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Numerical method for analyzing the nonlinear dynamic response of tubing running in a cased borehole

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Abstract. With the development of deeper and longer wells, the environment of the borehole becomes much more complex, leading to a higher risk of tubing failure. To accurately describe the contact force between the tubing and the borehole, a new method is developed to analyze the non-linear dynamic response of the tubing under complex loading condition. Based on the finite element method, the equilibrium equation of tubing segment is established, where the tubing is simulated by three-dimensional elastic beam elements and the contact between the tubing and borehole is represented by two no-linear springs. The element stiffness matrix was assembled under nodal coordinate system, and the modified square root algorithm was adopted to solve the large-scale banded systems of linear equations. Then, the contact force and position were solved by multi-step iterative method, and thus a new numerical method for analyzing the non-linear dynamic response of tubing was developed. Results show that, the new method could greatly improve the counting rate for analyzing the large deformation of tubing, and also could accelerate the convergence of the non-linear contact problem. This makes it describe well the non-linear dynamic behavior of the tubing, which is helpful for the optimization design and security evaluation of the tubular assembled in complex conditions. This method has been widely applied on the high angle deviated wells in the Shengli oilfields.

1. Introduction

During the development of non-conventional well, such as the deep well or super deep well, the geological condition is very complex, and the wellbore curvature, high temperature and high pressure usually occur. This makes the dynamic analysis of the downhole tubular get to be much more difficult, especially the non-linear contact state between the tubing and the borehole.

To provide the criterion for the optimization design of tubular, as well as, to avoid buckling failure, it is necessary to establish the dynamic equations of the elastic thin-walled tubing, and an efficient calculation method is also needed.

Considerable efforts have been carried out to analyze the deformation and dynamic responses of tubing running in a cased borehole, and many excellent works can be found in literature. Houliara et al. [1] focused on the structural stability of long uniformly pressurized thin elastic tubular shells subjected to in-plane bending. Using a special-purpose non-linear finite element technique, bifurcation on the
pre-buckling ovalization equilibrium path was detected, and the post-buckling path was traced. Fateh et al. [2] presented an evaluation of a curved spring element that may be utilized in a developed variable stiffness bracing system to confer the variable stiffness characteristic of the system. Mahmoudinezhad [3] developed an accurate spring-mass model, in the context of a three-dimensional finite element formulation for investigating the vibrational characteristics of single-walled carbon nanotubes. Yang [4] proposed an innovative drilling technique called slotted liner sheathing coiled tubing (SLSCT) to synchronize the jet drilling and liner running in one trip. Turner [5] developed a new tribometer for the measurement of sliding friction between a stationary flexible polymeric sleeve and a rigid metallic rod that moves longitudinally within the sleeve. Guana [6] used the method of combined numerical calculation and experimental validation to study coiled tubing buckling inside the pipeline or wellbore. Seide [7] investigated bifurcation of initially straight circular tubes under bending, assuming a linear (non-deformed) pre-buckling state, and a Ritz-type bifurcation solution in terms of trigonometric functions. Using a finite-difference discretization of shell stability equations, Stephens et al. [8] investigated bifurcation of finite length, initially-straight tubes under bending, considering prebucklingovalization, as well as end-effects, and calculated bifurcation moments for different levels of pressure. Chen [9] and Zhu [10] proceeded experimental and computational study on ways to reduce friction between coil tuber and riser to ensure the safety and reliability of the operation.

It can be noted that, for the existing methods for analyzing the non-linear dynamics of tubing, there always exists some assumptions and simplifications, which can be given as follows:

1. In the section of vertical well, the axis of the tubing is in coincidence with that of the casing. So the contact and friction between tubing and casing can be neglected.
2. In the section of horizontal well, the bottom of the tubing keeps contact with the casing.
3. The nonlinear deformation of the tubing that induced by bulking is ignored.

However, during the oilfield production, the tubing bulking usually occurs because of the wellbore curvature. Therefore, the assumptions listed above are indeed unreasonable. They make the non-linear contact between tubing and casing cannot be considered, giving rise to low analysis accuracy.

2. 3D beam-spring element coupled model

Tubing is simulated by three-dimensional beam element, as showed in Fig 1.

![Three-dimensional beam element](image)

**Fig. 1** Three-dimensional beam element

The three-dimensional beam element can bear axial tension/compression, bending and torsion. Every beam element has six degrees of freedom at each node: the displacements along the x, y, and z directions and the rotations around the axes x, y, and z.

