Weighing the spatial and temporal fluctuations of the dark universe

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A generic prediction of the standard cosmology, based on general relativity (GR), dark matter and the cosmological constant (and more generally, smooth dark energy), is that, the two gravitational potentials describing the spatial and temporal scalar perturbations of the universe are equivalent. Modifications in GR or dark energy clustering in general violate this relation. Thus this ratio serves as a smoking gun of the dark universe. We propose a method to extract this ratio at various cosmological scales and redshifts from a set of measurements, in a model independent way. The ratio measured by future surveys has strong discriminating power for a variety of dark universe scenarios.

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Introduction.— Predictions based on general relativity (GR) plus the Standard Model of particle physics are at odds with a variety of independent astronomical observations on galactic and cosmological scales, implying failures in particle physics or GR. There are various astrophysical tools to probe this dark side of the physical universe (e.g. 1,2). Combining them allows us to break parameter degeneracies, reduce statistical errors and diagnose possible systematics.

These multiple probes are also crucial to detect smoking guns of new physics. For example, combining probes of the expansion history of the universe and probes of the large scale structure, the relation between the expansion rate and structure growth rate can be checked for signs of deviation from GR 2. Indeed, one of the key questions in physics today is whether new particles/fields, such as dark matter and dark energy, or modifications to GR are needed to explain the observations.

On large scales, two features of gravity can distinguish between a dark sector and modified gravity 3,5,4,8. One is the effective Newton’s constant $G_{\text{eff}}$, which specifies the coupling between gravity and matter. In GR, $G_{\text{eff}}$ is equal to Newton’s constant, but modified gravity models often predict deviations. The other is the relation between the two gravitational potentials $\phi$ and $\psi$. Here, the two potentials are defined in the Newtonian gauge through $ds^2 = (1 + 2\psi)dt^2 - a^2(1 + 2\phi)dx^2$ where $a(t)$ is the scale factor. The ratio $\eta = -\phi/\psi$ weighs the relative ability of perturbations in matter-energy to distort the space-time.\footnote{Electronic address: pjzhang@shao.ac.cn, rbean@astro.cornell.edu, M.liguori@damtp.cam.ac.uk, dodelson@fnal.gov

\textsuperscript{1} Refer to other equivalent notations in 3, 4, 5, 6. An analogy of $\eta$ is the PPN parameter $\gamma$ (by forcing $\psi = -GM/r$ for point

on GR, dark matter and the cosmological constant (and more generally, smooth dark energy), predicts $\eta = 1$. Modifications from GR or emergence of intrinsic viscosity in dark energy fluid generally lead to $\eta$ deviating from unity. Therefore, identifying observations, or sets of observations, that will measure $G_{\text{eff}}$ and $\eta$ is of paramount importance 2, 4, 7, 8, 11.

In 4, we showed how to isolate the first key feature, feasibly testing the Poisson equation at ~ 1% accuracy level by combining weak lensing with galaxy redshift distortion. In this paper, we will show that the same surveys allow us to directly measure $\eta$, the second key feature, at cosmological scales. This can be done in a rather model independent manner.

Models with $\eta \neq -1$.— Here we consider three models which produce deviations from the standard prediction $\eta = 1 (\phi + \psi = 0)$.

Perturbations in the Dvali-Gabadadze-Porrati (DGP) model 12 have been carefully studied 13, 14. For a flat DGP model, $\eta = [1 - 1/3\beta_{\text{DGP}}]/[1 + 1/3\beta_{\text{DGP}}]$, where $\beta_{\text{DGP}} = 1 - 2r_c H(1 + H/3H^2) < 0$ and $H$ is the Hubble expansion rate. Here $r_c$ is the cross-over scale beyond which higher dimensional effects become important. In a flat model with matter density $\Omega_m$, $r_c = 1/H_0(1 - \Omega_m)$. Since $\beta_{\text{DGP}} < 0$, $\eta > 1$ in this model and the deviation from unity can be significant (Fig. 1).

Another modified gravity model (which aims to eliminate dark matter, not dark energy) is TeVeS 15, a relativistic version of MOND 16. Besides the gravitational metric, TeVeS contains a scalar $\phi_S$ and a vector field. It

\begin{align*}
\mathbf{a}^{\text{eff}} &= \frac{GM}{r} - \frac{\gamma \mathbf{a}}{c^2},
\end{align*}

and provided strong support of GR. Constraints at galactic size and sub-cluster scales are consistent with GR too 2, 11.
has been shown \cite{17,18} that the TeVeS vector field can source the evolution of cosmological perturbations and compensate for the lack of dark matter in the model. To fit observations, the TeVeS parameter $K_B$ should be small, in which case the vector perturbations $\alpha$ and $E$ become large. These vector perturbations then drive $\eta$ to deviate from unity \cite{17,18,19},

$$\phi + \psi = e^{4\bar{\phi}_S} \left[ \dot{\zeta} + 2 \left( \frac{\dot{a}}{a} + 2\bar{\phi}_S \right) \zeta \right] ,$$

