Earth matter effects in supernova neutrinos: optimal detector locations

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Abstract. A model-independent experimental signature for flavour oscillations in the neutrino signal from the next Galactic supernova (SN) would be the observation of Earth matter effects. We calculate the probability for observing a Galactic SN shadowed by the Earth as a function of the detector’s geographic latitude. This probability depends only mildly on details of the Galactic SN distribution. A location at the North Pole would be optimal with a shadowing probability of about 60\%, but a far-northern location such as Pyhäjärvi in Finland, the proposed site for a large-volume scintillator detector, is almost equivalent (58\%). We also consider several pairs of detector locations and calculate the probability that only one of them is shadowed, allowing a comparison between a shadowed and a direct signal. For the South Pole combined with Kamioka, this probability is almost 75\%; for the South Pole combined with Pyhäjärvi, it is almost 90\%. One particular scenario consists of a large-volume scintillator detector located in Pyhäsalmi to measure the geo-neutrino flux in a continental location and another such detector in Hawaii to measure it in an oceanic location. The probability that only one of them is shadowed exceeds 50\%, whereas the probability that at least one is shadowed is about 80\%. We provide an online tool to calculate different shadowing probabilities for the one- and two-detector cases.

Keywords: supernova neutrinos, neutrino detectors

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1. Introduction

Galactic core-collapse supernovae are rare (perhaps a few per century) [1, 2]. However, the large number of existing or future neutrino detectors with a broad range of science goals almost guarantees continuous exposure for several decades, so that a high-statistics supernova (SN) neutrino signal may eventually be observed. Such a measurement would provide a plethora of new insights that are crucial both for our astrophysical understanding of the core-collapse phenomenon [3] and for using SNe as particle-physics laboratories [4]. Of particular interest would be the observation of signatures for flavour oscillations, because this could address a key question of neutrino physics, i.e. whether the neutrino masses are ordered in a normal or inverted hierarchy [5]–[10].

The collapsed core of a SN emits neutrinos and anti-neutrinos of all flavours with comparable fluxes and spectra [11]–[14]. One expects differences between the spectra and fluxes of $\bar{\nu}_e$ and $\bar{\nu}_{\mu,\tau}$ or between $\nu_e$ and $\nu_{\mu,\tau}$ that are large enough to observe flavour oscillations, but it will be difficult to establish such effects solely on the basis of a $\bar{\nu}_e$ or $\nu_e$ ‘spectral hardening’ relative to theoretical expectations. Therefore, in the recent literature the importance of model-independent signatures has been emphasized, e.g. in association with the prompt $\nu_e$ neutronization burst [15] or with shock-wave propagation [16]–[19]. One unequivocal signature would be the observation of Earth matter effects, because they induce a characteristic energy-dependent modulation on the measured flux [6]–[8], [20, 21]. In a large-volume liquid scintillator detector such as the proposed 50 kt Low Energy Neutrino Astronomy (LENA) project [22], this modulation could be detected with high statistical significance [20]. Moreover, even if single detectors cannot resolve these modulations, comparing the signals from a shadowed detector with those from an unshadowed detector may allow one to diagnose Earth effects [8].

These intriguing possibilities have motivated us to investigate the probability for given detector locations to observe the next Galactic SN in an Earth-shadowed position. Most
of the Milky Way is in the southern sky, so a northern location is obviously preferred, but a quantitative determination of the probability distribution of Earth-shadowing as a function of geographic latitude is missing, except for a brief discussion in [6]. An additional motivation for our work is that large-volume scintillator detectors are being discussed for the purpose of geo-neutrino observations. After KamLAND’s pioneering measurement of the $\bar{\nu}_e$ flux from uranium and thorium $\beta$-decays in the Earth [23], performed in a complex geophysical environment with a large $\bar{\nu}_e$ background from power reactors, the exploration of different detector sites has become crucial. The choice of location could be influenced by the role of such detectors as excellent SN neutrino observatories, in particular because they need to operate for a long time to accumulate meaningful statistics for geo-neutrino studies. For example, if a large-volume scintillator detector were built in the Pyhäsalmi mine (Finland) [24] to measure the geo-neutrino flux mainly from the continental crust and another were built in Hawaii [25] to measure neutrinos from the oceanic crust, the ‘geographic complementarity’ of these locations with regard to SN shadowing is important.

