Energy dependence of femtoscopy properties of pion source in nuclear collisions

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In the paper energy dependence of femtoscopy characteristics of pion emission region at freeze-out is investigated for collisions of various ions and for all experimentally available energies. For the first time the normalized values of radii and volume of source are used for energy dependence. This approach allows to expand the set of interaction types, in particular, on non-symmetrical nucleus-nucleus collisions which can be studied in the framework of common approach. There are no the sharp changing of femtoscopy parameter values, in particular, \(R_o/R_s\) with increasing of \(\sqrt{s_{NN}}\) which were predicted by some phenomenological models as signature of first order phase transition in strongly interacting matter. The generalized parameterization for femtoscopy correlation function is suggested.

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I. INTRODUCTION

At the present time the presence of relationship between the geometry of 4-dimensional emission region of secondary particles and dynamics of final state creation is established reliably for different types of interactions. By the definition (see, for example, \[1-3\]) the field of research investigated the pair correlations for secondary particles (both identical and non-identical) with small relative momentum / velocity is called "correlation femtoscopy". The momentum quantum-statistical correlations (GGLP-effect) was observed for the first time in \(\bar{p}p\) annihilation for identical charged pions \[4\]. The bases of correlation femtoscopy are described in detail elsewhere (see, for example, \[1,3\]). The discussion below is focused on correlations in pairs of identical charged pions with small relative momenta – HBT-interferometry\(^1\) – in nucleus-nucleus collisions. Originally the GGLP-effect was interpreted in the framework of formalism of wave function \[4,6\], but the more general approach is based on the quantum field theory \[1\]. This approach was developed and used for the first time in quantum optics \[1\].

The space-time characteristics for emission region of secondary particles created in (heavy) ion collisions are important for study of deconfinement state of strongly interacting matter – strong-coupling quark-gluon plasma (sQGP): the significant increasing of emission duration was predicted as compulsory signature of the first order phase transition in strongly interacting matter from hadronic phase to quark-gluon one \[7,8\]. This effect should be shown experimentally as strong difference of emission region in transverse plane from azimuthal-symmetrical shape. The correlation femtoscopy allows to investigate the geometry of source at kinetic freeze-out, i.e. at late stage of space-time evolution of final state at transition from strongly coupling system to weakly interacting ensemble of secondary particles. Therefore the study of AA collisions in wide energy domain by correlation femtoscopy seems important for better understanding both of equation of state (EOS) of strongly interacting matter and general dynamic features of soft processes.

It should be emphasized that there is deep relationship between collective effects at various stages of space-time evolution of strong interaction processes. For example, the measurements of length scales and chaoticity of source with help of correlation femtoscopy can be used for estimation of multiplicity of hadron jets without application of specific algorithms for jet identification \[9\], for study of differences between quark and gluon jets \[10\]. The correlation femtoscopy seems the promising tool for investigation of fundamental discrete symmetries \[11\] and complex structure of quantum chromodynamic (QCD) vacuum. Moreover, the geometry of emission region is important for physics of cosmic rays and for search for signatures of physics beyond of Standard Model (SM) \[12\]. Therefore the studies in the field of the correlation femtoscopy have a inter-subject character.

The paper is organized as follows. In Sec. 2, definitions of main observables for correlation femtoscopy are described. The normalized characteristics for geometry of emission region are defined. The Sec. 3 devotes discussion of experimental energy dependence for the femtoscopy parameters of secondary particle source at freeze-out in various

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\(^1\) The detail explanation is shown in \[2\] for terminology and relationship between momentum GGLP correlations in physics of fundamental interactions and space-time correlations of photons – HBT-effect \[6\]. In accordance with the field of research of the paper and most popular terminology in corresponding literature the method of correlation femtoscopy for identical particles (pions) is called "HBT-interferometry" or briefly "HBT" also in this paper below.
ion collisions. In Sec. 4, the general case of Lévy stable distribution under study for correlation function. Some final remarks and conclusions are presented in Sec. 5.

