What can we learn from information-entropy about turbulence and Large-Eddy-Simulation?

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The Shannon entropy is a rigorous measure of information that can be used to evaluate the properties of information in turbulent flows. In this study, the behavior of the Shannon entropy is investigated for LES of an incompressible turbulent channel flow based on the classical DNS of Moser et al. The Shannon entropy is calculated from the velocity fluctuation fields and is compared to the turbulent kinetic energy. The quality and resolution of the LES calculations on different grids has been evaluated against DNS results. The Shannon entropy is calculated for velocities and the turbulent viscosity to observe how it behaves and how it is related to the simulation quality and grid resolution.

1 Introduction

Large-Eddy Simulation is gaining relevance in research and industry due to the constant increase of computational power. LES appears to be a promising concept as it uses most computational resources to resolve the energy-carrying eddies, which dictate the behavior of the flow, while the smaller dissipative eddies are expressed in a model. At the same time the numerical and modelling errors have to be investigated with care. To properly rate the quality of an LES calculation, we still rely on experimental or DNS data. While experimental and DNS data are available for only few flow configuration, a quality indicator solely based on the LES results is desirable. There is a handful of commonly used quality indicators available, however most of them fail to rate the true quality of an LES, especially when a poor numerical scheme is used [1, 2]. The present work aims to investigate an alternative quantity describing the turbulence resolved by the LES - the information entropy as introduced by Shannon.

2 Approach

For this study the Shannon-entropy as a well known form of information entropy for discrete random variables is used

\[ H(X) = - \sum_{i=1}^{N} p_i \cdot \log_2 (p_i) \] (1)

The Shannon-entropy – opposed to the variance – does not depend on physical values directly. However, the physics can be found in the probability distributions. The probability distributions are obtained for turbulent quantities such as the streamwise, wall-normal and spanwise velocity fluctuations \( u' \), \( v' \) and \( w' \) as well as the turbulent viscosity \( \nu_t \) to represent the influence of the turbulence model. The probability distributions are obtained by binning and histograms of the field with the turbulent quantity. To obtain a measure independant of the bins, the entropy is normalized with the binary logarithm of the number of bins \( \log_2(M) \) [3]. As a test case a simple incompressible, turbulent channel flow at \( Re_T = 395 \) has been chosen, based on the DNS of Moser et al. [4]. The LES has been performed with the in-house code PsiPhi of the chair of fluid dynamics, which has been used for LES and DNS calculations up to high resolution [5].

3 Results

In figure 1a the Shannon-Entropy of the streamwise velocity fluctuations \( H(u') \) versus the wall distance in viscous wallunits \( y^+ \) is shown. For the 1 mm and 2 mm grid a rising and falling slope can be observed, as the grid is fine enough to resolve one-digit wall distances. For coarser grids, the profile is less distinct. The entropy has higher values, if the grid is chosen coarser. Approaching the wall the influence of viscous friction starts to rise and damps out turbulent fluctuations, as at the wall the velocity must satisfy a no-slip condition. For a laminar case the velocity profile is deterministic and can be described analytically. Therefore, for a given wall distance there is a fixed velocity value, which remains constant in the steady state case. For this configuration, the fixed velocity occurs with no uncertainty. The probability distribution is given
as a one-event distribution, which leads to an entropy value of zero. For wall distances exceeding the laminar sublayer, small turbulent structures start to rise, which superimpose the deterministic velocity profile with velocity fluctuations. The velocity fluctuations lead to deviations of the deterministic value, generating more possible events at a given wall distance. The range of events forms a non-trivial probability distribution with actual uncertainty. Therefore, the entropy starts to increase.

The fluctuations will grow until the wall distance reaches the end of the shear layer, as in the shear layer the strongest production of turbulence can be found. For stronger velocity fluctuations the deviations of deterministic values are larger and therefore lead to a larger range of events, which leads to the growth of entropy. For wall distances beyond the shear layer the entropy starts to fall as the gradient of the mean velocity profile vanishes. The gradient of the mean turbulent velocity profile can be seen as a measure of sensitivity of the flow towards wall-normal perturbations. For large gradients, a fluid particle does not need to travel over a long distance normal to the wall to induce a large streamwise fluctuation, while stronger fluctuations hold the potential of larger entropy. However, for the strongest gradients at the wall, the friction damps out turbulent motion and hence inhibits high values of information entropy [6].

The Shannon entropy of the turbulent viscosity $H(\nu_t)$ is shown in figure 1b. For coarser grids, higher entropy is achieved. This is assumed to be caused by the influence of the turbulence model. For a coarse grid there are more neglected scales. Hence, the turbulence model has to provide a wider range of viscosity values to give the required dissipation representing the subgrid scales. For the finer grids on the other hand the resolution is better, hence the influence on the turbulence model is smaller. Thus, there is need for a smaller range of viscosity values, which will lead to a smaller information entropy.

## 4 Summary and outlook

The Shannon-entropy of streamwise velocity fluctuations and the turbulent viscosity has been studied. The behavior of the entropy has been observed and linked to physical phenomena in the turbulent flow. The entropy has been investigated in dependence of the grid size to create a link to the resolution. Differences in the behavior of the Shannon entropy for different resolution qualities have been found. It has been observed, that the entropy achieves higher values for coarser grids. An aim based on this finding might be the definition of a maximum entropy, that should not be exceeded for a simulation to be considered well resolved. However, to approach this aim the Shannon entropy has to be studied for several more test cases. The entropy of more turbulent quantities has to be calculated to identify a suitable indicator of resolution quality. Overall, we have found Information entropy to be an interesting quantity describing turbulence in LES. Many questions do remain open, though, and we believe that this quantity is a valuable topic for further research.

## References

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