Research Overview on Calculation of Shear Strength of Reinforced Concrete Bridge Piers

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Abstract. In this paper, the present research status on the shear capacity of piers at home and abroad was reviewed, and the principles and methods of the nine formulas for calculating the shear capacity presented by domestic and foreign codes and scholars were introduced in detail. The nine formulas were compared with the test data from Chenglan railway bridge piers to discuss the existing problems. The results show that the calculation formulas proposed by some scholars are more complicated and more factors are taken into account, while the calculation formulas proposed by the codes are simpler and more conservative. The existing formulas for calculating the shear capacity of bridge piers are mostly acquired by fitting of the experimental data and it is too simple to consider reduction effect of the stress distribution and shear capacity of section concrete with the increase of displacement ductility, so it is difficult to evaluate the applicability. Therefore, further research on the calculation formula of shear strength for bridge piers is needed.

1. Introduction

Destructive damage of earthquake at home and abroad shows that the piers in reinforced concrete bridges have extremely high seismic vulnerability. Because of the insufficient shear strength of the pier, the damage and even collapse of bridges' structures are reflected in the strong earthquake. In the San Fernando earthquake and the Kobe earthquake in Japan, there were examples of earthquake damage caused by shear failure of piers. One of the most typical examples, which occurred in the 1995 Hanshin earthquake in Japan, was a 18-span single-column reinforced concrete pier damaged and bent in Kobe, causing a section of the bridge about 500 meters to be completely laterally dumped. In the Tangshan earthquake in 1976, the Taiwan earthquake in 1999, and the Wenchuan earthquake in 2008, there were also examples of seismic damage caused by shear failure of reinforced concrete bridge piers.

The shearing problem of reinforced concrete structures is a classic subject. Scholars from various countries have carried out a lot of theoretical and experimental research on this problem, but there is no unified result for the mechanism of shear failure and the theory of calculation of shear strength. In order to study this problem, a detailed overview of the research status of reinforced concrete members' shear resistance and the calculation methods of existing main shearing formulas is given.
2. Research Status of Shear Strength of Reinforced Concrete

In the 19th century, scholars used the truss analysis method to propose the concept of providing shear bearing capacity with stirrups[1]. In 1906, Morsch proposed the average shear stress calculation formula, and the shear stress of the component is less than the ultimate shear strength. Since the 1950s, scholars have conducted in-depth research on the parameters affecting the shear strength, pointing out that the shear failure of reinforced concrete structures is a complex process with many influencing parameters[2], including shear span ratio, axial compression ratio, and volume distribution. Hoop rate, concrete strength, displacement ductility, etc. Taking these parameters as independent variables, a number of calculation formulas of shear capacity are given. The calculation of the pier shear resistance in the domestic and foreign codes refers to such a form.

There are many methods for the analysis of shear failure of reinforced concrete structures. In the early days, there were truss theory, limit equilibrium theory, plasticity theory, etc. In 1962, Ritter and Morsch[3] proposed the classical truss theory, which believed that the stirrups were tensioned to provide shear capacity. Subsequently, F.Leandardt made a correction based on this and proposed a modified truss model considering the influence of web stiffness.

In 1973, Bauman and Collins et al.[4] combined the stress balance equation, the molar strain coordination equation and the linear material constitutive relation in accordance with Hooke's law on the basis of the reinforced concrete truss theory, and proposed Moore. Strain coordination truss model. In 1976, Darwin et al.[5] proposed a model for analyzing the constitutive relationship of concrete subjected to two-way reciprocating loads and considered the compressive softening section of concrete. In the 1980s, Collins[6] and Vecchio[7] study the influencing factors of material softening effect under shear stress, and proposed soft truss theory and oblique pressure field theory. Karim et al.[8] proposed a statistical analysis method for analysis of shear failure of reinforced concrete structures.

In 1988, Hsu[9] and Belarbi based on the destruction test of a flat member under two-dimensional load, derived the model of the rotational angle truss. In the model, the tensile strength of concrete and the influence of the yield strength of the steel material due to the dispersion crack are considered, but the contribution of concrete to the shear strength of the pier cannot be reasonably calculated. In 1996, on the basis of the theory of rotational angle, Pang and Hsu[10] assumed that the crack was oriented to a fixed angle, and the theory of fixed angle was proposed to reasonably solve the problem of calculating the shear strength of concrete. From 1996 to 2001, the test was improved from the early force-controlled loading mode to the displacement-controlled loading mode. Hsu[11], Y. L. Mo et al. further studied the softening effect of materials under shear stress, and determined the normal and high-strength concrete constitutive considering the softening effect. In 2002, fixed angle theory analysis method can’t reasonably calculate the falling section of the structure after the peak. For this problem, the Poisson effect of fractured concrete is considered in the existing model, and the value of Poisson's ratio under different shear stress states is determined by Zhu and Hsu[12] based on the experiment, which solves the calculation problem of the falling section in the mechanical behaviour analysis of sheared structure. After that, the method for calculating the shear capacity of the structure of reinforced concrete wall is developed[13]. Lehman et al. proposed the index to measure the damage degree of reinforced concrete bridge piers by studying the damage condition and reaction characteristics of reinforced concrete columns after earthquake[14].

