Comment on “No–Signaling Condition and Quantum Dynamics”

In carefully worded paper [1], the authors tried to derive linearity (i.e. affnity on ‘density matrices’ := DM) and complete positivity (CP) of general quantum mechanical dynamics $g$ from usual (nonrelativistic) kinematics of quantum mechanics (QM), and from an additional “no–signaling condition” (NS). I shall try to show here that the declared goals of [1] were not attained there.

The authors consider a given system $A$ in an arbitrary state described by a DM $\rho_A$, as a subsystem of a composed system $A&B$ occurring in a pure state $|\psi_{AB}\rangle$. The subsystems $A$ and $B$ are spacelike separated. Different convex decompositions of the reduced DM $\rho_A = \sum_k q_k \rho_k$ are obtained by different choices of discrete measurements on $B$. ‘Measurements’ of $A&B$ give trivial decomposition of $\rho_B$, other non-maximal measurements give decompositions of $\rho_A$ to density matrices. The pure-state decompositions (corresponding to maximal measurements) are interpreted in [1] as representing the corresponding different “probabilistic mixtures” (PM) in the sense of (classical) statistical ensembles (of quantal systems), sometimes in literature called Gemenge, or also genuine mixtures in [2].

The time evolution transformation $g$ (“not necessarily linear”) of $A$ “is a priori defined only on pure states...” [2, p. 2–3]. An explicit extension of $g$ to all considered states of $A$, e.g. to all decompositions $\{\rho_k, q_k\}$ of $\rho_A$, is essential, however, for the forthcoming discussion: Effects of any deterministic (no collapse!) semi-group of time transformations $g$ are supposed to be uniquely determined in QM by its initial conditions (“$g(P_0)$ does not have to be a pure state” of $A$ (!)). The following observation will also support my criticism: (*) “...the results of measurement on $A$ will be completely determined by the reduced DM of the system.” [1, pp. 2–3].

Decisive for proving linearity of $g$ is: (**) “...every PM of pure states corresponding to the DM $\rho_A$ can be prepared via appropriate measurements on $B$” (this is supported by calculations of probabilities at $A$ conditioned by results of measurements on $B$): such a process is classified in [2] as the “reduction of the wave packet”, i.e. a use of the projection postulate (having an ontological meaning), what is, however, strongly rejected in [1]. The linearity of $g$ is then implied by:

\[ g(\rho_A) = g(\sum \rho_j p_j P_{\psi_j}) = \sum \rho_j p_j g(P_{\psi_j}), \]

(first): The necessity of (†) in [1] is given by mere “statics” of [1], without NS, since that kinematics (embracing all ‘state space points • appearing as initial conditions for $g$, and also its values $g(\bullet)$) does not contain in [1] any means (i.e. corresponding observables) to ascertain locally a distinction between different kinds of interpretation of $\rho_A$, cf. (*); then the value of $g(\rho_A)$ should be here the same for $\rho_A$ considered as an indecomposable quantity describing a quantum state of each single system $A$ in an ensemble of equally prepared couples $A&B$, as well as for $\rho_A$ representing a specific ensemble of subsystems $A$ each of which being in one of the states $\rho_k$ taken from the set composing the chosen convex decomposition $\{\rho_k, q_k\}$ of $\rho_A$.

It can be introduced, however, a state space for $A$ (as it was partly done implicitly in [1]) consisting of all probability measures on density matrices (interpreted as corresponding PM’s, and encompassing different decompositions of the same density matrix as different points) with observables distinguishing them; let us define then $g(\rho_A) := \sum k q_k g(\rho_k)$ for the case of PM $\{\rho_k, q_k\}$, and let $g(\rho)$ be ‘independently’ given for any (not decomposed!) density matrix $\rho$, cf. [2, 2.1-e]. Then the proof of linearity of $g$ in [1] (with a use of NS) depends on possibility of an empirical check of (**) (i.e. of existence of physical differences between different decompositions of $\rho_A$ at the instant of the measurements on $B$) without a use of results of measurements on $B$. Its negative result (due to NS) does not imply (†): All the physically indistinguishable “at a distance prepared PM’s” are described by $\rho_A$ and all of them evolve to $g(\rho_A) \equiv g(\sum k q_k \rho_k)$ [$\neq g(\{\rho_k, q_k\})$ for nonlinear $g$].

(second): Assuming linearity and positivity of each physical time evolution transformation $g$, authors infer CP of $g$ by applying these properties to extensions $A&B$ of the considered system $A$. Their arguments consist, however, of a rephrasing of the definition of CP and of its physical motivation published in [2, Sec. 9.2].

My conclusion is that the authors did not succeed in their effort to prove in [1] effectiveness of new quantum-mechanical axiom called the “no–signaling condition”, and the declared aims of the paper [1] were not achieved.

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[4] E. B. Davies, Quantum Theory of Open Systems (Academic Press, London – New York – San Francisco, 1976).

[5] Boldface in quotations are my emphases. P.B.