The main topic of this talk is the Hawking effect when the black holes in question are undergoing a uniform acceleration. The semiclassical effect of the acceleration is most striking when the Hawking temperature equals the acceleration temperature. Within the usual late-time approximation, the natural black hole vacuum is then equivalent to the asymptotically empty Minkowskian vacuum, and the accelerated black hole becomes semiclassically stable against the familiar thermal evaporation. An important application of this phenomenon is found in the problem of charged black hole pair-creation.

The problem I wish to address in this talk is essentially that of the Hawking effect in the presence of uniformly accelerated black holes. Without the acceleration, the natural vacua of the black holes are well known to be of thermal type with respect to the asymptotic inertial observers who must then find a blackbody radiation from the physical black holes. When the black holes are undergoing acceleration, however, two important facts invalidate this conventional semiclassical picture. First of all, the vacuum would be again thermal with respect to the asymptotic co-moving observers who are following the black hole at a large fixed distance, but they are no longer inertial observers. One must reidentify the asymptotic inertial observers and determine how they would perceive the black hole vacuum. Second, even for those asymptotic co-moving observers, the Hawking radiation could be modified because of another related, if less publicized, quantum effect known as Fulling-Davies-Unruh.

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effect or the acceleration heat bath [2][3].

A convenient starting point of this discussion would be this Fulling-Davies-Unruh effect. The statement is that, if any quantum particle detection device is undergoing a uniform acceleration, it will register the presence of a heat bath of particles at a temperature, call it $T_A$, proportional to the absolute acceleration. This is often expressed by saying that the usual Minkowski vacuum feels like a heat bath to any accelerated observers. However, one should keep in mind that the state is still the Minkowskian vacuum, so that, for example, the covariant energy-momentum expectation values are trivial despite the thermal characteristics.

With this in mind, we may consider the following gedanken experiment [4][5]: Let a black hole of the Hawking temperature $T_{BH}$ be accelerated uniformly such that the acceleration temperature $T_A$ equals $T_{BH}$. What happens to the semiclassical evolution of the black hole? Does the black hole still Hawking-radiate and thereby lose its mass continuously? Or will the acceleration heat bath counteract the Hawking flux and stabilize the system semiclassically? Going one more step, one may further ask if it is possible for the black hole to Hawking-radiate toward asymptotic inertial observers while being in an apparent thermal equilibrium with the acceleration heat bath.

A comparison to a classic puzzle illustrates well how nontrivial this problem is in principle. Suppose we consider a charged particle under a uniform acceleration $\mathbf{A}$. According to the classical electrodynamics, then, the charge emits the classical Bremmstrahlung, the power of which is given by the following formula,

$$\frac{dE}{dt} = \frac{2e^2}{3} \mathbf{A}^2.$$  \hspace{1cm} (1)

But the exactly same field configuration, if seen by co-moving accelerated observers, appears as the static Coulomb field of $1/r^2$ tail without a hint of radiative behavior. Furthermore, the uniformly accelerated charge does not experience the familiar radiation damping force that normally converts the kinetic energy of the charge to the radiation energy of the Bremmstrahlung:

$$\mathbf{F}_{\text{damping}} = \frac{2e^2}{3} \dot{\mathbf{A}} \to 0,$$  \hspace{1cm} (2)

and this leads to an apparent discrepancy with the energy conservation. This famous puzzle had been an outstanding issue for several decades in this century, until the resolution was brought under the light in a beautiful work by Boulware [6], where he explained how all these
observations are actually consistent with each other as well as with the energy-momentum conservation.

A quantum description [7] of this classical system makes the comparison even more compelling. Quantum mechanically, the co-moving observers simply find a detailed balance between the charge and its surroundings (that include the long range Coulomb field as well as the heat bath of thermal photons) just as their counterpart in the black hole problem may find a detailed balance between the two thermal behaviors owing to the fine-tuned temperatures $T_{BH} = T_A$.

As it turned out [4] and as will be discussed in detail below, however, the quantum problem associated with the accelerated black holes above is conceptually much simpler than this classical counterpart. In a nutshell, the Hawking radiation is indeed counteracted by the acceleration heat bath which then prevents thermal evaporation of the black hole. Furthermore, the asymptotic inertial observers agree with the co-moving observers that the black hole does not evaporate at all even though the acceleration heat bath is completely fictitious to the former [4]. The main purpose of this talk is to derive and illustrate this hitherto unknown quantum effect and also consider its implication.

