Hidden attractors in fundamental problems and engineering models. *

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Abstract. Recently a concept of self-excited and hidden attractors was suggested: an attractor is called a self-excited attractor if its basin of attraction overlaps with neighborhood of an equilibrium, otherwise it is called a hidden attractor. For example, hidden attractors are attractors in systems with no equilibria or with only one stable equilibrium (a special case of multistability and coexistence of attractors). While coexisting self-excited attractors can be found using the standard computational procedure, there is no standard way of predicting the existence or coexistence of hidden attractors in a system. In this plenary survey lecture the concept of self-excited and hidden attractors is discussed, and various corresponding examples of self-excited and hidden attractors are considered. The material is mostly based on surveys [1–4].

Keywords: hidden attractor, self-excited attractor, hidden oscillation, system with no equilibria, system without equilibria, multistability, coexistence of attractors, 16th Hilbert problem, Aizerman and Kalman conjectures, aircraft control system, phase-locked loop, Chua circuit

1 Analytical-numerical study of oscillations

An oscillation dynamical system can generally be easily numerically localized if the initial data from its open neighborhood in the phase space (with the exception of a minor set of points) lead to a long-term behavior that approaches the oscillation. Such an oscillation (or a set of oscillations) is called an attractor, and its attracting set is called the basin of attraction (i.e., a set of initial data for which the trajectories tend to the attractor).

When the theories of dynamical systems, oscillations, and chaos were first developed researchers mainly focused on analyzing equilibria stability, which can be easily done numerically or analytically, and on the birth of periodic or chaotic attractors from unstable equilibria. The structures of many physical dynamical

* International Conference on Advanced Engineering - Theory and Applications, 2015 (Ho Chi Minh City, Vietnam). [http://icaeta.org/] plenary lecture
systems are such that it is almost obvious that attractors exist because the trajectories can not tend to infinity and the oscillations are excited by an unstable equilibrium (see, e.g., the Rayleigh [5], Duffing [6], van der Pol [7], Tricomi [8], Beluosoizhbatinsky [9], and Lorenz [10] systems). This meant that scientists of that time could compute such attractors by constructing a solution using initial data from a small neighborhood of the equilibrium, observing how it is attracted and, thus, visualizes the attractor. In this standard computational procedure, computational methods and the engineering notion of a transient process were combined to study oscillations.

2 Self-excited and hidden attractors

From a computational perspective, it is natural to suggest the following classification of attractors, which is based on the connection of their basins of attraction with equilibria in the phase space:

**Definition 1.** An attractor is called a self-excited attractor if its basin of attraction intersects with any open neighborhood of a stationary state (an equilibrium), otherwise it is called a hidden attractor.

The first well-known example of a visualization of chaotic attractor in a dynamical system from the work of Lorenz [10] corresponds to the excitation of chaotic attractor from unstable equilibria. For classical parameters the Lorenz attractor is self-excited with respect to all three equilibria and could have been found using the standard computational procedure (see Fig. 1). Note that the chaotic attractor in the Lorenz system with other parameters may be self-excited with respect to zero unstable equilibrium only, and the possible existence in the Lorenz system of a hidden chaotic attractor is an open problem.

![Fig. 1. Numerical visualization of the classical self-excited chaotic attractor in the Lorenz system $\dot{x} = 10(y - x), \dot{y} = 28x - y - xz, \dot{z} = -8/3z + xy$. The attractor is self-excited respect to all three equilibria; it can be visualized by trajectories that start in small neighborhoods of any of unstable equilibria $S_{0,1,2}$. Here the separation of trajectory into transition process (green) and approximation of attractor (blue) is rough.](image-url)
At the same time in the generalized Lorenz system \( \dot{x} = -\sigma(x - y) - ayz, \dot{y} = rx - y - xz, \dot{z} = -bz + xy \) hidden chaotic attractors can be found [4, 13, 14] (see Fig. 2). For negative \( a < 0 \) the system corresponds to the Rabinovich system, which describes the interaction of plasma waves and was considered in 1978 [15, 16]; for positive \( a > 0 \) it corresponds to the Glukhovsky-Dolghansky system, which describes convective fluid motion and was considered in 1980 [17]; also it describes a rigid body rotation in a resisting medium and the forced motion of a gyrostat (see [18]).

![Fig. 2. Numerical visualization of hidden attractor (green trajectory) in the generalized Lorenz system. Outgoing separatrices of the zero saddle equilibrium are attracted to the stable equilibria \( S_{1,2} \) (blue trajectories).](image)

The basin of attraction for a hidden attractor is not connected with any equilibrium. For example, hidden attractors are attractors in systems with no equilibria or with only one stable equilibrium (a special case of the multistability and coexistence of attractors). Note that multistability can be undesired in various practical applications. At the same time the coexisting self-excited attractors in multistable systems (see, e.g. various examples of multistable engineering systems in famous book [19], and recent physical examples in [20]) can be found using the standard computational procedure, whereas there is no standard way of predicting the existence or coexistence of hidden attractors in a system.

