Weak decays of $J/\psi$: the non-leptonic case

Yu-Ming Wang$^1$, Hao Zou$^1$, Zheng-Tao Wei$^2$, Xue-Qian Li$^2$, and Cai-Dian Lü$^1$

$^1$Institute of High Energy Physics, P.O. Box 918(4), Beijing 100049, China and
$^2$Department of Physics, Nankai University, Tianjin 300071, China

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Abstract

In our previous study, we calculated the transition from factors of $J/\psi \to D^{(*)}_{(s)}$ using the QCD sum rules. Based on the factorization approximation, the obtained form factors can be applied to evaluate the weak non-leptonic decay rates of $J/\psi \to D^{(*)}_{(s)} + M$, where $M$ stands for a light pseudoscalar or vector meson. We predict that the branching ratio for inclusive non-leptonic two-body weak decays of $J/\psi$ which are realized via the spectator mechanism, can be as large as $1.3 \times 10^{-8}$, in particular, the branching ratio of $J/\psi \to D^*_s \pm + \rho^\mp$ can reach $5.3 \times 10^{-9}$. Such values will be marginally accessed by the ability of BESIII which will begin running very soon.
The decays of $J/\psi$ are dominated by strong and electromagnetic interactions via $c\bar{c}$ annihilating into intermediate gluons and photon at s-channel. By contrast, the weak decays, due to smallness of the strength of weak interaction, are rare processes. Under the spectator approximation, one of the charm quark or anti-charm quark in $J/\psi$ decays into light quarks, and the decay rate of a charm quark (anti-quark) is proportional to $G_F^2 m_c^5$ where $G_F$ is the Fermi coupling constant. Numerically the total branching ratio of weak decays was estimated to at the order of $10^{-8}$ [1]. Recently, due to remarkable improvements of experimental instruments and techniques people turn their interests onto these rare processes from both experiment [2, 3] and theory [4, 5, 6] sides. The forthcoming upgraded BESIII will be able to accumulate more than $10^{10}$ $J/\psi$ per year [7], which makes it possible to marginally measure such weak decays in near future. More important, such rare processes are also particularly interesting from the viewpoint of theory. On the one hand, it may provide further accurate examination of the mechanism which is responsible for the hadronic transition and fully governed by non-perturbative QCD effects. One can also expect that such decays may offer a unique opportunity to probe new physics beyond the standard model [8, 9], including the minimal supersymmetric standard model, the extra dimension model, the two-Higgs doublet, topcolor-assisted technicolor model etc, in the weak decay of vector mesons. The reason is that in such rare decays, weak coupling is rather weak and new physics may have a chance to show up.

In a previous study, we presented a detailed analysis of the semi-leptonic decays of $J/\psi$ [6], where the branching ratios for such channels were estimated to be at order of $10^{-10}$ and hence is almost impossible to be observed at BESIII.

The fundamental ingredients involved in the semi-leptonic processes are the transition form factors of $J/\psi \to D^{(*)}_{s}$, which are evaluated in terms of the three-point QCD sum rules (QCDSR) [10, 11, 12] in that work. Obviously, even though while deriving the form factors our goal was to estimate the branching ratios of semi-leptonic decays, under the factorization approximation, they can be applied to study the non-leptonic decays. Thus, we will take a step forward to investigate the exclusive non-leptonic decays with focusing on two-body processes.
In this study, we will explore the non-leptonic decays $J/\psi \rightarrow D^{(*)} + M$, where the final states contain a single charmed meson and a light meson $M$, such as $\pi$, $K$, $\rho$, $K^*$ etc.. These weak decays are realized via the spectator mechanism that one of the charm quark (anti-charm quark) acts as a spectator. In the Standard Model, at the quark level, the Feynman diagrams for charm quark decay are depicted in Fig. 1. The anti-charm quark decay can be obtained analogously by exchanging $c \leftrightarrow \bar{c}$. The effective theory for hadronic weak decays have been well formulated [13]. The most difficult work is to calculate the hadronic matrix elements which are governed by the non-perturbative QCD dynamics.

The non-relativistic QCD can simplify the picture by phenomenologically handling some non-perturbative QCD effects and has been widely applied to study some decay modes where heavy quarkonium are involved. However, it does not help much for the heavy-light mesons where relativistic effects may be significant.

