Ward identities and gauge flow for M-theory in $\mathcal{N}=3$ superspace

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Abstract

We derive the BRST symmetry, Slavnov-Taylor identities and Nielsen identities for the ABJM theories in $\mathcal{N}=3$ harmonic superspace. Further, the gauge dependence of one-particle irreducible amplitudes in such superconformal Chern-Simons theory is shown to be generated by a canonical flow with respect to the extended Slavnov-Taylor identity, induced by the extended BRST transformations (including the BRST transformations of the gauge parameters).

1 Introduction

In the recent literature there has been a lot of excitement in search of the superconformal Chern-Simons theory. The basic intention of doing so was to build a theory describing coincident M2-branes. M2 branes described by three-dimensional superconformal field theories have the structure of Chern-Simons-matter theory with $\mathcal{N}=6$ or $\mathcal{N}=8$ extended supersymmetry. In fact, the Aharony, Bergman, Jafferis and Maldacena (ABJM) theory which is the three-dimensional $\mathcal{N}=6$ superconformal theory was constructed to describe multiple M2 branes on the $\mathbb{C}^4/\mathbb{Z}_k$ orbifold [1]. In ABJM theory the Chern-Simons gauge connections interact with fermions and scalars in bifundamental representations. However, Bagger, Lambert and Gustavsson (BLG) theory [2] is a superconformal theory which follows $\mathcal{N}=8$ supersymmetry. Such three dimensional conformal field theories is also important in the sense that they describe conformal fixed points in condensed matter systems. From this point of view the highly supersymmetric versions are more solvable and, therefore, are more interesting models.

It is desirable to have a superfield description of the ABJM models with maximal number off-shell supersymmetries. As in other cases, such superfield formulations are expected to bring to light geometric and quantum properties of the theory. Here we are interested in harmonic superspace. The concept of harmonic superspace was developed by Galperin, Ivanov, Ogievetsky and Sokatchev in 1980s [3]. The $\mathcal{N}=2$ harmonic superspace, is standard superspace augmented by the two-dimensional sphere $S^2 \sim \text{SU}(2)/\text{U}(1)$. The $\mathcal{N}=2$ harmonic superspace has isospinor harmonics in addition to the usual one. By introduction of isospinor harmonics it is possible to SU(2)-covariantise the notion of Grassmann analyticity [4,5]. This helps enormously to the adequate off-shell unconstrained formulations, just like chirality [6], the simplest kind of Grassmann analyticity [7], is a basis in $\mathcal{N}=1$ supersymmetry. Such analyticity represents to build an analytic subspace of harmonic superspace whose odd dimension is half of that of the full superspace. Also a very similar analyticity underlies the $\mathcal{N}=3$ gauge theory [8,9]. The ABJM theory has been analysed, particularly, in harmonic superspace in Refs. [10,11].

Apart from such investigations, the BRST quantization of the superconformal Chern-Simons theories was subject of interest in recent past [12,13]. The BRST quantization materialize the gauge conditions described by gauge parameters. On the formal side, it has been known since a long time

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that gauge dependence of amplitudes can be studied algebraically through (generalized) Nielsen identities [16–18] having their origin in BRST symmetry. To be more precise these identities can be derived by extending the BRST differential to the gauge parameters. The BRST variation of the gauge parameters is given by classical anticommuting variables paired into a so-called BRST doublet. Recently, the BRST quantization is analysed for the ABJM theory in harmonic superspace in covariant gauges [19]. However, the Ward identities as well as flow of gauge parameters in such theories has not studied yet albeit the substantial progress made. The canonical flow in gauge parameters is studied recently for (non-conformal theory) Yang-Mills theory [20].

In this paper we consider \( \mathcal{N} = 3, d=3 \) harmonic superspace and their algebra. Furthermore, we analyse the superconformal ABJM theory in such superspace. Remarkably, we notice that the ABJM theory in harmonic superspace follows the gauge invariance. However, according to standard quantization methods, we need to fix the extra gauge freedom associated with gauge symmetry. We have, therefore, fixed it here at quantum level by adding suitable gauge-fixing as well as induced (super)ghost terms to the classical superconformal Chern-Simmons-matter parts. The resulting action remains invariant under fermionic rigid BRST symmetry. This BRST symmetry helps to compute the Slavnov-Taylor identities for the tree-level vertex functional. Further we extend the BRST symmetry by incorporating the variations of gauge parameters which help us to demonstrate the extended Slavnov-Taylor identities as well as Nielsen identity. Additionally, we show the gauge dependence of one-particle irreducible amplitudes in ABJM theory in harmonic superspace to be generated by a canonical flow with respect to the extended Slavnov-Taylor identity.

