BLACK HOLE BINARY DYNAMICS

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Abstract. We discuss a new \(N\)-body simulation method for studying black hole binary dynamics. This method avoids previous numerical problems due to large mass ratios and trapped orbits with short periods. A treatment of relativistic effects is included when the associated time-scale becomes small. Preliminary results with up to \(N = 2.4 \times 10^5\) particles are obtained showing systematic eccentricity growth until the relativistic regime is reached, with subsequent coalescence in some cases.

1. Introduction

The problem of the formation and dynamical evolution of a black hole (BH) binary with massive components is of considerable topical interest. Several previous efforts employed direct integration methods to elucidate the behaviour of such systems but applications to galactic nuclei pose severe limitations with regard to the particle number which can be investigated. The formation is usually envisaged as the end product of two separate galactic nuclei (or one being a dwarf galaxy) spiralling together by dynamical friction, but there are other scenarios.

Notwithstanding the particle number limitations imposed by the need for accurate numerical treatment, much can be learnt about the early evolution of BH binaries by studying smaller systems. It is well established that the presence of two massive bodies in a stellar system leads to their rapid inward spiralling and inevitable formation of a dominant binary. In fact, this development was already discussed 30 years ago (Aarseth 1972) for a relatively small system of \(N = 250\) particles. Here a massive binary absorbed about 90% of the total cluster energy after only 50 initial dynamical (or crossing) times. This calculation was also the earliest demonstration of two-body regularization in an \(N\)-body context.

Upon formation, the subsequent binary evolution is subject to a steady shrinkage of the semi-major axis and ejection of particles resulting from sling-shot interactions. Although the rate of shrinkage decreases significantly, the corresponding energy increase is fairly constant (Quinlan & Hernquist 1997, Milosavljević & Merritt 2001). These investigations employed two-body regularization in order to reduce the systematic errors associated with direct integration of hard binaries. Alternative studies based on a small softening of the Newtonian potential...
tial also yielded similar results (Makino 1997). The long-term evolution is characterized by significant depletion of the central region which ceases to be representative of a realistic system. However, even the use of chain regularization (Mikkola & Aarseth 1993) for treating compact subsystems is not sufficient to prevent numerical problems.

2. Special Binary Treatment

In view of the numerical problems outlined above, it is highly desirable to develop a more suitable integration method. A critical appraisal of chain regularization with two massive members reveals that their contribution to the Hamiltonian energy dominates and hence solutions of the equations of motion for any other members are subject to the loss of precision. This recognition led to the construction of a new method which is based on kinematical considerations (Mikkola & Aarseth 2002). Briefly, a special time transformation is combined with the standard leapfrog scheme, thereby avoiding a Hamiltonian formulation. This allows extremely close two-body encounters to be studied without significant loss of accuracy. The interested reader is referred to the published description for more details.

The new method is based on including all the $N(N-1)/2$ interaction terms and solving the equations of motion by the Bulirsch–Stoer (1966) integrator. However, all the solutions need to be advanced with the same time-step which limits the practical membership severely. Hence this formulation can only be used to describe the motions of a compact subsystem. The implementation of such a solution method into a large $N$-body simulation code is somewhat analogous to that for chain regularization and has been outlined elsewhere (Mikkola & Aarseth 1993, Aarseth 1999). In the following we comment on some special features of the BH scheme.

The vital question concerning binary BH evolution is whether a stage can be reached where the gravitational radiation time-scale is sufficiently short for coalescence or significant shrinkage to occur. Previous simulations did not address this issue, mainly because the calculations were terminated prematurely for technical reasons. In the present scheme we have included the 2.5 post-Newtonian approximation for the most critical two-body interaction (Soffel 1989). An estimate of the smallest semi-major axis which can be reached in a system of $N$ particles can readily be made for a given mass ratio. This size is several orders of magnitude outside the relevant range for reasonable system parameters. However, small two-body separations can also be achieved if the eccentricity becomes large enough.
Although large eccentricities were not reported by other investigators, more careful calculations do show significant eccentricity growth during the late stages. Hence this behaviour justifies the extra cost of including the relativistic terms, but only when the corresponding time-scale is less than the expected calculation time. We have \( \dot{a}/a \propto a^4(1 - e^2)^{7/2} \) for the decay time, where \( a \) is the semi-major axis and \( e \) is the eccentricity. Since the time-scale is quite long for circular orbits, we need large values of \( e \) before activating the relativistic treatment which is implemented at different levels of complexity. Thus we distinguish between the classical radiation term and two different expansion orders which describe the post-Newtonian acceleration and relativistic precession, respectively. Finally, coalescence is defined to take place if the BH separation becomes less than three Schwarzschild radii.

