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Multi-objective optimization of Nd:YAG laser cutting of nickel-based superalloy sheet using grey-fuzzy approach

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Abstract: A nickel-based superalloys sheet finds its applications in many industries. Laser cutting of these alloy sheets are very important from the quality point of view. Thus, it is important to have an optimal combination of parameters in order to achieve quality responses. Keeping this in view, grey relation analysis (GRA) aided with fuzzy logic is applied for multi-objective optimization of Nd:YAG laser beam cutting (LBC) of nickel-based superalloy (SUPERNI 718) sheet. The input parameters considered for this process are assist gas pressure, pulse width, pulse frequency, and cutting speed to optimize response parameters considered as kerf width, kerf deviation and kerf taper. The result found using this approach shows that for optimal response the input parameters need to be set as gas pressure at 2kg/cm$^2$, pulse width at 0.6μs, pulse frequency at 23Hz, and cutting speed at 20mm/min. The optimal parametric mix found using our approach is similar to that found by the previous researchers, which is also supported by technique for order preference by similarity to ideal solution (TOPSIS). Analysis of variance is also applied to identify the significance of each process parameters in the laser cutting process.

1. Introduction

Nickel-based super alloys have wide range of applications in aircraft and rocket industries [1]. These applications require stringent design and close tolerances. Laser beam cutting (LBC) is a cutting process being widely used for generating complex shapes and designs in many engineering materials such as metals, non-metals, ceramics and superalloys [2, 3]. Many researchers have studied the kerf qualities like $K_i$ and/or $K_o$ during LBC experimentally and found that the qualities can be further improved by properly controlling the process parameters [4, 5]. Thus, in order to exploit the full potential of the LBC process the control parameters are to be properly selected so as to obtain the optimum values of response parameters. Existing well known approaches such as grey relational analysis (GRA), analytic network process (ANP), preference ranking organization method for enrichment evaluation (PROMETHEE), VIKOR method etc. can be applied in these direction.

Grey system was first proposed by Deng [6]. It emerges to be a powerful tool that can deal with data related to poor, unknown and vague [7]. In recent years researchers has successfully used grey system in solving many multi conflicting criteria’s throughout various fields of manufacturing [8, 9]. Lin and Lin [10] have used GRA to optimize the process parameters of EDM process that deals with machining of SKD11 alloy steel with multiple responses. Zadeh [11] first introduced fuzzy sets which can effectively deal with improper, uncertain and vague data. While fuzzy logic aided with GRA can further improve its performance in solving multi-objective optimization problems. Many researchers have effectively employed grey fuzzy logic in optimizing multi-objective problems [7, 9]. Soepangkat et al. [12] applied gery fuzzy-logic in optimizing wire EDM process parameters while machining AISI D2 steel with optimal responses as surface roughness and layer thickness. Chakraborty et al. [13] applied grey-fuzzy technique to obtain the optimal parametric combination of three non-traditional
machining (NTM) processes, i.e. abrasive water-jet machining (AWJM), electrochemical machining (ECM) and ultrasonic machining (USM) processes.

In this paper GRA aided with fuzzy logic is applied to the problem of LBC process defined by Dubey et al. [14]. These researchers has already used a hybrid approach of Taguchi method (TM) and principal component analysis (PCA) in order to achieve optimal results. However, it was interesting to observe that the result obtained by our approach is similar to that obtained by previous researcher which verifies the potential of using our approach to obtain the optimal parametric settings and is also supported by TOPSIS. Analysis of variance is also applied to identify the significance of each process parameters in the laser cutting process.

