CP Violation in $t \rightarrow W^+b$ Decays in Two-Higgs Doublet Model

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ABSTRACT

Due to the large mass of top quark, CP violation in the top-quark decay is sensitive to the interactions mediated by Higgs bosons. We consider CP violation in $t \rightarrow W^+b$ decay by calculating consistently in unitary gauge the CP-violating up-down asymmetry of the leptons from $W$ boson decays in $t \rightarrow W^+b$, defined by Grządkowski and Gunion, in the two-Higgs doublet model with CP-violating neutral sector. The asymmetry is shown to be at most of the order of $(1 - 3) \times 10^{-4}$ for $\tan \beta = 1.0$, where $\tan \beta$ is the ratio of vacuum expectation values for the two neutral Higgs bosons.

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1. Introduction

The Kobayashi-Maskawa mechanism of CP violation is simple and very beautiful [1]. However, the origin is not yet finally verified by experiments, though that in the neutral kaon system seems to be explained well by the mechanism, and it is expected to be tested by B-meson decays in the coming B factories.

On the other hand, CP violation in the ordinary top-quark decay and top-pair production is estimated to be very small in the Kobayashi-Maskawa Standard Model [2-3]. This fact has given us an opportunity of investigating the non-standard origin of CP violation in the top decay and in the top-pair production [4-11].

There are some advantages in studying CP violation in the top-quark system; first, because of its large mass ($m_t \sim 180$ GeV [12]), the top-quark decays very fast, its lifetime being shorter than $10^{-23}$ s [13], so that the top would decay before it hadronizes and we do not have to worry about the hadronic effects, and secondly, for the same reason, the top decay interactions are sensitive to Higgs boson exchanges. This nature of top-quark has lead to many works of CP violation in the multi-Higgs doublet model [7,14-16]. Interesting among the Higgs models is two-Higgs doublet model of type II [17], in which CP violation is caused explicitly or spontaneously in its neutral sector [18]. Many authors have studied a variety of CP-violating observables in the top-quark productions [14] and decays [7,16] in this model.

In this paper, we investigate CP-violation in $t \rightarrow W^+b$ decay by calculating the up-down asymmetry of the leptons from $W$ decay with respect to the $t \rightarrow W^+b$ decaying plane, which is formed by $t$ production in $e^+e^-$ or hadron colliders, in the two-Higgs doublet model with CP-violating neutral sector. In the next section, the asymmetry of the lepton distributions is briefly expressed by the non-standard $t bW$-vertex form factor. In section 3,
two-Higgs doublet model is described in order to express the CP-violating part of the form factor as one-loop effects with CP-violating neutral scalar exchanges, and numerical results of the CP-violating asymmetry are given. Section 4 includes conclusions and discussions.

2. Asymmetry of lepton distributions in top decays

We here investigate CP violation in $t \rightarrow W^+b$ decays by the lepton distribution asymmetry from $W^+$ decay defined by Grządkowski and Gunion[7], assuming that the top quark is produced in $t\bar{t}$ pairs in $e^+e^-$ collision, for example.

The phase space for $e^+e^- \rightarrow t\bar{t} \rightarrow W^+W^-b\bar{b} \rightarrow \ell^+\ell^-\nu\bar{\nu}b\bar{b}$ is given by[19]

$$d\Phi = (2\pi)^{-4} d_s d_s W^+ d_s W^- d\Phi(e^+e^- \rightarrow t\bar{t}) d\Phi(t \rightarrow W^+b) d\Phi(W^+ \rightarrow \ell^+\nu)$$

$$\times d\Phi(\bar{t} \rightarrow W^-\bar{b}) d\Phi(W^- \rightarrow \ell^-\bar{\nu}),$$

where $s_x$ denotes the invariant mass of decaying particle $x$. In eq.(1), $d\Phi$’s are as follows

$$d\Phi(e^+e^- \rightarrow t\bar{t}) = (2\pi)^4 \frac{d^3 p_t}{(2\pi)^3 2E_t (2\pi)^3 2E_{\bar{t}}} \delta^{(4)}(P_{in} - p_t - p_{\bar{t}}),$$

$$d\Phi(t \rightarrow W^+b) = (2\pi)^4 \frac{d^3 p_{W^+}}{(2\pi)^3 2E_{W^+} (2\pi)^3 2E_b} \delta^{(4)}(p_t - p_{W^+} - p_b),$$

and so on, where $P_{in}, p_t, p_{\bar{t}}, p_{W^+}$ and $p_b$ are four-momenta of $e^+e^-, t, \bar{t}, W^+$ and $b$, respectively, $p_t, p_{\bar{t}}, p_{W^+}$ and $p_b$ their three-momenta, and $E_t, E_{\bar{t}}, E_{W^+}$ and $E_b$ their energies. The relevant matrix element for a given helicity final state is

