Non-Abelian Dark Forces and the Relic Densities of Dark Glueballs

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Our understanding of the Universe is known to be incomplete and new gauge forces beyond those of the Standard Model might be crucial to describing its observed properties. A minimal and well-motivated possibility is a pure Yang-Mills non-Abelian dark gauge force with no direct connection to the Standard Model. We determine here the relic abundances of the glueball bound states that arise in such theories and investigate their cosmological effects. Glueballs are first formed in a confining phase transition, and their relic densities are set by a network of annihilation and transfer reactions. The lightest glueball has no lighter states to annihilate into, and its yield is set mainly by $3 \rightarrow 2$ number-changing processes which persistently release energy into the glueball gas during freeze-out. The abundances of the heavier glueballs are dominated by $2 \rightarrow 2$ transfer reactions, and tend to be much smaller than the lightest state. We also investigate potential connectors between the dark force and the Standard Model that allow some or all of the dark glueballs to decay. If the connection is weak, the lightest glueball can be very long-lived or stable and is a viable dark matter candidate. For stronger connections, the lightest glueball will decay quickly but other heavier glueball states can remain stable and contribute to the dark matter density.
I. INTRODUCTION

Gauge invariance under the $SU(3)_c \times SU(2)_L \times U(1)_Y$ group of the Standard Model (SM) provides a remarkable description of the non-gravitational forces of Nature. Yet, our knowledge of the Universe is incomplete and new gauge forces beyond those of the SM may be crucial to describing the laws of physics. The existence of such forces is highly constrained if they couple significantly to SM matter unless they have an associated mass scale (such as from confinement or the Higgs mechanism) well above a TeV [1, 2]. In contrast new dark gauge forces, with only feeble connections to the SM, can exist at energy scales much less than the TeV scale (or even be in a massless phase) and still be fully consistent with existing experimental bounds [3–5]. Such dark forces may also be related to the cosmological dark matter [4–6].

Abelian dark forces have been studied in great detail and have the novel property that they can connect to the SM at the renormalizable level through gauge kinetic mixing [9, 10]. Limits on the existence of such a kinetically-mixed dark photon have been obtained from existing experimental searches and astrophysical and cosmological observations for a range of dark photon masses spanning many orders of magnitude [3–5]. An exciting dedicated experimental program to search for dark photons is also underway [4, 5].

Non-Abelian dark forces have received somewhat less attention. As gauge invariance forbids the simple kinetic-mixing interaction with the SM, it is less clear how they potentially connect to the SM. Even so, non-Abelian dark forces are well-motivated and arise in many contexts including string theory constructions [11], in models of dark matter [12–21], baryogenesis [22–24], theories of neutral naturalness [25, 26], and within the hidden valley paradigm [27–29]. Non-Abelian dark forces can also lead to very different phenomenological effects compared to their Abelian counterparts owing to the requisite self-interactions among the corresponding gauge bosons and their potential for a confining phase transition at low energies.

The minimal realization of a non-Abelian dark force is a pure Yang-Mills theory with simple gauge group $G_x$. Such a theory is expected to confine at the characteristic energy scale $\Lambda_x$, with the elementary dark gluons binding into a spectrum of colour-neutral dark glueballs of mass $m \sim \Lambda_x$ [30]. These dark states may have significant cosmological effects even when their connection to the SM is too small to be detected in laboratory experiments. For very small values of $\Lambda_x$, dark gluons can act as self-interacting dark radiation [31–34], and can be consistent with existing constraints provided their effective temperature is somewhat lower than the SM plasma. With larger $\Lambda_x$, the glueballs will contribute to the density of dark matter if they are long-lived [14, 18–21], or may lead to observable astrophysical or cosmological signals if they decay at late times [14, 18, 21].

Assessing the cosmological impact of massive dark glueballs requires a precise knowledge of their relic abundances. The primary goal of this work is to compute these abundances and map out the ranges of parameters where one or more dark glueball states might constitute all or some of the observed dark matter. We will focus mainly on $G_x = SU(3)$, but we will also comment on how our results can be applied to other non-Abelian gauge groups. In a future companion paper we will describe in detail the cosmological effects of both stable and unstable primordial glueball populations and use them to constrain the existence of non-Abelian dark forces [35].

Starting from an early Universe containing a thermal plasma of dark gluons with temperature $T_x > \Lambda_x$, typically different than the temperature of the SM plasma, dark glueballs will be formed in a phase transition as the temperature of the dark sector falls below the confinement scale, $T_x \lesssim \Lambda_x$. Since glueball number is not conserved, the number densities of the glueball states will then track their equilibrium values so long as their $2 \to 2$ and $n \to 2$ interaction rates are fast relative to the Hubble expansion rate. The key difference compared to standard freeze-out is that without direct annihilation or rapid decays to SM or lighter hidden states, the overall chemical equilibrium
of the dark glueballs will be maintained primarily by $3 \leftrightarrow 2$ number-changing reactions [36–38]. Moreover, if the hidden glueballs do not have a kinetic equilibration with the SM or a bath of relativistic hidden states, the energy released by the $3 \rightarrow 2$ annihilations will cause the remaining glueballs to cool much more slowly than they would otherwise [36]. Together, these two effects produce freeze-out yields with a much different dependence on the underlying parameters of the theory than the typical freeze-out paradigm of annihilation into light relativistic particles.

Previous works have studied the effects of $3 \rightarrow 2$ annihilation and self-heating in general massive self-coupled sectors [36–38]. The specific application of these processes to dark glueballs has also been studied in Refs. [19–21]. We expand upon these works in two ways. First, we investigate possible effects of the confining phase transition on the final glueball yields [1]. And second, we compute the freeze-out abundances of the heavier glueball states in addition to the lightest mode. We also show that when the glueballs are connected to the SM, the heavier relic glueball states can sometimes have a greater observational effect than the lightest mode.

Following this introduction, we discuss the general properties of dark glueballs in Section II. Next, we study the freeze-out of the lightest glueball in Section III and investigate the effects of the confining phase transition. In Section IV we extend our freeze-out analysis to include the heavier glueball states. The possibility of dark glueball dark matter is studied in Section V as well as additional constraints that may be placed on general dark forces when a connection to the SM is added. We give brief concluding remarks in Section VI.

II. GLUEBALL SPECTRUM AND INTERACTIONS

The spectrum of glueballs in pure $SU(N)$ gauge theories has been studied extensively using both analytic models and lattice calculations [40]. Stable glueballs are classified according to their masses and their quantum numbers under angular momentum ($J$), parity ($P$), and charge conjugation ($C$). The lightest state is found to have $J^{PC} = 0^{++}$ [41–43], as expected based on general grounds [44], but a number of stable states with other $J^{PC}$ values are seen as well. In this section we summarize briefly the expected spectrum of glueballs and we estimate how they interact with each other.

