Geometric optimization of magnetically actuated MEMS micromirrors

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Abstract. In this paper we develop a set of self-consistent equations describing the static and resonance characteristics of a rectangular micromirror actuated by driving a current into a multi-coil inductor fabricated on the micromirror surface. Under fairly general assumptions, these equations can be solved analytically for the most common design targets, such as the driving current, deflection angle, resonance frequency, and overall dimensions. We use this solutions to optimize the micromirror geometry by minimization of a linear objective function. The trade-offs that emerge from optimization data allow simple quantitative design of low current, low size mirrors.

1. Introduction

MEMS micromirrors are intensely researched as essential components in several applications, largely aimed at the market of portable devices (mobile phones, tablets, etc.) [1]. Among the most important are picoprojectors for imaging, and optical scanners for 3D object reconstruction and motion tracking. Both applications typically require scanning of one (or more, i.e. for color image projection) laser beams in two orthogonal directions.

The 2D scanning can be obtained with a single, 2-axis torsional MEMS micromirror or with two separate torsional 1-axis micromirrors (figure 1). The requirements for scanning frequency and angle are dependent on the application. Raster scanning, where the vertical (i.e. column) axis is scanned at a much slower frequency that the horizontal (row) axis, is by far the most common scanning strategy. For this reason, the vertical axis is typically driven in quasi-static conditions, i.e. at a frequency much lower than the typical resonance frequency for the vertical resonance mode, while the horizontal axis is driven at its resonance frequency. A 2-axis solution has the obvious advantage of a greater miniaturization, at the cost of a more complex kinematics, both in gimbal-less [2, 3] and gimbaled [4, 5] solutions, undesired coupling between the two axes, so that a 2-mirror solution is typically simpler to develop.

From the point of view of actuation mechanisms, the ones most pursued are electrostatic, electromagnetic, and, to a lesser extent, piezoelectric and electrothermal. Electrostatic actuation is the most common, as the most simple technologically, but also requires of high actuation voltages. Electromagnetic actuation requires integration of a multiloop coil (or a magnet) on the mirror, works at low voltages, and is intrinsically linear. The main drawback is the high current required, which is an important limit for portable applications. As the actuation torque is linked to the mirror area, electromagnetic actuation is more suitable for large mirrors.

In this paper, we present the geometric optimization of an electromagnetic mirror suitable for low-frequency (i.e. quasi-static) applications, actuated through a coil created on the mirror plate. A magnetic
field (typically provided by external magnets) is used to create a torque (and, consequently, an angular deflection) on the plate which is essentially proportional to the driving current.

2. Model
The structure of the mirror is shown in figure 2. The actuation coil has a rectangular spiral geometry. The external magnetic field is aligned along $y$. A number of loops in the form $n_c = (n + 1/2)$, with integer $n$, is assumed. The distance between adjacent metal lines $g_l$ is set by the technology, while the metal width $w_l$ is subject to optimization. Two torsional suspension springs, of rectangular cross-section (of width $w_S$ and thickness $t_S$), provide the returning torque. The same thickness $t_S$ is also assumed for the central plate. The material for the plate and springs is assumed to be silicon. Five design parameters, i.e. $n_c$, $w_l$, $w_S$, $t_S$, and the spring length $l_S$, completely define the mirror geometry. The design parameters and all the other dimensions and quantities are listed in table 1.

As the proposed magnetic micromirror is to be used as the slow mirror of a 2D scanner, an assumption of quasi-static actuation is made. Natural performance requirements for the mirror are: minimization of the actuation current at maximum angle, minimization of the mirror size, and a torsional resonance frequency much higher than the maximum driving frequency, to avoid undesired excitation of the resonance. While many loops result in a higher torque, and thus lower actuation current, they also increase the size of the mirror and its mass, which in turns lowers the resonance frequency. Several simultaneous trade-offs thus needs to be evaluated, and a generic optimization approach is consequently justified.

