A NEW METHOD FOR IDENTIFYING CHANGING MARITAL PREFERENCES FOR RACE AND EDUCATION LEVEL

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11/March/2021

In this paper, I generalize the Naszodi–Mendonca method in order to identify changes in marital preferences over multiple dimensions, such as the partners’ race and education level. Similar to the original Naszodi–Mendonca method, preferences are identified by the generalized method through estimating their effects on marriage patterns, in particular, on the share of inter-racial couples, and the share of educationally homogamous couples. This is not a simple task because marriage patterns are shaped not only by marital preferences, but also by the distribution of marriageable males and females by traits. The generalized Naszodi–Mendonca method is designed for constructing counterfactuals to perform the decomposition. I illustrate the application of the generalized Naszodi–Mendonca method by decomposing changes in the prevalence of racial and educational homogamy in the 1980s using US data from IPUMS.

JEL: J12, C02.

Keywords: Assortative Mating; Counterfactual Decomposition; Educational Homogamy; Racial Endogamy.

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Abstract

In this paper, I generalize the Naszodi–Mendonca method in order to identify changes in marital preferences over multiple dimensions, such as the partners’ race and education level. Similar to the original Naszodi–Mendonca method, preferences are identified by the generalized method through estimating their effects on marriage patterns, in particular, on the share of inter-racial couples, and the share of educationally homogamous couples. This is not a simple task because marriage patterns are shaped not only by marital preferences, but also by the distribution of marriageable males and females by traits. The generalized Naszodi–Mendonca method is designed for constructing counterfactuals to perform the decomposition. I illustrate the application of the generalized Naszodi–Mendonca method by decomposing changes in the prevalence of racial and educational homogamy in the 1980s using US data from IPUMS.
1. INTRODUCTION

There is a broadly shared view that the degree of marital sorting along race has been decreasing over the past five decades in the US\footnote{The typical finding of the empirical studies is that inter-racial marriages have become more and more acceptable in the American society since 1967, when the US Supreme Court decided that “anti-miscegenation” laws are unconstitutional (see: \cite{Wang2012}).}. By contrast, the views on the trend of sorting along the economic dimension are diverse. The lack of consensus among the empirical studies is partly due to the diversity in how the economic dimension is represented empirically (e.g., by wealth, current year income, permanent income, ability to generate income, etc.). However, there is no consensus even among those papers that study sorting along individual’s income generating ability proxied by their final educational attainment (see \cite{Rosenfeld2008}).

For instance, \cite{Greenwood2014} find that marital sorting along the educational had a positive trend in the 1980s. They quantify the trend with a regression coefficient obtained by regressing wives’ years of education on husbands’ years of schooling and using multiple cross-sectional data from different years. By contrast, \cite{Naszodi2021} find that marital sorting along the educational had a negative trend over the same decade. They quantify it by the matrix-valued generalized Liu–Lu measure projected on the scalar-valued share of educationally homogamous couples. \cite{Eika2019} also provide examples for the diverging empirical findings. They calculate the time trend of six measures commonly applied in the assortative mating literature. Some of the six measures suggest that marital sorting along the educational was somewhat decreasing, while some others suggest that it was unchanged in the 1980s.

\cite{Eika2019} study educational sorting for different races separately. However, they do not analyze marital sorting along race and education jointly. This paper contributes to the literature with developing a methodology suitable for such an analysis and illustrates the application of the methodology using US data from IPUMS on marriages (and cohabitations).
Studying marital sorting along race and education jointly is motivated by the example of a hypothetical society. In this society, all men and women marry someone from the opposite sex from their own generation. The education level of a new generation is not different from that of an old generation. People of a particular race tend to have higher educational attainment than people of another race both in the new generation and the old one. The members of the new generation are more picky about the education level of their spouses than the members of the old generation. However, they sort along race exactly as much as people in the old generation did.

One can mistakenly find that racism is on the rise in this hypothetical society by studying inter-racial marriages without controlling for the increase in the degree of sorting along the educational. Conversely, provided that racism is in decline, while educational sorting is not, one can mistakenly find the degree of educational sorting to be decreasing if not controlling for changes in sorting along race.

