Estimate of CP Violation for the LBNE Project and $\delta_{CP}$

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Abstract
Measurements of CP violation (CPV) and the basic $\delta_{CP}$ parameter are the goals of the LBNE Project, which is being planned. Using the expected energy and baseline parameters for the LBNE Project, CPV and the dependence of CPV on $\delta_{CP}$ are estimated, to help in the planning of this project.

1 Introduction
For several decades there have been many experimental and theoretical studies of CP violation (CPV). Recently we have studied time reversal violation (TRV) [1] and CPV [2] for neutrino oscillations in matter, using parameters of current neutrino oscillation experiments, MiniBooNE [3], JHF-Kamioka [4], MINOS [5], and CHOOZ [6]. See Refs. [1, 2] for details and references to earlier work on T and CP. When neutrino beams move through matter the CPT theorem is not valid (see, e.g., Ref [7]); therefore TRV and CPV are not directly related. There have been many other studies of CP asymmetries in weak decays: see Ref [8] for a recent study of $\bar{B}$ radiative decay with references to earlier work on CP violation in various weak decays. Also, in addition to the baseline, energy, and the matter density, there are a number of parameters associated with neutrino oscillations, as will be seen in our discussion of CP violation below. There have been a number of studies of
these parameters; see, e.g., Refs\[9, 10\], which contain references to earlier related publications.

In our present work we study CPV using the baseline and energies expected for the future LBNE Project\[11\]. See Ref\[12\] for a detailed report on the project, and the recent discussion of the LBNE parameters\[13\], with a baseline L=1300 km and energies in the range E=0.5-12 GeV. We use the expected baseline of 1300 km and five possible energies. There have also been a number of studies of matter effects for the LBNE Project\[12\]. See Davoudiasl et al\[14\] and Gonzalez-Garcia et al\[15\] for recent studies of matter effects and various parameters important for the baseline L expected in the LBNE Project, and references to earlier studies for the LBNE Project.

One of the main objectives of this future project is to measure the $\delta_{CP}$ parameter. In the present work we calculate CPV as a function of $\delta_{CP}$ for parameters being studied for the LBNE Project, to help in the design of this future experiment. In order to determine $\delta_{CP}$ via neutrino oscillations one also needs the angles $\theta_{12}, \theta_{23}$, which are well-known, and the angle $\theta_{13}$, which is being studied by a number of experiments: T2K\[16\], Daya Bay\[17, 18\], Double Chooz\[19, 20\], and RENO\[21\]. A very recent result from the Daya Bay project\[22\], which is consistent within errors of the recent RENO\[21\] result, finds that $s_{13} \simeq 0.15$, which we use in the present study.

As pointed out above, an essential aspect of the determination of CPV, as well as TRV and CPTV is the interaction of neutrinos with matter as they travel along the baseline. See, e.g., Refs\[23, 24, 25, 26\]. Since this was discussed in detail in Ref\[2\], some details are not given in the present work. In the present work we use the notation and formalism of Jacobson and Ohlsson\[7\], who studied possible matter effects for CPT violation.

CP violation in the $a \rightarrow b$ sector is given by the transition probability, denoted by $\mathcal{P}(\nu_a \rightarrow \nu_b)$, for a neutrino of flavor $a$ to convert to a neutrino of flavor $b$; and similarly for antineutrinos $\nu_a, \bar{\nu}_b$. The CPV (note that the C operator changes a particle to its antiparticle) is defined as

$$\Delta \mathcal{P}_{ab}^{CP} = \mathcal{P}(\nu_a \rightarrow \nu_b) - \mathcal{P}(\bar{\nu}_a \rightarrow \bar{\nu}_b). \quad (1)$$

In our present work we study $\mathcal{P}(\nu_\mu \rightarrow \nu_e)$ and $\mathcal{P}(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$, since the neutrino beams at LBNE are muon or anti-muon neutrinos.

