Shear strength and Characterization of Reinforced Concrete Deep Beams -A Review

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Abstract: Reinforced concrete deep beams are structural elements that primarily pass-heavy gravity loads to their supports by shearing action. High shear strength is a significant feature of deep beams. This is due to the inner tied-arch mechanism that transmits the load right away out of concrete compressive struts to the supports. Various researchers and codes of practice that vary in principle have suggested different meanings for RC deep beams. Several researchers have also developed different shear strength prediction models and codes for the calculation of the shear strength of deep beams by analytical models and comprehensive services for exams. There seems to be, however, no consensus on which description properly classifies what a deep beam is and which paradigm creates a great result that similar to experimental values, the value of shear strength of beams. The research compared the concepts and evaluated deep beam shear capacity models based on data from 210 deep beams according to different code provisions acquired in literature. The ACI-318 code description concept was found to provide the most practical characterization of a deep beam, while its shear strength model, based on the method of the strut-and-tie model, offers the best estimate for a deep beam's shear strength.

Keywords: Shear strength patterns, Deep beams, Average margin of safety, Strut-and-tie model.

1. Introduction

Reinforced deep concrete beams are structural elements that primarily pass heavy gravity loads to their supports by shearing action. High shear strength is a significant feature of deep beams. This is due to the inner tied-arch mechanism that transmits the load right away out of concrete compressive struts to the supports [1]. A great confrontation in any high rising structure is obtaining sufficient column-free space for parking or storage facilities on the lower level. In higher floors, the vertical feature or known as floating column is present to achieve spacious residence room size architectural design. The terminal level based on the girder of the transfer that serves as a point load. Deep beams are mainly advised as transfer girders in view of sufficient shear power. Such members pass loads in the transverse direction via the loading face to the supports. The deep horizontal components often struggle in shear instead of flexure. These beams are distinguished by a tiny percentage between duration and depth. Examples of deep RC beams are pile caps, corbels, braces, base walls and off-shore structures [2]. Reinforced concrete (RC) beams typically display
low shear resistance as the beam depth increases [1, 3-5]. The size effect is known as this phenomenon [5], [6]. However, it is also established by experiments that the shear strength of the beam rises regardless of the depth of the RC beam, when the ratio of \( a/d \) (shear span to effective depth) decreases [1,7]. The division of RC beams into deep and slender beams leads to this phenomenon [8]. Areas in RC beams where the shear span is less than twice the depth of the member are controlled by tied-arch action instead of beam action and are therefore classified as D-regions (disturbed or discontinuity regions) [9-11]. The presence of one or more D-regions characterizes deep beams [12]. Therefore, the RC deep beam design do not comply with the basic bending theory according to the Bernoulli theory, as the approach ignores the impacts of shear displacement and concentricity of stress, and supposes that before bending, divisions common to the neutral axis remained the same before and after bending. This means that (zero) is the transverse shear strain, which is not true for deep beams in operation. For example, the shear capacity of a deep beam can be two to three times higher than that anticipated using standard shear models used for slender beams, depending on the Euler-Bernoulli hypothesis, these assumption under-rates deflections in deep beams, otherwise shear deformation effects are very important and therefore underestimate their shear capacity [1,2]. Despite the difficulties created by deep beam design, the use of RC deep beams has become popular recently. They also notice their employment in the construction of transmitting girders in buildings that have a high-rise, and also the foundation walls, folded plate roof frameworks, bins, water tanks, floor diaphragms, pile caps, shear walls, and corbels or brackets, [4]. Over the years, several researchers have suggested a number of models and wide-ranging try expedition have been utilized on deep beams. Various concepts have also been suggested to describe RC deep beams by multiple studies and codes that may differ in concept. However, there seems to be no consensus on which description accurately classifies what a deep beam is, and which model produces best results similar to experimental values, the shear strength of this class of beams. The goal of the research was to compare the definitions and shear strength models provided by different code provisions for reinforced concrete deep beams, and to suggest the utmost reliable code to properly specify the shear strength of deep beams subjected to loading. The elastic solutions on deep beams give a great explanation of their behavior before the act of cracking. However, substantial redistribution of strains and stresses occurs after the development of a diagonal crack and, thus, the shear potential of the beam should be predicted by non-linear test. The top load and bottom interactions come up with broad compressive stresses perpendicular to the beam axis for a plain supported deep beam with a point load above. In order to form a complex stress field in the web, these stresses work along with shear stresses. The influence of such stresses results in a tied-arch action distinct in deep beams due to the limited horizontal space amidst top and bottom load points (small \( a/d \) ratios). This study regarding deep beams has become a special area of interest due to these dynamic stresses and strains.

