Optimal Preventive Maintenance Planning for Electric Power Distribution Systems Using Failure Rates and Game Theory

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Abstract: Current electric utilities must achieve reliability enhancement of considerable distribution feeders with an economical budget. Thus, optimal preventive maintenance planning is required to balance the benefits and costs of maintenance programs. In this research, the proposed method determines the time-varying failure rate of each feeder to evaluate the likelihood of future interruptions. Meanwhile, the consequences of feeder interruptions are estimated using interruption energy rates, customer-minutes of interruption, and total kVA of service areas. Then, the risk is assessed and later treated as an opportunity for mitigating the customer interruption costs by planned preventive maintenance tasks. Subsequently, cooperative game theory is exploited in the proposed method to locate a decent balance between the benefits of reliability enhancement and the costs required for preventive maintenance programs. The effectiveness of the proposed method is illustrated through case studies of large power distribution networks of 12 service regions, including 3558 medium-voltage distribution feeders. The preventive maintenance plans resulting from the proposed method present the best compromise of benefits and costs compared with the conventional approach that requires a pre-specified maintenance budget.

Keywords: cooperative game theory; time-varying failure rate; optimization; prioritization; risk assessment; reliability benefit; benefit–cost ratio; reliability-centered maintenance

1. Introduction

In most parts of the world, power distribution networks are still overhead feeders, prone to being affected by various outage causes, such as tree contact, animal contact, and equipment failure [1–3]. As a result, reliability concerns have been a critical issue among electric utilities for several decades. In a competitive environment, electric utilities are challenged to achieve reliability improvement of power distribution systems with tight maintenance budgets. Therefore, cost-effective preventive maintenance must be planned to mitigate power interruptions by fully exploiting data and information available in organizations.

A number of electric utilities prioritize their feeders so that the critical feeders receive appropriate and cost-effective maintenance [4–6]. To achieve this objective successfully, the available resources must be optimally allocated [7]. In [5,6,8–11], the reliability-centered maintenance (RCM) technique was employed to select cost-effective preventive maintenance tasks for electric power systems.

Several risk-based approaches for maintenance planning were well adopted in a wide range of power systems, for instance, hydroelectric power plants [12], generation units in combined heat and power systems [13], and power distribution systems [14]. In addition, ref. [15] explored long-term risk-based maintenance optimization for power distribution systems under hurricane hazards. In [16], a short-term maintenance strategy was successfully implemented based on a risk-cost analysis of power transmission and transformer equipment.
Most research works [4–6,8–11,17] employed failure rates assumed to be invariant with time in risk studies because of their simplicity and availability. However, such an assumption may sacrifice some helpful information for risk assessment, for example, a trend of failure rates [18–20]. Instead, the time-varying failure rate can be beneficial to the risk assessment since its trend, either increasing or decreasing, indicates the likelihood of future power interruptions [18,21]. This is very important since the effectiveness of preventive maintenance planning depends on the accuracy of risk assessment.

In [17,22–25], optimal preventive maintenance planning was conducted for power distribution systems under pre-specified maintenance budgets. In fact, policymakers often question the assigned budgets about a reasonable balance between utility investment and customer interruption costs. Admittedly, a larger budget will allow more reliability improvement in power distribution systems, but it will lessen profitability for utilities. As a result, finding the right figure for the budget can be problematic because an optimal balance of benefits between utilities and customers needs to be sought.

In multi-objective optimization, it is difficult to pick one optimal solution that can improve one objective without worsening the others. Recently, the cooperative game theory approach has been applied in various areas because it can effectively find a balanced solution with much less computational effort [26]. Some applications of game theory for solving power system problems can be found in [27]. Examples of recent studies are on allocating the transmission losses [28,29], allocating the small power producer losses [30], allocating unit start-up costs of generators [31], constructing prosumer coalitions for energy cost savings [32], maintenance planning [33], and applications in distribution system expansion planning, including distributed generation [34].

This research proposed a methodology for the preventive maintenance planning of power distribution systems using the time-varying failure rate and cooperative game theory. The concept of the method is described as follows.

First, the risk likelihood and consequences are determined to perform the risk assessment for individual distribution feeders. The time-varying failure rates, calculated from the three-year interruption record data based on the Weibull distribution, are exploited to indicate the likelihood of future interruptions. Subsequently, the consequences are evaluated using the interruption energy rates, customer-minutes of interruption, and total kVA of service areas. Accordingly, the risk of feeder interruptions can be assessed and later considered as an opportunity for mitigating customer interruption costs by appropriate preventive maintenance.

Secondly, preventive maintenance tasks corresponding to interruption causes are listed for each feeder. Then, the cost of each task can be estimated. Later, the benefit–cost ratio (BCR) of individual tasks is calculated to illustrate the cost-effectiveness. As a result, all maintenance tasks can be ranked according to their BCR. This step is essential because the tasks sorted by their BCR will simplify the problem formulation for optimal maintenance planning.

Lastly, the method employs cooperative game theory to locate the budget amount, resulting in an optimal balance between costs and benefits of preventive maintenance. Then, the list of selected tasks can be obtained accordingly.

To demonstrate the practicability and effectiveness of the proposed method, extensive overhead distribution networks of 12 service regions covering all provincial areas of Thailand [4,35] are used as case studies. The networks consist of 3558 medium-voltage distribution feeders with a total length of 318,349 circuit kilometers. The first case study illustrates that the proposed method can successfully find the preventive maintenance plan resulting in the optimal balance between costs and benefits. The second case study highlights the accuracy of the presented method by revealing the reduced accuracy of maintenance planning that neglects the time-varying failure rates.

