The Role of Ground State Correlations in the Single-Particle Strength of Odd Nuclei with Pairing.

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Abstract

A method based on the consistent use of the Green function formalism has been developed to calculate the distribution of the single-particle strength in odd nuclei with pairing. The method takes into account the quasiparticle-phonon interaction, ground state correlations and a "refinement" of phenomenological single-particle energies and pairing gap values from the quasiparticle-phonon interaction under consideration. The calculations for $^{121}\text{Sn}$ and $^{119}\text{Sn}$ that were performed in the quasiparticle⊗phonon approximation, have shown a reasonable agreement with experiment. The ground state correlations play a noticeable role and mostly improve the agreement with experiment or shift the results to the right direction.

\textit{Key words:} Single-particle strength, ground state correlations, pairing, Green functions.

1 Introduction

As is well known the role of the quasiparticle-phonon interaction is essential for the description of excitations in odd-mass nuclei \cite{1–3}. In magic \cite{1} and semi-magic \cite{4} nuclei it is possible to restrict ourselves to the approximation of the squared phonon creation amplitude $g^2$ in the propagators of the integral equations under study \cite{4}. In other words, it is necessary to take into account at least the complex configurations $1p\otimes\text{phonon}$ or $1h\otimes\text{phonon}$ (for non-magic nuclei we will use the unified notation $1qp\otimes\text{phonon}$). In the Green function

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(GF) language this means that for magic nuclei it is necessary to solve the Dyson equation with the mass operator

\[ M = \begin{array}{c}
\textcircled{1} \\
\textcircled{2}
\end{array} \]

where the circle is the phonon creation amplitude g.

The main difference of non-magic nuclei from magic ones is the necessity to take into account Cooper pairing in the nuclear ground state. This means that we should also consider the following simplest energy-dependent ”anomalous” mass operators [4,5]:

\[ M^{(1)} = \begin{array}{c}
\textcircled{1} \\
\textcircled{2}
\end{array}, \quad M^{(2)} = \begin{array}{c}
\textcircled{1} \\
\textcircled{2}
\end{array} \]  

where the lines with two ingoing and outgoing arrows denote the ”anomalous” GF \( F^{(1)} \) and \( F^{(2)} \) which are proportional to the gap. Pairing phonons are not included here because their contribution is probably small.

It is implied in the following that the initial quantities of our problem are ”observed” mean field described by a phenomenological potential, e.g. by the Woods-Saxon one, and pairing gap. The corresponding single-particle levels should be extracted from the observed excitation energies of non-magic nuclei (we will perform this procedure). The initial pairing gaps values are taken from experiment or solution of the BCS gap equation with the phenomenologically determined pp-interaction.

So far as the nuclear pairing problem is solved within the BCS approach as a rule, the quasiparticle-phonon interaction is not considered explicitly and quantitatively on this level. If we consider explicitly the contribution of the graphs (2) we must also take them into account in the pairing problem and , therefore, to avoid double counting, change the phenomenological pp-interaction entering the usual BCS problem. This question will be considered elsewhere but here we use a simpler phenomenological procedure of ”refining” the gap values from the quasiparticle-phonon interaction under consideration. It is analogous to refining the phenomenological single-particle energies in magic nuclei [6], [7].

In this work we study the role of the terms (2) and of complete taking into account ground state correlations to describe the excitations of \(^{119}Sn\) and \(^{121}Sn\). The important role of ground state correlations was shown long ago, e.g. for
the M1 excitations in $^{40}\text{Ca}$ [7] and $^{96}\text{Zr}$ [8]. For the excitations of odd non-magic nuclei this was considered in [9] but without any ”refining” procedure for the case of the phenomenological mean field. The above-mentioned procedure of ”refining” the gap and single-particle energies’ values will be also developed and realized here.

