Study on Transient Distributed Model of Frost on Heat Pump Evaporator

Zhiqiang Liu*,1, Hongtao Zhu2 and Hanqing Wang3

1 Associate Professor, School of Energy and Power Engineering, Central South University, P.R. China
2 Graduate Student, School of Energy and Power Engineering, Central South University, P.R. China
3 Professor, Department of Civil Engineering, Zhuzhou Institute of Technology, P.R. China

Abstract

As a building energy saving equipment, the air-source heat pump is used broadly in China. At present, the lumped parameter model of frost, which is based on experiences and depend heavily on the running conditions, is widely used in simulation study of air-source heat pumps. In this paper, a dynamic distributed parameter model is presented. The model takes the frost deposition on evaporator as one dimensional, transient formulation, whose mass and heat is transferred in porosity and has a moving boundary. Result shows that the proposed model favorably predicted the key parameters in frost formation and compared well with the experimental data.

Keywords: transient distributed model; frost; evaporator; simulation; heat and mass transfer

Introduction

As energy-saving equipment, the air-source heat pumps are broadly used in China especially in central-south areas and lack-of-water regions. The evaporator is an influential component of heat pumps, determining both the amount of heat extracted from the surrounds and the temperature at which it is extracted. The formation of frost on evaporator surface is a common problem when moist air comes into contact with cold evaporator surface whose temperature is below dew-point, condensation occurs. If the evaporator surface temperature is below freezing, frost occurs. Frost leads to the degradation of evaporator performance in terms of lower heat transfer and higher-pressure drop.

Many studies performed on the subject previously have been of experimental in nature with limited effort in the development of a model (O’Neal D.L., and Tree D.R. 1985). No general correlations were presented that would allow the designer to extrapolate their data to slightly different evaporator geometries or operation conditions (Hong Chen and Leena Thomas 2001). The experimental investigation is usually complicated, mainly due to the financial cost and the large number of variables involved. The use of numerical models can reduce the cost and also facilitate understanding the phenomena related to the problem. And they can simulate a wide range of operating conditions and account for time-variable operating conditions that are common in practice. Once validated, they can be used for the design and optimization of evaporator of heat pumps.

In this paper, a numerical transient distributed parameter model of frost growth on evaporator is presented. Results will be compared with the experimental data.

Simulation Model of Frost on Evaporator

For simplicity, we assume the distribution of refrigerant temperature and pressure in evaporator is constant.

1. Frost Growth Model

A substantial of literature is available on the frost properties and growth on evaporators (Sami S.M and Duong T. 1990) (Brian P.L.T, Reid R.C. and Shah Y.T. 1970). The majority of existing frost growth models average the frost density normal to the surface in direction of frost growth. No variation of frost density and temperature with frost layer is predicted. This is in part due to the complexity involved in formulating the problem, which requires accurate knowledge of transport phenomena within the frost layer, and a lack of sufficient experimental data on properties within the frost layer (Tao Y.X., Besant R.W. and Rezkllah K.S. 1993). Reviews of existing literature on frost growth concluded that there is inadequate understanding of the frost formation and growth phenomena. Therefore, the need arise for further investigation.

In recent development, reported by Tao et al (Tao Y.X. and Besant R.W. 1993) and Le Grill (Gall R.Le and Grillot M. 1997), attempts were made to establish a mathematical model that can predict both spatial and temporal variation of frost density and temperature. The frost thickness and the heat flux through the frost layer can also be predicted. A modified model is reported by Hong Chen (Hong Chen and Leena Thomas 2001) to model frost characteristics on heat exchanger fins in low...
temperature (air flow temperature range from –13˚C to –21˚C for the air supply, with a relative humidity over 90% but less than 100%). The frost growth model in this paper is quite similar to Hong Chen, except for the supply air temperature range and part of boundary conditions. Because the heat pumps supply temperature is always between –5˚C and 10˚C and the relative humidity is between 65% and 90%.