For nonautonomous systems, depending on the physical problem statement, the notion of self-excited and hidden attractors can be introduced with respect to the stationary states of the system at the fixed initial time or the corresponding system without time-varying excitations. For example, one of the classical examples of self-excited chaotic attractors was numerically found by Ueda in 1961 [21] in a forced Duffing system \( \ddot{x} + 0.05\dot{x} + x^3 = 7.5\cos(t) \). To construct a self-excited chaotic attractor in this system (Fig. 3) it was used a transient process from the zero equilibrium of the unperturbed autonomous system (i.e., without \( \cos(t) \)) to an attractor in the forced system. If the discrete dynamics of system are considered on a Poincare section, then we can also use stationary or
periodic points on the section that corresponds to a periodic orbit of the system (the consideration of periodic orbits is also natural for discrete systems). Note that if the attracting domain is the whole state space, then the attractor can be visualized by any trajectory and the only difference between computations is the time of the transient process.

One of the first well-known problems of analyzing hidden periodic oscillations arose in connection with the second part of Hilbert’s 16th problem (1900) on the number and mutual disposition of limit cycles in two-dimensional polynomial systems (see, e.g. resent results on visualization of nested limit cycles in quadratic systems: \( \dot{x} = \alpha_1 x + \beta_1 y + a_1 x^2 + b_1 xy + c_1 y^2 \), \( \dot{y} = \alpha_2 x + \beta_2 y + a_2 x^2 + b_2 xy + c_2 y^2 \)).

Later, in the 1950s-1960s, the study of the well-known Aizerman’s and Kalman’s conjectures on absolute stability led to the discovery of the possible coexistence of a hidden periodic oscillation and a unique stable stationary point in automatic control systems. In 1957 R.E. Kalman stated the following: “If \( f(e) \) in Fig. 1 [see Fig. 4] is replaced by constants \( K \) corresponding to all possible values of \( f'(e) \), and it is found that the closed-loop system is stable for all such \( K \), then it is intuitively clear that the system must be monostable; i.e., all transient solutions will converge to a unique, stable critical point.” Kalman’s conjecture is a strengthening of Aizerman’s conjecture [27], which considers nonlinearities belonging to the sector of linear stability. Note that these conjectures are valid from the standpoint of simplified analysis such as the linearization, harmonic balance, and describing function methods (DFM), what explains why these conjectures were put forward. Nowadays various counterexamples to these conjectures (nonlinear systems where the only equilibrium, which is stable, coex-
ists with a hidden periodic oscillation) are known (see [28–35]; the corresponding discrete examples are considered in [36,37]).

Similar situation with linear stability and hidden oscillations occur in the analysis of aircrafts and launchers control systems with saturation [38,39]. In [40] the crash of aircraft YF-22 Boeing in April 1992 caused by the difficulties of rigorous analysis and design of nonlinear control systems with saturation, is discussed and the conclusion is made that “since stability in simulations does not imply stability of the physical control system (an example is the crash of the YF22), stronger theoretical understanding is required”.

Corresponding limitations, caused by hidden oscillations, appear in simulation of various phase-locked loop (PLL) based systems [2,41–48]. PLL was designed to synchronize the phases of local oscillator and reference oscillator signals. Next example shows that the use of default simulation parameters in MATLAB Simulink for the study of two-phase PLL in the presence of hidden oscillation (see Fig. 5) can lead to the conclusions concerning the stability of the loop and the pull-in (or capture) range [4]. In Fig. 6 the model in Fig. 5 sim-

\[
x' = Ax + Bu
\]
\[
y = Cx + Du
\]

Fig. 5. Model of two-phase PLL with lead-lag filter in MATLAB Simulink. Lead-lag loop filter with the transfer function \( H(s) = \frac{1 + s\tau_1 + s^2\tau_1\tau_2}{1 + s\tau_1 + s^2\tau_1\tau_2} \), \( \tau_1 = 0.0448 \), \( \tau_2 = 0.0185 \) and the corresponding parameters \( A = -\frac{1}{\tau_1 + \tau_2} \), \( B = 1 - \frac{\tau_2}{\tau_1 + \tau_2} \), \( C = \frac{1}{\tau_1 + \tau_2} \), \( D = \frac{\tau_2}{\tau_1 + \tau_2} \).

ulated with relative tolerance set to “1e-3” or smaller does not acquire lock (black color), but the model with default parameters (a relative tolerance set to “auto”) acquires lock (red color). The same problems are also observed in SIMet-

rics SPICE model [41,45,46]. From a mathematical point of view, the above case corresponds to the existence of semistable periodic trajectory or co-existence of unstable and stable periodic trajectories (here the stable one is a hidden oscillation) [2,41,50,51]. Therefore, if the gap between stable and unstable periodic trajectories is smaller than the discretization step, the numerical procedure may slip through the stable trajectory [45,47].

3 http://www.youtube.com/watch?v=M6sy-xfIhFQ
4 See discussion of rigorous definitions in [48,49].
Fig. 6. Simulation of two-phase PLL. Loop filter output $g(t)$ for the initial data $x_0 = 0.1318$ obtained for default “auto” relative tolerance (red) — acquires lock, relative tolerance set to “1e-3” (green) — does not acquire lock.

