Scattering of Antiprotons by Nuclei (Atoms) in the Range of Low Energies $E \lesssim 10^5$ eV.

Mirror Reflection, Diffraction, and Channeling of Antiprotons in Crystals.

V.G. Baryshevsky

Research Institute for Nuclear Problems, Belarusian State University,
11 Bobruiskaya Str., Minsk 220030, Belarus

Abstract

Studying antiproton scattering by nuclei (atoms) in the range of low energies, we found out that the increase in the antiproton-nucleus scattering amplitude through Coulomb interaction provides the possibility to investigate spin-dependent processes accompanying the interaction of antiprotons with nuclei (polarized or unpolarized) by means of mirror reflection of antiprotons from the vacuum-matter boundary, diffraction and channeling (surface diffraction and channeling) of antiprotons in crystals.

Email address: bar@inp.bsu.by, v_baryshevsky@yahoo.com (V.G. Baryshevsky).
1 Introduction

The progress in development of the Facility for Low-Energy Antiproton and Ion Research (FLAIR) has spurred the rapid development of low-energy antiproton physics [1,2]. In particular, it was shown in [3] that Coulomb scattering leads to an increase in the real part of the amplitude of elastic scattering of antiprotons by nuclei as the energy of antiprotons decreases. As a result, the effective interaction energy between antiprotons and matter also rises with decreasing antiproton energy. This gives the possibility to observe antiproton spin rotation in a nuclear pseudo-magnetic field in matter with polarized nuclei. The observation of this phenomenon is not only of general physical interest but can also give information about the amplitude of coherent elastic zero-angle scattering in the range of low antiproton energies.

In this paper we show that the increase in the antiproton-nucleus scattering amplitude in the range of low energies through Coulomb interaction makes it possible to investigate many spin-dependent processes that accompany the interaction of antiprotons with nuclei (polarized or unpolarized) by means of mirror reflection of antiprotons from the vacuum-matter boundary, diffraction and channeling (surface diffraction and channeling [4]) of antiprotons in crystals.

2 Mirror Reflection of Antiprotons from the Surface of Matter

Before we start to consider mirror reflection of antiprotons from the surface formed by the vacuum-matter boundary, let us make some general remarks about mirror reflection of particles.
It is well known \[5,6\] that the mirror-reflection coefficient \( R \) is defined by the refractive index \( n \) of a wave in matter. For such particles as, say, neutrons (for light and \( \gamma \)-quanta, whose polarization is orthogonal to the mirror-reflection plane), the mirror-reflection coefficient has the form \[5,6\]

\[
R = |F|^2, \tag{1}
\]

\[
F = -\frac{\sqrt{n^2 - \cos^2 \varphi} - \sin \varphi}{\sqrt{n^2 - \cos^2 \varphi} + \sin \varphi}, \tag{2}
\]

where \( \varphi \) is the grazing angle (see Fig.1).

![Figure 1.](image)

The refractive index (of particles or \( \gamma \)-quanta) can be expressed as follows \[7,8\]:

\[
n^2 = 1 + \frac{4\pi \rho}{k^2} f(0), \tag{3}
\]

where \( \rho \) is the number of scatterers (nuclei, atoms) per cubic centimeter of matter and \( f(0) \) is the amplitude of zero-angle coherent elastic scattering. According to \[11-13\], the mirror-reflection coefficient is determined by the zero-angle scattering amplitude. But at the same time, the wave scattered in the mirror-reflection direction obviously makes a certain angle \( \vartheta \) relative to the incident direction. Thus, the amplitude of this wave is determined by the superposition of waves scattered at the angle \( \vartheta \), i.e., instead of \( f(0) \), the
amplitude $F$ is now determined by the amplitude $f(\vartheta)$ of coherent elastic scattering at the angle $\vartheta$.

Indeed, let us consider elastic scattering of a wave $e^{i\vec{k}\vec{r}} = e^{i\vec{k}_\perp \vec{r}_\perp} e^{ik_z z}$ by a set of scatterers located in the plane $z = z_0$. Here $\vec{k}_\perp$ is the wave vector’s component perpendicular to the $z$-axis (parallel to the plane where the scatterers are placed), $k_z$ is the component of the particle’s wave vector that is parallel to the $z$-axis. Upon summation of the spherical waves produced by the scatterers in the plane $z = z_0$, we obtain the following expression for the amplitude of a mirror-reflected wave $F_1$ [4]:

$$F_1 = \frac{2\pi i \rho'}{k_z} f(\vec{k}' - \vec{k}) e^{2ik_z z_0},$$

(4)

where $|\vec{k}'| = |\vec{k}|$, $\vec{k}'$ is the wave vector of the scattered particle, which has the components $\vec{k}'_\perp = \vec{k}_\perp'$ and $k'_z = -k_z$, and $\rho'$ is the density of scatterers in the considered plane (the number of scatters in cm$^2$ of the plane).