$$M \equiv (h_b, h_{\bar{b}}, h_{\ell^+}, h_{\ell^-}, h_{\nu}, h_{\bar{\nu}}) \equiv D_{t}(s_t) D_{\bar{t}}(s_{\bar{t}}) D_{W^+}(s_{W^+}) D_{W^-}(s_{W^-}) \sum_{h_{t}, h_{\bar{t}}} (h_t, h_{\bar{t}})$$

$$\times \sum_{h_{W^+}} (h_t, h_{W^+}, h_b)(h_{W^+}, h_{\ell^+}, h_{\nu}) \sum_{h_{W^-}} (h_t, h_{W^-}, h_b)(h_{W^-}, h_{\ell^-}, h_{\bar{\nu}}),$$

(2)
where $h_x$ denotes the helicity of particle $x$, $D_y$ the propagator of decaying particle $y$, and $(h_t, h_{\bar{t}})$, $(h_t, h_{W^+}, h_b)$ and $(h_{W^+}, h_{\ell^+}, h_{\nu})$ are the helicity amplitudes for $e^+e^- \to t\bar{t}$, $t \to W^+b$ and $W^+ \to \ell^+\nu$, respectively. In the above equation, the helicities of the initial $e^+e^-$ are omitted in the amplitude $(h_t, h_{\bar{t}})$.

We apply to $D_y$ the narrow-width approximation for the real particle decays of $t, \bar{t}, W^+$ and $W^-$,

$$D_y(s_y) \simeq \frac{\pi}{m_y \Gamma_y} \delta(s_y - m_y^2), \quad (3)$$

where $m_y$ and $\Gamma_y$ are mass and total decay width of particle $y$. By integrating over $d\Phi(\bar{t} \to W^-\bar{b})$ and $d\Phi(W^- \to \ell^-\bar{\nu})$ in the differential cross section

$$d\sigma_{\text{tot}} = |M|^2 d\Phi, \quad (4)$$

the differential cross section for $t \to W^+b \to \ell^+\nu b$ is given as

$$\frac{d\sigma_{\text{tot}}}{d\Phi(t \to W^+b)d\Phi(W^+ \to \ell^+\nu)} = \frac{\text{Br}(\bar{t} \to W^-\bar{b}) \text{Br}(W^- \to \ell^-\nu)}{2m_t \Gamma_t \cdot 2m_W \Gamma_W} d\sigma(e^+e^- \to t\bar{t})$$

$$\times \sum_{h_t h_{\bar{t}}}(h_t, h_{W^+}, h_b)(h_{W^+}, h_{\ell^+}, h_{\nu}) [\sum_{h_{W^+}'} (h_{W^+}', h_{W^+}, h_b)(h_{W^+}', h_{\ell^+}, h_{\nu})]^*, \quad (5)$$

where $\text{Br}(\bar{t} \to W^-\bar{b})$ and $\text{Br}(W^- \to \ell^-\nu)$ are the branching ratios for $\bar{t} \to W^-\bar{b}$ and $W^- \to \ell^-\nu$, respectively, $d\sigma(e^+e^- \to t\bar{t})$ the differential cross section for $e^+e^- \to t\bar{t}$, and $\rho_{h_t h_{\bar{t}}}$ is the normalized top-quark density matrix,

$$\sum_{h_t} (h_t, h_{\bar{t}})(h_t', h_{\bar{t}})^* = \rho_{h_t h_{\bar{t}}} \sum_{h_t, h_{\bar{t}}} |(h_t, h_{\bar{t}})|^2. \quad (6)$$

We introduce the up-down asymmetry of the lepton distributions from $W^+$ decays with respect to the $t \to W^+b$ decay plane in the top-quark laboratory frame for $e^+e^- \to t\bar{t}$, defined in [6] as

$$A' \equiv \frac{N^t}{D^t}, \quad (7)$$
where

\[ N^t = \int \Phi(t \to W^+ b) \int_1^{-1} d\cos \theta_{\ell^+} \left[ \int_0^\pi - \int_{-\pi}^0 \right] d\phi_{\ell^+} \frac{d\sigma_{\text{tot}}}{d\Phi(t \to W^+ b) d\Phi(W^+ \to \ell^+ \nu)}, \] (8)

\[ D^t = \int \Phi(t \to W^+ b) \int_1^{-1} d\cos \theta_{\ell^+} \left[ \int_0^\pi + \int_{-\pi}^0 \right] d\phi_{\ell^+} \frac{d\sigma_{\text{tot}}}{d\Phi(t \to W^+ b) d\Phi(W^+ \to \ell^+ \nu)}, \] (9)

where \( \theta_{\ell^+} \) and \( \phi_{\ell^+} \) are the polar and azimuthal angles of the lepton \( \ell^+ \) in the \( W^+ \) rest frame.

