Application of Synchrosqueezed Wavelet Transform in Microseismic Monitoring of Mines

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Abstract. Microseismic monitoring technique is an important means of ground pressure monitoring to ensure safe, high-efficient and sustainable development of mines. Microseismic data obtained by sensors in mines are easily influenced by non-stationary noises with a wide frequency band, resulting in the lack of available high-quality data for microseismic monitoring. Instead of traditional analysis in frequency domain, this paper introduces a new method, synchrosqueezed wavelet transform (SSWT), which provides a way to decompose data into time domain and frequency domain simultaneously. With higher time-frequency resolution of SSWT spectrum, purer microseismic signals can be extracted from raw data. Besides, two wavelet bases, Morlet wavelet and bump wavelet, are compared to match the microseismic signal in this paper. Two field data with different signal-noise rate (SNR) are used to show the application of the algorithm in the mine industry. The results of data graphical filtering method show that the SSWT has great practical value to extract the microseismic signal from raw data and improves SNR of signals effectively than traditional methods.

1. Introduction

The presence of dynamic disasters, such as roof fall, collapse and rock burst, is one of the main threats to the safety and efficient production of mine, especially in underground mines. [1-2] These disasters are usually caused by stress concentration during mining. Microseism is a small tremor caused by rock failure with low amplitude. Microseismic monitoring technology is to get and analyze the information of rocks for mine safety assessment. [3-5] Though this technology grows rapidly in recent years, the raw data collected in mines still cannot meet the requirements, since microseismic signals are easily mixed by environmental noises. Therefore, we need an effective way to analyze and obtain purer microseismic signals.

The traditional method is to analyze signals in frequency domain by Fourier transform (FT), but it cannot be used for non-stationary signals, such as the microseismic signal. Wavelet transform can not only make up this defect, but also provide a way to analyze signals in time-frequency domain instead of only in frequency domain. Although higher time-frequency resolution is obtained, continuous wavelet...
transform (CWT) has a serious defect, the information redundancy.[6] Synchrosqueezed wavelet transform (SSWT)[7] is a reassignment algorithm based on CWT, where the frequency variable is used to replace the scale variable to increase the spectral resolution.[8-10] The time-frequency spectrum obtained by SSWT is more readable and provides more information of signals and noises than the methods mentioned before.[11-13]

In this paper, SSWT is studied and applied into microseismic analysis in mine industry. First, the SSWT algorithm is introduced in section 2. Then we contrast and compare SSWT with CWT in section 3. Two typical kinds of wavelet bases, Morlet wavelet and bump wavelet, are also compared to match the microseismic signal. In section 4, two field data with different signal-noise rate (SNR) are used to testify the effectiveness and feasibility of SSWT, especially in aspects of signal analysis and filtering.

2. Synchrosqueezed wavelet transform (SSWT)

Traditionally, a time varying signal \( s(t) \) can be expressed in superposition form of multiple harmonic signals as[8-9]:

\[
s(t) = \sum_{k=1}^{K} A_k(t) \cos(\theta_k(t) + \eta(t))
\]

(1)

Where \( A_k(t) \) and \( \theta_k(t) \) are the instantaneous amplitude and instantaneous phase of \( k^{th} \) component, respectively. \( \eta(t) \) represents the noise signal and \( K \) is the number of components in the signal \( s(t) \).

By using CWT, the wavelet coefficient of \( s(t) \) can be written as:

\[
W_s(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} s(t) \psi^\prime \left( \frac{t-b}{a} \right) dt
\]

(2)

Where \( \psi^\prime \) is the complex conjugate of a mother wavelet, \( a \) is the scale variable, and \( b \) represents time shift. According to the Plancherel's theorem, (2) can be rewritten in frequency domain:

\[
W_s(a,b) = \frac{1}{2\pi} \int \frac{1}{\sqrt{a}} \hat{s}(\xi) \hat{\psi}^\prime(a\xi)e^{i\xi} d\xi
\]

(3)

Where \( \xi \) is the angular frequency, \( \hat{s}(\xi) \) and \( \hat{\psi}(\xi) \) are the Fourier transform of \( s(t) \) and \( \psi(t) \), respectively.

Consider the signal in a simple case \( s(t) = A \cos(\omega t) \), where \( \omega \) is the central frequency of the signal, whose Fourier pair is:

\[
\hat{s}(\xi) = \pi A \left[ \delta(\xi - \omega) + \delta(\xi + \omega) \right]
\]

(4)

Then, (3) can be transformed into

\[
W_s(a,b) = \frac{A}{2} \int \frac{1}{\sqrt{a}} \left[ \delta(\xi - \omega) + \delta(\xi + \omega) \right] \hat{\psi}^\prime(a\xi)e^{i\xi} d\xi = \frac{A}{2\sqrt{a}} \hat{\psi}^\prime(\omega)e^{i\omega a}
\]

(5)

In frequency domain, the wavelet \( \hat{\psi}^\prime(\xi) \) should be concentrated around \( \xi = \omega_b \), where \( \omega_b \) is the central frequency of wavelet, and \( W_s(a,b) \) should be concentrated around the scale \( a = \omega_b / \omega \). However,
the actual time spectrum is distributed in a certain range near $\omega_0$, leading to a blurred representation in time-frequency spectrum.

