CODEX clusters.

The Survey, the Catalog, and Cosmology of the X-ray Luminosity Function.

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ABSTRACT

Context. Large area catalogs of galaxy clusters constructed from ROSAT All Sky Survey provide the base for our knowledge on the population of clusters thanks to the long-term multiwavelength efforts on their follow-up.

Aims. Advent of large area photometric surveys superseding in depth previous all-sky data allows us to revisit the construction of X-ray cluster catalogs, extending the study to lower cluster masses and to higher redshifts and to provide the modelling of the selection function.

Methods. We perform a wavelet detection of X-ray sources and make extensive simulations of the detection of clusters in the RASS data. We assign an optical richness to each of the 24,788 detected X-ray sources in the 10,382 square degrees of SDSS BOSS area, using redMaPPer version 5.2. We name this survey COnstrain Dark Energy with X-ray (CODEX) clusters.

Results. We show that there is no obvious separation of sources on galaxy clusters and AGN, based on distribution of systems on their richness. This is a combination of increasing number of galaxy groups and their selection as identification of an X-ray sources found in the ROSAT All Sky Survey as galaxy clusters (see Piffaretti et al. [2011] for a summary of X-ray cluster catalogs). Given that those catalogs have been published a while ago and that they contain the brightest objects, most of the follow-up campaigns have concentrated on those clusters. In particular, the cluster weak lensing calibration for all currently published cosmological surveys are based on these samples. At the moment, a difference in the weak lensing calibration of cluster masses between redshifts below 0.3 and above have been revealed (Smith et al. [2016]), and the importance of the selection effects at $z > 0.3$ has been demonstrated (Kettula et al. [2015]). Thus, it is important to revisit the details of the cluster selection.

Conclusions. CODEX is the first large area X-ray selected catalog of Northern clusters reaching the fluxes of $10^{-13}$ ergs s$^{-1}$ cm$^{-2}$. We provide the modelling of the sample selection and discuss the redshift evolution of the high end of the X-ray luminosity function (XLF). Our results on $z < 0.3$ XLF are in agreement with previous studies, while we provide new constraints on the $0.3 < z < 0.6$ XLF. We find a lack of strong redshift evolution of the XLF and consider possibilities to explain it within a flat $\Lambda$CDM.

Key words. galaxy groups – galaxy evolution

1. Introduction

Many X-ray galaxy cluster catalogs rely on identification of X-ray sources found in the ROSAT All Sky Survey as galaxy clusters (see Piffaretti et al. [2011] for a summary of X-ray cluster catalogs). Given that those catalogs have been published a while ago and that they contain the brightest objects, most of the follow-up campaigns have concentrated on those clusters. In particular, the cluster weak lensing calibration for all currently published cosmological surveys are based on these samples. At the moment, a difference in the weak lensing calibration of cluster masses between redshifts below 0.3 and above have been revealed (Smith et al. [2016]), and the importance of the selection effects at $z > 0.3$ has been demonstrated (Kettula et al. [2015]). Thus, it is important to revisit the details of the cluster selection.

The abundance of galaxy clusters is a sensitive cosmological probe, and currently the focus of the research is to understand whether there is a tension in the reported constraints on the parameters of the $\Lambda$CDM model between clusters and CMB (Planck Collaboration et al. [2016]). It is therefore of primary im-
3. Since the naming convention for the merged sources is different among sub-projects, we present two IDs for each source, one is “CODEX” ID and the other as “SPIOPERS” ID, where SPIOPERS is the spectroscopic program on SDSS-IV (Dawson et al. 2014), which is useful for the weak lensing modelling (e.g. Ciolkaka et al. 2017). We fold the BOSS area mask into the selection of CODEX. The catalog requirements on the extragalactic sky adopted in BOSS (Dawson et al. 2013) are higher than the typical limitations considered in construction of X-ray extragalactic surveys and consequently, a variation of NH correction (computed using the data from Kalerba et al. 2005) within the survey area is small. The largest deviations in the sensitivity of the survey are driven by the variations in the exposure map of the RASS survey, which we illustrate in Fig. We show the histogram of the survey area as a function of sensitivity.

