A General Cardinality Estimation Framework for Subgraph Matching in Property Graphs

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Abstract—Many techniques have been developed for the cardinality estimation problem in data management systems. In this document, we introduce a framework for cardinality estimation of query patterns over property graph databases, which makes it possible to analyze, compare and combine different cardinality estimation approaches. This framework consists of three phases: obtaining a set of estimates for some subqueries, extending this set and finally combining the set into a single cardinality estimate for the query. We show that (parts of) many of the existing cardinality estimation approaches can be used as techniques in one of the phases from our framework. The three phases are loosely coupled, this makes it possible to combine (parts of) current cardinality estimation approaches. We create a graph version of the Join Order Benchmark to perform experiments with different combinations of techniques. The results show that query patterns without property constraints can be accurately estimated using synopses for small patterns. Accurate estimation of query patterns with property constraints require new estimation techniques to be developed that capture correlations between the property constraints and the topology in graph databases.

Index Terms—Cardinality estimation, selectivity estimation, query optimization, graph databases, property graph data model.

1 Introduction

Cardinality estimation can be defined as the task of estimating the number of results returned by a given query over a given database instance. This is a fundamental data management problem, important across many practical scenarios, e.g., in query planning, approximate query evaluation, and query execution progress estimation. The basic evaluation criteria for cardinality estimation techniques are: estimation accuracy, estimation time, memory cost, and preparation time. Different applications might value these dimensions differently, leading to trade-offs, e.g., between estimation accuracy and memory footprint. It has been shown that poor query plans are regularly produced by the query optimizers of current database management systems and that cardinality estimation errors are the main reason for these bad query plans [1], [2]. Hence, while a classical topic heavily studied for decades, cardinality estimation remains a significant challenge in practice.

In this work, we survey cardinality estimation solutions for query patterns on property graph databases. Property graphs are a popular data model in industry, part of the upcoming ISO graph database query language standard [3]. Essentially, a property graph is a graph where both nodes and edges can have labels (e.g., Person) and have a set of associated key-value pairs, which are called the properties, where the property key specifies the meaning of the property value (e.g., MemberSince = 2008-11-10). A query pattern (also known as a conjunctive graph query, or a subgraph matching query [4]) specifies a graph pattern of interest (e.g., Persons and the Movies they ActIn) and also possibly constraints on attributes (e.g., only those Persons who became a member after 1990-12-08). Query Patterns are fundamental in both theory and practice, forming the backbone of virtually all queries expressed in contemporary graph database query languages [3], [4], [5]. Cardinality estimation for query patterns over property graph databases gives rise to new challenges due to the schemaless design of graph databases and the correlation between the data (property values and labels) and the topology (connectivity) of the graph. The focus of our survey allows us to shed deep light on an important, timely, highly challenging, and broadly applicable subclass of the cardinality estimation problem. Further, many of the concepts of our survey generalize to other typical settings, for example conjunctive queries on relational databases.

There is a large and rich literature on cardinality estimation for queries on database instances which can be used for query patterns on property graphs. Each technique requires specialized statistics or indexes and uses a set of simplifying assumptions to overcome missing information or to improve on criteria of interest (i.e., accuracy, time, space, etc.). Hence, it is very difficult for practitioners, systems designers, and researchers to navigate the state of the art and understand which solutions work or not for a given application scenario. The most recent and complete survey here is the groundbreaking G-CARE framework [6]. G-CARE performs extensive experiments on different cardinality estimation approaches with queries of varying sizes, result sizes and topologies, on both real and synthetic data sets. This study addresses two limitations of G-CARE. First, G-CARE studies cardinality estimation techniques for query patterns without predicates on the properties. This study includes extensive experiments on property predicates. Our experiments show that properly handling property predicates is essential for accurate cardinality estimation. Second, the focus of G-CARE is on comparing different cardinality estimation approaches. In this study, we also focus on com-
bining parts of different cardinality estimation approaches with the aim of producing superior estimation techniques.

Our contribution in this survey is to introduce a framework that makes it possible to explain the state of the art and systematically analyze, compare and combine the current cardinality estimation techniques for query patterns on property graphs. We start by viewing a query pattern as a specification of a set of constraints (Section 2). We then highlight the basic ingredients of contemporary estimators and how they together are used to define concrete cardinality estimation solutions in a general framework (Section 3). The first phase of the framework consists of estimation techniques for individual constraints (Section 4) and of estimation techniques for multiple constraints (Section 5). The second phase consists of methods for extending partial estimates (Section 6). Finally, the third phase consists of methods for combining partial estimates in order to obtain a final estimate for the complete query (Section 7).

We leverage our framework to undertake a comprehensive comparative analysis of the state of the art cardinality estimators, shedding fresh light on their trade-offs and relative strengths and weaknesses (Sections 8-10). Given these new deep insights, we conclude the survey with recommendations for practitioners and researchers (Section 11). With this work we aim to help move the field forward towards more effective cardinality estimation solutions in practical graph data management.

2 PRELIMINARIES

2.1 Basic Property Graph Query Pattern

A basic property graph query pattern, or shortly query pattern, consists of a set of vertices and a set of edges. Vertices and edges have unique identifiers. Each edge is associated with exactly one source vertex and one target vertex. Vertices and edges can have a (possibly empty) set of label and a (possibly empty) set of key-value properties.

A query pattern \( Q \) is defined as the tuple \((E, V, I, \rho, \lambda, \sigma, \Theta, \text{Lab}, \text{Prop}, \text{Val})\).

The set of edge identifiers \( E \) and the set of vertex identifiers \( V \) are disjoint, and \( I = E \cup V \). The total function \( \rho : E \to (V \times V) \) maps each edge identifier to a pair of source-target vertex identifiers. The total function \( \lambda : I \to \mathcal{P}(\text{Lab}) \), where \( \text{Lab} \) is a finite set of labels and \( \mathcal{P}(\text{Lab}) \) is the powerset of \( \text{Lab} \), maps edge and vertex identifiers to a (possibly empty) set of labels. The partial function \( \sigma : (I \times \text{Prop} \times \Theta) \to \text{Val} \) defines the key-property constraints of vertices and edges, where \( \text{Prop} \) is a finite set of properties, \( \text{Val} \) is a set of values and \( \Theta \) is a set of binary predicates. 

Example 2.1. The formal description of the query pattern in Figure 1 is \( Q_{\text{example}} = (E_{\text{ex}}, V_{\text{ex}}, \rho_{\text{ex}}, \lambda_{\text{ex}}, \sigma_{\text{ex}}, \text{Lab}_{\text{ex}}, \text{Prop}_{\text{ex}}, \text{Val}_{\text{ex}}, \Theta_{\text{ex}}) \), where:

- \( E_{\text{ex}} = \{id1, id3, id5, id7\} \)
- \( V_{\text{ex}} = \{id0, id2, id4, id6, id8\} \)
- \( I_{\text{ex}} = \{id0, id1, ..., id8\} \)
- \( \rho_{\text{ex}}(id1) = \{id0, id2\}, ... \)

1. We use the dot notation to refer to an element in \( Q \) or \( G \), e.g. \( Q.\sigma \).

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Table 1: Notation for Query Patterns and Property Graph Instances

| \( Q \) | Query Pattern | \( G \) | Property graph notation |
|---|---|---|---|
| \( Q.\text{Lab} \) | Set of labels in \( Q \) | \( G.\text{Lab} \) | Set of labels in \( G \) |
| \( Q.\text{Prop} \) | Set of property keys in \( Q \) | \( G.\text{Prop} \) | Set of property keys in \( G \) |
| \( Q.\lambda \) | Set of binary predicates used in \( Q \) | \( G.\lambda \) | Set of binary predicates in \( G \) |
| \( Q.\rho \) | Set of edge identifiers in \( Q \) | \( G.\rho \) | Set of edge identifiers in \( G \) |
| \( Q.\sigma \) | Set of vertex identifiers in \( Q \) | \( G.\sigma \) | Set of vertex identifiers in \( G \) |
| \( Q.\tau \) | Set of constraints defined by \( Q \) | \( G.\tau \) | Set of constraints defined by \( G \) |
| \( \mathcal{P}(\text{Prop}) \) | Set of all mappings in \( Q.\text{Prop} \) | \( \mathcal{P}(\text{Prop}) \) | Set of all mappings in \( G.\text{Prop} \) |
| \( \mathcal{P}(\text{Lab}) \) | Set of all mappings in \( Q.\text{Lab} \) | \( \mathcal{P}(\text{Lab}) \) | Set of all mappings in \( G.\text{Lab} \) |
| \( \mathcal{P}(\text{Lab}) \) | 

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Table 2: Notation (section where notation is introduced)

| \( m_{D \to R} \) | A mapping that maps each element in the domain \( D \) to an element in range \( R \) |
|\( M_{D \to R} \) | Set of all possible mappings \( m_{D \to R} \) |
| \( \mathcal{S} \) | Cardinality of set \( \mathcal{S} \) |
| \( C \) | A constraint |
| \( C(Q) \) | Set of constraints defined by \( Q \) |
| \( C.I \) | Set of query ids contained in \( C \) |
| \( SA_{P D \to G.1} \) | Set of all mappings in \( M_{D \to G.1} \) that satisfy constraint \( c \) |
| \( Pr[SA_{P D \to G.1}] \) | Fraction of all mappings in \( M_{D \to G.1} \) that satisfy constraint \( c \) |
| \( PE.S \) | Selectivity value of \( PE \) |
| \( PE.C \) | Set of constraints in \( PE \) |
| \( PEI \) | Partial Estimation technique |
| \( PES \) | Partial Estimation Set |
| \( EPES \) | Extend PES technique |
| \( CPES \) | Complete PES |

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2.2 Property Graph Data Model

A property graph instance is defined as a tuple \( G = (E, V, I, \rho, \lambda, \sigma, \Theta, \text{Lab}, \text{Prop}, \text{Val}) \). Only the equality operator is allowed for \( \Theta \) in this definition, therefore \( \Theta \) is irrelevant. Let \( \ell_e(G) \subseteq G.\text{Lab} \) be the set of labels that occur on edges in graph \( G \) and \( \ell_v(G) \subseteq G.\text{Lab} \) be the set of all labels that occur on vertices in graph \( G \). Table 2 summarizes our notation.

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2.3 Semantics of Query Patterns

Consider a query pattern \( Q \) and a property graph instance \( G \). Let mapping \( m_{Q.1 \to G.1} \) be a total function from the edge and vertex identifiers in \( Q \) to edge and vertex identifiers in \( G \), i.e. \( m : Q.I \to G.I \). Let \( M_{Q.1 \to G.1} \) be the set of all possible mappings from \( Q \) to \( G \). Table 3 shows a summary of the notation used in this document.
A mapping $m$ is called a homomorphic match \[\text{from} Q \text{ on } G \text{ if and only if the following match requirements are satisfied:} \]

MR1: For each $v \in Q.V$: $m(v) \in G.V$;

MR2: For each $e \in Q.E$: if $Q.\rho(e) = (s, t)$, then $G.\rho(m(e)) = (m(s), m(t))$;

MR3: For each $v \in Q.I$ and $l \in Q.Lab$: if $l \in Q.\lambda(i)$, then $G.\lambda(m(i))$;

MR4: For each $i \in Q.I, k \in Q.Prop, \theta \in Q.\Theta$ and $v \in Q.Val$: if $Q.\sigma(i, k, \theta) = v$, then $G.\sigma(m(i), k) = w$ and $w \theta v$ holds for some $w \in G.Val$.

Here, MR1 and MR2 require that the mapping agrees with the topology defined in the query. Notice that MR1 is required when the query contains disconnected vertices. MR3 defines constraints on the existence of labels and MR4 defines constraints on the existence of properties.

