Joint short- and long-baseline constraints on light sterile neutrinos

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Recent studies have evidenced that long-base-line (LBL) experiments are sensitive to the extra CP-phases involved with light sterile neutrinos, whose existence is suggested by several anomalous short-base-line (SBL) results. We show that, within the 3+1 scheme, the combination of the existing SBL data with the LBL results coming from the two currently running experiments NOνA and T2K, enables us to simultaneously constrain two active-sterile mixing angles θ14 and θ24 and two CP-phases δ13 ≡ δ and δ14, albeit the information on the second CP-phase is still weak at the moment. The two mixing angles are basically determined by the SBL data, while the two CP-phases are constrained by the LBL experiments, once the information coming from the SBL setups is taken into account. We also assess the robustness/fragility of the estimates of the standard 3-flavor parameters in the more general 3+1 scheme. To this regard we find that: i) the indication of CP-violation found in the 3-flavor analyses persists also in the 3+1 scheme, with δ13 ≡ δ having still its best fit value around −π/2; ii) the 3-flavor weak hint in favor of the normal hierarchy becomes even less significant when sterile neutrinos come into play; iii) the weak indication of non-maximal θ23 (driven by NOνA disappearance data) persists in the 3+1 scheme, where maximal mixing is disfavored at almost the 90% C.L. in both normal and inverted mass hierarchy; iv) the preference in favor of one of the two octants of θ23 found in the 3-flavor framework (higher octant for inverted mass hierarchy) is completely washed out in the 3+1 scheme.

I. INTRODUCTION

The massive nature of neutrinos and their mixing have been established by a plethora of experiments performed in the last two decades with natural and artificial neutrino sources. The 3-flavor paradigm has been gradually recognized as the sole framework able to account for all the observations performed at baselines longer than ~ 100 meters. In contrast, the same scheme is not able to explain a series of results recorded at shorter distances dubbed as short-base-line (SBL) anomalies. One possible explanation of the SBL anomalies is provided by a flavor oscillation process mediated by new hypothetical light sterile neutrino states, albeit this hypothesis encounters some difficulties in explaining simultaneously all the existing data sets. In particular, a non-negligible statistical tension emerges when one compares the (positive) νμ → νe appearance signals with the joint disappearance results from the (positive) searches in the νe → νe channel and the (negative) ones in the νμ → νμ channel.

In the so-called 3+1 scheme, only one new (essentially sterile) mass eigenstate is introduced, with a squared-mass splitting of the order of ∆m2SBL ∼ 1 eV2 with respect to the three standard neutrinos. In such a scheme the active-sterile admixture is supposed to be small but large enough to explain the anomalies. The 3+1 scheme predicts by construction sizable effects at the short distances, where the oscillating factor ∆A SBL = ∆m2SBLL/4E (L being the baseline and E the neutrino energy) is of order one. A rich program of new SBL experiments is underway with the purpose of detecting the smoking gun of active-sterile neutrino oscillations, i.e., the characteristic L/E dependence of the events rate.

After a hypothetical discovery of a sterile neutrino at SBL experiments, the full exploration of the properties of these particles would need other types of experimental setups. In particular, the SBL experiments would not be able to provide any information on the CP-phases involved in the 3+1 scheme. In fact, the manifestation of CP-violation (CPV) is intimately related to the interference of two distinct frequencies. At the SBL setups only one frequency is observable (the new one), while the other two (atmospheric and solar) are undetectable. Therefore, the SBL searches are blind to the CP-phases involved in the 3+1 scheme.1

In the standard 3-flavor framework the CPV is encoded by the CP-phase δ which enters the leptonic mixing matrix. The 3-flavor CPV searches are performed at the long-base-line (LBL) experiments, which can observe the νμ → νe transition probability in both the neutrino and antineutrino channels. These setups are designed to maximize the amplitude of the interference between the solar and atmospheric oscillations, which embodies a dependency on the CP-phase δ. As a matter of fact, we have

1 In the 3 + Ns schemes with Ns > 1, CPV could be observed at SBL experiments. However, these setups can probe only a limited number of all the CP phases involved in the model. In contrast, LBL experiments have access to all such phases. For example, in the 3+2 scheme, the SBL experiments are sensitive only to one CP-phase over a total of five CP-phases.
already some intriguing indications on \( \delta \) coming from the partial \([1, 2]\) (including only LBL data) and global \([3, 4]\) analyses, which all point towards nearly maximal CPV with \( \delta \sim -\pi/2 \). This trend has been recently corroborated by the latest data released by NO\(\nu\)A \([7]\) and T2K \([8]\) at the Neutrino and ICHEP 2016 conferences.

