ORDER AND CHAOS IN A THREE-DIMENSIONAL BINARY SYSTEM OF INTERACTING GALAXIES

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ABSTRACT

We present a galactic gravitational model of three degrees of freedom in order to investigate and reveal the behavior of orbits in a binary quasar system. The two quasars are hosted in a pair of interacting disk galaxies. We study in detail the regular or chaotic character of motion in two different cases: the time-independent model in both two-dimensional (2D) and 3D dynamical systems, and the evolving 3D model. Our numerical calculations indicate that a large fraction of orbits in the 2D system are chaotic in the time-independent case. A careful analysis suggests that several Lindblad resonances are also responsible for the chaotic motion of stars in both host galaxies.

In the time-dependent system, we follow the evolution of 3D orbits in our dynamical model, as the two interacting host galaxies develop dense and massive quasars in their cores by mass transportation from the disks to their nuclei. In this interesting case, there are orbits that change their orbital character from regular to chaotic and vice versa; there are also orbits that maintain their characters during the galactic evolution. These results strongly indicate that the ordered or chaotic nature of 3D orbits depends not only on the galactic interaction but also on the presence of quasars in the galactic cores of the host galaxies. The outcomes derived from our dynamical model are compared with observational data. Some theoretical arguments to support the numerically derived outcomes are presented, both in 2D and 3D systems, and a comparison with similar earlier work is also made.

Key words: galaxies: interactions – galaxies: kinematics and dynamics

Online-only material: color figure

1. INTRODUCTION

Quasars were first discovered by Maarten Schmidt in 1963 as very distant, highly energetic stellar objects with a star-like appearance. Since the discovery of the first quasar, astronomers have been trying to reveal and understand the conditions of the birth and existence of these immensely powerful objects. Today, it is believed that quasars are the extremely luminous cores at the centers of distant galaxies, surrounding supermassive black holes (SMBHs). Binary quasars are pairs of quasars bound together by gravitational forces. Binary quasars, like other quasars, are thought to be the products of galaxy mergers. Recent observations indicate that a large number of bright quasars are hosted in massive spiral galaxies with very prominent disks (Letawe et al. 2006). In an early paper (Papadopoulos & Caranicas 2005), a simple dynamical model with a rotating disk and a massive nucleus was presented in order to study the properties of motion in a quasar. Furthermore, there is strong evidence (Letawe et al. 2007) that interacting galaxies host powerful quasars with strong emission lines. It is also found that about 50% of the observed host galaxies display signs of interaction, which is consistent with previous studies (see Hutchings & Neff 1992; Bahcall et al. 1997; Boyce et al. 1999; Kaspi et al. 2000; Floyd et al. 2004; Jahnke et al. 2004). Because most such mergers would have occurred in the very distant past, binary quasars and their associated galaxies are very far away and are therefore difficult for most telescopes to resolve.

The origin, growth, and evolution of massive galaxies and the supermassive black holes that host these galaxies represent a prime field of study in modern astrophysics. Today, the only well-known close binary pair of SMBHs is OJ 287 (see Sillanpää et al. 1988 and also later papers on OJ 287). Sillanpää et al. (1988) conducted numerical experiments using an N-body simulation code in order to study the mass flows. These simulations revealed that the inflow into the center of a black hole will produce an outburst. Tidal interaction and thus the luminosity of the binary system would be greatest in the last stages, which is consistent with OJ 287 being one of the brightest quasars. Moreover, under the tidal triggering hypothesis, binary galaxies with morphological signs of interaction in their disks should be more likely to display Seyfert or quasar activity in their cores (see Byrd et al. 1986; Sundelius et al. 1987, and references therein about accretion in interacting galaxies).

We know that galaxies regularly interact and merge (Toomre & Toomre 1972) and that SMBHs reside in the centers of most, if not all, galaxies (e.g., Richstone et al. 1998). These two facts alone suggest that binary SMBHs should be commonplace. Of course, one or both of the SMBHs in a binary system will only be detectable as quasars when they are actively accreting. One of the leading proposed mechanisms to trigger strong accretion (quasar) activity is galaxy mergers (see Hernquist 1989; Kauffmann & Haehnelt 2000; Hopkins et al. 2008, and references therein), so merging galaxies with binary quasars should also be common. Begelman et al. (1980) first discussed binary SMBH evolution, from galaxy merger to coalescence, as an explanation for the form and motion of radio jets in active galactic nuclei (AGNs). The “final parsec problem”—whether the coalescence of a binary SMBH ultimately stalls, proceeds to rapid coalescence (Escala et al. 2004), or instead recoils or is ejected (Madau & Quataert 2004)—has important implications for the detection of gravitational waves and for the spin and demography of SMBHs.

In a broader context, astronomers hypothesize that “feedback”—whereby dynamical interactions between galaxies trigger accretion onto their SMBHs—mediates the tight correlation between galaxy central black hole masses and the velocity dispersions of galaxy bulges (Ferrarese & Merritt 2000; Gebhardt et al. 2000). The resulting quasars grow in the galactic
cores until they blow out the very galactic gas that feeds them (Granato et al. 2004), choking off star formation and eventually leading to passive elliptical galaxies (Hopkins et al. 2007; Kormendy et al. 2009). This feedback paradigm dovetails with cosmological models of hierarchical structure formation if quasar activity is induced by massive mergers (Wyithe & Loeb 2002, 2005). Major mergers between gas-rich galaxies most efficiently channel large quantities of gas inward, fostering starbursts and feeding rapid black hole growth. Deep high-resolution imaging of quasar host galaxies (Bahcall et al. 1997; Guyon et al. 2006; Bennert et al. 2008) provides strong evidence for fine structure and tidal tails expected from previous gravitational interactions. Radio-loud quasar hosts tend to be found in gas-rich galaxy mergers that form intermediate-mass galaxies, while radio-loud QSOs reside in massive early-type galaxies, most of which also show signs of recent mergers of interactions (Wolf & Sheinis 2008). The far-infrared (FIR) emission of QSOs appears to follow a merger-driven evolution from FIR-bright to FIR-faint QSOs (Veilleux et al. 2009).

The measured excess of quasars with $R \lesssim 40$ kpc separations (Hennawi et al. 2006) over the extrapolated large-scale quasar correlation function may indeed be due to mutual triggering, but is also argued to arise naturally from their locally overdense environments (Hopkins et al. 2008). The dynamics and timescales of major mergers are therefore of the utmost interest.

Spatially unresolved systems are relatively easy to find in large spectroscopic samples. However, because of the lack of spatial information, the velocity offsets are open to a variety of interpretations depending on the relative strength and velocity of narrow and/or broad emission-line systems: small-scale gas kinematics, asymmetric or thermally inhomogeneous accretion disks, AGN outflows or jets, recoiling or orbiting SMBHs, or disturbed or rotating narrow-line regions (Smith et al. 2009). Furthermore, spectroscopic samples are biased against binary AGNs that are very close in redshift (i.e., unresolved in velocity space), or those that are more widely separated in the sky. For instance, in the Sloan Digital Sky Survey spectroscopic survey, the fiber diameter $3''$ and the minimum separation of fibers on a plate is $55''$ in the sky. Therefore, except in rare cases with multiple overlapping spectroscopic plates, any binary quasar with separation between these two values could only be found via dedicated follow-up spectroscopy.

Why are spatially resolved active nuclei in mergers so rare? First, they may be heavily shrouded and therefore only detectable as ultraluminous infrared galaxies (ULIRGs). ULIRGs have bolometric luminosities rivalling quasars, and by some (Hubble Space Telescope) estimates as many as 40% retain double active nuclei (Cui et al. 2001). In local ULIRGs, a binary fraction of at least 40% is also consistent with ground-based observational data (Veilleux et al. 2002, 2006). Among dust-reddened quasars, Urrutia et al. (2008) found that 85% show evidence of merging in images of their host galaxies. Second, detectable mergers may be rare, simply because the lifetime of the resolvable but unmerged interacting phase is extremely short (Mortlock et al. 1999; Foreman et al. 2009). Third, gas-rich major mergers should trace quasars and therefore should mainly have occurred near the “quasar epoch,” at higher redshifts ($z \gtrsim 1.5$; e.g., Khochar & Burkert 2001; Wolf et al. 2003; Silverman et al. 2005), where detection of extended host galaxy light is challenging.

The prevailing view in the literature (e.g., Djorgovski 1991; Kochanek et al. 1999; Mortlock et al. 1999) is that an excess of quasars with small ($R < 40$ kpc) separations is evidence for nuclear triggering in galaxies during dissipative mergers. According to Hopkins et al. (2007), the excess measured clustering (Hennawi et al. 2006) indeed represents compelling evidence for the merger-driven origin of quasars. However, they also note that attaching all quasars to moderately rich dark matter environments, in which mergers are most likely to occur, is sufficient to explain the observed excess of binary quasars at $R < 40$ kpc, even if they are not triggering each other in a bound orbit. That is, they just happen to be neighbors where the typical observed velocity differences could represent $\sim$Mpc separations, along the line of sight, rather than dynamical velocities. Their properties should be statistically indistinguishable from those of single quasars. The discovery of binary quasars whose hosts are clearly interacting thus presents a rare opportunity to study what merging/triggering really looks like and allows for derivation of important quantities associated with the interaction.

