Nonlocal quantum memory effects in a correlated multimode field

Steffen Wölfmann\textsuperscript{1} and Heinz-Peter Breuer\textsuperscript{1}

\textsuperscript{1}Physikalisches Institut, Universität Freiburg, Hermann-Herder-Straße 3, D-79104 Freiburg, Germany

We review the model of two qubits coupled locally to an environment which consists of nonlocally correlated field modes [Phys. Rev. Lett. \textbf{108}, 210402 (2012)]. We derive the correct expressions for the reduced dynamics of the two-qubit system and demonstrate that strong nonlocal memory effects are indeed present for suitable initial EPR-type Gaussian environmental states.

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I. INTRODUCTION

The study of non-Markovian effects in the dynamics of open quantum systems has attracted vast attention in recent years. Several suggestions for the quantification of non-Markovian behavior \cite{11,12} have been made, applied to different physical models \cite{13,14} and compared among each others \cite{15,16}. Moreover, several experiments \cite{17,18} have been performed quantifying non-Markovian behavior in terms of the flow of information between the open system and its environment \cite{19}.

Recently, it has been shown theoretically as well as experimentally that quantum memory effects can also be induced by nonlocal environmental correlations \cite{20,21}. The first model studied in Ref. \cite{21} illustrates this effect by means of an open two-qubit system coupled locally to an environmental multimode field in a nonlocally correlated initial state. The coherence factors and the assumption on the initial correlated environmental states used in Ref. \cite{21} are however not correct. Here, we provide the correct expressions for the quantum dynamical map and the conditions on two-mode Gaussian states and show that strong nonlocal memory effects indeed occur for particular two-mode Gaussian states whose covariance matrix is in standard form.

II. PHYSICAL MODEL

The first model studied in Ref. \cite{21} regarding nonlocal memory effects consists of two qubits coupled to a bosonic environment. Each of the two qubits interacts locally with its own multimode bosonic bath which is assumed to be a part of a correlated environment. For the state of the latter one chooses a product of two-mode Gaussian states correlating each pair of modes of the two bosonic baths.

The Hamiltonian of the total system is given by

\begin{equation}
H = \sum_{i=1}^{2}(H_S^i + H_E^i + H_{\text{int}}^i),
\end{equation}

where $H_S^i = \epsilon_i \hat{\sigma}_z^i$ and $H_E^i = \sum_k \omega_k \hat{b}_k^i \hat{b}_k^i$ with $\hat{b}_k^i(t)$ referring to the annihilation (creation) operator of the $k$th mode of bath $i$. The interaction Hamiltonian is build up by local interactions which obey

\begin{equation}
H_{\text{int}}^i = \chi_i(t) \sum_k \hat{\sigma}_z^i \otimes (g_k^+ \hat{b}_k^i + g_k^- \hat{b}_k^i),
\end{equation}

where $g_k^i$ denotes the coupling strength of the $i$th subsystem. Without loss of generality we assume that the coupling strengths are real-valued, i.e. $g_k^i \in \mathbb{R}$ for $i = 1, 2$ and all $k$. The function $\chi_i(t)$ is given by

\begin{equation}
\chi_i(t) = \Theta(t - t_i^*)\Theta(t_i^* - t) = \begin{cases} 1, & t \in [t_i^*, t_i^*'] \\ 0, & \text{else} \end{cases},
\end{equation}

for some $t_i^* > t_i^* > 0$. It simulates the turning-on and -off of the local interactions of subsystem $i$ at time $t_i^*$ and $t_i^*$', respectively. The duration of the local interactions and the inset can be varied independently for both subsystems so that it is possible to switch from simultaneous to a successive application of the interactions. Without loss of generality we may assume $t_i^* \leq t_i^*$.

