Dynamic analysis of the mechanical seals of the rotor of the labyrinth screw pump

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Abstract. A mathematical model of the work of the mechanical seal with smooth rings made from cast tungsten carbide in the condition of liquid friction is drawn up. A special feature of this model is the allowance for the thermal expansion of a liquid in the gap between the rings; this effect acting in the conjunction with the frictional forces creates additional pressure and lift which in its turn depends on the width of the gap and the speed of sliding. The developed model displays the processes of separation, transportation and heat removal in the compaction elements and also the resistance to axial movement of the ring arising in the gap caused by the pumping effect and the friction in the flowing liquid; the inertia of this fluid is taken into account by the mass reduction method. The linearization of the model is performed and the dynamic characteristics of the transient processes and the forced oscillations of the device are obtained. The conditions imposed on the parameters of the mechanical seal are formulated to provide a regime of the liquid friction, which minimizes the wear.

1. The introduction and the statement of task

The labyrinth screw pump (LSP, sealing assembly is shown in Figure 1 [1]) is installed in an oil well at a depth of about 1 km at the lower end of the pipeline, through which the vane pumps lift the oil to the surface of the earth; up to the specified mark, it flows under the influence of layer’s pressure.

![Figure 1](image-url)

(a) (b)

**Figure 1.** Mechanical seal of the rotor of the LSP: (a) – design; (b) – scheme. 1 – outlet chamber; 2 – spring; 3,4 – clips of rings; 5 – shaft; 6 – spring support.
The oil in the cavity of the LSP has the temperature of 120°C and the pressure of about 10 MPa. It is known [2] that there is a lot of gas which is dissolved in the layer’s oil and during the lifting along the well due to the pressure drop, about 2/3 of its mass is released in the form of bubbles and thus the oil becomes a two-phase medium. The task of LSP is to crush the bubbles and mix them with the liquid, which is necessary for a reliable operation of the vane pumps.

During the operation the LSP raises the oil pressure by 0.2 MPa, and this difference acts on the mechanical seal of the output chamber, shown in Figure 1. The presence of the pressure drop, as well as the specific physical properties of the working body and a very high purity of the sealing surfaces (roughness of the rings made of the wear-resistant alloy of two tungsten carbides – is only 0.03 micro mill.), create the prerequisites for maintaining the liquid friction regime in the gap what minimizes the wear. The task of the research was to determine the compaction parameters which ensure this mode.

2. The features of the sealed fluid

For every 100 m increase in depth, the temperature of the earth’s rock rises by 3 degrees. The depth of the layer from which oil is extracted reaches about 3 km and the pressure in the layer is close to 30 MPa. When the oil is being lifted along the well its temperature changes slightly and the pressure decreases threefold which is a result of the release of a large part of the dissolved hydrocarbon gas and nitrogen into the gas phase. In this case the initial viscosity of the liquid phase increases by 1.5 ... 2 times.

The rotation of the rotor leads to a separation of the oil in the outlet chamber in terms of density: the water which is present in the layer’s oil is pressed to the walls of the chamber and the gas phase remains in the center. As a result the liquid hydrocarbon fractions are fed to the compact but their viscosity does not remain constant and varies within rather wide limits. It is generally accepted that the average kinematic viscosity of the degassed and separated oil at a temperature of 120°C is about 4 cSt, but in the layer’s oil where there is a lot of gas is dissolved the viscosity is much lower; besides, some types of the oil have a higher viscosity [2].

In the layer, the viscosity of oil \( \nu \) (in cSt) obeys the Mayer equation

\[
\ln(\nu + 0.8)/\ln(\nu_{20} + 0.8) = \left(\frac{293}{(273 + t_{lay})}\right)^{3.5}
\]

(1)

\( \nu_{20} \) – the viscosity of the same oil at a temperature of 20°C; \( t_{lay} = 120°C \)

![Figure 2. Changing the index \( K_\nu \) depending on the viscosity \( \nu \) of the layer’s oil and temperature: \( K\nu_1 - t_{lay} = 120^0C \), \( K\nu_2 - t_{lay} = 70^0C \), \( K\nu_3 - t_{lay} = 20^0C \).](image)

Differentiating the both sides of equality (1) with respect to the temperature the coefficient of change (index) of the viscosity is obtained:

\[
K_\nu = \nu^{-1} \frac{d\nu}{dt_{lay}} = -3.5 (1 + 0.8/\nu) \ln(\nu + 0.8)/(273 + t_{lay})
\]

In order to take into account the partial degassing during the lifting of the oil through the well and its separation in the pump, the viscosity in the right side of this formula should be reduced by 2 times. As a result, the dependencies presented in Figure 2 are obtained. The kinematic viscosity \( \nu \) of oil in
the pump varies within the limits of 1 \ldots 4 \text{ cSt}; for further analysis an average value \( \nu_{med} = 2 \text{ cSt} \) will be taken.

