Spectral Variation across Pulsar Profile due to Coherent Curvature Radiation

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Abstract

The pulsar profile is characterized by two distinct emission components, the core and the cone. The standard model of a pulsar radio emission beam originating from dipolar magnetic fields places the core at the center surrounded by concentric layers of inner and outer conal components. The core emission is expected to have steeper spectra compared to the cones. We present a detailed analysis of the relative differences between the spectra of the core and conal emission from a large sample of 53 pulsars over a wide frequency range between 100 MHz and 10 GHz. The core spectra were seen to be much steeper than those of the cones, particularly between 100 MHz and 1 GHz, with a relative difference between the spectral index of $\Delta\alpha_{\text{core/cone}} \sim -1.0$. In addition, we found the spectra of the outer conal components to be steeper than those of the inner cone with a relative difference in the spectral index of $\Delta\alpha_{\text{in/out}} \sim +0.5$. The flattening of the spectra from the magnetic axis toward the edge of the open field line region with increasing curvature of the field lines is a natural outcome of the coherent curvature radiation from charged soliton bunches and explains the difference in spectra between the core and the cones. In addition, due to the relativistic beaming effect, the radiation is only visible when it is directed toward the observer over a narrow angle $\theta \lesssim 1/\gamma$, where $\gamma$ is the Lorentz factor of the outflowing plasma clouds. This restricts the emission particularly from outer cones that are associated with field lines with larger curvature, thereby making their spectra steeper than those of the inner cones.

Unified Astronomy Thesaurus concepts: Pulsars (1306); Radio pulsars (1353)

Supporting material: machine-readable tables

1. Introduction

The radio emission from normal-period pulsars ($P > 0.1$ s) with measured flux density over a wide frequency range, between 100 MHz and 10 GHz, exhibits an inherently steep power-law spectra with a typical spectral index of $\alpha \sim -1.8$ (S $\propto f^{-\alpha}$; Lorimer et al. 1995; Maron et al. 2000; Jankowski et al. 2018). In most cases, the spectral index is obtained from the total flux density estimated across the average pulse profile. The pulsar profiles have complex shapes, usually comprising of several Gaussian-like components, and detailed phenomenological studies suggest that the components have specific locations within the pulse window. The pulsar radio emission beam has a roughly circular shape and can be described by the so-called “core–cone” model. According to this model, the beam consists of a central core surrounded by two rings of nested conal emission, namely, the inner and outer cones (Rankin 1990, 1993; Mitra & Deshpande 1999). The width and shape of the pulse profile depends on the pulsar geometry and observers’ line of sight (LOS) traverse across the emission beam. Central LOS cuts form core–cone Triple ($T$) and core–double cone multiple ($M$) profile types, while the profile shape becomes shaped as a conal Quadruple ($Q$), conal Triple ($T$, and double ($D$) as the LOS traverses move progressively away from the magnetic axis. $T_{1/2}$ is another form of core–cone profile with two components, where a conal component on one side of the core is either missing or too weak to detect at certain frequencies. A core single ($S_t$) profile signifies a central LOS traverse where the conal components are absent, while a conal single ($S_o$) profile corresponds to peripheral LOS cuts. Table 1 summarizes the classification scheme of the pulsar profile types. It has also been shown that the distribution of both the core and the conal component widths with the period has similar lower boundary lines, which are proportional to $P^{-0.5}$ (Maciesiak et al. 2012; Skrzypczak et al. 2018), suggesting similar emission heights for both. The pulsar emission is usually highly polarized and shows a polarization position angle (PPA) swing across the profile that resembles an S-shaped curve. This is a result of the emission originating from a strictly dipolar magnetic field region (rotating vector model; Radhakrishnan & Cooke 1969). One of the most significant details about the origin of radio emission comes from constraints on its location at heights of 100–1000 km from the stellar surface. These are obtained from two independent estimates, the geometrical morphology described above (Rankin 1993; Kijak & Gil 1998; Mitra & Rankin 2002; Kijak & Gil 2003) and the aberration–retardation effect that shifts the PPA from the profile center, with the displacement being proportional to the emission height (Blaszkiewicz et al. 1991; Phillips 1992; Xilouris et al. 1996; Mitra & Li 2004; Weltevrede & Johnston 2008; Krzeszowski et al. 2009).

Once it is established that the emission arises from a narrow range of heights in the dipolar magnetic field region, a gradual variation in certain emission properties is also expected due to changes in curvature, from the central field lines near the axis toward the edge of the open field line region. This is reflected in the PPA, particularly in the central LOS traverses where the core component is associated with large PPA variations, while the PPA traverse is relatively flat across the cones (Rankin 1990, 1993).
Apart from having a specific location within the pulsar beam, the core and conal flux densities exhibit different spectra, with the core emission having a steeper spectra compared to the cones (Rankin 1983). This effect is once again best observed in T and M profiles, for which the core, inner, and outer cones are clearly visible across different frequencies. However, it is more complicated to measure the flux densities from pulsars as they require detailed instrumental calibration for scaling the flux level of the measured signal. The pulsar emission also shows variability due to scintillation in the intervening medium (Rickett 1990), and hence it is essential to average over multiple observing sessions to find a mean level. A way around the issues with the flux measurement has been suggested in the recent work of Basu et al. (2021), where it was noted that the different components in the profile are equally affected by the flux scaling and scintillation issues, and hence the ratio between their intensities is unaffected by these variations. The frequency evolution of the ratio between the components can be used to find the difference in spectra between the core and the different types of conal emission. The spectral difference between the core and the conal emission in 21 pulsars observed in the Meter-wavelength Single-pulse Polarimetric Emission Survey (MSPES) at 325 and 610 MHz (Mitra et al. 2016) were estimated, and the core was found to have a steeper spectrum than the cone with $\Delta \alpha_{\text{core/cone}} \sim -0.7$. In addition, it was found that the inner cones exhibited less steep spectra compared to the outer cones with $\Delta \alpha_{\text{fin/out}} \sim +0.3$.

Currently there are no physical models that can explain why the pulsar spectra varies across the emission beam. On the other hand, several observational features in pulsars, like the radio emission heights and polarization properties, suggest that the pulsar emission can be excited by curvature radiation from charge bunches in a relativistically flowing electron–positron plasma (Ruderman & Sutherland 1975; Melikidze et al. 2000; Gil et al. 2004; Mitra et al. 2009; Melikidze et al. 2014; Mitra 2017). These charge bunches are formed due to nonlinear growth of plasma instabilities that lead to the formation of relativistic, charged solitons (Gil et al. 2004; Melikidze et al. 2014; Lakoba et al. 2018; Rahman et al. 2020). Melikidze et al. (2000) also showed that there are significant differences between the spectrum of a single-particle curvature radiation and that of a charge bunch of finite length. Thus using the constraints from observations and the theory of coherent curvature radiation, it is now possible to address the origin of the pulsar spectral index and its variation across different components.