The dynamics of the model is conveniently solved in the interaction picture. Turning to this picture the interaction Hamiltonian $H_{\text{int}}^i(t)$ transforms into $H_{\text{int}}^i(t) = \exp (+iH_0t)H_{\text{int}}^i(t)\exp (-iH_0t)$ where $H_0 = \sum_i (H_S^i + H_E^i)$ which yields

\begin{equation}
H_{\text{int}}^i(t) = \sum_j \chi_j(t) \hat{\sigma}_z^i \otimes \sum_k (g_k^j e^{i\omega_k t} \hat{b}_k^i + g_k^j e^{-i\omega_k t} \hat{b}_k^i),
\end{equation}

as $[\hat{b}_k^i, \hat{b}_k^{i\dagger}] = \delta_{k}\delta_{ij}$. Applying again this commutation relation for the annihilation and creation operators it can be shown that the interaction Hamiltonian $H_{\text{int}}^i(t)$ at times $t$ and $t'$ obeys

\begin{equation}
[H_{\text{int}}^i(t), H_{\text{int}}^i(t')] = -2i\phi(t - t'),
\end{equation}

where $\phi(t - t') = \sum_{j,k} \chi_j(t)\chi_j(t')|g_k^j|^2 \sin[\omega_k^j(t - t')]$ is a scalar function. It is well known \cite{22} that the time evolution operator in the interaction picture is then given by

\begin{equation}
U_I(t) = T_\leftarrow \exp \left[ -i \int_0^t ds H_{\text{int}}^i(s) \right] = \exp \left[ i \int_0^t ds \int_0^s ds' \phi(s - s')\Theta(s - s') \right] \cdot \exp \left[ -i \int_0^t ds H_{\text{int}}^i(s) \right].
\end{equation}
The time evolution operator thus consists of a phase factor $d(t) \equiv \exp[i \int_0^t ds \phi(s - s') \Theta(s - s')]$ and a non-trivial operator $V(t) \equiv \exp[-i \int_0^t dsH_{int}(s)]$ which can be rewritten as

$$V(t) = \exp \left( \sum_{j,k} \beta_j^2(t) \beta_k^* \right),$$

which yields

$$\rho_{12}^{12}(t)$$

$$= \sum_{n,m,r,s=0} e^{i t \{ (-1)^n (-1)^r \} \kappa_1 + (-1)^m (-1)^s \kappa_2} a_{nm} a^{*}_{rs}$$

$$\cdot \langle \hat{n}_{12}^{nm}(t) | \hat{n}_{12}^{rs}(t) \rangle \cdot |nm \rangle \langle rs |$$

$$= \left( \begin{array}{ccc}
|a_{11}|^2 & a_{11}a_{10}^* & a_{11}a_{00}^* \\
|a_{10}|^2 & a_{10}a_{01} & a_{10}a_{00}^* \\
|a_{01}|^2 & a_{01}a_{00} & a_{01}a_{00}^* \\
\end{array} \right)$$

(16)

with

$$\kappa_1(t) = e^{-2i \epsilon_1 t} \langle \hat{n}_{12}^{10}(t) | \hat{n}_{12}^{00}(t) \rangle,$$

$$\kappa_2(t) = e^{-2i \epsilon_2 t} \langle \hat{n}_{12}^{01}(t) | \hat{n}_{12}^{00}(t) \rangle,$$

$$\tilde{\kappa}_1(t) = e^{-2i \epsilon_1 t} \langle \hat{n}_{12}^{11}(t) | \hat{n}_{12}^{01}(t) \rangle,$$

$$\tilde{\kappa}_2(t) = e^{-2i \epsilon_2 t} \langle \hat{n}_{12}^{11}(t) | \hat{n}_{12}^{01}(t) \rangle,$$

$$\kappa_{12}(t) = e^{-2i (\epsilon_1 + \epsilon_2) t} \langle \hat{n}_{12}^{10}(t) | \hat{n}_{12}^{00}(t) \rangle,$$

$$\Lambda_{12}(t) = e^{-2i (\epsilon_1 - \epsilon_2) t} \langle \hat{n}_{12}^{10}(t) | \hat{n}_{12}^{00}(t) \rangle,$$

(17) (18) (19) (20) (21) (22)

and

$$\langle \hat{n}_{12}^{10}(t) | \hat{n}_{12}^{00}(t) \rangle$$

$$= \prod_k \langle \hat{n}^{10}_{k} | [D((-1)^{n+1} \beta_k^1(t)) \otimes D((-1)^{m+1} \beta_k^2(t))]^{\dagger}$$

$$| \hat{n}_{12}^{10}(t) \rangle$$

$$= \prod_k \chi_k^{nmrs},$$

(23)

Using the identities $D(\alpha) = D(-\alpha)$ and $D(\alpha)D(\beta) = e^{-2i m (\alpha + \beta)} D(\alpha + \beta)$ for displacement operators one obtains for $\chi_k^{nmrs}$:

$$\chi_k^{nmrs}$$

$$= \langle \hat{n}^{10}_{12} | \exp \left[ \sum_{j=1}^2 \gamma_{k,nmrs}^j |b_k^j \rangle \langle b_k^j| \right] | \hat{n}_{12}^{10} \rangle,$$

