Construction of functional relationships providing cross-polarization equivalence of electromagnetic scatterers

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Abstract. Impedance coatings of cylindrical bodies’ synthesis in order to achieve their cross-polarization equivalence (coincidence of scattering diagrams for $E$ - and $H$ - polarizations of the incident wave) are considered in the work. The stationary model of electrodynamics’ two-dimensional equations with modified boundary conditions is used in order to construct dual distributions pairs of surface impedance. Using the Green’s function, a transition is made to the system of integral equations in the framework of the boundary element method. The operators’ appeal of solutions to direct problems made it possible to obtain functional relationships for the distribution of surface impedance at various polarizations. This made it possible to provide required ratios between the norms of dual coatings.

1. Introduction
Complex problems of mathematical modeling arise in the process of designing antenna systems and phased arrays with predetermined radar characteristics in different wave ranges [1-5]. We should mention cross-polarization equivalent electromagnetic scatterers, which the scattering diagram (SD) of the first $E$ - polarized incident wave coincides with the SD of the second at $H$ - polarized incident wave for. It should be noted that diffraction at $E$-polarization differs significantly from diffraction at $H$-polarization as a physical phenomenon. This belongs, in particular, to the directions of induced surface currents [6].

In addition, the mathematical models, adapted to these phenomena description, lead to integral equations with different orders of nuclear features. It is proposed to use a physical and mathematical model of plane electromagnetic waves diffraction by cylindrical bodies with an impedance surface coating to build cross-polarization equivalent scatterers [7].

Analysis of the considered computational algorithm showed that the dual sets’ synthesis of impedance coatings was a solution to the correctly posed problem [8].

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2. Materials and Methods

In the stationary case, the diffraction of a plane electromagnetic wave is described by the two-dimensional complex Helmholtz equation outside the cylindrical impedance surface $S$ for a nonzero field component $u$ [9]:

$$\nabla^2 u + k^2 u = 0,$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ — two-dimensional Laplace operator, with a modified boundary condition:

$$u = -iW_0 u_0/(kW_0) \quad \text{for } E\text{-polarization},$$

$$u_n = iW_0 u_0/W \quad \text{for } H\text{-polarization}.\quad (2)$$

here $W_0 = 120\pi$ — free space wave impedance, $u_n$ — derivative in the direction of the external normal to the contour, a solution $u_0$ (with zero surface impedance $W(S)=0$) corresponds to diffraction on a perfectly conducting surface.

The inverse problem formulation assumes that the surface distribution of $W(S)$ impedance is considered to be the desired function, and information on the values of the scattered electromagnetic field in the far or near zone is used as an additional condition.

The criterion for approaching a given SD $u_g^2$ is understood in the mean-square sense:

$$J = \sum_{m=1}^{m} |u - u_g|^2 \rightarrow \min.\quad (4)$$

3. Results. Transition to the solution using the boundary element method

The Green function of free space is considered (fundamental solution)

$$g = \frac{m}{2} H_0^{(1)}(kr),$$

satisfying the equation

$$\nabla^2 g + kr^2 g = -2\pi \delta(r_{M,P}).$$

The notations are introduced here: $\delta(r_{M,P})$ — Dirac Delta Function, $H_0^{(1)}(kr)$ — first-order Hankel function of the first kind. In its turn $H_0^{(1)} = J_0 + iN_0$, where $J_0$ — zero-order Bessel cylindrical function of the first kind and $N_0$ — zero-order second-order Bessel function, also called the Neumann function. In the future, the derivative of the fundamental solution in the normal direction will be used

$$g_n(r) = \frac{i}{2} (J_1(kr) + iN_1(kr)) \frac{k}{r} (r_\alpha \cos \alpha_n + r_\beta \cos \beta_n).$$

where $\cos \alpha_n$ and $\cos \beta_n$ — guide cosines of the inner normal to the contour $S$, a $J_1$ and $N_1$ — respectively, the Bessel and Neumann functions of the first order. The cylindrical functions calculation is realized in the form of convergent series (see [10]). The corresponding asymptotic formulas are taken for relatively large values of the argument.

We formulate the integral representation for the total electromagnetic field at an arbitrary point $M$, using the second Green formula and the Sommerfeld radiation condition:

$$u(M) = \frac{1}{2\pi} \int_S (u_n g - u g_n) dS + u_1(M),\quad (5)$$

here $u_1(M)$ — known field of the incident wave. The scattered field, associated with the full field, is easily found from here by the formula:

$$u^s(M) = u(M) - u_1(M).\quad (6)$$
Thus, the scattered field is represented as the sum of the integrals, which are the potentials of a simple and double layer with continuous distribution densities.

We examine the diffraction $H$-polarized electromagnetic wave, and perform the substitution $u_n$ from the boundary condition (3), then, taking into account the fulfillment of criterion (4), we come to the relation

$$u_n^s(M) = \frac{1}{2\pi} \int_S \left( i ku_0 gW/W_0 - u_n^s \right) dS_p.$$  

(7)

We should write down the integral equations for the functions under consideration. So, we lower the point $M$ onto $S$ surface in relation (5). The solution of the direct diffraction problem on the ideally conducting $u_0$ surface satisfies the equation:

$$u_0/2 + \int_S g_n u_0 dS/(2\pi) = u_1.$$  

(8)

In the case of solving the problem for $W(S)$ surface with an impedance coating with condition (3), we formulate the equation

$$u/2 + \int_S (g_n u - iu_0 g) dS/(2\pi) = u_1.$$  

(9)

where $\sigma = kW/W_0$.