2. A Deep Beam Concept

A beam is characterized as a deep beam in such way a clear span is equivalent to or smaller than 4 times the total depth or the concentrated loads are less or equivalent than twice the depth of the support face [9,13]. The ACI-318 describes deep beams as commissioners with an effective length \((Ln)\) not more than 4 times the total depth of the member or zones of beams loaded twice the depth of the member from the support face, with the beam placed on one side and supported on the opposite side in such manner that compression struts formed between the points of supports and loads [9].According to India Standard [14] a beam shall be deemed to be a deep beam when the ratio of effective span-to-overall depth, \(1/D\) is less than:2 for a beam supported simply; and 2.5 For a persistent beam, Although deep beams are described in various simple span-to-depth ratios by different codes, deep beams are known by their tiny span-to-depth ratio as a general rule [15]. The Canadian Code specifies that a deep beam must be defined as a flexural limb followed by a clear period \((Ln)\) to a ratio of total depth (h) of less than 2 and that the non-linear distribution of strains must be taken into account [16].
The authors categorize deep beams as transmitting beams and classified as horizontally aligned elements which possess a value of the ratio \( a / d \) lower from 2, which, by forming a diagonal crack, handle high gravity loads, primarily through shear [17]. A deep beam is described, according to Adinkrah and Adom [18], as an undeviatingly loaded beam with the value of \( a / d \) among 0.5 to 2.5. Shallow or slender beams are also described by the authors as beams with \( a / d \) exceeded than 2.5, while the beams that have a ratio of \( a / d \) smaller than 0.5 are named brackets and corbels. They clarify deep beams as the beams that show tied-arch attitude after the development of oblique cracking that permits the beam to have the acceptable alternate shear capacity. Shallow beams may be known as beams that may not well developed arching attitude and usually falls apart following the formation of diagonal cracking only if shear reinforcement is applied. Beams with a \( a / d \) ratio of less than 0.5 were considered to be corbels and brackets, according to the authors, with a low risk of developing inclined or diagonal cracking until they were defeated by a process of sliding or shear friction. The authors assume that deep beams are a switch between slender beams and brackets or corbels. There are two key principles for describing deep beams from the above definitions: the ACI-318 [9] based definition and the [14] based definition. A RC beam is defined by the ACI-318 [9] based description as a deep beam based on two distinctive parameters: geometry or loading state. In geometry terms, a horizontal structural member is known as a deep beam by the definition ACI-318 when the proportion of its sufficient length \( L \) to the amount of the whole depth \( h \) is lower than or equal to 4, as shown in Figure 1. For the state of loading, the ACI-318 code defines any beam is considered as a deep beam when the whole beam has been loaded with a concentrated load within the double value of the depth of the elements. This could also be defined as the existance of D-regions on the beam because of loading irregularities, despite of effective beam length, and the overall depth ratio [12]. A D-region is a part of an element within a range of the beam depth \( h \) from an abruption of force or a geometric abruption [9].

![Figure 1. Deep beam description on the basis of geometry (ACI-318, 2008).](image)

Based on the geometry specified by the ACI description, the percentage of the effective length to total depth within four means that, regardless of the loading state, there are 4 beams depth \( h \) which connect to form a beam length. It means that there would be 2 D-zones on each side of the load identified by 2 beams depth amidst load and supports \( (av = 2h) \) if a middle point load is put on the beam, as shown in Figure 2. This also means that between the load and each support that defines the 2 D-regions, a shear period of length two hour exists. On the other side, if 2 symmetrical point loads are positioned as shown in Figure 3 in the middle quarter of the beam, then there are 2 D-regions on either side of the shear loads described by less than 2 beam depths but more than one beam depth\( (h < av < 2h) \). Also, if in the first quarter of the beam the two symmetrical point loads are placed, there are 2 D-regions on either side of the loads identified by a shear span of smaller than single beam depth \( h \) amidst the loads and the supports \( (av < 2h) \) as shown in
Figure 4. This description implies that D-regions occur on the beam when the range is within 4 beam depths, regardless of the type of loading on the beam, as geometrically described by the ACI code, whether uniformly distributed or point loads, and can therefore be categorized as a geometry-based deep beam.

![Figure 2](image1)

**Figure 2.** D-Regions identified by the central-point load (ACI-318, 2008).

For the other alternative ACI identifies a deep beam, the focus upon the loading mode, regardless of the beam longitude. The type of load supposed to be point load and space among one support and load have to be two times the amount of the effective depth, which means that the value of the shear length is supposed to be at most double the depth of the beam \( (aV \leq 2h) \). It further means that on one end of a beam there should be at least two D-regions that interconnect to define it as a deep beam (Figure 5). This description is also true since non-linear strains are generated by the presence of D-regions on an RC beam, which is a main aspect of a deep beam.

![Figure 3](image2)

**Figure 3.** D-Regions described by (ACI-318, 2008) in the middle quarter regions with two symmetrical point loads.
Figure 4. D-Regions identified by (ACI-318, 2008) with two symmetric point loads in the area regions of the first quarter.

Figure 5. D-Regions described by (ACI-318, 2008) based on the condition of loading.

If the beam carries two symmetrical point loads, then, as shown in Figure 6, the D-regions are described by a space smaller than \( \frac{1}{2} \) the depth of the beam or by shear spans less than the depth of the beam between the loads and the supports (\( a_v < h \)).

Figure 6. D-Regions described by (IS-456, 2000) with two loads of symmetric points.
The IS-456 based description, while an RC beam classifies as a deep beam only in geometry terms. And, also regarding the geometry, the code considers an RC beam as a deep beam where the $a/d$ of the effective period to total depth ($h$) is lower than 2. The ratio of effective span to total depth of two therefore means that there are mainly two beam depths connected to define and determine the beam length, even in terms of geometry, as shown in Figure 7.