If the layer $[0, z]$ contains $m$ number of planes, then the amplitude of the wave reflected by these planes can be written in the form

$$F = \frac{2\pi i \rho'}{k_z} f(\vec{k}' - \vec{k}) \sum_m e^{2ik_z z_m},$$

(5)

Passing to a continuous distribution of planes in the layer $[0, z]$, i.e., replacing the summation in (5) by integration, we finally obtain [4]

$$F = -\frac{\pi \rho}{k_z^2} f(\vec{k}' - \vec{k}) = -\frac{4\pi \rho}{|\vec{k}' - \vec{k}|^2} f(\vec{k}' - \vec{k}).$$

(6)

According to (6), microscopic summation of waves scattered at the vacuum-matter boundary yields the formation of a wave reflected in the direction determined by the laws of classical optics. Against (1), its amplitude is defined
not by the amplitude $f(0)$, but by the amplitude of scattering at a nonzero angle $\vartheta$ equal to a double grazing angle $\varphi$, i.e., $\vartheta = 2\varphi$ [4]. Equation (4) is valid for thermal and ultracold neutrons because of prevailing isotropic S-scattering by nuclei ($f(\vartheta) = f(0)$). It is also valid for photons, because at wavelengths much greater than the size of the atom, dipole scattering, which is also isotropic under the considered polarization, occurs.

According to (5), the amplitude $F$ of mirror reflection rises with decreasing $k_z = k \sin \varphi$, i.e., with decreasing grazing angle $\varphi$. When $F$ reaches the values close to unity, equation (6) for $F$ is no longer valid. Mirror reflection for this range of angles is considered, e.g., in [4].

In view of (6), the amplitude $F$ is close to unity in the range where the grazing angles $\varphi$ satisfy the condition

$$\sin^2 \varphi \sim \frac{\pi \rho |f|}{k^2}.$$  
(7)

If in estimating the angle $\varphi$ we use a typical value of the amplitude of scattering via nuclear interaction, $|f| \sim 10^{-12}$ cm, then (7) readily yields that for antiproton energies from 100 to 1000 eV, the value of $F$ can be close to unity only when the grazing angles are very small: $\varphi \leq 10^{-4} \div 10^{-5}$. Further in our consideration we shall demonstrate that owing to the interference of Coulomb and nuclear interactions and the increase in the amplitude of nuclear scattering of antiprotons with decreasing energy [3], the coefficient of mirror reflection for antiprotons becomes noticeable even at much larger grazing angles $\varphi$, making it possible to use the phenomenon of mirror reflection to investigate scattering of slow antiprotons by nuclei.
According to (6), the mirror-reflection coefficient $R$ can be written as follows:

$$R = |F|^2 = \left| \frac{\pi \rho}{k_z^2} f(\vec{k}' - \vec{k}) \right|^2 = \frac{\pi^2 \rho^2}{k^4 \sin^4 \varphi} |f(\vec{k}' - \vec{k})|^2. \quad (8)$$

There are two interactions responsible for antiproton scattering by nuclei (atoms): Coulomb and nuclear (here we neglect the magnetic interaction of antiproton and electron spins). Consequently, the scattering amplitude $f$ can be presented as a sum of two amplitudes:

$$f = f_{\text{Coul}} + f_N, \quad (9)$$

where $f_{\text{Coul}}$ is the amplitude of a purely Coulomb scattering and $f_N$ is the amplitude related to nuclear interaction (it contains the contribution from Coulomb interaction that affects nuclear scattering [3]). Let us note that the contribution to the formation of a mirror-reflected wave comes from elastic scattering, in which the state of the target does not change. As is known, the scattering amplitude in this case can be presented as a product of the amplitude of elastic scattering by an infinite-mass nucleus (atom) (the reduced mass equals the mass of the incident particle) into the Debye-Waller factor $e^{-w(\vec{k}' - \vec{k})}$ describing the effect produced by thermal oscillations of nuclei (atoms) in matter on the process of scattering [5]. In subsequent consideration, by the amplitude $f$ we shall mean scattering by an infinite-mass nucleus. As a result, we have ($\varphi \ll 1$)

$$R = \frac{\pi^2 \rho^2}{k^4 \varphi^4} \left( |f_{\text{Coul}}(\vec{k}' - \vec{k})|^2 + 2 \text{Re} f_{\text{Coul}}(\vec{k}' - \vec{k}) f_N^* + |f_N|^2 \right) e^{-2w(\vec{k}' - \vec{k})}. \quad (10)$$

