Constraints on AdS/CFT Gravity Dual Models of Heavy Ion Collisions

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(Dated: June 22, 2009)

We show that the five-fold constraints due to (1) the observed nuclear modification of heavy quark jets measured via non-photonic electrons $R_{AA}(p_T \sim 6\text{ GeV})$ in central Au+Au collisions at 200 AGeV, (2) the “perfect fluid” elliptic transverse flow of low transverse momenta pions, $v_2(p_T \sim 1\text{ GeV})$ reported for noncentral collisions, (3) the global pion rapidity density $dN_\pi/dy$, (4) the lattice QCD entropy density deficiency, $S/S_{SB}$, of strongly coupled Quark-Gluon Plasmas (sQGP), and (5) a causal requirement are analytically correlated in a class of gauge/string gravity dual models of sQGP dynamics. Current RHIC/BNL and lattice QCD data are found to be remarkably compatible with these models if the t’Hooft and Gauss-Bonnet coupling parameters lie in the range $\lambda \approx 10^{-25}$ and $0 < \lambda_{GB} < 0.09$. In addition, the observed five-fold correlation appears to favor color glass condensate over Glauber initial conditions within current systematic errors.

PACS numbers: 25.75.-q, 11.25.Tq, 13.87.-a

The combined observations of the quenching of hard (high transverse momentum or high quark mass jets) processes and the nearly “perfect fluid” elliptic flow of soft (low momentum transfer) hadrons produced in Au+Au collisions at $\sqrt{s} = 200\text{ AGeV}$ at the Relativistic Heavy Ion Collider (RHIC) [1] have been interpreted as providing evidence for the formation of a new form of strongly interacting quark-gluon plasma (sQGP) [2]. Furthermore, bulk multiplicity (entropy) production systematics have been interpreted as evidence for gluon saturation of the initial conditions as predicted by the Color Glass Condensate (CGC) model [3, 4]. However, it has been a challenge to find a single consistent theoretical framework that can explain simultaneously both soft and hard phenomena. These phenomena include 1) the nuclear modification of high transverse momenta, $p_T > 5\text{ GeV}$, jet observables, 2) the bulk collective flow observables ($p_T < 1\text{ GeV}$), and 3) the sQGP thermodynamic equation of state entropy deficiency relative to the ideal Stefan-Boltzmann limit as predicted by nonperturbative Lattice QCD (LQCD). Attempts to explain these based on weak coupling (perturbative QCD) parton transport approaches [5-8], especially the surprising small viscosity needed to fit the elliptic flow data, require large coupling extrapolations $\alpha_s = g^2_{YM}/4\pi \rightarrow 0.5 - 0.6$. At such large gauge couplings, on the other hand, the t’Hooft parameter $\lambda = g^2_{YM}N_c \gg 1$ may already be large enough to validate string theory inspired AdS/CFT low energy approximations [9-11].

The great theoretical advantage and appeal of gravity/gauge dual models [2-24] is that they have led (for the first time) to analytic connections between a wide variety of thermodynamic and nonequilibrium dynamic variables at strong coupling that were not yet realized with traditional gauge theory techniques.

In this Letter, we focus on predicted analytic connections between three fundamental properties of the sQGP: (1) its equation of state (entropy=)$S$, (2) its long wavelength transport coefficients (viscosity=)$\eta$, and (3) coupling between long wavelength near-equilibrium “soft medium” properties and short-wavelength non-equilibrated “hard probes” (energy loss per unit length, $dE/dx$). We show that these three properties together with global entropy and causality restrictions can provide valuable phenomenological constraints of higher dimensional gravity dual models of sQGP in heavy ion collisions. We consider here the constraints imposed by current RHIC and LQCD data on a class of gravity dual models that include quadratic as well as quartic curvature corrections to the classical Einstein-Hilbert action for the effective 5 dimensional gravity action. Therefore, we implement and extend the suggestion made in [10] to include perturbatively both the lowest order $O(\lambda_{GB} \sim 1/N_c)$ Gauss-Bonnet $\mathcal{R}^2$ and $O(1/\lambda^{3/2})\mathcal{R}^4$ curvature corrections to the three properties above.

