Anti-short girth serial update decoding algorithm of GF(q) LDPC codes

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Abstract. A serial Fast Fourier Transform Belief Propagation (FFT-BP) algorithm with low complexity of non-binary LDPC codes decoding is presented in this paper to reduce the effects of short girth. During iteration, part of the variable nodes with short girths will just send messages to its neighbor, which are in the same short girth and avoid the messages from them. In the next iteration decoding, this part of the nodes updates and the newly updated node with short girth stops updating. Simulations show that the proposed algorithm achieves a better gain to the standard algorithm and reduces complexity at the same time.

1. Introduction

Impractical to implement when first developed by Gallager in 1963[1], LDPC codes were forgotten until his work was rediscovered in 1990[2]. Non-Binary LDPC (NB-LDPC) codes are an extension of binary LDPC codes constructed over $GF(q)(q > 2)$ [3]. NB – LDPC has better error-correcting performance than their binary counterpart [4]. The classical BP algorithm used in decoding NB-LDPC codes has a computational complexity dominated by $O(q^2)$ making the decoding over higher order fields computationally infeasible. The decoding complexity of the NB-LDPC code limits its widespread deployment in low power and low complexity constraint systems such as deep space exploration communication ultra long-distance wireless communication.

However, it has been shown that the belief propagation over $GF(q)$ can be conveniently transferred into frequency domain scaling down the complexity to $O(q \times \log^9 q)$, which is called FFT-BP [6]. In [7][8], log-domain decoding, extended min-sum (EMS) had also been introduced. But the performance of EMS is as good as FFT-BP. When the BP decoding algorithm is used, the short girth generated by the check matrix will make the message no longer independent during the transmission process, thus affecting the performance of the BP algorithm [9]. Various algorithms for calculating short girth distributions in graphs and optimized decoding algorithms against these short girths have been proposed [10][11].

In order to reduce the number of iterations required for BP decoding, a LDPC serial decoding algorithm with acceleration factor 2 is proposed in reference [12]. Serial algorithm is a good choice of power and complexity constraint applications because it only needs one VNU and CNU. In this paper, a serial FFT-BP decoding algorithm with loop cancellation is proposed, which suppresses the spread of error messages by stopping updating some variable nodes with a large number of short rings.

The rest of this paper is organized as follows. The second part gives a brief overview of NB-LDPC codes and FFT-BP algorithm. Section 3 we proposed low complexity anti-girth serial FFT-BP
decoding algorithm and discusses how it works. In Section 4 we present our results and discussion. Section 5 provides our concluding remarks.

2. Serial FFT-BP Decoding for non-binary LDPC codes

A NB-LDPC codes is a linear block code and can be described by a low-density parity check matrix \( H_{M \times N} \). Every element in \( H_{M \times N} \) is taken from \( GF(q) \). There are \( d_v \) and \( d_c \) respectively to represent the number of non-zero elements in each column and row of the matrix \( H \). If a row vector \( x \) of length \( N \) satisfies

\[
\sum_{n} h_{mn} x_n = 0, \quad m = 1, \ldots, M
\]

For each symbol \( a \in GF(q) \), \( a \) is the \( i \)th bit of the binary representation of \( a \), the likelihood of the \( n \)th received symbol \( y_n \) can be set as being equal to \( a \) as

\[
f_a = \prod_{j \in N(n) \setminus n} g_{aj}^a.
\]

For ease of description, define the following symbols:

- \( N(m) = \{ n : H_{mn} \neq 0 \} \) : symbols that participate in check \( m \).
- \( M(n) = \{ m : H_{mn} \neq 0 \} \) : checks that depend on symbol \( n \).
- \( q_{mn}^a \) : messages from \( n \)th symbol to \( m \)th check.
- \( r_{mn}^a \) : messages from the \( m \)th check to the \( n \)th symbol.

Row update step:

\[
r_{mn}^a = \sum_{x : x_n = a} P(z_n / x) \prod_{j \in N(m) \setminus n} q_{mj}^a
\]

The column update step:

\[
q_{mn}^a = P(x_n = a / M(n) \setminus m)
\]

In standard BP decoding algorithm, during each iteration, all the symbol nodes send messages to the checking node, and then all the check nodes send messages to the symbol nodes. While maintaining fast convergence, serial scheduling with each node update in a sequence is more efficient in decoding complexity and memory requirements. Compare with a binary LDPC decoder, the size of unit storage and message storage processing space required by NB-LDPC decoder expands with \( q = 2^p \). Serial decoding algorithm is more suitable for NB-LDPC decoder.

In [6], a normal Serial FFT-BP (SFFT-BP) decoder in log domain is proposed. Details of normal SFFT-BP decoder are shown in the following algorithm (NS-Algorithm).

