Hall effect in ferromagnetic nanomagnets: magnetic field dependence as an evidence of inverse spin Hall effect contribution. Supplement

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AMBIGUITY FOR THE EVALUATION OF THE AHE AND ISHE CONTRIBUTIONS.

AMBIGUITY FOR THE EVALUATION OF THE SPIN POLARIZATION.

The Hall angle \( \alpha_{HE} \), can be describe (See Eq.1 of main text) as

\[
\alpha_{HE} = \alpha_{OHE} \cdot H + \alpha_{AHE} + \alpha_{ISHE} \cdot \frac{P_{S}^{(0)} + \frac{H}{H_S}}{1 + \frac{P_{S}^{(0)}}{H_S}} \quad (S1)
\]

The ambiguity is originated from the fact that the functional dependence (1) does not change when the set of three initial fitting parameters \( \left( \alpha_{AHE} \quad \alpha_{ISHE} \quad P_{S}^{(0)} \right) \) is changed to a new set \( \left( \alpha_{AHE}^{*} \quad \alpha_{ISHE}^{*} \quad P_{S}^{(0)*} \right) \), which is related to the initial set as

\[
\alpha_{ISHE}^{*} = \frac{1 - P_{S}^{(0)*}}{1 - P_{S}^{(0)} \frac{P_{S}^{(0)*}}{P_{S} \left( 1 - P_{S}^{(0)} \right)}} \quad (S2)
\]

\[
\alpha_{AHE}^{*} = \alpha_{AHE} + \alpha_{ISHE} \frac{P_{S} - P_{S}^{*}}{P_{S} \left( 1 - P_{S}^{*} \right)} \quad (S3)
\]

In order to prove this fact, a comparison of Eq.(S1.1) for two sets of parameters gives

\[
\alpha_{AHE} + \alpha_{ISHE} \frac{1 + x \frac{1}{P_{S}}}{1 + x} = \alpha_{AHE}^{*} + \alpha_{ISHE}^{*} \frac{1 + x \frac{1}{P_{S}^{*}}}{1 + x} \quad (S4)
\]

where \( x = \frac{H}{H_S} \). Eq.(S1.4) is simplified as

\[
\alpha_{AHE} (1 + x) + \alpha_{ISHE} \left( 1 + x \frac{1}{P_{S}} \right) = \alpha_{AHE}^{*} (1 + x) + \alpha_{ISHE}^{*} \left( 1 + x \frac{1}{P_{S}^{*}} \right) \quad (S5)
\]

Comparison of the coefficients at \( x^0 \) gives

\[
\alpha_{AHE} + \alpha_{ISHE} = \alpha_{AHE}^{*} + \alpha_{ISHE}^{*} \quad (S6)
\]

Comparison of the coefficients at \( x^1 \) gives

\[
\frac{\alpha_{AHE} + \alpha_{ISHE}}{P_{S}} = \frac{\alpha_{AHE}^{*} + \alpha_{ISHE}^{*}}{P_{S}^{*}} \quad (S7)
\]

The solution of Eqs. (S6-S7) gives Eqs. (S2-S3).

The parameter \( H_S \) is obtained from fitting unambiguously. The \( 1/H_S \) describes the effectiveness of spin alignment along magnetic field. From LL equation, the alignment is faster when the Gilbert constant is larger. Also, the effectiveness is larger when the spin relaxation is smaller. The 1st and 2nd derivatives are larger when \( H_S \) is smaller.

The ambiguity is originated from the fact that the functional dependence (1) does not change when the set of three initial fitting parameters \( \left( \alpha_{AHE} \quad \alpha_{ISHE} \quad P_{S}^{(0)} \right) \) is changed to a new set \( \left( \alpha_{AHE}^{*} \quad \alpha_{ISHE}^{*} \quad P_{S}^{(0)*} \right) \), which is related to the initial set as

\[
\alpha_{ISHE}^{*} = \frac{1 - P_{S}^{(0)*}}{1 - P_{S}^{(0)} \frac{P_{S}^{(0)*}}{P_{S} \left( 1 - P_{S}^{(0)} \right)}} \quad (S2)
\]

\[
\alpha_{AHE}^{*} = \alpha_{AHE} + \alpha_{ISHE} \frac{P_{S} - P_{S}^{*}}{P_{S} \left( 1 - P_{S}^{*} \right)} \quad (S3)
\]

