STBC Identification for Multi-User Uplink SC-FDMA Asynchronous Transmissions Exploiting Iterative Soft Information Feedback of Error Correcting Codes

M. MAREY¹ (Senior Member, IEEE) and H. MOSTAFA² (Member, IEEE)

¹Smart Systems Engineering Laboratory, College of Engineering, Prince Sultan University, Riyadh 11586, Saudi Arabia (e-mail: mfmmarey@psu.edu.sa)
²Department of Information Technology, College of Computer and Information Sciences, Princess Nourah bint Abdulrahman University, P.O. Box 84428, Riyadh 11671, Saudi Arabia (e-mail: HFMostafa@psu.edu.sa)

Corresponding author: M. Marey (e-mail: mfmmarey@psu.edu.sa).

ABSTRACT With the advancement and widespread implementation of multiple-input multiple-output (MIMO) wireless communication systems over the last decade, space-time block coding (STBC) identification has become a critical task for intelligent radios. Previous examinations of STBC identification were focused on single-user transmissions over single-carrier and multi-carrier systems in combination with uncoded broadcasts. Practical systems, on the other hand, contain many users and employ error-correcting codes. For the first time in literature, this work explores the problem of STBC identification for multi-user uplink transmissions in single-carrier frequency division multiple access (SC-FDMA) systems. We take another step closer to real systems by addressing asynchronous transmissions and by conducting multi-user channel estimation. We also exploit the outputs of the channel decoder, which is usually used in many practical systems, to improve the identification and estimation processes. The mathematical analysis demonstrates that the maximum-likelihood (ML) solution of STBC identification, channel estimation, and synchronization can be executed by an iterative approach. The space-alternating generalized expectation-maximization (SAGE) algorithm is used to separate the overlaid signals arriving at the base-station (BS). The parameters under consideration for each user are then updated using an expectation-maximization (EM) processor. Simulation results show that the proposed architecture outperforms other identification methods published in the literature while maintaining a reasonable level of computational complexity.

INDEX TERMS STBCs identification, SC-FDMA, SAGE, ML estimation

I. INTRODUCTION

Analysis of wireless signals, aimed at determining the specific transmission parameters of the transmitter used, has been an prominent research area for decades. This analysis is generally referred to as signal identification with military and civilian implications. This has long been used in military applications such as signal interception, radio surveillance, interference detection and mitigation, jamming detection, and electronic warfare [1], [2]. The advent of intelligent radios, reconfigurable transceivers having the ability to alter their transmission settings such as modulation format [3]–[6] and channel coding rate [7]–[11], has heightened interest in signal identification systems in the context of recent civilian applications such as cellular mobile systems and WiFi networks [12], [13].

Signal identification for multiple-input multiple output (MIMO) systems poses unique technical issues that must be taken into account during identifier development. Not only is it difficult to estimate the number of transmit antennas and the type of transmit-side antenna arrangement in such systems, but there is also the issue of estimation and/or tracking of the signal characteristics and the wireless channel. Space-time block
coding (STBC) is a MIMO approach in which many copies of a data stream are broadcasted in different time slots via multiple transmit antennas, achieving diversity with a simple receiver structure.

In single-carrier transmissions contexts, a group of methods relying on the fourth-order moment is suggested to recognize between two STBC signals, Alamouti (AL) and spatial multiplexing (SM) over Nakagami frequency-flat channels in [14]. The authors of [15] investigate the usage of second-order cyclostationarity of two different received signals to classify among several STBC signals. The dispersive characteristics of multipath fading channels is utilized in [16] to discriminate between AL and SM STBC signals. A maximum-likelihood (ML) technique [17] and a Frobenius norm [18] are designed to distinguish between STBC signals. Recently, a convolutional neural network is used to create an STBC classification method [19]. The most significant drawback of [19] is the requirement for a large amount of data in order to accomplish training. In fact, it is not always possible to obtain training data from a source. For example, the categorization of military signals is an excellent illustration of this. Additionally, identifiers are typically implemented on small portable devices with less processing capability. Therefore, if and when it becomes required, retraining will be extremely difficult. This demonstrates the urgent necessity for proposing non-machine learning-based identification methods.

The existing standards’ high data rate requirements demand transmissions over wide-band frequency-selective channels. The combination of MIMO systems and multi-carrier (MC) transmissions offers a fascinating solution to the problem of inter-symbol interference, which is a key concern in these conditions. Several wireless communication standards, including WiMAX, LTE, 5G cellular communications, IEEE 802.11n, IEEE 802.11ac, and IEEE 802.11ax, have used MC-MIMO transmissions [20]–[22]. In the framework of MC transmissions, the authors of [23]–[26] use the time-domain correlation functions of two different received signals to classify between AL and SM signals.

Although the earlier studies focused on single-user transmissions, in most real communications systems, the STBC identification process should be carried out in the presence of many users’ signals. The main challenge in these multi-user scenarios is that different signals experience different unknown STBC signals, propagation delays, and channel coefficients. Also, the STBC identification process is hampered by multiple access interference (MAI). To the best of the authors’ knowledge, this work is the first of its kind to investigate the problem of STBC classification in an uplink multi-user scenario of single-carrier frequency division multiple access (SC-FDMA) systems. We provide a novel strategy in which the proposed identifier benefits from the soft information outputs of channel decoders, which are used in a variety of real SC-FDMA systems. The proposed identification algorithm operates also in uplink orthogonal frequency division multiple access (OFDMA) scenarios. Simply one disconnects the FFT and IFFT units from the transmitter and receiving sides, respectively.