3. The changes in the temperature of the liquid and the material of the ring along the length of the gap

It was noted in the work [3] that a radial temperature drop leads to a conicity of the gap, which in its turn entails a change in the lift force. To calculate the conicity, the heat conduction problem for the radial section of the ring is solved (Figure 3). The release of the heat is caused by the operation of the fluid friction force and the heat flux density \( q \) is described by the formula

\[
q(x) = \rho \nu ((R - x) \omega)^2 \nu (h_0 - \Delta h(x)) \tag{2}
\]

where
- \( \rho, \nu \) – is the density and kinematic viscosity of the oil;
- \( R, \omega \) – is the mean radius and the angular velocity of the rotation of the ring;
- \( h_0 \) – is the gap’s height on the outer radius of the ring;
- \( \Delta h \) – is the change of the gap;
- \( x \) – is the radial coordinate, directed along the flow of the fluid;
- \( x \in [0, H] \);
- \( \Delta h(0) = 0 \);
- \( H \) – is the width of the ring.

The task, after neglecting the radial contraction of the ring and the flow of heat from the pump shaft, takes the form of:

\[
\nabla^2 T = 0; \quad \frac{\partial T}{\partial x} \bigg|_{x=0} = 0; \quad \lambda_c \frac{\partial T}{\partial x} + \alpha T \bigg|_{x=H} = 0; \quad \frac{\partial T}{\partial y} \bigg|_{y=0} = 0; \quad \lambda_c \frac{\partial T}{\partial y} \bigg|_{y=H} = 0.5q \tag{3}
\]

and its solution was obtained by Fourier’s method of separation of variables [4] in the form of a series

\[
T(x, y) = 0.5H \lambda_c^{-1} \sum_{j=0,1,\ldots} A_j [\gamma_j \sinh(\gamma_j H y / H)] \cosh(\gamma_j y / H) \cos(\gamma_j (1 - x / H)) \tag{4}
\]

where the separation constants \( \gamma_j \) and the Fourier coefficients \( A_j \) are defined by the equalities:

\[
\gamma_j \tan \gamma_j = \alpha H \lambda_c^{-1}, \quad A_j = 2H \lambda_c^{-1} \int_0^H q(x) \cos(\gamma_j (1 - x / H)) dx; \quad \Delta \alpha = \alpha H \lambda_c^{-1}
\]

\( \nabla^2 [.\] – is Laplace operator;
\( \nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}; \)
- \( \alpha \) – is the coefficient of heat transfer to the liquid in the output chamber of the LSP (its temperature in the model (3) is considered zero);
- \( \lambda_c \) – is the average thermal conductivity of the material of the ring (in the calculation the thermal conductivity of the cast tungsten carbide \( \lambda_c = 29.5 \text{ W m}^{-1} \text{ K}^{-1} \) is used);
- \( H_y \) – is the height of the ring.
Integrating the series (4) over the height of the ring the following data is obtained:
\[ \int_{-H}^{H} T(x,y) \, dy = 0.5H^2 \lambda_c^{-1} \sum_{j=0}^{\infty} A_j \gamma_j^{-2} \cos(\gamma_j x / H) \]  
(5)

and, it is typical, that this result does not depend on the height of the ring \( H_r \). The change in the temperature of the liquid in the gap (i.e., the temperature of the channel wall) is determined by the equality
\[ T_i(x) = 0.5H \lambda_c^{-1} \sum_{j=0}^{\infty} A_j \left[ \gamma_j \text{th}(\gamma_j H_r / H) \right]^{-1} (1 - \cos [\gamma_j (1 - x / H)]) \]  
(6)

and when the ratio \( H_r > 0.5H \) is weakly dependent on the height \( H_y \).

