Fractal diffusers as noise barrier top-edge devices

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Abstract. With every passing decade, the issue of noise pollution has worsened in urban areas and the usefulness of noise barriers have increased. To improve the effectiveness of noise barriers in limited frequency bandwidth, a few researchers have reported that quadratic residue diffusers as noise barrier top-profiles have shown promising noise attenuation, albeit in their narrow frequency bandwidth. It has been reported for room acoustics that the frequency bandwidth of quadratic residue diffusers may significantly be extended by a nested or fractal design. Therefore, the aim of this work is to investigate the potential of fractal acoustic diffusers for noise barrier application. Upon conducting a numerical analysis of 1-D fractal diffuser as noise barrier top-profile using finite element method, it is observed that fractal diffusers outperformed T-barriers in both the shadow zone and the transition zone. Hence, fractal acoustic diffusers are promising candidates for our application as these have good diffusion properties in the integrated frequency bandwidth. If properly designed, the fractal diffuser top-edge device can be used to enhance noise attenuation over extended frequency bandwidth.

1. Introduction

The effect of top-edge of noise barriers in diffracted sound propagation into the shadow zone is known from past studies. Therefore, for the past several decades, extensive works have been published about the various designs of top-profiles in passive noise barriers. Shroeder diffusers are one such top-profile that have been reported by Monazzam and Lam\textsuperscript{[1]}, and Fard et. al\textsuperscript{[2]}\textsuperscript{[3]} to show improved attenuation over designs like T, Y, arrow, etc.

Schroeder diffusers use number-sequences to form surfaces with series of reflecting wells with the same width and different depths, separated by thin fins. The well depths are based on the residues of modulo arithmetic, which equate to fractional wavelength and wavelets of different phases. In 1979, Schroeder postulated that theoretically, the quadratic residue diffuser attempts to distribute the incident waves by scattering them into many wavelets. The scattered wavelets have definite phase relationships among them due the fact that each wavelet involved in scattering process experiences a different path length when it is reflected from the wells. The path length difference introduces phase differences which cause a wide angular range of diffusion\textsuperscript{[4]}\textsuperscript{[5]}\textsuperscript{[6]}. The acoustic pressure at any receiver location is given by the interference between all the reflected waves generated with different phases, diffracted and scattered sound. However, useful diffusion can be achieved in a narrow frequency bandwidth, which is limited at high frequencies by the well width and at low frequencies by the maximum depth. This limitation led to construction of nested or fractal diffusers, wherein the surface of the parent diffuser is integrated with progressively scaled-down diffusers. The surface consisted of nested self-similar diffusers, each generation of diffuser was designed for a specific frequency bandwidth\textsuperscript{[7]}. Konnert et. al\textsuperscript{[8]}, Cox et. al\textsuperscript{[7]}, Landerman et. al, Lock et. al\textsuperscript{[9]} have reported that
good diffusion can be achieved in the integrated bandwidth of the diffusers in a closed room. As fractal diffusers have been proven to improve the sound quality in room acoustics over the integrated bandwidth, an investigation has been conducted in this paper to study and report the performance of fractal diffusers as top-edge device of noise barriers.

1.1. Design parameters of diffuser
In this work, the variation of the depth of the wells is only in one direction, the resultant diffusers being called 1D Schroeder diffusers. The diffusers studied here are designed based on quadratic residue sequence and are hence called quadratic residue diffusers (QRD), as shown in Figure 1(a).

The well depths for a QRD are the non-negative residues or remainders of $n^2$ modulo $N$. Here, $N$ is a prime number and $n$ is the well position, i.e., 0, 1, 2, 3,...,N-1. Specification of bandwidth is limited by the fact that if the wavelength is too large in comparison with well dimensions, the the diffuser surface may act like a plain surface. Also, if the frequency is too high, achieving good diffusion may be difficult due to complex modes coming into play. Bandwidth is defined as the difference between design frequency $f_0$ and maximum frequency $f_{max}$, where maximum frequency $f_{max}$ is limited by the well width such that

$$f_{max} = \frac{c}{2w} \quad (1)$$

The depth sequence $d_n$ is therefore given by

$$d_n = \frac{s_n \lambda_0}{2N} \quad (2)$$

where $s_n = n^2 \mod N$. $\lambda_0$ corresponds to design frequency $f_0$ and is given by Equation 3.

$$\lambda_0 = \frac{c}{f_0} \quad (3)$$

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Figure 1: (a) Design variables of a quadratic residue diffuser and (b) illustration of parent and child diffusers.

2. Methodology
The acoustics solver of the commercial software Comsol Multiphysics 5.3, which uses finite element method to solve the Helmholtz equation, was used to obtain results for this numerical investigation. The problem has been solved in the two-dimensional frequency domain solver, and the exterior domain is modelled as air.

A schematic of the finite element model is shown in figure 2(a), the boundary conditions imposed at all exterior boundaries of the domain are the radiation boundary conditions, to ensure that the scattered waves are outgoing waves and satisfy Sommerfeld’s radiation condition. The surfaces of all barriers is assumed to be sound reflective, therefore, sound hard boundary
conditions are imposed in the finite element model. The monopole source in 2D represents a line source in 3D. In this study, this acoustic source is 5m away from the barrier. As shown in Figure 2(b), the overall height of the barriers is 4m, thickness of the stem is 0.03m and the width-span of the top profile is 1m for all barrier configurations. The maximum depth of the QRD is 0.3m.