The property of vacua around such uniformly accelerated black holes is also of some importance in another important context. As studied in recent years, there exist exact solutions to the Euclidean Einstein-Maxwell theory that describe a uniformly accelerated black hole and that play the role of the instanton for the pair-creation of non-extremal Reissner-Nordstrom black holes via quantum tunneling [8]. Such instanton solutions, if continued to the Lorentzian signature, correspond to the so-called Ernst spacetime where the oppositely charged Reissner-Nordstrom black holes are uniformly accelerated away from each other by an external electromagnetic field along a symmetry axis. Furthermore, provided that $T_{BH} \neq 0$, the Euclidean solution exists only when the accelerated temperature matches the Hawking temperature exactly. The effect of quantum fluctuation to the pair-creation process cannot be understood without the knowledge of the natural vacua for such accelerated geometry [9].

The rest of the talk is organized as follows. I will start with this Ernst metric [10] as an idealized model for the gedanken experiment. After identifying key features of this spacetime, the causal structure and the asymptotic inertial time coordinate are discussed in comparison with those of a freely falling black hole. A brief review of the Hawking effect...
and the related Bogolubov transformation is outlined, which are then applied to the current problem. The semiclassical physics of the black hole horizon and the acceleration horizon are shown to be surprisingly similar, and this will lead to the main conclusion of this talk. With this new insight, I will turn to the final topic of the talk: the one-loop WKB estimate of black hole pair-creation rate.

Let me first write down the Ernst metric:

\[
g = \frac{\Lambda^2}{(1 + r Ax)^2} \left\{ -F(r) \, ds^2 + F(r)^{-1} \, dr^2 + r^2 \, G(x)^{-1} \, dx^2 + \frac{r^2}{1 + r Ax} \, G(x) \, \Lambda^{-4} \, d\phi^2 \right\},
\]

\[
F(r) \equiv -A^2 r^2 G(-1/Ar) = (1 - \frac{r_+}{r})(1 - \frac{r_+}{r} - A^2 r^2),
\]

\[
\Lambda \equiv \left\{ 1 + \frac{Bx}{2} \sqrt{r_+ r_-} \right\}^2 + \frac{B^2 x^2}{4(1 + r Ax)^2} G(x).
\]

(3)

Although the detailed geometry does not enter the discussion below, it is important to identify a couple of key features. First of all, the metric (3) is written in a static coordinate system, as would be natural for the previously mentioned “co-moving” observers. In particular, the Killing coordinate \( s \) will be referred to as the Rindler time coordinate from the analogy with the Rindler spacetime.

**Figure 1:** A schematic diagram for a pair of uniformly accelerated black holes. The black holes are represented by two hyperbolic world lines in each Rindler wedges.
In the limit of vanishing external electromagnetic field \((A, B \to 0)\) with fixed \(r_{\pm}\), it is easy to see that the above metric reduces to the more familiar Reissner-Nordstrom metric with \(s\) as the asymptotic Minkowskian time coordinate. In this case, the geometry has two horizons at \(r = r_-\) and \(r = r_+\), the latter being the black hole event horizon.

With the external electromagnetic field that drives the acceleration, however, there is a third, so-called acceleration horizon. Note that the same quartic polynomial \(G\) appears in all components of the metric. Call the four roots of it, \(\xi_1, \xi_2, \xi_3, \xi_4\) in the ascending order. Then, the event horizon is now shifted to \(r = \tilde{r}_+ \equiv -1/\xi_2 A\), while \(r = r_A \equiv -1/\xi_3 A\) is the acceleration horizon. Define the surface gravities of the horizons:

\[
\kappa_{BH} \equiv \frac{F'(\tilde{r}_+)}{2}, \quad \kappa_A \equiv -\frac{F'(r_A)}{2},
\]

which are related to the aforementioned temperatures by \(T_{BH} = \frac{\hbar \kappa_{BH}}{2\pi}\) and by \(T_A = \frac{\hbar \kappa_A}{2\pi}\). See references \([11][5][9]\) for more details on the Ernst geometry.

As was emphasized above, I am primarily interested in the cases where the Hawking temperature \(T_{BH}\) is equal to the acceleration temperature \(T_A\):

\[
\kappa_{BH} = \kappa_A. \tag{5}
\]

In some cases, most notably when the black hole mass is much larger that its charge, \(\kappa_{BH} > \kappa_A\) is always true and this constraint can never be met. However, when the non-extremal RN black holes in question are sufficiently close to the extremality, it is possible to achieve this fine-tuning \([4]\). In fact, this constraint is naturally imposed if the two black holes are pair-created via the wormhole-type instanton \([8]\).

