Abstract—We show that noninterference and transparency, the key soundness theorems for dynamic IFC libraries, can be obtained “for free”, as direct consequences of the more general parametricity theorem of type abstraction. This allows us to give very short soundness proofs for dynamic IFC libraries such as faceted values and LIO. Our proofs stay short even when fully mechanized for Agda implementations of the libraries in terms of type abstraction.

1 Introduction

The goal of information flow control (IFC) research is to develop language-based techniques to ensure that security policies relating to confidentiality and integrity of data are followed, by construction. This paper is about a recent incarnation of this idea: IFC as a library. This appealing approach, pioneered by Li and Zdancewic [1] and championed by Russo et al. [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12] among others [13, 14, 15], promises to ease the integration of IFC techniques into existing software development pipelines, by replacing the specialized languages, compilers, and runtime systems traditionally needed for IFC applications, with libraries providing similar guarantees. Practically speaking, programming with an IFC library is similar to programming with a specialized IFC language, with one exception: Rather than being stand-alone, the library integrates with and uses the features of its host language to provide an interface that guarantees that all client programs are secure.

These libraries enforce IFC using two key language features:

Controlling side effects: most IFC libraries are implemented in safe [16] Haskell [17], a language that allows the library author to enforce type-based control over side effects (with the exception of non-termination).

Abstraction: all IFC libraries rely on type abstraction to provide data confidentiality and integrity.

But embedding IFC as a library risks leaving a gap in the soundness proofs, which usually do not cover how the library interacts with the host language. Indeed, the typical soundness proofs work by constructing a model of the library as a more-or-less standard IFC calculus, for which the authors prove some variant of the noninterference security property [18]. So although these noninterference proofs are sometimes mechanically verified [7, 10, 11, 13, 14], they give no guarantees about the realization of the calculus as a library.

The formal connection between the calculus and the library, which relies on host language type abstraction, was so far almost never investigated. We are the first to provide formal proofs explicitly covering the use of type abstraction for implementing dynamic IFC libraries, such as LIO. Moreover, the key to scaling security proofs to other advanced features of dynamic IFC libraries is employing better proof techniques.

In this paper we provide simpler proofs for Agda [19] implementations in terms of type abstraction of two different kinds of dynamic IFC libraries. On the one hand, we study the sequential LIO library [3], in which individual values can be labeled with metadata specifying their confidentiality and integrity levels and computations carry a “current label” that soundly over-approximates the level of already inspected labeled values. On the other hand, we study a library based on faceted values [7, 20], which are decision trees that can evaluate to different values based on the privilege level of the observer. These two styles reflect the most common ways to enforce dynamic IFC as a library.

To prove noninterference in a simple way, we give semantics controlling side effects to the libraries in terms of logical relations. Every type $T$ induces a binary relation $[T]$, such that every well-typed program $p : T$ is related to itself with respect to this relation. This connection between terms, types, and logical relations is called the fundamental lemma of logical relations, or the abstraction theorem [21], or parametricity. In his seminal work, Reynolds [21] uses this technique to show that users of an abstract type can never observe the details of its implementation. In this paper we apply this idea to dynamic IFC, showing that noninterference for dynamic IFC libraries is a direct consequence of the same parametricity theorem.¹

In practice, we implement the dynamic IFC libraries in a language with dependent types. This allows us to program our libraries and their clients in the same formalism we use to reason about the security of such programs. We also use the fact that (dependent) types are turned mechanically to logical relations and programs are turned mechanically to proofs of satisfaction of such relations, and everything remains neatly expressible within the framework of a language with dependent types [23]. This way, we do not need to prove any fundamental lemma for our logical relations, and additionally the junction between implementation and theory is watertight.

This proof technique was recently applied by Algehed and Bernardy [24] to show the noninterference of static IFC

¹Theorems directly obtained from parametricity are often called free theorems [22], hence the title of this paper.
libraries. The work in this paper differs in two crucial ways:

1. We show that the same technique can be applied to prove noninterference for dynamic IFC libraries. While this might seem counter-intuitive at first, since parametricity is a property of type abstraction, which is a static enforcement mechanism, our work shows formally that dynamic IFC libraries ultimately achieve their security also from type abstraction.

2. We show that the same proof technique can be used to prove more than just noninterference. Specifically, we use the parametricity theorem to also prove that our faceted values library is transparent [7, 25], ensuring that programs that are already secure do not have their semantics altered by our library.

More importantly, we inherit the simplicity of the parametricity-based proof technique. In particular, our completely mechanized proof of noninterference for a type abstraction-based implementation in Agda [19] of the state-of-the-art LIO library is an order of magnitude shorter than the previous partial Coq proof for a model of the same library [10]. Moreover, the simplicity of our proof technique allows us to cover more of our implementation of the LIO library than the previous Coq proof. Specifically, our mechanized LIO proof also covers the non-trivial mechanisms that enable soundly recovering from IFC and user-thrown exceptions [10, 26].

Concretely, we make the following contributions:

- We give Agda implementations of two core dynamic IFC libraries: LIO [3, 10] (Section 4.2) and a faceted values library inspired by the Faceted [20] and Multef [7] Haskell libraries (Section 2).
- We use the parametricity theorem (Section 3) for these dynamic IFC libraries to provide simple noninterference proofs for any client of each library (Section 4).
- We use the same proof technique to also show that our faceted values library satisfies transparency [7, 25], which ensures that this library does not alter the semantics of already secure programs. (Section 5).
- Finally, all our proofs have been fully mechanized in the Agda proof assistant, and are available as supplementary material for this paper.2 As a consequence of the use of parametricity for dependent types, the implementation of our libraries and the proofs are surprisingly short: our whole formal development for the two different libraries is less than one thousand lines of Agda.

While we use Agda as our language of choice in this paper, since it has a well worked out theory of parametricity (a generalization of Theorem 1 was already proved by Bernardy et al. [23]), we expect that our results are easily to apply to other languages with strong abstraction mechanisms and dependent types, like Coq and F∗.

2 Dynamic IFC as a Library

Before turning our attention to embedding dynamic IFC as a library, we give a quick primer on our host language, Agda: a total functional programming language with dependent types [19].

One characteristic of Agda is that terms, types, and propositions are unified. Thus, a single dependent arrow type-former (→) can be used to define function types and universal quantification. For example the type \((x : \text{Bool}) \rightarrow \text{Bool}\) is a function type whose domain and co-domain are Booleans. The type \((x : \text{Bool}) \rightarrow \text{true} \lor x \equiv \text{false}\) is an example of a quantified proposition over booleans. The type \((A : \text{Set}) \rightarrow \text{List} A \rightarrow \text{List} A\) is that of a polymorphic list-transformation. This last example illustrates quantification over types, which is seen as a dependent function whose domain is the type of types, written \(\text{Set}\) in Agda. (To avoid logical inconsistencies sets of sets are organized in a tower, such that \(\text{Set}_i : \text{Set}_{i+1}\), and \(\text{Set}\) is a shorthand for \(\text{Set}_0\).)

