Black hole qubit correspondence from quantum circuits

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We propose a black hole qubit correspondence (BHQC) from quantum circuits, taking into account the BHQC formulations of wrapped brane qubits. With base on BHQC, we implement the corresponding gate operations to realize any given quantum circuit. In particular, we implement cases of the generation of Bell states, quantum teleportation and GHZ states circuits. Finally, we give an interpretation of the BHQC from quantum circuits with base on the BHQC classification of entanglement classes.

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I. INTRODUCTION

One of the crucial ingredients to the scenario of extra dimensions is a brane on which particles of standard model are localized. In string theory, fields can naturally be localized on D-branes due their open string endings [1]. Although, extra dimensions are generally proposed in a compactified form, the localization mechanism for gravity can lead to new possibilities [2]. Due to gravitational interactions between brane in uncompactified 5D-space, a four dimensional Newtonian behavior can be achieved when the bulk cosmological constant and the brane tension are related. Additionally, gravitons can be localized on branes by separating two patches of AdS5 space-time [3]. Static 4D-brane universes can exist under the requirement that their tension is fine-tuned with the bulk cosmological constant. Interesting models in which extra dimensions can also take advantage in the AdS/CFT correspondence, where strongly coupled 4D theory to 5D warped dimensions can be related [4]. Extensions to charged branes and thermal strings are also possible scenarios [5, 6]. A more recent fact is the string-theoretic interpretation of the black holes in terms of Dp-branes wrapping around six compactified dimensions associated to qubits from quantum information (QI) [7]. This is the so-called black hole qubit correspondence (BHQC) [8]. It has lead to important achievements as the association between black hole entropy emerging from the solution of N = 2 supergravity STU model of string compactification and tripartite entanglement measurement [9], the association between the black hole configurations in STU supergravity and entanglement state classification [10] and the identification of the Hilbert space of the qubits associated to the wrapped branes inside the cohomology of the extra dimensions [11]. In fact, many important results were obtained [12–19] (see for a more complete review [20]). It is believed that this BHQC can also be extended to the context of supergeometries [25–28].

In this paper, we propose a BHQC from the point of view of quantum circuits. Following the previous proposes, we first associate the wrapped brane qubits in the BHQC according to an one-to-one association [7] and then build quantum gate operations to implement quantum circuits. These quantum gate operations reproduce the necessary set of gate operations for a quantum circuit and, as such, can be implemented in improved quantum computations [23, 24]. We apply these results to obtain specific quantum circuits as the generation of Bell states, quantum teleportation circuit [21, 22] and generation of GHZ states. Finally, we give the interpretation of these quantum circuits with base on the BHQC for the classification of entanglement classes [7, 10].

We organized the paper as follows: In Sec. II, we review the basic operations with base on the BHQC of wrapped brane qubits of extra dimensions and perform the necessary set of quantum gate operations to realize quantum circuits. In Sec. III, we implement specific examples of quantum circuits in BHQC. In Sec. IV, we give the corresponding interpretation of quantum circuits according to BHQC. In Sec. V we address our conclusions.

II. BHQC FOR QUBITS FROM EXTRA DIMENSIONS

The relations in the BHQC for qubits from extra dimensions start with the associations between one-forms and one-mode states $\Omega \leftrightarrow |0\rangle, \bar{\Omega} \leftrightarrow |1\rangle$, where nowhere vanishing can also be an holomorphic three-form in the Calabi-Yau space associated to the vacuum state and $K$ is the Kähler potential [11]. The orthonormality relations

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are written as
\[
\begin{align*}
\int_{\tau^2} \Omega \wedge \ast \Omega & \leftrightarrow (0|0) = 1, \\
\int_{\tau^2} \bar{\Omega} \wedge \ast \Omega & \leftrightarrow (1|1) = 1, \\
\int_{\tau^2} \bar{\Omega} \wedge \ast \Omega & \leftrightarrow (1|0) = 0, \\
\int_{\tau^2} \Omega \wedge \ast \Omega & \leftrightarrow (0|1) = 0.
\end{align*}
\]

The action of the Hodge star operator $\ast$, that introduces a phase term on $|0\rangle$, reads $\ast|0\rangle = -|0\rangle$, $\ast|1\rangle = |1\rangle$. In a superposed state, $\ast(1| \pm 0\rangle) = |1\rangle \mp |0\rangle$. On the other hand, the action of the flat Kähler covariant derivative $D_{\bar{z}} \Omega = (\bar{z}^\dagger - z^\dagger) \left( \partial_\tau + \frac{1}{2} \partial_\tau K \right) \Omega$ follow the rules $D_{\bar{z}} \Omega = \bar{\Omega}$, $D_{\bar{z}} \bar{\Omega} = 0$, and the flat adjoint covariant derivatives follows the correspondences $\bar{D}_{\bar{z}} \bar{\Omega} = 0$ and $\bar{D}_{\bar{z}} \Omega = \Omega$, leading to BHQC with bit-flippers
\[
\begin{align*}
D_{\bar{z}} \Omega & \leftrightarrow \uparrow |0\rangle = |1\rangle, \\
D_{\bar{z}} \bar{\Omega} & \leftrightarrow \uparrow |1\rangle = 0, \\
D_{\bar{z}} \Omega & \leftrightarrow \downarrow |0\rangle = 0, \\
D_{\bar{z}} \bar{\Omega} & \leftrightarrow \downarrow |1\rangle = 0.
\end{align*}
\]

