LIFETIMES OF HEAVY HADRONS*

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We review current status of theoretical predictions of lifetimes of heavy hadrons in heavy-quark expansion. We present a calculation of subleading $1/m_b$ corrections to spectator effects in the ratios of beauty hadron lifetimes. We find that these effects are sizable and should be taken into account in systematic analyses of heavy hadron lifetimes. In particular, the inclusion of $1/m_b$ corrections brings into agreement the theoretical predictions and experimental observations of the ratio of lifetimes of $\Lambda_b$-baryon and $B_d$ meson. We obtain $\tau(B_u)/\tau(B_d) = 1.09 \pm 0.03$, $\tau(B_s)/\tau(B_d) = 1.00 \pm 0.01$, $\tau(\Lambda_b)/\tau(B_d) = 0.87 \pm 0.05$.

1. Introduction

The hierarchy of lifetimes of heavy hadrons can be understood in the heavy-quark expansion (HQE), which makes use of the disparity of scales present in the decays of hadrons containing $b$-quarks. HQE predicts the ratios of lifetimes of beauty mesons\textsuperscript{1,2,3}, which agree with the experimental observations well within experimental and theoretical uncertainties. Most recent experimental analyses give\textsuperscript{4,5}

\begin{align*}
\tau(B_u)/\tau(B_d)|_{\text{ex}} & = 1.085 \pm 0.017, \\
\tau(B_s)/\tau(B_d)|_{\text{ex}} & = 0.951 \pm 0.038, \\
\tau(\Lambda_b)/\tau(B_d)|_{\text{ex}} & = 0.797 \pm 0.053.
\end{align*}

The most recent theoretical predictions show evidence of excellent agreement of theoretical and experimental results\textsuperscript{6,7}. This agreement also provides us with some confidence that quark-hadron duality, which states that

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smeared partonic amplitudes can be replaced by the hadronic ones, is expected to hold in inclusive decays of heavy flavors. It should be pointed out that the low experimental value of the ratio $\tau(Λ_b)/\tau(B_d)$ has long been a puzzle for the theory. Only recent next-to-leading order (NLO) calculations of perturbative QCD and $1/m_b$ corrections to spectator effects significantly reduced this discrepancy. Of course, the problem of $\tau(Λ_b)/\tau(B_d)$ ratio could reappear again if future measurements at Fermilab and CERN would find the mean value to stay the same with error bars shrinking. Upcoming Fermilab measurements of $Λ_b$ lifetime could shed more light on the experimental side of this issue.

This talk reports on the calculation of subleading contributions to spectator effects in the $1/m_b$ expansion to study their impact on the ratios of lifetimes of heavy mesons. We also discuss the convergence of the $1/m_b$ expansion in the analysis of spectator effects.

### 2. Formalism

The inclusive decay rate of a heavy hadron $H_b$ is most conveniently computed by employing the optical theorem to relate the decay width to the imaginary part of the forward matrix element of the transition operator:

$$\Gamma(H_b) = \frac{1}{2M_{H_b}} \langle H_b|T|H_b\rangle, \quad T = \text{Im} \int d^4x T \{ H_{\text{eff}}(x)H_{\text{eff}}(0) \}.$$  

Here $H_{\text{eff}}$ represents an effective $\Delta B = 1$ Hamiltonian,

$$H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{cb} \sum_{d',d,s,u,u'} V_{u'\mu}^* C_1(\mu) Q_{1}^{u'\mu} (\mu) + C_2(\mu) Q_{2}^{u'\mu} (\mu) + h.c.,$$

where the four-quark operators $Q_1$ and $Q_2$ are given by

$$Q_1^{u'\mu} = \bar{d}_L^\gamma \gamma_{\mu} u_L', \quad Q_2^{u'\mu} = \bar{c}_L \gamma_{\mu} u_L'. \quad (3)$$

In other words, the calculation of $\Gamma(H_b)$ is equivalent to computing the matching coefficients of the effective $\Delta B = 0$ Lagrangian with subsequent

$$\Gamma(H_b) = \frac{1}{2M_{H_b}} \sum_k \langle H_b|T_k|H_b\rangle = \sum_k \frac{C_k(\mu)}{m_{b}^k} \langle H_b|O_{\Delta B=0}^B(\mu)|H_b\rangle. \quad (5)$$

In other words, the calculation of $\Gamma(H_b)$ is equivalent to computing the matching coefficients of the effective $\Delta B = 0$ Lagrangian with subsequent
computation of its matrix elements. Indeed, at the end, the scale dependence of the Wilson coefficients in Eq. (5) should match the scale dependence of the computed matrix elements.

