The CST Bounce Universe model – A Parametric Study
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A bounce universe model with a scale-invariant and stable spectrum of primordial density perturbations was constructed using a consistent truncation of the D-brane dynamics from Type IIB string theory. A coupling was introduced between the tachyon field and the adjoint Higgs field on the D3-branes to lock the tachyon at the top of its potential hill and to model the bounce process, which is known as the Coupled Scalar and Tachyon Bounce (CSTB) Universe. The CSTB model has been shown to be ghost free, and it fulfills the null energy condition; in addition, it can also solve the Big Bang cosmic singularity problem. In this paper we conduct an extensive follow-up study of the parameter space of the CSTB model. In particular we are interested in the parameters that can produce a single bounce to arrive at a radiation-dominated universe. We further establish that the CSTB universe is a viable alternative to inflation, as it can naturally produce a sufficient number of e-foldings in the locked inflation epoch and in the post-bounce expansion to overcome the four fundamental limitations of the Big Bang cosmology, which are flatness, horizon, homogeneity and singularity, resulting in a universe of the current size.

1 Introduction

The four most fundamental problems in the Big Bang cosmology are Flatness, Horizon, Homogeneity and Singularity. In the 1980s inflation was proposed \cite{1} (with an update in \cite{2}) to address the first three of these problems. The predictions of the inflation paradigm are consistent with the high-precision cosmological observations, such as the highly isotropic and homogeneous cosmic microwave background radiation (CMBR) \cite{3}, and the scale invariance of primordial density perturbations attributed to the quantum fluctuations of the inflaton field at the end of inflation \cite{4}. Inflation, however, has its problems. For instance, the initial singularity does not appear but is delayed \cite{5}.

Building on the success of inflation models, cosmologists have been searching for alternative scenarios that can address the problem of the cosmic singularity. In 1993 a GR-compatible solution with a nonsingular bounce was proposed shortly after the first observation of current expansion \cite{6}. Meanwhile string theorists attempted to implement a nonsingular bounce in dilaton gravity, leading to the pre-big-bang (PBB) scenario within the string theory framework \cite{7}. A period of intensive string-inflation model building ensues, as reviewed by Linde \cite{8}, and Tye \cite{9}.

The bounce universe itself has a long history, and many ingenious models are reviewed in \cite{10}. A cyclic model in a con-
The CSTB model has several salient features, such as the generation of stable and scale invariant primordial density perturbations, absence of ghosts, and no violation of the null energy condition throughout the cosmic evolution [30]. It thus motivates us to investigate bounce universe models further and derive testable predictions in a model-independent manner. Dark matter creation and evolution is studied in the bounce universe [33-37]. When sufficient dark matter particles are not produced to reach kinematic equilibrium during contraction, relic abundance in the present universe then imposes a constraint on the dark matter mass and coupling constant. We can then produce numerical predictions for heavy dark matter in nuclear recoils experiments [35] and for light dark matter in electron-scattering experiments [37].

In the rest of this paper, after providing a short review of the CSTB model, we present a thorough study of the parameter space of this model. Our results verify that CSTB can produce a sufficient number of e-foldings to solve the problems related to flatness, horizon, homogeneity and Big Bang singularity, which will result in a universe of the current size. Furthermore this is achieved with a large volume of the parameter space with no limitation on the values of the free parameters in the model. In other words the problems related to the Big Bang, including the Big Bang problem, are solved naturally in the CSTB model.

2 The Single Bounce Criteria

In this section, we review briefly the CSTB model and discuss the essential physics of this string cosmology model based on D-brane and D-brane annihilations. In the effective theory, a D-¯D3-brane pair is described by the open string tachyon field action [28],

\[ \mathcal{L}_T = V_T \sqrt{1 + M_5^4 \partial_\mu T \partial^\mu T}, \quad V_T = \frac{V_0}{\cosh(\sqrt{2} M_5 \phi)} \] (1)

where \( T, M_5, V_0 \) are the tachyon field, the string mass, and the tension of D-¯D3-brane pair, respectively. The tachyon potential \( V_T \) has a maximum at \( T = 0 \) and two minima at \( T \to \pm \infty \). We take the background metric as \( (-, +, +, +) \), and we also assume that the tachyon field is spatially homogenous.

