Properties of Fluctuations
in Two Coupled Chains of Luttinger Liquids

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Abstract

Two chains of Luttinger liquids coupled by both interchain hopping and interchain interaction are investigated. A degeneracy of two kinds of dominant states with in phase and out of phase ordering in the absence of the hopping is removed since the transverse fluctuation of charge density is completely gapful and that of spin density has two kinds of excitation with and without gap. The crucial effect of the interchain hopping on the electronic properties is studied by calculating susceptibilities.
I. INTRODUCTION

It is well known that the ground state of one-dimensional interacting electron systems is non-Fermi liquid. The state, which is called as “Luttinger liquid”, is characterized by a separation of spin and charge degrees of freedom, and anomalous exponents of correlation functions which depend on the interaction [1–4]. The parallel chains of the interacting electrons coupled by the interchain hopping are basic models of quasi-one-dimensional electron systems. Theoretical understanding of these systems is fundamental for studying electronic properties of organic conductors [5], high temperature superconductors [6] and interacting electron systems applied to strong magnetic field (Fractional Quantum Hall Effects) [7]. In addition, the problem is important as a first step toward studying two-dimensional interacting electron systems. Consequently, there has been a growing interest in the system of the coupled chains, in particular two chains. Several work have been devoted to investigating the two chains coupled by the interchain hopping for the case of spinless Fermion [8–12] and for the case including spin degree of freedom [13–22]. In addition, Anderson localization in such a system has been studied based on the above investigation [23,24].

However, interchain interaction also plays a role of the interchain coupling [23,26]. Therefore, by extending the work by Finkel’stein and Larkin [15] and that by Schulz [20], we study the two chains of Luttinger liquids in the presence of both the interchain hopping and the interchain interaction. To our knowledge, much is not known about such a problem in the case including spin degree of freedom. We clarify the electronic properties originated from the interchain hopping by calculating excitation spectrum, phase diagram and susceptibilities. The gap appears in the excitation spectrum of the transverse fluctuations. This leads to splitting of the degenerated states in the absence of the hopping, i.e., “in phase” ordering states and “out of phase” ordering states between the chains. From the calculation of charge and spin susceptibilities with $q_x$ and $q_y$ being the longitudinal and transverse wavenumber, it is shown that both susceptibilities with $q_y = 0$ are the same as those in the absence of the hopping, and those with $q_y = \pi$ show remarkable dependence on $q_x$. 

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The plan of the paper is as follows. In II, the Hamiltonian of the model is given and is expressed by use of the phase variables. In III, we study the excitation spectrum and the phase diagram at $T = 0$ with $T$ being temperature. The charge and the spin susceptibilities are also calculated. IV is devoted to discussion on the present results.

### II. MODEL AND PHASE REPRESENTATION

#### A. Model Hamiltonian

We investigate the system where two chains of Luttinger liquids are coupled by both the interchain hopping and the interchain interaction. The Hamiltonian is given by

$$
\mathcal{H} = \mathcal{H}_k + \mathcal{H}_{\text{int}} + \mathcal{H}'_{\text{int}},
$$

where

$$
\mathcal{H}_k = \sum_{k,p,\sigma} \epsilon_k \{ a_{k,p,\sigma,1}^\dagger a_{k,p,\sigma,1} + (1 \to 2) \} - t \sum_{k,p,\sigma} \{ a_{k,p,\sigma,1}^\dagger a_{k,p,\sigma,1} + (1 \leftrightarrow 2) \},
$$

$$
\mathcal{H}_{\text{int}} = \frac{\pi v_F g^2}{L} \sum_{p,\sigma,\sigma'} \sum_{k_1,k_2,q} \{ a_{k_1,p,\sigma,1}^\dagger a_{k_2,-p,\sigma',1} a_{k_2+q,-p,\sigma',1} a_{k_1-q,p,\sigma,1} + (1 \to 2) \},
$$

$$
\mathcal{H}'_{\text{int}} = \frac{\pi v'_F g'^2}{L} \sum_{p,\sigma,\sigma'} \sum_{k_1,k_2,q} \{ a_{k_1,p,\sigma,1}^\dagger a_{k_2,-p,\sigma',2} a_{k_2+q,-p,\sigma',2} a_{k_1-q,p,\sigma,1} + (1 \leftrightarrow 2) \}.
$$

Equations (2.2), (2.3) and (2.4) express the kinetic energy, the intrachain interaction and the interchain interaction, respectively. Quantities $k$ and $\epsilon_{kp}(= v_F(pk - k_F))$ denote the momentum and the kinetic energy of a Fermion where $v_F$, $p = +(-)$ and $k_F$ are the Fermi velocity, the right-going (left-going) state of a Fermion and the Fermi momentum. The operator $a_{k,p,\sigma,i}^\dagger$ expresses creation of the Fermion with $k$, $p$, $\sigma$ and $i$ where $\sigma = +(-)$ and $i(= 1,2)$ are the spin $\uparrow$ ($\downarrow$) state and the index of the chains. The interchain hopping and the length of the chain are defined by $t$ and $L$, respectively. The normalized quantities, $g_2 (g'_2)$ denotes the matrix element of the interaction for the intrachain (interchain) forward scattering between particles moving oppositely. Note that the conventional definition of the elements are given by $g \to g/(2\pi v_F)$. 

\[3\]
B. Phase Representation

We represent the above Hamiltonian, Eqs. (2.2) \sim (2.4), and the field operators of Fermions in terms of phase variables based on bosonization method. The separation of the Fermi wavenumber due to the hopping is taken into account by use of the unitary transformation, \( c_{k,p,\sigma,\mu} = (-\mu a_{k,p,\sigma,1} + a_{k,p,\sigma,2})/\sqrt{2} (\mu = \pm) \). Then the kinetic term, \( \mathcal{H}_k \) is rewritten as

\[
\mathcal{H}_k = \sum_{k,p,\sigma,\mu} v_F (p k - k_{F\mu}) c_{k,p,\sigma,\mu}^\dagger c_{k,p,\sigma,\mu},
\]

where \( k_{F\mu} = k_F - \mu t/v_F \). The interaction terms are rewritten as follows,

\[
\mathcal{H}_{\text{int}} + \mathcal{H}'_{\text{int}} = \mathcal{H}_{\text{int}}^1 + \mathcal{H}_{\text{int}}^2 + \mathcal{H}_{\text{int}}^3,
\]

\[
\mathcal{H}_{\text{int}}^1 = \frac{\pi v_F}{2L} (g_2 + g'_2) \sum_{p,\sigma,\sigma',\mu,\mu'} \sum_q \rho_{p,\sigma,\mu}(q) \rho_{-p,\sigma',\mu'}(-q),
\]

\[
\mathcal{H}_{\text{int}}^2 = \frac{\pi v_F}{2} (g_2 - g'_2) \sum_{p,\sigma,\sigma',\mu} \int dx \left\{ \psi_{p,\sigma,\mu}^\dagger \psi_{-p,\sigma',\mu}^\dagger \psi_{-p,\sigma',-\mu} \psi_{p,\sigma,-\mu} \right\},
\]

\[
\mathcal{H}_{\text{int}}^3 = \frac{\pi v_F}{2} (g_2 - g'_2) \sum_{p,\sigma,\sigma',\mu} \int dx \left\{ \psi_{p,\sigma,\mu}^\dagger \psi_{-p,\sigma',-\mu} \psi_{-p,\sigma',\mu} \psi_{p,\sigma,-\mu} \right\},
\]

where \( \psi_{p,\sigma,\mu} \equiv (1/L) \sum_k e^{ikx} c_{k,p,\sigma,\mu} \). The operator \( \rho_{p,\sigma,\mu}(q)(= \sum_k c_{k+p,\sigma,\mu}^\dagger c_{k+p,\sigma,\mu}) \) expresses the density fluctuation around the new Fermi point \( k_{F\mu} \) and satisfies the commutation relations, \( [\rho_{p,\sigma,\mu}(-q), \rho_{p',\sigma',\mu'}(q')] = \delta_{\sigma\sigma'} \delta_{pp'} \delta_{qq'} \delta_{\mu\mu'} pqL/(2\pi) \). The Hamiltonian \( \mathcal{H}_{\text{int}}^1 \) denotes the interaction described in terms of the quadratic form of the density fluctuation. For the processes of Eqs. (2.8) and (2.9), the band index is not conserved in \( \mathcal{H}_{\text{int}}^2 \) and the index is exchanged in \( \mathcal{H}_{\text{int}}^3 \), respectively.

Here we define phase variables \( \theta_\pm(x), \phi_\pm(x), \tilde{\theta}_\pm(x) \) and \( \tilde{\phi}_\pm(x) \) as [24–28]

\[
\theta_\pm(x) = -\sum_{q \neq 0} \frac{\pi i}{\sqrt{2} q L} e^{(-\alpha q^2/2i)q} \sum_{\sigma,\mu} (\rho_{+,\sigma,\mu}(-q) \pm \rho_{-,\sigma,\mu}(-q)),
\]

\[
\phi_\pm(x) = -\sum_{q \neq 0} \frac{\pi i}{\sqrt{2} q L} e^{(-\alpha q^2/2i)q} \sum_{\sigma,\mu} (\rho_{+,\sigma,\mu}(-q) \pm \rho_{-,\sigma,\mu}(-q)),
\]

\[
\tilde{\theta}_\pm(x) = -\sum_{q \neq 0} \frac{\pi i}{\sqrt{2} q L} e^{(-\alpha q^2/2i)q} \sum_{\sigma,\mu} (\rho_{+,\sigma,\mu}(-q) \pm \rho_{-,\sigma,\mu}(-q)),
\]

