EXCESS ELLIPTICITY OF HOT AND COLD SPOTS IN THE WMAP DATA?

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Received 2012 November 22; accepted 2013 October 27; published 2013 November 22

ABSTRACT

We investigate claims of excess ellipticity of hot and cold spots in the Wilkinson Microwave Anisotropy Probe (WMAP) data. Using the cosmic microwave background (CMB) data from 7 yr of observations by the WMAP satellite, we find, contrary to previous claims of a 10σ detection of excess ellipticity in the 3 yr data, that the ellipticity of hot and cold spots is perfectly consistent with simulated CMB maps based on the concordance cosmology. We further test for excess obliquity and excess skewness/kurtosis of ellipticity and obliquity and find the WMAP7 data consistent with Gaussian simulated maps.

Key words: cosmic background radiation – cosmology: observations – methods: data analysis – methods: statistical

Online-only material: color figure

1. INTRODUCTION

Ever since the discovery of the cosmic microwave background (CMB) fluctuations in 1992 (Smoot et al. 1992), the availability and quality of CMB data have been steadily increasing through a series of experiments. The Wilkinson Microwave Anisotropy Probe (WMAP; Bennett et al. 2003) with its most recent 7 yr maps (Hinshaw et al. 2009) is currently the most important publicly available data set for the study of CMB fluctuations. In the standard model of cosmology the universe is asymptotically isotropic and homogeneous on large scales and the CMB consists of Gaussian fluctuations, making the fluctuation amplitude on different scales the only relevant information contained in the map.

Studies of CMB fluctuations have classically concentrated on this, achieved most commonly by the transformation of the map to spherical harmonic space and averaging over the directional modes to create the angular power spectrum. The power spectrum, while computationally more forgiving to work with than the original map, is only free of degeneracies under the assumption of perfect Gaussianity and isotropy.

In this paper we will study a property of CMB fluctuations directly related to the map, namely, the ellipticity of hot and cold spots. The aim of this study is to follow up on direct measures of ellipticity of spots in CMB data in Gurzadyan et al. (2003a, 2003b, 2005a, 2005b, 2007), where a significant deviation from the assumption of perfect Gaussianity and isotropy was detected for the WMAP data, that the ellipticity of hot and cold spots is perfectly consistent with simulated CMB maps based on the concordance cosmology. We further test for excess obliquity and excess skewness/kurtosis of ellipticity and obliquity and find the WMAP7 data consistent with Gaussian simulated maps.

1. INTRODUCTION

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We perform our own analysis of ellipticity on the WMAP 7 yr data and compare to results stated in the given references; in particular, we wish to test the claims of significantly higher ellipticity in WMAP data than in simulations and the claims for higher ellipticity on smaller scales. It is also claimed that there is no evidence of a preferred direction of the spot elongations, and we investigate this by obliquity measures of the CMB data.

In Section 2 we describe the data and masks used in the analysis. The methods used to assess the ellipticity and obliquity are outlined in Section 3. In Section 4 we show the results of the ellipticity and obliquity tests applied to the 7 yr WMAP data, and in Section 6 we conclude.

2. DATA

The analysis in this paper was performed using the 7 yr release of the WMAP data (publicly available at the Lambda Web site\(^1\)), as well as a statistical ensemble of 10,000 simulated maps of each of the channels Q (41 GHz), V (61 GHz), and W (94 GHz) (the map for each channel is obtained by taking the mean of all differencing assemblies for the given channel). All analyses are preformed on either the Q band or the co-added V + W band maps.

From all maps, we have subtracted the best-fit mono- and dipole. The mask used throughout was the WMAP QKQ5 galactic and point-source mask, leaving a sky fraction of 82%. Maps were simulated using WMAP noise and beam properties. All maps were pixelized in the HEALPix scheme\(^2\) (Gorski et al. 2005).

3. METHODOLOGY

As described in Gurzadyan et al. (2003a), the method of extracting anisotropic areas is to consider as relevant only those pixels for which the temperature value is above (or below) a given temperature threshold and as nonrelevant all pixels for which the temperature value is below (above) the threshold. A “spot” is then defined to be any set of relevant pixels such that

1. any pixel in the set may be path connected on the HEALPix map with any other within the set without moving through a nonrelevant pixel;
2. no pixel in the set may be path connected on the map with a relevant pixel outside the set without moving through a nonrelevant pixel.

