NEUTRINOS AS PSEUDO-ACOUSTIC ETHER PHONONS

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Abstract. Recently the author has proposed a condensed matter model which gives all fermions and gauge fields of the standard model of particle physics. In the model, the inertness of right-handed neutrinos is explained by an association with translational symmetry.

We argue that this association may be used as well to explain the small neutrino masses. They appear to be pseudo-Goldstone particles associated with an approximate translational symmetry of a subsystem.

Then we propose to explain the masslessness of $SU(3)_c \times U(1)_{em}$ with an unbroken $SU(3) \times U(1)$ gauge symmetry of the model.

We also detect a violation of a necessary symmetry property in the lattice Dirac equation and present a fix for this problem.

1. Introduction

In [1], a condensed matter model was proposed as a candidate for unification of the standard model of particle physics with gravity: It gives all the fermions and gauge fields of the standard model, together with a metric theory of gravity, as effective fields. Despite this, it is extremely simple: It consists of a lattice of elementary cells, with some other material between them, in classical Newtonian space. The three generations, three colors, and three generators of isospin become associated in different ways with spatial directions. Particles appear only as quantum excitations of effective fields (like phonons). Contrary to common wisdom, fermions have been obtained by canonical quantization of real fields: A potential with two minima gives an effective $\mathbb{Z}_2$-valued theory in its low energy sector, and this theory can be transformed into a fermionic lattice theory by some non-local transformation. Quarks define waves of linear deformations of the cells, while leptons define waves related with their translations. The doubling effect gives eight complex fields – a whole electroweak doublet – out of a single scalar field on the lattice. Gauge fields appear of two types: Wilson gauge fields give the strong interaction group $U(3)_c$, while deformations of the lattice lead to an effective gauge-like theory with maximal group $U(2)_L \times U(1)_R$. A requirement of ground state neutrality reduces the group to $G_{max} \cong S(U(3)_c \times U(2)_L \times U(1)_R)$, and the requirement of anomaly freedom gives a final reduction to the SM gauge group. Last but not least, the gravitational field is described in a variant of the ADM decomposition by the density $\rho$, three-velocity $v^i$ and the three-dimensional stress tensor $\sigma^{ij}$.

The aim of this paper is to present some progress in the development of the model. The main open problem is that the model, as presented in [1], is unable to explain the SM particle masses: It has full Euclidean symmetry. Because rotations of the lattice rotate the generations, such an isotropic model is unable to assign different masses to particles of the different generations. To explain the SM mass matrices, the $SO(3)$ rotational symmetry has to be broken, which has been left to future research.

Despite the fact that we do not present here a model with broken symmetry too, it appears possible to obtain some qualitative information about the mass terms: All we have to do is to apply the standard connection of exact symmetries with massless particles as well as approximate symmetries with particle with small masses. We identify the three neutrinos with three pseudo-Goldstone particles (or pseudo-acoustic phonons) associated with an approximate translational symmetry. It is especially remarkable that the association of the
neutrinos with translations has been already made in [1] for a completely different reason – the explanation of the inertness of the right-handed neutrinos.

We also argue that the $SU(3)_c \times U(1)_{em}$ massless subgroup of gauge group of the standard model may be explained by an exact $SU(3) \times U(1)$ lattice gauge symmetry of the lattice model. This is straightforward for the $SU(3)_c$ part, but nontrivial for the $U(1)$ part, because the $U(1)$ action on the fundamental level differs from the action of $U(1)_{em}$. So we have to assume some deformation of the action.

Another point is that the lattice Dirac equation as presented in [1] needs modification: To obtain translational symmetry of the weak $SU(2)_L$ action the direction of translation should be right-handed – an exact +1 eigenstate of the lattice $\gamma^5$ operator – but it isn’t. We present a “quick and dirty” fix of this problem, which has the disadvantage that it reduces the symmetry of the model: instead of a single type of elementary cells, we would have to introduce different types of cells on even and odd nodes of the lattice. But we find some hints which suggest that a better solution is possible.

2. Neutrinos as pseudo-acoustic phonons

A key postulate of the condensed matter model proposed in [1] was that all gauge groups have to preserve Euclidean symmetry. The consequence of this postulate for rotational symmetry was that the gauge groups have to preserve generations (we consider the massless case, thus, not the generations based on mass eigenstates) and act in the same way on all three generations. The consequence of translational symmetry was that each generation contains an inert direction preserved by all gauge fields. Given that the only inert particle in each generation is the right-handed neutrino, this leads to the identification of the direction of translation with some direction in the right-handed neutrino sector.

