Effect of thermal radiation and heat absorption of MHD Casson nanofluid over a stretching surface in a porous medium with convective heat and mass conditions

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Abstract. This article explores the impact of radiation and heat absorption on magneto-convective flow of Casson nanofluid over a stretching surface in a porous medium. The convective heat and mass boundary conditions are also taken into consideration in this study. The governing models are converted into nonlinear ODE models and they are solved analytically using homotopy analysis method (HAM). The significance of physical parameters on velocity, temperature and nano particle volume fraction are discussed graphically.

Keywords. Casson nanofluid, MHD, thermal radiation, suction/injection, convective heat and mass conditions.

1. Introduction
In heat transfer equipments, the thermal conductivity of a fluid plays a key role. Inherently, conventional heat transfer liquids, like water, oil, etc. have low thermal conductivity. So, many researchers are interested in their studies which aim to increase the thermal conductivity of the ordinary liquids. Nano sized particles are added to ordinary liquids to enhance the thermal conductivity. The liquids with nano particles are termed as nanofluids. The problem of boundary layer flow of a nanofluid with heat source/sink was investigated by Ahmed et al.[1]. Other studies on nanofluid flow in different physical situations are given in the Refs.[(2]-[8]). The Casson fluid, introduced by Casson [9], is one of the subclass of non Newtonian fluids which describes a shear thinning case. The viscosity of a Casson fluid is infinity when the stress rate is zero. However, the stress rate raises up to infinity, the viscosity comes down to zero level. Tomato sauce, honey, human blood, etc. are some of the examples. The Casson fluid flow behavior over an exponentially shrinking sheet was studied by Nadeem et al. [10]. Hayat et al. [11] analyzed the Casson nanofluid over a convective heated surface with chemical reaction.

Many engineering and industry problems are highly non-linear and very difficult to solve these problems analytically. HAM is one of the simplest method to solve highly non-linear problems, see ([12]-[16]). The major emphasis of this investigation is to discuss the radiation and heat absorption effects on magneto-convective flow of Casson nanofluid over a stretching surface in a porous medium with convective heat and mass boundary conditions.
2. Mathematical Formulation

Let us presume the 2D flow of a Casson nanoliquid over a stretching surface in a porous medium. The sheet is stretched with velocity $U_x = ax, \alpha > 0$ in the $x-$direction. The fluid phase is having heat absorption/generation and thermal radiation. A constant magnetic field with strength $B_n$ is applied on $y-$direction and the induced magnetic field is omitted because of small Reynolds number. In the above considerations, the governing equations are defined as, see [11],

\[ U_x + V_y = 0 \]  \hspace{1cm} (1)

\[ UU_x + VU_y = \nu \left(1 + \frac{1}{\beta}\right) U_{yy} - \frac{\sigma B^2}{\rho} + G\beta T (T - T_\infty) + G\beta C (C - C_\infty) \]  \hspace{1cm} (2)

\[ UT_x + vT_y = \alpha_T T_{yy} - \frac{16\sigma_s T^3}{3k_c \rho c_p} T_{yy} + \rho^* c_p \left[ DB C_y T_y + \frac{DT}{T_\infty} T_{yy} \right] + \frac{Q}{\rho c_p} (T - T_\infty) \]  \hspace{1cm} (3)

\[ UC_x + vC_y = DB C_{yy} + \frac{D_T}{T_\infty} T_{yy} \]  \hspace{1cm} (4)

where $(U & V), \beta, \sigma, (\rho & \rho^*), G, (\beta_T & \beta_C), \alpha_T, (\sigma_s & k_s), (C_p & C_p^*), (DB & D_T), Q$ velocity components, Casson fluid parameter, electrical conductivity, desity of fluid & nanoparticles, gravitational acceleration, thermal & concentration expansion coefficients, thermal diffusivity, Stefan-Boltzmann constant & mean absorption coefficient, specific heat of fluid & nanoparticles, Brownian diffusion & themophoretic diffusion coefficients and heat absorption/generation coefficient.

