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A new extended Rayleigh distribution with applications of COVID-19 data

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ABSTRACT

This paper aims to model the COVID-19 mortality rates in Italy, Mexico, and the Netherlands, by specifying an optimal statistical model to analyze the mortality rate of COVID-19. A new lifetime distribution with three-parameter is introduced by a combination of Rayleigh distribution and extended odd Weibull family to produce the extended odd Weibull Rayleigh (EOWR) distribution. This new distribution has many excellent properties as simple linear representation, hazard rate function, and moment generating function. Maximum likelihood, maximum product spacing and Bayesian estimation methods are applied to estimate the unknown parameters of EOWR distribution. MCMC method is used for the Bayesian estimation. A numerical result of the Monte Carlo simulation is obtained to assess the use of estimation methods. Also, data analysis for the real data of mortality rate is considered.

Introduction

One of the main tasks of statistics is to find efficient statistical modeling for natural life events in the form of known probability distributions. Probability distributions are used to model natural life phenomena that are characterized by uncertainty and riskiness. Many probability distributions have been derived because the natural life phenomena are complex and very hard to model by traditional distributions. However, known probability distributions remain unable to represent data for some natural phenomena accurately. These lead to the expansion and modification of generalized probability distributions. The addition of some new parameters to the known probability distributions improved the quality of suitability for the natural phenomena data and higher accuracy of describing the shape of the distribution’s tail.

Most of the Rayleigh probability distribution extensions have been derived because of their great importance in describing many natural life phenomena. The Rayleigh probability density function, attributed to Lord Rayleigh (1842–1919), is concerned with describing skewed data see Rayleigh [1] Many researchers consider one scale parameter Rayleigh, like Robert C.P. Diebolt and Robert [2] discussed deviation and distance measure in economic, which can be applied in another natural phenomena data. The extended probability distribution was originally introduced by Lehmann [3]. Kundu and Raqab [4] provided a generalization of the Rayleigh probability distribution and estimated its unknown parameters using several different methods. Voda [5] used the conservative technique to derive a new generalization of the Rayleigh probability distribution. Dey [6] presented Bayesian estimates of the Rayleigh probability distribution parameters using the lineex loss functions and square error loss function. Merovci [7] used the square ordinal transformation method in developing the transmuted Rayleigh probability distribution. Mahmoud and Ghazal [8] discussed the estimation of the exponentiated Rayleigh parameters based on type II censored data. Ateeq et al. [10] derived the Rayleigh-Rayleigh distribution (RMD) using the Transformed Transformer technique. El-Sherpieny and Almetwally [11] introduced Bivariate generalized Rayleigh distribution based on Clayton copula with various applications. Almetwally et al. [11] used the maximum likelihood and maximum product spacing estimates for generalized Rayleigh distribution based on the adaptive type-II progressive censoring schemes. Al-Babtain [13] proposed a new extension of the Rayleigh distribution with a two-parameter called type I half logistic Rayleigh distribution.

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Here we study a new model with three parameters, it is called extended odd Weibull Rayleigh (EOWR) distribution. The EOWR distribution is obtained based on the extended odd Weibull-G (EOW-G) family, which was introduced by Alizadeh et al. [14]. Let \( G(x; \delta) = 1 - G(x; \delta) \) and \( g(x; \delta) = \frac{dG(x; \delta)}{dx} \) denote the survival function (SF) and probability density function (PDF) of a baseline model with parameter vector \( \delta \) respectively, so the CDF of the EOW-G family is given by:

\[
F(x; \alpha, \beta, \delta) = \gamma \left( 1 + \beta \left[ \frac{G(x; \delta)^{\alpha}}{G(x; \delta)^{\alpha + 1}} \right] \right)^{\gamma}, x \in \mathbb{R}, \quad (1)
\]

The corresponding PDF of (1) is defined by

\[
f(x; \alpha, \beta, \delta) = \frac{\alpha g(x; \delta) G(x; \delta)^{\alpha - 1}}{\gamma \left( 1 + \beta \left[ \frac{G(x; \delta)^{\alpha}}{G(x; \delta)^{\alpha + 1}} \right] \right)^{\gamma + 1}}, x \in \mathbb{R}, \quad (2)
\]

where \( \alpha \) and \( \beta \) are positive shape parameters. The random variable with PDF (2) is denoted by \( X \sim \text{EOW-G}(\alpha, \beta, \delta) \). Affy and Mohamed [16] introduced a new flexible three-parameter exponential distribution called the extended odd Weibull exponential distribution. Alshenawy et al. [17] used the maximum likelihood estimation and maximum product spacing estimates to estimate the parameters of the extended odd Weibull exponential distribution under the progressive type-II censoring scheme with random removal.

Our goal in this paper is to study point estimation of the unknown parameters of EOWR by using two classical methods of estimation and the Bayes estimation method. A statistical comparison between these methods is conducted via simulation to assess these methods’ performance and to study how these estimators behave for several sample sizes and parameter values.

The rest of this paper is organized as follows. In Section ‘EOWR distribution’, we define EOWR distribution. Some EOWR statistical properties as a linear representation of its PDF is obtained in Section ‘Statistical properties’. Three methods of point estimation are studied in Section ‘Parameter estimation’. In Section ‘Simulation analysis’, a simulation study is conducted to compare the performance of these estimation methods. Three real data sets of COVID-19 from different life applications are used in Section ‘Applications to COVID-19 data’ to prove the efficiency of the EOWR distribution compared to other distributions. Finally, conclusions and major findings are given in Section ‘Conclusion’.

