Risk analysis of reservoir sedimentation under non-stationary flows

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Abstract
Reservoir sustainability is strongly impacted by the reservoir sedimentation processes. Most of the substantial sedimentation processes occur in non-stationary flows such as in the case of flash floods, surges and tidal waves. However, a stationary probability assumption is normally adopted to reduce mathematical model complexity. This work develops a non-stationary Gambler’s ruin model using the Monte-Carlo simulation method. Daily water-level data for the Xia-Yun station are used to predict the effective risk that the maximum capacity of the water treatment plant in the Shihmen Reservoir is reached. This non-stationary model yields fairly accurate probabilities of sedimentation by the transitional probability of a reservoir reaching different levels of turbidity, and the average time to reach a designated reservoir maximum handling turbidity. The extended capacity of the proposed model demonstrates the major particle processes during non-stationary flows. Such analytical results offer water resources agency to scientifically evaluate the dredging/remediation strategies with the existing reservoirs. Transport capacities of rivers and streams, and the potential consequences of flood risks in response to reservoir sedimentation can then be comprehensively estimated in order to allow effective contingency planning for public safety.

KEYWORDS
improved Gambler’s ruin model, Monte Carlo simulation, non-stationary flow, reservoir sedimentation risk analysis, reservoir sustainability, sediment capacity

1 INTRODUCTION

Sediment particle deposition and suspension processes are of substantial significance to the geomorphological change of a river and its water quality. Among others, reservoir sedimentation has gained significant awareness due to its longer-lasting impacts on water availability, public health and water resources planning and flood risk management (Lanzanova, Whitney, Shepherd, & Luedeling, 2019). Reservoir sustainability is strongly related to the effective control of the sedimentation process (Bertone, Stewart, Zhang, & O’Halloran, 2016). In particular, most of the substantial sedimentation processes occur under non-stationary (unsteady) flow conditions such as in the case of flash floods, tidal waves, and wind induced surges.
In August 2020, record floods hit the Three Gorges Dam on China’s Yangtze River. It is the world’s biggest dam, built to generate electricity and regulate the annual flooding, but the floods have affected millions of people. Hundreds of people have been killed, hundreds of thousands evacuated, and there were fears that the floods might even disrupt food supplies. These consequences make it evident the importance of managing the capacity of a reservoir through predicting the rate of sedimentation.

Despite more than six decades of research, sedimentation remains probably the most serious technical problem faced by the dam industry, and sediment trapping by reservoirs is now of primary global importance (McCully, 1996). This fact has significant consequences for both the downstream channels, and sustainability of reservoirs and, thus future water supplies. Evidence of channel erosion and impact on the ecosystem as a result of sediment starvation downstream of dams is increasing (Draut, Logan, & Mastin, 2011; Grams, Schmidt, & Topping, 2007; Kondolf, 1997; Ma, Huang, Nanson, Li, & Yao, 2012; Schmidt & Wilcock, 2008; Singer, 2010).

In Taiwan, most riverbeds and banks are subjected to erosion, leading to high sediment concentrations in most rivers. When sediment particles in a river are transported into a reservoir, a larger volume of sediment particles is deposited at the bottom of the reservoir as flow velocity is reduced. According to Taiwan Water Resources Agency, the Shihmen Reservoir has an effective storage capacity of nearly 309 million cubic meters, of which 31.6% is occupied by sediments. Sedimentation is one of the most important engineering problems that threaten reservoirs in Taiwan.

Structures such as reservoirs can exert substantial control over downstream hydrology. Conventional methods of estimation of reservoir sedimentation include sediment trap efficiency, and monitoring and measurements (such as Ackerman et al., 2009). Reservoir sediment management can be categorised into three approaches, sediment routing through or around the reservoir, removal of accumulated reservoir sediment to regain capacity, and reduction of sediment loading to reservoirs from upstream (Morris & Fan, 1998; Kantoush & Sumi, 2010).

Techniques for reservoir sediment reduction to preserve the capacity of the reservoir include sediment bypass, sluicing, drawdown flushing and hydraulics excavation (Kondolf et al., 2014). How to effectively predict the sedimentation process has been of critical significance to hydraulic engineers. As such, a non-conventional approach to estimate uncertainty of sediment loading and to quantify the risk of reservoir sedimentation under non-stationary flow conditions may serve as a viable alternative for water resources planning and management (Phatarfod, Unny, & Ponnamalam, 1990; Xu, Ito, Liao, & Wang, 1997; Jung, Niemann, & Greimann, 2018; Tsai, Hung, & Oh, 2018).

More recently, attention has been paid to stochastic approaches for estimation of reservoir sedimentation. Among others is the Gambler’s ruin problem which is a theoretical problem that describes how to estimate the probability of victory of a gambler who plays a series of games. The Gambler’s ruin problem is a special case of the discrete parameter Markov chain, which assumes that time and space are both discrete.

This GR model can estimate the risk of water turbidity fairly accurately but the model makes a simplified assumption of stationary probability, which may not hold for a time-varying flow. To overcome this shortcoming and enhance accuracy, the GR model is to accommodate non-stationary transition probabilities, which depend on the properties of the particles involved and time-varying flow conditions. The objectives of this paper are threefold.

1. To extend the capacity of the improved GR model to accommodate the non-stationary flows, which often-times occur in Taiwan due to its strong monsoon feature and steep sloped rivers and streams.
2. To account for particle deposition (control volume losing particles, in analogy to losing the game), particle entrainment (control volume gaining particles, in analogy to winning the game) processes and no deposition/entrainment (in analogy to a tie in a game) during the reservoir sedimentation process in the proposed GR model.
3. To acquire probabilistic predictions of the sedimentation process, including the transitional probability of a reservoir reaching different levels of turbidity, and the average time to reach a designated reservoir maximum handling turbidity etc.