II. METHOD AND VARIABLES

In general two-particle correlation function (CF) for secondary particles of type \( j \) with 4-momenta \( p_1, p_2 \) is defined as follows \[13\]:

\[
C_2^j(p_1, p_2) = \frac{\mathbf{P}_j^2(p_1, p_2)}{\mathbf{P}_j^1(p_1)\mathbf{P}_j^1(p_2)} = \sigma_{\text{in}} \frac{d^2\sigma^j(p_1, p_2)/dp_1 dp_2}{d\sigma^1/dp_1 \times d\sigma^1/dp_2},
\]

where \( \mathbf{P}_j^1 \) is (one)two-particle inclusive distribution density, \( \sigma_{\text{in}} \) – the total inelastic cross section for interaction under study. There is the following relationship between \(1 \) and normalized cumulant CF \[14\]

\[
K_2^j(p_1, p_2) = C_2^j(p_1, p_2) - 1.
\]

Functions \(1 \) and \(2 \) are studied depending on relative 4-momentum \( q \equiv (q_0, \mathbf{q}) = p_1 - p_2 \) and average 4-momentum of particles in pair \( K \equiv (K^0, \mathbf{K}) = (p_1 + p_2)/2 \) (pair 4-momentum). Phenomenological multidimensional parameterization for CF \(1 \) for standard simplest case can be written as (see, for example, \[8\]):

\[
C_2^{ph}(q, K) \propto 1 + \lambda(K)\mathbf{K}_2^{ph}(\mathbf{A}), \quad \mathbf{K}_2^{ph}(\mathbf{A}) = \prod_{i,j=1}^3 \mathbf{K}_2^{ph}(A_{ij}) = \exp \left( - \sum_{i,j=1}^3 q_i R_{ij}^2 q_j \right).
\]

Here \( \mathbf{A} \equiv \mathbf{q}^T \mathbf{R}^2 \mathbf{q}^T \) and \( \mathbf{R}^2 \) are the matrices \( 3 \times 3 \), \( \mathbf{q}^T \) – transposed vector \( \mathbf{q} \), \( \forall i, j : R_{ij}^2 = R_{ji}^2, R_{ii}^2 \equiv R_i^2 \), where \( R_i = R_i(K) \) are parameters derived by HBT method and characterized the linear scales of source part which can be studied at fixed \( K \), i.e. homogeneity region \[13\]: the products are taken on space components of vectors, \( \lambda(K) = \mathbf{K}_2(0, K), 0 \leq \lambda \leq 1 \) is the parameter which characterize the degree of source chaoticity\[2\]. Taking into account the hypothesis of cylindrical symmetry of source the volume of homogeneity region was derived as follows \[17\]

\[
V = (2\pi)^{3/2} \prod_{i=1}^3 R_i.
\]

The experimental correlation function is constructed as follows \[8\]

\[
C_2^{E}(q, K) = \zeta(q, K)D_E(q, K)D_B^{-1}(q, K),
\]

where \( D_E(q, K) \) is the pair distribution for particles measured in the same event, \( D_B(q, K) \) – background distribution – distribution for pairs of particles from different events. In an ideal case the background distribution is the same as \( D_E(q, K) \) with exception of presence of quantum-statistical correlations in the last case. The additional factor \( \zeta(q, K) \) takes formally into account all possible corrections. It was shown that the ratio \(5 \) is sensitive to the space-time extension of emission region \[18\].

The space component of pair 4-momentum \( \mathbf{K} \) is decomposed on longitudinal \( k_\parallel = (p_{\parallel,1} + p_{\parallel,2})/2 \) and transverse \( k_\perp = (\mathbf{p}_{\perp,1} + \mathbf{p}_{\perp,2})/2 \) parts of pair momentum. There are several version for decomposition of \( q \) \[7, 14, 21\]. In the paper decomposition of Pratt – Bertsch \[7, 19\] is used in which the \( \mathbf{q} \) is resolved into longitudinal component directed

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1 For discussion below the index for particle type is omitted for simplicity.