2 Equations for one-particle Green functions in non-magic nuclei

The general system of exact equations for the ”normal” GF’s G and $G^{(h)}$ and ”anomalous” GF’s $F^{(1)}$ and $F^{(2)}$ in a Fermi system with pairing has the form [10]:

\begin{align}
G &= G_0 + G_0 \Sigma G - G_0 \Sigma^{(1)} F^{(2)}, \\
F^{(2)} &= G_0^{(h)} \Sigma^{(h)} F^{(2)} + G_0^{(h)} \Sigma^{(2)} G,
\end{align}

where $G_0$ and $G_0^{(h)}$ are free GFs, i.e. the GFs of ideal gas, $\Sigma$, $\Sigma^{(1)}$, $\Sigma^{(2)}$ and $\Sigma^{(h)}$ are full irreducible self-energy parts (mass operators). Eq. (3) should be supplemented by the equations for $G^{(h)}$ and $F^{(1)}$. We use the symbolic form of equations very often.

It is very natural to single out explicitly known components of the mass operators. So we write

\begin{align}
\Sigma(\varepsilon) &= \tilde{\Sigma} + M(\varepsilon), \quad \Sigma^{(h)}(\varepsilon) = \tilde{\Sigma}^{(h)} + M^{(h)}(\varepsilon), \\
\Sigma^{(1)}(\varepsilon) &= \tilde{\Sigma}^{(1)} + M^{(1)}(\varepsilon) \equiv \tilde{\Delta}^{(1)} + M^{(1)}(\varepsilon), \\
\Sigma^{(2)}(\varepsilon) &= \tilde{\Sigma}^{(2)} + M^{(2)}(\varepsilon) \equiv \tilde{\Delta}^{(2)} + M^{(2)}(\varepsilon).
\end{align}

Here the first terms do not depend on the energy variable $\varepsilon$. The quantities $\tilde{\Sigma}$, $\tilde{\Sigma}^{(h)}$ correspond to a mean field and $\tilde{\Sigma}^{(1)}$, $\tilde{\Sigma}^{(2)}$ describe a pairing of the BCS type. The quantities $M$, $M^{(1)}$, $M^{(2)}$, $M^{(h)}$ (called further as $M^{(i)}$) are not defined so far. We mean that they contain the quasiparticle-phonon interaction. Due to the fact that, as was mentioned before, the single-particle energies $\varepsilon_{\lambda}$ and gaps $\Delta_{\lambda}$ are phenomenological, the quantities $M^{(i)}$ should give a contribution to $\varepsilon_{\lambda}$ and $\Delta_{\lambda}$. Therefore, to avoid the double counting of the quasiparticle-phonon interaction, we must ”refine” the first terms of the sums (4) and to obtain (see below) the corresponding $\tilde{\varepsilon}_{\lambda}$ and $\tilde{\Delta}_{\lambda}$ from $\varepsilon_{\lambda}$ and $\Delta_{\lambda}$. These ”refined” quantities are denoted by ”tilde”.

Taking Eqs. (4) into account the general system (3) can be transformed to the following equations (see the derivation in [4,5]):

3
\[ G = \tilde{G} + \tilde{G}MG - \tilde{F}^{(1)}M^{(h)}F^{(2)} - \tilde{G}M^{(1)}F^{(2)} - \tilde{F}^{(1)}M^{(2)}G \]  
\[ F^{(2)} = \tilde{F}^{(2)} + \tilde{F}^{(2)}MG + \tilde{G}^{(h)}M^{(h)}F^{(2)} - \tilde{F}^{(2)}M^{(1)}F^{(2)} + \tilde{G}^{(h)}M^{(2)}G, \]

(and the same for \( G^{(h)} \) and \( F^{(1)} \)). The bare GFs \( \tilde{G} \), \( \tilde{G}^{(h)} \) and \( \tilde{F}^{(1)} \), \( \tilde{F}^{(2)} \) are the known GFs of Gorkov. In the \( \lambda \) representation of single-particle wave functions they have the form:

\[ \tilde{G}_\lambda(\varepsilon) = \tilde{G}^{(h)}_\lambda(-\varepsilon) = \frac{\tilde{u}^2_\lambda}{\varepsilon - \tilde{E}_\lambda + i\delta} + \frac{\tilde{v}^2_\lambda}{\varepsilon + \tilde{E}_\lambda - i\delta} \]  
\[ \tilde{F}^{(1)}_\lambda = \tilde{F}^{(2)}_\lambda = -\frac{\Delta_\lambda}{2\tilde{E}_\lambda} \left( \frac{1}{\varepsilon - \tilde{E}_\lambda + i\delta} - \frac{1}{\varepsilon + \tilde{E}_\lambda - i\delta} \right) \]