As first evidenced in Ref. \([1]\), in the presence of light sterile neutrinos, the \( \nu_\mu \rightarrow \nu_e \) transition probability probed at the LBL facilities acquires a new interference term, arising from the interference between the atmospheric frequency and the new large frequency related to the sterile state. Although the fast oscillations driven by the new frequency are completely averaged out by the finite energy resolution of the detector, nonetheless, they can leave their footprints in the transition probability. This renders the LBL experiments sensitive also to the extra CP-phases involved in the 3+1 scheme. The recent 4-flavor analyses \([1, 2]\) of the data from NO\(\nu\)A and T2K have clearly shown that these two experiments are already sensitive to one of the new CP-phases provided that the active-sterile mixing angles are fixed at their best fit values determined by the SBL 3+1 fits \([9, 12]\). In addition, the prospective study performed in \([13]\) has shown that the sensitivity to the extra CP-phases is expected to improve when NO\(\nu\)A and T2K will reach their full exposures, and will further increase in the next-generation experiment DUNE \([14]\).

In this work, we stick to the real data and take a step forward with respect to the existing works. Instead of fixing the active-sterile mixing angles at their best fit values, we here incorporate a full analysis of all the existing SBL data in combination with the LBL results. In this way we are able to simultaneously constrain the two active-sterile mixing angles \( \theta_{14} \) and \( \theta_{23} \) and the two CP-phases \( \delta_{13} \equiv \delta \) and \( \delta_{14} \). The estimates of the two new mixing angles will be basically determined by the SBL data, while those of the two CP-phases will derive from the LBL experiments, once the information from the SBL setups is considered. In this work, we also assess the robustness/fragility of the estimates of the standard 3-flavor parameters in the more general 3+1 scheme, paying particular attention to the most important properties currently under scrutiny: the mass hierarchy, the CP phase \( \delta \), and the atmospheric mixing angle \( \theta_{23} \).

The rest of the paper is organized as follows. In Sec. II we briefly introduce the theoretical 3+1 framework. In Sec. III we recall the basic features of the flavor oscillations at short baselines. Section IV deals with the 4-flavor transition probability relevant for the LBL setups. In Sec. V we list the data used in the simulations and describe the details of their numerical analysis. In Sec. VI we present and discuss the details of the analysis. Finally, in Sec. VII we draw our conclusions.

II. THEORETICAL FRAMEWORK

In the 3+1 scheme, the flavor \( (\nu_e, \nu_\mu, \nu_\tau, \nu_s) \) and mass eigenstates \( (\nu_1, \nu_2, \nu_3, \nu_4) \) are related through a 4×4 mixing matrix, which we parametrize as

\[
U = \begin{pmatrix}
R_{34} & R_{31} & R_{32} & R_{33}
\end{pmatrix},
\]

where \( R_{ij} \) (\( R_{ji} \)) is a real (complex) 4×4 rotation in the \((i, j)\) plane, which contains the 2×2 matrix

\[
R_{ij}^{2×2} = \begin{pmatrix}
c_{ij} & s_{ij}
\end{pmatrix},
\]

in the \((i, j)\) sub-block, where we have introduced the definitions

\[
c_{ij} \equiv \cos \theta_{ij}, \quad s_{ij} \equiv \sin \theta_{ij}, \quad \tilde{s}_{ij} \equiv s_{ij} e^{-i \delta_{ij}}.
\]

The parameterization in Eq. ([1]) has the following features: i) When all the three mixing angles involving the fourth state vanish \( (\theta_{14} = \theta_{24} = \theta_{34} = 0) \) one recovers the 3-flavor matrix in its standard parameterization. ii) For small values of the mixing angles involving the fourth mass eigenstate, it is \( |U_{\nu s1}|^2 \approx s_{14}^2, |U_{\nu s4}|^2 \approx s_{24}^2 \), and \( |U_{\tau s4}|^2 \approx s_{34}^2 \), with an immediate physical interpretation of the new mixing angles. iii) With the leftmost positioning of the matrix \( R_{34} \), in vacuum, the LBL \( \nu_\mu \rightarrow \nu_e \) transition probability is independent of \( \theta_{34} \) and of the associated CP-phase \( \delta_{34} \).

III. FLAVOR CONVERSION AT SHORT BASELINES

Short-baseline experiments are sensitive only to the oscillations generated by the new squared-mass difference \( \Delta m_{34}^2 \sim 1 \text{eV}^2 \), which in the 3+1 framework is \( \Delta m_{34}^2 \sim \Delta m_{\text{SBL}}^2 = \Delta m_{41}^2 \sim \Delta m_{42}^2 \sim \Delta m_{43}^2 \), taking into account that the solar squared-mass difference \( \Delta m_{\text{SOL}}^2 = \Delta m_{12}^2 \approx 7.4 \times 10^{-5} \text{eV}^2 \) and the atmospheric squared-mass difference \( \Delta m_{\text{ATM}}^2 = |\Delta m_{31}^2| \approx |\Delta m_{32}^2| \approx 2.5 \times 10^{-3} \text{eV}^2 \) are much smaller (we use the notation \( \Delta m_{jk}^2 = m_j^2 - m_k^2 \)).

