Initialization of the Bell states of two qubits by unipolar pulses

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Abstract. Simulation of the entangled (Bell) states generation of two qubits by using of unipolar picoseconds pulses was performed. As an example, a system of two coupled superconducting flux qubits interacting with fluxons that integrated with a Josephson transmission line has been considered. The influence of the pulse shape and quantum noise on the accuracy of the Bell states initialization and the way to control of nonlocal entangled states is discussed.

1. Introduction
Recently the superconducting circuit quantum electrodynamics is an actively developing field, in which significant progress has been achieved in the manipulation of quantum bits (qubits). One of the interesting directions in this area is the use of new generation of energy-efficient logic family for fast read-out and control of quantum registers [1-8].

In this paper, we are going to study the simplest two qubits register whose transition frequencies are located in the micro- or millimeter ranges, and the times of longitudinal and transverse relaxation are microseconds [1]. It is well known that in this field of quantum logical manipulation usually the Rabi-technique is involved. We propose here a new approach for the implementation of quantum logic which is based on the control of the qubit system by unipolar sub-nanosecond solitary-like pulses of the rectangular shape. Such kind of magnetic field pulses may be generated by fluxons in transmission lines [2-4]. The analysis was carried out by numerical solution of the master equation for the density matrix operator and the populations of qubit levels were calculated. It was shown that the optimal way to control a system of two qubits can be achieved when two control pulses with proper delay may be applied. We found the parameters of the control pulses for the initialization of the entangled state with fidelity of 99%. Also, the initialization problem for the Bell states is analyzed and the fidelity 98% for these states is found.

2. The model of the system and the basic equations
Hamiltonian for two coupled flux qubits can be represented as

\[ H(t) = -\frac{1}{2}(\varepsilon_1(t)\sigma_z^{(1)} + \Delta_1\sigma_x^{(1)}) \otimes I^{(2)} - \frac{1}{2}I^{(1)} \otimes (\varepsilon_2(t)\sigma_z^{(2)} + \Delta_2\sigma_x^{(2)}) - \frac{1}{2}J\sigma_z^{(1)} \otimes \sigma_z^{(2)}, \]

(1)
where $\Delta_i$ is tunnel splitting in the $i$-th qubit $(i=1,2)$, $J$ is the interaction constant, $\epsilon_i(t)$ are the form for the pulses; $\sigma^{(i)} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $\sigma^{(i)}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ are Pauli matrixes, and $I^{(i)}$ is unit matrix of the $i$-th qubit, the symbol “$\otimes$” is used to indicate the Kronecker product.

We considered two coupled qubits with slightly different tunnel splitting: $\Delta_z = \Delta_i + \Delta \Delta$, where $\Delta \Delta \ll \Delta_i$. The two coupled qubits gives us a four-level system $(j=1,2,3,4)$. The eigenvalues $E_j$ and eigenvectors $\varphi_j$ in the stationary case $(\epsilon_i(t) = 0)$ can be determined from the stationary Schrödinger equation: $H(\epsilon_i(t) = 0)\varphi_j = E_j\varphi_j$. We assume that at the initial time $t = 0$ the qubits system is initialized in the ground state $(j=1)$ with energy $E_1 = \frac{1}{2}\sqrt{J^2 + (2\Delta_i + \Delta \Delta)^2}$ and eigenvector $\langle \varphi_1 \rangle = (2(k^2 + (2\Delta_i + \Delta \Delta)^2))^{-1/2}(k,2\Delta_i + \Delta \Delta,2\Delta_i + \Delta \Delta,k)$, where $k = J + \sqrt{J^2 + (2\Delta_i + \Delta \Delta)^2}$. The changes in the state of the system of coupled qubits occur due to interaction with unipolar pulses $\epsilon_i(t)$. Control pulses can act at different instants of time and have different parameters (amplitudes and durations) because of different magnetic coupling between a fluxon and a qubit and different shapes of fluxons in tunable transmission lines. For our calculations, we used square pulses with smoothed fronts of the following form:

$$\epsilon_i(t) = A_i \begin{cases} (t-t_{in,j})/t_0, & t_{in,j} \leq t < t_{in,j} + t_0, \\ 1, & t_{in,j} + t_0 \leq t \leq t_{off,j} - t_0, \\ (t_{off,j} - t)/t_0, & t_{off,j} - t_0 < t \leq t_{off,j}, \end{cases}$$

$$(2)$$

We found the parameters of the control pulses for the initialization of the entangled state with fidelity of 99%. Also, the initialization problem for Bell states is analyzed and the fidelity 98% for these states is found. where $A_i$ is an amplitude, $t_{in,j}, t_{off,j}$ are the turn-on and turn-off times for the pulse, which determine the duration $\tau_j = t_{off,j} - t_{in,j}$, and $t_0$ is time smoothed fronts $(t_0 \ll \tau_j)$. These parameters can be controlled in an experiment, thus controlling the evolution of the quantum system (by analogy with a single qubit [11]).

In conditions close to real experiments, there are many channels of decoherence that affect the measurement results of the coupled qubits. Following of general approach [9], we describe the relaxation in a system of two coupled qubits, assuming that each of them interacts with an infinite bosonic bath. The master equation of two qubits, which is averaged over the reservoir variables, has the form ($\hbar = 1$):

$$i\hbar\frac{\partial \rho}{\partial t} = [H(t), \rho] + \frac{\gamma_i}{2}(\sigma^{(i)} \otimes I^{(2)} - \rho) + \frac{\gamma_i}{2}(I^{(1)} \otimes \sigma^{(2)} - \rho)$$

$$(3)$$

where $\rho$ is the density matrix of two coupled qubits, $\gamma_i$ is the relaxation rate of the $i$-th qubit, $H(t)$ is the Hamiltonian in the form Eq. (1).