(24)

with

$$\gamma_{k,nmrs}^1(t) \equiv \{ (-1)^n - (-1)^r \} \beta_k^1(t),$$

$$\gamma_{k,nmrs}^2(t) \equiv \{ (-1)^m - (-1)^s \} \beta_k^2(t).$$

(25) (26)

Hence, $\chi_k^{nmrs}$ is the Wigner characteristic function of the pure state $| \hat{n}_{12}^{10} \rangle$, which is easily determined for two-mode Gaussian states.

**III. COHERENCE FACTORS FOR TWO-MODE GAUSSIAN STATES**

In the following we state the explicit expressions for the coherence factors [17]–[22] if the environmental state
Eq. (14) are a two-mode Gaussian state with zero mean. Without loss of generality one may assume that the Gaussian state has zero mean as this can always be achieved applying local operations \[15\,17\]. This does not change the correlations in the two-mode state we are mainly interested in.

We recall that a state of a continuous variable system $\rho \in \mathcal{S}(L^2(\mathbb{R}^n))$ is an $n$-mode Gaussian if and only if for all $\vec{x}, \vec{y} \in \mathbb{R}^n$ the observable $\vec{Y} \equiv \sum_{j=1}^{n} (x_j \hat{p}_j - y_j \hat{q}_j)$ has a normal distribution on $\mathbb{R}^n$ in the state $\rho$ \[17\] where

$$
\hat{q}_j = \frac{1}{\sqrt{2}} (\hat{b}_j + \hat{b}_j^\dagger), \quad \hat{p}_j = -i \frac{1}{\sqrt{2}} (\hat{b}_j - \hat{b}_j^\dagger),
$$

(27)
define the canonical position and momentum operators. That is, one has

$$
\chi_{\mathcal{W},\rho}(z) = \text{Tr}(\rho \exp[-i t \hat{Y}]) = \exp \left[ -it (\hat{X}^T \vec{x} - \vec{m}^T \vec{y}) - \frac{t^2}{2} \vec{u}^T \Sigma \vec{u} \right],
$$

(28)
where $\vec{u}^T = (y_1, x_1, \ldots, y_n, x_n)$ and $\vec{l}_i = \langle \hat{p}_i \rangle$, $\vec{m}_i = \langle \hat{q}_i \rangle$ denote the mean position and momentum. Moreover, the $2n \times 2n$-matrix $\Sigma$ is the covariance matrix of the operator $\vec{X}' = (\hat{q}_1 - \hat{p}_1, \ldots, \hat{q}_n - \hat{p}_n)$, i.e.

$$
\Sigma = \sigma_{\vec{X}'}, \quad \left( \Sigma \right)_{ij} = \langle \hat{X}'_i \hat{X}'_j \rangle - \langle \hat{X}'_i \rangle \langle \hat{X}'_j \rangle.
$$

(29)
For $t = \sqrt{2}$ the operator $\exp[-i t \hat{Y}]$ is the Weyl operator $\mathcal{W}(z)$

$$
\mathcal{W}(z) = \exp \left[ \sum_{j=1}^{n} (z_j \hat{b}_j - z_j^* \hat{b}_j^\dagger) \right],
$$

(30)
where $z_j = x_j + iy_j$ for all $j$. One can show \[17\,18\] that the right hand side of Eq. (28) defines the characteristic function of an $n$-mode Gaussian state for some $\vec{l}, \vec{m} \in \mathbb{R}^n$ and $\Sigma \geq 0$ if and only if

$$
\Sigma + \frac{i}{2} \Omega_n \geq 0,
$$

(31)
where the symplectic form $\Omega_n = \oplus_{k=1}^{n} \omega$ with $\omega = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ encodes the canonical commutation relations. This condition is sometimes called the Robertson–Schrödinger uncertainty relation and is a direct consequence of the Schrödinger uncertainty relation and Williamson’s theorem \[19\] which states that any real-valued, symmetric and positive matrix can be transformed into a diagonal form by an appropriate symplectic operation \[18\]. Note that Eq. (31) implies positivity of $\Sigma$ and that one has $\Sigma + (i/2) \Omega_n \geq 0$ if and only if $\Sigma = -(i/2) \Omega_n \geq 0$.