As a convenience feature, Agda offers implicit arguments. If a function domain is written with braces, for example \(f : \{A : \text{Set}\} \rightarrow \text{List} A \rightarrow \text{List} A\), then when calling the function \(f\) one will omit the corresponding argument, for example \(f\) \(\text{someList} : \text{List} \text{Bool}\) then \(A = \text{Bool}\). If doing so is impossible or ambiguous, Agda will report an error.

While Agda has many more features, we will only use a small time-tested subset. We also make use of records, which will be discussed below, and some limited use of data types, that work like in other functional languages.

Our first library is based on faceted values [20, 25] and is displayed in Figure 1. This is a straightforward port of the faceted values part of the Multef Haskell library [7] to Agda. Faceted values are binary decision trees that can evaluate to different values based on the privilege level of the observer. Formally, a faceted value \(f : \text{Fac} A\) can either be a regular value \(v : A\) that does not depend on who is observing it, in which case \(f = \text{return} v\), or it can depend on who is observing it, in which case it is a tree node containing a label \(\ell : \text{Label}\) (we assume a base-type \text{Label} of security labels explained below) and children \(f_0\) and \(f_1\) of type \text{Fac} A (i.e., \(f = \text{facet} \ell f_0 f_1\)). In the code in Figure 1, this tree structure is formalised by the \text{data _ where _ declaration}. Similarly to other functional languages like Haskell, it says that \text{Fac} : \text{Set} \rightarrow \text{Set} is a type-constructor mapping types to

```agda
module MultefImplementation where
data Fac : \text{Set} \rightarrow \text{Set} where
  return : \{A : \text{Set}\} \rightarrow A \rightarrow \text{Fac} A
  facet : \{A : \text{Set}\} \rightarrow \text{Label} \rightarrow \text{Fac} A \rightarrow \text{Fac} A \rightarrow \text{Fac} A

bind : \{A B : \text{Set}\} \rightarrow \text{Fac} A \rightarrow (A \rightarrow \text{Fac} B) \rightarrow \text{Fac} B
bind (return a) c = c a
bind (facet ℓ f₀ f₁) c = facet ℓ (bind f₀ c) (bind f₁ c)
```

Fig. 1. Faceted values part of Multef in Agda

2 Supplementary material available at https://github.com/MaximilianAlgehed/DynamicIFCTheoremsForFree
types, and that it has two constructors return and facet.

Returning to the meaning of facet ℓ₀ f₁, if an observer has access to level ℓ they will see f₀, otherwise f₁. For example, the faceted value facet Alice (return 0) (return 1) looks like 0 to anyone who is able to observe values with Alice’s confidentiality level, and looks like 1 to everyone else.

Combining two faceted values also yields a faceted value. For example, if we add

\[ f₀ + f₁ = \text{facet Alice (return } 0 + 2\text{) (return } 0 + 3\text{)} \]

\[ (\text{facet Bob (return } 1 + 2\text{) (return } 1 + 3\text{)} \]

By inspecting this value, we see that if the observer can observe both Alice’s and Bob’s data, they observe the sum f₀ + f₁ as 0 + 2, whereas an observer who can see only Bob’s data, not Alice’s, sees the sum as 1 + 2.

To formally define this addition operation on faceted values we follow the literature [7, 20] and show that faceted values form a monad [27, 28], which provides a general computational tool we can use to easily define operations like addition. A monad is a triple of a type former, \( M : \text{Set} \), and two operations, return : \( A \to M A \) and bind : \( \{A B : \text{Set}\} \to M A \to (A \to M B) \to M B \) that takes a monadic value \( m : M a \) and a continuation \( c : A \to M B \) and produces a new monadic bind \( m \circ c : M B. \)

In Figure 1, return is a constructor of the Fac type, and bind is defined as a function using pattern-matching. If \( f = \text{return } a \) then bind \( f c = c a \), and if \( f = \text{facet } \ell f₀ f₁ \) then bind \( f c \) is defined recursively as facet \( \ell \) (bind \( f₀ c \)) (bind \( f₁ c \)). In effect, this means that in bind \( f c \) the continuation \( c \) is computed once for every leaf of the faceted tree \( f \), potentially producing bigger faceted trees to replace the old leaves.

Using the monad operations, we can define addition and similar operations for Fac easily in the following manner:

\[
\text{bind } fx + fy = \text{bind } fx \lambda x \to \text{bind } fy \lambda y \to \text{return } (x + y)
\]

The syntactic form \( \lambda x \to \_ \) in the code above is simply Agda notation for lambda abstraction.

As illustrated by the addition example above, the bind operation from Figure 1 works by traversing faceted values in its operands and constructing a recursive faceted value, taking into account all possible observers.

Following the IFC literature [29], whether a level can be observed from another level is given by a partial order \(-\sqsubseteq -\) : \( \text{Label} \to \text{Label} \to \text{Bool} \). This order is also required to form a join semi-lattice, and thus it has a least element

\[ \bot : \text{Label} \text{ and the least-upper-bound always exists and is given by the function } \sqcap \_ : \text{Label} \to \text{Label} \to \text{Label} \].

Using this lattice structure, we can define project \( \ell f \), intuitively what a value of type \( f : \text{Fac } a \) will “look like” to an observer at level \( \ell \). We define project \( \ell f \) by recursion on \( f \):

\[
\text{project } : \{A : \text{Set}\} \to \text{Label} \to \text{Fac } A \to A
\]

\[
\text{project } \ell (\text{return } a) = a
\]

\[
\text{project } \ell (\text{facet } \ell' f₀ f₁) =
\]

\[
\text{if } \ell' \sqsubseteq \ell \text{ then }
\]

\[
\text{project } \ell f₀
\]

\[
\text{else }
\]

\[
\text{project } \ell f₁
\]

In turn, from the definition of project, we can define what it means for a program, for example a function \( p : \text{Fac } \text{Int} \to \text{Fac } \text{Int} \), to be secure. The program \( p \) is non-interfering if given any label \( \ell \) : \text{Label} and two faceted values \( f₀, f₁ : \text{Fac } \text{Int} \) such that project \( \ell f₀ \equiv \text{project } \ell f₁ \), we have that project \( \ell (p f₀) \equiv \text{project } \ell (p f₁) \). In other words, \( p \) does not reveal information from a different security level to an observer at level \( \ell \).

To see an example of this property in action, consider the Temp client module in Figure 2. When we give isCold the arguments \( f₀ \) and \( f₁ \) defined as:

\[
f₀ : \text{Fac } \text{Int}
\]

\[
f₀ = \text{facet Alice (return 10) (return 0)}
\]

\[
f₁ : \text{Fac } \text{Int}
\]

\[
f₁ = \text{facet Alice (return 30) (return 0)}
\]

we get:

\[
isCold f₀ = \text{facet Alice (return Cold) (return Cold)}
\]

\[
isCold f₁ = \text{facet Alice (return Hot) (return Cold)}
\]

If the observer level \( \ell \) is Bob, who cannot see Alice’s data (i.e., Bob \( \sqsubseteq \) Alice) we have that project \( f₀ Bob \equiv \text{project } f₁ Bob \) and also project (isCold \( f₀ \)) Bob \( \equiv \text{project } (\text{isCold } f₁) Bob \).

