Energy gain scaling with plasma length and density in the plasma wakefield accelerator

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Abstract. We present plasma wakefield acceleration experimental results showing that the energy gain by 28.5 GeV electrons scales with plasma length and reaches 14 GeV over a plasma with a density of \(2.6 \times 10^{17} \text{ cm}^{-3}\) and a length of 31 cm. At this plasma density the average accelerating gradient is 36 GeV m\(^{-1}\). These results are in good agreement with the numbers obtained from particle in cell simulations describing the experiment. The linear scaling is also observed both at lower and higher plasma densities, at which smaller energy gains and accelerating gradients are measured. The systematic measurements of energy gain show the reproducibility and control of the acceleration process.

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1. Introduction

One of today’s main challenges in accelerator physics is to produce intense beams of leptons with energy in the TeV range in an accelerator of reasonable length and cost. The length of a future electron/positron linear collider at the energy frontier of particle physics is foreseen to be in the tens of kilometers range, and is mainly determined by the achievable average accelerating gradient, the rate at which particles can be accelerated to their final energy. The maximum accelerating field in radio frequency (rf) cavities is limited to about 200 MV m\(^{-1}\) by electrical breakdown at the cavities’ metallic surfaces [1]. The excitation of plasma waves or wakes with ultra-high accelerating fields (\(\geq 10\) GV m\(^{-1}\)) has been demonstrated experimentally both with particle beam [2, 3] and laser beam drivers [4]. However, to effectively take advantage of these ultra-high gradients so as to reach large energy gains, the drive bunch must propagate stably and excite the wake over a long distance (centimeters to meters).

In this paper, we present systematic measurements of energy gain by the trailing electrons of a single bunch in a particle beam-driven plasma-based accelerator, known as the plasma wakefield accelerator (PWFA), with various plasma densities and lengths. We show that the energy gain increases linearly with plasma length and that the propagation and acceleration are stable and reproducible. We also show that in the nonlinear regime of the PWFA driven by a single bunch with length between \(\sim 15\) and \(\sim 50\) \(\mu\)m, the largest average accelerating gradient (\(\approx 36\) GV m\(^{-1}\)) is obtained at a plasma density of \(2.6 \times 10^{17}\) cm\(^{-3}\), larger than that predicted by the linear theory of the PWFA [5]. In these experiments, this gradient is sustained over 31 cm of plasma. The measured energy gains and optimum plasma density are in good agreement with the three-dimensional (3D) particle in cell simulations using the beam and plasma parameters of the experiment. The results presented here have been instrumental to the planning and success of the experiments in which the energy of incoming 42 GeV electrons was doubled over a plasma length of only 85 cm [3]. A review of the main PWFA results obtained at the SLAC National Accelerator Laboratory can be found in [6].

The PWFA is especially suitable for achieving large energy gain because in addition to the multi-GV m\(^{-1}\) accelerating fields, the transverse fields of the wake also provide the strong focusing [7, 8] that is necessary for the particle beam to propagate with a small size over long plasma distances. In particular, the beam and plasma parameters can be chosen such that the beam propagates with a constant transverse size along a plasma of density that is optimum for acceleration [9] and over distances much longer than that given by its natural vacuum divergence distance, or beta function. Moreover, plasmas have also been shown to focus [6], [10]–[12] and accelerate [13] positron bunches. Short, dense electron bunches similar to those envisaged for future colliders [14] have become available at SLAC. The bunch transverse field is large enough to ionize low ionization potential gases or vapors (such as lithium) over a timescale shorter than the bunch duration [15]. This creation of the plasma by the beam itself makes it possible to overcome the limitations in plasma length–density product imposed by pre-ionization processes, such as laser ionization [16] or capillary discharge [17]. It is important to note that in the PWFA the phase velocity of the wake is equal to that of the drive bunch, and independent of the plasma density: \(v_\phi = v_b = (1 - \frac{1}{\gamma^2})^{1/2}c\). For ultra-relativistic beams, there is therefore no dephasing between the beam particles over meter-long plasmas, and the acceleration process can in principle be sustained until the drive beam is nearly completely depleted of its energy. These unique characteristics of the PWFA allow one to contemplate the possibility of doubling
the energy of a future electron/positron linear collider over only a few tens of meters of plasma, as in the proposed afterburner concept [18]. Other concepts using the staging of meter-long plasma sections with energy gain per stage of the order of 25 GeV to reach 0.5–1 TeV have also been considered [22], and are being studied in detail. The PWFA may thus have the potential to extend the energy frontier of particle physics by significantly reducing the size and cost of a future collider. Necessary steps towards this goal include demonstrating that the beam can be propagated stably through long, dense plasmas, verifying that the energy gain can be scaled with the plasma length, and demonstrating that energy gains of the order of the incoming beam energy can be achieved. These are shown here. Further steps will include the acceleration of a witness bunch with a narrow final energy spread and a high energy transfer efficiency, as well as the acceleration of a positron beam in a high gradient PWFA. These will be addressed at the FACET facility currently under construction at SLAC (see [23]).

