Dynamic Logic of Legal Competences

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Abstract We propose a new formalization of legal competences, and in particular for the Hohfeldian categories of power and immunity, through a deontic reinterpretation of dynamic epistemic logic. We argue that this logic explicitly captures the norm-changing character of legal competences while providing a sophisticated reduction of the latter to static normative positions. The logic is completely axiomatizable, and we apply it to a concrete case in German contract law to illustrate that it can capture the distinction between legal ability and legal permissibility.

Keywords Hohfeldian Rights · Power · Immunity · Dynamic Logic · Logic and Law

The Hohfeldian typology of rights (Hohfeld 1913) distinguishes what one might call static and dynamic rights. Static rights encompass claims and privileges, as well as their respective correlatives duties and no-claim. They have also been called “normative positions” (Sergot 2001). On the dynamic side one finds legal competence.¹

¹ What we call here static and dynamic rights have been labelled in various ways in the formal literature. Kanger called static rights the “type of the states of affairs” and dynamic ones the “type of influence” (Kanger 1972). Makinson instead used the “deontic family” and the “legally capacitative family” for static and dynamic rights, respectively (Makinson 1986). Bentham, von Wright and Hart on the other hand used “legal validity” and “norm-creating action” (Lindahl 1977), while Lindahl (1977) called it “the range of action.”
power and immunity and their correlative liability and no-power. See Figure 1. In this paper we will be mainly focusing on legal competences.

| correlatives | opposites | correlatives | opposites |
|--------------|-----------|--------------|-----------|
| Claim        | Duty      | Power        | Liability |
| No-Claim     | Privilege | No-Power     | Immunity  |

Table 1 Legal Rights (Sergot 2013)

Although logical approaches to legal competences are scarcer than for static normative positions, existing theories can be divided into two broad families. The first formalizes power and immunity as (legal) permissibility, or absence thereof, to see to it that a certain normative position obtains (Kanger and Kanger 1966; Lindahl 1977). Lindahl (1977), for instance, defines j’s power to make it the case that i ought to see to it that φ as a permission that j sees to it that i ought to see to it that φ:

\[ P(Do_jO(Do_i\phi)) \]

We call such an approach reductive because it takes power and immunity as definable in the language of obligations, permissions, and actions, where claims and privileges are also defined. Non-reductive approaches, on the other hand, view power and immunity as position-changing actions (Makinson 1986; Jones and Sergot 1996; van Eijck and Ju 2016) or normative conditionals (Governatori et al. 2005) that are not reducible to static normative positions.

Both of these families have assets and drawbacks. Reductive approaches come with a rich logical theory of the relationship between static normative positions and legal competences, with the latter inheriting its logic from the former. Choosing to define power as above, however, obfuscates the changing potential of legal competence by reducing it to permissibility, a simple static legal relation. This potential for change was arguably crucial for Hohfeld who defined power as the ability to “change legal relations” (Hohfeld 1913, p. 44-45). The formalization above, in particular, furthermore conflates legal ability (rechtliches Können) with legal permissibility (rechtliches Dürfen), although these two concepts are distinct (Makinson 1986). Non-reductive approaches, on the other hand, do better justice to the dynamic character of legal competences by taking norm-changing actions or conditionals as first class citizens in the logic. This allows to distinguish legal ability and legal permissibility. The cost of this, however, is a logic of legal competences, which is, at least at the outset, independent from the logic of the static normative positions.

The terminology used by Hohfeld might not fully reflect how terms like “power”, “immunity” or “liability” have been used in all the English-speaking literature. The work following (Kanger 1972) has, however, in important parts used that terminology. In order to situate our contribution better in that tradition, we use it here as well. We thank one of the anonymous reviewers of the JLLI for pointing us to the importance of making this caveat.
The dynamic logic that we present in this paper provides a middle ground between these two types of approach. It is reductive, and as such comes with a rich set of principles of interaction between static and dynamic rights. It does so, however, while retaining both the dynamic character of legal competences and the distinction between legal ability and legal permissibility.

What we propose is a deontic re-interpretation of dynamic epistemic logic (van Ditmarsch et al. 2007; van Benthem 2011), and should be mainly seen as a contribution to that field. Indeed, the reader familiar with it will recognize both the modeling methodology and the axiomatization that we present here. What we show is that this formalism also yields interesting insights when interpreted in deontic terms, especially for theories of legal competence.

Our formalization of legal competences draws also from Markovich’s (2020) recent proposal for the analysis of powers. Like ours, her approach uses tools from dynamic epistemic logic, namely public announcement operators, to explicate Ho-hfeldian powers. Our underlying logic of static rights is furthermore taken directly from her work (Markovich 2016), as it constitutes one of the most sophisticated accounts of directed rights currently available. Our analysis, however, generalizes and complements hers in at least two ways. First, the theory we propose here uses general deontic action operators, for which public announcements are special cases. As we will see, this allows us to analyze “softer”, i.e. defeasible kinds of deontic actions. Second, our analysis of static rights is based on a logic with conditional obligation operators, in which standard operators of standard deontic logic are easily definable. Finally, our analysis addresses two points which are either implicit or only mentioned in passing in Markovich’s work, namely the problem of vacuous powers when formalized using DEL tools, and the distinction between legal permissibility and legal ability.

Another interesting recent contribution to which our proposal relates to is van Eijck and Ju (2016), which proposes a formalization of the Holfeldian categories using tools from propositional dynamic logic (Harel et al. 2000). The mathematical relationship between the two proposals needs to be investigated further. The following points should, however, be highlighted at the outset. On the one hand, van Eijck and Ju’s formalization allows for a more fine-grained description of complex actions than the language we use here, by capturing explicitly, for instance, concurrent or sequential actions as well as non-deterministic choices. They also explicitly include an epistemic component to address the question of knowledge-based obligations, which we leave out here. On the other hand they make two important simplifying assumptions. They first restrict their analysis of legal competences to abilities to change simple claim rights about atomic propositions. They furthermore explicitly leave out cases where a third party, for instance the judiciary, can change the legal relations obtaining between two other agents. As we will see below we consider these kinds of cases to be of primary importance for a logical model of legal competence.

3 The results presented in this paper build on and extend the work reported in Dong and Roy (2017). The present paper uses a more general condition for static conditional obligations, presents an improved version of the definitions of power and immunity, and a more in-depth conceptual discussion of the key points.
The rest of the paper is structured as follows. In Section 1 we present the underlying model of static rights, and then move on to dynamic modalities and legal competences in Section 2. We show how the two can be put together to capture the four Hohfeldian basic types of right, and we present a complete axiomatization. We then apply it to a concrete case in the German civil code in order to show that legal ability and legal permissibility can be naturally distinguished in this logic.

1 Static Rights

This paper is not about static rights, but rather about legal competences. Therefore, as to the former we stay as close as possible to the mainstream theory of normative positions stemming from the proposals developed by Kanger (1972); Lindahl (1977); Makinson (1986) and surveyed by Sergot (2013), but also incorporating directionality as proposed by Markovich (2016, 2020). This theory, at least in its non-directional form, has been thoroughly studied when it comes to claim rights and privileges. It has, of course, well-known drawbacks, which we inherit as well. The reader uncomfortable with this modeling choice should keep in mind that the dynamic tools that we deploy later are to a large extent modular. They may be used with different static theories. We point to some possible alternatives along the way.

