Electrically controlled spin-transistor operation in a helical magnetic field

P Wójcik and J Adamowski

AGH University of Science and Technology, Faculty of Physics and Applied Computer Science, al. A. Mickiewicza 30, Kraków, Poland

E-mail: pawel.wojcik@fis.agh.edu.pl

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Abstract
A proposal of an electrically controlled spin transistor in a helical magnetic field is presented. In the proposed device, the transistor action is driven by the Landau–Zener transitions that lead to a backscattering of spin polarized electrons and switching the transistor into the high-resistance state (off state). The on/off state of the transistor can be controlled by the all-electric means using Rashba spin–orbit coupling that can be tuned by the voltages applied to the side electrodes.

Keywords: spin transistor, spin–orbit interaction, helical magnetic field

(Some figures may appear in colour only in the online journal)

1. Introduction
An application of electron spin to conventional electronic devices is the basic concept lying at the heart of spintronics [1, 2]. The special role among the spintronic devices is played by the spin field effect transistor (spin-FET), the concept of which was proposed by Datta and Das [3]. According to this idea [3] the current of spin polarized electrons is injected from the ferromagnetic source into the two-dimensional electron gas (2DEG) and is electrically detected in the ferromagnetic drain. The resistance of the transistor channel depends on the electron spin, which is controlled by the Rashba spin–orbit interaction (SOI) [4] generated by the voltage applied to the gate attached close to the channel. Although the coherent manipulation of electron spin in semiconductors has been demonstrated by many experimental groups [5–7], the realization of the functional spin-FET still remains an experimental challenge of spintronics. This is due to the fundamental physical obstacles such as the low spin-injection efficiency from ferromagnet into the semiconductor (the resistance mismatch) [8], spin relaxation [2], and the spread of the spin precession angles. All these effects lead to the low electrical signal in the up-to date realized spin-FET [5, 9, 10].

Current research on the spin-FET is mainly focused in two directions. First, researchers try to overcome the physical obstacles by using the spin filters based on the semiconductors [10, 11]. In a very recent experiment [12], the quantum point contacts have been used as spin injectors and detectors and about 100, 000 times greater conductance oscillations have been observed as compared to the conventional spin-FET [5]. The second direction involves the proposals of the new spin transistor design, different from that proposed in [3] and free of its physical restrictions [13, 14].

Such an alternative spin-transistor design has been recently described by Žutić and Lee [15] and experimentally demonstrated by Bethuelsen et al in [16]. In this approach, the spin of electron flowing through the nanostructure is subjected to the combined homogeneous and helical magnetic fields. The latter is generated by ferromagnetic stripes located above the conduction channel. The transistor action is driven by the diabatic Landau–Zener transitions [17, 18] induced by the appropriate tuning of the homogeneous magnetic field. For the suitably chosen conditions the spin polarized electrons are scattered back, which gives rise to an increase in resistance. As shown in [16, 19, 20], by keeping the transport in the adiabatic regime, the proposed design is robust against the scattering on defects, while in the non-adiabatic regime, the additional conductance dips emerge which result from the resonant Landau–Zener transitions. Although this spin-FET [16] seems to be free of the above-mentioned physical obstacles, it requires the application of the external homogeneous magnetic field, which is difficult to apply in the integrated circuit.

Motivated by this experiment [16], in the present paper, we propose a spin transistor design based on the helical magnetic field, in which the spin transistor action is generated...
by all-electric means without the external magnetic field. For this purpose we use the Rashba spin–orbit interaction (SOI) induced by the lateral electric field between electrodes located on either side of the nanowire.

2. Model

The proposed device is based on an InSb nanowire grown in the [111] direction. Due to a strong spin–orbit interaction and a high effective g factor, InSb nanowires are natural candidates to create spintronic devices, in which the electron spin can be controlled by all-electric means [21]. For the planar InSb heterostructure the Rashba SOI coupling due to the structural inversion asymmetry in the growth direction has been determined to be as strong as $|\alpha| = 0.03$ eVÅ, while the Dresselhaus spin–orbit coupling constant is equal to 490 eVÅ$^{-2}$[22]. Moreover, the experiment [22] has shown that the cubic Dresselhaus term in the InSb two-dimensional electron gas is important and leads to pronounced anisotropy in the energy splitting induced by the spin–orbit interaction.