Finding all matches for a query pattern is also called subgraph pattern matching and is at the core of many query languages [7], e.g. SPARQL\[\text{Cypher}\] PGQL\[\text{G-CORE}\[9\].

The total number of mappings is $|\mathcal{M}_{Q,G}|$, where $|S|$ denotes the cardinality of set $S$. The number of these mappings that are matches is called the cardinality of $Q$ on $G$ and the fraction is called the selectivity.

2.4 Query Constraints

Given a query pattern $Q$, the set of constraints of $Q$, $C(Q)$, is defined as follows:

1) For each vertex id $v \in Q.V$: vertex($v$) $\in C(Q)$
2) For each edge id $e \in Q.E$: edge($e$) $\in C(Q)$
3) For each edge id $e \in Q.E$: if $Q.\rho(e) = (s, t)$, then src($s$, $e$) $\in C(Q)$ and trg($t$, $e$) $\in C(Q)$
4) For each $i \in Q.I$ and $l \in Q.Lab$: if $l \in Q.\lambda(i)$, then hasLabel($i$, $l$) $\in C(Q)$
5) For each $i \in Q.I, k \in Q.Prop, \theta \in Q.\Theta$ and $v \in Q.Val$: if $Q.\sigma(i, k, \theta) = v$, then hasPropKey($i$, $k$, $\theta$) $\in C(Q)$ and hasProp($i$, $k$, $\theta$, $v$) $\in C(Q)$
6) $C(Q)$ does not contain any other constraints

A single constraint is denoted by $c$, a set of constraints by $C$ and the set of all constraints of $Q$ by $C(Q)$. A constraint contains one or two identifiers from a query $Q$. Let $C.I$ represent the set of query identifiers contained in the constraints $C$.

Given a $c$ constraint, a mapping $m(c) \rightarrow G$ and a graph $G$, it is possible to verify if $m$ satisfies $c$ in $G$:

- If $c =$vertex($i$), then verify if $m(i) \in G.V$
- If $c =$edge($i$), then verify if $m(i) \in G.E$

2. Some systems require isomorphic matches, which can be obtained by adding the requirement MR5. For each $i, j \in Q.I$: if $i \neq j$, then $m(i) \neq m(j)$. See [7] for more advanced isomorphic-based semantics, like no-repeated-node and no-repeated-edge semantics.

3. Here, we use the infix notation of the binary predicate $\theta$
4. https://www.w3.org/TR/rdf-sparql-query/
5. https://neo4j.com/developer/cypher/

![Figure 1: JOB query 18a visualized as a query pattern](image)

- If $c =$src($v$, $e$), then verify if $G.\rho(m(e)) = (m(v), m(v))$ for some $v \in G.V$
- If $c =$trg($v$, $e$), then verify if $G.\rho(m(e)) = (m(v), m(v))$ for some $v \in G.V$
- If $c =$hasLabel($i$, $l$), then verify if $l \in \lambda(m(i))$
- If $c =$hasPropKey($i$, $k$), then verify if $G.\sigma(m(i), k) = w$ is defined for some $w \in G.Val$
- If $c =$hasProp($i$, $k$, $\theta$, $v$), then verify if $G.\sigma(m(i), k) = w$ and $w \theta v$ holds for some $w \in G.Val$

A mapping $m_{Q,I \rightarrow G,I}$ satisfies all constraints $C(Q)$ of query $Q$ in graph $G$, if and only if $m_{Q,I \rightarrow G,I}$ is a homomorphic match from $Q$ to $G$. Table\[\text{3}\] shows the categorization of constraints based on what they primarily deal with: topology or data.

Example 2.2. The constraints $C(Q)$ of the query pattern $Q$ in Figure\[\text{4}\] are given as follows:

- $Q_0$: vertex(id0),..., $Q_4$: vertex(id8),
- $Q_5$: edge(id1),..., $Q_7$: edge(id7),
- $Q_8$: src(id0, id1),..., $Q_{27}$: trg(id8, id7),
- $Q_{28}$: hasLabel(id0, title),..., $Q_{55}$: hasLabel(id8, person),
- $Q_{56}$: hasProp(id6, note, IN, ‘(producer)’, ‘(executive producer)’)
- $Q_{57}$: hasProp(id8, gender, =’m’),
- $Q_{58}$: hasProp(id8, name, CONTAINS, ‘Tim’),
- $Q_{59}$: hasPropKey(id6, note),
- $Q_{60}$: hasPropKey(id8, gender),
- $Q_{61}$: hasPropKey(id8, name).

2.5 Probability, Selectivity and Cardinality

Consider obtaining a random mapping $m_{Q,I \rightarrow G,I}$ from $M_{Q,I \rightarrow G,I}$ by throwing a fair $|G.I|$-sides dice for every $i \in Q.I$ to obtain $m(i)$. This gives a sample space of $G.I \times \cdots \times G.I$. Then, the probability of obtaining mapping $m = (\omega_1, \ldots, \omega_{|Q.I|})$ is

$$\Pr[(\omega_1, \ldots, \omega_{|Q.I|})] = \Pr[\omega_1] \cdots \Pr[\omega_{|Q.I|}] = (1/|G.I|)^{|Q.I|},$$

because the outcomes of the dice throws are mutually independent.
Table 3: Categorization of constraints

| Query Pattern Constraints | Property-value Constraints |
|---------------------------|----------------------------|
|                           | Labeled Topological Constraints | Topological Constraints | Data Constraints |
|                           | vertex(t) | edge(t) | src(t,e) | trg(t,e) | hasLabel(t,l) | hasPropKey(t,k) | hasProp(t,k,v) |

Figure 2: The set $S_C(Q)$ can be obtained by taking the intersection of the events $S_c$ for $c \in C(Q)$.

Subsets of the sample space are called events. Let $SAT_{Q,G.1}^{0.1}$ be the event that contains all mappings in $M_{Q,G.1}$ that satisfy the constraint $c$:

$$SAT_{Q,G.1}^{0.1} = \{ m \in M_{Q,G.1} | m \text{satisfies } conr c \}$$

Since each element from our sample set has the same probability, the probability of event $SAT_{Q,G.1}^{0.1}$ is defined as:

$$Pr[SAT_{Q,G.1}^{0.1}] = \frac{|SAT_{Q,G.1}^{0.1}|}{|M_{Q,G.1}|} = \frac{|SAT_{Q,G.1}^{0.1}|}{|M_{Q,G.1}|}$$

The event “$m$ satisfies all constraints in $C$” can be expressed as: $\bigcap_{C \in C(Q)} S_C$. This will be written shortly as $S_C$. Notice that the event $S_{C_{1} \subseteq C}$ represents the set of mappings that satisfy all constraints in $C_{1}$ and all constraints in $C_{j}$. This corresponds to $S_{C_{1}} \cap S_{C_{j}}$. Whereas the event $S_{C_{1} \cup C_{j}}$ represents the set of mappings that satisfies all constraints in $C_{i}$ or all constraints in $C_{j}$. Figure 2 shows an example with the notation introduced in this section.

Within the database literature, the term “selectivity of $c$” is usually used to represent the probability $Pr[S_C]$. In this document, we will use both terms interchangeably, since they mean the same thing. The goal in this document is to estimate the selectivity $Pr[S_C(Q)]$, since multiplying this with $|M_{Q,G.1}|$ (which is equal to $|G.I|^{Q,G.1}$) gives an estimate for the cardinality $|S_C(Q)|$.

3 Selectivity Estimation Framework

The task of obtaining an estimate for the selectivity of a query can be simplified by first obtaining estimates for subsets of the query constraints and then combining those estimates.

An estimate for a subset of the query constraints is called a partial estimate (PE). A PE contains a set of constraints (PE.C) and a selectivity estimate (PE.s) for that set of constraints, such that $Pr[PE.C]$ is approximated by $PE.s$. The set of all PEs for a query will be called its partial estimate set (PES).

Techniques that are able to obtain PEs are called Partial Estimation Techniques (PETs). A PET requires specific statistics or indexes (prerequisites) in order to obtain a PE for a specific set of constraints (targeted set of constraints) using a specialized procedure (estimation procedure). For each PET, these three elements will be described (summarized in Table 5). Techniques that rely on the existence of other partial estimates are called Extend PES Techniques (EPESTs). They add PEs to the PES by combining several PEs using specialized assumptions. The resulting set of PEs is called an Extended PES (EPES). Finally, a selectivity estimate for a query pattern is obtained by combining all PEs of a complete PES (CPES) using a general combine technique (CT).

Notice that a PE should be complete before it can be combined. A PES is complete w.r.t. a query $Q$, when every constraint from $C(Q)$ occurs at least one partial estimate from the PES. Therefore, it must hold that $\bigcup_{PE \in PES} PE.C = C(Q)$. A PES can be made complete by adding a PE of each missing constraint with a default selectivity value.

The general framework is illustrated in Figure 3, where the gray dashed boxes are considered as input and the green dashed box as output. Processes are shown as boxes with sharp corners and input and output are shown as boxes with rounded corners. The blue boxes represent the three phases of the framework, for which multiple techniques are available. Partially overlapping blue boxes mean that zero or more options can be chosen. Algorithm 1 shows the whole framework in more detail.

4 PETs for Individual Constraints

This section deals with PETs for obtaining estimates for individual constraints. The targeted set of constraints consists of individual constraints of a specific type. The following subsections will describe the estimation procedures and prerequisites for each type.

Vertex constraints. The constraint $\text{vertex}(i)$ is satisfied for every mapping that maps query id $i$ to an element in $G.V$.

7. We will refer to EPES and CPES usually as PES when it will not lead to ambiguity.

8. The statistics collection procedure, i.e. the step from graph instance to statistics, is separate from the estimation procedure. It can be executed after batch loading of a graph instance, at user’s request, at low system utilization, at periodic intervals or at demand. Some statistics can also be obtained using a feedback loop after executing queries.
Graph Instance

Data: \( Q \): a query pattern
Data: \( stats_G \): available statistics for instance \( G \)
Data: \( PETs \): A set of partial estimation techniques
Data: \( EPESTs \): A set of extend PES techniques
Data: \( CT \): Combination technique

Result: Selectivity estimate for \( Q \) on graph instance \( G \), i.e. an estimate for \( Pr_G[\mathcal{S}(Q)] \),

\[ PES_Q = \{\} \] for each partial estimation technique \( PET \) in \( PETs \) do
\[ cSets \leftarrow \text{find all sets of constraints (subsets of} \ C(Q) \text{)} \text{that belong to the targeted set of constraints of} \ PET \];
for each \( cSet \) in \( cSets \) do
if required stats are available in \( stats_G \) to obtain an estimate for \( cSet \) using \( PET \) then
\[ selEst \leftarrow \text{obtain selectivity estimate for} \ cSet \text{using statistics and estimation procedure from} \ PET \];
add \( (cSet, selEst) \) to \( PES_Q \);
end
\[ EPES_Q = \{\} \]
\[ EPES_Q.addAll(PES_Q) \]
for each Extend PES Technique \( EPEST \) in \( EPESTs \) do
\[ EPEs \leftarrow \text{obtain extended partial estimates using} \ EPEST(EPES_Q, Q) \]
\[ EPES_Q.addAll(EPEs) \]
end
\[ CPES_Q = \text{makeComplete}(EPES_Q, Q) \]
\[ selEst \leftarrow \text{combine all PEs in} \ CPES_Q \text{using} \ CT \]
return \( selEst \);

Algorithm 1: Selectivity Estimation Framework

![Selectivity Estimation Framework](image)

All other query ids can be mapped to any element in \( G.I \). This gives a total of \( |G.V| \cdot |G.I|^{|Q|.1^{|Q|-1}} \) mappings in event \( SAT_{vertex(i)}^{Q.1 \rightarrow G.1} \). Therefore, \( Pr[S_{vertex(i)}] = \frac{|G.V| \cdot |G.I|^{|Q|.1^{|Q|-1}}}{|G.I|} = \frac{|G.V|}{|G.I|} \). The required prerequisites are stored values for \( |G.V| \) and \( |G.I| \).

