Chiral symmetry breaking as open string tachyon condensation

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ABSTRACT: We consider a general framework to study holographically the dynamics of fundamental quarks in a confining gauge theory. Flavors are introduced by placing a set of (coincident) branes and antibranes on a background dual to a confining color theory. The spectrum contains an open string tachyon and its condensation describes the $U(N_f)_L \times U(N_f)_R \rightarrow U(N_f)_V$ symmetry breaking. By studying worldvolume gauge transformations of the flavor brane action, we obtain the QCD global anomalies and an IR condition that allows to fix the quark condensate in terms of the quark mass. We find the expected $N_f^2$ Goldstone bosons (for $m_q = 0$), the Gell-Mann-Oakes-Renner relation (for $m_q$ small) and the $\eta'$ mass. Remarkably, the linear confinement behavior for the masses of highly excited spin-1 mesons, $m_n^2 \sim n$ is naturally reproduced.

KEYWORDS: Gauge-gravity correspondence, Tachyon Condensation, QCD, Spontaneous Symmetry Breaking.
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Bibliography
1. Introduction

The original AdS/CFT conjecture [1, 2] states the existence of a stringy description of a conformal, highly symmetric, quantum field theory. The low energy limit on the string side (supergravity) corresponds to the strong coupling regime of the gauge theory. A natural question that has attracted a lot of attention is whether such a duality can be generalized in order to describe the strong coupling regime of more realistic gauge theories. In the last few years we have learned that the ideas of holography can indeed be applied to a certain extent in that direction. In particular, higher-dimensional duals of (non-supersymmetric) QCD-like theories have been built and many qualitative and, surprisingly, some quantitative features of real-world strong interactions can be described in such frameworks with reasonable accuracy.

One of the most important strong-coupling phenomena of QCD is spontaneous chiral symmetry breaking $U(N_f)_L \times U(N_f)_R \rightarrow U(N_f)_V$ by the formation of a non-perturbative quark condensate $\langle \bar{q}q \rangle \neq 0$ ($N_f$ denotes the number of quark flavors). The goal of this paper is the study of this symmetry breaking using holographic techniques.

We start by briefly reviewing the results obtained on this topic in the existing literature. This question was initially addressed in [3, 4] (see also [5]), by considering stacks of $N_f$ flavor branes in non-supersymmetric backgrounds created by $N_c$ color branes. The chiral symmetry is a $U(1)_A$ isometry of the geometry and is spontaneously broken due to the embedding of the flavor branes. Typically, these models are asymptotically supersymmetric in the UV. This allows to have good control on quantities like the quark condensate and the quark mass. Imposing regularity of the brane configuration, one obtains an IR condition which fixes the quark condensate in terms of the quark mass, as expected in QCD. Also, by introducing a small quark mass, the Gell-Mann, Oakes, Renner (GOR) relation for the pion masses

$$m^2_\pi = -2 m_q \langle \bar{q}q \rangle / f^2_\pi , \quad m_q \rightarrow 0$$

can be obtained. The $O(N_f)$ mass of the $\eta'$ in this kind of setup was studied in [6].

The limitation of this approach is that, even for massless quarks, only the abelian chiral symmetry is present, while the non-abelian chiral subgroup of the flavor symmetry is absent from the beginning, and its breaking cannot therefore be seen. From the field theory side, this is due to the existence of a $\bar{q}\Phi q$ Yukawa coupling (color and flavor branes are codimension four), where $\Phi$ is one of the (massive) scalars living on the brane world-volume [4].

A different approach was followed in [7, 8]. By considering $N_f$ D8-D8 pairs in a non-supersymmetric background corresponding to $N_c$ D4 branes, a $U(N_f) \times U(N_f)$ global symmetry was introduced. Contrary to the case described earlier, the chiral symmetry is now realized on the flavor brane world-volume rather than in the geometry. The breaking of the full non-abelian chiral symmetry is due to a smooth recombination of branes and antibranes in the IR. By computing the meson spectrum, $N_f^2$ massless Goldstone bosons are found. One of them, the $\eta'$, is massless only when $N_c \rightarrow \infty$ and the authors addressed the problem of finding its $O(N_f)$ mass. The chiral anomaly and associated WZW term
are nicely reproduced. Remarkably, quantities (some meson masses, decay constants and couplings) computed in this holographic setup match reasonably well the experimental values. Moreover, the model also incorporates a very natural picture of chiral symmetry restoration at high temperature: once a horizon is formed in the geometry, the branes and antibranes can fall into it and they do not need to recombine.

This construction however presents some shortcomings too. First, there is no parameter which can be associated to the quark bare masses. By modifying the flavor brane embedding, one can vary the masses of the massive mesons, which can be thought of as a modification of the constituent quark masses. However, the pions remain massless, implying that the bare quark masses are always vanishing (see [9, 10, 11, 12]). This is clearly not a physical feature of QCD. Related to this problem, there is no parameter that can be identified with the quark condensate, even though this is a crucial quantity to describe spontaneous chiral symmetry breaking in QCD. Another unphysical feature of the model is the absence of a tower of massive mesons with the quantum numbers of the pion.

Finally, there is an interesting alternative approach, that has been named AdS/QCD [13, 14]. It is a bottom-up construction in the sense that it does not emerge directly from some known string theory. Rather, it just assumes the existence of a dual description of QCD living in a five-dimensional, asymptotically AdS space. The five-dimensional bulk fields needed to dualize the quark bilinears are introduced by hand, and spontaneous chiral symmetry breaking is modeled by giving a vev to a bulk scalar by hand. The model incorporates the existence of Goldstone bosons, vector meson dominance [14] and the GOR relation for the pion masses [13]. In its simplest form, the five-dimensional space is taken to be just $\text{AdS}_5$ with a hard IR cutoff. However, modifications of the background have been considered in order to incorporate the effect in the geometry of the possible condensates or linear confinement [15]. A drawback of the model is that, since the condensate is not determined from some dynamical computation, there is an extra parameter compared to QCD. Also, being\textit{ ad hoc}, the models are less satisfying and it would be nice to understand how they are related to some (possibly non-critical) string theory which may help fix the background geometry or the potential for the bulk scalar. Some progress along this line seems imminent, [17].

The goal of this paper is to improve the present holographic description of chiral symmetry breaking by proposing a general string theory construction which accounts for all relevant phenomena in a natural and simple way. Flavors are introduced via a stack of $N_f$ overlapping brane-antibrane pairs. At the same time, we allow for a non-trivial profile for the open string tachyon field. This is generally the lightest string mode extending between branes and antibranes, and it transforms in the bifundamental representation of the $U(N_f)_L \times U(N_f)_R$ flavor symmetry group supported by the brane-antibrane pairs. This complex scalar is therefore the natural candidate to describe chiral symmetry breaking, as suggested in $\text{AdS}$ (see also [11, 18, 19]). A review on open string tachyon physics can be found in [20].

We will study the world-volume effective action for the brane-antibrane system, including the tachyon both in the Dirac-Born-Infeld and Wess-Zumino terms. To keep the
focus on the relevant dynamics, we will consider a simple setup in which only the tachyon and the world-volume gauge fields of the branes and antibranes are dynamical. This is the minimal setup to include the quark bilinears in the dual theory (up to spin one) and it matches the bulk field content of the AdS/QCD models [13, 14]. We will also not worry about the closed string physics. We will keep the discussion very general, without applying it to any particular model: we will only need to assume that the background metric is such that the color theory is confining (using a general prescription in [21]).

In a few places we will also perform some more explicit computations; in these cases we will assume that the space is asymptotically AdS in the UV. This will allow us to apply the general formalism developed in this paper to a relevant and interesting example, while hopefully clarifying our presentation.

1.1 Summary of results

The open string tachyon we introduce transforms in the bifundamental representation of the flavor symmetry group $U(N_f)_L \times U(N_f)_R$, and couples on the boundary to the quark scalar and pseudoscalar bilinears. Up to a normalization we will introduce later on, we have in fact:

$$T \leftrightarrow \bar{q} \frac{1 + \gamma^5}{2} q$$
$$T^\dagger \leftrightarrow \bar{q} \frac{1 - \gamma^5}{2} q$$

By considering brane-antibrane configurations, and including the dynamics of the complex open string tachyon scalar $T$, we have been able to provide a general framework to describe holographically the chiral dynamics of QCD and related theories, and to reproduce many features of QCD:

- The UV non-normalizable component of the tachyon corresponds to the quark’s bare (complex) mass matrix $m_q$. For simplicity, in this paper we will consider all quarks to have the same (possibly vanishing) mass $m_q$.

- The UV normalizable mode of the tachyon corresponds, in turn, to turning on an expectation value for the quark condensate.

- We show that in a confining background, even for $m_q = 0$, the tachyon profile cannot be trivial, which results in the chiral part $U(N_f)_A$ of the flavor symmetry being always broken, either explicitly or spontaneously. The construction of this paper, therefore, provides a holographic version of the Coleman-Witten theorem [22].

- The fact that the tachyon diverges in the IR (signalling the IR fusion of branes and antibranes) constrains the way it vanishes in the UV: this allows to fix the value of the quark condensate $\langle \bar{q}q \rangle$ in terms of the quark bare mass $m_q$.

- We can derive formulae for the anomalous divergences of flavor currents, when they are coupled to an external source.

- The WZ part of the flavor brane action gives the Adler-Bell-Jackiw $U(1)_A$ axial anomaly [23], and an associated Stuckelberg mechanism gives an $O \left( \frac{N_f}{N_c} \right)$ mass to the would-be Goldstone boson $\eta'$, in accordance with the Veneziano-Witten formula.
• When \( m_q = 0 \) (i.e. the tachyon has no UV non-normalizable mode), we find that
(in the \( N_c \to \infty \) limit) the meson spectrum contains \( N_f^2 \) massless pseudoscalars, the
\( U(N_f)_A \) Goldstone bosons.

• Studying the spectrum of highly excited spin-1 mesons, we find the expected property
of linear confinement: \( m_n^2 \sim n \).

• When we consider an asymptotically AdS space in the UV, we can precisely reproduce
the GOR relation for the mass of the pions in the case of a small but non-zero quark
bare mass.

On the other hand, the main limitations our approach still presents are the following:

• The details of the tachyon potential that one should insert in the DBI action are not
rigorously known. Despite this, its gross features suffice for extracting qualitative
information.

• A background in which the brane-antibranes are on top of each other (and with no
Wilson line turned on), would probably require large curvature, as we explain. Then,
a reliable background metric cannot be found by solving Einstein’s equations.

• We find that the slope of the linear confinement relation is not the same for vector
and axial mesons. This may be due to the fact that it is not clear whether the
DBI action we use to include the effects of the dynamics of the open string tachyon
remains valid in the region where the tachyon and its derivative diverge.

However we found encouraging that a large amount of expected QCD features can be
extracted from our construction despite the misgivings above.

2. The general construction

The general setup we consider consists of a system of \( N_f \) overlapping \( D_p-\overline{D_p} \) flavor brane-
antibrane pairs in the background associated to \( N_c \) Dq color branes. In critical, ten-
dimensional string theory, we will need to impose the condition \( p + q = 12 \) and require the
color and flavor branes to be codimension 6, in order to get the correct WZ couplings.\(^1\) For
instance, one can think of a D3-D9-\( \overline{D9} \) intersection as in \[18\] or a D4-D8-\( \overline{D8} \) intersection
similar to the Sakai-Sugimoto model \[7\]. Notice, though, that the general formalism we
develop can also be applied to lower dimensional setups provided the WZ couplings are
the obvious adaptation of those of section \[3\]. Interesting examples of this kind are the

\(^1\)The reason why is very simple to explain. In section \[3.2\], we will be looking for a coupling on the flavor
\( D_p-\overline{D_p} \) branes which involves 6-dimensional gauge field forms. This can only come from the Wess-Zumino
coupling of the \( p \)-branes to a magnetic \( k \)-form potential, where \( k = p + 1 - 6 = p - 5 \). Now Dq-branes
source a \( C_{q+1} \) RR potential, which is parallel to the source branes and generically depends on the radial
direction alone. Imposing 4-dimensional Lorentz invariance of the background ensures that the Hodge dual
of \( C_{q+1} \) is a magnetic \( \tilde{k} \)-form, with \( \tilde{k} = 10 - (q + 2) - 1 = 7 - q \). Imposing \( k = \tilde{k} \) finally gives the condition
\( p + q = 12 \).
AdS/QCD models or non-critical string constructions ($\mathcal{N} = 1$ theories with space-time filling brane-antibrane flavors have been discussed in [24, 19, 25] and $\mathcal{N} = 0$ in [19, 10]).

In the spectrum of stacks of overlapping brane-antibrane pairs there is a tachyonic, complex scalar, open string mode, which transforms in the bifundamental representation of the $U(N_f)_L \times U(N_f)_R$ flavor symmetry group supported by the brane-antibrane pairs. As we argued in the introduction, we can in general associate chiral symmetry breaking to a vacuum expectation value for this field. Near the UV region (in our coordinates this is around $z = 0$), the tachyon’s vev vanishes and the full $U(N_f)_L \times U(N_f)_R$ is present, as in the lagrangian of massless QCD. As we approach the IR, we prove in section 3.2 that if the theory is confining the tachyon must acquire a non-trivial vev, breaking the symmetry to its diagonal $U(N_f)_V$ subgroup, (see also section 2.2). Concretely, we show that the vev reaches infinity at a finite distance in the bulk, a point that we denote by $z_{IR}$. In a sense, this process can be thought of as brane-antibrane recombination. In the minimal construction that we consider in the rest of this paper, $z_{IR}$ coincides with the end of space. Figure 1 presents a general profile for the tachyon in the setup we just described.

![Figure 1](image)

**Figure 1:** A scheme of the setup. Generically, the tachyon vanishes in the UV where one has a stack of brane-antibrane pairs. In the IR, it diverges and the brane-antibrane pairs end smoothly. Here $\tau$ stands for the modulus of the tachyon field.

For the present discussion, it will be very important to consider the completion of the world-volume action of the brane-antibrane system to include also the couplings to the open string tachyon. As usual, this action is composed of a Dirac-Born-Infeld and a Wess-Zumino term. In the rest of this section and in section 3, we study the physics of the DBI piece, which yields the vacuum of the configuration and the meson mass spectrum. The WZ part will be studied in section 3. From it, we derive the holographic description of the parity and charge conjugation symmetries, the global anomalies, and a holographic Coleman-Witten theorem.

### 2.1 The degrees of freedom: a minimal setup

As proposed in [26, 27], the semiclassical action for a $Dp$-$\overline{Dp}$ system which includes the physics of the open string tachyon is a generalization of the usual Dirac-Born-Infeld action
for D-branes.

