The Analysis of Correlation between Anxiety and Students' Positive Attitudes toward Mathematic Using fuzzy Interval data and fuzzy correlation

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Abstract. In the present paper, we suggest the dimension that the statistical concept is based on uncertainty, in this case, the analysis of the correlation between anxiety and positive attitude towards Mathematics should employ a correlation analysis based on the fuzzy concept. However, far too little attention has been paid to this issue. Over past years, the anxiety and positive attitudes toward mathematics are usually explained using a crisp data. Both are usually using a correlation analysis based on statistical concepts. In certain conditions, a crisp data cannot expound a condition of a student; therefore the fuzzy interval data is used to overcome this problem. Fuzzy interval data is obtained from the data collection process using a fuzzy scale of mathematical anxiety and students' positive attitudes toward mathematics. Moreover, the participants were recruited from 132 Vocational High School students. This research has shown that: (1) fuzzy interval data can expound the flexible and real variables of mathematical anxiety and positive attitude instead of the crisp data; (2) mathematical anxiety has a negative correlation at a moderate level with students' positive attitudes toward mathematics; (3) the fuzzy correlation coefficients in the form of intervals provides benefits for researchers to determine the subjectivity of data analysis results, when compared to the correlation coefficient of Pearson correlation.

1. Introduction
As far as mathematical anxiety is concerned, the term is defined as a feeling of tension, fear, distraction that is troubling in manipulating numbers and mathematical problem solving [4]. Mathematical anxiety symptomizes negative emotions, mental and physical reactions to mathematical thinking processes and problem-solving [2]. The mathematical anxiety can be indicated by a faster heart beating and lower confidence in learning mathematics and even be avoiding the lessons [15]. Thus, mathematical anxiety can affect the performance of students in mathematics learning [3].

Mathematical attitudes are one of the affective variables that influence mathematics learning achievement [5]. The mathematical attitudes mean positive or negative emotions towards mathematics [17]. In other words, the positive attitude has a positive influence on mathematical performance and achievement, while a negative attitude applies the opposite.

Equally important, mathematical anxiety and attitude variables have an inseparable correlation. The mathematical anxiety has a negative correlation with students' positive attitudes toward mathematics in junior high school students [14], university students [1], secondary school [5] and vice versa.
The correlation between both variables is analyzed using a statistical concepts based-method using the crisp data. As stated by Copi [8], statistics are based on the concept of opportunity or uncertainty. This concept is not able to explain the detail correlation between the analyzed variables. Therefore, here which can explain the statistical weaknesses are the fuzzy logical concepts based on certainty developed in this paper. The fuzzy concept has advantages in the data analysis [7] that is; first, it does not depend on the data randomization required in statistics. Second, the fuzzy concept can analyze both a fuzzy and a crisp data, but on the contrary, statistics can only analyze a crisp data. Third Fuzzy logic can describe the obscurity in which the statistics are not able to do so. Fourth, classes with the fuzzy limits on statistics can be easily modeled using fuzzy set, and finally, the fuzzy set is more flexible so that it makes it easier to use expert knowledge and can be used to process the structured and consistent expert opinions.

Fuzzy correlation analysis is an alternative method for determining the correlation between two variables. The data used in fuzzy correlation and Pearson successively use the fuzzy and the crisp data. Cheng and Yang fuzzy correlation [6] determined the fuzzy correlation coefficient by adopting statistical correlation concepts using fuzzy interval data, triangular fuzzy data, and trapezoid fuzzy data. Hung and Wu [12] developed a fuzzy correlation with the expected-interval method using interval data. Hanafy, Salama, and Mahfouz [11] developed a fuzzy correlation on the neutrosophic set with the Centroid method. The data used is in the form of neutrosophic fuzzy consists of the degree of membership, not membership and indeterminacy.

If we now turn to Cheng and Yang, they provided convenience in the use of fuzzy correlations when compared with other fuzzy correlations. Researchers or data analyzers who do not explore the fuzzy concept can easily use it, as long as the data used is at least fuzzy interval data. This fuzzy correlation coefficient is in the form of intervals, which can explain two more real variable correlations [6].