In recent papers [23, 24] the SOI in InSb have been studied in 2D wires [23] and nanowires fabricated by the bottom-up technique [24]. As shown in [24], the magnetoconductance in the bottom-up grown nanowires is essentially different as compared with that of the nanowires based on 2DEG [24]. The Rashba spin–orbit coupling measured for the bottom-up grown nanowires has been reported to be $|\alpha| = 0.5 - 1$ eVÅ. Such a large value of $\alpha$ together with the high value of the $g$ factor makes InSb nanowires good candidates for the spin-transistor design presented in the paper. The proposed spin-transistor design (figure 1) consists of two building blocks: (i) ferromagnetic stripes (fabricated from dysprosium, Dy) located periodically near the conduction channel, which generate the helical magnetic field in the nanowire (this block has been realized in [16]) and (ii) two side gate electrodes ($g_1$ and $g_2$), which generate the lateral electric field $\mathbf{F} = (0, F_y, 0)$.

The lateral electric field $\mathbf{F}$, controlled by the voltages applied to the side gates, induces the Rashba SOI with the effective magnetic field $B_R$ directed along the $z$-axis. The possibility of using the SOI induced by the lateral electric field has been reported in many experiments [12, 25, 26]. The change in the SOI coupling constant $\alpha$ between the two electrodes has been obtained to be 0.04 – 0.5 eVÅ.

A recent theoretical study [27] on the Rashba spin–orbit coupling in InSb nanowires under transverse electric field has shown that for a small electron wave vector $k_z$, measured along the nanowire axis, the Rashba spin–orbit splitting energy is a linear function of $k_z$, similar to the quantum well case. Based on these results we have taken on the one-electron Hamiltonian in the 1D form

$$\hat{H} = \frac{\hbar^2 k_z^2}{2 m} I + \frac{1}{2} g_{\text{eff}} \mu_B B_0 \sigma \cdot \mathbf{k}_z + E_0,$$

where $m^*\hbar^2$ is the conduction-band electron mass, $\hbar k_z = -i \hbar \frac{d}{dt}$ is the momentum operator, $I$ is the 2 $\times$ 2 identity matrix, $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ is the vector of Pauli matrices, $g_{\text{eff}}$ is the effective $g$-factor, $\mu_B$ is the Bohr magneton, $\alpha$ is the Rashba spin-orbit coupling constant, $E_0$ is the energy of the lowest-subband corresponding to the lateral ($y, z$) confinement and $B_0(r)$ is the helical magnetic field, which has the form

$$B_0(r) = B_0 \left( \sin \frac{2\pi (x - x_0)}{a}, 0, \cos \frac{2\pi (x - x_0)}{a} \right).$$

where $B_0$ is the amplitude of the helical field, $a$ is the period of the magnetic field modulation determined by the distance between ferromagnetic stripes and $x_0$ is the localization of one of the ferromagnetic stripes. For simplicity, we study the nanowire with length $L$ equal to one period of the helical magnetic field modulation, i.e., $L = a$ and $x_0 = L/2$. We note that the InSb nanowire (figure 1) has a zinc-blende crystal structure, grown in the [111] direction, for which the Dresselhaus SOI is absent for momentum along the nanowire axis [24].

