Required sensitivity in the search of neutrinoless double beta decay in $^{124}$Sn

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Abstract: The INdia’s TIN (TIN.TIN) detector is under development in the search of neutrinoless double-$\beta$ decay ($0^{\nu}\beta\beta$) using 90% enriched $^{124}$Sn isotope as the target mass. This detector will be housed in the upcoming underground facility of the India-based Neutrino Observatory. We present the most important experimental parameters that would be used in the study of required sensitivity for the TIN.TIN experiment to probe the neutrino mass hierarchy. The sensitivity of the TIN.TIN detector in the presence of sole two neutrino double-$\beta$ decay ($2\nu\beta\beta$) background is studied at various energy resolutions. The most optimistic and pessimistic scenario to probe the neutrino mass hierarchy at 3$\sigma$ sensitivity level and 90% CL are discussed.

Keywords: Double beta decay; Nuclear matrix element; Neutrino mass hierarchy

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1. Introduction

Neutrinoless double-$\beta$ decay ($0^{\nu}\beta\beta$) is an interesting venue to look for the most important question whether neutrinos have Majorana or Dirac nature. The discovery of nonzero neutrino mass and mixing with neutrino oscillation experiments gives new motivation for more sensitive searches of $0^{\nu}\beta\beta$. The observation of $0^{\nu}\beta\beta$ would not only establish the Majorana nature of neutrinos, but also provide a measurement of effective mass and probe the neutrino mass hierarchy. There is no exact gauge symmetry associated with lepton number; therefore, there is no fundamental reason why lepton number should be conserved at all levels [1, 2]. The lepton number violates by two units in the case of $0^{\nu}\beta\beta$. This distinctive feature together with CP (charge, parity) violation supports the exciting possibility that neutrino plays an important role in the matter–antimatter asymmetry in the early universe. Primarily, studies of $0^{\nu}\beta\beta$ are searches for lepton number violation and to distinguish whether neutrinos are Majorana or Dirac particles [3]. The “Mass Mechanism” (standard interpretation) is one of the many possible channels to mediate $0^{\nu}\beta\beta$ [4], though this is the most studied.

The experimental search for $0^{\nu}\beta\beta$ is an attractive field of nuclear and particle physics. There are 35 isotopes where $0^{\nu}\beta\beta$ are energetically possible while single $\beta$-decays are kinematically prohibited. These are the ones with experimental significance [3]. Several experiments are focusing on different isotopes via utilizing various detector techniques such as GERDA (GERmanium Detector Array) [5], MAJORANA (Majorana Demonstrator) [6] and CDEX (China Dark matter EXperiment) with $^{76}$Ge-enriched high-purity Ge detectors [7]; EXO (Enriched Xenon Observatory) [8] and KamLandZen (Kamioka Liquid Scintillator Antineutrino Detector) with liquid $^{136}$Xe time projection chambers [9]; and CUORE (Cryogenic Underground Observatory for Rare Events) with $^{130}$Te bolometric detectors [10]. The next-generation experiments with tonne scale detectors such as LEGEND ($^{76}$Ge) (MAJORANA + GERDA) [11, 12], nEXO ($^{136}$Xe) [13], NEXT ($^{136}$Xe) [14],
CUPID ($^{130}$Te) [15], SuperNEMO (82Se, 150Nd) [16], AMoRE (100Mo) [17], COBRA (116Cd) [18], CANDLES-III (48Ca) [19], SNO+ ($^{130}$Te) [20], TIN.TIN ($^{124}$Sn) [21], MOON (100Mo) [22] and LUMINEU (100Mo) [23] have been proposed. Some of them will start taking data over the next few years and others are under construction phase. These large numbers of experiments, reveal the enthusiasm of the scientists working in this field worldwide.

The two neutrino double-β decay ($2\nu\beta\beta$) is a second-order weak process, in which two neutrons simultaneously transfer into two protons by emitting two electrons and two antineutrinos within the same nucleus [24]:

$$N A_{BB} \rightarrow N^{A-2} A + 2e^- + 2\bar{\nu}_e.$$  \hspace{1cm} (1)

The energy distribution of $2\nu\beta\beta$ process exhibits a continuous spectrum, ending at a well-defined endpoint which is determined by the $Q_{\beta\beta}$-value of the process, as depicted in Fig. 1. The $2\nu\beta\beta$ decay follows the conservation of lepton number and is also allowed by the standard model [24]. In the case of $0\nu\beta\beta$ process, no neutrino is emitted and both electrons carry the full energy equal to the $Q_{\beta\beta}$-value of the transition.

