Implications of the anomalous top quark couplings in $B_s - \bar{B}_s$ mixing, $B \to X_s \gamma$ and top quark decays

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Combined analysis of recent measured $B_s - \bar{B}_s$ mixing and $B \to X_s \gamma$ decays provides constraints on the anomalous $\bar{t}sW$ couplings. We discuss the perspectives to examine the anomalous $\bar{t}sW$ couplings through CKM-suppressed $t \to sW$ decays at the LHC.

I. INTRODUCTION

The standard model (SM) has been demonstrated to be remarkably successful in describing present data. Most parameters of the SM has been directly measured with high accuracy at various experiments. The only unobserved ingredient of the SM is the Higgs boson responsible for the electroweak symmetry breaking and a few top quark couplings are not measured directly. However, it is not believed that the SM is the final theory of our universe since there are still many theoretical and experimental problems which could not be explained in the SM framework. It is natural to expect that the hint of the new physics beyond the SM would be found at the unexamined part of the SM.

The top quark has been discovered at the Tevatron and its mass and production cross section are measured [1]. We will be able to study the top quark couplings with more than $10^8$ top quark pairs per year produced at the CERN Large Hadron Collider (LHC) [2, 3]. The dominant channel of the top quark decay is the $t \to bW$ channel in the SM and the $\bar{t}bW$ coupling will be measured at LHC with high precision to be directly tested. Other channels are highly suppressed by small mixing angles. The subdominant channel in the SM is the Cabibbo-Kobayashi-Maskawa (CKM) nondiagonal $t \to sW$ decay of which branching ratio is estimated as

$$Br(t \to sW) \sim 1.6 \times 10^{-3};$$

when $|V_{ts}| = 0.04$ is assumed in the SM. Although the branching ratio of this channel is rather small, the $t \to sW$ process may be detectable at the LHC due to the large number of top quark production and the $\bar{t}sW$ coupling be measured to provide a clue to new physics beyond the SM. Therefore the anomalous $\bar{t}sW$ coupling is worth examining at present. We do not specify the underlying model here but present an effective lagrangian to describe the new effects on the top quark couplings by introducing two parameters for each flavour. The relevant couplings are parametrized by the effective lagrangian as

$$\mathcal{L} = -\frac{g}{\sqrt{2}} \sum_{q=d,s,b} V_{tq}^{\text{eff}} \bar{t} \gamma^\mu (P_L + \xi_q P_R) q W^{\pm}_{\mu} + H.c.,$$

where $\xi_q$ are complex parameters measuring effects of the anomalous right-handed couplings while $V_{tq}^{\text{eff}}$ measures the SM-like left-handed couplings. Effects of the anomalous top quark couplings have been studied in direct and indirect ways [4, 5, 6, 7, 8, 9, 10, 11, 12, 13].

Particularly interesting is $b \to s$ transition in search of the anomalous top quark couplings. The radiative decay $B \to X_s \gamma$ is the first observation of $b \to s$ transition and provide strict constraints on the anomalous top quark couplings [4, 5]. Since no CP phase is involved in $V_{ts}$ and $V_{tb}$ in the SM, a large direct CP violation in $b \to s$ is an evidence of the new physics beyond the SM [6, 11]. Recently the first observation of the $B_s - \bar{B}_s$ mixing have been reported by the CDF [14] and D0 [15] collaborations with the results

$$\Delta M_s = (17.77 \pm 0.10 \pm 0.07) \text{ ps}^{-1} \quad \text{(CDF)},$$

$$17 \text{ ps}^{-1} \leq \Delta M_s \leq 21 \text{ ps}^{-1} \quad \text{at 90\% C.L.} \quad \text{(D0)},$$

\[ (3) \]
where the first error is statistical and the second is systematic. The $B_s - \bar{B}_s$ mixing arises through the box diagram with internal lines of $W$ boson and $u$-type quarks in the SM. Since the top quark loop dominates the $B_s - \bar{B}_s$ mixing might be also a testing laboratory for the study of $fW$ and $tbW$ couplings.

In this work, we concentrate on $tsW$ coupling and perform the combined analysis of $B_s - \bar{B}_s$ mixing and $B \to X_s \gamma$ to constrain the $V^\text{eff}_{ts}$ and $\xi_s$. $B_s - \bar{B}_s$ mixing depends upon $V^\text{eff}_{ts}$ and is insensitive to the right-handed couplings while $B \to X_s \gamma$ decay depends upon both of $V^\text{eff}_{ts}$ and $\xi_s$. If we measure the subdominant decay $t \to sW$ at the LHC or other future colliders, it will be the direct test of the CKM matrix element $V^\text{eff}_{ts}$ and we can determine the $tsW$ couplings. This paper is organized as follows: In section II, the effective $\Delta B = 1$ Hamiltonian formalism with anomalous $tsW$ couplings is given and the radiative $B \to X_s \gamma$ decays are studied. In section III, the analysis on the $B_s - \bar{B}_s$ mixing with anomalous $tsW$ couplings is presented. We discuss the top quark decays in section IV. Finally we conclude in section IV.