The following boundary conditions are

\[ U = U_w(x) = ax, \] \hspace{1cm} \[ V = V_w(x), \] \hspace{1cm} \[ -k_T \frac{\partial T}{\partial y} = h_T (T_f - T), \] \hspace{1cm} \[ -k_C \frac{\partial C}{\partial y} = h_C (C_f - C), \] \hspace{1cm} \[ \text{at } y = 0, \]

\[ U \to 0, V \to 0, T \to T_\infty, C \to C_\infty, \] \hspace{1cm} \[ \text{at } y \to \infty \] \hspace{1cm} (5)

The dimensionless variables are

\[ \eta = \sqrt{\frac{a}{\nu}} y, \] \hspace{1cm} \[ U = ax F' (\eta), \] \hspace{1cm} \[ V = -\sqrt{a \nu} F (\eta), \]

\[ \Theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}, \] \hspace{1cm} \[ \Phi(\eta) = \frac{C - C_\infty}{C_f - C_\infty}, \] \hspace{1cm} (6)

The equations (2)-(4) are converted as follows

\[ \left(1 + \frac{1}{\beta}\right) F'' + FF'' - F'r^2 + Ri \left[ \Theta + N \Phi \right] - \left[M + \Gamma\right] F_\eta = 0, \] \hspace{1cm} (7)

\[ \left(1 + \frac{4}{3} Rd\right) \Theta'' + Pr F \Theta' + Pr \left[Nb \Theta' \Phi + Nr \Theta'^2\right] + Pr Hg \Theta = 0 \] \hspace{1cm} (8)

\[ \Phi'' + Sc F' \Phi' + \frac{Nt}{Nb} \Theta'' = 0 \] \hspace{1cm} (9)

and boundary conditions are

\[ F(0) = F_w, \] \hspace{1cm} \[ F'(0) = 1, \] \hspace{1cm} \[ \Theta'(0) = -B_T [1 - \Theta(0)], \] \hspace{1cm} \[ \Phi'(0) = -B_C [1 - \Phi(0)], \]

\[ F'(\infty) = 0, \] \hspace{1cm} \[ \Theta(\infty) = 0, \] \hspace{1cm} \[ \Phi(\infty) = 0, \] \hspace{1cm} (10)

where $Ri, N, M, \Gamma, Rd, Nb, Nt, F_w, B_T, B_C$ are the Richardson number, buoyancy ratio parameter, magnetic field parameter, porosity parameter, radiation parameter, Brownian motion parameter, and other parameters.
parameter, themophoretic parameter, injuction/suction parameter, thermal Biot number and mass Biot number with fixed values of Prandtl number \((Pr = 2)\) and Schmidt number \((Sc = 1)\).

The skin friction coefficient and the Nusselt number are defined as follows,

\[
\frac{1}{2} C_f \sqrt{Re} = \left(1 + \frac{1}{B} \right) F'(0); \quad Nu = \frac{1}{2} \sqrt{Re} = -\left(1 + \frac{4}{3} Rd \right) \Theta'(0).
\]

3. HAM Solution
The HAM is applied to solve the equations (7) - (9) with boundary conditions (10). The initial approximations are

\[
F_0(\eta) = F_w + 1 - \exp(-\eta), \quad \Theta_0(\eta) = \frac{B_T e^{-\eta}}{1 + B_T}, \quad \Phi_0(\eta) = \frac{B_C e^{-\eta}}{1 + B_C}.
\]

These HAM solutions are having the parameters \(h_F, h_\Theta\) and \(h_\Phi\) and they maintain the convergence of the solution. The \(h_F, h_\Theta\) and \(h_\Phi\) curves are shown in figure 1 and indicates that the range values of \(h_F, h_\Theta\) and \(h_\Phi\) are \(-0.6 \leq h_F \leq -0.1, -0.8 \leq h_\Theta \leq 0.1\) and \(-0.7 \leq h_\Phi \leq 0.0\). We fix the \(h\) value from this range is \(h_F = h_\Theta = h_\Phi = -0.4\).