The only difference between the mainstream logic for static rights, including the work of Markovich (2016, 2020), and the one we use here is that ours contains conditional duties and permissions. This is a technical decision. Conditional duties allow for a simpler integration with the dynamic part. In its deontic version these conditional dynamic rights rest on the dyadic deontic logic developed by van Benthem et al. (2014), but goes back at least to Hansson (1969).

1.1 Language

On the surface, the language we use differs from mainstream normative positions in that it contains a family of Kripke modalities on an underlying preference order, along with the universal modality and the usual “seeing to it that” modality. Again, we use this language instead of the classical deontic one for technical reasons. It facilitates the axiomatization of the dynamic modalities. It is well known, however, that this language is expressive enough to define conditional obligations and permissions (Boutilier 1994; van Benthem et al. 2006). We will come back to this at the end of the section. The logic of conditional obligation can be completely axiomatized, both in the current language (van Ditmarsch et al. 2007) or by taking only conditional obligations as primitive (Baltag and Smets 2008; Parent 2014).

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4 There have been a number of different proposals for defining “seeing to it that”. One popular family of approaches uses either one Chellas (1980) or a pair of unary modalities Kanger (1972); Lindahl (1973); Makinson (1984). All of them satisfy the T axiom as well as the E rule for substitution of logical equivalence. More recent approaches, as for instance the so-called “Chellas STIT”, use a normal, S5 modality Chellas (1992); Horty (2003).
**Definition 1** Let $\text{Prop}$ be a countable set of atomic propositions and $i, j$ be elements of a given finite set $\mathcal{I}$ of agents. The language $\mathcal{L}$ is defined as follows:

$$\varphi := p | \neg \varphi | \varphi \land \varphi | [\leq_{i \rightarrow j}] \varphi | U \varphi | Do_i \varphi$$

where $p \in \text{Prop}$.

We write $(\leq_{i \rightarrow j}) \varphi$ for $\neg [\leq_{i \rightarrow j}] \neg \varphi$, and $E \varphi$ for $\neg U \neg \varphi$. A formula $U \varphi$ is read as “it is necessary that $\varphi$.” $Do_i \varphi$ indicates a non-deontic or ontic (van Ditmarsch and Kooi 2008) action of agent $i$, and should be read in the usual sense of “$i$ sees to it that $\varphi$.”

The modality $[\leq_{i \rightarrow j}]$ will be interpreted on a comparative ideality relation. Intuitively, in its undirected version, a formula $[\leq] \varphi$ would say that $\varphi$ is true in all the worlds at least as ideal as the current one. In its directed version, the modality $[\leq_{i \rightarrow j}] \varphi$ should be read as “$\varphi$ is true in the worlds that are, as far as the obligations of $i$ towards $j$ are concerned, at least as ideal as the current one.”

This directionality is important. Hohfeldian rights, both static and dynamic, are indeed “directed” (Sartor 2005) or, in Makinson’s words, “resolutely relational” (Makinson 1986). They specify both their subjects and their addressees. The subjects are the agents who hold the right, the addressees are those who have the correlative duty. The standard formalization of static rights follows the Kangarian tradition and leaves this directionality implicit. Indeed, with the notable exception of some early work (Kanger and Kanger 1966, Herrestad and Krogh 1995), in this tradition neither the subject $i$ appears in the formalization of claim rights, nor the addressee $j$ in the formalization of privileges. Instead, when writing $O(Do_j \varphi)$ for “$i$ has a claim against $j$ regarding $\varphi$”, it is assumed that $\varphi$ is something that concerns $i$ in a relevant manner. This modeling choice was already in place in Kanger’s work (Kanger 1972), but see a recent overview by Sergot (2013). Our language for static rights follows a recent proposal by Markovich (2016, 2020), in which this directionality is made explicit in the form of a family of directed obligation operators of the form $O_{i \rightarrow j}$. In the present language, as we explain below, both the conditional and the unconditional forms of these operators are definable using the $[\leq_{i \rightarrow j}]$ modality. There are, of course, alternative proposals for capturing directed rights, notably Governatori et al. (2005)’s work. We leave a full comparison for future work.

### 1.2 Semantics and Deontic Operators

The semantics for the static part is constituted by so-called preferential models (van Benthem et al. 2010, 2014), augmented with one binary relation for each $Do_i$ operator. We do not assume that the preference ordering is connected nor conversely well-founded. This will give rise to slight differences from, e.g. van Benthem et al. (2010, 2014)’s work.

**Definition 2** Let $\text{Prop}$ and $\mathcal{I}$ be as above. A preference-action model $\mathcal{M}$ is a tuple

$$(W, \{[\leq_{i \rightarrow j}, \sim_i], i \in \mathcal{I}, V\})$$

where:

- $W$ is a non-empty set of states
- $[\leq_{i \rightarrow j}]$ is a reflexive and transitive relation on $W$
- for each $i \in \mathcal{I}$, $\sim_i$ is an equivalence relation on $W$
– \( V : Prop \rightarrow \mathcal{P}(W) \) is a valuation function

The respective preference relations are interpreted in terms of comparative ideality or rather, in the present context, of comparative legal ideality. In its undirected version, \( w \preceq u \) would be read as saying that \( u \) is, from a legal point of view, at least as ideal as \( w \). This comparative legal ideality is now restricted to the legal relation of \( i \) towards \( j \). So the interpretation of \( w \preceq_{i \rightarrow j} u \) becomes “\( u \) is, from a legal point of view, at least as ideal as \( w \), as far as \( i \)’s obligations towards \( j \) are concerned.”

Preference-action frames are preference-action models minus the valuation. We assume that the relations \( \sim_i \) are equivalence relations to simplify the treatment of static rights and thus to put the emphasis on our dynamic extension. This assumption could be removed. Sentences of \( \mathcal{L} \) are interpreted in preference-action models.

**Definition 3** The truth conditions for sentence \( \varphi \in \mathcal{L} \) are defined as follows:

\[
\begin{align*}
\mathcal{M}, w \models p & \iff w \in V(p) \\
\mathcal{M}, w \models \neg \varphi & \iff \mathcal{M}, w \not\models \varphi \\
\mathcal{M}, w \models \varphi \land \psi & \iff \mathcal{M}, w \models \varphi \text{ and } \mathcal{M}, w \models \psi \\
\mathcal{M}, w \models [\preceq_{i \rightarrow j}] \varphi & \iff \mathcal{M}, w' \models \varphi \text{ for all } w' \preceq_{i \rightarrow j} w \\
\mathcal{M}, w \models U\varphi & \iff \mathcal{M}, w' \models \varphi \text{ for all } w' \in W \\
\mathcal{M}, w \models Do_i \varphi & \iff \mathcal{M}, w' \models \varphi \text{ for all } w' \sim_i w
\end{align*}
\]

The validity of the models and frames, and the classes thereof, is defined as usual. We define \( |||\varphi||| \), the truth set of \( \varphi \), as \( \{ w : \mathcal{M}, w \models \varphi \} \).

