Investigation of effect of process parameters on suddenly 
Expanded flows through an axi-symmetric nozzle for different 
Mach Numbers using Design of Experiments

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Abstract. Experiments are conducted to determine base pressure variation through micro-jets 
from a suddenly expanded axisymmetric passage. Four micro-jets having an orifice diameter of 
1mm are situated at 90° interims along the base at 6.5 mm from the geometrical axis of the 
main jet. The Mach number and L/D ratios were the process parameters employed in the study. 
The flow stream was extended all of a sudden into an axi-symmetric duct of 4.84 cross-
sectional area for all the Mach numbers and L/D ratios respectively. The test Mach numbers 
used in the study was 2, 2.5 and 3; length-to-diameter ratios selected for the sudden expansion 
tube were 4, 6 and 8. The jets were operated at an overexpansion level of (Pe/Pa = 0.277). The 
experiments are conducted as per Taguchi design of experiments. From this investigation, one 
will be able to identify the enlargement length to diameter ratio resulting in maximum 
increasing or decreasing base pressure. Mathematical models are also developed for base 
pressure based on Mach number and L/D ratio with a maximum error of ± 10%.

1. Introduction
In the area of Space flights and missile technology, base flows are deemed to be a core area of 
research due to high Reynolds numbers. Following these, the concern has been drifted towards 
hypersonic speed regime in prospect of heat transfer at the base and near-wake structure. However the 
understanding of base flows is incomplete due to the presence of strong pressure gradient and 
insufficient turbulence knowledge. Activated fundamentally by the necessities prevailing in 
technological developments, various inquisitive studies have been proclaimed in literature dedicated 
towards lowering the base drag penalty by implementing both active and passive techniques. 
Practically, these aspire in controlling the near-wake flow-field for the development of base pressure. 
The experimental studies involving sudden expansion with internal flows have various benefits when 
compared to external flows. The volume of air supply needed is reduced by discarding the 
requirement for tunnel with a cross-section considerable enough such that flow over the model is not
disturbed by wall interference. The static and surface pressure measurements are made not only through the entrance section; but also through the wake region. Because of such extensive applicability, suddenly expanded flows have been studied broadly [1]. Many researchers have made an attempt to control the pressure in wake area with passive control. However, to the best of the authors' knowledge there has been no work reported with active control of base pressure as per Taguchi design of experiments for high inertia levels and L/D ratio as control variables. Therefore, the present study involves control of base pressure control by active and passive means in the micro jets form under favorable pressure gradient as per Taguchi design of experiments.

Axi-symmetric expanded flow is a complicated aspect identified by the flow separation, flow recirculation and reattachment. This type of flow may be classified into two regions separated by a shear layer namely i) region of flow recirculation and ii) region of main flow [2]. The point where the segregated streamline touches the wall is called as reattachment point. The salient features of the flow field have been illustrated in Fig. 1. There is abundant data available in the case of sudden expansion problem which has been registered previously. The boundary layer effect was studied by experimentally by Wick [3], where the boundary layer was considered as the source of mass flow for corner flow. It was recognized that the pressure developed at the expansion corner was directly relevant to the expansion thickness upstream and boundary layer. However Hoerner [4] concluded that the layer of the boundary acts as insulating air thereby reducing the efficacy of the jet as pump. He finally concluded that the phenomenon of base pressure can be experimentally studied by use of internal flow. Khan et. al. [5-9] carried out experiments in order to study the response of micro jets influencing over, under and correctly expanded nozzles for controlling base pressure in axisymmetric ducts that were suddenly expanded. It was concluded that the micro jets do not adversely affect wall pressure distribution. Furthermore, it was also concluded that the NPR has a definitive role in controlling the variation the base pressure for both cases i.e. with and without control. Durst et. al. [10] concluded that symmetric flows exist in a 2-D plane for a limited range of Reynolds numbers. The higher Reynolds number generated minute disturbances at the tip of the sudden expansion which become amplified between the main flow and recirculation flow. Rathakrishnan et. al. [11] studied flows in pipe that were subjected to sudden enlargement. It was concluded that non-dimensional base pressure is an active function of expanded area ratios, NPR and L/D ratios of ducts. Thus for a given nozzle and enlargement area ratio, the duct has to have a minimum value in order to obtain a minimum value for base pressure. Thus for flows with sudden expansion, the total pressure also has to be taken into account along with base pressure minimization. From previous literature as stated above, it is evident that suddenly expanded flows with control can be useful in understanding many practices. It can be of incredible help for rocket and space program applications for thoroughly understanding the behavior of base pressure and a methodology which can be formulated for controlling base pressure. This is because; control of base pressure will result in the development of either increment or decrement of base pressure. Thus base pressure is minimized in case of a combustion chamber for maximizing the mixing, and maximized for the case of rockets and missiles for the purpose of base drag reduction.