![Figure 7. Deep beam definition according to (IS-456, 2000).](image)

Now, in case of a beam have a central point load, it will mean that, as shown in Figure 8, there is be 2 D-areas on either side of the load specified by half a beam depth or a shear span of single beam depth amidst load and supports($av = h$).

![Figure 8. D-Regions identified by (IS-456, 2000) with a load at the center point.](image)

For deep beams, the British Standards does not include precise design considerations, but only suggests contacting specialist literature [19]. It recognizes, however, the increase in the strength of concrete shear $vc$ in beams with parts near the face of a support or concentrated load with two times the effective depth, $d$ and continues to suggest an ability of shear enhancement from $vc$ to $2dvc$ / $av$. Similar to the India Standard [14] concept, the British Standards considers transparent span beams (l) to be less than twice the effective span capacity [20]. Similarly, the British Standards adopts an alternative approach by reducing the shear force by a factor of $av/2d$ due to loading within $2d$ of the support [21]. This identification of shear enhancement in parts near a support or concentrated load in RC beams conflicts the concept of a central point loaded deep beam by British Standards [19]. This is because a distance two times the effective depth of a central point load on both sides indicates that the total beam length shouldn’t be lower than 4 times the effective beam depth ($4d$) or four beam depths and not twice the effective beam depth as prescribed in the definition [20]. The India Standard based description implies that a deep beam contain $av$ / ($h = 1$) where
the beam have a central point load and \( \alpha v / (h < 1) \) in case of it has 2 symmetrical point loads [14]. This identification is limited as it eliminates different beam geometry likelihood where D-regions can happen on the beam, for example where four beam depths are collected of the beam span, as explained by the definition [9]. Moreover, several studies represent \( \alpha / d \) of approximately two times on a loaded RC beam causes nonlinear strikes in the beam as a result of the overriding of tied arch act that transfer the load from the point of loading to the support via struts of concrete; which makes the beam perform as a deep beam with greater shear capacity [7, 22-25].

3. The factors that affect the shear strength of Deep beams.
A number of variables influence a deep beam’s shear strength. Handful of most significant ones comprise compressive strength of concrete, Shear span to depth ratio, vertical and horizontal shear reinforcement, flexural reinforcement [26]. Affection of every constituent of deep beams regarding shear capacity is discussed.

3.1 The impact of Compressive Strength on deep beam concrete.
Compressive strength of concrete owns primary factor in the structural strength of deep beams. The value of the compressive strength of concrete raises the nominal shear stress [27], as shown in Figure 9.

![Figure 9. Impact of concrete's compressive strength on the nominal shear strength.](image)

The compression softening effect [28] in compression is all about cracked reinforced concrete with lesser strength and stiffness than uniaxial compressed concrete. The component strength, ductility and load-deformation curve are affected by this effect. The amount of cracking calculated by the key tensile strain was a parameter that affected the compression softening effect. Undertaking monotonic loadings, the load direction, crack orientation related to the reinforcement, crack rotation and form of reinforcing bar calculated zero impact on the softening effect. In case of the so-called effect of compression softening is overlooked or underestimated; the element's strength prediction can be overestimated. In addition, with the inclination angle of the strut, there will be a reduction in the strength reduction coefficient for the master strut [29]. According to Ahmed A K E Showed that 45% increase in \( f_{c'} \) is followed by a 10% increase in shear [30]. This rise is not proportional as the fractured aggregates at the ultimate load would produce lower friction in comparison to standard strength concrete at high strength or lightweight concrete. Similarly, deep beam research presents that \( f_{c'} \) has a significant effect regarding the ability to shear [26]. Their findings
showed that in the case of deep beams with high $f_{c}'$ and low web reinforcement, the shear potential is greater compared to a beam with low $f_{c}'$ and high web reinforcement. The tests performed, however, were limited to standard strength concrete only ($f_{c}' = 16$ to $23$ MPa). On the other hand, the compressive strength of concrete ($f_{c}' = 24$ to $37$ MPa) has been found to have a slight change on the shear ability of deep beams [17].

3.2 Impact of beam Web Reinforcement

Vertical web reinforcement considered to be an important variables which influence the shear strength of deep beams, apart from $a/d$. The fundamental intent of vertical web reinforcement is to supply containment for the concrete, that aids to upgrade capacity of deep beam shear. Vertical reinforcement also increases the shear strength of deep beams more efficiently than horizontal shear reinforcement, and vertical reinforcement helps the beam fail in a more ductile way in the case of shear failure [17, 31]. Several studies have shown that deep beam shear strength rises linearly with an increase in vertical shear strength [26, 32-37]. However, [26] found that as the a/d declines ($a/d < 1$), the contribution of the vertical shear reinforcement decreases. A similar study [38] found that the higher the $a/d$ ratio ($a/d > 0.75$), the higher the contribution to vertical web reinforcement. Contrariwise, [17] it has been found that the increment of the shear strength with vertical shear reinforcement up to a reinforcement proportion of 1.25 percent is recognized, higher from this ratio which has no influence there. This means that it is not infinite but has a limit to the participation of vertical shear reinforcement to the shear strength of a deep beam. Some studies have shown that deep beam shear strength has zero affect by horizontal shear reinforcement [33]. However, other studies have shown that, with the increase in horizontal shear reinforcement, there is a small increase in shear strength. This is particularly so when the vertical shear reinforcement is low. On the contrary, the rise in the shear reinforcement ratio in horizontal plane won’t has noticable participation to the shear strength of the beam in the presence of more vertical reinforcement [26].