A general qubit state in extra dimensions can then be represented by a non-normalized qubit $|\Gamma\rangle = \alpha|1\rangle + \beta|0\rangle$. The basic elements $\ast, \uparrow, \downarrow, |1\rangle, |0\rangle$ can be used to implement the BHQC to a large range of combinations. The Hodge star operators and the covariant derivatives can be combined to define operators
\[
\begin{align*}
\lambda_1 & = \ast \uparrow + \uparrow \ast, \\
\lambda_2 & = \ast \downarrow + \downarrow \ast, \\
\lambda_3 & = \ast \downarrow + \uparrow \ast, \\
\lambda_4 & = \ast \uparrow + \downarrow \ast.
\end{align*}
\]

These operators act on one-mode states $|0\rangle$ and $|1\rangle$ leading to the following relations $\lambda_1 |j\rangle = 0$, $\lambda_2 |j\rangle = 0$, $\lambda_3 |j\rangle = -|j \oplus 1\rangle$, $\lambda_4 |j\rangle = |j \oplus 1\rangle$, where $j = 0, 1 \in Z_2$. An immediate consequence of this operation is $\lambda_3^2 \leftrightarrow I$, $\lambda_4^2 \leftrightarrow I$. The action of these operators on qubit states are $\lambda_3 (\alpha|0\rangle + \beta|1\rangle) = - (\alpha|1\rangle + \beta|0\rangle)$ and $\lambda_4 (\alpha|0\rangle + \beta|1\rangle) = (\alpha|1\rangle + \beta|0\rangle)$. These operators are then equivalent to a NOT gate [24] and are related to each other by means of $\lambda_3 = -\lambda_3$. It follows, all the one-mode gate operations can be realized by combinations of the actions of the operator $\lambda_4$ and its square $\lambda_4^2 = I$. We can also have Hadamard gates by means of the operations $(I + \ast \lambda_4) |1\rangle = |1\rangle - |0\rangle$ and $(I + \ast \lambda_4) |0\rangle = |0\rangle + |1\rangle$ or $\lambda_4 (I + \ast \lambda_4) |1\rangle = |1\rangle - |0\rangle$ and $\lambda_4 (I + \ast \lambda_4) |0\rangle = |0\rangle + |1\rangle$. A $\sigma_2$-type gate can be obtained from $\lambda_4 \ast |0\rangle = -|1\rangle$ and $\lambda_4 \ast |1\rangle = |0\rangle$, or $\lambda_3 \ast |0\rangle = |1\rangle$ and $\lambda_3 \ast |1\rangle = -|0\rangle$. In similar fashion, other gate operations can be built from suitable applications, leading to a BHQC for a universal quantum computation.

The correspondence can be generalized to $n$-mode case. In the case of a two-mode space, whose basis is $\{|00\rangle, |10\rangle, |01\rangle, |11\rangle\}$, two mode gate operation can be obtained from one-mode ones. For instance,
\[
\begin{align*}
\ast \ast |ij\rangle & = \begin{cases} |ij\rangle, & \text{if } i = j; \\
-|ij\rangle, & \text{otherwise.}
\end{cases}
\end{align*}
\]
\[
\begin{align*}
\uparrow \ast \uparrow |ij\rangle & = \begin{cases} |11\rangle, & \text{if } i = j = 0; \\
|0\rangle, & \text{otherwise.}
\end{cases}
\end{align*}
\]
\[
\begin{align*}
\downarrow \ast \downarrow |ij\rangle & = \begin{cases} |00\rangle, & \text{if } i = j = 1; \\
|0\rangle, & \text{otherwise.}
\end{cases}
\end{align*}
\]