The contact and friction between tubing and casing were represented by 3D space spring elements. On the top of tubing is a tubing hanger, which is fixed. The bottom of the tubing can be twisted and can slip along its axis.
2.1. Equivalent nodal force

During the process of running the tubing into the hole, the tubing is suffered mainly by the gravity and the buoyancy force. The gravity can be given as:

\[ q_g = \rho g A \]  
(1)

where \( q_g \) is the gravity of the tubing per unit length, \( \rho \) the density of the tubing, \( g \) the gravitational acceleration, and \( A \) is the cross-section area of the tubing.

The buoyancy force is given by

\[ q_f = \rho g A \]  
(2)

where \( q_f \) is the buoyancy force per unit length, and \( \rho_f \) is the density of the well completion fluid.

As thus, in local coordinate system, the equivalent force of the node (\( p_{node1} \)) can be expressed by the gravity and buoyancy force, that is

\[ p_{node1} = \begin{bmatrix} p_{e11} & 0 & p_{e13} & 0 & p_{e15} & 0 & p_{e17} & 0 & p_{e19} & 0 & p_{e111} & 0 \end{bmatrix}^{T} \]  
(3)

where

\[ \begin{align*}
  p_{e11} &= \frac{l_i(q_g - q_f) \cos \alpha}{2}, & p_{e13} &= -\frac{l_i(q_g - q_f) \sin \alpha}{2}, & p_{e15} &= \frac{l_i^2(q_g - q_f) \sin \alpha}{12} \\
  p_{e17} &= \frac{l_i(q_g - q_f) \cos \alpha}{2}, & p_{e19} &= -\frac{l_i(q_g - q_f) \sin \alpha}{2}, & p_{e111} &= -\frac{l_i^2(q_g - q_f) \sin \alpha}{12} \\
  -\alpha &= \frac{\alpha_i + \alpha_j}{2}
\end{align*} \]  
(4)

Here, \( l_i \) is the length of the beam element, \( \alpha_i \) the deviation angle of the tubing at node \( i \), and \( \alpha_j \) is deviation angle of the tubing at node \( j \).

2.2. Contact analysis between tubing and casing

(1) The equilibrium equation of the spring element

The contact force and friction between tubing and casing at nodes \( i \) and \( j \) are schematically presented in Fig.2.

\[ \text{Fig. 2 Contact force and friction force} \]
For the node \( i \), the equivalent displacements of the spring along the direction of \( \vec{n} \) and \( \vec{b} \) are respectively marked as \( v_i \) and \( w_i \). The corresponding equivalent stiffness are marked as \( k_n \) and \( k_b \), respectively. As thus, the contact force can be given by

\[
N_i = k_n v_i \quad (6)
\]

\[
N_i = k_b w_i \quad (7)
\]

\[
N_i = \sqrt{(N_n)^2 + (N_b)^2} \quad (8)
\]

where \( N_i \) is the contact force at node \( i \), \( N_n \) and \( N_b \) are the components of the contact force along the direction of \( \vec{n} \) and \( \vec{b} \), respectively.

Then, the friction force between the tubing and casing can be achieved, namely

\[
f_i = \mu N_i \quad (9)
\]

where \( f_i \) is the friction at node \( i \), and \( \mu \) is the friction coefficient.

To facilitate the matrix assembling, we expand the Eqs. (6) and (7) as

\[
\{N_i\} = \begin{bmatrix} 0 & N_n & N_b & 0 & 0 & 0 \end{bmatrix}^T \quad (10a)
\]

\[
[k_i] = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & k_n & 0 & 0 & 0 & 0 \\
0 & 0 & k_b & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \quad (10b)
\]

Then, the equilibrium equation of the spring element can be expressed as

\[
\{N_i\} = [k_i]\{u_i\} \quad (11)
\]

where

\[
\{u_i\} = \begin{bmatrix} 0 & v_i & w_i & 0 & 0 & 0 \end{bmatrix}^T \quad (12)
\]

(2) Judgment criteria for contact

We set the inner diameter of the casing to be \( R_{ti} \), the outer diameter of the casing to be \( R_{yi} \), the inner diameter of the centralizer to be \( R_{fi} \), and the annular clearance to be \( \Delta R_i \), thus we have

\[
\Delta R_i = R_{ti} - R_{yi} \quad (13)
\]

Here, we introduce the displacement allowance to\( l \) to judge whether the contact between tubing and casing occurs.

