The Hubble diagram extended to $z \gg 1$: the $\gamma$–ray properties of GRBs confirm the $\Lambda$CDM model

C. Firmani$^{1,2,\ast}$, V. Avila–Reese$^2$, G. Ghisellini$^1$ and G. Ghirlanda$^1$

$^1$Osservatorio Astronomico di Brera, via E.Bianchi 46, I-23807 Merate, Italy
$^2$Instituto de Astronomía, Universidad Nacional Autónoma de México, A.P. 70-264, 04510, México, D.F.

20 March 2022

ABSTRACT

Tight constraints on cosmological parameters can be obtained with standard candles spanning a range of redshifts as large as possible. We propose to treat SNIa and long Gamma–Ray Bursts (GRBs) as a single class of candles. Taking advantage of the recent release of the Supernova Legacy Survey and the recent finding of a tight correlation among the energetics and other prompt $\gamma$–ray emission properties of GRBs, we are able to standardize the luminosities/energetics of both classes of objects. In this way we can jointly use GRB and SNIa as cosmological probes to constrain $\Omega_m$ and $\Omega_\Lambda$ and the parameters of the Dark Energy equation of state through the same Bayesian method that we have, so far, applied to GRBs alone. Despite the large disparity in number (115 SN Ia versus 19 GRBs) we show that the constraints on $\Omega_m$ and $\Omega_\Lambda$ are greatly improved by the inclusion of GRBs. More importantly, the result of the combined sample is in excellent agreement with the $\Lambda$CDM concordance cosmological model and does not require an evolving equation of state for the Dark Energy.

Key words: cosmological parameters — cosmology:observations — distance scale—gamma rays: bursts

1 INTRODUCTION

Cosmology passed from being mostly a theoretical science to be one of the most accurate physical sciences in the phenomenological sense. The recent high–precision measurements of cosmological parameters together with the spectacular advances in the understanding of cosmic structure formation, produced a coherent picture of the evolution of the Universe but, on the other hand, prompted new fundamental questions. One of them is related to the expansion of the Universe but, on the other hand, prompted new fundamental questions. One of them is related to the expansion history of the Universe and the possibility that a repulsive medium causes the accelerated expansion. The simplest interpretation of DE is the cosmic constant $\Lambda$, in which case $w = -1$ and $\rho_{DE} = \rho_{\Lambda} = \text{const.}$ However, more exotic models, with $w \neq -1$ and in general varying with $z$, have been proposed (e.g., quintessence, $k$–essence, Chaplygin gas, Brane-world models, etc.); even models that allow $w < -1$ (e.g., Phantom energy) have been considered (see for recent reviews Sahni 2004; Padmanabhan 2006).

The observations of SNIa demonstrated that the expansion of the Universe is accelerating (Riess et al. 1998; Perlmutter et al. 1999). The main explanation to this acceleration is the domination of DE in the current cosmological dynamics, though departures from conventional physics, like modified gravity theories, are also considered. DE is characterized mainly by its equation-of-state parameter, $w = p_{DE}/\rho_{DE}$, where $p_{DE}$ and $\rho_{DE}$ are the pressure and energy density of DE. For $w < -1/3$ the universe undergoes accelerated expansion. The simplest interpretation of DE is the cosmic constant $\Lambda$, in which case $w = -1$ and $\rho_{DE} = \rho_\Lambda = \text{const.}$ However, more exotic models, with $w \neq -1$ and in general varying with $z$, have been proposed (e.g., quintessence, $k$–essence, Chaplygin gas, Brane-world models, etc.); even models that allow $w < -1$ (e.g., Phantom energy) have been considered (see for recent reviews Sahni 2004; Padmanabhan 2006).