We start from the simpler configuration of a single brane-antibrane pair. In this case the proposed DBI action reads:

\[ S = -\int d^{p+1}x e^{-\phi} V(\tau^2, Y^I_L - Y^I_R, x) \left( \sqrt{-\det \mathbf{A}_L} + \sqrt{-\det \mathbf{A}_R} \right) \tag{2.1} \]

where (we will set \(2\pi\alpha' = 1\) from now on, and only reinsert \(\alpha'\) when needed):

\[
A^{(i)}_{MN} = P[g + B]_{MN} + F^{(i)}_{MN} + \partial_M Y^I_{(i)} \partial_N Y^I_{(i)} + \frac{1}{\pi} (D_M T)^* (D_N T) + \frac{1}{\pi} (D_N T)^* (D_M T) \tag{2.2}
\]

and \(V(\tau^2, Y^I_L - Y^I_R, x)\) is the tachyon potential. The complex tachyon is denoted by \(T \equiv \tau e^{i\theta}\), the indices \(i = L, R\) denote the brane or antibrane, the \(Y^I_{(i)}\) are the transverse scalars, and \(A^{(i)}\) is the world-volume gauge field. Here and in the following, capital latin characters, \(M, N\), denote world-volume directions, which include in particular the Minkowski directions, which we will denote by greek letters \(\mu, \nu\), and the holographic radius which will be referred to as \(z\).

For simplicity, we will not consider a background \(B\)-field. We will also ignore the transverse scalars living on the flavor branes. These modes, which are generally present in a critical string setup, do not have any obvious QCD interpretation and would not play any relevant role in the present discussion. Thus, we only consider the degrees of freedom coming from the open string tachyon and the world-volume gauge fields along the Minkowski or the holographic directions \(A^{(i)}_{\mu}, A^{(i)}_{z}\). This field content is enough to include the basic QCD quark bilinears up to spin 1 and coincides with the one of the AdS/QCD models.

Higher spin operators correspond to stringy excitations and cannot be addressed in this formalism. They could be introduced by hand as in \([28]\).

Since we are ignoring transverse scalars, for the tachyon potential we take the expression for overlapping brane-antibranes\(^2\) derived in boundary string field theory \([29]\), \([30]\):

\[ V(\tau^2) = T_p e^{-\tau^2} \tag{2.3} \]

One should keep in mind that this expression is not rigorously justified since there can be non-trivial field redefinitions between the two formalisms. Moreover the boundary string field theory computation is performed in flat space and it is not trivial at all that the result should apply unchanged to curved backgrounds. Remarkably, in section 4.2 we will find the nice feature of linear confinement using this gaussian behavior for the potential. Most of the rest of the discussion is insensitive to the precise expression of the tachyon potential.

\(^2\)If one considered more complicated setups with non-trivial brane profiles, the potential should depend also on the brane-antibrane distance. For a discussion in a related setup, see \([11]\).

\(^3\)Unfortunately, there are not unified conventions in the literature about the definition of \(T\). The unambiguous statement is that the mass squared of the open string tachyon in flat space is \(m^2_T = -\frac{1}{2\alpha'}\). Equation \((2.3)\) is thus consistent with \((2.1)\) and \((2.2)\) since, then, for small \(\tau\) the action is proportional to \(S \propto \int \left( \frac{1}{2} (\partial \tau)^2 - \frac{1}{2} \frac{1}{\alpha'} \tau^2 \right)\), where we have considered flat space and reinserted \(\alpha'\).
After presenting the simpler abelian case, we can now go to considering a stack of $N_f > 1$ brane-antibrane pairs, which is what we are eventually interested in. The natural non-abelian generalization of (2.1) is:

$$S = -\int d^{p+1}x \text{SymTr} \left[ e^{-\phi V(\mathcal{T}\mathcal{T}^\dagger)} \left( \sqrt{-\det A_L} + \sqrt{-\det A_R} \right) \right]$$  \hspace{1cm} (2.4)$$

where \text{SymTr} is a symmetric trace as defined in [31]. The analogous proposal for the action of a stack of non-BPS D-branes was made in [32] and can be related to systems of branes-antibranes along the lines of [27]. Although the symmetric trace prescription for the non-abelian action is known to be corrected at higher orders in $\alpha'$, this will not be relevant for our discussion. All that is important is that the potential appears in a single trace, and this is guaranteed by the large $N_c$ limit, [22].

The gauge group supported by the brane-antibrane system is $U(N_f)_L \times U(N_f)_R$. The gauge fields $A_{L,R}$ transform in the adjoint representation of $U(N_f)_L$ and $U(N_f)_R$ respectively, while the tachyon $T$ transforms in the bifundamental of $U(N_f)_L \times U(N_f)_R$, (that is, in the antifundamental of $U(N_f)_L$ and in the fundamental of $U(N_f)_R$). Its hermitian conjugate $T^\dagger$ transforms in the opposite way. For more details on our choice of conventions, we refer the reader to appendix A.

2.2 The vev for the tachyon field

In this section we want to determine the vacuum configuration for a stack of brane-antibranes over a confining vacuum. It is trivial to show that the equations of motion allow for the gauge fields to be consistently set to zero, $\langle A_L \rangle = \langle A_R \rangle = 0$. As we mentioned in the introduction, and we will explicitly show in section 3.2, anomaly considerations regarding the WZ part of the action of a stack of brane-antibranes over a confining vacuum require that $\langle \tau \rangle |_{z_{IR} \to \infty}$. This necessitates that we allow for a non-trivial profile of the tachyon in the vacuum configuration.

Four-dimensional Poincaré invariance determines the general form of the bulk five-dimensional metric. In the mostly plus signature this reads:

$$ds_5^2 = g_{xx}(z) \eta_{\mu\nu} dx^\mu dx^\nu + g_{zz}(z) dz^2$$  \hspace{1cm} (2.5)$$

The non-abelian action (2.4) is a very complicated object, where different components of the fields mix because of non-trivial commutation relations. We start from the simpler $N_f = 1$ case, where these complications do not arise. From (2.1), it is consistent to set the tachyon phase to a constant.

We are thus left with the world-volume action for the modulus of the tachyon $\tau$, which, upon reducing to five dimensions reads:

$$S = -2 \int d^4x dz e^{-\phi_{eff}} V(\tau^2) g_{xx}^2 \sqrt{g_{zz} + \frac{2}{\pi} (\partial_z \tau)^2}.$$  \hspace{1cm} (2.6)$$
The prefactor $e^{-\phi_{eff}(z)}$ is defined as the dilaton exponential times whatever comes from integrating the transverse, spectator, $p - 4$ dimensions. Schematically:

$$e^{-\phi_{eff}(z)} V(\tau^2(z)) g_{xx}^2 \sqrt{g_{zz} + (\partial_z \tau)^2} \equiv \int_{N^{p-3}} d^{p-4}y \ e^{-\phi} V(\tau^2, Y_L^I - Y_R^I, x) \left( \sqrt{-\det A_L} + \sqrt{-\det A_R} \right)$$ \hspace{1cm} (2.7)

where $N^{p-3}$ is the internal part of the flavor branes world-volume. 4

The Euler-Lagrange equation for $\tau$ obtained from (2.6) reads:

$$\partial_z^2 \tau + \frac{2}{\pi} \frac{\partial_z (g_{xx}^2 e^{-\phi_{eff}})}{g_{zz} g_{xx}^2 e^{-\phi_{eff}}} (\partial_z \tau)^3 + \left( \frac{\partial_z (g_{xx}^2 e^{-\phi_{eff}})}{g_{xx}^2 e^{-\phi_{eff}}} - \frac{\partial_z g_{zz}}{2 g_{zz}} \right) \partial_z \tau + 2 \tau \left( \frac{\pi}{2} g_{zz} + (\partial_z \tau)^2 \right) = 0$$ \hspace{1cm} (2.8)

This is a second order differential equation which therefore has two integration constants.

Since $\tau$ is dual to the quark bilinear, we know on general AdS/CFT grounds that, by looking at the UV behavior of $\tau$, these two constants can be related to the quark bare mass and the quark condensate. We now assume that the space is asymptotically AdS, that is $g_{xx}(z) \simeq g_{zz}(z) \simeq R^2_{AdS} / z^2 + \ldots$ and $e^{-\phi_{eff}} \simeq const \equiv e^{-\phi_0}$ for small $z$. The tachyon field is dual to the quark bilinear which has dimension 3. Thus, the usual relation between the mass of a five-dimensional scalar field and the conformal dimension of its dual operator, $\Delta(\Delta - 4) = m_T^2 R^2_{AdS}$, fixes $m_T^2 R^2_{AdS} = -3$, which in turn implies that, if $m_T^2 = -\frac{1}{2\alpha'} = -\pi$, the radius of AdS should be small: $R^2_{AdS} = 6\alpha' = \frac{\pi}{3}$. This leads to the following UV behavior (in order to simplify notation the vacuum expectation value $\langle \tau \rangle$ will be denoted just as $\tau$ from now on):

$$R^3_{AdS} \tau_{can} \equiv R^3_{AdS} \left( -\frac{4T_p}{m_T^2 e^{\phi_0}} \right)^{\frac{1}{2}} \tau = m_q (z + \ldots) + \sigma(z^3 + \ldots) \quad \text{(small } z) \right).$$ \hspace{1cm} (2.9)

We have written this expression in terms of the canonically normalized $\tau_{can}$, such that the boundary coupling is $R^3_{AdS} \int d^4x \tau_{can} z^{-1} \bar{q}q$. The parameter $\sigma$ can be related to the quark condensate $\langle \bar{q}q \rangle$, see appendix B for details. The UV behavior (2.9) is the same as the one for the scalar breaking the chiral symmetry in the AdS/QCD models [13, 14].

We now want to prove that the IR condition $\tau|_{zIR} \to \infty$ gives a condition on $\tau(z)$, fixing $\sigma$, and therefore the condensate, in terms of the mass $m_q$.

A general result of the gauge-gravity correspondence states that a sufficient condition for confinement is that at some $z_{div}$, $g_{zz}(z_{div}) \to \infty$ while $g_{xx}(z_{div}) \neq 0$ and $\partial_z g_{xx}(z_{div}) < 0$ [21]. We will assume this is the case for our background and in particular, after possibly redefining $z$, we fix the divergence to be a single pole $g_{zz} = b(z_{div} - z)^{-1}$ near $z = z_{div}$.

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4The reader might be puzzled by the approximation involved in discarding all dependence on the internal world-volume scalars. It should be noted, though, that our ansatz for the five-dimensional metric (2.5) is completely general, being constrained only by four-dimensional Poincaré invariance. The vacuum configuration of the internal scalars will only affect the explicit form of the metric coefficients and the radial dependence of the tachyon modulus and dilaton. Since our arguments are completely independent of the particular form of the confining, reduced, five-dimensional background, all the following results apply regardless what the critical string setup is.
This is the behavior, for instance, in Witten’s dual of Yang-Mills [33]. Inspecting the leading terms for large $\tau$ of (2.8), we see that the tachyon can only diverge at $z_{IR} = z_{div}$ and there it diverges as $\tau \propto (z_{IR} - z)^{-a}$, where $a = -\pi b \left[ \frac{g_2^2 e^{-\phi_{eff}}}{\partial_z (g_2^2 e^{-\phi_{eff}})} \right]_{z=z_{IR}}$ is a positive number since the derivative inside the bracket is typically negative.

We warn the reader that this result should be taken with a grain of salt because it relies on the behavior of (2.8) when $\tau, \dot{\tau} \to \infty$, which lies outside the regime of validity of the DBI action (2.6). Nonetheless, the general lesson one should learn from this computation is that the $\tau|_{z_{IR} \to \infty}$ consistency condition constrains the IR behavior of the tachyon. Physically, this fixes the quark condensate in terms of the quark mass [3, 4].

At this point we can consider the more general non-abelian case $N_f > 1$. Throughout the paper, we will consider for simplicity that all $N_f$ flavor fields have the same mass $m_q$. The symmetrized trace in the action (2.4) might seem to make it a lot more complicated to find a vacuum configuration for the tachyon matrix $T$. Fortunately we can show that there is a perturbatively stable configuration given by $N_f$ identical copies of the solution for the abelian case we described above

$$\langle T \rangle = \tau(z) \mathbb{I} \quad (2.10)$$

In fact, we can apply here a result that was derived in [4] for a different setup. The reasoning in [4] showing that the vacuum configuration where vevs are proportional to identity matrices, like (2.10) in our case, is a stable (up to quadratic order) solution of the non-abelian DBI action is very general. It only relies on having a single trace in front of the action and the fact that the difference between abelian and non-abelian DBI actions consists entirely of terms involving commutators. The argument does not prove that this is the global minimum. There could in principle be a different configuration with lower energy, but this is an unlikely possibility and we will assume that (2.10) describes the vacuum of the theory.

Once we have established that $\langle T \rangle$ is proportional to the identity matrix, the equation of motion for $\tau(z)$ is the same as (2.8) as well as the UV and IR conditions, since for the configuration (2.10) and up to the order we need here, the non-abelian action (2.4) is actually the sum of $N_f$ identical copies of the abelian action (2.1).

Looking at how the fields transform under flavor symmetry transformations (A.6), it is clear that the vacuum (2.10) spontaneously breaks $U(N_f)_L \times U(N_f)_R$ into its diagonal subgroup $U(N_f)_V$, where the vectorial gauge transformation is $V_L = V_R$.

\[^5\]It is natural that for a fixed $m_q$, the conditions stated above fix uniquely $\sigma$. However, within the present very general formalism, this uniqueness cannot be proved. We will come back to this in section 3.4.

\[^6\]From the field theory point of view the nature of the breaking depends on the asymptotic UV behavior of the tachyon. When $m_q \neq 0$, the vacuum configuration of the tachyon is not normalizable and we are adding a mass term for the quarks to the lagrangian: the breaking is explicit. If instead $m_q = 0$, the tachyon is normalizable, and we are turning on an expectation value for the quark bilinear in the original theory, which makes the symmetry breaking spontaneous. From the gravity point of view, instead, both for vanishing or non-zero $m_q$, the breaking is spontaneous since it is determined by a non-trivial expectation value.
When \( m_q = 0 \), this spontaneous breaking implies the existence of \( N_f^2 \) Goldstone bosons. We will explicitly find them in the spectrum in section 4.3 (see also section 5.3 for an indirect argument). We conclude this section by noting that the fact that the symmetry is broken down to \( U(N_f)_V \), and not further, is also related to a theorem by Vafa and Witten [34] that states that vectorial symmetries cannot be spontaneously broken.