Attractor in the systems without equilibria are the hidden attractors, according to the above definition. Systems without equilibria and with hidden oscillations appear naturally in the study of various electromechanical models with rotation and electrical circuits with cylindrical phase space. One of the first examples is from paper [52], published in 1902, in which Sommerfeld analyzed the vibrations caused by a motor driving an unbalanced weight and discovered the so-called Sommerfeld effect (see, e.g., [53, 54]). Another well-known chaotic system with no equilibrium points is the Nose–Hoover oscillator [55–57]. An example of hidden chaotic attractor in electromechanical model with no equilibria was reported in a power system in 2001 [58]. Recent examples of hidden attractors in the systems without equilibria can be found, e.g. in [59–62, 62–67].

Fig. 7. Hidden chaotic attractor (green) in Chua circuit: $\dot{x} = \alpha(y - x - m_1 x - \psi(x)), \dot{y} = x - y + z, \dot{z} = -(\beta y + \gamma z), \psi(x) = (m_0 - m_1)\text{sat}(x)$. Locally the stable zero equilibrium $F_0$ attracts trajectories (black) from stable manifolds $M_1^{st}$ of two saddle points $S_{1,2}$; trajectories (red) from the unstable manifolds $M_1^{unst}$ tend to infinity; $\alpha = 8.4562, \beta = 12.0732, \gamma = 0.0052, m_0 = -0.1768, m_1 = -1.1468.$
After the concepts of hidden chaotic attractors was introduced first in connection with discovery of the first hidden Chua attractors (see Fig. [7] [11][12][68][74], the hidden chaotic attractors have received much attention. Recent examples of hidden attractors can be found in [59][61][63][75][103][103][112].

See also The European Physical Journal Special Topics: Multistability: Uncovering Hidden Attractors, 2015 (see [113][124]).

3 Acknowledgments

This work was supported by Russian Scientific Foundation (project 14-21-00041, sec.2) and Saint-Petersburg State University (6.38.505.2014, sec.1).

