Nonlocal kinetic theory

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The short time behavior of a disturbed system is influenced by off-shell motion and best characterized by the reduced density matrix possessing high energetic tails. We present analytically the formation of correlations due to collisions in an interacting Fermionic system with and without initial correlation. After this short time regime the time evolution is controlled by small gradients. This leads to a nonlocal Boltzmann equation for the quasiparticle distribution and a functional relating the latter one to the reduced density matrix. The nonlocalities are presented as time and space shifts arising from gradient expansion and are leading to virial corrections in the thermodynamical limit.

1 Short time regime

The generalization of Boltzmann equation towards dense interacting systems is a still demanding and unsolved task. A huge variety of different attempts can be found in literature to incorporate modifications which lead to virial corrections in the equation of state, see citations in\textsuperscript{1,2,3,4,5,6}. These kinetic equations describe different relaxation stages. During the very fast first stage, correlations imposed by the initial preparation of the system are decaying\textsuperscript{7}. During this stage of relaxation the quasiparticle picture is established\textsuperscript{8,9}. After this very fast process the second state develops during which the one-particle distribution relaxes towards the equilibrium value with a relaxation time. During this relaxation state the virial corrections are established and can be consistently described by a nonlocal Boltzmann kinetic equation\textsuperscript{6}. We will present results for both stages here.

The formation of correlations is connected with an increase of the kinetic energy or equivalently the build up of correlation energy. This is due to rearrangement processes which let decay higher order correlation functions until only the one-particle distribution function relaxes. Because the correlation energy is a two-particle observable we expect that the relaxation of higher order correlations can be observed best within this quantity. Since the total energy of the system is conserved the kinetic energy increases on cost of the correlation energy $E_{\text{corr}}(t)$ and will be calculated from kinetic equations.

The kinetic equation for the reduced density distribution or Wigner function in Born approximation including memory effects but no damping is called
Levinson equation and reads in spatial homogeneous media:\(^{9,10,11}\)
\[
\frac{\partial}{\partial t} \rho(t, k) = 2 \sum_{b} s_{a} s_{b} \int \frac{dp dq}{(2\pi)^6} V_{ab}(|q|)^2 \int_{t_0}^{t} d\tau \cos \left( \left( \frac{k^2}{2m_a} + \frac{p^2}{2m_b} - \frac{(k - q)^2}{2m_a} - \frac{(p + q)^2}{2m_b} \right) (t - \tau) \right) \{ \rho(\tau, k - q) \rho(\tau, p + q) \times (1 - \rho(\tau, k) - \rho(\tau, p)) - \rho(\tau, k) \rho(\tau, p) (1 - \rho(\tau, p + q) - \rho(\tau, k - q)) \} \tag{1}
\]
with the spin (isospin, ... ) degeneracy \(s\). The Wigner distribution function is normalized to the density as \(s \int \frac{dp}{(2\pi)^3} f(p) = n\). The memory effect is condensed in the explicit retardation of the distribution function. This would lead to gradient contributions to the kinetic equation which can be shown to be responsible for the formation of high energetic tails in the distribution function\(^{12}\). This will be established on the second stage of relaxation and will lead to virial corrections in chapter 2.

The second effect is contained in the energy broadening or off-shell behavior in (1). This is exclusively related to the spectral properties of the two-particle propagator and therefore determined by the relaxation of two-particle correlation. On this time scale the memory in the distribution functions can be neglected but we will keep the spectral relaxation in the off-shell cos-function of (1). The resulting expression for (1) describes then how two particles correlate their motion to avoid the strong interaction regions. This very fast formation of the off-shell contribution to Wigner’s distribution has been found in numerical treatments of Green’s functions\(^{13,14}\).

Starting with a sudden switching approximation we consider Coulomb interaction and during the first transient time period the screening is formed. This can be described by the non-Markovian Lenard - Balescu equation\(^{15}\) instead of the static screened equation (1) leading to the dynamical expression of the correlation energy. To demonstrate its results and limitations, we use Maxwellian initial distributions with temperature \(T\) neglecting degeneracy.

