Charge-dependence of the $\pi NN$ coupling constant and charge-dependence of the $NN$ interaction

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Abstract. The recent determination of the charged $\pi NN$ coupling constant, $g_{\pi \pm}$, by the Uppsala Neutron Research Group implies that there may be considerable charge-splitting of the pion coupling constant. We investigate the consequences of this for the charge-independence breaking (CIB) of the $^1S_0$ scattering length, $\Delta a_{CIB}$. We find that $\Delta a_{CIB}$ depends sensitively on the difference between $g_{\pi \pm}$ and the neutral $\pi NN$ coupling constant, $g_{\pi 0}$. Moreover, if $g_{\pi \pm}^2$ is only about 3% larger than $g_{\pi 0}^2$, then the established theoretical explanation of $\Delta a_{CIB}$ (in terms of pion mass splitting) is completely wiped out.

1 Introduction

From 1973 to 1987, there was a consensus that the $\pi NN$ coupling constant is $g_{\pi \pm}^2/4\pi = 14.3 \pm 0.2$ (equivalent to $f_{\pi}^2 = 0.079 \pm 0.001$). This value was obtained by Bugg et al. from the analysis of $\pi^\pm p$ data in 1973, and confirmed by Koch and Pietarinen in 1980. Around that same time, the neutral-pion coupling constant was determined by Kroll from the analysis of $pp$ data by means of forward dispersion relations; he obtained $g_{\pi 0}^2/4\pi = 14.52 \pm 0.40$ (equivalent to $f_{\pi 0}^2 = 0.080 \pm 0.002$).

The picture changed in 1987, when the Nijmegen group determined the neutral-pion coupling constant in a partial-wave analysis of $pp$ data and obtained $g_{\pi 0}^2/4\pi = 13.1 \pm 0.1$. Including also the magnetic moment interaction between protons in the analysis, the value shifted to 13.55 $\pm$ 0.13 in 1990.

Triggered by these events, Arndt et al. reanalysed the $\pi^\pm p$ data to determine the charged-pion coupling constant and obtained $g_{\pi \pm}^2/4\pi = 13.31 \pm 0.27$. **Dedicated to Walter Glöckle on the occasion of his 60th birthday.

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CD of $\pi NN$ coupling constant and CD of $NN$ interaction

In subsequent work, the Nijmegen group also analysed $np$, $\bar{p}p$, and $\pi N$ data. The status of their work as of 1993 is summarized in Ref. [11] where they claim that the most accurate values are obtained in their combined $pp$ and $np$ analysis. The latest analysis yields $g_{\pi^0}^2/4\pi = 13.47 \pm 0.11$ (equivalent to $f_{\pi^0}^2 = 0.0745 \pm 0.0006$) and $g_{\pi^\pm}^2/4\pi = 13.54 \pm 0.05$ (equivalent to $f_{\pi^\pm}^2 = 0.0748 \pm 0.0003$). The latest analysis of all $\pi^\pm p$ data below 2.1 GeV conducted by the VPI group using fixed-$t$ and forward dispersion relation constraints has generated $g_{\pi^\pm}^2/4\pi = 13.75 \pm 0.15$ [12].

The VPI $NN$ analysis extracted $g_{\pi^0}^2/4\pi \approx 13.3$ and $g_{\pi^\pm}^2/4\pi \approx 13.9$ as well as the charge-independent value $g_{\pi}^2/4\pi \approx 13.7$ [13, 14].

Also Bugg and coworkers have performed new determinations of the $\pi NN$ coupling constant. Based upon precise $\pi^\pm p$ data in the 100–310 MeV range and applying fixed-$t$ dispersion relations, they obtained the value $g_{\pi^\pm}^2/4\pi = 13.96 \pm 0.25$ (equivalent to $f_{\pi^\pm}^2 = 0.0771 \pm 0.0014$) [15]. From the analysis of $NN$ elastic data between 210 and 800 MeV, Bugg and Machleidt [16] have deduced $g_{\pi^\pm}^2/4\pi = 13.69 \pm 0.39$ and $g_{\pi^0}^2/4\pi = 13.94 \pm 0.24$.