3.3 The Ratio of Shear Span-to-depth ($a/d$)

A deep beam's shear resistance is basically dependent on it’s $a/d$. It has been shown by numerous experimental studies that the $a/d$ is the most important parameter that impacts a deep beam's shear resistance, as the shear strength increases with a reduction in the amount of $a/d$ [17, 26, 36]. This is because the load is directly transferred to the supports by concrete struts, created as a consequence of diagonal cracks, as the $a/d$ ratio decreases. This process is known as the tied-arch or strut-and-tie effect in deep beams [7, 24]. In comparison to deep beams, beam-action or process dominates slender beams as the $a/d$ ratio rises above 2.5. The STM plot in Figure 9 depicts the tied-arch motion in deep beams and very well foretell the strength of deep beams when $a/d$ less or equall 2.5. Rather, traditional sectional approach foretell the shear potential of slender beams well when contrasted to the experimental plot, $a/d > 2.5$. [11] 3 specimens of a deep beam with a $a/d$ ratio of 0.94, 0.76, and 0.64 were tested under the condition of two-point loading. Results presents that the average value of the first crack load increased as the $a/d$ ratio decreased from 60.0, 69.5 to 78.5 KN, respectively. In the same manner, the stepback of loads increased from 140.5, 144.4 to 149.1 KN, which means as $a/d$ deep beams reduced ,while their shear strength rises.

3.4 Effective size (Beam Depth)

As the beam depth increases, the shear strength of a reinforced concrete beam decreases. The influence of the size is prominently known as as this phenomenon. The size effect occurrence was reaffirmed by many researchs tries on slender beams-beams with $a/d \geq 2.0$ [5, 6, 34, 41-43]. For the purpose of evaluation of the size-dependent shear strength of such beams, several analytical models have been suggested. [44] It was established the concrete beam’s critical strength reduced plus rise in beam depth, with a recognized
distinguishing depth of 225 mm where size influence was taking into consideration. The phenomenon is becoming apparent. [41] Additionally, a series of experiments revealed that the average shear stress causing the largest beam to fail was just around one-third of what caused the smallest beam to fail. This indicates that, when the beams depth is higher, shear strength of RC beam will decrease to as low as one-third of the beam strength at the lowest depth, with all other parameters remaining constant as shown in Figure 10.

![Figure 10. Different value of shear strength respected to $a/d$][45].

In order to investigate the shear properties with different variable beam depths, twenty one beam samples were tested [46]. It was the one an increase in beam depth has been found to result in a more brittle failure with large diagonal cracks and a high rate of energy dissipation associated with the size effect. In addition, deep beams of high strength concrete (HSC) have been found to show more severe size effects with brittle failure behavior. [47] Summarized that HSC beams show a significant shear size effect over normal strength concrete, which can be due to the fact that HSC failure usually occurs by aggregate particles, decreasing the shear strength of the concrete portion of the aggregate interlock. Several models of fracture mechanics have been suggested to explain concrete failure [48], [49] and [50]. Regardless of the structural design and scale, each of these models defines certain material fracture features. Concrete structures exhibit the impact of size, which has been explained by the randomness of material strength. A material point of lower strength is more likely to be reached in large structures. Nonetheless, the impact of size on concrete structures must be clarified by a non-linear type of fracture mechanics that considers the non-negligible size of the location of damage in the fracture process zone (FPZ). [51] Was the first to use infinite series to establish a size effect law and limit its applicability to the 1:32 size range. The size effect phenomenon has therefore been strongly confirmed in beams and has been considered in some predictive shear models [20].

### 3.5 Beam span to depth ratio ($l_s/d$)

The ACI Examined 12 deep beams with various ratios of span to depth and concluded that the ratio of ($l_s/d$) also plays a major effect on the shear strength of a deep beam, similar to $a/d$ ratio, with shear strength being inversely related to the ratio of ($l_s/d$) [20]. Due to a longer arch is shaped to hold the load to the support as the $l_s/d$ ratio increases and the mid span deflection increases at the same time, creating more flexural cracking than shear failure [36].