This work aims at establishing an improved GR model with non-stationary transition probabilities that can be used to estimate the risk of exceeding the pre-established turbidity standard. Elimination of the stationary probability assumption increases the applicability of the model. Accordingly, the Monte Carlo simulation is utilised to assist in the computation of a non-stationary GR model. The differences between the non-stationary and original models are discussed. The mean time spent in each Markov state is calculated using a transition matrix. Figure 1 presents a flow chart of our research.

The rest of this paper is organised as follows: Section 2 reviews relevant research. Section 3 introduces the Gambler’s ruin theory, and its potential application to water engineering problems.
resources engineering. The process of quantifying variables in GR, such as transition probability and number of units of sediment is defined. Section 4 presents a case study with the time series data for the Xia Yun hydrological station. Section 5 compares the original GR with the non-stationary GR. The last section presents concluding remarks.

2 | LITERATURE REVIEW

The frequency of extreme hydrological events is increasing owing to climate change. Occurrences of these extreme hydrological events such as typhoons in Taiwan have caused many reservoir sedimentation problems. Hydraulic researchers recently began paying more attention to effective reservoir management. In particular, numerous efforts have been made on deterministic sediment transport models of reservoir sedimentation (Amoudry & Souza, 2011; Armanini, Cavedon, & Righetti, 2015; Barry et al., 2013; Beilicci & Beilicci, 2015; Castro, Fernandez-Nieto, Ferreiro, & Pares, 2008; Czuba & Foufoula-Georgiou, 2014; Meyer-Peter & Muller, 1948; Parker, 1979; Perona, Molnar, Savina, & Burlando, 2009; Sabatine, Niemann, & Greimann, 2015; Yang, Mao, Huang, & Zhu, 2016; Zhou, Chen, Li, & Wang, 2015). Researchers in the field of sediment transport are interested in mechanisms of movement of sediment particles, and they have identified several such mechanisms, including pickup, rolling and suspending.

Einstein (1950) built the first stochastic framework of a sediment transport model after he proposed the innovative concept of sediment pickup probabilities. Subsequently, many researchers have considered the stochastic process of sediment transport, and some have proposed formulas for the probability of sediment movement by various mechanisms (Cheng & Chiew, 1999; Cinque, Morillo, Antenucci, Stevens, & Imberger, 2005; Dou, Yu, & Yang, 2009; Gorrick & Rodriguez, 2012; Kang & Tanaka, 2006; Klumpp & Greimann, 2004; Liu & Sato, 2005; Richards & Zeller, 1999; Scarlato & Lin, 1997; Schmelter, Hooten, & Stevens, 2011; Wu & Chou, 2003; Wu & Lin, 2002; Zech, Soares-Frazao, Spinewine, & Le Grelle, 2008) because such probability can be easily estimated from flow properties and some of the characteristics of sediment particles (Ettema & Mutel, 2004; Sholtes, Werbylo, & Bledsoe, 2014). It is also recognised that various sources of model parameter
uncertainty can contribute to predictive uncertainty of sediment runoff simulations.

Noticeably, stochastic sediment transport models as well as the morphological models of estimating reservoir sedimentation have been developed plentifully. A key parameter applied on morphological models is reservoir sediment trapping efficiency (TE). Churchill (1948) established the relationship between sediment load and sedimentation index in two curves model (Churchill, 1948). Brune (1953) followed the Churchill model and predicted the TE against the reservoir capacity to annual inflow ratio (C/I) (Brune, 1953). Further, Jothiprakash and Vaibhav (2008) utilised the concept of the Brune and Brown model to estimate the trapping efficiency and summed up the empirical model by regression analysis. Tan et al. (2019) proposed an improved model different from the previous method by considering the capacity of inflow and watershed, the effect of particle size, and settling velocity of the sediment (Tan et al., 2019).

Table 1 compares probabilities of various mechanisms of sediment movement. Several scholars found that the Markov chain process can effectively simulate sediment behaviour in the water column because this behaviour exhibits the memory-less property (Kuai, 2014; Lai, 2012; Tsai & Lai, 2014; Wu, 2015).

Sediment movement can be regarded as a Markov process because the motion of sediment particles depends only on the present state. Sun and Donahue (2000) simulated the exchange of non-uniformly sized sediment on a bed using a continuous-time Markov chain with two motion states (static and moving). Wu and Yang (2004) adopted a new definition of pseudo-four-state to improve the two-state Markov chain model that was developed by Sun and Donahue (2000), and compared experimental results from especially designed experiments with theory. Tsai and Yang (2013) developed a three-state continuous-time Markov chain model that considered bed material, bed load, and the suspended load layer.

Table 2 presents some sediment transport models that involve Markovian process with various transition states. The Gambler's ruin problem concerns a stochastic process that is used to describe a random walk, which is a mathematical formalisation of natural random processes (Abad & Kozak, 2015; Kim, Choe, & Kim, 2008; Tsai, Hsu, Lai, & Wu, 2014; Xu & Xu, 2011), such as the stock market index or the trajectory of bacteria. For example, Tsai et al. (2014) developed a Gambler's ruin model to estimate the risk that water quality exceeds the pre-established standard in a reservoir caused by high flow events, and obtained reasonable simulation results for the Shihmen Reservoir. Wu (2015) added a third state (sediment maintained in the water column) in the Gambler's ruin model that was developed by Tsai et al. (2014). Both studies provided case simulations of the Shihmen reservoir, and calculated the effective risk of reservoir sedimentation. In previous investigations, the assumption of stationary probability was made to derive an analytical solution to the Gambler’s ruin problem. Table 3 shows the evolution of the GR model.