We begin in section 2 with a discussion of the distribution of core-collapse SNe in the Galaxy, and then determine the SN probability distribution in the sky. In section 3 we determine the probability for Earth shadowing as a function of geographic location of one or two detectors. We also provide an online tool for the calculation of these probabilities [26]. We conclude our work in section 4.

2. Supernova distribution in the Milky Way

In this section we characterize the SN probability distribution in our Galaxy. In section 2.1 we present the models adopted for the SN volume distribution. In section 2.2 we discuss the probability of a core-collapse SN in terms of Galactic and Equatorial coordinates. Finally, in section 2.3 we estimate the distance distribution of SN events. The SN distribution in the Galaxy is expressed in cylindrical galactocentric coordinates $(r, z, \theta)$, where the origin corresponds to the Galactic centre, $r$ indicates the radial coordinate, $\theta$ is the azimuthal angle, and $z$ is the height with respect to the Galactic plane.

2.1. Models for the volume distribution

The probability distribution of core-collapse SNe in the Milky Way is not well known. These SNe mark the final evolution of massive stars and thus must occur in regions of active star formation, i.e. in the Galactic spiral arms. As proxies for the core-collapse SN distribution, one can use either observations of other galaxies, or in our Galaxy the distribution of pulsars and SN remnants (SNRs), the distribution of molecular hydrogen ($H_2$) and ionized hydrogen (HII) and the distribution of OB-star formation regions. (For a review, see [27] and references there.) These observables are either directly connected with core-collapse events (SNRs, pulsars) or with young, massive star formation activity and the related emission of ultraviolet light ($H_2$, HII, OB stars). All of these observables are consistent with a deficit of SNe in the inner Galaxy and a maximum of the probability at 3.0–5.5 kpc galactocentric distance.

The star formation activity is not smoothly distributed in the Galaxy. Besides generally following the spiral arms, it can be concentrated in small regions or spike-like complexes like the Cygnus OB association. Small regions of high star-forming activity
have also been found within 50 pc from the Galactic centre [28] that may contribute up to 1% of the total Galactic star formation rate, although this finding does not seem to contradict the overall picture of a reduced SN rate in the inner Galaxy.

However, we are only interested in the SN distribution projected on the sky, and the Earth’s rotation introduces an additional averaging effect. Therefore, it will be enough to consider a smooth distribution with azimuthal symmetry. In particular, we shall use the following common parametrization for the Galactic surface density of core-collapse (cc) events,

$$\sigma_{cc}(r) \propto r^\xi \exp(-r/u), \quad (1)$$

where $r$ is the galactocentric radius. For the birth location of neutron stars, a fiducial distribution of this form was suggested with the parameters [29]

Neutron stars: \[ \begin{align*} \xi &= 4, \\ u &= 1.25 \text{ kpc}. \end{align*} \quad (2) \]

These parameters are consistent with several SN-related observables, even though large uncertainties remain [29]. On the other hand, the pulsar distribution indicates [30]

Pulsars: \[ \begin{align*} \xi &= 2.35, \\ u &= 1.528 \text{ kpc}. \end{align*} \quad (3) \]

We will use the parameters of equation (2) as our benchmark values and those of equation (3) as an alternative model to illustrate the dependence of our results on different input choices.

In figure 1 we show the normalized Galactic surface density of core-collapse SNe according to equation (1) as a function of the galactocentric distance for the two choices.
of parameters given in equations (2) and (3). Our benchmark distribution shows a peak at $r = 5 \text{kpc}$, while for the parameters of equation (3) the surface density peaks at a lower distance, $r \approx 3 \text{kpc}$. The distributions are normalized as $\int dr \sigma(r) 2\pi r = 1$, i.e. the surface densities $\sigma$ are given in SNe per kpc$^2$. Of course, to obtain the number of SN events per Galactic unit surface and unit time, the quantities $\sigma$ would have to be multiplied by the integrated Galactic SN rate. However, since we are only interested in the relative probability of SNe in different regions of the sky, the overall rate is not important for our study.