The Euclidean version \((s \Rightarrow i\tau)\) of this metric with the condition \(\kappa_{BH} = \kappa_A\) is in fact the instanton that induces such a tunneling event. The leading WKB exponent from this instanton is first estimated by Garfinkle and Strominger \([8]\):

\[
-S_E = -\frac{\pi M^2}{\hbar |QB|} + \cdots \tag{6}
\]

where \(M\) and \(Q\) are the mass and the charge of the pair-created black holes respectively. The ellipsis denotes terms of higher power in \(QB\). When the size of the pair-created black holes are relatively small \((r_{\pm} A \ll 1)\), \(M\) and \(Q\) are approximately given by \(M \simeq (r_+ + r_-)/2\) and \(Q^2 \simeq r_+ r_-\). Also \(A \simeq \kappa_A\) can be regarded as the acceleration of the black hole and \(B\) as the external field strength that drives the acceleration. In the same limit, therefore, the
Newton’s equation for the black hole motion may be written as $MA \simeq |QB| (\ll 1)$, while the fine-tuned temperature requires $r_+ A \simeq (r_+ - r_-)/r_+ (\ll 1)$. Later on, I will come back to the leading one-loop correction to the leading WKB exponent, again in the weak field limit of small $QB$.

**Figure 2:** A schematic drawing of the pair-creation instanton. The acceleration horizon at AH and the black hole horizon at BH are both free of any conical singularity thanks to the fine-tuned acceleration $\kappa_A = \kappa_{BH}$. The cup-like region is essentially a Euclidean Reissner-Nordstrom black hole truncated beyond some large radial distance. See ref. [9] for more detail.

In addition to the above considerations, it is most essential for our purpose that we understand the causal structure. One of the more distinctive feature of the Ernst geometry when compared to the freely falling black hole is the existence of the acceleration horizon, which has the topology of $R^2$. Among other things, it complicates the asymptotic causal structure to such an extent that the Penrose diagram cannot be drawn on a plane. However, there is an easy way out for this extra complication: disregard the asymptotic infinities inside Rindler wedges. The reason one may do this is rather simple. Because the acceleration horizon extends toward infinity along all transverse direction, a typical future-directed quantum that originates near the black hole world line must cross either the acceleration horizon into the asymptotic future F or the event horizon into the black hole interior.

Thus one may visualize the causal structure relevant for the Hawking effect by drawing a truncated Penrose diagram as in figure 3 below. Now it becomes clear what one must do in
order to understand the difference between the freely falling and the accelerated black holes. In quantizing the matter field, one must take care to include the effect of the acceleration horizon and the new asymptotic region beyond it, which can be achieved with correct physical interpretation of various time coordinates. Also see figure 5. for a comparison with a freely falling black hole.

A convenient first step in quantizing matter fields is to consider the field equation in regions L and R. That is to say, I want to start with the eigenmodes of the field as appropriate for the co-moving Rindler-type observers. In case of a freely falling black hole (depicted in figure 5.), L and R are simply the asymptotic regions outside the black holes, and this is easily understood from the fact that in the absence of the acceleration the co-moving observers at large distances are also asymptotic inertial observers.

**Figure 3:** Penrose diagram of the Ernst spacetime with the Rindler infinities at \( x = \xi_3 = -1/Ar \) excised. The bold (straight) lines indicate the asymptotic infinities.
For this purpose, it is most convenient to introduce a new tortoise-like coordinate $z$ between the two horizons ($\tilde{r}_+ \leq r \leq r_A$):

$$z \equiv \int_{\tilde{r}_+}^{r} \frac{1}{F(\tilde{r})} d\tilde{r},$$

(7)

which logarithmically approaches $-\infty$ at the event horizon and $+\infty$ at the acceleration horizon.

Without loss of generality, one may consider a free scalar field with possible quadratic curvature coupling,

$$\nabla^2 \Psi = M^2 \Psi + \cdots.$$  

(8)

After rescaling the energy and angular-momentum eigenmodes $\Psi^{(w,m)}$ for each Rindler frequency $w > 0$ and the quantized angular momentum $m$,

$$\Psi^{(w,m)} = e^{\mp i w s} \frac{(1 + r A x)}{r} \left[ \Phi^{(w,m)}(r, x) e^{i m \phi} \right],$$

(9)

one finds the equation that must be solved for the eigenmodes:

$$w^2 \Phi^{(w,m)} + \frac{\partial^2}{\partial z^2} \Phi^{(w,m)} = F(r(z)) \left\{ \frac{1}{r^2} \left[ -\frac{\partial}{\partial x} G(x) \frac{\partial}{\partial x} + \frac{m^2 A^4}{G(x)} \right] + U_{\text{eff}} \right\} \Phi^{(w,m)}.$$  

(10)

Here, $U_{\text{eff}}$ is a bounded function of $z$ and $x$, and in particular contains the mass term and the possible curvature couplings.

