Blockchain Aided Privacy-Preserving Outsourcing Algorithms of Bilinear Pairings for Internet of Things Devices

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ABSTRACT

Bilinear pairing is a fundamental operation that is widely used in cryptographic algorithms (e.g., identity-based cryptographic algorithms) to secure IoT applications. Nonetheless, the time complexity of bilinear pairing is \(O(n^3)\), making it a very time-consuming operation, especially for resource-constrained IoT devices. Secure outsourcing of bilinear pairing has been studied in recent years to enable computationally weak devices to securely outsource the bilinear pairing to untrustworthy cloud servers. However, the state-of-art algorithms often require to pre-compute and store some values, which results in storage burden for devices. In the Internet of Things, devices are generally with very limited storage capacity. Thus, the existing algorithms do not fit the IoT well. In this paper, we propose a secure outsourcing algorithm of bilinear pairings, which does not require pre-computations. In the proposed algorithm, the outsourcer’s efficiency is significantly improved compared with executing the original bilinear pairing operation. At the same time, the privacy of the input and output is ensured. Also, we apply the Ethereum blockchain in our outsourcing algorithm to enable fair payments, which ensures that the cloud server gets paid only when he correctly accomplished the outsourced work. The theoretical analysis and experimental results show that the proposed algorithm is efficient and secure.

Index Terms—Bilinear Pairings; Cloud Computing; Blockchain; Internet of Things

I. INTRODUCTION

The Internet of Things plays an essential role in the new generation of information technology [15, 31]. It is known as the third wave of the world’s information industry after computers and the Internet. With the rise of 5G in recent years, its characteristics of high reliability, ultra-low latency and large-scale machine communication have driven the rapid development of the Internet of Things. Public-key cryptographic algorithms are widely applied in the IoT to safeguard the connected devices and the network, while some of the algorithms often involve time-consuming operations that the resource-constrained IoT devices cannot afford. For example, the bilinear pairing is a complex operation that is often applied in public-key cryptographic algorithms, such as the identity-based encryption algorithms and the elliptic curve cryptography algorithms. Thus, how to enable computationally weak IoT devices to accomplish complex operation is of great importance.

Over the past decade, the development of the cloud computing [1] has seen explosive growth. Cloud servers provide on-demand computing services on a pay-as-you-go basis [24], which allows users to delegate computation tasks to it and free themselves from the heavy computation workload. Taking advantages of the cloud computing, the computationally weak IoT devices can naturally accomplish complex cryptographic algorithms by outsourcing them to the cloud server. Nonetheless, outsourcing computation tasks to cloud servers also faces security challenges [20, 29, 28, 30]. First, the cloud server might be curious about the outsourced data, while the input and the output of an outsourced cryptography algorithm often involve sensitive information that should not be leaked to the cloud. For example, in the identity-based encryption, the input of a bilinear pairing involves the information about the private key, which should be kept secret from the untrustworthy cloud server. Thus, the privacy of the input and the output should be ensured during the outsourcing process. Second, the cloud server might return invalid computation results intentionally or unintentionally. The invalid result may be caused by software bugs or hardware failures. The cloud server might also return random results without executing the real computation tasks to save computation resources. Thus, IoT devices need to be able to verify if the returned results are correct during the outsourcing process. Also, the verification process and the workload to obscure the input and output should not involve any complex operation. The time cost of the outsourcing process on the client side should be unquestionably less than that of performing the original
Secure outsourcing algorithms for bilinear pairing have been studied, and many existing algorithms can ensure the privacy of the input and output. Nonetheless, most of the existing outsourcing algorithms require a significant amount of pre-computations that are used for encryption and verification purposes. The large amount of the pre-computation data will bring significant demand for storage of IoT devices, while in fact, IoT devices are generally equipped with very limited storage space. Thus, existing secure outsourcing algorithms for bilinear pairing cannot properly be applied in the Internet of Things.

Besides, in outsourcing computing, cloud/edge computing service providers should get paid only when correctly completing the computation task. Due to the lack of trust between outsourcers and users, traditional payment methods are difficult to ensure fairness. If cloud service providers get paid by the user first, it cannot be ensured that the cloud server will correctly perform the computation task; on the contrary, if the cloud service provider performs the computation task and returns the result to users first, it cannot guarantee that users will pay as they agreed on. Existing solutions to this trust problem usually rely on third parties such as banks, which will bring additional overhead, and trust is built on the basis of third parties. Fortunately, the emergence of blockchain and smart contract technology has made it possible to solve this problem. Based on the blockchain technology, the value can be directly transferred between the two parties in the form of cryptocurrencies without the need for a third party, which will provide a strong guarantee for the fairness of computing outsourcing services.

To address the above issues, in this paper, we explore how to securely outsource the bilinear pairing to untrustworthy cloud servers in a way that the IoT device does not need to perform pre-computations. Our contributions are summarized as follows:

- We propose a secure outsourcing algorithm for the bilinear pairing that does not require pre-computations. In the proposed algorithm, the input/output privacy of the client can be ensured so that the cloud server cannot obtain the original input/output.
- We leverage the secure outsourcing of scalar multiplications in our proposed algorithm so that our algorithm does not require any pre-computations, and therefore the client does not need additional storage space to save pre-computation results.
- We develop a fair payment scheme which ensures that the cloud server can get paid only when he has correctly performed the outsourced computation task. We develop fair payment smart contracts on the Ethereum blockchain.

This paper is an extension of our previous work [23], which was published on 2019 IEEE Conference on Dependable and Secure Computing (DSC). Compared with the conference version, in this paper, we present the comparison of our proposed algorithm and state-of-art algorithms, both theoretically and experimentally. Moreover, we enhance the proposed algorithm by applying the Ethereum blockchain to enable fair payments.

