Equivalent Strut Width for Partially Infilled Frames

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Abstract  The usual practice in the analysis of reinforced concrete frame structures is to analyze the frames with skeleton members comprising of only slabs, beams and columns. However, in reality the structures also possess masonry infills within most of the frames, but they are ignored in the models so as to minimize the computational works. Researchers have indicated that the frames comprising of masonry panels behave significantly stiffer as compared to bare frames. The infills contribute in stiffening the frames, but researches also show that the partial infills can cause adverse effect known as captive column effect. A lot of experimental evidences show that the captive column effect causes the partially infilled frames to damage during earthquakes. It is still a matter of interest to researchers to find out how much shear actually occurs at the location where the wall terminates. The shear force generated at the points where the wall terminates within the frame in the windward side causes the windward side column to fail. This study is done to identify the shear force values at such locations through analytical formula. The equivalent strut width as provided by various researchers is compared with the established formula for verification and further applied to obtain the shear forces at various locations in partially infilled frame. Equivalent strut width formulation is done in this paper, which may be used directly in the frame analysis wherever partially or fully infilled walls are provided.

Keywords  Equivalent Strut Width, Partially Infilled Frames, Lateral Resistance, Flexural Rigidity Model

1. Introduction

In the analysis of Reinforced concrete framed structures, there is a trend of ignoring the existence of brick infill mainly due to the reasons of complicated computations. Only the frame is considered in the analysis, which actually saves tedious calculation time and effort, but the real existence of bricks within the frames being ignored, actually underestimates the capacity of the structure. From the past studies done by various researchers, it has been found that the brick infills actually contribute in enhancing the strength of the structure by resisting the lateral deflection of frames applied to horizontal forces. Again, the contribution has been felt primarily during the earthquake events, where, most of the infilled framed structures remain less damaged as compared to the frames which are left bare. It is also necessary to examine whether the contribution of infilled frames remain equally good when some openings are provided within the panels. Some past studies also have indicated that the infills which include voids tend to be less effective, although, better than with the bare frames. The contribution of brick infill has been studied in this paper, particularly for partially infilled one because, the partially infilled frames in the past earthquakes have shown damaging effects, contrary to the completely filled walls or even with small openings. Most of the buildings of such kind have failed in the past earthquakes[1]. The short-column-effect or the captive-column-effect, are identified as the basic reason for the damage during the times of earthquakes. It is also understood that large shear force affects the location within the column where the partial infill height terminates.

Figure 1. Partially infilled framed structure

Studies from the past indicate that either some modifications[2] for full infill have been done to consider the partial infill condition or some experimental works have been done to point out the lateral resistance behaviour of partial infilled frames. This study is done mainly to find out the equivalent strut width and the lateral resisting capacity of partial infilled frame without any modification requirement using simple analytical formula. This will make the analysis work simpler than the usual methods. The flexural rigidities
of both infill and the frames have been taken into account for the analytical formulation and comparison of the result is done with various researchers’ findings for the case of fully infilled frames. The strut width value is then expressed for fully infilled and for partially infilled frames, particularly for reinforced concrete frames included with brick masonry infills.

2. Past Studies on Infilled Frames

Numerous studies have been done both for fully infilled frames and for infills containing openings. Thomas (1952) and Ockleston (1955) were one of the early major contributors in connection to the interaction between wall and frame[3]. Holmes (1961) studied experimentally on steel frames infilled with brick masonry and reinforced concrete walls and developed semi-empirical design method for laterally loaded infilled frames based on equivalent strut concept. His tests suggested that brick masonry walls increase the strength of frame by around 100%. The infill was considered to fail in compression. The load carried by infill at failure was calculated by multiplying the compressive strength of material by the area of equivalent strut. He states that the width of equivalent strut to be 1/3rd of the diagonal length of infill, which resulted in the infill strength being independent of frame stiffness (Table 1). Smith[5] has put up tremendous effort in finding out the interaction between frame and infill. He tested a number of infilled frames subjected to diagonal loading where he used the diagonal strut concept. His design curve gives the effective width of strut, the compressive failure load and the diagonal failure load as related to frame stiffness and infill aspect ratio. Mainstone[6] has given equivalent diagonal strut concept by performing tests on model frames with brick infills. His approach estimates the infill contribution both to the stiffness of the frame and to its ultimate strength. The strut width equation according to him is shown in Table 1. Liauw and Kwan[7] studied both experimentally and analytically the behavior of non-integral infilled frames. Finite Element method was adopted to find the effects of nonlinearities of the material and the structural interface, the initial lack of fit and friction at the interface was considered. Paulay and Priestley[8] gave the width of diagonal strut as 0.25 times the diagonal length of the strut (Table 1). Hendry[9] has also presented equivalent strut width that would represent the masonry that actually contributes in resisting the lateral force in the composite structure (Table 1). In addition to these studies, large numbers of researches have been done in the past for fully infilled frames with and without openings.

