Quantum-Classical Transition of Photon-Carnot Engine Induced by Quantum Decoherence

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We study the physical implementation of the Photon Carnot engine (PCE) based on the cavity QED system [M. Scully et al, Science, 299, 862 (2003)]. Here, we analyze two decoherence mechanisms for the more practical systems of PCE, the dissipation of photon field and the pure dephasing of the input atoms. As a result we find that (I) the PCE can work well to some extent even in the existence of the cavity loss (photon dissipation); and (II) the short-time atomic dephasing, which can destroy the PCE, is a fatal problem to be overcome.

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I. INTRODUCTION

Recently many investigations have been carried out to explore various possibilities of building Carnot (or Otto, etc.) heat engines in some “quantum way”. It is expected that, by taking the advantages of quantum coherence, such quantum heat engines (QHE) using quantum matter as working substance can improve work extraction as well as the working efficiency in a thermodynamic cycle [1, 2, 3, 4]. Marlan O. Scully and his collaborators proposed and studied a QHE based on a cavity QED system [5, 6, 7], namely, a photon-Carnot engine (PCE) [8]. The working substance of their PCE is a lot of single-mode photons radiated from the partially coherent atoms. In their model, the walls of the cavity are assumed to be ideal, i.e., the cavity loss are disregarded.

In practice, however, the walls can not perfectly reflect the photons (the cavity loss are not negligible), and the atomic dephasing of the input atoms are inevitable due to its coupling to the environment when passing through the cavity. To focus on the essence of the problem we only phenomenologically consider the pure dephasing effect in this paper. A question then follows naturally: How does the photon dissipation and atomic dephasing influence the efficiency of the PCE? In this paper, by analyzing a more realistic cavity QED system, we revised the PCE model proposed by Scully and his collaborators. We find the efficiency of the PCE decrease when the cavity quality $Q$ becomes smaller (cavity loss becomes more strong): when the atomic dephasing happens, though the atomic energy conserves, the quantum features of the PCE are demolished and then the QHE becomes a classical one.

Our investigation is significant in two aspects. On the one hand, our results confirm the robustness of the PCE proposed in Refs. [5, 6, 7], which can still work well even in the existence of not too strong cavity loss. On the other hand, our results demonstrate the quantum-classical transition of the PCE due to quantum dephasing, which agrees well with our intuition, the efficiency of the PCE decrease due to atomic dephasing. These predictions can not only help us better the understanding of the basic concepts of thermodynamics, statistical mechanics, but also help us to optimize the system parameters in future experiments of PCE. It is also of interest that the efficiency of the PCE in a Carnot cycle can be used to measure the quantum coherence of the input atoms and characterize the quantum-classical transition of the PCE.

II. CAVITY QED MODEL OF QHE REVISED

The PCE we consider here is similar to that proposed in Refs. [5, 6, 7] (see the schematic illustration in Fig. 1). In our PCE model, the ground states $|g_1\rangle$ and $|g_2\rangle$ are accurately degenerate. The atom-photon coupling is
described by the Hamiltonian \[ H_I = \hbar \lambda |e\rangle \langle g| + \frac{g_2}{\sqrt{2}} a + h.c. \] (1)

Here, \( \nu \) is the level spacing between the excited state \( |e\rangle \) and the ground states; \( a \) is the annihilation operator of the resonant photon field and \( \lambda \) is the atom-field coupling constant.

