Creep dynamics of a domain wall under an alternating driving field

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Abstract

Recently the thermally activated creep dynamics of a domain wall under an alternating driving field has been a focus of attention in the experiments of ultrathin ferromagnetic and ferroelectric films. With Monte Carlo simulations, we systematically investigate the creep dynamics in a two-dimensional driven random field Ising model at different temperatures, quenched disorders and driving fields. The creep exponent $\beta$, energy barrier exponent $\psi$ and roughness exponent $\zeta$ are determined, compatible with the experiments. According to numerical results, a temperature-independent scaling relation $\beta \sim \zeta/\psi$ is uncovered. In addition, two distinct growth stages of the creep correlation length are identified, one corresponding to the universality class of the random depositions and the other to the universality class of the quenched Edwards-Wilkinson (QEW) equation. For the later case, due to the dynamic effect of overhanging, the domain interface may exhibit intrinsic anomalous scaling behavior with $\zeta > \zeta_{loc} = 2/3$, different from the QEW equation.

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I. INTRODUCTION

In recent years, domain-wall motion has become a source of much experimental and theoretical research [1–8]. The dynamics of domain walls under an alternating driving field has attracted extensive interest in vortex lattices [9, 10], liquid crystals [11], ferromagnetic/ferroelectric materials [12–14] and crystalline solids [15]. In particular, the magnetic domain-wall dynamics is an important topic in nanomaterials, thin films and semiconductors, because of its potential technological applications including magnetic random access memory and logic devices [16–18]. In the experiments of ultrathin ferromagnetic and ferroelectric films, considerable attention is devoted to the complex susceptibility \( \chi = \chi' - i\chi'' \) [19–21], which depicts the motion of domain walls. In the Cole-Cole diagram of \( \chi' \) vs. \( \chi'' \), dynamic phase transitions can be detected between different states of the domain-wall motion, such as relaxation, creep, sliding and switching [2].

At low temperatures and low frequencies, the domain wall exhibits thermally activated creep motion even when the driving field is very weak [4, 5, 16]. At high frequencies, the segmental domain-wall relaxation occurs between local minima of the potential energy landscape but without macroscopic motion [12, 22, 23]. These two states are separated by the so-called relaxation-to-creep dynamic transition, theoretically predicted by Nattermann et al. around a decade ago [24]. Experimental evidences have been found recently not only in ultrathin ferromagnetic Pt/Co(0.5 nm)/Pt trilayer and ferroelectric films KTiOPO₄, Sr₀.₆₁Ba₀.₃₉Nb₂O₆, PbZr₀.₂Ti₀.₈O₃ [20, 21, 25, 26], but also in liquid crystals, ferroelastic materials and molecular ferrimagnets [11, 22, 27]. Very recently, an intermediate state between the relaxation and creep states has been revealed. It indicates that the transition occurs in two stages, somewhat similar to the scenario of the two-dimensional melting [28]. To our best knowledge, little attention has been paid to the dynamics of the relaxation-to-creep transition, especially for the creep dynamics under an alternating field.

In the creep regime, one observes an inverse power-law behavior for the complex susceptibility \( \chi(f) = \chi_\infty [1 + (i2\pi f \tau)^{-\beta}] \) [2]. Here \( \chi_\infty \) denotes the bulk background susceptibility at the frequency \( f \to \infty \), \( \tau \) is the characteristic relaxation time, and \( \beta \) is the creep exponent. Experimentally, \( 0.2 < \beta < 0.65 \) is measured in ultrathin ferromagnetic and ferroelectric films [20, 21, 26]. The deviation from the ideal conductivity \( \beta = 1 \) is attributed to the nonlinear dependence of the creep velocity on the driving field [4]. According to scaling
arguments, $\beta = (2 - 2\zeta)/z \approx 0.5$ is expected with the roughness exponent $\zeta \approx 2/3$ and dynamic exponent $z = 2(1 - \varepsilon/9) \approx 1.33^{21}$, but incompatible with experimental results. Hence, it remains very challenging to understand the creep exponent $\beta$ in experiments.

