Experimental study of the
$e^+e^- \rightarrow \pi^+\pi^-\pi^0$ reaction by SND detector
in the energy range $\sqrt{s} = 0.42 - 1.38$ GeV

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Abstract
The review of the SND results of the $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ process study
in the energy range $\sqrt{s} = 0.42 - 1.38$ GeV at VEPP-2M collider, based
on about $2 \times 10^6$ selected events, is presented. The total cross section,
parameters of the $\rho$, $\omega$, $\phi$ resonances, and $\omega'$, $\omega''$ states were obtained.
It was found that $\rho\pi$ and $\omega\pi^0$ intermediate states describe the reaction
dynamics. The experimental data cannot be described by a sum of only
$\omega$, $\phi$, $\omega'$ and $\omega''$ resonances contributions. This can be interpreted as a
manifestation of the $\rho \rightarrow 3\pi$ decay, suppressed by $G$-parity, with relative
probability $B(\rho \rightarrow 3\pi) = (1.01 \pm 0.04 \pm 0.034) \times 10^{-4}$.

1 Introduction
The $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ cross section at low energies is determined by the transitions
of light vector mesons $V (V = \omega, \phi, \omega', \omega'')$ into the final state: $V \rightarrow \rho\pi \rightarrow 3\pi$. The mesons with zero isospin have large branching ratios: $B(\omega \rightarrow 3\pi) \approx 0.9$,
$B(\phi \rightarrow 3\pi) \approx 0.15$, $B(\omega' \rightarrow 3\pi) \sim 1$, $B(\omega'' \rightarrow 3\pi) \sim 0.5$. The process

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can also proceed via mechanism suppressed by the G-parity: $V \rightarrow \omega \pi^0 \rightarrow 3\pi$ or $V \rightarrow \rho\pi \rightarrow 3\pi$ ($V = \rho, \rho', \rho''$). The study of the reaction allows to determine the vector mesons parameters and provide information on the OZI rule violation in the $\phi \rightarrow 3\pi$ decay and on the G-parity violation in the processes $\rho \rightarrow 3\pi$. The process $e^+e^- \rightarrow 3\pi$ in the energy region $\sqrt{s}$ below 2.2 GeV was studied in several experiments during the last 30 years [1, 2, 3]. Recently the process $e^+e^- \rightarrow 3\pi$ was also studied by the Spherical Neutral Detector (SND) [2, 7, 8, 9, 10], the process dynamics was analyzed and the cross section was measured in the energy region $\sqrt{s}$ from 420 to 1380 MeV. This talk is a review of the SND results.

### 2 Data processing

The Spherical Neutral Detector (SND) [12] has operated since 1995 up to 2000 at VEPP-2M [11] $e^+e^-$ collider in the energy range from 0.36 to 1.38 GeV. During six experimental years SND had collected data with integrated luminosity about 30 pb$^{-1}$. During the experimental runs, the first-level trigger [12] selects events with energy deposition in the calorimeter more than 180 MeV and with two or more charged particles. For analysis, events containing two charged and two or three neutral particles were selected. Extra photons in $e^+e^- \rightarrow 3\pi$ events can appear because of the overlap with the beam background or nuclear interactions of the charged pions in the calorimeter. Under these conditions the background sources are $e^+e^- \rightarrow e^+e^-\gamma\gamma$, $e^+e^-\gamma$, $\pi^+\pi^\mp\gamma$, $\mu^+\mu^-\gamma$, $2\pi\pm2\pi^0$, $K^+K^-$, $K_SK_L$ processes. To reject the collinear background, the cut on $\Delta\phi$ of the charged particles was imposed: $|\Delta\phi| > 5^\circ$. To suppress the $e^+e^-\gamma\gamma$ events an energy deposition of the charged particles in the calorimeter was required to be small: $E_{cha} < 0.5 \cdot \sqrt{s}$. The cut on dE/dx energy losses in the drift chamber rejected the $K^+K^-$ events in the vicinity of the $\phi$ meson peak: $(dE/dx) < 3 \cdot (dE/dx)_{min}$. Then a kinematic fit was performed under the following constraints: the charged particles are assumed to be pions, the system has zero total momentum, the total energy is $\sqrt{s}$, and the photons originate from the $\pi^0 \rightarrow \gamma\gamma$ decays. The cut on the $\chi^2_{3\pi}$ was applied: $\chi^2_{3\pi} < 20$ at $\sqrt{s} < 1030$ MeV, and $\chi^2_{3\pi} < 5$ at $\sqrt{s} > 1030$ MeV. In the energy region above 900 MeV for additional suppression of the $e^+e^- \rightarrow 2\pi\pm2\pi^0$ and $K_SK_L$ background, the events with exactly two photons were selected. The number of selected events is presented in the Table 1.