The first order approximation for the derivation is the factorization hypothesis, where the hadronic matrix element is factorized into a product of two matrix elements of single currents [14, 15, 16, 17, 18, 19]. In this scheme, one element can be written in terms of the decay constant of the concerned meson while the other is expressed by a few form factors according to the Lorentz structure of the current and meson (a pseudoscalar or vector, for example). The non-factorizable effects are incorporated into the effective coefficients which are usually assumed to be universal and determined by experiment (Only in some cases, they are pertubartively calculable. In reality, these coefficients depend on the concrete processes and differ case by case, but the variation may be not very drastic.). For the weak decays of heavy mesons, such factorization approach is verified to work very well for the color-allowed sub-processes. It is reasonable to believe that this conjecture would be valid for $J/\psi$, at least for the processes where the color-allowed sub-processes dominate. Thus, the study on two-body non-leptonic decays offer an ideal ground to testify the factorization hypothesis in the heavy quarkonium system and this test may be more appealing than in decays of $D^{(*)}$ because $J/\psi$ contains two heavy constituents. Moreover, they are of great importance to discriminate various theoretical tools for the evaluations of transition form factors.

The structure of this paper is as follows. In section II, the factorization approach for the non-leptonic decays is introduced and the formulations are given. In section III, after displaying the inputs involved in this work explicitly, the numbers of branching fractions for
FIG. 1: Quark diagrams for non-leptonic weak decays of $J/\psi$. (a) represents the color allowed processes; (b) represents the color suppressed processes; (c), (d) represent single-Cabibbo suppressed processes.

various $J/\psi \rightarrow D^{(*)}_{(s)} + M$ modes are presented and comparisons of our numerical results with that estimated in other theoretical models are also investigated at length in this section. The final section is devoted to the discussions and conclusions. It is noted that since most of the form factors applied in this work were obtained in our previous work, we generally refer the readers to it for some details of the derivation and how to achieve the numerical values.

II. NON-LEPTONIC DECAYS $J/\psi \rightarrow D_{(s)} + M$ IN FACTORIZATION APPROACH

For the non-leptonic weak decays of $J/\psi \rightarrow D_{(s)} + M$, the standard method is integrating out the heavy $W$-boson and obtaining a low energy effective Hamiltonian for $c$ quark decay which is given by

$$\mathcal{H}_{\text{eff}}(c \rightarrow qu\bar{q'}) = \frac{G_F}{\sqrt{2}} V_{cq}^* V_{uq'} (C_1 Q_1 + C_2 Q_2),$$

(1)
where \( q(q') \) represents the down type quarks \( s \) and \( d \); \( V^*_u(V_{uq'}) \) are CKM matrix elements; and the operators \( Q_1, Q_2 \) are respectively

\[
Q_1 = \bar{q}_a \gamma_\mu (1 - \gamma_5) c_\alpha \bar{q}_\beta \gamma^\mu (1 - \gamma_5) u_\beta, \quad Q_2 = \bar{q}_a \gamma_\mu (1 - \gamma_5) c_\beta \bar{q}_\beta \gamma^\mu (1 - \gamma_5) u_\alpha.
\] (2)

It should be pointed out that the penguin operators are neglected in this work due to the smallness of Wilson coefficients for such operators, which also indicates that CP symmetry is well respected within the accepted assumption.

With the free quark decay amplitude, we can proceed to calculate the transition amplitudes for \( J/\psi \to D + M \) at hadron level, which can be obtained by sandwiching the free-quark operators between the initial and final mesonic states. Consequently, the hadronic matrix elements \( \langle DM|Q_i|J/\psi \rangle \) which depend on the strong interactions need to be computed. The evaluation is indeed the main challenge in the heavy flavor physics due to our poor knowledge with respect to the non-perturbative QCD. Owing to the painstaking efforts in theory, several systemic approaches for hadronic \( B \) decays have been explored based on the expansion in small parameters \[20\]. However, a systematic theoretical method concerning the open-charm decays is still not available yet due to the fact that the accessible charm quark mass is not so heavy in reality. As the first order approximation, we may be able to apply the vacuum saturation approximation to factorize the four-quark operator matrix elements \( \langle DM|Q_i|J/\psi \rangle \). The consistency of the theoretical prediction with data (may be available in the future) will serve as an examination of such approximation in the heavy-quarkonium system as mentioned in the introduction. To be more specific, the factorization ansatz \[14\] states that the matrix elements can be factorized into a product of two single matrix elements of currents \( \langle M|J_1|0 \rangle \langle D|J_2|J/\psi \rangle \) where one is parameterized by the decay constant of the emitted light meson and the other is represented by the form factors responsible for the transition of \( J/\psi \) into the recoiled charmed meson.