A. Masses

Much of what is known about the spectrum of glueballs in $SU(N)$ gauge theories comes from lattice calculations. It is conventional to express these masses in terms of a length scale $r_0$ corresponding to where the gauge potential transitions from Coulombic to linear [45, 46], or in terms of the confining string tension $\sqrt{\sigma}$. Both of these quantities can be related to the energy scale $\Lambda_{\overline{MS}}$ where the running gauge coupling becomes strong [47]. For $SU(3)$ (with zero flavors), they are given by $r_0\Lambda_{\overline{MS}} = 0.614(2)(5)$ [47] and $r_0\sqrt{\sigma} = 1.197(11)$ [46, 48]. To facilitate connections with modern lattice calculations, we will express the glueball masses in terms of $1/r_0$ and define the strong coupling scale as the mass of the lightest $0^{++}$ glueball, $\Lambda_x \equiv m_0^{++}$.

Assuming conserved $P$ and $C$ in the dark sector, the dark glueballs will have definite $J^{PC}$ quantum numbers. In Table I we list the spectra of $SU(N)$ glueballs for $N = 2$ and $N = 3$ determined in lattice studies in units of $r_0$. The $N = 3$ glueballs in the table correspond to all the known stable states, with the masses listed taken from Ref. [41]. Listings for the $N = 2$ case are based on Ref. [48], and have significantly larger fractional uncertainties and may not give a

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[1] These effects were studied in a slightly different context in Ref. [39].
Table I. Masses of known stable glueballs in $SU(2)$ \cite{48} and $SU(3)$ \cite{41}.

| $J^{PC}$ | $mr_0(N=2)$ | $mr_0(N=3)$ |
|---------|-------------|-------------|
| 0++    | 4.5(3)      | 4.2(11)     |
| 2++    | 6.7(4)      | 5.8(2)      |
| 3++    | 10.7(8)     | 8.99(4)     |
| 0−     | 7.8(7)      | 6.33(7)     |
| 2−     | 9.0(7)      | 7.55(3)     |
| 1−     | −           | 7.18(3)     |
| 3−     | −           | 8.66(4)     |
| 2−     | −           | 10.10(7)    |
| 0−     | −           | 11.57(12)   |
| 1−     | −           | 9.50(4)     |
| 2−     | −           | 9.59(4)     |
| 3−     | −           | 10.06(21)   |

complete accounting of all the stable states. Note that the absence of $C$-odd states is expected for $SU(2)$ and other Lie groups with a vanishing $d^{abc}$ symbol \cite{28,40,49}. Glueball spectra for $SU(N > 3)$ have also been investigated on the lattice \cite{50,51}. The (lowest lying) glueball masses are found to scale with $N$ according to

$$r_0 m(N) \simeq P + Q/N^2,$$

with $P$ and $Q$ on the order of unity. These corrections are found to be numerically modest for $N > 3$, and the glueball spectrum for larger $N$ appears to be similar to $N = 3$. Extrapolations to large $N$ also find that $r_0 \sqrt{\sigma} \simeq 1.2$ remains nearly constant \cite{48} while the strong-coupling scale decreases smoothly to $r_0 \Lambda_{\overline{MS}} \simeq 0.45$ \cite{52}. A further variation on $SU(N)$ theories is the addition of a non-zero topological theta term. This violates $P$ and $T$ explicitly, and will shift the string tension $\sqrt{\sigma}$ and glueball masses \cite{53}, and induce mixing between glueball states with different $P$ quantum numbers \cite{53,54}.

The glueballs for other non-Abelian gauge groups have not been studied in as much detail on the lattice, but a few specific features are expected based on general arguments. As mentioned above, there are no $C$-odd states for $SU(2)$, $SO(2N + 1)$, or $Sp(2N)$ due to their vanishing $d^{abc}$ coefficient \cite{28,40,49}. For $SO(2N)$, $SO(4) \cong SU(2) \times SU(2)$ and $SO(6) \cong SU(4)$ reduce to previous cases, while for $2N > 6$ the $C$-odd states are expected to be significantly heavier than the lowest $C$-even glueballs \cite{28}. This follows from the fact that the minimal gluon operators giving rise to the $C$-odd states for the groups have mass dimension $2N$ \cite{49}, and higher-dimension gluon operators are generally expected to lead to heavier glueball states \cite{28,41,49}.

In this study we will concentrate on $SU(N)$ glueballs with $P$ and $C$ conservation in the dark sector. However, we will comment on more general scenarios when they lead to important phenomenological distinctions.

B. Interactions

Dark glueball freeze-out in the early Universe depends on the cross sections for $2 \rightarrow 2$ and $3 \rightarrow 2$ glueball reactions. Since glueball self-interactions are expected to be weak in the limit of very large $N$ \cite{55,56}, it should be possible to calculate these cross sections reliably in this limit.
in perturbation theory using the interactions specified by a glueball effective Lagrangian. These interactions have not been obtained in lattice calculations. Instead, we estimate them based on large-\(N\) scaling \[55, 56\] and naïve dimensional analysis (NDA) \[57, 58\].

The most important state for the freeze-out calculation is the lightest \(J^{PC} = 0^{++}\) glueball, \(\phi\). Using NDA and large-\(N\), we estimate its leading self-interactions to be

\[
\mathcal{L}_{\text{eff}} \supset \frac{1}{2} (\partial\phi)^2 - \frac{a_2}{2!} \Lambda^2 \phi^2 - \frac{a_3}{3!} \left(\frac{4\pi}{N}\right) \Lambda \phi^3 - \frac{a_4}{4!} \left(\frac{4\pi}{N}\right)^2 \phi^4 + \ldots
\]

(2)

where the coefficients \(a_i\) are expected to be of order unity. This form matches the NDA scaling of Ref. \[19\] as well as the \(1/N\) counting of Ref. \[21\].

Applying this form to \(2 \to 2\) elastic scatterings of the \(0^{++}\) state, we estimate

\[
\sigma_{2\to2} \simeq \frac{A}{4\pi} \left(\frac{4\pi}{N}\right)^4 \frac{\beta}{s},
\]

(3)

where \(A\) is dimensionless and close to unity, \(s = (p_1 + p_2)^2\) is the usual Mandelstam variable, and \(\beta = \sqrt{1 - 4m^2/s}\). The same arguments applied to \(3 \to 2\) processes at low momentum give

\[
\sigma_{3\to2} \simeq \frac{B}{(4\pi)^3} \left(\frac{4\pi}{N}\right)^6 \frac{1}{m^5},
\]

(4)

with \(B\) also close to unity. These cross sections are at the limit of perturbative unitarity for small \(N\), but become moderate for \(N \gtrsim 4\pi\) reflecting the expected transition to weak coupling in this regime \[56\]. In the analysis to follow we will set \(A = B = 1\), and we will generalize the cross section estimate for \(2 \to 2\) interactions to more general processes involving other glueball states using the same NDA and large-\(N\) arguments.

### III. FREEZE-OUT OF THE LIGHTEST GLUEBALL

Having reviewed the properties of glueballs, we turn next to investigate their freeze-out dynamics in the early Universe. In this section we study the thermodynamic decoupling of the lightest \(0^{++}\) glueball in a simplified single-state model. We also discuss the confining transition in which the glueballs are formed and investigate how it might modify the glueball relic density. The freeze-out of heavier glueballs will be studied in the section to follow.