Assessing how marital preferences over the partners’ race and education change is even more challenging in a society, where the race-specific and gender-specific educational distributions of married people can also vary across generations.

The main contribution of this paper is methodological. It proposes a new method for studying marital preferences over multiple dimensions while also controlling for changes in availability of marriageable men and women with various traits. The new method is the generalized version of the method developed by Naszodi and Mendonca (2021) (henceforth GNM method). The original Naszodi–Mendonca method (henceforth NM method) builds on Naszodi and Mendonca (2021).

The Appendix reports the result of the decomposition for the longer period between 1960 and 2015.

In addition, such an analysis is even more challenging once the possibility of remaining single is also taken into account. About this point see Naszodi and Mendonca (2019) and Appendix X of this paper.
the measure of assortative mating proposed by Liu and Lu (2006). The NM method works under the assumption that people sort along one dimension on the marriage market. This dimension can be, for instance, race (provided it is captured by a dichotomous variable), or education level (provided it is captured by an ordered categorical variable).

The seminal papers by Galichon and Salanié (2021), Chiappori, Oreffice, and Quintana-Domeque (2011), and Rosenfeld (2008) also belong to the multidimensional matching strand of the assortative mating literature. In the model by Galichon and Salanié (2021), the surplus from a marriage match depends on the partners’ income and education. Chiappori et al. (2011) provide a closed form solution of a multidimensional matching model and then test predictions of how spouses trade off education and non-smoking. Rosenfeld (2008) examines sorting along three dimensions (race, education, and religion) in the US by using odds-ratios.

This paper distinguishes itself from the existing literature by building on the Liu–Lu measure instead of using either of the conventional measures (the marital surplus, or the odds ratio). The choice of characterizing marital preferences at the societal level with the Liu–Lu measure is motivated by the empirical and analytical arguments of Liu and Lu (2006) and Naszodi and Mendonca (2021).

The empirical part of this paper confirms the commonly held view: racial segregation in the US has declined over the analyzed decade. Moreover, I find marital sorting along the educational to be also decreasing in the 1980s: young American adults in the 1990s (belonging to the generation of late boomers) are found to be less picky with respect to their spouses’ education level than young American adults in the 1980s (belonging to the early boomers). These trends are found to be robust to controlling for sorting along the other assorted trait. This robustness confirms that the declining trend identified by Naszodi and Mendonca (2021), is

4In particular, Liu and Lu (2006) claim that their measure can control for changes in the trait distribution, while other measures cannot. Naszodi and Mendonca (2021) and Naszodi (2021) show that survey evidence from the Pew Research Center are also in favor of measuring assortativity in preferences a la Liu and Lu.
not due to the omission of the racial factor.

This finding has the following significance. First, Naszodi and Mendonca (2021) used their findings for the 1980s for method selection. So, the robustness of their finding supports their method selection. Second, the robustness supports the view that segmentation of the marriage market along the economic dimension (proxied by the education level) cannot only increase, but it can decrease as well.

The rest of the paper is structured as follows. Section 2 presents some characterizations of the equilibrium in the marriage market. Section 3 presents the methodology suitable to identify changes in marital preferences: it introduces the decomposition scheme and the original Naszodi–Mendonca method (applicable to study sorting along a single dimension), and develops the generalized Naszodi–Mendonca method suitable for studying sorting along multiple dimensions. Section 4 illustrates the application of the generalized Naszodi–Mendonca method by using US data from 1980 and 1990. Finally, Section 5 concludes the paper.

2. CHARACTERIZING THE EQUILIBRIUM IN THE MARRIAGE MARKET

In this paper, I do not distinguish between officially married couples and couples in consensual union. Hereafter, by “marriage” I mean both types of unions. Accordingly, I define “wives” and husbands” broadly.