On the basis of the above-mentioned development of shear theory of reinforced concrete structures, foreign scholars have proposed shear strength calculation formulas.

3. Introduction to Calculation Formula for Shear Capacity of Reinforced Concrete Piers

3.1. Comparison of the Standard of Shearing of Piers in Various Countries and the Formula of Shearing Proposed by Scholars

In the capability-based seismic design method, in order to ensure sufficient displacement ductility of concrete piers, the shear strength of the piers are greater than the maximum shear force that the piers can withstand. The shear capacity of the pier is greater than the shearing demand, ensuring the
expected ductile failure mode of the pier. The formula for calculating the shear capacity of piers proposed by domestic and foreign codes and scholars will be introduced in detail below.

### 3.1.1. ACI 318M-05

The US ACI 318M-05 specification[15] was written by the American Concrete Association Committee based on corresponding reports and studies. This specification uses metric units. The checking conditions for the design section shear strength are:

\[
\phi V_u \geq V_n
\]  

In the formula, \( V_u \) is the shear force requirement at the design section and \( V_n \) is the nominal shear strength of the section. The calculation formula is as follows:

\[
V_n = V_c + V_s
\]

The shear strength provided by concrete and shear rebar respectively is:

\[
V_c = 0.17 \left( 1 + \frac{N_u}{14A_g} \right) \left( f'_c \right)^\frac{1}{2} b_n d
\]

\[
V_s = \left( A_v f'_s d \right) / S
\]

### 3.1.2. CALTRANS BRIDGE DESIGN SPECIFICATIONS

The US Caltrans specification[16] was promulgated and implemented by the California Transportation Bureau in February 2004. The checking conditions for the shear strength of the design section of ductile concrete members are:

\[
\phi V_u \geq V_n
\]

The nominal shear strength consists of two parts:

\[
V_n = V_c + V_s
\]

The shear strength provided by concrete needs to take the effects of bending and axial loading into account.

\[
V_c = v_c + A_c
\]

\[
A_c = 0.8 A_y
\]

In the zone of plastic hinge:

\[
v_c = \text{Factor1} \times \text{Factor2} \times \left( f'_c \right)^\frac{1}{2} \leq 0.33 \left( f'_c \right)^\frac{1}{2}
\]

Outside the area of the plastic hinge:

\[
v_c = 0.25 \times \text{Factor2} \times \left( f'_c \right)^\frac{1}{2} \leq 0.33 \left( f'_c \right)^\frac{1}{2}
\]

The values of the above two coefficients are:

\[
0.025 \leq \text{Factor1} = \frac{\rho f_y}{12.5} + 0.305 - 0.083 \mu_y < 0.25
\]

\[
\text{Factor2} = 1 + \frac{P_e}{13.8 \times A_y} < 1.5
\]
The contribution of shear rebar to shear strength is based on the truss principle. For rectangular and circular sections, respectively:

\[
V_s = \frac{A_{f_{sh}} d}{S} \quad \text{(13)}
\]

\[
V_s = \frac{\pi}{2} \left( \frac{A_{f_{sh}} D}{S} \right) \quad \text{(14)}
\]

Unlike the ACI 318M-05 calculation model, the Caltrans specification uses dimensionless coefficients to reflect the effect of the hoop rate, ductility and axial load on the shear capacity provided by concrete. Factor1 specifies a lower limit of 0.025 in the zone of plastic hinge and an upper limit of 0.25. Outside the area of plastic hinge, it is taken as 0.25. What Factor 2 reflects is the effect of axial compressive stress on shear strength improvement.