II. $B \to X_s \gamma$

The $\Delta B = 1$ effective Hamiltonian for $b \to s \gamma$ process is given by

$$
\mathcal{H}^\Delta B = \frac{4G_F}{\sqrt{2}} V^\ast_{ts} V_{tb} \sum_{i=1}^{8} (C_i(\mu)O_i(\mu) + C'_i(\mu)O'_i(\mu)),
$$

(4)

where the dimension 6 operators $O_i$ constructed in the SM are given in the Ref. [16], and $O'_i$ are their chiral conjugate operators. Matching the effective theory (5) and the lagrangian (4) at $\mu = m_W$ scale, we have the Wilson coefficients $C_i(\mu = m_W)$ and $C'_i(\mu = m_W)$. Although we will consider the anomalous $tsW$ couplings only, we present the full formalism including $tsW$ and $tbW$ couplings. In the SM, we have the Wilson coefficients

$$
\begin{align*}
C_2(m_W) &= -1, \quad C_7(m_W) = F(x_t), \quad C_8(m_W) = G(x_t), \\
C_i(m_W) &= C'_i(m_W) = 0, \quad \text{otherwise},
\end{align*}
$$

(5)

where $F(x)$ and $G(x)$ are the well-known Inami-Lim loop functions [16, 17]. Let us switch on the right-handed $tbW$ and $tsW$ couplings. Keeping the effects of $\xi_s$ in linear order, we obtain the modified Wilson coefficients

$$
\begin{align*}
C_7 &\to C_7^{\text{SM}} + \xi_s \frac{m_t}{m_b} F_R(x_t), \\
C_8 &\to C_8^{\text{SM}} + \xi_s \frac{m_t}{m_b} G_R(x_t),
\end{align*}
$$

(6)

and the new Wilson coefficients

$$
\begin{align*}
C'_7 &= \xi_s \frac{m_t}{m_b} F_R(x_t), \\
C'_8 &= \xi_s \frac{m_t}{m_b} G_R(x_t),
\end{align*}
$$

(7)

where the new loop functions

$$
\begin{align*}
F_R(x) &= \frac{-20 + 31x - 5x^2}{12(x-1)^2} + \frac{x(2-3x)}{2(x-1)^3} \ln x, \\
G_R(x) &= -\frac{4 + x + x^2}{4(x-1)^2} + \frac{3x}{2(x-1)^3} \ln x,
\end{align*}
$$

(8)

agree with those in Ref. [18].

The branching ratio of $B \to X_s \gamma$ process with the right-handed interactions at next-leading-order (NLO) is given by

$$
\text{Br}(B \to X_s \gamma) = \text{Br}(B \to X_s \ell \bar{\nu}) \frac{10.5\%}{10.5\%} \left[ B_{22}(\delta) + B_{77}(\delta) |r_7|^2 + |r'_7|^2 \right] + B_{88}(\delta) |r_8|^2 + |r'_8|^2 \\
+ B_{27}(\delta) Re(\tau_7) + B_{28}(\delta) Re(\tau_8) + B_{78}(\delta) \left[ Re(\tau_7 r'_7) + Re(\tau_8 r'_8) \right],
$$

(9)

where the ratios $r_i$ and $r'_i$ are defined by

$$
\begin{align*}
r_i &= \frac{C_i(m_W)}{C_i^{\text{SM}}(m_W)} = 1 + \xi_s \frac{m_t}{m_b} \frac{F_R(x_t)}{F(x_t)}, \\
r'_i &= \xi_s \frac{m_t}{m_b} \frac{F_R(x_t)}{F(x_t)},
\end{align*}
$$

(10)
The components $B_{ij}(\delta)$ depends on the kinematic cut $\delta$, of which numerical values are given in the Ref. [19]. We obtain the branching ratio in terms of $\xi_s$ and $\xi_b$ as

$$\text{Br}(B \to X_s\gamma) = \text{Br}^{\text{SM}}(B \to X_s\gamma) \left( \frac{|V_{ts}|^2|V_{tb}|^2}{0.0404} \right)^2 \left[ 1 + \text{Re}(\xi_s) \frac{m_t}{m_b} \left( 0.68 \frac{F_R(x_t)}{F(x_t)} + 0.07 \frac{G_R(x_t)}{G(x_t)} \right) ight. 
\left. + (|\xi_b|^2 + |\xi_s|^2) \frac{m_t^2}{m_b^2} \left( 0.112 \frac{F_R^2(x_t)}{F^2(x_t)} + 0.002 \frac{G_R^2(x_t)}{G^2(x_t)} + 0.025 \frac{F_R(x_t)G_R(x_t)}{F(x_t)G(x_t)} \right) \right], \quad (11)$$

