Charged Higgs mass bound from the $b \to s\gamma$ process in the minimal supergravity\textsuperscript{[1]}

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Abstract

The constraint on the mass of the charged Higgs boson from the recent measurement of the inclusive $b \to s\gamma$ decay is studied in a framework of the minimal supergravity. It is shown that the lower bound for the charged Higgs mass crucially depends on the sign of the higgsino mass parameter ($\mu$). For $\mu < 0$, the bound exceeds 400 GeV when the ratio of two Higgs vacuum expectation values ($\tan \beta$) is larger than 10. No strong bound is obtained for $\mu > 0$ due to cancellations between charged Higgs and supersymmetric particle contributions.

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§1. Introduction

Flavor changing neutral current (FCNC) processes play a unique role in searching for physics beyond the standard model (SM) of elementary particles. These processes are sensitive to virtual effects of new particles, since the FCNC processes in SM do not occur at the tree level. These processes can thus be more powerful than direct particle searches in putting constraints on the parameter space of various new physics. In particular, the radiative decay of the $b$ quark, $b \rightarrow s\gamma$, deserves a special attention. Recently, the CLEO group\(^1\) has reported the first measurement of the inclusive $b \rightarrow s\gamma$ branching ratio $\text{Br}(b \rightarrow s\gamma) = (2.32 \pm 0.57 \pm 0.35) \times 10^{-4}$, which is in good agreement with the SM prediction. It has been noticed that in a two Higgs doublet model (THDM) the charged Higgs boson can give a substantial contribution to the $b \rightarrow s\gamma$ rate\(^2,3\). In fact, this result constrains the mass of the charged Higgs boson in a certain type of THDM called Model II\(^2,3\) to be larger than $(244 + 63/(\tan \beta)^{1.3})$ GeV\(^1\) where $\tan \beta$ is the ratio of the vacuum expectation values of two Higgs fields.

The Higgs sector in the minimal supersymmetric (SUSY) extension of SM is a special case of THDM II. However, the above-mentioned limit cannot be directly applied, because SUSY particles can contribute to the $b \rightarrow s\gamma$ process in addition to the SM particles and the charged Higgs. It is a natural question to ask how the charged Higgs mass limit is modified in the SUSY extension of SM. Many authors have discussed the $b \rightarrow s\gamma$ process in SUSY\(^4,5,6,7,8,9,10,11,12,13,14,15,16\). In fact, the authors of Ref. 6 considered the constraints on the parameter space of the Higgs sector in the minimal supersymmetric standard model (MSSM) obtained from the $b \rightarrow s\gamma$ process and noted that this process is sensitive to the region where the Higgs search in the future hadron collider turns out to be the most difficult. It was, however, pointed out that the charged Higgs bounds given by them cannot be in general valid since they had only taken into account the charged Higgs effect and neglected the SUSY particle’s contributions\(^7\). Although the SM and the charged Higgs contributions to the $b \rightarrow s\gamma$ amplitude have the same sign the SUSY loop can interfere either constructively or destructively with them and the limit for the charged Higgs mass from this process can be weakened by the effects of the SUSY particles.

The minimal supergravity model provides an attractive framework for SUSY extension of SM. In this model, masses and mixing parameters for SUSY particles can be expressed by a few soft SUSY breaking parameters as well as gauge and Yukawa coupling constants. The $b \rightarrow s\gamma$ branching ratio thus depends on much smaller number of free parameters compared to that in general SUSY standard models. It has been noticed\(^8,13,16\) that the sign of the SUSY loop contributions with respect to those of the SM and charged Higgs is strongly
correlated with the sign of the higgsino mass parameter \( i.e. \) the \( \mu \) parameter in the minimal supergravity model.

