Determination of Geoid over South China Sea and Philippine Sea from Multi-satellite Altimetry Data

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Abstract A new computational procedure for derivation of marine geoid on a 2.5′ × 2.5′ grid in a non-tidal system over the South China Sea and the Philippine Sea from multi-satellite altimeter sea surface heights is discussed. Single-and dual-satellite crossovers were performed, and components of deflections of the vertical were determined at the crossover positions using Sandwell’s computational theory, and gridded onto a 2.5′ × 2.5′ resolution grid by employing the Shepard’s interpolation procedure. 2.5′ × 2.5′ grid of EGM96-derived components of deflections of the vertical and geoid heights were then used as reference global geopotential model quantities in a remove-restore procedure to implement the Molodensky-like formula via 1D-FFT technique to predict the geoid heights over the South China Sea and the Philippine Sea from the gridded altimeter-derived components of deflections of the vertical. Statistical comparisons between the altimeter-and the EGM96-derived geoid heights showed that there was a root-mean-square agreement of ±0.35 m between them in a region of less tectonically active geological structures. However, over areas of tectonically active structures such as the Philippine trench, differences of about −19.9 m were obtained.

Keywords satellite altimetry; sea surface height; deflection of the vertical; geoid height

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Introduction

The geoid, defined by Listing as that equipotential surface of the earth gravity field that best approximates the mean sea level (in the open oceans, ignoring oceanographic effects), is said to be determined for a given area with respect to a reference ellipsoid if for points in that area at reasonable intervals, its separation (or undulation) from that reference ellipsoid is known or can be estimated geometrically. Marine geoid undulations over the oceans are traditionally determined using tide gauge records, precise lev-
no attempt is made to review other methods for recovery of geoid undulation from satellite altimetry by previous researchers[1-6] as the focus is not on assessing their comparative advantages and/or disadvantages but rather on introducing a new different cost-effective computational procedure that could be implemented for efficient derivation of marine geoid from deflections of the vertical derived from altimeter-measured sea surface heights.

Undoubtedly, knowledge of the geoid will help hydrographers or bathymetrists in their prediction of ocean floor topography and charting of maritime routes, oceanographers to make precise predictions of ocean surface topography, sea level changes and weather patterns, and marine geophysicists to better understand the local geological structures and tectonic activities within the sea for commodity exploration and for effective predictions of tsunamis, etc.

1 Geographical location and geologic setting of study area

The study area is bound by latitude N0°-N30° and longitude E105°-E135°. Generally, the region is tectonically active with varying scale and type of volcanism from monogenetic cinder cones to large stratovolcanoes and calderas. Over 200 partially submerged islets, reefs, and rocks have been identified in the South China Sea with the majority located in the Xisha Islands and Nansha Islands chains. Main undersea trenches identified in the region include the Philippine trench, the Manila trench, and the Sulu trench. The South China Sea is rich in natural resources such as oil and natural gas, and has a proven oil reserves of around 1.2 km³ (7.7×10^9 barrels), with an estimate of 4.5 km³ (28×10^9 barrels) in total (http://en.wikipedia.org/wiki/South_China_Sea). Natural gas reserves are estimated to total around 7 500 km³ (266×10^12 cubic feet).

2 Data acquisition and processing

Multi-satellite altimeter sea surface heights consist of 11 years of T/P merged geophysical data records (392 cycles), 9.4 years of ERS-2 35-day-repeat data (96 cycles), 1.25 years of ERS-1 35-day-repeat data(13 cycles), 3.2 years of GEOSAT ERM data (63 cycles), and all the 1.5 years of GEOSAT GM data were subjected to rigorous editing criteria as recommended in the user handbooks of the various satellite altimeter missions and by Ohio State University. Using various empirical models, corrections to the edited sea surface heights were computed and applied to obtain the altimeter-derived corrected sea surface heights for both ascending and descending passes as follows.

For T/P, ERS-1 and ERS-2 datasets, the corrected sea surface height, $H_{cssh}$, for each data point was computed as:

$$H_{cssh} = H_{sat} - H_{Alt} - (C_{tides} + C_{wet} + C_{dry} + C_{iono} + C_{ib} + C_{ssb})$$

where $H_{sat}$ is the height of the satellite above the reference ellipsoid; $H_{Alt}$ is the 1-Hz altimeter range to the instantaneous sea surface; $C_{ssb}$ is the sea state bias correction; $C_{ib}$ is the inverse barometer correction per cycle; $C_{iono}$ is the ionospheric correction; $C_{dry}$ is the dry-tropospheric correction; $C_{wet}$ is the wet-tropospheric correction; and $C_{tides}$ is the sum of the solid earth tide, ocean tide, and loading tide corrections. For T/P dataset only, the $C_{tides}$ value includes pole tide correction.