It is also easy to check that $\uparrow \ast \downarrow |01\rangle = |10\rangle$ and $\downarrow \ast \uparrow |10\rangle = |01\rangle$, zero otherwise. As in the one-mode case, we can define new operators
\[
\begin{align*}
\Lambda_1 & = \ast \ast \uparrow + \uparrow \ast \ast, \\
\Lambda_2 & = \ast \ast \downarrow + \downarrow \ast \ast, \\
\Lambda_3 & = \ast \ast \uparrow + \downarrow \ast \ast, \\
\Lambda_4 & = \ast \ast \uparrow + \uparrow \ast \ast.
\end{align*}
\]

that lead to the following results
\[
\begin{align*}
\Lambda_1 |00\rangle & = -(|01\rangle + |10\rangle), \\
\Lambda_1 |11\rangle & = 0, \\
\Lambda_1 |01\rangle & = |11\rangle, \\
\Lambda_1 |10\rangle & = |11\rangle, \\
\Lambda_2 |00\rangle & = 0, \\
\Lambda_2 |01\rangle & = |01\rangle, \\
\Lambda_3 |00\rangle & = |00\rangle, \\
\Lambda_3 |01\rangle & = |01\rangle, \\
\Lambda_3 |10\rangle & = |11\rangle - |00\rangle, \\
\Lambda_4 |00\rangle & = |00\rangle, \\
\Lambda_4 |01\rangle & = |01\rangle, \\
\Lambda_4 |10\rangle & = |11\rangle - |00\rangle.
\end{align*}
\]

In particular, we can verify
\[
\begin{align*}
\Lambda_2 \Lambda_1 |00\rangle & = 2|00\rangle, \\
\Lambda_1 \Lambda_2 |11\rangle & = 2|11\rangle.
\end{align*}
\]

A controlled not (CNOT) gate can be implemented considering the conjugation rules
\[
\begin{align*}
\Omega \otimes \Omega & \rightarrow \Omega \otimes \Omega, \\
\Omega \otimes \bar{\Omega} & \rightarrow \Omega \otimes \bar{\Omega}, \\
\bar{\Omega} \otimes \bar{\Omega} & \rightarrow \bar{\Omega} \otimes \bar{\Omega}, \\
\bar{\Omega} \otimes \Omega & \rightarrow \Omega \otimes \bar{\Omega},
\end{align*}
\]

These rules corresponds to the operation
\[
|ij\rangle \rightarrow |i\rangle|j \oplus i\rangle,
\]
and then implement a CNOT gate, where \( i \) corresponds to the control and \( j \) to the target. As can be verified, this gate can be written as the action of the following operator \( U_{\text{CNOT}}(i) = (I \otimes \lambda_i)^j \).

### III. GENERATION OF BELL STATES AND QUANTUM TELPORTATION

As direct application of the BHQC, we can verify the usual quantum circuits implementing Bell states and quantum teleportation. Taking the input state |00⟩, we can apply a set of gate operations in order to generate all the Bell states. We apply the gate operation \((^* \otimes ^*)\Lambda_1\) on the input state we have the first Bell state

\[ |B_1⟩ = (^* \otimes ^*)\Lambda_1 |00⟩ = |01⟩ + |10⟩. \]  

(43)

The second Bell state |B_2⟩ can be generated by applying the \((I \otimes ^*)\) gate operation

\[ |B_2⟩ = (I \otimes ^*)|B_1⟩ = |01⟩ − |10⟩. \]  

(44)

The application of \((I \otimes \uparrow)\) leads to

\[ |B_3⟩ = (I \otimes \uparrow)|B_2⟩ = |00⟩ − |11⟩. \]  

(45)

Next, applying \((I \otimes ^*)\), results

\[ |B_4⟩ = (I \otimes \lambda_4 \lambda_3)|B_3⟩ = |00⟩ + |11⟩. \]  

(46)

![Quantum Circuit](image)

**FIG. 1.** (Color online) Quantum circuit for the generation of the bell states from qubits.

Let us consider now the qubit state (47) where the coefficients \( \alpha \) and \( \beta \) are unknown, we can realize a quantum state teleportation by means of a circuit teleportation [22] with the use of the gate operations realized on qubits. The initial state can be represented by

\[ |G⟩_a|B_4⟩_{b_1b_2}, \]  

(47)

where \( b_1 \) correspond to the first mode and \( b_2 \) to the second mode of the Bell state. The coefficients of \( |G⟩_a = \alpha |0⟩_a + \beta |1⟩_b \) are generally unknown.

