Covariance and Frames of Reference

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Abstract

A transition of focus from state space to frames of reference and their transformations is argued as being the appropriate setup for ensuring the covariance of physical laws. Such an approach can not only simplify and clarify aspects of General Relativity, but can possibly help in the development of a Grand Unified Theory as well.

"Physics may be not only too important, but also too deep, in order to be left alone to usual physical type thinking and arguments."

1. Frames of Reference versus State Space

1.1. State Spaces

Classical. At least since H Poincaré, we are accustomed to associate with the dynamics of a given physical system a corresponding state space which is the set of all possible points, each of which uniquely describing one of the possible situations the respective system can be. Consequently, a state space can be finite or infinite dimensional.

For instance, the dynamics of $n \geq 1$ material points in usual 3-dimensional Euclidian space $\mathbb{R}^3$ can be described by $6n$-dimensional
vectors which give their individual positions and momenta. Thus the corresponding state space is $\mathbb{R}^{6n}$. In case of a sufficiently laminar or smooth flow of a fluid in a domain $\Omega \subseteq \mathbb{R}^3$, the state space can be taken as $C^2(\Omega, \mathbb{R})$, each function $u : \Omega \rightarrow \mathbb{R}$, $u \in C^2(\Omega, \mathbb{R})$ describing the field of velocities at a certain moment in time.

In such classical situations time need not be included from the very beginning into the state space. Instead, it can be introduced as an outside and additional component. Namely, time is modelled by the real line $\mathbb{R}$, and then enters into consideration by a cartesian product with the state space. For instance, in the first case above, we shall have $\mathbb{R} \times \mathbb{R}^{6n}$, while in the second case $\mathbb{R} \times C^2(\Omega, \mathbb{R})$.

Quantum. Such an approach remains valid also in the non-relativistic quantum mechanics of a finite number of particles. Indeed, if we have $d \geq 1$ such particles, then the state space is the Hilbert space $L^2(R^d)$ which contains the wave functions $\psi$ as they are at any given moment of time, while the dynamics happens in $\mathbb{R} \times L^2(R^d)$.

Dynamics. The general pattern in the above cases is as follows.

There is a state space given by a certain set $\mathcal{S}$, and every specific dynamics of the system is described by a corresponding curve $C \subseteq \mathcal{S}$ which is parametrized by time, according for instance, to a mapping $I \ni t \mapsto x_t \in C$, where $I \subseteq \mathbb{R}$ is some time interval.

Alternatively, we can consider in such a situation the composite space

\begin{equation}
\Sigma = I \times \mathcal{S}
\end{equation}

in which case every specific dynamics of the system is described by a curve $\Gamma \subseteq \Sigma$, where $\Gamma = \{ (t, x_t) \mid t \in I \}$.

Relativity. In Special and General Relativity the situation is different, since from the start time has to be considered together with space. In this way, from the beginning the situation recalls or extends that in (1.1). For instance, for one point particle in Special Relativity, the state space still has the form (1.1), namely $\Sigma = \mathbb{R} \times \mathbb{R}^3$, while considered in General Relativity, the state space $\Sigma$ is a 4-dimensional so called space-time manifold, resulting from the Einstein equations.
1.2. Frames of Reference

In Newtonian Mechanics, Maxwell Electro-Magnetism, or for that matter, non-relativistic Quantum Mechanics of finite systems, the issue of frames of reference and transformations of such systems comes usually up in limited ways. What happens is that one assumes given a certain convenient coordinate system which automatically defines a frame of reference, and one then subsequently tends to remain within it. When a different coordinate system, thus frame of reference, has to be considered, the respective transformations are performed, including to the corresponding form of the physical laws of interest. And often such laws may take a different form, since the transformations used need not always be restricted to those which leave these laws covariant.

In this way, the stress is rather on the state spaces, than on the covariance of the laws under the transformations encountered.

This situation changes significantly starting with Special Relativity where one of the two fundamental principles is that the laws of Physics are the same in every inertial frame of reference, and this principle is frequently and essentially used in the development of the respective theory.

It follows that in Special and General Relativity frames of reference and their transformations are from the very beginning among the building blocks of the respective theories, thus in fact they acquire a new prominent position versus state spaces.

In this regard, it will be argued in the sequel that in pursuing both General Relativity and Grand Unified Theories, it may be appropriate to focus more on frames of reference and their transformations, as well as the requirement that the laws of Physics remain covariant under the respective transformations, than on the state spaces.

