QHC21 EQUATION OF STATE OF NEUTRON STAR MATTER – IN LIGHT OF 2021 NICER DATA

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ABSTRACT

The recent NICER measurement of the radius of the neutron star PSR J0740+6620, and the inferred small variation in neutron star radii from 1.4\(M_\odot\) to 2.1\(M_\odot\), suggest that the neutron star equation of state remains relatively stiff up to baryon densities \(n \sim 2-4\) times nuclear saturation density, \(n_0\). The region where we expect hadronic matter to be undergoing transformation into quark matter. To delineate the physics from the nuclear to the quark matter regimes we use the quark-hadron-crossover (QHC) template to construct an updated equation of state, QHC21. We include nuclear matter results primarily based on chiral effective field theory, but also note results of using nuclear matter variational calculations based on empirical nuclear forces, thus covering the range of uncertainties in the nuclear equation of state. To allow for a possible early transition to quark degrees of freedom we begin the crossover regime from nucleons to quarks at 1.5\(n_0\). The resulting equations of state are stiffer than our earlier QHC19 at \(\lesssim 2n_0\), predicting larger radii in substantial agreement with the NICER data, with accompanying peaks in sound velocity at 2-4\(n_0\). We discuss possible microscopic mechanisms underlying stiffening of the equation of state.

Subject headings: dense matter, equation of state, stars: neutron, quarks

1. INTRODUCTION

Recent data from the NICER observatory has yielded the important inference that the equatorial radius of the most massive neutron star known, PSR J0740+6620, with mass \(M/M_\odot = 2.08 \pm 0.07\) (Fonseca et al. 2021), is \(13.7^{+2.6}_{-1.7}\) km, as analyzed by Miller et al. (2021) combining NICER and XMM data for PSR J0740+6620. With nuclear physics constraints at low density, and gravitational radiation data from GW170817 added in, their inferred radius drops to \(12.35 \pm 0.75\) km for a 2.08 \(M_\odot\) neutron star. The analysis of Riley et al. (2021) using the NICER and XMM data for PSR J0740+6620 gives \(12.39^{+1.30}_{-1.08}\) km. These analyses lead to an updated radius of 1.4\(M_\odot\) neutron stars,\(^1\) \(R_{1.4} = 12.45 \pm 0.65\) km (Miller et al. 2021) and \(12.33^{+0.76}_{-0.81}\) km (Riley et al. 2021) to \(12.18^{+0.56}_{-0.79}\) km (Raaijmakers et al. 2021).

Our aim in this paper is to understand the implications of the new NICER data for the equation of state of neutron star matter within the Quark-Hadron Crossover (QHC) framework (Baym et al. 2018, 2019), which includes nucleonic degrees of freedom at lower densities, and allow for their transitioning to quark degrees of freedom at higher densities. QHC provides a useful baseline to describe the available neutron star data, in which variations such as weak first order phase transitions can be treated as perturbations. This approach lets one relate neutron star observables to nuclear physics as well as to quark physics, such as hadron spectroscopy and the quantum chromodynamics (QCD) phase diagram.\(^2\) We develop and show here the results of our new equation of state, QHC21, which well accommodates the NICER data.

The inferred NICER radii provide clues to the properties of dense matter at baryon densities beyond nuclear saturation density \(n_0\), and indicate the extent to which the central density of PSR J0740+6620 is high enough to contain quark matter. The relatively large

\(^1\) Here \(R_{M/M_\odot}\) denotes the radius of a star of mass \(M\).