Up to date, theoretical tools describing domain-wall motions are typically based on the Edwards-Wilkinson equation with quenched disorder (QEW) [6, 29–33]. With this equation, the creep dynamics under an constant field or zero field has been well understood [34–36]. It can be viewed as a thermally activated hopping movement from one local energy minimum to the next, dominated by the energy barrier $U_B$ that must be overcome. The energy barrier is expected to grow as a power law $U_B(\xi) \sim \xi^\psi$, responsible for the logarithmical growth of the correlated length $\xi(t) \sim (\ln t)^{1/\psi}$ [37, 38]. An effective energy barrier exponent $\psi \approx 0.49$ of the QEW equation has been numerically measured, rather different from the exactly known droplet exponent $1/3$ for the low-energy excitations above the ground state [39, 40]. The roughness exponent $\zeta \approx 2/3$ has also been measured, consistent with the one in equilibrium [34, 35]. However, few works deal with the creep dynamics under an alternating field, and detailed microscopic structures and interactions of real materials are not concerned in the phenomenological QEW equation [29].

To further understand the creep dynamics from a more fundamental viewpoint, we should build lattice models allowing a closer comparison between theory and experiment. The driven random-field Ising model (DRFIM) is a candidate, which has been used as a paradigmatic model to understand the dynamic transitions in ferroic systems [28, 41–44]. Very recently, the creep motion of a domain wall driven by a constant field has been studied in the DRFIM model [45], and a non-linear field-velocity relation has been revealed, comparable with experiments [4, 5, 16].

Since the creep dynamics is important in obtaining full understanding of the relaxation-to-creep transition, we conduct a comprehensive study on the creep dynamics under an alternating driving field in a two-dimensional (2D) DRFIM model. With Monte Carlo simulations, we accurately determine the scaling exponents $\beta, \psi$ and $\zeta$, and identify the universality classes, in comparison with those of the QEW equation and experiments. In Sec. II, the model and scaling analysis are described, and in Sec. III, the numerical results are presented. Finally, Sec. IV is devoted to the conclusions.
II. MODEL AND SCALING ANALYSIS

The DRFIM model is defined by the following Hamiltonian

\[ \mathcal{H} = -J \sum_{\langle ij \rangle} S_i S_j - \sum_i [h_i + H(t)] S_i, \]  

(1)

where \( S_i = \pm 1 \) is a Ising spin at site \( i \) of the lattice, \( \langle i, j \rangle \) denotes a nearest-neighbor pair of spins, and \( h_i \) is a quenched random field uniformly distributed within an interval \([-\Delta, \Delta]\). For convenience, we use a homogenous alternating driving field \( H(t) = H_0 \exp(i2\pi ft) \), and set the coupling constant \( J = 1 \) \([28]\). In order to make sure that the dynamic evolution of spins occurs at or around the domain wall, we restrict the temperature \( T \leq 0.66 \), the disorder strength \( \Delta \leq 2.0 \) and the alternating field \( H_0 \leq 0.5 \). Simulations are performed on a rectangle lattice \( L_x \times L_y \) with the antiperiodic and periodic boundary conditions along the \( x \) and \( y \) directions, respectively.

The initial state is a semiordered state with a perfect domain wall in the \( y \) direction. To eliminate the pinning effect irrelevant for the disorder, we rotate the lattice such that the initial domain wall orients in the (11) direction \([42, 43]\). After preparing the initial state, we update spins with the heat-bath algorithm \([46]\). As time evolves, the domain wall moves and roughens, while the bulk remains unchanged. Therefore, the domain wall can also be called as a domain interface \([47, 48]\). Main results of numerical simulations are presented with the lattice size \( L_x = 25 \) and \( L_y = 512 \), up to \( t_{max} = 400 \, 000 \) Monte Carlo step (MCS). Here MCS is defined by \( L_x \times L_y \) single-spin-flips attempts. For each set of model parameters \((T, \Delta, H_0, f)\), more than 10 000 samples are performed for average. Errors are estimated by dividing the samples into three or four subgroups. If the fluctuation of the curve in the time direction is comparable with or larger than the statistical error, it will also be taken into account. Additional simulations of \( L_x = 50 \) are performed to confirm that the finite-size effect is negligible.