Based on the knowledge gap observed from the previous studies, most of the substantial sedimentation processes occur in non-stationary flows, implying a non-stationary probability assumption is required to fit the physical problems in future work. In this study, an enhanced GR model is developed to estimate the probability that a reservoir reaches the maximum handling capacity of its water treatment plant (or the maximum capacity of the reservoirs). The Non-stationary models can estimate the transition probability for particle properties, risk of sedimentation, and time-varying flow conditions in different water depth.

### METHODOLOGY

#### 3.1 Modified Gambler’s ruin problem

The Gambler's ruin problem is a stochastic theoretical problem that concerns the situation in which a gambler...
plays a finite number of games, each with a probability \( p \) of winning one unit of wealth and \( q \) of losing one unit of wealth (Tsai et al., 2014).

Wu (2015) proposed the modified Gambler’s ruin problem, adding the probability \( r \) that a game results in neither a loss nor a gain \((p + q + r) = 1\). Each game is memory-less and independent. As such, the Markov type of theories can be used. The problem is to find the probability that a gambler who starts with initial wealth of \( i \) units reaches a pre-stated goal of \( N \) units before going bankrupt (0 unit).

In the modified Gambler’s ruin problem, the number of units of wealth of the gambler at time \( n \) is denoted by \( X_n \). \( \{X_n, n = 1, 2, 3, \ldots\} \) is a special case of a discrete-time Markov chain with transition probabilities.

\[
\begin{align*}
P_{i,i+1} &= p \\
P_{i,i-1} &= q \\
P_{i,i} &= r
\end{align*}
\] (for \( i = 1, 2, \ldots, N-1 \))

and \( P_{i,i} = P_{NN} = 1 \).

Such discrete-time Markov chains as presented in Equation (1) can be classified into three classes, which are \([0]\), \([1, 2, 3, \ldots, N-1]\) and \([N]\). The second class is transient, and the others are recurrent. If a process enters the recurrent class, it will never leave this state (so the gambler either reaches the pre-specified goal \([N]\) or goes bankrupt \([0]\)) so \( P_{00} = P_{NN} = 1 \).

A process in the transient class can always change state into a neighbouring or current state (i.e., the gambler wins, loses or neither win nor lose). Ultimately, any process will enter the recurrent class, so the gambler will either reach his goal or go bankrupt.

The probability, \( P_i \), of starting from \( i \) units and reaching \( N \) units can be denoted as:

\[
P_i = pP_{i+1} + rP_i + qP_{i-1}, \quad i = 1, 2, \ldots, N-1
\] (2)

since \( p + r + q = 1 \), Equation (2) can be written as

\[
P_{i+1} - P_i = \frac{q}{p}(P_i - P_{i-1})
\] (3)

If \( i = 1, 2, 3, \ldots, N-1 \), then given \( P_0 = 0 \), Equation (3) can be denoted as follows:

\[
P_{2} - P_1 = \frac{q}{p}(P_1 - P_0) = \frac{q}{p}P_1
\]

\[
P_{3} - P_2 = \frac{q}{p}(P_2 - P_1) = \left(\frac{q}{p}\right)^2 P_1
\] (4)

\[
P_{i} - P_{i-1} = \left(\frac{q}{p}\right)^{i-1} P_1
\]

Summing above equations based on the geometric series yields

\[
P_i = \begin{cases} 
1 - \left(\frac{q}{p}\right)^i & \text{if } p \neq q \\
iP_1, & \text{if } p = q
\end{cases}
\] (5)

Since \( P_N = 1 \),

\[
P_i = \begin{cases} 
1 - \left(\frac{q}{p}\right)^i & \text{if } p \neq q \\
\frac{1}{N}, & \text{if } p = q
\end{cases}
\] (6)

Hence,

\[
P_i = \begin{cases} 
1 - \left(\frac{q}{p}\right)^i & \text{if } p \neq q \\
iN, & \text{if } p = q
\end{cases}
\] (7)

If the parameters \( p \) and \( q \) are known, then the probability \( P_i \) can be estimated from state \( i \) to \( N \). Notably, as \( N \to \infty \),

\[
P_i = \begin{cases} 
1 - \left(\frac{q}{p}\right)^i, & \text{if } p > q \\
0, & \text{if } p \leq q
\end{cases}
\] (8)

### 3.2 Determination of transition probability

The parameters in the above Gambler’s ruin theories such as the probability of winning \( (p) \), the probability of losing \( (q) \), and the probability of having a tie \( (r) \) must first
be defined for various flow conditions and particle geometries. Tsai et al. (2014) presented an innovative hypothesis. They defined the number of sediment particles in the water column in state $i$, analogous to the number of coins that are owned by a gambler. Specifically, in analogy to a gambler playing games, sedimentation processes during stationary or non-stationary flows can include:

1. Particle deposition (control volume losing particles, in analogy to losing the game).
2. Particle entrainment (control volume gaining particles, in analogy to winning the game).
3. No deposition/entrainment (in analogy to a tie in a game) during the reservoir sedimentation process.

Such analogy is illustrated in Figure 2.

