Consistent quantization of massless fields of any spin

Alexander Gersten and Amnon Moalem
Department of Physics, Ben Gurion University of the Negev, Beer-Sheva, Israel

Abstract. Consistent quantization of massless fields of any spin is developed from first principles. Probability currents and Lagrangians are derived allowing a first quantized formalism. A simple procedure was derived of connecting the wavefunctions with potentials and gauge conditions. We emphasize the spin one and spin two cases, relating them to the quantum photon (via modified Maxwell’s equations) and to the quantum graviton respectively.

Key words: massless fields, any spin, quantization.

1. Introduction

Quantum theory was first formulated via the first quantization of Schroedinger’s, Klein-Gordon’s and Dirac’s equations. In the second quantization Maxwell’s equations were treated as classical field equations without referring to the first quantization. Here we present a consistent first quantization of zero mass fields of any spin. The second quantization can be done by an existing standard formalism. We present a detailed analysis of the spin one case, with which we give a first quantized version of Maxwell’s equations, and the spin two case relating it to the quantum graviton.

The present paper is a continuation of our previous work [1]. There we have used wavefunctions which formed a basis for the representations $D^{(s-\frac{1}{2}, \frac{1}{2})}$ of the Lorentz group. This representation describes the combination of two spins, spin $s$ and spin $s-1$. It is $(2s+1) + (2s-1) = 4s$ dimensional. By putting the spin $s-1$ components to zero we were able to get equations for spin $s$ with proper subsidiary conditions, which led to only two projections of the helicities, in the forward and backward directions of the momenta, as is required for free massless particles. In the present paper we used non-zero spin $s-1$ components, thus giving a unified formalism for both spins. This has important advantages. It simplifies the formalism. Only after the final derivations the spin $s-1$ components are put to zero for free particles, which generates the proper subsidiary conditions and helicity projections. Moreover it allows to define potentials and their relations to the wavefunctions. The potentials are important, as in the case of Maxwell’s equations, for defining interactions with other fields.

In the following we derive a consistent first quantization of massless fields of any spin. Detailed calculations are given for the spin 1 and spin 2 cases. For the spin 1 case generalized Maxwell’s equations are derived and potentials defined, relating them to the wavefunction. The Lorenz condition on the potentials (named after Ludvig Lorenz, which is frequently called the Lorentz condition because of confusion with Hendrik Lorentz, after whom the Lorentz transformation
and covariance are named) ensures that they describe a spin 1 field.

For the spin 2 case two-helicity wave equations are derived and potentials defined, relating them to the wavefunctions. Six gauge conditions are needed to ensure that they describe a spin 2 field.

2. Dirac’s equations for massless particles of any spin

Dirac [2] has derived equations for massless particles with spin \( s \), which in the ordinary vector notation are,

\[
\frac{1}{s} \left[ s \hat{p}_0 I^{(2s+1)} + S_x \hat{p}_x + S_y \hat{p}_y + S_z \hat{p}_z \right] \psi = \left[ \frac{\hat{E}}{c} I^{(2s+1)} + \frac{S}{s} \cdot \hat{p} \right] \psi^{(2s+1)} = 0, \tag{1}
\]

\[
\left[ s \hat{p}_x I^{(s)} + S_x \hat{p}_0 - iS_y \hat{p}_z + iS_z \hat{p}_y \right] \psi = 0, \tag{2}
\]

\[
\left[ s \hat{p}_y I^{(s)} + S_y \hat{p}_0 - iS_z \hat{p}_x + iS_x \hat{p}_z \right] \psi = 0, \tag{3}
\]

\[
\left[ s \hat{p}_z I^{(s)} + S_z \hat{p}_0 - iS_x \hat{p}_y + iS_y \hat{p}_x \right] \psi = 0, \tag{4}
\]

where \( \psi \) is a \((2s+1)\) component wave function and \( S_n \) are the \((2s+1) \times (2s+1)\) spin matrices which satisfy,

\[
[S_x, S_y] = iS_z, \quad [S_z, S_x] = iS_y, \quad [S_y, S_z] = iS_x, \quad S_x^2 + S_y^2 + S_z^2 = s(s+1)I^{(2s+1)}. \tag{5}
\]