Denoting a spin at site \((x, y)\) by \( S_{xy}(t) \), we first introduce the height function

\[ h(y, t) = \sum_{x=1}^{L_x} S_{xy}(t), \]  

(2)

and then define the position of the domain interface

\[ h(t) = \frac{1}{2} \langle h(y, t) \rangle + L_x. \]  

(3)
Here $\langle \cdots \rangle$ represents not only the statistic average over Monte Carlo samples but also the average in the $y$ direction. After the magnetic hysteresis loop is stable, the complex susceptibility can be calculated \[28, 29\],
\[
\chi(f, T) = \frac{1}{PH_0} \int_0^P h(t)e^{-i2\pi ft} dt,
\]
where $P = 1/f$ is the time period of the alternating driving field.

With the height function $h(y,t)$ at hand, the roughness function $\omega^2(t)$ and the correlation function $C(r, t)$ are derived,
\[
\omega^2(t) = \langle h(y,t)^2 \rangle - \langle h(y,t) \rangle^2,
\]
\[
C(r, t) = \langle h(y,t)h(y+r,t) \rangle - \langle h(y,t) \rangle^2.
\]
They describe the roughening of the domain interface in the $x$ direction and the growing of the spatial correlation in the $y$ direction, respectively. To reveal the characteristics of the creep dynamics, we introduce the creep susceptibility
\[
D\chi'(f, T) = \chi'(f, T) - \chi'(f, T = 0),
\]
and the pure roughness function
\[
D\omega^2(t) = \omega^2(t) - \omega^2(t, T = 0).
\]
To detect the overhangs generated in the creep motion, another two definitions of the height functions, $h^+(t)$ and $h^-(t)$, are introduced by the envelops of the positive and negative spins, respectively \[43\]. It is believed that the difference $Dh(t) = h^+(t) - h^-(t)$ is related to the average size of overhangs.

According to the phenomenological scaling arguments \[49\], a power-law dispersion of the creep susceptibility is obtained for the creep dynamics under an alternating driving field,
\[
D\chi'(f) \sim (1/f)^\beta.
\]
For the $\xi$-length domain-wall segments, a certain hopping time $t \sim \exp(U_B(\xi)/T)$ is required to overcome the energy barrier \[39\]. Assuming that the energy barrier scales as $U_B(\xi) \sim \xi^{\psi}$, one may deduce
\[
D\xi(t) \sim [T \ln(t)]^{1/\psi}.
\]
Here $D\xi(t) = \xi(t) - \xi(t, T = 0)$ is the so-called creep correlation length, and $\psi$ is the energy barrier exponent.

For a sufficiently large lattice $L \gg \xi(t)$, the dynamic behavior of $\xi(t)$ can be extracted from the standard scaling form of the correlation function [30, 50],

$$C(r, t) = \omega^2(t)\tilde{C}(r/\xi(t)),$$  \hspace{1cm} (10)

where $\tilde{C}(s)$ is the scaling function with $s = r/\xi(t)$, and $\omega^2(t)$ is the roughness function defined in Eq. (5). In the kinetic roughening of the domain interface, a power-law scaling behavior of the pure roughness function is expected with the roughness exponent $\zeta$,

$$D\omega^2(t) \sim [D\xi(t)]^{2\zeta}.$$  \hspace{1cm} (11)

Alternatively, one may determine the local roughness exponent $\zeta_{loc}$ by fitting $C(r, t)$ with an empirical scaling form [51],

$$C(r, t) = A [\tanh (r/\xi(t))]^{2\zeta_{loc}}.$$  \hspace{1cm} (12)