This paper is organized as follows: Section 2 carefully describes the risk assessment; in Section 3, the benefit–cost evaluation of preventive maintenance tasks is illustrated;
Section 4 explains the cooperative game theory exploited in the method; then, the results and discussion are illustrated in Section 5; and, finally, Section 6 provides the conclusions.

2. Risk Assessment

The risk assessment is the process to identify and evaluate hazards that deteriorate the distribution system reliability. This section is divided into three subsections to explain the presented concept. In the first subsection, power interruption causes are identified. The second part analyzes the time-varying failure rates of distribution feeders. The last subsection presents the evaluation of customer interruption costs.

2.1. Causes of Power Interruptions

As the overhead distribution feeders are exposed, they are vulnerable to damage from a variety of external factors. In addition, most overhead distribution feeders in Thailand are over 30 years old, so equipment failures due to aging and material degradation are likely to occur. The most common causes of power interruptions found in outage event records from 2004 to 2019 [36] are equipment failure, tree contact, and animal contact. Table 1 shows customer-minutes of interruption categorized by their causes regarding the overhead distribution systems in 2019 as calculated from [36]. According to the Pareto principle (80/20 rule), which states that 80% of consequences come from 20% of the causes, the most influential causes in Table 1 are equipment failure, tree contact, and animal contact, all resulting in over 80% of the total customer-minutes of interruption. Consequently, this research focuses solely on maintenance tasks that prevent outages caused by equipment failure, tree contact, and animal contact.

Table 1. Causes of power interruptions and customer-minutes of interruption.

| Rank | Cause of Interruptions | Customer-Minutes of Interruption | %  |
|------|------------------------|----------------------------------|----|
| 1    | Equipment failure      | 499,322,372                      | 33.34 |
| 2    | Tree contact           | 463,909,708                      | 30.97 |
| 3    | Animal contact         | 299,530,449                      | 20.00 |
| 4    | Environment            | 91,440,132                       | 6.10 |
| 5    | Vehicle                | 53,221,537                       | 3.55 |
| 6    | Foreign object         | 31,161,096                       | 2.08 |
| 7    | Others                 | 21,693,792                       | 1.45 |
| 8    | Human                  | 20,407,668                       | 1.36 |
| 9    | Natural disaster       | 16,073,329                       | 1.07 |
| 10   | Overload               | 1,069,261                        | 0.07 |
|      | Total                  | 1,497,829,344                    | 100.00 |

2.2. Feeder Failure Rates

The failure rate is one of the vital parameters describing the reliability characteristics of components or systems. It expresses the frequency with which a component or system fails. Superior to a constant or average failure rate [37], the time-varying failure rate can illustrate its trend that can be exploited for preventive maintenance planning. After excluding all feeders with no interruptions, this research determines the time-varying failure rates of 3112 distribution feeders with 3-year (2017–2019) interruption data based on the Weibull distribution. The Anderson–Darling test showed that 2654 feeders, approximately 75% of the total feeders, accepted the hypothesis, as categorized by the 12 service regions in Table 2.
## Table 2. Results of the Anderson–Darling test categorized by 12 service regions.

| Service Region | Number of Feeders | Accepted A.D. Test | Rejected A.D. Test | No Event |
|----------------|-------------------|--------------------|--------------------|----------|
| 1              | 258               | 214                | 18                 | 26       |
| 2              | 257               | 226                | 8                  | 23       |
| 3              | 198               | 177                | 12                 | 9        |
| 4              | 257               | 189                | 29                 | 39       |
| 5              | 253               | 213                | 32                 | 8        |
| 6              | 250               | 200                | 21                 | 29       |
| 7              | 533               | 388                | 61                 | 84       |
| 8              | 454               | 275                | 76                 | 103      |
| 9              | 389               | 258                | 56                 | 75       |
| 10             | 221               | 156                | 52                 | 13       |
| 11             | 286               | 203                | 58                 | 25       |
| 12             | 202               | 155                | 35                 | 12       |
| **Total**      | **3558**          | **2654**           | **458**            | **446**  |

The Weibull distribution is commonly used in a two-parameter version, including scale parameter $\alpha$ and shape parameter $\beta$. Both parameters are positive numbers [38]. From the statistics of power interruptions during the period 2017–2019, the data were analyzed to find parameters $\alpha$ and $\beta$. Then, the time-varying failure rate $\lambda(t)$ of each feeder is determined by Equation (1) [38], where $t$ is defined as the amount of time that the feeder continuously supplies its customers. Accordingly, every time the feeder is back in service after the last interruption, $t$ is reset to 0.

$$\lambda(t) = \frac{\beta t^{\beta-1}}{\alpha \beta}$$  \hspace{1cm} (1)

From Equation (1), it can be observed that the failure rate decreases with $t$ if the power of $t$ is negative (i.e., if $\beta < 1$). On the other hand, if $\beta > 1$, the failure rate increases with $t$. In the case of $\beta = 1$, the failure rate is constant with $t$.

In Figures 1 and 2, $t_1$ is the time when the maintenance planning is performed, usually by the end of each year. Additionally, $\lambda(t_1)$, the failure rate at $t = t_1$, implies the frequency with which the feeder fails currently. From a planning point of view, it needs to look ahead at least one year and consider a change in the failure rate. Hence, $\lambda(t_2)$, the failure rate at $t = t_2$, where $t_2$ is one year after $t_1$, presents the anticipated frequency. As a result, the failure rate tendency index $k_\lambda$ defined by Equation (2) can be used to predict how the risk likelihood will change in the following year.

$$k_\lambda = \frac{\lambda(t_2)}{\lambda(t_1)}$$  \hspace{1cm} (2)

![Figure 1. Increasing failure rate, $k_\lambda > 1$.](image-url)
Figure 1 shows the increasing failure rate, resulting in a $k \lambda$ value greater than 1. It implies that the reliability will deteriorate if maintenance plans are not upgraded. In Figure 2, the decreasing failure rate leads to $k \lambda$ being less than 1. This indicates that the reliability is likely to increase, although maintenance plans are not revised [38].

