Structural Damage Localization Based on Cloud Algorithm and Improved Modal Strain Energy Dissipation Ration Index

Huiyong Guo* and Xinyu Zhou
School of civil Engineering, Chongqing University, Chongqing 400045, P. R. China
*Email: ghy267@hotmail.com

Abstract. In order to solve structural multi-damage detection problem, a damage localization method based on cloud algorithm and modal strain energy index is presented. First, structural modal strain energy is considered as identification parameter and an improved modal strain energy dissipation ration index (IMSEDRI) is presented to identify structural damage locations. Then, cloud algorithm is introduced and a backward cloud generator is proposed to process the noisy modal data. The simulation results demonstrate that the calculated results of the proposed index are better than those of the basic modal strain energy dissipation ration index, and the method based on cloud algorithm and IMSEDRI can analyze damage problem with random noise interference.

1. Introduction
Structural damage detection has received increasing attention in the research community in recently two decades. Existence of structural damage leads to modifications of the dynamic characteristics. These modifications are manifested as changes in the vibration parameters (natural frequencies, mode shapes, modal strain energy etc.) which can be obtained from results of dynamic testing. In general, modal strain energy contains plenty of mode shape information, and the strain energy method is more sensitive than some mode shape-based methods for damage detection [1]. Shi et al [2] presented a damage detection method based on the elemental modal strain energy change before and after the occurrence of damage in a structure. Sazonov et al [3] utilized strain energy mode shapes to study vibration based damage detection method and tried to find the optimal spatial sampling interval for damage detection. Liu et al [4] proposed a structural damage detection method, which is based on the dissipation ratio theory of modal strain energy. They used the change of modal strain energy in each element before and after the occurrence of damage to obtain a modal strain energy dissipation ratio index (MSEDRI). However, the modal strain energy dissipation ratio index didn’t consider the modal strain energy change tendency to damage. Therefore, an improved modal strain energy dissipation ratio index will be proposed.

Cloud algorithm is a complete and efficient cognitive technique, which was developed by Li et al [5]. It is applicable to uncertain damage identification problem. In statistics, probability distributions can be used to model the uncertainty of variables representing random phenomena called randomness. In fuzzy mathematics, the membership function is used to model the uncertainty of membership to fuzzy concepts called fuzziness. Cloud algorithm can model both randomness and fuzziness with fixed parameters. The normal cloud model algorithm, based on the normal distribution and a membership function, has been widely applied in information science, including multi-criteria group decision making, intelligent control, image segmentation, and so on[5]. In this paper, cloud algorithm will be combined with modal strain energy for damage localization.
2. Improved modal strain energy dissipation ration index

2.1. Modal strain energy

Modal strain energy methods can efficiently identify structural damage locations. The modal strain energy of the \(j\)th element and the \(i\)th mode before and after the occurrence of damage is given by

\[
\begin{align*}
MSE_{ij}^u &= \frac{1}{2} \Phi_i^T K_j \Phi_i \\
MSE_{ij}^d &= \frac{1}{2} \Phi_i^{d^T} K_j \Phi_i^d
\end{align*}
\]

in which \(MSE_{ij}^u\) and \(MSE_{ij}^d\) are the undamaged and damaged modal strain energy, respectively; \(\Phi_i\) and \(\Phi_i^d\) are the \(i\)th undamaged and damaged mode shape, respectively; \(K_j\) is the stiffness matrix of the \(j\)th element for the undamaged system. When the first \(m\) mode shapes are used, modal strain energy of the \(j\)th element before and after the occurrence of damage can be written as

\[
\begin{align*}
MSE_{j}^u &= \frac{1}{2} \sum_{i=1}^{m} \Phi_i^T K_j \Phi_i \\
MSE_{j}^d &= \frac{1}{2} \sum_{i=1}^{m} \Phi_i^{d^T} K_j \Phi_i^d
\end{align*}
\]

(2a, b)

2.2. Improved modal strain energy dissipation ration

Modal strain energy is effective vibration parameter for damage detection. If

\[
u_j = MSE_{ij}^u \
d_j = MSE_{ij}^d
\]

Thus, the modal strain energy change of the \(j\)th element can be simply written as

\[
MSEC_j = d_j - u_j
\]

(4)

Damage will cause the change of modal strain energy. From energy dissipation viewpoint, damage can also be described as energy dissipation process. The energy change caused by the same damage should be equivalent. Therefore, the modal strain energy change caused by damage should be equivalent to the energy dissipation caused by the same damage.