We introduce the notation for the linear operators of direct problems for $E$- and $H$-polarizations:

$$A\sigma = \sigma/2 + \int_S g_n \sigma dS/(2\pi), \quad B\sigma = i \int_S g dS/(2\pi).$$

Therefore, the equations (8, 9) can be represented as:

$$Au_0 = u_1, \quad Au - B\sigma u_0 = u_1.$$  

Solving these equations, we obtain:

$$u_0 = A^{-1} u_1, \quad u = u_0 + A^{-1} B\sigma u_0.$$  

(10)

In order to solve the inverse problem, it is necessary to obtain the equation for the surface impedance $W(S)$ by substituting the found expression for $u$ in the equation (7):

$$u_n^s = \frac{1}{2\pi} \int_S \left( iku_0 gW/W_0 - (u_0 + A^{-1} B\sigma u_0) g_n^s \right) dS_p.$$  

After the transformations, we turn to the integral equation for the function $\zeta = iku_0 W/W_0$, which helps the impedance to be uniquely restored:

$$\int_S (g + ig_n A^{-1} B) \zeta dS = 2\pi u^s + 2\pi \int_S g_n A^{-1} u_1 dS.$$  

(11)

Modeling the diffraction of $E$-polarized electromagnetic wave, we perform substitution $u$ from the boundary condition (2), out of relation (5) and obtain:

$$u_n^s(M) = \frac{1}{2\pi} \int_S \left( u_n g - u_0 g_n^s W/(ikW_0) \right) dS_p.$$  

The solution of the direct problem of diffraction on an ideally conducting surface ($u_{0n}$ is taken as the derivative) here satisfies the equation:

$$\int_S u_{0n} g dS/(2\pi) = u_1.$$  

In the case of the problem solving for a surface with an impedance coating $W(S)$ of (2) condition, we formulate the equation

$$u_{0n} W/(2ikW_0) - \int_S (u_n g - u_0 g_n W/(ikW_0)) dS/(2\pi) = u_1.$$  

Similarly to the way of (11) equation is obtained, the validity of the equation is established:
where unlike the case $H$ - polarization function now appears $u_{in}$. Therefore, there is a functional relationship between the parameters: $\eta_E=\eta_H$, $u_{in}=u_{0H}ikW_H/W_0$. In the same way we get equality $\eta_H=\eta_E$, which corresponds to the formula:

$$u_H=u_{0H}W_E/(ikW_0). \quad (12)$$

4. Discussion. The equation for the scatterers synthesis

We consider the representation (10) for $u = u_H$ function, fixing $H$ - polarization of the incident wave and substituting the dual dependences (12):

$$u_{0H}W_E/(ikW_0)=u_{0H}+A^{-1}Bu_{0H}W_H/W_0. \quad (13)$$

Solving (13) relatively $W_E$, we obtain

$$W_E=ikW_0u_{0H}/u_{0H}+A^{-1}Bu_{0H}W_H/(u_{0H}). \quad (14)$$

The equivalent (14) functional dependence is derived:

$$W_H=W_0u_{0H}/(iku_{0H})+iB^{-1}Au_{0H}W_E/(ik^2u_{0H}). \quad (15)$$

Equations (14, 15) make it possible to obtain pairs of cross-polarization equivalent coatings for a scatterer. In particular, by setting a known distribution of $W_H$ impedance in advance, substituting it in the right-hand side of (14), one can calculate dual distribution $W_E$ in the final form of SR the diffuser with the first coating at probing with $H$ - polarized wave will coincide with SD of this diffuser with the second coating at probing with $E$ - polarized wave.

Such a property can be used for masking in the case when it is necessary to complicate the object recognition. As a camouflage, the SD of the same scatterer is set, when a transversely polarized (cross-polarized) wave is located. In addition, a change in the polarization of the incident wave is physically equivalent to an object turn by 90° in the transverse plane. This can be used if it is required the coating ensure SD preservation, changing the object orientation.

Often, one of the paired coatings must satisfy the specified properties due to structural and technological considerations, for example, its material or imaginary component has a certain sign or changes in a specific range. The advantage of the proposed approach is in simulation of impedance coating in the class of arbitrary complex-valued functions, which allows us to solve this problem. In addition, the presence of relations (14, 15) suggests the possibility of introducing an a priori relationship between the norms of dual coatings

5. Conclusion

Transition to the system of integral equations for both the main linear $E$ polarization and the transverse $H$ polarization of the incident wave was made, based on the integral representation for the scattered field recorded, using the fundamental solution.

Subsequent discretization allowed us to move to a well-conditioned system of linear algebraic equations in complex space, based on the boundary element method. As a result, an effective algorithm for constructing pairs of dual impedance coatings was obtained, where the simultaneous change in the coating and polarization of the probe signal kept the SR constant. The problem was solved in a wide class of arbitrary complex functions, which allowed us to use this approach to satisfy various design and technological requirements for the object.

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