\textit{B → φπ} and \textit{B^0 → φφ} in the Standard Model and new bounds on R parity violation

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We study the pure penguin decays \( B \rightarrow \phi \pi \) and \( B^0 \rightarrow \phi \phi \). Using QCD factorization, we find \( B(B^+ \rightarrow \phi \pi^+) = 2.0^{+0.3}_{-0.1} \times 10^{-5} \). For the pure penguin annihilation process \( B^0 \rightarrow \phi \phi \), analyzed here for the first time, \( B(B^0 \rightarrow \phi \phi) = 2.1^{+1.6}_{-0.3} \times 10^{-9} \). The smallness of these decays in the Standard Model makes them sensitive probes for new physics. From the upper limit of \( B \rightarrow \phi \pi \), we find constraints on R parity violating couplings, \( |\lambda''_{123}\lambda''_{i21}| < 6 \times 10^{-5} \). Our new bounds on \( |\lambda''_{123}\lambda''_{i21}| \) are one order of magnitude stronger than before. Within the available upper bounds for \( |\lambda''_{123}\lambda''_{i21}| \), we find that \( B(B \rightarrow \phi \phi) \) could be enhanced to \( 10^{-8} \sim 10^{-7} \). Experimental searches for these decays are strongly urged.

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Charmless two-body nonleptonic decays of \( B \) mesons provide tests for the Standard Model (SM) at both tree and loop levels. They also test hadronic physics and probe possible flavor physics beyond the SM. In past years, we have witnessed considerable progress in studies of these decays. Many such processes have been measured or upper-limited \footnote{1}. Theoretically, QCD factorization in which non-factorizable effects are calculable was presented \footnote{2}, while there is also progress in perturbative QCD approaches \footnote{3}.

In this letter, we will use QCD factorization to study \( B^{0,\pm} \rightarrow \phi \pi^{0,\pm} \) and \( B^0 \rightarrow \phi \phi \). In \( \phi \pi^0 \), dominated by electroweak penguins, were studied by employing naive factorization. By naive factorization, it means that the hadronic matrix elements of the relevant four-quark operators are factorized into the product of hadronic matrix elements of two quark currents that are described by form factors and decay constants. In contrast to color allowed processes, where naive factorization works reasonably well, this assumption is questionable for penguin processes. QCD factorization \footnote{4} can be used to calculate the non-factorizable diagrams. We will use this framework to improve the theoretical predictions for \( B^{0,\pm} \rightarrow \phi \pi^{0,\pm} \). It is interesting to note that \( B^{0,\pm} \rightarrow \phi \pi^{0,\pm} \) do not receive annihilation contribution, while \( B^0 \rightarrow \phi \phi \) is a pure penguin annihilation process.

To the best of our knowledge, there is no realistic theoretical study of \( B^0 \rightarrow \phi \phi \). With respect to the topology of non-factorizable penguin diagrams for charmless \( B \) decays, the decay \( B^0 \rightarrow \phi \phi \) is of interest. It can give us insight into the strength of the annihilation topology in nonleptonic charmless \( B \) decays which is still in dispute \footnote{5}, \footnote{6}. Experimentally, \( B^0 \rightarrow \phi \phi \) is relatively easy to identify. We find \( B(B^0 \rightarrow \phi \phi) = 2.1^{+1.6}_{-0.3} \times 10^{-9} \) and \( B(B^\pm \rightarrow \phi \pi^{\pm}) = 2.0^{+0.3}_{-0.1} \times 10^{-8} \) in the SM. The smallness of the SM predictions for these decays makes them sensitive probes for flavor physics beyond the SM. We use the recent Babar upper limits \( B(B^{0,\pm} \rightarrow \phi \pi^{0,\pm}) < 1.6 \times 10^{-6} \), at 90\% CL \footnote{7}, to obtain limits on the relevant R Parity Violating (RPV) couplings. We then use these limits to deduce the maximal possible enhancement of \( B(B^0 \rightarrow \phi \phi) \) in RPV supersymmetry. Note that currently \( B(B^0 \rightarrow \phi \phi) < 1.2 \times 10^{-5} \) at 90\% CL \footnote{8}.

