Contribution of hard photon emission to charge asymmetry in elastic (anti)lepton-proton scattering

A. Afanasev∗
Department of Physics, George Washington University
Washington, DC 20052 USA
A. Ilyichev†
Institute for Nuclear Problems
Belarusian State University, 220040 Minsk, Belarus

Abstract

The influence of the hard photon emission on the charge asymmetry in the (anti)lepton-proton scattering was estimated for the first time beyond the ultrarelativistic limit, while retaining the lepton mass at all steps of the calculation. This contribution, responsible for the charge asymmetry, is induced by interference between real photon emission from the lepton and the hadron. During the calculation any excited states in the intermediated proton are not considered that allow us to use the standard fermionic propagator for this particle. Another assumption consists in using the on-shell proton vertex within off-shell region. The infrared divergence extracted using Bardin-Shumeiko approach cancels by the corresponding soft part of the two-photon exchange contribution. Numerical analysis was performed within MUSE and JLab kinematic conditions.

∗E-mail: afanas@email.gwu.edu
†E-mail: ily@hep.by
1 Introduction

The elastic form factors of the proton play an essential role in our understanding the nucleon electromagnetic structure. However, $Q^2$-dependence of the ratio of the electric to magnetic proton form factors, $G_E(Q^2)/G_M(Q^2)$, obtained from the unpolarized and polarized electron elastic scattering data have an essential difference [1, 2]. Moreover, the uncertainty in the form factors can affect the determination of the proton radius [3].

One widely discussed and investigated source of this discrepancy consists in the correct account of the two-photon exchange contribution in the elastic lepton-proton scattering [4, 5, 6]. The challenge in computing two-photon exchange contribution is in the need for modeling nucleon’s structure, since the proton can go into all possible excited states in between the two virtual photon vertices. In the soft-photon exchange approximation, however, the two-photon correction is independent of proton’s internal structure. Naturally, this contribution is suppressed relative to the one-photon exchange (Born) contribution by an additional power of the fine structure constant $\alpha = 1/137$. An infrared divergence from this contribution is canceled with the interference of the real soft-photon emission from the lepton and hadron legs.

In many cases for the estimation of the high order QED effects to the exclusive processes the loop corrections (with the additional particle contributions) are calculated exactly or within ultrarelativistic approximation (with respect to lepton’s mass) while the real photon emission is considered within the soft photon approximation. For example, in the papers [7] and [8] for Möller and virtual Compton scattering processes, respectively, the virtual QED corrections have been calculated beyond the ultrarelativistic limit but only the soft part of the real photon emission was taken into account.

In this paper we computed for the first time the interference between the real hard photon emission from lepton and hadron legs as well as its influence on the charge asymmetry in elastic (anti)lepton-proton scattering. The infrared divergence is canceled with the corresponding soft part from the two-photon exchange using the Bardin-Shumeiko approach [9]. All calculations were performed beyond the ultrarelativistic limit, that allows to apply the obtained results for MUSE experiment [10] where the muon beam with the low momentum is used.
Figure 1: Feynman graphs for the lowest-order contribution to elastic $l^-p$ (a) and $l^+p$ (b) scattering.

## 2 Method of calculation

During our calculation we assume that there is no excited states of the intermediated proton. As a result, the proton propagator looks like a standard fermionic one. The second assumption is that the on-shell proton vertex,

$$\Gamma_\mu(q) = \gamma_\mu F_d(-q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2M} F_p(-q^2),$$

is applicable within off-shell region. Here $\sigma_{\mu\nu} = i[\gamma_\mu, \gamma_\nu]/2$, $M$ is a proton mass, $q$ is a four-momentum of the virtual photon. The Dirac and Pauli form factors can be expressed through the electromagnetic ones:

$$F_d(-q^2) = \frac{G_E(-q^2) + \tau G_M(-q^2)}{1 + \tau},$$

$$F_p(-q^2) = \frac{G_M(-q^2) - G_E(-q^2)}{1 + \tau},$$

where $\tau = -q^2/4M^2$.