Throughout our analysis, we assume that the dark glueballs are thermally decoupled from the SM during the freeze-out process but maintain a kinetic equilibrium among themselves. This implies that the entropy of the dark sector is conserved separately from the visible sector, up to a possible increase during a first-order confining phase transition. This motivates the definition

\[
R \equiv \frac{s_x}{s} = \text{constant},
\]

(5)

where \(s_x\) is the entropy density of the dark sector after the confining transition and \(s\) is that of the visible. The value of \(R\) is an input to our calculation, and may be regarded as an initial condition set by the relative reheating of the dark and visible sectors after inflation if they were never in thermal contact \[19\], or by the thermal decoupling of the sectors if they once were \[38\].
A. Single-State Model

Consider first a dark sector consisting of a single real scalar $\phi_x$ with mass $m_x$, $2 \rightarrow 2$ and $3 \rightarrow 2$ self-interaction cross sections given by Eqs. (3,4), and no direct connection to the SM. We will show below that this is often an accurate simplified model for the freeze-out of the lightest $0^{++}$ glueball, even when the heavier glueballs are included.

The freeze-out dynamics of this model coincide with the general scenario of Ref. [36]. Chemical equilibrium of the $\phi_x$ scalar is maintained by $3 \rightarrow 2$ transitions. These transitions also transfer energy to the remaining $\phi_x$ particles in the non-relativistic plasma causing them to cool more slowly than they would if there was a relativistic bath to absorb the input heat. Freeze-out occurs when the $3 \rightarrow 2$ transition rate becomes too slow to keep up with the Hubble expansion. While this happens, kinetic equilibrium is maintained by $2 \rightarrow 2$ elastic scattering of glueballs, which is parametrically much faster than the $3 \rightarrow 2$ processes at dark-sector temperatures below the scalar mass.

Kinetic equilibrium implies that the number density of $\phi_x$ particles takes the form

$$ n_x = \int \frac{d^3p}{(2\pi)^3} \left[ e^{(E_x - \mu_x)/T_x} - 1 \right]^{-1}, \quad (6) $$

where $E = \sqrt{\overrightarrow{p}^2 + m_x^2}$, and $T_x$ and $\mu_x$ refer to the temperature and chemical potential of the $\phi_x$ plasma. This number density evolves in time according to [36, 37]

$$ \dot{n}_x + 3Hn_x = -\langle \sigma_{32}v^2 \rangle (n_x^3 - n_x^2 \bar{n}_x), \quad (7) $$

where $H$ is the Hubble rate (sourced by both the visible and dark sectors), $\bar{n}_x = n_x(\mu_x \rightarrow 0)$ is the number density in the limit of zero chemical potential, and the thermally-averaged cross section is

$$ \langle \sigma_{32}v^2 \rangle = \frac{1}{n_x^3} \int d\Pi_1d\Pi_2d\Pi_3 \ e^{-(E_1+E_2+E_3)/T_x} \sigma_{32}v^2 $$

$$ \approx \frac{1}{(4\pi)^3} \left( \frac{4\pi}{N} \right)^6 \frac{1}{m_x^5}, \quad (8) $$

where we have used Eq. (4) in going to the second line. The dark-sector entropy is

$$ T_x s_x = \rho_x + p_x - \mu_x n_x, \quad (9) $$

with the energy density $\rho_x$ and pressure $p_x$ determined by the same distribution function as $n_x$ in Eq. (6). Together, Eqs. (5,7) provide two equations for the two unknowns $T_x(t)$ and $\mu_x(t)$ that can be solved in conjunction with the Friedmann equation for $H(t)$ [59].

While the results we present below are based on the numerical evaluation of Eqs. (5,7), it is instructive to derive an approximate solution for the non-relativistic freeze-out process [36]. For $m_x \gg T_x$, $\mu_x$, the dark-sector entropy density is

$$ s_x \approx \left( \frac{m_x}{T_x} \right) n_x, \quad (10) $$

This relation is maintained with zero chemical potential until freeze-out occurs, after which the number density just dilutes with the expansion of spacetime. Matching these limits and applying Eq. (5), the freeze-out yield is

$$ Y_x = \frac{n_x}{s} \approx x_x^{fo} R, \quad (11) $$
where \( x_f^{fo} = m_x/T_x^{fo} \) and \( T_x^{fo} \) is the dark temperature at which chemical equilibrium is lost. To determine \( x_f^{fo} \), we follow Ref. [36] and identify freeze-out with the point at which the equilibrium \( 3 \rightarrow 2 \) rate falls below the fractional rate of change of \( n_x a^3 \), which gives

\[
3H \simeq x_x \langle \sigma_{32} v^2 \rangle \bar{n}_x^2.
\]

Assuming visible radiation dominates the total energy density during freeze-out, this implies a visible-sector freeze-out temperature of

\[
T_x^{fo} \simeq \bar{n}_x \left( \frac{x_x^{fo} M_{Pl} \langle \sigma_{32} v^2 \rangle}{\sqrt{g_x^{fo} \pi^2 / 10}} \right)^{1/2}.
\]

Combining this with the entropy relation of Eq. (5) and the explicit form of \( \bar{n}_x \) in the non-relativistic regime, we find

\[
(x_x^{fo})^{5/2} e^{2 m_x^{fo}} = \frac{g_x^{fo} S}{180 \pi} R \left( \frac{m_x^4 M_{Pl} \langle \sigma_{32} v^2 \rangle}{\sqrt{g_x^{fo} \pi^2 / 10}} \right)^{3/2},
\]

which can be solved iteratively for \( x_x^{fo} \).

In Fig. 1 we show the mass-weighted relic yield of \( \phi_x \) with \( N = 3 \) as a function of the mass of the lightest glueball \( \Lambda_x = m_x \) and the dark-to-visible entropy ratio \( R \). We also indicate where \( \Omega_x h^2 = 0.1 \) and \( m_x Y_x = 10^{-13}, 10^{-5} \) with solid white lines. As found in Refs. [20, 21], masses
above \( m_x \gtrsim 1 \text{ MeV} \) require small entropy ratios to avoid overproducing the relic abundance. The dark shaded region at the lower right shows where \( x_f^o < 5 \). As will be discussed below, there is an additional uncertainty in the relic abundance in this region when this simplified model is applied to dark glueballs.

**B. Dynamics of the Confining Transition**

Dark glueballs are first formed in the early Universe in a confining phase transition. At dark temperatures much larger than the confinement scale, \( T_x \gg \Lambda_x \), the dark sector can be described as a thermal bath of weakly-interacting dark gluons with \( g_* = 2(N^2 - 1) \) degrees of freedom. As \( T_x \) cools below \( \Lambda_x \) a phase transition occurs with the gluons binding to form glueballs. Depending on the nature of the transition and the interaction rate of the resulting glueballs, this transition can affect the glueball relic density.

