Particle Swarm Algorithm Sliding Mode Control on Spacecraft’s Attitude with Switching Function Method Thorough Error Feedback

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Abstract. Small spacecraft requires capable processors with energy efficiency, low cost and low computational burden while maintaining the output tracking accuracy. This paper presents the extension of work in [1], to enhance the transient performance using particle swarm optimization (PSO) on decaying boundary layer and switching function thorough error feedback (DBLSF) in Sliding Mode Control (SMC). Generally, SMC is known for having chattering as the main drawback which can introduce wear and tear to moving mechanical parts. As a solution, a DBLSF proposed in [1] and capable of eliminating the chattering in SMC while considering the essential requirements for small spacecraft operation. Then, the extension implemented on spacecraft’s attitude, which is one-of-six subsystems in spacecraft, used to orient the spacecraft referred to reference objects and control the dynamics of a spacecraft time-to-time according to the needs. However, the SMC’s transient response can be tuned using some coefficients in the SMC algorithm. The parameters in [1] were tuned using outputs observation technique. In this paper, then, an improvement is introduced to optimize the outputs by adding a PSO in the SMC-DBLSF in term of transient performances and accuracy while reducing the chattering permanently.

Keywords: small spacecraft, spacecraft’s attitude, SMC, chattering, switching function error feedback, PSO, control accuracy

1. Introduction
A spacecraft or satellite is an object that is orbiting larger objects such as the earth. Currently, there are more than 1000 operational human-made spacecraft and satellites in orbit around earth [2]. One of the spacecraft operations in space is position control. Then, a spacecraft critically needs a motion control system to position and orientate itself correctly, mainly when disturbances and uncertainties occur. Hence, a robust control method is required to ensure that this task successfully is done.

A small spacecraft needs processors with energy efficiency, low cost and low computational burden while operating in space. Thus, Sliding Mode Control (SMC) is one of the robust control methods, capable of providing the requirements [3][4][5]. SMC, however, produces chattering in...
the controller inputs, specifically in the switching function (Eq. 1) which it can cause wear and tear to the actuator [6].

\[ u_n = \begin{cases} 
-1 & \text{for } s > 0 \\
0 & \text{for } s = 0 \\
1 & \text{for } s < 0 
\end{cases} \tag{1} \]

where \( s \) is a sliding surface as in Eq. 2.

\[ s = (\frac{d}{dt} + \lambda)^{n-1}e \tag{2} \]

with \( \lambda \) represents as a set of sliding surface coefficients, \( n \) denotes the order of the system and \( e \) is the error between the desired input and the measured output.

Thus many researchers proposed a modification in SMC techniques to overcome this problem [7][8][9][5]. One of the solutions is implementing a boundary layer around the sliding surface. A boundary layer technique is one of the most popular methods for chattering elimination in SMC. Initially, a constant boundary layer (CBL) introduced by the researchers, but the control output accuracy cannot be maintained [10]. Then, a decaying boundary layer (DBL) where the boundary layer existing is dependent on time is proposed to solve the accuracy issue. As results, the chattering only can be eliminated for finite time [11]. As a solution, thus, a decaying boundary layer and switching function thorough error feedback (DBLSF) is proposed in [1] to eliminate the chattering while maintaining the accuracy outputs. DBLSF, however, needs an optimization algorithm to enhance the transient performances by tuning the \( \lambda \) parameter (Eq. 2) in the DBLSF control algorithm. Hence, in this paper, a particle swarm optimization (PSO) is introduced to the DBLSF to improve the small spacecraft attitude and orientation (SAOM) transient performance.

PSO is a straightforward concept, requires only primitive mathematical operators and needs less memory and speed of computational load where can be coded using only a few lines in the program code. Besides, PSO also suitable implements on nonlinear functions [12]. In PSO, there are two primary components used to determine the optimization of the state, which is particle and swarm. Each particle updates their coordinates referred to the best solution (fitness) it has achieved so far, which known as \( p_{best} \). On the other hands, the swarm keeps tracking the best value and location so far among all the particles in the population, known as \( g_{best} \).

