The gluon-fusion uncertainty in Higgs coupling extractions

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We point out that the QCD corrections to the gluon-fusion Higgs boson production cross section at the LHC are very similar to the corrections to the Higgs decay rate into two gluons. Consequently, the ratio of these two quantities has a theoretical uncertainty smaller than the uncertainty in the cross section alone by a factor of two. We note that since this ratio is the theoretical input to analyses of Higgs coupling extractions at the LHC, the reduced uncertainty should be used; in previous studies, the full cross section uncertainty was employed.

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The benchmark for future new particle searches is the search for the Higgs boson at the LHC. It is the only particle predicted by the Standard Model (SM) that has not been observed, and its discovery will be an important milestone for high energy physics. Experiments at LEP and SLC established a lower limit on the mass of the SM Higgs of \( m_H > 114 \text{ GeV} \) through direct searches \(^\text{1}\). Indirect constraints from precision electroweak measurements indicate that a scalar Higgs boson with \( m_H < 200 \text{ GeV} \) exists \(^\text{2}\). This state will provide a powerful probe of the mechanism of electroweak symmetry breaking, which has motivated numerous analyses studying which of its properties can be measured: whether its \( CP \) properties can be determined, if its production and decay modes can be accurately measured, and a host of other issues \(^\text{3}\).

A large effort has been devoted to studying the extraction of Higgs boson partial widths at both the LHC and a future linear collider \(^\text{4}\). The starting point for these analyses is the following relation for each Higgs production and decay channel:

\[
\sigma_p(H) \times BR(H \rightarrow xx) = \frac{\sigma_{p}^{SM}(H)}{\Gamma_{SM}} \times \frac{\Gamma_{p}}{\Gamma} \times \frac{\Gamma_x}{\Gamma},
\]

where \( \Gamma_x \) denotes the partial width for the decay \( H \rightarrow xx \), \( \Gamma_p \) is the partial width for the Higgs production mode under consideration, and \( \Gamma \) is the total Higgs width. This equation is valid under two assumptions. First, the narrow-width approximation is used, which should hold up to intermediate Higgs boson masses. Second, it is assumed that a single, well-defined channel can be assigned to the Higgs production process. The observed rate furnishes a measurement of the product \( \Gamma_p \Gamma_x / \Gamma \), subject to experimental errors and the theoretical uncertainty in the normalization factor \( \sigma_{p}^{SM}(H) / \Gamma_{SM} \); the theoretical input and the experimental measurement are clearly separated using Eq. 1. The available production modes include gluon fusion, weak boson fusion, \( ttH \) associated production, and \( WH, ZH \) associated production. The accessible decay modes in each production channel are: \( gg \rightarrow H \rightarrow WW, ZZ, \gamma\gamma \); WBF with \( H \rightarrow WW, ZZ, \gamma\gamma, \tau\tau \); \( ttH \) production with \( H \rightarrow WW, \gamma\gamma, bb \); \( WH, ZH \) associated production with \( H \rightarrow WW, \gamma\gamma \). A combined analysis of the many channels available determines the various partial widths \( \Gamma_x \).

The largest theoretical error entering this analysis is the uncertainty in the gluon-fusion production cross section, \( gg \rightarrow H \). Next-to-leading order (NLO) QCD calculations of this cross section revealed corrections reaching nearly 70% and exhibiting large residual scale dependences \(^\text{5}\). This stimulated the calculation of the NNLO corrections. The inclusive NNLO result for \( gg \rightarrow H \) was
obtained in [6]; very recently the leading N3LO corrections to the inclusive cross-section were computed in [7]. The completely differential Higgs production cross section, which allows a realistic study of experimental acceptances at NNLO, was calculated in [8]. The partial width \( \Gamma_{gg}^{\text{SM}} \) was also calculated to NNLO in [9]. However, in spite of this significant effort, the \( gg \to H \) channel still suffers from large theoretical uncertainties. The residual scale dependences in both the cross section and partial width are at the 20% level. In the most recent study of Higgs couplings at the LHC [11], a \( \pm 20\% \) theoretical error was assigned to the \( gg \to H \) channel. The next largest error entering the analysis is a 15% uncertainty in the \( t\bar{t}H \) production cross section; the remaining theoretical errors are under 10%, as are the individual experimental systematics.