In the PWFA, a relativistic particle bunch travels in an initially neutral plasma. In the case of an electron bunch with a density \(n_b\) larger than the electron plasma density \(n_e\), the radial space charge field of the bunch expels all the plasma electrons from the bunch volume [19], to a maximum radius of the order of the plasma collisionless skin depth \(c/\omega_{pe}\). Here \(\omega_{pe} = (n_e e^2/\epsilon_0 m_e)^{1/2}\) is the plasma angular frequency, \(c\) the speed of light and \(e\) and \(m_e\) are the electron charge and mass, respectively. In the case of a single bunch, the electrons in the front and core of the bunch lose energy and excite the plasma wake. The positively charged ion column with a density \(n_i = n_e\) left behind the bunch head exerts a restoring force on the expelled plasma electrons, which rush back on the axis, approximately one plasma period or wavelength later. They create a local excess in negative charge and establish a longitudinal electric field. This field can then accelerate the electrons at the back of the same bunch. The positive ion column also partially neutralizes the relativistic bunch \(n_i < n_b\), which is therefore also focused. The uniform ion column has an electric field that increases linearly with radius and that is constant along the bunch length. This focusing field is free of geometrical aberrations and preserves the incoming beam emittance. The linear theory for a single bunch PWFA [20, 21] predicts that the accelerating field \(E_z\) scales as \(N/\sigma_z^2\), where \(N\) is the number of electrons in the Gaussian bunch with rms length \(\sigma_z\). This maximum value is reached when the wavelength of the plasma wake, \(\lambda_{pe} = 2\pi c/\omega_{pe}\), and the bunch length satisfy \(\lambda_{pe} \approx 4\sigma_z\). According to this theory, \(E_z\) exceeds 10 GV m\(^{-1}\) for bunches with \(\sigma_z < 60 \mu m\) and plasma densities > 1.5 \times 10^{16} cm\(^{-3}\). An intrinsic characteristic of the single bunch PWFA is that at the optimum density, the bunch fills all the phases of the accelerating structure. Since there is no relative motion between the ultra-relativistic beam particles and the wake, the energy change of the different longitudinal slices of the bunch reflects the local average field of the wake over the interaction distance [9, 13]. Therefore, the bunch energy spectrum after interaction with the plasma extends from that of the particle with the largest energy loss to that of the particle with the largest energy gain. In a practical PWFA, a high-charge, low-quality bunch shorter than the plasma wavelength will drive the plasma wake, while a shorter, very low emittance trailing witness bunch will be accelerated with a narrow energy spread [18]. It is important to note that the main challenge in accelerating a witness bunch resides in the generation of a suitable drive/witness bunch train, not in the wake formation or the acceleration process, and this is the subject of current research. External injection of the witness bunch allows for control of the accelerated bunch quality.
2. Experimental setup