3.1.3. Priestley et al. (UCSD). Priestley et al.[17] proposed that the shearing formula of reinforced concrete piers consists of three parts by the research results of ASCE/ACI codes[18], Ang/Wong[19,20] and Watanabe/Ichinose et al.[21] and the analysis of experimental data. In addition to the contribution of concrete and shear rebar to the shear strength of reinforced concrete piers, axial loads also have an effect on shear strength and separate them. The specific expression is:

\[
V_n = V_c + V_p + V_s \quad \text{(15)}
\]

The results show that the contribution of concrete to shear strength is related to the tensile strength of concrete and the ductility of piers. The contribution of axial load to shear strength is related to the axial load and shear span ratio. The contribution of shear rebar to shear strength relates with the reinforcement ratio of shear rebar. Nominal shear strength provided by concrete is:

\[
V_c = k \left( f_{c} \right)^{\frac{1}{2}} A_s \quad \text{(16)}
\]

The nominal shear strength provided by shear bars, for circular and rectangular sections, respectively are:

\[
V_s = \frac{\pi}{2} \left( \frac{A_{f_{sh}} D}{S} \right) \cot 30^\circ \quad \text{(17)}
\]

\[
V_s = \frac{A_{f_{sh}} D}{S} \cot 30^\circ \quad \text{(18)}
\]

Nominal shear strength provided by axial loads:

\[
V_p = \frac{D - c}{2a} P \quad \text{(19)}
\]

3.1.4. USC Model. Xiao Yan and Martirossyan et al.[22] based on the Priestley model to make it more accurate to predict the shear strength of high-strength concrete. The most obvious improvement is the use of a bilinear model to characterize the effect of displacement ductility on shear strength. The composition of the shear strength consists of concrete, shear rebars and axial loads.

\[
V_n = V_c + V_p + V_s \quad \text{(20)}
\]

The nominal shear strength provided by concrete, axial loads and shear rebar are:

\[
V_c = k \left( f_{c} \right)^{\frac{1}{2}} A_s \quad \text{(21)}
\]
\[ V_n = \frac{D - c}{2D(M/VD)} P \]  
(22)

\[ V_c = \frac{A_v f_{sh}(d - c)}{S} \cot(\theta) \]  
(23)

3.1.5. Sezen et al. (UCB). Sezen and Jack P. Moehe[23] proposed a different model from before, which is characterized by the fact that the displacement ductility of the pier has an effect on the shear strength provided by concrete and shear rebar, and is also believed that the axial load affects the shear strength of the pier indirectly by affecting the concrete. Its shear strength formula is:

\[ V_n = V_c + V_s \]  
(24)

The nominal shear strength provided by concrete and shear rebar is:

\[ V_c = k \left[ 0.5 \left( f_v \right)^{\frac{1}{2}} \left( 1 + \frac{P}{0.5 \left( f_v \right)^{\frac{1}{2}} A_v} \right) \right] 0.8A_v \]  
(25)

\[ V_s = \frac{k A_v f_{sh} d}{S} \]  
(26)

3.1.6. UH Model. Based on the research on the calculation model of shear capacity and the influence law of corresponding parameters, the scholars of the University of Houston in the United States[24,25,26,27] proposed the calculation model of the shear capacity of pier. Based on the Priestley model, the model refines the influence law of displacement ductility and performs simulation calculation based on SRCS program. Good results were obtained by comparison of the models.

\[ V_n = V_c + V_p + V_s \]  
(27)

The nominal shear strength provided by concrete, axial loads and shear rebar is:

\[ V_c = k \left( f_v \right)^{\frac{1}{2}} A_v \]  
(28)

\[ V_p = \frac{D - c}{2a} P \]  
(29)

\[ V_s = (3000 \rho_1^2 - 115 \rho_1^2 + 1.2 \rho_2) b_v f_{sh}(d - c) \cot(\theta) \]  
(30)

3.1.7. Guidelines for Seismic Design of Highway Bridges (H.G.). The 2008 edition of China's "Guidelines for Seismic Design of Highway Bridges" draws on the calculation formula for the shear capacity of the California California Seismic Design Guidelines, but it is simplified. The shear strength of the oblique section of the plastic hinge area of the pier shall be checked according to the following formula:

\[ V_{sh} \leq \phi \left( 0.0023 \left( f_v \right)^{\frac{1}{2}} A_v + V_c \right) \]  
(31)

Nominal shear capacity provided by stirrups:

\[ V_s = 0.1 \frac{A_b}{S_b} f_{sh} \leq 0.067 \left( f_v \right)^{\frac{1}{2}} A_v \]  
(32)
3.1.8. Guidelines for Seismic Design of Urban Bridges (U.G.). The 2011 edition of China's "Guidelines for Seismic Design of Urban Bridges" [28] draws on the calculation formula of the shear capacity of the American AASHTO specification. The shear strength of the oblique section of the plastic hinge area of the pier along the direction of the bridge and the transverse bridge shall be checked according to the following formula:

\[ V_{ch} \leq \phi (V_c + V_r) \]  

