Development of algorithms for the formation of steady-state modes based on the topology of electric power systems

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Abstract. Implementation of the new algorithm for the formation of the steady state mode based on the distribution coefficients of the driving currents in complex electrical networks. is proposed.

The advantage of the proposed approach, in contrast to previously known ones, is that it is not necessary to calculate all 2-trees, as is done in previously known topological methods, which reduces the number of operations by $n^2$ times.

The results of calculations by the proposed method on various test examples 99% coincided with the results of calculations obtained using the industrial software RastrWin3.

1. Introduction.

In this paper, an algorithm for the formation of the steady state of complex electrical networks using the distribution coefficients of driving currents is implemented.

In the paper [1], a comparative review of known topological methods for studying electric power networks is presented and a new algorithm for calculating current distribution coefficients in complex electric networks is proposed.

The steady state mode of electric power networks is described by a system of nonlinear equations, which can be solved by numerical methods. The difficulties of ensuring the scheduling and speed of iterative processes in the analysis of high-dimensional systems have led to the development of many programs that implement various methods.

Currently, various methods of calculating the steady state are used. Each of them has its advantages and disadvantages [2, 3, 4, 5, 6].

One of the authors of this work has developed an improved topological method, where the complex voltage of an arbitrary node of a complex electric power grid circuit is determined from the matrix equation [1, 7, 8]:

$$U = U_0 + C^T Z_d C U^{-1} S,$$  (1)

where $C$ is a rectangular complex matrix of current distribution coefficients; $Z_d$ is a diagonal matrix of branch resistances; $U$ is the complex-valued vector of nodal voltages; $S$ is a complex-
valued power vector of nodal loads, generators and transverse branches. The dash above means complex conjugation, and the “T” sign indicates transposition of the matrix.

From the expression (1) it can be seen that, mathematically, the properties of electrical network circuits are described by the natural parameters of the branches and the generalized parameter by the matrix of the distribution coefficients of the driving currents. Consequently, the formation of nodal voltages of a complex electric power grid for given generators and loads is reduced to the determination of the distribution coefficients of nodal currents. These current distribution coefficients are calculated on the basis of expressions obtained using matrix methods or methods of network circuit topology.

2. Formulation of the problem.
An electrical network (city-wide, country-wide) is represented as a replacement scheme with m - nodes and n - branches. It is required to determine the voltage in the nodes of the circuit at steady state, according to the formula (1).

3. Decision.
System (1) is solved by an iterative method. Moreover, the implementation of algorithms for calculating nodal voltages depends on the form of representation of the nodal parameters of the electric power network scheme. If the node parameters are specified by active and reactive powers, then the node voltage of the i-th node for the k-th iteration is defined as [8, 9]:

\[ U^{(k)}_i = U_0 + \sum_{j=1}^{n} Z_{ij} U_j^{-(k-1)} \left( P_j \cos(\delta_j^{k-1} + \psi_{ij}) + Q_j \sin(\delta_j^{k-1} + \psi_{ij}) \right), \]

\[ U^{(k)}_i = \sum_{j=1}^{n} Z_{ij} U_j^{-(k-1)} \left( P_j \sin(\delta_j^{k-1} + \psi_{ij}) - Q_j \cos(\delta_j^{k-1} + \psi_{ij}) \right), \]

where \( U^k_i \) - voltage module of the i-th node in the k-th iteration; \( \delta^k = \arctg \frac{U^{(k)}_i}{U^{(k)}_{ii}} \) - the voltage phase of the i-th node in the k-th iteration; \( Z_{ij} = \sqrt{(Re \sum_{j=1}^{m} C_{ij} L_{j} Z_j C_{ji})^2 + (Im \sum_{j=1}^{m} C_{ij} L_{j} Z_j C_{ji})^2} \) - the module of mutual knot resistance; \( \psi_{ij} = \arctg \frac{Im \sum_{j=1}^{m} C_{ij} L_{j} Z_j C_{ji}}{Re \sum_{j=1}^{m} C_{ij} L_{j} Z_j C_{ji}} \) - the phase of mutual knot resistance.