Under a CNOT gate where the qubit mode \( a \) is the control state, the state (47) is modified to

\[ (\alpha |00⟩_{ab_1} + \beta |11⟩_{ab_1}) |0⟩_{b_2} + (\alpha |01⟩_{ab_1} + \beta |10⟩_{ab_1}) |1⟩_{b_2}. \]  

(48)

Applying the Hadamard gate operation (??) in the mode \( a \), we arrive at

\[ | \alpha (|0⟩_a + |1⟩_a)|0⟩_{b_1} + \beta (|0⟩_a − |1⟩_a)|1⟩_{b_1} |0⟩_{b_2} + | \alpha (|0⟩_a + |1⟩_a)|1⟩_{b_1} + \beta (|0⟩_a − |1⟩_a)|0⟩_{b_1}|1⟩_{b_2}. \]  

(49)

and applying a \( \lambda_3 \) operation on the mode \( a \) we arrive in the opposed Hadamard operation

\[ | \alpha (|0⟩_a + |1⟩_a)|0⟩_{b_1} + \beta (|0⟩_a − |1⟩_a)|1⟩_{b_1}|0⟩_{b_2} + | \alpha (|0⟩_a + |1⟩_a)|1⟩_{b_1} + \beta (|0⟩_a − |1⟩_a)|0⟩_{b_1}|1⟩_{b_2}. \]  

(50)

Since the usual projection relations applies to the one-forms, we can project the total state into the state \( |00⟩_{ab_1} \), given by the application of the projector

\[ P_{ab_1}^{(0)} = (|00⟩⟨00|)_{ab_1}, \]  

(51)

on the whole state (50), the mode \( b_2 \) then assumes the state

\[ \alpha |0⟩_{b_2} + \beta |1⟩_{b_2}, \]  

(52)

which corresponds to the quantum state teleportation of the qubit from the mode \( a \) to \( b_2 \) (figure 2).

![Quantum Circuit](image)

**FIG. 2.** (Color online) Sequence of quantum gate operations in the quantum circuit teleportation for qubits.

### IV. INTERPRETATION OF QUANTUM CIRCUITS FOR BHQC

The association between the entropy of an STU black hole supergravity and a 3-tangle of a given tripartite state in one-to-one correspondence [7] leads to a clear association with a purely tripartite entangled state, the GHZ state. Considering a three qubit quantum circuit, this state is generated from a Bell state |B_4⟩_{a_1a_2} and a third state |0⟩_b, applying a CNOT gate in \( a_2b \), where \( a_2 \) is the control, we generate a GHZ state (figure 3). Alternatively, the same result is obtained if the control qubit is \( a_1 \). We can just write, \( k = 1, 2, \)

\[ U_{\text{CNOT}_{a_1 b}}|B_4⟩_{a_1a_2} \otimes |0⟩_b = |000⟩_{a_1a_2b} + |111⟩_{a_1a_2b}. \]  

(53)
FIG. 3. (Color online) Quantum circuit generating a GHZ state.

This state corresponds to four D3-branes intersecting over a string in the BHQC, according to the classification of three-qubit states as supersymmetric black holes [7]. In this classification, our quantum circuit BHQC can be used to generate different wrapping configurations of these intersecting D3-branes by the action of appropriate operators. In fact, if we consider a more general state corresponding to a STU black hole $|ABC\rangle = \sum_{ijk=0}^{1} \alpha_{ijk} |ijk\rangle$, in particular, the triality interchanges and class changes can be implemented by the use of quantum circuits. In particular, the classes $A - B - C$, $A - BC$, $W$ and $GHZ$ as described in [7] can be interconnected by gate operations. We can implement for instance the an interchange of $A - B - C$ to $A - BC$ by applying a Hadamard gate in the third qubit and then a CNOT gate operations between the second and the third as control

$$|000\rangle \rightarrow |000\rangle + |001\rangle \rightarrow |101\rangle + |110\rangle. \quad (54)$$

This change corresponds to modify the SUSY configuration of the small black hole from 1/2 preserved to 1/4 preserved (figure 4). On the other hand, the quantum circuit for the generation of a GHZ state corresponds to a passage from a small (non-attractor) to a large (attractor) black hole with SUSY 1/8 preserved or completely broken (figure 5). An implementation of STU black holes associated to four qubit systems [10, 13] is made in analogous way to the previous circuits, we can derive a quantum circuit for four entanglement to implement a BHQC applying adequate quantum gate operations in the presence of an auxiliary qubit, for example, $|GHZ\rangle \otimes |0\rangle$ under a CNOT gate. The extension to quantum circuits with a higher number of qubits can be made straightforwardly by the introduction of new auxiliary qubits. This procedure corresponds to deal with a given $n$-form in the cohomology class of wrapped D$p$-branes. Other mechanisms, as the process of moduli stabilization at the horizon associated to entanglement distillation to a GHZ state [14] can also be implemented in a more clear fashion from the perspective of quantum circuits.

V. CONCLUSIONS

We have showed quantum gate operations and quantum circuits implemented in the context of BHQC with qubits from wrapped branes. In particular, we have applied this BHQC to implement well-known quantum circuits as the generation of Bell states, GHZ states and quantum circuit teleportation. We interpreted the quantum circuits in terms of the previous results [7] associated to the relation between the classes of entanglement and the entropy and SUSY configurations of STU black holes. A consequence of this formulation is the possibility to the interchange of black hole configurations by using quantum circuits associated to any number of qubits.

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