We would like to compare the CLEO data with the prediction of the minimal supergravity model and determine the allowed region in the parameter space of the Higgs sector in the model. Namely, scanning the free parameter space extensively, we search for the constraint in the space of the charged Higgs mass and \( \tan \beta \). We study the whole range of \( \tan \beta \) which is consistent with the fact that all of the top, bottom and tau Yukawa coupling constants remain perturbative up to the grand unification scale. Although it is difficult to draw a general conclusion on the constraint in general SUSY standard models, we can derive a useful constraint if we restrict ourselves to the case of the minimal supergravity model. It will be shown that the lower bound of the charged Higgs mass crucially depends on the sign of \( \mu \). The bound becomes much larger than that in the non-SUSY THDM II for \( \mu < 0 \), while no strong bound is obtained for \( \mu > 0 \).

\section{b \to s\gamma in the minimal supergravity}

The calculation of the \( b \to s\gamma \) branching ratio has already been discussed extensively in the literature\cite{20,21}. The decay rate for \( b \to s\gamma \) normalized to the semileptonic decay rate is given by

\[
\frac{\Gamma( b \to s\gamma )}{\Gamma( b \to ce\nu )} = \frac{6\alpha_{\text{QED}}}{\pi g(m_c/m_b)} \frac{|V_{ts}V_{tb}|^2}{|V_{cb}|^2} \left| C_7^{\text{eff}}(Q) \right|^2 ,
\]

\[
C_7^{\text{eff}}(Q) = \eta^{16/23} C_7(M_W) + \frac{8}{3} \left( \eta^{14/23} - \eta^{16/23} \right) C_8(M_W) + C ,
\]

where \( \eta = \alpha_s(M_W)/\alpha_s(Q) \), \( Q \) being the scale of the order of the bottom mass, and \( g(z) = 1 - 8z^2 + 8z^6 - z^8 - 24z^4 \ln z \). \( C \) is a constant which depends on \( \eta \). The above formula takes the leading order QCD corrections into account. The \( C_7(M_W) \) and \( C_8(M_W) \) are coefficients of the magnetic and chromomagnetic operators at \( M_W \). The \( C \) term is induced by operator mixing in evolving from \( M_W \) to the low energy scale \( Q \).

Ambiguities in the calculation are discussed in detail in Ref. \cite{21}. The most important ambiguity comes from the choice of the renormalization scale \( Q \). Varying \( Q \) between \( m_b/2 \) and \( 2m_b \) induces an ambiguity of \( \pm 25 \% \) for the branching ratio in SM. Other ambiguities include the choice of \( m_c/m_b \) (which affects the semileptonic rate) and the value of \( \alpha_s(M_Z) \).

In MSSM, the coefficients \( C_7(M_W) \) and \( C_8(M_W) \) receive the following contributions at one loop:

1. the \( W \) and top quark loop (SM contribution);
2. the charged Higgs and top quark loop;
3. the chargino and up-type squark loops;
4. the gluino and down-type squark loops;
5. the neutralino and down-type squark loops.

The contribution from 5 is known to be very small, which we will ignore hereafter. THDM II amplitude is calculated with use of the first two contributions only. The charged Higgs contribution depends on its mass and the ratio of the vacuum expectation values of the two Higgs doublets, i.e. \( \tan \beta = \langle H^0_2 \rangle / \langle H^0_1 \rangle \), where \( H^0_1 \) and \( H^0_2 \) are the neutral components of the two Higgs doublets. The chargino and gluino loop contributions depend on the mass and mixing of the particles inside the loop. Although the squark mixing matrices are arbitrary parameters in a general SUSY standard model, they can be calculated from the flavor mixing matrix of quarks (the Cabibbo-Kobayashi-Maskawa matrix) in the minimal supergravity model by solving the renormalization group equations (RGEs) for various soft SUSY breaking terms.