The experimental upper limits could be improved in BaBar and Belle. Measurement of any of these decays with \( B \lesssim 10^{-7} \) will serve as an evidence for new physics.

In the SM, the relevant QCD corrected Hamiltonian is

\begin{equation}
H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{td} \sum_{i=3}^{10} C_i O_i.
\end{equation}

The operators in \( H_{\text{eff}} \) relevant for \( b \rightarrow d s \bar{s} \) are given in \footnote{9}, where at the scale \( \mu = m_b \), \( C_3 = 0.144 \), \( C_4 = -0.035 \), \( C_5 = 0.009 \), \( C_6 = -0.041 \), \( C_7 = -0.002/137 \), \( C_8 = 0.054/137 \), \( C_9 = -0.292/137 \), \( C_{10} = 0.262/137 \). Using the effective Hamiltonian and naive factorization

\begin{equation}
A(B^- \rightarrow \phi \pi^-) = \frac{G_F}{\sqrt{2}} V_{tb} V_{td} \left[ (a_3 + a_5) - \frac{1}{2} (a_7 + a_9) \right] \times f_\phi m_\phi F_{\pi^{-}}^B (m_\phi^2)\epsilon^\phi_\pi \cdot (p_B + p_\pi),
\end{equation}

and \( A(B^0 \rightarrow \phi \pi^0) = \frac{1}{\sqrt{2}} A(B^- \rightarrow \phi \pi^-) \) with \( a_i \equiv C_i + C_{i+1} / N_c \). The contributions of strong penguin operators arising from the evolution from \( \mu = M_W \) to \( \mu = m_b \) is very small due to the cancellations between them: \( C_3 (m_b) \approx -C_4 (m_b) / 3 \) and \( C_5 (m_b) \approx -C_6 (m_b) / 3 \). Obviously the amplitude is dominated by electroweak penguin. Using \( f_\phi = 254 \text{ MeV} \), \( |V_{td}| = 0.008 \), \( N_c = 3 \), and the form factor \( F_{\pi^{-}}^B (0) = 0.28 \) \footnote{10}, \footnote{11}, \footnote{12}, we get \( B(B^\pm \rightarrow \phi \pi^{\pm}) = 2B(B^0 \rightarrow \phi \pi^0) = 2.9 \times 10^{-9} \).

In the above calculations, non-factorizable contributions are neglected. However, this neglect is questionable for penguin dominated \( B \rightarrow \phi \pi \). The leading non-factorizable diagrams in Fig.1 should be taken into account. To this end, we employ the QCD factorization framework \footnote{13}, which incorporates important theoretical aspects of QCD like color transparency, heavy
quark limit and hard-scattering, and allows us to calculate non-factorizable contributions systematically. In this framework, non-factorizable contributions to $B^+ \to \pi^- \phi$ can be obtained by calculating the diagrams in Fig. 1. To leading twist and leading power, the amplitude for Fig. 1: Non-factorizable diagrams for $B^- \to \phi \pi^-$. $B^- \to \pi \phi$ is

$$A'(B^- \to \phi \pi^-) = -\frac{G_F}{\sqrt{2}} V_{tb} V_{td}^* f_{\phi} m_\phi F_0^{B+\pi}(m_\phi^2)2\xi_L \cdot p_B$$

$$\times \left\{ \left[(a_3 + a_5) - \frac{1}{2}(a_7 + a_9) \right] + \alpha_s(\mu) \frac{C_F}{4\pi} N_c \right. $$

$$\times \left[ (C_4 - \frac{1}{2} C_{10}) \Phi_{\phi} + (C_6 - \frac{1}{2} C_8)(-F_0 - 12) \right] \right\} \right\}, \quad (3)$$

and $A'(B^0 \to \phi \pi^0) = \frac{1}{\sqrt{2}} A'(B^- \to \phi \pi^-)$. The $\alpha_s$ term is the non-factorizable contribution with

