The variation of the gravitational constant inferred from the Hubble diagram of Type Ia supernovae

E. García–Berro†‡, Y. Kubyshin§, P. Loré–Aguilar¶‡ & J. Isern¶‡

† Departament de Física Aplicada, Universitat Politècnica de Catalunya,
Escola Politècnica Superior de Castelldefels,
Avda. del Canal Olímpic s/n, 08860 Castelldefels, Spain
‡ Institut d’Estudis Espacials de Catalunya,
Ed. Nexus, c/ Gran Capità 2, 08034 Barcelona, Spain
§ Institut de Tècniques Energètiques, Universitat Politècnica de Catalunya,
Edif. ETSEIB, Campus Sud, Avda. Diagonal, 647, 08028 Barcelona, Spain
¶ Institut de Ciències de l’Espai, C.S.I.C.,
Campus UAB, Facultat de Ciències, Torre C–5, 08193 Bellaterra, Spain

Abstract

We consider a cosmological model with a variable gravitational constant, $G$, based on a scalar-tensor theory. Using the recent observational data for the Hubble diagram of type Ia supernovae (SNeIa) we find a phenomenological expression describing the variation of $G$. The corresponding variation of the fine structure constant $\alpha$ within multidimensional theories is also computed and is shown not to support known constraints on $\Delta \alpha/\alpha$.

1 Introduction

The analysis of the observations of the Hubble diagram of distant type Ia supernovae [1, 2, 3] provide evidence that the universe has been accelerating recently, at $z < 0.5$, and decelerating at earlier stages [3, 4]. The Friedmann cosmology without cosmological constant and with zero curvature — as indicated by the recent CMB Boomerang and Maxima experiments [5, 6] — cannot explain such an evolution of the universe [7]. The accelerated behaviour can be attributed to a “dark energy” with negative pressure, the simplest possibility being the introduction of the cosmological constant in accordance with cosmic concordance model $\Omega_M \approx 0.3, \Omega_\Lambda \approx 0.7$ — see [8] for a review.

An alternative solution is to modify the gravitational theory, for example, by allowing a time variation of Newton’s constant. The possibility of a time variation of fundamental constants of nature, in particular of the fine structure constant, $\alpha$, and of the gravitational constant, $G$ — first considered by Dirac in the framework of his Large Number hypothesis [9] and later developed in [10] within an alternative theory of gravitation (see references [11] and [12] for more details) — has been recently a subject of numerous studies (see for example references [11, 13, 14, 15] for recent reviews and extensive bibliography). It is worth mentioning that many theoretical approaches, such as models with extra dimensions, string theories or scalar–tensor models of quintessence, contain a built–in mechanism for a possible time variation of the couplings. Astronomical measurements...
allow to constrain such hypothetical variations. As a matter of fact, local constraints on the rate of variation of $G$ can be derived, for example, from Lunar Laser ranging \cite{16,17}, whereas constraints at cosmological distances can be derived, amongst other methods, from the Hubble diagram of distant SNeIa \cite{18}.

On the other hand, in the framework of models with a varying gravitational constant it would be valuable to get a phenomenological expression for its variation. That is, to obtain an approximate form of the function $G(z)$, where $z$ is the redshift. Getting such description is the goal of the present paper. We would like to note that fitting the Hubble diagram of SNeIa within models with a variable gravitational constant for a particular parametrization and $\Lambda = 0$ was studied in \cite{19}. Using the considerably better observational data available now \cite{3} which extend to much larger distances we will find a more accurate approximation for $G(z)$ for a larger interval of look–back times. It is worth mentioning as well that a procedure of reconstruction of a general scalar–tensor model (the scalar field potential and the functional form of the scalar-gravity coupling) of dark energy from cosmological observational data, particularly the luminosity distance, was first developed in \cite{20}.