Thus, it may appear that recent determinations show a consistent trend towards a lower value for $g_\pi$ with no indication for substantial charge dependence.

Unfortunately this is not true. There is one recent determination that does not follow the current trend. Using the Chew extrapolation procedure, the Uppsala Neutron Research Group has deduced the charged-pion coupling constant from high precision $np$ charge-exchange data at 162 MeV [17]. Their latest result is $g_{\pi^\pm}^2/4\pi = 14.52 \pm 0.26$ [18]. We note that the method used by the Upppsala Group is controversial [4, 19].

If one tries to summarize the confusing current picture then one may state that recent determinations of the neutral-pion coupling constant are, indeed, consistently on the low side with a value of $g_{\pi^0}^2/4\pi = 13.6 \pm 0.3$ covering about the range of current determinations.

However, there is no such consistent picture for the charged-pion coupling constant with recent determinations being up to nine standard deviations apart. If we trust the Uppsala result of $g_{\pi^\pm}^2/4\pi = 14.52 \pm 0.26$, then large charge-splitting of $g_\pi$ exists.

This is the motive for the present paper in which we will investigate the impact of charge-splitting of $g_\pi$ on our established theoretical understanding of the charge dependence of the nuclear force. In particular, we will look into the charge-independence breaking (CIB) of the $^1S_0$ scattering length, $\Delta a_{CIB}$. We find that $\Delta a_{CIB}$ depends sensitively on the difference between $g_{\pi^\pm}$ and $g_{\pi^0}$. Moreover, if $g_{\pi^\pm}$ is only moderately larger than $g_{\pi^0}$, the established theoretical explanation of $\Delta a_{CIB}$ (in terms of pion mass splitting) is completely wiped out.

2 Conventional explanation of the charge-dependence of the $NN$ interaction

The equality between proton-proton ($pp$) [or neutron-neutron ($nn$)] and neutron-proton ($np$) nuclear interactions is known as charge independence—a symmetry that is slightly broken. This is seen most clearly in the $^1S_0$ nucleon-nucleon scattering lengths. The latest empirical values for the singlet scattering length $a$
and effective range $r$ are [20, 21]:

\[
\begin{align*}
\bar{a}_N^{pp} & = -17.3 \pm 0.4 \text{ fm}, & \bar{r}_N^{pp} & = 2.85 \pm 0.04 \text{ fm}, \\
\bar{a}_N^{nn} & = -18.8 \pm 0.3 \text{ fm}, & \bar{r}_N^{nn} & = 2.75 \pm 0.11 \text{ fm}, \\
\bar{a}_{np} & = -23.75 \pm 0.01 \text{ fm}, & \bar{r}_{np} & = 2.75 \pm 0.05 \text{ fm}.
\end{align*}
\]

The values given for $pp$ and $nn$ scattering refer to the nuclear part of the interaction as indicated by the superscript $N$. Electromagnetic effects have been removed from the experimental values, which is model dependent. The uncertainties quoted for $\bar{a}_N^{pp}$ and $\bar{r}_N^{pp}$ are due to this model dependence.

It is useful to define the following averages:

\[
\bar{a} \equiv \frac{1}{2}(a_N^{pp} + a_N^{nn}) = -18.05 \pm 0.5 \text{ fm},
\]

\[
\bar{r} \equiv \frac{1}{2}(r_N^{pp} + r_N^{nn}) = 2.80 \pm 0.12 \text{ fm}.
\]

By definition, charge-independence breaking (CIB) is the difference between the average of $pp$ and $nn$, on the one hand, and $np$ on the other:

\[
\Delta a_{CIB} \equiv \bar{a} - a_{np} = 5.7 \pm 0.5 \text{ fm},
\]

\[
\Delta r_{CIB} \equiv \bar{r} - r_{np} = 0.05 \pm 0.13 \text{ fm}.
\]

Thus, the $NN$ singlet scattering length shows a clear signature of CIB in strong interactions.

The current understanding is that the charge dependence of nuclear forces is due to differences in the up and down quark masses and electromagnetic interactions. On a more phenomenological level, major causes of CIB are the mass splittings of isovector mesons (particularly, $\pi$ and $\rho$) and irreducible pion-photon exchanges.