The vertical distribution of neutron stars at birth with respect to the Galactic plane can be approximated by the superposition of a thin Gaussian disk with a scale height of 212 pc and a thick disk with three times this scale height, containing 55% and 45% of the pulsars, respectively [27],

$$R_{cc}(z) \propto 0.79 \exp \left( -\left( \frac{z}{212 \text{ pc}} \right)^2 \right) + 0.21 \exp \left( -\left( \frac{z}{636 \text{ pc}} \right)^2 \right), \quad (4)$$

where $z$ is the height above the Galactic plane. We assume that this vertical distribution is independent of galactocentric distance [30] so that

$$n_{cc}(r, z) \propto \sigma_{cc}(r) R_{cc}(z) \quad (5)$$

is the volume distribution.

We stress that the distribution of core-collapse SNe differs significantly from the overall matter distribution, particularly in the inner part of the Galaxy. The distribution of Type Ia SNe—which are believed to originate from old stars in binary systems—more closely follows the matter distribution. It can be parameterized as [27]

$$n_{Ia}(r, z) = \sigma_{Ia}(r) R_{Ia}(z) \propto \exp \left( -\frac{r}{4.5 \text{ kpc}} \right) \exp \left( -\frac{|z|}{325 \text{ pc}} \right). \quad (6)$$

In figure 1 we show the monotonically falling SNe Ia surface density $\sigma_{Ia}(r)$ for comparison with the core-collapse case.

### 2.2. Projection on the sky

The probability of a core-collapse SN as a function of the Galactic longitude $l$ and latitude $b$ is given by an integration along the line of sight:

$$P(l, b) \propto \int_0^\infty ds \ n_{cc}[r(s, l, b), z(s, b)], \quad (7)$$

where

$$r = \left( s^2 \cos^2 b + d_\odot^2 - 2sd_\odot \cos l \cos b \right)^{1/2}, \quad (8)$$

$$z = s \sin b.$$
The transformation relating the two systems of coordinates is

\[
\begin{align*}
\sin \delta &= \sin b \sin \delta_{\text{NGP}} + \cos b \cos \delta_{\text{NGP}} \sin(l - l_0), \\
\cos(\alpha - \alpha_0) &= \cos(l - l_0) \cos b/\cos \delta, \\
\sin(\alpha - \alpha_0) &= \left[-\sin b \cos \delta_{\text{NGP}} + \cos b \sin \delta_{\text{NGP}} \sin(l - l_0)\right]/\cos \delta,
\end{align*}
\]

where, for the Julian epoch J2000, the coordinates of the north Galactic pole (NGP) are \(\alpha_{\text{NGP}} = 12 \, \text{h} \, 51.42 \, \text{m} \) and \(\delta_{\text{NGP}} = 27^\circ 07.8'\). Hence, the ascending node of the Galactic equator is at \(\alpha_0 = 282.86^\circ\) and \(l_0 = 32.93^\circ\). Further details on astronomical coordinate systems can be found, for example, in [31].

The sky map in Equatorial coordinates is shown in the upper panel of figure 2 for our benchmark distribution. However, since the neutrino detectors are fixed to the Earth, we only need this distribution averaged over time, i.e., over right ascension \(\alpha\). Therefore, all relevant information is contained in the ‘exposure probability function’,

\[
\omega(\delta) \propto \int_{-\pi}^{+\pi} d\alpha P(\alpha, \delta).
\]
Figure 3. The ‘exposure function’ $\omega(\delta)$ for different Galactic SNe distributions. Solid curve: Benchmark model based on equation (1) with the parameters of equation (2). Dashed curve: same with the alternative parameters of equation (3). Dotted curve: distribution of SNe Ia for comparison.

It provides the probability distribution of the arrival direction of a SN signal in terms of declination. We show the time-averaged sky map in the bottom panel of figure 2, normalized as $\int d\delta \omega(\delta) \cos \delta = 1$. This normalization also fixes the overall constant for the function $P(\alpha, \delta)$ plotted in the top panel of figure 2. Figure 2 clearly shows the preference of southern locations in the sky and also shows the polar regions that are nearly void of SNe because the Milky Way extends approximately between declinations of $-60^\circ$ to $+60^\circ$.

