Luttinger Liquid Renormalized by a Single Impurity

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Abstract. Transport through a spinless Tomonaga-Luttinger (TL) liquid is studied in the presence of a single impurity. A boundary bosonization technique is developed to treat an impurity of arbitrary strength. Our new bosonization formula (i) smoothly connects the ones for the strong and the weak impurity limits, and (ii) reproduces exact scaling equation for the transmission probability in the weak two-body interaction limit. With the use of our formula, TL parameter is determined from the long-distance correlation functions. The result indicates a possible renormalization of TL parameter due to the effect of the single impurity scattering.

1. Introduction

Interactions between particles introduce collective features into many-body systems. In one-dimensional (1D) systems, the effects are dramatic. Low energy physics of interacting 1D electrons are described by the collective excitations of density fluctuations, Tomonaga-Luttinger (TL) liquid \cite{1, 2, 3}. Once inhomogeneity is introduced to a TL liquid by a single impurity potential $V(x) \sim V\delta(x)$, interactions between the conduction electrons and the Friedel oscillations around the impurity, strongly changes the transmission probability of the carrier. Roughly speaking, as the temperature lowers, the potential barrier becomes more transparent (reflective) when the two-body interaction is attractive (repulsive). Tunneling in a TL liquid have been intensively studied in the context of the scaling of a single impurity potential \cite{3, 4}. These investigations are mostly based on perturbative renormalization group (RG) method with small expansion parameters $V/v_F$ for weak impurity limit ($v_F$ is the Fermi velocity) or $v_F/V$ for strong impurity limit. However, the scaling flows at arbitrary $V$ have not been accessible with some exceptions, the exactly solvable model for a special interaction parameter \cite{5}, or for weak interaction limits near Fermi-liquid fixed points \cite{6, 7}. Thus, the scaling flows for arbitrary $V$ and arbitrary two-body interaction remain unknown. Recently, the renormalization of the impurity potential was numerically checked by a path-integral Monte Carlo method \cite{8}.

In this paper, the boundary bosonization method \cite{9} is extended to the case of arbitrary value of $V$ \cite{10}. Our method to bosonize the TL liquid with a single impurity is explained in Sec. 2. With our formulation, the TL parameter ($K^*$) is calculated from the asymptotic behavior of the long-distance correlation functions in Sec. 3. The result indicates that the TL parameter is renormalized due to the presence of an impurity potential. Our interpretation of the renormalization of bulk parameter $K^*$ is given. The results are summarized in Sec. 4.

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2. Boundary bosonization for arbitrary barrier

Hamiltonian of the system consists of three parts: the kinetic energy $H_0$, the impurity potential $H_V$, and two-body interactions $H_{\text{int}}$. In our bosonization procedure [10], we take eigenstates for the $H_0 + H_V$ as the basis set. The solutions for the one-body scattering problem

$$(-\partial^2_{x}/2m + V\delta(x))\psi(x) = \varepsilon_k \psi(x), \quad (1)$$

are $\phi_{P=1,k}(x) = 2\sqrt{L}/\cos(kx-\theta_F)$ and $\phi_{P=-1,k}(x) = 2\sqrt{L}/\sin(kx)$ for $k > 0$. Here, $P = \pm1$ represents the parity, and the scattering phase shift $\theta_F = \tan^{-1}(V/v_F)$ is approximated at the Fermi level. The fermion field is given by

$$\psi(x) = \sum_{k > 0} \phi_{P,k}(x) c_{P,k},$$

where $c_{P,k} (\epsilon_{P,k}^\dagger)$ is annihilation (creation) operator of a state $(P,k)$. Noting that the parity must be conserved ($P\rightarrow -P$), the annihilation (creation) operator $\tilde{\psi}_{P,+} (\tilde{\psi}_{P,-})$ must satisfy the condition $\tilde{\psi}_{P,+}(x) = \tilde{\psi}_{P,-}(-x)$. It is convenient to express the four kinds of bosonic density operators in terms of pseudo spin defined by $(i = 0, 1, 2, 3)$