### 3.6 Longitudinal Reinforcement

[40] Examined 64 deep beams and found that the shear strength of a deep beam increased dramatically with the increase in longitudinal reinforcement. Similar studies in [38] and [17] have shown that longitudinal reinforcement has a linear association with shear strength for deep beams with no shear reinforcement up to
a certain limit and has zero impact beyond that. Longitudinal reinforcement improves the shear potential of a deep beam by diminishing the crack width, enhancing the interface shear transfer mechanism and developing the dowel action [17]. [31] Has demonstrated a linear increase in the average shear strength as the longitudinal reinforcement ratio rises up to 1.5 percent and reaches a plateau beyond that. [52] They have also shown that step back of longitudinally reinforced deep beams is lower than that suggested by [53] \( A_{ds, min} = 3 f'c f_s b_s d \) and, despite any inclined cracks, broad deflections follow. The authors further noted that as the flexural tensile steel reinforcement increases, the step back is because of cracking of the concrete at nodal zones is more noticeable.

3.7 Impact of side cover

There have been studies of the effect of side cover thickness on beam shear strength [54]. While, intact vertical side, the result regarding the thick side cover tension face corner section crushed. The thickness of the side cover, in contrast, boosts the number of diagonal cracks and does not affect the diagonal crack distance. Concrete spalling is expected to get through high strength beams and high thickness web cover. The truth is that no spelling would occur before the ultimate load is expected by the code, so it was not recommended to decrease the shear strength. Figure 11 display the effect of side cover on shear strength of beam.

![Figure 11. Influence of side cover on the value of shear strength of beams (Rahal, 2007).](image)

3.8 Type of Concrete

Research has been conducted for the purpose of comparison between the shear behavior and efficiency of deep beams of considerably higher-strength self-compacting concrete (SCC) with strength of 50 MPa and corresponding normal weight concrete (NWC) [55]. For both SCC and NWC, two groups of beams were evaluated, one group with crowded shear reinforcement (closely spaced) and the rest has no-crowded (widely spaced) shear reinforcement. Many authors complemented SCC deep beams with non-congested web strengthening demonstrated a substantially raised load carrying capacity regarding diagonal shear cracking and maximum load in comparison to NWC samples.[56] Studied the contribution of aggregate interlock to the shear strength of lightweight concrete in continuous deep beams with different aggregate sizes. Although the pattern of the step back surface of the samples examines showed merely affected by the maximum aggregate type as well as size, they found that with the increase in the maximum aggregate size of concrete, the diagonal crack size beside a reduction in the failure plane and was much lower in concrete with average weight than in lightweight concrete deep beams at the exact loading levels. It was also observed an incremental decrease in hardness following the diagonal crack are obvious in deep beams made of
lightweight concrete than in standard deep concrete beams and increased with the reduction of the overall aggregate size.

4. Shear Strength of RC Deep Beams

Research has shown that a tied-arch action or truss (beam) action is considered by deep beams to transfer shear (Figure 10). The ratio of \(a/d\) and the amount of transverse reinforcement are the two most influential parameters affecting the type of load transfer mechanism. It has been demonstrated that a higher load fraction is transferred as \(a/d\) decreases through tied-arch action, which shows higher beam’s shear strength as a consequence of immediate transfer of the load through compressive struts from the loading point to the support (Figure 12a). In comparison, higher \(a/d\) ratios transfer a large portion of loads by beam or truss action (Figure 10b) [57], [58]. Similarly, higher levels of transverse reinforcement contribute to a greater fraction of the load being transferred by truss action, although the effect of transverse reinforcement decreases at very low \(a/d\) ratios [38], [26]. On the other hand, to avoid splitting diagonal struts that form amidst loads and supports of deep beams that give them increased shear strength, a certain minimum amount of web reinforcement (vertical and horizontal) is required [5].

![Figure 12. The act of the loading transfer mechanisms in deep beams.](image)

Deep beam failure is normally due to concrete crushing in either the reduced compression zone area at the tip of the inclined cracks, known as shear compression failure, or by concrete fracturing along the crack, known as diagonal splitting failure [59]. In the post-cracking zone, reserved energy occurs in deep beam that have a shear span to depth ratio of lower than 2.5, resulting in less brittle failure [60]. There are four modes of failure for deep beams known as shear-compression failure, diagonal splitting shear failure and shear-flexure and flexure failure. The diagonal-splitting failure is brittle, sudden and for this reason treacherous, defined as shear failure. A critical diagonal crack develops from which failure occurs, joining the loading point at the top and the support point at the bottom of the beam. In the shear-compression failure mode, the concrete part between the load point and the support experiences high compression after the inclined crack appears and eventually fails. This mode of failure is an equally brittle mode of failure. A shear and flexure combination is the shear-flexure failure mode. First, flexural cracks, followed by partially diagonal cracks form. This is a ductile failure mode in which the beam deflects at the middle and at the time of failure, no explosive sound is heard [17].