(1)

Here $\zeta \equiv (e^{4\bar{\phi}_S} - 1)\alpha$. Since the background value $\bar{\phi}_S \ll 1$ as imposed by nucleosynthesis bounds, the deviation of $\eta$ from unity is mainly driven by the vector perturbation $(\phi + \psi \propto \bar{\phi}_S \alpha)$. A numerical evaluation of $\eta$ is shown in Fig. [1]. For this figure we adopted a model with $\Omega_b = 0.05, \Omega_c = 0.17, \Omega_{\Lambda} = 0.78$ and no dark matter.

A final possibility is that gravity is still GR, but dark energy has non-negligible anisotropic stress $\sigma$ and causes inequality in two potentials through \cite{20,21,22},

$$\phi + \psi = -12\pi G a^2 (1 + w) \bar{\rho}_{DE} k^{-2} \sigma .$$

(2)

Although quintessence models predict $\sigma = 0$, there are some dark energy models that predict $\sigma \neq 0$ and $\eta \neq 1$ \cite{21,22}. As a specific example, we consider an extrinsic shear stress of the form $\eta = 1/(1 + \omega)$, with $\omega = \omega_0 a^3 (1 - \Omega_m)/\Omega_m$ with $\omega_0$ constant, following \cite{20,21}. In general, $\eta$ varies not only with time, but also with scale. Richer physics encoded in the scale dependence of $\eta$ would allow better discrimination between such dark energy model from other scenarios.

The $\eta$ estimator.— To measure $\eta$, two independent measures of gravitational potentials are required. Both $\nabla \psi$ and $\nabla \phi$ source the particle acceleration. However, the contribution from $\phi$ is suppressed by a factor $v^2/c^2$, where $v$ is the particle velocity. For this reason, non-relativistic particles such as galaxies only respond to $\psi$. For the same reason, photons respond equally to both the potentials. Thus gravitational lensing measures the projected $\nabla^2 (\phi - \psi)$ along the line of sight. We propose an estimator consisting of the cross-correlation of each (the lensing field and the velocity field) with the galaxy distribution.

The first cross-correlation is the lensing measurement with galaxy over-density in a narrow redshift bin\cite{23,24}. We can then obtain the cross-power spectrum $P_{\nabla^2 (\phi - \psi) g}(k, z)$ between $\nabla^2 (\phi - \psi)$ and the galaxy number overdensity in the redshift bin associated with the galaxies.

The second cross-correlation power spectrum $P_{\theta g}$ can be obtained from the redshift distortions of the galaxy distribution in a spectroscopic survey \cite{23,24,25,26,27}. Here, $\theta_g = -\nabla \cdot \bar{\bar{v}}_g / H$ and $\bar{\bar{v}}_g$ is the comoving peculiar velocity. We show below that this cross-spectrum is directly related to $P_{\nabla^2 \psi g}$; but first let us assume that this is so, that $P_{\nabla^2 \psi g}$ can be extracted from the $\theta - g$ cross-correlation. In that case, the ratio of these two cross-spectra leads to an estimator for $\eta$:

$$\hat{\eta} = \frac{P_{\nabla^2 (\phi - \psi) g}}{P_{\nabla^2 \psi g}} - 1 .$$

(3)

To see that $\theta$ is related to $\psi$, recall that on large scales gravity is the only force accelerating galaxies, so $d(a\bar{v}_g^2)/dt = -\nabla \psi$, where $\bar{v}_g^2 = a\bar{v}_g^2$ is the proper motion. Taking the divergence of this leads to

$$\nabla^2 \psi = -\frac{d(a^2 \nabla \cdot \bar{v}_g)}{dt} = -\left( \ln [a^2 HD_\theta] \right)^' a^3 H \nabla \cdot \bar{v}_g(a) .$$

(4)

Here, $' \equiv d/da$ and $D_\theta$ is the growth factor of $\theta_g$. The last relation holds in the linear regime where different modes decouple. We then have

$$P_{\nabla^2 \psi g} = -\left( \ln [a^2 HD_\theta] \right)^' a^3 H^2 P_{\theta g} ,$$

(5)

the desired relation.

The proportionality factor relating the two cross-spectra in Eq. (5) requires knowledge of the expansion rate $H(z)$ and the growth factor $D_\theta(z)$. We assume that the former can be measured by other means; indeed our goal is to distinguish dark sector models which produce identical expansion histories. No such assumption is needed for the growth factor, because the same

![Graph](image.png)
survey that measures $P_{θθ}$ will also measure $P_{θη}$, which is proportional to $D_θ^2$ and thus measurement of $P_{θθ}$ in multiple redshift bins can be used to recover $(\ln a^2H D_θ)'$ (see the appendix for details). We adopt the minimum variance estimator to estimate errors in the reconstruction of $P_{θθ}$ and $P_{θη}$ [1, 24]. This reconstruction adopts no assumption on galaxy bias, so it is less affected by possible stochasticity or scale dependence in galaxy bias.