| $\sqrt{s}$ below 980 MeV | from 980 to 1060 MeV | from 1060 to 1380 MeV |
|-------------------------|---------------------|-----------------------|
| $N_{events}$            | $1.2 \times 10^6$   | $5 \times 10^5$       |
|                         |                     | $6 \times 10^3$       |

Table 1: The number of selected $e^+e^- \rightarrow 3\pi$ events.
10
10
10
10
10
10
10
10
10
10

Figure 1: The $e^+ e^- \to 3\pi$ cross section. The results of the SND \cite{8, 2, 10} and DM2 \cite{5} are shown. Dashed curve corresponds to the fit under assumption that a relative bias between the SND and DM2 data exists (DM2 data were scaled by a factor of 1.5). Solid curve is the result of the fitting to the SND data only.

Ref.\cite{13}. The obtained cross section is shown in Fig.1. The total systematic error $\sigma_{\text{sys}} \simeq 4 - 5\%$ includes the errors of detection efficiency, integrated luminosity, a model error, and an error due to background subtraction.

3 Data analysis

In analysis of the $e^+ e^- \to 3\pi$ reaction we took into account the $\rho\pi$ and $\omega\pi$ transition mechanisms, possible $\rho'\pi$ transition, and the interaction of the $\rho$ and $\pi$ mesons in the final state \cite{14}:

$$\frac{d\sigma}{d\Omega dm_{\pi^+}} = \frac{4\pi\alpha |\vec{p}_+ \times \vec{p}_-|^2}{s^{3/2} \cdot 12\pi^2 \sqrt{s}} m_0 m_+ \cdot |F|^2,$$

$$|F|^2 = |A_{\rho\pi}(s) \cdot \left( \sum_{i=+,-,0} \frac{g_{\rho\pi\pi}}{D_\rho(m_i) Z(m_i)} + a_{3\pi} \right) + A_{\omega\pi}(s) \frac{\Pi_{\rho\omega} g_{\rho\omega}}{D_\rho(m_0) D_\omega(m_0)}|^2,$$

A possible $\rho'\pi$ contribution was written as a constant term, because we searched it in the vicinity of the $\phi$ meson

$$a_{3\pi} = \frac{A_{\rho'\pi}(s)}{A_{\rho\pi}(s)} \sum_{i=+,-,0,-} \frac{g_{\rho'\pi\pi}}{D_{\rho'}(m_i)},$$

The $\rho\pi$ and $\omega\pi$ amplitudes are

$$A_{\rho\pi}(s) = \frac{1}{\sqrt{4\pi\alpha}} \sum_{V=\omega,\rho,\phi,\omega',\omega''} \frac{\Gamma_V m_V^2 \sqrt{m_V \sigma(V \to 3\pi)}}{D_V(s) W_{\rho\pi}(m_V)} e^{i\phi_V}, \quad A_{\omega\pi}(s) = \sum_{V=\rho,\rho',\rho''} \frac{g_{\rho\omega\pi}}{D_V(s)}.$$

From the dipion mass spectra analysis in the $\phi$-meson energy region \cite{9}, it was found that the experimental data can be described with $e^+ e^- \to \rho\pi \to 3\pi$.
transition only. The value of the constant term obtained by SND is consistent with zero and differs by 2σ from KLOE result [6]. The ρ meson mass and width were measured. The mass value agrees with the results obtained in other e⁺e⁻ experiments. The main results of this analysis are presented in Table 2. The analysis of the dipion mass spectra in the energy region above 1.1 GeV [2] has shown that for their description the e⁺e⁻→ωπ→3π mechanism is required. The phase between e⁺e⁻→ωπ and e⁺e⁻→ρπ processes amplitudes was measured.

The e⁺e⁻→3π cross section measured by SND [8, 2, 10] was analyzed together with the DM2 [5] results on the e⁺e⁻→3π and ωπ⁺π⁻ processes. The SND and DM2 measurements agree poorly. So, to take into account possible relative systematic shift between experiments, the DM2 cross section was multiplied by a factor of 1.5 [2, 10]. It was found that for the good description of the data, the ω, ρ, φ, ω', and ω'' contributions should be taken into account [10]. The measured ω and φ mesons parameters are shown in Table 2 and ω' and ω'' parameters in Table 3.

The conventional view on the OZI suppressed φ→3π decay is that it proceeds through φ→ω mixing, i.e. in the wave function of the φ-meson which is dominated by s quarks, there is an admixture of u and d quarks. An alternative to the φ→ω mixing is the direct decay [11]. Analysis of the Γ(φ→e⁺e⁻)/Γ(ω→e⁺e⁻) ratio and g_{φρπ} and g_{ωρπ} coupling constants obtained in SND experiments indicates that the direct transition is preferable to the φ→ω mixing as the main mechanism of the φ→3π decay [10].

