Massive gravitons from Extended Gravity to Effective Field Theories

S. Capozziello,$^{a,b}$ M. De Laurentis,$^{a,b}$ M. Paolella,$^{a,b}$ G. Ricciardi$^{a,b}$

$^a$Dipartimento di Fisica, Università di Napoli “Federico II”, Compl. Univ. di Monte Sant’Angelo, Via Cintia, 80126 Napoli, Italy
$^b$INFN Sez. di Napoli, Compl. Univ. di Monte Sant’Angelo, Via Cintia, 80126 Napoli, Italy

E-mail: capozziello@na.infn.it, felicia@na.infn.it, paolella@na.infn.it, ricciardi@na.infn.it

ABSTRACT: Massive gravitons in effective field theories can be recovered by extending General Relativity and taking into account generic functions of the curvature invariants not necessarily linear in the Ricci scalar $R$. In particular, adopting the minimal extension of $f(R)$ gravity, an effective field theory with a massive state is straightforwardly recovered. This approach allows to evade shortcomings like ghosts and discontinuities if a suitable choice of expansion parameters is performed. We show that the massive state can be identified with a massive graviton.

KEYWORDS: Modified gravity; effective field theories; massive graviton
1 Introduction

The long standing problem of graviton mass [1–7] has recently excited a renewed interest both at fundamental and cosmological level. From one side, massive spin-tensor states could be the signature of some effective theory quantization. From the other side, massive gravitons could be the natural candidates for dark matter capable of structuring self-gravitating astrophysical systems [8, 9].

Even though a quantum description of gravity has not been achieved yet [10, 11], it is possible to quantize gravity in the linear approximation of Minkowskian limit. Specifically, assuming General Relativity (GR) as the theory of gravitational interaction, the quantization in this limit gives rise to spin-2 massless bosons, i.e. the massless gravitons [13]. Starting from this result, a reasonable question to ask is whether gravitons could be massive in some alternative theory of gravity where GR is only a limiting or a particular case [7]. However, to give mass to gravitons pose some controversial issues that greatly complicate the formulation of self-consistent theories, such as the presence of ghost, instabilities, discontinuity and strong coupling effects at low energy scales [2, 3, 5, 6]. In any case, massive graviton solutions cannot be simply ruled out if one wants to face coherently the problem of gravitational interaction in the ultraviolet limit [4, 14, 15, 26].

On the other hand, a large amount of alternative theories of gravity has been recently developed in order to match the problem of cosmic acceleration in view of dark energy [16–20]. In all these approaches, the problem of massive gravitons emerges and has to be consistently considered also at infrared limit [21, 23]. The original motivation is related to the observational evidence of the accelerated expansion of the Universe at the present epoch. This accelerated expansion may be due to the cosmological constant, to new weakly interacting fields constituting some kind of dark energy. The problem is that the energy scale of dark energy appears to be smaller and smaller with respect to the energy scale of any known interactions. The unnatural smallness of dark energy density constitutes the cosmological constant problem. In this sense, infrared-modified gravity models could
be phenomenologically relevant as a possible alternative to dark matter and dark energy whose effects at large scales could be originated by geometry \cite{19}.

In theories with extra dimensions, gravitons are characterized by a whole family of massive excitations (e.g. the Kaluza-Klein modes).

Based on the previous, as well as other, motivations, there have been several experimental searches for massive gravitons, resulting in upper limits of the mass which differ by several orders of magnitude. For example, a limit on the graviton mass ($\sim 8 \times 10^4$ eV) has been achieved by measuring the decay of two photons \cite{24}. Besides, assuming that clusters of galaxies are bound by more-or-less standard gravity, it is possible to obtain an upper limit of $2 \times 10^{-29} h_0$ eV, where $h_0$ is the Hubble constant in units of 100 km s$^{-1}$ Mpc$^{-1}$ \cite{25}.