Indeed, the energy of the recoiling nucleus is negligible due to its high mass. Therefore, the experimental signature of $0\nu\beta\beta$ would manifest as a monoenergetic peak at the $Q_{\beta\beta}$-value detectable through the measurement of the total energy of the two emitted electrons:

$$N A_{BB} \rightarrow N^{A-2} A + 2e^-.$$  \hspace{1cm} (2)

The TIN.TIN (The INdia’s TIN) detector is under development for the search of $0\nu\beta\beta$ in $^{124}$Sn isotope. The TIN.TIN detector will use the cryogenic bolometer technique in closely packed module structure arrays [25]. This experiment will be housed at the India-based Neutrino Observatory, an upcoming underground laboratory [25, 26].

The natural isotopic abundance of $^{124}$Sn isotope is $\sim 5.8\%$ and its $Q_{\beta\beta}$-value is 2287.7 keV [27, 28]. The High Energy Physics experimental group of Tata Institute of Fundamental Research (TIFR), Mumbai, has tested the cryogenic Sn bolometers (size of mg scale). They found that these bolometers work very impressively with very good energy resolution at sub-Kelvin temperature [21, 29]. The R&D on approximately 1 kg natural Sn prototype and the enrichment of $^{124}$Sn is in progress [21]. The sensitivity of an experiment can be decided by the following five important parameters: (1) energy resolution ($\Delta E$) at $Q_{\beta\beta}$, (2) exposure ($\beta\beta\text{mass} \times \text{time}$) ($\Sigma$), (3) background rate ($\Lambda$), (4) isotopic abundance (IA) and (5) signal detection efficiency ($\varepsilon_{\text{exp}}$). The smearing of $2\nu\beta\beta$ events ($B_{2\nu}$) ($\tau_{2\nu}^0 = 0.8 - 1.2 \times 10^{21}$ year) [21] in the region of interest (ROI) is the irreducible background in the search of $0\nu\beta\beta$. This irreducible background can be minimized by using a detector with very good energy resolution. It follows that the cryogenic bolometers with their excellent resolution are optimal techniques in the search of $0\nu\beta\beta$ decay.

2. Neutrino parameters and $0\nu\beta\beta$ half-life

In the simplest case, $0\nu\beta\beta$ decay is mediated by the virtual exchange of a light Majorana neutrino in the absence of right-handed currents. The half-life ($\tau_{0\nu}^0$) of $0\nu\beta\beta$ isotopes can be expressed as [30]

$$\left[\tau_{0\nu}^0\right]^{-1} = G^{0\nu} g^0_{\nu} |M^{0\nu}|^2 \left[\langle m_{\beta\beta}^2 \rangle^{2}\right]^2 = G^{0\nu} |M^{0\nu}|^2 \left[\langle m_{\beta\beta}^2 \rangle^{2}\right]^2,$$  \hspace{1cm} (3)

where $G^{0\nu}$ is a known phase space factor, $G^{0\nu}$ is the product of weak axial vector coupling constant ($g^0_{\nu}$) and $G^{0\nu} |M^{0\nu}|$ is the nuclear matrix element, and $m_{\nu}$ is the mass of the electron. To avoid the ambiguity of $g_{\nu}$ in the presence of nuclear medium, its free nucleon value ($g_{\nu} = 1.269$) [31] is adopted. The effective Majorana neutrino mass is expressed as [32]

$$\langle m_{\beta\beta} \rangle = \sum_{k=1}^{3} m_{\nu_k} U_{e\nu k}^2,$$  \hspace{1cm} (4)

which depends on the neutrino masses ($m_{\nu_k}$ for eigenstates $\nu_k$) and PMNS (Pontecorvo–Maki–Nakagawa–Sakata) mixing matrix ($U$) [30, 32, 33]. Expansion of Eq. 4 will provide the $\langle m_{\beta\beta} \rangle$ as [33]

$$\langle m_{\beta\beta} \rangle = \left|c_{12} c_{13}^2 m_1 + s_{12} c_{13}^2 m_2 e^{i\xi} + s_{13}^2 m_3 e^{i(\beta - 2\delta)}\right|. $$  \hspace{1cm} (5)

The value of $\langle m_{\beta\beta} \rangle$ depends on sines ($s$) and cosines ($c$) of the leptonic mixing angles $\theta_{ij}$, the mass eigenvalues
(m), Majorana Phases $e^{i\varphi} = e^{i\beta} = \pm 1$ and the CP violating phase $e^{-i\varphi_{bb}} = 1$. The measurement of mass-squared splitting ($\Delta m_{21}^2 = \Delta m_{21}^2 + \Delta m_{31}^2/2$) and $\Delta m_{23}^2 = \frac{1}{2} \left| \Delta m_{21}^2 + \Delta m_{31}^2 \right|$) allows two hierarchy configurations for the mass eigenstates: either inverted hierarchy (IH) ($m_3 < m_1 < m_2$) or normal hierarchy (NH) ($m_1 < m_2 < m_3$) [32, 34]. The values of $\langle m_{bb} \rangle$ can vanish for NH at $m_{\text{min}} \in [10^{-3}, 10^{-2}]$ eV. However, this requires accidental correlations of the Majorana phases and has vanishing probabilities under the reasonable prior that the phases are uncorrelated [35]. In this analysis, we select $\langle m_{bb} \rangle$ at $m_{\text{min}} < 10^{-4}$ eV and use the best-fit values of the mass-mixing parameters with $1\sigma$ uncertainty range [32], such that