3. The D-brane Wess-Zumino sector

We now consider the WZ coupling of the \( N_f \) \( Dp-\overline{Dp} \) branes to the background RR fields, including the tachyon dependence. This part of the action is given by (the following expression was proposed in [35] and proved in [36, 37] using boundary string field theory):

\[
S_{WZ} = T_p \int_{\Sigma_{p+1}} C \wedge \text{Str} \exp [i2\pi\alpha' F]
\]

where \( \Sigma_{p+1} \) is the world-volume of the \( Dp-\overline{Dp} \) branes, \( C \) is a formal sum of the RR potentials \( C = \sum_n (-i)^{\frac{p+1}{2}} C_n \), and \( F \) is the curvature of a superconnection \( A \). In terms of the tachyon field matrix \( T \) and the gauge fields \( A_L \) and \( A_R \) living respectively on the branes and antibranes, they are (from now on, we will set \( 2\pi\alpha' = 1 \) again and use the notation of [36]):

\[
iA = \begin{pmatrix} iA_L & T^\dagger \\ T & iA_R \end{pmatrix}, \quad iF = \begin{pmatrix} iF_L - T^\dagger T & DT^\dagger \\ DT & iF_R - TT^\dagger \end{pmatrix}
\]

In appendix \( \Box \) we review the relevant definitions and properties of this supermatrix formalism (in particular, notice the definition of the supermatrix product).

The superconnection is defined as:

\[
F = dA - iA \wedge A
\]

and satisfies the Bianchi identity:

\[
dF - iA \wedge F + iF \wedge A = 0
\]

Using this identity and the cyclic property of the supertrace (\( \Box^2 \)) it is immediate to check that \( \text{Str} e^{iF} \) is a closed form and therefore that (3.1) is invariant under RR gauge transformations. At least locally, there exists then a form \( \Omega \) such that:

\[
d\Omega = \text{Str} e^{iF}
\]

The case of interest here is the WZ coupling on the \( Dp-\overline{Dp} \) flavor branes in the background of \( N_c \) \( Dq \)-color branes. As explained at the beginning of section 2, we have \( p+q = 12 \) and the electric RR potential sourced by the color branes is then \( C_{q+1} = C_{13-p} \), while its magnetic dual is \( C_{p-5} \). Formula (3.1) reduces to:

\[
S_{WZ} = iT_p \int_{\Sigma_{p+1}} C_{p-5} \wedge \text{Str} e^{iF}|_{6-\text{form}} = i(-)^p T_p \int_{\Sigma_{p+1}} F_{p-4} \wedge \Omega|_{5-\text{form}}
\]
When the branes worldvolume $\Sigma_{p+1}$ has boundaries, the two expressions in (3.6) differ by boundary terms, and are therefore not equivalent. In this case, which is the relevant one for this paper, it is argued in [38] (for intersecting branes) that the correct form for the action is the last one in (3.6). Since $\Omega^{5-\text{form}}$ is defined only up to an exact form, there is then a possible ambiguity in the definition of the WZ action (3.4). We will assume that the correct form for $\Omega^{5-\text{form}}$ respects the discrete symmetries of $\text{Str} e^{iF}|_{6-\text{form}}$, which we will discuss in the following subsection 3.1.

Notice that the potential sourced by the $D_q$-branes has indices parallel to the branes and generically depends on the radial direction. Moreover, 4-dimensional Lorentz invariance of the background (plus possibly a radial redefinition) prevents the metric from having non-zero off-diagonal terms in the directions $(0,1,2,3,z)$. These two facts ensure that $F_{p-4}$ does not have legs along the $(0,1,2,3,z)$ directions. Thus, the only components of the five-form $\Omega^{5-\text{form}} \equiv \Omega_5$ which we are interested in are those along $x^0, x^1, x^2, x^3$ and $z$ (we will call $M_5$ this five-dimensional space). Similarly to section 2, we assume at this point that $\Omega_5$ does not depend on the directions along which $F_{p-4}$ lies, as expected on physical grounds. Then we can integrate out those directions in (3.6) to obtain the 5-dimensional effective Chern-Simons action:

$$S_{CS} = i(-)^p T_p \int_{\Sigma_{p+1} \cap M_5^+} F_{p-4} \cdot \int_{M_5} \Omega_5 = (-)^p \frac{iN_c}{(2\pi)^\beta\alpha'} \int_{M_5} \Omega_5 (3.7)$$

where we have used the quantization condition $\frac{1}{2\kappa_{(10)}} \int F_{p-4} = N_c T_{12-p}$ and the explicit value of the D-brane tensions.

### 3.1 Discrete symmetries $(P,C)$

The six-form $\text{Str} e^{iF}|_{6-\text{form}}$ appearing in (3.4) is invariant under two discrete symmetries, $P$ and $C$, that we will present in this subsection. Thus, an $\Omega_5$ invariant under $P, C$ exists. It follows then from (3.7) that the CS action is invariant under the same symmetries, which acquire a fundamental physical interpretation in the dual field theory.

#### Parity

The six-form $\text{Str} e^{iF}|_{6-\text{form}}$ is left invariant by the change $P \equiv P_1 \cdot P_2$, where:

$$P_1: A_L \leftrightarrow A_R, \quad T \leftrightarrow T^\dagger \quad \text{and} \quad P_2: (x_1, x_2, x_3) \rightarrow (-x_1, -x_2, -x_3) \quad (3.8)$$

Notice that $P_1$ also transforms $DT \leftrightarrow DT^\dagger$, $F_L \leftrightarrow F_R$. Indeed we can easily check that:

$$\text{Str} e^{iF} = \text{Str} \left[ \exp \left( iF_L - T^\dagger T \quad DT \quad iF_R - TT^\dagger \right) \right] \overset{P_1}{\rightarrow} \text{Str} \left[ \exp \left( iF_R - TT^\dagger \quad DT \quad iF_L - T^\dagger T \right) \right] = -\text{Str} \left[ \exp \left( iF_L - T^\dagger T \quad DT \quad iF_R - TT^\dagger \right) \right]. (3.9)$$

For the last equality we have just swapped the ordering of the blocks inside the matrix, picking up a minus sign due to the definition of the supertrace (3.2). The overall minus
sign in (3.3) is compensated by the minus sign acquired by the form \(dx_1 \wedge dx_2 \wedge dx_3\) under the action of \(P_2\), and the CS action (3.7) is invariant under \(P\).

We conclude that four-dimensional parity is equivalent to charge conjugation for the D-branes (interchanging \(D \leftrightarrow \bar{D}\)).

**Charge conjugation**

The other discrete symmetry of Str\(_{e^iF}\)6–form, and consequently of the CS action (3.7), is:

\[
C: \quad A_L \to -A^t_R, \quad A_R \to -A^t_L, \quad T \to T^t, \quad T^\dagger \to \left(T^\dagger\right)^t \tag{3.10}
\]

where the superscript \(t\) denotes matrix transposition. Notice that under (3.10), the field strengths and tachyon covariant derivatives transform as

\[
F^L_L \to -F^t_R, \quad F^R_R \to -F^t_L, \quad DT \to DT^t, \quad DT^\dagger \to \left(DT^\dagger\right)^t.
\]

The supertrace transforms as:

\[
\text{Str } e^{iF} = \text{Str} \left[ \exp \left( \begin{array}{cc} iF^L_L - T^\dagger T & DT^\dagger \\ DT & iF^R_R - TT^\dagger \end{array} \right) \right] \subseteq \text{Str} \left[ \exp \left( \begin{array}{cc} -iF^t_R - \left(T^\dagger T\right)^t & DT^t \\ \left(DT^\dagger\right)^t & -iF^t_L - \left(TT^\dagger\right)^t \end{array} \right) \right] = -\text{Str} \left[ \exp \left( \begin{array}{cc} -iF^t_L - T^\dagger T & -iDT^\dagger \\ -iDT & -iF^t_R - TT^\dagger \end{array} \right) \right].
\]

For the identity in the second line we have again swapped the ordering of the blocks picking up a minus sign. In the equality of the following line we used the definition of pseudotranspose (C.4). Finally in the last one, we used property (C.3) and the fact that the supertrace does not change under pseudotransposition. The matrix of the last expression is not the same as the untransformed one of the first line of (3.11). Therefore, (3.10) does not leave the supertrace invariant. However, we are only interested in the 6-form within the full expression of the supertrace. Noticing that all two-forms \((F^L_L, F^R_R, DT \wedge DT^\dagger)\) inside the matrix of the last line have picked up a minus sign compared to the initial expression, we find that the 6-form (in fact, all the \(4k + 2\)-forms) is left invariant.

If one wanted to maintain invariant the \(4k\)-forms of the supertrace (which can be related to gauge theories in \(4k - 2\) dimensions in the same way as we relate the 6-form to a four-dimensional field theory), then the transformation would be \(A_L \to -A^t_L, \quad A_R \to -A^t_R, \quad T \to \left(T^\dagger\right)^t, \quad T^\dagger \to T^t\). This is in agreement with the fact that charge conjugation in \(4k - 2\) dimensions does not change chirality.

Table 1 summarizes the transformation properties of the different degrees of freedom. In particular, notice that the vacuum condensate (2.10) transforms as \(0^{++}\) as expected for the \(\langle \bar{q}q \rangle\) bilinear.

**3.2 The chiral anomaly: external currents**

We now want to study the anomaly of the chiral symmetry when the flavor currents are coupled to external sources, i.e. study the WZ term as an action for the flavor brane world-volume gauge fields \((A_L, A_R)\) which couple in the boundary to combinations of the
vector and axial currents, (see table \[\text{[1]}\]. As is usual for Chern-Simons terms, a gauge transformation does not leave the action \([\text{[2]}]\) invariant but produces a boundary term. As in previous cases, \([\text{[3]}]\), this term is matched with the global anomaly of the dual field theory.

We set the tachyon to its vacuum value \([\text{2.10]}\). It is a straightforward although lengthy computation to expand the supertrace in \([\text{3.6]}\).

\[
\text{Str } e^{i\mathcal{F}}|_{\text{6-form}} = \frac{1}{6} \text{Tr } e^{-\tau^2} \left\{ -i F_L \wedge F_L \wedge F_L + i F_R \wedge F_R \wedge F_R + 2i\tau d\tau \wedge (A_L - A_R) \wedge \left( F_L \wedge F_L + \frac{1}{2} F_L \wedge F_R + \frac{1}{2} F_R \wedge F_L + F_R \wedge F_R \right) + \tau^2 (A_L - A_R) \wedge (A_L - A_R) \wedge (F_L - F_R) + \tau^3 d\tau \wedge (A_L - A_R) \wedge (A_L - A_R) \wedge (F_L + F_R) + i\tau^4 (A_L - A_R) \wedge (A_L - A_R) \wedge (A_L - A_R) \wedge (F_L - F_R) + \frac{i}{10} \tau^5 d\tau \wedge (A_L - A_R) \wedge (A_L - A_R) \wedge (A_L - A_R) \wedge (A_L - A_R) \wedge (A_L - A_R) \right\}
\]

\[(3.12)\]

It is now not very hard to find a 5-form \(\Omega_5\) such that \(d\Omega_5 = \text{Str } e^{i\mathcal{F}}|_{\text{6-form}}\). We can write:

\[
\Omega_5 = \frac{1}{6} \text{Tr } e^{-\tau^2} \left\{ -i A_L \wedge F_L \wedge F_L + \frac{1}{2} A_L \wedge A_L \wedge A_L \wedge F_L + \frac{i}{10} A_L \wedge A_L \wedge A_L \wedge A_L + \right.
\]

\[
+ i A_R \wedge F_R \wedge F_R - \frac{1}{2} A_R \wedge A_R \wedge A_R \wedge F_R - \frac{i}{10} A_R \wedge A_R \wedge A_R \wedge A_R +
\]

\[
+ \tau^2 \left[ i A_L \wedge F_R \wedge F_R - i A_R \wedge F_L \wedge F_L + \frac{i}{2} (A_L - A_R) \wedge (F_L \wedge F_R + F_R \wedge F_L) + \frac{1}{2} A_L \wedge A_L \wedge A_L \wedge F_L - \frac{1}{2} A_R \wedge A_R \wedge F_R + \frac{i}{10} A_L \wedge A_L \wedge A_L \wedge A_L + \right.
\]

\[
- \frac{i}{10} A_R \wedge A_R \wedge A_R \wedge A_R \wedge A_R \right]
\]

\[
+ i\tau^3 d\tau \wedge \left[ (A_L \wedge A_R - A_R \wedge A_L) \wedge (F_L + F_R) + i A_L \wedge A_L \wedge A_L \wedge A_R + \right.
\]

\[
- \frac{i}{2} A_L \wedge A_R \wedge A_L \wedge A_R + i A_L \wedge A_R \wedge A_R \wedge A_R \wedge A_R \right]
\]

\[
+ \frac{i}{20} \tau^4 (A_L - A_R) \wedge (A_L - A_R) \wedge (A_L - A_R) \wedge (A_L - A_R) \wedge (A_L - A_R) \right\}
\]

\[(3.13)\]
We now consider a gauge transformation of (3.13). As we argued above, the tachyon vacuum configuration (2.11) we are considering breaks the flavor invariance to the diagonal (vectorial) subgroup of $U(N_f)_L \times U(N_f)_R$. Therefore, taking $V_L = V_R = V$, the gauge transformation of the fields involved in (3.13) reads (see appendix A for more details)

\[
\begin{align*}
\delta_\Lambda A_L &= -i D\Lambda = -i d\Lambda - A_L \Lambda + \Lambda A_L \\
\delta_\Lambda A_R &= -i D\Lambda = -i d\Lambda - A_R \Lambda + \Lambda A_R \\
\delta_\Lambda F_L &= [\Lambda, F_L] \\
\delta_\Lambda F_R &= [\Lambda, F_R]
\end{align*}
\]

(3.14)

Inserting (3.14) in (3.13) we find:

\[
\delta_\Lambda \Omega_5 = \frac{1}{6} \text{Tr} e^{-\tau^2} d\Lambda \wedge \left\{ (1 + \tau^2) \left[ -F_L \wedge F_L - i 2 (A_L \wedge A_L \wedge F_L + A_L \wedge F_L \wedge A_L + A_L \wedge A_L \wedge F_L + F_L \wedge F_L + A_L \wedge A_L \wedge A_L + A_L \wedge A_L \wedge F_L + F_R \wedge F_R + i 2 (A_R \wedge A_R \wedge F_R + A_R \wedge F_R \wedge A_R + F_R \wedge F_R + A_R \wedge A_R \wedge A_R + A_R \wedge A_R \wedge F_R + F_L \wedge F_L + A_R \wedge A_R \wedge A_R + A_R \wedge A_R \wedge A_L + A_R \wedge A_R \wedge A_L + A_R \wedge A_R \wedge A_R + A_R \wedge A_R \wedge A_R + A_R \wedge A_R \wedge A_R + A_R \wedge A_R \wedge A_R) \right] + \tau^3 d\tau \wedge \left[ (A_L - A_R) \wedge (F_L + F_R) + (F_L + F_R) \wedge (A_L - A_R) + i (A_L \wedge A_L \wedge A_L + A_L \wedge A_R \wedge A_L + A_L \wedge A_R \wedge A_R + A_R \wedge A_R \wedge A_R + A_R \wedge A_R \wedge A_R) \right] \right\}
\]