5. The models prediction of shear strength in deep beams

Different researchers have proposed several predictive models and several codes of practice have validated several of these models for RC deep beam design. In general, because deep beams are defined by small \(a/d\) ratios, failure usually occurs by shear compression or diagonal splitting in which the concrete fails to
compress after a diagonal crack between the support and the loading point. Thus, instead of beam action, the failure of deep beams is defined by tied-arch action, which makes deep beams exhibit greater shear strengths. Therefore, predictive models that account for the mode of failure or type of load transfer mechanism at failure are expected to generate more realistic observations. In the study, as listed in Table 1, seven different models were assembled for analysis to come up with easiest models for the prediction of the shear strength of deep RC beams. The performance of every model is calculated by making a comparison of each model’s results. Compared to the experimental findings collected from the data base in the deep beam literature, to find out their predictive strengths, Average Margin of Safety (AMS) as well as their coefficients of variation (CV). Shear equations supported by the following codes of practice have been evaluated: The American Concrete Institute’s proposed Strut-and-Tie (STM) model [9], and other standards [14,20,21,53,61,62]. The STM is the only method that predicts the shear strength of deep beams among the shear models considered, by recognizing the load transfer mechanism by taking into consideration the strength of the concrete struts that are created during deep beam failure. Instead, the remaining of the models depend on the combination of some of the individual beam component parameters such as shear span-to-depth($\frac{a}{d}$), beam size ($d$), beam span-to-depth($\frac{l_0}{d}$), concrete strength($f’_c$), and volume of flexural reinforcement ($\rho_t$) to estimate the shear strength of the beam without recourse to the load transfer apparatus.

| No. | Source | Model | Remarks |
|-----|--------|-------|---------|
| 1   | BS8110 (1997) | $v_c = \left(\frac{0.79}{\rho_{ym}}\left(\frac{100A_s}{bvd}\right)^{0.46}\left(d \frac{f’_c}{d}\right)^{0.46}\left(d \frac{A_{nt}}{\alpha}\right)^{0.46}\right)$ | $f_{cu} \leq 40\text{ MPa}$ |
| 2   | ACI-318 STM (2008) | $F_{ns} = f_{ce}A_{ce}, f_{ce} = 0.85f’c’, F_{nt} = A_{ts}f_{y}, F_{mn} = f_{ce}A_{nz}, f_{ce} = 0.85f’c’V_u =$ min of $(F_{nssin\theta}, F_{nssin\theta})$ | $A_{es}$ is the smallest effective cross-sectional area of a strut, $F_{ns}$ is nominal compressive strength of a nodal zone, $A_{es}$ is area of non-prestressed reinforcement, $A_{ns}$ is area of the face of the nodal zone, $\beta_s$ is strength reduction factors for struts and nodes respectively $F_{nt}$ is nominal strength of a tie, $f_{ce}$ is effective compressive strength, $\beta_s$, $F_{ns}$ is nominal compressive strength of concrete strut. |
| 3   | AIK(1998) | $v_n = 0.18\left(10 + \frac{ln}{d}\right)\sqrt{ckbhwd}$ | Where $2 \leq l_0d \leq 5.$ |
| 4   | EC2-2004 | $v_n = 0.18\left(1 + \frac{200}{d}\right)\left(\frac{100\rho f’c k}{0.33}\right)$ | $f_{y} \leq 410\text{ MPa}, A_{h}$ is the area of horizontal web reinforcement within a distance $S$ in inch; $A_{v}$ is the area of vertical web reinforcement within a distance $S$ in inch; |
| 5   | ACI-318 (2005) | $V = V_c + V_s$, where, $V_c$ | $V_s = (3.5 - 2.5 \frac{Mu}{V_{sd}})\left(0.16 \frac{f’c}{V_{sd}} + 17.8\frac{\rho t}{Mu}\right)\frac{v_{ud}}{bwd}$ |

where $V_{ud}Mu \leq 1; \text{ with } v_c \leq 0.29f’c\text{ \text{bw}d}$. |
\[ v_S = \frac{\Delta P_{sl}}{\Delta P_{sl}} \left( 1 + \frac{ln}{a} \right) + \frac{\Delta P_{sl}}{\Delta P_{sl}} \left( 1 - \frac{ln}{a} \right) = \]

6. Zsutty (1968)

\[ v_T = \left( \frac{2.5}{a} \right) (2.3f_{c} \rho_{d} \frac{d}{a}) 0.333 \]

Where \( \frac{a}{d} < 2.5 \).

