Soliton microcombs in whispering gallery mode crystalline resonators with broadband intermode interactions

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Soliton microcombs have shown great potential in a variety of applications ranging from chip scale frequency metrology to optical communications and photonic data center, in which light coupling among cavity transverse modes, termed as intermode interactions, are long-existing and usually give rise to localized impacts on the soliton state. Of particular interest are whispering gallery mode based crystalline resonators, which with dense mode families, potentially feature interactions of all kind. While effects of narrow-band interactions such as spectral power spikes have been well recognized in crystalline resonators, effects of broadband interactions remains unexplored. Here, we demonstrate soliton microcombs with broadband intermode interactions, in home-developed magnesium fluoride microresonators with an intrinsic Q-factor approaching $10^{10}$. Soliton combs with featured spectral tailoring effect have been observed, which is found determined by dispersive effects of the coupled mode family. Remarkably, we demonstrate a broadband power enhanced soliton comb whose spectrum is beyond the standard soliton profile and the power efficiency is largely increased to 45%. Our results not only contribute to the understanding of dissipative soliton dynamics in multi-mode or coupled resonator systems, but also open an access to stable yet efficient soliton combs in crystalline microresonators where mode control and dispersion engineering are hardly performed. © 2022 Optica Publishing Group

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1. INTRODUCTION

Soliton microcombs based on optical microresonators have triggered the rapid development of miniature and chip scale optical frequency combs in recent years [1–4], and have resulted in the emergence of fully integrated frequency comb chips, opening an access to high-performance, high-compactness, and high-volume laser sources for advanced optical metrology [5–9]. Indeed, a number of proof-of-concept applications have been demonstrated with soliton microcombs, such as massive parallel optical communications [10, 11], optical ranging [12–14], massive parallel LIDAR [15], low-noise microwave synthesis [16–18], astronomical spectral calibration [19, 20], and optical neuromorphic computing [21, 22].

While recent focus is on photonic integrated platforms where wafer scale, high quality and highly nonlinear optical microresonators are accessible [23], a parallel platform is whispering gallery mode (WGM) based crystalline resonators [24, 25]. In particular, crystalline fluoride resonators could have a record-high finesse beyond $10^7$ [26], which is suitable for the generation of ultra-narrow linewidth lasers [27–30] as well as soliton microcombs[1, 11, 31–33]. Given a weak thermo-refractive noise, such resonators could support solitons with low-noise repetition frequencies, serving as a photonic microwave synthesizer [16, 17]. Moreover, on the study of soliton physics, they represent an ideal platform which is almost free from high-order dispersive or nonlinear effects. A number of dissipative soliton dynamics have been demonstrated with high-level of agreement with the theory, such as the soliton double resonance [34], soliton pulse scaling [35], soliton breathers [36], soliton molecules [37], soliton crystals [38] and soliton behaviors to intermode interactions [39].

Indeed, effects of intermode interactions have been widely observed and studied in soliton microcombs, which in most cases are reflected as localized power enhancement on certain comb modes [40]. These modes were also referred as single-mode dispersive waves, which would help to stabilize the soliton state if located in a particular “quiet point” [41]. In the strong coupling condition, energy exchange between mode families was also observed, which leads to instability on the soliton state, including...
Fig. 1. Whispering gallery mode fluoride resonators. (a) Microscopic pictures of a home-made MgF$_2$ microresonator, in which a fundamental whispering gallery mode is numerically calculated and illustrated in the normalized form. The absolute amplitude at the peak of the mode field is normalized to unity. The positions on the cavity edge where the mode field amplitude is 1% of the peak are marked, which reveals an axial range of ca. 32.8 $\mu$m covering most of the light power in the cavity. (b) A fractional 3D surface profile of the resonator, at a selected measure position on the edge. The colorbar indicates the radial height of the profile. (c) The residual of the surface after being fitted and subtracted with a polynomial surface function, which in the area of $75 \times 75 \mu$m$^2$ (where the fit shows a high level of approximation) is within $\pm 15$ nm, and in $32.8 \times 32.8 \mu$m$^2$ (corresponding to the 1% mode field boundary) is within $\pm 5$ nm. The overall root mean square of the extracted residual is 3.5 nm. (d) A schematic of the experimental setup. AFG: arbitrary function generator, EDFA: Erbium-doped fiber amplifier, OSC: oscilloscope, ESC: electrical analyzer, OSA: Optical spectrum analyzer. (e) A soliton comb spectrum generated in a MgF$_2$ microresonator via the intermode pumping scheme, which is beyond the standard sech$^2$-profile. The pump power is $\sim 190$ mW.