We adopt the most general parametrization of \( tbW \) vertex for \( t \to W^+ b \) and \( \bar{t} \to W^- \bar{b} \) decays as follows,

\[ \Gamma^\mu = -\frac{g}{\sqrt{2}} V_{tb} \bar{u}(p_b) \left[ \gamma^\mu (f_1^L P_L + f_1^R P_R) - \frac{i\sigma_{\mu\nu} k^\nu}{m_W} (f_2^L P_L + f_2^R P_R) \right] u(p_t), \] (10)

\[ \bar{\Gamma}^\mu = -\frac{g}{\sqrt{2}} V_{tb} \bar{\bar{v}}(p_{\bar{b}}) \left[ \gamma^\mu (\bar{f}_1^L P_L + \bar{f}_1^R P_R) - \frac{i\sigma_{\mu\nu} k^\nu}{m_W} (\bar{f}_2^L P_L + \bar{f}_2^R P_R) \right] \bar{v}(p_{\bar{b}}), \] (11)

where \( P_{L/R} = (1 \mp \gamma_5)/2 \), \( k \) is the \( W \) momentum, \( V_{tb} \) is the \( (tb) \)-element of the Cabibbo-Kobayashi-Maskawa(CKM) mixing matrix and \( g \) is the \( SU(2) \) gauge coupling constant. Since \( W \) boson is on-shell, two other form factors do not contribute. The form factors of eqs.(10) and (11) are subject to [7,8]

\[ f_1^{L,R} = \pm \bar{f}_1^{L,R}, \quad \bar{f}_2^{L,R} = \pm \bar{f}_2^{R,L}, \] (12)

where upper(lower) signs are those for CP-conserving(-violating) interactions.

If we use the amplitudes \( (f_t, h_{W^+}, h_b) \) for \( t \to W^+ b \) decay derived from the vertex functions (10) and (11) [7], the following expressions for \( N^t \) and \( D^t \) of eqs.(8) and (9) are obtained,

\[ N^t = A 2\pi^3 m_t^2 - m_W^2 \bar{P}_\parallel \text{Im}(f_1^L f_2^{R*}), \] (13)

\[ D^t = A 16\pi^2 \left[ m_t^2 + 2m_W^2 \left| f_1^L \right|^2 + \frac{m_t^2}{m_W^2} \text{Re}(f_1^L f_2^{R*}) + \frac{2m_t^2 + m_W^2}{m_W^2} \left| f_2^R \right|^2 \right], \] (14)

where \( A \) is a common factor and \( P_\parallel^t \) is the longitudinal polarization of the top-quark which occurs in the density matrix \( \rho_{h_t \bar{h}_t} \) of eq.(6) [6,20]. If we
keep only the leading term in $D^t$, we obtain the following expression for the asymmetry $A^t$,

$$A^t = h(m_t) P^t_\parallel \text{Im}(f^L_1 f^R_2^*) / |f^L_1|^2,$$

(15)

where

$$h(m_t) = \frac{3\pi m^2_t - m^2_W}{8 m^2_t + 2m^2_W}.$$

It is important to note that the CP-violation phase of the CKM matrix does not appear in the asymmetry $A^t$ and that $f^L_1 = 1$ and $f^R_2 = 0$ at the tree level and $f^R_2$ does not have any contribution even at the one-loop level in the Standard Model [19]. So, $A^t$ is sensitive to the non-standard origin of CP violation.

Since $f^R_2$ has both CP-conserving and CP-violating parts, it is possible to have the only CP-violating contribution to the asymmetry by adding $A^{\bar{t}}$ for the anti-top decay ($\bar{t} \to W^- \bar{b}$) to $A^t$ as follows,

$$A \equiv A^t + A^{\bar{t}}.$$

(16)

The relations of eq.(12) and the fact that $P^t_\parallel = -P^t_\parallel$ at the tree level for the $t$ and $\bar{t}$ production mechanism [8] [24] lead to

$$A = 2h(m_t) P^t_\parallel \text{Im}(f^{R*}_{2\text{CPV}}),$$

(17)

where $f^L_1 = 1$ at the tree level and $f^{R*}_{2\text{CPV}}$ is the CP-violating contribution of $f^R_2$ given by

$$f^{R*}_{2\text{CPV}} = \frac{1}{2}(f^R_2 + \bar{f}^L_2).$$

(18)

3. The asymmetry $A^t$ in the two-Higgs doublet model

As stated in the previous section, since the asymmetry $A^t$ (or $A$) is sensitive to the non-standard origin of CP violation, we calculate this asymmetry
in the simplest extension of the Standard Model, that is, in the two-Higgs doublet model.

