Research on Solving Mathematical Model of Itinerary Problem Based on Recursive Method

Ni Ruan, Ming Qiu*

School of mathematics and information science, Guangxi Institute of Education, Nanning 530023, China

*Corresponding author e-mail: 33044810@qq.com

Abstract. A special type of itinerary problem is introduced in this paper, and by using the principle of recursion, the mathematical model is established under two conditions. The results obtained by MATLAB software: if all passengers wait for the car to pick them up and then they are sent to the railway station directly, the time calculated by model (1) is about 3.3 hours; in the process of each car sending passengers to the railway station, at the same time other passengers who have not reached the station should walk to the station, and the time calculated by model (2) - (4) is about 2.85 hours, 2.718 hours and 2.602 hours respectively. In the latter case, the time is less than 3 hours, which means that the model is feasible. An extended discussion is carried out on the itinerary problem at last.

Keywords: Recursion; Itinerary problem; Mathematical thinking; Status transition.

1. Introduction
Mathematics is a science with extensive application. Mr. Hua Luogeng, a famous mathematician in China, once said: "Mathematics exists everywhere from the big universe to the tiny particles, involving the speed of rocket, the ingenuity of chemical engineering, the change of the earth, the mystery of biology, and the complexity of daily use." Among the many methods of solving mathematical problems, recursion is the basic method to study the structure and nature of mathematics, which makes it an important part of mathematical thinking method and has a wide range of applications. For example, the famous tower of Hanoi[1], gives the recursive model of plate’s moving: 

\[ a_1 = 1, \quad a_n = 2a_{n-1} + 1; \]

Fibonacci sequence[2] is defined in a recursive way as follows

\[ F(1) = 1, \quad F(2) = 1, \quad F(n) = F(n-1) + F(n-2)(n \geq 3, \quad n \in N^*), \]

Which explained the problem of rabbits’ reproduction; The problem of extracting the root of the transcendental equation \( x = a + q \sin x(0 < q < 1) \), value is the root of the equation; To solve n-order determinant in linear algebra[4], according to the features and nature of determinant, the recursion relationship is sought for, the order is reduced step by step, to establish the relationship with low-order determinant, and the method of "back substitution" is adopted to calculate the value of n-order determinant.

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Principle of recursion[1]: Assume an unknown function \( f(n) \), its own known function \( g \) is used to define:

\[
\begin{align*}
  f(n) &= g(n, f(n-1)), n \neq 0 \\
  f(n-1) &= g(n-1, f(n-2)) \\
  f(n-2) &= g(n-2, f(n-3)) \\
  M &= f(0) = a, n = 0
\end{align*}
\]

Its features are: in order to define \( f(n) \), we must define \( f(n-1) \) first; in order to define \( f(n-1) \), we must first define \( f(n-2) \),...; this way of defining themselves with their own composition is called recursive definition. Its structure must be getting simpler step by step, and there must be an end at last, and there will be no infinite cycle. The execution process of recursion is divided into two stages: recursion and regression. In the stage of recursion, the solution of a more complex problem (Its scale is \( n \)) is pushed to the solution of a simpler problem (Its scale is smaller than \( n \)) than the original one. In the stage of regression, when the simplest case is solved, in turn the solution of the slightly more complicated problem is obtained step by step.

2. Itinerary Problem

The three elements of itinerary problem are distance (s), speed (v) and time (t). Based on the travel direction, the itinerary problem can be divided into encounter problem, chase problem; based on the travel route, it can be divided into straight-line problem and circular problem. Therefore, it is most interesting and changeable in solving the itinerary problem.

The itinerary problem that is discussed in this paper refers to: there are 12 passengers who have to rush to a railway station 40 km away from them, and the time before the train leaves is only 3 hours. Their walking speed is 4km per hour, so they can't walk there in time. The only means of transportation that can be used is a car. However, the car, including the driver, may take five people at most. The speed of the car is 60km per hour. Can these 12 passengers catch the train? If there were 15 passengers, or 20 passengers, could they still make it?

This is a special type of encounter problem, and the reason why it is called special is that when the car picks up and delivers passengers, the rest of the passengers wait in the original place or walk towards the station at the same time. If the latter is the case, there will be many encounters between people and cars during the whole process of transportation. The statuses before and after the encounter still changes with time, which can be understood as the problem of status transition. This kind of encounter problem has been studied by many scholars, and some solutions have been given, but most of them start from the equation of one variable without providing a very general method. Therefore, this paper will start from the angle of mathematical modeling, using the principle of recursion to explore its solution, which is of great significance to the study of itinerary problem.