(3.15)

As expected, it is straightforward to show that the 5-form $\delta_\Lambda \Omega_5$ is closed

\[
d(\delta_\Lambda \Omega_5) = 0
\]

(3.16)

The result (3.16) ensures that the gauge variation of the 5-dimensional CS action (3.7) is given by 4-dimensional boundary terms. The UV boundary term (where $\tau \to 0$) reproduces the field theory global anomaly [3], as we will see below. However, if we consider a background dual to a confining gauge theory, the gauge variation of the CS action (3.7) may receive a contribution also from a potential IR boundary. Because of the confinement property, the space-time ends smoothly in the IR, and the brane-antibrane world-volume must end before or at the IR end of space. The contribution to the gauge variation of the CS action coming from this potential IR boundary would give rise to an additional contribution to the gauge anomaly, and make the bulk theory inconsistent. It must therefore vanish. One is led then to the consistency condition $\tau \to \infty$ at the IR, which leaves us with the UV contribution alone:

\[
\delta_\Lambda S_{\text{CS}} = (-)^p \frac{i N_c}{(2\pi)^3 \alpha'} \int_{M_5} \delta_\Lambda \Omega_5 = \frac{i N_c}{24\pi^2} \int_{\text{Mink}^4} \text{Tr} \Lambda (\eta_L - \eta_R)
\]

(3.17)

where we have used the fact that the tachyon vanishes on the UV boundary and reinserted a $(2\pi\alpha')^3$ factor in (3.15). We have defined the 4-form $\eta$ as

\[
\eta \equiv (-)^{p+1} \left[ F \wedge F + i 2 (A \wedge A \wedge F + A \wedge F \wedge A + F \wedge A \wedge A) - \frac{1}{2} A \wedge A \wedge A \wedge A \right]
\]

(3.18)
Notice that the computation is rather similar to the one in [3], but here we have included the dependence on the tachyon. In fact, we have checked the anomalous behavior under a transformation within the $U(N_f)_V$ symmetry preserved by the vacuum. It is possible to find the anomaly in the full $U(N_f)_L \times U(N_f)_R$ chiral symmetry before it is spontaneously broken. For that purpose, we must consider a more general ansatz in which $T$ is proportional to a unitary matrix $TT^\dagger = T^\dagger T = \tau^2 \mathbb{I}_{N_f}$. With this milder assumption, the generalization of (3.12) reads:

$$
\text{Str} \ e^{iF}_{6-\text{form}} = \frac{1}{6} \left[ \text{Tr} e^{-T^\dagger T} \left\{ -i F_L \wedge F_L \wedge F_L + i F_R \wedge F_R \wedge F_R + DT^\dagger \wedge DT \wedge F_L \wedge F_L + DT^\dagger \wedge F_R \wedge DT \wedge F_L - DT \wedge DT^\dagger \wedge F_R \wedge F_R + \frac{i}{4} DT^\dagger \wedge DT \wedge DT^\dagger \wedge DT \wedge F_L - \frac{i}{4} DT \wedge DT^\dagger \wedge DT \wedge DT^\dagger \wedge F_R + \right. \\
\left. - \frac{1}{60} DT^\dagger \wedge DT \wedge DT^\dagger \wedge DT \wedge DT^\dagger \wedge DT \right\} \right]$$

(3.19)

One should now follow the same procedure as above and generalize the expressions (3.13)-(3.15). However, it proves a difficult task to find the five-form $\Omega_S$. In any case, the IR term will again vanish when $\tau \to \infty$ due to the overall exponential factor. Since in the UV $\tau \to 0$, one can find the UV contribution even without knowing the full expression for $\delta \Omega_S$. The result is of course (3.17) but now with independent parameters for the left and right gauge transformations.

$$
\delta A S_{CS} = \frac{iN_c}{24\pi^2} \int_{\text{Mink}^4} \text{Tr} \left[ \Lambda^L \eta^L - \Lambda^R \eta^R \right] 
$$

(3.20)

Possibly up to a sign, (which is a matter of conventions), (3.20) reproduces the QCD chiral anomaly, as we now check by using the general procedure sketched in [2] and further developed in [39]. The AdS/CFT conjecture [1, 2] states that $S_d[A] = W[A]$ where $W$ is the generating functional for current $(J^A = \frac{\delta W}{\delta A^\mu})$ correlators in the boundary theory. We can equate the gauge variation of both terms in the equality: $\delta A S_d[A]$ is given in (3.20) while $\delta A W[A] = \frac{4W}{s} \delta A^A = \int d^4x D_\mu A^A J^\mu = - \int d^4x A^A (D_\mu J^\mu)^A$. The anomalous divergences of the $U(N_f)_{L,R}$ flavor currents in the presence of sources are, thus:

$$
\partial \mu J^U_{L,R} = (-)^{h_L,R} \frac{N_c}{24\pi^2} \ast \text{Tr} (\eta_{L,R}) \\
\left( D_\mu J^\mu_{L,R} \right)^a = (-)^{h_L,R} \frac{N_c}{24\pi^2} \ast \text{Tr} (\lambda^a \eta_{L,R}) 
$$

(3.21)

Here $\ast$ stands for the four-dimensional Hodge dual, we have defined $h_L$ and $h_R$ to be 0 and 1 respectively, and the currents have been decomposed in their abelian and non-abelian components following the conventions of appendix [4]. Expressions (3.21) reproduce the known QCD results (see for example [40]).

For completeness, we report here the explicit expressions of the traces appearing in equations (3.21). They can be easily derived from the definition (3.18) of $\eta$:

$$
\text{Tr} (\eta) = (-)^{p+1} \left( \frac{1}{2} F^a \wedge F^a + N_f F^{U(1)} \wedge F^{U(1)} - \frac{1}{8} f^{abc} A^a \wedge A^b \wedge F^c \right) 
$$

(3.22)

7Notice the different definition of the gauge field with respect to [3]. $A = i\tilde{A}$, $F = i\tilde{F}$ where $\tilde{A}, \tilde{F}$ are the gauge field and field strength used in [3].
and

\[
\text{Tr}(\lambda^a \eta) = (-)^{p+1} \left[ \frac{1}{2} \varepsilon^{abc} F_b \wedge F_c + F^a \wedge F^{U(1)} - \frac{1}{8} t^{bc} a \left( 3 A^b \wedge A^c \wedge F^{U(1)} - 2 A^{U(1)} \wedge A^b \wedge F^c \right) + \frac{1}{8} \left( f^{ab} e^{cde} - f^{ac} e^{bde} + f^{bc} e^{ade} \right) A^b \wedge A^c \wedge F^d + \frac{1}{16} t^{bc} f^{de} d a b c \right] A^b \wedge A^c \wedge A^d \wedge A^e \right]
\]

(3.23)

3.3 The $U(1)_A$ axial symmetry

The first equation in (3.21) for the anomalous divergence of the abelian flavor currents in the presence of external sources is not complete. In the case of the $U(1)_A$ symmetry, an additional anomaly appears [23].

To study this $U(1)_A$ anomaly, we need to know what bulk field couples to $G \wedge G$ on the boundary (here $G$ denotes the field strength of the non-abelian color gauge field). We consider then a probe $Dq$ color brane. Since we are studying four-dimensional gauge theories, the $D_q$ brane wraps a $(q - 3)$-cycle $K_{q-3}$ of the internal geometry. Typically, for a background dual to a confining color gauge theory with no moduli space nor low-energy massive scalars, the $D_q$-brane embedding is stable only at the boundary, in the far UV region.

The action for this probe brane reads:

\[
S_{\text{probe}} = -T_q \int_{\Sigma_{q+1}} d^{q+1} y e^{-\phi} \sqrt{-\det (P[g] + G)} + T_q \int_{\Sigma_{q+1}} C \wedge \text{Tr} e^{iG}
\]

(3.24)

where $P[g]$ is the pull-back of the background metric on the world-volume $\Sigma_{q+1}$ of the $D_q$-brane, and as before we have set $2\pi \alpha' = 1$. The coupling to $G \wedge G$ we are interested in comes from the expansion of the exponential in the WZ term, and is given by $T_q \frac{\tau_q}{2} \int_{\Sigma_{q+1}} C_{q-3} \wedge \text{Tr}(G \wedge G)$.

Comparison of this term to the usual Yang-Mills action gives the holographic definition for the QCD $\theta$-angle (recall that $p + q = 12$)

\[
\theta_{\text{QCD}} = 4\pi^2 T_{12-p} \int_{K_{9-p}} C_{9-p}
\]

(3.25)

where the integral over $K_{9-p}$ is evaluated in the far UV (the boundary).

To study effects on the dynamics of flavors related to $\text{Tr} G \wedge G$, we will need, therefore, to turn on the RR potential $C_{9-p}$ in the background, and look at its interaction with the flavor branes. Using Hodge duality, this potential has the same degrees of freedom as $C_{p-1}$. The coupling of this potential to the $D_p$-$\overline{D}_p$ flavor branes can then be read from the WZ action (3.1)

\[
S_{WZ} = -i T_p \int_{\Sigma_{p+1}} C_{p-1} \wedge \text{Str} e^{iF}_{2-\text{form}}
\]

(3.26)

If we take $T$ to be proportional to a unitary matrix, $TT^\dagger = \tau^2 \mathbb{1}_N$, as we did at the end of subsection 3.2, it is easy to write from (3.2) an explicit formula for $\text{Str} e^{iF}_{2-\text{form}}$:

\[
\text{Str} e^{iF}_{2-\text{form}} = e^{-\tau^2} \text{Tr}(iF_L - iF_R - DT^\dagger \wedge DT)
\]

(3.27)
The coupling in (3.24) modifies the equation of motion of \( C_{p-1} \), or equivalently the Bianchi identity for the Hodge dual \( C_{9-p} \). Indeed, the terms of the overall action involving \( C_{p-1} \) are

\[
S = -\frac{1}{2\kappa_{(10)}^2} \int \frac{d^4x}{8} \epsilon_{\mu\nu} \partial^\mu F^\nu + i T_p \int_{\Sigma_{p+1}} C_{p-1} \wedge \text{Str} \, e^{iF} \big|_{2\text{-form}},
\]

from which we obtain

\[
dF_{10-p} = d * F_p = 2i\kappa_{(10)}^2 T_p \delta_{9-p} (\Sigma_{p+1}) \wedge \text{Str} \, e^{iF} \big|_{2\text{-form}}
\]

(3.28)

where \( F_{10-p} \) is the gauge-invariant (with respect to world-volume gauge transformations) \((10-p)\)-form RR field strength, and \( \delta_{9-p} (\Sigma_{p+1}) \) is the volume form for the space transverse to the flavor branes times a \( \delta \)-function localized at the position of the flavor branes.

From (3.28) we can write:

\[
F_{10-p} = dC_{9-p} + 2i(-)^{p+1} \kappa_{(10)}^2 T_p \delta_{9-p} (\Sigma_{p+1}) \wedge \Omega_1
\]

(3.29)

where \( \Omega_1 \) is defined by \( d\Omega_1 = \text{Str} \, e^{iF} \big|_{2\text{-form}} \) and can be explicitly obtained from (3.27):

\[
\Omega_1 = e^{-\tau^2} \text{Tr} \left( iA_L - iA_R + (\log T - \log T^\dagger) \tau d\tau \right)
\]

(3.30)

As we show in appendix D, the only gauge transformations under which \( \Omega_1 \) transforms non-trivially are the \( U(1)_A \) axial ones. We thus consider an infinitesimal such transformation \( \Lambda_L = -\Lambda_R = i\alpha \Pi_{N_f} \):

\[
\delta \Omega_1 = 2iN_f e^{-\tau^2} (d\alpha - \alpha d(\tau^2)) = d \left( 2iN_f e^{-\tau^2} \alpha \right) = d\omega_0
\]

(3.31)

where we defined \( \omega_0 = 2iN_f e^{-\tau^2} \alpha \).

Since \( F_{10-p} \) is gauge invariant, and as we just showed \( \delta \Omega_1 = d\omega_0 \), it follows from (3.29) that the RR-potential cannot be invariant under flavor brane world-volume gauge transformations [38]:

\[
\delta \Omega_1 = -2i\kappa_{(10)}^2 T_p \delta_{9-p} (\Sigma_{p+1}) \omega_0,
\]

(3.32)

which, inserted in (3.28) and using the identity \( 8\pi^2 T_{12-p} T_p \kappa_{10}^2 = 1 \) (in units of \( 2\pi\alpha' = 1 \)), results in the following transformation relation for the QCD theta angle under \( U(1)_A \) transformations:

\[
\delta \theta_{QCD} = -i \omega_0 |_{UV} = 2N_f \alpha.
\]

(3.33)

Because of the boundary coupling \( \int d^4x \frac{\theta_{QCD}}{8\pi^2} \text{Tr}(G \wedge G) \), the formula (3.21) for the divergence of the \( U(1)_A \) current \( J^{U(1)}_A \) has to be corrected in order to take into account the non-trivial transformation of \( \theta_{QCD} \) (the \( U(1)_V \) current \( J^{U(1)}_V \) is unaffected):

\[
\partial_\mu J^{U(1)}_A = \frac{N_c}{24\pi^2} \text{Tr} (\eta_L + \eta_R) + \frac{N_f}{16\pi^2} \epsilon^{\mu_1 \cdots \mu_4} \text{Tr}_{SU(N_c)} (G_{\mu_1\mu_2} G_{\mu_3\mu_4})
\]

(3.34)

\(^8\)The following formula has been computed for the case where all the quarks have the same (possibly vanishing) mass, which is the situation considered in this paper. Since in section 4.4 we will relate \( \Omega_1 \) to the would-be Goldstone boson \( \eta' \), we should expect that in a more general case \( \text{Tr}(\log T - \log T^\dagger) \) will be substituted by an expression of the form \( \log(\det T(T^\dagger)^{-1}) \). We will not pursue this computation here.
3.4 A holographic view of the Coleman-Witten theorem

In [22], Coleman and Witten proved that, under a few assumptions, in $N_c \to \infty$ massless QCD, chiral symmetry is spontaneously broken $U(N_f)_L \times U(N_f)_R \to U(N_f)_V$.

One of the main results of the previous section is that, if the theory is confining, the tachyon has to get a (diverging) vev near the IR even if $T \to 0$ in the UV. Since $T$ transforms in the bifundamental representation of the flavor group, $\langle T \rangle \neq 0$ means that the chiral symmetry is broken. We showed in section 2.2 that $\langle T \rangle$ is proportional to the identity matrix, and consequently the flavor symmetry is broken down to $U(N_f)_V$. Confinement, therefore, implies spontaneous chiral symmetry breaking, reproducing in a holographic language the result of [22].