where \( \tilde{u}^2_\lambda = 1 - \tilde{v}^2_\lambda = (\tilde{E}_\lambda + \tilde{\varepsilon}_\lambda)/(2\tilde{E}_\lambda) \), \( \tilde{E}_\lambda = \sqrt{\tilde{\varepsilon}^2_\lambda + \Delta^2_\lambda} \). The ”refined” quantities \( \tilde{\varepsilon}_\lambda \) and gaps \( \Delta_\lambda \) will be determined in the next Section.

The physical meaning of Eqs.(5) is that we have singled out explicitly the effects of mean field and Cooper pairing in a ”refined” form. In the hamiltonian approach, the latter corresponds to the Bogolubov’s transformation but without refining from the quasiparticle-phonon interaction in the case of taking it into account.

Let us define now the quantities \( M^{(i)} \). We will take them in the simplest \( g^2 \) approximation of Eqs. (1, 2). (In [4] it was shown that the dimensionless quantity \( g^2 \) is a small parameter for \(^{120}Sn\), i.e. \( g^2 < 1 \) for the most collective low lying \( 2^+_1 \) and \( 3^-_1 \) phonons.) Then the Eqs.(5) have the following graphical form:

\[ \begin{align*}
\begin{array}{c}
\begin{array}{c}
\text{original} \\
\text{approximation}
\end{array}
\end{array}
\end{align*} \]

\[ (\text{in our linearized version we need only two equations}). \] This corresponds to the approximation of \( 1qp + 1qp \otimes \text{phonon} \). Eqs.(7) take into account the ground
state correlation completely because we use two terms ("forward and backward
going graphs") both in Eq.(6) and in the formulae for all the mass operator
Eq.(1, 2) (see [4]), and, of course, in the QRPA calculations of the phonon
creation amplitude g. In the diagonal approximation for $M^{(i)}_{\lambda\lambda'}$, the secular equa-
tion, which determines the excitation energies of an odd nucleus with pairing,
is obtained from Eqs.(7) and has the form:

$$\Xi_{\lambda}(\varepsilon) \equiv \begin{vmatrix}
-\tilde{F}^{(2)}_{\lambda} M_{\lambda} - \tilde{G}^{(h)}_{\lambda} M_{\lambda}^{(2)} \\
-\tilde{F}^{(2)}_{\lambda} M_{\lambda} + \tilde{G}^{(h)}_{\lambda} M_{\lambda}^{(2)}
\end{vmatrix}
= 0 \quad (8)
$$

where our mass operators are given by the form

$$M_{1}(\varepsilon) = M^{(h)}_{1}(\varepsilon) = \sum_{s,2}(g_{12}^s)^2 \left( \frac{\tilde{u}_2^2}{\varepsilon - \tilde{E}_2 - \omega_s + i\gamma} + \frac{\tilde{v}_2^2}{\varepsilon + \tilde{E}_2 + \omega_s - i\gamma} \right),$$

$$M^{(1)}_{1}(\varepsilon) = M^{(2)}_{1}(\varepsilon) = -\sum_{s,2}(g_{12}^s)^2 \left( \frac{\tilde{\Delta}_2}{2\tilde{E}_2 (\varepsilon - \tilde{E}_2 - \omega_s + i\gamma) - \frac{1}{\varepsilon + \tilde{E}_2 + \omega_s - i\gamma}} \right),$$

here we have simplified the notations: index 1 $\equiv (\lambda_1) \equiv (n_1, j_1, l_1, m_1)$, $\omega_s$ is
the phonon energy and $g_{12}^s$ is the phonon creation amplitude.

