Goldstone and Higgs modes of photons inside a cavity

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Goldstone and Higgs modes have been detected in various condensed matter, cold atom and particle physics experiments. Here, we demonstrate that the two modes can also be observed in optical systems with only a few (artificial) atoms inside a cavity. We establish this connection by studying the $U(1)/Z_2$ Dicke model where $N$ qubits (atoms) coupled to a single photon mode. We determine the Goldstone and Higgs modes inside the super-radiant phase and their corresponding spectral weights by performing both $1/J$ expansion and exact diagonalization (ED) study at a finite $N$. We find nearly perfect agreements between the results achieved by the two approaches when $N$ gets down even to $N=2$. The quantum finite size effects at a few qubits make the two modes quite robust against an effectively small counterrotating wave term. We present a few schemes to reduce the critical coupling strength, so the two modes can be observed in several current available experimental systems by just conventional optical measurements.
setting by the Goldstone energy $E_G$. In both the photon and photon number correlation functions, we evaluate the low frequency Goldstone mode $E_G$, the high frequency Higgs mode $E_H$ and their corresponding spectral weights $G_G$ and $G_H$. The Higgs mode is a sharp mode protected by the $Z$-symmetry of the two atomic levels respectively, the Goldstone mode $E_G$ is the collective photon-atom coupling ($g$ is the individual photon-atom coupling). The $g = \sqrt{N}g'$ is the counter-rotating wave (CRW) term. We discuss several schemes to reduce the critical coupling, so the two modes can be observed in several experimental systems by conventional optical detection methods such as the florescence spectrum measurement in Eq. 5 and the Hanbury-Brown-Twiss (HBT) type of measurement on Eq. 6 respectively.

**Results**

**Reducing the $U(1)/Z_2$ to the $J = U(1)/Z_2$ Dicke model.** In the $U(1)/Z_2$ Dicke model, a single mode of photons couple to $N$ two level atoms with same coupling constants $g$ and $g'$. The two level atoms can be expressed in terms of 3 Pauli matrices $\sigma_{x,y,z}$, $\alpha = 1,2,3$. The $U(1)/Z_2$ Dicke model can be written as:

$$H_{U(1)/Z_2} = \omega_\alpha a_\alpha^\dagger a_\alpha + \frac{\omega_\beta}{2} \sum_{i=1}^N \sigma_i^z + \frac{g}{\sqrt{N}} \sum_{i=1}^N (a_\alpha^\dagger \sigma_i^- + h.c.)$$

$$+ \frac{g'}{\sqrt{N}} \sum_{i=1}^N (a_\beta^\dagger \sigma_i^+ + h.c.),$$

(1)

where the $\omega_\alpha$, $\omega_\beta$ are the cavity photon frequency and the energy difference of the two atomic levels respectively, the $g = \sqrt{N}g'$ is the collective photon-atom coupling ($g$ is the individual photon-atom coupling). The $g = \sqrt{N}g'$ is the counter-rotating wave term. It was demonstrated in Refs. 39 to 41 that in the thermal or cold atom experiments, the strengths of $g$ and $g'$ can be tuned separately by using circularly polarized pump beams in a ring cavity. In the qubit experiments or quantum dot experiments, the CRW terms and RW terms have the same strength at the bare level, however, the CRW term is usually much smaller than the RW term at the effective level as is the case in the experiment.

This is because the former violates the energy conservation, while the latter respects the energy conservation, while the latter respects the energy conservation. However, when the coupling strength gets close to the the transition frequency, the CRW term becomes comparable to the RW term as is the case in the experiment. In any case, the Hamiltonian Eq. (1) with independent $g$ and $g'$ is the most general Hamiltonian describing various experimental systems in various coupling regimes under the two atomic levels and a single photon mode approximation.