\begin{equation}
\left\{ \begin{array}{l}
\text{IH} : 1.8 \times 10^{-2} (\text{eV}) \leq \langle m_{bb} \rangle \leq 5.0 \times 10^{-2} (\text{eV}) \\
\text{NH} : 1.4 \times 10^{-3} (\text{eV}) \leq \langle m_{bb} \rangle \leq 4.1 \times 10^{-3} (\text{eV}).
\end{array} \right. \tag{6}
\end{equation}

The precise calculations of $G^{\text{bb}}$ and $|M^{\text{bb}}|$ are needed in order to translate the experimental values of the $0\nu\beta\beta$ half-lives into $\langle m_{bb} \rangle$. With an uncertainty of approximately 7%, $G^{\text{bb}}$ is well known [36]. On the other hand, the calculation of $|M^{\text{bb}}|$ is a difficult task involving the details of the underlying theoretical models. Several different theoretical models have been used to compute $|M^{\text{bb}}|$ for the different $A_{bb}$ such as interacting shell model (ISM) [37], quasiparticle random-phase approximation (QRPA) (and its variants) [38, 39], interacting boson model (IBM-2) [40], angular momentum projected Hartree–Fock–Bogoliubov method (PHFB) [41], generating coordinate method (GCM) and energy density functional method (EDF) [31, 42]. Deviations among their results are the main sources of theoretical uncertainties in the required sensitivity.

For $^{124}\text{Sn}$ isotope, $|M^{\text{bb}}|$ along with the corresponding theoretical models are listed in Table 1. In the given range of $|M^{\text{bb}}|$, the PHFB and SM are in the most optimistic and most conservative scenario, respectively. Therefore, the required sensitivity corresponding to other $|M^{\text{bb}}|$ will lie in between this range. Using the range of $\langle m_{bb} \rangle$ from Eq. 6 and $|M^{\text{bb}}|$ from Table 1, the corresponding benchmark sensitivities can be calculated in terms of with the help of Eq. 3. The value of combined function ($F_n$) for PHFB and SM models is adopted from Ref. [21]:

\begin{equation}
F_n = G^{\text{bb}} \cdot |M^{\text{bb}}|^2 = 8.6 \times 10^{-13} \text{ year}^{-1} (\text{PHFB})
= 1.4 \times 10^{-13} \text{ year}^{-1} (\text{SM}). \tag{7}
\end{equation}

Using Eqs. 3, 6 and 7, the required sensitivities in the form of $t_{1/2}^{0\nu}$ are

\begin{align*}
\text{PHFB} & \equiv \text{IH} : 1.2 \times 10^{26} (\text{year}) < t_{1/2}^{0\nu} < 9.8 \times 10^{26} (\text{year}) \\
\text{NH} & : 1.8 \times 10^{28} (\text{year}) < t_{1/2}^{0\nu} < 1.6 \times 10^{29} (\text{year}) \\
\text{SM} & \equiv \text{IH} : 7.6 \times 10^{26} (\text{year}) < t_{1/2}^{0\nu} < 6.1 \times 10^{27} (\text{year}) \\
\text{NH} & : 1.1 \times 10^{29} (\text{year}) < t_{1/2}^{0\nu} < 1.0 \times 10^{30} (\text{year}). \tag{8}
\end{align*}

The current generation of oscillation experiments may reveal Nature’s choice among the two hierarchy options. Moreover, the combined cosmology data may provide a measurement on the sum of $m_1$ [44, 45]. Thus, it can be expected that the ranges of parameter space in $0\nu\beta\beta$ searches will be further constrained.

From the experimental point of view, the measurement of $t_{1/2}^{0\nu}$ of $0\nu\beta\beta$ relies just on the observed signal ($S_{\text{expt}}$, $0\nu\beta\beta$-events)). The relationship between $t_{1/2}^{0\nu}$ and observed $S_{\text{expt}}$ can be derived from the law of radioactive decay

\begin{equation}
\left( t_{1/2}^{0\nu} \right)^{-1} = \left( \frac{\log_{2} A}{N_{A}} \right) \frac{1}{S_{\text{expt}}} \sum_{i} \frac{1}{\varepsilon_{\text{ROI}}}, \tag{9}
\end{equation}

where $A$ is the molar mass of the source $A_{bb}$, $N_{A}$ is the Avogadro Number and $\varepsilon_{\text{ROI}}$ is the efficiency of selected ROI.