In figure 3 we show the exposure probability function $\omega(\delta)$ for different assumptions about the Galactic SN distribution, i.e. our benchmark distribution (equation (1)) with the parameters of equation (2), the alternative parameters of equation (3) and, as an extreme case, the SN Ia distribution of equation (6) which, of course, is not realistic for core-collapse SNe. The normalization $\int d\delta \omega(\delta) \cos \delta = 1$ is adopted throughout. In this normalization of $\omega(\delta)$, an isotropic SN distribution would correspond to a horizontal line in figure 3. The exposure function depends only mildly on details of the assumed Galactic SN distribution, because the geometric effect dominates such that the SN probability is negligible outside the Galactic disk. The largest model variation occurs around a declination of $-30^\circ$ near the Galactic centre region. The edges at about $\pm 60^\circ$ correspond to the region where the Milky Way would end in the sky if it were infinitely thin. The tails beyond these declinations come from the vertical extension of the Galactic disk around us, where we have assumed the solar system to be located exactly in the Galactic plane.

2.3. Distance distribution

As an aside, we use our assumed core-collapse SN distribution in the Galaxy to evaluate the SN distance distribution relative to the solar system. The average distance is

$$\langle d_{cc} \rangle = \frac{\int n_{cc}(r, z) d(r, z, \theta) r \, dr \, dz \, d\theta}{\int n_{cc}(r, z) 2\pi r \, dr \, dz},$$

(13)
Figure 4. SN probability versus distance from the Sun. Solid curve: benchmark model based on equations (1) and (2). Dotted curve: SNe Ia assuming equation (6).

where

\[ d(r, z, \theta) = \left( (x - x_\odot)^2 + (y - y_\odot)^2 + (z - z_\odot)^2 \right)^{1/2} = \left[ r^2 + z^2 + d_\odot^2 - 2rd_\odot \cos \theta \right]^{1/2}. \]  

(14)

For the distribution \( n_{cc} \) of equation (5) with our benchmark parameters of equation (2), we find \( \langle d_{cc} \rangle = 10.7 \) kpc with a root mean square (rms) dispersion of 4.9 kpc. While the average distance agrees with the fiducial distance of 10 kpc that is frequently assumed in the literature, we note that the dispersion of distances is very large. This is clearly visible in figure 4, where we plot the probability distribution \( \Pi(d) \) of SN events as a function of the distance \( d \) from the Sun, so that \( \int_a^b \Pi(x) \, dx \) gives the probability that a SN happens between distance \( a \) and \( b \) from the Sun. The dip in the middle corresponds to the deficit of core-collapse SNe in the Galactic centre region. For comparison, we also show the probability distribution for SNe Ia according to equation (6). Their average distance is 11.9 kpc, with a rms dispersion of 6.0 kpc. On the other hand, the median distances are quite similar for the two distributions, and equal to about 10.9 kpc. The large-distance behaviour for both cases differs significantly due to the different scales in the exponential tails of equations (1) and (6). Of course, the scarcity of data at large galactocentric distances implies that the extrapolation may be unphysical.

3. Optimal detector location

3.1. One detector

Armed with the exposure function \( \omega(\delta) \) shown in figure 3, we now determine the probability that a detector located at a geographic latitude \( \lambda \) will observe the next Galactic SN below the Earth’s horizon. Of course, observable matter effects would require
a minimal path-length of a few thousand kilometers [20] so that our Earth-shadowing
criterion is somewhat schematic. Moreover, the phenomenological signature of Earth
matter effects also depends on whether or not the neutrinos cross the core [21,32,33]. We
use a core radius \( R_c = 3486 \text{ km} \) and an Earth radius of \( R_\oplus = 6371 \text{ km} \) [34]. The Earth
or core shadowing condition for a source with altitude \( a \) with respect to the horizon is
\[
\sin a < \kappa = \begin{cases} 
0, & \text{Earth shadowing,} \\
-\sin a_c, & \text{Core shadowing,}
\end{cases}
\]

where
\[
\sin a_c = \sqrt{1 - \left(\frac{R_c}{R_\oplus}\right)^2} = 0.837.
\]

