Stability analysis of slope in soil with strength mobilization

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Abstract. Considering the strength parameters cohesion $c$ and internal friction angle $\phi$ of the soil constitutive model are mobilized non-simultaneously. By gradually applying uniform vertical loads on the top of the slope, the progressive failure process of the slope is simulated by the FLAC3D software. The failure state of the slope is judged by monitoring the maximum unbalance force in the slope system, and the slope sliding surface is determined based on the maximum shear strain increment, and the safety factor is calculated by the slope critical height ratio method. The numerical simulation shows that when the strength parameters are not synchronicity come into play, the safety factor is less than that of two cases, which consider the synchronous mobilization of strength parameters and the traditional Strength Reduction Method (SRM). Finally, the influence of residual cohesion and plastic strain threshold on the position of sliding surface and safety factor are discussed by numerical analysis.

1. Introduction

The traditional slope stability analysis generally adopts the Mohr-Coulomb strength criterion, using the peak strength parameters or residual strength parameters. This criterion assumes that the cohesion strength and frictional strength are mobilized simultaneously throughout the deformation process, and the strength parameters are constant and independent of strain. However, if the peak strength is applied in the design and analysis, it will cause the potential safety hazard; On the other hand, accounting for residual strength may lead to conservative and expensive design. According to previous research results, many researchers have cast doubt on the validity of this approach, and studied the mechanism of the non-simultaneous mobilization of the cohesive strength and friction strength for natural clays, weak rocks, and hard rocks. Schmertmann[1] proposed the concept of Cohesion-Weakening Frictional-Strengthening (CWFS) based on experiments. The cohesion strength $c$ and the frictional strength $\sigma\tan\phi$ of hard clay were not assumed simultaneously to mobilize. The experimental results show that the cohesive strength decreases rapidly when shear strain increases, but the frictional strength is mobilized slowly and needs a large shear strain to reach strength. Schofield [2] pointed out that the cohesion and internal friction angle in the Mohr Coulomb criterion should be changed with strain or deformation displacement. Martin[3]obtained the law of cohesive strength loss and friction strength development based on strain by cyclic load test. Callisto & Rampello[4] conducted a series of true triaxial drainage tests on natural hard clay, obtained the stress-strain curves under different confining pressures, calculated the strength parameters under different strain levels, and obtained the distribution of cohesion force and friction angle with plastic shear strain, revealing that the two parts of strength did not work simultaneously. Hajjabdomajid[5] established the cohesion weakening and
friction strengthening model according to the results of the uniaxial rock test, and proposed the concept of \( c \text{ then } \phi \), that is, during the shear deformation process. The cohesive strength usually comes into work at early stage of deforming process, but the frictional strength needs large deformation to come into work, so two parameters of the strength are not fixed values and do not reach its peak simultaneously. Due to creep damage, water content change, mechanical erosion, chemical erosion, etc., the constitutive characteristics and strength parameters of geomaterials and structural surfaces are gradually changed. Therefore, the instability of the slope is not instantaneous, but a progressive failure process that gradually expands from partial failure to the formation of slip surface. The mechanism of progressive failure was recognized at an early date by Terzaghi and Peck [6]. Skempton[7] clearly evidenced its influence on slope stability, and Bjerrum [8] highlighted that such a failure process results in a non-uniform mobilization of the shear strength along the slip surface. Specifically, some portions of the slope fail first with the strains generally localized in a zone of limited thickness (shear zone). Considering that progressive failure is a strain-dependent process, hence, the conventional stability analyses based on the limit equilibrium approach are in unsuitable for dealing with such a phenomenon. In summary, the slope stability analysis method considering the strength mobilization is of great significance for the evaluation and treatment of slope. In this paper, the constitutive model considering the strength mobilization is used, and the vertical load of the slope is applied by the FLAC\textsuperscript{3D} software to realize the numerical simulation of the progressive failure process of the slope, and further determine the slip surface and safety factor of the slope.

