Accuracy of muon transport simulation

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Abstract. Chain of calculations which have to be performed to predict any kind of signal in a deep underwater/ice neutrino detector necessarily includes the lepton propagation through thick layers of matter, as neutrino can be observed only by means of leptons (muons, first of all, due to their large ranges) that are generated in $\nu N \rightarrow lN$ interactions. Thus, the muon propagation plays a key role when analyzing data and it is important to understand clearly how transportation part of simulation chain contributes to total inaccuracy of final results. Here we consider sources of uncertainties that appear in Monte Carlo algorithms for simulation of muon transport. The trivial but effective test is proposed to measure the propagation algorithm accuracy. The test is applied to three MC muon transport codes (PROPMU, MUSIC, MUM) and results are reported.

1 Introduction

The main challenge for existing and planned underwater/ice neutrino telescopes (Barwick \textit{et al}., 1991), (Ahrens \textit{et al}., 2001), (Aslanides \textit{et al}., 1999), (Carmona \textit{et al}., 2001), (Sokalski and Spiering, 1992), (Balkanov \textit{et al}., 2000), (Resvansis \textit{et al}., 1994), (Grieder \textit{et al}., 2001) is detection of neutrino of extraterrestrial origin. But neutrino is neutral weakly interacting particle and one can not observe it directly but has to detect it by means of lepton that appears in $\nu N \rightarrow lN$ interaction and passes a distance in medium before being detected. The case with $l = \mu^\pm$ is considered, first of all, because muon possesses the best ability to propagate large distances in contrast to $e^\pm$ (due to their short radiation length) and $\tau^\pm$ (due to their short life time) in a wide range of energies. On the other hand, atmospheric muons (that represent a penetrating component of atmospheric showers resulting from interaction of primary cosmic radiation with the Earth atmosphere) are the principal background for neutrino signal. To eliminate it one has to predict correctly the detector response on atmospheric muon flux. Also one should keep in mind the fact that atmospheric muons are of scientific interest themselves: for instance, from the point of view of charm production (Sinegovskaya and Sinegovsky, 2001). The last (but not the least) important point is detector calibration which has to be done, again by means of atmospheric muons that are the only more or less known intensive calibration source for deep underwater/ice neutrino telescopes.

Thus, when analyzing experimental data obtained with underwater/ice detectors one has to apply some model for muon transport through thick layers of matter, using an algorithm that gives the muon energy $E_1$ at the end of distance $D$ providing that its initial energy is equal to $E_0$. Commonly, Monte Carlo (MC) technique is used for this purpose (Aarnio \textit{et al}., 1983), (Lipari and Stanev, 1991), (Antonioli \textit{et al}., 1997), (Kudryavtsev \textit{et al}., 1999), (Sokalski \textit{et al}., 2001), (Rhode and Chirkin, 2001) being the most adequate to essentially stochastic nature of muon energy losses. Such a MC algorithm necessarily incorporates some uncertainties into the final result. These uncertainties can be divided into two parts:

(A) insurmountable uncertainties which relates to
\begin{itemize}
\item finite accuracy of formulae for muon cross-sections;
\item data on medium density and composition.
\end{itemize}

(B) “inner” errors that are produced by simulation algorithm itself due to
\begin{itemize}
\item finite accuracy of numerical procedures;
\item simplifications that are done to get reasonable computation time.
\end{itemize}

In case $(B) > (A)$ an algorithm is obviously too inaccurate and corrupts result more than uncertainties with the muon cross-sections and medium composition. $(B) \ll (A)$ is better but leads to unreasonable wasting of human and computer efforts, the tool for the muon transport is too “thin” comparing to insurmountable uncertainties of group $(A)$. Thus, the case $(B) < (A)$ seems to be an optimum equilibrium.
This report is aimed to get some quantitative parameters of muon transport algorithm which condition \((B) < (A)\) leads to.

2 How precisely do we know the muon energy losses?

The accuracy of existing formulae for muon cross-sections was analyzed in number of works (Kokoulin and Petrukhin, 1991), (Rhode and Cărloganu, 1998), (Kokoulin, 1999). The conclusion is that it is not higher than 1% for \(E_\mu < 1\) TeV and becomes the worse the higher muon energy is. We do not give here the complete analysis but consider only two examples that concern the muon photonuclear interaction which contributes the largest uncertainty to the total muon energy losses comparing with other kinds of muon interactions that represent purely electromagnetic (and hence, better known) processes: ionization, bremsstrahlung, and direct \(e^+e^-\)-pair production.

