CP violation in $\eta \to \pi + \pi$ by Higgs - $\eta$ mixing through two-loop quantum effects

X. Y. Pham

Université Pierre et Marie Curie, Paris VI
Université Denis Diderot, Paris VII
Physique Théorique et Hautes Énergies
Unité associée au CNRS : D 0280

Abstract

Strictly within the standard electro-weak interaction, CP violation in the flavour conserving process $\eta \to \pi + \pi$ could originate from the mixing of the $\eta$ meson with the virtual scalar Higgs $H^0$ via $W^+ + W^−$ and $Z^0 + Z^0$ exchange.

The parity-violation carried by these weak gauge bosons makes the mixing possible at two-loop level. Nowhere the Kobayashi-Maskawa (KM) phase mechanism is needed.

For the Higgs mass between 100-600 GeV, the $\eta \to \pi + \pi$ branching ratio is found to be $3 \cdot 10^{-26} - 2 \cdot 10^{-29}$, hence unconventional CP violation mechanisms are the only ones that could give rise to its observation at the existing or near future $\eta$ factories, unless the Higgs mass is improbably as light as 10 MeV.

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*Postal address: LPTHE, Tour 16, 1er Étage, Université Pierre et Marie Curie and Université Denis Diderot, 4 Place Jussieu, F-75252 Paris CEDEX 05, France.
Email address : pham@lpthe.jussieu.fr
To understand the origin and the nature of CP violation, in addition to the studies of flavour changing K and B mesons processes, investigations are also needed in flavour conserving ones for which $\eta \to \pi + \pi$ decay and electric dipole moment of baryons are some typical examples. Like the $K_L^0$, an eventual coexistence of both three and two-pion decay modes of the $\eta$ would imply that CP is violated in the flavour conserving sector. Therefore experimental searches for the two-pion decay mode of the $\eta$ is of great interest, and the purpose of this letter is to give a reliable estimate of its branching ratio, strictly within the standard electro-weak Higgs framework.

We do find indeed that CP violation in $\eta \to \pi + \pi$ simply can be generated by the neutral Higgs boson, independently of all other mechanisms.

In principle, an eventual role played by the standard neutral Higgs boson $H^0$ in CP odd interaction should be envisaged at first, since it lies in the heart of the standard model (SM). We even do not have to evoke the KM phase. The latter would be, in some sense, the next step to be considered in the studies of $\eta \to \pi + \pi$ decay, while the third one could be non conventional mechanisms of CP violation, for example the $\Theta$ vacuum in QCD or spontaneous breaking triggered by charged scalar fields.

With a touch of irony, the third stage - CP odd $\Theta$ term - as source of $\eta \to \pi + \pi$ decay was worked out long time ago before the second one for which the rate is recently computed by Jarlskog and Shabalin (JS) within the KM framework of the penguin effective lagrangian. As far as we know, the most straightforward implication of the standard Higgs mechanism on $\eta \to \pi + \pi$ reaction has never been noticed, although in principle it should be naturally considered as the first step to cross over. The purpose of this work is to fulfil the gap: we indeed show that this mode actually occurs because of the pseudoscalar-scalar (PS) mixing between the $\eta$ and the virtual neutral Higgs boson which subsequently decays into two pions; this PS mixing is due to quantum effects at two-loop level as shown in Figure 1.

Physically, it means that the parity-violation VA property of the weak bosons shifts the intrinsic $CP = -1$ of the $\eta$ into the $CP = +1$ of the $H^0$: parity-violation turns out to be the source of CP non-conservation, due to the Higgs and gauge bosons interplay. This observation is illustrated by explicit computation of the diagram, the relevant quantity to be considered is
\[ I(k^2) \equiv (-1) \int \frac{d^4q}{(2\pi)^4} \frac{d^4p}{(2\pi)^4} \frac{\text{Trace}\{\gamma_\mu(a - b\gamma_5)(\not{p} + m)\gamma_5(\not{p} + \not{k} + m)\gamma_\mu(a - b\gamma_5)(- \not{q} + m')\}}{(q^2 - m'^2)(p^2 - m^2)((p + k)^2 - m^2)((p + q + k)^2 - M^2)} \]

with \( a = b = V_{cs}, \ m = m_s, \ m' = m_c, \ M = M_W \) for W exchange.