The numerical calculations have been performed within the scattering matrix method using the Kwant package [28]. We have transformed Hamiltonian (1) into the discretized form on the grid $(x_i, \mu = 1, 2, \ldots, N)$ with mesh constant $\Delta x$ ($\mu = 1, 2, \ldots$). If we introduce the discrete representation of the spinor as follows: $|\Psi(x_i)\rangle = (|\psi^\uparrow(x_i)\rangle, |\psi^\downarrow(x_i)\rangle)^T \equiv |\Psi_i\rangle$, the Hamiltonian (1) is given by

$$H = \sum_{\mu} \left\{ 2t_1 + \frac{1}{2} g_{\text{eff}} \mu_B \mathbf{k}_z \cdot \mathbf{\sigma} \right\} |\Psi_i\rangle \langle \Psi_i| + \sum_{\mu} \left[ t (1 + i \alpha \sigma_t \mu) |\Psi_{i+1}\rangle \langle \Psi_i| + H. c. \right],$$

where $t = \hbar^2/(2\mu_{\text{e}} \Delta x^2)$ and $t' = 1/(2\Delta x)$.

The numerical calculations of the conductance have been performed on the lattice with $\Delta x = 2$ nm. We have considered the electron transport for the fixed Fermi energy $E_F$, assuming a small bias between the leads and calculated the conductance $G = \sum_{\mu} \sum_{\nu} T_{\mu\nu}$, where the summation runs
over the transmission probabilities $T_{i\rightarrow k}$ from the spin-dependent subband $i$ on the input to the spin-dependent subband $k$ in the output channel. In the calculations, the following values of the parameters have been used: the length $L = 1\mu m$, the conduction-band electron mass in InSb $m^* = 0.014m_e$ where $m_e$ is the free electron mass, and $g_{sf} = -51$. We assume that the electrons are injected into the lowest-energy subband taking on $E_0 = 4.2$ meV, which corresponds to the cylindrical nanowire radius $R \approx 50$ nm. The value of the helical magnetic field amplitude $B_h$ has been taken on the basis of the experimental report [16] and is equal to $B_h = 50$ mT.

3. Results and discussion

The behavior of the electron spin in the nanowire is determined by the superposition of the helical magnetic field $B_h$ generated by the ferromagnetic stripes and the Rashba effective magnetic field $B_R$ induced by the lateral electric field. The effective spin Zeeman energy for the parallel (+) and antiparallel (−) spin orientation (with respect to the effective magnetic field) depends on the electron position and is given by

$$E^\pm(x) = \pm \alpha k_x \sqrt{1 + \gamma^2 + 2\gamma \cos \left( \frac{2\pi (x - x_0)}{a} \right)},$$

(4)

where $\gamma = \frac{1}{2} g_{sf} \mu_B B_h / \alpha k_x$. Parameter $\gamma$ depends on the Rashba spin–orbit coupling $\alpha$, which can be tuned by changing the voltages applied to the side electrodes.

For electrons flowing through the nanostucture, the spatially varying Zeeman energy is seen as the time-dependent perturbation. Therefore, we can transform the $x$-dependence into a time dependence of a corresponding Hamiltonian (1). For the two level system subjected to the time-dependent perturbation, the Landau–Zener transitions can take place, the probability of which is derived in [17, 18] and is given by

$$P = \exp \left( -\frac{2\pi \varepsilon_{21}^2}{\hbar^2} \beta \right),$$

(5)

where $\varepsilon_{21} = \alpha k_x |\gamma - 1|$ corresponds to half the closest distance between the energies $E^\pm$ at the crossing point. Parameter $\beta = \frac{1}{\sqrt{\hbar}} \left( E^+ - E^- \right)$ determines how fast these eigenenergies approach each other in the electron rest frame. Using the relation $\partial E / \partial t = (\partial E / \partial x)(\partial x / \partial t) = (\partial E / \partial \xi)V_F$, where $V_F$ is the Fermi velocity, $\beta$ takes on the form

$$\beta = \frac{2\pi g_{sf} \mu_B B_h V_F}{a \hbar} \frac{2 \sin \left( \frac{2\pi (x - x_0)}{a} \right)}{\sqrt{1 + \gamma^2 + 2\gamma \cos \left( \frac{2\pi (x - x_0)}{a} \right)}}.$$

(6)

We note that the formula (5) was derived for the constant coupling between the states and for the detuning which is a linear function of time, under assumption of the infinite duration of the process $t \rightarrow \pm \infty$. Nevertheless, if the transition takes place in the short time interval around the crossing ($\varepsilon_{21} = 0$) and outside this region the system is far from resonance, then the error made by using the formula (5) is insignificant [29].