Our analysis is based on the following remarkably simple algebraic expressions relating these three fundamental sQGP properties:

$$S/S_{SB} = \frac{3}{4} \left( 1 + \lambda_{GB} + \frac{15 \zeta(3)}{8 \lambda^{3/2}} \right), \quad (1)$$

$$\eta/s = \frac{1}{4\pi} \left( 1 - 4\lambda_{GB} + \frac{15 \zeta(3)}{\lambda^{3/2}} \right), \quad (2)$$

$$\tau^{-1}_Q = \mu_Q \left( 1 + \frac{3}{2} \lambda_{GB} + \frac{15 \zeta(3)}{16 \lambda^{3/2}} \right). \quad (3)$$

where $s = S/V$ is the entropy density. The heavy quark jet relaxation rate, $1/\tau_Q$, is controlled by $\mu_Q = \sqrt{\lambda T^2/2M_Q}$ for a heavy quark with mass $M_Q$ in a plasma of temperature $T$. The relaxation time is related to the energy loss per unit length through $\tau_Q(\lambda, \lambda_{GB}) = -1/(d\log p/dt) = -1/(d\log E/dx)$, where $p = M_Q\gamma v$ and $v = p/E$.

We note that the $\mathcal{R}^4$ correction $O(1/\lambda^{3/2})$ to the heavy quark jet energy loss is a new result [25] reported in this Letter and it is needed for a consistent additive perturba-
The causality bound curve with $\alpha'/\lambda T \ll 1$ is the background metric, and $X^{\mu} = X^{\mu}(\tau, \sigma)$ is the induced worldsheet metric, $\sigma^a = (\tau, \sigma)$ are the internal worldsheet coordinates, $G_{\mu\nu}(X)$ is the background metric, and $X^{\mu} = X^{\mu}(\tau, \sigma)$ is the embedding of the string in spacetime. The trailing string ansatz (where $\tau = t, \sigma = u$ and $X^{\mu}(t, u) = (t, x_0 + v t + \xi(u), 0, 0, u)$) describes the asymptotic behavior of a string attached to a moving heavy quark (the string endpoint) with velocity $v$ in the $x$ direction and located at a fixed AdS radial coordinate $u_0 \gg u_0$. The black brane horizon coordinate $u_0 \propto T_0$ is determined by $G_{00}(u_0) = 0$. Using the ansatz above and the string’s classical equations of motion, one can show that the drag force $dp/dt = -C v/(2\alpha')$, where $C$ is a constant determined by the negativity condition that

$$g(u) = G_{uu} (G_{00} + v^2 G_{xx}) \left(1 + \frac{C^2 u^2}{G_{00} G_{xx}}\right)^{-1} < 0$$

for $u_h \leq u \leq u_m$. However, both the numerator and denominator in Eq. (4) change their sign simultaneously at a certain $u^* \geq \frac{\pi}{\sqrt{\lambda T}}$, given by the root of the equation $G_{00}(u^*) + v^2 G_{xx}(u^*) = 0$. This fixes $C = G_{xx}(u^*)$ and $dp/dt = -v G_{xx}(u^*)/(2\alpha')$. Neglecting higher-order derivative corrections in $N_c = 4$ SYM one finds $u^* = u_h \sqrt{\gamma}$, where $\gamma = 1/\sqrt{1 - v^2}$. The condition that $u^* \leq u_m$ leads to a maximum “speed limit” for the heavy quark jet to be consistent with this trailing string ansatz given by $\gamma_{\text{max}} \leq u_m^2/u_h^2$.