Edge constraints. The selectivity of \( edge \) (\( i \)) constraints, where \( i \in Q.I \), is \( |G.E|/|G.I| \). Required prerequisites are stored values for \( |G.E| \) and \( |G.I| \).

Src/Trg constraints. The constraint \( src \) \( (i_1, i_2) \) is satisfied if \( i_1 \) is mapped to an element \( v \in G.V \) and \( i_2 \) is mapped an element in \( G.E \) such that \( G.p(e) = (v, v') \) for some \( v' \in G.V \). Each edge has exactly one source vertex, therefore \( i_2 \) can be mapped to any element in \( G.E \), then there remains only one option for \( i_1 \). All other query ids can be mapped to any element in \( G.I \). This gives a total of \( |G.E| \cdot 1 \cdot |G.I|^{|Q|} \cdot 2 \) mappings in \( SAT_{vertex(i)}^{Q.1 \rightarrow G.1} \). Therefore, \( Pr[S_{src(i_1, i_2)}] = \frac{|G.E| \cdot 1 \cdot |G.I|^{|Q|}}{|G.E|} \cdot 2 = \frac{|G.E|}{|G.I|} \). Required prerequisites are \( |G.E| \) and \( |G.I| \). The selectivity of \( trg \) \( (i_1, i_2) \) is the same, because each edge also has exactly one target vertex.

Label constraints. When the number of different labels \( (|G.\text{Lab}|) \) is small, \( Pr[S_{hasLabel(i_1, l)}] \) can be precomputed for every \( l \in G.\text{Lab} \) (prerequisites). Otherwise, it is possible to treat them as property values with a special property key, e.g. ‘hasLabel’, and use one of the selectivity estimation methods for property-value constraints.

Property-key constraints. When the number of different property keys \( (|G.\text{Prop}|) \) is small, \( Pr[S_{hasProp(i_1, k)}] \) can be precomputed for every \( k \in G.\text{Prop} \) (prerequisites). Otherwise, it is possible to treat them as property values with a special property key, e.g. ‘hasKey’, and use one of the selectivity estimation methods for property-value constraints.

Property-value constraints. The remaining constraints are the property-value constraints, e.g. \( hasProp(i_1, \text{birthdate} \leq 2000) \). Storing the exact selectivity of every property constraint in a synopsis (prerequisites) requires only a synopsis lookup (estimation procedure) to obtain the exact selectivity. However, storing exact selectivities can be prohibitively expensive as the set \( G.V \) can be very large in property graph instances. This requires specialized techniques to obtain estimates for the selectivity of those constraints.

One way is to assign default selectivity values based on the operator used, i.e. property constraints with equality operator get a selectivity estimate of 1/10, with inequality get 9/10 and with other operators get 1/3 [10]. We will call this technique “Default values”. This technique has no prerequisites and the estimation procedure consist of applying simple rules, i.e. if operator is ‘=’, then selectivity is 1/10.

Another way is to store a small sample of the whole data and compute the selectivity of a constraint on that sample when needed. Use that selectivity as an approximation of the selectivity on the whole data. Prerequisite for this technique is a sample of \( G.I \). Sampling is very diverse, but is known to have problem with highly selective constraints and requires large estimation times [11].

Property constraints with a comparison operator, i.e. ‘\( = \), \( \neq \), \( < \), \( \leq \), \( > \), \( \geq \)’, can be approximated using histograms. A histogram summarizes the distribution of a collection of element by partitioning them into buckets. Different partitioning schemes have been introduced [12], [13] which makes it possible to choose between construction complexity and estimation accuracy. Assumptions are used to approximate the values and their frequencies within the buckets. Prerequisites are histograms for the property keys.

Property constraints with a substring operator, i.e. ‘CONTAINS’ or ‘LIKE’ in SQL, can be approximated using pruned suffix trees (PSTs) [14].
A summary of the different techniques is given in Table 4. The different approaches all have their own advantages and disadvantages, which makes them useful in different scenarios. Also, multiple techniques can be used. Most frequently occurring query constraints can be precomputed. Histograms can be constructed for property keys that occur in many constraints with comparison operators. While the remaining constraints can be estimated using sampling or using default values.

## 5 PETs for Multiple Constraints

Previous section showed techniques for estimating the selectivity of every individual constraint. The techniques described in this section are able to obtain estimates for multiple constraints (Table 5 shows a summary), but are not applicable in general to obtain an estimate for any possible query pattern. Section 4 shows how all estimates obtained for multiple constraints (in this section) and for each individual constraint (in the previous section) can be combined to obtain a final estimate for any query pattern.

When the estimation procedure obtains a cardinality estimate \( \hat{c} \) for the set of constraints \( C \), then the selectivity estimate \( \Pr[|S_C|] = c/|G.|^{|C|-1} \).

### 5.1 Estimation for Topological Constraints

The topology of a graph refers to its vertices, edges and their connectivity. Different topological patterns can be defined, which are subclasses of general query patterns:

- An **edge pattern**, \((v_1, e_1, v_2)\), consists of a single edge together with its source and target vertices.
- A **chain pattern**, \((v_1, e_1, v_2, \ldots, e_{m-1}, v_m)\), is a chain of edge patterns in one direction. Let \(e_1, e_2, \ldots, e_n\) be edge patterns. Then, \(e_1/e_2/\ldots/e_n\) is a chain pattern when the target vertex of \(e_i\) is the source vertex of \(e_{i+1}\) for all \(1 \leq j \leq n - 1\).
- A **star pattern** is a set of edge patterns that all have a common vertex. When all edge patterns have a common source vertex, then the pattern is also called a **source star pattern**, \( v_1, v_2, \ldots, v_n \). When all edge patterns have a common target vertex, then the pattern is also called a **target star pattern**, \( v_1, v_2, \ldots, v_n \).

Other frequently occurring patterns in the query workload can be used, e.g. triangles, cycles, trees, etc. See Bonifati et al. [5] for a study of frequently occurring query patterns.

The size of a topological pattern is defined as the number of edge patterns involved in that pattern.

**Example 5.1.** The query pattern in Figure 1 contains, among others, four subpatterns that are edge patterns, two subpatterns that are size-2 chain patterns and two subpatterns that are size-2 source star patterns.

### 5.1.1 Topological Synopsis Lookup

A topological synopsis contains the cardinalities of topological patterns. For example, it can contain the cardinalities of a chain pattern of size 2 and source star patterns of size 2 and 3. The cardinalities can be precomputed and stored at a small cost, when the patterns are small.

**Targeted Set of Constraints.** All topological subpatterns of the query pattern for which the synopsis stores its cardinality.

**Prerequisites.** The topological synopsis.

**Estimation Procedure.** The cardinality can be obtained using a lookup in the synopsis.

### 5.2 Estimation for Labeled Topological Constraints

Vertices and edges can have labels, which can have a large impact on the cardinality of a query.

Labeled topological patterns are an extension of topological patterns, where each query id is allowed to have at most one label e.g., a labeled edge pattern, labeled chain pattern, etc.

#### 5.2.1 Labeled Topological Synopsis Lookup

A labeled topological synopsis contains cardinalities of labeled topological patterns belonging to a specific class, e.g. labeled edge patterns or labeled chain patterns of size 2.

The specific pattern class and its size are considered the parameters of a labeled topological synopsis. For example, a labeled topological synopsis can be specified by LabTopSyn(chain, 2), which represents a synopsis which stores the cardinality of all labeled chain patterns of size 2.

Aboulnaga et al. [15] introduced a synopsis for labeled chain pattern up to size \( n \) and used that synopsis to obtain estimates for labeled chain patterns of size \( m > n \) using Markov Chains:

\[
\Pr[|S_{C(t_1/\ldots/t_m)}|] = \Pr[|S_{C(t_1/\ldots/t_n)}|] \cdot \prod_{i=1}^{m-n} \Pr[|S_{C(t_{i+1}/\ldots/t_{n+i-1})}|]
\]

where \( t_1/\ldots/t_j \) represents a chain of labeled edge patterns \( t_i \) for \( i \leq x \leq j \). This formula is a special case of the combination technique based on conditional independence (with sorting strategy based on maximum overlap, which is explained in detail in Section 7).

**Targeted Set of Constraints.** All subpatterns of the query pattern belonging to the pattern class of the labeled topological synopsis.

**Prerequisites.** The labeled topological synopsis.

**Estimation Procedure.** The cardinality can be obtained using a lookup in the synopsis.

#### 5.2.2 System R’s Join Size Estimation using Inclusion and Uniform Distribution Assumptions

Join size estimation in relational databases has its origin from System R [10]. Considering each labeled edge pattern as a relation, with an attribute for the source vertex id, edge id and target vertex id, allows us to apply System R’s join size estimation method in the context of this document.

System R uses the cardinality and column cardinality of each relation (i.e. number of edges and number of distinct source and target vertices for each labeled edge pattern), and three assumptions to estimate the cardinality of any
sequence of equality joins (i.e. any labeled topological pattern). The three assumptions are 1) the inclusion assumption (all values in the joining attribute with the lowest number of distinct values are included in the set of values of the other joining attribute), 2) the uniform distribution assumption (all distinct values of an attribute have the same frequency) and 3) the preservation of value set assumption (all distinct values in the non-joining attributes are preserved after the join).

**Targeted Set of Constraints.** Using all three assumptions, it is possible to obtain cardinality estimates for any labeled topological pattern. Using only the first two assumptions, it is possible to obtain estimates for any star pattern, which will be the focus of this section.

**Prerequisites.** Let $S_G(ep)$ and $T_G(ep)$ be the multiset of source and target vertices on the edges in $G$ that match labeled edge pattern $ep$. Let $S'_G(ep)$ and $T'_G(ep)$ be the set versions of $S_G(ep)$ and $T_G(ep)$, where duplicates are eliminated.

This approach can be applied to star pattern $S$ when the following statistics are available for every edge pattern $ep$ in $S$ for graph instance $G$:

- $n_G(ep)$: cardinality of $ep$ in $G$
- $|S'_G(ep)|$: the number of distinct source vertices of the edges in $G$ that match $ep$
- $|T'_G(ep)|$: the number of distinct target vertices of the edges in $G$ that match $ep$

**Estimation Procedure.** Let $f(M, v)$ represent the frequency of vertex $v$ in multiset $M$. Consider a labeled star pattern that consists of labeled edge patterns $\{ep_1, \ldots, ep_n\}$ and a center vertex $v$. Let $X_G(ep_i)$ be $S_G(ep_i)$ if $v$ is the source vertex of $ep_i$ and $T_G(ep_i)$ if $v$ is the target vertex of $ep_i$.

In general, the cardinality of the star pattern is

$$\sum_{v \in X_G(ep_1) \cap \cdots \cap X_G(ep_n)} \prod_{1 \leq i \leq n} f(X_G(ep_i), v)$$

Assuming a uniform distribution gives:

$$\approx \sum_{v \in X_G(ep_1) \cap \cdots \cap X_G(ep_n)} \prod_{1 \leq i \leq n} \frac{n_G(ep_i)}{|X'_G(ep_i)|}$$

The sum is independent of the value $v$:

$$= |X_G(ep_1) \cap \cdots \cap X_G(ep_n)| \prod_{1 \leq i \leq n} \frac{n_G(ep_i)}{|X'_G(ep_i)|}$$

Now, use the inclusion assumption to estimate the size of the intersection:

$$\approx \min(|X_G(ep_1)|, \ldots, |X_G(ep_n)|) \prod_{1 \leq i \leq n} \frac{n_G(ep_i)}{|X'_G(ep_i)|}$$

This makes it possible to estimate the cardinality of a star pattern $S$ using only $n_G(ep), |S'_G(ep)|$ and $|T'_G(ep)|$ for every edge pattern $ep$ in $S$.