In this paper, we aim to carry out an exhaustive study of the spectra of the pulsar profile. A large number of average profiles in a wide frequency range has been measured and made publicly available over many decades of pulsar studies (Lorimer et al. 1995; Seiradakis et al. 1995; von Hoenbroeck & Xilouris 1997; Gould & Lyne 1998; Kijak et al. 1998; Weisberg et al. 1999; Bilous et al. 2016; Mitra et al. 2016; Johnston & Kerr 2018). We have estimated the spectral variations of the different component types in 53 pulsars from archival profiles over a wide frequency range, between 100 MHz and 10 GHz. We also explore the variations expected in the spectra across the emission beam due to curvature radiation from charge bunches and compare with the measurements.

## 2. Average Profile Analysis

The primary analysis concerns with estimating the relative differences in spectra between the different component types—core, inner cones, and outer cones—within the pulsar average profile. Following the suggestion of Basu et al. (2021), the spectral difference between different component types is estimated as

$$
\frac{S_{\text{core}}}{S_{\text{cone}}} = \frac{S_1}{S_2} \rho^{\alpha_{\text{core/cone}}} \quad \Delta \alpha_{\text{core/cone}} = \alpha_{\text{core}} - \alpha_{\text{cone}}
$$

$$
\frac{S_{\text{cone}}}{S_{\text{in}}} = \frac{S_2}{S_3} \rho^{\alpha_{\text{cone/in}}} \quad \Delta \alpha_{\text{cone/in}} = \alpha_{\text{core}} - \alpha_{\text{in}}
$$

$$
\frac{S_{\text{cone}}}{S_{\text{out}}} = \frac{S_2}{S_4} \rho^{\alpha_{\text{cone/out}}} \quad \Delta \alpha_{\text{cone/out}} = \alpha_{\text{core}} - \alpha_{\text{out}}
$$

$$
\frac{S_{\text{in}}}{S_{\text{out}}} = \frac{S_3}{S_4} \rho^{\alpha_{\text{in/out}}} \quad \Delta \alpha_{\text{in/out}} = \alpha_{\text{in}} - \alpha_{\text{out}}
$$

(1)

It is expected that any variations due to scintillation or lack of flux calibration will affect all components of the profile in an identical manner and hence any properly formed average profile is useful for spectral difference studies. However, the pulsars need to have different component types, and these studies are viable in core–cone profiles, T and M, and conal profiles with clearly distinguished inner and outer cones, ,Q and . Profile classification was initiated in the pioneering works of Rankin (1990, 1993) and has been expanded in recent years (Mitra & Rankin 2011; Basu et al. 2019; Olszanski et al. 2019). We found around 150 pulsars in the literature that were classified as one of the four relevant profile types. The spectra studies can be carried out only if profile measurements are available at two or more well-separated frequency bands with sufficient temporal resolution to detect the individual components. The emission components also show frequency evolution due to various effects like radius to frequency mapping, LOS evolution with the frequency, relative spectral difference between components (the topic of our study), such that in many pulsars one or more components either merge together or vanish at certain frequencies. This left us with 53 pulsars, where profiles with distinct component types are available at more than one frequency.

Table 2 shows details of the 53 pulsars used for the spectra studies. The list consists primarily of T and T profiles and includes 43 such pulsars, while another 6 are of M type and 4 have ,Q profiles. The frequency coverage is wide, ranging

### Table 1

| Symbol | Type         | No. of Components | LOS Cut of Emission Beam |
|--------|--------------|-------------------|--------------------------|
| S_d   | Cone         | 1                 | Outer edge               |
| S_c   | Cone         | 1                 | Center                   |
| D     | Cone         | 2                 | Closer to the outer edge |
| T_{1/2}| Core–Cone    | 2                 | Center                   |
| T     | Cone         | 3                 | Middle                   |
| T_{Q} | Cone         | 4                 | Closer to the center     |
| M     | Core–Cone    | 5                 | Center                   |
from around 100 MHz up to 10 GHz in a few cases, with around 40% (21 pulsars) of the sample having measurements at more than three different frequencies. The table also reports the average spectral index for each pulsar, the sample showing typical steep spectra behavior with values ranging mostly between −1.0 and −2.5. The presence of the core component in most of the profiles also allowed us to estimate the magnetic inclination angle \((\dot{\theta})\), where 50% of the width of the core component \((W_{50})\) at 1 GHz can be associated with the emission geometry as \(W_{50} = 2.45P^{-0.5}/\sin \dot{\theta}\) (Rankin 1990). We also

