Analysis on vibration coupling characteristics of high-speed gears in two-shift AMT

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Abstract. In order to study the vibration coupling influence of no-load gear on the loaded gear with the driving motor speed increasing for pure electric vehicles, a two-shift automated mechanical transmission (AMT) is researched. Nonlinear dynamic models of the shift II gear pair and the coupled relationship between the two gear pairs are established. Vibration characteristics of the shift II gear pair under different speeds are analysed. The results show that the multi-gear coupling vibration makes the system easier to enter into complex multi-period motion or chaos at high speed, thus increasing the noise of the drive system.

1. Introduction
There have been many studies on the vibration characteristics and optimization of high-speed gear drive systems in the field of railway vehicles [1-4]. Many research results have shown that high speed gears often work in the cross resonance region [5] and its vibration is more severe than that in the medium or low speed condition [6-7]. But these researches are focused on either single gear pair or multi-stage gears, without considering the influence of no-load gears on loaded gears. Recently, with the development of electric vehicles, the demand on high-speed gear drive systems is increasing in order to make it compact and higher power density. AMT can be used to improve the power and economy of driveline and widely used in pure electric vehicles. However, when the transmission works in the high-speed condition, the vibration coupling mechanism of no-load gears on the loaded gear is important but still not clear. Therefore, it is of great significance to study these coupling vibration characteristics of multi-gear at high speed for the design and application of high-speed gears.

In this paper, two-shift AMT used in pure electric vehicle is researched. Nonlinear factors such as time-varying mesh stiffness and static transmission error are considered to establish nonlinear dynamic models of the gear system. The nonlinear vibration characteristics of coupling gear system and its affection among them are simulated and analysed.

2. Mathematical modelling
Figure 1 shows the simplified schematic diagram of the two-shift AMT in a pure electric vehicle. Figure 2 shows the two gear pairs coupled nonlinear dynamic model. The calculation parameters are shown in Table 1. Here, \( \bar{\theta}_m \) stands for vibration angular displacement of motor, \( \bar{\theta}_i \), \( \bar{\theta}_j \) stands for vibration angular displacement of gear 3 and gear 4, \( x_i (i = 1,2,3,4) \) stands for axial displacement of the
gears, \( y_i (i = 1, 2, 3, 4) \) stands for radial displacement of the gears, \( \beta_{b1}, \beta_{b2} \) stand for the helical angle of base circle, \( k_{ci} (i = 1, 2) \), \( c_i (i = 1, 2) \) and \( e_i (t) (i = 1, 2) \) stand for the meshing stiffness, meshing damping and static transmission error of the gear pair respectively, \( k_{ix}, k_{2x}, k_{iy}, k_{2y} \) stand for the support stiffness of bearings.

\[
y_i = y_i = \ldots
\]

![Schematic diagram of two-shift AMT.](image)

Figure 1. Schematic diagram of two-shift AMT.

![Nonlinear dynamic model at the shift II.](image)

Figure 2. Nonlinear dynamic model at the shift II.

**Table 1.** Calculation parameters of the gear transmission.

|                      | Shift I gear pair | Shift II gear pair |
|----------------------|-------------------|-------------------|
| Number of teeth      | 17                | 48                |
| Module (mm)          | 2.5               | 2.5               |
| Spiral angle (°)     | 17.08             | 17.08             |
| Lumped mass (kg)     | 0.32              | 1.98              |
| Concentrated inertia \((10^{-4} \text{kg} \cdot \text{m}^2)\) | —                 | 153.2             |
| Meshing stiffness \((\text{N/m})\) | \((4.8 + 0.4 \cos \omega_1 t) \cdot 10^8\) | \((5.1 + 0.2 \cos \omega_2 t) \cdot 10^8\) |
| Meshing damping \((\text{N} \cdot \text{s/m})\) | 3057.8            | 2643.7            |
| Static transmission error \((\text{m})\) | \(5 \cdot 10^{-6}\) | \(5 \cdot 10^{-6}\) |

|                      | Value             | Unit              |
|----------------------|-------------------|-------------------|
| Axial support stiffness of bearings | \(3.88 \cdot 10^9\) | N/m               |
| Radial support stiffness of bearings   | \(6.78 \cdot 10^8\) | N/m               |
| Rotational inertia of motor and input shaft | \(4.82 \cdot 10^{-2}\) | kg·m²            |
| Rotational inertia of output shaft     | \(2 \cdot 10^3\)   | kg·m²             |

The static transmission error \([8]\) and time-varying meshing stiffness of gears can be expressed as:
\[ e(t) = \sum_{j=0}^{N} e_j \sin(j \omega_t + \phi_j) \]

\[ k_n(t) = k_0 + \sum_{j=0}^{N} k_j \cos( j \omega_t + \phi_j) \]

Where, \( e_j \) and \( k_j \) are the \( j^{th} \) amplitude of static transmission error and stiffness. \( \phi_j, \phi_j \) is the phase of the \( j^{th} \) harmonic, \( k_0 \) is the average meshing stiffness. \( \omega_t \) is meshing frequency and its value equals to the product of the gear speed and the number of its own teeth. To simplify the calculation, the first harmonic amplitude is premeditated, and the phase is set to be zero.