The goal of library-based IFC is to ensure that all client code behaves securely, as isCold does. However, suppose that we could write the code in Figure 3. Function isAlicePos uses

\[
\begin{align*}
\text{data } \text{HotOrCold} & : \text{Set} \to \text{where} \\
\text{Hot} & : \text{HotOrCold} \\
\text{Cold} & : \text{HotOrCold}
\end{align*}
\]

\[
\text{isCold} : \text{Fac } \text{Int} \to \text{Fac } \text{HotOrCold}
\]

\[
isCold f = \text{bind } fint \lambda x \to
\]

\[
\text{if } x > 25 \text{ then return } \text{Hot}
\]

\[
\text{else return } \text{Cold}
\]

\[
\text{isAlicePos} : \text{Fac } \text{Int} \to \text{Fac } \text{Bool}
\]

\[
isAlicePos (\text{facet Alice (return } n \text{) } f) = \text{return } (n > 0)
\]

\[
isAlicePos f = \text{return } \text{False}
\]
The type of types is \( \text{Set} \) and inhabitants of this type can also be seen as propositions. Type \( \top : \text{Set} \) is inhabited by a trivial \( \texttt{tt} : \top \) inhabitant, and thus represents Truth as a proposition:

\[
\text{data } \top : \text{Set} \text{ where } \texttt{tt} : \top
\]

Conversely \( \bot : \text{Set} \) is not inhabited by any term and thus represents Falsity. Conjunction \( \land : \text{Set} \to \text{Set} \to \text{Set} \) and disjunction \( \lor : \text{Set} \to \text{Set} \to \text{Set} \) are implemented as product and sum types, respectively. Finally, as also explained in the previous section, quantification and implication correspond to the dependent function type \( (x : A) \to B \).

Since types are propositions, their inhabitants are proofs. Concretely, if \( P : \text{Set} \) is a proposition (type) then \( \texttt{t} : P \) is a proof (inhabitant) of the proposition (type). For example, the canonical proof of proposition \( A \land B \to B \land A \) is the following function:

\[
\text{swap} : A \land B \to B \land A
\]

\[
\text{swap} \ (a , b) = (b , a)
\]

With this background in place, we now introduce parametricity and data abstraction for dependently typed languages. This proof technique is based on logical relations [33, 34], which are an elegant tool to prove properties about programming languages, and in particular IFC. The key idea is that one interprets every type as a relation. For every type \( A \), one builds a relation \( [A] \) — thus \( [A] \ a_0 \ a_1 \) is a proposition given two values \( a_0, a_1 : A \) and so \( [A] : A \to A \to \text{Set} \). One then proves the fundamental lemma of logical relations, also known as abstraction theorem, or parametricity theorem:

**Proposition 1** (Parametricity). If \( t : A \) then \( [A] \ t \ t \). That is, every program \( t \) of type \( A \) satisfies the relational interpretation of its type \( [A] \).

One often uses a custom logical relation [35], but there is a general, most fundamental way to interpret dependent types as relations, given by Bernardy et al. [23], which we adopt below for the syntax of Agda types.

**Definition 1.** (Relational interpretation of types) This meta-level definition works by induction on the structure of types.

\[
\begin{align*}
\llbracket \text{Set} \rrbracket & A_0 \ A_1 = A_0 \to A_1 \to \text{Set}, \\
\llbracket (x : A) \to B \rrbracket & f_0 \ f_1 = (x_0 : A_0) \\
& \quad \to (x_1 : A_1) \\
& \quad \to (x_r : [A] x_0 x_1) \\
& \quad \to \llbracket B \rrbracket \ (f_0 \ x_0) \ (f_1 \ x_1) \\
\llbracket \text{record field } f^t : A_i \rrbracket & r_0 \ r_1 = \text{record field} \\
& \quad f^t : [A_i] \ (f^t \ r_0) \ (f^t \ r_1) \\
\llbracket [B] \rrbracket & b_0 \ b_1 = b_0 \equiv b_1
\end{align*}
\]

As mentioned above, the type of types \( \llbracket \text{Set} \rrbracket \) is interpreted as a function from two types to \( \text{Set} \) (i.e., a relation). A function type is interpreted as a relation requiring that inhabitants map related arguments to related results. In particular, the

\[4\]Their theory is for pure type systems with inductive families, covering all the features of Agda that we use in this paper.
dependent function type \( (x : A) \to B \) binds \( x \) as a variable in \( B \). The single bound variable \( x \) is turned into three bound variables in the translated type, \( x_0 \) and \( x_1 \) that bind elements of \( A \), respectively, and \( x_r \) that binds a proof that \( x_0 \) and \( x_1 \) are related by \( [A] \). A record type is interpreted as a relation relating two instances of the record by relating all their fields. Finally, base-types (\( B \)), like booleans, are interpreted as propositional equality. In short, propositional equality at the type \( A \) is interpreted as a relational interpretation \( J \) that bind elements in \( A \) are related by \( [A] \). This means that Agda will only equate two terms \( a_0 \), \( a_1 : A \) if it can prove that they both reduce to some term \( a : A \).

To prove the fundamental lemma, one proceeds by giving a relational interpretation \( [[t]] \) for every term \( t \): In the development of Bernardy et al. [23] that we use in this paper, this interpretation is as follows.

**Definition 2.** Relational interpretation of terms

\[
\begin{aligned}
[[u \ t]] &= [[u]] [[t]] \\
\lambda x \to t &= \lambda x_0 \to \lambda x_1 \to \lambda x_r \to [[t]] \\
\text{record } \{ f' = t \} &= \text{record } \{ f' = [[t]] \} \\
[[x]] &= x_r \\
[a] &= \text{refl} \\
[[A]] &= \lambda a_0 \to \lambda a_1 \to [[A]] a_0 a_1
\end{aligned}
\]

This interpretation mimics the behavior of the relational interpretation of types. In particular, if the term is a base type constant (the penultimate case) then the proof of relatedness is simply reflexivity of equality, and if the term is a type (the last case), we construct an explicit relation and fall back to the interpretation for types. Thus, the interpretation of types as relation and the interpretation of terms as proofs can be unified, hence the use of a single notation \( [t] \) for both purposes. Moreover, the translation of function types is mimicked in the translation of lambda terms, a single bound \( x \) is turned into three bound \( x_0 \), \( x_1 \), and \( x_r \). Finally, each use of a variable \( x \) is turned into a use of the bound proof \( x_r \) that \( x_0 \) and \( x_1 \) are appropriately related. We refer the reader to Bernardy et al. [23] for details.

This relational interpretation of terms and types provides an once and for all proof of the parametricity theorem:

**Theorem 1.** (*Parametricity [23]*)

If \( t : A \) then \( [[t]] : [A] \to t \).