Cheng and Chiew (1998, 1999) assumed a Gaussian distribution to derive formulas for the probability that a sediment particle is suspended from a bed load layer in the suspended load layer, and for the probability that a sediment particle is lifted from the river bed into the water column.

Wu and Lin (2002) improved the accuracy of the probability formulas by assuming a log-normal distribution. Wu and Yang (2004) improved the probability theory for sediment particles of mixed size. The probabilities of winning and losing in the above theories can be estimated from a dimensionless shear stress $\theta$ (Shields parameter).

The Rouse number $Z_R$ is utilised to classify sediment states (deposited sediment $Z_R \geq 2.5$; suspended sediment $Z_R \leq 2$; sediment maintained in water column $2 < Z_R < 2.5$). Here, $Z_R = 2.5$ is used as the threshold for sediment suspension, where $Z_R = w_s/(\kappa u^*)$; $w_s =$ settling velocity; $u^*$ = bed shear velocity; $\kappa = $ von Kármán constant; $\theta = \left( \frac{\sqrt{25 + 1.2 d^2 s} - 5}{(w_s/u^*)^2 d^3 s} \right)^3$

where $d_s = (\Delta g/\nu^2)^{1/3}$; $d_s$ is the dimensionless diameter of particles; $\Delta = (\rho_s - \rho)/\rho$; $\rho_s$ is the density of particles; $\rho$ is the density of fluid (e.g., water in this study); $g$ is acceleration due to gravity; and $\nu$ is kinematic viscosity of the fluid. $\theta_1$ and $\theta_{0.8}$ are used to represent and substitute $\omega_s/(u^*) = 1$ and $\omega_s/(u^*) = 0.8$ into Equation (9), since the von Kármán constant $\kappa = 0.4$ and $\theta_{0.8} = 1.5625 \times \theta_1$.

### 3.2.1 Increasing probability, $p$

In the Gambler’s ruin model, sediment lifting probability is an increasing probability (Cheng & Chiew, 1998). The lifting probability can be represented as a first-order Taylor series expansion (Wu & Lin, 2002). An increasing probability ($p$) can be estimated in terms of a dimensionless shear stress by Equation (10).

$$p = 0.5 - 0.5 \ln(0.3819 + \theta) / \ln(0.3819 + \theta) \left[ 1 - \exp \left\{ -\frac{2}{\pi} \left( \frac{\ln(0.3819 + \theta)}{0.724} \right)^2 \right\} \right]\]$$

Figure 2: Schematic illustration of the improved GR model
3.2.2 | Decreasing probability, $q$

The deposition of a sediment particle is defined as the event that occurs when $ω_s/(u_*) > 1$ to estimate the decreasing probability. The probability of the depositing event equals $P(θ < θ_1)$, and the decreasing probability can be represented as $q = 1 - P(θ > θ_1)$

$$q = 0.5 + 0.5 \cdot \frac{\ln(0.2444 + θ)}{\ln(0.2444 + θ)} \cdot \left[1 - \exp\left\{-\frac{2}{π} \cdot \frac{\ln(0.2444 + θ)}{0.724}\right\}\right]$$

(11)

3.2.3 | Maintaining probability, $r$

The formula for the probability of no particle number change ($r$), in other words, having a tie, is shown in Equation (12). The Gambler's ruin problem can be easily obtained as $p + q + r = 1$. Figure 3 displays the probabilities of different states.

$$r = 0.5 \cdot \frac{\ln(0.3819 + θ)}{\ln(0.3819 + θ)} \cdot \left[1 - \exp\left\{-\frac{2}{π} \cdot \frac{\ln(0.3819 + θ)}{0.724}\right\}\right]$$

$$-0.5 \cdot \frac{\ln(0.2444 + θ)}{\ln(0.2444 + θ)} \cdot \left[1 - \exp\left\{-\frac{2}{π} \cdot \frac{\ln(0.2444 + θ)}{0.724}\right\}\right]$$

(12)

3.3 | Characteristics of flow data

Before the probability of sediment movement can be estimated, the dimensionless shear stress must be obtained ($θ = u_c^2/Δgd$ where $u_c$ = shear velocity; $d$ = diameter of particle). The dimensionless shear stress is a parameter that characterises the flow data and geometric features of the sediment particles.

Several methods are available for estimating the shear velocity of the flow filed. Wu (2015) applied the energy gradient method in the modified Gambler's ruin model for steady and uniform open channel flow ($u_c = \sqrt{gRS}$; $R$ is hydraulic radius, and $S$ is friction slope).

Wu (2015) further estimated the bed friction slope by curve fitting, assuming that the discharge can be obtained by multiplying the depth-average velocity by the cross-sectional area ($Q = A \cdot \bar{u}_d$ where $A$ is cross-sectional area, and $\bar{u}_d$ is the depth-averaged velocity).

Notably, the flow is assumed to follow the logarithmic velocity profile. The depth-averaged velocity $\bar{u}_d$ can be evaluated by integrating the velocity profile

$$\bar{u}_d = \frac{1}{h} \cdot \int_{z_0}^{h} \bar{u}_i(z) dz$$

(13)

$$\bar{u}_d = \frac{1}{h} \cdot \int_{z_0}^{h} \frac{1}{\kappa} \ln\left(\frac{z}{z_0}\right) dz$$

(14)

FIGURE 3 Probabilities associated with different states in Gambler’s ruin model
where $\bar{u}_r(z)$ is the temporal mean velocity at height $z$ above the bed; $u_\kappa$ = shear velocity; $\kappa$ = von Kármán constant; $h$ is the depth of the water; $z_0$ is the zero-velocity level ($z_0 = k_\kappa/30$); $k_\kappa$ is the equivalent roughness height of Nikuradse, and is assumed to be $2d$ (Wu & Chou, 2003).