Therefore, it seems very challenging to construct a gravitational model of three degrees of freedom (3D) in order to study the dynamical properties in a pair of interacting disk galaxies hosting quasars. The present paper is organized as follows: In Section 2, we present our gravitational dynamical model, which describes the motion in the binary quasar stellar system. In Section 3, we provide an analysis of the 2D system, considering orbits in the galactic plane ($z = 0$). In the next section, we study the character of motion in the 3D system using different kinds of dynamical methods. Some interesting semi-numerical results are also provided in the same section. In Section 5, we use a 3D time-dependent model in order to follow the evolution of orbits as the host galaxies develop massive and dense quasars in their centers. We conclude with Section 6, where the discussion is presented and a comparison between the present theoretical results with observational data is made.

2. PRESENTATION OF THE DYNAMICAL MODEL

Our gravitational model consists of a pair of disk galaxies, each having a dense, massive and spherically symmetric nucleus. The potential that describes the motion in the first host galaxy (hereafter galaxy G1) is given by the equation

$$V_1(r, z) = V_{n1}(r, z) + V_{d1}(r, z) = -\frac{M_{n1}}{\sqrt{r^2 + z^2 + c_{n1}^2}} - \frac{M_{d1}}{\sqrt{b_1^2 + r^2 + (a_1 + \sqrt{h_1^2 + z^2})^2}},$$

where $r^2 = x^2 + y^2$ and $M_{n1}, M_{d1}$ are the mass of the nucleus and the disk of galaxy 1, respectively, $a_1$ is the disk’s scale length, $h_1$ is the disk’s scale height, $b_1$ is the core radius of the disk halo, and $c_{n1}$ is the scale length of the nucleus. The second host galaxy (hereafter galaxy G2) is described by the potential

$$V_2(r, z) = V_{n2}(r, z) + V_{d2}(r, z) = -\frac{M_{n2}}{\sqrt{r^2 + z^2 + c_{n2}^2}} - \frac{M_{d2}}{\sqrt{b_2^2 + r^2 + (a_2 + \sqrt{h_2^2 + z^2})^2}},$$

where again $r^2 = x^2 + y^2$ and $M_{n2}, M_{d2}$ are the mass of the nucleus and the disk of galaxy 2 respectively, $a_2$ is the disk’s...
scale length, $h_2$ is the disk’s scale height, $b_2$ is the core radius of the disk halo, and $c_{e2}$ is the scale length of the nucleus. The disk of the two host galaxies are represented by the well-known Miyamoto–Nagai model (Miyamoto & Nagai 1975). The Plummer sphere we choose to describe each nucleus has been used many times in the past to study the effects of the introduction of a central mass component in the core of a galaxy (see Hasan & Norman 1990; Hasan et al. 1993).

In our study, we use the theory of the circular restricted three body problem (see Caranlicas & Inanen 2009; Caranlicas & Papadopoulos 2009; Caranlicas & Zotos 2009). The two bodies move in circular orbits in an inertial frame OXYZ, with the origin at the center of mass of the system, with a constant angular frequency $\Omega_p > 0$, given by Kepler’s third law

$$\Omega_p = \sqrt{\frac{G M_t}{R^3}},$$  

where $M_t = M_{a1} + M_{d1} + M_{a2} + M_{d2}$ is the total mass of the system and $R$ is the distance between the centers of the two galaxies. A clockwise rotating frame Oxyz is used with axis Oz coinciding with the axis OZ and the axis Ox coinciding with the straight line joining the two bodies. In this frame, which rotates with angular frequency $\Omega_p$, the two galactic centers have fixed positions $C_1(x, y, z) = (x_1, 0, 0)$ and $C_2(x, y, z) = (x_2, 0, 0)$, respectively. The total potential that is responsible for the motion of a star in the dynamical system of the binary quasar is

$$\Phi(x, y, z) = \Phi_{G1}(x, y, z) + \Phi_{G2}(x, y, z) + \Phi_{tot}(x, y, z),$$

where

$$\Phi_{G1}(x, y, z) = -\frac{M_{a1}}{\sqrt{r_1^2 + c_{a1}^2}} - \frac{M_{d1}}{\sqrt{b_1^2 + r_1^2 + (a_1 + b_1 + c_1)^2}},$$

$$\Phi_{G2}(x, y, z) = -\frac{M_{a2}}{\sqrt{r_2^2 + c_{a2}^2}} - \frac{M_{d2}}{\sqrt{b_2^2 + r_2^2 + (a_2 + b_2 + c_2)^2}},$$

$$\Phi_{tot}(x, y, z) = -\frac{\Omega_p^2}{2} \left[ \frac{M_{a1} r_1^2 + \epsilon r_1^2 - R^2 M_{a1}}{M_t} \right],$$

and

$$r_1^2 = (x - x_1)^2 + y^2, \quad r_2^2 = (x - x_2)^2 + y^2,$$

$$r_1^2 = r_1^2 + z^2, \quad r_2^2 = r_2^2 + z^2,$$

with

$$x_1 = -\frac{M_{d1}}{M_t} R, \quad x_2 = R - \frac{M_{d2}}{M_t} R = R + x_1,$$

$$M_t = M_{a2} + M_{d2}, \quad \epsilon = 1 - \frac{M_{a1}}{M_t}.$$  

The angular frequency $\Omega_p$ is calculated as follows: The two bodies move about their common mass center of the system with angular frequencies $\Omega_{p1}$ and $\Omega_{p2}$, given by

$$\Omega_{p1} = \sqrt{\frac{1}{x_1} \left( -\frac{d V_1(r)}{d r} \right)_{r=R}},$$

$$\Omega_{p2} = \sqrt{\frac{1}{x_2} \left( \frac{d V_2(r)}{d r} \right)_{r=R}}.$$  

As the two bodies are not mass points, the two angular frequencies are not equal, in general. However, this issue can easily be resolved. The angular frequencies of the two bodies can become equal under reasonable assumptions. This is justified if the final set of the parameters has physical meaning and represents satisfactorily the dynamical system. The equation $\Omega_{p1} = \Omega_{p2}$ leads to an eight-fold infinity of solutions in the eight unknowns $(a_1, a_2, b_1, b_2, h_1, h_2, c_{a1}, c_{a2})$. If one chooses proper values (representing the dynamical system), let us say for the seven parameters $(a_2, b_1, b_2, h_1, h_2, c_{a1}, c_{a2})$, then equation $\Omega_{p1} = \Omega_{p2}$ gives only two values for the parameter $a_1$. One value is positive, while the other is negative and is rejected. The author would like to make clear that, after properly choosing the parameters, the deviation between the two angular frequencies is negligible, so that $\nu = (|\Omega_{p1} - \Omega_{p2}|)/\Omega_{p1}$ is of the order of $10^{-8}$ or even smaller and $\xi = |\Omega_{p1} - \Omega_{p2}|$ or $\xi = |\Omega_{p2} - \Omega_{p1}|$ is of the order of $10^{-6}$. Therefore, we consider the two angular frequencies practically equal, that is $\Omega_{p1} = \Omega_{p2} = \Omega_p$. Moreover, the treatment of large bodies with spherical symmetry as mass points is very common in celestial modeling. This method is quite familiar to those measuring the masses of disk galaxies. The fact that the two bodies (host galaxies) are sufficiently apart from each other allows us to assume that the tidal phenomena are very small and therefore negligible.

In this rotating frame the equations of motion are

$$\ddot{x} = -\frac{\partial \Phi}{\partial x} - 2\Omega_p y,$$

$$\ddot{y} = -\frac{\partial \Phi}{\partial y} + 2\Omega_p x,$$

$$\ddot{z} = -\frac{\partial \Phi}{\partial z},$$

where the dot indicates derivative with respect to the time. The only integral of motion for the system of differential equation (9) is the Jacobi integral given by the equation

$$J = \frac{1}{2}(p_x^2 + p_y^2 + p_z^2) + \Phi(x, y, z) = E_J,$$

where $p_x, p_y, p_z$ are the momenta per unit mass conjugate to $x, y, z$, while $E_J$ is the numerical value of the Jacobi integral.

All numerical outcomes of the present work are based on the numerical integration of the equations of motion (9), which was made using a Bulirsch–Stoer routine in Fortran 95, with double precision in all subroutines. The accuracy of the calculations was checked by the consistency of the Jacobi integral (10), which was conserved up to the 18th significant figure.

In this paper, we use a system of galactic units in which the unit of length is 20 kpc, the unit of mass is $10^{11} M_\odot$, and the unit of time is $0.99 \times 10^8$ yr. The velocity unit is 197 km s$^{-1}$, while $G$ is equal to unity (see Vozikis & Caranlicas 1992). In these units, we use the following values: $a_1 = 0.15, b_1 = 0.2542, h_1 = 0.00925, c_{a1} = 0.0125, a_2 = 0.175, b_2 = 0.0789, h_2 = 0.00875$, and $c_{a2} = 0.01$. The values of the above quantities of the dynamical system remain constant during this research, while the values of $M_{a1}, M_{d1}, M_{a2}, M_{d2}$, and $R$ are treated as parameters. The above numerical values of the constant dynamical quantities of the system secure positive density everywhere and free of singularities.

Figures 1(a)–(c) show the contours of the projections of the iso-potential curves $\Phi(x, y, z) = E_J$, on the $(x, y), (x, z), (y, z)$ planes, respectively. The values of the parameters are $M_{a1} = 0.08, M_{d1} = 2.0, M_{a2} = 0.04, M_{d2} =$
In this section, we investigate the properties of motion in the Hamiltonian system of two degrees of freedom (2D). This can be derived from Equation (10) if we set \( z = p_z = 0 \). Then, the corresponding Hamiltonian is

\[
J_2 = \frac{1}{2} (p_x^2 + p_y^2) + \Phi(x, y) = E_{J2},
\]

where \( E_{J2} \) is the numerical value of \( J_2 \). As the dynamical system is now two dimensional, we can use the classical method of the \( x - p_x, y = 0, p_y > 0 \) Poincaré surface of section in order to determine the regular or chaotic character of motion. The results obtained from the study of the 2D system will be used in order to help us understand the structure of the more complicated phase space of the 3D system, which will be presented in the following section.

