Random Seismic Response Analysis of Long-Span Steel-Concrete Composite Structure

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Abstract. Considering the randomness of seismic input and the nonlinear characteristic of long-span structures, the probability density evolution method is used to study the dynamic response of the long-span steel-concrete composite structure under random earthquake. Three-dimensional structural model is established in the finite element software SAP2000, and then the displacements between layers and the response rule of vertical displacements of truss roof of the composite structure are obtained. The results show that under the input of three-dimensional earthquake, the vertical displacement of the grid roof of the isolated long-span steel-concrete composite structure is an order of magnitude larger than the horizontal direction of each layer. The probability density evolution method can offer comprehensive probability information of nodal displacements, and provide a reasonable way for the seismic reliability analysis of long-span isolated structures.

1. Introduction
The deterministic ground motions can’t reflect the strong random nature of ground motions. The randomness of the ground motion has a great influence on the response of the structure as the input of the structure. If the deterministic analysis method is adopted, it can’t fully reflect structural nonlinear response characteristics. There must be a definite law of probability density evolution for the nonlinear response of the structure. Therefore, it’s more in line with the actual situation to study the nonlinear state of the engineering structure from the point of view of probability.

The long-span isolated structures have characteristics of both spatial structures and seismic isolated structures. Due to the excellent performance in light weight, high ductility, reasonable force, rich modeling styles, effective vibration reduction and so on, they’re more and more used in public buildings [1, 2]. For such special and important structures with obvious nonlinear characteristics, the random vibration method should be adopted to analyze the seismic response to obtain the structural probabilistic statistical information under the input of random ground motion. Then, the seismic behavior of the long-span isolated structures will be mastered accurately. In addition, the researches of the scholars at home and abroad have already shown that the ground motion is multi-dimensional. Hence, it’s of great necessity to adopt multi-dimensional seismic input when the seismic analysis of long-span structures is conducted [3-5].

In this paper, a long-span steel-concrete composite structure with the based isolation is taken as the research object, and the probabilistic density evolution method proposed creatively by Professor Li and his team [6, 7] is adopted to conduct the dynamic response analysis of the structure under...
three-dimensional random seismic input. Then, the abundant probability information of structural displacement response is obtained, which provides a new way for the seismic analysis of the long-span isolated structures.

2. Structural Overview
The model is a long-span steel-concrete composite structure with a seismic fortification intensity of 8 degrees, and a site category of class II. The main direction of ground motion input is along the structural span. The upper part of the structure is the steel double-layer grid and the lower part is a two-layer concrete frame. The Geometry sizes of the structural model are shown in table 1. All the dead loads and live loads which applied to the upper layer and lower chord layer of the grid are valued according to the Load Code for the Design of Building Structures, and then are added to each node of the roof equivalently in the form of concentrating mass. The function of the second layer of the frame requires hollow, and the supporting columns are around boundary of the roof of the grid. Base isolation is adopted for the structure, and the bottom of supporting columns use isolation bearings of the type LRB400, the rest ones use isolation bearings of the type NRB400. The overall model is shown in figure 1.

![Figure 1](image_url)

**Figure 1.** The 45 m×90 m model of isolated long-span steel-concrete composite structure.

| Structural span | Depth length | Height of grid | Height of frame |
|-----------------|--------------|----------------|-----------------|
| 90              | 45           | 2.44           | 5               |

The structure adopts Rayleigh damping which means $C = \alpha M + \beta K, \alpha = \omega_1\omega_2, \beta = 2\omega / (\omega_1 + \omega_2)$ in this paper, where $\omega_1, \omega_2$ are the first and the second circular frequencies of the long-span isolated structure, respectively. The model is the joint action of steel roof and lower concrete structure. According to the Code for Seismic Design of Buildings, the damping ratio $\xi$ should be between 0.025 and 0.035, hence, the ratio of the structure $\xi$ is 0.03.

3. PDEM of Structure Based on Random Seismic Response
The isolated long-span steel-concrete composite structure is a multi-degree-of-freedom structure system, and its equation of motion is:

$$M(\Theta)\ddot{X} + C(\Theta)\dot{X} + f(\Theta, X) = F(\Theta, t)$$

where, $\Theta$ is basic random vector of the input, and $\Theta = (\Theta_1, \Theta_2, \cdots, \Theta_s)$, $s$ represent the numbers of random variables; $M(\Theta), C(\Theta)$ are the mass matrix and damping matrix, respectively; $X, \dot{X}, \ddot{X}$ are the acceleration vector, velocity vector and displacement vector; $f(\Theta, X)$ represents the nonlinear restoring force vector; $F(\Theta, t)$ represents the external input vector.