### 3.4 Quantification of sediment concentration

After the probabilities in the Gambler’s ruin model are estimated, concentrations in the water column must be quantified. The number of sediment particles in state $i$ in the water column above a unit area of the bed can be calculated by multiplying the mean concentration by the depth of the water. Wu (2015) assumed that the sediment concentration was given by the Rouse profile, and used the logarithmic profile assumption for the flow velocity.

Most researchers express sediment concentration as solid volume per unit fluid volume ($\text{m}^3/\text{m}^3$) or the solid mass per unit fluid volume ($\text{kg}/\text{m}^3$). The Rouse profile gives the concentration $c$ as a function of height $z$.

$$c = c_a \cdot \left( \frac{h - z}{a} \right)^{\frac{h - z}{h - z_0}}$$

where $c_a$ is the reference concentration, $Z_R$ is the Rouse number, and $a$ is the reference height ($a = 2.75d$), which is thickness of the load on the bed (Cheng & Chiew, 1999).

The depth-integrated suspended load transport ($q_{s,p}$) is obtained by integrating the product of flow velocity $\bar{u}_r(Z)$ and suspended concentration $c(z)$ from $a$ to $h$ as follows:

$$q_{s,p} = \int_a^h \bar{u}_r(z)c(z)dz = \int_a^h c_a \cdot \left( \frac{h - z}{a} \right)^{\frac{h - z}{h - z_0}} \left[ \frac{u_\kappa}{\kappa} \ln \left( \frac{z}{z_0} \right) \right] dz$$

The mean volumetric concentration $c_{\text{mean}}$ (kg/m$^3$) is defined as

$$c_{\text{mean}} = q_{s,p}/q_f$$

where $q_{s,p}$ is the suspended sediment particle discharge per unit width (kg/s/m), and $q_f$ is the flow discharge per unit width (cmsg/s/m) (Figure 4).

State $i$ can be specified as the number of particles in the water column at different times, which can be expressed as the number of particles above a unit bed area ($1/\text{m}^2$), as follows.

$$N_s = \frac{c_{\text{mean}} \cdot h}{V_{s,p}/\rho_s}$$

where $V_{s,p}$ is the volume of each suspended sediment particle ($V_{s,p} = \pi d^3/6$), and $\rho_s$ is the density of the particles ($\text{kg}/\text{m}^3$).

In the Gambler’s ruin model, $N_i$ is set to the initial number, $i$, in Equation (18). Moreover, we defined per 1,000 particles as one unit since $i$ must be reduced to increase computational efficiency.

### 3.5 Mean time spent in each state

The time required for the concentration to reach a pre-specified threshold is useful to know, and this time is an important factor in water management. The Gambler’s ruin problems involve estimating not only the probability that the gambler reaches a pre-specified gain but also the mean time spent in each state. If the relationship between the game and the time for each game can be accurately determined, then the mean “real time” spent can be predicted. Prior to computing the mean time spent in each state, the transition probabilities need to be quantified, as follows.

#### 3.5.1 Matrix of transition probabilities

The transition probability matrix comprises probabilities of state transitions. The Gambler’s ruin model is a finite-state Markov chain whose finite number of transition states are numbered $T = \{1, 2, 3, ...l\}$ (Lai, 2012). The transition probability matrix is as follows.

$$P_T = \begin{bmatrix} P_{11} & \cdots & P_{1l} \\ \vdots & \ddots & \vdots \\ P_{l1} & \cdots & P_{ll} \end{bmatrix}$$

This matrix may not include state 0 and state $N$. The value $s_{ij}$ is the expected number of periods that the Markov chain is in transient state $j$, given that it starts in transient state $i$. When $i = j$, let $\delta_{ij} = 1$; otherwise $\delta_{ij}$ equals 0. Now,

$$s_{ij} = \delta_{ij} + \sum_k P_{ik}s_{kj} = \delta_{ij} + \sum_{k=1}^l P_{ik}s_{kj}$$
Let $S$ denote the matrix of elements $s_{ij}$

$$S = \begin{bmatrix} s_{11} & \cdots & s_{1T} \\ \vdots & \ddots & \vdots \\ s_{1T} & \cdots & s_{TT} \end{bmatrix}$$

(21)

The above equations can be expressed in matrix form as

$$S = (I - P_T)^{-1}$$

(22)

Finally, Equation (22) can be used to estimate the mean time spent in each state in units of rounds (of the game). In engineering applications, the simulation interval is a second or a minute. The relationship between time and the number of rounds (of the game) will be defined in the next section.

### 3.5.2 Correlation between time and one round of the game

In the Gambler’s ruin theory, the number of rounds of a game differs from real time. In engineering applications of risk analysis, the real time needs to be determined. Before the effect of time can be added to the developed model, the relationship between the number of rounds of the game and real time must be established. The ratio between the number of rounds of the game and time is assumed to be constant. This ratio is influenced by flow conditions and characteristics of the sediment. This ratio is used herein to estimate the real time spent, and an appropriate probability can be determined from the time spent.

$$C_{\text{time}} = \frac{D(\text{day})}{R(\text{round})}$$

(23)

where $C_{\text{time}}$ is the ratio between the real time $D(\text{day})$ and the number of the round of the game $R(\text{round})$.