The educational trait variable of husbands and wives is an ordered categorical variable that can take three possible values. Its value $L$ stands for “low level of education” corresponding to not having completed the high school; $M$ denotes “medium level of education” corresponding to having a high school degree, but neither a college degree nor a university

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5This decade broke the empirical equivalence of the NM method and some highly popular conventional methods, such as the Iterative Proportional Fitting (IPF) algorithm and the Choo–Siow model based method.
degree; and \( H \) stands for “high level of education” corresponding to holding at least a BA diploma.

The race variable can take two possible values. In the benchmark specification, race is either Black (\( B \)) or White (\( W \)). (In the alternative specifications, race can be Black (\( B \)) or Not-Black (\( NB \)), or it can be White (\( W \)) or Not-White (\( NW \)).

Accordingly, in the benchmark case, we characterize the matching outcome by the contingency Table 1. Let us denote this table by \( K \). Its element \( N_{h,w} \) is the number of \( h, w \) type marriages with \( h, w \in \{ WL, WM, WH, BL, BM, BH \} \), where \( h \) denotes the husbands’ type and \( w \) denotes the wives’ type. Ones’ type is given by ones’ race and education level.

### Table 1: The contingency table

| Husband/male partner | Wife/female partner | Edu. | L | M | H | Total |
|----------------------|---------------------|------|---|---|----|-------|
| Black                | L                   | \( N_{BL, BL} \) | \( N_{BL, BM} \) | \( N_{BL, BH} \) | \( N_{BL, WL} \) | \( N_{BL, WM} \) | \( N_{BL, WH} \) | \( N_{BL, \cdot} \)
|                     | M                   | \( N_{BM, BL} \) | \( N_{BM, BM} \) | \( N_{BM, BH} \) | \( N_{BM, WL} \) | \( N_{BM, WM} \) | \( N_{BM, WH} \) | \( N_{BM, \cdot} \)
|                     | H                   | \( N_{BH, BL} \) | \( N_{BH, BM} \) | \( N_{BH, BH} \) | \( N_{BH, WL} \) | \( N_{BH, WM} \) | \( N_{BH, WH} \) | \( N_{BH, \cdot} \)
|                     | L                   | \( N_{WL, BL} \) | \( N_{WL, BM} \) | \( N_{WL, BH} \) | \( N_{WL, WL} \) | \( N_{WL, WM} \) | \( N_{WL, WH} \) | \( N_{WL, \cdot} \)
|                     | M                   | \( N_{WM, BL} \) | \( N_{WM, BM} \) | \( N_{WM, BH} \) | \( N_{WM, WL} \) | \( N_{WM, WM} \) | \( N_{WM, WH} \) | \( N_{WM, \cdot} \)
|                     | H                   | \( N_{WH, BL} \) | \( N_{WH, BM} \) | \( N_{WH, BH} \) | \( N_{WH, WL} \) | \( N_{WH, WM} \) | \( N_{WH, WH} \) | \( N_{WH, \cdot} \)
| Total               | \( N_{BL} \)       | \( N_{BM} \)    | \( N_{BH} \)    | \( N_{WL} \)    | \( N_{WM} \)    | \( N_{WH} \)    | \( N_{\cdot} \)    |

Once, we know the contingency table \( K \) (i.e., the joint distribution of wives and husbands by race and education level), we can characterize the equilibrium on the marriage market by some scalar-valued indicators. For instance, we can characterize it by the share of educationally homogamous couples calculated as \( \text{SEHC}(K) = (N_{BL, BL} + N_{BL, WL} + N_{WL, BL} + N_{WL, WL} + N_{BM, BM} + N_{BM, WM} + N_{WM, BM} + N_{WM, WM} + N_{BH, BH} + N_{BH, WH} + N_{WH, BH} + N_{WH, WH})/N_{\cdot, \cdot} \), where \( N_{\cdot, \cdot} \) denotes the total number of couples.

Also, we can characterize the equilibrium by the share of inter-racial marriages: \( \text{SIRM}(K) = \)

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\( ^6 \)The reason for not considering more than two races is that I want to keep the number of parameters low.
1 - \( (N_{B,B} + N_{W,W})/N \)

It is worth to note that \( SEHC(K) \) and \( 1 - SIRM(K) \) characterize the “prevalence of educational and racial homogamy”, not the “preference for educational and racial homogamy”. The next section describes how preferences can be characterized/identified.