The decay constants for pseudoscalar \((P)\) and vector \((V)\) mesons are defined as follows

\[
\langle P(q)|A_\mu|0 \rangle = -i f_P q_\mu, \\
\langle V(q, \epsilon)|V_\mu|0 \rangle = f_V m_V \epsilon^*_\mu,
\] (3)

where the axial vector current \( A_\mu \) represents \( \bar{q}_1 \gamma_\mu \gamma_5 q_2 \) and the vector current \( V_\mu \) represents \( \bar{q}_1 \gamma_\mu q_2 \); \( \epsilon \) is the polarization vector of \( V \). The matrix elements \( \langle D|\bar{q}_\gamma(1 - \gamma_5)c|J/\psi \rangle \) are
parameterized in terms of various form factors as [6]:

\[
\langle D(p_2) | q\gamma_\mu(1-\gamma_5)c | J/\psi(\epsilon_\psi, p_1) \rangle \\
= -\epsilon_{\mu\nu\alpha\beta}\epsilon_\nu^\alpha p_1^\alpha p_2^\beta \frac{2V(q^2)}{m_\psi + m_D} + i(m_\psi + m_D) \left[ \epsilon_\mu^\psi - \frac{\epsilon_\psi \cdot q}{q^2} q_\mu \right] A_1(q^2) \\
+ i\frac{\epsilon_\psi \cdot q}{m_\psi + m_D} A_2(q^2) \left[ (p_1 + p_2)_\mu - \frac{m_\psi^2 - m_D^2}{q^2} q_\mu \right] + 2im_\psi \frac{\epsilon_\psi \cdot q}{q^2} q_\mu A_0(q^2),
\]

(4)

\[
\langle D^*(\epsilon_D^*, p_2) | q\gamma_\mu(1-\gamma_5)c | J/\psi(\epsilon_\psi, p_1) \rangle \\
= -i\epsilon_{\mu\nu\alpha\beta}\epsilon_\nu^\alpha \epsilon_\nu^\beta \left[ (p_1^\nu + p_2^\nu - \frac{m_\psi^2 - m_D^2}{q^2} q^\nu) \tilde{A}_1(q^2) + \frac{m_\psi^2 - m_D^2}{q^2} q^\nu \tilde{A}_2(q^2) \right] \\
+ i\frac{m_\psi^2 - m_D^2}{q^2} \epsilon_{\mu\nu\alpha\beta} p_1^\alpha p_2^\beta [\tilde{A}_3(q^2) \epsilon_\nu^\psi \epsilon_D^* \cdot q - \tilde{A}_4(q^2) \epsilon_\nu^\psi \epsilon_\psi \cdot q] \\
+ (\epsilon_\psi \cdot \epsilon_D^*)[-(p_1^\mu + p_2^\mu) \tilde{V}_1(q^2) + q_\mu \tilde{V}_2(q^2)] \\
+ \frac{\epsilon_\psi \cdot q}{m_\psi^2 - m_D^2} (\epsilon_D^* \cdot q) \left[ (p_1^\mu + p_2^\mu - \frac{m_\psi^2 - m_D^2}{q^2} q_\mu) \tilde{V}_3(q^2) \right] \\
+ \frac{m_\psi^2 - m_D^2}{q^2} q_\mu \tilde{V}_4(q^2) \right] - (\epsilon_\psi \cdot q) \epsilon_D^* \cdot \tilde{V}_5(q^2) + (\epsilon_D^* \cdot q) \epsilon_\psi \mu \tilde{V}_6(q^2),
\]

(5)

where \( q = p_1 - p_2 \) and the convention \( \text{Tr}[\gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma \gamma_5] = 4i\epsilon_{\mu\nu\rho\sigma} \) is adopted. For the transition of \( J/\psi \) into a charmed pseudoscalar meson which is induced by the weak current, there are four independent form factors: \( V, A_0, A_1, A_2 \); while there are ten form factors for \( J/\psi \) transiting into a charmed vector meson which are parameterized as \( \tilde{A}_i(i = 1, 2, 3, 4) \), \( \tilde{V}_j(j = 1, 2, 3, 4, 5, 6) \).

According to the quark diagrams in Fig. [1] the decays are classified into two categories: color allowed and suppressed processes. For the color allowed processes, the decay amplitudes are proportional to

\[
a_1 = C_1 + C_2/N_c,
\]

with \( N_c \) being the color number of QCD. Because \( C_1 \sim 1 \) and \( C_2 \sim \alpha_s \), \( a_1 \) is estimated to be of order 1. While for the decays shown in Fig. [1(b)], the amplitude is proportional to

\[
a_2 = C_2 + C_1/N_c.
\]

(6)

(7)

As \( a_2/a_1 \sim 1/N_c \), this kind of decays are usually named as color-suppressed processes.

