Modeling of interaction of long-wave disturbances with a shock wave on a flat plate with allowance for real gas effects

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Abstract. The problem of the long-wave disturbance interaction with a shock wave on a wedge at an angle of attack is considered. Numerical simulations are performed by solving two-dimensional unsteady Navier–Stokes equations with the module addition created by the user and allowing taking into account the vibrational nonequilibrium of carbon dioxide molecules within the two-temperature model of relaxation flows. A parametric study on the effect of Mach numbers, Reynolds numbers and angles of attack on the disturbance transformation coefficients is carried out for the following gases: equilibrium air, equilibrium vibrationally excited CO₂, and nonequilibrium CO₂. The calculation results on the pressure pulsations on the model surface were compared with the data obtained for long-wave disturbance from the general solution of the inviscid problem of the disturbance interaction with a shock wave on a wedge [1]. It is shown that the obtained long wavelength disturbance transformation coefficients are practically the same as for the inviscid case in the studied range of Mach numbers (M∞=4.5÷8.9).

1. Introduction
The receptivity to the incoming flow disturbances plays an important role in instability evolution of boundary layers. Recently there has been an interest in studying the processes of disturbance evolution, stability and laminar-turbulent transition in the boundary layers of models in hot-short high-enthalpy wind tunnels (such as the pulsed wind tunnel IT-302M ITAM SB RAS). Such wind tunnels can reproduce a wide range of stagnation parameters, including full-scale stagnation temperatures. However, the disturbance characteristics and type (mode composition) for the flow in such tubes require additional investigations. It is required to determine the transformation coefficients of external perturbations to the pulsations on the model surface located in the wind tunnel for the developing methods and devices for separating the total pulsation field of an external supersonic flow into various modes (vortex, entropy, and acoustic). The transformation coefficients can be obtained from the problem solution of the disturbance interaction with an oblique shock wave, arising at supersonic gas flow around bodies.

The problem of the interaction of perturbations with an oblique shock wave was analytically solved within an inviscid formulation in [1]. The general analytic solution for disturbances was defined in the region between the shock wave and the wedge surface and is given in the form of the infinite set of terms that significantly complicates analysis of this solution for specific applied problems. Numerical simulation of the short-wave disturbance interaction problem (λ<<L, where λ is the wavelength and L is the distance from the leading model edge along its surface) with shock wave on a wedge in a viscous formulation is presented in [2]. It was shown in [2] that there is the range of the propagation angles of the external disturbances, where pressure pulsations on the wedge surface are not observed at all. This corresponds to transformation coefficients equal to zero. It is required that all transformation
coefficients are non-zero for the mode decomposition problem. This is observed in the case of long-wave perturbations that allows determination of the disturbance type and amplitude in the entire propagation angle range, i.e. solution of the problem of mode decomposition.

The transformation coefficients of the external long-wave disturbances in flows without taking into account the real gas properties were obtained in [3] for the first time. These transformation coefficients have been presented in the form of analytical dependences on the angle of the shock wave inclination, propagation angle of perturbations, and Mach number of the incoming flow. Under the conditions of a real flight at the high flow stagnation temperatures, the real gas properties (in particular, vibrational relaxation) have a significant influence on the flow characteristics and stability. Carbon dioxide (CO$_2$) is an ideal object for modeling physical and chemical processes in high-temperature hypersonic flows, since it is a gas with relatively low excitation temperature of the vibrational molecules freedom degrees (~100K). Therefore, in this paper the problem of the long-wave disturbance evolution is considered on a plate at an angle of attack in the flow of vibrational excited CO$_2$ for the experiment conditions in the wind tunnel IT-302M ITAM SB RAS.

2. Numerical Simulation

The numerical simulation was carried out with the aid of the ANSYS Fluent package using a density-based solver, explicit difference scheme of the second-order accuracy in spatial variables with the Roe-FDS method for splitting convective flows and explicit Runge–Kutta method in time.

2.1. The computational domain and boundary conditions

The calculated domain is a rectangle, whose part of the lower side coincides with the surface of the plate, where the no-slip condition and constant surface temperature (300K) are set. The conditions of the incoming flow (pressure $- p_{i\infty}$, Mach number $- M_{i\infty}$, temperature $- T_{i\infty}$, vibrational energy of carbon dioxide molecules $- e_{v\infty}$) are established on the left and upper bounds. The expiration condition is set on the right and on the lower boundaries, which are not occupied by the plate surface. The calculated domain height is chosen from the condition that the bow shock does not interact with the upper boundary. The calculated grid consists of rectangular cells of 0.2 mm × 0.1 mm in the amount of 118000 for a plate with a length of 0.1 m. The calculated area and the regular computational grid are constructed by using the ANSYS Meshing programm.