Now, suppose the environmental states $|\eta_{12}\rangle$ of Eq. (14) are a two-mode Gaussian state with zero mean. According to \[28\, Eq. (24) is then given by

$$
\chi_{k, nmr}(\gamma^1_{k,nmr}(t), \gamma^2_{k,nmr}(t)) = \exp \left[ -\bar{\lambda}_{k,nmr}(t)^\dagger \Sigma_k \bar{\lambda}_{k,nmr}(t) \right],
$$

(32)
where

$$
\bar{\lambda}_{k,nmr}(t) = \begin{pmatrix} \text{Im}(\gamma^1_{k,nmr}(t)) \\ \text{Re}(\gamma^1_{k,nmr}(t)) \\ \text{Im}(\gamma^2_{k,nmr}(t)) \\ \text{Re}(\gamma^2_{k,nmr}(t)) \end{pmatrix},
$$

(33)
and $\Sigma$ satisfies \[31\]. We point out that the covariance matrix which is considered in Ref. \[13\] violates Eq. (31) for any $c \neq 0$.

For any two-mode covariance matrix $\Sigma$ there exist local symplectic operations such that the expectation values $\langle \hat{q}_i, \hat{p}_j \rangle$ are removed \[20\,21\] so that $\Sigma$ is transformed into the so called standard form

$$
\Sigma_{\text{st}} = \begin{pmatrix} a & 0 & c_+ & 0 \\ 0 & a & c_- & 0 \\ c_+ & 0 & b & 0 \\ 0 & c_- & 0 & b \end{pmatrix},
$$

(34)
where $a, b, c_\pm \in \mathbb{R}$ and $a, b \geq 1/2$.

The coherence factors defined in Eq. \[17\,22\] can now be written as products of characteristic functions, i.e.

$$
\kappa_1(t) = e^{-2i \epsilon_1 t} \cdot \exp \left[ -\sum_k \bar{\lambda}_{k,1000}(t)^\dagger \Sigma_k \bar{\lambda}_{k,1000}(t) \right],
$$

(35)
$$
\kappa_2(t) = e^{-2i \epsilon_2 t} \cdot \exp \left[ -\sum_k \bar{\lambda}_{k,0100}(t)^\dagger \Sigma_k \bar{\lambda}_{k,0100}(t) \right],
$$

(36)
$$
\kappa_{12}(t) = e^{-2i(\epsilon_1 + \epsilon_2) t} \cdot \exp \left[ -\sum_k \bar{\lambda}_{k,1100}(t)^\dagger \Sigma_k \bar{\lambda}_{k,1100}(t) \right],
$$

(37)
$$
\Lambda_{12}(t) = e^{-2i(\epsilon_1 - \epsilon_2) t} \cdot \exp \left[ -\sum_k \bar{\lambda}_{k,1001}(t)^\dagger \Sigma_k \bar{\lambda}_{k,1001}(t) \right],
$$

(38)
and $\bar{\lambda}_j(t) = \kappa_j(t)$. Henceforth, we assume that the Gaussian states are identical for all modes, i.e. $\Sigma = \Sigma_k$ for all $k$. For a general covariance matrix in standard form, the exponentials in Eqs. (35)-(38) can be evaluated employing Laplace transforms. After performing the continuum limit for an ohmic spectral density $J = \alpha_j \omega \exp[-\omega/\omega_c]$ with equal cutoff frequency $\omega_c$ but different couplings $\alpha_j$ for the two bosonic baths, one obtains expressions containing the Laplace transform of $(1 - \cos(yt))/t$ and sinc-modulated functions in the exponentials which can be evaluated using standard techniques. For a covariance
matrix in standard form with real-valued coefficients $a$, $b$ and $c$, one then obtains for the coherence factors (35)-(38):