The effective oscillation probabilities of the flavor neutrinos in short-baseline experiments are given by

\[
P_{\alpha\beta}^{(\text{SBL})} \approx \left| \delta_{\alpha\beta} - \sin^2 2\theta_{\alpha\beta} \sin^2 \left( \frac{|\Delta m_{jk}^2| L}{4E} \right) \right|,
\]

where \( \alpha, \beta = e, \mu, \tau, s \), \( L \) is the source-detector distance and \( E \) is the neutrino energy. The short-baseline oscillation amplitudes depend only on the absolute values of the elements in the fourth column of the mixing matrix:

\[
\sin^2 2\theta_{\alpha\beta} = 4 |U_{\alpha 1}|^2 \left| \delta_{\alpha\beta} - |U_{\beta 4}|^2 \right|.
\]

Hence, the transition probabilities of neutrinos and antineutrinos are equal and it is not possible to measure
any CPV effect generated by the complex phases in the mixing matrix in short-baseline experiments.

The short-baseline anomalies in favor of the existence of active-sterile neutrino oscillations are:

1. The LSND observation of an excess of $\bar{\nu}_e$-induced events in a $\bar{\nu}_\mu$ beam [16, 17].
2. The Gallium neutrino anomaly [15, 18-22], consisting in the disappearance of $\nu_e$ measured in the Gallium radioactive source experiments GALLEX [23] and SAGE [24].
3. The reactor antineutrino anomaly [25], which is a deficit of the rate of $\bar{\nu}_e$ observed in several reactor neutrino experiments in comparison with that expected from the calculation of the reactor neutrino fluxes [20, 27].

IV. FLAVOR CONVERSION AT LONG BASELINES

Let us now come to the transition probability relevant for the LBL experiments T2K and NO$\nu$A. In Ref. [1], it has been shown that the probability can be written as the sum of three terms

$$P_{\mu e} \simeq P_{\text{ATM}} + P_{\text{INT}} + P_{\text{M}}.$$

The first term represents the positive definite atmospheric transition probability, which can be expressed as

$$P_{\text{ATM}} \simeq 4s_{12}^2 s_{13}^2 \Delta \sin^2 \theta,$$

where $\Delta \equiv m_2^2 - m_1^2 - m_3^2 / 2E$ is the atmospheric oscillating frequency. The second term is related to the interference between the oscillations driven by the solar and atmospheric squared-mass splittings. This term, apart from higher order corrections, coincides with the standard interference term, which makes the transition probability sensitive to the CP-phase $\delta \equiv \delta_{13}$. It can be written as

$$P_{\text{INT}} \simeq 8s_{13} s_{12} c_{12} s_{23} c_{23} (\alpha \Delta) \sin \Delta \cos(\Delta + \delta_{13}).$$

The third term is due to 4-flavor effects and is driven by the interference between the atmospheric frequency and the new large frequency introduced by the fourth mass eigenstate. It takes the form

$$P_{\text{M}} \simeq 4s_{14} s_{24} s_{13} s_{23} \sin \Delta \sin(\Delta + \delta_{13} - \delta_{14}).$$

This term does not depend on $\Delta m_{31}^2$ because the fast oscillations are averaged by the finite resolution of the detector. The transition probability depends on the three small mixing angles $s_{13}, s_{14}, s_{24} \sim 0.15$, which can be all assumed to be of the same order $\epsilon$. Another small quantity is the ratio of the solar and atmospheric squared-mass splittings $\alpha \equiv \Delta m_{12}^2 / \Delta m_{13}^2 \simeq 0.03$, which is of order $\epsilon^2$. Remarkably, for values of the mixing angles indicated by the current global 3-flavor analyses [3-5] (for $\theta_{13}$) and the 3 + 1 fits [9, 12] (for $\theta_{14}$ and $\theta_{24}$), the size of the new (atmospheric-sterile) interference term is basically identical to that of the standard (solar-atmospheric) interference term [2]. This implies that T2K and NO$\nu$A are sensitive to both CP-phases $\delta_{13}$ and $\delta_{14}$.