The soft SUSY breaking parameters at the GUT scale are the universal scalar mass \( (m_0) \), a parameter in the trilinear coupling of scalars \( (A_X) \), a parameter in the two Higgs coupling \( (B_X) \) and the gaugino mass \( (M_{gX}) \). We are assuming the GUT relation for the three gaugino masses i.e. the SU(3), SU(2) and U(1) gaugino mass parameters are equal at the GUT scale. Besides these soft SUSY breaking parameters, the superpotential contains the Yukawa coupling constants and the \( \mu \) parameter. Given a set of values for the quark and lepton masses, CKM matrix elements and \( \tan \beta \), we determine all the particle masses and mixings at the weak scale by solving relevant RGEs with initial conditions at the GUT scale specified by the above parameters. We compute the Higgs effective potential at the weak scale and require that the electroweak symmetry is broken properly (the radiative breaking scenario). We include the one loop corrections to the effective potential induced by the Yukawa couplings for the third generation. The condition for radiative breaking with the correct scale reduces the number of free parameters to three for given \( \tan \beta \) and \( M_t \). We can think of these parameters as the charged Higgs mass \( (m_{H^\pm}) \), SU(2) gaugino mass \( (M_2) \) and \( \mu \) at the weak scale. For a given set of these five parameters, all other masses and mixings are calculated. For the detail of the calculation, see Ref. [22].

\[\text{§3. Numerical results}\]

We now present the results of the \( b \to s \gamma \) branching ratio. Besides the radiative breaking condition we require the following phenomenological constraints[23]:
1. The mass of any charged SUSY particle is larger than 45 GeV;
2. The sneutrino mass is larger than 41 GeV;
3. The gluino mass is larger than 100 GeV;
4. Neutralino search results at LEP\textsuperscript{24}, which require $\Gamma(Z \to \chi\chi) < 22$ MeV, $\Gamma(Z \to \chi'\chi') < 5 \times 10^{-5}$ GeV, where $\chi$ is the lightest neutralino and $\chi'$ is any neutralino other than the lightest one;
5. The lightest SUSY particle (LSP) is neutral;
6. The condition for not having a charge or color symmetry breaking vacuum\textsuperscript{25}.

In Fig. 1, we show the $b \to s\gamma$ branching ratio for and tan $\beta = 5$. The top quark mass is fixed to $m_t = 175$ GeV\textsuperscript{24} in the present calculations. Each point in the figure corresponds to the value of the $b \to s\gamma$ branching ratio for each scanned point in the parameter space compatible with the above conditions. This branching ratio includes the chargino and gluino loop contributions as well as the SM and the charged Higgs loop. The line in the figure represents the branching ratio when only the SM and charged Higgs contributions are retained. We notice that the points are divided by this line. In fact, the points above and below this line correspond to $\mu < 0$ and $\mu > 0$ cases respectively\textsuperscript{25}. This confirms the assertion\textsuperscript{8, 13, 16} that the sign of $\mu$ determines whether the SUSY contribution enhances or suppresses the $b \to s\gamma$ branching ratio.

We show the excluded region in the tan $\beta$ and $m_{H^\pm}$ space in Fig. 2. The range of the tan $\beta$ we have scanned are $2 < \tan \beta < 55$. For the values of tan $\beta$ larger or smaller than this range the Yukawa coupling constant for top or bottom/tau blows up below the GUT scale. The two branches $\mu > 0$ and $\mu < 0$ are separately plotted. The excluded region is determined using the CLEO result $1 \times 10^{-4} < \text{Br}(b \to s\gamma) < 4 \times 10^{-4}$. In order to take account