The rest of the paper is organized as follows: We introduce some background knowledge, including bilinear pairings and elliptic curves in section II. In section III, we present the system model and provide some security definitions. We illustrate our developed secure outsourcing algorithms for bilinear pairings in section IV. In section V, we analyze the security, efficiency and verifiability of the proposed algorithms. In section VI, we propose a blockchain-based fair payment method which ensures that the server gets paid only when he correctly performed the outsourced computation task. In section VII, we compare our proposed algorithms with state-of-art algorithms. We conduct both theoretical analysis and experiments to evaluate the performance of the proposed algorithms. In Section VIII, we review the related works. Finally, we conclude the paper in Section IX.

II. PRELIMINARIES

In this section, we provide some background knowledge, including bilinear pairings, elliptic curves, and basic operations on elliptic curves.

A. Bilinear Pairings

Bilinear pairing is a complex operation which is widely applied in cartographic protocols (e.g., one-round three-party key agreement, identity-based encryption).

\( \mathbb{G}_1 \) and \( \mathbb{G}_2 \) are two cyclic additive groups, in which \( \mathbb{P}_1 \) and \( \mathbb{P}_2 \) are generators of \( \mathbb{G}_1 \) and \( \mathbb{G}_2 \), respectively. \( p \), a large prime number, is the order of \( \mathbb{G}_1 \) and \( \mathbb{G}_2 \). Let \( \mathbb{G}_T \) be a cyclic multiplicative group with the same order \( p \). A bilinear pairing is a map \( e : \mathbb{G}_1 \times \mathbb{G}_2 \rightarrow \mathbb{G}_T \) which satisfies the following properties:

1. Bilinear: \( e(aR, bQ) = e(R, Q)^{ab} \) for any \( R \in \mathbb{G}_1 \), \( Q \in \mathbb{G}_2 \) and \( a, b \in \mathbb{Z}_p^* \).
2. Non-degenerate: There are \( R \in \mathbb{G}_1 \) and \( Q \in \mathbb{G}_2 \) such that \( e(R, Q) \neq 1 \).
3. Computable: There is an efficient algorithm to compute \( e(R, Q) \) for all \( R \in \mathbb{G}_1 \) and \( Q \in \mathbb{G}_2 \).

B. Elliptic Curve

Suitable bilinear pairings can be constructed from the pairing for specially chosen elliptic curves. Assume \( E \) is an elliptic curve defined on a finite field \( \mathbb{F}_p \). The elliptic curve can be described as follows:

\[
    y^2 = x^3 + bx + c, \quad b, c \in \mathbb{F}_p
\]  

(1)

The set of points \( E(\mathbb{F}_p) \) is a finite abelian group. We use \( E = \{b, c, p\} \) to denote an elliptic curve. A point in \( E(\mathbb{F}_p) \) can be written as \((x, y)\).
C. Basic Operations on Elliptic Curve

(1) Point Addition: Let $P$ and $Q$ be two points on the elliptic curve, point addition describes the addition of $P$ and $Q$, which is denoted as $\text{point_add}(P, Q)$ on elliptic curve. The $\text{point_add}(P, Q)$ function first draws a straight line between the point $P$ and $Q$. The line intersects the elliptic curve at another point $-R$. The output of $\text{point_add}(P, Q)$ is the point $R$, which is the reflection of the point $-R$ with respect to the X-axis.

(2) Point Doubling: Let $P$ be a point on the elliptic curve, point doubling describes the double of the point $P$, which is denoted as $\text{point_double}(P)$. The function $\text{point_double}(P)$ draws one tangent line to the elliptic curve at the point $P$. The line intersects the elliptic curve at the point $-R$. The output of $\text{point_double}(P)$ is the point $R$, which is the reflection of the point $-R$ with respect to the X-axis.

(3) Scalar Multiplication: Let $P$ be a point on the elliptic curve, scalar Multiplication describes the operation that $P$ is multiplied by a scalar $n$, which is denoted as $\text{scalar_multi}(P, n)$. The function $\text{scalar_multi}(P, n)$ repeatedly adds the point $P$ to the result, which is denoted as $nP = P + P + P + ... + P$.

(4) Double-and-add is the widely used approach to conduct the scalar multiplication operation on an elliptic curve. Let $P$ be a point on the elliptic curve, and let $n$ be a scalar. The binary form of scalar $n$ can be expressed as $n = n_mn_{m-1}n_{m-2}...n_0$, where $m$ is the binary bits of $n$. The output is $Q = nP$. This algorithm works as follows:

\[
\text{Point } Q \\
\text{for } i = 0 \text{ to } m \\
\quad \text{if } n_i = 1 \text{ then} \\
\quad \quad Q \leftarrow \text{point_add}(Q, P) \\
\quad \text{end if} \\
\quad P \leftarrow \text{point_double}(P) \\
\text{end for} \\
\text{return } Q
\]

III. SYSTEM MODEL AND SECURITY DEFINITIONS

A. System model

Figure 1 shows the workflow of outsourcing bilinear pairings. As shown in the figure, the IoT device $T$ and the cloud or edge server $U$ are the two parties involved in this framework. The IoT device $T$ needs to perform the complex bilinear pairing operation. However, $T$ is limited in computation power that he cannot afford such a time-consuming operation. The cloud or edge server $U$ is an entity with sufficient computation power that provides computation services at costs. Thus, $T$ plans to delegate the complex bilinear pairing to $U$. Before outsourcing the inputs ($A$ and $B$), $T$ transforms $A$ into $A'$ and $B$ into $B'$ respectively, to protect the privacy of the input and output. The server $U$ conducts the bilinear pairing operation with the obscured inputs ($A'$ and $B'$) and returns $e(A', B')$ to $T$. $T$ verifies the correctness of the returned result before recovering the original result.