Finally, we mention that the matter effects slightly modify the transition probability, leaving unaltered its decomposition in the sum of three contributions. We refer the reader to [1] for a detailed treatment of matter effects in the 3+1 scheme. Here, we just recall that they introduce a dependency on the dimensionless quantity

$$v = \frac{2VE}{\Delta m_{31}^2},$$

where

$$V = \sqrt{2}G_FN_e,$$

is the constant matter potential along the neutrino trajectory in the Earth crust. We have $v \simeq 0.05$ in T2K and $v \simeq 0.17$ in NO$\nu$A at the energy corresponding to the first oscillation maximum ($E \simeq 0.6$ GeV in T2K, $E \simeq 2$ GeV in NO$\nu$A). Therefore, the matter effects are appreciable only in NO$\nu$A and confer to this experiment an enhanced sensitivity to the neutrino mass hierarchy.

V. DATA USED AND DETAILS OF THE ANALYSIS

For the determination of the two active-sterile mixing angles $\theta_{14}$ and $\theta_{24}$ we use the update of the analysis in Ref. [28] presented in Ref. [29]. We considered the data of the following three groups of experiments:

(A) The $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ appearance data of the LSND [17], MiniBooNE [30, 31], BNL-E776 [32], KARMEN [33], NOMAD [34], ICARUS [35], and OPERA [36] experiments. The two last ones have been treated following the approach described in [37]. We did not consider the anomalous low-energy bins of the MiniBooNE experiment [30, 31], according to the “pragmatic” approach advocated in Ref. [9].

(B) The following $\nu_e$ disappearance data: 1) the data of the Bugey-4 [38], RNOV91 [39], Bugey-3 [40], Gosgen [41], ILL [42], Krasnoyarsk [43], Rovno88 [44], SRP [45], Chooz [46], Palo Verde [47], Double Chooz [48], and Daya Bay [49] reactor antineutrino experiments with the new theoretical fluxes [25-27, 50]; 2) the data of the GALLEX [23] and SAGE [24] Gallium radioactive source experiments with the statistical method discussed in Ref. [21], considering the recent $^{71}$Ga($^{3}$He, $^{3}$H)$^{73}$Ge cross section measurement in Ref. [61]; 3) the solar neutrino constraint on $\sin^2 2\theta_{13}$ [22, 62, 65]; 4) the KARMEN [56] and LSND [57] $\nu_e + ^{12}$C $\rightarrow ^{12}$N$_{\text{g.s.}} + e^-$. 
scattering data [58], with the method discussed in Ref. [59].

(C) The constraints on $\nu_e$ disappearance obtained from the data of the CDHSW experiment [60], from the analysis [61] of the data of atmospheric neutrino oscillation experiments, from the analysis [62, 63] of the MINOS neutral-current data [64] and from the analysis of the SciBooNE-MiniBooNE data neutrino [65] and antineutrino [66] data. We have not included the IceCube results recently reported in [67]. However, this has no impact in our results, because, as already noted in [29], these data modify the upper bounds on $\theta_{24}$ only for values of $\Delta m_{31}^2$ which are lower then $\sim 1$ eV$^2$. This conclusion is corroborated by the numerical analysis performed in [12].

Concerning the LBL experiments, we use the preliminary data released at the Neutrino 2016 and ICHEP 2016 conferences by the NOνA [7] and T2K [8] collaborations, considering the neutrino and antineutrino datasets, and including both the appearance and disappearance channels. In order to calculate the theoretical expectation for the total number of events and their binned spectra in the reconstructed neutrino energy, we use the software GLoBES [68, 69]. As input information we use the unoscillated $\nu_\mu$ and $\bar{\nu}_\mu$ fluxes extrapolated at the far detector from Ref. [70, 71] for T2K and from Ref. [72] for NOνA. The analysis for the appearance channel is performed using the total rate information as in [1, 2], which presents very small differences with respect to the analysis done using the full energy spectrum. This is due to three factors: i) the off-axis configuration of NOνA and T2K, which leads to a narrow energy spectrum peaked around the first oscillation maximum; ii) the limited statistics currently available in the appearance channel both in NOνA and T2K; iii) the smearing induced by the finite energy resolution of the far detectors. Differently, for the disappearance channel we perform a full spectral analysis of the far detector event distribution, since in this case the energy information has a crucial role.

In the standard 3-flavor case, the free parameters in the analysis are the atmospheric mass splitting $\Delta m_{32}^2$, the two mixing angles $\theta_{13}$, $\theta_{23}$ and the CP-phase $\delta_{13}$. In the 4-flavor analysis, in addition, we consider as free parameters $\Delta m_{31}^2$, $\theta_{14}$, $\theta_{24}$ and the CP-phase $\delta_{14}$. We fix $\theta_{34} = 0$, because the perturbations induced by non-zero $\theta_{34}$ are very small in T2K and NOνA. We have explicitly checked numerically that for non-zero values of $\theta_{34}$ currently allowed by data, the oscillation probabilities in both the appearance and disappearance channels are almost indistinguishable from those calculated with $\theta_{34} = 0$. Finally, we mention that both in the 3-flavor and 4-flavor analyses we fix the solar mass-mixing parameters at their best fit values obtained in the global 3-flavor analysis [9].