The nature of the confining transition in pure \( SU(N) \) gauge theories has been studied in detail on the lattice \([60–69]\) and in a number of semi-analytic models (e.g. Refs. \([70–75]\)). The transition is found to be second-order for \( N = 2 \), weakly first-order for \( N = 3 \), and increasingly first-order for \( N \geq 4 \) \([62, 63]\). The critical temperature \( T_c \) for \( N = 2 – 8 \) is fit well by the relation \([68]\)

\[
\frac{T_c}{\sqrt{\sigma}} = 0.5949(17) + 0.458(18)/N^2 ,
\]

where \( \sqrt{\sigma} \simeq 1.2/r_0 \) \([46]\). Note that this is about a factor of five smaller than the mass of the lightest glueball in Tab. I. For \( N > 2 \) where the transition is found to be first-order, the latent heat \( L_h \) scales according to \([66]\)

\[
\frac{L_h}{(N^2 - 1)T_c^4} = 0.388(3) - 1.61(4)/N^2 ,
\]

while the interface tension between the phases is consistent with \([63]\)

\[
\frac{\sigma_{cd}}{T_c^3} = 0.0138(3)N^2 - 0.104(3) .
\]

In the confined phase just below the critical temperature, \( 0.7T_c \lesssim T_x < T_c \), the entropy and pressure are significantly larger than what is predicted from the known glueball states \([67, 76]\). Interestingly, this discrepancy can be explained by additional glueball states with a Hagedorn spectrum corresponding to the excitations of a bosonic closed string \([76–78]\), in agreement with the model of Ref. \([79]\). The lattice studies of Refs. \([80, 81]\) also suggest that the lowest-lying glueball pole masses persist nearly unchanged up to \( T_c \) (although see Ref. \([82]\) for a different conclusion).

Much less is known about the non-equilibrium properties of the \( SU(N) \) confining transition such as the nucleation temperature and rate. An estimate of the nucleation rate in the early Universe for \( SU(3) \) valid in the limit small supercooling is given in Ref. \([39]\). For supercooling by an amount \( T_x = (1 - \delta)T_c \), they find

\[
\Gamma/V \simeq T_x^4 e^{-\Delta F_c/T_x} ,
\]

with

\[
\frac{\Delta F_c}{T_x} \simeq \frac{16\pi}{3} \frac{\sigma_{cd}^3}{L_h^2 T_c} \delta^{-2}
\]

\[
\simeq 2.92 \times 10^{-4} \delta^{-2} N^2 \frac{(1 - 7.54/N^2)^3}{(1 - 4.15/N^2)^2} ,
\]

\([19, 20]\).
where in the second line we have generalized the result of Ref. [39] to $SU(N \geq 3)$ using the central lattice values of $L_h$ and $\sigma_{cd}$ listed above. For moderate $N$, this suggests that nucleation occurs at $T_x$ extremely close to $T_c$ (provided $T_x \sim R^{1/3}T$ is not too small) with only a very small injection of entropy. However, significant supercooling appears to be possible at large $N$.

To apply these results to the calculation of relic glueball abundances, we assume that the phase transition completes with $T_x = T_c$ [33] and that the mass spectrum of stable glueballs just after the transition is the same as at $T_x \to 0$. The simplified model discussed above can then be used with initial conditions at $x_x = x^c_x \equiv m_x/T_c$, which can be specified completely in terms of $R = s_x/s$, and $\mu_x(x^c_x)$. If the $3 \to 2$ depletion process is fast relative to the Hubble rate at $x_x = x^c_x$, the initial chemical potential relaxes quickly to zero and the final relic density is specified completely by the choice of $R$. However, if full chemical equilibration does not occur at $x_x = x^c_x$ a range of $\mu(x^c_x)$ values can be consistent with the equilibration rate relative to Hubble, and there is an additional uncertainty in the final glueball relic density for a given value of the entropy ratio $R$.

To investigate the potential dependence of the relic yield on the initial glueball density following the phase transition, we repeat the freezeout calculation described in the previous section for $G_x = SU(3)$ with different initial glueball densities at $T = T_c$ defined by the ratio $f = Y_x(x_x^c)/Y_x(x_x^c, \mu_x = 0)$. Our results are shown for a range of values of $\Lambda_x$ with $R = 10^{-9}$ and $N = 3$ in Fig. 2. For most of the range of $\Lambda_x$ and $R$ of interest, dark freezeout occurs with $x_x^{f_0} > x_x^c \simeq 5$ and the final glueball relic density is insensitive to the initial value after the phase transition. Even when $x_x^{f_0} < x_x^c$, some residual annihilation (or creation) typically occurs and the final density tends to be similar to $f = 1$. The region in the $\Lambda_x$–$R$ plane in which this modest additional uncertainty is present is indicated by the shaded area in Fig. 1.
IV. FREEZE-OUT WITH MULTIPLE GLUEBALLS

We turn next to the heavier glueballs above the lightest state. Recall from Section II that multiple stable glueballs are expected in a confining Yang-Mills theory, with the spectra found for $SU(2)$ and $SU(3)$ groups listed in Table I. These heavier states lead to new annihilation channels involving the lightest glueball, and their relic densities can be of cosmological interest.

The freezeout of the full glueball spectrum involves many states and a network with numerous reaction channels. Despite this complexity, we find that the glueball relic densities follow a relatively simple pattern with three main features. First, the relic density of the lightest glueball is described very well by the simplified one-state model presented above provided it freezes out while it is significantly non-relativistic. Second, the relic densities of the heavier $C$-even states are typically extremely small relative to the lightest glueball. And third, the total relic density of $C$-odd states (for $SU(N \geq 3)$ gauge groups) is dominated by the lightest $C$-odd mode, and is much smaller than the lightest $0^{++}$ state but typically larger than all the other $C$-even states. This significant difference arises from conserved $C$-number in the dark sector, which allows coannihilation of the heavier $C$-even states with the lightest glueball but forbids it for $C$-odd states.

In this section we investigate the relic densities of the full set of glueballs for the dark gauge group $SU(3)$. We begin by determining which $2 \rightarrow 2$ glueball reactions are allowed by $J^{PC}$ conservation in the dark sector and we estimate their rates. Next, we study a simplified reaction network of $C$-even states that we argue captures the most important features of the full dynamics. Finally, we perform a similar analysis for the $C$-odd states.

A. Glueball Reactions

To discuss glueball reactions for $G_x = SU(3)$, it will be convenient to label the modes in the spectrum by $i = 1, 2, \ldots, 12$ as in Table II. This table also lists their relative masses and $J^{PC}$ quantum numbers.

The specific interactions between glueballs are not known, but all possible processes consistent with dark-sector $J$, $P$, and $C$ conservation are expected to be present. For a $2 \rightarrow 2$ glueball...
reaction of the form \( i + j \rightarrow k + l \), conservation of \( C \) requires
\[
C_j C_j = C_k C_l .
\] (21)

This is trivial to apply and rules out a number of reactions. Conservation of \( P \) implies
\[
P_i P_j = (-1)^L P_k P_l ,
\] (22)
where \( L \) is the relative orbital angular momentum of the reaction channel. When identical particles are present, they must also be symmetrized. In general, it can be shown that there always exists a value of \( L \) such that both parity and total angular momentum are conserved unless either \( J_i = J_j = 0 \) or \( J_k = J_l = 0 \).