**EOWR distribution**

The three-parameter EOWR distribution is a special model of EOW-G family with Rayleigh distribution as a baseline function. The Rayleigh distribution under consideration has respectively PDF and CDF of the form \( g(x; \delta) = 2\delta x e^{-\delta x^2} \) and \( G(x; \delta) = 1 - e^{-\delta x^2}, x > 0, \delta > 0 \). By substituting the CDF and PDF of the Rayleigh model in Eq. (1) and Eq. (2) respectively, we obtain the CDF and PDF of the EOWR distribution respectively as:

\[
F(x; \alpha, \beta, \delta) = 1 - \left\{ 1 + \beta \left[ e^{\delta x^2} - 1 \right] \right\}^{\gamma}, x > 0, \alpha, \beta, \delta > 0. \quad (3)
\]
\[
\begin{align*}
\mu_x(\alpha', \delta) &= E(x') = \int_0^\infty x' h(x; \alpha', \delta) dx \\
&= \alpha' \delta \sum_{\ell=0}^{\infty} \frac{\alpha' - 1}{l} \left( -1 \right)^l \Gamma\left( \frac{q}{2} + 1 \right) (\delta(l + 1))^{\frac{q}{2} - 1}, \\
M_\ell(t; \alpha', \delta) &= E(e^{t x'}) = \int_0^\infty e^{t x'} h(x; \alpha', \delta) dx \\
&= \alpha' \delta \sum_{l=0}^{\infty} \frac{\alpha' - 1}{l} \left( -1 \right)^l \Gamma\left( \frac{q}{2} + 1 \right) (\delta(l + 1))^{\frac{q}{2} - 1}.
\end{align*}
\]

Moments and moment generating functions

The \( r \)th moment of the EOWR distribution follows directly from Eqs. (6) and (7)

\[
\mu_r = \sum_{j=0}^{\infty} a_j \mu_{r,j}(\alpha j + k, \delta).
\]

Referring to Eq. (6) and Eq. (8), the MGF of the EOWR distribution is given by:

\[
M(t) = \sum_{j=0}^{\infty} a_j M_j(t, \alpha j + k, \delta).
\]

Quantile function and median

The quantile function of EOWR distribution is used in the theoretical aspect of probability theory for this model, like statistical applications, and simulations. Simulation algorithm used quantile function to produce simulated random samples. The quantile function (Q) of the EOWR distribution is given by

\[
Q(q) = \left\{ \frac{1}{\delta} \log \left[ 1 + \left( \frac{1}{\beta} \right) [(1 - q)^{\frac{q}{2}} - 1] \right] \right\}^{\frac{1}{q}}.
\]

In particular, the median of EOWR distribution can be derived from Eq. (11) by setting \( q = 0.5 \). Then, the median is given by

\[
\text{median} = \left\{ \frac{1}{\delta} \log \left[ 1 + \left( \frac{1}{\beta} \right) [2^q - 1] \right] \right\}^{\frac{1}{q}}.
\]

Parameter estimation

In this section, we use different point estimation methods to estimate the unknown parameters of the EOWR. We use maximum likelihood estimator (MLE), the maximum product of spacing estimator (MPS) and Bayesian estimation methods. In the last few years, parameter estimation using different estimation methods got great attention from many authors such as Almetwally and Almongy [19], Haj Ahmad and Almetwally [20], Basheer et al. [21], and Afify and Mohamed [16].

Maximum likelihood method

Let \( x_1, \ldots, x_n \) be a random sample from the EOWR distribution with parameters \( \alpha, \beta, \) and \( \delta \). The likelihood function can be written as:

\[
f(x; \alpha, \beta, \delta) = 2a\delta x e^{\delta x (e^{\beta x} - 1)} \sum_{k=0}^{\infty} a_k h_k(x; \alpha, \beta, \delta).
\]
\[
L(\Theta) = 2^n a^\alpha b^\beta \sum_{i=1}^{n} x_i \left[ \frac{e^{\alpha x_i} - 1}{1 + \beta (e^{\alpha x_i} - 1)^n} \right]^{\frac{1}{2n}},
\]

(12)

and \(\Theta = (\alpha, \beta, \delta)\) is a vector of the EOWR parameters, the log-likelihood function is

\[
\ell(\Theta) = \log[\log(\alpha) + \log(\beta)] + \delta \sum_{i=1}^{n} x_i^\alpha + (\alpha - 1) \sum_{i=1}^{n} \log(e^{\alpha x_i} - 1) - 1 - \left[ \frac{1 + \beta}{\beta} \right] \sum_{i=1}^{n} \log[1 + \beta (e^{\alpha x_i} - 1)^n],
\]

(13)

The MLE are obtained by solving the following normal equations,

\[
\frac{\partial \ell(\Theta)}{\partial \alpha} = a + \sum_{i=1}^{n} \left[ e^{\alpha x_i} - 1 \right] \left( \frac{\alpha + 1}{1 + \beta (e^{\alpha x_i} - 1)^n} \right),
\]

(14)

\[
\frac{\partial \ell(\Theta)}{\partial \beta} = \sum_{i=1}^{n} \left[ e^{\alpha x_i} - 1 \right] \left( \frac{\alpha - 1}{1 + \beta (e^{\alpha x_i} - 1)^n} \right),
\]

(15)

and

\[
\frac{\partial \ell(\Theta)}{\partial \delta} = \sum_{i=1}^{n} \left[ e^{\alpha x_i} - 1 \right] \left( \frac{\alpha - 1}{1 + \beta (e^{\alpha x_i} - 1)^n} \right).\]

(16)

These Eqs. (14)-(16) cannot be solved explicitly. Hence a nonlinear optimization algorithm such as Newton Raphson method is used by equating them to zero.