In D-brane inflation [38] the attractive potential of the D-¯D pair takes the following form:

\[ V_\phi = \frac{1}{2} m_\phi^2 \phi^2 + V_0 - \frac{V_0^2}{4 \pi^2 \phi^2}, \quad \phi = \sqrt{V_0} y, \] (2)

where \( y \) being the distance between the D-¯D3-branes.

The CSTB model [30-32] introduces a scalar-tachyon coupling term \( \lambda \phi^2 T^2 \), and the tachyon condensation comes about naturally after a period of the locked inflation, driven by the tension of D-¯D3-branes, and another period of rolling inflation while rolling down its potential hill. The coupling of between the scalar and the tachyon plays a crucial role in the cosmic evolution by greatly extending the time the tachyon spends at the top of its potential hill and hence increasing the number of the e-foldings obtained in the periods of locked inflation and rolling inflation. This is other impossible [27-29].

The CSTB Lagrangian takes the form:

\[ \mathcal{L} = \mathcal{L}_T - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m_\phi^2 \phi^2 - \lambda \phi^2 T^2, \] (3)

in addition to the Hilbert-Einstein term, in a FRW background with \( k = 1 \),

\[ ds^2 = -dt^2 + a^2(t)\left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right]. \]

Since each component of this model has the correct sign of kinetic energy term and positive potential, we have \( \rho_{tot} > 0 \) and \( \omega_{tot} = \frac{\rho_{tot}}{p_{tot}} \geq -1 \), where \( \omega_{tot}, \rho_{tot} \) and \( p_{tot} \) being, respectively, the Equation of State, energy density and pressure derived from the model. The CSTB universe model is thus free of ghosts and satisfies the null energy condition \( p + \rho \geq 0 \) throughout the cosmic evolution [39].
2.1 Cosmic evolution in the CSTB model

The CSTB cosmos starts at action at $C'$ as it undergoes a period of tachyon matter dominated contraction, from $C'$ to $B'$, during which the tachyon behaves like cold dark matter \[28, 40]. From $B'$ to $A$, it undergoes a period of accelerating contraction as the potential energy comes into play. A locked inflation era takes place from $A$ to $B$ (about the bounce point $\dot{a} = 0$ and $\ddot{a} > 0$) as the universe undergoes a period of exponential expansion driven by the vacuum energy of the tachyon (due to the brane tension). This period is extended by the effective coupling of tachyon and Higgs fields, and it solves the problems with tachyon inflation pointed out by \[41]. Quantum mechanically the tachyon is prevented from decaying (tachyon condensation) by the fast oscillations of the Higgs fields. The tachyon condensation eventually takes place at $B$, where all the energy of tachyon field transfers to tachyon matter \[28, 40]. The phase transition at $B$ is followed by a tachyon matter dominated period, $B\rightarrow C$. At point $C'$, reheating may happen as the universe undergoes another phase transition, during which the energy of tachyon matter transfers to radiation. This is depicted schematically in Fig. 2.

The background cosmology evolves according to the Friedman equations,

$$H^2 = \frac{1}{a^2} + \frac{8\pi}{3M_p^2} \left[ \frac{V(T)}{\sqrt{1 - M_s T^2}} + \left( \frac{m_T^2}{2} + \lambda T^2 \right) \phi^2 + \frac{1}{2} \dot{\phi}^2 \right];$$

whereas the equations of motion for $T$ and $\phi$ are governed by

$$\ddot{T}^2 + (1 - \dot{T}^2) \left[ 3H\dot{T} + \frac{V'(T)}{V(T)} + \frac{2\lambda \dot{\phi}^2 T}{V(T)} \sqrt{1 - T^2} \right] = 0,$$

$$\ddot{\phi} + 3H\dot{\phi} + (m_T^2 + 2\lambda T^2)\phi = 0;$$

with the string mass, $M_s$, being suppressed in the above equations.

