Quark-Gluon Jet Differences at LEP

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Abstract
A new method to identify the gluon jet in 3-jet “Y” decays of $Z^0$ is presented. The method is based on differences in particle multiplicity between quark jets and gluon jets, and is more effective than tagging by leptonic decay. An experimental test of the method and its application to a study of the “string effect” are proposed. Various jet-finding schemes for 3-jet events are compared.

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1 Introduction

$e^+e^-$ interactions at LEP offer a bountiful supply of hadronic final states at the $Z^0$ mass $\sqrt{s} = 91.2$ GeV. The subject of this paper is a particularly interesting subset of these final states that look like the letter “\text{\textbf{Y}}”: three-jet events with angles between the jet axes $\psi_{12} \approx 60^\circ$ and $\psi_{13} \approx \psi_{23} \approx 150^\circ$. They correspond to $qqG$ final states in leading order QCD perturbation theory.

The jet energies, based on massless kinematics, are given by

$$E_1 = \frac{\frac{1}{2} \sqrt{s}}{1 + \cos \frac{1}{2} \psi_{12} \cos \frac{1}{2} \psi_{13} / \cos \frac{1}{2} \psi_{23}} \quad (1)$$

and its cyclic permutations. For $\text{\textbf{Y}}$ events, $E_1 \approx E_2 \approx 24.4$ GeV while $E_3 \approx 42.4$ GeV. The larger energy of jet 3 ensures that it almost always comes from a quark or antiquark. We will further reduce the small fraction of events in which jet 3 is a gluon by a mild cut on its particle multiplicity. The remaining two jets consist of one $q$ jet (or $\bar{q}$ jet, which is equivalent for our purposes) and one $G$ jet. These two softer jets can be used to compare properties of $q$ jets and $G$ jets at the same energy, and to study the regions “between” jets — \textit{provided} a way can be found to identify which of them is the gluon. The OPAL group\cite{1}, for example, has used the presence of electrons or muons from $c$ or $b$ decays to identify the soft quark jet in a small fraction of $\text{\textbf{Y}}$ events.

Quarks and gluons produce QCD showers by repeated branching $q \rightarrow qG$ and $G \rightarrow GG, q\bar{q}$. The showers are observed as jets of hadrons. Since the branching of gluons is favored over the branching of $q$ or $\bar{q}$ by a color factor $9/4$, one expects gluon jets to be broader in angle and to include more particles than quark jets of similar energy. According to QCD Monte Carlo simulations, the differences at hadronic level, for modest jet $E_T$, are not so large as the naive $9/4$ ratio might suggest; but they are nevertheless large enough to be useful for distinguishing $q$ jets from $G$ jets in $p\bar{p}$ and $e^-p$ collisions\cite{2}.

The OPAL tagged jet data show some significant differences between quark jets and gluon jets, but only a small difference in average particle multiplicity. The purpose of this paper is to show that multiplicity differences can nevertheless be used to identify the gluon jet in $\text{\textbf{Y}}$ events, with a greater accuracy than lepton tagging and in a much higher proportion of the events. The method can be tested by comparison with lepton
tagging, and also by an application proposed here to study the “string effect” \[3\].

2 Event Simulation, Jet Finding, and Cuts

Simulated hadronic decays of \(Z^0\) were generated using PYTHIA 5.6\[4\], which has been found to describe the jets observed at LEP rather accurately\[1, 5\]. Default parameter settings in PYTHIA were used. For theoretical clarity at the expense of realism, no measurement errors were assumed for the particle momenta. Neutral particles and even neutrinos were taken to be observable. The significance of the neutrinos will be discussed below.

Three-jet \(Y\) events were selected by a special-purpose procedure in which each event is viewed as if it is close to the desired form, and accepted if it lies in the region

\[
\begin{align*}
|\psi_{23} - 60^\circ| &< 8^\circ \\
|\psi_{12} - \psi_{13}| &< 5^\circ.
\end{align*}
\]

This region has \(\psi_{12}, \psi_{13}\) within 6.5° of 150°, with a tighter limit on \(\psi_{12} - \psi_{13}\) to make \(E_2\) and \(E_3\) nearly equal. According to the simulation, \(Y\) events defined in this tightly-restricted way make up 0.88% of hadronic \(Z^0\) decays. Other jet-finding algorithms such as JADE\[3\] or \(k_\perp\) (“Durham”)\[7\] yield similar rates and estimates for the jet axes, as will be discussed in Section 4.