2. Definition of safety factor

Traditional rigid body Limit Equilibrium Method (LEM) is often used in slope stability analysis, such as Swedish Slice Method, Bishop Slice Method, Janbu Slice Method, Sarma Method, Spencer Method, etc.. But these methods ignore the fact that the rock and soil mass of the slope is a deformation body and the failure process of the slope is the process of the continuous adjustment of its internal stress distribution and deformation. In addition, the sliding surface must be assumed in advance. Various numerical analysis methods based on elasto-plastic theory, the safety factor analysis methods represented by the Strength Reduction Method are more and more widely adopted in engineering. However, the Strength Reduction Method does not adjust the load applied to the calculation model according to the load step, but continuously changes the value of the strength parameters \( c, \phi \) of the geotechnical material. The mechanical state of the potential sliding surface and landslide body obtained after the strength reduction is not a real state; it is only a certain virtual state. A safety factor is usually defined as the ratio of the algebraic sum of resistance forces to the algebraic sum of driving forces of every section along a certain slip surface. This coefficient has no practical physical meaning because it is a particular material artificially defined and the strength of this material is reduced. A more appropriate approach should be based on the current real state of mechanics. Without the iterative process, the safety factor can be calculated at one time, and the results obtained are more reasonable and straightforward. Since the critical height of the slope is intuitive and more familiar to engineers, it is used as the safety measure of the slope, and the safety factor of the slope is calculated by Eq. (1):

\[
F_s = \frac{H_c}{H}
\]  

(1)

Where \( F_s \) is the safety factor of slope, \( H_c \) is the critical height of the slope; \( H \) is the real height of the slope. The calculation of the critical height of the slope is relatively complicated. It is necessary to regrid whenever the slope height is adjusted. Therefore, the method of applying the vertical uniform load on the top of the slope is used to equivalent to the increased height of the slope. When the slope reaches the limit equilibrium state under the \( q \) load, the safety factor can be calculated by Eq. (2).

\[
F_s = \frac{H + q/\gamma}{H}
\]  

(2)

Where \( \gamma \) is density weight of soil.
3. The constitutive model of soil based on Strength Mobilization

Mobilized Strength = Mobilized Cohesive Strength + Mobilized Frictional Strength

\[ f(\sigma) = f(c, \varepsilon^p) + f(\sigma_n \tan \phi, \varepsilon^p) \]  

(3)

Where \( f(c, \varepsilon^p) \) and \( f(\sigma_n \tan \phi, \varepsilon^p) \) are the mobilized cohesive and frictional strength components respectively. The expression of the Mohr-Coulomb strength criterion for the slope body considering the non-simultaneous mobilization of strength parameters is as follows:

\[ F(\sigma, \varepsilon^p) = \sigma_1 \left( \frac{1}{1+\sin \phi(\varepsilon^p)} \right) \left( \frac{1}{\sin \phi(\varepsilon^p)} \right) \sigma_3 - \frac{2c_1 \sigma_3 \cos \phi(\varepsilon^p)}{1+\sin \phi(\varepsilon^p)} = 0 \]  

(4)

where \( \sigma_1, \sigma_3 \) are the major and minor principal stresses, respectively; \( c \) and \( \phi \) are the cohesion and the internal friction angle, respectively, and they are both a function of plastic strain \( \varepsilon^p \). \( \varepsilon^p \) is used to represent the plastic strain. This parameter represents the current and the history of plastic strain (accumulated damage), expressed as follows:

\[ \varepsilon^p = \int \left( \frac{2}{3} (d\varepsilon^p) d\varepsilon^p + d\varepsilon^p d\varepsilon^p + d\varepsilon^p d\varepsilon^p) \right) dt \]  

(5)

Where \( d\varepsilon^p, d\varepsilon^p \) and \( d\varepsilon^p \) are the increments of principal plastic strain. The relationship between strength parameters and plastic shear strain is as shown in Fig. 1.

Fig. 1. Relationship between strength parameters and plastic shear strain

e and \( \phi \) can be expressed as the form of the following piecewise function:

\[ c = \begin{cases} c_i - \frac{c_i - c_c}{\varepsilon_c} \varepsilon^p, & \varepsilon^p \leq \varepsilon_c^p \\ c_c, & \varepsilon^p > \varepsilon_c^p \end{cases} \]  

(6)

\[ \phi = \begin{cases} \phi_i - \frac{\phi_i - \phi_c}{\varepsilon_c} \varepsilon^p, & \varepsilon^p \leq \varepsilon_c^p \\ \phi_c, & \varepsilon^p > \varepsilon_c^p \end{cases} \]  

(7)

Where \( c, c_c \) are the initial value and the residual value of the cohesion respectively when cohesion is mobilized; Similarly, \( \phi, \phi_c \) is the peak value of the internal friction angle. \( \varepsilon_c^p, \varepsilon_c^p \) are the plastic shear strain when the cohesion and the internal friction angle are constant.
4. Numerical simulation of progressive failure of slope

4.1. Principle of progressive damage of slope

When the slope is sliding, the shear strength of the slip mass varies between the peak strength and the residual strength, and the shear strength will be changed with the slip mass sliding, that is, the cohesion decreases while the angle of internal friction increases. When stress unevenness or stress concentration occurs somewhere inside the slope, considerable strain is easily generated there, resulting in local failure, which leads to stress release, redistribution, causing a large strain in more areas. Finally, these areas are connected to form a shear zone, leading to the destruction of the slope (Jiang 1998). The following is an analysis and explanation by an example.