If the process \( i + j \leftrightarrow k + l \) is allowed, it contributes to the collision term in the Boltzmann equation for glueball \( i \) according to
\[
\Delta \bar{n}_i = - \langle \sigma v \rangle_{ijkl} n_i n_j - \langle \sigma v \rangle_{kljl} n_k n_l ,
\] (23)
where \( \langle \sigma v \rangle_{ijkl} \) is the thermally-averaged cross section and \( n_i \) refers to the number density of the \( i \)-th species. Assuming kinetic equilibrium is maintained among the glueballs, we have
\[
n_i = g_i e^{\mu_i/T_x} \frac{(4\pi)^2 T_x^2}{2}(m_i/T_x) K_2(2m_i/T_x) / \pi,
\] (24)
\[
\simeq g_i \frac{(m_i T_x)^{3/2}}{2\pi} e^{-(m_i-\mu_i)/T_x} ,
\] (25)
where \( T_x \) is the temperature of the glueball bath and \( g_i \), \( m_i \), and \( \mu_i \) are the number of degrees of freedom, mass, and chemical potential of the type-\( i \) glueball. The thermally-averaged cross-section is given by
\[
\langle \sigma v \rangle_{ijkl} = \frac{1}{n_i n_j} \int \frac{d^3p_i}{(2\pi)^3} \int \frac{d^3p_j}{(2\pi)^3} g_i e^{(\mu_i-E_i)/T_x} g_j e^{(\mu_j-E_j)/T_x} \langle \sigma v \rangle_{ijkl}
\] (26)
\[
= g_i g_j \frac{1}{\bar{n}_i \bar{n}_j} \int \frac{d^3p_i}{(2\pi)^3} \int \frac{d^3p_j}{(2\pi)^3} e^{-(E_i+E_j)/T_x} \langle \sigma v \rangle_{ijkl} ,
\] (27)
where \( E_i = \sqrt{m_i^2 + p_i^2} \) and \( \bar{n}_i = n_i(\mu_i = 0) \). Note that the chemical potentials cancel in this expression.

The reaction \( i + j \rightarrow k + l \) is either exothermic \( (m_i + m_j \geq m_k + m_l) \) or endothermic \( (m_i + m_j < m_k + m_l) \). Equilibration of this process implies \( \mu_i + \mu_j = \mu_k + \mu_l \). Combined with detailed balance, we must have
\[
\langle \sigma v \rangle_{ijkl} \bar{n}_i \bar{n}_j = \langle \sigma v \rangle_{kljl} \bar{n}_k \bar{n}_l .
\] (28)
Using these relations, the thermally-averaged rates of endothermic reactions can be estimated based on those of exothermic reactions.

Thermal averaging of cross sections was studied in detail in Refs. [84, 85]. Generalizing their results slightly and using the large-\( N \) and NDA estimates of interaction strengths, we estimate the thermally-averaged cross section of an exothermic process \( i + j \rightarrow k + l \) that proceeds at lowest orbital angular momentum level \( L \) by
\[
\langle \sigma v \rangle_{ijkl} \simeq \frac{(4\pi)^3}{N^4} \frac{\beta_{ijkl}}{s_{ij}} c_L \left( \frac{2}{\frac{x_i}{x_i} + \frac{x_j}{x_j}} \right)^L ,
\] (29)
where $x_i = m_i/T$,

$$s_{ij} = \left(1 + \frac{3}{x_i + x_j}\right)(m_i + m_j)^2,$$  \hspace{1cm} (30)

along with

$$\beta_{ijkl} = \frac{2p_{kl}'}{\sqrt{s_{ij}}},$$  \hspace{1cm} (31)

$$= \frac{1}{s_{ij}} \left(s_{ij}^2 + m_k^4 + m_l^4 - 2s_{ij}m_k^2 - 2s_{ij}m_l^2 - 2m_k^2m_l^2\right)^{1/2},$$

and the coefficients $c_L$ are \[ 84 \]

$$c_0 = 1, \quad c_1 = 3/2, \quad c_2 = 15/8, \quad c_3 = 35/16, \quad c_4 = 315/128.$$  \hspace{1cm} (32)

The first factor in Eq. (29) contains the couplings, the second factor describes the kinematics near threshold in the non-relativistic limit, while the third is the velocity suppression for a process that goes at the $L$-th partial wave.

The cross-section estimates of Eq. (29) can be used to judge which reactions are most significant during freezeout. The relative effect of the process $i + j \rightarrow k + l$ (with $j, k, l \neq i$) on the number density of glueball species $i$ is

$$\left|\Delta \dot{n}_i\right|/n_i = \langle \sigma v \rangle_{ijkl} \dot{n}_j.$$  \hspace{1cm} (33)

In general, this reaction is cosmologically active for $\left|\Delta \dot{n}_i\right|/n_i > H$. Scanning over all possible $2 \rightarrow 2$ reactions of $SU(N = 3)$ glueballs, we find that in full equilibrium with $x_e > 5$ and for every glueball species $i > 1$ there exist multiple number-changing $2 \rightarrow 2$ reactions down to lighter states with $\left|\Delta \dot{n}_i\right|/n_i$ significantly larger than the corresponding quantity for $3 \rightarrow 2$ annihilation of the lightest glueball. This implies that relative chemical equilibrium is maintained among the glueballs during and for some time after $3 \rightarrow 2$ freeze-out, with

$$\frac{n_i}{n_j} = \frac{\bar{n}_i}{\bar{n}_j} \simeq \frac{g_i}{g_j} \left(\frac{m_i}{m_j}\right)^{3/2} e^{-\left(m_i - m_j\right)/T_e}.$$  \hspace{1cm} (34)

Equivalently, the number densities of all species immediately after $3 \rightarrow 2$ freeze-out are given by their equilibrium values with a common chemical potential.

Relative chemical equilibrium after $3 \rightarrow 2$ freezeout implies further that the relative importance of different $2 \rightarrow 2$ reactions on the subsequent freezeout of the heavier glueballs can be estimated using their equilibrium number densities. This allows us to greatly simplify the set of reaction networks by keeping only the dominant processes and concentrating exclusively on a few key states. It turns out to be consistent and convenient to study the $C$-even and $C$-odd states independently, and this is the approach we take below.