III. MONTE CARLO SIMULATIONS

A. Numerical results

In Fig. 1, the spectra of the creep susceptibility $D\chi'(f)$ defined in Eq. (6) are plotted on a log-log scale for different temperatures $T$ at the strength of the disorder $\Delta = 1.5$ and driving field $H_0 = 0.01$. In order to obtain stationary results, the waiting time $t_0 = 20$ periods is used to calculate $\chi(f, T)$. Power-law behaviors are observed but with certain corrections to scaling. Direct measurements from the slopes give the exponents $\beta = 0.23(1), 0.23(1), 0.26(1), 0.38(1)$ and $0.55(2)$ for $T = 0.025, 0.05, 0.1, 0.2$ and $0.33$, respectively. The power-law correction $y = a x^\beta(1 - c/x)$ extends the fitting to the early times, and yields the same results.

Taking the set of model parameters ( $T = 0.33, \Delta = 1.5, H_0 = 0.01$ and $f = 10^{-4}$ Hz ) as an example, the correlation function $C(r, t)$ is displayed as a function of $r$ in Fig. 2(a). According to Eq. (12), a perfect fitting to the numerical data is performed with the dash-dotted line, and the local roughness exponent $\zeta_{loc} = 0.65(1)$ is measured. Based on the scaling form of $C(r, t)$ in Eq. (10), data of different time $t$ nicely collapse to the curve of $t' =$
400 000 MCS by rescaling $r$ of another $t$ to $[\xi(t')/\xi(t)] r$ and $C(r, t)$ to $[\omega^2(t')/\omega^2(t)] C(r, t)$. With this data collapse technique, we extract the nonequilibrium correlation length $\xi(t)$ from the correlation function $C(r, t)$. In the inset, the dynamic evolution of $\xi(t)$ is displayed on a log-log scale. Significantly deviation from the power-law behavior indicates that the correlation length $\xi(t)$ at $T = 0.33$ does not grow as $\xi(t) \sim t^{1/z}$ [39, 52]. Additionally, a time-independent correlation length $\xi = L_c$ is observed at $T = 0$ but irrelevant to the creep dynamics. Therefore, we introduce the creep correlation length $D\xi(t)$ by subtracting the relaxation length $L_c$, and use a dimensionless variable $D\xi(t)/L_c$ in the following.

In Fig. 2(b), the creep correlation $D\xi(t)/L_c$ is displayed as a function of $\ln t$ at different $T$ on a log-log scale. Power-law behaviors are observed, and the effective energy barrier exponents $1/\psi = 0.98(1), 0.98(1), 1.16(1), 1.73(2), 1.84(2)$ and $1.75(3)$ are estimated from the slopes of the curves for $T = 0.025, 0.05, 0.1, 0.2, 0.33$ and $0.66$, respectively. To confirm the scaling form in Eq. (9), $D\xi(t)/L_c$ vs. $T \ln t$ is plotted in Fig. 3(a) on a double-log scale. Data of different $T$ nicely collapse to a master curve, and two distinct scaling regimes are found with the slopes $1/\psi = 1.00(1)$ and $1.80(2)$, respectively. Between them, a dynamic crossover occurs at $T \ln t \approx 1$ and $D\xi(t) \approx 0.5L_c$. Then we extract the characteristic of the energy barrier

$$U_B \sim T \ln t \sim \begin{cases} 2D\xi/L_c, & \text{when } D\xi(t) \ll 0.5L_c, \\ 1.5 (D\xi/L_c)^{0.56}, & \text{when } D\xi(t) \gg 0.5L_c, \end{cases}$$ (13)

in the small-$D\xi$ and large-$D\xi$ scaling regimes, respectively.

With the creep correlation length $D\xi(t)/L_c$ at hand, we measure the roughness exponent $\zeta$ according to Eq. (11). In Fig. 3(b), $D\omega^2(t)$ is displayed against $D\xi(t)/L_c$ on a log-log scale. Similarly, data collapse of different $T$ is performed with different symbols. In the small-$D\xi$ regime, the slope of the master curve yields the roughness exponent $\zeta = 0.53(1)$, close to $1/2$. It indicates that the domain interface belongs to the universality class of the random depictions [53, 56]. In the large-$D\xi$ regime, $\zeta = 0.68(1)$ is estimated, in good agreement with $\zeta = 2/3$ of the QEW equation [35, 37, 39, 57]. Hence, it belongs to the universality class of the QEW equation. In the inset, however, a noticeable deviation of $\zeta$ is observed at a higher temperature $T = 0.66$, and the asymptotic value $\zeta = 1.00(2)$ is measured.