2.2. Problem formulation

The system of two-dimensional nonstationary Navier-Stokes equations (1-3), which is supplemented by the equation of state of the ideal gas (4), is solved in this paper. The viscosity is specified according to the Sutherland law, thermal conductivity is specified by a formula from the kinetic theory, and heat capacity is defined as a function of temperature within the thermally perfect gas model.

\begin{equation}
\frac{\partial \rho}{\partial t} + \nabla (\rho \vec{u}) = 0
\end{equation}

\begin{equation}
\frac{\partial}{\partial t} (\rho \vec{u}) + \nabla (\rho \vec{u} \vec{u}) = -\nabla p + \nabla \tau_{\mu}
\end{equation}

\begin{equation}
\frac{\partial}{\partial t} (\rho e_I) + \nabla [\vec{u} (\rho e_I + p)] = (\lambda \nabla T_{tr} + \tau_{\mu} \vec{u}) + \nabla \tau_{\mu} - \sum_{n=1}^{4} q^n_{tv}
\end{equation}

\begin{equation}
p = \rho R T_{tr}
\end{equation}

\begin{equation}
\frac{\partial}{\partial t} (\rho e^n_v) + \nabla (\vec{u} \rho e^n_v) = q^n_{iv}, \quad n = 1, ..., 4
\end{equation}

here $\rho$, $p$, $\lambda$ are density, pressure, thermal conductivity; $\vec{u}$ is the velocity vector; $T_{tr}$ is temperature of translational-rotational freedom degrees; $n$ is the number of the mode of the vibrational freedom degree; $\tau_{\mu}$ is the viscous stress tensor; $e_I$ is the energy of gas; $R$ is the universal gas constant.
Four conservation equations of the vibrational energy (5) for each vibrational freedom degree of CO₂ molecules are added to the Navier-Stokes equations in order to take into account the real properties (vibrational relaxation) of CO₂ molecules. The energy exchange between the vibrational and translational-rotational CO₂ molecules freedom degrees with a finite relaxation time [4] is taken into account by adding the source terms calculated by the Landau-Teller equation. At low Reynolds numbers \( \text{Re} \approx 4 \times 10^3 \), the rarefaction parameter has a value greater than 0.1, therefore it is required setting the slip conditions and temperature jump on the model surface. Acoustic perturbations are simulated by specifying superposition of the steady flow and plane monochromatic acoustic wave, as described in [3].

3. Results

The transformation coefficients of the incoming flow long-wave disturbances are determined behind the shock wave in a wide range of angles of attack \( (\alpha = 5 \text{°} - 20\text{°}) \), Reynolds numbers \( \text{Re}_L \), calculated over the plate length \( \text{Re}_L = 4 \times 10^3 \), and Mach numbers \( (M_c = 4.5 - 8.9) \). The acoustic waves amplitude of the fast mode is \( A = 0.03 p_\infty \), frequencies are \( f = 10 - 1000 \text{ kHz} \), propagation angle is equal to the angle of attack \( \theta = \alpha \). The perturbations transformation coefficients are determined for the values of parameter \( x/\lambda = 0.646 \) as the ratio of root-mean-square pressure pulsation amplitudes on the plate surface to the root-mean-square pressure amplitude of the incoming flow perturbation \( g_A = p'_w / p'_\infty \). The long-wave disturbance transformation coefficients of the flow in pressure pulsations on the plate surface are obtained by decomposition of the small parameter in general solution of the inviscid problem of the disturbance interaction with a shock wave in a wedge [1]. We compared these disturbance transformation coefficients and interaction problem calculations of the long-wave perturbations with a shock wave on the plate at an angle of attack flowed around by CO₂.

3.1. Mach number effect

The Mach number effects on disturbance transformation coefficients for equilibrium air and nonequilibrium vibrationally excited carbon dioxide are calculated parametrically in the paper. In these calculations, a change in the Mach number is accompanied by a change in the temperature of the incoming stream (see Table 1), and the pressure in the freestream and unit Reynolds number are constant \( (p_\infty = 494.3 \text{Pa}, \text{Re}_1 = 4 \times 10^6 \text{m}^{-1}) \).

| Table 1. The parameters of the incoming flow |
|-----------------|-----------------|-----------------|
| \( M_\infty \)  | \( T_\infty \) (Air), K | \( T_\infty \) (CO₂), K |
|-----------------|-----------------|-----------------|
| 8.9             | 106.7           | 153.9           |
| 7.8             | 97.8            | 141.8           |
| 6.7             | 88.65           | 129             |
| 5.6             | 79.1            | 115.7           |
| 4.5             | 68.9            | 101.5           |

