Applying the Concepts of “Community” and “Social Interaction” from Vygotsky’s Sociocultural Theory of Cognitive Development in Math Teaching to Develop Learner’s Math Communication Competencies

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ABSTRACT
As an integral skill in Math learning at high schools, Math communication competencies are formed and developed throughout the process of Math learning in the classroom environment through student-teacher, student-student as well as student-learning material and instrument interactions. Vygotsky’s Sociocultural Theory of Cognitive Development acts as the foundation and guideline for the teaching and learning process at school, emphasizing the role of social interaction in cognitive development. The study presents the two concepts of ‘community’ and ‘social interactions’ from the theory by Vygotsky for application in Math teaching. The article starts with clarifying the two concepts above in the context of Math classrooms, then proposes several sample activities of teaching Algebra in high schools with the application of the two concepts to develop learners’ Math communication competence.

1. INTRODUCTION
Lev Vygotsky is considered the father of sociocultural theories. His theory on human development highlights the influence of an individual’s social and cultural environment on their personal development (de Abreu, 2000). It is believed that Vygotsky’s psychology is an application of Marx’s theories into learning, providing a framework whereby the sociocultural roots of thought become internalised in the individual (Lerman, 2001). Thus, he has been widely acknowledged as the founder of sociocultural theories, researching the relationships between learning and development, raising issues in general education as well as education for children in the 21st century (Mahn, 1999). The theory is applicable throughout one’s lifetime and across different cultures. Specifically, children’s social world is guided by language which they use to interpret and experience their own world. Eventually, the language they hear (together with other cultural tools, sign systems and practice) become internalised and help them to control their behaviour. When children grow up, they use language to convey cultural values and language acts as a tool to educate cultural standards for children. Vygotsky believes that development and education are social (Tavassolie & Winsler, 2018). In other words, people’s cognitive development can be considered a consequence of their communication with others (Alkhudiry, 2022).

There are a number of studies exploiting Lev Vygotsky’s theory or applying his theory in Math teaching. Forman (2013) encouraged his students to actively participate in Math practice through group work and collaboration to identify, and arrange data in open Math questions which accept multiple possible solutions; and justify their ideas for their partners. Thus, it is important to improve students’ competencies to communicate and collaborate with others (Forman, 2013).

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In Vietnam, the Math subject at high schools aims to promote learners’ reasonable thinking, presenting (in spoken and written forms) the identification and solutions to simple and familiar problems in life. The ultimate objective of Math education at school is the formation and development of Math competencies, including these component competencies: Math thinking and reasoning competency, Math modelling competency, Math problem-solving competency, Math communication competency, competency of using Math learning instruments (Ministry of Education and Training, 2018). These component competencies are inextricably connected. Particularly, Math communication competency plays a critical role in deciding the quality of Math teaching and learning as well as developing learners’ Math competencies. In this study, the core arguments of the constructivist theory, the role of society in learners’ cognitive process and the application of Vygotsky’s Sociocultural Theory of Cognitive Development are highlighted to propose suggestions for Math teaching at high schools in Vietnam and to further clarify the interaction in Math classrooms based on preliminary findings by Luong et al. (2021). Specifically, the article analyses the interactive environment in Math classrooms, Math communicative activities based on Vygotsky’s two concepts of ‘community’ and ‘social interactions’.

2. LITERATURE REVIEW

Math communication is the process of expressing Math ideas and knowledge by using figures, images and words to various subjects including teachers, colleagues, groups or classes (Ontario Ministry of Education, 2005). In other words, Math communication is the process of expressing Math ideas and knowledge verbally, or visually with text, figures, symbols, images, graphs, diagrams, and words.

Math communication is a necessary skill in Mathematics, which allows one to coherently express his mathematical ideas to friends, teachers, and others through spoken and written language. Math communication skills can involve the following sub-skills: (1) connecting real objects, models and diagrams into Math ideas; (2) explaining Math ideas, situations and relationships orally or with written texts with real objects, images, graphs and algebra; (3) using Math terms and symbols; (4) listening to, discussing and writing about Math; (5) reading with proper comprehension a written Mathematical presentation; (6) hypothesizing, arguing, and formulating definitions and generalisations; (7) depicting and questioning learnt Math knowledge (Qohar & Sumarmo, 2013). Alternatively, Greenes and Schulman (2003) mention four main components of the skill, namely (1) expressing Mathematical ideas through oral or written forms, visually presenting and illustrating them in various ways; (2) understanding, justifying and evaluating ideas presented orally or in writing or visually; (3) developing, justifying and connecting the representation of ideas and relationships.