Figure 3(a) shows the \( x - p_x \) phase plane, when the distance between the centers of the two galaxies is \( R = 2.5 \). The values of the other parameters are \( M_{d1} = 0.08, M_{d2} = 2.0, M_{n2} = 0.04, M_{n2} = 0.6, \) and \( \Omega_p = 0.417229 \). The value of the Jacobi integral is \( E_{J2} = -2.0 \). We observe that the regular motion is confined around the stable retrograde periodic point in galaxy 1 (G1), while in galaxy 2 (G2) there are regular regions around both the direct (i.e., in the same direction as the rotation) and the retrograde periodic points. The rest of the phase plane is occupied by a large unified chaotic sea surrounding both host galaxies. There are also some small islands corresponding to secondary resonances. Another interesting point in Figure 3(a) is the high velocities observed near the centers of the two host galaxies. These result from the presence of the dense nucleus in each galactic core and are characteristic of nuclear galactic activity. Figure 3(b) is similar to Figure 3(a), but when \( M_{d1} = 2.08 \) and \( M_{d2} = 0.64 \). As the total mass of the system is conserved, this means that now the mass is on the disks of the two galaxies. In this case, the pattern has one main difference from the pattern shown in Figure 3(a). Here, the chaotic regions are low near the center of each galaxy and the velocities near the two galactic cores are smaller compared to those in Figure 3(a). Therefore, one can say that the structure of chaos in both galaxies is not only a result of galactic interaction, but is also a result of nuclear galactic activity. In other words, the presence of the quasars in each galactic core affects drastically the character of motion in the host galaxies. Moreover, the absence of the quasars reduces the velocities near the centers of the two host galaxies.

Figure 4(a) is similar to Figure 3(a) but when the distance between the centers of the two galaxies is \( R = 3 \) and \( \Omega_p = 0.317397 \). In this case, the two galaxies do not communicate.

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**Figure 1.** (a)–(c) Contours of the projections of the iso-potential curves \( \Phi(x, y, z) = E_J \) on the \((x, y)\), \((x, z)\), and \((y, z)\) planes.

**Figure 2.** Contours of the projections of the iso-potential curves \( \Phi(x, y, z) = E_J \) on the \((x, y)\) plane. The five Lagrange equilibrium points are indicated as \( L_1, L_2, L_3, L_4, \) and \( L_5 \); while \( C_1 \) and \( C_2 \) are the centers of the two host galaxies at a distance \( R = 2.5 \). }

"Figure 4(a) is similar to Figure 3(a) but when the distance between the centers of two the galaxies is \( R = 3 \) and \( \Omega_p = 0.317397 \) in this case, the two galaxies do not communicate. Some additional details regarding the stability and the structure of the dynamical system. Moreover, \( L_1, L_2, L_3, L_4, L_5 \) are the five Lagrange equilibrium points, while \( C_1 \) and \( C_2 \) are the centers of the two galaxies at a distance \( R = 2.5 \). At these equilibrium points we have

\[
\frac{\partial \Phi_x}{\partial x} = \frac{\partial \Phi_y}{\partial y} = \frac{\partial \Phi_z}{\partial z} = 0. 
\]

\( L_1, L_2, L_3 \) are the unstable saddle equilibrium points, while \( L_4 \) and \( L_5 \) are the triangular points (see Binney & Tremaine 2008).
directly in the $x-p_x$ phase plane; therefore, we can observe two separate regions of motion around each galaxy. The majority of orbits around the direct and retrograde periodic points in each galaxy are ordered. There is a considerable chaotic region, mainly near the central regions of each host galaxy. High velocities are once more observed near the galactic cores of the two host galaxies, while some secondary resonances are also present. Figure 4(b) is similar to Figure 3(b) but when the two host galaxies (G1 and G2) are quiet. Here, the secondary resonances look more prominent. Furthermore, as the quasars are absent, the velocities near the center of the two galactic cores are smaller than those observed in Figure 4(a).

Figure 5(a) is similar to Figure 2(a) but when the distance between the centers of the two active host galaxies is $R = 3.5$ and $\Omega_p = 0.251830$. Once more, there are two separate regions of motion around each host galaxy. There is a relatively small chaotic layer, confined mainly near the central region of each galaxy, while the rest of the phase plane is covered by regular orbits circulating around the stable direct and retrograde periodic points. Some small sticky regions are also present in both galaxies. Figure 5(b) is similar to Figure 3(b), but when the two host galaxies (G1 and G2) are quiet. All orbits in both galaxies seem to be regular. Chaotic motion was not observed and, if present, is negligible. Therefore, we can say that the two interacting host galaxies, for large separations, do not show chaotic motion when the quasars are not present in their galactic cores.

Our numerical results suggest that there are two kinds of chaotic orbits: (1) chaotic orbits approaching both galaxies and (2) chaotic orbits that approach only one of the two host galaxies. Later in this section, we will see that whether a chaotic orbit approaches the two galaxies or only one of them strongly depends on the integration time of the particular orbit. On the other hand, the ordered orbits circulate around only one of the two host galaxies. It would be of particular interest to find regular orbits that could circulate around both host galaxies. Unfortunately, this kind of ordered orbit does not exist in our gravitational model, which describes a system of two interacting galaxies that host quasars in their cores.

Figure 6(a) shows the percentage $A\%$ of the $x-p_x$ phase planes covered by chaotic orbits as a function of the distance between the centers of the two active host galaxies. The values of the parameters are $M_{n1} = 0.08, M_{d1} = 2.0, M_{n2} = 0.04,$ and $M_{d2} = 0.6,$ while the value of the Jacobi integral is $E_{J2} = -2.0$. In this case, the quasars are present in the cores of two host galaxies G1 and G2. The value of the
angular frequency $\Omega_p$ is calculated for each particular value of the distance $R$ from Equation (3). The range of values regarding the distance between the centers of the two galaxies is $2.58 \leq R \leq 3.5$. This particular range was chosen so the two galaxies do not communicate with each other directly, producing two separate phase planes, in order to be able to calculate more easily the chaotic percentage of each phase plane. The lower limit of the above range is the minimum distance between the centers of the two galaxies, for which we have two separate phase planes as shown in Figures 4(a) and 5(a). We observe from Figure 6(a) that the percentage $A\%$ decreases exponentially as the distance $R$ increases for both active galaxies G1 and G2. Figure 6(b) is similar to Figure 6(a), but it illustrates the case when the quasars are not present in the cores of the two galaxies. The values of the parameters are $M_{n1} = 0$, $M_{d1} = 2.08$, $M_{n2} = 0$, and $M_{d2} = 0.64$, while the value of the Jacobi integral is $E_{J2} = -2.0$. Here again the percentage $A\%$ decreases exponentially as the distance $R$ increases for both quiet galaxies G1 and G2. A more detailed view of Figures 6(a) and (b) reveals that for each particular value of the distance $R$, the chaotic percentage $A\%$ is always smaller when the quasars are not present, in both host galaxies G1 and G2. We must point out that the percentage $A\%$ is calculated as follows: We choose 1000 orbits with different and random initial conditions $(x_0, p_{0x})$ in each phase plane and then divide the number of those that produce chaotic orbits by the total number of orbits.

Figure 7(a) shows a plot of the average value of the maximum Lyapunov characteristic exponent (LCE; see Lichtenberg & Lieberman 1992) as a function of the distance between the centers of the two active host galaxies. The values of the parameters are $M_{n1} = 0.08$, $M_{d1} = 2.0$, $M_{n2} = 0.04$, and $M_{d2} = 0.6$, while the value of the Jacobi integral is $E_{J2} = -2.0$. In this case, the quasars are present in the cores of two host galaxies G1 and G2. The value of the angular frequency $\Omega_p$ is calculated for each particular value of the distance $R$ from Equation (3). One can see in Figure 7(a) that $\langle LCE \rangle$ decreases linearly as the distance $R$ increases for both cases (G1 and G2). Figure 7(b) is similar to Figure 7(a) but for the case when the quasars are not present in the cores of the two galaxies. The values of the parameters are $M_{n1} = 0$, $M_{d1} = 2.08$, $M_{n2} = 0$, and $M_{d2} = 0.64$, while the value of the Jacobi integral is $E_{J2} = -2.0$. Here again $\langle LCE \rangle$ decreases linearly as the distance $R$ increases, for both quiet galaxies G1 and G2. Once more, as we have pointed out in Figures 6(a) and (b), for each particular value of the distance $R$, the average value $\langle LCE \rangle$ is always smaller when the quasars are not present, for both galaxies G1 and G2. Here we must note that it is well known that the value of LCE is different in each chaotic component (see Saito & Ichimura 1979). As we have in all cases regular regions and only one unified chaotic sea in
each $x-\rho_x$ phase plane, we calculate the average value of LCE by taking 500 orbits with different and random initial conditions $(x_0, p_{x0})$ in the chaotic sea in each case. Note that all calculated LCEs are different on the fifth decimal point in the same chaotic region.