As mentioned, directed conditional obligation, understood in terms of “truth in all the most ideal worlds” is definable in this language. In fact this language can define a stronger notion, that of conditional directed obligation. This definability argument is standard. Indeed, let \( O_{i \rightarrow j}(\psi / \varphi) \) be defined as follows:

\[
\begin{align*}
\mathcal{M}, w \models O_{i \rightarrow j}(\psi / \varphi) & \iff \\
\exists u \preceq_{i \rightarrow j} \forall v \preceq_{i \rightarrow j} \forall s \preceq_{i \rightarrow j} u(\mathcal{M}, u \models \varphi \land \forall s \models \varphi \Rightarrow \mathcal{M}, s \models \psi) \Rightarrow \\
\exists v \models \psi(\mathcal{M}, v \models \varphi \Rightarrow \mathcal{M}, v \models \psi)
\end{align*}
\]

This directed conditional obligation operator is then definable as:

\[
O_{i \rightarrow j}(\psi / \varphi) \equiv df [\preceq_{i \rightarrow j}](\varphi \rightarrow [\preceq_{i \rightarrow j}](\varphi \land [\preceq_{i \rightarrow j}](\varphi \rightarrow \psi)))
\]

The definition essentially states the truth condition of the conditional obligation. Recall that the latter says, in finite models, that all most ideal \( \varphi \)-world, as far as \( i \)’s obligations towards \( j \) are concerned, are also \( \psi \)-worlds. In the general case it says that for every \( \varphi \)-world \( v \) that is at least as ideal as the current one, there is a (possibly different) \( \varphi \)-world \( v' \) that is at least as ideal as \( v \), from which any weakly more ideal world makes \( \varphi \rightarrow \psi \) true.

\[5\text{ The complexity of the formula below is due to the fact that there might not be states that are maximal according to the relation } \preceq_{i \rightarrow j}, \text{ i.e. states that have no other states strictly above them. When such states are guaranteed to exist, for instance finite models, this definition reduces to the more familiar definition in terms of truth in all maximal states.}\]
In Hohfeldian terminology, given that duties correspond to a correlative claim, the formula $O_{i \rightarrow j}(\psi/\varphi)$ can also be read as “Given $\varphi$, $j$ has a claim against $i$ regarding $\psi$.” Unconditional directed obligations $O_{i \rightarrow j}(\varphi)$ are defined as $O_{i \rightarrow j}(\varphi/T)$, while permission is “weak permissions”, i.e. $P_{i \rightarrow j}(\varphi)$ iff $\neg O_{i \rightarrow j}(\neg \varphi)$. A routine argument shows that all these modalities are normal. They furthermore satisfy the following qualified form of the $D$ axiom.

$$\langle \leq_{i \rightarrow j} \rangle \varphi \rightarrow (O_{i \rightarrow j}(\psi/\varphi) \rightarrow \neg O_{i \rightarrow j}(\neg \psi/\varphi))$$

It states that conditional directed obligations with the same conditions are consistent unless their condition itself is not satisfiable in any world accessible from the current one.

1.3 Claims Rights and Privileges

With this in hand we have the machinery required to define claims and privileges. We do so, again, using the directed Kangerian approach put forward by Markovich (2020), which also contains an in-depth discussion of this modeling of static rights in view of Hohfeld’s original proposal and the broader legal theory context:

- Given $\psi$, agent $i$ has a claim against $j$ regarding $\varphi$: $O_{j \rightarrow i}(Do_j \varphi/\psi)$
- Given $\psi$, agent $i$ has a privilege against $j$ regarding $\varphi$: $\neg O_{i \rightarrow j}(Do_i \neg \varphi/\psi)$ or, equivalently, $P_{i \rightarrow j}(\neg Do_i \neg \varphi/\psi)$

1.4 Claims Rights and Privileges: An Example

Here is an example to illustrate these notions, to which we will return later. Ivy ($i$) can park her car in a space that requires the display of a parking permit in her windshield for monitoring by a city official. Call $d$ the fact that the permit is displayed, and $p$ the fact that the car is parked. The city ($c$) has a claim right against Ivy to display the permit given that she has parked. In our setting this is represented by the fact that the state where both $d$ and $p$ hold is strictly better, with respect to $i$ towards $c$, than the state where the car is parked but the permit is not displayed ($p, \neg d$). On the other hand, given that she hasn’t parked, the city has no such right. Rather, Ivy has the freedom to display the permit or not. This is illustrated by the double arrow between the two states to the right of Figure 1. Suppose the current state is $w_1$, where Ivy has parked her car without a permit. Now, the city has a conditional claim against Ivy to display the permit, $O_{i \rightarrow c}(Do_i d/p)$, but, on the other hand, Ivy has the privilege both to display and not to display the permit conditional on not parking her car, $\neg O_{i \rightarrow c}(Do_i d/\neg p)$ and $\neg O_{i \rightarrow c}(\neg Do_i d/\neg p)$. Note, furthermore, that Ivy has no unconditional obligation towards the city to display the permit or to park her car.

Parking without displaying the permit can of course have legal consequences for Ivy, for instance that it is obligatory that she pays a fine ($f$). The crucial point of this example, however, is that this only happens if a parking officer with the legal power to, first, witness that Ivy is parked without a permit and, second, issue a parking ticket.
Absent these two deontic actions, or more precisely this combination of deontic observation and action, the city has no claim against Ivy regarding the payment of a fine. This is illustrated here by \( f \) being false everywhere in the model.

![Diagram](image)

**Fig. 1** A static model of Ivy’s example. All arrows represent the relation \( \leq_{i,c} \).

2 Legal Competences

We now turn to our proposal for modeling legal competences. It essentially follows the so-called “event models” methodology developed by Baltag and Smets (2008) for epistemic modalities. The key idea, in this epistemic environment, is to model the structure of a particular learning event using the same tools as for an agent’s static information, that is Kripke models. The result of updating one’s knowledge or belief in the light of new information is then computed using some form of restricted product of these models. See the textbook (van Ditmarsch et al. 2007) for details.

Transposed into our deontic context, the proposal is to model explicitly the structure of deontic actions or legal competences using what we call deontic action models.

2.1 Deontic Action Models

We start with the definition of deontic action models, which are our main modeling tools for legal competences.

**Definition 4** A deontic action model \( A_i \) for agent \( i \) is a tuple

\[
\langle A, \{ \leq_{j,k} \}_{j,k \in \mathbb{Z}}, \text{Pre}, \text{Post} \rangle
\]

where:

- \( A \) is a non-empty finite set of deontic actions.
- \( \leq_{j,k} \) is a reflexive and transitive relation on \( A \)
- \( \text{Pre} : A \rightarrow \mathcal{L} \) is a precondition function.
- \( \text{Post} : A \rightarrow (\text{Prop} \rightarrow \mathcal{L}) \) is a post-condition function that assigns to each action and each atomic proposition a formula in \( \mathcal{L} \). We assume that, for all \( a \), \( \text{Post}(a) \) differs only, at most, in finitely many elements of Prop from the identity function.
A is the set of deontic actions or the bases for legal competences. The reader should be careful, however, in equating deontic actions with legal competences. There will be typically no one-to-one correspondence between the elements of a deontic action model and the legal competences of a given agent. Often, the latter will be better represented using sets of actions in A, together with the relation \( \preceq_{j \rightarrow k} \). Our running example, which we present below, is a case in point. Deontic actions are thus components of legal competences, but not equal to them.

Typically, legal competences can only be enacted in specific circumstances. A sale, for instance, requires that the seller legally owns the goods. These provisions are modeled by the preconditions function \( \text{Pre} \). It specifies for each action \( a \) the conditions in the underlying static models that need to obtain for \( a \) to be executable in the first place.