**Maximum product spacing**

According to Cheng and Amin [22], the MPS method is an efficient estimation method that have some advantages compared to other point estimation methods. So we use MPS in this section to have point estimation of the unknown parameters of EOWR distribution. This can be obtained by solving the normal equations resulting from taking partial derivatives of logarithm of product spacing function \(G(\Theta)\) which is written as:

\[
G(\Theta) = \left\{ \left[ 1 - \{ 1 + \beta \Delta(x_1)^n \}^{\frac{1}{n}} \right] \left[ 1 - \{ 1 + \beta \Delta(x_n)^n \}^{\frac{1}{n}} \right] \right\}^{\frac{1}{n}},
\]

\[
\times \prod_{i=2}^{n} \left[ \left[ 1 + \beta \Delta(x_i)^n \right]^{\frac{1}{n}} + \left[ 1 + \beta \Delta(x_i)^n \right]^{\frac{1}{n}} \right],
\]

where \(\Delta(x_i) = (e^{\alpha x_i} - 1)\) and the logarithmic function of \(G(\Theta)\)

\[
\log G(\Theta) = \frac{1}{n+1} \left( \log \left[ 1 - \{ 1 + \beta \Delta(x_1)^n \}^{\frac{1}{n}} \right] + \log \left[ 1 - \{ 1 + \beta \Delta(x_n)^n \}^{\frac{1}{n}} \right] \right) + \frac{1}{n+1} \sum_{i=2}^{n} \left( \log \left[ 1 + \beta \Delta(x_{i-1})^n \right] \right) + \log \left[ 1 + \beta \Delta(x_{i})^n \right].
\]

(17)

The MPS estimators of \(\Theta\) are obtained by differentiating the log-product Eq. (17) with respect to each parameter, then we solve the nonlinear system of equations found by using any iterative procedure techniques such as Newton Raphson algorithms. This method was developed in the last few years to estimate parameter of the model under censoring scheme see Ng et al. [23], Basu et al. [24], Almetwally and Almongy [25], Almetwally et al. [19,26], El-Sherpieny et al. [27], and Alshenawy et al. [18].

**Bayesian estimation**

Bayesian methods is a statistical inference that depends on the choice of the prior distribution and the loss function. In this method all parameters are considered as random variables with certain distribution called prior distribution. If prior information is not available, we need to select one. Since the selection of prior distribution plays an important role in estimation of the parameters, our choice for the priors are the independent gamma distributions. On the other hand, the loss function is important in Bayesian methods. Most of the Bayesian inference procedures are developed under the symmetric and asymmetric loss functions. One of the most common symmetric loss function is the squared error loss (SEL) function. The independent joint prior density function of \(\Theta\) can be written as follows:

\[
\pi(\Theta) = \frac{\beta_0^r}{\Gamma(s_1)} \frac{\beta_1^r}{\Gamma(s_2)} \frac{\beta_0^{r+1}}{\Gamma(s_3)} e^{-(\beta_0^r \gamma_0^s + \beta_1^r \gamma_1^s + \beta_0^{r+1} \gamma_0^s)}.
\]

(18)

The joint posterior density function of \(\Theta\) is obtained from (12) and (18)

\[
\pi(\Theta|\mathcal{X}) = \frac{L(\mathcal{X}|\Theta)}{\int_L L(\mathcal{X}|\Theta) d\Theta}.
\]

(19)

can be rewritten as follows

\[
\pi(\Theta|\mathcal{X}) \propto \alpha e^{\alpha x_1 - 1} e^{\frac{\gamma_0^s}{s_1}} \prod_{i=1}^{n} \left[ \frac{e^{\alpha x_i} - 1}{1 + \beta (e^{\alpha x_i} - 1)^n} \right]^{a+1} \left[ 1 + \beta (e^{\alpha x_i} - 1)^n \right] \frac{1}{2n},
\]

(20)

The conditional distribution can obtained as follows

\[
\pi(\delta|\alpha, \beta, \mathcal{X}) \propto \alpha e^{\alpha x_1 - 1} e^{\frac{\gamma_0^s}{s_1}} \prod_{i=1}^{n} \left[ \frac{e^{\alpha x_i} - 1}{1 + \beta (e^{\alpha x_i} - 1)^n} \right]^{a+1} \left[ 1 + \beta (e^{\alpha x_i} - 1)^n \right] \frac{1}{2n},
\]

(21)

\[
\pi(\beta|\alpha, \delta, \mathcal{X}) \propto \beta e^{\beta x_1 - 1} e^{\frac{\gamma_0^s}{s_1}} \prod_{i=1}^{n} \left[ \frac{e^{\alpha x_i} - 1}{1 + \beta (e^{\alpha x_i} - 1)^n} \right]^{a+1} \left[ 1 + \beta (e^{\alpha x_i} - 1)^n \right] \frac{1}{2n}.
\]

(22)

and

\[
\pi(\alpha|\beta, \delta, \mathcal{X}) \propto \alpha e^{\alpha x_1 - 1} e^{\frac{\gamma_0^s}{s_1}} \prod_{i=1}^{n} \left[ \frac{e^{\alpha x_i} - 1}{1 + \beta (e^{\alpha x_i} - 1)^n} \right] \frac{1}{2n}.
\]