Note that the right-hand-side of Eq. (10) has the overall factor $F(r(z))$ that vanishes exponentially at either horizon. Near the black hole horizon ($z \to -\infty$), $F \sim e^{2\kappa_{BH} z}$ while near the acceleration horizon ($z \to \infty$), $F \sim e^{-2\kappa_A z}$. The universal nature of the Hawking effect is related to the fact that only those modes localized near the horizon matters, and thus we may safely ignore such exponentially small $|z|$ dependence. Effectively, then, a separation of variables occurs that allows one to find all the relevant eigenmodes.

Introducing two null coordinates $u = s - z$ and $v = s + z$, one finds the behavior (near each horizon) of all the positive-frequency Rindler eigenmodes $\Psi^{(w)}_L$ and $\Psi^{(w)}_R$ that have respective supports in either L or R:

$$\Psi^{(w)}_L \sim e^{-iw u} \text{ or } e^{-iw v} \text{ in L}, \quad \Psi^{(w)}_L = 0 \text{ in R},$$

(11)

$$\Psi^{(w)}_R \sim e^{+iw u} \text{ or } e^{+iw v} \text{ in R}, \quad \Psi^{(w)}_R = 0 \text{ in L}.$$  

(12)

The positive sign in (12) is because $(u, v)$ grow toward past rather than toward future in region R. Suppressed here are the dependences on the transverse coordinates $x$ and $\phi$, for these details do not enter the discussion below.
The second step is to expand the quantum field in terms of such eigenmodes,

$$\Psi = \sum_{w>0} a_w \Psi^{(w)} + a_w^{\dagger} \Psi^{(-w)}$$

and declare that $a_w$ and $a_w^{\dagger}$ span a harmonic oscillator algebra for each eigenmode. From this one may construct the Fock space and define the ground state or the vacuum $|0\rangle$ by requiring that it be annihilated by all annihilation operators $a_w$.

$$a_w |0\rangle_R \equiv 0$$

Now the field theoretical reason behind the Hawking radiation is easy to explain. Although the above eigenmodes are natural to certain class of observers, others may find these modes unphysical. This ambiguity is particularly pronounced when there exists a horizon in the spacetime, in which case a particle state in one mode expansion may look like an anti-particle state in another. In the present case of the Ernst spacetime, there are at least two more set of natural time coordinates, each of which are to be called the Kruskal coordinates.

![Figure 4: Various null coordinates near the horizons. $U_1 = 0$ or $V_1 = 0$ at the event horizon, while $U_2 = 0$ or $V_2 = 0$ at the acceleration horizon. All Kruskal coordinates increase toward future. The Rindler-type null coordinates $(u, v)$, however, increase toward future only in L, and actually increase toward past in R.](image)

The approximate form of such coordinates near the respective horizons, are completely determined by the surface gravities alone: Calling the Kruskal coordinates near the event
horizon \((U_1, V_1)\), we find,

\[
\kappa_{BH} u \simeq -\ln(-\kappa_{BH} U_1) \quad \text{in L}, \quad \kappa_{BH} u \simeq -\ln(+\kappa_{BH} U_1) \quad \text{in R},
\]

\[
\kappa_{BH} v \simeq +\ln(+\kappa_{BH} V_1) \quad \text{in L}, \quad \kappa_{BH} v \simeq +\ln(-\kappa_{BH} V_1) \quad \text{in R}.
\]

For the other Kruskal coordinates \((U_2, V_2)\) near the acceleration horizon, we simply replace \(\kappa_{BH}\) by \(\kappa_A\) and reverse every single sign on the right-hand-side. Already the parallel between the two horizons is manifest.