As pointed out in Ref. [1], in the 4-flavor scenario, the analysis has to deal with the fact that the near detectors in the long-baseline experiments T2K and NOνA are sensitive to the oscillations induced by the extra sterile neutrino. The neutrino fluxes used in the standard analysis are constrained with the information extracted from the near detectors under the assumption of no oscillation at short baselines, which is true only for three flavors. With the addition of an extra neutrino with $\Delta m_{31}^2 \sim O(1$ eV$^2$), the survival probability for $\nu_\mu$ at the near detector can be approximated as

$$P_{\mu\mu}^{\text{ND}} \simeq 1 - 4 \sin^2 \theta_{24} \sin^2 \left( \frac{\Delta m_{31}^2 L}{4E} \right).$$

(12)

Therefore, a suppression of the fluxes that depends on the parameters $\Delta m_{31}^2$, $\theta_{24}$ and on the energy is expected. A precision analysis of the LBL data in the 3+1 scheme would require the simultaneous treatment of the near and far detector. However, the spectrum of events expected at the near detector is problematic to reproduce, since many details are not accessible from outside the collaborations. To circumvent this problem we have incorporated the effects of the oscillation at the near detector using the following approximate procedure. We have corrected the expected distribution of events at the far detector multiplying it by the energy dependent factor $1/P_{\mu\mu}^{\text{ND}}$, taking its averaged value in each energy bin. In this way, we approximately untie the far detector fluxes from their dependency on the oscillations occurred at the near detector. We have checked that these corrections introduce small modifications on our final results.

Hence, our approximate approach is justified. However, we stress that a more detailed analysis in the 3+1 scheme should incorporate the simultaneous fit of the native neutrino fluxes with the near and far detector data for varying values of the parameters $\Delta m_{31}^2$ and $\theta_{24}$. At the moment, this is possible only from inside the experimental collaborations.
FIG. 1: Regions allowed by the combination of the SBL and LBL data (T2K and NOνA) together with the $\theta_{13}$-sensitive reactor results for the NH case. The left-bottom panel reports the projection on the plane of the two mixing angles ($\theta_{14}, \theta_{24}$). The other two panels display the constraints in the plane formed by each one of these two mixing angles and the new CP-phase $\delta_{14}$. The confidence levels correspond to 68%, 90% and 95% for 2 d.o.f ($\Delta \chi^2 = 2.3, 4.6, 6.0$), and the best-fit points are marked with a red point.

VI. NUMERICAL RESULTS

A. Constraints on the new mixing angles ($\theta_{14}, \theta_{24}$) and the new CP-phase $\delta_{14}$

Figure 1 and 2 represent the bidimensional projections of the $\Delta \chi^2$ for normal hierarchy (NH) and inverted hierarchy (IH) in the planes [$\sin^2 \theta_{14}, \delta_{14}$], [$\sin^2 \theta_{14}, \sin^2 \theta_{24}$] and [$\delta_{14}, \sin^2 \theta_{24}$] for the top left, bottom left and bottom right panels respectively. The three contours are drawn for $\Delta \chi^2 = 2.3, 4.6, 6.0$, corresponding to 68%, 90% and 95% for 2 d.o.f. The allowed regions in the [$\sin^2 \theta_{14}, \sin^2 \theta_{24}$] plane are almost the same of those (not shown) that we obtain from the fit of the SBL data taken alone. This finding can be understood by observing that the SBL experiments currently dominate over the LBL ones in the determination of the two new mixing angles. We find that the overall goodness of fit is satisfactory (GoF = 24%), while the parameter goodness of fit (see [76] for its definition), which measures the statistical compatibility between the (discordant) appearance and disappearance data sets, is lower (GoF = 7%). This implies that even if the closed contours presented for the two new mixing angles $\theta_{14}$ and $\theta_{24}$ exclude zero with high significance (more than six standard deviations), one cannot naively interpret this circumstance as an evidence for sterile neutrinos. In addition, we mention that light sterile neutrinos, unless dressed with new properties, are in strong tension with cosmological data (see for example [77–81]).