For the decays of \( c \to \bar{sud} \) which is the Cabibbo-favored process, the CKM element \( V_{cs}V_{ud} \) responsible for these modes is close to 1. For the Cabibbo-suppressed transitions of \( c \to \bar{dud} \)
Lastly, the expressions for $q^{2}$ are written as a function of $q$. These expressions are for the decay of $J/\psi$ decays into two-body decays of $D\bar{D}$, $D^{*}P$, and $D^{*}V$, where contributions in our case are neglected, the annihilation decay modes are both color allowed and Cabibbo favored ones. The less dominant modes are the color suppressed but Cabibbo favored or color allowed but Cabibbo suppressed processes. These are the processes we will focus on in this study. Note that there is no annihilation type contributions in our case at all.

Now, we are able to write down the decay amplitudes associating with the non-leptonic two-body decays of $J/\psi$ explicitly based on the information we achieved before. In light of the characters of final states, three different types of processes $J/\psi \to DP$, $DV$, $D^{*}P$ and $D^{*}V$ will be investigated one by one in the following sections. As for $J/\psi$ decaying into two pseudoscalars where one is a $D$ meson and the other is a light meson $P$, the decay amplitude is written as

$$A(J/\psi \to DP) = \langle DP | \mathcal{H}_{eff} | J/\psi \rangle = \frac{G_{F}}{\sqrt{2}} V_{cq}^{*} V_{uq} a_{i} f_{V} m_{V} \left\{ -\epsilon_{\mu\alpha\beta} \epsilon_{\nu}^{*} \epsilon_{\psi}^{\nu} p_{\psi}^{\alpha} p_{D}^{\beta} \frac{2 V(q^{2})}{m_{\psi} + m_{D}} + i(m_{\psi} + m_{D}) (\epsilon_{\psi}^{*} \epsilon_{\nu}^{\ast} A_{1}(q^{2}) + i \frac{(\epsilon_{\nu} \cdot q) [\epsilon_{\psi}^{\nu} \cdot (p_{1} + p_{2})]}{m_{\psi} + m_{D}} 2 A_{2}(q^{2}) \right\}; \quad (8)$$

where $q$ denotes the momentum of light emitted meson; $a_{i}$ is the effective coefficients with $a_{1}$ for color allowed process and $a_{2}$ for color suppressed process.

The decay amplitude of $J/\psi \to DV$ decay is given as

$$A(J/\psi \to DV) = \frac{G_{F}}{\sqrt{2}} V_{cq}^{*} V_{uq} a_{i} f_{V} m_{V} \left\{ -\epsilon_{\mu\alpha\beta} \epsilon_{\nu}^{*} \epsilon_{\psi}^{\nu} p_{\psi}^{\alpha} p_{D}^{\beta} \frac{2 V(q^{2})}{m_{\psi} + m_{D}} + i(m_{\psi} + m_{D}) (\epsilon_{\psi}^{*} \epsilon_{\nu}^{\ast} A_{1}(q^{2}) + i \frac{(\epsilon_{\nu} \cdot q) [\epsilon_{\psi}^{\nu} \cdot (p_{1} + p_{2})]}{m_{\psi} + m_{D}} 2 A_{2}(q^{2}) \right\}; \quad (9)$$

the decay amplitude of $J/\psi \to D^{*}P$ decay can be shown as

$$A(J/\psi \to D^{*}P) = \frac{i G_{F}}{\sqrt{2}} V_{cq}^{*} V_{uq} a_{i} f_{P} \left\{ 2 i \epsilon_{\mu\alpha\beta} p_{1}^{\mu} p_{2}^{\nu} \epsilon_{\psi}^{*} \epsilon_{\psi}^{\nu} \epsilon_{D}^{\beta} \tilde{A}_{1}(q^{2}) + (\epsilon_{\psi} \cdot \epsilon_{D}^{\ast}) \left[ (m_{\psi}^{2} - m_{D}^{2}) \tilde{V}_{1}(q^{2}) - q^{2} \tilde{V}_{2}(q^{2}) \right] \right\}; \quad (10)$$

