Variance gamma for stock model performance with excess kurtosis

A Hoyyi, Tarno, D A I Maruddani, R Rahmawati
Department of Statistics, Faculty of Science and Mathematics, Diponegoro University, Jl. Prof. Soedharto, SH, Tembalang, Semarang 50275, Indonesia
Corresponding author: abdulhoyyi@lecturer.undip.ac.id

Abstract. The Geometric Brownian Motion (GBM) model is used widely for model the dynamic of asset price movement. One of the company's assets is a stock. The distribution of stock data that is normally distributed can be modeled using the Geometric Brownian Motion model. However, the distribution of stock data showed excess kurtosis and tail when using the Brown Geometry Motion model was less precise. One model for data showing excess kurtosis and tail was Variance Gamma (VG). In this research, the sample used was the stock data of PT Bank Danamon Indonesia Tbk for the period April 25th, 2018 to April 24th, 2020. The data sample was divided into two parts, namely training data and testing data. Based on the result of the stock description statistics, the value of skewness = -2.105417 and kurtosis = 22.16438 was obtained, while hypothesis testing concluded that the stock distribution did not spread normally. The resulting parameters for the VG model were σ = 0.08071, ν = 8.00500 and θ = 0.01976. Based on the results of testing on the last 38 observations, the MAPE value was = 6.97560%. These results gave the conclusion that the VG model provided excellent forecasting results.

1. Introduction
The capital market is one of the ways to get funding which has been proven by many companies that use the capital market as a medium to seek investment funds and the media to strengthen their financial position. The existence of a capital market makes investment not only possible in real assets such as house, land, gold, and others but also in financial assets or financial securities such as deposit, stock, bond, mutual funds, and others. Research related to asset price modeling has been done a lot and the development is very fast (especially modeling using stochastic differential equations).

The Geometric Brownian Motion (GBM) model is used widely for model the dynamic of asset price movement. The model is very simple, so it is easy to understand and easy to apply in modeling the prices of various company assets as well as various measuring tools in risk management, such as Value-at-Risk (VaR). The model assumes that the log return of assets is normally distributed. The normal distribution is known not only by mathematicians but is widely used by researchers, scientists and practitioners of various fields of science. Another reason is that the normal distribution has a special characteristic that make it easier to derive various theories or concepts, namely the weighted sum of random variables that are normally distributed. Research using data from the prices of various assets traded in Indonesia shows the existence of excess kurtosis and tail in the log return distribution so that the performance of the Geometric Brownian Motion model is not good enough to describe the dynamic of asset prices.
A research according to [1], there is an assumption that is not quite right to use in practical investment in bond, namely company asset data does not follow the Normal distribution, in this case it has extreme data which is shown by the existence of jump. To capture a jump in company asset data, Geometric Brownian Motion (GBM) with jump diffusion is the right model. According [2], They have conducted research on bond valuation. The bond valuation obtained is the Black-Scholes model bond valuation plus the equation associated with the third and fourth moments, namely skewness and kurtosis.

The Levy process has the ability to capture excess kurtosis and tail in a log return distribution. One of the members of the Levy Process, namely the Variance Gamma (VG) process, has a much better performance in capturing the characteristics shown by real data [3]. A three-parameter stochastic process, called the Variance Gamma process, which generalizes Brownian motion was developed as a model for the dynamic of asset log prices. This process is obtained by evaluating Brownian motion with drifts at a random time given by the Gamma process. Two additional parameters are drift of Brownian movement and time change volatility. This additional parameter provides control over the tail and kurtosis. This VG process was introduced by [4].

2. Methodology

2.1. Stochastic Process

The set of random variables \(\{X(t), t \in T\}\) with time events and \(X(t)\) events occurring at time \(t\) is called a stochastic process. The set \(T\) is called the index set of a stochastic process. The set \(T\) is the set \(t \in [0, T]\) it is said to be a stochastic process of discrete time, and it is expressed in the form \(\{X(t); t = 0, 1, 2, \ldots\}\). While the set \(T\) as a time interval \(t \in [0, T]\), it is said to be a continuous time stochastic process, and it is expressed in the form \(\{X(t); t \geq 0\}\). The set of all possible values of the random variable \(X(t)\) in a process is defined as the stochastic process state space [5]. A continuous time stochastic process is said to have a stationary increase if \(X(t + s) - X(t)\) has the same distribution for all \(t\), it is said to have an independent increase if for all \(t_0 < t_1 < \cdots < t_n\) random variable \(X_{t_1} - X_{t_0}, \ldots, X_{t_n} - X_{t_{n-1}}\) independent [6].