If there were no photon dissipation and atomic dephasing, \( H_I \) would completely govern the pure quantum evolution. Let \( |m\rangle \), \( m = 0, 1, 2, \ldots \) denote the Fock state of the photon field. In every invariant subspace \( V_m \), which is spanned by the ordered basis vectors \( \{|e\rangle \otimes |m - 1\rangle, |g_1\rangle \otimes |m\rangle, |g_2\rangle \otimes |m\rangle\} \), the evolution operator \( U(\tau) = \exp(-i H_I t) \) can be expressed as a quasi-diagonal matrix with the diagonal blocks

\[
U_m(\tau) = \begin{bmatrix}
C_m(\tau) & -\frac{i}{\sqrt{2}}S_m(\tau) & -\frac{1}{\sqrt{2}}S_m(\tau) \\
\frac{i}{\sqrt{2}}S_m(\tau) & C_m^{2}(\tau) & -S_m^{2}(\tau) \\
-\frac{1}{\sqrt{2}}S_m(\tau) & S_m^{2}(\tau) & C_m^{2}(\tau)
\end{bmatrix}
\] (2)

Here, \( C_m(\tau) = \cos(\lambda \sqrt{m} \tau) \) and \( S_m(\tau) = \sin(\lambda \sqrt{m} \tau) \).

In obtaining above explicit expressions for the matrix elements of \( U_m(\tau) \), we have used the following technique. By writing

\[ |G\rangle = |g_1\rangle + |g_2\rangle \]

we find

\[ H_I = \hbar \lambda |e\rangle \langle G| a + h.c. \]

is, in fact, the Hamiltonian for the Jaynes-Cummings model at resonance, and then we can exactly solve the problem of time evolution in each subspace \( V_m \) following the method in Refs. [10, 11].

We denote the initial density matrix of the photon field and the atom ensemble as \( \rho_{PL}(t_i) \) and \( \rho_A(t_i) \). The reduced density matrix \( \rho_L(t) \) of the radiation field will evolve according to the “super-operator” \( M(\tau) \) defined by

\[ M(\tau) \rho_L(t_i) = Tr_A[U(\tau) \rho_L(t_i) \otimes \rho_A(t_i) U^\dagger(\tau)] \] (4)

where \( Tr_A \) means tracing over the atomic variable. The atoms pass through the cavity at the rate \( r \). Then we can write the known master equation at zero temperature as

\[ \frac{d}{dt} \rho_L(t) \approx r[M(\tau) - 1] \rho_L(t) + L \rho_L(t) \] (5)

Here, we have made the approximation \( \ln[M(\tau)] \approx M(\tau) - 1 \) for a short time \( \tau \) (the reliability of this approximation can be seen from later parameter estimation \( \tau \sim 10^{-10} \) s), and the cavity loss term is defined as

\[ L \rho_L(t) = (\frac{\nu}{2Q})[2a \rho_L(t)a^\dagger - (a^\dagger a \rho_L(t) + h.c.)] \] (6)

with the cavity quality factor \( Q \).

Inspired by the concept of “phasonium” in Ref. [9], we prepare the atoms initially in a partially coherent state

\[ \rho_A(0) = p_e |e\rangle \langle e| + |g\rangle \langle g| \] (7)

Here, \( |g\rangle \langle g| \) contains the superposition of the ground states

\[ |g\rangle \langle g| = |c_1|^2 |g_1\rangle \langle g_1| + |c_2|^2 |g_2\rangle \langle g_2| + c_1c_2^* |g_1\rangle \langle g_2| \]

where \( p_e \) is the probability distribution in the excited state \( |e\rangle \).

Then we obtain the equation of motion for the average photon number \( \langle n(t) \rangle \)

\[ \frac{d}{dt} \langle n(t) \rangle = \mu [(2p_e - \theta) \langle n(t) \rangle + 2p_e] - \frac{\nu}{Q} \langle n(t) \rangle \] (13)

in the isothermal compression process the input atoms, prepared in a state slightly deviating from the thermal equilibrium with some quantum coherence, serves as a high temperature energy source, while during the isothermal compression process the input atoms, prepared in thermal equilibrium state, serves as the entropy sink. For a short interaction time \( \tau \), we have (the reliability of this approximation can also be seen from later parameters estimation \( \sqrt{\tau} \sim 10^{-10} \) s)

\[ C_m(\tau) \approx 1 - \frac{1}{2} m \lambda^2 \tau^2 \] (12)

\[ S_m(\tau) \approx \sqrt{m \lambda^2 \tau} \]