$F_\phi = -12 \ln \frac{\mu}{m_B} - 18 + V + S$, \quad (4)

$$V = \int_0^1 du \Phi_\phi(u) \left( \frac{3}{2} - \frac{2u}{1 - u} \ln u - 3i\pi \right), \quad (5)$$

$$S = \frac{4\pi^2}{N_c} \frac{f_{\pi} f_B}{F_0^{B+\pi}(0)} \int_0^1 d\xi dudv \frac{\Phi_{\phi}(\xi) \Phi_{\phi}(u) \Phi_{\phi}(v)}{\xi} \frac{u}{v}, \quad (6)$$

where $\xi = l_+ / M_B$ is the momentum fraction carried by the spectator quark in the $B$ meson. The $\Phi$'s are the leading twist light-cone distribution amplitudes of $\pi$, $\phi$ and $B$ mesons. They describe the long-distance QCD dynamics of the matrix elements of quarks and mesons, which is factorized out from the perturbative short-distance interactions in the hard scattering. These distribution amplitudes can be found in $[14, 13]$. In our calculation, we use the model proposed in $[14]$

$$\Phi_\phi^B(l_+) = \sqrt{\frac{2}{\pi\lambda^2 \lambda^2}} \exp \left[ -\frac{l_+^2}{2\lambda^2} \right], \quad (7)$$

$$\Phi_{\phi\pi}^B(l_+) = \sqrt{\frac{2}{\pi\lambda^2 \lambda^2}} \exp \left[ -\frac{l_+^2}{2\lambda^2} \right], \quad (8)$$

where $\lambda$ is the momentum scale of the light degrees of freedom in the $B$ and taken to 350 MeV. To show model dependence of our prediction, we vary $\lambda$ from 150 MeV to 550 MeV, we get $B(B^+ \to \phi \pi^\pm) = 2B(B^0 \to \phi \pi^0) = 2.0^{+0.3}_{-0.1} \times 10^{-8}$. From Eq. (3) we see that non-factorization is dominated by strong penguin due to the absence of $C_9$. We also note that non-factorizable contributions dominate these decays and there is no isospin symmetry breaking because annihilation contributions are absent.

The decay $\bar{B}^0 \to \phi \phi$ is also of interest to study. Firstly it is a pure penguin process. Secondly it is a pure annihilation and thirdly its experimental signature is very clean. By naive factorization, the amplitude for this decay mode is

$$A(B^0 \to \phi\phi) = -\frac{G_F}{\sqrt{2}} V_{tb} V_{td}^* \left[ \left( a_3 - \frac{1}{2} a_9 \right) \right.$$

$$\times \langle \phi\phi | \bar{s} \gamma_\mu L s | 0 \rangle \langle 0 | d \bar{y}^\mu L b | B^0 \rangle$$

$$\left. + \left( a_5 - \frac{1}{2} a_7 \right) \langle \phi\phi | \bar{s} \gamma_\mu L s | 0 \rangle \langle 0 | d \bar{y}^\mu L b | B^0 \rangle \right]$$

$$= -i \frac{G_F}{\sqrt{2}} V_{tb} V_{td}^* \frac{M_B}{F_0^{B+\pi}(0)} \left[ \left( a_3 - \frac{1}{2} a_9 \right) \langle \phi\phi | \bar{s} \gamma_\mu L s | 0 \rangle$$

$$\left. + \left( a_5 - \frac{1}{2} a_7 \right) \langle \phi\phi | \bar{s} \gamma_\mu L s | 0 \rangle \right] \right\}, \quad (9)$$

This amplitude vanishes for $m_s \to 0$. The $\alpha_s$ order matrix $\langle \phi\phi | \bar{s} \gamma_\mu (1 - \gamma_5) s | 0 \rangle$ also vanishes due to the cancelation between the amplitudes of $F \phi\pi^\pm (c)$ and (d). Non-factorizable contributions can be obtained by calculating the amplitudes of Fig. 2.(a) and Fig. 2.(b). They are
FIG. 2: (a) and (b) are non-factorizable diagrams for $B^0 \to \phi\phi$ decays. (c) and (d) are factorizable diagrams at $\alpha_s$ order.

virtually of gluon, $b$ and $d$ quark propagators, respectively. As in [16], we meet end point divergence when $t_+ = 0$. Instead of a cut-off treatment [17], we use an effective gluon propagator [18]

$$\frac{1}{k^2} \Rightarrow \frac{1}{k^2 + M_g^2(k^2)}, \quad M_g^2(k^2) = m_g^2 \left[ \frac{\ln \left( \frac{k^2 + 4m_s^2}{\Lambda^2} \right)}{\ln \left( \frac{200}{\Lambda^2} \right)} \right].$$