2.2. Gamma Process

Definition:

The Gamma process \(G(t; \nu)\) is a Levy process where the addition of \(G(t + h; \nu) - G(t; \nu) = g\) has the density Gamma with mean \(h\) and variance \(\nu h\):

\[
f_h(g) = \frac{h^{h-1}}{\Gamma(h)} \exp\left(-\frac{g}{\nu}\right).
\]

Its characteristic function is:

\[
\phi_G(u) = \left(\frac{1}{1-\nu u}\right)^{h/\nu},
\]

And for \(x > 0\), the density Lévy is:

\[
k_G(x) = \frac{\exp\left(-\frac{x}{\nu}\right)}{\nu x}.\]

Process \(X_{VG}(t; \sigma, \nu, \theta)\) is defined

\[
X_{VG}(t; \sigma, \nu, \theta) = \theta g(t; \nu) + \sigma W(g(t; \nu))
\]

The characteristic function of \(VG\) is evaluated by a conditional Gamma process. This is because \(g(t; \nu)\) is known, \(X_{VG}\) is Gaussian, so \(\theta\)

\[
E[\exp(iuX_{VG}(t)) | g(t; \nu)] = \exp\left(iu \theta g(t; \nu) - \frac{\sigma^2 u^2}{2} g(t; \nu) \right).
\]

Then the process characteristics function is obtained \(VG\phi_{X_{VG}}(u)\) by means of unconditional expectations, namely,
\[
\phi_{X_{VG}}(t; u) = E[(iuX_{VG})]_{t/v} = \frac{1}{1 - iu\theta v + \frac{\sigma^2 v}{2}u^2}.
\]

The moment of the VG process is calculated by deriving the characteristic function. The parameters \(\theta\) and \(v\) each of it controls the skewness and kurtosis. Parameter estimation is done by moment method. For more details, the estimated VG parameter can be seen in [7].

A Variance Gamma \(X_{VG}(t)\) process can be determined from the difference between two independent Gamma processes, that is \(G_p(t)\) dan \(G_n(t)\) namely,
\[
X_{VG}(t) = G_p(t) - G_n(t)
\]

\(G_p(t)\) and \(G_n(t)\) are two independent Gamma processes with mean rates of each \(\mu_p\) and \(\mu_n\) and each the variance rates of \(v_p\) and \(v_n\) [4],

\[
\mu_p = \frac{1}{2}\sqrt{\theta^2 + \frac{2\sigma^2}{v} + \frac{\theta}{2}},
\]
\[
\mu_n = \frac{1}{2}\sqrt{\theta^2 + \frac{2\sigma^2}{v} - \frac{\theta}{2}},
\]
\[
v_p = \mu_p^2 v, \quad v_n = \mu_n^2 v.
\]

A sample path from the VG process can be obtained by simulating \(G_p(t)\) with the shape parameter is \(\mu_p^2 v_p\) and the scale parameter is \(\mu_p v_p\); and \(G_n(t)\) with the shape parameter is \(\mu_n^2 v_n\) and the scale parameter is \(\mu_n v_n\). Mean Absolute Percentage Error (MAPE) to determine the accuracy of data. The MAPE value is determined by the following equation:

\[
MAPE = \frac{1}{n} \sum_{t=1}^{n} |\epsilon_t| \times 100\% \quad \text{where} \quad |\epsilon_t| = \frac{|A_t - P_t|}{A_t}
\]

\(A_t\) is the actual value for period \(t\). \(P_t\) is the forecast value at time \(t\). \(n\) represents the amount of observed data.

**Table 1. MAPE Accuracy Rating Scale**

| MAPE Value | Forecasting Accuracy                      |
|------------|------------------------------------------|
| < 10%      | Very good forecasting accuracy            |
| 11% - 20%  | Good forecasting accuracy                 |
| 21% - 50%  | Forecasting accuracy is still within reasonable limits |
| >51%       | Forecasting accuracy is not accurate      |

### 2.3. Research Stages

The steps that will be carried out in the implementation of the research are as follows.

