Application of time-varying heat transfer model

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Abstract. The process of heat conduction has important practical significance in industrial production. This paper takes the central temperature of electronic components as the research object, uses the Fourier one-dimensional heat conduction equation and the separation variable method, uses Newton's cooling law to deal with the boundary conditions, studies the heat transfer model of electronic components, and three-dimensional problems become one dimensional problem, thus providing some feasible ideas for improving industrial production.

Keywords: Time-varying partial heat transfer model, Fourier one-dimensional heat transfer equation, separation of variables method.

1. Introduction
During the production of integrated circuit board, the printed circuit board needs to be placed in the rewelding furnace, and the electronic components are heated to make it automatically welded to the circuit board. Through the temperature sensor, the central temperature of the welding area can be measured to form a temperature curve, reflecting the heat transfer of electronic components under different temperatures in the furnace. We need to establish a thermodynamic physical model to study the influence of the temperature change of the rewelding furnace on the temperature of electronic components. This paper takes "Furnace temperature Model" as the research background in question A of China Mathematical Modeling Contest in 2020 as the research background. When electronic components pass through different temperature environments, heat exchange continuously takes place with the outside world, so as to achieve balance.

2. The establishment process of heat conduction model
2.1. Propose the heat conduction model
We study the changes of furnace temperature curve, that is, the temperature change of the circuit board welding area. Due to the circuit board with the conveyor belt moved to the area within the scope of different temperature, so with the passage of time, the environment air temperature change, heat is passed to the welding area also produces change, so the welding area temperature change over time, we set up the unsteady heat transfer model. At the same time, the information shows that the welding area has a certain thickness, and the temperature sensor is located at the center of the welding area, so we need to study the heat conduction process inside the circuit board welding area.
First, we idealize the soldering area of the circuit board as a thermally conductive block with a thickness of $2\delta$. We assumed that the temperature $u = u(x,t)$ inside the block was a two-dimensional function of space and time, and the temperature outside the block was the air temperature $T$ in the furnace obtained from the previous model. We make a schematic diagram of the ideal heat conduction block in the welding area, as shown in Figure 1.

![Figure 1. Simplified diagram of welding area](image)

After simplifying the welding area, we analysed the heat conduction inside the welding area. The essence of heat conduction is that the thermal motions of molecules collide with each other, so that energy is transferred from the high temperature part to the low temperature part of the object. The heat conduction process is mainly calculated based on Fourier law [1]. We only study the heat transfer in the $x$ direction in Figure 1, and the Fourier law is written as:

$$ q = -\lambda \frac{du}{dx} $$  \hspace{1cm} (1)

$q$ denotes the heat flux, $\lambda$ denotes the thermal conductivity. This formula shows that the heat flux is proportional to the temperature gradient, but the direction of heat flux is opposite to the direction of temperature gradient, which is also consistent with our basic understanding of heat transfer. Energy flows from the high temperature part to the low temperature part.

Meanwhile, according to the first law of thermodynamics $Q = \Delta U$, energy is conserved in the process of transfer and conversion, and the net heat inflow and outflow is equal to the increase of heat energy. We can obtain the time-varying one-dimensional partial differential heat transfer model as:

$$ \rho c \frac{\partial u}{\partial t} = \frac{\partial}{\partial x} \left( \lambda \frac{\partial u}{\partial x} \right) $$  \hspace{1cm} (2)

$\rho$ denotes the density, $c$ denotes the specific heat capacity, which can be obtained after finishing $\frac{\partial u}{\partial t} = \frac{\lambda}{\rho c} \frac{\partial^2 u}{\partial x^2}$, and because $\alpha = \frac{\lambda}{\rho c}$, then the partial differential equation of heat conduction in the welding area is $\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$.

In addition, there is thermal convection between the boundary and the air in the welding area, so we need to study the thermal convection equation to determine the boundary conditions of the model. The thermal convection process is the heat transfer process in which the relative displacement of particles in the fluid occurs, which is mainly calculated based on Newton's cooling law [2]. We analyze the Newton cooling formula on the boundary of the welding area as follows $q = h\Delta u$. Then we analyze the heat
input from the object at the boundary is all from the convection tropics, and combining with Fourier's law, we get the boundary condition is \(-\lambda \frac{\partial u}{\partial x} = h\Delta t\).