Cheng and Yang have tested their fuzzy correlation model with the data generated using the Monte Carlo method and the management data. This fuzzy correlation method has never been applied to analyze the correlation of students’ psychological elements in published literature. Therefore, based on the framework above, the researchers are intent on explaining mathematical anxiety and students’ positive attitudes using the interval data, which these two variables are explained using the crisp data. Furthermore, the correlation between both variables will be determined using Cheng and Yang fuzzy correlations and compared to the Pearson correlation.

2. Methodology

2.1. Participant
Participants were recruited from 132 vocational high school grade, 11 students. Further, the age from participants was between 15 and 17 years.

2.2. Instrument
The mathematical Anxiety and positive attitudes are measured using a fuzzy Likert scale, this scale is modified from the fuzzy liqueur scale [10]. Each scale consists of 15 items. Each item has five answer options in language terms namely Strongly disagree, Disagree, Neutral, Agree, and Strongly Agree. Each term has successive intervals of [0, 15), [15, 25), [25, 35), [35, 45), [45, 60). The procedure for using this scale is that students first choose answers like the Classic Likert scale. Both students determine the appropriate confidence intervals based on the choice in the first step. The crisp data is obtained from the first step, and the fuzzy interval data is obtained from the second step. The crisp data scoring is based on 10 to strongly disagree, 30 to be neutral, 40 to agree, and 50 to strongly agree, then to find out the level of each variable in each student is based on a hypothetical statistical concept. Student mathematics anxiety and positive attitude levels are determined using categorization in table 1.
Table 1. Level of mathematical anxiety and positive attitude categorization.

| Level language terms | Level interval |
|----------------------|---------------|
| Very high            | x > 550       |
| High                 | 500 < x < 550 |
| Moderate             | 400 < x < 500 |
| Low                  | 350 < x < 400 |
| Very Low             | x < 350       |

2.3. Fuzzy number and correlations.
Fuzzy sets are mathematical objects, generalizing crisp sets, which are designed to model the vagueness present in natural language when observe or describe phenomena that do not have sharply depend on boundaries [13].

2.3.1. Fuzzy data. In order to model one-dimensional fuzzy data, the best up-to-date mathematical model is so-called fuzzy numbers [16]. Gil, Lubiano, saa & Sinova [9] has written the fuzzy number definition as follows:

**Definition 1.** A fuzzy number \( x^* \) is a fuzzy subset of the space of real numbers, that is, a mapping \( \tilde{U} : R \rightarrow [0,1] \) which is normal, convex and has compact levels (i.e., the \( \alpha \)-level sets given by

\[
\tilde{U}_\alpha(x) = \{ x \in R : \tilde{U}(x) \geq \alpha \} \text{ if } \alpha \in (0,1)
\]

\[
\tilde{U}_0(x) = \text{cl}\{ x \in R : \tilde{U}(x) > 0 \}
\]

The fuzzy number application is written as interval numbers which are finite numbers of \( \delta \)-Cut, hereinafter referred to as fuzzy interval numbers. Viertl [16] provided the following fuzzy intervals.

**Definition 2.** A fuzzy number is called a fuzzy interval if all its \( \delta \)-cuts are non-empty closed bounded intervals.

The fuzzy interval data in this article is taken directly by using a fuzzy scale and written with \( x^*_i = [a_{1i}, a_{2i}] \), for \( a_{1i}, a_{2i} \in R \). This interval shows the student’s confidence interval in choosing a language term which corresponds to the fuzzy Likert scale. Fuzzy interval data can be transformed into triangular fuzzy data

\[
x^*_i = [a_{1i}, \frac{a_{1i}+a_{2i}}{2}, a_{2i}]
\]

or trapezium fuzzy data.