2.2. Source identification
We run the redMaPPer version 5.2 (Rykov et al. 2014) on the position of every source (24788 sources in the BOSS footprint), identifying the maximum richness red sequence between redshifts 0.05 and 0.8. We search for the best optical center within 400 kpc from the X-ray center. We report the cluster richness both at X-ray and the optical positions, required corrections for the masked area of SDSS and photometric depths, which affect the error calculation for the richness. We calculate the probability of the center to be correct (Rykov et al. 2014), which is useful for the weak lensing modelling (e.g. Ciolkaka et al. 2017). We fold the BOSS area mask into the selection of CODEX. The catalog

"Throughout the whole sky, we have surveyed an area of 100,000 for sources at faint limit (Voges et al. 1999, Boller 2017). Exploration of RASS sources for the purpose of identification of galaxy clusters has been primarily concentrated on the bright sub-sample (Bohringer et al. 2013, 2017). The main purpose of CODEX is to extend the source catalog down to the lowest fluxes accessible to RASS, reaching $10^{-14}$ ergs s$^{-1}$ cm$^{-2}$. This requires an in-depth understanding of source detection and characterization. We therefore carry out the source detection ourselves and accompany it with a detailed modeling.

RASS data is available in the form of sky images in several bands, background images and exposure maps. We use those of Data Release 4 which contain only the photons with reliable attitude restoration (for more details see Boller 2017). The DR3 data consists of count maps covering an area of $\sim 41$ square degree each, having some overlap between the tiles. For the source detection we use the wavelet decomposition method of Vichitvishal et al. (1998). We run several scales of wavelet decomposition, starting from 2 pixels, which corresponds to 1.5$'$ and extending the search for the X-ray emission to the scales of 12$''$. Larger spatial scales are important only for nearby ($z < 0.1$) clusters, and within CODEX are only used for the flux refinements. Even use of the adopted scales at RASS depths tend to connect several sources (see e.g. Mirkazemi et al. 2015) and in order not to miss sources, we identify the small scales separately from the large scales and latter merge the identifications. The catalog using all scales is called C1 and the catalog derived using small scales is called C2. Given a large fraction of duplicates between the two catalogs, we merge the sources with the offsets below.

1. Unless explicitly noted otherwise, all observed values quoted throughout this paper, are calculated adopting a $\Lambda$CDM cosmological model, with $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$, $\Omega_M = 0.3$, $\Omega_r = 0.0$. We quote X-ray flux in the observer’s 0.5–2.0 keV band and rest-frame luminosity in the 0.1–2.4 keV band and provide the confidence intervals on the 68% level. FKS Epoch J2000.0 coordinates are used throughout.

2. http://www.xray.mpe.mpg.de/rosat/survey/rass-3/main/help.html
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Fig. 1. Aitoff projection of the sensitivity of the ROSAT All-Sky Survey data within BOSS footprint. Nominal sensitivity in the 0.5–2 keV band towards 4 counts is plotted. The units are ergs s\(^{-1}\) cm\(^{-2}\). The grid shows Equinox J2000.0 Equatorial coordinates.

Fig. 2. Cumulative (\(\Omega(S)\), solid curve) and differential (\(\frac{d\Omega}{dS}\), dashed curve, using \(\Delta S = 0.05\) dex bins in the flux) survey area as a function of flux, based on the sensitivity of the ROSAT All-Sky Survey data within BOSS footprint. Nominal sensitivity in the 0.5–2 keV band towards 4 counts is plotted.

of cluster member galaxies has been released as a target catalog of SPIDERS (Clerc et al. 2016) and can be found online.

We have completed the spectroscopic follow-up campaign of CODEX clusters down to a richness of 10 through a number of programs on SDSS-II, III and IV, as well as using the Nordic Optical Telescope. The first results are presented in Clerc et al. (2016) and Kirkpatrick et al. (in prep.), and include a full characterization of the uncertainty in the photometric redshift estimate. We report on the 100% success rate in identification of clusters at \(z < 0.3\), which required 5 spectroscopic members to achieve. At higher redshift the depths of the follow-up drive the identification success and success of it reaches 100% once we are able to target \(> 7\) member galaxies. The same galaxies form the bulk of the estimate of the cluster richness. Fig. 3 shows the photometric depth correction factor used in calculating the reported SDSS cluster richness, \(\zeta\), which has an exponential increase towards high-\(z\). This is a ratio between the richness of the cluster and the actually observed part of it. High values of \(\zeta\) imply that only the tip of the cluster galaxy luminosity function is observed. It also serves a description for spectroscopically confirmed sub-sample, which is a combination of the threshold for spectroscopic cluster confirmation and success rate of cluster member targeting. As the robustness of photometric identification relies on the actual number of galaxies used, consistently with other redMaPPer catalogs, we have chosen to use at least 10 member galaxies richness limit with redshift. While we also correct the richness for the masked area, the fraction of clusters with masking correction exceeding 20% is 3% of the sample and therefore does not require additional modelling, apart from the tests performed for galaxy cluster clustering (Lindholm et al. in prep.).

The resulting redshift range of CODEX clusters is 0.05–0.65. Below a redshift of 0.1 performance of the redMaPPer has not been calibrated, and systematic offset between the photometric and spectroscopic redshifts is found (Clerc et al. 2016). Large projection effects and large size of X-ray sources also require additional care. We will therefore not discuss the properties of the \(z < 0.1\) part of the catalog. A comparison of literature red-
shifts and redMaPPer measurements is also discussed in [Rozo et al. 2015].