The mathematical study in this work reveals that the actual ML solution to STBC identification of multi-user SC-FDMA uplink scenarios is too sophisticated for real applications. Therefore, we resort to a new technique that acts iteratively. The overlaid signals arriving at the base-station (BS) are detached using the space-alternating generalized expectation-maximization (SAGE) algorithm at each iteration [27], [28]. The SAGE algorithm’s expectation step uses the soft information of the channel decoders to reduce MAI created by other asynchronous users. This replaces the sophisticated multi-dimensional search with a series of one-dimensional searches. The resulting design is evocative of parallel STBC identification for multiple users, in which MAI is re-constructed and eliminated from the received signal to optimize each user’s identification process. Channel estimation and timing synchronization algorithms are also designed to complement the proposed identification technique. Notably, the feedback provided by an uncoded data detector has been used to solve several difficulties that have arisen in multi-carrier systems, such as equalization [29], [30] and frequency synchronization [27], [28]. However, this is the first time it has been used in conjunction with coded transmissions in STBC classification for such systems.

The remainder of the work is broken down into the following sections. The problem formulation and system model are discussed in Section II. The proposed identification algorithm is described in Section III. Practical considerations and interpretations are reported in Section IV. Simulation results are discussed in Section V. Concluding remarks are presented in Section VI.

II. SIGNAL MODEL AND PROBLEM FORMULATION

We consider a wireless uplink multi-user SC-FDMA system with $K$ active users as shown in figure 1. The total number of subcarriers $M$ is split into $U \geq K$ subgroups. Each subgroup has $M_k = M / U$ subcarriers which are uniquely allocated to an active user, not to other users in the same time slot. Mathematically speaking, the set of subcarriers reserved to user $k$, $S^{(k)}$, satisfies $\bigcup_{k=0}^{K-1} S^{(k)} = \{0, 1, \ldots, M - 1\}$ and $S^{(k)} \cap S^{(k')} = \emptyset$ for $k \neq k'$. Here $\emptyset$ refers to the null set. Each user $k$ has $p^{(k)}$ transmit antenna elements.

A. Transmitter
A sequence of binary information digits of user $k$ passes through a channel encoder of rate $c^{(k)}$ which adds redundancy bits to correct errors produced in the transmission. The coded bits are interleaved and then mapped to complex data symbols which are selected from a given signal constellation $\Phi^{(k)}$ of unit energy. Here, we do not impose any constraints on modulation, coding, and interleaving parameters of each user. A few pilots are encapsulated into data symbols to initialize the identification process as shown later on. The resulting sequence is split into $Z^{(k)}$ vectors, each has $M_s$ symbols. Let $\mathbf{a}_z^{(k)} = \left[ a_z^{(k)}(0), \cdots, a_z^{(k)}(M_s - 1) \right]$ be the $z$th vector, with $a_z^{(k)}(m)$ being its $m$th element. After the $M_s$-point fast Fourier transform (FFT) operation, the corresponding frequency-domain samples $\mathbf{d}_z^{(k)} = \left[ d_z^{(k)}(0), \cdots, d_z^{(k)}(M_s - 1) \right]$ are expressed as

$$d_z^{(k)}(l) = \sum_{m=0}^{M_s-1} a_z^{(k)}(m) \exp \left( -j2\pi ml/M_s \right), \quad (1)$$

where $d_z^{(k)}(l)$ is the $l$th sample of $\mathbf{d}_z^{(k)}$ and $j = \sqrt{-1}$. The elements of $\mathbf{d}_z^{(k)}$ modulate $M_s$ subcarriers accord-
ing to interleaved mapping defined as
\[
d_{z}^{(k)}(m) = \begin{cases} 
  d_{z}^{(k)}(l) & m = k + lU \\
  0 & \text{otherwise,}
\end{cases}
\] (2)
and localized mapping characterized as
\[
d_{z}^{(k)}(m) = \begin{cases} 
  d_{z}^{(k)}(l) & m = kM_{s} + l \\
  0 & \text{otherwise,}
\end{cases}
\] (3)
where \(0 \leq k < K - 1\) and \(0 \leq l, m \leq M_{s} - 1\). The conversion of \([d_{z}^{(k)}(0), \ldots, d_{z}^{(k)}(M - 1)]\) into an SC-FDMA symbol is performed by using an M-point inverse FFT (IFFT) with including a cyclic prefix of length \(v\). We write the \(m\)th sample of \(z\)th SC-FDMA symbol \(x_{z}^{(k)}\) as
\[
x_{z}^{(k)}(m_1) = \sum_{m_2=0}^{M - 1} d_{z}^{(k)}(m_2) \exp(j2\pi m_2 M / M),
\] (4)
where \(m_1 = -v, \ldots, M - 1\). SC-FDMA symbols of user \(k, x^{(k)} = [x_{0}^{(k)}, \ldots, x_{Z_{k} - 1}^{(k)}]\), are fed to a space-time encoder which transmits each \(Z_{k}\) SC-FDMA symbols through \(p^{(k)}\) antennas in \(Z_{2}\) time slots. For example, the STBC \(\{Z_{1} = 2, Z_{2} = 2, p^{(k)} = 2\}\) called Alamouti code [31] conveys a block of two SC-FDMA symbols \(x_{z}^{(k)}, x_{z+1}^{(k)}\) through two antenna elements in two consecutive time slots. In the first time slot, \(x_{z}^{(k)}\) and \(x_{z+1}^{(k)}\) are broadcasted from the first and second antenna elements, respectively. In the subsequent time slot, \(x_{z+1}^{(k)}\) and \(x_{z}^{(k)}\) SC-FDMA symbols are transmitted from the first and second antenna elements, respectively. Here, \(^*\) refers to conjugate operation.

Each user \(k\) selects a STBC scheme, denoted as \(\omega^{(k)}\), from a pool of candidates. The transmitted signal from antenna \(p\) is created by concatenating all time-domain vectors broadcasted in different time slots, \(\tilde{c}^{(k,p)}_{\omega^{(k)}} = [c_{0}^{(k,p)}, \ldots, c_{N^{(k)}}^{(k,p)}]\), where \(N^{(k)} = Z_{k} Z_{1} / Z_{2}\) and \(c_{\omega^{(k)}}^{(k,p)}\) is related to \(x^{(k)}\) through the specific structure of \(\omega^{(k)}\). Note that we attach \(\omega^{(k)}\) to \(c^{(k,p)}_{\omega^{(k)}}\) as a subscript to emphasize that the structure of vector \(c^{(k,p)}_{\omega^{(k)}}\) depends on STBC \(\omega^{(k)}\). Finally, each transmit antenna of user \(k\) communicates with the BS through unknown \(L\)-path wireless channel, \(h^{(k,p)} = [h^{(k,p)}(0), \ldots, h^{(k,p)}(L - 1)]\).