We organize the paper as following. In Sec. 2, we provide the general \( \mathcal{N} = 3 \) harmonic superspace conventions and setup. Also we review the ABJM theory in \( \mathcal{N} = 3 \) harmonic superspace with their gauge symmetry. In section 3, we compute the BRST symmetry along with various identities, namely, Slavnov-Taylor identities and Nielsen identity. These identities helps us to study the behaviour of gauge parameters. Further, in section 4, we discuss the canonical flow of gauge parameters in ABJM theory in harmonic superspace. In the last section we draw concluding remarks.

## 2 ABJM Theory in \( \mathcal{N}=3 \) harmonic superspace

In this section we mainly recapitulate the conventions and algebra followed by \( \mathcal{N}=3 \) harmonic superspace [21,22]. We also embed the ABJM theory in this setup and discuss their gauge symmetry [10].

### 2.1 Harmonic superspace

Let us start by reviewing the \( \mathcal{N}=3, d=3 \) harmonic superspace as originally advocated in [21,22] along with the field models [10]. Here \( \mathcal{N}=3 \) superspace is described by the following real coordinates:

\[
    z = (x^m, \theta_{ij}^{\alpha}), \quad \overline{x^m} = x^m, \quad \overline{\theta_{ij}^{\alpha}} = \theta_{ij\alpha}.
\]

Now, the covariant spinor derivatives and supercharges are given by

\[
    D^{kj}_{\alpha} = \frac{\partial}{\partial \theta_{kj}^{\alpha}} + i\theta^{kj\beta}(\gamma^m)_{\alpha\beta} \frac{\partial}{\partial x^m}, \quad Q^{kj}_{\alpha} = \frac{\partial}{\partial \theta_{kj}^{\alpha}} - i\theta^{kj\beta}(\gamma^m)_{\alpha\beta} \frac{\partial}{\partial x^m}.
\]

The notations are setted as follows: the Greek letters \( \alpha, \beta, \ldots \) denote the spinorial indices corresponding to the \( \text{SO}(1, 2) \simeq \text{SL}(2, \mathbb{R}) \) Lorentz group.
In order to construct the harmonic superspace parametrized by the new bosonic coordinates (known as the harmonics in superspace) $u^\pm_i$ one should write a matrix belonging to the coset $SU(2)/U(1)$.

Now, the harmonic superspace can now be described by the following coordinates:

$$\zeta_A = (x^{a\beta}_A, \theta^{++}_\alpha, \theta^{--}_\alpha, \theta^0_{\alpha}, u^\pm_i),$$

where

$$x^{a\beta}_A = (\gamma_m)^{a\beta} x^m_A = x^{a\beta} + i(\theta^{++}_\alpha \theta^{--}_\beta + \theta^{++\beta} \theta^{--\alpha}),$$

and $\theta^{++}_\alpha$, $\theta^{--}_\alpha$, $\theta^0_{\alpha}$, are harmonic decompositions of the anticommuting coordinates $\theta^{ij}_\alpha$, given by

$$(\theta^{++}_\alpha, \theta^{--}_\alpha, \theta^0_{\alpha}) = (u^+_i u^+_j \theta^{ij}_\alpha, u^-_i u^-_j \theta^{ij}_\alpha, u^+_i u^-_j \theta^{ij}_\alpha).$$

Now the harmonic derivatives in the above coordinates are

$$D^{++} = u^+_i \frac{\partial}{\partial u^+_i} + 2i\theta^{++\alpha} \partial^{A}_{\alpha} + \theta^{++\alpha} \frac{\partial}{\partial \theta^{++\alpha}} + 2\theta^0_{\alpha} \frac{\partial}{\partial \theta^{--\alpha}},$$

$$D^{--} = u^-_i \frac{\partial}{\partial u^-_i} - 2i\theta^{--\alpha} \partial^{A}_{\alpha} + \theta^{--\alpha} \frac{\partial}{\partial \theta^{--\alpha}} + 2\theta^0_{\alpha} \frac{\partial}{\partial \theta^{++\alpha}},$$