3. Numerical Results

The initial conditions consist or two cuspy dwarf galaxy models with \( N_0 \) equal-mass particles of mass \( \bar{m} \) at the apocentre of an eccentric orbit (\( e = 0.8 \)) having a separation of \( 8r_h \), where \( r_h \) is the local half-mass radius. A single BH of mass \( m_{\text{BH}} = (2N_0)^{1/2}\bar{m} \) is placed at the centre of each system. The availability of the special-purpose GRAPE-6 supercomputer together with a fast workstation host allows quite large particle numbers to be studied. Here we report briefly on two recent simulations with \( N_0 = 6 \times 10^4 \) and \( 1.2 \times 10^5 \) particles, making a total of \( 1.2 \times 10^5 \) and \( 2.4 \times 10^5 \) members, respectively. The two mass distributions soon combine into one slightly elongated system, with the dominant binary already formed at the centre after only about 20 crossing times in both models. Then follows a period of constant energy gain where the BH binary is advanced by standard two-body regularization. A switch is made to the new method when the binary becomes super-hard; i.e. \( a \leq 10^{-4}r_h \). The subsequent slow evolution necessitates a large number of perturbed binary orbits to be studied.

The increase of the binding energy, \( E_{\text{BH}} = -m_{\text{BH}}^2/2a \), is illustrated in Fig. 1. As a result of the scaling procedure for two subsystems, the initial total energy is \( E_{\text{tot}} = -1.05 \). Not shown on the plot is the final value \( E_{\text{BH}} = -61 \) with the corresponding semi-major axis \( a = 2.7 \times 10^{-7} \) for the smallest system, compared to \( r_h \approx 1 \) initially. Scale factors \( r_h = 4 \text{ pc} \) and \( \bar{m} = 1 \text{ M}_{\odot} \) were chosen. In other experiments we have demonstrated that a separation of three Schwarzschild radii can be reached without numerical problems. However, the end result of coalescence is ensured once the eccentricity starts to decline significantly, in which case the present purpose is achieved.
Figure 1. Binding energy of BH binaries. The upper curve (dashed line) is for $N = 2.4 \times 10^5$ and the lower curve (solid line) for $N = 1.2 \times 10^5$. Time is in scaled $N$-body units where one initial crossing time is 2.8 or about 1 Myr.

The strong binary evolution gives rise to the ejection of high-velocity particles by the slingshot mechanism. Although the effective mass ratio is about 1000 in the largest system, these ejections still result in significant recoil velocities acquired by the binary. Hence the typical velocity of the central object exceeds the standard equipartition value by a considerable factor, which has implications for the so-called loss-cone effect. However, it should be emphasized that the present results cannot be scaled directly to systems with much larger mass ratios.

The eccentricity evolution of the two systems is shown in Fig. 2. Although the initial eccentricities are relatively high, the trend is for a gradual increase superimposed on fluctuations due to external perturbations. Two subsidiary maxima, $e_{\text{max}} \simeq 0.998$ and 0.997, are first reached in the smaller system, followed by temporary declines before the final stage where coalescence sets in. The eccentricity growth is more pronounced in the second model. Note that $\dot{e} < 0$ during the final approach to coalescence.

The above examples should be considered as tests of the method rather than giving definite results. In this respect the outcome was highly successful, demonstrating that the numerical scheme is both efficient and accurate. Needless to say, the very large number of binary
periods involved ($\sim 10^7$) represents a massive computational effort. However, the binary BH problem is a fundamental one and its study by the direct numerical approach is bound to be fruitful.

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