Critical buckling load analysis on spherical shells under various geometric imperfections

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Abstract. The strength of shell structures is mainly determined by their ability to resist buckling. However, the presence of geometric imperfections – a condition which the geometry is no longer the same as the original condition – significantly deteriorates the critical load and often leads to catastrophic failure. Ratio between the actual critical buckling load to the theoretical prediction is called knockdown factor. The study aims to analyse the critical buckling load on spherical dome elastic shells under various geometric imperfections. This study was conducted into several thicknesses (10, 16, 20, 37, 50 mm) and fixed base radius of 4000 mm. Analyses of linear buckling and nonlinear static are performed and aided with finite element program using MIDAS FEA. The result of this study reveals that the critical buckling load decreases sharply in the beginning of imperfection, then reach for quite slow progression until a constant value of Kd = 0.1. In addition, the higher minimum knockdown factor is obtained as the shell thickness increases.

1. Introduction
The strength of shell structure is mainly determined by their excellent ability to resist buckling. However, if the applied load exceeds its critical buckling load, the structure undergoes a catastrophic failure. This is caused by the critical buckling load has dropped into certain point due to its unstable post-buckling behavior. Deterioration of critical buckling load do not only happen during the post-buckling stage, but could also happen in the same buckling situation. The geometric imperfections play a significant role in decreasing the critical load, as the structure may fail in such lower load than it should be.

Acknowledging that geometric imperfections are not such thing that can be avoided, it is important to study about their effect on the critical buckling load of shell. This will lead to the aim of this study, to analyze the critical buckling load on spherical elastic shells under various geometric imperfections. The study focuses on a configuration of hemispherical as known as dome structure. This study will contribute in ensuring a better and safer guideline design of shell structure, especially domes, due to its critical buckling load to prevent from a catastrophic failure.

2. Shell stability
Buckling of shell occurs when the applied load exceeds the critical buckling load. The buckling of shell may lead to a catastrophic failure since shells are classified as unstable post-buckling behaviour [1]. During this stage, the post-buckling buckling load is much lower than the critical buckling load during
the buckling stage; while at the same time, the applied load has exceeded the critical load. Most shell structures may undergo a plastic deformation in post-buckling stage. The plastic deformation occurs as a result of the post-buckling deformation, yet may not be associated with the initial buckling or the near post-buckling deformation. This state allows the prediction of buckling using the assumption of elastic material behaviour [2].

Prediction of buckling was started on a thin spherical shell under uniform external pressure in 1915 and was proposed by Zoelly [3], who followed a linear buckling analysis to obtain

\[ p_{cr} = \frac{2E}{\sqrt{12(1-\mu^2)}} \eta^{-2} \]  

(1)

where \( E \) is the Young’s modulus; \( \mu \) is the Poisson’s ratio; and \( \eta \) is radius-to-thickness ratio of shell. But this theoretical prediction was found to be in disagreement with the experimental results in the following researches, which have been compared by Lee et al [4]. The results were to be found ranging in a large variation in \( K_d = 0.17 – 0.9 \). These results are presented in figure 1 as knockdown factor, a comparison actual obtained buckling pressure to the theoretical prediction, versus the radius-to-thickness ratio. Note that most of the previous experiments were conducted with shallow spherical caps, however Lee et al themselves used hemispherical configuration.

![Figure 1](image.png)

**Figure 1.** Comparison of experimental results based on the previous studies. (Lee et al, 2016)

Based on the research of Babcock [2], he proved that among different type of imperfections, the geometric imperfections play the most significant role in decreasing the buckling strength of structure. Moreover, the knockdown factor is more highly sensitive for shells with configuration of H/R (height-to-base radius of shell), which to be found hemispherical, H/R = 1, has the most significant drop in the beginning of imperfections [5]. The higher immediate drop in beginning can also occur as the shells have bigger imperfections and the knockdown factor graph has a perpendicular be like shape rather than smaller imperfections with a smooth curving progression [6]. The comparison of both conditions can be seen in the figure 2.

In finite element modelling, the geometric imperfections can be modelled with several methods. One of the methods is the perturbation cutout, where a cutout with certain depth is given in the apex of the shells, which represents the most realistic imperfections on shells. This method has a major advantage since there is a characteristic threshold for the minimum buckling pressure and can be used for parametric studies [5].

Based on the work of [7], it is important to take into account of the transverse shear deformation effect to the buckling strength of shells. They made a comparison between a previous study (using thin shell theory) and their work (using Mindlin shell theory) and found to be both had quite difference as the thickness increases. They found that in \( \eta \) ratio of 50, the Mindlin shell theory was 0.7 percent lower; \( \eta \) of 25, was 2.4 percent lower; and \( \eta \) of 10, was 5.2 percent lower. Note that the attained buckling pressure was normalized with the Young’s modulus of structures.
3. Methodology
The analyses are conducted using FEM and performed using MIDAS FEA program. Shell elements are considered as membranes, which only can resist axial deformation (2 DOF of in-plane translation). However, as the shells need to take account into transverse shear deformation, the elements are also treated as DKMQ (Discrete Kirchoff-Mindlin Quadrilateral) elements, which consider the normal translation and also rotation about plane axes (x and y). The in-plane and out-of-plane deformation will be analyzed separately since the elements are considered as discrete elements.