In the experiment, the 28.5 GeV electron beam of the SLAC final focus test beam (FFTB) line [24] is focused near the entrance of a lithium (Li) vapor column [25] (see figure 1) produced in a heat-pipe oven [26]. Lithium was chosen because of the low ionization potential of its first electron (5.392 eV) and the high ionization potential of its second electron (75.638 eV). The first electron is therefore relatively easy to ionize, whereas the second is not, thereby ensuring that in this experiment the plasma density is equal to the neutral Li vapor density along the beam path. Lithium also has a low atomic number ($Z = 3$), which minimizes beam particle scattering. Schematically, the Li heat-pipe oven consists of a stainless steel tube with 25 µm beryllium windows at each end that isolate it from the high vacuum of the accelerator. The oven is filled with helium (He) at a given pressure and the tube is heated near its middle, where the solid Li is initially placed. When the power delivered to the oven heater is high enough for the vapor pressure of the melted Li to equal the pressure of the He, the Li vapor completely replaces the He in the hot zone of the oven. The preset He pressure therefore determines the Li density (through its temperature), and the heating power determines the Li column length. A stainless steel mesh lining the tube inside ensures the return of the condensed Li to the oven center. A thermocouple probe is pulled along the oven axis prior to the experiment to measure the Li vapor temperature profile for various He pressures, heating powers and numbers of heating elements. The Li column density and longitudinal profile are calculated from the well-known expression for the vapor pressure of Li as a function of temperature [27]. For this experiment the oven is built with three heaters, each approximately 10 cm long. Figure 2(a) shows three measured Li neutral density profiles, each with a maximum density $n_e = 2.6 \times 10^{17} \text{ cm}^{-3}$. The corresponding plasma lengths defined as the full-width at half-maximum (FWHM) of the profiles are: 13, 22 and 31 cm. Similar profiles were obtained for the other densities quoted later: $n_e = 1.0$ and $3.0 \times 10^{17} \text{ cm}^{-3}$. The beam parameters near the plasma entrance are: energy $E_0 = 28.5 \text{ GeV}$; bunch length $\sigma_z$ variable between $\sim 15$ and $\sim 50 \mu \text{m}$; radius at its waist in vacuum $\sigma_r \simeq 10 \mu \text{m}$; normalized emittances $\epsilon_{N_x} = 50 \times 10^{-6}$, $\epsilon_{N_y} = 5 \times 10^{-6} \text{ m rad}$ and number of electrons $N \simeq 1.8 \times 10^{10}$. The peak radial electric field of an ultra-relativistic Gaussian electron bunch is given by $E_{\text{peak}} \approx 5.2 \times 10^{-10} N / \sigma_r \sigma_z$ and occurs in the middle of the bunch and
Figure 2. (a) Density of the neutral Li vapor deduced from the Li vapor column temperature measurements. These three density profiles have FWHM of 12 (blue circles), 22 (green squares) and 31 cm (red diamonds). (b) Four energy spectra measured with plasma off (labeled I), and after the three profiles of panel (a), and labeled accordingly. The vertical energy scale is nonlinear because over a large range the beam deflection is inversely proportional to energy. Numerical attenuators have been used to evidence the particles in the different slices of the beam: $\div 120$ for the head particles that remain around the initial $\approx 28.5$ GeV, $\div 7$ for the energy loss particles ($<28.0$ GeV) and $\div 1$ for the energy gain particles ($>29.5$ GeV). The peak energy gains are 4, 8 and 14 GeV for three respective plasma lengths. The number of Cherenkov photons is directly proportional to the number of electrons and independent of their energy. The CCD camera response is linear with the number of photons and so the number of counts on the image is proportional to the number of electrons at all energies.
at \( r \approx 1.6\sigma_r \). With the above parameters, this field reaches more than 18 GV m\(^{-1}\) (\( \sigma_z = 50 \mu\text{m} \)) and greatly exceeds the 6 GV m\(^{-1}\) necessary to field ionize the first Li electron over a timescale shorter than the bunch duration [28]. In these experiments, the single bunch thus creates its own plasma and accelerating structure. The field ionization process cannot be turned off; therefore ‘no plasma’ measurements are obtained by replacing the lithium column by a bypass line filled only with the oven He buffer gas at a pressure of \(< 40 \text{Torr} \). The ionization potential of He (24.587 eV) is too high for the beam to ionize the He in the ‘no plasma’ condition.