References

1. Leonov, G.A., Kuznetsov, N.V.: Analytical-numerical methods for investigation of hidden oscillations in nonlinear control systems. IFAC Proceedings Volumes (IFAC-PapersOnline) 18(1) (2011) 2494–2505
2. Leonov, G.A., Kuznetsov, N.V.: Hidden attractors in dynamical systems. From hidden oscillations in Hilbert-Kolmogorov, Aizerman, and Kalman problems to hidden chaotic attractors in Chua circuits. International Journal of Bifurcation and Chaos 23(1) (2013) art. no. 1330002.
3. Kuznetsov, N., Leonov, G.: Hidden attractors in dynamical systems: systems with no equilibria, multistability and coexisting attractors. IFAC Proceedings Volumes (IFAC-PapersOnline) 19 (2014) 5445–5454
4. Leonov, G., Kuznetsov, N., Mokaev, T.: Homoclinic orbits, and self-excited and hidden attractors in a Lorenz-like system describing convective fluid motion. Eur. Phys. J. Special Topics 224(8) (2015) 1421–1458
5. Rayleigh, J.W.S.: The theory of sound. Macmillan, London (1877)
6. Duffing, G.: Erzwungene Schwingungen bei Veranderlicher Eigenfrequenz. F. Vieweg u. Sohn, Braunschweig (1918)
7. Van der Pol, B.: On relaxation-oscillations. Philosophical Magazine and Journal of Science 7(2) (1926) 978–992
8. Tricomi, F.: Integrazione di unequazione differenziale presentatasii in elettrotechnica. Annali della R. Scuola Normale Superiore di Pisa 2(2) (1933) 1–20
9. Belousov, B.P.: A periodic reaction and its mechanism. In: Collection of short papers on radiation medicine for 1958. Moscow: Med. Publ. (in Russian) (1959)
10. Lorenz, E.N.: Deterministic nonperiodic flow. J. Atmos. Sci. 20(2) (1963) 130–141
11. Leonov, G.A., Kuznetsov, N.V., Vagaitsev, V.I.: Localization of hidden Chua’s attractors. Physics Letters A 375(23) (2011) 2230–2233
12. Leonov, G.A., Kuznetsov, N.V., Vagaitsev, V.I.: Hidden attractor in smooth Chua systems. Physica D: Nonlinear Phenomena 241(18) (2012) 1482–1486
13. Leonov, G., Kuznetsov, N., Mokaev, T.: Homoclinic orbit and hidden attractor in the Lorenz-like system describing the fluid convection motion in the rotating cavity. Communications in Nonlinear Science and Numerical Simulation 28(doi:10.1016/j.cnsns.2015.04.007) (2015) 166–174
14. Kuznetsov, N., Leonov, G.A., Mokaev, T.N.: Hidden attractor in the Rabinovich system. arXiv:1504.04723v1 (2015) http://arxiv.org/pdf/1504.04723v1.pdf
15. Rabinovich, M.I.: Stochastic autooscillations and turbulence. Uspehi Physicheskikh
   125(1) (1978) 123–168
16. Pikovski, A.S., Rabinovich, M.I., Trakhtengerts, V.Y.: Onset of stochasticity in
decay confinement of parametric instability. Sov. Phys. JETP 47 (1978) 715–719
17. Glukhovskii, A.B., Dolzhanskii, F.V.: Three-component geostrophic model of
convection in a rotating fluid. Academy of Sciences, USSR, Izvestiya, Atmospheric
and Oceanic Physics 16 (1980) 311–318
18. Leonov, G.A., Boichenko, V.A.: Lyapunov’s direct method in the estimation of
the Hausdorff dimension of attractors. Acta Applicandae Mathematicae 26(1)
(1992) 1–60
19. Andronov, A.A., Vitt, E.A., Khaikin, S.E.: Theory of Oscillators (in Russian).
ONTI NKTP SSSR (1937) (English transl. 1966, Pergamon Press).
20. Pisarchik, A., Feudel, U.: Control of multistability. Physics Reports 540(4) (2014)
   167-218
21. Ueda, Y., Akamatsu, N., Hayashi, C.: Computer simulations and non-periodic
   oscillations. Trans. IEICE Japan 56A(4) (1973) 218–255
22. Hilbert, D.: Mathematical problems. Bull. Amer. Math. Soc. (8) (1901-1902)
   437–479
23. Leonov, G.A., Kuznetsova, O.A.: Lyapunov quantities and limit cycles of twodimen-
   sional dynamical systems. Analytical methods and symbolic computation.
   Regular and Chaotic Dynamics 15(2-3) (2010) 354–377
24. Leonov, G.A., Kuznetsov, N.V., Kuznetsova, O.A., Seledzhi, S.M., Vagaitsev, V.I.:
   Hidden oscillations in dynamical systems. Transaction on Systems and Control
   6(2) (2011) 54–67
25. Kuznetsov, N.V., Kuznetsova, O.A., Leonov, G.A.: Visualization of four normal
   size limit cycles in two-dimensional polynomial quadratic system. Differential
   equations and dynamical systems 21(1-2) (2013) 29–34
26. Kalman, R.E.: Physical and mathematical mechanisms of instability in nonlinear
   automatic control systems. Transactions of ASME 79(3) (1957) 553–566
27. Aizerman, M.A.: On a problem concerning the stability in the large of dynamical
   systems. Uspekhi Mat. Nauk (in Russian) 4 (1949) 187–188
28. Pliss, V.A.: Some Problems in the Theory of the Stability of Motion. Izd LGU,
   Leningrad (in Russian) (1958)
29. Fitts, R.E.: Two counterexamples to Aizerman's conjecture. Trans. IEEE AC-
   11(3) (1966) 553–556