In the equations given in Table 1, H is the height of the frame, θ is the angle made by the strut with the horizontal, $E_c$ and $I_c$ are the Young’s modulus and Moment of inertia of column respectively and $E_m$, $t$ and $h_m$ are the Young’s modulus, thickness and height of masonry infill respectively. In Hendry’s equation, $α_h$ and $α_t$ are the contact length between wall and column and beam respectively at the time of initial failure of wall.

![Table 1. Equations for strut width value for full infill by various researchers](image)

Very few literatures are available regarding partial masonry infilled framed structures so far. Ghassan[2] has given an approach to calculate the equivalent strut width for partial infilled frames, in which Mainstone’s[6] approach is modified with reduction factors for partial opening and for existing damages prevailing in the structure. Huang et al.[10] have tested six reinforced concrete frames with and without infill (including partial infill) under horizontal cyclic loads. Similarly, Taher and Afefy[11] have done investigation on partial infill structures for various percentage openings. Their system consists of homogeneous continuum for the reinforced concrete members braced with unilateral diagonal struts for each bay, which activate only in compression. The results reflect the significance of infill in increasing the strength, stiffness, and frequency of the entire system depending on the position and amount of infilling. Subramanian and Jayaguru[12] have conducted experimental study on behaviour of partial reinforced concrete frames with masonry infills using 1/3 scaled model for lateral load. The partially infilled masonry wall induced captive column effect and led to a severe failure of the column.

3. Analytical Approaches

This study commences with the hypothesis that the flexural rigidity of infill should be equal to the flexural rigidity of frame in order to have equilibrium in the structure. Since the wall height is not full, the partial height wall will be providing significant resistance to the columns when the frame is laterally loaded until the wall gets initial crack. The partially infilled wall restrains the lateral deflection of the frame up to the height of the wall, but the lateral force which actually is acting on the top node of the frame equal to the base shear will try to deflect the void portion of the frame. In this instant, the top of the frame gets deflected while the junction between the column and wall will remain less...
affected until the wall cracks. So, the effort has been done to find out how the columns get sheared due to the shear force action by the restraining diagonal at the junction. From the fundamental concept of deflection theories when a cantilever is loaded at free end, in order to avoid deflection at free end, large opposing force need to be applied by props and this phenomenon is analogous to the case where the masonry wall behaves like prop to the frame elements. Wherever the wall terminates, the vertical component of the diagonal strut will be the propping force which is actually the shear force acting in the column which causes the column to fail during the lateral force application to the portal frame.

3.1. Formulation

When the wall height doesn’t extend up to the top beam level, the lateral force applied to the composite structure will cause deflection of the concrete frame laterally, which further compresses the infill frame diagonally as shown by Rs (Fig. 3a). Since the void portion above the wall is not compressed by the beam, only the wall adjacent to the column will be compressed, which indicate that the strut effectively working will be as shown in the figure (Fig. 3b).

If the flexural rigidities of frame and masonry wall are compared the equivalent width of the wall contributing in the resistance against deflection can be obtained. According to elastic strip theory, the contact length between frame and masonry $k_x$ is given by:

$$k_x = \frac{\pi}{2} * h_y$$

where $h_y$ is the equivalent length of wall (Fig.2) that contributes in compression[13].