2 It should be emphasized that this title of \( \lambda \) is the historical and can be used with some carefulness because it is valid for fully chaotic source without any other features of dynamics (contribution of long-lived resonances etc.) and experiment. The set of effect contributions included in \( \lambda \) depends on certain investigation. As usual the \( \lambda \) takes into account the partial coherence of source and long-lived resonance decays in theoretical studies. In the framework of phenomenological and experimental investigations the \( \lambda \) includes both the effects indicated above and contributions of weak decays and particle misidentification. The effects of final state interactions (see below) can be accounted in \( \lambda \) in some rare case. The separation of influence of source coherence on \( \lambda \) from contribution of other effects can be made with help of three-pion correlation only. These correlations suppress contributions from long-lived resonance decays and particle misidentification significantly \[14\]. The physical analysis of three-particle correlations allow to define the true degree of source coherence and, as consequence, influence just of the coherence on the \( \lambda \) values estimated with help the study of \( C_2(q, K) \).
along the beam axis, \( q_1 \), outward component directed parallel to the pair transverse momentum, \( q_o \), and a sideward component directed perpendicular to those two, \( q_s \). For the identical particle pairs the longitudinal co-moving system (LCMS) frame is chosen as the reference frame. The LCMS moves together with pair in longitudinal direction, thus the \( k_\parallel = 0 \) in this frame. In general case for nuclear beam collisions \( A_1 + A_2 \) one can write for length scales of homogeneity region \( R_i = f(\sqrt{s_{NN}}, A_1, A_2, |\vec{b}|, \phi, y, |k_\perp|, m) \), where \( i = 1, o, s \), \( \vec{b} \) is the impact parameter vector, \( \phi \), \( y \) \( - \) azimuthal angle and rapidity. The main part of measurements for correlation femtoscopy in field of heavy ion interactions was made for pairs of identical charged pions in central collisions. This allows to simplify the theoretical formalism significantly due to azimuthal symmetry \([18, 22]\) and to reach the maximum for energy density and linear sizes of emission region.

Conservation laws and interactions of particles in final state (FSI) influence on quantum-statistical correlations, moreover just the FSI is most important for nuclear collisions \([23]\). The methods to take into account of FSI in general case are described in \([24]\). For the pairs of charged hadrons the main contribution is Coulomb FSI but correction due to strong interactions influences much weaker in the correlation peak domain \( |q| \leq 0.1 \, \text{GeV}/c \) at the same time \([23]\). As known the Coulomb repulsion in the pairs of same-sign charged particles leads to decreasing of amount of real pairs at small \( q \) and consequent decreasing of peak amplitude of CF. Therefore the function

\[
C_2^{\text{ph}}(q, K) = \left[D_B(q, K)D_B^{-1}(q, K)\right]P_{\text{coul}}^{-1}(q).
\]  

(6)
is called corrected experimental CF in the field of correlation femtoscopy, where \( P_{\text{coul}}(q) \) is the correction on the Coulomb FSI. It would be noted that \( P_{\text{coul}}(q) \) is introduced either in phenomenological parameterization \([3]\) or in experimental CF. In the first case the \( C_2^{\text{ph}}(q, K) \) is multiplied on \( P_{\text{coul}}(q) \) but in the last case the experimental CF is divided on the correction on the Coulomb FSI and the equation (6) is derived. The main procedures for accounting for Coulomb FSI are described in \([25, 27]\). The standard procedure is suggested the iteration procedure for calculation of Coulomb correction \( P_{\text{coul}}^{(1)}(q) \) for extended source. The model approach for the source is the static spherically-symmetrical Gaussian source with fixed radius \( R \) in the rest frame of pair \([25]\). Historically this procedure was suggested as first and was used in the many experiments, in particular, for the first HBT study of \( \text{Au} + \text{Au} \) collision at RHIC energy \( \sqrt{s_{NN}} = 130 \, \text{GeV} \) \([28]\). However this procedure some overestimates the value of correction on the Coulomb FSI due to suggestion that all pairs in \( D_B(q, K) \) are primary and should be corrected \([25]\). This aspect is taken into account in framework of the second procedure \([26]\) by excluding the pairs which are formed by pions from resonance decays and participate in the Coulomb interaction at the same time. The such exclusion leads to some attenuation of Coulomb correction and the following relation is derived for \( P_{\text{coul}}(q) \) in the framework of second procedure

\[
P_{\text{coul}}^{(2)}(q) = (1 - f_2) + f_2 P_{\text{coul}}^{(1)}(q),
\]

where \( f_2 \) is the fraction of primary pions and \( 0 \leq f_2 \leq 1 \). This procedure is called the dilution procedure respectively. In the framework of third procedure it is suggested that only pairs followed by Bose–Einstein statistics participate in the Coulomb FSI \([27]\). These pairs are formed by the particles which are close to each other in the center of mass of pair system. In the third procedure it is suggested that there are no misidentified pairs and corresponding correction is defined as follows:

\[
P_{\text{coul}}^{(3)}(q) = (1 - f_3)[1 + K_2^{\text{ph}}(A)]^{-1} + f_3 P_{\text{coul}}^{(1)}(q),
\]

where \( f_3 \) is the fraction of pairs followed by Bose–Einstein statistics and \( 0 \leq f_3 \leq 1 \). In the equation above the first term corresponds to the pairs which do not participate in the Coulomb FSI, the second term – to the pairs which follows by Bose–Einstein statistics and participate in the Coulomb FSI at the same time. The choice \( \forall m = 2, 3 \) : \( f_{(m)} = \lambda \) seems reasonable for fully chaotic source and the specific physical meaning of the \( \lambda \) corresponds to the certain procedure for Coulomb FSI. As seen from the relations for \( P_{\text{coul}}^{(m)}(q) \) at \( m = 2, 3 \) all procedure for accounting of Coulomb FSI are identical at \( \lambda = 1 \). Fig. 1 shows the dependence of various Coulomb corrections \( P_{\text{coul}}^{(m)} \) on \( q_{\text{inv}} = \sqrt{-q^2} \), the corrections were obtained for model of static spherically-symmetrical Gaussian source with \( R = 6 \, \text{fm} \). As expected the difference between various procedures decrease with \( \lambda \) increasing. Therefore the following general equation can be written for phenomenological parameterization of CF with taking into account all forms of corrections on Coulomb FSI under consideration

\[
C_{2,(m)}^{\text{ph}}(q, K) = \epsilon P_{\text{coul}}(q)\left[\epsilon^{-1} + K_2^{\text{ph}}(A)\right], \epsilon = \begin{cases} \lambda, & \text{at } m = 1, 2; \\ 1, & \text{at } m = 3. \end{cases}
\]  

(7)

Because of complex dynamics and space-time structure of emission region in AA interactions both at intermediate and high \( \sqrt{s_{NN}} \) the difference is possible between pair ensembles which participate in Bose–Einstein correlations and Coulomb FSI. Therefore the third procedure for calculation of Coulomb correction seems most adequate for study of heavy ion collisions. The correction \( P_{\text{coul}}^{(3)} \) was used for femtoscopy analysis in energy domain from SPS to LHC in various experiments (see below Sec. 3).

In the paper the following set of femtoscopy observables \( G = \{G_4 \}_{i=1}^5 = \{\lambda, R_s, R_o, R_t, V\} \) is under consideration. This set of parameters characterizes the chaoticity of source and its 4-dimensional geometry at freeze-out stage completely. Moreover in the paper the using of normalized values of HBT-radii and volume of homogeneity region is
suggested in order to extend the set of types of collisions which can be studied in the framework of general approach. The normalized femtoscopy parameters $G_i$, $i = 2 - 5$ are calculated as follows:

$$R_i^0 = R_i / R_A, \quad i = s, o, l; \quad V^n = V / V_A.$$  

Here $R_A = r_0 A^{1/3}, V_A = 4 \pi R_A^3 / 3$ are radius and volume of spherically-symmetrical nucleus, $r_0 = (1.25 \pm 0.05) \text{ fm}$ \cite{20}. In the case of non-symmetrical nucleus-nucleus collisions the factors for normalization in (8) is defined by $\langle R_A \rangle = 0.5 (R_A^1 + R_A^2)$ is the arithmetical mean value of radii of colliding ions, where $\forall i: R_A^i$ is calculated based on the equation above for spherically-symmetrical nucleus.

### III. ENERGY DEPENDENCE FOR THE FEMTOSCOPY PARAMETERS

The investigation of energy dependence of femtoscopy parameters from set $G$ seems important, in particular, for search of creation of deconfinement state of strongly interacting matter (sQGP) in A+A collisions and for study of corresponding phase transitions. In accordance to some theoretical predictions the methods of correlation femtoscopy allow to search for qualitatively new physical effects in RHIC energy domain. Therefore the comparison of femtoscopy results at various initial energies is important for energy range as wide as possible. The dependencies of $\{G_i\}_{i=1}^4$ and $R_o / R_e$ on $\sqrt{s_{NN}}$ for secondary pions were shown elsewhere, for example, \cite{28, 30, 31}. The corresponding dependencies for charged kaons were discussed in \cite{17} for the first time. It would be noted that some new experimental results were obtained for the last years, in particular, the range of collision energy was extended in the TeV-region for secondary pions. Therefore in the paper the dependencies of set $G$ of femtoscopy parameters on $\sqrt{s_{NN}}$ are studied based on the all available experimental results which were obtained in the framework of approach for gaussian shape of correlation function.

