Evaluation of dielectric materials properties using microwave enhanced metamaterials sensor

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Abstract. The paper proposes the use a metamaterial structure for improving the transmission properties of open waveguide probe, developing a frequency selective sensor in microwave frequency X band sensitive to changing of properties of investigated dielectric material. This assembly can be used for properties of dielectric material investigation. The numerical simulations of electromagnetic wave propagation in rectangular waveguide with metamaterial structure were performed using the FEM and FDTD. The numerical results were proved with experiments, testing two types of biological materials.

1. Introduction
The mathematical evidence of electromagnetic waves propagation in circular and rectangular waveguides, and the concept of critical frequency, was introduced by Lord Rayleigh in 1893, but Heaviside [1] was attempting to transfer the wave to closed tubes. Since then, waveguides have been used to transmit electromagnetic waves in antennas, radar and microwave applications.

Nowadays, the trend is miniaturization of microwave circuits [2] with the use of planar technology such as microprocessor lines. However, there are a number of other applications where the use of waveguide properties is necessary, as field antennas, multi-frequency antennas, certain types of radar, satellite communications and radio. Waveguides offer few benefits compared to coaxial or microstrip lines. The advantages of waveguides are their ability to isolate the signal from noised signals, low loss, high performance and a large range of transferred frequency bands [3]. These performances can be increased using metamaterials structures placed at the opening of waveguide [4].

The dielectric properties of materials can be determined by measuring the modifications in resonant circuits [5], electronic bridge [6], transient methods in DC [7], etc. Microwaves (MW) measurements can be carried out by velocity-modulated tubes, cavities, waveguide technologies and free space configurations [8]. MW waveguide sensors [9 ] measure dielectric properties of materials due to MW interactions with them.

Metamaterials (MM) are structures which behave unusual under specific electromagnetic (EM) excitation [10 ], allow the focusing of evanescent waves [11] and their enhancement, leading to the improvement of near field sensors. Recently, the MM structures have been employed in improvement of MW waveguide sensor sensitivity by embedding them at the opening of classical MW guides [12].

The paper presents an approach for determination of the dielectric properties of dielectric materials by embedding a metamaterial (MM) structure at the opening of waveguide sensor in order to increase its sensing properties.
2. Design and analysis of waveguide system

EM waves can be described using the Maxwell theory of equations, with adequate boundary conditions on the perfectly conductive (PEC) walls of the waveguide. The waveguide is a metal tube with any cross-sectional shape, most often used are rectangular or circular waveguides. The metallic casing of the waveguide transforms EM field into so-called modes. Each modes have its limit frequency. The energy is propagated by the waveguide only if it is fulfilled the condition that the frequency of the transmitted mode is greater than its limit frequency. At the EM field excitation in the waveguide below the cut-off frequency, evanescent modes occur, meaning that these modes are exponentially damped along the whole of the waveguide length. From the theory of the EM field it is known that the energy is a waveguide propagates like an EM wave with a precisely defined distribution. The three basic ones types of waves led:

- TE (transversal electric wave) - the electric field only has transverse components, the magnetic field also has a longitudinal component. That is why the transmission structure is representing a waveguide (e.g., rectangular, circular, coaxial, etc.)
- TM (Transverse Magnetic Wave) - The magnetic field has only transverse components, the electric field also has a longitudinal component. Here also the transmission structure is a waveguide.
- TEM (transverse electromagnetic wave) - electrical components and the magnetic field are perpendicular to the propagation direction, there is no longitudinal component of the field. In this case, the transmission structure is a line.

Thus, only TE and TM waves can be propagated by the rectangular waveguide, presenting an infinite number of modes. These modes correspond to the maximum number of standing waves perpendicular to the propagation direction and are dependent by the waveguide geometry. The rectangular waveguide is used to transmit EM energy in frequency range from about 1 GHz up to frequencies above 100 GHz.