Structural damage can also be described as the energy dissipation process. The strain energy dissipation ratio at time \(t\) is given by

\[
\varphi(t) = \frac{-\dot{c}(t)}{[1-c(t)]^2} \int_{t'} (\sigma^T \varepsilon) \text{d}t'
\]

(5)

where \(\sigma\) and \(\varepsilon\) are the stress vector and the strain vector, respectively; \(V\) is the total volume of the structure; \(c\) is damage coefficient; \(t\) is time. In general, structural damage can be considered as the energy dissipation process of undamaged element. So, the strain energy dissipation ratio of the \(j\)th element can be written as

\[
\varphi_j(t) = \frac{-\dot{c}_j(t)}{[1-c_j(t)]^2} u_j
\]

(6)

When the time \(t=0\), no damage occurs, i.e. \(c_j(0)=0\); when \(t=t^d\), damage occurs. It can be hypothesized that there is a linear relationship between \(c_j(t)\) and \(t\). Thus, the derivative \(\dot{c}_j(t)\) is a constant. If a damage \(c_j\) occurs in the \(j\)th element, the energy dissipation of the \(j\)th element can be given by

\[
\int_0^{t^d} \varphi_j(t) \text{d}t = u_j \int_0^{t^d} \frac{-\dot{c}_j(t)}{[1-c_j(t)]^2} \text{d}t = u_j \frac{-c_j}{1-c_j}
\]

(7)
The energy change caused by the same damage should be equivalent. Therefore, the modal strain energy change caused by the damage should be equivalent to the energy dissipation caused by the same damage. According to equations (4) and (7), we have

$$|d_j - u_j| = |u_j - c_j|$$

Thus, the modal strain energy dissipation ration index of the $j$th element is given by [4]

$$MSEDRI_j = \frac{\text{MSE}^d_j - \text{MSE}^u_j}{\left|\text{MSE}^d_j - \text{MSE}^u_j\right| + \text{MSE}^u_j}$$

(9)

In general, damage will lead to the reduction of the element stiffness, and it will also lead to the increase of modal strain energy [6]. Therefore, an improved modal strain energy dissipation ration index(IMSEDRI) is given by

$$\text{IMSEDRI}_j = \max\left[0, \frac{\text{MSE}^d_j - \text{MSE}^u_j}{\left|\text{MSE}^d_j - \text{MSE}^u_j\right| + \text{MSE}^u_j}\right]$$

(10)

Thus, incorrect index value can be directly excluded. From equation (10), we can find the suspected damage elements. In general, the measurement noise cannot be neglected in the damage identification. Cloud model will be utilized to analyze noise problem.

3. Cloud algorithm

In cloud algorithm, three numerical characteristics are used to describe the information uncertainty, Expectation $Ex$, Entropy $En$, and Excess Entropy $He$ [7]. In this paper, we use the three numerical characteristics to construct a cloud generator to realize the transformation between the qualitative concept and the quantitative numeric.

The normal cloud generator algorithm:

a) Generating a normal random number $X_r$ according to the three numerical characteristic ($Ex$, $En$, $He$), and the expectation of $X_r$ is $Ex$ and the standard deviation is $En$.

b) Generating a normal random number $En$, of which the expectation is $En$ and the standard deviation is $He$.

c) Generating the cloud droplet $(x_r, \mu_r)$ by

$$\mu_r = \exp\left[-\left(x_r - Ex\right)^2 / \left(2En^2\right)\right]$$

(11)

d) Repeat the above procedure until N cloud droplets were generated.