### B. Relic Densities of $C$-Even States

The lightest $C$-even glueballs above the lowest mode have $J^{PC} = 2^{++}, 0^{-+}, 2^{-+}$ and correspond to $i = 2, 4, 5$, in our labelling scheme. Scanning over all possible reactions for these states and estimating their relative effects on the number densities as above, the dominant interactions near
relative equilibrium are found to form a minimal closed system. The reaction network for the system is described by

\[ \dot{n}_1 + 3H n_1 = -\langle \sigma v \rangle_{2111} \left( \frac{n_2}{n_1} \right) n_1 n_2 - n_2^2 \]  
\[ -\frac{1}{2} \langle \sigma v \rangle_{2111} \left( \frac{n_2}{n_1} \right)^2 n_1 n_2 - n_2^2 \]  
\[ -\frac{1}{2} \langle \sigma v \rangle_{2211} \left( \frac{n_2}{n_1} \right)^2 n_1 n_2 - n_2^2 \]  
\[ n_1 n_2 - n_2^2 \]  
\[ n_1 n_4 - n_2^2 \]  
\[ n_1 n_5 - n_2 n_4 \]  
\[ n_1 n_5 - n_2 n_4 \]  
(35)

\[ \dot{n}_2 + 3H n_2 = +\frac{1}{2} \langle \sigma v \rangle_{2111} \left( \frac{n_2}{n_1} \right) n_1 n_2 - n_2^2 \]  
\[ +\langle \sigma v \rangle_{2211} \left( \frac{n_2}{n_1} \right)^2 n_1 n_2 - n_2^2 \]  
\[ +\langle \sigma v \rangle_{2214} \left( \frac{n_2}{n_1} \right)^2 n_1 n_2 - n_2^2 \]  
\[ +\langle \sigma v \rangle_{2415} \left( \frac{n_2}{n_1} \right)^2 n_1 n_2 - n_2^2 \]  
(36)

\[ \dot{n}_4 + 3H n_4 = -\frac{1}{2} \langle \sigma v \rangle_{2214} \left( \frac{n_2}{n_1} \right)^2 n_1 n_4 - n_2^2 \]  
\[ +\frac{1}{2} \langle \sigma v \rangle_{2415} \left( \frac{n_2}{n_1} \right)^2 n_1 n_4 - n_2^2 \]  
(37)

\[ \dot{n}_5 + 3H n_5 = -\frac{1}{2} \langle \sigma v \rangle_{2415} \left( \frac{n_2}{n_1} \right)^2 n_1 n_5 - n_2 n_4 \]  
\[ +\frac{1}{2} \langle \sigma v \rangle_{1512} \left( \frac{n_2}{n_1} \right)^2 n_1 n_5 - n_2 n_4 \]  
(38)

The factors of 1/2 appearing here are symmetry factors for initial states that are not included in the standard definition of the thermally-averaged cross section. They ensure that the summed number density \( n_1 + n_2 + n_4 + n_5 \) is conserved in the absence of 3 \( \rightarrow \) 2 reactions. In addition to these evolution equations, the ratio of entropies \( R = s_x / s \) is conserved after the confining transition at \( T'_x \approx m_x / 5 \), with the dark sector entropy now extended to include all (relevant) glueball modes.

Numerical solutions of this system of equations for SU(3) dark glueballs are shown in Fig. 3 for the parameter values \( (\Lambda_x / \text{GeV}, R) = (1, 10^{-9}), (10^5, 10^{-9}), (1, 10^{-3}), (10^5, 10^{-3}) \). In each panel, the evolution of the mass-weighted yields \( m_x Y_x \) with \( x_x = m_x / T_x \) are given by the solid lines, while the dashed lines show the number densities of each species with \( \mu_i = 0 \). In all four panels, the lightest 0++ mode is seen to dominate the total glueball relic abundance for \( x_x \gtrsim 10 \). This abundance is found to match closely with the value determined by the one-glueball simplified model discussed above. The much smaller relic abundances of the heavier glueball modes is due to the efficient coannihilation reactions they experience. Since these 2 \( \rightarrow \) 2 processes
FIG. 3. Mass-weighted relic yields of the four lightest $C$-even glueballs in $SU(3)$ as a function of the dark glueball temperature variable $x_x = m_x / T_x$. The solid lines show the yields derived from the reaction network while the dashed lines indicate the yields expected if the states were to continue following equilibrium with $\mu_i = 0$. Top left: $(\Lambda_x / \text{GeV}, R) = (1, 10^{-9})$. Top right: $(\Lambda_x / \text{GeV}, R) = (10^5, 10^{-9})$. Bottom left: $(\Lambda_x / \text{GeV}, R) = (1, 10^{-3})$. Bottom right: $(\Lambda_x / \text{GeV}, R) = (10^5, 10^{-3})$.

are parametrically faster than the $3 \to 2$ annihilations setting the $0^{++}$ density, relative chemical equilibrium is maintained to large values of $x_x$. This implies a strong exponential suppression of the heavier glueball densities as in Eq. (34).

Let us also point out that the $0^{-+}$ state freezes out (of relative chemical equilibrium) well before the $2^{++}$ and $2^{-+}$ modes, even though it is heavier than the $2^{++}$. This can be understood by examining the relative rates of the depletion reactions for the $0^{-+}$ state; for $x_x \gtrsim 20$ it is found to be $0^{-+} + 0^{++} \to 2^{++} + 2^{++}$. Comparing masses, this reaction is found to be endothermic and thus it receives an additional rate suppression as discussed in Ref. [86]. The dependence of the $0^{-+}$ ($i = 4$) glueball relic density on $\Lambda_x = m_x$ and $R$ is also shown in Fig. 4.
FIG. 4. Mass-weighted relic yields of the $0^{-+}$ dark glueball in $SU(3)$ as functions of $\Lambda_x = m_x$ and $R$.

C. Relic Densities of $C$-Odd States

Freezeout of the $C$-odd glueballs is qualitatively different from that of the $C$-even modes due to the conservation of $C$-number in the dark sector. This forbids coannihilation reactions of the $C$-odd states with the relatively abundant lightest $0^{++}$ glueball into final states with only $C$-even modes, and can lead to a significant relic density for the lightest $C$-odd $1^{+-}$ state.

To see how this comes about, let us split up the labels of the state indices defined in Table II according to

$$i, j = 1, 2, 3, 4, 5 = C\text{-even} , \quad a, b = 6, 7, \ldots 12 = C\text{-odd} ,$$

and let us also define the net $C$-odd density by

$$n_- = \sum_{a=6}^{12} n_a .$$

The net collision term in the Boltzmann equation for $n_-$ is

$$\Delta \dot{n}_- = \sum_a \Delta \dot{n}_a$$

$$= - \sum_{ab,ij} \langle \sigma v \rangle_{abij} n_a n_b + \sum_{ij,ab} \langle \sigma v \rangle_{ijab} n_i n_j .$$

The key feature of this expression is that all processes contributing to the rate of change of $\dot{n}_-$ have either two $C$-odd particles in the initial or the final state [55]. Using detailed balance, we can rewrite Eq. (42) in the form

$$\Delta \dot{n}_- = - \langle \sigma v \rangle_{6611} n_-^2 \left[ \sum_{abij} \left( \frac{\langle \sigma v \rangle_{abij}}{\langle \sigma v \rangle_{6611}} \frac{n_a n_b}{n_-^2} \Theta_+ + \frac{\langle \sigma v \rangle_{ijab}}{\langle \sigma v \rangle_{6611}} \frac{n_i n_j}{n_-^2} \Theta_- \right) \right]$$

$$+ \langle \sigma v \rangle_{6611} n_1^2 \left( \frac{\bar{n}_-}{\bar{n}_1} \right)^2 \left[ \sum_{abij} \left( \frac{\langle \sigma v \rangle_{abij}}{\langle \sigma v \rangle_{6611}} \frac{\bar{n}_a \bar{n}_b \bar{n}_i \bar{n}_j}{\bar{n}_-^2 \bar{n}_1^2} \Theta_+ + \frac{\langle \sigma v \rangle_{ijab}}{\langle \sigma v \rangle_{6611}} \frac{n_i n_j n_1^2}{\bar{n}_1^2 n_-^2} \Theta_- \right) \right] ,$$
where \( \Theta_+ = \Theta(m_a + m_b - m_i - m_j) \) and \( \Theta_- = \Theta(m_i + m_j - m_a - m_b) \) are step functions to select out exothermic reactions as appropriate.