In this work, various values of $C_{\text{time}}$ are tested. Eventually, a rule for fitting $C_{\text{time}}$ will be required. The value of $C_{\text{time}}$ can be determined experimentally for various flow conditions and time scales.

### 3.6 Monte Carlo simulation of non-stationary model

The Gambler’s ruin model is used for a number of related statistical ideas (Harik, Cantu-Paz, Goldberg, & Miller, 1997; Lai, 2012; Tsai et al., 2014; Wu, 2015). For instance, probabilities are hypothesized to be stationary...
in each game. The ideas have specific relevance for gamblers; however, they are also general theorems with wide application and many related results in probability and statistics. The main purpose of this work is to construct a non-stationary probability model. Analytical solutions are not available when probabilities are non-stationary.

In this investigation, the Monte Carlo simulation is carried out to construct a numerical model. Additionally, a relationship between time and the number of rounds of the game will be obtained to define non-stationary transition probabilities. The accuracy of a numerical model will be demonstrated by application to a stationary flow case.

4 | CASE STUDY: SHIHMEN RESERVOIR BASIN

In this study, uncertainty analysis is introduced to evaluate the effective risk variation of violating the pre-established water quality standard when considering the variability of the daily water level.

4.1 | Properties of research region

The water system of the Shihmen Reservoir watershed originates from the Xueshan Mountains, and the mainstream Dahan Stream is the largest tributary of the Danshui River, with a total length of about 94.01 km. Most of the exposed rock layers in the watershed are sedimentary rocks and lightly metamorphic rocks. The fault structures are mostly northeast-southwest strikes, with slopes of grade 6 (slopes between 55% and 100%) accounting for the highest proportion. The data from the Xia Yun hydrological station (No.1140H054) is used in this investigation. The Xia Yun hydrological station is located at the upstream side (X: 285813, Y: 2740336, TWD67) of the Shihmen Reservoir (No.20405) basin, as shown in Figure 5. Hydrographical data (water level, flow discharge, sediment concentration and cross-sectional area) in year 2008 were obtained from the Northern Region Water Resources Office (NRWRO).

4.2 | Model calibration and validation

The proposed model must be calibrated to determine the friction slope $S$. Sixty-one daily flow discharge data were used for calibration and validation. These data were separated into two groups. The first group, used for calibration, consists of 41-day data, and the second group, used for validation, is composed of 20-day data. The resulting friction slope $S$ was obtained as 0.00135 using the least squares method. Figure 6 plots the result of calibrating and validating the flow discharge rating curve. The $R$-square in the verification of the flow discharge rating curve is 0.964.

After curve fitting in Section 3.4, Figure 7 shows the sediment discharge rating curve by the computed and the verification result, solid line is the fitting result in Section 3.4, dashed line is the fitting result by power function fitting in Tsai et al. (2014). In Figure 7 when flow discharge is less than about 1,000 cms, present study modelling suspended load discharge result is higher than Tsai et al. (2014) which is previous Gambler’s ruin problem modelling study in Shihmen reservoir basin. The overestimating (flow discharge is less than about 1,000 cms) causes some initial state problems, would be discussed later in chapter summary. Moreover, both sediment discharge rating curves estimate not good when flow discharge comes very large (Wu, 2015, p.22).

4.3 | Risk analysis

The maximum standard operating turbidity of a water treatment plant (2,000NTU) is used in this study to define the threshold to calculate the risk. We used an empirical formula that was presented by Rao and Hsu (2008) for relating sediment concentration in the Shihmen Reservoir basin and turbidity, as follows.

$$Q_{SL,PPM} = 1.124Q_{NTU}$$  \(24\)

where $Q_{SL,PPM}$ is the sediment concentration [PPM], and $Q_{NTU}$ is water turbidity.

The Gambler’s ruin model is utilised herein to analyse the risk that water turbidity reaches its maximum allowed value in the water treatment plant ($Q_{NTU} = 2,000$). The probability of failure is the probability that $Q_{max} < Q_{SL,PPM}$, so the risk can be given as follows:

$$Risk = P(Q_{max} < Q_{SL,PPM})$$  \(25\)

where $Q_{max}$ is the maximum sediment capacity of water treatment plant [PPM], which equals 2,248 PPM in this study, and $Q_{SL,PPM}$ is the estimated sediment concentration [PPM].

4.4 | Uncertainty analysis

In the Gambler’s ruin model, the flow data are important. The use of daily water levels to calculate flow data may come with uncertainty. One of the uncertainties arises
from using daily mean water level to represent the time varying flow depth on each day. In this work, uncertainty in the variable water level is considered, and the point estimate method (PEM) is used to perform an uncertainty analysis.

Rosenblueth (1975) presented a two-point method to calculate the uncertainty of an input variable of a functional relationship; in 1981, he improved the method to accommodate asymmetric probability distributions (Rosenblueth, 1981). The point estimate method (PEM) uses statistical moments (26) and (27) to present the preservation of statistical moments, which are used to obtain representative points and their weights.

