Resonance in Magnetostatically Coupled Transverse Domain Walls

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Abstract

We have observed the eigenmodes of coupled transverse domain walls in a pair of ferromagnetic nanowires. Although the pair is coupled magnetostatically, its spectrum is determined by a combination of pinning by edge roughness and dipolar coupling of the two walls. Because the corresponding energy scales are comparable, the coupling can be observed only at the smallest wire separations. A model of the coupled wall dynamics reproduces the experiment quantitatively, allowing for comparisons with the estimated pinning and domain wall coupling energies. The results have significant implications for the dynamics of devices based on coupled domain walls.

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Interactions between nano-scale ferromagnetic domains or domain walls (DWs) can become significant as separations approach the dimensions of the individual structures. These interactions can modify existing resonant behavior [1, 2], lead to the appearance of new coupled modes [3–5], or result in a combination of these [6–8]. The coupling of DWs has also been proposed as a scheme towards increasing the output power of spin-torque oscillators [9], and will have an impact on the design of DW-logic-based devices as well. Understanding resonant behavior in these and similar systems is thus of great interest, as resonances provide a direct probe of the underlying coupling. The system of two interacting transverse domain walls (TDWs) in parallel nanowires (NWs) [Fig. 1(b)] is one of the simplest manifestations of coupled DWs. Despite great interest in this particular coupling both experimentally and numerically [4, 5, 10], no experimental observation of the coupled mode has yet been observed.

In this Letter, we report on the observation of the resonant excitation of coupled TDWs in adjacent NWs. By comparing such coupled excitations with those from single TDWs and micromagnetic simulations, we show the experimental results may only be understood with the inclusion of intrinsic pinning due to NW roughness: an effect predominantly neglected when considering dynamic DW excitation. Extending simple 1-D analytical models to describe phenomenologically both inter-DW coupling and intrinsic DW pinning reproduces the observed spectra and provides insight into the interaction. Finally, we demonstrate that intrinsic pinning and DW coupling remain comparable even at small inter-wire separations, therefore pinning through roughness must always be considered when investigating such dynamical DW experiments.

We use time-resolved Kerr microscopy to measure the DW dynamics. The samples discussed in this Letter are made by evaporating a 10 nm film of Permalloy (Ni$_{81}$Fe$_{19}$) through an electron-beam lithography defined resist mask followed by a subsequent lift-off step. Figure 1(a) shows a scanning electron microscope (SEM) image of the nanowire pair geometry. Pairs of semicircular NWs with radii of 5 $\mu$m were studied for NW widths $w$ of 70, 85, 140, and 190 nm, and closest separations $d$ in the range of 40–140 nm. This design allows for repeatable DW nucleation in the region of closest NW separation (here onwards referred to as the interaction region) by the application of a saturation field along the $y$-direction [11]. Based on the thickness and widths of the NWs, the stable configuration is a TDW [12, 13]. This is also seen in Fig. 1(b) using micromagnetic simulations [14] with a cell size...
of $2 \times 2 \times 10 \text{ nm}^3$, saturation magnetization $M_s = 800 \text{ kA/m}$, exchange constant $A = 13 \text{ pJ/m}$, and Gilbert damping $\alpha = 0.01$.

In order to measure the devices, the samples are mounted above the center conductor of a co-planar waveguide (CPWG). The sinusoidal output of a microwave synthesizer (Agilent N5183A) is phase-locked to the 76 MHz repetition rate of a pulsed Ti:Sapphire laser and sent through the center conductor of the CPWG. The resulting in-plane Oersted field ($\sim 6 \text{ Oe}$) produces a torque on the DW magnetization and drives the motion. To probe the dynamic magnetization of the coupled TDWs, a 810 nm wavelength pulse train is focused with an optical spot size of $\sim 400 \text{ nm}$ onto the interaction region using a 100$\times$ oil immersion microscope objective. The out-of-plane component of the dynamic magnetization is then detected stroboscopically using the polar Kerr effect. Time resolved (TR) signals are acquired at a fixed sample position by sweeping the phase between the Oersted field and the laser probe pulse, while TR images are acquired by scanning the sample beneath the laser spot at a fixed phase shift. Figure 1(c) shows the spatial map of the sample reflectivity localized to the interaction region of a $w = 85 \text{ nm}$, $d = 130 \text{ nm}$ pair of NWs. The solid lines show the NW positions. In the interaction region, both NWs contribute to the reflected signal and so the total reflectivity approximately doubles. Figure 1(d) — acquired simultaneously with Fig. 1(c) — shows the corresponding map of the $\theta_{Kerr}$ response for an on-resonance excitation. Clearly the excited response is localized in the interaction region, indicating that the observed spectra are due solely to the dynamic response of the DWs. This is further verified by applying a saturating field in the $x$-direction. Initializing in this manner, no DWs are present, and no resonance is detected.