We need further observations to unveil the true nature of what we call DE. A strong effort is being done now for developing the next–generation SNIa experiments (see e.g. Linder & Huterer 2003) aimed mainly to reduce random uncertainties. However, it is also crucial to reduce systematic uncertainties as well as to break model degeneracies (e.g., Weller & Albrecht 2002; Linder & Huterer 2003; Nesseris & Perivolaropoulos 2003, Ghisellini et al. 2005, Firmani et al. 2005b). The two latter papers illustrate how some degeneracies in the cosmological parameter space can be reduced if the standard candles used to construct...
the HD span a wide $z$ range. In this sense, long GRBs have been proposed as a class of objects able to extend the HD up to very high redshifts in a natural manner (Schaefeli 2005; Firmani et al. 2004b).

Despite the large dispersion of the long GRB energetics (Frail et al. 2000; Bloom, Frail & Kulkarni 2003), the discovery that their energetics correlate with observable quantities like the peak energy $E_{\text{pk}}$ (in $\nu L_{\nu}$) of the time-integrated prompt emission spectrum and the achronic “break-time” in the afterglow light curve (Ghirlanda et al. 2004a; Liang & Zhang 2005; Nava et al. 2006), has been used to “standardize” it. This allowed to employ GRBs as truly cosmological tools (Ghirlanda et al. 2004b, 2006; Dai, Liang & Xu 2004; Firmani et al. 2005; Xu, Dai & Liang 2005; Firmani et al. 2006). A new GRB correlation whose tightness, in the framework of the standard fireball scenario, is explained by its scalar nature. Since such new correlation has the same shape in the observer and in the comoving frame, the influence of the $\Gamma$ relativistic factor on the observed scatter becomes negligible. The correlation is based on prompt $\gamma$-ray observables only (besides the redshift), by-passing the need of measuring afterglow quantities as is the case of the “Ghirlanda” relation (Ghirlanda et al. 2004a). The quantities involved in the new correlation are the isotropic peak luminosity $L_{\text{iso}}$, $E_{\text{pk}}$, and the “high signal” timescale $T_{0.45}$, previously used to characterize the variability behavior of bursts. In Firmani et al. 2006b, we have found that by varying the cosmology, the data points present a minimum scatter around their best fit line in correspondence of the so-called $\Lambda$ cold dark matter ($\Lambda$CDM) concordance cosmology: a flat geometry Friedmann-Robertson-Walker-Lemaître model with the cosmological constant dominating today. This result shows that, indeed, the $L_{\text{iso}}-E_{\text{pk}}-T_{0.45}$ relation can be used to derive cosmological constraints.

Due to the lack of low $z$ GRBs for calibrating a given correlation independently from cosmology, the use of a statistical approach to jointly calibrate the correlation and constrain the cosmological parameters is required. Firmani et al. 2006b have presented an iterative Bayesian method to deal with this, so-called, ‘circularity problem’. The same method can be used for SNIa. Note that SNIa are not perfect standard candles: their luminosities vary with the shape of the light–curve (the brighter–slower relation) and with color (the brighter–bluer relation) (e.g., Guy et al. 2005, and the references therein). Due to several high $z$ systematic effects, a better calibration of these relations is obtained if higher $z$ SNIa are included. The latter makes these relations cosmology dependent. Therefore, the best fit to these relations has to be carried out jointly within the same cosmological fit (Astier et al. 2005, hereafter A05). This approach has been applied to the ‘Supernova Legacy Survey’ (SNLS) of 115 SNIa (A05). We apply here our Bayesian method to this sample to improve the constraints given by A05, who used a simple multi–parametric $\chi^2$ minimization method.