### B. Receiver

Because users are placed at different positions from the BS, their received signals are subjected to distinct propagation delays. The propagation delay of each user is divided into an integer part and a fractional part with respect to the sampling interval. The fractional part can be incorporated into the channel impulse response (CIR) of each user as reported in [28], therefore, it does not be included in the following analysis. Denoting \(\mu^{(k)}\) as the integer part of the propagation delay of user \(k\), the received signal at the BS is expressed as
\[
r' = \sum_{k=0}^{K-1} \sum_{p=0}^{p^{(k)}-1} c^{(k,p)}_{\omega^{(k)}} (\mu^{(k)}) \odot h^{(k,p)} + w',
\] (5)
where \(c^{(k,p)}_{\omega^{(k)}} = [0_{1 \times \mu^{(k)}} \odot \bar{c}^{(k,p)}_{\omega^{(k)}}]\) with \(0_{1 \times \mu^{(k)}}\) being all zero sequence of length \(\mu^{(k)}\), \(\odot\) refers to the convolution operation, and \(w'\) is the corresponding additive white Gaussian noise (AWGN). The aim is to utilize the received signal \(r\) to identify the type of STBC \(\omega^{(k)}\) under unavailability of \(h^{(k,p)}\) and propagation delay \(\mu^{(k)}\) for \(k = 0, \ldots, K - 1\), and \(p = 0, \ldots, p^{(k)} - 1\). This is a prerequisite for performing multi-user data detection.

### III. PROPOSED ALGORITHM

For the sake of mathematical convenience, the expression of (5) is written in a matrix form as
\[
r = \sum_{k=0}^{K-1} \sum_{p=0}^{p^{(k)}-1} \tilde{c}^{(k,p)}_{\omega^{(k)}} (\mu^{(k)}) h^{(k,p)} + w,
\] (9)
where \(r = r^{T}\) and \(w = w'^{T}\). Here \((\cdot)^{T}\) refers to vector transpose operator and \(\tilde{c}^{(k,p)}_{\omega^{(k)}} (\mu^{(k)})\) is given as
\[
\tilde{c}^{(k,p)}_{\omega^{(k)}} (\mu^{(k)}) = \begin{bmatrix}
0_{\mu^{(k)} \times (L-1)} \\
C^{(k,p)}_{\omega^{(k)}} (\mu^{(k)})
\end{bmatrix},
\] (10)
where \(\mu_{\text{max}}\) is the maximum possible integer propagation delay\(^1\), \(0_{1 \times 2}\) is all-zero matrix of size \(1 \times 2\), and \(C^{(k,p)}_{\omega^{(k)}} (\mu^{(k)})\) is an \((M + v) Z^{(k)} + L - 1\) \((L - 1)\) matrix created as
\[
C^{(k,p)}_{\omega^{(k)}} (v_1, v_2) = \begin{cases}
\tilde{c}^{(k,p)}_{\omega^{(k)}} (v_1 - v_2) & \text{for } v_1 = 0, \ldots, s (M + v) Z^{(k)} + L - 1, \\
0 & \text{otherwise,}
\end{cases}
\] (11)
where \(\tilde{c}^{(k,p)}_{\omega^{(k)}} (v_1, v_2)\) is the element located at row \(v_1\) and column \(v_2\) of matrix \(\tilde{c}^{(k,p)}_{\omega^{(k)}}\) and \(\tilde{c}^{(k,p)}_{\omega^{(k)}} (v_1 - v_2)\) is the \((v_1 - v_2)\)th element of vector \(\tilde{c}^{(k,p)}_{\omega^{(k)}}\).

Bear in mind (9), one writes the ML estimates of the unknown parameters as shown in (6), (7), and (8). Here \(\diamond\) is the trial value of variable \(\odot\) and \(\Pr (\text{close} | \diamond)\) is the probability density function of given \(\diamond\). A closer look at (7) reveals that the ML algorithm performs

\(^1\)In practice, \(\mu_{\text{max}}\) is expressed as a function of the cell radius as \(\mu_{\text{max}} \approx \text{cell radius} / \text{speed of light}\). Since \(\mu_{\text{max}}\) does not depend on users locations, user superscript \(k\) is dropped from \(\mu_{\text{max}}\) without the loss of generality.
that is required to maximize the likelihood function

\[
[\hat{\omega}^{(0)}, \ldots, \hat{\omega}^{(K-1)}, \hat{\mu}^{(0)}, \ldots, \hat{\mu}^{(K-1)}, \hat{\mathbf{h}}^{(0)}, \ldots, \hat{\mathbf{h}}^{(K-1,p(K-1))}] = \text{arg max} \Pr \left( \mathbf{r} \left| \omega^{(0)}, \mu^{(0)}, \mathbf{h}^{(0)}, \ldots, \omega^{(K-1)}, \mu^{(K-1)}, \mathbf{h}^{(K-1,p(K-1))} \right. \right),
\]

where,

\[
\Pr \left( \mathbf{r} \left| \omega^{(0)}, \mu^{(0)}, \mathbf{h}^{(0)}, \ldots, \omega^{(K-1)}, \mu^{(K-1)}, \mathbf{h}^{(K-1,p(K-1))} \right. \right) \propto \sum \Pr \left( \mathbf{r} \left| \hat{C}^{(0)}_{\omega^{(0)}}(\hat{\mu}^{(0)}), \mathbf{h}^{(0)}, \ldots, \hat{C}^{(K-1,p(K-1))}_{\omega^{(K-1)}}(\hat{\mu}^{(K-1)}), \hat{\mathbf{h}}^{(K-1,p(K-1))} \right. \right).
\]

