GE vs GM: Efficient side-channel security evaluations on full cryptographic keys

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Thanks Christ, the UPB team and Virgil Gligor from CMU

(The UPB campus – left: our Church; right: the rector offices)
Side-channel attack security evaluations

Images from https://medium.com/@charles.guillemet/ledger-donjon-3e04e0ce49a9

SCA evaluations necessary:

- During product manufacturing to assess security of products
- For governments, to establish some required standards
- For security industry (e.g. automotive, banking) to ensure that third-party products (e.g. smartcards) have a sufficient level of security
- To obtain a uniform level of security certification (e.g. Common Criteria EAL4+)
SCA security evaluation tools for short data (e.g. key byte)

- Commonly used security level estimation metrics: Success Rate (SR), Guessing Entropy (GE) aka Rank
- Less common (yet...): Massey’s Guessing Entropy (GM)
- A mess of guessing entropy measures and notations
  - 1994: James Massey proposes $E[G]$
  - 1997: Christian Cachin terms it ‘Guessing Entropy’ $E[G(X)]$ and present conditional version $E[G(X|y)]$
  - 2007: Köpf and Basin use the conditional guessing entropy in the context of side-channel attacks
  - 2009: FX Standaert et al. present (empirical) Guessing Entropy in framework for SCA evaluations
- Bigger problem: GE and GM both run in $O(N \log N)$
  - Do not directly scale for large keys (impractical for $N > 2^{16}$)
  - We need special methods for full-key security evaluations
Two main approaches for full-key security evaluations:

- **Key enumeration for large keys** ([Charvillon et al. 2012, Poussier et al. 2016])
- **Security level estimation for large keys:**
  - Empirical Guessing Entropy (Rank) estimation ([Charvillon et al. 2013, Glowacz et al. 2015, Zhang et al. 2020])
  - Massey’s Guessing Entropy (GM) bounds ([Choudary and Popescu 2017])
SCA security evaluation tools for full keys (e.g. 128-bit AES key, 4096-bit RSA key)

Our main goal – comparing full-key SCA evaluation tools:

- **FSE’15 rank estimation** [Glowacz et al. 2015]
  - One of the fastest GE estimation methods to date
  - Works well up to 256 key bytes, with good precision

- **GM bounds** [Choudary and Popescu 2017]
  - Mathematical, rigorous bounds for GM
  - Fastest and most scalable full-key evaluation method to date
  - Works with 1024-byte keys and beyond

- **GEEA rank estimation** [Zhang et al. 2020]
  - One of the newest methods for GE estimation on large keys
  - Lower STD than FSE’15
GM vs GE computation

\[
\text{(Massey’s)GM} = \frac{1}{N} \sum_{q=1}^{N} \sum_{i=1}^{|S|} i \cdot P(k_i|X = X_q)
\]

\[
\text{(Empirical)GE} = \frac{1}{N} \sum_{q=1}^{N} \{\text{rank of } k* \text{ in experiment } q\}
\]

\[
(P(k_1|X_q) \geq \ldots \geq P(k_i|X_q) = P(k*|X_q) \geq \ldots \geq P(k_{|S|}|X_q))
\]

Observations:

- Same complexity (need to sort all the list of probabilities)
- Both dependent on acquired datasets \((X_q)\)
- Different use of probabilities
- GE requires knowledge of correct key, GM does not
GM vs GE simple example

\[ \text{GM} \quad 3.63 \quad \text{GE} \quad 3 \]

\[ \text{GM} \quad 3.63 \quad \text{GE} \quad 4 \]

\[ \text{GM} \quad 3.63 \quad \text{GE} \quad 5 \]
GM vs GE simple example

\[ \begin{align*}
\text{GM} & \quad 3.63 & \quad 3.63 & \quad 3.63 \\
\text{GE} & \quad 3 & \quad 4 & \quad 5 \\
\rightarrow & \text{GE provides actual (empirical) estimation of rank} \\
\rightarrow & \text{GM is generally a lower bound for GE [KB’07]} 
\end{align*} \]
Experimental datasets

- We used three different datasets:
  - *Simulated* dataset (Hamming weight of AES S-box output mixed with Gaussian noise): \( x_i = \text{HW}(S\text{-box}(k \oplus p_i)) + r_i \)
  - *XMEGA* dataset (AVR XMEGA AES engine)
  - *SoC* dataset (ChipWhisperer-Lite with STM32F303 32-bit ARM)

- We used Template Attacks to obtain lists of probabilities for each AES key byte \((p_1, p_2, \ldots, p_{256})\)
On the utility of GM

Observation 1: GM is generally a lower bound for GE
→ Can be used to confirm security is above a certain threshold

Observation 2: we may combine both measures to determine the quality of a leakage model

GM close to GE → good model (e.g. in Simulated dataset)
GM departs from GE → bad model (e.g. in SoC dataset)
We focus on the three representative methods

- FSE’15 (Glowacz et al. 2015)
- GM Bounds (Choudary and Popescu 2017)
- GEEA (Zhang et al. 2020).
Introduction

GM vs GE

Full key evaluation tools

Precision analysis on 128-bit data (16-byte results)

XMEGA

SoC

Simulated

Guessing Entropy (LOG2)

FSE

FSE-STD

FSE+STD

GMLB

GMLB-STD

GMLB+STD

GMUB

GMUB-STD

GMUB+STD

GEEA

GEEA-STD

GEEA+STD

Median STD

FSE’15 1.84
GM Bounds 0.74
GEEA 0.56

FSE 2.67
GMLB 2.89
GMUB 2.22
GEEA 1.77

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Scalability and usability analysis on larger data (128 bytes)

Guessing Entropy (log2) for SoC

- FSE
- GMLB
- GMUB
- GEEA

nr attack traces
Scalability and computation analysis on large data (16/128/1024-byte results)

|                  | 16 bytes | 128 bytes       | 1024 bytes       |
|------------------|----------|-----------------|------------------|
| FSE’15           | 29/60/172| 1027/5336/4689  | Not practical    |
| GM Bounds        | 1/1/1    | 2/6/6           | 40               |
| GEEA (M = 10^4, 10^6) | 17/18/26 | 432/415/473     | Not practical    |
Overall analysis and usability guidelines

- FSE’15:
  - Good approximation of GE
  - Works well for up to 256 key bytes
  - Slow computation for large keys
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- **GM Bounds:**
  - Guaranteed, tight bounds for GM
  - (Typically) Lower bound for GE/FSE
  - Can be used with very large keys
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- **GEEA:**
  - High accuracy (low STD)
  - Deviates from GE/FSE within similar computation time
  - Needs more analysis to provide some guarantees
Overall analysis and usability guidelines

Conclusions:

- Use GM Bounds for a very fast security evaluation (lower bound) – works with very large keys
  https://gitlab.cs.pub.ro/marios.choudary/gmbounds
- Use FSE’15 or other GE estimation algorithm for accurate estimate of key rank
- (Optionally) Use a key enumeration algorithm to output list of keys in decreasing probability

Greetings from the UPB (GM Bounds) Team
GM Bounds (log2) on 1024-byte key (SoC data)
GEEA with varying amount of data (SoC, 16 bytes)

- GEEA computation on large keys uses random selection of subkey computations (comparison vectors)
- Needs very large $M$ (large computation) to approach GE/FSE
- May not be able to follow GE within given computing power