Non-fragile $H_\infty$ control for LPV-based CACC systems subject to denial-of-service attacks

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Abstract
This paper is concerned with a non-fragile $H_\infty$ state feedback control issue for linear parameter-varying collaborative adaptive cruise control systems subject to denial-of-service attacks. The dynamics of the collaborative adaptive cruise control system is described by a linear model where the deviation of the position and the velocity are selected as the state variables. The attack model is utilized, thereby better reflecting the randomly occurring phenomenon of the denial-of-service attacks based on a sequence of binary random variables. The main objective of this note is to develop a non-fragile state feedback control scheme such that, for denial-of-service attacks and possible parameter variations in controller gains, the exponential mean-square stability and the predefined performance index for the system states are guaranteed simultaneously. By using the matrix analysis techniques and Lyapunov stability theory, sufficient conditions for the desired controller are established and solved based on the solutions to the linear matrix inequality conditions. Finally, a three-car model is provided to check the feasibility of the designed control scheme.

1 | INTRODUCTION
During the past few decades, the connected automated vehicles (CAVs) technology has become a research frontier. Enabled by advanced sensing and communication, the CAVs technologies have been widely used to overcome the traffic jam problems (see [1–3] for details). Among them, the cooperative adaptive cruise control (CACC) technology has become one of the most promising technologies for CAVs due primarily to its ability to thoroughly improve traffic throughput and stability through real-time control of vehicles’ longitudinal motion. In a CACC system, CAVs share their own information (position, velocity, acceleration) with other CAVs by vehicle-to-vehicle networks communications which are realized in an autonomous manner. Inspired by this merit, much research effect has been devoted to this aspect and some initial results have been reported [4–7]. For example, model predictive control issues have been addressed in [8] for a class of CAVs with CACC systems by minimizing a quadratic function. In [9], the consensus control problems have been investigated for CAVs with CACC system where a vehicular platooning issue is taken into account, and a velocity-dependent spacing strategy is realized. Moreover, some applications of CACC technology have been addressed, such as platoon vehicles [6, 10], eco-driving on signalized corridors [11], collaborative driving on the highway [12] and so on.

On the other research frontier, LPV systems have gained much research enthusiasm because of its wide applications in CAVs. LPV systems are often subject to uncertainty for measurable time varying parameters in the real model. In this case, the measurement of the parameters can reflect the real-time system characteristics change. Exhausting the existing literature, it should be pointed out that there are two basic approaches to cope with this unique system. The first basic approach was pioneered by the authors in [13], in which the state variables depend affinely on the time-varying parameters. A quadratic Lyapunov function is used to solve the problems of analysis and synthesis on the basis of quadratic stability. On the other hand, following the pioneering work in [14], it is assumed that the uncertain real parameters and their rate of change vary within a given range. Inspired by this idea, the stability analysis and control
synthesis are implemented based on the notion of affine stability by using parameter-dependent quadratic Lyapunov functions. To date, enormous research enthusiasm has been available for this kind of systems [15–19]. For instance, LPV-based slip-controller has been designed in [15] for two-wheeled vehicles. In [20], a data-based approach is developed for LPV-ARMAX models to the modelling and analysis of vehicle collision. Moreover, the issue of model predictive control is addressed for constrained LPV systems where both the disturbance and noise are bounded in [21]. Nevertheless, the aforementioned results are concerned with the determined gain parameter. The non-fragile feedback control problems for LPV-based CAVs with CACC have not been completely investigated yet, which makes one of the main motivations of this article.

From the perspective of the attack mechanism and mathematical model, main streams of the cyber-attacks are often divided into the following categories, such as deception attacks [22], replay attacks [23] and denial-of-service (DoS) attacks [24, 25]. Among them, DoS attacks which attempt to block the transmissions between the receiver and the delivery, are more natural to be launched by the adversaries and reachable in the attack space. In the existing literature, there are some elegant results concerned with filtering and control problems for DoS attacks [25–28]. For example, the problem of filtering has been investigated in [29], for the power systems subject to the DoS attacks describing by the Bernoulli variables with the help of recursive filtering method. In [30], a decentralized adaptive fuzzy secure controller is developed for the non-linear uncertain interconnected systems subject to intermittent DoS attacks. The controller parameters are derived by the solutions to a set of linear matrix inequalities (LMIs). In addition, the resilient/non-fragile control and detection and estimation problems are addressed where the DoS attacks are taken into account, respectively, in [28, 31]. However, state feedback control problems for CAVs with CACC system has not been investigated, yet still has been an open topic, not to mention a non-fragile case. It would be an interesting problem by addressing the issue of non-fragile $H_{\infty}$ state feedback control (NFSFC) for LPV-based CACC system subject to DoS attacks, thereby shortening the gap.