\[
\tilde{\phi}_\pm(x) = -\sum_{q \neq 0} \frac{\pi i}{\sqrt{2} q L} e^{(-\alpha q^2/2i)q} \sum_{\sigma,\mu} (\rho_{+,\sigma,\mu}(-q) \pm \rho_{-,\sigma,\mu}(-q)),
\]
where $\alpha^{-1}$ is a cutoff of the large momentum corresponding to the band width $v_F\alpha^{-1}$. Equations (2.10) ~ (2.13) satisfy commutation relations given by $[\theta_+(x), \theta_-(x')] = [\phi_+(x), \phi_-(x')] = [\tilde{\theta}_+(x), \tilde{\theta}_-(x')] = \ln \{1 + i(x - x')\alpha^{-1}\} - \ln \{1 - i(x - x')\alpha^{-1}\} \approx i\pi \text{sgn}(x - x')$ and zero for the others. The variables, $\theta_\pm$ and $\phi_\pm$ express the fluctuations of the total charge density and that of the total spin density, respectively, while $\tilde{\theta}_\pm$ and $\tilde{\phi}_\pm$ express the transverse fluctuation of the charge density and that of the spin density, respectively. Actually these properties are understood from the facts that $\partial_x\theta_+ = (\pi/\sqrt{2})\sum_{p,\sigma,i} \psi_p^\dagger \psi_{p,\sigma,i}$, $\partial_x\phi_+ = (\pi/\sqrt{2})\sum_{p,\sigma,i} \sigma \psi_p^\dagger \psi_{p,\sigma,i}$, $\partial_x\tilde{\theta}_+ = (-\pi/\sqrt{2})\sum_{p,\sigma} \psi_p^\dagger \psi_{p,\sigma,2} + h.c.$ and $\partial_x\tilde{\phi}_+ = (-\pi/\sqrt{2})\sum_{p,\sigma} \sigma \psi_p^\dagger \psi_{p,\sigma,2} + h.c.$ where $\psi_{p,\sigma,i} = (1/\sqrt{L})\sum_k e^{ikx} a_{k,p,\sigma,i}$. In terms of the phase variables, the field operator of Fermions, $\psi_{p,\sigma,\mu}(x)$, is expressed as [29,30],

$$
\psi_{p,\sigma,\mu}(x) = \frac{1}{\sqrt{2\pi\alpha}} \exp [i p k_{F\mu} x + i \Theta_{p,\sigma,\mu} + i \pi \Xi_{p,\sigma,\mu}] \\
eq \psi'_{p,\sigma,\mu}(x) \exp (i\pi \Xi_{p,\sigma,\mu}), \quad (2.14)
$$

with

$$
\Theta_{p,\sigma,\mu} = \frac{1}{2\sqrt{2}} \left\{ p\theta_+ + \theta_+ + \mu(\hat{\rho}_+ + \hat{\rho}_-) + \sigma(\rho\phi_+ + \phi_-) + \mu \rho \phi_+ + \phi_- \right\}. \quad (2.15)
$$

In Eq.(2.14), the phase factor, $\pi \Xi_{p,\sigma,\mu}$, is added so that the Fermion operators with different indices satisfy the anticommutation relation [1]. The factor $\Xi_{p,\sigma,\mu}$ is given by $\Xi_1 = 0$ and $\Xi_i = \sum_{j=1}^{i-1} \hat{N}_j$, $(i = 2 \sim 8)$ with $\hat{N}_i$ being the number operator of the Fermions with indices $i$ where the index $(p, \sigma, \mu)$ corresponds to $(+, +, +) = 1$, $(+, +, -) = 2$, $(+, +, -) = 3$, $(+, -,-) = 4$, $(-, +, +) = 5$, $(-, +, -) = 6$, $(-, -, -) = 7$ and $(-, -, -) = 8$, respectively. Note that the above choice of $\Xi_{p,\sigma,\mu}$ is not unique.

By substituting Eqs.(2.14) and (2.15) into Eqs.(2.8) and (2.9), and by using the fact that $\mathcal{H}_k = (\pi v_F/L) \sum_{p,\sigma,\mu} \sum_q \rho_{p,\sigma,\mu}(q) \rho_{p,\sigma,\mu}(-q)$ [29,31], the Hamiltonian $\mathcal{H} = \mathcal{H}_T + \mathcal{H}_R$ can be expressed as,

$$
\mathcal{H}_T = \frac{v_p}{4\pi} \int dx \left\{ \frac{1}{\eta_\rho} (\partial_x \theta_+)^2 + \eta_\rho (\partial_x \theta_-)^2 \right\} + \frac{v_F}{4\pi} \int dx \left\{ (\partial_x \phi_+)^2 + (\partial_x \phi_-)^2 \right\}, \quad (2.16)
$$
\[ \mathcal{H}_R = \frac{v_F}{4\pi} \int dx \left\{ \tilde{A}_{\theta,+}(\partial_x \tilde{\theta}^+)^2 + \tilde{A}_{\theta,-}(\partial_x \tilde{\theta}^-)^2 + \tilde{A}_{\phi,+}(\partial_x \tilde{\phi}^+)^2 + \tilde{A}_{\phi,-}(\partial_x \tilde{\phi}^-)^2 \right\} \\
+ \frac{v_F(g_2 - g_2')}{\pi \alpha^2} \int dx \left\{ \cos \sqrt{2} \tilde{\theta}_- - \cos(2q_0 x - \sqrt{2} \tilde{\theta}_+) \right\} \left\{ \cos \sqrt{2} \tilde{\phi}_- + \cos \sqrt{2} \tilde{\phi}_+ \right\}, \quad (2.17) \]

where \( v_\rho = v_F \sqrt{(1 + 2g_2 + 2g_2')(1 - 2g_2 - 2g_2')}, \quad \eta_\rho = \sqrt{(1 - 2g_2 - 2g_2')/(1 + 2g_2 + 2g_2')}, \)
\( \tilde{A}_{\theta,\pm} = \tilde{A}_{\phi,\pm} = 1 \) and \( q_0 = 2t/v_F \). Here \( \mathcal{H}_T \) expresses fluctuations of total charge and spin densities, whose excitations are given by \( v_\rho |k\rangle \) and \( v_F |k\rangle \), respectively. On the other hand, \( \mathcal{H}_R \), which expresses the transverse fluctuation, includes the complex nonlinear terms.

The nonlinear terms including \( \cos \sqrt{2} \tilde{\theta}_- \) result from \( \mathcal{H}_{\text{int}}^2 \) and those with \( \cos(2q_0 x - \sqrt{2} \tilde{\theta}_+) \) are due to \( \mathcal{H}_{\text{int}}^3 \). In deriving Eq.\((2.17)\), we chose a Hilbert space with the even integers for respective numbers \( N_1 + N_3, N_2 + N_4, N_5 + N_7, N_6 + N_8, N_1 + N_5, N_1 + N_6 \) and \( N_1 + N_2 \) where \( N_i \) is the eigenvalue of \( \tilde{N}_i \). The negative sign of \( \cos(2q_0 x - \sqrt{2} \tilde{\theta}_+) \) is due to such a choice of the phase factor, \( \pi \Xi_{\rho_\sigma \mu} \). It is noticed that the interchain interaction does not change the structure of the Hamiltonian, i.e., the parameters in the presence of \( g_2' \) are obtained by rewriting as \( g_2 \to g_2 + g_2' \) in \( \mathcal{H}_T \) and \( g_2 \to g_2 - g_2' \) in \( \mathcal{H}_R \).

The Hamiltonian, \( \mathcal{H}_R \) can be rewritten as

\[ \mathcal{H}_R = v_F \int dx \left\{ \tilde{\psi}_1^\dagger(-i \partial_x) \tilde{\psi}_1 - \tilde{\psi}_2^\dagger(-i \partial_x) \tilde{\psi}_2 + \tilde{\psi}_3^\dagger(-i \partial_x) \tilde{\psi}_3 - \tilde{\psi}_4^\dagger(-i \partial_x) \tilde{\psi}_4 \right\} \\
+ \pi v_F(g_2 - g_2') \int dx \left\{ i \tilde{\psi}_1^\dagger \tilde{\psi}_4^\dagger - i \tilde{\psi}_2^\dagger \tilde{\psi}_3^\dagger - \tilde{\psi}_4^\dagger \tilde{\psi}_3 e^{-i2q_0 x} - \tilde{\psi}_3^\dagger \tilde{\psi}_4 e^{i2q_0 x} \right\} \\
\times \left\{ i \tilde{\psi}_1^\dagger \tilde{\psi}_2 - i \tilde{\psi}_2^\dagger \tilde{\psi}_1 + \tilde{\psi}_2^\dagger \tilde{\psi}_1 + \tilde{\psi}_1^\dagger \tilde{\psi}_2 \right\}, \quad (2.18) \]

where \( \tilde{\psi}_i \) (\( i = 1 \sim 4 \)) are new Fermion fields defined by

\[ \tilde{\psi}_1 = \frac{1}{\sqrt{2 \pi \alpha}} e^{i \frac{\sqrt{2}}{v_F} (\tilde{\theta}_+ + \tilde{\phi}_-)} e^{i \frac{\sqrt{2}}{v_F} (\tilde{N}_1 + \tilde{N}_2)} \equiv \tilde{\psi}_1^\dagger e^{\frac{i \sqrt{2}}{v_F} (\tilde{N}_1 + \tilde{N}_2)}, \quad (2.19) \]

\[ \tilde{\psi}_2 = \frac{1}{\sqrt{2 \pi \alpha}} e^{-i \frac{\sqrt{2}}{v_F} (\tilde{\theta}_- - \tilde{\phi}_+)} e^{-i \frac{\sqrt{2}}{v_F} (\tilde{N}_1 + \tilde{N}_2)} \equiv \tilde{\psi}_2^\dagger e^{-i \frac{\sqrt{2}}{v_F} (\tilde{N}_1 + \tilde{N}_2)}, \quad (2.20) \]

\[ \tilde{\psi}_3 = \frac{1}{\sqrt{2 \pi \alpha}} e^{i \frac{\sqrt{2}}{v_F} (\tilde{\theta}_+ - \tilde{\phi}_-)} e^{i \frac{\sqrt{2}}{v_F} (\tilde{N}_3 + \tilde{N}_4) + i \pi (\tilde{N}_1 + \tilde{N}_2)} \equiv \tilde{\psi}_3^\dagger e^{\frac{i \sqrt{2}}{v_F} (\tilde{N}_3 + \tilde{N}_4) + i \pi (\tilde{N}_1 + \tilde{N}_2)}, \quad (2.21) \]

\[ \tilde{\psi}_4 = \frac{1}{\sqrt{2 \pi \alpha}} e^{-i \frac{\sqrt{2}}{v_F} (\tilde{\theta}_- + \tilde{\phi}_+)} e^{-i \frac{\sqrt{2}}{v_F} (\tilde{N}_3 + \tilde{N}_4) + i \pi (\tilde{N}_1 + \tilde{N}_2)} \equiv \tilde{\psi}_4^\dagger e^{-i \frac{\sqrt{2}}{v_F} (\tilde{N}_3 + \tilde{N}_4) + i \pi (\tilde{N}_1 + \tilde{N}_2)} \quad (2.22) \]