| PSR | Period (s) | Class | Spectral Index | Incl. Angle (°) | Freq. Range (MHz) | Nfreq | Method |
|-----|------------|-------|----------------|----------------|------------------|-------|--------|
| B0203-40 | 0.631 | T\text{\textsubscript{1/2}} | ... | 72 | 325–1400 | 2 | Peak |
| B0329+54 | 0.715 | T | −1.6 | 70 | 140–4850 | 9 | Total |
| B0450+55 | 0.341 | T | −1.2 | 32 | 325–4850 | 7 | Avg |
| B0621-04 | 1.039 | M | −1.0 | 32 | 410–1408 | 3 | Peak |
| B0626+24 | 0.477 | T\text{\textsubscript{1/2}} | −1.5 | 30 | 170–4850 | 4 | Peak |
| B0844-35 | 1.116 | Q | −2.0 | 26 | 325–610 | 2 | Avg |
| B0919+06 | 0.431 | T | −1.8 | 48 | 135–610 | 3 | Avg |
| B0940-55 | 0.664 | T | −2.6 | 34 | 1400–3100 | 2 | Total |
| J1034-3224 | 1.151 | Q | −1.6 | 20 | 325–1400 | 4 | Total |
| B1046-58 | 0.124 | T | −0.5 | ... | 1400–8356 | 2 | Total |
| J1141-3322 | 0.291 | T | −1.2 | 36 | 436–1400 | 2 | Total |
| B1154-62 | 0.401 | T | −2.4 | 17 | 1400–3100 | 2 | Total |
| B1237+25 | 1.382 | M | −1.8 | 53 | 120–610 | 5 | Avg |
| B1323-58 | 0.478 | T\text{\textsubscript{1/2}} | −1.8 | 52 | 1400–8356 | 3 | Peak |
| B1325-49 | 1.479 | M | ... | ~90 | 325–610 | 2 | Total |
| B1353-62 | 0.456 | T | −1.8 | 27 | 1400–3100 | 2 | Total |
| B1508+55 | 0.740 | T | −2.3 | 45 | 140–325 | 3 | Total |
| B1541+09 | 0.748 | T | −2.1 | 5 | 140–1418 | 5 | Avg |
| J1557-4258 | 0.329 | T | ... | 67 | 610–1400 | 2 | Total |
| B1556-44 | 0.257 | T | −2.3 | 32 | 325–1560 | 3 | Peak |
| B1600-49 | 0.327 | T | −1.6 | ~90 | 610–1400 | 2 | Total |
| J1625-4048 | 2.355 | T | ... | 29 | 436–1400 | 3 | Total |
| B1642-03 | 0.388 | T | −2.3 | 70 | 610–10550 | 5 | Avg |
| B1700-32 | 1.212 | T | −1.5 | 48 | 325–610 | 2 | Total |
| B1732-07 | 0.419 | T | −1.8 | 64 | 325–1400 | 3 | Avg |
| B1737+13 | 0.803 | M | −1.4 | 41 | 325–1418 | 5 | Total |
| B1738-08 | 2.043 | Q | −2.5 | 26 | 325–610 | 2 | Avg |
| B1758-29 | 1.082 | T | −2.0 | 36 | 325–1400 | 3 | Total |
| B1804-08 | 0.164 | T | −1.4 | 63 | 610–3100 | 4 | Total |
| B1821+05 | 0.753 | T | −1.7 | 32 | 325–4850 | 7 | Total |
| B1826-17 | 0.307 | T | −1.7 | 39 | 925–1408 | 2 | Total |
| B1831-03 | 0.687 | T | −2.8 | 54 | 925–1400 | 2 | Peak |
| B1831-04 | 0.290 | M | −1.3 | 10 | 325–610 | 2 | Avg |
| B1839+09 | 0.381 | T | −2.0 | 83 | 130–4850 | 4 | Peak |
| B1857-26 | 0.612 | M | −1.2 | 25 | 325–1400 | 5 | Total |
| B1859+03 | 0.655 | T\text{\textsubscript{1/2}} | −2.7 | 35 | 925–1642 | 3 | Peak |
| B1907+00 | 1.017 | T | −1.8 | 69 | 610–1418 | 2 | Total |
| B1907+10 | 0.284 | T\text{\textsubscript{1/2}} | −1.9 | 49 | 610–1400 | 2 | Peak |
| B1907-03 | 0.505 | T | −2.6 | 33 | 610–1420 | 2 | Avg |
| B1911+13 | 0.521 | T | −1.4 | 52 | 606–1418 | 2 | Peak |
| B1914+09 | 0.270 | T\text{\textsubscript{1/2}} | −2.3 | 52 | 325–1642 | 6 | Total |
| B1917+00 | 1.272 | T | −2.3 | 81 | 325–1642 | 5 | Total |
| B1918+26 | 0.786 | T\text{\textsubscript{1/2}} | −1.3 | 44 | 170–1418 | 2 | Peak |
| B1920+21 | 1.078 | T\text{\textsubscript{1/2}} | −2.4 | 44 | 610–1642 | 4 | Peak |
| B1929+10 m | 0.227 | T | −1.7 | ~90 | 120–10550 | 10 | Peak |
| B1946+35 | 0.717 | T | −2.2 | 34 | 925–4850 | 4 | Peak |
| B1952+29 | 0.427 | T | ... | 30 | 610–1418 | 2 | Peak |
| B2002+31 | 2.111 | T | −1.5 | 49 | 610–1642 | 3 | Total |
| B2045-16 | 1.962 | T | −1.7 | 36 | 325–4850 | 7 | Total |
| B2111+46 | 1.015 | T | −2.0 | 9 | 408–4850 | 7 | Total |
| B2210+29 | 1.005 | T | −1.4 | 41 | 130–1418 | 4 | Total |
| B2224+65 | 0.683 | T\text{\textsubscript{1/2}} | −1.7 | 16 | 325–1642 | 7 | Peak |
| B2327-20 | 1.644 | T | −2.1 | 60 | 325–610 | 2 | Total |

(This table is available in machine-readable form.)
list the frequency range over which average profile measurements were available in each pulsar as well as the total number of such measurements (Nfreq).

In Figure 1, two examples of the frequency evolution of the profile components are shown. The left panel corresponds to the M-type pulsar B1237+25 with four profiles at 180 MHz, 325 MHz, 610 MHz, and 1400 MHz, while the right panel shows the T-type pulsar B2111+46 at 408 MHz, 610 MHz, 1408 MHz, and 4850 MHz. In both cases, the central core component has higher relative intensities at the lower frequencies compared to the cones. It is apparent from the figure that the different component types show significant evolution with the frequency, and in many cases the components merge together at one or more frequencies (see core emission at 1400 MHz for B1237+25). Hence, it becomes difficult to estimate the total power in the components for spectra calculations at certain frequencies, and the peak or the average intensities in the components provide better estimates in such cases. The last column in Table 2 lists the method by which the intensities of the components were estimated in each pulsar. The peak intensities are estimated by Gaussian approximation of suitably selected points around the expected peak. In instances where the average intensities are used, the adjacent components are not clearly separated. The longitude corresponding to the minimum intensity between these components is considered the boundary for the measurements. The intensity ratio between the components is used to estimate the relative spectra. As a result, no additional scaling is required when using the peak or the average intensities as they are expected to affect both components in a similar manner.

3. Relative Spectral Difference Measurements

We have measured the ratio of the intensities between the core and the cone ($S_{\text{core}}/S_{\text{cone}}$) in 49 pulsars, the core and inner and outer cones, ($S_{\text{core}}/S_{\text{in}}$ and $S_{\text{core}}/S_{\text{out}}$) in five pulsars, and between the inner and outer cones ($S_{\text{in}}/S_{\text{out}}$) in nine pulsars, from profiles in different frequencies. These measurements are reported in Table 3 along with the difference in the spectral index between the different component types and the frequency range over which the spectra are calculated. The spectral difference between the core and conal components ($\Delta S_{\text{core}/\text{cone}}$) shows a relatively wide spread between $-0.2$ and $-2.0$ with a fairly uniform distribution, as shown in Figure 2. A slight peak around $\Delta S_{\text{core}/\text{cone}} \sim -0.7$ is visible, which is consistent with earlier results (Basu et al. 2021). No clear correlation is seen between $\Delta S_{\text{core}/\text{cone}}$ and different physical parameters like the period, characteristic age, spin-down energy loss, and average spectral index. The spectral difference between the inner and outer cones ($\Delta S_{\text{in}/\text{out}}$) in nine pulsars varies between $+0.1$ and $-0.8$ with a mean value of $+0.4$. The spectra variation in terms of the relative intensities of different component pairs of 19 pulsars where measurements were available over a relatively wide frequency range are shown in Figures 3, 4, 5. We briefly discuss below the profile characteristics and component frequency evolution in all these cases.