However, for GEOSAT GDRs, the corrected sea surface heights are computed by:

$$C_{cssh} = H - (C_{tides} + C_{wet} + C_{dry} + C_{iono} + C_{ib} + C_{ssb} + C_{glob} + C_{hcal} + C_{uso})$$

where $H$ is the 1-Hz sea surface height provided in the GDRs; $C_{glob}$, $C_{uso}$, and $C_{hcal}$ are respectively global inverse barometer correction, ultra-stable-oscillator correction, and the internal temperature calibration correction, termed refined “secondary” corrections meant to augment their previous values given in the GDRs. Table 1 shows descriptive statistics of the corrected sea surface heights before crossovers were performed.

Single- and dual-satellite crossovers were performed, and at the crossover positions, north and east components of deflections of the vertical were determined from the corrected sea surface heights, using Sandwell’s computational philosophy[7] expressed as follows:
Table 1  Statistics of corrected sea surface heights before crossovers

| Mission     | No. of points | Max/m | Min/m | Mean/m | RMS/m | STD/m |
|-------------|---------------|-------|-------|--------|-------|-------|
| ERS1        | 338 961       | 77.360| -24.167| 35.928 | ±41.794 | ±21.353 6 |
| ERS2        | 2 102 098     | 79.159| -25.215| 35.576 | ±41.519 2 | ±21.403 9 |
| GEOSAT-GM   | 299 821       | 78.626| -24.148| 35.676 | ±41.295 7 | ±20.797 0 |
| GEOSAT-ERM  | 528 073       | 78.191| -24.069| 34.188 | ±39.884 7 | ±20.540 4 |
| T/P         | 2 073 723     | 77.870| -36.753| 40.112 | ±44.794 0 | ±19.936 7 |

East component of deflection of the vertical:

\[ \eta = -\frac{1}{2\lambda R\cos\varphi}(\ddot{H}_a + \ddot{H}_d) \]

North component of deflection of the vertical:

\[ \xi = -\frac{1}{2|R|\varphi}(\ddot{H}_a - \ddot{H}_d) \]

where \( \ddot{H} \) denotes derivative of corrected sea surface height, subscripts \( a \) and \( d \) representing ascending and descending tracks, respectively; \( \lambda \) and \( \varphi \) are the respective longitudinal and latitudinal components of the satellite’s ground track velocity; and \( R \) is the mean radius of the earth.

The altimeter-derived components of deflections of the vertical were then grided onto a 2.5°×2.5° resolution grid by employing the Shepard’s interpolation procedure; see References [8, 9] for details. Table 2 presents descriptive statistics of the grided altimeter-derived components of deflections of the vertical. Using GMT xyz2grd, grdcontour and pscoast programs, the altimeter-derived components of deflections of the vertical were “contoured” at 1° interval and 5′×5′ spatial resolution (for display purposes) as exemplified in Fig.1 and Fig.2 for ERS1.

Table 2  Statistics of gridded altimeter-derived components of deflections of the vertical

