Loop-Less Electric Dipole Moment of the Nucleon in the Standard Model

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Abstract

We point out that the electric dipole moment of the neutron in the Standard Model is generated already at tree level to the second order in the weak interactions due to bound-state effects, without short-distance Penguin loops. The related contribution has a regular nonvanishing chiral limit and does not depend on the mass splitting between s and d quarks. We estimate it to be roughly $10^{-31}$ e·cm and expect a more accurate evaluation in the future. We comment on the connection between $d_n$ and the direct CP-violation in $D$ decays.
Electric Dipole Moments (EDMs) of elementary particles and of composite objects allow one to probe CP and T violation at the fundamental level. The Standard Model (SM) of particle physics has room for only two sources of CP violation. The first one roots in the strong interaction proper, described by the so-called $\vartheta$-term in the QCD Lagrangian \[1\]. Described by a dimensionless coupling $\vartheta$, this term is flavor-diagonal and induces a large electric dipole moment of the neutron $d_n \approx \vartheta \times 3 \times 10^{-16} e \cdot cm$, which by far exceeds the experimental limit unless $\vartheta$ is extremely small, the notorious strong-CP problem. It is then natural to assume that $\vartheta$ is zero or nearly vanishes due to a symmetry or a dynamic mechanism \[2\].

The second source of CP violation lies in the electroweak sector of the SM and is expressed through the irreducible phase in the CKM matrix $V_{jk}$ describing the flavor-nondiagonal weak transition amplitudes between quarks:

$$
\frac{G_F}{\sqrt{2}} C^w = \frac{G_F}{\sqrt{2}} J^\mu J^\mu \text{ with } J^\mu = \sum_{j,k=1}^3 V_{jk} \bar{u}_j \Gamma^\mu d_k, \quad \Gamma^\mu = \gamma^\mu (1 - \gamma^5). \tag{1}
$$

As a result, it primarily manifests itself as CP violation in decays of kaons, $B$ and, potentially, $D$-mesons. The CKM parameterization of flavor dynamics in the SM has been successfully tested in detail over the last ten years in $K$ and $B$ mesons, in particular through its CP violating thread.

Being rooted in the intricacies of flavor-changing transitions, the CKM-CP violation leads to extremely small effects in flavor-diagonal amplitudes such as EDMs; therefore EDMs in general and the neutron EDM $d_n$ in particular are a very sensitive probe of the underlying source of CP violation. Yet calculating the observable effect for hadrons in terms of the fundamental parameters of the SM is often a difficult task requiring control of the complicated hadronic dynamics in the grossly nonperturbative regime.

Following the success of the quark-gluon picture of the hadronic world and of the qualitative understanding of the properties of hadrons based on the constituent quark model, the early estimates aimed at calculating the EDMs of quarks as the source of the nucleon EDM. It turns out that for quarks the SM prediction is particularly suppressed: it emerges first at the three-loop level where an additional loop with at least a gluon must be included \[3\]. On top of this, the quark EDM has to be proportional to the quark mass; this yields an additional suppression for the light quarks. The same applies to the color dipole moments of quarks considered as the simplest induced CP-odd strong force generated through weak interactions at small distances.

It has been noted a while ago \[4\] that the severe chiral suppression intrinsic to $u$ and $d$ quarks can be vitiated in composite hadronic systems like nucleons. The transition dipole moments changing $d$-quark into $s$-quark, electromagnetic or color, are suppressed by the strange quark mass $m_s$, and such flavor-changing transition without a quark charge change are mediated by the loop-induced short-distance renormalization of the bare weak interaction at some level via the so-called Penguin diagrams \[5\]. A similar mechanism is believed to dominate the nucleon EDMs in spite of the suppression associated with the heavy quark loops, although it is notoriously difficult to account for the long-range part of the strong interactions it depends upon \[6\].
In this Letter we point out that the SM in fact generates the nucleon EDMs already without any loop corrections, at the tree level of the second-order weak interaction Lagrangian. This contribution has no chiral suppression neither vanishes if strange and down quarks become nearly degenerate in mass. Being loop-free, the induced EDM does not suffer from typically small numerically perturbative factors characteristic to Penguin-induced effects in spite of the accompanying logarithmic factors.