It is customary to make predictions for the ratios of lifetimes (widths), as many theoretical uncertainties cancel out in the ratio. In addition, since the differences of lifetimes should come from the differences in the “brown mucks” of heavy hadrons, at the leading order in HQE all beauty hadrons with light spectators have the same lifetime. Thus, all ratios converge to unity in the heavy quark limit.

The difference between meson and baryon lifetimes first occurs at order $1/m^2$ and is essentially due to the different structure of mesons and baryons. In other words, no lifetime difference is induced among the members of the meson multiplet at this order. The ratio of heavy meson and baryon lifetimes receives a shift which amounts to at most $1-2\%$, not sufficient to explain the observed pattern of lifetimes\(^2\).

The main effect appears at the $1/m^3$ level and comes from the set of dimension-six four-quark operators, whose contribution is enhanced due to the phase-space factor $16\pi^2$. They are thus capable of inducing corrections of order $16\pi^2(\Lambda_{QCD}/m_b)^3 = O(5-10\%)$. These operators, which are commonly called Weak Annihilation (WA) and Pauli Interference (PI), introduce a difference in lifetimes for both heavy mesons and baryons. Their effects have been computed\(^2,8\) at leading order in perturbative QCD, and, more recently, including NLO perturbative QCD corrections\(^6\) and $1/m_b$ corrections\(^7\). The contribution of these operators to the lifetime ratios are governed by the matrix elements of $\Delta B = 0$ four-fermion operators

$$T_{\text{spec}} = T_{\text{spec}}^u + T_{\text{spec}}^{d'} + T_{\text{spec}}^s'$$

where the $T_i$ are

$$T_{\text{spec}}^u = \frac{G_F^2 m_b^2 |V_{bc}|^2 (1 - z)^2}{4\pi} \left\{ \left( c_1^2 + c_2^2 \right) O_1^u + 2c_1 c_2 \tilde{O}_1^u + \delta_1^{u/m} \right\},$$

$$T_{\text{spec}}^{d'} = \frac{G_F^2 m_b^2 |V_{bc}|^2 (1 - z)^2}{4\pi} \left\{ c_1^2 \left[ (1 + z)O_1^{d'} + \frac{2}{3} (1 + 2z)O_2^{d'} \right] 
+ (N_c c_2^2 + 2c_1 c_2) \left[ (1 + z)\tilde{O}_1^{d'} + \frac{2}{3} (1 + 2z)\tilde{O}_2^{d'} \right] + \delta_1^{d'/m} \right\},$$

$$T_{\text{spec}}^s' = \frac{G_F^2 m_b^2 |V_{bc}|^2 \sqrt{1 - 4z}}{4\pi} \left\{ c_1^2 \left[ O_1^{s'} + \frac{2}{3} (1 + 2z)O_2^{s'} \right] 
+ (N_c c_2^2 + 2c_1 c_2) \left[ \tilde{O}_1^{s'} + \frac{2}{3} (1 + 2z)\tilde{O}_2^{s'} \right] + \delta_1^{s'/m} \right\}.$$
Here the terms $\delta_i^{1/m}$ refer to $1/m_b$ corrections to spectator effects, which we discuss below. Note that we include the full $z = m_2^2/m_1^2$ dependence, which is fully consistent only after the inclusion of higher $1/m_b$ corrections. The operators $O_i$ and $\tilde{O}_i$ in Eq. (7) are defined as