We first investigate the response of a DW in a single NW. We expect, based on DW propagation experiments, that pinning from intrinsic defects such as edge or surface roughness will play a role in the dynamics. In DW resonance experiments however, the effects of intrinsic pinning on the resonance have not been thoroughly explored except in confined structures such as a disk. For vortex domain walls (VDWs) in NWs, studies have reported findings only on the characterization of a single intrinsic or patterned pinning site. Thus, it cannot be determined what influence intrinsic pinning (common to both cases) has on the resonant frequency. A similar situation exists for TDWs in NWs, however in this case an intrinsically pinned TDW has not been investigated until now.
FIG. 1. (color online) (a) Scanning electron micrograph of a $w = 85$ nm, $d = 40$ nm device. (b) Micromagnetic simulation of two coupled DWs in a $w = 100$ nm, $d = 20$ nm pair of nanowires. Arrows indicate the in-plane direction of $M$, and colors the magnitude of $M_x (+M_s$ red, $-M_s$ blue). (c) 2D optical reflectivity map and (d) Kerr rotation map of a $w = 85$ nm, $d = 130$ nm device. The solid lines in (c) are the nanowire positions. The Kerr rotation map is shown for the maximum response on resonance.

Figure 2(a) shows recorded spectra for isolated semicircular NWs that reveal DW resonances in the range of 1–2 GHz. The spectra are obtained by plotting the mean-squared-amplitude of the TR $M_z$ signals acquired for different frequencies of the driving Oersted field. As shown, different resonant frequencies are found when testing different NWs and when testing different DW nucleation sites in the same NW. These observations are suggestive of a pinned mode (PM), in which the DW oscillates in a local energy minimum created by the inherent roughness of the NW. The variations in frequency in such a case are due to the random distribution of pinning site depths and shapes. Figure 2(b) shows the average PM frequency of four devices at each width $w$. The error bars indicate the dispersion of observed frequencies. In the tested range of $w$, we see no dependence of the PM frequency on the width.

To confirm the origin of the PM, we perform dynamical micromagnetic simulations. A
FIG. 2. (color online) (a) DW spectra for $w = 85$ nm nanowires taken at two DW nucleation sites and in a separate device. Spectra are offset vertically for clarity. The solid lines are Lorentzian fits from which the central frequency is extracted. (b) Averaged pinning site frequency as a function of nanowire width for experiment (squares) and micromagnetic simulation (circles). Error bars indicate dispersion of pinned mode frequencies over the range of tested devices in experiment and simulation.

DW is prepared in the simulated NW, and a 6 Oe, 100 ps wide Gaussian field pulse is applied in the $x$-direction. The decay of the response is then recorded, and resonances are found by taking the Fourier transform of the volume-averaged $M_z(t)$. We begin by simulating an ideal NW having zero roughness. As expected, the DW moves freely along the NW and no PM is observed. To include pinning, edge roughness, characterized by a root-mean-square roughness $\sigma_{rms}$ and correlation length $\lambda$, is added to the NW. In these simulations, a PM is observed that is a collective oscillation of the DW spins, and results in the oscillating translational motion of the DW about its equilibrium position \[22\]. From SEM images of the NWs, we estimate $\sigma_{rms}$ and $\lambda$ to be in the range of 1–4 nm and 2–6 nm respectively. We find that the PM frequency is only weakly dependent on these two parameters over these ranges, due to an effective convolution of the edge roughness with the larger DW profile ($\sim$100 nm). Figure 2(b) shows the average PM frequency of six simulated NWs versus width for $\sigma_{rms} = 2$ nm and $\lambda = 4$ nm, which match the data well. Typical
pining site depths for these values are $\sim 1$ eV. We have also observed modes internal to the DW in simulation that are characterized by a spatially non-uniform amplitude and phase. These have also been seen in previous simulations \cite{23}. Due to the non-uniformity of these modes, coupling to the uniform Oersted field is weak and the net out-of-plane component of the dynamic magnetization is small. Therefore, we expect our measurements to largely be insensitive to internal modes.