(23)

The Bayes estimators of \(\Theta\), say \((\hat{\alpha}_B, \hat{\beta}_B, \hat{\delta}_B)\) based on SEL function is given by

\[
\hat{\Theta}_{B-SEL} = \left\{ \hat{\alpha}_B, \hat{\beta}_B, \hat{\delta}_B \right\}
\]

(24)

where,

\[
\hat{\alpha}_B = \int_{0}^{\infty} \alpha d\alpha \int_{0}^{\infty} \int_{0}^{\infty} \pi(\Theta|\mathcal{X}) d\beta d\delta,
\]

(25)

\[
\hat{\beta}_B = \int_{0}^{\infty} \beta d\beta \int_{0}^{\infty} \int_{0}^{\infty} \pi(\Theta|\mathcal{X}) d\alpha d\delta,
\]

(26)
In this section, the Monte-Carlo simulation procedure is performed to compare the classical estimation methods: MLE, MPS, with Bayesian estimation method under square error loss function based on MCMC, for estimating parameters of EOWR distribution by R programming language. Monte-Carlo experiments are carried out based on data generated from the distribution. Ten thousand random samples from EOWR distribution were generated, where X has EOWR lifetime for different actual values of parameters and different sample sizes n = (50, 100, and 200). We could define the best estimator methods as which minimizes the bias and root mean squared error (RMSE) of estimators.

Tables 1, 2 summarizes the simulation results of point estimation methods proposed in this paper. We consider the bias and the RMSE values in order to perform the needed comparison between different point estimation methods. The following remarks can be noted from these tables:

Then, the BEs of \( a, \beta, \delta \) using MCMC under SE loss function

\[
\bar{u}_{\text{SE}} = \frac{1}{N} \sum_{i=1}^{N} u(a^{(i)}, \beta^{(i)}, \delta^{(i)}).
\]

Simulation analysis

Table 1

| \( \alpha = 0.5 \) | \( \beta \) | \( \delta \) | n | MLE Bias | RMSE | MPS Bias | RMSE | Bayesian Bias | RMSE |
|---|---|---|---|---|---|---|---|---|---|
| 0.5 | 0.5 | 50 | \( a \) | 0.0183 | 0.1240 | -0.0059 | 0.0087 | 0.0062 | 0.0414 |
| | | | \( \beta \) | 0.0105 | 0.8714 | 0.1152 | 0.3386 | 0.1075 | 0.2227 |
| | | | \( \delta \) | 0.0269 | 0.3340 | 0.0587 | 0.0665 | 0.0907 | 0.1791 |
| 100 | \( a \) | 0.0025 | 0.0683 | -0.0062 | 0.0040 | 0.0048 | 0.0371 |
| | | | \( \beta \) | -0.0189 | 0.6002 | 0.0741 | 0.1938 | 0.1272 | 0.2294 |
| | | | \( \delta \) | 0.0177 | 0.2677 | 0.0422 | 0.0348 | 0.0732 | 0.1392 |
| 200 | \( a \) | 0.0042 | 0.0460 | -0.0030 | 0.0020 | 0.0043 | 0.0318 |
| | | | \( \beta \) | 0.0413 | 0.3988 | 0.0685 | 0.1171 | 0.1079 | 0.2137 |
| | | | \( \delta \) | 0.0289 | 0.1856 | 0.0342 | 0.0203 | 0.0608 | 0.1189 |
| 2 | 50 | \( a \) | 0.0095 | 0.1031 | -0.0155 | 0.0078 | 0.0078 | 0.0418 |
| | | | \( \beta \) | -0.0498 | 0.5912 | -0.0046 | 0.1853 | 0.0772 | 0.1711 |
| | | | \( \delta \) | -0.0202 | 0.7408 | 0.0011 | 0.3149 | 0.1607 | 0.3494 |
| 100 | \( a \) | 0.0020 | 0.0632 | -0.0057 | 0.0035 | 0.0078 | 0.0373 |
| | | | \( \beta \) | -0.0483 | 0.5419 | 0.0356 | 0.1069 | 0.0751 | 0.1651 |
| | | | \( \delta \) | 0.0008 | 0.9164 | 0.0654 | 0.2222 | 0.1581 | 0.3380 |
| 200 | \( a \) | 0.0017 | 0.0467 | -0.0028 | 0.0019 | 0.0050 | 0.0325 |
| | | | \( \beta \) | 0.0017 | 0.3634 | 0.0485 | 0.0559 | 0.0745 | 0.1608 |
| | | | \( \delta \) | 0.0411 | 0.6169 | 0.0772 | 0.1242 | 0.1478 | 0.3097 |