Then we obtain the equation of motion for the average photon number \( \langle n(t) \rangle \)

\[ \frac{d}{dt} \langle n(t) \rangle = \mu [(2p_e - \theta) \langle n(t) \rangle + 2p_e] - \frac{\nu}{Q} \langle n(t) \rangle \] (13)
where
\[ \mu = \frac{r\lambda^2\tau^2}{2}, \] (14)
\[ \theta = |c_1|^2 + |c_2|^2 + 2\text{Re}(\xi c_1 c_2^*), \]
and \( \text{Re}(\xi c_1 c_2^*) \) is the real part of \( \xi c_1 c_2^* \).

In the thermal equilibrium state, the atomic probability distributions \( p_c, |c_1|^2 \) and \( |c_2|^2 \) satisfy
\[ \frac{p_c}{|c_1|^2} = \frac{p_c}{|c_2|^2} = \exp\left(-\frac{\hbar\nu}{kT}\right), \] (15)
where \( k \) is the Boltzmann constant and \( T \) is the temperature of the thermalized atoms. Since the relaxation time of the radiation field is very short, in the following analysis, we will use the equilibrium state solution
\[ \langle n_E \rangle = \frac{n}{1 + \zeta(T)} \] (16)
of Eq. (13) to replace the time-dependent average photon number \( \langle n(t) \rangle \). Here, \( n \) is the average photon number
\[ n = \frac{2p_c}{|c_1|^2 + |c_2|^2 - 2p_c} \] (17)
in the absence of atomic coherence and cavity loss, and
\[ \zeta(T) = \frac{\hbar}{|c_1|^2 + |c_2|^2 - 2p_c} \] (18)
is a temperature dependent parameter concerning the cavity loss as well as the atomic dephasing.

We imagine the radiation field also obeys a virtual Bose distribution
\[ \langle n_E \rangle = \frac{1}{\exp[\hbar\nu/(kT)] - 1} \] (19)
with an effective temperature \( T' \). In high temperature limit, we approximately have (for the microwave cavity QED system and circuit QED, this approximation is reliable when the effective temperature \( T' \) is at room temperature or higher)
\[ \langle n_E \rangle \approx \frac{KT'}{\hbar\nu}, \quad n \approx \frac{KT}{\hbar\nu}. \] (20)
Therefore \( T' \) can be approximately determined as
\[ T' = \frac{T}{1 + \zeta(T)}. \] (21)

It can be seen that the effective temperature \( T' \) being different from \( T \) is due to the atomic coherence as well as the cavity loss. Obviously, when \( Q \to \infty \) and \( \text{Re}(\xi c_1 c_2^*) = 0 \), the effective temperature \( T' \) approaches \( T \). I.e., the effective temperature becomes equal to the temperature of the input atoms when the atomic coherence cancels and the cavity is perfect.

\[ \text{FIG. 2: (color online) Temperature-entropy diagram for the photon-Carnot cycle. Here we consider neither the cavity loss nor the atomic dephasing. i.e., } \xi = 1, Q \to \infty. \text{ In addition, we consider the special case of the phase angle } \text{Arg}(c_1c_2^*) = \pi. \text{ As a result the effective temperature } T' \text{ is higher than } T \text{ by the amount } \Delta T_h = n_h T_h |c_1^h c_2^{*h}| / P_h. \]

\[ \text{IV. THERMODYNAMIC CYCLE OF THE QHE WITH QUANTUM MATTER AS THE WORKING SUBSTANCE} \]