The strength of the transition to the excited state $\lambda\eta$ under consideration
(spectroscopic factor) is given by

$$S^{\pm}_{\lambda\eta} = \frac{1 + q_{\lambda\eta})(E_{\lambda\eta} \pm \varepsilon_{\lambda\eta})}{\Theta_{\lambda}(E_{\lambda\eta})} \quad (9)$$

where

$$E_{\lambda\eta} = \sqrt{\varepsilon_{\lambda\eta}^2 + \Delta_{\lambda\eta}^2}, \quad \varepsilon_{\lambda\eta} = \frac{\tilde{\varepsilon}_\lambda + M_{(even)}(E_{\lambda\eta})}{1 + q_{\lambda\eta}}, \quad \Delta_{\lambda\eta} = \frac{\tilde{\Delta}_{\lambda}^{(1,2)} + M_{(even)}^{(1,2)}(E_{\lambda\eta})}{(1 + q_{\lambda\eta})}, \quad (10)$$

$$q_{\lambda\eta} = - \frac{M_{(odd)}(E_{\lambda\eta})}{E_{\lambda\eta}}, \quad \Theta_{\lambda}(\varepsilon) = (\varepsilon - \tilde{\varepsilon}_\lambda - M_{\lambda}(\varepsilon))(\varepsilon + \tilde{\varepsilon}_\lambda + M_{\lambda}^{(h)}(\varepsilon)) - (\tilde{\Delta}_\lambda + M_{\lambda}^{(1)}(\varepsilon))^2 \quad (11)$$
$M_{\text{even}}$ and $M_{\text{odd}}$ are the even and odd terms of the mass operator $M$ [11]. The denominator in Eq. (9) is the energy-derivative. Eqs.(8-12) determine the characteristics of odd nuclei with pairing in our approximation.

3 The refinement of phenomenological single-particle energies and gaps

The initial general system (3) can also be transformed to another form (see the derivation in [4]). Let us introduce the GF

$$
\tilde{G} = G_0 + G_0(\tilde{\Sigma} + \tilde{M})G = \tilde{G} + \tilde{G}MG
$$

where $\tilde{G}$ determines "refined" quasiparticle energies $\tilde{\varepsilon}_\lambda$ (we mean Landau’s quasiparticles here). Then the system (3) can be written as one equation for $G$:

$$
G = \tilde{G} - \tilde{G}\Sigma^{(1)}\tilde{G}^{(b)\Sigma^{(2)}G}.
$$

In Eq.(14) we use the approximation which is diagonal in the single-particle index $\lambda$. Let us represent the mass operator $M$ as a sum of its odd $M_{\text{odd}}$ and even $M_{\text{even}}$ terms [11] and determine the energies $E_\lambda$ of the observable quasiparticle levels as dominant solutions of Eq.(14). Then we obtain the general formulae which connect the observable $\{\varepsilon_\lambda, \Delta_\lambda\}$ and refined $\{\tilde{\varepsilon}_\lambda, \tilde{\Delta}_\lambda\}$ quantities:

$$
\varepsilon_\lambda = \tilde{\varepsilon}_\lambda + M_{\text{even} \lambda}(E_\lambda) / (1 + q_\lambda E_\lambda),
$$

$$
\Delta_\lambda \equiv \Delta^{(1,2)} = \tilde{\Delta}^{(1,2)}_\lambda + M^{(1,2)}_\lambda(E_\lambda) / (1 + q_\lambda E_\lambda)
$$

where $E_\lambda = \sqrt{\varepsilon^2_\lambda + \Delta^2_\lambda}$, $q_\lambda = -M_{\text{odd} \lambda}(E_\lambda)/E_\lambda$. Thus, if the phenomenological quantities $\varepsilon_\lambda$ and $\Delta_\lambda$ are known we can find the bare ones, which enter Eqs.(5), (6), (7), from the solution of the nonlinear relations (15).

4 Calculations of single-particle strength in $^{119}Sn$ and $^{121}Sn$.

At first we developed and realized the procedure to extract the phenomenological $\varepsilon_\lambda$ and $\Delta_\lambda$ from the observed excited quasiparticle levels $E_\lambda$. We used
Table 1
Refined neutrons single-particle energies $\tilde{\epsilon}_\lambda$ and gaps $\tilde{\Delta}_\lambda$ for $^{120}\text{Sn}$.

