Evolution of Shape in the Field

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Abstract.

The evolution in shape of an isolated density enhancement in the early universe is studied through numerical simulations. The formation scenarios of a cold dissipationless collapse and that of a slow accumulation of gas in a dark matter halo are examined. In the later case the conversion of gas to stars and the accompanying energy feedback from stellar winds and SNII are taken into account.

It is found that with the more realistic initial conditions the radial orbit instability (ROI) is able to operate at higher values of the virial coefficient than previously believed. In the cold dissipationless collapse the system becomes fully axisymmetric on a timescale that is less than one collapse time. A prolate or oblate figure is obtained for low and high rotations, respectively. The inclusion of gas and star formation leads to halos that are more triaxial, while the stellar systems that form in them are oblate, independent of the initial rotation.

1. Introduction

The transformation of an isolated primordial gas cloud into a galaxy may proceed through either a short phase of cold dissipationless collapse or a longer period of gas accumulation. A dissipationless collapse requires an early high efficiency “burst” of star formation which converts most of the gas to stars. In contrast the energy feedback from stellar winds and supernovae may produce a self-regulated slow accumulation of gas and conversion into stars.

A sufficiently cold initially spherical system was found by Merritt & Aguilar (1985) not to retain its spherical shape during collapse. The symmetry breaking mechanism is a still not fully understood form of the radial orbit instability (ROI). A further work by Aguilar & Merritt (1990) showed that the final shape may be either prolate or oblate depending on the initial amount of rotation. Other studies have examined the role played by a variety of different parameters (Cannizzo & Hollister 1992; Udry 1993; Hozumi, Fujiwara, & Kan-ya 1996; Theis & Spurzem 1999).

In this study the shape evolution of a $2\sigma$ density enhancement in an Einstein-de Sitter universe is followed through N-body/SPH simulations. In the dissipative scenario, the conversion of gas to stars, along with the accompanying mass and energy feedback is included in the model. The evolution in the shape of the dark matter halo and of the stellar component that forms in it will be examined.
2. Dissipationless Evolution

2.1. Methods

_N-body Code_ The numerical simulations are performed with an updated version of the N-body algorithm described in Heller (1995) and Heller & Shlosman (1994). The integrator used is a second-order Runge-Kutta, which also provides an estimate of the required time-step for each particle (Navarro & White 1993). A hierarchical set of time bins is employed, an important consideration given the large range of dynamical timescales present in the models. The gravitational forces are computed using a surface harmonic method combined with a logarithmic radial grid (Sellwood 1997). For this study terms up to \( l = 6 \) are retained in the expansion.

_Shape Determination_ To determine the intrinsic shape of the particle distribution we start by removing any residual net velocity from the system, followed by rejection of any unbound particles, and iterating this procedure as required (Aguilar & Merritt 1990). Next we locate the density center as defined by Casettano & Hut (1985),

\[
\rho_i = \frac{\sum \rho_i r_i}{\sum \rho_i},
\]

where \(\rho_i\) is the local mass density at the position of each particle. This local density is evaluated using a kernel based expectation value as used in SPH methods (Monaghan 1992),

\[
\rho_i = \sum m_j W(r_{ij}, h_i),
\]

where \(2h_i\) is the radius of a sphere centered on particle \(i\) which contains a fixed number of neighboring particles, taken here as \(N = 96\). The kernel \(W\) is given by a normalized spline function (Hernquist & Katz 1989).

In the reference frame defined by the so determined velocity centroid and density center, we compute the eigenvalues of the moment of inertia tensor. From these we may determine the axes \(a > b > c\) of a uniform spheroid with the same eigenvalues. The ratios of these axes may then be used to characterize the shape of the system. Typically the ratio of the shortest to longest axis, \(c/a\), is given for different fractions of the particles sorted by binding energy.