In the search of $0\nu\beta\beta$ decay, the ROI around the $Q_{\beta\beta}$ value could be symmetric and asymmetric. The symmetric FWHM ROI at $Q_{\beta\beta}$-value is most often selected to quantify experimental sensitivities. The ROI in the current study is taken to be the FWHM window centered at $Q_{\beta\beta}$-value, such that the efficiency $\varepsilon_{\text{ROI}}\sim 76\%$. Presence of ambient background $\gamma$-lines near to the $Q_{\beta\beta}$-value of isotopes leads to the contamination of ROI. Therefore, the selection of asymmetric ROI would be an appropriate choice for the detector with poor resolution. The major sources of ambient $\gamma$-radiations around the $Q_{\beta\beta}$-value of $^{124}\text{Sn}$ are $^{210}\text{Bi}$ line at 2.2 MeV and $^{208}\text{TI}$ line at 2.6 MeV. The excellent resolution (0.5% or $\sigma = 4.9$ keV at $Q_{\beta\beta}$) expected for TIN.TIN makes the ROI many $\sigma$ away from all such background structures. For example, $^{210}\text{Bi}$ line and the Compton edge of $^{208}\text{TI}$ line at 2.4 MeV are 17$\sigma$ and 19$\sigma$ away from the $Q_{\beta\beta}$ value of $^{124}\text{Sn}$, respectively. Thus, the selection of symmetric ROI ($Q_{\beta\beta} \pm \text{FWHM}_{2/3}$) would be an appropriate choice for the TIN.TIN experiment.

Every experiment needs to use an enriched isotope for obtaining better sensitivity. Therefore, for simplicity and being easily convertible, both the IA of the $0\nu\beta\beta$ isotopes in the target and the other experimental efficiencies ($\varepsilon_{\text{expt}}$)

| Theoretical model (scheme) | $|M^{bb}|$ |
|-----------------------------|---------|
| Projected Hartree–Fock–Bogoliubov (PHFB) | 6.0     |
| Generating coordinate method (GCM) | 4.8     |
| Interacting boson model (IBM) | 3.5     |
| Shell model (SM) | 2.6     |

| Table 1 Nuclear matrix elements for $^{124}\text{Sn}$ isotope extracted from the references [21, 30, 43]. |
are taken to be 100%. In practice, the required combined exposure $\Sigma$ of $A_{bb}$ can be converted from the ideal $\Sigma$ of the present work via $\Sigma = \Sigma / (1A \cdot \epsilon_{expt})$.

3. Results and discussion

In this section, we present the sensitivity of the TIN.TIN experiment with respect to the various experimental parameters such as irreducible background from $2\nu\beta\beta$ decay, ambient background, $\Sigma$ and $\Delta$. We also discuss the statistical significance of signal in the reference of background fluctuation, which may be crucial at the design stage of experiment to set the road map. Lastly, we focus on the required $\Sigma$ at various benchmark background levels to explore the IH and NH.

3.1. Influence of $2\nu\beta\beta$ background

The background events are always present in realistic experiments which degrade the sensitivities of the identifying spectral peaks at $Q_{\beta\beta}$-value. The source of background in the search of $0\nu\beta\beta$ can be divided into two categories: intrinsic and ambient. The ambient background is mostly induced by external $\gamma$-rays, especially from trace radioactivity present in the experimental hardware and cosmogenically activated isotopes in the vicinity of target volume.

The total ambient background counts $N_a$ in the $0\nu\beta\beta$ ROI can be obtained from the following expression:

$$N_a = A_a \cdot \Sigma \cdot [\Delta \cdot Q_{\beta\beta}],$$

where $A_a$ (counts/tonne-year-keV (t/tyk)) is considered as the flat ambient background rate, because the dominant ambient $\gamma$-line of $^{214}\text{Bi}$ and the Compton edge of $^{208}\text{TI}$ line are more than 15 eV away from the $Q_{\beta\beta}$ value of $^{124}\text{Sn}$. The intrinsic background in the search of $0\nu\beta\beta$ comes from the $2\nu\beta\beta$ decay process. It is therefore inherently associated with the $A_{bb}$ and directly proportional to $\Sigma$. The finite detector resolution leads the irreducible $B_{2\nu}$ events which contaminates the $0\nu\beta\beta$ ROI. Therefore, the sum $(B_0 = B_{2\nu} + N_a)$ would be the total background counts in the selected ROI.