| $\lambda$   | $\epsilon_\lambda$, Mev | $\tilde{\epsilon}_\lambda$, 21 phon. | $\Delta_\lambda$, MeV | $\tilde{\Delta}_\lambda$, 21 phon. | $\gamma_\lambda$, % |
|------------|--------------------------|--------------------------------------|-----------------------|-------------------------------------|------------------|
| 2d5/2     | -2.67                    | -3.69                                | -3.41                 | 1.35                                | -0.23            |
| 1g7/2     | -1.36                    | -3.27                                | -2.54                 | 1.58                                | 0.76             |
| 2d3/2     | -0.09                    | -0.17                                | -0.24                 | 1.36                                | 1.09             |
| 3s1/2     | 0.13                     | 0.37                                 | 0.20                  | 1.27                                | 0.89             |
| 1h11/2    | 0.42                     | 0.86                                 | 0.91                  | 1.62                                | 1.41             |

the formula $E_\lambda = \tilde{\Delta} + E^\lambda_{ex}$, where $\tilde{\Delta}$ is the odd-even difference and $E^\lambda_{ex}$ are the excitation energies of $^{119}\text{Sn}$ and $^{121}\text{Sn}$, and determined the quantities $\epsilon_\lambda$ and $\Delta_\lambda$ from an iteration procedure. This procedure was organized in such a way that the $E_\lambda$ values could be reproduced by the formula $E_\lambda = \sqrt{\epsilon^2_\lambda + \Delta^2_\lambda}$. The $\epsilon_\lambda$ and $\Delta_\lambda$ obtained are given in Table 1 (for details see [4]).

Further, it is necessary to perform the "refining" procedure, i.e. to find $\tilde{\epsilon}_\lambda$ and $\tilde{\Delta}_\lambda$ from the known $\epsilon_\lambda$ and $\Delta_\lambda$. To do it the non-linear Eqs.(15) with our $g^2$ choice of $M^{(i)}$ have been solved. The results are given in Table 1 where $\gamma_\lambda$ and $\bar{\gamma}$ quantities were defined as follows

$$\gamma_\lambda = \frac{\Delta_\lambda - \tilde{\Delta}_\lambda}{\Delta_\lambda}, \quad \bar{\gamma} = \frac{\sum_\lambda \gamma_\lambda(2j_\lambda + 1)}{\sum_\lambda (2j_\lambda + 1)},$$

These values give the contribution of the quasiparticle-phonon pairing mechanism caused only by the retarded pp-interaction, i.e. by the $M^{(1)}$, $M^{(2)}$ contribution in Eq.(15). In Table 1 the calculations using 3 low-lying collective $2^+_1$, $3^-_1$, $4^+_1$ phonons and 21 phonons are given for 5 levels near to the Fermi surface. The phonons were calculated within the standard theory of finite Fermi systems with phenomenological $\epsilon_\lambda$ and $\Delta_\lambda$ which is permissible in our $g^2$ approximation because we omitted the $g^4$ terms. We obtained that the $\gamma_\lambda$ values depend rather strongly on $\lambda$ and the mean value $\bar{\gamma} = 32\%$, which was calculated for all the 8 levels under consideration.

At last, using the $\tilde{\epsilon}_\lambda$ and $\tilde{\Delta}_\lambda$ values as the initial data we solved Eq.(8) and calculated the spectroscopic factors, Eqs. (9, 12). The results and their comparison with the experiment [12], [13] are given in Table 2 for all the 5 levels below and above the Fermi surface. We obtained a reasonable agreement with experiment except for the $2d5/2$ and $1g7/2$ levels in $^{121}\text{Sn}$ where strength is very fragmented and small, i.e. it was not observed very reliably.