The EM field propagates inside the waveguide if the waveguide cross-sectional width is equal at least half of the critical wavelength EM wave in the waveguide

\[ \lambda_{crit} = 2a \]

If the waveguide is filled with air,

\[ f_{crit} = c / 2a \]

Figure 1 shows a waveguide with a rectangular cross-section with internal dimensions a × b and length d. The waveguide walls are made of conductive material (silver, copper, brass or aluminum). To achieve a minimum of attenuation electromagnetic waves, the inner walls of the waveguide are metallic, with thickness of walls must be five to ten times the effective penetration depth [13].

Figure 1. Rectangular waveguide

2.1. Electromagnetic wave propagation by waveguide

Vectors EM field \( \vec{E} \) and \( \vec{H} \) are the solution of the homogeneous vector wave equation

\[
\nabla^2 \vec{E} + k^2 \vec{E} = 0
\]

\[
\nabla^2 \vec{H} + k^2 \vec{H} = 0
\]

where \( k \) is the wave number for which \( k = 2\pi f \sqrt{\mu/\varepsilon} \), where \( f \) is the EM wave frequency, \( \mu \) is
permeability and \( \varepsilon \) is the permittivity of the environment that fills the waveguide. TE mode EM has \( E_z = 0 \). Constant propagation (phase constant) of the EM wave \( \beta \) is given by:

\[
\beta = \sqrt{k^2 - k_c^2}
\]

where \( k_c \) is the critical (threshold) wavelength given by

\[
k_c = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}
\]

(4)

When \( k > k_c \), the phase constant is a real number. Critical (limit) frequency is given by the relationship

\[
f_{\text{crit}} = \frac{1}{2\pi\sqrt{\mu_0\varepsilon_0}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}
\]

(5)

where \( a \) and \( b \) are guide dimensions (Figure 1), \( m \) - the number of the half-wave variations of the field along \( a \) direction, \( n \) is the number of half-wave variations of the field along \( b \) direction, \( \varepsilon_0 \) is the permittivity vacuum, \( \mu_0 \) is the vacuum permeability.

At frequencies lower than the cut-off frequency, an EM field is generated, the propagation constant \( \beta \) is an imaginary number. Such an EM field is exponentially dampened.

The dominant mode in the rectangular waveguide is mode TE. In closed waveguides, two types of losses are formed: dielectric losses and losses due to the final conductivity of the waveguide walls.

Losses caused by dielectrics in rectangular waveguide \( \alpha_d \) with TE or TM mode are given by

\[
\alpha_d = \frac{k^2 \tan \delta}{2\beta}
\]

(6)

Losses due to the final conductivity of the conductive guides of the waveguide (\( \alpha_c \)) for the base TE are

\[
\alpha_c = \sqrt{\frac{2\sigma}{a'b\beta k\eta}} (2b\pi^2 + a'k^2)
\]

(7)

Total damping constant - related to waveguide losses for rectangular waveguide is the sum of both of these losses

\[
\alpha = \alpha_d + \alpha_c
\]

(8)

When is about waveguide impedance, two terms are used wave impedance and the characteristic impedance \( Z_0 \). For any propagation mode, the ratio of cross-sections \( E \) and \( H \) field are defined as wave impedance. The characteristic impedance of the material is \( Z_0 = \sqrt{\frac{\mu}{\varepsilon}} \).

The TE mode has impedance

\[
Z_{\text{TE}} = \frac{j\omega\mu}{\gamma}
\]

(9)

The wave impedance differs from the characteristic impedance in this case.

The rectangular waveguide is characterized by impedance defined in terms of voltage, current and power (\( Z_{VI} \), \( Z_{PV} \) and \( Z_{PI} \)). All three definitions are given by the same equation which is distinguishes only by the constant. Therefore, [14]

\[
Z_{PI} = Z_{TE} \frac{\pi^2 b}{8a}
\]

(10)

2.2. Design and analysis of waveguide system

The designing and analyzing of the system is carried out in CST Microwave Studio. For the base of the system, a rectangular waveguide BJ-100 with dimensions \( a = 22.86 \text{ mm}, b = 10.16 \text{ mm} \) is used. The length of the waveguide used is 60mm, the waveguide walls are PEC, the end of the waveguide is
a metallic grounding square shape of size 40mm x 50mm, the optimal operating frequency is 12 GHz. Boundary conditions must be set, so that the properties of guide are minimum affected by the presence of waveguide wall. Figure 2 shows the settings and symmetry axes determined with FEM in CST Microwave Studio. The blue color shows the magnetic symmetry of the plane YZ and the green color displays electrical symmetry in XZ plane.