In Eqs.(2.19)~(2.22), the phase factors with the number operator are introduced again to satisfy the anticommutation relation. Equation \((2.18)\) is derived by choosing the Hilbert
space with both $N_1 + N_2$ and $N_3 + N_4$ being even integers. It is noted that the phase variables in Eqs. (2.19) ∼ (2.22) are expressed as

$$\tilde{\theta}_\pm(x) = -\sum_{q \neq 0} \sqrt{2\pi} \frac{q}{qL} e^{i(-\alpha|q|/2+iqx)} (\tilde{\rho}_3(-q) \pm \tilde{\rho}_4(-q)),$$  \hspace{1cm} (2.23)

$$\tilde{\phi}_\pm(x) = -\sum_{q \neq 0} \sqrt{2\pi} \frac{q}{qL} e^{i(-\alpha|q|/2+iqx)} (\tilde{\rho}_1(-q) \pm \tilde{\rho}_2(-q)),$$  \hspace{1cm} (2.24)

where $\tilde{\rho}_j(q) \equiv \sum_k \tilde{\psi}_j^\dagger(k+q) \tilde{\psi}_j(k)$ with $\tilde{\psi}_j = 1/\sqrt{L} \sum_k \tilde{\psi}_j(k) e^{ikx}$ ($j = 1 \sim 4$).

### III. PROPERTIES AT LOW TEMPERATURES

By using renormalization group method, Finkel’stein and Larkin [15] showed that the nonlinear terms in Eq. (2.17) with $g_2 \neq g'_2$ tend to the strong coupling in the limit of low energy and the terms without (with) the misfit parameter, $2q_0$ become relevant (irrelevant). Schulz insisted that the transverse charge excitation is completely gapful and that of spin excitation has two kinds of excitation with gap and gapless from the symmetry of the Hamiltonian. Here we explicitly show the results by use of the mean field approximation in which the terms including the misfit parameter are neglected. This method is expected to be effective in the limit of strong coupling and has an advantage of the straightforward calculation of the several quantities. It should be noted that the break of the balance between $\tilde{\theta}_+$ and $\tilde{\theta}_-$ due to the misfit parameter may lead to the renormalization of $\tilde{\eta}_\rho$ defined by $(\tilde{A}_{\theta,-}/\tilde{A}_{\theta,+})^{1/2}$. We neglect such an effect as zeroth-approximation since the system tends to strong coupling and the gap appears. On the other hand, $\tilde{\eta}_\sigma(\equiv (\tilde{A}_{\phi,-}/\tilde{A}_{\phi,+})^{1/2})$ remains unity due to the balance between $\tilde{\phi}_+$ and $\tilde{\phi}_-$. The present method is effective for the energy lower than the hopping, and then the large momentum cutoff $\alpha^{-1}$ must be read as $t/v_F$.

#### A. Excitation Spectrum

By making use of the mean-field approximation, $\mathcal{H}_R$ with $g_2 \neq g'_2$ is rewritten as

$$\mathcal{H}_R = \mathcal{H}_M^{12} + \mathcal{H}_M^{34} - \Delta \Delta' L/(2\pi v_F(g_2 - g'_2)),$$  \hspace{1cm} (3.1)
where

\[ \mathcal{H}_{\text{MF}}^{12} = v_F \int \, dx \left\{ \bar{\psi}_1^i (-i \partial_x) \psi_1 - \bar{\psi}_2^i (-i \partial_x) \psi_2 \right\} + \frac{\Delta}{2} \int \, dx \left\{ i \bar{\psi}_1 \psi_2 \bar{\psi}_1^i - i \bar{\psi}_2 \psi_1 + \bar{\psi}_2 \psi_1^i + \psi_1 \bar{\psi}_2^i \right\}, \quad (3.2) \]

\[ \mathcal{H}_{\text{MF}}^{34} = v_F \int \, dx \left\{ \bar{\psi}_3^i (-i \partial_x) \psi_3 - \bar{\psi}_4^i (-i \partial_x) \psi_4 \right\} + \Delta' \int \, dx \left\{ i \bar{\psi}_3 \psi_4 \bar{\psi}_3^i - i \bar{\psi}_4 \psi_3^i \right\}. \quad (3.3) \]

The quantities, \( \Delta \) and \( \Delta' \) in Eqs. (3.2) and (3.3) are gap parameters determined by the following self-consistent equations,

\[ \frac{\Delta}{2} = \pi v_F (g_2 - g'_2) \left\{ i \left\langle \bar{\psi}_3^i \psi_4^i \right\rangle - i \left\langle \bar{\psi}_4 \psi_3 \right\rangle \right\}, \quad (3.4) \]

\[ \Delta' = \pi v_F (g_2 - g'_2) \left\{ i \left\langle \bar{\psi}_3 \psi_4 \bar{\psi}_3^i \right\rangle - i \left\langle \bar{\psi}_4 \bar{\psi}_3 \right\rangle + \left\langle \bar{\psi}_2 \psi_4 \bar{\psi}_2^i \right\rangle \right\}. \quad (3.5) \]

The eigenvalues, \( \omega_{12} \) of Eq. (3.2) and \( \omega_{34} \) of Eq. (3.3) are calculated as

\[ \omega_{12} = \left\{ \begin{array}{c} \pm E_k \equiv \pm \sqrt{(v_F k)^2 + \Delta^2}, \\ \pm v_F k, \end{array} \right\}, \quad (3.6) \]

\[ \omega_{34} = \pm E'_k \equiv \pm \sqrt{(v_F k)^2 + \Delta'^2}. \quad (3.7) \]

The excitations of Eqs. (3.6), whose spectral weights are 1/2, are obtained by \( \mathcal{H}_{\text{MF}}^{12} \) in terms of Majorana Fermion as

\[ \mathcal{H}_{\text{MF}}^{12} = \frac{v_F}{2} \int \, dx \left\{ C_1 (-i \partial_x) C_1 - C_2 (-i \partial_x) C_2 \right\} + i \Delta \int \, dx C_1 C_2 \]

\[ + \frac{v_F}{2} \int \, dx \left\{ C_0 (-i \partial_x) C_0 - C_3 (-i \partial_x) C_3 \right\}, \quad (3.8) \]

where \( C_0 = \frac{i}{\sqrt{2}} \left( e^{i \pi/4} \bar{\psi}_1 - e^{-i \pi/4} \bar{\psi}_4 \right), C_1 = \frac{1}{\sqrt{2}} \left( e^{i \pi/4} \bar{\psi}_1 + e^{-i \pi/4} \bar{\psi}_4 \right), C_2 = \frac{1}{\sqrt{2}} \left( e^{-i \pi/4} \bar{\psi}_2 + e^{i \pi/4} \bar{\psi}_4 \right) \) and \( C_3 = \frac{i}{\sqrt{2}} \left( e^{-i \pi/4} \bar{\psi}_2 - e^{i \pi/4} \bar{\psi}_4 \right) \). Finkel’stein and Larkin [15] have already obtained the same form as the first line in Eq. (3.8) by replacing \( \cos \sqrt{2} \bar{\theta} \) in Eq. (2.17) with a value of a fixed point. However they did not show the gapless excitation which contributes to low energy properties e.g., specific heat [20].

The gap equations of Eqs. (3.4) and (3.5) are calculated (see Appendix A) as

\[ \Delta = -2 \Delta' (g_2 - g'_2) \log \frac{\xi_c + \sqrt{\xi_c^2 + \Delta'^2}}{|\Delta'|}, \quad (3.9) \]

\[ \Delta' = -\Delta (g_2 - g'_2) \log \frac{\xi_c + \sqrt{\xi_c^2 + \Delta^2}}{|\Delta|}, \quad (3.10) \]
where \( \xi_c \) is cut-off energy of the order of \( t \). From the the gap equations, it is found that both \((-\Delta, -\Delta')\) and \((\Delta, \Delta')\) are solutions, and that \( \text{sgn}(\Delta\Delta') = -1 \) for \( g_2 - g'_2 > 0 \) and \( \text{sgn}(\Delta\Delta') = 1 \) for \( g_2 - g'_2 < 0 \). The solutions of Eqs. (3.9) and (3.10) in the case of \( \Delta' > 0 \) is shown in Fig. [4].