If the ambient background reduces to a minimum level ($N_a = 0$), the irreducible background $B_{2\nu}$ would remain in the ROI. The contamination of $B_{2\nu}$ events mainly depends on the detector’s $\Delta$ and $\Sigma$. The numbers of $B_{2\nu}$ events are counted in the selected ROI which is smeared by the finite energy resolution of detector. This contamination of ROI due to the $B_{2\nu}$ events is shown in Fig. 2 for $\Sigma = 0.1$ and 1.0 tonne-year (ty) by the continuous and dotted lines, respectively. These lines represent that the $\tau_{0.1}^{0\nu}$ sensitivity of $0\nu\beta\beta$ would be limited by the contamination of $B_{2\nu}$ background events alone which varies with the energy resolution of detector. The IH and NH bands corresponding to the SM and PHFB $|M^{0\nu}|$ are also superimposed (From Eqs. 6 and 8) to get the prospects of $B_{2\nu}$ for future $^{124}\text{Sn}$ isotope-based experiments. As an example, if we consider that $B_{2\nu} = S_{0\nu}$ then, for $\Sigma = 0.1$ ty and $\Delta = 2\%$, the $\tau_{0.1}^{0\nu} = 2.9 \times 10^{28}$ year would face the contamination of ROI with $B_{2\nu}$ events in probing the NH (for both the SM and PHFB $|M^{0\nu}|$). Although, probing the IH region would be approximately free from $B_{2\nu}$ events (more specifically for PHFB $|M^{0\nu}|$), start contaminating moderately for SM $|M^{0\nu}|$.

The conversion of $\tau_{0.1}^{0\nu}$ in $\langle m_{bb} \rangle$ sensitivity faces the theoretical uncertainty of $|M^{0\nu}|$ (Eq. 3). It follows that the $\langle m_{bb} \rangle$ sensitivity due to $B_{2\nu}$ background alone forms the band structure (apart from the IH and NH bands) in the $\langle m_{bb} \rangle$ and $\Delta$ parameter space as shown in Fig. 3. The upper and lower line of $|M^{0\nu}|$ uncertainty band arises due to the $|M^{0\nu}|$ of SM and PHFB, respectively. Consequently, the $|M^{0\nu}|$ of SM would impose more severe requirements on experimental sensitivity in comparison to the PHFB.

With the maximum range of $|M^{0\nu}|$ uncertainty for $\Sigma_0 = 1.0$ ty to cover the NH, the intact region from the contamination of $B_{2\nu}$ events begins at $\Delta < 1.6\%$ for SM and $\Delta < 2.2\%$ for PHFB. This region for IH case begins from $\Delta < 3.9\%$ for SM and $\Delta < 5.3\%$ for PHFB. Therefore, the TIN.TIN experiment would be very less affected by the $B_{2\nu}$ events ($\sim 3.1 \times 10^{-6}$ counts) if it reaches to the energy resolution $\Delta_0 = 0.5\%$ at $Q_{\beta\beta}$, which is close to the achieved energy resolution $= 0.31\%$ at $Q_{\beta\beta}$ of the CUORE experiment (Bolometric detector using $^{130}\text{Te}$) [10].

3.2. Statistical significance of signal

Rare event physics searches like $0\nu\beta\beta$ and dark matter naturally demand very low background experiment [46].

![Fig. 2 Contamination of $B_{2\nu}$ events in ROI is shown by the continuous and dotted lines for $\Sigma = 0.1$ and 1.0 ty, respectively. To get the expectations of $B_{2\nu}$ events in the maximum range of $|M^{0\nu}|$ uncertainty, the IH and NH bands are also superimposed (From Eqs. 6 and 8) to get the prospects of $B_{2\nu}$ for future $^{124}\text{Sn}$ isotope-based experiments. As an example, if we consider that $B_{2\nu} = S_{0\nu}$ then, for $\Sigma = 0.1$ ty and $\Delta = 2\%$, the $\tau_{0.1}^{0\nu} = 2.9 \times 10^{28}$ year would face the contamination of ROI with $B_{2\nu}$ events in probing the NH (for both the SM and PHFB $|M^{0\nu}|$). Although, probing the IH region would be approximately free from $B_{2\nu}$ events (more specifically for PHFB $|M^{0\nu}|$), start contaminating moderately for SM $|M^{0\nu}|$. The conversion of $\tau_{0.1}^{0\nu}$ in $\langle m_{bb} \rangle$ sensitivity faces the theoretical uncertainty of $|M^{0\nu}|$ (Eq. 3). It follows that the $\langle m_{bb} \rangle$ sensitivity due to $B_{2\nu}$ background alone forms the band structure (apart from the IH and NH bands) in the $\langle m_{bb} \rangle$ and $\Delta$ parameter space as shown in Fig. 3. The upper and lower line of $|M^{0\nu}|$ uncertainty band arises due to the $|M^{0\nu}|$ of SM and PHFB, respectively. Consequently, the $|M^{0\nu}|$ of SM would impose more severe requirements on experimental sensitivity in comparison to the PHFB. With the maximum range of $|M^{0\nu}|$ uncertainty for $\Sigma_0 = 1.0$ ty to cover the NH, the intact region from the contamination of $B_{2\nu}$ events begins at $\Delta < 1.6\%$ for SM and $\Delta < 2.2\%$ for PHFB. This region for IH case begins from $\Delta < 3.9\%$ for SM and $\Delta < 5.3\%$ for PHFB. Therefore, the TIN.TIN experiment would be very less affected by the $B_{2\nu}$ events ($\sim 3.1 \times 10^{-6}$ counts) if it reaches to the energy resolution $\Delta_0 = 0.5\%$ at $Q_{\beta\beta}$, which is close to the achieved energy resolution $= 0.31\%$ at $Q_{\beta\beta}$ of the CUORE experiment (Bolometric detector using $^{130}\text{Te}$) [10].}