1. Exploring data. This step is necessary to determine the distribution of the data. After that, test the normality of the stock data of PT Bank Danamon Indonesia Tbk
2. Model daily stock price movements using the Variance Gamma model
3. Calculating the daily stock price prediction of PT Bank Danamon Indonesia Tbk
4. Calculating the MAPE value

3. Research Variable
The data used in this research is secondary data, namely the daily stock price data of PT. Bank Danamon Indonesia Tbk totals 516 data, April 25th, 2018 to April 24th, 2020. The training data used is from April 25th, 2018 to February 28th, 2020, while the testing data used is from March 2nd to April 24th, 2020. The data sourced from http://finance.yahoo.com. The research was conducted in the laboratory of the Department of Statistics UNDIP.

4. Result and Discussion
The first step is doneto display descriptive visual statistics, namely the histogram and summary descriptive statistics. The log returns daily stock price histogram of PT Bank Danamon Indonesia Tbk is as follows:

![log returns daily stock price histogram](image)

**Figure 1.** Stock Histogram of PT Bank Danamon Indonesia Tbk

Figure 1 shows the histogram shape of the asymmetric and leptokurtic data distribution. These results provide a rough conclusion that the data distribution is not normal. Descriptive statistics in the form of a summary of the data as follows:

| N   | Mean  | Variance       | Minimum  | Median | Maximum | Skewness | Kurtosis |
|-----|-------|----------------|----------|--------|---------|----------|----------|
| 477 | -0.001413 | 0.000583563 | -0.220323 | 0.00000 | 0.079481 | -2.105417 | 22.16438 |

In Table 2, Descriptive Statistics shows a Skewness value equal to -2.105417, which indicates asymmetric data distribution (heavy tail). Kurtosis value equal to 22.16438 (greater than 3) indicates that the shape of the curve height (excess kurtosis) does not show the shape of a normal distribution. Based on the results of the descriptive analysis, it can be concluded that the log returns data distribution of PT Bank Danamon Indonesia Tbk stock does not spread normally. To strengthen the results obtained in descriptive analysis, formal testing is necessary. The following is the test using the Kolmogorov-Smirnov normality test, the significance level used is 5%:

**Hypothesis:**

*H₀*: The log returns daily stock sample data of PT Bank Danamon Indonesia Tbk spreads normally.
H1: The log returns daily stock sample data of PT Bank Danamon Indonesia Tbk does not spread normally.

Based on the calculation results, the statistical value of the test D = 0.11964 and the p-value = 2.347x10^{-6} which is smaller than 0.05. These results provide the decision to reject $H_0$ hypothesis, so that the conclusion is the log returns daily stock price sample data of PT. Bank Danamon Indonesia Tbk does not spread normally. Plot of opportunities for daily stock sample data of PT. Bank Danamon Indonesia Tbk as follows:

**Figure 2.** The q-q plot of log return daily stock price for PT. Bank Danamon Indonesia Tbk

In Figure 2. The q-q plot shows a pattern that tends to be non-linear, which indicates that the distribution of the sample data does not match its theoretical (normal) distribution. Based on the results of descriptive analysis and formal testing, the conclusion is that the log returns daily stock data sample of PT Bank Danamon Indonesia Tbk is not normally distributed. The next stage is to model log returns daily stock prices that are not normally distributed with the Variance Gamma model.

**Variance Gamma Modeling (VG)**

The Variance Gamma Model has three parameters, namely $\sigma$, $\nu$, and $\theta$,

$$X_{VG}(t; \sigma, \nu, \theta) = \theta g(t; \nu) + \sigma W(g(t; \nu))$$

The parameter estimation results are as follows:

**Table 3.** Estimated Value of Variance Gamma Model Parameters

| Stock Name                                      | $\hat{\sigma}$ | $\hat{\nu}$ | $\hat{\theta}$ |
|------------------------------------------------|-----------------|--------------|-----------------|
| PT Bank Danamon Indonesia Tbk                  | 0.08071         | 8.00500      | 0.01976         |

