Mathematical Modeling, Moisture Diffusion, Energy consumption and Efficiency of Thin Layer Drying of Potato Slices

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Abstract

In this research, drying characteristics, energy requirement and drying efficiency for microwave drying of potato slices were reported. The drying experiments were carried out at 200, 250, 300, 350, 400, 450 and 500 W with slice thickness of 5 mm. In this study, measured values were compared with predicted values obtained from Page’s thin layer drying semi-empirical model according to R2, χ2 and RMSE. The increase in microwave power significantly reduced the drying time from 9.5 to 3.25 min of the potato slices. Experimental drying curves showed only a falling drying rate period. A third order polynomial relationship was found to correlate the effective moisture diffusivity (D_eff) with moisture content. The values of drying rate constant (k) and D_eff increased from 0.105 to 0.322 (1/min) and 1.013×10^-8 to 3.799×10^-8 m^2/s with the increase of microwave power level, respectively. Also, the effective moisture diffusivity increased with decrease in moisture content of samples. The activation energy for the moisture diffusion was determined to be 14.945 W/g. Drying efficiency increased with increase in microwave power and moisture content. The minimum and the maximum specific energy consumption and drying efficiency for drying of potato slices were determined as 4.645 (MJ/kg water) and 48.59% for 500 W and 5.882 (MJ/kg water) and 38.37% for 300 W, respectively.

Keywords: Microwave drying; Modeling; Moisture diffusivity; Energy consumption; Drying efficiency; Potato slice

Introduction

Potatoes represent the fourth most important vegetable crop for human nutrition in the world and an important source of carbohydrate for people in Asia [1]. Potato is a food with high initial water content and approximately 12% are dehydrated products [1-2]. Potato drying is a highly energy-consuming process. Also, the drying methods have significant effects on the dried potato quality such as nutritional values, color, shrinkage and other organoleptic properties. So far, many works have been performed to study hot air, tray dryer with and without air circulation, fluidized bed, and superheated steam drying of potato pieces of various shapes [1-15]. But commonly used hot air techniques are limited by high energy consumption, long drying times, low energy efficiency and high costs, which is not desirable for the food industry. Due to these difficulties, more rapid, safe and controllable drying methods are required. Also, it is necessary to dry the product with minimum cost, energy and time. In microwave drying, drying time is shortened due to quick absorption of energy by water molecules, causes rapid evaporation of water, resulting in high drying rates of the food [16-18].

One of the most important aspects of drying technology is the modeling of the drying process. There are various studies at the research level about drying of vegetables. For example; Bakal et al. [4] and Senadheera et al. [5] reported that the Page model best described the drying behaviour of potato. As little research has been performed effect of microwave power on energy consumption and drying efficiency in microwave drying method [7]. The present research is focused on this issue. The aim of this study was to (i) describe the influence of microwave output power on drying kinetics and energy efficiency, and (ii) compare the measured findings obtained during the drying of potato with the predicted values obtained with Page’s semi-empirical equation for the purpose of simulation and scaling up of the process.

Materials and Methods

Material

Potatoes were purchased from a local market, in Tehran, Iran, and were stored in the refrigerator at temperature of 4 ± 1°C until the experiments were carried out. The potatoes were washed with tap water, peeled and cut into slices with thickness of 5 mm. The initial moisture content of the samples found about 75 ± 1.5% (w.b.), and was determined by drying in an air convection oven at 103 ± 1°C till the weight did not change any more [14].

Experimental procedure

A domestic microwave oven (M945, Samsung Electronics Ins) with maximum output of 1000 W at 2450MHz was used for the drying experiments. The oven has a fan for air flow in drying chamber and cooling of magnetron. The moisture from drying chamber was removed with this fan by passing it through the openings on the right side of the oven wall to the outer atmosphere. The microwave dryer was operated by a control terminal which could control both microwave power level and emission time. For measuring the weight of the samples during experimentation without taking them out of the oven, the tray with sample was suspended on the balance with a nylon wire through a
ventilation hole in the center of chamber ceiling. Experiments were performed at five microwave powers of 200, 250, 300, 350, 400, 450 and 500 W. The moisture losses of samples were recorded at 15 s intervals during the drying process by a digital balance (GF-600, A and D, Japan) and an accuracy of ± 0.01 g. Drying was carried out until the final moisture content reaches to a level less than 4% (w.b.). All measurements were carried out in triplicate.

**Data analysis**

The moisture ratio (MR) was calculated using the following equation:

$$MR = \frac{X_t - X_e}{X_0 - X_e}$$

where, MR is the moisture ratio (dimensionless); $X_t$, $X_e$ and $X_0$ are the moisture content at any time, the equilibrium moisture content, the initial moisture content (kg water/kg dry mater), respectively. The values of $X_e$ are relatively small compared to $X_t$ and $X_0$, hence the error involved in the simplification by assuming that $X_e$ is equal to zero is negligible.

