Modeling of a high-power heating unit with pulse-width modulated control

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Abstract. We constructed a heating unit that can be used for provide the desired temperature regime of the technological equipment. Such type of a heating unit allow us to obtain a higher efficiency which is ensured by using a high-frequency switching power electronic converter with pulse-width modulated control. In this paper we also describe the transitions from regular periodic mode to chaos that can be observed in this high-power heating unit. The behavior of such a system can be described by a set of two coupled differential equations with discontinuous right-hand sides. We show that the transitions to chaos in this system are associated with the interplay between classical smooth bifurcations on one side and different forms of border-collision bifurcations on the other.

1. Introduction

The control of temperature is one of the important task in many heat technology processes, such as building materials production, colored glass production, crystal growing and so on. Well known that even a small deviation from the required temperature may lead to violation of the technological process and reduce the finished product quality [1].

In the modern industry are widely used the temperature regulators with thyristor converters. However, this type of converters may leads to the distortion of the waveform of the input current. High total harmonic distortion (THD) due to low power quality can provoke serious problems on the AC line. In order to reduce the total harmonic distortion of the input current and improve a power factor we have developed an energy-efficiency heating unit based on a high-frequency switching power electronics converter with pulse-width modulated control [2, 3].

The normal operation regime for the considered pulse-width modulated control system is the regime of period-1 mode dynamics. Different types of feedback correctors may be used in order to obtain a fast response and an accurate control. However, in practice it is not easy to decide on the proper choice of the feedback corrector and its parameters that can guarantee an operating mode with the desired dynamic characteristics. This becomes even more complicated by the fact that under realistic conditions of operation, smaller and larger parameter changes will always take place. Such variations may lead to the loss of stability of the normal operation mode and to the appearance of chaotic dynamics [4-6].

The main purpose of the present paper is the numerical study of bifurcation phenomena in the pulse modulated control system of the proposed high-power heating unit. The paper is organized as follows. In section 3 we describe a model of the heating unit with pulse-width modulated control. The behavior
of such a system may be represented by a two dimensional piecewise-smooth set of nonautonomous differential equations. We reduce the investigation of this system to studying the dynamics of a two-dimensional piecewise-smooth mapping. Hereafter we discuss the bifurcation phenomena occurring in the considered system. The obtained results are summarized in Conclusions.

2. Technical implementation of the control unit

The circuit diagram of the heating unit is presented in Fig. 1. In the control circuit the industrial controller SMH 2010 is used [1, 2]. The current temperature of the heater is measured with a thermocouple, the signal from which through a normalizing transducer is fed to the analog input module of the controller. To provide the high capacity of the power unit for operating in the industrial supply network a three-phase transformer is used. The peculiarity of field transistor operation in switching mode, which consists in the decrease of source-drain resistance of a field transistor at the short duration of a control impulse, has been applied.

Figure 1. Energy-efficient heating unit:
1-three-phase transformer, 2-three-phase AC network, 3-diode rectifier, 4-capacitor filter, 5-resistive heater, 6, 7-two groups of high-power field-effect transistors, 8-inverting driver, 9-non-inverting driver, 10-waiting multivibrator, 11-DC power supply, 12-industrial controller, 13-normalizing converter, 14-thermocouple.

The operation of field transistor in switching mode can be provided by setting the duration time of control impulses, fed to the transistor gate. For transistor IRF3205 according to the dependence of source current on source-drain voltage and the duration of the control pulse, fed to the field transistor gate, the commutation time of control impulses 10 µs ... 100 µs was selected. As an industrial controller can not commutate control impulses of field transistor about 10 µs, it is necessary to apply an additional pulse converter, converting the impulses from its discrete output, into short control impulses of IRF3205 field
transistors. This is allowed by an integral timer, operating in self-oscillating mode. The application of timer’s microcircuit together with external RC-circuits makes it possible to set the required duration and repetition frequency of control impulses to provide the half-duplex operation of two sets of transistors. To provide the required capacity the number of paralleled field transistors is increased proportionally in each set. At that, the power consumption from the supply electrical network becomes continuous.

The energy-efficient heating unit under study allows expanding the time range of operating currents and increasing the controlled electric heating power by adding a second set of field transistors, antiphase to the first one, and by the additional application of controlling inverting and non-inverting drivers, which provide the complete control period of commutation moments of two groups of high-power field transistors, as well as by their proportional increase in number at the parallel switching in each set, limited only with loading capacitance of the used drivers, which makes it possible to feed all the power from the three-phase alternating current network to a resistance-type heater [2].

3. Mathematical model and bifurcation analysis

The purpose of the present paper is to investigate some of the mechanisms that are involved in the transitions from a period-1 mode to chaos in a heating unit with pulse-width modulated control. With this purpose, we consider a system, whose continuous linear part is described by the transfer function

$$W(s) = \frac{K}{(T_1 \cdot s + 1)(T_2 \cdot s + 1)}$$

which has been obtained experimentally [3-5]. Here $K$, $T_1$, $T_2$ are the transfer and time constants of the plant, respectively.