For further consideration we need to compare the amplitudes of Coulomb and nuclear elastic scattering of antiprotons by nuclei (atoms).
3 The Amplitude of Antiproton Scattering by a Nucleus (Atom) at Low Temperatures

The scattering amplitude relates to the $T$-matrix as \[7,8\]

\[
f_{ba} = -\frac{m}{2\pi\hbar^2} \langle \Phi_b | T | \Phi_a \rangle, \tag{11}
\]

where $|\Phi_{a(b)}\rangle$ is the wave function describing the initial (final) state of the system "incident particle–atom (nucleus)". The wave functions $|\Phi_a\rangle$ are the eigenfunctions of the Hamiltonian $H_0 = H_p(\vec{r}_p) + H_A(\vec{\xi}, \vec{r}_{\text{nuc}})$, i.e., $H_0 |\Phi_a\rangle = E_a |\Phi_a\rangle$; $H_p(\vec{r}_p)$ is the Hamiltonian of the particle incident onto the target; $\vec{r}_p$ is the particle coordinate; $H_A(\vec{\xi}, \vec{r}_{\text{nuc}})$ is the atomic (nuclear) Hamiltonian; $\vec{\xi}$ is the set of coordinates of the atomic electron; $\vec{r}_{\text{nuc}}$ is the set of coordinates describing the atomic nuclei.

The Hamiltonian $H$ describing the particle–nucleus interaction can be written as:

\[
H = H_0 + V_{\text{Coul}}(\vec{r}_p, \vec{\xi}, \vec{r}_{\text{nuc}}) + V_{\text{nuc}}(\vec{r}_p, \vec{r}_{\text{nuc}}), \tag{12}
\]

where $V_{\text{Coul}}$ is the energy of Coulomb interaction between the particle and the atom, and $V_{\text{nuc}}$ is the energy of nuclear interaction between the particle and the atomic nucleus.

According to the quantum theory of reactions \[7,8\], in the case of two interactions, the matrix element of the operator $T$, which describes the system’s transition from the initial state $|\Phi_a\rangle$ into the final state $|\Phi_b\rangle$, can be presented
as a sum of two terms:

\[ T_{ba} = T_{ba}^{\text{Coul}} + T_{ba}^{\text{N}} = \langle \Phi_b | T_{\text{Coul}} | \Phi_a \rangle + \langle \varphi_b^{(-)} | T_{\text{N}} | \varphi_a^{(+)} \rangle , \]  

(13)

where the first term, \( T_{ba}^{\text{Coul}} \), describes the contribution to the T-matrix that comes from the Coulomb scattering alone, the operator

\[ T_{\text{Coul}} = V_{\text{Coul}} + V_{\text{Coul}} (E_a - H_0 + i\varepsilon)^{-1} T_{\text{Coul}} , \]  

(14)

and the second term describes the contribution to the T-matrix that comes from nuclear scattering and accounts for the distortion of waves incident onto the nucleus, \( \varphi^{(\pm)} \), which is caused by the Coulomb interaction. The operator

\[
T_{\text{N}} = V_{\text{nuc}} + V_{\text{nuc}} (E_a - H_0 - V_{\text{Coul}} + i\varepsilon)^{-1} T_{\text{N}} \\
= V_{\text{nuc}} + V_{\text{nuc}} (E_a - H_0 - V_{\text{Coul}} - V_{\text{nuc}} + i\varepsilon)^{-1} V_{\text{nuc}} ,
\]  

(15)

and the wave functions \( \varphi_a^{(\pm)} \) describe the interaction between particles and atoms via the Coulomb interaction alone \((V_{\text{nuc}} = 0)\) [7,8,9]:

\[ \varphi_a^{(\pm)} = \Phi_a + (E_a - H_0 \pm i\varepsilon)^{-1} V_{\text{Coul}} \varphi_a^{(\pm)} , \]  

(16)

the wave function \( \varphi_a^{(+)} \) at large distances has the asymptotics of a diverging spherical wave, and the wave function \( \varphi_a^{(-)} \) at large distances has the asymptotics of a converging spherical wave [7,8,9].