Figures 8(a)–(h) show eight representative regular orbits of the 2D dynamical system. Figure 8(a) shows an orbit circulating around host galaxy 1 with initial conditions $x_0 = 0.2, y_0 = 0$, and $p_{x0} = 0$, while the value of $p_{y0}$ is obtained from the Jacobi integral (12) for all orbits. The value of Jacobi integral is always $E_{J2} = -2.6$. The values of all the other parameters are the same as those in Figure 3(a). Figure 8(b) shows a quasi-periodic orbit moving around galaxy 1, with initial conditions $x_0 = 0.52, y_0 = 0$, and $p_{x0} = 0$, while the values of all other parameters are the same as those in Figure 3(a). This orbit is characteristic of the 3:3 resonance. In Figure 8(c), an ordered orbit circulating around galaxy 2 is shown. The initial conditions are $x_0 = 2.2, y_0 = 0$, and $p_{x0} = 0$, while the values of all other parameters are the same as those in Figure 3(a). Figure 8(d) shows a quasi-periodic orbit moving around galaxy 1 with initial conditions $x_0 = -0.595, y_0 = 0$, and $p_{x0} = 0$, while the values of all the other parameters are the same as those in Figure 3(b). This orbit is characteristic of 3:4 resonance. In Figure 8(e), a complicated orbit with initial conditions $x_0 = 1.53, y_0 = 0$, and $p_{x0} = 0.76$ moving around galaxy 2 is presented. The values of all other parameters are the same as those in Figure 3(b). In Figure 8(f) we see a quasi-periodic orbit around galaxy 1, with initial conditions $x_0 = -1.22, y_0 = 0$, and $p_{x0} = 0.92$. The values of all other parameters for this figure are the same as those in Figure 4(a). Figure 8(g) shows a figure-eight-type orbit circulating around galaxy 1, with initial conditions $x_0 = -0.9, y_0 = 0$, and $p_{x0} = 3.305$, while the values of all the other parameters are the same as those in Figure 5(a). A regular orbit starting at the center of galaxy 1, with initial conditions $x_0 = -0.8235, y_0 = 0$, and $p_{x0} = 0$, is given in Figure 8(h). The values of all other parameters for this orbit are the same as those in Figure 5(b). All regular orbits were calculated for a time period of 150 time units.

Figures 9(a)–(d) show four different chaotic orbits of the 2D dynamical system. In Figure 9(a), we can see a chaotic orbit with initial conditions $x_0 = -1.92, y_0 = 0$, and $p_{x0} = 0$, approaching only host galaxy 1. The time interval of the numerical integration for this orbit is 180 time units. Figure 9(b) shows a chaotic orbit approaching only host galaxy 2. The initial conditions are $x_0 = 1.92, y_0 = 0$, and $p_{x0} = 0$, while the time of the numerical integration is 110 time units. In Figure 9(c), we observe a chaotic orbit approaching both galaxies. The initial conditions are $x_0 = -1.2, y_0 = 0$, and $p_{x0} = 0$, while the time of the numerical integration is 300 time units. So far, one may conclude that there are two kinds of chaotic orbits, as we have mentioned previously. However, numerical simulations of a large number of chaotic orbits (about 1000) with different initial conditions $(x_0, p_{x0})$ show that all chaotic orbits inevitably will approach both galaxies after a certain time interval. Therefore, the crucial factor is the time interval of the numerical integration. This can be shown more clearly in Figure 9(d). This chaotic orbit has the same initial conditions as the orbit shown in Figure 9(a), but the time of the numerical integration this time is equal to 250 time units. We can see that now the orbit approaches both host galaxies. The values of all other parameters for the orbits shown in Figures 9(a)–(d) are the same as those in Figure 3(a). Of course, this phenomenon is observed only in the case in which the $x-\rho_x$ phase planes of the two galaxies are connected (see Figures 3(a) and (b)). When the centers of the two galaxies are located at distances $R$ such that they do not communicate directly in the $x-\rho_x$ phase plane (see Figures 4(a) and (b) and 5(a) and (b)), there are obviously chaotic orbits approaching only one of the two galaxies, regardless of the time interval of numerical integration.

In what follows we shall present some semi-theoretical results in order to give a more detailed picture of the structure and behavior of the 2D dynamical system. The forces acting on a test particle along the $x$- and $y$-axes are given by the equations

$$F_x = -\frac{M_{d1}(x - x_1)}{(r_{a1}^2 + c_{a1}^2)^{3/2}} - \frac{M_{d2}(x - x_2)}{(r_{a2}^2 + c_{a2}^2)^{3/2}} - \frac{M_{d3}(x - x_3)}{(r_{a3}^2 + c_{a3}^2)^{3/2}} + \Omega_p^2 x - 2\Omega_p^2 y,$$

$$F_y = -\frac{M_{a1}y}{(r_{d1}^2 + c_{d1}^2)^{3/2}} - \frac{M_{a2}y}{(r_{d2}^2 + c_{d2}^2)^{3/2}} - \frac{M_{a3}y}{(r_{d3}^2 + c_{d3}^2)^{3/2}} + \Omega_p^2 y + 2\Omega_p^2 x.$$

(13)

It is obvious from Equation (13) that the strength of both forces increases as the masses of the nuclei or the disks increase or their scale lengths decrease. Figure 10 shows the contours of $F_x = \text{const}$ together with the contours $F_y = \text{const}$. The values

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**Figure 7.** Plot of the average value of the maximum Lyapunov characteristic exponent (LCE) vs. distance $R$ for (a) the two active host galaxies and (b) the two quiet host galaxies.
of all other parameters are the same as those in Figure 2. The contours of $F_x = \text{const}$ look like ellipses, while the contours of $F_y = \text{const}$ have a figure eight shape. Inside the first and third "ellipse" from left to right we have $F_x > 0$, while inside the second and the fourth "ellipse" we have $F_x < 0$. On the other hand, inside the lower figure eight curve we have $F_y > 0$, while inside the upper figure eight curve we have $F_y < 0$. Looking carefully near each galactic center, we observe that there are areas where $F_x$ is positive and $F_y$ is negative at the same time and vice versa. There are also areas where both forces are positive or negative at the same time. Therefore, we conclude that near each galactic center there are strong attractive and repulsive forces acting on the test particle (star). These forces are responsible for the chaotic scattering of the star near each galactic center, leading to chaotic orbits.

4. STRUCTURE OF THE 3D HAMILTONIAN SYSTEM

In this section, we investigate the regular or chaotic nature of motion in the 3D Hamiltonian system described by Equation (10). In order to keep things simple, we use our experience gained from the study of the 2D dynamical system to obtain a clear picture regarding the properties of motion in the 3D dynamical model. We are particularly interested in locating
the initial conditions in the 3D dynamical system, producing regular or chaotic orbits. A convenient way to obtain this is to start from the $x - p_x$-phase planes of the 2D system with the same value of the Jacobi integral used in the 2D system and described in the previous section. Specifically, the regular or chaotic nature of the 3D orbits is found as follows: We choose initial conditions $(x_0, p_{x0}, z_0), y_0 = p_{y0} = 0$, such that $(x_0, p_{x0})$ is a point on the phase planes of the 2D system. The points $(x_0, p_{x0})$ lie inside the limiting curve

$$\frac{1}{2}p_{x0}^2 + \Phi_l(x) = E_{J2},$$

which is the limiting curve containing all the invariant curves of the 2D system. Thus, we take $E_{J2} = E_{J2}$. For this purpose a large number of orbits (about 1000) was computed with initial conditions $(x_0, p_{x0}, z_0)$, where $(x_0, p_{x0})$ is a point in the chaotic regions of the $x-p_x$ phase planes of Figures 3(a) and (b), 4(a) and (b), and 5(a) and (b), with all permissible values of $z_0$ and $p_{y0} = 0$. Remember that, as we are on the phase plane, we have $y_0 = 0$, while in all cases the value of $p_{y0}$ was obtained from the Jacobi integral (10). All tested orbits were found to be chaotic. Therefore, one concludes that orbits with the initial conditions $(x_0, p_{x0})$ are points in the chaotic regions of the 2D phase planes, and for all permissible values of $z_0$ they remain chaotic in the 3D system as well.

Figure 11(a) shows a plot of the average value of the maximum Lyapunov characteristic exponent (LCE) of the 3D system as a function of the distance between the centers of the two host galaxies. The values of the parameters are $M_{n1} = 0.08, M_{d1} = 2.0, M_{n2} = 0.04$, and $M_{d2} = 0.6$, while the value of the Jacobi integral is $E_{J} = -2.0$. In this case, the quasars are present in the cores of two host galaxies, G1 and G2. The value of the angular frequency $\Omega_p$ is calculated for each particular value of the distance $R$ from Equation (3). One can observe in Figure 11(a), that (LCE) decreases linearly as the distance $R$ increases for both cases (G1 and G2). Figure 11(b) is similar to Figure 11(a) but for the case when the quasars are not present in the cores of the two galaxies. The values of the parameters are

![Figure 9. (a)–(d) Four different chaotic orbits of the 2D dynamical system. See the text for details.](image)

![Figure 10. Contours of the $F_x = \text{const}$ (elliptical shaped) together with the contours $F_y = \text{const}$ (figure eight shaped). Details are given in the text.](image)
$M_{s1} = 0, M_{d1} = 2.08, M_{s2} = 0$, and $M_{d2} = 0.64$, while the value of the Jacobi integral is $E_J = -2.0$. Here again (LCE) decreases linearly as the distance $R$ increases for both quiet galaxies G1 and G2. Once more, as we point out in Figures 7(a) and (b) regarding the results of the 2D system, for each particular value of the distance $R$, the average value (LCE) of the 3D system is always smaller when the quasars are not present, in both galaxies G1 and G2. The method we use in order to compute the (LCE) in each case is the same as that described in Figures 7(a) and (b). For all tested chaotic orbits in the 3D system the initial value of $z_0$ is common and equal to 0.1. Here we must point out that if we compare the plots of the 3D system shown in Figures 11(a) and (b) with those of the 2D system shown in Figures 7(a) and (b), we observe that in each case (active and quiet galaxies, respectively) the average values of the LCEs in the 3D system are smaller than the corresponding values in the 2D system.