Application of the $η$ estimator in Eq. (3) relies on the condition of linear evolution such that Eq. (4) and therefore (6) hold. For this reason, we restrict our discussion to the linear regime. This approach is robust against several uncertainties: (1) It does not suffer uncertainty induced by the galaxy bias, whose effect cancels when taking the ratio in Eq. (4). (2) It is not susceptible to possible galaxy velocity bias, defined with respect to peculiar velocity of dark matter or dark energy, since we directly measure $(\ln a^2H D_θ)'$, instead of relying on a theory to calculate it. (3) It is applicable to general dark energy models and modified gravity models. It does not require dark energy to be smooth, nor gravity to be minimally coupled, nor scale-independent $D_θ$.

Forecast.—In order to measure $η$ in this way, the lensing and redshift surveys must be sufficiently deep and wide. The proposed spectroscopic galaxy survey ADEPT or 21cm survey HSHS [23], combined with a lensing survey such as LSST, would be sufficient. Alternatively, SKA alone would be able to provide both suitable lensing through cosmic shear [24] and cosmic magnification [27], and galaxy redshift measurements, as potentially would the Euclid mission. So we focus on SKA projections. $D_θ$ can be measured by SKA at multiple bins of redshift and scale to impressive accuracy (Fig. 2). We then infer $(\ln a^2H D_θ)'$ from the above measurements.

Projections for the errors on $η$ from SKA in a variety of $(k, z)$ bins are shown in Fig. 1. One example of the power of this measurement is in constraining the DGP model. The $E_G$ measurement proposed in [44] can only marginally distinguish the $Ω_m = 0.2$ flat DGP model from ΛCDM. Fig. 1 shows, though, that these models have significantly different predictions for $η$. The TeVeS model adopted has been shown to produce a good fit of CMB and LSS data [17]. However, with large deviation from $η = 1$, this model can be unambiguously distinguished from ΛCDM. Thus $η$ and $E_G$ are highly complementary to probe the dark universe. Modifications in gravity or dark energy viscosity often lead to stronger scale dependence in $η$ than what is shown in Fig. 1. Our $η$ estimator could have stronger discriminating power for these models.

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2 http://sci.esa.int/science-e/www/area/index.cfm?fareaid=102
3 Errors in $η$ and in $E_G$ are partly correlated. Future work should take this into account by fitting $η$ and $E_G$ simultaneously, while marginalizing all other parameters.
from the inclusion of such observations. (3) Furthermore, measurements of \( \eta \) at \( z > 2 \) can be made feasible by the inclusion of CMB lensing and 21cm background lensing.

We have shown that future precision imaging surveys of weak gravitational lensing and spectroscopic surveys of galaxy redshift distortions provide highly complementary methods to probe the dark universe. In combination they allow us to isolate two key features of the dark universe, the effective Newton’s constant \( G_{\text{eff}} \) and \( \eta \equiv -\dot{\phi}/\psi \), from many astrophysical complexities, and distinguish competing scenarios of the dark universe robustly.

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Appendix.—To infer \( \ln(a^2H D_0) \) from \( D_0 \) measured in limited redshift bins, a parametrization of \( D_0 \) is required. Since \( D_0 \) evolve smoothly, \( \ln(a^2H D_0) \) should not be strongly dependent on the precise form of the parametrization. In this paper, we extend a widely used parameterization for \( D_0 \) in standard gravity. For gravity models minimally coupled to matter, \( D_\theta = D' a = f D \), where \( f = d \ln D/d \ln a \) and \( D \) is the linear density growth factor. One approximation adopted in the literature is \( f \approx (\Omega_m a^{-3}/E^2) \gamma \) (e.g. [29]). Here, \( E \equiv H/H_0 \) is the normalized Hubble parameter. This approximation works well not only for CDM (\( \gamma = 5/9 \) for \( \Omega_\Lambda + \Omega_m = 1 \) and \( \gamma = 0.6 \) for \( \Omega_\Lambda = 0 \)), but also for some modified gravity models such as DGP (\( \gamma = 2/3, \Omega_\Lambda = 0 \)). We thus propose to fit a parameterization

\[
\gamma_* \equiv \left( \frac{\Omega_m a^{-3}}{E^2} \right) \gamma.
\]

Here, both \( \Omega_* \) and \( \gamma_* \) are parameters to be fitted for each \( k \) bin. \( D \) and \( D_\theta \) are then obtained by the relation \( f = d \ln D/d \ln a \) and \( D_\theta = f D \).

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