Once the phenomenological form of $G(z)$ is obtained it can be compared with predictions of cosmological models and/or contrasted with other astrophysical observational constraints. For instance, models with extra dimensions incorporate a natural mechanism for the space and time variation of various fundamental constants, which was apparently studied for the first time in \cite{21} and later on in a number of articles. It should be also mentioned that in \cite{22} the relation between the time variation of the fine structure constant and that of the gravitational constant was studied for three classes of theories—namely, for the pure Kaluza–Klein theory, for Einstein–Yang–Mills theories and for Randall–Sundrum type models. Using the relation between $\alpha$ and $G$ in a given model and the phenomenological expression for $G(z)$ one can then obtain an estimate of the variation of the fine structure constant. This prediction can therefore be then contrasted with the observational constraints on the variation of $\alpha$, which has recently been a subject of intensive studies. Particularly, using the many multiplet method it has been claimed \cite{23,24,25} that the fine structure constant $\alpha$ was smaller in the past. However, a similar analysis carried out in \cite{26} and in \cite{27} using a different line fitting code and data sample of better quality shows that the measurements are consistent with zero variation within the observational uncertainties and, consequently, these results do not support the claims by previous authors.

The plan of the paper is the following. In Sect. 2 we outline a theoretical scheme for a variable gravitational “constant” and derive a generalization of the Hubble law for this case. A phenomenological description of the function $G(z)$ which fits the Hubble diagram of SNeIa is then found in Sect. 3. In the next section the correlated variations of the fine structure constant and of the gravitational constant for models with extra dimensions is discussed. Finally, in Sect. 5 our main conclusions are presented, followed by some discussion of our most important results.
2 Variation of $G$ in scalar-tensor theories

Theories of gravity in which the gravitational “constant” $G$ varies with time and cosmological models based on them have been extensively studied in the literature (see, for example, [28] and references therein). One of the most natural and relativistic covariant ways to describe the variation of the gravitational constant is to interpret it as a scalar field $\phi$. This can be done self-consistently in the framework of scalar–tensor theories of gravity of the Jordan–Brans–Dicke type [10, 29] with the action given by

$$ S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left( \phi R + \frac{w(\phi)}{\phi} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + 16\pi L_m \right), $$

where the function $w = w(\phi)$ determines the coupling between the scalar field and gravity.

In considering the cosmic evolution of the scale factor $a(t)$ of the Friedmann–Robertson–Walker metric and of the scalar field $\phi$ in Eq. (1) we assume for simplicity that $w$ is constant. Then, the Hubble parameter $H \equiv \dot{a}/a$ is determined by the Friedmann equation [28]

$$ H^2 \equiv \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi}{3\phi} \rho - \frac{k}{a^2} - \frac{\dot{\phi}}{\phi} + \frac{w}{6} \phi^2 + \frac{\Lambda}{3}, $$

where $\Lambda$ is the cosmological constant and $k$ is the curvature parameter. We furthermore assume that the universe contains a simple perfect fluid described by the equation of state

$$ p = (\gamma - 1)\rho. $$

Eqs. (2) and (3) and the energy conservation condition $\dot{\rho} + 3\gamma H\rho = 0$ have to be complemented with the acceleration equation for $\phi$ [28]:

$$ \ddot{\phi} + 3H \dot{\phi} = \frac{8\pi \rho}{2w + 3}(4 - 3\gamma). $$

In what follows we will consider $a$ and $\phi$ to be functions of the redshift $z$. To convert time derivatives to the derivatives with respect to $z$ we use the standard relation:

$$ \frac{d}{dt} \equiv -H(1 + z) \frac{d}{dz}. $$

Denoting the $z$-derivatives with prime we get relations of the type $\dot{\phi} = -H(1 + z)\phi'$.

By considering the weak–field limit in the scalar–tensor theories the following relation between the gravitational constant and the scalar field $\phi$ can be established [10]

$$ G(z) = \frac{4 + 2w}{3 + 2w} \frac{1}{\phi(z)}. $$

Using these expressions, the Hubble law — given by Eq. (2) — can be written after some algebra in the following form

$$ H^2 = H_0^2 g_0 \frac{G(z) \Omega_M G(z) (1 + z)^3 + \Omega_R (1 + z)^2 + \Omega_\Lambda}{g(z)}. $$

3
The function $g(z)$ is given by

$$g(z) = 1 + (1 + z) \frac{G'}{G} - \frac{w}{6} (1 + z)^2 \left( \frac{G'}{G} \right)^2,$$