The calculation of $k \lambda$ is performed for all feeders accepting the Weibull distribution hypothesis. It is found that 1509 feeders have increasing failure rates ($k \lambda > 1$) and 720 feeders have decreasing failure rates ($k \lambda < 1$). However, the other 425 feeders have almost constant failure rates ($k \lambda \approx 1$). As a result, the number of feeders in different types of failure rate tendencies are shown in Table 3. In addition, $k \lambda = 1$ is assumed for the 458 feeders rejecting the hypothesis.

Table 3. Number of feeders with different types of failure rate tendencies in each service region.

| Service Region | $k \lambda < 1$ | $k \lambda > 1$ | $k \lambda \approx 1$ | Total |
|----------------|-----------------|-----------------|---------------------|-------|
| 1              | 63              | 103             | 48                  | 214   |
| 2              | 13              | 196             | 17                  | 226   |
| 3              | 29              | 137             | 11                  | 177   |
| 4              | 58              | 101             | 30                  | 189   |
| 5              | 21              | 155             | 37                  | 213   |
| 6              | 50              | 124             | 26                  | 200   |
| 7              | 163             | 133             | 92                  | 388   |
| 8              | 98              | 123             | 54                  | 275   |
| 9              | 59              | 170             | 29                  | 258   |
| 10             | 77              | 56              | 23                  | 156   |
| 11             | 53              | 115             | 35                  | 203   |
| 12             | 36              | 96              | 23                  | 155   |
| Total          | 720             | 1509            | 425                 | 2654  |

2.3. Evaluation of Customer Interruption Costs

This subsection explains the concept of evaluating the impacts of power interruptions. The customer-minutes of interruption (CMI) is a quantity indicating the effects of power interruptions in terms of the number of affected customers and interruption duration in minutes. Although CMI is a rational index to express the reliability impacts, it is preferable to convert this index into the customer interruption costs, which is more understandable from an economic perspective.

First, the CMI in each zone of a feeder due to three influential causes (equipment failure, tree contact, and animal contact) can be calculated by Equation (3) [39].

$$CMI_{z,f}^c = \sum_{i=1}^{N_c} C_{zi}^c \times d_i^c$$

(3)
where
\( CMI_{z,f} \) is the customer-minutes of interruption in zone \( z \) of feeder \( f \) due to cause \( c \);
\( C_{zi}^c \) is the number of customers in zone \( z \) affected by incident \( i \) due to cause \( c \);
\( d_i^c \) is the interruption duration of incident \( i \) due to cause \( c \) (minutes);
\( N_c^c \) is the number of interruptions due to cause \( c \).

Subsequently, the outage rate of customers (THB/customer-minute) is proposed to assess the impacts on customers when experiencing a power interruption. This value depends on customer loads and interruption energy rates of customers in different zones (industrial, metropolitan, urban, suburban, and rural) as presented in Table 4. Accordingly, the outage rate of customers in each zone of a feeder can be calculated by Equation (4).

\[
O_{z,f} = \frac{kVA_{z,f} \times PF \times UF}{C_{z,f}} \times \frac{IER_z}{60}
\]  

(4)

where
\( O_{z,f} \) is the outage rate of customers in zone \( z \) of feeder \( f \);
\( kVA_{z,f} \) is the installed kVA in zone \( z \) of feeder \( f \);
\( PF \) is the power factor;
\( UF \) is the utilization factor;
\( C_{z,f} \) is the number of customers in zone \( z \) of feeder \( f \);
\( IER_z \) is the interruption energy rate of customers in zone \( z \) (THB/kWh) as presented in Table 4 (Exchange rate: THB 1 = USD 0.028).

### Table 4. Interruption energy rate of customers in different zones of each service region.

| Service Region | Zones      |  |  |  |
|----------------|------------|---|---|---|
| 1              | 125.12     | 65.52 | 72.65 | 103.07 | 94.25 |
| 2              | 154.54     | 80.92 | 89.74 | 127.30 | 116.40 |
| 3              | *          | 71.56 | 79.35 | 112.57 | 102.93 |
| 4              | *          | 48.26 | 53.52 | 75.92  | 69.42  |
| 5              | *          | 45.25 | 50.17 | 71.18  | 65.09  |
| 6              | 121.42     | 63.58 | 70.50 | 100.02 | 91.45  |
| 7              | 79.83      | 41.80 | 46.35 | 65.76  | 60.13  |
| 8              | 77.64      | 40.66 | 45.08 | 63.96  | 58.48  |
| 9              | 104.34     | 54.63 | 60.59 | 85.95  | 78.59  |
| 10             | 110.61     | 57.92 | 64.23 | 91.12  | 83.32  |
| 11             | *          | 44.80 | 49.68 | 70.47  | 64.44  |
| 12             | 105.16     | 55.07 | 61.07 | 86.63  | 79.21  |

* No industrial zone in service regions.