The ω' and ω'' parameters obtained from the fits (Table 3) should be considered as rather approximate estimation of the ω' and ω'' resonances main parameters. To measure the parameters of these states precisely new data above 1.4 GeV required.

It was found that the experimental data cannot be described by a sum of ω, φ, ω', and ω'' resonances contributions. This can be interpreted as a manifestation of the ρ→3π decay suppressed by G-parity. The obtained parameters of the decay B(ρ→3π) = (1.01 ±0.36 ±0.34) × 10⁻⁴ and φₚ = −135 ±13 ±9

|                | SND            | Other data              |
|----------------|----------------|-------------------------|
| mₚ, MeV       | 775.0 ± 1.3    | 775.9 ± 0.5 (PDG-2002)  |
| Γₚ, MeV       | 150.4 ± 3.0    | 147.9 ± 1.3 (PDG-2002)  |
| mₚ±-mₚ₀, MeV  | -1.3 ± 2.3     | 0.4 ± 0.7 ± 0.06 (KLOE[3])|
| | 0.01 ± 0.34    | 0.7 ± 0.1 (KLOE[6])      |
| mₐ, MeV       | 782.79 ± 0.08 ± 0.09 | 782.57 ± 0.12 (PDG-2002) |
| Γₐ, MeV       | 8.68 ± 0.04 ± 0.24 | 8.44 ± 0.09 (PDG-2002)   |
| σ(ω→3π), nb   | 1615 ± 9 ± 57   | 636 ± 27 (PDG-2002)      |
| | 657 ± 10 ± 37   | 1484 ± 29 (PDG-2000)     |
| φₐ, degree    | 163 ± 3 ± 6     | 158 — 172 [15]           |

Table 2: The results of the dipion mass spectra analysis [9], results of ω→3π and φ→3π decays study [10, 8].
\[ V \quad \omega' \quad \omega'' \]

|        | \( m_V \), MeV | \( \Gamma_V \), MeV | \( \sigma(V \rightarrow 3\pi) \), nb | \( \sigma(V \rightarrow \omega\pi^+\pi^-) \), nb | \( B(V \rightarrow e^+e^-) \), eV | \( B(V \rightarrow 3\pi) \) | \( B(V \rightarrow \omega 2\pi) \) |
|--------|----------------|-------------------|-----------------|-----------------|-------------------|------------|-----------------|
| \( \omega' \) | 1400 ± 50 ± 130 | 870 ± 300 ± 450  | 4.9 ± 1.0 ± 1.6 | 4.9 ± 0.2 ± 3.9 | \( \sim 6.5 \times 10^{-7} \) | \( \sim 1 \) | \( \sim 0.35 \) |
| \( \omega'' \) | 1770 ± 50 ± 60  | 490 ± 200 ± 130  | 5.4 ± 0.7 ± 3.9 | 1.9 ± 0.4 ± 0.6 | \( \sim 1.6 \times 10^{-6} \) | \( \sim 0.65 \) | \( \sim 0.65 \) |

Table 3: The \( \omega' \) and \( \omega'' \) parameters obtained from the fit \[10\] \[2\] of SND and DM2 data.

degree are in agreement with the theoretical values expected from the \( \rho-\omega \) mixing \( B(\rho \rightarrow 3\pi) = (0.4 - 0.6) \times 10^{-4} \) and \( \phi_\rho \simeq -90 \) degree.

Using the \( e^+e^- \rightarrow 3\pi \) cross section obtained by SND detector, the contribution to the anomalous magnetic moment of the muon due to the \( \pi^+\pi^-\pi^0 \) intermediate state was calculated \( a_\mu(3\pi, \sqrt{s} < 1.38 \text{GeV}) = (458 \pm 2 \pm 17) \times 10^{-11} \).

4 Conclusion

The \( e^+e^- \rightarrow 3\pi \) cross section was measured in the SND experiment at the VEPP-2M collider in the energy region \( \sqrt{s} \) below 1380 MeV. The experimental data were analyzed in the framework of the generalized vector meson dominance model. It was found that the \( \omega \pi \) and \( \rho\pi \) intermediate states describe the process dynamics. The \( \omega \) and \( \phi \) mesons parameters were obtained and parameters of the \( \omega' \), \( \omega'' \) resonances were estimated. Experimental data cannot be described by a sum of \( \omega \), \( \phi \), \( \omega' \) and \( \omega'' \) contributions. This can be interpreted as a manifestation of the \( \rho \rightarrow 3\pi \) decay. The SND study of the \( e^+e^- \rightarrow 3\pi \) process was reported in Ref. [8, 9, 2, 10]. Now the VEPP-2000 collider with the maximum center-of-mass energy 2 GeV is under construction [17]. The \( e^+e^- \rightarrow 3\pi \) process study will be continued in future experiments with SND detector at VEPP-2000.

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