B. Security Definitions

We now introduce the security model for secure outsourcing of a cryptographic algorithm, which was proposed in [12]. We use Alg to denote the outsourced computation task. This security model contains three components: a resource constrained client $T$, an untrusted server $U$ and an untrusted environment $E$. $U$ and $E$ together play the role of the adversary $A$, in which the environment $E$ writes programs for the server $U$ and submits adversarial chosen inputs to Alg. The resource constrained client $T$ outsources complex computations to $U$. $E$ and $U$ can communicate with each other only through $T$ once the program developed by $E$ is deployed on the client $T$. Suppose that instead of $U$, $T$ is given oracle access to a malicious $U'$. The adversary $A = \{E, U'\}$ tries to learn some information about the computation task Alg, including the input and output. Therefore, the goal of the client $T$ is to complete the computation task Alg with the help from $U$. Informally, we say that $T$ securely outsources the computation task $Alg$ to the cloud server $U$ if: (a) $T$ and $U$ correctly implement Alg, i.e., $Alg = T^{U}$. (b) even if the $U'$ is given oracle access to all the inputs and outputs of previous computation tasks, $U'$ still not able to obtain the input and output of $Alg$. Formally, we define an algorithm with outsource-I/O as follows:

Definition 1:(Algorithm with outsource-I/O).

Based on the level of secrecy, we categorize the inputs and outputs into 3 categories: (a) Secret: The information is only known to $T$, and is kept secret from $U$ and $E$. (b) Protected: The information is known to $T$ and $E$, and is kept secret from $U$. (c) Unprotected: The information is known to $T$, $U$ and $E$. Besides, based on whether the input is honest, we categorize the inputs into (a) honest and (b) adversarial.
Then, based on above categorizations, we divide inputs and outputs of $Alg$ as follows:

**Five types of inputs:**
1) The honest, secret input: The input that is generated by $T$, and is unknown to both $E$ and $U'$.
2) The honest, protected input: The input that is generated by $T$, which is known to the environment $E$, but is protected from $U'$.
3) The honest, unprotected input: The input that is generated by $T$, and is known to both $E$ and $U'$.
4) The adversarial, protected input: The input that is generated by $E$, which is known to $T$, but protected from $U'$.
5) The adversarial, unprotected input: The input that is generated by $E$, which is known by $T$ and $U'$.

**Three types of outputs:**
1) The secret output: The output that is only known to $T$, but is unknown to both $E$ and $U'$.
2) The protected output: The output that is known to $T, E$ but is unknown to $U'$.
3) The unprotected output: The output that is known to $T, E$ and $U'$.

**Definition 2:** (Outsource-security). Assume that an algorithm $Alg$ with above five types of inputs and three types of outputs, $(T, U)$ is an outsourcing implementation of $Alg$. We say that $(T, U)$ is outsource-secure if it meets the following requirements:
1) Correctness: $T^U$ correctly implements $Alg$.
2) Security: Assume $S_1$ and $S_2$ are polynomial-time simulators. The following pairs of random variables are computationally indistinguishable.

**Pair One:** $EVIEV_{real} \sim EVIEV_{ideal}()$:

$EVIEV_{real}$ is $E$'s view at the end of the following real process.

$EVIEV_{real}^i = (i_{state}^i, x_{hs}^i, x_{hp}^i, x_{hu}^i) \leftarrow \begin{cases} I(1^k, i_{state}^{i-1}) \quad & \text{if stop}^i = \text{TRUE} \\ E(1^k, EVIEV_{real}^{i-1}, x_{hp}^i, x_{hu}^i) \quad & \text{if stop}^i = \text{FALSE} \end{cases}$

The real process runs in rounds. In round $i$, an honest, stateful process $I$ first generates honest inputs, including honest, secret input $(x_{hs}^i)$, honest, protected input $x_{hp}^i$, and honest, unprotected input $x_{hu}^i$. Based on $x_{hp}^i, x_{hu}^i$ and its view from last round $EVIEV_{real}^{i-1}$, the environment $E$ then generates the following 5 outputs:

1) $estate_i$: the internal state of current round.
2) $j_i$: specifies which previously generated honest input to be given to $T^U$.
3) $x_{ap}^i$: the adversarial, protected input.
4) $x_{au}^i$: the adversarial, unprotected input.
5) $stop^i$: the boolean variable which decides whether to stop the process in the current round.

Then the algorithm $T^U$ takes as input $T$'s state of last round $s_{state}^{i-1}$, $U$'s state of last round $u_{state}^{i-1}$ and previously generated 5 inputs $(x_{hs}^i, x_{hp}^i, x_{hu}^i, x_{ap}^i, x_{au}^i)$. The algorithm outputs $T$'s state of the current round $s_{state}^i$, $U$'s state of the current round $u_{state}^i$, the secret output $y_i^i$, the protected output $y_p^i$ and unprotected output $y_u^i$. ($estate_i, y_p^i, y_u^i$) is the view of the real process in round $i$. $EVIEV_{real}$, the final view of $E$ in the real process is the view of the last round.

$EVIEV_{ideal} = EVIEV_{ideal}^i$ if $stop^i = \text{TRUE}$.