3. IDENTIFYING MARITAL PREFERENCES

Identifying changes in “preference for educational homogamy” and “preference for racial homogamy” is a challenging task. The challenge stems from the fact that preferences are not directly observable. Therefore, we need to identify them through their effects on observable variables, such as \( SEHC(K) \) and \( SIRM(K) \).

Both \( SEHC(K) \) and \( SIRM(K) \) are driven by multiple factors. In this paper, I control for the effects of some factors, other than preferences, with a decomposition. For the decomposition, we have to construct counterfactuals and apply a decomposition scheme.

Naszodi and Mendonca (2021) propose a method for constructing counterfactuals in a simple setup. Their method, the NM method, is suitable for studying sorting when the assorted trait is a one-dimensional categorical variable. The categorical assorted trait variable has to be either dichotomous (e.g. Black/White, or Black/Not-Black, or White/Not-White), or, if it can take more than two possible values, it has to be ordered (e.g. the level of educational attainment, or income quantile, or skier ability level, etc.).

In Subsection 3.3, I generalize the NM method to two assorted traits after I describe the decomposition scheme in Subsection 3.1 and briefly introduce the NM method in Subsection 3.2.

3.1. Decomposition scheme

In the empirical analysis, I use the decomposition scheme promoted by Biewen (2012). It is the path-independent version of the Oaxaca–Blinder decomposition scheme. It is free from
any assumption on the sequence of changes in the factors.

For two factors, it is given by

\[
f(A_1, P_1) - f(A_0, P_0) = \underbrace{[f(A_1, P_0) - f(A_0, P_0)]}_{due to \Delta A} + \underbrace{[f(A_0, P_1) - f(A_0, P_0)]}_{due to \Delta P} + \underbrace{[f(A_1, P_1) - f(A_1, P_0) - f(A_0, P_1) + f(A_0, P_0)]}_{due to the joint effect of \Delta A and \Delta P}.
\] (1)
For three factors, it is

\[
f(A_1, PR_1, PE_1) - f(A_0, PR_0, PE_0) = \underbrace{[f(A_1, PR_0, PE_0) - f(A_0, PR_0, PE_0)]}_{\text{due to } \Delta A (\text{availability})} \\
+ \underbrace{[f(A_0, PR_1, PE_0) - f(A_0, PR_0, PE_0)]}_{\text{due to } \Delta PR (\text{preferences toward race})} \\
+ \underbrace{[f(A_0, PR_0, PE_1) - f(A_0, PR_0, PE_0)]}_{\text{due to } \Delta PE (\text{preferences toward education})} \\
+ \underbrace{[f(A_1, PR_1, PE_1) - f(A_0, PR_1, PE_0) + f(A_0, PR_0, PE_0) - f(A_0, PR_1, PE_0) + f(A_0, PR_0, PE_0)]}_{\text{interaction term capturing the joint effect of } \Delta PR \text{ and } \Delta A} \\
+ \underbrace{[f(A_1, PR_0, PE_1) - f(A_0, PR_0, PE_0) - f(A_1, PR_0, PE_0) + f(A_0, PR_0, PE_0) - f(A_0, PR_1, PE_0) + f(A_0, PR_0, PE_0)]}_{\text{interaction term capturing the joint effect of } \Delta PE \text{ and } \Delta A} \\
+ \underbrace{[f(A_0, PR_1, PE_1) - f(A_0, PR_0, PE_0) - f(A_0, PR_1, PE_0) + f(A_0, PR_0, PE_0) - f(A_0, PR_0, PE_1) + f(A_0, PR_0, PE_0)]}_{\text{interaction term capturing the joint effect of } \Delta PE \text{ and } \Delta PR} \\
+ \underbrace{[f(A_1, PR_1, PE_1) - f(A_0, PR_1, PE_0) + f(A_0, PR_0, PE_0) - f(A_0, PR_1, PE_0) + f(A_0, PR_0, PE_0)]}_{\text{residuum}}
\]

In our specific setting, function \( f(A_t, PR_t, PE_t) \) denotes either the observable share of educationally homogmous couples \( \text{SEHC}(K_t) \), or the observable share of inter-racial marriages \( \text{SIRM}(K_t) \) at time \( t \in \{0,1\} \). Under the assumption that the search and matching mechanism is frictionless, these shares are functions of the following three factors: (i) the observable availability \( A_t \), i.e., the educational and racial distributions of marriageable men
and women at time \( t \); (ii) the unobservable preferences over the partners’ race \( PR_t \); and (iii) the unobservable preferences over the partners’ education level \( PE_t \).

