Thermoeconomical analysis of a non-endoreversible Novikov power plant model under different regimes of performance

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Abstract. In this work, we study the thermoeconomics of a non-endoreversible simplified thermal power plant model, the so-called Novikov engine. Our study is made by means of the maximization of objective functions defined by the quotient of the characteristic functions (power output, efficient power and ecological function) and the total costs considered in the performance of the power plant. In our study three different costs are considered: a capital cost that is proportional to the investment and, therefore, to the size of the plant, a fuel cost that is proportional to the fuel consumption and a cost associated to maintenance of the power plant; that is, proportional to the power output of the plant. It is shown that under ecological conditions the plant dramatically reduces the amount of heat rejected to the environment, and a loss of profits is translated in an usage of fuels that dramatically reduces the heat rejected towards the environment in comparison to that obtained by means of maximum power regime.

1. Introduction

In 1995 [1], Alexis De Vos introduced a thermoeconomical analysis of an irreversible heat engine model, the so-called Novikov plant model (see figure 1) [2], in terms of the maximization of an objective function defined by the ratio of the power output and the running costs of the plant exploitation. De Vos [1] assumed that the running costs of the plant consist of two parts: a capital cost that is proportional to the investment and, therefore, to the size of the plant and a fuel cost that is proportional to the fuel consumption. De Vos [1] showed how the optimal efficiency (in the economical sense) smoothly increases from the maximum-power point (Curzon-Ahlborn efficiency [3]) corresponding to energy sources where the investment is the preponderant cost up to the Carnot value (Carnot efficiency), that is, for energy sources where the fuel is the predominant cost when the heat fluxes in the Novikov model are given by a linear Newtonian heat transfer law, that is, $\eta_{MP} < \eta_{opt} < \eta_{C}$, where the subscripts $MP$, $opt$ and $C$ mean maximum power, optimal efficiency and Carnot respectively. De Vos [1] found for the Novikov’s model that $\eta_{opt}$ is given by:
\[ \eta_{\text{opt}}(\tau, f) = 1 - \frac{f}{2} \tau - \frac{\sqrt{4(1-f)\tau + f^2\tau^2}}{2}, \]

where \( f \) is the fractional fuel cost (see Table 1), which is defined as the ratio of the cost of the fuel consumption and the running costs of the power plant; \( \tau = T_L/T_H \) with \( T_H \) and \( T_L \) the temperatures of the hot and cold thermal reservoirs respectively (see Figure 1). Later, Barranco-Jiménez and Angulo-Brown [4, 5] also studied the Novikov engine model following the thermoeconomical approach used by De Vos, but by means of the so-called ecological optimization criterion. The ecological optimization criterion consist in maximizing the well known ecological function \[ E = W - \epsilon T_L \sigma, \]

where \( W \) is the plant’s power output, \( \epsilon \) is a parameter depending of the heat transfer law [7] and \( \sigma \) is the total entropy production of the endoreversible power plant model. Barranco-Jiménez and Angulo-Brown [4, 5] also obtained a thermoeconomical efficiency \( \eta_E^{\text{opt}} \) between \( \eta_M^P \) and \( \eta_C \) that drastically reduces the entropy production of the engine. Besides the optimal efficiency satisfy the following inequality \( \eta_M^P < \eta_{\text{opt}} < \eta_E^{\text{opt}} < \eta_C \).

| Fuel          | \( f \) (%) |
|---------------|-------------|
| Renewable     | 0           |
| Uranium       | 25          |
| Coal          | 35          |
| Gas           | 50          |

Table 1. Relative fuel cost (in percent) for various energy sources

In this work, we extend the thermoeconomical analysis following the De Vos’s approach but maximizing three benefit functions defined by the ratio of the characteristic functions (Power Output [1], Efficient Power [8, 9] and Ecological Function [6]) and the total costs involved in the performance of the plants, that is: \( F_M^P = \frac{W}{C_i}, F_E^P = \frac{\eta W}{C_i} \) and \( F_{ME} = \frac{W - \epsilon T_L \sigma}{C_i} \). Besides in the maximization of the benefit functions, the total costs \( C_i \) are taken as \( C_i = aQ_{\text{max}} + bQ_H + cW \), where the proportionality constants \( a, b \) and \( c \) have units of \$/Joule (\$/currency), the last term, is a cost associated to maintenance of the power plant that is proportional to the power output of the plant [10], and \( Q_{\text{max}} = g(T_H - T_L) \) is the maximum heat that can be extracted from the heat reservoir without supplying work [1].

2. Thermoeconomical optimization under different regimes of performance for a non-endoreversible Novikov plant model

Applying the first law of thermodynamics to Fig. 1, the power output is given by,

\[ W = Q_H - Q_L, \]

Figure 1. Novikov’s model for a thermal power plant.
where \(Q_H\) and \(Q_L\) are the heat transfer supplied by the hot source to working fluid and the heat transfer from the working fluid to cold source respectively. On the other hand, the internal efficiency of the engine is given by [5],

\[
\eta = \frac{W}{Q_H} = 1 - \frac{\tau}{R \theta},
\]

