Find the twins: Evidence for a population of coalescing binary black holes with nearly equal-mass components

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The coalescing binary black hole (BBH) systems were likely formed in several channels and the identification of them is a challenging task. Previously, people have found out that at the mass of ∼ 34M⊙ there is a distinct Gaussian-like peak superposed on the power-law mass function of the primary components. In this work, with the refined Power Law + Peak primary mass distribution models, we identify a population of BBHs with a much stronger preference for equal-mass binaries, which dominates the Peak, for the first time. These BBHs have a pairing function with a β = 8.56±3.68, or with a σm = 3.14±4.59M⊙, where σm represents the width of the distribution of m1 − m2 (i.e., the difference between and the primary and secondly masses). The BBHs in the Power Law have a much flatter pairing function with a β = −0.11±2.48. All these parameter ranges are reported for 90% credibility. Our finding likely points towards the chemically homogeneous evolution channel featured by a high mass ratio for the BBHs with a total mass above ∼ 55M⊙. As shown in our simulation, the increase of the GWTC-3 sample by a factor of several would formally establish the presence of the high q population of massive BBHs.

Introduction. The first successful detection of gravitational wave (GW) signal from a coalescing binary black hole (BBH) in September 14, 2015 [1] has launched an era of GW astronomy. Very recently, LIGO-Virgo-KAGRA Collaborations report the second part of the GW events detected in the third observing run (O3) [2], and up to now, about 90 detections have been reported [2–5]. The number of detections may even reach one thousand once the GW detector network is running at the design sensitivity [6]. However, the origins of these compact objects still remain uncertain. Several evolutionary channels have been proposed (see [7, 8] for recent reviews), including for instance the isolated binary evolution and dynamical capture, and these formation channels can leave imprints on the properties of the compact binary population [9, 10]. Therefore, studying the rapidly increasing sample of GW events enables us to investigate how compact binaries are formed. Various studies were carried out, based on some analytical models (e.g., [11–14]) and some non-parametric approaches (e.g., [15–17]), some formation/evolution characteristics of the compact binaries are being revealed (e.g. [18–26]).

The pairing function may also carry some information about the formation and evolution mechanism of the BBHs. For instance, Fishbach and Holz [27] found that the two objects in the merging BBHs prefer to be of comparable mass rather than randomly paired, consistent with the predictions of some formation channels [28–32]. In particular, Mandel and de Mink [31] (see also [32, 33]) proposed a route towards merging massive BHs, i.e., the chemically homogeneous evolution, formed through which, the BBHs are expected to strongly prefer equal mass components, because the binary systems were in contact (shared mass) on the main sequence, before disengaging during subsequent phases of chemically homogeneous evolution. The traditional isolated evolution channel, i.e., the common envelope evolution, is also predicted to produce BBHs with comparable mass components. Anyhow, the chemically homogeneous evolution can only produce massive binaries with total BH mass above ∼ 55M⊙ [31, 32], which exceeds the majority of mass range for the common envelope evolution. Additionally, this mass range are consistent with the previously identified peak located at ∼ 34M⊙ [13, 14, 18], which may be caused by the pulsational pair-instability supernovae [34, 35]. It is therefore possible to identify the chemically homogeneous evolution channel via examining the primary mass-dependent pairing function. Motivated by such a prospect, in this Letter, we perform a hierarchical Bayesian inference to explore the features in the distribution of mass ratio of the BBH events.

Method. Our analysis focuses on the GW data of BBHs reported in the Gravitational-wave Transient Catalog 3 (GWTC-3; [2]). Following The LIGO Scientific Collaboration et al. [14], we choose the false-alarm rate (FAR) of 1yr⁻¹ as the threshold to select the events. We exclude GW190814 in the analysis, because it may belong to a subpopulation of highly asymmetric binaries [14], and is disconnected from the BBH population but potentially connected to the recently-identified population of NSBH [21, 23]. Therefore, we adopt a total of 69 BBHs under the guidance of FAR < 1yr⁻¹. The posterior samples for each BBH event are adopted from Gravitational Wave Open Science Center (https://www.gw-openscience.org/eventapi/html/GWTC/). For the (new) events in the GWTC-1 [3], GWTC-2 [4], GWTC-2.1 [5] and GWTC-3 [2], we use the ‘Overall posterior’ samples, the ‘PublicationSamples’
samples, the ‘PrecessingSpinIMRHM’ samples and the ‘C01:Mixed’ samples, respectively.