$$D^0 = u^+_i \frac{\partial}{\partial u^+_i} - u^-_i \frac{\partial}{\partial u^-_i} + 2\theta^{++\alpha} \frac{\partial}{\partial \theta^{++\alpha}} - 2\theta^{--\alpha} \frac{\partial}{\partial \theta^{--\alpha}},$$

and the harmonic decompositions of spinor derivatives are

$$D^{++}_{\alpha} = u^+_i u^+_j D^{ij}_{\alpha} = \frac{\partial}{\partial \theta^{--\alpha}}, \quad D^{--}_{\alpha} = u^-_i u^-_j D^{ij}_{\alpha} = \frac{\partial}{\partial \theta^{++\alpha}} + 2i\theta^{--\beta} \partial^{A}_{\beta},$$

$$D^0_{\alpha} = u^+_i u^-_j D^{ij}_{\alpha} = \frac{1}{2} \frac{\partial}{\partial \theta^{0\alpha}} + i\theta^{0\beta}(\gamma_m)^{\alpha\beta}\partial/\partial x^m_A.$$

The algebra satisfied by these derivatives are

$$[D^{++}, D^{--}] = D^0, \quad [D^{++}_{\alpha}, D^{--}_{\beta}] = 2i\theta^{AB}_{\alpha\beta}, \quad [D^0_{\alpha}, D^0_{\beta}] = -i\theta^{AB}_{\alpha\beta}, \quad [D^{\pm}_{\alpha}, D^{0}_{\beta}] = 0,$$

$$[D^{\pm}_{\alpha}, D^{\pm}_{\alpha}] = 2D^{0}_{\alpha}, \quad [D^0_{\alpha}, D^{\pm}_{\alpha}] = \pm 2D^{\pm}_{\alpha}, \quad [D^{\pm}_{\alpha}, D^{\pm}_{\alpha}] = D^{\pm}_{\alpha}.$$

The full and analytic integration measures are given conveniently by

$$d^3z = -\frac{1}{16} i^3 x^i_A (D^{++})^2 (D^{--})^2 (D^0)^2, \quad d\zeta^{(-4)} = \frac{1}{4} i^3 x^i_A d\zeta (D^{--})^2 (D^0)^2.$$

### 2.2 ABJM theory

In this section we sketch briefly the superconformal Chern-Simons-matter theory with $N = 6$ supersymmetry in harmonic superspace as in Ref. [10]. As the component content of the ABJM theory is given by four complex scalar fields and four complex spinor fields, we first consider the two gauge superfields for ABJM theory in this harmonic superspace $V^{++}_{AB}$ and $V^{++A}_{AB}$ ($A, B = 1, 2, ..., N$) which are $N \times N$ matrices. These gauge superfields transform under the gauge group $U(N)_k$ and $U(N)_{-k}$, respectively. To define the ABJM theory we first write the gauge part of the action as [10]

$$S_{\text{gauge}} = \frac{i k}{16\pi} \text{Tr} \int d\zeta^{(-4)} [V^{++}_{AB} D^{++A} V^{++}_{AB} - V^{++}_{AB} D^{++A} V^{++}_{AB}],$$

$$d\zeta^{(-4)} = \frac{1}{4} \alpha^3 x^i_A d\zeta (D^{--})^2 (D^0)^2.$$
with non-analytic gauge superfields

\[
V_L^{-} = \sum_{n=1}^{\infty} (-1)^n \int du_1 \ldots du_n \frac{V_L^{++}(z, u_1) V_L^{++}(z, u_2) \ldots V_L^{++}(z, u_n)}{(u^+ u_1^+)(u_1^+ u_2^+) \ldots (u_n^+ u^+)},
\]

\[
V_R^{-} = \sum_{n=1}^{\infty} (-1)^n \int du_1 \ldots du_n \frac{V_R^{++}(z, u_1) V_R^{++}(z, u_2) \ldots V_R^{++}(z, u_n)}{(u^+ u_1^+)(u_1^+ u_2^+) \ldots (u_n^+ u^+)},
\]

(11)

We also define matter fields \( q^+_a, \bar{q}^+_a \) (\( a = 1, 2 \)), transform under the bifundamental representation of the group \( U(N)_k \times U(N)_{-k} \). The gauge invariant generalization of the hypermultiplet action is [10]

\[
S_M[q^+, \bar{q}^+] = \text{Tr} \int d\zeta (-\bar{q}^+_a \nabla^{++} q^+_a),
\]