We now turn our investigation to the case of coupled DWs. Resonant frequencies obtained from the measured spectra are plotted in Fig. 3(a) for different separations $d$. As only slight ($< 1$ GHz) width dependence is expected $\cite{4}$ and no trend with respect to NW width is observed, we make no distinction in $w$ in this plot. Of the 43 devices tested, 20 show two peaks in their spectra and two devices show three peaks. The inset of Fig. 3(a) is a sample spectrum that shows two modes corresponding to the two starred points. The lowest frequency resonance for each device is plotted as an open square, the second lowest as a closed square, and the two third modes as crosses. Also included in this plot is a hatched region that indicates the dispersion of PM frequencies from the single NW case. While we may statistically correlate the first mode with a PM, the frequency of the second mode is well above this region. In addition, the frequency of the second mode decreases with increasing separation, as would be expected for a mode dependent on the inter-DW coupling.

To further explore these modes, we utilize micromagnetics. We start by testing the case of zero edge roughness. In contrast to the zero-roughness single NW simulations, a translational mode is observed in the double NW system due to the inter-DW coupling. Figure 3(b) shows the results of these simulations for a $w = 100$ nm pair, plotted as a solid line. Comparing to Fig. 3(a), the zero-roughness case cannot explain the higher frequencies and the observation of multiple modes. We thus proceed to test the system with the inclusion of edge roughness, using $\sigma_{rms} = 2$ nm and $\lambda = 4$ nm as previously characterized from the single NW data. In these simulations the lowest two modes (as higher frequency internal modes are again present) are translational, and plotting them gives the open and closed circles in Fig. 3(b), which shows good agreement with trends observed in experiment. Simulations also show a qualitative similarity in the phase response of the higher frequency mode compared to that of the the zero-roughness DW-DW mode.

We have shown that the addition of edge roughness leads to a quantitative agreement between simulation and experiment. We now look for an analytical understanding of this
FIG. 3. (color online) Separation dependence of (a) experimental and (b) simulated coupled DW resonances. Open symbols correspond to the lowest frequency resonance observed in each spectrum, closed symbols correspond to the next lowest frequency, and a cross designates the two third modes seen in experiment. Data is included for all nanowire widths. Hashed regions in (a) and (b) are the average plus or minus one standard deviation of the single nanowire resonances from Fig. 2.

Inset in (a) is the spectrum for a $w = 85$ nm, $d = 130$ nm nanowire pair with two resonances at 1.5 and 2 GHz. In (b) the solid line is the simulated DW-DW resonance for the case of a $w = 100$ nm pair of nanowires with zero edge roughness.

![Graph showing separation dependence of coupled DW resonances.](image)

The system using the 1-D equations of motion [4, 24]. In these equations the DW is described by $x$ and $\phi$, the central coordinate and angle with respect to the $xy$ plane respectively. Letting $i = 1$ or 2 denote the top or bottom DW, equations of motion are

\[ Q_i \ddot{x}_i - \alpha \dot{\phi}_i = \frac{\gamma}{M_s} K_s \sin(2\phi_i), \]

\[ Q_i \ddot{\phi}_i + \frac{\alpha}{\Delta} \dot{x}_i = \frac{\gamma}{2M_s S} \left[ -\frac{\partial U_i}{\partial x_i} \right]. \]
FIG. 4. (color online) (a) Simplified model of the coupled DWs: two oscillating masses coupled together with spring constant $k$. (b) Energy profiles for the edge roughness of a $w = 100$ nm nanowire (lower) and the DW interaction (upper) for a $w = 100$ nm, $d = 60$ nm pair, as a function of the position of the DWs in the nanowires, offset for clarity. Nanowires shown in the background share the x-axis scale. (c) Numerically solved $M_z^2$ response of the 1-D coupled DW equations of motion [Eq. (1)] as a function of drive frequency and plotted for multiple nanowire separations, shown for the case of two $w = 100$ nm nanowires. Values used were $k_1 = 0.21$ erg/cm$^2$, $k_2 = 0.40$ erg/cm$^2$, and $k = 0.03 - 0.22$ erg/cm$^2$. Insets in bottom left (upper right) are representations of the lower (higher) frequency mode.

