The Stellar Population and Orbit of the Galactic Globular Cluster Palomar 3

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Deep stellar photometry of one of the most distant Galactic globular clusters, Palomar 3, based on frames taken with the VLT in Johnson-Cousins broadband V and I filters is presented, together with medium-resolution stellar spectroscopy in the central region of the cluster obtained with the CARELEC spectrograph of the Observatoire de Haute Provence and measurements of the Lick spectral indices for the integrated spectrum. Computations of the orbital parameters of Palomar 3 and nine Galactic globular clusters with similar metallicities and ages are also presented. The orbital parameters, age, metallicity, and distance of Palomar 3 are estimated. The interstellar absorption is consistent with and supplements values from the literature. The need to obtain more accurate data on the proper motions, ages, and chemical compositions of the cluster stars to elucidate the origin of this globular cluster is emphasized.

1. INTRODUCTION

Like other Palomar globular clusters [1], Palomar 3 (Pal 3) was discovered in the 1950s on photographic plates of the Palomar Sky Survey [2]. The main observational characteristics of this cluster from the literature are listed in Table 1. Pal 3 is ~96 kpc from the Galactic center ([1], [3], [4]) in the Galactic halo, and is not a member of any known stellar streams, such as the Sagittarius stream ([5], [6]).

There are only seven globular clusters at distances of 40–120 kpc from the Galactic center [1]: AM 1, Eridanus, Pyxis, Pal 3, Pal 4, Pal 14, and NGC 2419. Their origin is not clear (see, e.g., [7]). Apart from NGC 2419, all have red horizontal branches, low stellar densities, and similar masses and metallicities ([1], [8]). The range of their masses is \((1.8 \pm 0.6) \times 10^4 M_\odot\) [8], and the range of their metallicities is \(-1.3 < [\text{Fe/H}] < -1.8 \text{ dex}\). NGC 2419 is a more massive object \((1.6 \times 10^6 M_\odot)\) with a low metallicity \([\text{Fe/H}] < -2.1 \text{ dex}\) and a blue horizontal branch.

The first estimate of the metallicity of Pal 3 was obtained by Gratton and Ortolani [11] based on the object’s color-magnitude diagram (CMD):  

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1 The iron content in solar units is \([\text{Fe/H}] = \log(N_{\text{Fe}}/N_{\text{H}}) - \log(N_{\text{Fe}}/N_{\text{H}})_\odot\), where \(N_{\text{Fe}}/N_{\text{H}}\) is the ratio of the abundances of iron and hydrogen in terms of numbers of atoms, or in terms of mass, which is related to the mass fraction of elements heavier than helium \(Z\) by an empirical formula (see, e.g., [9]). The solar mass fractions of hydrogen \(X\), helium \(Y\), and metals \(Z\) are given in [10]. Obviously, \(X + Y + Z = 1\).
Table 1. Main properties of Pal 3 from the literature: (1, 2) right ascension and declination, (3) color excess in magnitudes, (4) metallicity [dex], (5) distance from the Sun in kpc, (6, 7) apparent and absolute magnitudes, (8) central surface brightness in mag/arcsec², (9) half-luminosity radius in pc, (10) heliocentric radial velocity in km/s, (11) HBR, and (12) age in billions of years. The first row gives values from [13] and the second row data from the catalog [1].

| RA(2000) | DEC(2000) | E(B − V) | [Fe/H] | Dist | Vt | µV | rh | MV | Vh | HBR | T |
|----------|-----------|----------|--------|------|----|-----|----|----|----|-----|---|
| 10:05:31.9 | +00:04:18 | 0.04     | 1.74   | 91.9 | 14.91 | 23.84 | 19.3 | 5.03 | –  | –   | – |
| 10:05:31.4 | +00:04:17 | 0.04     | 1.66   | 92.7 | 14.26 | 23.08 | 17.8 | 5.70 | 83.4 | −0.5 | – |

[Fe/H] = −1.4 ± 0.4 (this value is [Fe/H] = −1.57 ± 0.19 [14]) on the scale of Zinn and West [12]). Subsequent estimates of the metallicity of Pal 3 based on its CMD range from [Fe/H] = −1.4 ± −1.7 ([13] and references therein). Lee et al. [14] used the CMD of the cluster to determine the horizontal branch ratio, (HBR = (B − R)/(B + V + R)), which serves as an index characterizing the numbers of stars in different parts of the horizontal branch, which they found to be -0.82 for Pal 3. Harris [1] presents another value in his catalog (Table 1). Catelan et al. [19] found that the dispersion of the masses of horizontal-branch stars in Pal 3 was significantly lower than for the cluster M3. According to estimates of the age of Pal 3 based on its CMD ([15], [16], [13], [17]), this cluster is approximately one to two billion years younger than clusters in the Galaxy belonging to the old halo subsystem, such as M3 and M13²

² There is currently no unique system for dividing globular clusters among various Galactic subsystems (see, e.g., [4], [3]). It is believed that clusters of the young halo were members of dwarf galaxies in the past. Such clusters are ~1-2 billion years younger than objects of the old halo (13–14 billion years). Absolute ages in billions of years are known for only some objects. Only relative ages are known for most globular clusters (e.g., [18]). Clusters of the disk rotate at a modest height above the Galactic plane (≤1 kpc) with roughly the same rotational speed as the disk, and are, on average, younger than old-halo objects. Clusters of the bulge have ages comparable to those of old-halo objects, but are located near the Galactic center, at Galactocentric distances of Rgc ≤ 3 − 4 kpc. Globular clusters of the halo are conventionally separated into inner-halo and outer-halo objects, with the tentative division between them being at Rgc = 15 kpc [4]. Clusters located in the young and old halo can be members of either the outer or the inner halo. Clusters located in the bulge can belong to the inner halo.

Borissona et al. [20] discovered a Type II Cepheid in Pal 3. Such Cepheids are usually encountered in massive globular clusters with blue horizontal branches. Based on his deep CCD photometry of stars in Pal 3 with the Very Large Telescope (VLT, Paranal, Chile) in Johnson-Cousins broadband B and V filters, Hilker [13] distinguished stars that were probable cluster members and estimated the cluster’s metallicity and age (Table 1). Sohn et al. [21] carried out photometry of stars in the broad vicinity of Pal 3 using large-scale frames of a mosaic CCD of the Canada-France-Hawaii telescope. They detected
an elongation in the distribution of the cluster stars along the direction of the Galactic antecenter.

There is no unified opinion about whether or not Pal 3 belongs to the young or the old halo. Koch et al. [22] determined the elemental abundances for four red giants in the cluster via high-resolution spectroscopy. They also presented the results of the best deep B and V photometry of Pal 3 currently available, obtained with the LRIS instrument on the Keck I telescope; the main-sequence turn-off point is just barely visible in these data. Koch et al. [22] fitted the observed distribution of stars in the CMD using an isochrone with an enhanced abundance of alpha-process elements ($[\alpha/Fe]$), $[Fe/H] = -1.6$ dex and an age of 10 billion years, developed by Pietrinferni et al. [23]. Based on high-resolution spectra obtained for four of the cluster stars and model stellar atmospheres, it was concluded that the metallicity of the cluster indicated by the Ca II lines of the CaT (8498, 8542 and 8662 Å) and the Mg I lines was $[Fe/H] = -1.58 \pm 0.02\text{(stat.)} \pm 0.13\text{(syst.)}$ dex. Koch et al. [22] also concluded that the abundances of the alpha-process elements Mg, Si, and Ca and of iron-peak and neutron-capture elements were enhanced. This same picture was observed for old halo globular clusters with the same metallicity. Koch et al. [22] suggested that the chemical composition of Pal 3 does not resemble those for stars in dwarf galaxies. The heavy elements in Pal 3 were produced mainly in slow nucleosynthesis processes. Koch et al. [22] did not detect any variations in the elemental abundances in the cluster stars.