![Figure 2. Contact length between frame and masonry](image)

The equivalent strut width $w$ which contributes in resisting the lateral deflection is:

$$w = k_x * \sin \Psi$$

where $\Psi$ is the angle subtended by the equivalent strut’s upper dimension with the vertical. Thus, the strut width equation suggested by the author will be;

$$w = \frac{\pi}{2} * 2.29 * \left(\frac{E_c * l_c * h_m}{E_m * t * h_c}\right)^{1/2} * \frac{L_m}{L_m^2 + (h_m - k_x)^2} \quad (1)$$

where, $E_c$ is the Young’s modulus of frame, $I_c$ is the Moment of inertia of column, $h_c$ is the center to center height of frame, $E_m$ is the Young’s modulus of masonry, $t$ is the thickness of masonry, $L_m$ is the length of masonry and $h_m$ is the height of partial infill. The $k_x$ is the product of first three terms and $\sin \Psi$ is the last term (4th term) of equation (1). The diagonal force ($R_s$) on the equivalent strut can be obtained by using the following relation;

$$R_s = w * t * f'_m$$

where, $f'_m$ is the allowable compressive strength of masonry unit. The horizontal component of the force in diagonal strut will give the lateral resisting force ($V_m$) of wall, which is the shear force that acts on the column where the wall height terminates.

$$V_m = R_s \cdot \cos \theta \quad (2)$$

where, $\theta$ is the angle subtended by the diagonal with the horizontal.

![Figure 3(a). Effect in partial infilled frame](image)

![Figure 3(b). Analytical model for equivalent strut](image)

Various researchers have suggested the strut width as the product of some coefficient and the diagonal length of the wall. Using the equation 1, the strut widths for various aspect ratios between frame height and frame width may be calculated. For example, the diagonal strut width will be 1.462m for aspect ratio 0.63:1 for a particular model with data of Table 2. Here, coefficient 0.251 is obtained if the strut width value is divided by the diagonal strut length. This coefficient value again changes for various aspect ratios and if the aspect ratio is equal to or more than 1, then the coefficient increases. This phenomenon coincides with the works done by Liauw and Kwan[7] and Hendry[9], where the strut width value varies with the variation in aspect ratios. The equations are also tallied with various researchers’ findings and are shown in Table 3.

The allowable compressive strength of masonry wall ($f'_m$) depends on many parameters like mortar strength, mortar
thickness, brick height, age of mortar etc. In general, the strength of masonry is considered 25 to 50% of the brick strength and so, the strength of masonry wall lies within a large range between 1 MPa to 50 MPa\[3\]. According to FEMA 273, for masonry in good condition up to 6.3 N/mm\(^2\) (900psi), for fair condition up to 4.13N/mm\(^2\) (600psi) and for poor condition, the value can be taken up to 2.07 N/mm\(^2\) (300psi). For this study it is suggested to provide \(f_m'\) as 5.6 N/mm\(^2\), within the tolerance suggested by FEMA 273.

4. Results and Discussions

4.1. Comparisons with Various Researchers’ Findings for Full Infill

The formulation for strut width has been compared with the formulae suggested by various researchers. The Table 2 indicates the data assumed to calculate the equivalent strut width for the comparison purpose. It is observed that the equivalent strut width obtained by the Flexural model is very much close to the results compared to those suggested by Paulay and Priestley\[8\], Holmes\[4\], Liauw and Kwan\[7\] and Hendry\[9\]. The observation from Fig. 5 is regarded as the condition for validity of the formulated equation 1.

![Figure 4](image)

**Figure 4.** Coefficient for strut width for aspect ratio 0.63:1

![Figure 5](image)

**Figure 5.** Equivalent strut width by various researchers

### Table 2. Data considered for verification study

| Parameters                                                | Data | Units |
|-----------------------------------------------------------|------|-------|
| Characteristic strength of concrete \((f_{ck})\)          | 25   | MPa   |
| Young’s mod. of concrete, \(E_c = (5000√f_{ck})\)        | 25000| MPa   |
| Beam width \(B_w\)                                       | 0.25 | m     |
| Beam depth \(B_d\)                                       | 0.4  | m     |
| MOI beam \(I_b\)                                        | 0.001333| m\(^4\) |
| Column width \(C_w\)                                    | 0.4  | m     |
| Column depth \(C_d\)                                     | 0.4  | m     |
| MOI column \(I_c\)                                      | 0.002133| m\(^4\) |
| Young’s modulus of masonry wall \(E_m\)                  | 2750 | MPa   |
| Wall thickness \(t\)                                     | 0.225| m     |
| Height of infill wall \(h_m\)                            | 3    | m     |
| Length of infill wall \(L_m\)                            | 5    | m     |
| Angle made by strut with horizontal \(θ\)                | 30.96| degrees |
| Height of frame \(c/c H\)                                | 3.4  | m     |
| Diagonal length of infill \(d_m\)                        | 5.8309| m     |