![Figure 2. Waveguide symmetry settings.](image)

The distribution of EM field for the waveguide has been simulated in FDTD (figure 3). Even if FEM and FDTD are different solution techniques, one in frequency and one in time, that generally require the same computational power (computer power) to solve problems and each of these solutions discretizes the problem and requires a radiation volume, generally their problem sizes are comparable, the preferred simulation in FDTD due to its simple way of design.

![Figure 3. The distribution of EM field at 12GHz.](image)

3. Metamaterial structure

The metamaterial layer consists in 5x5 split ring resonators (SRR) whose resonant frequency corresponds to the operating frequency of the system. The metamaterial structure was etched on the ROGERS RT / DUROID 5870 substrate with dielectric thickness 0.508mm and 35μm rolled copper layers with dielectric constant $\varepsilon = 2.33$ and loss factor $\tan \delta = 0.0012$ in working frequency range. Figure 4a shows the unit cell. The dimensions of this resonator are $l=h=3mm$, $t = g = 0.3mm$, with substrate thickness 0.508mm, considered as strip gratings [15].

The frequency dependency of the studied structure has been simulated using FEM. The properties of the used structure were simulated in the input waveguide and output port. The waveguide walls must meet the boundary conditions. For the two of waveguide walls the boundary conditions for a perfect electric conductor - PEC, respectively for the other two walls perfect magnetic conductor (PMC) have been chosen. If a single ring model is considered and only its first order resonance, the resonant frequency for the equivalent circuit is

$$\omega_r = \frac{1}{\sqrt{LC}}$$  \hspace{1cm} (11)
where \( L \) represents the overall inductance and \( C \) the overall capacity of the ring. If it is possible to control the resonance of each element of the SRR array, the investigation of sample can be carried out.

The gap of the SRR can be described as a capacitor with effective permittivity \( \varepsilon_{\text{eff}} \) due to fringing fields around the gap.

If a sample, e.g. biological tissue, is placed to the vicinity of the sensitive area of any SRR it interacts with the gap of SRR, it changes the capacity of the ring due to a change of the effective permittivity [16]. According to equation (11) the change of the capacity causes the shift of the resonant frequency. This phenomenon is a principle of described sensor.

The behavior of metamaterial structure were simulated for the frequency range of 3-16GHz and \( S \) parameters can be defined [17]

\[
\begin{bmatrix}
    b_1 \\
    b_2
\end{bmatrix} =
\begin{bmatrix}
    S_{11} & S_{12} \\
    S_{21} & S_{22}
\end{bmatrix}
\begin{bmatrix}
    a_1 \\
    a_2
\end{bmatrix}
\]

(12)

The independent variables \( a_1 \) and \( a_2 \) represent the amplitudes of incident waves at port 1 and 2, normalized to characteristic impedance of two-ports network viewed as a transmission line

\[
a_1 = \frac{\text{amplitude of the wave incident at port 1}}{\sqrt{Z_0}}
\]

\[
a_2 = \frac{\text{amplitude of the wave incident at port 2}}{\sqrt{Z_0}}
\]

(13)

The dependent variables \( b_1 \) and \( b_2 \) are normalized reflected amplitude wave

\[
b_1 = \frac{\text{amplitude of the wave reflected at port 1}}{\sqrt{Z_0}}
\]

\[
b_2 = \frac{\text{amplitude of the wave reflected at port 2}}{\sqrt{Z_0}}
\]

(14)

The \( S \)-parameters are

\[
S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2 \to 0} = \text{input reflection coefficient with the output port terminated by a matched load}
\]

\[
S_{22} = \left. \frac{b_2}{a_2} \right|_{a_1 \to 0} = \text{output reflection coefficient with the input port terminated by a matched load}
\]
\[
S_{21} = \left| \frac{b_2}{a_1} \right|_{a_1 = 0} \quad = \text{forward transmission gain with the output part terminated in a matching load}
\]

\[
S_{12} = \left| \frac{b_1}{a_2} \right|_{a_2 = 0} \quad = \text{reverse transmission gain}
\]

The dielectric sample was placed in the near electromagnetic field area of waveguide sensor tuned with 2D metamaterial structure, figure 5. The purpose is to find the shift of resonant frequency due to changing of sample dielectric properties.