_Initial Conditions_ The initial conditions are that of a spherically symmetric density enhancement in an Einstein-de Sitter (\(\Omega = 1\)) universe as given by Thoul & Weinberg (1995). The initial density profile is that of the average density around a \(2\sigma\) peak in a Gaussian random density field with power spectrum \(P(k) = Ak^{-2} \exp \left( -k^2 R_f^2 \right) \). The particles are initially moving outward with the Hubble flow, with consecutive shells of mass stopping, turning around and falling back inward. The model is conveniently defined by the filter mass \(M_f\), which defines the mass contained within a sphere of radius \(2R_f\), and a collapse redshift \(z_c\), which defines the redshift at which this shell reaches the center. In addition to these the initial conditions are set by a virial coefficient \(q = 2T_{\text{rad}}/|U|\), where \(T_{\text{rad}}\) is the kinetic energy in random motions and \(U\) is the potential energy, and
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Figure 1. Comparison of results for different combinations of integrator and gravity solver. Shown is the axis ratio $c/a$ of the most bound 60% of the particles vs. time in units of the collapse time. The integrators used are a second-order Runge-Kutta (RK) and a standard leap-frog (LF). The gravity solvers are a spherical harmonic grid, a BH-tree, and a Grape-3Af.

![Graph showing axis ratio $c/a$ vs. time for different integrators and gravity solvers.]

A typical run conserves energy to about one percent.

A spin parameter $\lambda = J|E|^{1/2} G^{-1} M^{-5/2}$, where $J$, $E$, and $M$ are respectively the total angular momentum, energy, and mass of the system.

As a standard model we adopt a non-rotating model with $M_f = 1$, $z_c = 2$, $q = 0.05$, and $\lambda = 0.0$. Following Thoul & Weinberg (1995) all models are started at an initial redshift of $z_i = 36$ to ensure they begin in the linear regime. For reference, with mass and distance units of, respectively, $10^{10} M_\odot$ and 1 kpc, and a Hubble constant of 75 km s$^{-1}$ Mpc$^{-1}$, the model would have a filter radius $R_f = 3.2$ kpc and a circular velocity of $v_c = 54$ km/s.

Tests Many tests were performed to check the sensitivity of the results to the algorithm and its parameters. Tests with half and twice the number of particles give essentially identical results. Changing the grid resolution only affects the very inner region, while making no noticeable difference to the overall evolution. A typical run conserves energy to about one percent.

As a check of the gravity solver the standard model was also run using a BH-tree (Barnes & Hut 1987) with quadrupole terms (Hernquist 1987) and the Grape-3Af special purpose hardware (Sugimoto et al. 1990). The integration was also checked against that of a standard leap-frog. The results are shown in Fig. 1 where the axis ratio $c/a$ is given for the most bound 60% of the particles as a function of time in units of the collapse time $t_c$. It should be noted that the evolution is independent of both the mass of the model and the collapse
redshift when given in these units. The two integrators give similar results, though the leap-frog shows perhaps slightly less fluctuations. The tree and Grape results also match each other closely, but compared to the grid produce a greater maximum elongation. The difference is likely due to the higher central resolution provided by the grid.

2.2. Results

Standard Model The evolution of the shape for the standard model (collisionless) is shown in Fig. 2, where the axis ratio $c/a$ is given for different fractions of the most bound particles, $f_b$, as a function of time. The time at which maximum elongation is obtained increases with the binding fraction, in support of the previous finding that the instability does not operate until around the time of maximum collapse (Hozumi, Fujiwara, & Kan-ya 1996; Theis & Spurzem 1999). In addition to this the maximum value obtained decreases with the binding fraction. After the initial collapse, between $1 - 3 t_c$, the elongation becomes slightly less, reaching an nearly equilibrium value of around $c/a = 0.48$ for most of the system. The deviations displayed by the $f_b = 0.8$ curve are attributed to both edge effects introduced by truncating the initial distribution at $r = 2 R_f$ and the longer relaxation time in the outer parts. The final relaxed shape is highly prolate, with the ratio of the two shorter axes $c/b = 0.98$. The corresponding triaxiality index $\tau \equiv (b - c)/(a - c)$ is around 0.02.
Figure 3. Normalized surface density profile of the relaxed standard model. The abscissa is given in units of the effective radius obtained from the best-fit $r^{1/4}$ law, which is indicated by the solid line.

The surface density profile of the standard model at $t = 4t_c$ is given in Fig. 3. Shown is the normalized surface density as a function of radius. The surface density is measured in logarithmically spaced elliptical annuli aligned with the principal axes of the inertia tensor and with the same ellipticity. Also shown is a fit to the functional form $\Sigma \propto (r/r_e)^{1/4}$. The radius is given in units of this effective radius $r_e$, which in terms of the filter radius has a value of about $0.05R_f$. As can be seen the surface density of the relaxed model is well fit by an $r^{1/4}$ law over a range in radius of $2 - 50r_e$. Inside of this range, numerical resolution has affected the results, while outside, the system is still not fully relaxed.