Using positions of random sources, we have estimated a probability of the chance identification of a richness 20 source to be 10% in each 0.1 width bin of redshift in the range 0 \( < z < 0.6 \). In Fig. 3 we compare the completeness limits of the RASS and SDSS surveys towards detection of a galaxy cluster. We used the scaling relation of [Capasso et al. 2019a] to express the survey mass limits in terms of (true) richness. Identification of RASS sources using the DES survey has been considered in [Klein et al. 2019]. While this survey covers a different area, the strategy for source identification is similar. The 10% sensitivity curve of the CODEX survey matches well the definition of low (5%) contamination subsample in [Klein et al. 2019], which we verify using an overlap area of two surveys, located in the Stripe 82 [Capasso et al. 2019b]. In order to reduce the effect of contamination down to 5%, we need to remove the sources with richness below the curve, and propagate this selection into the modelling. The analytical form of the selection reads:

\[
\exp(\lambda_{50\% \text{ cont}}) > 22 \left( \frac{z}{0.15} \right)^{0.8},
\]

where \( \lambda \) denotes a natural logarithm of richness to simplify the notation for the log-normal distribution:

\[
\lambda_{50\%}(z) = \ln(17.2 + e^{\frac{15 - z}{0.12}})
\]

obtained using the tabulations of [Rykoff et al. 2014]. We use an error function with the mean of \( \lambda_{50\%}(z) \) and a \( \sigma = 0.2 \), which reproduces the 75% and 90% percentiles of the distribution tabulated in [Rykoff et al. 2014]. We use the probability of the optical detection of the cluster in SDSS data as

\[
p_{\text{SDSS}}(\lambda|z) = 1 - 0.5\text{erfc}\left(\frac{\lambda - \lambda_{50\%}}{0.2\sqrt{2}}\right)
\]

which is discussed further in §3.

2.3. CODEX catalog

Once an X-ray source has an optical counterpart, we can assign a redshift to it. This allows us to compute the source rest-frame properties, such as luminosity. We apply the procedure of [Finoguenov et al. 2007] to iteratively retrieve the X-ray luminosity. We obtain an initial guess on cluster mass, using \( M - L_X \) relation and compute the missing source flux correction (\( A \)), taking into account the flux extraction aperture and the expected surface brightness profile of the source, given the mass [Kafner et al. 2019]. In performing mass and temperature estimates, we use the XXL \( M - T \) [Lieu et al. 2016] and \( L_X - T \) [Giles et al. 2016] relations, which is also consistent with CODEX weak lensing calibration of [Kettula et al. 2015]. For small apertures, we use the PSF correction. Applying these corrections we obtain a new estimate of luminosity. We iterate this procedure 100 times. The resulting catalog of cluster properties is presented in Tab. 4 with column (1) listing CODEX source ID, (2) frequently used SDSS ID, columns 3-4 providing the coordinates of the X-ray center, column (5) providing the redMaPPer redshift, column (6) providing the richness estimate and its error, column (7-8) providing the probability of correct center to be correct. This allows us to compute the source rest-frame properties, such as luminosity. We apply the procedure of [Finoguenov et al. 2007] to iteratively retrieve the X-ray luminosity. We obtain an initial guess on cluster mass, using \( M - L_X \) relation and compute the missing source flux correction (\( A \)), taking into account the flux extraction aperture and the expected surface brightness profile of the source, given the mass [Kafner et al. 2019]. In performing mass and temperature estimates, we use the XXL \( M - T \) [Lieu et al. 2016] and \( L_X - T \) [Giles et al. 2016] relations, which is also consistent with CODEX weak lensing calibration of [Kettula et al. 2015]. For small apertures, we use the PSF correction. Applying these corrections we obtain a new estimate of luminosity. We iterate this procedure 100 times. The resulting catalog of cluster properties is presented in Tab. 4 with column (1) listing CODEX source ID, (2) frequently used SDSS ID, columns 3-4 providing the coordinates of the X-ray center, column (5) providing the redMaPPer redshift, column (6) providing the richness estimate and its error, column (7-8) providing the coordinates of the best optical center, column (9) gives the probability of this center to be correct. Column (10) lists the X-ray luminosity in the rest-frame 0.1-2.4 keV, col (11) lists the temperature used in estimating the K-correction, (12-14) list A, B, C, respectively.

The table will be published electronically upon acceptance of the paper. Spectroscopic properties of CODEX clusters are released as a part of SDSS-IV DR16 under SPIDERS cluster catalog.
Poorly studied redshift range of 0.25. Patrick et al. (in prep.).