As to the non-special engineering problems, equation (1) has a unique solution if the initial condition is given which depends on the random parameter vector $\Theta$. The physical quantities to be
examined by the system \( Z = (Z_1, Z_2, \ldots, Z_m)^T \) can be determined by the structural state quantities, such as displacement, velocity, etc.

The formal solution of the displacement and the velocity is:

\[
X(t) = G(\Theta, t), \dot{X}(t) = H(\Theta, t)
\]

Then\[
Z(t) = H(\Theta, t), \dot{Z}(t) = h(\Theta, t)
\]

This article studies the law of structural seismic response in the vertical direction, and selects the vertical displacement as the investigation quantity. The probability of the augmented system \( Z(t) \) remains unchanged on the basis of the probability conservation principle. Suppose the joint probability density function (PDF) of \( Z(t) \) is \( P_{Z_0}(z, \Theta) \), and it satisfies the following generalized probability density evolution equation:

\[
\frac{\partial P_{Z_0}(z, \Theta, t)}{\partial t} + \sum_{j=1}^{n} \frac{\partial P_{Z_0}(z, \Theta, t)}{\partial z_j} = 0
\]

The initial condition of equation (4) is:

\[
P_{Z_0}(z, \Theta, t_0) = P_{Z_0}(z, \Theta, t_0)\delta(z - z_0)
\]

where, \( z_0 \) represents initial deterministic value; \( \delta(\cdot) \) represents Dirac function.

By solving equation (4), the PDF of \( Z(t) \) can be obtained by performing \( \theta \) integral on \( P_{Z_0}(z, \Theta, t) \).

\[
P_z(z,t) = \int_{\Omega_{\theta}} P_{Z_0}(z, \Theta, t) d\Theta
\]

where, \( \Omega_{\theta} \) is the distribution space of \( \Theta \).

4. Structural Response Analysis Steps

An isolated long-span steel-concrete composite structure is established in the finite element software SAP000. The precise integral method is used to solve the structural nonlinear response under the three-dimensional random ground motion. PDAM analysis of the response is conducted, and then the time-varying information such as mean value, variance and probability density curve and so on are obtained finally. Considering the characteristics of long-span isolated structures and combining with the basic theoretical knowledge of PDEM, the probability density evolution analysis of the structure subjected to random seismic input is conducted. Here are the basic steps:

(1) Choose the discrete representative points based on probability space division. The inherent circular frequency of the site \( \omega_0 \), the equivalent damping ratio of the site \( \xi_q \) and the Fourier spectrum of the base input \( S_g \) are used to consider the randomness of the ground motion, and it is assumed that all the three random parameters obey the log normal distribution. The number theory method is adopted to select the discrete representative points with the quantity of \( n_{se} \), that is, \( \Theta_q = (\theta_{q1}, \theta_{q2}, \ldots, \theta_{qn}) \), \( q = 1, 2, \ldots, n_{se} \), and assign the probability of each representative point is \( P_q = \int_{\Omega_{\theta}} P_{\Theta} d\Theta \).

(2) Synthesize random ground motion artificially. A physics-based random ground motion model is used to synthesize the random ground motion based on the FFT technology according to the discrete representative points of the equation (1).
(3) Solve the deterministic seismic response of long-span isolated structures subjected to multi-dimensional earthquake.

(4) Obtain the solution of the generalized probability density evolution equation. The finite difference method is used to solve the probability density evolution equation, and the differential format is Lax-Wendroff (LW) or Total variation diminishing (TVD).

(5) Obtain the solution of the PDF of response quantity. The numerical solution of the PDF of the structural response can be obtained by summing up the results of equation (4).

5. The Numerical Calculation Results
Adjust the ratio between the peaks of acceleration time histories of there-dimensional random ground motion to horizontal X direction: horizontal Y direction: vertical Z direction =1:0.85:0.65. The three-dimensional nonlinear response of the long-span structure is calculated, and then substitute the time derivative of the displacement response which is the velocity information into the probability density evolution equation. Finally, the two differential formats, LW and TVD, are used to solve the numerical value of the probability density evolution equation.