Dependencies of femtoscopy parameters $G_i(\sqrt{s_{NN}})$, $i = 1 - 4$ and $R_o / R_e(\sqrt{s_{NN}})$ are shown in Figs. 2a – d and Fig. 3, respectively. The experimental results have been obtained for identical charged pion pairs with low $k_\perp$ and midrapidity in (quasi)symmetric heavy ion collisions. For correct comparison with results from previous measurement at intermediate energies the values of femtoscopy parameters at RHIC are shown for standard gaussian approximation of $C_i^2(q, K)$ and Coulomb correction $P_c^{(1)}(q)$. The more careful correction of correlation function on Coulomb final state interaction with help of $P_c^{(3)}(g)$ leads to some decreasing of $\lambda, R_c$ and smaller changing of other parameters from $G$ for Au+Au collisions at $\sqrt{s_{NN}} = 200 \text{ GeV}$ in comparison with standard procedure. These features are observed both in STAR \cite{21} and in PHENIX \cite{32} experiments at RHIC. Therefore accounting for results at intermediate energies it can be suggested that correction type on Coulomb FSI does not depend on general trends of energy dependencies of femtoscopy parameters in energy domain, at least $\sqrt{s_{NN}} \approx 17 - 200 \text{ GeV}$. As seen there is increasing of HBT radii (Figs. 2a – d) at growth of collision energy from $\sqrt{s_{NN}} \sim 20 \text{ GeV}$ up to maximum available LHC energy $\sqrt{s_{NN}} = 2.76 \text{ TeV}$. The chaoticity parameter $\lambda$ shows the weak changing at $\sqrt{s_{NN}} > 4 \text{ GeV}$ (Fig. 2). One can see that pion source is far from fully chaotic at LHC energy ($\lambda \approx 0.5$). Taking into account the results at RHIC and type of secondary particles under study (pions) one can suggest that decays of various resonance states influences on the chaoticity parameter even at LHC energy and leads to amplification of coherence of source. The study of multipion correlations is necessary for more definite physical conclusion. There is no significant increasing of ratio $R_o / R_e$ in all experimentally available energy domain (Fig. 2) which was predicted in the framework of ideal hydrodynamics for first order transition from hadronic to quark-gluon matter. Therefore it should be emphasized that one of the possible signatures of first order phase transition to the deconfinement state of strongly interacting matter is absent for soft observables in wide energy range.

The volume of homogeneity region in heavy ion collisions is calculated based on \cite{41} and known HBT-radii which are shown in Figs. 2a – d. The pion pairs with low $k_\perp$ are used in these calculations. Thus the homogeneity region volume obtained for such pairs can be considered as the estimation of volume of all emissions region. The energy dependence of estimations of emission region volume is shown in Fig. 3. As seen from the figure, the increasing of $V$ with growth of collision energy starts with $\sqrt{s_{NN}} \approx 5$ and it is close to the (quasi)linear behavior with $\ln(s / s_0)$, $s_0 = 1 \text{ GeV}^2$. It should be noted that the similar functional dependence was observed for $v_2(\sqrt{s_{NN}})$ for similar collision energy range, where $v_2$ is the elliptic flow \cite{33}. The increasing of both HBT-radii and source volume observed in Figs. 3 is explained by the growth of pion multiplicity for larger $\sqrt{s_{NN}}$. The $V$ increases significantly (about 1.5 – 2 times) at transition from the largest SPS energy ($\sqrt{s_{NN}} = 17.3 \text{ GeV}$) to the highest RHIC energy for heavy-ion mode ($\sqrt{s_{NN}} = 200 \text{ GeV}$). On the other hand linear sizes of pion source change weaker in this range of $\sqrt{s_{NN}} = 200$ (Figs. 2a – d). The HBT-radii, especially, $R_1$ increase substantially with growth of collision energy from RHIC to LHC. Perhaps, this behavior of $G_i, i = 2 - 4$ parameters can be explained as follows. The absolute increasing of $\sqrt{s_{NN}}$ for transition from SPS to RHIC is significantly smaller than that for further change from RHIC to LHC. Thus in the first case the energy range under consideration is not enough for substantial increasing of radii of pion source. On
the other hand the increasing of $\sqrt{s_{NN}}$ on $\approx 2.5$ TeV in the last case leads to clear growth all geometric parameters of emission region.