Finally, \( f(A_1, PR_1, PE_0), f(A_1, PR_0, PE_1), f(A_0, PR_1, PE_1), f(A_1, PR_0, PE_0), f(A_0, PR_1, PE_0), f(A_0, PR_0, PE_1) \) denote the shares under each possible counterfactuals, where the three factors are not measured contemporaneously.

Apparently, the challenge of identifying the changes in the unobservable factors through their effects on the observable shares boils down to determining the counterfactual shares in Equation 2. The next section introduces how the corresponding counterfactual contingency tables can be constructed.

3.2. The original Naszodi–Mendonca method for constructing counterfactual contingency tables

The NM method transforms a contingency table observed at time \( t_p \) into another contingency table representing the equilibrium matching outcome under a counterfactual. Under the counterfactual, marital preferences are the same as at time \( t_p \), while the structural availability (i.e., the marginal distributions) are measured at time \( t_a \neq t_p \).

For the NM transformation, marital preferences are captured by the scalar-valued measure developed by Liu and Lu (2006) if the assorted trait is a dichotomous variable (taking the values \( L \) or \( H \)) and the contingency table \( Z_{2-by-2} \) is a 2-by-2 matrix. If sorting is positive (i.e., the share of homogamous couples is higher than it would be under random matching) then the Liu–Lu measure is given by the simplified Liu–Lu measure:

\[
LL_{\text{sim}}(Z_{2-by-2}) = \frac{N_{H,H} - Q^-}{\min(N_{H,:}, N_{:,H}) - Q^-}.
\]

The NM transformation method is implemented in Excel, Visual Basic, and R. It can be downloaded from [http://dx.doi.org/10.17632/x2ry7bcm95.2](http://dx.doi.org/10.17632/x2ry7bcm95.2)
where

\[
Z^{2\times2} = \begin{bmatrix}
N_{L,L} & N_{L,H} \\
N_{H,L} & N_{H,H}
\end{bmatrix},
\]

(4)

\(N_{H,H} (N_{L,L})\) denotes the number of couples where both spouses are high (low) educated. \(N_{L,H} (N_{H,L})\) stands for the number of couples where the husband (wife) is low educated, while the wife (husband) is high educated. \(N_{L,H} = N_{H,H} + N_{H,L}\), similarly, \(N_{H,H} = N_{L,H} + N_{H,L}\) and \(N_{H,H} = N_{L,H} + N_{H,L}\). While \(Q = N_{H,H} / N_{H,H}\) is the expected number of \(H,H\)-type couples under random matching. Furthermore, \(Q^-\) is the biggest integer being smaller than or equal to \(Q\).

In the general case, the one-dimensional assorted trait distribution can be gender-specific. For instance, it is possible that the market distinguishes between \(n\) different education levels of men, and \(m\) different education levels of women. In addition, the joint distribution of the gender-specific assorted traits can vary over time. So, in the general case, the market equilibrium at time \(t\) is represented by the contingency table \(Z_t\) of size \(n \times m\).