Gravitational waves are described by the transverse-traceless gauge, which is a spin-2 tensor under rotations. It is possible to construct consistent models where Lorentz invariance is broken and the masses of scalar, vector and tensor perturbations are different. A direct limit on the mass of graviton can be obtained from gravitational waves by binary stellar systems and from the inspiral rate inferred from the timing of binary pulsars. This bound is about $7.6 \times 10^{-20}$ eV for the binary pulsar PSR B1913+16 \cite{26}. The same limit can be also obtained by studying binary systems in $f(R)$-gravity \cite{27}. An estimate of the graviton mass upper limit of about $7 \times 10^{-32}$ eV, is obtained by considering the effect of gravitons on the power spectrum of weak lensing, with assumptions about dark energy and other parameters \cite{12}.

From a genuine theoretical point of view, the study of gravitons is challenging, due to the problem of reconciling gravity and the quantum field theories describing fundamental interactions. A bridge is represented by effective field theories (EFT), that allow to analyze different energy regimes separately (see e.g. \cite{28, 29}). In general, since the effective Lagrangians is non-renormalizable due to an infinite number of counterterms, one retains only a few of them, in a phenomenological approach where only leading terms are necessary. This means that the determination of the effective degrees of freedom is a crucial point for any effective theory and this fact is even more important in connection with gravity.

Technically, a way to build up an effective Lagrangian is to identify some expansion parameters and classify terms in the Lagrangian according to such parameters. Without knowing the underlying fundamental theory, the coefficients of the expansion are necessarily unknown, and their values have to be determined, in principle, by experiments.

In this paper, we take into account an effective theory of the gravity that follows naturally from Extended Theories of gravity (ETG) (see e.g. \cite{17}). The action can be expanded in powers of the Ricci curvature scalar $R$ satisfying a massive Klein Gordon field equation. In particular, by linearizing $f(R)$ gravity, the Lagrangian describes a massive scalar field where a mass scale $m$ emerges naturally. The theory does not predict the value of this mass, but it does predict its connection with parameters of the ETG Lagrangian. It is possible to identify correlations between the coefficients of the effective Lagrangian, which may, in turn, induce correlations among observables at different scales. A first important result is that the assumption of an effective Lagrangian derived from $f(R)$ gravity allows to escape the problem of scalar ghosts in massive theories, as pointed out in \cite{2}. In the limit where $m \gg \Lambda$ (being $\Lambda$ the cosmological constant), we achieve a physically acceptable
scalar field satisfying a homogeneous Klein Gordon equation.

The paper is organized as follows. Sec. 2 is devoted to the construction of the field equations for analytical $f(R)$ gravity models. Here we put in evidence the main parameters of the theory defining an effective mass. The linearized theory is discussed in Sec. 3. In particular, we derive massive modes that can be interpreted as massive gravitons. The effective Lagrangian is considered in Sec. 4. Conclusions are drawn in Sec. 5.

2 Field equations for $f(R)$ gravity

Let us consider a 4-dimensional action in vacuum for a generic function $f(R)$ of the Ricci scalar \[ S = \int d^4x \sqrt{-g} f(R), \] (2.1)

where the Ricci scalar is defined as $R = g^{\mu\nu} R_{\mu\nu}$, and $g$ is the determinant of the metric. The only assumption at this stage is that $f(R)$ is an analytic function (i.e. Taylor expandable) in term of the Ricci scalar, that is

\[ f(R) = \sum_n \frac{f^n(R_0)}{n!} (R - R_0)^n = f_0 + f'_0 R + \frac{1}{2} f''_0 R^2 + ..., \] (2.2)

where we recover the flat-Minkowski background as soon as $R = R_0 = 0$. Here $f'(R) = \frac{df(R)}{dR}$ and $f''(R) = \frac{d^2f(R)}{dR^2}$ indicate the derivative with respect to the Ricci scalar $R$. We have defined $f_0 = f(R)|_{R=R_0}$, $f'_0 = f'(R)|_{R=R_0}$, and so on. At the second order of approximation in term of $R$, the above action (2.1) becomes:

\[ S = \int d^4x \sqrt{-g} \left[ f_0 + f'_0 R + \frac{1}{2} f''_0 R^2 \right]. \] (2.3)

