Super-radiance in Nuclear Physics

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Abstract. The theory of the super-radiant mechanism as applied to various phenomena in nuclear physics is presented. The connection between super-radiance and the notion of doorway is presented. The statistics of resonance widths in a many-body Fermi system with open channels is discussed. Depending on the strength of the coupling to the continuum such systems show deviations from the standard Porter-Thomas distribution. The deviations result from the process of increasing interaction of the intrinsic states via the common decay channels. In the limit of very strong coupling this leads to super-radiance.

1. Introduction
The possibility of forming a “super-radiant” (SR) state in a gas of atoms confined to a volume of a size smaller than the wave length of radiation was proposed by Dicke [1]. In the absence of a direct interaction, the atoms are coupled through their common radiation field. The indirect interaction through the continuum leads to a redistribution of decay widths among unstable intrinsic states. A short life time SR state is created at the expense of the rest of the states of the system that are “robbed” of their decay width and become very narrow. This mechanism of general origin and analogous phenomena should appear in many quantum systems when quasi-bound states are strongly coupled through common decay channels. The SR approach has been used in many different fields. For example, it was applied in chemistry [2], atomic physics [3], condensed matter physics [4, 5], intermediate energy nuclear physics [6, 7], in particle physics [8], in the theory of nuclear reactions [5, 9, 10]. In the next section we develop the SR formalism for some simple cases in order to arrive in a heuristic fashion to the idea of a super-radiant state.

2. The effective Hamiltonian
Using the projection formalism [6, 11], we divide the Hilbert space of nuclear states into two parts, the \{Q\}—subspace involving complicated many-body states \(|q\rangle\), and the subspace \{P\} of open channels \(|c\rangle\). We use the notations Q and P for the corresponding projection operators onto the above subspaces. The wave function of the total system,

\[ |\Psi\rangle = Q |\Psi\rangle + P |\Psi\rangle \]  

obeys the Schrödinger equation

\[ H |\Psi\rangle = E |\Psi\rangle \]
which can be written as a set of coupled equations,

\[(E - H_{QQ}) Q |\Psi\rangle = H_{QP} P |\Psi\rangle, \quad \text{and} \quad (E - H_{PP}) P |\Psi\rangle = H_{PQ} Q |\Psi\rangle.\]  

We use the notations $H_{AB} = AHB$. Eliminating the part $P |\Psi\rangle$, we obtain an equation in the $Q$–space,

\[\left( E - \tilde{H}_{QQ} \right) Q |\Psi\rangle = 0 \]  

with the effective Hamiltonian

\[\tilde{H}_{QQ} = H_{QQ} + H_{QP} \frac{1}{E^{(+)} - H_{PP}} H_{PQ}\]  

here $E^{(+)} \equiv E + i0$ contains the infinitesimal imaginary term $+ i0$ ensuring correct asymptotic conditions for the continuum wave functions. The second term of the effective Hamiltonian (6) contains a real and imaginary part of the propagator

\[G^{(+)}(E) = \frac{1}{E^{(+)} - H_{PP}}\]  

arising from the principal value and the delta-function $\delta(E - H_{PP})$, respectively. The imaginary part of the effective Hamiltonian is:

\[-(i/2) W, \quad \text{with} \quad W = 2\pi \sum_{c} H_{QP} |c\rangle \langle c| H_{PQ}\]  

thus, the effective Hamiltonian (7) in $Q$–space is anti-Hermitian,

\[\tilde{H}_{QQ} = H_{QQ} - \frac{i}{2} W\]  

where $H_{QQ}$ is a symmetric and real matrix that includes, apart from the original Hamiltonian in the $Q$–space, $H_{QQ}$, the principal value contribution of the $PQ$–coupling. The second part is anti-Hermitian. The eigenvalues of $\tilde{H}$, $\tilde{E} = E - (i/2) \Gamma$, are complex poles of the scattering matrix corresponding to the resonances in the cross sections.

