Main Results:

- An $l_1$, Total Variation (TV), regularized Maximum Likelihood (ML) method to segment a time series with respect to changes in the mean or in the variance.
- We show that, in this setting, variance estimation of $\{y_t\}$ is equivalent to mean estimation of $\{y_t^2\}$.

Why?

- Estimating means, trends and variances in time series data are of fundamental importance in a variety of areas. Typically done to pre-process data before estimation of, for example, parametric models.
- For non-stationary data it is important to detect changes in the mean and the variance in order to segment the data into stationary subsets.

How?

Traditionally: Moving window sample mean and variance estimation. Hypothesis test based change detection.

Here: The $l_1$ sparseness approach. Penalize the difference between consecutive variables.

Mean Estimation

Data: $\{y_1, \ldots, y_N\}$

Model: $y_t \sim N(m_t, 1)$, where $m_{t+1} = m_t$ often

Method: $\min_{m_t} \left[ \frac{1}{2} \sum_{t=1}^{N} (y_t - m_t)^2 + \lambda \sum_{t=1}^{N} |m_t - m_{t-1}| \right]$

ML+TV: Related to fused lasso, $l_1$ trend filtering and total variation denoising

Equivalence

The variance estimation problem for $\{y_t\}$ has same sub-gradient (first order) optimality conditions as the mean estimation problem for $\{y_t^2\}$.

Proof idea:

$$\frac{d}{d\eta_t} \left[ \sum_{t=1}^{N} \ln(-\eta_t) - \sum_{t=1}^{N} 2\eta_t y_t^2 \right] = \frac{1 - 2y_t^2}{\eta_t} = 2(\sigma_t^2 - y_t^2)$$

$$\sigma_t - \sigma_{t-1} = \frac{\sigma_t^2 - \sigma_{t-1}^2}{2\sigma_t^2 \sigma_{t-1}} \Rightarrow \text{Ordering is preserved} \Rightarrow$$

$$\frac{d}{d\sigma_t^2} \left[ \sigma_t^2 - \sigma_{t-1}^2 \right] = \frac{d}{d\sigma_t^2} \left[ |\eta_t - \eta_{t-1}| + |\eta_{t+1} - \eta_t| \right]$$

Ongoing and Future Work

- The vector valued covariance matrix case: $(n + 1)n/2$ variables per $n$-dimensional sample.
- Alternating Direction Method of Multipliers (ADMM) convex optimization algorithm with linear complexity.
- Statistical analysis and applications.

Concave + Convex! Standard trick: $\eta_t = -1/(2\sigma_t^2)$

Method: $\min_{\eta_{t < 0}} \left[ \frac{1}{2} \sum_{t=1}^{N} \ln(-\eta_t) - \sum_{t=1}^{N} 2\eta_t y_t^2 + \lambda \sum_{t=1}^{N} |\eta_t - \eta_{t-1}| \right]$

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