PARTON DISTRIBUTIONS IN IMPACT PARAMETER SPACE

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Parton distributions in impact parameter space, which are obtained by Fourier transforming GPDs, exhibit a significant deviation from axial symmetry when the target and/or quark is transversely polarized. In combination with the final state interactions, this transverse deformation provides a natural mechanism for naive-T odd transverse single-spin asymmetries in semi-inclusive DIS. The deformation can also be related to the transverse force acting on the active quark in polarized DIS at higher twist.

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1. Impact Parameter Dependent PDFs and SSAs

The Fourier transform of the GPD \( H_q(x,0,t) \) yields the distribution \( q(x,b_\perp) \) of unpolarized quarks, for an unpolarized target, in impact parameter space [1]

\[
q(x,b_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} H_q(x,0,-\Delta_\perp^2) e^{-ib_\perp \cdot \Delta_\perp},
\]

with \( \Delta_\perp = p'_\perp - p_\perp \). For a transversely polarized target (e.g. polarized in the \(+\hat{x}\)-direction) the impact parameter dependent PDF \( q_{+\hat{x}}(x,b_\perp) \) is no longer axially symmetric and the transverse deformation is described by the gradient of the Fourier transform of the GPD \( E_q(x,0,t) \) [2]

\[
q_{+\hat{x}}(x,b_\perp) = q(x,b_\perp) - \frac{1}{2M} \frac{\partial}{\partial y} \int \frac{d^2 \Delta_\perp}{(2\pi)^2} E_q(x,0,-\Delta_\perp^2) e^{-ib_\perp \cdot \Delta_\perp}
\]

\( E_q(x,0,t) \) and hence the details of this deformation are not very well known, but its \( x \)-integral, the Pauli form factor \( F_2 \), is. This allows to relate the average transverse deformation resulting from Eq. (2) to the contribution from
Fig. 1. Distribution of the $j^+$ density for $u$ and $d$ quarks in the $\perp$ plane ($x_{Bj} = 0.3$ is fixed) for a proton that is polarized in the $x$ direction in the model from Ref. [2]. For other values of $x$ the distortion looks similar. The signs of the distortion are determined by the signs of the contribution from each quark flavor to the proton anomalous magnetic moment.

the corresponding quark flavor to the anomalous magnetic moment. This observation is important in understanding the sign of the Sivers function.

In a target that is polarized transversely (e.g. vertically), the quarks in the target nucleon can exhibit a (left/right) asymmetry of the distribution $f_{q/p}^T(x_B, k_T)$ in their transverse momentum $k_T$ [3,4]

$$f_{q/p}^T(x_B, k_T) = f_1^q(x_B, k_T) - f_{1T}^q(x_B, k_T^2) \frac{(\hat{P} \times k_T) \cdot S}{M}, \quad (3)$$

where $S$ is the spin of the target nucleon and $\hat{P}$ is a unit vector opposite to the direction of the virtual photon momentum. The fact that such a term may be present in (3) is known as the Sivers effect and the function $f_{1T}^q(x_B, k_T^2)$ is known as the Sivers function. The latter vanishes in a naive parton picture since $(\hat{P} \times k_T) \cdot S$ is odd under naive time reversal (a property known as naive-T-odd), where one merely reverses the direction of all momenta and spins without interchanging the initial and final states. The momentum fraction $x$, which is equal to $x_{Bj}$ in DIS experiments, represents the longitudinal momentum of the quark before it absorbs the virtual photon, as it is determined solely from the kinematic properties of the virtual photon and the target nucleon. In contradistinction, the transverse momentum $k_T$ is defined in terms of the kinematics of the final state and hence it represents the asymptotic transverse momentum of the active quark after it has left the target and before it fragments into hadrons. Thus the Sivers function for semi-inclusive DIS includes the final state interaction between
struck quark and target remnant, and time reversal invariance no longer requires that it vanishes.