(12)

where the gauge covariant harmonic derivative \( \nabla^{++} = \mathcal{D}^{++} + V_L^{++} - V_R^{++} \). Now, the classical action for the ABJM theory in harmonic superspace can now be given by

\[
S_{ABJM} = S_{gauge} + S_M,
\]

(13)

which remains invariant under following gauge transformations [10]:

\[
\delta q^+_a = \Lambda_L q^+_a - q^+_a \Lambda_R, \quad \delta \bar{q}^+_a = \Lambda_R \bar{q}^+_a - \bar{q}^+_a \Lambda_L, \\
\delta V_L^{++} = \nabla_L^{++} q^+_a - q^+_a \Lambda_L, \\
\delta V_R^{++} = \nabla_R^{++} q^+_a - q^+_a \Lambda_R,
\]

(14)

where \( \Lambda_L \) and \( \Lambda_R \) are the gauge parameters. This model is also invariant under the following extra \( \mathcal{N} = 3 \) supersymmetric transformations [10]:

\[
\delta \epsilon q^+_a = i \epsilon^{(ab)} \nabla_0 q^+_b, \\
\delta \epsilon \bar{q}^+_a = i \epsilon^{(ab)} \nabla_0 \bar{q}^+_b, \\
\delta \epsilon V_L^{++} = \frac{8 \pi}{k} \epsilon^{(ab)} \theta_0 \theta_0 q^+_a q^+_b, \\
\delta \epsilon V_R^{++} = \frac{8 \pi}{k} \epsilon^{(ab)} \theta_0 \theta_0 \bar{q}^+_a \bar{q}^+_b,
\]

(15)

where

\[
\nabla_0 q^+_b = D_0 q^+_b - \frac{1}{2} D^{++} V_L^{--} q^+_b + \frac{1}{2} \bar{q}^+_b D^{++} V_R^{--} + \theta^- \bar{q}^+_b (W_L^{++} q^+_b - q^+_b W_R^{++}).
\]

(16)

Thus, together with the original manifest \( \mathcal{N} = 3 \) supersymmetry, this model has \( \mathcal{N} = 6 \) supersymmetry.

3 BRST symmetry and Ward identities

In this section we analyse the BRST symmetry of ABJM theory in \( \mathcal{N} = 3 \) harmonic superspace. As the ABJM model in harmonic superspace is gauge invariant it contains some spurious degrees of freedom. These extra degrees of freedom give rise to constraints in the canonical quantization and
divergences in the path integral quantization [24]. To get rid of such redundancy of degrees of freedom we restrict the gauge superfields to follow the certain gauge-fixing conditions:

\[ \mathcal{F}_L = D^{++}V_L^{++} = 0, \quad \mathcal{F}_R = D^{++}V_R^{++} = 0. \]  
(17)

The effect of above gauge conditions can be incorporated at quantum level in the theory by adding appropriate gauge-fixing terms in the classical action [13]. Here the (linearized) gauge-fixing terms are [4]

\[ S_{gf} = \int d\zeta (-4)\text{Tr} \left[ -\alpha b_L \mathcal{F}_L + \alpha b_R \mathcal{F}_R \right], \]  
(18)

where \( b_L \) and \( b_R \) are the multiplier superfields. According to the Faddeev-Popov quantization, the gauge-fixing terms induce the ghost terms in the functional integral. Here the gauge-fixing terms [18] induce following ghost terms in the path integral:

\[ S_{gh} = \int d\zeta (-4)\text{Tr} \left[ \alpha \bar{c}_L s \mathcal{F}_L - \alpha \bar{c}_R s \mathcal{F}_R \right], \]
\[ = \int d\zeta (-4)\text{Tr} \left[ \alpha \bar{c}_L D^{++} \nabla^{++} c_L - \alpha \bar{c}_R D^{++} \nabla^{++} c_R \right], \]  
(19)

where \( c_L, c_R \) and \( \bar{c}_L, \bar{c}_R \) are ghost superfields and corresponding antighost superfields respectively and \( s \) denote the BRST variation. The BRST transformations for the superfields are defined by

\[ s V_L^{++} = \nabla^{++} c_L, \quad s V_R^{++} = \nabla^{++} c_R, \]
\[ s c_L = -\frac{1}{2} [c_L, c_L], \quad s c_R = -\frac{1}{2} [c_R, c_R], \]
\[ s \bar{c}_L = b_L, \quad s \bar{c}_R = b_R, \]
\[ s b_L = 0, \quad s b_R = 0, \]
\[ s q^{+a} = c_L q^{+a} - q^{+a} c_R, \quad s \bar{q}^{+a} = c_R \bar{q}^{+a} - \bar{q}^{+a} c_L. \]  
(20)

Under the above transformations the effective action \( S_{ABJM} + S_{gf} + S_{gh} \) is invariant.