2. Governing Equations of Frost as a Porous Media.

The governing equations for frost growth and discussion of the assumptions are similar to those of Hong Chen (Hong Chen and Leena Thomas 2001). Within the frost layer, the following equations apply at every point (z, t).

Energy equation:

\[ \rho_f c_f \frac{\partial T}{\partial t} + m h_{\infty} = \frac{\partial}{\partial z} \left( \lambda_{\text{eff}} \frac{\partial T}{\partial z} \right) \] (1)

Water vapor diffusion equation:

\[ \frac{\partial (\varepsilon_f \rho_f)}{\partial t} - m = \frac{\partial}{\partial z} \left( D_{\text{eff}} \frac{\partial \varepsilon_f}{\partial z} \right) \] (2)

Ice phase continuity equation:

\[ \frac{\partial \varepsilon_f}{\partial t} + \frac{m}{\rho_f} = 0 \] (3)

where:

\[ \varepsilon_v + \varepsilon_f = 1.0 \] (4)

\[ \rho_f = \varepsilon_f \rho_s + \varepsilon_v (\rho_v + \rho_s) \] (5)

\[ \varepsilon_f = \varepsilon_f \rho_f c_i + \varepsilon_v (c_i \rho_v + c_v \rho_s) \] (6)

\[ D_{\text{eff}} = \varepsilon_f D_{\text{in}} \] (7)

\[ \lambda_{\text{eff}} = 0.02422 + 7.214 \times 10^{-4} \rho_f + 1.1797 \times 10^{-6} \rho_f^2 \] (8)

The thermodynamic properties of water vapor and air can be calculated by the ideal gas state equation. The saturation pressure can get from the Clapeyron equation. The Hirchfelder’s correlation is used to obtain the \( D_{\text{eff}} \) (Xu W.Q. 1999).

3. Boundary Conditions and Initial Conditions for Frost Growth.

At the frost-air interface, the mass transfer boundary condition is:

\[ h_{\text{ref}} \left[ W_0 - W (z = \delta_f, t) \right] = D_{\text{eff}} \frac{\partial \varepsilon_f (z = \delta_f, t)}{\partial z} + \rho_f \frac{d\delta_f}{dt} \] (9)

The boundary conditions for heat transfer are:

\[ h_f [T_0 - T (z = \delta_f, t)] = \lambda_{\text{eff}} \frac{\partial T (z = \delta_f, t)}{\partial z} - h_{\infty} \rho_f \frac{d\delta_f}{dt} \] (10)

\[ T (z = 0, t) = T_i \] (11)

The boundary conditions for ice volume fraction are (Hong Chen and Leena Thomas 2001):

\[ \varepsilon_f (z = 0, t) = 0.3 \] (12)

The initial height of frost over the entire fin is assumed to be (Sami S.M and Duong T. 1990):

\[ \delta_f (t = 0) = 0.1 \times 10^{-3} \] (13)

The initial temperature within the frost layer is:

\[ T (t = 0, z) = T_i \] (14)

From trial and error process, the initial condition for volume fraction of ice within the frost layer is taken to be constant and the following value is acceptable:

\[ \varepsilon_f (t = 0, x, y, z) = 0.015 \] (15)

In equations 9 and equation 10, the air flow temperature \( T_0 \), and the humidity ratio \( W_0 \) are defined as the average airflow bulk temperature and humidity ratio with finned section and are obtained by combination equation 16 through equation 19.

\[ T_0 = \left( T_{\text{in}} + T_{\text{out}} \right) / 2 \] (16)

\[ W_0 = \left( W_{\text{in}} + W_{\text{out}} \right) / 2 \] (17)

\[ \rho_f Q (c_{\text{in}} \Delta T + 2500 \Delta W) = q_{\text{loss}} \] (18)

\[ \rho_f Q \Delta W = \int h_{\text{in}} \rho_f (W_0 - W_f) dx \] (19)