| | 2 | 50 | \( a \) | 0.0179 | 0.0883 | -0.0225 | 0.0065 | 0.0034 | 0.0402 |
| | | | \( \beta \) | 0.0107 | 0.6120 | -0.0195 | 0.1984 | 0.0508 | 0.0853 |
| | | | \( \delta \) | 0.0381 | 0.1775 | 0.0150 | 0.0288 | 0.0561 | 0.1332 |
| 100 | \( a \) | 0.0137 | 0.0667 | -0.0102 | 0.0035 | 0.0045 | 0.0384 |
| | | | \( \beta \) | 0.2856 | 1.3758 | -0.0040 | 0.0876 | 0.0482 | 0.0834 |
| | | | \( \delta \) | 0.0718 | 0.3392 | 0.0045 | 0.0112 | 0.0342 | 0.0912 |
| 200 | \( a \) | 0.0034 | 0.0425 | -0.0093 | 0.0017 | 0.0013 | 0.0320 |
| | | | \( \beta \) | 0.0785 | 0.5627 | -0.0209 | 0.0384 | 0.0454 | 0.0819 |
| | | | \( \delta \) | 0.0217 | 0.1403 | -0.0016 | 0.0052 | 0.0251 | 0.0706 |
| 2 | 50 | \( a \) | 0.0176 | 0.1017 | -0.0213 | 0.0073 | 0.0062 | 0.0382 |
| | | | \( \beta \) | 0.0172 | 1.0997 | -0.0480 | 0.1761 | 0.0400 | 0.0757 |
| | | | \( \delta \) | 0.0687 | 0.7861 | -0.0452 | 0.1742 | 0.0940 | 0.3061 |
| 100 | \( a \) | 0.0109 | 0.0658 | -0.0111 | 0.0038 | 0.0053 | 0.0376 |
| | | | \( \beta \) | 0.0967 | 0.8821 | -0.0236 | 0.0966 | 0.0394 | 0.0748 |
| | | | \( \delta \) | 0.1147 | 0.9710 | -0.0105 | 0.0960 | 0.0968 | 0.2719 |
| 200 | \( a \) | 0.0025 | 0.0470 | -0.0094 | 0.0018 | 0.0017 | 0.0307 |
| | | | \( \beta \) | 0.0254 | 0.5610 | -0.0232 | 0.0602 | 0.0365 | 0.0719 |
| | | | \( \delta \) | 0.0346 | 0.4849 | -0.0115 | 0.0543 | 0.0841 | 0.2240 |

\[
\bar{\delta}_{\text{B-SE}} = \int_{0}^{\infty} \int_{0}^{\infty} \int_{0}^{\infty} \pi(\Theta | x) \, d\theta d\alpha d\beta.
\]
Concluding remarks on simulation

1. By increasing the sample size the values of the parameters tend to the true values of the parameters.
2. By increasing the sample size the RMSE of the parameters decreases.
3. Bayesian estimation method under squared error loss function has the least RMSE among all estimation methods.
4. The MPS provides better estimation values compared with the MLE referring to the value of the RMSE.

Table 2
Bias and RMSE of EOWR distribution for MLE, MPS, and Bayesian when $\alpha = 2$

| $\beta$ | $\delta$ | n | MLE Bias | RMSE | MPS Bias | RMSE | Bayesian Bias | RMSE |
|--------|----------|---|----------|------|----------|------|--------------|------|
| 0.5    | 0.5      | 50 | 0.0763   | 0.4108 | -0.0188  | 0.1031 | 0.0037       | 0.0208 |
| -0.0087 | 0.4392  | 0.0655  | 0.1490 | 0.0605  | 0.1722 |
| -0.0024 | 0.0589  | 0.0063  | 0.0034 | 0.0116  | 0.0454 |
| 100    | a        | 0.0250 | 0.2283  | -0.0174 | 0.0449  | 0.0036 | 0.0265       |
| -0.0189 | 0.2722  | 0.0337  | 0.0669 | 0.0336  | 0.1440 |
| -0.0014 | 0.0390  | 0.0043  | 0.0015 | 0.0067  | 0.0312 |
| 200    | a        | 0.0238 | 0.1901  | -0.0031 | 0.0208  | 0.0048 | 0.0325       |
| -0.0055 | 0.1994  | 0.0347  | 0.0280 | 0.0299  | 0.1179 |
| 2      | 50       | 0.0848 | 0.4883  | 0.0110  | 0.1802  | 0.0040 | 0.0220       |
| -0.0053 | 0.5195  | 0.0884  | 0.2352 | 0.0557  | 0.1710 |
| -0.0185 | 0.2605  | 0.0229  | 0.0624 | 0.0372  | 0.1659 |
| 100    | a        | 0.0360 | 0.3164  | 0.0031  | 0.0869  | 0.0022 | 0.0273       |
| 0.0009  | 0.3449  | 0.0647  | 0.1137 | 0.0381  | 0.1486 |
| -0.0052 | 0.1806  | 0.0222  | 0.0309 | 0.0235  | 0.1232 |
| 200    | a        | 0.0119 | 0.2015  | -0.0014 | 0.0383  | 0.0015 | 0.0306       |
| -0.0078 | 0.2116  | 0.0351  | 0.0445 | 0.0183  | 0.1150 |
| -0.0067 | 0.1138  | 0.0116  | 0.0126 | 0.0094  | 0.0881 |
| 2      | 0.5      | 50   | 0.0933   | 0.4091 | -0.0698 | 0.1299 | 0.0010       | 0.0210 |
| -0.0039 | 0.8778  | 0.0208 | 0.4735 | 0.0448  | 0.1068 |
| 0.0078  | 0.0917  | 0.0005 | 0.0060 | 0.0190  | 0.0621 |
| 100    | a        | 0.0665 | 0.3711  | -0.0348 | 0.0763  | 0.0024 | 0.0265       |
| 0.0732  | 0.7833  | 0.0243  | 0.3001 | 0.0392  | 0.1178 |
| 0.0027  | 0.0625  | -0.0007 | 0.0033 | 0.0101  | 0.0416 |
| 200    | a        | 0.0450 | 0.2147  | -0.0192 | 0.0342  | 0.0032 | 0.0295       |
| -0.0078 | 0.4542  | 0.0291  | 0.1509 | 0.0410  | 0.1219 |
| -0.0045 | 0.0433  | 0.0006  | 0.0017 | 0.0066  | 0.0293 |
| 2      | 50       | 0.3318 | 0.9574  | -0.0362 | 0.1857  | 0.0013 | 0.0214       |
| 0.7134  | 2.4516  | 0.1022  | 0.7926 | 0.0379  | 0.1031 |
| 0.1614  | 0.6438  | 0.0328  | 0.1295 | 0.0745  | 0.2226 |
| 100    | a        | 0.1235 | 0.4749  | -0.0302 | 0.0936  | 0.0025 | 0.0257       |
| 0.2302  | 1.1271  | 0.0382  | 0.4345 | 0.0353  | 0.1108 |
| 0.0471  | 0.3434  | -0.0017 | 0.0659 | 0.0369  | 0.1543 |
| 200    | a        | 0.0427 | 0.2836  | -0.0292 | 0.0532  | 0.0020 | 0.0291       |
| 0.0653  | 0.6103  | -0.0026 | 0.2270 | 0.0297  | 0.1154 |
| 0.0138  | 0.2215  | -0.0075 | 0.0362 | 0.0270  | 0.1238 |

