Imprint of anisotropic primordial non-Gaussianity on halo intrinsic alignments in simulations

Kazuyuki Akitsu,1, * Toshiaki Kurita,2,† Takahiro Nishimichi,3,† Masahiro Takada,† and Satoshi Tanaka3

1Kavli Institute for the Physics and Mathematics of the Universe (WPI), UTIAS
The University of Tokyo, Kashiwa, Chiba 277-8583, Japan
2Department of Physics, The University of Tokyo, Bunkyo, Tokyo 113-0031, Japan
3Center for Gravitational Physics, Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan

(Dated: July 8, 2020)

Using N-body simulations for the first time, we show that the anisotropic primordial non-Gaussianity (PNG) causes a scale-dependent modification, given by $1/k^2$ at small $k$ limit, in the three-dimensional power spectra of halo shapes (intrinsic alignments), whilst the conventional power spectrum of halo number density field remains unaffected. We discuss that wide-area imaging and spectroscopic surveys observing the same region of the sky allow us to constrain the quadrupole PNG coefficient $f_{NL}^{s=2}$ at a precision comparable with that of the cosmic microwave background.

Introduction – An observational exploration of non-Gaussianity in the primordial perturbations, which are the seeds of cosmic structures, gives a powerful test of the physics in the early universe such as inflation [1–3]. The cosmic microwave background (CMB) anisotropies and wide-area galaxy surveys can be used to pursue the primordial non-Gaussianity (PNG) from their observables [4–8] and these two carry complementary information.

Suppose that $\Phi(x)$ is the primordial potential field. The simplest PNG model is a local-type one, and its bispectrum is generally, as in given by Refs. [8, 9]:

$$B_{\Phi}(k_1, k_2, k_3) = 2 \sum_{\ell=0, 1, 2, \cdots} f_{NL}^{s=2} \left[ L_{\ell}(\hat{k}_1 \cdot \hat{k}_2) P_\phi(k_1) P_\phi(k_2) + 2 \text{ perms.} \right],$$

(1)

where $\hat{k} \equiv k/k$, $P_\phi(k)$ is the power spectrum of a Gaussian field, denoted by $\phi(x)$, and $L_\ell$ is the Legendre polynomial of order $\ell$; $L_0(\mu) = 1$ and $L_2(\mu) = (3\mu^2 - 1)/2$. The coefficient, $f_{NL}^{s=2}$, is a parameter to characterize the amplitude of the local PNG at each order $\ell$. Due to the orthogonality of the Legendre polynomials $L_\ell$, the PNG modes of different $\ell$ are independent with each other, and are expected to carry complementary information on the physics in the early universe, if detected or constrained separately. The isotropic PNG model with $s = 0$ has been well studied in the literature [2, 6]. The reality condition of $\Phi(x)$ ensures that the odd multipoles should vanish in the squeezed limit, where one of wavevectors is much smaller than the other two. Thus, in this Letter we focus on the anisotropic PNG described by the $s = 2$ term in the above bispectrum, which is the leading-order anisotropic PNG model among PNGs that have greater amplitudes in the squeezed limit.

The anisotropic PNG can be generated in several inflationary scenarios: the solid inflation [10], the non-Bunch-Davies initial states [11], and the existence of vector fields [8, 12–15] and higher-spin fields [3, 16, 17] in the inflationary epoch. Although the predicted bispectrum generally has a particular scale dependence such as $L_\ell(k_1, k_2) \rightarrow (k_1/k_2)^{3\ell} L_\ell(k_1, k_2)$ in Eq. (1), we consider a model with $\Delta_2 = 0$ for simplicity.

Nonlinear transformation from anisotropic PNG – To realize the PNG given by the $s = 2$ term in the bispectrum Eq. (1), we consider the following nonlinear transformation of $\phi$:

$$\Phi(x) = \phi(x) + \frac{2}{3} f_{NL}^{s=2} \sum_{ij} \left[ (\psi_{ij})^2(x) - \langle (\psi_{ij})^2 \rangle \right],$$

(2)

where $\psi_{ij}$ is the trace-less tensor that has the same dimension as $\phi$, defined as

$$\psi_{ij}(x) = \frac{3}{2} \left[ \frac{\partial_i \partial_j}{\partial^2} - \frac{1}{3} \delta_{ij} \right] \phi(x) = \int \frac{d^3k}{(2\pi)^3} \frac{3}{2} \left( \delta_{ij} - \frac{1}{3} \delta^{jk}_i \right) \phi(k)e^{ik\cdot x},$$

(3)

where $\delta^{jk}_i$ is the Kronecker delta function. One can easily confirm that the non-Gaussian field $\Phi(x)$ leads to the bispectrum with $s = 2$ in Eq. (1).