Step1: for \( H_{mn} \neq 0 \), initialization \( lq_{0n}^0, lq_{mn} \) as:

\[
lq_{0n}^0 = \{ \log(f_n^0), \log(f_n^1), \ldots, \log(f_n^p) \}
\]

\[
lq_{mn} = lq_{0n}^0; \quad lr_{mn} = 0; \quad n = 1;
\]

Step2: calculate \( lr_{mn} \):

\[
lr_{mn} = \text{IFFT} \left[ \sum_{n \in N(m) \setminus n} \text{FFT}[lq_{mn}] \right] \quad m \in M(n)
\]

Step3: calculate \( lq_{mn} \):

\[
lq_{mn} = lq_{0n}^0 + \sum_{m \in M(n) \setminus m} lr_{mn} \quad m \in M(n)
\]

Step4: \( n = n + 1 \), if \( n > N \), go to Step 5; else back to Step 2.

Step5: For \( m = 1, 2, \ldots, M \), calculate \( e_m \):
If the syndrome $c_m$ is zero or iterations is reached maximum number, decoding stopped. Otherwise, go to Step 2.

3. **Anti-short girth Serial FFT-BP Decoding Algorithm**

*SFFT-BP* traverses variable nodes sequentially during decoding. For each variable node, it receives all messages from neighbor nodes and passes the updated messages to all neighbor nodes. For medium and short length block codes, there will inevitably be a large number of short rings. If the messages from independent variable nodes are wrong, after updating the short rings back and forth, these errors will affect the confidence of other variable nodes. The standard *SFFT-BP* decoding algorithm isn’t dealing with the influence of short loops.

In order to reduce the influence of short-loop in serial update algorithm, an anti-loop serial update decoding algorithm is proposed. During each iteration, some variable nodes connected with many short-loop will stop updating their own messages, but still transmit messages outward, so as to reduce the impact of short-loop on decoding process.

Let $\Theta$ be the set of variable nodes in short girth, $\text{Iter}$ represents the number of iterations. The computation of $\Theta$ is done before the decoding process. The improved *SFFT-BP* decoding algorithm (IS-Algorithm) with anti-short girth messages transmission schedule is as followed.

**Step1:** for each $m, n$ $H_{mn} \neq 0$, initialization:

$$lq_n^0 = \log(f^{0}_n), \log(f^{1}_n), \cdots, \log(f^{s-1}_n)$$

$$lq_{mn} = lq_n^0; \quad lr_{mn} = 0; \quad n = 0; \quad \Theta = \{\Theta_1, \Theta_2\}; \quad \text{Iter} = 0$$

**Step2:** $\text{Iter} = \text{Iter} + 1$.

**Step3:** if $n \in \Theta_1$ and $\text{mod}(\text{Iter}, 2) = 1$, go to Step 6; if $n \in \Theta_2$ and $\text{mod}(\text{Iter}, 2) = 0$, go to Step 6

**Step4:** For each $m \in M(n)$, calculate $lr_{mn}$:

$$lr_{mn} = \text{IFFT} \left( \sum_{n \in \Theta(n) \setminus n} \text{FFT}[lq_{mn}] \right)$$

(7)

**Step5:** For each $m \in M(n)$, calculate $lq_{mn}$:

$$lq_{mn} = lq_n^0 + \sum_{n \in \Theta(n) \setminus m} lr_{mn}$$

(8)

**Step6:** $n = n + 1$, If $n \leq N$, go to Step 3, else go to Step 7.

**Step7:** For $m = 1, 2, \ldots, M$, calculate $c_m$:

$$lq_a = lq_n^0 + \sum_{m \in M(n)} lr_{mn}$$

$$\hat{x}_n = \text{arg max}_a lq_n^a$$

$$c_m = H_{mn}\hat{x}_n$$

(9)

If the syndrome $c_m$ is zero or iteration times is reached maximum number, decoding stopped. Otherwise, go to Step 2.
4. Simulation Results
In this section, we compare the simulated BER performance of normal FFT-BP decoding algorithms and SFFT-BP decoding algorithms with that of the anti-girth serial FFT-BP decoding algorithm. A random constructed regular \((3, 6)\) NB-LDPC code over \(GF(8)\) was used. The maximum iteration number is set to 50. The channel model is AWGN with BPSK-modulated code words.

The average iteration times of three decoding algorithm were shown in Figure 1. At low signal-to-noise ratio region, the convergence speed of IS-Algorithm is the lowest compare to other algorithms because part of variable nodes with sort girth stopped updating temporally. We observe that for higher signal-to-noise ratios the iteration times of NS-Algorithm and IS-Algorithm needed are about half of the NM-BP Algorithm. The parallel schedules have more average iteration times than that of serial schedule.

![Fig.1 Convergence speed comparisons](image1)

![Fig.2 BER performance comparisons](image2)

Figure 2 shows the BER performance of NM-BP Algorithm, NS-Algorithm and IS-Algorithm respectively. When the bit error rate is equal to \(10^{-6}\), the IS-Algorithm has 0.15 dB gains compare to the NM-BP Algorithm and 0.1 dB compared to NS-Algorithm. Moreover, the performance gains is achieved with reducing delay and decoding complexity. Compared with NM-BP Algorithm and NS-Algorithm, the IS-Algorithm improves the decoding performance and reduces the computational complexity at the same time.

5. Conclusions
In this paper, we proposed a anti-short girth serial FFT-BP decoding algorithm for short regular NB-LDPC codes, which outperforms previous algorithms based on standard FFT-BP. IS-Algorithm could improve performance and reduce complexity at the same time for these codes with sort girth.

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