In general, for a neutrino path-length \( L \) in the Earth, one has
\[
\kappa = -\sin a_L = -\frac{L}{2R_\oplus}.
\]

The altitude of an object of Equatorial coordinates \((\alpha, \delta)\) is
\[
\sin a = \sin \lambda \sin \delta + \cos \lambda \cos \delta \cos H,
\]
where \( H = t - \alpha \) is the hour angle and \( t \) the local sidereal time. Since no time information
on the next SN is available in advance, \( H \) has to be taken as a random variable.

For polar locations \((\lambda = \pm \pi/2)\) the shadowing condition simplifies, because \( \cos \lambda = 0 \)
and the time dependence disappears. The geometrical probability \( p_\kappa(\lambda, \delta) \) for a trajectory
to satisfy one of the shadowing conditions is
\[
p_\kappa \left(\pm \frac{\pi}{2}, \delta\right) = \Theta(\kappa \mp \sin \delta),
\]

where \( \Theta(x) \) is the usual step function. A similar simplification holds for objects located
at the celestial poles \((\delta = \pm \pi/2)\) for which \( \sin a = \pm \sin \lambda \) and
\[
p_\kappa \left(\lambda, \pm \frac{\pi}{2}\right) = \Theta(\kappa \mp \sin \lambda).
\]

Equation (20) is related to equation (19) by the symmetry \( p_\kappa(\lambda, \delta) = p_\kappa(\delta, \lambda) \). This is a
general property following directly from equation (18).

Apart from these special cases, we have both \( \cos \lambda > 0 \) and \( \cos \delta > 0 \), so that the
general shadowing condition (equation (15)) can be recast as
\[
\cos H < -\tan \lambda \tan \delta + \frac{\kappa}{\cos \lambda \cos \delta},
\]
or equivalently
\[
\cos H > \tan \lambda \tan \delta - \frac{\kappa}{\cos \lambda \cos \delta} \equiv h_\kappa(\lambda, \delta).
\]

Note that we have replaced \( H + \pi \) with \( H \) in the argument of the cosine, since both are
random variables. Unless equation (22) is always or never satisfied, one has
\[
-\arccos h_\kappa(\lambda, \delta) < H \text{(mod } 2\pi) < \arccos h_\kappa(\lambda, \delta).
\]
The general solution is then
\[
p_\kappa(\lambda, \delta) = \begin{cases} 
\Theta[1 - h_\kappa(\lambda, \delta)][\Theta[-h_\kappa(\lambda, \delta) - 1] & \text{for } |h_\kappa(\lambda, \delta)| \geq 1, \\
\frac{1}{\pi} \arccos h_\kappa(\lambda, \delta) & \text{otherwise.}
\end{cases}
\]
(24)

This follows because all values of the random variable \(H(\mod 2\pi)\) in the interval \((-\pi, \pi]\) are equally likely. As a check of equation (24), we note that, for a detector at the equator (\(\lambda = 0\)), it simplifies to
\[
p_\kappa(0, \delta) = \frac{1}{\pi} \arccos \left( \frac{\kappa}{\cos \delta} \right),
\]
(25)
reproducing the trivial result \(p_0(0, \delta) = 1/2\).

In order to obtain the Earth-shadowing probability for the neutrinos from the next Galactic SN, given the detector latitude \(\lambda\), we must convolve with the distribution of source declination angles, i.e. with the exposure function, so that
\[
P_\kappa(\lambda) = \int \cos \delta \omega(\delta) d\delta,
\]
(26)
where we have used the normalization \(\int \cos \delta \omega(\delta) = 1\).