To make the analogy between these two different approaches more evident, we can compare the assumptions of [22] with those we made in the present holographic setup. In order to do this, we start by reviewing the five assumptions of [22]: 1) The large $N_c$ limit of QCD exists. 2) In that limit the theory is confining. 3) There is a single order parameter for the breaking that is a quark bilinear and transforms in the bifundamental of $U(N_f)_L \times U(N_f)_R$. 4) Its vev can be found by minimizing some potential. 5) The potential is assumed not to have degenerate minima.

For the holographic setup, assumptions 1) and 2) are also necessary, and of course, one must further assume the (non-trivial) fact of the existence of a holographic string dual of large $N_c$ QCD which fits in the general framework we have introduced. It is appealing that assumption 3) is automatic in the setup of our work since it is a well established fact that the lowest state of a brane-antibrane system is the open string tachyon. Concerning assumption 4), the string theory dual provides, in principle, a way to determine the vev of the quark bilinear: the bulk vev of the tachyon field is determined by solving the bulk field equations with the UV boundary condition corresponding to $m_q = 0$ (2.9) and the IR consistency condition $\tau|_{IR} \to \infty$. Then, one can read the value of the quark bilinear vev $\sigma$ from the UV behavior of $\tau(z)$ (2.9). This computation requires the DBI part of the action, and is similar to that in [3, 4]. It was discussed in section 2.2.

In principle, one expects that the UV and IR conditions determine uniquely the bulk vev of the tachyon and therefore the $\langle \bar{q}q \rangle$ condensate of the dual theory. Nevertheless, in the very general setup we are considering, we cannot prove that the solution for the $\langle \bar{q}q \rangle$ is indeed unique, so this must be taken as a further assumption, analogous to the non-degeneracy 5) listed above.

We remind the reader that in section 2.2 we also made the (reasonable) assumption that the stable minimum given by the vacuum (2.10) is the real vacuum of the theory.

The demonstration presented in this paper, namely that confinement is a sufficient condition for spontaneous chiral symmetry breaking, can be thought of as a reformulation of the geometrical picture of [9]. There it was argued that if one places branes and antibranes in a background that smoothly ends at some point of the radial coordinate (and therefore is confining), they must necessarily recombine in the IR, sparking the breaking of chiral symmetry. The advantage of the present formulation stands in the fact that the tachyon, the scalar responsible for the symmetry breaking, is explicitly taken into account which,
for instance, allows to introduce a bare quark mass. Notice that, if one is in the conformal window of $\mathcal{N} = 1$ or $\mathcal{N} = 0$ QCD, then there is no IR boundary and it is consistent to take a vanishing vev for the tachyon as in [24, 19]. However, as in [22] or [9], our argument does not rule out having spontaneous chiral symmetry breaking without confinement.

### 3.5 The effects of a non-trivial quark mass

In [39], it was explained how an expression like (3.21) can be related to the anomalous three-point function in the case of $\mathcal{N} = 4$ SYM where the global symmetry associated to the current $J$ is $SU(4)_R$. Analogously, in our case one can relate (3.21) to the anomalous massless QCD three-point function. This anomaly equation in the large $N_c$ limit was used by Coleman and Witten [22] to prove the existence of massless Goldstone bosons, and therefore the spontaneous breakdown of the symmetry $U(N_f)_L \times U(N_f)_R \rightarrow U(N_f)_V$. Clearly, if there is a non-zero bare mass for the quarks, there cannot be spontaneous breaking nor massless Goldstone bosons, so (3.21) must be modified when $m_q \neq 0$. In section 4.3, we will show by analyzing the equations that define the mesonic spectrum that there are massless Goldstones if and only if $m_q = 0$. The goal of this section is to describe holographically the modification of (3.21).

The axial current is classically non-conserved when $m_q \neq 0$. In the holographic picture this shows up because a gauge transformation of the bulk gauge fields is accompanied by a gauge transformation of the tachyon (A.6), which contributes to (3.21) iff $m_q \neq 0$. In the rest of this section we make these statements more precise, assuming that the space is asymptotically AdS:

\[
g_{xx}(z) \simeq R_{\text{AdS}}^2/z^2 + \ldots \quad g_{zz}(z) \simeq R_{\text{AdS}}^2/z^2 + \ldots \quad (3.35)
\]

The antihermitian part of the tachyon matrix $\frac{T_T - T_T^*}{2}$ is dual to ($-i$ times) the pseudoscalar current, see table 1. We consider a perturbation around the vacuum $T = \tau e^{i\theta}$ where $\theta$ is proportional to the (hermitian) pion matrix, $\theta \propto \sqrt{2N_f} \mathbb{I} + \pi^a \lambda^a$. At first order, $T^2 - T_T^* = i\tau \theta$. There must exist a boundary coupling contributing in the $W$ generating functional $\int \! d^4x \left( -i \phi^A_0 J^A_5 \right)_{z=0}$, where $J_5$ is the pseudoscalar (pion) current $J_5 = i\bar{q}\gamma^5 q + i\bar{q}\gamma^5 \lambda^a q$ and $\phi_0$ is proportional to $i\tau \theta$. As argued in section 2.2, we require that the mass of the tachyon, at least near the boundary, is $m_T^2 R_{\text{AdS}}^2 = -3$ since $\bar{q}q$ and $i\bar{q}\gamma^5 q$ are operators of dimension 3. Following Witten’s prescription [2], one has to couple to the pseudoscalar current in the boundary, $\phi_0 = R_{\text{AdS}}^2 \frac{1}{z} i\tau \phi_{\text{can}}$, since for a scalar of the above cited mass, the non-normalizable behavior is $\sim z$. Inserting the value of $\tau_{\text{can}}$ given in (2.3), the contribution to the generating functional is:

\[
\int \! d^4x \left( m_q \theta^A J^A_5 \right) \quad (3.36)
\]

We now compute how this expression transforms under an axial gauge transformation $\Lambda_L = -\Lambda_R = \Lambda = i\Lambda^A \lambda^A = i\alpha \mathbb{I} + i\Lambda^a \lambda^a$. From (A.6), the infinitesimal gauge transformation $V \equiv e^{\alpha A}$ of the tachyon around the vacuum (2.10) is given by $\delta_A T = \Lambda_RT - T\Lambda_L = \tau (\Lambda_R - \Lambda_L) = -2\Lambda \tau$, which in turn can be expressed as a transformation of the pion matrix $\theta$.
as: $\delta \chi^A = (2i\Lambda)^A = -2A^A$, which, inserted in (3.36), yields \[ \int d^4x \, m_q \left(-2\alpha J^{(1)}_5 - 2\Lambda^a J^a_5\right). \]

This leads to a modification of (3.21) and (3.34):

\[ \partial^\mu J^{(1)}_5 A^\mu = -2m_q J^{(1)}_5 A^\mu + \frac{N_c}{24\pi^2} \epsilon^\mu_1...\epsilon^\mu_4 \text{Tr}_{SU}(N_c)(G_{\mu_1\mu_2} G_{\mu_3\mu_4}) \]

\[ (D^\mu J^a_5) = -2m_q J^a_5 A^\mu + \frac{N_c}{24\pi^2} \epsilon^\mu_1...\epsilon^\mu_4 \text{Tr}(\lambda^a(\eta_L + \eta_R)) \]

(3.37)

One can now relate these expressions to the three-point function in the presence of a non-trivial quark mass. The term with $m_q$ in (3.37) results in an extra term for the three-point function. The presence of this new term invalidates the argument that Coleman and Witten used in the $m_q = 0$ case to show that the three-point function has a pole at zero momentum. Thus, as expected, the three-point function can be analytic at zero momentum, consistent with the absence of massless Goldstone bosons.

4. Features of the mesonic mass spectrum

4.1 Equations determining the spectrum

We now study small fluctuations of the different fields around the vacuum configuration. They correspond to the mesons of the field theory. We will just consider quadratic terms in the action, which are enough to find the mass spectrum, but not the couplings. At this level, the non-abelian action (2.4) is just the sum of $N^2_f$ copies of the abelian one (2.1). Thus, we use (2.1) in the following and there are $N^2_f$ copies of each of the towers of resonances that we will study.

We start by considering excitations for the fields $\theta, A^L, A^R$ around the vacuum of section 2.4. Fluctuations of the modulus of the tachyon $\tau$ will be briefly mentioned below. We expand the simplified version of (2.1) (neglecting the scalars $Y^I$ and assuming there is no $B$-field). We define:

\[ e^{-\tilde{\phi}} = e^{-\phi_{\text{eff}} V(\tau^2)}, \quad \tilde{g}_{zz} = g_{zz} + \frac{2}{\pi}(\partial_z \tau)^2 \]

(4.1)

In order to use a notation similar to the one in [13], we choose a gauge:

\[ A^L_z = A^R_z = 0 \]

(4.2)

and define:

\[ V_M = \frac{A^L_M + A^R_M}{2}, \quad A_M = \frac{A^L_M - A^R_M}{2}, \quad v = \frac{2\sqrt{2}}{\sqrt{\pi}} \tau, \]

(4.3)

and $V_{\mu\nu}, A_{\mu\nu}$ as the (abelian) field strengths of $V_M, A_M$. Expanding (2.1), we find:

\[ S = -\int d^4x \, d\tau \, e^{-\tilde{\phi}} \left[ \frac{1}{2}g_{zz} (V_{\mu\nu} V^{\mu\nu} + A_{\mu\nu} A^{\mu\nu}) + g_{xx} \tilde{g}_{xx}^{-\frac{1}{2}} (\partial_x V_\mu)^2 + (\partial_x A_\mu)^2 + \frac{1}{4}g_{xx} \tilde{g}_{zz}^{-\frac{1}{2}} v^2 (\partial_x \theta)^2 \right] + \frac{1}{4}g_{xx} \tilde{g}_{zz}^{-\frac{1}{2}} v^2 (\partial_x \theta)^2 \]

(4.4)
Vector sector

The vector (1−−) sector is decoupled from the rest in the action (4.4). The equation of
motion for $V_\mu$ can be solved by expanding $V_\mu$ in modes:

$$V_\mu = \sum_n \psi_n(z) V_\mu^{(n)}(x^\mu)$$  \hspace{1cm} (4.5)

with:

$$\partial_z (e^{-\phi} g_{xx} g_{zz}^{1/2} \partial_z \psi_n(z)) + m_n^2 e^{-\phi} g_{zz}^{1/2} \psi_n(z) = 0$$  \hspace{1cm} (4.6)

The physical modes are those which yield a finite action when integrating in $z$. Substituting
(4.5) in (4.4), one finds a tower of massive vectors:

$$S = -\int d^4x \sum_n \left[ \frac{1}{2} V_\mu^{(n)} \gamma_\mu^{(n)} + m_n^2 V_\mu^{(n)} V^{(n)}_\mu \right]$$  \hspace{1cm} (4.7)

provided the normalization conditions:

$$\int_0^{z_{1R}} dz e^{-\phi} g_{xx} g_{zz}^{1/2} (\partial_z \psi_n(z))^2 = \frac{1}{2}$$

$$\int_0^{z_{1R}} dz e^{-\phi} g_{xx} g_{zz}^{1/2} (\partial_z \psi_n(z))^2 = m_n^2$$  \hspace{1cm} (4.8)

Integrating by parts the second expression and using (4.6), we consistently obtain the first
line of (4.8).

Axial vector sector

The vector field fluctuation $A_\mu$ can be split in a transverse and a longitudinal part,
$A_\mu = A_\mu^\perp + A_\mu^\parallel$, with $\partial^\mu A_\mu^\parallel = 0$. We first consider the transverse part, corresponding to 1−− excitation. We expand it as:

$$A_\mu^\perp = \sum_n A_\mu^{(n)}(z) B_\mu^{(n)}(x^\mu)$$  \hspace{1cm} (4.9)

which results in a tower of massive axial vectors:

$$S = -\int d^4x \sum_n \left[ \frac{1}{2} B_\mu^{(n)} B_\mu^{(n)} + m_n^2 B_\mu^{(n)} B^{(n)}_\mu \right]$$  \hspace{1cm} (4.10)

subject to the normalization conditions:

$$\int_0^{z_{1R}} dz e^{-\phi} g_{xx} g_{zz}^{1/2} (\partial_z A_\mu^{(n)})^2 = \frac{1}{2}$$

$$\int_0^{z_{1R}} dz \left[ e^{-\phi} g_{xx} g_{zz}^{1/2} (\partial_z A_\mu^{(n)})^2 + e^{-\phi} g_{xx} g_{zz}^{1/2} (\partial_z A_\mu^{(n)})^2 \right] = m_n^2$$  \hspace{1cm} (4.11)

9 We have chosen this normalization such that when the fields are promoted to their non-abelian
generalization $V_\mu^{(n)} \rightarrow V_\mu^{(n)} v^a \lambda^a$, the components $V_\mu^{(n)} v^a$ are normalized in the standard way, since we are using
$\text{Tr} \lambda^a \lambda^b = \frac{1}{2} \delta^{ab}$. We apply this prescription to all the different towers of modes.
and the second order differential equation:

\[ \partial_z (e^{-\tilde{\phi}} g_{xx} \tilde{g}_{zz}^{\frac{1}{2}} \partial_z A^\perp_{(n)}) + m_n^2 e^{-\tilde{\phi}} \tilde{g}_{zz}^{\frac{1}{2}} A^\perp_{(n)} - e^{-\tilde{\phi}} g_{xx} \tilde{g}_{zz}^{\frac{1}{2}} v^2 A^\perp_{(n)} = 0 \] (4.12)

It is interesting to notice at this point a difference with respect to constructions like the Sakai-Sugimoto model \[7\]. In that kind of models, the vector and axial vector mesons are solutions of a single second order differential equation with different matching conditions at the IR. In this case, they satisfy different equations (4.6), (4.12). If the chiral symmetry were not broken \((v = 0)\), the equations would become degenerate and vector and axial vector mesons would have the same mass spectrum due to the unbroken symmetry. The differential equations found in this construction coincide with those of the AdS/QCD models \[13, 14\] up to the \(z\)-dependent dilaton, metric factors and tachyon vev, which we have kept general.

