Triple junction at the triple point resolved on the individual particle level

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At the triple point of a repulsive screened Coulomb system, a face-centered-cubic (fcc) crystal, a body-centered-cubic (bcc) crystal and a fluid phase coexist. At their intersection, these three phases form a liquid groove, the triple junction. Using confocal microscopy, we resolve the triple junction on a single particle level in a model system of charged PMMA colloids in a nonpolar solvent. The groove is found to be extremely deep and the incommensurate solid-solid interface to be very broad. Thermal fluctuations hence appear to dominate the solid-solid interface. This indicates a very low interfacial energy. The fcc-bcc interfacial energy is quantitatively determined based on Young’s equation and, indeed, it is only about 1.3 times higher than the fcc-fluid interfacial energy close to the triple point.

According to the traditional Gibbs phase rule of thermodynamics [1], in a one-component system up to three phases can coexist. Their coexistence is represented by a triple point in the temperature-pressure phase diagram and a triple line in the temperature-density phase diagram. At triple conditions, the three phases are in mutual mechanical, thermal and chemical equilibrium. The three possible interfaces only occur at the same time if the interfacial energies are similar; if an interfacial energy is larger than the sum of the other two, this interface is unstable and the third phase intervenes. When the three interfaces intersect, they form an interfacial line, the triple junction line (which is a point in the slice shown in Fig. 1). Triple junctions have been studied on the macroscopic level, for example in metals where liquid lenses form on top of a crystallite surrounded by coexisting vapor [2–3]. Another classical example is the triple point of water where vapor, liquid water and ice coexist. Such a gas-liquid-solid triple point involves two disordered and one ordered phase and can exist in systems governed by sufficiently long-ranged attractive interparticle interactions [4–6]. In contrast, the coexistence of a fluid and two different solids involves two ordered structures and hence an interface between two crystallites that are not commensurate. The corresponding phase behavior has been studied in suspensions of charged colloids [7–10], star polymers [11], microgels [12] or dusty plasmas [13–15]. In these systems, a face-centered cubic (fcc) crystal, a body-centered cubic (bcc) crystal and a fluid can coexist. Colloids have long been used as model systems to explore such situations, including nucleation [16–19], crystallization [20–23], melting [24–26], defects [27], glass transition [28–30], solid-liquid interfaces [31–34], solid-solid phase transformations [35–38], as well as the microscopic response to external forces [39–41].

Using charged colloids [24,25], we investigate the triple junction at the triple point on an individual-particle level, i.e. including the smallest relevant length scale. At the triple point, we find a deep and tight fluid groove between the two solid phases and a very broad solid-solid interface. This indicates a small solid-solid interfacial energy and hence a considerable effect of thermal fluctuations. Indeed, a quantitative determination of the interfacial energy using Young’s equation confirms this suggestion.

The interactions of highly charged colloids in the presence of small ions can be described by a purely repulsive screened Coulomb (or Yukawa) effective pair interaction

$$U(r) = (Q^2/4\pi\epsilon\epsilon_0 r) \exp(-r/\lambda)$$

with the particle charge $Q = Ze$, the elementary charge $e$, the permittivity of the vacuum $\epsilon_0$, the relative permittivity $\epsilon$ and the Debye screening length $\lambda$ [11–13]. Phase space is completely parameterized by the Coulomb coupling parameter $\Gamma = Q^2/(4\pi\epsilon\epsilon_0 k_B T) = Z^2\lambda_B/a$ and the screening parameter $\kappa = a/\lambda$, where $k_B T$ is the thermal energy, $\lambda_B = \epsilon^2/(4\pi\epsilon\epsilon_0 k_B T)$ the Bjerrum length, $a = r^{-1/3}$ the mean interparticle distance and $\rho$ the particle number density. For this system, using Molecular Dynamics simulations the triple point was located at $\Gamma \approx 3500$ and $\kappa = 6.90$ [13].

We use charged fluorescently-labelled poly-methylmethacrylate (PMMA) spheres coated with poly-hydroxysteaer acid with a radius $R \approx 0.80 \mu m$, as determined by dynamic
light scattering. They are suspended in a nonpolar solvent mixture of decalin ($\epsilon = 2.1$, density $\rho = 0.881$ g/mL, refractive index $n = 1.48$) and tetrachloroethylene ($\epsilon = 2.5$, $\rho = 1.623$ g/mL, $n = 1.51$) with a ratio of 6:5 (by volume). This mixture has a viscosity $\eta = 1.29$ mPa s as determined by rheology, a relative permittivity $\epsilon = 2.3$, a density which closely matches the particle density and a refractive index very similar to the one of the particles.

