159\textsuperscript{Dy} electron-capture: new candidate for neutrino mass determination

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The ground-state to ground-state electron-capture $Q$ value of 159\textsuperscript{Dy} (3/2\textsuperscript{−}) has been measured directly utilizing the novel phase-imaging ion-cyclotron resonance technique. The $Q$ values for allowed Gamow-Teller transition to 5/2\textsuperscript{−} and the third-forbidden unique transition to 11/2\textsuperscript{+} state with excitation energies of 363.5449(14) keV and 362.050(40) keV in 159\textsuperscript{Tb} were determined to be 1.18(19) keV and 2.68(19) keV, respectively. The high-precision $Q$ value of transition 3/2\textsuperscript{−} → 5/2\textsuperscript{−} from this work, revealing itself as the lowest electron-capture $Q$ value, is utilized to unambiguously characterise all the possible lines that are present in its electron capture spectrum.

We performed atomic many-body calculations for both transitions to determine electron-capture probabilities from various atomic orbitals, and found an order of magnitude enhancement in the event rates near the end-point of energy spectrum in the transition to the 5/2\textsuperscript{−} excited state, which can become very interesting once the experimental challenges of identifying decays into excited states are overcome. The transition to the 11/2\textsuperscript{+} state is strongly suppressed and found unsuitable for measuring the neutrino mass. These results show that the electron capture in the 159\textsuperscript{Dy} atom, going to the 5/2\textsuperscript{−} state of the 159\textsuperscript{Tb} nucleus, is a new candidate which may open the way to determine the electron-neutrino mass in the sub-eV region by studying EC.

Further experimental feasibility studies, including coincidence measurements with realistic detectors, will be of great interest.

The neutrino is perhaps the most mysterious particle of all elementary particles. The problem of the overall scale of neutrino masses is a matter of paramount importance in the search for generalizations of the Standard Model, as well as for cosmology. Numerous modern experiments on neutrino oscillations \cite{1–3} allow extracting non-zero differences between the neutrino masses-squared and also the oscillation parameters. These experiments are insensitive to the overall scale of neutrino masses. However, they limit the effective mass of electron antineutrino to be at least 0.048 eV/c\textsuperscript{2} and 0.0085 eV/c\textsuperscript{2} for the inverted and normal mass orderings, respectively \cite{2}. Indirect measurements of the neutrino mass, which also allow to clarify the Dirac or Majorana nature of neutrinos, are conducted in the search of neutrinoless double-$\beta$ decay with a sensitivity of about 0.1 eV/c\textsuperscript{2} \cite{4–7} and neutrinoless double-electron capture \cite{8}. The only direct and model-independent methods for measuring the mass of neutrinos are based on the study of single-electron capture (EC) \cite{9}, while the mass of antineutrinos is measured in single-$\beta$\textsuperscript{−} decays \cite{10}.

Presently, the most stringent upper limit of 0.8 eV/c\textsuperscript{2} (90\% Confidence Level (C.L.)) for the effective electron anti-neutrino mass $m_{\nu_e}$ originates from very recent data obtained with the KATRIN (KArlsruhe TRitium Neutrino) experiment, by investigating the emitted electron spectrum endpoint of tritium $\beta^–$ decay \cite{10}. The most stringent upper limit of the effective electron neutrino mass $m_{\nu_e}$ is as large as 150 eV/c\textsuperscript{2} \cite{11} (95\% C.L.), derived from the analysis of the EC endpoint of 163\textsuperscript{Ho}, which is being utilized for next-generation direct neutrino-mass determination experiments such as EChO \cite{9} and HOLMES \cite{12}.