The significant distortion of parton distributions in impact parameter space (2) provides a natural mechanism for a Sivers effect. In semi-inclusive DIS, when the virtual photon strikes a $u$ quark in a $\perp$ polarized proton, the $u$ quark distribution is enhanced on the left side of the target (for a proton with spin pointing up when viewed from the virtual photon perspective). Although in general the final state interaction (FSI) is very complicated, we expect it to be on average attractive thus translating a position space distortion to the left into a momentum space asymmetry to the right and vice versa (Fig. 2) [5]. Since this picture is very intuitive, a few words of caution are in order. First of all, such a reasoning is strictly valid only in mean field models for the FSI as well as in simple spectator models [6]. Furthermore, even in such mean field or spectator models there is in general no one-to-one correspondence between quark distributions in impact parameter space and unintegrated parton densities (e.g. Sivers function) (for a recent overview, see Ref. [7]). While both are connected by an overarching Wigner distribution [8], they are not Fourier transforms of each other. Nevertheless, since the primordial momentum distribution of the quarks (without FSI) must be symmetric we find a qualitative connection between the primordial position space asymmetry and the momentum space asymmetry (with FSI). Another issue concerns the $x$-dependence of the Sivers function. The $x$-dependence of the position space asymmetry is described by the GPD $E(x, 0, -\Delta^2_\perp)$. Therefore, within the above mechanism, the $x$ dependence of the Sivers function should be related to the $x$ dependence of $E(x, 0, -\Delta^2_\perp)$. However, the $x$ dependence of $E$ is not known yet and we only know the Pauli form factor $F_2 = \int dx E$. Nevertheless, if one makes the additional assumption that $E$ does not fluctuate as a function of $x$ then the contribution from each quark flavor $q$ to the anomalous magnetic mo-
ment \( \kappa \) determines the sign of \( E^q(x,0,0) \) and hence of the Sivers function. With these assumptions, as well as the very plausible assumption that the FSI is on average attractive, one finds that \( f_{1T}^{+u} < 0 \), while \( f_{1T}^{+d} > 0 \). Both signs have been confirmed by a flavor analysis based on pions produced in a SIDIS experiment by the HERMES collaboration [9] and are consistent with a vanishing isoscalar Sivers function observed by COMPASS [10].

1.1. Evolution

At increasing \( Q^2 \), the nucleon is increasingly likely to be in a higher Fock component. In particular, quarks at low \( x \) are likely to have dressed themselves with perturbative gluons, which also changes the color of the quark. For example, a quark that was red before emitting the gluon may e.g. become green by emitting a red-antigreen gluon. As a result, that quark will no longer be attracted by the remaining spectators (which are still antired) but repelled (this repulsion is weaker than the attraction between red and antired and vanishes in the large \( N_C \) limit). Of course, that (now) green quark will still be attracted by the red-antigreen gluon. The main effect is that the ‘chromodynamic lensing’ mechanism described in the previous section essentially disappears after the active quark has radiated off a gluon (the perturbatively radiated off gluon will be at almost the same transverse position as the quark that radiated it off and hence the \( \perp \) impulse exerted on the active quark will be small (and correlated with the quark spin and not the nucleon spin).

Of course, the active quark getting dressed by a gluon is not the only QCD correction. For example, the gluon that provides the FSI can also split, or the spectators can dress themselves with gluons, but neither one of these processes changes the momentum of the active quark. Therefore, while including the latter is essential for a proper determination of the \( Q^2 \) dependence of the running coupling, it is less important when considering the \( Q^2 \) evolution of the quark Sivers function at smaller values of \( x \) — a regime which is dominated by dressing of quark lines with gluons. As a result, at high \( Q^2 \) and small \( x \) one would not expect a significant Sivers effect. In particular, it is plausible that COMPASS measurements of the Sivers function are already in that regime where the active quark is more likely than not to already have radiated off a gluon. Therefore, particularly at smaller \( x \), the Sivers function at COMPASS i.e. a significant decrease of the Sivers function compared to what one would expect in the \( (x,Q^2) \) regime dominated by valence quarks.
2. Transverse Force on Quarks in DIS

The chirally even spin-dependent twist-3 parton distribution \( g_2(x) = g_T(x) - g_1(x) \) is defined as

\[
\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS|\bar{\psi}(0)\gamma^\mu\gamma_5\psi(\lambda n)|Q\rangle = 2 \left[ g_1(x, Q^2) p^\mu (S \cdot n) + g_T(x, Q^2) S_\perp^\mu + M^2 g_3(x, Q^2) n^\mu (S \cdot n) \right].
\]

neglecting \( m_q \):

\[
g_2(x) = g_W^2(x) + \bar{g}_2(x), \quad \text{with} \quad g_W^2(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y).
\]