Lastly, the expressions for $J/\psi \to D^{*}V$ can be readily derived from Eqs. (3) [4] as

$$A(J/\psi \to D^{*}V) = \frac{G_{F}}{\sqrt{2}} V_{cq}^{*} V_{uq} a_{i} f_{V} m_{V} \left\{ -i \epsilon_{\mu\alpha\beta} \epsilon_{\psi}^{*} \epsilon_{D}^{\beta} \epsilon_{V}^{\mu} [p_{1}^{\nu} + p_{2}^{\nu} - \frac{m_{\psi}^{2} - m_{D}^{2}}{q^{2}} q_{\nu}^{*} \tilde{A}_{1}(q^{2}) + \frac{m_{\psi}^{2} - m_{D}^{2}}{q^{2}} q_{\nu}^{*} \tilde{A}_{2}(q^{2}) \right\}$$
\[ + \frac{i}{m_\psi^2 - m_{D*}^2} \epsilon^{\mu
u\alpha\beta} p_1^\mu p_2^\nu \epsilon_V^\mu [\bar{A}_3(q^2) \epsilon_\psi' \epsilon_\psi^\mu \epsilon_{D*} (q - \bar{A}_4(q^2) \epsilon_\psi' \epsilon_\psi \cdot q]
\]
\[- (\epsilon_\psi \cdot \epsilon_{D*}') [\epsilon_V^\mu \cdot (p_1 + p_2) \bar{V}_1(q^2)] + \frac{(\epsilon_\psi' \cdot q)(\epsilon_{D*}' \cdot q)}{m_\psi^2 - m_{D*}^2} [\epsilon_V^\mu \cdot (p_1 + p_2) \bar{V}_2(q^2)]
\]
\[- (\epsilon_\psi' \cdot \epsilon_{D*}') \epsilon_V^\mu \bar{V}_5(q^2) + (\epsilon_{D*}' \cdot q) \epsilon_\psi \cdot \epsilon_V^\mu \bar{V}_6(q^2) \}\),

where \(\epsilon_V^\mu\) denotes the polarization vector of light emitted mesons.

### III. DECAY RATES FOR NON-LEPTONIC WEAK DECAYS OF \(J/\psi\)

#### A. Input parameters

The decay rates of the non-leptonic decays \(J/\psi \rightarrow D + M\) are written as

\[
\Gamma_{\psi\rightarrow DM} = \frac{1}{3} \frac{1}{8\pi} |A(J/\psi \rightarrow DM)|^2 \frac{|P_D|}{m_\psi^2},
\]

where \(P_D\) denotes the three-momentum of the final \(D\) meson in the rest frame of \(J/\psi\) and the factor \(\frac{1}{3}\) is due to the spin average of \(J/\psi\). In order to calculate the decay rates, the input parameters including the CKM parameters, effective Wilson coefficients, decay constants and transition form factors are necessary. The CKM parameters are taken from ref. [21]

\[
V_{ud} = 0.974, \quad V_{us} = 0.227, \quad V_{cd} = 0.227, \quad V_{cs} = 0.973.
\]

The effective Wilson coefficients are determined as [5]

\[
a_1 = 1.26, \quad a_2 = -0.51,
\]

which are extracted from the isospin analysis for \(D \rightarrow K\pi\) decays with the help of the factorization ansatz [22].

The decay constants for light mesons are taken as [21, 23]

\[
f_\pi = 0.131 \text{ GeV}, \quad f_K = 0.160 \text{ GeV},
\]
\[
f_\rho = 0.209 \pm 0.002 \text{ GeV}, \quad f_{K^*} = 0.217 \pm 0.005 \text{ GeV},
\]

where the pseudoscalar decay constants are determined from the combined rate for \(P \rightarrow l^\pm \nu_l\) and \(P \rightarrow l^\pm \nu_l\gamma\) experimentally and vector meson longitudinal decay constants are extracted from the data on \(\tau^- \rightarrow (\rho^-, K^{*-})\nu_\tau\) [21].
TABLE I: The form factors $A_0$ and $A_1$ at $q^2 = 0$ responsible for the decays of $J/\psi \to D_{(s)}$ in BSW model \cite{4} and QCDSR \cite{6} approach.

| Models  | $A_0^{\psi\to D}$ | $A_0^{\psi\to D_s}$ | $A_1^{\psi\to D}$ | $A_1^{\psi\to D_s}$ |
|---------|-------------------|-------------------|-----------------|-----------------|
| BSW     | 0.61              | 0.66              | 0.68            | 0.78            |
| QCDSR   | 0.27              | 0.37              | 0.27            | 0.38            |

TABLE II: Branching ratios of non-leptonic decays of $J/\psi \to D_{(s)}P$ (in units of $10^{-10}$).