2.3.2. The mathematical operation for fuzzy interval data. The addition of two fuzzy interval data more detail see ([16], [6]). Let \( x^*_1 = [a_{11}, a_{21}] \) and \( x^*_2 = [a_{12}, a_{22}] \) be two fuzzy number. Then

\[
x^*_1 + x^*_2 = [a_{11}, a_{21}] + [a_{12}, a_{22}] = [a_{11} + a_{12}, a_{21} + a_{22}]
\]

The multiplication of two fuzzy interval data more detail see [10]. Let \( x^*_1 = [a_{11}, a_{21}] \) and \( x^*_2 = [a_{12}, a_{22}] \) be two fuzzy number. Then

\[
x^*_1 \times x^*_2 = [\min\{a_{11}a_{12}, a_{11}a_{22}, a_{21}a_{12}, a_{21}a_{22}\}, \max\{a_{11}a_{12}, a_{11}a_{22}, a_{21}a_{12}, a_{21}a_{22}\}]
\]

2.3.3. Mean value of fuzzy interval data. Mean value of fuzzy interval data more detail see [16], [9]. Let \( n \) fuzzy \( x^*_i = [a_{1i}, a_{2i}] \), \( i = 1, 2, \ldots, n \) then the average of the data can be written as follows.
\[
\bar{x}^* = \left[ \frac{\sum_{i=1}^{n}a_{1i}}{n} - \frac{\sum_{i=1}^{n}a_{2i}}{n} \right]
\] (5)

2.3.4. The median value of fuzzy data interval. The addition of two fuzzy interval data more detail see [9]. Fuzzy \(x'_i = [a_{1i}, a_{2i}], i = 1, 2, ..., n\) thus, the average of the data can be written as follows.

\[
Me' = \left[ Me(a_{12}, a_{13}, ..., a_{1n}), Me(a_{21}, a_{22}, ..., b_{2n}) \right]
\] (6)

2.3.5. Fuzzy correlation, Fuzzy correlation developed in [6] uses the following definition.

Let \(x'_j = [a_{1j}, a_{2j}], y'_i = [b_{1i}, b_{2i}], be\ a\ sequence\ of\ paired\ fuzzy\ sample\ on\ populations \(\Omega\).

\[
r_{jk} = \sum_{i=1}^{n} (a_{ji} - \bar{a}_j)(b_{ki} - \bar{b}_k) \sqrt{\sum_{i=1}^{n} (a_{ji} - \bar{a}_j)^2} \sqrt{\sum_{k=1}^{n} (b_{ki} - \bar{b}_k)^2}
\] (7)

Then fuzzy correlation is \(r_{l,w}, r_{u,p}\), with \(r_{l,w} = \bar{r} - s_r\) and \(r_{u,p} = \bar{r} + s_r\) where

\[
\bar{r} = \frac{\sum_{j=1}^{n} \sum_{k=1}^{n} r_{jk}}{4} \text{ and } s_r = \frac{\sum_{j=1}^{n} \sum_{k=1}^{n} (r_{jk} - \bar{r})^2}{4}
\] (8)

The definition has a correlation coefficient value at the interval of [-1,1] and the correlation level of the inter-observed variables can be determined by the following definition.

a. When \(r_{l,w}, r_{u,p}\) \(\in [-0.10,0.10]\), the fuzzy correlations is not significant.

b. When \(r_{l,w}, r_{u,p}\) \(\in [-0.39, -0.11] or [0.11, 0.39]\), the fuzzy correlations is low value.

c. When \(r_{l,w}, r_{u,p}\) \(\in [-0.69, -0.40] or [0.40, 0.69]\), the fuzzy correlations is middle value.

d. When \(r_{l,w}, r_{u,p}\) \(\in [-0.99, -0.70] or [0.70, 0.99]\), the fuzzy correlations is High value.