and

\[
\Pr \left( \mathbf{r} \left| \hat{C}^{(0)}_{\omega^{(0)}}(\hat{\mu}^{(0)}), \mathbf{h}^{(0)}, \ldots, \hat{C}^{(K-1,p(K-1))}_{\omega^{(K-1)}}(\hat{\mu}^{(K-1)}), \hat{\mathbf{h}}^{(K-1,p(K-1))} \right. \right) \propto \exp \left( \sum_{k=0}^{K-1} \sum_{p=0}^{p(K-1)} \left\| \mathbf{r} - \hat{C}^{(k)}_{\omega^{(k)}}(\hat{\mu}^{(k)}), \hat{\mathbf{h}}^{(k,p)} \right\|^2 \right).
\]

averaging over the transmission matrices of all users. This is because the original data symbols are unknown at the BS. However, the real implementation of (7) is not possible because it demands huge computations, which are highly undesirable in practical systems.

The expectation-maximization (EM) procedure is useful in this context as it provides an iterative technique to estimate the ML solution in the presence of nuisance parameters. The procedure updates all unknown variables simultaneously, resulting in a time-consuming and complicated search procedure due to the large number of dimensions involved. In contrast, the SAGE methodology separates the unknown variables into numerous non-overlapping groups and then utilizes the EM algorithm to modify each group one at a time. Thus, the SAGE technique can be thought of as an upgraded version of the EM algorithm, which improves the convergence rate significantly with preserving the advantages of numerical simplicity and stability. The SAGE approach has been widely utilized to tackle parameter estimate problems in multcarrier systems, such as synchronization and channel estimation [27, 28]. It is the first time that the SAGE algorithm has been utilized to identify STBC for uplink SC-FDMA systems using channel coding outputs, which is a departure from its traditional context of parameter estimation with uncoded transmissions.

Each iteration of SAGE comprises of cycles rather than estimating all parameters at once. By maximizing the conditional expectation of the log-likelihood of the augmented data corresponding to a cycle, the parameter subset associated with this cycle is updated. As a result, the complex multidimensional search problem that is required to maximize the likelihood function is reduced to several one-dimensional effortless search problems.

The mathematical details of the proposed SAGE procedure for computing the parameters under consideration are provided as follows. We divide the unknown parameters into K non-overlapping subgroups \(\{\omega^{(0)}, \mu^{(0)}, \mathbf{h}^{(0,p(0))}, \ldots, \omega^{(K-1)}, \mu^{(K-1)}, \mathbf{h}^{(K-1,p(K-1))}\}\). A single user’s parameters are updated at a time. This means that an iteration is made up of K cycles, each of which updates the user’s settings. Given initial estimates, the \((i+1)\)th iteration consists of the following steps:

- During the \(k\)th cycle, we update the parameters of user \(k\)’ while the other users’ parameters remain unaltered.
- Subtracting all other users’ MAI from the total received signal produces

\[
\mathbf{y}_{k'} = \mathbf{r} - \sum_{k=0, k \neq k'}^{K-1} \sum_{p=0}^{p(k)} \Omega^{(k,p)} \left( \hat{\omega}^{(k)}(i), \hat{\mu}^{(k)}(i) \right) \hat{\mathbf{h}}^{(k,p)}(i),
\]

where \(\mathbf{y}_{k'}\) is the received signal component of user \(k'\), \(\Omega^{(k,p)} \left( \hat{\omega}^{(k)}(i), \hat{\mu}^{(k)}(i) \right)\) is the a posteriori expectation of the transmission matrix \(\mathbf{C}^{(k,p)}_{\omega^{(k)}}(\mu^{(k)})\) given in (10), and \(\hat{\mu}^{(k)}(i), \hat{\omega}^{(k)}(i), \hat{\mathbf{h}}^{(k,p)}(i)\) and \(\hat{\mathbf{h}}^{(k,p)}(i)\) are the estimates of \(\mu^{(k)}, \omega^{(k)}, \mu^{(k)}\), and \(\mathbf{h}^{(k,p)}\), respectively, at iteration \(i\). Note that the transmission matrix of \(\mathbf{C}^{(k,p)}_{\omega^{(k)}}(\mu^{(k)})\) is inaccessible since the information symbols are unknown at the BS. As a result, \(\Omega^{(k,p)} \left( \hat{\omega}^{(k)}(i), \hat{\mu}^{(k)}(i) \right)\) is used in (14) instead of \(\mathbf{C}^{(k,p)}_{\omega^{(k)}}(\mu^{(k)})\). Mathemati-
In the EM processor’s expectation step, given the parameters of user $k$, one rewrites (17) as

$$
\sum_{p=0}^{p(k')-1} \left\{ 2R \left( y_k^H \Omega \left( \mu(k'), \sigma(k') \right) \right) - h(k',p) H (y_k^H \sigma(k')) \right\}
$$

To compute the updated values of the parameter set of user $k'$, we describe the log-likelihood function as

$$
\mathcal{L} = \log \text{Pr} \left( y_k' \mid \hat{C}^{(k',p)}_{\mu(k')} \left( \mu(k') \right), h(k',p) \right).
$$

Bearing in mind that

$$
\text{Pr} (y_k' \mid \hat{C}^{(k',p)}_{\mu(k')} \left( \mu(k') \right), h(k',p)) \propto \exp \left( \frac{-1}{2} \sum_{p=0}^{p(k')-1} \| y_k' - \hat{C}^{(k',p)}_{\mu(k')} \left( \mu(k') \right) h(k',p) \|^2 \right),
$$

then, after eliminating the useless elements, we rewrite (17) as

$$
\mathcal{L} \propto 2 \sum_{p=0}^{p(k')-1} \left\{ R \left( y_k^H \hat{C}^{(k',p)}_{\mu(k')} \left( \mu(k') \right) h(k',p) \right) - h(k',p) H \hat{C}^{(k',p)}_{\mu(k')} \left( \mu(k') \right) h(k',p) \right\}.
$$