|                  | other works | this study     |
|------------------|-------------|----------------|
| $BR(J/\psi \to D_s\pi)$ | $17.4$ \cite{4} | $2.0^{+0.4}_{-0.2}$ |
|                  | $10.0$ \cite{5} |                |
| $BR(J/\psi \to D_sK)$   | $1.10$ \cite{4} | $0.16^{+0.02}_{-0.02}$ |
| $BR(J/\psi \to D\pi)$   | $1.10$ \cite{4} | $0.080^{+0.02}_{-0.02}$ |
| $BR(J/\psi \to DK)$     | —           | $0.36^{+0.10}_{-0.08}$ |

Besides, the values of all the relevant transition form factors $F_i(q^2)$ are taken from our earlier study \cite{6} where the detailed expression are presented. In the literature, the form factors $A_0$ and $A_1$ have been calculated by various authors \cite{4, 5}, which are grouped in the Table I together with the numbers obtained in the QCD sum rules \cite{6}. We can see that the form factors at zero momentum transfer predicted in the BSW model \cite{24} are approximately greater than that in the QCD sum rules by a factor 2.

B. Branching ratios of non-leptonic decays

The numerical results of branching ratios for non-leptonic decays of $J/\psi \to D_{(s)}P$ are presented in Tables II where the numbers obtained in ref. \cite{4, 5} are also collected together for a comparison. Here, the results are given for decays which including the charge conjugate process, for instance, $BR(J/\psi \to D_s\pi)$ is the branching ratio for decays of $J/\psi \to D_s^+\pi^- + D_s^-\pi^+$. Table II shows that the decay rate for color allowed and Cabibbo favored channel $J/\psi \to D_s\pi$ calculated in this work is five times smaller than that given in Ref. \cite{5}. Such discrepancy
may be attributed to two aspects: Firstly, an $SU(4)f_{ijk}$ rotation matrix is employed in Ref. [5] to relate the $J/\psi \to D_s$ transition to $D \to K^*$ decay, and the form factor $A_0(0)$ is estimated as $0.7 \sim 0.8$, which is almost twice as that computed in the QCD sum rules. Secondly, the experimental data on total decay width of $J/\psi$ used in Ref. [5] is 67.0 keV, however, this value has been updated to 93.4 $\pm$ 2.1 keV [21].

As for the Cabibbo suppressed but color-allowed mode $J/\psi \to D_sK$, the following relation

$$R_1 \equiv \frac{BR(J/\psi \to D_sK)}{BR(J/\psi \to D_s\pi)} \approx \left| \frac{V_{us}f_K}{V_{ud}f_\pi} \right|^2 \approx 0.081$$

(16)
is achieved in the factorization assumption. Similarly, we can define a parameter $R_2$ as

$$R_2 \equiv \frac{BR(J/\psi \to D\pi)}{BR(J/\psi \to D_s\pi)} \approx \left| \frac{V_{cd}A_{\psi D}\left(m_\pi^2\right)}{V_{cs}A_{\psi D_s}\left(m_\pi^2\right)} \right|^2 \approx 0.032,$$

(17)
which is also in agreement with that listed in Table II, as long as the phase space is properly considered for these two channels.

Now, we move on to the discussions of color suppressed mode $J/\psi \toDK$. A ratio of decay rates between it and $J/\psi \to D_s\pi$ can be estimated as

$$R_3 \equiv \frac{BR(J/\psi \to DK)}{BR(J/\psi \to D_s\pi)} \approx \left| \frac{a_2A_{\psi D}^D\left(m_\pi^2\right)}{a_1A_{\psi D_s}^D\left(m_\pi^2\right)} \right|^2 \approx 0.18,$$

(18)
which is consistent with that collected in Table II. In addition, we should emphasize that the ratio $R_3$ is quite sensitive to the effective Wilson coefficient $a_2$, which can receive considerable corrections [24, 26, 27, 28, 29, 30, 31, 32] due to uncertainties of the renormalization scale, higher order effects together with non-factorizable contributions, where we also refer to [20] for a recent comment.