2. Methodology

2.1. Grey relational analysis

In grey system, the data in the decision matrix are normalized (data pre-processing) so as to bring range between 0 and 1 to make the data dimensionless and comparable. The following expressions are utilized for normalizing based on the type of the characteristics of the data, i.e. equation (1) is used for data with larger-the-better characteristics and equation (2) for smaller-the-better type [10, 11].

\[
x'_i (k) = \frac{x_i (k) - \min x_i (k)}{\max x_i (k) - \min x_i (k)} \quad i = 1,2,\ldots,m \text{ and } k = 1,2,\ldots,n
\]

\[
x'_i (k) = \frac{\max x_i (k) - x_i (k)}{\max x_i (k) - \min x_i (k)}
\]

where \(x_i (k)\) and \(x'_i (k)\) are the observed and normalized data respectively for \(i^{th}\) alternative and \(k^{th}\) criterion. The grey relational coefficient (GRC) is calculated using equation (3).

\[
\xi_i (k) = \frac{\Delta_{\text{min}} + \zeta \Delta_{\text{max}}}{\Delta_{\text{max}} (k) + \zeta \Delta_{\text{max}}}
\]

where \(\Delta_{\text{min}}(k)\) is the difference between \(x_i^0 (k)\) and \(x'_i (k)\) (\(x_i^0 (k)\) is the ideal sequence). The distinguishing coefficient \(\zeta\) lies between 0 and 1, usually considered as 0.5. \(\Delta_{\text{min}} = \forall j \in \text{max} \min \left\| x_0 (k) - x_j (k) \right\|\) is the smallest value of \(\Delta_{ij}\); and \(\Delta_{\text{max}} = \forall j \in \text{max} \max \left\| x_0 (k) - x_j (k) \right\|\) is the largest value of \(\Delta_{ij}\). A higher value of GRC for an alternative signifies it to be closer to the optimal solution with respect to a particular criterion. The grey relational grade (GRG) for an alternative is computed by averaging the corresponding GRC values of criterion using equation (4).

\[
\gamma_i = \frac{1}{n} \sum_{k=1}^{n} \xi_i (k)
\]

where \(n\) is the number of criterion. A higher value of GRG indicates that the corresponding alternative is the best choice for the said application.

2.2. Fuzzy logic in grey relational analysis

Fuzzy set deals with imprecise and inadequate information in an efficient way to arrive at a logical conclusion for any decision making problem. Fuzzy set theory states that, in a universe of discourse \(X\), a fuzzy subset \(\tilde{A}\) of \(X\) is defined by a membership function \(f_{\tilde{A}} (x)\), which takes each element \(x\) in \(X\) to a real number \(R\) in unit interval \([0,1]\). The function value \(f_{\tilde{A}} (x)\) represents the grade of membership of \(x\) in \(\tilde{A}\). The larger the the value of \(f_{\tilde{A}} (x)\), the stronger is the grade of membership for \(x\) in \(\tilde{A}\).

In GRA, the use of lower-the-better and higher-the-better characteristics results in some uncertainty in the derived results which can be effectively controlled using fuzzy logic. A fuzzy logic unit consists of a fuzzifier, fuzzy membership functions, a fuzzy rule base, an inference engine and a defuzzifier. In the fuzzy logic, the membership functions are the inputs to the fuzzifier in order to fuzzify the GRC values. The inference engine performs a fuzzy reasoning of the developed fuzzy rules...
to generate a fuzzy value. The defuzzifier finally converts the fuzzy value into an understable value known as grey fuzzy reasoning grade (GFRG). A fuzzy rule base comprising a set of if-then control rules is developed to represent the inference relationship between the input and output. A set of such fuzzy rules is provided as below:

Rule 1: if \( x_1 \) is \( a_1 \) and \( x_2 \) is \( b_1 \) and \( x_3 \) is \( c_1 \) and \( x_4 \) is \( d_1 \), then output \( (G) \) is \( e_1 \), else

Rule 2: if \( x_1 \) is \( a_2 \) and \( x_2 \) is \( b_2 \) and \( x_3 \) is \( c_2 \) and \( x_4 \) is \( d_2 \), then output \( (G) \) is \( e_2 \), else

Rule \( n \): if \( x_1 \) is \( a_n \) and \( x_2 \) is \( b_n \) and \( x_3 \) is \( c_n \) and \( x_4 \) is \( d_n \), then output \( (G) \) is \( e_n \).

where \( A_1, B_1, C_1 \) and \( D_1 \) are the fuzzy subsets defined by the corresponding membership functions, \( \mu_{A_i}, \mu_{B_i}, \mu_{C_i} \) and \( \mu_{D_i} \) respectively. The inference engine performs fuzzy reasoning on fuzzy rules while taking max-min inference to generate a fuzzy value, \( \mu_{v_0} (G) \).