The Page’s model is an empirical modification of the simple exponential model to overcome its shortcomings. It was successfully used to describe the drying characteristics of a variety of biological materials. Therefore, the semi-empirical Page’s equation (Eq. (2)) was used to describe the thin layer drying kinetics of samples [4-5]:

$$MR = \frac{X_t}{X_0} = \exp\left(-ct^n\right)$$

where $k$ is the drying constant (1/min); and $n$ is the dimensionless exponent. Statistical test using the coefficient of determination ($R^2$), reduced chi-square ($\chi^2$) and root mean square error (RMSE) were calculated to evaluate the goodness of fit of Page’s model. The reduced $\chi^2$ and RMSE were calculated according to the following equation [14]:

$$\chi^2 = \frac{\sum_{i=1}^{N} (MR_{pre,i} - MR_{exp,i})^2}{N - z}$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N} (MR_{pre,i} - MR_{exp,i})^2}{N}}$$

where $MR_{exp}$ is the experimental dimensionless moisture ratio, $MR_{pre}$ is the predicted dimensionless moisture ratio by Page model, $N$ is the number of experimental data points, and $z$ is the number of parameters. The values of $\chi^2$ and RMSE were low.

**Activation energy:** In as much as temperature is not precisely measurable inside the microwave drier, the activation energy is found as modified from the revised Arehnious equation. In this method it is assumed as related to effective moisture diffusion and the ratio of microwave output power to sample weight (m/p) instead of to air temperature. Then Equation (9) can be effectively used as follows [16-17]:

$$D_{eff} = \frac{F_0}{(\pi L)^2}$$

where $F_0$ is the activation energy (W/g), $m$ is the mass of raw sample (g), $D_{eff}$ is the pre-exponential factor (m$^2$/s) and $P$ is the microwave power (W).

**Energy consumption and efficiency:** The energy consumption of microwave could be calculated as follows:

$$E_c = P \times t$$

The specific energy consumption is calculated using Eq. (14) [16].

$$E_s = \frac{P \times t}{m_w}$$

where $E_s$ is the specific energy consumption (J/kg water); $P$ is the microwave power (W); and $m_w$ is the total mass of evaporated water (kg).

The microwave drying efficiency was calculated as the ratio of heat energy utilised for evaporating water from the sample to the heat supplied by the dryer [16-18].

$$\eta = \frac{m_w \times \lambda_w}{P \times t}$$

where $\eta$ is the microwave drying efficiency (%) and $\lambda_w$ is the latent heat of vapourisation of water.
Results and Discussion

The moisture content versus drying time curves for microwave drying of potato slice samples as affected by various microwave powers are shown in figure 1. The time required to dry potato samples from initial moisture content of 75 ± 1.5% (w.b.) to the final moisture content of 4 ± 1% (w.b.) was 9.5, 8.5, 6.5, 5.75, 5, 3.75 and 3.25 min at 200, 250, 300, 350, 400, 450 and 500 W, respectively. Drying microwave power had an important effect on drying time. The results indicated that mass transfer within the sample was more rapidly during higher microwave power heating because more heat was generated within the sample creating a large vapor pressure difference between the center and the surface of the product due to characteristic microwave volumetric heating.

The moisture content data obtained from the drying experiments was fitted to the Page model. The statistical results from the models such as $R^2$, $\chi^2$ and RMSE values are shown in table 1. As it is seen, the $R^2$, $\chi^2$ and RMSE values range from 0.996 to 0.999, 0.00018 to 0.00023 and 0.01294 to 0.01477, respectively. The high values of $R^2$ and the low values of $\chi^2$ and RMSE indicate in the Page model a good fit. Based on the multiple regression analysis, the Page model, the constants and coefficients were as follows:

$$k = 0.044 \exp(0.0041P) R^2 = 0.961$$  \hspace{1cm} (16)

$$n = 1.5006 - 0.0015P + 10^{-6} P^2 R^2 = 0.945$$  \hspace{1cm} (17)

Thus:

$$MR = \exp(-0.044 \exp(0.0041P) t^{1.5006 - 0.0015P + 10^{-6} P^2})$$  \hspace{1cm} (18)

It is determined that the value of the drying constant $k$ increased with the increase in microwave power. This data points out that following the increase in microwave output power, drying curve becomes steeper, indicating faster drying of the product. Figure 2 shows the comparison between experimental moisture ratio at different drying powers and that predicted by the Page’s model. As can be seen, the dots in figure 2 are closely banding around a 45° straight line - a very good agreement between calculated and experimental data, which indicates that the Midilli model could adequately describe the drying behavior of potato slices.