But this is not the only consequence of translational symmetry. The other one is standard knowledge in condensed matter theory, and leads to the notion of “acoustic phonons” being different from “optical phonons”: Acoustic phonons have no mass gap, while optical phonons have a mass gap (see, for example, sec. 3.7 of [3]). This distinction is explained by translational symmetry: Essentially, the acoustic phonons are the Goldstone bosons related with Galilean symmetry, which is broken by the velocity of the background [2]. As a consequence, we have exactly three acoustic phonons – the Goldstone bosons of the three-dimensional group of Galilean boosts. All other phonon modes have a mass gap – one needs some minimal energy to excite them.

Now, the acoustic phonons themself are density waves, and density $\rho$, velocity $v^i$ and the stress tensor $\sigma^{ij}$ have been identified in the model with the gravitational field. Thus, the acoustic phonons themself are gravitons. And, in nice correspondence with the prediction that the acoustic phonons have to be massless, the mass of the graviton is yet below the level of detection and much smaller than all masses of fermions.

Nonetheless, the neutrinos are associated with translations of a subsystem of the medium – the lattice of cells. This subsystem interacts with the other parts of the medium – the material between the cells. So we have no translational symmetry for the neutrinos. But, if we assume that the interaction with the surrounding medium is sufficiently weak, we obtain an approximate translational symmetry for the subsystem of the cells. And such an approximate symmetry leads to corresponding Goldstone particles with low masses.

But this is exactly what we need: The masses of the three neutrinos are many orders smaller than those of the other SM fermions, but not zero. This mass difference is large enough to require an explanation, and the standard way to explain such small masses is an approximate symmetry – exactly what we have found, with the translational symmetry of the lattice of cells.

With similar arguments one may hope to explain the higher masses of the quarks in comparison with the leptons, and the higher masses of the upper in comparison with the lower quarks: the higher masses correspond to modes which are in some sense more far away from the translational symmetry. But this is, of course, much too vague, and clearly requires future research.
3. Why is the photon massless?

In [1] we have identified two different mechanisms which lead to effective gauge fields in the large distance limit: First, terms compensating inhomogeneous influences of the material between the cells on the equations of the cells. These are essentially described by Wilson gauge fields, with the minor difference that the time direction remains continuous. Therefore, the interaction term has an exact lattice gauge symmetry. Once the Wilson gauge group acts pointwise, and electroweak doublets are described by a scalar complex field on the lattice, Wilson gauge fields have to have the same charge on all parts of an electroweak doublet. Combined with the requirement of preservation of Euclidean symmetry, this reduced the maximal possible Wilson gauge field to $U(3)_c$ acting on the colors of the quarks. Following Volovik [4], we have argued in [1] that the ground state (the Dirac sea) should be uncharged. That means, the sum of the charges of all particles has to be zero. While this holds for the special subgroup $SU(3)_c$, it does not hold for the diagonal $U(1)_B$, which has the baryon charge as its charge. Therefore, the diagonal $U(1)_B$ symmetry has to be suppressed. What survives is only the strong gauge field $SU(3)_c$.

The other mechanism which leads to effective gauge fields are deformations of the lattice itself. Different from the Wilson-like gauge fields, these deformations do not have an exact gauge invariance on the lattice – the symmetry properties hold only modulo small shifts, which become irrelevant in the large distance limit, but nonetheless do not allow exact lattice gauge symmetry. As a consequence, we should not expect that the resulting effective gauge field becomes massless. Thus, a simple correspondence principle between microscopic and macroscopic gauge symmetry appears sufficient to explain why strong interaction is massless but weak interaction not.

But what about the electromagnetic field? Observation tells that if it is not exactly massless, then its mass should be extremely small. This seems to require an exact symmetry for explanation. On the other hand, it appears as an effective combination of the $U(1)_B$ field and a subgroup of the group $U(2)_L \times U(1)_R$, which we obtain as the maximal gauge group for weak interactions, thus, contains something which does not have to show exact lattice gauge invariance. So, why is it massless? This cries for an explanation.