2. Experimental set-up

Fig. 2 depicts the experimental setup used for the present investigation. Eight holes are marked at the exit rim of the nozzle. Out of these four are marked ‘c’ utilized for blowing; four are marked ‘m’ and utilized for measuring base pressure. The pressure at the base region was controlled by blowing through ‘c’ marked gapes by means of pressure that was generated in the settling chamber. The experiments were administered for Mach numbers 2.0, 2.5 and 3.0. Since active controls are used in the present study, L/D ratios greater than or equal to 10 can be employed. However for this particular study L/D ratios of 4, 6 and 8 have been employed. The results presented here are for an over expanded case of $(P_0/P_e = 0.277)$ and therefore the corresponding NPR’s for Mach number 2.0, 2.5 and 3.0 are 2.16, 4.64 and 10 respectively. A PSI system 2000 pressure transducer was implemented for
base pressure measurement. It consisted of 16 channels with optimum pressure ranging from 0 to 300 psi. It displayed reading by averaging 250 samples per second.

3. Plan of experiments
Taguchi design of experiments for two factors at three levels was used for exercising the experiment. The orthogonal array selected was L9 (23) as shown in Table 1. The experiments were carried out considering Mach number and L/D ratio for a common expansion area ratio of 4.84. Each experiment has been repeated for two times and average values of base pressure have been recorded.

![Fig. 1. Sudden expansion flow field.](image1)

![Fig. 2. Experimental Set up.](image2)
Table 1. Allocation of factors an L₀(2³) Orthogonal Array.

| Experiment No. | Mach number (M) | L/D Ratio | Non-dimensional base pressure without control (P₀/Pₐ) | Non-dimensional base pressure with control (P₀/Pₐ) |
|----------------|----------------|-----------|--------------------------------------------------------|--------------------------------------------------|
| 1              | 2.0            | 4         | 0.808                                                  | 0.806                                            |
| 2              | 2.0            | 6         | 0.795                                                  | 0.793                                            |
| 3              | 2.0            | 8         | 0.802                                                  | 0.802                                            |
| 4              | 2.5            | 4         | 0.68                                                   | 0.675                                            |
| 5              | 2.5            | 6         | 0.64                                                   | 0.64                                             |
| 6              | 2.5            | 4         | 0.637                                                  | 0.637                                            |
| 7              | 3.0            | 4         | 0.575                                                  | 0.577                                            |
| 8              | 3.0            | 6         | 0.505                                                  | 0.506                                            |
| 9              | 3.0            | 8         | 0.484                                                  | 0.486                                            |

4. Results and Discussions

The obtained data consisted measured base pressure (P₀), across the length of the augmented pipe and nozzle pressure ratio (NPR) defined as the proportion of stagnation pressure (P_{Stagnation}) to the back pressure (P_{atm}). The registered pressure values were non-dimensionalized by subdividing them with the surrounding ambient pressure. Fig. 3(a) presents results of base pressure variation with respect to L/D ratios for Mach number M = 2.0. It is observed that base pressure continues to decrease with L/D attaining a minimum value at L/D = 6. For an L/D greater than 6, it tends to increase again. This minimum base pressure obtained at L/D = 6 are compatible with the results Khan and Rathakrishnan [5]. However the influence of control is only marginal over base pressure. Fig. 3(b) presents results of base pressure variation Mach number M = 2.5. The behavior is distinctive when compared to M=2.0. Once again in this case, control marginally influences the base pressure. Furthermore, for an L/D = 8; did observe minimum base pressure. Fig. 4 illustrates the results for Mach number M = 3.0. Here again an L/D upto 8 only plays a compelling effect on minimizing base pressure. Thus for all three cases control marginally influences base pressure. This minimum base pressure achieves the reattachment length for the corresponding Mach number. Thus for Mach number 2.0, the reattachment length is L/D = 6 and Mach numbers 2.5 and 3, the reattachment length is L/D = 8. This is because the quality of the vortex at the base area which is high for higher Mach numbers prompted for bigger suction at the base. For higher Mach numbers i.e. 2.5 and 3.0, a decrement in the base pressure is observed as compared to Mach 2.0; for which the physical reason is the impacted oblique stun or development fan that is positioned at the spout lip [12]. The base pressure values for Mach 3.0 show distinct behavior when control is applied thus making control effective. The values for control are imperceptibly higher when compared to without control which is vice-versa otherwise in the case 2.0 and 2.5. In the existing study the position of microjets are fixed. Therefore, for a particular value of area ratio, the microjets are fixed to a specific distance away from the base corner. Hence, for higher Mach numbers, the microjets will tend to produce a shock effect which in turn contributes to increase in the base pressure values more effectively. Hence, the vortex at the base will generate more suction at the base. This appears to be the purpose for control becoming more effective at Mach number 3.0 than Mach numbers 2.5 and 2.0.

4.1. Experimental Model for Failure Index

A Statistical model based on multiple linear regression equations were developed for non-dimensional base pressure (P₀/Pₐ) using the experimental parameters for both cases of with and without control are given below. The standard commercial statistical software MINITAB 17 has been used to derive. The polynomial model disposed below represents the non-dimensional base pressure (P₀/Pₐ) as a function of Mach number (M) and L/D ratio. The regression equations are

\[
P₀/Pₐ \text{ (With Control)} = 1.4078 - 0.2803 \text{ (M)} - 0.01167 \text{ (L/D)} \tag{1}
\]

\[
P₀/Pₐ \text{ (Without Control)} = 1.4178 - 0.2773 \text{ (M)} - 0.01108 \text{ (L/D)} \tag{2}
\]
Where M=Mach number; L/D= length to diameter ratio.