\(^2\) QHC employs the picture of quark-hadron continuity – a smooth crossover from baryonic to quark degrees of freedom with increasing density, suggested in the context of the color superconductivity by Schäfer & Wilczek (1999); Hatsuda et al. (2006) and of quantum percolation by Fukushima et al. (2020). Neutron star phenomenology in the QHC framework was originally introduced in (Masuda et al. 2013a,b) to study the stiffening of the equation of state at high density in the presence of the strongly interacting quark matter.
radii, compared with earlier equations of state based on variational calculations of nuclear matter starting from empirical nuclear forces (Akmal et al. 1998; Togashi et al. 2017; Baym et al. 2019), imply that neutron star matter is relatively stiff up to ~ 2-4 $n_0$, a result confirmed in the equations of state of Miller et al. (2021) used to fit the data. The radii of 1.4$M_\odot$ neutron stars, for which the central baryon density is ~ 2-3$n_0$, contain information on nuclear descriptions for 1-3$n_0$. Furthermore, the lack of significant variation of the radii from 1.4$M_\odot$ to ~ 2.1$M_\odot$ illuminates the properties of dense matter at densities ~ 3$n_0$, and rules out substantial softening of matter (as would result from a strong first order phase transition) between 2-3$n_0$ and 4-5$n_0$ (Drischler et al. 2021a); the radial shrinkage in the radius such softening would induce is incompatible with the new NICER data.

The radii inferred from NICER are about 0.7-0.8 km larger than those predicted by our earlier QHC19 equation of state (Baym et al. 2019). There, for example, $R_{1.4} \simeq 11.6$ km, a value essentially determined by the nuclear equation of state at densities in the range of 2$n_0$. Similarly, 2.1$M_\odot$ neutron stars were predicted in QHC19 to have radii ~ 11.3-11.5 km. The challenge is how QHC can allow stiffer equations of state in order to yield such larger radii. The ways in which matter, up to ~ 2-4 $n_0$, can be stiffer than in the QHC19 description include stiffening in the nucleonic regime and possible early onset of the transition to quark degrees of freedom. Indeed, the NICER data provide the first observational test of the relatively soft equation of states for neutron star matter obtained by microscopic variational calculations (Akmal et al. 1998; Togashi et al. 2017).

The NICER analysis of Miller et al. (2021) assumes the Togashi equation of state (Togashi et al. 2017) for the crust up to baryon densities $n \lesssim 0.5n_0$, and parameterized equations of state – a polytrope model, a model with a parameterized speed of sound, and a Gaussian model – beyond that density. A more microscopically motivated approach is to use nuclear equations of state at all densities below ~ 1.5$n_0$, where they are reasonably constrained by controlled uncertainty estimates. The chiral effective field theory ($\chi$EFT) formalism (Lonardoni et al. 2020; Drischler et al. 2021a,b), an expansion in the effective momentum transfers, provides such a control. As discussed by Drischler et al. (2021a) (see their Fig. 1), the uncertainty in the pressure within $\chi$EFT is small at nuclear matter saturation density, and grows to ~ 25% at ~ 1.4$n_0$; the uncertainties at 2$n_0$ are too large to continue to use this approach at such density. The analyses of Riley et al. (2021); Raaijmakers et al. (2021) use $\chi$EFT and polytrope or constant speed of sound models.

QHC19 used the Togashi nuclear equation of state at $n \leq 2n_0$, quark matter equations of state calculated within the Nambu-Jona-Lasinio (NJL) model for $n \geq 5n_0$, and a smooth interpolation between these equations of state. The Togashi equation of state, based on precision two-body nuclear forces and empirical three-body nuclear forces, computed using variational techniques for nuclear many-body problems, as in the Akmal-Pandharipande-Ravenhall (APR) equation of state (Akmal et al. 1998); the same nuclear forces are used consistently in the nuclear liquid and crust equations of state.

Figure 1 shows the range of the pressure in nuclear equations of state vs. baryon density in $\chi$EFT approaches (labeled Lynn, Hebeler, Tews and Drischler) and variational approaches (labeled APR and Togashi). To allow for stiffer matter, we focus on the nuclear equation of states constructed by Drischler et al. (2021a) based on beyond-leading-order (N$^3$LO) $\chi$EFT, and the Togashi equation of state, thus covering the uncertainty range of existing nuclear calculations at ~ 1.5$n_0$. Specifically we take the central value of the N$^3$LO $\chi$EFT pressure at given density. This pressure is larger than Togashi by ~ 30-40%, leading to larger radii.