In detail, our recipe for 3-jet analysis is as follows. Step (1): The main axis \(\hat{n}\) of the event is found by a sum over the momenta of all final particles:

\[
P = \sum_a \vec{p}_a \left\{ \begin{array}{cl} +1 & \text{if } \vec{p}_a \cdot \hat{n} > 0 \\ -1 & \text{if } \vec{p}_a \cdot \hat{n} < 0 \end{array} \right\}.
\]

\(\hat{n}\) is set equal to the direction of \(\vec{P}\) and the sum is recalculated, iterating from an arbitrary initial direction until \(\hat{n}\) stops changing. Step (2): One of the two directions \(\hat{n}\) or \(-\hat{n}\) corresponds approximately to the “hard” jet 3 that will form the stem of the \(Y\). The correct choice is found by calculating the total momentum of particles within 25° of \(\hat{n}\) and of \(-\hat{n}\). That angle is small enough to exclude much of the two “soft” jets,
so the larger sum corresponds to the hard jet direction. Step (3): The estimate of the hard jet direction is improved by defining it as the direction of the sum of all particle momenta within 60° of the jet axis, and iterating this up to 10 times. Step (4): The two soft jets are desired to make angles of $\approx 150^\circ$ with the hard jet axis, so angles of $147^\circ$ and $153^\circ$ are tried for each, along with all possible choices for the azimuthal angle of the 3-jet plane in increments of $3.6^\circ$. Each particle is associated with the nearest assumed jet axis, if it is within $45^\circ$ of that axis, and the resulting total momentum along each axis direction is computed. The choice of axes which yields the largest value for the minimum of the two soft-jet momenta is taken to be the correct one. Step (5): Each particle is associated with the nearest of the three jet axes. The momentum of each jet is defined as the sum of the momenta of the particles assigned to it, to obtain a new estimate of the jet axis directions. This step is repeated until the axes stop changing. Step (6): the angular cuts in Eq. (3) are applied.

Parton quantum numbers of the jets from the Monte Carlo were identified as follows. Each event begins as $e^+e^- \rightarrow q\bar{q}$. It is easy to find a “final” $q$ (or $\bar{q}$) with the same flavor as the original and with momentum close to the direction of the high energy jet 3. The presence of a “final” $\bar{q}$ (or $q$) with the opposite flavor in the direction of jet 1 or 2 identifies the lower energy quark jet. The remaining jet is the gluon. This identification procedure is unambiguous for at least 98% of our final event sample.

### 3 Quark/Gluon jet discrimination

As a measure of particle multiplicity, we mainly use the combination

$$n = n_{\text{ch}} + n_{\text{neut}}/2$$  \hspace{1cm} (4)

This treats charged particles and neutral ones on a roughly equal footing because neutral particles are mostly photons from $\pi^0$ decay. It is possible instead to use the experimentally more convenient $n_{\text{ch}}$ alone, at the expense of some accuracy.

We impose a mild cut $n_{\text{jet},3}^{\text{jet}} \leq 16$ on the hard jet multiplicity, to eliminate events in which it results from a gluon. This cut keeps 70% of the Y events. When discussing $n_{\text{ch}}$, we use $n_{\text{ch}}^{\text{jet},3} \leq 11$ instead, which keeps 75%.
The soft quark jet can be recognized in a fraction of the events by the OPAL lepton tag method. In our sample, 6.8% of events contain $e^\pm$ with energy $> 2 \text{ GeV}$ or $\mu^\pm$ with energy $> 3 \text{ GeV}$ in jet 1 or jet 2. The presence of such a lepton identifies the quark jet correctly 88% of the time. The lepton tag method has two problems, however: (1) it is available in only a small fraction of events, and (2) the analysis of real events containing a lepton suffers from the undetected neutrino that accompanies leptonic decay. The energy carried by neutrinos in events with the above lepton tag is characterized by a mean of 7.0 GeV and a standard deviation of 6.7 GeV. This problem is not included in our analysis, which takes all final particles to be observable, so the lepton tag is accurate less than 88% of the time in practice — OPAL finds about 79%. We therefore set about to distinguish between the soft quark and gluon jets using multiplicity differences.

The jet 1 and jet 2 multiplicity distributions show a rather small difference: mean = 13.8, S.D. = 3.7 for $n^q$ and mean = 11.1, S.D. = 3.9 for $n^G$. The ratios $n^G/n^q = 1.24$ or $n^G_{ch}/n^q_{ch} = 1.26$ are similar to Monte Carlo predictions by OPAL[1], whose measured ratios are closer to 1.1 as a result of jets which are misidentified.