Table 3
MLE, SE, KS test, P-values, W*, and A* for COVID-19 data of Italy.

| Italy | $\alpha$ | $\beta$ | $\delta$ | $\lambda$ | W* | A* | KS | P-value |
|-------|----------|---------|----------|-----------|----|----|----|---------|
| EOWR  | 2.9019   | 15.8688 | 0.0551   | 0.0653    | 0.3685 | 0.0828 | 0.7819 |
| R     | 6.5829   | 0.1328  | 0.8044   | 0.1360    | 0.2056 |
| MOR   | 0.7578   | 7.0444  | 0.0170   | 0.0138    | 0.2215 | -0.0075 | 0.0362 | 0.0270 | 0.1238 |
| KER   | 0.0131   | 1.6751  | 3.1031   | 0.0116    | 0.1753 | -0.0116 | 0.0823 | 0.1239 | 0.3004 |
Applications to COVID-19 data

In this section, three real data of COVID-19 mortality rates from Italy, Mexico, and the Netherlands (see https://covid19.who.int/) are given to test the EOWR distribution’s goodness of fit. The EOWR model is compared with other related models such as Rayleigh (R), Marshall-Olkin Rayleigh (MOR) [28], Kumaraswamy exponentiated Rayleigh (KER) Rashwan [29] and extended odd Weibull exponential (EOWE) distribution see Afify et.al. [16]. Tables 3–5 provide values of Crammer-von Mises ($W^*$), Anderson–Darling ($A^*$), and Kolmogorov-Smirnov (KS) statistic and its P-value for all models fitted based on three real data sets. For more information about Covid-19 data see, Abdel-Rahman [31–36,40,30].

The first data represents a COVID-19 mortality rates data belongs to Italy of 59 days, that is recorded from 27 February to 27 April 2020. The data are as follows: 4.571 7.201 3.606 8.479 11.410 8.961 10.919 10.908 6.503 18.474 11.010 17.337 16.561 13.226 15.137 8.697 15.787 13.333 11.822 14.242 11.273 14.330 16.046 11.950 10.282 11.775 10.138 9.037 12.396 10.644 8.646 8.905 8.906 7.407 7.445 7.214 6.194 5.452 4.859 4.408 4.253 4.011 3.564 3.827 3.134 2.780 2.881 3.341 2.686 2.814 2.508 2.450 1.518.

The second data represents a COVID-19 mortality rate data belongs to Mexico of 108 days, that is recorded from 4 March to 20 July 2020. This data formed of rough mortality rate. The data are as follows: 8.826 6.105 10.383 7.267 13.220 6.015 10.855 6.122 10.685 10.035 5.242 7.630 14.604 7.903 6.327 9.391 14.962 4.730 3.215 16.498 11.665 9.284 12.878 6.566 3.440 5.854 8.813 10.043 7.260 5.985 4.244 3.440 5.143 9.935 7.840 9.550 6.968 6.370 3.537 2.386 10.158 8.108 6.697 7.151 6.560 2.988 3.336 6.814 8.325 7.854 8.551 3.228 3.499 3.751

The second data represents a COVID-19 mortality rate data belongs to Mexico of 108 days, that is recorded from 4 March to 20 July 2020. This data formed of rough mortality rate. The data are as follows: 8.826 6.105 10.383 7.267 13.220 6.015 10.855 6.122 10.685 10.035 5.242 7.630 14.604 7.903 6.327 9.391 14.962 4.730 3.215 16.498 11.665 9.284 12.878 6.566 3.440 5.854 8.813 10.043 7.260 5.985 4.244 3.440 5.143 9.935 7.840 9.550 6.968 6.370 3.537 2.386 10.158 8.108 6.697 7.151 6.560 2.988 3.336 6.814 8.325 7.854 8.551 3.228 3.499 3.751