In the CWT spectrogram, for any time-scale location $(a,b)$, if $W_r(a,b) \neq 0$, then the candidate instantaneous frequency corresponding here is

$$\omega(a,b) = -iW_r(a,b) \cdot \frac{\partial W_r(a,b)}{\partial b}$$

With the $(\omega(a,b),b)$ corresponding to points $(a,b)$, the time-scale spectrum can be rearranged into time-frequency spectrum, this operation is called synchrosqueezing.

In this way, $W_r(a,b)$ can only be computed at binned discrete scale variable $a$ and frequency variable $\omega$, where $\Delta a_k = a_k - a_{k-1}$. And the SSWT $T_s(\omega_k, b)$ is determined by the centers $\omega_k$ of the interval $[\omega_k - \frac{1}{2} \Delta \omega, \omega_k + \frac{1}{2} \Delta \omega]$, where $\Delta \omega = \omega_k - \omega_{k-1}$:

$$T_s(\omega_k, b) = \frac{1}{\Delta \omega} \sum_{a, |(a,b) - a| \leq \frac{\Delta \omega}{2}} W_r(a_k, b) \delta^{\frac{3}{2}} \Delta a_k$$

The inverse transform of the SSWT is:

$$s(t) = \text{Re} \left[ C_s \sum_{\omega} T_s(\omega, b)(\Delta \omega) \right]$$

Where $C_s = \int_0^\infty \psi^*(\xi) \frac{d\xi}{\xi}$ is the Fourier transform of the conjugate of wavelet function, and $\text{Re}$ is the real part of the component.

3. Parameter selection

The choice of the wavelet basis is a key issue in SSWT. Two wavelet bases, Morlet wavelet [9] and bump [12] wavelet, which have been widely used in seismic, are tested to match the microseismic signal.

Morlet wavelet has no scale function and it’s non-orthogonal decomposition, which is defined by:

$$\psi(x) = e^{\pi x^2} \cos(5x)$$

In the Fourier domain, bump wavelet is defined with parameters $\mu$ and $\sigma$:

$$\hat{\psi}(s) = e^{\left(1 - (s/\sqrt{(\mu-\mu_0)^2})/\sigma^2\right)} \left[1_{((\mu-\sigma)/\sigma, (\mu+\sigma)/\sigma)}\right]$$

Where $1_{[\mu-\sigma)/\sigma, (\mu+\sigma)/\sigma]}$ is the indicator function for the interval $[(\mu-\sigma)/\sigma, (\mu+\sigma)/\sigma]$. Smaller the $\sigma$ is, much better the frequency localization is, while the time localization is poorer. And vice versa.

The number of voices per octave in CWT used here is 32. We get a microseismic signal from a metal mine in XinJiang as it shown in Figure 1.
Figure 1. The microseismic signal of a metal mine in XinJiang.

The field microseismic data shown in Figure 1 has high SNR, where noise has little effect on the microseismic signal.

Figure 2. CWT spectrum with Morlet wavelet.

Figure 3. CWT spectrum with bump wavelet.

Figure 4. SSWT spectrum with Morlet wavelet.

Figure 5. SSWT spectrum with bump wavelet.

The CWT spectrum based on Morlet wavelet is shown in Figure 2, and the spectrum that bump wavelet is used in CWT is shown in Figure 3. In time-frequency spectrum, both wavelet bases
characterize the dominant frequency band with their corresponding time window. They have a similar energy distribution in time domain, while in frequency domain, the spectrum in Figure 2 is less clear than that in Figure 3, because it has a wider range of frequency leakage.

Based on CWT, the time-frequency spectrum in SSWT also shows corresponding characteristics in Figure 4 and Figure 5. With the operation of synchrosqueezing, the spectrum is clearer in SSWT than in CWT both in time and frequency axis. And the trend of energy shows the position of microseismic signal.

Another important parameter is the wavelet threshold $\gamma$. It decides the lowest magnitude of CWT spectrum $W(a,b)$. The hard threshold is set as $10^{-8}$ in ideal non-noise condition. While in real cases, where the field noise is unknown, median absolute deviation of the finest scale of the wavelet decomposition is used to estimate the noise variance $\delta_n^2$:

$$\delta_n = \frac{\text{median} (|W(a_{v_0},b) - \text{median}(W(a_{v_0},b))|)}{0.6754}$$

Where $W(a_{v_0},b)$ are the coefficients of the finest scale wavelet. The threshold is

$$\gamma = \sqrt{2\log n \cdot \delta_n}$$

### 4. Field data filtering examples

According to the different value of SNR, there are two examples of signal filtering shown here. The first example is the field data mentioned in section 3, which has a high SNR value. In the other example, we select a low SNR microseismic signal, which also comes from the metal mine in XinJiang. In these examples, we use graphical method to analyze and filter the signal in SSWT spectrum based on the bump wavelet.