The search for potential isotopes for possible future long-term and high-sensitivity (anti)neutrino-mass determination experiments \cite{7,17–20} in the pursuit of sub-eV sensitivity, is of great interest. For $\beta^–$ decay spectra, the neutrino mass sensitivity depends on the fraction of events close to the endpoint, where the cumulative decay rate is proportional to the phase-space factor and scales with $Q^{-3}$. For EC, the cumulative event rate near the endpoint is proportional to $Q^{-2}$, and it increases when the electron orbitals have an ionization energy close to the value of $Q$. Nuclides, favored for such direct neutrino mass experiments, are the ones with a small $Q$ and the electron orbitals close to the threshold. 159\textsuperscript{Dy}, studied here, decays only by EC and its ground-to-ground state $Q$ value ($Q_{EC}^{\text{gs}}$) 365.2(12) keV \cite{15,16} is close to the excitation energies ($E_i^\ast$) \cite{13} of two candidate excited states having
spín-parity 5/2− and 11/2+ in the daughter nucleus 159Tb, see Table I. The EC Q values to the excited states are expected to be very small. Especially EC to the 5/2− state is of significant interest since it is of Gamow-Teller type and has been experimentally confirmed to exist with a branching ratio of 1.9(5) × 10−6 [21]. Branching ratio of EC to the 11/2+ state is tiny compared to 5/2− state and this decay branch has not been observed. The total energy of the neutrino emitted in EC decay is determined by the atomic binding energies of the possible allowed atomic shells of the captured electron. In the present case, captures of electrons occupying the K and L shells for the transition 159Dy(3/2−) → 159Tb+(5/2−) are energetically forbidden. Only electrons from s and p1/2, levels from the third and higher shells (M1, M2, N1, N2, O1, O2, and P1) can possibly be captured due to angular momentum conservation and the finite overlap of their wave function with the nucleus. This makes the EC energy even smaller, as tabulated in Table I. The nuclear excitation energies of the two daughter states are already rather accurately known (< 40 eV). The main uncertainty in the Q value is due to the 1.2 keV uncertainty in the ground-to-ground state Q value, which is primarily determined from 159Dy(3/2−) → 159Tb+(5/2−) decay data [13, 15, 21]. With this large uncertainty it is impossible to model the EC spectrum shape, especially near the endpoint where the decay rate is extremely sensitive to the Q value. The current precision does not even allow an order-of-magnitude scale estimate.

In this letter, we report on the first direct 159Dy ground-to-ground state EC Q-value determination. Based on the results, we performed atomic many-body calculations in order to determine the partial EC rates from different atomic shells for the two discussed EC transitions: the allowed Gamow-Teller transition 3/2− → 5/2− and the third-forbidden unique transition 3/2− → 11/2+. We have also determined the partial half-lives of the captures from different atomic shells for the Gamow-Teller transition by normalizing to the measured total EC branching to the 5/2− state.

The measurements were conducted at the Ion Guide Isotope Separator On-Line facility (IGISOL) using the double Penning trap mass spectrometer JYFLTRAP [22] in the accelerator laboratory of University of Jyväskylä, Finland [23].

To produce 159Dy+ ions, a proton beam of 40 MeV in energy from the K-130 cyclotron was used to bombard dysprosium target with natural abundance. Ions of stable daughter 159Tb+ were separately produced with an offline glow-discharge ion source.

The phase-imaging ion-cyclotron resonance (PI-ICR) technique [24, 25] was used to measure the cyclotron frequencies νc = q/mB, where q/m is the charge-to-mass ratio of the measured 159Dy+ and 159Tb+ ions and B the magnetic field. We used the scheme that allows direct determination of νc via the sideband coupling frequency νc = νc + νs, where νs is the trap-modified cyclotron frequency and νs the magnetron frequency. Phase accumulation time t = 514 ms was chosen for both 159Dy+ and 159Tb+ ions to ensure that the spot of interest was resolved from any leaked isobaric, isomeric and molecular contamination. No contaminating ions were observed.

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![Graph](image-url)
The probability depends on the wave function of the electrons and modelling of their shape possible. Furthermore, $^{159}$Dy and $^{159}$Tb ions being mass doublets cancel many of the systematic uncertainties in the cyclotron frequency ratio [27].

The $Q_{EC}^{0\beta}$ is obtained from the mass difference of $^{159}$Dy and $^{159}$Tb utilizing the mass-energy equivalence formula $E = mc^2$:

$$Q_{EC} = (M_i - M_f)c^2 = (R - 1)(M_i - m_e)c^2 + \Delta B_d,$$  

where $M_i$ and $M_f$ are the atomic masses of the parent and daughter atoms, respectively, and $R = v_{c,i}/v_{c,i}$ is their cyclotron frequency ratio obtained in charge state $1^+$. The value $\Delta B_d$ describes the contribution from electron binding energy differences of the parent and daughter atoms (here 0.07525(60) eV for $^{159}$Dy$^+$. and $^{159}$Tb$^+$ [28]). $m_e$ is the mass of electron. Three sets of data were collected (see Fig. 1).