Now, we restricted to the pure gauge sectors of the theory where only gauge and ghost fields have non-linear BRST variations, so their renormalization requires the introduction of external sources known as anti-superfields. These anti-superfields are coupled to the BRST variation of the corresponding superfields as follows

\[ S_{af} = \int d\zeta (-4)\text{Tr} \left[ V_L^{+++} s V_L^{++} - c_L^* s c_L - V_R^{+++} s V_R^{++} + c_R^* s c_R \right], \]  
(21)

where \( U(1) \) charge of the antifields \( c_L^* \) and \( c_R^* \) is +4. These antifields are introduced to analyse theory at general ground. However, for a linear covariant gauge such terms vanishes because the the antifields \( c_L^* \) and \( c_R^* \) do not exist for such case. Here minus signs in front of the terms \( c_L^* s c_L \) and \( V_R^{+++} s V_R^{++} \) are introduced for consistency with the Batalin-Vilkovisky (BV) bracket conventions. Now, we are able to define the tree-level vertex functional as follows,

\[ \Sigma^{(0)} = S_{ABJM} + S_{gf} + S_{gh} + S_{af}. \]  
(22)

This vertex functional obeys the following Slavnov-Taylor identity:

\[ S(\Sigma^{(0)}) = \int d\zeta (-4)\text{Tr} \left[ \frac{\delta \Sigma^{(0)}}{\delta V_L^{+++}} \frac{\delta \Sigma^{(0)}}{\delta V_L^{++}} - \frac{\delta \Sigma^{(0)}}{\delta c_L^*} \frac{\delta \Sigma^{(0)}}{\delta c_L} + b_L \frac{\delta \Sigma^{(0)}}{\delta c_L} + \frac{\delta \Sigma^{(0)}}{\delta V_R^{+++}} \frac{\delta \Sigma^{(0)}}{\delta V_R^{++}} - \frac{\delta \Sigma^{(0)}}{\delta c_R^*} \frac{\delta \Sigma^{(0)}}{\delta c_R} + b_R \frac{\delta \Sigma^{(0)}}{\delta c_R} \right] = 0. \]  
(23)
Notice that the linearity of the BRST transformation of the antighosts $\bar{c}_L, \bar{c}_R$ do not require the introduction of corresponding anti-superfields. The above identity holds irrespectively of the particular form of the gauge-fixing chosen. For instance, some specific choices of the gauge (e.g. linear covariant gauges or the Landau gauge) further identities (the auxiliary-field and the ghost equations) arise \cite{25}.

Now, one can extend the BRST symmetry to act on the gauge parameters to derive an extended Slavnov-Taylor identity, leading to the Nielsen identities \cite{26,27}. The BRST variation of gauge parameters are

$$s\lambda_i = \theta_i, \quad s\theta_i = 0, \quad s\alpha = \theta, \quad s\theta = 0,$$

where $\lambda_i, \theta_i, \alpha$ and $\theta$ are the (arbitrary) gauge parameters. So, the extended BRST transformations can now be given by expressions (20) and (24) collectively. Under this extended BRST transformation the gauge-fixing fermion introduces additional terms into the BRST exact parts

$$S_{gf} + S_{gh} = s\int d\zeta (-4)\text{Tr} \left[ -\alpha \bar{c}_L F_L + \alpha \bar{c}_R F_R \right],$$

$$= \int d\zeta (-4)\text{Tr} \left[ -\alpha b_L F_L + \alpha \bar{c}_L s F_L - \theta \bar{c}_L F_L + \alpha \bar{c}_L \left( \frac{\partial F_L}{\partial \lambda_i} \theta_i + \frac{\partial F_L}{\partial \theta} \theta \right) \right] + \alpha b_R F_R - \alpha \bar{c}_R s F_R + \theta \bar{c}_R F_R - \alpha \bar{c}_R \left( \frac{\partial F_R}{\partial \lambda_i} \theta_i + \frac{\partial F_R}{\partial \theta} \theta \right) \right] \right].$$

