Electrically-controllable RKKY interaction in semiconductor quantum wires

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We demonstrate in theory that it is possible to all-electrically manipulate the RKKY interaction in a quasi-one-dimensional electron gas embedded in a semiconductor heterostructure, in the presence of Rashba and Dresselhaus spin-orbit interaction. In an undoped semiconductor quantum wire where intermediate excitations are gapped, the interaction becomes the short-ranged Bloembergen-Rowland super-exchange interaction. Owing to the interplay of different types of spin-orbit interaction, the interaction can be controlled to realize various spin models, e.g., isotropic and anisotropic Heisenberg-like models, Ising-like models with additional Dzyaloshinsky-Moriya terms, by tuning the external electric field and designing the crystallographic directions. Such controllable interaction forms a basis for quantum computing with localized spins and quantum matters in spin lattices.

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All-electrical quantum manipulation of the spin degree of freedom of electrons and/or magnetic ions in semiconductors is a central issue in the fields of spintronics and quantum information processing.¹²,¹³,¹⁴,¹⁵,¹⁶,¹⁷,¹⁸,¹⁹,²⁰,²¹,²²,²³,²⁴,²⁵. Electron spins in semiconductors have long coherence time and cost very low energy to flip. These features have obvious advantages for solid-state quantum information processing where spins of electrons or magnetic ions have been proposed as a candidate of qubits. The Heisenberg-like exchange interaction between two electrons confined in neighboring quantum dots can be controlled electrically via changing the wavefunction overlap of the two electrons. The distance between the neighboring dots is crucial for the strength of the exchange interaction. In order to achieve lower accuracy thresholds for quantum error correction, the implementation of coherent long-distance interaction between two qubits is desirable. Optical field can provide a practical way to realize the remote coupling between two local spins via exchange interaction mediated by the cavity modes, and optically generated excitons and/or electrons. Optical control can be realized in femtosecond processes and made robust against decoherence. The limitation of the spot size of laser beams, however, hinders the integration of qubits under optical control. It is therefore legitimate to design quantum gates based on electrically tunable remote coupling between two spins.

The RKKY interaction is an indirect exchange interaction between localized spins mediated by itinerant electrons in semiconductors or metals.¹³,¹⁴,¹⁵,¹⁶,¹⁷,¹⁸,¹⁹,²⁰,²¹,²²,²³,²⁴,²⁵. The local spins can be magnetic ions or electron spins in quantum dots. A particularly interesting system is quantum dots doped with itinerant electrons. Since the RKKY interaction is mediated by itinerant electrons, the effects of spin-orbit interaction (SOI) are inevitable in conventional zinc-blende semiconductors due to breaking of the crystal inversion symmetry, i.e., Dresselhaus SOI (DSOI), and the structural symmetry, i.e., Rashba SOI (RSOI). The SOI is one of the major sources of spin decoherence and leads to anisotropy in the relevant exchange interaction. Such anisotropy in the RKKY interaction arising from the SOIs is a resource to be exploited in this paper for electrical control of various types of spin interactions, which is not available in systems without SOIs.

In this work, we wish to draw attention to the possibility of creating spin chains or lattices on semiconductor heterostructures. State-of-the-art e-beam lithography makes it possible to fabricate such structures. We demonstrate theoretically that the SOIs can be used to manipulate electrically the symmetry type of the spin-spin interaction. The analytical expression of the RKKY interaction shows the possibility of implementing different quantum spin models by changing the strengths of the RSOI and DSOI, e.g., isotropic and anisotropic Heisenberg models and Ising-like model. A man-made spin lattice or chain mediated by this spin-spin interaction would exhibit rich quantum phases.