Observations of SNIa are accurate and the current samples comprise more than one hundred objects, but they are detected only at relatively low $z$’s, which introduces the degeneracy problem mentioned above. The GRBs useful as distance indicators span a large redshift range (up to $z = 4.5$) but they are still scarce (19 bursts for the $L_{\text{iso}}-E_{\text{pk}}-T_{0.45}$ relation). Thus, a promising strategy to partially overcome the problems that each family of objects individually suffer of, is to combine both in the same Hubble diagram and use them jointly for constraining the cosmological parameters. This is the goal of this Letter. Our main result is that the concordance $\Lambda$ cosmology (minimal DE model) is fully consistent with the joint GRB and SNLS SNIa data spanning the redshift range from $z = 0.015$ to 4.5. Previous results with the so-called SNIa ‘gold-set’ ($z < 1.7$) showed a marginal inconsistency with the concordance model (Riess et al. 2004; Alam, Sahni, & Starobinsky 2004; Choudhury & Padmanabhan 2004; Fassal, Bagla & Padmanabhan 2005; Neßers & Perivolaropoulos 2006), suggesting the possibility of alternative DE models. In §2 we describe the SNIa and GRB samples used here. Section 3 deals with the method while the results are presented in §4. The conclusions of this work are given in §5.

2 THE SAMPLES

2.1 Type Ia supernovae

We analyze here the SNIa sample presented in A05. This sample consists of 44 nearby ($0.015 < z < 0.125$) SNIa assembled from the literature, and of 73 distant SNIa (0.249 < $z < 1.010$) discovered and followed during the first year of SNLS\(^1\). The same light–curve fit method (Guy et al. 2005) was applied to all SNIa in the sample. For each SN, the reported quantities along with their errors are the fitted rest–frame $B$–band magnitude $m_B$ and the values of the parameters $s$ (light–curve stretch) and $c$ (normalized $B - V$ color at the maximum of the light–curve). The magnitude $m_B^*$ refers to observed brightness, and therefore does not account for the brighter–slower and the brighter–bluer correlations. As mentioned in the Introduction, A05 suggest to empirically calibrate these correlations using all objects (either at low or high $z$). For the cosmological fits, two SNIa out of the 117 SNLS sample objects were excluded because they are outliers in the HD.

We follow A05 and include in the SNIa dispersions the contribution of a peculiar velocity of 300 km/s (this dispersion depends on $z$ and on cosmology) and an intrinsic dispersion of SN absolute magnitudes of 0.132. The latter is adjusted to obtain a SN reduced errors are the fitted rest–frame $B$–band magnitude $m_B^*$ and the values of the parameters $s$ (light–curve stretch) and $c$ (normalized $B - V$ color at the maximum of the light–curve). The magnitude $m_B^*$ refers to observed brightness, and therefore does not account for the brighter–slower and the brighter–bluer correlations. As mentioned in the Introduction, A05 suggest to empirically calibrate these correlations using all objects (either at low or high $z$). For the cosmological fits, two SNIa out of the 117 SNLS sample objects were excluded because they are outliers in the HD.

2.2 GRBs and the $L_{\text{iso}}-E_{\text{pk}}-T_{0.45}$ relation

The sample of 19 GRBs with known redshifts and with the observational information required to establish the $L_{\text{iso}} = CE_{\text{pk}}^{n}T_{0.45}^{-n}$ correlation was presented in Firmani et al. 2006a. The rest frame quantities entering in this correlation are the bolometric corrected (in the range of $1-10^4$ keV in the rest frame) isotropic-equivalent luminosity, $L_{\text{iso}}$, the spectrum peak energy, $E_{\text{pk}}$, and the time spanned by the brightest 45% of the total lightcurve counts above the background, $T_{0.45}$. The assumed energy range for calculating $T_{0.45}$ is $50 - 300$ keV in the rest frame. We use the recipe proposed by Reichart et al. 2003 to pass from the observed energy range to the assumed rest one, and the light-curve time binning of HETE-II, 164–ms (see Firmani et al. 2006a).

Besides the redshift, the observables required to estimate $L_{\text{iso}}$, $E_{\text{pk}}$ and $T_{0.45}$ are: the peak flux $P$, the fluence $F$, the spectrum peak energy $E_{\text{pk}}$, and its overall shape (as described by Band et al. 1993 model for most cases), and the time scale of the brightest 45% of the total light–curve counts above the background.