For galaxy surveys, the mass density fluctuation field, $\delta(x)$, instead of the primordial potential $\Phi(x)$, is more relevant for observables. These fields in the linear regime are related to each other via $\delta(k) = M(k, z)\Phi(k)$, where $M(k, z) \equiv (2/3)k^2T(k)D(z)/(\Omega_{\text{m}}H_0^2)$, with $T(k)$ and $D(z)$ denoting the transfer function and the linear growth factor, respectively. As discussed in Ref. [9], in the presence of the above PNG, the amplitude of the local small-scale power spectrum at $x$ has a modulation depending on the long-wavelength potential $\psi_{ij}^L$ as

$$P_\delta(k|x) |_{\psi_{ij}^L} = \left[ 1 + 4 f_{NL}^{s=2} \sum_{ij} \psi_{ij}^L(x) \hat{k}_i \hat{k}_j \right] P_\delta(k),$$

(4)

1 Our notation $f_{NL}^{s=2}$ is different from the notation $A_2$ used in Ref. [9]; the relation is $A_2 = 4f_{NL}^{s=2}$. 
where $k$ is a short-wavelength mode and $P_b(k)$ is the global matter auto-power spectrum. Since $\psi_{ij}^L$ is the trace-less tensor, $\psi_{ij}^L$ causes a quadrupolar modulation in the power of short mode fluctuations.

Intrinsic alignment and PNG – The linear intrinsic alignment (IA) model \cite{9, 18, 19} predicts that the shapes of galaxies originate from the gravitational tidal field as

$$\gamma_{ij}(\mathbf{x}) = b_K K_{ij}(\mathbf{x}), \quad (5)$$

where $\gamma_{ij}$ is the $(3 \times 3)$-tensor to characterize the shape of each galaxy and $K_{ij}$ is the tidal field at the galaxy’s position. We define the tidal field as $K_{ij} = (\partial \sigma_j / \partial^2 - \delta_{ij} \partial^2 \delta_{ij}) \delta$ so that $K_{ij}$ has the same dimension as that of the mass density fluctuations. This relation holds on scales sufficiently larger than the reach of galaxy and halo for-density fluctuations. This relation holds on scales sufficiently larger than the reach of galaxy and halo formation physics. Here $b_K$ is the linear shape “bias” coefficient, which can be interpreted as a response of the galaxy shape to the long-wavelength tidal field, whereas the linear “density” bias parameter $b_1$ gives a response of the galaxy number density to the long-wavelength mass density fluctuation \cite{20–22}. For adiabatic, Gaussian initial conditions, $b_K$ takes a constant value at the limit of a sufficiently large smoothing scale or $k \to 0$ in Fourier space, and the value varies with the type of galaxies. However, the anisotropic PNG breaks the condition, and causes a characteristic scale-dependent modification in $b_K$, as the isotropic PNG does for the density tracers \cite{6}.

As we discussed in Eq. (4), the anisotropic PNG induces the coupling between the local tidal field, $K_{ij}$, and the long-wavelength quadrupole potential, $\psi_{ij}$. Similarly to the effect of isotropic PNG on the density distribution of galaxies, this mode-coupling leads to a scale-dependent modification in the IA of galaxy shapes as pointed out by Ref. [9]:

$$\gamma_{ij}(k) \simeq [b_K + 12 b_1 f_{NL}^2 M^{-1}(k)] K_{ij}(k), \quad (6)$$

where $b_\psi$ is a parameter to characterize the response of galaxy shapes to the long-wavelength quadrupole potential, defined as $b_\psi \equiv \partial \gamma_{ij}/\partial (2 f_{NL}^2 \psi_{ij})$. The second term on the r.h.s. shows that the anisotropic PNG induces a scale-dependence of $1/k^2$ in the IA effect at very small $k$, as in the effect of the local-type isotropic PNG on the galaxy density bias parameter \cite{6}. In the following we treat $b_K$ and $b_\psi$ as free parameters, and then estimate their values (the value of $b_\psi$ for the first time) for a sample of halos from $N$-body simulations adopting the Gaussian and the anisotropic PNG initial conditions, respectively. If we use the peak theory for the nearly random, Gaussian field, extending the formula in Refs. [23, 24], we might be able to estimate a relation between $b_K$ and $b_\psi$ for halos. However, this is beyond the scope of this Letter, and will be our future work. We also note that an apparent infrared divergence at the limit $k \to 0$ should be restored if properly taking into account the finite survey region and relativistic effects \cite{e.g. see Refs. 25–27}, for the discussion on the density bias parameter. Since we are interested in the IA effect on subhorizon scales, we can safely ignore the relativistic effect.