Then, the customer interruption cost can be obtained by multiplying the CMI by the outage rate of the customers. Where \( N_z \) is the number of zones, the customer interruption cost of feeder \( f \) due to cause \( c \), \( CIC_{c}^{f} \), is evaluated by using Equation (5).

\[
CIC_{c}^{f} = \sum_{z=1}^{N_z} CMI_{z,f}^{c} \times O_{z,f}
\]  

(5)

### 3. Benefit–Cost Evaluation of Preventive Maintenance Tasks

This section illustrates the cost–benefit evaluation of preventive maintenance tasks. As the three influential causes, including equipment failure, tree contact, and animal contact, were identified previously, corresponding preventive maintenance tasks must be carefully planned for reliability enhancement. In this research, the patrol and condition-based maintenance, tree trimming, and installation of animal guards are selected accordingly.
This section is divided into three subsections. The first and the second subsections explain the evaluation of benefits and costs of preventive maintenance tasks, respectively. The third part describes the benefit/cost analysis.

3.1. Benefits of Preventive Maintenance Tasks

Preventive maintenance can help to reduce the risk of power interruptions and lessen customer interruption costs. When the preventive maintenance benefits are evaluated in monetary terms, it is easier to understand and helpful for economic analysis. To achieve this objective, it requires the maintenance effectiveness of each maintenance task. In this research, the effectiveness factors of preventive maintenance tasks are obtained from the field surveys. Hundreds of engineers, technicians, maintenance crews, patrolmen, and skilled operators answered the questionnaires based on maintenance records, interruption records, and field experience. In addition, the survey conclusions were confirmed by the results of the maintenance model in [40]. Associated with the statistical comparison of before and after performing each task, the effectiveness factors of preventive maintenance tasks to prevent power interruptions due to equipment failure $\eta^e$, tree contact $\eta^p$, and animal contact $\eta^a$ are 0.05, 0.70, and 0.30, respectively.

The proposed method uses the effectiveness factor of maintenance tasks preventing power interruptions due to cause $c$, $\eta^c$, and the failure rate tendency index, $k_{\lambda,f}$, to assess the opportunity for the corresponding maintenance task to mitigate the interruptions due to cause $c$ in the following year. Thus, the benefits of preventive maintenance tasks corresponding to cause $c$ for feeder $f$ or $bpm^c_f$ can be described by Equation (6).

$$bpm^c_f = k_{\lambda,f} \times \eta^c \times CIC^c_f$$  \hspace{1cm} (6)

3.2. Costs of Preventive Maintenance

For each feeder, the costs of planned preventive maintenance tasks can be estimated from the historical costs. Alternatively, the unit cost per kilometer of these preventive maintenance tasks can be adopted to estimate the costs. Such information can be found in the yearly maintenance budget plans [41] and the actual maintenance expenses recorded in one fiscal year. In this study, the maintenance cost per feeder kilometer for patrol and condition-based maintenance, $m^e$, is approximately THB 160 per kilometer. The unit costs of tree trimming, $m^p$, and installing animal guards, $m^a$, are approximately THB 3200 and 1200 per kilometer, respectively.

Then, the costs of preventive maintenance tasks corresponding to cause $c$ for feeder $f$ or $cpm^c_f$ can be estimated from the unit cost per kilometer, $m^c$, multiplied by the length of feeder $f$, $L_f$, as expressed by Equation (7).

$$cpm^c_f = L_f \times m^c$$  \hspace{1cm} (7)

3.3. Benefit/Cost Analysis of Maintenance Tasks

Benefit/cost analysis of preventive maintenance tasks is an approach for evaluating the cost-effectiveness of maintenance tasks by comparing the benefits with the costs in terms of the benefit/cost ratio, BCR. Hence, the BCR of preventive maintenance tasks corresponding to cause $c$ for feeder $f$ or $BCR^c_f$ can be given by Equation (8) [5]:

$$BCR^c_f = \frac{bpm^c_f}{cpm^c_f}$$  \hspace{1cm} (8)

From the analysis of 3112 distribution feeders, 7587 or $H$ preventive maintenance tasks were initially suggested, as shown in Table 5. Then, the $BCR^c_f$ value of each task was evaluated and all were sorted in descending order. According to the BCR ranking, accumulating $bpm^c_f$ and $cpm^c_f$ of each task in a one-by-one fashion from $j = 1$ to $H$ will generate $BPM_j$ and $CPM_j$ as members of the datasets $BPM$ and $CPM$, respectively, in Table 6.
While BPM\textsubscript{j} and CPM\textsubscript{j} are defined as the benefit and cost of maintenance strategy \( j \), the method of cooperative game theory examines strategies for \( j = 1 \) to \( H \) to determine the optimal preventive maintenance program, as explained in the next section.

### Table 5. Number of preventive maintenance tasks categorized by maintenance type.

| Service Region | Patrol and Condition-Based Maintenance | Tree Trimming | Installing Animal Guards | Total |
|----------------|----------------------------------------|---------------|--------------------------|-------|
| 1              | 200                                    | 204           | 201                      | 605   |
| 2              | 213                                    | 222           | 220                      | 655   |
| 3              | 169                                    | 176           | 168                      | 513   |
| 4              | 126                                    | 107           | 109                      | 342   |
| 5              | 224                                    | 246           | 230                      | 700   |
| 6              | 137                                    | 132           | 140                      | 409   |
| 7              | 311                                    | 303           | 311                      | 925   |
| 8              | 294                                    | 302           | 293                      | 889   |
| 9              | 263                                    | 249           | 260                      | 772   |
| 10             | 184                                    | 182           | 182                      | 548   |
| 11             | 237                                    | 221           | 243                      | 701   |
| 12             | 176                                    | 176           | 176                      | 528   |
| Total          | 2534                                   | 2520          | 2533                     | 7587  |