The SM branching ratio is predicted to be $\text{Br}(B \to X_s\gamma) = (3.15 \pm 0.23) \times 10^{-4}$ for $E_\gamma > 1.6$ GeV at next-to-next-to-leading order (NNLO) [20]. The current world average value of the measured branching ratio is given by [21]

$$\text{Br}(B \to X_s\gamma) = (3.55 \pm 0.24^{+0.09}_{-0.10} \pm 0.03) \times 10^{-4}, \quad (12)$$

with the same photon energy cut. The allowed parameter sets of $(|\xi_s|, |V_{ts}^{\text{eff}}|)$ are depicted in Fig. 1 by green (grey) area at 95% C.L..

### III. $B_s - \bar{B}_s$ MIXING

A $B_s$ meson can oscillate into its antiparticle $\bar{B}_s^0$ via flavour-changing processes of $B_s - \bar{B}_s$ mixing. The oscillation is represented by the mass difference between the heavy and light $B_s$ states,

$$\Delta M_s \equiv M_{B_s} - M_{\bar{B}_s} = 2|\text{Re}(\xi_s)|, \quad (13)$$

where the $\Delta B = 2$ transition amplitudes given by

$$\langle B_s^0 | H^{\Delta B=2}_s | \bar{B}_s^0 \rangle = M_{12}^s, \quad (14)$$
is obtained by the box diagrams with internal lines of $W$ boson and up-type quarks in the SM. The new contributions to $B_s - \bar{B}_s$ mixing with anomalous top quark couplings given in Eq. (1) would be examined with the $B_s - \bar{B}_s$ mixing data. The $B_s - \bar{B}_s$ mixing is also described by the width difference of the mass eigenstates

$$\Delta \Gamma_s \equiv \Gamma^s_L - \Gamma^s_H = 2 \text{Re} \frac{\Gamma_{12}^s}{M_{12}^s},$$

(15)

where the decay widths $\Gamma_L$ and $\Gamma_H$ are corresponding to the physical eigenstates $B_L$ and $B_H$. Since the decay matrix elements $\Gamma^s_{12}$ is derived from the SM decays $b \rightarrow cc\bar{s}$ at tree level, it is hardly affected by the new physics. We consider the new effects of the anomalous top couplings only in $M_{12}^s$. Since $\xi_q$ are complex parameters, the new physics effects arise in both magnitude and phase of $M_{12}^s$ in general. In this analysis, we just consider the mass difference. Effects of the phase and CP violation in $M_{12}^s$ have been measured [22], although not very accurately, and discussed in several literatures [23].

Including the odd number of right-handed couplings in the box diagram does not contribute to the transition amplitude $M_{12}^s$ due to vanishing the loop integral of the odd number of momentum. Thus the leading contribution of the anomalous right-handed top couplings to the $B_s - \bar{B}_s$ mixing is quadratic order of $\xi_q$. Calculating box diagrams including the anomalous couplings, the transition amplitude is given by

$$M_{12}^s = \frac{G_F^2 m_b^2}{12\pi} m_{B_s} \eta_B \delta \hat{B}_{B_s} \bar{B}_{B_s} S_0(x_1) \left( \frac{|V^*_{ts}| |V^*_{tb}|}{0.0404} \right)^2 \times \left( 1 + \frac{S_3(x_1)}{S_0(x_1)} \frac{\xi_2^2}{4} \frac{(bP_R)(\bar{b}P_R)}{(b\gamma\mu P_L)(b\gamma\mu P_L)} + \frac{\xi_3^2}{2} \frac{(bP_L)(\bar{b}P_R)}{2(b\gamma\mu P_L)(b\gamma\mu P_L)} + \frac{\xi_6^2}{4} \frac{(bP_L)(\bar{b}P_L)}{4(b\gamma\mu P_L)(b\gamma\mu P_L)} \right),$$

(16)

where $\eta_B$ is the perturbative QCD correction to the $B - \bar{B}$ mixing [24]. The Inami-Lim loop functions are given by

$$S_0(x) = \frac{4x - 11x^2 + x^3}{4(1 - x)^2} - \frac{3x^3}{2(1 - x)^2} \log x,$$

$$S_3(x) = 4x^2 \left( \frac{2}{(1 - x)^2} + \frac{1 + x}{(1 - x)^3} \log x \right).$$

(17)