The CP-violating neutral sector of two-Higgs doublet model is caused by the soft breaking in the Higgs potential of the discrete symmetry imposed on the Yukawa-coupling Lagrangian. The discrete symmetry avoids the flavor-changing neutral current by coupling the charge $-\frac{1}{3}$ quarks and $+\frac{2}{3}$ quarks to the Higgs doublets $\phi_1$ and $\phi_2$, respectively (the so-called “type II” model), and their masses are generated through the vacuum expectation values $v_1$ and $v_2$ of $\phi_1$ and $\phi_2$, respectively. Relevant CP violation may be either explicit or spontaneous.

We will not use a specific model but adopt the parametrization for the Higgs scalars defined by Weinberg[21], which is formulated in unitary gauge through the unitarity gauge condition [22], and Goldstone boson do not appear in this model. The three neutral scalars in the model are parametrized as

$$
\phi_1^0 = \frac{v_1}{\sqrt{2}|v_1|} \left[ \Phi_1 - i \frac{|v_2|}{v} \Phi_3 \right], \quad \phi_2^0 = \frac{v_2}{\sqrt{2}|v_2|} \left[ \Phi_2 + i \frac{|v_1|}{v} \Phi_3 \right]
$$

(19)

where $v \equiv \sqrt{|v_1|^2 + |v_2|^2}$, and the real new fields $\Phi_1, \Phi_2$ and $\Phi_3$ are subject to the canonical kinetic Lagrangian in unitary gauge,

$$
L_{\text{kin}} = -\frac{1}{2} \sum_{n=1}^{3} (\partial_\mu \Phi_n)(\partial^\mu \Phi_n).
$$

(20)

The two neutral scalars $\Phi_1$ and $\Phi_2$ are of CP-even and the third scalar $\Phi_3$ is of CP-odd, as is evident from eq.(19), so that CP violation shows up in the scalar exchange between quarks and/or gauge bosons through the imaginary parts of the following four quantities[21],

$$
\frac{<\phi_1^0 \phi_i^0>}{v_1^2 v_2^2}, \quad \frac{<\phi_2^0 \phi_i^0>}{v_1 v_2}, \quad \frac{<\phi_1^0 \phi_i^0>}{(v_1)^2}, \quad \frac{<\phi_2^0 \phi_i^0>}{(v_2)^2},
$$

(21)

where $<\phi_i^0 \phi_j^0(\pm)>$ denotes the propagator of neutral Higgs scalars (see Appendix). If we adopt the approximation of taking the effect of neutral scalar
exchange to be dominated by a single scalar particle of mass \( m_\phi \), the four quantities are expressed as \[ 21, 7 \]
\[
\begin{align*}
\frac{< \phi_2^0 \phi_1^0 >}{v_1 v_2} &= \frac{\sqrt{2} G_F Z_0}{m_\phi^2 - q^2}, \\
(\frac{v_1}{v_2})^2 &= \frac{\sqrt{2} G_F \tilde{Z}_0}{m_\phi^2 - q^2}, \\
\frac{< \phi_1^0 \phi_1^0 >}{(v_1)^2} &= \frac{\sqrt{2} G_F Z_1}{m_\phi^2 - q^2}, \\
< \phi_1^0 \phi_2^0 > &= \frac{\sqrt{2} G_F Z_2}{m_\phi^2 - q^2},
\end{align*}
\tag{22}
\]
where we use the formalism of Feynman rules designed to minimize the number of times that the imaginary unit \( i \) appears \[ 23 \]. By using eq.(19), \( \text{Im}Z_0, \text{Im}\tilde{Z}_0, \text{Im}Z_1 \) and \( \text{Im}Z_2 \) of eq.(22) are expressed as
\[
\begin{align*}
\text{Im}Z_0 &= (1 + \cot^2 \beta)^{1/2} u_1 u_3 + (1 + \tan \beta)^{1/2} u_2 u_3, \\
\text{Im}\tilde{Z}_0 &= (1 + \cot^2 \beta)^{1/2} u_1 u_3 - (1 + \tan^2 \beta)^{1/2} u_2 u_3, \\
\text{Im}Z_1 &= -2(\tan^2 \beta + \tan^4 \beta)^{1/2} u_1 u_3, \\
\text{Im}Z_2 &= 2(\cot^2 \beta + \cot^4 \beta)^{1/2} u_2 u_3, \tag{23}
\end{align*}
\]
where \( \tan \beta = |v_2|/|v_1| \), and the three real numbers \( u_1, u_2 \) and \( u_3 \) are the coefficients of \( \phi \) component of the \( \Phi_1, \Phi_2 \) and \( \Phi_3 \) states, respectively, and they are subject to the constraint, \( u_1^2 + u_2^2 + u_3^2 = 1 \). Only two of the four quantities of eq.(23) are independent and there are the following two relations among them,
\[
\begin{align*}
|v_1|^2 \text{Im}Z_1 + |v_2|^2 (\text{Im}\tilde{Z}_0 + \text{Im}Z_0) &= 0, \tag{24} \\
|v_1|^2 (\text{Im}\tilde{Z}_0 - \text{Im}Z_0) + |v_2|^2 \text{Im}Z_2 &= 0. \tag{25}
\end{align*}
\]
Now, we will compute the asymmetry \( A^t \) in eq.(13). In this two-Higgs doublet model, we have \( f_1^L = 1 \) at the tree level in the same way as in the Standard Model. The CP-violating part of the form factor \( f_2^R \) can be obtained from the five one-loop diagrams of Fig.1, since we use the unitary gauge.