In each of these Kruskal coordinate systems, the natural mode expansion of the quantum field \(\Psi\) is distinct from the above Eq. (13): For \((u, v), (U_1, V_1)\) and \((U_2, V_2)\) respectively, one must find the corresponding natural bases of eigenmodes,

\[
\Psi = \sum_{w>0} a_w \Psi^{(w)} + a_w^\dagger \Psi^{(-w)} = \sum_{w'>0} b_{w'} \Psi^{(w')}_{B} + b^\dagger_{w'} \Psi^{(-w')}_{B} = \sum_{w''>0} c_{w''} \Psi^{(w'')}_{A} + c^\dagger_{w''} \Psi^{(-w'')}_{A},
\]

which lead to the natural vacuum appropriate for each coordinate system:

\[
a_w |0\rangle_R \equiv 0, \quad b_{w'} |0\rangle_B \equiv 0, \quad c_{w''} |0\rangle_A \equiv 0.
\]

The “black hole vacuum” \(|0\rangle_B\) is the one that allows smooth event horizon(s), while the “asymptotic vacuum” \(|0\rangle_A\) allows smooth acceleration horizon(s). Depending on the precise initial conditions to be imposed on the quantum field, the physical vacuum would be given by either the former only or a certain composite of the two. The static vacuum composed of \(|0\rangle_B\) type only is the physical vacuum for the Euclidean instanton geometry and in the limit of \(\kappa_A = 0\) corresponds to the so-called Hartle-Hawking vacuum.

In general, different mode expansions are related by unitary transformations. When the unitary transformation in question do not mix creation operators with annihilation operators, the transformation would act trivially on the vacuum state itself. However, in the present case with horizons, the above three vacua are expected to be inequivalent, for the relevant unitary transformation, often called Bogolubov transformations, mixes in negative and positive modes rather indiscriminately. For instance, following Unruh \(\tilde{3}\) one may choose the following unitary transformation rule to construct eigenmodes \(\Psi_B^\prime\)’s that are appropriate near the black hole event horizon,

\[
\Psi^{(w)}_{BL} \simeq N_w(\kappa_{BH})[\Psi^{(w)}_L + e^{-\pi w/\kappa_{BH}} \Psi^{(-w)}_R], \quad \Psi^{(w)}_{BR} \simeq N_w(\kappa_{BH})[\Psi^{(w)}_R + e^{-\pi w/\kappa_{BH}} \Psi^{(-w)}_L],
\]

(19)
with \( N_w(\kappa_{BH}) \equiv 1/\sqrt{1 - e^{-2\pi w/\kappa_{BH}}}. \) This leads to a specific relationship between the two vacua:

\[
|0\>_B = S(\kappa_{BH}) |0\>_R.
\]  

(20)

The Bogolubov transformation \( S(\kappa_{BH}) \) depends on the single parameter \( \kappa_{BH} \) or equivalently \( T_{BH} \), and excites the Rindler modes \( \Psi_L \) and \( \Psi_R \) in a pairwise and thermal fashion.

**Figure 5:** Penrose diagram of the “freely falling” Reissner-Nordstrom black hole with a positive Hawking temperature \( (\kappa_A = 0, \kappa_{BH} \neq 0) \). The Hawking effect induces a thermal radiation toward the asymptotic future infinities \( I^+ \).

For an ordinary nonaccelerated black holes \( (\kappa_A = 0) \), this would be the end of the story, since \( (u, v) \) are themselves the asymptotic Minkowskian coordinates. In this limit of \( \kappa_A = 0 \), one finds \( \Lambda \equiv 1 \) and \( F \to 1 \) at large distances, so that the metric is approximately given by \( g \simeq -du dv + \cdots \). The second Kruskal coordinate system \( (U_2, V_2) \) is irrelevant since the
acceleration horizon does not exist, as illustrated in figure 5. For physical black holes with smooth future event horizon, then, the physical vacuum known as the Unruh vacuum is such that the asymptotic inertial observers find outward thermal radiation at $T_{BH} = \hbar \kappa_{BH}/2\pi$.

However, with the uniformly accelerated black holes, $(u,v)$ are not asymptotic inertial coordinates. Rather, $(U_2,V_2)$ are, or more precisely certain coordinates that behaves as $(U_2,V_2)$ near the acceleration horizon. It is particularly easy to see this in the limit of small $QB$ when the black hole size is relatively small compared to the Schwinger length. Introducing a new set of coordinates far away from the black hole:

$$A^2 \zeta^2 \simeq \frac{1 - r^2 A^2}{(1 + r A x)^2}, \quad A^2 \rho^2 \simeq \frac{1 - x^2}{(1 + r A x)^2},$$

it leads to the following approximate form of the Ernst metric far away from the black holes,

$$g \simeq \Lambda^2 (-A^2 \zeta^2 \, ds^2 + dc^2) + \Lambda^2 \, d\rho^2 + \Lambda^{-2} \rho^2 \, d\phi^2, \quad \Lambda \simeq 1 + \frac{B^2 \rho^2}{4}$$

Here the null Minkowskian coordinates $(U,V)$ are defined as $U = +\zeta e^{+A s}$ and $V = -\zeta e^{-A s}$ in region L and to be analytically continued everywhere else outside the black hole. Recalling that $A \simeq \kappa_A$ in this weak field limit and that $2\kappa_A z \simeq -\ln F \simeq -2 \ln A \zeta$ as $\zeta \to 0$, one finds that the null Minkowskian coordinate system $(U,V)$ coincides with $(U_2,V_2)$ near the acceleration horizon. Since $\Lambda^2$ is never zero, this observation implies among other thing that $|0\rangle_A$ is not only a natural vacuum near the acceleration horizon but the asymptotically empty, Minkowskian vacuum. More general form of the coordinates $\zeta$ and $\rho$ suitable for all values of $QB$, can be found in Ref. [5].