4. Model for Heat Conduction In a Fin

The model of heat transfer in the frost layer is coupled with the heat conduction in a fin through the boundary conditions. Therefore, the governing equation for heat conduction within the fins is

\[ \rho_{\text{fin}} c_{\text{fin}} \frac{\partial T}{\partial t} = \lambda_{\text{fin}} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + S \] (20)

\[ S = \frac{2.0}{\delta_{\text{fin}}} \frac{\lambda_{\text{eff}}}{\delta_{\text{eff}}} \frac{\partial T (z = 0, x, t)}{\partial z} \] (21)

Boundary condition and initial condition are

\[ T_i (x^2 + y^2 = R^2) = T_i \] (22)

\[ \frac{\partial T}{\partial x} (x = h_{\text{fin}}, y) = 0 \] (23)

\[ \frac{\partial T}{\partial y} (x, y = l_{\text{fin}}) = 0 \] (24)

\[ T_i (x, y, t = 0) = T_0 \] (25)

5. Airside Correlation for Evaporator in Frost Conditions

The Chi-Chuan Wang correlation is chosen to calculate the convective heat and mass transfer coefficient and pressure drop (Wang C.C, Lin Y.T. and Lee C.J. 2000). The correlation is developed by using a total of 31 samples of fin-tube heat exchangers under wet conditions. The proposed heat transfer correlation can describe 93.4% of the test data with ±15% with a mean
deviation of 6.33% while the proposed friction correlation can describe 83.5% of results with ±15% with a mean deviation of 9.51%. The heat transfer performance is in terms of the Coburn j factor, i.e.

\[ h_j = \frac{1.5 \times C_x \times G_{\text{max}}}{Pr^2} \]  

(26)

where:

\[ f = 19.36 \times Re_{\text{tub}}^{0.6795} \left( \frac{P_f}{P_t} \right)^{1.5} \left( \frac{D_f}{D_t} \right)^{0.012} \]  

(27)

\[ f_l = 0.3745 - 1.554 \left( \frac{F_p}{D_0} \right)^{0.24} \left( \frac{P_t}{P_f} \right)^{0.62} \]  

(28)

\[ Re_{\text{tub}} = \frac{G_{\text{max}} \times D_0}{\mu_{\text{tub}}} \]  

(29)

The convective mass transfer coefficient is obtained from equation 30.

\[ h_j = \frac{0.372 \times \text{Re}_{\text{tub}}^{0.6795} \left( 0.6 + 0.624 \times \text{Re}_{\text{tub}}^{0.012} \right)^{0.012}}{\mu_{\text{tub}}} \]  

(30)

The reduction of the friction factor of the evaporator is evaluated from the pressure drop equation proposed by Kays and London (Kays W.M. and London A.L.) as

\[ f = \frac{A_r \rho_{\text{sat}}}{A_r \rho_t} \left[ 2 \Delta \rho \rho_t \left( 1 - b^2 \rho_t \rho_2 \right)^{1.117} \right] \]  

(31)

where:

\[ f = 16.55 \times Re_{\text{tub}}^{0.1} \left( 0.7 Re_{\text{f, steam}} \right)^{0.5} \left( \frac{A_0}{A_t} \right)^{0.9} \]  

(32)

\[ f_1 = -0.7339 + 7.187 \left( \frac{F_p}{P_t} \right) \log_{10} \left( 9 Re_{\text{f, steam}} \right) \]  

(33)

\[ f_2 = -0.5417 \log_e \left( \frac{A_0}{A_t} \right)^{0.9} \]  

(34)

\[ f_3 = 0.02722 \log_e \left( 6 \times Re_{\text{f, steam}} \right)^{1.2} \]  

(35)

\[ f_4 = 0.2973 \log_e \left( \frac{A_0}{A_t} \right)^{0.9} \]  

(36)

6. Calculation for Total Heat Rate

The heat rate through each fin with frost growth is calculated using

\[ q_{f, m} = \int_{0}^{D_0} \frac{\partial T_r}{\partial x} \, dx \]  