$$J_i^\tau(x) = \frac{1}{\sqrt{2}} \tilde{c}_i^\tau(x) \gamma_i \tilde{c}_\tau(x), \quad (3)$$

where $\gamma_0$ is a $2 \times 2$ unit matrix, and $(\gamma_1, \gamma_2, \gamma_3)$ are Pauli matrices in the pseudo spin space. Noting that the parity must be conserved during a two-body scattering ($P_1P_2\rightarrow P_3P_4$), the interaction Hamiltonian of the $g_2$-model [11] becomes

$$H_{\text{int}} = \frac{g_2}{2L} \sum_{\tau,q} \left( J_\tau^0(q) J_{-\tau}^0(-q) + \sin^2(\theta_F) J_\tau^1(q) J_{-\tau}^1(-q) - \cos^2(\theta_F) J_\tau^2(q) J_{-\tau}^2(-q) \right). \quad (4)$$

$H_{0+}$ is diagonalized by Bogoliubov transformation with the rotation angles

$$\tanh(2\phi_0) = -\tilde{g}_2, \quad \tanh(2\phi_1) = -\tilde{g}_2 \sin^2(\theta_F),$$

$$\tanh(2\phi_2) = \tilde{g}_2 \cos^2(\theta_F), \quad \tanh(2\phi_3) = 0, \quad (5)$$

where $\tilde{g}_2 = g_2/2\pi v_F$. The four-component TL liquid is characterized by four TL parameters $K_i = e^{2\phi_i} (i = 0, 1, 2, 3)$. The parameter for the charge $(i = 0)$ $K_0$ is exactly the TL parameter in a homogeneous system. For clarity, we shall call $K(=K_0)$ a bare TL parameter.

The chiral field is expressed by the four component bosons $(b_i(q) = \sqrt{2\pi/\xi q^2} J_i^\tau(q)$ for $\tau q > 0)$ as

$$\tilde{\psi}_\tau(x) \sim \sqrt{K_F/L} e^{i\tau q x} \chi_\tau(x),$$

$$\chi_\tau(x) = \frac{1}{\xi} \sum_{q > 0} \sqrt{\frac{\pi}{q L}} e^{i\tau q x} \left[ \tau b_{0}(q) + \tau \text{sgn}(x) \sin(\theta_F) b_{1}(q) - \cos(\theta_F) b_{2}(q) \right] - \text{h.c.}. \quad (6)$$

We have used here the boundary condition $\tilde{\psi}_{P,+}(x) = \tilde{\psi}_{P,-}(-x)$ i.e. $b_i(q) = b_i(-q)$. It can be checked easily that this formula recovers the known expression in the weak and strong barrier limits [3, 9] by putting $\theta_F = 0$ and $\pi/2$, respectively.

Using Eq. (6), one can calculate any physical quantity by performing the imaginary-time path-integral with the Gaussian action. The local density of states $N_{\text{bulk}}$ far from the impurity $(|x| \gg v_F\beta)$, and $N_{\text{edge}}$ near the impurity $(|x| \ll v_F\beta)$ are given

$$N_{\text{bulk}} \propto (1/\beta\lambda)^{\lambda_{\text{bulk}}-1}, \quad N_{\text{edge}} \propto (1/\beta\lambda)^{\lambda_{\text{edge}}-1} |k_{F,x}|^{\lambda_{\text{edge}}-\lambda_{\text{bulk}}}. \quad (7)$$
which reproduces the known results for two cases: \( \lambda_{\text{bulk}} = \lambda_{\text{edge}} = (K + 1/K)/2 \) for \( V = 0 \), and \( \lambda_{\text{edge}} = 1/K \) for \( V = \infty \). Renormalization of the tunneling probability is obtained under a conjecture: the tunneling probability should be defined as the ratio of particle number which passed the impurity potential \( \propto N_{\text{edge}} \) over injected particle number \( \propto N_{\text{bulk}} \). The RG equation for \( \xi(l) = \cos^2(\theta_F(l)) \) is

\[
d\xi/dl = 2(\lambda_{\text{bulk}} - \lambda_{\text{edge}})\xi(l),
\]

where \( l = \ln \beta \Lambda \) with a high energy cutoff \( \Lambda \sim \varepsilon_F \). By expanding to the linear order in \( \tilde{g}_2 \), Eq. (8) reproduces the exact equation in the weak interaction limit [6]

\[
d\xi/dl = -2\tilde{g}_2(1 - \xi)\xi.
\]

To summarize this section, our bosonization formula (i) smoothly connects the well-known formulae for the strong and the weak impurity limits, and (ii) reproduces exact scaling equation for the transmission probability in the weak two-body interaction limit.