In this article, our aim is to develop a non-fragile $H_{\infty}$ state feedback control for LPV-based CACC system subject to DoS attacks. The addressed problem poses the following significant challenges: (1) how to construct the comprehensive LPV-based CACC model which contains gain variations and DoS attacks; (2) how to design the controller which is able to effectively handle the adverse effects caused by gain variations and DoS attacks; and (3) how to guarantee the feasibility of the algorithm and the asymptotic stability of the closed-loop system. The main novelties of this article are emphasized as follows: (1) the first of few attempts is made to investigate the design problem of NFSFC for a class of LPV-based CACC system subject to DoS attacks; (2) a modified non-fragile $H_{\infty}$ state feedback controller is developed such that, for all introduction phenomena such as parameter variations in gain parameters as well as the DoS attacks, the system is exponentially mean-square stable and the performance index is satisfied as well; and (3) an effective NFSFC algorithm is proposed, which represents a simple and computational effective algorithm that is tested in a platoon of three-car model, and embedded in a simulation model, built using MATLAB/Simulink. At last, the simulation is utilized to validate the usefulness of the proposed NFSFC algorithm.

The remaining part of this paper is summarized as follows: In Section 2, the problem formulation under consideration is set up. Sufficient conditions for stability analysis of the CACC system under a prescribed $H_{\infty}$ performance index are firstly established, and then controller parameter is obtained by the solutions to the LMI in Section 3. A three-car model is exploited to validate the effectiveness of the developed scheme in Section 4. Finally, we draw the conclusion in Section 5.

Notation: In this paper, the notations mentioned are reasonable unless stated otherwise. $\mathbb{R}$, $\mathbb{R}^m$, $\mathbb{R}^{n \times m}$, respectively, denote the $n$-dimensional Euclidean space and $n \times m$ real matrix. $I$ means the identity matrix. Moreover, diag{·} denotes the block-diagonal matrix. $\text{col}_N$ denotes a matrix consisting of $N$ single column elements. For symmetric matrices $u$ and $v$, $u \geq v \ persecuiwes(>)$ means that $u - v$ is a positive semi-definite (positive definite) matrix. $\text{Pf}$ denotes the occurrence probability of the event $\omega$. $N^T$ and $N^{-1}$ represent the transposition and the inverse of matrix $N$, respectively. $\text{tr}[M]$ stands for the trace of the matrix $M$. $E\{\cdot\}$ is the mathematical expectation of the stochastic variable $x$. $\lambda_{\text{min}}(M)$ is the minimum eigenvalue of matrix $M$. The subscript $t$ and $k$ are, respectively, used for continuous-time and discrete-time.

2 | PRELIMINARIES

2.1 | The model of CACC

Assumption 1. Throughout the paper, the following assumption is considered:

i. Controlled vehicles monitor positions and velocities of other vehicles without any delay.

ii. Controlled vehicles are fully automated and cooperative: the information is shared via vehicle-to-vehicle and vehicle-to-infrastructure communications, so that human drivers and non-cooperative behaviours are not considered.

iii. Control decisions are updated at constant time intervals.

iv. Control vehicles implement control decisions simultaneously.

Consider CAVs equipped with the CACC system implemented in a platoon system for improving traffic safety and maintaining the road capacity. The structure of the CAVs with a CACC system is shown in Figure 1, and the relevant parameter variables are defined in Table 1, and the target system.
\( x(t) \in \mathbb{R}^{2n_x} \) can be defined as (see, [1, 10, 32])

\[
\tilde{x}_i = \text{col}_k\{x_{i,j}\}, \quad i = 1, 2, \ldots, n_x \tag{1}
\]

where \( x_{i,j} = [\Delta s_{i,j}^T, \Delta v_{i,j}^T]^T \), \( \Delta s_{i,j} = s_{i,j} - s_{i,j}^* \) is the deviation of the \( i \)-th vehicle from the equilibrium spacing, \( s_{i,j} \) is the spacing of the \( i \)-th vehicle with its ahead vehicle, \( s_{i,j}^* = \tau_i \times v_{i,t} + d_i \) represents the equilibrium spacing, \( \tau_i \) is the \( i \)-th vehicle’s desired time gap, \( d_i \) is the minimum secure spacing, \( \Delta v_{i,j} \) represents the deviation between the desired velocity \( v \) and the current velocity \( v_{i,t} \). Then, according to the analysis in [10, 32], the dynamics of \( x_i \) is formulated as

\[
\dot{x}_i = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & \cdots \\
0 & -\frac{\mu_k}{m_k} & 0 & 0 & 0 & 0 & \cdots \\
0 & 0 & 0 & 0 & 0 & 1 & \cdots \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{bmatrix}
\begin{bmatrix}
x_i \\
\Delta s_{i,j} \\
\Delta v_{i,j} \\
\end{bmatrix}
\tag{2}
\]

with control input \( u_j = \text{col}_k\{f_{i,j}\} \) \((j = 1, 2, \ldots, n_u)\), where \( f_{i,j} \) is the external force for the \( j \)-th vehicle to adjust the deviation of the spacing and velocity, and \( m_j \) and \( \tau_j \), respectively, represent the mass and time gap of the \( j \)-th vehicle.