As an illustration, we show how to use Theorem 1 to prove properties about an abstract module and its clients. Consider the following (restricted) interface for Booleans:

```agda
record Booleans where
  field
  Bool : Set
  true : Bool
  false : Bool
  _\&_ : Bool \to Bool \to Bool
```

The above declares an abstract type, \( \text{Bool} \), two constants \( \text{true} \) and \( \text{false} \) of type \( \text{Bool} \), and a binary operation \( \& \) over \( \text{Bool} \). We instantiate this interface in the standard way:

```agda
module Impl where
data Impl where
  true : Bool
  false : Bool
  _\&_ : Bool \to Bool \to Bool
```

What may be surprising about this \( \text{Booleans} \) interface is that when we use it, we always end up writing monotonic functions. More precisely, if we write a function \( o : (\text{imp} : \text{Booleans}) \to \text{Bool} \to \text{Bool} \) imp, it is possible to prove, using parametricity, that if \( b_0, b_1 : \text{Impl} \). Bool are such that \( b_0 \) implies \( b_1 \) then \( o \text{ booleans } b_0 \) implies \( o \text{ booleans } b_1 \). Intuitively, this is because the \( \text{Booleans} \) interface gives the \( o \) function no way to do negation. This means that all \( o \text{ imp} \) can do to its \( b : \text{Bool} \) imp argument is to either discard it and return some other boolean or take its conjunction with some other boolean. These other booleans are either constants, or obtained by calling functions, which are themselves parametric in \( \text{imp} \). However, “by induction”, these functions are also monotonic and so \( o \) is monotonic.

Making this “by induction” phrase precise demands an argument based on logical relations. In Agda we can let Theorem 1 do the ground work, making the proof feel nearly automatic. To see how this works formally, we need to understand two things. Firstly, the **standard** relational interpretation of the Booleans interface ([Booleans], obtained mechanically by using the meta-level function from Definition 1), tells us how to relate two implementations of Booleans:

```agda
record [Booleans] (m₀ m₁ : Booleans) : Set where
  field
    Bool₀ : Bool₀ m₀ → Bool m₀ → Set
    true₀, (true m₀) (true m₁)
    false₀, (false m₀) (false m₁)
    _\&_, ∀ a₀ a₁ → Bool, a₀ a₁ →
    ∀ b₀ b₁ → Bool, b₀ b₁ →
    Bool₀ (\& m₀ a₀ b₀) (\& m₁ a₁ b₁)
```

This relation contains a custom logical relation \( (\text{Bool}_0)_r \), such that each method in the interface respects this relation (and \( \text{true}_r, \text{false}_r, \_\&_r \) are proofs witnessing this).

Secondly, because \( o \) is parameterized by \( \text{imp} : \text{Booleans} \), we have that \( [o] \) is parameterized over \( [\text{Booleans}] \):

```agda
[0] : (imp₀, imp₁, Booleans)
  → (imp₀, [Booleans] imp₀ imp₁)
  → (b₀, Bool imp₀) (b₁ : Bool imp₁)
  → (b₀, Bool, imp₀ b₀ b₁)
  → Bool₀ (o imp₀ b₀) (o imp₁ b₁)
```

Now all it takes to prove our theorem is to realize that:

3 Accessing a record field is done by treating the field name as a function with the analyzed record as an additional first argument.
1) We care about two Bool booleans, so we have to take
\[ \text{imp}_0 = \text{imp}_1 = \text{booleans}, \]
and
2) \( \text{imp}_r : [\text{Booleans}] \rightarrow \text{booleans} \) is an argument
to \([\text{o}]\) that we get to pick, and
3) the final thing we want to prove is that if
\[ b_0 \Rightarrow b_1, \text{ then } o \ldots b_0 \Rightarrow o \ldots b_1, \]
so we want that
\[ \text{Bool} \circ b_0 \circ b_1 = b_0 \Rightarrow b_1. \]
With insight (2) and (3) and a definition of \( \Rightarrow \) we can define the fields of
\[ \text{Booleans} \]
that each operation preserves these relations. In Section 4 we
define the relations necessary to prove noninterference and show that each operation in the library respects the relations.

4 Two Proofs of Noninterference

In this section we use the parametricity technique outlined above to prove noninterference for two dynamic IFC libraries. The first proof (Section 4.1) is for the faceted values part of the Multef Haskell library, which we have already introduced in Section 2. The second proof is for the significantly more complex LIO library (Section 4.2).

4.1 Noninterference for Faceted Values

Recall the faceted values interface in Figure 4 from Section 2. It exports a type \( \text{Fac} \) for faceted values and operations \( \text{facet}, \text{return}, \text{and bind} \) for manipulating them. The goal in this section is to show that any client library of this abstract interface obeys noninterference.

Intuitively, this means that any client function of \( \text{MultefImplementation} \) needs to take \( \ell \)-equivalent inputs to \( \ell \)-equivalent outputs. In order to make the above statement explicit, we recall the \( \text{sec} \) instantiation of \( \text{MultefInterface} \) and the definition of \( \text{project} \) from Section 2 and provide the following definition of \( \ell \)-equivalence: If \( A \) is a base-type and \( f_0, f_1 : \text{Fac} A \) we say that \( f_0 \) and \( f_1 \) are \( \ell \)-equivalent, written \( f_0 \sim(\ell) f_1 \), when:
\[ f_0 \sim(\ell) f_1 = \text{project} f_0 \ell \equiv \text{project} f_1 \ell \]
Where \( \equiv \) is propositional equality.

In order to prove noninterference, we need to prove that for a given base-type \( A \) (say \( \text{Bool} \)), function \( o : \text{Fac} A \rightarrow \text{Fac} A \) and faceted values \( f_0, f_1 : \text{Fac} A \) such that \( f_0 \sim(\ell) f_1 \) we have that \( o \circ f_0 \sim(\ell) o \circ f_1 \). However, this proposition only holds if \( o \) is a function in a client of the Multef library (more accurately, its abstract interface). In other words, noninterference only needs to hold for a function \( o : (m : \text{MultefInterface}) \rightarrow \text{Fac} m A \rightarrow \text{Fac} m A \).

Recall from the Booleans example in Section 3 that we can reason about \( o \) by providing a suitable \( \text{sec}_r : [\text{Multef}] \text{sec} \text{sec} \), a \( \text{proof} \) that \( \text{sec} \) from Section 2:
\[ \text{sec} : \text{MultefInterface} \]
\[ \text{sec} = \text{record} \{ \text{MultefImplementation} \} \]
satisfies the relational interpretation of its type. Formally, parametricity requires us to construct a \( f_r : [\text{A}] f f \) for each function \( f : A \) in the MultefInterface record type. Concretely, this means that to construct \( \text{sec}_r \), we need to construct Agda terms inhabiting the four types in Figure 5.

Picking the implementation of \( \text{sec}_r : [\text{MultefInterface}] \text{sec} \text{sec} \) that we will use in our noninterference proof is straightforward given our formulation of the \( f_0 \sim(\ell) f_1 \) relation above. Given \( \ell * \) as the attacker-level that we are concerned about, we pick:
\[ \text{Fac} r A_0 A_1 A_r f_0 f_1 = \text{project} f_0 (*) (\text{project} f_1 \ell *) \]
Note that if $A$ is a base type, like $\text{Bool}$, then $[A] = \Xi\Xi$ and so $\text{Fac}_{-} A \text{ Bool Bool} f 0 f 1$ is equivalent to $f 0 \sim\langle^*\rangle f 1$. The definitions of $\text{facet}_{-}$, $\text{return}_{-}$, and $\text{bind}_{-}$ are easy to fill out, and can be looked up in the Agda mechanization.

