Operation mode optimization of centrifugal pumps of a water supply system booster pumping station with a reserve tank

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Abstract. A technique of the optimal selection of power units and their operation mode in the booster pumping station of the water supply system with a reserve tank is proposed. The optimization mathematical model of pumping station in the form of a non-linear, multi-criteria, partially integral mathematical programming problem is developed. Necessary data for optimization problem definition: the structure of daily water consumption by consumers and resistance of the water-supply system were found on the base of a full-scale experiment results. The adopted approach has provided both mathematical rigor of the problem formulation and solution, and real data recording of consumer behavior and water supply system conditions. The processing of the full-scale experiment data was carried out by statistical methods of analysis of non-stationary random processes. The optimization procedure was realized with the help of genetic algorithm method. Comparison of booster pumping station operation optimal plans on the base of different centrifugal pumps has allowed choosing such a type of pump, that when using it, maximum energy efficiency of the water supply system is achieved and operation of pumping station with minimum number of pumps with maximum possible efficiency is provided. Minimum essential dimensions of a reserve tank, optimal operation speed of the centrifugal pumps are determined.

1. Objectives, tasks and research procedure
The development and improvement of urban infrastructure is impossible without upgrade of life support systems, one of which is the water supply system. It belongs to the most energy – consuming facilities in housing and utility, industrial sector. The main electric power consumers in the water supply system are power units of pumping stations (PS), i.e. centrifugal pumps (CP). The purpose of PS operation is to provide the required pressure and volume of water supply for consumers, as regulated by the requirements of the SNiP (Civil Design Code). This objective fulfillment requires certain material, energy and financial resources expenses. Solving of the problem of water supply provision with minimum recourses expenses is profitable for the state in the whole and individual consumers as well. The optimization of the parameters of functioning of water supply system individual elements cannot solve the overall problem of its efficient operation in the whole, as the main elements of water supply system (PS, distribution pipeline system and consumers) significantly effect on each other, causing a number of interrelated deterministic and stochastic processes. This special feature is consistent with the basic principle of system analysis, i.e. irreducibility of response of a compound system to the sum of responses of its individual elements. The posed problem requires a comprehensive solution using both the methods of exact and natural sciences and engineering, economic and sociological approaches.
The objective of this work is to optimize the choice of power units and their operation mode in the water supply system booster PS (BPS) with a reserve tank.

In the process of work fulfillment the following problems have been solving:
- fulfillment of a full-scale experiment for studying of process regularity of a daily water consumption of residential community;
- determination of the averaged curve of water consumption random process and water supply system resistance, with the help of this full-scale experiment;
- development of non-linear, multi-criteria, optimal minimization model of an electric power consumption daily cost, water supply process insuring, with minimum number of centrifugal pumps, with their maximum possible efficient operation;
- justification method of optimization model solving and numerical calculation fulfillment that allows to choose the centrifugal pump type, their number and optimal operation;
- determination of a reserve tank minimum volume.

The statistical methods of analysis of non-stationary random processes were used for processing of the full-scale experiment data. STATISTICA software was used for implementation of statistical methods. Optimization procedure considered the use of numerical methods of non-linear programming problems solving and was realized by MATLAB software.

2. Full-scale experiment
The optimization mathematical model of the water supply system booster pumping station (1) through the water supply system (2) with a reserve tank (3) (Figure 1) was developed in the work.

![Figure 1. Residential community water supply plan by the booster pumping station with a reserve tank.](image)

A set of initial data based on the mathematical model contains 80 implementation processes of a booster pumping station daily operation. Within each implementation, with 1-hour discrete interval, pumping station head $Q$, water volume in a reserve tank $V$, and pressure in the water supply system $H$, were recorded. Data were recorded by a pressure sensor (4), by a flow meter (5) intended for conducting of real time measurements of acoustically transparent liquids in pressure pipelines and by a liquid level gage (6) installed in a reserve tank (Fig. 1). Digitized and averaged data obtained during one hour measurements were input into a personal computer (7) (Fig. 1).

For each time interval value of pumping station head is given by the following expression:
\[ Q = Ras + \Delta V \]  \hspace{1cm} (1)

where \( \Delta V \) - water volume change in a reserve tank; \( Ras \) - water consumption volume which can be found by the expression (1), by use of experiment data.

Water consumption volumes that were found for identical time moments of different water consumption implementation processes had distinct meanings. This is due to the fact that value of water consumption volume depends on consumers’ aggregate behavior changing water supply network resistance \( A_c \). Consumer motivation is effected by many internal and external factors, control over the majority of which within this model is difficult. It’s easier to model consumers’ behavior in this situation using non-deterministic and stochastic approaches. \( Ras(t) \) and \( A_c(t) \) dependencies can be considered as random processes, different implementations of which do not coincide with each other; and a set of values of various implementations at a fixed time \( t = t_i; i = 1,24 \) (a random process cross-section) – is a random value with its own statistical point features (mathematical expectation, mean square deviation).