Since SMC and PSO sharing similar important specifications (simple mathematical operators, required low computational load and inexpensive), thus, the combination of the PSO-DBLSF is suitable to implement on small spacecraft operation.

The paper is organised as follows. Section 2 describes the orbits relative to the earth with possible disturbances and uncertainties in state-space form. Section 3 describes the SMC general model and introduces the DBLSF control algorithm. Section 4 elaborates the PSO techniques and the development control strategies for SAOM alongside the PSO-DBLSF. Section 5 demonstrates the comparison results between DBLSF and PSO-DBLSF. In Section 6, brief conclusions are given along with proposals for future work, that is where improvements of the control strategies are possible.

2. Spacecraft’s Attitude and Orientation Model around Earth
In this section, the angular velocity of the spacecraft’s attitude is designed and translated into state-space form. Figure 1(a) represents a rigid body spacecraft, orbiting the earth concerning Earth Centered Inertial (ECI) at an angular velocity, \( \omega_Q \) with three rotational degrees of freedom. The general dynamics equation [13] for Figure 1(a) as in Eq. 3.
Figure 1. (a) Spacecraft’s attitude in moving frame $B$ with respect to an orbiting reference frame $O$ and both are moving in ECI
(b) Sequence of Euler’s angles, $(R_x(\psi) \rightarrow R_y(\theta) \rightarrow R_x(\psi))$, according moving frame $B$ orientation relative to an orbiting frame $O$

\[ J\dot{\omega} = J\omega \times \omega + \tau \]  

where $J = \text{diag}(J_x, J_y, J_z)$ represents the constant inertia matrix in the body-fixed frame, $\tau = \text{diag}(\tau_x, \tau_y, \tau_z)$ is the applied torque and $\omega$ is the spacecraft angular velocity orbiting around the Earth.

The kinematics of the rigid body (Figure 1(a)) are designed using Euler’s angles with the sequence rotation (Figure 1(b)) as in Eq. 4 with each axis denoted as angular velocity for $\dot{\psi}$ (roll), $\dot{\theta}$ (pitch) and $\dot{\phi}$ (yaw).

\[
Q = \begin{bmatrix}
c\phi c\theta & c\psi s\phi + c\phi s\theta & s\psi s\phi - c\psi c\phi s\theta \\
c\phi s\theta & c\phi c\theta - s\psi s\phi & c\psi s\theta + c\phi s\phi \\
-s\theta & -c\theta s\phi & c\psi c\theta \\
c\phi c\theta & c\phi s\theta + c\psi s\phi & c\psi c\theta - s\psi c\phi s\theta \\
-s\theta & -c\theta s\phi & c\psi c\theta \\
-c\theta s\phi & c\psi c\theta - s\psi c\phi s\theta & c\psi s\phi + c\phi s\theta
\end{bmatrix}
\]

with $(c\psi \ c\theta \ c\psi)$ and $(s\psi \ s\theta \ s\psi)$ denote the $sin$ and $cos$ for each axis respectively.

Finally, the spacecraft’s attitude model [1] in state-space form as in Eq. 5.

\[
\begin{bmatrix}
x_1(t) \\
x_2(t) \\
x_3(t) \\
x_4(t) \\
x_5(t) \\
x_6(t)
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
h & 0 & 0 & i & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & j & k & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x_1(t) \\
x_2(t) \\
x_3(t) \\
x_4(t) \\
x_5(t) \\
x_6(t)
\end{bmatrix} +
\begin{bmatrix}
\tau_x \\
\tau_y \\
\tau_z
\end{bmatrix}
\]