The analysis showed that in the sums (4) - (6) one can confine the first term; then all the thermal characteristics of the ring are proportional to one temperature \( T_0 \):

\[ \Delta T_r = Kp(\Delta \alpha) T_0; \quad \Delta T_i = Ks(\Delta \alpha) T_0; \quad \Delta T_w = Kn(\Delta \alpha) T_0; \quad \text{grad}_r T = 2Kp(\Delta \alpha) T_0 / H \]

where
\[ T_0 = 0.5q_{med} H / \lambda_r \]

- is the calculated value of the temperature obtained under the condition of the transfer of the heat with density \( q_{med} = (\rho v (R_{med} \omega)^2 / h_0) \) from the inner radius of the ring to the outer radius;
\[ R_{med} = R - 0.5H \]

- \( \Delta T_r \), \( \Delta T_i \) – are the radial temperature differences of the ring and the liquid;
\[ T_{in} \] – is the temperature of the material in the initial section of the ring; the corresponding thermal elongations in the inlet \( \Delta H_{in} \) and the outlet sections \( \Delta H_{out} \) are determined by the formulas
\[ \Delta H_{in} = \alpha_c T_{in} H; \quad \Delta H_{out} = \alpha_c \Delta T_{out} H, \]  
(7)

\( \alpha_c \) – is the coefficient of thermal linear expansion of the cast tungsten carbide, \( \alpha_c = 5.8 \times 10^{-6} \text{ K}^{-1} \); the dependence of the coefficients \( Ks, Kp, Kn \) on the parameter \( \Delta \alpha \) is given in the graphs of Figure 4.

With a good organization of the heat removal from the rings, the values \( \Delta \alpha \) are located in the interval 2 ... 3.

In [3], the restriction on the gradient of the expansion of the material of the ring is indicated; it is \( 6 \times 10^{-5} \), that is why, the condition \( Kp(\Delta \alpha) \alpha_c T_0 < 6 \times 10^{-5} \) must be satisfied. From this point, as well as from the graphs in Figure 4, it follows that for a ring from the cast tungsten carbide, the value \( \Delta T_i \) of...
the rise in the temperature of the liquid in the channel should not exceed 6.5K.

4. The change in ring’s temperature in the transient process
If the gap or other parameters used in the right side of the equation (2) are being changed during the time $t$, then a stationary task (3) is transformed into a non-stationary task:

$$\lambda_c \nabla^2 T = c_r \rho_l \partial T / \partial t; \partial T / \partial x \bigg|_{x=0} = 0; \ldots; \lambda_c \partial T / \partial y \bigg|_{y=-H} = 0.5q(x,t)$$

where

$c_r$, $\rho_l$ – is the specific heat and the density of the material of the rings.

Applying the Laplace transformation and the Fourier method, the following equalities are obtained:

$$\int_0^H T(x,y,s)dy = 0.5H^2\lambda_c^{-1} \sum_{j=0,1,\ldots} A_j(s)(\gamma_j^2 + \alpha_p H^2 s)^{-1} \cos[\gamma_j(1-x/H)]$$

$$A_j(s) = 2H^{-1} \int_0^H q(x,s) \cos[\gamma_j(1-x/H)]dx$$

where

$s$ – is the Laplace variable; as a result for thermal deformations $\Delta H_{in}, \Delta H_{out}$ the solutions for images in the form of generalized Fourier series are obtained:

$$\Delta H_{in} = 0.5H^2\lambda_c^{-1} \sum_j A_j(s)(\gamma_j^2 + \alpha_p H^2 s)^{-1} \cos \gamma_j, \Delta H_{out} = 0.5H^2\lambda_c^{-1} \sum_j A_j(s)(\gamma_j^2 + \alpha_p H^2 s)^{-1}$$

$$A_j(s) = 2H^{-1} \int_0^H q(x,s) \cos(\gamma_j x / H)dx, \alpha_p = c_r \rho_l \lambda_c^{-1}$$

Returning to the originals and using the first terms of the series (the others are small), the previous proportionalities are obtained

$$\Delta H_{in}(t) = K_{in}HT_0(t); \Delta H_{out}(t) = K_{out}HT_0(t)$$

and the differential equation for the design temperature $T_0(t)$:

$$\alpha_p H^2\gamma_0^{-2} dT_0(t)/dt + T_0(t) = 0.5q_0(t)H\lambda_c^{-1}; q_0(t) = \rho \nu (R_{med} \alpha(t))^2 / h_0(t)$$