\[ a = \left(-\frac{1}{\sqrt{2}} + 4 \sin^2 \theta_W / 3\right) \frac{1}{\sqrt{2}}, \ b = \frac{-1}{\sqrt{2}} \cos \theta_W, \ m = m' = m_s, M = M_Z \) for Z exchange.

In Eq.(1) we have taken, as an illustrative example, the contribution of the \( s\bar{s} \) component of the \( \eta \) to the loops. We first integrate over \( d^4p \) using the \( x, y, z \) Feynman parametrization, and then over \( d^4q \), the result is:

\[ I(k^2) = C \frac{m a b}{32 \pi^4} \frac{k^2}{M^2} \left[ 1 + O\{\frac{k^2}{M^2}, \frac{m^2}{M^2}, \frac{m'^2}{M^2}\} \right] \]

where:

\[ C \equiv \int_0^1 dx \int_0^x dy \int_0^y dz \frac{(z(1 - y) - y(1 - x))}{(x - z)^3(1 - x + z)^2} = 1/4 \]

The expression (2) we obtain for the two-loop integration is impressively simple, because higher orders in \( \frac{k^2}{M^2}, \frac{m^2}{M^2}, \frac{m'^2}{M^2} \) (beyond the linear term \( \frac{k^2}{M^2} \)) are neglected in the course of our \( d^4q \) integration. In the \( d^4p \) one, everything is kept however. Without this legitimate approximation, we would obtain an avalanche of unnecessary and numerically negligible-complicated expressions involving, among others, dilogarithmic (or Spence) function frequently met in such circumstance.

It remains two questions to be settled: the first one concerns the effective point-like coupling constant \( g_{\eta Q\bar{Q}} \) of the \( \eta \) meson with quarks Q assumed in Fig.1. Is it justified? If yes what is its numerical values? The second point deals with the off-shell (virtually light mass \( k^2 = m_\eta^2 \)) Higgs decay amplitude into two pions.

1- The justification for the \( \eta \)-quarks coupling can be traced back to the famous antecedent Adler, Bell, Jackiw (ABJ) chiral anomaly\[\text{[7]}\] inherent to \( \eta \rightarrow \gamma + \gamma \) (or \( \pi^0 \rightarrow \gamma + \gamma \)) decay for which the same triangle one-loop is involved where external real photons replace internal virtual gauge bosons of our two-loop diagram in Fig.1. Such \( \eta \) (or \( \pi \)) coupling to light quarks seems to stand on a firm ground and is intimately connected to its Goldstone nature, to the partially conserved axial current (PCAC) and its consequence: the Goldberger-Treiman (GT) relation. These properties hold\[\text{[8, 9]}\] also in the Nambu-Jona-Lasinio model\[\text{[9]}\], a prototype of low-energy effective lagrangian suitable for studying Goldstone particles.
However there is an important difference between the external on-shell photons in $\pi^0 \rightarrow \gamma + \gamma$ decay\footnote{For simplicity we take $\pi$ as an example, the $\eta$ case is similar although the situation is getting complicated by the SU(3) flavour singlet-octet $\eta - \eta'$ mixing laterly included.} and the internal off-shell weak gauge bosons considered in Fig.1. For $\pi^0 \rightarrow \gamma + \gamma$, in order to get the right answer in agreement with data and with the ABJ anomaly i.e. the amplitude is proportional to $\frac{g_{\pi\eta\eta}}{m_Q} = \frac{1}{f_\pi}$, two conditions have to be satisfied:

(i) $k^2 \ll 4m_Q^2$, such that the triangle loop integration\footnote{Incidently, the same condition is satisfied by the Steinberger old calculation of $\pi^0 \rightarrow \gamma + \gamma$ (Phys.Rev, 76, 1180, (1949)) in which quark was proton at that time; his work fits naturally with the constituent u, d mass.} yields term proportional to $\frac{1}{m_Q}$ ($m_Q$ in the denominator).