(25)

Therefore, the tree-level classical action satisfies the following extended Slavnov-Taylor identity:

$$\hat{S}(\Sigma^{(0)}) = \sum_i \theta_i \frac{\partial \Sigma^{(0)}}{\partial \lambda_i} + \theta \frac{\partial \Sigma^{(0)}}{\partial \alpha} + S(\Sigma^{(0)}) = 0.$$ (26)

In case of non-anomalous theories this equation holds up to the full vertex functional $\Sigma$:

$$\hat{S}(\Sigma) = \sum_i \theta_i \frac{\partial \Sigma}{\partial \lambda_i} + \theta \frac{\partial \Sigma}{\partial \alpha} + S(\Sigma) = 0.$$ (27)

Now, we compute the Nielsen identity by taking derivative of $\Sigma$ with respect to $\theta$ and then setting $\theta, \theta_i$ equal to zero

$$\left. \frac{\partial \Sigma}{\partial \alpha} \right|_{\theta=\theta_i=0} = -\int d\zeta (-4)\text{Tr} \left[ \frac{\delta^2 \Sigma}{\delta \theta \delta V_L^{++}} \frac{\delta \Sigma}{\delta V_L^{++}} - \frac{\delta \Sigma}{\delta \partial \delta c_L} \frac{\delta \Sigma}{\delta \partial \delta c_L} - \frac{\delta \Sigma}{\delta \partial \delta c_L} \frac{\delta \Sigma}{\delta \partial \delta c_L} \right] + b_L \frac{\delta^2 \Sigma}{\delta \theta \partial \delta c_L} + b_R \frac{\delta^2 \Sigma}{\delta \theta \partial \delta c_L} \right|_{\theta=\theta_i=0}.$$ (28)

In the same fashion we can get an expression for the derivative of $\Sigma$ with respect to $\lambda_i$ by taking the derivative of the extended ST identity with respect to $\theta_i$.

The quantum action principle (QAP) \cite{25} describes the structure of the ward identities at the quantum level. For QAP is applicable to the theories which are local, Lorentz invariant and power counting renormalizable. To prove the renormalizability of the ABJM theory in harmonic superspace
we have to show stability. For that we split the effective action at first order in loop expansion in to two parts: a finite part and a divergent part

\[ \Sigma^{(1)} = \Sigma^{(1)}_{\text{fin}} + \Sigma^{(1)}_{\text{div}}. \]  

Due to linearity of Slavnov-Taylor identity \( S \Sigma^{(1)}_{\text{div}} = 0 \). Now the divergence occuring in the quantum level can be reabsorbed by introduction of local counter terms obtained by redefining the fields. In this way, we are able to prove the algebraic renormalization of the ABJM theory in harmonic superspace.

4 Canonical flow of gauge parameters

In this section we analyse the canonical flow of gauge parameters. Let us begin the section by defining the antibracket (BV bracket) as follows

\[ \{ X, Y \} = \int d\zeta (-4) \text{Tr} \sum_{\phi} \left[ (-1)^{\epsilon_X + 1} \frac{\delta_X \delta_Y}{\delta \phi \delta \phi^*} - (-1)^{\epsilon_{X^*}} \frac{\delta_{X^*} \delta_{Y^*}}{\delta \phi \delta \phi^*} \right]. \]  

(30)

where collective superfield \( \phi \equiv (V_L^{++}, c_L, b_L, V_R^{++}, c_R, b_R) \) and collective anti-superfields \( \phi^* \equiv (V_L^{+++}, c_L^*, b_L^*, V_R^{+++}, c_R^*, b_R^*) \). Here \( \epsilon_\phi \) and \( \epsilon_{\phi^*} \) denote the statistics of the superfields \( \phi \) and the anti-superfields \( \phi^* \). However \( \epsilon_X \) refers the statistics of the functional \( X \).