To demonstrate in a simple way the role of the anti-Hermitian term we assume that only one channel is open. Then the matrix $W$, eq. (8), has a completely separable form,

\[\langle q|W|q'\rangle = 2\pi A_c^c A_{q'}^c\]  

where

\[A_c^c = \langle q|H_{QP}|c\rangle.\]  

The rank of the $W$ matrix is 1, so that all the eigenvalues of this matrix are zero except one that has the value equal to the trace of the matrix:

\[\Gamma_s = \sum_q \langle q|W|q\rangle = 2\pi \sum_q \sum_{q'} |A_q^{c}|^2 \equiv \sum_q \Gamma_q^+\]  

with $\Gamma_q^+$ denoting the escape width of the individual levels before the $W$–matrix is diagonalized. The special unstable state with width $\Gamma_s$ is often referred to as the super-radiant (SR), in analogy to the Dicke coherent state [1] of a set of two-level atoms coupled through the common
radiation field. Here the coherence is generated by the common decay channel. The stable states are trapped and decoupled from the continuum.

In the more general case of \( N \) intrinsic states and \( N_c \) open channels, with \( N_c \ll N \), the super-radiant mechanism survives if the mean level spacing \( D \) of internal states and their decay widths \( \Gamma_q \) obey:

\[
\kappa_c = \frac{\Gamma_q^+}{D} > 1
\]

in this case we have \( N_c \) broad states, while the rest of states \( N - N_c \) become very narrow.

3. Doorway States

Often only a subset of intrinsic states \( \{Q\} \) connects directly to the \( \{P\} \) space of channels. The rest of states in \( \{Q\} \) will connect to \( \{P\} \) only through admixtures of these selected states. The special states directly coupled to continuum are the *doorways*, \( |d\rangle \). They form the doorway subspace \( \{D\} \) within \( \{Q\} \), and the corresponding projection operator will be denoted here as \( D \). The remaining states in \( \{Q\} \) will be denoted as \( |\tilde{q}\rangle \) and the subspace as \( \{\tilde{Q}\} \).

The full Hamiltonian can be decomposed the following way:

\[
H = \left[ H_{Q\tilde{Q}} + H_{DD} + H_{Q\tilde{Q}} + H_{D\tilde{Q}} \right] + \left[ H_{PP} + H_{DP} + H_{PD} \right]
\]  

(14)

Note that the terms \( H_{P\tilde{Q}} \) and \( H_{\tilde{Q}P} \) are missing because in accordance with the doorway hypothesis they are zero. Also note that diagonalizing the operator in the upper line of Eq. (14) would give back the states \( |q\rangle \) with the components of \( |d\rangle \) mixed with \( |\tilde{q}\rangle \) states. The two last terms in the above equation couple the doorway states, and therefore all the \( |q\rangle \) states to the open channels.

3.1. The case of a single doorway

In the case when there is only one important doorway state \( |d\rangle \) the matrix elements of the effective operator \( W \) in the intrinsic space are given by:

\[
\langle q|W|q'\rangle = 2\pi \sum_{c=1}^{N_c} \langle q|H_{DP}|c\rangle \langle c|H_{PD}|q'\rangle
\]

(15)

with the doorway assumption:

\[
\langle q|H_{DP}|c\rangle = \langle q|d\rangle \langle d|H_{DP}|c\rangle
\]

(16)

where \( \langle q|d\rangle \) is the amplitude of the admixture of the doorway into the \( |q\rangle \) state. Eq. (16) becomes:

\[
\langle q|W|q'\rangle = 2\pi \langle q|d\rangle \langle d|q'\rangle \sum_c |\langle d|H_{DP}|c\rangle|^2
\]

(17)

again we have separable matrix elements of the matrix \( W \), this time irrespective of the number of open channels—\( N_c \). Diagonalizing this matrix one finds again a single state that has the widths:

\[
\langle q|W|q'\rangle = 2\pi \sum_q |\langle q|d\rangle|^2 \sum_c |\langle d|H_{DP}|c\rangle|^2
\]

(18)

this width is the decay width of the doorway \( \Gamma^+_d \).
As discussed above, the criterion of validity of the SR mechanism is that the average spacing between the levels in \( \{ Q \} \) is smaller than the decay width of such a state “before” the SR mechanism takes effect. This can be expressed, in the case of doorways, the following way. Consider the spreading width, \( \Gamma^\downarrow_{d} \) of the doorway state representing the fragmentation of \( |d\rangle \) into compound states \( |\tilde{q}\rangle \). If \( N_q \) is the number of compound states in the interval covered by the spreading width, their average energy spacing is:

\[
\overline{D} \approx \frac{\Gamma^\downarrow_{d}}{N_q}
\]