The ultra-short electron bunches are obtained in three stages of energy chirping followed by magnetic compression [29], and the bunch length and current profile are determined by the accelerator parameters. The energy spectrum of the bunch is measured before the plasma using synchrotron radiation emitted by the bunch in a weak vertical magnetic chicane placed in a high horizontal dispersion region of the FFTB [30] (see figure 1). The bunch length can be retrieved from these spectra, as explained in [2] and detailed in [31], and can be varied between \( \sim 15 \) and \( \sim 50 \mu\text{m} \). The bunch spectrum is measured again downstream from the plasma location to determine the effect of the plasma fields. The beam travels through quadrupole and dipole magnets that image it to a small size, and disperse it in the vertical plane according to its energy, at a distance of \( \approx 25 \text{m} \) from the plasma exit (see figure 1). At that location the electrons traverse a 1 mm-thick piece of aerogel and emit Cherenkov radiation. The visible Cherenkov light is imaged onto a CCD camera to obtain the time-integrated spectrum of each bunch. Within the resolution limits, the spectra before and after the plasma location are identical when there is no plasma in the beam path. The magnetic spectrometer ensures that at the measurement location the transverse beam size \( \sigma_y \) in the dispersive \( y \)-plane is dominated by energy dispersion \( \eta_y \) and not by the beam divergence resulting from its finite geometric emittance \( \epsilon_y \); \( \sigma_y = (\beta_y \epsilon_y + (\eta_y \Delta E / E_0)^2)^{1/2} \approx |\eta_y \Delta E / E_0| \), where \( \beta_y = \sigma_y^2 / \epsilon_y \) is the beam beta function in the \( y \)-direction, and \( \Delta E \) is the bunch energy content. It also removes the ambiguity in the energy gain signal that could result from the possible large transverse momentum that the strong focusing force of the plasma can impart to beam tail particles entering the plasma offset from the beam axis. The spectra measured before the plasma are used to identify similar incoming bunches and to compare the PWFA interaction with plasmas of various lengths and densities. The effect of transverse wakefield on the bunch size (\( \sigma_x \)) can be measured in the perpendicular \( x \)-plane.

The relative bunch length is monitored on a bunch-to-bunch basis by recording the coherent transition radiation (CTR) energy emitted by individual bunches when entering a 1 \( \mu\text{m} \)-thick titanium foil located before the plasma (not shown in figure 1). For a fixed bunch longitudinal profile this energy is inversely proportional to the bunch length (\( \propto 1 / \sigma_z \) for a Gaussian profile). However, the compression process used in these experiments does not preserve the bunch profile as the parameters are varied. Nevertheless, measurement of peak energy loss by beam electrons versus CTR energy (with short, low-density plasmas) shows a very strong correlation with the energy loss, which is proportional to CTR energy. This relative bunch length diagnostic proved very useful to systematically acquire the energy gain data presented hereafter.