B. Possible States

We examine phase diagram which shows the most divergent state. Since the dominant contribution in the low energy limit is given by \( \mathcal{H}_{\text{int}}^2 \) in Eq. (2.8), we rewrite the term as

\[
\mathcal{H}_{\text{int}}^2 = \frac{\pi v_F}{4} (g_2 - g'_2) \sum_{p,\sigma,\sigma'} \int dx \left\{ \left( \sum_{\mu'} \psi_{p,\sigma,\mu'}^{\dagger} \psi_{p,\sigma',\mu'}^{\dagger} \right) \left( \sum_{\mu} \psi_{p,\sigma',\mu} \psi_{-p,\sigma,\mu} \right) - \left( \sum_{\mu'} \psi_{-p,\sigma,\mu'}^{\dagger} \psi_{p,\sigma',\mu'}^{\dagger} \right) \left( \sum_{\mu} \psi_{p,\sigma',\mu} \psi_{-p,\sigma,\mu} \right) \right\}
\]

\[
= -\frac{\pi v_F}{4} (g_2 - g'_2) \sum_{p,\sigma,\sigma'} \int dx \left\{ \left( \sum_{\mu'} \psi_{p,\sigma,\mu'}^{\dagger} \psi_{p,\sigma',\mu'}^{\dagger} \right) \left( \sum_{\mu} \psi_{p,\sigma',\mu} \psi_{-p,\sigma,\mu} \right) - \left( \sum_{\mu'} \psi_{-p,\sigma,\mu'}^{\dagger} \psi_{p,\sigma',\mu'}^{\dagger} \right) \left( \sum_{\mu} \psi_{p,\sigma',\mu} \psi_{-p,\sigma,\mu} \right) \right\}. \tag{3.11}
\]

Since the states should be selected so as to gain the energy from Eq. (3.11), the possible states in the case from \( g_2 - g'_2 > 0 \) are given by

\[
S_{-\sigma',\sigma}^{\sigma} \equiv \sum_{\mu} \mu \psi_{p,\sigma,\mu} \psi_{-p,\sigma',\mu} = - \left\{ \psi_{p,\sigma,1} \psi_{p,\sigma',2} + (1 \leftrightarrow 2) \right\}
\]

\[
\sim \frac{i}{\pi \alpha} e^{\sqrt{2} \phi} e^{i \theta} e^{i \pi (\sigma - \sigma') \phi} \sin \left( \frac{\tilde{\theta} + p + \sigma - \sigma'}{2} \phi \right) \sin \left( \frac{\tilde{\theta} - p + \sigma + \sigma'}{2} \phi \right), \tag{3.12}
\]

\[
DW_{-\sigma',\sigma}^{\sigma} \equiv \sum_{\mu} \psi_{p,\sigma,\mu}^{\dagger} \psi_{-p,\sigma',\mu} = - \left\{ \psi_{p,\sigma,1}^{\dagger} \psi_{p,\sigma',1} - (1 \rightarrow 2) \right\}
\]

\[
\sim -\frac{i}{\pi \alpha} e^{-i 2 k_F x} e^{i \theta} e^{i \pi (\sigma + \sigma') \phi} \sin \left( \frac{\tilde{\theta} + p + \sigma - \sigma'}{2} \phi \right) \sin \left( \frac{\tilde{\theta} - p + \sigma + \sigma'}{2} \phi \right), \tag{3.13}
\]

and those in the case of \( g_2 - g'_2 < 0 \) are given by

\[
S_{\sigma',\sigma}^{\sigma} \equiv \sum_{\mu} \psi_{p,\sigma,\mu} \psi_{-p,\sigma',\mu} = \psi_{p,\sigma,1} \psi_{p,\sigma',1} + (1 \rightarrow 2)
\]

\[
\sim \frac{1}{\pi \alpha} e^{\sqrt{2} \phi} e^{i \theta} e^{i \pi (\sigma - \sigma') \phi} \cos \left( \frac{\tilde{\theta} + p + \sigma - \sigma'}{2} \phi \right) \cos \left( \frac{\tilde{\theta} - p + \sigma + \sigma'}{2} \phi \right), \tag{3.14}
\]

9
$DW^{-\sigma,\sigma'} \equiv \sum_{\mu} \mu \psi^\dagger_{p,\sigma,\mu} \psi_{-p,\sigma',-\mu} = - \left\{ \psi_{p,\sigma,1}^\dagger \psi_{-p,\sigma',2} - (1 \leftrightarrow 2) \right\}$

\[
\sim \frac{1}{\pi \alpha} e^{-i2pFx} e^{-i\theta_+} e^{-i\phi_+} e^{-i\theta_-} e^{-i\phi_-} \cos \left( \frac{\sigma - \sigma'}{2\sqrt{2}} \phi_+ + \frac{\sigma + \sigma'}{2\sqrt{2}} \phi_- \right).
\]

(3.15)

Here $DW^{-\sigma,\sigma'}$ ( $DW^{+\sigma,\sigma'}$ ) expresses density wave with interchain and out of phase ordering (with intrachain and out of phase ordering), while $S^{-\sigma,\sigma'}$ ( $S^{+\sigma,\sigma'}$ ) expresses superconductivity with interchain and in phase ordering (with intrachain and in phase ordering). The most dominant state between $S^{-\sigma,\sigma'}$ and $DW^{+\sigma,\sigma'}$ for $g_2 > g_2'$ ($S^{+\sigma,\sigma'}$ and $DW^{-\sigma,\sigma'}$ for $g_2 < g_2'$) is determined by the total charge and spin fluctuations. By noting that the correlation functions for the total fluctuations are calculated as,

\[
\left\langle e^{-i\sqrt{2} \theta_-} \frac{1}{e^{i\sqrt{2} \phi_+}} \frac{1}{e^{i\sqrt{2} \phi_-}} \right\rangle \sim \left( \frac{\alpha}{|x|} \right)^{\frac{1}{2} + \frac{\eta}{\eta'}}
\]

(3.16)

\[
\left\langle e^{i\sqrt{2} \theta_+} \frac{1}{e^{i\sqrt{2} \phi_+}} \frac{1}{e^{i\sqrt{2} \phi_-}} \right\rangle \sim \left( \frac{\alpha}{|x|} \right)^{\frac{1}{2} + \frac{\eta}{\eta'}}
\]

(3.17)

we obtain the phase diagram in Fig.2. The exponent of the correlation function of CDW is the same as that of SDW, and the exponent of singlet superconductivity is the same as that of triplet superconductivity, respectively. This comes from the symmetry of the interaction for the spin degree of freedom, as is also seen in spin dependent Tomonaga model with the isotropic interaction [4]. Therefore the phase diagram shown in Fig.2 is essentially the same as that of spinless Fermion studied previously [20]. In the repulsive case of $g_2 + g_2' > 0$, the most dominant state is the density wave. However, the interchain hopping leads to the superconducting state being subdominant even in such a region.

### C. Susceptibilities

In this subsection, we calculate the charge susceptibilities, $\chi_\rho(q_x, q_y; i\omega_n)$, and the spin susceptibilities, $\chi_\sigma(q_x, q_y; i\omega_n)$, in the case of $q_x \ll 2k_F$ and $q_y = 0$ or $\pi$ which are defined as
\[ \chi_{\nu}(q_x, q_y; i\omega_n) = \int_0^\beta d\tau \int d(x - x') e^{i\omega_n \tau} e^{-iq_x(x-x')} \chi_{\nu}(x - x', q_y; \tau), \]  

(3.18)

and \( \nu = \rho \) or \( \sigma \). In Eq. (3.18),

\[ \chi_{\rho}(x - x', q_y; \tau) = \frac{1}{2} \left\langle T_\tau \left\{ \rho(x, 1; \tau) + e^{iq_y \rho(x, 2; \tau)} \right\} \left\{ \rho(x', 1; 0) + e^{iq_y \rho(x', 2; 0)} \right\} \right\rangle, \]  

(3.19)

\[ \chi_{\sigma}(x - x', q_y; \tau) = \frac{1}{2} \left\langle T_\tau \left\{ m(x, 1; \tau) + e^{iq_y m(x, 2; \tau)} \right\} \left\{ m(x', 1; 0) + e^{iq_y m(x', 2; 0)} \right\} \right\rangle, \]  

(3.20)

where \( \rho(x, i; \tau) = \sum_{p,\sigma} \psi_{p,\sigma,i}^\dagger(x; \tau) \psi_{p,\sigma,i}(x; \tau) \) and \( m(x, i; \tau) = \sum_{p,\sigma} \sigma \psi_{p,\sigma,i}^\dagger(x; \tau) \psi_{p,\sigma,i}(x; \tau) \) denote operators of the charge and spin densities at the \( i \)-th chain, respectively.

At first we consider the case of \( q_y = 0 \). From Eq. (2.10), both \( \chi_{\rho}(x - x', 0; \tau) \) and \( \chi_{\sigma}(x - x', 0; \tau) \) are calculated as

\[ \chi_{\rho}(x - x', 0; \tau) = \frac{1}{\pi \eta_{\rho}^{1/2}} \left\langle T_\tau \partial_+ x, \partial_+ x' \right\rangle \]  

(3.21)

\[ = \frac{2 \eta_{\rho}}{\pi \eta_{\rho}^{1/2}} \left\langle \left( \frac{v_F q_x}{\omega_n^2} \right)^2 e^{iq_x(x-x') - i\omega_n \tau} \right\rangle, \]

\[ \chi_{\sigma}(x - x', 0; \tau) = \frac{1}{\pi \eta_{\rho}^{1/2}} \left\langle T_\tau \partial_+ \phi_+ x, \partial_+ \phi_+ x' \right\rangle \]  

(3.22)

\[ = \frac{2 \eta_{\rho}}{\pi \eta_{\rho}^{1/2}} \left\langle \left( \frac{v_F q_x}{\omega_n^2} \right)^2 e^{iq_x(x-x') - i\omega_n \tau} \right\rangle. \]

Therefore \((i\omega_n \rightarrow \omega)\), one obtains

\[ \text{Re} \chi_{\rho}(q_x, 0; \omega) = \frac{2 \eta_{\rho}}{\pi \eta_{\rho}^{1/2}} \left( \frac{v_F q_x}{\omega_n^2} \right)^2 \]  

(3.23)

\[ \text{Re} \chi_{\sigma}(q_x, 0; \omega) = \frac{2 \eta_{\rho}}{\pi \eta_{\rho}^{1/2}} \left( \frac{v_F q_x}{\omega_n^2} \right)^2 \]  

(3.24)

which are familiar to Luttinger liquid [1].