Above the \( \hat{p}_n \) are the momenta, \( \hat{p}_0 = \hat{E}/c, \hat{E} \) the energy, and \( I^{(2s+1)} \) is a \((2s+1) \times (2s+1)\) unit matrix. Eqs. (1-4) were analyzed extensively by Bacry [3], who derived them using Wigner’s condition [4] on the Pauli-Lubanski vector \( W^\mu \) for massless fields,

\[
W^\mu = \hat{s}\hat{p}^\mu, \quad \mu = x_0, x, y, z. \tag{6}
\]

Dirac has suggested to use Eq. (1) as the basic helicity equation and substitute from it the \( \hat{p}_0 \psi \) into the other 3 equations (2-4). Free massless particles may have only two helicity projections (forward and backward along the momentum vector). Using the Dirac procedure one obtains the \( 2s+1 \) component Eq. (1) and \( 2s-1 \) independent subsidiary conditions which reduces the number of helicity projections to 2,

\[
\left[ \hat{E} I^{(2s+1)} + \frac{c}{s} S \cdot \hat{p} \right] \psi^{(2s+1)} = 0, \tag{7}
\]

\[
(\mathbf{\Pi} \cdot \hat{p}) \psi^{(2s+1)} = 0, \tag{8}
\]

where \( \mathbf{\Pi} \) is a \((2s-1) \times (2s+1)\) vector matrix of the subsidiary conditions. These two sets of equations can be combined into \((2s+1) + (2s-1) = 4s\) equations [1],
\[
\left[ \hat{E} I^{(4s)} + c \Gamma^{(4s)} \cdot \hat{p} \right] \Phi^{(4s)} = 0,
\]

(9)
equivalent to the former equations, provided that the rows of the \( \Pi \) matrices are normalized so that the eigenvalues of \( \Gamma^{(4s)} \cdot \hat{p} \) are \( \pm p \) (two helicities) and,

\[
\left( \hat{E} I^{(4s)} - c \Gamma^{(4s)} \cdot \hat{p} \right) \left( \hat{E} I^{(4s)} + c \Gamma^{(4s)} \cdot \hat{p} \right) = \left( \hat{E}^2 - c^2 \hat{p}^2 \right) I^{(4s)},
\]

(10)
i.e. they factorize the d’Alembertian. The wave function \( \Phi^{(4s)} \) is of the form [1] (with 2s-1 zeros),

\[
\Phi^{(4s)} = \begin{pmatrix}
0 \\
\vdots \\
0 \\
\psi_{2s+1}^{(2s+1)} \\
\vdots \\
\psi_{-2s-1}^{(2s+1)}
\end{pmatrix},
\]

(11)
and the \( \Gamma^{(4s)} \) vector matrices have the form [1]:

\[
\Gamma_0^{(4s)} = I^{(4s)} \quad (\text{the unit matrix})
\]

\[
\Gamma_k^{(4s)} = \frac{1}{s} \begin{pmatrix}
\begin{pmatrix}
-\tilde{S}_{k}^{(2s-1)} \\
\Pi_k^H \\
\end{pmatrix} & \begin{pmatrix}
\Pi_k \\
\tilde{S}_{k}^{(2s+1)}
\end{pmatrix}
\end{pmatrix}, \quad k = x, y, z
\]

(12)
Lomont [5] has proved that Eq. (9) is Lorentz covariant and the zeros of the wave function are unchanged under the Lorentz transformation. We can interpret this result as follows, the wave functions \( \Phi^{(4s)} \) of Eq. (11) belong to the basis of the \( D\left(\frac{1}{2}, s - \frac{1}{2}\right) \) representation of the Lorentz group, which is 4s dimensional and its angular momentum decomposition is

\[
4s = (2s - 1) + (2s + 1),
\]

(13)
i.e. a combination of spin \( s - 1 \) and \( s \). The absence of the spin \( s - 1 \) components indicates that Eq. (9) is a pure spin \( s \) equation with two helicities. This should be correct for free particles, but interactions with other fields may introduce non-zero components, therefore the wavefunction \( \Phi^{(4s)} \) of Eq. (11) should be complemented with its spin \( s - 1 \) components,
\[ \Phi^{(4s)} = \begin{pmatrix} \psi^{(2s-1)}_{2s-1} \\ \vdots \\ \psi^{(2s-1)}_{2s+1} \\ \psi^{(2s+1)}_{2s+1} \\ \vdots \\ \psi^{(2s+1)}_{-2s-1} \end{pmatrix}. \] (14)