The clusters with richness greater than 60 in the previously published simulations correspond to a level of one background count in the zone of interest. The background level is reduced at high redshifts, typically, the background level in COSMIC X-ray Background to the background counts and use the exposure maps as a model for its spatial distribution. Note that strong flares in RASS data have been filtered out by removing the corresponding time intervals.

We simulate the cluster detection as a function of total number of detected cluster counts, performing a simulation of each value of the cluster shape parameter grid 1000 times and trying 1000 realizations of background for each simulated cluster. The grid of the cluster shape parameters samples the parameter of the β-profile of clusters, with surface brightness distributed with radius r as

$$ S_r(r) = \left(1 + \left(\frac{r}{\rho}\right)^2\right)^{-\beta+0.5} $$

(normalized to the total count, $\eta_{true}$, and sampling the distribution of core radii for clusters of given T, derived using a fixed $\beta - T$ relation (Käfer et al. 2019). We sample mass range in the 13.5-15.5 in $\log_{10}(M_{200})$ to predict the shape of the cluster ($\beta, \rho$), and the redshift range 0.1-0.6. We perform a Poisson realization of the simulated image, and store the results based on observed count rate ($\eta^{ob}$) from 3 to 30. As we will be using multivariate log-normal distributions throughout this paper, we conveniently define the quantities $r_c = \ln(R_c), \nu = \ln(L_c), \mu = \ln(M_{200}), \lambda = \ln(Richness)$. We denote the obtained grid as a probability of detection given the detected counts of the source ($\eta^{ob}$), and shape parameters $\beta$ and $r_c$: $P(\eta^{ob}|\beta, r_c)$. To use the results of Käfer et al. (2019) we substitute $T$ with $\rho$ using $M - T$ relation of Kettula et al. (2015).

As has been pointed out by Käfer et al. (2019), the cluster shape is covariant with the scatter in $M - L_X$ relation, and we will use their tabulations of the multivariate Gaussian distribution, $P(r_c, I|\mu)$. While Käfer et al. (2019) characterize the cluster population at $z < 0.1$, the importance of the considered effects is reduced at $z > 0.3$, where all our clusters are nearly point-like for RASS and we deem this data sufficient. The effect of the covariant change in the core radius results in an even larger detectability of the cool core clusters. Not only do they have larger $L_X$ for a given mass, their peaked shape has a better chance of being detected (Eckert et al. 2011).

In addition to covariance of X-ray luminosity and shape, we include a covariance of X-ray luminosity and richness, based on the results of Farahi et al. (2019), leading to

$$ P(\nu^{true}, r_c, I|\mu, z) = \frac{1}{(2\pi)^{1/2}} \exp \left[-\frac{1}{2} X^T \Sigma^{-1} X \right] $$

where the vector

$$ X = \left[ \begin{array}{c} \nu^{true} - \langle \nu, \mu \rangle \\ r_c - \langle r_c, \mu \rangle \\ \lambda - \langle \lambda, \mu \rangle \end{array} \right] $$

is defined using the scaling relations of Käfer et al. (2019), Mulroy et al. (2019), Capasso et al. (2019a). The covariance matrix $\Sigma$ reads:

$$ \left( \begin{array}{cccc} \sigma_{\nu}^2 & \rho_{\nu, r_c} \sigma_{\nu} \sigma_{r_c} & \rho_{\nu, \lambda} \sigma_{\nu} \sigma_{\lambda} \\ \rho_{\nu, r_c} \sigma_{\nu} \sigma_{r_c} & \sigma_{r_c}^2 & \sigma_{r_c} \sigma_{\lambda} \\ \rho_{\nu, \lambda} \sigma_{\nu} \sigma_{\lambda} & \sigma_{\lambda} \sigma_{\lambda} & \sigma_{\lambda}^2 \end{array} \right) $$

(6)

To calculate $\Sigma$ we adopt the following values $\rho_{\nu, r_c} = -0.3, \sigma_{r_c} = 0.36$ (Käfer et al. 2019), $\sigma_{\lambda} = 0.2$ (Capasso et al. 2019a, Mulroy et al. 2019), $\rho_{\nu, \lambda} = -0.3$ (Farahi et al. 2019).

3. Modelling of X-ray cluster detection

We model the source detection as a probability given the number of detected source counts ($\eta_{true}$) and the shape parameters of the surface brightness ($S_B$) distribution of the source $P(I|\eta^{ob}, S_B)$, where $I$ denotes the selection.

We measure the background level in the 0.5–2 keV RASS images, using source-free zones. Typically, the background level corresponds to a level of one background count in the zone of detection. In the model we assume a dominant contribution of Cosmic X-ray Background to the background counts and use the exposure maps as a model for its spatial distribution. Note that strong flares in RASS data have been filtered out by removing the corresponding time intervals.