For last twenty years one has been using the formula for photonuclear muon cross-section that was developed in the frame of generalized vector dominance model (GVDM) (Bezrukov and Bugaev, 1981a). In such an approach the differential cross-section (and, consequently, the muon energy losses due to photonuclear interactions) is proportional to the total cross-section for absorption of a real photon by a nucleon, \(\sigma_{tot}^{\gamma p}\). There are several parameterizations for \(\sigma_{tot}^{\gamma p}\) that were obtained by trying to get the best fit to experimental data (Bezrukov and Bugaev, 1981a), (Abramowicz et al., 1991), (Donnachie and Landshoff, 1992), (Breitweg et al., 1999), (Butkevich and Mikheyev, 2001). Two of them are shown in fig. 1 along with the last results from the ZEUS experiment (Breitweg et al., 1999). \(\sigma_{tot}^{\gamma p}\) is presented as a function of \(W^2\), where \(W\) is the center-of-mass energy of the \(\gamma p\) system. Parameterizations (Bezrukov and Bugaev, 1981a) (BB) and (Breitweg et al., 1999) (ZEUS) both fit more or less the experimental data but even within “experimentally known” range \(W^2 \leq 10^5 \text{ GeV}^2\) the difference between two parameterizations reaches 4%. At \(W^2 = 10^8 \text{ GeV}^2\) the BB curve is higher than the ZEUS one by 15%. The same data are presented in fig. 2 in terms of muon energy losses due to photonuclear interaction in pure water as obtained using (a) BB parameterization for \(\sigma_{tot}^{\gamma p}\) (solid line) and (b) ZEUS parameterization (dashed line).

It became clear in last few years that at very high lepton energies the essential part of photonuclear cross-section is due to nonGVDM contribution. It appears, that this additional part is well described by QCD perturbation theory. Recent new calculations (Dutta et al., 2000), (Butkevich and Mikheyev, 2001), (Bugaev and Shlepin, 2001) show an essential increase of the muon energy losses due to photonuclear interactions. In (Dutta et al., 2000) the ALLM formulae (Abramowicz et al., 1991) (that is based on Regge approach and on H1 and ZEUS data) is used for parameterization of nucleon structure function \(F_2\). In (Butkevich and Mikheyev, 2001) the CKMT Regge model (Capella et al., 1994) is applied for a description of \(F_2\) at low and intermediate \(Q^2\) (squared four-momentum transfer) and the fit of parton distribution functions given by MRS group (Martin et al., 2000) at high \(Q^2\).
Muon energy losses become higher when accounting for perturbative QCD part in photonuclear interactions by $1\pm2\%$ at $E_{\mu} = 100$ TeV and by $20\pm30\%$ at $E_{\mu} = 1$ EeV. Fig. 4 shows changes in muon survival probabilities in sea water and standard rock which incorporation of nonGVDM QCD-corrections results in. It is seen that for extremely high energies ($E_{\mu} > 1$ PeV) effect is very significant.

Table 2. Differences in terms of total muon energy losses in standard rock (in percents) between two works that account for QCD part (Dutta et al., 2000), (Bugaev and Shlepin, 2001) and Bezrukov-Bugaev formula based on GVDM (Bezrukov and Bugaev, 1981a).

| $E_{\mu}$       | Dutta et al. | Bugaev & Shlepin |
|-----------------|--------------|------------------|
| 100 TeV         | $+10\%$      | $+16\%$          |
| $1$ PeV         | $+19\%$      | $+26\%$          |
| $10$ PeV        | $+34\%$      | $+42\%$          |
| $100$ PeV       | $+53\%$      | $+71\%$          |
| $1$ EeV         | $+81\%$      | $+119\%$         |

Fig. 3. Muon energy losses in standard rock due to photonuclear interaction with- (Dutta et al., 2000), (Butkevich and Mikheyev, 2001), (Bugaev and Shlepin, 2001) and without (Bezrukov and Bugaev, 1981a) accounting for QCD part (dashed lines and open circles). Curves for bremsstrahlung (Bezrukov and Bugaev, 1981b), (Andreev et al., 1994), (Kelner et al., 1995), direct $e^+e^-$-pair production (Kokoulin and Petrukhin, 1969), (Kokoulin and Petrukhin, 1971), (Kelner, 1997), (Kelner et al., 1999) (dotted lines) are shown, as well. Solid lines stand for total energy loss (brems + pair + nuc, ionization not included) with- (Dutta et al., 2000), (Bugaev and Shlepin, 2001) and without (Bezrukov and Bugaev, 1981a) accounting for QCD part in photonuclear interaction.