30. Barabanov, N.E.: On the Kalman problem. Sib. Math. J. 29(3) (1988) 333–341
31. Bernat, J., Llibre, J.: Counterexample to Kalman and Markus-Yamabe conjectures
   in dimension larger than 3. Dynamics of Continuous, Discrete and Impulsive
   Systems 2(3) (1996) 337–379
32. Leonov, G.A., Bragin, V.O., Kuznetsov, N.V.: Algorithm for constructing coun-
   terexamples to the Kalman problem. Doklady Mathematics 82(1) (2010) 540–542
33. Bragin, V.O., Vagaitsev, V.I., Kuznetsov, N.V., Leonov, G.A.: Algorithms for
   finding hidden oscillations in nonlinear systems. The Aizerman and Kalman con-
   jectures and Chua's circuits. Journal of Computer and Systems Sciences Interna-
   tional 50(4) (2011) 511–543
34. Leonov, G.A., Kuznetsov, N.V.: Algorithms for searching for hidden oscillations
   in the Aizerman and Kalman problems. Doklady Mathematics 84(1) (2011) 475–
   481
35. Kuznetsov, N.V., Leonov, G.A., Seledzhi, S.M.: Hidden oscillations in nonlinear
   control systems. IFAC Proceedings Volumes (IFAC-PapersOnline) 18(1) (2011)
   2506–2510
36. Alli-Oke, R., Carrasco, J., Heath, W., Lanzon, A.: A robust Kalman conjecture for first-order plants. In: Proc. IEEE Control and Decision Conference. (2012)
37. Heath, W.P., Carrasco, J., de la Sen, M.: Second-order counterexamples to the discrete-time Kalman conjecture. Automatica 60 (2015) 140 – 144
38. Andrievsky, B.R., Kuznetsov, N.V., Leonov, G.A., Pogromsky, A.Y.: Hidden oscillations in aircraft flight control system with input saturation. IFAC Proceedings Volumes (IFAC-PapersOnline) 5(1) (2013) 75–79
39. Andrievsky, B.R., Kuznetsov, N.V., Leonov, G.A., Seledzhi, S.M.: Hidden oscillations in stabilization system of flexible launcher with saturating actuators. IFAC Proceedings Volumes (IFAC-PapersOnline) 19(1) (2013) 37–41
40. Lauvdal, T., Murray, R., Fossen, T.: Stabilization of integrator chains in the presence of magnitude and rate saturations: a gain scheduling approach. In: Proc. IEEE Control and Decision Conference. Volume 4. (1997) 4404–4005
41. Kuznetsov, N., Leonov, G., Yuldashev, M., Yuldashev, R.: Nonlinear analysis of classical phase-locked loops in signal's phase space. IFAC Proceedings Volumes (IFAC-PapersOnline) 19 (2014) 8253–8258
42. Kuznetsov, N., Kuznetsova, O., Leonov, G., Neittaanmaki, P., Yuldashev, M., Yuldashev, R.: Simulation of nonlinear models of QPSK Costas loop in Matlab Simulink. In: 2014 6th International Congress on Ultra Modern Telecommunications and Control Systems and Workshops (ICUMT), Volume 2015-January., IEEE (2014) 66–71
43. Kuznetsov, N., Kuznetsova, O., Leonov, G., Seledzhi, S., Yuldashev, M., Yuldashev, R.: BPSK Costas loop: Simulation of nonlinear models in Matlab Simulink. In: 2014 6th International Congress on Ultra Modern Telecommunications and Control Systems and Workshops (ICUMT). Volume 2015-January., IEEE (2014) 83–87
44. Kudryasoha, E.V., Kuznetsova, O.A., Kuznetsov, N.V., Leonov, G.A., Seledzhi, S.M., Yuldashev, M.V., Yuldashev, R.V.: Nonlinear models of BPSK Costas loop. ICINCO 2014 - Proceedings of the 11th International Conference on Informatics in Control, Automation and Robotics I (2014) 704–710
45. Kuznetsov, N., Kuznetsova, O., Leonov, G., Neittaanmaki, P., Yuldashev, M., Yuldashev, R.: Limitations of the classical phase-locked loop analysis. In: International Symposium on Circuits and Systems (ISCAS), IEEE (2015) 533–536 [http://arxiv.org/pdf/1507.03468v1.pdf]
46. Best, R., Kuznetsov, N., Kuznetsova, O., Leonov, G., Yuldashev, M., Yuldashev, R.: A short survey on nonlinear models of the classic Costas loop: rigorous derivation and limitations of the classic analysis. In: Proceedings of the American Control Conference, IEEE (2015) 1296–1302 art. num. 7170912, [http://arxiv.org/pdf/1505.04288v1.pdf]
47. Bianchi, G., Kuznetsov, N., Leonov, G., Yuldashev, M., Yuldashev, R.: Limitations of PLL simulation: hidden oscillations in MATLAB and SPICE. In: 2015 7th International Conference on Ultra Modern Telecommunications and Control Systems and Workshops (ICUMT), (2015) 79–84 [http://arxiv.org/pdf/1506.02484.pdf] [http://www.mathworks.com/matlabcentral/fileexchange/52419-hidden-oscillations-in-pll]
48. Leonov, G., Kuznetsov, N., Yuldashev, M., Yuldashev, R.: Hold-in, pull-in, and lock-in ranges of PLL circuits: rigorous mathematical definitions and limitations of classical theory. IEEE Transactions on Circuits and Systems-I: Regular Papers 62(10) (2015) 2454–2464
49. Kuznetsov, N., Leonov, G., Yuldashev, M., Yuldashev, R.: Rigorous mathematical definitions of the hold-in and pull-in ranges for phase-locked loops. IFAC-PapersOnLine 48(11) (2015) 710 – 713