For free particles equation Eq. (9), with \( \Phi^{(4s)} \) of Eq. (14), will have a physical solution if,

\[ \psi^{(2s-1)}_{x} = 0, \quad \alpha = -2s + 1, ..., 2s - 1, \] (15)

corresponding to a pure spin \( s \) state, moreover all the subsidiary conditions will be satisfied.

We should note that for spin \( \frac{1}{2} \) Eq. (10) becomes,

\[ [E\sigma_0 - c\mathbf{p} \cdot \sigma] [E\sigma_0 + c\mathbf{p} \cdot \sigma] \psi = (E^2 - c^2 \mathbf{p}^2) \sigma_0 \psi, \]

where the \( \sigma \) are the Pauli matrices

\[ \sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \] (16)

thus the \( \Gamma \) matrices form a representation of the Pauli matrices.

3. First quantization

As is in the case of the Klein-Gordon or Dirac equations, we substitute in Eq. (9)

\[ \hat{E} \rightarrow i\hbar \frac{\partial}{\partial t}, \quad \hat{p} \rightarrow -i\hbar \nabla, \] (17)

and use the wavefunction of Eq. (14),

\[ i\hbar \left( \Gamma_0^{(4s)} \frac{\partial}{\partial t} - c\Gamma^{(4s)} \cdot \nabla \right) \Phi^{(4s)} = 0. \] (18)

Similarly to the Schrödinger and Dirac equations we define the Hamiltonian \( \mathcal{H} \) from Eq. (18),

\[ i\hbar \frac{\partial}{\partial t} \Phi^{(4s)} = i\hbar c \Gamma^{(4s)} \cdot \nabla \Phi^{(4s)} = \mathcal{H} \Phi^{(4s)}, \] (19)

and find a conserved probability current (superscript \( H \) denotes the Hermitian conjugate),
\[ \partial_t \left( \left( \Phi^{(4s)} \right)^H \Phi^{(4s)} \right) + \frac{c}{s} \nabla \cdot \left( \left( \Phi^{(4s)} \right)^H \tilde{S} \Phi^{(4s)} \right) = 0. \] \tag{20}

Thus the probability density (which should be normalized) is,

\[ \rho = \left( \Phi^{(4s)} \right)^H \Phi^{(4s)}. \] \tag{21}

Having this result we can find a Lagrangian density,

\[ \mathcal{L} = i\hbar \left( \Phi^{(4s)} \right)^H \left( \Gamma_0^{(4s)} \frac{\partial}{\partial t} - c \Gamma_1^{(4s)} \cdot \nabla \right) \Phi^{(4s)}, \] \tag{22}

and using the definition of the energy momentum tensor \( T^{\mu\nu} \), we find [1],

\[ \int \int \int dx dy dz T^{00} = \int \int \int dx dy dz \left( \Phi^{(4s)} \right)^H \mathcal{H} \Phi^{(4s)} = \langle \mathcal{H} \rangle, \] \tag{23}

\[ \int \int \int dx dy dz T^{0k} = \int \int \int dx dy dz \left( \Phi^{(4s)} \right)^H (c \hat{p}_k) \Phi^{(4s)} = \langle c \hat{p}_k \rangle, \quad k = 1, 2, 3, \] \tag{24}

i.e. consistent with the expectation values of energy and momentum.