We simulate the cluster detection as a function of total number of detected cluster counts, performing a simulation of each value of the cluster shape parameter grid 1000 times and trying 1000 realizations of background for each simulated cluster image. The grid of the cluster shape parameters samples the parameter of the β-profile of clusters, with surface brightness distributed with radius r as
$\sigma_{\mu\gamma} = 0.46(1-0.61z)$ (Mantz et al. 2016), while no measurement of $\rho_{\mu\gamma}$ is published and which is set to 0 in our work.

The effect of richness on the selection is only by offsetting the distributions of $I$ and $r_c$, and so for the purpose of determining the mass-richness relation, it is convenient to store just the effect of covariance on the selection function, replacing richness with $\nu = d-I(\nu_{\mu\gamma})$, storing $P(I|\mu, z, \nu)$ and transforming $P(r_c, I, \nu_{\mu\gamma}, z)$ to $P(r_c, I, \nu_{\mu\gamma}, z) P(\nu_{\mu\gamma}, z)$, where only the $P(\nu_{\mu\gamma}, z)$ is varied (in the external to this paper) scaling relation work. Given the freedom in treatment of the covariance $\rho_{\mu\gamma}$, we set it to zero, which makes $\Sigma$ a block diagonal matrix, with two $2 \times 2$ elements (we could also explain this by noting that $P(r_c, I, \nu_{\mu\gamma}, z) = P(r_c, I, \nu_{\mu\gamma}, z) P(\nu_{\mu\gamma}, z)$): $\Sigma_{\mu\mu}$ and $\Sigma_{\nu\nu}$, whose inversion is analytical, resulting in:

$$
N \left( r_c - \langle r_c \rangle _{\mu_{\mu}, \nu_{\nu}} - \rho_{\mu\nu} (I-\langle I \rangle _{\nu_{\nu}}) \sigma_{\mu\gamma} \sqrt{1 - \rho_{\mu\nu}^2} \right) . 
$$ (8)

The usefulness of this formula is a demonstration that covariance results in the offset of the distribution (for a graphical illustration of the offset, see also Capasso et al. 2019). Denoting the survey area $\Omega$ (deg$^2$) and sensitivity $S$ (ergs s$^{-1}$cm$^{-2}$), which includes the effects of exposure and $n_H$, we define the survey selection function as

$$
P(I|\mu, z) = \int dS \frac{d\Omega}{dS} 
$$

$$
\iint d\nu_{\mu\gamma} dr_c d\eta_{\mu\gamma} \chi_{\mu\gamma} \eta_{\mu\gamma} P(I|\mu, z) P(\nu_{\mu\gamma}|I, \nu_{\mu\gamma}, z) P(r_c, \nu_{\mu\gamma}, \nu_{\mu\gamma}, z) 
$$ (9)

where

$$
P(\eta_{\mu\gamma}|I, \nu_{\mu\gamma}, z) = \frac{(\eta_{\mu\gamma}|I, \nu_{\mu\gamma}, z)}{\eta_{\mu\gamma}} . 
$$ (10)

Conversion of luminosity to counts uses the luminosity distance to the object $d_L(z)$, sensitivity $S$ (counts per flux in ergs cm$^{-2}$ s$^{-1}$) and K-correction $K(T|\mu, z)$, and

$$
\eta_{\mu\gamma} = \frac{L_{\mu\gamma} \Sigma}{4 \pi d_L(z)^2 K(T|\mu, z)} . 
$$ (11)

Fig 6 illustrates the resulting calculation for two values of $\nu$: i) $\nu = 0$, i.e. clusters following the scaling relation ($\nu_{\mu\gamma}$) and ii) $\nu = 1.5$, i.e. clusters deviating by $+1.5 \sigma_{\mu\gamma}$ from the mean relation. It demonstrates the reduced sensitivity of the survey towards clusters deviating up in richness. This matrix is used to fit the richness-mass relation (Kiiveri et al. in prep.) and to constrain cosmology using the richness function (Ider Chitham et al. subm.).