To explain the unexpectedly large roughness exponent $\zeta = 1.00(2)$, we examine the existence of the overhangs in the creep motion [42, 43]. As shown in Fig. 4, black and red lines represent the time evolutions of the height functions $h^+(t)$ and $h^-(t)$, respectively. The
coincidence and noncoincidence of these two curves in the upper and lower panels suggest that the contribution of overhangs is negligible at $T = 0.2$ but important at $T = 0.66$. Besides, the snapshots of the domain walls at the time $t = 4 \times 10^5$ MCS are also shown in the insets. The overhangs can be observed directly in the lower panel while not in the upper panel. Consequently, it is convincing that the overhangs affect the dynamic evolution of the spin configuration and play an essential role in the deviation of the roughness exponent from $\zeta = 2/3$.

Besides the temperature, the effects of the quenched disorder and driving field are also investigated in this paper. In Fig. 5(a), the creep correlation length $D\xi(t)$ is displayed as a function of $\Delta^{-\delta} \ln t$ at $T = 0.33$ on a log-log scale. Taking $\delta = 0.58(1)$ as input, data collapse of different disorders $\Delta = 0.5, 0.7, 1.0$ and $1.5$ is demonstrated, and $1/\psi = 1.85(2)$ is measured, close to $1.80(2)$ in Fig. 3(a). With the scaling relation $\delta \approx \psi \approx 0.56$, one may derive the scaling form of $U_B$ in the large-$D\xi$ regime,

$$U_B \sim T \ln t \sim (D\xi \Delta)^{\psi}. \quad (14)$$

In the inset, the scaling function $\Delta^{-\varepsilon} D\omega^2(t)$ is plotted as a function of $D\xi(t)$ on a log-log scale. Data of different $\Delta$ nicely collapse together with the parameter $\varepsilon = 0.22(1)$ as input. The abnormal increase of the roughness exponent together with the parameter $\varepsilon = 0.22(1)$ as input.

The abnormal increase of the roughness exponent from $\zeta = 0.68(1)$ to $0.78(1)$ is also induced by the dynamic effect of the overhangs.

In Fig. 5(b), the creep dynamics of the domain wall for different frequencies $f$ is presented at the driving field $H_0 = 0.1$ on a log-log scale. If the frequency is sufficiently low, e.g., $f = 10^{-4}$ Hz, a power-law behavior of $\xi(t)$ can be observed with the exponent $1/\psi = 1.90(2)$, somewhat larger than the one $1.80(2)$ at $H_0 = 0.01$. Additional simulations at $\Delta = 0.05, 0.2$ and $0.5$ show that the energy barrier exponent $\psi$ is $H_0$-dependent, and data of different $H_0$ are unlikely to collapse together. For a higher frequency, e.g., $f = 1$ Hz, $D\xi(t)$ drops at the tail of the curve. It means that the spatial correlation length in equilibrium is finite and frequency-dependent, consistent with the arguments in Ref.[21].