The necessary condition for the contact can be given as
\[
\Delta R_i - \text{tol} \leq \sqrt{(v_i)^2 + (w_i)^2} \leq \Delta R_i + \text{tol}
\] (14)

The contact can be neglected if

\[
0 \leq \sqrt{(v_i)^2 + (w_i)^2} < \Delta R_i - \text{tol}
\] (15)

The tubing cannot be injected when

\[
\sqrt{(v_i)^2 + (w_i)^2} \leq \Delta R_i + \text{tol}
\] (16)

Therefore, judgment criteria for contact at node \(i\) can be expressed as

\[
\begin{cases}
0 \leq \sqrt{(v_i)^2 + (w_i)^2} < \Delta R_i - \text{tol}, & kw_{ni} = 0, \ kW_{bi} = 0 \\
\Delta R_i - \text{tol} \leq \sqrt{(v_i)^2 + (w_i)^2} \leq \Delta R_i + \text{tol}, & kw_{ni} > 0, \ kW_{bi} > 0
\end{cases}
\] (17)

2.3. Matrix assembling

The contact force and friction are unknown, and they could be expressed easily in nodal Coordinate system. So, the beam element stiffness matrix could be obtained in nodal coordinate system by coordinate conversion.

\[
K\U = P
\] (18)

where \(K\) is the total stiffness matrix of the node, \(\U\) the total deformation matrix, \(P\) the total load matrix. \(K\) and \(P\) are defined as follows

\[
K = K_{node} - K_s
\] (19)

\[
P = P_{node} + P_{nodef} + P_{nodeN}
\] (20)

where \(K_{node}\) is the node stiffness matrix of the beam element, \(K_s\) the node stiffness matrix of the spring element, \(P_{node}\) the load matrix for gravity and buoyancy force, \(P_{nodef}\) the load matrix for friction, \(P_{nodeN}\) the load matrix for contact force.

3. Results

Table 1. Model parameters of tubing

| Upper tubing          | Insulated tubing | Lower tubing          | Plain tubing          |
|-----------------------|------------------|-----------------------|-----------------------|
| Outer diameter /mm    | 114.3            | Outer diameter /mm    | 73.0                  |
| Inner diameter /mm    | 62.0             | Inner diameter /mm    | 62.0                  |
| Top depth /mm         | 2676.0           | Top depth /mm         | 1576750.0             |
| Bottom depth /mm      | 1576750.0        | Bottom depth /mm      | 1908830.0             |
| Element number        | 165.0            | Element number        | 35.0                  |
| Element length /mm    | 9393.9           | Element length /mm    | 9488.0                |
Take one well in Shengli oil field for example, the deformation and the dynamics parameters of the tubing are analyzed when running tubing in/out the borehole using the numerical method presented above.

The upper casing size is 13.375 inch (\(\phi 339.725\text{mm} \times 12.21\text{mm}\)), and the lower casing size is 9 inch (\(\phi 228.65\text{mm} \times 13.53\text{mm}\)). The tubing is composed by two parts, i.e., the upper isolated one and the lower common one. Table 1 shows the model parameters of the tubing.

### 3.1. Contact force and friction of tubing

The evolution of friction with well depth is given in Fig. 3. It can be fund that, for the well depth less than 500 m, the contact force and friction force are large when running tubing out the cased hole. This is caused by the gravity and buoyancy of the tubing.

Influenced by the dogleg, the contact force and friction reach their peak values at the place where the well depth equals to 215 m. It indicates that the wellbore curvature is the predominant factor affecting the contact.

At the well depth of 1576 m, where the tubing size changes, the contact force and friction vary greatly.

![Fig. 3 Friction along well depth](image)

![Fig. 4 Axial force along well depth](image)

### 3.2. Axial force of tubing

Fig. 4 gives the axial force of tubing at different well depth when running tubing in/out the cased hole. The results show that the axial force decreases nonlinearly with increasing the well depth, and it becomes almost constant at the horizontal segment of the well.

### 4. Conclusion

Based on this method, the non-linear dynamic responses of tubing under complex loading conditions were analyzed. The results can be concluded as follows:

1. The method developed in this work could greatly improve the counting rate for analyzing the large deformation of tubing, and could accelerate the convergence of the non-linear contact problem. This makes it describe well the non-linear dynamic behavior of the tubing.

2. The contact and friction between the tubing and casing are significant at the bending section and slant section. Thus the block accident usually occurs at these places. So, a reasonable style and a proper size of the tubing should be designed according to the wellbore curvature.

3. The buckling deformation and dynamics parameters, such as contact force, friction and axial force, increase rapidly at the place where the diameter of tubing changes suddenly. So, for this pipe section, the centralizer is suggested to be used.
This numerical method is helpful for the optimization design and security evaluation of the tubular that assembled in complex conditions. This method has been successfully applied on the high angle deviated well in the Shengli oilfields, with good effect.

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