IV. PRACTICAL CONSIDERATIONS AND INTERPRETATIONS

The following practical concerns with the suggested iterative identification, estimation, and data detection structure are worth mentioning:

A. EXPECTATION OF TRANSMISSION MATRICES

As observed from (15), (20), and (21), the proposed design relies on determining the expectation of each user’s transmission matrix $\Omega \left( \hat{\sigma}(k) (i), \hat{\mu}(k) (i) \right)$. How to compute this matrix in practice is a question that emerges. According to (15), $\Omega^{(k,p)} \left( \hat{\sigma}(k) (i), \hat{\mu}(k) (i) \right)$ can be calculated by substituting each matrix element with its a posteriori expectation. Bearing (4) in mind,
the a posteriori expectation of transmitted sample \( x_z^{(k)}(m_1) \) is expressed as
\[
E \left[ x_z^{(k)}(m_1) \bigg| y_{k, \hat{\omega}^{(k)}(i)}, \hat{\mu}^{(k)}(i) \right] = \\
\sum_{m_2=0}^{\Lambda-1} E \left[ d_z^{(k)}(m_2) \bigg| y_{k, \hat{\omega}^{(k)}(i)}, \hat{\mu}^{(k)}(i) \right] \times \\
\exp \left( j2\pi m_1 m_2 / \Lambda \right),
\]
where
\[
E \left[ d_z^{(k)}(m_2) \bigg| y_{k, \hat{\omega}^{(k)}(i)}, \hat{\mu}^{(k)}(i) \right] = \\
\sum_{d_z^{(k)}(m_2) \in \Phi^{(k)}} d_z^{(k)}(m_2) \Pr \left( d_z^{(k)}(m_2) \bigg| y_{k, \hat{\omega}^{(k)}(i)}, \hat{\mu}^{(k)}(i) \right).
\]
As noted from (23), the key issue is to compute the a posteriori probability of \( \Pr \left( d_z^{(k)}(m_2) \bigg| y_{k, \hat{\omega}^{(k)}(i)}, \hat{\mu}^{(k)}(i) \right) \). Fortunately, the decoders of modern error-correcting codes include convolutional, turbo, and low-density parity check codes computes this probability during their iterative nature [32], [33]. As a result, we exploit this probability to support the proposed identification and estimation algorithm without causing extra overhead on the decoding process. The conceptual block diagram of the proposed design is shown in figure 2.

**B. CHANNEL DECODER UPDATE**

We must re-compute the a posteriori probability \( \Pr \left( d_z^{(k)}(m_2) \bigg| y_{k, \hat{\omega}^{(k)}(i)}, \hat{\mu}^{(k)}(i) \right) \) every time we update \( \hat{\omega}^{(k)}(i), \hat{\mu}^{(k)}(i), \text{and} h^{(k,p)} \) for every user \( k \). This necessitates the resetting of the channel decoder, which results in numerous iterations. To reduce this overhead, we employ the embedded estimation technique [34], in which the channel decoder is not reset when the parameters \( \hat{\omega}^{(k)}(i), \hat{\mu}^{(k)}(i), \text{and} h^{(k,p)} \) are updated, but the extrinsic and a priori probabilities from the previous iteration of the channel decoder are kept unchanged. The overhead associated with the suggested iterative
procedure becomes tolerable in this instance.

C. COMPUTATIONAL COMPLEXITY

We analyze the computational complexity of the proposed SAGE-based identification algorithm in terms of the number of floating operations (flops), with 6 and 2 flops required for multiplication and addition of two complex numbers, respectively. We also assume that the $M$-point FFT algorithm requires $5M \log_2 M$ flops, that multiplication of two complex matrices with dimensions of $q_1 \times q_2$ and $q_2 \times q_3$ requires $8q_1q_2q_3$ flops, that addition/subtraction of two complex matrices with dimensions of $q_1 \times q_2$ each requires $2q_1q_2$ flops, and that the inverse of a complex matrix with dimensions of $\psi_3 \times \psi_3$ requires $\psi_3^3$ flops [35], [36]. As shown in Table 1, the precise computations of the required computational complexity $\psi$ per iteration per user is

$$
\psi = 16\lambda (L-1)^2 + 8(L-1)^3 + 8\lambda (L-1)
+ 8\mu_{\text{max}}Z(L-1)(\lambda + 2),
$$

where $\lambda = (M + \nu)g + L - 1$ with $g$ being the maximum number of input STBC blocks and $Z$ is the number of STBC candidates. As a numerical example, we consider system specifications of $M = 1024$, $\nu = 7$, $g = 10$, $L = 6$, $\mu_{\text{max}} = 30$, $Z = 5$. This provides $8.3 \times 10^6$ flops, which yield a run-time of 66.4 $\mu$s with a central processing unit of 1 Teraflops per second [3]. This runtime is clearly appropriate in terms of actual execution.

D. INITIAL ESTIMATES

Users send a few pilot symbols to the BS in order to initialized the proposed SAGE-based algorithm. As the number of pilot symbols grows, the first estimates $\hat{\phi}^{(k)}(0)$, $\hat{\mu}^{(k)}(0)$, and $\hat{\mathbf{h}}^{(k,p)}(0)$ improve. Increasing the number of pilot symbols, on the other hand, reduces the amount of energy available for data symbols and increases the needed bandwidth. As a result, the number of pilot symbols to data symbols must be remain as low as feasible. In the sense that it produces good identification performance with minimal throughput loss, the suggested algorithm takes advantage of the data symbols’ soft information supplied by the channel decoders. Without the use of supplemental pilot symbols, this iteratively improves the first estimates. The starting values of $\omega^{(k)}(0)$, $\mu^{(k)}(0)$, and $\mathbf{h}^{(k,p)}(0)$ are extracted from (15) by setting the entries in $\Omega \left( \mu^{(k)}, \omega^{(k)} \right)$ to simply the contribution of pilot symbols.