With the help of above antibrackets the extended Slavnov-Taylor identity (26) is writte by

\[ \tilde{S}(\Sigma) = \sum_i \theta_i \frac{\partial \Sigma}{\partial \lambda_i} + \theta \frac{\partial \Sigma}{\partial \alpha} + \frac{1}{2} \{ \Sigma, \Sigma \} = 0. \]  

(31)

By taking a derivative with respect to \( \theta \) we get

\[ \frac{\partial \Sigma}{\partial \alpha} \bigg|_{\theta=\theta_i=0} = - \left( \frac{\partial \Sigma}{\partial \theta}, \Sigma \right) \bigg|_{\theta=\theta_i=0}. \]  

(32)

Here we note that the argument goes in the same way if one takes a derivative with respect to \( \theta_i \). The expression (32) shows that the derivative of the vertex functional with respect to \( \alpha \) is obtained by a canonical transformation (with respect to the antibracket) induced by the generating functional \( \frac{\partial \Sigma}{\partial \theta} \). One cannot solve (32) by simple exponentiation because the RHS in general depends on \( \alpha \) and therefore one needs to make recourse to a Lie series. To achieve this goal, we introduce the following operator:

\[ \Delta_\Psi = \{ \cdot, \Psi \} + \frac{\partial}{\partial \alpha}. \]  

(33)

Now in terms of the Lie series the vertex functional \( \Sigma \) is given by

\[ \Sigma = \sum_{n \geq 0} \frac{1}{n!} \alpha^n \left[ \Delta_\Psi^n \Sigma^{(0)} \right]_{\alpha=0}, \]  

(34)

where \( \Sigma^{(0)} \) refers the vertex functional at \( \alpha = 0 \). Here we remark that the above equation holds irrespectively of the form of the gauge-fixing (and in particular is independent of the existence of a
auxiliary field equation and of a ghost equation, validating the stability of the gauge-fixing in certain cases).

Now, for illustration purpose, we choose the Lorentz covariant gauges

\[ F_L = D^{++}V^{++}_L, \quad F_R = D^{++}V^{++}_R. \]  

(35)

For these particular choices, the extended BRST-exact terms are given by

\[ S_{gf} + S_{gh} = \int d\zeta (-4) \text{Tr} \left[ -\alpha b_L D^{++}V^{++}_L + \alpha c_L D^{++}V^{++}_L + \alpha c_R D^{++}V^{++}_R + \theta \bar{c}_L D^{++}V^{++}_L \right. \]

\[ + \left. \alpha b_R D^{++}V^{++}_R - \theta \bar{c}_R D^{++}V^{++}_R \right]. \]  

(36)

For these gauges, the auxiliary superfield equation and the superghost equation hold:

\[ \frac{\delta \Sigma}{\delta b_L} = -\alpha D^{++}V^{++}_L, \quad \frac{\delta \Sigma}{\delta b_R} = -\alpha D^{++}V^{++}_R, \]

\[ \frac{\delta \Sigma}{\delta c_L} = \alpha D^{++}, \quad \frac{\delta \Sigma}{\delta c_R} = \alpha D^{++} - \theta D^{++}V^{++}_R, \]  

(37)

where the first two of the above equations imply that the auxiliary superfields-dependence are confined at tree level. However, the last two of the above equations in turn imply that at higher orders \((n \geq 1)\) \(\Sigma\) can depend on \(\bar{c}_L, \bar{c}_R\) only through the combinations

\[ \tilde{V}^{++}_L = V^{++}_L - D^{++}\bar{c}_L, \quad \tilde{V}^{++}_R = V^{++}_R - D^{++}\bar{c}_R. \]  

(38)

Implying the above redefinitions we define the reduced functional as follows,

\[ \bar{\Sigma} = \Sigma - \int d\zeta (-4) \text{Tr} \left[ -\alpha b_L D^{++}V^{++}_L + \alpha b_R D^{++}V^{++}_R \right]. \]  

(39)

