The Reduced Stress Method (EN-1993-1-5) In Designing of The Plated Structural Elements

Al-Aqbi Ameer Tawfeeq¹ and Nicolae Popa²

¹ Ph.D. Degree candidate at FILS, Technical University of Civil Engineering, Bulevardul Lacul Tei 124, Bucharest, 020396 Romania
² Department of the Strength of Materials, Bridges and Tunnels, Technical University of Civil Engineering, Bulevardul Lacul Tei 124, Bucharest, 020396 Romania

ameeraqbi@yahoo.com

Abstract. The paper describes the main features and advantages of European standard EN 1993-1-5 in the design of plated structures. Compared to other types of steel structures, plated structures are more prone to local buckling and therefore required design rules to cover such a phenomenon. The concept of the article focuses on The Reduced Stress Method. A steel box girder chooses for numerical example was fabricated of S235 steel, has 7 panels with transverse and longitudinal stiffeners and was tested in 1985 at the Testing Laboratory of the Research Institute for Transportation Engineering, in Bucharest.

1. Introduction

The modern steel structures are characterized by a lightweight, slender and fabrication optimized design. The progress in welding technology since the 1930s has facilitated the increased application of steel plated structures, (Fig.1), especially for bridges. Compared to other types of steel structures, the plated structures are more prone to local buckling and therefore required design rules to cover such a phenomenon. In the last 50 years, after four serious accidents, bridges collapsed, in Austria, Great Britain, Australia and Germany (1969-1972), [1] important research programs were developed with the main aim to provide knowledges for designers of plated structures. The results of these research programs have clearly influenced the design as well as the development of the design standards. A good example is the Eurocodes which harmonized the European rules of design in civil engineering. In Eurocode 3 the rules used to prevent local buckling in steel plated structures are collected in part 1-5: Plated structural elements (EN1993-1-5). Based on this part of Eurocode 3 the designer can choose, mainly between two different types of design methods: Effective width method and Reduced stress method. If the effective width method needs four steps for resistance verification the reduce stress method needs only one step.
The effective width method comprises resistance models for bending moment, shear force, transverse force and interaction criterion. This method is very efficient for standard geometries of structural elements because it accounts not only for post-critical reserve in a single plate element but also for load shedding between cross section elements.

The general form of the Reduced Stress Method simplified its use for serviceability verifications and for the design of non-uniform members such as hunched beam, beam webs with openings and plates with non-orthogonal stiffeners. This method assumes a linear stress distribution up to the stress limit of the plate element which buckles first. Until this stress limit has been reached, the cross section is fully effective. The cross section verified according to section 10, EN 1993-1-5, can be categorized as class 3 members, assuming no load shedding from among cross section elements. This method does not consider load shedding from highly stressed to less stressed plate elements. As a result the weakest plate element in a steel plated cross section governs the resistance of the entire cross section.

2. Basic procedure and numerical example (EN 1993-1-5, section 10)
The Reduced Stress Method may be used to determine the stress limits for the stiffened or unstiffened plates. This method uses von Mises criterion to take into account the interaction between different stress types. For unstiffened or stiffened panels subjected to combined stress $\sigma_x$, $\sigma_z$, $\tau$, class 3 section properties may be assumed. The basic procedure of the Reduced Stress Method is found in the section 10, and Annex A1 of EN 1993-1-5.

2.1. Description of the girder model
A “Fabricated and Tested of the Steel Box Girder” PhD Thesis submitted to the UTCB, [3] box steel girder was chosen, in the numerical example, for application of the reduced stress method for a plated structure (Fig.2). The characteristic cross sections of the girder are found in (Fig.3).

Figure 1. A modern steel plated structure of bridges during construction. [2]
Figure 2. Lateral view of the box steel girder. [3]

Figure 3. Characteristic cross section of the box girder. [3]

The box girder was fabricated of S235 steel and has 7 panels out of which 6 panels, between the support sections (bearings) and the application sections of the concentrated external forces P, are provided with longitudinal stiffeners and the thickness of the webs (walls) is 4,0 mm. The central panel has webs of 10 mm thickness and a joint with calibrated prestress high strength bolts.