Palma et al. [5] proposed that Pal 3 was earlier a member of the Phoenix dwarf irregular galaxy, since the orbit of the cluster is coplanar with the orbit of this galaxy. Pal 3 is now located near the pericenter of its orbit, with the apocenter located at a distance of 445 kpc [5]. This distance coincides with the distance to the Phoenix galaxy.

Orbits for Pal 3 were computed by Dinescu et al. [24] and Balbinot and Gieles [25]. Both analyses used the proper motions of [26], but the two obtained different orbital parameters.

In our current study, we have carried out photometric and spectroscopic studies of Pal 3, and also computed the orbit of this object. This study was motivated by the lack of a unified opinion about the structure of the horizontal branch, the metallicity, and the age of this cluster, which are required to study the relationship between the morphology of the horizontal branch and the cluster’s evolutionary parameters. Another motivation was the contradictory conclusions about the character of the motion and the origin of Pal 3 encountered in the literature.
Figure 1. Illustration of the alignments of the slits. A 4′ × 4′ fragment of a VLT frame in the V filter is shown (North is up, East is to the right). Stars for which photometry was carried out are indicated.

Table 2. Journal of VLT observations. The columns give for the two chips the right ascension and declination of the centers of the CCD frames, exposure time in seconds for the V and I frames, mean FWHM of stellar images in arcseconds for the V and I frames, and the air masses for the V and I observations.

| Chip | RA(2000) | DEC(2000) | $T_{exp}$ | FWHM | Airmass |
|------|----------|-----------|-----------|-------|---------|
|      | hh:mm:ss | gr:mm:ss | V | I | V | I |
| 1    | 10:05:31.4 | +00:06:25.9 | 30 30 | 1.0 0.88 | 1.205 1.201 |
|      |          |           | 0.3 0.3 | 0.75 1 | 1.207 1.202 |
| 2    | 10:05:31.4 | +00:02:26.0 | 30 30 | 0.98 0.95 | 1.205 1.201 |
|      |          |           | 0.3 0.3 | 0.97 0.85 | 1.207 1.202 |

2. CHARACTERISTICS OF THE OBSERVATIONAL DATA

The frames from the archive of the European Southern Observatory (ESO)\textsuperscript{3} used for our photometry were obtained on February 23, 2006 on the Very Large Telescope (Paranal, Chile). Pal 3 was observed with exposures of 0.3 and 30 s in the V and I filters using the camera of the FORS2 spectrograph. The total size of the two-chip CCD array was 4000 × 4000 pixels 0.25 × 0.25 in size. A journal of the observations is given in Table 2.

\textsuperscript{3} http://archive.eso.org/cms.html#PRODUCTS
Table 3. Journal of CARELEC spectral observations (Observatoire de Haute Provence). The columns give the object, RA and DEC, date, exposure, and the seeing (the mean FWHM of stellar images in arcsec).

| Object | RA(2000) | DEC(2000) | Date      | $T_{exp}$ | FWHM |
|--------|----------|-----------|-----------|-----------|------|
| Pal 3  | 100531.4 | +000417   | 02.12.08  | 3x1800, 2.7 | 2.7  |
| Pal 3  | 100531.4 | +000417   | 03.12.08  | 4x1800, 2.5 | 2.5  |
| HD93521| 104823.5 | +373413   | 02.12.08  | 15        | 2.7  |
| HR1544 | 045036.7 | +085401   | 01.12.08  | 2         | 2.6  |
| HR1805 | 052419.8 | +342607   | 02.12.08  | 2         | 2.7  |
| HR2002 | 054556.7 | +243309   | 01.12.08  | 2         | 2.6  |
| HR2600 | 065538.5 | +380723   | 02.12.08  | 5         | 2.7  |
| HR3418 | 083608.7 | +033105   | 03.12.08  | 5         | 2.5  |
| HR3422 | 083734.2 | +460039   | 01.12.08  | 2         | 2.6  |
| HR3905 | 094955.4 | +261436   | 03.12.08  | 2         | 2.5  |
| HD201626| 210748.3 | +262438   | 02.12.08  | 20        | 2.7  |
| HR8924 | 232657.0 | -044819   | 01.12.08  | 1         | 2.6  |

The spectral data for Pal 3 were kindly presented by E. Davoust. The spectral observations were obtained using the CARELEC spectrograph [28] mounted on the 1.93-m telescope of the Observatoire de Haute Provence (OHP). A journal of these observations is given in Table 3. The slit size was 5.5$'$x2$''$. A grating with 300 lines/mm was used, which provided a spectral resolution of approximately 1.78 Å/pixel over the spectral range 3700–6800 Å. Exposures of He and Ne lamps were obtained at the beginning and end of each night, for subsequent calibration of the wavelength scales of the spectra. Spectrometric standards and stars from the list of Worthey [29] (so-called Lick standards) were observed over the course of the night to enable translation of the measured absorption indices into the standard Lick system and control of the radial-velocity measurements of the observed globular clusters. Table 3 lists the standards observed on December 1-3, 2008. The stars HD93521 and HR1544 are spectrophotometric standards.

3. REDUCTION OF THE OBSERVATIONAL DATA

The ESO archive contains direct frames that have passed through a standard preliminary reduction, including subtraction of the electron zero level, correction for dark current, and flat fielding. We removed cosmic-ray traces using the MIDAS program package [27] filter/cosmic.

Our stellar photometry was carried out using the DAOPHOT-II program in the MIDAS package [30]. The seeing in arcseconds estimated by fitting two-dimensional Gaussians to stellar images is presented in Table 2. We constructed growth curves for roughly a dozen isolated stars with medium brightnesses in each frame, to determine the aperture corrections. These corrections were roughly 0.4$''$ lower for the I than for the V frames, due to the different forms of their point spread functions. The magnitudes obtained as a result of applying the aperture corrections were corrected for atmospheric extinction, reduced to a 1 s exposure time, and translated to the international Johnsons-Cousins photometric system using the zero points and color factors...
presented at the ESO site.

The alignment of the CARELEC spectrograph slits during the spectral observations is shown in Fig. 1. The process used to reduce the spectral observations was analogous to the one described, for example, in [31]. The reduction of the long-slit spectra was carried out in the MIDAS [27] and IRAF [32] packages. The dispersion relation provided a mean wavelength calibration accuracy of about 0.16Å. The one-dimensional spectra were extracted using the IRAF procedure *apsum*. We removed sky emission lines using the IRAF procedure *background*.