We show the Earth-and core-shadowing probability as a function of detector latitude in figure 5. The solid lines refer to our Galactic benchmark distribution (equation (2)) with parameters (equation (3)), whereas the dashed lines refer to the alternative parameters (equation (3)). Once more we find that the dependence on details of the Galactic distribution is mild. Note that, for the Earth-shadowing case, \(P_0(\lambda) + P_0(-\lambda) = 1\), because all information on the longitude is lost and two locations at \(\lambda\) and \(-\lambda\) are complementary: if one detector is shadowed, the antipodal one is certain not to be shadowed.
Optimal detector locations for SN neutrinos

Table 1. Representative locations of proposed or existing SN neutrino detectors and neutrino shadowing probabilities, assuming our benchmark Galactic distribution.

| Location              | Latitude | Longitude | Earth Core |
|-----------------------|----------|-----------|------------|
| Pyhäsalmi, Finland    | 63.66° N | 26.04° E  | 0.581 0.116 |
| Soudan, USA           | 47.82° N | 92.23° W  | 0.572 0.112 |
| Fréjus, France–Italy  | 43.43° N | 6.73° E   | 0.568 0.110 |
| Kamioka, Japan        | 36.27° N | 137.3° E  | 0.560 0.104 |
| Hawaii, USA           | 19.70° N | 156.30° W | 0.528 0.082 |
| Sydney, Australia     | 33.87° S | 151.22° E | 0.445 0.069 |
| South Pole            | 90° S    | —         | 0.414 0.064 |

The largest probability for Earth shadowing is at the North pole, but the Pyhäsalmi site in Finland is almost equivalent. The behaviour for the core-shadowing condition is similar, although the advantage of a northern site is somewhat boosted: for \( \kappa = -\sin \alpha_c \), the probability \( P_\kappa(\lambda) \) varies by almost a factor of two moving from a far-southern location to a far-northern location, where it reaches almost 12%. If one asks for a minimal neutrino path-length of \( L = 3000 \text{ km} \) [20], the shadowing probability is about 10% less than the shown Earth-crossing case. However, the minimum path-length to detect Earth signatures depends on the detailed features of the flavour-dependent neutrino fluxes and on the detector properties. Therefore, to keep our discussion simple and general, we restrict ourselves to the illustrative cases of Earth and core crossing. Note that the generic case can be calculated with the online tool [26].

Some representative locations are tabulated in table 1. The list is not exhaustive, but is rather meant to provide representative and complementary geographic locations. It includes two existing locations of high-statistics experiments sensitive to SN neutrinos, i.e. Super-Kamiokande in Japan [35] and IceCube, which is under construction at the South Pole [36], and a few possible locations of next-generation detectors proposed in the literature. In particular, the possibility of a Megatonne water-Cherenkov detector is discussed worldwide, including the Underground nucleon decay and Neutrino Observatory (UNO), with a possible location in the Soudan mine (Minnesota, USA) [37], the Hyper-Kamiokande (HK) detector in the Tochibora Mine (Kamioka region, Japan) [38], and the MEgaton class PHYSics (MEPHYS) detector at the Fréjus site (France–Italy) [39]. Different locations are under study for the large-volume scintillator detector LENA. The Pyhäsalmi mine in Finland and Hawaii are of interest for geo-neutrino studies. Hawaii and Australia have also been discussed as favoured sites for detecting the cosmic diffuse SN neutrino background, because of the low reactor background [40].

3.2. Two detectors

Next we consider simultaneous SN neutrino detection by two different detectors. A single detector can observe Earth matter effects only if it has good energy resolution and, of course, enough statistics. A scintillator detector like the 50 kton LENA project has good energy resolution and can detect these signatures unambiguously, whereas water-
Cherenkov detectors have an intrinsically poorer energy resolution so that Megaton-scale masses would be necessary \cite{20}. However, smaller detectors, even if they cannot resolve the modulations directly, could still reveal Earth effects by comparison of the signals from a shadowed detector with an unshadowed detector. Of particular interest are Super-Kamiokande in Japan and IceCube at the South Pole, which will be completed in a few years. In this case, the Earth effect will be detected as a difference in the signal normalization between the two detectors \cite{8}. To this end, it would be crucial that one of the detectors is shadowed whereas the other is not, i.e. we are interested in the probability for exactly one of them to be shadowed.