In nonpolar solvents charges do not readily dissociate. However, they can be stabilized in the cores of reverse surfactant micelles [55]. They favor the dissociation of charges from the particle surface resulting in negatively charged particles and charge screening by the charged reverse micelles, similar to the mechanism in polar solvents. Furthermore, two neutral micelles can reversibly interact to yield two oppositely charged micelles; roughly one in $10^5$ micelles acquires a single elementary charge in this way [55]. We use 20 mM dioctyl sodium sulfosuccinate (AOT), which is well above the estimated critical micelle concentration of about 1 mM [55,57], and yields reverse micelles with an essentially concentration-independent radius $R_m \approx 1.5$ nm [55,58]. At this AOT concentration the conductivity $\kappa \approx 80$ pS/cm, as measured using an immersion probe, and thus the number density of charged micelles, and hence ions, is estimated to be $p_m = 6\pi \eta R_m c/\epsilon^2 \approx 10^{19}$ m$^{-3}$ (and the number density of all, that is charged and uncharged, micelles is higher by a factor of about $10^5$). This results in an estimate of the screening length $\lambda = 1/\sqrt{4\pi \kappa \eta p_m} \approx 0.5$ µm with $\lambda_B \approx 24$ nm. Electrokinetic light scattering measurements yield the normalized zeta potential $|e\zeta/k_B T| \approx 3.6$. The particle charge number can be estimated within the DLVO theory, $|Z| = (e^{R/\lambda}(1 + R/\lambda)|e\zeta/k_B T| \approx 900$ [59]. Due to charge saturation, this is only a crude upper estimate [60].

We use a sample at the triple point to determine the particle charge with high accuracy. A concentrated sample is prepared and less dense samples obtained by adding supernatant. The sample with three coexisting phases is identified using confocal microscopy [61]. The three coexisting phases have slightly different particle number densities $\rho \approx 0.030$ µm$^{-3}$ (fluid), 0.035 µm$^{-3}$ (bcc) and 0.040 µm$^{-3}$ (fcc, Fig. 2 filled stars), which is consistent with the small size of the coexistence region predicted for highly charged particles [11]. At the triple point, $\Gamma \approx 3500$ and $\kappa = 6.90$ [14]. This suggests a screening length $\lambda \approx 0.45$ µm consistent with the estimated $\lambda \approx 0.5$ µm and a particle charge $|Q| \approx 670$ e which is below the crude upper estimate $|Q| \approx 900$ e. Thus, the values determined from the sample at the triple point are consistent with those based on the sample composition. It also confirms that the particles are highly charged. This can be quantified by the reduced contact value of the pair potential consisting of repulsive Yukawa and hard core interactions, $\Gamma^* = Z^2(\lambda_B/2R)(1 + R/\lambda)^{-2} \approx 900$ [8,11]. Thus $\Gamma^* \gg 20$ and hard core interactions are negligible [11]. The particles hence can be treated as highly charged, point-like particles with purely repulsive screened Coulomb interactions.

Having determined the particle charge $Q \approx -670$ e and the screening length $\lambda \approx 0.45$ µm, using these parameters we can locate the samples in the phase diagram (Fig. 2). The observed phase behavior is consistent with previous experimental findings and theoretical predictions for a repulsive screened Coulomb system [9,11,23,62,67].

The sample with three coexisting phases is investigated in more detail in the following. Confocal microscopy images taken 20 µm from the cover slip show the coexistence of two crystalline solids and a fluid (Fig. 3) and thus fluid-solid interfaces as well as a solid-solid interface. At the solid-solid-fluid triple line a fluid groove starts which is many crystal layers deep. It has a small dihedral angle that appears slightly asymmetric, reflecting the two different crystalline solids. This is different in grain-boundary grooves that are formed between crystallites of the same structure, which have experimentally been observed in hard sphere systems [50,68].

Images of the groove are quantitatively analyzed to retrieve a profile of the groove and to determine the crystal struc-
fractures. This requires to determine whether a particle belongs to the fluid or one of the solids. The particle locations are extracted from the image stacks by standard algorithms [69] and for each particle the local bond order parameters $q_4$ and $q_6$ [34, 70] are calculated (Fig. 4). The distribution of $q_6$ and $q_4$ values indicate three populations which can also be identified in the histogram of $q_6$ (Fig. 4, right). The $q_6$ values hence can be used to guide the classification of the particles [70]. Particles with $q_6 \leq 0.19$ are likely to belong to the fluid, particles with $0.19 < q_6 < 0.28$ to the bcc crystal and particles with $q_6 \geq 0.28$ to the fcc crystal. A similar assignment is obtained based on the number of neighbors instead of the local bond order parameter $q_6$. If the particles are labelled accordingly, the groove and interface separating the fcc and bcc crystals are clearly visible (Fig. 5, bottom). Furthermore, the orientations of the fcc and bcc bulk crystals can be determined; in both cases the (111) plane is oriented horizontally. This analysis also confirms the presence of the triple junction at the intersection of the fcc-fluid, bcc-fluid and fcc-bcc interfaces (Figs. 3, 5). While the three phases can be distinguished on a mesoscopic level, this is not the case on a microscopic level; individual particles are observed to spread into the neighboring regions (Fig. 5, top). Fluid particles significantly penetrate into the two crystalline regions, in particular into the interfaces and the bcc crystal. Moreover, individual particles with bcc structure are found in the fcc and fluid regions. In contrast, the fcc particles are essentially confined to the fcc crystal with only very few particles with an fcc-like structure inside the bcc crystal. This distribution of particles is attributed to defects and fluctuations and partially could be due to the ambiguity in the link between crystal structure and $q_6$ value, but also the difficulty to assign a crystal symmetry to an individual particle at finite temperature. Furthermore, the fcc-fluid interface shows a high degree of bcc-like ordering, as predicted by simulations [71].