Fortunately there seems to be a natural route to such an explanation. The basic ingredient is the thesis that the $U(1)_B$ symmetry does not simply disappear. If the mechanism which enforces the charge neutrality of the ground state is not a mechanism which destroys the symmetry – which may seem plausible – it will have to deform its action. It is reasonable to expect that the deformation will be the minimal possible one. But in this case the action of $U(1)_{em}$ is the straightforward candidate for the result of the deformation. Indeed, it fulfills the following properties:

1. The symmetry group of the exact gauge symmetries itself remains unchanged – it is $SU(3) \times U(1)$.
2. The action of $U(1)$ remains to be a vector action.
3. The action is among the ones which are compatible with all other requirements, in particular with charge neutrality of the ground state.

And it is easy to see that these properties already uniquely identify the $U(1)_{em}$ action as the result of the deformation. Indeed, all other $U(1)$ subgroups of the maximal possible group $G_{max} \cong S(U(3)_c \times U(2)_L \times U(1)_R)$ are chiral (violating condition 2) or do not commute with $SU(3)_c$ (violating condition 1).

But if this works, and the action of $U(1)_{em}$ is simply the result of a deformation of the action of $U(1)_B$, then $U(1)_{em}$ has an exact lattice gauge symmetry on the fundamental level and therefore has to be massless. Thus, the condensed matter model also promises a way to explain the masslessness of the photon. There remains, of course, much to be left to future research about the mechanism of deformation.

4. Is the translational direction right-handed?

Let’s consider now the question if the translational direction is right-handed. This is necessary to explain the action of $SU(2)_L$: The translational direction has to be preserved by all gauge fields as a consequence
of the basic postulate of Euclidean symmetry, and there is no left-handed direction preserved by the whole group $SU(2)_L$.

To solve this problem we have to take a more careful look at some of the formulas of [1]. The reader unfamiliar with [1] may have already wondered how one can transform the $Z_2$-valued lattice field theory with commuting operators on different nodes (denoted by $\sigma^i_n$) into a fermionic lattice theory with anticommuting field operators (denoted by $\psi^i_n$). This transformation is defined in the following way:

\[
\begin{align*}
\psi^{1/2}_n &= \sigma^{1/2}_n \prod_{m>n} \sigma^3_m, & \psi^3_n &= \sigma^3_n, \\
\sigma^{1/2}_n &= \psi^{1/2}_n \prod_{m>n} \psi^3_m, & \sigma^3_n &= \psi^3_n.
\end{align*}
\]

and it is easy to see that it transforms the $\sigma^i_n$ (with $[\sigma^i_n, \sigma^j_m] = 0$ for $m \neq n$) into anticommuting fermion operators $\psi^i_n$ ($\{\psi^i_n, \psi^j_m\} = 0$ for $m \neq n$). This formula depends on the choice of an order between the lattice nodes. The particular choice of the order proposed in [1] is described in figure 1. This choice has been justified there by the preservation of some nice algebraic properties. The Hamilton operator, as expressed in the local operators $\sigma^i_n$, is commuting with the basic lattice shifts $\tau_i$. The symmetry of the Dirac operator expressed in the $\psi^i_n$ is smaller: It is generated by the three lattice shifts $\tau_1^2, \tau_2^2, \tau_3$, while $\tau_1$ and $\tau_2$ are no longer symmetries.

The result is the staggered lattice Dirac operator we need, with doubling factor eight (because discretized only in spatial directions) and therefore giving two Dirac fermions (identified with an electroweak doublet) for a scalar lattice field. The approach presented in [1] further simplifies this picture, because the Hamilton operator as expressed in the $\sigma^i_n$ is not even a staggered one – a remarkable and beautiful property.