The key to our modeling of deontic or legal dynamics is the relation \( \preceq_{j \rightarrow k} \) and the post-condition function \( \text{Post} \). The first encodes \( i \)'s potential for changing the ideality ordering concerning \( j \)'s obligations towards \( k \), in states satisfying certain preconditions. It thus captures, so to speak, the deontic potential or deontic effectivity of \( i \)'s actions in \( A \) on the different directed ideality relation. The second captures the legal ability to render certain legal facts true (Herzig et al. 2011). For instance, in our example, issuing a parking ticket thereby makes it obligatory that Ivy pays a fine. The situation here involves both deontic changes on the legal potential as well as on the truth of legal facts.

Note that if we were to adopt the same strategy as in dynamic epistemic logic to interpret the relations \( \preceq_{j \rightarrow k} \), we would read \( a_1 \preceq_{j \rightarrow k} a_2 \) in terms of comparative deontic or legal ideality. That is we would say that \( a_2 \) is at least as ideal, legally speaking, as \( a_1 \), as far as \( j \)'s obligations towards \( k \) are concerned. In the epistemic interpretation the ordering between events is indeed interpreted in the same way as the ordering between states in the static models, that is in terms of comparative plausibility (Baltag and Smets 2008). If, in that context, we have for instance \( e_1 \succ e_2 \), it means that \( e_1 \) is strictly more plausible than \( e_2 \).

This reading, however, can be misleading in the present deontic interpretation. Saying that a deontic action \( a_2 \) is at least as, or even more ideal than another action \( a_1 \), legally speaking, strongly suggests that performing \( a_2 \) is at least as good as performing \( a_1 \). If we accept the principle that everything at least as ideal as a permissible action is also permissible, we would then arrive at \( a_2 \) becoming legally permissible as soon as \( a_1 \) is. As we will see in Section 3 this is not the case when legal ability and legal permissibility come apart. In such cases some agents may be in a position to perform two actions, say \( a_1 \) and \( a_2 \), the effects of which would be to make all states that satisfy the precondition of \( a_2 \) strictly higher in the relation \( \preceq_{j \rightarrow k} \) than all states that satisfy the precondition of \( a_1 \). This is naturally encoded by putting \( a_1 \preceq_{j \rightarrow k} a_2 \). But when this legal ability does not entail legal permissibility, it could be that \( a_1 \) is permissible while \( a_2 \) is not. In such cases it is not clear to what extent one can read \( a_1 \preceq_{j \rightarrow k} a_2 \) as “\( a_2 \) is legally more ideal” or “better than \( a_1 \)”.

For this reason we use the more neutral reading of \( \preceq_{j \rightarrow k} \) in terms of deontic potential or legal effectivity. In most cases, of course, legal ability coincides with legal permissibility, and when this happens one could add the additional evaluative
layer to the interpretation of the relation \( \leq_{A_i}^{A_j,k} \). The reader should keep in mind, however, that this interpretation only works in those cases.

In order to improve readability we index deontic-action models with particular agents. This is simply a mnemonic device, and plays no role in the mathematical definitions below. Conceptually, however, this allows us to distinguish between different legal competences of different agents, over the same static model, i.e. in a given legal situation. Indeed, some agents, for instance the judiciary, might have, in a given situations, competences that are different than those of a layperson in that same situation. These could in principle affect the legal relation of the same individuals, only in different ways. A layperson might have the competence to enter or renew a contract between her and someone else, but only the judiciary might have the competence to rule a contract as invalid. In that case we would model this using two different action models, one for the judiciary and one for the layperson, and index them accordingly. In Section 2.4 we present an example with two different action models for two different agents, which can be executed in the same circumstances. Once again, however, the reader should keep in mind that these particular indices on action models do not play any role in the definitions below.

Observe, furthermore, that the relation \( \leq_{A_i}^{A_j,k} \) is indexed by a pair of agents \( j, k \) which might differ from \( i \). The reason for this is that it is possible for some agents to change the legal relations between two different parties. A common example is civil servants with the legal power to marry couples. A specific deontic action model \( A_i \) for such an agent \( i \) thus captures not only her potential to change the legal relations where she is the subject or the addressee, i.e. where either \( i \rightarrow j \) or \( j \rightarrow i \) for some \( j \), but also the potential of \( i \) to change the legal relations of other pairs \( j, k \) of agents different from her. This modeling choice is in line with a number of recent proposals to capture legal relations (Lorini and Longin 2008; Herzig et al. 2011), but notably generalizes (van Eijck and Ju 2016), where the legal competences of an agent \( i \) are restricted to the relation that \( i \) has towards others.

### 2.2 Deontic Action Models: an example

Let us now turn to an example to illustrate this. John is a parking officer. He can confirm an illegal parking by issuing a parking ticket, if indeed a car is parked without a permit. To model this we use two deontic actions, \( a_1 \) and \( a_2 \). The first encodes confirming that a car is parked without a permit displayed. This is expressed, on the one hand, in the precondition \(-d \land p \) and, on the other hand, in the post-condition that after \( a_1 \) is executed \( f \) becomes true. The second action \( a_2 \) captures the case where he does not confirm a violation of the parking regulations. This deontic-action model is illustrated in Figure 2.

### 2.3 Updating with Deontic Action Models

The effect of executing a deontic action in a particular state is computed by the so-called lexicographic update. The intuitive idea is the following. First, a particular
deontic action can only be executed in states where its preconditions hold. Second, the normative or legal status of a particular state after the update is a function of the normative status of that state before the update, together with the deontic potential or legal effectivity of the actions that can be executed in that state. The update gives priority to the effect of the deontic actions, hence the label “lexicographic.” This reflects the idea that successfully enacting a legal competence means effectively changing the underlying legal relations. Third, and finally, executing a deontic action only changes legal relation and legal fact. The non-deontic actions remain unchanged after the update. Formally, this gives the following:

Definition 5 Let $M$ be a preference-action model and $A_i$ be a deontic action model. The preference-action model $M \otimes A_i$ is defined as follows, for all agents $j, k$.

- $W' = \{(w, a) \mid M, w \models \text{Pre}(a), \text{where } a \in A\}$.
- $(w, a) \preceq_{j \to k} (w', a')$ iff either $a \preceq_{j \to k} a'$ or $a \succeq_{j \to k} a'$ and $w \preceq_{j \to k} w'$.
- $(w, a) \sim_{j} (w', a')$ iff $w \sim_{j} w'$.
- $V'(p) = \{(w, a) \in W' : M, w \models \text{Post}(a)(p)\}.$

The lexicographic update thus takes pairs of preference-action models and deontic action models as input and returns an updated model $M \otimes A_i$. The domain of this new model is the set of pairs $(w, a)$ such that $M, w$ satisfies the precondition of $a$. This captures the idea that actions are only executed in states where their preconditions hold. As mentioned, the adjective “lexicographic” comes from the update rule for the preference order $\preceq_{j \to k}$.