### 5.2.3 Bound Sketch

Consider a size-2 labeled chain pattern $ep_i/ep_j = (i_1, i_2) / (j_1, j_2)$ where $ep_i = (i_1, i_2, i_3, i_4)$ and $ep_j = (j_1, j_2, j_3, j_4)$ are labeled edge patterns, $i_1, i_2, \ldots$ are ids and $j_1, j_2, \ldots$ are labels.

If the cardinalities of $ep_i$ and $ep_j$ are known, then the cardinality of $ep_i/ep_j$ is at most $|ep_i| \cdot |ep_j|$. This upper bound occurs if every mapping from $ep_i$ matches with every mapping from $ep_j$, which can be visualized as $\begin{array}{c} 0 \\ \hline \end{array}$.

Let $d_{ep}$ be the maximum degree of vertices in $m(v)$ in the mappings from edge pattern $ep$. Above is a special case where $d_{ep} = |ep|$ and where $d_{ep} = |ep|$.

When $d_{ep}$ is known, then each mapping from $ep$ can be matched with at most $d_{ep}^3$ mappings from $ep_j$, which leads to the upper bound $|ep_i| \cdot d_{ep}^3$. Similar, when $d_{ep}$ is known, then this leads to the upper bound $d_{ep}^3 \cdot |ep_j|$ for the cardinality of $ep_i/ep_j$. When both

### Table 5: Summary of different PETs for obtaining estimates for multiple constraints.

| Technique | Targeted Set of Constraints | Prerequisites | Estimation Procedure |
|-----------|----------------------------|--------------|---------------------|
| Topological Synopsis | Topological Patterns | Topological Synopsis | Synopsis Lookup |
| Labeled Topological Synopses | Labeled Topological Patterns | Labeled Topological Synopsis | Synopsis Lookup |
| System R’s Join Size Estimation | Labeled Star Patterns | Cardinality and the number of distinct source and target vertices of every labeled edge pattern | Apply inclusion and uniform distribution assumptions |
| BoundSketch | Any Labeled Topological Pattern | Cardinality and maximum degree for every vertex in every partition | Use upper bound as cardinality estimate. Obtain upper bound by using degree statistics and partitioning |
| Characteristic Sets | Source/Target Labeled Topological Patterns | For each CS in the data: the number of vertices that have that CS and the count for each label in that CS | Find the CSs in the data that are supersets of the query CS and sum their contributions |
| Multidimensional Histograms | Property-value constraints to same query id | The multidimensional histograms | Sum the fractions of the buckets in the histogram that satisfy the constraints |
| Sampling | Any set of query pattern constraints on a specific pattern type | A random sample of all patterns in the data that belong to a specific pattern type | Use the fraction of the sample that satisfies the constraints as selectivity estimate |
| Wander Join | Any set of constraints such that a valid walk plan (ordering of the query edges) exists | Indexes to efficiently perform a random walk | Perform random walks. If the walk satisfies the constraints, then add $1/$probability of obtaining that walk) to the result. Divide the result by the number of random walks performed to obtain the cardinality estimate |
\(d^2_{ep_i}\) and \(d^3_{ep_i}\) are known, the upper bound can be obtained by taking the minimum of both upper bounds.

Cai et al. [16] improve upon this kind of upper bounds, by partitioning the data of the join attributes. Their data summary is called a Bound Sketch. For example, above upper bound can be improved by partitioning the mappings of \(ep_i\) and \(ep_j\) on their values for \(m(i,j)\), and storing the cardinality and maximum degree for each partition. Then, the cardinality estimate of the pattern \(ep_i/ep_j\) is the sum of the upper bound obtained from each partition. The upper bound for the cardinality can be used as an estimate for the cardinality.

Targeted Set of Constraints. Any labeled topological pattern.

Prerequisites. Cardinality and degree statistics for each partition.

Estimation Procedure. Obtain upper bounds for each bounding formula. Use the lowest one as cardinality estimate. Obtain an upper bound by applying a bounding formula to each partition and sum the results.

5.3 Estimation for Query Pattern Schema Constraints

The class of schema constraints extends the class of labeled topological constraints with ‘has property key’ constraints.

5.3.1 Characteristic Sets

Frequently, in graph instances, many vertices have the same property keys and the same labels on adjacent edges. This insight was exploited by Neumann and Moerkotte [17] and will be discussed in this section.

The set of labels on all outgoing edges of a vertex \(v\) is called the Characteristic Set (CS) of \(v\), denoted by \(CS(v)\) [17]. This definition was defined on the RDF data model [21], [22]. In the RDF data model, properties are encoded as edges, with the property key on the edge and the property value on the target vertex. Since we are focusing on property graphs, we extend the definition of \(CS(v)\) to also include all property keys of vertex \(v\).

Targeted Set of Constraints. For each labeled source star pattern, \(P_{SS}\), this approach can produce an estimate for the set of constraints that define \(P_{SS}\), i.e. \(C(P_{SS})\), together with all ‘has property key’ constraints on its center vertex.

Prerequisites. Let \(CS(G)\) be the set of all different CSs that occur in the graph instance \(G\). In theory, each vertex can have a different CS, but, in practice, the number of CSs in an instance is usually much smaller [17].

For each \(CS' \in CS(G)\) in the graph instance \(G\), store:

- \(|CS'|\): The number of vertices \(v\) in the data, such that the CS of \(v\) is equal to \(CS'\) (i.e. \(CS(v) = CS'\))
- For each edge label \(l \in CS'\):
  - \(CS'.count(l)\): The total number of outgoing edges with label \(l\) from the vertices with CS \(CS'\)

Each query id can have only one value for a property key in our model. Therefore, for each property key \(k \in CS'\), \(CS'.count(k) = |CS'|\). Therefore, this does not need to be stored explicitly for every property key in a CS.

A large number of different characteristic sets in a graph instance will lead to slow estimation time, since finding all superset of the CS of the query will lead to a noticeable overhead (see Estimation Procedure). Neumann and Moerkotte [17] propose to merge a CS into a CS that is a superset to limit to total number of different CSs. They suggest to keep the 10000 most frequent CSs and merge the remaining CSs. Merging is first attempted directly by finding the smallest possible superset (use the most frequent one as tiebreaker). When no superset can be found, the set is first split (largest possible set that is a subset of another CS, and the rest) and then merging both parts individually.

Estimation Procedure. Let \(P'_{SS}\) denote the a labeled sourced star pattern extended with the ‘has property key’ constraints on its center vertex. Let \(CS(P'_{SS})\) be the CS of the (sub)query pattern \(P'_{SS}\). Estimate the cardinality of \(P''_{SS}\) as:

\[
\text{cardEst}(P''_{SS}) \approx \sum_{\{CS' \in CS(G) | CS' \supseteq CS(P'_{SS})\}} \left( |CS'| \cdot \prod_{e \in CS(P'_SS)} \frac{CS'.count(e)}{|CS'|} \right)
\]

Here, CSs in \(CS(G)\) that includes statistics of interest to estimate of the cardinality of \(P''_{SS}\) are those that are superset of \(CS(P''_{SS})\). The first part \(|CS'|\) gives the number of vertices with CS \(CS'\). Each vertex with CS \(CS'\) can have multiple edges with the label \(l\) \(\in CS(P'_{SS})\). A uniform distribution is assumed, i.e. each vertex with CS \(CS'\) has approximately \(\frac{CS'.count(l)}{|CS'|}\) number of edges with label \(l\). If \(e\) is a property key, then \(CS'.count(e) = |CS'|\).

Example 5.2. Our example query from Figure 1 has three CSs with at least two elements: \(C_{Q1} = \{\text{budget}, \text{notes}\}\), \(C_{Q2} = \{\text{cast\_info\_person}, \text{cast\_info\_movie}, \text{note}\}\) and \(C_{Q3} = \{\text{gender}, \text{name}\}\).

With minor differences, this technique can also be used for labeled target star patterns.

5.4 Estimation for Property-value Constraints

All previous approaches focused on obtaining estimates for the topology of the query and extensions that also include labels and ‘has property key’ constraints. Combination of property-value constraints will be the focus of this section.

5.4.1 Multidimensional Histogram

Section 4 showed that the distribution of property values for a specific property key can be approximated using a histogram. Extension are developed to approximate joint distributions of values from more than one property key [18], [19].

A difficult question is for which set of property keys to construct a multidimensional histogram. Bruno and Chaudhuri [23] follow a workload driven approach to find the combinations of constraints that will benefit most from additional statistics.

Targeted Set of Constraints. A set of property-value constraints referring to the same query id.

Prerequisites. A multidimensional histogram for a combination of all property keys of the property-value constraints.
Estimation Procedure. Find the buckets in the multidimensional histogram that contain elements that satisfy the constraints and estimate the fraction of the element within those buckets that satisfy the constraint. The sum of those values will be the cardinality estimate. Intra bucket estimation is done using assumptions like the uniform spread and uniform distribution assumption.

5.5 Estimation for Query Pattern Constraints

This section will focus on estimation techniques for sets of constraints that can include any combination of query pattern constraints.

5.5.1 Sampling

A sample of all graph instance ids can be used to obtain a cardinality estimate for any query pattern by evaluating the query pattern on the sample and scaling up the number of results obtained. However, evaluating a query pattern over a sample can make the estimation time large, especially when the sample size needs to be large in order to obtain accurate estimates (e.g. for queries containing very selective constraints). Therefore, we focus on using sampling to obtain estimates for small patterns together with all data constraints on those patterns.

Each graph pattern in the graph instance that belongs to a specific pattern type is sampled with a specific probability. As pattern type, id, (labeled) vertex pattern, (labeled) edge pattern, etc. The probability defines a trade-off between estimation time and estimation accuracy. A larger probability, is expected to lead to a larger sample size, a larger estimation time and a better estimation accuracy.

Targeted Set of Constraints. All subpatterns of the query pattern that belong to the pattern type pt together with all the data constraints on those subpatterns.

Prerequisites. A random sample of all patterns in the graph instance of the type pt, where each pattern is chosen with a probability of pr.

Estimation Procedure. The selectivity can be estimated by the fraction of the patterns in the sample that satisfy all the data constraints. This approach assumes that the sample is representative for the whole collection of patterns, i.e. the fraction of patterns that satisfy the constraints is the same in the sample and in the whole collection.

Example 5.3. $S(id, 0.001)$ can obtain an estimate for the constraints vertex(id8), hasLabel(id8, person), hasProp(id8, gender, =, m), hasProp(id8, name, CONTAINS, Tim) from Figure 1 because they all refer to the same query id, namely id8.

5.5.2 Wander Join

Wander Join [20] is a sampling technique that obtains samples of the results of a (sub)query by performing random walks over the topology of the (sub)query.

Targeted Set of Constraints. Any set of constraints on the subpatterns of the query pattern, such that there exists an ordering of the query edges $(e_1, e_2, \ldots, e_n)$ where $e_i$ has a common vertex with some $e_j$ where $j < i$ and $e_i$ has an index on that vertex.

Prerequisites. Indexes to efficiently obtain all edges with a specific source or target vertex, in order to perform efficient random walks.

Estimation Procedure. Perform $x$ random walks according to the valid order, where $x$ is a parameter that defines the trade-off between estimation time and estimation accuracy. Each random walk returns a value. For cardinality estimation this would be either 1 or 0 (i.e. the random walk satisfies all constraints or it does not). Divide the value by the probability of occurrence of the walk that was obtained. Finally, take the average over the values obtain by the $x$ random walks to obtain a cardinality estimate.

6 Extend PES Techniques

Techniques that can obtain partial estimates for query patterns that rely on the existence of other partial estimates are covered in this section.