5.1. Probability Density Calculation of Story Drift
Figure 2 shows the evolution process of mean value and standard deviation (std.D) value of inter-layer displacements of the isolation layer over time. It demonstrates that when subjected to an 8-degree (0.3 g) rare earthquake, the maximum values of the mean value and std.D of inter-layer displacements of the isolation layer calculated by LW format and TVD format are about 0.364 m and 0.582 m, 0.358 m and 0.550 m, respectively. The std.D of the horizontal displacement of the isolation layer is about 6 times that of the two floors of the superstructure and the roof truss roof, which illustrates that the degree of random dispersion of the horizontal response of the isolation layer is greater. According to the specification, the inter-layer displacements of the isolation layer should be less than 0.55 Dmin, which is the minimum diameter of the isolation bearing (0.8 m), and it comes out that the result meets the requirement. The mean value results of LW and TVD are basically the same, but the std.D results are not different. The main reason is that the LW format has a small dissipation, so there are often oscillations in the mutation, while the oscillation of TVD format is caused by its dispersion.

Figure 3 shows the PDF of inter-layer displacements of the isolation layer at three typical time intervals. It shows that at the time of 2 s, the maximum probability of the isolation layer displacement is between -0.2 and 0.1 m, and the density function is steep which indicates that the structure is in the vibration initiation stage. At the time of 5.5 s, the probability interval of the displacement distribution of the isolation layer is wider than that at 2 s, and the probability is the largest between -0.4 and 0.2 m. At the time of 14 s, the probability of the isolation layer displacement in each position is relatively large with the largest probability between ±0.02 m. It is noted that the structure is in the strong vibration stage at the last two moments with irregular function. Figures 4 and 5 show the probability density surface (PDF curved surface, PDS) and probability density contour (contour of PDF surface, PDC) of the inter-layer displacement of the isolation layer respectively. It can be clearly seen that under the multi-dimensional input of 8-degree (0.3 g) rare earthquake, the evolution process of the PDS is like continuous mountain range, and the probability density appears a number of distinct ‘peaks’ in the interval of 4 to 5 seconds. The PDC line is like the flow of a river, because the probability is flowing in the state space.

Figure 6 shows the evolution process of the mean value and std.D of the inter-layer displacement over time on the upper layer of the isolation layer. It shows that under the action of 8-degree (0.3 g) multi-dimensional rare earthquake, the mean value and std.D of the inter-layer displacement have their maximum values at around 3.8 s. The maximum values of the mean value and std.D solved by LW format and TVD format are around 1.52×10⁻² m and 1.04×10⁻¹ m, 1.50×10⁻² m and 0.97×10⁻¹ m, respectively. In Ref. [8], it is suggested that the inter-layer displacement of the superstructure of an isolated structure in the basic operating state should be less than H/200 (H is the height). The height of the structure is 5m which meets the requirement. The std.D of inter-layer displacement is one order
of magnitude larger than its mean value, which is due to the reason that the high randomness of ground motion leads to a large uncertainty of displacement response. The mean displacement is one order of magnitude smaller than the isolation layer, and the variance value of the displacement is smaller than that of the isolation layer.

Figure 7 shows the PDF of the first inter-layer displacement at three typical moments. It demonstrates that when subjected to 8-degree (0.3 g) multidimensional rare earthquake, the positions of probability peaks at three different times are slightly different, and are basically distributed within 2.00×10^{2} m. Due to the limited length of the paper, the probability density evolution information of the second inter-layer displacement is not given, because their laws are very similar with those of the first floor and the isolation layer. The difference between them is that the response curves at the two later moments of the first floor have a high degree of coincidence, while those of the second floor still do not coincide. After 5.5 s, the structural response of the first floor has tended to be stable, while that of the second floor is influenced by the vibration of the truss layer with a small stiffness. Figures 8 and 9 give the PDS and the PDC of the first inter-layer displacement.

Figure 2. Mean and Std. D of isolation layer’s displacement.  
Figure 3. Typical PDF of isolation layer’s displacement at three time points.  
Figure 4. The PDS of isolation layer’s displacement.  
Figure 5. The PDC of isolation layer’s displacement.

Figure 10 shows the distribution positions of 10 key nodes selected in the grid roof, where points 1-7 are the points on the lower chord layer and the rest ones are the points on the upper chord layer.