B0329+54. The pulsar profile has three distinct components with a bright core and has been classified as a T-type profile (Rankin 1993). The pulsar is very bright with average flux density $>200\, \text{mJy}$ at 1 GHz (Lorimer et al. 1995) and has been observed over a wide frequency range from 40 MHz to around 30 GHz. The core component is asymmetrical with elongation near the leading edge. The pulsar emission is scattered, and the components are indistinguishable at frequencies below 100 MHz, while one or more components vanish at the high-frequency range above 10 GHz. The components also exhibit radius to frequency mapping with the two conal components being well separated from the core at lower frequencies, while the components merge at frequencies above 1 GHz but are still distinguishable up to 10 GHz. $S_{\text{core}}/S_{\text{cone}}$ shows a power-law frequency dependence between 400 MHz and 2 GHz (see Figure 3, first row, left panel) with $\Delta S_{\text{core}/\text{cone}} \sim -0.7$. The spectral difference are flatter at lower frequencies, while at the high-frequency end the relative intensities become more irregular.

B0450+55. The pulsar has an asymmetric profile with three merged components, with a trailing component showing a prominent tail. The profile was classified as T type and is part of the partial cone sample (Mitra & Rankin 2011). The leading component is not distinguishable from the core at frequencies below 300 MHz, and as a result these profiles are not used. Due to the components being merged together, the average intensities are used to estimated the evolution of the relative spectra between the components, which show a power-law dependence between 300 MHz and 4.8 GHz with $\Delta S_{\text{core}/\text{cone}} \sim -0.4$ (see Figure 3, first row, right panel).

J1034-3224. The pulsed emission is relatively wide occupying around 30% of the profile window and can be classified into two distinct parts, the main pulse and a precursor component (Basu & Mitra 2018). The precursor becomes more prominent at higher frequencies (>1 GHz) and is characterized by large fractional linear polarization and flat PPA traverse across the component (Basu et al. 2015). The main pulse consists of four distinct conal components and is classified as a Q profile type. The pulsar was observed between 300 MHz and 1.4 GHz, and the main pulse shows distinct evolution over this range. A large plateau is seen between the inner conal components at 325 MHz. The separation between the inner cones narrows with the frequency, and at 1.4 GHz the two components merge. The intensity of the outer cones increases in comparison with that of the inner cones with decreasing frequency and has a relative difference in spectral index below 1 GHz as $\Delta S_{\text{in}/\text{out}} \sim +0.78$ (see Figure 3, second row, left panel). The 1.4 GHz measurements are not used for the relative spectral index estimate.

B1237+25. The central core in this M-type profile is relatively weak and is clearly visible only at frequencies of 325 MHz and below (see Figure 1). The spectra of the relative intensities between the core and the cones show a power-law dependence below 1 GHz (see Figure 3, second row, right panel), while at higher frequencies the measurements become less clear due to the core merging with the inner cone and the conal components overlapping each other. As a result, only the leading side of the core is used to estimate the relative intensities at all frequencies. In this pulsar, the spectral differences are relatively flat with $\Delta S_{\text{core}/\text{cone}} \sim -0.3$ while $\Delta S_{\text{in}/\text{out}} \sim +0.15$, which are near the upper and lower ends of the distributions, respectively.

B1541+09. The pulsar has a wide profile, which encompasses almost half the rotation period and has a T-type classification. The conal emission is very low at frequencies of 400 MHz and below but becomes more prominent at higher frequencies. The relative intensities show a power-law spectrum from 100 MHz to 1.4 GHz with a relatively steep
Figure 1. Evolution of the average profile over four different frequency bands in two different pulsars, B1237+25 (left panel) with an M-type profile and B2111+46 (right panel) with a T profile type. The references for each measurement are listed in Table 3.
## Table 3

Component Spectral Difference

| PSR    | Type | Freq (MHz) | $S_{\text{core}}/S_{\text{cone}}$ | $\Delta \alpha^a$ | $S_{\text{core}}/S_{\text{in}}$ | $\Delta \alpha^\beta$ | $S_{\text{core}}/S_{\text{out}}$ | $\Delta \alpha^\gamma$ | $S_{\text{in}}/S_{\text{out}}$ | $\Delta \alpha^\delta$ | Ref. |
|--------|------|------------|-----------------------------------|-------------------|---------------------------------|-------------------|---------------------------------|-------------------|--------------------------------|-------------------|------|
| 2      | B0329+54 | 140        | $4.076 \pm 0.007$                | $-0.72 \pm 0.06$  | $...$                           | $...$             | $...$                           | $...$             | $...$                           | $...$             | [3] |
|        |       | 408        | $6.830 \pm 0.006$                |                   | $...$                           | $...$             | $...$                           | $...$             | $...$                           | $...$             | [4] |
|        |       | 610        | $5.555 \pm 0.002$                |                   | $...$                           | $...$             | $...$                           | $...$             | $...$                           | $...$             | [4] |
|        |       | 925        | $3.926 \pm 0.004$                |                   | $...$                           | $...$             | $...$                           | $...$             | $...$                           | $...$             | [4] |
|        |       | 1410       | $2.339 \pm 0.002$                |                   | $...$                           | $...$             | $...$                           | $...$             | $...$                           | $...$             | [5] |
|        |       | 1642       | $2.707 \pm 0.005$                |                   | $...$                           | $...$             | $...$                           | $...$             | $...$                           | $...$             | [4] |
|        |       | 2250       | $2.190 \pm 0.004$                |                   | $...$                           | $...$             | $...$                           | $...$             | $...$                           | $...$             | [6] |
|        |       | 4850       | $3.265 \pm 0.004$                |                   | $...$                           | $...$             | $...$                           | $...$             | $...$                           | $...$             | [5] |
|        |       | 8500       | $3.5 \pm 0.1$                    |                   | $...$                           | $...$             | $...$                           | $...$             | $...$                           | $...$             | [6] |
|        |       | 10550      | $1.72 \pm 0.02$                  |                   | $...$                           | $...$             | $...$                           | $...$             | $...$                           | $...$             | [8] |
| 13     | B1237+25 | 120        | $1.044 \pm 0.003$                | $-0.32 \pm 0.04$  | $1.413 \pm 0.006$               | $-0.46 \pm 0.06$  | $0.950 \pm 0.003$               | $-0.29 \pm 0.04$  | $0.673 \pm 0.002$               | $0.15 \pm 0.03$   | [9] |
|        |       | 180        | $0.966 \pm 0.007$                | $...$             | $1.23 \pm 0.01$                | $...$             | $0.880 \pm 0.006$               | $...$             | $0.715 \pm 0.006$               | $...$             | [9] |
|        |       | 325        | $0.727 \pm 0.002$                |                   | $0.822 \pm 0.002$              | $...$             | $0.680 \pm 0.001$               | $...$             | $0.827 \pm 0.001$               | $...$             | [1] |
|        |       | 610        | $0.648 \pm 0.004$                |                   | $0.716 \pm 0.005$              | $...$             | $0.612 \pm 0.004$               | $...$             | $0.855 \pm 0.003$               | $...$             | [1] |
|        |       | 1400       | $0.709 \pm 0.002$                |                   | $0.637 \pm 0.002$              | $...$             | $0.758 \pm 0.002$               | $...$             | $1.190 \pm 0.002$               | $...$             | [8] |

### Notes
Table 3 is published in its entirety in the machine-readable format. A portion is shown here for guidance regarding its form and content.