We will be interested in the dynamics of the diagonal elements $\rho_{ii}$ of the density matrix, which correspond to the level populations $W_i$. We introduce the concept of concurrence for a two-qubit system [10]. The concurrence is an entanglement monotone defined for a mixed state of two qubits as:
\[ C(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\} . \] (4)

In Eq. (4) \( \lambda_j \) is the non-negative eigenvalue of the Hermitian matrix \( R = \sqrt{\rho^* \rho} \) in decreasing order. Here, \( \rho = (\sigma_1 \otimes \sigma_2) \rho^* (\sigma_1 \otimes \sigma_2) \), and \( \rho^* \) are the complex conjugate of \( \rho \), \( \sigma_j \) is Pauli matrix.

3. Results and discussion

For definiteness we will assume that the duration of the first pulse \( e_1(t) \) is fixed value \( r_1 \). The second unipolar pulse \( e_2(t) \) has a delay \( \delta T \) relative to the first one \( t_{on,2} = t_{on,1} + \delta T \) and the pulse duration \( r_2 \) may be changed. The numerical analysis of Eq. (4) showed that there are ranges of amplitudes and durations of unipolar action, when the system is initialized in nonlocal entangled states (with the population of the 2nd and the 3d levels) with a fidelity of 98%. The characteristic dynamics of excitation of entangled states (the level populations) \( W_j(t) \) is shown in Fig. 1 by vertical arrows. Note that the ranges for the adjustable parameters of the controllable pulse \( A_2, r_2 \) are spaced, which makes it possible to realize an “isolated” initialization in each of the states with large fidelity. We note that the effect of noise affects the process at times \( \tau_i = 1/\gamma_i \).

**Figure 1.** The time-dependant dynamics of the level populations \( W_j(t) \) of two coupled qubits when the parameters were chosen as: \( A_{1,2} = 1.58 \text{ GHz} \) and \( t_{off,2} = 6 \text{ ps} \) (a); \( A_{1,2} = 4.1 \text{ GHz} \) and \( t_{off,2} = 2.75 \text{ ps} \) (b). The blue curve characterizes the behavior of \( W_1(t) \), the red curve – \( W_2(t) \), the green – \( W_3(t) \), the black – \( W_4(t) \). Vertical arrows show the range of turn-on and off for unipolar pulses. The parameters of the pulses and qubits are as follows: \( \Delta_1 = 0.01 \text{ GHz} \), \( J = 0.01 \text{ GHz} \), \( \delta \alpha = 0.02 \text{ GHz} \), \( \gamma_{1,2} = 0.0001 \text{ GHz} \), \( t_0 = 0.1 \text{ ps} \), \( t_{on,1} = 1 \text{ ps} \), \( \delta T = 1 \text{ ps} \), \( t_{off,1} = 3 \text{ ps} \) (a) and \( t_{off,1} = 2.5 \text{ ps} \).

It is interesting to study the initialization of Bell’s states, which are very important for the implementation of two-qubit quantum logic. The Bell states are four specific maximally entangled quantum states for two qubits. In this case, the intermediate states of the qubit system have the same level populations \( W_j(t) = W_j(t = 0.5) \). The degree of entanglement (the concurrence) of the individual qubits states, which can be described by the quantity Eq. (4), is maximal: \( C(\rho) \rightarrow 1 \) [10].

In Fig. 2 it is shown that it is possible to select parameters for the Bell states generation. At the same time, the degree of entanglement reaches its maximum value.
Figure 2. Evolution of the level populations of two coupled qubits in the recording of Bell states (a) and the dynamic of the concurrence (b). The colors of the curves on (a) are identical to Fig. 1. The parameters of the pulses and qubits are as follows: $\Delta_1 = 0.01$ GHz, $J = 0.065$ GHz, $\delta \Delta = 0.02$ GHz, $\gamma_{1,2} = 0.0001$ GHz, $A_{1,2} = 3.15$ GHz, $t_0 = 0.1$ ps, $t_{on,1} = 1$ ps, $\delta T = 3$ ps, $t_{off,1} = 3$ ps and $t_{off,2} = 5$ ps.

4. Conclusion
In this paper, we showed that a system of two coupled superconducting flux qubits can be controlled by means of unipolar sub-nanosecond pulses. It is shown that by varying the parameters of such impacts (amplitude, duration), one can realize initialization in nonlocal intermediate as well as Bell states. These ranges of operating parameters were obtained numerically, based on the solution of the master equation. The effects of quantum noise on these manipulations were studied and it was found that decoherence affects only when the duration of the signal is increased (i.e., at times $1/\gamma \sim 0.1$ μs). The degree of entanglement (the concurrence) was estimated and it was demonstrated that for Bell states it can be realized with accuracy of up to 98%, and in nonlocal states with an accuracy of 99%.

Acknowledgments
This work was supported by the Ministry of Education of the Russian Federation (contract No. 3.3026.2017), by the RFBR (grants No. 18-07-01206 and 16-07-01012), and by the RNF No. 18-72-00158.

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