The dynamic transmission error of the shift II gear meshing line in axial and radial directions can be described as

\[ \delta_{x_2} = x_2 - x_1 - \tan \beta_{b_2} (r_z \theta_m + y_2 - r_t \theta_3 - y_3) - e_2(t) \sin \beta_{b_2} \]

\[ \delta_{y_2} = r_z \theta_m + y_2 - r_t \theta_3 - y_3 - e_2(t) \cos \beta_{b_2} \]

Now, the value of gear backlash is set to be \( 2b \). So, the nonlinear function of gear backlash in the axial and radial direction (\( f_x(\delta) \) and \( f_y(\delta) \)) can be expressed as

\[
\begin{aligned}
  f_x(\delta) &= \begin{cases} 
  \delta + b \sin \beta_b, & \delta < -b \sin \beta_b \\
  0, & -b \sin \beta_b \leq \delta \leq b \sin \beta_b \\
  \delta - b \sin \beta_b, & \delta > b \sin \beta_b 
  \end{cases} \\
  f_y(\delta) &= \begin{cases} 
  \delta + b \cos \beta_b, & \delta < -b \cos \beta_b \\
  0, & -b \cos \beta_b \leq \delta \leq b \cos \beta_b \\
  \delta - b \cos \beta_b, & \delta > b \cos \beta_b 
  \end{cases}
\end{aligned}
\]

The meshing force of shift II gear pair in axial and radial directions can be expressed as

\[ F_{x_2} = \sin \beta_{b_2} c_{b_2} \delta_{x_2} + \sin \beta_{b_2} k_{b_2}(t) f_x(\delta_{x_2}) \]

\[ F_{y_2} = \cos \beta_{b_2} c_{b_2} \delta_{y_2} + \cos \beta_{b_2} k_{b_2}(t) f_y(\delta_{y_2}) \]

Also, the meshing force of shift I gear pair (\( F_{x_1} \) and \( F_{y_1} \)) can be expressed in the same way.

The 6-DOF dynamic vibration model (Model S) of the shift II gear pair can be expressed as Equation (1), where \( m_i (i = 2,3) \), \( I_i \) and \( r_i (i = 2,3) \) denote the lumped mass, the lumped inertia, and the base circle radius of the corresponding gears in Figure 2 respectively. \( I_m \) denotes the total lumped inertia of motor and input shaft. \( T_m \) and \( T_l \) are the drive and load torques acting on gears 2 and gear 3, respectively.

\[
\begin{align*}
  m_2 \ddot{x}_2 &= -k_{1x} x_2 - F_{x_2} \\
  m_2 \ddot{y}_2 &= -k_{1y} y_2 - F_{y_2} \\
  m_3 \ddot{x}_3 &= -k_{2x} x_3 + F_{x_2} \\
  m_3 \ddot{y}_3 &= -k_{2y} y_3 + F_{y_2} \\
  I_m \dddot{\theta}_m &= T_m - r_2 F_{y_2} \\
  I_l \dddot{\theta}_l &= r_2 F_{y_2} - T_l
\end{align*}
\] (1)

When the transmission works in the shift II, the shift II gear pair bears the load and its meshing force is much larger than that of the shift I gear pair which is no-load. The location of shift II is set at \( x = c \) while the shift I location is set at \( x = a \). Based on the mode superposition principle, when a dynamic concentrated force works at \( x = c \), the deflection at the points of ‘a’ and ‘c’ can be expressed as
\[ x_i = \chi_x \cdot x_2 \]
\[ y_i = \chi_y \cdot y_2 \]

Where, \( \chi_x \) and \( \chi_y \) are axial and radial displacement ratios, respectively. If we know the location of the two gears, they are constant. So, the 11-DOF dynamic coupled model (Model C) of the two gear pairs can be expressed as

\[
\begin{align*}
    m_1 \ddot{x}_1 &= -k_{1x} x_1 - F_{s1} \\
    m_1 \ddot{y}_1 &= -k_{1y} y_1 - F_{s1} \\
    m_2 \ddot{x}_2 &= -k_{2x} x_2 + F_{s2} \\
    m_2 \ddot{y}_2 &= -k_{2y} y_2 + F_{s2} \\
    I_m \ddot{\theta}_m &= T_m - r_1 F_{y1} - r_2 F_{y2} \\
    I_I \ddot{\theta}_I &= r_1 F_{yI} - T_I \\
    I_4 \ddot{\theta}_4 &= r_2 F_{y4} - T_4 \\
    x_i &= \chi_x \cdot x_2 \\
    y_i &= \chi_y \cdot y_2 \\
    x_4 &= \chi_x \cdot x_3 \\
    y_4 &= \chi_y \cdot y_3
\end{align*}
\]