If the two gender-specific assorted trait variables are ordered categorical polytomous variables, then Naszodi and Mendonca characterize marital preferences at time \(t\) by the matrix-valued generalized Liu–Lu measure. Its \((i, j)\)th element is

\[
LL_{i,j}^{\text{gen}}(Z_t) = LL_{i,j}^{\text{sim}}(V_i Z_t X_j^T),
\]

(5)

where \(V_i\) is the \(2 \times n\) matrix

\[
V_i = \begin{bmatrix}
1 & \cdots & 1 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 1 & \cdots & 1
\end{bmatrix}
\]

and \(X_j^T\) is the \(m \times 2\) matrix given by

The statistical interpretation of \(LL^{\text{sim}}(Z)\) is that it is the normalized distance between the realized matching outcome \(K\) and a benchmark outcome where individuals are randomly matched. \(\text{Shen (2019)}\) proposes the so called “Perfect-Random normalization measure”, a slightly modified version of the Liu–Lu measure. In her formula \(Q^-\) is replaced by \(Q\).
The NM-transformed contingency table is \( \text{NM}(Z_{tp}, Z_{ta}) = Z^*_{tp,ta} \), where the preferences are measured at time \( t_p \): \( \text{LL}^{\text{gen}}(Z^*_{tp,ta}) = \text{LL}^{\text{gen}}(Z_{tp}) \). While the availability (i.e., the target marginals) are measured at time \( t_a \): \( Z^*_{tp,ta} e^T_m = Z_{ta} e^T_m \), and \( e_n Z^*_{tp,ta} = e_n Z_{ta} \), where \( e_m \) and \( e_n \) are all-ones row vectors of size \( m \) and \( n \), respectively.

The assortative mating literature offers a number of alternatives to the NM transformation method. Naszodi and Mendonca (2021) reviews some of them and compares them with the NM method. These methods differ from each other in the way marital preferences are characterized.

The NM method distinguishes itself from its alternatives not only by characterizing marital preferences with the matrix-valued generalized Liu–Lu measure, but also by its attractive analytical properties and the analytical properties of its transformed table. These properties are discussed by Naszodi and Mendonca (2021) in detail. Here, I just enumerate them. The NM-transformed table \( Z^*_{tp,ta} \) is (i) unique, (ii) deterministic, \(^{10}\) and (iii) given by a closed-form formula. \(^{11}\) While the NM-transformation (i) commutes with the operation of merging categories, \(^{12}\) (ii) works even if the \( K_{ts} \) table contains zeros, \(^{13}\) (iii) can signal with a negative entry in the transformed table that the counterfactual scenario is not realistic. \(^{14}\)

\(^{10}\)In contrast to the NM method, one of its alternatives, the method proposed by Eika et al. (2019), produces a stochastic transformed table.

\(^{11}\)In contrast to the NM method, the transformed table produced by its most popular alternative, the IPF algorithm, cannot be expressed with a closed-form formula in general.

\(^{12}\)The significance of this analytical property is that any sensitivity of an empirical decomposition to the number of categories is an empirical property and not the shortcoming of the NM method.

\(^{13}\)In contrast to the NM method, the IPF algorithm fails to work if some entries of the \( K_{ts} \) are zeros.

\(^{14}\)This property makes the NM method similar to the linear probability model. The latter


3.3. The generalized Naszodi–Mendonca transformation method

In this section, I generalize the NM transformation method for the case where individuals sort along two dimensions. The generalized Naszodi–Mendonca (GNM) method works under the assumption that sorting along the two dimensions is sequential: each individual is assumed to sort along the same dimension first and then along the other dimension.

To be consistent with the empirical part of this paper, let us call one of the dimensions race (with possible values B and W), and the other dimension education (with possible values L, M and H). In principle, these dimensions can be many other pairs of traits as well.

Let us denote the time at which marital preferences over partners’ race is measured by $t_r$. Analogously, $t_a$ denotes the time when availability is observed, and $t_e$ denotes the time when preferences over partners’ education level is measured. We observe the matching outcome at $t_r$, $t_a$, and $t_e$ by observing $K_{t_r}$, $K_{t_a}$, and $K_{t_e}$ in the form given by Table 1.