4. PHOTOMETRIC RESULTS

A table with the results of our photometry of stars in the field of Pal 3 in the VLT frames is presented at the site ftp: ftp://ftp.sao.ru/pub/sme/Pal3. The CMD of the cluster obtained using these stellar photometry results is shown in Fig. 2. This diagram has been constructed using the absolute I magnitude and \((V - I)_0\), corrected for absorption in our Galaxy\(^5\). The large symbols show stars within the visible boundary of the cluster (within 1.5′ of the center of Pal 3). The stars falling into slits "1" and "2" of the spectrograph are shown by different symbols. The left panels shows in two different colors the two isochrones from [33] that best fit the distribution of stars in the cluster CMD. The optimal position of the isochrone was found using the method described in Section 3 of [31]. The isochrone of [33] corresponding to an age of 11.2 billion years and metallicity \(Z = 0.0004\) \((\text{[Fe/H]} \sim -1.74\), according to the empirical calibration of [9]), has parameters similar to those of the isochrone used by Hilker [13] to describe his CMD for the B and V filters.

The right-hand panel of Fig. 2 presents three isochrones. The isochrone of [9] with \(Z = 0.0004\), \(Y = 0.23\), and \(T = 11.2\) billion years is repeated in light gray (green in the electronic version). The isochrone computed by Kim et al.[35] with \(Z = 0.0004\), \([\alpha/\text{Fe}] = 0.3\), \(T = 10\) billion years used by Hilker [13] is shown by the black curve (dark blue in the electronic version). Finally, the isochrone computed by Pietrinferni et al. [23] with \([\text{Fe/H]} = -1.6\) dex, \([\alpha/\text{Fe}] = 0.3\), and \(T = 10\) billion years used by Koch et al. [22] is shown in dark gray (red in the electronic version). We can see that the red-giant and subgiant branches and the main-sequence turn-off points for the three isochrones are shifted relative to one another along the color (temperature) axis. There is no horizontal branch in the isochrones of Kim et al. [35]. The positions of the horizontal branches are close for the isochrones with \(Z = 0.0004\), \(Y = 0.23\), and \(T = 11.2\) billion years [33] and with \([\text{Fe/H]} = -1.6\) dex, \([\alpha/\text{Fe}] = 0.3\), and \(T = 10\) billion years [23].

Hilker [13] used other ESO data and the

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\(^4\) http://www.eso.org/observing/dfo/quality/FORS2/qc/photoeff/photoeffsorfors2.html

\(^5\) \((V - I)_0 = (V - I) - E(V - I)\), where the color excess is given by \(E(V - I) = A_V - A_I\), and \(A_V\) and \(A_I\) are the absorptions in magnitudes in the V and I filters.
Figure 2. Color-magnitude diagram for Pal 3 constructed using the VLT photometry data. Various isochrones are shown in the two panels. The left panel shows the isochrones from [33] for (1) $Z = 0.0004$, $Y = 0.23$, $T = 11.2$ billion years (green) and (2) $Z = 0.001$, $Y = 0.26$, $T = 10$ billion years (red). The left panel shows (1) the isochrone from [33] for $Z = 0.0004$, $Y = 0.23$, $T = 11.2$ billion years (green), (2) the isochrone of Kim et al. [35] for $Z = 0.0004$, $[\alpha/\text{Fe}] = 0.3$, $T = 10$ billion years (the red-giant branch passes to the left of isochrone (1)), and (3) the isochrone of Pietrinferni et al. [23] $Z = 0.0004$, $[\alpha/\text{Fe}] = 0.3$, $T = 10$ billion years (the red-giant branch passes to the right of isochrone (1)). Stars within 1.5 of the cluster center are shown by large filled circles, and the remaining stars in the field by small circles. The squares and asterisks denote stars that fell into the two spectrograph slits during the OHP observations (Fig. 1).

Figure 3. Comparison of our photometry of stars in the field of Pal 3 with the results of [13].
isochrones computed by Kim et al. [35]. They observed three exposures in each of their filters \(T_{\text{exp}} = 25\) s in B and 10 s in V), enabling effective removal of cosmic-ray traces and fairly deep photometry. The seeing was slightly better during the observations of Hilker [13] than in our observations; his stellar photometry was roughly 1 m deeper in V than our results (Fig. 2 in this paper and Fig. 4 in [13]). Nevertheless neither study was able to confidently resolve stars at the main-sequence turn-off point in Pal 3. Therefore, the results for the integrated spectrum of the cluster presented in the previous section, which yield independent estimates of the age and metallicity of the cluster, are needed to better estimate the distance to Pal 3 and the interstellar absorption in the direction of the cluster. In both our CMD and the CMD of Hilker [13], there is a deficit of bright stars near the tip of the red-giant branch. Figure 3 shows the difference between our V photometry and the data of Hilker [13] as a function of magnitude (left) and color (right). The dispersion of the differences \(dV_{\text{our-Hilker}}\) is, on average, \(\sim 0.05^m\). There is a small systematic shift of \(0.03^m\) between the two datasets, possibly due to small inaccuracies in the photometric zero points used. Several stars that strongly deviate from the general trend are either variable or have had their brightnesses distorted by remnants of cosmic-ray traces on the CCD frames.

With some caution, since the main-sequence turn-off point is not visible in Fig. 2 we can conclude from Fig. 2 that the isochrone of [33] corresponding to an age of 10 billion years and a metallicity of \(Z = 0.001\) \([\text{[Fe/H]} \sim -1.33\), according to [9]) best describes the horizontal branch of the cluster CMD. In contrast to many other isochrones, the isochrones of [33] include the horizontal-branch stage, and enable studies of how the temperature and luminosity ranges of the horizontal-branch stars vary with their ages, metallicities, and helium abundances. It is primarily the helium abundance, as well as the age

| Index | \(a, \text{std}\) | \(b, \text{std}\) | Range | Unit |
|-------|-----------------|-----------------|-------|------|
| \(H_\delta\) | 1.02, 0.11 | -1.03, 0.60 | [-8.2, -2] | Å |
| \(H_\gamma\) | 0.99, 0.06 | -0.36, 0.67 | [-8.2, -2] | Å |
| \(H_\gamma\) | 0.69, 0.08 | -4.06, 0.72 | [-12, 0] | Å |
| \(H_\gamma\) | 0.49, 0.08 | -1.66, 0.23 | [-4, -0.5] | Å |
| CN1 | 1.04, 0.07 | -0.007, 0.02 | [0.1, 0.4] | mag |
| CN2 | 1.12, 0.07 | -0.03, 0.02 | [0.07, 0.4] | mag |
| Ca4227 | 1.20, 0.16 | 0.22, 0.21 | [-1.8, 3] | Å |
| Fe4384 | 1.05, 0.11 | 0.31, 0.61 | [2, 9.3] | Å |
| Ca4455 | 1.14, 0.15 | 0.43, 0.24 | [0, 2.8] | Å |
| Fe4531 | 1.08, 0.20 | 0.17, 0.75 | [1, 5.4] | Å |
| Fe4668 | 1.13, 0.06 | -0.50, 0.58 | [4, 17] | Å |
| H\(\beta\) | 0.81, 0.16 | 0.22, 0.21 | [0.5, 2] | Å |
| Fe5015 | 1.13, 0.07 | 0.05, 0.38 | [0, 8] | Å |
| Mg\(\beta\) | 0.80, 0.04 | 1.08, 0.13 | [0, 6] | Å |
| Mg\(\delta\) | 0.96, 0.07 | 0.03, 0.01 | [0.05, 0.25] | mag |
| Mg\(\gamma\) | 1.04, 0.06 | 0.03, 0.01 | [0.1, 0.4] | mag |
| Fe5270 | 1.43, 0.19 | -0.34, 0.54 | [1, 4.5] | Å |
| Fe5335 | 1.09, 0.11 | 0.28, 0.28 | [0.5, 4] | Å |
| Fe5406 | 1.22, 0.06 | -0.18, 0.12 | [0.2, 3] | Å |
| Fe5782 | 1.25, 0.09 | -0.07, 0.09 | [-0.1, 1.5] | Å |
| Na5895 | 0.95, 0.03 | 0.23, 0.10 | [0.2, 5.2] | Å |
| TiO\(\delta\) | 0.95, 0.08 | -0.01, 0.004 | [-0.05, 0.1] | mag |
and metallicity, that most influence the morphology of the horizontal branch [34]. The values of
the distance modulus \((m-M)_0 = 19.82\) (Dist. = 91.9 kpc) and color excess \(E(B-V) = 0.07\) we
have obtained via our fitting of the observed cluster CMD using theoretical isochrones are in rea-
sonable agreement with data from the literature (Table 1).