\(^1\) see cftt.hawaii.edu/SNLS/
3 THE METHOD

The ‘circularity problem’ mentioned in the Introduction in principle should not be a problem for SNIa because the brighter–slower and brighter–bluer correlations could be calibrated with a low $-z$ sample. However, the distance estimates to high $-z$ SNIa improve if the parameters of these correlations are empirically determined along with the $\chi^2$ minimization, from which the cosmological parameters are also constrained (A05; note that multi–band photometric data are necessary to apply this technique). The situation is therefore, at least mathematically, similar to the circularity problem of GRBs. This suggests us to use our Bayesian method for improving the SNLS cosmological constraints estimated by A05.

The basic idea of our approach is to find the best–fitted correlation on each point $\bar{\Omega}$ of the explored cosmological parameter space [for instance $\bar{\Omega} = (\Omega_m, \Omega_\Lambda)$] and estimate with such correlation the scatter $\chi^2(\bar{\Omega}, \bar{\Omega})$ on the HD for any given cosmology $\Omega$. The conditional probability $P(\bar{\Omega}|\bar{\Omega})$ inferred from the $\chi^2(\bar{\Omega}, \bar{\Omega})$ statistics provides the probability for each $\bar{\Omega}$ given a possible $\bar{\Omega}$–defined correlation. By defining with $P'(\bar{\Omega})$ an arbitrary probability for each $\bar{\Omega}$–defined correlation, the total probability of each $\bar{\Omega}$, using the Bayesian formalism, is given by

$$ P(\bar{\Omega}) = \int P(\bar{\Omega}|\bar{\Omega})P'(\bar{\Omega})d\bar{\Omega}, \quad (1) $$

where the integral is extended on the available $\bar{\Omega}$ space. Because the observations give a correlation for each cosmology, $P'(\bar{\Omega})$ is actually the probability of the cosmology. Consequently such probability is obtained putting $P'(\bar{\Omega}) = P(\bar{\Omega})$ and solving the integral Eq. 4. This method based on the use of an iterative Monte Carlo technique has been presented in Firmaniet et al. (2005,2006b), and we refer the reader to such papers for more details.

In what follows, we apply the Bayesian method to (i) the SNLS SNIa sample alone, and (ii) to the combination of both the SNIa and GRB samples.

4 RESULTS

Figure 1 shows the HD assuming the concordance cosmology ($\Omega_m=0.28, \Omega_\Lambda=0.72$ and $h=0.71$) for the 117 SNIa ($0.015 < z < 1.010$) reported in A05 (red symbols), as well as for the 19 GRBs ($0.17 < z < 4.5$) from our sample (blue symbols). From this plot one sees that GRBs are a natural extension of SNIa to high redshifts. The observational uncertainties for GRBs are still significantly larger than for SNIa. The residuals of both samples with respect to the assumed cosmology (solid curve) are shown in the bottom panel of Fig. 1. The average of the absolute values of the residuals and its uncertainty for the SNIa and GRB samples are $0.15 \pm 0.01$ and $0.26 \pm 0.05$, respectively.

In Fig. 2 we show the 1$sigma$ confidence levels (CL's) for only the SNIa sample (green line) and for the combined SNIa+GRB sample (red shaded region) in the $\Omega_m$–$\Omega_\Lambda$ space. The use of the Bayesian method for analyzing the first–year SNLS SNIa dataset improves somewhat the cosmological constraints, especially for the large $\Omega_\Lambda$ part of the CL, as can be appreciated by comparing Figs 2 with Fig. 5 in A05. From Fig. 2, we also see that the inclusion of GRBs greatly improves the constraints given by SNIa alone. Because GRBs span a wide range of $z$‘s, the degeneracy between $\Omega_m$ and $\Omega_\Lambda$ is less severe for them than for the SNIa (see the discussion in Firmaniet al. 2006b). This achievement is obtained despite the small number of GRBs and their relatively large observational uncertainties.