(ii) the validity of the GT relation at the quark level, i.e. $\frac{g_{\pi\eta\eta}}{m_Q} = \frac{1}{f_\pi}$.

These two conditions are naturally fulfilled with the Goldstone nature of the pion. The case $k^2 > 4m_Q^2$ (not chosen by Nature) would lead to catastrophic disagreement with data and with the ABJ anomaly, since instead of $\frac{1}{m_Q}$ one would get $\frac{m_Q}{\frac{4}{m_Q} - 4m_Q^2 - i\pi}^2$.

The independence of the ABJ anomaly on the internal quark mass, a well known fact, has its origin in the $k^2 \ll 4m_Q^2$ constraint:\footnote{The virtually light ($k^2 = m^2_{\eta}$) Higgs boson coupling to two pions can be reliably estimated from the so-called conformal anomaly\cite{13} i.e. the trace of the energy-momentum tensor in QCD \cite{13} $\Theta^\mu = -\beta_0 \frac{\alpha_s}{8\pi} G_{\mu\nu} G^{\mu\nu}$ ($\beta_0 = 9$ is the first coefficient of the QCD $\beta$ function). The crucial point - as explained in \cite{14} - is that the matrix element of the operator $\alpha_s G_{\mu\nu} G^{\mu\nu}$ between the two-pion state and vacuum is nonvanishing in the chiral limit, it even does not depend on}$: the $\pi^0 \rightarrow \gamma + \gamma$ amplitude depends neither separately on $g_{\pi\eta\eta}$ nor on $m_Q$ but only on their ratio $\frac{g_{\pi\eta\eta}}{m_Q}$ which is fixed by $\frac{1}{f_\pi}$. As a consequence, the $\pi^0 \rightarrow \gamma + \gamma$ rate is helpless for indicating which values of $g_{\pi\eta\eta}$ or $m_Q$ to be employed: current or constituent mass of the light quarks?

In our case with both internal off-shell gauge bosons, the loop integration no longer produces $\frac{1}{m_Q}$ but instead $m_Q$ in the numerator, as explicitly shown in Eq.(2). Our PS mixing is then proportional to $m_Q g_{\eta\eta\eta} = \frac{m^2_{\eta}}{16}$ such that the choice of $m_Q$ is unavoidable. We have seen that the real photons rate cannot help, nevertheless the $k^2 \ll 4m_Q^2$ lesson must not be forgotten. This condition favourably hints to the choice of constituent mass, as also found by other authors \cite{3, 12} in a different context.

2- The virtually light ($k^2 = m^2_{\eta}$) Higgs boson coupling to two pions can be reliably estimated from the so-called conformal anomaly\cite{13} i.e. the trace of the energy-momentum tensor in QCD \cite{13} $\Theta^\mu = -\beta_0 \frac{\alpha_s}{8\pi} G_{\mu\nu} G^{\mu\nu}$ ($\beta_0 = 9$ is the first coefficient of the QCD $\beta$ function). The crucial point - as explained in \cite{14} - is that the matrix element of the operator $\alpha_s G_{\mu\nu} G^{\mu\nu}$ between the two-pion state and vacuum is nonvanishing in the chiral limit, it even does not depend on
\( \alpha_s \); the (virtually light \( k^2 \)) Higgs decay amplitude into two pions is found to be\[14\]

\[
f_{H\pi\pi}(k^2) = -\frac{g}{\beta_0} \cdot \frac{k^2 + 5.5 \cdot m_{\pi}^2}{M_W}
\]

where \( g = e/\sin\theta_W \) is the standard SU(2) gauge coupling which enters also in the three other vertices of Fig.1. Putting altogether the ingredients, we obtain for the \( \eta \rightarrow \pi + \pi \) decay amplitude the following result:

\[
A_{\eta^{+}\pi^{-}} = A_{\eta^{0}\pi^{0}} = \frac{1}{6\sqrt{3}} \left( \frac{G_F M_W^2}{4\pi^2} \right)^2 \cdot \frac{m_{\eta}^2}{M_W^2} \cdot \frac{m_{\eta}^2 + 5.5m_{\pi}^2}{m_H^2} \cdot \frac{1}{f_{\eta}} \left( X_W + \frac{X_Z}{4\cos^2\theta_W} \right)
\]

with

\[
X_W = m_{\pi}^2 (\sqrt{2}\cos\theta_P + \sin\theta_P) - (m_u^2 + m_d^2) \left( \frac{\cos\theta_P}{\sqrt{2}} - \sin\theta_P \right)
\]

\[
X_Z = \left( 1 - \frac{4}{3}\sin^2\theta_W \right) \left[ m_{s}^2 (\sqrt{2}\cos\theta_P + \sin\theta_P) - m_d^2 \left( \frac{\cos\theta_P}{\sqrt{2}} - \sin\theta_P \right) \right]
\]

\[- \left( 1 - \frac{8}{3}\sin^2\theta_W \right) m_{u}^2 \left( \frac{\cos\theta_P}{\sqrt{2}} - \sin\theta_P \right)
\]

from which:

\[
\Gamma(\eta \rightarrow \pi^+\pi^-) = 2 \cdot \Gamma(\eta \rightarrow \pi^0\pi^0) = \frac{|A_{\eta^{+}\pi^{-}}|^2}{16\pi m_{\eta}} \cdot \sqrt{1 - \frac{4m_{\pi}^2}{m_{\eta}^2}}
\]

In Eqs. (5) - (6), the GT like relation \( g_{\eta Q\bar{Q}} = \frac{m_{Q}}{f_{\eta}} \) is used, quark color indices are summed up, and \( \theta_P \simeq -19^\circ \) is the flavour SU(3) \( \eta - \eta' \) mixing angle determined from their two photon rates. We take \( f_{\eta} = f_{\pi} \simeq 93 \text{ MeV} \). Surprisingly enough, it turns out that the numerical values of the quantity \( Y \equiv \frac{(X_W + (X_Z/4\cos^2\theta_W))}{f_{\eta}} \) entering in Eq. (5) is relatively insensitive to the choices of quark masses: for the constituent ones \( m_s = 500 \text{ MeV}, m_u = m_d = 300 \text{ MeV} \), we have \( Y = 1.02 \text{ GeV} \); for the current ones \( m_s = 200 \text{ MeV}, m_u = m_d = 8 \text{ MeV} \), we get \( Y = 0.52 \text{ GeV} \).

With the constituent mass choice, we obtain:

\[ Br(\eta \rightarrow \pi + \pi) = 3.10^{-26} \left( \frac{100 \text{ GeV}}{M_H} \right)^4 \]

such that for the Higgs mass between 100 GeV and 600 GeV, the branching ratio into both charged and neutral pions of the \( \eta \) meson varies in the range \( 3 \cdot 10^{-26} - 2 \cdot 10^{-29} \), which is similar although (for the Higgs mass \( \leq 250 \text{ GeV} \)) somewhat larger than the JS result.

Therefore the standard model predicts that existing as well as future \( \eta \) factories (Saturne, Celsius, Daphne) could not detect the \( \eta \rightarrow \pi + \pi \) mode (unless the Higgs mass is improbably as light as 10 MeV), implying that unconventional CP violation mechanisms are the only ones
that could give rise to its eventual observation. This conclusion is not as negative as it seems, since as noted by JS, New CP violation mechanisms, what ever they may be, will have a golden opportunity to show up in the $\eta \rightarrow \pi + \pi$ decay.

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Figure Caption :

Figure 1 : $\eta$ - Higgs mixing by two-loop quarks-gauge bosons exchange
Figure 1