### 4. First quantized Maxwell’s equations without sources

For spin=1, Eq. (18) becomes,

\[ i\hbar \left( \Gamma_0^{(4)} \frac{\partial}{\partial t} - c \Gamma_1^{(4)} \cdot \nabla \right) \Phi^{(4)} = 0, \quad \Phi^{(4)} = \begin{pmatrix} \Psi_0 \\ \Psi_x \\ \Psi_y \\ \Psi_z \end{pmatrix}, \] \tag{25}

\[ \Gamma_0^{(4)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \Gamma_1^{(4)} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & \langle S_x \rangle & 0 \\ 0 & \langle S_y \rangle & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \] \tag{26}

\[ \Gamma_2^{(4)} = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & \langle S_y \rangle \\ -1 & \langle S_x \rangle & 0 & -1 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad \Gamma_3^{(4)} = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & \langle S_z \rangle & 0 \\ 0 & \langle S_y \rangle & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \]
If we substitute,

$$\Phi^{(4)} = \begin{pmatrix} \Psi_0 \\ \Psi_x \\ \Psi_y \\ \Psi_z \end{pmatrix} = \mathcal{N} \begin{pmatrix} E_0 - iB_0 \\ E_x - iB_x \\ E_y - iB_y \\ E_z - iB_z \end{pmatrix},$$

and assume $\Psi_0 = 0$, the free field Maxwell’s equations are obtained for the electric $\mathbf{E}$ and magnetic field $\mathbf{B}$. Above $\mathcal{N}$ is a normalization factor. The subsidiary condition is equivalent to $\Psi_0 = 0$, but Maxwell’s Eqs. with sources may induce $\Psi_0 \neq 0$.

Four vectors belong to the $D(\frac{3}{2}, \frac{3}{2})$ representation of the Lorentz group, its basis combines the spin 0 and spin 1 bases. Therefore $\Psi_0 = 0$ eliminates the spin 0 component.

The probability density becomes:

$$\rho = \left( \Phi^{(4)} \right)^H \Phi^{(4)} = \mathcal{N}^2 \left( E_0^2 + B_0^2 + E^2 + B^2 \right),$$

and the energy density:

$$\left( \Phi^{(4)} \right)^H \mathcal{H} \Phi^{(4)} = \left( \Phi^{(4)} \right)^H \left( i\hbar \mathbf{\Gamma}^{(4)} \cdot \nabla \right) \Phi^{(4)}.$$  \tag{29}

Let us introduce the potentials $A^\mu$,

$$A^{(4)} = \begin{pmatrix} A_0 \\ A_x \\ A_y \\ A_z \end{pmatrix},$$

Their relation to the wavefunction $\Phi^{(4)}$ can be obtained by requiring that in the Lorenz gauge they will satisfy the wave equation,

$$\partial_\mu \partial^\mu A^{(4)} = 0.$$  \tag{30}

Using Eq.(10) we find that,

$$\Phi^{(4)} = \mathcal{N} \begin{pmatrix} E_0 - iB_0 \\ E_x - iB_x \\ E_y - iB_y \\ E_z - iB_z \end{pmatrix} = -\mathcal{N} \begin{pmatrix} \Gamma_0^{(4)} \partial_t + i \mathbf{\Gamma}^{(4)} \cdot \nabla \end{pmatrix} \begin{pmatrix} A_0 \\ A_x \\ A_y \\ A_z \end{pmatrix}$$

$$= -i\hbar \mathcal{N} \begin{pmatrix} \partial_0 A_0 + \partial_1 A_x + \partial_2 A_y + \partial_3 A_z \\ \partial_0 A_x + \partial_1 A_0 - i \left( \partial_2 A_z - \partial_3 A_y \right) \\ \partial_0 A_y + \partial_2 A_0 + i \left( \partial_1 A_z - \partial_3 A_x \right) \\ \partial_0 A_z + \partial_3 A_0 - i \left( \partial_1 A_y - \partial_2 A_x \right) \end{pmatrix},$$  \tag{31}
from which the relation between the fields and the potentials (in the Lorenz gauge) is,

\[ E_0 = -\partial_0 A_0 - \nabla \cdot A, \quad \text{(33)} \]

\[ B_0 = 0, \quad \text{(34)} \]

\[ E = -\nabla A_0 - \partial_0 A, \quad \text{(35)} \]

\[ B = \nabla \times A. \quad \text{(36)} \]

The Lorenz condition

\[ \partial_0 A_0 + \partial_1 A_x + \partial_2 A_y + \partial_3 A_z = 0, \quad \text{(37)} \]

eliminates the spin 0 component from the wave function.