\[
\begin{align*}
\kappa_1(t) &= e^{-2i\epsilon_1 t} \left( 1 + \omega_c^2 t_1(t)^2 \right)^{-4\alpha_1} , \\
\kappa_2(t) &= e^{-2i\epsilon_2 t} \left( 1 + \omega_c^2 t_2(t)^2 \right)^{-4\alpha_2} , \\
\kappa_{12}(t) &= \frac{e^{-2i(\epsilon_1+\epsilon_2) t}}{(1 + \omega_c^2 t_1(t)^2)\, 4\alpha_1 (1 + \omega_c^2 t_2(t)^2)^{4\alpha_2}} \left( 
\begin{split}
(1 + \omega_c^2 (t_1(t) - t_2(t))^2) (1 + \omega_c^2 (t_1(t) - t_2(t))^2) \\
(1 + \omega_c^2 (t_2(t) + t_2(t) + t_1(t))^2) \\
(1 + \omega_c^2 (t_2(t) + t_2(t) + t_1(t))^2) \\
(1 + \omega_c^2 (t_2(t) + t_2(t) - t_1(t))^2) \\
\end{split}
\right)^{2(\epsilon_1 - \epsilon_2) / \sqrt{\alpha_1 \alpha_2}} , \\
\Lambda_{12}(t) &= \frac{e^{-2i(\epsilon_1 - \epsilon_2) t}}{(1 + \omega_c^2 t_1(t)^2)\, 4\alpha_1 (1 + \omega_c^2 t_2(t)^2)^{4\alpha_2}} \left( 
\begin{split}
(1 + \omega_c^2 (t_1(t) - t_2(t))^2) (1 + \omega_c^2 (t_1(t) - t_2(t))^2) \\
(1 + \omega_c^2 (t_2(t) + t_2(t) + t_1(t))^2) \\
(1 + \omega_c^2 (t_2(t) + t_2(t) + t_1(t))^2) \\
(1 + \omega_c^2 (t_2(t) + t_2(t) - t_1(t))^2) \\
\end{split}
\right)^{2(\epsilon_1 - \epsilon_2) / \sqrt{\alpha_1 \alpha_2}} ,
\end{align*}
\]

where we have set $t_1 = 0$ for simplicity. The time $t_2$, at which the interaction of the second spin with its bath is turned on, remains however arbitrary.

### IV. EPR-TYPE INITIAL STATE

A particular candidate for a two-mode Gaussian state whose covariance matrix $\sigma_X$ for $\hat{X} = (\hat{q}_1, \hat{p}_1, \hat{q}_2, \hat{p}_2)$ is in standard form (34) is given by the EPR-type state (22)

\[|\psi_u\rangle = \sqrt{1 - u^2} \sum_{n=0}^{\infty} u^n |n\rangle \otimes |n\rangle , \quad (43)\]

where $u = \tanh(r)$, This state is the analog of a maximally entangled state for continuous variable systems as it corresponds to the Schmidt-decomposition of a maximally entangled state for $r \to \infty$. The state represents the physical realization of the model used by Einstein, Podolski and Rosen in their famous Gedankenexperiment (24). The variable $r \in \mathbb{R}$ denotes the squeezing parameter and is a reminder that this state is obtained by squeezing the two-mode vacuum. Values about $r = 5$ can be realized in experiments (22).

In the position representation the wave function takes the form

\[\psi_u(q_1, q_2) = \frac{1}{\sqrt{\pi}} \exp \left[ -\frac{1}{4} \left( \frac{1 - u}{4} \right) (q_1 + q_2)^2 - \frac{1}{4} \left( \frac{1 + u}{4} \right) (q_1 - q_2)^2 \right] , \quad (44)\]

We see that the exponent is dominated by the second term in the limit $r \to +\infty$ ($u \to +1$), yielding a wave function with strong positive correlations of the positions of the particles. In the opposite limit $r \to -\infty$ ($u \to -1$) the wave function describes strong anti-correlations between the particle positions.

The EPR-type state is also referred to as twin-beam or squeezed vacuum state and defines a two-mode Gaussian state with zero means as one can show by a direct calculation of the characteristic function $\chi^{(2)}_{Y, \rho}$ (28). Its covariance matrix for $\hat{X}$, which can be also easily derived, is given by

\[\sigma^{\text{EPR}}_{\hat{X}, r} \equiv \frac{1}{2} \left( \begin{array}{cccc}
\cosh(2r) & 0 & \sinh(2r) & 0 \\
0 & \cosh(2r) & 0 & \sinh(2r) \\
\sinh(2r) & 0 & \cosh(2r) & 0 \\
0 & -\sinh(2r) & 0 & \cosh(2r) \\
\end{array} \right) \quad . \quad (45)\]

We remark that according to our conventions the expressions for the wave function (14) and the covariance matrix (45) differ from what one typically finds in the literature (see, e.g., (22)). The commonly stated covariance matrix of the twin-beam state lacks an overall factor of 1/2 and a factor of 2 in the arguments of the sinh- and cosh-terms. We also note that within the conventions used in Ref. (13) the correct covariance matrix is obtained from (45) by omitting the overall factor of 1/2 and by reversing the signs of the sinh-terms.