4. Result and Discussion
In this section we show the influence of physical parameters, like, Casson fluid parameter \((\beta)\), magnetic field parameter \((M)\), injection/suction parameter \((F_w)\), porosity parameter \((\Gamma)\), heat absorption/generation parameter \((H_g)\), Richardson number \((R_i)\), radiation parameter \((R_d)\), Brownian motion parameter \((N_b)\), themophoretic parameter \((N_t)\), thermal Biot number \((B_T)\) and mass Biot number \((B_C)\). Table 1 portrays the different order of approximation of HAM and found that 15\(^{th}\) order is sufficient for both distributions.

Variations of the skin friction coefficient and the local Nusselt number with different values of \(\beta, M, R_i, F_w, \Gamma, R_d, H_g, N_b, N_t, B_T & B_C\) are shown in Table 2. It is found that the skin friction coefficient enhances with rising values of \(\beta, R_d, H_g, N_b & B_T\) and the reverse trend is obtained for large values of \(F_w, N_t & B_C\). The heat transfer rate boosted up when increasing the values of \(F_w, R_d & B_T\). However, it suppresses with enhancing the values of \(\beta, H_g, N_b, N_t & B_C\). Figures 2(a-b) delineate the impact of \(\beta, M, F_w & \Gamma\) on velocity distribution \(F_\eta(\eta)\). It is seen that the fluid velocity enhances with enhancing the values of \(\Gamma\) and compress with growing the values of \(\beta, M & F_w\). The temperature distribution \(\Theta(\eta)\) for various values of \(H_g, N_t, N_b & B_T\) are exhibited in figures 3(a-d). We found that the fluid temperature is upgrading with escalating the values of \(H_g, N_t, N_b & B_T\).

5. Conclusion
The present investigation analyzed the effect of magnetic field, radiation and heat absorption of Casson nanofluid over a stretching surface in a porous medium with convective heat and mass boundary conditions. The governing models are converted into a nonlinear ODE models and are solved analytically using homotopy analysis method (HAM). The following are the findings of our study

- The fluid velocity rises with enhancing the \(\Gamma\) values and reduces with increasing the \(\beta, M & F_w\)
- The fluid temperature is an enlarging function of \(H_g, N_t, N_b & B_T\).
- The skin friction coefficient is high with ascending values of \(\beta, R_d, H_g, N_b & B_T\) and quite opposite for big values of \(F_w, N_t & B_c\).
- The local Nusselt number enhances with increasing the values of \(F_w, R_d & B_T\) and it suppresses with enhancing the values of \(\beta, H_g, N_t, N_b & B_C\).
Table 1. Order of approximations

| Order | $-F''(0)$ | $-\Theta'(0)$ | $-\Phi'(0)$ | Order | $-F''(0)$ | $-\Theta'(0)$ | $-\Phi'(0)$ |
|-------|-----------|---------------|-------------|-------|-----------|---------------|-------------|
| 1     | 0.73031   | 0.23017       | 0.21515     | 25    | 0.74058   | 0.23472       | 0.16896     |
| 5     | 0.73932   | 0.23477       | 0.18309     | 30    | 0.74058   | 0.23472       | 0.16896     |
| 10    | 0.74063   | 0.23477       | 0.16988     | 35    | 0.74058   | 0.23472       | 0.16896     |
| 15    | 0.74058   | 0.23472       | 0.16896     | 40    | 0.74058   | 0.23472       | 0.16896     |
| 20    | 0.74058   | 0.23472       | 0.16896     | 45    | 0.74058   | 0.23472       | 0.16896     |

Table 2. The skin friction coefficient and the local Nusselt number with different values of $\beta, Fw, Rd, Hg, Nb, Nt, BT \& BC$.