Modelling is started by assigning material and geometry parameters as mentioned in Table 1. Multiple FE modellings are conducted to apply various geometric imperfections on the shells. Sets of geometric imperfections for each thickness are displayed in table 2. Shell with thickness of 50 mm is given additional cutouts to search the constant value of its knockdown factor; and additional cutouts for thickness of 37 mm is used to make an exploration for the progression of knockdown factor curve. The illustration of modelling and geometry of the imperfection are as in figure 3 and figure 4.

Table 1. Material and geometry parameter.

| Material Parameter          | Value     |
|----------------------------|-----------|
| Young’s modulus, E (MPa)   | 200000    |
| Poisson’s ratio, μ          | 0.3       |

| Geometry Parameter         | Value     |
|----------------------------|-----------|
| Radius, R (mm)             | 4000      |
| Thickness, t (mm)          | 10, 16, 20, 37, 50 |
| Boundary condition         | Fixed at base |

Table 2. Various sets of perturbation cutouts.

| Group | Depth of cutout (mm) | CR/R       | Thickness |
|-------|----------------------|------------|-----------|
|       |                      |            | 10 mm     | 16 mm     | 20 mm     | 37 mm     | 50 mm     |
| I     | 2.5 – 80             | 0.035 – 0.199 | √         | √         |           |           |           |
| II    | 100 – 600            | 0.222 – 0.527 | √         | √         |           |           |           |
| III-A | 0.2 – 2.4            | 0.01 – 0.035 |           |           | √         |           |           |
| III-B | 85 – 500             | 0.205 – 0.484 |           |           |           | √         |           |
The simulation is conducted into 2 analyses. First, models should be analyzed in linear buckling to obtain the linear buckling pressure and mode shape of the buckled shell. Second, the buckled shape is used as the modelled with linear buckling pressure and ran into nonlinear static analysis. From this second analysis, the load factor should be obtained and processed with the linear buckling pressure. The product is the actual buckling pressure with the corresponding imperfections. Knockdown factor is also obtained, expressed as

\[ Kd = \frac{p_{\text{exp}}}{p_{\text{cr}}} \]  

(2)

4. Result and discussion

4.1. Knockdown factor

Figure 5 shows the knockdown factor graph versus CR/R (cutout to base radius of shell), obtained from all types of thickness under various geometric imperfections. It is shown that even at a small imperfection, CR/R ≤ 0.05, the knockdown factor has decreased sharply. This represents that the critical buckling load of this shell structure is highly sensitive to imperfections. The knockdown factor continues decreasing as the imperfection getting larger, yet with rather slow progression. It is also found that shell with greater thickness produces a slightly higher knockdown factor than the smaller thickness. Also, in the same figure, it can be seen that for t = 10, 16 and 20 mm, each graph starts to coincide at CR/R ≥ 0.18 and obtaining a minimum Kd about 0.1. Both shells with t = 37 and 50 mm also coincides with these 3 graphs at the higher CR/R value and reach for the similar minimum Kd about 0.1. According to these minimum knockdown factors, the exact value and its corresponding \( \eta \) (radius to thickness of shell) will be displayed in Table 3 and the relationship will be plotted in Figure 4. These results can lead to basic guideline for designing a domed spherical shell, the critical buckling load as calculated from equation (1) has to be reduced to 10% to prevent buckling to happen on structures.

| R (mm) | t (mm) | \( \eta \) | Min Kd |
|--------|--------|----------|--------|
| 4000   | 10     | 400      | 0.0795 |
| 4000   | 16     | 250      | 0.0914 |
| 4000   | 20     | 200      | 0.1021 |
| 4000   | 37     | 108      | 0.0877 |
| 4000   | 50     | 80       | 0.0920 |
4.2. Exploration of knockdown factor graph

In order to give a better result, an exploration was also made thoroughly to the shell with thickness of 37 mm. The exploration was made in two regions; first, extra detailing in the very small imperfection; then, extending the graph from the previous result. The exploration result is as shown in figure 6.
It is clear to see that in the very beginning of imperfection (CR/R), there is a sudden drop of the knockdown factor and undergoes for an upsurge to a local peak, then the graph continues to decrease slowly as what is shown in the previous section. Observing at \( CR/R \approx 0.25 \), the progression of the graph starts to rise for a slightly increasement, then just proceeds for more slowly movement, and finally it coincides with the linear condition at \( CR/R \geq 0.4 \). The presence of linear knockdown factor curve is to show the coincidence point with the nonlinear condition, and also to show that linear analysis provides a result which beyond safety, especially in the very beginning of imperfection. The exploration also proves what cannot be obtained on small experimental shells in small imperfections [4]. This inclination shows a good agreement as what was mentioned earlier by Wagner et al [5] and Hutchinson [6].

5. Conclusions

Finite element simulations have been done for analyzing the critical buckling load of domed spherical shells under various geometric imperfections. The result shows that the models are highly sensitive to geometric imperfections, as can be seen even at a tiny imperfection, there has been already a sudden drop of the buckling strength. Every thickness shows a similar inclination, both in the beginning of sharp deteriorations and slow progression until they reach for the minimum knockdown factor. Result shows that dome structure with any imperfection, the critical buckling load should be reduced to 10% of its classical predicted load, as this could grant a conservative yet safe basic guideline for designing spherical shell structures.

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