Optical forces from an evanescent wave on a magnetodielectric small particle

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Abstract

We report the first study on the optical force exerted by an evanescent wave on a small sphere with both electric and magnetic response to the incident field, immersed in an arbitrary non-dissipative medium. New expressions and effects from their gradient, radiation pressure, and curl components are obtained due to the particle induced electric and magnetic dipoles, as well as to their mutual interaction. We predict possible dramatic changes in the force depending on either the host medium, the polarization and the nature of the surface wave.
The expression for the optical force on a small magnetodielectric particle \[1\] consist of three terms: an electric, \(< F_e >\), a magnetic, \(< F_m >\), and an electric-magnetic dipolar interaction component, \(< F_{e-m} >\), whose physical meaning, associated to the differential scattering cross section, was given \[2\] on the basis of the formal analogy between the conservation of the momentum, (the optical force), and the energy, (the optical theorem). Studies on photonic forces in the near field are of importance if one wishes to enter in the subwavelength scale \[3-12\]. This involves evanescent waves. However, as far as we are aware, the effects of such waves on magnetodielectric objects, have not been yet investigated.

Like for non-magnetic particles \[13, 14\], each of those above mentioned three terms of the optical force admit a decomposition into a gradient, a scattering (radiation pressure) and a curl component. Let \(e^{(i)}(s_{xy})\) and \(b^{(i)}(s_{xy})\) denote the complex amplitude of each angular plane wave component of the electric and magnetic vectors incident on the particle \[15, 16\], inducing the electric and magnetic dipole moments \(p\) and \(m\), respectively; the wavevector of this wave being: \(k = k(s_{xy}, s_z)\), \(k = n\omega/c\), \(\omega\) representing the frequency and \(n = \sqrt{\epsilon\mu}\), \((\epsilon\) and \(\mu\) stand for the dielectric and magnetic constants of the lossless surrounding medium), \(s_z = \sqrt{1 - s_{xy}^2}\) for propagating waves: \(s_{xy}^2 \leq 1\), and \(s_z = \sqrt{s_{xy}^2 - 1}\) for evanescent waves: \(s_{xy}^2 > 1\). A time dependence \(\exp(-i\omega t)\) is assumed throughout.

Let the particle be a small sphere of radius \(a\), with constants \(\epsilon_p, \mu_p, n_p = \sqrt{\epsilon_p\mu_p}\), such that its scattering is accurately described (see details in \[17\]) by the electric and magnetic Mie coefficients \(a_1\) and \(b_1\) \[18\]. Then the electric and magnetic polarizabilities are: \(\alpha_e = i\frac{3}{2k}a_1\) and \(\alpha_m = i\frac{3}{2\mu k}b_1\), respectively \[2\]. The induced dipole moments are expressed in terms of the amplitude of the incident field components: \(p = \alpha_e e^{(i)}; m = \alpha_m b^{(i)}\). We write \[2, 19\]: \(\alpha_e = \alpha_e^{(0)}\left(1 - i\frac{2}{3}k^3\alpha_e^{(0)}\right)^{-1}, \alpha_m = \alpha_m^{(0)}\left(1 - i\frac{2}{3}\mu k^3\alpha_m^{(0)}\right)^{-1}, \alpha_e^{(0)}\) and \(\alpha_m^{(0)}\) being the corresponding static polarizabilities. Notice that in particular, in the Rayleigh limit \(ka \ll 1, k|n_p|a \ll 1\) one has \[2, 18\]: \(\alpha_e^{(0)} = \epsilon a^3\frac{\epsilon_p - \epsilon}{\epsilon_p + 2\epsilon}, \alpha_m^{(0)} = \mu^{-1}a^3\frac{\mu_p - \mu}{\mu_p + 2\mu}\). Such magnetodielectric spheres have recently been shown to be available of \(Si, Ge\) and \(TiO_2\) in the near infrared \[17\], as well as high refractive index spheres and rods in the microwave region (see Refs. 17 - 21 in \[17\]).