### Table 6. BPM\textsubscript{j} and CPM\textsubscript{j} generated for strategy \( j \) from \( j = 1 \) to \( H \).

| BCR Ranking | bpm\textsubscript{j} | cpm\textsubscript{j} | Strategy No. | BPM | CPM |
|-------------|----------------------|----------------------|--------------|-----|-----|
| 1           | bpm\textsubscript{1} | cpm\textsubscript{1} | 1            | BPM\textsubscript{1} = bpm\textsubscript{1} | CPM\textsubscript{1} = cpm\textsubscript{1} |
| 2           | bpm\textsubscript{2} | cpm\textsubscript{2} | 2            | BPM\textsubscript{2} = bpm\textsubscript{2} + BPM\textsubscript{1} | CPM\textsubscript{2} = cpm\textsubscript{2} + CPM\textsubscript{1} |
| 3           | bpm\textsubscript{3} | cpm\textsubscript{3} | 3            | BPM\textsubscript{3} = bpm\textsubscript{3} + BPM\textsubscript{2} | CPM\textsubscript{3} = cpm\textsubscript{3} + CPM\textsubscript{2} |
| ...         | ...                   | ...                 | ...          | ...                     | ...                     |
| 4 \( j \)   | bpm\textsubscript{j} | cpm\textsubscript{j} | \( j \)      | BPM\textsubscript{j} = bpm\textsubscript{j} + BPM\textsubscript{j-1} | CPM\textsubscript{j} = cpm\textsubscript{j} + CPM\textsubscript{j-1} |
| ...         | ...                   | ...                 | ...          | ...                     | ...                     |
| H           | bpm\textsubscript{H} | cpm\textsubscript{H} | \( H \)      | BPM\textsubscript{H} = bpm\textsubscript{H} + BPM\textsubscript{H-1} | CPM\textsubscript{H} = cpm\textsubscript{H} + CPM\textsubscript{H-1} |

### 4. Cooperative Game Theory

For multi-objective optimization, it is difficult to pick one optimal solution that can improve one objective without worsening the others. According to game theory, optimization seeks to find a solution that results in a Pareto optimum and satisfies the requirements of all players. As investigated by previous research [42], the game theory approach is an effective technique to find the best compromise answer with lower computing time than combinatorial optimization.

In this work, cooperative game theory was employed to search for the most effective preventive maintenance program that resulted in a decent balance between the preventive maintenance costs and the benefits of reliability enhancement. The objectives to be optimized are modeled as game players \( i = (1 \ldots P) \), where \( P \) is the number of players. Thus, this optimization has two players, which are BPM and CPM, called \( P_1 \) and \( P_2 \), respectively.

Each player has decisions \( d = (1 \ldots D) \), where \( D \) is the number of decisions. In addition, decision \( d \) has a strategy set \( s = \{ s_{d,1}, \ldots, s_{d,j}, \ldots, s_{d,H} \} \) that presents all strategies available to play. The number of strategies \( H_d \) may or may not be equal for each decision, as shown in Figure 3.
For multi-objective optimization, it is difficult to pick one optimal solution that can satisfy the requirements (9) [26].

\[ K = \prod_{d=1}^{D} H_d \]  

(9)

In this research, the decisions represent the 12 service regions, so \( H_d \) is the total number of preventive maintenance tasks for service region \( d \). According to the number of preventive maintenance tasks for each service region in Table 5, the number of possible scenarios \( K \) as calculated by Equation (9) is up to \( 2.56 \times 10^{33} \). By constructing datasets for game-playing strategies, consistent with Table 6, this large number can be significantly reduced to only 7575 game scenarios. The number of game scenarios or matches, \( M \), can be calculated by Equation (10), where \( D \) is the number of decisions, i.e., 12.

\[ M = \sum_{d=1}^{D} H_d - D \]  

(10)

Scenario \( S_k \) as a set of strategies played by service regions \( r = (1 \ldots R) \) creates a particular function value for player \( i \), \( F_i(S_k) \). Therefore, the function values for players \( P_1 \) and \( P_2 \) resulting from \( S_k \) can be determined by Equations (11) and (12), respectively.

\[ F_1(S_k) = \sum_{r=1}^{R} BPM_{r,k} \]  

(11)

\[ F_2(S_k) = \sum_{r=1}^{R} CPM_{r,k} \]  

(12)

where

- \( F_1(S_k) \) is the function value for player 1 in scenario \( S_k \);
- \( F_2(S_k) \) is the function value for player 2 in scenario \( S_k \);
- \( S_k \) is a set of strategies \( s_{r,k} \) played by service regions \( r = (1 \ldots R) \);
- \( R \) is the number of service regions;
- \( BPM_{r,k} \) is the regional benefit of preventive maintenance tasks in strategy \( s_{r,k} \);
- \( CPM_{r,k} \) is the regional cost of preventive maintenance tasks in strategy \( s_{r,k} \).

If function values for players are not in the same unit, they can be normalized to a per-unit value using the sigmoid function. The normalized value is defined as a utility.
value that presents a player’s profit for a considered scenario. For player $i$, a profit in scenario $S_k$, $u_i(S_k)$ can be evaluated from the utility function shown in Equation (13) [42].

$$u_i(S_k) = \frac{1}{1 + e^{\epsilon F_i(S_k) - F_i(S_0)}}$$  

(13)

The adjustment multiplier for the sigmoid function, $\epsilon$, should be a near-zero value. If the objective of player $i$ is maximized, $\epsilon$ must be a positive value. On the other hand, if the objective is minimized, $\epsilon$ must be a negative value. However, they must have the same magnitude value for all players involved [26]. At the beginning of the game, the base scenario has been defined as the collection of the first strategies from all individual service regions as $S_0 = \{s_{1,1}, s_{2,1}, \ldots, s_{r,1}, \ldots, s_{R,1}\}$. As a result, the utility function of each player in the base scenario produces $u_i(S_0) = 0.5$ by using Equation (13). For each successive match, service region $r$ is allowed to shift its strategy step by step from $j = 2$ to $H_r$, as shown in Table 7, where $S_{r(j)}$ is the scenario in which service region $r$ plays its strategy $j$, while the other service regions use their strategies given in $S_0$.