Mathematical communication skills allow students’ Math understanding through thinking, discussing and decision-making (Viseu & Oliveira, 2012); and also facilitate students in expressing their mathematical ideas in different ways (Utami, 2015). Syaban (2008) argues that only when students talk and write about what they do, can they truly learn Mathematics. In addition, students would actively engage in Math learning when required to think through their ideas, and talk and listen to others in proposing ideas, strategies and solutions. Through Mathematical communication, students can express, explain, describe, and listen to deepen their Mathematical understanding. Mathematical communication is an integral part of Mathematics and Mathematics education. It is a way to share ideas and clarify understanding. Through communication, ideas become the objects of reflection, selection, discussion and modification. The communication process also establishes the significance and sustainability of ideas and their external expressions (National Council of Teachers of Mathematics, 2000).

Vygotsky’s Sociocultural Theory of Cognitive Development focuses on the fundamental role of social interaction in the development of cognition. Vygotsky (1978) emphasises that the community is at the centre of the knowledge construction process, social learning precedes development, learning is guided within the zone of proximal development and the majority of vital issues are learnt through social interaction with instructors. Instructors may act as role models of behaviours or provide children with oral instructions, which is referred to by Vygotsky as cooperative or collaborative dialogues. The child tries to make sense of instructors’ behaviour or instructions (usually parents or teachers), then transform information, and use it to navigate/adjust their work. In Vygotsky’s theory, there are two important concepts namely More Knowledgeable Other (MKO) and Zone of Proximal Development (ZPD). MKO refers to someone who has a better understanding or higher ability level than the learner, in regard to a specific task, process or concept. ZPD is defined as the gap between the level of actual development, what a child can achieve on his own, and the level of potential development, what a child can achieve with the support of more advanced and
competent individuals. Thus, social interaction supports children’s cognitive development within the ZPD, leading to a higher level of reasoning (Vygotsky, 1978).

The continuous mutual impact between thinking and language in learners’ Math learning across school levels in the form of student-teacher and student-student Mathematical communication gradually develops Math knowledge and Math communication competence. The concepts of ‘community’ and ‘social interaction’ from Vygotsky’s Sociocultural Theory of Cognitive Development are the theoretical backgrounds for the role of Math communication in Math classrooms where learners construct Mathematical knowledge through the process of Mathematical communication with teachers and other students. In other words, the process of communication with the ‘community’ in the Math classroom is the process of Mathematical communication and ‘social interaction’ in the Math classroom is the activity of Mathematical communication.

3. MATERIAL AND METHODS

In the research, all the keywords ‘community’, ‘social interaction’ and ‘Vygotsky’s Sociocultural Theory of Cognitive Development’ were used to look for relevant literature, which subsequently was analysed in search for justification of the role of Math communication in learners’ process of Math learning, which was defined as a cognitive process of children. Then, the General Education Program for Mathematics was investigated, focusing on the expressions of Math communication competence and connecting to the justifications in the previous stage in order to design some sample teaching sessions in the light of Vygotsky’s two concepts ‘community’ and ‘social interaction’. The Math textbook published based on the 2016 Math Curriculum was adopted in the research due to the fact that prior to 2022, teachers and students in high schools all over Vietnam still used these as supplementary materials for their teaching and learning.

4. RESULTS AND DISCUSSION

4.1. The concepts of ‘community’ and ‘social interaction’ in social constructivism theory-based teaching

Vygotsky asserts the importance of social and cultural context in learning. Cognitive development roots in social interactions with guided learning within the zone of proximal development where children and their skilled partner’s co-construct knowledge. According to Vygotsky, the environment where children grow up affects the way they think and what they think about. Accordingly, the researcher would propose the following pedagogical implications: (1) teachers create a social environment for learners to develop themselves and their independence, learn from their peers and receive support from more superior peers or teachers, (2) learners gradually develop their ability level through learning, supporting and practising. Through language and culture, teachers and learners discuss, present, raise their opinion and experience the activities in the community that teachers create.

Clements and Battista (1990) assert that ‘learning is a social process in which children gradually blend in with the intellectual activities of surrounding ones. The concepts and truths, from either literal or practical perspectives, are co-created by all members of a ‘culture’. Therefore, in constructivist-based teaching, learners not only participate in the exploration and discovery but also the social process, including explanation, exchange, negotiation and evaluation’. In short, learning is an interactive social process in the educational context, which basically involves teachers and learners.