Initial conditions, simulations, and IA measurements – To generate the initial conditions for $N$-body simulations with the anisotropic PNG, we modified 2LP1c, developed in Ref. [28, 29]. First, in Fourier space we generate a Gaussian random field $\phi(k)$ using the assumed $P_b(k)$, and prepare the auxiliary field $\psi_{ij}(k)$ according to Eq. (3). Then Fourier transforming $\phi(k)$ and $\psi_{ij}(k)$ to real space, we construct the non-Gaussian field $\Phi(\mathbf{x})$ following Eq. (2). We solve the Lagrangian dynamics up to the second order based on the non-Gaussian field $\Phi$ and the matter transfer function computed by CLASS \cite{30}. Throughout this Letter we employ a flat $\Lambda$CDM cosmology consistent with the Planck satellite \cite{31}. We confirmed that the bispectrum measured from the $\Phi$ field generated with this procedure is consistent with the $s = 2$ term of Eq. (1).

We then evolve the particle distribution using a newly developed $N$-body solver based on the Tree Particle-Mesh (PM) scheme \cite{32}. It is based on a general-purpose framework for particle methods, FDPS \cite{33, 34}, with the PM part originally implemented in GReNe \cite{35–37}. We further accelerate the calculation of gravitational force term with a 512-bit SIMD instruction set in a similar manner as in the Phantom-GRAPE library \cite{38–40} and optimize the memory footprint for efficient execution in high-performance parallel environments. The final accuracy of the code is tuned such that it reproduces the matter power spectrum from a Gadget2 \cite{41} run started from an identical initial condition with the accuracy parameters used in [42], to within one percent up to the particle Nyquist frequency. We adopt $N_{\text{part}} = 2048^3$ particles and 4.096 $h^{-1}$ Gpc for the comoving simulation box size. The particle mass is $m_p \simeq 7.0 \times 10^{11} h^{-1} M_\odot$. For comparison, we also run simulations for a Gaussian initial condition and the isotropic ($s = 0$) PNG model, using the same initial seeds. In summary, we run 6 simulations in total; one Gaussian simulation and 5 simulations with $f_{NL} = 500$ and $f_{NL} = \pm 100$ and $\pm 500$. We study the shapes of halos identified by Rockstar \cite{43}, as a proxy of the galaxy IA effect. We use the Rockstar output to infer the virial mass of each halo, denoted as $M_{\text{vir}}$ \cite{44}.

To measure the IA correlations from simulations, we use a novel method developed in Ref. [44]. First we measure the inertia tensor defined by member particles of each halo according to $I_{ij} = \sum w(r_p) \Delta x_p^i \Delta x_p^j$, where $\Delta x_p^i$ is the relative position of each member particle from the halo center, and $w(r_p)$ is the $1/r_p^2$ radial weight; that is, we up-weight inner member particles assuming that those are better proxies of stellar particles if a galaxy forms at the center. Taking the $z$-axis to the line-of-sight direction, we define the two ellipticity components, $e_1^h, e_2^h$, for each halo from the $(2 \times 2)$ sub-matrix of $I_{ij}$ in the $xy$-plane as an observable halo
shape on the sky. After that, we use the Nearest-Grid-Point assignment [45] to define the 3-dimensional ellipticity fields, $\epsilon_1(x)$, $\epsilon_2(x)$, as well as the matter and halo density fluctuation fields, $\delta(x)$ and $\delta_b(x)$. Since the two ellipticity components form spin-2 fields in the $xy$-plane, we can perform the $E/B$-mode decomposition in Fourier space: $E(k) \equiv \epsilon_1(k) \cos 2\varphi_k + \epsilon_2(k) \sin 2\varphi_k$, where $\varphi_k$ is the azimuthal angle of $k$. We finally estimate the IA power spectra from each simulation, and in this Letter we mainly focus on the IA cross-power spectrum, defined as

$$\langle \delta(k) E(k') \rangle \equiv (2\pi)^3 \delta_D^3(k + k') P_{\delta E}(k, \mu), \quad (7)$$

where $\mu \equiv \hat{k} \cdot \hat{z}$. The linear alignment model with the anisotropic PNG predicts $P_{\delta E}(k, \mu) = (b_K + \Delta b_K)(1 - \mu^2) P_{\delta}(k)$ and $P_{EE}(k, \mu) = (b_K + \Delta b_K)^2 (1 - \mu^2)^2 P_{\delta E}(k)$ at very small $k$, where $\Delta b_K$ is the second term in the brackets of Eq. (6). The $\ell$-th multipole moments of the power spectrum, in each wavenumber bin, are defined as

$$P_{\delta E}^{(\ell)}(k) = \frac{2\ell + 1}{2} \int_{-1}^{1} d\mu \mathcal{L}_\ell(\mu) P_{\delta E}(k, \mu). \quad (8)$$