Let us give a more detailed consideration of the matrix element \( \langle \varphi_b^{(-)} | T_{\text{N}} | \varphi_a^{(+)} \rangle \).

Because nuclear forces are short–range, the radius of the domain of integration in this matrix element is of the order of the nuclear radius (of the order of the radius of action of nuclear forces in the case of the proton). The Coulomb interaction, \( V_{\text{Coul}} \), in this domain is noticeably smaller than the energy of
nuclear interaction, $V_{\text{nuc}}$. We can therefore neglect the Coulomb energy in the first approximation in the denominator of \((15)\), as compared to $V_{\text{nuc}}$.

As a result, the operator $T_N$ is reduced to the operator describing a purely nuclear interaction between the incident particle and the nucleus. The effect of Coulomb forces on nuclear interaction is described by wave functions $\varphi_{ba}^{(\pm)}$ (distorted-wave approximation [8]).

In the range of antiproton energies of hundreds of kiloelectronvolts and less, the de Broglie wavelength for antiprotons is larger than the nuclear radius. Therefore, in \((13)\) for $T_{ba}^N$, one can remove the wave functions $\varphi_{a(b)}^{(\pm)}$ outside the sign of integration over the coordinate of the antiproton center of mass, $\vec{R}_p$, at the location point of the nuclear center of mass, $\vec{R}_{\text{nuc}}$. As a result, one may write the following relationship [3]:

$$
T_{ba}^N = g_{ba} T_{ba}^{\text{nuc}} = \langle \varphi_b^{(-)}(\vec{R}_p = \vec{R}_{\text{nuc}}) | \varphi_a^{(+)}(\vec{R}_p = \vec{R}_{\text{nuc}}) \rangle T_{ba}^{\text{nuc}},
$$

(17)

where $T_{ba}^{\text{nuc}}$ is the matrix element describing a purely nuclear interaction (in the absence of Coulomb interaction) between the incident particle and the nucleus. The factor $g_{ba} = \langle \varphi_b^{(-)}(\vec{R}_p = \vec{R}_{\text{nuc}}) | \varphi_a^{(+)}(\vec{R}_p = \vec{R}_{\text{nuc}}) \rangle$ appearing in Eq. \((17)\) defines the probability to find the antiproton (the negative hyperon, e.g. $\Omega^-$, $\Sigma^-$) at the location of the nucleus.

From \((11)\), one can derive the following expression for scattering amplitude:

$$
f_{ba}^{N} = -\frac{m}{2\pi\hbar} g_{ba} T_{ba}^{\text{nuc}} = g_{ba} f_{ba}^{\text{nuc}},
$$

(18)

where $f_{ba}^{\text{nuc}}$ is the amplitude of particle scattering by the nucleus in the absence of Coulomb interaction.
Thus the Coulomb interaction leads to a change in the value of the amplitude of antiproton-nucleus scattering. Let us estimate the magnitude of this change.

In what follows we shall be primarily concerned with elastic scattering. In this case $|\vec{k}| = |\vec{k'}|$, and it follows that from the expression given in [9] for the wave functions $\varphi_b^{(-)}$ and $\varphi_a^{(+)}$, which describe particle scattering in the Coulomb field, we can derive the below relationships for $g_{ba}$:

- for the case of repulsion, i.e., elastic scattering of similarly charged particles
  \[ g_{ba}^{\text{rep}} = \frac{2\pi}{\kappa(e^{2\pi\kappa} - 1)}, \quad \kappa = \frac{v}{Z\alpha c}, \]  
  \[ g_{ba}^{\text{rep}} = \frac{2\pi\alpha Zc}{v} e^{-\frac{2\pi\alpha Zc}{v}}, \quad g_{ba}^{\text{att}} = \frac{2\pi\alpha Zc}{v}. \]  

  With decreasing particle energy (velocity), $\kappa$ diminishes, and for such values of $\kappa$ when $\frac{2\pi}{\kappa} \geq 1$, one can write
  \[ g_{ba}^{\text{rep}} = \frac{2\pi\alpha Zc}{v} e^{-\frac{2\pi\alpha Zc}{v}}, \quad g_{ba}^{\text{att}} = \frac{2\pi\alpha Zc}{v}. \]

According to [18], with decreasing energy of positively charged particles, the amplitude $f_{ba}^N$ diminishes rapidly because of repulsion. For negatively charged particles, the amplitude grows with decreasing particle energy (velocity).