(7)

where $G_0 = G(0)$ is the present-day value of the gravitational constant, $g_0 = g(0)$, and the parameters $\hat{\Omega}_M$, $\hat{\Omega}_R$ and $\hat{\Omega}_\Lambda$ are related to the standard ratios

$$\Omega_M \equiv \frac{8 \pi G_0 \rho_0}{3 H_0^2}, \quad \Omega_R \equiv -\frac{k}{a_0^2 H_0^2}, \quad \Omega_\Lambda \equiv \frac{\Lambda}{3 H_0^2}$$

through the following relations

$$\hat{\Omega}_M = \frac{\Omega_M}{g_0} 3 + 2w, \quad \hat{\Omega}_R = \frac{\Omega_R}{g_0}, \quad \hat{\Omega}_\Lambda = \frac{\Omega_\Lambda}{g_0}.$$

(8)

From Eq. (6) it follows that

$$\hat{\Omega}_M + \hat{\Omega}_R + \hat{\Omega}_\Lambda = 1.$$

(9)

We would like to note at this point that a particular case of Eqs. (6)–(9) was discussed in [30].

Finally, the luminosity distance $d_L$ is calculated via the standard formula, which in the flat case has the form

$$d_L = c(1 + z) \int_0^z \frac{du}{H(u)}.$$

(10)

Similar to the way in which it was done in [18], for the calculation of the Hubble diagram of SNeIa we will use the Chandrasekhar mass model for the SNeIa light curve. According to this, the peak luminosities of SNeIa are proportional to the Chandrasekhar mass ($L \propto M_{Ch}$) and, therefore, scale as $L \propto G^{-3/2}$. This result has been validated by detailed numerical calculations of exploding white dwarfs. As a result the apparent magnitude is given by

$$m(z) = M_0 + 5 \log \frac{d_L(z) H_0}{c} + 25 + 15 \log \frac{G(z)}{G_0},$$

(11)

where $M_0$ is the absolute magnitude. We will use Eqs. (10) and (11) to fit the Hubble diagram of distant SNeIa with a certain parametric representation for $G(z)$.

## 3 A phenomenological description of the variation of the gravitational constant

As it has been already commented before, our goal is to obtain an empirical description of the variation of the gravitational constant as inferred from the observational data of SNeIa, including the most recent and reliable datasets [3]. For this purpose we use a simple phenomenological expansion of the function $G(z)$ in powers of $z$:
\[
G(z) = G_0(1 + p_1 z + p_2 z^2 + p_3 z^3 + O(z^4)).
\]  

(12)

Such approximation is in the spirit of phenomenological descriptions of the scale factor, deceleration parameter or equation of state employed previously by other authors [3, 4, 31].

The first coefficient in Eq. (12), \(p_1\), is determined from experimental bounds on the time derivative of the gravitational constant at the present time \((\dot{G}/G)_0 \equiv (\dot{G}/G)_{t=t_{\text{now}}}\). For convenience we translate these bounds in the ones on \(\eta \equiv G'(0)/G_0\) using the relation

\[
\frac{G'(0)}{G_0} = -\frac{1}{H_0} \left(\frac{\dot{G}}{G}\right)_{t=t_{\text{now}}}.
\]

For this estimate we will take the present value of the Hubble constant to be \(H_0 = 63 \text{ km/s/Mpc}\).

There are a number of constraints on \((\dot{G}/G)_0\) obtained from very different observations and methods — see, for instance, [11] and [14] and references therein. For example, the Lunar Laser ranging experiments yield \(|\dot{G}/G|_0 < 8 \times 10^{-12} \text{ yr}^{-1}\) [16, 17], whereas the improved constraints on the post–Newtonian parameters give an upper bound of \(10^{-14} \text{ yr}^{-1}\) [32]. Summarizing, one can see that the parameter \(\eta\) can take values satisfying roughly \(|\eta| \leq 0.01\), which is similar to the estimate obtained in [19]. Actually, as far as \(\eta\) is small enough, the values of the other coefficients depend very weakly on its precise value. Moreover, we have checked that our final result is not sensitive to the value of \(\eta\) within the interval \(-0.01 < \eta < 0.01\). Since \(\eta = p_1\) the bound on the local rate of variation of \(G\) determines the linear term in Eq. (12). For the forthcoming analysis we adopt \(p_1 = \eta = -0.01\).