(19)

before the SR mechanism is turned on, the average decay width of a typical \( |q\rangle \) state is:

\[
\Gamma^\uparrow_{q} = 2\pi \sum_c |\langle q|H_{QP}|c\rangle|^2
\]

(20)

that can be estimated as:

\[
\Gamma^\uparrow_{q} = \frac{\Gamma_s}{N_q}
\]

(21)

thus

\[
\frac{\Gamma^\uparrow_{q}}{\overline{D}} \approx \frac{\Gamma_s}{\Gamma^\downarrow_{d}} \approx \frac{\Gamma^\uparrow_{d}}{\Gamma^\downarrow_{d}}
\]

(22)

We conclude that the requirement for the SR doorway mechanism to be valid can be formulated as:

\[
\frac{\Gamma^\uparrow_{d}}{\Gamma^\downarrow_{d}} > 1
\]

(23)

4. Examples

4.1. Isobaric Analog States

The isobaric analog state (IAS), \( |A\rangle \), is obtained from the parent state with certain isospin \( T \rightarrow T - 1 \), by acting with the isospin lowering operator, \( T_- \) that changes a neutron into a proton:

\[
|A\rangle = \text{const} \cdot T_- |\pi\rangle
\]

(24)

In a compound nucleus, the IAS is surrounded by many compound states \( |q\rangle \) of lower isospin \( T_- = T - 1 \). The Coulomb interaction violates the isospin symmetry fragmenting the strength of the IAS over many states \( |q\rangle \), giving rise to the spreading width \( \Gamma^\downarrow_{A} \) of the IAS. If located above thresholds, the IAS can also decay into several continuum channels that gives rise to the decay width \( \Gamma^\uparrow_{A} \). In medium and heavy mass nuclei the condition the in Eq. (23) is satisfied. For example in \(^{208}\text{Pb}\) the spreading width is about 80 keV while the escape width is about 160 keV \([12]\), thus \( \Gamma^\uparrow_{A}/\Gamma^\downarrow_{A} \approx 2 \).

The SR mechanism is therefore relevant to this case providing a straightforward explanation why the IAS appears as a single resonance with the decay width given by that of \( |A\rangle \):

\[
\Gamma^\uparrow_{A} = 2\pi \ |\langle A|H_{QP}|P\rangle|^2
\]

(25)
4.2. Giant resonances

Giant resonances in nuclei (or atomic clusters) present another example of similar physics [13], [14]. Usually, the giant resonances are discussed in terms of $p-h$ configurations with identical spin-parity quantum numbers. The residual interactions form a correlated state that carries much of the transition strength of a corresponding multipole operator. The giant resonances are mostly located in the particle continuum decaying via particle emission to the ground and excited states in the daughter nucleus.

Since the $p-h$ giant resonance $|G\rangle$ is surrounded usually by a dense spectrum of $2p-2h$ and more complex configurations, the residual strong interaction will mix $|G\rangle$ with this background. Each of the resulting states, denoted as $|b\rangle$, will contain the admixture $\langle G|b\rangle$ of the giant resonance. The mixed states $|b\rangle$ couple to the continuum. If we assume that the dominant coupling is through the admixture of the giant state, then $|G\rangle$ serves as a doorway, and the matrix $W$ is separable, given by:

$$\langle b|W|b'\rangle = \langle b|G\rangle \langle G|b'\rangle \sum_{c=1}^{N_c} \langle G|V|c\rangle \langle c|V|G\rangle$$

(26)

The matrix $W$ is again of rank one and the non-zero eigenvalue is:

$$\Gamma^\uparrow_G = 2\pi \sum_{c} |\langle G|V|c\rangle|^2.$$  

(27)

This holds under the assumption that the energy spread of the background states $|b\rangle$ is small compared to their decay width:

$$\Gamma^\uparrow_b = 2\pi \sum_{c} |\langle b|V|c\rangle|^2.$$  

(28)

As before, this condition can be expressed in this case as:

$$\frac{\Gamma^\uparrow_G}{\Gamma^\uparrow_G} > 1$$

(29)

the spreading width of the giant resonance is smaller than its total decay width. When this condition is not satisfied, so that the energy intervals between the $2p-2h$ states are larger than their decay widths $\Gamma^\uparrow_b$, the situation of a single decay peak for the giant resonance might not hold. Still one could expect some bunching of background states into groups that have spacing within the group smaller than their decay widths $\Gamma^\uparrow_b$. Each such group then can be treated separately using the SR mechanism and appear as a single peak in the decay (or excitation) curve. Then one will observe intermediate structure resonances in the GR energy domain.