This box girder was tested in 1985 at the Testing Laboratory of the Research Institute for Transportation Engineering, from Bucharest (Fig. 4). The experimental research was a part of studies developed for a new standard for design of steel structures for railway bridges. The choosing of this box girder for the numerical example of this paper was due its complexity as a plated structure and the value limit for loads applied by 4 hydraulic press in the field of the elastic behavior. By measurements during testing it was established a value of P= 350 KN, for each hydraulic press, as a limit load for elastic behavior. At these values of loads (2x2x350 kN) the first permanent deformation was noticed in the webs of Panel 0-1 (Fig. 5)
2.2. Statically schematic of the girder, loads and stresses diagram, at ULS.

The statically schematic of the girder, the established value during testing of 2P= 700 KN, as a limit load for elastic behavior. The shear force, bending moment diagrams is shown in (Fig. 6)

2.3. Main parameters of the cross-section

2.3.1. Panel 2-3.

\[ M_y^{2-3} = 700 \times 2500 = 1.75 \times 10^6 \text{ KN.mm} \]

\[ V_z = 700 \text{ KN} \]
Panel 2-3 was chosen because this panel has the maximum bending moment and maximum shear force acting together (simultaneous) (Fig.6). The gross cross-section of this panel is shown in (Fig.7):

- **Elastic shear lag in the flange (EN1993-1-5, section 3):**

  The effective width $b_{eff}$ for the shear lag under elastic conditions should be determined from equation 3.1, EN1993-1-5(6):
  \[
  b_{eff} = \beta b_o
  \]
  \[L_e = L = 7000 \text{ mm}\]
  Shear lag in the flange maybe neglected if:
  \[b_o < L_e/50\]

  **Upper flange**
  \[b_o = 100 \text{ mm} \text{ (see Fig.9)}\]
  \[b_{eff} = \beta b_o\]
  \[b_o = 100 \text{ mm} < 7000/50 = 140 \text{ mm}\]
  \[b_{eff} = \beta b_o\]
  \[b_{o2} = 350 \text{ mm} > 7000/50 = 140 \text{ mm}\]

  To calculate the effective width factor $\beta$ Table 3.1, EN1993-1-5 should be used:
  \[
  \alpha_o = \sqrt{1 + \frac{As_l}{b_o.1}}
  \]
  \[\alpha_o = \sqrt{1 + \frac{150 + 6}{350 + 12}} = 1.1\]
  \[k = \frac{\alpha_o b_o}{L_e}\]

  (see EN1993-1-5 table 3.1)
  \[k = \frac{1.1 \times 350}{7000} = 0.055\]
  \[0.02 < k < 0.055 < 0.7, \text{ hogging bending}\]
  \[\beta = \frac{1}{1 + 6 \left( k - \frac{1}{2500 - k} \right) + 1.6 k^2}\]
  \[\beta = \frac{1}{1 + 6 \left( 0.055 - \frac{1}{2500 - 0.055} \right) + 1.6 \times 0.055^2} = 0.774\]
  \[b_{o2} = b_{eff} = 0.774 \times 350 = 270.9 \text{ mm}\]

  **Bottom flange**
  \[b_o = 125 \text{ mm} < 7000/50 = 140 \text{ mm}\]

- Effective cross-section taking into account of the elastic shear lag in the flanges is shown in (Fig.8).

- **Center of gravity ($Z_{CG}$) for the effective cross section**
  \[Z = \frac{\sum x_i A_i}{\Sigma A_i}\]
where:

\[ z_i : \text{is the vertical distance from the middle of each element cross section to the axis } y \]

\[ A_i : \text{is the area of cross section} \]

\[ Z_{CG} = \frac{1033 \times (370.9 \times 12) + 527 \times (1000 + 4) + 21 (250 + 12) + 777 \times (70 + 10) + 277 \times (70 + 10) + 7.5 (200 + 15)}{(370.9 + 12) + (1000 + 4) + (250 + 12) + 3 \times (70 + 10) + (200 + 15)} \]

\[ = 477.19 \text{ mm} \]

**Figure 8. Effective cross-section in the panel 2-3 and normal stress } \sigma_{x,Ed} \]

- **The moment of inertia (} I_y \)**

\[ I_y = \sum I_{yi} + A_i \times z_i^2 \]  \hspace{1cm} (3)

where:

For rectangular: \( I_y = \frac{bh^3}{12} \)

\[ z_i : \text{the vertical distance from the middle of each element cross section to the center of gravity CG} \]

\[ I_y = \left( \frac{370.9 \times 12^3}{12} + (370.9 \times 12) \times 555.81^2 + \frac{4 \times 1000^3}{12} + (1000 \times 4) \times 49.81^2 + \frac{250 \times 12^3}{12} + (250 \times 12) \times 456.19^2 + \frac{70 \times 10^3}{12} + (70 \times 10) \times 299.81^2 + \frac{70 \times 10^3}{12} + (70 \times 10) \times 49.81^2 + \frac{70 \times 10^3}{12} + (70 \times 10) \times 200.19^2 + \frac{200 \times 15^3}{12} + (200 \times 15) \times 469.69^2 \right) \times 2 = 6.18 \times 10^9 \text{ mm}^4 \]