For pumping station (PS), on the basis of \( n \) identical centrifugal pumps (CP), operating point \((H,Q)\) can be found from the equation [1]:

\[ H(t) = A_c i^2 + B_i q + C_i q^2(t) = H_o + A_c(t) \cdot Q^2(t) \]  \hspace{1cm} (2)

In the equitation (2) \( A, B, C \) – coefficients that are calculated by interpolation method of CP head characteristic; \( i = i_i/i_o = \text{const} \) - relative speed variation ratio; \( i_i \) and \( i_o \) - real and nominal speed; \( q = Q/n \) - one CP head.

Using the full-scale experiment data, resistance value of the water supply network for various implementations and cross-sections of a random water consumption process can be found from the equation (2):

\[ A_c(t) = (H(t) - H_o)/Q^2(t) \]  \hspace{1cm} (3)

In this paper work, the authors guess that the overall consumers’ stochastic effect on the value of water supply network is much stronger than deterministic effect of pressure variation value – in the mathematical model the network resistance is considered as a random process \( A_c(t) \).

For use within optimization mathematical model, the random processes \( Ras(t) \) and \( A_c(t) \) have been displaced by their deterministic analogs in the form of mathematical expectations \( \overline{Ras(t)} \) and \( \overline{A_c(t)} \). Mathematical expectations and their confidence intervals have been found with the help of data averaging by the random process cross-sections, by use of known statistic formulas [2]. Figure 2 shows the plot of mathematical expectation of a random process of a water supply system resistance daily variation that was done by use of STATISTICA software.

Mathematical expectation of the network resistance \( \overline{A_c(t)} \) is a nonrandom time function and amplitude of their value changes points at the non-stationary process [3]. Further statistical analysis of the full-scale experiment data has shown that the random process cross-sections \( Ras(t) \) and \( A_c(t) \) are distributed by the normal law.
3. Mathematical model

Optimization mathematical model of PS water supply system with a reserve tank is developed with taking into account the following basic provisions and assumptions:

- the problem of PS operation mode optimization is considered as a multi-criteria one; and as optimality criteria – the minimum cost of a daily water supply, taking into account the possible changes of electricity rates during a day, pump equipment efficiency maximization, centrifugal pumps number and a reserve tank dimensions minimization are considered. The basic principle of system constrains of optimization problem is a requirement for BSP customs’ needs meeting at a time and the physical conditions of system operation ensuring;

- as one of the possible ways of a multi-criteria optimization problem statement [4] ε-constraint method is suggested, where one of the possible objective functions is chosen and the rest of them are stated as the optimization problem constrains:

\[
\min_{\varepsilon} \left( f_i \left( \bar{x} \right) \right), \quad f_i \left( \bar{x} \right) \leq \varepsilon, \quad i = 1, 2, ..., r - 1, r + 1, ..., k ,
\]

in the expression (4) ε, values specify acceptable levels of initial function values which are not chosen as the objective functions;

- pressure frequency stabilization of centrifugal pump (CP) is not used;
- only identical CP are used in the pumping station (PS);
- by the authors electricity price was chosen as an objective function for a daily water supply ensuring;

- a daily time interval of PS operation is divided into k intervals;
- within each j-time interval, the water supply system state is characterized by a set of parameter constant values: \( \{ H_j, Q_j, A_{j\alpha}, n_j, \bar{Ras}_j, h_j ; j = 1, k \} \), where: \( n_j \) - number of PS identical pumps in operation; \( h_j \) - water column height above \( H_0 \) mark at the end of \( j \)-time interval; \( A_{j\alpha} \bar{Ras}_j \) - averaged values of certain random process mathematical expectations at \( j \)- interval; parameter entry can be considered as variables of phase space of water supply system states;
two possible consumers’ water supply modes are in the water supply system with a reserve tank: either PS flow is used for water consumption and accumulation in a reserve tank, or tank storage capacity is used for shortage covering of PS flow. For any described operation mode the following condition is fulfilled:

\[(H_j - H_0 - h_j) \cdot \Delta V_j > 0, \quad (5)\]

where \[\Delta V_j = (Q_j - Ras_j) \cdot \Delta t_j\] - change of water volume in a reserve tank;

\[\Delta t_j - j \cdot \text{interval duration}. \] Inequality (5) is a condition of the model physical correctness.

