Reconciling the nonrelativistic QCD prediction and the $J/\psi \to 3\gamma$ data

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Abstract

It has been a long-standing problem that the rare electromagnetic decay process $J/\psi \to 3\gamma$ is plagued with both large and negative radiative and relativistic corrections. To date it remains futile to make a definite prediction to confront with the branching fraction of $J/\psi \to 3\gamma$ recently measured by the CLEO-c and BESIII Collaborations. In this work, we investigate the joint perturbative and relativistic correction (i.e. the $O(\alpha_s v^2)$ correction, where $v$ denotes the characteristic velocity of the charm quark inside the $J/\psi$) for this decay process, which turns out to be very significant. After incorporating the contribution from this new ingredient, with the reasonable choice of the input parameters, we are able to account for the measured decay rates in a satisfactory degree.

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Calculating the ortho-positronium (o-Ps) annihilation decay into three photons, together with the hyperfine splitting of the positronium, has played a pivotal role in developing the bound-state QED formalism, particularly in establishing the modern theoretical framework such as nonrelativistic QED (NRQED) \(^1\). After incorporating a number of higher order corrections \(^2\)–\(^4\), the theoretical prediction to the decay rate for o-Ps→3γ is consistent with the measurement \(^5\) to an impressive accuracy, which can be viewed as a great triumph of NRQED.

The QCD analogue of the o-Ps, the \(J/\psi\) meson, has also occupied a special stage in the making of the Standard Model. The numerous decay channels of the \(J/\psi\) have been intensively studied in the past four decades, among which the electromagnetic decay \(J/\psi \rightarrow 3\gamma\) is of special interest. Despite considerable efforts, experimentalists have not observed this rare decay channel until quite recently. In 2008, the CLEO-c collaboration measured the branching fraction for this rare decay to be about \(10^{-5}\) and obtained \(\text{Br}(J/\psi \rightarrow 3\gamma) = (1.2 \pm 0.3 \pm 0.2) \times 10^{-5}\). With much larger \(J/\psi\) samples that come from the pionic transition of \(\psi'\), BESIII Collaboration recently refined the earlier measurement and gave \(\text{Br}(J/\psi \rightarrow 3\gamma) = (11.3 \pm 1.8 \pm 2.0) \times 10^{-6}\) \(^7\). Both experiments are compatible with each other, nailing down the branching fraction for this rare decay to be about \(10^{-5}\).

The theoretical investigation on \(J/\psi \rightarrow 3\gamma\) preceded the experimental discovery long earlier. By the early 1980s, both the first-order radiative and relative corrections have already been calculated in the context of the potential model \(^8\)–\(^10\). Just analogous to the very successful application of NRQED to positronium, nowadays the nonrelativistic QCD (NRQCD) effective field theory \(^1\) becomes the standard tool to analyze the quarkonium spectrum, decay and production \(^11\). In particular, the influential NRQCD factorization approach \(^12\) has superseded the phenomenological potential model as a modern and systematic framework. To the best of our knowledge, up to the error of relative order \(v^4\), the decay rate of the process \(J/\psi \rightarrow 3\gamma\) in the NRQCD formalism can be written as

\[
\Gamma(J/\psi \rightarrow 3\gamma) = \frac{8(\pi^2 - 9)e^6e^3}{9m_c^2} \left| \langle 0 | \bar{\psi} \cdot \epsilon | J/\psi(\epsilon) \rangle \right|^2 \left\{ 1 - 12.630 \frac{\alpha_s}{\pi} + \left[ \frac{132 - 19\pi^2}{12(\pi^2 - 9)} + \left( \frac{16}{9} \ln \frac{m_c^2}{m_\epsilon^2} + G \right) \frac{\alpha_s}{\pi} \right] \langle v^2 \rangle_{J/\psi} + \cdots \right\},
\]

where \(e_\epsilon = \frac{2}{3}\) and \(m_c\) represent the electric charge and the mass of the charm quark. \(\langle 0 | \bar{\psi} \cdot \epsilon^* \psi | J/\psi(\epsilon) \rangle\) is the lowest order (LO) NRQCD \(J/\psi\)-to-vacuum matrix element, where \(\epsilon\) represents the polarization vector of \(J/\psi\). Up to an error of relative order \(v^2\), this matrix element can be approximated by \(\sqrt{\frac{3}{2\pi}} R_{J/\psi}(0)\), where \(R_{J/\psi}(0)\) denotes the wave function at the origin for the \(J/\psi\) in the potential model. A useful ingredient of this NRQCD formula is the inclusion of the leading relativistic correction, characterized by the ratio of the following NRQCD matrix elements:

\[
\langle v^2 \rangle_{J/\psi} \equiv \frac{\langle 0 | \bar{\psi} \cdot \epsilon^* \left( - \frac{i}{2} \hat{D} \right)^2 \psi | J/\psi(\epsilon) \rangle}{m_c^2 \langle 0 | \bar{\psi} \cdot \epsilon^* \psi | J/\psi(\epsilon) \rangle},
\]

where \(\hat{D}\) stands for the left-right asymmetric covariant derivative. The Grems-Kapustin relation \(^13\), which stems from the NRQCD equation of motion, can be employed to express \(\langle v^2 \rangle_{J/\psi}\) in terms of the \(J/\psi\) mass and the charm quark pole mass. Empirically, one may
FIG. 1: The renormalization scale dependence of the branching fraction for $J/\psi \to 3\gamma$. The LO prediction is represented by the black solid line, the LO result plus the $O(v^2)$ correction by the dashed line, the LO result plus the $O(\alpha_s)$ correction by the dotted curve, the prediction incorporating both the $O(\alpha_s)$ and $O(v^2)$ corrections by the dot-dashed curve. Finally, the complete NRQCD prediction incorporating the new $O(\alpha_s v^2)$ correction is represented by the red solid curve. For the sake of comparison, the BESIII measurement of the decay branching fraction is represented by the green band, with statistical and systematic errors added in quadrature.

expect that $\langle v^2 \rangle_{J/\psi} \approx 0.3$, compatible with the typical velocity of charmonium deduced from the potential model.

The leading term in (1) is translated from the analogous formula for the ortho-positronium decay to 3 photons, originally derived by Ore and Powell in 1949 [14]. The $O(\alpha_s)$ correction [10] was adapted from the $O(\alpha)$ correction to o-Ps$\to$3$\gamma$, first correctly calculated by Caswell, Lepage and Sapirstein in 1977 [2], or from the $O(\alpha_s)$ correction to $J/\psi \to 3g$, first calculated by Mackenzie and Lepage in 1981 [8]. Note at that time the $O(\alpha_s)$ short-distance coefficient can be obtained only with very limited precision. The relative order-$v^2$ correction was originally obtained by Keung and Muzinich in 1982 [9]. With the aid of the Gremm-Kapustin relation, their result is consistent with the value given in (1),

$$\frac{132 - 19\pi^2}{12(\pi^2 - 9)} \langle v^2 \rangle_{J/\psi} \approx -5.32 \langle v^2 \rangle_{J/\psi} [15].$$

Our current understanding of $J/\psi \to 3\gamma$ is essentially not much better than three decades ago, since to date the $O(\alpha_s)$ and $O(v^2)$ corrections remain to be the only known corrections. Upon confronting (1) with the $J/\psi \to 3\gamma$ data, the agreement seems far less satisfactory than that for its ortho-positronium cousin. The situation is most clearly illustrated in Fig. 1. With some reasonable choices of the NRQCD matrix elements, the LO prediction is several times greater than the new BESIII measurement. After including the $O(\alpha_s)$ correction, the decay rate becomes significantly lower than the measured one, and even turns negative for most plausible range of the renormalization scale entering the $\alpha_s$. The situation deteriorates desperately after the $O(v^2)$ correction is further included, in which for virtually all the
plausible input parameters, the decay rate becomes deeply negative thus loses physical significance.

The utter failure of predicting the decay rate for $J/\psi \rightarrow 3\gamma$ may imply the breakdown of the NRQCD for this decay process. The symptom seems to be rooted in the uncomfortably large negative radiative and relativistic corrections. In 1997, Braaten and Chen estimated the asymptotic behavior of the higher order radiative corrections to the coefficient associated with the LO NRQCD matrix element, and found the series are badly divergent due to the large residue of the $u = -\frac{1}{2}$ infrared renormalon [16].