**Remark 1.** In a CACC system, position, velocity and acceleration are the three fundamental elements in keeping the vehicle platoon working in a good manner. This paper chooses the state variable (which contains the velocity and position information) as \( x_i = \text{col}_k\{x_{i,j}\} \), where \( x_{i,j} = [\Delta s_{i,j}^T, \Delta v_{i,j}^T]^T \). The control output is the external force \( f_{i,j} \) (which contains the acceleration information). The desired position from the equilibrium position and the desired velocity from the equilibrium velocity can be expressed by \( \Delta s_{i,j} = l_j - l_{j+1} - s_{i,j}^* \) and \( \Delta v_{i,j} = v_{i,j} - v^* \), respectively. With the help of kinematic equations \( \frac{d}{dt} \Delta s_{i,j} = \Delta v_{i,j} \) and \( m_j \frac{d}{dt} \Delta v_{i,j} = f_{i,j} \), and augmenting the state variables, which eventually equals to Eq. (2).

In reality, the system parameters are not always exactly known, whilst the real system depends on uncertain measurable time-varying parameters because of the varying external environment and the disturbance. In this case, the continuous-time system (2) is discretized as the following LPV system

\[
\tilde{x}_{k+1} = A(\tilde{\varphi}_k)\tilde{x}_k + B(\tilde{\varphi}_k)\tilde{u}_k + D(\tilde{\varphi}_k)\tilde{w}_k, \quad (3)
\]

where \( \tilde{w}_k \in \mathbb{R}^{\hat{m}_w} \) is the received control signal, \( \tilde{w}_k \in (l(0), \infty), \mathbb{R}^{\hat{m}_w} \), the system parameter matrices \( A(\cdot), B(\cdot) \) and \( D(\cdot) \) are the functions of time-varying parameter \( \tilde{\varphi}_k \), the varying parameter \( \tilde{\varphi}_k \) is the scheduling parameter of the LPV system. At each time \( k \), the parameter \( \tilde{\varphi}_k \) is exactly known. The parameter vector \( \tilde{\varphi}_k = [\tilde{\varphi}_{1,k}, \tilde{\varphi}_{2,k}, \cdots, \tilde{\varphi}_{N,k}]^T \in \mathbb{R}^N \), which belongs to a convex polytope set \( \Omega \) defined by

\[
\Omega = \left\{ \tilde{\varphi}_k \in \mathbb{R}^N \mid \sum_{i=1}^N \tilde{\varphi}_{i,k} = 1, \tilde{\varphi}_{i,k} \geq 0 \right\},
\]

where \( N \) is the number of vertex matrices. The system parameters \( [A(\tilde{\varphi}_k), B(\tilde{\varphi}_k), D(\tilde{\varphi}_k)] = \sum_{i=1}^N \tilde{\varphi}_{i,k}[A_i, B_i, D_i] \) and \( [A_i, B_i, D_i] \) are the submodels of the LPV system. For notation simplicity, we denote \( \tilde{\varphi} \) for \( \tilde{\varphi}_k \) in the rest of the paper.

**Remark 2.** In ideal conditions, the dynamics of CACC system is formulated as \( \frac{d}{dt} \tilde{x}_i = A^T \tilde{x}_i + B^T \tilde{u}_i \) in [10]. In the discretization process, we have \( A = e^{A'T}, B = \int_0^T e^{A'T}dA'B' \), \( T \) is the sampling interval, such that the discrete-time system for CACC can be transformed into \( \tilde{x}_{k+1} = A^T \tilde{x}_k + B^T \tilde{u}_k \). Unfortunately, the system parameters are not always accurate and invariant due mainly to the external varying environment and undesired disturbance (such as wet pavement, the wind action, tire friction etc). Actually, the system parameters usually change with the evolution of the states. It should be pointed out that the system is often subject to unknown, yet measurable parameters, which provide real-time information on the variations of the plant’s characteristics, thereby better reflecting the practical engineering. As such, an LPV-based CACC system model with
external disturbance, $\xi_{k+1} = A(\bar{\theta})\xi_k + B(\bar{\theta})\hat{u}_k + D(\bar{\theta})\hat{w}_k$, is adopted.

### 2.2 Design of the controller

In a CACC system, in order to keep the vehicle platoon works in a good manner, the $\Delta f$ and $\Delta v$ should converge to zero in a reasonable interval. In view of this, an effective and convenient controller is of great importance.

As shown in Figure 2, for open-loop system (3), a non-fragile state feedback controller $\bar{u}_k \in \mathbb{R}^{x_k}$ is established by

$$\bar{u}_k = (F(\bar{\theta}) + \Delta F)\xi_k,$$

where the controller gain $F(\bar{\theta}) = \sum_{i=1}^{N} \bar{\theta}_i F_i$, and the $\Delta F$ is the controller gain variation which satisfies

$$\Delta F = M\Theta N', \Theta' \Theta \leq I,$$

where $M$ and $N'$ are matrices of known appropriate dimensions.