One can introduce the total “spin” of the $N$ two level atoms $J = \sum_i \sigma_i^z$, $J^+ = \sum_i \sigma_i^+$, $J^- = \sum_i \sigma_i^-$, When all the $N$ atoms are in the ground state, then $J = N/2$, $J_z = -N/2$, because the total spin $J^2 = \sum_i \sigma_i^2$ is a conserved quantity, by confining the Hilbert space only to $J = N/2$, one reduces the Hilbert space from $2^N$ to $2J + 1 = N + 1$. One can call the resulting model as the $J = U(1)/Z_2$ Dicke model.

One main advantage of this reduction is that one can study the $J = U(1)/Z_2$ model by using Holstein-Primakoff (HP) representation of the angular momentum operator $J_z = b^\dagger b - J$, $J_z = b^\dagger b$, $J_z = 2J^2 - b^\dagger b^\dagger$, therefore treat photon and atom on the same footing. This advantage will enable us to bring out many new and important results hard to retrieve from the $1/N$ expansion. Very fortunately, this reduction will not change the most important physics of the original $U(1)/Z_2$ model Eq. (1). As argued in the Methods section, except the $U(1)/Z_2$ Dicke model contains some additional energy levels, both models share the same other physical quantities to be studied in this article.

If $g' = 0$, the Hamiltonian Eq. (1) has the $U(1)$ symmetry $a \rightarrow ae^{i\theta}$, $\sigma^- \rightarrow \sigma^- e^{i\theta}$. The CRW $g'$ term breaks the $U(1)$ to the $Z_2$ symmetry $a \rightarrow -a$, $\sigma^- \rightarrow -\sigma^-$. If $g' = 0$, it becomes the $Z_2$ Dicke model studied in Ref. 41. In this article, we focus on the $U(1)$ Dicke model, but will also consider the effects of the small counter-rotating wave term $g' < g$ in the experimental detection section and the Methods section. The $g'$ = $g$ and the $g' = -g$ cases will be studied in Ref. 41. The $U(1)$ Dicke model was solved in the thermodynamic limit $N = \infty$ by various methods. In the normal phase $g < g_c = \sqrt{\frac{\omega_\alpha}{\omega_\beta}}, \langle a \rangle = 0$, the $U(1)$ symmetry is respected. In the super-radiant phase $g > g_c, \langle a \rangle \neq 0$, the $U(1)$ symmetry is spontaneously broken.
Goldstone and Higgs modes in the super-radiant phase by $1/J$ expansion. In the super-radiant phase $g > g_c$, and also not too close to the quantum critical point (QCP) (if too close, then $a' < a < b'$, a direct $1/J$ expansion is needed and will be performed elsewhere), it is convenient to write both the photon and atom in the polar coordinates $a = \sqrt{\lambda_+^2 + \delta \rho_+ e^{i\theta_+}}$, $b = \sqrt{\lambda_-^2 + \delta \rho_- e^{i\theta_-}}$ where $\lambda_+^2 \sim \lambda_-^2 \sim J$. When performing the controlled $1/J$ expansion, we keep the terms to the order of $\sim J$, $\sim 1$, and $\sim 1/J$, but ignore orders of $1/J^2$. We first minimize the ground state energy at the order $J$, and then saddle point values of $\lambda_+$ and $\lambda_-$: 

\[
\lambda_+ = \frac{g}{\omega_a} \sqrt{\left[\frac{1}{2} - (1 - \mu^2)\right]}, \quad \lambda_- = \sqrt{\left[\frac{1}{2} - (1 - \mu^2)\right]}
\]

where $\mu = g^2 / g_0^2 = \omega_a \omega_b / g^2$. It holds only in the superradiant phase $g > g_c$.