Since the large negative relativistic correction appears to be even more troublesome than the radiative correction, in this work we attempt to investigate the $\mathcal{O}(\alpha_s v^2)$ effect, by computing the first-order radiative correction to the short-distance coefficient associated with $\langle v^2 \rangle_{J/\psi}$. Thus far, the $\mathcal{O}(\alpha_s v^2)$ correction has only been available for a few reactions involving $S$-wave quarkonium, e.g., $J/\psi \rightarrow e^+e^-$ [17, 18], $B_c \rightarrow l\bar{v}$ [19], $\eta_c \rightarrow \gamma\gamma$ [20, 21], $\eta_c \rightarrow$ light hadrons [21, 22], $e^+e^- \rightarrow J/\psi + \eta_c$ [23]. The $\mathcal{O}(\alpha_s v^2)$ correction turns out to be insignificant for most of the aforementioned processes. Nevertheless, for the hadronic decay of $\eta_c$, the $\mathcal{O}(\alpha_s v^2)$ correction is found to be sizable [21]. Therefore, it is also of some interest to assess the impact of the $\mathcal{O}(\alpha_s v^2)$ correction for $J/\psi \rightarrow 3\gamma$.

The $\mathcal{O}(\alpha_s v^2)$ coefficient is represented by the entity in the parenthesis in (1). The first term that depends on the NRQCD factorization scale $\mu_f$, $\frac{16}{9} \ln \frac{\mu^2}{m_c^2}$, can be inferred solely based on the knowledge of the one-loop UV divergence of the lowest-order NRQCD matrix element [17] and the Ore-Powell result. Therefore, our central task is to calculate the unknown constant $G$.

We proceed to briefly describe our calculation of the desired NRQCD short-distance coefficients. The technical details will be presented in a long write-up. Rather than invoking the standard perturbative matching approach, we resort to a shortcut by directly extracting the hard region contribution of a loop diagram in the spirit of method of region [24].

For the process at hand, the NRQCD factorization also holds at the amplitude level. We start by considering the on-shell amplitude for $c\bar{c} \rightarrow 3\gamma$ through $\mathcal{O}(\alpha_s)$. At the tree level, there are 6 diagrams for this process. At next-to-leading order in $\alpha_s$, there are 12 self-energy diagrams, 12 outer vertex diagrams, 6 inner vertex diagrams, 12 double vertex diagrams, together with 6 ladder diagrams. The MATHEMATICA package FeynArts [25] is utilized to generate the Feynman diagrams and the corresponding amplitude.

We use Dimensional Regularization to regularize both UV and IR divergences. It is convenient to employ the covariant spin projection technique [26] to enforce the $c\bar{c}$ pair to be in the color-singlet and spin-triplet state. The color and Dirac trace algebra are handled by the package FeynCalc [27]. Prior to carrying out the loop integration, we expand the amplitude in powers of relative momentum between $c$ and $\bar{c}$ through the second order, followed by the projection of the $S$-wave orbital angular momentum state.

We then employ two different versions of the self-written MATHEMATICA codes [28] to carry out the partial fractions, to reduce the general higher-point tensor one-loop integrals into a minimal set of scalar integrals up to the four-point. Up to this step, some scalar integrals may contain propagator of cubic power. The package FIRE [29] is then employed to reduce these unconventional scalar integrals to the standard ones. All the required one-loop master integrals can be found in Ref. [3].

After summing the contributions from all the diagrams, and taking into account the wave function renormalization of the charm quark, the ultimate QCD amplitude becomes completely UV finite, nevertheless contains a piece of unremoved logarithmic IR divergence.
The occurrence of the IR divergence in the hard region at this order is just as expected, physically because of the breakdown of the color transparency once beyond the leading order in $\psi$. This IR divergence can be absorbed in the $\overline{MS}$-renormalized NRQCD matrix element, and consequently, the $O(\alpha_s v^2)$ short-distance coefficient now explicitly depends on the NRQCD factorization scale $\mu_f$, with a natural range from $m_c v$ to $m_c$.

Since the differential short-distance coefficients inferred at the amplitude level are IR finite, one can safely return to 4 spacetime dimensions. Squaring the amplitude, summing over the photon polarizations and averaging upon the $J/\psi$ spin, integrating over the entire three-photon phase space and multiplying a symmetry factor $1/3$, we are able to determine each of the short-distance coefficients tabulated in (1). We readily confirm the existing $O(\alpha_s)$ and $O(v^2)$ coefficients $[10, 15]$.

