Primacy analysis of the system of Bulgarian cities

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Abstract

We study the primacy in the Bulgarian urban system. Two groups of cities are studied: (i) the whole Bulgaria city system that contains about 250 cities and is studied in the time interval between 2004 and 2011; and (ii) A system of 33 cities, studied over the time interval 1887 till 2010. For these cities the 1946 population was over 10 000 inhabitants. The notion of primacy in the two systems of cities is studied first from the global primacy index of Sheppard [¹]. Several (new) additional indices are introduced in order to compensate defects in the Sheppard index. Numerical illustrations are illuminating through the so called ”length ratio”.

Key words: city sizes, Zipf’s law, primacy, primacy indices

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1 Introduction

Nonlinearity [²−⁵] and complexity [⁶−⁹] are common features of a large number of systems studied in modern science [¹⁰−¹²]. Such systems are much investigated by nonlinear dynamics methods, and time series analysis [¹³−¹⁷]. In the last decade or so, these methods have been applied also to many social, economic, and financial systems [¹⁸−²⁰]. In many cases, researchers have detected the existence of power laws, for different characteristic quantities of these complex systems. Power laws are useful tools in studying complex systems because scaling relations may indicate that the system is controlled by a few rules that propagate across a wide range of scales [²¹,²²].

Below we analyze data from a specific nonlinear complex system where power laws can be observed: the city population system of a specific country. In the course of time, the cities in a country develop a hierarchy. An expression of this hierarchy is the city population size distribution that can be easily constructed for any urban system. Zipf [²³] suggested that a large number of observed city population size distributions could be approximated by a simple scaling (power) law \( N_r = N_1/r \), where \( N_r \) is the population of the \( r \)-th largest city. A more flexible equation, with two parameters, reads \( N_r = N_1/r^\beta \), is called the rank-size scaling law. Zipf suggested that the particular case \( \beta = 1 \) represents a desirable situation,
in which forces of concentration balance those of decentralization. Such a case is called the rank-size rule. The urban population size distribution of developed countries, like the USA, fits very well the rank-size rule over several decades \[24,25\].

In this paper we discuss the human population of Bulgaria. In Bulgaria exist about 250 cities and about 4000 villages. The human population of the country reached almost 9 million in 1985 but after this it has decreased steadily in the last 25 years reaching 7.3 million in 2011. Below, we examine two sets of urban population data. The first set is the yearly count of the population of whole Bulgarian cities from 2004 till 2011, as recorded by the National Statistical Institute of the Republic of Bulgaria (http : www.nsi.bg). The second data set is the yearly population count in 1887, 1910, 1934, 1946, 2000 and 2011 for the 33 Bulgarian cities which had a population over 10 000 citizens in 1946. The data for 1887, 1910, 1934, 1946 is taken from from \[26\] while the data from 2000 and 2010 are from the National Statistical Institute of Republic of Bulgaria.

2 Analysis of primacy

An important problem connected to the real city size distributions is the problem of primacy. It has been seen that, in a few cases, the city size distributions can be close to the rank-size relationship of Zipf. In most cases these distributions are primate distributions \[1\], i.e. when one or very few but very large cities (the capital and several other cities) predominate the distribution; convex distributions that correspond to presence of number of large cities; or distributions with some mix of primacy and convexity, leading to S-shape like or more complicated structure.

Measures of primacy can be of the kind

\[
P_{r(k)} = \frac{N_1}{\sum_{r=2}^{k} N_r}, k = 2, 3, \ldots \tag{1}
\]

Eq.(1) gives a numerical value for the primacy of the largest city with respect to the next \(k - 1\) cities if the cities are ordered by decreasing number of inhabitants. If a power law of the kind \(N_r = N_1/r^\beta\) is substituted into each of these measures, it is obvious that the corresponding index of primacy depends on \(\beta\), whence the rank-size relationships with different slopes will have different levels of primacy. Then, it will be not possible to discriminate between a country where a primate city dominates a city size distribution, which otherwise may have a low and fairly consistent negative slope, from a country exhibiting a rank-size relationship with steep slope \(\beta\). Sheppard \[1\] tried to avoid this puzzle by formulating a primacy index that is independent of \(\beta\), i.e., he defined

\[
P_{r_N} = \frac{1}{N-2} \sum_{r=1}^{N-2} \left[ \frac{\ln(N_r + 1) - \ln(N_r)}{\ln(N_{r+2}) - \ln(N_{r+1})} \right] \left[ \frac{\ln(r + 2) - \ln(r + 1)}{\ln(r + 1) - \ln(r)} \right] \tag{2}
\]
Figure 1: Evolution of the first 4 primacy indices for Sofia, the capital of Bulgaria, from 2004 till 2011. Remember that $P_{r(k)} = N_1/(\sum_{i=2}^{k} N_k)$, for $k = 2, 3, \ldots$, Eq. (1). Till 2006, the values of the primary indices increase; later, till 2009, the values of the indices decrease. Next, a relatively sharp increase is observed in 2010 and 2011.