Using the metric derived in [10] to $O(\alpha'^3)$, one can compute the effects of quartic corrections on the drag force [13] and determine $u^*$ perturbatively to $O(\lambda^{-3/2})$ as [26] $u^* = u_h \sqrt{\gamma} \left[1 + \frac{15(3)}{16} \left(5 + \frac{5}{\gamma} \sqrt{1 - \gamma}\right)\right]$ and the drag force

$$\frac{dp}{dt} = -\sqrt{\lambda} T^2 \frac{\pi \gamma}{2} \left[1 + \frac{15(3)}{16} \left(1 - \frac{197}{24\gamma} + \frac{67}{24\gamma^2}\right)\right].$$

The heavy quark mass at $T = 0$ is $M_Q = u_m/(2\alpha')$ and, to leading order in $1/\lambda$, $u_m^2/u_h^2 \approx \frac{4M_Q^2}{M_{\text{YM}}^2}$. Thus, the corrected $u^*$ displayed above defines a new speed limit $\gamma_{\text{m}} \approx \frac{4M_Q^2}{M_{\text{YM}}^2} \left[1 - \frac{1}{\gamma} \left(\frac{4M_Q^2}{M_{\text{YM}}^2} - 1\right)\right]$, after neglecting terms of $O(1/\gamma, 1/N_c)$. Note that $\gamma_{\text{m}}$ and $dp/dt$ decrease with increasing $\eta/s$.

For our applications, we consider the range $\lambda \sim 5 - 30$ and $|\lambda_{GB}| \lesssim 0.1$ with fixed $N_c = 3$. In this parameter range the $1/\lambda^{3/2}$ and $\lambda_{GB} \sim 1/N_c$ corrections are comparable. We neglect known (but formally) higher-order terms $O(\sqrt{N_c}/N_c^2)$ in this first attempt to test predicted dynamical correlations between hard and soft phenomena in high energy A+A collisions.

The small Gauss-Bonnet parameter $\lambda_{GB} = (c - a)/4c_e$ is related to the central charges $c$ and $a$ (related to the conformal anomaly in curved spacetime) of the dual CFT as noted in Eq. (2.14) of Ref. [10]. Varying $\lambda_{GB}$ provides a parametric way to explore deformations of the original $N_c = 4$ SYM theory. Interest in Gauss-Bonnet deformations was heightened when Kats and Petrov [21] argued that for $N_c = 2 Sp(N_c)$, $\lambda_{GB} = 1/8N_c$, the KSS viscosity bound on $\eta/s \geq 1/4\pi$ was violated by 17% for $N_c = 3$. As further shown in [16], a large class of other effective CFTs are now known to lead to similar $\lambda_{GB} \propto 1/N_c$ effects. However, the analysis of Refs. [18-20] revealed that $\lambda_{GB}$ deformations are limited by requirements of causality and positive energy flow to a narrow range $-7/36 < \lambda_{GB} < 9/100$.

In order to convert $T_Q$ into the observed nuclear modification of single non-photonic electrons, $R_{AA}^{p_t}(pt = 5.5 \text{ GeV})$, from quenched heavy quark jets, we follow and extend [27] by using here the generalized drag force
FIG. 2: (Color online) Five fold phenomenological constraints in the t’Hooft and Gauss-Bonnet parameter space ($\lambda, \lambda_{GB}$). (1) The green region with black dashed contours is from lattice QCD constraints on $0.8 < S/S_{SB} < 0.9$. (2) The cyan region is determined from noncentral elliptic flow $v_2(p_T = 1, 20 – 60\%) = 0.11 \pm 0.01$ [33]. Blue dashed contours correspond to fixed $4\pi\eta/s = 1, 1.5, 2$. The inversion $\lambda_{GB}(\lambda, v_2(\eta/s))$ is based on minimum bias viscous hydro results of [31] assuming CGC initial eccentricities scaled by a factor 1.1 in panel (a) and unscaled 1.0 in panel (b). The entropy is constrained by the (3) $dN_{\pi}/dy = 1000$ pion rapidity density in central collisions. (4) The gray region and contours are determined from central $R^{AA}_{min}(p_T = 5.5\text{ GeV}) = 0.25 \pm 0.07$ data [32]. (5) The horizontal red line constraint is the causality upper bound for $\lambda_{GB}^{max} = 0.09$ [13, 20]. The yellow trapezoidal region with the purple boundary is the intersection of the five fold constraint bands. Note the red circled five fold conjunction area in panel (a) ($\lambda \approx 13, \lambda_{GB} \approx 0.08$) that is absent in the unscaled panel (b).