If the fluctuation range of the testing data is unknown, the cloud algorithm can be established by a backward cloud generator with the sampling discrete testing levels. Statistical parameters can be given by:

Sample mean

$$\bar{X} = \frac{1}{N} \sum_{r=1}^{N} x_r,$$

(12)

First order absolute central moment

$$M_1^X = \frac{1}{N} \sum_{r=1}^{N} |x_r - \bar{X}|.$$

(13)

Sample variance

$$S^2 = \frac{1}{N-1} \sum_{r=1}^{N} (x_r - \bar{X})^2.$$

(14)

Thus, the backward cloud generator algorithm is as follows:

$$\bar{Ex} = \bar{X}.$$
\[
\hat{E}_n = \sqrt{\frac{\pi}{2N}} \sum_{r=1}^{N} |x_r - \hat{E}x|.
\]

\[
\hat{H}e = \sqrt{|S^2 - \hat{E}n^2|}.
\]

where, \(N\) is the total number of testing samples.

Here, we mainly utilized the backward cloud generator to process the measurement noise interference problem. Besides, modal strain energy is also utilized to detect the damage.

4. Numerical examples

![Two-dimensional truss structure](image)

**Figure 1.** Two-dimensional truss structure

In order to investigate the effectiveness of the two-stage method, a two-dimensional truss structure has been used. The size of the truss structure is depicted in Figure 1. The material property of the structure is: \(\rho=2800\) kg/m\(^3\), \(E=72\) GPa, and \(A=0.001\) m\(^2\). The truss model consists of 30 bar elements, 14 nodes, and 24 degrees of freedom. Damage is simulated by a reduction in the stiffness of individual bars in the structure. Damage cases are shown in Table 1.

| Case1 | Case2 |
|-------|-------|
| Element No. | Damage | Element No. | Damage |
| (1) | (2) | (3) | (4) |
| 5 | 35% | 7 | 30% |
| 28 | 15% | 25 | 25% |

4.1. Damage case 1

In case 1, damage occurs in elements 5 and 28 with 35% and 15% stiffness reduction, respectively. The improved modal strain energy dissipation ratio index is applied to identify damage locations. In order to compare with the proposed IMSEDRI, the MSEDRI is also applied to identify damage locations. The identification results of the numerical calculations are shown in Figure 2 with the element number plotted against the element value. A relatively higher value means that the assumed damage is closely correlated to the true damage state, and this element is more probable damage element. From Figure 2, it can be seen that the mean value localization results of the IMSEDRI are better than those of the MSEDRI. The locations of the highest two values are damage locations.
Figure 2. Localization results when damages occur in elements 5 and 28

Figure 3. Membership function values of IMSEDRI when damages occur in elements 5 and 28

The membership function of the IMSEDRI is depicted in Figure 3. Here, we mainly considered the membership function values of damaged elements 5 and 28. From Figure 3, it can be seen that the measurement noise has an effect on membership function values.

4.2. Damage case 2
In case 2, damage occurs in elements 7, 25 and 28 with 30%, 25% and 25% stiffness reduction, respectively. The improved modal strain energy dissipation ratio index is applied to identify damage locations. The MSEDRI is also used to detect damage locations. The identification results of the numerical calculations are shown in Figure 4 with the element number plotted against the element value. A relatively higher value means that the assumed damage is closely correlated to the true damage state, and this element is more probable damage element. From Figure 4, it can be seen that the mean value localization results of the IMSEDRI are better than those of the MSEDRI. The locations of the highest three values are damage locations.

The membership function of the IMSEDRI is depicted in Figure 5. Here, we mainly considered the membership function values of damaged elements 7, 25 and 28. From Figure 5, it can be seen that the measurement noise has an effect on membership function values.
5. Conclusions
We mainly used the modal shapes with random noise to identify structural damage problem. A damaged detection method based on modal strain energy and cloud model is presented to detect the damage locations. First, structural modal strain energy is considered as information source and an improved modal strain energy dissipation ration index (IMSEDRI) is proposed to identify suspected damage elements. Then, cloud model theory is introduced and a backward cloud generator is proposed to analyze uncertain damage problem. The simulation results demonstrate that the calculated results of the proposed IMSEDRI are better than those of the basic modal strain energy dissipation ratio index. However, the identification results aren’t good enough. We think the method can be improved by integrating some other dynamic parameters, for example, if some time-domain dynamic parameters are added into damage localization identification process, the identification results may become better.

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7. References

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