In this process, the stateful simulator $S_1$ simulates the view of $E$, who does not have the access to $x_{hs}^i$'s, but has the access to non-secret outputs of $Alg$. In round $i$, $S_1$ takes as input its state from last round $s_{state}^{i-1}$, $U$'s state from last round $u_{state}^{i-1}$, non-secret inputs of $Alg$ $(x_{hp}^i, x_{hu}^i, x_{ap}^i, x_{au}^i)$, and non-secret outputs of $alg$ $(y_p^i, y_u^i)$, and outputs its state of the current round $s_{state}^i$, $U$'s state of the current round $u_{state}^{i}$, fake output values $(Y_p^i, Y_u^i)$, and an indicator variable $replace^i$. The indicator variable $replace^i$ indicates whether to replace the original outputs $(y_p^i, y_u^i)$ with fake outputs $(Y_p^i, Y_u^i)$, i.e., if $replace^i$ equals 1, the final output $(z_p^i, z_u^i)$ equals $(Y_p^i, Y_u^i)$. If $replace^i$ equals 0, the final output $(z_p^i, z_u^i)$ equals $(y_p^i, y_u^i)$. The final view of $E$ in the ideal process $EVIEV_{ideal}$ is $(estate_i, z_p^i, z_u^i)$.

**Pair Two:** $UVIEW_{real} \sim UVIEW_{ideal}()$:

The view that the untrusted software $U$ obtains by participating in the real process is described in **Pair One**.

$UVIEW_{real} = u_{state}^i$ if $stop^i = \text{TRUE}$.
Fig. 2: Outsourcing workflow

The **IDEAL** process:

\[
\begin{align*}
UVIEW_{ideal} &= (istate^i, x_{hs}^i, x_{hp}^i, x_{hu}^i) \leftarrow I(1^k, istate^{i-1}); \\
(estate^i, j, x_{ap}^i, x_{au}^i, stop^i) &\leftarrow E(1^k, estate^{i-1}, x_{hp}^i, x_{hu}^i, y_{p}^{i-1}, y_{u}^{i-1}); \\
(astate^i, y_{s}^i, y_{p}^i, y_{u}^i) &\leftarrow Alg(astate^{i-1}, x_{hs}^i, x_{hp}^i, x_{hu}^i, x_{ap}^i, x_{au}^i); \\
(sstate^i, ustate^i) &\leftarrow S_{2}^{U'}(sstate^{i-1}, x_{hu}^i, x_{au}^i); \\
(ustate^i) &
\end{align*}
\]

\[
SVIEW_{ideal} = SVIEW_{ideal}^{i} \text{ if } stop^i = TRUE.
\]

The stateful simulator \(S_2\) in the ideal process is similar with \(S_1\) in Pair One. \(S_2\) only takes the unprotected \((x_{hu}^i, x_{au}^i)\) to query \(U'\).

**Definition 3** (\(\beta\)-checkable [12]): Checkability requires that the client \(T\) could detect the invalid results from the cloud server \(U\) with high probability. An algorithm \((T,U)\) is said to be \(\beta\)-checkable if (a) the client \(T\) and the cloud server \(U\) perform the algorithm correctly and (b) for \(\forall x\), if the server \(U\) misbehaves during execution of \(T^U(x)\), \(C\) could detect it with probability greater than or equals \(\beta\).

**Definition 4** (\(\alpha\)-efficient [12]): Efficiency requires that the workload carried by \(T\) should be significantly less than conducting the original computation on its own. An algorithm \((T,U)\) is \(\alpha\)-efficient if (a) \(T\) and \(U\) correctly implement the algorithm and (b) for \(\forall x\), the execution time of \(T^U\) is less than or equal to an \(\alpha\)-multiplicative factor of the execution time of \(F\).

IV. THE PROPOSED SECURE OUTSOURCING ALGORITHM

A. Design Rationale

We now describe our underlying thoughts when we design the proposed algorithm. The overall objective is to design an algorithm that can protect the input/output privacy and the checkability when outsourcing the bilinear pairing. Meanwhile, the proposed scheme should not require extra storage space. The existing secure outsourcing algorithms often require a significant amount of pre-computations, which would bring huge demand for storage space of IoT devices. Thus, when we design the outsourcing algorithm, we focus on developing a strategy that avoids the pre-computation. To obscure the input and output of bilinear pairings, we consider multiplying the input points \(A\) and \(B\) by random scalars. Notice that the scalar multiplication itself is a time-consuming operation. Thus, we consider outsourcing the scalar multiplication as well. Figure 2 shows the overall workflow of the proposed algorithm. The local resource-constrained IoT device obscures the scalar multiplication and sends it to the cloud server. The cloud server returns the result after conducting the scalar multiplication, and the user decides whether to accept it after verifying the correctness of the result. After recovering the result of the scalar multiplication, the user sends the calculated points to the cloud. On receiving the inputs, the cloud server calculates the bilinear pairing and returns the calculation result to the user. After the user receives the result, the correctness
verification is performed. If the returned result passes the verification, the user recovers the original result from it.

B. SM: The secure outsourcing algorithm for scalar multiplications

As introduced above, when obscuring the inputs of the bilinear pairing, the client needs to outsource the scalar multiplications to the cloud server. The state-of-art algorithm to outsource the scalar multiplication was proposed by Zhou et al. in 35. In their proposed scheme, they outsource two relevant scalar multiplications to a single cloud server. Since we have two cloud servers in our system model, we leverage their strategy but adjust the strategy to fit the one-malicious version of the two-untrusted-program model. We now introduce the algorithm to outsource the scalar multiplication in our system.