The logics behind this index is as follows. Let us substitute here the power law rank-size relationship $N_r = N_1 r^{-\beta}$. The result is $P_{rN} = (1/(N-2)) \sum_{r=1}^{N-2} 1 = 1$. Thus, for a perfect power law rank-size relationship, the index $P_{rN}$ has a value of 1, irrespective of the slope of the relationship.

We have applied the Sheppard index, Eq. (2), to study the primacy (or "hierarchy") of Bulgarian cities in the years between 2004 and 2011. Figure 1 shows the changes in the first 4 primacy indices $P_{r(1)}, \ldots, P_{r(4)}$ for the largest city (and capital) of Bulgaria: Sofia. A decreasing of primacy is observed between 2006 and 2009. One reason for this is economic: the good economic development before the crisis (that appeared in Bulgaria in 2009). Because of favorable economic conditions, there was enough inflow of people to the second, third, and the fourth largest city, thereby decreasing the primacy of the capital, Sofia. However the subsequent economic crisis worsened the job perspectives in the above mentioned large cities which led to an increased inflow of people back to Sofia. This led to increasing the primacy of the capital in the last few years.

Figure 2 (a)-(c) shows the evolution of the population of the capital Sofia within the class of the 33 cities (with population exceeding 10 000 in 1946). It is seen that the primacy in 1887 was below 1. Note that Sofia was the capital but not the largest city in Bulgaria up to 1890. Figure 6 (d) shows how the advantage
to be a capital was favorable for the population increase. In several more words, in 1879, almost an year after creation of the Third Bulgarian state a new capital had to be selected. There were two contestant cities: Sofia and Veliko Tarnovo (the capital of the Second Bulgarian state). Sofia was selected to be the capital of Bulgaria (Sofia won by 1 vote over the old capital Veliko Tarnovo). At this time, the population of Sofia was about twice larger than the population of Veliko Tarnovo. The concentration process led to a situation in which the population of Sofia became 25 times larger than the population of Veliko Tarnovo. In the last 25 years, the total country population as well as the urban population have decreased but the population of these two cities has further increased: and the rate of increase of the Veliko Tarnovo population is larger that the rate of increasing rate of Sofia population. Thus in 2010 the population of Sofia is about 15 times larger than the population of Veliko Tarnovo. This is a strong evidence for the fact that the population growth of the Bulgarian cities is size dependent.

The primacy index of Sheppard contains a variance of two logarithms in the denominator. When two cities have almost the same number of citizens this variance can be very small thus leading to large value of the Sheppard index. Actually, this happened: when we analyzed primacy in the (large) system of about 250 Bulgarian cities; there were two cities for which number of citizens differs from each other by 1 only. In order to avoid such a kind of problems we propose to consider
two other local primacy measures where the variance of logarithms is present only in the numerator, as follows. Let the cities be ranked, in the each studied city system, according to the population, i.e. \( N_r \geq N_{r+1} \). The measures are:

\[
V_r = \frac{\ln(N_{r-1}) - \ln(N_r)}{\ln(r) - \ln(r-1)}
\]

and

\[
W_r = \frac{\ln(N_r) - \ln(N_{r+1})}{\ln(r+1) - \ln(r)} - \frac{\ln(N_{r+1}) - \ln(N_{r+2})}{\ln(r+2) - \ln(r+1)} \equiv V_{r+1} - V_{r+2}
\]

For the case of power law relationship \( N_r = N_1 r^{-\beta} \) (\( r = 1, \ldots, N \)) for each \( \beta \geq 0 \) the values of the measures \( V_i \) and \( W_i \) are as follows: \( V_r = \beta \) (\( r = 2, \ldots, N - 1 \)) and \( W_r = 0 \) (\( r = 2, \ldots, N - 2 \)).