\( \bar{g}_2(x) \) involves quark-gluon correlations, e.g. [11,12]

\[
\int dxx^2 \bar{g}_2(x) = \frac{d_2}{6} (4)
\]

\[ g \langle P,S|\bar{q}(0)G^+y(0)\gamma^+q(0)|P,S\rangle = MP^+P^x d_2 \] (5)

At low \( Q^2 \), \( g_2 \) has the physical interpretation of a spin polarizability, which is why the matrix elements (note that \( \sqrt{2}G^+y = B^z - E^y \))

\[
\chi_E 2M^2 \tilde{S} = \langle P,S|q^\dagger \vec{a} \times g\vec{E}q|P,S\rangle \quad \chi_B 2M^2 \tilde{S} = \langle P,S|q^\dagger g\vec{B}q|P,S\rangle (6)
\]

are sometimes called spin polarizabilities or color electric and magnetic polarizabilities [13]. In the following we will discuss that at high \( Q^2 \) a better interpretation for these matrix elements is that of a ‘force’.

As Qiu and Sterman have shown [14], the average transverse momentum of the ejected quark (here also averaged over the momentum fraction \( x \) carried by the active quark) in a SIDIS experiment can be represented by the matrix element

\[
\langle k_\perp \rangle = -\frac{1}{2P^x} \left( P,S \left| \bar{q}(0) \int_0^\infty dx^- G^{+y}(x^+ = 0, x^-) \gamma^+ q(0) \right| P,S \right) (7)
\]

which has a simple physical interpretation: the average transverse momentum is obtained by integrating the transverse component of the color Lorentz force along the trajectory of the active quark — which is an almost light-like trajectory along the \( -\hat{z} \) direction, with \( z = -t \): The \( \hat{y} \)-component of the Lorentz force acting on a particle moving, with (nearly) the speed of light \( \vec{v} = (0, 0, -1) \), along the \( -\hat{z} \) direction reads

\[
g\sqrt{2}G^{+y} = g \left( E^y + B^z \right) = g \left[ \vec{E} + \vec{v} \times \vec{B} \right]^y.
\]

We now rewrite Eq. (7) as an integral over time

\[
\langle k_\perp \rangle = -\frac{\sqrt{2}}{2P^x} \left( P,S \left| \bar{q}(0) \int_0^\infty dt G^{+y}(t, z = -t) \gamma^+ q(0) \right| P,S \right) (9)
\]
in which the physical interpretation of $-\frac{\sqrt{2}}{2p^+} \langle P, S | \bar{q}(0)G^{+y}(t, z = -t)\gamma^+ q(0) | P, S \rangle$ as being the averaged force acting on the struck quark at time $t$ after being struck by the virtual photon becomes more apparent:

$$F^y(0) \equiv -\frac{\sqrt{2}}{2p^+} \langle P, S | \bar{q}(0)G^{+y}(0)\gamma^+ q(0) | P, S \rangle \quad (10)$$

$$= -\frac{1}{\sqrt{2}}MP^+S^x d_2 = -\frac{M^2}{2}d_2,$$

where the last equality holds only in the rest frame ($p^+ = \frac{1}{\sqrt{2}}M$) and for $S^x = 1$, can be interpreted as the averaged transverse force acting on the active quark in the instant right after it has been struck by the virtual photon.

Lattice calculations of the twist-3 matrix element yield [15]

$$d_2^{(u)} = 0.010 \pm 0.012 \quad d_2^{(d)} = -0.0056 \pm 0.0050 \quad (11)$$

renormalized at a scale of $Q^2 = 5$ GeV$^2$ for the smallest lattice spacing in Ref. [15]. Here the identity $M^2 \approx 5$ GeV/fm is useful to better visualize the magnitude of the force.

$$F_{(u)} = -25 \pm 30 \text{ MeV/fm} \quad F_{(d)} = 14 \pm 13 \text{ MeV/fm}. \quad (12)$$

In the chromodynamic lensing picture, one would have expected that $F_{(u)}$ and $F_{(d)}$ are of about the same magnitude and with opposite sign. The same holds in the large $N_C$ limit. A vanishing Sivers effect for an isoscalar target would be more consistent with equal and opposite average forces. However, since the error bars for $d_2$ include only statistical errors, the lattice result may not be inconsistent with $d_2^{(d)} \sim -d_2^{(u)}$.

