STRONG ANOMALIES IN KAON DECAYS

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We point to a potentially large contribution of the QCD trace anomaly to the isoscalar pion pair production in Kaon decays. This contribution arises from the long-distance evolution of a 7-dimensional quark-gluon operator generated by short-distance QCD corrections to the \( \Delta S = 1 \) Fermi Hamiltonian.

1 Introduction

In addition to the well-known anomaly in the global \( U(1) \) axial symmetry, massless QCD also exhibits a breaking of the invariance under dilatations at the quantum level. This is known as the trace anomaly. While the axial anomaly is related to a pseudoscalar combination of two gluon fields, the trace anomaly is associated with a scalar combination of two such fields.

Their implications in the hadronic world are numerous. For example, in the quarkonium transitions \( \Psi' \to J/\Psi \eta \) and \( \Psi' \to J/\Psi \pi \pi \), it has been shown that the gluon conversions into \( \eta \) and \( \pi \pi \) are governed by the axial and trace anomalies respectively.

Similarly, the possibility of trace anomaly dominance has been investigated in weak K decays.\(^2\) In this talk, we show how an enhancement of the isoscalar pion pair production due to trace anomaly effects may be induced by strong interaction corrections to the \( \Delta S = 1 \) Fermi Hamiltonian.

2 A trace anomaly contribution to the \( \Delta I = 1/2 \) rule in Kaon decays

The lowest order \( \Delta S = 1 \) quark-gluon \( (qG) \) effective Hamiltonian that contains the trace anomaly \( (TA) \) operator reads:

\[
H_{TA}^{(qG)} \sim \bar{s}_R d_L G^{a}_{\mu \nu} G^{\mu \nu \alpha},
\] (1)
where $G^{a}_{\mu\nu}$ denotes the gluon field strenght tensor and the subscript $L(R)$ refers to left(right)-handed spinors.

This 7-dimensional operator is generated by leading order short-distance (SD) QCD corrections to the $\Delta S = 1$ Fermi Hamiltonian, at next-to-leading order in the momentum expansion. Yet the bulk of its contribution is expected to come from long-distance (LD) physics, as the anomaly is a non-perturbative phenomenon. Its effects on the $\Delta I = 1/2$ amplitudes could thus be larger than first expected from power counting arguments.

The LD evolution is computed in a truncated non-linear sigma model approach. The initial operator (1) is bosonized into $U^{ds}$ at the confining scale. The unitary matrix $U$ is related to the octet of light pseudoscalar fields $\pi^a$ through $U = \exp(i\sqrt{2}\Pi/F_\pi)$, $\Pi = \sum_{a=1}^{8} \lambda^a \pi^a / \sqrt{2}$. $F_\pi$ is the neutral pion decay constant and $\lambda^a$ are the Gell-Mann matrices with normalization $\text{tr} \lambda^a \lambda^b = 2 \delta^{ab}$. Strong interaction corrections are modelled by the usual $O(p^2)$ non-linear Lagrangian $L_{NL} = (F^2_\pi/4) \text{tr}(\partial_\mu U \partial^\mu U^\dagger) + r(F^2_\pi/4) \text{tr}(MU^\dagger + UM)$, where $M$ is the real, diagonal, light quark mass matrix.

Using the background field method with a cut-off regularization, we obtain the following result at the one-loop level: $^a$\footnote{Note that a similar computation has already been performed in another context.}

$$U^{ds} \rightarrow (1 + a) U^{ds} + (b/2F^2_\pi) U^{ds} \text{tr}(\partial_\mu U \partial^\mu U^\dagger + r(MU^\dagger + UM)) + (3b/2F^2_\pi) (\partial_\mu U \partial^\mu U^\dagger + r(MU^\dagger + UM))^{da} U^{qs} + O(p^6)$$

with $a = -3\Lambda^2/(4\pi F_\pi)^2$, $b = \ln(\Lambda^2/m^2_\pi)/32\pi^2$ and $\Lambda$ is the ultra-violet cut-off.

We see that the lowest order non-linear realization of the TA operator, obtained by factorizing the quark from the gluon fields in (1), does indeed show up in the second line of Eq. (2).

3 Conclusion

We have indicated how a potentially important contribution of the QCD trace anomaly to the $\Delta I = 1/2$ Kaon decay amplitudes can arise from strong interaction corrections to the $\Delta S = 1$ Fermi Hamiltonian. A more quantitative analysis will require the investigation of the matching between the SD and LD evolutions. A previous study of $K \rightarrow \pi\pi$ decay amplitudes with gluons in the intermediate state (but without any reference to the trace anomaly) predicts an enhancement of $14 \pm 6\%$ using QCD sum rules. The consistency between the two methods would be worth checking.

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References

1. M. Voloshin and V. Zakharov, Phys. Rev. Lett. 45, 688 (1980); V.A. Novikov and M.A. Shifman, Z. Phys. C 8, 43 (1981).
2. J.-M. Gérard and J. Weyers, Phys. Lett. B 503, 99 (2001).
3. A.A. Penin and A.A. Pivovarov, Phys. Rev. D 49, 265 (1994).
4. T. Hambye et al., Phys. Rev. D 58, 014017 (1998).
5. A.A. Penin and A.A. Pivovarov, Nuovo Cim. A 107, 1211 (1994).
6. J.-M. Gérard and S. Trine, in preparation.