6.1 Implied Constraints

If the constraint src$(i_1, i_2)$ from query pattern $Q$ is satisfied for a mapping $m$ to graph $G$, then it must be the case that the constraints vertex$(i_1)$ and edge$(i_2)$ are also satisfied. This implication holds for any $i_1, i_2 \in Q.I$, due to the definition of the source constraint. Namely, if src$(i_1, i_2)$ is satisfied in $G$, then $G.p(i_2) = (i_1, v')$ for some $v' \in G.V$. Since $p$ is defined as $E \rightarrow (V \times V)$, it must hold that $i_2 \in E$ and $i_1 \in V$, which is specified by the constraints edge$(i_2)$ and vertex$(i_1)$.

If the PES contains an estimate for src$(i_1, i_2)$, for any $i_1, i_2 \in Q.I$, then a new PE will be added to the PES consisting of the set of constraints $\{\text{src}(i_1, i_2), \text{vertex}(i_1), \text{edge}(i_2)\}$ with the selectivity of the PE for src$(i_1, i_2)$. More precisely, $Pr[S_{\text{src}(i_1, i_2), \text{vertex}(i_1), \text{edge}(i_2)}] = Pr[S_{\text{src}(i_1, i_2)}]$, for any $i_1, i_2 \in Q.I$.

Similarly, trg$(i_1, i_2)$ implies vertex$(i_1)$ and edge$(i_2)$ for any $i_1, i_2 \in Q.I$. Also, hasProp$(i, k, \theta, v)$ implies hasPropKey$(i, k, \theta, \phi, v)$, for any $i \in Q.I, k \in Q.Prop, \theta \in Q.\Theta, v \in Q.Val$.

6.2 Implied Constraints Assumptions

Each query id can have many property constraints associated with it. Neumann and Moerkotte [17] noticed that usually one property-value constraint is extremely selective, which (nearly) implies the other property-value constraints. This means that, when the most selective property-value constraint on a query id is satisfied, then all other property-value constraints on that query id are (usually) satisfied too.

Whenever detailed statistics about combinations of those constraints are not available, instead of assuming independence between those constraints, assuming an implication from the most selective one to the others might be more appropriate.

Example 6.1. Consider a graph $G$ to which the query from Figure 1 is executed. If $G$ has less ids that have the property key "name" that contains the word “Tim” ($c_{28}$ from
Example than ids that have the property key “gender” with the value “m” (c_{27}), i.e. \( \Pr[S_{c_{27}}] < \Pr[S_{c_{28}}] \), then constraint \( c_{28} \) is assumed, by this approach, to imply \( c_{27} \). This gives \( \Pr[S_{\{c_{27},c_{28}\}}] = \Pr[S_{c_{28}}] \).

This idea can be extended to a more general form, which requires as input a pattern class and a class of constraints. Above description has ‘Id’ as pattern class and ‘Property-value constraints’ as class of constraints, which will be identified as \( IP(Id, PropValueConstrs) \).

The generalized procedure works as follows. Let the defined pattern class be \( x \) and the class of constraints be \( y \). For every pattern \( x_i \) in the query pattern that belongs to the class \( x \), find \( PE_i(x_i,y) \). The set \( PE(x_i,y) \) consists of all PEs in the PES where all constraints belong to the class \( y \) and where the query ids in the constraints of the PE is a subset of the query ids in the constraints of \( x_i \), i.e. \( (PE.C).I \subseteq C(x_i).I \). An implication assumption is applied from the constraints in the PE in \( PE(x_i,y) \) with the lowest selectivity estimate (PE.s) to the union of all constraints in the PEs in \( PE(x_i,y) \). More precisely, \( PE_{\text{min}}(x_i,y).C \) is assumed to imply \( \bigcup_{PE \in PE(x_i,y)} PE.C \), where \( PE_{\text{min}}(x_i,y) \) is the PE in \( PE(x_i,y) \) that has the lowest selectivity.

7 COMBINE TECHNIQUES

This section will show different techniques to combine the partial estimates of complete PESs into a single selectivity estimate. The different approaches that will be discussed are summarized in Table 6. Notice that these techniques do not depend on the availability of specific statistics or indexes, therefore they can be applied in all cases (on PESs that are complete). Statistical assumptions are used to combine PEs. This part of the estimation procedure can introduce a large estimation error. Therefore, it is important to make the best out of the available partial estimates, since they are obtained using available statistics instead of rough assumptions.

Table 6: General Combine Techniques

| Technique | Assumptions made |
|-----------|------------------|
| CondIndep | Conditional Independence between sets of constraints |
| MaxEnt    | Most uniform probability mass function that satisfies all constraints from the PES |
| Upper Bound | Worst case scenario (from comp. perspective) |
| Lower Bound | Best case scenario (from comp. perspective) |

7.1 Conditional Independence Assumptions

The most simple way of combining all partial estimates for the individual constraints (using techniques from Section 6) is by assuming independence between the different constraints. This makes it possible to obtain an estimate by multiplying the individual selectivity estimates:

\[
\Pr[S_{C(Q)}] = \prod_{c \in C(Q)} \Pr[S_c]
\]  

(1)

This formula implies independence between all constraints and does not make use of the partial estimates obtained for multiple constraint using techniques from Section 5.

If a partial estimate set \( PES_Q \) is complete, w.r.t. query \( Q \), then the intersection of all events \( S_{PE,C} \) for \( PE \in PES_Q \) gives the event \( S_{C(Q)} \). Therefore, the probability of all query constraints can be defined in terms of a complete partial estimate set \( PES_Q \):

\[
\Pr[S_{C(Q)}] = \Pr[\bigcap_{PE \in PES_Q} S_{PE,C}].
\]

Assuming some order of the partial estimates in \( PES_Q \), makes it possible to apply the chain rule of probability:

\[
\Pr[\bigcap_{PE \in PES_Q} S_{PE,C}] = \prod_{i \in [1,\ldots,n]} \Pr[S_{PE_i,C} | \bigcap_{j=1}^{i-1} S_{PE_j,C}].
\]

This formula performs a product over all PEs in the PES. The values of the product consist of the probability that the set of constraints (of the current PE) hold, given that all previously considered constraints hold. If the PES is complete, then all constraints are considered in the end.

We are limited to the PEs that are available in the PES. Therefore, the probability will be approximated using conditional independence where necessary. This gives:

\[
\Pr[S_{C(Q)}] \approx \prod_{i \in [1,\ldots,n]} \Pr[S_{PE_i,C'} | S_{\bigcup_{j=1}^{i-1} PE_j,C'} \cap PE_i,C']
\]

where \( S'' \) represents the set of all constraints in \( C(Q) \) implied by \( S \), which can be obtain from the set \( S \) as follows:

- add all elements from \( S \) to \( S'' \);
- for each \( src(i_1,i_2) \) (or \( trg(i_1,i_2) \)) where \( i_1,i_2 \in Q.I \), add vertex \( (i_1) \) and edge \( (i_2) \) to \( S'' \);
- for each \( G.\theta(i_1,k,\theta)v = v \) where \( i_1 \in Q.I, k \in Q.Prop, \theta \in Q.\Theta, v \in Q.Val \), add \( hasPropKey(i_1,k) \).

The probability of a subset of the constraints in a partial estimate might be needed due to the intersection in the ‘given’ part of the conditional probability. This selectivity might not be available. Therefore, the general selectivity estimation process is called recursively on this subproblem. The lower bound for this selectivity is set to \( \Pr[S_{PE_i,C}] \), because any subset of \( PE_i.C \) imposes no more constraints than \( PE_i.C \) does.

Notice that the resulting formula leads to (1) when the PES includes only PEs for individual constraints and no constraints imply other constraints.

Different orderings of the PES can lead to different selectivity estimates. Some possible options that can be used as sorting strategy are:

1) Primary sort on the number of constraints in the partial estimates (descending). Secondary sort on the selectivity value (ascending).
2) Primary sort on the deviation from the independence assumption (descending). Deviation
from independence for partial estimate \(PE\) is defined as: \(\max(PES/\prod_{c \in PE}Pr[S_c], \prod_{c \in PE}Pr[S_c]/PES)\). This gives the value 1 if the constraints in \(PE\) are mutually independent and a higher value the more dependent they are. Secondary sort on the selectivity value (ascending).

3) Primary sort on the overlap between its constraints and the previously considered constraints (descending). Secondary sort on the deviation from independence (descending).

Algorithm 2 shows the general procedure of combining all partial estimates using the chain rule of probability and conditional independence assumptions. The algorithm uses \(sort\), \(impl\) (short for implied) and \(selEst\) as subroutines. The first two are explained in the text above and the last one is the general selectivity estimation process, which is shown in Algorithm 1. The parameters involving the (sets of) techniques of choice and the available graph instance statistics are shown as dots to keep the description clean.

**Data:** CPES: a complete partial estimate set

**Result:** Selectivity estimate for the conjunction of all constraints in the PEs of CPES

\[
\text{sort}(\text{CPES});
\]

\[
\text{constraintsDone} = \emptyset;
\]

\[
\text{currentEst} = 1.0;
\]

\[
\text{for each partial estimate } PE \text{ in CPES do}
\]

\[
c\text{DoneAndImplied} = \text{impl}(\text{constraintsDone});
\]

\[
c\text{mpl} = \text{impl}(PE.C);
\]

\[
\text{intersection} = c\text{DoneAndImplied} \cap c\text{mpl};
\]

\[
\text{if intersection is empty then}
\]

\[
\text{currentEst} *= PE.s;
\]

\[
\text{else if intersection != cmpl then}
\]

\[
\text{currentEst} *= \max(PE.s, \text{selEst}(\text{intersection}...));
\]

\[
\text{constraintsDone} = \text{constraintsDone} \cup PE.C;
\]

\[
\text{return currentEst};
\]

**Algorithm 2:** combineIndep(CPES, \ldots)

7.2 The Maximum Entropy approach

Each constraint \(c_i \in C(Q)\) can be modeled as a Bernoulli variable, \(X_i\), where \(Pr[X_i] = Pr[S_{c_i}]\). Then, \(Pr[S_{C(Q)}]\) corresponds to the joint probability \(Pr[X_1 = True, X_2 = True, \ldots, X_n = True]\), where \(n = |C(Q)|\).

Instead of estimating only the joint probability \(Pr[X_1 = True, X_2 = True, \ldots, X_n = True]\), Markl et al. [24] approximate the whole joint probability mass function \(Pr[X_1 = x_1, \ldots, X_n = x_n]\) where \(x_i\ is True or False for 1 \leq i \leq n\). The idea is to assign the probabilities such that:

- each probability is \(\geq 0\) and \(\leq 1\);
- the sum of all probabilities adds up to 1;
- for each partial estimate PE it holds that: the sum of all items, where \(X_i = True\ for all \ i\ such that \ c_i \in PE.C\, is\ equal\ to\ PE.s\).

Since many probability mass functions might satisfy above requirements, the one which maximizes the Entropy function will be chosen. This is the most uniform probability mass function (i.e. the one with the largest uncertainty) that is consistent with the information from the complete partial estimate set (CPES).

7.2.1 Constraint Optimization Problem

Let \(S \subseteq C(Q)\), then \(Pr[S \land \neg(C(Q) - S)]\ is the probability that all variables \(X_i\ are True\ for all \ i\ such that \ \ c_i \in S\ and all variables \(X_j\ are\ false\ for all \ j\ such that \ c_j \in (C(Q) - S)\).

The task is now to find the joint probability \(Pr[S \land \neg(C(Q) - S)]\, for\ each \ S\ where \ S \subseteq C(Q),\ such that\)

\[
\sum_{\{S|S \subseteq C(Q)\}} Pr[S \land \neg(C(Q) - S)] = 1
\]

and for each partial estimate \(PE\)

\[
\sum_{\{S|PE.C \subseteq S \subseteq C(Q)\}} Pr[S \land \neg(C(Q) - S)] = PE.s
\]

that maximizes the entropy function:

\[
- \sum_{\{S|S \subseteq C(Q)\}} Pr[S \land \neg(C(Q) - S)] \log Pr[S \land \neg(C(Q) - S)]
\]

which sums all probabilities in the joint probability mass function where all variables \(X_i\ are True\ for all \ i\ such that \ c_i \in V'.\ The remaining variable can be either True or False.