$$O_1^q = \bar{b}_i \gamma^\mu (1 - \gamma_5) b_j q_j \gamma_\mu (1 - \gamma_5) \bar{q}_j, \quad O_2^q = \bar{b}_i \gamma^\mu \gamma_5 b_j q_j \gamma_\mu (1 - \gamma_5) \bar{q}_j,$$

$$\tilde{O}_1^q = \bar{b}_i \gamma^\mu (1 - \gamma_5) b_j q_j \gamma_\mu (1 - \gamma_5) \bar{q}_j, \quad \tilde{O}_2^q = \bar{b}_i \gamma^\mu \gamma_5 b_j q_j \gamma_\mu (1 - \gamma_5) \bar{q}_j. \quad (8)$$

In order to assess the impact of these and other operators, parameterizations of their matrix elements must be introduced. The meson matrix elements are

$$\langle B_q | O_1^q | B_q \rangle = f_{B_q} m_{B_q}^2 (2\epsilon_1 + B_1) \frac{B_1}{N_c}, \quad \langle B_q | \tilde{O}_1^q | B_q \rangle = f_{B_q} m_{B_q}^2 B_1,$$

$$\langle B_q | O_2^q | B_q \rangle = -f_{B_q}^2 m_{B_q}^2 \left[ \frac{m_{B_q}^2}{(m_b + m_q)} \right] 2 \epsilon_2 + \frac{B_2}{N_c} \left( \frac{B_1}{N_c} \right) + \frac{1}{2} \left( \frac{2\epsilon_1 + B_1}{N_c} \right), \quad (9)$$

$$\langle B_q | \tilde{O}_2^q | B_q \rangle = -f_{B_q}^2 m_{B_q}^2 \left[ \frac{m_{B_q}^2}{(m_b + m_q)} \right] \frac{B_2}{N_c} + \frac{1}{2} B_1.$$

Here the parameters $B_i$ and $\epsilon_i$ are usually referred to as “singlet” and “octet” bag parameters. Expressed in terms of these parameters, the lifetime ratios of heavy mesons can be written as

$$\tau(B_u)/\tau(B_d) = 1 + 16\pi^2 \frac{f_{B_q}^2 m_B}{m_{B_q}^2 c_3(m_b)} |G_{1s}^o(m_b)B_1(m_b) + G_{1s}^o(m_b)\epsilon_1(m_b) + G_{2s}^o(m_b)B_2(m_b) + G_{1o}^o(m_b)\epsilon_2(m_b)| + \delta_{1/m}, \quad (10)$$

where the coefficients $G$ were computed at NLO. $\delta_{1/m}$ represents spectator corrections of order $1/m_b$ and higher, which we discuss below.

Calculations of the matrix elements of four-fermion operators in baryon decays are not straightforward. Similar to the meson matrix elements above, we relate them to the value of $\Lambda_b$-baryon wave function at the origin,

$$\langle \Lambda_b | O_1^q | \Lambda_b \rangle = -\tilde{B} \langle \Lambda_b | \tilde{O}_1^q | \Lambda_b \rangle = \frac{B}{6} f_{B_q}^2 m_{B_q} m_{\Lambda_b} r,$$

$$\langle \Lambda_b | O_2^q | \Lambda_b \rangle = -\tilde{B} \langle \Lambda_b | \tilde{O}_2^q | \Lambda_b \rangle = \frac{B}{6} f_{B_q}^2 m_{B_q} m_{\Lambda_b} \delta. \quad (11)$$