**Theorem 2** (Noninterference for Faceted Execution). Given: $o : (m : \text{MultefInterface}) \rightarrow \text{Fac} m \text{ Bool} \rightarrow \text{Fac} m \text{ Bool}$ We know that for all $f 0$, $f 1$: $\text{Fac} sec \text{ Bool}$, if $f 0 \sim\langle^*\rangle f 1$ then $o \text{ sec } f 0 \sim\langle^*\rangle o \text{ sec } f 1$

**Proof.** From Theorem 1, we know that the following holds:

$[0] : (m_0, m_1 : \text{MultefInterface}) \rightarrow (m_0, m_1 : \text{MultefInterface}) m_0 m_1$

$\rightarrow (f_0 : \text{Fac} m_0 \text{ Bool}) \rightarrow (f_1 : \text{Fac} m_1 \text{ Bool})$

$\rightarrow (f : \text{Fac} m_0 \text{ Bool Bool} f_0 f_1)$

$\rightarrow \text{Fac} f m_0 \text{ Bool Bool} f_0 f_1$

This means that:

$[0] \text{ sec } f_0 f_1$ assume : $o \text{ sec } f_0 \sim\langle^*\rangle o \text{ sec } f 1$

And is thus a valid Agda proof term for our theorem. □

### 4.2 Noninterference for Core LIO

Next we turn our attention to LIO. We first explain our Agda port of LIO and then how we use parametricity to prove noninterference for our port. Our code covers both the original LIO library [3] and a more recent extension to allow recovering from user-defined and IFC exceptions [10], but leaves out state and clearance (the latter a concern not directly related to IFC that could also be easily added). The interface of the core library can be seen in Figure 6, but most of the implementation has been elided for space. Instead on focusing on the code, we give the intuition behind this library.

The interface first defines $\text{Labeled } A$, the abstract type of labeled values of type $A$. If $v$ is a value of type $A$ and $\ell$ is an IFC label, then we can use the label $\ell$ $v$ operation to classify value $v$ at level $\ell$, which results in a labeled value of type $A$

It may be helpful to recall that $\text{Label}$ is a base-type, so the $\text{[Label]} \ell_0 \ell_1$ argument to $\text{facet}_{-}$ is equivalent to $\ell_0 \equiv \ell_1$.

Labeled $A$. Once we have a $\text{Labeled } A$ we can use $\text{label } 10 f$ to obtain its label, which witnesses the fact that, in LIO, the label on data is public information. Unlike the label, the value of a $\text{lv } : \text{Labeled } A$ is not public, but protected precisely by $\text{label } 10 f$ $\text{lv}$. This means that it would not be secure to simply extract the value of $\text{lv}$, so the LIO interface provides no such operation. Instead, to work with labeled values in a way that ensures IFC we need to turn to $\text{labeled LIO}$ computations.

An LIO computation keeps track of a $\text{current label}$, which is the upper bound of all labeled values already inspected by the computation. LIO threads through the current label, and in our case we also produce an explicit proof that the current label can only increase in the IFC lattice, or stay the same. This (monotonic) state passing makes LIO a monad, with $\text{return}$ and $\text{bind}$ operations having analogous type signatures to those for faceted execution. In addition, the LIO monad provides an $\text{unlabel}$ operation that soundly returns the value inside a...
labeled value \( \ell v \) by increasing the current label by \( \text{label}0 \! \ell \! v \):

\[
\text{unlabel} : \{ A : \text{Set} \} \rightarrow \text{Labeled} A \rightarrow \text{LIO} A
\]

This allows \( \text{LIO} \) computations to process labeled data, while using the current label to track both explicit and implicit information flows (i.e., flows through the control flow of the program [29]). Once we are done computing based on labeled values we can label the result and restore the current label to what it was at the beginning of the current computation. To prevent leaking information via the label of the final result, this label has to be chosen in advance before inspecting any labeled data. This functionality is implemented by the following operation:

\[
\text{toLabeled} : \{ A : \text{Set} \} \rightarrow \text{Label} \rightarrow \text{LIO} A \rightarrow \text{LIO} (\text{Labeled} A)
\]

The expression \( \text{toLabeled} \ell \text{lio} \) runs the \( \text{lio} \) computation and if at the end the current label is below \( \ell \) the result is labeled \( \ell \) and the current label restored. On the other hand, if at the end the current label is not below \( \ell \) we have to signal an IFC error. The original \( \text{LIO} \) [3] treated such errors as fatal and stopped execution, however a more recent extension of \( \text{LIO} \) [10] makes IFC errors recoverable. In the case of a wrongly annotated \( \text{toLabeled} \) though, throwing an exception would not be sound: we can restore the current label at the end only if that is a control-flow join point. To preserve this property, \( \text{LIO} \) returns instead a delayed exception [26], which is another kind of labeled value, labeled with the originally chosen level \( \ell \). When unlabeling it the delayed exception is re-thrown, which is sound because, as explained above, unlabeling a value raises the current label.

In addition to re-throwing delayed exceptions on \( \text{unlabel} \), \( \text{LIO} \) provides standard primitives to throw user exceptions and to catch arbitrary ones. In order to achieve soundness, though, \( \text{LIO} \) also has to delay any such exceptions at the end of \( \text{toLabeled} \). To make debugging easier, all exceptions carry information such as the current label at the time the exception was originally thrown together with a stack trace. In addition, IFC exceptions record additional information about the involved labels, when this information can be securely revealed (e.g., when the label check fails for \( \text{toLabeled} \ell \) it is secure to reveal the label \( \ell \), but not the current label that is not below \( \ell \)).

With this intuition in place, we can look at the actual definitions of the \( \text{Labeled} \) and \( \text{LIO} \) types. The definition of \( \text{Labeled} \) is straightforward: a labeled value is a pair of a \( \text{Label} \) and either a result of type \( A \), or a delayed exception of type \( E \).

\[
\text{Labeled} A = (A \sqcup E) \times \text{Label}, \text{where} \sqcup \text{denotes the tagged sum, or “disjunctive union”}:
\]

\[
\text{data \_\_ : Set} \rightarrow \text{Set} \rightarrow \text{Set} \text{where} \\
\text{inj}_1 : \{ A B : \text{Set} \} \rightarrow A \rightarrow A \sqcup B \\
\text{inj}_2 : \{ A B : \text{Set} \} \rightarrow B \rightarrow A \sqcup B
\]

The definition of \( \text{LIO} \) meanwhile, is more involved. A term of type \( \text{LIO} A \) is a function that takes a current label \( \ell c : \text{Label} \) and produces a result that we call a \textit{configuration}. A configuration is an output label together with either a result of type \( A \) or a delayed exception of type \( E \) (that’s either a user exception or an IFC exception whose details are omitted here). However, as noted above we make the \( \text{LIO} \) computation also produce a proof that the output label is at least as restrictive as the input label \( \ell c \). This addition is opaque to the programmer, who programs against the abstract interface of \( \text{LIO} \), but is useful for simplifying our proofs, since it prevents the logical relation below from being cluttered with this monotonicity proof. Consequently, the definition of \( \text{LIO} \) is as follows:

\[
\text{-- Configuration} \\
\text{Cfg A} = (A \sqcup E) \times \text{Label} \\
\text{-- LIO computation} \\
\text{LIO A} = (\ell c : \text{Label}) \rightarrow \Sigma (\text{Cfg A}) \rightarrow \text{Result} \\
\quad (\lambda r \rightarrow \ell c \sqsubseteq \pi_2 r) \rightarrow \text{Useful proof}
\]

This definition uses a generalized sum type, or \( \Sigma \)-type, to connect the proof of monotonic labels to the computation. The \( \Sigma \) type-former is given by the following record type:

\[
\text{record} \Sigma (A : \text{Set}) (B : A \rightarrow \text{Set}) : \text{Set where} \\
\quad \text{field} \quad \pi_1 : A \\
\quad \quad \pi_2 : B \pi_1
\]

and we additionally have the notation \( (a , b) \) for record \( \{ \pi_1 = a , \pi_2 = b \} \). Finally, to avoid confusion we note that the \( \_,\_\)-syntax is shared between \( \Sigma \)-types and simple product types \( A \times B \), indeed the latter is an instantiation of the former: \( A \times B = \Sigma A (\lambda _{-} \rightarrow B) \).