In practice we use a discrete summation, instead of the integral, in Fourier space over grids in each $k$-bin, spaced by the fundamental mode $k_f = 2\pi/L$ ($L$ is the side length of a simulation box). We similarly estimate, from each simulation, the matter-halo power spectrum, $P_{\delta h}$, and the multipole components for the auto-spectrum of halo

\[ FIG. 1. \] The matter-halo power spectrum (left panel), the monopole moment of the cross-power spectrum of matter and halo shapes (middle), and the monopole moment of shape-shape auto-spectrum (right) for various initial conditions: Gaussian (blue), isotropic PNG (orange) and anisotropic PNG (green) initial conditions, respectively. Here we assume $(f_{NL}^0, f_{NL}^{s=2}) = (500, 0)$ or $(-500, 0)$ for the isotropic or anisotropic PNG case Eq. (1), respectively. These are measured for the halo sample with $M_{vir} > 10^{14} h^{-1} M_\odot$ at $z = 0$. The errorbars denote the Gaussian errors for a volume of $V = 69 (h^{-1} \text{Gpc})^3$.

\[ FIG. 2. \] Similar to the middle panel of the previous figure, but the plot shows $P_{\delta E}^{(0)}(k)$ for the anisotropic PNG model with $f_{NL}^{s=0} = -500, -100, 100$ or $500$. For comparison, the gray points show the result for the Gaussian initial condition. The solid lines show the best-fit model predictions Eq. (6).

\[ FIG. 3. \] The best-fit IA parameters $b_K$ and $b_\psi$ for different mass-threshold samples of halos, selected with $M_{vir} > M_{th}$, at redshifts $z = 0$ (solid line), $0.5$ (dot-dashed) and $1.0$ (dotted), respectively. For comparison, the gray points show $b_K$ for lower-mass halos that are measured from the higher-resolution simulations with Gaussian initial conditions in Ref. [44].
shape $E$-field, $P_{EE}$, and for the cross-power spectrum between the $E$-field and the halo number density field, $P_{hi}$. We set the minimum wavenumber, $k_{\text{min}} = 0.002 \ h\,\text{Mpc}^{-1}$, and adopt the bin width; $\Delta \ln k = 0.26$ (10 bins in one decade of $k$). In this Letter we do not include the redshift-space distortion effect due to peculiar velocities of halos for simplicity.

Results – The middle and right panels of Fig. 1 show the main result. The PNG simulation confirms that the anisotropic ($s = 2$) PNG induces a scale-dependent modification in the IA power spectra in small $k$ bins in the linear regime, but does not change the matter-halo power spectrum, $P_{hi}$ shown in the left panel. On the other hand, the isotropic ($s = 0$) PNG does not alter the IA power spectra, but does alter $P_{hi}$ as shown in Ref. [6]. Thus, the scale-dependent bias of the IA power spectra gives a smoking gun evidence of the $s = 2$ PNG, if detected.

In Fig. 2 we compare the best-fit model predictions with the simulated IA power spectra for different values of $f_{\text{NL}}^{s=2}$. To estimate the best-fit model, we first estimate $b_K$ in Eq. (5) by comparing $P_{hi}$ and $P_\delta$ up to $k = 0.05 \ h\,\text{Mpc}^{-1}$ for the Gaussian simulation assuming the Gaussian covariance. Then we estimate $b_\psi$ in Eq. (6) in the same way by using the simulated spectra measured from all the PNG simulations with different $f_{\text{NL}}^{s=2}$ values up to $k = 0.05 \ h\,\text{Mpc}^{-1}$, varying $b_\psi$ as the only free parameter. The figure shows that the best-fit model predictions well reproduce the data points. Fig. 3 shows the estimated $b_K$ and $b_\psi$ for different mass-threshold samples of halos at different redshifts. We find $b_\psi/b_K \sim 0.18$ for all the samples and redshifts. For comparison, we also show the $b_k$ parameters that are measured from high-resolution simulations of $1 \ h^{-1}\text{Gpc}$ box in Ref. [44].