Let us now discuss cases when deviations from the power law occur. Let us consider the \((\ln(r), \ln(N_r))\)-plane. Suppose first that \( \ln(N_{r-1}) \) is fund over the straight line formed between the points \( \ln(N_r) \) and \( \ln(N_{r+1}) \), a case of local primacy. In such a case, \( V_r > V_{r+1} \) and \( W_{r-1} > 0 \). However, if \( \ln(N_{r-1}) \) is below the straight line formed by the points \( \ln(N_r) \) and \( \ln(N_{r+1}) \), a case of local convexity, then \( V_r < V_{r+1} \) and \( W_{r-1} < 0 \). Of course, if \( \ln(N_{r-1}) \) lies on the straight line formed by the points \( \ln(N_r) \) and \( \ln(N_{r+1}) \), i.e. the power law case, then \( V_r = V_{r+1} \) and \( W_{r-1} = 0 \). Thus for the power law case \( V \) will form a straight line as a function of \( r \) and the deviation of \( V \) from a straight line will be a signal for deviation of city size distribution from a power law function.

Fig.3 shows the \( W \)-measures for the system of all Bulgarian cities in 2004 and 2011. If a single power law was present in the rank-size relationship then \( W_r = 0 \) and \( W_r \) would be a straight line as a function of \( r \). As easily observed, this is
Table 1: Length ratio $R_N$ of the $V$-measure line for different number $N$ of BG cities, ranking from $r = 1$ till $N$, from 2004 till 2011.

| year: | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 |
|-------|------|------|------|------|------|------|------|------|
| N:    |      |      |      |      |      |      |      |      |
| 50    | 2.1171 | 2.2677 | 2.1652 | 2.2441 | 2.3363 | 2.3823 | 2.4169 | 2.4442 |
| 100   | 2.3458 | 2.3564 | 2.2130 | 2.1390 | 2.2488 | 2.3856 | 2.2360 | 2.2755 |
| 150   | 2.3987 | 2.5607 | 2.3736 | 2.3122 | 2.4431 | 2.7221 | 2.4188 | 2.3798 |
| 240   | 3.2635 | 3.5600 | 2.9865 | 2.7529 | 3.2125 | 3.4156 | 3.2255 | 3.0221 |

not the case for the system of Bulgarian cities, since what is observed in the Fig. 3 is a mix of regions of local primacy and regions of local convexity. In order to characterize the deviation of the system of cities from a system with the same number of cities, but obeying a power law, one can e.g. measure the length of the curves corresponding to the $V$-measures. Indeed, let us consider $N$ cities. If the rank-size distribution of these cities is a single power law, then the $W$-measure of each 3 neighboring cities is equal to 0. For a system of $N$ cities, there will be $N - 2$ points in the $(r, W_r)$ plane with coordinates $(j, 0)$ where $j = 1, \ldots, N - 2$. These $N - 2$ points connect $N - 3$ segments of the $W$-curve and each segment has the same length 1. (Remember that we discuss the case when a power law holds for the distribution of cities populations). Then, the total length of the $W_r$ line in the $(r, W_r)$-plane is $L_\beta = N - 3$.

Let now consider the case when the distribution of the cities population does not behave according to a power law. Then, the $W_r$ curve is not a straight line (see Fig. 3 for an example); the length of such a curve is bigger than $L_\beta$. Next, let us define the length ratio

$$R_N = \frac{L_N}{L_\beta}$$

where $L_N$ is the length of the line associated with the corresponding $W_r$-index:

$$L_N = \sum_{r=1}^{N-3} \sqrt{1 + (W_{r+1} - W_r)^2}$$

The results for the length ratio $R_N$ for several classes of Bulgarian cities are shown in Table 1. For a given number of cities, the evolution of the deviation of the city size distribution from a power law$^1$ can be calculated. For example, for the 50 largest cities, $R_{50}$ increases steadily since 2006. This means that the populations of cities change in such a manner that the corresponding rank-size distribution deviates more and more from a single power law. The evolution with respect to $R_N$ of the 100 largest cities is even more interesting, since between 2006 and 2008 the distribution appears to be more like a single power law than for 2005.

$^1$ for a single power law $R_N = 1$
Finally, note that the length ratio $R_N$ can be generalized in order to investigate the distribution deviation from a power law for any sub-class of cities, e.g. ranking between $N_1$ and $N_2$. One can define, e.g.,

$$R_{N_1,N_2} = \frac{1}{L_\beta} \sum_{r=N_1}^{N_2-3} \sqrt{1 + (W_{r+1} - W_r)^2}$$

(7)

As concluding remark we note the following. In this paper, the city primacy has been investigated for the two so defined groups of Bulgarian cities on the basis of the conventional index of Sheppard. However, we have indicated that other measures should be useful and have given definitions and subsequent numerical results. In particular we have defined and discussed results obtained by the measures $V_r$, $W_r$, and $R_N$. These measures can be used to quantify the deviation of the rank-size distribution of a system of cities from a power-law rank-size relationship. These measures can be applied not only for a group of cities but also for any any group of objects that can be ranked on the basis of some quantitative characteristics.

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