The preferred values of $\sin^2 \theta_{14}$ and $\sin^2 \theta_{24}$ lie in the range [0.01-0.03], which means that the new mixing angles $\theta_{14}$ and $\theta_{24}$ are of the same order of magnitude of the standard mixing angle $\theta_{13}$ (we recall that $\sin^2 \theta_{13} \approx 0.025$). A quick estimate of the amplitude of the new interference term in Eq. (9) reveals that its size is similar to that of the standard interference term in Eq. (6). Therefore, it is quite natural to expect that the LBL data will posses some sensitivity to the new CP-phase $\delta_{14}$. This qualitative conclusion is validated by our numerical results displayed in the top left and bottom right panels of Figs. 1 and 2. It is important to observe that the input from the SBL experiments is essential for the extraction of the information on $\delta_{14}$ from the LBL setups, since these last ones have a very scarce sensitivity to $\theta_{14}$ and $\theta_{24}$, and therefore are unable to constrain the amplitude of the new interference term in Eq. (9). In addition, we underline that also the precise determination of $\theta_{13}$ attained independently by the reactor experiments Daya Bay and RENO, plays a relevant...
Inverted hierarchy

![Normal hierarchy vs Inverted hierarchy](image)

Figure 3: Regions allowed by the combination of the SBL and LBL data (T2K and NOνA) together with the $\theta_{13}$-sensitive reactor results for NH (left panel) and IH (right panel) in the plane spanned by the two CP phases $\delta_{13}$ and $\delta_{14}$. The confidence levels are the same ones used in Fig. 1.

B. Correlation between the two CP-phases $\delta_{13}$ and $\delta_{14}$

Figure 3 shows the constraints in the plane of the two CP-phases $[\delta_{14}, \delta_{13}]$ for NH (IH) left panel (right panel). Also in this figure, the regions are obtained combining the SBL data, the LBL results from NOνA and T2K, and the data from Daya-Bay and RENO. In both mass hierarchies the CP-conserving cases $\delta_{13} = 0, \pi$ are disfavored at $\Delta \chi^2 \approx 2.7$. The best fit value $\delta_{13} \approx -\pi/2$, is basically the same obtained in the 3-flavor case (see the analyses 3,4). This preference comes from the observation of an excess (deficit) of $\nu_e$ ($\bar{\nu}_e$) events with respect to the expectations for the appearance channel $\nu_\mu \rightarrow \nu_e$ ($\bar{\nu}_\mu \rightarrow \bar{\nu}_e$), when assuming a value of $\theta_{13}$ equal to the best fit point of reactor experiments. In fact, Eq. 3 shows that, around the first oscillation maximum ($\Delta = \pi/2$), the standard interference term is proportional to $\sin \delta_{13}$. This implies that this term is maximized (minimized) for neutrinos (antineutrinos) for $\delta_{13} = -\pi/2$ in agreement with the observed pattern. Our numerical analysis in the 3+1 scheme reveals that the presence of the new interference terms does not spoil this picture. This behavior can be explained by observing that at the first oscillation maximum ($\Delta = \pi/2$) the new interference term is proportional to $\cos(\delta_{13} - \delta_{14})$, and therefore (in contrast to the standard term) its sign is the same for neutrinos and antineutrinos. We observe that for $\delta_{13} \approx \delta_{14} \approx -\pi/2$, the new interference term assumes its maximal positive value (for both neutrinos and antineutrinos). In the fit the neutrino dataset dominates over the (lower statistics) antineutrino data set and, as a consequence, the excess of $\nu_e$’s wins over the deficit of $\bar{\nu}_e$’s, driving the new CP-phase to a best fit value close to $\delta_{14} \approx -\pi/2$. Finally, we note that the constraints on the new CP-phase $\delta_{14}$ are very weak. This is imputable to the smaller amplitude of the new interference term when compared to the standard interference term.

C. Impact of sterile neutrinos on the standard neutrino properties

In the previous subsections we have focused our discussion on the new parameters of the 3+1 scheme and to the correlation among the two CP-phases. However, it is of interest to see what happens to the estimates of the standard parameters, which were marginalized in the figures shown until now. In particular, it seems of particular interest to assess the robustness/fragility of the estimate of the CP-phase $\delta \equiv \delta_{13}$, the mass hierarchy and the mixing angle $\theta_{23}$, which all are at the center of current investigations.

Figure 4 displays the regions allowed in the plane $[\sin^2 2\theta_{13}, \sin^2 2\theta_{13}]$ by the joint analysis of all the SBL experiments and the two LBL experiments T2K and NOνA. The two upper panels correspond to the 3-flavor framework,3 while the two lower ones are obtained in the 4-flavor scheme. The two left (right) panels refer to NH (IH). The interval of $\theta_{13}$ identified by the reactor experiments at 68% C.L. (represented by the green vertical band) is displayed for the sake of comparison. In all cases $\Delta 2 \chi^2$ and the mixing angle $\theta_{23}$ are marginalized away. In addition, in the 4-flavor case, we marginalize over the two mixing angles $(\theta_{14}, \theta_{24})$ and the CP-phase $\delta_{14}$. The contours represented in the plots correspond to the same

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2 We recall that the when passing from neutrino to antineutrino probability one has to invert the sign of all the CP-phases.