Our next step is to study the character of orbits with initial conditions $(x_0, p_{x0}, z_0)$, $y_0 = p_{y0} = 0$, such that $(x_0, p_{y0})$ is a point in the regular regions of Figures 3(a) and (b), 4(a) and (b), and 5(a) and (b). The phase space of a conservative system of three degrees of freedom has six dimensions, i.e., in Cartesian coordinates $(x, y, z, x, y, z)$. For a given value of the Jacobi integral, a trajectory lies on a five-dimensional manifold. In this manifold the surface of section is four dimensional. This does not allow us to visualize and interpret directly the structure and the properties of the phase space in dynamical systems of three degrees of freedom. One way to overcome this problem is to project the surface of the section to space with lower dimensions.

In fact, we apply the method introduced by Pfenniger (1984; see also Revaz & Pfenniger 2001). We take sections in the plane $y = 0, p_y > 0$ of 3D orbits whose initial conditions differ from the plane parent periodic orbits only by the $z$-component. The set of the resulting four-dimensional points in $(x, p_x, z, p_z)$ phase space are projected on the $(z, p_z)$ plane. If the projected points lie on a well-defined curve, let us call it an “invariant curve,” then the motion is regular; if not, the motion is chaotic.

The projected points on the $(z, p_z)$ plane show nearly invariant curves around the periodic points at $z = 0, p_z = 0$, as long as the coupling is weak. When the coupling is stronger, the corresponding projections in the $(z, p_z)$ plane shows increasing departure of the plane periodic point, up to making a direct orbit a retrograde one and vice versa. Here we must define what one means by direct and retrograde 3D orbit. If consequents in the $(z, p_z)$ section of the 3D orbit drop in one of the two domains of the corresponding section of 2D orbits at the same value of the Jacobi integral $E_J$, we can distinguish between direct and retrograde motion. Orbits that visit both domains are intermittently direct or retrograde.

Figures 12(a)–(d) show four typical $(z, p_z)$ sections of 3D orbits, starting with initial conditions close to different periodic points on the $(x, p_x)$ phase planes of the 2D system. In order to obtain the results shown in Figure 12(a) we have taken the point $(x_0, p_{x0}) = (-0.05, 0)$, representing approximately the position of the retrograde periodic point in the $(x, p_x)$ phase plane of Figure 3(a) for the active host galaxy 1. Similarly, in Figure 12(b) we can observe the $(z, p_z)$ projections near the retrograde periodic point of quiet galaxy 2, as shown in Figure 3(b). The position of the periodic point is $(x_0, p_{x0}) = (2.25, 0)$. Moreover, in Figure 12(c) we see the $(z, p_z)$ projections near the direct periodic point of active host galaxy 1, as shown in Figure 4(a). The position of the periodic point is $(x_0, p_{x0}) = (-1.48, 0)$.

One more example is given in Figure 12(d), where one can see the $(z, p_z)$ projections near the retrograde periodic point of active host galaxy 1, as shown in Figure 5(a). The position of the periodic point this time is $(x_0, p_{x0}) = (-0.28, 0)$. Note that in all cases the numerical results indicate that for small values of $z_0$ the motion is regular, while for larger values of $z_0$ the motion becomes chaotic. Numerical calculations not given here suggest that the above method can be applied in all regular regions around each retrograde or direct stable periodic point of Figures 3(a) and (b), 4(a) and (b), and 5(a) and (b). We must emphasize that the results shown in Figures 12(a)–(d) are rather qualitative and can be considered an indication that the transition from regularity to chaos in 3D orbits occurs as the value of $z_0$ increases. In order to form a more complete and accurate view of the phase space in the 3D system, we compute a large number of 3D orbits (approximately 1000) near each periodic point of the $(x, p_x)$ phase planes of the 2D system for different initial conditions $(x_0, p_{x0})$ and for different values of $z_0$. Our target is to determine the average minimum value of $z_0$ for which the nature of a 3D orbit changes from regular to chaotic. Table 1 shows the value $(z_{0\text{min}})$ near the direct and retrograde stable periodic points of Figures 3(a) and (b), 4(a) and (b), and 5(a) and (b) for three different values of distance between the centers of the two host galaxies $R$. From Table 1 we can deduce three important results: (1) In every case (active or quiet galaxy) the minimum value of $z_0$ in the regions near the retrograde stable periodic points is always larger than the corresponding value in the regions of the direct periodic points. (2) If we compare each set of galaxies regarding their nuclear activity, (active G1,2 with quiet 1,2) for the same distance $R$,
Figure 12. (a)–(d) Projections of the sections of 3D orbits with the plane $y = 0$ when $p_y > 0$. The set of the four-dimensional points $(x, p_x, z, p_z)$ is projected on the $z - p_z$ plane.

Table 1
Average Value of Minimum $z_0$ Near the Direct and Retrograde Periodic Points, for Different Values of the Distance $R$

| Distance | Case     | Region | $(z_{\text{min}})$ |
|----------|----------|--------|---------------------|
| $R = 2.5$ | Active G1 | Direct  | ...                |
|          |          | Retrograde | 0.125          |
|          | Active G2 | Direct  | 0.042               |
|          |          | Retrograde | 0.117          |
|          | Quiet G1  | Direct  | ...                |
|          |          | Retrograde | 0.128          |
|          | Quiet G2  | Direct  | 0.047               |
|          |          | Retrograde | 0.086          |
| $R = 3.0$ | Active G1 | Direct  | 0.092               |
|          |          | Retrograde | 0.131          |
|          | Active G2 | Direct  | 0.105               |
|          |          | Retrograde | 0.121          |
|          | Quiet G1  | Direct  | 0.098               |
|          |          | Retrograde | 0.136          |
|          | Quiet G2  | Direct  | 0.109               |
|          |          | Retrograde | 0.124          |
| $R = 3.5$ | Active G1 | Direct  | 0.127               |
|          |          | Retrograde | 0.138          |
|          | Active G2 | Direct  | 0.122               |
|          |          | Retrograde | 0.126          |
|          | Quiet G1  | Direct  | 0.132               |
|          |          | Retrograde | 0.141          |
|          | Quiet G2  | Direct  | 0.129               |
|          |          | Retrograde | 0.137          |

we observe that when the quasars are not present at the galactic cores (quiet—non-active galaxies), the 3D orbits can approach higher values of $z_0$ and remain regular. On the other hand, when the quasars are present, the value of $\langle z_{\text{min}} \rangle$ is smaller. (3) As the distance between the centers of the galaxies increases, the minimum initial value of $z_0$ for which a 3D orbit can remain regular increases in both cases (active and quiet galaxies). This means that when the two galaxies are in large distances, their mutual interactions are weak enough that majority of 3D orbits are ordered. Moreover, in large distances one can conclude that the main factor responsible for the observed chaotic motion is the nuclear activity of the quasars in the cores of each host galaxy.

So far we have seen that 3D orbits with initial conditions $(x_0, p_{x0}, z_0)$, such that $(x_0, p_{x0})$ is a point in the chaotic regions of the 2D system, for all permissible values of $z_0$ are chaotic. On the other hand, the nature (ordered or chaotic) of 3D orbits with initial conditions $(x_0, p_{x0}, z_0)$, such that $(x_0, p_{x0})$ is a point in the regular regions around the stable direct or retrograde periodic points of the 2D system, depends on the particular value of $z_0$, as shown in Table 1. We did not feel that it was necessary to try to define the values of $\langle z_{\text{min}} \rangle$ for each regular region of the 2D system corresponding to secondary resonances that are represented by multiple small islands in the $x - p_x$ phase planes, as shown in Figures 3(a) and (b), 4(a) and (b), and 5(a) and (b). Numerical results indicate that 3D orbits with initial conditions $(x_0, p_{x0}, z_0)$, such that $(x_0, p_{x0})$ is a point in the
regular regions corresponding to secondary resonances of the 2D system, remain regular for \(z_{\text{min}} \geq 0.047\), while for larger values of \(z_0\) they change their characters from regular to chaotic.