Observe that (1) in the superradiant phase $g > g_c$, $\lambda_+^2 \sim \lambda_-^2 \sim J$, (2) it is convenient to get to the $+\,$ modes: $\theta_+ = (\theta_+ + \theta_-)/2$, $\delta \rho_+ = \delta \rho_-$; $\lambda_+ = \lambda_- \approx \lambda$. (3) paying a special attention to the crucial Berry phase term in the $\theta_+$ sector, (4) after shifting $\theta_+ \rightarrow \theta_+ + \pi/2$, then one can get the effective action up to the order of $1/J$:

\[
\mathcal{L}_{U(1)}[\delta \rho_+, \theta_+] = i(\lambda_+^2 + \delta \rho_+ \hat{\psi}_+ \hat{\psi}_+) + \frac{D}{2} (\delta \rho_+)^2 + D_- (\delta \rho_- + \gamma \rho_+)^2 + 4 \omega_a \lambda_+^2 \sin^2 \theta_+ - \frac{1}{\omega_a^2} (1 - \frac{\mu^2}{g_0^2})
\]

where the first line are the crucial Berry term in the $\theta_+$ and $\theta_-$ respectively, $D = 2 \omega_a g_0^2 / E_{H}$ is the phase diffusion constant, $D_- = E_{H}^2 / 16 \lambda_+^4 \omega_a^2 + E_{H}^2 = (\omega_a + \omega_b)^2 + 4 g_0^2 \lambda_+^2 / N$. The $\gamma = \omega_a^2 / E_{H} (1 - \frac{\mu^2}{g_0^2})$ is the coupling between the $+\,$ and $-\,$ sector. Under the $U(1)$ transformation $\theta_+ \rightarrow \theta_+ + \gamma_0$, $\theta_- \rightarrow \theta_+ + \gamma_0$, $\theta_- \rightarrow \theta_-$, the $\theta_-$ is neutral under the $U(1)$ transformation. There is a mass term for $\theta_-$, but no mass term for $\theta_+$. The conjugate pair $(\theta_+, \delta \rho_+)$ leads to the Goldstone mode $E_0$ as shown in Eq. (5). While the conjugate pair $(\theta_-, \delta \rho_-)$ leads to the Higgs mode $E_0$ as shown in Eq. (6) (See also the Methods section).

Defining the Berry phase in the $+\,$ sector as $\lambda_+^2 = P + \pi$ where $P = 1, 2, \cdots$ is the closest integer to the $\lambda_+^2$, so $-1/2 < \pi < 1/2$. In fact, $P = a' a + b' b$ is just the conserved total excitations number. Redefine $\delta \rho_+ = N - P$, then one can write the corresponding Hamiltonian of Eq. (2) as:

\[
H_{U(1)}[\delta \rho_+, \theta_+] = \frac{D}{2} (\delta \rho_+ - x)^2 + D_- (\delta \rho_- + \gamma \delta \rho_+)^2 + 4 \omega_a \lambda_+^2 \sin^2 \theta_+ - \frac{1}{\omega_a^2} \frac{x^2}{E_{H}^2}.
\]

Because the $\theta_-$ is very massive, after pinning $\theta_- \rightarrow 0$, one can approximate $\sin^2 \theta_- \approx \theta_-^2$, so the total wavefunction is $\psi_{l,m}(\theta_+ , \theta_-) = \frac{1}{\sqrt{2\pi}} \frac{\Gamma(m + j + l + m + 0)}{\Gamma(m - j + l + m) \Gamma(j + l + m)} e^{i\theta_+ + \theta_-}$ where the $l = 0, 1, \cdots$ are the Landau level indices, the $m = -y, -y + 1, \cdots$ are the magnetic indices at a given sector $P, 0 < 0_a < 2\pi, 0 < 0 < \theta_- < \infty$ and the $\psi(\theta_+)$ is just the $l$-th the wavefunction of a harmonic oscillator.