It has been known for a long time that the difference between the charged and neutral pion masses in the one-pion-exchange (OPE) potential accounts for about 50% of $\Delta a_{CIB}$. Based upon the Bonn meson-exchange model for the $NN$ interaction [22], also multiple pion exchanges have been taken into account. Including these interactions, about 80% of the empirical $\Delta a_{CIB}$ can be explained [23, 24]. Ericson and Miller [25] obtained a very similar result using the meson-exchange model of Partovi and Lomon [26].

The CIB effect from OPE can be understood as follows. In nonrelativistic approximation [27] and disregarding isospin factors, OPE is given by

\[
V_{1\pi}(g_\pi, m_\pi) = -\frac{g_\pi^2}{4M^2} \frac{(\sigma_1 \cdot k)(\sigma_2 \cdot k)}{m_\pi^2 + k^2} \left( \frac{A^2 - m_\pi^2}{A^2 + k^2} \right)^n
\]

with $M$ the average nucleon mass, $m_\pi$ the pion mass, and $k$ the momentum transfer. The above expression includes a form factor with cutoff mass $\Lambda$ and exponent $n$.

For $S = 0$ and $T = 1$, where $S$ and $T$ denote the total spin and isospin of the two-nucleon system, respectively, we have

\[
V_{01}^{\text{OPE}}(g_\pi, m_\pi) = \frac{g_\pi^2}{m_\pi^2 + k^2} \frac{k^2}{4M^2} \left( \frac{A^2 - m_\pi^2}{A^2 + k^2} \right)^n.
\]
Table 1. Predictions for $\Delta a_{CIB}$ as defined in Eq. (4) in units of fm without and with the assumption of charge-dependence of $g_\pi$.

|                  | No charge-dependence of $g_\pi$ | Charge-dependent $g_\pi$: |
|------------------|---------------------------------|---------------------------|
|                  | Ericson & Miller [25] | Li & Machleidt [24] | $g_\pi^2/4\pi = 13.6$ |
| $1\pi$           | 3.50                          | 3.24                      | -1.58                    |
| $2\pi$           | 0.88                          | 0.36                      | -1.94                    |
| $\pi\rho, \pi\sigma, \pi\omega$ | —                              | 1.04                      | -0.97                    |
| **Sum**          | 4.38                          | 4.64                      | -4.49                    |
| **Empirical**    | 5.7 ± 0.5                     |                           |                          |

where the superscripts 01 refer to $ST$. In the $^1S_0$ state, this potential expression is repulsive. The charge-dependent OPE is then,

$$V_{1\pi}^{01}(pp) = V_{1\pi}^{01}(g_{\pi^0}, m_{\pi^0})$$

for $pp$ scattering, and

$$V_{1\pi}^{01}(np) = 2V_{1\pi}^{01}(g_{\pi^\pm}, m_{\pi^\pm}) - V_{1\pi}^{01}(g_{\pi^0}, m_{\pi^0})$$

for $np$ scattering.

If we assume charge-independence of $g_\pi$ (i.e., $g_{\pi^0} = g_{\pi^\pm}$), then all CIB comes from the charge splitting of the pion mass, which is $28$

$$m_{\pi^0} = 134.976\text{MeV},$$

$$m_{\pi^\pm} = 139.570\text{MeV}.$$ (10) (11)

Since the pion mass appears in the denominator of OPE, the smaller $\pi^0$-mass exchanged in $pp$ scattering generates a larger (repulsive) potential in the $^1S_0$ state as compared to $np$ where also the larger $\pi^\pm$-mass is involved. Moreover, the $\pi^0$-exchange in $np$ scattering carries a negative sign, which further weakens the $np$ OPE potential. The bottom line is that the $pp$ potential is more repulsive than the $np$ potential. The quantitative effect on $\Delta a_{CIB}$ is about 3 fm (cf. Table 1).