#### 4.1. High SNR field data

The field data shown in Figure 1 is a high SNR signal. Its SSWT spectrum can be seen in Figure 5. It is clear that the frequency bandwidth of microseismic signal is between 100 and 600Hz. So we divide the frequency domain into two parts to filter the signal: non-signal part and signal with noise part. In non-signal part, we use the empirical value, obtained by estimating the mine microseismic signals, as the hard threshold. In signal with noise part, we use the adaptive threshold with noise variance $\delta_n^2$, where $\delta_n^2$ should be computed only in 100–600Hz. With the inverse transform of SSWT, the filtered microseismic data shown in Figure 6. (b) is obtained and its SSWT spectrum is shown in Figure 7.

![Figure 6. SSWT spectrum with Morlet wavelet](image1)

![Figure 7. The SSWT spectrum of filtered data](image2)
4.2. Low SNR field data

This field data (shown in Figure 8. (a)) has a lower SNR than the signal in 4.1, where microseismic signal is weak under strong noise. Using Fourier transformation, the Fourier spectrum is shown in Figure 9, and its SSWT spectrum is shown in Figure 10.

![Figure 8. Original data and its filtered data](image1)

![Figure 9. The Fourier spectrum of field data](image2)

This field data is quite complicated since its noise signal is not only in all time domain, but also in all frequency domain. From Fourier spectrum, it’s hard to tell exactly where the dominant frequency band of microseismic signal is. While in SSWT spectrum, we can easily figure out the distribution of the noise and the position of microseismic signal. Using graphical method with threshold mentioned above, we can obtain the filtered microseismic signal as shown in Figure 8. (b), and spectrum shown in Figure 11.

![Figure 10. The SSWT spectrum of field data](image3)

![Figure 11. The SSWT spectrum of filtered data](image4)

5. Conclusions

This paper introduces SSWT analysis in microseismic monitoring of mines. Compared with traditional methods, SSWT can decompose the signal into time domain and frequency domain simultaneously. The analysis based on time-frequency domain presents more information and details of signals. Different from CWT, SSWT has a higher resolution and strong readability. By using frequency scale, it prevents the information redundancy and makes energy more concentrate on its frequency scale. In addition, bump wavelet seems more compatible with microseismic signals than Morlet wavelet. Based on SSWT
with bump wavelet, the results of data graphical filtering method with time-frequency spectrum show that high SNR filtered data is achieved.

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References
[1] Xuan D, Xu J, Zhu W.. Dynamic disaster control under a massive igneous sill by grouting from surface boreholes. International Journal of Rock Mechanics and Mining Sciences, 2014, 71: 176-187.
[2] Zheng X, Xu K, Wei Y.. Study on the disaster-causing mechanism of the tailings dam falling [J]. Journal of Safety Science and Technology, 2008, 5.
[3] Maxwell, Shawn C., et al. Petroleum reservoir characterization using downhole microseismic monitoring. Geophysics, 2010, 75.5: 75A129-75A137.
[4] Warpinski, Norm, et al. Microseismic monitoring: Inside and out. Journal of Petroleum Technology, 2009, 61.11: 80-85.
[5] Sun J.. Mine safety monitoring and control technology and system. Coal Science and Technology, 2010, 38.10: 1-4.
[6] Qiu H., et al. Wavelet filter-based weak signature detection method and its application on rolling element bearing prognostics. Journal of sound and vibration, 2006, 289.4-5: 1066-1090.
[7] Daubechies I., Lu J., Wu H. T.. Synchrosqueezed wavelet transforms: An empirical mode decomposition-like tool. Applied and computational harmonic analysis, 2011, 30.2: 243-261.
[8] Mousavi S. M., Langston C. A., Horton S. P.. Automatic microseismic denoising and onset detection using the synchrosqueezed continuous wavelet transform. Geophysics, 2016, 81.4: V341-V355.
[9] Herrera R. H., Han J., Van der Baan M.. Applications of the synchrosqueezing transform in seismic time-frequency analysis. Geophysics, 2014, 79.3: V55-V64.
[10] Auger F., Flandrin P., et al. Time-frequency reassignment and synchrosqueezing: An overview. IEEE Signal Processing Magazine, 2013, 30.6: 32-41.
[11] Wang P., Gao J., Wang Z.. Time-frequency analysis of seismic data using synchrosqueezing transform. IEEE Geoscience and Remote Sensing Letters, 2014, 11.12: 2042-2044.
[12] Herrera R. H., Tary J. B., Van der Baan M.. Time-frequency representation of microseismic signals using the synchrosqueezing transform. arXiv preprint arXiv:1301.1295, 2013.
[13] Qin X., Song W.. Weak signal extraction method of microseismic data based on synchrosqueezing transform. Geophysical Prospecting for Petroleum, 2016, 55.1: 60-66.