The corresponding eigen-energy is

\[
E_0(l,m) = (l+1/2) \frac{h}{E_{H}} + \frac{D}{2} (m+l-x)^2.
\]

The ground state energy is at $l = 0$, $m = 0$. One can see that the energy spectrum Eq. (4) has a Landau-level structure: the Landau level energy scale is given by the Higgs energy $E_{H}$, the intra-Landau level is set up by the Goldstone energy scale $E_0 \sim 1/J$. In the large $j$ limit, there is a wide separation of the two energy scales $E_{H} \sim 1 \gg E_0 \sim 1/J$. When the excitation number $P$ reaches the order of $N$, then the intra-Landau levels with $|m| \approx P$ will start to overlap with the inter-Landau levels. These analytical results explain precisely the ED energy level structures shown in Fig. 2 for the resonant case $\omega_a = \omega_b$.

Away from the QCP, one can write down the $1/J$ expansion of the photon operator: $a = \left[ \lambda_+ + \frac{\delta \rho_+}{\Delta \delta} - \frac{(\delta \rho_+)^2}{8 \lambda_+^4} + \cdots \right] e^{i\theta_+}$. At a finite $N$, due to the restoration of the $U(1)$ symmetry by the phase diffusion in the $\theta_+$ sector, any $U(1)$ non-invariant correlation functions vanish $\langle a \rangle = 0, \langle a^\dagger a(0) \rangle = 0$ so we need only focus on the $U(1)$ invariant correlation functions. By using both canonical quantization and path integral approaches, we find the single photon correlation function:

\[
\langle T a(t) a^\dagger(0) \rangle = C_G e^{-E_0 t} + C_0 e^{-E_0 t} + O(1/j),
\]

\[
C_G = \lambda_+^2 - \lambda_+ - (1 - \gamma x)/2, \quad E_0 = D \left( \frac{1}{2} - x \right),
\]

\[
C_0 = \frac{\omega_a}{4 E_{H}} \left( \frac{\omega_a + \omega_b}{E_{H}} + 1 \right)^2, \quad E_{G} = E_{H} + E_{G},
\]

where $E_{G} = D \left( \frac{1}{2} - x \right)$ is the Goldstone mode with the corresponding spectral weight $C_G$, while $E_{H} = E_{H} + E_{G}$ is the optical mode with the corresponding spectral weight $C_0$. All these quantities can be directly measured by the florescence spectrum measurement.

The $E_0$, $C_G$ and $E_0$, $C_0$ are compared with the ED results in Fig. 3 and Fig. 4 respectively. One can see that except at the first few $P \leq N$ steps, the ED in $E_0$ match the analytical relation $E_0 = E_{H} + E_{G}$ in Eq. (5) well. The discrepancy at the first few steps is not surprising, as said previously, if too close to the QCP, a direct $1/J$ expansion is needed and will be performed elsewhere. However, the agreement between the analytical and ED results in $C_0$ holds in all couplings even near the QCP.

One can also compute the photon number correlation function:

\[
\langle T n_0(t) n_0(0) \rangle - \langle n_0 \rangle^2 \approx \langle \delta \rho_+ \delta \rho_0 \rangle = \frac{\omega_a \lambda_+^2}{E_{H}} e^{-E_0 t},
\]

Figure 2 | The ED results (See the Methods section) of the energy levels $E$ measured by subtracting the ground-state energy versus $g/g_c$ at resonance $\omega_a = \omega_b$ with $N = 5$ atoms. Different colors of the energy curves correspond to several smallest numbers of total excitations number $P = a' a + b' b$. The dashed vertical lines correspond to the critical values of $g$ where the number of total excitations $P$ in the ground state increases by one.
where $\langle n_a \rangle = \lambda_a^2$. The Higgs energy $E_H$ and the corresponding spectral weight $C_H = \omega_0 \lambda_a^2 / E_H$ are compared with the ED results in Fig. 5. Note that the sharpness of the Higgs mode is protected by the conservation of $\delta \rho_+$ in Eq. (3). Both $C_H$ and $E_H$ can be directly measured by the Hanbury-Brown-Twiss (HBT) type of measurement on two photon correlation functions.46

From the Eq. (6), one can see that $\langle (\delta \rho_+)^2 \rangle = \omega_0^2 \lambda_a^2 / E_H$, so one can find the Mandel Q factor: $Q_M = -1 + \omega_0 / E_H$, which was compared with the ED result in the Fig. 1(b). For $\omega_0 = \omega_b$, one can see $-1 < Q_M < -1/2$. So it is always in a number squeezed state. As $g \rightarrow \infty$ limit, $Q_M \rightarrow -1$, so it approaches a photon Fock state. It is known that number squeezed states could be very important in quantum information processing and also in high-resolution and high sensitivity measurements. Very similarly, one can evaluate the atom correlation functions.