The lowest-order contribution to the elastic $l^\pm p$ scattering is presented by Feynman graphs in Fig. 1 and it can be described by the following matrix elements:

$$M_{b^-} = \frac{ie^2}{Q^2} \bar{\psi}(k_2)\gamma^\mu u(k_1)\bar{U}(p_2)\Gamma_\mu(q)U(p_1),$$

$$M_{b^+} = \frac{ie^2}{Q^2} \bar{\psi}(-k_1)\gamma^\mu u(-k_2)\bar{U}(p_2)\Gamma_\mu(q)U(p_1),$$

where $\Gamma_\mu(q) = \gamma_\mu F_d(-q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2M} F_p(-q^2)$.
where $Q^2 = -q^2 = -(k_1 - k_2)^2$ and $e = \sqrt{4\pi\alpha}$. Since the squares of these two matrix elements are identical and are insensitive to the sign of lepton’s charge, it is not possible to distinguish the lepton-proton from antilepton-proton scattering processes at the one-photon exchange level. Their contribution to the cross section can be written as

$$d\sigma_B = \frac{1}{2\sqrt{\lambda_S}}|\mathcal{M}_b^\mp|^2d\Gamma_2,$$

(4)

where $\lambda_S = S^2 - 4m^2M^2$, $S = 2p_1k_1$, $m$ is a lepton mass and the phase space reads:

$$d\Gamma_2 = (2\pi)^4\delta^4(p_1 + k_1 - p_2 - k_2)\frac{d^3k_2}{(2\pi)^32k_20}\frac{d^3p_2}{(2\pi)^32p_20} = \frac{dQ^2}{8\pi\sqrt{\lambda_S}}.\quad(5)$$

The Feynman graphs with the real photon emission both from the lepton and proton legs are shown in Fig 2 (a-d) for $l^-p$ scattering. The matrix elements corresponding to these processes as well as the real photon emission in $l^+p$ scattering read:

$$\mathcal{M}_{lR} = i\frac{e^3}{t}\tilde{u}(k_2)\varepsilon_\alpha\Gamma_{lR}^{\mu\alpha}u(k_1)\bar{U}(p_2)\Gamma_\mu(q - k)U(p_1),$$

(6)

$$\mathcal{M}_{lR}^+ = -i\frac{e^3}{Q^2}\tilde{u}(-k_1)\varepsilon_\alpha\Gamma_{lR}^{\mu\alpha}u(-k_2)\bar{U}(p_2)\Gamma_\mu(q - k)U(p_1),$$

$$\mathcal{M}_{hR} = -i\frac{e^3}{Q^2}\tilde{u}(k_2)\gamma_\mu u(k_1)\bar{U}(p_2)\varepsilon_\alpha\Gamma_{hR}^{\alpha\mu}U(p_1),$$

$$\mathcal{M}_{hR}^+ = -i\frac{e^3}{Q^2}\tilde{u}(-k_1)\gamma_\mu u(-k_2)\bar{U}(p_2)\varepsilon_\alpha\Gamma_{hR}^{\alpha\mu}U(p_1),$$

where $t = -(q - k)^2 = -(p_2 - p_1)^2$, $\varepsilon_\alpha$ is the photon polarized vector and

$$\Gamma_{lR}^{\mu\alpha} = \left(\frac{k_1\alpha}{kk_1} - \frac{k_2\alpha}{kk_2}\right)\gamma^\mu - \frac{\gamma^\mu\hat{k}\gamma^\alpha}{2k_1k} - \frac{\gamma^\alpha\hat{k}\gamma^\mu}{2k_2k},$$

$$\bar{\Gamma}_{lR}^{\mu\alpha} = \left(\frac{k_1\alpha}{kk_1} - \frac{k_2\alpha}{kk_2}\right)\gamma^\mu - \frac{\gamma^\alpha\hat{k}\gamma^\mu}{2k_1k} - \frac{\gamma^\mu\hat{k}\gamma^\alpha}{2k_2k},$$

$$\Gamma_{hR}^{\mu\alpha} = \Gamma^\mu(q)\frac{\hat{p}_1 - \hat{k} + m}{2p_1k}\Gamma^{\alpha}(-k) - \Gamma^\alpha(-k)\frac{\hat{p}_2 + \hat{k} + m}{2p_2k}\Gamma^\mu(q).\quad(7)$$