* Unless otherwise specified, all frequencies were used for the spectral calculation. In cases where a narrower range of frequencies was used, the range is provided in parenthesis below the relative spectral index calculation or in distinct columns in the machine-readable format.

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18. Wu et al. (1993);
19. Stairs & Thorsett (1999).

(This table is available in its entirety in machine-readable form.)
The spectral difference of $\Delta \alpha_{\text{core/cone}} \sim -1.2$ (see Figure 3, third row, left panel).

**B1642-03.** The pulsar is classified as a T-type profile with a prominent core emission and has measurements over a wide frequency range between 100 MHz and 10 GHz. However, the conal emission is relatively weak and undetectable at frequencies below 600 MHz. At higher frequencies, the conal emission merges with the core without clear boundaries between them and hence the average intensities are used for estimating the relative spectra evolution. The estimated spectra between 610 MHz and 10 GHz are $\Delta \alpha_{\text{core/cone}} \sim -1.4$ (see Figure 3, third row, right panel).

**B1737+13.** The pulsar profile has a prominent core emission with the profile classified as an M type. However, the pulsar profile is asymmetric with only one conal component seen in the trailing side for most of the observing frequencies. The core emission shows a steeper spectrum compared to the cone, with the relative intensities showing a power-law nature between 325 MHz and 1.4 GHz with $\Delta \alpha_{\text{core/cone}} \sim -0.8$ (see Figure 3, fourth row, left panel). The outer cones also exhibit a steeper spectrum compared to the inner cone with $\Delta \alpha_{\text{in/out}} \sim +0.3$.

**B1804-08.** The pulsar has a T-type profile with multiple reported measurements between 400 MHz and 5 GHz. At frequencies lower than 900 MHz, the components are merged together and cannot be measured separately. Between 900 MHz and 3 GHz, the relative intensities show a power-law spectrum (see Figure 3, fourth row, right panel) with $\Delta \alpha_{\text{core/cone}} \sim -0.9$. The components are once again indistinguishable in the 5 GHz profile.

**B1821+05.** This pulsar also has a T profile type with measurements available from 100 MHz to 5 GHz. The conal emission is not seen around 100 MHz, while at frequencies above 1 GHz the core merges with the trailing cone. The relative intensities between the core and conal components show a power-law spectrum between 300 MHz and 1 GHz with a spectral index of $\Delta \alpha_{\text{core/cone}} \sim -1.55$ (see Figure 4, first row, left column). The spectrum becomes flatter at the higher-frequency range likely due to mixing of component intensities. As a result, the measurements at 1.4 GHz, 1.6 GHz, and 4.8 GHz are not used for the relative spectral index estimate.

**B1857-26.** This pulsar has an M profile type with a prominent core as well as pairs of inner and outer cones between 300 MHz and 1 GHz. The components merge together at higher frequencies, and although it is still possible to distinguish the core at 1.4 GHz, the inner and outer cones merge together and cannot be measured separately. The evolution of the spectrum of relative intensity between the core and the cones can be constrained using a power-law nature between 300 MHz and 1.4 GHz with $\Delta \alpha_{\text{core/cone}} \sim -1.0$ (see Figure 4, first row, right column). The outer cone has a steeper spectrum than the inner cone between 300 MHz and 1 GHz with $\Delta \alpha_{\text{in/out}} \sim +0.35$.

**B1914+09.** The pulsar has two components in the profile, with the leading component being the core and the trailing component the cone, and is classified as T$_{1/2}$. The components merge together and cannot be distinguished at frequencies around 100 MHz and 5 GHz. In the intervening frequency range of 325 MHz and 1.4 GHz, the relative intensity of the core and the conal components shows a power-law spectral dependence with $\Delta \alpha_{\text{core/cone}} \sim -0.8$ (see Figure 4, second row, left column).

**B1917+00.** The pulsar is characterized by a T-type profile with the conal components not clearly separated from the core. In many profiles, the time resolutions are insufficient to resolve the components. The relative intensity between the core and the cone shows a power-law dependence between 325 MHz and 610 MHz, which flattens at 1.4 GHz (see Figure 4, second row, right column). The estimated $\Delta \alpha_{\text{core/cone}}$ between 325 and 610 MHz is around $-0.6$. The high-frequency measurements at 1.4 GHz and 1.6 GHz are not used for the relative spectral index estimate.

**B1920+21.** The pulsar has a T$_{1/2}$ profile with a prominent core emission along with a trailing conal component. At frequencies around 100 MHz, the profile is affected by scattering. Below 600 MHz, the conal emission is very weak and cannot be clearly measured, while between 600 MHz and 1.6 GHz the relative intensities between the core and the cone have a power-law spectral nature with $\Delta \alpha_{\text{core/cone}} \sim -1.0$ (Figure 4, third row, left column).

**B1929+10.** The pulsar has a diverse profile where, in addition to the main pulse, an interpulse emission is seen at a phase separation of approximately 180° in longitude. There is also the presence of a wide postcursor component between the main pulse and the interpulse (Rankin & Rathnasree 1997; Basu et al. 2015; Kou et al. 2021). The main pulse also shows high levels of linear polarization with almost 100% fractional polarization and hence is often used to calibrate the polarization response in telescopes (Mitra et al. 2016). As a result, the pulsar has been extensively observed over a wide frequency range between 100 MHz and 20 GHz. The main pulse is classified as a T-type profile with three merged components seen around 1.4 GHz. At lower frequencies, the leading conal component is not clearly visible and hence we have used the peak intensities of the central core and the trailing cone to estimate the relative spectral evolution (see Figure 4, third row, right column). The relative intensity shows a power-law spectral dependence between 100 MHz and 8.5 GHz with a comparatively low $\Delta \alpha_{\text{core/cone}} \sim -0.2$. At 10 GHz, a flattening in the spectra is seen, which may be likely due to the merging of the two components as they become indistinguishable at still higher frequencies.

**B1946+35.** The pulsar emission is affected by scattering, and the profiles show prominent elongated tails below 900 MHz, where the individual components cannot be identified. At higher frequencies, the presence of a T-type profile is seen with a prominent core emission. The leading conal component is
Figure 3. Spectra of the relative intensities between the core and conal components within the pulsar profile in eight pulsars between 100 MHz and 10 GHz. The spectral evolution between the inner and outer cones is also estimated for PSR B1237+25 and B1737+13 with M profile types as well as for PSR J1034-3224 with a Q profile.
Figure 4. Spectra of the relative intensities between the core and conal components within the pulsar profile in eight other pulsars between 100 MHz and 10 GHz.
merged with the core around 1 GHz but becomes clearly visible at 5 GHz. The peak intensities of the core and trailing core have been used to estimate their relative spectral evolution. Between 900 MHz and 5 GHz, the relative intensity shows a power-law dependence with $\Delta \alpha_{\text{core/cone}} \sim -1.5$ (Figure 4, fourth row, left column).