As is known to all, during the process of the transmission, the vehicle-to-vehicle communication is often subject to the DoS attacks because of the cyber vulnerability. Actually, in a cyber-physical system, the transmitted packets will undergo a vast risk of attacking by a malicious adversary. In other words, the information may be blocked randomly or intermittently during the transmission, and therefore, the actual control signal can be formulated by

$$\tilde{u}_k = \pi_k \bar{u}_k + (1 - \pi_k)\hat{u}_k,$$

where $\pi_k$ is a sequence of binary random variables with the following distribution:

$$\Pr[\pi_k = 1] = \bar{\pi}, \Pr[\pi_k = 0] = 1 - \bar{\pi},$$

where $\bar{\pi} > 0$ is a given scalar.

**Remark 3.** Although communication between vehicles are often with certain security protection, the open and assailable media/equipment may lead to communication intermittence because of the increased capability of adversary [33]. The attackers attempt to destroy the reliability of network communication maliciously. For DoS attacks, the adversaries often block the transmission of useful control signals by occupying the communication channel with the transmission of large amounts of invalid data. Generally, the effect of the DoS attacks, either implemented by forging or blocking packets transmitted on the vehicle-to-vehicle network or by installing malicious hardware or software on a vehicle, is to make vehicle receive the attacked control signal $\bar{u}_k$ instead of the real control signal $u_k$. In this case, a Bernoulli random variable $\pi_k$ is utilized to describe the phenomenon of the attack. If $\pi_k = 1$, which means no attacks occur. Otherwise attacks occur.

### 2.3 The problem statement

The controlled output of the system is described by

$$\tilde{z}_k = C(\bar{\theta})\xi_k,$$

where $\tilde{z}_k$ is the controlled output, $C(\bar{\theta}) = \sum_{i=1}^{N} \bar{\theta}_i C_i$ represents the matrix with compact dimensions.

By setting $Y_k = [\tilde{z}_k^T \tilde{u}_k^T]^T$, substituting (4) and (6) into (3), one has the closed-loop LPV system

$$Y_{k+1} = \tilde{A}(\bar{\theta})Y_k + \tilde{D}(\bar{\theta})\tilde{u}_k$$

$$\tilde{z}_k = \tilde{C}(\bar{\theta})Y_k,$$

where

$$\tilde{A}(\bar{\theta}) = \tilde{A}(\bar{\theta}) + \pi_k \tilde{A}(\bar{\theta}), \tilde{D}(\bar{\theta}) = \left[\sum_{i=1}^{N} \bar{\theta}_i D_i, 0\right],$$

with

$$\tilde{A}(\bar{\theta}) = \sum_{i=1}^{N} \bar{\theta}_i A_i + \bar{\pi} \sum_{i=1}^{N} \bar{\theta}_i B_i \left(\sum_{j=1}^{N} \bar{\theta}_j F_j + \Delta F\right)$$

$$\pi_k \left(\sum_{i=1}^{N} \bar{\theta}_i F_i + \Delta F\right)$$

$$(1 - \bar{\pi}) \sum_{i=1}^{N} \bar{\theta}_i B_i$$

$$(1 - \bar{\pi})I$$

$$\tilde{A}(\bar{\theta}) = \sum_{i=1}^{N} \bar{\theta}_i B_i \left(\sum_{j=1}^{N} \bar{\theta}_j F_j + \Delta F\right) - \sum_{i=1}^{N} \bar{\theta}_i B_i$$

$$\sum_{i=1}^{N} \bar{\theta}_i F_i + \Delta F - I.$$
The purpose of this article is to develop a non-fragile $H_\infty$ state feedback controller (4) for closed-loop LPV system (9), such that for all non-zero $\bar{w}_k \in (\ell_2[0, \infty), \mathbb{R}^{n_x})$ under zero initial conditions, the following two requirements are met simultaneously.

1. The closed-loop LPV system (9) is exponentially mean-square stable with disturbance $\bar{w}_k = 0$.
2. Under the condition of zero-initial, the controlled output $\zeta_k$ satisfies

$$
\sum_{k=0}^{\infty} \mathbb{E}\left[\zeta_k^T \zeta_k\right] \leq \gamma^2
$$

(10)

for all non-zero $\bar{w}_k$, where $\gamma > 0$ is a given scalar.

3. MAIN RESULTS

In this section, the system stability, as well as the $H_\infty$ performance of the closed-loop LPV system (9), is analyzed and guaranteed with sufficient conditions, with the help of the Lyapunov stability theory, and then the control gain parameter is derived by the solutions to the LMI condition. The following lemmas will do a favour for further developing the article.

Lemma 1 ([34]). Given constant matrices $F_1$, $F_2$ and $F_3$, where $F_1 = F_1^T < 0$ and $F_3 = F_3^T > 0$. The inequality $F_1 + F_2 F_3^{-1} F_2^T < 0$, if and only if

$$
\begin{bmatrix}
F_1 & F_2^T \\
F_2 & F_3
\end{bmatrix} < 0 \text{ or } 
\begin{bmatrix}
-F_3 & F_2 \\
F_2^T & F_1
\end{bmatrix} < 0
$$

is satisfied.