(11)

Typically $m_g = 500 \pm 200$ MeV, $\Lambda = \Lambda_{QCD} = 300$ MeV. Our use of this gluon propagator instead of imposing a cut-off, is supported by lattice [18] and field theoretical solutions [19] which indicate that the gluon propagator cut-off, is supported by lattice [18], and field theoretical virtualities of gluons, respectively. As in [16], we meet end point divergence when $t_+ = 0$. Instead of a cut-off treatment [16], we use an effective gluon propagator [17].

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(11)

Potentially $m_g$ may be enhanced by rescattering $B \to \eta'(\eta') \to \phi\phi$ or by $\omega - \phi$ through the channel $B^0 \to \omega \phi \to \phi\phi$. However, $\eta'$ and $\eta$ contributions are almost completely canceled [20], and $\phi$ is nearly a pure $\bar{s}s$ state, so the mixing mechanism is also negligible. Furthermore in the language of QCD factorization framework, such kinds of soft final state interactions are leading and suppressed by power of $O(\Lambda_{QCD}/m_b)$ [21], although it is hard to be calculated reliably. Lastly, strong interaction annihilation is negligible since at least two gluons should be exchanged. Thus, any unexpected large branching ratio observed, will indicate new physics.

As an example for new physics, we will discuss the effects of the trilinear $\lambda'$ and $\lambda''$ terms in the RPV superpotential $W_{\phi}$ [22, 23, 24] on the process $b \to d\bar{s}s$. We are therefore interested in

$$W_{\phi} = \varepsilon^{abc} \delta^{\alpha\beta} \lambda'_{ijb} L_{ia} q_{jba} D_{\bar{k}\beta} + \frac{1}{2} \varepsilon^{\alpha\beta\gamma} \lambda''_{[ijk]} U_{\alpha} D_{\beta} D_{\gamma},$$

(12)

where $a, b$ are $SU(2)$ indices, $i, j, k$ are generation indices, $\alpha, \beta, \gamma$ are $SU(3)$ color indices and $c$ denotes charge conjugation. The $L$ ($Q$) are the lepton (quark) $SU(2)$ doublet superfields, and $U$ ($D$) are the up- (down-) quark $SU(2)$ singlet superfields. Then we have

$$\mathcal{L}_{\phi} = \frac{1}{2} \varepsilon^{\alpha\beta\gamma} \left[ \lambda''_{ijb} \bar{u}_{R\alpha} \left( \bar{d} \gamma^\mu Rb \gamma - \{ j \leftrightarrow k \} \right) \right] + \lambda'_{ijb} \bar{\nu}_{L\alpha} \bar{d} k \lambda^\mu + h.c.$$ 

(13)

From $\mathcal{L}_{\phi}$, we get the effective Hamiltonian for $b \to d\bar{s}s$

$$\mathcal{H}_{\phi} = -\frac{2 m_{\nu}}{m_{d_{ij}}} \eta^{-4/30} \lambda''_{ijb} \lambda''_{ijc} \left[ (\bar{s}_\alpha \gamma^\mu R s_\beta) (\bar{d}_\beta \gamma^\mu R b_\gamma) - (\bar{s}_\beta \gamma^\mu R s_\alpha) (\bar{d}_\alpha \gamma^\mu R b_\gamma) \right]$$

$$-\frac{1}{2 m_{\nu}} \eta^{-8/30} \left[ \lambda'_{ijb} \lambda'_{ijc} (\bar{s}_\alpha \gamma^\mu R b_\gamma) (\bar{d}_\beta \gamma^\mu R s_\alpha) + \lambda'_{ijb} \lambda'_{ijc} (\bar{d}_\alpha \gamma^\mu R b_\gamma) (\bar{s}_\beta \gamma^\mu R s_\alpha) + \lambda'_{ijb} \lambda'_{ijc} (\bar{d}_\gamma \gamma^\mu R b_\gamma) (\bar{s}_\alpha \gamma^\mu R s_\alpha) \right].$$

(14)

where $\eta = \frac{\alpha_s(m_{\nu})}{\alpha_s(m_b)}$ and $\beta_0 = 11 - \frac{2}{3} n_f$. The coefficients $\eta^{-4/30}$ and $\eta^{-8/30}$ are due to running from the sfermion mass scale $m_{\nu}$ (100 GeV assumed) down to the $m_b$ scale.