Table 4

| Mexico | α      | β      | δ      | λ      | $W^*$ | $A^*$ | KS    | P-value |
|--------|--------|--------|--------|--------|-------|-------|-------|---------|
| EOWR   | 1.9711 | 6.6509 | 0.0633 | 0.1777 | 0.0293|       |       | 0.9915  |
| R      | 4.6719 | 3.8516 | 0.0227 | 0.0449 |       |       |       |         |
| MOR    | 0.6115 | 5.3157 | 0.0502 | 0.5155 | 0.0822|       |       | 0.8283  |
| KER    | 0.2670 | 0.1734 | 0.9477 | 1.3805 | 0.0955|       |       | 0.0773  |
| EOWE   | 2.1998 | 1.1979 | 0.1406 | 0.5128 | 0.0908|       |       | 0.6017  |

Table 5

| Netherlands | α      | β      | δ      | λ      | $W^*$ | $A^*$ | KS    | P-value |
|-------------|--------|--------|--------|--------|-------|-------|-------|---------|
| EOWR        | 1.3172 | 2.7624 | 0.0335 | 0.1807 | 0.0262|       |       | 0.9932  |
| R           | 4.9985 | 2.2505 | 0.0170 | 0.0734 |       |       |       |         |
| MOR         | 0.6136 | 5.6754 | 0.0170 | 0.0832 | 0.0506|       |       | 0.9746  |
| KER         | 0.0115 | 3.2253 | 2.3294 | 0.1046 | 0.2301|       |       | 0.8646  |
| EOWE        | 2.0538 | 1.0192 | 0.1274 | 0.1873 | 0.0272|       |       | 0.9758  |

Fig. 3. Estimated PDF, PP-plot and QQ-plot of EOWR for COVID-19 data of Italy.
7.486 6.625 6.140 4.909 4.661 1.867 2.838 5.392 12.042 8.696 6.412 3.395 1.815 3.327 5.406 6.182 4.949 4.089 3.359 2.070 3.298 5.317 5.442 4.557 4.292 2.500 6.535 4.648 4.697 5.459 4.120 3.922 3.219 1.402 2.438 3.257 3.632 3.233 3.027 2.352 1.205 2.077 3.778 3.218 2.926 2.601 2.065 1.041 1.800 3.029 2.058 2.326 2.506 1.923.

The third data represents a COVID-19 data belonging to the Netherlands of 30 days, which recorded from 31 March to 30 April 2020. This data formed of rough mortality rate. The data are as follows: 14.918 10.656 12.274 10.289 10.832 7.099 5.928 13.211 7.968 7.584 5.555 6.027 4.097 3.611 4.960 7.498 6.940 5.307 5.048 2.857 2.254 5.431 4.462 3.883 3.461 3.647 1.974 1.273 1.416 4.235.

Concluding remarks on real data

From Tables 3–5 it is obvious that EOWR distribution has minimum values of all information criteria as W*, A*, and KS compared with other competitive distributions. Also the P-value for KS has the highest value. This leads us to conclude that EOWR is the best fit for the three real data sets. Empirical, Q-Q and P-P plots are shown in Figs. 3–5, indicate that our distribution is a superior model for modeling the above real data.

Conclusion

In this paper, we presented a new generalization of Rayleigh and Weibull distributions called EOWR distribution. We studied its statistical properties and obtained a linear representation for its pdf, which was efficient in finding moments, generating function. Different classical and Bayes estimation methods were considered to find point estimation of EOWR unknown parameters $\alpha$, $\beta$, and $\delta$. A comparison was conducted via simulation analysis using the R package to distinguish the performance of different estimation methods. MCMC method was used for that purpose, and the Bayesian estimation method was better than classical estimation methods referring to the value of RMSE. Also, three real data sets of COVID-19 mortality rates were considered, and they showed that EOWR provides the best fit for these kinds of data compared with other competitive distributions.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