The part of the cross section with the interference between the real photon emissions from hadron and lepton lines reads:

$$d\sigma_R = \frac{1}{2\sqrt{\lambda_S}}(\mathcal{M}_{lR}^\mp \tilde{\mathcal{M}}_{hR}^\mp + \tilde{\mathcal{M}}_{hR}^\mp \mathcal{M}_{lR}^\mp)\Gamma_3,$$

(8)
Figure 2: Feynman graphs for the real photon emission from the lepton (a,b) and proton (c,d) legs as well as the direct (e) and cross (f) two-photon exchange within $l^{-}p$-scattering. The similar graphs for $l^{+}p$ scattering processes have an opposite direction for the leptonic arrows and a negative sign for its momenta.

where the phase space has a form:

$$d\Gamma_3 = (2\pi)^4 \delta^4(p_1 + k_1 - p_2 - k_2 - k) \frac{d^3k}{(2\pi)^3} \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3}$$

$$= \frac{dQ^2 dv dtd\phi_k}{2^8 \pi^4 \sqrt{\lambda_S Q^2 (Q^2 + 4M^2)}}. \tag{9}$$

Here $\nu = (p + q)^2 - M^2$ is a photonic variable defining inelasticity, $\phi_k$ is an angle between $(k_1,k_2)$ and $(k,q)$ planes in the rest frame $(p_1 = 0)$. Performing the direct calculation, one can find that the interference terms
for $l^- p$ and $l^+ p$ have opposite signs,
\[ d\sigma_R^\pm = -d\sigma_R^\mp. \quad (10) \]

In order to estimate this contribution to the elastic process, it is necessary to integrate $d\sigma_R^\pm$ over three photonic variables: $v$, $t$ and $\phi_k$. However, since the expressions (8) contain the infrared divergent terms at $v = 0$, it is not possible to perform the integration in the straightforward way. Following to the Bardin-Shumeiko approach [9] for the extraction of the infrared (IR) divergence, the identity transformation has to be performed:
\[ d\sigma_R^\pm = d\sigma_R^\pm - d\sigma_R^\pm_{IR} + d\sigma_R^\pm_{IR} = d\sigma_R^\pm_{IR}. \quad (11) \]

In the infrared-free term $d\sigma_R^\pm_{IR}$, the integration can be performed over three photonic variables without any restrictions. The infrared term $d\sigma_R^\pm_{IR}$ can be written in the factorized form:
\[ \frac{d\sigma_R^\pm_{IR}}{dQ^2} = \mp \frac{\alpha}{\pi} (\delta_S + \delta_H) \frac{d\sigma_B}{dQ^2}. \quad (12) \]

The quantities $\delta_S$ and $\delta_H$ appear after splitting the integration region over inelasticity $v$ by the introduction of the infinitesimal parameter $\bar{v}$
\[ \delta_S = -\frac{1}{\pi} \int_0^\bar{v} dv \int \frac{d^3k}{k_0} F_{IR} \delta((p_1 + q - k)^2 - M^2), \]
\[ \delta_H = -\frac{1}{\pi} \int_0^{v_m} dv \int \frac{d^3k}{k_0} F_{IR} \delta((p_1 + q - k)^2 - M^2), \quad (13) \]

where
\[ F_{IR} = \frac{1}{4} \left( \frac{S}{(k_2 k)(p_2 k)} + \frac{S}{(k_1 k)(p_1 k)} - \frac{X}{(k_1 k)(p_2 k)} - \frac{X}{(k_2 k)(p_1 k)} \right) \quad (14) \]

and $X = S - Q^2$.

The upper limits of integration with respect to the variable $v$ is defined as
\[ v_m = \frac{1}{2m^2} \left( \sqrt{\lambda S} \sqrt{Q^2(Q^2 + 4m^2)} - 2m^2Q^2 - Q^2S \right). \quad (15) \]

In practice, however, the influence of the hard real photon emission to the asymmetry can be essentially reduced by applying a cut $v_{cut}$ on the
inelasticity which is also a measured quantity in the elastic lepton-proton scattering.