Here $Q_1 = 1$, $Q_2 = -1$ are the effective magnetostatic charges of the two DWs, $\Delta$ is the DW width, $K_s$ is the out-of-plane anisotropy, $\gamma$ is the gyromagnetic ratio, and $S$ is the cross-sectional area of the nanowire. $U_i$ is the total energy of a DW and is given by $U_i = U_0 - Q_i M_s S H x_i + k(x_1 - x_2)^2 + k_i x_i^2$, where the terms on the right hand side are the internal energy, Zeeman energy, interaction energy, and pinning energy due to roughness respectively. We have assumed a harmonic potential for the roughness. In the limit of small $\phi_i$, Eq. (1) can be diagonalized and the natural frequencies of the resulting two modes are
given by
\[ \omega_0^2 = \frac{k_1 + k_2 + 2k}{m_D} \pm \frac{\sqrt{(k_1 - k_2)^2 + (2k)^2}}{m_D}, \]
where \( m_D = (1 + \alpha^2)M_s^2S/\gamma^2 \Delta K_s \) is the DW Döring mass [25]. Figure 4(a) shows the simplified description of this system as a coupled oscillator problem in which each mass (DW) sits in its own potential (pinning site). We note here that an in-phase response of the DW displacements \( x_i \) is accompanied by an out-of-phase response of \( \phi_i \) and vice versa, due to the coupling of \( x \) and \( \phi \) in Eq. (1). From Eq. (2), as \( k \) goes to zero \((d \to \infty)\) the two modes approach the respective PM frequencies. As \( k \) becomes large, the lower mode goes to the root-mean-square of the two PM frequencies, while the upper mode goes to the expected DW-DW resonance. Estimates of the spring constants can be found from energy landscapes calculated using micromagnetics which are illustrated in Fig. 4(b). To find the energy profile due to edge roughness, we translate a DW profile through the length of a NW and compute the total energy of the system at each position. We find the energy of the DW coupling by separating the two DWs laterally in zero-roughness NWs and computing the sum of the magnetostatic and exchange energies for different values of these separations. Comparing the DW interaction potential to various pinning sites in Fig. 4(b), we see similar curvatures despite very different depths. Previous work [4] has shown that at the smallest separations, \( k \) does not continue increase with decreasing separation. As the pinning site shape is set by the convolution of the DW with the local roughness, these devices are always in a regime where the effective spring constants of the pinning sites are comparable to those describing the coupling. Pinning will therefore important when considering any coupled DW resonance.

Using the estimates for the \( k_i \) and \( k \) obtained from micromagnetics, we numerically solve Eq. (1) and plot the power spectrum of the net \( M_z \) component for multiple \( d \) [Fig. 4(c)]. At large separations, the two DWs tend to their independent PM frequencies and phases. As \( d \) decreases, the lower mode, illustrated in the bottom left inset, is suppressed due to the unfavorable driving force and the increasingly out-of-phase response of the \( M_z \) components. These observations can explain the detection of only a single mode in some of the spectra: at large separations two accidentally degenerate PM frequencies will be unresolvable, while at small separations the lower frequency mode may be below the detection threshold. Looking at the upper mode (depicted in the top right inset) we see an increase in amplitude and
frequency with decreasing $d$. The overall behavior of the system agrees with the micromagnetics and also makes it clear that the first two modes in Fig. 3(a) can be attributed to the two eigenmodes of Eq. (2), given a random distribution of pinning sites in the NWs.

In summary, the system of two magnetostatically coupled TDWs has been investigated using time-resolved Kerr microscopy. We have measured the spectra of coupled TDWs for the first time, and the observed resonances have been shown to agree with a model that includes both pinning and magnetostatic coupling. The experimental results are reproduced in micromagnetic simulations with the addition of edge roughness and thus DW pinning. Experiment and simulation indicate that the curvature due to pinning will always be significant compared to DW coupling or other dipolar magnetostatic pinning schemes and thus must be considered in future dynamical experiments.

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