5. The condition of the static equilibrium of the sealing ring
If the gap exceeds the size of the roughness by 3 times or more then the liquid friction regime is observed. Here, the rotating ring does not have a contact with the stationary ring and its equilibrium is achieved when the oppositely directed forces are equal $F_{spr}(h_0) = F_{spr}(h_0) + F_{gr}$ – the force of the spring $F_{spr}(h) = F_{spr,0} + z_{spr} h$, the force of gravity $F_{gr}$ (in which Archimedes’ law is taken into account, as well as the third part of the mass of the moving coils of the spring) and the fluid pressure force in the gap

$$F_p(h_0) = \pi \int_0^H (R - x) p(x)dx$$

where

$h_0$ – is the initial clearance on the outer radius $R$ of the ring;

$z_{spr}$ – the coefficient of the spring’s stiffness;

$x$ – the radial coordinate of the section;

$p(x)$ – the diagram of the oil overpressure in the gap, which satisfies the known differential equation $dp/dx = -12\rho_u v u / h^2$ or if the flow velocity $u$ passes to the mass flow $\dot{m} = 2\pi(R-x)hu$,,
\[ \frac{dp}{dx} = -\left[6\pi n v_0 / (\pi R h_0^2)\right] \left[\delta_\ell(x) / (\delta_\epsilon(x) \delta_h^3(x))\right] \]  

(11)

where

\[ \delta_\ell = (R - x) / R, \quad \delta_h = (h - h_0) / h_0, \quad \delta_\epsilon = v / v_0 \]  

– the relative values of the radius, the gap and the kinematic viscosity in the section \( x \); \( v_0 \) – is the value of the viscosity at the inlet to the seal.

For the differential equation (11), the boundary value task with boundary conditions is solved

\[ p(0) = \Delta p, \quad p(H) = 0, \]  

where \( \Delta p \) – is the preset pressure drop on the slits, from which the correction for the centrifugal force of Coriolis is subtracted \( \Delta p_c = 0.5 \rho_{med} V_{med}^2 \); \( V_{med} \) – is an average tangential velocity of fluid in the gap (in the investigated LSP at the operating mode \( V_0 \approx 5.5 \text{m} / \text{s} \) ); \( \rho_{med} \) – is its average density. Then the result of the solution is substituted into the integral (10).

If it is assumed that the gap and the viscosity along the length of the gap do not change, then integration gives the value of the force

\[ F_p(h_0) = f_R k_R \Delta p / 2 \]

where

\[ f_R \]  

– is the area of the ring; and the correction \( k_R \approx 1 + 0.5 H / R_{med} \) takes into account the taper, caused by the annular form of the gap; it is present in all further results.

It is taken into account that the values \( \delta_\nu(x) \) and \( \delta h(x) \) vary with the change in the temperature:

\[ \delta_\nu(x) = 1 + K_\nu (T_i(x) / \Delta T_i) ; \quad \delta h(x) = 1 + \alpha_\nu (2H / h_0) k_H (T_i(x) / \Delta T_i) \]

where

\[ k_H = Kp(\Delta a) / Ks(\Delta a) \approx 0.8 ; \quad \Delta T_i = T_i(H) \]

\( K_\nu \) is the viscosity index at \( t_\text{avg} = 120^\circ \text{C} \) (Figure 2).

And the amendment \( \delta_\nu(x) \) differs slightly from 1, therefore

\[ (\delta_\nu(x))^3 \approx \Psi(x) \text{, where} \]

\[ \Psi(x) = (1 + \delta T_i / \Delta T_i)^3 ; \quad \delta = \Delta T_i (\alpha_\nu (2H / h_0) k_H - K_\nu / 3) \]

The use of the function \( \Psi(x) \) in equation (11), and then in the integral (10), leads to the following result:

\[ F_p(h_0) = f_R k_R (\Delta p / 2) \left[1 + 0.45 \delta / (1 - 0.4 \delta)\right] \]  

(12)

In particular, if \( \delta \approx 1 \), i.e. the taper of the gap leads to an almost complete overlap of the flow near the inner radius of the ring, then \( F_p(h_0) \approx f_R \Delta p \) and this is the maximum tearing force which can be obtained during the liquid friction regime. However, if in the operating mode of the LSP this result is obtained (due to the corresponding tightening of the spring), this can lead to overheating of the sealing ring.