Although the average multiplicities for $q$ and $G$ are not very different, the jet 1 or 2 that has the higher multiplicity is quite likely to be the gluon. This can be seen clearly using the asymmetry variable

$$A = \frac{|n^{\text{jet}1} - n^{\text{jet}2}|}{(n^{\text{jet}1} + n^{\text{jet}2})}.$$  \hspace{1cm} (5)

Fig. 1 shows that for events in which this asymmetry is sufficiently large, $q/G$ discrimination on the basis of multiplicity becomes very accurate. Quantitatively, a cut $A > 0.20$ keeps 42% of the events, with the larger multiplicity correctly indicating the gluon in 85% of the survivors. A stronger cut $A > 0.30$ keeps 21% of the events, and correctly tags the gluon in 90%. If only charged particle multiplicities are used, the asymmetry $A_{ch} = \frac{|n^{\text{jet}1}_{ch} - n^{\text{jet}2}_{ch}|}{(n^{\text{jet}1}_{ch} + n^{\text{jet}2}_{ch})}$ still allows a useful degree of $q/G$ discrimination as shown in Fig 2. A cut $A_{ch} > 0.25$ keeps 43% of the events and correctly tags the gluon in 81% of them.

The fraction of events for which the soft $q$ and $G$ are correctly assigned by the multiplicity difference is shown in Fig. 3 as a function of the fraction of events kept, for all possible cuts on the asymmetry $A$. Note that discrimination based on $n =$
$n_{ch} + n_{neut}/2$ is significantly more effective than $n_{ch}$ alone.

4 Jet Finder Dependences

The predictions of Section 3 are based on the jet finder of Section 2, which was specially designed to analyze $Y$ events. This algorithm adopts the simple point of view taken by other jet finders for $e^+e^-$ scattering, that particles belongs to one and only one jet. It rather logically assigns each particle to the nearest jet axis as measured by the angle in the $Z^0$ rest frame, and defines the jet axis to be the direction of the total momentum of particles assigned to it.

It is interesting to compare with results obtained using the standard jet algorithms JADE and “$k_\perp$”. To make the comparison, we re-analyze our events according to those schemes, choosing the resolution parameter $y_{\text{cut}}$ in these schemes for each event to obtain three jets. (Fixed values $y_{\text{cut}} = 0.02$ for the $k_\perp$ algorithm or $y_{\text{cut}} = 0.04$ for JADE will almost always do.)

The different algorithms agree rather well on the hard jet 3 axis: both JADE and $k_\perp$ differ from our determination by less than 3° for > 90% of the events. In a small fraction of events (0.8% for $k_\perp$, 2.8% for JADE), the determinations differ by $\approx 180^\circ$. Determinations of the two soft-jet axes are also very consistent: omitting the few events where the jet 3 axes are opposite, the jet 1 and jet 2 axes differ from our determination by an average of 3.0° for $k_\perp$ and 4.0° for JADE.

When particle multiplicities are used to distinguish between $q$ and $G$ jets using the asymmetry $A$, there is a sizable difference between the three analysis methods. As shown in Fig. 3, the special purpose jet finder of Section 4 performs substantially better than the $k_\perp$ algorithm, which in turn performs substantially better than the JADE algorithm. It is reasonable to conclude that the special purpose finder does a better job of assigning particles to the jets.

An intermediate possibility for jet finding would be to use $k_\perp$ or JADE to find an initial set of axes, and then associate particles with the nearest axis in place of their assignment by the jet algorithm. One could then recompute the axes and iterate as in the algorithm of Section 2. This yields better $q/G$ discrimination than either $k_\perp$ or
JADE alone, but is not as good as the method of Section 2. It also finds fewer events that pass the angular cut (Eq. (3)), by a factor 0.85.

5 Experimental tests

A direct test of our $q/G$ discrimination could be made by the lepton tag method: when the quark is $c$ or $b$, there is a significant probability of semi-leptonic decay. For events with the cuts advocated here ($A > 0.20$), a tag of $e^\pm$ with $E > 2\text{ GeV}$ or $\mu^\pm$ with $E > 3\text{ GeV}$ will be available $7\%$ of the time.

An indirect test, which is at the same time an interesting application of our method, involves studying the azimuthal structure of the hard jet. Fig. 4 shows the dependence of the multiplicity in jet 3 on azimuthal angle $\phi$, for various regions in polar angle $\theta$ with respect to the jet axis. The coordinates are defined so that $\phi = 0$ is the azimuthal direction of the soft quark jet and $\phi = 180^\circ$ is the azimuthal direction of the gluon. One sees that jet 3 is not azimuthally symmetric: there is a pronounced tendency for particles to be produced on the side of the gluon. (This effect is reversed in the smallest region of $\theta$, simply because the jet 3 axis is defined as the centroid of the momenta within $60^\circ$ of itself.) The asymmetry shown in Fig. 4 is traditionally known as the “string effect.”

It is important to note that the $\phi$ asymmetry is not a trivial result of our $q/G$ discrimination method, which identifies the gluon azimuthal direction as the side with higher multiplicity in the opposite hemisphere from jet 3. This is checked by showing that a stronger cut on the asymmetry used for $q/G$ separation (dashed curve) does not change the azimuthal structure of jet 3.