Table III collects the numerical results for $J/\psi \to D_{(s)}^*P$ and $D_{(s)}V$ decays. As one can see, the branching ratio of $J/\psi \to D_{s}\rho$ computed in [4] is 5.8 times larger than that evaluated in this work. It is shown in [4], the dominant contributions for decay width of $B \to D^{(*)}P$ are from the form factor $A_1(q^2)$ corresponding to the S partial wave in the final states. As listed in Table II, the number of form factor $A_1$ derived in the BSW model is 2.1 times greater than that in terms of the QCD sum rules, which can indeed result in an enormous discrepancy for the branching fraction of $J/\psi \to D_s$ obtained in two different approaches. Moreover, the ratio

$$R_4 \equiv \frac{BR(J/\psi \to D_s\rho)}{BR(J/\psi \to D_s\pi)},$$

(19)
TABLE III: Branching ratios of non-leptonic decays of $J/\psi \rightarrow D^{*} P$ and $D_{s} V$ (in units of $10^{-10}$).

| Decay                  | Other works | This study |
|------------------------|-------------|------------|
| $BR(J/\psi \rightarrow D_{s} \rho)$ | 72.6 [4]    | $12.6^{+3.0}_{-1.2}$ |
| $BR(J/\psi \rightarrow D_{s} K^{*})$ | 4.24 [4]    | $0.82^{+0.22}_{-0.10}$ |
| $BR(J/\psi \rightarrow D \rho)$     | 4.40 [4]    | $0.42^{+0.18}_{-0.08}$ |
| $BR(J/\psi \rightarrow D K^{*})$   | —           | $1.54^{+0.68}_{-0.38}$ |
| $BR(J/\psi \rightarrow D^{*}_{s} \pi)$ | —           | $15.0^{+1.2}_{-0.4}$ |
| $BR(J/\psi \rightarrow D^{*}_{s} K)$ | —           | $1.1^{+0.08}_{-0.04}$ |
| $BR(J/\psi \rightarrow D^{*} \pi)$  | —           | $0.60^{+0.04}_{-0.04}$ |
| $BR(J/\psi \rightarrow D^{*} K)$    | —           | $2.6^{+0.2}_{-0.2}$ |

TABLE IV: Branching ratios of non-leptonic decays of $J/\psi \rightarrow D^{*}_{(s)} V$ (in units of $10^{-10}$).

| Channels                | This study |
|-------------------------|------------|
| $BR(J/\psi \rightarrow D^{*}_{s} \rho)$ | $52.6^{+7.2}_{-6.2}$ |
| $BR(J/\psi \rightarrow D^{*}_{s} K^{*})$ | $2.6^{+0.4}_{-0.4}$ |
| $BR(J/\psi \rightarrow D^{*} \rho)$    | $2.8^{+0.6}_{-0.4}$ |
| $BR(J/\psi \rightarrow D^{*} K^{*})$   | $9.6^{+3.2}_{-2.2}$ |

is usually introduced from a viewpoint of experiment, whose value is estimated as 6.3 and 4.2 respectively in the framework of QCDSR and BSW model. Therefore, the decay of $J/\psi \rightarrow D_{s} \rho$ is more detectable than the corresponding pseudoscalar channel $J/\psi \rightarrow D_{s} \pi$ in experiment. Moreover, it can be seen that the ratio of decay rates is not sensitive to the absolute magnitude of the transition form factors on account of the large cancelations of the non-perturbative effects. The relative magnitude of decay widths for the Cabibbo-suppressed as well as color suppressed processes to the mode of $J/\psi \rightarrow D_{s} \rho$ can be readily derived by following the discussions on $J/\psi \rightarrow D_{(s)} P$ and will not be repeated again.

Furthermore, we group the decay rates for dominant channels of $J/\psi \rightarrow D^{*}_{(s)} V$ in Table [IV] from which we can observe that $BR(J/\psi \rightarrow D^{*}_{s} \rho)$ is as large as $5.3 \times 10^{-9}$ and stands as the most promising mode to be measured at BESIII. Such finding presents a striking con-
contrast to the argument given by the authors of Ref. [1] where the authors claimed that a specific non-leptonic decay channel like $J/\psi \to D_s^{(*)} M$ ($M = \pi, \rho...$) is hardly to be detected owing to the tiny branching factions for these processes. In addition, it is also helpful to define the following two ratios

$$R_5 \equiv \frac{BR(J/\psi \to D_s^{*}\pi)}{BR(J/\psi \to D_s\pi)}, \quad R_6 \equiv \frac{BR(J/\psi \to D_s^{*}\rho)}{BR(J/\psi \to D_s\rho)},$$

(20)

which characterize the relative size of branching fractions to distinguish the final states with vector and pseudoscalar ones respectively in the non-leptonic two-body weak decays of $J/\psi$. The numbers of $R_5$ and $R_6$ are evaluated as 7.5 and 4.2 in the QCD sum rules, while they are determined as 3.5 and 1.4 respectively with the ISGW model in the framework of heavy quark spin symmetry [1]. Such discrepancies can be attributed to the different values of form factors employed in the numerical calculations.