(33)

\[ V_c = 0.1 v_c A_c \]  

(34)

\[ V_r = \begin{cases} 
0, & P_r \leq 0 \\
\lambda \left(1 + \frac{P_r}{1.38 \times A_r}\right)^{1/2} & \frac{0.355(f_{cd})^{1/2}}{\lambda}, \quad P_r > 0 \\
\frac{1.47\lambda(f_{cd})^{1/2}}{P_r} & P_r > 0 
\end{cases} \]  

(35)

The shear forces provided by circular and rectangular sections of shear rebar is:

\[ V_c = 0.1 \times \pi \frac{A_v f_{cd} D}{S} \leq 0.08 \left( f_{cd} \right)^{1/2} A_c \]  

(36)

\[ V_r = 0.1 \times \frac{A_v f_{cd} h_0}{S} \leq 0.08 \left( f_{cd} \right)^{1/2} A_c \]  

(37)

3.1.9. Specifications for Highway Bridges. Same Explanation. In the Japanese Code for Seismic Design of Bridges in 2002. The lateral shear capacity of pier section is provided by concrete and transverse reinforcement.

\[ P_t = S_c + S_s \]  

(38)

\[ S_c = c_c c_p \tau_b d \]  

(39)

\[ S_s = \frac{A_s \sigma_s d (\sin \theta + \cos \theta)}{1.15a} \]  

(40)

3.2. Analysis and Evaluation of Shearing Formula

3.2.1. Analysis of influencing factors in shear strength formula The calculated results of the multiple shearing formulas mentioned in the previous section are different. This paper summarizes the factors affecting the shear strength considered in each shearing formula, as Table 1 shows. For ACI 318M-05, the shear capacity of the pier is considered to be constant and does not change with the increase of displacement ductility, which is obviously inconsistent with the corresponding theoretical and experimental studies. The calculation formula for shear capacity in many national codes is too conservative, which is related to the lack of research on the shear mechanism and the lack of a consistently accepted computational model. It leads to unscientific design and waste of materials.
| Formula          | Sectional form | Sectional area | Concrete strength | Longitudinal reinforcement ratio | Stirrup ratio | Stirrup strength | Axial compression ratio | Shear span ratio | Ductility coefficient |
|------------------|----------------|----------------|-------------------|----------------------------------|---------------|------------------|------------------------|----------------|----------------------|
| ACI 318-08       |                |                |                   |                                  |               |                  |                        |                |                      |
| Caltrans         | √              |                |                   |                                  |               |                  |                        |                |                      |
| Priestley       |                |                |                   |                                  |               |                  |                        |                |                      |
| USC              |                |                |                   |                                  |               |                  |                        |                |                      |
| Sezen            |                |                |                   |                                  |               |                  |                        |                |                      |
| UH               |                |                |                   |                                  |               |                  |                        |                |                      |
| U.G.             |                |                |                   |                                  |               |                  |                        |                |                      |
| Japanese         |                |                |                   |                                  |               |                  |                        |                |                      |

Compared with ACI 318M-05, the Caltrans formula considers the effects of stirrups, axial and displacement ductility on the shear strength provided by concrete by introducing the factors Factor1 and Factor2. It can be seen from the formula (11) that as the displacement ductility increases, the value of Factor1 gradually becomes smaller. It causes the decrease of shear resistance provided by the concrete to be weakened, which is consistent with the experimental observation. Factor 2 is used to consider the effect of axial pressure on the shear strength provided by concrete. The lower and upper limits of the values are 1.0 and 1.5 respectively, indicating that the maximum contribution of axial pressure to the shear capacity provided by concrete is 150%. The shear capacity calculation formula provided by the shear rebar is basically the same as the ACI 318M-05 specification. The most important feature of the calculation formula proposed by Priestley et al. is to investigate the reduction effect of the shear capacity with the increase of displacement ductility through a dimensionless coefficient. Another feature is that it separately considers the contribution of shaft pressure to shear resistance. The calculation of shear capacity provided by the shear rebar takes the development direction of the oblique crack into account by introducing a coefficient.

The USC model is more accurate in predicting the shear resistance of high-strength concrete columns. Similarly, a dimensionless coefficient is used to investigate the reduction effect of displacement ductility on the shear resistance of concrete, but the value of this coefficient is slightly different. In calculating the shear resistance provided by the shear rebar, an angle was introduced to consider the direction of the different oblique cracks, instead of introducing a definite coefficient as in the previous example.