Now one must perform another Bogolubov transformation near the acceleration horizon at $\zeta \simeq 0$. In fact the situation is exactly parallel to the above, except that the relative positions of L and R are switched. For the asymptotic inertial modes $\Psi^{(w)}_A$, we find near the acceleration horizon:

$$\Psi^{(w)}_{AR} \simeq N_w(\kappa_A)[\Psi^{(w)}_R + e^{-\pi w/\kappa_A} \Psi^{(-w)}_L], \quad \Psi^{(w)}_{AL} \simeq N_w(\kappa_A)[\Psi^{(w)}_L + e^{-\pi w/\kappa_A} \Psi^{(-w)}_R].$$

This again leads to the following relationship between the two vacua:

$$|0\rangle_A = S(\kappa_A) |0\rangle_R.$$  

A comparison with (20) makes immediate the special nature of fine-tuned geometry $\kappa_A = \kappa_{BH}$. As seen by the co-moving Rindler observers (to whom $|0\rangle_R$ is the natural empty
vacuum), both the black hole vacuum $|0\rangle_B$ and the asymptotic $|0\rangle_A$ appear thermal, which results in an equilibrium when the two temperatures happen to be equal. On the other hand, for the asymptotic inertial observers (to whom $|0\rangle_A$ is the natural empty vacuum), the natural black hole vacuum may be written as

$$|0\rangle_B = S(\kappa_{BH}) |0\rangle_R = S(\kappa_{BH}) S(\kappa_A)^{-1} |0\rangle_A \quad \rightarrow \quad |0\rangle_A$$

if $\kappa_A = \kappa_{BH}$. (24)

Therefore, unlike in the case of a freely falling black hole, the black hole vacuum $|0\rangle_B$ here is in fact equivalent to the asymptotically empty vacuum $|0\rangle_A$ and there is no possibility of a Hawking-type radiation toward the asymptotic inertial observers.

A couple of remarks are in order. First of all, I have used the eternal Ernst geometry where both future horizons and past horizons are taken seriously, while in more realistic geometries the past horizons would be absent. However, we believe that this idealization does not alter the final conclusion, in much the same way one may derive the Bogoliubov transformation responsible for the Hawking radiation using eternal black holes rather than realistic ones without the past event horizon. Note that the matter of the initial condition is obviated in the present problem because of the null Bogoliubov transformation.

This can be seen more clearly by considering the transformation between the two Kruskal coordinates:

$$U_2 \sim -\frac{1}{U_1}, \quad V_2 \sim -\frac{1}{V_1}.$$  (25)

This transformation is easily seen to preserve the lower-half-planes of the Kruskal time coordinates, being an $SL(2, R)$ generator, which in turn explains why the Bogoliubov transformation thereof is trivial to the leading approximation. In fact, this transformation (25) is exactly what we would have found if we had been considering a freely falling extremal RN black hole that has zero Hawking temperature and thus no late-time Hawking radiation, provided that we again identify $U_2$ and $V_2$ as the asymptotic inertial time coordinates.

For the extremal black hole, however, the above argument that utilizes the property of $SL(2, R)$ transformations is not entirely correct since the relevant range of $U_1$ for instance should be confined to the negative real line. Nevertheless, the fact that the extremal black hole is of zero Hawking temperature remains true: While the resulting Bogoliubov transformation is not entirely trivial, it does not involve a steady flow of radiation energy either. In a similar vein, we expect the present coordinate transformation in (25) lead to the vanishing Hawking radiation for the single accelerated black hole formed by gravitational collapse,
although only half-lines of the Kruskal coordinates would overlap with each other in that case.

Also, this along with the fact the above transformation laws are valid only near the respective horizons, tells us that there could be certain transient behavior: More careful analysis could predict some residual one-loop effect even when the leading Hawking flux vanishes. For instance, in the absence of unbroken extended local supersymmetry, the extremal Reissner-Nordstrom black hole of vanishing Hawking temperature may suffer a finite energy loss, which shifts its mass by a small amount $\sim \hbar/Q$ [13]. Similarly one should expect a similar mass shift for the present non-extremal black holes under the uniform acceleration, which actually manifests itself in the one-loop corrected tunneling rate of the black hole pair-creation as will be shown shortly.