Algorithm 1 SM

Input: \( E = \{a, b, p\} \), \( P(x, y) \in E(\mathbb{F}_p) \) and \( c \in \mathbb{F}_p \)

Output: \( R = cP \)

1) Problem Transformation
   
   a) The client generates a random prime \( q \) and computes \( N = pq \).
   
   b) The client selects random integers \( r_1, r_2, r_3, r_4, r_5, r_6, t_1, t_2 \) and calculates:

   \[
   x' = (x + r_1p) \mod N \\
   y' = (y + r_2p) \mod N \\
   a' = (a + r_3p) \mod N \\
   b' = (b + r_4p) \mod N \\
   c_1 = (c + r_5p) \mod N \\
   c_2 = (t_1c + t_2 + r_6p) \mod N \\
   P' = (x', y')
   \]

2) Computation
   
   a) Client queries \( U_1 \) as: \( Q_1 = c_1P' Q_3 = r_1Q_1 + r_2P' \)
   
   b) Client queries \( U_2 \) as: \( Q_2 = c_2P' \)
   
   c) \( U_1 \) returns \( Q_1 \) and \( Q_3 \) to Client, \( U_2 \) returns \( Q_2 \) to Client.

3) Verification
   
   a) Client verifies the results by verifying \( Q_3 = Q_2 \mod p \).

4) Recovery
   
   a) Client recovers the result \( R = Q_1 \mod p \).

The algorithm is named \( SM \), which is shown in Algorithm 1. The input of the algorithm includes the elliptic curve \( E = \{a, b, p\} \), a point on the curve \( P(x, y) \in E(\mathbb{F}_p) \), and a scalar \( c \in \mathbb{F}_p \). The algorithm outputs a point \( R = cP \) on the elliptic curve. The client first select a random prime number \( q \), which is with the same length of \( p \), and calculates \( N = pq \). Then, the client selects random integers \( r_1, r_2 \) calculates \( x' = (x + r_1p) \mod N, y' = (y + r_2p) \mod N \) to obscure the point \( P \). The client selects random integers \( r_3, r_4 \) and calculates \( a' = (a + r_3p) \mod N, b' = (b + r_4p) \mod N \) to obscure the parameters of the elliptic curve \( E \). The client selects a random integer \( k_5 \) and calculates \( c_1 = (c + r_5p) \mod N \) to blind the scalar \( c \). For verification purpose, the client also selects a random integer \( k_6 \) and calculates \( c_2 = (t_1c + t_2 + r_6p) \mod N \). Then the client queries \( U_1 \) and obtain \( Q_1 = c_1P', Q_3 = r_1Q_1 + r_2P' \). The client queries \( U_2 \) and obtains \( Q_2 = c_2P' \). On receiving the returned results, the client verifies if \( Q_3 = Q_2 \mod p \).

If the results pass the verification, the client recovers original result \( R = Q_1 \mod p \).

C. BPSM: Our proposed secure algorithm

Algorithm 2 BPSM

Input: \( A \in G_1 \), \( B \in G_2 \) and \( E = \{a, b, p\} \).

Output: \( e(A, B) \)

1) Client randomly selects four integer \( a_1, a_2, b_1, b_2 \), such that \( a_1a_2 + b_1b_2 = 1 \). Client selects a random integer \( x \).

2) Client runs \( SM() \) to obtain:

\[
\begin{align*}
a_1A &= SM(a_1, A) \\
b_1A &= SM(b_1, A) \\
a_2B &= SM(a_2, B) \\
b_2B &= SM(b_2, B) \\
xb_1A &= SM(x, b_1A) \\
xo_2B &= SM(x, o_2B)
\end{align*}
\]

3) Client queries \( U_1 \) in random order as:

\[
\begin{align*}
H_1 &= U_1(a_1A, a_2B) = e(A, B)^{a_1a_2} \\
L_1 &= U_1(xb_1A, b_2B) = e(A, B)^{xb_1b_2}
\end{align*}
\]

Similarly, Client queries \( U_2 \) in random order as:

\[
\begin{align*}
H_2 &= U_2(b_1A, b_2B) = e(A, B)^{b_1b_2} \\
L_2 &= U_2(xa_1A, a_2B) = e(A, B)^{xa_1a_2}
\end{align*}
\]

4) Finally, Client verifies the results by checking

\[
L_1L_2 = H_1H_2^x. \text{ If the equality does not hold, client outputs "error". Otherwise, because } a_1a_2 + b_1b_2 = 1, \text{ client can compute } e(A, B) = H_1H_2.
\]

We now introduce our proposed secure algorithm for bilinear pairings, which is named \( BPSM \). \( BPSM \) takes the elliptic curve \( E \) and two points on the curve \( A \) and \( B \) as the input, and output \( e(A, B) \). The algorithm runs as follows:

The client first generates random integers \( a_1, a_2, b_1, b_2, s.t., a_1a_2 + b_1b_2 = 1 \). The client also generates a small integer \( x \). With the generated parameters, the client calls the \( SM \) function to calculate a set of scalar multiplications and obtains \( a_1A = SM(a_1, A), a_2A = SM(a_2, A), b_1A = SM(b_1, A), b_2B = SM(b_2, B) \). Then, the client queries \( U_1 \) and \( U_2 \) in random order as:

\[
\begin{align*}
H_1 &= U_1(a_1A, a_2B) = e(A, B)^{a_1a_2} \\
L_1 &= U_1(xb_1A, b_2B) = e(A, B)^{xb_1b_2}
\end{align*}
\]

Similarly, Client queries \( U_2 \) in random order as:

\[
\begin{align*}
H_2 &= U_2(b_1A, b_2B) = e(A, B)^{b_1b_2} \\
L_2 &= U_2(xa_1A, a_2B) = e(A, B)^{xa_1a_2}
\end{align*}
\]