The model proposed by Sezen et al. considers that the ductility of displacement has a reduction effect on the shear resistance provided by concrete and shear rebar, and that the existence of axial pressure is considered to improve the shear resistance of concrete.

The UH model introduces a dimensionless coefficient to consider the reduction effect of displacement ductility on shear resistance, but the calculation is slightly more complicated, considering the influence of the stirrup ratio. The calculation formula of shear capacity provided by the shear rebar is expressed in the form of stirrup ratio.

The calculation formulas for the shear capacity of the plastic hinges of bridge piers in China's "Code for Seismic Design of Highway Bridges" and "Code for Seismic Design of Urban Bridges" draw on the Caltrans and AASHTO specifications of the United States and make some improvements. The former calculates the shear strength provided by concrete and rebar respectively and combines them by coefficient reduction. As with the ACI 318M-05 specification, the calculations are conservative. The latter considers the axial pressure to enhance the shear resistance of the concrete. The calculation formula is relatively complicated, but the reduction factor is also introduced.

In the calculation formula of the shear capacity proposed by the Japanese Bridge Seismic Design Code, the three non-dimensional coefficients are used to consider the reciprocating load, the pier
section and the tensile reinforcement to correct the shear capacity of the pier provided by the concrete. Similar to the previous model, the direction of the oblique crack is considered.

3.2.2. Evaluation of shear strength formula In order to better evaluate the above nine calculation formulas of shear capacity, the results of the calculation of nine formulas are compared with the test data of four shear destruction piers in the Chenglan Railway Engineering Test, and then compare the accuracy of the listed formulas and discuss the rationality of each calculation model. The parameters of the four piers are shown in Table 2:

| Name of piers | Shear span ratio | Axial compression ratio | Stirrup ratio | Destruction mode          |
|---------------|------------------|------------------------|---------------|---------------------------|
| A1            | 2                | 0.99%                  | 0.12%         | Bending shear failure     |
| A2            | 1.5              | 0.99%                  | 0.12%         | Bending shear failure     |
| A3            | 2                | 1.98%                  | 0.12%         | Bending shear failure     |
| A4            | 2                | 0.99%                  | 0.08%         | Bending shear failure     |

According to the nine calculation formulas of shear capacity listed in Section 3.1, the curves of shear capacity of the piers are calculated separately and compared with the demand curves obtained by the test. The results are shown in Figure 1 and Figure 4.

**Figure 1.** Comparison of shear capacity with shear demand of A1

**Figure 2.** Comparison of shear capacity with shear demand of A2

**Figure 3.** Comparison of shear capacity with shear demand of A3

**Figure 4.** Comparison of shear capacity with shear demand of A4
As can be seen from Figures 1~4, the results obtained by the calculation formulas of shear capacity for the same pier are quite different. The calculation formula of shear capacity proposed by domestic and foreign scholars overestimates the actual shear capacity of the pier and that proposed by national codes is conservative. Considering the reduction factor, the calculation results are lower than the test results. And the formula itself cannot accurately reflect the change rule of shear capacity of pier with the increase of ductility, which leads to the difference between the calculated results and the actual shear capacity of the test pier in the later stage of loading. Moreover, the existing formulas often use a coefficient independent of pier parameters to consider the change trend of shear capacity of pier with the increase of displacement ductility, which cannot fully reflect the relationship between the change of shear capacity and the increase of displacement ductility of the pier.

Generally speaking, the calculation formula proposed by scholars is more complex and considers more factors and the formula proposed in the code is simple and conservative.

4. Conclusions
The principles and methods of the nine calculation formulas of shear capacity presented by domestic and foreign codes and scholars are compared in the paper, and the calculation results of each formula are compared with the test results of four shear failure piers in the Chengan Railway Engineering Test. The following conclusions are drawn:
(1) The existing calculation formulas are mostly acquired by fitting of test data, stress distribution of corresponding cross-section concrete of single stress value is often used, and constant reduction coefficient is used to investigate the reduction effect of shear capacity with increase of displacement ductility without considering the influence of design parameters (ACI 318M-05 does not consider the reduction of shear capacity of piers with the increase of displacement ductility), which does not match the actual situation. The influence parameters of each formula are different, and the calculation results are obviously different and the dispersion is large. China's Seismic codes directly draw on foreign codes. Due to material characteristics and regional differences, it is difficult to evaluate their applicability.
(2) According to the results of calculation and test comparison, the calculation formulas proposed by scholars are more complex and more factors are taken into account. The calculation formulas proposed by the code are simple and conservative.

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