\[
\begin{bmatrix}
\dot{x}_1(t) \\
\dot{x}_2(t) \\
\dot{x}_3(t) \\
\dot{x}_4(t) \\
\dot{x}_5(t) \\
\dot{x}_6(t)
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
h & 0 & 0 & i & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & j & k & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\dot{x}_1(t) \\
\dot{x}_2(t) \\
\dot{x}_3(t) \\
\dot{x}_4(t) \\
\dot{x}_5(t) \\
\dot{x}_6(t)
\end{bmatrix}
\]

where
\[
h = \frac{(J_x-J_y)}{J_z}; \quad i = \frac{(J_x-J_z)}{J_y}; \quad k = \frac{-J_y}{J_z}; \quad \tau_x = \frac{J_z}{J_y} \omega_y; \quad \tau_y = \frac{J_z}{J_x} \omega_x; \quad \tau_z = \frac{J_y}{J_x} \omega_x;
\]

\[
[x_1(t) \ x_2(t) \ x_3(t) \ x_4(t) \ x_5(t) \ x_6(t)]^T = [\psi \ \dot{\psi} \ \theta \ \dot{\theta} \ \phi \ \dot{\phi}]^T
\]

\[
[\dot{x}_1(t) \ \dot{x}_2(t) \ \dot{x}_3(t) \ \dot{x}_4(t) \ \dot{x}_5(t) \ \dot{x}_6(t)]^T = [\ddot{\psi} \ \dot{\psi} \ \ddot{\theta} \ \dot{\theta} \ \ddot{\phi} \ \dot{\phi}]^T
\]
3. Sliding Mode Control

In this section, the general SMC’s approach is elaborated. Then, the advantages and disadvantages of the DBLSF are discussed with a possible solution. In SMC, a compensated system guaranteed to achieve the robustness once hitting around a sliding surface (determine the transient performance) and achieve the equilibrium point by the switching surface \( (u_n) \) expression. The switching function, however, produces chattering in control input as a drawback. Hence, many modifications in switching function developed and proposed to overcome this issue. Then, a decaying boundary layer and switching function thorough error feedback (DBLSF) [1] is proposed and can solve the chattering problem. The DBLSF used outputs observation techniques to determine the \( \lambda \) parameter (Eq. 8) in the sliding surface, which influenced the transient output characteristics.

Consider a linear system with matching uncertainties in (6).

\[
\dot{x}(t) = Ax(t) + B(u(t) + d(t))
\]

with \( x(t) \in R^n \) is the system state, \( u(t) \) is the scalar input, \( A \in R^{n \times n} \) and \( B \in R^m \) are the nominal system matrices satisfying the controllability condition and \( d(t) \) an unknown bounded disturbance.

In general, SMC’s control input algorithm \( u_{smc} \) (7) is construct by two parts; sliding surface (continuous, \( u_{eq} \)) and switching surface (discontinuous, \( u_n \)).

\[
u_{smc} = u_{eq} + u_n\]

with \( u_{eq} \) is an equivalent estimation control (Eq. 8) derived from Eq. 6 [1] and \( u_n \) from Eq. 1 which can be simplified into Eq. 9. However, Eq. 9 generates chattering in the control inputs.

\[
u_{eq} = -(\lambda B)^{-1}(\lambda Ax(t)) - d(t)
\]

where \( \lambda \) is a set of sliding surface coefficients.

\[
u_n = \frac{s}{|s|}
\]

Initially, the CBL in SMC can eliminate the chattering, however, reduce the output accuracy, which depends on the boundary layer width [1]. Thus, the DBLSF (Eq. 10) is introduced to solve this disadvantage where the boundary layer width is dependent on the error, \( d_0 \). In DBLSF, when the time converges to finite, then the \( d_0 \approx 0 \). As a result, the output accuracy is guaranteed [1].

\[
u_{ndlsf} = \frac{se^{-\frac{|d_0|}{s + \epsilon_0e^{-\frac{|d_0|}{s}}}}}{|s + \epsilon_0e^{-\frac{|d_0|}{s}}|}
\]

where \( |d_0| \) is the error between actual output and desired output and \( \epsilon_0 \) is the boundary layer width.