**Pseudoscalar sector**

The modes coming from \(\theta\) and the longitudinal part of \(A_\mu\) combine to give a single tower of resonances with the quantum numbers of pions \(0^{-+}\). The equations of motion from (4.4) can be solved by expanding:

\[
A^\parallel_\mu = -\sum_n \varphi_{(n)}(z) \partial_\mu (\alpha^{(n)}(x^\nu)) \\
\theta = 2 \sum_n \vartheta_{(n)}(z) \alpha^{(n)}(x^\nu) \tag{4.13}
\]

where the functions \(\varphi_{(n)}\) and \(\vartheta_{(n)}\) satisfy the following coupled differential equations

\[
\partial_z (e^{-\tilde{\phi}} g_{xx} \tilde{g}_{zz}^{\frac{1}{2}} \partial_z \varphi_{(n)}) + e^{-\tilde{\phi}} g_{xx} \tilde{g}_{zz}^{\frac{1}{2}} v^2 (\vartheta_{(n)} - \varphi_{(n)}) = 0 \\
g_{xx} v^2 \partial_z \vartheta_{(n)} - m_n^2 \partial_z \varphi_{(n)} = 0 \tag{4.14}
\]

Inserting (4.13) in (4.4) and normalizing:

\[
\int^{z_{IR}}_0 dz \left[ e^{-\tilde{\phi}} g_{xx} \tilde{g}_{zz}^{\frac{1}{2}} (\partial_z \varphi_{(n)})^2 + e^{-\tilde{\phi}} g_{xx} \tilde{g}_{zz}^{\frac{1}{2}} v^2 (\vartheta_{(n)} - \varphi_{(n)})^2 \right] = 1 \tag{4.15}
\]

\[
\int^{z_{IR}}_0 dz \left[ e^{-\tilde{\phi}} g_{xx} \tilde{g}_{zz}^{\frac{1}{2}} v^2 (\partial_z \vartheta_{(n)})^2 \right] = m_n^2 \tag{4.16}
\]

one finds a tower of pseudoscalars in the four-dimensional theory:

\[
S = -\int d^4 x \sum_n \left[ (\partial_\mu \alpha^{(n)})^2 + m_n^2 (\alpha^{(n)})^2 \right] \tag{4.17}
\]

**Scalar sector**

The scalar mesons \((0^{++})\) come from excitations around the vacuum of the form \(\delta \tau = S(x_\mu, z)\). Expanding (2.6) up to quadratic order in \(S\), one finds an action:

\[
S = -2 \int d^4 x dz e^{-\tilde{\phi}} \left[ \frac{1}{2} \frac{\partial^2 V}{\partial \tau^2} \bigg|_{\tau=v/2} g_{xx} \tilde{g}_{zz}^{\frac{1}{2}} \frac{1}{2} \left( \partial_z S \right)^2 + \frac{\sqrt{2}}{2 \sqrt{\pi}} \frac{\partial_z V}{\partial \tau} \bigg|_{\tau=v/2} \tilde{g}_{xx} \tilde{g}_{zz}^{\frac{1}{2}} (\partial_z v) S (\partial_z S) + \frac{1}{4} \sqrt{\frac{2}{\pi}} \frac{\partial^2 V}{\partial \tau^2} \tilde{g}_{xx} \tilde{g}_{zz}^{\frac{1}{2}} (\partial_z v)^2 \right] + \frac{1}{4} (\partial_z v)^2 (\partial_z S)^2 + \frac{1}{2} g_{xx} \tilde{g}_{zz}^{\frac{1}{2}} (\partial_\mu S)^2 \tag{4.18}
\]
from which one can straightforwardly extract the linear equation of motion and normalizability condition. Since we will not need them in the following, we do not present them explicitly.

4.2 On “linear confinement” (for highly excited mesons)

In a theory with linear confinement such as QCD, one expects that the squared masses $m_n^2$ of highly excited hadrons grow linearly with the excitation number $n$: $m_n^2 \propto n$. This behavior is difficult to find from holographic models, that typically yield $m_n^2 \propto n^2$. In [16] it was shown that, by appropriately tuning the behavior of the metric and dilaton in the IR, one can indeed find the expected behavior. Moreover, it was conjectured that such behavior could come from closed string tachyon condensation, but no concrete model producing such IR asymptotics of metric and dilaton has been found.

We will show here that the relation $m_n^2 \propto n$ is automatic in our construction. Therefore, the linear confinement relation for mesons comes from open string tachyon condensation on the flavor brane world-volume. In this section we just state the ideas and results while details are relegated to appendix E.

The argument relies on the fact that the meson excitations feel the effective open string dilaton and metric (see eq. (4.1)) rather than just the closed string dilaton and metric. In the IR, $\tau \to \infty$ so $\tilde{g}_{zz} = g_{zz} + \frac{2}{\pi} (\partial_z \tau)^2 \approx \frac{2}{\pi} (\partial_z \tau)^2$ even if $g_{zz}$ has a single pole. Moreover, $g_{xx}$ goes to a constant, which, on general grounds, can be identified with the QCD string tension. Reinserting $\alpha'$ and defining a new radial variable $u = \sqrt{\frac{2}{\pi T_{QCD}}} \tau(z)$, we have, near the IR, where $u, \tau \to \infty$:

$$ds^2 = 2\pi \alpha' \left( T_{QCD} dx_1^2 + \frac{2}{\pi} (\partial_z \tau)^2 dz^2 \right) = 2\pi \alpha' T_{QCD} \left( dx_{1,3}^2 + du^2 \right)$$

$$e^{-\tilde{\phi}} \sim V(\tau^2) \sim e^{-\frac{\pi T_{QCD}}{2} u^2} \Rightarrow \tilde{\phi} \sim \frac{\pi T_{QCD}}{2} u^2 \quad (4.19)$$

It turns out that the natural radial variable in the IR is proportional to $\tau$. The quadratic growth of the dilaton (4.19) is the IR behavior advocated in [16] to account for linear confinement. In fact, using the WKB approximation to compute the masses of highly excited bound states one obtains, for the vector meson tower (see appendix E):

$$m_n^2 \approx 2\pi T_{QCD} n \quad \text{(large } n, \text{ vector mesons)} \quad (4.20)$$

This is a quite general result that only relies on having a confining background and therefore diverging tachyon, on the large $\tau$ gaussian behavior of the tachyon potential and on the DBI action. We find very encouraging that our construction naturally accounts for the physically expected relation of linear confinement, including the correct multiplicative factor.

However, there is an important caveat: if one repeats the same computation for axial vector mesons, eq. (4.12), one finds:

$$m_n^2 = 2\pi \sqrt{1 + \frac{16}{\pi^2} T_{QCD} n} \quad \text{(large } n, \text{ axial vectors)} \quad (4.21)$$
This cannot be physically correct since vector and axial mesons should asymptotically have equal masses at large $n$, as chiral symmetry is restored for excited hadrons [42]. The problem may arise from the fact that we are using the DBI action outside its range of validity. This point definitely deserves a better understanding.

Finally, it is important to stress that in view of the confining IR behavior, there is no need to impose an arbitrary IR boundary condition. The IR condition for physical excitations is simply the normalizability of the action [16].

4.3 Goldstone bosons

Large $N_c$ QCD with massless quarks has a set of $N_f^2$ massless pseudoscalars which are the Goldstone bosons of the spontaneous breaking of the $U(N_f)_A$. We will generically call them pions. In this section we show how they appear in this construction$^{10}$. They are the solutions to equations (4.14)-(4.16) with $m_n = 0$. Notice that, as expected, there cannot be massless (axial) vectors due to (4.8), (4.11).

We generalize the analysis in (4.13)-(4.17) with $m_n = 0$ to the non-abelian $N_f > 1$ case. Define the pion matrix as the generalization of the $\alpha(0)(x)$ in (4.13):

$$\pi(x) = \frac{\eta'(x)}{\sqrt{2N_f}} I + \pi^a(x) \lambda^a$$

(4.22)

To lowest order, the action for the pions is just the one for a set of massless scalars:

$$S = - \int d^4x \text{Tr} (\partial_\mu \pi \partial^\mu \pi) = - \int d^4x \left( \frac{1}{2} \partial_\mu \eta' \partial^\mu \eta' + \frac{1}{2} \partial_\mu \pi^a \partial^\mu \pi^a \right)$$

(4.23)

Due to (4.14), $\vartheta(0)$ must be a constant. We will shortly see that it is related to the pion decay constant. We define a quantity:

$$\xi(z) = \varphi(0)(z) - \vartheta(0)$$

(4.24)

which, regarding (4.14), (4.15), satisfies:

$$\partial_z (e^{-\tilde{\phi}} g_{xx} \tilde{g}_{zz} \frac{1}{4} \partial_z \xi(z)) - e^{-\tilde{\phi}} g_{xx} \tilde{g}_{zz} \frac{1}{4} \xi(z) = 0,$$

$$\partial_z \xi(z) \bigg|_{z=0} = 1$$

(4.25)

(4.26)

For the second line, we have integrated by parts (4.15) and taken into account that for $\xi(z)$ we must choose the normalizable mode in the IR.

The question of whether there are massless modes boils down to the existence of a function $\xi(z)$ satisfying (4.25), (4.26). From (1.20), it is obvious that the answer depends on the UV behavior of $\xi(z)$, which, due to (4.25) depends on the UV behavior of $v = 2\sqrt{\frac{2}{\pi}} \tau$.

We now give a heuristic argument suggesting that if quarks are massless, there are indeed massless Goldstone bosons. Since $\tau$ is dual to $\bar{q}q$, the UV behavior of $\tau$ is related to the quark mass and condensate. If there is no quark mass, $\tau$ is normalizable, i.e. it vanishes

$^{10}$One of the $N_f^2$ pseudoscalars is massless only in the strict $N_c \to \infty$ limit. This is described in section 4.4.
fast enough in the UV. It is natural to think then that the second term in (4.25) can be neglected near the UV, allowing for a solution to (4.25) and (4.26), which is, asymptotically,

$$\xi(z) = -\vartheta(0) + \vartheta^{-1}(0) f(z)$$  \hspace{1cm} (4.27)

The constant $\vartheta(0)$ has to be determined by the IR condition, we have defined $f(z)$ such that

$$\partial_z f(z) = (e^{-\tilde{\phi}} g_{xx} g_{zz}^{-\frac{1}{2}})^{-1}$$

and used the residual gauge invariance to fix $\varphi(0) = 0$ and therefore $f(0) = 0$. If the quarks are, instead, massive, $\nu$ is larger in the UV and the second term in (4.25) cannot be neglected, which hinders the existence of a solution as the one just described corresponding to a massless Goldstone.

Just as a clarifying example, we consider a five-dimensional space which is asymptotically AdS. In this case, all expressions asymptote in the UV to those of $[13]$, where one can explicitly check that the heuristic reasoning above is valid.

The pion decay constant

Consider $m_q = 0$ so that there are indeed massless pions. The pion decay constant can be related to the pole of the axial current two-point function at zero momentum:

$$\Pi_A(q^2) = \sum_n \frac{f^2_{A_n}}{q^2 + M^2_{A_n}} + \frac{f^2_\pi}{q^2}$$  \hspace{1cm} (4.28)

One can compute this correlator following the AdS/CFT prescription, as in $[13, 14]$. This is done by computing the on-shell action giving the appropriate boundary condition to the field dual to the axial current, i.e. $A^\perp_\mu$. When $q^2 = 0$, the bulk equation for $A^\perp_\mu(z)$ is (4.12):

$$\partial_z (e^{-\tilde{\phi}} g_{xx} g_{zz}^{-\frac{1}{2}} \partial_z A^\perp_{(n)}) - e^{-\tilde{\phi}} g_{xx} g_{xx}^{\frac{1}{2}} v^2 A^\perp_{(n)} = 0$$  \hspace{1cm} (4.29)

Substituting in (4.4) and deriving twice with respect to $B^{(n)}_\mu$ (as defined in (4.9)), one gets:

$$f^2_\pi = \lim_{\epsilon \to 0} \left( - e^{-\tilde{\phi}} g_{xx} g_{zz}^{-\frac{1}{2}} A^\perp(z) \partial_z A^\perp(z) \right)$$  \hspace{1cm} (4.30)

where $A^\perp(z)$ is a solution to eq (4.29) subject to the UV condition $A^\perp|_{z=\epsilon} = 1$ and the IR normalizability condition. Notice there is an extra factor of 1/2 in (4.30) with respect to what one would get from (4.4). This is again because we use conventions suitable for the non-abelian generalizations of the fields, see footnote 9.

Two remarks are in order: even if the $f_\pi$ in (4.30) depends on quantities at $z = 0$, the value of $f_\pi$ depends on the full metric, since one has to select the well behaved IR mode. This typically involves a non-trivial numerical integration. Notice also that $f^2_\pi$ is of order $N_c$ since inside $e^{-\tilde{\phi}}$ there is a D-brane tension which scales as $g^{-1}_s \sim N_c$.

Comparing (4.25), (4.26), (4.29), (4.30) and using the fact that both $\xi(z)$ and $A^\perp(z)$ must follow the normalizable behavior in the IR, it is straightforward to conclude that up to an unimportant sign, $\xi(z) = -f_\pi^{-1} A^\perp(z)$. This equality at $z = 0$ yields:

$$\vartheta(0) = \frac{1}{f_\pi}$$  \hspace{1cm} (4.31)
The Gell-Mann-Oakes-Renner relation

We now show how, considering asymptotically AdS, one can obtain the GOR relation \([43]\), which gives the masses of the pions when the quark mass is small but non-vanishing. The argument is very similar to the one in \([13]\) but details are different. In the following we assume \(m_q \neq 0\) but only keep terms linear in \(m_q\).

We have to solve (4.14)-(4.16) for small \(m_n \equiv m_\pi\). This solution can be obtained as a perturbation of the \(m_n = 0\) case studied at the beginning of this section. As opposed to that case, \(\vartheta\) cannot be a constant and, from (4.16), one can show that \(\vartheta|_{z=0} = 0\) is needed. Then, using (4.14):

\[
\vartheta(z) = m_\pi^2 \int_0^z dz \frac{\partial_z \varphi(z)}{g_{xx} v^2} = m_\pi^2 f_\pi^2 \int_0^z dz \frac{z^3}{(m_q z + \sigma z^3)^2}
\]

For the second equation we have used that the integral is dominated by the small \(z\) region so we can substitute the asymptotic value of the different functions. To obtain \(\partial_z \varphi(z)\) we have used its value in (4.27) and for \(v^2\) we have substituted (4.3) and (2.9). As a consistency check, notice that the region where the integral above has significant support is around \(z \sim \sqrt{m_q/\sigma}\), so taking \(m_q\) small enough, the integral only probes the asymptotically AdS region.

For \(z \gg \sqrt{m_q/\sigma}\), the function \(\vartheta(z)\) goes to a constant which has to be the one of the massless case (4.31). Putting everything together, we get the known expression:

\[
m_\pi^2 = \frac{4m_q \sigma}{f_\pi^2} = \frac{2m_q \langle \bar{q}q \rangle}{f_\pi^2}, \quad (m_q \to 0)
\]

where we have substituted (B.10).