Now we estimate the precision of a wide-area galaxy survey for constraining the anisotropic PNG amplitude, using the Fisher information matrix:

$$
\frac{1}{\sigma^2(f_{\text{NL}}^{s=2})^2} = \sum_{i,l} \sum_{k_i,k_i} \left( \frac{\partial P_{hi}^{(l)}(k_i)}{\partial f_{\text{NL}}^{s=2}} \right)^2 \left[ \mathbf{C}^{-1} \right]_{ij} \left( \frac{\partial P_{hi}^{(l)}(k_j)}{\partial f_{\text{NL}}^{s=2}} \right),
$$

where $\mathbf{C}$ is the covariance matrix between $P_{hi}^{(l)}(k_i)$ and $P_{hi}^{(l)}(k_j)$ for which we assume a Gaussian covariance taking into account the shot noise and the intrinsic shape noise measured from the simulations [44]. Here we employ the maximum wavenumber $k_{\text{max}} = 0.1 \ h\,\text{Mpc}^{-1}$ in the above summation, and consider only the cross-power spectrum because the signal-to-noise ratio for the $E$-mode auto-power spectrum is smaller than that of $P_{EE}$ and a measurement of $P_{EE}$ is contaminated by cosmic shear due to foreground structures. Notice that, although the scale-dependent bias in $P_{hi}$ can arise from both $s = 0$ and $s = 2$ PNGs, the two parameters ($f_{\text{NL}}^{s=0}$ and $f_{\text{NL}}^{s=2}$) can be simultaneously constrained from the combined measurements of $P_{hi}$ and $P_{hi}$. We consider a sample of halos with $M_{\text{vir}} > 10^{13} \ h^{-1}\text{M}_\odot$ taken from a hypothetical survey covering a comoving volume of $V_c = 69 \ (h^{-1}\text{Gpc})^3$ that corresponds to a spectroscopic survey with sky coverage $f_{\text{sky}} \approx 0.7$ in the redshift range $z = [0.5, 1.4]$. This sample roughly corresponds to luminous early-type galaxies in massive hosts with a number density of $2.5 \times 10^{-4} \ (h^{-1}\text{Mpc})^{-3}$ for the Planck cosmology. As can be found from Fig. 3, such a halo sample has $b_K = -0.29$ at $z \approx 1$, and we assume $b_\psi = -0.05$, assuming the same ratio $b_\psi/b_K$ as for halos with $M_{\text{vir}} > 10^{14} \ h^{-1}\text{M}_\odot$ at $z = 1$ in Fig. 3. We obtain the expected precision $\sigma(f_{\text{NL}}^{s=2}) \approx 5$ or $\sigma(b_\psi f_{\text{NL}}^{s=2}) \approx 0.3$. Note that, if we change the minimum wavenumber to $k_{\text{min}} = 0.005 \ h\,\text{Mpc}^{-1}$ from our default choice of $k_{\text{min}} = 0.002 \ h\,\text{Mpc}^{-1}$, the precision is slightly degraded to $\sigma(f_{\text{NL}}^{s=2}) \approx 6$. This precision is much better than the forecast in Ref. [9] which is based on the angular IA power spectrum instead of the three-dimensional IA power spectrum. This result is only slightly worse than the current CMB constraint, $\sigma(f_{\text{NL}}^{s=2}) \approx 1$ [5]. In any case it should be noted that the IA method constrains the anisotropic PNG at different redshifts and for different length scales compared to the CMB constraints, and the two methods are complementary to each other.

Discussion – We showed that the IA power spectra, measured from the wide-area spectroscopic and imaging surveys of galaxies for the same region of the sky, can be used to constrain the anisotropic PNG at a similar precision to the current CMB constraint. Here an imaging survey is needed to measure shapes of individual galaxies, while a spectroscopic survey is needed to obtain their three-dimensional positions. A further improvement can be obtained, e.g. by having a larger volume covering up to a higher redshift, combining the bispectrum information of both the number density [46] and IA, combining the IA power spectra of different density and shape tracers (i.e. multi-tracer technique in Refs. [47, 48]), and also including the redshift-space distortion effect. In addition, it is important to investigate effects of the anisotropic PNGs with a particular scale dependence [49]. These are all interesting, and worth exploring in more detail.

Acknowledgments – We thank Mareshu Sekiya and Jingjing Shi for useful discussions. This work was supported in part by World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan, and JSPS KAKENHI Grant Numbers JP15H05887, JP15H05889, JP15K21733, JP17K14273, JP19H00677, JP19J12254 and JP20J22055, and by JST AIP Acceleration Research Grant Number JP20317829, Japan. K.A. and T.K. are supported by JSPS Research Fellowship for Young Scientists. Numerical computation was carried out on Cray XC50 at Center for Computational Astrophysics, National Astronomical Observatory of Japan.