With this background in place, we turn to stating noninterference for \( \text{LIO} \). As in the previous subsection, we could do this for any client function that is parametric in the \( \text{LIO} \) interface, takes a labeled boolean input, and performs a \( \text{LIO} \) computation returning a boolean as result:

\[
\text{o' : (m:LIOMeasure)} \rightarrow \text{Labeled} m \text{ Bool} \rightarrow \text{LIO} m \text{ Bool}
\]

While the soundness of IFC libraries such as \( \text{LIO} \) was sometimes only proved with respect to such observers [3], such statements are too specialized to provide a sufficiently useful reasoning principle for most clients of the library. Instead, we generalize “\( \text{Labeled} m \text{ Bool} \)” and “\( \text{LIO} m \text{ Bool} \)” above to a first-order subset of \( \text{Agda} \) types, which includes base types, \( \text{LIO} \)-specific types and sum and products. We define this subset syntactically, as a new inductive type in \( \text{Agda} \):

\[
\text{data Univ : Set where} \\
\text{bool : Univ} \\
\text{nat : Univ} \\
\text{error : Univ} \\
\text{labeled : Univ} \rightarrow \text{Univ} \\
\text{plus_ : Univ} \rightarrow \text{Univ} \rightarrow \text{Univ} \\
\text{times_ : Univ} \rightarrow \text{Univ} \rightarrow \text{Univ}
\]

We interpret such syntax as the corresponding \( \text{Agda} \) type:

\[
\text{El : LIOMeasure} \rightarrow \text{Univ} \rightarrow \text{Set} \\
\text{El m bool} = \text{Bool} \\
\text{El m nat} = \text{N} \\
\text{El m error} = \text{E} \\
\text{El m (labeled u)} = \text{Labeled} m \text{ (El m u)} \\
\text{El m (lio u)} = \text{LIO} m \text{ (El m u)} \\
\text{El m (u0 plus u1)} = \text{El m u0} \sqcup \text{El m u1} \\
\text{El m (u0 times u1)} = \text{El m u0} \times \text{El m u1}
\]
Together, the above two constructions form a so-called “universe” of types — hence the name $\text{Univ}$ for the inductive set. This allows us to define a more general type of observers as functions taking first-order values as inputs (“$\text{El} \ m \ u_i$”) and returning first-order values (“$\text{El} \ m \ u_o$”):

\[
o : (u_i, u_o : \text{Univ}) \\
\rightarrow (m:\text{LIOInterface}) \rightarrow \text{El} \ m \ u_i \rightarrow \text{El} \ m \ u_o
\]

To state noninterference for $o$ we define $\ell^*$-equivalence by induction on the structure of our universe of first-order types. For $\text{Bool}$, $\mathbb{N}$, and $E$ we define $\ell^*$-equivalence simply as equality. For pair types $a \times b$ and sum types $a \sqcup b$ we define $\ell^*$-equivalence pointwise. In other words, two pairs $(a_0, b_0)$ and $(a_1, b_1)$ are $\ell^*$-equivalent if $a_0 \sim (\ell^*) a_1$ and $b_0 \sim (\ell^*) b_1$ and two sums $\inj_j x$ and $\inj_j y$ are $\ell^*$-equivalent if $i \equiv j$ and $x \sim (\ell^*) y$. We say that two labeled values $\ell v_0$, $\ell v_1 : \text{Labeled } a$ are indistinguishable at an observer label $\ell$, written $\ell v_0 \sim (\ell^*) \ell v_1$, if and only if:

1. $\ell \ell \text{labelOf} \ell v_0 \equiv \ell \ell \text{labelOf} \ell v_1$
2. if $\ell \ell \text{labelOf} \ell v_0 \sqsubseteq \ell$ then $\ell \ell \text{f} \ell v_0 \sim (\ell^*) \ell \ell \text{f} \ell v_1$

Point 1 says that the labels of $\ell v_0$ and $\ell v_1$ may never diverge from each other, since they are public information. Point 2 says that if $\ell v_0$ (and therefore also $\ell v_1$) is a public level, then the payloads of $\ell v_0$ and $\ell v_1$ have to be $\ell^*$-equivalent. The recursive call into $\ell^*$-equivalence happens at the sum type $a \sqcup E$, which ensures that the payloads are either $\ell^*$-equivalent or equal errors.

Next we turn our attention to the relation for LIO computations. Recall that $c_0$ and $c_1$ are internally state-passing computations where the state is the current label: $\text{Label} \rightarrow \Sigma ((a + E) \times \text{Label}) ...$. Because the current label is not necessarily public, but instead protects itself, the standard way to define $\ell^*$-equivalence for the current label is the following:

$\ell c_0 \sim (\ell^*) \ell c_1 = (\ell c_0 \sqsubseteq \ell^* \lor \ell c_1 \sqsubseteq \ell^*) \rightarrow \ell c_0 \equiv \ell c_1$

Two current labels are $\ell^*$-equivalent if they are equal whenever one of them is observable at level $\ell$. We can leverage this definition to define $\ell^*$-equivalence for final configurations of type $(a + E) \times \text{Label}$:

\[
(x_0, \ell c_0) \sim (\ell^*) (x_1, \ell c_1) = \\
(\ell c_0 \sim (\ell^*) \ell c_1) \\
(\ell c_0 \sqsubseteq \ell^* \land \ell c_1 \sqsubseteq \ell^*) \rightarrow x_0 \sim (\ell^*) x_1
\]

That is, for two configurations to be $\ell^*$-equivalent, we require that the current labels are $\ell^*$-equivalent and if they are public then the results of the computation should also be $\ell^*$-equivalent. With the above extensions of $\ell^*$-equivalence in place, it is easy to define $\ell^*$-equivalence for LIO computations:

$\ell c_0 \sim (\ell^*) \ell c_1 = (\ell c_0 \circ \ell c_1 : \text{Label}) \rightarrow \ell c_0 \sim (\ell^*) \ell c_1 \rightarrow \\
\ell \pi_1 (\ell c_0 \circ \ell c_1 \sim (\ell^*) \pi_1 (\ell c_1 \circ \ell c_1))$

In words, the above states that for any $\ell^*$-equivalent initial current labels we obtain $\ell^*$-equivalent final configurations (the $\pi_1$ projections are needed to ignore the proof part of the LIO type). This completes the definition of $\ell^*$-equivalence for our universe, and we can now state our noninterference theorem:

**Theorem 3 (Noninterference).** Given:

\[
o : (u_i, u_o : \text{Univ}) \\
\rightarrow (m:\text{LIOInterface}) \rightarrow \text{El} \ m \ u_i \rightarrow \text{El} \ m \ u_o
\]

For all $v_0$, $v_1 : \text{El} \ s e c \ u_i$, if

$v_0 \sim (\ell^*) v_1$

then

$\ell s e c \ v_0 \sim (\ell^*) \ell s e c \ v_1$

The general strategy of the proof is the same as for Multef. While out Agda proofs covers all the cases for $u_i$ and $u_o$, for simplicity, here we show only the special case where “$\text{El} \ m \ u_i = \text{Labeled m Bool}$” and “$\text{El} \ m \ u_o = \text{LIO m Bool}$”. In particular we use Theorem 1 to obtain:

\[
o : (m_0, m_1 : \text{LIOInterface}) \\
\rightarrow (m : ([\text{LIOInterface}] m_0 m_1)) \\
\rightarrow (\ell_0 : \text{Labeled m Bool}) \rightarrow (\ell_1 : \text{Labeled m Bool}) \\
\rightarrow (\ell_0, \ell_1 : \text{LIO m Bool Bool}) ([\text{LIO}] \ell_0 \ell_1)
\]

To use this result we first need to pick two relations $\text{Labeled} : ([\text{Set} \rightarrow \text{Set}] \text{Labeled})$ and $\text{LIO} : ([\text{Set} \rightarrow \text{Set}] \text{LIO})$ and prove that relatedness at these relations is respected by the LIO operations. Moreover, to be useful for proving noninterference these relations have to specialize to the $\ell^*$-equivalence relations used in the noninterference statement above, for all the types in our $\text{Univ}$ universe (e.g., for $\text{Labeled Bool}$).

For example, to define $\text{Labeled}$, we generalize the definition of $\ell^*$-equivalence to ensure that when $A_r$ coincides with $\ell^*$-equivalence, then $\text{Labeled}$, $A_0$, $A_1$, $A_r$ does too: $\text{Labeled} : A_0, A_1, A_r. \exists v_1 : (\text{Labeled m Bool Bool}) (\ell_0 \ell_1)(\ell_1 \ell_1)$

Where the relation $A_r : [\text{Set}] \text{Set}$ relates two values if they are either $\text{inj}_j$ and related by $A_r$ or $\text{inj}_j$ and related by $B_r$. In the second conjunct, we require that if the level of the two labeled values is public then either they are both errors related at $E_r$, or they both carry values of type $A$ that are related at $A_r$. The relation $E_r : \text{Set} \rightarrow \text{Set}$ relates two delayed exceptions if and only if they are the same. We do a similar generalization types in the $\text{Univ}$ universe and $\ell^*$-equivalence to arbitrary types for defining $\text{LIO}$.

The main part of our Agda proof of Theorem 3 is showing that the LIO operations respect the $\text{Labeled}$ and $\text{LIO}$ relations. Some of the LIO operations have straightforward proofs (label, return, labelOf, unlabel, and throw), while the higher-order operations (bind, toLabeled, and catch), have slightly more interesting proofs that rely on the monotonicity of the current label. Fortunately, all these proofs are pleasantly...
short adding up to around 360 lines of Agda for the complete noninterference proof in our supplementary material.

The short and fully mechanized Agda proof that we describe above can be contrasted with the previous partially mechanized proof for LIO [10]. This previous proof shows noninterference for an abstract calculus without exception handling and state, covering a strict subset of the features of the library implementation that we have verified here. Their proof technique, to show a simulation between evaluation of an LIO term with secrets and the same term with the secrets erased, is standard but cumbersome. Consequently, their proof amounts to over 3000 lines of Coq, even if it is not fully mechanized and it only covers a small calculus, not a library implementation in terms of type abstraction. Their proof could probably be finished and made shorter by using more tactic automation or a better proof strategy [26], yet it seems hard to match the conceptual simplicity and compactness of our parametricity-based proof.

5 Transparency

One of the primary justifications for faceted semantics is the so-called transparency theorem [25]. In short, transparency states that: for any program p that is noninterfering under a non-faceted, “standard” semantics, the behavior of p is preserved when p is run with faceted semantics. Intuitively, this means that there are no false alarms with faceted execution: if the program is noninterfering to begin with, facets don’t change anything. This is unlike systems like LIO, where false alarms are a problem that the programmer has to work around by adhering to proper programming style.

In our setting, this transparency property can be reformulated in terms of one of the key lemmas used to prove both noninterference and transparency for traditional faceted calculi: faceted evaluation simulates standard evaluation [25, 36].

To make sense of what this means in our context we need to explain the distinction between faceted and standard evaluation. Luckily, it is straightforward for us to define what we mean by different semantics for the same program: we simply give different implementations of the MultefInterface! In particular, the faceted semantics was already defined as the MultefImplementation module in Figure 1 from Section 2.

To define the standard semantics, we first introduce the Maybe (also known as “option”) type former, as a special case of the $\sqcup$ type, in Figure 7. With this in place, we define the standard semantics of Multef in Figure 8. Most of the definitions are unsurprising: Fac A is Maybe A, while return and bind are standard for the Maybe monad. The only potentially surprising definition is facet $f_0 f_1 = \text{nothing}$. To understand it, note that the standard phrasing of the transparency property in the literature makes reference to facet-free programs [7, 25, 36]. In our setting, we cannot easily talk about such “facet-free” programs, because all programs we study are clients of the MultefInterface, so instead we make do by talking about programs that do not return nothing under evaluation in the standard semantics.

**Theorem 4 (Transparency).** Fix a label $\ell*$ : Label. Given any $b : \text{Bool}$, define the faceted value $f^b$ as having value $b$ for observers that can see data labeled $\ell*$, and value false otherwise as:

$f^b = \text{facet sec Bool } \ell*(\text{return sec } b) \\
\quad \text{(return sec false)}$

For any client function $o$, parametric in MultefInterface:
\[ o : (m : \text{MultefInterface}) \rightarrow \text{Fac } m \text{ Bool } \rightarrow \text{Fac } m \text{ Bool}, \]
which does not crash under the standard semantics (std) when given the non-faceted constant $\text{b}$ as input:

\[ o \text{ std } (\text{just } b) \neq \text{nothing}, \]

then running $o \text{ std } (\text{just } b)$ yields the same result from the point of view of an observer at level $\ell*$ as running $o$ with the faceted semantics (sec) on input $f^b$:

\[ o \text{ std } (\text{just } b) \equiv \text{just Bool } (o \text{ sec } f^b (\ell*)) \]

To prove Theorem 4 we need to relate the execution of $o \text{ std}$ with the execution of $o \text{ sec}$. According to our setup, Theorem 1 gives us that if $o : (m : \text{MultefInterface}) \rightarrow \text{T}$ for some type T, then:

\[ [o] : (m_0 m_1 : \text{MultefInterface}) \\
\rightarrow (m_r : [\text{MultefInterface}] m_0 m_1) \\
\rightarrow [T] (\text{o } m_0) (\text{o } m_1) \]
In particular, if we can provide some \( \text{std-sec} : \text{MultefInterface} \to \text{std sec} \), then parametricity lets us relate \( \text{o std} \) and \( \text{o sec} \).