From (1) we find with \(\frac{\partial}{\partial t} E_{\text{corr}} = - \sum_{a} \int \frac{dk}{(2\pi)^3} \frac{k^2}{2m_a} \frac{\partial}{\partial t} \rho_a\) for the one-component plasma
\[
\frac{\partial}{\partial t} E_{\text{corr}}^{\text{static}}(t) = - \frac{e^2 \kappa T}{2\hbar} \text{Im} \left[ (1 + 2z^2)e^{z^2} (1 - \text{erf}(z)) - \frac{2z}{\sqrt{\pi}} \right]
\]
\[
\frac{\partial}{\partial t} E_{\text{corr}}^{\text{dynam}}(t) = - \frac{e^2 \kappa T}{\hbar} \text{Im} \left[ e^{z^2} (1 - \text{erf}(z_1)) \right] \tag{2}
\]
where we used \(z = \omega_p \sqrt{t^2 - \frac{\hbar t}{2}}\) and \(z_1 = \omega_p \sqrt{2t^2 - \frac{\hbar t}{2}}\). This is the analytical quantum result of the formation of correlation for statically as well
as dynamically screened potentials. The long time limit of (2) leads to the quantum Montroll correlation energy \(16\) for the dynamical case.

The situation of sudden switching of interaction may be considered as an artificial one. In the simulation experiment we have initial correlations which are due to the set up within quasiperiodic boundary condition and Ewald summations. Therefore we consider now initial correlations which lead to an additional collision term \(19\). This term can be written into the form of (1) with the only difference that the occurring Wigner functions are not time dependent but the initial ones and the interaction \(V_0\) represents initial correlations \(17\). The additional collision term, \(I_0\), cancels therefore exactly the Levinson collision term (1) in the case that we have initially the same interaction as during the dynamical evolution \((V_0 = V)\) and if the system starts from the equilibrium \(\rho(t) \equiv \rho_0\). We choose a model interaction of Debye potential \(V_i(q) = 4\pi e^2 \left/ \left(q^2 + \kappa_i^2\right)\right.\) with fixed parameter \(\kappa_i = \kappa_D\) and for the initial correlations \(\kappa_i = \kappa_0\) to obtain the kinetic energy from the kinetic equation:

\[
E_{\text{kin}}(t) = E_{\text{total}}(t) - E_{\text{init}}(t) - E_{\text{coll}}(t).
\] (3)

For the classical limit the time dependent collisional energy reads

\[
\frac{E_{\text{coll}}(t)}{nT} = -\frac{\sqrt{3}}{4x} \frac{\Gamma^{3/2}}{t \omega_p} \partial_y \left(y \mathcal{F}(y)\right)_{y=x\tau},
\] (4)

where \(\mathcal{F}(y) = 1 - e^{y^2} \text{erfc}(y), \tau = t \omega_p / \sqrt{2}, x = \kappa_D / \kappa\) and \(\kappa^2 = 4\pi e^2 n / T = \omega_p^2 T / m\). The plasma parameter is given as usually by \(\Gamma = \frac{\omega_p^2}{\tau}, \) where \(a_e = \left(\frac{3}{4\pi n}\right)^{1/3}\). In Fig. upper panel, we compare the analytical results of (3) with MD simulations using the Debye potential \(V_i\) as bare interaction. The evolution of kinetic energy is shown for three different ratios \(x\). The agreement between theory and simulations is quite satisfactory, in particular, the short time behavior for \(x = 2\). The stronger initial increase of kinetic energy observed in the simulations at \(x = 1\) may be due to the finite size of the simulation box which could more and more influence the results for increasing range of the interaction. Now we include the initial correlations which leads to

\[
\frac{E_{\text{init}}(t)}{nT} = -\frac{\sqrt{3}}{2(x_0^2 - x^2)} \left[ x \mathcal{F}(x\tau) - x_0 \mathcal{F}(x_0\tau) \right],
\] (5)