in Eq. (4) to compute the path length, $L$, dependent heavy quark fractional energy loss $\epsilon(L)$. The heavy quark jet nuclear modification factor is then $R^{AA} = \langle 1 - \epsilon \rangle_{R^{AA}}$, where $\epsilon(p_T)$ is the flavor dependent spectral index $n_{Q} + 1 = -\frac{d}{dp_T} \ln \left( \frac{\sigma_{NN}}{\sigma_{np}} \right)$ obtained from FONLL production cross sections [28] as used in [27]. The path length average of the nuclear modification at impact parameter $b$ is computed using a Woods-Saxon nuclear density profile with Glauber profiles $T_A(\vec{x})$ with $\sigma_{NN} = 42$ mb. For $0-10\%$ centrality triggered data both Glauber and CGC geometries lead to similar numerical results [29]. The distribution of initial hard jet production points at a given $\vec{x}$ and azimuthal direction $\phi$ is taken to be proportional to the binary parton collision density, $T_{AA}(\vec{x}, b)$. We assume a longitu-
sion. We find that a 10% reduction of the viscous hydrodynamic $\nu_2(q/s)$ (as for minimum bias centrality) virtually eliminates the yellow overlap region and could falsify the AdS/CFT gravity dual model. Parameters could be achieved. We close by emphasizing that future comparison of the nuclear modification of identified bottom and charm quark jets at RHIC and LHC combined with the fivefold (hard/soft) constraints considered in this Letter will provide especially stringent tests of AdS/CFT gravity dual phenomenology applied to high energy heavy ion reactions.

We thank A. Dumitru, S. Gubser, W. Horowitz, A. Poszanzer, B. Cole, and W. Zajc for useful comments. J.N. and M.G. acknowledge support from US-DOE Nuclear Science Grant No. DE-FG02-93ER40764. G.T. acknowledges support from the Helmholtz International Center for FAIR within the framework of the LOEWE program (Landesoffensive zur Entwicklung Wissenschaftlich-Ökonomischer Exzellenz) launched by the State of Hesse.

[1] I. Arsene et al. [BRAHMS Collaboration], Nucl. Phys. A 757, 1 (2005); B. B. Back et al., [PHOBOS Collaboration] Nucl. Phys. A 757, 28 (2005); J. Adams et al. [STAR Collaboration], Nucl. Phys. A 757, 102 (2005); K. Adcox et al. [PHENIX Collaboration], Nucl. Phys. A 757, 184 (2005).

[2] M. Gyulassy and L. McLerran, Nucl. Phys. A 750, 30 (2005); E. V. Shuryak, Nucl. Phys. A 750, 64 (2005).

[3] D. Kharzeev, E. Levin and M. Nardi, Phys. Rev. C 71, 054903 (2005); L. D. McLerran and R. Venugopalan, Phys. Rev. D 49, 2233 (1994).

[4] A. Dumitru, E. Molnar, and Y. Nara, Phys. Rev. C 76, 024910 (2007).

[5] P. Danielewicz and M. Gyulassy, Phys. Rev. D 31, 53 (1985).

[6] D. Molnar and M. Gyulassy, Nucl. Phys. A 697, 495 (2002) [Erratum-ibid. A 703, 893 (2002)].

[7] A. Majumder, B. Muller and X. N. Wang, Phys. Rev. Lett. 99, 192301 (2007).

[8] O. Fochler, Z. Xu and C. Greiner, arXiv:0806.1169 [hep-ph].

[9] J. M. Maldacena, Adv. Theor. Math. Phys. 2, 231 (1998); E. Witten, Adv. Theor. Math. Phys. 2, 253 (1998); S. S. Gubser, I. R. Klebanov and A. M. Polyakov, Phys. Lett. B 428, 105 (1998).

[10] S. S. Gubser, I. R. Klebanov and A. A. Tseytlin, Nucl. Phys. B 534, 202 (1998).