Finally, the SMC-DBLSF control strategy as in Eq. 11.

\[
u_{smc} = -(\lambda B)^{-1}(\lambda Ax(t)) - d(t) - \frac{se^{-\frac{|d_0|}{s + \epsilon_0e^{-\frac{|d_0|}{s}}}}}{|s + \epsilon_0e^{-\frac{|d_0|}{s}}|}
\]

Besides, the transient response of the SAOM outputs depends on the \( \lambda \) value in Eq. 11. In [1], an output observation technique is used to determine the \( \lambda \) value, which did not optimize the outputs transient. As a solution, the \( \lambda \) can be tuned using the PSO approach to enhance transient optimization.
4. Particle Swarm Optimization
In this section, the PSO concept is elaborated, and the control strategies for PSO-DBLSF on SAOM is developed. In PSO, a swarm represents by a population while a particle denotes by an individual [14]. Each particle produces two parameters, which are position and velocity. The relationship between the current position and next iteration position, and velocity as in Eq. 12.

\[ s_i(k + 1) = s_i(k) + v_i(k + 1) \]  

where \( i \) is the particle number, \( k \) refers to the number of iteration, \( s_i(k + 1) \) and \( v_i(k + 1) \) represent the next iteration value for particle’s position and velocity respectively and \( s_i \) is the current iteration for particle’s position.

Then, the \( v_i(k + 1) \) term can be obtained using Eq. 13 [14].

\[ v_i(k + 1) = w v_i(k) + c_1 r_1 (pbest_i - s_i(k)) + c_2 r_2 (gbest_i - s_i(k)) \]  

where \( w \) denotes the inertia weight for current particle’s velocity, \( v_i(k) \) represents the current iteration for velocity of particle, \( c_1 \), and \( c_2 \) are the cognitive and social component which are known as learning factors and \( r_1 \) and \( r_2 \) are random number between 0 and 1.

In this paper, the objective function in the SAOM model is to minimize the error value for each state (\( \psi, \theta, \phi \)). Hence, the PSO used the error values from the simulation to tuning the \( \lambda \), which act as the particle to improve the transient performance.

5. Results
In this section, the spacecraft’s position model (Eq. 5) is analyzed using DBLSF and PSO-DBLSF. Then, the transient performances and the outputs accuracy for both approaches are compared. The numeric parameters are set up for the SAOM, as shown in Table 1.

| Parameter | Value | Unit     |
|-----------|-------|----------|
| \( \omega_0 \) | 0.0011 | rads\(^{-1}\) |
| \( J_x \) | 35 | kgm\(^2\) |
| \( J_y \) | 16 | kgm\(^2\) |
| \( J_z \) | 25 | kgm\(^2\) |
| \( \tau_{x,y,z} \) | 0.001 | Nm |
| \( d(t) \) | \( \sin(t) \) | – |
| \( c_1, c_2 \) | 1.42 | – |
| \( w \) | 0.9 | – |

Next, the \( \lambda \) values for all axis (\( \psi, \theta \) and \( \phi \)) using PSO (13) and output’s observation technique are represent in Table 2. There are two periodic inputs given to each axis with the state’s initial condition as in Table 3.

| \( \lambda \) | DBLSF | PSO-DBLSF |
|-------|-------|-----------|
| \( \lambda_\psi \) | 0.2500 | 2.4567 |
| \( \lambda_\theta \) | 0.2500 | 2.1442 |
| \( \lambda_\phi \) | 0.2500 | 1.6229 |
Table 3. Inputs characteristics on the SAOM with states initial condition

| Initial Condition | Inputs |
|-------------------|--------|
| $0 \leq t < 75\, \text{s}$ | $75 \leq t \leq 150\, \text{s}$ |
| Roll ($\psi$) | -0.50 | 0.50 | 1.00 |
| Pitch ($\theta$) | -0.87 | 1.05 | 0.25 |
| Yaw ($\phi$) | 0.35 | -1.57 | -0.25 |