2001) use for calculation of QCD perturbative part the color dipole model in version of (Forshaw et al., 1999), with parameters founded from latest DESY data.

The results of these three works are presented in fig.3 and tables 1,2 in terms of the muon energy losses in standard rock. (Butkevich and Mikheyev, 2001) and (Bugaev and Shlepin, 2001) are close to each other while the muon energy losses obtained in (Dutta et al., 2000) are essentially lower. We believe that this is due to the fact that old data (Abramowicz et al., 1991) were used in (Dutta et al., 2000) to fit the parameters of model. Anyway, the total muon energy losses obtained in (Dutta et al., 2000), (Bugaev and Shlepin, 2001) and without (Bezrukov and Bugaev, 1981a) accounting for QCD part (dashed lines and open circles) are close to each other while the muon energy losses in standard rock (in percents) between two works that account for QCD part (Dutta et al., 2000), (Bugaev and Shlepin, 2001) and Bezrukov-Bugaev formula based on GVDM (Bezrukov and Bugaev, 1981a).

Table 1. Differences in terms of the muon energy losses in standard rock due to photonuclear interaction (in percents) between three works that account for QCD part (Dutta et al., 2000), (Butkevich and Mikheyev, 2001), (Bugaev and Shlepin, 2001) and Bezrukov-Bugaev formula based on GVDM (Bezrukov and Bugaev, 1981a).

| $E_{\mu}$ (GeV) | Dutta et al. | Bugaev & Shlepin | Butkevich & Mikheyev |
|-----------------|--------------|------------------|----------------------|
| 100 TeV         | $+10\%$      | $+16\%$          | $+22\%$              |
| 1 PeV           | $+19\%$      | $+26\%$          | $+29\%$              |
| 10 PeV          | $+34\%$      | $+42\%$          | -                    |
| 100 PeV         | $+53\%$      | $+71\%$          | -                    |
| 1 EeV           | $+81\%$      | $+119\%$         | -                    |
can be measured precisely. But since the main technique to select the neutrino induced events is looking for up-going muons from the lower hemisphere, one should know also the density and composition of sea bed rock. When this is unknown one has to use a standard rock model ($\rho = 2.65$ g/cm$^3$, $A = 22$, and $Z = 11$). At the same time the measured rock densities at Gran Sasso and Frejus underground laboratories are equal, for instance, to $\rho_{\text{Gran Sasso}} = 2.70$ g/cm$^3$ and $\rho_{\text{Frejus}} = 2.74$ g/cm$^3$, respectively (Ambrosio et al., 1995), (Rhode, 1993) which is $2 \pm 3\%$ higher. Investigation of the Baikal lake bed (NT-200 neutrino telescope) showed a complex structure with several layers of different composition and density that varies with increase the depth from $\rho_{\text{Baikal}} = 1.70$ g/cm$^3$ to $\rho_{\text{Baikal}} = 2.90$ g/cm$^3$ ($\pm 25\%$ variations) (Panfilov, 2000). Thus, error that is incorporated to muon transport calculations by uncertainties with the rock density/composition lies, at least, at a few percent level.

All this have led us to a conclusion that we never know the muon energy losses better than with $1\%-accuracy$ (actually, worse) and, consequently it makes no sense to waste efforts trying to reproduce energy losses with accuracy better than $10^{-2}$ with an algorithm for the muon transport. So, there is a reasonable quantitative criterium for a muon propagation algorithm goodness: algorithm whose accuracy is better than $1\%$ can be accepted as a good one, otherwise it is bad.