\[
\begin{align*}
\mu_{v_0} (G) &= (\mu_{A_1} (x_1) \land \mu_{B_1} (x_2) \land \mu_{C_1} (x_3) \land \mu_{D_1} (x_4) \land \mu_{E_1} (G)) \lor \ldots \ldots \\
&= (\mu_{A_2} (x_1) \land \mu_{B_2} (x_2) \land \mu_{C_2} (x_3) \land \mu_{D_2} (x_4) \land \mu_{E_2} (G)) \lor \ldots \ldots \\
&= (\mu_{A_n} (x_1) \land \mu_{B_n} (x_2) \land \mu_{C_n} (x_3) \land \mu_{D_n} (x_4) \land \mu_{E_n} (G))
\end{align*}
\]

where \( \land \) is the minimum and \( \lor \) is the maximum operation. Finally, a centric fuzzification method is utilized to transform the fuzzy multi-response output, \( \mu_{v_0} (G) \) into a crisp value of GFRG \( (G_0) \).

\[
G_0 = \frac{\sum G\mu_{v_0} (G)}{\sum \mu_{v_0} (G)}
\]

The GFRG values are then arranged in descending order. The alternative with the maximum value of GFRG signifies it to the best choice with respect to a set of criteria/attributes.

3. Results and discussion

3.1 Design of experiments

LBC machining of sheet with three kerf qualities such as kerf width \( (K_w) \), Kerf deviation \( (K_d) \), and Kerf taper \( (K_t) \) as response parameters are considered with control parameters are gas pressure \( (GP) \), pulse width \( (PW) \), pulse frequency \( (PF) \), and cutting speed \( (CS) \) each with three levels as shown in Table 1. The workpiece material is 0.7mm thick nickel based superalloy SUPERNI 718. The experiments are designed based on L9 orthogonal array \( (OA) \) and the corresponding responses are observed as shown in Table 2. Finally, analysis of variance \( (ANOVA) \) was used to identify the most influential LBC parameter for multiple responses.

| Table 1: Machining parameters and their levels [14] |
|-----------------|---------|--------|--------|--------|
| Process Parameter | Symbol | Level 1 | Level 2 | Level 3 |
| Oxygen pressure (kg/cm²) | A | 2 | 3 | 4 |
| Pulse width (µs) | B | 0.6 | 1 | 1.4 |
| Pulse frequency (Hz) | C | 18 | 23 | 28 |
| Cutting speed (mm/min) | D | 20 | 40 | 60 |

| Table 2: Experimental layout using L9 orthogonal array [14] |
|-----------------|---------|--------|--------|--------|
| Exp. No | A | B | C | D | \( K_w \) (mm) | \( K_d \) (mm) | \( K_t \) (°) |
| 1 | 1 | 1 | 1 | 1 | 0.234 | 0.03 | 0.4092 |
| 2 | 1 | 2 | 2 | 2 | 0.406 | 0.05 | 0.8185 |
| 3 | 1 | 3 | 3 | 3 | 0.416 | 0.12 | 1.2278 |
| 4 | 2 | 1 | 2 | 3 | 0.328 | 0.03 | 0.8185 |
| 5 | 2 | 2 | 3 | 1 | 0.438 | 0.03 | 0.6139 |
| 6 | 2 | 3 | 1 | 2 | 0.438 | 0.12 | 1.0231 |
| 7 | 3 | 1 | 3 | 2 | 0.39 | 0.04 | 1.2278 |
| 8 | 3 | 2 | 1 | 3 | 0.38 | 0.07 | 1.2278 |
| 9 | 3 | 3 | 2 | 1 | 0.464 | 0.02 | 0.4092 |
3.2 Grey- fuzzy system

The normalized value of experimental results are calculated using equation 1 and 2, the GRC and GRG values using equation 2 and 3 respectively for each of the parametric combination and are presented in Table 3. For all the responses, ‘lower-the-better’ criterion is preferred. On the other hand, in order to obtain an improved quality in the performances and to decrease the vagueness in the data, fuzzy logic is aided with GRA for computing the grey-fuzzy reasoning grade (GFRG).