We now turn to the CIB created by the $2\pi$ exchange (TPE) contribution to the $NN$ interaction. There are many diagrams that contribute (see Ref. [24] for a complete overview). For our qualitative discussion here, we pick the largest of all $2\pi$ diagrams, namely, the box diagrams with $N\Delta$ intermediate states, Fig. 1. Disregarding isospin factors and using some drastic approximations [27], the amplitude for such a diagram is

$$V_{2\pi}(g_\pi, m_\pi) = -\frac{g_\pi^4}{16M^4} \frac{72}{25} \int \frac{d^3p}{(2\pi)^3} \frac{[\sigma \cdot k S \cdot k]^2}{(m_\pi^2 + k^2)^2(E_p + E_p^\Delta - 2E_q)\Lambda^2 + k^2} \left(\frac{\Lambda^2 - m_\pi^2}{\Lambda^2 + k^2}\right)^{2n},$$ (12)

where $k = p - q$ with $q$ the relative momentum in the initial and final state (for simplicity, we are considering a diagonal matrix element); $E_p = \sqrt{M^2 + p^2}$ and $E_p^\Delta = \sqrt{M_\Delta^2 + p^2}$ with $M_\Delta = 1232$ MeV the $\Delta$-isobar mass; $S$ is the spin...
Figure 1. $2\pi$-exchange box diagrams with $N\Delta$ intermediate states that contribute to (a) $pp$ and (b) $np$ scattering. The numbers below the diagrams are the isospin factors.
CD of $\pi NN$ coupling constant and CD of $NN$ interaction

transition operator between nucleon and $\Delta$. For the $\pi N\Delta$ coupling constant, $f_{\pi N\Delta}$, the quark-model relationship $f_{\pi N\Delta}^2 = \frac{72}{25} f_{\pi NN}^2$ is used [22].

For small momentum transfers $k$, this attractive contribution is roughly proportional to $m^{-4}_\pi$. Thus for TPE, the heavier pions will provide less attraction than the lighter ones. Charged and neutral pion exchanges occur for $pp$ as well as for $np$, and it is important to take the isospin factors carried by the various diagrams into account. They are given in Fig. 1 below each diagram. For $pp$ scattering, the diagram with double $\pi^\pm$ exchange carries the largest factor, while double $\pi^\pm$ exchange carries only a small relative weight in $np$ scattering. Consequently, $pp$ scattering is less attractive than $np$ scattering which leads to an increase of $\Delta a_{CIB}$ by 0.79 fm due to the diagrams of Fig. 1. The crossed diagrams of this type reduce this result and including all $2\pi$ exchange diagrams one finds a total effect of 0.36 fm [24]. Diagrams that go beyond $2\pi$ have also been investigated and contribute another 1 fm (see Table 1 for a summary).

In this way, pion-mass splitting explains about 80% of $\Delta a_{CIB}$.

3 Charge-dependence of the pion coupling constant and charge-dependence of the singlet scattering length

In this section, we will consider also charge-splitting of $g_\pi$, besides pion mass splitting.

As discussed in the Introduction, some current determinations of $g_\pi$ may suggest the values

$$g_{\pi^0}^2/4\pi = 13.6,$$  
$$g_{\pi^\pm}^2/4\pi = 14.4.$$  

Accidentally, this splitting is—in relative terms—about the same as the pion-mass splitting; that is

$$\frac{g_{\pi^0}}{m_{\pi^0}} \approx \frac{g_{\pi^\pm}}{m_{\pi^\pm}}.$$  

From the discussion in the previous section, we know that (for zero momentum transfer)

$$\text{OPE} \sim \left( \frac{g_\pi}{m_\pi} \right)^2$$  

and

$$\text{TPE} \sim \left( \frac{g_\pi}{m_\pi} \right)^4,$$

which is not unexpected, anyhow. On the level of this qualitative discussion, we can then predict that any pionic charge-splitting satisfying Eq. (15) will create no CIB from pion exchanges. Consequently, a charge-splitting of $g_\pi$ as given in Eqs. (13) and (14) will wipe out our established explanation of CIB of the $NN$ interaction.