50. Shakhtarin, B.: Study of a piecewise-linear system of phase-locked frequency control. Radiotechnica and electronika (in Russian) (8) (1969) 1415–1424

51. Belyustina, L., Brykov, V., Kiveleva, K., Shalfeev, V.: On the magnitude of the locking band of a phase-shift automatic frequency control system with a proportionally integrating filter. Radiophysics and Quantum Electronics 13(4) (1970) 437–440

52. Sommerfeld, A.: Beiträge zum dynamischen ausbau der festigkeitslehre. Zeitschrift des Vereins deutscher Ingenieure 46 (1902) 391–394

53. Blekhman, I., Indeitsev, D., Fradkov, A.: Slow motions in systems with inertially excited vibrations. IFAC Proceedings Volumes (IFAC-PapersOnline) 3(1) (2007) 126–131

54. Eckert, M.: Arnold Sommerfeld: Science, Life and Turbulent Times 1868-1951. Springer (2013)

55. Nose, S.: A molecular dynamics method for simulations in the canonical ensemble. Molecular Physics 52(2) (1984) 255–268

56. Hoover, W.: Canonical dynamics: Equilibrium phase-space distributions. Phys. Rev. A 31 (1985) 1695–1697

57. Sprott, J.: Some simple chaotic flows. Physical Review E 50(2) (1994) R647–R650

58. Venkatasubramanian, V.: Stable operation of a simple power system with no equilibrium points. In: Proceedings of the 40th IEEE Conference on Decision and Control. Volume 3. (2001) 2201–2203

59. Wei, Z., Wang, R., Liu, A.: A new finding of the existence of hidden hyperchaotic attractors with no equilibria. Mathematics and Computers in Simulation 100 (2014) 13–23

60. Pham, V.T., Jafari, S., Volos, C., Wang, X., Golpayegani, S.: Is that really hidden? The presence of complex fixed-points in chaotic flows with no equilibria. International Journal of Bifurcation and Chaos 24(11) (2014) art. num. 1450146.