- **Calculate of normal and shear stresses:**

\[ \sigma_{x,Ed} (\text{MAX +}) = \frac{M_i \times Z_{CG,i}}{I_y} \]  \hspace{1cm} (4)

\[ \sigma_{x,Ed} (\text{MAX +}) = \frac{1.75 \times 10^9 \times 477.19}{6.18 \times 10^9} = 135.12 \text{ N/mm}^2 \]

\[ \sigma_{x,Ed} (\text{MAX -}) = \frac{M_i \times Z_{CG,s}}{I_y} \]  \hspace{1cm} (5)
\[ \sigma_{x,Ed} = \frac{1.75 \times 10^9 \times 561.81}{6.18 \times 10^9} = 159.08 \text{ N/mm}^2 \]

\[ \sigma_1 = \frac{1.75 \times 10^9 \times 549.81}{6.18 \times 10^9} = -155.69 \text{ N/mm}^2 \]

\[ \sigma_2 = \frac{1.75 \times 10^9 \times 450.19}{6.18 \times 10^9} = 127.4 \text{ N/mm}^2 \]

\[ \tau_{Ed} = \frac{V_{Ed}}{2h_w \cdot t_w} \]

\[ \tau_{Ed} = \frac{700 \times 10^3}{2 \times 1000 \times 4} = 87.5 \text{ N/mm}^2 \]

The stress field action, at the considered web panel 2-3, are shown in Figure 9.

**Figure 9.** Stress field action on web panel 2-3

- **Determining of \( \alpha_{ult,k} \) from Equation (10.3) from EN 1993-1-5[6] result:**

Because there is no transverse stress the equation (10.3) will become:

\[
\frac{1}{\alpha^2_{ult,k}} = \left( \frac{\sigma_{x,Ed}}{f_y} \right)^2 + 3 \times \left( \frac{\tau_{Ed}}{f_y} \right)^2
\]

\[ \frac{1}{\alpha^2_{ult,k}} = \left( \frac{-155.69}{235} \right)^2 + 3 \times \left( \frac{87.5}{235} \right)^2 = 0.853 \]

\[ \alpha_{ult,k} = 1.08 \]

- **Determining of reduction factors \( \rho_x \) and \( \chi_w \)**

The reduction factor \( \rho_x \) is a function of the plate slenderness \( \lambda_p \) which can be obtained from equation (10.2, SR EN1993-1-5). For the value of \( \alpha_{cr,s} \) it was used equation (10.6, SR EN 1993-1-5). As for the web panel 2-3 the parameter \( \psi \) is less than 0.5

According to the Annex A1, Note 1 in EN1993-1-5, the buckling coefficient \( k_{x,p} \) can be obtained using appropriate charts for plates with smeared stiffeners or relevant computer simulations. In this paper we chose the first solution using charts for plates with smeared stiffeners [4].

Taking the following parameters of panel 2-3:

\[
\alpha = \frac{a}{b} = \frac{1250}{1000} = 1.25 > 0.5
\]

\[ n^b = 3 \]

\[ \delta^b = \frac{f_L}{b \cdot a} = \frac{70 \times 10}{1000 \times 4} = 0.175 \]

\[ n^b \delta^b = 3 \times 0.175 = 0.525 \]
\[ \gamma^L = \frac{E I^L}{N b} = \frac{E I^L}{\frac{12(1 - \nu^2)}{b} b} = 118.58 \]

\[ n^L \gamma^L = 3 \times 118.58 \approx 356 \]

where:
- a, b, t are defined in Figure 9
- \( F^L \) is the gross area of an individual longitudinal stiffeners
- \( n^L \) is the number of the longitudinal stiffeners
- \( I^L \) is the second moment of area of an longitudinal stiffener about the Z-Z axis (see Figure 5.3(b), EN 1993-1-5)
- \( N \) is the second moment of area for bending of the plate

The buckling factor \( k_{\sigma,p} = k_\sigma \) result from the diagrams of Table V/2.2, Page 99, of the book [4]. (Fig. 10). It is obtained \( k_{\sigma,p} = 250 \).