In this note, we point out that the appropriate uncertainty in the \( gg \to H \) channel which enters the analysis of Higgs couplings should instead be \( \pm 5\% \), which is smaller by a factor of four. This reduction relies upon the observation that the theoretical input for the Higgs coupling determination is the ratio \( \sigma_{gg}^{\text{SM}}/\Gamma_{gg}^{\text{SM}} \). The QCD corrections to \( \sigma_{gg}^{\text{SM}} \) and \( \Gamma_{gg}^{\text{SM}} \) track each other, and a large portion of the uncertainty cancels when the ratio is taken. We believe that the \( \pm 20\% \) error assumed in the analysis of [11] is overly conservative. The NNLO Higgs production itself suffers from an uncertainty of only \( \pm 10\% \), and as we will argue later, the imprecise knowledge of the gluon distribution function is smaller than the residual scale uncertainty. Since this is the largest single error entering the analysis of Higgs couplings, its reduction may have an important effect on the precision of the \( \Gamma_x \).

We now briefly explain why the theoretical uncertainty in the ratio is reduced, and then present a numerical proof. The perturbative expansions for the cross section and partial width have the forms

\[
\sigma_{gg}^{\text{SM}} \sim \alpha_2^2(\mu_R)C^2 \left\{ 1 + \alpha_s(\mu_R)X_1(\mu_R, \mu_F) + \alpha_2^2(\mu_R)X_2(\mu_R, \mu_F) + \ldots \right\},
\]

\[
\Gamma_{gg}^{\text{SM}} \sim \alpha_2^2(\mu_R)C^2 \left\{ 1 + \alpha_s(\mu_R)Y_1(\mu_R) + \alpha_2^2(\mu_R)Y_2(\mu_R) + \ldots \right\}.
\]

(2)

\( \mu_R \) and \( \mu_F \) respectively denote the renormalization and factorization scales, while \( C \) denotes the Wilson coefficient obtained from integrating out the top quark. The width has no \( \mu_F \) dependence, as no initial-state mass-factorization is required. An immediate consequence of these formulae is that both \( \sigma_{gg}^{\text{SM}} \) and \( \Gamma_{gg}^{\text{SM}} \) are proportional to the same factor of \( \alpha_2^2(\mu_R)C^2 \). This quantity contributes a large renormalization scale dependence to both \( \sigma_{gg}^{\text{SM}} \) and \( \Gamma_{gg}^{\text{SM}} \). The same scale \( \mu_R \) can be chosen in both calculations, which completely removes the large renormalization scale uncertainty when the ratio \( \sigma_{gg}^{\text{SM}}/\Gamma_{gg}^{\text{SM}} \) is taken. When integrating out the top quark to obtain the Wilson coefficient \( C \), the relevant scale is the top quark mass \( m_t \); this choice removes all large logarithms from the expression. This dependence is identical in both \( \sigma_{gg}^{\text{SM}} \) and \( \Gamma_{gg}^{\text{SM}} \), and therefore cancels exactly. The observation that \( \mu_R \) is approximately the same for both \( \sigma_{gg}^{\text{SM}} \) and \( \Gamma_{gg}^{\text{SM}} \) in the remaining pieces relies upon the fact that the Higgs boson production is dominated by partonic threshold. This implies that the partonic center-of-mass energy squared \( s \), relevant for the \( gg \to H \) process, is \( s \sim m_H^2 \). Since \( m_H \) is also the only mass scale that enters the decay rate, the kinematic scales for the production and decay processes must be similar. This indicates that identical \( \mu_R \) should be chosen for both processes, and also implies that the \( \mu_R \) dependence of the expansion coefficients for the cross section and width should be similar, \( X_i \sim Y_i \). This further reduces the renormalization scale dependence. Since the factorization scale uncertainty turns out to be small, the cancelation of the renormalization scale dependence leads to a reduced theoretical error in the ratio.

We now present numerical proof of this assertion. We estimate the theoretical errors by varying both scales within the range \( m_H/2 \leq \mu_R, \mu_F \leq 2m_H \). We vary each scale in the range \( m_H/2 \leq \mu_R, \mu_F \leq 2m_H \), and fix the other scale to \( m_H \). The \( \mu_R \) variation of the cross section is 20% to 25% even at NNLO (see Fig. 1), while the \( \mu_F \) variation is less than 5% at NNLO and is therefore negligible compared with \( \mu_R \) scale variation. The \( \mu_R \) dependence is reduced to the 10% level when the ratio is taken, as shown in Fig. 2, while the \( \mu_F \) dependence is unchanged. We note that because of the complete cancellation of the \( \mu_R \) dependence at LO, we do not obtain a reliable estimate of the theoretical uncertainty until the NNLO corrections are included. We use the program FEHiP [8] to obtain cross section numbers, and we take the width results from [9]. We believe that the scale variation bands presented here are a good estimate of the theoretical uncertainty: the NLO and NNLO bands overlap, the
fixed order results are in good agreement with those obtained with threshold resummation \[10\], and the leading N^3LO corrections do not significantly change the NNLO results \[7\].