V. SIMULATION RESULTS

The proposed STBC identification technique was investigated using Monte Carlo simulations. If not stated differently, we considered a SC-FDMA system with the following parameters.

- The number of active users was $K = 8$.
- The number of total subcarriers was $M = 1024$.
- The number of allocated subcarriers per user was $M_s = 128$.
- The number of cyclic prefix samples was $\nu = 16$.
- The interleaved sub-carrier assignment was used.
- The allocated signal constellation $\phi^{(k)}$ for each user was randomly selected from a pool of eight higher order QAM constellations, 4-QAM, 8-QAM, 16-QAM,..., 512-QAM. Similar results can be accomplished with ease for PSK signals.
- A convolutional code of rate 1/2, constraint length 5, and generator polynomials $(23)_8$ and $(35)_8$ was employed for each user.
- Pilots symbols of length $P_s = 40$ were inserted to initialized the identification process.
- Each wireless channel, $\mathbf{h}^{(k,p)}$ between antenna $p$ of user $k$ and the BS was generated using 15 paths where each one has an exponential power delay profile as [24], [37]:

$$
\sigma^2_{\text{ch}}(l) = \Xi_{\text{ch}} \exp \left( -l/10 \right), \quad l = 0, \ldots, 14
$$

where $\Xi_{\text{ch}}$ was selected in such a way that the average energy was equal to one.

- The maximum propagation delay normalized to the sampling duration was $\mu_{\text{max}} = 50$, and each user’s propagation delay was chosen at random.
- Each user was assigned an STBC at random from the list of $\{\text{ST1, ST2, ST3, ST4, ST5}\}$ where those candidates’ transmission matrices are shown in (26a-26e) [15], [22]. It is worth mentioning that the proposed identifier can be employed with any number of STBCs. Those five codes are offered differently, we considered a SC-FDMA system with the following parameters.

$$
\sigma^2_{\text{ch}}(l) = \Xi_{\text{ch}} \exp \left( -l/10 \right), \quad l = 0, \ldots, 14
$$

where $\Xi_{\text{ch}}$ was selected in such a way that the average energy was equal to one.

- The maximum propagation delay normalized to the sampling duration was $\mu_{\text{max}} = 50$, and each user’s propagation delay was chosen at random.
- Each user was assigned an STBC at random from the list of $\{\text{ST1, ST2, ST3, ST4, ST5}\}$ where those candidates’ transmission matrices are shown in (26a-26e) [15], [22]. It is worth mentioning that the proposed identifier can be employed with any number of STBCs. Those five codes are offered solely for the purpose of simulating various scenarios.
TABLE 1. Computational complexity per iteration.

| Equation | Required flops |
|----------|----------------|
| Channel update (20) | $16\lambda (L - 1)^3 + 8 (L - 1)^2 + 8\lambda (L - 1)$ |
| Code and synchronization update (21) | $8f_{\text{max}} Z^2 (L - 1)(M + 2)$ |

![Comparison of identification performance between the proposed algorithm and the algorithm presented in [38] for single-user transmission](image)

**FIGURE 3.** Comparison of identification performance between the proposed algorithm and the algorithm presented in [38] for single-user transmission ($K = 1$), code set = {SM, AL}.

![Identification performance of the proposed algorithm as a function of the number of users at different SNR values. The number of iterations is seven and STBC set is {ST1, ST2, ST3, ST4, ST5}](image)

**FIGURE 4.** Identification performance of the proposed algorithm as a function of the number of users at different SNR values. The number of iterations is seven and STBC set is {ST1, ST2, ST3, ST4, ST5}.

![Proposed algorithm](image)

![Algorithm of [38]](image)

- The probability of incorrect identification $P_f$ was utilized as a figure of merit for the suggested identifier, probability mass function $P_m$ was employed to evaluate the proposed synchronizer, and the mean square estimation error (MSE) was used to assess channel estimation performance.

Figure 3 compares the STBC identification performance of the proposed algorithm to that of [38] for a single-user transmission over a wide range of signal-to-noise ratios (SNR). As far as the authors’ knowledge, the mentioned reference is the sole study in the literature dedicated to STBC identification for SC-FDMA systems, and it is restricted to the classification of AL and SM STBC signals over a single-user transmission.

To be fair, the proposed algorithm’s identification performance is also limited to these signals. As can be observed, the proposed algorithm’s identification improves with iterations. This is in line with the theoretical analysis presented in Section III. Furthermore, the suggested approach significantly outperforms [38]. This is due to the fact that we employ the channel decoder outputs to refine the identification process, whereas [38] does not.

Figure 4 describes the proposed algorithm’s STBC identification performance as a function of the number of users at different values of SNR, with the number of iterations being seven. Hereafter, each user selects a STBC signal among the five candidates shown in (26a-26e). It is worth noting that the values of $P_f$ at $K = 1$ of this figure are slightly greater than that in the preceding one. This is due to the fact that we are classifying among five STBC signals in this figure. However, in the prior one, we were limited to only two STBC signals. It has been observed from figure 4 that there is a performance loss in $P_f$ when compared to a single-user transmission. This is the result of MAI associated with the circumstance of multiple users. Despite the fact that we developed a method to eliminate this interference as indicated in (14), this deterioration, which is caused by residual interference, has a slight detrimental influence at high values of SNR and a large number of users. However, it almost vanishes otherwise. This is due to the fact that in the former case, the residual interference dominates with a lesser influence of AWGN.

In order to access iterative algorithm’s convergence rate, figure 5 evaluates the identification performance...
and channel estimation with perfect estimation of propagation delays. Third scenario performs STBC identification and propagation delays with perfect channel estimation. The last one has STBC identification with perfect estimation of propagation delays and channel impulse responses. The results show that there are no significant variations in the identification performance of the four scenarios. This validates the proposed STBC identification algorithm with channel estimates for asynchronous uplink SC-FDMA transmissions.