Lemma 2 ([35]). For any matrices $M$, $N$, $\Theta$ with $\Theta^T \Theta \leq I$, and an arbitrary scalar $\epsilon > 0$, the following inequality holds

$$
M \Theta^T N^T + N \Theta M^T \leq \epsilon M \Theta M^T + \epsilon^{-1} N \Theta N^T.
$$

Theorem 1. Consider system (1) and the given scalar $\gamma > 0$. Given controller gain $P_f (i = 1, 2, \ldots, N)$, the closed-loop LPV system (9) is asymptotically stable and has an $H_\infty$ performance level less than $\gamma$, if there exist positive matrix function $P = P^T > 0$, scalar $\delta > 0$, satisfying the following matrix inequality condition:

$$
\Gamma = 
\begin{bmatrix}
-P & * & * & * \\
0 & -\gamma^2 I & * & * \\
\bar{A}(\Theta) & D(\Theta) & -P^{-1} & * \\
\bar{A}(\Theta) & 0 & 0 & \frac{-P^{-1}}{(1-\bar{\pi})\bar{\pi}} \\
\bar{C}(\Theta) & 0 & 0 & -I
\end{bmatrix} < 0,
$$

(11)

where $\bar{A}(\Theta), \bar{A}(\Theta), D(\Theta), \bar{C}(\Theta)$ are defined in (9).

Proof. First, we are going to analyze the asymptotic stability of the closed-loop LPV system (9), and the following Lyapunov function candidate is selected

$$
\mathcal{V}_k = Y_k^T P Y_k.
$$

(12)

Then, the difference of the Lyapunov function candidate is calculated by

$$
\Delta \mathcal{V}_k = Y_{k+1}^T P Y_{k+1} - Y_k^T P Y_k.
$$

(13)

From (9) and (12) with $\bar{w}_k = 0$, taking the mathematical expectation, one has

$$
\mathbb{E}\{\Delta \mathcal{V}_k\} = \mathbb{E}\left\{Y_k^T \bar{A}^T(\Theta) P (\bar{A}(\Theta) + \pi \bar{A}(\Theta) Y_k) - Y_k^T P Y_k\right\}
$$

$$
= \mathbb{E}\left\{Y_k^T \bar{A}^T(\Theta) P (\bar{A}(\Theta) Y_k + (1-\pi)Y_k^T \bar{A}^T(\Theta)) \times P \bar{A}(\Theta) Y_k - Y_k^T P Y_k\right\}.
$$

(14)

By noting (14) and Lemma 1, we obtain

$$
\mathbb{E}\{\Delta \mathcal{V}_k\} = Y_k^T \Gamma Y_k,
$$

(15)

where

$$
\Gamma = 
\begin{bmatrix}
-P & * & * \\
0 & -\gamma^2 I & * \\
\bar{A}(\Theta) & D(\Theta) & -P^{-1} \\
\bar{A}(\Theta) & 0 & 0 \\
\bar{C}(\Theta) & 0 & 0 & -I
\end{bmatrix}.
$$

With the help of Lemma 1 and noting (11), it can be concluded that $\Gamma < 0$, and subsequently

$$
\mathbb{E}\{\Delta \mathcal{V}_k\} \leq Y_k^T \Gamma Y_k \leq \lambda_{\min}(\Gamma) \|Y_k\|^2.
$$

(16)

Then, we are going to handle the $H_\infty$ performance of the closed-loop LPV system (9). We introduce the following auxiliary index

$$
J_n = \sum_{k=0}^{n} \mathbb{E}\{\|\zeta_k\|^2\} - \gamma^2 \sum_{k=0}^{n} \mathbb{E}\{\|\bar{w}_k\|^2\},
$$

(17)

where $n \geq 0$ is an integer. Apparently our goal is to validate $J(n) < 0$ under the zero-initial condition.

Combining (9), (13) and (17), one has

$$
J_n = \sum_{k=0}^{n} \left(\mathbb{E}\{\|\zeta_k\|^2\} - \gamma^2 \mathbb{E}\{\|\bar{w}_k\|^2\} + \mathbb{E}\{\Delta \mathcal{V}_k\}\right) - \mathbb{E}\{\mathcal{V}_{k+1}\}
$$

$$
\leq \sum_{k=0}^{n} \left(\mathbb{E}\{\|\zeta_k\|^2\} - \gamma^2 \mathbb{E}\{\|\bar{w}_k\|^2\} + \mathbb{E}\{\Delta \mathcal{V}_k\}\right) + \mathbb{E}\{\mathcal{V}_{k+1}\}
$$