The Carnot cycle of our QHE consists of two isothermal and two adiabatic processes (see Fig. 2). We use the subscripts (or superscript) \( h \) and \( l \) to indicate the isothermal expansion and the isothermal compression processes respectively. E.g., \( n_h \) and \( n_l \) represent \( n \) in the isothermal expansion and the isothermal compression processes respectively, i.e.,
\[ n_i = \frac{2p_i^h}{|c_1|^2 + |c_2|^2 - 2p_c}, \quad i = h, l. \] (22)

During the isothermal expansion process, from a thermal state 1 of the photon field to another 2, the input three-level atoms are prepared with quantum coherence of the ground states \( \rho_A^h (0) \equiv \rho_A (0) \). But during the isothermal compression process from 3 to 4, the input atoms are prepared in a thermalized state, i.e.,
\[ \rho_A^l (0) = p_c^h |e\rangle \langle e| + |c_1|^2 |g_1\rangle \langle g_1| + |c_2|^2 |g_2\rangle \langle g_2|. \] (23)

From above facts and Eq. (13) we know, in the isothermal expansion process from 1 to 2
\[ \zeta_h(T_h) = \frac{n_h}{p_c^h} \text{Re}(\xi c_1^h c_2^{*h}) + \frac{\hbar \nu}{2|c_1|^2 |c_2|^2 - 2p_c^h}. \] (24)

but in the isothermal compression process
\[ \zeta_l(T_l) = \frac{\nu n_l}{2|c_1|^2 |c_2|^2 - 2p_c^h}. \] (25)

We apply the entropy expression
\[ S_i = k \ln \left( \langle n_E^i \rangle + 1 \right) - \frac{\hbar\nu\langle n_E^i \rangle}{T_i} \quad (i = h, l) \] (26)
of the radiation field to calculate the heat transfer. In a Carnot circle, the work done by the radiation field is
\[ \Delta W = Q_{in} - Q_{out}. \] Here
\[ Q_{in} = T_h [S_h (2) - S_h (1)], \]
and
\[ Q_{out} = T_l [S_l (3) - S_l (4)] \]
are respectively the heat absorbed into the cavity during the isothermal expansion process from 1 to 2 and the heat released out of the cavity during the isothermal compression process from 3 to 4.

Similar to the cycle in Ref. 9, the frequency of the radiation field, i.e., the resonant mode of the cavity, is assumed to change slightly form 1 to 2, i.e.,
\[ \frac{[\nu (1) - \nu (2)]}{\nu (1)} \ll 1. \]

Namely, we can make the approximations
\[ \nu (1) \approx \nu_l \approx \nu (2), \]
\[ \nu (3) \approx \nu_h \approx \nu (4). \]

In the adiabatic process, the average photon number does not change, i.e.,
\[ \langle n_e^1 (2) \rangle = \langle n_e^1 (3) \rangle, \]
\[ \langle n_e^1 (4) \rangle = \langle n_e^1 (1) \rangle. \]

Form Eq. 24, it follows that
\[ \frac{\nu (1)}{T_h} = \frac{\nu (4)}{T_l}, \quad \frac{\nu (2)}{T_h} = \frac{\nu (3)}{T_l}. \]

From these observations and Eq. 240 we find that
\[ S_h (2) - S_h (1) = S_l (3) - S_l (4). \]

Therefore, in the high temperature limit, the PCE efficiency \( \eta = (Q_{in} - Q_{out})/Q_{in} \) can be explicitly expressed as \( \eta = 1 - T_l / T_h \) or
\[ \eta = 1 - \left[ \frac{1 + \zeta_h (T_h)}{1 + \zeta_l (T_l)} \right] \frac{T_l}{T_h}. \]

Based on above results we are now able to analyze the effects of the two decoherence mechanisms separately.