[1] Rayleigh L. Xii. on the resultant of a large number of vibrations of the same pitch and of arbitrary phase. London Edinburgh Dublin Philos Mag J Sci 1880;10(60): 73–8.
[2] Diebold J, Robert CP. Estimation of finite mixture distributions through Bayesian sampling. J Roy Stat Soc: Ser B (Methodol) 1994;56(2):363–75.
[3] Lehmann EL. The power of rank tests. Ann Math Stat 1953;24:23–43.
[4] Kundu D, Raqab MZ. Generalized Rayleigh distribution: different methods of estimations. Comput Stat Data Anal 2005;49(1):187–200.
[5] Voda VG. A new generalization of the Rayleigh distribution. Reliability: Theory Applications 2007;2(2):47–56.
[6] Dey S. Comparison of Bayes estimators of the parameter and reliability function for Rayleigh distribution under different loss functions. Malays J Math Sci 2009;3(2):247–64.
[7] Merovci F. Transmuted rayleigh distribution. Austrian J Stat 2013;42(1):21–31.
[8] Merovci F, Elbatal I. Weibull-Rayleigh distribution: theory and applications. Appl Math Inf Sci 2015;9(3):1–11.
[9] Mahmoud AW, Ghazal MGM. Estimations from the exponentiated lindley distribution based on generalized Type-II hybrid censored data. J Egypt Math Soc 2017;25(1):71–84.
[10] Ateeq K, Qasim TR, Abi AR. An extension of Rayleigh distribution and applications. Cogent Math 2019;6(1):e1625191.
[11] Almetwally EM, Almongy HM, ElSherpieny EA. Adaptive type-II progressive censoring schemes based on maximum product spacing with application of generalized Rayleigh distribution. J Data Sci 2019;17(4):802–31.
[12] El-Sherpieny ES, Almetwally EM. Bivariate Generalized Rayleigh Distribution Based on Clayton Copula. In: Proceedings of the Annual Conference on Statistics (54rd). Cairo University: Computer Science and Operation research, Faculty of Graduate Studies for Statistical Research; 2019. p. 1–19.
[13] Albaatni AA. A new extended rayleigh distribution. J King Saud Univ Sci 2020;32(5):2576–81.
[14] Alizadeh M, Altun E, Afify AZ, Gamze OZEL. The extended odd Weibull-G family: properties and applications. Communications Faculty of Sciences University of Ankara Series A1 Mathematics and Statistics 2018;68(1):161–86.
[15] Almongy HM, Almetwally EM, Alharbi R, Alnagar D, Hafez EH, El-Din Mohie, Marwa M. The Weibull generalized exponential distribution with censored sample: estimation and application on real data. Complexity 2021;2021:1–15. https://doi.org/10.1155/2021/6653534.
[16] Afify AZ, Mohamed OA. A new three-parameter exponential distribution with variable shapes for the hazard rate: estimation and applications. Mathematics 2020;8(11):135.
[17] Ashenawy R, Al-Alwan A, Almetwally EM, Afify AZ, Almongy HM. Progressive type-II censoring schemes of extended odd Weibull exponential distribution with applications in medicine and engineering. Mathematics 2020;9(10):1679.
[18] Ashenawy R, Saeby MA, Almetwally EM, Almongy HM. Product spacing of stress–strength under progressive hybrid censored for exponentiated-gumbel distribution. Comput Mater Continua 2021;66(3):1973–95.
[19] Almetwally EM, Almongy HM. Estimation methods for the new Weibull-Pareto distribution: simulation and application. J Data Sci 2019;17(3):610–36.
[20] Haj Ahmad H, Almetwally E, Marshall-Olkin generalized pareto distribution: bayesian and non bayesian estimation. Pakistan J Stat Oper Res 2020;21:23–33.
[21] Badeer AM, Almetwally EM, Okasha HM. Marshall-Olkin alpha power inverse Weibull distribution: non-bayesian and bayesian estimations. J Stat Appl Pro 2020;9(2):1–21.
[22] Cheng RCH, Amin NAK. Estimating parameters in continuous univariate distributions with a shifted origin. J Roy Stat Soc: Ser B (Methodol) 1983;45(3):394–403.
[23] Ng HKT, Luo L, Hu Y, Duan F. Parameter estimation of three-parameter Weibull distribution based on progressively type-II censored samples. J Stat Comput Simul 2012;82(11):1661–78.
[24] Basu S, Singh SK, Singh U. Estimation of inverse lindley distribution using products of spacings function for hybrid censored data. Methodol Comput Appl Prob 2019;21(4):1377–94.

[25] Almetwally EM, Almongy HM. Maximum product spacing and bayesian method for parameter estimation for generalized power Weibull distribution under censoring scheme. J Data Sci 2019;17(2):407–44.
[26] Almetwally EM, Almongy HM, Rastogi MK, Ibrahim M. Maximum product space estimation of Weibull distribution under adaptive type-II progressive censoring schemes. Ann Data Sci 2020;7(2):257–79.
[27] El-Sherpieny ES, Almetwally EM, Nabil MM. Progressive Type-II hybrid censored schemes based on maximum product spacing with application to Power Lomax distribution. Phys A: Stat Mech Appl 2020;124251.
[28] MirMontafar SMKR, Mahmoudzadeh M, Lemonte AJ. The Marshall-Olkin extended generalized Rayleigh distribution: properties and applications. Commun Stat Theory Methods 2017;46(2):653–71.
[29] Rashwan NI. A note on Kumaraswamy exponentiated Rayleigh distribution. J Stat Theory Appl 2016;15(3):286–95.
[30] Almetwally EM, Alharbi R, Alnagar D, Hafez EH. A New Inverted Topp-Leone Distribution: Applications to the COVID-19 Mortality Rate in Two Different Countries. Axioms 2021, 10, 25. https://doi.org/10.3390/axioms10010025.
[31] Abdel-Rahman MAM, El-Gomati E, Al-Atiay H, Alaziz SN. Optimal Discrete Search for a Randomly Moving COVID19. J Stat Appl Prob 93(3):2020; 473–481.
[32] Hafez EH, El-Din Mohie, Marwa M. The digital transformation effects in education in light of the epidemics (COVID-19) in Egypt. Inf Sci Lett 2021;10(1):101–160.
[33] Mohie El-Din MM, Amin MM, El-Attiay EH, Hafez EH. Symmetric and asymmetric bayesian estimation for Lindley distribution based on progressive first failure censored data. Math Sci Lett 2017;6:255–61.
[34] El-Din MM, El-Attiay EH, Hafez EH. Bayesian estimation for Lindley distribution based on progressive first failure censored data. J Stat Appl Pro 2020;9:181–91.
[35] Almahmood AA, Al-Hamadi MS, Hafez EMM. Estimation of stress-strength from data attributable to the exponential distribution under progressive type-II censoring. J Appl Stat Pro Lett 2020;7:109–26.
[36] Almumuni AA, Alhazmi AA, Al-Durairi AM. Estimation of stress-strength parameter for Rayleigh distribution based on progressive type-II censoring. Inf Sci Lett 2021;10(1):101–10.
[37] Abd El-Raheem AM, Abu-Moussa MI, Mohied El-Din, Marwa M, Hafez EH. Accelerated Life Tests under Pareto-IV Lifetime Distribution: Real Data Application and Simulation Study Mathematics 8(10):2020;1786.https://doi.org/10.3390/math8101786.