The average transverse momentum from the Sivers effect is obtained by integrating the transverse force to infinity (along a light-like trajectory) $\langle k^y \rangle = \int_0^\infty dt F^y(t)$. This motivates us to define an ‘effective range’

$$R_{eff} \equiv \frac{\langle k^y \rangle}{F^y(0)}. \quad (13)$$

Note that $R_{eff}$ depends on how rapidly the correlations fall off along a light-like direction and it may thus be larger than the (spacelike) radius of a hadron. Of course, unless the functional form of the integrand is known, $R_{eff}$ cannot really tell us about the range of the FSI, but if the integrand in (10) does not oscillate, (13) provides a reasonable estimate for the range over which the integrand in (10) is significantly nonzero.
Fits of the Sivers function to SIDIS data yield about $|\langle k^y \rangle| \sim 100$ MeV [17]. Together with the (average) value for $|d_2|$ from the lattice this translates into an effective range $R_{\text{eff}}$ of several fm. It would be interesting to compare $R_{\text{eff}}$ for different quark flavors and as a function of $Q^2$, but this requires more precise values for $d_2$ as well as the Sivers function.

Note that a complementary approach to the effective range was chosen in Ref. [18], where the twist-3 matrix element appearing in Eq. (10) was, due to the lack of lattice QCD results, estimated using QCD sum rule techniques. Moreover, the ‘range’ was taken as a model input parameter to estimate the magnitude of the Sivers function.

A measurement of the twist-4 contribution $f_2$ to polarized DIS allows determination of the expectation value of different Lorentz/Dirac components of the quark-gluon correlator appearing in (5)

$$f_2 M^2 S^\mu = \frac{1}{2} \langle p, S | \bar{q} G^{\mu \nu} \gamma_\nu q | p, S \rangle,$$

In combination with (5) this allows a decomposition of the force into electric and magnetic components using

$$F_E^y(0) = -\frac{M^2}{8} \chi_E \quad F_B^y(0) = -\frac{M^2}{4} \chi_B$$

for a target nucleon polarized in the $+\hat{x}$ direction, where [13,16]

$$\chi_E = \frac{2}{3} (2d_2 + f_2) \quad \chi_M = \frac{1}{3} (4d_2 - f_2).$$

A relation similar to (10) can be derived for the $x^2$ moment of the twist-3 scalar PDF $e(x)$. For its interaction dependent twist-3 part $\bar{e}(x)$ one finds for an unpolarized target [19]

$$4MP^+ P^+ e_2 = g \langle p | \bar{q} \sigma^{+i} G^{+i} q | P \rangle,$$

where $e_2 \equiv \int_0^1 dx x^2 \bar{e}(x)$. The matrix element on the r.h.s. of Eq. (17) can be related to the average transverse force acting on a transversely polarized quark in an unpolarized target right after being struck by the virtual photon. Indeed, for the average transverse momentum in the $+\hat{y}$ direction, for a quark polarized in the $+\hat{x}$ direction, one finds

$$\langle k^y \rangle = \frac{1}{4P^+} \int_0^\infty dx^- g \langle p | \bar{q}(0) \sigma^{+y} G^{+y}(x^-)q(0) | p \rangle.$$

A comparison with Eq. (17) shows that the average transverse force at $t = 0$ (right after being struck) on a quark polarized in the $+\hat{x}$ direction reads

$$F_y^y(0) = \frac{1}{2\sqrt{2} P^+} g \langle p | \bar{q} \sigma^{+y} G^{+y} q | p \rangle = \frac{1}{\sqrt{2}} MP^+ S^x e_2 = \frac{M^2}{2} e_2.$$
where the last identity holds only in the rest frame of the target nucleon and for \( S^x = 1 \).

The impact parameter distribution for quarks polarized in the \(+\hat{x}\) direction \([20]\) was found to be shifted in the \(+\hat{y}\) direction \([21,22]\). Applying the chromodynamic lensing model implies a force in the negative \( \hat{y} \) direction for these quarks and one thus expects \( e_2 < 0 \) for both \( u \) and \( d \) quarks. Furthermore, since \( \kappa_{\perp} > \kappa \), one would expect that in a SIDIS experiment the \( \perp \) force on a \( \perp \) polarized quark in an unpolarized target on average to be larger than that on unpolarized quarks in a \( \perp \) polarized target, and thus \( |e_2| > |d_2| \).

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