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Rare event physics searches like $0\nu\beta\beta$ and dark matter naturally demand very low background experiment [46].
Understanding of the background and its suppression would significantly improve the experimental sensitivity. In the design stage of experiments, the averaged $N_0$ and $B_{2e}$ can be approximately estimated from the prior knowledge of the most relevant sources of background and simulation studies [47-50]. The low background counts in the ROI are subjected to the Poisson fluctuation. Excess of counts from expected background may originate from the upward fluctuations of the background channels. The discovery potential (DP) and sensitivity level (SL) can be expressed in the frame of background fluctuation. In order to get the strong evidence, we have calculated the signal counts with 3$\sigma$ SL and 5$\sigma$ is expressed for DP.

Following earlier studies [35], the continuous representation of the Poisson distribution [51] is adopted in this work to define the signal-to-background criteria. This choice avoids the complications of discrete steps at small $B_0$ but captures the essence sufficient for the required accuracies at the stage of experimental design studies. The continuous representation of the Poisson distribution is obtained by normalized upper incomplete gamma function and it gives the probability distribution [51]:

$$F(k) = \frac{\Gamma(k+1, \lambda)}{\Gamma(k+1)}, \quad k > 0, \quad \text{with}$$

$$\Gamma(k, \lambda) = \int_{\lambda}^{\infty} e^{-t}t^{k-1}dt \quad \text{and} \quad \Gamma(k) = \int_{0}^{\infty} e^{-t}t^{k-1}dt,$$

where $\lambda$ is the mean value of distribution, $k$ is the number of counts, $\Gamma(k, \lambda)$ is the upper incomplete gamma function, and $\Gamma(k)$ is the ordinary gamma function. The variation in sensitivity becomes free from the discrete steps (with Eq. 11). For completeness, the signal counts at 90% CL are also calculated from the Poisson distribution as illustrated in Fig. 4.

For very low expected background, the minimum requirement of $S_{0\nu}$ for an experiment is chosen to be 1 event. This leads to the same sensitivity at the background-free level. The background-free criteria depend on the chosen statistical scheme.

The background-free scenario is shown in Fig. 4 from the horizontal line limiting at $S_{0\nu} = 1$ event. It is not a fixed value and depending upon the chosen statistics and the presence of $B_0$ in experiment, this criteria can be set to either 1 event or 2, 3 etc. In our criteria, it is chosen to be the minimum value of $S_{0\nu} = 1$ event. As the background decreases, the significance of $S_{0\nu}$ increases. This increment in significance is shown in Fig. 4 by flattened line. On reaching the background-free criteria, the extension of 90% CL is extended up to the 2$\sigma$ level while the 3$\sigma$ SL is extended up to the 5$\sigma$ DP. The zoomed part of this extension for very low background values is shown in the inset of Fig. 4.

After using these schemes for the identification of $S_{0\nu}$, the $\tau_{0\nu}$ sensitivity of Eq. 9 would take the following form in background-free scenario:

$$\left[ \frac{S_{0\nu}}{\tau_{0\nu}} \right]^{-1} = \left[ \log_2(2) \right]^{-1} \left[ \frac{A}{N_A} \right] \frac{1}{\Delta} \left[ \frac{S_{0\nu} | S_{3\sigma} | S_{5\sigma}}{S_{90\%} \Delta S_{R}} \right], \quad \text{and} \quad (13)$$

for background dominated situation it would take the following form (using Eqs. 3, 9, 10)

$$\left[ \frac{S_{0\nu}}{\tau_{0\nu}} \right]^{-1} = \left[ \log_2(2) \right]^{-1} \left[ \frac{A}{N_A} \right] \frac{1}{\Delta_{ROI}} \frac{1}{n_{\sigma}} \sqrt{\frac{A \cdot \Delta}{\Sigma}}, \quad (14)$$

where $S_{0\nu}$ of Eq. 9 is replaced by $S_{90\%}$, $S_{3\sigma}$ and $S_{5\sigma}$ to obtain the $\tau_{0\nu}$ sensitivity at 90% CL, 3$\sigma$ SL and 5$\sigma$ DP level, respectively, and $n_{\sigma}$ represents the number of

![Fig. 3](image-url) Uncertainty of $|M|^{0\nu}$ leads to the uncertainty in $m_{bb}$ due to the $B_{2e}$ events, which is shown in the form of the band (other than the hierarchy bands)

![Fig. 4](image-url) Variation of $S_{0\nu}$ corresponding to $B_0$ under the 3$\sigma$ SL, 5$\sigma$ DP and at 90% CL schemes of signal identification
standard deviations corresponding to a given confidence level (such as 1.64σ for 90% CL). Under these two schemes (90% CL and $3\sigma$ SL), the required sensitivity for $^{124}$Sn isotope is studied in terms of required $\Lambda$, $\Sigma$ at the $\Delta_0 = 0.5\%$ at $Q_{\beta\beta}$. These sensitivities are calculated with the aim to reach the most conservative (min.) and most optimistic (max.) regime of IH and NH (see Eq. 6).