Lexicographic updates capture changes at the legal or deontic level, taken in isolation. Borrowing from the epistemic interpretation of this formalism, we could call them “pure” deontic actions (van Ditmarsch et al. 2007). This is encoded in the condition defining the valuation $V'$ and the equivalence relation in the updated model:

$$(w, a) \in V'(p) \iff w \in V(\text{Post}(a)(p)),$$

$$(w, a) \sim_{j} (w', a') \iff w \sim_{j} w'.$$

---

Our definition of the update rule is slightly different from the standard one, used for instance in the proposals of Baltag and Smets (2008); van Benthem et al. (2014). Here two pairs $(w, a)$ and $(w', a')$ are connected in the updated model as soon as $a \succeq_{j \to k} a'$ or the other way around. This is so irrespective of whether $w$ and $w'$ were initially connected by $\preceq_{j \to k}$. We made this modeling choice because it allowed us to capture more naturally some of the examples that we present here.
In other words, executing a deontic action, or enacting a legal competence, first does not change the non-deontic powers of the agents. This is what the second line above specifies. Second, it changes the facts only to the extent specified by the deontic actions themselves. This is what the first line encodes. One can take such pure deontic actions to be actions that are explicitly defined by the legislator, for instance entering into a contract or getting married. These are the “purely legal” actions, which Hohfeld himself took care to distinguish from other types of changes:

At the very outset it seems necessary to emphasize the importance of differentiating purely legal relations from the physical and mental facts that call such relations into being. (Hohfeld 1913, p.20)

Pure deontic actions will, of course, usually be generated by non-deontic ones, in Goldman’s sense of “generation” (Goldman 1970). Changes brought about by deontic actions will usually supervene on non-deontic changes. Entering a contract might require signing certain documents, or getting married uttering some words. The models proposed by Jones and Sergot (1996); Governatori et al. (2005); Grossi et al. (2006) capture such conventional or legal action generation. The focus here is on the changes at the level of the generated deontic actions, in isolation from their non-deontic counterparts. A full comparison and combination between our model of deontic actions and those other models is left for future work.

2.4 Updating with Deontic Action Models: An Example

With this in hand we can return to our running example. The legal effects of enacting John’s legal competence in Ivy’s situation are represented by the updated model in Figure 3. John’s confirmation of the fact that Ivy has parked illegally leads to the removal of all arrows from the state where she is parked without a permit, i.e. \((w_3, a_1)\) to all others. That action also changes the truth value of the legal facts that Ivy pays the fine in the now most ideal worlds, legally speaking. In \((w_3, a_1)\) the city has now an unconditional claim against her to pay a fine. In all other states, i.e. where Ivy either does not park or else displays the permit, her duties and privileges, both conditional and unconditional, remain unchanged.

This analysis of John’s legal power is close to the one provided by Markovich (2020) in terms of public announcement, with one important difference. Here John’s actions are “soft” in the sense that they are reversible. Suppose for instance that Mary,

\[7\] The semantics of conditional obligation that we use also yields, after the update, that at \((w_3, a_1)\) the city has an unconditional claim against Ivy to park and not display a permit. This counter-intuitive prediction is a consequence of the fact that this semantics validates \(O_{i \rightarrow \phi}(\phi/\psi)\), for any \(\psi\). This means that, when the current state can only see itself according to the relation \(\leq_{i \rightarrow \psi}\), we obtain that \(O_{i \rightarrow \psi}\) whenever \(\phi\) is true. This is an issue that is shared by most preferential accounts of conditional obligations (Zvolenszky 2002), and that also affects some approaches based on default logic (Fuhrmann 2017). Addressing this thoroughly would require using a completely different approach to static conditional obligations, which would go beyond the scope of this paper. Note, however, that this is a defect that only affects the static conditional obligations. One could also interpret “given \(\phi\), \(\psi\)” dynamically, for instance using in terms of public announcements of the form \([\phi]\psi\), which are a special cases of the updates that we define here (Baltag and Smets 2008). It is well known that \([\phi]\psi\) is not a valid formula (van Ditmarsch et al. 2007).
John’s superior at the town hall, has the power to cancel John’s decision, i.e. to nullify his observation, and by that very fact restore Ivy’s original situation, which in particular cancels the city’s claim against her to pay a fine. This can be represented by the deontic action model in Figure 4. It is straightforward to check that updating $M \otimes \mathcal{A}_{John}$ with $A_{Mary}$ yields a model which is isomorphic to $M$. This kind of reversibility is ruled out by public announcements since they completely remove states.

To express the effect of deontic action the language $L$ is extended with a dynamic, unary operator $\mathcal{R}^A_i\ell_{s\ell_2}$, with the following semantics:

- $M, w \models [\mathcal{A}_i, a]\varphi$ iff if $M, w \models \text{Pre}(a)$ then $M \otimes \mathcal{A}_i, (w, a) \models \varphi$.

A formula $[\mathcal{A}_i, a]\varphi$ thus is read as “$\varphi$ holds after executing $a$” or, more precisely, “whenever $a$ is executable, i.e. its preconditions hold, $\varphi$ holds after executing $a$”. These dynamic modalities have duals, which we write $\mathcal{X}^A_i\ell_{s\ell_2}\varphi$. A formula of the form $\langle \mathcal{A}_i, a \rangle \varphi$ should be read as “$a$ is executable and $\varphi$ holds after one of its executions.” So the only difference between the dynamic “box” $[\mathcal{A}_i, a]$ and its “diamond” $\langle \mathcal{A}_i, a \rangle$ is that the latter states that the preconditions of $a$ hold in the current state, while the former only describes what happens if they hold. In cases where the preconditions are false we obtain $[\mathcal{A}_i, a]\varphi$ true for every $\varphi$, while every such $\langle \mathcal{A}_i, a \rangle\varphi$ turns out false. In particular, $\langle \mathcal{A}_i, a \rangle \perp$ can be read as saying that $a$ is executable in the current state, while $[\mathcal{A}_i, a] \perp$ that $a$ is not executable. Also, importantly, reading $\langle \mathcal{A}_i, a \rangle \varphi$ as “some executions of $a$ result in $\varphi$” might be misleading. This is so because the result
of updating any preference-action model with a deontic action model is unique. In other words, updates are deterministic.

2.5 Power and Immunity

We define power and immunity using these dynamic modalities. To do this we use Lindhal’s notation (Lindahl 1977) and write $T(i, j, \psi/\varphi)$ for an arbitrary normative position, static or dynamic.

**Definition 6 (Power and Immunity: Global Version)** Let $T(j, k, \psi/\varphi)$ be a normative position, $\mathcal{M}$ a preference-action model and $w$ any state in $W$. Then:

- $i$ has a power against $j, k$ regarding $T(j, k, \psi/\varphi)$ at $\mathcal{M}, w$:
  $$\mathcal{M}, w \models \bigvee_{a \in A} \langle A_i, a \rangle T(j, k, \psi/\varphi)$$

- $j, k$ have an immunity against $i$ regarding $T(j, k, \psi/\varphi)$ at $\mathcal{M}, w$:
  $$\mathcal{M}, w \models \neg \bigvee_{a \in A} \langle A_i, a \rangle T(j, k, \psi/\varphi)$$

Before we discuss the local/global distinction, three comments regarding this definition are in order. First, we define legal competences as tripartite relations. We say that $i$ has a power against two agents $j$ and $k$ regarding the normative position $T(j, k, \psi/\varphi)$ whenever there is an executable – hence the use of the modality $\langle A_i, a \rangle$ – deontic action of $i$ which would result in $T(j, k, \psi/\varphi)$. On the other hand we define the fact that $j, k$ have an immunity against $i$ regarding $T(j, k, \psi/\varphi)$ if $i$ does not have any action that would result in the position $T(j, k, \psi/\varphi)$. The reason for this tripartite definition is, again, that we want to be able to capture cases where a particular agent, for instance the judiciary or an institutional representative, might have the competence to change the legal relations between other agents, for instance by declaring someone guilty and thereby creating a claim to reparation. This is indeed the case in our running example above. Of course, this definition allows for cases where, for instance, $i$ has the legal power to create a claim right herself against $k$. In that case we simply have $i = j$ in the above definition.