We explore a greater range of physics above nuclear matter density, permitting possibly stiffer interpolated equations of state, by using explicit nuclear degrees of freedom only up to $n \sim 1.5n_0$. At the same time we also explore quark equations of state at lower densities, $n \sim 3-4n_0$, than in QHC19 ($3n_0$). In the crossover regime between the nucleonic and quark regimes we carry out a smooth interpolation. By lowering the density for onset of quark matter can be stiffer at lower densities in the interpolation region.

Although theoretical calculations in the crossover region are difficult, possibly involving baryon-like and quark-like degrees of freedom at the same time (McLerran & Reddy 2019; Kojo 2021a), the physics in this regime is highly constrained by having to match the nuclear and quark matter equations of state at the boundaries of the crossover region. The constraints that the sound velocity not exceed the speed of light, and that the matter in the interpolation region be thermodynamically

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3 Recent analyses similar to NICER but with nuclear constraints from $\chi$EFT (Fang et al. 2021) yield somewhat lower estimates for the radii $R_{1.4} = 11.9^{+0.77}_{-0.82}$ km and $R_{2.08} = 11.96^{+0.03}_{-0.05}$ km than the NICER analyses. Previously the same group (Dietrich et al. 2020) gave $R_{1.4} = 11.75^{+0.01}_{-0.01}$ km without the data of PSR J0740+6620; the new NICER data leads to an increase in the estimate by ~ 0.2 km, illustrating how data for 2$M_\odot$ neutron stars constrains neutron stars of lower mass and the equation of state in the core.
stable further constrain possible interpolations. The interpolating curves reveal the general trends (Masuda et al. 2013a,b, 2016; Kojo et al. 2015; Annala et al. 2020).

### 2. PHYSICS OF QHC21

We detail here the construction of QHC21. We specify the thermodynamics in terms of the pressure $P$ as a function of baryon chemical potential $\mu_B$. Nuclear equations of state are used up to a density $n^L$, and quark equations of state above a density $n^U$ ($> n^L$). In the crossover between these regimes, we smoothly interpolate the pressure, the baryon density $n = \partial P/\partial \mu_B$, and the susceptibility $\chi_B = (\partial^2 P/\partial \mu_B^2)$ at the matching boundaries. Thermodynamic stability imposes the requirement that the interpolated pressure leads to non-negative $\chi_B$, and causality that the adiabatic speed of sound $c_s = \sqrt{\partial P/\partial \epsilon}$ does not exceed the speed of light, $c$. Here $\epsilon = -P + \mu_B n$ is the energy density.

In QHC21 we consider primarily the $\chi$EFT equation of state in the nucleonic regime, but also note results using the Togashi equation of state there (labeled QHC21TI). To describe quark matter we employ the Nambu–Jona-Lasinio (NJL) model. Although the NJL model does not include confining effects it is a good framework in which to capture dynamical effects, e.g., chiral symmetry breaking, concisely (Hatsuda & Kunihiro 1994), and in addition it reproduces quantities that are not very sensitive to confinement such as effective quark masses and low energy constants in the chiral effective Lagrangian for pions. QHC thus incorporates general features of quark matter by exploring a wide range of the NJL model parameters consistent with neutron star observables. The coupling constant $G$ and the ultraviolet cutoff $\Lambda$ of the NJL model, specified in Baym et al. (2018), are the relevant parameters describing chiral symmetry breaking. In addition, we include the short range density-density repulsion in QCD (Kunihiro 1991; Song et al. 2019), of strength measured by a coupling constant $g_\psi$, and an attraction between quarks, proportional to a coupling $H$, which governs the color superconductivity pairing gap.\(^4\)

The quark equations of state, parametrized by given $(g_\psi, H)$, and the nuclear equation of state are first interpolated in a given range $(n^L, n^U)$ by using fifth order interpolating polynomials in $\mu_B$ for which the coefficients are uniquely fixed by six boundary conditions. We then determine the ranges of $(g_\psi, H)$ that are consistent with stability, causality, and the existence of the known high mass neutron stars. This procedure predicts $M$ vs. $R$, thermodynamic quantities such as baryon density as a function of $\mu_B$, as well as more microscopic quantities such as effective quark masses and diquark pairing gaps.