| $\beta$ | $Fw$ | $Rd$ | $Hg$ | $Nb$ | $Nt$ | $BT$ | $BC$ | $\frac{1}{2}C_f\sqrt{Re}$ | $Nu/\sqrt{Re}$ |
|---------|------|------|------|------|------|------|------|-----------------|---------------|
| 0.3     | 0.2  | 0.3  | -0.2 | 0.2  | 0.2  | 0.3  | 0.3  | -2.26472        | 0.33210       |
| 0.6     |      |      |      |      |      |      |      | -2.10094        | 0.32817       |
| 0.9     |      |      |      |      |      |      |      | -1.88102        | 0.32720       |
| 1.2     |      |      |      |      |      |      |      | -1.76010        | 0.32656       |
| 0.5     | -0.6 | 0.3  | -0.2 | 0.2  | 0.2  | 0.3  | 0.3  | -1.81445        | 0.25801       |
|         | -0.3 |      |      |      |      |      |      | -1.96332        | 0.28966       |
|         | 0.0  |      |      |      |      |      |      | -2.11650        | 0.31503       |
|         | 0.3  |      |      |      |      |      |      | -2.27597        | 0.33456       |
|         | 0.6  |      |      |      |      |      |      | -2.44348        | 0.34922       |
| 0.5     | 0.2  | 0.3  | -0.2 | 0.2  | 0.2  | 0.3  | 0.3  | -2.23062        | 0.24445       |
|         | 0.4  |      |      |      |      |      |      | -2.21922        | 0.35559       |
|         | 0.8  |      |      |      |      |      |      | -2.20870        | 0.45882       |
|         | 1.2  |      |      |      |      |      |      | -2.19885        | 0.55577       |
| 0.5     | 0.2  | 0.3  | -0.2 | 0.2  | 0.2  | 0.3  | 0.3  | -2.22955        | 0.34269       |
|         | -0.3 |      |      |      |      |      |      | -2.22440        | 0.33294       |
|         | 0.0  |      |      |      |      |      |      | -2.21522        | 0.31722       |
|         | 0.3  |      |      |      |      |      |      | -2.19778        | 0.28753       |
|         | 0.6  |      |      |      |      |      |      | -2.09699        | 0.14327       |
| 0.5     | 0.2  | 0.3  | -0.2 | 0.5  | 0.2  | 0.3  | 0.3  | -2.21144        | 0.32540       |
|         | 1.0  |      |      |      |      |      |      | -2.20542        | 0.31969       |
|         | 1.5  |      |      |      |      |      |      | -2.20111        | 0.31362       |
|         | 2.0  |      |      |      |      |      |      | -2.19700        | 0.30718       |
| 0.5     | 0.2  | 0.3  | -0.2 | 0.2  | 0.0  | 0.3  | 0.3  | -2.20854        | 0.33087       |
|         | 0.5  |      |      |      |      |      |      | -2.24208        | 0.32516       |
|         | 1.0  |      |      |      |      |      |      | -2.27538        | 0.31905       |
|         | 1.5  |      |      |      |      |      |      | -2.30834        | 0.31257       |
| 0.5     | 0.2  | 0.3  | -0.2 | 0.2  | 0.2  | 0.001 | 0.3 | -2.23837        | 0.00014       |
|         | 0.1  |      |      |      |      |      |      | -2.23229        | 0.12833       |
|         | 1.0  |      |      |      |      |      |      | -2.19915        | 0.71383       |
|         | 5.0  |      |      |      |      |      |      | -2.16708        | 1.16260       |
| 0.5     | 0.2  | 0.3  | -0.2 | 0.2  | 0.2  | 0.3  | 0.001 | -2.21148        | 0.33031       |
|         | 0.1  |      |      |      |      |      |      | -2.21576        | 0.32963       |
|         | 1.0  |      |      |      |      |      |      | -2.23276        | 0.32687       |
|         | 5.0  |      |      |      |      |      |      | -2.24431        | 0.32495       |
Figure 1. \( h \) curves of \( F''(0), \Theta'(0) \) and \( \Phi'(0) \).

Figure 2. Velocity distribution for different values of \( \beta, M, F_w \) & \( \Gamma \)

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Figure 3. Temperature distribution for various values of $H_g$, $N_t$, $N_b$ & $B_T$.

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