Finally, Client verifies the results by checking

\[
L_1L_2 = H_1H_2^x. \text{ If the equality does not hold, client outputs "error". Otherwise, } a_1a_2 + b_1b_2 = 1, \text{ client can compute } e(A, B) = H_1H_2.
\]
$SM(b_1, A)$, $b_2 A = SM(b_2, A)$, $x b_1 A = SM(x, b_1 A)$, 
$xa_2 A = SM(x, a_2 A)$. $a_1 A, a_2 A, b_1 A, b_2 A, x b_1 A$ and $x b_2 A$ are now the obscured inputs to conduct the bilinear pairings. The client queries $U_1$ in a random order and obtains $H_1 = U_1(a_1 A, a_2 B) = e(A, B)^{a_1 a_2}$ and $L_1 = U_1(x b_1 A, b_2 B) = e(A, B)^{x a_1 b_2}$. The client queries $U_2$ in a random order and obtains $H_2 = U_2(b_1 A, b_2 B) = e(A, B)^{b_1 b_2}$ and $L_2 = U_2(x a_1 A, a_2 B) = e(A, B)^{x a_1 a_2}$. Based on the returned results, the client verifies if $L_1 L_2 = (H_1 H_2)^x$. If the results pass the verification, the client recovers original result $e(A, B) = H_1 H_2$. Otherwise, the client outputs "error".

V. Security Analysis

Theorem 1. The algorithms $(T,(U_1, U_2))$ of BPSM are an outcome-secure implementation, where the input $(A, B)$ may be honest, secret, or honest, protected, or adversarial, protected.

PROOF. According to Definition 2, we need to prove the correctness and the security. We first prove the correctness:

$$H_1 H_2 = U_1(u_1 A, u_2 B) U_2(v_1 A, v_2 B)$$
$$= e(A, B)^{u_1 v_1} e(A, B)^{v_1 v_2}$$
$$= e(A, B)^{u_1 v_2}$$
$$= e(A, B)$$

We now prove the security.

- **Pair One**: $EVIEW_{real} \sim EVIEW_{ideal}$.
  If the input $(A, B)$ is not honest and secret, the performance of $S_1$ is the same with that in the real execution, and $EVIEW_{ideal}$ is therefore same with $EVIEW_{real}$. Thus, we suppose that the input $(A, B)$ is honest and secret. In round $i$, the stateful simulator $S_1$ behaves as follows. When receiving the inputs, $S_1$ ignores the inputs and randomly selects points $(w_1 P_1, w_2 P_2, w_3 P_1, w_4 P_2)$ and $(w_5 P_1, w_4 P_2, w_6 P_1, w_2 P_2)$ and makes random queries to $U_1'$ and $U_2'$ as follows:

$U_1(w_1 P_1, w_2 P_2) \rightarrow H_1$
$U_1(w_3 P_1, w_4 P_2) \rightarrow L_1$
$U_2(w_5 P_1, w_4 P_2) \rightarrow H_2$
$U_2(w_6 P_1, w_2 P_2) \rightarrow L_2$

Based on the the results from $U_1'$ and $U_2'$, $S_1$ performs as follows:

- If there exists an error, $S_1$ outputs $Y_1 = "ERROR")$, $Y_2 = \emptyset$, replace = 1. $S_1$ saves the state of the current round.
- If no error is detected, $S_1$ outputs $Y_1 = \emptyset$, $Y_2 = \emptyset$, rep = 0. $S_1$ saves the state of the current round.

The inputs to $(U_1', U_2')$ between the real process and ideal process are computationally indistinguishable. The reason is that in the ideal process, the inputs are chosen randomly by $S_1$. In the real process, the inputs of $U_1$ and $U_2$ are independently re-randomized. We have the following possible cases to be considered:

- $U_1$ and $U_2$ perform honestly in the round $i$. In this case, $U_1, U_2$ correctly implements BPSM in the real experiment and $S_1$ will not replace the output in the ideal experiment, then $EVIEW_{real} \sim EVIEW_{ideal}$.
- One of $U_1$ and $U_2$, or both of them behave dishonestly, the misbehavior will be detected by $S_1$ and $T$, the algorithm outputs "ERROR". In this process, $EVIEW_{real} \sim EVIEW_{ideal}$.

Thus, we can conclude that whether $U_1$ and $U_2$ misbehave or not, $EVIEW_{real} \sim EVIEW_{ideal}$.

- **Pair Two**: $UVIEW_{real} \sim UVIEW_{ideal}$.
  In round $i$, the stateful simulator $S_2$ performs similar with $S_1$. When receiving the input, $S_2$ ignores it and randomly selects points and makes random queries to $U_1$ and $U_2$. $S_2$ then saves the state of $U_1$ and $U_2$ and also its own state. The inputs generated by $T$ are randomized and independent. Thus, we can conclude that $UVIEW_{real} \sim UVIEW_{ideal}$.

Theorem 2. The algorithms $(T,(U_1, U_2))$ of BPSM is 1-checkable.

PROOF. When receiving the results form the cloud servers, the client checks if the equation $L_1 L_2 = (H_1 H_2)^x$ holds. If $U_1$ and $U_2$ misbehave, The client will detect the misbehavior with probability 1. Thus, according to definition 3, the algorithms $(T,(U_1, U_2))$ of BPSM is 1-checkable.

Theorem 3. The algorithms $(T,(U_1, U_2))$ of BPSM is $\frac{1}{n \log p}$-efficient.

PROOF. The algorithm BPSM requires 1 modular exponentiation and 2 multiplication in $\mathbb{G}_T$. The original bilinear pairing requires roughly $\log p$ multiplications in resulting finite field $\mathbb{G}$. Thus, according to definition 4, the algorithms $(T,(U_1, U_2))$ of BPSM is $\frac{1}{n \log p}$-efficient.