The selectivity of query \(Q\) is the probability in the estimated joint probability mass function where all variable are true.

Markl et al. [24] solve this constraint optimization problem using an iterative scaling algorithm based on Lagrange multipliers.

7.3 Obtaining Cardinality Bounds

The above methods all construct a single value as an estimate for the cardinality of a query. The real cardinality can be very different from this estimate. Therefore, this section will introduce a technique to obtain a lower and an upper bound such that the real cardinality is likely to fall within that range, depending on the error guarantees of the PEs in the PES.

7.3.1 Finding the Upper Bound

The selectivity of all constraints in \(C(Q)\) is less or equal than the selectivity of a subset of \(C(Q)\). The PES of query \(Q\) \((PES_Q)\) stores estimates for subsets of \(C(Q)\). Therefore, an upper bound can be obtained directly from \(PES_Q:\)

\[
Pr[S_{C(Q)}] \leq \min_{PE \in PES_Q} PE.s
\]

Now consider a special case: Let \(PE_k\ and \ PE_l\ be elements of \ PES_Q.\ If \ (PE_k.C).I\ and \ (PE_l.C).I\ are disjoint, then the events \(S_{PE_k,C}\ and \ S_{PE_l,C}\ are independent.\ This makes it possible to obtain the selectivity of the intersection of both events as follows: \(Pr[S_{PE_k,C} \cap S_{PE_l,C}] = Pr[S_{PE,k}]. Pr[S_{PE_l,C}] = PE_k.s \cdot PE_l.s\).
This can be generalized to any subset $S$ of $PES_Q$, such that the query ids in the constraints of all pairs of PEs from $S$ are disjoint. Let $\text{indep}(PES_Q)$ represent all those sets. This gives

$$Pr[S_c(Q)] \leq \min_{I \in \text{indep}(PES_Q)} \prod_{PE \in I} PE.s$$ \hspace{1cm} (3)

Note that (3) is an improvement upon (2)9.

### 7.3.2 Finding the Lower Bound

For most PESs, there is not enough evidence that a query return at least one result. This means that a lower bound of zero for $Pr[S_c(Q)]$ is common. Only when the selectivity values of the PEs are large, it might be possible to obtain a lower bound larger than 0.

**Example 7.1.** For example, $PES_Q = \{PE_3.s = 0.8, PE_2.s = 0.7, PE_3.s = 0.9\}$ gives a lower bound 0.4 for the selectivity $Pr[S_{PK,C\cup PK_2,C\cup PK_3,C}]$.

By making the sets of mappings that satisfy the different PEs are disjoint as possible, it is possible to obtain the smallest possible set of mappings that must satisfy all partial estimates. This can be computed using the formula $lbSel(PES_Q) = 1 - \sum_{PE \in PES_Q} (1 - PE.s)$, where $(1 - PE.s)$ represent the fraction of mappings that do not satisfy the constraints in $PE$. The final lower bound is $\min(0, lbSel(PES_Q))$. A lower bound larger than 0 will only be obtained when $\sum_{PE \in PES_Q} PE.s > |PES_Q| - 1$, where $|PES_Q|$ is the number of PEs in the PES.

Notice that, in order to obtain the best lower bound, all PEs $PE_i$ need to be removed for which there exists another PE $PE_j$ where $PE_i,C \subseteq PE_j,C$.

### 8 Experimental Setup

The Join Order Benchmark (JOB) was introduced by Leis et al. [1], [2]. It consists of a fixed database instance and a fixed query workload. The database instance is a real instance of the Internet Movie Data Base (IMDB), which is full of correlations and non-uniform distributions. The query workload consists of 113 analytical SQL queries that were manually constructed to represent questions that could have been asked by a movie enthusiast. The queries contain 33 structures, each with several variants that differ in selections (i.e. data constraints) only.

We have translated the relational database instance into a property graph database instance and each SQL query in the workload into an openCypher query. See Figure 4 for the graph schema. Table 9 shows the abbreviations that will be used in the experiment results and descriptions.10

9. Each set that consists of a single $PE$ from $PES_Q$ is by definition an element of $\text{indep}(PES_Q)$. Also $PE.s \leq 1$. Therefore, (3) cannot lead to a larger upper bound than (2).

10. A sorting strategy will be abbreviated by a combination like $SaNd$, where $Sa$ defines the primary sorting criteria and $Nd$ the secondary sorting criteria.

### 8.1 Translation from relational to graph instance

For the IMDB schema, we refer to Figure 2 in [2]. We partitioned the relation into three sets $I, E$ and $V$, i.e. Identifying relations, Edge relations and Vertex relations. A relation belongs to $I$ if it is only used to identify a string value, i.e. it only has two attributes, the primary key and the string value. A relation belongs to $E$ if it contains exactly two foreign key attributes to relations not in $I$ and it does not have any references from other relations to its primary key: $movie\_companies$ to title and company, $movie\_link$ twice to title and $movie\_keyword$ to title and keyword.

The remaining relations belong to $V$. $I = \{\text{link\_type}, \text{role\_type}, \text{complete\_cast\_type}, \text{company\_type}, \text{kind\_type}, \text{info\_type}\}$ $E = \{\text{movie\_companies}, \text{movie\_link}, \text{movie\_keyword}\}$ $V = \{\text{title}, \text{complete\_cast}, \text{keyword}, \text{company\_name}, \text{movie\_info}, \text{movie\_info\_idx}, \text{cast\_info}, \text{char\_name}, \text{name, aka\_name, person\_info}\}$

We created a vertex for each tuple in the relations in $V$. We assigned the name of the relation as a label to the vertex. If a relation in $V$ contains a foreign key (e.g. $movie\_id$ in aka_title), then an edge will be created between the vertices associated with the tuple in the relation and the tuple to which it refers.

For each tuple in the relations in $E$, we created an edge between the vertices associated with the tuples to which both foreign keys refer. If the relation contains only one attribute that is not a foreign key or primary key, then the value of that attribute is used as edge label (e.g. $company\_type$ for $movie\_companies$). Otherwise, a label based on the name of the relation is assigned, e.g. $\text{has\_keyword}$ for $movie\_keyword$.

The remaining attributes are added as key-value properties to the vertices and edges, where the key is the attribute name.

### 8.2 Basic Query and Data Statistics

Several statistics from Section 5 are tailored to specific patterns. If those patterns do not occur frequently in the data and the queries, then those statistics lose their benefit. Table 7 shows the number of occurrences of several patterns in the JOB query workload. There are much more source star patterns than chain patterns and much more chain patterns than target star pattern. Therefore, statistics for source star patterns have the potential to improve the estimation accuracy for many queries. Table 8 shows some basic statistics of the graph version of the JOB dataset.
Table 7: Types of patterns in the OpenCypher version of the JOB queries and tics of the graph version of the JOB dataset.

| Pattern | Occ. |
|---------|------|
| Query edges | 464 |
| Chains length 2 | 201 |
| Chains length > 2 | 0 |
| Source stars size 2 | 356 |
| Source stars size 4 | 130 |
| Source stars size > 4 | 20 |
| Target stars size 2 | 15 |
| Target stars size > 2 | 0 |

Table 8: Basic statistics of the JOB dataset.

| JOB data stats |
|----------------|
| (E,V)  | 119343754 |
| (E,V)  | 52639796  |
| (E,V)  | 171983590 |
| (E,V)  | 12 |
| (E,v)  | 1086 |

8.3 Notes on Disjunctions in JOB Queries

Some queries in the JOB contain disjunctions, e.g. [hasLabel(e1,'production companies') OR hasLabel(e1,'distributors')]. The way we handled them is by generating a set of queries without disjunctions and finally unifying their results, by assuming that the sets of results are disjoint.

9 Example of the Estimation Process

Figure [5] shows an example of the estimation process for the query from Figure [11] on the property graph version of the JOB dataset.

The PET for the individual constraints that is used is synopsis lookup, where the synopsis contains the precomputed selectivities of the required individual constraints. The PETs for multiple constraints that are used are: SysR and EF. The EPEST IP[p,c] is used, which assumes implied constraints between property constraint referring to the same query id.

Using those PETs, EPEST and the combination technique condIndep(NdSa), the cardinality of the query from Figure [11] is estimated to be almost 4 in the JOB dataset. The real cardinality of this query in the JOB dataset is 410.

Each PE contains a selectivity estimate ̂. For analysis, we also added the exact selectivity values s for each pattern for which a PE is available. All estimates in green do not introduce any error. Estimates in yellow introduce a very small errors (q-errors less than 2). Estimates in orange introduce some errors (q-errors between 2 and 10). Finally, estimates in red introduce large errors (q-errors larger than 10).

The errors introduced during the estimation process will be investigated in the following subsections.

System R’s assumptions. The inclusion assumption and uniform distribution assumption caused a very small overestimation error in the PE sysR1. The PE sysR2 does not introduce any error, since each cast_info vertex (i.e. vertex with the label cast_info) has exactly one outgoing edge with label cast_info_movie to a title vertex and exactly one outgoing edge with label cast_info_person to a person vertex. Therefore, inclusion and uniform distribution assumptions hold.

Implied constraints assumptions. The implied constraints assumption (an element whose name contains ‘Tim’ determines that the element has gender ‘m’) in ip1 caused a small overestimation error.

Independence between labeled topological patterns. Pr[SC(sysR2) | SC(sysR1)] is approximated by Pr[SC(sysR2) | SC(EPEST)]. This assumes conditional independence between the pattern Pbv and the combination technique sysR1. This assumes Pr[SC(sysR1)] is approximated by Pr[SC(sysR2)] given Pm. This did not introduce an estimation error. The reason for this is that every cast_info vertex in the dataset has exactly one outgoing edge with label cast_info_person to a person vertex and exactly one outgoing edge with label cast_info_movie to a title vertex. Therefore Pbv only depends on Pm and not on Pbv.

Independence between property constraints and labeled topological pattern. Pr[SC(ip1) | SC(sysR1):C(sysR2)] (which has real s = 3.55 · 10⁻³) is approximated by Pr[SC(ip1)] (which has real s = 5.27 · 10⁻³). The leads to a large q-error of 67.4. Notice that Pr[SC(ip1)] itself is approximated using implied constraints assumptions. This approximation causes a small overestimation, which cancels out a part of the underestimation for using Pr[SC(ip1)] as an approximation for Pr[SC(ip1) | SC(sysR1):C(sysR2)]. This eventually leads to a q-error of 23.7.

Approximating Pr[SC(ip1) | SC(sysR1):C(sysR2)] by Pr[SC(ip1) | SC(sysR1):C(sysR2)] (which has real s = 1.72 · 10⁻⁴) would improve estimation accuracy (q-error of 20.6). However, approximating it by Pr[SC(ip1) | SC(sysR1):C(sysR2)] (which has real s = 2.17 · 10⁻³) would lead to a very accurate estimate (q-error of 1.6).

This shows a strong positive correlation between property and label constraints.
beled topological pattern and other property constraints. \( Pr[S_{C_{(yp2)}} | S_{C_{(sysR2)}}[C_{(sysR2)}],C_{(yp1)}] \) (which has real \( s = 6.30 \cdot 10^{-2} \)) is approximated by \( Pr[S_{C_{(yp2)}}] \) (which has real \( s = 1.38 \cdot 10^{-2} \)). This leads to a q-error of 4.6.