The solutions to these equations of motion have been studied \[30] in detail. It turns out, by introducing the coupling term $\lambda T^2 \dot{\phi}^2$, CSTB model suggests a novel feature: for a non-vanishing value of $\phi$, the coupling term pulls back the vacuum expectation value of this model at finite, $(T, \phi) = (T_c, 0)$, as shown in Figure 2. In contrast to the single tachyon field model which tachyon keep rolling towards infinity \[27], the tachyon in this model is stabilised around the effective vacuum with quasi-harmonic oscillations. This feature in turn pave the way for the studying phase transition and matter generation in the bounce universe context \[42, 43].

A detailed analysis of the cosmological perturbations of this model has been carried in \[31, 32]. The spectrum of the primordial matter perturbations takes the form (c.f Eq.(3.18) in \[31, 32]):

$$P_{\phi} = \left( \frac{H}{T_c} \right)^2 P_{\delta T} = \frac{C^2}{16\pi}\kappa^2 \eta^0$$

with $k$ being the wave vector and $\eta$ being the conformal time. $P_{\delta T}$ is the spectrum of tachyon field whose perturbations are responsible for primordial matter generation \[31, 32]. Both $C$ and $\kappa$ are constant parameters whose values are determined by $\lambda$ and $\frac{M_s}{T_c}$. Therefore, the primordial curvature perturbation of this model is scale invariant and stable. Furthermore, taking the sub-leading effects into account, the spectral index of the primordial curvature perturbation is also obtained in Eq.(3.22) of \[31, 32]. It is thus natural obtain the value of $n_s = 1 \pm 0.04$ around a few percents in the CSTB model. A detailed analysis presented in \[31, 32] shows that a large parameter space in CSTB model satisfies the current observational constraints of spectral index.

2.2 How to make a successful bounce: Initial conditions

In this section we study the constraints on parameters from having enough e-foldings. In the locked inflation era the universe is dominated by vacuum energy. The Hubble parameter is nearly constant as long as the tachyon is locked at its peak value, $0 < V(\phi) < V_{\text{max}}$, as the universe undergoes a period of vacuum energy domination. In the locked inflation era, the tension of D-D-brane is so large that it dominates the energy density in this era.

The scalar field, $\phi$, oscillates while being redshifted

$$\phi \sim a^{-1} e^{\sqrt{m_T^2 - 2kT}i}.$$

We can calculate the number of e-foldings $N_L$ in this era determined by the critical value of $\langle \phi_0^2 \rangle = V_0/4\lambda M_s^2$:

$$N_L = \frac{1}{6} \ln \frac{32\lambda^3 M_s^4 \langle \phi_0^2 \rangle^2 (T_0^2)}{m^2 V_0^2}.$$  

Let us now turn to observational constraints on CSTB model:

- A period of vacuum energy domination: The first constraint comes from vacuum energy domination. In the locked inflation era, the tension of D-D-brane is so large that it dominates the energy density in this era.

$$V_0 > \frac{1}{2}m^2 \langle \phi_0^2 \rangle + \lambda (T_0^2) \langle \phi_0^2 \rangle$$

\[10\]
\( \langle \phi_0^2 \rangle < \frac{V_0}{4M_p^2} \) \hspace{1cm} (12)

- **The bounce point**: For an extended period of locked inflation about \( T \sim 0 \), the initial value \( \langle \phi_0^2 \rangle \) should be larger than \( \langle \phi_1^2 \rangle \). And the locked inflation ends when

\[ 2\lambda T_0^2 > m_\phi^2 > 2\lambda T_1^2 \] \hspace{1cm} (13)


- The effective mass of \( \phi \): The effective mass of \( \phi \) is given by \( m_{eff} = m_\phi^2 + 2HT_1^2 \). During locked inflation, \( T \) red-shifts and its vev \( T_1^2 \) diminishes dramatically. We assume that at the beginning of the locked inflation era \( T_0 \) is very large and that the \( T \) field is smaller than the bare mass of scalar field, \( m_\phi \) at the end. Therefore we have

\[ 2\lambda T_0^2 > \frac{6\pi}{M_p^2} (V_0 + 2\lambda T_1^2 \langle \phi_0^2 \rangle) \]. \hspace{1cm} (14)