Fig. 4 shows the energy dependence of $\lambda$ (a), normalized HBT-radii (b – d) and $R_{\rho}/R_{\rho}$ ratio (e) both for symmetrical and non-symmetrical collisions of various nuclei. The corresponding dependence for $V^n$ is demonstrated in Fig. 5. As usual the festosity parameters from the set $\mathcal{G}$ depend on sign of electrical charge of secondary pions weakly. Thus the results for $\pi^{+}\pi^{+}$ pairs obtained in the experiments E802 for Al+Si collisions $[26]$ and NA44 for S+Pb $[34]$ collisions are shown in Figs. 4 and 5 also. As seen these results are in a good agreement with common trends. The large errors in Fig. 5 for strongly non-symmetrical nuclear collisions is dominated by large difference of radii of colliding moderate and heavy nuclei and corresponding large uncertainty for ($R_{\Lambda}$). The energy dependencies for set $\mathcal{G}$ of femtoscopy parameters shown in Figs. 4 and 5 demonstrate the reasonable agreement between the values of normalized parameters $[35]$ obtained for (quasi)symmetrical collisions of moderate nuclei and for strongly asymmetrical nuclear-nuclear interactions Si+Au, S+Pb with results for (quasi)symmetrical heavy ion collisions. Therefore the method suggested in the paper for normalized femtoscopy parameters allows to unite the study both symmetrical and non-symmetrical nuclear collisions in the framework of general approach. It seems this approach allows to obtain the general energy dependencies of femtoscopy parameters both for nucleus-nucleus and proton-(anti)proton collisions. This investigation is in the progress at the present time.

IV. GENERALIZED PARAMETERIZATION FOR THE CORRELATION FUNCTION

The shape of peak of the correlation function contains the unique experimental information about space-time structure of secondary particle source at freeze-out. The some physics investigations confirm the importance of detail study of shape of two-particle correlation function (see, for example, $[31]$). The parameterization of $K_{2}(A)$ depends on type of distribution which was chosen for emission region. In general there is rich class of random processes with additive stochastic variables for which (i.e. for these processes) there are finite distributions but the Central Limit Theorem (CLT) in the traditional (Gaussian) formulation is not valid. The class of random processes under considered are characterized by large fluctuations, power-law behavior of distributions in the range of large absolute values of random variables, non-analytic behavior of characteristic function of the probability distribution for small values of its arguments $[36]$. In mathematical statistics and probability theory the class of such distributions are called as stable (on Lévy) distributions$^1$ $[37]$. The general stable distribution is described by four parameters: an index of stability (or Lévy index) $\alpha \in (0, 2]$, a skewness parameter $\beta$, a scale parameter $\gamma$ and a location parameter $\delta$. These distributions satisfy with requirements of generalized Central Limit Theorem (gCLT) and self-similarity$^2$. Therefore the detail investigation of the shape of correlation peak have to do with verification of hypothesis of possible self-affine fractal-like geometry of emission region. At present the study of Lévy – Feldgeim distributions is the advanced region of mathematics but the specific case of central-symmetrical stable distributions is known in more detail $[38]$. Just this subclass of stable distributions is most important on the point of view of correlation femtoscopy. In this case the application of subset of non-isotropic central-symmetrical Lévy – Feldgeim distributions $[39]$ seems reasonable because the projections of $\vec{q}$ are independent random variables.