| Satellite altimeter data type | No. of grid nodes with values | Component of vertical deflection | Max/ (″) | Min/ (″) | Mean/ (″) | RMS/ (″) | STD/ (″) |
|------------------------------|-------------------------------|---------------------------------|----------|----------|-----------|----------|----------|
| ERS1                         | 439 774                       | North ξ^ERS1\_{\text{AH}}    | 36.938   | -18.377 0| 3.640 13  | ±5.550 69 | ±4.190 41 |
| ERS1                         | 447 205                       | North η^ERS1\_{\text{AH}}    | 66.963   | -29.777 0| -3.780 45 | ±8.298 88 | ±7.387 80 |
| ERS2                         | 455 596                       | North ξ^ERS2\_{\text{AH}}    | 37.457   | -34.155 0| 3.515 83  | ±5.545 33 | ±4.288 31 |
| ERS2                         | 455 596                       | North η^ERS2\_{\text{AH}}    | 62.415   | -48.779 0| -3.941 50 | ±8.128 79 | ±7.109 27 |
| GEOSAT                       | 455 596                       | North ξ^GEOSAT\_{\text{AH}}  | 42.477   | -44.759 0| 3.104 96  | ±5.448 55 | ±4.477 26 |
| GEOSAT                       | 455 596                       | North η^GEOSAT\_{\text{AH}}  | 99.702   | -43.176 0| -3.666 80 | ±8.053 01 | ±7.169 77 |
| T/P                          | 356 936                       | North ξ^T/P\_{\text{AH}}     | 28.579   | -28.904 0| 2.671 97  | ±5.951 20 | ±5.317 64 |
| T/P                          | 356 936                       | North η^T/P\_{\text{AH}}     | 64.300   | -56.088 0| -2.962 26 | ±9.803 24 | ±9.344 97 |
| ERS(1+2)                     | 447 230                       | North ξ^ERS(1+2)\_{\text{AH}}| 37.827   | -34.018 0| 3.521 11  | ±5.542 97 | ±4.280 92 |
| ERS(1+2)                     | 447 230                       | North η^ERS(1+2)\_{\text{AH}}| 62.580   | -48.226 0| -3.936 09 | ±8.125 87 | ±7.108 94 |
| GEOSAT-ERS(1+2)              | 456 747                       | North ξ^GEOSAT-ERS(1+2)\_{\text{AH}}| 37.819   | -44.918 0| 3.386 04  | ±5.447 81 | ±4.267 71 |
| GEOSAT-ERS(1+2)              | 456 747                       | North η^GEOSAT-ERS(1+2)\_{\text{AH}}| 101.606  | -49.677 0| -3.734 14 | ±7.800 06 | ±6.848 15 |
| T/P-ERS(1+2)                 | 366 092                       | North ξ^T/P-ERS(1+2)\_{\text{AH}}| 35.632   | -36.985 0| 3.180 57  | ±6.094 76 | ±5.199 04 |
| T/P-GEOSAT                   | 375 786                       | North η^T/P-GEOSAT\_{\text{AH}}| 88.470   | -46.982 0| -2.605 52 | ±8.310 83 | ±7.891 85 |

Fig.1  5′×5′ ERS1-derived east component of deflection of vertical at 1° contour interval

Fig.2  5′×5′ ERS1-derived north component of deflection of vertical at 1° contour interval
2.5′×2.5′ grid of EGM96-derived components of deflections of the vertical and geoid heights, depicted in Figs. 3, 4 and 5, were then used as reference global geopotential model quantities in a remove-restore procedure to implement the Molodensky’s formula via 1D-FFT technique to predict the 2.5′×2.5′ geoid heights from the gridded altimeter-derived components of deflections of the vertical.

The EGM96-derived components of deflections of the vertical $\xi_q^\text{GM}$ and $\eta_q^\text{GM}$ were first subtracted from the altimeter-derived components of the deflections of the vertical to obtain the residual north $\delta \xi_q$ and east $\delta \eta_q$ components of deflections of the vertical as:

$$
\begin{align*}
\delta \xi_q &= \xi_q^\text{Alt} - \xi_q^\text{GM} \\
\delta \eta_q &= \eta_q^\text{Alt} - \eta_q^\text{GM}
\end{align*}
$$

With the residual components of vertical deflection as input data, the Molodensky’s formula was invoked to compute the residual geoid heights, through the 1D-FFT technique, as:

$$
\delta N(\varphi, \lambda) = -\frac{R}{4\pi} F_1^{-1} \left\{ F_1 [\delta \xi(\varphi, \lambda) \cos \varphi] \cdot F_1 [Q_\xi(\varphi, \lambda)] + F_1 [\delta \eta(\varphi, \lambda) \cos \varphi] \cdot F_1 [Q_\eta(\varphi, \lambda)] \right\} d\varphi.
$$

Table 3 gives the statistical information on the final altimeter-derived geoid heights $N$ via 1D FFT from deflections.

| Mission        | Data points | Max/m | Min/m | Mean/m | RMS/m | STD/m | CPU time |
|----------------|-------------|-------|-------|--------|-------|-------|----------|
| ERS1           | 394 655     | 76.812| -27.727| 34.239 | ±41.115| ±22.761| 5 min0 s |
| ERS2           | 400 671     | 76.715| -27.922| 34.151 | ±41.149| ±22.956| 4 min8 s |
| GEOSAT         | 407 331     | 76.101| -28.923| 33.737 | ±40.967| ±23.240| 4 min6 s |
| T/P            | 327 006     | 77.354| -13.303| 39.372 | ±44.022| ±19.693| 5 min5 s |
| ERS(1+2)       | 400 696     | 76.723| -27.921| 34.152 | ±41.150| ±22.956| 4 min19 s|
| GEOSAT-ERS(1+2)| 408 032     | 77.050| -28.964| 33.687 | ±40.956| ±23.290| 5 min30 s|
| T/P-ERS(1+2)   | 334 961     | 77.410| -14.523| 38.969 | ±43.822| ±20.045| 4 min52 s|
| T/P-GEOSAT     | 343 609     | 77.725| -14.360| 38.639 | ±43.611| ±20.221| 5 min10 s|