Performing the integration in $\delta_S$ (using the photon mass $\lambda$ for the infrared divergence regularization) and $\delta_H$ we can find that

\[
\delta_S = \delta_S^1 - 2(SL_S - XL_X) \log \left[ \frac{\bar{v}}{M\lambda} \right],
\]
\[
\delta_H = \delta_H^1 - 2(SL_S - XL_X) \log \left[ \frac{v_m}{\bar{v}} \right],
\]

where

\[
L_S = \frac{1}{\sqrt{\lambda_S}} \log \frac{S + \sqrt{\lambda_S}}{S - \sqrt{\lambda_S}},
\]
\[
L_X = \frac{1}{\sqrt{\lambda_X}} \log \frac{X + \sqrt{\lambda_X}}{X - \sqrt{\lambda_X}},
\]

and $\lambda_X = X^2 - 4M^2m^2$. The quantities $\delta_S^1$ and $\delta_H^1$ have a rather complicated structure and depend neither on $\bar{v}$ nor on $\lambda$. As a result, the sum of $\delta_S$ and $\delta_H$ is free from the separated parameter $\bar{v}$ but it contains dependence on the fictitious photon mass $\lambda$.

As shown in Fig. 2 (e,f), the matrix elements with two-photon exchange contribution to the elastic $l^\mp p$ scattering can be separated into the direct $M_{\text{box}}$ and cross $M_{\text{zbox}}$ terms that can be presented through the loop integration in a following way:

\[
M_{\text{box}}^- = \frac{e^4}{(2\pi)^4} \int \frac{d^4l}{l^2(l-q)^2} \bar{u}(k_2) \gamma^\nu \hat{\gamma}^1 - \hat{l} + m \frac{l^2}{l^2 - 2k_1l} \gamma^\mu u(k_1) \times U(p_2) \Gamma\nu(q - l) \frac{\hat{p}_1 + \hat{l} + M}{l^2 + 2p_1l} \Gamma\mu(l) U(p_1),
\]
\[
M_{\text{box}}^+ = \frac{e^4}{(2\pi)^4} \int \frac{d^4l}{l^2(l-q)^2} \bar{u}(-k_1) \gamma^\nu \hat{\gamma}^1 - \hat{l} + m \frac{l^2}{l^2 - 2k_1l} \gamma^\mu u(-k_2) \times U(p_2) \Gamma\nu(q - l) \frac{\hat{p}_1 + \hat{l} + M}{l^2 + 2p_1l} \Gamma\mu(l) U(p_1),
\]
\[
M_{\text{zbox}}^- = \frac{e^4}{(2\pi)^4} \int \frac{d^4l}{l^2(l-q)^2} \bar{u}(k_2) \gamma^\nu \hat{\gamma}^1 - \hat{l} + m \frac{l^2}{l^2 - 2k_1l} \gamma^\mu u(k_1) \times U(p_2) \Gamma\mu(l) \frac{\hat{p}_2 - \hat{l} + M}{l^2 - 2p_2l} \Gamma\nu(q - l) U(p_1),
\]
\[ M_{\text{box}}^+ = \frac{e^4}{(2\pi)^4} \int \frac{d^4 l}{l^2(l - q)^2} \bar{u}(-k_1) \gamma^\mu \hat{l} - \hat{k}_1 + m\frac{\gamma^\nu u(-k_2)}{l^2 - 2k_1 l} \times \bar{U}(p_2) \Gamma_\mu (l) \hat{p}_2 - \hat{l} + M\frac{l_\nu(q - l) U(p_1)}{l^2 - 2p_2 l}. \]  

(18)

Note that all of these matrix elements contain the infrared divergence at \( l = 0 \) and \( l = q \) points.