This can be viewed an EFT Lagrangian, naturally emerging in the context of ETG. In a bottom-up approach, from the point of view of unconstrained EFT, there is no rationale, like symmetries or renormalizability, for choosing the gravitational action proportional to $R$ like in GR, except indications that the curvature $R$ is rather small. Moreover, there are infinite terms allowed by general coordinate invariance, such as $R_{\mu\nu} R^{\mu\nu}$, where $R_{\mu\nu}$ is the Ricci tensor, $R_{\mu\nu;\lambda;\sigma} R^{\mu\nu;\lambda;\sigma}$, where $R_{\mu\nu;\lambda;\sigma}$ is the Riemann tensor, derivatives of $R$, and so on. Where one has to truncate the expansion is somehow a matter of choice, and the coefficients are completely unknown from a theoretical point of view. Instead, the terms in the action (2.3) follow from the underlying ETG, which can also give indications on the coefficients and the order of the series. Here we are choosing the simplest possibility considering an analytical $f(R)$ theory of gravity.

By varying the action (2.3) with respect to the metric, we obtain the field equations

\[ - \frac{f_0}{2} g_{\mu\nu} + f'_0 G_{\mu\nu} - f''_0 \left[ \nabla_\mu \nabla_\nu R - g_{\mu\nu} \square R + R \left( \frac{1}{4} R g_{\mu\nu} - R_{\mu\nu} \right) \right] = 0, \] (2.4)
where
\[ G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R \]  
(2.5)
is the Einstein tensor and \( \Box = \nabla_\sigma \nabla^\sigma \) is the d’Alembert operator with \( \nabla_\sigma \) that indicates the covariant derivative. It is interesting to note that if we rewrite the ETG Lagrangian in the form:
\[ L = \sqrt{-g} \left[ \frac{f_0}{f_0'} + R + \frac{1}{2} \frac{f''_0}{f'_0} R^2 \right] f'_0, \]
(2.6)
we can identify the cosmological constant term as \( \frac{f_0}{f_0'} = -2\Lambda \). We are working in Planck units, therefore we assume that the Lagrangian in Action (2.6) is multiplied by \( \frac{1}{16\pi G} \), where \( G \) is the Newton’s constant. From now on, we will work in in ”modified” Planck units, that is we will assume a multiplicative factor \( \frac{1}{16\pi \tilde{G}} \), with \( \tilde{G} = G/f_0' \), that reduces to the standard one as soon as \( f_0' = 1 \). Immediately, we have
\[ \Lambda g_{\mu\nu} + G_{\mu\nu} - \frac{f''_0}{f'_0} \left[ \nabla_\mu \nabla_\nu R - g_{\mu\nu} \Box R + R \left( \frac{1}{4} Rg_{\mu\nu} - R_{\mu\nu} \right) \right] = 0. \]
(2.7)
The trace of above field equations gives
\[ \Box R - \frac{f'_0}{3f_0'} (R - 4\Lambda) = 0, \]
(2.8)
Obviously, setting \( f_0 = 0 \), that is discarding the \( 0^{th} \) term, is equivalent to set to zero the cosmological constant, and the trace equation becomes
\[ \Box R - \frac{f'_0}{3f_0'} R = 0. \]
(2.9)
We observe that Eqs. (2.8) and (2.9) are analogous to the Klein-Gordon equation; indeed, by assuming that the ratio \( f'_0/f''_0 \) is negative, we can define a mass
\[ m^2 \equiv - \frac{f'_0}{3f_0'} \]
(2.10)
so that they become
\[ \Box R + m^2 R = 0, \]
(2.11a)
\[ \Box R + m^2 (R - 4\Lambda) = 0. \]
(2.11b)
It follows that the curvature \( R \) can be considered formally analogous to a massive scalar field [31]. We can neglect the non-homogeneous equation as soon as the condition
\[ R \gg \Lambda \]
(2.12)
holds. Let us study now the linearized version of such a theory, to be interpreted in the context of EFT.
\section{Linearized $f(R)$ gravity}