**Effects of the CRW term and experimental detections of the Goldstone and Higgs modes.** The effects of the CRW terms on system’s energy Eq. (4), photon correlation function Eq. (5) and the number correlation function Eq. (6) are examined in the Methods section. Their effects were found to be much smaller than those of the finite size for a few qubits $N \approx 2 - 5$ if $g' / g < 1 / 3$. Recent experiments reached the $Z_2$ super-radiant regime with the help of a transverse pumping. In this transverse pumping scheme, the CRW terms in Eq. (1) are as important as the RW ones, so only the $Z_2$ super-radiant phase can be realized. However, it was

**Figure 3** | (a) The analytical Goldstone mode at $\alpha = -1/2$, $E_G(\alpha = -1/2) = D(g) = 2\omega_0 g^2 / E_H N$ (red line) are contrasted with the ED result $E_G = E_1^{>0} - E_1^0$ (blue lines) at $N = 5, 3, 2, 1$ respectively. It is remarkable that the analytical result can even map out broad peaks at small $P$ in the ED results very precisely. (b) The analytical spectral weight (red) of the Goldstone mode $C_G$ against the ED result (blue) at $N = 3$.

**Figure 4** | (a) The analytical relation $E_o = E_H + E_G$ (red line) is satisfied by the ED optical mode $E_o = E_1^{>0} - E_0^0$ (blue lines) at $N = 3$ except at the first few steps. (b) The analytical spectral weight (red) of the optical mode $C_o$ against the ED result (blue) at $N = 3$. 

\[
E_H(g) = \sqrt{\omega_0^2 + \omega_b^2 + 4g^2\lambda_a^2 / N} \\
E_o(g) = E_{P+1, l=1} - E_{P, l=0}
\]
Figure 5. (a) The analytical Higgs energy $E_H$ (red) against the ED result $E_H = E_P^a - E_P^b$ (blue) at $N = 3$. (b) The analytical spectral spectral weight $C_H$ (red) for the Higgs mode against the ED result (blue) at $N = 3$.

Demonstrated in\cite{48,49} that the strengths of $g'$ and $g$ can be tuned independently by using circularly polarized pump beams in a ring cavity. So we expect that $g'/g < 1/3$ can be achieved in this transverse pumping scheme, then the system can be tuned to the $U(1)$ superradiant regime. It is also promising to reach the $Z_2$ superradiant regime “simultaneously” (namely without any transverse pumping) with artificial atoms such as superconducting qubits inside micro-wave circuit cavity\cite{33,34} and quantum dots inside a semi-conductor nano-cavity engraved in a photonic crystal in Fig. 1(a)\cite{35–37}. Indeed, very recently, by enhancing the inductive pumping scheme, then the system can be tuned to the $U(1)$ super-radiant regime against the ED result (blue) at $N = 3$. (b) The analytical spectral spectral weight $C_H$ (red) for the Higgs mode against the ED result (blue) at $N = 3$.