Figures 13(a)–(h) depict eight orbits of the 3D dynamical system. Figure 13(a) shows a regular quasi-periodic orbit circulating around active galaxy 1, with initial conditions \(x_0 = 0.2, y_0 = 0, z_0 = 0.01\), and \(p_0 = p_0 = 0\); while the value of \(p_0\) is always found from the Jacobi integral (Equation (10)). In Figure 13(b), we observe a 3D quasi-periodic orbit characteristic of the 3:3 resonance. This orbit has initial conditions \(x_0 = 0.52, y_0 = 0, z_0 = 0.03\), and \(p_0 = p_0 = 0\); it is going around active galaxy 1. Figure 13(c) shows a 3D orbit with initial conditions \(x_0 = 2.2, y_0 = 0, z_0 = 0.03\), and \(p_0 = p_0 = 0\), circulating around active galaxy 2. In Figure 13(d) we see quasi-periodic orbit, with initial conditions \(x_0 = -0.595, y_0 = 0, z_0 = 0.015\), and \(p_0 = p_0 = 0\); it is going around quiet galaxy 1. A complicated 3D resonant orbit with initial conditions \(x_0 = 1.53, y_0 = 0, z_0 = 0.01\), \(p_0 = 0.767\), and \(p_0 = 0\); moving around quiet galaxy 2, is shown in Figure 13(e). Figure 13(f) shows a quasi-periodic orbit circulating around active galaxy 1. This orbit has initial conditions \(x_0 = -1.22, y_0 = 0, z_0 = 0.01\), \(p_0 = 0.92\), and \(p_0 = 0\). In Figure 13(g) we observe a 3D chaotic orbit with initial conditions: \(x_0 = -1.2, y_0 = 0, z_0 = 0.1\), and \(p_0 = p_0 = 0\), approaching arbitrarily both active galaxies. It is interesting to note that near the more massive host galaxy (G1), the orbit is deflected to higher values of \(z\), while near the less massive host galaxy (G2), the orbit stays close to the disk. Figure 13(h) shows an orbit with the same initial conditions as those in Figure 13(a) but for \(z_0 = 0.2\). The orbit has now become chaotic and goes arbitrarily close to active host galaxy 1. This orbit shows, from another point of view, that 3D orbits starting near the stable periodic points (direct or retrograde) of the 2D system remain regular only for small values of \(z_0\). We must note that in all 3D orbits shown in Figures 13(a)–(g) the initial conditions \((x_0, p_0, 0)\) the values of all the other parameters are as in the corresponding 2D orbits shown in Figures 8(a)–(f) and 9(c). Moreover, we observe that all regular 3D orbits shown in Figures 13(a)–(f) stay near the galactic plane and therefore support the disk of each host galaxy. The numerical integration time for all regular 3D orbits shown in Figures 13(a)–(f) is 200 time units, while for the chaotic 3D orbit shown in Figure 13(g) it is 500 time units. We should also mention that the phenomenon regarding the time integration and its influence on the structure of an orbit, which was discussed in detail in the 2D system in the previous section, was also observed in the 3D system. For instance, the chaotic orbit shown in Figure 13(h) was integrated for a time interval of 220 time units, and it moves only around host active galaxy 1. But if we integrate numerically this orbit for a longer time interval, we will observe that the orbit will gradually form a shape like the one shown in Figure 13(g), which means that it will approach both host galaxies.

The physical parameter playing an important role in the orbital behavior of the stars is the \(L_z\) component of the total angular momentum. From our previous experience, we know that low angular momentum stars, upon approaching a dense, massive nucleus, are scattered off the galactic plane, displaying chaotic motion (Caranicolas & Innanen 1991; Caranicolas & Papadopoulos 2003, Caranicolas & Zotos 2010; Zotos 2011). Of course, in 3D phase space, things are more complicated than in axially symmetric dynamical models, where the \(L_z\) component was conserved. As the motion takes place in a rotating non-axially symmetric system, the \(L_z\) component, which is given by

\[
L_z = x \dot{y} - y \dot{x} - \Omega_p(x^2 + y^2),
\]

is not conserved. Nevertheless, we can compute numerically the mean value \(\langle L_z \rangle\) of \(L_z\) using the formula

\[
\langle L_z \rangle = \sum_{i=0}^{n} L_{zi}.
\]

Our numerical calculations suggest that the chaotic orbits have low values of \(\langle L_z \rangle\), while regular orbits obtain high values of \(\langle L_z \rangle\). Figure 14(a) shows a plot of the evolution of the \(\langle L_z \rangle\) component with the time for the regular orbit of Figure 13(a). In this case we observe that \(\langle L_z \rangle\) is nearly a periodic function of time, while \(\langle L_z \rangle\) is 0.6274. Figure 14(b) is similar to 14(a), but it is for the chaotic orbit shown in Figure 13(g). Here one can see abrupt changes of \(\langle L_z \rangle\) during the chaotic motion, while for this chaotic orbit we have \(\langle L_z \rangle\) = -1.8266. In both cases the time interval of the numerical integration is 500 time units, while \(n = 10^5\).

It would be of particular interest to study the structure of the velocity profile, that is, the plot of the total velocity of a test particle (star), \(v = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}\), as a function of time, for ordered and chaotic motion. Figure 15(a) shows the velocity profile for a time period of 500 time units corresponding to the regular 3D orbit shown in Figure 13(a). Here the velocity profile is quasi-periodic and the maximum value of the velocity is about 490 km s\(^{-1}\). This suggests that regular 3D motion is made in small velocities. Figure 15(b) shows the velocity profile for a time period of 500 time units corresponding to the chaotic 3D orbit shown in Figure 13(g). In this case we can discuss two aspects. First, the velocity obtains high values up to 790 km s\(^{-1}\); second, the velocity profile appears to be highly asymmetric, displaying abrupt changes and large deviations between the maxima and also between the minima in the \([v-t]\) plot. Therefore, we can conclude that the chaotic 3D motion is made in high and abruptly changing velocities.

At this point, we present some semi-theoretical results in order to give a more detailed and complete picture of the structure and behavior of the 3D dynamical system. The method is similar to that used in the 2D system in the previous section. The force acting on a test particle along the \(z\)-axis is given by the equation

\[
F_z = - \frac{M_{n1}z}{(r_1^2 + c_1^2)^{3/2}} - \frac{M_{d1}(a_1 + r_{1z})}{r_1[b_1^2 + r_{1z}^2 + (a_1 + r_{1z})^2]^{3/2}} - \frac{M_{n2}z}{(r_2^2 + c_2^2)^{3/2}} - \frac{M_{d2}(a_2 + r_{2z})}{r_2[b_2^2 + r_{2z}^2 + (a_2 + r_{2z})^2]^{3/2}},
\]

where \(r_{1z} = \sqrt{\dot{r}_1^2 + \dot{z}^2}\) and \(r_{2z} = \sqrt{\dot{r}_2^2 + \dot{z}^2}\). It is obvious from Equation (17) that the strength of the \(F_z\) force increases as the masses of the nuclei or the disks increase or their scale lengths decrease. Figure 16(a) shows a 3D plot of the value of the \(F_z\) force on the \((x, z)\) plane. Figure 16(b) depicts the contours of the projections \(F_z = \text{const.}\) on the \((x, z)\) plane. We observe that for positive values of \(z\) the \(F_z\) force is negative, while for negative values of \(z\) the value of the \(F_z\) is positive, near both galactic cores. Therefore, lighter colors on the lower half of the \((x, z)\) plane indicate higher values of the \(F_z\) force, while darker colors on the upper half of the same plane indicate lower values of the \(F_z\) force. One can observe that near each active galactic core
Figure 13. (a)–(h) Eight orbits of the 3D dynamical system. The initial conditions and details regarding the values of all other parameters are given in the text.
Figure 14. Plot of $L_z$-component of the total angular momentum vs. time for (a) a regular 3D orbit and (b) a chaotic 3D orbit.

Figure 15. (a) The total velocity profile of the 3D orbit shown in Figure 13(a). We observe a nearly periodic pattern. (b) The total velocity profile for the chaotic 3D orbit shown in Figure 13(g). In this case, there are abrupt changes in the profile’s pattern, indicating chaotic motion.

Figure 16. (a) A 3D plot of the value of the $F_z$ force on the $(x, z)$ plane. (b) Contours of the projections $F_z = \text{const}$ on the $(x, z)$ plane. Lighter colors indicate higher values of $F_z$. For positive values of $z$ the $F_z$ force is negative, while for negative values of $z$ the $F_z$ force is positive.

(A color version of this figure is available in the online journal.)
that hosts a quasar, the test particle experiences a very strong $F_z$ force. The values of all the parameters in Figures 16(a) and (b) are the same as those in Figure 2.

It also is interesting to investigate the behavior of the velocity near each galactic core as a function of the distance $x$. In order to do that, we consider the limiting curve, that is, the curve containing all the invariant curves on the $x - p_x$ phase plane. This can be obtained if we set $y = z = p_y = p_z = 0$ in Equation (10), yielding

$$\frac{1}{2}p_x^2 = \frac{1}{2}v^2 = E_J - \Phi(x),$$

where we have set $p_x = v$ because at the limiting curve the $p_x$ velocity is the total velocity (see also Papadopoulos & Caranicolias 2006). In Figure 17 we can observe two plots of the velocity $v$ as a function of $x$, derived using Equation (18). The values of all the parameters are the same as those in Figure 2. The dashed line corresponds to the case of quiet galaxies, while the solid line corresponds to the case in which the galaxies host the quasars in their cores and therefore are active. We see that the velocity is about the same for all values of $x$, except near each galactic core, where higher velocities correspond to active galactic centers, hosting massive and dense quasars.

5. EVOLUTION OF 3D ORBITS IN THE TIME-DEPENDENT MODEL

Let us now follow the evolution of 3D orbits, as mass is transported from the disks of the quiet galaxies to their centers. By this procedure a massive and dense quasar is developed in the central regions of each galaxy. The mass transport is linear, following the equations:

$$M_{n1f} = M_{n1i} + k_1t, \quad M_{d1f} = M_{d1i} - k_1t,$$
$$M_{n2f} = M_{n2i} + k_2t, \quad M_{d2f} = M_{d2i} - k_2t,$$

where $M_{n1i} = M_{n2i} = 0$, and $M_{d1i} = 2.08, M_{d2i} = 0.64$ are the initial values of the mass of the nuclei and the disks, respectively, while $k_1$ and $k_2$ are positive parameters. As in Caranicolias & Innanen (2009), we assume that the linear rate described by relation (19) is slow compared to the orbital period of the binary system and is therefore adiabatic. This is true because the mass transportation period is 100 time units, while the orbital period is about three orders of magnitude smaller. It is also assumed that the transportation stops when the mass of the quasar in each galactic core takes the value $M_{n1f} = 0.08$ or $M_{n2f} = 0.04$. It is well known that the shape of 3D orbits is sometimes inconclusive or misleading. In order to overcome this drawback, we have decided to use a highly accurate method, the maximum LCE. The main advantage of this method is that it uses certain and objective numerical thresholds beyond which we can distinguish between ordered and chaotic motion. This method is, however, very time consuming, as it needs time intervals of numerical integration of order of $10^5$ time units, in order to give reliable and definitive results regarding the nature of a 3D orbit. However, the results are well worth the time commitment.