The price to pay is that the effect is inversely proportional to the square of the charm quark mass. Since the latter is not excessively large in the hadronic mass scale, the related suppression comes out relatively mild and this effect may well dominate \( d_n \) in the SM. It is described by multiquark effective operators involving simultaneously \( u, d \) and \( s \) quark fields.

The numeric estimate of \( d_n \) mediated in this way is presently rather uncertain depending on the corresponding nucleon matrix elements. We do not even speculate on its sign. Applying the natural counting rules we expect \( |d_n| \) between \( 10^{-31} \) and \( 10^{-30} \, \text{e} \cdot \text{cm} \). At the same time we expect that definite estimates can be elaborated in the future owing to the special form the effective CP-odd operators assume in the CKM case.

Since the contribution we focus on does not require loop effects, in the following we neglect the well known short-distance renormalization of the weak interaction Lagrangians altogether. This simplification makes the reasoning transparent and all the expressions compact. Including the gluon-mediated short-distance effects is straightforward, and we briefly comment on this.

Let us note that the similar ideas regarding the role of long-distance effects in \( d_n \) were discussed in the 1980s by I. Khriplovich with collaborators [7] as an extension of the original proposal by Gavela et al. [4], and then in the 1990s by I. Khriplovich and A. Vainshtein. Unfortunately, the latter analysis was not published and its main message remains largely unknown. Reportedly, the authors focused on developing a consistent low-energy effective theory for the corresponding CP-odd effects, and therefore the loop renormalization occupied an important place in the analysis. We are grateful to A. Vainshtein for the information and for useful comments.

1 Effective low-energy Lagrangian

Through the neutron EDM we are looking into the CP-odd amplitudes that are flavor-diagonal in every quark flavor. We assume that the renormalized effective \( \vartheta \)-term vanishes exactly and the only source of CP nonconservation is the general CKM \( 3 \times 3 \) unitary mixing matrix \( V_{ij} \) describing the interaction of \( W \) bosons with quarks. Any first-order term in the conventional four-fermion weak interaction Lagrangian with \( \Delta F = 0 \) is automatically CP-invariant due to its Hermiticity. The observable CP-odd effects appear from the second order in \( \mathcal{L}_w \) and thus are proportional to \( G_F^2 \), being embodied in

\[
\mathcal{L}_2 = \frac{G_F^2}{2} \int d^4x \, \frac{1}{2} i T \{ \mathcal{L}_w(x) \mathcal{L}_w(0) \}. \tag{2}
\]
The generalized GIM-CKM mechanism ensures that the CP-odd piece of \( \mathcal{L}_2 \) is finite in the local four-fermion approximation.

Descending to a low normalization point we first integrate out top quark and at the second stage, below \( m_b \) the bottom quark as well. At the tree level integrating them out essentially means that all the terms containing \( t \) or \( b \) fields are crossed out of \( \mathcal{L}_w \).

Within this setup we have effectively nearly a two-family weak Lagrangian

\[
\mathcal{L}_w = J_\mu^I J^\mu_I \quad \text{with} \quad J_\mu = V_{cs} \bar{c} \Gamma_\mu s + V_{cd} \bar{c} \Gamma_\mu d + V_{us} \bar{u} \Gamma_\mu s + V_{ud} \bar{u} \Gamma_\mu d ,
\]

yet not quite so, since the four \( V_{kl} \) do not form a unitary matrix; in particular, it is not CP-invariant. The phases in the four CKM couplings cannot all be removed simultaneously by a redefinition of the four quark fields, as quantified by

\[
\Delta \equiv \text{Im} U_4 = \text{Im} V^*_c V^*_d V^*_u V^*_s .
\]

\( \mathcal{L}_w \) above contains altogether 16 terms bilinear in \( V \) and \( V^* \). The product \( \mathcal{L}_2 \) of two \( \mathcal{L}_w \) in Eq. (2) already consists of 256 terms. Most of them change quark flavors – only 64 are flavor-diagonal. Of these most still contain the two pairs of complex conjugate CKM factors and therefore are automatically CP-even. Hence a handful of terms are relevant to flavor-diagonal CP: only two different operators are not CP-invariant driven by the same CKM product \( U_4 \), plus their Hermitian conjugated partners. These non-local 8-quark operators include both \( q \) and \( \bar{q} \) fields for each of the four quark flavors. This is readily understood: the CP-odd invariant \( \Delta \) (as well as CP-violation altogether) vanishes wherever any single CKM matrix element becomes zero.