5. SPECTROSCOPY RESULTS

The Lick system of absorption spectral indices\(^6\) ([36], [29], [37], [38], [39]) is widely used
to distinguish the influence of age and metallicity on integrated spectra of repre-
sentative of old stellar populations (with ages of order a billion years or more). The Lick system is based
on indices measured in the spectra of standard stars\(^7\). Measurements of absorption line indices
are reduced to the standard system by observing objects from a list of standard stars and
constructing the relationship between the instrumental and standard indices for these stars. The
coefficients of these relationships for our observa-
tions are presented in Table 4. Tables 5 and 6
present measurements of the Lick indices in the integrated spectra of stars in the central region
of Pal 3, reduced to the standard system. Unfortunately, because the signal-to-noise ratio per
resolution element is only \(S/N \sim 30\) in our spec-
tra at the most sensitive wavelength of 5000Å the
uncertainties in the measured indices are large. Our measured indices for the hydrogen lines \(H_\beta, H_\delta\) and \(H_\gamma\) are too weak compared to the corre-
spanding values for globular clusters with similar metallicitis and ages (see, e.g., [40], [41]). One
possible reason for this is a systematic overesti-
mation of the continuum level due to the contri-
bution of night-sky absorption lines. We note,
however, that the index ratio \(H_\delta F/H_\beta = 0.94\)
un-
ambiguously demonstrates that the cluster has a
red horizontal branch [42].

Schiavon et al. [42] found that Galactic clusters with blue horizontal branches \((HBR \sim 1)\)
have higher values for this ratio than those for ob-
jects with red horizontal branches \((HBR \sim -1)\),
on average, by \(\sim 0.15\). Further, the indices and
combinations of indices in Tables 5 and 6 that are
sensitive to variations in the metal abundances –

\[
\langle Fe \rangle = \frac{(Fe5270 + Fe5335)}{2} = 0.63 \pm 0.28,
\]

\[
[Mg/Fe]' = \sqrt{(Mgb \cdot (0.72Fe5270 + 0.28Fe5335))}
\]

\(= 0.95 \pm 0.35\), \(Mgb\), \(Mg_1\), \(Mg_2\) and others – have
values characteristic for Galactic clusters with
\(\langle Fe/H \rangle \sim -1.4 \div -1.6\) dex.

So-called diagnostic diagrams (see, e.g., [41]
and references therein), two of which are shown
in Fig. 4, enable a comparison of the Lick indices
for globular clusters with metallicities \(\langle Fe/H \rangle \sim
-1.6\) dex and \(\langle Fe/H \rangle \sim -1.3\) dex with the mod-
els of Thomas et al. [43], [44]. These models
for simple stellar populations contain Lick indices

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\(^6\) http://astro.wsu.edu/worthey/html/index.table.html

\(^7\) http://astro.wsu.edu/worthey/html/system.html, astro.wsu.edu/ftp/W097/export.dat
Table 5. Lick indices ($\lambda \leq 5015\,\text{Å}$) measured in the integrated spectrum for several stars in the central region of Pal 3 (Fig. 1).

| $H_{\delta}$ | $H_{\beta}$ | CN$_1$ | CN$_2$ | Ca4227 | Ca4455 | Fe4531 | Fe4668 | H$_\beta$ | Fe5015 |
|-------------|-------------|--------|--------|--------|--------|--------|--------|--------|--------|
| (Å)         | (Å)         | (mag)  | (mag)  | (Å)    | (Å)    | (Å)    | (Å)    | (Å)    | (Å)    |
| 1.53$^{+1.03}_{-1.04}$ | 0.91$^{+0.74}_{-0.75}$ | -0.097$^{+0.027}_{-0.088}$ | 0.040$^{+0.010}_{-0.075}$ | 0.39$^{+0.39}_{-0.40}$ | 0.49$^{+0.37}_{-0.37}$ | 1.71$^{+0.40}_{-0.40}$ | 0.74$^{+0.49}_{-0.49}$ | 0.97$^{+0.22}_{-0.22}$ | 1.88$^{+0.31}_{-0.31}$ |

Table 6. Lick indices ($\lambda > 5015\,\text{Å}$), measured in the integrated spectrum for several stars in the central region of Pal 3 (Fig. 1).

| Mg$b$ | Mg$_1$ | Mg$_2$ | Fe5270 | Fe5335 | Fe5406 | Fe5782 | Na5895 | TiO$_1$ | TiO$_2$ |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| (Å)   | (Å)    | (Å)    | (Å)    | (Å)    | (Å)    | (Å)    | (Å)    | (Å)    | (Å)    |
| 1.31$^{+0.19}_{-0.20}$ | 0.099$^{+0.005}_{-0.010}$ | 0.075$^{+0.005}_{-0.022}$ | 0.78$^{+0.20}_{-0.20}$ | 0.48$^{+0.20}_{-0.20}$ | 0.89$^{+0.15}_{-0.15}$ | 0.16$^{+0.11}_{-0.11}$ | 1.31$^{+0.12}_{-0.12}$ | 0.032$^{+0.008}_{-0.006}$ | 0.004$^{+0.002}_{-0.001}$ |

Figure 4. Age-metallicity (left) and metallicity-abundance of $\alpha$-elements (right) diagnostic diagrams of the Lick indices for Pal 3 (filled circles) and Galactic globular clusters (red points) with indices from [40] and with $[\text{Fe/H}] \sim -1.6$ dex (NGC 1904, 3201, 5286, 5946, 5986, 6254, 6333, 6752, and 7089) and $[\text{Fe/H}] \sim -1.3$ dex (NGC 5904, 5946, 6218, 6235). The curves show models for simple stellar populations [43], [44].
computed for various ages and metallicities. The Lick indices of Pal 3 that are sensitive to variations in the metal content correspond to the model with $[Fe/H] \sim -1.35 \pm 0.2$ dex and an age of $T = 11 \pm 2$ billion years. The results of [22], [13] and the diagram in the right panel of Fig. 4 indicate a high content of $\alpha$ elements in the cluster ($[\alpha/Fe] \geq 0.3$).