Using the vacuum insertions, we calculate

$$\frac{\langle B_s^0 | (\bar{b}P_R)(\bar{b}P_R) | \bar{B}_s^0 \rangle}{\langle B_s^0 | (b\gamma\mu P_L)(b\gamma\mu P_L) | B_s^0 \rangle} = \frac{5}{8} \left( \frac{m_{B_s}}{m_b + m_s} \right)^2,$$

$$\frac{\langle B_s^0 | (b\gamma\mu P_L)(b\gamma\mu P_L) | \bar{B}_s^0 \rangle}{\langle B_s^0 | (bP_R)(bP_R) | B_s^0 \rangle} = \frac{3}{4} \left( \frac{1}{6} - \left( \frac{m_{B_s}}{m_b + m_s} \right)^2 \right),$$

$$\frac{\langle B_s^0 | (b\gamma\mu P_L)(b\gamma\mu P_L) | B_s^0 \rangle}{\langle B_s^0 | (bP_R)(bP_R) | B_s^0 \rangle} = \frac{\langle B_s^0 | (b\gamma\mu P_L)(b\gamma\mu P_L) | B_s^0 \rangle}{\langle B_s^0 | (b\gamma\mu P_L)(b\gamma\mu P_L) | B_s^0 \rangle},$$

(18)

and

$$\frac{\langle B_s^0 | (\bar{b}P_R) | (b\gamma\mu P_L) | \bar{B}_s^0 \rangle}{\langle B_s^0 | (bP_R) | (b\gamma\mu P_L) | B_s^0 \rangle} = \frac{8}{3} m_{B_s} \delta \hat{B}_{B_s} f_{B_s}^2,$$

(19)

where $\hat{B}_{B_s}$ is the Bag parameter and $f_{B_s}^2$ the decay constant.

We show the allowed parameter sets $(|\xi_3|, V^*_{ts})$ in Fig. 1 by black area at 95% C.L. We use the SM prediction $\Delta m_s = 19.3 \pm 6.74 \text{ps}^{-1}$ given in Ref. [23]. The conservative bounds $|\xi_3| < 0.027$ and $|V^*_{ts}| > 0.017$ are obtained from this analysis. The correlated results between observables, $\text{Br}(B \rightarrow X_s \gamma)$ and $\Delta M_s$ are shown in Fig. 2 with allowed parameters given in Fig. 1 (black area).

IV. TOP QUARK DECAYS

The flavour-diagonal $t \rightarrow bW$ decay dominates, $\text{Br}(t \rightarrow sW) \approx 1$. The branching ratio of the CKM-suppressed decays are given by

$$\text{Br}(t \rightarrow sW) = |V^*_{ts}|^2 (1 + |\xi_3|^2).$$

(20)
FIG. 2: Correlation of $\text{Br}(B \to X_s\gamma)$ and $\Delta M_s$ with allowed values of $(|\xi_s|, |V_{ts}^{\text{eff}}|)$.

Since there is no enhancement factor involved, the branching ratio is insensitive to $\xi_s$ and determined by $V_{ts}^{\text{eff}}$. The predictions of $\text{Br}(t \to sW)$ is depicted in Fig. 3 with respect to the allowed values of $\xi_s$. We find that large deviation of $\text{Br}(t \to sW)$ from the SM prediction is possible. The correlation between $\Delta M_s$ and $\text{Br}(t \to sW)$ are shown in Fig. 4 with allowed parameters given in Fig. 1 (black area). Both observables of $\Delta M_s$ and $\text{Br}(t \to sW)$ crucially depend on $V_{ts}^{\text{eff}}$ but are insensitive to $\xi_s$. Since the value of $V_{ts}^{\text{eff}}$ will be strongly constrained by $\text{Br}(t \to sW)$, the right-handed coupling $\xi_s$ will be also constrained through $B \to X_s\gamma$ decay if we measure the branching ratio of $t \to sW$ at the LHC or the future colliders.

V. CONCLUDING REMARKS

We consider the anomalous top quark coupling which are not direct measured yet. The $\bar{t}sW$ coupling is parametrized by $V_{ts}^{\text{eff}}$ and $\xi_s$. Combined analysis of $B_s - \bar{B}_s$ mixing and $B \to X_s\gamma$ decay gives strong constraints on by $V_{ts}^{\text{eff}}$ and $\xi_s$. The prediction of the branching ratio of the top decay $\text{Br}(t \to sW)$ is given and it is shown that both of $\Delta M_s$ and $\text{Br}(t \to sW)$ depend only on $V_{ts}^{\text{eff}}$. In conclusion, we can examine the anomalous $\bar{t}sW$ coupling through $B_s - \bar{B}_s$ mixing and $B \to X_s\gamma$ decay and will test it more by the $t \to sW$ decay in the future colliders.
FIG. 3: Prediction of $\text{Br}(t \rightarrow sW)$ with respect to $|\xi_s|$.  

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FIG. 4: Correlation of $\Delta M_s$ and $\text{Br}(t \to sW)$ with allowed values of $(|\xi_s|, |V_{ts}^{\text{eff}}|)$. 

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