Finally we are in position to discuss the pair-creation problem. As was briefly mentioned above, the Euclidean version of the above geometry is the instanton that mediates this quantum tunneling. While general one-loop WKB tunneling rate would be rather difficult to obtain owing to the uncertainty in the gravitational sector, it is in principle possible to calculate the one-loop contribution from the matter fluctuations. Then, the problem reduces to estimating certain matter partition functions in the background of Euclidean Ernst metric with $T_{BH} = T_A$.

In principle one would try to perform a (Euclidean) mode expansion of the relevant functional determinant, but this approach is unlikely to be effective. Even for the far simpler case of freely falling Reissner-Nordstrom black hole geometry, such a program was carried out only very recently [14]. Alternatively one may concentrate on the weak field behavior of one-loop correction and consider just the leading $QB$-dependence of the additive one-loop correction $W$ to the leading WKB exponent $-S_E/\hbar$. Varying with respect to the background geometry and using the definition for the energy-momentum expectation values, one gets

$$-\delta W = \int dx^4 \sqrt{g} \delta g^{\alpha\beta} \langle 0|T_{\alpha\beta}|0\rangle_{\text{one-loop}}.$$  \hspace{1cm} (26)

It is most convenient to keep the external field $B$ fixed (or equivalently the temperature $T_{BH} = T_A$) and vary with respect to the black hole charge $Q$. A crucial point here is that the instanton geometry has only two independent (dimensionful) parameters.

This integral picks up a trivial factor of $\hbar/T_{BH} \simeq 1/B$ from the periodicity of the Euclidean
time spanned by the Killing coordinate $\tau = -is$, and therefore the remaining spatial part of the integral determines the leading $QB$-dependence of $W$. Thus the behaviors of one-loop energy-momentum expectation values in various spatial regions become a matter of essential importance.

First of all, since the local Euclidean geometries are given by $S^2 \times D^2$ near the black hole horizon and by $R^2 \times D^2$ near the acceleration horizon, one should not expect any singular behavior of $\langle 0|T_{\alpha\beta}|0 \rangle_{\text{one-loop}}$ there.\footnote{As it turned out, there is some subtlety in going to the $QB \to 0$ limit, which was shown to be harmless at least for conformally coupled fluctuations.}

What about the asymptotic region? While one do not expect singular $\langle 0|T_{\alpha\beta}|0 \rangle_{\text{one-loop}}$ there either, even such a mild behavior as $\langle 0|T_{\alpha\beta}|0 \rangle_{\text{one-loop}} \to \text{constant}$, is dangerous due to the infinite volume associated with the region. In fact, if we were considering a “freely falling” Euclidean black hole geometry, the vacuum with smooth black hole horizon would be intrinsically thermal at large spatial distances: The asymptotically constant energy-momentum expectation value thereof would induce a huge gravitational backreaction that distorts the geometry at large distances. And this is where the main result (24) makes the difference.

Unlike the case of a “freely falling” Euclidean black hole, the instanton geometry is such that the natural Hartle-Hawking type vacuum with smooth horizons is asymptotically trivial: $\langle 0|T_{\alpha\beta}|0 \rangle_{\text{one-loop}}$ vanishes rapidly far away from the Euclidean black hole horizon. It is only inside the truncated Euclidean black hole region (see figure 2.) that the vacuum state appears thermal. At the moment it is unclear how rapidly the energy-momentum vanishes at large distances. But the point is, the possible gravitational backreaction to the one-loop quantum effect is a far less serious problem than one might have anticipated otherwise.

Now a reasonable conjecture would be that the behavior of the one-loop energy-momentum is asymptotically insensitive to the truncated Euclidean black hole at the center and thus is asymptotically identical to that of the one-loop energy-momentum in the background Melvin space without any black hole. Then it suffices to consider the above integral over the truncated Euclidean black hole, as argued in Ref. [9], for it is the difference between two partition functions on the instanton and the background geometry that enters the pair-creation rate. In the weak field limit $QB \to 0$ where the black hole mass $M$ is equal to the charge $|Q|$ to the leading order in $QB$, a simple dimensional argument can be used to show
that
\[ W \sim \frac{1}{QB} \Rightarrow -\frac{S_E}{\hbar} + W \simeq -\frac{\pi M^2}{\hbar|QB|} \left\{ 1 + \sigma \frac{\hbar}{Q^2} \right\} + \cdots \] (27)

The proportionality constant \( \sigma \) was explicitly calculated for the chargeless sector of Callan-Rubakov modes, each of which contributes \(-1/36\pi\). See Ref. [9] for more detail.