### 3. NEUTRON STAR STRUCTURE

Figures 2-4 display, for $(n^L, n^U)/n_0 = (1.5, 3.5)$, the results of QHC21 in the $(g_\psi, H)$ plane for the maximum mass and the corresponding central density, as well as the radius and central density of 2.08 $M_\odot$ and 1.4 $M_\odot$.

\(^4\) For simplicity we assume that the effective coupling constants $g_\psi$ and $H$ are independent of density. However, one expects from QCD that $g_\psi$ falls with increasing density (Song et al. 2019); see Appendix A. The quark pairing interaction, $H$, should behave similarly.

| $n^L$ | $n^U$ |
|-------|-------|
| $A_\psi$ | $B_\psi$ | $C_\psi$ | $D_\psi$ | $A_T$ | $B_T$ | $C_T$ | $D_T$ |
| $(1.5, 3.5)$ | $(1.5, 3.5)$ |

\(^5\) This mass is in practice a lower bound on the absolute maximum, since we use only a minimal interpolation between the nuclear and quark regimes. As noted in Appendix B, narrowing the interpolation window can lead to larger maximum mass, owing to small differences of the resultant $P(\mu_B)$ from that with a simple interpolation.
These radii are by at least 0.5 $M_n$ matter, is sensitive to how close the mass is to the maximum where pure hadronic calculations are applicable. Notice that the central density is where pure hadronic calculations are applicable. Notice that the central density is where pure hadronic calculations are applicable.

The maximum mass $M_{\text{max}}$ of the neutron star, allowed by the equation of state, since near $M_{\text{max}}$ the radius shrinks rapidly, by 0.5-1 km (see upper panel of Fig. 5). The radius of PSR J0740+6620 being comparable to $R_{1.4}$ indicates that it mass is not near the maximum; we expect that $M_{\text{max}}$ is limited to at least $2.08 M_\odot$. For $(g_\nu, H)$ satisfying this condition, $n_c \lesssim 5 n_0$.

The range of radii of 1.4$M_\odot$ neutron stars, $\lesssim 11.8-12.7$ km, is consistent with the NICER result $R_{1.4} = 12.4 \pm 0.6$ km. The corresponding central density is $2.1-3.2 n_0$. These radii are $\sim 0.3-1.2$ km larger than in QHC19 ($R_{1.4} \approx 11.5$ km, with $(n_c, n_U)/n_0 = (2.0, 5.0)$), a consequence primarily of using the stiffer $\chi$EFT nuclear equation of state. The Togashi equation of state, even if one allows more freedom for the interpolated pressure in the density range 1.5-2.0 $n_0$, leads to radii $\approx 11.6-12.2$ km, on the smaller side of the NICER band.

Having described the overall trend of the masses, radii, and central densities in the parameter space, we look more closely at results for the parameter sets $A_\chi-D_\chi$ for QHC21. The $M-R$ relation for these parameters is shown in the upper panel of Fig. 5; the lower panel shows the corresponding results for QHC21T for the parameter sets $A_T-D_T$. The numbers next to the heavy dots indicate the central density of the star, in units of $n_0$, for the given mass. As we have matched interpolating functions with nuclear equations of state up to second derivatives at $n = 1.5 n_0$, the deviation of the $M-R$ curves from what the pure, i.e., without quark matter, $\chi$EFT or Togashi equations of state would give appears only after $n$ increases considerably beyond the matching point $n = 1.5 n_0$. (This trend can also be seen in the speed of sound in Fig. 7.) The central $\chi$EFT equation of state in Fig. 1, which is available only for $n \lesssim 2.2 n_0$, predicts overall larger radii for low core densities, leading to better agreement of QHC21 with the NICER data. As we
see in Fig. 6, in QHC21 equations of state become stiffer than QHC21T and pure Togashi at $\sim 2n_0$, producing $1.4M_\odot$ neutron stars at a lower central density than with Togashi alone.