Let’s consider now the question if the direction of translation (that means, the constant solution) is an eigenstate of the operator $\gamma^5$. The operator $\gamma^5$ can be computed as $-i\alpha^1\alpha^2\alpha^3$ from the lattice Dirac operator. The operators $\alpha_i$ are compositions of the basic lattice shifts $\tau_i$ and multiplication operators with scalar $\pm 1$-valued lattice functions. Thus, $\gamma^5$ appears to be a composition of the diagonal shift $\tau_1\tau_2\tau_3$ and a multiplication operator:

\[
\gamma^5 = c_n \tau_1 \tau_2 \tau_3; \quad c_n \in \{-1, 1\}, \quad c_{n+2h_i} = c_n.
\]
Unfortunately, in the representation presented in \( \text{I} \) \( c_n \) is not constant. This can be easily seen: \( \gamma^5 \) has to commute with the lattice Dirac operator (which is the three-dimensional one \( D_3 = \alpha^i \partial_i \) in our approach, and \( [\alpha^i, \gamma^5] = 0 \)), but the diagonal shift \( \tau_1 \tau_2 \tau_3 \) (which has the constant solution as a +1 eigenstate) taken alone does not, because the diagonal shift does not preserve the order presented in fig. \( \text{I} \). Thus, the constant is not an eigenstate of \( \gamma^5 \). It has therefore non-zero left-handed and right-handed components. As a consequence, the weak gauge group \( SU(2)_L \) would not preserve translations and violate the basic axiom of Euclidean symmetry of the gauge action.

Fortunately the problem is a solvable one. Let’s simply note that there are simple eigenstates of \( \gamma^5 \): All functions \( \phi_n \) such that \( \phi_{n+h_1+h_2+h_3} = c_n \phi_n \). We have to choose one such \( \phi_n \) as the true direction of translation and accordingly redefine the operators \( \psi^i_n \). As a consequence, following \( \text{I} \) the operators \( \sigma^i_n \) also obtain factors \( c_n \).

This has an unfortunate consequence for the symmetry of the fundamental Hamiltonian in terms of the \( \sigma^i_n \): It becomes a staggered operator too, with coefficients which are different on even and odd nodes. The remarkable property of the approach presented in \( \text{I} \) that the staggered operator in terms of the fermion operators \( \psi^i_n \) leads to a non-staggered operator in terms of the more fundamental \( \sigma^i_n \) would be lost. We would have to introduce different types of cells on even and odd nodes into the model. This is something we would not like to do. But, on the other hand, it is not a serious threat for the viability of the approach.

Nonetheless, this leads to a nice and interesting puzzle: Is there another Dirac operator, say, on another lattice, constructed with another point order, which does have \( \gamma^5 \) as a pure translation? A hint that this may be possible is that the Dirac operator as presented in \( \text{I} \) gives almost exactly what we need: The operator \( \tau_3 = 2I_3 \gamma^5 \) is a pure translation, and therefore the direction of translation is a +1 eigenstate of \( 2I_3 \gamma^5 \). This is something like a near hit. Note also that to have this property for \( \gamma^5 \) alone would be sufficient: This is all we need to obtain the \( U(2)_L \times U(1)_R \) we need as the maximal possible gauge group. Then, whatever the direction of translation, there exists an isospin operator which preserves it, and this operator can then be identified with \( I_3 \).

5. Discussion

Given the results of this paper, it seems plausible that the condensed matter model presented in \( \text{I} \) is not only able to give the observed up to now particle content of the standard model of particle physics (all three generations of fermions, together with the whole gauge group) and metric gravity. It may also allow the explanation of the most characteristic properties of the mass parameters of the standard model. In particular, the neutrinos have to be identified with the three pseudo-acoustic phonons of the model related with translations of the lattice of cells. This explains why their masses are extremely small in comparison with the masses of the other fermions. Moreover, the exact \( SU(3)_c \times U(1)_{em} \) lattice gauge symmetry of the fundamental model provides a good reason to expect that the \( SU(3)_c \times U(1)_{em} \) gauge fields will be massless, while the weak bosons, which do not have any associated lattice gauge symmetry, appear to be massive.

After this, all the massless particles of the SM are associated with an exact symmetry, and all the particles with extremely small masses associated with an approximate symmetry. This is already an almost ideal situation: One cannot reasonably hope for much more, say, an exact derivation of the mass matrices from first principles. It is much more reasonable to expect that there will be a class of condensed matter models with lots of different parameters which allow to fit the mass parameters and interaction constants of the standard model. One may, of course, hope to identify more details of the condensed matter model, but large distance universality puts strong limits on such hopes.

Nonetheless, the details have to be worked out: It is a strong lesson from the fermion doubling problem and the regularization problem of chiral gauge theory that the consideration of details of microscopic models can lead to unexpected problems and insights. Thus, a lot has been left to future research.
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