Second, this definition captures power as potential to change legal relations, and thus does not conflate this potential with the actual exercising of the power. To be sure, our definition ensures the precondition necessary for a power to hold. We are indeed using the “diamond” $\langle A_i, a \rangle$, instead of the “box” $[A_i, a]$ to define power. This does not mean, however, that the power is actually exercised in that state. The formula only describes what would be the potential consequences of exercising it.

Third, this definition captures the Hohfeldian square of legal competences (Table 1, page 2). The correlative of agent $i$’s power against $j, k$ regarding $T(j, k, \varphi)$ is $j$ and $k$’s liability against $i$ regarding the same static normative position. In classical formalizations of Hohfeldian rights, for instance (Kanger and Kanger 1966), lateral moves in the diagrams of Table 1 are modeled as changes in the directionality of
the corresponding rights. Liability is modeled in the same way as its correlative, i’s power:

\[ j, k \text{ have a liability against } i \text{ regarding } T(j, k, \varphi) \iff \bigvee_{a \in A} \langle A_i, a \rangle T(j, k, \varphi) \]

Vertical moves in the same diagrams, i.e. moving to opposites, are modeled by changing the polarity of the corresponding right. So the opposite of j, k’s liability corresponds to

\[ \neg \bigvee_{a \in A} \langle A_i, a \rangle T(j, k, \varphi) \]

This is exactly our definition of j and k’s immunity against i regarding \( T(j, k, \varphi) \). Pushing the negation inside we obtain:

\[ \bigwedge_{a \in A} [A_i, a] \neg T(j, k, \varphi) \]

This states that no deontic action of i would result in \( T(j, k, \varphi) \). Again, since directionality is left implicit, the same formula corresponds to moving left in the diagram of page [3] and gives a natural formalization of i’s no-power against j, k regarding \( T(j, k, \varphi) \).

This definition of power and immunity has, however, an important shortcoming. It predicts that i has a power against j, k regarding \( T(j, k, \psi/\varphi) \) as soon as i has an executable deontic action that preserves, or, in other words, does not change \( T(j, k, \psi/\varphi) \). This can be seen in our running example by looking at what happens in states other than \( w_1 \). There Ivy’s conditional obligation to display her permit whenever she parks remains unchanged by John issuing a ticket. \( O(DO_{t_{wp}}/p) \) holds in all states except \( w_1 \) before and after the update. So the global definition of power yields that John has a power against Ivy regarding her obligation to display a ticket, which might be seen as counter-intuitive, since John’s actions do not change that right. We furthermore obtain the reverse predictions for immunity. Cases where none of i’s actions result in \( T(j, k, \psi/\varphi) \) are compatible with some of them effectively changing \( T(j, k, \psi/\varphi) \) to \( \neg T(j, k, \psi/\varphi) \). This, again, appears rather counter-intuitive.

A natural way to make up for this shortcoming is to enforce that power induces genuine changes in the underlying normative positions, while immunity genuinely preserves them. To do this we move to a local definition of legal competences, in the sense that it is relative to a given preference-action model \( M \) and state \( w \) in it.

**Definition 7 (Power and Immunity: Local Version)** Let \( M \) be a preference-action model and \( w \) a state in it such that \( M, w \models T(j, k, \psi/\varphi) \).

- i has a power against j, k regarding \( T(j, k, \psi/\varphi) \) at \( M, w \) iff

  \[ M, w \models \bigvee_{a \in A} \langle A_i, a \rangle \neg T(j, k, \psi/\varphi) \]

- j, k have an immunity against i regarding \( T(j, k, \psi/\varphi) \) at \( M, w \) iff:

  \[ M, w \models \bigwedge_{a \in A} [A_i, a] T(j, k, \psi/\varphi) \]
This definition is local in the sense that it is relative to a given pointed preference-action model. It requires checking whether the given right $T(j, k, \psi/\varphi)$ holds at $w$ in the underlying preference-action model. For $i$ to have a power regarding that right we require that $i$ has some executable deontic action that, when executed, would change the polarity of that right. So $i$ has a power to establish a position $T(j, k, \psi/\varphi)$ if it doesn’t hold in the given situation, but $i$ can make it true. Similarly, $i$ has a power to cancel a position if that position holds in the given situation, though $i$ can make it false. For immunity we require the contrary, namely that no executable action changes the static normative position in question. Both notions are context dependent. Whether they hold depends on the specifics of the underlying static legal relations, hence the local definition, relative to a given model-state pair.

This alternative definition is again tripartite and keeps the correlative and opposite relation of the Hohfeldian diagram. To see this latter point, observe that negating the fact that $i$ has a power against $j, k$ regarding $T(j, k, \psi/\varphi)$ at $M, w$ in a situation where $T(j, k, \psi/\varphi)$ does hold gives us:

$$M, w \models \bigwedge_{a \in A} [A_i, a] T(j, k, \psi/\varphi)$$

which is precisely the definition of immunity. This states that no executable action of $i$ will change the status of $T(j, k, \psi/\varphi)$, which gives the “no-power” correlative of immunity.

This new definition avoids the shortcoming of the global definition. On the one hand it rules out cases of trivial powers that do not change the underlying legal relations. In our running example it yields the intuitively correct prediction that John has no power against Ivy regarding the unconditional obligation to display her parking permit or, equivalently, that Ivy has an immunity against John regarding that obligation.

The local definition is of course context- or model-dependent, in the sense that it is relative to a concrete legal situation, as opposed to constituting a general definition of what “having a legal power towards $T(j, k, \psi/\varphi)$” means. It provides an analysis of tokens of legal abilities, as opposed to types.

As such, the local definition comes with stronger logical relations between different powers than the global one. The latter is, for instance, compatible with an agent $i$ having both a power regarding $T(j, k, \psi/\varphi)$ and a power regarding its negation. This combination is inconsistent under the local definition. These two local powers require that we consider situations where $T(i, j, \psi/\varphi)$ is, respectively, false and true, which of course cannot occur simultaneously. This highlights the fact that this local definition of power boils down to modeling power as the ability to establish or cancel a certain legal relation. The former assumes that the legal relation does not hold before the power is enacted, and the latter that it does.

A welcome corollary of this is that abstaining from enacting a power is distinct, in this logic, from having a power to the contrary. As we have just argued, a locally

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8 This view of legal powers as potential to change or “flip”, so to speak, the truth value of certain legal facts is also used by [van Eijck and Ju (2016)](vanEijckAndJu2016). Note, however, that in that paper they restrict to the case where the agents have control over atomic propositions, while where the legal powers extend to any right $T(j, k, \psi/\varphi)$. 


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defined power regarding $T(i, j, \psi/\varphi)$ is incompatible, in this model, with a power regarding $\neg T(i, j, \psi/\varphi)$. This is so because the former models a case of establishing $T(i, j, \psi/\varphi)$ and the latter of canceling $T(i, j, \psi/\varphi)$, and the two have inconsistent preconditions. Either of these competences, however, are compatible with the ability to refrain from exercising them, i.e. to do nothing at all. This ability can be modeled by adding to any action model an action unconnected by $\leq_{i,j}^{A,k}$ to all others except itself, and with $\top$ as pre condition. Updating with that larger deontic action model will create a copy of the original model alongside the updated one, and of course in that model the original legal relations remain unchanged.