Figure 2 shows the SAOM’s outputs using the DBLSF (Figure 2(a)) and the PSO-DBLSF (Figure 2(b)) techniques. It clearly can be seen that the transient response of the PSO-DBLSF improved compared to the DBLSF. The rise time for the PSO-DBLSF’s first periodic input ($0 \leq t < 75\, \text{s}$) are 2.746s, 3.912s and 3.207s for $\psi$, $\theta$ and $\phi$ respectively but the DBLSF shows over than 8s for all states. In term of accuracy, the outputs error compared to the desired inputs are slightly comparable for both methods. The PSO-DBLSF recorded less maximum-error among the states (0.2095%) compared to the DBLSF (2.0200%) algorithm. On the other hand, both methods effectively eliminate the chattering in the control inputs (Figure 3(b) and Figure 3(c)) compared to the classical SMC algorithm (Figure 3(a)). The detailed summary for both observations in term of the outputs accuracy comparison and transient characteristics between the DBLSF and the PSO-DBLSF is shown in Table 4 and Table 5.

![SAOM Model with SMC DBLSF](a)

![SAOM Model with SMC PSO-DBLSF](b)

Figure 2. The SAOM’s outputs using the DBLSF (a) and the PSO-DBLSF (b)

Table 4. The outputs accuracy comparison between the DBLSF and the PSO-DBLSF compared to the target output

| Output Parameters | Time (s) | Desired Outputs (rad) | DBLSF | PSO-DBLSF |
|-------------------|----------|-----------------------|-------|-----------|
|                   |          |                       | Output (rad) | Error (%) | Output (rad) | Error (%) |
| Roll ($\psi$)     | 75       | 0.5000                | 0.4899 | 2.0200    | 0.4991 | 0.1800 |
|                   | 150      | 1.0000                | 0.9997 | 0.0300    | 0.9999 | 0.0100 |
| Pitch ($\theta$)  | 75       | 1.0500                | 1.0402 | 0.9333    | 1.0478 | 0.2095 |
|                   | 150      | 0.2500                | 0.2503 | 0.1200    | 0.2504 | 0.1600 |
| Yaw ($\phi$)      | 75       | -1.5700               | -1.5628 | 0.4586    | -1.5678 | 0.1401 |
|                   | 150      | -0.2500               | -0.2521 | 0.8400    | -0.2496 | 0.1600 |
Figure 3. The SAOM’s control inputs using the classical SMC (a), the DBLSF (b) and the PSO-DBLSF (c)

Table 5. The transient comparison between the DBLSF and the PSO-DBLSF

| SMC Approaches | DBLSF | PSO-DBLSF |
|----------------|-------|-----------|
| Roll ($\psi$)  | 8.713 | 2.746     |
| Pitch ($\theta$)| 8.753 | 3.912     |
| Yaw ($\phi$)   | 8.753 | 3.207     |

6. Conclusions and Future Recommendations
SMC approaches can produce high control accuracy, but the occurrence of chattering phenomena is a significant drawback. The proposed DBLSF method in [1] can eliminate chattering; however, did not offer optimization in the transient characteristics. Implementing a PSO on a DBLSF, then, capable of optimizing the performance of the output while maintaining the accuracy and eliminate the chattering in the control input.

In space, however, some scenarios and applications need drastic changing in the inputs, for instance, debris (encompasses by natural and artificial particles produced from meteoroid and human-made respectively) avoidance in space [15], spacecraft formation [16] and spacecraft rendezvous and docking manoeuvres (SRDM) [17]. Then, PSO-DBLSF is proposed to validate the robustness and capability to encounter the challenges on these scenarios for future research.

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