**Virial Coefficient** In Fig. 4 the evolution of the axis ratio $(c/a)_{0.6}$ is given for different initial values of the virial coefficient. As the value of $q$ is increased the maximum elongation attained decreases slightly and the time at which it occurs is increasing delayed. However, the final shape is rather insensitive to the value of $q$. This is true even for the relatively hot model with $q = 0.3$ which reaches the same elongation as the colder models after about three collapse times. It is also found (not shown) that the instability is damped to progressively larger radii as $q$ is increased. Only with a remarkably high value of the virial coefficient is the ROI fully damped.

**Rotation** The non-rotating standard model shown in Fig. 2 may be directly compared with a rotating model ($\lambda = 0.07$) in Fig. 5. The rotating model is rounder and the difference between maximum elongation and the final shape
Figure 4. Evolution of the axis ratio of the most bound 60% of the particles for different values of the initial virial coefficient. The time is given in units of the collapse time.

Figure 5. Evolution of the shape of a model with initial spin parameter $\lambda = 0.07$. Shown is the axis ratio $c/a$ for different fractions of the most bound particles as a function of time in units of the collapse time.
significantly reduced. There is also a trend in the outer 50% of the mass of decreasing elongation with binding fraction. The model is also significantly flattened, with \((c/b)_{0.6} = 0.6\), with again a slight trend of increasing with binding fraction. The corresponding triaxiality index is \(\tau = 0.99\), indicating a highly oblate figure. While the number of escapers increases with the introduction of rotation, the value is still rather small, with only about 2% of the particles having positive energies at \(t = 4t_c\).

3. Dissipative Component

3.1. Methods

The gas component is simulated using the smooth particle hydrodynamic (SPH) method, where the continuous fields (e.g. velocity, density, internal energy, etc.) are evaluated from a set of points or particles which move with the fluid. The gravitational forces and SPH neighborhoods are computed using the Grape-3Af special hardware. The initial model consists of 5000 SPH particles representing 10% of the mass, distributed with the same radial profile as the collisionless particles which represent the dark matter.

Heating and Cooling Sources of heating and cooling in the energy equation include adiabatic, viscous, and radiative. The net rate to the specific internal energy, along with the ionization fractions (H, H+, He, He+, He++, e) and mean molecular weight are computed as a function of density and temperature for an assumed optically thin primordial composition gas (Katz et al. 1996).

Star Formation Star formation takes place in regions which are not expanding, are at least moderately self-gravitating in the background of stars and dark matter, and where the collapse would continue unhindered if given sufficient numerical resolution. Specifically, the criteria are given by,

1. \(\nabla \cdot \vec{v} \leq 0\),
2. \(\rho_{HI} > \frac{1}{4}\rho_{vir}\),
3. \(\tau_{cool} \ll \tau_{dyn} < \tau_{sound} \Rightarrow \rho_{HI} > \rho_{crit}\),

where the virial density \(\rho_{vir} = \rho_{gas} + \rho_{star} + \rho_{dm}\) includes the combined mass of gas, stars and dark matter. The critical density is taken as \(7 \times 10^{-26}\ \text{g/cm}^3\). Gas is converted into stars at a rate

\[\dot{\rho}_{gas} = -\frac{\rho_{HI}}{\tau_c}\],

which implies a time scale for star formation of

\[\tau_* = -\tau_c \frac{\dot{\rho}_{gas}}{\rho_{HI}} \ln (1 - \eta),\]

where \(\eta\) is a star formation efficiency parameter which controls the amount of mass converted in each star formation event, and therefore determines the
number of events per time $\tau_s$ required to obtain the rate given by Eqn. [3]. The efficiency and star formation collapse timescale is taken here as $\eta = 0.3$ and $\tau_c = 10 \tau_{\text{dyn}}$, respectively.

Metal enriched mass (40%) is returned to the surrounding gas by the newly formed stellar particles. Also energy from stellar winds and SNII is injected into the gas, added a little at a time at every timestep based on a Salpeter IMF and lifetime vs. mass relation. This feedback energy is injected in both mechanical (1%) and thermal form (1-10%), dependent on the local gas density and metallicity (Thornton, 1998).

### 3.2. Results

A slow rotating model with $\lambda = 0.02$ and fast rotating model with $\lambda = 0.07$ were run. The other parameters are the same as the standard model without gas defined in Sec. 2.1.