We next determine the action exerted by one of those aforementioned components: an evanescent wave, on the sphere. Such a wave may be produced by e.g. total internal reflection (TIR) at a plane dielectric interface, or by a surface plasmon-polariton (SPP) at
a metal film surface. In either situation, the particle is in the rarer medium of constants \( \epsilon, \mu \) and \( n = \sqrt{\epsilon \mu} \), in the half space \( Z > 0 \). \( XZ \) is the plane of incidence, (see the inset of Fig.1). We shall consider the broad variety of cases, delimited in e.g. [12,13], in which both the particle scattering cross section and distance to the surface allow to neglect multiple wave interactions between the sphere and the plane. The electric and magnetic vectors of the generic evanescent wave, created in the medium \( Z > 0 \) containing the particle, are:

\[
\mathbf{E}^{(i)}(K) = (-\frac{iq}{k} T_{\perp}, \frac{K}{k} T_{\parallel}) \exp(iKx - qz),
\]

\[
\mathbf{B}^{(i)}(K) = n \left( -\frac{iq}{k} T_{\perp}, -T_{\parallel}, \frac{K}{k} T_{\perp} \right) \exp(iKx - qz).
\]

For TE or s (TM or p) - polarization , i.e. \( \mathbf{E} \) (\( \mathbf{B} \)) perpendicular to the plane of incidence \( XZ \), only those components with the transmission coefficient \( T_{\perp}, (T_{\parallel}) \) are chosen in the incident fields \( \mathbf{E}^{(i)} \) and \( \mathbf{B}^{(i)} \). Now \( K \) denotes the component of the wavevector \( k = k(s_{xy}, s_{z}) \), parallel to the interface; i.e. \( k = (K, 0, iq) \), \( q = \sqrt{K^2 - k^2}, k^2 = K^2 - q^2 \).

Introducing the above equations into Eqs. (42) - (44) of [2], the resulting electric dipole force components then are

\[
< \mathbf{F}_e >_x = \frac{3\alpha_e}{2} K \exp(-2qz)|T_{\perp}|^2 \\
+|T_{\parallel}|^2 \left( \frac{2K^2}{k^2} - 1 \right), \quad (1)
\]

\[
< \mathbf{F}_e >_z = -\frac{3\alpha_e}{2} q \exp(-2qz)|T_{\perp}|^2 \\
+|T_{\parallel}|^2 \left( \frac{2K^2}{k^2} - 1 \right). \quad (2)
\]

Whereas the magnetic dipole forces become

\[
< \mathbf{F}_m >_x = n \frac{3\alpha_m}{2} K \exp(-2qz)|T_{\parallel}|^2 \\
+|T_{\perp}|^2 \left( \frac{2K^2}{k^2} - 1 \right), \quad (3)
\]

\[
< \mathbf{F}_m >_z = -n \frac{3\alpha_m}{2} q \exp(-2qz)|T_{\parallel}|^2 \\
+|T_{\perp}|^2 \left( \frac{2K^2}{k^2} - 1 \right). \quad (4)
\]
And the electric-magnetic dipolar interaction forces are given by

\[ \langle \mathbf{F}_{e-m} \rangle_x = -\frac{k^4}{3} \sqrt{\frac{\mu_1}{\varepsilon_k}} \Re(\alpha_e \alpha_m^*) K \exp(-2qz) |T_\perp|^2 \\
+ |T_\parallel|^2 (2\frac{K^2}{k^2} - 1), \quad (5) \]

\[ \langle \mathbf{F}_{e-m} \rangle_z = -\frac{k^4}{3} \sqrt{\frac{\mu_1}{\varepsilon_k}} \Im(\alpha_e \alpha_m^*) q \exp(-2qz) |T_\perp|^2 \\
+ |T_\parallel|^2 (2\frac{K^2}{k^2} - 1). \quad (6) \]

\( \Re \) and \( \Im \) mean real and imaginary parts, respectively. Once again, for \( s \) (\( p \)) polarization, only the terms with \( T_\perp \), \( (T_\parallel) \) are taken in these equations. Out of a Mie resonance, for pure electric dipole or pure magnetic dipole particles, with positive \( \varepsilon_p \) and \( \mu_p \) and with little absorption, \( \Re \alpha > \Im \alpha \), and Eqs. (1)-(4) show that the \textit{Z-component}, i.e. the \textit{gradient force}, of \( \langle \mathbf{F}_e \rangle \) and \( \langle \mathbf{F}_m \rangle \) is larger than the \textit{X-component}, i.e. than the \textit{radiation pressure} or \textit{scattering force}. This would confirm some of the observations in the experiment of [12] for purely dielectric particles, and predicts an analogous effect in \( \langle \mathbf{F}_m \rangle \) for magnetodielectric particles. However, this behavior competes with the opposite one of the interaction force \( \langle \mathbf{F}_{e-m} \rangle \), for which Eqs. (5) and (6) show that the \textit{X-component} is larger than the \textit{Z-component}. On the other hand, in a Mie resonance: \( \Re \alpha << \Im \alpha \) and, hence, the opposite effect should occur in its neighborhood.