With regard to aforementioned considerations, the operation optimization model of booster PC with a reserve tank is defined as follows:

\[F = \rho \cdot g \cdot \sum_{j=1}^{k} \frac{H_j \cdot Q_j \cdot St_j \cdot \Delta t_j}{(1 - \eta_j)} \cdot 1^{0.25} \rightarrow \min \quad (6.1)\]

\[h_0 < h_j < h_{\text{max}}; H_0 < H_j < H_{\text{max}}; (H_j - H_0 - h_j) \cdot \Delta V_j > 0; j = \overline{1,k} \quad (6.2)\]

\[\sum_{j=1}^{k} \left[ Q_j - (\overline{Ras_j} + \mu \cdot \sigma_{b_j}) \right] \cdot \Delta t_j = h_0 \cdot S; 0 \leq \mu \leq 3 \quad (6.3)\]

\[\Delta V_j = (Q_j - Ras_j) \cdot \Delta t_j; \Delta h_j = \Delta V_j / S; h_j = h_{j-1} + \Delta h_j; j = \overline{1,k} \quad (6.4)\]

\[Q_j = \frac{n_j \cdot \sqrt{\frac{(B \cdot i)^2 - 4 \left(A \cdot i^2 - H_0\right) \left(C - (\overline{A_{ij}} - \mu \cdot \sigma_{a_{ij}}) n_j^2\right)}}}{2 \left(C - (\overline{A_{ij}} - \mu \cdot \sigma_{a_{ij}}) n_j^2\right)}; n_j \in Z; j = \overline{1,k} \quad (6.5)\]

\[\eta_j = D \left(\frac{Q_j}{n_j}\right)^2 + E \frac{Q_j}{n_j}; H_j = H_0 + (\overline{A_{ij}} - \mu \cdot \sigma_{a_{ij}}) Q_j^2; 0 \leq \mu \leq 3; n_j \in Z; j = \overline{1,k} \quad (6.6)\]

In expressions (6):

\[\rho - \text{water density};\]

\[g - \text{gravitational acceleration};\]

\[St_j - \text{electricity rate};\]

\[\eta_j - \text{CP efficiency};\]

\[h_0, S - \text{water column height and a reserve tank bottom area};\]

\[\sigma_{b_j}, \sigma_{a_{ij}} - \text{average square deviations of certain random values};\]

\[\Delta h_j - \text{water column height increment};\]

\[D, E - \text{interpolation curve factors of CP efficiency}.\]

The expression (6.1) specifies an objective function of F problem which defines the overall electricity cost expended for water supply provision; inequalities (6.2) specify the limit values in a reserve tank and PS pressure, determine conditions of the model physical correctness; equation (6.3) ensures equality of water storage capacity in a reserve tank in the beginning and at the end of a daily water consumption; equations (6.4 – 6.6) give an opportunity to determine parameters of the water supply system state, they are used in the objective function and constraints of the described above problems of mathematical programming. PS head (flow) (6.5) is found by solving of the equation (2).
relative to \( Q(t) \). The first equation (6.6) is an adopted equation of efficiency dependence from the PS head in the CP theory; and the second one - pressure formula in the water supply network.

Random process cross-section random values \( Ras(t) \) and \( A_j(t) \), as they have normal distribution, can differ from their mathematical expectations, but not more than three average square deviations. In the model (6.3, 6.5 and 6.6) the possibility of the network consumption and resistance determination is foreseen – in the form of a sum (difference) of their mathematical expectations and \( \mu \) prescribed number of average square deviations. It allows PS operation optimizing both with average random \((\mu = 0)\) and maximum values \((\mu = 3)\).

Optimization model (6) is a non-linear, partially mixed-integer problem of the mathematical programming with control variables \( i, n_j; j = 1, k \). By decision results, for any \( j \)-interval, optimal values for \( n_j \) - number of pumps in operation and identical relative speed of CP (i) must be determined for all intervals. The number of time intervals effects on the dimension of a problem of the mathematical programming, and to a great extent, defines its complexity, time period for solving and accuracy of obtained results. In this work a daily time period of water supply process was divided into eight intervals – as a result – a mathematical programming problem has been solved with nine control variables.

4. Optimization results and analysis

For optimization problem solving (6) genetic algorithm numeric method was chosen [5], it is used for a search of non-linear problems solving in the multidimensional spaces. For these reasons generic algorithm method in Optimization Tool package of MATLAB software was used.

Software optimization is realized in the form of three interrelated files – the function call script of generic algorithm method and two M-functions one of which specifies constraints of the problem and the other one – calculation method of an objective function at predetermined set of values of control variables.