4.2. An example analysis of the progressive failure of slope

Considering a regular single-step slope and it’s geometric dimensions is shown in Fig. 2. The mechanical parameters are shown in Table 1. The model is divided into 9830 units and 11981 nodes.

![Fig. 2. Computational model for slope (unit: m)](image)

The above-mentioned Mohr-Coulomb model of the non-simultaneous mobilization is used for analysis. In order to further compare and analyze the influence of the strength parameters on the slope stability, we change the value of the $c_i, c_r, \varepsilon_i^p, \varepsilon_r^p$ separately.

| Scheme | $\rho$/kg/m$^3$ | $c_i$ | $c_r$ | $\varepsilon_i^p$ | $\varphi_i$ | $\varphi_r$ | $\varepsilon_r^p$ | $E$/MPa | $\Psi$(°) | B/MPa | G/MPa |
|--------|----------------|------|------|------------------|-------------|-------------|------------------|--------|---------|--------|-------|
| A      | 2000           | 35   | 8    | 0.02             | 20          | 0           | 0.05             | 40     | 20      | 30     | 20    |
| B      | 2000           | 35   | 15   | 0.02             | 20          | 0           | 0.05             | 40     | 20      | 30     | 20    |
| C      | 2000           | 35   | 8    | 0.04             | 20          | 0           | 0.05             | 40     | 20      | 30     | 20    |
| D      | 2000           | 35   | 8    | 0.02             | 20          | 0           | 0.1              | 40     | 20      | 30     | 20    |

FLAC$^{3D}$ simulates the evolution of nonlinear systems over time. In order to grasp and analyze this evolution process, FLAC$^{3D}$ evaluates the state of numerical simulation by using such indexes as unbalanced force, mesh node velocity, plastic zone identification and the duration curve of some important variables. For the slope stability analysis, it is judged whether the slope is stable by
observing the maximum imbalance force of the system. Theoretically, when the imbalance force at each point in the slope is zero, the slope is in a static equilibrium state, otherwise, it is considered to be in a plastic flow state. Because the numerical calculation itself is an interpolation approximation, the truncation error inevitably exists. As long as the maximum unbalanced force is small enough compared with the external force acting on the system, it can be considered that the system has reached the state of force equilibrium and the slope is stable. However, there is no quantitative criterion for the magnitude of the maximum unbalanced force of the node at the point of slope instability. In this paper, as long as the maximum unbalance force of the landslide body in the system under different loads is increases suddenly, the failure of the slope is considered to occur under the current load.

4.3. FLAC3D Numerical Simulation

This section takes the calculation data of scheme A as an example. Considering that the strength parameters are mobilized non-simultaneously, different vertical loads are applied to the top of the slope to monitor the maximum unbalanced force after stabilization. As shown in Fig.3, when the load changes from $16\, kPa$ to $22\, kPa$, the maximum unbalanced force changes linearly and slowly. When $24\, kPa$ is loaded, the maximum unbalance force changes suddenly, and the slope is considered to be in a plastic flow state under the load. After that, when the plastic strain reaches the limit strain at every point on the slip surface, the slope will be destroyed. Fig.4 shows the maximum unbalance force curve when $24\, kPa$ is applied. From the diagram, it can be concluded that the maximum unbalance force of the nodes in the slope is stabilized from the maximum value $6.52\, KN$ to that of the final value $5.98\, KN$. Fig.5 shows the expansion process of the shear zone when $24\, kPa$ is applied, revealing the progressive failure process of the slope.

![Fig. 3. Maximum unbalance force under different loads](image)

![Fig. 4. Maximum unbalance force](image)

As can be seen from Fig.5, at step 1700 (Fig.5(a)), the strain is concentrated at the foot of the slope, and then at step 3800 (Fig.5(b)), plastic yielding begins to occur inside the slope. At 7600 (Fig.5(c)),
the strain concentration zone continues to expand from the inside of the slope to the top of the slope, and the strain rate concentration zone also widens. By 10300 steps (Fig. 5(d)), the strain rate has been completely concentrated in a transfixion belt, and ultimately forms a shear band (Wang 2005). From the above analysis, it can be concluded that the instability process of the slope is in essence a progressive failure process from part to whole.