The term with factor \( 2q^2/k^2 = (2K^2/k^2) - 2 \) in the \textit{X-component} of \( \langle \mathbf{F}_e \rangle \) and \( \langle \mathbf{F}_m \rangle \), Eqs. (1) and (3), is due to the \textit{curl component} of the decomposition [cf. Eqs. (42)-(44) in [2]] of \( \langle \mathbf{F}_e \rangle \) and \( \langle \mathbf{F}_m \rangle \), respectively. On the other hand, the contribution of this term to the \textit{Z-component} of \( \langle \mathbf{F}_e \rangle \) and \( \langle \mathbf{F}_m \rangle \), Eqs. (2) and (4), is totally due to the gradient component.

For example, for a perfectly conducting Rayleigh sphere: \( \alpha_e^{(0)} = \varepsilon a^3 \), \( \alpha_m^{(0)} = -a^3/2\mu \), so that:

\[ \langle \mathbf{F}_x \rangle = \frac{K}{2} \exp(-2qz)k^3 a^6 \varepsilon \{|T_\perp|^2 \left[ \frac{2}{3} + \frac{1}{6} (2\frac{K^2}{k^2} - 1) \right] \\
+ \frac{1}{3} |T_\parallel|^2 \left[ \frac{2}{3} (2\frac{K^2}{k^2} - 1) + \frac{1}{6} + \frac{1}{3} (2\frac{K^2}{k^2} - 1) \right] \}, \quad (7) \]

\[ \langle \mathbf{F}_z \rangle = -\frac{q}{2} \exp(-2qz) a^3 \varepsilon \{|T_\perp|^2 \left[ 1 - \frac{1}{2} (2\frac{K^2}{k^2} - 1) \right] \\
+ |T_\parallel|^2 \left[ (2\frac{K^2}{k^2} - 1) - \frac{1}{2} \right] \}. \quad (8) \]
In the special case of a particle small enough to neglect terms \( k^6 a^6 \) and higher, \( \langle \mathbf{F}_{e-m} \rangle \) does not contribute to \( \langle \mathbf{F}_z \rangle \). In Eqs. (7) - (8) the first and second terms within the square bracket that multiplies the corresponding \(|T|^2\) transmission coefficient, are due to \( \langle \mathbf{F}_e \rangle \) and \( \langle \mathbf{F}_m \rangle \), respectively. The force \( \langle \mathbf{F}_{e-m} \rangle \) is the third term in these brackets of Eq. (7). The force \( X \)-component differs from the \( Z \)-component by a factor \( k^3 a^3 \).

If the medium containing the particle were a hypothetical quasi-lossless *left-handed* fluid (LHM) \([20]\), \( \epsilon < 0, \mu < 0 \) which conveys \( n < 0 \), then instead of \( i q \) one has \( -i q \) in all equations and, within the regularization conditions for the wavefunction in LHM's \([16]\), the forces exponentially increase with the distance to the surface.

The scattering components of \( \langle \mathbf{F}_e \rangle \) and \( \langle \mathbf{F}_m \rangle \), Eqs. (1) and (3), change sign with that of \( n \), (i.e. with that of \( \epsilon \) and \( \mu \)); thus as regards the sign of the radiation pressure in LHM's, one obtains the same result as for an incident plane propagating wave \([2, 20]\). The sign of the scattering component of \( \langle \mathbf{F}_{e-m} \rangle \), Eq. (5), and of the gradient components, Eqs. (2), (4) and (6), depends on the relative values of the particle constitutive constants with respect to those of the embedding medium. Notice that for the polarizabilities of these small particles, a change of sign in \( n \), (i.e., in \( \epsilon \) and \( \mu \)), is equivalent to a change of sign in \( n_p \), (namely, in \( \epsilon_p \) and \( \mu_p \)).