In order to compute the GFRG values fuzzy toolbox of MATLAB (R2013A) is used. For the present problem three inputs and one output (GFRG) fuzzy-logic system is used [15]. Mamdani inference engine is used which performs fuzzy reasoning with fuzzy rules for generating a fuzzy value. The fuzzy rules are developed using ‘if-then’ control rule. GRC values of K_w, K_d and K_t are the inputs to the fuzzy system. Triangular membership function each with five fuzzy subsets as lowest (LT), low (L), medium (M), high (H) and highest (HT) are used to represent the input variables of K_w, K_d and K_t. However, for output GFRG nine fuzzy subset as lowest (LT), very Low (VL), low (L), medium low (ML), medium (M), medium high (MH), high (H), very high (VH), highest (HT) are considered as shown in Figure 1 and 2 respectively. In total 9 fuzzy rules are developed to define the inference relation between the input GRC and output GFRG. The rule-based fuzzy-logic reasoning is shown in Figure 3. Maximum–minimum compositional operation by tracking the fuzzy reasoning yields a fuzzy output. Finally, the defuzzifier converts the fuzzy predicted value into GRFG which are presented in Table 3. High value of GFRG indicates experiment number 1 to be the most preferred. A validation test is done with technique for order preference by similarity to ideal solution (TOPSIS) which confirms that experimental number 3 gives the optimal parametric setting for the considered EDM process. It is a popular method that seeks to identify the best solution with shortest distance to positive ideal solution and longest distance from negative ideal solution. The computed TOPSIS scores are provided in Table 3.

**Table 3: Normalized data, grey relational coefficients and grey-fuzzy reasoning grade**

| Exp.No. | Normalised Data | Grey relational coefficient | GRG | GFRG | TOPSIS Score |
|---------|-----------------|-----------------------------|-----|------|--------------|
|         | K_w  | K_d  | K_t  | K_w  | K_d  | K_t  |       |       |            |
| 1       | 1.0000 | 0.9000 | 1.0000 | 1.0000 | 0.8333 | 1.0000 | 0.9444 | 0.922 | 0.9201 |
| 2       | 0.2522 | 0.7000 | 0.5000 | 0.4007 | 0.6250 | 0.5000 | 0.5086 | 0.500 | 0.5980 |
| 3       | 0.2087 | 0.0000 | 0.0000 | 0.3872 | 0.3333 | 0.3333 | 0.3513 | 0.376 | 0.0637 |
| 4       | 0.5913 | 0.9000 | 0.5000 | 0.5502 | 0.8333 | 0.5000 | 0.6279 | 0.648 | 0.7336 |
| 5       | 0.1130 | 0.9000 | 0.7499 | 0.3605 | 0.8333 | 0.6666 | 0.6201 | 0.648 | 0.7193 |
| 6       | 0.1130 | 0.0000 | 0.2501 | 0.3605 | 0.3333 | 0.4000 | 0.3646 | 0.377 | 0.1197 |
| 7       | 0.3217 | 0.8000 | 0.0000 | 0.4244 | 0.7143 | 0.3333 | 0.4907 | 0.500 | 0.5413 |
| 8       | 0.3652 | 0.5000 | 0.0000 | 0.4406 | 0.5000 | 0.3333 | 0.4246 | 0.425 | 0.3882 |
| 9       | 0.0000 | 1.0000 | 1.0000 | 0.3333 | 1.0000 | 1.0000 | 0.7778 | 0.796 | 0.7476 |

![Figure 1: Input membership function](image)

![Figure 2: Output membership function](image)
Table 4 and Figure 4 show the response table and corresponding graph for GFRG. It is obtained by calculating the average of each input machining parameter at its corresponding level which reveals that oxygen pressure, pulse width and cutting speed must maintained at level 1 while pulse frequency at level 2 in order to obtain the optimal responses. The max–min column indicates that cutting speed is the most significant among the three input parameters.

Obtained optimum parametric settings using the proposed approach is A1 B1 C2 D1 which exactly matches with the parametric settings obtained by the past researchers. This verifies that the said approach is very much effective in obtaining the optimal parametric settings.