Figures 18(a)–(d) show the evolution of four different 3D orbits, as the total mass distribution of the dynamical system changes with time, following the first set of Equation (19), regarding host galaxy 1. For all orbits shown in Figures 18(a)–(d), the initial value of the Hamiltonian (10) is $E_J = -2.0$, while $k_1 = 0.0008$. In Figure 18(a) we can see the evolution of the maximum LCE for a 3D orbit and for a time period of $10^5$ time units, as galaxy 1 evolves following the first set of Equation (19). The initial conditions are $x_0 = -0.46, y_0 = 0, z_0 = 0.06$, and $p_{x0} = 0, p_{y0} = 0$, while the value of $p_{z0}$ is found from the Hamiltonian (10) in all cases. The values of all other parameters are the same as those in Figure 4(b). When $t = 100$ time units, the mass of the developed quasar in the core of galaxy 1 is $M_{n1f} = 0.08$, and the evolution stops. The value of the Hamiltonian is now $E_J = -2.00032$. The profile of the LCE clearly indicates that this orbit starts regular and remains regular during the galactic evolution. Figure 18(b) is similar to Figure 18(a). This orbit has initial conditions $x_0 = -0.88, y_0 = 0, z_0 = 0.12, p_{x0} = 1.25$, and $p_{z0} = 0$. The values of all other parameters are the same as those in Figure 4(b). When $t = 100$ time units, the mass of the developed quasar in the core of galaxy 1 is $M_{n1f} = 0.08$, and the evolution stops. The value of the Hamiltonian is now $E_J = -2.00044$. In this case, the profile of the LCE clearly indicates that this orbit starts as a chaotic 3D orbit and remains chaotic during the galactic evolution. In Figure 18(c), we observe the evolution of the LCE of a 3D orbit with initial conditions $x_0 = -0.64, y_0 = 0, z_0 = 0.03, p_{x0} = 0$, and $p_{z0} = 0$. The values of all other parameters are the same as those in Figure 4(b). After a time interval of 100 time units, the mass of the quasar in the core of galaxy 1 is $M_{n1f} = 0.08$, and the galactic evolution stops. The Hamiltonian settles to the value $E_J = -2.00027$. The profile of the LCE given in Figure 18(c) shows that the orbit starts as a regular 3D orbit, but after the galactic evolution it becomes chaotic. It is evident that, if mass transport were not present, the orbit would have remain regular. The presence of the quasar in the core of galaxy 1 has changed the character of the 3D orbit from regular to chaotic. Figure 18(d) depicts the evolution of the LCE of a 3D orbit with initial conditions $x_0 = -0.93, y_0 = 0, z_0 = 0.045, p_{x0} = 0$, and $p_{z0} = 0$. The values of all other parameters are the same as those in Figure 5(b). After a time interval of 100 time units, the mass of the quasar in the core of galaxy 1 is $M_{n1f} = 0.08$, and the galactic evolution stops. The Hamiltonian settles to the value $E_J = -2.00015$. The profile of the LCE given in Figure 18(d) indicates that this orbit starts as a chaotic 3D orbit, but after the galactic evolution it becomes an ordered one. Therefore, it is evident that, if mass transport were not present, the orbit this time would have remain chaotic. In this case, the presence of the quasar in the core of galaxy 1 has changed the nature of the 3D orbit from chaotic to regular.
Figure 18. Evolution of the maximum LCEs of four different 3D orbits, following the first set of Equation (19). By this procedure a massive quasar is formed in the core of host galaxy 1. (a) The orbit starts regular and remains regular, (b) the orbits starts chaotic and remains chaotic, (c) a 3D orbit that starts regular but after 100 time units, when the evolution stops, it becomes chaotic, and (d) a 3D orbit that starts chaotic but after the evolution changes its nature to regular. Details are given in the text.

In Figures 19(a)–(d), one can observe the evolution of four different 3D orbits, as the total mass distribution of the dynamical system changes with time, following the second set of Equation (19), regarding host galaxy 2. For all orbits shown in Figures 19(a)–(d), the initial value of the Hamiltonian (10) is \( E_J = -2.0 \), while \( k_2 = 0.0004 \). In Figure 19(a) we can see the evolution of the maximum LCE for a 3D orbit and for a time period of \( 10^5 \) time units, as galaxy 2 evolves following the second set of Equation (19). The initial conditions are \( x_0 = 2.25, y_0 = 0, z_0 = 0.035, p_{x0} = 0, \) and \( p_{z0} = 0 \), while the value of \( p_{y0} \) is found from the Hamiltonian (10) in all cases. The values of all the other parameters are the same as those in Figure 4(b). When \( t = 100 \) time units, the mass of the developed quasar in the core of galaxy 2 is \( M_{n2f} = 0.04 \), and the evolution stops. The value of the Hamiltonian is now \( E_J = -2.00021 \). The profile of the LCE clearly indicates that this orbit starts regular and remains regular during the galactic evolution. Figure 19(b) is similar to Figure 19(a). This orbit has initial conditions: \( x_0 = 2.05, y_0 = 0, z_0 = 0.10, p_{x0} = 1.68, \) and \( p_{z0} = 0 \). The values of all the other parameters are the same as those in Figure 4(b). When \( t = 100 \) time units, the mass of the developed quasar in the core of galaxy 2 is \( M_{n2f} = 0.04 \), and the evolution stops. The value of the Hamiltonian is now \( E_J = -2.00051 \). In this case, the profile of the LCE clearly indicates that this orbit starts as a chaotic 3D orbit and remains chaotic during the galactic evolution. Figure 19(c) shows the evolution of the LCE of a 3D orbit with initial conditions \( x_0 = 2.6764, y_0 = 0, z_0 = 0.057, p_{x0} = 0, \) and \( p_{z0} = 0 \), while the values of all other parameters are the same as those in Figure 6(b). After a time interval of 100 time units, the mass of the quasar in the core of host galaxy 2 is \( M_{n2f} = 0.04 \), and the galactic evolution stops. The Hamiltonian settles to the value \( E_J = -2.00034 \). The profile of the LCE given in Figure 19(c) shows that the orbit starts as a regular 3D orbit, but after the galactic evolution it becomes a chaotic one. It is evident that if mass transport were not present, the orbit would have remained regular. The presence of the massive and dense quasar in the core of galaxy 2 has transformed the character of 3D orbits can change either from regular to chaotic or vice versa or not change at all as the mass is transported or vice versa or not change at all as the mass is transported and the massive and dense quasars are developed in the central disks. Numerical results, not provided here, suggest that the character of 3D orbits can change either from regular to chaotic or vice versa or not change at all as the mass is transported and the massive and dense quasars are developed in the central
regions of the host galaxies. In particular, from the sample of the 1000 tested orbits in the case of the time-dependent model, we conclude that 59% of the orbits altered their nature from regular to chaotic, 23% remained chaotic, 16% remained regular, and only 2% changed their characters from chaotic to regular. Whether or not the nature of a 3D orbit will change during the galactic evolution described by one of the sets of Equation (19) strongly depends on the initial conditions \((x_0, p_{x0}, z_0)\) of each orbit. Moreover, as the change of the value of the Hamiltonian (10) is negligible \((\Delta E_T \simeq 10^{-4})\), we can say that the phase space is transformed to itself during the galactic evolution. In order to make this statement more clear, we present an example: If we suppose that the evolving time-dependent model describes the 2D system, then as the change of the Hamiltonian is negligible, we could say that in the phase plane of the quiet galaxy 1 shown in Figure 5(b) chaotic regions would appear in the central regions, and it would be transformed to the phase plane shown in Figure 5(a), as the quasar is formed in the core of host galaxy 1.

6. DISCUSSION

In the present paper, we have constructed a simple 3D gravitational model in order to study the character of motion in a binary quasar dynamical model. In particular, we present a model for a binary pair of galaxies, following a circular orbit with galactic disks that are aligned with the orbital plane. Each galaxy is assumed to host a massive black hole at its core. The motion of a test particle (star) in the gravitational field of this system is studied, and various techniques are used to identify regular and chaotic motion. We have chosen the host galaxies in our dynamical model to be disk galaxies due to the fact that the belief that bright quasars reside in massive ellipticals (Dunlop et al. 2003) must be revised, as recent observations indicate that the majority of bright quasars are hosted in disk galaxies (Letawe et al. 2006). Observational data regarding radial velocities have shown that the masses of the large spirals are about \(2 \times 10^{11}\) solar masses. This mass is consistent with the mass of our disk galaxy 1. As observational data reveal that about 50% of the host galaxies show signs of gravitational interactions, we have decided to present a binary quasar dynamical model of three degrees of freedom, based on a pair of two interacting host galaxies.