A comparison of the summed stellar spectrum in the central region of Pal 3 with the best-fit model spectrum is presented in the upper panel of Fig. 6. The best fit is provided by a simple stellar-population model with metallicity $[Fe/H] = 1.35$ dex and age 10 billion years, computed by Vazdekis et al. [46] using the MILES spectral library [47]. The lower panel shows the difference between the observed and model spectra, calculated using the program ULySS$^8$ ([48], [49]). The use of the PEGASE.HR grid of models [50] with the Elodie library of stellar spectra [51] slightly increases the age and metallicity: $12\pm2$ billion years and $[Fe/H] = -1.2 \pm 0.25$ dex.

Thus, the results shown in this section demonstrate that fitting observed spectra with model spectra and comparing the Lick indices with those for other Galactic clusters and with model Lick indices give consistent results.

$^8$ http://ulyss.univ-lyon1.fr
6. ORBITS OF PAL 3 AND NINE MASSIVE GLOBULAR CLUSTERS

In this section, we consider the orbit of Pal 3 and compare it with the orbits of several massive Galactic globular clusters with similar metallicities and ages, [Fe/H] ∼ −1.6 dex, T ∼ 11.2 billion years ([41] and references therein): NGC 1904, 5139, 5286, 6254, 6273, 6656, 6681, 6752, and 7089. The main characteristics of the clusters related to the computation of their orbits are given in Table 7. The cluster masses from [8] and half-luminosity radii from [56] are also given.

6.1. Method for Orbit Computation

Data on distances, radial velocities, and the proper-motion components were taken from the catalogs of Harris [1] and Kharchenko et al. [57]. The proper motions for Pal 3 were taken from [26]. The positions of the clusters (apart from Pal 3, NGC 1904, and NGC 6273) were taken from [58]. The positions of Pal 3, NGC 1904, and NGC 6273 were taken from [59] (Pal3 = UGC 05439), [60], and [61], respectively.

We considered an axially symmetrical potential for the Galaxy consisting of potentials for the disk, bulge, and halo. The potential of the bulge Φ_b(r) was described using a spherically symmetrical Plummer distribution [62],

\[ \Phi_b(r) = -\frac{M_b}{(r^2 + b_b^2)^{1/2}}, \]  

and the potential of the disk Φ_d(R, Z) was taken to have the form proposed by Miyamoto and Nagai [63]:

\[ \Phi_d(R, Z) = -\frac{M_d}{\left[ R^2 + (a_d + \sqrt{Z^2 + b_d^2})^2 \right]^{1/2}}, \]  

where M_b and M_d are the mass of the bulge and disk, and b_b, a_d, and b_d are scaling parameters of the components in kpc. We used a logarithmic potential to describe the dark-matter halo [64]:

\[ \Phi_h(r) = \frac{v_0^2}{2} \ln(r^2 + R_h^2), \]  

with the asymptotic rotational velocity \( v_0 = 220 \) km/s and radius \( R_h \), determined from the condition that the circular velocity in the plane of the disk attain the value \( v_0 \) at \( R = 8.5 \) kpc when the full potential is taken into account, i.e., including both the bulge (1) and the disk (2).

The parameters of the bulge and disk potentials (distance from the Sun to the Galactic center, local circular rotational velocity of the Galaxy, scaling parameters of the disk, bulge, and halo) were taken from [65].

The translation from the coordinates in the International Celestial Reference Frame to Galactocentric coordinates was done using a rotation matrix based on fitting of a catalog of objects observed by Hipparcos [66]. The position and velocity of the Sun that we used were \( X_\odot = 8.33 \) kpc [67], \( Z_\odot = 27 \) pc [68], \((U, V, W)_\odot = \)
**Table 7.** Detailed characteristics for Pal 3 and nine Galactic globular clusters: (2-4) right ascension and declination at epoch J2000.0 and corresponding uncertainties in milliarcsec (mas), (5, 6) proper motions, (7) distance from the Sun, (8) radial velocity, (9) mass, (10) half-luminosity radii. The literature sources for data in this table are indicated in Sections 6 and 6.1.

| Object | RA [hh mm ss] | DEC [gg mm ss] | δRA [mas] | µa* [mas/yr] | µd [mas/yr] | Dist [kpc] | Vr [km/s] | M [M⊙] | rh [pc] |
|--------|----------------|----------------|------------|--------------|-------------|------------|----------|--------|--------|
| Pal3   | 10 05 31.9     | +00 04 18.0    | 2200       | 0.33±0.23    | 0.30±0.31   | 91.90±9.19 | 83.4±8.4 | 6.38e4 | 17.8   |
| NGC1904| 05 24 11.09    | -24 31 29.0    | 1000       | 0.20±0.43    | 1.13±0.43   | 12.89±1.42 | 232.1±0.1| 2.64e6 | 6.44   |
| NGC5139| 13 26 47.28    | -47 28 46.1    | 100        | -0.01±0.25   | -5.02±0.25  | 5.20±0.57  | 232.1±0.1| 2.64e6 | 6.44   |
| NGC5286| 13 46 26.81    | -51 22 27.3    | 100        | -6.24±0.83   | -3.68±0.83  | 11.71±1.29 | 57.4±1.5 | 4.80e5 | 2.21   |
| NGC6254| 16 57 09.05    | -04 06 01.1    | 100        | -6.88±0.36   | -8.28±0.36  | 11.71±1.29 | 57.4±1.5 | 4.80e5 | 2.21   |
| NGC6273| 17 02 37.80    | -26 16 04.7    | 100        | -3.89±0.43   | -1.48±0.43  | 8.79±0.97  | 135.0±4.1| 1.56e6 | 3.13   |
| NGC6656| 18 36 23.94    | -23 54 17.1    | 800        | 4.72±0.34    | -3.59±0.34  | 3.20±0.35  | 146.3±0.2| 5.36e5 | 3.03   |
| NGC6681| 18 43 12.76    | -32 17 31.6    | 100        | 3.84±0.47    | -5.78±0.47  | 9.00±0.99  | 220.3±0.9 | 1.89e6 | 2.43   |
| NGC6752| 19 10 52.11    | -59 59 04.4    | 100        | -4.79±0.62   | -4.84±0.62  | 4.00±0.44  | -26.7±0.2| 3.64e5 | 2.72   |
| NGC7089| 21 33 27.02    | -00 49 23.7    | 100        | 6.77±0.42    | -4.54±0.42  | 11.51±1.27 | -5.3±2.0| 8.81e5 | 3.11   |

**Figure 6.** Model orbit for Pal 3 shown in projection onto the X, Y, Z, and R axes in Galactocentric coordinates. X, Y, and Z are directed toward the center, in the direction of rotation, and toward the north Galactic pole, respectively. The current position of the object, initial position 12 billion years in the past (black curve), and final position 12 billion years in the future (gray curve) are marked by points. (11.1, 12.24, 7.25) km/s [69], \( V_{LSR} = 218 \text{ km/s} \) [70].