3.3. Experimental potential of TIN.TIN

The accessible physics with $0\nu\beta\beta$ experiments is the effective mass of Majorana neutrinos $\langle m_{\beta\beta} \rangle$, which is the linear combination of neutrino mass eigenstates. The minimal desired experimental sensitivity of the TIN.TIN experiment is to probe the IH mass region. The $t^{0\nu}_{12}$ is inversely proportional to the $\langle m_{\beta\beta} \rangle$. The variation of $t^{0\nu}_{12}$ at $3\sigma$ SL and 90% CL as a function of $\Sigma$ at a fixed $\Delta_0$ with various background rates ($\Lambda$) is depicted in Figs. 5 and 6, respectively. The hierarchy bands arise from uncertainty of $|M^{0\nu}|$, and range of $\langle m_{\beta\beta} \rangle$ (Eqs. 6 and 8) is also superimposed.

In order to enter the hierarchy, the required sensitivity in terms of exposure $\Sigma$ and the benchmark background rate $\Lambda = (0, 0.1, 1.0, 10)/tyk$ is summarized in Table 2. It is found that to enter the IH$_{PHFB}$ with $\Lambda = 0.1/tyk$, the TIN.TIN experiment needs $\Sigma = 0.12$ ty ($4.8 \times 10^{-2}$ ty), while entering to the IH$_{SM}$ requires $\Sigma = 1.7$ ty (0.62 ty) for $3\sigma$ SL (90% CL). It is obvious from Figs. 5 and 6 that the access to the NH mass region appears for larger exposure in comparison to the IH. It follows that entering to the NH$_{PHFB}$ demands $\Sigma = 5.5 \times 10^4$ ty ($1.7 \times 10^2$ ty), whereas in order to enter the NH$_{SM}$ needs $\Sigma = 2.0 \times 10^4$ ty ($6.0 \times 10^3$ ty) for $3\sigma$ SL (90% CL). The uncertainty of $|M^{0\nu}|$ leads the uncertainty in required sensitivity. Therefore, precise calculation of $|M^{0\nu}|$ from a different model is the main requirement.

The potential of background improvement is explained in the parameter space of $\Sigma$ and $\Lambda$ at $\Delta_0$ in conjunction with the uncertainty bands of $|M^{0\nu}|$ for both the IH and NH (Figs. 7, 8). It can be seen that the background suppression is a necessity for the experiment. Improvement in the background plays crucial role in order to cover the hierarchy region completely. The required sensitivity in the terms of $\Sigma$ at $\Delta_0$ to completely cover both the hierarchy is summarized in Table 3 for both $|M^{0\nu}|$ at $3\sigma$ SL and 90% CL.

At the prior chosen background rate of $\Lambda = 0.1/tyk$, the coverage of IH$_{PHFB}$ requires $\Sigma = 2.6$ ty (0.91 ty) while for IH$_{SM}$ this requirement becomes $\Sigma = 66$ ty (21 ty) for the signal to be identified at $3\sigma$ SL (90% CL). It is evident from Figs. 7 and 8 that the coverage of NH claims for larger exposure in comparison to the IH. It follows that to cover the NH$_{PHFB}$ needs $\Sigma = 4.3 \times 10^4$ ty ($1.3 \times 10^4$ ty) and the coverage of NH$_{SM}$ demands an exposure of $\Sigma = 1.8 \times 10^6$ ty ($5.1 \times 10^3$ ty) for $3\sigma$ SL (90% CL).

The value of minimum exposure $\Sigma_{\text{min}}$ corresponding to 1 $S_0$ event is obtained at very low background (close to $\Lambda = 0/tyk$) and is shown by the left flattened region in Figs. 7 and 8. $\Sigma_{\text{min}}$ is an important parameter where each related experiment wants to reach by making improvement in the background rate such that $\Lambda = 0/tyk$. The value of $\Sigma_{\text{min}}$ gives clear indication about the enhancement of required sensitivity in terms of $\Sigma$ with $\Lambda$ in the experiment. It has explicitly come out and shown in Table 2 that just entering the IH$_{PHFB}$ (IH$_{SM}$) requires $\Sigma_{\text{min}} = 4.8 \times 10^{-2}$ ty (0.30 ty) and entering the NH$_{PHFB}$ (NH$_{SM}$) demands $\Sigma_{\text{min}} = 7.1$ ty (44 ty). To cover the IH$_{PHFB}$ (IH$_{SM}$), the TIN.TIN experiment needs $\Sigma_{\text{min}} = 0.38$ ty (2.4 ty) and the

![Fig. 5](image_url) Signal identification at $3\sigma$ SL in $t^{0\nu}_{12}$ versus $\Sigma$ at $\Delta_0$ for $\Lambda = (0, 0.1, 1.0, 10)/tyk$. The IH and NH bands are superimposed for both PHFB and SM $|M^{0\nu}|$ models.