An important feature of the QHC21 equation of state is the behavior of the sound velocity. As seen in Fig. 7, $c_s^2$ exceeds the conformal limit $c_s^2 = c_T^2/3$ at $n \approx 2.1 - 2.3n_0$, a considerably lower density than $\sim 2.8n_0$ with Togashi alone, and also less than the $\sim 2.6n_0$ in QHC19. The peaks in $c_s^2$ for the QHC21 set $A_X$-$D_X$ or $A_T$-$D_T$ are at $\approx 2.1-3.0n_0$, a considerably density than in QHC19 they are in the range $\approx 3.0-4.0n_0$.

The peak in the sound velocity is a novel feature of dense QCD matter at finite baryon density, as originally pointed out phenomenologically by Masuda et al. (2013b) and recently analyzed theoretically by Kojo & Suenaga (2021). It does not exist in a hot QCD plasma (Bazavov et al. 2014), where the speed of sound develops a dip instead of a peak in the crossover region. The finite temperature transition from a hadron resonance gas to a quark gluon plasma is largely driven by non-relativistic resonances which are energetically disfavored but are important due to their large entropies. Such entropic effects are absent in cold dense matter.

Finally, we compare the pressure in QHC21 with constraints deduced by the NICER analyses. Figure 8 shows the pressure in QHC21 as a function of the baryon mass density, $\rho = m_A n$, where $m_A = 1.67 \times 10^{-24}$ g is the nucleon mass. Then Fig. 9 compares the pressure as a function of baryon chemical potential $\mu_B$, with the constraint “All Measurements (Gaussian Process)” in Miller et al. (2021) at a 95% CL. The QHC21 equations of state are quite consistent with the NICER inferences.

4. DISCUSSION

The NICER analyses strengthen the evidence for a peak in the sound velocity, reflecting the stiffening of the equation of state beyond the nucleonic regime. In the quark regime, matter can be stiffened by several physical effects (as discussed in greater detail in Appendix...
A). First, the vector repulsion between quarks, which allows quark matter to support neutron stars above $2M_\odot$, stiffens the matter in general, and produces sound velocities above the conformal limit, $c/\sqrt{3}$. Furthermore, quark pairing correlations leading to color superconductivity (Alford et al. 2005)\(^7\) or baryonic correlations in quarkyonic matter (McLerran & Reddy 2019) can also stiffen the equation of state.\(^8\)

Within QHC21, the central density of PSR J0740+6620, $\lesssim 5n_0$, is well above densities where pure hadronic calculations are valid, and is entering the transition regime to strongly interacting quark matter. The cores of higher mass neutron stars, possibly to be discovered in the future, could reach the transition regime and beyond. Whether fully developed quark matter exists in the cores will have to await fully microscopic calculations of matter undergoing the transition from nucleonic to quark degrees of freedom. Nonetheless the properties of quark matter strongly influence physics in the transition, possibly even slightly above saturation density.

Tables of the equations of state QHC21 for parameter sets $A_\chi$-D$\chi$ and $A_T$-D$_T$ are available on the CompOSE
A. THE CONFORMAL LIMIT FOR THE SOUND VELOCITY

In QCD at ultrahigh densities in the absence of significant length scales, the velocity approaches the conformal limit, $c_s = c/\sqrt{\mathbb{S}}$, from below; this limiting value is the conformal bound. We focus in this Appendix on two questions: The first is how the equation of state determines whether the sound velocity can exceed the conformal bound. In considering quark matter in neutron stars we are particularly interested in the domain where quark descriptions are natural but the matter is not dense enough for perturbative treatments (valid at $\lesssim 40n_0$). While such a domain, above $5n_0$, may not show up in the cores of neutron stars, understanding $c_s$ there should give important constraints on the behavior of equations of state in the range of baryon density $\sim 2-5n_0$ which is only weakly constrained by perturbative results alone. The second question we discuss is the behavior of the sound velocity in QED, a theory that is not asymptotic free. As we show, in the perturbative treatment of the massless electron gas, the sound velocity exceeds $c/\sqrt{\mathbb{S}}$. We set $c = 1$ here for simplicity.