### Table 3. Strut width and coefficient for various aspect ratios

| Researchers            | Aspect ratio, A.R. = frame height: frame length | 0.63:1 | 1:1 | 1.5:1 |
|------------------------|-----------------------------------------------|-------|-----|-------|
|                        | strut width (m) | coefficient | strut width (m) | coefficient | strut width (m) | coefficient |
| Mainstone[6]           | 0.635           | 0.109          | 0.462           | 0.109          | 0.387           | 0.109        |
| Hendry[9]              | 1.842           | 0.316          | 1.616           | 0.381          | 1.523           | 0.429        |
| Paulay and Priestley[8]| 1.458           | 0.250          | 1.061           | 0.250          | 0.888           | 0.250        |
| Holmes[4]              | 1.942           | 0.333          | 1.413           | 0.333          | 1.183           | 0.333        |
| Liauw and Kwan[7]      | 1.439           | 0.247          | 1.187           | 0.280          | 0.898           | 0.253        |
| Flexural model         | 1.462           | 0.251          | 1.367           | 0.322          | 1.204           | 0.339        |
4.2. Shear Forces and Strut Widths for Partial Infilled Frame

In this study, a single bay, single storey reinforced concrete frame is considered as given in Table 4. It is assumed that if a partial height infill is included within a frame and if the frame is applied with some lateral load, the wall will tend to resist the deflection due to the resistance provided by the frame in composite manner. While \( V \) is the force applied to the composite structure, we consider \( V_m \) is the resistance offered by the wall up to which it has been constructed. The force \( V \) tends to deflect the structure laterally while the resisting force \( V_m \) tends to resist the whole structure from being deformed. During this phenomenon, the structure obviously gets benefited by the resistance offered by the wall. When there is resistance at the junction of the wall and column, it is possible that the junction will face significant moment. This moment will be sufficient for damaging effect in the concrete column if designed brittle or disregarding ductility. Various parameters are considered as per Table 4 for the study.

The formula by Flexural model holds well until pre-cracking stage. A study is done for a particular model considered by Agarwal and Shrikhande [15], and it is observed by using the equation 1, that the equivalent strut width for 2550 mm tall masonry infill (full infill) of length 4550 mm will be 805.222 mm while the strut width used by Hendry’s equation is 778.4 mm. This indicates that the strut width may be taken as 0.154 times the diagonal length in this case. Further it indicates that the strut width value depends upon Young’s modulus of the materials as well as the aspect ratio between the frame height and span. The strut width value and shear forces at various levels of infill for various aspect ratios are computed and tabulated in Table 5.

| Table 4. Parameters chosen for shear force and strut-width calculations |
|-------------------------|-------------------|----------|
| Parameters               | Data              | Unit     |
| Grade of concrete        | 20                | MPa      |
| Young’s Modulus of concrete \( E_c \) | 22360.67         | MPa      |
| Young’s Modulus of masonry \( E_m \) | 13800            | MPa      |
| Depth of column \( C_D \) | 450               | mm       |
| Width of column \( C_b \) | 300               | mm       |
| Moment of inertia of column \( I_c \) | 2278125000       | mm\(^4\) |
| Depth of Beam \( B_D \)  | 450               | mm       |
| Width of beam \( B_b \)  | 300               | mm       |
| Moment of inertia of beam \( I_b \) | 2278125000       | mm\(^4\) |
| Thickness of infill \( t \) | 230              | mm       |
| Height of infill \( h_m \) | 2550             | mm       |
| Length of masonry \( L_m \) | 4550             | mm       |
| Height of frame \( c/c \) | 3000              | mm       |
| Length of frame \( c/c \) | 5000              | mm       |