In order to linearize the field equation (2.4) at first order in $h_{\mu\nu}$, we have to expand around the flat spacetime metric $\eta_{\mu\nu}$ \cite{32-34}. Therefore we have

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad \Rightarrow \quad ds^2 = g_{\mu\nu}dx^\mu dx^\nu = (\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu,$$

with $h_{\mu\nu}$ small ($O(h^2) \ll 1$). It is important to stress that the perturbation $h_{\mu\nu}$ is a symmetric tensor. The Ricci scalar, at the first order in metric perturbation, reads

$$R = \partial^\sigma \partial^\tau h_{\sigma\tau} - \Box h,$$

(3.2)

where $h \equiv h^\mu_{\mu}$ is the trace of $h_{\mu\nu}$ and $\Box = \partial_\sigma \partial^\sigma$ that is now the standard d’Alembert operator defined on the underlying Minkowski spacetime where gravity is assumed as a perturbation. Assuming the harmonic gauge condition\footnote{Such a condition is also called Hilbert, or De Donder or Lorentz gauge. In general, the harmonic gauge is defined in a curved background by the condition $\partial_{\nu} (g^{\mu\nu}\sqrt{-g}) = 0$. Writing $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ and expanding to linear order, the harmonic gauge reduces to the standard Lorentz gauge.}

$$\partial^\mu h_{\mu\nu} = 0$$

(3.3)

we find

$$R = -\Box h.$$

(3.4)

The fluctuation of the metric around the background represent the field mediating the gravitational interaction. Our aim is now to identify its properties by setting the corresponding field equations. Let us consider the homogeneous Klein-Gordon Eq. (2.11a). Substituting the expression for $R$ given by Eq. (3.4), we find

$$\Box (\Box h + m^2 h) = 0,$$

(3.5)

We can choose the trivial solution

$$\Box h + m^2 h = 0,$$

(3.6)

and find the condition

$$\Box h = -m^2 h.$$

(3.7)

We can also consider Eq. (2.11b). As it is well known, a general solution is the sum of the field satisfying the associated homogeneous Eq. (2.11a) plus a particular solution $R'$, that we can formally write as

$$R'(x) = 4\Lambda m^2 \int G(x, x')dx'.$$

(3.8)

Here $G(x, x')$ is a non-local Green function satisfying the field equation

$$(\Box + m^2)G(x, x') = \delta(x, x').$$

(3.9)
Only the scale $m^2$ appears in Eqs. (3.7) and (3.9), while $R'$ is suppressed by $\Lambda$. We can reasonably assume that $R'$ can be neglected with respect to the solutions of Eq. (2.11a), as far as the approximation $\Lambda \ll m^2$ holds.

Let us now rewrite the Lagrangian (2.6) in term of the perturbation. It is

$$L = \sqrt{-g} \left[ f_0 + R + \frac{f''_0}{2f'_0} R^2 \right] = \sqrt{-\det (\eta_{\mu\nu} + h_{\mu\nu})} \left( R - 2\Lambda - \frac{1}{6m^2} R^2 \right). \quad (3.10)$$

where the determinant is indicated with det. Substituting $R \rightarrow -\Box h$, we find

$$L = \sqrt{-\det (\eta_{\mu\nu} + h_{\mu\nu})} \left[ -2\Lambda - \Box h - \frac{1}{6m^2} (\Box h)^2 \right], \quad (3.11)$$

and using the condition (3.7), it becomes

$$L = \sqrt{-\det (\eta_{\mu\nu} + h_{\mu\nu})} \left[ m^2 h - 2\Lambda - \frac{m^2}{6} h^2 \right]. \quad (3.12)$$