With the updated data from the GWTC-3, the simple Power Law + Peak model is still acceptable [14]. Therefore, our models are constructed by modifying the Power Law + Peak model. For the first mass distribution model, we assume all the BBH populations share one common pair function (hereafter Model I),

\[ \pi(m_1, m_2|\alpha, m_{min}, m_{max}, \delta_m, \beta, \lambda, \mu, \sigma) = ((1 - \lambda)P(m_1|\alpha, m_{min}, m_{max}, \delta_m) + \lambda G(m_1|\mu, \sigma))P(m_2|\beta, m_{min}, m_1, \delta_m), \]

where \( m_1 \) and \( m_2 \) are the primary and secondary mass of the BBHs, \( -\alpha, \delta_m, m_{min}, \) and \( m_{max} \) are the spectral index, smoothing scale, low-mass and high-mass cut-off, \( \lambda, \mu, \) and \( \sigma \) are the mixing fraction, mean, and width of the Peak component, \( \beta \) is the power-law index of the secondary mass. The first modification is that the Power Law component \( P \) is normalized after the smooth treatment on its lower boundary as suggested by Wang et al. [18]:

\[ P(m|\alpha, m_{min}, m_{max}, \delta_m) = A m^{-\alpha} S(m|m_{min}, \delta_m), \]

where \( A \) is the normalization constant and \( S \) is the smoothing function (see [13] and [14] for details). Secondly, the Peak component \( G \) is truncated on its lower bound at \( m_1 = 27M_\odot \),

\[ G(m|\mu, \sigma) = \Phi e^{-\frac{(m-\mu)^2}{2\sigma^2}}, \]

where \( \Phi \) is the normalization constant. And this modification is motivated by the \( m_1 - q \) plane of the observation data (See the Web App for LIGO-Virgo Compact Binary Catalogue from Gravitational Wave Open Science Center, http://catalog.cardiffgravity.org), where a large fraction of BBHs with \( m_1 \geq 27M_\odot \) have a trend to be equal-mass. Meanwhile, this feature is also predicted by the chemically homogeneous evolution, through which the BBHs are expected to have a total mass of \( \gtrsim 55M_\odot \) [33]. To quantitatively examine the presence of a population of BBHs with a distinctive pairing function, we introduce another power-law index \( \beta_g \) for the secondary masses of BBHs with the primary masses in the Peak component. Therefore the second mass distribution model reads (hereafter Model II):

\[ \pi(m_1, m_2|\alpha, m_{min}, m_{max}, \delta_m, \beta_{pl}, \lambda, \mu, \sigma, \beta_g) = (1 - \lambda)P(m_1|\alpha, m_{min}, m_{max}, \delta_m)P(m_2|\beta_{pl}, m_{min}, m_1, \delta_m) + \lambda G(m_1|\mu, \sigma)G(m_2|\beta_g, m_{min}, m_1, \delta_m), \]

where \( \beta_{pl} \) and \( \beta_g \) are the power-law index of the secondary for the BBHs in the Power Law and Peak components, respectively.