The modelling of the sample takes a mass function of clusters (for a discussion of our choice of cluster mass functions see Ider Chitham et al., subm.); predicts a covariant distribution of $L_X$ and richness for each mass value; associates shape parameters of the cluster with distribution of $L_X$; performs a calculation of the probability of cluster detection in each element of effective exposure and finally obtains a corresponding effective area, which is added to the total area. In the above equations, $\lambda$ corresponds to the true parameter, so only the intrinsic scatter in the $\mu - \lambda$ relation is taken into account, which for CODEX has been measured to be 0.2 (Capasso et al. 2019, Mulroy et al. 2019). In generating the SDSS richness, we need to account for the depth of the SDSS survey, using the scale value $\zeta$ (see Fig 5 for its redshift evolution), as discussed in Capasso et al. (2019a). This extra scatter is not covariant with X-ray properties, so we use $P(\nu_{SDSS}|A, \lambda, \nu_{SDSS}) = N(\nu_{SDSS} - A, \sqrt{\zeta^2})$, where we are accounting for an additional detail that the scatter in the observed richness is a function of true richness and not the mean true richness.

The expected number of clusters within a given photon count, $\Delta\eta_{\mu\gamma}$, and redshift, $\Delta z_c$, bin is:

$$
\langle N(\Delta H_{\mu\gamma}, \Delta A_{\nu_{\mu\gamma}}) \rangle = \frac{\int \Delta z_c dV d\Omega}{\int \Delta A_{\nu_{\mu\gamma}} dS \int \Delta H_{\mu\gamma} d\nu_{\mu\gamma} d\eta_{\mu\gamma}} \frac{d\eta_{\mu\gamma}}{\eta_{\mu\gamma}} . 
$$ (12)

where $\Omega$ is the geometric survey area in steradians and ($dV/d\Omega$) is the comoving volume element, and we will be using the calculation of Hogg (1999) for the flat Universe. The halo number density as a function of the observed photon counts $\eta_{\mu\gamma}$ - ($d\eta_{\mu\gamma}/d\nu_{\mu\gamma}$) - can be related to the theoretical halo mass function, $n(\mu, z)$, through:

$$
\frac{d\eta_{\mu\gamma}}{d\nu_{\mu\gamma}} = \frac{d\Omega}{d\nu_{\mu\gamma}} \frac{d\eta_{\mu\gamma}}{d\nu_{\mu\gamma}} . 
$$ (13)

where

$$
P(\lambda|\mu, z) = \int d\lambda_{\nu_{\mu\gamma}} dS_{\nu_{\mu\gamma}} P(\nu_{\nu_{\mu\gamma}}|A_{\nu_{\nu_{\mu\gamma}}}, \mu, z) . 
$$ (14)
RASS analysis is presence of the background, and our adopted procedure for the measurement of the observed count is not possible, due to the
we simulate the detection, we use the full count produced by a
⟨ distribution:
L which is a complementary error function (erfc). We also add an the selection threshold needed to clean the RASS data (Eq.1),
corresponds to a probability of the observed richness to exceed
richness, going from 10% for richness of 20 to 30% for richness
of 10.

The probability of the identification of a cluster predominantly through its AGN activity is driven by the probability of a cluster to host an AGN, which is given by the AGN halo occupation distribution (HOD), and a probability of AGN to have a certain luminosity, which is given by the AGN X-ray luminosity function (AGN XLF). We use the HOD results of [Allevato et al. 2012] and the [Ebrero et al. 2009] luminosity function for 0.5–2 keV to perform the calculation. The typical luminosity of AGN, calculated using the AGN XLF, is $10^{44} \text{ erg s}^{-1}$, with the probability of finding such an AGN in a cluster of 0.05x$(1+z)^{1.5}$. There is no dependence on halo mass at $M_{500c} > 10^{13} M_{\odot}$, predicted by the model. This modelling allows us to conclude that AGNs only provide a modest contamination to cluster luminosity, important only at lower redshifts, where our sensitivity is below the typical AGN luminosity. According to this modelling, the main contribution to cluster counts are AGNs detected in galaxy groups. High X-ray luminosity and low optical richness systems are therefore regarded as AGNs in groups or chance identification.

In Fig[7] we compare the measured cluster richness function with the prediction based on CDM and our AGN contamination model. AGN luminosity produces an additional component which at zero order is simply a fraction of all clusters of a given richness that we have not yet detected. The evolution of the fraction of the detected clusters as a function of redshift is due to two competing effects, evolution of the AGN XLF, and evolution of the threshold luminosity of AGN, which leads to a decreased AGN detectivity per cluster.

In addition to detection of new systems, AGNs can contribute to the total luminosity of the clusters, selected primarily by the ICM luminosity. This contribution is discussed in [Clerc et al. 2016] and is below the 10% level.

So, how do these results compare with contamination calculation of [Klein et al. 2019]? As we have mentioned the contaminated zone outlined in [Klein et al. 2019] corresponds to the 10% X-ray completeness curve in our calculation. There, truly detected clusters are 10%, while the rest 90% can be identified by chance (richness-dependent process), or by AGN activity (nearly richness independent). As we mentioned the AGN activity yields 2% identification, chance identification is at most 10% for richness of 20. The fractional importance of the contamination is therefore 50% of the total at lowest value of richness considered here, and drops to 9% at high redshift where it is dominated by AGN HOD while chance identifications are rare as number of clusters of high richness is low. This consideration allows us to conclude that contamination is indeed driven by the lack of real detections.