Being interested in the nucleon amplitudes, we need to eventually integrate out the charm field as well. Here the distinction between the above two terms becomes important. If the two charm fields belong to the same four-fermion vertex in the product Eq. (2) as in Fig. 1a, they can be contracted into the short-distance loop yielding, for instance, the usual perturbative Penguins. These are the conventional source of the long-distance CP-odd effects [4, 7]. The loop cannot be formed for the alternative possibility where \( c \) and \( \bar{c} \) belong to different \( \mathcal{L}_w \), Fig. 1b, since the charm quark must propagate between the two vertices. Such contributions therefore are routinely discarded.

On the contrary, our interest lies in the latter term: it does not involve short-distance loops, and has a single charm propagator, although highly virtual in the hadronic scale. Each weak vertex contains a flavorless quark-antiquark pair, but these are light down-type quarks \( d \) and \( s \) and are not contracted via a perturbative loop, instead going into the nucleon wavefunction. The corresponding operator is

\[
\frac{G_F^2}{2} V_{cs} V_{cd} V_{ud} V^*_s \int d^4x \ i T\{ (\bar{d} \Gamma_\mu c) (\bar{u} \Gamma_\mu d) (\bar{c} \Gamma_\mu s) (\bar{s} \Gamma_\mu u) \} \ + \text{H.c.} \quad (5)
\]

The Hermitian conjugate, apart from complex conjugation of the CKM product, is simply the exchange between \( s \) and \( d \), \( s \leftrightarrow d \).

\[\text{This general reasoning does not depend on the tree level assumption: we can exclude } V_{tj} \text{ and } V_{ib} \text{ from the pair products using the CKM unitarity.}\]
Figure 1: Two types of CP-odd terms. Weak vertices must be off-diagonal in flavor, either for down-type (a) or up-type (b) quark. Solid dots denote the four-quark vertices. Light lines correspond to $u$, $d$ or $s$ quarks, thicker lines stand for charm.

As the space separation $x$ in Eq. (5) is of order $1/m_c$, eliminating charm results in a local OPE; the expansion parameter $\mu_{\text{hadr}}/m_c$ is not too small and we need to keep a few first terms. The tree-level OPE is particularly simple here and amounts to the series

$$c(0)c(x) = \left( \frac{1}{m_c-i\not{D}} \right) = \frac{1}{m_c}\delta^4(x) + \frac{1}{m_c^2}\delta^4(x) \not{D} + \frac{1}{m_c^3}\delta^4(x) \not{D}^2 + \ldots$$

(6)

valid under the $T$-product. For purely left-handed weak currents in the SM the odd powers of $1/m_c$ in Eq. (6) are projected out, including the leading $1/m_c$ piece. We then retain only the $1/m_c^2$ term and obtain the local effective CP-odd Lagrangian

$$\tilde{L}_- = -i \frac{G_F^2}{2m_c^2} \Delta (\tilde{O}_{uds} - \tilde{O}_{uds}^\dagger),$$

(7)

$$\tilde{O}_{uds} = (\bar{u}\gamma^\mu(1-\gamma_5)u)(\bar{s}\gamma^\nu\gamma^\alpha\gamma^\beta(1-\gamma_5)d)(\bar{d}\gamma_\nu(1-\gamma_5)u) + \text{c.c.};$$

in the last expression the covariant derivative acts only on the $s$-quark field immediately following it.