\(^1\) http://lambda.gsfc.nasa.gov/
\(^2\) http://healpix.jpl.nasa.gov/
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where two pixels in the spot, $A_1$ and $A_2$, with the greatest angular distance between them; this angular distance is the major axis $a$. 3. Determine the shortest angular distance to this axis for all pixels in the spot. The two pixels (on either side) with the largest distance to the axis are referred to as $B_1$ and $B_2$ and their angular distance to the major axis as $b_1$ and $b_2$, respectively.

4. The minor axis $b$ is then defined as

$$ b = b_1 + b_2. \tag{1} $$

5. Calculate ellipticity, defined as

$$ \epsilon \equiv \frac{a}{b}. \tag{2} $$

6. Calculate the obliquity $\epsilon$, defined as the smallest angle between the great circle traced by the major axis and a chosen equator.

The first step is a purely computational one made to avoid cases where all pixels in a spot fall on a line, causing a potentially near-infinite ellipticity as all pixels return close to zero distance to the major axis. This was seen as superior to the use of vertex positions as a means for less skewed results for smaller spots. Note that any of the three geodesics $a$, $b_1$, and $b_2$ may trace areas not covered by the spot as the major axis does in Figure 1. Except for a test of obliquity measures against 12,288 chosen equators to check for differences, obliquity was measured against galactic equator. Step 6 differs from Gurzadyan et al. (2003a), as it only measures obliquity from 0 to 90 deg to avoid ambiguity on polar spots. Finally, the results for one map is returned as the average over all spots and statistical uncertainties with no weighting with regard to position or size except that spots smaller than certain values are excluded from some of the analysis. Statistical analysis is carried out in frequentist manner to obtain mean, variance, skewness, and kurtosis.

To obtain a result against which to compare the ellipticity and obliquity values of the WMAP maps, an ensemble of 10,000 maps were simulated using modules included in the HEALPix package (as explained below, 9000 of these are used to calculate correlations, and 1000 are used to quantify significance). The analysis was performed on simulated and WMAP data in the $Q$ and combined $V + W$ bands with thresholds ranging from $-500 \mu K$ to $-40 \mu K$ for negatively defined spots and then from positive $40 \mu K$ to $500 \mu K$ at every 20th $\mu K$. The ellipticity and obliquity were calculated for

1. All spots (including the ones with only two pixels).
2. Spots with >2 pixels.
3. Spots with >3 pixels.
4. Spots with >8 pixels.
5. Spots with >20 pixels.
6. Spots with >50 pixels.
7. Spots with >100 pixels.
8. Spots with >300 pixels.

Single pixel “spots” were ignored throughout.

In order to evaluate the significance of the results, we performed a $\chi^2$ test,

$$ \chi^2 = \sum_{ip} \sum_{i'p'} (x_{ip} - \langle x_{ip} \rangle) C_{ip,i'p'}^{-1} (x_{i'p'} - \langle x_{i'p'} \rangle), \tag{3} $$

where $x_{ip}$ is the value (mean over all spots for that map) for the ellipticity or obliquity for the map to be tested at a temperature threshold $t$ and for a spot size $p$. $\langle x_{ip} \rangle$ is the mean over all simulated maps and $C_{ip,i'p'}$ is the covariance matrix obtained from 9000 Gaussian simulations based on the WMAP best-fit power spectrum and noise model.

4. RESULTS

Results gathered for the $V + W$ map are shown in Figure 2 (for number of spots) and Figure 3 (ellipticity). Note that the data seem to be systematically above the mean for several thresholds and spot sizes in the figures for spot numbers. This is caused by the strong correlations between thresholds and between spots at different sizes and is also seen in simulations. These correlations are taken into account in the $\chi^2$ analysis as described above, showing that the values for the data are in perfect agreement with the simulations.
Figure 2. Number of spots in the $V+W$ map as a function of threshold for a given size of spot. Data points are shown as crosses and the mean value from simulations as a dotted line. The confidence intervals are given to $2\sigma$. Note that we only show the points for which we had sufficient spot statistics to be used in the analysis. Note also that the points are highly correlated.

### Table 1

$V+W$ Band $\chi^2$ Expressed as Percentage of Simulated Maps with Greater $\chi^2$ than the Corresponding WMAP Values

| Spot Size | Number of Spots | Ellipticity | Obliquity |
|-----------|-----------------|-------------|-----------|
| >3        | 76              | 25 (41)     | 19        |
| >8        | 71              | 9 (38)      | 28        |
| >20       | 42              | 85 (68)     | 52        |
| >50       | 16              | 11 (44)     | 79        |
| >100      | 63              | 24 (14)     | 44        |
| >300      | 30              | 76 (44)     | 83        |

Note. Numbers in parentheses are based on the Gurzadyan et al. (2003a) method for calculating ellipticity as explained in detail in the text.