Multiplicities outside the center of the jet are rather small, as is shown in Table I, which gives the multiplicity integrated over $\phi$ for various ranges of $\theta$. The $\phi$-asymmetry predicted by Fig. 4 will therefore not show itself in individual events, but will appear when the $\phi$ distribution is summed over events. Note that the multiplicity begins to rise only in the last line of the table ($75^\circ < \theta < 90^\circ$), as a result of proximity to jets 1 and 2 at $\theta = 150^\circ$. The region $\theta < 75^\circ$ explored in Fig. 4 is thus not directly influenced by the wings of jets 1 and 2.
One might expect a further test of the method based on comparing the multiplicity asymmetry distribution of Fig. 1 with an uncorrelated distribution obtained by taking $n_{\text{jet}1}$ and $n_{\text{jet}2}$ from different events. It is worth trying this with experimental data, although very little difference between the two is predicted by the simulation.

6 Conclusions

We have shown that the gluon jet can be identified reliably in a sizable fraction of $Z^0 \rightarrow q\bar{q}G$ decays using particle multiplicity. For example, 85% accuracy is possible in 40% of the $Y$ events. This is potentially more useful than the previous lepton tagging approach\cite{1}, which is available in only 7% of events and is subject to uncertainties caused by unobserved neutrinos.

With the cuts described here, one could obtain $\approx 2600$ samples each of identified $q$ and $G$ jets at 24 GeV from every $10^6$ hadronic $Z^0$ decays. A much larger sample could be obtained by opening up the tight angular cuts of Eq. (3). Many studies, e.g., of polarization effects, could be done with such events.

Our method depends on associating particles with the “correct” jet. In doing this, a jet finder that was specially designed to analyze $Y$ events outperforms the $k_\perp$ algorithm, and by even more the JADE\cite{6} algorithm.

Experimentally, the first need is to test our QCD Monte Carlo prediction on real data. This could be done simply by using lepton-tagged events to generate histograms corresponding to the dotted and dashed curves of Fig. 1 or Fig. 2. According to the simulation, $q/G$ multiplicity differences for events containing a lepton are even somewhat larger than shown in these figures.

A second experimental step should be to look for the azimuthal asymmetry of jet 3 predicted in Fig. 4. This is a way to study the “string effect” in a large sample of events. In addition, it provides an implicit test of the $q/G$ discrimination, since an azimuthal preference for the gluon side can only be seen if the gluon jet is correctly identified in a good fraction of the events.

At a more basic level, comparing Figs. 1, 2, or 4 with experiment will provide a test of QCD and its representation by the shower Monte Carlo.
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Table I
Angular dependence of multiplicity in jet 3

| $\theta$  | $n_{ch}$ | $n_{ch} + n_{neut}/2$ | $dn/d\cos\theta$ |
|------------|----------|------------------------|-----------------|
| $0^\circ - 10^\circ$ | 3.48 | 5.21 | 342.9 |
| $10^\circ - 20^\circ$ | 1.35 | 2.13 | 47.2 |
| $20^\circ - 30^\circ$ | 0.70 | 1.12 | 15.2 |
| $30^\circ - 45^\circ$ | 0.71 | 1.13 | 7.1 |
| $45^\circ - 60^\circ$ | 0.51 | 0.83 | 4.0 |
| $60^\circ - 75^\circ$ | 0.45 | 0.77 | 3.2 |
| $75^\circ - 90^\circ$ | 0.53 | 0.86 | 3.3 |

FIGURE CAPTIONS

1. **Solid curve**: Asymmetry (Eq. (5)) of the multiplicity $n = n_{ch} + n_{neut}/2$. **Dashed curve**: Contribution from $n^G > n^q$; **Dotted curve**: Contribution from $n^G < n^q$.

2. Similar to Fig. 1, for the charged particle multiplicity $n_{ch}$.

3. **Solid curve**: Fraction of events with $q$ jet and $G$ jet correctly identified (“purity”) versus fraction of events kept (“efficiency”) by cuts on the multiplicity asymmetry $A$ for $n_{ch} + n_{neut}/2$. Markers indicate $A > 0.4, 0.3, 0.2, 0.1$. **Dotted curve**: Similar but using $n_{ch}$ only. **Long-dashed curve**: $n_{ch} + n_{neut}/2$ is determined using the “$k_\perp$” jet algorithm. **Short-dashed curve**: $n_{ch} + n_{neut}/2$ is determined using the JADE algorithm.

4. Predicted azimuthal distribution of $n_{ch} + n_{neut}/2$ in the high energy quark jet 3, for various ranges in angle $\theta$ from the jet 3 axis. $\phi = 0$ lies in the 3-jet plane on
the side of the low energy $q$ jet, while $\phi = 180^\circ$ lies in that plane on the side of the $G$ jet. Quarks are distinguished from gluons using $A > 0.20$ (Solid curve) or $A > 0.35$ (Dashed curve).