![Figure 5. Rectangular waveguide with metamaterial sensor.](image1)

Figure 5. Rectangular waveguide with metamaterial sensor.

Figure 6(a) shows the results of simulation of the frequency response of the reflection \(S_{11}\) and the transmission \(S_{21}\) coefficients. From the S parameters you can use the program calculate the effective permittivity (\(\text{Re (}\varepsilon)\)) and the permeability (\(\text{Re (}\mu)\)) shown in figure 6(b), respectively, the refractive index of the metamaterial structure in figure 6(c).

![Figure 6. Results of simulation: a) S parameters; b) effective permeability; c) effective permittivity](image2)
Figure 7 presents the distribution of EM field in the presence of the metamaterial structure. It can be shown the interaction of the metamaterial layer with the incident electromagnetic wave.

\[ \hat{\varepsilon}_r = \varepsilon'_r - j\varepsilon''_r \]  

(13)

where \( \varepsilon'_r \) is a dielectric constant and measure of how much energy from an external field is stored in the material. \( \varepsilon''_r \) accounts for the frequency dependency of the dielectric which contributes to losses. The loss tangent - tan \( \delta \) is defined as the ratio of the imaginary part to the real part of the relative permittivity [18].

When the microwave pulse reaches the dielectric sample it sends a changed reflected signal which depends on its dielectric properties. According to shift between resonant frequency of empty waveguide port and waveguide port with dielectric sample, the relative permittivity of the sample can be determined.

The most important dosimetric parameter to assess exposure to high frequency EM field is SAR (Specific Absorption Rate). Dosimetric studies quantify the interaction of EM fields with biological materials. Usually SAR value can be evaluated from energy absorbed per unit mass of biological material

\[ SAR = \frac{(\sigma + \omega\varepsilon_0\varepsilon'_r)E^2}{\rho} \]  

(14)

in (W/kg), where \( \rho \) is density of tissue. SAR depends by induced electric field and electric properties of biological material. Knowing the field intensity and measuring SAR, the dielectric properties of samples can be determined [19].
The studied dielectric materials are biological tissue having 3 mm thickness and conductivity of 8 S/m and bone sample with 20 mm width and conductivity of 3.9 S/m [20].

The first measurement was performed without dielectric sample. In the next step the dielectric samples with unknown permittivity were placed in the waveguide. Figure 8 presents the experimental results from measuring the transmission coefficient S21 with dielectric and without dielectric in the waveguide.

![Figure 8. Comparison of resonant characteristics for empty and with dielectric filled port of waveguide.](image)

It can be seen that the amplitude of parameter S21 changing due to the new value of relative permittivity of investigated sample. The shift of resonant frequency due to the change of relative permittivity can be also observed. The sensor with metamaterial placed in front of the waveguide has been simulated and then realized in order to evaluate the dielectric properties of biological materials without intervening over them. The simulation of cell units number and configuration of the cell unit structure have followed the construction of the matrix that assure adequate frequency range and high quality factor for each element. The most visible change can be found at frequency 6 GHz and this can became a precursor for measuring dielectric properties of biological materials using SAR measurements.

5. Conclusions
The results of simulation and experimental measurements had shown that waveguide sensor tuned with metamaterial structure allow the permittivity estimation in comparison with classical open waveguide sensor, because the evanescent wave is amplified by using metamaterial structure. The metamaterial structure has been designed for requested resonant frequency up to value of investigated dielectric sample - more or less loss dielectrics. The sensor tuned with designed metamaterial structure has also the frequency selective properties. The resonant response of sample can be evaluated by measurement of scattering parameters. The designed waveguide sensor tuned with metamaterial structure was employed in measurement of biological samples permittivity.

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