The only possible interpretation of this one-loop correction seems to be that the effective mass of the pair-created black hole is shifted from \( M \simeq |Q| \) to \( M_{\text{semi}} \simeq |Q| \{ 1 + \sigma \hbar/2Q^2 \} \).

It is as if the external magnetic field \( B \) creates a pair of particles with the charges \( \pm Q \) but the mass \( M_{\text{semi}} \) instead of \( M \). How does this fit into the previously known one-loop effects on Reissner-Nordstrom black holes?

We have found that the pair-created black holes do not suffer from the usual Hawking radiation as long as the fine-tuned acceleration is maintained. However, as briefly mentioned above, this does not mean that the one-loop effect is completely absent, rather this implies that the one-loop effect may induce at most a finite shift of the black hole mass.

A similar circumstance exists for a freely falling extremal Reissner-Nordstrom black hole. The leading late-time Hawking radiation vanishes due to the vanishing Hawking temperature but there are in general subleading transient radiation of finite integrated flux. Again the mass shift was explicitly calculated [13] for chargeless sector of the Callan-Rubakov modes:

\[ \frac{\Delta M}{M} = -\frac{Nh}{72\pi Q^2}, \] (28)

where \( N \) is number of the chargeless Callan-Rubakov modes. This result is easily seen to be consistent with the above value of \( \Delta \sigma = -1/36\pi \) from each chargeless Callan-Rubakov modes in the black hole pair-creation rate.

(It is worthwhile to recall that the sum total of such one-loop effects and thus the total \( \sigma \) are expected to vanish identically if the theory is embedded in certain unbroken extended supergravity, due to the Bogomol’nyi bound interpretation of the extremal black hole [14].)

The point I want to emphasize here is not so much that these corrections are found, as that the naive WKB procedure for the tunneling rate seems to work pretty well. Not only the potential problem from the gravitational backreaction turned out to be rather benign, but the resulting one-loop corrections above are quite consistent with another one-loop effect, which was already found and estimated rigorously in Ref. [13] and which involves the far simpler background geometry of freely falling extremal black hole. This renders more weight
to the validity of the semiclassical method in the pair-creation process.

Before closing, it is appropriate to return to the classic puzzle of the Bremmstrahlung from a uniformly accelerated charge. Clearly the nature of the radiations in the two problems are too different to allow any naive comparison to be made as above. But at the same time, it is instructive to understand exactly where the key differences lie.

The energetics part of the puzzle was in fact first understood by Coleman [15] almost twenty years before Boulware’s conclusive work. The crucial observation by Coleman was that one must take care to include the energy associated with the (boosted) Coulomb field around the moving charge. After an appropriate regularization of the point-like charged particle, the total energy of the system may be split into three pieces: the kinetic energy of the charged particle, the radiation energy of the Bremmstrahlung, the electromagnetic energy of the Coulomb field. In effect, the last acts as a sort of energy reservoir that mediates the energy transfer from the first to the second and in the special case of uniform acceleration provides all the radiation energy without extracting any from the charged particle.

Furthermore, whenever the acceleration lasts only for a finite duration, the initial and the final Coulomb fields are identical up to a boost and the energy conservation between the charged particle and the Bremmstrahlung is well maintained:

$$\Delta E = \int_i^f dt \frac{d}{3} \mathbf{A}^2 + \int_i^f dv \cdot \mathbf{F}_{\text{damping}} = \frac{2e^2}{3} \int_i^f dt \frac{d}{dt} \mathbf{v} \cdot \mathbf{A} = \frac{2e^2}{3} v \cdot \mathbf{A} \Big|_i^f \to 0. \quad (29)$$

Thus the existence of the Bremmstrahlung in the inertial frame is perfectly consistent with the energy conservation. I refer the reader to Ref. [6] for the complete resolution of this classic problem.

In comparison, a nonvanishing Hawking radiation with $T_{BH} = T_A$ would have been very difficult to explain despite the (superficial) similarity from the viewpoint of co-moving observers. Simply put, there is no such intermediary as the Coulomb field that could explain the different energy flows that should have been seen by different classes of observers. There was always a logical possibility that the subtlety in defining the black hole mass in the asymptotically nonflat geometry of the Ernst metric might play a role, but it is gratifying to know that the simplest possible answer is also true.

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