(16)
\[
\sum_{k=0}^{\infty} \mathbb{E} \{ Y_k^T \bar{A}^T(\vartheta) P \bar{A}(\vartheta) Y_k \} + \bar{\pi}(1 - \bar{\pi}) Y_k^T \bar{A}^T(\vartheta) P \bar{A}(\vartheta) Y_k
\]
\[- Y_k^T P Y_k + Y_k^T \bar{C}^T(\vartheta) \bar{C}(\vartheta) Y_k
\]+ \bar{w}_k^T \bar{D}^T(\vartheta) P \bar{D}(\vartheta) \bar{w}_k
\]+ 2Y_k^T \bar{A}^T(\vartheta) P \bar{D}(\vartheta) \bar{w}_k - \gamma^2 Y_k^T \bar{w}_k \}
\[
= \sum_{k=0}^{\infty} \mathbb{E} \{ \phi_k^T \Gamma \phi_k \}, \tag{18}
\]
where
\[
\phi_k = [Y_k^T \bar{w}_k^T]^T, \quad \Gamma = \begin{bmatrix} \Theta_1 & \ast \\ \Theta_2 & \Theta_3 \end{bmatrix},
\]
with
\[
\Theta_1 = \bar{A}^T(\vartheta) P \bar{A}(\vartheta) + \bar{\pi}(1 - \bar{\pi}) \bar{A}^T(\vartheta) P \bar{A}(\vartheta)
\]+ \bar{C}^T(\vartheta) \bar{C}(\vartheta) - P,
\]
\[
\Theta_2 = \bar{D}^T(\vartheta) P \bar{A}(\vartheta), \quad \Theta_3 = \bar{D}^T(\vartheta) P \bar{D}(\vartheta) - \gamma^2 I. \tag{19}
\]

Now using Lemma 1 again, we conclude from (11) that \( f_\infty < 0 \). Letting \( n \to \infty \), one has \( \sum_{k=0}^{\infty} [\bar{\mathbb{E}}[\phi_k^T \Gamma \phi_k]] \leq \gamma^2 \), which completes the proof. \( \square \)

**Theorem 2.** Consider the closed-loop LPV-based CACC system (9). For any given scalar \( \gamma > 0 \), if there exist scalars \( \varepsilon_i > 0 \) (\( i = 1, 2, 3, 4 \)), positive-definite matrices \( Q_i = \text{diag}(Q_{1i}, Q_{2i}) > 0 \), matrix \( Y_i \) such that, for \( i = 1, 2, \ldots, N \), the following LMI holds:
\[
\begin{bmatrix}
\Xi_{11} & \ast & \ast \\
\Xi_{21} & \Xi_{22} & \ast \\
\Xi_{31} & \Xi_{32} & \Xi_{33}
\end{bmatrix} < 0, \tag{20}
\]
where
\[
\Xi_{11} = \begin{bmatrix} -Q_i & \ast & \ast \\ 0 & -Q_i & \ast \\ 0 & 0 & -\gamma^2 I \end{bmatrix},
\]
\[
\Xi_{21} = \begin{bmatrix}
A_i + \bar{\pi} B_i \bar{V}_i & (1 - \bar{\pi}) B_i Q_{2i} & D_i \\
\bar{\pi} \bar{Y}_i & (1 - \bar{\pi}) Q_{2i} & 0 \\
B_i \bar{Y}_i & -B_i Q_{2i} & 0
\end{bmatrix},
\]
\[
\Xi_{22} = \text{diag}(\Phi_1, \Phi_2, \Phi_3, \Phi_4, \Phi_5),
\]
\[
\Xi_{31} = \text{diag}( -\varepsilon_1 I, -\varepsilon_2 I, -\varepsilon_3 I, -\varepsilon_4 I),
\]
\[
\Xi_{32} = \begin{bmatrix} N Q_{1i} & 0 & 0 \\ N Q_{1i} & 0 & 0 \\ N Q_{1i} & 0 & 0 \end{bmatrix}, \quad \Xi_{33} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},
\]
\[
\Xi_{32} = \begin{bmatrix} N Q_{1i} & 0 & 0 \\ N Q_{1i} & 0 & 0 \\ N Q_{1i} & 0 & 0 \end{bmatrix}, \quad \Xi_{33} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},
\]
\[
\Phi_1 = -Q_{1i} + \varepsilon_1 \bar{\pi}^2 B_i M M^T B_i^T,
\]
\[
\Phi_2 = -Q_{2i} + \varepsilon_2 \bar{\pi}^2 M M^T,
\]
\[
\Phi_3 = \frac{-Q_{1i}}{\bar{\pi}(1 - \bar{\pi})} + \varepsilon_3 B_i M M^T B_i^T,
\]
\[
\Phi_4 = \frac{-Q_{2i}}{\bar{\pi}(1 - \bar{\pi})} + \varepsilon_4 M M^T, \quad \Phi_5 = -I.
\]
Moreover, the corresponding feedback gains in the control law (4) are given by
\[
F_i = \mathcal{Y}_i Q_{1i}^{-1}. \tag{21}
\]

**Proof.** Choose the following Lyapunov function candidate
\[
\mathcal{V}_i = \mathcal{Y}_i Q_{1i}^{-1}. \tag{22}
\]

It follows from (8), (9), (10) and (13) and using Lemma 1 again, one has
\[
\Delta \mathcal{V}_i = \phi_k^T \begin{bmatrix}
-\mathcal{P} & \ast & \ast & \ast \\
0 & -\gamma^2 I & \ast & \ast \\
\bar{A}(\vartheta) & \bar{D}(\vartheta) & -\mathcal{P}^{-1} & \ast \\
\bar{A}(\vartheta) & 0 & 0 & \left( \frac{1}{1 - \bar{\pi}} \right) \bar{\pi} \ast
\end{bmatrix} \phi_k.
\]