The relevant CP-violating propagators of the neutral scalars are the four quantities of eq.(23), \( < \phi_2^0 \phi_1^{0*} >, < \phi_2^0 \phi_2^0 >, < \phi_1^0 \phi_1^0 >, < \phi_2^0 \phi_2^0 > \) and their
complex conjugates. The propagator of $W$ gauge boson in the unitary gauge is given as \((g_{\mu\nu} - \frac{q_{\mu}q_{\nu}}{m_W^2})/(q^2 - m_W^2 + i\varepsilon)\) and that of the charged Higgs boson $H^+$ is given as \(1/(m_H^2 - q^2 - i\varepsilon)\), where $m_H$ is the charged Higgs boson mass. The other necessary Feynman rules for our calculation are summarized in the Appendix. The loop integrals for the diagrams in Fig.1 are expressed by the three-point functions defined in [24],

\[
- \int \frac{d^nq}{(2\pi)^n} \frac{[1, q_{i\nu}, q_{\mu}q_{\nu}, q_{\mu}q_{\nu}q_{\alpha}]}{(q^2 - m_1^2)(q + k)^2 - m_2^2((q + k + p)^2 - m_3^2)} \\
= \frac{1}{(4\pi)^4} [C_0, k_{\mu} C_{11} + p_{\mu} C_{12}, k_{\mu} k_{\nu} C_{21} + p_{\mu} p_{\nu} C_{22} + (k_{\mu} p_{\nu} + p_{\mu} k_{\nu}) C_{23} + g_{\mu\nu} C_{24}, \\
k_{\mu} k_{\nu} k_{\alpha} C_{31} + p_{\mu} p_{\nu} p_{\alpha} C_{32} + \{pkk\}_{\mu\nu\alpha} C_{33} + \{kp\}_{\mu\nu\alpha} C_{34} + \{kg\}_{\mu\nu\alpha} C_{35} \\
+ \{pg\}_{\mu\nu\alpha} C_{36}],
\]

(26)

where \(\{pkk\}_{\mu\nu\alpha} = p_{\mu} k_{\nu} k_{\alpha} + k_{\mu} p_{\nu} k_{\alpha} + k_{\mu} k_{\nu} p_{\alpha}\), \(\{kg\}_{\mu\nu\alpha} = k_{\mu} g_{\nu\alpha} + k_{\nu} g_{\mu\alpha} + k_{\alpha} g_{\mu\nu}\).

All the $C$’s in eq. (26) have the arguments $C(k, p, m_1, m_2, m_3)$. In terms of these functions, we can calculate the CP-violating contributions to the form factor $\text{Im} f_2^R$ for the five diagrams in Fig.1, consistently in the unitary gauge, as

\[
\text{Im} f_2^R |_{1} = \frac{1}{(4\pi)^2} \frac{g_G}{v} m_t^2 m_t |v_1|^2 (-\text{Im} Z_1 + \text{Im} \tilde{Z}_0)(C_{12} + C_{23}) - |v_1|^2 \text{Im} Z_0 (C_0 + C_{11}) \\
+ |v_2|^2 \text{Im} Z_0 (C_{11} + C_{21} - C_{12} - C_{23}),
\]

(27)

where $\ldots = C \ldots (-p_t, p_{W^+}, m_b, m_H, m_\phi)$,

\[
\text{Im} f_2^R |_{2} = -\frac{1}{(4\pi)^2} \frac{g_G}{v} m_t^2 m_t |v_2|^2 (-\text{Im} Z_2 + \text{Im} \tilde{Z}_0)(C_{12} + C_{23} - C_{11} - C_{21}) \\
+ m_t^2 |v_1|^2 \{ (\text{Im} \tilde{Z}_0 - \text{Im} Z_2)(C_0 + C_{11}) + \text{Im} Z_0 (C_{12} + C_{23}) \},
\]