**B2045-16.** The profile is classified as a T type with the central core component shifted toward the trailing cone. The emission has been measured over a wide frequency range between 325 MHz and 10 GHz, with the core vanishing at the highest frequencies. The relative intensity between the core and the cones shows a power-law dependence between 325 MHz and 5 GHz with $\Delta \alpha_{\text{core/cone}} \sim -0.5$ (Figure 4, fourth row, right column).

**B2111+46.** The pulsar has a relatively wide profile that occupies more than 20% of the period and shows three distinct emission components with T classification (see Figure 1, right column). The pulsar emission is scattered at frequencies around 100 MHz. The power-law dependence is seen for the frequency evolution of the relative intensity between 400 MHz and 5 GHz with $\Delta \alpha_{\text{core/cone}} \sim -0.8$ (Figure 5, upper left panel).

**B2210+29.** The pulsar has a T-type profile and has many measurements between 100 MHz and 1.6 GHz. However, some of these profiles have lower detection sensitivities where the components cannot be measured. The relative intensity between the core and the cone has a power-law dependence between 100 and 600 MHz with $\Delta \alpha_{\text{core/cone}} \sim -0.3$, which signifies a relatively flatter behavior (Figure 5, upper right panel), and shows further flattening at 1.4 GHz.

**B2224+65.** The pulsar has a profile with two emission components and was classified as T$_{1/2}$, with the leading component being the core and the trailing one the cone (Basu et al. 2015). The profiles have been measured over multiple frequencies between 100 MHz and 1.6 GHz, but the cone vanishes around 100 MHz. The detection sensitivities show large variations as a result of which we used the peak intensities for the relative intensity estimates. The relative intensity of the core and the cone shows a power-law dependence between 325 MHz and 1.6 GHz with $\Delta \alpha_{\text{core/cone}} \sim -1.0$ (see Figure 5, lower panel).

### 4. Curvature Radiation

The radio emission in pulsars arises due to coherent curvature radiation from relativistic charge bunches that are formed due to two-stream instabilities developing in the outflowing plasma (Asseo & Melikidze 1998; Melikidze et al. 2000; Gil et al. 2004). Two-stream instabilities can develop in overlapping clouds of plasma moving along the open magnetic field lines. The presence of an inner acceleration region (IAR), with unscreened electric field, above the polar cap is required to generate a nonstationary flow resulting in the overlapping clouds of plasma. Electron–positron pairs are formed due to magnetic pair creation from high-energy photons in the IAR, which gets separated and accelerated into opposite directions by the electric field (Ruderman & Sutherland 1975;
Gil et al. 2003). A sparking discharge ensues as a result of additional pair production from curvature radiation and/or inverse Compton scattering of high-energy photons from the initial charges (Medin & Lai 2007; Szary et al. 2015). The potential difference along the IAR vanishes over timescales of hundreds of nanoseconds due to the excess charges that inhibit further pair formation. The positrons are accelerated away from the surface to relativistic energies, with Lorentz factors of $\gamma_p \sim 10^6$, forming the primary particles. Once the IAR becomes sufficiently empty due to outflow of the excess charges, the electric field reappears starting the sparking process once again to generate the nonstationary plasma flow.

The primary particles continue to radiate high-energy photons as they move along the curved magnetic field lines beyond the IAR, resulting in a pair cascade that forms the secondary plasma comprising of both electron and positron streams. The primary particles outside the IAR has the corotation Goldreich–Julian density, $n_{GJ}$ (Goldreich & Julian 1969), while the secondary plasma has a Lorentz factor of $\gamma_{fs} \sim 10^2$ and density $n_s = \kappa n_{GJ}$, where $\kappa \geq 10^2$ is the multiplicative factor (Sturrock 1971) and $n_{GJ}$ can be obtained from

$$n_{GJ} = -\langle \mathbf{\Omega} \cdot \mathbf{B} \rangle / 2 \pi c e$$

$$= 5.6 \times 10^5 \left( \frac{P_{15}}{P} \right)^{1/2} R_{50}^{-3} \text{ cm}^{-3}.$$  (2)

Here $\mathbf{\Omega}$ is the angular velocity of stellar rotation, $\mathbf{B}$ is the magnetic field at a specific location, $P$ is the pulsar period $P_{15} = P / 10^{15}$ with $P$ being the rate of period slowdown, $R_{50} = r / 50 R_S$, where $r$ is the radial distance, and $R_S = 10 \text{ km}$ is the radius of the neutron star. The plasma instability develops in the secondary plasma, and its linear and nonlinear growth results in the formation of relativistic charge bunches around heights of $100–1000$ km from the surface. As these charge bunches accelerate in the curved magnetic field lines, they radiate curvature radiation, which is seen as the radio emission.

The curvature of the magnetic field lines at any emission height increases from the magnetic axis toward the edge of the open field line region. The centrally located core component in the emission beam is expected to originate from the field lines close to the magnetic axis, while the inner and outer cones from field lines further away from the axis. In this section, we explore the effect of the different magnetic field curvature on the spectra of the curvature radiation. Below we consider two different cases, the first corresponds to spectra of incoherent emission from a distribution of charged particles emitting curvature radiation, and the second case deals with curvature radiation from charged solitons. We are primarily concerned with the total spectra obtained from the radiation energy of these sources. However, quantities like the brightness temperature require estimation of the radiation intensity per unit solid angle (see Roy & Gangadhara 2019; Gangadhara et al. 2021). The formation of charged solitons in the pulsar magnetosphere has been worked out in more detail in several works (Melikidze et al. 2000; Lakoba et al. 2018; Yang & Zhang 2018; Rahaman et al. 2020) and is beyond the scope of this paper. We use the results from these earlier works to simulate the variation in the spectral nature across the different profile components.

![Figure 6](image)

**Figure 6.** Schematic representing the intersection of the line of sight (LOS) along the radial direction (r) with the magnetic field lines. The wavevector of the radiation ($k$) is along the tangent to the magnetic field lines, and the LOS makes different angles with the wavevector emitted from different heights, e.g., $\theta_1$ and $\theta_2$, as shown here. The curvature radiation around the characteristic frequency is highly beamed and can only be detected over a small angular radius $\theta < 1/\gamma$, where $\gamma$ is the Lorentz factor of the relativistic particles.

