Calibration of a 3-D Digital Image Correlation system for large deformation contact problems

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Abstract. In this work, the non-contact full-field Digital Image Correlation (DIC) technique has been utilized for measuring and analyzing the contact behaviour of soft materials undergoing large deformation. A vulcanized silicone rubber in contact with a wedge-shaped rigid indenter was investigated. In order to provide confidence in the measured results from the DIC system, an in-plane strain calibration procedure was conducted. Further, a procedure of out-of-plane displacement calibration was also deduced basing on the law of propagation of uncertainty.

1. Introduction
Over the past century, contact problems have occupied an important place in Solid Mechanics[1,2]. A practical application is a rigid indenter pressed onto an elastic body. It is difficult to get accurate analytical solutions for this kind of contact problem, so asymptotic methods were commonly implemented in theoretical analysis [3]. Since an indenter with blunted apex is more applicable to real problems, Korsunsky illustrated the influence of small variations in punch shape on the contact behaviour for a blunted Hertzian indenter and a rounded cone [4]. Then Hills and his co-workers have addressed a series of contact problem with various profiles [5]. It can be seen that the previous methods are useful to resolve some engineering problems, however, it should be noted that many of the following assumptions were employed in majority of the solutions for contact problems: the strains are small and within the elastic limit; each body were considered as elastic half-plane; there are no shearing tractions present, which requires the surfaces to be frictionless; the contacting bodies are elastically similar; the external angle of the wedge or cone must be very small.

The focus of this study is to remove the idealization of previous researchers, and investigate contact properties of a soft material indented by a rigid wedge in real conditions. Contact mechanics of soft materials is rather complex due to the geometrical and material nonlinearities and changing boundary conditions. Relatively little experimental work has been carried out so far because it is difficult to measure contact behavior by traditional techniques such as strain gauges or photo-elasticity [6,7]. Therefore, in this paper, the non-contact full-field Digital Image Correlation (DIC) technique has been utilized for measuring contact behavior of soft materials undergoing large deformation. Specifically, a vulcanized silicone rubber in contact with a rounded-tip rigid wedge was investigated.

2. Experimental setup and procedure
The specimen used in the experiment was cured from a Room Temperature Vulcanization (RTV) Rubber. It was cast into a rectangular block with dimensions 60mm×60mm×30mm. A very thin coat of quick-drying white paint (Matt Super White 1107, Plasti-kote, UK) was sprayed onto the surface using an aerosol can and then a fine coating of black speckle (Matt Super Black 1102, Plasti-kote, UK) was sprayed on top of the white coating. During the DIC experiment, the surface deformation of specimen can be deduced from the randomly distributed small paint dots.

The wedge indenter was manufactured using 2024 aluminum with a tip radius $R=1.68\text{mm}$ and external angle $\theta=73.45^\circ$. It had the same thickness as the rubber specimen (30mm). The specimen was not pre-stressed before loading and placed in the centre of an aluminum disc which was mounted on the bottom grip of an Instron “Electropuls” machine, while the indenter was mounted in the top grip of the machine. Afterwards, the loading system and DIC system (Q-400, Dantec Dynamics GmbH, Germany) were triggered simultaneously. The test machine control software (Bluehill, Instron, UK) was set to apply the load at a constant rate of 1 mm/min up to the maximum load of 34.5N, and the DIC software (ISTRAS 4D) was programmed to record specimen images every 30 seconds.

After the experimental measurements, data processing was performed using the DIC software. In order to save data-processing time and get better results near the contact interface, a small rectangular area around the contact interface (mask area) was analyzed. It was found that the best results could be obtained with a facet size of 25 pixels and grid spacing of 3 pixels. A close-up of the evaluated information near contact area is shown in Figure 1.

\[\text{Figure 1: Displacement field in the loading direction evaluated by DIC system, overlapping with experimental image, which was captured by the left camera. For interpretation of the references in colour in this and all other figures, the reader is referred to the electronic version of this paper.}\]

3. Calibration of DIC system

3.1. In-plane strain measurement calibration

According to the procedure described in the SPOTS\(^3\) guideline, a successful calibration of a strain measurement system requires a known strain field to compare with the results generated by the measurement system. This can be achieved by using a Reference Material (RM). For an in-plane strain measurement calibration, the Reference Material is designed as a central horizontal beam loaded in symmetric four-point bending via an outer loading frame (shown in Figure 2). The RM used in this paper was manufactured from a single piece of 5 mm thick 2024-T351 Aluminum. The beam depth ($W$) in the loading direction was 15mm; the distance between the loading points ($a$) was 45mm; and the distance between inner loading point and beam centre ($c$) was 15mm. The specifications and methodologies for its use in calibration can be found in the SPOTS guideline [8].

According to SPOTS guideline [8], there are three sources of total measurement uncertainty of a system. The first uncertainty primarily comes from the manufacture of Reference Material (RM), including material property uncertainty and geometric uncertainty. These values can be found by repeating the measurement enough times to get a good estimate of the standard deviation of the values.