As we have seen with the previous proofs in this paper, the key to getting this to work is picking the correct instantiation of \( \text{Fac} \) in \( \text{std-sec} \). In this case the choice is clear from the theorem that we are trying to prove. We want that, when the standard (non-faceted) result is not nothing, the projection at \( \ell \) of the value resulting from the faceted execution is related to the standard value. In other words, we pick the following definition of \( \text{Fac} \) in \( \text{std-sec} \):

\[
\text{Fac}_r \triangleq \lambda \; A_0 \; A_1 \; A_r \; f_0 \; f_1 \to f_0 \not\equiv \text{nothing} \to \text{Maybe} \; A_0 \; A_1 \; A_r \; f_0 \; (\text{just} \; (\text{project} \; f_1 \; \ell *))
\]

From this definition, filling out \( \text{facet} \), \( \text{return} \), and \( \text{bind} \), is straightforward. With the definition of \( \text{std-sec} \) in place, we can tackle the proof of Theorem 4.

**Proof of Theorem 4.** From:

\[
o : (m : \text{MultefInterface}) \to \text{Fac}_r \; m \; \text{Bool} \to \text{Fac}_r \; m \; \text{Bool}
\]

We obtain by parametricity:

\[
[0] : (m_0 \; m_1 : \text{MultefInterface}) \to (m_0 \; m_1 : \text{MultefInterface} \to \text{m0 m1}) \to (f_0 : \text{Fac}_r \; m_0 \; \text{Bool}) \to (f_1 : \text{Fac}_r \; m_1 \; \text{Bool}) \to (f_r : \text{Fac}_r \; \text{m0 m1 Bool Bool Bool} \; (o \; m_0 \; f_0) \; (o \; m_1 \; f_1))
\]

We pick \( m_0 = \text{std} \), \( m_1 = \text{sec} \), \( m_r = \text{std-sec} \), \( f_0 = \text{just b} \), \( f_1 = f^b \), and let \( f_r \) be some (omitted) proof that \( f_0 \) and \( f_1 \) are appropriately related\(^9\)

\[
f_r : \text{Fac}_r \; \text{std-sec} \; \text{Bool Bool Bool} \; f_0 \; f_1,
\]

whose type becomes this after unfolding definitions:

\[
f_r : \text{just b} \not\equiv \text{nothing} \to \text{Maybe} \; \text{Bool Bool}_{\equiv} \; (\text{just b}) \to (\text{just} \; (\text{project} \; f^b \; \ell *))
\]

This gives us the following instantiation for \([0]\):

\[
[0] \; \text{std sec std-sec} \to (\text{just b}) \; f^b \; f_r : \text{o std} \; (\text{just b}) \not\equiv \text{nothing} \to \text{Maybe} \; \text{Bool Bool}_{\equiv} \; (\text{o std} \; (\text{just b})) \to (\text{just} \; (\text{project} \; (\text{o sec} \; f^b) \; \ell *))
\]

This is sufficient to easily establish the theorem. \( \square \)

The key takeaway from this proof is the generality of parametricity as a proof technique. While previous proofs have focused on the connection between noninterference and parametricity [24, 37, 38], the proof above shows that parametricity can also be useful for proving other interesting meta-theoretical properties about security libraries.

## 6 Related and Future Work

Dynamic IFC libraries, like LIO [3] and Multef [7], promise to provide noninterference guarantees without the need for a specialized tool-chain. Embedding such libraries in existing languages allows programmers to reuse the functionality and library ecosystem of the host language. Case studies show that this is a promising direction for IFC [3, 13, 39].

Stefan et al. [10] provide a noninterference proof for LIO partially mechanized in Coq, over which we improve in several ways. First, we cover a larger subset of LIO, by additionally supporting recoverable exceptions [10, 26], thrown by clients of the IFC mechanism itself. Second, our complete proof fits in just 360 lines of Agda, while the proof of noninterference of Stefan et al. [10] is more than 3000 lines of Coq. We achieve this order-of-magnitude conciseness improvement by using logical-relations to reason about LIO and by relying on Theorem 1 to automatically derive the relational interpretation of the “standard” parts of the language (crucially \( \lambda \)-abstraction): a large part of this proof one would traditionally need to carry out manually.

Third, and importantly, we certify an implementation of the library in terms of type abstraction, rather than a model of the library that ignores it—an issue which affects much of the library-based IFC literature. This issue is not surprising, because in order to be practical the libraries are implemented in languages such as Haskell which are lacking a formal semantics based on logical relations. Formalizing library-based IFC would require developing such a semantics first—a daunting task for a full-featured language. In contrast, we use a less mainstream language, Agda, but, in return, there is no informal, unverified, modeling step remaining in our approach.

To be sure, this means that we also do not claim to provide guarantees for the Haskell implementations of the libraries, but rather the Agda implementations in this paper. Specifically, this means that we only protect against Agda attackers, not the potentially more powerful Haskell attackers that previous approaches claim to defend against. However, we believe this paper demonstrates that as techniques for reasoning about parametricity for languages like Haskell are developed, the necessary meta-theory for practical IFC libraries will come within reach of our techniques.

Developing tools to cover the full Haskell language requires some innovation over the Agda parametricity theorems we rely on in this paper. Specifically, Haskell has a number of features not covered by the existing theory for Agda, including IO, concurrency, lazyness, and mutable state. Correctly reasoning about these features may require using a more expressive ambient logic than what is afforded by having just a dependently typed language like Agda. For example, separation logic has been used in previous work to provide parametricity-like reasoning for a subset of the language features needed to cover the full Haskell language [40].

Another difference between Haskell and Agda is that Agda is a total language, so all code written in it is guaranteed to terminate and so the termination channel [41] is a non-issue. Extending our techniques to deal with non-termination, and specifically to deal with libraries that protect against termination leaks (e.g., [7, 42]), is interesting future work. In particular, we hope to build on various monadic representations of non-termination in dependent type theory [43, 44].

Last but not least, as already mentioned in the introduction,
Algehed and Bernardy [24] introduced the technique of using parametricity for dependent types to show noninterference for a security library. They are concerned with static IFC libraries, while we show that the technique extends naturally to libraries for dynamic IFC libraries. Furthermore, in Theorem 4 we show that the parametricity proof-technique works well for proving meta-theoretical properties other than noninterference for security libraries using this approach.

7 Conclusion

This paper shows the versatility of parametricity as a proof technique for language-based security. Specifically, we show that parametricity can be used to prove noninterference for two different dynamic IFC libraries as well as transparency for the faceted values one. Ours are the first proofs of noninterference for the implementation of dynamic IFC libraries in terms of type abstraction. Furthermore, using parametricity allows us to give compact yet fully machine-checked proofs.

We believe that the simplicity of the proof techniques used in this paper will be key to scaling noninterference proofs to cover the actual code of feature-rich IFC libraries, including for instance mutable state [10, 45], concurrency [8, 42, 46], parallelism [47], and cryptography [4].

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