Next we examine the case of \( q_y = \pi \), for which Eqs. (3.19) and (3.20) are expressed as

\[ \chi_{\rho}(x - x', \pi; \tau) \]  

(3.25)

\[ = \frac{1}{2} \left\langle T_\tau \left( \sum_{p',\sigma',\mu'} \psi_{p',\sigma',\mu'}^\dagger(x; \tau) \psi_{p',\sigma',\mu'}(x; \tau) \right) \left( \sum_{\rho,\sigma,\mu} \psi_{\rho,\sigma,\mu}^\dagger(x') \psi_{\rho,\sigma,\mu}(x') \right) \right\rangle, \]

\[ \chi_{\sigma}(x - x', \pi; \tau) \]  

(3.26)

\[ = \frac{1}{2} \left\langle T_\tau \left( \sum_{p',\sigma',\mu'} \sigma \psi_{p',\sigma',\mu'}^\dagger(x; \tau) \psi_{p',\sigma',\mu'}(x; \tau) \right) \left( \sum_{\rho,\sigma,\mu} \sigma \psi_{\rho,\sigma,\mu}^\dagger(x') \psi_{\rho,\sigma,\mu}(x') \right) \right\rangle. \]
After some manipulations (Appendix B), the static susceptibilities, \( \text{Re}\chi_{\rho}(q_x, \pi; 0) \) and \( \text{Re}\chi_{\sigma}(q_x, \pi; 0) \) at \( T = 0 \) are respectively obtained as

\[
\text{Re}\chi_{\rho}(q_x, \pi; 0) = \frac{1}{2\pi} \int dk \left\{ \frac{1}{E_k + E_{k-q_x-q_0}} \left( 1 - \frac{\xi_k}{E_k} \frac{\xi_{k-q_x-q_0}}{E_{k-q_x-q_0}} + \frac{\Delta}{E_k E_{k-q_x-q_0}} \frac{\Delta'}{E_{k-q_x-q_0}} \right) + \frac{1}{E_k + E'_{k+q_x-q_0}} \left( 1 - \frac{\xi_k}{E_k} \frac{\xi_{k+q_x-q_0}}{E_{k+q_x-q_0}} + \frac{\Delta}{E_k E_{k+q_x-q_0}} \frac{\Delta'}{E_{k+q_x-q_0}} \right) \right\}, \tag{3.27}
\]

\[
\text{Re}\chi_{\sigma}(q_x, \pi, 0) = \frac{1}{2\pi} \left\{ P \int_{q_x+q_0}^{q_x+q_0} dk \left( 1 + \frac{\xi_k}{E_k} \right) \frac{1}{E'_k - \xi_k + v_F(q_x + q_0)} + P \int_{q_x+q_0}^{q_x-q_0} dk \left( 1 - \frac{\xi_k}{E_k} \right) \frac{1}{E'_k + \xi_k - v_F(q_x + q_0)} + P \int_{q_x+q_0}^{-q_x+q_0} dk \left( 1 + \frac{\xi_k}{E_k} \right) \frac{1}{E'_k - \xi_k + v_F(-q_x + q_0)} + P \int_{q_x-q_0}^{-q_x+q_0} dk \left( 1 - \frac{\xi_k}{E_k} \right) \frac{1}{E'_k + \xi_k - v_F(-q_x + q_0)} \right\} = \frac{1}{\pi v_F} \left\{ 2 - \frac{\Delta'^2}{2\epsilon_+^2} \ln \left( 1 + \frac{\epsilon_+^2}{\Delta'^2} \right) - \frac{\Delta^2}{2\epsilon_-^2} \ln \left( 1 + \frac{\epsilon_-^2}{\Delta^2} \right) \right\}, \tag{3.28}
\]

where \( P \) denotes the principal value, \( \xi_k = v_F k \) and \( \epsilon_{\pm} = v_F (\pm q_x + q_0) \). In Fig.3 and Fig.4, we show the normalized quantities \( \tilde{\chi}_{\rho}(q_x, \pi; 0) \) and \( \tilde{\chi}_{\sigma}(q_x, \pi; 0) \) which are defined by \( \text{Re}\chi_{\rho}(q_x, \pi; 0)/(2/\pi v_F) \) and \( \text{Re}\chi_{\sigma}(q_x, \pi, 0)/(2/\pi v_F) \), respectively. The cutoff energy, \( \xi_c \), defined in Eqs.(3.9) and (3.10) is taken as 2t. Note that Eqs.(3.27) and (3.28) are valid in the case of \( |q_x \pm q_0| \lesssim q_0 \).

Equation (3.27) shows that the value of \( \text{Re}\chi_{\rho}(q_x, \pi; 0) \) in the case of \( g_2 - g'_2 < 0 \) is larger than that in the case of \( g_2 - g'_2 > 0 \) because \( g_2 - g'_2 < 0 \) \((>0)\) leads to \( \text{sgn}(\Delta \Delta') = 1 (-1) \). Such dependence on \( g_2 - g'_2 \) is also found in the absence of the hopping (see Eq.(C14)). On the other hand, \( \text{Re}\chi_{\sigma}(q_x, \pi; 0) \) is independent of sign of the relative interaction. This result seems to correspond to the fact that \( \text{Re}\chi_{\sigma}(q_x, \pi; 0) \) in the absence of the hopping is independent of \( g_2 - g'_2 \) (see Eq.(C13)).

In Fig.3, \( \text{Re}\chi_{\rho}(q_x, \pi; 0) \) takes a minimum around \( q_x = q_0 \) in the case of \( g_2 - g'_2 > 0 \) and has a small dependence on \( q_x \) and \( g_2 - g'_2 \), i.e., being nearly unity for \( g_2 - g'_2 < 0 \). On the other hand, \( \text{Re}\chi_{\sigma}(q_x, \pi; 0) \) in Fig.3 takes the minimum for both \( g_2 - g'_2 > 0 \) and \( g_2 - g'_2 < 0 \). The characteristic dependence is due to the separation of Fermi wavenumber i.e., \( k_{F-} - k_{F+} = q_0 \) and the gap in the transverse fluctuation, both of which result from the
interchain hopping. Note that the $q_x$-dependence of these $\text{Re}\chi_\rho(q_x, \pi; 0)$ and $\text{Re}\chi_\sigma(q_x, \pi; 0)$ does not change qualitatively by the choice of $\xi_c/t$.

IV. DISCUSSION

In the present paper, we studied the low temperature properties of two chains coupled by the interchain hopping and the interchain interaction where the interactions of only the forward scattering between oppositely moving particles were taken into account as a simplest model of Luttinger liquid.

There are four kinds of excitations originated from the fluctuations of total charge, total spin, transverse charge and transverse spin respectively. The total fluctuations which show the gapless excitation are the same as those in the absence of the interchain hopping (Appendix C). On the other hand, the transverse fluctuations of both charge density and spin density which are expressed by the complicated non-linear terms are crucial in the presence of the interchain hopping. By utilizing the mean field approximation, it was shown that the transverse fluctuation of the charge is completely gapful and that of the spin has the two kinds of excitations with and without gap.

The most dominant states are obtained as $DW_{\sigma,\sigma'}^+\sigma$, $DW_{\sigma,\sigma'}^-\sigma$, $S_{\sigma,\sigma'}^+\sigma$ for $g_2 > |g'_2|$ and $S_{\sigma,\sigma'}^-\sigma$ for $g'_2 > -|g_2|$, respectively where density wave (superconductivity) belongs to out of phase (in phase) ordering between the chains, i.e., the transverse wavenumber being $\pi(0)$. We note that in the case of quasi one-dimensional electron system with only the hopping of pairs \[32\], the density wave with the transverse wave number $(\pi, \pi)$ or superconductivity with $(0, 0)$ has maximum critical temperature. It is worth while noting that the states of the superconductivity remain subdominant even for the repulsive interaction, i.e., $g_2 + g'_2 > 0$. Such a result may be the important point toward understanding of the competition of superconductivity and SDW observed in quasi one-dimensional conductors, $(\text{TMTSF})_2\text{X} \ [22]$.

Possible states in the absence of the interchain hopping are obtained by calculating
correlation functions for $S_{\sigma,\sigma'}^{\parallel} (= \psi_{p,\sigma,1}^\dagger \psi_{-p,\sigma',1}$ or $\psi_{p,\sigma,2} \psi_{-p,\sigma',2}$), $S_{\perp}^{\sigma,\sigma'} (= \psi_{p,\sigma,1}^\dagger \psi_{-p,\sigma',2}$ or $\psi_{p,\sigma,2} \psi_{-p,\sigma',1}$), $DW_{\parallel}^{\sigma,\sigma'} (= \psi_{p,\sigma,1}^\dagger \psi_{-p,\sigma',1}$ or $\psi_{p,\sigma,2} \psi_{-p,\sigma',2}$) and $DW_{\perp}^{\sigma,\sigma'} (= \psi_{p,\sigma,1}^\dagger \psi_{-p,\sigma',2}$ or $\psi_{p,\sigma,2}^\dagger \psi_{-p,\sigma',1}$) which express the order parameters of the intrachain superconductivity, the interchain superconductivity, the intrachain density wave and the interchain density wave, respectively (Appendix C). The phase diagram in the absence of the interchain hopping is shown in Fig.5. By comparing the phase diagram in Fig.2 and that in Fig.5, it is found that the energy gain due to the interchain hopping removes the degeneracy of “in phase” and “out of phase” ordering.

The $q_x$-dependence of charge and spin susceptibilities were calculated for both $q_y = 0$ and $q_y = \pi$. The susceptibilities in the case of $q_y = 0$ are the same as those in the absence of the hopping since the total dynamics is not affected by the hopping. On the other hand, the static susceptibilities, $\text{Re} \chi_{\rho}(q_x, \pi; 0)$ for $g_2 - g'_2 > 0$ and $\text{Re} \chi_{\sigma}(q_x, \pi; 0)$ in the case of $q_y = \pi$ show the minimum around $q_x = q_0$ which is ascribed to the separation of the Fermi wavenumber and the excitation gaps of the transverse fluctuation in the presence of the interchain hopping. The fact that $\text{Re} \chi_{\rho}(q_x, \pi; 0)$ in the case of $g_2 - g'_2 < 0$ is larger than that in the case of $g_2 - g'_2 > 0$ is found also in the absence of the interchain hopping.

We treated the system with the interaction processes of the forward scattering between the oppositely moving particles as the simplest model of Luttinger liquid. However, the two chains coupled by the interchain hopping have been known to show the various properties depending on the parameters of the system. The repulsive backward scattering becomes relevant in the low energy limit and opens the gap in the excitations of the total spin, and thus modifies the electronic properties [20]. This fact is different from the strictly one-dimensional case, where the repulsive backward scattering is renormalized to zero. The two chains of Hubbard Model shows the richer phases depending on the magnitude of the intrachain interaction, the interchain hopping and the filling [14,21]. In addition, it has been reported that the two chains coupled by both the Coulomb repulsion and the exchange interaction show the superconductivity [33]. Therefore further investigations are needed to
identify the ground state of two coupled chains as a crossover from one dimension to higher
dimension.