In Fig. 2 we estimate the remaining theoretical error in \(O = \frac{\Gamma_{gg}}{\Gamma_{gg}^{SM}}\) by first setting \(\mu_{R} = \mu_{F} = \mu\), and defining \(\Delta O = |O(\mu = 2m_{H}) - O(\mu = m_{H}/2)|/2O_{\text{average}}\). We can then find the appropriate error band by taking \(\pm \Delta O\). In both cases there is a slight cancellation between the \(\mu_{R}\) and \(\mu_{F}\) dependences; we conservatively round the error up and estimate \(\Delta \sigma \approx \pm 10\%\) and \(\Delta (\sigma/\Gamma) \approx \pm 5\%\). As claimed, a large portion of the theoretical uncertainty cancels in the ratio.

We also point out that the cancellation of the QCD uncertainties between \(\sigma_{gg}\) and \(\Gamma_{gg}\) is expected even after mild cuts on the final states are imposed. For example, the calculation of \[8\] shows that the QCD corrections to \(\sigma(gg \to H \to \gamma\gamma)\) are quite similar to that of the total \(gg\to H\) cross section even if the standard cuts used by ATLAS and CMS for the identification of the two-photon signal \[12\] are applied. We also expect a reduction of the theoretical uncertainty in quantities of the type \(\sigma(gg \to H + \text{jets})/\Gamma_{gg}\). However, this reduction should be less effective than in the total cross section because \(\sigma(H+\text{jets})\) depends on additional kinematic factors, such as the \(p_{T}\) of the jets, that control the magnitude and scale dependence of QCD effects.

Finally, we note that the Higgs boson production cross section suffers from imprecise knowledge of parton distribution functions (PDFs). The resulting uncertainties strongly depend on the production mode and the mass of the Higgs boson; they were recently analyzed in \[13\]. For \(gg \to H\) and a Higgs boson with mass \(m_{H} \sim 200\text{ GeV}\), the PDF errors lead to an uncertainty in the Higgs boson production cross section of about \(\pm 2.5\%\), smaller than the scale variations studied here and negligible when added in quadrature. In contrast to the renormalization and factorization scale uncertainties, the PDF error is reducible, since measurements of Standard Model processes at the LHC may lead to more accurate PDF determinations. For these reasons, we do not consider the PDF uncertainty in what follows.

The \(\pm 20\%\) uncertainty on the gluon fusion production cross section is the dominant uncertainty in the analysis of \[11\] for \(m_{H} > 150\text{ GeV}\). While the full re-analysis is required to access the impact of the uncertainty reduction that we advocate, some simple estimates can be made. Consider the determination of \(\Gamma_{gg}\) from the production of \(W\)-bosons in the gluon fusion, \(gg \to H \to W^{+}W^{-}\). The uncertainty in \(\Gamma_{gg}\) is given by

\[
\frac{\delta \Gamma_{gg}}{\Gamma_{gg}} = \sqrt{\left(\frac{\delta N}{N}\right)^{2} + \left(\frac{\delta T}{T}\right)^{2} + \left(\frac{\delta \Gamma}{\Gamma}\right)^{2} + \left(\frac{\delta \Gamma_{W}}{\Gamma_{W}}\right)^{2}},
\]

where \(N\) is the number of events, \(T = \sigma_{gg}^{SM}/\Gamma_{gg}^{SM}\) is the theoretical normalization factor and \(\Gamma_{W}\) is the decay rate for \(H \to WW\). For the purpose of this estimate we assume that the errors are not correlated. Following \[11\], we use \(\delta N/N \approx 10^{-1}\), \(\delta T/T \approx 1.4 \times 10^{-1}\) and \(\delta \Gamma_{W}/\Gamma_{W} \approx 10^{-1}\), for \(m_{H} \approx 160\text{ GeV}\). Using \(\delta T/T \approx 2 \times 10^{-1}\), we find that the uncertainty on \(\Gamma_{gg}\) is thirty percent. As stressed here, it is more appropriate to use \(\delta T/T \approx 5 \times 10^{-2}\); the uncertainty on \(\Gamma_{gg}\) then becomes about twenty percent. An analysis is underway to determine the effect of the error reduction on the extraction of Higgs properties at the LHC \[14\].

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