According to equation (5), the probability of the Landau–Zener transitions $P \rightarrow 1$ if the subbands $E^\pm(x)$ cross over, i.e., the energy separation $\varepsilon_{21} \rightarrow 0$. In figure 2, we present $\varepsilon_{21}$ as a function of the position in the nanowire and wave vector $k_x$ for two different values of $\alpha$.

We observe that for electrons with a positive wave vector, i.e., flowing from the left to the right, $\varepsilon_{21}$ reaches zero for the well-defined $k_x$, exactly in the middle of the nanowire, i.e., for $x = 0.5 \mu m$, for which $B_{k_x} = 0$. The value of $k_x$, for which $\varepsilon_{21} = 0$, increases with the decreasing Rashba spin–orbit coupling $\alpha$. Based on these results, we conclude that for the specified Fermi wave vector (Fermi energy), we can tune the Rashba coupling $\alpha$ in order to reach the Landau–Zener transition probability $P = 1$. Probability $P$ as a function of $\alpha$ is depicted in figure 3 for the two different Fermi wave vectors. Figure 3 shows that the maximum of the Landau–Zener transition probability shifts towards the higher value of $\alpha$ with decreasing Fermi wave vector $k_F$.

The analysis of the Landau–Zener transition probability presented above is needed to understand the spin-transistor action in the proposed spin-FET. In order to demonstrate this operation in a quantitative manner, in figure 4 we present the conductance as a function of the Rashba spin–orbit coupling $\alpha$ calculated for the Fermi energy $E_F = 4.2$ meV.

As we can see, in the presence of the spin–orbit interaction the conductance exhibits a sharp distinct dip at a certain value of the spin–orbit coupling $\alpha$ (point (b)), i.e. the
device is switched into the high-resistance state. Since the changes of $\alpha$ can be induced by the voltages applied to the side electrodes, this allows us to have full electrical control of the transistor state, i.e. the device can be switched between the high resistance (off state) and the low resistance (on state) states. The appearance of the conductance dip can be explained based on the analysis of the electron conduction subbands, which vary with position in the nanowire. The dispersion relations $E^{\pm}(k)$ at the point $x$ are determined by combining the Rashba effective magnetic field $B_R$ generated by the lateral electric field $F$ and directed along the z-axis in the entire nanostructure, and the helical magnetic field, which varies with the position. While the helical magnetic field generates the Zeeman spin splitting of the conduction subbands, the spin–orbit interaction shifts the dispersion relations $E^{\pm}(k)$ in the wave vector space, in a manner that depends on the relative alignment of $B_R$ and $B_h$. In figure 5, we present the lowest-energy subbands in the left lead ($x=0$), in the middle of the nanowire ($x=L/2$), where the ferromagnetic stripe is located, and in the right lead ($x=L$), for different values of spin–orbit strength $\alpha$ marked by the points (a-c) in figure 4.