This formalization of dynamic rights has two assets in comparison with classical, reductive approaches. First, it explicitly captures, both semantically and syntactically, the dynamic character of power and immunity. Second, as we will see below, this clear static-dynamic distinction allows for a natural distinction between legal ability and legal permissibility.

2.6 Axiomatization

Axiomatizing the set of validities for the frames and updates just defined proceeds in two modular steps. First the validities for the static modalities of $\mathcal{L}$ are axiomatized, and second provide an axiomatization of the dynamic extension. For the static part the axiomatization proceeds in a standard manner. We use $S4$ for $[\leq_{i,j}]$ and $S5$ both for $U$ and $Do_i$. Interaction between $[\leq_{i,j}]$, $Do_i$ and $U$ can be captured by standard inclusion axioms. Of course, each modality satisfies the necessitation rule. See Table 2.

### Table 2

| Axioms for preference-action models |
|-------------------------------------|
| **S4 for $\leq$:** |
| $\vdash [\leq_{i,j}](\varphi \rightarrow \psi) \rightarrow [\leq_{i,j}]\varphi \rightarrow [\leq_{i,j}]\psi$ |
| $\vdash [\leq_{i,j}]\varphi \rightarrow \varphi$ |
| $\vdash [\leq_{i,j}]\varphi \rightarrow [\leq_{i,j}][\leq_{i,j}]\varphi$ |
| From $\vdash \varphi$ infer $\vdash [\leq_{i,j}]\varphi$ |
| **S5 for $U$, and similarly for $Do_i$:** |
| $\vdash U(\varphi \rightarrow \psi) \rightarrow U\varphi \rightarrow U\psi$ |
| $\vdash U\varphi \rightarrow \varphi$ |
| $\vdash U\varphi \rightarrow UU\varphi$ |
| $\vdash \neg U\varphi \rightarrow U\neg U\varphi$ |
| From $\vdash \varphi$ infer $\vdash U\varphi$ |
| **Interaction axioms:** |
| $\vdash U\varphi \rightarrow [\leq_{i,j}]\varphi$ |
| $\vdash U\varphi \rightarrow Do_i\varphi$ |

Axiomatizing the dynamic part uses the well-known “reduction axioms” methodology (Baltag and Smets 2008; van Benthem et al. 2014). Formulas containing dynamic modalities are shown to be semantically equivalent to formulas of $\mathcal{L}$, that is without dynamic modalities. The formulas in Table 3 indeed show how to “push”
dynamic modalities inside the various connectives and modal operators of the static language, until they range over atomic propositions where they can be eliminated.

Table 3 Reduction Axioms for Lexicographic Update

These formulas are sound with respect to the lexicographic update over preference-action models. Taking them as axioms thus makes formulas containing dynamic modalities provably equivalent to formulas of $\mathcal{L}$. Completeness for the extended language then follows from completeness of the static part with respect to the class of preference-action models. This is a standard technique to prove completeness for dynamic extensions of static languages. See e.g. (van Ditmarsch et al. 2007, p.196) for details.

Theorem 1 The axioms in Table 2 together with the reduction axioms in Table 3 all propositional tautologies and Modus Ponens are sound and complete with respect to the class of preference-action frames and lexicographic updates.

2.7 Reduction of Legal Competences to Static Legal Relations

Through the soundness of the reduction axioms, together with the fact that conditional obligations are definable in $\mathcal{L}$, we obtain that power and immunity are also reducible to the language where static normative positions are defined. So the approach presented here is reductive. This reduction, however, is complex, especially in comparison with the reduction proposed for instance by Lindahl (1977).

The only difference comes from our slightly non-standard clause in the lexicographic update rule, which results in the use of the universal modality in the first group of conjuncts in the second equivalence from below. See again footnote 6 on page 11 for details.
The complexity of this formula results from it essentially encoding syntactically the lexicographic update rule in combination with the specific semantic definition of obligations as truth in all the most ideal worlds. For unconditional obligations we obtain the following, slightly more readable formula.

\[
[A_i, a]O_{j\rightarrow k} \psi \leftrightarrow \bigwedge_{b \in \text{max}([a, \varphi])} \bigwedge_{d \in [b]} \bigwedge_{c \in [c]} \bigwedge_{e \in [c]} \left( \left( \text{Pre}(d) \rightarrow \langle A_i, d \rangle \psi \right) \lor \left( \text{Pre}(e) \right) \bigwedge_{d \in [b]} \left( \text{Pre}(d) \rightarrow \langle A_i, d \rangle \psi \right) \lor \left( \text{Pre}(e) \right)
\]

Each formula shows that the effects of changes in legal relations are reducible to statements describing legal relations holding before the deontic action takes place.

In view of the complexity of the resulting formulas we cannot, however, claim that the present approach yields much insight into the relation between static and dynamic rights. Unlike non-reductive approaches, this deontic re-interpretation of dynamic-epistemic logic does come with a rich theory combining the static and the dynamics parts. Whether this richness can be cashed out into concrete insights for the Hohfeldian typology remains however to be seen.

3 Legal Ability and Legal Permissibility

Although legal ability and legal permissibility often go together, they are conceptually distinct notions (Makinson 1986; Jones and Sergot 1996). The German Civil Code (Bürgerliches Gesetzbuch) illustrates this through “Missbrauch der Vertretungsmacht”, which we translate freely here as “abuse of the terms of representation”. This is defined as follows:

A failure to comply with restrictions from the legal relationship on which the power of representation is based when undertaking a legal transaction on behalf of the representative. Since the power of representation is abstracted from the underlying legal relationship, such restrictions do not lead to a restriction of the power of representation, so that even a declaration made in abuse of the power of representation remains within the scope of the power of representation.\(^\text{10}\)

\(^{10}\) Our emphasis. Freely translated from “Nichteinhaltung von Beschränkungen aus dem der Vertretungsmacht zugrunde liegenden Rechtsverhältnis bei Vornahme eines Geschäftes in Stellvertretung durch den Vertreter. Da die Vertretungsmacht von dem zugrunde liegenden Rechtsverhältnis abstrakt ist, führen solche Beschränkungen nicht zu einer Einschränkung der Vertretungsmacht, so dass sich auch eine unter Missbrauch der Vertretungsmacht abgegebene Erklärung grundsätzlich im Rahmen der Vertretungsmacht hält.” Source: http://rechtslexikon.net/d/missbrauch-der-vertretungsmacht/missbrauch-der-vertretungsmacht.htm
The idea being that provision appears to protect the counter-party of a transaction that violates the terms of a representation contract with a principal. Let us illustrate this by a simple example.

Suppose that $i$ contracts $j$ to buy supplies for her house. Our agent $i$ adds the condition not to spend more than 500 euro, if something is bought at all. Agent $j$ goes to the shop $k$ and buys goods for 600 euro. The sale is a deontic action made by $j$ in $i$’s name and is valid although it violates the “legal relationship” between $i$ and $j$, i.e. the contract. After the sale $i$ owns the good and owes 600 euro to $k$. Of course, the German Civil Code might have additional provisions regulating reparations in case of breach of contract. We bracket them here, as our goal is to illustrate the difference between legal ability and legal permissibility. So $j$’s buying from $k$ is not legally permissible, because excluded by the contract between $i$ and $j$, while being legally feasible for $j$. We now show that our logical model of legal competences can capture this case in a simple manner.

3.1 Agent $j$’s Power to Buy

Ownership rights involve a number of different obligations and permissions, but for simplicity here we represent them as the fact that the owner of a given good is in possession of it. So let $p$ be the fact that $i$ is in possession of the goods for sale at $k$, and $\neg p$ the fact that $k$ is in possession of those goods. Let furthermore $f$ be the fact that $i$ pays the 600 euro to $k$.