For the test compaction, the connection between the viscosity \( \nu \), the outer gap \( h_0 \) and the maximum oil heating temperature \( \Delta T_i \) in the gap is determined by the equation:

\[ \Delta T_i = Ks(\Delta a) [\rho \nu (R_{med} \omega)^2 / (\lambda_c / H)] / (h_0 - 2k_H \alpha_c H \Delta T_i) \]  

(13)

hereof it is easy to express the height of the gap through the values of the viscosity and the temperature:

\[ h_0 = Ks(\Delta a) [\rho \nu (R_{med} \omega)^2 H / \lambda_c] / \Delta T_i + 2k_H \alpha_c H \Delta T_i \]

Then the formula (12) was used, which was rewritten in the following form:

\[ F_p(h_0) = f_R k_R (\Delta p / 2) [1 + \delta F] \]
where

$$\delta F = \Delta T_A_A; \quad A_A = [0.9(\alpha_e H k_H / h_0 - K_e / 6)]/(1 - 0.8 \alpha_e H k_H \Delta T_e)$$

(14)

The coefficient $\delta F$ indicates the fraction of the repulsive force which was acquired after the ring was heated. In Figure 5 (a), the graphs corresponding to the dependences (13), (14) for the parameters of the analyzed compaction are plotted at the maximum permissible value $\Delta T_e = 6.5^\circ$N, and also at $5^\circ$C and $8^\circ$C. With a viscosity $\nu = \nu_{med} = 2$ cSt, the gap $h_0$ is 0.65 $\mu$m, and at the inner radius it decreases to 0.45 $\mu$m, 0.5 $\mu$m and 0.4 $\mu$m accordingly.

If the tightening of the spring is selected and remains unchanged then the coefficient $\delta F$ is not changed. As we can see, for equal values $\delta F$ an increase in the viscosity leads to an increase in the steady temperature drop and following this rule the tightening must be made at the highest viscosity, where $\delta F \approx 1$ and it is difficult to make such a fine adjustment. But if the viscosity fluctuation takes a short time, then, as we shall show later the temperature difference will also vary little. Therefore the tightening of the spring should be calculated based on the average viscosity of the oil.

Figure 5. The coefficients $\delta F$ of increase in the lift force and the initial gaps $h_i$, $\mu$m with temperature difference $8^\circ$C (curves 1), $6.5^\circ$C (curves 2) and $5^\circ$C (curves 3) and the oil temperature:

(a) – $120^\circ$C, (b) – $20^\circ$C.

In the formula (14), two factors affect the change in the lift force: the thermal expansion of the rings increases this force and the decrease in viscosity with the increasing of the temperature reduces on the contrary. With the viscosity $\nu = \nu_{med} = 2$ cSt, the first factor is stronger than the second one by an order of magnitude, but at high viscosities (due to the increase in the gap) this gap is reduced. If the temperature is significantly reduced (Figure 5 (b)), then with increased viscosities $\nu > 6$ cSt, it is going to be difficult to adjust the seal to work in liquid friction mode.

6. The modeling the movement of the ring in the transient modes

As it follows from (9) the time of the thermal inertia of the rings is $\tau_e = \alpha_e H^2 \gamma_0^{-2} \approx 1.6$ s, but when the ring moves the stronger factors which make slower the transient processes are acting. It is considered to observe the one dimensional non stationary motion of a viscous incompressible fluid along the gap of the variable and the varying height $h(x,t)$, described by the well-known equations (here $S = 2\pi(R - x)h(x)$ – this is the area of the annular section of the gap):

$$\partial m / \partial t + u \partial m / \partial x + 8 \partial p / \partial x + 12 \nu \partial t / h^2 = 0; \quad \partial (\rho S) / \partial t + u \partial (\rho S) / \partial x + \partial m / \partial x = 0$$

The velocity of the flow is small, so the convective terms are not considered:
\[ \frac{\partial p}{\partial x} = -12\nu \dot{m}/(Sh^2) - \rho \ddot{u}; \quad \frac{\partial \dot{m}}{\partial x} = -(\ddot{\rho} + \ddot{h}/h)S \]

And the acceleration \( \dot{u} \), by virtue of the non-compressibility of the liquid is considered to be proportional to the axial acceleration of the ring:

\[ \dot{u} = [(x - 0.5H)/h]d^2h/dt^2 \]

The integration of the equation

\[ dp/dx = -\rho[(x - 0.5H)/h] d^2h/dt^2 \]

under the boundary conditions \( p(0) = 0 \), \( p(H) = 0 \) and substitution of the result into the integral (10) leads to the appearance of inertia force \( F_m = m_{red} \frac{d^2h}{dt^2} \), where \( m_{red} \) – is the mass of fluid in the gap, reduced to the ring,

\[ m_{red} = (1/3) \rho_{red} f_k k_h h (0.5H/h)^2 \]

Note that the mass \( m_{red} \) increases in the proportion to the reduction of the gap and in the working modes of the compaction exceeds the mass \( m_R \) of the moving ring (to which, according to the known rule, the inertial mass of the spring’s turns is shown) in several times.