In order to prove this fact, a comparison of Eq.(S1.1) for two sets of parameters gives

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\alpha_{AHE} + \alpha_{ISHE} \frac{1 + x \frac{1}{P_{S}}}{1 + x} = \alpha_{AHE}^{*} + \alpha_{ISHE}^{*} \frac{1 + x \frac{1}{P_{S}^{*}}}{1 + x} \quad (S4)
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where \( x = \frac{H}{H_S} \). Eq.(S1.4) is simplified as

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\alpha_{AHE} (1 + x) + \alpha_{ISHE} \left( 1 + x \frac{1}{P_{S}} \right) = \alpha_{AHE}^{*} (1 + x) + \alpha_{ISHE}^{*} \left( 1 + x \frac{1}{P_{S}^{*}} \right) \quad (S5)
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\alpha_{AHE} + \alpha_{ISHE} = \alpha_{AHE}^{*} + \alpha_{ISHE}^{*} \quad (S6)
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Comparison of the coefficients at \( x^1 \) gives

\[
\frac{\alpha_{AHE} + \alpha_{ISHE}}{P_{S}} = \frac{\alpha_{AHE}^{*} + \alpha_{ISHE}^{*}}{P_{S}^{*}} \quad (S7)
\]

The solution of Eqs. (S6-S7) gives Eqs. (S2-S3).

The important requirement for our measurements is the monodomain nature of the specimen which ensures stability of the localized moment in the external magnetic field. This stability is very important to exclude any possible contribution to the Hall effect due to a realignment of localized moments. In our measurements the existence of static domains can be clearly identified.

Figure 1a shows measured hysteresis loop of the Hall angle \( \alpha_{Hall} \). Figure 1b shows the same loop but for \( \alpha_{Hall} \) vs \( |H| \). It is noticeable that different parts of loop scan does not coincide with each other. Additionally, \( \partial \alpha_{Hall}/\partial H \) has is not monotonic and has a sharp sparks (Fig.S1c). Both features indicate the existence of static domains. During its movement, a domain wall constantly overcomes obstacles, which causes the sharp changes of the magnetization and therefore sparks in \( \partial \alpha_{Hall}/\partial H \). Movement of the domain wall also depends on the movement direction of domain wall and therefore the \( H \) scanning direction, which makes a slight difference of data for different scan parts of a hysteresis loop.

Figure 2 shows similar data for monodomain nanomagnet in absence of static domains. All parts of hysteresis loop nicely coincide with each other. There is no sharp sparks in \( \partial \alpha_{Hall}/\partial H \).
FIG. S1: (color online) Influence of static domain on the measurement in case of a large nanomagnet (3 µm x 3 µm). (a) Hysteresis loop (b) absolute value of the Hall angle $\alpha_{\text{Hall}}$. (c) Derivative $\partial \alpha_{\text{Hall}} / \partial H$. There are spark changes of the derivative in the regions of existence of static domains. In the regions of absence of static domains the change of derivative is monotonic. Different line color corresponds to different part of scan of the magnetic field.

FIG. S2: (color online) Measurement of a smaller nanomagnet (200nm x 200nm) in absence of static domain. (a) Hysteresis loop (b) absolute value of the Hall angle $\alpha_{\text{Hall}}$. (c) Derivative $\partial \alpha_{\text{Hall}} / \partial H$. All dependencies are monolithic without sparks. Data of all scan parts of hysteresis loop coincide with each other.

COMPARISON OF MEASUREMENTS OF DIFFERENT NANOMAGNETS AND MEASUREMENTS AT A DIFFERENT CURRENT

In order to prove that the magnetic moment of the studied ferromagnetic nanomagnets with a strong PMA is in the single-domain state and is not realigned when an external magnetic field is applied along it, we performed the measurement of HE in multiple devices at different current densities leading to a slight temperature change. A typical comparison for different devices and temperatures is given in Fig. S3 and Fig. S4, respectively. Note, the first and second derivatives remain nearly the same showing that the OHE and ISHE contributions are nearly the same in different samples and their temperature dependence is weak. This nearly-identical dependence of derivatives in different nanomagnets excludes the possibility of existence of any magnetic domains, because the movement of the domain wall should be individual and different for each nanomagnet due to different distributions of fabrication defects and edge irregularities and a slight difference in shape of the nanomagnets.