The mean square error of the proposed channel estimator is shown in Figure 7 as a function of SNR and the number of iterations. The MSE performance is low in the initialization step. However, by utilizing soft information of channel decodes as mentioned in (20), the suggested estimator’s performance improves until it converges at the seventh iteration.

Figure 8 illustrates the probability mass function of the propagation delay error, $\tau_\Delta = \frac{1}{K} \sum_{k=0}^{K-1} \left( \hat{\omega}(k) - \omega(k) \right)$, of the proposed synchronizer at SNR = 14 dB. As previously explained, the performance improves with iterations. This is consistent with the theoretical conclusions in Section III.

VI. CONCLUSIONS

This work investigated the problem of space-time block coding (STBC) identification for multi-user uplink asynchronous transmissions in single-carrier frequency division multiple access (SC-FDMA) systems. The mathematical analysis revealed that a space-alternating expectation-maximization (SAGE) approach can be used to implement the maximum-likelihood (ML) solution of STBC identification, channel estimation, and synchronization. The channel decoder’s a posteriori
probabilities were exploited to improve the quality of identification and estimation processes in an iterative manner. Simulation results indicated that the proposed design outperforms the existing identification algorithms reported in the literature, with a reasonable processing time. Despite the fact that the presented strategy has recognized to be an effective technique for STBC identification, it is constrained by the requirements of the modulation type and error correcting codes being used. The process of jointly identifying all of these factors will be undertaken in the future.

ACKNOWLEDGMENT

Princess Nourah bint Abdulrahman University Researchers Supporting Project number (PNURSP2022R137), Princess Nourah bint Abdulrahman University, Riyadh, Saudi Arabia. The authors would like to acknowledge the support of Prince Sultan University for paying the Article Processing Charges (APC) of this publication.

REFERENCES

[1] A. Serbes, H. Cukur, and K. Qaraqe, “Probabilities of False Alarm and Detection for the First-Order Cyclostationarity Test: Application to Modulation Classification,” IEEE Communications Letters, vol. 24, no. 1, pp. 57–61, Jan. 2020.

[2] T. Huynh-The, Q.-V. Pham, T.-V. Nguyen, T. T. Nguyen, R. Ruby, M. Zeng, and D.-S. Kim, “Automatic Modulation Classification: A Deep Architecture Survey,” IEEE Access, vol. 9, pp. 142,950–142,971, 2021.

[3] M. Marey and H. Mostafa, “Turbo Modulation Identification Algorithm for OFDM Software-Defined Radios,” IEEE Communications Letters, vol. 25, no. 5, pp. 1707–1711, May 2021.

[4] M. Marey, H. Mostafa, S. Alshebeili, and O. A. Dobre, “Blind Modulation Identification Algorithm For Two-Path Successive Relaying Systems,” IEEE Wireless Communications Letters, vol. 10, no. 11, pp. 2369–2373, Nov. 2021.

[5] Z. Xing and Y. Gao, “A Modulation Classification Algorithm for Multipath Signals Based on Cepstrum,” IEEE Transactions on Instrumentation and Measurement, vol. 69, no. 7, pp. 4742–4752, Jul. 2020.

[6] R. Gupta, S. Kumar, and S. Majhi, “Blind Modulation Classification for Asynchronous OFDM Systems Over Unknown Signal Parameters and Channel Statistics,” IEEE Transactions on Vehicular Technology, vol. 69, no. 5, pp. 5281–5292, May 2020.

[7] Z. Wu, L. Zhang, Z. Zhong, and R. Liu, “Blind Recognition of LDPC Codes Over Candidate Set,” IEEE Communications Letters, vol. 24, no. 1, pp. 11–14, Jan. 2020.

[8] T. Xia and H.-C. Wu, “Novel Blind Identification of LDPC Codes Using Average LLR of Syndrome a Posteriori Probability,” IEEE Transactions on Signal Processing, vol. 62, no. 3, pp. 632–640, Feb. 2014.

[9] ——, “Blind Identification of Nonbinary LDPC Codes Using Average LLR of Syndrome a Posteriori Probability,” IEEE Communications Letters, vol. 17, no. 7, pp. 1301–1304, July 2013.

[10] R. Swaminathan and A. S. Madhukumar, “Classification of Error Correcting Codes and Estimation of Interleaver Parameters in a Noisy Transmission Environment,” IEEE Transactions on Broadcasting, vol. 63, no. 3, pp. 463–478, Sep. 2017.

[11] D. Cabric, “Novel Blind Encoder Parameter Estimation for Turbo Codes,” IEEE Communications Letters, vol. 16, no. 12, pp. 1917–1920, Dec. 2012.

[12] Y. Huang, Y. Chen, Y. T. Hou, W. Lou, and J. H. Reed, “Recent Advances of LTE/WiFi Coexistence in Unlicensed Spectrum,” IEEE Network, vol. 32, no. 2, pp. 107–113, Apr. 2018.

[13] L. Wang, M. Zeng, J. Guo, Q. Cui, and Z. Fei, “Joint Bandwidth and Transmission Opportunity Allocation for the Coexistence Between NR-U and WiFi Systems in the Unlicensed Band,” IEEE Transactions on Vehicular Technology, vol. 70, no. 11, pp. 11881–11893, Nov. 2021.

[14] Y. Eldemerdash, M. Marey, O. A. Dobre, G. K. Karagiannidis, and R. Inkol, “Fourth-Order Statistics for Blind Classification of Spatial Multiplexing and Alamouti Space-Time Block Code Signals,” IEEE Transactions on Communications, vol. 61, no. 6, pp. 2420–2431, Jun. 2013.

[15] M. Marey, O. A. Dobre, and R. Inkol, “Classification of Space-Time Block Codes Based on Second-order Cyclostationarity with Transmission Impairments,” IEEE Transactions on Wireless Communications, vol. 11, no. 7, pp. 2574–2584, Jul. 2012.