[11] A. Buchel, J. T. Liu and A. O. Starinets, Nucl. Phys. B 707, 56 (2005); A. Buchel, Nucl. Phys. B 803, 166 (2008).

[12] A. Buchel, Nucl. Phys. B 803, 166 (2008); R. C. Myers, M. F. Paulos and A. Sinha, Phys. Rev. D 79, 041901 (2009); A. Buchel, R. C. Myers, M. F. Paulos and A. Sinha, Phys. Lett. B 669, 364 (2008).

[13] C. P. Herzog, A. Karch, P. Kovtun, C. Kozcaz and L. G. Yaffe, JHEP 0607, 013 (2006); S. S. Gubser, Phys. Rev. D 74, 126005 (2006); J. Casalderrey-Solana and D. Teaney, Phys. Rev. D 74, 085012 (2006).

[14] J. Casalderrey-Solana and D. Teaney, JHEP 0704, 039 (2007); S. S. Gubser, Nucl. Phys. B 790, 175 (2008).

[15] J. F. Vazquez-Poritz, arXiv:0803.2890 [hep-th].

[16] A. Buchel, R. C. Myers and A. Sinha, JHEP 0903, 084 (2009).

[17] A. Karch and E. Katz, JHEP 0206, 043 (2002).

[18] M. Brigante, H. Liu, R. C. Myers, S. Shenker and S. Yaida, Phys. Rev. D 77, 126006 (2008).

[19] M. Brigante, H. Liu, R. C. Myers, S. Shenker and S. Yaida, Phys. Rev. Lett. 100, 191601 (2008).

[20] D. M. Hofman and J. Maldacena, JHEP 0805, 012 (2008).

[21] Y. Kats and P. Petrov, arXiv:0712.0743 [hep-th].

[22] P. K. Kovtun, D. T. Son and A. O. Starinets, Phys. Rev. Lett. 94, 111601 (2005).

[23] R. G. Cai, Phys. Rev. D 65, 084014 (2002).

[24] K. B. Fadafan, JHEP 0812, 051 (2008).

[25] J. Noronha et al, to be published.

[26] The drag force in this case had been computed numerically in Ref. [15] but the analytical expression in Eq. (5) is, as far as we know, a new result.

[27] W. A. Horowitz and M. Gyulassy, Phys. Lett. B 666, 320 (2008).

[28] M. Cacciari, P. Nason, R. Vogt, Phys. Rev. Lett. 95, 122001 (2005); M. L. Mangano, P. Nason, G. Ridolfi, Nucl. Phys. B 373, 295 (1992).

[29] A. Adil, H. J. Drescher, A. Dumitru, A. Hayashigaki and Y. Nara, Phys. Rev. C 74, 044905 (2006).

[30] S. S. Gubser, Phys. Rev. D 76, 126003 (2007).

[31] M. Luzum and P. Romatschke, Phys. Rev. C 78, 034915 (2008); Erratum-ibid. 79, 039903 (E) (2009).

[32] P. F. Kolb, U. W. Heinz, P. Huovinen, K. J. Eskola, and K. Tuominen, Nucl. Phys. A 696, 197 (2001).

[33] K. Adcox et al. [PHENIX Collaboration], Phys. Rev. Lett. 89, 212301 (2002); D. d’Enterria, private communication.

[34] A. Bazavov et al., arXiv:0903.4379 [hep-lat]; M. Cheng et al., Phys. Rev. D 77, 014511 (2008); C. Bernard et al., Phys. Rev. D 75, 094505 (2007); Y. Aoki, Z. Fodor, S. D. Katz and K. K. Szabo, JHEP 0601, 089 (2006).

[35] S. S. Adler et al. [PHENIX], Phys. Rev. Lett. 96, 032301 (2006); A. Adare et al. [PHENIX], Phys. Rev. Lett. 98, 172301 (2007).

[36] B. I. Abelev et al. [STAR], Phys. Rev. Lett. 98, 192301 (2007).

[37] A. Afanasiev et al. [PHENIX Collaboration], arXiv:0905.1070 [nucl-ex].