The behavior of the considered system is described in accordance with this transfer function by the second-order differential equation with discontinuous right-hand side:

$$T_1 T_2 \frac{d^2 T}{dt^2} + (T_1 + T_2) \frac{dT}{dt} + T = \Gamma \cdot K_p (\xi),$$

$$\Gamma = K \cdot U, \quad K_p (\xi) = \frac{1}{2N} [1 + \text{sign}(\xi)], \quad \xi = \alpha (V_{\text{ref}} - \beta T) - V_0 \left(t/a - \left[t/a \right]\right),$$

where $T$ and $U$ represent, respectively, the temperature in a heating unit and input voltage; $\xi$, $K_p(\xi)$ are the input and output signals of the modulator, respectively. The function $\lfloor \cdot \rfloor$ is defined as the largest integer number not greater than $t/a$ (i.e., the integer part, or floor, of $t/a$); $V_{\text{ref}}$ is the reference signal, and $\beta$ is referred to as the temperature sensor sensitivity parameter; $\alpha$ is the corrector gain factor; $V_0$ determines the amplitude of the ramp signal and $a$ is the period of this signal.

The study of the dynamic system (2) was reduced to studying the properties of a piecewise-smooth map:

$$x_{k+1} = e^{a_1} x_k - e^{a_1} e^{a_2 (1-z_k)}, \quad y_{k+1} = e^{a_2} y_k - e^{a_2} e^{a_1 (1-z_k)}, \quad k = 1, 2, 3, \ldots$$

The time continuous dynamical system (2) was reduced to a piecewise-smooth discrete time map [6, 7]:

$$z_k = \begin{cases} 0, & \varphi_k (0) < 0, \\ 1, & \varphi_k (0) > 0, \varphi_k (1) > 0, \\ z^*, & \varphi_k (0) > 0, \varphi_k (1) < 0. \end{cases}$$

Here $z^*$ is the smallest non-negative root of the equation:

$$\varphi_k (z) = \theta z - q - 1 = 0,$$

with

$$\lambda_1 = -\frac{1}{T_1}, \quad \lambda_2 = -\frac{1}{T_2}, \quad \theta = \lambda_1 / \lambda_2, \quad q = \frac{\lambda_2 - \lambda_1}{\beta \cdot K \cdot U}, \quad P = V_{\text{ref}}/q.$$
\[ T = \frac{K \cdot U}{\lambda_1} \cdot (x - \beta y), \quad \frac{dT}{dt} = \frac{K \cdot U}{\lambda_2} \cdot (x - y). \]

In the following simulations, we shall use: \(T_1/T_2=10240\ s^2;\ T_1+T_2=352\ s;\ K=328,7\ \degree\text{C/V};\ a=10\ s;\ 2<U<24\ \text{V};\ \beta=0,01\ \text{V/}\degree\text{C};\ V_0=5\ \text{V};\ V_{\text{ref}}=5\ \text{V};\ \alpha>0.\)

Figure 2 displays the bifurcation diagram obtained through direct simulation for \(U=10\ \text{V}\) and \(50.0<\alpha<104.0\) illustrating the mechanisms of formation of the coexisting attractors and transition to chaos through a finite sequence of period-doubling bifurcations.

Recall that pulse-width modulated control systems can generally be modeled as piecewise smooth dynamical systems \([5, 6, 8]\). Such models are characterized by the fact that their phase space is divided into regions with different dynamics, separated from each other by so-called switching manifolds \([9]\).

In addition to the bifurcations occurring in smooth systems, piecewise smooth systems also show a special class of nonlinear dynamic phenomena known as border-collision bifurcations \([10-16]\), which occur when an invariant set such as, for example, a cycle, collides with a switching manifold.

A simple type of border-collision bifurcation consists in the direct transition from one fixed point to another. This is sometimes referred to as a persistence border-collision \([8, 9]\). However, more complicated phenomena are also possible, including non-smooth fold, period-doubling, period-multiplying bifurcations, multiple-choice bifurcations, and direct transition from a stable fixed point to quasiperiodic or chaotic attractor \([4-16]\).

The bifurcation diagram showing in Fig.2 contains two branches. The first branch begins from the 1-cycle, which represents the normal operation regime. As the gain factor increases, the switching 1-cycle undergoes a period-doubling bifurcation, leading to the appearance of the stable 2-cycle.

With further increase of \(\alpha\), the period–doubling is truncated and we may observe a sudden transition from a stable 2-cycle to a chaos in a border-collision bifurcation. The second branch begins with the gain factor value \(\alpha_1\) where the stable and unstable 3-cycles appear via a border-collision fold bifurcation. As the parameter \(\alpha\) increases the stable 3-cycle transforms into a stable 6-cycle through a smooth period-doubling transition. With future increase of \(\alpha\), the stable 6-cycle undergoes a border collision bifurcation. This leads to the abrupt transition to a six-band chaotic attractor. The domain between the points \(\alpha_1\) and \(\alpha_2\) is a region of multistability where the stable period-1 orbit coexists with chaotic or high-periodic attractors.

4. Conclusion
The purpose of this paper was to study some of the complex dynamic phenomena that can arise in the proposed heating unit based on a high-frequency switching power electronics converter with pulse-width modulated control. Such a system allows us to reduce the total harmonic distortion of the input current and improve a power factor.
The mathematical model of this system was represented as a two-dimensional set of differential equations with discontinuous right-hand sides. The first step in our investigation was to reduce this system to a two-dimensional piecewise-smooth map. We showed that the transitions to chaos in this system are associated with the interplay between classical smooth bifurcations on one side and different forms of border-collision bifurcations on the other.

Acknowledgments
The authors would like to thank Prof. Zh.T. Zhusubaliyev for his careful reading and constructive suggestions to the first draft of this manuscript.

The work was supported by the development program of the Base University on the basis of BSTU named after V.G. Shukhov.

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