The equation \( \frac{\partial p}{\partial x} = -12\nu \dot{m}/(Sh^2) \) coincides with (11), but the magnitude of the mass \( \dot{m} \) flow due to non-stationarity of the process for different cross sections can be different. Note that on the right-hand side of the second equation the density of the liquid varies with its temperature, and

\[ \dot{p} = -[\alpha_{oil} + \alpha_e (2H/h)(k_{h} T_i(x)/\Delta T_i + K_{in})]\rho \Delta T_i \]

where \( \alpha_{oil} \) – is the coefficient of thermal expansion of oil, \( \alpha_{oil} \approx 0.012 K^{-1} \), and \( \dot{h}(0,t) = dh/dt \) – is the speed of the axial movement of the ring; \( K_{in} = Kn(\Delta a)/Kp(\Delta a) \approx 0.6...0.8 \) (Figure 4).

In order to determine the value \( \dot{m} \) the second equation was integrated:

\[ \dot{m}(x) = \dot{m}(0) - \int_0^x [\dot{\rho}(z) + \dot{h}(z)/h(z)]S(z) \, dz \]

then this function was inserted into the equation (11) and the integral (10) using the established procedure. As a result the following formula for detachment strength was obtained:

\[ F_p = f_k k_r (\Delta P/2)[1+A_T \Delta T_i] - m_{red} \frac{d^2h}{dt^2} - 12m_{red} (\nu/h^2)dh/dt + 12m_{red} (\nu/h^2)B_T \Delta T_i \]

where

\[ B_T = K_p(\alpha_{oil} + \alpha_e (2H/h)k_{h}); \]

\[ K_p = 1/15 + K_{in}/12. \]

Taking into account the forces of the spring and the inertia of the ring, the equation of axial oscillations is obtained:

\[ (m_R + m_{red}) \frac{d^2h}{dt^2} + (12\nu/h^2)m_{red} dh/dt + z_{spr} h = f_k k_r (\Delta P/2)[1+A_T \Delta T_i] + 
\]

\[ + (12\nu/h^2)m_{red}B_T \Delta T_i + F_{spr} - F_{spr'} - 2z_{spr} K_m \alpha_e H \Delta T_i. \]  \hspace{1cm} (15) \]

The last term on the right-hand side of (15), which takes into account the deformation of the spring caused by the thermal expansion of the sealing rings can be neglected.

Note that the mass \( m_{red} \) increases in the proportion to the change in the gap and on the workers modes of compaction exceeds the mass \( m_R \) in several times.

The varying part of the force depends on the temperature and the rate of the change of the
temperature, which is determined from the equation (9). Equations (9), (16) are supplemented by an
equation describing the angular oscillations \( \psi(t) \) of the sealing ring relatively to the shaft:

\[
j_{m,\text{tors}} \frac{d^2 \psi}{dt^2} + f_R (\rho v h) R_{\text{med}}^2 \frac{d\psi}{dt} + z_{qpr,\text{tors}} \psi = f_R (\rho v h) R_{\text{med}}^2 \omega
\]

where \( j_{m,\text{tors}}, z_{qpr,\text{tors}} \) is the moment of the inertia of the ring and the torsional rigidity of the spring
and the formula (9) for the heat flux is refined:

\[
q_0(t) = \rho v (R_{\text{med}} \omega(t) + d\psi / dt)^2 / h_0(t)
\]

Thus, the dynamics of the compaction is described by a system of three nonlinear differential
equations and is of the 5th order.

7. The frequency analysis and the model simplification

The linearization of the system of equations (9), (15), (16) and the analysis of the frequencies of its
free oscillations showed that in the transient condensation process there are damped periodic and
aperiodic oscillations. The periodic angular vibrations of the rotating ring have a frequency of about
12 Hz and the attenuation decrement that increases as the gap decreases; these quantities are mainly
determined by the coefficients of equation (16). The aperiodic process of changing the radial
temperature difference in the gap has a time constant \( \tau \approx 1.6 \) s which is mainly determined by the
coefficients of equation (9).