From Equation (26),

\[
\begin{align*}
    P_+ + P_- &= 1 \\
    P_+ x_+ + P_- x_- &= \bar{X} \\
    P_+ (x_+ - \bar{X})^2 + P_- (x_- - \bar{X})^2 &= \sigma_X^2 \\
    P_+ (x_+ - \bar{X})^3 + P_- (x_- - \bar{X})^3 &= \sigma_X^3 \nu_X^3
\end{align*}
\]  

(26)

From Equation (27),

\[
\begin{align*}
    P_+ &= 0.5 \left[ 1 \pm \sqrt{1 - \frac{1}{1 - (\nu_X/2)^2}} \right] \\
    P_- &= 1 - P_+ \\
    x_\pm &= \bar{X} \pm \sigma_X \sqrt{P_+/P_\pm}
\end{align*}
\]  

(27)
If the statistical moment of the input variables is known, then those of the model outputs can be obtained using Equation (28).

\[
E(Y^n) = P_+ y_+^n + P_- y_-^n
\]

\[
\text{Var}(Y) = E(Y^2) - E(Y)^2
\]

Although the point estimate method (PEM) can be used to calculate the statistical moments of the model output, its probability distributions cannot be directly determined, which can be obtained using the Monte Carlo method. The advantage of the point estimate method (PEM) is its computational efficiency.

5 | RESULTS AND DISCUSSION

5.1 | Original Gambler’s ruin model

The limitations of the existing GR model (Tsai et al., 2014) are threefold. First, the transition probabilities are built primarily for stationary flows. Second, either the particle deposition or particle entrainment needs to take place. However, the probability of having no deposition/entrainment processes (i.e., a tie) can be accommodated. Third, the uncertainty analysis cannot be directly coupled with the proposed GR model for probabilistic predictions and risk assessment.

The probability of the event that the sediment concentration exceeds 2,248 PPM can be calculated using the original Gambler’s ruin model. In the model, only the present flow data and not the future change in the flow data are considered to obtain the transition probabilities. Figure 8a shows the results obtained using the original Gambler’s ruin model. Figure 8b compares these results with the daily concentration data for the Shihmen Reservoir. The risk of excessive water turbidity is high in June to October, but the original concentration data reveal only three events in which water turbidity in the Shihmen Reservoir exceeded 2,248 PPM from July to October.

Figure 9 shows the sediment lifting probability. Before the peak probability is reached, the probability decreases very fast. The time scale of extreme flow events in Taiwan is typically not very long so the lifting probability may not remain high for a long time. When a
stationary transition probability is assumed, the predicted water turbidity is prone to increase. In this work, the stationary assumption is relaxed and the Gambler's ruin model can then consider time-varying flow conditions.

5.2 Gambler’s ruin model with non-stationary probability

The non-stationary Gambler’s ruin model is developed using Monte Carlo simulation. The proposed analytical solution to the stationary Gambler’s ruin model is validated. The special case of the stationary Monte Carlo simulation is demonstrated to yield non-distinguishable results with the analytical solution. Section 3 introduces the non-stationary probability model, in which $C_{\text{time}}$ values are used to account for the transition probability that is estimated using future flow condition. Figure 10 presents results obtained using the Gambler’s ruin model with non-stationary probability and consideration of flow conditions 2–3 days into the future. These results are closer to actual daily concentration data. The non-stationary model depends on predicted flow conditions. Figure 10 displays the obtained results when the ratio $C_{\text{time}}$ is 1/500.

Figure 11 displays the mean time spent of each date in each state, which is found to vary between 200 and 1,200 rounds. To consider flow conditions for 2–3 days in the model, the ratio must be set to 1/500 (number of rounds: 200–1,200; number of days: 2 ~ 3; $C_{\text{time}} = 1/500$). In this non-stationary probability model, predicted flow conditions are considered by adjusting $C_{\text{time}}$ values. However, $C_{\text{time}}$ must be obtained as a function of flow and particle conditions. In this case, $C_{\text{time}}$ is determined from the mean time spent. The mean time spent is the number of rounds in the simulation. Figure 11 shows the mean time spent in the original stationary GR model, estimated using the transition probability matrix $P_T$.

For non-stationary probabilities, $C_{\text{time}}$ is defined in terms of the predicted flow conditions, which may be simulated using a hydraulic channel flood model, such as the HEC-RAS. In this study, the predicted flow conditions herein are referred to as the non-stationary flow conditions 2–3 days into the future. The periods of high risk are July and September, as the concentration data (Figure 8b) reveal events of high water turbidity (over 2,248 PPM) are identified then.

Typhoons attacked Taiwan on dates close to those of high risk. Table 4 reveals that the original GR overestimates the risk since the time scale of typhoon events in 2008 was not very long. A model with non-stationary probability is more suitable for estimating the risk of high water turbidity than is the original model, according to the results from the comparison of both Figures 8a and 11 with Figure 8b.

The Gambler’s ruin model is an innovative method to quantify the probability of sediment transition. That means it can be considered as a rough estimate but relatively accurate method. Compared to deterministic model and empirical model, it also can provide uncertainty that makes the scenario closer to the reality owing to the transition probability in the non-stationary flows. However, the proposed model requires a number of flow and concentration data. If the data cannot satisfy the requirement for analysis, reliable results may not be guaranteed. Particularly, in the Gambler’s ruin model $C_{\text{time}}$ is a critical parameter that should be determined with caution because it depends on the time scale characteristic of extreme event.

![Figure 8](image-url) Risk that water quality exceeds standard, estimated using original Gambler’s ruin model
**FIGURE 9**  Increasing sediment lifting probability on different date

**FIGURE 10**  Risks obtained using non-stationary Gambler’s ruin model, with $c_{\text{time}} = 1/500$
Results using Rosenblueth method

Hourly data were used to estimate the uncertainty of the non-stationary model by the Rosenblueth point estimate method (Rosenblueth, 1975). If the temporal variability of extreme flow events is not very high, then the flow can be viewed as approximately stationary, and averaged stationary transition probabilities can be used. The mean value and the standard deviation of the output were thus obtained. They provide such information as the times when the flow data are most uncertain. Figure 12 displays the results concerning the uncertainty of the non-stationary model. The continuous line in Figure 12 indicates the mean output and the dashed lines represent the plus- and minus-one standard deviation of the output away from the mean of the output.