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APPENDIX A: GREEN FUNCTIONS OF EQUATIONS (3.2) AND (3.3)

We calculate Green functions of Eqs. (3.2) and (3.3). By using the solutions of Majorana Fermions which are calculated from Eq. (3.3), the Green functions corresponding to the transverse spin fluctuations are calculated as

\[ -\left<T_r \tilde{\psi}_1(x, \tau) \tilde{\psi}^\dagger_1(x', \tau')\right> = \frac{1}{2\beta L} \sum_{k, \epsilon_n} e^{ik(x-x') - i\epsilon_n(\tau-\tau')} \left\{ \frac{1}{i\epsilon_n - \xi_k} + \frac{i\epsilon_n + \xi_k}{(i\epsilon_n + E_k)(i\epsilon_n - E_k)} \right\}, \quad (A1) \]

\[ -\left<T_r \tilde{\psi}_2(x, \tau) \tilde{\psi}^\dagger_2(x', \tau')\right> = \frac{1}{2\beta L} \sum_{k, \epsilon_n} e^{ik(x-x') - i\epsilon_n(\tau-\tau')} \left\{ \frac{1}{i\epsilon_n + \xi_k} + \frac{i\epsilon_n - \xi_k}{(i\epsilon_n + E_k)(i\epsilon_n - E_k)} \right\}, \quad (A2) \]

\[ -\left<T_r \tilde{\psi}_1(x, \tau) \tilde{\psi}_1(x', \tau')\right> = \frac{1}{2\beta L} \sum_{k, \epsilon_n} e^{ik(x-x') - i\epsilon_n(\tau-\tau')} \left\{ \frac{1}{i\epsilon_n - \xi_k} - \frac{i\epsilon_n + \xi_k}{(i\epsilon_n + E_k)(i\epsilon_n - E_k)} \right\}, \quad (A3) \]

\[ -\left<T_r \tilde{\psi}_2(x, \tau) \tilde{\psi}_2(x', \tau')\right> = \frac{i}{2\beta L} \sum_{k, \epsilon_n} e^{ik(x-x') - i\epsilon_n(\tau-\tau')} \frac{\Delta}{(i\epsilon_n + E_k)(i\epsilon_n - E_k)}, \quad (A4) \]

\[ -\left<T_r \tilde{\psi}_1(x, \tau) \tilde{\psi}_2(x', \tau')\right> = \frac{1}{2\beta L} \sum_{k, \epsilon_n} e^{ik(x-x') - i\epsilon_n(\tau-\tau')} \frac{\Delta'}{(i\epsilon_n + E'_k)(i\epsilon_n - E'_k)}, \quad (A5) \]

where \( T_r \) is the time ordering operator, \( \xi_k = v_F k \), \( 1/\beta = T \) and \( \epsilon_n = (2n + 1)\pi T \). On the other hand, by diagonalizing Eq. (3.3), the Green functions expressing the transverse charge fluctuations are calculated as

\[ -\left<T_r \tilde{\psi}_3(x, \tau) \tilde{\psi}^\dagger_3(x', \tau')\right> = \frac{1}{\beta L} \sum_{k, \epsilon_n} e^{ik(x-x') - i\epsilon_n(\tau-\tau')} \frac{i\epsilon_n + \xi_k}{(i\epsilon_n + E'_k)(i\epsilon_n - E'_k)}, \quad (A7) \]

\[ -\left<T_r \tilde{\psi}_4(x, \tau) \tilde{\psi}^\dagger_4(x', \tau')\right> = \frac{1}{\beta L} \sum_{k, \epsilon_n} e^{ik(x-x') - i\epsilon_n(\tau-\tau')} \frac{i\epsilon_n - \xi_k}{(i\epsilon_n + E'_k)(i\epsilon_n - E'_k)}, \quad (A8) \]

\[ -\left<T_r \tilde{\psi}_3(x, \tau) \tilde{\psi}_4(x', \tau')\right> = \frac{i}{\beta L} \sum_{k, \epsilon_n} e^{ik(x-x') - i\epsilon_n(\tau-\tau')} \frac{\Delta'}{(i\epsilon_n + E'_k)(i\epsilon_n - E'_k)}. \quad (A9) \]

From these Green functions, one obtains in the limit of \( T = 0 \),

\[ \left<\tilde{\psi}_1 \tilde{\psi}_2\right> = -\left<\tilde{\psi}_2 \tilde{\psi}_1\right> = -i \left<\tilde{\psi}_1 \tilde{\psi}_2\right> = -i \left<\tilde{\psi}_2 \tilde{\psi}_1\right> \]

\[ = \{i\Delta/(4\pi v_F)\} \log\left\{ (\xi_c + \sqrt{\xi_c^2 + \Delta^2})/|\Delta| \right\}, \quad (A10) \]

\[ \left<\tilde{\psi}_3 \tilde{\psi}_4\right> = -\left<\tilde{\psi}_4 \tilde{\psi}_3\right> = \{i\Delta'/(2\pi v_F)\} \log\left\{ (\xi_c + \sqrt{\xi_c^2 + \Delta'^2})/|\Delta'| \right\}. \quad (A11) \]

By substituting Eqs. (A10) and (A11) into Eqs. (3.4) and (3.5), one obtains Eqs. (3.9) and (3.10).
APPENDIX B: CALCULATION OF SUSCEPTIBILITIES, EQS. (3.27) AND (3.28)

Neglecting the constant whose absolute value is unity, the quantity $F_{\rho(\sigma)} \equiv \sum_{p,\sigma,\mu}(\sigma)\psi^\dagger_{p,\sigma,\mu}\psi_{p,\sigma,-\mu}$ is expressed as,

$$
F_{\rho(\sigma)} = \sum_{p,\sigma,\mu} p\mu(\sigma)\psi^\dagger_{p,\sigma,\mu}\psi_{p,\sigma,-\mu}
$$

$$
= \frac{1}{2\pi\alpha} \sum_{p,\sigma,\mu} p\mu(\sigma)e^{ip\mu\alpha x} \exp\left\{\frac{-ip\mu}{\sqrt{2}}(\tilde{\phi}_+ + p\tilde{\phi}_-}\right\} \exp\left\{\frac{-ip\mu\sigma}{\sqrt{2}}(\tilde{\phi}_+ + p\tilde{\phi}_-)\right\}
$$

$$
= e^{iq_0x}\left\{\overline{\psi}_3^\dagger\psi_1^\dagger + (-)\overline{\psi}_3^\dagger\psi_4^\dagger + \overline{\psi}_4^\dagger\psi_2^\dagger + (-)\overline{\psi}_4^\dagger\psi_2^\dagger\right\}
$$

$$
+ e^{-iq_0x}\left\{-\overline{\psi}_1\overline{\psi}_3^\dagger - (+)\overline{\psi}_1\overline{\psi}_4^\dagger - \overline{\psi}_2\overline{\psi}_4^\dagger - (+)\overline{\psi}_2\overline{\psi}_4^\dagger\right\}
$$

$$
= e^{iq_0x}\left\{\overline{\psi}_3^\dagger\left[\overline{\psi}_1^\dagger - (+)e^{-i\pi/2}\overline{\psi}_1^\dagger\right] - \overline{\psi}_4^\dagger\left[\overline{\psi}_2^\dagger - (+)e^{-i\pi/2}\overline{\psi}_2^\dagger\right]\right\}
$$

$$
- e^{-iq_0x}\left\{\left[\overline{\psi}_1^\dagger - (+)e^{i\pi/2}\overline{\psi}_1^\dagger\right]\overline{\psi}_3 - \left[\overline{\psi}_2^\dagger - (+)e^{i\pi/2}\overline{\psi}_2^\dagger\right]\overline{\psi}_4\right\}, \quad (B1)
$$

which is derived by calculating $\langle T_\tau F^\dagger_{\rho(\sigma)}(x;\tau)F_{\rho(\sigma)}(x';0)\rangle$ with the precise treatment of the negative sign originated from the phase factors in terms of $\hat{N}_{p,\sigma,\mu}$ and $\hat{N}_i$ ($i = 1 \sim 4$). Equation (B1) shows that the susceptibilities with $q_y = \pi$ are expressed by only the transverse degree of freedoms.

By use of Eqn. (A1) $\sim$ (A3), the quantity $\langle T_\tau F^\dagger_{\rho(\sigma)}(x;\tau)F_{\rho(\sigma)}(x';0)\rangle$ is calculated as

$$
\langle T_\tau F^\dagger_{\rho(\sigma)}(x;\tau)F_{\rho(\sigma)}(x';0)\rangle
$$

$$
= e^{-iq_0(x-x')}\langle T_\tau \left\{\left[\overline{\psi}_1(x;\tau) - (+)e^{i\pi/2}\overline{\psi}_1^\dagger(x;\tau)\right]\overline{\psi}_3(x;\tau) - \left[\overline{\psi}_2^\dagger(x;\tau) - (+)e^{i\pi/2}\overline{\psi}_2(x;\tau)\right]\overline{\psi}_4^\dagger(x;\tau)\right\}\right. 
$$

$$
\times \left\{\left[\overline{\psi}_3^\dagger(x') - (+)e^{-i\pi/2}\overline{\psi}_3^\dagger(x')\right] - \overline{\psi}_4(x')\left[\overline{\psi}_2^\dagger(x') - (+)e^{-i\pi/2}\overline{\psi}_2^\dagger(x')\right]\right\} 
$$

$$
+ (x \leftrightarrow x', \tau \rightarrow -\tau)
$$

$$
\equiv e^{-iq_0(x-x')}H_{\rho(\sigma)}(x - x', \tau) + (x \leftrightarrow x', \tau \rightarrow -\tau), \quad (B2)
$$

where

$$
H_{\rho}(x - x', \tau) = 2 \left(\frac{1}{\beta L}\right)^2 \sum_{k,k',\epsilon_n,\epsilon_n'} e^{i(k+k')(x-x')-i(\epsilon_n+\epsilon_n')\tau}
$$