Letting \( \mathcal{P}^{-1} = Q \), pre- and post-multiplying (22) by \( \text{diag}(\mathcal{P}^{-1}, I, I, I, I) \) and its transpose, we arrive at
\[
\begin{bmatrix}
-\mathcal{Q} & \ast & \ast & \ast \\
0 & -\gamma^2 I & \ast & \ast \\
\bar{A}(\vartheta) Q & \bar{D}(\vartheta) & -\mathcal{Q} & \ast \\
\bar{A}(\vartheta) Q & 0 & 0 & \left( \frac{1}{1 - \bar{\pi}} \right) \bar{\pi} \ast
\end{bmatrix} < 0. \tag{23}
\]

Therefore, by noting the definitions of \( \bar{A}(\vartheta), \bar{A}(\vartheta), \bar{C}(\vartheta), Q \), Lemma 1 and Lemma 2, the inequality (23) is guaranteed by (20) after a tedious calculation, which ends the proof. \( \square \)

**Remark 4.** To show the relationship between the attack probability \( 1 - \bar{\pi} \) (According to our definition before, we have \( \text{Pr}(\bar{\pi} = 0) = 1 - \bar{\pi} \)) and the sub-optimal value \( \gamma \), we add Table 2 for...
further illustrating where the results are derived by solving the following optimization problem:

$$\min \gamma^2,$$

s.t.  LMI (20),

$$\varepsilon_i > 0 \ (i = 1, 2, 3, 4), \ \mathcal{Q} > 0,$$

with different attack probabilities $1 - \tilde{\pi} = 0.2, 0.4, 0.6$ and 0.8. By noting the fact that the term $\tilde{\pi}(1 - \tilde{\pi})$ occurs in the denominator in (20), which infers that $\tilde{\pi}$ can not be taken as 0 or 1. Therefore, we choose four values uniformly distributed between 0 and 1 as the reference. The detailed results will be validated in the simulation part.

4 | EXPERIMENTAL RESULTS

In this section, a non-fragile $H_{\infty}$ state feedback control method is implemented on the LPV-based CACC system with a three-car scenario (including a leading car and two followers).

4.1 | Parameter setting

During the discretization process, we set $T = 0.05 \text{ s}$ as the sampling period, and set the time gap $\tau$ as $\tau_1 = \tau_2 = \tau_3 = 1 \text{ s}$. According to the model in [32], the corresponding LPV-based CACC system parameters after discretization is given as

$$
\begin{bmatrix}
{x}_{1,k+1} \\
{x}_{2,k+1} \\
{x}_{3,k+1} \\
{x}_{4,k+1} \\
{x}_{5,k+1} \\
{x}_{6,k+1}
\end{bmatrix} =
\begin{bmatrix}
1 & \frac{\mu_k}{m_1} & 0 & 0 & 0 & 0 \\
0 & 1 - 0.5 \frac{\mu_k}{m_1} & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & \frac{\mu_k}{m_2} & 0 & 0 \\
0 & 0 & 0 & 1 - 0.5 \frac{\mu_k}{m_2} & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & \frac{\mu_k}{m_3} \\
0 & 0 & 0 & 0 & 0 & 1 - 0.5 \frac{\mu_k}{m_3}
\end{bmatrix}
\begin{bmatrix}
x_{1,k} \\
x_{2,k} \\
x_{3,k} \\
x_{4,k} \\
x_{5,k} \\
x_{6,k}
\end{bmatrix} +
\begin{bmatrix}
-0.05 & 0 & 0 \\
-0.05 & 0 & 0 \\
0 & -0.05 & 0 \\
0 & -0.05 & 0 \\
0 & 0 & -0.05 \\
0 & 0 & -0.05
\end{bmatrix}
\begin{bmatrix}
x_{1,k} \\
x_{2,k} \\
x_{3,k} \\
x_{4,k} \\
x_{5,k} \\
x_{6,k}
\end{bmatrix}
\begin{bmatrix}
0.02 \\
0.03 \\
0.05 \\
-0.05 \\
-0.03 \\
0.05
\end{bmatrix}
\begin{bmatrix}
y_k \\
p_k
\end{bmatrix}
$$

where the state $x_{i,k}$ (containing velocity and position information) and $u_{i,k}$ (containing acceleration information) are, respectively, representing the state and control signal, which both are detailedly defined in Preliminaries.