The lowest-order two-photon exchange contribution to the elastic \( l^\pm p \) cross section has a form

\[ d\sigma_{\text{box}}^\pm = \frac{1}{2\sqrt{\lambda_S}} [M_b (M_{\text{box}}^+ + M_{\text{box}}^-) + (M_{\text{box}}^- + M_{\text{box}}^+) M_b^+ d\Gamma_2. \]  

(19)

Once again we can find that

\[ d\sigma_{\text{box}}^+ = -d\sigma_{\text{box}}^- \]  

(20)

The infrared divergence extracted from two-photon exchange contribution reads

\[ \frac{d\sigma_{\text{box}}^{\mp}}{dQ^2} = \mp \frac{\alpha}{\pi} \left( \delta_{12}^1 + (S L_S - X L_X) \log \left[ \frac{Q^2}{\lambda^2} \right] \right) \frac{d\sigma_B}{dQ^2}, \]  

(21)

where the quantity \( \delta_{21}^1 \) has a rather complicated structure and does not depend on the photon mass \( \lambda \).

The sum Eq. (12) with Eq. (21)

\[ \frac{d\sigma_{\text{box}}^{\mp}}{dQ^2} + \frac{d\sigma_{\text{box}}^{\mp}}{dQ^2} = \mp \frac{\alpha}{\pi} \delta_{VR}(Q^2) \frac{d\sigma_B}{dQ^2} \]  

(22)

is infrared free since

\[ \delta_{VR}(Q^2) = \left( \delta_S^1 + \delta_H^1 + \delta_{21}^1 + (S L_S - X L_X) \log \left[ \frac{Q^2 M^2}{v_m^2} \right] \right) \]  

(23)

does not depend on \( \lambda \).

A physical requirement coming from vanishing asymmetry at \( Q^2 \to 0 \) can be provided by the difference \( \delta_{VR}(Q^2) \) and its value at \( Q^2 = 0 \): \( \hat{\delta}_{VR} = \delta_{VR}(Q^2) - \delta_{VR}(0) \).

Finally, the lowest order of the charge-odd contribution to the elastic lepton-proton cross section reads:

\[ \frac{d\sigma_{\text{odd}}^\pm}{dQ^2} = \frac{d\sigma_{\text{odd}}^\pm}{dQ^2} \mp \frac{\alpha}{\pi} \hat{\delta}_{VR} \frac{d\sigma_B}{dQ^2}. \]  

(24)
3 Numerical results

The dependence of the charge asymmetry

\[ A = \frac{\frac{d\sigma_{\text{odd}}^+}{dQ^2} - \frac{d\sigma_{\text{odd}}^-}{dQ^2}}{\frac{d\sigma_B}{dQ^2}}, \]

on the value of the inelasticity cut for different lepton beams and \( Q^2 \) is presented in Fig. 3 at MUSE kinematic conditions [10]. It can be seen that at the fixed lepton momentum the asymmetry decreases with growing \( v_{\text{cut}} \), i.e. when the unobserved photon becomes harder. It can be seen that the magnitude of the asymmetry is reduced with increasing \( Q^2 \). Moreover, the value of this asymmetry is higher for the lighter lepton.

Another important quantity is the ratio of \( e^+p/e^-p \) cross sections that can be defined as

\[ R = \frac{\frac{d\sigma_B}{dQ^2} + \frac{d\sigma_{\text{odd}}^+}{dQ^2}}{\frac{d\sigma_B}{dQ^2} + \frac{d\sigma_{\text{odd}}^-}{dQ^2}}. \]

The dependence of this ratio on the virtual photon polarization \( \varepsilon \) at JLab kinematic conditions is shown on Fig. 4. For the fixed electron beam energies (solid lines) the small \( \varepsilon \) corresponds to the hard photon contribution where the ratio is closer to one. Similar to the experimental observation [4], the cross section ratio with the soft photon emission (dashed line) decreases with growing \( \varepsilon \).

4 Conclusion

The contribution of the hard photon emission to the charge asymmetry in lepton-proton scattering was estimated for the first time beyond the ultrarelativistic limit, while keeping lepton mass during the entire calculation.

Only two assumptions were used in the calculation: I) We did not consider excitations of the intermediated proton and used a standard fermionic propagator for it; II) The on-shell proton vertex with the Dirac and Pauli form factors was used in the off-shell region.