Let us assume that individuals sort along the dichotomous race variable first. In order to obtain the racial distribution of couples under the counterfactual, I apply the NM transformation $NM(Z_{t_r}, Z_{t_a}) = Z^*_{t_r,t_a}$, where $Z_{t_r} = RK_{t_r}Y^T$, $Z_{t_a} = RK_{t_a}Y^T$, and $R$ is the $2 \times 2n$ matrix $R = \begin{bmatrix} n & \cdots & 1 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 1 & \cdots & 1 \end{bmatrix}$ and $Y^T$ is the $m \times 2$ matrix given by the transpose of $Y = \begin{bmatrix} m & \cdots & 1 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 1 & \cdots & 1 \end{bmatrix}$. So, table $Z^*_{t_r,t_a}$ represents the racial distribution of couples under the counterfactual.

signals with a larger than one predicted probability or with a negative predicted probability that the question we asked cannot be answered or does not make sense. The Choo and Siow (2006) model-based method and the IPF method are “well-behaving” similar to the logit model and probit model: the former two methods never produce contingency tables with negative entries, while the latter two models never predict probabilities outside the [0,1] interval. However, none of these “well-behaving”, conservative methods/models is able to warn us if we happen to reach the limitation of its applicability.
As a second step of the GNM method, we calculate the educational distribution of couples under the counterfactual set of conditions that includes the racial distribution $Z^*_{t_r,t_a}$. The challenge here is that we know only the gender-specific and race-specific educational target distributions of the population. But we do not know the educational target distributions of husbands and wives for each of the four marriage types $BB, BW, WB, WW$ defined by racial composition. If we would know these target distributions then applying the NM method for the four types of marriages separately, could provide us the unique counterfactual joint distribution of wives and husbands along both race and education.

Let us denote the type-specific target distributions as follows. The educational distribution of husbands of race $B$, married to a woman of race $B$, is denoted by $v^\text{race}_{\text{male},t_a}$, a column vector of size $n$. The educational distribution of wives of race $i$, married to a man of race $j$ ($i, j \in B, W$) is denoted by $v^\text{race}_{\text{female},t_a}$ a row vector of size $m$.

The corresponding four NM transformations are these. For couples, where both the wife and the husband are of race $B$, it is

$\text{NM}(Z_{t_r,1..n,1..m}, v^\text{race}_{\text{female},t_a}, v^\text{race}_{\text{male},t_a}) = Z^*_{t_r,t_a,\text{race } B,B}$, where $Z_{t_r,1..n,1..m} = K_{t_r,1..n,1..m}$, and the condition defined by the target marginals is

$Z^*_{t_r,t_a,\text{race } B,B}^T v^\text{race}_{\text{male},t_a} = v^\text{race}_{\text{female},t_a}$, and $e_n Z^*_{t_r,t_a,\text{race } B,B} = v^\text{race}_{\text{male},t_a}$.

For couples, where both the wife and the husband are of race $W$, it is

$\text{NM}(Z_{t_r,n+1..2n,m+1..2m}, v^\text{race}_{\text{female},t_a}, v^\text{race}_{\text{male},t_a}) = Z^*_{t_r,t_a,\text{race } W,W}$, where $Z_{t_r,n+1..2n,m+1..2m} = K_{t_r,n+1..2n,m+1..2m}$, and the condition defined by the target marginals is $Z^*_{t_r,t_a,\text{race } W,W}^T v^\text{race}_{\text{female},t_a} = v^\text{race}_{\text{male},t_a}$, and $e_n Z^*_{t_r,t_a,\text{race } W,W} = v^\text{race}_{\text{male},t_a}$.

For inter-racial marriages, where the wife is of race $B$ and the husband is of race $W$, it is

$\text{NM}(Z_{t_r,n+1..2n,1..m}, v^\text{race}_{\text{female},t_a}, v^\text{race}_{\text{male},t_a}) = Z^*_{t_r,t_a,\text{race } W,B}$, where $Z_{t_r,n+1..2n,1..m} = K_{t_r,n+1..2n,1..m}$, and the condition defined by the target marginals is $Z^*_{t_r,t_a,\text{race } W,B}^T v^\text{race}_{\text{female},t_a} = v^\text{race}_{\text{male},t_a}$, and $e_n Z^*_{t_r,t_a,\text{race } W,B} = v^\text{race}_{\text{male},t_a}$.