$$
\times \left\{\left(-\frac{1}{\epsilon_n - E_k'}\frac{1}{i\epsilon_{n'} - E_{k'}} + \frac{1}{\epsilon_n + E_k'}\frac{1}{i\epsilon_{n'} + E_{k'}}\right) (u_k'u_{k'}' + v_k'u_{k'}')^2
$$

$$
\right\}.
$$
\[ H_\sigma(x - x', \tau) = 2 \left( \frac{1}{\beta L} \right)^2 \sum_{k, k'} \sum_{\epsilon_n, \epsilon_{n'}} e^{i(k + k')(x - x') - i(\epsilon_n + \epsilon_{n'})\tau} \]
\[ \times \left\{ \left( \frac{u''_k^2}{\epsilon_n - E_k'} + \frac{v''_k^2}{\epsilon_{n'} - E_k'} \right) \frac{1}{\epsilon_n - \xi_k'} + \left( \frac{v''_k^2}{\epsilon_n - E_k'} + \frac{u''_k^2}{\epsilon_{n'} + E_k'} \right) \frac{1}{\epsilon_{n'} + \xi_k'} \right\}, \] (B4)

and the factors, \( u_k, v_k, u'_k, \) and \( v'_k \) are defined by
\[ u_k = \sqrt{\frac{1}{2} \left(1 + \frac{\xi_k}{E_k}\right)}, \]
\[ v_k = \sqrt{\frac{1}{2} \left(1 - \frac{\xi_k}{E_k}\right)} \text{sgn}\Delta, \]
\[ u'_k = \sqrt{\frac{1}{2} \left(1 + \frac{\xi_k}{E_k}\right)}, \]
\[ v'_k = \sqrt{\frac{1}{2} \left(1 - \frac{\xi_k}{E_k}\right)} \text{sgn}\Delta'. \] (B8)

From Eqs. (B3) and (B4), \( \chi_\rho(q_x, \pi, i\omega_n) \) and \( \chi_\sigma(q_x, \pi, i\omega_n) \) are calculated as,
\[ \chi_\rho(q_x, \pi, i\omega_n) = \frac{1}{L} \sum_k \left\{ \left( \frac{f(E_k) - f(-E_k')}{E_k + E_k' - i\omega_n} - \frac{f(-E_k) - f(E_k')}{-E_k - E_k' - i\omega_n} \right) (u_k u'_k + v_k v'_k)^2 \right\}_{k' = -k + q_0 + q_x} \]
\[ + \left( \frac{f(-E_k) - f(-E_k')}{-E_k + E_k' - i\omega_n} - \frac{f(E_k) - f(E_k')}{E_k - E_k' - i\omega_n} \right) (u_k v'_k - v_k u'_k)^2 \right\}_{k' = -k - q_0 + q_x} \]
\[ + (\omega_n \rightarrow -\omega_n, q_x \rightarrow -q_x) \} \}, \] (B9)
\[ \chi_\sigma(q_x, \pi, i\omega_n) = \frac{1}{L} \sum_k \left\{ u''_k^2 \left( \frac{f(E_k) - f(-\xi_k)}{E_k' + \xi_k' - i\omega_n} - \frac{f(-E_k') - f(-\xi_k')}{-E_k' - \xi_k' - i\omega_n} \right) \right|_{k' = -k + q_0 + q_x} \]
\[ + v''_k^2 \left( \frac{f(-E_k) - f(-\xi_k)}{-E_k' + \xi_k' - i\omega_n} - \frac{f(E_k) - f(\xi_k')}{E_k' - \xi_k' - i\omega_n} \right) \right|_{k' = -k - q_0 + q_x} \]
\[ + (\omega_n \rightarrow -\omega_n, q_x \rightarrow -q_x) \} \}, \] (B10)

where \( f(z) = 1/(e^{\beta z} + 1) \) is a Fermi distribution function. Equations (B9) and (B10) in the limit of \( T = 0 \) lead to Eqs. (3.27) and (3.28).

**APPENDIX C: PROPERTIES IN THE ABSENCE OF THE HOPPING**

In this case, by using density operators, \( \rho_{p,\sigma,\nu}(q) = \sum_k a_{k+q,\rho,\sigma,\nu}^+ a_{k,\rho,\sigma,\nu} \) with \( \nu = + \) and \( \nu = - \) being the chain index \( i = 1 \) and \( i = 2 \) respectively, Eqs. (2.2) \( \sim \) (2.4) in the absence
of the hopping can be written as,

\[ \mathcal{H}_k = \frac{\pi v_F}{L} \sum_{p,\sigma,\nu} \sum_q \rho_{p,\sigma,\nu}(q) \rho_{p,\sigma,\nu}(-q), \]  

(C1)

\[ \mathcal{H}_{\text{int}} = \frac{\pi v_F g_2}{L} \sum_{p,\sigma,\nu} \sum_q \left\{ \rho_{p,\sigma,\nu}(q) \rho_{-p,\sigma,\nu}(-q) + \rho_{p,\sigma,\nu}(q) \rho_{-p,-\sigma,\nu}(-q) \right\}, \]  

(C2)

\[ \mathcal{H}'_{\text{int}} = \frac{\pi v_F g_2'}{L} \sum_{p,\sigma,\nu} \sum_q \left\{ \rho_{p,\sigma,\nu}(q) \rho_{-p,\sigma,-\nu}(-q) + \rho_{p,\sigma,\nu}(q) \rho_{-p,-\sigma,-\nu}(-q) \right\}. \]  

(C3)

Then the Hamiltonian is rewritten in terms of the phase variables as

\[ \mathcal{H} = \frac{v_F}{4\pi} \int \text{d}x \left\{ \frac{1}{\eta} (\partial_x \theta) + \eta \eta' \left( \partial_x \theta' \right)^2 + \frac{v_F}{4\pi} \int \text{d}x \left\{ \frac{1}{\eta} (\partial_x \theta) + \eta \eta' \left( \partial_x \theta' \right)^2 \right\} \]  

\[ + \frac{v_F}{4\pi} \int \text{d}x \left\{ (\partial_x \phi) + (\partial_x \phi')^2 + (\partial_x \phi')^2 \right\}, \]  

(C4)

where

\[ \theta_\pm(x) = -\sum_{q \neq 0} \frac{\pi i}{2qL} e^{(-\alpha|q|/2i)q} \sum_{\sigma,\nu} \left( \rho_{-\sigma,\nu}(q) \pm \rho_{-\sigma,\nu}(q) \right), \]  

(C5)

\[ \theta'_\pm(x) = -\sum_{q \neq 0} \frac{\pi i}{2qL} e^{(-\alpha|q|/2i)q} \sum_{\sigma,\nu} \left( \rho_{-\sigma,\nu}(q) \pm \rho_{-\sigma,\nu}(q) \right), \]  

(C6)

\[ \phi_\pm(x) = -\sum_{q \neq 0} \frac{\pi i}{2qL} e^{(-\alpha|q|/2i)q} \sum_{\sigma,\nu} \left( \rho_{-\sigma,\nu}(q) \pm \rho_{-\sigma,\nu}(q) \right), \]  

(C7)

\[ \phi'_\pm(x) = -\sum_{q \neq 0} \frac{\pi i}{2qL} e^{(-\alpha|q|/2i)q} \sum_{\sigma,\nu} \left( \rho_{-\sigma,\nu}(q) \pm \rho_{-\sigma,\nu}(q) \right), \]  

(C8)

and \( \eta, \eta' = \sqrt{(1 - 2g_2 - 2g_3)/(1 + 2g_2 + 2g_3)} \). The quantities, \( \theta_\pm \) and \( \phi_\pm \) defined here are the same as those of Eqs.(2.10) and (2.11), while \( \theta'_\pm \) and \( \phi'_\pm \) are different from \( \tilde{\theta}_\pm \) and \( \tilde{\phi}_\pm \) of Eqs.(2.12) and (2.13). Since the parts including \( \theta_\pm \) and \( \phi_\pm \) in Eqs.(C4) is the same as Eq.(2.16), the dynamics of total fluctuation does not depend on the interchain hopping. By making use of field operators, \( \psi_{p,\sigma,\nu} \) expressed as

\[ \psi_{p,\sigma,\nu} = \frac{e^{ipk_Fx}}{\sqrt{2\pi\alpha}} \exp \left[ \frac{i}{2\sqrt{2}} \left\{ p\theta_+ + \theta_- + \nu(p\theta'_+ + \theta'_-) + \sigma(p\phi_+ + \phi_-) + \sigma\nu(p\phi'_+ + \phi'_-) \right\} \right], \]  

(C9)

correlation functions of the order parameters are calculated as,
\[ \langle S_{\parallel}^{\sigma,\sigma'}(x)S_{\parallel}^{\sigma,\sigma'}(0) \rangle \sim \left( \frac{\alpha}{|x|} \right)^{1+\frac{1}{2}(\frac{1}{\eta_{\rho}^*} + \frac{1}{\eta_{\nu}^*})}, \]  
(C10)

\[ \langle S_{\perp}^{\sigma,\sigma'}(x)S_{\perp}^{\sigma,\sigma'}(0) \rangle \sim \left( \frac{\alpha}{|x|} \right)^{1+\frac{1}{2}(\nu_{\rho}^* + \eta_{\nu}^*)}, \]  
(C11)

\[ \langle DW_{\parallel}^{\sigma,\sigma'}(x)DW_{\parallel}^{\sigma,\sigma'}(0) \rangle \sim \left( \frac{\alpha}{|x|} \right)^{1+\frac{1}{2}(\eta_{\rho}^* + \eta_{\nu}^*)}, \]  
(C12)

\[ \langle S_{\perp}^{\sigma,\sigma'}(x)S_{\perp}^{\sigma,\sigma'}(0) \rangle \sim \left( \frac{\alpha}{|x|} \right)^{1+\frac{1}{2}(\nu_{\rho}^* + \frac{1}{\eta_{\rho}^*})}, \]  
(C13)

where \( S_{\parallel}^{\sigma,\sigma'} = \psi_{p,\sigma,\nu}^\dagger \psi_{-p,\sigma',\nu}, \) \( S_{\perp}^{\sigma,\sigma'} = \psi_{p,\sigma,\nu}^\dagger \psi_{-p,\sigma',-\nu}, \) \( DW_{\parallel}^{\sigma,\sigma'} = \psi_{p,\sigma,\nu}^\dagger \psi_{-p,\sigma',\nu} \) and \( DW_{\perp}^{\sigma,\sigma'} = \psi_{p,\sigma,\nu}^\dagger \psi_{-p,\sigma',-\nu} \) express the order parameters of the intrachain superconductivity, the interchain superconductivity, the intrachain density wave and the interchain density wave, respectively. In Eq.(C9), we neglect the phase factor given by \( \hat{N}_{p,\sigma,\nu} \), because the particle number of each branch is conserved in the absence of the hopping. The exponent of the correlation functions of “in phase” ordering is the same as that of the “out of phase” ordering in addition to degeneracy in spin degree of freedom. The phase diagram obtained from Eqs.(C10) \( \sim \) (C13) is shown in Fig.5.