4.4 The mass of the \(\eta'\)

We now return to the \(m_q = 0\) case, where, in principle there are \(N_f^2\) massless Goldstones. In section 3.3 it was shown how our setup correctly reproduces the \(O(N_f)\) anomaly of the \(U(1)_A\) axial symmetry \([23]\). This anomaly implies that the (generalization of the) \(\eta'\) meson, the would-be Goldstone boson corresponding to the diagonal \(U(1)_A\) subgroup of the spontaneously broken \(U(N_f)_A\), has a mass of order \(N_f N_c\), and is therefore massless only in the strict \(N_c \to \infty\) limit.

The \(\eta'\) mass appears in the present holographic setup via a Stuckelberg mechanism. The reasoning we will follow is very similar to the one of \([7]\) (see \([3, 44]\) for related work and \([45]\) for a recent discussion in deconfined theories with broken chiral symmetry).

The \(C_{9-p}\) can only appear in the action in the gauge invariant combination (3.29). Integrating this expression in the space composed of the \(9 - p\) non-Minkowski dimensions that the color branes wrap plus the radial direction we obtain:

\[
(2\pi)^2 T_{12-p} \int_{\mathcal{M}_{10-p}} \frac{F_{10-p}}{2} = \theta_{QCD} + i(-)^{p+1} \int (\Omega_1)_{z} dz = \theta_{QCD} + (-)^p \frac{\sqrt{2N_f}}{f_\pi} \eta'. \quad (4.34)
\]

We have used (3.23) and \(\int_{\mathcal{M}_{10-p}} dC_{9-p} = \int_{K_{9-p}} C_{9-p}\) where the last integral is evaluated in the UV. For a 3+1 confining theory defined on \(Dq\)-branes with \(q \geq 4\) as in \([33, 46]\) this
equality holds automatically because the space closes off smoothly at the IR and \( K_{9-p} \) is the boundary of \( M_{10-p} \). On the other hand, if one wants to build some kind of five-dimensional model in the spirit of \([13, 14, 15, 16, 17]\), one would have to impose that the RR-potential vanishes in the IR boundary to find the same condition.

The integral in (4.34) is computed substituting \( T = \tau e^{i\theta} = \tau e^{i2\theta} = \tau e^{i\frac{2\pi(x^\nu)}{f_\pi}} \) in (3.30):

\[
\int (\Omega_1)_{x,z} = \int e^{-\tau^2} \text{Tr} \left( \left( \log T - \log T^\dagger \right) \tau d\tau \right) = \frac{4i}{f_\pi} \text{Tr}(\tau) \int_0^\infty e^{-\tau^2} \tau d\tau = i\sqrt{2N_f} \eta' \tag{4.35}
\]

The contribution of \( \theta_{QCD} \) to the vacuum energy density can be computed by integrating the kinetic term of the RR \( C_{9-p} \)-form \([17]\), and must appear through the gauge invariant combination (4.34). Thus:

\[
S = -\frac{\chi}{2} \int d^4x \left( \theta_{QCD} + (-)^H \sqrt{2N_f} \eta'/f_\pi \right)^2 \tag{4.36}
\]

where the topological susceptibility of the vacuum is, by definition, the second derivative of the vacuum energy density in the glue theory without flavors \( \chi = \left( \frac{d^2 E}{d\theta_{QCD}} \right)_{\text{no quarks}} \). It can be computed from the supergravity action \([17]\). The expression (4.36) reproduces the Veneziano-Witten formula for the mass of the \( \eta' \) [18] \(^{11}\):

\[
m_{\eta'}^2 = \frac{2N_f}{f_\pi^2} \chi \tag{4.37}
\]

5. Comments and discussion

Compared to the models \([3] ([4])\) where flavors where introduced with D7 (D6)-branes in the background of D3 (D4)-branes, our construction has the advantage that the breaking of the full non-abelian chiral symmetry is described. In fact, one may think of the models presented in those papers as the result of a \( Dp \)-\( \overline{Dp} \) system in which the tachyon has fully condensed leaving a \( D(p-2) \) brane as a vortex. Since a non-trivial tachyon breaks the \( U(N_f) \), this is not present in the final configuration and only a remnant \( U(1)_A \) is left as the rotation symmetry around the vortex (for a related discussion, see \([18]\)).

The model of Sakai and Sugimoto \([7, 8]\) and its generalizations share many properties with our setup, but there are also important differences. In \([7, 8]\), it does not seem possible to include a bare mass for the quark, so clearly there are aspects of the QCD symmetry breaking that are not present in that setup. Another phenomenon that we have shown to be intimately related to the physics of the tachyon is the existence of massive resonances with the quantum numbers of pions \( 0^{-+} \). These are not present in the model of \([7, 8]\). A heuristic way of understanding the differences is that the description of \([7, 8]\) is done already in the broken phase (with the brane-antibrane pair reconnected), so that information about

\(^{11}\)The difference in a factor of 2 with respect to \([18]\) is due to a factor \( \sqrt{2} \) in the definition of \( f_\pi \). In our conventions \( \langle 0 \left| J_{(1)}^{(1)^A} \right| \eta' \rangle = \sqrt{2N_f} \eta' f_\pi \) and \( \eta' = f_\pi \) up to suppressed \( O(N_c^{-1}) \) corrections.
the order parameter is lost. It would be interesting to try to include the tachyon in such
type of models, generalizing the formalism of this paper to non-overlapping brane-antibrane
with excited transverse scalars. For some progress in this direction, see [11].

In a sense, the model presented in this paper can be thought of as a string theory
construction in which the AdS/QCD models [13, 14] are embedded. We have seen that the
five-dimensional field spectrum is the same. Expanding the square roots for small values
of \( A_{M} \) and \( T \), one gets the same kind of action as in [13, 14]. In fact, the equations to
determine the spectrum (4.6), (4.12), (4.14) reduce to those in [13, 14] if one considers AdS
metric and constant dilaton. On the other hand, the equation for the scalar mesons (4.18)
is different from its analog in [49]. Small \( T \), as argued above, corresponds to the UV. The
successful features of [13, 14] come from the UV. We therefore conclude that a model built
along the lines described in this paper in asymptotically AdS space can capture the good
physical features of [13, 14] while the IR arbitrariness of those models is lifted due to the
condition \( \tau |_{\text{IR}} \to \infty \). This condition fixes the quark condensate in terms of the qua rk
mass and removes the extra non-physical parameter appearing in [13, 14]. Moreover, the brane
physics automatically provides the 5d Chern-Simons term.

By applying the formalism described in this paper to a concrete model (i.e. to particular
expressions of the metric and dilaton), one could find numerical estimates of QCD
observables such as meson masses or couplings. Nevertheless, as stressed several times,
enforcing a well behaved asymptotically AdS space, implies \( m_{T}^{2}R_{\text{AdS}}^{2} = -3 \) because the
quark bilinear has dimension 3. Since \( m_{T}^{2} = -\frac{1}{2\alpha'} \), we need \( R_{\text{AdS}}^{2} = 6\alpha' \) and the space has
large curvature, of the order of the string scale. The complete dual background, then, is
not a solution to just Einstein equations, but higher derivative corrections to the equations
of motion have to be introduced. This is a very difficult, if at all possible, task.

The drawback of large curvature implies that meson masses or couplings which would
be numerical results obtained from the DBI action cannot be considered trustworthy. This
same problem has arisen when trying to build non-critical holographic models. Neverthe-
less, Einstein-like equations have been used to extract qualitatively correct results [24, 10,
30]. Considering the impressive quantitative precision of predictions in models like [7, 8,
13, 14], it would be interesting to build at least phenomenological models incorporating
the tachyon physics. Constructions like [15, 17] could be a starting point.

In any case, apart from quantitative computations, we expect the general features
derived from the DBI action to hold. Moreover, since the WZ term is topological, the
results derived from its analysis in section 3 hold even if the curvature is large.

We end by commenting on some additional open problems. First of all, it would be of
major interest to add finite temperature and describe the deconfinement phase transition.
The physics of fundamental matter in this regime is very rich and has been studied holo-
graphically (see [21] and references therein). It may be possible to generalize the analysis
to the kind of setup described in this paper. It would also be nice to understand the physics
of rotating strings and Wilson loops. Another possible generalization is to introduce a large
number of flavors \( N_{f} \sim N_{c} \), to go beyond the quenched approximation. This may be done
along the lines of [24, 19, 52]. In fact, the general form of our expressions should guarantee
that they can account for any such backreaction, as long as the theory with fundamentals
is still confining. Finally, it would also be interesting to make contact with holographic $\mathcal{N} = 1$ theories built on the cigar, where some exact world-sheet computations can be done despite the large curvature, $^{23}$.

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APPENDIX

A. Conventions

For the $U(N_f)$ generators $\lambda^A$, $A = 0, \ldots, N_f^2 - 1$, we take

$$ (\lambda^A)^\dagger = \lambda^A \quad \text{Tr}(\lambda^A \lambda^B) = \frac{1}{2} \delta^{AB} \quad (A.1) $$

This in particular fixes the normalization of the $U(1)$ generator: $\lambda^0 = \frac{1}{\sqrt{2N_f}} \mathbb{I}$, where $\mathbb{I}$ is the $N_f \times N_f$ identity matrix. For the non-abelian $SU(N_f)$ generators $\lambda^a$, $a = 1, \ldots, N_f^2 - 1$, we have

$$ [\lambda^a, \lambda^b] = i f^{abc} \lambda^c \quad \text{Tr}(\lambda^a \{\lambda^b, \lambda^c\}) = d^{abc} \quad (A.2) $$

where $f^{abc}$ and $d^{abc}$ are, respectively, the structure constants and the normalized anomaly Casimir for $SU(N_f)$. Because of (A.1) $f^{abc}$ and $d^{abc}$ are real numbers.

We define gauge fields to be hermitian

$$ A_\mu = A^A \lambda^A = A^{U(1)}_{\mu} \mathbb{I} + A^a_{\mu} \lambda^a \quad (A.3) $$

In differential form notation, the field strength and covariant derivative read then

$$ F = dA - iA \wedge A \quad D = d - iA \quad (A.4) $$

where $A \cdot$ indicates the representation-dependent action of the gauge algebra. In particular the Bianchi identity reads $DF = 0$, and the covariant derivative of the tachyon is given by

$$ DT = dT + iT A_L - i A_R T \quad DT^\dagger = dT^\dagger - i A_L T^\dagger + i T^\dagger A_R \quad (A.5) $$

Under gauge transformations, the left and right gauge potentials, and tachyon transform in the following way

$$ A_L \to V_L A_L V_L^\dagger - id V_L V_L^\dagger, \quad A_R \to V_R A_R V_R^\dagger - id V_R V_R^\dagger $$

$$ T \to V_R TV_L^\dagger, \quad T^\dagger \to V_L T^\dagger V_R^\dagger, \quad V_L V_L^\dagger = V_R V_R^\dagger = \mathbb{I} \quad (A.6) $$

An infinitesimal gauge transformation is defined as $V_\epsilon(x) = e^{\epsilon A(x)} \simeq 1 + \epsilon A$. From (A.6) we have then:

$$ \delta A = -i D\Lambda = -i d\Lambda + [\Lambda, A] $$

$$ \delta F = [\Lambda, F] $$

$$ \delta T = -T\Lambda_L $$

$$ \delta T^\dagger = \Lambda_R T \quad (A.7) $$

Notice that the generators of gauge transformations are antihermitian. When we decompose them in their $U(1)$ and $SU(N_f)$ parts, we will write then

$$ \Lambda = i\Lambda^A \lambda^A = i\alpha \mathbb{I} + i\lambda^a \lambda^a \quad (A.8) $$
with $\alpha$ and $\Lambda^a$ real parameters. In particular we have, from (A.3) and (A.8)

$$\delta A^U_{\mu} = \partial_\mu \alpha \quad \text{and} \quad \delta A^a_{\mu} = (D_\mu \Lambda)^a$$

(A.9)

Currents are defined to be hermitian. We decompose a $U(N_f)$ flavor current as

$$J_\mu = J_0^0 \lambda^0 + J_\mu^a \lambda^a.$$  

(A.10)

This decomposition corresponds to defining the $A^{4\text{th}}$ component as

$$J^A_{L,R \mu} = \text{Tr}_{\text{colors}}(i\bar{q}\gamma_\mu 1 \pm \gamma_5 \lambda^A q).$$

(A.11)

For the $U(1)$ component $J^0$, this would be a strange normalization

$$J^0_{L,R \mu} = \frac{1}{\sqrt{2N_f}} \text{Tr}(i\bar{q}\gamma_\mu 1 \pm \gamma_5 q).$$

(A.12)

We therefore define a rescaled $J^U(1) = \sqrt{2N_f} J^0$, such that $J$ now reads

$$J_\mu = J^U_{L,R \mu} = \frac{1}{2N_f} J^U(1) \lambda^0 + J_\mu^a \lambda^a.$$  

(A.13)

Notice that the normalization of $A^U(1)$ in (A.3) has been chosen in such a way that the boundary coupling of the current to the gauge field reads

$$2 \int d^4x \text{Tr}(J_\mu A^\mu_\mu) = \int d^4x \left( J^U(1) \mu A^U(1) + J^a_\mu A^a_\mu \right).$$

(A.14)

### B. Determination of $\langle \bar{q}q \rangle$ from holographic renormalization

When solving the equation for the modulus of the tachyon in asymptotically AdS space, we found that it depends on two integration constants $m_q$ and $\sigma$, see (2.9). The constant associated to the non-normalizable mode $m_q$ can be immediately identified with the quark mass. On the other hand, $\sigma$ is related to the quark condensate $\langle \bar{q}q \rangle$ in, in principle, a non-trivial way. Schematically, $\langle \bar{q}q \rangle = -\frac{\delta S}{\delta m_q}$, where $S$ denotes the on-shell action. However, the on-shell action is UV divergent, and must be renormalized by adding covariant counterterms. This is done by following the so-called holographic renormalization procedure (for a review, see [53]). In the following, we adapt to our case the method of [54], where holographic renormalization was applied to probe flavor branes. In fact, it will be enough for our purposes to consider a simple case in which the scalar $\tau$ does not depend on the Minkowski $x^\mu$-coordinates. Also for simplicity, we consider that the metric and dilaton only depart from their AdS values at an order that does not contribute to UV divergences, i.e.

$$g_{xx}(z) = R_{\text{AdS}}^2/z^2(1 + O(z^5)),$$

$$g_{zz}(z) = R_{\text{AdS}}^2/z^2(1 + O(z^5))$$

and $e^{-\phi_{eff}} = e^{-\phi_0}(1 + O(z^5))$.