3 It should be noted that at the SBL experiments the 3-flavor effects are completely negligible. Consequently, one can adopt two different approaches when considering the 3-flavor scheme: i) include the SBL data in the fit, ii) exclude them from the fit. What changes between the two approaches is only the value of the absolute minimum of the $\chi^2$. Following the first option, one obtains a much higher value than following the second one. This just corresponds to the fact that in the 3+1 scheme the goodness of fit increases, because the sterile oscillations are able to fit the SBL data. However, when one is interested in parameter estimation, only the expansion of the $\chi^2$ around its absolute minimum matters and the choice of including or not including the SBL data in the fit is irrelevant.
is lower, so the neutrino data sets dominate. In addition, as discussed in Sec. IV, the NOνA νe data are more sensitive than the T2K νe data to the matter effects. More specifically, the following differences among the two hierarchies emerge, which are present both in the 3-flavor and 4-flavor schemes. The regions obtained for the case of IH are shifted towards larger values of \( \theta_{13} \) and are slightly wider in the variable \( \theta_{13} \) with respect to those obtained in the NH case. In addition, in the IH case, the fit tends to prefer (reject) values of \( \sin \delta_{13} < 0 \) (\( \sin \delta_{13} > 0 \)) in a more pronounced way.

After marginalizing over all parameters we can calculate the \( \Delta \chi^2 \) difference between normal and inverted hierarchy

\[
\Delta \chi^2 (\text{IH-NH}) = \chi^2_{\text{min}}(\text{IH}) - \chi^2_{\text{min}}(\text{NH}).
\]

For the 3-flavors (4-flavors) analysis of the LBL data alone we obtain \( \Delta \chi^2 \) \( \approx 1.0 \) (0.8). Therefore this data are (still) not sensitive to the mass hierarchy. The situation sensibly changes when the reactor experiments sensitive to \( \theta_{13} \) are included in the fit. In fact, the combination of LBL and reactor provides a slight preference for NH: \( \Delta \chi^2 \) \( \approx 2.0 \) (1.3) in the 3-flavor (4-flavors) case. The reduced value obtained in the 3+1 framework is due to the inevitable widening of the parameter space in the presence of an additional neutrino. The preference for the NH case can be understood comparing the allowed regions from T2K and NOνA with the constraint on \( \sin^2 2\theta_{13} \) from reactor experiments (vertical green band in Fig. 4). One notes that there is a better agreement for NH, whereas for IH the separation between the two best fit points is at the level of about \( \sim 1 \sigma \).

Let us now come to the estimate of the standard mixing angle \( \theta_{23} \). Recently, the disappearance analysis of the NOνA collaboration \(^{[7]}\) has reported a preference for non-maximal \( \theta_{23} \) at the level of 2.5\( \sigma \). The latest 3-flavor global fits \(^{[14,15]}\) have shown that this feature persists at the level of about 2\( \sigma \) even when other datasets are included in the analysis. Given the important role of the atmospheric angle \( \theta_{23} \) in the context of model building, it seems opportune to assess the estimate of such a parameter in the enlarged 3+1 scheme.

We recall that, in the 3-flavor framework, the disappearance channel is sensitive to possible deviations from maximal mixing but it is blind to the octant of \( \theta_{23} \). This occurs because the \( \nu_\mu \rightarrow \nu_\mu \) disappearance probability is proportional to \( \sin^2 2\theta_{23} \). Therefore, if only the disappearance channel data are included in the analysis, the allowed ranges are symmetrical with respect to \( \sin^2 \theta_{23} = 0.5 \). This symmetry is broken when one considers also the appearance channel. This happens because the \( \nu_\mu \rightarrow \nu_e \) transition probability is octant sensitive since its leading term depends on \( \sin^2 \theta_{23} \). In the 4-flavor scheme, the disappearance probability remains basically unaltered, so one expects that the sensitivity to potential deviations from maximal mixing remains unaltered.

In contrast, the appearance probability is profoundly affected by the new interference term, which, as recently
2.8

\[ m_2 \times 10^2 \text{eV}^2 \]

\[ |m_{23}| \times 10^2 \text{eV}^2 \]

\[ \sin^2 \theta_{23} \]

\[ \chi \Delta \]

\[ \Delta \chi^2 \]

\[ \nu_3 \]

\[ \nu_4 \]

FIG. 5: Regions allowed in the plane \( \sin^2 \theta_{23}, \Delta m_{32}^2 \) by the joint analysis of all the SBL, the LBL data (T2K and NO\( \nu\)A), together with the \( \theta_{13} \)-sensitive reactor results. The left (right) panels represent the NH (IH) case. The upper (lower) panels refer to the 3-flavor (4-flavor) scheme. The confidence levels are the same reported in Fig. 1.

shown in [82], leads to a loss of sensitivity to the octant of \( \theta_{23} \).