Working out the square root up to the second order in $h_{\mu\nu}^2$, we find

$$\sqrt{-\det (\eta_{\mu\nu} + h_{\mu\nu})} \simeq 1 + \frac{1}{2} h + \frac{1}{8} h^2 - \frac{1}{4} h_{\mu\nu} h^{\mu\nu}, \quad (3.13)$$

and the Lagrangian becomes

$$L = \left( 1 + \frac{1}{2} h + \frac{1}{8} h^2 - \frac{1}{4} h_{\mu\nu} h^{\mu\nu} \right) \left( -2\Lambda + m^2 h - \frac{m^2}{6} h^2 \right)$$

$$= -2\Lambda + (m^2 - \Lambda) h + \left( \frac{m^2}{3} - \frac{\Lambda}{4} \right) h^2 + \frac{1}{2} \Lambda h_{\mu\nu} h^{\mu\nu} + \frac{m^2}{24} h^3 - \frac{m^2}{48} h^4$$

$$- \frac{m^2}{4} h_{\mu\nu} h^{\mu\nu} + \frac{m^2}{24} h^2 h_{\mu\nu} h^{\mu\nu}. \quad (3.14)$$

By truncating up to the second order in $h$, we get

$$L = -2\Lambda + (m^2 - \Lambda) h + \left( \frac{m^2}{3} - \frac{\Lambda}{4} \right) h^2 + \frac{1}{2} \Lambda h_{\mu\nu} h^{\mu\nu} \quad (3.15)$$

This is the expected Lagrangian that describe a spin-0 massive particle and a spin-2 massive particle. The term proportional to $h$ does not affect the calculation of perturbative observables, since it is linear in the creation and destruction operators and vanishes when it is inserted between vacuum states. Let us recall that a massive spin-$s$ particles has $2s + 1$ degree of freedom, while a massless particle with a quantum number $s$ has only two degree of freedom if $s > 0$. If $s = 0$, there is only one degree of freedom. Such a result is fully in agreement with a Riemann theorem stating that a gravitational field in 4 dimension has 6 degrees of freedom [21, 22]. In our case, the multiplet is given by the 5 states related to $s = 2$ and the singlet related to $s = 0$.

\footnote{We expand $\sqrt{-\det (\eta_{\mu\nu} + h_{\mu\nu})}$ at the second order in $h_{\mu\nu}$, in agreement with the order of expansion of $f(R)$ in $R$.}
4 The effective field Lagrangian

Eqs. (3.10) and (3.15) can be considered as effective Lagrangians written in different variables. In [30], the EFT is used to select the low energy modes, that are those of the GR, and contributions from quantum physics are analyzed. As in Eq. (3.10), the gravitational action is chosen proportional to powers of the curvature $R$, but the motivation of this choice is the physical smallness of $R$, motivation that we consider somewhat arbitrary. Instead, the expansion in $R$ comes out naturally from the ETG, as well as fixed and verifiable relations between the coefficients of the effective Lagrangian.

Let us compare the effective Lagrangian from linearized $f(R)$ gravity Eq. (3.15) with a Lagrangian derived on purely phenomenological ground. The free part of the Lagrangian for a massless spin-2 field can be written as

$$L_0 = \frac{1}{2} \partial^\lambda (h_{\lambda\mu} + h_{\mu\lambda}) \partial^\mu h - \frac{1}{4} \partial^\lambda (h_{\lambda\mu} + h_{\mu\lambda}) \partial_\nu (h^{\mu\nu} + h^{\nu\mu}) + \frac{1}{8} \partial_\lambda (h_{\mu\nu} + h_{\nu\mu}) \partial^\lambda (h^{\mu\nu} + h^{\nu\mu}) - \frac{1}{2} \partial_\lambda h \partial^\lambda h.$$  \hfill (4.1)