VI. Blockchain-Based Fair Payment Scheme

In outsourcing computing, cloud/edge computing service providers should get paid only when correctly completing the computation task. Due to the lack of trust between service providers and users, traditional payment methods are difficult to ensure fairness. If cloud service providers get paid by the user first, it cannot be ensured that the cloud server will correctly perform the computation task; on the contrary, if the cloud service provider performs the computation task and returns the result to users first, it cannot guarantee that users will pay as they agreed on. Existing methods to solve this trust problem are based on traditional electronic cash, relying on third parties such as banks, which will bring additional overhead, and trust is built on the basis of third parties. Fortunately, the emergence of blockchain and smart contract technology has made it possible to solve this problem. Based on the blockchain technology, the value can be directly transferred between the two parties in the form of cryptocurrencies without the need for a third party, which
TABLE I: Comparison of Pre-computations

|       | BPSM | Alg[7] | Alg[22] | Alg[10] |
|-------|------|--------|---------|---------|
| PA    | 0    | 5      | 9       | 2       |
| ME    | 3    | 0      | 2       | 1       |
| SM    | 3    | 3      | 9       | 2       |
| PE    | 0    | 0      | 0       | 0       |
| M     | 0    | 0      | 0       | 0       |

TABLE II: Comparison of Client’s Workload

|       | BPSM | Alg[7] | Alg[22] |
|-------|------|--------|---------|
| PA    | 0    | 5      | 9       |
| ME    | 1    | 0      | 2       |
| SM    | 0    | 0      | 3       |
| PE    | 0    | 0      | 0       |
| M     | 2    | 4      | 3       |

In this section, we apply the blockchain technology to enable fair payment for the secure outsourcing process. Figure 3 shows the workflow of the blockchain-based fair payment system. As we can see, the client first generates some random parameters and obscure the inputs. Then the client runs the uploadSM() smart contract function to upload the computations task onto the blockchain-based platform. Meanwhile, T pays a service fee to the smart contract. When the cloud server S wants to take a task from the platform, he runs the getSM() smart contract function and gets the inputs of scalar multiplication. Meanwhile, S needs to make a deposit to the payment system. Then S conducts the scalar multiplication and returns the result. T verifies the correctness of the scalar multiplication. Then T obscures the inputs of bilinear pairings and calls uploadTask() function to upload the task to the blockchain. Next, S runs getTask() function to get the bilinear paring task. S conducts the bilinear pairing operation and returns the result to the blockchain. The blockchain verifies the correctness of the calculation result and makes the judgment accordingly. If the result passes the verification, the smart contract will pay the service fee and return the deposit to the cloud server. If the result failed to pass, the smart contract would send the deposit and the service fee back to the client.

VII. COMPARISON

A. Numeric Analysis

In this section, we compare our algorithm BPSM with algorithms [7], [10] and [22]. In the following tables, we denote point addition in $G_1$ (or $G_2$) as PA, modular exponentiation as ME, scalar multiplication as SM, pairing evaluation as PE, point multiplication in $G_T$ as M. Table I presents the comparison of pre-computations. Table II compares the client’s computation. Table III and Table IV show the storage space required to store the pre-computation results. As we can observe from Table I the client in BPSM does not need to perform any expensive operations since there is no pre-computation in our algorithm BPSM. Alg[7] requires 9 SM and 3 PE. Alg[22] requires $5(k + h - 3)$ PA, where the value $k$ is the size of a set $S_1$. We can learn from [19] that $k$ is around 20 and the value $h$ is less than 10. Thus, Alg[22] requires about 135 PA, 2 ME and 3 SM. Table II shows that the client only needs to conduct one modular exponentiation and two-point multiplications. And the power of modular exponentiation is less than the security parameter $m$. The computation conducted by the client in our algorithm BPSM is less than that in the other algorithms, which include the time-consuming point addition. Thus, our algorithm BPSM is more efficient on the client side than the other algorithms.

Table III and Table IV show the required storage space for pre-computation results on the client side with different elliptic curves and different times of calculation. Note that the labels of elliptic curves are from the PBC library [17] we used to implement the algorithms. As we can observe from Table III the pre-computation will occupy a large storage size of the client when conduct algorithm [7] many times. For example, with the $d1003 - 291 - 247$ elliptic curve, when the client conducts 10M times of pre-computations, it will require 16.26GB storage space, which is too much for resource-constrained IoT devices. Although the algorithm in Table IV has been improved on the basis of algorithm [7], it still requires a lot of storage space. In contrast, Our algorithm does not require any pre-computation, which is more suitable to be applied on IoT devices.

B. Performance Evaluation

We implement our algorithms to demonstrate the practical efficiency. All data is the average value obtained by experimenting 500 rounds. In our experiment, the computations of the client and the cloud servers are conducted on the computer with Xeon E5-2620 processor running @2.4GHz with 1024MB RAM. The operating system is Ubuntu 17.10 x64. The program is developed in C++ with two open source libraries: GMP library and PBC library. We do not consider the communication consumption between the client and the cloud servers in our experiment. Our goal is to test the computation efficiency of our algorithm.