These results for the amplitude $f_{ba}^N$ generalize a similar, well-known relationship for taking account of the Coulomb interaction effect on the cross section of inelastic processes, $\sigma_r$, [9].
So in the range of low energies, the amplitude \( f(\vec{k}' - \vec{k}) \) of antiproton (negative hyperon) scattering by a nucleus can be presented in the form (for antiproton scattering in ferromagnets, the magnetic interaction of antiprotons with electrons in atoms should also be considered):

\[
f(\vec{k}' - \vec{k}) = f_{\text{Coul}}(\vec{k}' - \vec{k}) + f_N(\vec{k}' - \vec{k}),
\]

where

\[
f_N(\vec{k}' - \vec{k}) = \frac{2\pi\alpha Z e}{v} f_{\text{nuc}}(\vec{k}' - \vec{k}).
\]

Using (23) and the expression for \( f_{\text{Coul}} \) in [9], we can obtain the following expression for the modulus of the Coulomb (Rutherford) scattering amplitude in the range of small scattering angles (\( \vartheta \ll 1, \vartheta > \frac{1}{kR} \)):

\[
|f_{\text{Coul}}(\vec{k}' - \vec{k})| = \frac{2Ze^2}{mv^2\vartheta^2} = \frac{Ze^2}{2mv^2\varphi^2}
\]

and then write the ratio for these amplitudes as

\[
\frac{|f_N|}{|f_{\text{Coul}}|} = 4\pi k\varphi^2|f_{\text{nuc}}|.
\]

As a result, using for the characteristic nuclear amplitude the estimate \(|f_{\text{nuc}}| \approx 10^{-12} \text{ cm}, k \leq 10^{10} \div 10^{11}, \varphi \sim 10^{-1}\), we can estimate the ratio \(|f_{\text{nuc}}|/|f_{\text{Coul}}|\) as

\[
|f_{\text{nuc}}|/|f_{\text{Coul}}| \leq 3 \cdot 10^{-4} \div 10^{-2}.
\]

In view of the above estimate, we can recast the expression for the coefficient of reflection as follows:

\[
R = R_{\text{Coul}} \left( 1 + 2 \frac{Re\beta \bar{f}_{\text{nuc}}}{|f_{\text{Coul}}|} \right) e^{-2\vartheta(\vec{k}' - \vec{k})},
\]
where $R_{\text{Coul}} = \frac{\pi^2 e^2}{2\kappa^2} \left| f_{\text{Coul}}(\vec{k}' - \vec{k}) \right|^2$ is the coefficient of mirror reflection due to a purely Coulomb interaction, $f_{\text{Coul}} = |f_{\text{Coul}}| e^{i\beta}$, and the term in (10) that is proportional to $\frac{|f_{\text{Coul}}|^2}{|f_{\text{Coul}}|}$ is dropped for its smallness. Let us recall that (26) holds true for such values of $R_{\text{Coul}}$ that are much less than unity.

Thus the coefficient of mirror reflection contains two contributions: one comes from a purely Coulomb interaction and the other is due to the Coulomb-nuclear interference. This makes it possible to obtain data about the amplitude of nuclear scattering of antiprotons by the nucleus from the experiments investigating angular and energy dependence of $R$ on the grazing angle and the energy of antiprotons.

4 Spin Polarization of Antiprotons Reflected from the Vacuum-Matter Boundary

As is well known, spin-orbit interaction during scattering causes the initially unpolarized particle beam to become polarized \cite{9}. In this case, the polarization vector of particles appears to be orthogonal to the scattering plane, i.e., the plane containing vectors $\vec{k}$ and $\vec{k}'$. If the particle beam had a nonzero polarization vector before the interaction, then a left-right asymmetry in the intensity of particle scattering is observed. For slow neutrons, the spin-orbit interaction is caused by the interaction $V_{so}$ of the neutron magnetic moment and the nuclear electric field \cite{10}

$$V_{so}^{\text{nuc}} = i\mu_n \hbar \frac{\vec{\sigma}}{mc} \vec{E} (\vec{r}) \nabla \vec{r},$$

(27)
where $\mu_n$ is the neutron magnetic moment, $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli spin matrices, $\vec{E}$ is the electric field at point $\vec{r}$ where the neutron is located, and $m$ is the neutron’s mass. First experiments to observe the effect of spin-orbit interaction on neutron scattering were performed by C.G. Shull. (For further experiments see, e.g., [11]).