(28)

where $\ldots = C \ldots (-p_b, -p_{W^+}, m_t, m_H, m_\phi)$,

\[
\text{Im} f_2^R |_{3} = -\frac{1}{(4\pi)^2} \frac{g_G}{v} m_t^2 m_t |v_2|^2 \text{Im} Z_0 (C_0 + 2C_{11} + C_{21}),
\]

(29)

where $\ldots = C \ldots (-p_t, p_{W^+}, m_b, m_W, m_\phi)$,

\[
\text{Im} f_2^R |_{4} = \frac{1}{(4\pi)^2} \frac{g_G}{v} m_t |v_1|^2 \text{Im} Z_0 m_H^2 C_{31} + (m_t^2 - m_b^2 - m_W^2) C_{33} + m_W^2 C_{34} - \frac{1}{6} m_t^2 |v_1|^2 \text{Im} Z_0 (C_0 + 2C_{11} + C_{21}) + m_t^2 |v_2|^2 \text{Im} Z_0 (C_{12} + C_{23} - C_{11} - C_{21}),
\]

(30)

where $\ldots = C \ldots (-p_b, -p_{W^+}, m_t, m_W, m_\phi)$.
\[ +6C_{35} + 2m_b^2C_{21} + m_W^2C_{22} + (m_t^2 - 2m_b^2 - m_W^2)C_{23} + 4C_{24} \\
+ (m_b^2 - 2m_W^2)(C_{11} - C_{12}) + m_t^2(C_{12} + C_{23}) \],
\tag{30}
\]

where \( C \ldots = C \ldots (-p_b, -p_{W^+}, m_t, m_W, m_\phi) \),

\[
\text{Im} f_2^R \big|_5 = \frac{1}{(4\pi)^2} \frac{gG_F}{\nu} \nu^2 m_t^2 m_t [\text{Im} Z_0(C_{11} - C_{12}) + \text{Im} \tilde{Z}_0(C_{11} - C_{12} + C_{21} - C_{23})],
\tag{31}
\]

where \( C \ldots = C \ldots (-p_b, -p_{W^+}, m_\phi, m_b, m_t) \).

Now, we will evaluate the asymmetry \( A^t \) in eq. (13). The free parameters are \( u_1, u_2, u_3, \tan \beta(\equiv |v_2|/|v_1|), m_\phi \) and \( m_H \). The neutral scalar boson mass \( m_\phi \) has been restricted to \( m_\phi = 100 - 1000 \text{ GeV} [25] \) or \( m_\phi \leq 900 \text{ GeV} [26] \) through the analyses of radiative corrections to \( Z \) and \( W \) boson masses in the Standard Model, and supersymmetric models constrain as \( 100 \leq m_\phi \leq 200 \text{ GeV} \). ALEPH Collaboration has recently set a lower limit on the neutral Higgs boson mass of 63.9 GeV [27]. So, we choose \( 50 \leq m_\phi \leq 250 \text{ GeV} \) in our calculations, and we will take \( 100 \leq m_H \leq 1000 \text{ GeV} \) for the charged scalar boson mass. The dependence of the absolute value \( |A^t|/|P_\parallel| \) on the parameter \( u_1, u_2 \) and \( u_3 \) is obtained in Table 1 for \( m_H = 200 \text{ GeV}, m_\phi = 100 \text{ GeV}, \tan \beta = 1.0 \) and top-quark mass of \( m_t = 180 \text{ GeV} \). As seen from Table 1, the maximum value of the asymmetry \( A^t \) proves to be of the order of \( (1 - 2) \times 10^{-4} \), which is roughly consistent with the results of Grz\'adkowski and Gunion [7]. In the following calculations, we will take \( u_1 = -u_2 = u_3 = 1/\sqrt{3} \), and assume \( |P_\parallel| = 1 \) in order to estimate the maximum value of \( A^t \). The magnitude of longitudinal polarization of \( t \) quark depends on its production mechanism. The functions \( C \)’s develop imaginary parts for \( m_H < m_t - m_b \) and always for the third diagram in Fig. 1. In this case, \( \text{Im} f_2^R \) involves CP-
Table 1: Dependence of $\text{Im}f_{2}^{R}$ and $|A^t|/|P_{||}|$ on the parameter set $(u_1, u_2, u_3)$ under the constraint of $u_1^2 + u_2^2 + u_3^2 = 1$. 