Recent studies have also shown that with an induced auxiliary mode, an intermode pumping wave may cause the thermal equilibrium in the cavity, and recover the access of soliton microcombs with reduced thermal instability [48–50].

Usually, such coupled mode or coupled resonator systems require a dedicated design to apply proper intermode interactions. In this regard, crystalline resonators with a high number of mode families would equally support a high volume of intermode interactions of different kind, and serve as an alternative platform in the study of emergent features of cavity solitons. Nevertheless, most reported effects of intermode interactions in crystalline resonators were narrow-band and involving few comb modes, and the effects of broadband interactions remains unexplored.

In this paper, we demonstrate soliton microcombs in crystalline magnesium fluoride (MgF$_2$) resonators with the intermode pumping scheme. Effects of broadband intermode interactions are experimentally evidenced, which would not only tailor the soliton comb spectrum, but also enhance the soliton power efficiency. We also demonstrate soliton combs in company with the Raman lasing, including both narrow-band lasers and Raman-Kerr combs [51–54], and a suspect Raman-soliton comb [55, 56] around the anti-stokes mode.

| Granule size (nm) | Time (min) | RMS Roughness (nm) | Q |
|-------------------|------------|--------------------|---|
| 1300              | 35-40      | 125-160            | $10^5$ |
| 250               | 20-30      | 12-20              | $10^8$ |
| 50                | 15-20      | 3-5                | $10^9$ |
Fig. 2. Cavity Q-factors and soliton comb generation. (a) The measured transmitted power trace of one resonance of a MgF₂ resonator, which shows a ring-down profile at the ending edge upon a laser tuning speed of 350 GHz/s. Fitting of this resonance reveals: \( Q_0 = 8.44 \times 10^9 \), \( Q_e = 1.1 \times 10^{10} \), and the loaded Q-factor is \( Q = 4.75 \times 10^9 \). (b) Extracted maxima and minima from the resonance trace in (a), as the function of time, which reveal a decreased linear proportion of the photon decay rate over time. The fitted photon life time (\( e^{-1} \) level) is 4.08 \( \mu \)s, corresponding to a loaded Q-factor of 4.95 \( \times 10^9 \). (c) The measured transmission of the same resonance in (a), upon the laser tuning speed of 2.5 GHz/s. The probe laser is phase modulated at a fixed frequency of 3 MHz, resulting in side-band signals as markers on the transmission trace. The fitted and estimated resonance linewidth by these markers is ca. 48 kHz, corresponding to the loaded Q-factor of 4.02 \( \times 10^9 \). (d) Assessment of the ideality of three resonant modes in the MgF₂ resonator. The plot shows the distribution of the central transmission of these resonances, upon tuning the external coupling rate. (e) The transmission of the resonance that is pumped and scanned by an intense cw laser, in which the stair-like pattern (soliton steps) indicates the formation of dissipative solitons in the cavity. (f) The measured frequency comb spectrum corresponding to the single soliton state in the MgF₂ resonator. The left inset shows the spectra of both an MI comb and a soliton comb, in which the central portion including the transmitted pumping wave is blocked by more than 20 dB by using an FBG. The right inset shows the low frequency rf spectrum of the filtered combs, indicating the noisy and the low-noise nature of the MI and the soliton comb states, respectively.