Susceptibilities are calculated as follows. From Eq.(C4), it is found that \( \text{Re} \chi_{\rho}(q_x, 0, \omega) \) and \( \text{Re} \chi_{\sigma}(q_x, 0, \omega) \) are the same as Eqs.(3.23) and (3.24). By noting that

\[ \chi_{\rho}(x - x', \pi, \tau) = \frac{1}{\pi^2} \left( T_{\tau} \partial_x \theta_{\tau}^*(x, \tau) \partial_{x'} \theta_{\tau}^*(x', 0) \right), \]

\[ \chi_{\sigma}(x - x', \pi, \tau) = \frac{1}{\pi^2} \left( T_{\tau} \partial_x \phi_{\tau}^*(x, \tau) \partial_{x'} \phi_{\tau}^*(x', 0) \right), \]

we obtain \( \text{Re} \chi_{\rho}(q_x, \pi, \omega) \) and \( \text{Re} \chi_{\sigma}(q_x, \pi, \omega) \) written as

\[ \text{Re} \chi_{\rho}(q_x, \pi, \omega) = \frac{2\eta_{\rho}^* (v_{\rho}^* q_x)^2}{\pi v_{\rho}^* (v_{\rho}^* q_x)^2 - \omega^2}, \]  
(C14)

\[ \text{Re} \chi_{\sigma}(q_x, \pi, \omega) = \frac{2 (v_{F} q_x)^2}{\pi v_{F} (v_{F} q_x)^2 - \omega^2}, \]  
(C15)

which are the forms familiar to the Luttinger liquid. Note that \( \text{Re} \chi_{\rho}(q_x, \pi, \omega) \) is suppressed (enhanced) in the case of \( g_2 - g_2' > 0 \) \( (< 0) \) while \( \text{Re} \chi_{\sigma}(q_x, \pi, \omega) \) is independent of \( g_2 - g_2' \). Thus it turns out that the two chains coupled only by the interchain interaction remain as the Luttinger liquid.
REFERENCES

[1] J. Solyom, Adv. Phys. 28, 201 (1979).

[2] V. J. Emery, in Highly Conducting One-Dimensional Solids, edited by J. T. Devreese, R. P. Evrard and V. E. van Doren (Plenum, New York, 1979), p.247.

[3] F. D. M. Haldane: J. Phys. C 14, 2585 (1981).

[4] H. Fukuyama and H. Takayama: in Electronic Properties of Inorganic Quasi One-Dimensional Compounds Part 1, ed. P. Monceau (D. Riedel Pub. Comp, New York, 1985) p 41.

[5] For example, see T. Ishiguro and K. Yamaji: Organic Superconductors, Springer Series in Solid-State Sciences 88 (Springer-Verlag, Berlin, 1990).

[6] For example, see S. Chakravarty, A. Sudbø, P. W. Anderson and S. Strong: Science 261, 337 (1993).

[7] X. G. Wen: Int. J. Mod. Phys. B 6, 1711 (1992).

[8] X. G. Wen: Phys. Rev. B 42, 6623 (1990).

[9] F. V. Kusmartsev, A. Luther and A. Nersesyan: JETP Lett. 55, 724 (1992).

[10] V. M. Yakovenko: JETP Lett. 56, 510 (1992).

[11] H. Yoshioka and Y. Suzumura: Synth. Met. 71, 1967 (1995).

[12] H. Yoshioka and Y. Suzumura: J. Phys. Soc. Jpn. 63, 4298 (1994).

[13] C. Castellani, C. Di Castro and W. Metzner: Phys. Rev. Lett. 69, 1703 (1992).

[14] M. Fabrizio: Phys. Rev. B 48, 15838 (1993).

[15] A. M. Finkel’stein and A. I. Larkin: Phys. Rev. B 47, 10461 (1993).

[16] D. G. Clarke, S. P. Strong and P. W. Anderson: Phys. Rev. Lett. 72, 3218 (1994).
[17] K. Yamaji and Y. Shimoi: Physica C 222, 349 (1994).

[18] Y. Shimoi, K. Yamaji and T. Yanagisawa: Synth. Met. 70, 1017 (1995); T. Yanagisawa, Y. Shimoi and K. Yamaji: Phys. Rev. B 52, R3861 (1995).

[19] N. Nagaosa: Solid State Commun. 94, 495 (1995); N. Nagaosa and M. Oshikawa: J. Phys. Soc. Jpn. 65, 2241 (1996).

[20] H. J. Schulz: Phys. Rev. B 53, R2959 (1996), 1996 cond-mat preprint 9605075.

[21] L. Balents and M. P. A. Fisher: Phys. Rev. B 53, 12133 (1996).

[22] H. Yoshioka and Y. Suzumura: to be published in Phys. Rev. B.

[23] T. Kimura, K. Kuroki and H. Aoki: Phys. Rev. B 51, 13860 (1995).

[24] E. Orignac and T. Giamarchi: Phys. Rev. B 53, R10453 (1996).

[25] A. Nersesyan, A. Luther and F. V. Kusmartsev: Phys. Lett. A 176, 363 (1993).

[26] H. Yoshioka and Y. Suzumura: J. Phys. Soc. Jpn. 64, 3811 (1995).

[27] Y. Suzumura: Prog. Theor. Phys. 61, 1 (1979).

[28] Correspondence between notations used by us and that by Schulz [20] are as follows,
\[ \theta_+ = \sqrt{2}\phi_{\rho^+}, \quad \theta_- = -\sqrt{2}\theta_{\rho^-}, \quad \phi_+ = \sqrt{2}\phi_{\sigma^+}, \quad \phi_- = -\sqrt{2}\theta_{\sigma^-}, \quad \tilde{\theta}_+ = \sqrt{2}\phi_{\sigma^-}, \quad \tilde{\theta}_- = -\sqrt{2}\theta_{\sigma^-}, \]
\[ \tilde{\phi}_+ = \sqrt{2}\phi_{\sigma^+} \quad \text{and} \quad \tilde{\phi}_- = -\sqrt{2}\theta_{\sigma^-}. \]

[29] A. Luther and I. Peschel: Phys. Rev. B 9, 2911 (1974).

[30] A. Luther and V.J. Emery: Phys. Rev. Lett. 57, 589 (1974).

[31] D.C. Mattis and E. Lieb: J. Math. Phys. 6, 304 (1965).

[32] Y. Suzumura and H. Fukuyama: J. Low Temp. Phys. 31, 273 (1978).

[33] D. G. Shelton and A. M. Tsvelik: Phys. Rev. B 53, 14036 (1996).
FIGURES

FIG. 1. Solutions of the self-consistent equations, $\Delta/\xi_c$ and $\Delta'/\xi_c$ as a function of $g_2' - g_2^*$, which are obtained from Eqs. (3.9) and (3.10). Note that $(-\Delta, -\Delta')$ is also the solution as well as $(\Delta, \Delta')$.

FIG. 2. Phase diagram in the presence of the interchain hopping. Here $DW_{\sigma,\sigma'} (DW_{\sigma,\sigma'}^*)$ and $S_{\sigma,\sigma'} (S_{\sigma,\sigma'}^*)$ express density wave with interchain (intrachain) ordering with out of phase and superconductivity with interchain (intrachain) ordering with in phase, where $\sigma$ and $\sigma'$ express spin indices.

FIG. 3. Normalized charge susceptibility, $\bar{\chi}_\rho(q_x, \pi, 0) (\equiv \text{Re} \chi_\rho(q_x, \pi; 0)/(2/\pi v_F))$ at $T = 0$ as a function of $v_F q_x / \xi_c$ in the case of $g_2' - g_2^* = -0.2, 0.2, 0.3$ and $0.4$ where $\xi_c = 2t$.

FIG. 4. Normalized spin susceptibility, $\bar{\chi}_\sigma(q_x, \pi, 0) (\equiv \text{Re} \chi_\sigma(q_x, \pi; 0)/(2/\pi v_F))$ at $T = 0$ as a function of $v_F q_x / \xi_c$ in the case of $|g_2' - g_2^*| = 0.2, 0.3$ and $0.4$ where $\xi_c = 2t$.

FIG. 5. Phase diagram in the absence of the interchain hopping. Here $DW_{\perp,\sigma'} (DW_{\parallel,\sigma'})$ and $S_{\perp,\sigma'} (S_{\parallel,\sigma'})$ express density wave with interchain (intrachain) ordering and superconductivity with interchain (intrachain) ordering, where $\sigma$ and $\sigma'$ express spin indices.
Fig. 1
\begin{align*}
g_2 &= -g_2' \\
g_2' &= g_2 \\
g_2 &= g_2'
\end{align*}

Fig. 2
$\bar{\chi}_p(q_x, \pi, 0)$

$\chi_2 = \chi_2' = 0.4$

$V_F q_x / \xi_c$

Fig. 3
\[ \chi_\sigma(q_x, \pi, \theta) \]

\[ |g_2 - g_2'| = 0.2 \quad 0.3 \quad 0.4 \]

Fig. 4
$g_2 = -g'_2$  $g_2 = g'_2$

\[DW_{\perp}^{\sigma,\sigma'}\]
\[S_{\perp}^{\sigma,\sigma'}\]
\[S_{\parallel}^{\sigma,\sigma'}\]

interchain

\[\text{intrachain}\]

Fig.5