The normalized $\chi^2$ for the sets were 1.2, 0.9 and 1.0. The uncertainty of the first was expanded with the square-root of it, marginally affecting the final weighted mean ratio $\bar{R}$. The final weighted mean frequency ratio $\bar{R}$ is 1.000 002 463 8(13), which results in $Q_{EC}^{0\beta} = 364.73(19)$ keV.

The obtained $Q_{EC}^{0\beta}$ from this work is more than six times more precise and 0.47 keV smaller than the AME2020 value, which was derived primarily from an EC decay measurement of $^{159}$Dy(EC)$^{159}$Tb [15]. The newly measured high-precision $Q_{EC}^{0\beta}$, together with the accurate nuclear energy level data, yields $Q_{EC}^{0\beta}$ values of 1.18(19) keV and 2.68(19) keV for the $5/2^-$ and $11/2^+$ states in $^{159}$Tb, respectively. $Q$ values of different atomic electron shell captures are tabulated in Table I. Which orbital electrons take part in the EC process and the absolute $Q$ values of the decays are crucial for modelling the spectrum shape near the endpoint. In this work, M2 capture to the $5/2^-$ state is confirmed to be energetically forbidden at 3.3$\sigma$ level, revealing N1 to be the first energetically possible capture at 4.0$\sigma$ level. In addition, the M1 capture to the $11/2^+$ state is confirmed to be positive at 3.7$\sigma$ level and captures can proceed from M1 and higher orbits. The unambiguous characterization of all the possible lines in the EC spectrum at a significance level of at least 3$\sigma$ for the transitions, makes the modelling of their shape possible.

To estimate the EC partial half-lives and the distribution of energy released in the decays, we have performed Dirac-Hartree-Fock atomic many-body calculations. The EC capture rate is determined by the standard $\beta$-decay Hamiltonian. The probability depends on the wave function of the electrons inside the nucleus, on the exchange-and-overlap factor of the spectator electrons due to the non-orthogonality of the atomic shells of the parent and daughter atoms, as well as the nuclear matrix element.

| TABLE II. Normalized partial half-lives for the Gamow-Teller EC transition $3/2^- \rightarrow 5/2^-$. The first line lists the atomic orbitals $x$ for a positive EC $Q$ value, the second line shows the corresponding electron binding energies of the daughter $^{159}$Tb atom [14]. The last line lists the resulting partial EC half-lives after normalizing to the total half-life 2.08$\times 10^5$ years of the Gamow-Teller transition from [21]. The level P1 is calculated using the GRASP2018 software package [29] for an isolated atom of $^{159}$Tb$^+$ in the configuration $[\text{Xe}(4f)^{10}(6s)^2]$. |
|----------|--------|--------|--------|--------|--------|--------|
| $x$      | N1     | N2     | O1     | O2     | P1     |
|----------|--------|--------|--------|--------|--------|--------|
| $\epsilon_x$ [eV] | 396    | 322.4  | 45.6   | 28.7   | 9.5    | 9.5    |
| $t_{1/2}$ [year]    | 3.0$\times 10^5$ | 5.8$\times 10^5$ | 8.9$\times 10^5$ | 2.6$\times 10^7$ | 1.3$\times 10^7$ |

The energy distribution of EC events is represented as the incoherent sum of the contributions of individual orbitals:

$$\rho(E) = \frac{G^2_\beta}{(2\pi)^2} \sum_x n_x \beta_x \Gamma_{x}^i/\Gamma_{x} \epsilon_x, (2)$$