Since the power law \( (P) \) may not be an appropriate pairing function to describe the population with nearly equal-mass component, we also consider another Parametric model, where the deviation between the primary mass and the secondary mass of the BBHs in the Peak component is described by a half Gaussian, i.e., \( G'(m_2|m_1, \sigma_m) = \frac{2}{\sqrt{2\pi}\sigma_m} e^{-(m_2-m_1)^2/2\sigma_m^2} \) for \( m_2 \leq m_1 \), and \( \sigma_m \) is the width of the \( m_1 - m_2 \) distribution. Hence the mass distribution model (hereafter Model III) becomes:

\[ \pi(m_1, m_2|\alpha, m_{min}, m_{max}, \delta_m, \beta, \lambda, \mu, \sigma, \sigma_m) = (1 - \lambda)P(m_1|\alpha, m_{min}, m_{max}, \delta_m)P(m_2|\beta, m_{min}, m_1, \delta_m) + \lambda G(m_1|\mu, \sigma)G'(m_2|m_1, \sigma_m), \]

Considering the possibility that the BBHs with primary masses in the Peak component may not belong to a single population, we therefore assume their secondary masses follow a mixture pairing function with the mixture ratio \( r \). Therefore the Model III (hereafter the extended Model III) is extended as:

\[ \pi(m_1, m_2|\alpha, m_{min}, m_{max}, \delta_m, \beta, \lambda, \mu, \sigma, r) = (1 - \lambda)P(m_1|\alpha, m_{min}, m_{max}, \delta_m)P(m_2|\beta, m_{min}, m_1, \delta_m) + \lambda G(m_1|\mu, \sigma)G'(m_2|m_1, \sigma_m) + (1 - r)P(m_2|\beta, m_{min}, m_1, \delta_m). \]

Note that the extended Model III can reduce to Model I and Model III with \( r \) of 0 and 1, respectively.

Since there is mass–spin degeneracy, we fit the distribution of primary and secondary masses jointly with the spin distribution, and the Default spin model as defined in [14] is adopted.

We perform a hierarchical Bayesian inference to fit the data of the observed events \{d\} with the population models described above. Following the framework described in [13] and [14], for the given data \{d\} from \( N_{det} \) GW detections, the likelihood of the hyperparameters \( \Lambda \) can be expressed as

\[ L(\{d\} | \Lambda) \propto N_{det}^{-N_{det}} e^{N_{det} \xi(\Lambda)} \prod_{i=1}^{N_{det}} \int L(d_i|\theta_i) \pi(\theta_i | \Lambda) d\theta_i, \]
where \( N \) is the expected number of mergers over the observation period, and \( \xi(\Lambda) \) means the detection fraction. The single-event likelihood \( L(d_i|\theta_i) \) can be estimated using the posterior samples (see [13] for detail), and \( \xi(\Lambda) \) is estimated using a Monte Carlo integral over detected injections as introduced in the Appendix of [13]. We assume that the merger rate density increases with redshift, \( \mathcal{R} \propto (1+z)^{2.7} \) as was obtained by [14]. The injection campaigns can be adopted from [36], where they combine O1, O2 and O3 injection sets ensuring a constant rate of injections across the total observing time. Following [13], we do not account for the spin-dependent selection bias, which may slightly over-emphasise high positive effective spins relative to the true distribution, while this shift is likely to be small, as Galaudage et al. [20] suggested. We apply the sampler Pymultinest [37] for the hierarchical Bayesian inference of posteriors.

### Results

As mentioned above, the Model I contains a truncated PEAK (Gaussian) component with a lower cut at \( 27M_\odot \). Such a case is preferred over the absence of a cut off at \( 27M_\odot \) by a \( \ln \mathcal{B} \) (i.e., the logarithmic Bayes factor) of 0.5, suggesting that our modification is rational. The fit results of the four models are summarized in Tab. I. The Model II and Model III are preferred over Model I with \( \ln \mathcal{B} = 2.3 \) and \( \ln \mathcal{B} = 2.8 \), which provide evidence for the two component pairing functions of the BBHs. Since there is a lower mass cut in the PEAK components of our models, the range of the central value is wider than the one without a cut-off (where we obtain \( \mu = 33.58 \pm 4.65 M_\odot \)), it is nearly identical with that of [14]).