The results of calculations on pressure pulsations on the model surface are compared with data obtained from the general inviscid problem solution of the disturbance interaction with a shock wave on a wedge for long-wave disturbances (Figure 1). In the investigated range of Mach and Reynolds numbers the transformation coefficients of long-wave perturbations obtained for \( x/\lambda < 0.2 \) practically coincide with the coefficients obtained from the analytical solution of the inviscid problem in approximation \( x/\lambda << 1 \). The difference is not more than 3% with an increase in parameter \( x/\lambda \) to 0.37.
Figure 1. Transformation coefficients of fast acoustic wave at different Mach numbers ($Re_L=4\times10^4$, $\alpha=10^\circ$, $x/\lambda=0.2$): 1 – viscous equilibrium Air; 2 – viscous nonequilibrium CO$_2$; 3 – inviscid theory [1].

3.2. Reynolds number effect

The change in the Reynolds number is modeled by a change in the plate length (see Table 2) with a change in perturbation frequency, since long-wave perturbations ($\lambda>\ell$) are investigated in this paper.

| $L$, mm | $Re_L$    | $f$, kHz |
|---------|-----------|----------|
| 100     | 400000    | 10       |
| 10      | 40000     | 100      |
| 1       | 4000      | 1000     |

Figure 2. Coefficients of transformation of a fast acoustic wave for different Reynolds numbers (viscous Air, $M_\infty=7.8$, $x/\lambda=0.2$): 1 – $x/\lambda=0.2$; 2 – $x/\lambda=0.37$, 3 – inviscid theory [1].

Figure 2 shows the perturbation transformation coefficients obtained in numerical simulation for different Reynolds numbers. It is seen that the obtained coefficients of long-wave perturbations practically coincide with the coefficients for the inviscid case at $Re_L \leq 40000$. The difference between
the transformation coefficients and inviscid interaction theory is about 13% at \( \text{Re}_L = 4000 \). This is probably because the flow parameters correspond to a strong viscous-inviscid interaction.

### 3.3. Attack angle effect

Figure 3 shows the ratio between the calculated coefficients of the transformation of acoustic disturbances of the fast mode (the viscous case) to the coefficients obtained from the general solution of the inviscid problem of interaction of perturbations with a shock wave on the wedge [1] for air as a function of the angle of attack. It is seen that the normalized transformation coefficients in the flow of nonequilibrium CO\(_2\) coincide with the data for viscous air for all angles of attack.

![Figure 3](image)

**Figure 3.** The normalized transform coefficients fast acoustic wave for different angles of attack \( \text{Re}_L = 4\times10^4 \), \( M_e = 7.8 \), \( x/\lambda = 0.2 \). 1 - viscous equilibrium Air, 2 - viscous nonequilibrium CO\(_2\).

Further, deviations values \( \delta \gamma_a = \frac{\gamma_a \text{vis} - \gamma_a \text{invis}}{\gamma_a \text{invis}} \) of the transformation coefficients (viscous case) from the inviscid theory are obtained for all the variants in dependence on the hypersonic parameter \( \chi = \frac{M_e^3}{\sqrt{\text{Re}_L}} \) (Figure 4), where \( M_e \) is the value of Mach number behind the bow shock. It is seen that all deviations do not exceed 2% for \( \chi < 2 \). This indicates that it is necessary to take into account the viscous-inviscid interaction for values of hypersonic parameter \( \chi > 2 \).

![Figure 4](image)

**Figure 4.** Deviation of the transformation coefficients from the inviscid case as a function of the hypersonic interaction parameter for \( x/\lambda = 0.2 \). 1 - viscous equilibrium Air, 2 - viscous nonequilibrium CO\(_2\), 3 - approximation curve.
4. Conclusion
The development of long-wave disturbances on the plate at the angle of attack in flows of air and CO$_2$ has been simulated numerically for the conditions of experiments in the wind tunnel IT-302M at ITAM SB RAS.

The two-temperature model of relaxation flows is used to take into account the real properties of CO$_2$.

The transformation coefficients of external long-wave disturbances were determined behind the shock wave for different angles of attack, Reynolds numbers and Mach numbers. The values of deviations of the transformation coefficients from the inviscid analytical solution in the long-wave approximation have been calculated for all variants of calculation as a function of the hypersonic parameter $\chi$ characterizing the degree of viscous-inviscid interaction.

In a wide range of Mach and Reynolds numbers, the transformation coefficients for viscous flows of air and carbon dioxide practically coincide with the values of the coefficients for inviscid air flows. A significant influence of the viscous-inviscid interaction on the transformation coefficients has been observed for $\chi>2$.

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