Figure 3 shows how the drying rate of potato samples was changed with increased drying time under various drying conditions. The drying rates increased with the increasing microwave power levels. The moisture content of the material was very high during the initial phase of the drying which resulted in a higher absorption of microwave power and higher drying rates due to the higher moisture diffusion. As the drying progressed, the loss of moisture in the product caused a decrease in the absorption of microwave power and resulted in a fall in the drying rate. Similar results were obtained by several researchers in other foods such as tomato pomace [20], apple pomace [21], Gundelia tournefortii [22], okra [23], peach [24], parsley [18,25], garlic cloves [26], spinach [27], fish and shrimp [16,28].

Effective moisture diffusivity

Variation in effective moisture diffusivity of potato slices with moisture content at different microwave power levels is shown in figure 4. The effective moisture diffusivity increased with decrease in moisture content. However, the moisture diffusivity further was higher at any level of moisture content at higher microwave power level, resulting into shorter drying time. This may indicate that as moisture content decreased, the permeability to vapour increased, provided the pore structure remained open. The temperature of the product rises rapidly in the initial stages of drying, due to more absorption of microwave heat, as the product has a high loss factor at higher moisture content. This increases the water vapour pressure inside the pores and results in pressure induced opening of pores. In the first stage of drying, liquid diffusion of moisture could be the main mechanism of moisture transport. As drying progressed further, vapour diffusion could have been the dominant mode of moisture diffusion in the latter part of drying. Pickles [29], Sharma and Prasad [30]; Reyes et al. [6], Caglar et al. [31] and Sharma et al. [32] also reported similar trend in the variation in the moisture diffusivity with moisture content.

A third order polynomial relationship was found to correlate the effective moisture diffusivity with corresponding moisture content of samples and is given by Eq. (19)

$$D_{eff} = \left(A + BX + CX^2 + DX^3\right) \times 10^{-8}$$  \hspace{1cm} (19)

where A, B, C and D is the constants of regression, and X is moisture content (d.b.). Regression constants for microwave drying of potato slices under different powers are presented in table 2. The high values of $R^2$ are indicative of good fitness of empirical relationship to represent the variation in effective moisture diffusivity with moisture content of potato slices.

The variation in ln (MR) and drying time (t) for different powers have been plotted in figure 5 to obtain the slope S (Eq. 9) which can give the average effective moisture diffusivity ($D_{eff}$). The determined average values of $D_{eff}$ for different microwave powers are given in table 3. The values lie within the general range of $10^{-10}$ to $10^{-12}$ m²/s for food materials. It can be seen that the values of $D_{eff}$ increased with increasing microwave power. This might be explained by the increased heating energy, which

**Table 1:** Results of statistical analysis on the modeling (Page's model) of moisture content and drying time for potato slices.

| P(W) | Model constants | $R^2$ | $\chi^2$ | RMSE |
|------|----------------|------|--------|------|
| 200  | k=0.105, n=1.383 | 0.997 | 0.0023 | 0.01477 |
| 250  | k=0.125, n=1.412 | 0.997 | 0.0041 | 0.01972 |
| 300  | k=0.130, n=1.457 | 0.997 | 0.0032 | 0.01742 |
| 350  | k=0.174, n=1.642 | 0.999 | 0.0056 | 0.02269 |
| 400  | k=0.256, n=1.564 | 0.998 | 0.0018 | 0.01294 |
| 450  | k=0.289, n=1.762 | 0.996 | 0.0048 | 0.02069 |
| 500  | k=0.322, n=1.922 | 0.998 | 0.0023 | 0.01404 |
power. $5.612 \times 10^{-4}$ to $1.317 \times 10^{-8}$ m$^2$/s for fluidized bed drying of potatoes [4], $4.606 \times 10^{-4}$ to $7.065 \times 10^{-8}$ m$^2$/s freeze-drying of sweet potato cubes with far-infrared [15], $3.17 \times 10^{-7}$ to $15.45 \times 10^{-7}$ m$^2$/s for thin-layer drying of potato slices in length of continuous band dryer [14], and $2.90 \times 10^{-4}$ to $4.88 \times 10^{-8}$ m$^2$/s, and $3.15 \times 10^{-5}$ to $5.36 \times 10^{-5}$ m$^2$/s for convective, microwave and combined drying of potato cylinders, respectively [7]. The differences between the results can be explained by effect of method drying, type, composition, and tissue characteristics of the potatoes and the proposed model used for calculation.