3. Result

The fuzzy data needed to use Cheng and Yang correlation, namely fuzzy interval data, this data is obtained from the data collection process using fuzzy anxiety scale, and fuzzy attitude scale. Mathematical anxiety and attitudes towards mathematics scores are the sums of fuzzy interval data scores using formulas above.

| Table 2. Descriptive statistics of fuzzy data and crisp data |
|-------------------------------------------------------------|
| **fuzzy Anxiety data** | **fuzzy Positive Attitude data** | **Crisp anxiety data** | **crisp positive Attitude data** |
|------------------------|---------------------------------|-----------------------|-------------------------------|
| Min                    | [65 177]                        | [43 168]              | 160                           | 150                           |
| Max                    | [607 707]                       | [625 835]             | 660                           | 700                           |
| Mean                   | [393.59 495.96]                 | [428.17 529.96]       | 433                           | 483                           |
| Median                 | [407 508]                       | [430 528.5]           | 440                           | 485                           |

Table 1 shows that the successive fuzzy data mean of each variable are at the level of moderate to high. In the meanwhile, the data mean of the crisp data from each variable is at a moderate level.

4. Data analysis

The Fuzzy data analysis is manually carried out due to the fact that no present software application can be used. Further, the crisp data analysis is calculated using SPSS. The adjusted significance level is 5%.
Table 3. The Pearson and fuzzy Correlation’s Coefficient of Math Anxiety and Positive Attitude

| Data used          | Correlation Methods | Correlation Coefficients | P-value      |
|-------------------|---------------------|--------------------------|--------------|
| crisps data       | Pearson Correlation | -0.374                   | 0.00001      |
| Fuzzy interval data | Fuzzy correlation   | [-0.3256 -0.32302]       | [0.000139 0.000158] |

Table 3 shows that the coefficient value of the correlation inter-crisp data is at -0.374. the p-value is <0.05, which means that the correlation coefficient between mathematical anxiety and student positive attitudes toward mathematics is meaningful. Furthermore, the correlation coefficient inter-fuzzy data forms intervals of [-0.3256 -0.32302]. The significance test of fuzzy correlation is carried out by using two-way t-test as in the Pearson correlation concept. It is caused by the absence of a concept which discusses the significance test of the correlation coefficient using the fuzzy data. The p-value of the fuzzy data significance test is obtained in the form of intervals namely [0.000139 0.000158]. Each p-value value at the interval is <0.05.

5. Discussion

This fuzzy score has unique characteristics compared to the classical data score, that is a student can simultaneously, at two levels, have the anxiety level and positive attitude, for instance low to moderate, moderate to high, and so on. In the meanwhile, the crisp score can only show one level. The uniqueness of fuzzy interval data implies that fuzzy interval data can expound conditions which cannot be explained by the crisp data.

The crisp data conceptualizes the anxiety and students’ positive attitude condition constantly and inflexibly. Interestingly, on the contrary, that the fuzzy interval data provides a flexible and more real overview of students' anxiety and positive attitude condition. Logically, students’ anxiety and positive attitudes are very likely to be in a certain value condition or are in several conditions at once. Fuzzy interval data can describe both of them, while vise versa for the crisp data, which can not describe these conditions.

Referring to the data analysis in table 3, mathematical anxiety and positive attitudes toward mathematics with the crisp data have a negative correlation at a moderate level. Likewise, the correlation between both variables with fuzzy interval data shows a negative correlation at the moderate level. The advantage of Cheng and Yang [6] fuzzy correlation coefficients in the form of intervals is to provide the researcher choice to determine the correlation coefficient subjectivity that is used based on students’ characteristics considerations. Unfortunately, the researcher cannot determine its subjectivity in Pearson correlation coefficient because it obtained an absolute coefficient.

Fuzzy correlation coefficients in the form of intervals are unique similarly to the uniqueness of fuzzy interval data discussed above. Thus, the correlation between two variables is very likely to be located at different levels at once. For instance, in this case, the correlation between anxiety and positive attitudes of students may have a correlation at moderate and high levels at once. For more extremely, the interval of fuzzy correlation coefficients can cover from insignificant values (having no correlation) to having a low correlation.

6. Conclusions

Base on the research finding and analysis, those can be concluded that: 1) fuzzy interval data can expound more flexible and real mathematical anxiety and positive attitude variables compared to the crisp data; 2) mathematical anxiety has a negative correlation at a moderate level with students' positive attitudes toward mathematics; and 3) the fuzzy correlation coefficients in the form of intervals provide advantages for researchers to determine the subjectivity of the data analysis results when compared to the correlation coefficient of Pearson correlation.
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