3. Acceleration results

Figure 2(a) shows three measured Li vapor density profiles with lengths \( L_p = 13, 22 \) and \( 31 \text{ cm} \), and a peak density \( n_e = 2.6 \times 10^{17} \text{ cm}^{-3} \). Note that even though in the shortest Li column length \( (L = 13 \text{ cm}) \) the density is at its peak only over a very short distance (unlike the two
longer lengths for which the density is constant over a length longer than the rise of the density at the column ends), the measured energy gains are consistent with the profiles FWHM (see below). Figure 2(b) shows bunch energy spectra measured downstream from the plasma location for the case of no plasma (bI), and three increasing plasma lengths: (bII), (bIII) and (bIV), respectively. These four events were chosen because they have very similar incoming energy spectra and CTR energies, and therefore bunch lengths and current distributions. This plasma density gave the largest energy gain at all plasma lengths and is higher than that satisfying the linear criteria for maximum accelerating field: \( \lambda_{pe} \approx 4\sigma_z \approx 20 \) [20]. Figure 2(bI) shows the spectrum of the incoming bunch that has a full width of \( \Delta E \approx 0.5\% \) around the \( E_0 = 28.5 \) GeV mean incoming energy. Figure 2(bII) shows that after propagating through a plasma with a FWHM length of \( \approx 13 \) cm, a large fraction of the bunch charge has lost energy. The energy loss is the sum of the energy lost to the plasma wake excitation and to synchrotron radiation resulting from the betatron motion of the beam electrons along the plasma ion column [32]. Figure 2(bII) also shows that the energy of a significant fraction of the bunch charge is not affected by the plasma. The bunch current profile retrieved from the incoming bunch energy spectrum shows that the beam enters the plasma with a trunk that corresponds to a low beam current in the head of the bunch. This trunk only weakly contributes to the ionization and wake excitation processes [2] and is therefore not affected by the plasma. Because of the energy chirp, the trunk is at the high energy end of the incoming bunch spectrum. Most importantly, figure 2(bII) shows that some of the beam charge has gained an energy of 4 GeV after only 13 cm of plasma. Figure 2(bIII) displays similar features after 22 cm of plasma, with particle energy up to 8 GeV. With the longest plasma, \( L_p = 31 \) cm (figure 2(bIV)), the peak energy gain reaches \( \approx 14 \) GeV, or about half the energy of the incoming beam. The accelerating gradient inferred from the peak energy gain is \( \approx 45 \) GeV m\(^{-1}\) and is sustained over 31 cm. Note also that the images in figures 2(bII)–(bIV) show no sign of hosing instability [33]. This beam/plasma instability would manifest itself by wiggles along the energy spectrum (visible in the x-plane), could potentially limit the achievable energy gain and possibly destroy the drive bunch [34]. While it is clear that the measured energy gain increases with plasma length, the peak energy loss appears not to increase in figures 2(bII)–(bIV). Calculations of the transport of the large energy spread bunch along the beam line between the plasma and the energy diagnostic indicate that particles with energies smaller than about 24 GeV intercept the beam line walls and the protection collimators before reaching the energy diagnostic. This is due to the energy dependence of the focusing/defocusing strength of the quadrupole: \( \kappa = q B_0 / \gamma m_e c a \), where \( B_0 \) is the magnet pole tip field, \( a \) is the pole tip distance and \( \gamma \), \( q \) and \( m_e \) are the particle relativistic factor, charge and mass. However, the number of counts per pixel in the energy loss region decreases from figures 2(bII)–(bIV) showing that the number of particles per unit energy decreases (i.e. the electrons are spread over a larger energy range), which is an indication that, as expected, the energy loss also increases with the plasma length. The spectra with the plasma illustrate an important characteristic of single-bunch PWFA experiments: the energy spectra (after the plasma) are very broad. It is important to realize that this is not due to any physical limitation of the PWFA, but merely a consequence of the external injection of electrons at all phases of one cycle of the wakefield field, as mentioned earlier. This will be overcome by injecting distinct drive and witness bunches.

Figure 3 shows the result of the systematic measurement of peak energy gain (from images similar to those of figure 2(b)) for the three plasma densities and lengths mentioned earlier and plotted as a function of the CTR energy corresponding to each bunch. The CTR energy is varied.
Figure 3. Maximum energy of the particles measured at three plasma densities ($n_e = 1, 2.6$ and $3 \times 10^{17}$ cm$^{-3}$) and each at three plasma lengths ($L_p = 13$ (red symbols), 22 (green symbols) and 31 (blue symbols) cm) (see figure 2(a)) as the bunch compression, and therefore the CTR energy, is varied. Higher CTR energy corresponds to higher peak current, shorter bunches. The events are binned by CTR energy, and the error bars represent the rms energy measurement variation in each bin.