(3)

where \(\theta = T_i/T_i\) and the parameter \(R = \Delta S_i/|\Delta S_2|\) is the non-endoreversibility parameter [11-13] (which characterizes the degree of internal irreversibility that comes from the Clausius inequality) \(\Delta S_i\) being the change of the internal entropy along the hot isothermal branch and \(\Delta S_2\) the entropy change corresponding to the cold isothermal compression. This parameter in principle is within the interval \(0 < R \leq 1\) (\(R = 1\) for the endoreversible limit, see [11]). If we consider that the heat transfer between the hot source and the working fluid obey a Newton heat transfer law, and applying the first and second laws of thermodynamics we can write the benefit functions defined in the previous section as [10],

\[
a F_{MP} = \frac{\left(1 - \frac{\tau}{R \theta}\right)(1-\theta)}{(1-\tau) + \beta(1-\theta) + \gamma \left(1 - \frac{\tau}{R \theta}\right)(1-\theta)},
\]

(4)

\[
a F_{EP} = \frac{\left(1 - \frac{\tau}{R \theta}\right)^2 (1-\theta)}{(1-\tau) + \beta(1-\theta) + \gamma \left(1 - \frac{\tau}{R \theta}\right)(1-\theta)},
\]

(5)

\[
a F_{ME} = \frac{\left[(1+\varepsilon) \left(1 - \frac{\tau}{R \theta}\right) - \varepsilon(1-\tau)\right](1-\theta)}{(1-\tau) + \beta(1-\theta) + \gamma \left(1 - \frac{\tau}{R \theta}\right)(1-\theta)},
\]

(6)

where \(\beta = b/a\) and \(\gamma = c/a\). Fig. 2 shows that exists an optimal efficiency value which depends on the parameter \(R\) and the optimum value of \(\theta\). We can also observe that the benefit diminishes as the internal irreversibilities (parameter \(R\)) increase. Besides, for the case of maximum power conditions, the optimum value of \(\theta\) does not change the maximum of benefits when the maintenance costs (parameter \(\gamma\), dashed lines) are present in the optimization. Therefore, taking the derivatives of \(a F_{MP}\), \(a F_{EP}\) and \(a F_{ME}\) with respect to \(\theta\) and setting them equal to zero we obtain the optimum working fluid temperature \((\theta' = T_i/T_i)\) for each value that maximize equations (4), (5) and (6) respectively, and by using equation (3), the optimal efficiencies are obtained at maximum power output, efficient power and ecological function conditions respectively:

\[
\eta_{MP}(\tau, f, R) = 1 - \frac{f}{2R} - \frac{\sqrt{4(1-f)R\tau + f^2\tau^2}}{2R},
\]

(7)

\[
\eta_{EP}(\tau, f, \gamma = 0, R) = 1 - \frac{(1+f)}{4R} - \frac{\sqrt{8(1-f)R\tau + (1+f)^2\tau^2}}{4R},
\]

(8)
\[
\eta_{ME}(\tau, f, \gamma, R) = 1 - \frac{2\tau(f - 1)^{\gamma - 1}R + (R - \gamma - 1)\sqrt{\tau}}{Rf\tau + 2\gamma(1 - f)\sqrt{\tau} + f\sqrt{\tau} + (f + \sqrt{\tau})[4(f - 1)((f - 1)\gamma - 1)R] + \Lambda(\tau, f, \gamma, R)}
\]  

(9)

where \( \Lambda(\tau, f, \gamma, R) = [4(R - \gamma) + f(f - 4R + 4\gamma(2 - f))]\sqrt{\tau} + f^2\tau \). In figure 3 we show the behavior of the optimal efficiencies (equations (7), (8) and (9)). Figure 3 also shows how the optimal efficiencies smoothly vary from the value \( f = 0 \), corresponding to energy sources where the investment is the preponderant cost up to the Carnot value for \( f = 1 \), that is, for energy sources where the fuel is the predominant cost, in an analogous way to De Vos-efficiency [1].

**Figure 2.** Comparison between the three thermoeconomic objective functions with respect to the reduced temperature \( \theta \).

**Figure 3.** Optimal efficiencies for ecological regime, efficient power and maximum power output conditions.

3. Environmental impact

Applying the first and second laws of thermodynamics to the engine model shown in figure 1, we obtain the expression for the heat rejected \( Q_L \) to the environment by the power plant given by [10],

\[
Q_L(R, \tau, \theta^*) = gT_{H}\left[\frac{\tau}{R \theta^*}\right](1 - \theta^*). 
\]  

(10)

From equation (10) we can calculate the heat rejected to the environment for each value of \( f \) and under different ways of operation of the power plant. In Fig. 4, it can be seen how the heat rejected under ecological function conditions is lower than the heat rejected under both maximum power output and maximum efficient power conditions. In Fig. 5 we show the ratio between the amounts of rejected heat considering the case when \( \gamma \neq 0 \) (red dashed line).
We also observe in figure 5 that for the case of \( \gamma = 0 \) (blue line), we obtain the result recently reported in [5] and previously reported by Velasco et al [14]. In addition, we can calculate the total entropy production and we also analyze the effect on the reduction of power output in terms of an internal irreversibility for the Novikov model for both ecological function and maximum power conditions [10].

### 3. Concluding Remarks

In this work, we have made a thermoeconomic study of a non-endoreversible simplified thermal power plant model (the so-called Novikov engine). This non-endoreversible case improves the results obtained by means of an endorreversible model due to the inclusion of the engine’s internal dissipation through the lumped parameter \( R \). In our study, we also take into account a cost associated to maintenance of the power plant; that is, proportional to the power output of the plant.

In our study we consider different regimes of performance: Maximum Power Output, Maximum Efficient Power and Maximum Ecological Regime. We found that when the Novikov model maximizes the ecological function, it reduces the rejected heat to the environment down to about 55\% (see figure 5) of the rejected heat in the case of a power plant model working under maximum power conditions. Besides, we analyze the effect on the reduction of power output in terms of an internal irreversibility.

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