The sound velocity involves a microscopic interplay between the kinetic energy density, $\varepsilon_{\text{kin}}$, and the interaction energy, $\varepsilon_{\text{int}}$. We first show schematically how the conformal bound can be violated in relativistic matter. The quark number density, $n_q$, is given in terms of the quark Fermi momentum, $p_F$, for equal mass quarks, by $n_q = N_f p_F^3/\pi^2$, with $N_f$ the number of quark flavors present; the kinetic energy density for massless quarks, is thus $\varepsilon = ap_F^4 (1 + h(p_F))$, where $a = 3N_f/4\pi^2$ and $h(p_F) = \varepsilon_{\text{int}}/\varepsilon_{\text{kin}}$ is the interaction energy relative to the kinetic energy. Then the pressure is

$$P = \frac{a}{3} p_F^4 (1 + h + p_F h'),$$

(A2)

where primes denote derivatives with respect to $p_F$. Equations. (A1) and (A2) imply

$$\frac{\partial \varepsilon}{\partial p_F} = ap_F^3 (4(1 + h) + p_F h'), \quad \frac{\partial P}{\partial p_F} = \frac{a}{3} p_F^3 (4(1 + h) + 6p_F h' + p_F^2 h''),$$

(A3)

so that

$$c_s^2 = \frac{1}{3} \left[ 1 + \frac{5p_F h' + p_F^2 h''}{4(1 + h) + p_F h'} \right].$$

(A4)

In the absence of $h'$ and $h''$, the $h$ term does not affect the sound velocity.

Let us first consider $h$ having a power-law (PL) behavior, $h = b p_F^b$, where $b$ is a constant. Then,

$$c_s^2(\text{PL}) = \frac{1}{3} \left[ 1 + \frac{(4 + \zeta) b}{4 + (4 + \zeta) h} \right].$$

(A5)

We see that a repulsive interaction, $h > 0$, with $\zeta > 0$ increases the sound velocity above the conformal bound. An attractive interaction, $h < 0$, can also stiffen the equation of state if $\zeta < 0$. Such density dependence is typical in correlation effects near the Fermi surface. Pairing correlations in color superconductivity (Alford et al. 2005) or baryonic correlations in quarkyonic matter (McLerran & Reddy 2019) are in a regime with $\zeta \sim 2$ to within the density dependence of the gap (which is not known well in the non-perturbative regime). For example, for the repulsive vector

9 Tews et al. (2018), also Greif et al. (2019), assuming the validity of nuclear calculations up to $n \sim 1.5n_0$ where $c_s^2$ is only $\sim 0.1 - 0.2$, point out that to have sufficient stiffening at higher density in order to reach neutron stars over two solar masses, it is necessary that $c_s^2$ exceed $1/3$ there. On the other hand, Annala et al. (2020) construct stiff equations of state by allowing $c_s^2$ to reach $\sim 1/3$ from below at $n = 1.1\sim 1.5n_0$, a velocity well above the nuclear results; such early stiffening results in stiff equations of state that do not violate the conformal limit.

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interaction between quarks in the NJL model, \( \sim g^\_\alpha n^\_Q \), one has \( \zeta = 2 \), and in the three-flavor NJL model employed in the text, we have \( h^\_\text{NJL} = (4/\pi^2)g^\_p p^\_p^\_e^2 \). Then

\[
c^\_s^2(\text{NJL}) = \frac{1}{3} \left[ 1 + \frac{6h^\_\text{NJL}}{2 + 3h^\_\text{NJL}} \right],
\]

which always exceeds 1/3 and approaches to 1 in the high density limit.