2. RESULTS

High-Q WGM crystalline resonators

First, we prepared ultra-high-Q crystalline MgF₂ resonators, by means of mechanical machining[57–59]. The preform of the cavity is diamond turned from a cylindrical bulk material, followed by surface polishing processes. Diamond slurries with three particle sizes (i.e. 1300 nm, 250 nm, and 50 nm) are used in the polishing in an order to improve the surface smoothness, see Tab. 1. During the polishing, the surface profile as well as the roughness is also monitored by a commercial optical interferometric profiler (Sensofar Metrology, system’s noise < 1 nm). Figure 1(a,b,c) showcase a measure of the cavity surface. In particular, by having the detailed geometric structure, the transverse mode field of cavity’s WGMs is numerically calculated, by which an axial range of care is defined to cover an area where the absolute mode field amplitude is greater than 1% of its central peak, see Fig. 1(a). The measured profile is also fitted by a 2D polynomial function, and the surface roughness is extracted as the residual of the fit, see. Fig. 1(b,c). Remarkably, the estimated RMS roughness can be as low as a few nano-meters, which indicates a high quality surface polishing and an ultra-high-Q factor (typ. > \( 10^9 \)) of the resonator. In practice, such a surface assessment is carried out at 18 positions along the circumference of the resonator (spaced by every 20 degree), indicating a variation of 5% on the cavity diameter (i.e. 0.25 mm over a diameter of ca. 5 mm). The curvature on the edge of the cross-section is ca. 35 \( \mu \)m.

The MgF₂ resonator is then coupled with a tapered fiber for the assessment of the Q-factor and for soliton microcomb generation, using an experimental setup shown in Fig. 1(d). The transmission of the cavity resonance is measured by scanning the cw diode laser over it. For ultra-high-Q resonators, the resonance linewidth is already comparable with that of the probe laser (\( \mathcal{O}(10 \text{kHz}) \)), and the laser frequency tuning within the linewidth could be faster than the intracavity photon lifetime.
This leaves a ring-down profile at the ending edge of the resonance, see Fig. 2(a), which is also theoretically derived in Ref [60]. In detail, the time evolution of the intracavity field of one resonant mode could be described as a a simple harmonic oscillator model, i.e.:

\[
\frac{\partial A}{\partial t} = i \omega_0 A - \frac{\kappa}{2} A + \sqrt{\kappa} s_{\text{in}} \]

where \( \omega_0 \) is the angular frequency of the resonance, \( \kappa \) is the overall loss rate of the system which consists of both the intrinsic loss rate \( \kappa_0 \) and the coupling loss rate \( \kappa_{\text{ex}} \), i.e. \( \kappa = \kappa_0 + \kappa_{\text{ex}} \). \( s_{\text{in}} \) stands for the external source, which in the frequency tunable cw mode can have the variation \( s_{\text{in}} = s_0 \exp(i \phi(t)) \). In the stationary form, i.e. the laser frequency tuning with respect to its initial value \( \omega_s \) is slowly changed compared with the intracavity photon lifetime, we have \( \phi(t) = \omega_s t \), and the solution of Eq. 1 gives a standard Lorentz profile as a function of the laser frequency. Hence, a variation of Eq. 1 could be:

\[
\frac{\partial a}{\partial t} = i (\delta \omega - V_0 t) a - \frac{\kappa}{2} a + \sqrt{\kappa_{\text{ex}}} s_0
\]

where \( a = A \exp(i \phi(t)) \). The integration of Eq. 2 further gives:

\[
a = \sqrt{\kappa_{\text{ex}}} s_0 \exp \left( i \delta \omega - \frac{\kappa}{2} \right) \left[ f(t) - \frac{1}{i \delta \omega - \frac{\kappa}{2}} \right]
\]

with

\[
f(t) = - \sqrt{\frac{i \pi}{2 V_s}} \exp \left( i \frac{(\delta \omega - \kappa/2)^2}{2 V_s} \right)
\times \left[ \text{erf} \left( -i \frac{\delta \omega - V_0 t - \kappa/2}{\sqrt{2V_s}} \right) - \text{erf} \left( -i \frac{\delta \omega - \kappa/2}{\sqrt{2V_s}} \right) \right]
\]

where \( \text{erf}(z) \) is the complex error function, i.e. \( z \in C \). The cavity transmission is then calculated as:

\[
T = \left| \frac{s_0 - \sqrt{\kappa_{\text{ex}}} a}{s_0} \right|^2
\]

Note that fitting with Eq. 4 would deterministically return the coupling rate \( \kappa_{\text{ex}} \) as part of the overall loss rate \( \kappa \), such that the coupling regime of the cavity (over- or under-coupling) can be distinguished. As a result, an intrinsic Q-factor \( Q_0 = \omega_0/\kappa_0 = 8.44 \times 10^9 \) is obtained with respect to the transmission profile shown in Fig. 2(a).