### 3. Mapping to a single-component TL liquid

In the previous section, 1D spinless fermions are found to be described as the four component TL liquid. To discuss the impurity effects on the bulk properties, it is worthwhile to determine the effective mapping of the four component model to one component TL model with an effective TL parameter. It should be determined from the asymptotic behavior of the long-distance correlation functions. The correlation functions of density wave \( O_{\text{dw}} = \tilde{\psi}_+^\dagger \tilde{\psi}_- \) and superconductivity \( O_{\text{sc}} = \tilde{\psi}_+^\dagger \tilde{\psi}_- \) are calculated as \( \langle O_j^\dagger(x)O_j(x') \rangle \propto |x - x'|^{-\kappa_j} \) for \( |x|, |x'| \gg v_F\beta \). The exponents are

\[
\kappa_{\text{dw}} = (K_0 + \sin^2(\theta_F)K_1 + \cos^2(\theta_F)(K_2)^{-1})/2 \equiv A_{+1},
\]

\[
\kappa_{\text{sc}} = ((K_0)^{-1} + \sin^2(\theta_F)(K_1)^{-1} + \cos^2(\theta_F)K_2)/2 \equiv A_{-1},
\]

Comparing them with the result for a homogeneous case [3], the new TL parameter \( K^* \) of an effective one-component TL liquid must satisfy

\[
(K^*)^\nu = A_\nu,
\]

for \( \nu = \pm 1 \). Clearly, Eq. (12) has no exact solutions since it contains larger number of equations than that of indeterminate parameters. Generally, mapping the four-component TL liquids to one-component model is impossible. However, a solution of a set of two equations derived from Eq. (12) gives the reasonable value, which is exact at least to the linear order in \( \tilde{g}_2 \),

\[
K^* = \sqrt{1 + \left( \frac{A_{+1} - A_{-1}}{2} \right)^2 + \frac{A_{+1} - A_{-1}}{2}}.
\]

We accept \( K^* \) in Eq. (13) as the effective TL parameter. \( K^* \) is plotted as a function of \( \xi \) in Fig. 1. The result indicates that the TL parameter, which characterizes the bulk quantity of the lowest energy scale in a TL liquid, is possibly renormalized due to the presence of a single impurity. This could be understood as the simple result since our calculation starts from a scattering problem. In other words, all the single-particle states are under the effects of the single impurity potential especially in one dimension. It is worthwhile to note that \( K^* \) approaches 1 for the intermediate value of \( \xi \) in Fig. 1. To the linear order in \( \tilde{g}_2 \), it is approximated as

\[
K^* = 1 - \left( \left( \xi - \frac{1}{2} \right)^2 + \frac{3}{4} \right) \tilde{g}_2.
\]
This indicates that the partial transmission (or the partial reflection) destabilizes the two-body interaction effectively from the bare value \( K = K^*(\xi = 0) = K^*(\xi = 1) \). This could be understood as follows. The impurity scattering causes the phase mismatch between the density waves. Hence, the overlapping of the wave functions decreases which results in the weakening of the two-body interaction.

4. Summary and discussions

In summary, we have studied a tunneling problem in a spinless TL model using our new bosonization method. 1D interacting fermions with a single impurity can be described as four component TL liquid. We obtain the new bosonization formula which connects the existing formulae valid only near \( \theta_F = 0 \) or \( \pi/2 \). Effective TL parameter which characterizes the long distance correlation functions is possibly renormalized by the impurity potential. The two-body interaction seems to be destabilized when the partial transmission occurs. This may be understood as the result of phase mismatch between the density waves.

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