In what follows we truncate the expansion given in Eq. (12) at a certain order, substitute this polynomial expression into Eq. (6), calculate the apparent magnitudes of SNeIa as given by Eq. (11) in terms of the coefficients \(p_i\) and compare them with the Hubble diagram based on the observational data of SNeIa.

Let us consider the case of the flat and matter dominated universe without the cosmological constant. Namely, we set \(\Omega_R = \Omega_\Lambda = 0\) and \(\gamma = 1\) in the Hubble law, Eq. (6). An important observation is that with such assumptions the value of the ratio \(\Omega_M\) turns out to be fixed by the value of \(p_1\). Indeed, from (7) one gets

\[
\Omega_M = \frac{4 + 2w}{3 + 2w} g_0 = \frac{4 + 2w}{3 + 2w} \left(1 + p_1 - \frac{w}{6}p_2^2\right).
\]  

(13)

The value of the Brans–Dicke parameter \(w\) depends on the specific model. For example, in the case of multidimensional Einstein–Yang–Mills models with \(d\) extra dimensions, discussed in the next section, \(w = (d - 1)/d\) [33]. Having in mind this class of models we take \(w \sim 0.5 \div 1\). The exact value of this parameter does not affect our final result in a significant way. In the case of models obtained by dimensional reduction from multidimensional theories with six extra dimensions, which can be motivated by the string theory, from Eq. (13) one gets \(\Omega_M \approx 1.2\).

Fitting the Hubble law to the supernova dataset with a quadratic polynomial for \(G(z)\) — i.e., with only one free parameter, \(p_2\) — does not give satisfactory results. To get a
Figure 1: The observational Hubble diagram of distant supernovae (dots with their corresponding error bars), the best fit curve to it in the model with the variable gravitational constant $G(z)$ as given by Eq. (12) (solid line).

better phenomenological approximation we consider a cubic polynomial as a parametrization of the function $G(z)$ in Eq. (12). Varying $p_2$ and $p_3$ we obtained that the best fit to the Hubble diagram of SNeIa is achieved when the values of the parameters $p_2$ and $p_3$ in Eq. (12) are $p_2 \approx 0.34$ and $p_3 = -0.17$, respectively.

Fig. 1 shows the observational Hubble diagram of distance moduli for SNeIa based on the data of [3] (their gold sample). Overplotted is the best fit curve for the predicted distance modulus

$$\mu_{th}(z) \equiv 5 \log d_L(z) + 25 + \frac{15}{4} \log \frac{G(z)}{G_0},$$

(14)

calculated in our model with the variable gravitational constant $G(z)$, Eq. (12). For these calculations we used the present Hubble parameter $H_0 = 63 \text{ km/s/Mpc}$. As it can be seen from Fig. 1 the agreement between the theoretical fit and the observations is quite good.

The likelihood for the parameters $p_2$ and $p_3$ is determined from the $\chi^2$-analysis with

$$\chi^2(p_2, p_3) = \sum_i \frac{[\mu_{th}(z_i) - \mu_{obs}^i]^2}{\sigma_{\mu}^2},$$

(15)
Figure 2: Joint confidence intervals for the coefficients \( p_2 \) and \( p_3 \) of the polynomial fit to \( G(z) \) as given in Eq. (12).

where \( \mu_{\text{th}}(z) \) is given by Eq. (14), \( \mu_{\text{obs}}^i \) are the observational data for the distance moduli and \( \sigma_{\mu}^i \) are the uncertainties in the individual distance moduli. Definition (15) of \( \chi^2 \) is analogous to the one in Ref. [3]. We would like to add that the best fit cubic polynomial \( G(z) \) yields \( \chi^2 = 199 \), for comparison the value of \( \chi^2 \) obtained for the cosmic concordance model, which is \( \chi^2 = 178 \).