| $(u_1, u_2, u_3)$ | $\text{Im}f_{2}^{R} \times 10^{-3}$ | $|A^t|/|P_{||}| \times 10^{-3}$ |
|-------------------|-----------------|-----------------|
| $1/\sqrt{2}$ 0 $1/\sqrt{2}$ | -0.16 | 0.11 |
| $1/\sqrt{2}$ 0 $-1/\sqrt{2}$ | 0.16 | 0.11 |
| 0 $1/\sqrt{2}$ $1/\sqrt{2}$ | 0.35 | 0.24 |
| 0 $1/\sqrt{2}$ $-1/\sqrt{2}$ | -0.35 | 0.24 |
| $1/\sqrt{3}$ $1/\sqrt{3}$ $1/\sqrt{3}$ | 0.13 | 0.084 |
| $1/\sqrt{3}$ $1/\sqrt{3}$ $-1/\sqrt{3}$ | -0.13 | 0.084 |
| $1/\sqrt{3}$ $-1/\sqrt{3}$ $1/\sqrt{3}$ | -0.34 | 0.23 |
| $1/\sqrt{3}$ $-1/\sqrt{3}$ $-1/\sqrt{3}$ | 0.34 | 0.23 |

conserving part of $Z$’s, that is, Re$Z_0$, Re$\tilde{Z}_0$, etc., and we do not include this part for the calculation of CP-violating $\text{Im}f_{2}^{R}$.

The numerical results are shown in Figs.2 and 3. As seen in Fig.2, $m_t$-dependence of $|A^t|$ is strong, as expected from its dependence of Higgs scalar coupling to the fermions. On the contrary, as seen in Fig.3, the dependence on the neutral and charged scalars masses $m_{\phi}$ and $m_H$ is moderate, since their dependence is only of $\log(m_{\phi})$ and $\log(m_H)$ in the one-loop amplitudes.

Fig.4 shows the dependence of $|A^t|$ on $\tan \beta$. Consequently, the CP-violating up-down asymmetry of $W$-decaying leptons in $t \to W^+b$ decay is of the order of $10^{-3} - 10^{-5}$ for $0.5 \leq \tan \beta \leq 10$ for reasonable Higgs scalars mass range, $50 \leq m_{\phi} \leq 250$ GeV and $100 \leq m_H \leq 1000$ GeV.

4. Conclusions and discussions

We have investigated CP violation in the top-quark decay by studying the asymmetry ($A^t$) of lepton distributions from the subsequent decay $W^+ \to \ell^+\nu$ in $t \to W^+b$ in the two-Higgs doublet model with CP-violating neutral sector. As expected, due to the large coupling of Higgs scalars to the top-
quark, the asymmetry is significantly large \( ((1 - 3) \times 10^{-4}) \) for the one-loop effects, for the typical parameter values of \( m_{H^+} = 200 \text{ GeV} \), \( m_\phi = 100 \text{ GeV} \) and \( \tan \beta = 1.0 \), though this magnitude of the asymmetry may be hard to detect by the experiments. This value goes up to \( 1 \times 10^{-3} \) for \( \tan \beta = 0.5 \) for the same \( m_{H^+} \) and \( m_\phi \) values.

If CP-conserving contributions are included, this asymmetry shows values \( (\sim 10^{-3}) \) larger by one order of magnitude, as seen in Fig.5. It is due to the third diagram of Fig.1 which develops a large imaginary part for the vertex amplitude. Therefore, the magnitude of CP-violating contribution to the asymmetry \( A_t \) is about \( 1/10 \) of the CP-conserving contribution. Experimentally, the CP-violating contribution could be extracted by adding the asymmetries from \( t \) and \( \bar{t} \) decays as stated in the section 2.

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Appendix

In this Appendix we summarize the Feynman rules which are necessary for the calculations in this paper in the two-Higgs doublet model with CP-violating neutral sector. The model we adopted here is the so-called "type II", in which one Higgs doublet $\phi_0^1$ couples exclusively to the "down"-type quarks and another doublet $\phi_0^2$ to the "up"-type quarks. Our formalism for constructing the Feynman rules is the one designed to minimize the number of times that the imaginary unit $i$ appears\[23\]. In the following, we denote the $tbW^-$-vertex, for example, as $t\bar{b}W^-$, which means that all of the three particles $t$, $\bar{b}$ and $W^-$ enter the vertex, and the Feynman rule for this vertex is $g/\sqrt{2}\gamma\mu P_L$ according to our formalism.