Then we analyze the partial differential equations of the interior and boundary of the welding area, solve the temperature change in the center of the welding area, and obtain the unsteady heat transfer model of the welding area.

\[
\begin{align*}
\frac{\partial u}{\partial t} &= \alpha \frac{\partial^2 u}{\partial x^2} \\
u &= u_0, t = t_0 \\
\frac{\partial u}{\partial x}, x = 0 \\
-\lambda \frac{\partial u}{\partial x} &= h(T - u), x = \delta
\end{align*}
\]

### 2.2. Solve the heat conduction model

In order to solve the above model, we introduce the concept of a residual temperature, namely as an object of arbitrary shape is placed in a fluid of constant temperature \(T(t)\), when the object is cooled to \(t\), we define the excess temperature \(\theta = t - t(\infty)\). We need to absorb heat from the air, then the object is heated, finally we concluded that the temperature is \(\theta = T - u(x, t)\), the use of more than a equations of temperature we can above to the following form:

\[
\begin{align*}
\frac{\partial \theta}{\partial t} &= \alpha \frac{\partial^2 \theta}{\partial x^2}, 0 < x < \delta, t > 0 \\
\theta &= \theta_0, t = 0 \\
\frac{\partial \theta}{\partial x}, x = 0 \\
-\lambda \frac{\partial \theta}{\partial x} &= h\theta, x = \delta
\end{align*}
\]

The partial differential equation is solved by the separation of variables method, and the original equation is decomposed into several more easily solved equations. By using the separation of variables method, its analytical solution can be obtained as: \(\frac{\theta(x, t)}{\theta_0} = \sum_{n=1}^{\infty} C_n \exp(-\mu_n^2 F_n) \cos(\mu_n \eta)\), and

\[
C_n = \frac{2\sin \mu_n}{\mu_n + \sin \mu_n \cos \mu_n}, F_n = \frac{\alpha t}{\delta^2}, \eta = \frac{x}{\delta}.
\]

\(\mu_n\) is the root of the transcendental equation \(\tan \mu_n = \frac{B_n}{\mu_n}\), \(n = 1, 2, \ldots\), which is called the eigenvalue, which we're going to get \(\cot \mu_n = \frac{\mu_n}{h\delta / \lambda}\) by deforming the transcendental equation, \(B_n = \frac{h\delta}{\lambda}\) is called the Beward criterion number.

Time-varying heat transfer temperature field depends on two aspects: first, the speed of heat transfer between the medium and the surface of the object is determined by the heat resistance \(1/h\) of the
surface heat transfer. The other is that the speed of thermal conductivity of the object itself is determined by its internal thermal resistance $L/\lambda$. The Beword criterion $B_i = \frac{L/\lambda}{1/h}$ indicates the relative size of the thermal conduction resistance inside the object and the heat transfer resistance on the surface, $L$ is one half of the thickness of the block [3].

Next, we discuss the influence of the values on the model solution. When $B_i \rightarrow 0$, that is, the convective thermal resistance is much greater than the thermal resistance, and the internal temperature of the object is almost uniform, that is to say, the temperature field of the object is only a function of time and has nothing to do with the spatial coordinates. We call such an unsteady thermal conductivity system the lumped heat capacity.

The applicable condition of lumped parameter method is that, on this basis $0 < B_i < 0.1$, excess temperature is introduced, and the temperature field of lumped heat capacity object can be obtained through integral operation: $\theta = \theta_0 \exp\left(-\frac{hA}{\rho c V} t \right)$, $V$ is the volume of the block, $A$ is the heat conduction area of the block. In fact, we consider the temperature curve of drawing is considered $u(0,t) = T - \theta = T - \theta = T - \theta_0 \exp\left(-\frac{hA}{\rho c V} t \right)$. $\theta_0 = T_0 - u_0(0,t)$ is the temperature difference between inside and outside the block when the initial temperature change is made, at the same time as the air temperature in the furnace is about one dimensional linear piecewise function of the space, we need to convert it to about the function of time, because of $x_1 = vt$, $v$ says the speed of conveyor belt, then can be converted into one dimensional linear piecewise function $T(t)$ about time, then said furnace temperature curve function is:

$$u(t) = T(t) - (T_0(t) - u_0(t))e^{-kt}$$

We took the experimental data to obtain the unknown variable $k$, and then fitted the experimental data according to the analytical formula [4], and found that the calculated value and the experimental value were poorly fitted. Therefore, it was unreasonable for us to consider that the unsteady thermal conductivity system was lumped heat capacity.