With such introduction of the reduced functional the antibrackets can be restricted to the variables \((V^{++}_L, V^{++}_R), (c_L, c_R)\) and \((c_L, c_R^*)\) and therefore flow equation reads

\[ \frac{\partial \bar{\Sigma}}{\partial \alpha} \bigg|_{\theta = \theta_i = 0} = -\int d\zeta (-4) \text{Tr} \left[ \frac{\delta \Psi}{\delta V^{++}_L} \frac{\delta \bar{\Sigma}}{\delta V^{++}_L} + \frac{\delta \Psi}{\delta V^{++}_R} \frac{\delta \bar{\Sigma}}{\delta V^{++}_R} + \frac{\delta \bar{\Sigma}}{\delta c_L} \frac{\delta \bar{\Sigma}}{\delta c^*_L} - \frac{\delta \bar{\Sigma}}{\delta c^*_L} \frac{\delta \bar{\Sigma}}{\delta c_L} \right. \]

\[ + \left. \frac{\delta \bar{\Sigma}}{\delta c_R} \frac{\delta \bar{\Sigma}}{\delta c^*_R} - \frac{\delta \bar{\Sigma}}{\delta c^*_R} \frac{\delta \bar{\Sigma}}{\delta c_R} \right]. \]  

(40)

Now, the Lie operator \(\Delta_\Psi\) is given by

\[ \Delta_\Psi(X) = \{X, \Psi\} + \frac{\partial X}{\partial \alpha}, \]

\[ \Delta_\Psi(X) = \int d\zeta (-4) \text{Tr} \left[ \frac{\delta X}{\delta V^{++}_L} \frac{\delta \Psi}{\delta V^{++}_L} + \frac{\delta X}{\delta V^{++}_R} \frac{\delta \Psi}{\delta V^{++}_R} + \frac{\delta X}{\delta c_L} \frac{\delta \Psi}{\delta c_L} - \frac{\delta X}{\delta c^*_L} \frac{\delta \Psi}{\delta c_L} \right. \]

\[ + \left. \frac{\delta \Psi}{\delta c_R} \frac{\delta \Psi}{\delta c^*_R} - \frac{\delta \Psi}{\delta c^*_R} \frac{\delta \Psi}{\delta c_R} \right] + \frac{\partial X}{\partial \alpha}. \]  

(41)
By expression (36) we see that at tree-level gauge-fixing fermion reduces to
\[ \Psi = O(h). \]  
(42)

The Lie series of the vertex functional \( \Sigma \) given in (34) then allows one to express the coefficients of the \( \alpha \)-expansion of one-point irreducible amplitudes in the Lorentz-covariant gauge in terms of one-point irreducible Landau gauge amplitudes plus an \( \alpha \)-dependent contribution, arising from the gauge-fixing fermion \( \Psi \).

5 Conclusion

In this paper we have considered the ABJM theory in \( \mathcal{N} = 3 \) harmonic superspace. The field content of the ABJM model is given by four complex scalar and spinor fields which live in the bifundamental representation of the \( U(N) \times U(N) \) gauge group. Besides their general setup the gauge symmetry of the theory are also presented. As we know a gauge theory can’t be quantized without getting rid of spurious gauge freedom. Therefore, we have chosen the covariant gauges in order to fix the the gauge freedom. We have achieved this at quantum level by adding suitable gauge-fixing and the ghost actions to the classical action. Resulting effective action admits the BRST symmetry. Furthermore, we have computed the Ward identities for such theory in \( \mathcal{N} = 3 \) superspace. With the help of this set of Ward identities we have established the algebraic renormalizability of the ABJM theory in harmonic superspace. To see the behaviour of gauge parameters we have derived the Nielsen identities by extending the quantum action. Such an extended quantum action remains symmetric under larger set of BRST transformations. This identities will be helpful to demonstrate the gauge independence of the gauge self-energy, and of the matter mass shell in case of ABJM theory. It was demonstrated in the case of ABJM theory that the identities lead to results complementary to those of the usual Ward identities. As with the Ward identities, the Nielsen identities offer possibilities to check ones calculations, however, they also allow us to see where physical meaning may be found in apparently gauge dependent Greens functions.

Further, the existence of a canonical flow in the space of gauge parameters and the related solution in terms of a Lie series provide a way to analyse the results within an algebraic framework. As the generating functional of the canonical flow depends on the gauge parameters, we are unable to get the full solution by a naive exponentiation. The results is bound to hold even beyond perturbation theory (as far as the ST identity is valid). Such a solution can be expressed only through an appropriate Lie series. Knowing such a Lie series makes the comparison between computations carried out in different gauges for ABJM theory in harmonic superspace easy. The relations derived here are particularly in the perturbative sector. However, it will be interesting to analyse such discussion in the non-perturbative regime. The two point function for gauge connection, under the assumption that analyticity in the gauge parameter around \( = 1 \) holds, a closed formula interpolating between the Landau and the other suitable covariant gauge can be obtained.

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