The axial movements of the ring have two time constants:

\[
\tau_{h,1} = (1 + m_R / m_{rod})^{-1} h^2 / (12 \nu) \approx h^2 / (12 \nu) < 10^{-6} \text{s}, \quad \tau_{h,2} = 12 (m_{rod} / z_{qpr}) \nu / h^2 \approx 10^2 ... 10^3 \text{s}
\]

As one can see, the first process is too short-lived and it is not necessary to take it into account when
modeling it. To eliminate this process it is sufficient to exclude the force of inertia from equation (15).
As a result, the equation of the motion has the first order:

\[
dh / dt = [h^2 / (12 \nu m_{rod})] \left( f_R k_R (\Delta p / 2) [1 + A_T \Delta T_i] + F_{gr} - F_{qpr,0} - z_{qpr} h \right) + B_i \Delta T_i
\]

8. The results of solving the test problems

Figures 6-9 show the results of the calculation of transient processes caused by the various factors.
The change in values is shown in a relative form:

\[
\delta h = h(t) / h(0), \quad \delta \psi = \psi(t) / \psi(0), \quad \delta T = \Delta T_i / \Delta T_i(0), \quad \delta Fd = Fd(t) / F_{qpr,0}
\]

where \( Fd \) – is the sum of the active forces applied to the ring (without accounting the frictional force);
the argument \( t \) as it was before designates the time and is measured in seconds.

In Figure 8, the values \( \delta h \) and \( \delta T \) are shown in the percentages and the fractions of a percent.
Analyzing the graphs, one can come to the conclusion that the frictional forces effectively dampen
the action of the perturbations so that the integrity remains. The leakage of oil through the end gap does
not exceed 0.5 G h^{-1} what is considered permissible for this device.
Figure 6. Fluctuation of viscosity (in 2 times): (a) – into the large, (b) – into the smaller side.

Figure 7. Fluctuation of the pressure drop (in 2 times): (a) – into the large, (b) – into the smaller side.

Figure 8. Forced oscillations with a frequency of 50 Hz: (a) – axial vibration of the housing with an amplitude of 0.2 mm, (b) – unevenness of the pump feed 2.5%. The value $\delta h$ is shown as a percentage, and $\delta T$ – in tenths of a percent (Ppm). Time $t$ is counted from the beginning of the cause of the oscillations.
Moreover, there is no significant overheating of the rings in these processes.

9. Conclusion
In order to ensure a stable operation of the mechanical seal in the liquid friction mode the designer needs to create an additional hydrodynamic force which tends to increase the gap between the rings by the increasing of the speed of the shaft or the pressing force. The authors have left the traditional view which says that the cause of this force is a roughness or an undulation of the surfaces, and there are explanations about this case.

The maximum size of irregularities on the ground surface of a ring made from cast tungsten carbide does not exceed 0.03 μm, which is an order of magnitude smaller than the working gap. In addition, cast tungsten carbide – this is 96% (by weight) wolfram and its high firmness prevents an increase in roughness during the usage what occurs on the contrary with the seals which made from soft composite materials.

It turned out that the appearance and the change of the additional lifting force can be explained by the uneven temperature deformation of the rings which lead to the appearance of the taper of the gap. The lack of empiricism in the formula of the lifting force made it possible to explain the static and the dynamics of the compaction from a unified position. The calculated equations reflect the factors known from the literature but in a closed form they are considered and used for the first time.

Further on, the authors plan to specify the model: take into account the heat flowing from the shaft as well as the regular plastic deformations of the sealing surfaces arising during operation. Then they will look for the ways to identify the model but if to take into account the extreme values of oil pressure and temperature it will be difficult to do this.

References
[1] Andrenko P M and Lebedev A Y 2017. Labyrinth screw pumps (Kharkiv: Panov) p 156
[2] Eygelson A S and Shaykhali D M 1989. Calculation of density and viscosity of layer oil by data of surface degassing. (Moscow: Geology of oil and gas) 11
[3] Mayer E 1978 Mechanical seals (Moscow: Mechanical Engineering) p 288
[4] Farlow S 1983 Partial differential equations for researchers and engineers (Moscow: Mir) p 381
[5] Loitsyansky L G 1978 Mechanics of liquids and gases (Moscow: Nauka) p 736