For $\alpha = 5 \text{ meVnm}$ the electrons with Fermi energy are injected into the conduction channel from the lowest-energy subband (see figure 5(a)). In this case, the Rashba spin–orbit coupling $\alpha$ is so small that the helical magnetic field $B_h$ is much stronger than the Rashba field $B_R$. Since the Zeeman spin-splitting is only slightly affected by the spin–orbit interaction, the electrons are transmitted through the nanostructures without the scatterings giving rise to the conductance $G \approx e^2/h$ (see figure 4). For $\alpha = 26 \text{ meVnm}$ (figure 5(b)) a degeneracy point emerges in the middle of the waveguide, i.e., for $x = L/2$. At this point, for the chosen Fermi energy, the Rashba field $B_R$ compensates the helical field $B_h$ and the total magnetic field vanishes. Since the distance between spin-split eigenenergies approaches zero, the Landau–Zener transmission probability $P \to 1$. As a result the electrons are transmitted to the upper Zeeman subband (blue curve in figure 5(b)). However, the energy of this subband for the positive wave vector in the right lead is above the Fermi energy (see figure 5(b), $x = L$), which means that there are no available electronic states in the right lead. This leads to the backscattering of electrons. As a result, the transport is blocked, which gives rise to the conductance dip presented in figure 4. A further increase of the Rashba coupling constant $\alpha$ results in the energy minima of both spin-split subbands being below Fermi energy and both the subbands conduct electrons. The conductance increases up to $G = 2e^2/h$ (see figure 4, point (c)).
The spin-transistor action in the proposed device is generated by the Landau–Zener transitions between spin-split subbands, which occurs when the Rashba effective field compensates the helical field at some region of the nanostructure. For the fixed Fermi energy this can be realized by applying the appropriate voltages to the side gate electrodes. Nevertheless, the spin–orbit interaction strength depends not only on the electric field but also on the Fermi energy. This means that for different Fermi energies the conditions for the Landau–Zener transitions are fulfilled for different Rashba coupling constant. This dependence is depicted in figure 6, which presents the conductance as a function of coupling \( \alpha \) and Fermi energy \( E_F \). In figure 6, we mark the range in which the Landau–Zener transitions occur. We see that the values of \( \alpha \), for which the spin-transistor action is expected, decreases with increasing Fermi energy \( E_F \).

\[ \text{Figure 6. Conductance } G \text{ as a function of Rashba spin–orbit coupling } \alpha \text{ and Fermi energy } E_F. \]

4. Summary

In summary, we have proposed an electrically controlled spin-transistor setup, which is based on the effect of the helical magnetic field generated by ferromagnetic stripes and the lateral Rashba spin–orbit interaction with the strength tuned by the voltages applied to the side electrodes. We have shown that the appropriate tuning of the Rashba spin–orbit interaction results in the blocking of the electron transport due to the Landau–Zener transition. This allows us to have all-electric control of the device and the ability to switch it between the low-resistance (on state) and the high resistance state (off state). It is worth noting that the proposed setup combines the blocks already tested in recent experiments: the helical magnetic field in 2DEG has been generated by the ferromagnetic stripes used in [16], while the lateral Rashba spin–orbit interaction has been intensively studied as potential spin filters [25, 26]. Therefore, we expect that the proposed spin transistor will be realized in the near future.

Recent studies [15, 16, 19, 20] clearly indicate that the spin transistor action induced by the Landau–Zener transition is very a promising alternative to the original concept proposed by Data and Das [3]. Although in the previous experiment [16] the Landau–Zener transition was induced by the external magnetic field, which is difficult to apply in the integrated circuit, the new concepts, including the present one, demonstrate that the spin-transistor operation can be generated by the Landau–Zener transition controlled by all-electric means. In the supplementary material of [16], the authors propose the application of the magnetic gate switched by the spin transfer torque. According to this proposal, the magnetic gate based on a Fe-Co/Fe-Co-B layer structure is located between the ferromagnetic stripes, while their relative magnetization is controlled by the current flowing through the stack. For the parallel magnetization the gate acts as a single ferromagnet, which generates a stray field leading to the degeneracy point in which the Landau–Zener transition probability is equal to 1. For the antiparallel magnetization the magnetic field in the 2DEG vanishes and the transistor is switched into the low resistance state. Nevertheless, this architecture assumes that the magnetization direction of the hard magnets stripes can be adjusted in such a way that it leads to the degeneration of the spin state at some point of the nanostructure. This can be done by the external magnetic field during the fabrication process and cannot be changed after this process. The spin transistor design proposed in the present paper is based on the effective magnetic field stemming from the Rashba spin–orbit interaction which can be controlled during the operation and does not require any adjustment of the magnetization of the ferromagnetic stripes which could be difficult in the integrated circuits. Moreover, the design proposed in the paper does not require the flow of the current in order to change the transistor state.

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