Here the parameter $\tilde{B}$ accounts for the deviation of the $\Lambda_b$ wave function from being totally color-asymmetric ($\tilde{B} = 1$ in the valence approximation), and the parameter $r = \left| \psi_{bq}^{\Lambda_b}(0) \right|^2 / \left| \psi_{bq}^{B_q}(0) \right|^2$ is the ratio of the wave functions at the origin of the $\Lambda_b$ and $B_q$ mesons. Note that $\delta = O(1/m_b)$, which
follows from the heavy-quark spin symmetry. It needs to be included as we consider higher-order corrections in $1/m_b$. While these parameters have not been computed model-independently, various quark-model arguments suggest that the meson and baryon matrix elements are quite different. Thus a meson-baryon lifetime difference can be produced. In general, one can parametrize the meson-baryon lifetime ratio as

$$\tau(\Lambda_b)/\tau(B_d) \simeq 0.98 - (d_1 + d_2 \bar{B}) r - (d_3 \epsilon_1 + d_4 \epsilon_2) - (d_5 B_1 + d_6 B_2)$$

$$\simeq 0.98 - m_b^2 (d'_1 + d'_2 \bar{B}) r - m_b^2 [(d'_3 \epsilon_1 + d'_4 \epsilon_2) + (d'_5 B_1 + d'_6 B_2)], \quad (12)$$

where in the last line we scaled out the coefficient $m_b^2$ emphasizing the fact that these corrections are suppressed by $1/m_b^3$ compared to the leading $m_b^5$ effect. The scale-dependent parameters are $d_i(m_b) = \{0.023, 0.028, 0.16, -0.16, 0.08, -0.08\}$ at NLO$^2$.

It is interesting to note that in the absence of $1/m_b$ corrections to spectator effects, it would be equally correct to substitute the $b$-quark mass in Eq. (12) with the corresponding meson and baryon masses, so

$$\tau(\Lambda_b)/\tau(B_d) \simeq 0.98 - m_{\Lambda_b}^2 (d'_1 + d'_2 \bar{B}) r$$

$$- m_{B_d}^2 [(d'_3 \epsilon_1 + d'_4 \epsilon_2) + (d'_5 B_1 + d'_6 B_2)], \quad (13)$$

which reflects the fact that WS and PI effects occur for the heavy and light quarks initially bound in the $B_d$ meson and $\Lambda_b$ baryon, respectively. While correct up to the order $1/m_b^3$, these simple substitutions reduce the ratio of lifetimes by approximately 3-4%! We take this as an indication of the importance of bound-state effects on the spectator corrections, represented by subleading $1/m_b$ corrections to spectator operators.

### 3. Subleading corrections to spectator effects

We computed the higher order corrections, including charm quark-mass effects, to Eq. (7) in the heavy-quark expansion, denoted below as $\delta_{1/m}^q$.

The $1/m_b$ corrections to the spectator effects were computed$^7$ by expanding the forward scattering amplitude of Eq. (2) in the light-quark momentum and matching the result onto the four-quark operators containing derivative insertions (see Fig. 1). The resulting $\delta_{1/m}^q$ contributions can be
Figure 1. Kinetic corrections to spectator effects. The operators of Eqs. (14) are obtained by expanding the diagrams in powers of spectator’s momentum.

written in the following form:

\[
\delta_{1/m}^{u} = -2 \left( c_1^2 + c_2^2 \right) \frac{1 + z}{1 - z} R_1^{u} - 4c_1c_2 \frac{1 + z}{1 - z} R_0^{u},
\]

\[
\delta_{1/m}^{d'} = c_2^2 \left[ \frac{8z^2}{1 - z} R_0^{d'} + \frac{2}{3} \frac{1 + z + 10z^2}{1 - z} R_1^{d'} + \frac{2}{3} (1 + 2z) \left( R_2^{d'} - R_3^{d'} \right) \right],
\]

\[
+ (Nc_2^2 + 2c_1c_2) \left[ \frac{8z^2}{1 - z} R_0^{d'} + \frac{2}{3} \frac{1 + z + 10z^2}{1 - z} R_1^{d'} + \frac{2}{3} (1 + 2z) \left( R_2^{d'} - R_3^{d'} \right) \right]
\]

\[
\delta_{1/m}^{s'} = c_2^2 \left[ \frac{16z^2}{1 - 4z} R_0^{s'} + \frac{2}{3} \frac{1 - 2z + 16z^2}{1 - 4z} R_1^{s'} + \frac{2}{3} (1 + 2z) \left( R_2^{s'} - R_3^{s'} \right) \right],
\]