In accordance with discussion above the generalization is made in the paper of the experimental results for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV $[31]$ and model-independent approach for study of shape of correlation peak $[40]$. In general case the following multidimensional phenomenological parameterization of n-th order for CF $[1]$ is suggested:

$$C_{2}^{ph,n}(q, K) = \xi_{1}(q, K) \left[ 1 + \xi_{2}(q, K)K_{2}^{ph,n}(A) \right],$$

$$K_{2}^{ph,n}(A) = K_{2}^{ph,0}(A) \prod_{i=1}^{3} \prod_{j=1}^{3} \left[ 1 + \sum_{m=1}^{n} g_{mn} h_{mn}(A_{ij}) \right], \text{ at } n \geq 1.$$  \hspace{1cm} (9a)

where $K_{2}^{ph,n}$ – phenomenological parameterization of n-th order for cumulant correlation function $[?]$, functions $\xi_{1,2}(q, K)$ take into account formally all corrections on degree of source chaoticity, final state interactions, etc. The experimental and theoretical investigations in the field of correlation femtoscopy allow to derive some approach for

\footnotesize
$^1$ In literature for physics and mathematics the multidimensional distributions included in the class are called as Lévy – Feldgeim distributions.

$^2$ The applications of stable distributions in the physics of fundamental interactions and, in particular, for correlation femtoscopy are described, for example, in $[3]$. 

$\therefore$




cumnal two-particle function (???) in the lowest order. In the framework of the subset of non-isotropic central-symmetrical Lévy–Feldgeim distributions the most general parameterization of \( K_{2}^{\phi,0} \) can be given by

\[
K_{2}^{\phi,0}(A) = \prod_{i=1}^{3} \prod_{j=1}^{3} K_{2}^{\phi,0}(A_{ij}) = \exp\left( - \sum_{i,j=1}^{3} |A_{ij}|^{\alpha/2} \right), \quad K_{2}^{\phi,0}(z) = \exp(-|z|^{\alpha/2}).
\]  

Here were take into account that \( x \equiv (q_{i} R_{i})^{2}, i = 1, o, s \) for correlation femtoscopy, the products are on the space components of vectors. The \( \{h_{n}(x)\}_{n=0}^{\infty} \) is the closed system of orthogonal polynomials in the Hilbert space \( \mathcal{H} \):

\[
\int dx K_{2}(x)h_{n}(x)h_{m}(x) = \delta_{nm}, \quad g_{n} = \int dx K_{2}(x)h_{n}(x).
\]

The system \( \{h_{n}(x)\}_{n=0}^{\infty} \) for exponential weight function can be derived with the help of the following recurrent relations \( a_{1} h_{1}(x) = (x - b_{0})h_{0}(x), \quad a_{n+1}h_{n+1}(x) = (x - b_{n})h_{n}(x) - a_{n-1}h_{n-1}(x), \quad n = 1, 2, ... \) [41] and moments \( \mu_{n} = \int \int \int dx_{1}x_{2}x_{3} \exp\left( -|x|^{\gamma} \right) = 2^{\gamma-1}\Gamma(\gamma^{-1}[n+1]), \quad n \geq 0, \quad \gamma > 0 \) [42]. Here \( \forall n \geq 0 : b_{n} = \tilde{H}_{n+1}H_{n+1}^{-1} - \tilde{H}_{n}H_{n}^{-1}; \quad \forall n > 0 : a_{n} = H_{n+1}^{-1}\sqrt{H_{n+1}H_{n+1}} \), and \( \tilde{H}_{n}, H_{n} \) are the following determinants:

\[
H_{n} = \begin{vmatrix}
\mu_{0} & \ldots & \mu_{n-1} \\
\vdots & \ddots & \vdots \\
\mu_{n-1} & \ldots & \mu_{2n-2}
\end{vmatrix}, \quad \tilde{H}_{n} = \begin{vmatrix}
\mu_{0} & \ldots & \mu_{n-2} & \mu_{n} \\
\vdots & \ddots & \vdots & \vdots \\
\mu_{n-1} & \ldots & \mu_{2n-3} & \mu_{2n-1}
\end{vmatrix}.
\]

\( H_{0} = 1 \) and \( \tilde{H}_{0} = 0 \), the \( h_{0}(x) = \text{const} > 0 \) is defined by normalization which is chosen for system \( \{h_{n}(x)\}_{n=0}^{\infty} \) under consideration. The specific case \( \alpha = 1 \) and \( \alpha = 2 \) correspond to Cauchy and Gauss distributions respectively which are widely used in the correlation femtoscopy. For the first case the Laguerre polynomials, \( L_{n}(x) \), are used as \( \{h_{n}(x)\}_{n=0}^{\infty} \); the Hermite polynomials, \( H_{n}(x) \), are chosen as the closed system of orthogonal polynomials for the second specific case [40].