In contrast to the theoretical fitting, reading out the decay rate of the ring-down profile would also reveal the photon lifetime of the resonator, as well as the overall loaded Q-factor (i.e. \( Q = \omega_0/\kappa \)) [26, 61], see Fig. 2(b). As a comparison, the loaded Q-factor via the theoretical fitting is \( 4.75 \times 10^9 \) and that from the decay rate is \( 4.95 \times 10^9 \). By slowing down the laser scan speed, the same resonance would recover to have a single-dip Lorenzian profile, see Fig. 2(c). We introduced a weak modulation to the laser to create side-band markers (at the frequency of \( \pm 3 \text{ MHz} \)) on the transmission trace. In this way, the linewidth of the resonance in the unit of frequency is measured, and the corresponding loaded Q-factor is \( 4.02 \times 10^9 \), similar to the value with the ring-down trace.

Therefore, we have demonstrate ultra-high-Q MgF₂ resonators, with an convincing Q-factor above \( 10^9 \). In addition, the coupling ideality with respect to selected resonant mode of the resonator is characterized, see Fig. 2(d), which is on a high-level indicating almost no loss at the coupling junction [62, 63].

Soliton microcomb generation

We next carried out experiments for soliton microcomb generation in MgF₂ resonators. By conventional laser tuning scheme with respect to an isolated high-Q resonance, we could reproduce the classic transmittance of the system as a function of the laser-cavity detuning, which reveals soliton steps on the red-detuned side of the resonance[1], see Fig. 2(e). In addition, the detuning region corresponding to the modulation instability (MI) regime and to the soliton breathers are also noted. Tuning the laser from the blue-detuned side and landing on the soliton step, we could observe soliton microcomb generation, especially the single soliton state that has an overall smooth sech²-shape spectral envelope, see Fig. 2(f). Moreover, the stability of the soliton comb is assessed by measuring its low-frequency rf spectrum, where the residual pump wave was removed by a fiber notch filter before the spectrum was photo-detected, see insets in Fig. 2(f). As a result, while the comb in the MI regime shows a low-frequency spectral profile in the rf domain, recognized as a noisy state, the soliton comb is in a stable state with no contents in this spectral region.

Soliton combs with broadband intermode interactions

In fact, there is a large number of mode families in our crystalline resonators, each corresponding to a transverse eigen-mode field (in the \( rz \) plane). As a consequence, there is a high probability to observe resonances that are in proximity, such that the intermode pumping scheme can be investigated. Figure 3(a) showcases a transmission of two resonances in proximity, via the laser tuning at the power of 3.8 mW. When the pump power is increased to \( \sim 250 \text{ mW} \), which is sufficient to excite the comb generation, a complex transmittance of the system is detected, see Fig. 3(b), and a soliton step situated in between the two resonances is observed. Indeed, tuning the laser frequency from the blue detuned side to stop on the soliton step, a soliton comb spectrum is observed, see Fig. 3(d). Intuitively, the soliton state is stemming from pumping the primary (left) resonance and is on the slope of the auxiliary (right) resonance, such that it may feature interactions in between. As previously reported, one effect is that the pumping wave being coupled to the second resonant mode would cause the thermal equilibrium of the system [49, 50]. This effect also applies to our system, where we noted that the soliton comb would be running for hours free from additional feedback control (despite that the laser drift would surpass the soliton existence range during the time).

Moreover, we noted that the generated soliton comb in our resonators usually features a strongly tailored spectral envelope at the centre. This includes not only two comb spectra (i.e. comb 1 and comb 2 in Fig. 3(d)) as two separated cases when applying an intermode pumping, but also the comb spectrum in Fig. 1(e), where the spectral center feature a broadband enhancement such that the overall envelope is beyond the standard sech² profile. The conversion efficiency of the comb can be estimated from its monitored spectrum, i.e.:

\[
\eta = \frac{\sum_{\mu} P_{\mu} - P_0}{\sum_{\mu} P_{\mu}}
\]

where \( P_{\mu} \) indicates the power of the comb mode and \( \mu \) is the mode index, \( P_0 \) is the power of the central mode which contains
Fig. 3. Wide-span soliton comb with Raman lasing with the intermode pumping scheme. (a) Measured transmission of two resonances in proximity, probed by the laser at low power (ca. 3.8 mW). (b) The transmission of the same resonances probed at high power (ca. 246.4 mW), which reveals a soliton step. (c) Low noise rf spectrum of two combs (spectra shown in (d)) via the intermode pumping. (d) The measured soliton comb spectra. Comb 1 is generated corresponding to the soliton step region in (b), while Comb 2 is pumped regarding a second pair of resonances and is in a changed polarization state. Both combs feature intermode interactions on the central portion, and strong Raman lasing at both stokes and anti-stokes bands. In particular, a suspect anti-stokes soliton comb are observed in comb 2.