Returning to the case of ordinary surrounding media, i.e. \( n > 0 \), the factor \( (2(K^2/k^2) - 1) \) associated to the polarization that appears in Eqs. (1) - (8), may be large when \( K^2 >> k^2 \), a situation that occurs in the *electrostatic approximation*. In TIR, this can be attained either by illuminating at large angles of incidence, by employing a large contrast interface, or by a combination of both. However, these possibilities are hindered by a consequent drastic decrease of the Fresnel transmission coefficient. Fig. 1 illustrates these facts for the electric force, Eqs. (1) and (2), and for the electric-magnetic interaction force, Eqs. (5) - (6), for an index contrast: 2.58. Even so, this figure also shows that the ratio \(|T||2(K^2/k^2) - 1||T\perp|^2\) can be almost 6 near the critical angle \( \theta_i = 22.8^\circ \). These results indicate a similar interplay of the factor \( (2K^2/k^2 - 1) \) and \(|T\perp|^2\) for the magnetic force, Eqs. (3) and (4).

An alternative, more efficient than TIR to enhance the optical force at optical wavelengths with \( p \)-polarization, and hence with the contribution of the factor \( 2(K^2/k^2) - 1 \), is when the evanescent wave is a *SPP* emerging at the plane \( Z = 0 \) from a noble metal film in \( Z < 0 \). For instance, for silver in the Kretschmann-Raether configuration (cf. \([21]\) and Fig. 1(b) of \([22]\)), the transmission coefficient of the evanescent wave in \( Z > 0 \) reaches a value as large
FIG. 1: T.I.R.: With reference to Eqs. (1) - (2) and (5) - (6), $F_s(x)$ denotes the force component factor: $|T_\perp|^2$, $F_p(x)$ means the same for the factor: $|T_\parallel|^2(2K^2/k^2-1)$, $T_p(x)$ stands for $|T_\parallel|^2$, $Ratio(x) = F_p(x)/F_s(x)$, $p(x) = 2K^2/k^2 - 1$. $\theta(x)$ is the angle of incidence in degrees, $\theta_i$, from the denser medium of relative index $n = 2.58$ in $Z < 0$, (see inset geometry).

As $|T_\parallel|^2 \approx 100$ at $\lambda = 600nm$, and $|T_\parallel|^2 \approx 50$ at $\lambda = 450nm$ for an angle of incidence on the metal layer (of thickness: $500nm$) from the quartz prism: $\theta_i = 45.2^\circ$, [cf. Ref. 22, Section 2.4: Figs. 2.12 and 2.13 and Eqs. (2.27) - (2.30)], when the Fabry-Perot resonance of the metal film is excited. Then, $2(K^2/k^2) - 1 = 2\sin^2\theta_t - 1 = 2n_0^2\sin^2\theta_i - 1$, $\theta_t$ denoting the (complex) angle of transmission into the medium $Z > 0$ containing the particle, (which we assume of unity refractive index), and $n_0 = 2.2$ standing for the quartz refractive index.

Thus, $2(K^2/k^2) - 1 = 2(2.2)^2\sin^2 45.2^\circ - 1 = 3.87$. Namely, $|T_\parallel|^2(2K^2/k^2 - 1) = 387$ for $\lambda = 600nm$ and $193.5$ for $\lambda = 450nm$. These values are more than one order of magnitude larger than those obtained by TIR, as seen from a comparison with Fig. 1.

Concerning the magnetic dipole force on a magnetodielectric particle, Eqs. (3) and (4), the role played by $E$ for the electric force is now played by $B$. A strong surface wave under s-polarization, constituting a magnetic SPP, may now be excited if the medium in $Z < 0$ is magnetodielectric instead of a metal. A particular case is that of a left-handed metamaterial in $Z < 0$ [23]. Further work, both theoretical and experimental, is necessary to estimate the strength and feasibility of such excitation.
In conclusion, we have reported what we believe are the first predictions on effects of
the optical force exerted by an evanescent, or surface, wave on a magnetodielectric sphere,
immersed in an arbitrary lossless uniform medium. Since such particles are now
known to be available [17], future experimental work should be feasible, observing these
findings and possible additional phenomena.

Work supported by grants of the Spanish MEC: Consolider NanoLight CSD2007-00046,
FIS2006-11170-C01-C02 and FIS2009-13430-C01-C02.

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