2.1. Problem Analysis

It is obviously impossible for all passengers to walk to the railway station in this itinerary problem. And the passengers who arrive at the railway station first can not get on the train first, but have to wait for all 12 passengers to arrive at the station before getting on the train. If the total time of passengers exceeds 3 hours, it is said that all passengers are late and can not catch the train. Considering that the only car with a speed of 60km/h and the maximum capacity of 5 passengers, including the driver, can be used to pick up 12 passengers, during the process of car shuttle, other passengers who have not arrived at the railway station should walk in the direction of the station at the same time. Under such background of the problem, the time required for all passengers to arrive at the railway station is explained in two conditions, and the time required is calculated in three ways in the second condition.
2.2. Model Assumption
(1) The waiting time for passengers to get on and off the car is ignored;
(2) The car is in good condition with sufficient oil and the maximum loading capacity is 5 people;
(3) The driver is not included in the 12 passengers;
(4) The traffic condition is good, with no traffic jams and no traffic accidents. Both the passengers taking the car and walking passengers can keep the maximum and uniform speed.

2.3. Model Establishment and Solution
The marks are given as follows: $S_i$: the distance between the passenger on foot and the railway station when stopping for the $i$ time, $i = 0, 1, 2, ..., s$ in which: $S_0 = 40$; $P_i$: the distance of the passenger walking for the $i$ time, $i = 0, 1, 2, ..., s$; $v_1$: the speed of the car; $v_2$: the speed of the walking passengers; $t_i$: the driving time of the car, $i = 0, 1, 2, ..., s$; $i$ is odd number which means the journey to the destination, and $i$ is even number which means the return journey.

2.3.1. All the passengers are waiting for the car to pick them up and return to the railway station.
To send all the 12 passengers to the railway station requires three going trips and two returning trips. Since the time required for each trip is the same, the total time model (1) is as follows:

$$ t = 5t_1 = \frac{5S_0}{v_1} \tag{1} $$

It can be directly calculated that $t = \frac{10}{3} \approx 3.3 > 3$, which means it’s over 3 hours left before the train leaves, that is, 12 passengers waiting for the car transfer in situ is not feasible.

2.3.2. Every time when the car takes passengers to the railway station, other passengers who have not arrived at the station should walk to the station at the same time.
Case 1: After each batch of passengers are delivered to the railway station, the car will turn around to pick up the next batch of passengers, and deliver all 12 passengers to the railway station. It takes three going trips and two returning trips. The time model required (2) is as follow

$$\begin{align*}
t_1 &= \frac{S_0}{v_1}, \\
t_2 &= \frac{S_1}{v_1 + v_2}, \\
t_3 &= \frac{S_2}{v_1}, \\
t_4 &= \frac{S_3}{v_1 + v_2}, \\
t_5 &= \frac{S_4}{v_1},
\end{align*}$$

$P_1 = v_2 \cdot t_1, \quad S_1 = S_0 - P_1;$
$P_2 = v_2 \cdot t_2, \quad S_2 = S_0 - (P_1 + P_2) = S_1 - P_3;$
$P_3 = v_2 \cdot t_3, \quad S_3 = S_0 - (P_1 + P_2 + P_3) = S_2 - P_3;$
$P_4 = v_2 \cdot t_4, \quad S_4 = S_0 - (P_1 + P_2 + P_3 + P_4) = S_3 - P_3;$

MATLAB software is used to calculate that $t = \sum_{i=1}^{5} t_i \approx 2.85 > 3$, which means the 12 passengers can catch the train, indicating that this method is feasible.

Case 2: After the car will deliver the first batch of passengers to 20 km from the midrange of the whole journey and then the passengers get off the train, and the car will turn around to pick up the next batch of passengers immediately. After the car has delivered the rest of the passengers to the station, if the first batch of passengers are still walking but not arriving at the station, the car will continue to pick
up the passengers; it will take 4 going trips and 3 returning trips to deliver all 12 passengers to the railway station, and the required time model (3) is as follows:

$$\begin{align*}
    t_i &= \frac{S_i - 20}{v_i}, \quad P_i = v_i \cdot t_i, \quad S_i = S_0 - P_i \\
    t_j &= \frac{S_j - 20}{v_i + v_j}, \quad P_j = v_j \cdot t_j, \quad S_j = S_0 - (P_i + P_j) = S_i - P_j \\
    t_k &= \frac{S_k}{v_i}, \quad P_k = v_i \cdot t_k, \quad S_k = S_0 - (P_i + P_j + P_k) = S_j - P_k \\
    t_{jick} &= \frac{S_{jick} - 20 - (P_i + P_j + P_k)}{v_i + v_j}, \quad P_{jick} = v_j \cdot t_{jick}, \quad S_{jick} = S_0 - (P_i + P_j + P_k) + 20 = (S_i - 20 + P_j) - P_k \\
    t_{songk} &= \frac{S_{songk}}{v_i}
\end{align*}$$

(3)

Among them, $t_{jick}$: the time when the car turns around to pick up the walking passengers, $t_{songk}$: the time for the car to send the walking passengers to the railway station, $k = 1, 2, ...$

MATLAB software is used to calculate the time required for the third going trip to the station:

$$\sum_{i=1}^{5} t_i \approx 2.344 \quad \sum_{i=1}^{5} t_i \approx 2.344$$

and the time for the first batch of passengers to get off at 20km, the midrange of the whole journey, and continue walking towards the railway station is:

$$\sum_{i=2}^{5} t_i \approx 2.011,$$

that is, covering about 8.044km, which is 11.956km away from the railway station, so the car still needs to turn around and pick up the rest of passengers, and the total time required is:

$$t = \sum_{i=1}^{5} t_i + t_{jick} + t_{songk} \approx 2.718 < 3,$$

that is, the 12 passengers can catch the train, which shows that this method is feasible.