We have also conducted accurate numerical calculations based upon the Bonn meson-exchange model for the $NN$ interaction [24]. The details of these calculations are spelled out in Ref. [24] where, however, no charge-splitting of $g_\pi$ was
considered. Assuming the $g_\pi$ of Eqs. (13) and (14), we obtain the $\Delta a_{CIB}$ predictions given in the last column of Table 1. It is seen that the results of an accurate calculation go even beyond what the qualitative estimate suggested: the conventional CIB prediction is not only reduced, it is reversed. This is easily understood if one recalls that the pion mass appears in the propagator \((m_\pi^2 + k^2)^{-1}\). Assuming an average \(k^2 \approx m_\pi^2\), the 7% charge splitting of \(m_\pi^2\) will lead to only about a 3% charge-dependent effect from the propagator. Thus, if a 6% charge-splitting of \(g_\pi^2\) is used, this will not only override the pion-mass effect, it will reverse it.

Based upon this argument and on our numerical results, one can then estimate that a charge-splitting of \(g_\pi^2\) of only about 3% (e.g., \(g_\pi^2 / 4\pi = 13.6\) and \(g_\pi^2 / 4\pi = 14.0\)) would erase all CIB prediction of the singlet scattering length that is based upon the conventional mechanism of pion mass splitting.

4 Conclusions

All current determinations of the neutral-pion coupling constant seem to agree on a ‘low’ value, like \(g_\pi^2 / 4\pi = 13.6 \pm 0.3\). However, for the charged-pion coupling constant, there is no such agreement. While some recent determinations of \(g_\pi^2 / 4\pi\) come up with a value close to \(g_\pi^2 / 4\pi\), the Uppsala group \([8]\) obtains \(g_\pi^2 / 4\pi = 14.52 \pm 0.26\) which implies a large charge-dependence of \(g_\pi\).

In this paper, we have investigated the consequences of such a large charge-dependence of \(g_\pi\) for the conventional explanation of the charge-dependence of the \(^1S_0\) scattering length, \(a_s\). We find that a charge-splitting of the coupling constant, defined by \(\Delta g_\pi^2 / 4\pi \equiv (g_\pi^2 - g_\pi^0) / 4\pi\), of \(\Delta g_\pi^2 / 4\pi = 0.4\) would wipe out the effect of the conventional mechanism (namely, pion mass splitting) and a splitting of \(\Delta g_\pi^2 / 4\pi = 0.8\) would even reverse the charge-dependence of \(a_s\) \([29]\).

Besides pion mass splitting, we do not know of any other essential mechanism to explain the charge-dependence of \(a_s\). Therefore, it is unlikely that this mechanism is annihilated by a charge-splitting of \(g_\pi\). This may be taken as an indication that there is no significant charge splitting of the \(\pi NN\) coupling constant.

Consequently, charge-dependence of \(g_\pi\) is most likely not the resolution of the large differences in recent \(g_\pi\) determinations; which implies that we are dealing here with true discrepancies. The reasons for these discrepancies may be large (unknown) systematic errors and/or a gross underestimation of the errors in essentially all present \(g_\pi\) determinations.

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1. Using \(\pi NN\) Lagrangians as defined in the authoritative review \([3]\), the relevant relationships between the pseudoscalar pion coupling constant, \(g_\pi\), and the pseudovector one, \(f_\pi\), are

\[
\frac{g_{\pi^0 pp}^2}{4\pi} = \left(\frac{2M_p}{m_{\pi^0}}\right)^2 f_{\pi^0 pp}^2 = 180.773 f_{\pi^0 pp}^2
\]  

\[(18)\]
and
\[
\frac{g_{\pi\pi}^2}{4\pi} = \left(\frac{M_p + M_n}{m_{\pi\pi}}\right)^2 f_{\pi\pi}^2 = 181.022 f_{\pi\pi}^2 .
\]  \hspace{1cm} (19)

with \( M_p = 938.272 \) MeV the proton mass, \( M_n = 939.566 \) MeV the neutron mass, and \( m_{\pi\pi} = 139.570 \) MeV the mass of the charged pion.

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29. We note that a negative value for $\Delta g_\pi^2 / 4\pi$ would enhance $\Delta a_{CIB}$. Since pion mass splitting explains only 80% of $\Delta a_{CIB}$, a charge-dependent coupling constant splitting of $\Delta g_\pi^2 / 4\pi \approx -0.1$ (e.g., $g_\pi^2 / 4\pi = 13.6$ and $g^2_\pi/4\pi = 13.5$) would explain the remaining 20%.