First we consider two local spins S₁ and S₂ located at R₁ and R₂, mediated by electrons occupying the lowest subband of a quantum wire in the presence of both the RSOI and DSOI (see Fig. 1(a)). The Hamiltonian of the system contains the single-particle part H₀ and the s-d exchange interaction H₁ as

\[
H = H₀ + H₁, \quad \text{(1a)}
\]

\[
H₀ = \sum_{\mathbf{k},\mathbf{q}} E_{k\delta} c^\dagger_{\mathbf{q}\eta} c_{\mathbf{k},\eta}, \quad \text{(1b)}
\]

\[
H₁ = J \sum_{\mathbf{k},\mathbf{q},\mathbf{q}′} e^{-i\mathbf{q}\mathbf{r}} c^\dagger_{\mathbf{k}+\mathbf{q},\eta} c_{\mathbf{k}+\mathbf{q}′,\eta'} \sigma_{\mathbf{q}'} \cdot \mathbf{S}_i, \quad \text{(1c)}
\]

where \(c_{\mathbf{q}}\) annihilates an electron with quasi-momentum \(\mathbf{k}\) and spin \(\eta, \sigma\) denotes the Pauli matrices, and \(J\) is the strength of the s-d exchange interaction between itinerant electrons and the local spins.

The non-interacting electron energy \(E_{k\delta}\), determined by the single-particle Hamiltonian \(H₀ = \hbar^2 k^2/2m^* + V(\mathbf{y}) + H_{\text{SO}}\), is spin-dependent due to the SOI. Above \(V(\mathbf{y})\) is the transverse confining potential along the y axis for electrons in the heterostructure, \(m^*\) is the electron effective mass, and \(H_{\text{SO}} = B_{\text{SO}}(k) \cdot \sigma\) is the SOI which is equivalent to a momentum-dependent effective magnetic field \(B_{SO}(k)\). The direction of the effective magnetic field depends on the crystallographic plane and its strength is proportional to the quasi-momentum in the linear SOI regime. For example, for typical crystallographic planes (001), (110), and (111), the effective field \(B_{\text{eff}} = k(\beta, -\alpha, 0), k(0,-\alpha, -\beta/2), \) and \(k(0,-\alpha + 2\beta/\sqrt{3}, 0)\) in...
The processes underlying different terms in the RKKY interaction is clearly identified: The first term in the bracket of Eq. (5) arises from the spin-conserving scattering within each spin-split band and hence has exactly the same form as in systems without the SOI. The second and third terms correspond to the spin-flip scattering between different spin-split bands and the phase factor $e^{i2\theta} = e^{i0(R_{12})}$ is the phase shift accumulated over $R_{12}$ by the extra momentum transfer $Q$ that separates the minima of the two bands. The RKKY interaction in Eq. (5) is general for arbitrary crystallographic planes and quantum wire orientations.

Back into the laboratory coordinate systems, the RKKY interaction is transformed to

$$H_{RKKY}^{R_{12}} = F_1(q_F|R_{12}) \left\{ \langle S_1 \cdot S_2 - 2S_{1z}S_{2z} + 2 \left[ S_1 \cdot \mathbf{n}(\pi + \phi, \theta) \right] [S_2 \cdot \mathbf{n}(\phi, \theta)] \right\},$$

where $\mathbf{n}(\phi, \theta) = (\cos \theta \sin \phi, \sin \phi \sin \theta, \cos \theta)$. The angle $\theta$ and angle $\phi$ are given in Table I depend on the crystallographic planes where the quantum wire is embedded. The angle $\theta$ describes how both the RSOI and DSOI twist the two local spins away from the z axis and the angle $\phi$ determines the in-plane twist of the spin orientation (see Fig. 1(b)). If the Dresselhaus interaction is absent, our result is reduced to the previous work.

The system with both the DSOI and RSOI has a great extent of controllability, owing to the interplay of the two types of SOIs and the sensitive dependence of DSOI on the crystallographic plane and the quantum wire orientation. Eq. (7) works for all but the (110) crystallographic plane. For a quantum wire embedded in the heterostructure grown along the [110] direction, the RKKY interaction becomes $H_{RKKY}^{R_{12}} = F_1(q_F|R_{12}) \left\{ S_1 \cdot S_2 - 2S_{1z}S_{2z} + 2 \left[ S_1 \cdot \mathbf{n}(\pi + \phi, \theta) \right] [S_2 \cdot \mathbf{n}(\phi, \theta)] \right\}$. The RKKY interaction shows isotropic behaviour at $\alpha = 2\beta/\sqrt{3}$ when the quantum wire is embedded on the (111) crystallographic plane, because the SOSI and DSOI have the same dependence on the in-plane momentum (see Table I). It means it is possible to switch on/off the SOIs. When the quantum wire is embedded in arbitrary crystallographic directions on the (001) plane, i.e. in the direction with an angle $\theta$ respect to the [100] direction, the formalism of RKKY interaction remains the same and the angle $\theta$ and angle $\phi$ should be redefined as $\theta \equiv q_F|q_F|/h^2$, $\phi \equiv \arctan(2x_1\sin 2\theta + 2y_1\cos 2\theta)/|x_1|$. Note that when the quantum wire is embedded along the [110] direction on the (001) plane, $\theta \equiv \alpha - \beta$ and $\phi = 0$, it would give us another way to switch on/off the SOIs by tuning the strength of the external electric field.