We do not show plots for the $Q$ band as they are very similar to the $V+W$ results. The mean ellipticity value for spots in a perfectly Gaussian map with infinite resolution can be shown to be $\epsilon \approx 1.648$ (Aurich et al. 2011), so this is the naively expected result. Ellipticity significantly in excess of this number is only evident for spots smaller than 8 pixels; for larger spots, the ellipticity is close to the theoretical value. This is due to a higher probability for smaller spots of consisting only of pixels placed vertically, horizontally, or diagonally on a row, resulting in such spots having ellipticity in excess of 10, and hence skewing the results toward a higher mean ellipticity.

### Table 2

$Q$ Band $\chi^2$ Expressed as Percentage of Simulated Maps with Greater $\chi^2$ than the Corresponding WMAP Values

| Spot Size | Number of Spots | Ellipticity | Obliquity |
|-----------|-----------------|-------------|-----------|
| >3        | 38              | 17 (17)     | 1         |
| >8        | 74              | 10 (33)     | 36        |
| >20       | 94              | 86 (77)     | 40        |
| >50       | 40              | 96 (36)     | 99        |
| >100      | 54              | 87 (47)     | 54        |
| >300      | 72              | 83 (29)     | 1         |

Note. Numbers in parentheses are based on the Gurzadyan et al. (2003a) method for calculating ellipticity as explained in detail in the text.

Figure 4 shows a map of the spots found in the case of the $V+W$ map with a $+100$ $\mu$K threshold. The spots have been color-coded to highlight their ellipticity in the range from 1 to 4 with zero (dark blue) representing nonrelevant pixels. The measured ellipticity and obliquity were compared to an ensemble of 1000 simulated Gaussian maps and values compared to its $\chi^2$ distribution, results for which are shown in Table 1 ($V+W$ data) and Table 2 ($Q$ data). In Table 3 we show the total $\chi^2$ summed also over all pixel numbers for ellipticity and obliquity, as well as the skewness and kurtosis of these. No significant detection of excess ellipticity could be found for any
Figure 3. Spot ellipticity in the V+W map as a function of threshold for a given size of spot. Data points are shown as crosses and the mean value from simulations as a dotted line. Confidence intervals are given to 2σ. Note that we only show the points for which we had sufficient spot statistics to be used in the analysis. Note also that the points are highly correlated.

Table 3

| Spot Size | V+W Band | Q Band |
|-----------|----------|--------|
| Number of spots | 75 | 94 |
| Ellipticity | 7 (18) | 29 (23) |
| Obliquity | 18 | 3 |
| Skewness ellipticity | 40 | 1 |
| Kurtosis ellipticity | 14 | 51 |
| Skewness obliquity | 53 | 6 |
| Kurtosis obliquity | 62 | 62 |

Note. Numbers in parentheses are based on the Gurzadyan et al. (2003a) method for calculating ellipticity as explained in detail in the text.

5. DISCUSSION

The spots in this paper were defined in the same manner as Gurzadyan et al. (2003b), as was the formula for calculating ellipticity

\[ \epsilon = \frac{a}{b} \]  

where \( a \) and \( b \) are major and minor axis, respectively. The definition of semi-major axis, however, was slightly different from the one presented in Gurzadyan et al. (2003a), where the semi-major axis is found by defining the spot center and letting the semi-major axis be the line to the center of the spot from the pixel farthest away from that center. This difference in calculation of semi-major axis should disappear when averaged over many spots, and for sufficiently large spots the ellipticity should be close to the theoretical mean value of \( \epsilon \approx 1.648 \) (Aurich et al. 2011) in both cases. We have shown above that this is indeed the case for our algorithm, whereas in Gurzadyan et al. (2007, Figure 7) the mean ellipticity for simulated maps is always above 2 for any pixel size. Note that our results are based on the 7 yr release of the WMAP data, whereas Gurzadyan et al. (2007) used the 3 yr release. Small differences in noise fluctuations between the two data releases will cause tiny differences in the absolute numbers for ellipticity, but this is unlikely to be the reason for the 10σ detection of excess ellipticity in Gurzadyan et al. (2007).