We use the built-in function of multi-dimensional global adaptive integration algorithm in MATHEMATICA to compute the one-loop coefficients. We finally nail down the value of the desired constant to be

$$G = 68.913,$$

with the fractional error estimated to be smaller than $10^{-4}$.

Since the logarithmic term in the $O(\alpha_s v^2)$ coefficient is overwhelmed by the $G$, from now on we will set $\mu_f = m_c$ in (1). The full NRQCD prediction is then proportional to $1-4.02 \alpha_s(\mu)+[2.62+39.94 \alpha_s(\mu)]\langle v^2 \rangle_{J/\psi}$. It is clear to see that the new piece of correction yields a surprisingly significant and positive contribution. With a reasonable choice of $\mu$, it has strong potential to counterbalance the known large negative corrections.

In the numerical analysis, we take the values of the NRQCD matrix elements from Ref. [30]:

$$\langle 0|\chi^\dagger \cdot \epsilon^* \psi|J/\psi(\epsilon)\rangle|^2 = 0.446 \, \text{GeV}^3, \quad \langle v^2 \rangle_{J/\psi} = 0.223,$$

(4)

which are fitted through the decay $J/\psi \to e^+e^-$ accurate through the relative order $v^2$. We take the charm quark pole mass as $m_c = 1.4$ GeV. The Gremm-Kapustin relation $[13]$ then implies that the value of $\langle v^2 \rangle_{J/\psi}$ is consistent with that given in (1).

We take the fine structure constant $\alpha$ to be 1/137. There exists some ambiguity in choosing the strong coupling constant $\alpha_s(\mu)$. Presumably, one may think that any scale between $2m_c/3$ and $2m_c$ GeV could be equally acceptable.

Taking $\mu = m_c$ in (1), consequently $\alpha_s(m_c) = 0.388$, which is calculated through the two-loop renormalization group equation with $\Lambda_{QCD}^{(n_f=3)} = 390$ MeV, we then obtain the NRQCD predictions for the decay rate of $J/\psi \to 3\gamma$ at various level of accuracy. The LO prediction is 6.01 eV. After including $O(\alpha_s)$ correction, the prediction drops to $-3.36$ eV. If further including the $O(v^2)$ correction, we then get $-10.48$ eV. However, once the new $O(\alpha_s v^2)$ correction is added, we end up with the reasonable value of 0.91 eV. Dividing these predicted partial widths by the total width of $J/\psi$, whose latest value is 92.9 keV $[31]$, we then find the $\text{Br}[J/\psi \to 3\gamma]$ to be $6.46 \times 10^{-5}$, $-3.61 \times 10^{-5}$, $-11.28 \times 10^{-5}$, and $0.98 \times 10^{-5}$, respectively. It is amazing that after incorporating the new $O(\alpha_s v^2)$ correction, the full NRQCD prediction agrees with both the CLEO-c $[6]$ and BESIII $[7]$ data quite well.

One might be curious about the sensitivity of our predictions to the renormalization scale $\mu$. To address this question, in Fig. 1 we explicitly illustrate the scale dependence of various NRQCD predictions for $\text{Br}[J/\psi \to 3\gamma]$ in the range between 1 and 3 GeV. For
reader’s convenience, the BESIII measurement is also juxtaposed in the plot. One readily observes that the full NRQCD prediction exhibits much flatter μ-dependence than that only incorporating the \( O(\alpha_s) \) correction. Finally, it is interesting to remark that, the agreement between the full NRQCD prediction and the data appears to be rather stable in a window of between 1.1 ≤ μ ≤ 1.5 GeV.

In summary, in this work we investigate the \( O(\alpha_s v^2) \) correction to the rare decay process \( J/\psi \to 3\gamma \) in the NRQCD framework. Thus far, the large negative first-order radiative and relativistic corrections have prevented one from making any sensible prediction for the corresponding decay branching fraction. It is very encouraging that this new ingredient of correction turns out to be significant and positive. Including this \( O(\alpha_s v^2) \) correction appears to be crucial to reconcile the NRQCD prediction and the recent CLEO-c and BESIII experiments. The satisfactory agreement between the data and theory may lend some support to the values of the NRQCD matrix elements obtained in Ref. [30]. Finally, we remark that the future exploration of the \( O(v^4) \) and \( O(\alpha_s^2) \) corrections for this process, may turn out to be quite enlightening.

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