The geometrical, electromagnetic, and mechanical description of the mirror is contained in equations (1)-(4):

$$\begin{align}
p_l &= g_l + w_l \\
l_x &= d_x + (1 + 2n_c)p_l \\
l_y &= d_y + 2n_c p_l \\
l_{Tx} &= 2l_s + l_x
\end{align} \tag{1}$$

$$\begin{align}
T_x &= 2k_\phi \phi_{max} \\
k_\phi &= G_{Si} \frac{l_{Sw}}{l_S} \\
I_{Sw} &= w_ST^2 \left( \frac{1}{3} - \frac{21}{100} \frac{t_S}{w_S} \right)
\end{align} \tag{2}$$

**Figure 1.** Schematic structure of a 1-mirror (left) and a 2-mirror raster scanner. Adapted from [1].
Figure 2. Structure of the mirror with the most relevant geometrical parameters. Design variables are marked in red. The value of \( n_c \) is 2 ½.

\[
\begin{align*}
\left\{ \begin{array}{l}
\frac{f_0^2}{4\pi^2 J_{PX}} = \frac{2k\varphi}{4\pi^2 J_{PX}} \\
J_{PX} = \frac{1}{12} l_y^2 M_p \\
M_p = l_x l_y t_S \mu_S
\end{array} \right. \\
(3)
\end{align*}
\]

\[
T_x = A_c B_Y I_c \\
A_c = \sum_{k=1,5,9} \frac{d_y + 2k p_l + w_l}{2} \left( d_y + 2k p_l + w_l \right) + \\
+ \sum_{k=3,7,11} \frac{d_y + (2k+1) p_l + w_l}{2} \left( d_y + 2k p_l + w_l \right)
\]

Equations in (1) give obvious geometric constraints among the various geometric parameters; equations in (2) describe the torque-angle static characteristics of the mirror; equations in (3) give expression for the resonance frequency. Finally, equations in (4) define the relationship between torque and current. An assumption of linearity was made for the behavior of the suspending springs. The magnetic field was assumed uniform. Most equations are applications of basic physics of mechanical and electromagnetic systems and do not need special comments.

Additional remarks are only required for two of the equations. First, the sum for \( A_c \), the equivalent area of the loop, is extended over the loop branches parallel to \( x \), that are the only ones contributing to the torque. While it is not shown for simplicity, a closed form for this sum as a function of \( n_c \) exists. Secondly, a closed expression for the torsional constant for rectangular cross-sections does not exist, and approximate formulas are commonly given [6]. We chose a simplified expression which allows a closed solution for equations (1)-(4), but is only valid for \( w_S > t_S \), and also has a reduced accuracy for aspect ratios \( t_S/w_S \) close to one.

3. Optimization

Because of the specific form used, equations (1)-(4) can be solved analytically to obtain a closed expression for any involved quantity as a function of the five design parameters \( n_c \), \( w_l \), \( w_S \), \( t_S \), and \( l_S \). The solution was derived by using a software for algebraic manipulation, and is too lengthy to report here.
Table 1. Definition and values of the quantities used in the text.

| Symbol | Parameter | Value |
|--------|-----------|-------|
| $d_x$  | width of mirror surface (along main rotation axis $x$) | 2 mm |
| $d_y$  | width of mirror surface (along other axis $y$) | 1 mm |
| $l_x$  | total width of plate (including the coil) along $x$ | - |
| $l_y$  | total width of plate (including the coil) along $y$ | - |
| $w_l$  | metal width | - |
| $g_l$  | distance between metal lines | 20 $\mu$m |
| $p_l$  | metal line pitch ($g_l + w_l$) | - |
| $l_S$  | spring length (along $x$) | - |
| $t_S$  | thickness of mirror central plate and springs | - |
| $w_S$  | spring width (along $y$) | - |
| $l_{Ty}$ | total occupancy along $y$ | - |
| $\phi$ | [maximum] rotation along main axis $x$ | $[10^\circ \text{ max}]$ |
| $n_c$  | number of coils (counting half-coil) | - |
| $T_x$  | total torque on the plate | - |
| $I_c$  | coil current | - |
| $A_C$  | equivalent area of coil (i.e. area such that $T_i = B_y A_C I_c$) | - |
| $B_y$  | magnetic field | 100 mT |
| $k_\phi$ | torsional spring constant of a suspension spring | - |
| $J_{sp}$ | torsional constant of the spring cross-section | - |
| $f_\phi$ | torsional resonance frequency | - |
| $M_p$  | plate mass | - |
| $J_{p-x}$ | moment of inertia of the plate along rotation ($x$) axis | - |
| $G_{Si}$ | shear modulus of silicon along $x$ | 79.5 GPa |
| $\mu_{Si}$ | density of silicon | 2330 kg/m$^3$ |