B. Discussion

The measurements of scaling exponents at different $T$ are summarized in Table II at $H_0 = 0.01$ and $\Delta = 1.5$. As $T$ increases, the creep exponent $\beta$ changes from $0.23(1)$ to $0.55(2)$,
compatible with experimental results in the ferromagnetic and ferroelectric films [26], e.g., 
$\beta = 0.6(1)$ in the ultrathin Pt/Co(0.5nm)/Pt trilayer and $\beta = 0.35(2)$ in the periodically
poled KTiOPO$_4$ [20, 21]. The exponent $\beta = 0.52(2)$ measured at the highest temperature
$T = 0.66$ is also consistence with the prediction of the scaling relation $\beta = (2 - 2\zeta)/z \approx 0.5$
[21]. According to the general scaling arguments in Ref.58, the complex susceptibility
$D\chi(f) \sim \ln(1/f)^{\zeta/\psi} \sim (1/f)^{k\zeta/\psi}$ is deduced with an effective coefficient $k$. Then a novel
scaling relation $\beta = k\zeta/\psi$ is proposed for the creep dynamics of a domain wall under an
alternating driving field. As shown in Table. I, $k \approx \beta\psi/\zeta_{loc} = 0.45(3)$ holds quite well for
the whole temperature range. With this scaling relation, one can predict the creep exponent
$\beta$ by only measuring $\psi$ and $\zeta$ at a certain frequency.

Two distinct growth stages of the creep correlation length $D\xi(t)$ are found with the
scaling exponents $1/\psi = 1.00(1), \zeta = 0.53(1)$ in the small-$D\xi$ scaling regime and $1/\psi =
1.80(2), \zeta = 0.68(1)$ in the large-$D\xi$ scaling regime. The exponents indicate that the former
belongs to the universality class of the random depositions, while the later belongs to the
universality class of the QEW equation. They are separated by the so-called Larkin length
$L_p$ at which the effects of the quenched disorder and domain-wall elasticity are of the same
order [12, 49, 59, 60]. According to Eq. (13), $L_p \approx 0.5L_c$ can be estimated for the creep
dynamics. Now let us recall the growth process of the creep correlation length. At the
beginning, the elasticity wins over the disorder. The kinetic roughening of domain interface
is dominated by thermal fluctuations with $\zeta_T = 1/2$ [31, 34], and the energy barrier is
linear with $D\xi(t)$ based on the perturbation analysis [33, 60], the same as our numerical
results in the small-$D\xi$ regime. After $D\xi(t)$ reaches $L_p$, the quenched disorder overcomes
the elasticity and becomes dominant. Then the domain-wall motion can be described by the
QEW equation with the nontrivial exponents $\psi = 1/2$ and $\zeta_{eq} = 2/3$ [39, 57], compatible
with the ones in the large-$D\xi$ regime.

In Table. I, the effects of $\Delta$, $H_0$ and $f$ are uncovered in the large-$D\xi$ regime with the
fixed sets of model parameters $(T = 0.33, H_0 = 0.01, f = 10^{-4}$Hz), $(T = 0.33, \Delta = 1.5, f =
10^{-4}$Hz) and $(T = 0.33, \Delta = 1.5, H_0 = 0.1)$, respectively. For moderate disorder, i.e.,
$0.5 \leq \Delta \leq 1.5$, a robust value $\psi = 0.54(1)$ is determined, close to $\psi = 1/2$ of the QEW
equation. According to Eq. (14), the hopping time $t \sim \exp\left(\left[D\xi/L_p\right]^{\psi/\zeta}/T\right)$ is derived with
$L_p \sim 1/\Delta$ [12, 37], consistent with Refs.39, 61. The factor $\Delta^{\psi/T}$ shows that the hopping
process is determined by the competition between the quenched disorder and thermal noise
Since $1/\psi$ increases monotonically with $H_0$ and $D\xi(t)$ drops at the tail of curve, further studies are needed to derive the functional form of the hopping time on $H_0$ and $f$.

For the kinetic roughening of the domain interface, a robust value $\zeta = 0.68(1)$ is determined in the large-$D\xi$ scaling regime at different $T$, $\Delta$, $H_0$ and $f$. The scaling relation $\zeta = \zeta_{loc} < 1$ indicates that the domain interfaces belongs to the Family-Vicsek universality class \[62\]. When $D\xi(t)$ exceeds a certain threshold, however, $\zeta$ differs from $\zeta_{loc}$ by more than 20 percent not only at $T = 0.66$, but also at $\Delta = 0.1, 0.5$, $H_0 = 0.2, 0.5$ and $f = 10^{-1}, 1$ Hz. It suggests that the domain interface is no longer single-valued and one-dimensional. As a consequence, the domain interface belongs to a new universality class with intrinsic anomalous scaling and spatial multiscaling \[42, 62\].