residual pump power. As a result, the soliton comb in Fig. 1(e) shows an efficiency of ca. 45.1%, while that of the standard dissipative Kerr soliton comb in Fig. 2(f) is ca. 1.6%.

In addition, the Raman lasing is observed in our MgF$_2$ resonators, in company with the soliton comb, see Fig. 3(d). The noted frequency shift for both the stokes and anti-stokes modes is ca. 7.35 THz (comb 1), and ca. 5.61 THz (comb 2 with a different polarization), which are much smaller compared with that of reported Raman bonds of fluoride materials, and are attributed to Raman bonds of isolated MgF$_2$ oligomers on the surface of the resonator [64]. We observed not only isolated Raman lasing and narrow-band Raman-Kerr combs as usually states [51], but also a suspect Raman-soliton comb [55] on the anti-stokes side of the soliton comb, which has a wide spectral span and has a similar sech$^2$-shape envelope to the soliton comb. The Raman combs are expected to be based in the auxiliary mode family or in other high-order modes rather than being in the soliton-based mode family, and are expected to feature the same repetition frequency to the soliton comb. Unfortunately, at the moment we are not able to measure the repetition frequency of the generated soliton combs or Raman combs, with a lack of high-speed photodetectors.

The stability of such soliton and Raman composites microcombs is assessed again by measuring the low-frequency rf spectrum, see Fig. 3(c). It can be noted that the soliton comb 1 accompanied with narrow-band Raman-Kerr combs could feature a low-noise figure, indicating that the comb is overall in a stable state. The comb 2 features an outstanding rf tone at ca. 6 MHz, together with certain noise, which indicates that the comb is in a transition from the MI to the breathing state.

Numerical simulations
We also performed numerical simulations to investigate the soliton comb with intermode interactions. The modelling is
Fig. 4. Simulation of soliton combs with intermode interactions. (a, b, c) Dispersion profiles of two coupled mode families, in which the primary mode family (blue dots) is unchanged and the auxiliary mode family (orange dots) shows three different profiles, corresponding to anomalous, normal and close-to-zero dispersion with respect to the central mode ($\mu = 0$). The generated comb modes (light blue dots) at a certain detuning ($\delta_{\mu}$) is reflected as a straight line in these mode maps. (d, e, f) Intracavity field spectra corresponding to their dispersion profiles, including fields in the primary and the auxiliary mode families (blue and orange lines), and a composited comb spectrum (light blue drop lines).

Based on a pair of coupled Lugliato-Lefever equations [39]:

$$\frac{\partial \hat{A}_p(\mu, t)}{\partial t} = \left( \frac{\kappa_p}{2} + i\delta_p + iD_{\text{int}}(\mu) \right) \hat{A}_p - i\kappa_c \hat{A}_{\text{aux}}$$

$$- i\gamma_p F \left| \hat{A}_p \right|^2 \hat{A}_p - i\gamma_c F \left| \hat{A}_{\text{aux}} \right|^2 \hat{A}_p + \sqrt{\kappa_{\text{exc}} \gamma_{\text{in}}} \tag{6}$$

$$\frac{\partial \hat{A}_{\text{aux}}(\mu, t)}{\partial t} = \left( \frac{\kappa_{\text{aux}}}{2} + iD_{\text{aux}}(\mu) - i\kappa_c \hat{A}_p \right) \hat{A}_{\text{aux}} - i\kappa_p \hat{A}_p$$

$$- i\gamma_{\text{aux}} F \left| \hat{A}_{\text{aux}} \right|^2 \hat{A}_{\text{aux}} - i\gamma_c F \left| \hat{A}_p \right|^2 \hat{A}_{\text{aux}} \right|_\mu \tag{7}$$