As discussed above, there are four angles in the matrix relating a neutrino with flavor to neutrinos with mass, the basis for neutrino oscillations. The two angles under current study are $\delta_{CP}$ and $\theta_{13}$. In order for the LBNE proposed project to be successful in determining $\delta_{CP}$, $\theta_{13}$ must be known. One might expect that the experiments Daya Bay, RENO, and Double Chooz could not achieve their goal of determining $\theta_{13}$ to an accuracy of 1% without knowing $\delta_{CP}$. As is discussed below, fortunately, this is not the case.
2 CP Violation $\Delta \mathcal{P}^\text{CP}_{\mu e}$

We use the time evolution matrix in flavor space to derive CPV. The neutrino state at time $= t$ is obtained from the state at time $= t_0$ from the matrix, $S_f(t, t_0)$, for neutrino flavor $f$. See Ref[7] for a detailed derivation of $S_f(t, t_0)$.

Using the notation $S_{ab}$ and $\bar{S}_{ab}$ for the flavor a,b matrix element for neutrinos and antineutrinos, the CPV probability is given by

$$\Delta \mathcal{P}^\text{CP}_{\mu e} = \mathcal{P}(\nu_\mu \to \nu_e) - \mathcal{P}(\bar{\nu}_\mu \to \bar{\nu}_e)$$

$$= |S_{12}|^2 - |\bar{S}_{12}|^2$$

$$S_{12} = c_{23}\beta - is_{23}ae^{-i\delta_{CP}}A$$

$$\bar{S}_{12} = c_{23}\bar{\beta} - is_{23}ae^{i\delta_{CP}}\bar{A}.$$  

The parameters in Eq(2) are

$$a = s_{13}(\Delta - s^2_{12}\delta)$$

$$\delta = \delta m^2_{12}/(2E)$$

$$\Delta = \delta m^2_{13}/(2E)$$

$$A \simeq f(t)I_\alpha^*$$

$$I_\alpha^* = \int_0^t dt'\alpha^*(t')f(t')$$

$$\alpha(t) = \cos\omega t - isin2\theta sin\omega t$$

$$f(t) = e^{-i\Delta t}$$

$$2\omega = \sqrt{\delta^2 + V^2} - 2\delta V cos(2\theta_{12})$$

$$\beta = -isin2\theta sin\omega L$$

$$\bar{\Delta} = \Delta - (V + \delta)/2$$

$$sin2\theta = s_{12}s_{13}$$

where the neutrino mass differences are $\delta m^2_{12} = 7.6 \times 10^{-5}(eV)^2$ and $\delta m^2_{13} = 2.4 \times 10^{-3}(eV)^2$. The neutrino-matter potential $V = \sqrt{2}G_F n_e$, where $G_F$ is the universal weak interaction Fermi constant, and $n_e$ is the density of electrons in matter. Using the matter density $\rho=3$ gm/cc, which is the expected average density for LBNE experiment, $V = 1.13 \times 10^{-13}$ eV. We use $s_{12} = 0.56$ and $s_{23} = 0.707$; and $s_{13} = 0.15$, as recently found in the antineutrino disappearance Daya Bay[22] and RENO[21] experiments. Note that for antineutrinos $\delta_{CP} \to -\delta_{CP}$. $\bar{\beta} = \beta(V \to -V)$ and $\bar{A} = A(V \to -V)$. 

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For example, 

\[ 2 \tilde{\omega} = \sqrt{\delta^2 + V^2 + 2\delta V \cos(2\theta_{12})} \text{ and } \tilde{\Delta} = \Delta + (V - \delta)/2. \]

Using conservation of probabiltiy\[7\], \(|\tilde{A}|^2 = |\tilde{A}|^2\), from Eq\[2\]

\[ \Delta \mathcal{P}_{\mu e}^{CP} = c_{23}(|\beta|^2 - |\bar{\beta}|^2) - 2c_{23}s_{23}aIm[\beta e^{-i\delta_{CP}} A^* - e^{i\delta_{CP}} \bar{\beta} \bar{A}^*] . \quad (14) \]