Numerical results shown that at the fixed lepton momentum the asymmetry decreases with growing energy of the unobserved photon. This asymmetry is sensitive to the lepton mass: its value is higher for the lighter lepton.
Figure 3: Charge asymmetry defined in (25) vs value of the inelasticity cut for elastic $e^\mp p$ and $\mu^\mp p$ scattering with the beam momenta 115 MeV, 153 MeV and 210 MeV.
Figure 4: Ratio of $e^+p/e^-p$ cross sections defined in (26) as a function of $\varepsilon$ at $Q^2 = 1.45$ GeV$^2$. Solid lines correspond to the different electron beam energies $E_{\text{beam}}$. A dashed (dotted) line presents a soft (hard) photon emission with $v_{\text{cut}} = 0.05v_{\text{max}}$ ($v_{\text{cut}} = v_{\text{max}}$) for $1.5$ GeV $< E_{\text{beam}} < 4$ GeV.

The next planned step consists in implementation of the obtained results into Monte-Carlo generator ELRADGEN [11] for the simulation of hard photon emission in future experiments.

References

[1] Jones M.K. et al. [Jefferson Lab Hall A Collaboration], $G(E(p))/G(M(p))$ ratio by polarization transfer in polarized $ep \rightarrow e$ polarized $p$. Phys. Rev. Lett. 2000. Vol. 84, P. 1398-1402 DOI:10.1103/PhysRevLett.84.1398

[2] Gayou O. et al. [Jefferson Lab Hall A Collaboration], Measurement of $G(Ep)/G(Mp)$ in polarized $ep \rightarrow e$ polarized $p$ to $Q^2 = 5.6$ GeV$^2$. Phys. Rev. Lett. 2002. Vol. 88, 092301 DOI:10.1103/PhysRevLett.88.092301
[3] R. Pohl et al., “The size of the proton,” Nature 2010. Vol. 466, P. 213-216. DOI:10.1038/nature09250

[4] D. Adikaram et al. [CLAS Collaboration], Towards a resolution of the proton form factor problem: new electron and positron scattering data. Phys. Rev. Lett. 2015. Vol. 114, 062003 DOI:10.1103/PhysRevLett.114.062003

[5] O. Koshchii and A. Afanasev, Charge asymmetry in elastic scattering of massive leptons on protons. Phys. Rev. D. 2017. Vol.96, No. 1. 016005 DOI:10.1103/PhysRevD.96.016005

[6] A. Afanasev, P.G. Blunden, D. Hasell, and B.A. Raue, Two-photon exchange in elastic electron-proton scattering. Prog. Part. Nucl. Phys. 2017. Vol. 95, 245. DOI:10.1016/j.ppnp.2017.03.004

[7] N. Kaiser, Radiative corrections to lepton-lepton scattering revisited. J. Phys. G. 2010. Vol.37. 115005. DOI:10.1088/0954-3899/37/11/115005

[8] M. Vanderhaeghen, J. M. Friedrich, D. Lhuillier, D. Marchand, L. Van Hoorebeke and J. Van de Wiele, QED radiative corrections to virtual Compton scattering. Phys. Rev. C. 2000. Vol. 62, 025501. DOI:10.1103/PhysRevC.62.025501

[9] D. Y. Bardin and N. M. Shumeiko, An Exact Calculation of the Lowest Order Electromagnetic Correction to the Elastic Scattering. Nucl. Phys. B. 1977. Vol. 127, P. 242-258. DOI:10.1016/0550-3213(77)90213-9

[10] R. Gilman et al. [MUSE Collaboration], Studying the Proton ”Radius” Puzzle with $\mu p$ Elastic Scattering. arXiv:1303.2160 [nucl-ex].

[11] I. Akushevich, O. F. Filoti, A. N. Ilyichev and N. Shumeiko, Monte Carlo Generator ELRADGEN 2.0 for Simulation of Radiative events in Elastic ep-Scattering of Polarized Particles. Comput. Phys. Commun. 2012. Vol. 183, P. 1448-1467. DOI:10.1016/j.cpc.2012.01.015