Consider the relative sizes of the individual terms in Eq. (43) when relative equilibrium is maintained. In the first line, the first term is on the order of unity for \( a = b = 6 \) but has an exponential suppression otherwise from the factor of \( n_a n_b / n_i^2 \), while the second term has an additional exponential suppression from the factor \( \bar{n}_a \bar{n}_j / \bar{n}_a \bar{n}_b (m_i + m_j > m_a + m_b) \). Similar arguments also apply to the terms in the second line of Eq. (43), noting that \( \bar{n}_i \bar{n}_j n_i^2 = n_i \bar{n}_j n_i^2 \) in relative equilibrium, and only the \( a = b = 6 \) portion of the first term avoids an exponential suppression. Indeed, a numerical evaluations of these contributions, assuming relative equilibrium and moderate \( x_x \gtrsim 10 \), confirms that the \( a = b = 6 \) terms of the \( \Theta_+ \) pieces dominate the collision term.

The total \( C \)-odd density begins to deviate appreciably from the relative equilibrium value for \( \langle \sigma v \rangle_{6611} \bar{n}_1^2 (n_1 / \bar{n}_1)^2 \sim H \). This occurs well before the \( C \)-even states freeze out, and also well before \( C \)-odd transfer reactions, such as \( 7 + 1 \leftrightarrow 2 + 6 \), turn off. The latter result implies that the relative densities of \( C \)-odd states is maintained among themselves (but not the \( C \)-even states) even after the net \( C \)-odd density has frozen out. Therefore we also expect \( n_6 / n_- \to 1 \) and \( n_{a>6} / n_- \to 0 \) provided these processes turn off at moderate \( x_x \gtrsim 10 \).

The net result of this analysis is that it is generally a good approximation to compute the freezeout of the \( C \)-odd density using a simplified two-state system consisting only of the \( i = 1, 6 \) (\( 0^{++} \) and \( 1^{+-} \)) glueballs. Correspondingly, the system of Boltzmann equations is

\[
\dot{n}_1 + 3H n_1 = -\langle \sigma v \rangle_{32} n_1^2 (n_1 - \bar{n}_1) + \langle \sigma v \rangle_{6611} n_6^2 - \left( \frac{n_1}{\bar{n}_1} \right)^2 \bar{n}_6^2 \tag{44}
\]

\[
\dot{n}_6 + 3H n_6 = -\langle \sigma v \rangle_{6611} n_6^2 - \left( \frac{n_1}{\bar{n}_1} \right)^2 \bar{n}_6^2 \tag{45}
\]

Corrections to this estimate are expected to be of order unity, which is well within the uncertainties on the cross sections.

The mass-weighted yields of the lightest \( 1^{+-} \) \( C \)-odd \( SU(3) \) glueball based on this analysis are shown in Fig. 5. Like for the \( C \)-even states, the inclusion of additional heavier \( C \)-odd glueballs generally has a negligible effect on the final abundance of the lightest \( 0^{++} \) mode, which dominates the total glueball density. The relic abundance of the \( 1^{+-} \) state is much smaller than the \( 0^{++} \), but can be considerably larger than any of the \( C \)-even states, even though it is heavier than the \( 2^{++} \) and \( 0^{--} \) glueballs. As discussed above, this can be understood by the absence of relevant coannihilation reactions involving the much more abundant \( 0^{++} \) glueball. Let us also point out that \( C \)-odd dark glueballs provide an explicit realization of the scenario discussed in Ref. [37] consisting of a stable dark matter state freezing out in the background of a massive bath.

V. DARK MATTER SCENARIOS AND CONNECTIONS TO THE SM

Stable dark glueballs will contribute to the dark matter (DM) density of the Universe. However, if the dark sector has a connection to the SM, some or all of the dark glueballs will be able to decay [13]. We outline here a number of different connector scenarios and describe their implications for dark matter and cosmology. A more detailed investigation of the cosmological effects of dark glueballs will be presented in Ref. [35].
FIG. 5. Mass-weighted relic yields of the $1^{--}$ dark glueball in $SU(3)$ as a function of $\Lambda_x = m_x$ and $R$.

A. No Connection: Stable Glueballs

With no connection to the SM, all the states in the glueball spectrum discussed in Section II will be stable and contribute to the net DM density. As reported in Sections III and IV, the total glueball contribution will be dominated by the lightest $0^{++}$ state. The DM scenario in this case coincides with the glueball scenarios considered of Refs. [19–21] in which only the lightest glueball was considered. Avoiding overclosure by the glueball relic density bounds $\Lambda_x$ and $R$ from above, as can be seen in Fig. 1. If the lightest glueball makes up all the DM, $\Lambda_x$ is bounded from below by the requirement that its self-interaction cross section not be too large, $\sigma_{2\rightarrow2}/m \lesssim 10 \text{ cm}^2/\text{g}$, which translates into

$$\Lambda_x \gtrsim 100 \text{ MeV} \left( \frac{3}{N} \right)^{4/3}. \quad (46)$$

Smaller $\Lambda_x$ can also interfere with cosmic structure formation [21, 88, 89].

B. Charged Matter Connection: Unstable Glueballs

A minimal connection to the SM consists of heavy matter charged under both the dark and SM gauge groups. For example, integrating out massive vector-like fermions at the scale $M \gg \Lambda_x$ generates operators of the form

$$\mathcal{L}_{\text{eff}} \supset \frac{\alpha_x \alpha_i}{M^4} \text{tr}(W_i W_i) \text{tr}(G_x G_x) + \frac{\alpha_x^2 \alpha_Y^2}{M^4} B_{\mu\nu} \text{tr}(G_x G_x G_x)^\mu_{\nu}, \quad (47)$$

where $\alpha_x = g_x^2/(4\pi)^2$ refers to the dark gauge coupling evaluated at the perturbative matching scale $M \gg \Lambda_x$ and $\alpha_i (i = Y, 2, 3)$ to the SM gauge coupling. The operators written in Eq. (47)

\footnote{Decays to gravitons are possible, but the corresponding lifetime is much longer than the age of the Universe for $\Lambda_x \lesssim 10^7 \text{ GeV}$ [21].}
are schematic, and represent a set of many different Lorentz contractions; full expressions can be found in Refs. [28, 29].

The operators of Eq. (47) connect the dark glueballs to the SM vector bosons after glueball confinement at $\Lambda_x$. Together, they allow all the would-be stable glueballs discussed in Sec. II to decay either directly to the SM, or to a lighter glueball state and a set of SM particles [28]. The parametric dependences of the decay rates are

$$\Gamma \sim \left\{ \alpha_x^2 \alpha_i^2, \alpha_x^3 \alpha_Y \right\} N^2 \frac{\Lambda_x^9}{M^8},$$

with significant additional suppression possible if the final-state phase space is constrained [28].