Where, \( I_i \) and \( r_i (i = 1, 4) \) denote the lumped inertia, and the base circle radius of the corresponding gears respectively. In Model C, meshing force of the shift II gear pair will cause lateral vibrations of the shift I gear pair, and meshing process of the latter will have effect on torsional vibration of the former on the contrary.

Introduce the dimensionless variables

\[ X = \{ X_1, X_2, \ldots, X_{11} \}^T \]

Where, \( b_x = b \sin \beta_x \), \( b_y = b \cos \beta_y \), \( X_1 = \frac{x_1}{b_x} \), \( X_2 = \frac{x_2}{b_x} \), \( X_3 = \frac{x_3}{b_x} \), \( X_4 = \frac{y_3}{b_y} \), \( X_5 = \frac{r_1 \theta_m}{b_y} \), \( X_6 = \frac{r_2 \theta_4}{b_y} \), \( X_7 = \frac{r_2 \theta_4}{b_y} \), \( X_8 = \frac{x_1}{b_x} \), \( X_9 = \frac{y_3}{b_y} \), \( X_{10} = \frac{x_3}{b_x} \), \( X_{11} = \frac{y_4}{b_y} \), \( X_{11+1} = \frac{dx_i}{d\tau} (i = 1, 2, \ldots, 11) \). The fundamental meshing frequency of the shift II gear pair is defined as \( \omega_2 = \sqrt{\frac{k_{s2}}{m_{s2}}} \) and \( m_{s2} \) denotes the equivalent mass of the shift II gear pair. Then the dimensionless meshing frequency is given by \( \omega = \frac{\omega_2}{\omega_0} \). And the dimensionless time is defined as \( \tau = \omega_0 \cdot t \). The dimensionless dynamic meshing errors of the shift II in axial and radial directions gear pair can be deduced as

\[
\Delta_{x2} = X_1 - X_3 - (X_2 + X_4 + X_5 - X_6) - \frac{e_3(\tau)}{b}
\]
\[
\Delta_{y2} = X_2 + X_4 - X_5 - X_6 - \frac{e_3(\tau)}{b}
\]

Similarly, the dimensionless dynamic meshing errors of the shift I gear pair in axial and radial directions could be expressed respectively as

\[
\Delta_{x1} = X_8 - X_{10} - (X_9 + \frac{r_1}{r_2} X_5 - X_{11} - X_7) - \frac{e_4(\tau)}{\omega_2 b}
\]
\[
\Delta_{y1} = X_9 + \frac{r_1}{r_2} X_5 - X_{11} - X_7 - \frac{e_4(\tau)}{\omega_2 b}
\]
Where, the dimensionless static transmission error is  
\[ e_i(\tau) = e_i \sin \omega \tau \quad (i = 1, 2) \] 
So, the dimensionless form of Equation (2) could be deduced as

\[
\begin{align*}
\dot{X}_i &= X_{s,i} (i = 1, 2, \ldots, 11) \\
\ddot{X}_{12} &= -\frac{k_{12}}{m_2 \omega_0^2} X_1 - \frac{\sin \beta_2 c_{12}}{m_2 \omega_0} \dot{\Delta}_{12} - \frac{\sin \beta_2 k_{12}}{m_2 \omega_0^2} f(\Delta_{12}) \\
\dot{X}_{11} &= -\frac{k_{11}}{m_2 \omega_0^2} X_2 - \frac{\cos \beta_2 c_{12}}{m_2 \omega_0} \dot{\Delta}_{12} - \frac{\cos \beta_2 k_{12}}{m_2 \omega_0^2} f(\Delta_{12}) \\
\ddot{X}_4 &= \frac{\sin \beta_2 c_{12}}{m_2 \omega_0} \dot{\Delta}_{12} + \frac{\sin \beta_2 k_{12}}{m_2 \omega_0^2} f(\Delta_{12}) \\
\ddot{X}_5 &= \frac{\cos \beta_2 c_{12}}{m_2 \omega_0} \dot{\Delta}_{12} + \frac{\cos \beta_2 k_{12}}{m_2 \omega_0^2} f(\Delta_{12}) \\
\ddot{X}_6 &= \frac{r T_m}{b_1 I_m \omega_0^2} - \frac{r_1^2 \cos \beta_2 c_{12}}{I_m \omega_0^2} \dot{\Delta}_{12} - \frac{r_1^2 \cos \beta_2 k_{12}}{I_m \omega_0^2} f(\Delta_{12}) - \frac{r_2 T_s}{b_1 I_s \omega_0^2} \dot{\Delta}_{j} - \frac{r_2 \cos \beta_2 c_{12}}{I_s \omega_0^2} \dot{\Delta}_{j} - \frac{r_2 \cos \beta_2 k_{12}}{I_s \omega_0^2} f(\Delta_{j}) \\
\ddot{X}_7 &= \frac{r_1^2 \cos \beta_2 c_{12}}{I_s \omega_0^2} \dot{\Delta}_{j} + \frac{r_1^2 \cos \beta_2 k_{12}}{I_s \omega_0^2} f(\Delta_{j}) \\
\ddot{X}_8 &= \frac{r_1^2 \cos \beta_2 c_{12}}{I_m \omega_0^2} \dot{\Delta}_{j} + \frac{r_1^2 \cos \beta_2 k_{12}}{I_m \omega_0^2} f(\Delta_{j}) \\
\ddot{X}_9 &= \chi_s \cdot \dot{X}_{12} \\
\ddot{X}_{10} &= \chi_s \cdot \dot{X}_{11} \\
\ddot{X}_{11} &= \chi_s \cdot \dot{X}_{14} \\
\ddot{X}_{12} &= \chi_s \cdot \dot{X}_{15}
\end{align*}
\]