To address the electric dipole moments we need to incorporate the electromagnetic interaction. One photon source lies in the covariant derivative in the operator $\tilde{O}_{uds}$, which includes electromagnetic potential along with the gluon gauge field. It is proportional to the up-type quark electric charge $+\frac{2}{3}$. The corresponding photon vertex is local and is given by the Lorentz-vector six-quark operator which we denote as $O^\alpha_{uds}$:

$$O^\alpha_{uds} = (\bar{u}\gamma^\mu(1-\gamma_5)s)(\bar{s}\gamma^\nu\gamma^\alpha\gamma^\beta(1-\gamma_5)d)(\bar{d}\gamma_\nu(1-\gamma_5)u).$$

(8)

Another, non-local contribution is the $T$-product of the pure QCD part of $\tilde{O}_{uds}$

$$O_{uds} = (\bar{u}\gamma^\mu(1-\gamma_5)s)(\bar{s}\gamma^\nu\gamma^\alpha\gamma^\beta(1-\gamma_5)d)(\bar{d}\gamma_\nu(1-\gamma_5)u)$$

(9)

with the light-quark electromagnetic current.\footnote{In general only the sum of the two terms yields the transverse electromagnetic vertex; however, when projected on the dipole moment Lorentz structures they separately conserve current.} The total photon vertex is thus given by
the effective CP-odd Lagrangian

\[ A_\alpha \mathcal{L}_\alpha^\alpha = -e i \frac{G_F^2}{m_e^2} A_\alpha \left[ \frac{2}{3} O_{uds}^\alpha + \int d^4 x \: i T \{ O_{uds}(0) \: J^\alpha_{em}(x) \} - \text{H.c.} \right], \quad J^\mu_{em} = \sum_q e_q \bar{q} \gamma^\mu q, \]

(10)

where \( A_\mu \) is the electromagnetic potential and \( e \) is the unit charge.

In principle, the local and non-local pieces above correspond to distinct physics: one has photon emitted from distances of order \( 1/m_e \) while the latter senses charge distribution over the \( 1/\mu_{\text{hadr}} \) range. The latter usually dominates, however the specifics of the left-handed weak interactions in the SM makes them of the same \( 1/m_e^2 \) order.

An interesting feature of the considered contribution is that it remains finite in the chiral limit and it does not vanish if \( d \) and \( s \) quarks become nearly degenerate, at first glance contradicting the origin of the KM mechanism. This in fact is fully consistent, since the external state, the neutron, is explicitly \( s \leftrightarrow d \) non-symmetric. This highlights the difference with the short-distance effects for light quarks where severe GIM-type suppression would arise and the EDM is proportional to the powers of the light quark masses.

The CP-odd operators contain strange quark fields. This means that the induced effects would vanish in a valence approximation to nucleon where only \( d \) and \( u \) quarks are active. It is known, however, that even at low normalization point the strange sea in nucleon is only moderately suppressed. The large-\( N_c \) perspective on the nucleons paralleling the picture of the baryon as a quantized soliton of the pseudogoldstone meson field makes this explicit: the weight of the operators with strange quarks in the chiral limit is generally determined simply by the operator-specific Clebsh-Gordan coefficients of the \( SU(3) \) group. This differs from the perturbative Penguin effects which yield small coefficients whenever considered in the truly short-distance regime. The neutron dipole moment induced by the operator in Eq. (10) does not need to vanish even in the quenched approximation to QCD.

### 2 Matrix elements

The CP-odd operators \( O_{uds}^\alpha - O_{uds}^{\alpha \dagger} \) and \( O_{uds} - O_{uds}^{\dagger} \) have high dimension which is routinely regarded as an evidence for being poorly defined for practical applications. However, these particular operators possess very special symmetry properties, including antisymmetry in respect to \( s \leftrightarrow d \), which prohibit mixing with lower-dimension operators, and make them a suitable object for the full-fledged nonperturbative analysis.

The neutron EDM is obtained by evaluating the hadronic operator in Eq. (10) over the neutron state. Since \( \mathcal{L}_\alpha^\alpha \) is T-odd, the matrix element vanishes for zero momentum transfer and the linear in \( q \) term describes \( d_n \):

\[ \langle n(p+q) | \mathcal{L}_\mu^\mu | n(p) \rangle = -d_n \: q_\nu \bar{u}_n(p+q) \sigma^{\mu\nu} \gamma_5 u_n(p). \]

(11)

\[^3\text{It would vanish if charm and top become degenerate; considering the cases of degenerate bottom and strange quarks, or charm and up makes no sense in this context since it has been assumed as the starting point that } m_b, m_c \gg \mu_{\text{hadr}} \text{ while } u, d \text{ and } s \text{ are light quarks.}\]
Neither of the matrix elements are easy to evaluate, although one may hope that just this particular contribution may eventually be determined without major ambiguity, including the definitive prediction for the overall sign. Clearly only the $P$-violating part of $O_{uds}^{(\alpha)}$ contributes, but we keep them in the original form for the sake of explicit symmetry and compactness.