The most important difference between our result and the previous work is that the interplay between the SOIS and DSOI offers us a new way to control the spin-spin interaction. Due to the interplay of the DSOI and RSOI, the RKKY interaction presents, in addition to the usual Heisenberg-like exchange term, not only an Ising-type anisotropic term, but also a twisted Dzyaloshinsky-Moriya (DM) -like term which twists the local spins (see the third term in Eq. (7)).

Tuning the parameters $\theta$ and $\phi$, we can rotate the local spins in a spin space (see Fig. 1(b)), and construct various quantum spin systems.
kinds of quantum spin models. From Eq. (7), we can re-
gates spin operations, constitute the complete set of gates for uni-
the exchange coupling be done at that rate.
cosecond cycle would be possible if the electrical control can
of-plane directions, the spin correlations, being ferromagnetic
or antiferromagnetic, are different.

The conditions mentioned in Table I e.g., \( \theta = k \pi \), are re-
izable in a narrow bandgap semiconductor InSb quantum
well with 10 nm thickness at a specific perpendicular electric
field \( E \approx 50 \text{ kV/cm} \). The SOI is strong in narrow bandgap
semiconductor quantum wells, e.g., HgCdTe QWs, in which
\( \alpha \) ranges from \( 10^{-13} \text{ eV m} \) to \( 10^{-10} \text{ eV m} \) depending on the
external gate voltage, thickness of QW, and electron density. 
Choosing a proper external electric field, one can realize the
switching between different spin models.

All-electrical two-qubit gates can be implemented with the
RKKY interaction, being either the Heisenberg-like interac-
tion or the Ising-like interaction. The controllability of the
interaction symmetry in the SOI systems gives us further flex-
ibility of realizing various types of two-qubit gates such as the
\text{VSWAP} gate and the phase gate, either of which, plus one-
spin operations, constitute the complete set of gates for uni-
versal quantum computing. In particular, the isotropic Heisen-
berg interaction can be used for both one-qubit and two qubit
gates. For an estimation of the operation rate, we notice that
the exchange coupling \( J \) can be tuned to 1 meV by external
electric fields, which indicates that two-qubit gates with a pi-
osecond cycle would be possible if the electrical control can
be done at that rate.