The method of Social constructivism-based teaching consists of three aspects: (1) Constructing aspect (involving the subject of the learning process: learner), (2) the Social aspect (involving relevant partners: other learners and teachers – ‘community’); (3) Interactive aspect (involving interactive activities between learners and community related to learning contents – ‘social interactions’). The concepts ‘community’ and ‘social interaction’ from Vygotsky’s Sociocultural Theory of Cognitive Development refer to the communicative environment and learners’ Mathematical communication activities respectively. Learners construct Math knowledge through the process of Mathematical communication with teachers and other learners. In other words, the process of communicating with the ‘community’ in the Mathematics classroom is a Mathematical communication process, while ‘social interactions’ in the Math classroom are Mathematical communication activities.

4.2. Mathematical communication activities in the Math classroom

Learners’ Mathematical communication activities in the classroom can be simply put as learners’ activities (speaking, listening, reading, writing) while interacting with teachers or other learners during Math lessons. Two
requirements for the conduct of Math communication activities are the Environment for communicating and Activities of communication (in different communicative forms). Mathematical communication environment refers to one of the Math classrooms which involves teachers, learners and the ‘interactive atmosphere’ that the class creates. All learners are supposed to be open and active in sharing their thoughts and solutions as well as questions, misunderstandings or concerns. At first, it would be challenging for learners when transforming from the passive learning method to the active communicative method, especially ones with incompetent Math knowledge, insufficient Math communication skills and a lack of willingness to converse with teachers and other students. In those cases, teachers need to direct the lesson step-by-step, create a positive and open interactive environment for lively discussions, engage learners in Math communicative situations and build up their activeness and initiative in raising their voices.

Diversity of Math communicative activities in the Math classroom: Reading and listening activities: Learners carry out reading and listening Math communicative activities when reading materials such as textbooks, workbooks, and reference books, dealing with any Math problems; or listening to teachers, and other students or any MKO talk about Math topics. While both traditional and communicative classrooms expose learners to these kinds of activities, the latter focuses on what learners perceive through reading and listening. Teachers collect information by requiring learners to reconstruct the Math knowledge they have perceived (in spoken, written or presented forms). Learners need to discuss, exchange and reflect by speaking or writing down the perceived information. According to Luong (2021), the diversity of Math communicative activities with the focus on interaction and creating positive and open exchanging environments is a direction for Math communication competency-based teaching. Speaking activities are conducted by presenting Math ideas and solutions; discussing Math contents, and debating for agreement or disagreement with other ideas and solutions. Writing activities involve presenting solutions for a Math problem or arguments for such an answer and ideas, etc.

4.3. Applying the concepts of ‘community’ and ‘social interactions’ in Math communication competence-based teaching: Teaching Samples.

In the following samples, the activities related to the concept of ‘community’ are written in bold while ones related to ‘social interactions’ or Math communication activities are in italics.

Sample 1: Introducing the concept ‘proposition’ and ‘propositional variables’

With the aims of establishing and consolidating the concepts of ‘proposition’ and ‘propositional variables’, Activity 1 can exploit the teaching techniques focusing on helping learners to construct Math language knowledge and enhance their Math language proficiency. Activity 1 includes the following steps:

Learners are divided into groups to complete the assigned tasks by the teacher individually and then discuss in groups; finally, they present their group results on A4 papers.

| Task 1: Each group writes down random statements (either about Mathematics or not) |
| Task 2: Write down next to each statement the letter ‘Đ’ if the statement is true, and ‘S’ if the statement is false. |
| Task 3: Categorize the written statements into groups based on their similar features. |

(In case there is no group with any proposition containing variables, teachers may do Task 1 like a student with the statement ‘with every real number, \( x, x^2 \geq 0 \))

The teacher monitors learners’ interactive process in groups and supports if required by eliciting and giving suggestions and instructions. The teacher requires learners to be a reporter of their group work results and direct the activities as follows:

| **Teacher/Learners** | Math Speaking/Writing content |
|----------------------|-----------------------------|
| Teacher              | A learner is randomly selected to report his/her group work results (the results are stuck on the board) |
| Learner 1            | Report results More |
| Learner 2            | Report results Trình bày kết quả |
| Learner 3            | Report results |

(121226)
Teacher: Are there any questions about the results of all the groups? You can raise questions about anything you do not understand in your friends’ presentations.