ADDITIONAL EXPERIMENTAL VERIFICATION OF A SINGLE-DOMAIN STATE OF STUDIED NANOMAGNETS

The size of nanomagnets was intentionally fabricated smaller than a minimum size of a static domain in FeB and FeCoB, which ensures the single-domain state of the nanomagnets. Additionally to three above-described methods, the absence of static magnetic domains was verified by two experimental methods.

In the first method, an external magnetic field $H_{||}$ is applied in plane (along the magnetic hard axis) and in-plane component of magnetization $M_{||}$ was measured. Figure S5 shows the measured dependence which is a perfect line. The linear dependence of $M_{||}$ vs. $H_{||}$ is only possible when the nanomagnet is in a single-domain state and the magnetic moments inclines towards
FIG. S3: (color online) Hall angle in similar devices (nanomagnets), which are fabricated in different parts of the same wafer.

FIG. S4: (color online) Hall angle measured in the same device at different current densities.

$H_{\|}$ coherently over whole nanomagnet. In the case of existence of static domains, the dependence is non-linear due to a complex movement of domain wall in a magnetic field $H_{\|}$.

In the second method, the size of nucleation domains was measured. For different nanomagnets, the size varies between 40 and 90 nm [1]. A nucleation domain is an unstable magnetic domain, which exists for a very short time (a few milliseconds) during magnetization switching. The very existence of the nucleation domain confirms that the sample is monodomain because such nucleation domain can exist only in a single domain nanomagnet in absence of static domains.

**FABRICATION PROCEDURE OF NANOMAGNETS AND DATA ON SPECIMENS CHARACTERISTICS**

The samples were fabricated on a Si/SiO$_2$ substrate by sputtering. The Ta is used to ensure a sufficient adhesion of metal layers and to smooth the film surface. A set of samples with Ta thickness between 2 and 10 nm were fabricates and studied. The Ta thickness should be thicker than 2 nm. Otherwise, the top surface of FeB is rough and there is no PMA. In order to minimize influence of OHE in Ta on the measurement, we keep the Ta thickness thinner than 10 nm.

For our measurements, we have used the FeB and Fe$_{0.4}$Co$_{0.4}$B$_{0.2}$ nanomagnets with thickness between 0.8 and 1.4 nm. A sputtered FeB film thinner than 0.8 nm is not continuous and therefore paramagnetic. The magnetization of a FeB film thicker than 1.4 nm is in-plane. A thinner nanomagnet is magnetically harder and a thinner nanomagnet is magnetically softer. The FeB was covered by a MgO (10 nm) layer in order to induce a sufficient PMA and a thick SiO$_2$ (100 nm) to isolate it from top contact electrodes.

An electron-beam lithography and an Ar ion milling were used for nano fabrication. Etching material was monitored in-situ by a Secondary-ion-mass-spectroscopy (SIMS) detector to ensure the required precision of etch-
ing depth. Four steps of nano fabrication were used. At first step, the Hall-bar was etched till the substrate. At second step, the nanomagnet was etched until Ta. At third step, the contact area was opened. At forth step, the contact electrodes were fabricated. The measured magnetization of a FeB film is 1.2 T. The Curie temperature of FeB and FeCoB nanomagnets is classically high and is substantially higher than the measurement temperature [2–7].

For each nanomagnet, the coercive field $H_c$, parameter delta, size of nucleation domain and anisotropy field were measured $H_a$. For our magnetically-softest nanomagnet $H_c = 20 G$ and $H_a = 2kG$ were measured. For our magnetically-hardest nanomagnet $H_c = 550 G$ and $H_a = 11kG$ were measured. For every studied monodomain nanomagnet we have achieved a perfect fitting by Eq.1 of main text similarly as shown in Fig.3. The scaling relaxation field $H_S$ is evaluated to be 4-5 kG for the magnetically-softer nanomagnets and 9-10 kG for the magnetically-harder nanomagnets.

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