Approximating it using \( Pr[S_{C_{(yp2)}} | S_{\{\text{vertex(id6)}\}]} \) (which has real \( s = 4.52 \cdot 10^{-2} \)) improves estimation accuracy (q-error of 1.4). Approximating it using \( Pr[S_{C_{(yp2)}} | S_{\{\text{vertex(id6),hasLabel(id6,cast}_\text{info)}\}]} \) (which has real \( s = 6.56 \cdot 10^{-2} \)) would lead to an almost perfect estimate (q-error of 1.0). This, again, shows a strong positive correlation between property and label constraints.

**Summary.** The illustrated estimation process in Figure 5 introduces the largest errors by assuming independence between each property constraint and the labeled topological constraints. Therefore, the selectivity estimate can be improved by adding partial estimates that contain combinations of property constraints, edge or vertex constraints and label constraints.

## 10 Experiments

Section 9 identified different source of errors that can be introduced within the estimation process. In the following sections, these error sources are investigated further. All experiments are performed on the graph version of the JOB dataset and using the OpenCypher version of the JOB query workload.

For all experiments, a synopsis lookup for individual constraints is used. This allows us to focus on the estimation errors introduced by PETs for multiple constraints, EPESTs and CTs. In a real world scenario, individual constraints might be estimated using histograms or sampling.

Several experimental results are visualized using boxplots. The boxplot is drawn, such that 50% of the values fall within the box, 90% within the whiskers. The remaining 10% is shown as dots (top 5% and bottom 5%). The median is drawn as an orange line within the box.

Section 10.1 shows experiments regarding the errors introduced by some (extend) partial estimation techniques. Experiments related to the errors that are introduced by the combination techniques are shown in Section 10.2.

**Key observations from the experiments:**

- for labeled star patterns, a combination of a labeled edge pattern synopsis (PET EP) and conditional independence assumptions (CT condIndep) requires less statistics than System R’s join size estimation (PET SysR) and its largest Q-error is \( 7 \times \) lower. The median Q-errors are similar. (Section 10.1.1);
- for labeled source star patterns with property-key constraints, the PET CS has better estimation accuracy than structural synopses combined with condIndep. Its median Q-error is \( 60 \times \) lower. (Section 10.1.2);
- using implication assumptions instead of independence assumptions leads to a median Q-error that is \( 20 \times \) lower for estimating multiple property-value constraints referring to the same query id. (Section 10.1.3);
- Section 10.2.2 also supports this claim for general query patterns, independence assumptions can lead to Q-errors that are \( 10^{24} \times \) larger than using implication assumptions on the JOB subqueries with less than 7 edges;
- sorting strategies (for CT condIndep) with primary sort SD or NA should be avoided (Section 10.2.1);
- condIndep outperforms maxEnt and bounds (Section 10.2.1);
- in the absence of property constraints, labeled structural synopses with CT condIndep(Mobli) can obtain accurate estimates efficiently (Section 10.2.1);
- sampling improves estimation accuracy (median Q-error is \( 16 \times \) lower), but increases estimation time (median estimation time is \( 135 \times \) higher), compared to the best alternative using implication assumptions, on the JOB subqueries with less than 7 edges (Section 10.2.2).

### 10.1 Estimation Error Experiments for PETs and EPESTs

Partial estimation techniques like c2 can obtain partial estimates that are exact. However, PETs like CS or SysR can obtain partial estimates that are not exact. Those techniques rely on a combination of specialized statistics and assumptions. The assumptions can introduce errors in the partial estimates. This section investigates on the estimation errors introduced by PETs and EPESTs.

#### 10.1.1 System R Join Size Estimation

The PET SysR (Section 5.2.2) can obtain estimates for labeled star patterns, using labeled edge pattern statistics, inclusion assumptions and uniform distribution assumptions. This section focuses on the accuracy of estimating labeled star patterns, using SysR and alternative approaches.

**Theoretical Analysis.** The full estimation error introduced by SysR on star patterns is due to the inclusion assumptions and uniform distribution assumptions. The inclusion assumption can only lead to an overestimate. The uniform distribution assumption can lead to both over- and under-estimations.

The PET SysR underestimates in the case that 1) the degree distributions are skewed, 2) vertices with a relative high number of edges in one edge pattern also have a relatively high number of edges in other edge patterns and 3) the inclusion assumption is approximately satisfied.

The PET SysR overestimates in the case that in inclusion assumption does not hold (for example when \( |T_{G}(ep1) \cap S_{G}(ep2)| \) is much smaller than \( \min(|T_{G}(ep1), S_{G}(ep2)|) \) in a pattern \( ep1,ep2) \), or when 1) the degree distributions are skewed and 2) vertices with a relative high number of edges in one edge pattern have a relatively low number of edges in other edge patterns.

**Analysis on Star Patterns in JOB Queries.** For each labeled star pattern in the JOB queries with size \( \geq 2 \), the exact selectivity and estimates using SysR and alternative approaches are obtained. Figure 6 shows a summary of these results.
by plotting the values estimate/real in boxplots partitioned by the size of the star patterns. In order to compare the errors obtained using the PET $\text{SysR}$, alternative approaches (labeled topological synopses) are also illustrated in Figure 6. The PES will be combined using condIndep with sort strategy $\text{ModI}$. For the alternative approaches, all PEs are exact. Therefore, the full estimation error is due to (conditional) independence assumptions that are made during combining the PES using CT condIndep. For the PET $\text{SysR}$, combing the PES is not necessary, since $\text{SysR}$ can obtain an estimate for every star pattern. Therefore, the full estimation error is due to the assumptions made by $\text{SysR}$.

From Figure 6, it can be seen that

- PEs obtained using $\text{SysR}$ will introduce estimation errors up to 3 orders of magnitude for the JOB queries on the JOB dataset;
- larger star patterns lead to larger underestimations of the selectivity;
- when statistics about labeled topological pattern (larger than edge patterns) are available, then it is better to use PEs that use those statistics (e.g. $c_2, s_2, s_3$) and combine using conditional independence assumptions, instead of using $\text{SysR}$;
- the approach EP is performing comparable to $\text{SysR}$.

Figure 5: Example of an estimation process of the query from Figure 1.
Figure 6: Estimation error for all star patterns in the JOB queries, partitioned on the size of the star patterns, using different PETs. The CT \text{condIndep} \text{(MoDi)} is used when the PETs are not able to produce an estimate for the whole star pattern.

Figure 7: Estimation error for all csPatterns in the JOB queries, with at least two edge pattern, partitioned on the size of the csPatterns, using different PETs. The CT \text{condIndep} \text{(MoDi)} is used when the PETs are not able to produce an estimate for the whole csPattern.

(and in many cases, slightly better), while it requires less statistics than \text{SysR}.

The PET \text{EP} with CT \text{condIndep} \text{(MoDi)} gives a median and max Q-error of 2.0 and 205, where \text{SysR} gives 2.1 and 1515, on the star patterns in the JOB query workload. The max Q-error is 7× larger for \text{SysR}, while it requires more statistics than \text{EP} with \text{condIndep}. As a result, PET \text{SysR} is not recommended.

10.1.2 Characteristic Sets

The PET \text{CS} (Section 5.3.1) can obtain estimates for labeled source star patterns with 'has property key' constraints on their center vertices. A source star pattern with exactly one label on each edge together with the 'has property key' constraints on its center vertex will be called a \text{csPattern}.

For each csPattern in the JOB queries, which contains at least two edge patterns, the exact selectivity and an estimate using \text{CS} are obtained. Figure 7 shows a summary of these results together with alternative techniques, similar as was done in Section 10.1.1.

Figure 7 shows that \text{CS} performs very well compared to \{\text{EP}, s2\} and \{\text{EP}, s3\}. The approaches \{\text{EP}, s2\} and \{\text{EP}, s3\} have to assume independence between the labeled topological pattern and the 'has property key' constraints, also between all of the 'has property key' constraints. Those independence assumptions lead to many large underestimations. \text{CS} gives median and max Q-error of 1.1 and 184, where \{\text{EP}, s3\} gives 68.0 and 4762. Therefore, the median Q-error of \text{CS} is 60× lower than for \{\text{EP}, s3\} on all csPatterns in the JOB query workload.

Figure 8: Estimation error for the set of all property-value constraints that refer to the same query id, for each query id in the JOB queries.

Instead of assuming independence between all 'has property key' constraints, it is possible to assume implications using IP(id,p). Figure 8 shows that the estimates using implied 'has property key' constraints perform better than assuming independence between the 'has property key' constraints. However, the selectivity of most csPatterns is still underestimated by more than an order of magnitude. Another source of underestimation is the independence assumption between the 'has property key' constraints and the labeled topological constraints. This source of underestimation can be removed by assuming implications using IP(id,a). Figure 8 shows that most csPatterns are estimated very accurately with techniques \text{EP} and IP(id,a). The addition of s2 and s3 helps to further improve the selectivity estimation accuracy.

The advantage for the \text{CS} approach compared to the approaches based on implication assumptions is that \text{CS} is based on real statistics. Therefore, it is expected to perform well even in cases where implication and independence assumptions do not hold. However, the price that \text{CS} pays is an increase in memory cost, preparation time and estimation time.

10.1.3 Implied Constraints Assumptions

The EPEST IP(id,pv) can obtain an estimate for all property-value constraints that refer to the same query id. We obtained for each query id in each JOB query all property-value constraints. With only the exact selectivities of the individual constraints (no additional PETs and EPESTs), the full estimation error is caused by the CT. Figure 8 shows this using CT \text{condIndep} \text{(MoDi)}. From the boxplot above (_ 0, 4), it can be seen that assuming independence mostly leads to underestimations. However, it can also lead to a large overestimation.

With the EPEST IP(id,pv), an estimate for all constraints can be obtained. Therefore, the full estimation error is caused by the implication assumptions within IP(id,pv). Clearly, IP(id,pv) cannot lead to an underestimation of the selectivity (when the PES contains only exact selectivity values, which is the case for these experiments). However, it can lead to a large overestimation of the selectivity. Here, IP(id,pv) gives a median and max Q-error of 1.6 and 1.1·10^5, where independence gives 32.7 and 8142. Therefore, the median Q-error of implication assumptions is 20× lower than for independence assumptions, but the max Q-error is 13× larger.
For both approaches, the set \( s_1 = \{ \text{hasProp(id0, note, CONTAINS, \text{('producer')})}, \text{hasProp(id0, role, =,'actor')} \} \) caused the largest overestimation. A total of 1436536 graph ids satisfy the first constraint, 12670688 graph ids satisfy the second constraint, but only 13 graph ids satisfy both constraints.

### 10.2 Estimation Error Experiments for CTs

PETs can obtain selectivity estimates for a specific subclass of query patterns (e.g. only for star patterns, or only for all property constraints referring to the same query id). In order to obtain a selectivity estimate for every possible query pattern, a CT will combine all PEs into a single selectivity estimate for the query pattern.

Section 10.2.1 focuses first on the estimation error introduced by combining PEs consisting of only labeled topological constraints. After that, Section 10.2.2 focuses on the estimation error introduced by combing all PEs.

#### 10.2.1 Combining Labeled Topological Constraints

For this section, we use all the subqueries, that have at most six edge patterns, from all JOB queries. For each such subquery, the property constraints have been removed. This allows us to focus on the error that is introduced during combination of PEs consisting of labeled topological constraints.

In order to focus on the estimation error caused by the combination techniques, this section uses only PETs that add PEs that are exact (e.g. labeled topological synopses).

**Conditional Independence Assumptions.** The CT CondIndep requires a specific sorting strategy. Table 10 shows the median (and max) q-error of the selectivity estimates for different combinations of sorting strategies and PETs. Those results clearly show that a primary sort on the selectivity value of the PEs in descending order or a primary sort on the number of conjunct of the PEs in ascending order should be avoided. Those sorting strategies have the effect that essentially all partial estimates involving more than one constraint are ignored. All other sort strategies perform much better and show that adding more PEs, that use more advanced statistics, lead to better estimation accuracy.