4.2. Validation of Experimental Results
To validate the base pressure results, for non-dimensional base pressure (\(P_b/P_a\)) was computed theoretically by standard Eq. 1 and Eq. 2 developed by for arbitrary experimental conditions and validated with modeling base pressure results. It was found from Table 2 and Table 3 that; the percentage of error is within 9.56% for the case of with control and 6.07% for the case of without control respectively. The instrument errors in measurement system and lack of surface finish at the base region produce uneven rate of change base pressure. This may be the plausible reason for the error, which is experimentally reasonable.

Fig. 3. Variation in Base Pressure with L/D Ratio for (a) M=2.0, \(P_e/P_a=0.277\); (b) M=2.5, \(P_e/P_a=0.277\).

Fig. 4. Variation in Base Pressure with L/D Ratio for a) M=3.0, \(P_e/P_a=0.277\).
4.2 Analysis of Variance (ANOVA)

ANOVA helps in formally testing the implication of the two factors i.e., Mach number and L/D ratio by correlating the mean square against the approximate calculation of the base pressure errors at specific confidence levels. The objective of analysis of variance is to inspect the factors which considerably affect the variation of base pressure. Table 4 and Table 5 illustrate the analysis of variance for non-dimensional base pressure results for the case of with and without control respectively. It is observed that Mach number has the highest significance of 95.91% for the case of with control and 95.21% for the case of without control respectively. This is further followed by L/D ratio having less significance of 2.63% and 2.43% for both cases of without and with control respectively. The dependence of base pressure on Mach number observes that in the Mach numbers in the supersonic regime have a substantial influence on base pressure when compared to L/D ratio. This is due to the account of these flows which are highly overexpanded; the shock at the nozzle exit becomes stronger. These shocks possess a larger shock angle owing to smaller flow deflection angles. Thus, the base pressure is managed by the recycling flow stream instead of the flow behind the shock [5]. Thus, these waves are to be considered while analyzing base flows with Mach numbers in the
supersonic regime which become the prime aspect for influencing base pressure. It is apparently clear from the outcome that, the L/D has a lesser effect on control of base pressure when compared to Mach number. It can be declared that, the pressure at base is predominated by reattachment length; defined as span of flow from its inception at the expansion to the location where in the nozzle’s shear layer adheres with the wall duct. This can be plausible only by placing a duct of definitive length. However for supersonic Mach numbers employed in the current study, the minimum length for reattachment as per experimental results was L/D = 6. Thus supersonic Mach numbers associate themselves with higher L/D ratios thus minimizing their significance in controlling the expanded flows.

| Source | DF | Adj SS | Adj MS | F-Value | P-Value | %P |
|--------|----|--------|--------|---------|---------|----|
| A      | 1  | 0.117880 | 0.117880 | 263.97  | 0.000   | 95.91 |
| B      | 1  | 0.003267 | 0.003267 | 7.32    | 0.035   | 2.63 |
| Error  | 6  | 0.002679 | 0.000447 |         |         | 1.46 |
| Total  | 8  | 0.12382  |         |         |         | 100 |

Table 4. ANOVA results for non-dimensional base pressure with control.

| Source | DF | Adj SS | Adj MS | F-Value | P-Value | %P |
|--------|----|--------|--------|---------|---------|----|
| A      | 1  | 0.115371 | 0.115371 | 242.96  | 0.000   | 95.21 |
| B      | 1  | 0.002948 | 0.002948 | 6.21    | 0.047   | 2.43 |
| Error  | 6  | 0.002849 | 0.000475 |         |         | 2.36 |
| Total  | 8  | 0.121168  |         |         |         | 100 |

Table 5. ANOVA results for non-dimensional base pressure without control.

5. Conclusions

Controls of base flows for various process parameters as per Taguchi design of experiments have been studied. The following conclusions have been made:

- The reattachment length for Mach number 2.0 is L/D=6 and for Mach numbers 2.5 and 3, the reattachment length is L/D=8. This is mainly due to the high strength vortex at the base region which is high for higher Mach numbers leading to larger suction at the base.
- The control becomes effective for Mach number 3.0 since the microjets fixed to a specific distance away from the base corner tend to produce a shock effect in turn contributing to increase in the base pressure for higher Mach numbers.
- Linear polynomial models representing the non-dimensional base pressure (Pb/Pa) as a function of Mach number (M) and L/D ratio were developed for both the cases of with and without control. The models agreed well with the experimental results with percentage error of 9.56% for the case of with control and 6.07% for the case of without control respectively.
- Analysis of variance is also conducted to inspect the factors which considerably affect the variation of base pressure. It is observed that Mach number has the highest significance of 95.91 % and 95.21% in cases of with control and without control respectively. This is further followed by L/D ratio having less significance of 2.63% and 2.43% for both cases of with and without control respectively.

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