Due to the large exponents in Eq. (48), the decay rates of glueballs through the operators of Eq. (47) can span an enormous range. For lifetimes beyond the age of the Universe the glueballs will contribute to the DM density and the considerations discussed above apply here as well. In addition, for lifetimes $\tau \lesssim 10^{26}$ s there will also be constraints from energy injection into the CMB near recombination [90–92], x-ray and gamma-ray fluxes [21, 93], and energy release during primordial nucleosynthesis [91, 94–96]. Given the parametrically similar decay rates and the much larger relic density of the lightest $0^{++}$ glueball relative to the others, these bounds apply primarily to this state.

The operators of Eq. (48) can also be relevant for the glueball freezeout abundances. At high temperatures they can lead to the thermalization of the dark and visible sectors, although the specific details depend on the reheating history after primordial inflation. They may also help to further populate the dark sector through inverse decays [91], or induce decays before freezeout occurs, although this typically requires relatively larger values of $\Lambda_x/M$.

### C. Higgs Portal Connection

A second type of SM connector is a scalar $\Phi_x$ charged only under $G_x$ with a Higgs portal coupling to the SM,

$$\mathcal{L} \supset M^2|\Phi_x|^2 + \lambda_x |\Phi_x|^2 |H|^2.$$  

(49)

Integrating out the scalar at its mass $M$, assumed to be much larger than both $\Lambda_x$ and $\langle H \rangle = v$, produces an operator of the form

$$\mathcal{L}_{\text{eff}} \supset \frac{\lambda_x \alpha_x}{M^2} |H|^2 \text{tr} (G_{x\mu\nu} G^{\mu\nu}).$$

(50)

With $C$ and $P$ conservation in the dark sector, this is the only Lorentz structure generated at the dimension-six level.

After glueball confinement, the operator of Eq. (50) gives rise to a Higgs portal coupling between the SM Higgs boson and the $0^{++}$ dark glueball. The implications of this mixing were studied in detail in Ref. [29], where it was shown that it allows nearly all the heavier glueballs to decay radiatively to lighter glueballs or directly to the SM. If $\Delta m_x \sim \Lambda_x$ is the mass splitting for radiative decays or the mass of the decaying glueball for direct decays to the SM, the parametric decay width for $\Delta m_x$ less than twice the Higgs boson mass is [29]

$$\Gamma \sim N^2 \lambda_x^2 \alpha_x^2 \frac{\Lambda_x^6}{v^2 M^4} \Gamma_h (m_h = \Delta m_x), \quad (\Delta M_x < 2m_h)$$

(51)

where $\Gamma_h$ is the width the SM Higgs would have if its mass were $m_h = \Delta m_x$. For $\Delta m_x$ greater than twice the Higgs mass, the parametric width becomes

$$\Gamma \sim N^2 \lambda_x^2 \alpha_x^2 \frac{\Lambda_x^5}{M^4}, \quad (\Delta m_x > 2m_h),$$

(52)
and is dominated by Higgs final states.

The only two ($SU(3)$) glueballs unable to decay through the operator of Eq. (50) are the $0^{-+}$ and $1^{+-}$ states [29]. If the sole connection to the SM is the Higgs portal, the dark sector has an independent charge conjugation symmetry that implies that the lightest $C$-odd glueball is stable (up to gravitational effects), which is $1^{+-}$ for $G_x = SU(3)$. The situation for the $0^{-+}$ state is more model-dependent, and decays with rates on the order of Eqs. (51,52) can arise if the visible Higgs sector contains two doublets or if there is additional parity violation [29].

The DM picture that arises in this scenario consists of stable $1^{+-}$ (and possibly $0^{-+}$) glueballs in a bath consisting mostly of metastable $0^{++}$ states, and is an explicit realization of the general scenario presented in Ref. [87]. The $0^{++}$ glueballs can still dominate the dark-sector contribution to the DM density if they are long-lived. In contrast, if the $0^{++}$ glueballs decay to the SM early enough, only the $1^{+-}$ (and possibly $0^{-+}$) will contribute to the DM density. The subleading relic densities of the heavier glueballs computed in Sec. III then become essential to the net DM abundance.

**D. Yukawa Connection**

An intermediate scenario relative to the previous two can arise from vector-like fermions $\Psi$ and $\chi$ with both $G_x$ and SM quantum numbers that couple to the SM Higgs field according to

$$\mathcal{L} \supset M\overline{\Psi}\Psi + M'\overline{\chi}\chi + y_x\overline{\Psi}H\chi + (h.c.) \, .$$

Integrating out these massive states at $M \sim M'$, assumed to be much larger than $\Lambda_x$ and $\langle H \rangle$, generates the operators of Eq. (47) as before along with the operator of Eq. (50) with $\lambda_x \to y_x^2$.

This scenario opens a broad range of phenomenological possibilities. Most of the glueballs, including the lightest $0^{++}$, can decay through either the dimension-six operator of Eq. (50) or the dimension-eight operators of Eq. (47). However, the $1^{+-}$ (and possibly the $0^{-+}$) glueball is only able to decay at dimension eight. When the ratio $\Lambda_x/M$ is large and $y_x^2/4\pi$ is not strongly suppressed relative to $\alpha_x$, the $1^{+-}$ glueballs are parametrically long-lived relative to the $0^{++}$ and other glueballs. Thus, depending on the relative lifetimes the strongest constraints on this scenario can come from the very late decays of the subleading relic abundance of the $1^{+-}$ glueballs.

**VI. CONCLUSIONS**

In this paper we have investigated the freezeout dynamics of dark glueballs in the early Universe. Such glueballs arise from confinement in theories with a new non-Abelian gauge force decoupled from the SM and all charged matter significantly heavier than the confinement scale. Our results expand upon previous studies of the cosmological history dark glueballs in two key ways [19–21]. First, we studied potential new effects on the glueball relic density due to the confining phase transition itself. And second, we performed a detailed analysis of the freezeout dynamics of the heavier glueballs in the spectrum. We also discussed connections to the SM as well as some of the implications of the heavier glueballs on dark matter, astrophysics, and cosmology, with a more detailed study to appear in Ref. [35].

In the absence of glueball decays through connectors to the SM (or other lighter states), we find that the lightest $0^{++}$ state dominates the total glueball relic abundance, and the abundance we calculate is in agreement with previous studies that only considered the lightest state [20,21]. The relative relic densities of heavier glueballs in the spectrum are orders of magnitude smaller, with the largest contributions coming from the $0^{-+}$ and $1^{+-}$ modes (for $SU(3)$). Even though the
abundances of these states are much smaller than the lightest 0++, they can also be parametrically long-lived compared to the 0++. This opens the possibility of the 0++ mode decaying away early, and the heavier modes making up the DM density today or leading to the most stringent constraints on dark Yang-Mills theories. A detailed study of these effects based on the results of this paper is underway [35].

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