### 3 Choice of $v_{\text{cut}}$

Among all simplifications that may be done when developing an algorithm for the muon propagation (and that affect the algorithm accuracy) one is obligatory. As number of muon interactions per unit of path is practically infinite it is impossible to simulate all acts and one has to set some threshold for relative energy transfer $v_{\text{cut}}$ ($v = \Delta E_{\mu}/E_{\mu}$, where $\Delta E_{\mu}$ is energy which is passed by muon of energy $E_{\mu}$ to either real or virtual photon in a single interaction) above which muon energy losses are treated by the direct simulation of $\Delta E$ for each interaction (“stochastic” part) and below which energy losses are treated by means of stopping-power formula (“soft” part) that can be obtained by integration of differential cross-sections for the muon interactions:

$$\frac{dE_{\mu}}{dx}(E_{\mu}) = \frac{N_A}{A} E_{\mu} \int_{0}^{v_{\text{cut}}} \frac{d\sigma(E_{\mu}, v)}{dv} v dv$$

($A$ is a mass number of the target nucleus, $N_A$ is the Avogadro number). Number of interactions to be simulated per unit of muon path depends on $v_{\text{cut}}$, growing roughly as $N_{\text{int}} \propto v_{\text{cut}}^{-1}$ along with computation time. So, it would be desirable to choose $v_{\text{cut}}$ as large as possible. But, on the other hand, increase of $v_{\text{cut}}$ affects the simulation accuracy since some part of statistical fluctuations in energy losses goes out of simulation and is not accounted for. Thus, the question is how large value of $v_{\text{cut}}$ may be chosen to keep result within $1\%-accuracy$? This problem was discussed in literature (Nau-mov et al., 1994), (Lipari and Stanev, 1991), (Antonioli et al., 1997), (Lagutin et al., 1998) but, in our opinion, more careful analysis is lacking.

Let’s firstly consider propagation of monoenergetic muon beams. Fig.5 shows results on propagation (survival probabilities $p \text{ vs } v_{\text{cut}}$) for four beams with initial energies $1\text{ TeV}$, $10\text{ TeV}$, $100\text{ TeV}$ and $10\text{ PeV}$ through slant depths $3.2\text{ km}$, $12\text{ km}$, $23\text{ km}$ and $40\text{ km}$, respectively, in pure water. Closed circles represent results obtained with knock-on electrons (that appear as a result of ionization with large energy transferred to an atomic electron) included in direct simulation, open circles stand for completely continuous ionization that is treated simply by Bethe-Bloch formula. Solid lines on each panel correspond to $v_{\text{cut}} = 10^{-4}$ and all energy losses multiplied by factors 0.99 (upper line) and 1.01 (lower line), thus these lines limit “±1\%-energy-losses-error-band”. Horizontal dotted lines correspond to $v_{\text{cut}} = 10^{-4}$ and cross section for absorption of a real photon by a nucleon parametrized according to (Breitweg et al., 1999) instead of parameterization (Bezrukov and Bugaev, 1981a) that was mainly applied at presented simulations.

One can conclude as follows:

(a) In most cases with except for lower row and the left column (that corresponds to low survival probabilities and low muon initial energies, respectively) survival probabilities lie mainly within ±1\%-band.

(b) The difference between survival probabilities for two models of ionization - partially fluctuating (above $v_{\text{cut}}$)
and completely “soft” - is the less appreciable the larger muon energy is.

(c) Parameterizations (Bezrukov and Bugaev, 1981a) and (Breitweg et al., 1999) do not differ noticeably from each other in terms of survival probabilities.

(d) For $v_{cut} \leq 0.02 \div 0.05$ there is almost no dependence of survival probability on $v_{cut}$ with except for very last part of muon path where survival probability becomes small.

Now let’s go to the more realistic case with atmospheric muons. Fig.6 represents intensities of vertical atmospheric muon flux at 8 depths $D$ of pure water as functions of $v_{cut}$ as obtained by simulation with muons sampled according to sea level spectrum (Klimushin et al., 2001):

$$\frac{dN}{dE} = \frac{0.175 E^{-2.72}}{cm^2 s sr GeV} \left( \frac{1}{1 + \frac{E}{103 GeV}} + \frac{0.037}{1 + \frac{E}{810 GeV}} \right)$$

(2)

Meaning of closed and open circles, as well as solid and dotted lines is the same as for fig.5. Dashed lines on panels for $D \leq 5$ km correspond to intensities which were computed for all energy losses treated as completely “continuous” (no fluctuations included). Dash-dotted lines show intensities of vertical muon flux simulated with muons sampled according to the Gaisser sea level spectrum (Gaisser, 1990):

$$\frac{dN}{dE} = \frac{0.14 E^{-2.7}}{cm^2 s sr GeV} \left( \frac{1}{1 + \frac{E}{104.6 GeV}} + \frac{0.054}{1 + \frac{E}{772.7 GeV}} \right)$$

(3)

General conclusions are qualitatively the same as for mono-energetic muon beams but quantitatively the influence of $v_{cut}$ and model of ionization is much weaker. One can conclude the following:

(a) Computed muon flux is strongly affected by accounting for fluctuations in energy losses: muon flux intensity computed by means of stopping-power formula is less comparing with MC simulated flux by $\approx 10\%$ at 3 km and by $\approx 20\%$ at 5 km. At the depth of 20 km muon flux computed with ignorance of fluctuations is only 10 \% of simulated flux.