### TABLE I. Posteriors and Bayes Factors of the mass models with their priors.

| Parameters | Prior | Model I | Model II | Model III | extended Model III |
|------------|-------|---------|----------|-----------|-------------------|
| \( m_{\text{min}}[M_\odot] \) | U(2,10) | \( 4.69_{-1.61}^{+1.13} \) | \( 4.66_{-1.53}^{+1.10} \) | \( 4.61_{-1.47}^{+1.13} \) | \( 4.63_{-1.46}^{+1.12} \) |
| \( m_{\text{max}}[M_\odot] \) | U(30,100) | \( 86.73_{-13.03}^{+11.44} \) | \( 86.82_{-13.65}^{+11.62} \) | \( 87.14_{-13.01}^{+11.29} \) | \( 87.40_{-13.57}^{+10.95} \) |
| \( \alpha \) | U(-4,12) | \( 3.67_{-0.55}^{+0.71} \) | \( 3.43_{-0.62}^{+0.70} \) | \( 3.45_{-0.62}^{+0.71} \) | \( 3.49_{-0.64}^{+0.74} \) |
| \( \delta_{m}[M_\odot] \) | U(0,10) | \( 5.94_{-3.21}^{+3.14} \) | \( 6.01_{-3.29}^{+3.17} \) | \( 6.17_{-3.27}^{+3.13} \) | \( 6.05_{-3.25}^{+3.05} \) |
| \( \beta_{\text{pl}}(\beta) \) | U(-4,12) | \( 2.00_{-1.64}^{+0.24} \) | \( -0.11_{-1.55}^{+2.48} \) | \( -0.14_{-1.65}^{+2.38} \) | \( -0.12_{-1.70}^{+2.47} \) |
| \( \lambda \) | U(0,1) | \( 0.08_{-0.04}^{+0.06} \) | \( 0.01_{-0.03}^{+0.05} \) | \( 0.07_{-0.03}^{+0.04} \) | \( 0.08_{-0.03}^{+0.05} \) |
| \( \mu[M_\odot] \) | U(20,50) | \( 31.48_{-8.32}^{+3.97} \) | \( 29.52_{-7.58}^{+4.68} \) | \( 28.77_{-7.20}^{+5.13} \) | \( 29.54_{-7.45}^{+4.75} \) |
| \( \sigma[M_\odot] \) | U(0.5,10) | \( 6.01_{-3.90}^{+3.52} \) | \( 6.43_{-3.85}^{+3.10} \) | \( 6.63_{-3.89}^{+2.93} \) | \( 6.69_{-4.08}^{+2.79} \) |
| \( \beta_{\text{k}} \) | U(-4,12) | - | \( 8.56_{-5.62}^{+3.08} \) | - | - |
| \( \sigma_{m}[M_\odot] \) | U(1,8) | - | - | \( 3.14_{-1.76}^{+4.99} \) | \( 3.01_{-1.74}^{+3.82} \) |
| \( r \) | U(0,1) | - | - | - | \( 0.72_{-0.25}^{+0.47} \) |
| \( \ln \mathcal{B} \) (FAR < 1/yr) | - | 2.3 | 2.8 | 2.6 |

**Note.** Here, ‘U’ means the uniform distribution.

For Model II, we have a large \( \beta_{\text{k}} = 8.56_{-5.62}^{+3.08} \) (which corresponds to a high-\( q \) population) but a small \( \beta_{\text{pl}} = -0.11_{-1.55}^{+2.48} \), suggesting significantly different pairing functions of the BBHs (see also panel (a) and (b) of Fig.1). As for Model III, we have \( \sigma_{m} = 3.11_{-1.83}^{+5.14} M_\odot \) at 90% confidence level, which is narrow and suggests that each of these BBH systems has nearly equal-mass components (see also panel (c) of Fig.1). Consequently, we calculate that 95% of BBHs in the PEAK (POWER LAW) component have \( q \geq 0.78_{-0.25}^{+0.10} (0.59_{-0.15}^{+0.12}) \) at 90% credibility. The extended Model III is designed to contain mixed populations in the PEAK component. As shown in the Fig.2, the mixture ratio \( r \), though is not tightly constrained, has a preference to be high (i.e., > 50%). This indicates that the high-\( q \) population may dominate over the PEAK component in the primary mass function.