This form is derived on the basis of Lorentz invariance and gauge transformations as

$$h_{\mu\nu} \rightarrow h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \eta_\mu$$  \hfill (4.2)

where $\xi_\nu$ and $\eta_\nu$ are eight arbitrary functions. Generic mass terms can be added

$$L_m = -a_1 h^2 - a_2 h_{\mu\nu} h^{\mu\nu} - a_3 h_{\mu\nu} h^{\nu\mu}$$  \hfill (4.3)

with $a_1$, $a_2$ and $a_3$ being arbitrary coefficients. In our case, $h_{\mu\nu}$ is symmetric, therefore the second and the third term coincide, which is equivalent to set, for instance, $a_3 = 0$. The Lagrangian $L_0 + L_m$ describes an effective theory with two massive particles of spin-zero and spin-two, as the Lagrangian in Eq. (3.15). It has been demonstrated that, when $a_2 \neq a_3$, the condition of null divergence of $h$ is not generally respected by the scalar field, resulting in negative energy, or indefinite metric, which are not physically acceptable [2]. In order to recover null divergence, the coefficients and the masses of Eq. (4.3) need to respect fixed relations among them.

It is not obvious to build sensible descriptions of the gravitational interaction with this characteristic. A standard way is to use the Einstein-Hilbert action for the massless gravitational field together with mass terms respecting the symmetries leading to correct Ward identities. In [2], it is observed that such mass terms do not respect the relations necessary to a physically acceptable Lagrangian $L_0 + L_m$, and we are forced to conclude that this description of massive gravity is not satisfactory.

In our case, the effective Lagrangian (3.15) evades the condition $a_2 \neq a_3$ assumed in [2]. In fact, the Lagrangian contains, at leading order, only terms proportional to powers of $h$, which correspond to $a_2 = a_3 = 0$. Additional contributions are suppressed by $\Lambda$, in the limit $\Lambda \ll m^2$, which is the same limit where dynamics of the scalar is described by the physical Klein Gordon equation (3.7). In other words, we can say that starting from an analytical $f(R)$ gravity model, it is quite natural to recover an EFT where massive gravitons emerge as scalar or tensor modes.
5 Conclusions

The issue of the consistency of a field theory for massive gravitons can be settled by extending the Einstein gravity through generic functions of curvature invariants. The minimal extension is $f(R)$ gravity where the standard Hilbert-Einstein action, linear in the Ricci scalar $R$ is substituted by a generic function. From a dynamical viewpoint, this means that further degrees of freedom of gravitational field have to be taken into account and the possibility of massive gravitons naturally comes out.

In this paper, we have confronted the principal features of the Lagrangian resulting from the linearization of $f(R)$ gravity with the ones of the effective phenomenological Lagrangian previously discussed in [2]. The main result is that it is possible to obtain massive terms which indeed emerge naturally if one breaks spontaneously the diffeomorphism invariance of GR, and, in this case, for a certain interval of parameters, it is possible to evade ghosts and discontinuities.

Furthermore, it is possible to identify a natural mass scale $m$ directly related to the expansion parameters of the theory. This fact could avoid to fix by hand the graviton mass since it comes directly from the structure of the theory and then fixing upper limits (or mass ranges) by experimental constraints (see e.g. [34]). Finally, in the limit $m \gg \Lambda$ (or the less restrictive one $m^2 \gg \Lambda$), the theory results naturally regularized and the massive scalar satisfies a physically acceptable Klein Gordon equation. In a forthcoming paper, we will extend this approach to more general theories involving also the other curvature invariants. Quantitative constraints to the massive modes resulting from the present analysis will be included.

Acknowledgments

SC and MDL acknowledge partial support of INFN Sez. di Napoli (iniziativa specifica QGSKY and TEONGRAV). MDL is supported by MIUR (PRIN 2009). GR acknowledges partial support by MIUR under project 2010YJ2NYW. GR and MP acknowledge partial support by INFN Sez. di Napoli (iniziativa specifica RM21).

References

[1] M. Fierz and W. Pauli, “On Relativistic Wave Equations For Particles Of Arbitrary Spin In An Electromagnetic Field”, Proc. Roy. Soc. Lond. A 173, 211 (1939).

[2] H. van Dam and M. J. G. Veltman, “Massive And Massless Yang-Mills And Gravitational Fields”, Nucl. Phys. B 22, 397 (1970).