The median estimation time is between 1 and 15 milliseconds for the different sort strategies and different PETs, and the maximum estimation time lies between 10 and 100 milliseconds (due to page limit restrictions, we omit the full table). The estimation time increases slightly when more PETs are used. The estimation times for different sort strategies are comparable. Only the strategies based on maximum overlap take a bit more time (compared to other sorting strategies), which is required to find the overlap between the different PEs.

Figure 9 shows the boxplots for the sorting strategy Modi, where the subqueries are partitioned on the number of edge patterns. This shows very accurate estimates for \( \{ \text{EP}, c_2, s_2, t_2 \} \), which have mean and max Q-error of 1.0 and 54.7 and becomes even better for labeled structural synopses for larger patterns.

**Maximum Entropy.** The combination technique based on the maximum entropy principle is computational expensive (exponential in the number of constraints). Therefore, a forced partitioning of the constraints is used, as described by Markl et al. [24]. The number for the maximum partition size (mps) is given as a parameter.

The MaxEnt technique is performed for every partition and assumes independence between the different partitions to obtain the final selectivity estimate.

Notice that the effect of partitioning the constraints is that all PEs are lost that have constraints that belong to different partitions. Therefore, it is expected that larger mps values lead to selectivity estimates with higher accuracy, but require larger estimation times.

MaxEnt using mps values larger than 8 made the estimation time of some subqueries larger than a second. Experiments (omitted due to page limit) show indeed that larger mps values improve estimation accuracy and increase estimation times. More PETs, that use more advanced statistics, lead to better estimation accuracy on average. However, it did not show a big improvement on the worst case errors. Figure 10 also shows this phenomenon, where for large subqueries, adding more advanced PETs does not help estimation accuracy. The PEs obtain by those PETs typically include many constraints, which could not all fit into a single partition. Therefore, due to forced partitioning, those PEs are lost.

Comparing the results of the CTs CondIndep and

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Table 10: The median (and max) q-error of the selectivity estimates using CT CondIndep with different sort strategies and different PETs.

| PETs | \{ EP \} | \{ EP, c_2, s_2, t_2 \} | \{ EP, c_2, s_4, t_2 \} |
|------|----------|----------------|----------------|
| SnDi | 4.54e+09 (4.29e+24) | 1.96 (204.48) | 1.0 (67.64) |
| Sd   | 4.54e+09 (4.29e+24) | 1.96 (204.48) | 1.0 (624.75) |
| NaSa | 4.54e+09 (4.29e+24) | 4.54e+09 | 4.54e+09 |
| NaNd | 4.54e+09 (4.29e+24) | 4.54e+09 | 4.54e+09 |
| NsNd | 4.54e+09 (4.29e+24) | 4.54e+09 | 4.54e+09 |
| D1   | 4.54e+09 (4.29e+24) | 1.96 (204.48) | 1.0 (15.59) |
| Modi | 4.54e+09 (4.29e+24) | 4.54e+09 | 1.0 (108.01) |
| MoNd | 4.54e+09 (4.29e+24) | 4.54e+09 | 1.0 (64.4) |
| MoSa | 4.54e+09 (4.29e+24) | 4.54e+09 | 1.0 (54.72) |
| SaNd | 4.54e+09 (4.29e+24) | 4.54e+09 | 1.0 (28.23) |
maxEnt, shows that condIndep leads to more accurate estimates and requires lower estimation times, which makes condIndep preferable over maxEnt.

**Upper Bound.** The combination approach based on lower and upper bounds gives the guarantee that the real cardinality is in between the lower and upper bound (when the PEs did not introduce any error). Whenever the PES does not contain a PE that includes all constraints for the query, then the lower bound was 0. Therefore, our focus is on the upper bound. Figure 11 shows how the upper bounds improve when more advanced statistics become available.

Table 11 shows the median (and max) q-errors and estimation times. The q-error clearly decrease with the amount of detailed statistics that become available. However, the actual errors are very large. Obtaining all independent set of PEs by considering all subsets required time that is exponential in the number of PEs in the PES. This caused the estimation time of some subqueries to take more than 10 seconds to obtain its upper bound, which makes it impractical for applications of cardinality estimation like query planning.

10.2.2 **Combining Query Pattern Constraints**

For this section we use all the subqueries, that have at most six edge pattern, from all JOB queries (same as last section). For each such subquery, all constraints are considered (different from last section, where the property constraints were removed).

From last section, we observe that the CT CondIndep outperforms the CTs MaxEnt and upper bounds on both estimation time and estimation accuracy. Therefore, this section focuses on the CT CondIndep.

**Labeled Topological Synopses** First we repeat the same experiment as in Figure 9 but now all subqueries keep their property constraints. The results are shown in Figure 12. The labeled topological synopsis for edge patterns improves the estimation accuracy. However, adding labeled topological synopses for chain, source star and target star patterns do not further improve the accuracy.

Notice that none of the PETs include PEs for combination of property constraints or combination of property and labeled topological constraints. This means that the error is dominated by the correlation between those combinations of constraints.

**Implied Constraints Assumptions.** Using IP EPESTs can add PEs that contain combinations of property and labeled topological constraints. Therefore, the CT condIndep will not perform independence assumptions between those constraints as was done in the previous section.

Figure 13 shows the accuracy results of experiments with different IP EPESTs when only the PET EP is used. For this scenario, IP \( \{id,a\} \) with IP \( \{ep,p\} \) performs the best. Here, \( \{EP, IP \{id,a\} \, IP \{id,p\}\} \) has a median and max Q-error of 101 and 3.5 \( \times 10^{13} \), where \{EP\} has 1.1 \( \cdot 10^{11} \) and 3.0 \( \cdot 10^{15} \). The maximum Q-error of independence assumptions is \( 10^{24} \times \) larger than for implication assumptions.

The IP type IP \( \{ep, a\} \) is too strong. It essentially ignores the effect of many constraints that are actually restricting the cardinality of the subqueries. The other IP types are more conservative, i.e. they make less implication assumptions (as a result, the CT might have to make more independence assumptions).
Table 12: The median (and max) estimation time (in ms) for the experiments from Figure 14.

| PETs and EPEs | #EdgeIds: 1 | #EdgeIds: 2 | #EdgeIds: 3 | #EdgeIds: 4 | #EdgeIds: 5 | #EdgeIds: 6 |
|---------------|-------------|-------------|-------------|-------------|-------------|-------------|
| {}            | 2.20 (36.4) | 6.14 (91.3) | 12.4 (126)  | 25.4 (266)  | 38.3 (299)  | 50.3 (221)  |
| {EP}          | 2.86 (44.5) | 8.83 (93)   | 19.1 (185)  | 36.4 (314)  | 62.8 (463)  | 91.1 (468)  |
| {EP,S(id,0.001)} | 711 (8.61e+3) | 1.26e+3 (1.77e+4) | 1.86e+3 (2.08e+4) | 2.77e+3 (2.43e+4) | 3.66e+3 (2.56e+4) | 4.37e+3 (2.72e+4) |
| {EP,S(id,0.001),S(ep,0.001)} | 132e+3 (1.16e+04) | 2.86e+3 (3.63e+4) | 4.68e+3 (6.28e+4) | 6.75e+3 (8.25e+4) | 8.12e+3 (9.61e+4) | 9.49e+3 (5.42e+4) |
| {EP,IP(id,a),IP(ep,p)} | 5.45 (108) | 17.7 (226) | 38.0 (372) | 73.8 (613) | 123 (939) | 185 (901) |

Figure 14: Estimation accuracy experiment for different sampling techniques.

Sampling techniques: Sampling PETs are able to obtain PEs about combinations of property and topological constraints. Figure 14 shows the results of accuracy experiments for different sampling techniques. For the pattern parameter of sampling, we experimented with id and ep and for the probability we used 10^{-3}. For comparison reasons, we also added the best technique using IP EPESTs from the previous section. Here, {EP,S(id,0.001),S(ep,0.001)} gives as median and max Q-error 6 and 4.4 \cdot 10^{12}, where {EP,IP(id,a),IP(ep,p)} gives 101 and 3.5 \cdot 10^{13}. Therefore, the median Q-error of sampling is 16x better and max Q-error is 8x better than the best approach based on implication assumptions.

Clearly, sampling techniques improve the estimation accuracy, which highlights the importance of capturing correlations between constraints referring to the same id or ep. Assuming independence can lead to large underestimations. Assuming implications can lead to large overestimations.

The median and max estimation time for sampling is 135x higher and the max estimation time is 102x higher. This raises the question whether or not the large estimation time of sampling techniques is worth the improvement in estimation accuracy. Table 12 shows the estimation times of the experiments in Figure 14.

11 Conclusions

Many cardinality estimation techniques have been proposed in the literature which require specific statistics or indexes to be available and use a certain set of simplifying assumptions. The framework introduced in this document makes it possible to compare, and even combine, different techniques with the goal of producing superior solutions.

The framework consists of obtaining estimates for subqueries (partial estimates) using techniques (partial estimate techniques) that have specific prerequisites, e.g.: specialized statistics or indexes. All partial estimates are stored in a partial estimate set (PES). This set can be extended and made complete, such that every constraint from the query pattern is available in at least one partial estimate in the PES. Finally, all partial estimates in the PES are combined using a general combine technique (CT). A CT is a general technique that uses simplifying assumption to combine partial estimates and therefore can be applied for any complete PES, i.e. it does not require any form of statistics.

Extensive experiments show that synopsis consisting of the cardinality of small labeled topological patterns makes it possible to obtain accurate estimates for query patterns that do not include property constraints. For query patterns that include property constraints, capturing correlations between property constraints and labeled topological constraints is essential. When this information is insufficiently available, then, making implication assumptions typically performs better than making independence assumptions.

12 Future Work

12.1 Research Perspective

The framework proposed in this document allows new PETs, EPESTs and CTs to be developed in isolation and finally used in combination with other existing techniques. Future work consists of proposing new PETs that are able to capture most important correlations between property and labeled topological constraints, while keeping the estimation time low. For example, is it possible to find subclasses of query patterns where sampling can be efficient and accurate?

Another type of future work is to improve upon the combination techniques. For example, defining metrics to obtain an optimal sorting strategy for CT condIndep. The CT MaxEnt might be more valuable as an EPEST.

Instead of only obtaining a cardinality estimate, it would also be very useful to obtain some sort of error guarantee, e.g. there is a 95% chance that the actual cardinality is between [100, 250].

Extensions w.r.t. the query class needs to be considered in the future, e.g. estimate the cardinality for UCRPQs [4]. UCRPQs is an extension of query patterns that allows path navigation of arbitrary length, which are specified using regular expressions.

12.2 Practitioners Perspective

In order to benefit from recently developed cardinality estimation techniques, it is possible to implement our general cardinality estimation framework, i.e. Algorithm 1. This makes it possible to implement current cardinality estimation techniques as PETs. Our framework then allows to combine different techniques by selecting the PETs of interest (depending on trade-offs, e.g. estimation time and estimation accuracy). For each query, a decision about what
combination of techniques to be used can be made. For example, complex analytical queries that are expected to run for a long time might afford to spend more time in the query optimizer, therefore using PETs with higher estimation times might be allowed.

For the CT, \texttt{condIndep} is recommended since it outperformed the other CIs in both estimation accuracy and estimation time.

Considering a system with a classical setup for cardinality estimation, e.g. some (multidimensional) histograms and some form of sampling. Then those implementations can be transformed into PETs, which allows them to be used in combination with each other.

Experimental evaluation over the JOB dataset showed that instead of using the PET \texttt{Sysr} (which uses the uniform distribution and inclusion assumptions), it is recommended to use \texttt{labTopSyn(EP)} in combination with \texttt{condIndep} (which uses conditional independence assumptions).

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