Perhaps, the generalized parameterization [40] contain the important physical information concerning the possible high irregular geometry of emission region and dynamics of its creation which is additional with respect to information derived for set \( \mathcal{G} \) of femtoscopy parameters based on traditional Gauss parameterization. It seems the future development of theoretical formalism is essential to definition of presence of this new physical information.

V. SUMMARY

The following conclusions can be obtained by summarizing of the basic results of the present study.

The dependencies of femtoscopy characteristics of emission region on \( \sqrt{s_{NN}} \) are studies for collisions of various ions. These dependencies are obtained for range of all experimentally available initial energies and for estimations of femtoscopy parameters from set \( \mathcal{G} \) derived in the framework of Gauss approach. For the first time the normalized values of radii and volume of source are suggested to use for energy dependence. This suggestion allows to expand the set of interaction types, in particular, on non-symmetrical nuclear-nuclear collisions which can be studied in the framework of common approach. There are no the sharp changing of femtoscopy parameter values, in particular, \( R_{0}/R_{s} \) with increasing of \( \sqrt{s_{NN}} \) which were predicted by some phenomenological models as signature of first order phase transition in strongly interacting matter.

The generalized parameterization for \( C_{2}^{K}(q, K) \) is suggested. This parameterization takes into account the expansion in closed system of orthogonal polynomials for general case of non-isotropic central-symmetrical Lévy–Feldgeim distribution.

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FIG. 1: The values of correction on Coulomb FSI $P_{\text{coul}}^{(m)}$ depend on $q_{\text{inv}}$ for different values of $\lambda$, where $m$ is the number of procedure for definition of Coulomb correction. The corrections are calculated for static spherically-symmetrical Gaussian source with $R = 6$ fm. The dashed line corresponds for $m = 1$, dotted line $- m = 2$, solid line $- m = 3$. 
FIG. 2: Dependencies of chaoticity parameter (a), HBT-radii (b – d) and ratio $R_o/R_s$ (e) on initial energy for central heavy ion Au+Au, Au+Pb, Pb+Pb collisions at midrapidity and $\langle k_{\perp} \rangle \simeq 0.2$ GeV/c [26, 28, 31–34]. Experimental results are demonstrated for pairs of $\pi^-$ mesons (in the case of ALICE – for $\pi^\pm\pi^\pm$ pairs) and for standard Coulomb correction $P_C^{(1)}(q)$ (in cases of ALICE, NA44, NA45, PHOBOS and STAR at $\sqrt{s_{NN}} = 62.4$ GeV – for correction $P_C^{(3)}$). Statistical errors are shown (for NA44 – total uncertainties).
FIG. 3: Energy dependence of volume of emission region at freeze-out for secondary charged pions in central heavy ion collisions Au+Au, Au+Pb, Pb+Pb in midrapidity region and at $\langle k_\perp \rangle \simeq 0.2$ GeV/c. Experimental results are shown for the same particle types and Coulomb corrections as well as in Fig. 2. Error bars are only statistical (for NA44 – total uncertainties).
FIG. 4: Energy dependence of $\lambda$ parameter (a), normalized HBT-radii (b – d) and ratio $R_o/R_s$ (e) in various nuclear-nuclear collisions at $\langle k_{\perp} \rangle \simeq 0.2$ GeV/c [26, 28, 31–34]. Experimental results are shown for central collisions (for minimum bias event in the case of E802 for Al+Si), for pairs of $\pi^-$ mesons (in cases ALICE and STAR for Cu+Cu – for $\pi^+\pi^-$ pairs, E802 for Al+Si, NA44 for S+Pb – for pairs of $\pi^+$ mesons) and for standard Coulomb correction $P_C^{(3)}(q)$ (in cases ALICE, NA44, NA45, PHOBOS, STAR both for Cu+Cu and for Au+Au at $\sqrt{s_{NN}} = 62.4$ GeV – for correction $P_C^{(3)}$). Statistical errors are shown (for NA44 – total uncertainties).
FIG. 5: Energy dependence of normalized volume of emission region at freeze-out for secondary charged pions in various nuclear-nuclear collisions at \( \langle k_\perp \rangle \approx 0.2 \text{ GeV}/c \). Experimental results are shown for the same collision, particle and Coulomb correction types as well as in Fig. 4. Error bars are only statistical (for NA44 – total uncertainties).