where $|\hat{A}_p(\mu, t)|^2$ stands for the photon number in the primary mode family (where the soliton formation is expected), and $|\hat{A}_{\text{aux}}(\mu, t)|^2$ is that in the auxiliary mode family, $A_p(\theta, t)$ and $A_{\text{aux}}(\theta, t)$ are amplitudes in the Fourier domain with respect to the mode index $\mu$, loss factors $\kappa_p$ and $\kappa_{\text{aux}}$ physically reveal the spectral linewidth of resonances in the primary and in the auxiliary mode families, respectively, and $\kappa_c$ is the linear coupling strength between the two mode families. The laser-cavity detuning is $\delta_p = \omega_p(0) - \omega_1$, where $\omega_p$ is the angular frequency of resonances in the primary mode family, and $\omega_1$ is that of the pumping wave. The resonant mode $\mu = 0$ is the central pumped mode and $s_{\text{in}}$ is the pump amplitude. The integrated dispersion of the primary mode family is $D_{\text{int}}(\mu) = \omega_p(\mu) - \omega_p(0) - \mu D_{1,p}$, where $D_{1,p}/(2\pi)$ is the free spectral range (FSR) around the pumped mode. In the same reference frame, the dispersion profile of the auxiliary mode family is $D_{\text{aux}}(\mu) = \omega_{\text{aux}}(\mu) - \omega_{\text{aux}}(0) - \mu D_{1,p}$. Coefficients $g_p$ and $g_{\text{aux}}$ indicate the single-photon induced nonlinear frequency shift by means of the self-phase modulation, and $g_c$ is that by the cross-phase modulation. Indeed, $g_p(\mu) \approx n_2 / V_{\text{eff},p}(\mu)$ and $g_c(\mu) \approx n_2 / V_{\text{eff},c}(\mu)$, where $n_2$ is the nonlinear refractive index of the material, $V_{\text{eff},p}(\mu)$ is the effective mode volume of the primary mode family and $V_{\text{eff},c}(\mu)$ is the effective overlapped mode volume between the primary and the auxiliary mode families.

In simulations, we fixed the dispersion of the primary mode family to support the soliton comb generation, and fixed the frequency offset ($\Delta \omega$) between the central modes. The relative dispersion profile of the auxiliary mode family could be very different given the fact that all mode families in the resonator may have a different FSR as well as different dispersion. The nonlinear self-phase modulation and cross phase modulation regarding $A_{\text{aux}}$ are neglected, i.e. $g_{\text{aux}} = 0$ and $g_c = 0$, provided that the field pattern in the auxiliary mode family is usually much weaker compared with the soliton pattern in the primary mode family.

As such, we defined three types of the auxiliary mode dispersion, which by featuring interactions with the soliton comb would cause a tailored spectral envelope in a broadband scale, see Fig. 4. A complete list of parameters used in the simulations is shown in Tab. 2.

Indeed, these simulated soliton combs are qualitatively similar to our experimental results in Fig. 1(e) and in Fig. 3(d). In detail, the first dispersion profile of the auxiliary mode family is set identical to that of the primary mode but with an frequency...
Table 2. Parameters for numerical simulations.