(37)

The heat rate on the base plate between the fins is calculated from the frost growth model using the equation

\[ q_{b, m} = A_{b, m} \lambda_{\text{off}} \frac{\partial T}{\partial x} \bigg|_{x=0} \]  

(38)

The total heat rate is

\[ q_{\text{tot}} = N \times q_{f, m} + q_{b, m} \]  

(39)

Numerical Solution Procedure

A finite difference formulation is used to solve the governing partial differential equations for frost growth and fin models. The frost growth model is solved by the upwind difference scheme for the time deviations, and the central-difference scheme for internal nodes and backward or forward difference scheme for the boundary nodes and the number of grid is fixed as 15. Therefore, for each time step, the spatial coordinates are updated based in the new boundary position. The spatial distribution of each variable at the last time is also updated to fit the new coordinate system, using linear interpolation method. To minimum the errors introduced by interpolation and keep the calculation stable, a small time step of 6E-2 second is chosen in the calculation. The two dimensional fin surface is divided into a number of continuous non-overlapping control volume and the fin surface temperature is solved at the central point of each control volume.

The whole simulation is calculating the frost growth model, couple with the fin model and airside correlations. The relaxation iteration scheme is chosen to solve the non-linear differential partial equations.

Typical Simulation Results

For convenience, we choose tube-fin exchange as evaporator and assume it equivalent a refrigerant tube with fin (See Fig. 1). The equivalent square fin is 0.025x0.025m² and the out diameter of refrigerant tube is 0.0095m. Fig. 2 shows the frost height profile on the fin. From the figure, the fact that the frost height near the cold base is higher shows that in lumped model, the uniform frost height assumption is not reasonable. Therefore in Fig. 3, the distribution of density is not uniform, and the frost density is higher near the cold base also.

We can see from Fig. 4 that the frost growth rate at every point of the fin is not uniform and that there is the more time the frost grows, the more different of frost height in different point is. It is an interesting phenomenon for us to design defrost controller. As we know, for general time-temperature defrost controller, we need design a defrost temperature to energy it to defrost. So we should take that where the sample temperature is and how much it is into consideration. Fig. 5 is the frost growth profile versus relative humidity and Fig. 6 is the frost growth profile versus surround temperature. We can see from that in the range of frost, the lower surround temperature and the higher relative humidity, the more frost accumulated. This is consists with many reports (Oskarsson S.P. and Krakow K.I. 1990) (Wang J.F. and Zhang S.Z.2001) (Mao Y. and...
Fig. 1. The Scheme of Equivalent Evaporator

Fig. 2. Frost Height Contours on Fin Surface (t=48 min)

Fig. 3. Frost Density Profile within Frost Layer (X=7.28mm, Y=6.25mm, T=48min)

Fig. 4. Frost Growth Profile versus Time (Y=7.8mm, T=272K, RH=70%)

Fig. 5. Frost Growth Profile versus Relative Humidity (Y=6.25mm, T=273K, t=48min)

Fig. 6. Frost Growth Profile versus Surround Temperature (Y=6.25, RH=80%, t=48min)

Fig. 7. Transient Pressure Drop Across the Fin Section (T=273K, RH=70%)

Fig. 8. Transient Heat Rate Profile through Evaporator (T=273K, RH=70%)

Besant R.W. 1999).

Fig. 7 shows the transient pressure drop versus time. We can see that the pressure drop increases dramatically with time due to frost growing on the fins. Figure 8 shows that the heat rate through the evaporator significantly decreases with time. This is verified by many experimental data (Oskarsson S.P. and Krakow K.I. 1990) (Amen F.R. 1993).
Experimental Validation

It is difficult to measure the frost height directly and some way like laser method just measure the local height. In this experiment, we measure the humidity change and get the frost accumulation indirectly. Within a time interval of experiment, the frost accumulation is, \( \Delta m = m(W_0 - W_{fs}) \Delta t \).