Optimization procedure fulfillment allows founding of the following parameters: \( F_{\text{min}} \) - minimum electricity costs for a daily water supply ensuring; \( n_{\text{max}} \) - maximum number of such type of CP being used during a daily cycle; \( i \) – CP optimal operation speed; \( h_{\text{max}} \) - maximum liquid column height in a reserve tank. With the help of \( \eta(t) \) function, an average time-weighted estimate of CP operation efficiency can be obtained:

\[
\bar{\eta} = \left( \sum_{j=1}^{k} \eta_j \cdot \Delta t_j \right) / \left( \sum_{j=1}^{k} \Delta t_j \right)
\]  

(7)

By results of optimization method, software builds time dependence diagrams of water supply consumption basic parameters – pressure, water supply and rate, efficiency, water storage capacity in a reserve tank. As an example in Figures 3 and 4 diagrams \( Q(t), Ras(t), V(t) \) are represented from an optimal plan for CP, based on CP \( NB/NK \) 65/217.
Vector in a phase space with components \( \left( F_{\min}, n_{\max}, \bar{\eta}, i, h_{\max} \right) \) represents the optimal plan of PS operation. With the help of software optimization, PS optimal plans based on different CP were obtained. Comparison of optimal plans has allowed finding of an optimal type of CP, for obtained water rate function. \( h_{\max} \) value defines the required minimum dimensions of a reserve tank, that is equal to a daily maximum stored water volume in it - \( V_{\eta_{\max}} = h_{\max} \cdot S \), thus, a reserve tank (from an available standard set) with the closest to optimal dimensions can be chosen for the water supply system.

Data for PS optimal plans based on different CP are listed in the Table 1.

| Pump type         | \( F_{\min}, \text{RUB} \) | \( n_{\max}, \text{each} \) | \( \bar{\eta} \) | \( i \) | \( V_{\eta_{\max}}, \text{m}^3 \) |
|-------------------|-----------------------------|-----------------------------|-----------------|-------|-----------------------------|
| Wilo 5205         | 1574                        | 4                           | 0.74            | 0.95  | 922                         |
| MVI 9503/2        | 1607                        | 2                           | 0.75            | 1.02  | 963                         |
| MVI 9503/1        | 1614                        | 2                           | 0.75            | 0.97  | 963                         |
| NB/NK 65/217      | 1473                        | 2                           | 0.83            | 1     | 790                         |
| NL 100/225        | 1783                        | 1                           | 0.66            | 0.99  | 480                         |
| NL 100/235        | 1682                        | 1                           | 0.72            | 0.98  | 963                         |
| NL 80/235         | 1650                        | 2                           | 0.7             | 0.99  | 922                         |

Table 1 data can be used for the following conclusions:

\( - \) practically nominal speed of CP is considered in the operation optimal mode;

\( - \) minimum electricity cost for a daily water supply provision is found for CP NB/NK 65/217; the second - as for costs, with 7% approximate difference – is CP Wilo 5205; and the third is CP MVI 9503/2 and MVI 9503/1 – cost increase - about 9%.

It should be noted that CP NB/NK 65/217 shows the best results not only by an objective function value of optimization problem, but also by majority of values of the other optimality criteria that are used in its constraints (average-weighted efficiency – with respect to time, required volume of a reserve tank). Only by one criterion – minimum number of CP – the best result is shown by CP NL 100/235. But by the criterion of CP minimum cost NL is already worse by 14%.
From comparison of PS operation optimal plants it follows that within the model under consideration, without use of second thoughts, a choice in behalf of one of them cannot be made definitely. The reason of such ambiguity is initial multicriteriality of a stated problem of the mathematical programming for which availability of both unidirectional and multidirectional criteria is typical. Competing criteria are criteria of CP number minimization in the operation plan and their operation efficiency maximization. From the type of an objective function of optimization problem it follows that CP efficiency maximization leads to minimization of a daily electricity cost, i.e. mentioned above criteria are unidirectional. Maybe change of a reserve tank volume and design can effect on the final values of optimizable parameters as well.

5. Conclusions
The authors have developed a mathematical optimization model of the water supply system booster pumping station (BSP) with a reserve tank operation, with use of one type CP. For optimization problem solving the method of genetic algorithms was used.

In optimization problem solving the full-scale experiment results, by the random recording of basic parameters dynamics variation of non-stationary water consumption, were obtained and used. It’s indicated that:

- network resistance $A_c(t)$ and water flow rate $R_{as}(t)$ changes during a daily water consumption cycle can be considered as interrelated non-stationary random processes, their source is a great number of independent consumers unpredictable behavior, that depends on many uncontrolled factors, and directly effects on the value of network overall resistance;
- random process cross-sections are normally distributed random values; their mathematical expectations and average square deviations are determined.

Optimal BPS operation plans based on different CP have been developed. Comparison of obtained optimal plans helps in choosing of the best BPS version with power units in regards to daily expenses minimization for electricity payment, CP number and a reserve tank dimensions minimization, CP efficiency maximization.

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