The presence of a charge in antiprotons appreciably affects the dependence of spin-orbit interaction on their magnetic moment. The energy of spin-orbit interaction of antiprotons with nuclei is determined by the electric field and has the form [10]

$$V_{sp} = - \left( \mu' + \frac{e\hbar}{4mc} \right) \left( \vec{\sigma} \left[ \vec{E} \hat{p} \right] \right),$$

(28) where $\mu'$ is the anomalous part of the antiproton’s magnetic moment, $e = -|e|$ is the antiproton charge, $|e|$ is the electron charge, and $\hat{p} = -i\hbar \nabla$ is the momentum operator of the antiproton.

The energy dependence of the amplitude of spin-orbit scattering also changes noticeably through the interference of Coulomb and spin-orbit interactions. The energy dependence of the contribution coming from the antiproton-nucleus strong interaction to the amplitude of spin-orbit scattering changes, too.

The expression describing the amplitude of spin-orbit scattering in a general form reads:

$$F_{so} = F_{so} \vec{\sigma}[\vec{k'} \times \vec{k}] = F_{so}\vec{\sigma}[\vec{q} \times \vec{k}],$$

(29) where $\vec{q} = \vec{k'} - \vec{k}$ is the momentum transfer. It should be noted that the general form of (29) is clear even without calculations and follows from the symmetry
considerations, being valid for all spin particles.

Thus, the amplitude of mirror reflection from matter with unpolarized nuclei is a sum of three amplitudes: the amplitude of Coulomb scattering (or the amplitude of magnetic scattering by electrons in ferromagnets), the particle-spin-independent amplitude of nuclear scattering, and the amplitude of spin-orbit scattering. Hence, the coefficient of mirror reflection contains the contributions coming from these amplitudes and their interference. Let us consider the contribution coming to the coefficient of mirror reflection from the interference of Coulomb and spin-orbit interactions. Obviously, it is proportional to $\bar{\sigma}[\vec{k}' \times \vec{k}]$.

Let us choose the quantization axis to be parallel to the reflecting plane (the plane containing vectors $\vec{k}'$ and $\vec{k}$). It follows from (10) and (29) that due to Coulomb-nuclear interference, the reflection coefficients $R_{\uparrow\uparrow}$ and $R_{\downarrow\uparrow}$ for antiprotons with spins parallel and antiparallel to the direction of the axial vector $[\vec{k} \times \vec{k}']$ will differ. Let an unpolarized antiproton beam be incident on the target. Such beam is represented by a coherent sum of two beams with spins parallel and antiparallel to $[\vec{k} \times \vec{k}']$. A mirror-reflected beam appears partially polarized, and the degree to which the beam is polarized is determined by the difference $R_{\uparrow\uparrow} - R_{\downarrow\uparrow}$ of mirror-reflection coefficients:

$$p = \frac{R_{\uparrow\uparrow} - R_{\downarrow\uparrow}}{R_{\uparrow\uparrow} + R_{\downarrow\uparrow}}$$  \hspace{1cm} (30)

When estimating the magnitude of the effect, we should take into account that in the Born approximation, the amplitude $F_{so}$ is purely imaginary, while the amplitude $F_{Coul}$ is real, and so it is important that the imaginary part of $F_{Coul}$ be considered. It can be readily found, since we know that in the range of small scattering angles, $\text{Im}F_{Coul}$ can be set equal to $\text{Im}F_{Coul}(0)$ (Im$F_{Coul}(0)$
is the amplitude of Coulomb scattering by a nucleus (atom) at zero angle).

According to the optical theorem, \( \text{Im} F_{\text{Coul}}(0) = \frac{k}{4\pi} \sigma_{\text{tot}} \). The total cross section of scattering by a screened Coulomb potential, \( V_{\text{Coul}} = \frac{Z e^2}{r} e^{-\frac{r}{R_A}} \), can be written as

\[
\sigma = 16\pi \left( \frac{m Z e^2 R_A^2}{\hbar^2} \right)^2 \frac{1}{1 + \frac{8mE R_A^2}{\hbar^2}}, \tag{31}
\]

where \( E \) is the particle energy.