The contact operator $O_{uds}^{\alpha}$ is a product of three left-handed flavor currents; $O_{uds}$ instead of the $\bar{s}d$ current has a flavor non-diagonal left-handed partner of the quark energy-momentum tensor in the chiral limit. Therefore it seems plausible that the required matrix elements can be directly calculated within the frameworks like the Skyrme model \[9, 8\], or in its dynamic QCD counterpart \[10\] derived in the large-$N_c$ limit from the instanton liquid approximation. This is the subject of the ongoing study.

In the absence of better substantiated calculations we apply the simple dimensional estimates to assess the expected size of $d_n$. We denote
\[
\langle n(p+q) | \bar{u} L^\alpha s L \rangle (\bar{s} L^\gamma u L) - (d \leftrightarrow s) | n(p) \rangle = -2iK_{uds} q \bar{u}(p+q)\sigma^{\mu\nu}\gamma_5 u(p)
\]
for the local piece given by the operator $O_{uds}^{\mu}$. The reduced matrix element $K_{uds}$ has dimension of mass to the fifth power. We estimate it as
\[
|K_{uds}| \approx \kappa \mu_{hadr}^5,
\]
where $\mu_{hadr}$ is a typical hadronic momentum scale and $\kappa$ stands for the ‘strangeness suppression’ to account for the fact that neutron has no valence strange quarks; $\kappa \approx 1/3$ is taken as a typical guess.

The estimate for $d_n$ depends dramatically on the assumed value of $\mu_{hadr}$. It is known that the typical momentum of quarks in nucleon is around 600 MeV or higher. Yet six powers of mass in Eq. \([13\) would come from the product of two local light quark currents each intrinsically containing factors $N_c/8\pi^2$ when converted into the conventional momentum representation. This is illustrated by the magnitude of the vacuum quark condensate where such a factor effectively reduces $\mu_{hadr}^3$ down to $\sim (250 \text{ MeV})^3$.

To account for this essential difference we assign a factor of $(0.25 \text{ GeV})^3 \equiv \mu_\psi^3$ to each additional quark current in the product, while the remaining dimension will be made of the powers of $\mu_{hadr}$. Then this contribution to $d_n$ becomes
\[
|d_n| = \frac{32}{3} e \Delta \frac{G_F^2}{m_c^2} |K_{uds}| \approx 3.3 \cdot 10^{-31} e \cdot \text{cm} \times \kappa \left(\frac{\mu_\psi}{0.25 \text{ GeV}}\right)^6 \left(\frac{0.5 \text{ GeV}}{\mu_{hadr}}\right),
\]
where $\Delta \approx 3.4 \cdot 10^{-5}$ has been used. A potential enhancement may come from summation over the Lorentz indices in the currents.

The most naive estimates for the $d_n$ induced by the non-local piece in Eq. \([10\) would yield a similar dimensional scaling except that no explicit factor $e_c = 2/3$ appears: the dimension of the non-local $T$-product is the same as of $O_{uds}^{\mu}$ itself. Within the more careful way advocated above the result is literally different:
\[
|d_n|^{\text{non-loc}} \approx e \Delta \frac{G_F^2}{m_c^2} 32\kappa \mu_\psi \mu_{hadr}^9 \approx 1.2 \cdot 10^{-31} e \cdot \text{cm} \times \kappa \left(\frac{\mu_\psi}{0.25 \text{ GeV}}\right)^9 \left(\frac{0.5 \text{ GeV}}{\mu_{hadr}}\right)^4,
\]
although numerically is not too far away.