4.1. Incoherent Curvature Radiation from Distribution of Charges

We first consider the case of the incoherent curvature radiation from a distribution of charge particles with Lorentz factors equivalent to the secondary plasma, moving along the open dipolar magnetic field lines. This requires suitably adding the radiation energies from all particles visible within the line of sight for all frequencies. The emitted radiation energy ($I$) at a given frequency ($\omega$) per unit solid angle ($\Omega$) by a relativistically charge particle moving along a curved trajectory can be obtained as (Jackson 1998)

$$\frac{d^2I}{d\omega d\Omega} = I_0 \left( \frac{\omega}{\omega_c} \right)^2 (1 + \gamma_{fs}^2 \theta^2)^2 \times \left[ K_{2/3}^2(\xi) + \left( \frac{\gamma_{fs}^2 \theta^2}{1 + \gamma_{fs}^2 \theta^2} \right) K_{1/3}^2(\xi) \right],$$  (3)

where $I_0$ is the energy at the characteristic frequency ($\omega_c$) emitted along the tangential direction, $\gamma_{fs}$ is the Lorentz factor of the secondary plasma particle, $\theta$ is the angle between the line of sight and the tangential direction of particle trajectory (see Figure 6), $K_{1/3}$ and $K_{2/3}$ are modified Bessel functions, and

$$\omega_c = \frac{3}{2} \frac{\gamma_{fs}^2 \rho_c}{\omega_c} \left( \frac{c}{\rho_c} \right), \quad \xi = \frac{\omega}{2 \omega_c} (1 + \gamma_{fs}^2 \theta^2)^{3/2},$$  (4)

with $\rho_c$ being the radius of curvature of the curved trajectory at the point of emission. The above spectrum peaks around $\omega \sim \omega_c$ and falls sharply on either side.
In order to estimate the spectra for any particular LOS, specified by the angle \( \theta \), the directed emission from all relevant charges needs to be added for all frequencies. The radio emission is expected to arise between heights of \( r = 100 \) km and \( r = 1000 \) km. We consider the angular size of the core, inner cone, and outer cone as defined below and average the spectra of all LOSs within each component. These estimates can be compared with the observed spectra, as shown in Section 3. We have used the following scheme to implement the measurement of the spectra in each component.

1. A power-law function, \( n(\gamma) \propto \gamma^a \), is considered for the relativistic charge particle distribution. For these estimates, \( \gamma_i \) is confined between 50 and 300 with an index of \( a = -0.3 \).

2. The emission is constrained to originate between heights of \( 10R_\xi < r < 100R_\xi \). The radius of curvature at any given point is estimated in Basu et al. (2020a; see Appendix).

3. At any given height, the beam opening angle \( (\theta_b) \) is obtained as
\[
\sin \theta_b = \left( \frac{r}{R_{LC}} \right)^{1/2} \frac{R_{LC}}{R} = P_c/2\pi.
\]

4. The radiation energies corresponding to all relevant particles within the angular boundaries of each component type and over the relevant emission heights are added up to form their respective spectra, as shown in Figure 7. The simulated spectra show a consistent evolution across the profile with the core component having a steeper spectrum compared to the cones. This shows that the variations in the relative spectra between the core and the cones (see Section 3) can be naturally explained from curvature radiation over narrow emission heights due to a change in the radius of curvature from the axis toward the edge of the open field lines. However, as seen in Figure 7, the spectrum is much steeper than the observed spectrum with the spectral index being around \(-4\) to \(-6\) between 100 MHz and 1 GHz compared to being around \(-2.0\) in the observed cases (Maron et al. 2000). In order to obtain the measured spectra, one requires \( \gamma \sim 10^{-3}-10^{-4} \), which is too high for the secondary plasma in pulsars. It should also be noted that, for a distribution of particles, incoherent emission from individual particles is possible only when the separation between them is more than the wavelength of the emitted radiation. For longer wavelengths, the particles will interact with each other, resulting in absorption of radiation. This makes it impossible for the density of plasma in the pulsar magnetosphere to emit incoherent emission, where the average separation between particles \( \propto n^{-1/3} \) is much smaller than radio wavelengths. Nonetheless, the purpose of this exercise is to demonstrate the effect of radius of curvature on the spectra of curvature radiation.

\[\frac{d^2I}{d\omega d\Omega} = \frac{Q^2}{c} \omega \omega_p F\left(\frac{\omega}{\omega_p}\right) \left[ 1 - \cos \left( a \frac{\omega}{\omega_p} \right) \right]^2.\]

Here \( Q \) is the total charge in solitons, \( \omega_p = \frac{1}{2} \gamma_o^3 / \rho_p \), and
\[F\left(\frac{\omega}{\omega_p}, \theta\right) = \left( \frac{\omega}{\omega_p} \right)^2 \left[ 1 + \gamma_o^2 \frac{\theta^2}{2} \left( K_{-1/2}(\xi) \right) \right].\]

In contrast with the single-particle curvature radiation, the soliton spectra have an extra term \( 1 - \cos (a(\omega / \omega_p))^2 \), which shifts the peak frequency by a factor of \( 4 \) from \( \omega_p \) and makes the curve wider and more symmetrical (see Figure 4 in MGP00). In addition, the Lorentz factors of solitons \( \gamma_o \) correspond to the group velocities of the wave packets and are usually greater than the Lorentz factors of the secondary plasma particles, \( \gamma = \gamma_x \), with \( \gamma \approx 2 \). As a result, the characteristic frequencies of solitons \( \omega_p \) are almost an order of magnitude greater than that of the single particle \( \omega_p \). This further highlights the applicability of the soliton model as a viable mechanism for pulsar emission as the peak frequencies
of soliton spectra are around 100–1000 MHz, which are between 1 to 2 orders of magnitude higher than that of the single-particle case. Using basic pulsar parameters, MGP00 estimated the spectra of the coherent curvature radiation energy from each soliton as

\[ \frac{dI}{d\omega d\Omega} = A_1 \gamma_2 \kappa_4 R_{50}^{2.5} \chi^{3} P^{-3/7} \hat{P}_{-15}^{-9/14} F \left( \frac{\omega}{\omega_0}, \theta \right) \times \left[ 1 - \cos \left( \frac{\omega}{\omega_0} \right) \right]^2 \text{ergs Hz}^{-1} \text{sr}^{-1}, \]  

(9)

and the total number of solitons contributing to the pulsar emission at any instant to be

\[ N_s \sim 10^5 \gamma_2^{-0.5} \kappa_4 R_{50}^{0.5} \Delta_\chi^{-1} \chi^{-0.5} P^{-1/4} \hat{P}_{-15}^{-1/4}. \]  

(10)