Moreover, we also mention that

$$R_7 \equiv \frac{BR(J/\psi \to D_s^{*}K^{*})}{BR(J/\psi \to D^{*}\rho)} \cdot \frac{BR(J/\psi \to D\rho)}{BR(J/\psi \to D_sK^{*})}$$

(21)

should be equal to 1 in the heavy quark limit. However, this ratio is estimated as 0.48 in the QCD sum rules owing to a serious suppression factor from the phase space for the decay of $J/\psi \to D_s^{*}K^{*}$ for the limited charm quark mass.

Combining the Table II, III and IV, we find that the branching ratio for inclusive weak decay of $J/\psi$ can be as large as $1.3 \times 10^{-8}$, which is also in remarkable agreement with the naive estimation

$$BR(J/\psi \to X_c + ... ) \approx \frac{2\Gamma_{D^\pm}}{\Gamma_{J/\psi}} \approx 1.4 \times 10^{-8}.$$

(22)

IV. DISCUSSIONS AND CONCLUSIONS

Since $J/\psi$ mainly decays via strong and electromagnetic interactions, its weak decays usually take small fractions which cannot be measured by available experimental apparatus. On other aspect, however, because $J/\psi$ contains two heavy constituents, its weak decay may possess a unique character. Indeed weak decays of $J/\psi$ may offer an ideal platform to examine the mechanism which governs the hadronization process, without possible contamination from the light spectator as well as one may determine its fundamental parameters such as
the CKM matrix which can be a complementary test to the values obtained in $D$ decays. It is lucky for high energy physicists that a tremendous database on $J/\psi$ will be available in the forthcoming BESIII and the measurements on the weak decays of $J/\psi$ may become possible.

As is well known, the essential challenge in the theoretical calculations on the rates of weak decays of $J/\psi$ is to disentangle the underlying weak-interaction transitions from the notorious effects owing to strong interactions reasonably $[34]$. In our previous paper $[6]$, the transition form factors in the semi-leptonic weak decays of $J/\psi$ have been investigated to the leading order of $\alpha_s$ based on QCD sum rules, where the non-perturbative QCD dynamics is characterized by a few universal parameters. The branching ratios for dominant exclusive processes are evaluated and their order of magnitude is typically at $10^{-10}$. Obviously based on the factorization assumption, the form factors obtained for the semi-leptonic decays can be applied to study the non-leptonic decays.

This paper can be viewed as a continuation of our earlier work $[6]$. We present a comprehensive study of non-leptonic decays of $J/\psi \to D(s) + M$ based on the factorization assumption and apply the transition form factors calculated in the QCD sum rules. It is observed that the sum of the branching fractions for the dominant non-leptonic decays of $J/\psi \to D^-_s\pi$, $D^-_s\rho$, $D^{*-}_s\pi$, and $D^{*-}_s\rho$ as well as their charge conjugate channels can reach as large as $0.82 \times 10^{-8}$, a special decay mode $J/\psi \to D^{*-}_s\rho$ can even arrive at $5.3 \times 10^{-9}$, which is hopefully to be marginally detected in the $e^+e^-$ colliders in view of the large database of the BESIII. Our results are in agreement with the finding in Ref.$[1]$ that $J/\psi$ decays to vector charmed meson $D^{*-}_s$ more favorably than to the pseudoscalar one, however, the ratios of these two channels calculated in this work is twice or three times larger than that given by Ref.$[1]$, where heavy quark spin symmetry and the non-recoil approximation were adopted and the ISGW model was employed to compute the single form factor $\eta_{12}$.

As there is a light to see a possibility of measuring weak decays of $J/\psi$ which have obvious advantages for getting insight to the physics picture, we strongly urge our experimental colleagues to search for vector charmed mesons productions in $J/\psi$ decays at BESIII $[2, 3]$.

Moreover, from the theoretical side, it should be emphasized that Coulomb-type corrections for the heavy quarkonium system $[35, 36, 37, 38]$ are not included in the computations of form factors in the QCD sum rules, which could induce additional uncertainties to the
evaluation of the branching fractions for non-leptonic two body decays of $J/\psi$. However, one can still trust the order of magnitude gained in this work, since while calculating the form factors which need to deal with the three-point correlations, most uncertainties originating from Coulomb-like corrections are canceled by that in the two-point correlations for evaluating the decay constant of $J/\psi$. Apart from weak decays of $J/\psi$ presented in this paper, weak decays of $\Upsilon$ are suggested to be explored seriously in a complementary fashion from both the theoretical and experimental point of view.

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