With these constraints we deduce two critical relations amongst the key parameters \( m_\phi, M_s \) and \( \lambda \):

\[ 0 < \frac{m}{\lambda M_p} < \frac{4(\sqrt{7} - 1)}{\sqrt{3\pi}} \sim 0.54 \] \hspace{1cm} (15)

\[ \lambda > \frac{1}{8} \] \hspace{1cm} (16)

We thus see that these values cover a wide volume of the parameter space, and it is easy to pick a generic set of initial conditions for a successful bounce model resulting in a realistic universe. In particular this only requires that \( N_L \sim 6 \) e-foldings are generated in the locked inflation era. This is to be contrasted with the fine tuning one needs for successful inflation modelling.

### 3 The contraction era in CSTB Universe

We shall show that CSTB universe is free of the Horizon problem by calculating the e-foldings in the contraction phase of the CSTB universe.

#### 3.1 CSTB model solves the horizon problem

Bounce universe models solve the horizon problem because there exists a phase of contraction—prior to the bounce—long enough to put the entire universe back into thermal contact and re-establish causality. In CSTB model, particle horizon is therefore nonzero. Considering that particle horizons in expansion and contraction phases are symmetric, we only calculate the particle horizon in expansion phase.

In the era of locked inflation, the universe is vacuum energy dominated: the Friedman equation becomes equation 6 and the universe undergoes an exponential expansion

\[ a = a_0 e^\sqrt{\frac{2k}{3\pi}} \lambda T_0 \] \hspace{1cm} (17)

The particle horizon in locked inflation is given by

\[ d_{p_1} = a_1 \int_{a_0}^{a_1} \frac{da}{H a^2} = \frac{1}{H_0} \left( \frac{1}{a_0} - \frac{1}{a_{max}} \right) \]. \hspace{1cm} (18)

Shortly after the bounce the Hubble parameter changes from 0 to \( \sqrt{\frac{2k}{3\pi}} \lambda T_0 \), and \( H_0 a_0 = 1 \), which in turn implies

\[ d_{p_2} = a_1 - a_0 \].

In the tachyon matter dominated era, the growth of the particle horizon is

\[ d_{p_2} = a_1 \int_{a_0}^{a_{max}} \frac{da}{H a^2} = \frac{2a_0 a_{max}}{a_1} \left( \sqrt{a_{max}} - \sqrt{a_1} \right) \].

All in all the total particle horizon at the end of expansion would be

\[ d_{p_{tot}} = d_{p_1} + d_{p_2} = a_1 - a_0 + \frac{2a_0 a_{max}}{a_1} \left( \sqrt{a_{max}} - \sqrt{a_1} \right) \]

and thus \( \frac{d_{p_{tot}}}{a_0} \gg 1 \). Since the maximum size of universe is larger than current size \( a_0 \) at the bounce point the particle horizon is much larger than the scale factor. So bounce universe models guarantee that the particle horizon be larger than the current homogeneous regions.

#### 3.2 E-foldings of the universe

In Big Bang cosmos the flatness problem is a cosmological fine-tuning problem. CMB data [3] determines the current universe to be flat up to \( \sim 1\% \) percent level, \( \Omega = 1.00 \pm 0.01 \),

\[ \frac{1}{H_0^2 a_0^2} \sim 0.01 \].

As shown in Fig. ?? we can read off the required e-foldings from the end of locked inflation to the present:

\[ N_L = \ln \left( \frac{a_1}{a_{now}} \right) = \ln \left( \frac{\rho_{now}}{\rho_D} \right) \left( \frac{\rho_D}{\rho_C} \right)^{\frac{1}{3}} \left( \frac{\rho_C}{\rho_B} \right)^{\frac{1}{3}} \hspace{1cm} (19) \]

\[ = \ln \left( \frac{T_{now}}{T_D} \right)^{\frac{1}{3}} \left( \frac{T_D}{T_C} \right)^{\frac{1}{3}} \left( \frac{T_C}{T_B} \right)^{\frac{1}{3}} \].