\[
+ (Nc_2^2 + 2c_1c_2) \left[ \frac{16z^2}{1 - 4z} R_0^{s'} + \frac{2}{3} \frac{1 - 2z + 16z^2}{1 - 4z} R_1^{s'} + \frac{2}{3} (1 + 2z) \left( R_2^{s'} - R_3^{s'} \right) \right].
\]

where the following operators contribute

\[
R_0^{q} = \frac{1}{m_b^2} \bar{b}_i \gamma^\mu \gamma_5 \bar{D}^\alpha b_i q_j \gamma_\mu (1 - \gamma_5) \bar{D}_\alpha q_j,
\]

\[
R_1^{q} = \frac{1}{m_b^2} \bar{b}_i \gamma^\mu (1 - \gamma_5) \bar{D}^\alpha b_i q_j \gamma_\mu (1 - \gamma_5) \bar{D}_\alpha q_j,
\]

\[
R_2^{q} = \frac{1}{m_b^2} \bar{b}_i \gamma^\mu (1 - \gamma_5) \bar{D}^\nu b_i q_j \gamma_\nu (1 - \gamma_5) \bar{D}_\nu q_j,
\]

\[
R_3^{q} = \frac{m_q}{m_b} \bar{b}_i (1 - \gamma_5) b_i \bar{q}_j (1 - \gamma_5) q_j.
\]

Here \( \tilde{R}_i^{q} \) denote the color-rearranged operators that follow from the expres-
sions for $R^q_i$ by interchanging color indexes of $b_i$ and $q_j$ Dirac spinors. Note that the above result contains full QCD $b$-fields, thus there is no immediate power counting available for these operators. The power counting becomes manifest at the level of the matrix elements. In order to include above corrections into the prediction of lifetime ratios a calculation of meson and baryon matrix elements of the operators in Eq. (15) must be performed. We use factorization to guide our parameterizations of $\Lambda_b$ and meson matrix elements, but keep matrix elements which vanish in factorization. This is important, as the Wilson coefficients of these operators are larger than the ones multiplying the operators whose matrix elements survive in the $N_c \to \infty$ limit.

Our parameterizations for meson and baryon matrix elements can be found in\textsuperscript{7}, where it was shown that a set of $1/m_b$-corrections to spectators effects can be parametrized by eight new parameters $\beta_i$ and $\tilde{\beta}_i$ ($i = 1, \ldots, 4$) for heavy mesons and eight new parameters $\beta_{\Lambda_i}$ and $\tilde{\beta}_{\Lambda_i}$ for heavy baryons. Although model-independent values of these parameters will not be known until dedicated lattice simulations are performed, we presented an estimate of these parameters based on quark model arguments. In our numerical results we assume the value of the $b$-quark pole mass to be $m_b = 4.8 \pm 0.1$ GeV and $f_B = 200 \pm 25$ MeV, as well as lattice-inspired values of $B_i$ and $\epsilon_i$ parameters\textsuperscript{10}.

Numerically, the set of $1/m_b$ corrections does not markedly affect the ratios of meson lifetimes, changing the $\tau(B_u)/\tau(B_d)$ and $\tau(B_s)/\tau(B_d)$ ratios by less than half a percent. The effect is more pronounced in the ratio of $\Lambda_b$ and $B_d$ lifetimes, where it constitutes a $40 - 45\%$ of the leading spectator (WA plus PI) contribution, or an overall correction of about $-3\%$ to the $\tau(\Lambda_b)/\tau(B_d)$ ratio. While such a sizable effect is surprising, the main source of such a large correction can be readily identified. While the individual $1/m_b$ corrections to WS and PI are of order $20\%$, as expected from the naive power counting, they contribute to the $\Lambda_b$ lifetime with the same (negative) sign, instead of destructively interfering as do WS and PI\textsuperscript{3,7}. This conspiracy of two $\sim 20\%$ effects produces such a sizable shift in the ratio of the $\Lambda_b$ and $B$-meson lifetimes.