The cluster orbits were characterized by the mean parameter values and their deviations for 1000 versions of the orbits computed 12 billion years into the past and 1000 versions computed 12 billion years into the future. The integration was carried out using a 5th-order Runge-
Table 8. Parameters of the orbits of Pal 3 and nine other globular clusters with similar metallicities and ages: (2) mean rotation period in millions of years, (3) mean orbital inclination in degrees, (4) and (5) mean radii of pericenter and apocenter in kpc, (6) maximum distance of the cluster in the directions of the Galactic poles, $Z_{\text{max}}$, in kpc, (7) eccentricity, (8) time required to reach the orbit pericenter and apocenter in millions of years, and (9) time required to reach the cluster's maximum distance from the Galactic plane in millions of years. A negative mean rotation period, $T_{\text{rot}}$, denotes rotation in the direction opposite to the direction of the Galactic rotation.

| Object      | $T_{\text{rot}}$ | $\text{Incl}$ | $R_{\text{apo}}$ | $R_{\text{peri}}$ | $Z_{\text{max}}$ | $\text{ecc}$ | $T_{\text{ap}} = T_{\text{per}}$ | $T_{Z_{\text{max}}}$ |
|-------------|------------------|----------------|------------------|-------------------|------------------|-------------|-------------------------------|----------------------|
| Pal 3       | 12056±7425       | 72±9           | 272±135          | 86±15             | 182±101          | 0.5±0.2     | 3584±1498                     | 12873±7425           |
| NGC1904     | 512±57           | 24±2           | 25±3             | 12±2              | 6.8±1            | 0.34±0.03   | 344±142                       | 899±124              |
| NGC5139     | -141±3.5         | 34±2           | 6.5±0.1          | 1.7±0.2           | 1.4±0.3          | 0.59±0.03   | 93±2                         | 164±14               |
| NGC5286     | 333.5±97         | 32±6.5         | 15±6             | 8±1               | 5.3±2.2          | 0.29±0.11   | 216±64                       | 552±198              |
| NGC6254     | 123±6.6          | 57±8           | 5.4±0.3          | 1.5±0.16          | 2.5±0.4          | 0.57±0.03   | 83±4                         | 192±9                |
| NGC6273     | -37.8±38         | 71±12          | 2.3±0.8          | 0.9±0.2           | 1.4±0.2          | 0.42±0.21   | 35±12                        | 86±13                |
| NGC6656     | 187±6.7          | 17±2           | 8.9±0.3          | 2.9±0.2           | 0.8±0.1          | 0.51±0.02   | 120±4                        | 164±7                |
| NGC6681     | 116±209          | 82±4           | 11±3             | 1.3±0.4           | 5±1              | 0.76±0.11   | 135±28                       | 359±60               |
| NGC6752     | 153±7            | 35±4           | 5.8±0.2          | 3.4±0.4           | 2.1±0.2          | 0.27±0.05   | 99±4                         | 207±8                |
| NGC7089     | -815±285         | 69±2           | 45±17            | 6.7±1.2           | 25±8             | 0.73±0.04   | 514±182                      | 1596±568             |

Figure 7. Model orbit for NGC 7089 shown in projection onto the X, Y, Z, and R axes in Galactocentric coordinates. X, Y, and Z are directed toward the center, in the direction Galactic rotation, and toward the north Galactic pole, respectively. The current position of the object, initial position 12 billion years in the past (black curve), and final position 12 billion years in the future (gray curve) are marked by points.

The input parameters were randomly varied 1000 times in accordance with the uncertainties in the observational parameters (the set of initial conditions for each cluster was generated based on a normal distribution bounded by $2\sigma$; we did not use the Kutta-Feldberg method with a variable step [71].
include systematic errors associated with uncertainty in the position and velocity of the Sun relative to the Galactic center. It is important that the acceleration of the cluster in the Galactic potential depends on the cluster mass only in the case of dynamical friction. This effect is usually important only for massive globular clusters whose trajectories pass close to the Galactic center. We did not include the effect of dynamical friction in our study. The results of constructing model orbits for Pal 3 and nine other Galactic clusters are presented in Table 8. The models of the orbits are shown graphically in Fig. 6 for Pal 3, in Fig. 7 for NGC 7089, and at the ftp site ftp://ftp.sao.ru/pub/sme/V220_plot/ for the other clusters.

6.2. Results of Orbit Computations

The tables and figures illustrating the computed orbit models show that the orbit of Pal 3 clearly does not resemble any of the orbits of the other nine massive Galactic clusters with similar ages and metallicities, in terms of either its shape or its characteristics. Pal 3 is moving around the Galactic center along a complex trajectory. The period for the cluster to reach its maximum distance from the Galactic center is comparable to the age of the Universe. The cluster emerged from a position close to this most distant point roughly 12 billion years ago, and the object is currently located near the pericenter of its orbit. We obtained a lower value for the maximum distance of Pal 3 than was found in [5]. According to our computations, this distance is about 340 kpc. The cluster has approached the Galactic center to a minimum distance of 70 kpc. Therefore, only relaxation as a result of stellar encounters exerts an important influence on the dynamical and structural evolution of this cluster [8]. The effects of dynamical friction and collisions with the disk and bulge are not important due to the cluster’s appreciable distance from the Galactic plane.

The orbits of the other nine clusters are close to the Galactic plane (in the Z direction, Table 8). In contrast to Pal 3, these nine clusters are some of the most massive in the Galaxy. They have high central densities and unusual properties for their stellar populations, such as extremely blue horizontal branches, multiple evolutionary sequences, and anticorrelations between elemental abundances (see [41] and references therein). Some of these objects correspond to stellar flows. The clusters have probably passed through dense gaseous layers many times, becoming closer and closer to the Galactic center and the Galactic plane due to the effects of dynamical friction and collisions with the disk and bulge [8]. The high masses and high central densities of these clusters have hindered their total disruption during the course of their evolution.

The parameters of the orbit models for the massive clusters from the literature overall resemble our own values ([24], [25] and [72]). Di-
nescu et al. [24] and Balbinot and Gieles [25] also computed elements of the orbit of Pal 3. The differences are larger for more distant clusters, for which the uncertainties in their proper motions and other parameters are higher. The mean rotation periods of four of the clusters have large uncertainties: Pal 3, NGC 5286, NGC 6681, and NGC 7089. The case of Pal 3 is the most complex. Its proper motion is known only with large uncertainties: $\mu_a = 0.33 \pm 0.23$, $\mu_d = 0.30 \pm 0.31$ [26]. Differences in the methods used to construct the models and large uncertainties in the observed parameters lead to differing orbital parameters. Dinescu et al. [24] used the proper motions of [26] and obtained the following elements of the orbit: mean rotation period in units of $10^6$ years $P \sim 9740$, radius of apocenter $R_{apo} \sim 420$ kpc, radius of pericenter $R_{peri} \sim 83$ kpc, orbital eccentricity $ecc \sim 0.7$, and maximum distance of the cluster in the direction of the Galactic poles $Z_{max} \sim 308$ kpc. Balbinot and Gieles [25] also used the proper motions of Pal 3 from [26], as did we. However, the radii of apocenter and pericenter and the orbital eccentricities they obtained were $R_{apo} \sim 1985$ kpc, $R_{peri} \sim 94$ kpc, $ecc \sim 0.9$. Our own derived orbital parameters are closer to the values of [24].