After evaluation of the data, to determine the relationship between drying rate constant and effective moisture diffusivity, linear model, Eq. (20), which was derived by Ozbek and Dadali [17] were used with the coefficient of determination of (R$^2$) statistical value of 0.974. The value of constant (A) was obtained as $8.77 \times 10^6$ (s/min.m$^2$). The fitness of the data with Eq. (10) was illustrated in figure 6.

$$k = Y D_{eff}$$

(20)

![Figure 2: Comparison of experimental and calculated dimensionless moisture content values by the Page’s model.](image)

![Figure 3: Variation of drying rate with time for the potato slices.](image)

![Figure 4: Variation in effective moisture diffusivity with moisture content at different microwave powers.](image)

![Figure 5: Relationship between D$_{eff}$ and sample weight/microwave operating power for potato slices.](image)

would increase the activity of the water molecules leading to higher moisture diffusivity when samples were dried at higher microwave powers.

![Table 2: Average effective diffusivity values for microwave drying of potato slices.](image)

| P (W) | Effective diffusivity (m$^2$/s) |
|------|-------------------------------|
| 200  | 1.013 ($\pm 0.053$) $\times 10^{-8}$ |
| 250  | 1.317 ($\pm 0.094$) $\times 10^{-8}$ |
| 300  | 1.520 ($\pm 0.121$) $\times 10^{-8}$ |
| 350  | 2.127 ($\pm 0.085$) $\times 10^{-8}$ |
| 400  | 2.659 ($\pm 0.183$) $\times 10^{-8}$ |
| 450  | 3.343 ($\pm 0.203$) $\times 10^{-8}$ |
| 500  | 3.799 ($\pm 0.163$) $\times 10^{-8}$ |

![Table 3: Regression coefficients of effective moisture diffusivity for different microwave powers.](image)

| P (W) | A          | B          | C           | D           | R$^2$ |
|------|------------|------------|-------------|-------------|-------|
| 200  | 1.3075     | -1.1399    | 0.658       | -0.1679     | 0.999 |
| 250  | 1.6120     | -1.4102    | 0.7967      | -0.1910     | 0.999 |
| 300  | 2.1517     | -2.2570    | 1.3288      | -0.2998     | 0.992 |
| 350  | 2.4086     | -2.0564    | 1.099       | -0.2658     | 0.995 |
| 400  | 3.3906     | -4.179     | 2.9204      | -0.7306     | 0.965 |
| 450  | 4.0522     | -4.6663    | 3.0721      | -0.7915     | 0.985 |
| 500  | 4.5821     | -4.7721    | 2.7606      | -0.6376     | 0.994 |
Activation energy

The activation energy was calculated by plotting the natural logarithm of $D_{\text{eff}}$ versus sample amount/power ($m/P$) as presented in figure 7. The plot was found to be a straight line in the range of microwave power studied, indicating Arrhenius dependence. Then, the dependence of the effective diffusivity of potato samples on the microwave power can be represented by the following equation:

$$D_{\text{eff}} = 8.29 \times 10^{-8} \exp \left(\frac{-14.945}{m/P}\right) R^2 = 0.939 \quad \text{(21)}$$

The activation energy for potato samples was found to be 14.945 W/g. This value is higher than that corresponding to okra (5.54 W/g) [23], mint leaves (12.284 W/g) [17], sardine fish (14.1383 W/g) [16], shrimp (12.834 W/g) [28], pandanus leaves (13.6 W/g) [33], but lower than the value obtained for sweet and sour pomegranate (16.675 and 24.222 W/g) [34]. The lower activation energy translates to higher moisture diffusivity in the drying process [30].

Energy consumption and efficiency

Figure 8 shows the variation of energy efficiency whit drying time for microwave drying of potato samples. The energy efficiency was very high during the initial phase of the drying which resulted in a higher absorption of microwave power. Following moisture reduction, the energy absorbed by the product decreased and reflected power increased. The best result with regard to energy efficiency was obtained from 250 W microwave power levels among all microwave power. Similar trends were also observed by Soysal et al. [18] and Darvishi et al. [16].

The average energy needed for drying 1 kg of samples can be seen from figure 9. The values ranged from 4.645 to 5.882 MJ/kg water. Average energy efficiency of potato slice samples ranged from 38.37% to 48.59% for the output microwave power. The minimum specific energy consumption (4.645 MJ/kg) obtained at microwave power of 500 W.

Conclusion

The effect of microwave drying technique on moisture content, drying rate, drying time, effective moisture diffusivity, energy consumption and drying efficiency of potato slices were investigated. The changes of moisture content have been described by using Page’s model. The value of the drying rate constant, $k$, increased with the increase in microwave output power. Effective moisture diffusivity depends on the moisture content, and increases with decrease in moisture content. The values of effective diffusivity for microwave drying of potato ranged from $1.013 \times 10^{-8}$ to $3.799 \times 10^{-8}$ m²/s and activation energy was found 14.945 W/g. We concluded that 500 W is...
the optimum microwave power level in the microwave drying of potato with respect to drying time, specific energy consumption and drying efficiency. At the later stages of the drying process, drying efficiency values decreased considerably to a value as low as 10%.

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