A binary system of interacting galaxies hosting quasars in their cores is very complex; therefore, we need to assume some necessary simplifications and assumptions in order to be able to study the orbital behavior of such a complicated stellar system. Thus, our model is simple and contrived, in order to give us the ability to study different aspects of the dynamical model. Nevertheless, contrived models can provide insight into more realistic stellar systems, which unfortunately are very difficult to study if we take into account all the astrophysical aspects. Here, we must point out the main restrictions and limitations of our gravitational model: (1) The two host galaxies are assumed to be coplanar, orbiting each other in the same plane on circular orbits. (2) Our dynamical model only deals with the non-dissipative components of the host galaxies, stars, or possibly dark matter particles. It may be that gas accretion would be even more important to the question addressed in the present study. (3) The potentials we use are rigid and do not respond to the evolving density distribution in a more realistic way. This is because
our gravitational model that describes the binary system of the host galaxies is not self-consistent. Self-consistent models are usually deployed when conducting N-body simulations. Obviously, this is out of the scope of the present paper. Once more, we have to state that the above restrictions and limitations of our model are necessary; otherwise, it would be extremely difficult or even impossible to apply the extensive and detailed dynamical study presented in our study.

In our research, two different cases were investigated: the time-independent model and the evolving model, that is, the case when mass is transported from the disks of the galaxies to their centers forming a massive and dense quasar in each galactic core. Our numerical calculation indicates that there are several factors responsible for the observed chaotic motion in the time-independent model: (1) the galactic interaction, (2) the galactic activity, that is, the presence of the quasars, and (3) the Lindblad resonances. Furthermore, the presence of the quasars increases the velocities near the central regions of the host galaxies. The value of the velocity depends on the mass of the dense nucleus and the value of its scale length. Regular motion corresponds to low central velocities, while chaotic motion is characterized by high velocities. All of the above strongly indicate that in the centers of active galaxies chaotic motion in high velocities is expected. On the other hand, it was found that the two interacting galaxies, for large values of the distance between their centers, do not present chaotic motion when the massive quasars are not present in their cores.

As we have mentioned previously, one of the factors responsible for the chaotic motion and other resonance phenomena, such as islandic motion, is several inner Lindblad resonances

$$\Omega_p = \Omega - \frac{n}{m} \kappa,$$  \hspace{1cm} (20)

where $\Omega$ and $\kappa$ indicate the circular and the epicycle frequency of the star, respectively, while $m$ and $n$ are integers. The main resonances for the two host galaxies in both cases (active and quiet) together with the corresponding resonance radii $r_1$ and $r_2$, when $R = 2.5$ and $\Omega_p = 0.417229$ are given in Tables 2 and 3.

Figure 20(a) shows a plot of the curves $\Omega - n \kappa/m$ for the active host galaxy 1, as a function of the radius $r$. The numbers 1, 2, 3, and 4 indicate the curves $\Omega - 2\kappa/3$, $\Omega - 3\kappa/4$, $\Omega - 5\kappa/8$, and $\Omega - 7\kappa/9$, respectively. The straight lines are the curves

| Case      | Resonance | Region  | $r_1$  | $r_2$  |
|-----------|-----------|---------|--------|--------|
| Active G1 | $\Omega_p = \Omega - 2\kappa/3$ | Direct  | 0.0151 | ...    |
|           | $\Omega_p = \Omega - 3\kappa/4$ | Retrograde | 0.0147 | 0.2372 |
|           | $\Omega_p = \Omega - 5\kappa/8$ | Direct  | 0.0222 |        |
|           | $\Omega_p = \Omega - 7\kappa/9$ | Retrograde | 0.0212 | 0.3550 |
| Quiet G1  | $\Omega_p = \Omega - 2\kappa/5$ | Direct  | 0.0273 | ...    |
|           | $\Omega_p = \Omega - 3\kappa/4$ | Direct  | 0.0132 | 0.5268 |
|           | $\Omega_p = \Omega - 3\kappa/7$ | Direct  | 0.0226 | 0.3618 |
|           | $\Omega_p = \Omega - 4\kappa/9$ | Direct  | 0.0117 | 0.6054 |

| Case      | Resonance | Region  | $r_1$  | $r_2$  |
|-----------|-----------|---------|--------|--------|
| Active G2 | $\Omega_p = \Omega - 2\kappa/3$ | Direct  | 0.0120 | 0.5529 |
|           | $\Omega_p = \Omega - 3\kappa/4$ | Direct  | 0.0117 | 0.1346 |
|           | $\Omega_p = \Omega - 5\kappa/8$ | Retrograde | 0.0116 | 0.2348 |
|           | $\Omega_p = \Omega - 7\kappa/9$ | Direct  | 0.0097 | 0.6781 |
| Quiet G2  | $\Omega_p = \Omega - 2\kappa/5$ | Direct  | 0.0241 | 0.5561 |
|           | $\Omega_p = \Omega - 3\kappa/4$ | Direct  | 0.0098 | ...    |
|           | $\Omega_p = \Omega - 3\kappa/7$ | Direct  | 0.0180 | 0.2935 |
|           | $\Omega_p = \Omega - 4\kappa/9$ | Direct  | 0.0087 | 0.4756 |

$\Omega_p = \pm 0.417229$. The values of all other parameters are the same as those in Figure 2. Figure 20(b) is similar to Figure 20(a) but for the active host galaxy 2. As we can see, there is a considerable number of resonance radii for both the direct and retrograde orbits in both host galaxies. Details regarding the resonances and the resonance radii can be obtained from Tables 2 and 3. In other words, all the Lindblad resonances given in Tables 2 and 3 are also responsible for the chaotic motion in the two host galaxies. Things are very similar when the quasars are not present in the cores of the two host galaxies. It is also interesting to note that the above resonances produce large chaotic regions for small values of the distance $R$, while for larger values of $R$ (see Figures 5(a) and (b)) the chaotic regions are small, although the resonance radii are still present. This means that in this case the distance between the two galaxies precedes the Lindblad resonances.

In order to explore and understand the nature of orbits in the 3D dynamical system, we have used knowledge obtained from the study of the 2D system. Of particular interest was the determination of the regions of initial conditions in the $(x, p_x, z)$, $p_z > 0$, $(y - p_z = 0)$ phase space that produce regular or chaotic 3D orbits. As the value of $p_{z0}$ was found from the Jacobi integral (10), we used the same value of $E_j$, as in the 2D system and took initial conditions $(x_0, p_{z0}, z_0)$ such that $(x_0, p_{z0})$ lies in the chaotic regions of the 2D system. It was found that the motion is chaotic for all permissible values of $z_0$. On the other hand, in the case when $(x_0, p_{z0})$ is inside a regular region around the direct and retrograde periodic points, the corresponding 3D orbits are regular for small values of $z_0$, while for larger values of $z_0$ the orbits become chaotic. The particular values of $z_0$ for which the transition from regularity to chaos in 3D orbits occurs are different for each regular region of the 2D system. Of particular interest are the results given in Table 1, where we define the average minimum value of $z_0$ for each case near the direct and retrograde periodic points in both host galaxies.

An important role is played by the $L_z$-component of the test particle’s angular momentum. It was found that the values of $(L_z)$ for regular 3D orbits are larger than those corresponding to chaotic 3D orbits. Thus, the $L_z$-component of the angular momentum is a significant dynamical parameter connected with...
the regular or chaotic character of orbits in both 2D and 3D dynamical systems.

In order to estimate the degree of chaos in the 2D and 3D dynamical systems, we have computed the average value of the maximum LCE for a large number of orbits with different initial conditions in the chaotic regions in each case, for a time period of $10^5$ time units. Note that all the calculated LCEs were different on the fifth decimal point in the same chaotic region. The numerical results indicate that the degree of chaos in the 3D binary quasar system is smaller than that in similar 2D systems.

It is of great interest to follow the evolution of orbits as the quasars are formed in the central regions of the host disk galaxies. In this procedure, mass is transported from the disks to the nuclei of both galaxies; therefore, the quiet galaxies become gradually active, following Equation (19). We observe that the final character of the 3D orbits strongly depends on the particular initial conditions $(x_0, p_0, z_0)$. Therefore, regular orbits can turn to chaotic and vice versa, or they can maintain their characters (ordered or chaotic) during the quasar’s formation and after the quasars are formed. It was observed that a number of regular quasi-periodic orbits, starting near the central regions of each host galaxy, become chaotic. This can be seen in a velocity versus time plot, where the asymmetric profile of the total velocity is in agreement with observational data (see Grosbøl 2002), in which an increase of the stellar velocity is expected in regions with significant chaoticity. Moreover, observations show that an asymmetric velocity profile indicates chaotic motion.

An interesting question is whether interactions are essential to trigger the mass transportation and therefore the galactic activity. This question remains unsolved, as observational astronomers have found binary systems with disk galaxies (see Letawe et al. 2006) showing no signs of interaction but harboring active nuclei.

Forty years ago, galactic activity and interactions between galaxies were viewed as unusual and rare. Now they seem to be segments in the lives of many galaxies. From the astrophysical point of view, in the present work we have tried to connect galactic activity and galactic interactions with the nature of orbits (regular or chaotic) and with the behavior of the velocities of stars in host disk galaxies. We consider the outcomes of the present research to be an initial effort in exploring the dynamical structure of the 3D binary quasar system in more detail. As results are positive, further investigation will be initiated to study all the available phase space, including orbital eccentricity of the small host galaxy (the galaxy with the smallest value of the total mass) and its inclinations to the primary host galaxy. Moreover, we shall use the outcomes obtained from this initial dynamical study to conduct computer N-body simulations in a system of interacting galaxies in order to reveal changes in their orbital properties through merger processes and tidal effects, which are out of the scope of our present work.

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