Discussion
Quantum mechanics describes the motion of a single or a few particles\cite{34–38}. Condensed matter physics studies various emergent quantum phenomena of macroscopic number of interacting particles. Ultracold atom systems and optical cavity systems can provide unprecedented experimental systems to study quantum phenomena ranging from a few particles to a million number of interacting particles. Due to the tremendous tunability of all the parameters in these systems, they can be tuned to scale up from the isolated quantum mechanics systems to macroscopic condensed matter systems. In this article, we show that the many body theory developed to study the emergent phenomena of condensed matter systems can also be a very powerful tool to study the physical phenomena from millions of particles down even to a few particles. Especially, we study how the emergent Goldstone and Higgs modes evolve as the number of particles gets less and less, even down only a few particles in quantum optical systems. The discrete natures of both modes shown in Fig. 3 and Fig. 5 at a finite $N$ are due to the Berry phase effects $-1/2 < \chi < 1/2$. Both modes are well defined sharp quasi-particle excitations with no damping. Especially, the sharpness of the Higgs mode is the $U(1)$ symmetry protected at any finite $N$. We also found that the finite size effects at a few qubits making the $U(1)$ super-radiant phase quite robust against the counter rotating wave (CRW) term. Both modes can be detected even with a few qubits or ions inside a QED microwave cavity by conventional optical measurements\cite{45,46,59}. Considering it is difficult to scale up these systems to large number of qubits at the present technologies, this feature becomes experimentally appealing. Our theoretical works should provide a solid foundation for various ongoing and upcoming systems with a small number of particles to observe the novel phenomena due to strong light-matter interactions explored in this report.

Methods
Exact diagonalization (ED) study. For simplicity, in the following, we limit our ED study only to the resonant case $\omega_{q} = \omega_{a}$. We assume $P \leq N$. The $P > N$ case can be similarly addressed by changing $P + 1$ to $N + 1$. The ground state in the given $P$ Hilbert space is:
where the coefficients $A^{l+1}_{i l}$ can be determined by the ED. From Eq. (8), one can evaluate the Mandel Q factor $Q_{M} = -1 + \left(\langle \delta n_{l} \rangle / \langle \delta \bar{n}_{l} \rangle \right)$ which was compared with the analytical result in Fig. 1(b).

for the $l$-th eigenstate in the $P + 1$ sector with the energy- 

$$
(P + 1) J_{a l} \equiv \sum_{n=1}^{l} A^{l+1}_{a l} \sqrt{n + 1 - \delta } \langle P + 1 | \sqrt{n + 1} | \langle P | G C | G \rangle^{2},
$$

$$
\langle P | G c | G \rangle = \sum_{n=0}^{l} A^{l+1}_{a l} A^{0}_{c l} \delta (P - s) = \delta^{2}, \text{ where and the Higgs mode } E_{H} = E_{H}^{2} = E_{H}^{2} \text{ with the spectral weight } C_{0} = \langle P, l | 1 | \langle P | G C | G \rangle \rangle^{2}. \text{ Very similarly, one can evaluate the atom correlation functions.}

where $C_{l} = \langle P, l | 1 | \langle P | G C | G \rangle \rangle^{2}$. The effects of the counter-rotating wave term at $N = \infty$ and at a finite $N$. Now we consider the effects of the counter-rotating wave (CRW) terms in Eq. (1). Following the same procedures in the main text, we find $\lambda_{1,2} = \frac{g^{2}}{\alpha_{g}} \left( \frac{1}{2} - (1 - \mu)^{2} \right)$, $i_{g} = \sqrt{1 - \mu}$ with $\mu = \alpha_{g} \lambda_{1} / (g^{2} + \mu^{2})$, so the QCP is shifted to $g + \lambda_{1} = g_{c} / \alpha_{g}$. The Hamiltonian to the order of 1 is: $H_{U(1)/Z_{2}} = D_{1} (\delta \rho_{\perp} - \delta \rho_{\parallel})^{2} + D_{2} (\lambda_{1} \delta \rho_{\perp} + \delta \rho_{\parallel})^{2}$, 

$$
\Lambda_{G} = \frac{4}{g^{2} + \frac{g^{2}}{c_{G}^{2}}} [ (g + \lambda_{1})^{2} - g_{c}^{2} ] . \text{ Obviously, this gap vanishes at the QCP } g + \lambda_{1} = g_{c}. \text{ In the following, we discuss its effects at a finite } N.