IV. CONCLUSION

With Monte Carlo simulations, we have explored the dynamics of the relaxation-to-creep transition in the experiments of the ultrathin ferromagnetic and ferroelectric films. Since the QEW equation is phenomenological and contains little microscopic information, lattice models based on microscopic structures and interactions are considered. Taking the 2D DRFIM model as an example, we have systematically investigated the creep dynamics of a domain wall under an alternating driving field, and identified the universality classes, in comparison with those of the QEW equation and experiments. Scaling exponents $\beta, \psi$ and $\zeta$, which characterize the complex susceptibility $\chi(f)$, the spatial correlation length $\xi(t)$ and the roughness function $\omega^2(t)$, respectively, are extracted, and the results are summarized in Table. II and III.

(i) As the temperature increases, the creep exponent $\beta$ changes from $0.23(1)$ to $0.55(2)$, compatible with the experiments \[20, 21, 26\]. The scaling relation $\beta = k\zeta/\psi$ is then observed with a temperature-independent coefficient $k = 0.45(3)$.

(ii) Two distinct growth stages of the creep correlation length $D\xi(t)$ are identified. The small-$D\xi$ one corresponds to the universality class of the random depositions with $\psi = 1$ and $\zeta = 1/2$, while the large-$D\xi$ one corresponds to the universality class of the QEW equation with $\psi = 1/2$ and $\zeta = 2/3$.

(iii) In the large-$D\xi$ regime, due to the dynamic effect of overhangs, the roughness exponent significantly deviates from $\zeta = 2/3$ of the QEW equation, but comparable with
experimental measurements [63, 64]. The relation $\zeta > \zeta_{\text{loc}} = 2/3$ indicates that the domain interface belongs to a new universality class with intrinsic anomalous scaling.

(iv) The hopping time $t \sim \exp \left( [D\xi \Delta]^\beta / T \right)$ is identified as a function of the temperature $T$ and quenched disorder $\Delta$. Besides, the effects of the driving field $H_0$ and frequency $f$ are also uncovered, more complicated than the ones of $T$ and $\Delta$. It is a challenge to derive the deterministic functional form of the hopping time on $H_0$ and $f$.

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TABLE I: Scaling exponents of the creep dynamics at different temperatures \( T \). The scaling relation \( \psi\beta/\zeta_{\text{loc}} = 0.45(3) \) holds quite well within error bars. The roughness exponents \( \zeta = 0.53(1) \) and 0.68(1) are measured in the small-\( D_\xi \) regime (centered columns) and large-\( D_\xi \) regime (right columns), respectively, in good agreement with the local ones \( \zeta_{\text{loc}} \).

| \( T \)  | 0.025 | 0.05 | 0.1   | 0.2   | 0.33  | 0.66  |
|---------|-------|------|-------|-------|-------|-------|
| \( \beta \) | 0.23(1) | 0.23(1) | 0.26(1) | 0.38(1) | 0.55(2) | 0.52(2) |
| \( 1/\psi \) | 0.98(1) | 0.98(1) | 1.16(1) | 1.73(2) | 1.84(2) | 1.75(3) |
| \( \zeta_{\text{loc}} \) | 0.50(1) | 0.51(1) | 0.50(1) | 0.51(1) | 0.65(1) | 0.69(1) |
| \( \psi\beta/\zeta_{\text{loc}} \) | 0.47(3) | 0.46(3) | 0.45(3) | 0.43(3) | 0.46(3) | 0.43(3) |
| \( \zeta \) | 0.53(1) | | | | | 0.68(1) |

TABLE II: Scaling exponents in the large-\( D_\xi \) regime at different disorders \( \Delta \), driving fields \( H_0 \) and frequencies \( f \). As the creep correlation length \( D_\xi(t) \) grows, significant deviation of the roughness exponent \( \zeta \) from \( \zeta_{\text{loc}} = 2/3 \) is observed in the right columns.