In Figs.1,2 and Table 2 the results "GSC−" obtained without taking into
| $\lambda$ | $E_x, MeV$ | $S_\lambda$ | $v^2_\lambda$ |
| --- | --- | --- | --- |
| | Exp. | GSC+ | GCS− | Exp. | GSC+ | GCS− | $u^2_\lambda$ |
| $2p^{\&}$ | 7.10 | 7.50 | 6.91 | 2.42 | 2.13 | 3.56 | 1 |
| $1g9/2$ | 5.41 | 5.90 | 5.40 | 0.53 | 0.43 | 0.43 | $\approx 1$ |
| $2d5/2$ | 1.10 | 1.69 | 1.69 | 0.43 | 0.46 | 0.3 | 0.95 |
| | 1.14 | 1.41 | 1.66 | 0.11 | 0.04 | 0.02 | 0.05 |
| $1g7/2$ | 0.79 | 1.81 | 0.95 | 0.75;0.6 | 0.66 | 0.77 | 0.83 |
| | 2.85 | 1.23 | 0.94 | 0.15 | 0.04 | 0.16 | 0.17 |
| $2d3/2$ | 0.02 | 0.01 | 0.02 | 0.4;0.45 | 0.4 | 0.48 | 0.52 |
| | 0 | $\approx 0$ | $\approx 0$ | 0.44;0.65 | 0.35 | 0.43 | 0.48 |
| $3s1/2$ | 0 | $\approx 0$ | $\approx 0$ | 0.26;0.32 | 0.36 | 0.42 | 0.45 |
| | 0.06 | $\approx 0$ | $\approx 0$ | 0.3 | 0.43 | 0.51 | 0.55 |
| $1h11/2$ | 0.09 | 0.18 | 0.15 | 0.29 | 0.26 | 0.34 | 0.37 |
| | 0.06 | 0.17 | $\approx 0$. | 0.49 | 0.43 | 0.56 | 0.63 |
| $2f7/2$ | | | | | | | $\approx 0$ |
| | 2.83 | 3.11 | 2.7 | 0.35 | 0.39 | 0.59 | $\approx 1$ |
| | | | | | | | (0.0-4.0) |
| $3p3/2$ | | | | | | | $\approx 0$ |
| | 3.73 | 4.97 | 4.9 | 0.54 | 0.42 | 0.4 | 1 |
| | | | | | | | (0.0-5.0) |
| $1h9/2$ | | | | | | | 0 |
| $+1i13/2^{(\&)}$ | 7.5 | 7.47 | 6.61 | $\approx 24$ | 19.92 | 22.23 | 1 |

*For the states without strong dominancy the energy intervals of summation are given. The energies $E_x$ correspond to the dominant peak position or to the mean energy.

$^{\&}$The quantities $S_\lambda \ast (2j + 1)$ are given.
$E_x^{exp} = 5.41 \text{MeV}$

Fig. 1. Strength distributions for the $1g9/2$ neutron state in $^{119}\text{Sn}$. The solid line gives the results with taking into account all the ground state correlations, the dashed line gives the results without them. The averaging parameter is 1.0MeV. The experimental data (histogram) are taken from [12].

$E_x^{exp} = 2.83 \text{MeV}$

Fig. 2. The same as in Fig.1 but for the $2f7/2$ neutron state in $^{121}\text{Sn}$ The experimental data (histogram) taken from [13].

account all the ground state correlations (except for the QRPA ones in our phonons) are also given. We see that the difference is rather noticeable. The inclusion of GSC mostly gives a correct trend, both for the levels near to the Fermi surface and for those beyond this surface, and improves the agreement with experiment. One can see from Fig.1,2 that it is more manifested for the ”differential” results than for the integral ones presented in Table 2.
We also investigated the role of the terms $M^{(1)}$, $M^{(2)}$ Eq.(2) of the anomalous mass operators both for the ground state ("refining" the gap) and for the description of excited states. It turned out that they give small contributions only for states corresponding to the single-particle levels which are very far from the Fermi surface. For the rest of the states considered, they have a contribution and improve the agreement with the experimental data rather often.

In conclusion, in order to calculate the distribution of single-particle strength in odd nuclei with pairing we formulated the method which takes consistently into account all ground state correlations (within the approximation used) and new terms, i.e. Eq.(2), of the "anomalous" mass operators which are specific for non-magic nuclei. For the first time the method includes also the refinement of the phenomenological pairing gap values from the quasiparticle-phonon interaction under study. A more exact than usual receipt of determining phenomenological single- particle energies for non-magic nuclei was realised here. The first calculation for $^{119}Sn$ and $^{121}Sn$ performed in the $1qp + 1qp \otimes phonon$ approximation showed a reasonable enough agreement with the available experimental data.

The mean value of the contribution of the quasiparticle-phonon pairing mechanism caused only by the retarded pp-interaction was obtained for the first time, it is about 32% for $^{120}Sn$. A noticeable numerical role of ground state correlations, although not so dramatic as in [9], was obtained.

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