In some continuum RPA calculations it was found that high lying giant resonances have substantial decay widths [14] larger than the spreading widths. For example the isovector monopole resonances calculated in heavy nuclei had decay widths of the order of $8-10$ MeV, which is larger than the spreading width which is of the order of several MeV. In these cases the condition eq. (29) for super-radiance is satisfied.

We should emphasize that the SR approach with respect to doorways is not in any way contradicting the more conventional approach to the notion of doorways [11, 12]. It presents a different view of the same physical phenomenon, stressing the collective nature of the decay width. The SR formalism in the present application is analogous to the calculation of the collective states in nuclear structure. However in the SR mechanism one deals with the imaginary
Figure 1. Distribution of widths for one open channel ($M = 1$) and for two channels ($M = 2$) for an intrinsic Gaussian Orthogonal Ensemble Hamiltonian and for different continuum coupling strength, $\kappa$. The numerical results are given by histograms; the PTD for $M = 1$ and the $\chi^2_{\nu=2}$ distribution for $M = 2$ are shown as a smooth curve. The full squares stand for the best fit to a $\chi^2_{\nu}$ distribution.

part of the effective Hamiltonian and not with the real part. This analogy can bridge nuclear structure and nuclear reactions, especially when dealing with weakly bound exotic nuclei. In certain situations the SR mechanism may provide an understanding of the appearance of unexpectedly narrow resonances. As a consequence of the creation of a SR resonance the widths of the other resonances coupled to the same channel, become narrow.

5. Porter-Thomas Distribution and the Super-Radiant Mechanism

We now discuss an example that illustrates the use of the super-radiant approach to a complicated problem in nuclear physics. One the best examples in which the interplay between intrinsic dynamics and decay channels is manifested are the low-energy neutron resonances. These resonances are interpreted as quasi-stationary levels of the compound nucleus formed after the neutron is captured. These resonances were described, with certain degree of success, by the theory of random matrices [15]. With exceedingly complicated wave functions of the compound states their components are Gaussian distributed. This applies also to the specific component related to the channel, thus the neutron in the continuum and the residual nucleus in the ground state. The decay width of the neutron resonance is proportional to the amplitude squared of this component and the width distribution is therefore given by the $\chi^2_{\nu}$ function with $\nu = 1$ that is the Porter-Thomas distribution (PTD). Recently experiments found significant departures from the PTD [16]. Attempts to use the $\chi^2_{\nu}$ distribution in order to fit the data require $\nu < 1$. This result has been interpreted as a breakdown of the random matrix theory.

In a paper published recently [17] it was shown that a proper description of unstable quantum states using the super-radiant mechanism can in fact produce deviations from the PTD of the same type as observed in experiments. The coupling to open channels is essential in the
redistribution of the decay widths of the compound states. The details of the calculations are described in reference [17]. When the strength of the coupling between compound states and the open channel(s) (denoted by $\kappa$) is very small ($\kappa \ll 1$) one finds that the distribution of the calculated widths follows the PTD. With increasing coupling the deviations become more pronounced. At $\kappa \sim 1$ the super-radiant stage is reached and the distribution of widths becomes singular. In Figure 1 are shown some the results from reference [17].

We should mention that when a realistic value for $\kappa$ is used, the effect of the super-radiance is not sufficiently large to explain the experimental deviations from PTD. At this time there is no satisfactory explanation of the experimental findings. Additional measurements could clarify the situation.

In summary, the interpretation of the widths of compound resonances as a strength of the pure neutron component in the wave function fails due to the coupling to the continuum that has to be taken into account in a proper statistical description. This phenomenon is of general nature and might influence many other quantum systems.

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