In Fig[8] we compare the richness distributions of CODEX clusters, its subsamples, based on flux and redshift and the literature sample, MCXC [Piffaretti et al. 2011], matched to CODEX clusters in order to obtain a richness estimate. The literature sample is primarily composed of the bright X-ray clusters and its distribution does not significantly change by imposing a cut on the flux of $10^{-12} \text{ ergs cm}^{-2} \text{ s}^{-1}$. We imposed a similar flux cut to the CODEX sample in order to illustrate the effect of a different flux. We also test a redshift cut of $z < 0.3$ on the CODEX sample to eliminate the effect of the noise in the optical data. The comparison points out that the literature sample of clusters systematically lacks identification of clusters of richness below 70 (obtained by restricting the comparison to both low z and changes in the chance identification rate with redshift, there is a strong increase in the chance identification towards low values of richness, going from 10% for richness of 20 to 30% for richness of 10.

4. Modelling of the association between X-ray source and optical cluster

We consider the following processes to result in the association between an X-ray source and the optical cluster
1. Chance association between an X-ray source and an optical cluster.
2. Detection of the optical cluster due to AGN activity of its member galaxies.
3. Detection of the optical cluster due to the thermal emission of ICM

The probability of chance identification has been calculated by placing random points on the sky and running the redMaPPer algorithm on them. We obtain the probability of chance identification as a function of redshift for two cuts in detected optical richness, 10 and 20. While within a factor of 1.5, there are no
Fig. 7. Richness function of CODEX clusters in the $0.1 < z < 0.3$ (left panel) and $0.1 < z < 0.6$ range (right panel). The solid grey histogram shows the data, the dotted histogram shows the contribution to the total counts from the clusters detected through their AGN activity, the dashed histogram show the contribution from clusters detected through their thermal emission and the solid histogram show the total expected number of detection, which provides a good match to the data. The dotted grey histogram shows the data with excision of points deviating beyond the $2\sigma$ from the richness–$L_X$ relation.

Fig. 8. Richness distribution of CODEX clusters (solid black histogram), compared to a subsample of CODEX clusters with flux above $10^{-12}$ ergs cm$^{-2}$ s$^{-1}$ (dashed black histogram), $z < 0.3$ CODEX clusters (dotted black histogram) and the matched MCXC clusters (solid grey histogram). The comparison shows a deficiency in the low richness identifications present in the MCXC catalog.

5. Evolution of cluster X-ray Luminosity Function

One of the direct measurements that CODEX provides is the evolution of the XLF of galaxy clusters. The evolution of XLF combines a decrease in the number of clusters of given mass with higher X-ray luminosity per given mass at higher redshifts. While, we are using red sequence redshifts in calculation of the X-ray luminosity, we have verified that use of spectroscopic redshift does not change the XLF (Clerc et al. subm.) and so we can omit the integration over the redshift uncertainties.

In Fig.9 we present CODEX constraints on the evolution of the cluster XLF measured in the redshift interval $0.1 < z < 0.6$ using bins of redshift of 0.1. We only show the data without strong (exceeding a factor of 2) completeness correction, as those are sensitive to both the adopted scaling relations and impact of the assumed distance-redshift relation on detection statistics. The completeness correction is calculated by rationing the predicted distribution of clusters on luminosity accounting for the sample properties, described above (Eqs[13] and the one assuming no selection effects and infinite statistics:

$$\langle N(\Delta l_{\text{true}}, \Delta z) \rangle = \int_{\Delta z} d\zeta \frac{dV(\Omega)}{dz} \int_M d\mu_{\text{true}} \int d\mu P(\mu_{\text{true}}|\mu, z) \frac{dn(\mu, z)}{dV d\mu}$$

(18)

The main results seen in Fig.9 are i) an agreement with the XLF determined from other low-$z$ ($z < 0.3$) studies, and ii) a lack of strong evolution of the XLF with redshift.

In order to compare the observed luminosity function with the expectations of different cosmological models, we need to adopt a mass calibration. At the moment, the $0.3 < z < 0.6$ part of the CODEX scaling relations have not yet tightly constrained, while the available studies extending to a redshift of one argue in favor of the self-similar scaling for these relations outside of the cool core (McDonald et al. 2019). So, here we will consider...
calibration for changing the $\Omega_M$ is not required in the scaling relations based on dynamical mass estimates done with respect to the critical density, as $M_{200c} \sim \sigma^3 \sqrt{\rho_c}$; assumption of the geometry of Universe in calculation of $L_X$ cancels out by the calibration procedure, which establishes the link to the mass and so does not need to be updated on average. And the mass is both measured and used in defining the mass function with the same scaling for the Hubble constant. However, since our modelling of the high-z sample considers a self-similar evolution of scaling relations, instead of direct determination, we need to allow for an effect of geometry of the Universe on $L_X$.