Star formation in both models starts near the center at $t \approx 0.3 - 1.0 t_c$ with 1-3 bursts of $0.5 - 3.0 M_\odot/yr$, before transitioning to a low continuous rate of around $0.1 - 0.2 M_\odot/yr$. In both models almost all of the star formation occurs within $0.3 R_f$ of the center. The large gas disk ($R \approx 1.5 R_f$) that forms in the high rotation model remains smooth and axially symmetric, never reaching a high enough surface density to clump or form spiral features which would promote star formation.

The shape evolution of the halo in the slow rotating model is shown in Fig. 6. The maximum elongation of the inner regions has been significantly reduced by the mass accumulation and ejections at the center. The overall shape after $2 t_c$ is rounder than in the non-dissipative case and with a larger spread in axis ratios, $c/a \approx 0.6 - 0.8$. To determine if this spread will tighten up with time, the model must be continued longer than what is shown here. The halo is also less prolate with a triaxiality index $\tau = 0.33$. The stellar component that has formed at the center of the halo is in contrast oblate ($\tau = 0.90$), with the short axis lying perpendicular to the halo long axis.

The elongation of the faster rotating halo, shown in Fig. 7, is in contrast less affected by the gas inflow. This is because the higher rotation has reduced the central concentration of gas and stars. The stellar bulge that has formed at the center of the halo and large gas disk contains only about a third of the mass as the slow rotating model at any given time. However, similar to the slow model, the halo has become more triaxial, with $\tau = 0.58$ for $f_b \lesssim 0.2$ and $\tau = 0.85$ for $f_b \gtrsim 0.4$. The stellar bulge is oblate ($\tau = 0.84$) with the short axis aligned with the short axis of the halo.

### 4. Discussion

Most previous studies of isolated shape evolution have adopted initial conditions in which the collapse starts from a configuration with no net radial motions. This has been justified under the assumption that it represents a reasonable approximation to a cloud in the early universe which as decoupled from the Hubble flow and is just starting to turn-around after having reached its maximum extent. However, the collapse of a density enhancement as described by a typical cosmological fluctuation spectrum is intrinsically different, in that at no time are
Figure 6. Evolution in shape of a slowly rotating dark matter halo in a model which includes gas and star formation. The initial spin parameter is $\lambda = 0.02$. Shown is the axis ratio $c/a$ for different fractions of particles as a function of time in units of the collapse time.

Figure 7. Evolution in shape of a fast rotating dark matter halo in a model which includes gas and star formation. The initial spin parameter is $\lambda = 0.07$. Shown is the axis ratio $c/a$ for different fractions of particles as a function of time in units of the collapse time.
all mass shells at rest with each other. This property produces a unique profile of relative collapse times.

In the dissipationless collapse the system reaches full axisymmetry at around $0.5t_c$, while the time of maximum elongation varies with the mass shell from about $0.5 - 1.0t_c$. This is in contrast to the models of Theis & Spurzem (1999) which first go through a phase of maximal triaxiality before becoming axisymmetric over a period of many dynamical times. The decrease in elongation which occurs between $0.5 - 4.0t_c$ operates on a timescale which is much shorter than that of the two-body relaxation timescale and may be due to a collective effect. The phase of violent relaxation is very efficient as evidenced by the $r^{1/4}$ surface density profile and lack of any core-halo structure. The final shape of the system is either prolate or oblate, respectively, in the cases of low or high rotation, in agreement with Aguilar & Merritt (1990).

A difference with previous studies is found in the relatively large values of the virial coefficient at which the ROI can still operate. Aguilar & Merritt (1990) found at values $q \gtrsim 0.1$ the instability was fully damped. In contrast the dissipationless non-rotating model produced the same overall final shape at values as high as $q = 0.3$. Such high values makes the existence of isolated round dark matter halos unlikely.

The presence of gas and star formation produces a more triaxial final shape. In terms of elongation, the halo with low rotation shows a greater response to the dissipative component, particularly the inner regions where it is significantly reduced. In both the low and high rotation models, the stellar component that forms in the dark matter halo has an oblate shape, with the short axis lying perpendicular to the halo long axis. This result is unlikely to be sensitive to the adopted star formation recipe as long as the potential is dominated by the dark matter. For a baryonic dominated potential the shape of the stellar component that is formed may depend on several different parameters, such as the star formation feedback efficiency, IGM pressure, or the presence of an external UV field.

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