It follows from (31) that for antiproton energies greater than the characteristic energy \( E_A = \frac{\hbar^2}{4\pi m R_A^2} \approx 10^{-2} \text{ e V} \), the scattering cross section \( \sigma \sim \frac{1}{E} \sim \frac{1}{v} \). As a result, \( \text{Im} F_{\text{Coul}} \approx \text{const} \). The amplitude of spin-orbit interaction contains the term \( \frac{2\pi \alpha Z e}{v} \) [see (18), (23)] that increases the amplitude of spin-orbit scattering of antiprotons by nuclei (atoms), making it grow as \( \sim \frac{1}{v} \) in the range of small grazing angles (the momentum transfer in this case is required to be \( q > \frac{1}{R_A} \)). As a result, the contribution of spin-orbit scattering to the coefficient of mirror reflection can be estimated as

\[
R_{so} \sim \frac{\pi \rho p^2}{k_A^4 \phi^4} |\text{Im} f_{\text{Coul}}|^2 \frac{f_{so}}{\text{Im} f_{\text{Coul}}} \approx 10^{-1} \div 10^{-2}.
\]

Consequently, this process can be used to obtain polarized antiprotons in this low energy range.

Let us note here that when a polarized antiproton beam falls on the surface, the particles’ polarization vector rotates about the quantization axis, i.e., about the direction \( [\vec{k'} \times \vec{k}] \). The phenomenon described here also occurs during the diffraction reflection of antiprotons from a crystal’s surface and is caused by the interference between the spin-orbit amplitude and the real part of
the Coulomb amplitude in the case when crystal’s cells lack the center of symmetry. (Compare with a similar phenomenon for slow neutrons [4], in which case the effect occurs through the interference of the spin-independent part of nuclear scattering and the amplitude of spin-orbit scattering.)

Now, let us suppose that a particle beam is incident on the boundary between vacuum and matter with polarized nuclei. The elastic scattering amplitude \( \hat{f}(\vec{k}' - \vec{k}) \) in this case depends on spin orientations of the incident particle, \( \vec{S} \), and the nucleus, \( \vec{J} \), i.e., the scattering amplitude is the operator in the spin space of particle and nucleus. Investigating refraction and mirror reflection, we are interested in coherent elastic scattering, in which the nuclear spin state remains unchanged. The scattering amplitude \( \hat{f}_N(\vec{k}' - \vec{k}) \), describing such scattering, is obtained by averaging the total amplitude \( \hat{T} \) using the nuclear spin density matrix \( \hat{\rho}_J \): \( \hat{f}_N(\vec{k}' - \vec{k}) = \text{Tr} \hat{\rho}_J \hat{T}(\vec{k}' - \vec{k}) \). (The general expression for the amplitude \( \hat{T} \) of scattering of a particle with spin \( S = \frac{1}{2} \) by a nucleus with spin \( J = \frac{1}{2} \) is given, e.g., in [9].)

Consequently, in this case the contribution from nuclear scattering to the amplitude of a mirror-reflected wave can be written in the form:

\[
\hat{F}_{\text{pol}} = -\frac{\pi \rho}{k^2} \hat{f}_N(\vec{k}' - \vec{k})
\]  \hspace{1cm} (32)

As a result, the amplitude of a mirror-reflected wave can be presented in the form

\[
\hat{F}(\vec{k}' - \vec{k}) = F_{\text{Coul}}(\vec{k}' - \vec{k}) + \hat{F}_{\text{so}}(\vec{k}' - \vec{k}) + \hat{F}_{\text{pol}}(\vec{k}' - \vec{k}),
\]  \hspace{1cm} (33)

where \( F_{\text{Coul}} \) is the amplitude of the mirror-reflected wave that is due to Coulomb interaction, \( \hat{F}_{\text{so}}(\vec{k}' - \vec{k}) \) is the amplitude of the reflected wave that
is due to antiproton-nucleus (or antiproton-atom) spin-orbit interaction, and 
\( \hat{F}_{\text{pol}}(\vec{k}' - \vec{k}) \) is the amplitude of antiproton scattering by a polarized nucleus (except for those terms in the amplitude \( \hat{F}_{\text{pol}} \) that describe spin-orbit interactions).

Using (33), we can find the intensity and polarization of reflected particles. For example, the intensity of reflected particles is related to spin orientation of incident particles by the expression of the form

\[
I_{\text{ref}} = I_0 \text{Tr} F \rho_0 F^+ = I_0 \text{Tr} F^+ F \rho_0, \tag{34}
\]

where \( \rho_0 \) is the spin density matrix of the incident beam and \( I_0 \) is the beam’s intensity.