As a final test of consistency, we also implemented the method to calculate ellipticity in Gurzadyan et al. (2003a; described above). We found that for a given spot, their and our ellipticity measures were correlated at the 70%–80% level depending on...
spot size. However, as anticipated, the mean ellipticity between the two methods agrees very well with a difference of typically 1%–2% depending again on the spot size.

Obliquity was also calculated slightly differently, but here our results agree with Gurzadyan et al. (2003b) in that no significant detection could be found.

Residual foregrounds were not considered, and confidence was placed on the mask. The mask used (KQ85) is fairly liberal, and this may create problems, especially in the Q band; if so, the effect is small enough not to be detectable. In the tables, there is a noticeable difference between the results for the V+W band and the Q band. These differences are due to the difference in beam and noise properties of the two channels as similar differences are seen in simulated maps. Note also two small percentages for the Q-channel obliquity (Table 2). As the total \( \chi^2 \) for Q-band obliquity is normal (Table 3), these numbers are shown to be consistent with statistical fluctuations found in simulations and are not detections of excess obliquity.

In Tables 1 and 2 we presented \( \chi^2 \) for ellipticity and obliquity for different spot sizes. In these tables there are no indications for excess ellipticity or obliquity for any spot size. In order to check whether such numbers are expected given the number of data points, we also calculated the total \( \chi^2 \) taking into account all spot sizes. The results are shown in Table 3, where we also show \( \chi^2 \) values for the skewness and kurtosis of ellipticity and obliquity. All values seem to indicate that the ellipticity and obliquity of spots in the WMAP data are consistent with simulated data sets based on the concordance cosmological model.

### 6. CONCLUSIONS

Gurzadyan et al. have, in several papers, claimed strong evidence for an abnormally high ellipticity in the hot and cold spots of CMB fluctuations as measured by both BOOMERanG and WMAP and to a certain extent also COBE (Gurzadyan et al. 2003a, 2003b, 2005b). Here, the WMAP 7 yr data were examined in the Q and combined V+W band maps to look for such an effect. No extraordinary ellipticity was found, and the results obtained here also disagree with the reported substantial difference in ellipticity for spots greater than 50 pixels compared to results when spots of 20–50 pixels also are included.

Gurzadyan et al. (2003a, 2003b, 2005b) also report that no preferred direction can be found to any statistical significance on the CMB spots. Our results agree that no such direction can be found to within a satisfactory statistical confidence.

Redoing the analysis on higher resolution Planck maps would determine the extent of pixel effects and noise in the heightened ellipticity for the smallest spots. Probing smaller scales will not only increase the statistics overall but make it possible
to compare ellipticity of large spots to small ones given the increased sensitivity. But previous claims of excess ellipticity, which is what we investigate in this paper, were based on the WMAP data, and we are not able to reproduce the excess ellipticity reported for these data.

F.K.H. is thankful for a grant from the Norwegian Research Council. We acknowledge the use of the HEALPix package and of the Legacy Archive for Microwave Background Data Analysis (LAMBDA). Support for LAMBDA is provided by the NASA Office of Space Science. Support for HEALPix is provided by JPL at CalTech and the international HEALPix crew.

REFERENCES

Aurich, R., Janzer, H. S., Lustig, S., & Steiner, F. 2011, IJMPD, 20, 2253
Bennett, C. L., Halpern, M., Hinshaw, G., et al. 2003, ApJS, 148, 1
Gorski, K. M., Hivon, E., Banday, A. J., et al. 2005, ApJ, 622, 759
Gurzadyan, V. G., Ade, P. A. R., de Bernardis, P., et al. 2003a, IJMPD, 12, 1859
Gurzadyan, V. G., Ade, P. A. R., de Bernardis, P., et al. 2003b, NCimB, 118, 1101
Gurzadyan, V. G., Ade, P. A. R., de Bernardis, P., et al. 2005a, MPLA, 20, 491
Gurzadyan, V. G., Bianco, C. L., Kashin, A. L., Kuloghlian, H., & Yegorian, G. 2007, PRL, 5, 363, 121
Gurzadyan, V. G., de Bernardis, P., de Troia, G., et al. 2005b, MPLA, 20, 813
Gurzadyan, V. G., & Kocharyan, A. A. 2009, A&A, 493, L61
Hinshaw, G., Weiland, J. L., Hill, R. S., et al. 2009, ApJS, 180, 225
Smoot, G. F., Bennett, C. L., Kogut, A., et al. 1992, ApJ, 396, L1