5. First quantization of spin 2 (graviton?) equations

We proceed as in the section of first quantization of Maxwell’s equations. The wavefunction is,

\[ \Phi^{(8)} = \begin{pmatrix} \Psi_1^{(1)} \\ \Psi_0^{(1)} \\ \Psi_2^{(1)} \\ \Psi_0^{(2)} \\ \Psi_1^{(2)} \\ \Psi_2^{(2)} \\ \Psi_{-1}^{(2)} \\ \Psi_{-2}^{(2)} \end{pmatrix}, \quad \text{(38)} \]

and the wave equation (see Eq.(18)) is,

\[ i\hbar \left( \Gamma^{(8)}_0 \frac{\partial}{\partial t} - c\Gamma^{(8)} \cdot \nabla \right) \Phi^{(8)} = 0. \quad \text{(39)} \]

The \( \Gamma^{(8)} \) matrices are given in Eq.(12) with the subsidiary submatrices \( \Pi_k \),

\[ \Pi_x = \begin{pmatrix} \sqrt{3} & 0 & 0 & 0 \\ 0 & \sqrt{\frac{3}{2}} & 0 & -\sqrt{\frac{3}{2}} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\sqrt{3} \end{pmatrix}. \quad \text{(40)} \]
\[ \Pi_y = i \begin{pmatrix} -\sqrt{3} & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & -\sqrt{\frac{3}{2}} & 0 & -\sqrt{\frac{3}{2}} & 0 \\ 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & -\sqrt{3} \end{pmatrix}, \tag{41} \]

\[ \Pi_z = \begin{pmatrix} 0 & -\sqrt{3} & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & -\sqrt{3} & 0 \end{pmatrix}. \tag{42} \]

If we impose,

\[ \Psi^{(1)}_1 = \Psi^{(1)}_0 = \Psi^{(1)}_{-1} = 0, \tag{43} \]

the spin 1 contribution is eliminated and the free field equations for the the spin 2 field are obtained with the proper subsidiary conditions.

Let us introduce the potentials,

\[ A^{(8)} = \begin{pmatrix} A^{(1)}_1 \\ A^{(1)}_0 \\ A^{(1)}_{-1} \\ A^{(2)}_2 \\ A^{(2)}_1 \\ A^{(2)}_0 \\ A^{(2)}_{-1} \\ A^{(2)}_{-2} \end{pmatrix}. \tag{44} \]

Their relation to the wavefunction \( \Phi^{(8)} \) can be obtained by requiring that in a certain gauge they will satisfy the wave equation,

\[ \partial_\mu \partial^\mu A^{(8)} = 0. \tag{45} \]

This will happen if,

\[ \Phi^{(8)} = \mathcal{N} \left( \Gamma^{(8)}_0 \frac{\partial}{\partial t} + c \Gamma^{(8)} \cdot \nabla \right) A^{(8)}, \tag{46} \]

where \( \mathcal{N} \) is a normalization constant. The result is,
which ensure the pure spin 2 properties.

\[
\frac{\mathcal{A}}{A} = \begin{pmatrix}
4\partial_0^2 A^{(1)}_1 - \partial_x A^{(1)}_1 + \frac{1}{\sqrt{2}} \partial_x A^{(2)}_0 - \sqrt{3} \partial_x A^{(2)}_2 + \sqrt{3} \partial_x A^{(2)}_1 - \frac{1}{\sqrt{2}} \partial_x A^{(1)}_0 \\
2\partial_x A^{(2)}_0 + 4\partial_0 A^{(1)}_0 - \frac{1}{\sqrt{2}} \partial_x A^{(1)}_1 - \frac{1}{\sqrt{2}} \partial_x A^{(1)}_1 - \sqrt{3} \partial_x A^{(2)}_1 + \sqrt{3} \partial_x A^{(2)}_2 \\
4\partial_0 A^{(1)}_0 + \partial_x A^{(1)}_0 + \sqrt{3} \partial_x A^{(2)}_2 + \sqrt{3} \partial_x A^{(2)}_1 - \frac{1}{\sqrt{2}} \partial_x A^{(1)}_0 \\
4\partial_x A^{(2)}_2 + \partial_x A^{(2)}_1 + \sqrt{3} \partial_x A^{(2)}_1 - \sqrt{3} \partial_x A^{(2)}_2 \\
4\partial_0 A^{(2)}_0 = \frac{1}{2} \partial_0 A^{(2)} - \frac{1}{\sqrt{2}} A^{(1)}_0 + \sqrt{3} A^{(2)}_2 \\
\frac{\sqrt{3}}{2} A^{(2)}_0 + \frac{1}{\sqrt{2}} A^{(1)}_1 + \frac{1}{\sqrt{2}} A^{(1)}_1 + \frac{1}{\sqrt{2}} A^{(1)}_1 \\
\frac{1}{\sqrt{3}} A^{(1)}_0 + \frac{1}{\sqrt{3}} A^{(1)}_0 + \sqrt{3} A^{(2)}_2 \\
A^{(2)}_0 - \sqrt{3} A^{(1)}_1 - A^{(1)}_2 \\
A^{(2)}_1 - \sqrt{3} A^{(1)}_0 - A^{(1)}_0 \\
\sqrt{3} A^{(1)}_1 + \sqrt{3} A^{(1)}_1 - A^{(1)}_1 \\
\end{pmatrix}
\]