On the other hand, in the perturbative QCD (pQCD) regime at ultrahigh densities, beyond the range one actually encounters in neutron stars, \( \zeta = 0 \), and an additional physical effect comes into play, the strong coupling constant \( \alpha^\_s \) in dense matter runs with \( p^\_e \) (or equivalently the chemical potential) as \( \sim 1/\ln(p^\_e/\Lambda^\_\text{QCD}) \) with \( \Lambda^\_\text{QCD} \) the QCD scale parameter (\( \simeq 340 \text{ MeV for } N_f = 3 \)). To first order in \( \alpha^\_s \), this dependence leads explicitly to

\[
\varepsilon^\_\text{pQCD} = \frac{3N_f}{4\pi^2}p^\_e^4 \left( 1 + \frac{2\alpha^\_s}{3\pi} \right);
\]

thus \( h^\_\text{pQCD} = 2\alpha^\_s/3\pi \). To lowest order

\[
p^\_e \frac{\partial \alpha^\_s}{\partial p^\_e} = -\frac{33 - 2N_f}{24\pi} \alpha^\_s^2,
\]

while \( h^\prime \) is of order \( \alpha^\_s^3 \). From Eq. (A4) we then find for three flavors,

\[
c^\_s^2(\text{pQCD}) \simeq \frac{1}{3} \left[ 1 + \frac{5}{4}p^\_e h^\prime \right] = \frac{1}{3} \left[ 1 - \frac{5(33 - 2N_f)}{144\pi^2} \alpha^\_s^2 \right]^{N_f \rightarrow 3} \frac{1}{3} \left( 1 - \frac{15}{16} \left( \frac{\alpha^\_s}{\pi} \right)^2 \right),
\]

approaching the asymptotic value 1/3 from below. We see here that the asymptotic approach to the conformal limit is the result of a competition between the gluonic factor 33, which produces an approach from below, and the fermionic term 2\( N_f \), which produces an approach from above. In QCD the gluonic term wins, and the approach is from below.

Equation (A6), valid at intermediate densities relevant to neutron star interiors, and Eq. (A9) for asymptotically high density, are smoothly joined by a density dependent vector interaction, \( g^\_V^\_c \), as introduced in Song et al. (2019),

\[
g^\_V^\_c(p^\_e) = \frac{4\pi}{3} \frac{\alpha^\_s(p^\_e)}{9m^\_c^2 + 8p^\_e^2}.
\]

Here \( m^\_c \approx 400 \text{ MeV} \) is a non-perturbative scale proportional to \( \Lambda^\_\text{QCD} \), and the running coupling for \( p^\_e < m^\_c \) is frozen at \( \alpha^\_s(m^\_c) \) with \( g^\_c = 4\pi\alpha^\_s(m^\_c)/27m^\_c^2 \). With this density dependent vector coupling, \( h = (4/\pi^2)g^\_V^\_c m^\_e^2 \), which approaches \( h^\_\text{NJL} \) at \( p^\_e < m^\_c \) and \( h^\_\text{pQCD} \) at \( p^\_e \gg m^\_c \). The Fermi momentum at which \( c^\_s^2 = 1/3 \) changes sign from positive to negative is \( \simeq 1.5\Lambda^\_\text{QCD} \), corresponding to a baryon density \( n \sim 10n^\_0 \).

Quantum electrodynamics, in contrast to QCD, has no “charged” gluons, so that it is not asymptotically free and the sound velocity exceeds 1/3. To see this in detail, we note that the energy density in the relativistic limit to first order in \( \alpha^\_e \) is

\[
\varepsilon^\_\text{QED} = \frac{1}{4\pi^2}p^\_e^4 \left( 1 + \frac{\alpha^\_e}{2\pi} \right),
\]