by changing the parameters that affect the bunch compression, in this case the phase at which the bunch rides the rf wave in the linac (i.e. the chirp for the second compression stage). For a fixed current profile (e.g. Gaussian) and charge, the CTR energy increases proportionally to the bunch peak current (CTR $\sim I_{\text{peak}}$) or inversely proportionally to the bunch length (CTR $\sim 1/\sigma_z$). Simulations of the compression process [2] show that over the range covered here ($\sigma_z \sim 50–15$ µm rms width), the current profile is not exactly preserved. In particular, at the highest compression (i.e. the shortest bunch with the highest CTR energy) charge is transferred from the bunch high-current core to the head of the bunch where it forms a low-current (<2 kA) trunk that only very weakly contributes either to the CTR energy or to the wake excitation. However, even in this situation, the general scaling is preserved and the peak current increases and the bunch length decreases with increasing CTR energy. Each data set for one length and one density consists of 200–800 events. These events were binned by CTR energy. The error bars represent the standard deviation of the peak energy measurement for the events in each bin. At all three densities of figure 3 the energy gain increases with plasma length, as expected. The energy gain is modest at the lowest plasma density ($1.0 \times 10^{17}$ cm$^{-3}$), peaks at $2.6 \times 10^{17}$ cm$^{-3}$ and is lower at the highest density ($3.0 \times 10^{17}$ cm$^{-3}$). Note that non-systematic data were taken around $n_e = 2.6 \times 10^{17}$ cm$^{-3}$ to verify that it is indeed the optimum density. Qualitatively, at the lowest plasma density, the wakefield wavelength ($\lambda_{\text{pe}} = 2\pi c/\omega_{\text{pe}} = 106$ µm) is too long for the bunch lengths of the experiments and therefore the wakefields are small. In addition, there are too few trailing electrons at the peak accelerating field location to be measured ($\sigma_z < \lambda_{\text{pe}}$). These two factors contribute to the low-energy gains of figure 3(a). At $n_e = 2.6 \times 10^{17}$ cm$^{-3}$, the plasma wavelength and bunch length combination leads to a maximum energy gain. This determines experimentally the best match between the bunch length (available in these experiments) and the plasma wavelength or density. For shorter bunches (higher CTR energy) the peak wakefield should monotonically increase. However, it is likely that there are again too few to be measured
Figure 4. Average maximum energy gain obtained from the systematic measurement of energy gain as a function of bunch length for various plasma lengths and densities (see figure 3). The average accelerating gradient for each plasma density is given by the slope of the linear fits of the energy gain as a function of plasma length: 8.5 GeV m$^{-1}$ at $n_e = 1.0 \times 10^{17}$ cm$^{-3}$ (red points and line), 36.1 GeV m$^{-1}$ at $n_e = 2.6 \times 10^{17}$ cm$^{-3}$ (green points and line), and 27 GeV m$^{-1}$ at $n_e = 3.0 \times 10^{17}$ cm$^{-3}$ (blue points and line). The error bars are the same as in figure 3.

bunch particles experiencing these large fields. At the higher density the bunches are too long or the plasma wavelength ($\lambda_{pe} = 63 \mu$m $> \sigma_z$) is too short for the beam to efficiently drive the wakefield.

Even though in practice the bunches can be selected to have similar incoming energy spectra and CTR energy, and therefore similar longitudinal phase space, their transverse phase space can vary independently. The electrons are not always aligned on the beam axis and the bunches are produced with various tilts along their length. Once entering the plasma this transverse displacement of the particles in the back of the bunch with respect to its head causes transverse oscillation of the accelerated particles in the wake fields [35]. When averaged over a population of events, this results in a lower-energy gain than in the ideal beam case (figure 2(b)). This effect contributes to the error bars of figure 3.

Figure 4 shows the average measured energy gain as a function of the plasma length for three plasma densities and for the beam parameters that gave the largest energy gain in figure 3 (CTR energy $\sim 250$ au). As expected, the average energy gain increases linearly with the plasma length in the three density cases. The slope of the linear fits to the energy gain points gives the average accelerating gradient driven by the bunch at each plasma density. The largest gradient is 36 GeV m$^{-1}$ at $n_e = 2.6 \times 10^{17}$ cm$^{-3}$. This gradient is the equivalent of the unloaded gradient in an rf accelerating cavity since the number of accelerated particles is much smaller than the number of particles that drive the wake. The accelerated electrons are at the back of the bunch profile and their charge does not exceed 150 pC out of the total $\approx 3$ nC charge; thus no beam loading effect is expected. The variations in energy gain observed in the experiment and indicated by the error bars in figures 3 and 4 are attributed mostly to variations in the transverse distribution of the bunch charge at the plasma entrance. The small size of these error bars...
demonstrates that the PWFA process is stable and reproducible. This is the direct consequence of the generation of the plasma by the bunch itself through field ionization. The plasma density is identical for all electron bunches interacting with the Li vapor column over a time scale smaller that the thermal time constant of the heat-pipe oven (many minutes). This time constant and stability could be further enhanced by standard temperature regulation systems. Better control of the beam parameters will lead to smaller variations in energy gain and can easily be achieved with standard feedback systems.

Note that experiments with 90 and 120 cm long plasmas and a 42 GeV beam showed that the increase in energy with plasma length stopped at \( \approx 85 \text{ cm} \) [3]. This was attributed to the erosion of the bunch head that propagates in the neutral lithium vapor, is therefore not focused by the plasma and diverges according to its emittance. However, this length limit can be extended by using a lower emittance beam. In these proof-of-principle experiments, the beam emittance was increased by scattering in metallic foils that will not be present in a future PWFA-based collider. Another method to extend this limit is to use a pre-formed low-density plasma to guide the bunch head further than in the field-ionized plasma.