To determine the solid-solid, i.e. fcc-bcc, interfacial energy $\gamma_{\text{fcc-bcc}}$, the groove is quantitatively analyzed. The rendered three-dimensional stack is divided into 25 quasi-two-dimensional $x$-$y$ planes with a thickness of about 2.2 $\mu$m (Fig. 5). These planes are essentially parallel to the crystal planes and represent one particle layer (Fig. 5, bottom). They show grooves with slightly different depths (Fig. 5b). The tip of a groove, i.e. the triple junction, is defined as the particle that is most distant from the bulk fluid but still connected to the bulk fluid through other fluid particles. The structural parameters of the grooves (Fig. 1b) are quantitatively determined for each plane (Fig. 6c,d). The grooves are slightly more than 100 $\mu$m wide and about 80 $\mu$m deep, which corresponds to about 50 particle diameters or 35 interlayer spacings. Towards the fcc crystal the depth of the groove, on average $h_{\text{fcc}} \approx 84 \mu$m, is slightly larger than towards the bcc crystal, on average $h_{\text{bcc}} \approx 72 \mu$m. Similar is found for the angles; $\beta_{\text{fcc}} \approx 8.2^\circ$ is slightly larger than $\beta_{\text{bcc}} \approx 4.9^\circ$, but both are very small. This results in a small dihedral angle $\beta = \beta_{\text{bcc}} + \beta_{\text{fcc}} \approx 13^\circ$. This is in contrast to findings for grain boundaries between crystallites with the same symmetry.
where the dihedral angles are larger, about 100° [50].

Mechanical stability of the triple junction line requires Young’s condition to hold [68, 72–74]. It links the interfacial free energies, γ, to the interface intersection angles β (Fig. [1]):

$$\gamma_{\text{fcc-bcc}} = \gamma_{\text{fcc}} \cos(\beta_{\text{fcc}}) + \gamma_{\text{bcc}} \cos(\beta_{\text{bcc}}) \ ,$$

(1)

where the subscripts fcc, bcc and fcc-bcc refer to the fcc-fluid, bcc-fluid and fcc-bcc interfaces with \( \gamma_{\text{bcc}} \approx 0.12k_B T/\alpha^2 \) and \( \gamma_{\text{fcc}} \approx 0.40k_B T/\alpha^2 \) [46], with an extended comparison of experimental and theoretical values provided by [45]. The angles refer to the directions of the interfaces close to the triple junction. This equation is based on the assumption that the interfacial energy is isotropic. It was shown to adequately describe the fluid-bcc interfaces in Yukawa systems close to the triple conditions, where the interfacial energy is almost isotropic [46]. In addition, the fluid-fcc interface energy in hard sphere systems has been found to show only a modest dependence on the orientation [75]. Since the fcc-bcc interface has a fluid-like structure with many defects, it appears reasonable to assume that its interfacial energy is not very anisotropic.

Using Young’s equation (Eq. [1]), the fcc-bcc, i.e. solid-solid, interfacial energy \( \gamma_{\text{fcc-bcc}} \) can be calculated. We find \( \gamma_{\text{fcc-bcc}} \approx 0.52k_B T/\alpha^2 = 1.3\gamma_{\text{fcc}} \) (Fig. [1]). Thus, the interfacial energy is very small, although it involves two solid phases. The small interfacial energy is consistent with the pronounced fluctuations observed at the fcc-bcc interface (Fig. [3]); the interface is several layers wide and hence astonishingly broad. The value of the interfacial energy is considered to be reliable as it shows only a weak sensitivity to uncertainties in the angles \( \beta_{\text{fcc}} \) and \( \beta_{\text{bcc}} \) due to the small size of the angles and the dependence of \( \gamma_{\text{fcc-bcc}} \) on the cosine of the angles, which is close to 1. Thus, there is qualitative support as well as semi-quantitative and quantitative evidence for a very small fcc-bcc interfacial energy.

To conclude, we investigated suspensions of charged colloids under triple conditions, where a fcc crystal, a bcc crystal and a fluid coexist. The fcc-bcc interfacial energy was found to be about 1.3 times higher than the fcc-fluid interfacial energy close to the triple point with \( \gamma_{\text{fcc-bcc}} \approx 0.52k_B T/\alpha^2 \). Thus, the fcc-bcc interfacial energy is very small, despite the fact that two solid phases are involved. This is consistent with the observation of broad interfaces and indicates the importance of surface fluctuations also for solid-solid interfaces. Our quantitative findings and qualitative arguments suggest that a small solid-solid interfacial energy not only occurs in systems with charged particles but also in other systems with soft interactions exhibiting a triple point. Furthermore, also in atomic or molecular systems, e.g. metals [76, 79], similar values might be found if expressed in dimensionless units. Our finding hence might in general apply to triple points involving a fluid and two solids. They might also be extended to more complex conditions, such as the presence of shear or other external fields [80, 82].

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