### Simulation

We expect to have hundreds more BBH events by the end of the O4 observing run of the LIGO-Virgo-KAGRA detector network [6]. Here we evaluate how significant the feature of mass ratio distribution in BBH populations can be revealed using an artificially constructed mock population. The simulated population are described by a simplified model:

\[
\pi(m_1, m_2|\alpha, m_{\text{min}}, m_{\text{max}}, \beta_{\text{pl}}, \lambda, \mu, m_{\text{low}}, \sigma, \beta_{\text{k}}) = (1 - \lambda) \mathcal{P}'(m_1|\alpha, m_{\text{min}}, m_{\text{max}}) \mathcal{P}'(m_2|\beta_{\text{pl}}, m_{\text{min}}, m_1) + \lambda \mathcal{G}'(m_1|\mu, \sigma, m_{\text{low}}, m_{\text{max}}) \mathcal{G}'(m_2|m_1, \sigma_m, m_{\text{min}}, m_1), \tag{8}
\]

where \( \mathcal{P}'(m_1|\alpha, m_{\text{min}}, m_{\text{max}}) \) is a normalized truncated power law function with index of \( -\alpha \), and boundary of \( (m_{\text{min}}, m_{\text{max}}) \), and is not smoothed at lower bound. \( \mathcal{G}'(m_1|\mu, \sigma, m_{\text{low}}, m_{\text{max}}) \) is a normalized truncated Gaussian with center and width of \( (\mu, \sigma) \), and boundary of \( (m_{\text{low}}, m_{\text{max}}) \). The parameters of the simulated population are assumed
as $m_{\text{min}} = 5M_\odot$, $m_{\text{max}} = 80M_\odot$, $m_{\text{low}} = 27M_\odot$, $\alpha = 3$, $\mu = 32M_\odot$, $\sigma = 4M_\odot$, and $\lambda = 0.1$. The significance of the feature in the distribution of mass ratio depends mainly on the true values of $\beta_{\text{pl}}$ and $\sigma_m$, therefore we perform simulations for several cases with series sets of $(\beta_{\text{pl}}, \sigma_m)$. To obtain the mock detections, we assume that the underlying population follows a uniform in comoving volume and source frame time merger rate, with isotropic sky positions and inclinations. The primary and the secondary masses are drawn from the assumed population described above. The spins of both components are assumed to be zero, as suggested by [27]; because the spins distribution can be fit jointly with mass distribution and marginalized over [12]. The identification of a GW signal is based on a single-detector S/N threshold $\rho = 8$ and the design sensitivity noise curves (https://dcc.ligo.org/LIGO-T2000012/public; [6]).

To generate the mock posteriors for each ‘detected’ event, we follow the procedure that performed by [27], i.e., the uncertainty on the source-frame chirp mass follows $\sigma_M/M = 8(0.01 + (0.2/(1 + z))^2)/\rho$, and the uncertainty on the symmetric mass ratio $\eta \equiv \frac{m_1m_2}{(m_1 + m_2)^2}$ follows $\sigma_\eta = 0.03 \times 8/\rho$. We randomly draw $\ln M_{\text{obs}}$ and $\eta_{\text{obs}}$ from $\mathcal{N}(\ln M, \sigma_{\ln M})$ and $\mathcal{N}(\eta, \sigma_\eta)$ respectively, where $M$ and $\eta$ are the true values of each ‘detected’ event. Then the mock posterior samples for the component masses $m_1$ and $m_2$ are generated from $\mathcal{N}(\ln M_{\text{obs}}, \sigma_{\ln M})$ and $\mathcal{N}(\eta_{\text{obs}}, \sigma_\eta)$, with flat priors. We apply the user-friendly software Bilby [38] with sampler Pymultinest [37] for sampling.