| \( \Delta \) | 1.5   | 1.0   | 0.7   | 0.5   | 0.1   |
|--------------|-------|-------|-------|-------|-------|
| \( 1/\psi \) | 1.84(1) | 1.89(2) | 1.82(2) | 1.85(3) | 3.07(3) |
| \( \zeta_{\text{loc}} \) | 0.65(1) | 0.66(1) | 0.66(1) | 0.67(1) | 0.66(1) |
| \( \zeta \) | \( \zeta = 0.68(1) \) | \( \zeta = 0.78(1) \) |
| \( H_0 \) | 0.01   | 0.05   | 0.1   | 0.2   | 0.5   |
| \( 1/\psi \) | 1.84(2) | 1.86(2) | 1.90(2) | 2.11(2) | 2.62(2) |
| \( \zeta_{\text{loc}} \) | 0.65(1) | 0.65(1) | 0.66(1) | 0.68(1) | 0.66(1) |
| \( \zeta \) | \( \zeta = 0.68(1) \) | \( \zeta = 0.94(2) \) |
| \( f \) (Hz) | \( 10^{-4} \) | \( 10^{-3} \) | \( 10^{-2} \) | \( 10^{-1} \) | \( 10^{0} \) |
| \( 1/\psi \) | 1.90(2) | 1.90(3) | 1.88(3) | 1.87(3) | 1.84(3) |
| \( \zeta_{\text{loc}} \) | 0.66(1) | 0.66(1) | 0.65(1) | 0.65(1) | 0.65(1) |
| \( \zeta \) | \( \zeta = 0.69(1) \) | \( \zeta = 0.84(1) \) |
FIG. 1: The spectra of creep susceptibility $D\chi'(f)$ are displayed on a log-log scale, for different temperatures $T$ at the strength of the disorder $\Delta = 1.5$ and driving field $H_0 = 0.01$. Dashed lines represent power-law fits, and solid lines include corrections $y = ax^\beta(1 - c/x)$. 
FIG. 2: (a) The correlation function $C(r, t)$ at different times $t$ are plotted with solid lines. Symbols show data collapse, and the dash-dotted line represents a fitting according to Eq. (12). In the inset, the rescaled spatial correlation length $\xi(t)/L_c$ is plotted for $T = 0$ and 0.33 on a log-log scale. (b) The creep correlation length $D\xi(t) = \xi(t) - \xi_{T=0}$ rescaled by $L_c$ is displayed as a function of $\ln t$.

FIG. 3: (a) The creep correlation length $D\xi(t)/L_c$ vs. $T\ln t$ and (b) the pure roughness function $D\omega^2(t)$ vs. $D\xi(t)/L_c$ are plotted at different $T$. In the inset, the asymptotic behavior of $D\omega^2(t)$ is shown at $T = 0.66$. In both (a) and (b), data collapse is demonstrated. Dashed lines represent power-law fits, and vertical dotted lines indicate a crossover at $D\xi(t) \approx 0.5L_c$. 
FIG. 4: (Color on-line) Time evolutions of the height functions $h^+(t)$ and $h^-(t)$ are plotted with black and red lines, respectively. In the insets, the snapshots of the domain interfaces at the time $t = 4 \times 10^5$ MCS are displayed for $T = 0.2$ and 0.66.

FIG. 5: (a) The creep correlation length $D\xi(t)/L_c$ is displayed against $\Delta^{-\delta}\ln t$ with $\delta = 0.58(1)$ as input, for different strengths of the disorder $\Delta$ on a log-log scale. In the inset, $\Delta^{-\epsilon}D\omega^2(t)$ is plotted with $\epsilon = 0.22(1)$ as input. The vertical dotted line indicates a crossover. (b) Log-log plot of $D\xi(t)/L_c$ is shown as a function of $\ln t$ for different frequencies $f = 10^{-4}$ and $10^0$ Hz at the driving field $H_0 = 0.1$. In both (a) and (b), dashed lines represent power-law fits.