Both the SNLS SNIa sample and the GRBs sample, show each one that the best–fit values of $\Omega_m$ and $\Omega_\Lambda$ are close to the flat–geometry case: the concordance model is actually well inside the corresponding 1$sigma$ CL constraints (A05, Nessers & Perivolaropoulos 2006, Firmaniet al. 2006b). Now, the combined set makes the constrain even more restrictive. If one forces $\Omega_m=1$, our statistical analysis gives $\Omega_\Lambda = 0.273^{+0.027}_{-0.024}$. This range intersects the range of $\Omega_m$ values allowed by dynamical determinations (e.g., Hawkins et al. 2003; Schuecker et al. 2003). Thus the constraints to the $\Lambda$ cosmology parameters obtained here are consistent with several other independent cosmological measurements.

4.1 Flat cosmology with alternative DE

We now relax the assumption $w = -1$, which was implicit up to now, and explore the possibility of $w = w_0$, where $w_0$ is a free parameter. The limited number of objects in our two samples and the current accuracies do not allow to have more than two free parameters. Therefore, we fix $\Omega_m+\Omega_\Lambda=1$, and find the CL contours in the...
Figure 2. Constraints on the $(\Omega_m, \Omega_A)$ plane from the SNIa HD using our Bayesian approach to circumvent the circularity problem (green line) and from the combined SNIa+GRB HD (red line and shaded region). Both lines are contours at 68.3% CL’s. The star shows where $\chi^2$ reaches its minimum, while the cross indicates the concordance cosmology. The line corresponds to the flat geometry cosmology, the upper curve is the loitering limit between Big Bang and no Bing Bang models.

Figure 3. Constraints on the $(\Omega_m, w_0)$ plane for a flat cosmology with static DE, using the same convention of Fig. 2. $w_0 - \Omega_m$ plane. Figure 4 shows the 1σ CL’s in the $w_0 - \Omega_m$ plane using only the SNLS SNIa sample (green line), and the combined SNIa+GRB sample (red line and shaded region). Again, the SNIa constraints obtained with our iterative Bayesian method are tighter than the ones obtained in A05 (compare Fig. 3 with their Fig. 5). When using the combined SNIa+GRB sample, we obtain a tight constraint on $w_0$ for reliable values of $\Omega_m$. For values of $\Omega_m$ in the range 0.236–0.286, the $\Lambda$ model ($w = -1$) is consistent at the 1σ. Assuming the prior $\Omega_m = 0.28$, we obtain $w_0 = -1.055^{+0.073}_{-0.049}$ while for $\Omega_m = 0.26$, we obtain $w_0 = -1.000^{+0.055}_{-0.073}$. By combining independent cosmological probes that are sensitive to $\Omega_m$ (e.g., [Allen et al. 2004; Eisenstein et al. 2005]), with our joint GRB+SNIa probe, better constraints on $w_0$ could be obtained.

In order to explore the constraints on a possible evolution of $w$, based on the same arguments given in Firmani et al. (2006b), we use the parametrization (Rapetti et al. 2005)

$$w(z) = w_0 + w_1 \frac{z}{z_t + z}.$$  

(2)

Constraints on $(w_0, w_1)$ for flat geometry dynamical DE models with the priors $\Omega_m=0.28$ and $z_t=1$ are plotted in Fig. 4. As discussed in Firmani et al. (2006b), current observational data do not allow yet to determine well $z_t$ ($z_t = 1$ corresponds to a popular parametrization introduced by [Chevallier & Polarski 2001]. The green line and the red shaded region represent the 1σ CL’s for the SNIa dataset alone and for the combined SNIa+GRB sample, respectively. The encapsulated panel shows the corresponding 1σ CL for the evolution of $w(z)$ using the SNIa dataset alone (green plus red region) and the SNIa+GRB datasets (red shaded region). Again the $\Lambda$ case ($w_0=-1$ and $w_1=0$, which reduces to the concordance model because of the assumption that $\Omega_m = 0.28$), is within the 1σ CL’s for both SNIa and SNIa+GRB constraints. Our results allow
at the 1σ CL for models that avoid crossing the phantom dividing line ($w[z] = -1$). Notice that although Eq. 3 describes the evolution of $w$ up to any arbitrary large $z$ once its parameters were determined, the changes on $w$ with $z$ suggested by the observational constraints are formally valid only within the redshift range of the observational data, $z < 4.5$ in our case.