The polarization vector \( \vec{p} \) of mirror-reflected particles has the form

\[
\vec{p} = \frac{1}{I_{\text{ref}}} \text{Tr} \rho_0 F^+ \hat{\vec{S}} F, \tag{35}
\]

where \( \hat{\vec{S}} \) is the spin operator of particles; the spin of antiprotons equals \( 1/2 \), hence \( \hat{\vec{S}} = \frac{1}{2} \hat{\vec{\sigma}} \).

It follows from (33), (34), and (35) that \( I_{\text{ref}} \) and \( \vec{p} \) depend on the interference of the nuclear amplitude \( \hat{F}_{\text{pol}}(\vec{k}' - \vec{k}) \) and the Coulomb and spin-orbit amplitudes.

The amplitude \( \hat{f}_N \) that determines the reflection amplitude \( \hat{F}_{\text{pol}} \) has quite a complicated structure. For slow antiprotons scattered at the angle \( \vartheta \ll 1 \) (the grazing angle \( \varphi \ll 1 \), the amplitude \( \hat{f}_N \) coincides with a zero-angle scattering amplitude and has the form

\[
\hat{f}_N = A_0 + A_1(\vec{S} \hat{\vec{p}}) + A_2(\vec{S} \hat{\vec{e}})(\vec{e} \hat{\vec{p}}), \tag{36}
\]
where \( \vec{p}_t \) is the polarization vector of the target. Using (34), (35), and (36), we can find the intensity and polarization of reflected particles for each particular case. These expressions yield that when unpolarized antiprotons are incident on a polarized target, the reflected antiprotons appear to be polarized. If the incident antiproton beam is polarized, then the spin of the mirror-reflected beam undergoes rotation (the rotation angle is estimated at the order of \( 10^{-1} \div 10^{-2} \)), and the intensity of the reflected beam depends on the mutual orientation between the spin of incident particles and the target polarization. The degree of polarization that the initially unpolarized beam acquires through reflection has the order of magnitude about \( 10^{-1} \div 10^{-2} \).

5 Conclusion

The analysis performed in this paper shows that the effects described here can be used to obtain polarized beams of low-energy antiprotons from unpolarized beams of low-energy antiprotons and to study the polarization thereof. By investigating the magnitudes of arising polarization and the angle of spin rotation during mirror reflection, diffraction, or channeling (surface diffraction and channeling), we can experimentally measure the contributions coming from the amplitudes \( A_1 \) and \( A_2 \).

References

[1] E. Klempt, F. Bradamante, A. Martin, J.-M. Richard, Antinucleon-nucleon interaction at low energy: scattering and protonium, Phys. Rep., Vol. 368 (2002) pp. 119–316.
[2] E. Widmann, FLAIR, A next-generation facility for low-energy antiprotons, in: 8th International Conference on Nuclear Physics at Storage Rings, STORI11 October 9-14, 2011 Frascati (Rome) Italy, PoS(STORI11)035.

[3] V. G. Baryshevsky, Growth of nuclear spin precession frequency of antiprotons (negative hyperons) under deceleration in matter with polarized nuclei, Phys. Lett. B, Vol. 711, Issue 5 (2012) pp. 394–397.

[4] V.G. Baryshevsky, High-Energy Nuclear Optics of Polarized Particles, World Scientific, Singapore, 2012.

[5] I. Gurevich and V. Tarasov, Low-Energy Neutron Physics, North-Holland, Amsterdam, 1968.

[6] L. D. Landau, L. P. Pitaevskii and E.M. Lifshitz Electrodynamics of Continuous Media, 2nd ed. Vol. 8 (Course of Theoretical Physics) Butterworth-Heinemann, 1979.

[7] M. L. Goldberger and R. M. Watson, Collision Theory, Wiley, New York, 1984.

[8] A. S. Davydov, Quantum Mechanics, Pergamon Press, Oxford, 1965.

[9] L. D. Landau and E. M. Lifshitz, Quantum Mechanics: Non-Relativistic Theory, 3rd ed. Vol. 3 (Course of Theoretical Physics), Pergamon Press, 1977.

[10] V.B. Berestetskii, L.P. Pitaevskii and E.M. Lifshitz, Quantum Electrodynamics, 2nd ed. Vol. 4, (Course of Theoretical Physics), Butterworth-Heinemann, 1982.

[11] C.G. Shull, Neutron spin-neutron orbit interaction with slow neutrons Phys. Rev. Lett. Vol. 10 (1963) p. 297.