This approach of constructing electrical-controllable spin-
spin interaction outlined above can be extended to more com-
plicated structures. Here we propose that a single pair of spin
qubits be replaced with an array of local spins, i.e., a spin lat-
tice or chain, which is defined on quantum wires embedded
in semiconductor heterostructures. Spin lattices are platforms
of a wealth of many-body physics and quantum phenomena
such as quantum phase transitions and may also be a com-
puting resource such as in quantum simulation of condensed
matter systems. The RKKY interaction is a long-ranged inter-
action since its asymptotic behavior \( \lim_{R \to \infty} F_1 (q_{F} R_{12}) \sim \cos(2q_{F}R_{12})/R_{12} \)
that is inversely linear in distance \( R_{12} \) with an oscillation superimposed. In practice, precise positioning
of spins for realizing an artificial spin lattice proposed here
is still a great challenge. The long-range interaction would
make the quantum physics richer and more complicated. Such
systems often manifest quantum phase transitions governed by
parameters such as the external field and concentration of
impurities. In order to realize a short-ranged spin-spin in-
teraction, we could use a one-dimensional intrinsic narrow
bandgap semiconductor quantum wire in which the virtual ex-
itations between the valence and conduction bands, in lieu of
iterater electrons in doped semiconductors, mediate the in-
teraction. The range function of the spin-spin interaction be-
comes \( F_1 (q_{F} R) \sim e^{-3R} \) from Eq. (1a) utilizing the Keldysh
Green’s function. The interaction length \( \lambda \approx h / \sqrt{2m_\alpha} \),
mostly determined by the electron effective mass considering
the large mass of the holes, can be tuned from 10 nm to in-
finity by adjusting the bandgap \( \Delta \) of, e.g., a HgCdTe quantum
well from 0.1 eV to zero where a quantum phase transition takes place. It provides us a new way to control the range of the
spin-spin interaction. Using virtual excitations to mediate
the spin-spin interaction also largely avoids the fast optical
decoherence. In the spin lattice, one can also control the
spin-spin interaction spatially, and realize different spin mod-
els in a spin lattice electrically, i.e., anisotropic Heisenberg
model and Ising-like model with an additional DM term. The
DM-like term can induce the interesting spiral phase in the
spin chain in which the spins rotate along the virtual axis.
Using the twisted DM term induced by the SOIs, one can use
an electric field pulse, which propagates along the spin chain,
to generate a propagating spin wave along the spin chain, and
this spin wave excitation is actually a low power consumption
spin current since one only needs to flip the neighboring spins
without drifting of electrons.

However, the SOI in semiconductor low-dimensional elec-
tron gases is a double-edged sword, since the spin relaxation
is typically dominated by the D’yakonov-Perel’ (DP) mecha-
nism, and is enhanced with increasing the SOI. The spin
decoherence induced by the SOIs is strongly suppressed in
this spin lattice due to the quasi-one-dimensional geometry of
quantum wires, since only one single point in the k-space sat-
fies the momentum and energy conservation conditions for
real excitations.

In summary, we propose all-electrical manipulation of the
spin-spin interaction via the RSOI and DSOI of electrons lo-
calized in quantum wires. This RKKY interaction can be
controlled in both magnitude and symmetry-tuned heavily by
adjusting the strength of SOIs. Fermi energy and crystallo-
graphic planes, and display different types of spin-spin inter-
actions. Both isotropic and anisotropic Heisenberg models
and Ising-like models with additional DM terms could be re-
alized. The anisotropy and twisted term in the RKKY inter-
action caused by the SOIs can be removed by adjusting the
strength of SOIs. The parameters related to constructing the

| Crystallographic planes | \( H_0 \) | \( \varphi \) | \( \theta \) |
|------------------------|--------------|-----------|----------|
| (001)                  | \( H_0^{(01)} = \frac{\mu_0^2}{2m_e} + \alpha (\mathbf{k} \times \mathbf{z}) \cdot \sigma + \beta (\mathbf{k} \times \mathbf{z}) (\sigma \mathbf{z}) \) | \( \arctan(\frac{\beta}{\alpha}) \) | \( \frac{\pi}{2} \sqrt{\alpha^2 + \beta^2} | R_{12} | \) |
| (110)                  | \( H_0^{(10)} = \frac{\mu_0^2}{2m_e} + \alpha (\mathbf{k} \times \mathbf{z}) \cdot \sigma - \frac{\beta}{2} (\mathbf{k} \times \mathbf{z}) (\sigma \mathbf{z}) \) | \( \pi - \arctan(\frac{\alpha}{\beta}) \) | \( \frac{\pi}{2} \sqrt{\alpha^2 + \frac{\beta^2}{4}} | R_{12} | \) |
| (111)                  | \( H_0^{(11)} = \frac{\mu_0^2}{2m_e} + \alpha (\mathbf{k} \times \mathbf{z}) \cdot \sigma \) | 0 | \( \frac{\pi}{2} (\sqrt{\beta^2 - \alpha^2}) | R_{12} | \) |

TABLE I: The angles \( \theta \) and \( \varphi \) describing the effect of RSOI and DSOI in different crystallographic planes.
spin models can be electrically controlled. Such in-situ controllability may be used for observing quantum phase transitions in spin lattices without external magnetic fields. The short-ranged spin-spin interaction can be realized utilizing the virtual interband excitations in narrow bandgap semiconductors.

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