The special location of IceCube makes this kind of problem a very simple generalization of the previous one-detector case. The shadowing condition of equation (15) is satisfied or missed at the South Pole with a probability given by the lower or upper sign of equation (19), and at any other location according to equation (24) or its complement to 1. Note that we can safely use the formulae of section 3.1, since the location of one detector at the South Pole implies that no longitude difference enters the problem.

In the general case of two detectors at geographic latitudes $\lambda$ and $\lambda'$ with a difference in longitude of $\Delta$, the altitudes $a$ and $a'$ of an object located at $(\alpha, \delta)$ are
\[
\begin{align*}
\sin a &= \sin \lambda \sin \delta + \cos \lambda \cos \delta \cos H, \\
\sin a' &= \sin \lambda' \sin \delta + \cos \lambda' \cos \delta \cos(H + \Delta).
\end{align*}
\] (27)

To maximize the chance to detect Earth matter effects, the most interesting case for two general detectors is that exactly one of them sees a SN from a shadowed position, thus allowing one to compare a shadowed signal with an unshadowed signal. The probability for detector I to be shadowed and detector II not to be shadowed is: (i) zero if one of the two probabilities vanishes when calculated as described in the previous section; (ii) trivially equal to the one-detector case when the probability of one of the single-detector conditions is 1; (iii) in all remaining cases, it can be evaluated from the following system of inequalities, where we omit (mod $2\pi$):

\[
-\arccos h_\kappa(\lambda, \delta) < H < \arccos h_\kappa(\lambda, \delta), \\
\arccos h_\kappa(\lambda', \delta) < H + \Delta < 2\pi - \arccos h_\kappa(\lambda', \delta).
\] (28)

The complementary situation, i.e. detector II is shadowed and detector I not, is

\[
\arccos h_\kappa(\lambda, \delta) < H < 2\pi - \arccos h_\kappa(\lambda, \delta), \\
-\arccos h_\kappa(\lambda', \delta) < H + \Delta < \arccos h_\kappa(\lambda', \delta),
\] (29)
apart for the trivial cases (i) and (ii) described above.

It is also worthwhile considering the situation that two detectors with excellent energy resolution are available, e.g. two scintillator detectors. We have already mentioned that next-generation scintillator detectors may be built at two different sites, e.g. one in Europe and another in Hawaii, for the purpose of geo-neutrino research. In this case, the most interesting case is that at least one detector sees a shadowed signal. The generalization of the previous formulae to this case is straightforward.

Our results are reported in table 2 for the Earth-crossing case and in table 3 for core-crossing case. The quantities listed in each cell at row $i$ and column $j$ are the following:
Table 2. Shadowing probability for two detectors. The quantities reported in each cell are explained in equation (30).

| Locations   | South Pole | Sydney | Hawaii | Kamioka | Fréjus | Soudan |
|-------------|------------|--------|--------|---------|--------|--------|
| Pyhäsalmi   | 0.519      | 0.457  | 0.285  | 0.179   | 0.065  | 0.148  |
|             | 0.353      | 0.315  | 0.231  | 0.157   | 0.052  | 0.139  |
|             | 0.062      | 0.130  | 0.296  | 0.400   | 0.516  | 0.433  |
|             | 0.475      | 0.437  | 0.187  | 0.221   | 0.162  |        |
| Pyhäsalmi   | 0.317      | 0.310  | 0.142  | 0.208   | 0.158  |        |
|             | 0.307      | 0.361  | 0.285  | 0.220   |        |        |
|             | 0.097      | 0.135  | 0.386  | 0.351   | 0.410  |        |
|             | 0.461      | 0.484  | 0.326  | 0.230   |        |        |
| Soudan      | 0.290      | 0.164  | 0.152  | 0.124   | 0.281  | 0.375  |
|             | 0.124      | 0.281  | 0.375  |        |        |        |
|             | 0.362      | 0.278  | 0.184  |        |        |        |
| Fréjus      | 0.249      | 0.168  |        |         |        |        |
|             | 0.165      | 0.277  |        |         |        |        |
|             | 0.160      |        |        |         |        |        |
| Kamioka     | 0.129      |        |        |         |        |        |
|             | 0.285      |        |        |         |        |        |

For example, in table 2 in the row ‘Pyhäsalmi’ and column ‘South Pole’ we read that the probability that a SN will be shadowed at Pyhäsalmi but not at the South Pole is 0.519, and for the reverse case it is 0.353, whereas the chance that both detectors see it shadowed is 0.062.