5 CONCLUSIONS

We have combined a sample of potential standard candles that includes 115 SNIa of the SNLS dataset (A05) and 19 long GRBs. The latter were “standardized” on the basis of a tight correlation among prompt γ-ray properties alone (Firmani et al. 2006a). Exploiting some similarities between the energetic calibrations of the SNLS SNe and the GRBs, we use for both populations of objects the same method to constrain cosmological parameters, namely a Bayesian approach described in Firmani et al. (2005,2006b). GRBs may be conceived as the natural extension of SNIa to large $z$’s in the HD. The advantage of this extension by using GRBs is that the heavy degeneracy in the space of the cosmological parameters due to a narrow and low $z$ range (as is the case of SNIa) is remarkably reduced. Our main results are as follow:

- The cosmological constraints obtained with the Bayesian method for the SNLS sample alone are in agreement with those reported in A05. However, with the Bayesian method we obtain somewhat tighter constraints on $\Omega_m$, $\Omega_\Lambda$, and $w_0$ than with the $\chi^2$ minimization method used in A05. The values of $\Omega_m$ and $\Omega_\Lambda$ of the best fit (using SNIa only) are in good agreement with the flat–geometry case. Moreover, the best fitting values of $\Omega_m$ and $\Omega_\Lambda$ do not disagree with those obtained by using other cosmological probes (e.g., Spergel et al. 2006), re-affirming the reliability of the cosmological concordance model.

- We have presented in a previous paper (Firmani et al. 2006b) cosmological constraints derived by using GRBs only, whose energetics is standardized with the $L_{iso}-E_{pk}-T_{0.45}$ relation. We have shown here that the combined SNIa+GRB sample is able to largely reduce the allowed region of the parameter space with respect to the case where a single population is used. The resulting values of the best fits, once again, are in agreement with the concordance $\Lambda$CDM model.

- As a consistency check, we have explored DE models with $w = w_0 = \text{const.}$ (i.e. relaxing the assumption of $w = -1$), but assuming flat geometry. Furthermore, for completeness, we have allowed also $w$ to change with $z$ according to a parametric law and assuming $\Omega_m=0.28$. In both cases we find that $w = -1 = \text{const.}$ is within the 1σ CL from the best fits.

Finally, we emphasize the next two general conclusions:

1) GRBs and SNIa as standard candles should not be considered as competing cosmological probes but as complementary methods. Besides, both GRBs and high-$z$ SNIa suffer from the circularity problem concerning their calibration in such a way the same Bayesian method can be applied for both samples.

2) According to our results there is no need for DE “exotic” equations of state. The flat Friedmann-Robertson-Walker-Lemaître $\Lambda$ cosmology is fully consistent with the HD constructed for the joint sample of SNIa and GRBs up to $z = 4.5$. Similar conclusions were obtained recently on the basis of other high–precision cosmological probes (Spergel et al. 2006).

ACKNOWLEDGMENTS

We acknowledge the anonymous referee for constructive comments and Giuseppe Malaspina for technical support. V.A-R. acknowledges the hospitality extended by INAF–OAB. This work was supported by the italian INAF and MIUR (Cofin grant 2003020775_002), and PAPIIT-UNAM grant IN107706-3.