\textsuperscript{*)} This top quark mass is the \textsc{ms} running mass at $Q = M_Z$. This mass coincides with the pole mass within a few percent\textsuperscript{27}.
\textsuperscript{**)} Our convention of the sign of $\mu$ is opposite to those in Refs.\textsuperscript{13} and \textsuperscript{40}.
of the theoretical uncertainties we have calculated the $b \to s\gamma$ branching ratio by varying the renormalization scale $Q$ between $m_b/2$ and $2m_b$. There are other sources of theoretical ambiguities which are expected to be minor compared to the choice of the renormalization scale. These includes the choice of the $m_c/m_b$, $\alpha_s(m_Z)$, CKM matrix elements, etc. In the standard model, the unitarity of the CKM matrix and the smallness of the quantity $V_{us}^*V_{ub}$ guarantees the $b \to s\gamma$ amplitude to be proportional to a single factor $V_{ts}^*V_{tb} \approx -V_{cs}^*V_{cb}$, so that the uncertainty from the CKM matrix element is small. The same situation occurs in the minimal supergravity model where the flavor mixing is essentially determined by the Yukawa couplings. Consequently the uncertainty from the choice of CKM matrix elements is small in our case. In the calculation we have used $\alpha_s(m_Z) = 0.116$, $m_c(m_c) = 1.27$ GeV, $m_b(m_b) = 4.25$ GeV, $|V_{us}| = 0.221$, $|V_{cb}| = 0.041$, $|V_{ub}|/|V_{cb}| = 0.08$ and $\delta_{13} = \pi/3$ where $\delta_{13}$ is the CP phase with the convention used in Ref. 23. Taking into account of these ambiguities other than the renormalization scale dependence, we allow an additional 10% uncertainty in order to obtain a conservative bound. We regard a point in $(\tan \beta, m_{H^\pm})$ space excluded when the branching ratio cannot be within the CLEO bound for any choice of soft SUSY breaking parameters even if we consider the above-mentioned theoretical ambiguities. We also show in these figures the lower bound of the charged Higgs mass when only the SM and the charged Higgs contributions are retained. In comparison, the excluded region in minimal supergravity becomes larger when $\mu < 0$. The bound reaches 400 GeV for $\tan \beta > 10$. For $\mu > 0$, the $b \to s\gamma$ process is not very effective in constraining the charged Higgs mass, because the charged Higgs contribution can be completely cancelled by SUSY particle contributions. It is also interesting to see the $\tan \beta$ dependence of the charged Higgs mass bound for $\mu < 0$. The bound becomes strongest around $\tan \beta \sim 35$ and becomes weaker for larger values of $\tan \beta$.

The reason for the strong dependence on $\mu$ can be understood as follows. For the chargino and up-type squark loop the most important contribution comes from the loop diagram with the top and bottom Yukawa coupling constants. This diagram is proportional to a product of the left-right mixing parameter of the stops, i.e. $A_t + \mu \cot \beta$, and the higgsino mixing parameter $\mu$. The parameter $A_t$ is determined by $A_X$ and $M_{gX}$ through the RGEs. An interesting observation is that for such a high value of the top quark mass as considered here $A_t$ is almost independent of $A_X$ and proportional to $M_{gX}$. Moreover, the $\mu \cot \beta$ term is suppressed for $\tan \beta > 3$. Therefore, the amplitude from this sector is proportional to a product of the gaugino mass and the higgsino mass parameter in a very good approximation. A similar consideration applies to the gluino and down-type squark loop. In this case a

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***If we reduce the theoretical uncertainty by fixing the renormalization scale $Q = m_b$, the lower bound of $m_{H^\pm}$ for $\mu < 0$ is raised by $\sim 100$ GeV.
sizable contribution can arise when the graph involves the left-right mixing in the sbottom sector, especially for a large value of $\tan\beta$. Then, the amplitude is proportional to a product of the gluino mass ($M_3$) and the sbottom mixing parameter, i.e. $A_b + \mu \tan\beta$. For a large value of $\tan\beta$ the latter parameter is governed by the $\mu \tan\beta$ term. Here again, the contribution to the amplitude is almost proportional to the product of the gaugino mass and the higgsino mass parameter. In both cases, the SUSY contribution enhances (suppresses) the SM amplitude when $\mu < 0$ ($\mu > 0$). This will explain the tendency seen in Fig. 1.