![Fig. 6](image_url) Signal identification at 90% CL in $t^{0\nu}_{12}$ versus $\Sigma$ at $\Delta_0$ for $\Lambda = (0, 0.1, 1.0, 10)/tyk$. The IH and NH bands are superimposed for both PHFB and SM $|M^{0\nu}|$ models.
Next-generation neutrinoless double-β decay experiments like TIN.TIN have a primary aim to probe the IH region. We have investigated the experimental parameters such as energy resolution, exposure and background rate to meet this goal in reference of background fluctuation sensitivity at 3σ SL and 90% CL. This background fluctuation sensitivity study can be straightforward extended to the discovery potential for any experiment.

Our present study shows that the energy resolution of 0.5% at $Q_{\beta\beta}$ for TIN.TIN detector is good enough to diminish the two neutrino double-β decay background events ($B_{2\nu} \sim 3.1 \times 10^{-6}$ counts) to probe the IH. In order to probe the NH region, the two neutrino double-β decay background events start moderately contributing in the total background. Further improvement in energy resolution would be beneficial to completely suppress the contribution of $2\nu\beta\beta$ background events.

The ambiguity of nuclear matrix elements leads to severe uncertainty in the required experimental sensitivity. It is observed that using PHFB model, the required sensitivity in terms of energy resolution, exposure and background rate is in the optimistic scenario in comparison to the SM model. The accurate knowledge of the nuclear matrix element is required to minimize the uncertainty in the required sensitivity. Furthermore, it is an essential parameter for determining the effective mass of Majorana neutrino once this $0\nu\beta\beta$ process is observed.

The optimistic region of required sensitivity in terms of the background rate to enter the hierarchy starts from $\Lambda \leq 0.1$/tyk and the pessimistic region starts from $\Lambda \geq 0.1$/tyk for both nuclear matrix elements at 3σ SL and 90% CL. Although entering the IH$_{\text{PHFB}}$ can tolerate the background

### Table 2: Required $\Sigma$ corresponding to the benchmark $\Lambda = (0, 0.1, 1.0, 10)$/tyk in the most optimistic scenario to just enter the hierarchy

| $\Lambda$ (/tyk) | $[\theta^0]_{\text{IH}}$ | $[\theta^0]_{\text{NH}}$ |
|-----------------|-----------------|-----------------|
| 0.0             | 0.30            | 44              |
| 0.1             | 1.7             | 2.2 x 10^4      |
| 1.0             | 10              | 2.5 x 10^6      |
| 10              | 92              | 2.5 x 10^6      |

**Fig. 7** Signal identification to cover completely and to just enter the hierarchy with 3σ SL in $\Sigma$ versus $\Lambda$ space for $^{124}\text{Sn}$ at $\Delta_0$, in complete regime (min. to max.) of $\langle m_{\beta\beta}\rangle$ for IH and NH, using the $|M^0|$ of SM and PHFB models

**Fig. 8** Signal identification to cover completely and to just enter the hierarchy with 90% CL in $\Sigma$ versus $\Lambda$ space for $^{124}\text{Sn}$ at $\Delta_0$, in complete regime (min. to max.) of $\langle m_{\beta\beta}\rangle$ for IH and NH, using the $|M^0|$ of SM and PHFB models

The coverage of NH$_{\text{PHFB}}$ (NH$_{\text{SM}}$) demands $\Sigma_{\text{min}} = 64$ ty (4.0 x 10^2 ty) as listed in Table 3.

### 4. Conclusions
rate up to $\Lambda = 10$/tyk at 90% CL, for NH_{PHFB} requires $\Lambda \sim 0.1$/tyk.

The TIN.TIN experiment with energy resolution 0.5% at $Q_{\beta\beta}$ needs a minimum exposure of $\Sigma_{\text{min}} = 0.38$ ty to cover the IH_{PHFB} completely and in a conservative scenario to cover the IH_{SM} requires $\Sigma_{\text{min}} = 2.4$ ty. Similarly, the coverage of NH_{PHFB} requires $\Sigma_{\text{min}} = 64$ ty whereas for NH_{SM} this needs $\Sigma_{\text{min}} = 4.0 \times 10^2$ ty. This $\Sigma_{\text{min}}$ is necessary to observe the minimum 1 signal event at background-free level. Though $\Sigma_{\text{min}}$ is an ideal case, this will provide a lower limit to the required exposure.

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