where $E = Q_{EC}^{0\beta} - E_x$. $Q_{EC}^{0\beta}$ is the $Q$ value of the decay, $E_x$ is the neutrino energy, $\lambda(E)$ is the total decay probability in the interval ($Q_{EC}^{0\beta} - m_e$): $G^2_\beta = G_F \cos \theta_C$, $G_F$ is the Fermi constant and $\theta_C$ is the Cabibbo angle; $p_x(E_x) = \sqrt{E_x^2 - m_e^2}$ is the neutrino momentum, $\epsilon_x$ is the energy of the electron hole with quantum numbers $x = (n,l,j)$ of the daughter atom, and $n_x$ is the occupation fraction of electrons in a partially filled shell $x$ of the parent atom ($n_x = 1$ for closed shells). The shape factor $C_x$ contains the nuclear-structure information in terms of nuclear form factors [30]. $\Gamma_x$ is the intrinsic linewidth of the Breit-Wigner resonance centered at the energies $\epsilon_x$. The amplitudes $\beta_x$, which characterize the electron wave functions inside the nucleus, and the exchange-and-overlap factors $\beta_x$ are given for a broad set of atomic numbers and orbitals, e.g., in [31] and here calculated for all orbitals of $^{159}$Dy and $^{159}$Tb$^+$ by using the atomic structure software package GRASP2018 [29]. The nuclear charge density is given by the Fermi distribution with the root mean square radius of $R_{nucl} = 5.1$ fm and thickness 2.3 fm. The parent $^{159}$Dy atom is in the ground state, while the daughter atom $^{159}$Tb$^+$ is described by the electron wave functions depending on the hole $x$. Electrons of the daughter atom inherit quantum numbers from the configuration $[\text{Xe}(4f)^{10}(6s)^2]$ of the parent $^{159}$Dy atom. The exchange-and-overlap factors $\beta_x$ calculated in the Vatai approach [31] deviate from unity by 25% or less.

The total decay constant $\lambda \equiv \lambda(Q_{EC}^{0\beta} - m_e)$ is calculated from

$$\lambda(E) = \int \rho(E')dE'.$$  

In the narrow-width approximation $\lambda \approx \sum_x \lambda_x$; the partial decay constants equal

$$\lambda_x = \frac{G^2_\beta}{(2\pi)^2} n_x \beta_x \Gamma_{x}^i/\Gamma_{x} (Q_{EC}^{0\beta} - \epsilon_x)/(Q_{EC}^{0\beta} - \epsilon_x).$$  

For the presently discussed transitions to the $5/2^-$ state and $11/2^+$ state the shape factor contains only one nuclear form factor in the leading order.
For the EC to the $5/2^-$ state the shape factor can be written as $C_t=\left(A_{101}^{F(0)}\right)^2$, with the nuclear form factor given in terms of the Gamow-Teller nuclear matrix element as

$$A_{101}^{F(0)} = -\frac{g_A}{\sqrt{2J+1}}M_{GT}. \quad (5)$$

Here $g_A$ is the strength of the weak axial coupling, $J$ the angular momentum of the initial state, and $M_{GT}$ the Gamow-Teller nuclear matrix element [32]. In fact, for this decay transition we do not need the value of the form factor $A_{101}^{F(0)}$ since we normalize $\lambda$ by the available half-life for the Gamow-Teller transition, derived from the measured branching [21] and the total half-life [13]. For this transition the experimental binding energies and normalized partial half-lives are listed in Table II.

The summation in Eq. (2) runs over the electron orbitals shown in Table II, as well as over the M1 and M2 orbitals. Although M1 and M2 are outside the kinematically accessible energy region, the tails of their Breit-Wigner amplitudes have a significant effect on the number of events for $E \lesssim Q_{EC}$. The electromagnetic decay widths $\Gamma_x$ of the N1, N2, M1 and M2 electron holes in $^{159}$Tb atom are taken from Ref. [33]; the data for $x = O1, O2, P1$ are not available, so we assume $\Gamma_{O1, O2, P1} = \Gamma_{N2} = 5.26$ eV. The widths of the levels N1, N2, M1 and M2 closest to the threshold are known with an accuracy of 10%, 15-15%, 5% and 5-10%, respectively [33]. The corresponding uncertainties in the spectrum do not exceed 30%, while the integral over the spectrum is almost independent of the level widths. The experimental error in $Q_{EC}$ introduces through the phase space volume about 50% uncertainty in the half-life estimates.

The computed calorimetric $^{163}$Ho spectrum of Fig. 2 takes the electron orbitals M1, M2, N1, N2, O1 and O2 into account with the parameters given in Ref. [9]. The distances from the endpoint to the nearest peak for dysprosium (N1) and holmium (M2) are almost the same. Proximity of the M1 and M2 orbitals of dysprosium to the endpoint partly compensates the difference between the absolute EC rates of dysprosium and holmium at $E \lesssim Q_{EC}$. The normalized cumulative distribution of the EC events near the endpoint equals

$$(\lambda - \lambda(E))/\lambda \approx C_V p_0(E_V), \quad \text{where } C_V = 0.0061/\text{keV}^3 \text{ for dysprosium and } 0.00056/\text{keV}^3 \text{ for holmium.}$$