From Eqs\[2\]-\[14\] it follows that \(\Delta \mathcal{P}_{\mu e}^{CP}\) as a function of energy \(E\) and \(\delta_{CP}\) is

\[ \Delta \mathcal{P}_{\mu e}^{CP}(E, \delta_{CP}) = c_{23}^2 s_{12}^2 s_{13}^2 (s_{23}^2 - \tilde{s}_{23}^2) + 2c_{23} s_{23} c_{12} s_{13} \delta (\Delta - \delta s_{12}^2) \]

\[ \times (\sin\delta_{CP} (s_{23}^2 - s_{23}^2) (\tilde{\Delta} - \omega \cos 2\theta) + \tilde{s}_{23}^2 (\tilde{e} - \cos \tilde{\Delta} L) \tilde{\Delta} - \tilde{\omega} \cos 2\tilde{\theta} ) \]

\[ + \cos \delta_{CP} (s_{23}^2 - s_{23}^2) (\sin \tilde{\Delta} L \tilde{\Delta} - \omega \cos 2\tilde{\theta} ) + \tilde{s}_{23}^2 (\tilde{\Delta} \cos 2\tilde{\theta} - \omega) ) \]

\[ - (\tilde{s}_{23}^2 - s_{23}^2) (\sin \tilde{\Delta} L \tilde{\Delta} - \omega \cos 2\tilde{\theta} ) + \tilde{s}_{23}^2 (\tilde{\Delta} \cos 2\tilde{\theta} - \omega) ) \] . \quad (15)

As is clear from Eq\[15\], \(\Delta \mathcal{P}_{\mu e}^{CP}(E, \delta_{CP})\) depends on the value of \(s_{13}\) as well as the known \(s_{12}, s_{23}\). Fortunately, as shown in Ref\[2\] \(\mathcal{P}(\nu_{\mu} \to \nu_e)\) and the anti-electron neutrino dissappearance being studied at Daya Bay, RENO, and Double Chooz is almost independent of \(\delta_{CP}\), and we can use the value \(s_{13} = 0.15\), consistent with Refs\[22, 21\].

We calculate \(\Delta \mathcal{P}_{\mu e}^{CP}(E, \delta_{CP})\) for \(L=1300\) km, the expected baseline in the proposed LBNE project\[13\]. From the possible energy range \(E=0.5-12\) GeV for the LBNE project\[13\] we estimate \(\Delta \mathcal{P}_{\mu e}^{CP}(E, \delta_{CP})\) a function of \(\delta_{CP}\) for energies within the expected range.

The dependence of on \(\Delta \mathcal{P}_{\mu e}^{CP}(E, \delta_{CP})\) on \(\delta_{CP}\) for \(L=1300\) km is estimated for LBNE energies \(E=1, 2, 3, 5, 10\) GeV, as shown in Fig. 1.

### 3 Conclusions

In the LBNE Report\[12\] results of extensive studies have shown that future experiments can extend our knowledge of neutrino oscillations beyond present and planned experiments. Since there will be both \(\nu_{\mu}\) and \(\bar{\nu}_{\mu}\) beams, the LBNE Project can test CPV.

We have estimated CP violation for the LBNE Project, with a baseline \(L=1300\) km as a function of \(\delta_{CP}\) for \(\delta_{CP} = 0\) to \(\pi/2\) for energies of 1, 2, 3, 5, and 10 GeV, as shown in Fig. 1. We found CPV over 3% with \(\delta_{CP} = \pi/2\) for some energies, which the Project should be able to measure. For higher energies, \(E=5, 10\) GeV, \(\Delta \mathcal{P}_{\mu e}^{CP}\) is smaller than at lower energies, and would be
harder to measure. We find that even for $\delta_{CP} = 0$, for which CPV is entirely a matter effect, CPV probabilities of over 1% were found for $E= 1$ GeV, so the Project should be able to measure CPV for any expected values of $\delta_{CP}$. Fortunately, the value of $\theta_{13}$ has been determined, which should enable the LBNE project attain the goal of measuring $\delta_{CP}$.

We believe that our calculations should be useful in planning the future LBNE Project.

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Figure 1: $\Delta P(\nu_\mu \rightarrow \nu_e)$ as a function of $\delta_{CP}$ for expected LBNE L and E