Comparisons with the Higgs mode and pseudo-Goldstone mode in one gap and two gaps superconductors. It is constructive to compare the Goldstone and Higgs mode of the atom–photon system studied in this report with those in (charge neutral) superconductors (so one can ignore the Anderson-Higgs mechanism for the sake of explaining physical concepts). In a one gap superconductor, as explicitly demonstrated in the last reference in Ref. 21–22, when integrating out the fermions, the amplitude and phase of the pairing order parameter $\psi = \Delta^{*}$ emerges as two independent degree of freedoms, instead of being conjugate to each other. Its phase fluctuation in $\theta$ leads to the Goldstone mode, while its amplitude fluctuation in $\Delta$ leads to the Higgs mode.

In the following, we discuss its effects at a finite $N$. If we ignore the CRW term, all the results achieved in the main text on the system's energy (Eq. 4), the photon correlation function (Eq. 4) and the photon number correlation function (Eq. 6) remain intact after making the corresponding changes in the parameters. Then for small $g^{2} / \lambda_{1}$ at a finite $N$, we can treat the CRW term by the perturbation theory. Here, we only list the main results. Obviously, the high energy Higgs mode is insensitive to this CRW term, so we only need to focus on its effect on the low energy Goldstone mode. Then the sole dimensionless small parameter is $\delta = 2 g^{2} / \sqrt{g_{c}^{2} + \frac{g^{2}}{c_{G}^{2}}} / D_{1}$ (1). For the Berry phase $\pi \neq 0$, non-degenerate perturbation leads to the correction to the system's eigen-energy Eq. (4) at the second order $\sim \delta^{2}$. Note that although at $x = -1/2$, the energy is doubly degenerate with $\delta \rho_{\perp} = m, \delta \rho_{\parallel} = -m - 1)$, but $m$ and $-m - 1$ carry opposite parities, so they will not be mixed by the CRW term. So the nondegenerate perturbation theory is valid. For the Berry-phase $x = 0$, because the two degenerate states $(m, -m)$, $m > 0$ carry the same parity, one need to use the degenerate perturbation theory to treat their splitting. The pair $(m, -m)$ will split only at the $m$-the order degenerate perturbation, so the splitting $\Delta_{m} \sim \delta^{2}$ (2). The normal photon correlation function Eq. (5) receives a correction $\sim \delta^{2}$ in both energy and spectral weight. Most importantly, there appears also an anomalous photon correlation function ($\langle \delta a_{l}^{\dagger} (\delta a_{l}^{\dagger}) \rangle \sim \delta^{2}$). So the detection of a small non-anomalous photon correlation function by phase sensitive homodyne experiments $\varphi_{as,\varphi_{0}}$ could be used to determine the strength of the CRW term. One can see that the corrections to all the physical quantities are at the second order $\sim \delta^{2}$ or higher. From the $N = 2$ quibits in the Fig. 3(a), one can see that $D_{1} \sim \alpha_{g} / A_{2} \lambda_{1} \sim 1$ near the QCP, then when $g^{2} / \Delta_{1} < 1/3$, the corrections due to the CRW term is
suppressed compared to the finite size effects. Physically, at $N = \infty$, any CRW term will transform the gapless Goldstone mode into a pseudo-Goldstone mode whose gap is proportional to the strength of the CRW term. In contrast, at a finite $N$, the quantum finite size effects already opened a gap to the Goldstone mode which is of the size $D - 1/N$. This gap make the Goldstone in a finite system $N = 2 - 5$ quite robust against the CRW term if $|g| < 1/\epsilon$.

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How to cite this article: Yu, Y.-X., Ye, J.W. & Liu, W.-M. Goldstone and Higgs modes of photons inside a cavity. Sci. Rep. 3, 3476; DOI:10.1038/srep03476 (2013).