7. DISCUSSION AND CONCLUSION

We have attempted to improve our knowledge of the origin of Pal 3 – one of the most distant globular clusters – by analyzing various observational data available to us: direct VLT frames, integrated spectra of stars in the central region of the cluster, and catalog information about the object’s kinematics.

We carried out stellar photometry using previously unused archival VLT data, and estimated the distance to Pal 3, $Dist. = 91.9$ kpc, and its color excess, $E(B-V) = 0.07$. We have found that the isochrone of [33] corresponding to an age of 10 billion years, metallicity $Z = 0.001$ ([Fe/H] $\sim -1.33$, according to the empirical calibration of [9]), and helium abundance $Y = 0.26$ fits the distribution of stars on the horizontal branch of the cluster better than other isochrones. Deeper direct frames are needed to carry out photometry stars at the main-sequence turn-off point, and to refine the age, metallicity, and helium abundance of the cluster. Deep spectral observations of stars in Pal 3 could answer the question of whether two or three stars on the continuation of the red horizontal branch toward the blue are cluster members. This will make it possible to refine the cluster’s horizontal-branch index HBR and evolutionary parameters. The absence of stars near the tip of the red-giant branch of Pal 3 and the closeness of the luminosities of stars at the main-sequence turn-off point to the photometric limit has hindered precise estimation of the cluster’s age and metallicity based on its CMD.

We have carried out spectroscopic studies of stars in the central region of the cluster based on archival data from the CARELEC spectrograph.
of the 1.93-m telescope of the OHP; the total exposure time for stars in the central region of the cluster was 3.75 hours under good observing conditions. Our fitting of a model to the observed spectrum at wavelengths $[3900:6800]\, \text{Å}$ and comparison with model measurements of the Lick indices indicates that Pal 3 has an enhanced abundance of $\alpha$-process elements, $[\alpha/\text{Fe}] \geq 0.3$, an age of $T = 11 \pm 2$ billion years, and metallicity $[\text{Fe}/\text{H}] = -1.4 \pm 0.2$ dex. The values of $[\alpha/\text{Fe}]$ and $T$ derived through our spectroscopic and photometric analyses are in good agreement with data from the literature. Our value of $[\text{Fe}/\text{H}]$ is about 0.2 dex higher than previously determined values in the literature.

Our constructed model orbit for Pal 3 indicates that this cluster is located near its closest distance from the Galactic center. In its path from the most distant point on its orbit, $R_{gc} \sim 340$ kpc, the cluster has approached the Galactic center to a minimum distance of $\sim 70$ kpc only once. Our computed model orbit suggests a probable extragalactic origin for Pal 3.

It is difficult to determine which of the nearby existing galaxies originally hosted the cluster based on the available observational data. The Phoenix dwarf galaxy has a transitional morphological type between a gas-rich irregular galaxy and a decaying spheroidal galaxy (dIrr/dSph, $Dist. = 0.43$ Mpc [73]), and an orbit that is coplanar with the orbit of Pal 3, according to Palma et al. [5]. The stellar population of Phoenix is characterized by a wide range of metallicities and an appreciable metallicity gradient ([74] and references therein). For most evolved stars in Phoenix, $[\text{Fe}/\text{H}]$ lies in the range from $-1$ dex to $-2$ dex. The mean metallicity of this galaxy is $\langle [\text{Fe}/\text{H}] \rangle = -1.49 \pm 0.04$ dex. No observational data about the proper motion of Phoenix are available.

Palma et al. [5] undertook a statistical study of the distribution of orbital moments for galaxies in the Local Group and globular clusters. They distinguished a number of dynamical subgroups of objects. Pal 3 is a member of the subgroup of dwarf galaxies: the Small Magellanic Cloud and Ursa Minor. In addition to these two galaxies, this subgroup also contains the cluster NGC 5024.

Peebles et al. [73] constructed a dynamical numerical model for the orbit of the Phoenix dwarf galaxy. The input parameters for the model were the position and radial-velocity components for galaxies in the Local Group at distances of less than 1.5 Mpc from the Galaxy. Simulations enabled estimation of the masses and proper motions of the galaxies. It was found that Phoenix was the first to approach this close to the Milky Way, and its motion, like those of most other dwarf galaxies, originated in the outskirts of the Local Group. Thus, more accurate proper-motion data should help elucidate whether or not the cluster Pal 3 was ejected from this galaxy, and, if it was, what was the origin of this event.

Future observations using large ground and space telescopes will help clarify the origin of
Pal 3, and also of five other globular clusters with similar ages and metallicities located far from the Galactic center.