Fig. 2 shows the joint confidence intervals for the fit to the SNeIa observational data. The analysis was done in the region of the parameters \( p_2, p_3 \) such that \( H^2(z) > 0 \) for all redshifts in the interval \( 0 < z < 1.7 \). The bottom boundary of this region is seen in Fig. 2. To be more precise, for values of these parameters below the boundary the function \( g(z) \) defined by Eq. (7) becomes negative as \( z \) approaches to \( z = 1.7 \). The best fit values of \( p_2, p_3 \) are in fact rather close to this boundary.

The plot of the cubic polynomial for \( G(z) \) with the coefficients found above is shown in Fig. 3. The phenomenological expression for \( G(z) \) based on the SNeIa data suggests that the value of the gravitational constant was larger in the past. To be more precise, \( G(z) > G_0 \) in the interval from \( z \approx 0.03 \) to \( z \approx 1.97 \). It can be seen from Fig. 3 that for \( z > 0.03 \) as one moves towards larger redshifts the function \( G(z) \) first grows, reaches its maximum \( G_{\text{max}} = 1.19 G_0 \) at \( z = 1.32 \) and then steadily decreases. Of course, the phenomenological expression for \( G(z) \) obtained here is approximate and presumably makes sense as far as the last, cubic term in Eq. (12) is smaller than the previous quadratic.
The function $G(z)$ represented by cubic polynomial of Eq. (12) with the coefficients determined from the best fit to the SNeIa Hubble diagram.

The form obtained here for the function $G(z)$ should be contrasted with existing constraints and/or compared with results on the variation of $G$ in various cosmological models. As an illustration, in the following section $G(z)$ will be considered as a phenomenological input in a multidimensional model and a prediction for the variation of the fine structure constant will be derived and analyzed.

4 Variation of $G$ and $\alpha$ in multidimensional models

The time variation of the fundamental constants within models with extra dimensions has been considered in a number of papers [21], [34]–[38], just to mention a few of them. Additionally, in [22] the correlated variations of the fine structure constant and of the gravitational constant were analyzed. In this paper it was shown that in the framework of certain multidimensional models there exists a robust relation between the time derivatives of $\alpha$ and $G$ which quite generically can be written as

$$\frac{\dot{\alpha}}{\alpha} = \beta \frac{\dot{G}}{G},$$

where the factor $\beta$ is model dependent and, in general, may depend on the scale (or size)
\( R \) of the space of extra dimensions. For the case of constant \( \beta \) a similar relation between the derivatives with respect to the redshift holds:

\[
\frac{\alpha'}{\alpha} = \beta \frac{G'}{G}.
\]

Integrating this equation one gets

\[
\frac{\Delta \alpha}{\alpha}(z) \equiv \frac{\alpha(z) - \alpha_0}{\alpha_0} = \left( \frac{G(z)}{G_0} \right)^\beta - 1,
\]

where \( \alpha_0 = \alpha(0) \) is the present day value of the fine structure constant.

To be specific, let us consider the case of multidimensional Einstein–Yang–Mills theories. In this case it can be shown that \( \beta = 1 \). Using the cubic polynomial of Eq. (12), with the coefficients determined in the previous section, the redshift dependence of \( \alpha \) can be obtained and, from it, the behaviour of \( \Delta \alpha/\alpha \) predicted for such theory can be derived. In particular, it can be seen that at \( z = 0.5 \) this ratio is \( \Delta \alpha/\alpha \approx 0.06 \). This theoretical prediction is at odds with the known constraints on the rate of variation of the fine structure constant. More specifically, the latest analysis of a Keck/Hires sample of quasar absorption lines using the many multiplet method gives

\[
\frac{\Delta \alpha}{\alpha} = (-0.54 \pm 0.12) \times 10^{-5}
\]

for \( z \) in the range \( 0.5 < z < 3 \) \cite{25}. Note, moreover, that these authors obtained that the value of \( \Delta \alpha \) is negative, whereas the results derived from the Hubble diagram of SNeIa predict \( \Delta \alpha > 0 \) for the same range of redshifts in the cosmological theories under consideration. Moreover, as already mentioned in Sect. 1, the observational results leading to a non-zero rate of variation of \( \alpha \) have been challenged recently \cite{26,27}. These studies provide much tighter constraints on the rate of variation of the fine structure constant. In particular, the constraint \( \Delta \alpha/\alpha = (-0.06 \pm 0.06) \times 10^{-5} \) was obtained, which is consistent with zero variation. In summary, neither the sign of our theoretical prediction nor its order of magnitude coincide with the above bounds. In fact, a similar conclusion was formulated in \cite{22}.