The total energy of coherent curvature radiation emitted at any frequency can be obtained from \( N_s dI/d\omega \). We are primarily interested in the relative difference of the spectra and the absolute values of the energy contained in the constant term \( A_1 \). The soliton spectra obtained from Equations (9) and (10) is dependent on a number of parameters that are likely to have a range of values, e.g., the variability of soliton lengths, the Lorentz factors of solitons, the distribution of secondary pair plasma, the optimal emission heights for the plasma instabilities to develop and their frequency dependence, the growth factor of the plasma waves, the soliton charge densities. It is unlikely that the full extent of how these parameters vary can be estimated from observational constraints alone and would require more detailed modeling of the nonlinear plasma instabilities (see, e.g., Lakoba et al. 2018; Rahaman et al. 2020), which is outside the scope of this paper. It is expected that most of these parameters will vary as a function of distance as well as across different field lines. However, our objective in this exercise is to determine if it is possible to obtain the observed variation in spectra seen across different components within the pulsar emission beam by using coherent curvature radiation from solitons. A similar process as explained for the

incoherent case was followed to estimate the total radiation energy spectra across each component type, the core, the inner cone, and the outer cone. We describe below the steps for estimating the spectra for different components using the above setup.
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1. The emission is once again constrained to originate between heights of $10R_e < r < 100R_e$.
2. The emission beam model is identical to the one used for the single-particle curvature radiation described earlier, comprising of a central core between $0 < \phi_{cr} < \theta_b/5$, an inner cone between $\theta_b/5 < \phi_{in} < 3\theta_b/5$, and an outer cone with $3\theta_b/5 < \phi_{out} < \theta_b$, where $\theta_b$ is defined in Equation (5). We consider a central LOS traverse across the beam and approximate the components to have Gaussian shapes.
3. For most parameters, we use a fixed set of values as prescribed in MGP00. The only variable parameter is the Lorentz factors of secondary plasma $\gamma_r$ which is allowed to have different sets of values around $\gamma \sim 100$ in order to vary the shape of spectra across the components. We once again consider a power-law distribution for the Lorentz factors with index $\alpha = -0.3$. The range of $\gamma$ used for the core is between 50 and 170, the inner cone between 50 and 280, and the outer cone with variations from 5 to 130.

We are primarily interested in the behavior of the spectra within the frequency range 100 MHz and 1 GHz, where the relative component spectra are well constrained. The region from where a particular frequency in emitted depends on $\omega_\nu$, which is a function of the radius of curvature and $\gamma_r$. The radius of curvature further depends on the emission height as well as the distance from the magnetic axis. For any given $\gamma_r$, the emission along any field line is directed toward the observer when the angle between the radial vector ($\vec{r}$) and the tangent to the field line at the point of emission is less than $1/\gamma_r$ (see Figure 6). This introduces an additional constraint on the emission between the inner magnetic field lines, closer to the magnetic axis and the outer field lines further away, particularly those for the outer cones where the curvature is higher resulting in larger angles. Figure 8 shows the area of the open field line region where the radio emission arises from at different frequencies of 200, 300, and 600 MHz for our selected setup of emission parameters. We divided the open field line region between emission heights of 100–1000 km into small grids and separately estimated the spectra in each grid. For any given frequency, if significant emission was seen at a grid, i.e., more that 1% of power of the grid with maximum energy for that frequency, it was marked with a blue point and shown in the figure. At the lower frequencies, the emission can originate from much higher $r$, while the emission is restricted to much lower $r$ in the outer cones across all frequencies. Figure 9 shows the estimated spectra for the core, inner cone, and outer cones. The core emission is steeper than that of the cones due to variation in the curvature, which is also seen for the single-particle case. The outer cones also have steeper spectra than the inner cones as a result of the beaming effect, such that emission from certain field lines at higher frequencies is not directed toward the observer making the spectra steeper. The spectral index for the inner cone between 100 MHz and 1 GHz is $\sim -2.0$ (Figure 9, left panel), which is consistent with observations. In addition, $\Delta\alpha_{core/cone} \sim -1.2$ and $\Delta\alpha_{in/out} \sim +0.6$ (9, right panel), which matches the observational results in Section 3. It should be noted that the physical parameters used in these simulations of spectra are demonstrative in nature and do not represent unique solutions regarding the physical conditions in the pulsar magnetosphere.

5. Conclusion

The relative spectral index between the core and the conal emission has been estimated by Basu et al. (2021) in 21 pulsars over a relatively narrow frequency range between 325 MHz and 610 MHz. They found the core emission to be relatively steeper than that of the cones with a mean relative spectral index of $\Delta\alpha_{core/cone} \sim -0.7$. The outer cone also showed a steeper index than the inner core, such that $\Delta\alpha_{out/in} \sim +0.4$. In this work, we have used average profile measurements from archival observations to expand the above studies in a larger sample of 53 pulsars with a much wider frequency coverage between 100 MHz and 10 GHz. In all cases, we found the core component to have a steeper spectrum compared to the core, although the differences in spectra show a relatively uniform distribution between $-0.2$ and $-2.0$. In nine pulsars, we were able to measure the spectral difference between the inner and the outer cones, with the spectrum of the inner cone being less steep than that of the outer cone.
cone with a relative spectral index between +0.1 and +0.8. In a few cases, the differences in spectra show flattening at higher frequencies above 1 GHz. However, in almost all such cases, we found that some of the components merged together at higher frequencies, thereby affecting the relative intensity estimates. Our analysis clearly highlights the evolution of the pulsar spectra within the emission window in the entire pulsar population as no clear correlation is seen between the relative spectral index and different pulsar parameters.

The radio emission in pulsars is expected to originate within a narrow range of heights, a few hundred kilometers from the stellar surface, due to coherent curvature radiation. This requires the formation of charged bunches in the form of charge-separated solitons due to nonlinear instabilities in the outflowing plasma. A natural steepening of the spectra is expected due to an increase in the curvature of the magnetic field lines from the axis toward the edge of the open field lines. Further, the relativistic beaming effect restricts the emission at the outermost field lines from being directed toward the observer. This mostly affects the higher frequencies, which are supposed to be emitted by particles with higher Lorentz factors. As a result, additional steepening of the spectra is expected for the outermost field lines compared to the relatively inner ones. The above mechanism can explain the variations in spectra seen in the pulsar emission window if one considers the core to arise from field lines close to the magnetic axis and the inner cones from field lines further away from the core, while the outer cones are furthest away and located near the edge of the open field line region.

The plasma populating the open field line region originates due to sparking discharges from an IAR above the polar cap. The sparking process is thermally regulated due to the discharge of ions from the heated polar cap surface forming a partially screened gap (Gil et al. 2003; Szary et al. 2015). Differences in the expected potential drop along the IAR can arise from the center of the IAR toward the polar cap edge due to the distribution of sparks. The center is populated by sparks from all sides and is likely to have a larger potential drop, while the thermal regulation requires the presence of a spark always near the rim of the polar cap, which likely has reduced potential difference along the gap (Basu et al. 2020a; Gil & Sendyk 2000). As a result, the distribution function of the outflowing plasma generated from the sparks can show variations between the central regions and the outer field lines. Our studies show that such variations are likely to affect the relative differences in spectra across the profile. However, the emission mechanism is dependent on a number of parameters that are not well constrained. More detailed modeling of the nonlinear plasma processes are required to narrow the range of these parameters before such association between the variations in spectra and the plasma distribution function can be studied in more detail.

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