If the three upper components (for the real and imaginary parts) will be equal to zero, the spin 1 contribution will be eliminated including the correct subsidiary conditions. Thus we have six gauge conditions,

\[
4\partial_0 A^{(1)}_1 - \partial_x A^{(1)}_1 + \frac{1}{\sqrt{2}} \partial_x A^{(2)}_0 - \sqrt{3} \partial_x A^{(2)}_2 + \sqrt{3} \partial_x A^{(2)}_1 - \frac{1}{\sqrt{2}} \partial_x A^{(1)}_0 = 0,
\]

\[
2\partial_x A^{(2)}_0 + 4\partial_0 A^{(1)}_0 - \frac{1}{\sqrt{2}} \partial_x A^{(1)}_1 - \frac{1}{\sqrt{2}} \partial_x A^{(1)}_1 - \sqrt{3} \partial_x A^{(2)}_1 + \sqrt{3} \partial_x A^{(2)}_2 = 0,
\]

\[
4\partial_0 A^{(1)}_0 + \partial_x A^{(1)}_0 + \sqrt{3} \partial_x A^{(2)}_2 + \sqrt{3} \partial_x A^{(2)}_1 - \frac{1}{\sqrt{2}} \partial_x A^{(1)}_0 = 0,
\]

\[
\partial_y \left( \frac{1}{\sqrt{2}} A^{(2)}_0 - \frac{1}{\sqrt{2}} A^{(1)}_0 + \sqrt{3} A^{(2)}_2 \right) = 0,
\]

\[
\partial_y \left( \sqrt{\frac{3}{2}} A^{(2)}_1 + \sqrt{\frac{3}{2}} A^{(2)}_1 - \frac{1}{\sqrt{2}} A^{(1)}_1 + \frac{1}{\sqrt{2}} A^{(1)}_1 \right) = 0,
\]

\[
\partial_y \left( \frac{1}{\sqrt{2}} A^{(1)}_0 + \frac{1}{\sqrt{2}} A^{(2)}_0 + \sqrt{3} A^{(2)}_2 \right) = 0,
\]

which ensure the pure spin 2 properties.
6. Summary

Wavefunction equations were derived for massless fields of any spin. Consistent first quantization was worked out.

A simple procedure was derived of connecting the wavefunctions with potentials and gauge conditions.

Detailed calculations were worked out for the spin 1 and spin 2 cases. The results of the spin 1 case lead to a new quantization of Maxwell’s equations. The Lorenz condition on the potentials ensures that they describe a spin 1 field.

We are continuing to work on the spin 2 case in order to connect it with quantum gravity. Six gauge conditions on the potentials were found to ensure that they describe a spin 2 field.

[1] A. Gersten and A. Moalem, Journal of Physics: Conference Series 330 (2011) 012010.
[2] P.A.M. Dirac, Proc. Roy. Soc. A155, 447-59 (1936).
[3] H. Bacry, Nuovo Cimento, A32, 448-60 (1976).
[4] E.P. Wigner, Ann. Math., 40, 149 (1939).
[5] J.S. Lomont, Phys. Rev. 111, 1710-1716 (1958)