Figures 11 and 12 show the evolution process of the mean value and std.D over time at point 7 (the maximum inter-layer displacement of the lower chord layer) and point 10 (the maximum inter-layer displacement of the upper chord layer) of the grid roof respectively. It can be seen from that: (1) the mean value of the horizontal relative displacement of the grid roof layer all reaches the maximum value at 6.2 s, while the std.D all reaches it at 3.8 s. For the key point 7, the maximum mean values
calculated by LW format and TVD format are $1.65 \times 10^{-2}$ m and $1.56 \times 10^{-2}$ m respectively. For the key point 10, the maximum mean values calculated by LW format and TVD format are $1.68 \times 10^{-2}$ m and $1.60 \times 10^{-2}$ m respectively. It means that the mean value and std.D of the displacement of the upper chord point 10 is larger than that of point 7. In addition, the maximum value of horizontal displacements of the grid appear at the angle point of the upper chord layer (point 8), the geometric midpoint of the upper layer and the lower chord layer (point 7, point 10) and the angle point of the lower chord layer (point 1), which agrees with our previous cognition; (2) the std.D of the horizontal displacement of key points is one order of magnitude larger than the mean value, which reflects the uncertainty of the grid’s horizontal displacement caused by the earthquake randomness.

**Figure 6.** Mean and Std. D of 1st layer’s displacement.

**Figure 7.** Typical PDF of 1st layer’s displacement at three time points.

**Figure 8.** The PDS of 1st layer’s displacement.

**Figure 9.** The PDC of 1st layer’s displacement.

**Figure 10.** The key points layout of grid roof.
Figures 11 and 12 show the PDF of horizontal relative displacement (relative to the second ground floor) of key point 7 and point 10 of grid roof layer at three typical moments. At any time, the PDF is steep which indicates that the dispersion of the horizontal displacement response of grid roof is well controlled. The response curves of the latter two moments are still not coincident, but they are close to each other. This is because the stiffness of the grid layer is not enough and there is a certain time of stable vibration in the horizontal direction after an earthquake. Figures 13-18 give the PDS and the PDC of the horizontal relative displacement of the key points 7 and 10 of the grid layer, respectively.

Figure 11. Mean and Std. D of grid layer’s horizontal displacement (Point 7).

Figure 12. Mean and Std. D of grid layer’s horizontal displacement (Point 10).

Figure 13. Typical PDF of grid layer’s horizontal displacement at three time points (Point 4).

Figure 14. Typical PDF of grid layer’s horizontal displacement at three time points (Point 10).

Figure 15. The PDF surface of grid layer’s horizontal displacement (Point 7).

Figure 16. The contour of PDF surface of grid layer’s horizontal displacement (Point 7).
5.2. Probability Density Calculation of Roof’s Vertical Displacement

The maximum vertical displacement of grid roof is obtained at the key point 4, which is at the support where is near the middle-span of the lower chord layer along the long side, and the probability an information result are shown in figures 19-22. It shows that, compared with the inter-layer displacement results of each layer (except the isolated layer), the mean value of the maximum vertical displacement is one order of magnitude larger than the horizontal displacements of the first floor, the second floor and the grid roof itself, which are up to 8 to 14 times. During the whole seismic process, the vertical displacement response of the grid is more discrete. In the period from 4 s to 5 s, the probability density evolution of the vertical displacement of the key points on the upper chord layer
and the lower chord of the grid shows a more significant multimodal characteristic, that is, the non-stationary probability flow phenomenon.

6. Conclusions

Based on the physics-based random ground motion model and the probability density evolution method (PDEM), the dynamic response analysis of the long-span steel-concrete composite structure with base isolation under the action of three-dimensional random earthquakes are carried out, and the following main conclusions were obtained:

1) Probability density evolution analysis accurately gives the evolution process of the random displacement response of the isolated long-span steel-concrete structure. The mean value of the inter-layer displacement of the isolation layer under three-dimensional input is an order of magnitude larger than that of the two floors of the upper structure and the grid roof. The standard deviation is about 6 times larger than that of the two floors of the superstructure and the grid roof, indicating that the random response of the horizontal response of the isolation layer is greater.

2) Under the three-dimensional seismic input, the vertical vibration of the grid roof is obvious whose maximum value of vertical displacement is 8 to 14 times the value of horizontal displacements of the first floor, the second floor and the grid roof itself. The probability density evolution of the vertical response of the grid layer is more frequent than that of the horizontal direction, indicating that the horizontal vibration of the superstructure is relatively stable, while the random fluctuation of the vertical response characteristics is more obvious.

3) Through probabilistic density evolution analysis, you can obtain abundant probabilistic statistical information and evolution process of the response volume, which can grasp the performance status of long-span isolated steel-concrete composite under the excitation of random earthquakes more efficiently and accurately. It can also provide a new way for the structural reliability analysis under earthquake.

Acknowledgment

The authors gratefully acknowledge the support from the National Natural Science Foundation of China (Grant No.51808298 & 11802145), and the Research Fund of Xinglin College of Nantong University (Grant No.2018K125).

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