Finally, for inter-racial marriages, where the wife is of race $W$ and the husband is of race $B$, it is $\text{NM}(Z_{t_r,n+1..2n,1..m}, v^\text{race}_{\text{female},t_a}, v^\text{race}_{\text{male},t_a}) = Z^*_{t_r,t_a,\text{race } B,W}$, where $Z_{t_r,n+1..2n,1..m} = K_{t_r,n+1..2n,1..m}$.
\( K_{t_r,n+1..2n,m+1..2m} \), and the condition defined by the target marginals is \( Z^*_{t_r,t_a,race B,W} e_m^T = v_{male,t_a}^{race B,W} \) and \( e_n Z^*_{t_r,t_a,race B,W} = v_{female,t_a}^{race B,W} \).

These transformation problems are under-determined: in general, there can be multiple set of counterfactual tables of \( Z^*_{t_r,t_a,race B,B} \), \( Z^*_{t_r,t_a,race B,W} \), \( Z^*_{t_r,t_a,race W,B} \), \( Z^*_{t_r,t_a,race W,W} \) that meet the counterfactual conditions on preferences and marginal distributions simultaneously.

Fortunately, lack of uniqueness does not prevent us to perform counterfactual decompositions. For the decompositions, we do not need to know what would be the joint distribution of traits under the counterfactual. It is sufficient to know a particular moment of it: this moment is either the share of educationally homogamous couples \( SEHC(Z^*) \), or the share of inter-racial marriages \( SIRM(Z^*) \). The set of counterfactual conditions determine a finite number of possible counterfactual tables. So, these conditions determine an interval for any scalar-valued moment. These intervals allow us to calculate the intervals of the components of our interest, i.e., the contributions of changing preferences over the partners’ race and education to the share of educationally homogamous couples, and the share of inter-racial marriages.

For each component-interval, we have to solve a maximization problem and a minimization problem with a scalar-valued objective function (either \( SEHC(Z^*) \), or \( SIRM(Z^*) \)) and multiple constraints over the \( \mathbb{N}^{2(n-1+m-1)} \) space spanned by the independent elements of the vectors \( v_{male,t_a}^{race i,j} \) and \( v_{female,t_a}^{race j,i} \) (\( i,j \in \{B,W\} \)). In the empirical part of the paper \( n = m = 3 \), so the space is 8 dimensional.

The GNM method can be applied not only when sorting along race precedes sorting along education, but also when sorting along race follows sorting along education. Allowing this possibility widens the component-intervals.

\(^{15}\)Any extension of the model that increases the number of races, or the number of traits considered increases the number of parameters to be estimated and make the optimization problems challenging to solve.
3.3.1 A numerical example with the GNM-transformation method

TO BE WRITTEN

4. EMPIRICAL ANALYSIS

TO BE COMPLETED

For the empirical analysis, decennial census data of the United States are used from two waves: 1980 and 1990. The census wave specific contingency tables are presented in Tables X and Y. Details on the construction of the data used are left to the Appendix.

Our sample covers those heterosexual young couples where the men are aged 30 to 34 years.\footnote{Another sample covers those heterosexual couples where the women are aged 30 to 34 years. I use this sample in one of our robustness checks (see Appendix).}

Our variable on the highest level of education can take three values: “less than high school”, “high school completed”, and “university completed”\footnote{Choo and Siow (2006) and Naszodi and Mendonca (2021) use the same three categories as we do.}

4.1. Stylized facts

TO BE WRITTEN

4.2. The empirical application of the GNM method for 1980s

TO BE COMPLETED

Figure ?? presents the outcome of our main decompositions. Specifically, it reports the maximum and minimum extent to which certain drivers contributed to the changes in the share of educationally homogamous couples in the US between 1980 and 1990.
Figure ??a shows the results for the maximum, while Figure ??b for the minimum under the assumption that individuals sort along race first. Figures ??c and d show the results for the maximum, and minimum, respectively under the assumption that individuals sort along education first.

Figure ?? suggests that ...

5. CONCLUSION

TO BE WRITTEN
APPENDIX X

TO BE WRITTEN

Applying the GNM method for the period between 1960–2015

Robustness checks

Data
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