(b) 1%-uncertainty in muon cross sections plays the principal role for resulting error (all simulated data lie within $\pm 1\%$-band with no exceptions). This error has a tendency to grow with depth from $\pm 2.5\%$ at 1 km to $\sim \pm 15\%$ at 20 km.

(c) Difference between muon spectra (Klimushin et al., 2001) and (Gaisser, 1990) leads to uncertainty from $-4\%$ at 1 km to $+16\%$ at 20 km.

(d) Error which appears due to simplified, entirely “continuous” ionization lies, commonly, at the level of $2\div 3\%$.

(e) Dependence of simulated muon flux intensity upon $v_{cut}$ is the most weak one comparing with other error sources.

Thus, we can make the following conclusions: to reproduce muon energy losses with an accuracy better than $10^{-2}$ it is quite enough to account only for fluctuations in energy losses with fraction of energy lost being as large as $v > v_{cut} \approx 0.05 \div 0.1$ (commonly, values $v_{cut} = 10^{-3} \div 10^{-2}$ are incorporated in majority of algorithms for the muon propagation by now (Lipari and Stanev, 1991), (Antonioli et al., 1997), (Rhode and Chirkin, 2001)). At least, in case with atmospheric muons only radiative losses may be simulated while ionization may be treated as an entirely continuous process without any lost of accuracy. In spite of remarkable dependency of simulated survival probabilities upon accounting or not accounting for fluctuations in ionization (see fig.5) it seems to be correctly to treat ionization by completely continuous way also for neutrino-produced muons, as (a) differences in survival probabilities appear only for relatively low energies; (b) they are of few percents only; (c) they are of both signs and so, are expected to be partially self-compensated; (d) the uncertainties with $\nu N$ cross-sections are still much larger, in any case (Agrawal et al., 1996), (Gaisser, 2001), (Lipari, 2001), (Naumov, 2001). With such parameters there are only several muon interactions to be simulated per 1 km w.e.

We would like to emphasize that uncertainties with atmospheric muon flux intensity that are caused by only $\pm 1\%$ variations in energy losses are much higher than 1\% (see item (b) above). This is due to sharp power-law shape of surface atmospheric muon spectrum and due to the fact that source of atmospheric muons is far away from an underwater/ice
detector and their flux may only decrease when passing from the sea level down to detector depth. The source of muons that are produced in $\nu N$ interactions is uniformly distributed over water/ice and/or rock both out- and inside the array. So, uncertainties with energy losses should affect the results for neutrino-generated muons weaker. Indeed, intensity of the muon flux which accompanies the neutrino flux in a medium is proportional to the muon range that, in turn, is inversely proportional to energy losses and, consequently, an error in energy losses leads to an equal error with an opposite sign in simulated flux of muons which are born in $\nu N$-interactions.

4 “Inner accuracy” of muon transportation algorithm

Any muon MC propagation algorithm consists of a set of procedures on numerical solution of equations, interpolation and integration. All these procedures are of finite accuracy and, consequently, the incoming model for muon interactions is somewhat corrupted by them. Having a set of formulae for the muon cross-sections for bremsstrahlung, $e^+ e^-$-pair production, photonuclear process, and knock-on electron production it is easy to obtain the formula for the total averaged muon energy losses by integration of differential cross-sections over all range of kinematically allowed $\nu$:

$$\frac{dE}{dx}(E_\mu) = N A E_{\mu} \int_{\min}^{\max} d\sigma(E_{\mu},\nu) \nu dv$$  \(4\)

But energy losses as they are simulated by an algorithm will differ from (4) due to “corrupting” application of numerical procedures. Two algorithms with the same formulae for cross-sections at the input will reproduce the muon energy losses differently. The difference between simulated and calculated energy losses collects all errors that are contributed by each step of algorithm and thus, is a good quantitative criterium for its inner accuracy, whose contribution to the resulting error must not exceed 1%, as it was shown in the Sec. 2.