In Fig.10 we compare the observed number of clusters with the results of the modelling presented in this paper. In Fig.11 we present the associated cosmological parameters. We use two redshift bins to simplify the presentation of the results: low redshift bin $0.1 < z < 0.3$ and high redshift bin, $0.3 < z < 0.6$. We estimate the errors ($\sigma_i$) based on the measured number of clusters. Given the large area of the survey and the large redshift bins used, we can ignore the sample variance term in the mass-function calculation. In Fig.10 we plot the models that satisfy both high-z and low-z sample.

Smaller values of $\Omega_M$ predict slower evolution of the mass function and larger volume, partially compensated by the slower evolution of the scaling relation, and smaller cluster X-ray detected count-rates, which would be converted into smaller $L_X$ under a fixed cosmology. The normalization of the XLF can be adjusted by changing $\sigma_8$, but it is constrained by the slope of the XLF, which in our case is well measured only at low-z.

In calculating the best fit, we used a grid of models, covering values of $\Omega_m$ in the $0.1 - 0.4$ interval and the values of $\sigma_8$ in the $0.7 - 1.0$ interval. We compute the likelihood of the solution using $\chi^2$. The minimum of the

$$\chi^2 = \sum \frac{(N_i^{ob} - N(D_i^{ph}, \Delta_i))}{\sigma_i^2}$$

is comparable with the number of degrees of freedom and thus the solutions are statistically acceptable. To compute the error intervals on cosmological parameters, we use the deviations from the minimum. We quote the errors associated with two parameters of interest, so $1\sigma$ corresponds to a $\Delta \chi^2 = 2.3$.

The cosmology of low-z sample is comparable with previous similar studies (Bohringer et al. 2014), with the slight differences in the best-fit values primarily due to the adopted mass calibration, well within the reported associated uncertainties. Thus, our revision in the cluster identification did not result in the change of the cosmological constraints coming from low-z RASS surveys, with a possible exception of the work of Mantz et al. (2016), where we disagree on the adopted $M - L_X$ relation. Our relation has a 10% lower normalization in mass, coming from LoCuSS and CCCP studies (for a careful discussion of the problem, see Smith et al. 2016), and confirmed by the CODEX mass calibration efforts (Capasso et al. 2019) and Pfrikee et al. 2019). We deem our low-z calibration to be good to 5% in mass, which results in the associated systematic uncertainty in $\Omega_m$ of 0.015. For the discussion of comparison of low and high-z XLF, this uncertainty would shift the final solution, but does not result in a larger overlap in the solutions and so, we do not show it in Fig.11. The main result, seen in Fig.11 consists in finding a solution for the lack of evolution of the XLF, implied by our data. Within a flat $\Lambda$CDM and an assumption of self-similar evolution of scaling relations, the required cosmological parameters are $\Omega_m = 0.240 \pm 0.014 \pm 0.015$(syst.) and $\sigma_8 = 0.853 \pm 0.014 \pm 0.015$(syst.), quoting 68% confidence interval,
with a combined $\sigma_{\Omega_m}(0.3)^{0.2} = 0.815 \pm 0.013 \pm 0.015 \text{(syst.)}$. Comparison to the literature on galaxy clusters, our solution overlaps with the parameter space in overlap with all cluster surveys \cite{Boehringer2014, Henry2009, Bocquet2019, Vikhlinin2009} and Fig[11] can serve as a forecast for the importance of the calibration of scaling relation work at high-z.

6. Conclusions

We present a new large catalog of X-ray clusters detected in the SDSS area. The catalog is constructed with the aim of efficient spectroscopic follow-up program of SDSS, which has completed data acquisition process in March 2019 and this paper describes the catalog construction to accompany the final (DR16) data release.

Despite the low photon statistics of RASS, we show that we can convincingly model the survey selection function. We point out that low-richness clusters are underrepresented in the identification process of X-ray clusters, and this needs to be included in the modelling of the sample selection. We provide the forward modelling of such a selection and apply it to the sample to construct the X-ray luminosity functions of the survey. Our main result consists in the lack of evolution of cluster XLF, but the required solution in terms of adopted scaling relations and cosmology seems likely. As with most new cluster samples, more work on understanding the properties of clusters will serve towards improving the robustness of the results and uniqueness of CODEX consists in its largest calibration database on cluster dynamics, which is yet to be fully explored.

Acknowledgements

References

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