For the diagrams of table V/6, Page 106, of the book [4] (Fig. 11) it is obtained the buckling factor \( k_\tau = 190 \).

With the value \( k_{\sigma,p} = 250 \) will be calculate \( \sigma_{cr,x}, \alpha_{cr,x} \). With the value \( k_\tau = 190 \) will be calculate \( \tau, \alpha_{cr,x}, \tau \). With the \( \alpha_{cr,x} \) and \( \alpha_{cr,x} \). Can be obtaiend \( \alpha_{cr} \) from [10.6 of EN1993-1-5]

\[ \sigma_{cr,x} = k_{\sigma,p} \sigma_E \]

The elastic critical plate buckling stress should be taken taken as in Annex A.1, EN1993-1-5:

\[ \sigma_E = \frac{\pi^2 E t^3}{12(1 - \nu^2) b^2} = 190000 \left( \frac{b}{L} \right)^2 \]

\[ \sigma_E = 190000 \times \left( \frac{4}{1000} \right)^2 = 3.04 \text{ N/mm}^2 \]

\[ \sigma_{cr,x} = 250 \times 3.04 = 760 \text{ N/mm}^2 \]
\[ \alpha_{cr,x} = \frac{\sigma_{cr,x}}{\sigma_1} = \frac{760}{-155.69} = -4.88 \]
\[ \tau_{cr} = k \cdot \sigma_E = 190 \times 3.04 = 577.6 \text{ N/mm}^2 \]
\[ \alpha_{cr,t} = \frac{\tau_{cr}}{\tau_{Ed}} = \frac{577.6}{87.5} = 6.6 \]

Using relation (10.6), EN1993-1-5 it is obtained:
\[
\frac{1}{\alpha_{cr}} = \frac{1}{\alpha_{cr,x}} + \left[ \left( \frac{1+\psi_x}{4\alpha_{cr,x}} \right)^2 + \frac{1-\psi_x}{2\alpha_{cr,x}} + \frac{1}{\alpha_{cr,t}} \right]^{1/2}
\]
\[
\frac{1}{\alpha_{cr}} = \frac{1}{4 \cdot -4.88} + \left[ \left( \frac{1-0.819}{4 \cdot -4.88} \right)^2 + \frac{1+0.819}{2 \cdot -4.88} + \frac{1}{6.6^2} \right]^{1/2} = 0.42
\]
\[ \alpha_{cr} = 2.33 \]

The plate slenderness \( \lambda_{p} \) should be taken from (10.2) NE1993-1-5:
\[
\lambda_{p} = \sqrt{\frac{\sigma_{ult}}{\alpha_{cr}}} (11)
\]
\[ \lambda_{p} = \sqrt{\frac{108}{23.3}} = 0.68 \]

For internal compression components as \( \lambda_{p} = 0.68 < 0.86 \) result the reduction factor \( \rho_s = 1.0 \).

(4.4 (2) of EN1993-1-5)\[6].

The slenderness parameter \( \lambda_{w} \) can be obtain from equation (5.6) from EN1993-1-5 which is:
\[
\lambda_{w} = \frac{h_{w}}{37.4 \cdot f_{y}/\gamma_{M1}} (12)
\]
\[ \lambda_{w} = \frac{1000}{37.4 \cdot 1.1 - 190} = 0.48 \]
as \( \lambda_{w} = 0.48 < \frac{0.83}{\eta} = \frac{0.83}{1.2} = 0.69 \)

the reduction factor \( \chi_{w} = \eta = 1.2 \) (Table 5.1 of EN1993-1-5)\[6].

with \( \rho_s = 1.0 \) and \( \chi_{w} = 1.2 \) the verification formula 10.5, EN1993-1-5 will be \[6]:
\[
\left( \frac{\sigma_1}{\rho_s f_y/\gamma_{M1}} \right)^2 + 3 \left( \frac{\tau_{Ed}}{f_y/\gamma_{M1}} \right)^2 \leq 1.0
\]
\[
\left( \frac{155.69}{1+235/1.1} \right)^2 + 3 \left( \frac{87.5}{1.2+235/1.1} \right)^2 = 0.88 < 1.0 \quad \text{(OK)}
\]

3. Conclusions:

- The web panel 2-3 of the chosen steel box girder is safe against buckling phenomenon at ULS (Ultimate Limit State).
- Any other panels of the webs girder and upper compression flange can be verified in the same manner using Redused Stress Method.

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