where \(x_0 = \kappa_0 / \kappa\) characterizing the strength of the initial correlations with the Debye potential \(V_0\) which contains \(\kappa_0\) instead of \(\kappa_D\). Besides the kinetic energy (5) from initial correlations, the total energy \(E_{\text{total}}\) now includes the correlation energy at the beginning which is the total energy content of
The formation of correlation energy $-E_{\text{corr}} \equiv E_{\text{total}} - E_{\text{init}} - E_{\text{coll}} = E_{\text{kin}}$ in a plasma with Debye interaction $V_i$. The upper panel compares the analytical results (4) with MD simulations from 18 for three different ratios of $\kappa_D$ to the inverse Debye length $x = \kappa_D/\kappa$. In the lower panel we compare theoretical predictions for the inclusion of Debye initial correlations characterized by $x_0 = \kappa_0/\kappa$ where $x = \kappa_D/\kappa = 1$.

Figure 1: The formation of correlation energy $-E_{\text{corr}} \equiv E_{\text{total}} - E_{\text{init}} - E_{\text{coll}} = E_{\text{kin}}$ in a plasma with Debye interaction $V_i$. The upper panel compares the analytical results (4) with MD simulations from 18 for three different ratios of $\kappa_D$ to the inverse Debye length $x = \kappa_D/\kappa$. In the lower panel we compare theoretical predictions for the inclusion of Debye initial correlations characterized by $x_0 = \kappa_0/\kappa$ where $x = \kappa_D/\kappa = 1$.

the system and can be calculated from the long time limit of (4) leading to

$$\frac{E_{\text{total}}}{nT} = \frac{\sqrt{3} \Gamma^{3/2}}{2(x + x_0)}.$$  \hspace{1cm} (6)

The result (6) is seen in Fig. 1, lower panel. We observe that if the initial correlation is characterized by a potential range larger than the Debye screening length, $x_0 < 1$, the initial state is over–correlated, and the correlation energy starts at a higher absolute value than without initial correlations relaxing towards the correct equilibrium value. If, instead, $x_0 = 1$ no change of correlation energy is observed, as expected. Similar trends have been observed in numerical solutions [19].

The characteristic time of formation of correlations at high temperature limit is given by the inverse plasma frequency $\tau_c \approx \frac{1}{\omega_p} = \frac{\sqrt{x}}{v_{th} \kappa}$. This confirms the numerical finding of 19 that the correlation or memory time is proportional to the range of interaction. In the low temperature region, i.e., in a highly degenerated system $\mu \gg T$, one finds 9 that the build up time is the inverse Fermi energy, $\hbar/\epsilon_f$ in agreement with the quasiparticle formation time known as Landau’s criterion. Indeed, the quasiparticle formation and the build up of correlations are two alternative views of the same phenomenon.

2 Virial regime

Now we proceed and investigate how the relaxation into equilibrium is performed by the system. We will convince ourselves firstly that the direct use of reduced density matrix as done so far for short time regimes is not possible.
On larger time scales we expect smoothed time-space gradients and expand the Levinson equation for the reduced density matrix up to first order memory to obtained from (7)

\[ \frac{\partial}{\partial t} \rho = I_{\text{Boltz}} + \frac{\partial}{\partial t} I_{\text{corr}} \]  

where besides the usual Boltzmann term, \( I_{\text{Boltz}} \), there appears an additional off-shell term, \( I_{\text{corr}} \), which describes the correlation in the reduced density matrix. For the density balance we see from (7) that the correlation part from the right side \( \int dp I_{\text{corr}} = n_c \) which is known as the Beth-Uhlenbeck equation of state [20, 21, 22] cancels exactly with the left hand side of (7) according to the definition \( \int dp \rho = n_f + n_c \). So we have to face the exact cancelation of \( n_c \) in the balance equation and the incorrect conservation of \( n_f \). We will show that the virial corrections and the correlated density will appear instead from internal gradients. That the correlated part \( I_{\text{corr}} \) does not provide the virial corrections but a relation between reduced density matrix and quasiparticle distribution has not been noted in the literature.