In Fig. 4 we show the time cost on the client in our algorithm and the time required by computing the bilinear pairing locally. It is obvious that the time spent by the client in our algorithm is far less than that of computing the bilinear pairing locally. Fig. 5 shows the time costs of the client in the three phases in our algorithm including blinding, verification and recovery. As shown in Fig. 6 algorithm [7] and algorithm [22] both need to conduct time-consuming pre-computation. In IoT system, the pre-computation cannot be applied to IoT devices due to the insufficient storage resources of IoT devices. So the pre-computation of algorithm [7] and algorithm [22] should be calculated by the IoT device in real time. In Fig. 7 we compare the time costs of client among algorithm [7], algorithm [22] and our algorithm in IoT system. We can see that the time cost of the client in our algorithm is
Fig. 3: Blockchain-Based Fair Payment Scheme

TABLE III: Precomputation Storage of Alg[7]

| Elliptic Curves       | 10K    | 100K   | 1M     | 10M    |
|-----------------------|--------|--------|--------|--------|
| d11499 − 85 − 82     | 4980KB | 48MB   | 486MB  | 4.75GB |
| d277699 − 175 − 167  | 10MB   | 100MB  | 1001MB | 9.78GB |
| d496659 − 224 − 224  | 13MB   | 128MB  | 1.25GB | 12.52GB|
| d1003 − 291 − 247    | 17MB   | 167MB  | 1.63GB | 16.26GB|

TABLE IV: Precomputation Storage of Alg[22]

| Elliptic Curves       | 10K    | 100K   | 1M     | 10M    |
|-----------------------|--------|--------|--------|--------|
| d11499 − 85 − 82     | 2905KB | 28MB   | 284MB  | 2.77GB |
| d277699 − 175 − 167  | 5981KB | 58MB   | 584MB  | 5.70GB |
| d496659 − 224 − 224  | 7656KB | 75MB   | 748MB  | 7.30GB |
| d1003 − 291 − 247    | 9946KB | 97MB   | 971MB  | 9.49GB |

significantly less than that in the other two algorithms. So the efficiency in client side of our algorithm is much higher than that of algorithm [7] and algorithm [22] in IoT system.

VIII. RELATED WORKS

Researchers have spent numerous efforts studying how to securely outsource varieties of computations so that resource-constrained devices can reduce the local workload. Gentry et al. [11] proposed a fully homomorphic encryption (FHE) algorithm so that users can achieve the goal of secure outsourcing with the help of an ideal lattice. Then, multiple general structures for secure outsourcing computation like [9, 4, 3] have been proposed to realize various functions of computation, which are usually based on FHE algorithm. But these algorithms are usually inefficient as a result of their unpractical feature. Also, there are researchers who focus on solving specific problems. This kind of algorithm is usually much more efficient. For instance, Wei et al. [26] first proposed a signature based on identity to achieve unforgeability against selected message attacks without random predictions. They designed two outsourcing algorithms for exponential operations, which is both secure and efficient, and it can reduce the computation cost of clients.

In cryptographic algorithm based on discrete logarithm, modular exponentiation is one of research hotspots due to its wide use. But it consumes time due to the large scale of the modular. Therefore, many studies have taken outsourcing modular exponentiation securely into consideration. Hohenberger et al. [12] presented the first solution. Then Chen et al. [6] proposed a more efficient algorithm in two untrusted program model, they also designed the first algorithm aimed at outsourcing the calculation of simultaneous modular exponentiations. Zhou et al. [35] solved the problem of how to outsource exponentiation operations securely in one single untrusted program model. Their approach provides a secure verification scheme in which the checkability is about 1. Ren et al. [21] proposed two algorithms about how to outsource modular exponentiation, which can detect
Liu et al. [16] designed an innovative outsourcing solution about shareable functions, which is for modular exponentiation. It is secure even if there are adaptive adversaries. There are also some research works on outsourcing applications of modular exponentiation. For example, Zhang et al. [32] designed a secure outsourcing algorithm for RSA Decryption, in which modular exponentiation is involved.

In many fields, such as image processing and machine learning, matrix operations are fundamental operations. Atallah et al. [2] first proposed a scheme of outsourcing matrix multiplication, which can be proved to be safe under the new calculation assumptions related to secret sharing. Zhang et al. [34] proposed algorithms designed to outsource matrix multiplications, which is verifiable in both malicious and rational adversary model. Zhang et al. [33] also focused on this subject and presented another public verifiable algorithm for matrix multiplication. Using random matrix blinding the original matrix, Mohassel et al. [18] designed for matrix inversion, while Lei et al. [13] proposed a scheme by using the matrix transformation technique. As for Xiao et al. [27], they designed a neural network for the inversion of the time matrix solving in the complex field.

In mathematics, the definition of a linear system means it contains two or more linear equations with the same variable. Wang et al. [25] first presented a secure outsourcing proposal using an iterative method for solving large linear equations. And Chen et al. [5] proposed a new protocol using special linear transformations. Different from the previous protocols, there is no homomorphic encryption and interaction between the client and the cloud.

Outsourcing bilinear pairing securely is also a hotspot because of the wide usage of bilinear pairing. Chevallier-Mames et al. [8] designed a scheme for bilinear pairing. Devices which has constrained resource can detect malicious behaviors of servers. But many expensive operations still need to be executed by clients, which are time-consuming. Chen et al. [7] first considered algorithms for bilinear
pairing by using precomputation. Tian et al. [22] proposed a more efficient project-based on [7] under the same assumption with the same checkability. By introducing the pre-computation, Dong et al. [10] presented two sufficient secure schemes on the basis of a single untrusted server. But these pre-computation results depend on the large storage space of the client. Hence it’s difficult for clients with limited computation and storage resources to realize all these algorithms. Lin et al. [14] focused on it and proposed a novel blockchain-based system designed to efficiently solve the problem.

IX. Conclusion

In this paper, we explore how to delegate the bilinear pairing to untrustworthy cloud servers in a secure, fair, and efficient way. Existing algorithms cannot fit the IoT scenarios since they require extra storage space for the client. Our proposed algorithm solves this problem by coming up with new strategies to obscure the inputs. To ensure the fairness of payment, we construct a fair payment framework on the Ethereum blockchain. Our developed smart contract ensures that the cloud server gets paid only when he correctly performed the bilinear pairing for the client. We also evaluate our proposed algorithm through theoretical analysis and experiments in which privacy, fairness, and efficiency are justified.

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