(3)

3. Dynamic response of the loaded gear pair

In order to verify the vibration coupling influence of no-load gear pair on the loaded gear, the difference between Model S and Model C will be shown and discussed here. The AMT works in shift II condition with \( T_m = 43.8 \text{N} \) and \( T_s = 80 \text{N} \) for research. All the other parameters needed will be set to be the same for comparison. The dimensionless meshing frequency \( \tilde{\omega} \) is expressed as

\[ \tilde{\omega} = \omega m z_2 / \omega_0 \]

Where, \( \omega_0 \) is rotating speed of the motor and \( z_2 \) is the teeth number of gear 2. The drive motor speed ranges from 4000r/min to 15000r/min, while the dimensionless meshing frequency \( \tilde{\omega} \) varies from 0.705 to 2.126.

Radial meshing error can reflect torsional vibration of gear pair. Bifurcation diagrams of the shift II gear pair’s radial meshing error vibration are shown in Figure 3. The same feature of the two models are that vibration displacement is small in low frequency while large in high frequency, and there is an obvious resonance phenomenon. But there are also some differences. The resonance frequency of Model C is 1.262 which is larger than that of Model S (1.198). At the same time, the vibration of Model S is mainly periodic or quasi-periodic while Model C appears more chaotic states.

Figure 4 shows the radial vibration characteristics of Model S. When dimensionless meshing frequency \( \tilde{\omega} = 1.198 \), time history diagram shows a sine wave. Phase diagram of radial vibration is a single closed circle and Poincare map has only a single discrete point. Meantime FFT spectrum consists of only two frequency components, as shown in Figure 4(a). So, the shift II gear pair is in single-period motion at the resonance point. When \( \tilde{\omega} = 1.262 \), time history diagram also shows a periodic curve as shown in Figure 4(b), but the amplitude is less than that at the resonance point. Phase diagram approximately has a cluster of concentric circles and Poincare map has a set of points that
gather around value 2.5. Moreover, FFT spectrum has more frequency components than that at $\omega = 1.198$. So, the system turns to quasi-periodic motion.

![Bifurcation diagrams of radial dynamic meshing error of the shift II gear pair.](image1)

**Figure 3.** Bifurcation diagrams of radial dynamic meshing error of the shift II gear pair.

![Vibration characteristics of Model S.](image2)

**Figure 4.** Vibration characteristics of Model S.

![Vibration characteristics of Model C.](image3)

**Figure 5.** Vibration characteristics of Model C.
Similarly, the shift II gear pair of Model C is also in quasi-periodic motion at $\omega = 1.198$, as shown in Figure 5(a). Whereas when Model C reaches the resonance point, the time history fluctuates drastically in Figure 5(b). Circles of phase diagram are disorder and unsystematic and meantime Poincare map has no obvious gathering points. So, it is considered that the radial vibration of the shift II gear pair is in a chaotic state. Therefore, the no-load shift I gear pair make torsional vibration of the loaded gear pair more complicated.

4. Conclusions
In addition to the resonance phenomenon, the nonlinear vibration of gear transmission system is more severe at high speed than that at low speed in vehicles. Compared with the single-stage decelerator, the no-load gear pair of the multi-gear transmission will make the vibration of the loaded gear pair more complicated, or even make it instability, and entering into chaos working condition, which will increase the vibration and noise of the transmission system.

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