### Table 2. The samples of deep beams not includes shear reinforcement

| Properties           | No   | d (mm) | Min | Max | Min | Max | Min | Max | Min | Max | Min | Max | Min | Max | Min | Max |
|----------------------|------|--------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Moody et al (1954)   | 12   | 533    | 533 | 610 | 610 | 178 | 178 | 18  | 25  | 1.1 | 1.7 | 4.3 |
| Londhe (2011)        | 19   | 125    | 375 | 150 | 400 | 100 | 100 | 24.4| 36.7| 1.1 | 2.0 | 0.3 | 2.4 |
| Placas (1969)        | 3    | 272    | 272 | 305 | 305 | 152 | 152 | 34  | 34  | 2.0 | 0.8 | 2.2 |
| Chen et al (2001)    | 4    | 444    | 1559| 500 | 1750| 140 | 140 | 39  | 44  | 1.5 | 1.6 | 2.6 | 2.6 |
| Sagasta (2008)       | 2    | 438    | 438 | 500 | 500 | 135 | 135 | 68  | 80  | 1.1 | 3.3 | 3.3 |
| Clarke (1951)        | 5    | 390    | 390 | 457 | 457 | 203 | 203 | 24  | 25  | 1.7 | 1.0 | 1.0 |
| Reyes de Oritz (1993)| 5    | 326    | 363 | 400 | 400 | 150 | 150 | 32  | 51  | 1.1 | 1.4 | 2.1 |
| Vollum & Tay (2001)  | 10   | 180    | 180 | 200 | 200 | 100 | 100 | 25  | 44  | 1.1 | 1.2 | 1.3 | 2.2 |
| Smith & Vantsiotis (1982) | 3   | 305    | 305 | 356 | 356 | 102 | 102 | 20  | 22  | 1.2 | 1.8 | 1.9 |
| Salamy et al (2005)  | 12   | 400    | 1400| 475 | 1505| 240 | 720 | 23  | 37  | 0.5 | 1.5 | 2.1 |
| de Cossio & Siess (1960) | 1   | 360    | 660 | 400 | 700 | 250 | 250 | 21  | 21  | 1.4 | 1.3 | 1.3 |
| Ramadan et al (2011) | 18   | 360    | 660 | 400 | 700 | 250 | 250 | 37.5| 37.5| 1.0 | 2.0 | 0.7 |
| Tan et al (1997)     | 1    | 443    | 443 | 500 | 500 | 102 | 102 | 78  | 78  | 1.4 | 1.4 | 1.0 |
| Leonhardt & Walter (1964) | 1   | 270    | 270 | 320 | 320 | 190 | 190 | 21  | 21  | 1.1 | 1.5 | 1.5 |
| Mathey & Watsein (1963) | 16 | 403    | 403 | 457 | 457 | 230 | 230 | 22  | 27  | 1.3 | 1.3 | 0.8 |
| Lehwalter (1988)     | 4    | 160    | 390 | 200 | 1000| 250 | 250 | 14  | 19  | 1.3 | 1.7 | 2.1 |

### Table 3. The samples of deep beams includes shear reinforcement.

| Properties           | No   | d (mm) | h (mm) | b (mm) | fc’ (MPa) | a/d | ρw (%) | ρf (%) |
|----------------------|------|--------|--------|--------|-----------|-----|--------|--------|
| Lehwalter (1988)     | 1    | 460    | 460    | 600    | 600       | 250 | 17     | 1.3    |
| Regan (1971)         | 6    | 254    | 272    | 305    | 305       | 152 | 17     | 1.1    |
| Clarke (1951)        | 27   | 314    | 390    | 381    | 457       | 152 | 14     | 1.3    |
| Sagasta (2008)       | 6    | 438    | 438    | 500    | 500       | 135 | 68     | 1.1    |
| Vollum & Tay (2001)  | 2    | 180    | 180    | 200    | 200       | 100 | 34     | 1.1    |
| Rao & Prasad (2010)  | 12   | 252    | 1105   | 300    | 1500      | 150 | 29.2   | 0.8    |
| Brena & Roy (2009)   | 12   | 303    | 581    | 356    | 635       | 152 | 27     | 3.54   |
| Kong & Rangan (1998) | 2    | 292    | 292    | 350    | 350       | 250 | 89     | 1.1    |
| Tan et al. (1997)    | 2    | 433    | 433    | 500    | 500       | 110 | 35     | 1.4    |
| Salamy et al (2005)  | 8    | 400    | 1400   | 475    | 1505      | 240 | 840    | 3.78   |
| Ahmad et al (2011)   | 6    | 319    | 468    | 375    | 525       | 150 | 34     | 0.6    |
| Londhe (2010)        | 9    | 375    | 375    | 400    | 400       | 100 | 32.1   | 3.22   |
| Tan et al. (1995)    | 1    | 463    | 463    | 500    | 500       | 110 | 51     | 1.3    |
Table 4. The average of the capacity prediction for RC deep beams in terms of Margin of safety for the beams not includes shear reinforcement.

| No. | Pattern | Number of beams utilized for assessment | Standard Deviation | (AMS) Average Margin of Safety | Coeff. of Variation |
|-----|---------|-----------------------------------------|--------------------|-------------------------------|---------------------|
| 1   | ACI 318-05 | 116                                      | 0.5                | 0.8                           | 0.50                |
| 2   | Zsutty (1968) | 116                                      | 0.2                | 0.4                           | 0.5                 |
| 3   | EC2-04 | 116                                      | 0.8                | 1.9                           | 0.42                |
| 4   | ACI 318-08 (STM) | 116                                      | 0.4                | 1.1                           | 0.36                |
| 5   | BS 8110, 1997 | 116                                      | 1.8                | 4.4                           | 0.41                |
| 6   | IS:456 (2000) | 116                                      | 1.7                | 4.5                           | 0.38                |

Table 5. The average of the capacity prediction for RC deep beams in terms of Margin of safety for the beams that includes shear reinforcement.

| No. | Pattern | Number of beams utilized for assessment | Standard Deviation | (AMS) Average Margin of Safety | Coeff. of Variation |
|-----|---------|-----------------------------------------|--------------------|-------------------------------|---------------------|
| 1   | ACI 318-05 | 94                                       | 0.5                | 0.6                           | 0.83                |
| 2   | Zsutty (1968) | 94                                       | 0.2                | 0.5                           | 0.40                |
| 3   | EC2-04 | 94                                       | 1.3                | 2.9                           | 0.45                |
| 4   | ACI 318-08 (STM) | 94                                       | 0.5                | 1.3                           | 0.38                |
| 5   | BS 8110, 1997 | 94                                       | 2.5                | 6.0                           | 0.41                |
| 6   | IS:456 (2000) | 94                                       | 2.5                | 6.4                           | 0.39                |