![Figure 2. Experimental data fitting diagram](image-url)
Since lumped parameter method is not applicable, we can only analyze $B_i > 0.1$, we introduce the excess temperature and solve the analytical solution with the separation variable method is 
\[
\frac{\theta(x,t)}{\theta_0} = \sum_{n=1}^{\infty} C_n \exp(-\mu_n^2 F_0 \sin(\mu_n \eta / \delta)) \cos(\mu_n \eta),
\]
we know 
\[
C_n = \frac{2 \sin \mu_n}{\mu_n + \sin \mu_n \cos \mu_n}, \quad F_0 = \frac{\alpha t}{\delta^2}, \quad \eta = \frac{x}{\delta}.
\]
Because the Fourier number $F_0 = \frac{\alpha t}{\delta^2}$, we know through data access general metal thermal diffusion coefficient around in $10^{-9}$ [5], and the thickness of 0.15 mm of content block, then $\delta = 7.5 \times 10^{-5}$ m, we calculate the Fourier number $F_0 > 0.2$, so just take the first item in the series has enough precision, 
\[
\frac{\theta(x,t)}{\theta_0} = \frac{2 \sin \mu_1}{\mu_1 + \sin \mu_1 \cos \mu_1} \exp(-\mu_1^2 F_0 \sin(\mu_1 \eta)),
\]
because we need to solve the core temperature is the temperature of welding area, namely the temperature $x = 0$, because of $\eta = \frac{x}{\delta}$, so the original formula can be reduced to 
\[
\frac{\theta(0,t)}{\theta_0} = \frac{2 \sin \mu_1}{\mu_1 + \sin \mu_1 \cos \mu_1} \exp(-\mu_1^2 F_0),
\]
then the said furnace temperature curve function is:
\[
u(t) = T(t) - (T_0(t) - u_0(t))k_1e^{-k_2t}
\] (6)

3. Test the heat conduction model
In this problem, we mainly establish the unsteady one-dimensional partial differential heat transfer model, and the function expression obtained contains unknown parameters $k_1$ and $k_2$. In order to solve the unknown parameters, we use the experimental data for fitting, and find the corresponding function parameter values in the best fitting case, $k_1 = 0.991, k_2 = -0.004795$.

The graph of furnace temperature curve function is made according to the model established by us. At this point, the graph drawn by the unsteady heat transfer model under initial conditions is compared with the graph drawn by the original data in the attachment, and the following figure is drawn:

![Figure 3. Simulation calculation data fitting diagram](image)

It can be seen that the fitting effect of the two is good. The correlation coefficient R2 of calculated value and measured value fitting is 0.9972, RMSE is 2.777 and SSE is 5421. This further illustrates the applicability of our model to this problem.
Part of the data fitted by the model is compared with the actual measured data, and the following table is obtained:

**Table 1. Simulation data and actual data**

| Simulation data (℃) | actual data (℃) |
|---------------------|-----------------|
| 30.35               | 30.03           |
| 31.45               | 30.48           |
| 32.55               | 30.95           |
| ......               | ......           |
| 151.99              | 149.26          |
| 152.16              | 149.52          |
| 152.32              | 149.78          |
| ......               | ......           |
| 139.59              | 145.01          |
| 138.66              | 144.4           |
| 137.72              | 143.79          |

4. Conclusion

Through theoretical analysis, the research and the establishment of the model, can get electronic components welding temperature distribution center, according to the analytic expression for fitting the experimental data, found that the calculated value and experimental value fitting degree good, according to the time-varying partial differential heat transfer model, assuming that the temperature zone 1~5 temperature is 173℃, the temperature zone 6 temperature is 198℃, temperature zone 7 temperature is 230℃, the temperature zone 8 ~ 9 temperature is 257℃, we first consider the air temperature in the furnace after reaching stability diagram function change.

![Figure 4. Air temperature change diagram in furnace](image)

Python software was used to bring in the unsteady partial differential heat transfer model for calculation, and the temperature change in the center of the welding area was obtained, and the corresponding furnace temperature curve was made, as shown in the figure below.
Figure 5. Diagram of furnace temperature curve

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