Fig. 9 shows the experiment schematic diagram that includes two parts: the evaporator and the differential enthalpy method test equipment. The valve II is used to keep the evaporating pressure constant and satisfy the constant evaporating temperature. And the variable fan 3 and 8 are used to keep constant airflow rate. FLUKE DAS is installed to get all of the data and put them into computer to analyze. Dimension of heat pump evaporator is showed in Table 1 and the comparison result between the experiment and simulation is in Fig. 10.

Table 1. Dimensions of Heat Pump Evaporator

| Description                               | Value  |
|-------------------------------------------|--------|
| Coil total air side area                  | 6.7 m² |
| Coil face area                            | 0.092  |
| Coil depth                                | 100 mm |
| Tube external diameter                    | 9.5 mm |
| Tube external area                        | 0.46 m²|
| Tube internal area                        | 0.41 m²|
| Tube material                             | copper |
| Fin thickness                             | 0.12 mm|
| Fin surface area                          | 6.4 m² |
| Fin height                                | 0.3 m  |
| Number of fins                            | 118    |
| Fin material                              | aluminum |

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Nomenclature

- \( A_{base} \): area of tube [m²]
- \( A_{c} \): minimum flow area [m²]
- \( A_{fin} \): area of fin [m²]
- \( A_t \): bare tube outside surface area [m²]
- \( A_o \): total air-side surface area [m²]
- \( b \): fin thickness [m]
- \( c \): specific heat [J/kg.K]
- \( D_{ab} \): binary diffusion coefficient for water vapor in air at the film temperature [m²/s]
- \( D_{eff} \): effective diffusion coefficient [m²/s]
- \( D_o \): out diameter of tube [m]
- \( F_p \): fin pitch [m]
- \( G_{max} \): maximum mass flux of air [kg/m².s]
- \( h_{s} \): enthalpy of sublimation [J/kg]
- \( h_f \): convective heat transfer coefficient [W/m².K]
- \( h_{mf} \): convective mass transfer coefficient [m/s]
- \( j \): the colburn factor
- \( l_{fin}, h_{fin} \): length and height of fin [m]
- \( L_e \): Lewis number
- \( m \): phase change rate [kg/m³.s]
- \( \Delta m \): frost accumulation [kg]
- \( N \): number of fins
- \( P \): Prandtl number
- \( P_{Re} \): longitudinal tube pitch [m]
- \( Re_{D} \): transverse tube pitch [m]
- \( Re_{fin} \): Reynolds number
- \( Q \): mean condensate film Reynolds number
- \( q_{total} \): volume flow rate [m³/s]
- \( q_{fin} \): total heat rate [W]
- \( q_{base} \): heat rate through fin [W]
- \( S \): heat rate through the tube [W]
- \( T \): source term [kg/m³.s]
- \( T_e \): frost temperature [T]

Conclusion

The development of an improved transient distributed parameter model of frost on evaporator to predict frost formation characteristic has been presented. Numerical results revealed that the proposed model favorably predicted the key parameters in frost formation and compared well with the experimental data.
$T_c$ fin temperature [°C]
$T_b$ temperature of base fin [°C]
$t$ time [s]
$W_e$ average air flow humidity ratio [kg/kg]
$W$ humidity ratio of air [kg/kg]
$x, y$ coordinate axis of fin [m]
$z$ coordinate axis of frost [m]
$\varepsilon_a$ volume fraction of air within frost
$\varepsilon_p$ volume fraction of ice within frost
$\lambda$ thermal conductivity [W/m·K]
$\delta$ frost thickness [m]
$\mu$ dynamic viscosity of air [Pa·s]
$\rho$ density [kg/m$^3$]

Subscripts:
$f$ frost property
$a$ air property
$v$ vapor property
$i$ ice property
$eff$ effective
$fs$ surface property
$T_0$ average air flow temperature [°C]
$\Delta t$ time interval [s]

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