2. Starting with Frames of Reference and Covariance

With hindsight, we can now realize that in General Relativity by far the most important fact is that the equations of Physics must be covariant under arbitrary $C^2$-diffeomorphisms.
In other words, just as much as from the point of view of Physics as such it is absolutely irrelevant whether a book on the subject is written in English or in any other sophisticated enough language, so it must be absolutely irrelevant for their form in which coordinate system the laws of Physics are written, see arXiv:physics/0505045.

As it happens, however, the way A Einstein pursued from Special Relativity to General Relativity during the decade 1905-1915, or for that matter, the last moment involvement of D Hilbert, did not have as starting point this argument of very general type of covariance, see Kaku. Instead, a variety of arguments of physical nature had been involved, some of them leading nowhere, even if the initial insight of A Einstein related to what is nowadays called the Equivalence Principle proved to be correct and fundamental.

Here it may be important to note that such an approach which brings in an abundant variety of physical type arguments, often regardless of their effective relevance, is rather typical in the development of Physics. Yet, such an approach can often lead to unnecessary prolixity, and thus possible lack of clarity or confusion, and in addition, also to the risk of missing some really important points. A nontrivial example in this regard is the way quantum physicists, for instance, tend so often to argue and obtain the celebrated Bell inequalities. Indeed, most of the rather endless arguments ranging around the Bell inequalities, arguments in which any amount of physical insights may be marshalled, happen to miss the point that the Bell inequalities - as inequalities - belong to a family of classical inequalities which were already known to George Boole back in 1856, see arXiv:quant-ph/0406004.

Needless to say, such a situation should serve as a good warning, and not only in Quantum Mechanics.

And then, there are only two issues left to consider:

• which laws of Physics we talk about,
• what do we mean by frames of reference.

When it comes to frames of reference, however, one usually proceeds quite automatically, and without much thinking. Namely, one starts
with a state space, with corresponding coordinate systems, and then
one associates frames of reference and talks about smooth enough and
invertible transformations of the state space one may happen to use.

Therefore, one may have to revise such an approach as follows:

- first, reconsider deeply enough what we really mean by frames
  of reference, and what such systems may indeed be like,
- then, simply ask what are the covariant laws of Physics in such
  frames of reference.

The advantage of such an approach may be twofold:

- to simplify the story of General Relativity,
- to help us in setting up a Grand Unified Theory.

As for the second point above, it is indeed obvious that the present
rather automatic and unquestioned approach to the concept of frames
of reference is one of the major factors which lead us to the impossibil-
ity of bridging the divide between the ”continuum” aspect of General
Relativity, and on the other hand, of the ”discrete” aspect of Quan-
tum Theory.

In his recent book ”Einstein’s Cosmos”, M Kaku stresses that two very
simple but fundamental physical ideas had led Einstein to his setting
up relativity. For Special Relativity he used his teenage idea of racing
along a beam of light, while for General Relativity he used what came
to be called the ”Equivalence Principle”. Further, M Kaku suggests
that the last three decades of Einstein’s life, when he attempted a
Grand Unified Theory, did not lead to a success since he could not
find a third such a simple but equally fundamental physical idea to
start with and then build upon.

It is also suggested that Special Relativity was considered by Einstein
insufficient, since it did not include gravitation.

As for finding a Grand Unified Theory, Einstein kept talking about
”marble and wood”, of turning ”wood into marble”, of creating a the-
ory of ”pure marble”
The fact however is that the need not to stop at Special Relativity, and instead go further for General Relativity has, in terms of frames of reference and covariance, a very simple and most obvious direct reason, namely:

Why should covariance be limited only to Lorentz transformations of frames of reference?

In this way, such a simple question, arXiv:physics/0505045, is all alone and all in itself sufficient in order to get us going from Special Relativity towards General Relativity. And once one realizes that, one can of course use arguments of Physics in order to set up the respective theory.

However, on the level of motivation for not remaining with Special Relativity, one does not need to be much concerned about any Physics at all, except to remember that there are far more general frames of reference and coordinate transformations than the Lorentzian ones. Thus covariance cannot and should not be limited to what it means in Special Relativity.

As for creating a "pure theory of marble", that is, creating a Grand Unified Theory, it may well happen that an argument which for the time being is equally overlooked is in fact needed. An argument which, as it happens at present, is replaced by endless and not particularly effective physical considerations. Namely, and as mentioned above, it may well happen that we shall first have to work out much more general concept of frames of reference than used in General Relativity, a concept which can indeed bring properly together the "marble" and the "wood" ...

3. Precedents

The idea of abandoning usual state spaces in Physics already has precedents, one of them as famous as String Theory.

In another direction, however along the same intent, categories and toposes have been used, see for instance Coecke, Isham, Butterfield & Isham and the references cited there, for some of the more recent such approaches.
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