The lower bound of the charged Higgs mass for $\mu < 0$ is determined by the minimum value of the branching ratio for a fixed choice of $m_{H^\pm}$ and $\tan\beta$. From the above discussion the

*) The reason for the strong dependence on the sign of $\mu$ is analyzed for example in Ref. [19] where they concentrated the case of small and very large values of $\tan\beta$ motivated by the Yukawa unification.

Fig. 2. Excluded region in the $\tan\beta$ and $m_{H^\pm}$ space for either sign of $\mu$. Each line represents the lower bound for the charged Higgs mass; solid line: all constraints included; dashed line: without $b \to s\gamma$ constraint; dot-dashed line: THDM II with $b \to s\gamma$ constraint.

Fig. 3. Allowed region in $\mu$–$M_2$ space for $\tan\beta = 3$ and 30.

The lower bound of the charged Higgs mass for $\mu < 0$ is determined by the minimum value of the branching ratio for a fixed choice of $m_{H^\pm}$ and $\tan\beta$. From the above discussion the
chargino and stop loop contribution is approximately proportional to the product of the top and bottom Yukawa coupling constants, namely \( \frac{m_b m_t}{\sin \beta \cos \beta} \). Also the branching ratio depends on the mass of stop/chargino. To determine the minimal contribution we need to know the maximum value of the stop/chargino mass for a given set of \( m_{H^\pm} \) and \( \tan \beta \). For each given set, we can scan the remaining two parameters \((\mu, M_2)\) allowed by the phenomenological constraints and the condition for the electroweak radiative breaking (see Fig. 3). In general the allowed region becomes a strip for each sign of \( \mu \). Up to \( \tan \beta \sim 35 \) the maximum value of the lighter stop mass relative to the charged Higgs mass does not strongly depends on the value of \( \tan \beta \). Then the \( \tan \beta \) dependence of the minimum branching ratio is essentially determined by \( \frac{m_b m_t}{\sin \beta \cos \beta} \). Close investigations show that beyond \( \tan \beta \sim 35 \) where the bottom and tau Yukawa coupling constants becomes as large as the top Yukawa coupling constant the condition for the radiative symmetry breaking allows the larger values of the stop mass, therefore the contribution from the chargino and stop loop can be much smaller. This explains why the lower bound becomes smaller for the value of \( \tan \beta > 35 \). Note that the lower bound of the charged Higgs mass exceeds that of the Model II THDM even in the case of \( \tan \beta > 35 \) since the chargino and stop loop effect does not change its sign. In fact, the bound approaches the line of the THDM II for the maximal value of \( \tan \beta \).

\section{Conclusion}

To summarize, we have shown that for \( \mu < 0 \) the lower bound of the charged Higgs mass increases compared to that in the THDM II. On the other hand, for \( \mu > 0 \), the \( b \to s\gamma \) process cannot put useful constraints on the allowed range of the charged Higgs mass because it is possible that the charged Higgs contribution is completely cancelled by other SUSY contributions. We have also pointed out that a parameter region corresponding to the charged Higgs mass less than 180 GeV and \( 3 \lesssim \tan \beta \lesssim 5 \) is excluded by the \( b \to s\gamma \) process irrespective of the sign of \( \mu \). The constraints obtained here are important for the SUSY Higgs search in the future colliders. For example the parameter region in the \((m_{H^\pm}, \tan \beta)\) space where no signal could be found at LHC roughly corresponds to the region near 150 GeV \( \lesssim m_{H^\pm} \lesssim 300 \) GeV, \( 5 \lesssim \tan \beta \lesssim 10 \) although details of the Higgs search limits depend on other parameters like the stop mass or expected detector performance, etc.\cite{3}. It is interesting to see that most of these region are already excluded for \( \mu < 0 \). On the other hand if LHC or linear colliders find the MSSM Higgs and investigation of its property shows that the Higgs parameters are, say, \( \tan \beta \sim 10 - 20 \) and charged Higgs mass \( \sim 300 \) GeV, we can conclude that the \( \mu \) should be positive since otherwise the \( b \to s\gamma \) constraint cannot be satisfied.
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