|               | Case-I | Case-II | Case-III |
|---------------|--------|---------|----------|
| \( \kappa_p \) | \( 2\pi \times 100 \text{ kHz} \) | \( 2\pi \times 200 \text{ kHz} \) | \( 2\pi \times 10 \text{ kHz} \) |
| \( \kappa_{\text{aux}} \) | \( 2\pi \times 100 \text{ kHz} \) | \( 2\pi \times 200 \text{ kHz} \) | \( 2\pi \times 10 \text{ kHz} \) |
| \( \kappa_{\text{ex},p} \) | \( \kappa_p / 2 \) | \( \kappa_p / 2 \) | \( \kappa_p / 2 \) |
| \( \kappa_{\text{ex,aux}} \) | \( \kappa_p / 2 \) | \( \kappa_p / 2 \) | \( \kappa_p / 2 \) |
| \( \kappa_c \) | \( \kappa_p \) | \( \kappa_p \) | \( \kappa_p \) |
| \( |s_{\text{in}}|^2 \) | 120 mW | 200 mW | 120 mW |
| \( D_{\text{int}}(\mu) \) | \( \mu^2 D_{2,p} / 2 + \mu^3 D_{3,p} / 6 \) | \( \mu^2 D_{2,p} / 2 + \mu^3 D_{3,p} / 6 \) | \( \mu^2 D_{2,p} / 2 + \mu^3 D_{3,p} / 6 \) |
| \( D_{2,p} \) | \( 2\pi \times 2 \text{ kHz} \) | \( 2\pi \times 2 \text{ kHz} \) | \( 2\pi \times 2 \text{ kHz} \) |
| \( D_{3,p} \) | 0 | 0 | 0 |
| \( D_{2,\text{aux}} \) | \( 2\pi \times 2 \text{ kHz} \) | \( 2\pi \times 300 \text{ Hz} \) | \( 2\pi \times 200 \text{ Hz} \) |
| \( D_{3,\text{aux}} \) | 0 | 0 | 0 |
| \( \delta_p \) | \( 2\pi \times 10^{-4} \text{ Hz} \) | \( 2\pi \times 10^{-4} \text{ Hz} \) | \( 2\pi \times 10^{-4} \text{ Hz} \) |
| \( \xi_{\text{aux}} \) | 0 | 0 | 0 |
| \( \xi_c \) | 0 | 0 | 0 |
| \( \delta_{\omega} \) | \( 44 \times \kappa_p \) | \( 44 \times \kappa_p \) | \( 44 \times \kappa_p \) |

* The integrated dispersion profile is expressed by a polynomial function up to the third order.

The third dispersion profile of the auxiliary mode family features close-to-zero dispersion around the central mode, see Fig. 4(b). In this case, \(| \tilde{A}_{\text{aux}} |^2 \) would always feature a single-peak, narrow-bandwidth waveform, and the combined comb spectrum is tailored accordingly, see Fig. 4(f). This comb is similar to the comb 1 in Fig. 3(d).

Therefore, numerical simulations could confirm our experimentally observed soliton combs with a spectral tailoring effect via intermode interactions. Nevertheless, we realized that the missing nonlinear effects in the simulation could be critical, which may lead to instability of the soliton state if the laser frequency is tuned into the MI region of the coupled auxiliary mode. This would explain e.g. the comb 2 in Fig. 3(d) as an unstabilized state.

3. CONCLUSION

In conclusion, we have demonstrated soliton combs in ultra-high-Q crystalline fluoride resonators. The home-developed MgF\(_2\) resonators have an intrinsic Q-factor approaching \(10^{10}\), which is on par with the best performances of the WGM-based microresonators. Cavity dissipative soliton state can be excited in these resonators, both with the conventional pumping scheme regarding an isolated cavity resonance, and with the intermode pumping scheme that allows for long-term running and spectral tailored soliton comb via thermal and optical intermode interactions. Indeed, given a large number of transverse mode families in the resonator, the relative dispersion profile of the auxiliary mode family can be quite different in normal, anomalous, or close-to zero dispersion regimes. Therefore, soliton combs with featured spectral envelope can be implemented accordingly. Of most importance, a soliton state with broadband power enhancement were obtained, whose spectrum is beyond the standard \(\text{sech}^2\) profile and the power efficiency is largely increased. We experimentally monitored a soliton comb with 45% power efficiency, together with other featured soliton comb spectra, and performed numerical simulations to confirm the spectral tailoring effect via broadband intermode interactions. In addition, the Raman lasing was observed in company to the soliton comb in our crystalline resonators, which in contrast to previous understandings could be in a low-noise and stabilized state. A suspect Raman-soliton comb was also observed around the anti-stokes side, rather than being on the stokes side as previously reported in high-Q silica microresonators \([55, 56]\).

Overall, our work reveals certain insights of cavity dissipative structures in nonlinear microresonator systems, and would contribute to the understanding of rich even complex soliton dynamics in the presence of dense mode families. It also opens an alternative approach to access stable yet efficient soliton microcombs in crystalline resonators where dispersion engineering is challenging. Nevertheless, the results would be supplementary to recent advances in the engineering of intermode interactions in photonic integrated platforms, which has enabled properly tailored soliton comb spectra \([42]\) and super efficient soliton combs \([47]\).

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