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1. W.E. Harris, Astron. J. **112**, 1487 (1996)(2010 edition).
2. G.O. Abell, Publications of the Astron. Society of the Pacific **67**, 258 (1955).
3. T. V. Borkova and V. A. Marsakov, Astronomy Reports **44**, 665 (2000).
4. E. Carretta, A. Bragaglia, R. G. Gratton, A. Recio-Blanco, S. Lucatello, V. D’Orazi, and S. Cassisi, Astron. and Astrophys. **516**, 55 (2010).
5. C. Palma, S. R. Majewski, and K.V. Johnston, Astrophys. J. **564**, 736 (2002).
6. M. Bellazzini, F. R. Ferraro, and R. Ibata, Astron.J. **125**, 188 (2003).
7. T.K. Fritz et al., Astrophys. J. **840**, 30 (2017).
8. O.Y. Gnedin, J.P. Ostriker, Astrophys. J. **474**, 223 (1997).
9. G. Bertelli, A. Bressan, and C. Chiosi, Astron. and Astrophys. Suppl. **106**, 275 (1994).
10. M. Asplund, N. Grevesse, A.J. Sauval, P. Scott, Ann. Review Astron. Astrophys. **47**, 481 (2009).
11. R.G. Gratton and S. Ortolani, Astron. Astrophys. Suppl. Ser. **57**, 177 (1984).
12. R. Zinn & M.J. West, Astrophys. J. Suppl. **55**, 45 (1984).
13. M. Hilker, Astron. and Astrophys. **448**, 171 (2006).
14. Y.-W. Lee, P. Demarque, R. Zinn, Astrophys. J. **423**, 248 (1994).
15. P.B. Stetson, M. Bolte, and W.E. Harris, Astron.J. **117**, 247 (1999).
16. D. A. VandenBerg, Astrophys. J. Suppl. **129**, 315 (2000).
17. A. Dotter, A. Sarajedini, S.-C. Yang, Astron. J. **136**, 1407 (2008).
18. A. Marin-Franch et al., Astrophys. J. **694**, 1498 (2009).
19. M. Catelan, F.R. Ferraro, and R.T. Rood, Astrophys. J. **560**, 970 (2001).
20. J. Borissova, V. D. Ivanov, and M. Catelan, Information Bulletin on Variable Stars **4919**, 1 (2000).
21. Y.-J. Sohn, J.-H. Park, and S.-Ch. Rey, Astron.J. **126**, 803 (2003).
22. A. Koch, P. Cote, and A. McWilliam, Astron. and Astrophys. **506**, 729 (2009).
23. A. Pietrinferni, S. Cassisi, M. Salaris, and F. Castelli, Astrophys. J. **612**, 168 (2004).
24. D.I. Dinescu, T.M. Girard, and W.F. van Altena,
25. E. Balbinot, M. Gieles, Monthly Notices Royal Astron. Soc. 474, 2479 (2018).
26. S.R. Majewski, K.M. Cudworth, K. M., Publ. Astron. Soc. Pacific 105, 987 (1993).
27. K. Banse, Ph. Crane, Ch. Ounnas, and D. Ponz, in Proceedings of the DECUS Europe Symposium (Digital Equipment Corporation, Maynard, MA, USA), p. 87 (1983).
28. G. Lemaitre, D. Kohler, D. Lacroix, et al., Astron. Astrophys. 228, 540 (1990).
29. G. Worthey, S. M. Faber, J.J. Gonzalez, and D. Burstein, Astrophys. J. Suppl. Ser. 94, 687 (1994).
30. P.B. Stetson, Publ. Astron. Soc. Pacific 99, 191 (1987).
31. D. A. Khamidullina, M. E. Sharina, V. V. Shimansky, and E. Davoust, Astrophysical Bulletin 69, 409 (2014).
32. D. Tody, in Astronomical Data Analysis Software and Systems II, ed. R. J Hanisch, R. J. V. Bris senden, and J. Barnes (San Francisco, CA: ASP), ASP Conf. Ser. 52, 173 (1993).
33. G. Bertelli, L. Girardi, P. Marigo, and E. Nasi, 2008, Astron. and Astrophys. 484, 815 (2008).
34. Gratton R. G., Carretta E., Bragaglia A., Lucatello S., D’Orazi V., Astron. and Astrophys. 517, 81 (2010).
35. Y. Kim, P. Demarque, S. K. Yi, & D. R. Alexander, Astrophys. J. Suppl. 143, 499 (2002).
36. D. Burstein, S. M. Faber, C. M. Gaskell, N. Krumm, Astrophys. J. 287, 586 (1984).
37. G. Worthey, Astrophys. J. Suppl. Ser. 95, 107 (1994).
38. G. Worthey & D. L. Ottaviani, Astrophys. J. Suppl. Ser. 111, 377 (1997).
39. S. C. Trager, G. Worthey, S. M. Faber, D. Burstein, J. J. Gonzalez, Astrophys. J. Suppl. Ser. 116, 1 (1997).
40. R. P. Schiavon, N. M. Caldwell, H. P. Heather, S. Courteau, L.A. MacArthur, G. J. Graves, Astron. J. 143, 14 (2012).
41. M. E. Sharina, V. V. Shimansky, and A. Y. Kni azev, Monthly Notices Royal Astron. Soc. 471, 1955 (2017).
42. R. P. Schiavon, J.A. Rose, S. Courteau, L.A. MacArthur, Astrophys. J. 608, L33 (2004).
43. D. Thomas, C. Maraston, R. Bender, Monthly Notices Royal Astron. Soc. 343, 279 (2003).
44. D. Thomas, C. Maraston, A. Korn, Monthly Notices Royal Astron. Soc. 351, L19 (2004).
45. S.C. Trager, S.M. Faber, G. Worthey, J.J. Gonzalez, Astron. J. 119, 1645 (2000).
46. A. Vazdekis, P. Sanchez-Blazquez, J. Falcon-Barroso, A. J. Cenarro., M. A. Beasley, N. Cardiel, J. Gorgas, R. F. Peletier, Monthly Notices Royal Astron. Soc. 404, 1639 (2010).
47. P. Sanchez-Blazquez, et al., Monthly Notices Royal Astron. Soc. 371, 703 (2006).
48. M. Koleva, P. Prugniel, P. Ocvirk, D. Le Borgne, C. Soubiran, Monthly Notices Royal Astron. Soc. 385, 1998 (2008).
49. M. Koleva, P. Prugniel, A. Bouchard, Y. Wu, Astron. and Astrophys. 501, 1269 (2009).
50. D. Le Borgne, B. Rocca-Volmerange, P. Prugniel, A. Lancon, M. Fioc, C. Soubiran, Astron. and Astrophys. 425, 881 (2004).
51. P. Prugniel, C. Soubiran, Astron. and Astrophys. 369, 1048 (2001).
52. E. E. Salpeter, Astrophys. J. 121, 161 (1955).
53. P. Kroupa, Monthly Notices Royal Astron. Soc. 322, 231 (2001).
54. G. Chabrier, Publ. Astron. Soc. Pacific 115, 763 (2003).
55. M. E. Sharina, V. V. Shimansky, and E. Davoust, Astronomy Reports 57, 410 (2013).
56. A.D. Mackey, S. van den Bergh, Monthly Notices Royal Astron. Soc. 360, 631 (2005).
57. N.V. Kharchenko, A.E. Piskunov, E. Schilbach, S. Röser, and R.-D. Scholz, Astron. Astrophys. 558, A53 (2013).
58. R. Goldsbury, H.B. Richer, J. Anderson, A. Dotter, A. Sarajedini, K. Woodley, Astron. J. 140, 1830 (2010).
59. W. D. Cotton, J. J. Condon, E. Arbizzani, Astrophys. J. Suppl. 125, 409 (1999).
60. B. Lanzoni et al., Astrophys. J. 663, 1040 (2007).
61. A. Picard, H. M. Johnston, Astron. Astrophys. Suppl. 112, 89 (1995).
62. H. C. Plummer, Monthly Notices Royal Astron. Soc. 71, 460 (1911).
63. M. Miyamoto and R. Nagai, Publ. Astron. Soc. Jpn. 27, 533 (1975).
64. V.C. Rubin, W. K., Jr Ford, N. Thonnard, Astrophys. J. 238, 471 (1980).
65. C. Allen and A. Santillan, Rev. Mexicana Astron. Astroc. 22, 255 (1991).
66. ESA, ESA Special Publication 1200, The Hipparcos and Tycho catalogues. ESA, Noordwijk (1997).
67. S. Gillessen, F. Eisenhauer, S. Trippe, T. Alexander, R. Genzel, F. Martins, T. Ott, Astrophys. J. 692, 1075 (2009).
68. B. Chen et al., Astrophys. J. 553, 184 (2001).
69. R. Schöierich, J. Binney, W. Dehnen, Monthly Notices Royal Astron. Soc. 403, 1829 (2010).
70. Jo Bovy et al., Astrophys. J. 759, 131 (2012).
71. E. Fehlberg E., Klassische Runge-Kutta-Formeln fünfter und siebenter Ordnung mit Schrittweitenkontrolle. Computing, 4, 93 (1969).
72. Pouliasis, P. Di Matteo, and M. Haywood (2017) Astron and Astrophys. 598, 66 (2017).
73. P.J.E. Peebles, R.B. Tully, E.J. Shaya, eprint arXiv:1105.5596 (2011).
74. N. Kacharov et al., Monthly Notices Royal Astron. Soc. 466, 2006 (2016).

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