The trivial test may be suggested to check inner accuracy of any muon transport algorithm: let’s simulate propagation of $n$ muons (we used $n = 10^5$ for test reported below) with energy $E_0$ over a short distance $\Delta l$ “measuring” the final energy $E_i$ for each $i$-th muon at the end. Then, simulated energy loss may be computed as

$$\frac{dE}{dx}(E_\mu) = \frac{1}{n \Delta l} \sum_{i=1}^{n} (E_0 - E_i)$$  \(5\)

Comparing (5) with (4) it is possible to determine a goodness of the given algorithm applying, for example, criterium...
abs((5) - (4)) < 1%.

Fig.7, fig.8, and fig.9 show results of such a comparison for three muon transport codes: PROPMU (Lipari and Stanev, 1991), MUSIC (Antonioli et al., 1997), and MUM (Sokalski et al., 2001). Results for pure water are presented in fig.7. In case with MUM the difference abs((5) - (4)) never exceeds 1%, for MUSIC it reaches 2±3%. Inaccuracy of the PROPMU code is much worse and in a range $E_{\mu} < 10$ TeV is up to 25%. We try 2 versions of PROPMU with different values of $v_{cut}$ and different random generators (fig.8). In some cases inner accuracy becomes better but, in any case it stays worse than 10% that is unacceptable according to conclusion of Sec.2. Fig.9 reports the results of accuracy test for MUM and PROPMU with standard rock. Qualitatively, these results are the same.

In fig.10 differential spectra for atmospheric muons at different depths in pure water are presented as simulated with PROPMU, MUSIC and MUM. Muons at the sea level were sampled according to the same spectrum (Klimushin et al., 2001). Also parameterizations (Klimushin et al., 2001) and (Okada, 1994) are shown. Simulations with MUSIC and MUM give practically the same results because both codes reproduce almost the same energy losses (see fig.7). MUSIC’s and MUM’s spectra coincide with parameterization (Klimushin et al., 2001) that is based on the same sea level muon spectrum as was used for simulation presented on the plot and on muon propagation with MUM. Parameterization (Okada, 1994) is lower than KBS, MUM and MUSIC (up to 18% in terms of integral muon flux at $D = 1$ km) at relatively shallow depths and becomes higher starting with $D \sim 5$ km. The reason is that it is based on very hard sea-level muon spectrum with index $\gamma = 2.57$ (Miyake, 1973), while KBS parameterization adopts a spectrum with $\gamma = 2.72$. As a result, there are less muons with relatively low energies (which contribute the main bulk to the muon flux at low depths) and more muons of high energies which contribute to the muon flux at larger depths. The reduced muon energy losses in PROPMU result in the depth muon spectra which (a) are significantly higher (31%, 30%, 27% and 17% in terms of integral muon flux at the depths $D = 1$, 3, 6 km and 10 km, correspondingly) and (b) are expanded to the low energies. The spectrum for $D = 1$ km was simulated also with “corrupted” version of MUM: all muon cross-sections along with Bethe-Bloch ionization formula were multiplied by a factor 0.9, thus energy losses were reduced by 10% and became close to ones that are reproduced by PROPMU.

The result is given on upper left panel of fig.10 by open circles. A good agreement between PROPMU and MUM+ represents a good quantitative cross-check for results.

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3The majority of muons which compose the spectrum at 1 km depth have the energy in a range $E < 1$ TeV at the sea level. For this range PROPMU reduces the simulated energy losses by $\sim 10\%$ as can be seen in upper panel of fig.7.
5 Conclusions

The optimum value for inner accuracy of a MC code for the muon transport is close to 1%. In any case we never know the muon energy losses better. To keep such inner accuracy it is quite enough to simulate the muon interactions with a fraction of energy lost that are as large as $v > v_{cut} = 0.05 \div 0.1$, treating the rest of energy losses (soft part) by means of stopping-power formula.

The simple test may be applied to measure the accuracy of an algorithm for the muon transport by comparison of reproduced energy losses and incoming model for muon cross-sections.

Such a test has been applied to three muon transportation codes. According to “1% goodness criterium” codes MUM and MUSIC can be accepted as accurate enough. Accuracy of the PROPMU algorithm is insufficient (down to 25%) and depends on medium, version of code and simulation parameters. It leads, for example, to $\approx 30\%$ overestimation of atmospheric muon fluxes at depths $1 \div 5$ km w.e.

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