The global utility is a value that represents the outcome of the cooperation among players. The global utility in scenario $S_k$, $u_G(S_k)$ is the summation of individual profits of all players divided by the number of players, $P$, as expressed by Equation (14) [42].

$$u_G(S_k) = \frac{\sum_{i=1}^{P} u_i(S_k)}{P}$$  

(14)

In Table 7, the best strategy for service region $r$, $s_{r,B_r}$, can be found when its global utility $u_G(S_{r(j)})$ is maximized. Accordingly, the best scenario, $S_B$, is the collection of the best strategies for all individual service regions, i.e., $S_B = \{s_{1,B_1}, s_{2,B_2}, \ldots, s_{r,B_r}, \ldots, s_{R,B_R}\}$ as summarized in the last row of Table 7. Then, the global utility value of the best scenario, $u_G(S_B)$, can be calculated by Equation (15) [42].

$$u_G(S_B) = \frac{\sum_{i=1}^{P} u_i(S_B)}{P}$$  

(15)

Finally, the optimal balance between the benefits and costs of preventive maintenance tasks resulting from the best scenario $S_B$ produces the function values of players 1 and 2, as expressed in Equations (16) and (17), respectively.

$$F_1(S_B) = \sum_{r=1}^{R} BPM_{r,B_r}$$  

(16)

$$F_2(S_B) = \sum_{r=1}^{R} CPM_{r,B_r}$$  

(17)

where

$F_1(S_B)$ is the function value of player 1 in the best scenario $S_B$;

$F_2(S_B)$ is the function value of player 2 in the best scenario $S_B$;

$S_B$ is the collection of the best strategies for all individual service regions;

$R$ is the number of service regions;

$BPM_{r,B_r}$ is the regional benefit of preventive maintenance tasks in strategy $s_{r,B_r}$;

$CPM_{r,B_r}$ is the regional cost of preventive maintenance tasks in strategy $s_{r,B_r}$. 

| Match | Scenario | Service Regions | Utility Values of Players | Global Utilities |
|-------|----------|-----------------|--------------------------|-----------------|
|       | Base     | $s_{0}$         | $u_{1}(S_{0})$           | $u_{C}(S_{0})$  |
| 1     | $S_{1(2)}$ | $s_{1,2}$       | $u_{1}(S_{1(2)})$       | $u_{C}(S_{1(2)})$ |
| 2     | $S_{1(3)}$ | $s_{1,3}$       | $u_{1}(S_{1(3)})$       | $u_{C}(S_{1(3)})$ |
|       | $S_{1(i)}$ | $s_{1,i}$       | $u_{1}(S_{1(i)})$       | $u_{C}(S_{1(i)})$ |
|       | $S_{1(H_{1})}$ | $s_{1,H_{1}}$ | $u_{1}(S_{1(H_{1})})$ | $u_{C}(S_{1(H_{1})})$ |
|       | $S_{1(2)}$ | $s_{1,2}$       | $u_{1}(S_{1(2)})$       | $u_{C}(S_{1(2)})$ |
|       | $S_{1(3)}$ | $s_{1,3}$       | $u_{1}(S_{1(3)})$       | $u_{C}(S_{1(3)})$ |
|       | $S_{2(1)}$ | $s_{2,1}$       | $u_{1}(S_{2(1)})$       | $u_{C}(S_{2(1)})$ |
|       | $S_{2(2)}$ | $s_{2,2}$       | $u_{1}(S_{2(2)})$       | $u_{C}(S_{2(2)})$ |
|       | $S_{2(3)}$ | $s_{2,3}$       | $u_{1}(S_{2(3)})$       | $u_{C}(S_{2(3)})$ |
|       | $S_{2(2,H_{2})}$ | $s_{2,H_{2}}$ | $u_{1}(S_{2(2,H_{2})})$ | $u_{C}(S_{2(2,H_{2})})$ |
|       | $S_{r(2)}$ | $s_{r,2}$       | $u_{1}(S_{r(2)})$       | $u_{C}(S_{r(2)})$ |
|       | $S_{r(3)}$ | $s_{r,3}$       | $u_{1}(S_{r(3)})$       | $u_{C}(S_{r(3)})$ |
|       | $S_{r(i)}$ | $s_{r,i}$       | $u_{1}(S_{r(i)})$       | $u_{C}(S_{r(i)})$ |
|       | $S_{r(H_{r})}$ | $s_{r,H_{r}}$ | $u_{1}(S_{r(H_{r})})$ | $u_{C}(S_{r(H_{r})})$ |
|       | $S_{R(2)}$ | $s_{R,2}$       | $u_{1}(S_{R(2)})$       | $u_{C}(S_{R(2)})$ |
|       | $S_{R(3)}$ | $s_{R,3}$       | $u_{1}(S_{R(3)})$       | $u_{C}(S_{R(3)})$ |
|       | $S_{R(i)}$ | $s_{R,i}$       | $u_{1}(S_{R(i)})$       | $u_{C}(S_{R(i)})$ |
|       | $S_{R(H_{r})}$ | $s_{R,H_{r}}$ | $u_{1}(S_{R(H_{r})})$ | $u_{C}(S_{R(H_{r})})$ |
|       | $S_{g}$     | $s_{g}$         | $u_{1}(S_{g})$           | $u_{C}(S_{g})$   |