**Table 4: Response table for GFRG**

| Process Parameter          | Level 1  | Level 2  | Level 3  | Max-Min | Rank |
|----------------------------|----------|----------|----------|---------|------|
| Oxygen pressure (kg/cm²)   | 0.5993   | 0.5576   | 0.5736   | 0.0417  | 4    |
| Pulse width (μs)           | 0.6900   | 0.5243   | 0.5163   | 0.1737  | 1    |
| Pulse frequency (Hz)       | 0.5746   | 0.6480   | 0.5080   | 0.1400  | 3    |
| Cutting speed (mm/min)     | 0.7887   | 0.4590   | 0.4830   | 0.3297  | 1    |

**Figure 3:** Fuzzy logic rule viewer

**Figure 4:** Response graph for GFRG
3.3 ANOVA for GFRG

To find the significance of different input parameters the grey-fuzzy reasoning grade obtained is subjected to ANOVA as is shown in Table 5. As the degrees of freedom for residual error is zero, it does not provide enough data for calculation of f and p values. It usually happens when four input parameters with three level values are considered and an L₉ orthogonal array because of which pooling is performed. It is done mainly because of two reasons. First when a number of factors are included in an experiment, the laws of nature make it probable that half of them would be more influential than the rest and the second one is in statistical predictions, which encounters two types of mistakes: alpha and beta mistakes. When something is important but actually is not, that is called alpha mistake and the beta mistake is just the reverse of the alpha mistake. Unfortunately, the test of significance can be done only when the error term has non zero DoF. Pooling is started with the factor which has the least influence. In this analysis, oxygen pressure is having the least influence hence it is pooled as shown in Table 6. From table 6 it can be seen that cutting speed has a p-value less than 0.05 which indicates it to be a significant parameter at 95% confidence level.

| Source                  | DoF | SS   | MSS   | F value | P value |
|-------------------------|-----|------|-------|---------|---------|
| Oxygen pressure (kg/cm²) | 2   | 0.0026 | 0.0013 | **      |         |
| Pulse width (ms)        | 2   | 0.0576 | 0.0288 | **      |         |
| Pulse frequency (Hz)    | 2   | 0.0294 | 0.0147 | **      |         |
| Cutting speed (mm/min)  | 2   | 0.2026 | 0.1013 | **      |         |
| Error                   | 0   | *     | *     |         |         |
| Total                   | 8   | 0.2924 |       |         |         |

Table 5: ANOVA before pooling

| Source                  | DOF | SS   | MSS   | F value | P value |
|-------------------------|-----|------|-------|---------|---------|
| Pulse width (ms)        | 2   | 0.0576 | 0.0288 | 21.75   | 0.044   |
| Pulse frequency (Hz)    | 2   | 0.0294 | 0.0147 | 11.10   | 0.083   |
| Cutting speed (mm/min)  | 2   | 0.2026 | 0.1013 | 76.46   | 0.013   |
| Error                   | 2   | 0.0026 | 0.0013 |         |         |
| Total                   | 8   | 0.2924 |       |         |         |

Table 6: ANOVA after pooling

4 Conclusion

In this paper GRA aided with fuzzy logic is applied to LBC process in machining of nickel-based superalloy (SUPERNI 718) sheet. It was found that oxygen pressure of 2 kg/cm², pulse width of 0.6μs, pulse frequency of 23Hz and cutting speed of 60 mm/min is the optimal combination of parameters to generate quality output. The optimal parametric mix found using the proposed approach is similar in compared to the parametric mix found by the past researchers using hybrid Taguchi and PCA method. Thus, it verifies the efficient use of our proposed approach in obtaining the optimal parametric mix. ANOVA statistics is applied which reveals that cutting speed is the most influencing factor in effecting the response parameters. Therefore, it is concluded that the optimization approach proposed in this paper can significantly improve the machining of nickel-based superalloy sheet for LBC process. Furthermore, it will be interesting to see the effectiveness of the proposed approach in parametric optimization of other NTM processes so as to obtain the optimal parametric mix which may result in improved quality of response parameters.

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