Since all three heavy mesons belong to the same $SU(3)$ triplet, their lifetimes are the same at order $1/m_b^2$. The computation of the ratios of heavy meson lifetimes is equivalent to the computation of $U$-spin or isospin-violating corrections. Both $1/m_b^2$-suppressed spectator effects and our corrections computed in the previous sections arise from the spectator interactions and thus provide a source of $U$-spin or isospin-symmetry
Figure 2. Histograms showing the random distributions around the central values of the $f_{B^q}$, $m_b$, $R$, $\delta$, $\epsilon$, $\beta_1$, and $\tilde{\beta}_1$ parameters contributing to $\tau(\Lambda_b)/\tau(B_d)$. Three histograms are shown for the scales $\mu = m_b/2$ (a), $\mu = m_b$ (b), and $\mu = 2m_b$ (c).

Figure 3. Same as Fig. 2 for $\tau(B_u)/\tau(B_d)$.

breaking. We shall, however, assume that the matrix elements of both $1/m_b^2$ and $1/m_b^4$ operators respect isospin. The ratio of lifetimes of $B_s$ and $B_d$ mesons involves a breaking of $U$-spin symmetry, so the matrix elements of dimension-6 operators could differ by about 30%. We shall introduce different $B^-$ and $\epsilon$-parameters to describe $B_s$ and $B_d$ lifetimes.

In order to obtain numerical estimate of the effect of $1/m_b$ corrections to spectator effects, we adopt the statistical approach for presenting our results and generate 20000-point probability distributions of the ratios of lifetimes obtained by randomly varying the parameters describing matrix elements within a $\pm 30\%$ interval around their “factorization” values for three different scales $\mu$. The decay constants $f_{B^q}$ and $b$-quark pole mass $m_b$ are taken to vary within $1\sigma$ interval indicated above. The results are presented in Figs. 2, 3, and 4. These figures represent our main result.

We also performed studies of convergence of $1/m_b$ expansion by computing a set of $1/m_b^2$-corrections to spectators effects and estimating their size in factorization. The expansion appears to be well-convergent for the $b$-flavored hadrons. Due to the relative smallness of $m_c$ (and thus vicinity
of the sector of QCD populated by the light quark resonances$^{11}$) it is not clear that the application of these findings to charmed hadrons will produce quantitative, rather than qualitative results.

4. Conclusions

We computed subleading $1/m_b$ and $1/m_b^2$ corrections to the spectator effects driving the difference in the lifetimes of heavy mesons and baryons. Thanks to the same $16\pi^2$ phase-space enhancement as $1/m_b^3$-suppressed spectator effects, these corrections constitute the most important set of $1/m_b^4$-suppressed corrections.

The main result of this talk are Figs. 2, 3, and 4, which represent the effects of subleading spectator effects on the ratios of lifetimes of heavy mesons and baryons. We see that subleading corrections to spectator effects affect the ratio of heavy meson lifetimes only modestly, at the level of a fraction of a percent. On the other hand, the effect on the $\Lambda_b$-$B_d$ lifetime ratio is quite substantial, at the level of $-3\%$. This can be explained by the partial cancellation of WS and PI effects in $\Lambda_b$ baryon and constructive interference of $1/m_b$ corrections to the spectator effects.

There is no theoretically-consistent way to translate the histograms of Figs. 2, 3, and 4 into numerical predictions for the lifetime ratios. As a useful estimate it is possible to fit the histograms to gaussian distributions and extract theoretical predictions for the mean values and deviations of the ratios of lifetimes. Predictions obtained this way should be treated with care, as it is not expected that the theoretical predictions are distributed according to the gaussian distribution. This being said, we proceed by fitting the distributions to gaussians and, correcting for the small scale uncertainty, extract the ratios $\tau(B_u)/\tau(B_d) = 1.09 \pm 0.03$, $\tau(B_s)/\tau(B_d) = 1.00 \pm 0.01$, and $\tau(\Lambda_b)/\tau(B_d) = 0.87 \pm 0.05$. This brings the experimental
and theoretical ratios of baryon and meson lifetimes into agreement.

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