The second puzzle is a more obvious hint that the Levinson equation fails to describe the long time evolution of a system. We can iterate the time derivative inside the collision integral on the right hand side of (7) to create an infinite series. Neglecting backscattering terms we estimate

\[ \frac{\partial}{\partial t} f \geq \zeta[f] I_{\text{Boltz}}[f] \]  

with \( \zeta = \sum_{n=0}^{\infty} \eta^{2n} \) and \( \eta \) is similar to the backflow (second order response) [12] and is a positive number but not every time smaller than one. Therefore \( \zeta \) appears to be divergent at certain momentum situations. Therefore we conclude that the gradient expanded Levinson equation leads not to a stationary solution in any case. This has also been observed in numerical solutions of these non-Markovian kinetic equations, see figure 4 of [25]. We will show that a proper balance between off-shell terms in the reduced density and the kinetic equation for the pole part is necessary to ensure stationary solutions.

The approach presented now is based on the real-time Greenfunction technique. We consider the two independent correlation functions for Fermionic creation operators \( G^>(1,2) = \langle a(1)a^+(2) \rangle \) and \( G^<(1,2) = \langle a^+(2)a(1) \rangle \), where cumulative variables means time, space, spin, etc \( 1 = t, x, s, \ldots \). The time diagonal part of \( G^<= \) yields the reduced density matrix according to \( \rho(x_1, x_2, t) = G^<=1,2)_{t_1=t_2} \). For the latter one the time diagonal Kadanoff and Baym equation of motion reads [26].
\[ i[G_0^{-1}, \rho](x_1, x_2, t) = \int_0^t dt' \int dx' \left[ G^<(x_1, x', t, t') \Sigma^>(x', x_2, t', t) + \Sigma^>(x_1, x', t, t') G^<(x', x_2, t', t') - G^>(x_1, x', t, t') \Sigma^<(x', x_2, t', t) - \Sigma^<(x_1, x', t, t') G^>(x', x_2, t', t') \right] \]

(9)

with the Hartree-Fock drift term \( G_0^{-1} \). The right hand side contains a non-Markovian collision integral and can be considered as a precursor of Levinson equation \( \text{(9)} \). Using gradient approximation we obtain a kinetic equation of the structure \( \text{(7)} \) and see that the time derivative of the Wigner function on the left side of \( \text{(9)} \) combines with the time derivative of the off-shell part of right hand side into a time derivative of a quasiparticle distribution

\[
\rho \approx z f - \int \frac{d\omega}{2\pi} \frac{\mathcal{P}'}{\omega - \epsilon} \sigma^<(\omega)
\]

(10)

with \( \mathcal{P}' \) the derivative of the principal value. The wave function renormalization has been abbreviated as \( z = (1 - \partial_\omega \sigma)^{-1} \approx 1 + \partial_\omega \sigma \) which can be confirmed using the spectral decomposition \( \sigma(\omega) = \int \frac{d\omega'}{2\pi} \mathcal{P} \gamma(\omega') \) with the imaginary part \( \gamma = \sigma^> + \sigma^< \). We have fulfilled the task and give with \( \text{(10)} \) a connection between reduced density matrix \( \rho \) and the quasiparticle distribution \( f \). We extrapolate the ansatz also for the correlation functions

\[
g^>(\omega) = 2\pi z \delta(\omega - \epsilon) \left( \frac{1}{f} - \frac{\mathcal{P}'}{\omega - \epsilon} \sigma^>(\omega) \right).
\]

(11)

The spectral identity \( a = g^> + g^< \) proofs that this ansatz is consistent with the extended quasiparticle picture which is obtained for small imaginary parts of the self-energy \( \gamma \).