6. Discussions and Results
The literature collected a total of 120 reinforced concrete deep beams, 116 of them with no shear reinforcement, while 94 had shear reinforcement (Tables 2 and 3). Table 4 reveals the effects of the assumption of different models relative to the exploratory findings of deep beams with no shear reinforcement, while Table 5 shows the results of deep beams with shear reinforcement. The large value mediocre margin of safety (AMS) is the most conservative model; the lowest standard deviation (STD) indicates a substantial improvement in prediction uniformity, while the lowest variance coefficient (CV) indicates that the lowest variance in accuracy is observed in a single data set, thus the higher uniformity in prediction [63]. The [9] STM model created an AMS that was most economical and safe for the deep beams without reinforcement for the results presented. Compared to [14,20,21], which produced AMS values of 1.9, 4.4 and 4.5, they produced an AMS of 1.1. The [9] STM model thus proved superior to the rest of the models, which can be defined as overly conservative because, in terms of accuracy and protection, their AMS values are much greater than 1. The [9] STM also recorded the lowest CV of 0.36 compared to 0.38, 0.41 and 0.42 recorded for each of the other three models [14,20,21] respectively. This implies that, as a result of its lowest CV, the [9] STM proves superior among these models in terms of prediction uniformity. On the other hand, the design equations [64], [62] and [61] with AMS of 0.8, 0.4 and 0.4 respectively overestimate the shear strength of deep beams without shear reinforcement and, in terms of protection, are not accurate for design. This AMS results pattern is close to the values obtained in a study conducted by [17]. Again, the [9] STM provides the best estimate in terms of precision, economy and protection for deep beams that have shear reinforcement (Table 5). With the deepest mediocre integrity of 1.3, the [9] STM
model has been shown to offer closer average experimental average predictions compared to the [21], [20], and [14] AMS generators of 2.9, 6.0, and 6.4 respectively. As they developed very high AMS in excess of 1, these other models can be said to be excessively conservative. Besides, the [9] STM continued to produce the deepest CV of 0.38 in terms of shear strength prediction uniformity, compared to [14], [20], and [21], which registered CV values of 0.39, 0.41, and 0.45 for shear beams, respectively (Table 5). This shows that the [9] STM is more accurate in terms of estimation uniformity than the rest of the models for shear reinforcement beams. Once again, the design models [53], [61] and [62] overrate the shear strength of deep shear beams, as they provided AMS of 0.6, 0.6 and 0.5 respectively, which are below unity. This means that the predicted values of strength are much higher than the real strength achieved from experiments that pose a risk to design. The high precision and uniformity of the deep beam shear strength predictions, with and without shear reinforcement, produced by the STM [9] provision could be due to the fact that, unlike the others, the model measures the actual strength of the compressive struts created by the internal tied-arch mechanism, which transmits the load directly to the failure supports. The conservativeness of the three code models for the prediction of both beams with and without shear reinforcement, namely [21], [20] and [14], can be attributed to the fact that they reflect equations developed for slender beams modified with a $\frac{2}{g_{1853}}$ shear enhancement factor. This is viable when the employed point load is, as given by these codes, within two effective depths of the face of the beam support. Efficacy of the [9] STM shear strength provision over the other models is therefore well established.

7. Conclusions and the recommendations

To find the most practical description for the characterization of deep beams, the meaning of a deep beam, as captured by a number of distinct codes of practice and standards, was discussed. Finding the predictive efficiency of each model, a comparative analysis is also carried out among six models to predict the shear resistance of deep beams in the analysis. Depending on the analysis of the main conclusions were derived:

- Factors such as shear span-to-depth ratio, beam depth, beam web reinforcement, concrete compressive strength, and beam span-to-depth ratio, type of concrete and longitudinal reinforcement influence the shear strength of a deep beam. However, the most important factor affecting the shear strength of a deep beam traced by the vertical shear reinforcement is the shear span-to-depth ratio.
- Two bases are usually classified as deep beams: the [9] concept and the [14] concept. The definition of the concept [9] is based on two parameters: geometric consideration and loading state, while the definition of the concept [14] only accounts for geometry.
- The strut-and - tie (STM) model [9] showed to be the most economical and secure model in the study for the prediction of deep beam shear strength, with or without shear reinforcement. This is because it provided the lowest estimate of the Average margin of safety just above 1 and the lowest Co-efficient of Variance, providing the highest uniformity predictions. This means that the strut-and - tie model [9] is the most efficient model for the prediction of RC deep beam shear strength and must therefore be used in its design.
- In terms of the type of concrete, the SCC deep beams owning standard shear reinforcement performed a slightly more ultimate capacity than that of NC deep beams.
- According to the statistical analysis, it was found that the ACI models have the most reliable description of deep beams and from that, it is supposed to rely upon the description of deep beams
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