[16] M. Marey, O. A. Dobre, and B. Liao, “Classification of STBC Systems over Frequency-selective Channels,” IEEE Transactions on Vehicular Technology, vol. 64, no. 5, pp. 2159–2164, May 2015.

[17] V. Choqueuse, M. Marazin, L. Collin, K. Yao, and G. Burel, “Blind Recognition of Linear Space Time Block Codes: A Likelihood-Based Approach,” IEEE Transactions on Signal Processing, vol. 58, no. 3, pp. 1260–1299, Mar. 2010.

[18] V. Choqueuse, K. Yao, L. Collin, and G. Burel, “Hierarchical Space-Time Block Code Recognition Using Correlation Matrices,” IEEE Transactions on Wireless Communications, vol. 58, no. 9, pp. 3526–3534, Sep. 2008.

[19] Y. Zhang, W. Yan, L. Zhang, and L. Ma, “Automatic Space-Time Block Code Recognition Using Convolutional Neural Network With Multi-Delay Features Fusion,” IEEE Access, vol. 9, pp. 79994–80005, 2021.

[20] L. Xiao, D. Chen, I. Hemadeh, P. Xiao, and T. Jiang, “Generalized Space Time Block Coded Spatial Modulation for Open-Loop Massive MIMO Downlink Communication Systems,” IEEE Transactions on Communications, vol. 68, no. 11, pp. 6858–6871, Nov. 2020.

[21] V. Abbasi, M. G. Shayesteh, and M. Ahmadi, “An Efficient Space-Time Block Code for LTE-A System,” IEEE Signal Processing Letters, vol. 21, no. 12, pp. 1526–1530, 2014.

[22] G. Giannakis, Z. Liu, X. Ma, and S. Zhou, Space-Time Coding for Broadband Wireless Communications. Wiley, 2007.

[23] M. Marey, O. A. Dobre, and R. Inkol, “Blind STBC Identification for Multiple Antenna OFDM Systems,” IEEE Transactions on Communications, vol. 62, no. 5, pp. 1556–1567, May 2014.

[24] M. Marey and O. A. Dobre, “Automatic Identification of Space-Frequency Block Coding for OFDM Systems,” IEEE Transactions on Wireless Communications, vol. 16, no. 1, pp. 117–128, Jan. 2017.

[25] E. Karami and O. A. Dobre, “Identification of SM-OFDM and AL-OFDM Signals Based on Their Second-order Cyclostationarity,” IEEE Transactions on Vehicular Technology, vol. 64, no. 3, pp. 942 – 953, Mar. 2015.

[26] Y. A. Eldemerdash, A. O. Dobre, and J. B. Liao, “Blind Identification of SM and Alamouti STBC-OFDM Signals,” IEEE Transactions on Wireless Communications, vol. 14, no. 2, pp. 972–982, Feb. 2015.
[27] J.-H. Lee and S.-C. Kim, “Detection of Interleaved OFDMA UplinkSignals in the Presence of Residual Frequency Offset Using theSAGE Algorithm,” IEEE Transactions on Vehicular Technology, vol. 56, no. 3, pp. 1455–1460, May 2007.

[28] M.-o. Pun, M. Morelli, and C.-c. J. Kuo, “Iterative Detection andFrequency Synchronization for OFDMA Uplink Transmissions,”IEEE Transactions on Wireless Communications, vol. 6, no. 2, pp. 629–639, Feb. 2007.

[29] N. Iqbal, A. Zerguine, and M.-S. Alouini, “A Robust Frequency Do-main Decision Feedback Equalization System for Uplink SC-FDMA Systems,”IEEE Transactions on Wireless Communications, vol. 20, no. 12, pp. 8110–8118, Dec. 2021.

[30] N. Iqbal, N. Al-Dhahir, A. Zerguine, and A. Zidouri, “Adaptive Frequency-Domain RLS DFE for Uplink MIMO SC-FDMA,”IEEE Transactions on Vehicular Technology, vol. 64, no. 7, pp. 2819–2833, 2015.

[31] S. Alamouti, “A Simple Transmit Diversity Technique for WirelessCommunications,”IEEE Journal on Selected Areas in Communications, vol. 16, pp. 1451–1458, Oct. 1998.

[32] Z. Yang et al., “Analysis and Optimization of Tail-Biting Spatially Coupled Protograph LDPC Codes for BICM-ID Systems,”IEEE Transactions on Vehicular Technology, vol. 69, no. 1, pp. 390–403, Jan. 2020.

[33] A. Chindapol and J. Ritcey, “Design, Analysis and Performance Evaluation for BICM-ID with Square QAM Constellations inRayleigh Fading Channels,”IEEE Journal on Selected Areas in Communications, vol. 19, no. 5, pp. 944–757, May 2001.

[34] V. Lottici and M. Luise, “Embedding Carrier Phase Recovery Into Iterative Decoding of Turbo-Coded Linear Modulations,”IEEE Transactions on Communications, vol. 52, no. 4, pp. 661–668, Apr. 2004.

[35] M. Marey, M. Samir, and O. A. Dobre, “EM-based joint channel es-timation and IQ imbalances for OFDM systems,”IEEE transactions on broadcasting, vol. 58, no. 1, pp. 106–113, Mar. 2012.

[36] M. Marey, H. Mostafa, O. A. Dobre, and M. Ahmed, “Data Detection Algorithms for BICM Alternate-Relaying Cooperative Systems with Multiple-Antenna Destination,”IEEE Transactions on Vehicular Technology, vol. 65, no. 5, pp. 3802–3807, Jun. 2016.

[37] M. Marey and O. A. Dobre, “Iterative Receiver Design for UplinkOFDMA Cooperative Systems,”IEEE Transactions on Broadcasting, vol. 62, no. 4, pp. 936–947, Dec. 2016.

[38] Y. A. Eldemerdash and O. A. Dobre, “On the Identification of SM and Alamouti-Coded SC-FDMA Signals: A Statistical-Based Approach,”IEEE Transactions on Vehicular Technology, vol. 65, no. 12, pp. 10079–10084, Dec. 2016.