\(^3\) SPOTS: Standardization Project for Optical Techniques of Strain measurement, www.opticalstrain.org
The evaluated poisson’s ratio of RM was $\nu = 0.33$ and its corresponding uncertainty was $u(\nu) = 0.005$ in this study, while the geometric uncertainties were $u(W) = 0.017$, $u(a) = 0.017$ and $u(c) = 0.017$ respectively.

![Figure 2: Schematics showing the central horizontal beam of RM. The axis system is located at the beam’s geometric centre.](image)

The second uncertainty primarily comes from the optical instrument, which means the DIC system in this study. For all data points $(i, j)$ within the gauge area of Reference Material, this value can be found from the evaluation of mean square residual deviation after the linear fit of $d_k(i, j)$, which is the map of the difference between the predicted values and the measured values of strain. For each increment of load step $k$, the measurement uncertainty is:

$$u^2(d_k) = \frac{1}{N} \sum_{i,j} [d_k(i, j)]^2 - \alpha_k^2 - \beta_k^2 \frac{1}{N} \sum_{i,j} y_j^2$$  \hspace{1cm} (1)

where $\alpha_k$ and $\beta_k$ are the fit parameters for a linear least-squares fit of the deviation $d_k(i, j)$.

The third source of uncertainty is the displacement measurement and the constraint imposed by the attachment to the monolithic frame. In standard practice, the applied displacement should include at least 3 increments over a range between 10% and 90% of the maximum allowable load. In this study, the average of the measurement values in three displacement steps were $v_1 = 112 \, \mu m$, $v_2 = 152 \, \mu m$ and $v_3 = 214.5 \, \mu m$ respectively. Their associated uncertainty was provided by the calibration certificates purchased with digital displacement indicators (Mitutoyo 543-392, Kawasaki-shi, Japan), which was $u(v) = 0.89 \, \mu m$. The strain expressions for an ideal beam should be modified due to the constraint imposed by whiffle-trees which support the beam, so the more accurate expression is:

$$\varepsilon_{xx} = \frac{\bar{v}}{6W^2} (\kappa y + \eta), \quad \varepsilon_{yy} = -\frac{u\bar{v}}{6W^2} (\kappa y + \eta) \quad \text{and} \quad \varepsilon_{xy} = 0$$  \hspace{1cm} (2)

The correction factors $\kappa$ and $\eta$ and the corresponding uncertainties were determined using a pair of strain gauges on the top and bottom surfaces of the beam. The values obtained in this study were $\kappa = 0.986$, $\eta = -0.068 \, \mu m$, and their associated uncertainty values were $u(\kappa) = 0.006 \, \mu m$, $u(\eta) = 0.006 \, \mu m$.

After all of the uncertainty values mentioned above were determined, the combined uncertainty of the RM for each displacement step were calculated using the following equation:

$$u^2_{\text{RM}}(\varepsilon_{xx}) = (\varepsilon_{xx})^2 \left( \frac{u^2(v)}{v^2} + \frac{u^2(\kappa)}{K^2} + \frac{16u^2(W) + u^2(a) + u^2(c)}{4W^2} \right) + \left( \frac{v}{6W^2} \right)^2 u^2(\eta)$$  \hspace{1cm} (3)

These values were used to obtain the expanded uncertainty in the RM, $U_{\text{RM}} = 2u_{\text{RM}}$. Finally, for each of the three displacement load steps, the mean residual deviations $(\alpha_k + \beta_k y) - 2u(d_k)$ were plotted along with the expanded uncertainty of the calibration specimen, $\pm U_{\text{RM}}$ in Figure 3.
In an ideal system, the measured strain value should be the same as the predicted one, that is, the deviation \( \Delta y_{i,j} = \alpha_k + \beta_k y \) should be zero and no adjustments would be necessary. It can be observed from Figure 3 that, for every load step, the scatter band \( \pm 2u(d_k) \) on \( \alpha_k + \beta_k y \) and the area bounded by \( \pm U_{RM} \) are completely overlapped, which means the measurement values coincide well with the predicted values, and \( \Delta y_{i,j} \) contains random noise only. In this case, no correction is needed for the DIC system.

### 3.2. Out-of-plane displacement calibration

A homogeneous isotropic steel cantilever beam was utilized as a Reference Material to calibrate the out-of-plane displacement. Although the simple cantilever was designed by ADVISE\(^4\) to calibrate full-field optical systems for transient and non-linear deformation cases, it can also be used for three-dimensional deformation fields induced by static loading. Specifications of the simple cantilever can be found in ADVISE deliverable D3.5 [9]. The design is fully parametric based on the cantilever thickness \( T \), which is 8mm in this study. Other dimensions can be expressed as length 20T and width 5T, the enlarged portion at one end was thickness 3T and length 10T, giving the cantilever an overall length of 30T. During calibration, the enlarged portion of the cantilever was clamped on a rigid, rigid.