The mean output is very close to the deterministically obtained result, as the deterministic model uses mean daily flow data. One should note that the deterministic model in this study is referred to as a functional relationship without considering the uncertainty of its independent variables. For the deterministic model, the mean values of the measured data are used to quantify the independent variables. However, the standard deviations are very high, indicating high changes in hourly flow data. The following figures include error bars in flow values for 4 months.

Figure 13a shows uncertainty in the results obtained using the developed model from January to March, when the standard deviations are close to zero. Uncertainties of the flow data from January to March are insignificant. In Taiwan, the rainfall is concentrated between May and November. Rainfall dominates the change of hourly flow data, which is therefore insignificant from January to March. Figure 13b presents the uncertainty in the results obtained using the non-stationary model estimated using Rosenblueth method from for April to June. The standard deviation becomes high in June. The uncertainty of flow increases significantly, owing to the onset of the Asian rainy season.

Figure 13c presents uncertainty in the results obtained using the model from July to September. The standard deviation fluctuates considerably. The uncertainty in flow changes frequently, perhaps because of frequent typhoons in summer (Kalmaegi: July 16–July 19; Fung-Wong: July 26–July 29; Sinlaku: September 11–September 16; Jang-Mi: September 26–September 29) as
presented in Table 4. Figure 13d shows uncertainty in the results obtained using the model from October to December. There is a typhoon event in September 26–29, and the uncertainties are negligible in December. The uncertainty analysis revealed high-uncertainty events such as typhoons attack and leave. This work considers only uncertainty in the hourly change in the flow data.

The results indicate that typhoons account for most of the uncertainty in the hourly flow data, which alter dramatically during typhoon events. Tsai et al. (2014) demonstrated that typhoon events bring significant rainfall, and higher sediment concentrations. High-risk events typically accompany high uncertainty in the flow. Uncertainty in the model is not caused only by changes in the flow data. If the temporal variability of extreme flow events is not very high, then the flow can be viewed as approximately stationary. In this research, only water level is considered to generate uncertainty because a simple flow condition is assumed in this study. If a more complex flow condition is assumed, then various uncertainty factors can be considered in the uncertainty analysis.

6 | CONCLUSIONS

Sediment transport processes such as scouring and deposition are critical in rivers. Such phenomena affect the turbidity of a reservoir, and the geomorphology and water quality of a river. Particular attention should be paid to reservoir sedimentation under non-stationary flow conditions such as in the case of flash floods, wind-induced surges and tidal waves. This work establishes an improved Gambler's ruin model with non-stationary transition probabilities that can be used to estimate the risk when the reservoir turbidity standard is violated. The extended capacity of the improved GR model to accommodate non-stationary flows, which oftentimes occur in Taiwan due to its strong monsoon feature and steep sloped rivers and streams, is demonstrated herein. Specific hydrologic insights that can be potentially gained from probabilistic predictions of the sedimentation processes are summarised as follows:

1. The transition probability of a reservoir reaching different levels of turbidity (i.e., mean time spent in transient states) can be determined, even under the non-stationary flow conditions.
2. The average time to reach a designated reservoir maximum handling turbidity can be quantified.
3. The risk of reservoir sedimentation for both stationary and non-stationary flow conditions can be calculated.

The original GR overestimates the risk when the time scale of extreme flow events is relatively short. If the temporal variability of extreme flow events is insignificant, then the flow can be viewed as approximately stationary, and averaged stationary transition probabilities can be
used. In the case herein, when extreme flow events in Taiwan change dramatically over time, non-stationary probabilities in GR should be used. Although the non-stationary GR can more accurately estimate the risk of extreme flow events, the ratio $C_{\text{time}}$ must be carefully determined for the predicted flow conditions. The ratio $C_{\text{time}}$ is determined herein from the mean time spent. $C_{\text{time}}$ was found to depend on the characteristic time scale of extreme flow events, which changes over time. Therefore, $C_{\text{time}}$ must be related to the characteristic time scale of extreme flows, which needs to be further investigated.

The results using Rosenblueth method also reveal that both risk and uncertainty are high in summer, mainly because of typhoons. In this research, the transition probability is defined in terms of flow conditions. The size of particles is assumed herein to be uniform. This work focuses on flow conditions, but transition probabilities may also depend on many other variables, such as rainfall intensity, bed form and erosion. Discussions on the aforementioned variables are desirable. The proposed model can be extended to more complex flow conditions.

In conclusion, it is critical to identify the critical factors that affect the rate of sedimentation, so that the probability and severity of losses in future flood events can be more comprehensively estimated. This study provides a quick estimate for the lifespan of a reservoir, as well as a risk assessment of reservoir sedimentation as a function of time. Such information can render policy makers to effectively and scientifically manage the dredging/remediation with existing reservoirs. Research outcomes will provide useful information for policy decision-making by federal and state agencies to evaluate transport capacities of rivers and streams, to estimate environmental impacts of flood damages, and to understand the potential

![Figure 13](image-url)  
Figure 13: Uncertainty of the flow data from (a) January to March; (b) April to June; (c) July to September; and (d) October to December
consequences of flood risks when accounting for river sedimentation processes in order to allow effective contingency planning for public safety.

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CONFLICT OF INTEREST
The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

DATA AVAILABILITY STATEMENT
The data used are listed in the references and tables.

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