After the tachyon condenses the single tachyon field rolls towards infinity with almost zero effective mass. When \( \langle T \rangle \) becomes large, the tachyon field can be replaced by an equivalent scalar field \( \sigma = \frac{\sqrt{8k}}{M_s} e^{-\frac{2\langle T \rangle}{M_s}} \), which approaches zero.
when \( T \to \infty \). With the new field the action can be expanded around \( \sigma \sim 0 \) and to first order in \( \lambda \),
\[
L_{\psi-\sigma} = -\frac{1}{2}\sigma^2 - \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \lambda\phi^2\sigma^2,
\] (20)
i.e. at point \( C' \) in Fig. 22, with
\[
\left(\frac{\dot{a}}{a}\right)^2 = -\frac{1}{a^2} + \frac{8\pi}{3M_p^2}\left[\frac{1}{2}\sigma^2 + \frac{1}{2}m^2\phi^2 - \lambda\sigma^2\phi^2 + \frac{1}{2}\phi^2 + \frac{1}{2}\sigma^2\right].
\] (21)
Scalar fields, \( \sigma \) and \( \phi \), take on similar equations of motion:
\[
\ddot{\sigma} + 3H\dot{\sigma} + \left[\left(\frac{M_p}{2}\right)^2 - 2\lambda\phi^2\right]\sigma = 0
\] (22)
\[
\ddot{\phi} + 3H\dot{\phi} + \left(m^2 - 2\lambda\sigma^2\right)\phi = 0.
\] (23)
From (21) we can deduce that at \( C' \), \( \langle \sigma^2 \rangle \) and \( \langle \phi^2 \rangle \) are at their minima, and divide the energy density equally:
\[
\langle \phi^2 \rangle_{min} = \frac{3M_p^2}{8\pi a^2}.
\] (24)
E-foldings can be computed accordingly,
\[
N_e = \frac{1}{3}\ln\frac{\langle \phi^2 \rangle}{\langle \phi^2 \rangle_{min}},
\] (25)
because \( \langle \phi^2 \rangle \propto a^3 \). Upon substituting the current size of universe \( a_{\text{current}} \sim 5.4 \times 10^{61}l_p \) into \( N_e \), we push back the current universe to the beginning of the deflation era:
\[
N_e = \frac{1}{3}\ln\frac{8\pi m^2 a^2}{843M_p^2} \sim 118,
\] (26)
where \( m \sim 1/19713 \), \( a \sim 1/100M_p \), \( \lambda \sim 0.25 \). It is persuasive to say that, with a contraction which can generate 120 e-foldings, the expansion era after the bounce can enjoy an analogous number of e-foldings [10]. We can thus conclude that CSTB cosmology model solves the flatness problem by the 118 e-foldings in the contraction phase.

4 Conclusion and outlook

We present a viable alternative to inflation based on string theory utilising coupled scalar and tachyon fields in the D-D-brane system [30]. The CSTB cosmos undergoes a period of tachyon matter dominated contraction in which a stable and scale invariant spectrum of primordial density perturbations is produced [31,32]. The model has a signature out-of-equilibrium production of dark matter which can be tested by the future array of dark matter detections [33-37], independently of cosmological observations [44]. Exploration on matter production and baryon asymmetry genesis is well underway [42,43]. We expect our success can inspire further studies on bounce models built from string theory or other quantum gravity theories.

Recently an interesting attempt is made to use AdS/CFT correspondence to study the evolution of matter through the bounce point [46] which is inspired by an earlier attempt of using AdS/CFT to study cosmic universe [45]. Pioneer works on studying AdS singularity can be found in [47-54], together with earlier initiatives of using AdS/CFT to study cosmic singularities [55-66]. A thorough study of physics in the bounce universe is in sight.

Another interesting question to ask if the CSTB universe model can be imbedded in higher dimensions, a la [67], and to study the stabilisation of the extra moduli in this context. Further explorations shall also exploit the symmetries and dualities of the mother string theory as string-cosmology models utilising T-duality [68,69]. This direction is expected to hold more promise of modulus stabilisation [70-72].

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Locked Inflation

Tachyon Matter Dominated Era

Radiation Dominated Era

Matter Dominated Era

Current Point

\( (T_c, 0) \)

\( T \)

\( \phi \)

\( a(t) \)

\( t \)