We write the action in terms of a canonically normalized, rescaled tachyon, as defined in (2.3):

$$\tau = c \tau_{\text{can}}, \quad c^2 \equiv \frac{e^{\phi_0} \pi}{4T_p}.$$  

(B.1)
The action (with a UV cutoff $\epsilon$) in terms of these quantities reads:

$$S_{\text{reg}} = -\frac{\pi R_{\text{AdS}}^5}{2c^2} \int d^4x \int_\epsilon^{z_{1R}} dz \frac{1}{z^5} e^{-c^2 z^2} \tau_{\text{can}} \sqrt{1 + \frac{2}{3} c^2 z^2 (\partial_z \tau_{\text{can}})^2}$$  \hspace{1cm} (B.2)

In the asymptotically AdS region, the equation of motion for $\tau_{\text{can}}$ reads:

$$-3\tau_{\text{can}} - z^5 \partial_z(z^{-3} \partial_z \tau_{\text{can}}) + \frac{c^2}{3} z^2 (\partial_z \tau_{\text{can}}) \partial_z \left( \frac{z^2 (\partial_z \tau_{\text{can}})^2}{1 + \frac{2}{3} c^2 z^2 (\partial_z \tau_{\text{can}})^2} \right) = 0$$  \hspace{1cm} (B.3)

and can be solved in series for small $z$ as:

$$\tau_{\text{can}} = z \left[ \Phi(0) + \frac{1}{3} c^2 z^2 \log z \Phi^3(0) + z^2 \Phi(2) + O(z^4) \right]$$  \hspace{1cm} (B.4)

Inserting back this result in the on-shell action, one finds UV divergencies as $\epsilon \to 0$. They have to be subtracted by adding the following counterterms localized at a $z = \epsilon$ slice:

$$S_{\text{ct0}} = \frac{\pi R_{\text{AdS}}}{8c^2} \int d^4x \sqrt{-\gamma}$$

$$S_{\text{ct1}} = -\frac{\pi R_{\text{AdS}}}{6} \int d^4x \sqrt{-\gamma} \tau_{\text{can}}^2$$

$$S_{\text{ct2}} = -\frac{\pi R_{\text{AdS}c^2}}{18} \int d^4x \sqrt{-\gamma} (\log \epsilon) \tau_{\text{can}}^4$$  \hspace{1cm} (B.5)

where $\gamma$ is the determinant of the induced metric in the $z = \epsilon$ slice, i.e. $\sqrt{-\gamma} = R_{\text{AdS}}^4 \epsilon^{-4}$.

As pointed out in [54], one can also add a finite counterterm:

$$S_{\text{ct3}} = \int d^4x \alpha \sqrt{-\gamma} \tau_{\text{can}}^4$$  \hspace{1cm} (B.6)

where $\alpha$ is some constant. It was argued in [54] that different values of $\alpha$ correspond to different renormalization schemes. Defining:

$$S_{\text{sub}} = S_{\text{reg}} + S_{\text{ct0}} + S_{\text{ct1}} + S_{\text{ct2}} + S_{\text{ct3}}$$  \hspace{1cm} (B.7)

the quark condensate is given by:

$$\langle \bar{q}q \rangle = \lim_{\epsilon \to 0} \left[ -R_{\text{AdS}}^{-\frac{3}{2}} \frac{\delta S_{\text{sub}}}{\delta \tau_{\text{can}}(\epsilon)} \right] = -\frac{2}{3} \pi R_{\text{AdS}}^{\frac{7}{2}} \Phi(2) + R_{\text{AdS}}^{\frac{5}{2}} \Phi^3(0) \left( \frac{c^2 \pi R_{\text{AdS}}}{3} - 4\alpha \right)$$  \hspace{1cm} (B.8)

Noticing from (2.8), (B.4) that $\Phi(0) = R_{\text{AdS}}^{-\frac{3}{2}} m_q$; $\Phi(2) = R_{\text{AdS}}^{-\frac{3}{2}} \sigma$ and substituting $R_{\text{AdS}}^2 = 6\alpha' = \frac{3}{\pi}$, we find:

$$\langle \bar{q}q \rangle = -2\sigma + \frac{\pi}{3} m_q^3 e^2 \sqrt{\frac{\pi}{3}} - 4\alpha$$  \hspace{1cm} (B.9)

In section 4.3 we have used the value of the condensate for small $m_q$ which is unambiguously given by:

$$\langle \bar{q}q \rangle \approx -2\sigma \hspace{1cm} (m_q \to 0)$$  \hspace{1cm} (B.10)
C. Comments on the superconnection formalism

In section 3 we have worked with the WZ world-volume action for stacks of brane-antibrane pairs and used a superconnection formalism which makes the notation quite compact. In this appendix we review some definitions and properties of this construction. The supermatrices are \(2N_f \times 2N_f\) matrices of differential forms \(M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}\) where the blocks \(A, B, C, D\) are \(N_f \times N_f\) matrices. We deal with two kinds of supermatrices: if the blocks in the diagonal \(A, D\) are composed of odd (even) differential forms, then, the off-diagonal ones \(B, C\) consist of even (odd) forms. For instance, \(\mathcal{A} (\mathcal{F})\) defined in (3.2) are matrices of each kind.

The multiplication of supermatrices is defined as \([36]\):

\[
M \cdot M' = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \cdot \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix} = \begin{pmatrix} AA' + (--)^{c} BC'^* & AB' + (--)^{d} BD' \\ DC' + (--)^{a} CA'^* & DD' + (--)^{b} CB'^* \end{pmatrix} \quad (C.1)
\]

where \(x'\) is 0 if \(X\) is a matrix of even forms or 1 if \(X\) is a matrix of odd forms. The associative property of this product can be easily checked.

The supertrace is defined as

\[
\text{Str} M = \text{Tr} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} M \quad \Rightarrow \quad \text{Str} \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \text{Tr} A - \text{Tr} D \quad (C.2)
\]

It is straightforward to prove the cyclic property of the supertrace, where the \(\mathcal{A}\)-type \((\mathcal{F}\)-type) supermatrices behave as odd (even) forms:

\[
\text{Str}(a b) = -\text{Str}(b a) ; \quad \text{Str}(a f) = \text{Str}(f a) ; \quad \text{Str}(f g) = \text{Str}(g f) . \quad (C.3)
\]

We have denoted by \(a, b, f, g\) generic supermatrices of the \(\mathcal{A}\)-type \((\mathcal{F}\)-type).

We also define a pseudotransposition operation:

\[
\begin{pmatrix} A & B \\ C & D \end{pmatrix}^{pt} \equiv \begin{pmatrix} A^t & IC^t \\ iB^t & D^t \end{pmatrix} \quad (C.4)
\]

where \(t\) denotes usual matrix transposed. This generalizes the usual transposition in the sense that (using the same notation of \((C.3)\)):

\[
(a b)^{pt} = -b^{pt} a^{pt} ; \quad (a f)^{pt} = f^{pt} a^{pt} ; \quad (f g)^{pt} = g^{pt} f^{pt} . \quad (C.5)
\]

Nevertheless, unlike the usual transposition, pseudotransposing twice does not yield the initial matrix. Obviously, the supertrace does not change under pseudotransposition.

D. Gauge transformation of the 1-form \(\Omega_1\)

In this appendix we show that the 1-form \(\Omega_1\) defined in (3.30) in section 3.3 transforms non-trivially only under \(U(1)_A\). This is an important point in the understanding of the \(U(1)_A\)
axial anomaly and it is therefore worth being explicitly shown. For clarity of exposition, we report here the expression (3.30) for $\Omega_1$

$$\Omega_1 = e^{-\tau^2} \text{Tr} \left( iA_L - iA_R + (\log T - \log T^\dagger) \tau d\tau \right) \quad (D.1)$$

It is obvious that the first two terms transform only under $U(1)_A$ transformations. The remaining log terms, instead, require a little more effort. First of all notice that we can write a general variation of $\text{Tr}(\log T - \log T^\dagger)$ as

$$\delta \text{Tr}(\log T - \log T^\dagger) = \text{Tr} \left( T^{-1} \delta T - (T^\dagger)^{-1} \delta T^\dagger \right) = \frac{1}{\tau} \text{Tr} \left( T^\dagger \delta T - T \delta T^\dagger \right) = \frac{2}{\tau} \text{Tr} \left( T^\dagger \delta T \right) \quad (D.2)$$

where in the second equality we used the same condition on $T$ we imposed in section [3.3], $TT^\dagger = T^\dagger T = \tau^2 I_{N_f}$, from where it also follows that $T \delta T^\dagger = -\delta TT$, which we used in the last equality of (D.2).

From (A.7), we may show that for a vectorial ($\Lambda_L = \Lambda_R = \Lambda_V$) gauge transformation $\delta \Lambda_V \text{Tr}(\log T - \log T^\dagger)$ vanishes. For an axial infinitesimal transformation ($\Lambda_L = -\Lambda_R = \Lambda_A$), from (A.7) and (D.2), we find, instead:

$$\delta \Lambda_A \text{Tr}(\log T - \log T^\dagger) = -4 \text{Tr} \Lambda_A \quad (D.3)$$

This is clearly non-zero for abelian $U(1)_A$ axial gauge transformations only, as we wanted to show.

### E. On the WKB approximation and linear confinement

In order to use the WKB approximation to determine the spectrum, we start by mapping the standard eigenvalue problem to a Schrödinger-like equation. We follow [41]. We start from an equation (notice that $\lambda_n$ is always proportional to the square of the four-dimensional mass):

$$-\frac{1}{\Gamma(z)} \partial_z \left[ \frac{\Gamma(z)}{\Sigma^2(z)} \partial_z \psi(z) \right] + B(z) \psi(z) = \lambda_n \psi(z) \quad (E.1)$$

and define a new radial variable $u$ and a rescaled wave function $\alpha$:

$$\frac{du}{dz} = \Sigma(z), \quad \alpha(u) = \Xi(u) \psi(z(u)), \quad \Xi(u) \equiv \sqrt{\frac{\Gamma(z(u))}{\Sigma(z(u))}} \quad (E.2)$$

Then, equation (E.1) is rewritten as

$$-\alpha''(u) + V(u) \alpha(u) = \lambda_n \alpha(u) \quad (E.3)$$

with:

$$V(u) = \frac{\Xi''(u)}{\Xi(u)} + B(z(u)) \quad (E.4)$$

---

In this appendix, we define $V$ as the Schrödinger-like potential that appears in equation (E.3). It should not be confused with the tachyon potential used in the rest of the paper.
One can use the WKB approximation to estimate the mass of high excitations:

\[
\frac{d\lambda_n}{dn} = 2\pi \left[ \int_{u_1}^{u_2} \frac{du}{\sqrt{\lambda_n - V(u)}} \right]^{-1}
\]

(E.5)

where \(u_1\) and \(u_2\) are the classical turning points. In [11] it was observed that, quite generically, the range in the variable \(u\) is finite. Therefore, for large \(\lambda_n\), the problem is similar to an infinite potential well: the integral behaves as \(\lambda_n^{-1/2}\) and \(\lambda_n \sim m_n^2 \sim n^2\). In [16] it was remarked that the behavior of the functions defined above can be tuned near the infrared such that the IR turning point in equation (E.5) goes to infinity in such a way that the desired relation \(m_n^2 \sim n^2\) is obtained. We may naively expect that highly excited masses should depend mainly on the UV behavior of different functions. However, linear confinement emerges from the IR dynamics and the size of highly excited hadrons grows with the excitation number. Therefore, highly excited mesons are still affected by IR physics.

We now apply this formalism to the vector meson equation (4.6). Comparing to (E.1), we define \(\lambda_n \equiv m_n^2\), \(B(z) \equiv 0\), \(\Gamma(z) \equiv e^{-\tilde{\phi}} g_{zz}\), \(\Sigma(z) \equiv \sqrt{\tilde{g}_{zz} - g_{xx}}\). The change of variable to \(u\) would involve a complicated integral, but in order to estimate the large excitation spectrum, it is enough to study the leading UV and IR behavior of the functions.

Near the UV, we consider an asymptotically AdS space \(\tilde{g}_{zz} \approx g_{xx} \approx R_{AdS}^2/z^2\), \(e^{-\tilde{\phi}} \approx \text{const}\), so \(\Sigma \approx 1\) and \(u_{UV} = z\). The potential (E.4) is \(V_{UV}(u) \approx 3/4 u^2\). Thus, for large \(\lambda_n\), the classical turning point is at \(u_1 = \sqrt{3/(4\lambda_n)}\) so \(u_1\) remains finite as \(\lambda_n \to \infty\) and, as expected, the UV contribution to the integral in (E.5) decreases as \(\lambda_n^{-1/2}\).

The intermediate region when one cannot apply the IR nor the UV asymptotics has finite size in the \(u\)-variable so its contribution to the integral also decreases as \(\lambda_n^{-1/2}\).

Near the IR, the vev of the open string tachyon is diverging and \(g_{xx} \approx T_{QCD}, \tilde{g}_{zz} \approx \frac{2}{\pi} (\partial_z \tau)^2\) and \(e^{-\tilde{\phi}} \approx e^{-\tau^2}\). By substituting in (E.2), we find that the \(u\)-coordinate near the IR is just \(u_{IR} \approx \sqrt{\frac{2}{\pi T_{QCD}}} \tau(z)\) and the potential is \(V_{IR} \approx (\frac{\pi}{4} T_{QCD} u^2)\). The classical turning point is therefore at \(u_2 = \frac{2}{\pi T_{QCD}} \frac{1}{\sqrt{\lambda_n}}\). Thus, \(u_2\) grows to infinity as \(\lambda_n\), unlike the cases considered in [41]. The IR contribution to the integral is:

\[
\int_{u_1}^{u_2} \frac{du}{\sqrt{\lambda_n - (\frac{\pi}{4} T_{QCD} u^2)}} = \frac{1}{T_{QCD}} + \ldots
\]

(E.6)

where the dots stand for a piece that vanishes as \(\lambda_n \to \infty\). Therefore, the integral in (E.5) is dominated by the IR region and:

\[
\lim_{\lambda_n \to \infty} \frac{d\lambda_n}{dn} = 2\pi T_{QCD}
\]

(E.7)

which, reinserting \(\lambda_n = m_n^2\) yields (4.20) as we wanted to show. For the axial vectors, (see (4.12)), the \(z\)-dependent mass for \(A_\perp\) (such that \(B(z) = g_{xx} v(z)^2\)) adds an extra term to the potential near the IR \(V_{IR} = (\pi^2/4 + 4) T_{QCD}^2 u^2\), leading to (4.21).

Figure 3 depicts a sketch of the Schrödinger-like potential in the \(u\)-variable.
Figure 2: Qualitative behavior of the Schrödinger-like potential in the $u$-variable. Near the UV, it grows as $V \propto u^{-2}$ while near the IR $V \propto u^2$. In the middle, it may present more complicated features which do not affect the leading behavior of the spectrum for large excitation number.

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