so that \( h = \alpha^\_e/2\pi \). The evolution of \( \alpha^\_e \) in QED is equivalent to dropping the 33, letting \( N_f = 1 \) and \( \Lambda^\_\text{QCD} \rightarrow m^\_e \) in \( \alpha^\_s \), and multiplying by a factor 2 which arises from the algebraic difference of the interactions in QED and QCD. Then, to lowest order, we have \( p^\_e \partial \alpha^\_e / \partial p^\_e = \alpha^\_e^2/6\pi \). The resulting sound speed, to order \( \alpha^\_e^2 \), is given by

\[
c^\_s^2(\text{QED}) = \frac{1}{3} \left( 1 + \frac{5}{48} \left( \frac{\alpha^\_e}{\pi} \right)^2 \right),
\]

which exceeds 1/3 (and grows as the Fermi momentum increases). In the low density non-relativistic limit, the sound speed of the electron gas is \( c^\_s^2(\text{QED}) = (p^\_e/m^\_e)^2/3 \), which is smaller than 1/3.

B. EXPLORING OTHER INTERPOLATION RANGES

While we take the interpolation range \( n^\_L = 1.5n^\_0 \) and \( n^\_U = 3.5n^\_0 \) for our main results in the text, we examine in this Appendix the dependence of the NJL parameters and the resulting mass-radius relations on the matching densities \( n^\_L \) and \( n^\_U \). We consider \( n^\_L = (1.3, 1.7)n^\_0 \) and \( n^\_U = (3.5, 4.5)n^\_0 \) in the four possible combinations, continuing to use the \( \chi^\EL \) equation of state of the nuclear regime. Figure 10 shows the allowed ranges of \( (g^\_V, H) \), with the parameter sets \( \Lambda^\_\chi^\EL \) in the main text marked as guides. As we can see, changing the interpolation range modifies the domains of \( (g^\_V, H) \) by only 10-20%.

Increasing the size of interpolating interval should in principle monotonically extend the window for \( (g^\_V, H) \), as well as allow a larger mass range. For example, the results of \( (n^\_L, n^\_U) = (1.3, 4.5)n^\_0 \) should contain the results
of (1.7, 3.5)n₀ as special cases. However in our modeling where we use simple polynomials which cannot cover all allowable curves, this is not the case. We use simple structureless polynomials so as not to introduce exotic structures by hand, and yet the interpolation results in a highly nontrivial structure in the equation of state, e.g., the peak in sound velocity. Owing to our not exploring all possible interpolation curves, the maximum mass we find in a given interpolation window should be understood as a lower bound of the maximum mass.

Shown in Fig. 11 are c₂ vs. n (upper panels), M vs. R (middle panels), and M vs. n (lower panels), for the matching densities n_L = 1.5n₀ and n_U = (3.5, 4.5)n₀. For given g_v = 1.1, and 1.2G, the lower (bold lines) and upper (thin lines) bounds on H are as shown in Fig. 10. The plots for intermediate values of H lie between these lines.

There are clear correlations between the locations of the peaks in the sound velocity, the radii of 1.4M⊙ neutron stars, and the growth of the neutron star mass as a function of the central density. A peak in sound velocity at lower density indicates rapid stiffening which results in a larger radius. In the upper panels of Fig. 11, the locations of the peaks change by ~1-1.5n₀ as H is varied at given g_v, and the radii change by ~0.3-0.7 km, as shown in the middle panels. The central value of H leads to R_{1.4} ≈ 12.2-12.4 km, and with the variation of H, the M-R relations remain largely consistent with the NICER data.

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Fig. 11.— The relations among the sound velocity, mass-radius, and core density of neutron stars; \( c_s^2 \) vs. \( n \) (upper panels), \( M \) vs. \( R \) (middle panels), and \( M \) vs. \( n \) (lower panels), for matching densities \( n_L = 1.5 \, n_0 \) and \( n_U = 3.5, 4.0, \) and 4.5 \( n_0 \). For given \( g_V = 1.1 \), and 1.2 \( G \), the coupling \( H \) has a lower (shown as bold) and an upper (shown as thin) bound (see Fig. 10). The plots for intermediate \( H \) lie basically between the bold and thin lines. The error bars are as those shown in Figs. 5 and 7.

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