Alternatively, to account for the specifics of the correlator with the electromagnetic current in the non-local contribution we can consider the contribution of the lowest resonant state, the $\frac{1}{2}^-$ nucleon resonance $N(1535)$ referred to below as $\tilde{N}$. Denoting

$$\langle n(p')|J_{em}^\mu(0)|\tilde{N}(p)\rangle = \rho_{\tilde{N}}\bar{u}_n\gamma_5\sigma^{\mu\nu}q_\nu u_{\tilde{N}}, \quad \langle \tilde{N}(p')|O_{uds}(0)-O_{uds}(0)^{\dagger}|n(p)\rangle = 16iN_{uds}\bar{u}_n\tilde{N}u_n$$

we obtain for the Feynman diagrams in Fig. 2

$$d_n^{(\tilde{N})} = -e\Delta\frac{32G_F^2}{m_c^2}\left(\frac{\rho_{\tilde{N}}N_{uds}}{M_{\tilde{N}}-M_N}\right).$$

(17)

![Figure 2: Nonlocal contribution to $d_n$ with the intermediate $\tilde{N}$. Solid block denotes the CP-odd part of the operator $O_{uds}$.](image)

The operator $O_{uds}$ has dimension ten, and the similar dimensional estimate reads

$$|N_{uds}| \approx \kappa \mu_\psi^6 \mu_{\text{hadr}}.$$  

(18)

$\rho_{\tilde{N}}$ can be estimated from the measured transition $\tilde{N} \to n + \gamma$ approximating the rate with the dipole expression

$$\Gamma(\tilde{N} \to n\gamma) \approx \frac{1}{2}\alpha_{em}\rho_{\tilde{N}}^2M_{\tilde{N}}^3\left(1-\frac{M_n^2}{M_{\tilde{N}}^2}\right)^3 \approx (380 \pm 180) \text{ keV}$$

(19)

yielding $\rho_{\tilde{N}} \approx (0.34 \pm 0.08) \text{ GeV}^{-1}$. Finally this estimate would read

$$|d_n^{(\tilde{N})}| \approx e\Delta\frac{32G_F^2}{m_c^2}\kappa \mu_\psi^6 \mu_{\text{hadr}} \rho_{\tilde{N}}M_{\tilde{N}}-M_n \approx 1.4 \times 10^{-31} \text{ e}\cdot\text{cm} \times \kappa \left(\frac{\mu_\psi}{0.25 \text{ GeV}}\right)^6 \left(\frac{\mu_{\text{hadr}}}{0.5 \text{ GeV}}\right).$$

(20)

This value appears quite consistent with the direct dimensional estimate of the non-local contribution, in particular considering the fact that the lowest excited state alone may not necessary saturate it.

Therefore, our estimate for $d_n$ in the SM literally centers around $10^{-31} \text{ e}\cdot\text{cm}$ although even the values 5 to 10 times larger may not be excluded.
3 Discussions and conclusions

So far we have neglected short-distance loop effects. They can be included in the standard way. Importantly, the gluon corrections do not change the symmetry properties and therefore cannot radically modify the structure of the result.

The evident impact is renormalization of the individual $\mathcal{L}_w$ modifying the strength and inducing different color flow. This generates different flavor allocation between the brackets in the operators $O_{uds}$ and $O^\alpha_{uds}$, yet does not change their properties. In particular, the overall antisymmetry with respect to $s$ and $d$ is never modified. Qualitative difference neither arises from the Penguin-induced operators with right-handed currents. Most of these changes affect the conventional contributions belonging to Fig. 1a.

It turns out the situation neither changes where an external magnetic photon or gluon attaches to the $\bar{Q}Q$ loop from a single $\mathcal{L}_w$: the sum of all the diagrams vanishes or is proportional to the external light quark field mass. In any case such renormalization emerges only at the two-loop level and apparently is too suppressed numerically.

As a result, the short-distance loops should not radically affect $d_n$ obtained already at the tree level, unless the latter happens to suffer from accidental numeric cancellations. All the potential corrections have counterparts in the gluon-induced renormalization of the operators $O_{uds}$ and $O^\alpha_{uds}$ themselves. Therefore we may a priori expect a moderate overall enhancement of $d_n$ due to increased size of $c_-$ down from the $W$-boson scale; the final conclusion would require evaluating the matrix elements accounting, in particular, for the alternative color contractions in the operators.