The constraints studied before correspond to a redshift \( z \sim 0.5 \). The best local \((z \sim 0.0)\) bound on the time variation of \( \alpha \) is that obtained from the Oklo natural nuclear reactor:

\[
\frac{\Delta \alpha}{\alpha} = (0.15 \pm 1.05) \times 10^{-7}
\]

at \( z \approx 0.15 \) \cite{39}. The obtained phenomenological formula yields \( \Delta \alpha/\alpha \approx 0.006 \). Actually, as one can see from Eq. (17), for \( \Delta \alpha/\alpha \) to fit the Oklo constraint at \( z = 0.15 \) the value of the parameter \( \beta \) should be of the order of \(|\beta| \sim 10^{-5}\), which is the same to say that \( \alpha(z) \) must be practically independent of \( G(z) \).

5 Conclusions and discussion

We have studied the possibility of fitting the observational Hubble diagram of SNeIa assuming cosmological models of a flat universe without cosmological constant but with
a varying gravitational “constant”, $G(z)$. The function $G(z)$ was represented by a cubic polynomial, Eq. (12), parametrized with three coefficients, $p_1$, $p_2$ and $p_3$. The linear order coefficient was fixed by the constraints on the present day rate of the time variation of $G$. The other two coefficients were determined from the best fit to the Hubble diagram. Finally we arrived at the following phenomenological expression:

$$G(z) = G_0 \left(1 - 0.01z + 0.34z^2 - 0.17z^3\right), \quad (19)$$

where $G_0 = G(0)$ is the present day value of the gravitational constant. It is important to mention that the sharp boundary at the bottom of the joint confidence intervals in Fig. 2 is due to the restriction that no negative values of $H^2(z)$ occur in the range of redshifts $0 < z < 1.7$. Values of $p_3$ smaller that those of the bottom boundary produce $g(z) < 0$ (see Eq. (6)) for some values of $z$ in this interval and are consequently discarded.

In this paper we limited ourselves to a cubic polynomial for $G(z)$. Such choice of approximation is motivated by the results on two–parametric descriptions of astrophysical characteristics obtained from the same datasets in [3], [4] and [31], which suggest that the available data does not allow a good determination of higher order parameters. The phenomenological determination of $G(z)$ from SNeIa data suggests that the value of the gravitational constant was higher than the present day one for $0.03 < z < 1.97$ with the maximal value $G_{\text{max}} = 1.19 G_0$ reached at $z = 1.32$. This change from the growing to the decreasing behaviour of $G(z)$ is a manifestation of the change in the behaviour of the observational data with the redshift. The latter feature was studied in Ref. [3] in terms of the deceleration $q(z)$ represented by a linear polynomial with two parameters. Using the observational data for distant SNeIa the change of sign of $q(z)$ at $z = 0.46 \pm 0.13$ was discovered. This is interpreted as an indication of the change from the epoch of acceleration of the evolution of the Universe to the epoch of deceleration as $z$ increases.

The features of the approximation $G(z)$ obtained here should definitely be compared, within the domain of its validity, with other cosmological and astrophysical bounds and restrictions. The discrepancy between the theoretical prediction for the variation of the fine structure constant obtained in the models with extra dimensions and the existing observational constraints, provided that the latter are solid and confirmed, indicates that either the multidimensional models considered here are phenomenologically unsatisfactory or the very hypothesis of the variability of $G$ is not correct.

**Acknowledgements**

This work has been partially supported by the Spanish MEC grants AYA2005–08013–C03–C01 and C02, by the AGAUR and by the European Union FEDER funds. The work of Y. K. was supported by the UR.02.03.028 grant of the Programme “Universities of Russia” and grant 04-02-16476 of the Russian Fund for Basic Research.

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