4. Numerical simulations

Numerical simulations provide a very important tool to explore the parameters considered for future experiments and to help understand and corroborate the latest experimental results. The PWFA interaction is simulated numerically using the 3D particle-in-cell code QuickPIC [36]. QuickPIC uses a reduced algorithm and a simulation window moving with the electron beam to efficiently reduce the computation time when compared to traditional full-scale PIC simulation codes. The input parameters for the electron bunch are similar to those of the experiment. The code also uses the measured longitudinal neutral density profiles of the experiment as shown in figure 2(a). The plasma formation is described using the ADK field-ionization model [28] for Li. The presence of the He buffer gas in the regions where the Li density ramps up to its constant value is not included in these simulations. Ionization trapping of He electrons may occur in these regions [37]. The size of the moving simulation box is \( 300 \times 300 \times 300 \mu\text{m}^3 \). The number of grid points is \( 256 \times 256 \times 512 \), i.e. more cells along the \( z \)-propagation axis. The quasi-static time step is \( \approx \frac{44}{\omega_p} \). The beam is described by \( 8.4 \times 10^6 \), and each plasma slice by \( 2.6 \times 10^5 \) simulation particles. The energy gain as a function of the propagation distance into the plasma is shown in figure 5 for the three longitudinal experimental profiles of figure 2(a). The peak energy gain is evaluated by recording the value of the highest energy simulation macroparticle. As expected, the energy gain is significant only in the regions of constant plasma density. After leaving the plasma, the particles energy remains constant and can be compared to the values measured in the experiment. The final energy gains in the simulation are 8, 15 and 18 GeV for the three plasma lengths and \( n_e = 2.6 \times 10^{17} \text{ cm}^{-3} \). These are larger than the largest energy gain obtained at each length in the experiment (figure 2(b)). The average gradient obtained from these simulation results is about 61 MeV m\(^{-1}\) and is also larger than that measured experimentally (\( \approx 36 \text{ GeV m}^{-1} \)). Note that the simulations do not include energy loss from the emission of synchrotron radiation by the electrons due to their betatron oscillations along the plasma. This energy loss affects all electrons and scales as \( \gamma^2 \langle r_0^2 \rangle \), where \( r_0 \) is the oscillation amplitude of the individual electrons. The energy loss to radiation can be a few GeV m\(^{-1}\) (see [32]). Another effect not described by the reduced model is the trapping of plasma electrons by the large-amplitude wake. Trapped electrons have been observed in these
Figure 5. Highest energy of the simulation particles (empty symbols, right-hand side axis) along the experimental plasma density profiles (filled symbols, left-hand side axis). The final energy gains are 8 GeV (blue symbols, 13 cm plasma), 15 GeV (green symbols, 22 cm plasma) and 18 GeV (red symbols, 31 cm plasma) and $n_e = 2.6 \times 10^{17} \text{ cm}^{-3}$.

experiments with energies comparable to the energy gain by bunch particles. However, at this point it is not possible to quantify experimentally their reduction of the energy gain of the wake through possible loading of the accelerating wake field. Preliminary simulation results indicate that the energy gain reduction is relatively low, of the order of 10–15% of the corresponding trapped particle free case. Also, in the simulations the bunch is perfectly symmetric around the beam axis.

5. Conclusions

The results presented in this paper show that the energy gain by ultra-relativistic electrons scales linearly with the plasma length. The largest average accelerating gradient was measured at a plasma density of $2.6 \times 10^{17} \text{ cm}^{-3}$, reached 36 GeV $\text{m}^{-1}$ and was sustained over a 31 cm plasma length. The measured energy gain and gradient is comparable to that obtained in numerical simulations. Demonstration of large energy gain and scaling with plasma length are two important steps toward the realization of a plasma-based electron/positron collider. These results were instrumental in the design of the experiment that demonstrated an energy gain of 42 GeV over a plasma length of only 85 cm [3]. The next experimental steps include the demonstration of the acceleration of a witness bunch with a narrow energy spread and the acceleration of positrons in a multi-GeV $\text{m}^{-1}$ PWFA module. These issues will be addressed at the FACET facility currently under construction at the SLAC National Accelerator Laboratory [23].

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