a stationary (unmoving) mode of laser pulse propagation (if \( \nu = 0 \)), boundary conditions (29), (30) correspond to a homogeneous shift of the soliton phase along the \( z \)-coordinate.

Figure 2 illustrates the luminescence wave propagation in the bright elliptic soliton mode corresponding to an unmoving (stationary) solution \( (\nu = 0, \text{figure } 2a) \) and a travelling wave solution \( (\nu = 1, \text{figure } 2b) \). The incident pulse shapes for both waves are shown in figure 3a, the chirp distributions for both waves are the same. If \( \nu = 0 \), the pulse shapes and chirp distributions remain unchanged, while the phase distributions undergo the homogeneous shift along the \( z \)-coordinate according to the eigenvalues (figure 4a). If \( \nu = 1 \), the maximal intensity positions of both waves move to the left, and the pulses shapes satisfy formula (23) at each section of the medium (figures 3a-c). The chirp distributions for both waves are the same and relate to the intensity distributions according to formula (11). The phase distributions undergo homogeneous shifts along the time- and the \( z \)-coordinate according to the velocity of the travelling wave (along the time-coordinate) and the eigenvalues (along the \( z \)-coordinate); the phase distributions for the travelling mode are shown in figure 4b for \( z = 0 \) and \( z = 10 \).
Figure 4. Phase distributions in the bright elliptic soliton mode for $\nu=0$ (a) and $\nu=1$ (b). Black lines correspond to the luminescence wave, red lines – to the basic wave, blue lines indicate the coincident phase distributions for both waves at $z=0$.

The luminescence wave propagation in the dark (singular) elliptic soliton mode is shown in figure 5. In the unmoving mode ($\nu=0$), the pulses shapes and chirp distributions remain unchangeable as the pulses propagate through the medium (figure 6a). The phase distributions undergo the homogeneous shift along the z-coordinate according to the eigenvalues (figure 7, dashed lines). In the travelling wave mode ($\nu=0.2$), the pulses shapes shift to the left according to $\nu=0.2$ (figure 6b); the chirp distributions are related with high accuracy to the pulses shapes according to formula (11); the phases distributions undergo homogeneous shifts along the time- and z-coordinate according to parameter $\nu$ and the eigenvalues (figure 7, solid lines).

Figure 5. Evolution of the luminescence wave for $\nu=0$ (a) and $\nu=0.2$ (b) in the dark (singular) elliptic soliton mode.

5. Conclusions
We investigated numerically and analytically a bistable mode of the luminescence wave and the basic wave propagation in the media with induced photoluminescence. We have shown that two families of nonlinear chirped soliton solutions – bright elliptic solitons at the CW background and dark (singular) elliptic solitons – can simultaneously exist in the considered system. Both soliton families are characterized by the same period along the time coordinate and can be realized in an unmoving (stationary) soliton mode or in a travelling wave mode depending on the boundary conditions. We have demonstrated the realization of obtained solutions for both families in computer experiment which was carried out using conservative difference scheme.

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Figure 6. Pulse shapes for luminescence wave (black lines) and basic waves (red lines), and chirp distributions for the luminescence wave (solid blue lines) and the basic wave (dashed blue lines) in the dark (singular) elliptic soliton mode for \( \nu = 0 \) (a) and \( \nu = 0.2 \) (a,b).

Figure 7. Phase distributions in the dark (singular) elliptic soliton mode for \( \nu = 0 \) (dashed lines) and \( \nu = 0.2 \) (solid lines). Black lines correspond to the luminescence wave, red lines – to the basic wave, blue lines indicate the coincident phase distributions for both waves at \( z=0 \).

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