The flowcharts, as shown in Figures 4 and 5, summarize the proposed approach.
Figure 4. Flowchart for constructing the datasets for game-playing strategies.
5. Results and Discussions

Two situations were stipulated for optimal preventive maintenance planning to investigate the effectiveness of the proposed methodology. In the first situation, called BCR prioritization, the cost-effective preventive maintenance tasks were selected according to their benefit-per-cost ratios under pre-specified different maintenance budgets. In the other situation, the proposed method was employed to find the most appropriate preventive maintenance tasks that balanced the benefits and costs.
According to the BCR prioritization, preventive maintenance plans can be achieved when a maintenance budget is assigned. This technique cost-effectively spends the budget to reach the most benefits of preventive maintenance. When the budget amount is varied with a step of THB 100 million (MB) from 100 MB to 700 MB, the most cost-effective plan for each amount is obtained as its CPM and BPM are plotted in Figure 6. However, one remaining question for the management is how much the budget should be specified to achieve a decent balance between the costs and benefits.

![Figure 6. The benefits and costs of preventive maintenance plans.](image-url)

To answer the question above, the proposed method extended from the BCR prioritization exploits cooperative game theory to locate the budget amount, resulting in an optimally balanced solution. It can be found where the global utility plotted against CPM reaches the maximum, as shown in Figure 6. In this case, the maximum global utility value or the global utility value of the best scenario \( u_G(S_B) \) is 0.51074848, and the total cost of the preventive maintenance plan equals 201.85 MB. To highlight the results, three comparable plans (budget \( \leq 100 \) MB, 200 MB, and 300 MB) of the BCR prioritization are selected to illustrate their global utility values against the proposed method in Table 8. In addition, the benefits, \( BPM_r \), costs, \( CPM_r \), and the number of selected tasks, \( B_r \), by service region are also compared among those plans in Table 9.

### Table 8. Comparisons of results between BCR prioritization and the proposed method.

| Approach          | Best Strategies by Service Region | Utility Values | Global Utility |
|-------------------|----------------------------------|----------------|----------------|
|                   |                                  | \( u_1(S_k) \) | \( u_2(S_k) \) | \( u_G(S_k) \) |
| BCR prioritization| CPM \( \leq 100 \) MB             | 1,310          | 0.522597       | 0.497490       | 0.510044       |
|                   | CPM \( \leq 200 \) MB             | 1,383          | 0.526493       | 0.495001       | 0.510747       |
|                   | CPM \( \leq 300 \) MB             | 1,461          | 0.528242       | 0.492503       | 0.510372       |

| Proposed method   | Best scenario                    | 1,384          | 0.526543       | 0.494954       | 0.510748       |
According to Table 9, when the budget rises from 100 MB to 300 MB, or triples, the benefit BPM increases by only 25.16% ((1130.87–903.54)/903.54). Such a result shows that an increase in preventive maintenance budgets may not offer worthy reliability improvement. On the other hand, the plan with the budget of 100 MB results in the highest BCR, but the increased reliability may be unsatisfactory for the customers. Consequently, the plan resulting from the proposed method, which is located by the maximum global utility, is the optimally balanced solution.

After the budget amount of the preventive maintenance plan is obtained, the percentage of allocated CPM for each service region is shown in Figure 7. The results show a wide range of variation in CPM among all regions because individual regions have different degrees of risks to mitigate. As presented in the risk assessment section, the time-varying failure rates of feeders can illustrate the likelihood of future interruptions. The proposed method utilizes the failure rate tendency index $k_\lambda$ as introduced in Equation (2) to describe a change in the failure rate at one year ahead. To be more explicit, $k_\lambda > 1$ presents the increasing failure rate, and $k_\lambda < 1$ indicates the decreasing failure rate. In the case of $k_\lambda = 1$, the failure rate does not change with time or is constant.

**Table 9.** Preventive maintenance plans obtained from the BCR prioritization and the proposed method.

| Service Region | Budget ≤ 100 MB | Budget ≤ 200 MB | Budget ≤ 300 MB | Best Scenario |
|----------------|----------------|----------------|----------------|---------------|
|                | BPMr | CPMr | Br | BPMr | CPMr | Br | BPMr | CPMr | Br | BPMr | CPMr | Br |
| 1              | 120.77 | 17.10 | 310 | 131.78 | 23.91 | 383 | 141.11 | 37.21 | 461 | 131.78 | 23.91 | 384 |
| 2              | 1.20 | 0.17 | 50 | 1.86 | 0.64 | 72 | 3.85 | 3.61 | 116 | 1.86 | 0.64 | 72 |
| 3              | 28.77 | 4.04 | 117 | 42.93 | 13.54 | 181 | 51.61 | 25.85 | 264 | 42.93 | 13.54 | 182 |
| 4              | 18.35 | 3.24 | 102 | 33.63 | 13.15 | 172 | 38.90 | 21.51 | 229 | 33.64 | 13.16 | 173 |
| 5              | 23.09 | 4.21 | 131 | 42.43 | 16.51 | 213 | 53.80 | 33.01 | 328 | 42.77 | 16.85 | 219 |
| 6              | 46.37 | 5.28 | 158 | 55.06 | 11.13 | 203 | 61.64 | 20.15 | 264 | 55.29 | 11.36 | 206 |
| 7              | 0.59 | 0.11 | 62 | 0.98 | 0.35 | 126 | 1.48 | 1.02 | 203 | 1.01 | 0.38 | 131 |
| 8              | 233.08 | 17.60 | 526 | 247.09 | 26.42 | 621 | 251.87 | 33.58 | 699 | 247.29 | 26.63 | 626 |
| 9              | 84.01 | 6.22 | 325 | 93.64 | 12.60 | 411 | 97.11 | 17.34 | 492 | 93.67 | 12.62 | 414 |
| 10             | 131.32 | 20.00 | 234 | 171.59 | 44.35 | 358 | 180.34 | 56.64 | 430 | 172.11 | 44.86 | 363 |
| 11             | 44.31 | 5.68 | 250 | 49.07 | 8.85 | 323 | 52.46 | 13.57 | 413 | 49.38 | 9.15 | 327 |
| 12             | 171.68 | 16.34 | 220 | 190.74 | 28.52 | 301 | 196.70 | 36.41 | 363 | 190.98 | 28.75 | 307 |
| Total          | 903.54 | 99.99 | 2485 | 1060.80 | 199.97 | 3364 | 1130.87 | 299.90 | 4262 | 1062.71 | 201.85 | 3404 |