The quasiparticle energies \( \epsilon(p\omega r t) \) are a solution of the dispersion relation \( \omega - \frac{p^2}{2m} - \sigma(p\omega r t) = 0 \). It is noteworthy to remark that \( \text{(12)} \) fulfills the spectral sum rule \( \text{[2]} \) and the energy weighted sum rule \( \text{[6]} \). The limit of small scattering rates has been first introduced by Craig \( \text{[3]} \). An inverse functional \( f[\rho] \) has been constructed in \( \text{[4]} \). For equilibrium nonideal plasmas this approximation has been employed by \( \text{[5]} \) and under the name of the generalized Beth-Uhlenbeck
approach has been used by in nuclear matter for studies of the correlated density. The authors in have used this approximation with the name extended quasiparticle approximation for the study of the mean removal energy and high-momenta tails of Wigner’s distribution. The non-equilibrium form has been derived finally as modified Kadanoff and Baym ansatz. Using this ansatz from the Kadanoff-Baym equations the known Landau-Silin equation for the quasiparticle distribution \( f \) follows
\[
\frac{\partial}{\partial t} f + \nabla_k \epsilon \nabla_r f - \nabla_r \epsilon \nabla_k f = z \left( (1 - f) \sigma^< - f \sigma^> \right). \quad (13)
\]
We repeat that the Landau-Silin equation is coupled with a functional that specifies a relation between the quasiparticle distribution \( f \) and Wigner’s function \( \rho \) via \( \sigma \). Using different approximations for the self energy we obtain all known kinds of kinetic equations with the generalization that the internal gradients of collision integrals will yield the nonlocal or virial corrections.

The quality of the extended quasiparticle approximation can be seen in the figure of the contribution by P. Lipavsky. One can see that the off-shell contribution given by the difference between the Wigner and the Fermi-Dirac distributions is not small, in particular at the high momenta region where the power-law off-shell tails always dominate over the exponentially falling quasiparticle distribution. Formula provides inevitable and sufficiently precise off-shell corrections.

In order to show the nonlocal character of the kinetic equation explicitly, we choose as an approximation for the self energy the ladder or \( T \)-matrix approximation appropriate for dense interacting systems of short range potentials
\[
\Sigma^<(1, 2) = T^R(1, \bar{3}; 5, \bar{6})T^A(\bar{7}, \bar{8}; 2, \bar{4})G^> (\bar{4}, \bar{3})G^< (\bar{5}, \bar{7})G^< (\bar{6}, \bar{8}). \quad (14)
\]
For non-degenerate systems, the gradient expansion has been carried through by Bärwinkel. One can see in Bärwinkel’s papers, that the scattering integral is troubled by a large set of gradient corrections. This formal complexity seems to be the main reason why most authors either neglect gradient corrections at all or provide them buried in multi-dimensional integrals. For a degenerate system, the set of gradient corrections to the scattering integral is even larger than for rare gases studied by Bärwinkel, see. The quasiclassical limit with all linear gradients kept is a tedious but straightforward algebraic exercise. It results in one nongradient and nineteen gradient terms that are analogous to those found within the chemical physics. All these terms can be recollected into a nonlocal and noninstantaneous
scattering integral that has an intuitively appealing structure of a nonlocal Boltzmann equation
\[
\frac{\partial f_1}{\partial t} + \frac{\partial \varepsilon_1}{\partial k} \frac{\partial f_1}{\partial r} - \frac{\partial \varepsilon_1}{\partial r} \frac{\partial f_1}{\partial k} = \sum_b \int \frac{dp dq}{(2\pi)^5} \delta (\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4 + 2\Delta E) z_1 z_2 z_3 z_4
\times \left| T_{ab}(\varepsilon_1 + \varepsilon_2 - \Delta E, k - \frac{\Delta_K}{2}, p - \frac{\Delta_K}{2}, q, r - \Delta_r, t - \frac{\Delta_t}{2}) \right|^2 \left(1 - \frac{1}{2} \frac{\partial \Delta_2}{\partial r} - \frac{\partial \varepsilon_2}{\partial r} \frac{\partial \Delta_2}{\partial \omega}\right)
\times \left[f_3 f_4 (1 - f_1)(1 - f_2) - (1 - f_3)(1 - f_4)f_1 f_2\right],
\] (15)

with Enskog-type shifts of arguments
\[f_1 \equiv f_a(k, r, t), \quad f_2 \equiv f_b(p, r - \Delta_2, t), \quad f_3 \equiv f_a(k - q - \Delta_K, r - \Delta_3, t - \Delta_t), \quad \text{and} \quad f_4 \equiv f_b(p + q - \Delta_K, r - \Delta_4, t - \Delta_t).\]