REFERENCES

Alam U., Sahni V., Saini T.D. & Starobinsky A.A., 2004, JCAP, 06, 008
Allen S. W., Schmidt R. W., Ebeling H., Fabian A. C. & van Speybroeck L., 2004, MNRAS, 353, 457
Astier P., Guy J., Regnault N. et al., 2006, A&A, 447, 31 (A05)
Band D., Matteson J., Ford L., et al., 1993, ApJ, 413, 281
Bloom J. S., Frail D. A. & Kulkarni S. R. 2003, ApJ, 594, 674
Chevallier M. & Polarski D., 2001, Int. J. Mod. Phys. D10, 213
Choudhury T.R. & Padmanabhan T., 2004, A&A, 429, 807
Dai Z.G., Liang E.W. & Xu D. 2004, ApJ, 612, L101
Eisenstein D. J.,Zehavi I., Hogg D. W. et al., 2005, ApJ, 633, 550
Firmani C., Ghisellini G., Ghirlanda G. & Avila-Reese V., 2005, MNRAS, 360, L1
Firmani C., Ghisellini G., Avila-Reese V. & Ghirlanda G., 2006a, MNRAS, in press (astro-ph/0605073)
Firmani C., Avila-Reese V., Ghisellini G. & Ghirlanda G., 2006b, MNRAS, submitted
Frail D. A., Kulkarni S.R., Sari, R., et al., 2001, ApJ, 562, L55
Ghirlanda G., Ghisellini G. & Lazzati D., 2004a, ApJ, 616, 331
Ghirlanda G., Ghisellini G., Lazzati D. & Firmani C., 2004b, ApJ, 613, L13
Ghirlanda G., Ghisellini G., Firmani C., Nava, L., Tavecchio, F. & Lazzati, D. 2006, A&A, in press (astro-ph/0511359)
Ghisellini G., Ghirlanda G., Firmani C., Lazzati D. & Avila-Reese V., 2005, Il Nuovo Cimento C, 2006
Guy J., Astier P., Nobili S., Regnault N. & Pain R., 2005, A&A, 443, 781
Jassal H.K., Bagla J.S., & Padmanabhan T., 2005, Phys. Rev. D, 72, 103503
Hawkins E., Maddox S., Cole S., et al., 2003, MNRAS, 346, 78
Liang E. & Zhang B., 2006, ApJ, in press (astro-ph/0504404)
Linder E.V. & Huterer D., 2003, Phys. Rev. D, 67, 081303
Nava L., Ghisellini G., Ghirlanda G., Lazzati D. & Avila-Reese V., 2005, Phys. Rev. D, 72, 043509
Nava L., Ghisellini G., Ghirlanda G., Tavecchio F. & Firmani C., 2006, A&A, 450, 471
Nesseris S. & Perivolaropoulos L., 2005, JCAP, 10, 001
Östman L. & Mörtell E., 2005, 02, 005
Padmanabhan T., 2006, preprint (astro-ph/0603114)
Perlmutter S., Aldering G., Goldhaber G., et al. 1999, ApJ, 517, 565
Rapetti D., Allen S. W., & Weller J., 2005, MNRAS, 360, 555
Reichart D., Lamb D. Q., Fenimore E. E., Ramirez-Ruiz E., Cline Th. L. & Hurley K., 2000, ApJ, 552, 57
Riess A.G., Filippenko A.V., Challis P., et al. 1998, AJ, 116, 1009
Riess A.G., Strolger L.-G., Tonry J., et al., 2004, ApJ, 607, 665
Sahni V., 2004, in “The Physics of the Early Universe”, Ed. E. Linder (Springer) p.141 (astro-ph/0403324)
Scheuer B.E. 2003, ApJ, 583, L67
Scheuuer P., Caldwell R.R., Böhringer H., et al., 2003, A&A, 402, 53
Firmani et al.

Spergel D.N., Bean R., Dorè O., et al., 2006. preprint (astro-ph/0603449)

Weller J. & Albrecht A., 2002, Phys. Rev. D, 65, 103512

Xu D., Dai Z. G. & Liang E. W., 2005, ApJ, 633, 603