One model of the initial situation, before the sale, is depicted in Figure 5. For simplicity we assume that $i$ and $k$ are passive in this case, so $\sim i$ and $\sim k$ is set to $W \times W$. We consider three situations: 

- $w_1$, in which the goods are in $k$’s possession and $i$ doesn’t pay the 600 euro to her 
  as a result of some action by $j$. One can think of this case as one where $j$ sees to it that goods within the 500 euro budget get into $i$’s possession;
- $w_4$ in which the goods are in $i$’s possession and she pays the 600 euro to $k$, again 
  as a result of some action by $j$;
- $w_2$ and $w_3$, which together represent the situation where $j$ abstains from doing anything regarding $p$ and $f$.

The facts that $k$ initially owns the goods is represented by the relation $\leq_{i \rightarrow k}$. It ranks $w_1$ highest, followed by $w_2$ and $w_3$ together, and $w_4$ lowest. Since $i$ is $j$’s principal in this case, we assume as well that this legal ideality is the same for $j$ as far as $i$’s legal obligations are concerned, i.e. $\leq_{i \rightarrow j} = \leq_{j \rightarrow i}$. This gives, as desired, that $j$ ought to see to it that she doesn’t put $i$ in the position of having to pay the 600 euro to $k$: $O_{j \rightarrow i} \rightarrow O_{i \rightarrow k} f$.

Agent $j$’s power to buy the 600 euro’ worth of goods from $k$ is simply the power to transfer possession of those goods from $k$ to $i$, and creates a claim for the latter that the former pays the 600 euro. We model this as an action that results in making it the case that $i$ ought to be in possession of the goods and hence pays the 600 euro to $k$. The deontic action model of Figure 5 has precisely that effect.

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11 The German Civil Code of course explicitly voids this provision if there was proven collusion between buyer and seller, or when the seller should have known the terms of the agent’s contract.
The preference-action model before the sale. The solid lines represent both $\leq_{i\rightarrow k}$ and $\leq_{j\rightarrow i}$. The dashed rectangles represent $j$’s relation $\sim_j$.

$$D_0 j \neg p \quad a_1 \quad a_2 \quad a_3 \quad D_0 j p$$

$\neg D_0 j p \wedge \neg D_0 j \neg p$

Fig. 6 The deontic action model $A_j$ for $j$’s buying from $k$.

The result of updating the original model with the deontic action model in Figure 6 is represented in Figure 7. It essentially reverts to the order of the strict preference relations $\leq_{i\rightarrow k}$ and $\leq_{j\rightarrow i}$, leaving the indifference relation between $w_2$ and $w_3$ constant. This gives $O_{i\rightarrow k} f$, as desired, as well as that $j$ ought to see to it that title to the property now goes to their new legitimate owner $i$: $O_{j\rightarrow i} D_0 j p$. Since none of those claim rights held before, we obtain as desired that $j$ has indeed the (local) power to buy the goods from $k$: both $\bigvee_{a \in A} (A_j, a) O_{i\rightarrow k} f$ and $\bigvee_{a \in A} (A_j, a) O_{j\rightarrow i} D_0 j p$ hold in the original model.

$$\neg p, \neg f \quad \begin{array}{c} w_1 \\ \uparrow \\ \downarrow \\ w_4, a_3 \end{array} \quad \begin{array}{c} p, f \\ \uparrow \\ \downarrow \\ \end{array}$$

$$\neg p, \neg f \quad \begin{array}{c} w_2, a_2 \end{array} \quad \begin{array}{c} \uparrow \\ \downarrow \\ \end{array} \quad \begin{array}{c} (w_3, a_2) \quad p, f \end{array}$$

Fig. 7 The preference-action model after the sale. The solid lines represent both $\leq_{i\rightarrow k}$ and $\leq_{j\rightarrow i}$, after the update.

3.2 Impermisibility of Buying

The language $L$, even extended with the dynamic modalities, cannot directly express the notion of legal permissibility of deontic actions. This language is designed to describe the effects of deontic action or, by having deontic modalities scoping over dynamic expressions, the normative status of those effects. But this is still different from saying that a certain deontic action is obligatory, permitted or forbidden. In our
example it is arguably the case that it ought to be that $i$ owns the goods $j$ has bought in her name, even though this transaction constitutes a violation of the contract’s conditions.

Our modeling of the legal permissibility of deontic action is inspired by the Anderson-Kanger reduction of deontic modalities to a combination of alethic ones with a “violation” or “sanction” constant, c.f. [Kanger 1970, Meyer 1988]. Let $V$ be that constant, and $a$ one of $i$’s deontic actions, then this reduction goes as follows:

$$P(a) =_{df} \langle A_i, a \rangle \sim \neg V$$

Im permis sible deontic actions can be straightforwardly represented using the post-condition function, since there being a violation is itself a normative, or at least an evaluative notion, which might be changed by the execution of certain deontic actions. A higher court can, for instance, invalidate a guilty verdict from a lower one, effectively changing whether a violation has occurred or not. In our example, for instance, we can simply set

$$\text{Post}(a_i)(V) := \top$$

for all $i = 1, 2, 3$. This immediately leads to the result that buying the goods from $k$ is not legally permissible, i.e. $\bigwedge_{a \in A} \neg P(a)$ will be true in all states in the original model. By setting $V$ false everywhere before the update we furthermore obtain the welcome consequence that, before the sale, $j$ is not in a state of violation, in line with the fact that $O_{j \sim} Do_j f$ is false in all states before the update.

4 Conclusion

This paper has studied the potential of a deontic re-interpretation of a formalism developed in dynamic epistemic logic to model Hohfeldian legal competences. We have shown that it allows for a model of Hohfeldian power and immunity that, unlike Lindahl’s (1977) approach, explicitly captures the norm-changing or dynamic character of legal competences. It does so both at the semantic level, through the explicit update mechanism, and at the syntactic level, by using an explicit dynamic modality to express the effects of deontic actions. The approach we propose here is, however, also reductive in the sense that formulas with dynamic modalities are semantically and provably reducible to formulas without. As a result, it comes with a rich set of interaction principles between static and dynamic rights, captured by the so-called “reduction axioms.” We have pointed out in Section 2, however, that in view of its complexity, the reduction of dynamic rights to static language only yields limited insights on the relation between both levels. Finally, we have shown that this system can capture the distinction between legal ability and legal permissibility in a more auspicious way than reductive approaches, without paying the price of thorough-going non-reductionism.

We take this to be a starting point for the methodology we propose, but of course it also raises a number of questions that could not be addressed in this paper. The natural next step is to study the theory of normative positions stemming from our models of dynamic rights. See the thesis [Dong 2017] for some steps in that direction. Equally important in our view is to study the theory of legal competences.
that would result from extending a static base that is different from standard deontic logic. In the epistemic context a wide variety of static logics of knowledge and belief have been “dynamified” using the action model methodology. More radical departures from standard deontic logic have been proposed to capture actual legal reasoning, and the question remains whether they would yield a plausible theory of power and immunity once augmented with a dynamic module as we have done here. Finally, there is of course an interesting connection between legal competences as we have represented them and a dynamic analysis of the notion of strong permission (von Wright 1963), as for instance suggested by van Benthem et al. (2014). This, however, is the topic of another paper.

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