In agreement with [33, 34], all gradient corrections result proportional to derivatives of the scattering phase shift \(\phi = \text{Im} \ln T_{ab}^{R}(\Omega, k, p, q, t, r)\),

\[\Delta_2 = \left. \left(\frac{\partial \phi}{\partial p} - \frac{\partial \phi}{\partial q} - \frac{\partial \phi}{\partial k}\right)\right|_{\varepsilon_3 + \varepsilon_4}, \quad \Delta_3 = \left. -\frac{\partial \phi}{\partial k}\right|_{\varepsilon_3 + \varepsilon_4}, \quad \Delta_4 = \left. -\left(\frac{\partial \phi}{\partial k} + \frac{\partial \phi}{\partial q}\right)\right|_{\varepsilon_3 + \varepsilon_4},
\]

\[\Delta_\varepsilon = \left. \frac{\partial \phi}{\partial \Omega}\right|_{\varepsilon_3 + \varepsilon_4}, \quad \Delta_K = \left. \frac{1}{2} \frac{\partial \phi}{\partial r}\right|_{\varepsilon_3 + \varepsilon_4}, \quad \Delta_E = \left. -\frac{1}{2} \frac{\partial \phi}{\partial t}\right|_{\varepsilon_3 + \varepsilon_4}, \quad \Delta = \left. (\frac{1}{2} - \frac{1}{2})\right|_{\varepsilon_3 + \varepsilon_4},
\] (16)

and \(\Delta_r = \frac{1}{2}(\Delta_2 + \Delta_3 + \Delta_4)\). After derivatives, \(\Delta\)'s are evaluated at the energy shell \(\Omega \rightarrow \varepsilon_3 + \varepsilon_4\).

The \(\Delta\)'s in arguments of distribution functions in (15) remind non-instant and non-local corrections in the scattering-in integral for classical particles. The displacements of the asymptotic states are given by \(\Delta_{2,3,4}\). The time delay enters in an equal way the asymptotic states 3 and 4. The momentum gain \(\Delta_K\) also appears only in states 3 and 4. Finally, there is the energy gain which is discussed in [35]. We remind that the non-localities should be viewed as a compact form of gradient corrections.

Despite its complicated form it is possible to solve this kinetic equation with standard Boltzmann numerical codes and to implement the shifts [36]. Therefore we have calculated the shifts for different realistic nuclear potentials [37]. The numerical solution of the nonlocal kinetic equation has shown an observable effect in the dynamical particle spectra of around 10%. The high energetic tails of the spectrum is enhanced due to more violent collisions in the early phase of nuclear collision. Therefore the nonlocal corrections lead to an enhanced production of preequilibrium high energetic particles.
3 Thermodynamic properties

The meaning of nonlocal shifts can be best seen on thermodynamic observables like density $n_a$ of particles $a$, density of energy $E$, and the stress tensor $\mathcal{J}_{ij}$ which conserve within the nonlocal and noninstantaneous kinetic equation (15). Integrating (15) over momentum $k$ with factors $\varepsilon_1$, $k$, and unity one finds that each observable has the standard quasiparticle part following from the drift

$$E_{qp} = \sum_a \int \frac{dk}{(2\pi)^3} \frac{k^2}{2m} f_1 + \frac{1}{2} \sum_{a,b} \int \frac{dkdp}{(2\pi)^6} T_{ab}(\varepsilon_1 + \varepsilon_2, k, p, 0) f_1 f_2,$$

$$\mathcal{J}_{ij}^{qp} = \sum_a \int \frac{dk}{(2\pi)^3} \left( k_j \frac{\partial \varepsilon_1}{\partial k_i} + \delta_{ij} \varepsilon_1 \right) f_1 - \delta_{ij} E_{qp},$$