Quark couplings to neutral Higgs scalars;

- $D_j D_j \phi_0^0 : -\frac{1}{v_1} m_{D_j} P_R$,
- $D_j D_j \phi_0^0* : -\frac{1}{v_1^*} m_{D_j} P_L$,
- $U_i \bar{U}_i \phi_0^0 : -\frac{1}{v_2} m_{U_i} P_L$,
- $U_i \bar{U}_i \phi_0^0* : -\frac{1}{v_2^*} m_{U_i} P_R$,

where $D_j$ and $U_i$ are "down"-type quark and "up"-type quark, respectively, $m_{D_j}$ and $m_{U_i}$ their masses, $P_{R/L} = (1 \pm \gamma_5)/2$, and $v_1$ and $v_2$ are vacuum expectation values for $\phi_0^1$ and $\phi_0^2$, respectively.

Quark couplings to charged Higgs scalar $H^\pm$;

- $U_i D_j H^- : \frac{1}{v} V_{ij} (\frac{v_2}{v_1^*} m_{D_j} P_L + \frac{v_1}{v_2^*} m_{U_i} P_R)$,
- $D_j U_i H^+ : \frac{1}{v} V_{ij} (\frac{v_1^*}{v_1} m_{D_j} P_R + \frac{v_2^*}{v_2} m_{U_i} P_L)$,

where $v \equiv \sqrt{|v_1|^2 + |v_2|^2}$ and $\sqrt{2}G_F = 1/(2v^2)$. $V_{ij}$ is the $(ij)$-element of Cabibbo-Kobayashi-Maskawa mixing matrix. $H^\pm$ are the charged Higgs scalars. Goldstone bosons do not appear in our formalism, since we use the unitary gauge.
Gauge boson pair couplings to neutral Higgs scalars;

\[ W^+_\mu W^-_\nu \phi_{1,2}^0 : \frac{1}{2} g^2 v^*_1 g_{\mu\nu}, \quad W^+_\mu W^-_\nu \phi_{1,2}^{0*} : \frac{1}{2} g^2 v^*_2 g_{\mu\nu}, \]

where \( g \) is the SU(2) gauge coupling constant and \( g = \sqrt{2} M_W/v \).

Gauge boson couplings to charged Higgs scalar and neutral Higgs scalar;

\[ W^+_\mu H^-_\mu \phi_1^0 : \frac{g}{\sqrt{2}} v^*_2 (p - q)_\mu, \quad W^-_\mu H^+_\mu \phi_1^{0*} : - \frac{g}{\sqrt{2}} v^*_2 (p - q)_\mu, \]

\[ W^+_\mu H^-_\mu \phi_2^0 : - \frac{g}{\sqrt{2}} v^*_1 (p - q)_\mu, \quad W^-_\mu H^+_\mu \phi_2^{0*} : \frac{g}{\sqrt{2}} v^*_1 (p - q)_\mu, \]

where \( p \) and \( q \) are the momenta of the incoming \( H^\pm \) and \( \phi_{1,2}^{0(\ast)} \), respectively.

We should mention that the propagator \( < \phi_i^0 \phi_j^0 > \) means that \( \phi_i^0 \) and \( \phi_j^0 \) are both entering the vertices and directed back to back on the propagator.

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Figure captions

Fig.1. The five one-loop diagrams contributing to the CP-violating part of the form factor $f_R^R$ in $t \to W^+ b$ decay in the two-Higgs doublet model in the unitary gauge. The propagators with $\phi^0$ are the ones of $<\phi_i^0, \phi_j^{0(*)}>$ of CP-violating neutral Higgs-boson exchange.

Fig.2. The dependence of the maximum absolute value of the asymmetry $A_t$ on the top-quark mass $m_t$ for various sets of charged and neutral scalar boson masses; $(m_H, m_\phi) = (100, 50)\text{GeV}$ (dashed-dotted), $(200, 100)\text{GeV}$ (solid), $(500, 100)\text{GeV}$ (dashed) and $(1000, 100)\text{GeV}$ (dotted curve).

Fig.3. The dependence of the maximum absolute value of the asymmetry $A_t$ on the neutral scalar boson mass $m_\phi$ for the charged scalar boson mass, $m_H = 100\text{GeV}$ (solid), $200\text{GeV}$ (dashed-dotted), $500\text{GeV}$ (dashed) and $1000\text{GeV}$ (dotted curve).

Fig.4. The dependence of the maximum absolute value of the asymmetry $A_t$ on the ratio of vacuum expectation values of the two neutral Higgs fields, $\tan \beta (\equiv |v_2|/|v_1|)$, for $(m_H, m_\phi) = (200, 100)\text{GeV}$.

Fig.5. The dependence of the maximum absolute value of the asymmetry $A_t$ with the inclusion of CP-conserving contributions to the form factor $f_R^R$ on the neutral scalar boson mass $m_\phi$ for the charged scalar boson mass, $m_H = 100\text{GeV}$ (solid), $200\text{GeV}$ (dashed-dotted), $500\text{GeV}$ (dashed) and $1000\text{GeV}$ (dotted curve).
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