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\(^4\) ADVISE: Advanced Dynamic Validation by Integrated Simulation and Experimentation, www.dynamicvalidation.org
immovable table and five displacement increments were applied to the tip of cantilever. One of the experimental images captured by DIC system is shown in Figure 4.

The tip deflection $\bar{v}$ was measured using a calibrated displacement transducer (the same one used in in-plane strain calibration). The deflection $\delta$ in the cantilever can be related to the tip deflection $\bar{v}$ using theory of elasticity:

$$\delta = \frac{\bar{v}x^2}{2a^3} (3a - x) \quad (0 \leq x \leq a)$$  \hspace{1cm} (4)

where $a$ is the length from the fixed end to loading point. By repeating the measurement twenty times, the average value of $a = 147.05 \text{mm}$ was determined and the associated uncertainty $u^2(a) = 0.483 \text{mm}^2$ was estimated from the standard deviation of the measured values. The expression for the combined uncertainty of the Reference Material deflection values, $u_{\text{RM}}$, can be deduced based on the law of propagation of uncertainty using the root-sum-of-squares (RSS) method [10]. It is given in the following form:

$$u_{\text{RM}}^2 = \delta^2\left(\frac{u^2(\bar{v})}{\bar{v}^2} + \left(\frac{a}{3a-x} - 1\right)^2 u^2(a)\frac{a^2}{a^2}\right)$$  \hspace{1cm} (5)

Three displacements steps were chosen for analysis, which were 428, 1280, and 2760 µm. Ideally the deflection at any section at a distance, $x$ from the cantilever end should be constant for a given applied load. The differences $d_k(i,j)$ between the predicted values and the measured values of deflection were calculated over the gauge area. Similar to in-plane strain measurement calibration, a linear least-squares fit was applied to the field of deviations $d_k(i,j)$, which yields fit parameters $\alpha_k$ and $\beta_k$ for each load step.

The uncertainty values obtained from the calibration experiment are summarized in Table 1. The resulting calibration uncertainty of the system $u_{\text{cal}}$ was found using the maximum displacement value from each of the load steps and the results from the combination of the uncertainties of the Reference Material and of the displacement measurements. For each of the three displacement load steps, the mean residual deviations $(\alpha_k + \beta_k y) - 2u(d_k)$ were plotted along with the expanded uncertainty of the calibration specimen, $\pm U_{\text{RM}}$ in Figure 5.

| $\bar{v}_k$ (µm) | Measurement Uncertainty | $u(d_k)$ | $u_{\text{RM}}$ | $u_{\text{cal}}$ | $w_{y=\text{max}} - w_{y=\text{min}}$ |
|----------------|------------------------|----------|----------------|----------------|-------------------------------|
| 428            | 0.89 µm                | 0.24     | 0.65           | 0.69           | 4.38                          |
| 1280           | 0.89 µm                | 0.39     | 0.64           | 0.75           | 1.82                          |
| 2760           | 0.89 µm                | 0.28     | 0.65           | 0.70           | -2.16                         |
\[ (\alpha_1 + \beta_1 y) + 2u(d_i) \]

\[ (\alpha_1 + \beta_1 y) - 2u(d_i) \]

\[ (\alpha_2 + \beta_2 y) + 2u(d_i) \]

\[ (\alpha_2 + \beta_2 y) - 2u(d_i) \]

\[ (\alpha_3 + \beta_3 y) + 2u(d_i) \]

\[ (\alpha_3 + \beta_3 y) - 2u(d_i) \]

Figure 5: Plots of expanded uncertainty of Reference Material, \( \pm U_{RM} \) (red solid lines) as a function of \( y \), the distance from the geometric centre of the cantilever in the direction of width together with the mean residual deviations \( (\alpha_k + \beta_k y) - 2u(d_i) \) (blue long dash lines) for each load step: 428 \( \mu m \) (top left), 1280 \( \mu m \) (top right), and 2760 \( \mu m \) (bottom). \( \pm 2u(d_i) \) indicates the width of the scatter band around the long dash double dot line \( (\alpha_k + \beta_k y) \).

For every load step, a complete overlap between the scatter band \( \pm 2u(d_i) \) on \( \alpha_k + \beta_k y \) and the area bounded by \( \pm U_{RM} \) can be observed in Figure 5, which indicates the measurement values coincide well with the predicted values, and \( d_i \) \((i, j)\) contains random noise only. In this case, the quality of the measured out-of-plane displacement made by the DIC system is satisfactory and no correction is needed.

4. Conclusion
The 3-D Digital Image Correlation technique has been applied to an investigation of contact problem in soft materials. Two calibration experiments demonstrate that the DIC system is appropriately calibrated, and the results obtained from the calibrated system were accurate and reliable. The experimental data will be used to compare with theoretical solutions in our future work.

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