We can now write down the contributions of $\mathcal{H}_{\phi}$ to $B^+ \to \phi\pi^-$ and $B^0 \to \phi\phi$ decays,
In the numerical results, we assume that only one sfermion contributes at a time and that they all have a mass of 100 GeV. The uncertainties of the theoretical predictions, due mainly to the B meson distribution function, are displayed as thickness of curves in Fig.3

Our results for the RPV contributions to $B \to \phi \pi$ are summarized in Fig.3. From the BaBar upper limit $[6] B(B^{\pm} \to \phi \pi^{\pm}) < 1.6 \times 10^{-6}$, we obtain the following constraints (90%CL)

$$\begin{align*}
|\lambda''_{23} \lambda'_{21}| &< 6 \times 10^{-5} \left( \frac{m_{\text{DR}}}{100} \right)^2 , \quad (17) \\
|\lambda'_{32} \lambda'_{21}| &< 4 \times 10^{-4} \left( \frac{m_{\text{DR}}}{100} \right)^2 , \quad (18) \\
|\lambda'_{21} \lambda'_{23}| &< 4 \times 10^{-4} \left( \frac{m_{\text{DR}}}{100} \right)^2 . \quad (19)
\end{align*}$$

We note that our constraints on $\lambda''_{23} \lambda'_{21}$ are more than one order of magnitude stronger than the limits obtained recently $[24]$. For $\lambda'_{32} \lambda'_{21}$ and $\lambda'_{21} \lambda'_{23}$, our bounds are comparable with the present upper limits $[24], 25]$. Within the available upper bounds for these couplings, $\lambda''_{23} \lambda'_{21}$, RPV terms could enhance $B(B^0 \to \phi \phi)$ to $10^{-8}$, while $\lambda'_{21} \lambda'_{23}$ and $\lambda'_{32} \lambda'_{21}$ RPV terms could enhance $B(B^0 \to \phi \phi)$ to $10^{-7}$ which may be measurable at Belle and Babar.

In summary, we have studied the pure penguin processes $B^{\pm,0} \to \phi \pi^{\pm,0}$ and $B^0 \to \phi \phi$ by using QCD factorization for the hadronic dynamics. We estimate that in the SM $B(B^- \to \phi \pi^-) = 2.0^{+0.3}_{-0.1} \times 10^{-8}$ and $B(B^0 \to \phi \phi) = 2.1^{+1.6}_{-0.3} \times 10^{-9}$. The smallness of these decays in the SM makes them sensitive probes of flavor physics beyond the SM. Using the BaBar result $B(B^- \to \phi \pi^-) < 1.6 \times 10^{-6}$, we have obtained new bounds on some products of RPV coupling constants. In the case of $\lambda''_{23} \lambda'_{21}$, our limits are better than previous bounds. Given the available bounds on $\lambda''_{23} \lambda'_{21}$, $\lambda'_{23} \lambda'_{21}$ and $\lambda'_{32} \lambda'_{21}$, the decay $B^0 \to \phi \phi$ could be enhanced to $10^{-8} \sim 10^{-7}$. Due to the clear signatures of $\phi$ and $\pi^\pm$, the experimental sensitivity of these decay modes is high. Babar and Belle could reach very low upper limits on these decays if not measured. Searches for these decays are strongly urged.

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FIG. 3: The branching ratio of $B^- \to \phi \pi^-$ as a function of the RPV couplings $|\lambda_{23}^{''} \lambda_{12}^{''*}|$ (upper curve), $|\lambda_{23}^{'} \lambda_{21}|$ and $|\lambda_{12}^{'} \lambda_{12}^{''}|$ (lower curve) respectively. The thickness of curves represent our theoretical uncertainties. The horizontal lines are the upper limits and the SM prediction as labeled respectively. The thicknesses of the curves and the line labelled as SM are theoretical uncertainties.