Firstly, to focus on the cavity loss (photon dissipation), we consider the ideal case with no atomic dephasing, \( \xi = 1 \). In the ideal case the cavity loss is negligible, i.e., \( Q \to \infty \), the efficiency 33 is reduced to
\[ \eta = 1 - [1 + \frac{n_h}{p_e^h} \text{Re}(c_1^h c_2^{h*})] \frac{T_l}{T_h}, \]
which agrees with the result in Ref. 3. It seems that, in principle, the PCE can extract positive work from a single heat bath if we control the phase angle \( \theta = \text{Arg}(c_1^h c_2^{h*}) \) properly, e.g., \( \theta = \pi \). This shows the advantage of the “quantum fuel” – we can extract more work from the “quantum fuel” than from the classical fuel. However, in the case the cavity is “extreme bad”, i.e., the cavity quality factor is vanishingly small \( Q \to 0 \), the efficiency decrease to zero (this can be verified easily)
\[ \eta \to 1 - \frac{\nu_h n_h p_e^h}{\nu_l n_l p_e^l} \approx 0. \]

Namely, the PCE is destroyed by the strong loss of the cavity. This can also be understood intuitively from the Eq. 21. When the quality factor of the cavity becomes so small that few photons can stay stably in the cavity. Accordingly, both the two effective temperatures \( T_h \) and \( T_l \) becomes vanishingly small, and thus no work can be done by the “working substance”. As a result, the efficiency of the PCE decrease to zero. Mathematically, it can be verified that \( \eta \) is a monotonically increasing function of \( Q \), and the efficiency of the PCE decrease to zero when \( Q \) becomes vanishingly small.

Secondly we consider the pure atomic dephasing. From Refs. 11, 12, 13 we know that the dephasing time is much shorter than the atom and cavity lifetimes. After the atom interacting with the environment for a short time \( \tau \), the term \( \text{Re}(\xi c_1^e c_2^{e*}) \) concerning atomic coherence becomes vanishingly small. We can properly assume \( \xi \) decrease to zero in the short time \( \tau \). Then the efficiency 33 of the PCE becomes
\[ \eta = 1 - \left[ \frac{1 + \nu_h n_h (2 \mu p_e^h Q)}{1 + \nu_l n_l (2 \mu p_e^l Q)} \right] \frac{T_l}{T_h}. \]

In principle, if the cavity is ideal, \( Q \to \infty \), we regain the maximum classical Carnot efficiency \( \eta = 1 - T_l / T_h \) from the efficiency 33 of the PCE. It turns out that, without cavity loss, the complete dephasing of the atoms makes the PCE become an ideal (reversible) classical heat engine. This demonstrate the quantum-classical transition due to the atomic dephasing. Similarly, in the case of “extreme bad” cavity, \( Q \to 0 \), the efficiency 33 decrease to zero.

Finally we analyze the positive-work condition of this PCE 14, 15, 16, under which positive-work can be extracted. From Eq. 33 we know that the positive-work condition \( \Delta W = Q_{in} - Q_{out} > 0 \) of this QHE is
\[ T_h \geq \frac{1 + \zeta_h (T_h)}{1 + \zeta_l (T_l)} T_l, \]

where \( [1 + \zeta_h (T_h)]/[1 + \zeta_l (T_l)] \) is either less or greater than unit. It is counterintuitive when \( [1 + \zeta_h (T_h)]/[1 + \zeta_l (T_l)] \) is smaller than unit, and the same novel result occurs in Ref. 35. These novel results originate from the fact that the atoms are not prepared in the thermal equilibrium state in the isothermal expansion process. In other words, the initial state with partial quantum coherence is out of thermal equilibrium, and thus the “temperature” \( T_h \) of the input atoms in the isothermal expansion process should not be regarded as a real thermodynamic temperature essentially. We also would like to emphasize
that the second law of thermodynamics is not violated when
\[
\frac{1 + \zeta_h(T_h)}{1 + \zeta_l(T_l)} < 1,
\]
for it would take energy from an external source to prepare the atomic coherence \( \rho \). In an overall consideration, the extra energy cost for preparing the atomic coherence would prevent the second law from being violated.