The EGM96-derived geoid heights $N^\text{GM}$ were then “restored” to the residual altimeter-derived geoid heights $\delta N^\text{Alt}$ to obtain the final altimeter-derived geoid heights $N$ over the South China Sea and the Philippine Sea:

$$
N = \delta N^\text{Alt} + N^\text{GM}
$$
altimeter-predicted geoid heights (excluding the EGM96-derived geoid heights on land) over the study area via 1D-FFT while Fig.6 and Fig.7 present examples of the predicted geoid heights contoured at an interval of 1 m on a grid of $5^\prime \times 5^\prime$ spatial resolution (for display purposes).

### 3 Results and discussions

Each of the satellite altimeter-predicted geoid heights is compared with the EGM96-derived geoid heights and the residuals computed. Table 4 shows the descriptive statistics of the comparisons while Fig.8 and Fig.9 present the contour map and a scatter plot respectively of the differences.

The differences between the altimeter-predicted geoid heights and the EGM96-derived geoid heights actually represent the residual geoid heights, which are the short-wavelength component of the geoidal height signal attributed to the local geological structures. From geodetic point of view, these residuals only indicate the level of agreement or fitness between the

### Table 4  Statistics of differences between EGM96-derived and altimeter-derived geoid heights via 1D FFT over South China Sea and Philippine Sea

| Mission           | Data points | Max/m | Min/m | Mean/m | RMS/m | STD/m |
|-------------------|-------------|-------|-------|--------|-------|-------|
| ERS1              | 394 655     | 6.320 | -18.829 | -0.113 501 | ±1.123 6 | ±1.117 8 |
| ERS2              | 400 671     | 6.335 | -18.941 | -0.111 201 | ±1.118 4 | ±1.112 9 |
| GEOSAT            | 407 331     | 5.838 | -18.330 | -0.114 874 | ±1.089 4 | ±1.083 4 |
| T/P               | 327 006     | 6.441 | -17.613 | -0.140 413 | ±1.210 4 | ±1.202 2 |
| ERS1-2            | 400 696     | 6.334 | -18.896 | -0.111 151 | ±1.118 1 | ±1.112 6 |
| GEOSAT-ERS(1+2)   | 408 032     | 5.955 | -18.898 | -0.111 886 | ±1.099 1 | ±1.093 3 |
| T/P-ERS(1+2)      | 334 961     | 6.328 | -18.668 | -0.134 930 | ±1.214 6 | ±1.207 1 |
| T/P-GEOSAT        | 343 609     | 6.296 | -18.043 | -0.129 541 | ±1.195 0 | ±1.188 0 |
EGM96-generated geoid and the satellite-altimeter-derived geoids. From the scatter plot it could also be seen that high values of the differences occur around longitude E127°, thus suggesting a “bad” agreement between the satellite-altimeter derived geoid and the EGM96-derived geoid around that particular longitude. But interestingly, that is the region where the Philippine trench occurs, and is tectonically active.

When the data was restricted to longitude E126°, the recomputed statistics of the differences as shown in Table 5 below indicate that over a less tectonically active region, there is a comparatively better agreement between the satellite altimeter- and the EGM96-derived geoids.

Table 5  Statistics of recomputed differences between EGM96-generated and ERS1-derived geoid heights

| Mission Data points | Max/m | Min/m | Mean/m | RMS/m  | STD/m  |
|---------------------|-------|-------|--------|--------|--------|
| ERS1                | 265   | 631   | 2.295  | -1.817 | 0.007  |
|                     |       |       | 46±0.358 |       | 4±0.358 |

4  Conclusions

In this paper, a $2.5' \times 2.5'$ grid of marine geoid in a non-tidal system over the South China Sea and the Philippine Sea is derived from multi-satellite altimeter sea surface heights. The results indicate there is a root-mean-square agreement of about $\pm 0.35$ m between the altimeter-derived and the EGM96-derived geoid heights in the region bound by latitude N1°–N29° and longitude E106°–E126° where there are comparatively less pronounced geological structures. Over areas known to host geological structures which are tectonically active, higher differences up to $-18.9$ m are found between the altimeter-derived and the EGM96-derived geoids. Thus it could be concluded that satellite altimetry could be used as a geological mapping tool and/or reconnaissance tool for geophysical commodity prospecting.

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