For the special case where one of the sites is the South Pole, it is intuitively obvious that Pyhäsalmi is the best location among the proposed locations, being furthest north. For the most conservative scenario, in which no new large detectors will be built, the combination of IceCube at the South Pole plus the Super-Kamiokande detector already existing in Japan offers a 73% probability that a comparison between a shadowed neutrino signal and an un-shadowed neutrino signal could be observed, and a 17% probability for this comparison in the case of core shadowing. Notice that the case of a detector in Pyhäsalmi and another one in Hawaii, which is of importance for geo-neutrino research, also offers a nice opportunity for Earth effect detection in SN neutrinos. The probability that exactly one of them is in a shadowed position exceeds 50% (20% for core shadowing), whereas the probability that one or both are shadowed is about 80% (20% for core shadowing).
4. Conclusions

The possibility of detecting Earth matter effects in the neutrino signal from the next Galactic SN is a powerful tool for probing the neutrino mass hierarchy. Next-generation large-volume detectors with excellent energy resolution would offer the opportunity to detect directly the specific signature of an energy-dependent modulation of the measured neutrino flux. Even if current detectors could not reconstruct these modulations directly, the comparison of the neutrino signal in a shadowed detector with an unshadowed detector could allow one to detect Earth effects. Of course, it is assumed that the location of the SN in the sky can be determined by observations in the electromagnetic spectrum. In the unlikely case that this is not possible, neutrinos alone can be enough to determine the SN location with sufficient precision [41, 42].

Motivated by these opportunities, we have provided the first detailed study of the probability that a detector at a given geographic latitude will observe a Galactic SN in an Earth-shadowed or core-shadowed position. We have shown that this probability is rather insensitive to detailed assumptions about the uncertain distribution of core-collapse events in the Galaxy. The main effect is the simple geometric constraint that SNe occur within the Galactic disk.

We find that a far-northern location such as the Pyhäsalmi mine in Finland, the preferred site for the LENA scintillator detector, is almost optimal for observing a SN signal shadowed by the Earth. The shadowing probability is close to 60%, against an average of 50% for a random location on the Earth. The shadowing probability $P_0(\lambda)$ depends only mildly on the latitude $\lambda$ for $\lambda > 40^\circ$, a condition fulfilled by most of the northern locations proposed for next-generation experiments. The behaviour for core locations is typically lower than for Pyhäsalmi.
shadowing is similar, although the advantage of a northern site is more pronounced. The core-crossing probability varies by almost a factor of two between a far-southern and a far-northern site, where it reaches almost 12%.

We have also studied the case of two detectors. It is of interest because one may be able to compare a shadowed SN neutrino signal with an unshadowed SN neutrino signal and thus diagnose Earth effects, even if both detectors lack the energy resolution that is necessary to observe a modulated signal. On the other hand, if both detectors see a shadowed signal, one may perform Earth tomography because the observed neutrinos would cross different geophysical layers [43]. For the South Pole, where IceCube will be completed in a few years, combined with Kamioka, the probability that at least one of them is shadowed is almost 75%; for the South Pole combined with Pyhäsalmi, it is almost 90%.

One particular scenario consists of a large-volume scintillator detector located at Pyhäsalmi to measure the geo-neutrino flux in a continental location and another such detector in Hawaii to measure it in an oceanic location. The probability that exactly one of them is shadowed exceeds 50%, whereas the probability that one or both are shadowed is about 80%. Therefore, Pyhäsalmi and Hawaii are not only complementary for the purpose of geo-neutrino observations, but also for observing Earth matter effects in SN neutrinos.

The various probabilities relevant for two detectors at arbitrary locations are difficult to represent in a useful figure or table. Therefore, based on our benchmark Galactic SN distribution, we provide an online tool that allows one to calculate the various Earth and core-shadowing probabilities for one or two detectors at arbitrary geographic locations [26].

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