Figure 5 reports the allowed regions in the plane \( \sin^2 \theta_{23}, \Delta m_{32}^2 \), all the other parameters having being marginalized away. The left (right) panels refer to normal (inverted) hierarchy, while the upper (lower) panels refer to the 3-flavor (4-flavor) case. In both schemes we have included in the analysis all the SBL data, the LBL results from T2K and NO\( \nu\)A (both appearance and disappearance channels) and the \( \theta_{13} \)-sensitive reactor experiments. The results reported in the upper panels show a weak preference for non-maximal mixing in the 3-flavor scenario. We note that there is a change in the preferred octant when switching from normal to inverted hierarchy. This is a consequence of the anticorrelation between \( \theta_{13} \) and \( \theta_{23} \), introduced by the appearance data set: the lower \( \theta_{13} \) the higher the value of \( \theta_{23} \). In NH we find a negligible preference for the lower octant (\( \theta_{23} < 45^0 \)). In IH the effect is more pronounced and the higher octant (\( \theta_{23} > 45^0 \)) is favored at a non-negligible statistical level. The two lower panels depict how the situation changes in the 4-flavor scheme. We can observe that the allowed regions becomes basically symmetric around maximal mixing. As expected from the discussion above, in the 4-flavor scheme, the sensitivity to the \( \theta_{23} \) octant gets lost.

In order to clarify this picture, we present in Fig. 6 the marginalized \( \Delta \chi^2 \) as a function of \( \sin^2 \theta_{23} \). The left (right) panel corresponds to NH (IH). The black solid line indicates the 3-flavor case, while the red dashed line refers to the 3+1 scheme. In both NH and IH cases, in the 4-flavor scheme non-maximal mixing is disfavored approximately at \( \Delta \chi^2 \simeq 2.5 \) (corresponding almost to 90% C.L. for 1 d.o.f.). Therefore, the weak preference for non-maximal \( \theta_{23} \) originating from (part of) the disappearance channel data is a stable feature, which is independent of the scheme adopted (3-flavor or 4-flavor). In contrast, we see that the preference for \( \theta_{23} > 45^0 \) found in IH completely disappears in the 3+1 scheme. We finally note that this behavior is in line with the results of the sensitivity study performed in [82], where it has been shown that even in a future experiment like DUNE, which will make use of a high-intensity broad-band neutrino beam, the sensitivity to the octant drastically decreases in the 3+1 scheme. Our analysis performed with the real data confirms such a general behavior, showing that the indication on the octant of \( \theta_{23} \) becomes a fragile feature in the 3+1 framework.

FIG. 6: Marginalized \( \Delta \chi^2 \) for the parameter \( \sin^2 \theta_{23} \) for NH (left panel) and IH (right panel). The black solid line indicates the 3-flavor case, while the red dashed line refers to the 3+1 scheme.

VII. CONCLUSIONS

We have shown that, within the 3+1 scheme, the combination of the existing SBL data with the LBL results coming from the two currently running experiments NO\( \nu\)A and T2K, enables us to simultaneously constrain two active-sterile mixing angles \( \theta_{14} \) and \( \theta_{24} \) and two CP-phases \( \delta_{13} \equiv \delta \) and \( \delta_{14} \), although the constraints on this last CP-phase are still weak. The two mixing angles are basically determined by the SBL data, while the two CP-phases are identified by the LBL experiments, once the information coming from the SBL setups is taken into account. We have also assessed the robustness/fragility of the estimates of the standard 3-flavor properties in the more general 3+1 scheme. To this regard we found that: i) the indication of CP-violation found in the 3-flavor analyses persists also in the 3+1 scheme, with \( \delta_{13} \equiv \delta \) having still its best fit value around \(-\pi/2\); ii) the 3-flavor weak hint in favor of the normal hierarchy becomes even less significant when sterile neutrinos come into play; iii)
the weak indication of non-maximal $\theta_{23}$ (driven by NO$\nu$A disappearance data) persists in the 3+1 scheme, where maximal mixing is disfavored at almost the 90% C.L. in both normal and inverted mass hierarchy; iv) the preference in favor of one of the two octants of $\theta_{23}$ found in the 3-flavor framework (higher octant for inverted mass hierarchy) is completely washed out in the 3+1 scheme. We hope that our joint analysis of SBL and LBL data in the 3+1 scheme may serve as a guide for more comprehensive analyses and may increase the awareness of the neutrino community towards the important role of LBL experiments in the search of CP violation induced by light sterile neutrinos.

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