Figure 1(b) depicts a nested or fractal diffuser, which consists of a parent diffuser and a child diffuser. The parent diffuser, designed for lower frequencies is denoted as LFD (acronym for low frequency diffuser) in this paper. Similarly, as the child diffuser is designed for higher frequencies it is referred to as HFD (high frequency diffuser). For the purpose of comparison between child diffusers, the parent diffusers are the same in every model. The parent-diffuser common for all fractal designs analyzed in this paper has the number of wells $N$ as 7, design frequency $f_0$ of 400Hz, width of each well $w$ as 12cm, and maximum depth $d_{\max}$ as 0.25m.

Keeping in mind the undulations in spatial pressure response, the values represent an average over three receivers spaced 0.1m apart. Sound field was measured at receiver locations behind the barrier, in the geometrical shadow zone and near the shadow boundary, as can be seen in Figure 3. The geometrical shadow boundary is an imaginary boundary obtained by drawing a line connecting the source and the top of the barrier, shielding the receivers from direct incident waves. In this study, the shadow boundary calculated by geometry is at $123^\circ$.

3. Results and discussions
The objective of this study is to investigate whether the sound measured is lower when a noise barrier with fractal diffuser as top-profile is in place than when shielded by a T-barrier. To do so, measurements are conducted in the shadow zone and near the shadow boundary. A schematic of receivers is represented in the Figure 3, the lines $60^\circ$ and $90^\circ$ being in the shadow zone whereas the line $120^\circ$ being near the shadow boundary.

The results are reported in terms of additional attenuation achieved against a reference T-barrier.

$$\text{Additional attenuation} = SPL_{\text{barrier}} - SPL_{\text{model-barrier}}$$

where SPL stands for sound pressure level in dB. Therefore, positive additional attenuation
implies that our test cases are performing better than T-barrier.

Keeping the max depth and total width fixed, several designs of nested diffusers featuring a variation of number sequence, N and design frequency, $f_0$ were studied in this exercise. The selected results shown here are compare the designs LFD, modelA and modelB. LFD denotes QRD without fins, whereas modelA and modelB are nested diffusers , as shown in Figure 2(b). The child diffuser in both modelA and modelB have N7 sequence but design frequency of 1000 Hz and 800 Hz, respectively. The design parameters of LFD can be found in section 2.

The lowest design frequency, of the parent diffusers, is 400Hz and hence the results are presented from 400Hz onwards.

As seen in Figure 4, the blue line assigned to model LFD has higher attenuation than the other two designs. Although the results are presented only along the $60^0$ line, this characteristic was observed along $60^0$, $90^0$ and $120^0$ lines, i.e., in the shadow zone as well as near the shadow boundary. This behaviour could be attributed to the difference in the diffusion pattern of a parent diffuser with and without the inclusion of child diffusers, which have a diffraction pattern of their own. However, as it can be seen in the figures below, this behaviour is observed only at the design frequency of 400Hz of LFD.

As seen in the Figures 5(a) and 5(b), the red and yellow lines denoting modelA and modelB have an average of 6dB more attenuation than the blue line denoting the barrier LFD. These
figures support the claim in this sample frequency size that at multiples of design frequency of parent and child diffusers, i.e., at 800, 1000, 1200, 1600 and 2000Hz, \textit{model}A and \textit{model}B outperformed the LFD model. Therefore, at these frequencies, the response of the barriers having fractal diffusers as top-profiles is similar to their response in Figure 5.

For further proof, this response has been clearly captured in Figure 6. In the Figure 6, instead of attenuation, the sound pressure levels are directly plotted 5.5m behind the barrier at a height of 4m. This receiver position is a representative of other positions tested and in the shadow zone.
and the has similar response. Thus, higher SPL implies that the barrier is performing worse than others and vice versa. The SPL of LFD, model$_A$ and model$_B$ with respect to frequency has been plotted at a resolution of 50Hz. Closer to the design frequency of 400Hz, the LFD barrier has lower SPL than the other two models. However, it can be seen that between 550Hz and 1650Hz, with the exception of 1500Hz, the barriers with fractal diffusers as their top-profiles have lower SPL. From 1700Hz onwards, the difference is negligible and all three seem to perform as well as each other.

However, there is an uncertainty of numerical measurement of ±1dB and hence any similar gain in attenuation may not be considered significant.

4. Conclusions

Nested diffusers offered promising response and improved the performance of barriers over simple quadratic residue diffusers at the tested frequencies. T-barrier has the lowest attenuation in the shadow zone and near the geometric shadow boundary when compared with the quadratic residue diffuser and the fractal diffusers at all tested frequencies. The difference is particularly evident at the design frequencies of diffusers and integer multiples of design frequencies. The diffusive noise barrier caps have an average additional attenuation of 6dB over the T-barrier at these frequencies.

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