$$n_a^{qp} = \int \frac{dk}{(2\pi)^3} f_1,$$

and the $\Delta$-contribution following from the scattering integral

$$\Delta E = \frac{1}{2} \sum_{a,b} \int \frac{dkdpdq}{(2\pi)^9} P(\varepsilon_1 + \varepsilon_2) \Delta t,$$

$$\Delta \mathcal{J}_{ij} = \frac{1}{2} \sum_{a,b} \int \frac{dkdpdq}{(2\pi)^9} P \left[ (p+q)_j \Delta_{4j} + (k-q)_i \Delta_{3j} - p_i \Delta_{2j} \right],$$

$$\Delta n_a = \sum_b \int \frac{dkdpdq}{(2\pi)^9} P \Delta t,$$

where $P = |T_{ab}|^2 2\pi \delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4) f_1 f_2 (1 - f_3 - f_4)$.

The density of energy $E = E_{qp} + \Delta E$ alternatively results from Kadanoff and Baym formula, $E = \sum_a \int \frac{dk}{(2\pi)^3} \int \frac{d\omega}{2\pi} \frac{1}{2 \sqrt{2m}} \left( \omega + \frac{k^2}{2m} \right) G_\omega^\ast(\omega, k, r, t)$, with $G_\omega^\ast$ in the extended quasiparticle approximation (11). Its complicated form, however, shows that $E$ cannot be easily inferred from an eventual experimental fit of the kinetic equation as it has been attempted in 38. The conservation of $E$ generalizes the result of Bornath, Kremp, Kraeft and Schlange restricted to non-degenerated systems. The particle density $n_a = n_a^{qp} + \Delta n_a$ is also obtained from (11) via the definition, $n_a = \int \frac{d\omega}{2\pi} \frac{dk}{(2\pi)^3} G_\omega^\ast$. This confirms that the extended quasiparticle approximation is thermodynamically consistent with the nonlocal and noninstantaneous corrections to the scattering integral.

For equilibrium distributions, formulas (17) and (18) provide equations of state. Two known cases are worth to compare. First, the particle density $n_a^{qp} + \Delta n_a$ is identical to the quantum Beth-Uhlenbeck equation of state 22.
where \( n_{qp} \) is called the free density and \( \Delta n_a \) the correlated density. Second, the virial correction to the stress tensor has a form of the collision flux contribution known in the theory of moderately dense gases.

### 4 Summary

While the short time behavior is described by off-shell transport condensed in the reduced density matrix, the later stage of evolution is described by a nonlocal Boltzmann equation. This equation is derived in the quasiclassical limit and leads to consistent thermodynamics in equilibrium including binary correlations. The latter ones represent the second virial coefficients in the low density limit. The presented kinetic equation unifies achievements of dense gases and the kinetic theory of quasiparticle transport in quantum systems. The equation has been shown applicable in recent simulation codes.