Based on the results of parameter estimation in Table 3, the mean rate and variance rate are as follows:

**Table 4.** Value of Mean Rate and Variance Rate of Gamma Process

| Gamma Process | Parameter          | Value       |
|---------------|--------------------|-------------|
| $G_p(t)$      | $\mu_p = 0.03234089$ | $\nu_p = 0.008372695$ |
| $G_n(t)$      | $\mu_n = 0.01258089$ | $\nu_n = 0.001267022$ |
The estimated value of the mean rate and variance rate parameters in Table 4 is used to estimate the shape parameter and scale parameter in the Gamma process. The estimated values for shape parameters and scale parameters are as follows:

| Gamma Process | Parameter | $G_p(t)$ | shape = 0.1249219 | scale = 0.2588888 | $G_n(t)$ | shape = 0.1249219 | scale = 0.1007100 |
|---------------|-----------|----------|-------------------|------------------|----------|-------------------|------------------|

The estimation results of the shape and scale estimates are used to generate the Gamma $G_p(t)$ process and the Gamma $G_n(t)$. The VG process is obtained by setting aside $G_p(t)$ with $G_n(t)$. After estimating the parameters, the next step is to predict the daily stock price. The predicted value of the daily stock price of PT Bank Danamon Indonesia Tbk based on the Variance Gamma model is as follows:

| Date | Actual | Prediction |
|------|--------|------------|
| March 02nd, 2020 | 3170 | 3090 |
| March 03rd, 2020 | 3260 | 3132 |
| March 04th, 2020 | 3260 | 3176 |
| March 05th, 2020 | 3170 | 3396 |
| March 06th, 2020 | 3120 | 3085 |
| March 09th, 2020 | 2800 | 3036 |
| March 10th, 2020 | 2780 | 2726 |
| March 11th, 2020 | 2570 | 2691 |
| March 12th, 2020 | 2320 | 2502 |
| March 13th, 2020 | 2320 | 2664 |
| March 16th, 2020 | 2320 | 2870 |
| March 17th, 2020 | 2010 | 2609 |
| March 18th, 2020 | 1870 | 1964 |
| March 19th, 2020 | 1740 | 1820 |
| March 20th, 2020 | 1760 | 1553 |
| March 23rd, 2020 | 1710 | 1709 |
| March 24th, 2020 | 1675 | 1682 |
| March 26th, 2020 | 1950 | 1629 |
| March 27th, 2020 | 2060 | 2117 |

| Date | Actual | Prediction |
|------|--------|------------|
| March 20th, 2020 | 1945 | 2005 |
| March 31st, 2020 | 2090 | 1890 |
| April 01st, 2020 | 2110 | 2093 |
| April 02nd, 2020 | 2010 | 1887 |
| April 03rd, 2020 | 2030 | 1957 |
| April 06th, 2020 | 2300 | 1956 |
| April 07th, 2020 | 2240 | 2239 |
| April 08th, 2020 | 2090 | 2180 |
| April 09th, 2020 | 2100 | 2034 |
| April 13th, 2020 | 2240 | 2043 |
| April 14th, 2020 | 2250 | 2180 |
| April 15th, 2020 | 2160 | 1796 |
| April 16th, 2020 | 2090 | 2033 |
| April 17th, 2020 | 2250 | 2018 |
| April 20th, 2020 | 2240 | 2190 |
| April 21st, 2020 | 2230 | 2181 |
| April 22nd, 2020 | 2290 | 2078 |
| April 23rd, 2020 | 2410 | 2219 |
| April 24th, 2020 | 2420 | 2575 |

Evaluation of the forecasting results used MAPE value. Based on the forecasting results and the actual value in Table 6, the MAPE value is equal to 0.069756 or 6.9756%. These results provide the conclusion that the VG model has very good prediction accuracy.

The plot between the actual value and the predicted value of the daily stock price of PT Bank Danamon Indonesia Tbk during the period March 02nd, 2020 to April 24th, 2020 is as follows:
5. Conclusion
In this case, the stock sample data of PT Bank Danamon Indonesia Tbk were not normally distributed, so the model used is the Variance Gamma model. The Variance Gamma Model provides excellent prediction results for PT Bank Danamon Indonesia Tbk stocks with a MAPE value = 6.9756%.

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