If the node parameters are set by active powers and voltages, then for reactive power of the i-th node, for k-th iteration, the expression [10, 11] is true:

\[ Q_i^k = \frac{U_i^2 \cos \delta_i^{k-1} - U_0 U_i - Z_{ii} P_i \cos(\delta_i^{k-1} + \psi_{ii})}{Z_{ii} \sin(\delta_i^{k-1} + \psi_{ii})} - \sum_{j \neq i, j=1}^{n} Z_{ij} U_j^{-(k-1)} \left( P_j \cos(\delta_j^{k-1} + \psi_{ij}) + Q_j \sin(\delta_j^{k-1} + \psi_{ij}) \right) \]

\[ Z_{ii} U_i^{-1} \left( P_i \sin(\delta_i^{k-1} + \psi_{ii}) - Q_i^k \cos(\delta_i^{k-1} + \psi_{ii}) \right). \]

The imaginary part of the voltage of the given node, respectively, is equal to:

\[ U^{(k)}_i = \sum_{j=1, j \neq i}^{n} Z_{ij} U_j^{-(k-1)} \left( P_j \sin(\delta_j^{k-1} + \psi_{ij}) - Q_j \cos(\delta_j^{k-1} + \psi_{ij}) \right) + \]

\[ Z_{ii} U_i^{-1} \left( P_i \sin(\delta_i^{k-1} + \psi_{ii}) - Q_i^k \cos(\delta_i^{k-1} + \psi_{ii}) \right). \]
From here, the phase value of the i-th node voltage for the k-th iteration is:

\[ \delta_k^i = \arcsin \frac{U_i^{(k)}}{U_i}. \]

Further, the calculations are carried out similarly to obtain results for a given accuracy.

In the present work, the algorithm described above is implemented. The developed program was tested on the example of the design scheme of a dedicated part of the network of a real power system of the Republic of Kazakhstan with a voltage of 220 kV (Figure 1).

![Figure 1. Substitution scheme](image)

In order to verify the correctness of obtained results, we calculated voltages in the network’s nodes at steady state using both the proposed method and industrial software RastrWin3. The results of comparison are presented in Table 1.

| Node | Voltage (V) | Proposed Method | RastrWin3 |
|------|-------------|-----------------|-----------|
| 1    | 6.62 + j128.31 | 6.62 + j128.31 | 6.62 + j128.31 |
| 2    | 1.74 + j17.77 | 1.74 + j17.77 | 1.74 + j17.77 |
| 3    | 6.73 + j30.04  | 6.73 + j30.04  | 6.73 + j30.04  |

The deviation of the calculated values by the proposed method from using the industrial program RastrWin3 is less than 1%.

The counting time of our program was \(10^{-1}\) seconds. This means that there is a real possibility to eliminate the main drawback of the existing software systems - the lack of the ability to dynamically analyze the power system.

4. Conclusion.

The paper implements an algorithm for modeling the steady-state mode of complex electric power networks based on the distribution coefficients of the driving currents. It is shown that the software implementation of the algorithm for the formation of a steady state significantly increases the efficiency of research as the number of nodes and branches of the power grid’s electric network increases.
Table 1. Results of comparative calculations of node voltages.

| Node number | Calculations using matrix C | RASTR verifications calculations | Deviations |
|-------------|----------------------------|---------------------------------|------------|
|             | $U, kV$                    | $U, kV$                         | $\Delta U, kV$ | $\Delta U, %$ | $\Delta \delta, \text{grad}$ |
| 1           | 241.43                     | -4.8315                         | 240.65      | 0.78         | 0.32         | -0.04         |
| 2           | 241.53                     | -4.8433                         | 240.78      | 0.75         | 0.31         | -0.03         |
| 3           | 240.89                     | -4.3086                         | 240.36      | 0.53         | 0.22         | -0.03         |
| 4           | 237.38                     | -3.0902                         | 237.1       | 0.28         | 0.12         | -0.01         |
| 5           | 235.60                     | -1.8466                         | 235.45      | 0.15         | 0.06         | -0.01         |
| 6           | 236.27                     | -1.9230                         | 236.12      | 0.15         | 0.06         | 0.00          |
| 7           | 239.60                     | -6.2297                         | 238.33      | 1.27         | 0.53         | -0.06         |
| 8           | 239.12                     | -6.5359                         | 238.03      | 1.09         | 0.46         | -0.05         |

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