We fit the mock data with the simplified Model I and Model III, i.e., the models are not smoothed at the lower bound. As summarized in Tab. II, with 200 detections we can rule out the single pairing function model with $\ln B$ of

FIG. 1. Posterior distributions and posterior population distributions. (a): The blue histogram represents the distribution of $\beta$ obtained by the Model I, the orange and green histograms are the distribution of $\beta_{\text{pl}}$ and $\beta_g$ fitted by the Model II. (b): The blue (green) region is the mass ratio distribution of BBHs in the Gaussian (power law) component that obtained by Model II, and the solid lines represent the mean values. (c): Similar to (b), but for Model III.
\[ \beta_{pl} = -0.1 \pm 0.2 \]

\[ \sigma_m = 3.5 \pm 0.3 \]

\[ r = 0.72 \pm 0.25 \]

**FIG. 2.** Posterior distribution of the extended Model III, the vertical lines represent the corresponding 90% confidence intervals.

**TABLE II.** Bayes Factors of Model III over Model I under the different simulated populations

| Number of detections | \( \beta_{pl}, \sigma_m \) of the simulated population |
|----------------------|--------------------------------------------------|
| \( N_{\text{obs}} = 200 \) | \( \beta_{pl} = 0, \sigma_m = 3M_\odot \) | \( \beta_{pl} = 0, \sigma_m = 6M_\odot \) | \( \beta_{pl} = 0.5, \sigma_m = 3M_\odot \) | \( \beta_{pl} = 0.5, \sigma_m = 6M_\odot \) |
| | 9.3 | 6.5 | 6.2 | 3.8 |

*Note.* The \( N_{\text{obs}} \) detections are randomly generated from the assumed procedure, so Bayes factors may also depend on the fluctuations of the ‘detected’ events.

\( \sim 9.3 \) (\( \sim 3.8 \)) in an optimistic (pessimistic) case. Therefore we conclude that the presence of a subpopulation of BBHs with nearly equal-mass components will be formally established in the next Observing run of LIGO-Virgo-KAGRA detector network [6].

**Conclusion and Discussion.** We have investigated the population of BBHs with some parameterized models that are modified to properly address a potential feature in the mass ratio distribution. With the current coalescing BBH sample [2], we find that the BBHs with a primary mass in the Peak component have a significantly different pairing function from the rest events. The former has a much stronger trend to be equal-mass while the latter has a relatively flat \( q \) distribution. The features of the high-\( q \) population are consistent with the prediction of the chemically homogeneous evolution channel for “massive” stellar-mass BBHs [31–33]. The other interesting possibility, though speculative, is that these nearly equal-mass components might be the primordial black holes [39]. For the rest BBHs, i.e., that in the Power Law component, 95% of BBHs have \( q > 0.52^{+0.11}_{-0.15} \) at 90% credibility, consistent with the prediction that common envelope evolution tends to result in mergers with \( q \gtrsim 0.5 \) [40].

Note that besides the chemically homogeneous evolution, the common envelope evolution can produce BBHs in the Peak component range as well [41–44], especially in the case of lower metallicities [45, 46]. To address such a possibility, in this work we have developed the extended Model III, and shown that a population of BBHs with a typical \( \sigma_m \sim 3M_\odot \), which corresponds to \( q \gtrsim 0.9 \), and is likely to be from the chemically homogeneous evolution channel, takes a fraction (\( \sim 70\% \)) of the Peak component. With the mixing fraction of Peak component \( \sim 0.07 \), the BBHs via chemically homogeneous evolution channel may be a fraction of \( \sim 0.05 \) of the total BBH populations.

The BBH sample is expected to increase rapidly in the near future. With a significantly extended sample, besides confirming the mass ratio distribution feature in the Gaussian peak, new structures may be revealed in other mass range and then shed new light on the BBH formation channels.

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