Case 3: The car will send the first batch of passengers to 20km, the midrange of the whole journey, and then turn around to pick up the second batch of passengers and deliver them to 30km of the whole journey. After the car has delivered the rest of the passengers to the station, if the first and second batch of passengers are still walking on the way and have not arrived at the station, the car will continue to pick them up and deliver all 12 passengers to the railway station, which requires 5 going trips and 4 returning trips, so the required time model (4) is as follows:
\[ t_1 = \frac{S_0 - 20}{v_1}, \quad P_1 = v_2 \cdot t_1, \quad S_1 = S_0 - P_1; \]
\[ t_2 = \frac{S_1 - 20}{v_1 + v_2}, \quad P_2 = v_2 \cdot t_2, \quad S_2 = S_0 - (P_1 + P_2) = S_1 - P_2; \]
\[ t_3 = \frac{S_2 - 10}{v_1}, \quad P_3 = v_2 \cdot t_3, \quad S_3 = S_0 - (P_1 + P_2 + P_3) = S_2 - P_3; \]
\[ t_4 = \frac{S_3 - 10}{v_1 + v_2}, \quad P_4 = v_2 \cdot t_4, \quad S_4 = S_0 - (P_1 + P_2 + P_3 + P_4) = S_3 - P_4; \]
\[ t_{jie1} = \frac{S_6 - 20 - (P_2 + P_3 + P_4)}{v_1 + v_2}, \quad P_6 = v_2 \cdot t_{jie1}, \quad S_6 = (S_5 - 20 + P_1) - P_6; \]
\[ t_{song1} = \frac{S_6}{v_1}, \quad P_7 = v_2 \cdot t_{song1}, \quad S_7 = S_6 - P_7; \]
\[ t_{jie2} = \frac{S_8 - 30 - (P_2 + P_3 + P_4)}{v_1 + v_2}, \quad P_8 = v_2 \cdot t_{jie2}, \quad S_8 = (S_7 - 10 + P_1 + P_2 + P_3) - P_8; \]
\[ t_{song2} = \frac{S_8}{v_1}. \]

MATLAB software is used to calculate the time required for the third going trip for the car to arrive at the station: \( \sum_{i=1}^{5} t_i \approx 2.052 \), and the time for the first batch of passengers to get off at 20km from the midrange of the whole journey to continue walking towards the railway station is \( \sum_{i=2}^{5} t_i \approx 1.719 \), that is, the passengers have walked about 6.876km, which is 13.124km away from the railway station and the car still needs to turn around to pick up the first batch of passengers. At this time, the time for the second batch of passengers to get off the train at 30km and continue walking towards the railway station is: \( \sum_{i=3}^{5} t_i + t_{jie1} + t_{song1} \approx 1.837 \), covering about 7.348km, which is 2.652km away from the railway station, so the car still needs to return to pick up the second batch of passengers, so the total time required is: \( t = \sum_{i=1}^{5} t_i + t_{jie1} + t_{song1} + t_{jie2} + t_{song2} \approx 2.602 < 3 \). These 12 passengers can catch the train, which shows that this method is feasible.

3. Conclusion

From the discussion mentioned above, it can be seen that adopting the principle of recursion to establish the corresponding solution models, the process is simple and clear, and the total time can be calculated in a quick and effective way. Except for model (1), the rest of the models can help the 12 passengers to catch the train. In model 2 and model 3, the first or second batch of passengers are sent to a certain place in a quick and effective way. Except for model (1), the rest of the models can help the 12 passengers to catch the train. In model 2 and model 3, the first or second batch of passengers are sent to a certain place in a quick and effective way. Except for model (1), the rest of the models can help the 12 passengers to catch the train. In model 2 and model 3, the first or second batch of passengers are sent to a certain place in a quick and effective way. Except for model (1), the rest of the models can help the 12 passengers to catch the train. In model 2 and model 3, the first or second batch of passengers are sent to a certain place in a quick and effective way. Except for model (1), the rest of the models can help the 12 passengers to catch the train. In model 2 and model 3, the first or second batch of passengers are sent to a certain place in a quick and effective way. Except for model (1), the rest of the models can help the 12 passengers to catch the train. In model 2 and model 3, the first or second batch of passengers are sent to a certain place in a quick and effective way. Except for model (1), the rest of the models can help the 12 passengers to catch the train. In model 2 and model 3, the first or second batch of passengers are sent to a certain place in a quick and effective way.
trip, the time taken is: \( \sum_{i=1}^{7} t_i \approx 3.3 > 3 \); When 6 going trips and 5 returning trips are required, after the car arrives at the station on the fourth trip, the time taken is: \( \sum_{i=1}^{7} t_i \approx 3.04 > 3 \), from which it can be judged that the aforementioned model is not feasible to the case of more than 12 passengers.

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