Fig. 5. Relationship between shear strain increment and progressive destruction progress of slope

4.4. Determination of the slip surface of the slope
A series of equidistant vertical lines are arranged along the horizontal direction inside the slope model. Then the points with the maximum equivalent plastics strain on each vertical line can be found out. These points form the sliding surface of the slope, and the sliding surface can be smoothed by the least square method as shown in Fig.6.

Fig. 6. Sliding surface based on maximum shear strain increment

4.5. Discussion on safety factor
Taking scheme A of single-step slope as an example, taking into account non-simultaneous mobilization of \( c, \phi \), the magnitude of uniformly distributed load \( q \) applied on the top of the slope is determined when the maximum unbalance force increases suddenly and the slope is beginning to failure under the current load. The increment of slope height can be derived from the gravity of the soil. When the load applied to the top of the slope is \( q = 24 \text{kPa} \), the maximum unbalance force of the mesh node increases suddenly. At this time, a clearly plastic connecting zone is formed in the slope, which can be considered as non-convergence. The maximum shear strain increment is shown in Fig. 5 (d). The safety factor is calculated by Eq. (8) and Eq. (9).

\[
\Delta h = \frac{q}{\gamma} = \frac{2.4 \times 10^4}{2000 \times 9.8} = 1.22 \text{m}
\]
When taking into account simultaneous mobilization of $c, \varphi$, the cohesion and the angle of internal friction are constant values, $c = 35kPa$, $\varphi = 20^\circ$. The slope failure is occur when the uniformly distributed load $q = 7.4 \times 10^4 Pa$. The safety factor $F_s = 1.32$. The safety factor obtained by the traditional Strength Reduction Method (SRM) is 1.37. The safety factor results calculated by each method are shown in Table 2.

\[ F_s = \frac{H_c}{H} = \frac{1.22 + 12}{12} = 1.10 \] (9)

Table 2. Calculation result of $F_s$

| Strength Parameter | Non-simultaneous | simultaneous | SRM |
|--------------------|------------------|--------------|-----|
| $F_s$              | 1.10             | 1.32         | 1.37|

It can be seen that the safety factor considering non-simultaneous mobilization of strength parameters is smaller than that considering the simultaneous mobilization of strength parameters. From the perspective of soil constitutive model, it also explains why the actual slope is still unstable when the safety factor obtained by traditional analysis method is greater than 1.

5. Parametric Study

The above is to take scheme A as an example to analyze the progressive failure process of slope and the calculation of safety factor by loading method. The following is mainly to study the effect of slope parameter variation on slope slide surface and safety factor. The parameters include the residual cohesion $c_r$ and its corresponding plastic strain threshold $\varepsilon^p_c$, $\varepsilon^p_f$, and the loading method is used in the scheme B,C,D. Table 3 shows the safety factor of the scheme A, B, C, D, and figure 7 shows the slope slip surface determined by different schemes.

Table 3. $F_s$ obtained from different experimental schemes

| Scheme | A     | B     | C     | D     |
|--------|-------|-------|-------|-------|
| $F_s$  | 1.10  | 1.130 | 1.017 | 1.091 |

Fig. 7. Critical slip surfaces of slope at different schemes

By comparing the sliding surface of the slope corresponding to scheme A and B, it can be seen that with other parameters unchanged, when the only residual cohesion force $c_r$ is increased, the position of the slip surface of the slope develops towards the deep of the slope and the safety factor is increased. When $\varepsilon^p_c$ is increased, the position of the slip surface of the slope also moves to the deep of the slope, and the safety factor is increased accordingly. When $\varepsilon^p_f$ is increased, the slip surface position of the slope moves to the shallow of the slope, and the safety factor is decreased.
6. Conclusion

1. Considering that the strength parameters $c, \varphi$ of soil constitutive model are mobilized non-simultaneously, by gradually applying loads on the top of the slope, the progressive failure process of the slope is simulated by the FLAC3D software.

2. The instability state of the slope is judged by monitoring the sudden change of the maximum unbalance force of the grid node. The numerical simulation shows that when the strength parameters are mobilized non-simultaneously, the safety factor obtained by the method is less than that of other two methods, which consider the simultaneous mobilization of strength parameters and the traditional Strength Reduction Method. The rationality and reliability of the loading method based on non-simultaneous mobilization of strength parameters are verified.

3. Through numerical analysis, the effects of residual cohesion and plastic strain threshold $e^r_c$ and $e^r_f$ on the position of the slip surface and safety factor are discussed.

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