1. S. Chapman and T. G. Cowling, *The Mathematical Theory of Non-uniform Gases* (Cambridge University Press, Cambridge, 1990), third edition Chap. 16.
2. J. O. Hirschfelder, C. F. Curtiss, and R. B. Bird, *Molecular Theory of Gases and Liquids* (Wiley, New York, 1964), chapt. 6.4a and 9.3.
3. Y. L. Klimontovich, *Kinetic Theory of Nonideal Gases and Nonideal Plasmas* (Pergamon Press, Oxford, 1982).
4. K. Bärwinkel, Z. Naturforsch. a 24, 38 (1969), Z. Naturforsch. a 24, 22 (1969).
5. T. Bornath, D. Kremp, W. D. Kraeft, and M. Schlanges, Phys. Rev. E 54, 3274 (1996).
6. V. Špička, P. Lipavský, and K. Morawetz, Phys. Lett. A 240, 160 (1998), K. Morawetz, habilitation thesis (1998) University Rostock.
7. M. Bonitz et. al., J. Phys.: Condens. Matter 8, 6057 (1996).
8. P. Lipavský, F. S. Khan, A. Kalvová, and J. W. Wilkins, Phys. Rev. B 43, 6650 (1991).
9. K. Morawetz, V. Špička, and P. Lipavský, Phys. Lett. A 246, 311 (1998), K. Morawetz and H. Köhler, Eur. Phys. J. A 4, 291 (1999), K. Morawetz, Phys. Lett. A 199, 241 (1995).
10. I. B. Levinson, Fiz. Tverd. Tela Leningrad 6, 2113 (1965), Zh. Eksp. Teor. Fiz. 57, 660 (1969), [Sov. Phys.–JETP 30, 362 (1970)].
11. A. P. Jauho and J. W. Wilkins, Phys. Rev. B 29, 1919 (1984).
12. V. Špička and P. Lipavský, Phys. Rev. B 52, 14615 (1995), Phys. Rev. Lett 73, 3439 (1994).
13. P. Danielewicz, Ann. Phys. (NY) 152, 305 (1984).
14. H. S. Köhler, Phys. Rev. C 51, 3232 (1995).
15. K. Morawetz, Phys. Rev. E 50, 4625 (1994).
16. W. D. Kraeft, D. Kremp, W. Ebeling, and G. Röpke, Quantum Statistics of Charged Particle Systems (Akademie Verlag, Berlin, 1986).
17. K. Morawetz et al., Phys. Rev. Lett., sub., physics/9905024
18. G. Zwicknagel, Contrib. Plasma Phys. 39 (1999) 1-2,155, and private communications
19. D. Semkat, D. Kremp, and M. Bonitz, Phys. Rev. E 59, 1557 (1999).
20. G. E. Beth and E. Uhlenbeck, Physica 4, 915 (1937).
21. H. Stolz and R. Zimmermann, Phys. Status Solidi B 94, 135 (1979), R. Zimmermann and H. Stolz, Phys. Status Solidi B 131, 151 (1985).
22. M. Schmidt and G. Röpke, Phys. stat. sol. (b) 139, 441 (1987), M. Schmidt, G. Röpke, and H. Schulz, Ann. Phys. (NY) 202, 57 (1990).
23. R. A. Craig, Ann. Phys. 40, 416 (1966), Ann. Phys. 40, 434 (1966).
24. B. Bezzerides and D. F. DuBois, Phys. Rev. 168, 233 (1968).
25. D. Kremp, M. Bonitz, W. Kraeft, and M. Schlages, Ann. of Phys. 258, 320 (1997).
26. L. P. Kadanoff and G. Baym, Quantum Statistical Mechanics (Benjamin, New York, 1962).
27. D. Kremp, W. D. Kraeft, and A. D. J. Lambert, Physica A 127, 72 (1984).
28. H. S. Köhler and R. Malfliet, Phys. Rev. C 48, 1034 (1993).
29. P. Danielewicz, Ann. Phys. (NY) 152, 239 (1984).
30. W. Botermans and R. Malfliet, Phys.Rep. 198, 115 (1990).
31. V. G. Morozov and G. Röpke, Physica A 221, 511 (1995).
32. D. Loos, J. Stat. Phys. 59, 691 (1990), J. Stat. Phys. 61, 467 (1990).
33. P. J. Nacher, G. Tastevin, and F. Laloe, Ann. Phys. (Leipzig) 48, 149 (1991), Journal de Physique I 1, 181 (1991).
34. M. de Haan, Physica A 164, 373 (1990), H. de Haan, Physica A 165, 224 (1990), H. de Haan, Physica A 170, 571 (1991).
35. P. Lipavský, V. Špička, and K. Morawetz, Phys. Rev. E 52, R1291 (1999).
36. K. Morawetz et al., Phys. Rev. Lett. 82, 3767 (1999).
37. K. Morawetz, P. Lipavský, V. Špička, and N. Kwong, Phys. Rev. C 59, 3052 (1999).
38. H. Stöcker and W. Greiner, Phys. Rep. 137, 277 (1986).