| Table 5. Shear on columns and strut widths |
|-------------------------------------------|
| \( h_m \) (mm) | 0.6:1 | 1:1 | 1.5:1 | 0.6:1 | 1:1 | 1.5:1 |
|----------------|-------|-----|-------|-------|-----|-------|
| shear on column (kN) | shear on column (kN) | shear on column (kN) | shear on column (kN) | shear on column (kN) | shear on column (kN) |
| 2550          | 904.729 | 652.032 | 222.230 | 805.222 | 715.925 | 453.156 |
| 2295          | 907.855 | 688.421 | 258.835 | 789.444 | 719.080 | 483.027 |
| 2040          | 904.131 | 721.011 | 303.398 | 769.291 | 716.883 | 514.719 |
| 1785          | 892.314 | 747.036 | 356.898 | 744.195 | 707.976 | 546.516 |
| 1530          | 871.005 | 763.066 | 418.980 | 713.455 | 690.901 | 574.886 |
| 1275          | 838.528 | 765.057 | 486.075 | 676.108 | 664.099 | 593.650 |
| 1020          | 792.645 | 748.371 | 548.675 | 630.681 | 625.792 | 593.896 |
| 765           | 729.824 | 707.514 | 589.112 | 574.587 | 573.499 | 565.905 |
| 510           | 643.052 | 634.512 | 582.039 | 502.391 | 502.389 | 502.378 |
| 255           | 512.539 | 510.241 | 494.480 | 398.559 | 398.126 | 395.072 |
| 0             | 0.000   | 0.000   | 0.000   | 0.000   | 0.000   | 0.000   |
The same model gives strut width as 676.108mm from equation 1 and then equation 2 gives the resistance of masonry as 838.52 kN at 1275mm level (50% opening) of wall, but the full infill wall gives resistance of 904.72 kN (higher than the force provided by partial infill) for aspect ratio between height of frame and span of frame as 0.6:1. It indicates that as the wall height reduces, the shear exerted on the partial infill reduces and the column thus gets more deflected during the lateral loading to the composite structure. However, as the aspect ratio increases, the shear on column start to increase which means that the partial masonry infill tries to provide more resistance to high aspect ratio structures and in turn the column gets comparatively greater shear than that by full infilled, which further means that the column is liable to get damaged at the junction. As the infill reduces to zero height, it resembles a bare frame and thus, the loading has to be taken only by the bare frame. Thus, the study suggests that the partially infilled frame is more susceptible to damage at the location up to where the wall is extended when the wall height is reduced for aspect ratio 1:1 or more. This is due to the fact that the resisting capacity reduces and the column gets deflected at the void portion as the lower portion is resisted by the wall. The void portion shortens the column length and so the deflection is also reduced but, on the other hand due to the lateral loading being constant, the resistant portion within the column activates in resisting the lateral deflection, which damages the joint.

As the height of infill reduces the masonry wall’s resisting capacity reduces for aspect ratio less than 1:1, however, the resisting capacity increases with lowering the height up to 30% of wall height (70% opening), and then declines for higher aspect ratios. This phenomenon is shown in Figure 6.

The study shows that when the masonry infill height is reduced forming a captive column effect, then the resistance capacity of masonry gradually decreases at aspect ratio 0.6:1, however as the aspect ratio increases, the initial resistance capacity is reduced but the capacity keep on increasing up to 70% opening percentage of opening in the wall. When the aspect ratio is 1.5:1, it is observed that the initial strength of full wall is reduced but as the wall height gets reduced the strength increases drastically. This indicates that when the aspect ratio is more than or equal to one, the resistance offered by the partial wall is significant, which ironically allows more shear in the column and further, the column will be damaged at the junction up to where the wall is constructed. The study is done up to the level of initial cracking of the wall.

5. Conclusions

The suggested formula (1) gives an idea that the infill would be resisting the deflection of the frame and thus, the global structure gets strengthened against lateral deflection. However, when the infill height gets reduced, then the action against the columns would be different at different aspect ratios between the frame height and the frame span. It is found that the strut width for various masonry heights within concrete frames can be determined analytically and then it can be used for analyzing infilled frames conveniently. The value of strut width should be assigned separately for various percentage openings, but beyond 70% opening, the strut width will be almost same for all aspect ratios and the composite frame will behave just similar to bare frame. The strut width value depends much upon the aspect ratio as well as other material parameters. The lateral strength offered by the infill depends on the height of the wall and as the height of wall reduces, it is found that the resistance capacity gradually increases till nearly 70% of the wall is left open. This shows the short column effect on the frame offered by the partial infill. It is also to be noted that the shear force offered by partial infills where the wall terminates can be obtained using the equation 2 as shown in Table 5.

The capacity of composite frame increases due to the additional resistance offered by infill when lateral load is applied. The partial infilled frames are liable to damage during lateral load application due to the shear force at columns transferred by the partial wall in higher aspect ratios (more than or equal to 1:1). When the infill is not provided the masonry infill’s capacity will not be active and only the bare frame has to resist the lateral force.
This study limits to static loading condition, however, further study may be done for dynamic case. With the help of the equivalent strut width obtained from this research, computer analyses for structures inclusive of partial infills will be possible by incorporating them in the analytical models. The realistic responses of partially infilled frames may thus be obtained.

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