**Figure 7.** The percentage of allocated preventive maintenance budget by service region.
Assuming the failure rate is constant when it is really not eases the failure rate computation. Yet, it may sacrifice some helpful information for risk assessment, such as the likelihood of future interruptions. To clarify this statement, the failure rates of all feeders are assumed constant \( k_\lambda = 1 \). Then, the proposed method is employed to reproduce the new results for \( k_\lambda = 1 \) to compare with the previous results in Figure 7. The comparison of CPM by service region shows differences between both cases, which imply the reduced accuracy of preventive maintenance planning when the time-varying failure rates are disregarded. For instance, CPM is excessively allocated to service region 3, while service region 10 should have received more expenditure. Similar to Figure 7, the numbers of selected preventive maintenance tasks by service region are compared in percentage terms in Figure 8. The differences between both cases are consistent with those presented in Figure 7. In addition, the number of tasks by service region does not necessarily depend on the allocated CPM because the costs of individual preventive maintenance tasks can be widely different due to a variety of circuit lengths and unit costs of various preventive maintenance activities.

![Figure 8. The percentage of the number of selected preventive maintenance tasks by service region.](image)

6. Conclusions

This research presents a methodology of optimal preventive maintenance planning for power distribution systems using the time-varying failure rates and cooperative game theory. For each feeder, the time-varying failure rate was determined, and its trend was analyzed to assess the likelihood of future interruptions. Meanwhile, the risk consequences were estimated using the interruption energy rates, customer-minutes of interruption, and total kVA of service areas. Then, the assessed risk of feeder interruptions was considered as an opportunity for mitigating the customer interruption costs by proper preventive maintenance tasks. Thus, it was defined as the benefits of preventive maintenance tasks. Subsequently, the BCR of each task was calculated so that all tasks could be ranked according to their BCR in descending order.

According to the BCR prioritization, the most cost-effective plan can be obtained by gathering the top-ranked tasks as many as possible under a given budget. However, a challenge for the management is how much the budget should be specified to achieve a decent balance between the costs and benefits. In this research, the proposed method extended from the BCR prioritization exploits cooperative game theory to set the budget amount that results in an optimally balanced solution.

To illustrate its practicability and effectiveness, the proposed method was applied to the extensive distribution networks of the 12 service regions covering all provincial
areas of Thailand. The time-varying failure rates were computed from three-year historical outage records for more than three thousand overhead distribution feeders. Then, the costs and benefits of preventive maintenance tasks corresponding to the outage causes were evaluated.

The BCR prioritization was performed by varying the budget from 100 MB to 700 MB. The most cost-effective plan for each amount was obtained, as its CPM and BPM were plotted in Figure 6. However, those results could not show how well the CPM and BPM were balanced. On the other hand, the proposed method successfully located the best-balanced solution where the global utility value was maximum. This global utility value was compared with those resulting from the BCR prioritization to confirm the results.

In the proposed method, cooperative game theory was directly applied to the strategies obtained from the ranked maintenance tasks. This problem formulation substantially reduced the search space or the number of scenarios, so it required less computational effort to explore the optimal result.

Furthermore, the case study assuming all the failure rates were constant \((k_\lambda = 1)\) was conducted to illustrate the other contribution of the proposed method. By comparing the allocated CPM by service region for \(k_\lambda = 1\) with the proposed method using the time-varying failure rates, the differences between both cases implied the reduced accuracy of preventive maintenance planning that neglects the time-varying failure rates.

The highlighted results discussed above have strengthened the contributions of the proposed method in aspects of practicability and effectiveness for preventive maintenance planning. Moreover, the presented method fully exploits utilities’ available information to reach the best decision making with light computational effort. Therefore, this method appeals to current electric utilities that strive to achieve efficient investment and become data-driven organizations.

**Author Contributions:** Conceptualization, D.R.; methodology, N.T.-a. and D.R.; software, N.T.-a.; validation, D.R.; formal analysis, N.T.-a.; investigation, N.T.-a. and D.R.; resources, D.R.; data curation, N.T.-a.; writing—original draft preparation, N.T.-a.; writing—review and editing, D.R.; visualization, N.T.-a. All authors have read and agreed to the published version of the manuscript.

**Funding:** This research received no external funding.

**Institutional Review Board Statement:** Not applicable.

**Informed Consent Statement:** Not applicable.

**Data Availability Statement:** Not applicable.

**Conflicts of Interest:** The authors declare no conflict of interest.

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