The solution was used the starting point for subsequent optimization. As already mentioned, the three optimization targets were: a small current $I_c$ at maximum deflection, a small total length $l_{Ty}$, and a high resonance angular frequency $\omega_\phi$. A maximum angular deflection $\phi_{\text{max}}$ of $\pm 10^\circ$, compatible with a standard application, was chosen. A natural choice for the objective function to be minimized is then the linear combination:

$$f(n_c, w_l, t_S, l_S, w_S) = w_1 I_c - w_2 \omega_\phi + w_3 l_{Ty}$$

where the $w_i$ are positive weights to be tuned in dependence of the importance of each target variable. Without loss of generality, we can always set $w_1 = 0$. The objective function was minimized numerically with reasonable constraints for the design variables (shown in table 2) and for several different values of $w_2$ and $w_3$. Specifically, $w_2$ was varied in the range $[10^{-5}, 2 \cdot 10^{-4}]$ and $w_3$ in the range $[50, 1000]$. Larger ranges also led to possible optimal designs, but they were ignored because of the resulting unpractical values of the target quantities for real applications.

Table 2. Allowed optimization ranges for the design variables.

| Parameter | min   | max   |
|-----------|-------|-------|
| $n_c$     | 1.5   | 79.5  |
| $w_l$     | 20 $\mu$m | 500 $\mu$m |
| $t_S$     | 30 $\mu$m | 50 $\mu$m |
| $l_S$     | 0.3 mm | 2 mm  |
| $w_S$     | 40 $\mu$m | 200 $\mu$m |
Interestingly but not unexpectedly, all optimum solutions led to the minimum possible cross-section for the suspension springs ($t_S = 30 \mu m$, $w_S = 40 \mu m$), so that the hypothesis of an aspect ratio higher than one is always verified. As the actual torsional constant of a rectangular cross-section is invariant for a swap of the two sides, the choice of the simplified expression for $J_S$ does not exclude possible optimal solutions.

Results for the simulations are shown in figure 3, which summarizes the results for a set of 36 different optimizations. Data points corresponding to the same weight given to the resonance frequency are given the same colour. Inside each colour group, each point corresponds to a different weight given to the mirror size (i.e. its total length $l_{Tx}$). The plot on the left, which shows the relationship between the current required to drive the mirror at maximum deflection and the size of the mirror, demonstrates a clear trade-off between the two. As a matter of fact, larger mirrors in the plot correspond to longer springs and a higher number of coils, which both reduce the required current. Our procedure ensures that the increase in occupancy caused by the coils and springs is optimally allocated between the two. The plot on the right shows the relationship between the current and the resonance frequency. Again, an evident trade-off exists between low currents and high frequencies. High resonance frequencies plainly require high stiffness of the springs, which in turn requires a high current. However, the data show that for practical applications, where the scanning frequency of the slow mirror is of a few tens of Hertz, the constraint on the frequency is not very important.

4. Conclusions
In this paper we started from the descriptive equations for the static and resonance behaviour of a torsional MEMS micromirror to develop a semi-analytical procedure to design mirrors with optimal performances. The optimization data allow the extraction of optimum frontiers between the requirement for low driving current and small size, and between current and resonance frequency. The same approach can be easily extended to evaluate the trade-offs between different target parameters, and adapted to different geometries. Further generalizations can include the size of external magnets in the occupancy of the device.

![Figure 3](image1.png)

**Figure 3.** Total length of the mirror (left) and resonance frequency (right) as a function of the maximum driving current. Data are grouped by colour, each group corresponding to a different weight $w_2$ for the resonance frequency.
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