To summarize, we have described the contribution to the EDM of nucleons proportional to $\Delta G_2^2 \mu^5_{\text{had}/mc^2}$ that does not require loop effects associated with heavy flavors or short-distance gluon corrections. It is finite in the chiral limit for light quarks and would not vanish even if $m_s$ and $m_d$ were very close. Free from typically small perturbative loop factors it plausibly dominates $d_n$ in the SM, yet it is interesting regardless of the precise magnitude.

Within the effective theory describing the SM below the bottom quark scale, charm quark must necessarily appear for the CP-violation to show up. Therefore, if we avoid perturbative charm Penguin loops, they are replaced by the terms suppressed by powers of $1/m_c$. We had encountered similar nonperturbative charm effects dubbed ‘Intrinsic charm’ in the inclusive beauty decays; they were distinct, yet not too significant and mattered only as long as high precision was aimed at.

The situation is different for $d_n$ and evidently in an important aspect: here we deal with ‘tree’ nonperturbative charm and it is suppressed by powers of $1/m_c$ vs. powers of $1/(2m_c)$ for charm Penguins. Moreover, the absence of the loop factor here apparently leads to a notably milder suppression. For instance, no $\alpha_s(2m_c)$ enters as a prefactor.

The left-handed structure of the charged currents in the SM yields the second power of the charm mass suppression and, when considering the photon interaction, the local and the non-local contribution of the same order. In this situation the local piece possibly dominates; this can be verified having more elaborate estimates of the nucleon matrix elements. Although rather uncertain at the moment, we believe the proposed contribution
to the \( d_n \) in the SM can eventually be evaluated without a too significant uncertainty, owing to a rather peculiar form of the effective CP-odd operator.

The chiral properties of the SM forbid the effective CP-violating scalar pion-to-nucleon coupling at small momenta, which is often important for the atomic EDMs. This applies as well to the terms power-suppressed in \( 1/m_c \).

Since any CKM-generated contribution to \( d_n \) must be proportional to \( \Delta G_F^2 \), the considered effect is not too far from the top benchmark \( \Delta G_F^2 \mu^2_{\text{had}} \): it is moderated by \( 1/m_c^2 \), and may be somewhat numerically suppressed through explicitly containing the strange quark field, a feature likewise unavoidable for CKM. It also does not generate chiral enhancement \( \ln \mu^2_{\text{had}}/m_c^2 \) which would be possible with the right-handed currents. Therefore we conclude that \( d_n \) in the SM is moderately suppressed compared to the intrinsic smallness of the flavor-diagonal CP violation built in the CKM ansatz.

The discussed mechanism generating \( d_n \) roots in the CP-odd interference of the \( \Delta C = 1, \Delta S = 0 \) amplitudes in the usual weak decay Lagrangian. This is precisely what induces the mixing-free ‘direct’ CP asymmetry in \( D \) decays. Therefore, New Physics (NP) phenomena in the latter would directly affect \( d_n \) at the stated level \([12]\). It turns out that, barring contrived cancellations, the NP \( d_n \) enhancement is greater than its effect on the \( D \)-decay asymmetries. This indicates that, in a certain sense, the particular SM implementation of the CKM ansatz nearly minimizes the intrinsic size of \( d_n \).

The increase in the EDM depends on the details of the underlying NP interaction. For instance, the amplitudes with the right-handed charm field lead to \( 1/m_c \) scaling of \( d_n \), and the right-handed light-quark currents tend to generate an overall enhancement. The latter also induce the CP-odd \( \pi NN \) coupling. Taking the reported asymmetry in \( D \to K^+K^- \) and \( D \to \pi^+\pi^- \) at face value \([13]\) we typically find an enhancement in \( d_n \) by a factor of 10 to 100, with the case where the NP amplitude is described by \( \bar{c}\sigma_{\mu\nu}F^{\mu\nu}\gamma_5 u \) alone leading to \( d_n \) up to \( 5 \cdot 10^{-27} \text{e}\cdot\text{cm} \). Such an extreme possibility does not look too natural from the perspective of speculating about the underlying flavor theory, though.

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