61. Pham, V.T., Rahma, F., Frasca, M., Fortuna, L.: Dynamics and synchronization of a novel hyperchaotic system without equilibrium. International Journal of Bifurcation and Chaos 24(06) (2014) art. num. 1450087.

62. Pham, V.T., Volos, C., Gambuzza, L.: A memristive hyperchaotic system without equilibrium. The Scientific World Journal 2014(04) (2014) art. ID 368986.

63. Pham, V.T., Volos, C., Jafari, S., Wei, Z., Wang, X.: Constructing a novel no-equilibrium chaotic system. International Journal of Bifurcation and Chaos 24(05) (2014) 1450073.

64. Tahir, F., Jafari, S., Pham, V.T., Volos, C., Wang, X.: A novel no-equilibrium chaotic system with multiwing butterfly attractors. International Journal of Bifurcation and Chaos 25(04) (2015) 1550056

65. Vaidyanathan, S., Volos, C.K., Pham, V.: Analysis, control, synchronization and SPICE implementation of a novel 4-D hyperchaotic Rikitake dynamo system without equilibrium. Journal of Engineering Science and Technology Review 8(2) (2015) 232–244

66. Cafagna, D., Grassi, G.: Fractional-order systems without equilibria: The first example of hyperchaos and its application to synchronization. Chinese Physics B 24(8) (2015) 080502

67. Chen, G.: Chaotic systems with any number of equilibria and their hidden attractors (plenary lecture). In: 4th IFAC Con-
68. Leonov, G.A., Kuznetsov, N.V.: Localization of hidden oscillations in dynamical systems (plenary lecture). In: 4th International Scientific Conference on Physics and Control. (2009) http://www.math.spbu.ru/user/leonov/publications/2009-PhysCon-Leonov-plenary-hidden-oscillations.pdf.

69. Kuznetsov, N.V., Leonov, G.A., Vagaitsev, V.I.: Analytical-numerical method for attractor localization of generalized Chua’s system. IFAC Proceedings Volumes (IFAC-PapersOnline) 4(1) (2010) 29–33

70. Kuznetsov, N., Vagaitsev, V., Leonov, G., Seledzhi, S.: Localization of hidden attractors in smooth Chua’s systems. International Conference on Applied and Computational Mathematics (2011) 26–33

71. Kuznetsov, N.V., Kuznetsova, O.A., Leonov, G.A., Vagaytsev, V.I.: Hidden attractor in Chua’s circuits. ICINCO 2011 - Proceedings of the 8th International Conference on Informatics in Control, Automation and Robotics 1 (2011) 279–283

72. Leonov, G.A., Kuznetsov, N.V.: IWCFTA2012 Keynote speech I - Hidden attractors in dynamical systems: From hidden oscillation in Hilbert-Kolmogorov, Aizerman and Kalman problems to hidden chaotic attractor in Chua circuits. In: IEEE 2012 Fifth International Workshop on Chaos-Fractals Theories and Applications (IWCFTA). (2012) XV–XVII

73. Kuznetsov, N., Kuznetsova, O., Leonov, G., Vagaitsev, V.: Analytical-numerical localization of hidden attractor in electrical Chua’s circuit. Informatics in Control, Automation and Robotics, Lecture Notes in Electrical Engineering, Volume 174, Part 4 174(4) (2013) 149–158

74. Leonov, G., Kiseleva, M., Kuznetsov, N., Kuznetsova, O.: Discontinuous differential equations: comparison of solution definitions and localization of hidden Chua attractors. IFAC-PapersOnLine 48(11) (2015) 408–413

75. Kiseleva, M., Kuznetsov, N., Leonov, G., Neittaanmaki, P.: Hidden oscillations in drilling system actuated by induction motor. IFAC Proceedings Volumes (IFAC-PapersOnline) 5 (2013) 86–89

76. Kiseleva, M., Kondratyeva, N., Kuznetsov, N., Leonov, G., Solovyeva, E.: Hidden periodic oscillations in drilling system driven by induction motor. IFAC Proceedings Volumes (IFAC-PapersOnline) 19 (2014) 5872–5877

77. Leonov, G.A., Kuznetsov, N.V., Kiseleva, M.A., Solovyeva, E.P., Zaretskiy, A.M.: Hidden oscillations in mathematical model of drilling system actuated by induction motor with a wound rotor. Nonlinear Dynamics 77(1-2) (2014) 277–288

78. Leonov, G.A., Kuznetsov, N.V.: Hidden oscillations in dynamical systems. 16 Hilbert’s problem, Aizerman’s and Kalman’s conjectures, hidden attractors in Chua’s circuits. Journal of Mathematical Sciences 201(5) (2014) 645–662