[3] V. I. Zakharov, “Linearized Gravitation Theory and the Graviton Mass”, JETP Lett. 12, 312 (1970)

[4] A. I. Vainshtein, “To The Problem Of Nonvanishing Gravitation Mass”, Phys. Lett. B 39, 393 (1972).

[5] D. G. Boulware and S. Deser, “Can Gravitation Have A Finite Range?”, Phys. Rev. D 6, 3368 (1972).
[6] N. Arkani-Hamed, H. Georgi and M. D. Schwartz, “Effective field theory for massive gravitons and gravity in theory space”, Annals Phys. 305, 96 (2003) [arXiv:hep-th/0210184].

[7] V. A. Rubakov and P. G. Tinyakov, “Infrared-modified gravities and massive gravitons”, Phys. Usp. 51, 759 (2008) [arXiv:0802.4379 [hep-th]].

[8] H. W. Lee, K.Y. Kim, and Y. S. Myung, Mod. Phys. Lett. A 27, 1250146 (2012).

[9] L. Bellagamba, R. Casadio, R. Di Sipio, V. Viventi, Eur.Phys.J. C 72, 1957 (2012).

[10] Smolin, L. 2003, arXiv:hep-th/0303185

[11] Kiefer, C. 2006, Annalen der Physik, 15, 129

[12] Will, C. 2006, Living Reviews in Relativity, 9, 3

[13] Gupta, S. N. 1952, Proceedings of the Physical Society A, 65, 161

[14] Visser, M. 1998, General Relativity and Gravitation, 30, 1717

[15] Defayet, C., Dvali, G., Gabadadze, G., & Vainshtein, A. 2002, PRD, 65, 044026

[16] Dvali, G., Gabadadze, G., & Porrati, M. 2000, Physics Letters B, 485, 208

[17] S. Capozziello, M. De Laurentis, Phys. Rept. 509, 167 (2011).

[18] S. Nojiri, S.D. Odintsov, Phys. Rept. 505, 59 (2011).

[19] S. Capozziello, M. Francaviglia, Gen. Rel. Grav. 40, 357, (2008).

[20] A. De Felice, Tsujikawa, Living Rev.Rel. 13, 3 (2010).

[21] S. Capozziello, G. Basini, M. De Laurentis, Eur. Phys. Jou. C 71, 1679, (2011).

[22] G. F. B. Riemann, Über die Hypothesen, welche der Geometrie zu Grunde liegen , Abhand. K. Ges. Wiss. Gottingen, 13, 133 (1868); English translation by Clifford, W. K. Nature 8, 14, (1873); reprinted and edited by Weyl, H., Springer, Berlin, 1920. Included in its Gesammelte Mathematische Werke, wissenschaftlicher Nachlass und Nachträge, eds. Weber, H., Dedekind, R., Teubner, B. G., Leipzig, (1892); 2d ed. Dover Publ., New York.

[23] C. Bogdanos, S. Capozziello, M. De Laurentis, S. Nesseris, Astrop. Phys. 34, 236, (2010).

[24] M. G. Hare, Canadian Journal of Physics, 51, 431 (1973).

[25] Goldhaber, A. S., & Nieto, M. M. 1974, PRD, 9, 1119

[26] Finn, L. S., & Sutton, P. J. 2002, PRD, 65, 044022

[27] M. De Laurentis, I. De Martino Mon. Not. Roy. Astr. Soc. 431, 741 (2013).

[28] J. F. Donoghue, gr-qc/9512024.

[29] C. P. Burgess, Living Rev. Rel. 7 (2004) 5 [gr-qc/0311082].

[30] J. F. Donoghue, AIP Conf. Proc. 1483 (2012) 73 [arXiv:1209.3511 [gr-qc]].

[31] A.A. Starobinsky, Phys. Lett. B 91, 99 (1980).

[32] M. Maggiore, Gravitational Waves: Theory and Experiments (Oxford Univ. Press, Oxford, 2007).

[33] M. De Laurentis, The Open Astronomy Journal 4, 1874 (2011).

[34] M. De Laurentis and S. Capozziello, Astropart. Phys. 35, 257 (2011).