Learner 4: Why is this statement of your group true?

Teacher: Can you categorise the statements written down by all the groups based on the criteria ‘True/False’? Is there any statement belonging to either of the two groups?

Learner 5: Yes, there is a statement which is neither true nor false.

Teacher: Good job. Can someone emphasize again the results we have achieved?

Learner 6: Given the results from 3 groups, there are 3 kinds of statements: True statement, False Statement, Neither-True-Nor-False Statement

Teacher: Student 7, do you agree with Student 6?

Learner 7: Respond

Teacher: Introduce the concept of ‘proposition’ and elicit to introduce characteristics of propositions with variables.

Therefore, learners construct knowledge by talking about the concepts of ‘proposition’ and ‘propositions with variables’.

In the sample activity above, ‘interactive environment’ – ‘community’ is created by teachers in order that learners can construct the concepts of ‘proposition’ and ‘propositions with variables’ by themselves: Learners are organised in groups. In their group, learners actively interact and discuss to create a shared group result; meanwhile, teachers act as a guide, a facilitators to ensure the groups’ smooth cooperation. In such a community-like interactive environment, learners are required to engage in Math communicative activities actively, with analysis, and critical thinking towards their friends’ results and ideas within a group or as a whole class; and eventually to report their group work results with confidence.

Sample 2: Teaching reading skills when instructing students’ self-study on the topic ‘the concept of permutation’ – Algebra 10 (currently in-use curriculum)

Activity 1: Teacher: ‘You will read a text mentioning the concept of permutation to identify the Mathematical information mentioned in the text.’ (Teacher distributes the handouts with the text to students.)

Learners: Read the text, focusing on the important Mathematical information in the text (learners underline with Mathematical terms in the text while reading.) Figure 1 represents a handout completed by a student after the Activity.

2. Chỉnh hợp

a) Chỉnh hợp là gì?

Ví dụ 3. Trong trận chung kết bóng đá phải phân định thắng thua bằng đà lâu lưu 11 mét. Huấn luyện viên của mỗi đội cân tính với trọng tài một danh sách sắp thứ tự 5 cầu thủ trong số 11 cầu thủ để đà lâu lưu 5 quả 11 mét.

Một danh sách có xếp thứ tự 5 cầu thủ được coi là một chỉnh hợp chấp 5 cầu thủ 11 cầu thủ.

Một cách tổng quát, ta có

Chọn tập hợp \( A \) gồm \( n \) phần tử và số nguyên \( k \) với \( 1 \leq k \leq n \). Khi lấy ra \( k \) phần từ \( A \) và sắp xếp chúng theo một thứ tự ta được một chỉnh hợp chấp \( k \) của \( A \) (gọi tất cả một chỉnh hợp chấp \( k \) của \( A \))

**Figure 1. Illustration of learners’ Mathematical information perception**
The Mathematical terms and symbols in the text include:
+ List of the 5 players with orders
+ Permutations for 11 players taken 5 at a time
+ Set A, n elements, integer $k, 0 \leq k \leq n$
+ Permutations for n elements from set A taken k at a time
+ Permutations for set A taken k at a time

The teacher presents the file showing collected Mathematical terms and symbols (or the Teacher may organise groups and have group representatives write their groups’ identified terms and symbols on the board).

- Activity 2: Ask students to say any mathematical content they perceived while reading, which would allow connecting separated mathematical information from Activity 1 into Mathematical knowledge (preliminary forms of speaking, writing or presenting). The teacher asks students to give peer feedback and self-check their own results to identify the key content of the activity: ‘the concept of permutations for n elements taken k at a time’.

- Activity 3: Through peer discussion, students self-check again the mathematical information and achieve the precise concept of permutations for n elements taken k at a time with the teacher’s support.

This is an example of reading and comprehending the information in a text, selecting necessary information and re-construct the provided mathematical knowledge (by writing, speaking or presenting). Mathematical communicative activities are designed to support learners in constructing knowledge.

5. CONCLUSION

Mathematical communication in certain aspects is the manifestation of the concept of ‘social interaction’ in the context of a Math classroom. In the journey of self-constructing Math knowledge, learners can not depart from mathematical communicative activities. Thus, Mathematical communication is the means and also the results of the learning process. With the search for effective mathematical communicative activities, it is vital that learners enjoy a ‘community’ with active, open, and engaging members. Regular social interactions in an open community, it’s highly likely for students to not only be Math knowledge perceivers but also proactive owner.

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