The M1 and M2 orbitals increase the number of events in the endpoint region by an order of magnitude. The same absolute numbers of events near the endpoint are provided by the ratio between the numbers of dysprosium atoms decaying to Tb ($5/2^-$) and holmium atoms: $R(159\text{Dy}/163\text{Ho}) = T_{1/2}(159\text{Dy}) \rightarrow 159\text{ Tb}^*(5/2^-)/T_{1/2}(163\text{Ho})C_V(163\text{Ho})/C_V(159\text{Dy}) = 4.2$, while the total numbers of atoms before the filtering are in the ratio $R_0(159\text{Dy}/163\text{Ho}) = 4.2/1.9 \times 10^{-6} = 2.2 \times 10^6$. The smallness of $C_V$ values limits, due to statistical requirements, the sensitivity of EC experiments measuring the mass of electron neutrino. To improve sensitivity, reliable parameterization of the energy spectrum away from peaks is also necessary, taking into account the dependence of the electron level widths and decay constants on energy. Decays accompanied by shake-up and shake-off excitations with the associated formation of multiple holes in the electron shell generate a fine structure of the spectrum [34–39], which is experimentally visible in holmium EC and is described well theoretically [40, 41].

The decay to the $11/2^+$ state gathers contributions from the M1-M5, N1-N7, O1-O3, and P1 atomic orbitals. The decay rate involves one nuclear form factor which we have computed using the microscopic interacting boson-fermion model (IBFM-2). In this manner, we obtain an estimate of the half-life of $t_{1/2} \sim 25$ years for this transition, thus excluding it as a candidate for electron-neutrino mass measurements. There is also no experimental evidence for the existence of this transition.

The transition to the $5/2^-$ state has an experimentally measured half-life of $2.08 \times 10^5$ years [21]. This measured half-life can be used, together with the computed partial decay constants $\lambda_x$, to determine the normalized partial half-lives of the dominant EC channels. Using the computed decay constants and the IBFM-2 computed nuclear matrix element one obtains a theoretical half-life which is consistent with the measured one. Figure 2 shows the calculated EC spectrum. For comparison, the spectrum is also given for $^{163}$Ho. Both spectra are normalized to unity. It is clear that a larger fraction of events lands near the endpoint for $^{159}$Dy decay, $Q_{EC}$ is the difference in energy of the parent and daughter atoms. A larger fraction of events lands near the endpoint for $^{159}$Dy than for $^{163}$Ho. The inset in the lower left part of the figure shows on an enlarged scale the energy spectra of dysprosium and holmium close to the threshold value to illustrate the effect of the neutrino masses of 1 and 0 eV/c$^2$.

In conclusion, our findings reveal that the $Q_{EC}$ of $1.18(19)$ keV for the transition $^{159}$Dy($3/2^-$) $\rightarrow$ $^{159}$Tb$^*(5/2^-)$...
is lower than the ground-to-ground state $Q_{EC}$ of $^{163}$Ho, which is utilized in presently running or planned direct neutrino mass experiments. Therefore, this allowed transition, with a universal spectral shape driven by a single decay matrix element and known branching ratio, becomes a potential candidate for effective electron neutrino mass measurements. Proximity of $Q_{EC}$ and atomic lines N1, M1, and M2 with values of 0.79(19) keV, -0.78(19) keV, and -0.58(19) keV, respectively, indicates a significant potential of this EC transition for a self-calibrated and high-sensitivity EC experiment in the direct neutrino mass determination. The background from the EC to other states of $^{159}$Tb can be suppressed by coincident registration of de-excitation gamma-rays from the $5/2^-$ state of the nucleus. Such event selection is used in the search for neutrinoless double electron capture accompanied by nuclear excitations \[8\]. Decay to the $5/2^-$ level has a branching ratio of only $1.9 \times 10^{-6}$. In order to achieve sub-eV sensitivity, the measurement of the neutrino mass requires reliable coincidence measurements between the calorimeter and the $\gamma$ detector to identify only a very small fraction of total events, as well as a low background and a high counting rate of microcalorimeters.

We also want to point out that the Gamow-Teller EC transition to the $5/2^-$ state also serves as one of the most prospective transitions for a possible relic anti-neutrino capture experiment [42]. Here the very small $Q$ value, reported in this work, implies a promingly high sensitivity to relic neutrinos requiring orders of magnitude less active material than needed for other suggested candidate nuclei like $^{163}$Ho or $^{157}$Tb.

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