79. Leonov, G.A., Kuznetsov, N.V.: Prediction of hidden oscillations existence in nonlinear dynamical systems: analytics and simulation. Advances in Intelligent Systems and Computing 210 AISC (2013) 5–13

80. Leonov, G.A., Kiseleva, M.A., Kuznetsov, N.V., Neittaanmäki, P.: Hidden oscillations in drilling systems: torsional vibrations. Journal of Applied Nonlinear Dynamics 2(1) (2013) 83–94

81. Kiseleva, M., Kondratyeva, N., Kuznetsov, N., Leonov, G.: Hidden oscillations in drilling systems with salient pole synchronous motor. In: IFAC-PapersOnLine. Volume 48., Elsevier (2015) 700–705

82. Sprott, J., Wang, X., Chen, G.: Coexistence of point, periodic and strange attractors. International Journal of Bifurcation and Chaos 23(5) (2013) art. no. 1350093.
83. Wang, X., Chen, G.: Constructing a chaotic system with any number of equilibria. Nonlinear Dynamics 71 (2013) 429–436
84. Zhusubaliyev, Z., Mosekilde, E.: Multistability and hidden attractors in a multilevel DC/DC converter. Mathematics and Computers in Simulation 109 (2015) 32–45
85. Wang, Z., Sun, W., Wei, Z., Zhang, S.: Dynamics and delayed feedback control for a 3d jerk system with hidden attractor. Nonlinear Dynamics (2015)
86. Sharma, P., Shrimali, M., Prasad, A., Kuznetsov, N., Leonov, G.: Controlling dynamics of hidden attractors. International Journal of Bifurcation and Chaos 25(04) (2015) art. num. 1550061.
87. Dang, X.Y., Li, C.B., Bao, B.C., Wu, H.G.: Complex transient dynamics of hidden attractors in a simple 4d system. Chin. Phys. B 24(5) (2015) art. num. 050503.
88. Kuznetsov, A., Kuznetsov, S., Mosekilde, E., Stankevich, N.: Co-existing hidden attractors in a radio-physical oscillator system. Journal of Physics A: Mathematical and Theoretical 48 (2015) 125101
89. Pham, V.T., Volos, C., Jafari, S., Wang, X., Vaidyanathan, S.: Hidden hyperchaotic attractor in a novel simple memristive neural network. Optoelectronics and Advanced Materials - Rapid Communications 8(11-12) (2014) 1157–1163
90. Li, C., Sprott, J.C.: Coexisting hidden attractors in a 4-D simplified Lorenz system. International Journal of Bifurcation and Chaos 24(03) (2014) art. num. 1450034.
91. Wei, Z., Moroz, I., Liu, A.: Degenerate Hopf bifurcations, hidden attractors and control in the extended Sprott E system with only one stable equilibrium. Turkish Journal of Mathematics 38(4) (2014) 672–687
92. Pham, V.T., Volos, C., Vaidyanathan, S., Le, T., Vu, V.: A memristor-based hyperchaotic system with hidden attractors: Dynamics, synchronization and circuitual emulating. Journal of Engineering Science and Technology Review 2 (2015) 205–214
93. Chen, M., Yu, J., Bao, B.C.: Finding hidden attractors in improved memristor-based Chua’s circuit. Electronics Letters 51 (2015) 462–464
94. Chen, M., Li, M., Yu, Q., Bao, B., Xu, Q., Wang, J.: Dynamics of self-excited attractors and hidden attractors in generalized memristor-based Chua’s circuit. Nonlinear Dynamics (2015) doi: 10.1007/s11071-015-1983-7.
95. Wei, Z., Zhang, W., Wang, Z., Yao, M.: Hidden attractors and dynamical behaviors in an extended Rikitake system. International Journal of Bifurcation and Chaos 25(02) (2015) art. num. 1550028
96. Burkin, I., Khien, N.: Analytical-numerical methods of finding hidden oscillations in multidimensional dynamical systems. Differential Equations 50(13) (2014) 1695–1717
97. Wei, Z., Zhang, W.: Hidden hyperchaotic attractors in a modified Lorenz-Stenflo system with only one stable equilibrium. International Journal of Bifurcation and Chaos 24(10) (2014) art. num. 1450127.
98. Li, Q., Zeng, H., Yang, X.S.: On hidden twin attractors and bifurcation in the Chua’s circuit. Nonlinear Dynamics 77(1-2) (2014) 255–266
99. Zhao, H., Lin, Y., Dai, Y.: Hidden attractors and dynamics of a general autonomous van der Pol-Duffing oscillator. International Journal of Bifurcation and Chaos 24(06) (2014) art. num. 1450080.
100. Lao, S.K., Shekofteh, Y., Jafari, S., Sprott, J.: Cost function based on Gaussian mixture model for parameter estimation of a chaotic circuit with a hidden attractor. International Journal of Bifurcation and Chaos 24(1) (2014) art. num. 1450010.
101. Chaudhuri, U., Prasad, A.: Complicated basins and the phenomenon of amplitude death in coupled hidden attractors. Physics Letters A 378(9) (2014) 713–718
102. Pham, V.T., Volos, C., Jafari, S., Wang, X.: Generating a novel hyperchaotic system out of equilibrium. Optoelectronics and advanced materials rapid communications 8(5-6) (2014) 535–539
103. Kingni, S., Jafari, S., Simo, H., Woafo, P.: Three-dimensional chaotic autonomous system with only one stable equilibrium: Analysis, circuit design, parameter estimation, control, synchronization and its fractional-order form. The European Physical Journal Plus 129(5) (2014)
104. Li, C., Sprott, J.: Chaotic flows with a single nonquadratic term. Physics Letters A 378(3) (2014) 178–183
105. Molaie, M., Jafari, S., Sprott, J., Golpayegani, S.: Simple chaotic flows with one stable equilibrium. International Journal of Bifurcation and Chaos 23(11) (2013) art. num. 1350188.
106. Jafari, S., Sprott, J.C., Pham, V.T., Golpayegani, S.M.R.H., Jafari, A.H.: A new cost function for parameter estimation of chaotic systems using return maps as fingerprints. International Journal of Bifurcation and Chaos 24(10) (2014) art. num. 1450134.
107. Jafari, S., Sprott, J.: Simple chaotic flows with a line equilibrium. Chaos, Solitons and Fractals 57(0) (2013) 79 – 84
108. Dudkowski, D., Prasad, A., Kapitaniak, T.: Perpetual points and hidden attractors in dynamical systems. Physics Letters A 379(40-41) (2015) 2591 – 2596
109. Wei, Z., Yu, P., Zhang, W., Yao, M.: Study of hidden attractors, multiple limit cycles from hopf bifurcation and boundedness of motion in the generalized hyper-chaotic rabinovich system. Nonlinear Dynamics (2015)
110. Zhusubaliyev, Z.T., Mosekilde, E., Rubanov, V.G., Nabokov, R.A.: Multistability and hidden attractors in a relay system with hysteresis. Physica D: Nonlinear Phenomena (0) (2015) –
111. Bao, B., Hu, F., Chen, M., Xu, Q., Yu, Y.: Self-excited and hidden attractors found simultaneously in a modified Chua’s circuit. International Journal of Bifurcation and Chaos 25(05) (2015) 1550075
112. Danca, M.F., Feckan, M., Kuznetsov, N., Chen, G.: Looking more closely to the Rabinovich-Fabrikant system. International Journal of Bifurcation and Chaos (2016) (accepted, http://arxiv.org/pdf/1509.09206v1.pdf).
113. Shahzad, M., Pham, V.T., Ahmad, M., Jafari, S., Hadaeghi, F.: Synchronization and circuit design of a chaotic system with coexisting hidden attractors. European Physical Journal: Special Topics 224(8) (2015) 1637–1652
114. Brezetskyi, S., Dudkowski, D., Kapitaniak, T.: Rare and hidden attractors in Van der Pol-Duffing oscillators. European Physical Journal: Special Topics 224(8) (2015) 1459–1467
115. Jafari, S., Sprott, J., Nazarimehr, F.: Recent new examples of hidden attractors. European Physical Journal: Special Topics 224(8) (2015) 1469–1476
116. Zhusubaliyev, Z., Mosekilde, E., Churilov, A., Medvedev, A.: Multistability and hidden attractors in an impulse Goodwin oscillator with time delay. European Physical Journal: Special Topics 224(8) (2015) 1519–1539
117. Saha, P., Saha, D., Ray, A., Chowdhury, A.: Memristive non-linear system and hidden attractor. European Physical Journal: Special Topics 224(8) (2015) 1563–1574
118. Semenov, V., Korneev, I., Arinushkin, P., Strelkova, G., Vadivasova, T., Anishchenko, V.: Numerical and experimental studies of attractors in memristor-
based Chua’s oscillator with a line of equilibria. Noise-induced effects. European Physical Journal: Special Topics 224(8) (2015) 1553–1561

119. Feng, Y., Wei, Z.: Delayed feedback control and bifurcation analysis of the generalized Sprott B system with hidden attractors. European Physical Journal: Special Topics 224(8) (2015) 1619–1636

120. Li, C., Hu, W., Sprott, J., Wang, X.: Multistability in symmetric chaotic systems. European Physical Journal: Special Topics 224(8) (2015) 1493–1506

121. Feng, Y., Pu, J., Wei, Z.: Switched generalized function projective synchronization of two hyperchaotic systems with hidden attractors. European Physical Journal: Special Topics 224(8) (2015) 1593–1604

122. Sprott, J.: Strange attractors with various equilibrium types. European Physical Journal: Special Topics 224(8) (2015) 1409–1419

123. Pham, V., Vaidyanathan, S., Volos, C., Jafari, S.: Hidden attractors in a chaotic system with an exponential nonlinear term. European Physical Journal: Special Topics 224(8) (2015) 1507–1517

124. Vaidyanathan, S., Pham, V.T., Volos, C.: A 5-D hyperchaotic Rikitake dynamo system with hidden attractors. European Physical Journal: Special Topics 224(8) (2015) 1575–1592