The friction coefficient $\mu_k$ is assumed to be a time-varying function

$$\mu_k = 0.1(x_{i,k})^2.$$ 

Consider the safety issues in platoon vehicles, it is assumed that the state constraint $|x_{i,k}| \leq 0.5$ is imposed. It can be verified that $\mu_k \in [0, 0.025]$. Then, introducing the parameters $\vartheta_{1,k} = 1 - \frac{\mu_k}{0.025}$ and $\vartheta_{2,k} = 1 - \vartheta_{1,k}$, and the system matrices $A_1, B_1, D_1$ are as follows

$$A_1 =
\begin{bmatrix}
1 & 0.025 & 0 & 0 & 0 & 0 \\
0 & 0.9875 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.025 & 0 & 0 \\
0 & 0 & 0 & 0.9875 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1.025 \\
0 & 0 & 0 & 0 & 0 & 0.9875
\end{bmatrix},

A_2 =
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix},

B_1 = B_2 =
\begin{bmatrix}
-0.05 & 0 & 0 \\
-0.05 & 0 & 0 \\
0 & -0.05 & 0 \\
0 & -0.05 & 0 \\
0 & 0 & -0.05 \\
0 & 0 & -0.05
\end{bmatrix},

D_1 = D_2 =
\begin{bmatrix}
0.02 \\
0.03 \\
0.05 \\
0.05 \\
0.03 \\
0.05
\end{bmatrix}.$$

The controller gain variations in (4) are described by $\mathcal{M} = [0.1 0.1 0.1 0.1]^T$, $\mathcal{N} = [0.1 0.1 0.1 0.1 0.1 0.1 0.1]$ and $\mathcal{Q} = 0.1$. We set $\gamma = 4$ as the disturbance attenuation level. Moreover, the probability of $\tilde{\pi}$ is 0.8, the disturbance $\nu_k = \sin(8k)/k$ and the other parameters are chosen as $C_1 = C_2 = I_6$. By solving the LMI (20), the state-space matrices of the controller (21) are

| $1 - \tilde{\pi}$ | 0.2 | 0.3 | 0.4 | 0.5 | $\cdots$ | 0.8 |
|------------------|-----|-----|-----|-----|---------|------|
| $\gamma_{\text{min}}$ | 2.619 | 2.742 | 11.213 | $\text{infeasible}$ | $\cdots$ | $\text{infeasible}$ |

TABLE 2 The relation between $1 - \tilde{\pi}$ and $\gamma_{\text{min}}$
FIGURE 3  The deviation from equilibrium position for vehicle 1

computed as

\[
P = \begin{bmatrix}
16.8342 & 3.5893 & 0.0039 \\
-0.0266 & 0.0259 & 17.0814 \\
0.0201 & -0.0196 & -0.1859 \\
0.0039 & -0.0031 & 0.0031 \\
3.3486 & -0.1982 & 0.1930 \\
0.1811 & 16.9839 & 3.4435
\end{bmatrix}.
\]

4.2  Results

The initial value is set as \( \tilde{x}_0 = [0.5 \ 0.5 \ 0.5 \ 0.5 \ 0.5]^T \). With the help of the solutions to the LMI (20) and the parameters defined before, we have the following results from Figures 3–12. In Figure 3, the first subplot shows the trajectories of the \( \Delta s_{i,k} \) for vehicle 1 with non-fragile control, the second and third subplots depict the trajectories of the \( \Delta s_{i,k} \) for vehicle 1 without control and with regular control (means controller designing without considering the gain variations and DoS attacks), respectively. In Figure 4, trajectories of the \( \Delta v_{i,k} \) are shown in three subplots respectively for the system with non-fragile control, without control and regular control. The same simulation results for vehicles 2 and 3 are also obtained in Figures 5–8. The control signals for vehicles 1, 2, 3 are depicted in Figure 11. Moreover, to show the superiority of the proposed algorithm in this paper, we do the comparison in Figures 9 and 10. It is obvious that our algorithm has a faster response time. Figure 12 plots the occurrence of the DoS attacks.

It can be easily seen from Table 2 that with the increasing of the attack probability \( 1 - \pi \), the sub-optimal value \( \gamma_{\text{min}} \) is also increasing, until it is bigger than 0.4. Moreover, in order to more intuitively explain the impact of attack probability on...
FIGURE 7  The deviation from equilibrium position for vehicle 3

FIGURE 8  The deviation from equilibrium velocity for vehicle 3

FIGURE 9  The deviation from equilibrium position

FIGURE 10  The deviation from equilibrium velocity

FIGURE 11  The control signals for vehicles 1, 2, 3

FIGURE 12  The occurrence of the DoS attacks
system performance, some comparisons are shown in Figure 13, which validate that the attack do have a significant impact on system performance. Nevertheless, the three-car scenario works well by using the scheme proposed in this paper.

5 | CONCLUSION

This paper addresses a non-fragile \( H_{\infty} \) state feedback control scheme for the LPV-based CACC system subject to DoS attacks. It is assumed the parameters in the model to be measurable but unknown. The DoS attacks has been taken into account by utilizing a set of Bernoulli-distributed sequences, thereby better reflecting the reality of the vehicle-to-vehicle communication. The structure of the non-fragile \( H_{\infty} \) state feedback controller has been designed such that the closed-loop system has been exponentially mean-square stable and the \( H_{\infty} \) performance requirement has been satisfied as well. Then controller parameters have been obtained by recurring to the matrix inequality technology. Finally, a three-car model has been presented to validate the feasibility of the proposed NFSC algorithm. In our future topics, we will introduce our main results into more complex control issues [36–38].

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