### V. MAXIMAL BACKFLOW OF INFORMATION

An established measure for the degree of non-Markovianity of the dynamics of an open quantum system is given by (21, 22)

\[\mathcal{N}(\Phi) \equiv \max_{\rho_1, \rho_2} \int_{t' > 0} dt \, \sigma(t, \rho_1, \rho_2) , \quad (46)\]
where $\rho_1$, $\rho_2$ are two orthogonal states of the open system and
\[
\sigma(t, \rho_1, \rho_2) \equiv \frac{d}{dt} D(\Phi_t(\rho_1), \Phi_t(\rho_2))
\]
describes the dynamical change of the trace distance $D$ of these states. Moreover, the set $\Phi = \{\Phi_t|0 \leq t \leq T\}$ denotes the one-parameter family of dynamical mappings which describe the dynamics of the open system. Hence, the measure $\mathcal{N}$ determines the maximal increase of the trace distance for any pair of orthogonal input states.

Employing this tool to quantify memory effects in our pure dephasing dynamics for a combined state of the two spin-$\frac{1}{2}$ subsystems \cite{Breuer2009} one observes that a backflow of information is signified by an increase of the coherences. Of particular importance are the coherences $\kappa_{12}(t)$ \cite{Yuan2009} and $\Lambda_{12}(t)$ \cite{Schaudt2010} as they describe the nonlocal features of the joined state of the two two-level systems. The time evolution of the modulus squared of these coherence factors are connected to the trace distance of the (orthogonal) Bell-states
\[
|\Psi_{I}^{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle),
\]
\[
|\Psi_{II}^{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle).
\]
More precisely, the trace distance of these states at time $t$ in the considered model is given by
\[
D(|\Psi_{I}^{+}(t)\rangle, |\Psi_{I}^{-}(t)\rangle) = |\kappa_{12}(t)|^2,
\]
\[
D(|\Psi_{II}^{+}(t)\rangle, |\Psi_{II}^{-}(t)\rangle) = |\Lambda_{12}(t)|^2.
\]

Choosing the EPR-state for the two-mode Gaussian state, determined by the covariance matrix \cite{Yuan2009}, one can study the occurrence of memory effects for the combined two-level dynamics for subsequently applied interactions of equal length, similar to the analysis done in Ref. \cite{Ciccarello2010}. Performing the maximization included in the measure $\mathcal{N}$ \cite{Ciccarello2010} numerically, one shows that the maximal increase is given by the orthogonal pair of states $|\Psi_{I}^{\pm}\rangle$ for positive values of the squeezing. Due to the structure of the coherence factors and the covariance matrix $\sigma_{X,r}^{\text{EPR}}$ it is clear that changing the sign of the squeezing parameter $r$ transform the functions $|\kappa_{12}(t)|$ and $|\Lambda_{12}(t)|$ into each other.

Fig. 1 shows the dynamics of $|\Lambda_{12}(t)|^2$ for $\alpha_{1,2} = 1$ and local interactions of equal length $\omega_c \Delta t = 2.5 \cdot 10^{-2}$ which are in addition turned on and off subsequently. One sees that the rephasing is almost complete for this setup. For $r = 4, 5$ the non-Markovianity quantified by $\mathcal{N}$ is about 0.8. Hence, there are indeed non-local memory effects in this model which are in addition experimentally accessible. Moreover, going to larger squeezings while reducing the interaction length the effect is amplified yielding full rephasing.

VI. CONCLUSIONS

In this paper we have studied the model introduced in Ref. \cite{Ciccarello2010} with respect to the emergence of non-Markovian effects induced by nonlocal environmental correlations. We have derived the correct expressions for the dynamical map of this model for the case of an environmental state which is given by a product of correlated two-mode Gaussian states with a covariance matrix in standard form. Our results demonstrate that strong nonlocal memory effects can be observed if one chooses EPR-type Gaussian initial states. Thus, the phenomenon of nonlocal memory effects indeed exists in correlated multimode fields.

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