V. EXPERIMENTAL FEASIBILITY

Finally, we would like to estimate the efficiency \( \eta \) according to a set of experimentally accessible parameters. For different material and cavity parameters \([17]\) the discussions are listed below.

Firstly, for the optical cavity QED system \([17]\), the resonance frequency \( \nu_h (\nu_l) \sim 10^{14} \text{ Hz} \), the atom-field coupling constant \( \lambda \sim 10^{8} \text{ Hz} \) and the quality factor is less than \( 10^8 \). We take a set of reasonable values \([3, 4]\): \( n_h (n_l) \sim 10^2 \), \( \lambda \sim 10^{-11} \). Thus we get order of magnitude of the interaction time \( \tau \sim 10^{-10} \text{ s} \). To require only one atom in the cavity once, we have \( r \lesssim (1/\tau) \sim 10^{10} \text{ /s} \). Based on above estimation, we get \( \mu = \tau \lambda^2 \tau^2 / 2 \lesssim 10^6 \text{ /s} \). Hence, the term \( \nu_h / (2\mu Q) \) in Eq. \( \text{(24)} \) has the order of magnitude greater than \( 10^{-1} \), but the order of the other part \( \text{Re}(\zeta_1 \zeta_3^* \zeta_2^* \lambda^2) \) in Eq. \( \text{(24)} \) is less than \( 10^{-1} \) due to the atomic dephasing \( \zeta_1 \leq 1 \). Thus the conclusion for optical cavity system remains valid for microwave cavity system.

Thirdly, we consider the circuit QED based on superconducting Josephson junction systems \([17, 18]\). Here, the coupling between the charge qubit \([\text{CPB}]\) and a quantum transmission line is in the similar way as the photon-atom coupling in cavity QED system. The passage of atoms through the cavity can be simulated by the periodic switch-off and -on of the on chip-interaction \([19, 20]\). The experimental parameters can also be found in Ref. \([17]\): the resonance frequency \( \nu_h (\nu_l) \sim 10^{10} \text{ Hz} \); the atom-field coupling constant \( \lambda \sim 10^8 \text{ Hz} \) and the quality factor is less than \( 10^4 \). After similar analysis, we find the value of the term \( \nu_h / (2\mu Q) \) in Eq. \( \text{(24)} \) has the order of magnitude greater than \( 10^0 \), but the order of magnitude of the other part \( \text{Re}(\zeta_1 \zeta_3^* \zeta_2^*) \) in Eq. \( \text{(24)} \) is less than \( 10^{-1} \) due to the atomic dephasing \( \zeta_1 \leq 1 \). Therefore, under current experimental capability, the Circuit QED system can not also be used to implement the QHE, either.

We therefore conclude, based on the present experimental accessibility, we will have to improve the quality factor of cavity \( Q \) to a much higher level before we can implement the PCE as a practical QHE.

VI. CONCLUSIONS

In summary, we revised the PCE proposed in Ref. \([3]\), and we found that the PCE efficiency decrease monotonically with \( Q \) due to the cavity loss. In the ideal case, \( Q \rightarrow \infty \), we regained the result of Ref. \([3]\), where the improvement of the working efficiency is due to the non-equilibrium state preparation. Thus the obtained efficiency, which is beyond the classical Carnot efficiency, does not imply the violation of the second law of thermodynamics. This observation has been made in Refs. \([14, 15]\) for different physical system. We also phenomenologically considered atomic dephasing and found that a short time dephasing may make the PCE become a classical one. From these heuristic discussions, we conclude that both the photon dissipation and atomic dephasing can diminish the efficiency of the QHE. Considering the continue improvement of the cavity quality factor \( Q \), we believe the crucial issue in future experiments will be to control the atomic coherence. We also would like to point out that the dissipation mechanism of atoms due to its coupling with the environment, e.g., the vacuum modes, are not considered microscopically in this paper. Detailed investigation of the PCE with atomic dissipation will be presented in our forthcoming paper.

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