Cosmological bubble friction in local equilibrium

Shyam Balaji
School of Physics, The University of Sydney, NSW 2006, Australia

Michael Spannowsky
Institute for Particle Physics Phenomenology, Department of Physics, Durham University, Durham DH1 3LE, U.K.

Carlos Tamarit
Physik-Department T70, Technische Universität München, James-Franck Straße 1, D-85748 Garching, Germany

Abstract
In first-order cosmological phase transitions, the asymptotic velocity of expanding bubbles is of crucial relevance for predicting observables like the spectrum of stochastic gravitational waves, or for establishing the viability of mechanisms explaining fundamental properties of the universe such as the observed baryon asymmetry. In these dynamic phase transitions, it is generally accepted that subluminal bubble expansion requires out-of-equilibrium interactions with the plasma which are captured by friction terms in the equations of motion for the scalar field. We show that subluminal propagation can still happen in the case of local equilibrium, in which the total entropy remains conserved but bubbles slow down due to hydrodynamic effects across the bubble wall associated with the field-dependence of the entropy density. These effects can by accounted for by simply imposing local conservation of stress-energy and including field dependent thermal contributions to the effective potential. We illustrate this with explicit calculations of dynamical and static bubbles for a first-order electroweak transition in a Standard Model extension with additional scalar fields. The results qualitatively match with recent analyses of friction forces in local equilibrium, which discard runaway behaviours, although we find corrections from the temperature and velocity gradients across the bubble. Even if local equilibrium is violated for some particle species, the effects described here will apply for the background plasma of the species that remain equilibrated, thereby leading to smaller velocities.

I. INTRODUCTION

The hot plasma in the early universe may have gone through different phase transitions which contributed to forge the properties of the world around us. Classical examples are the phase transition in Quantum Chromodynamics and, if the temperature at early times was large enough, the electroweak phase transition. Though both of the former are of the crossover type in the Standard Model (SM) [1,2], first-order phase transitions remain an intriguing possibility which can be realized in SM extensions. Such transitions, which proceed through the nucleation and subsequent expansion of bubbles of the thermodynamically preferred phase, are particularly interesting due to the enhanced deviations from equilibrium during the transition. The loss of spatial homogeneity and isotropy due to the colliding bubble walls can source a stochastic background of gravitational waves [3,4] (see Ref. [5] for a review) amenable to experimental confirmation by future space-borne interferometers like the Big Bang Observer (BBO) [6], the Deci-hertz Interferometer Gravitational Wave Observatory (DECIGO) [7] and LISA [8]. On the other hand, if the electroweak phase transition were to be of first-order, the former inhomogeneities coupled with novel CP-violating interactions could lead to the generation of the observed baryon asymmetry through the mechanism of electroweak baryogenesis [9] (for a review, see Ref. [10]).

The predictions of the physical effects of a first-order phase transition, such as the power emitted in gravitational waves or the generated baryon asymmetry, crucially depend on the velocity reached by the bubbles expanding through the plasma. While gravitational wave emission is enhanced if the velocity becomes nearly luminal, the generation of the baryon asymmetry requires slow bubbles that allow for the diffusion of the particles reflected in a CP-violating manner by the advancing bubble. This enables the CP excess in front of the bubble wall to be converted into baryon number asymmetry by sphaleron interactions [11].

For these reasons the estimation of bubble velocities has been the subject of intense study, centered on the understanding of the friction effects between the bubbles and the plasma which may slow the advance of the former. Studies based on kinetic theory [12,15], fluctuation-dissipation arguments [16,17] or non-equilibrium quantum field theory [18] suggest a velocity-dependent friction force caused by deviations from equilibrium interactions in the vicinity of the bubble wall. While most analyses are based on evaluating the rate of momentum transfer integrated across the bubble wall, the ensuing friction force is usually incorporated into the local equation of motion of the scalar field. The kinetic-theory approach or equivalent methods provide first-principle estimates of friction effects by using Boltmann equations to estimate...
the out-of-equilibrium effects. Investigations mostly focusing on SM extensions have recently been performed [18, 22]. Many studies consider an effective friction term proportional to a phenomenological friction parameter $\eta$ [23, 25], which is sometimes fixed to match the results from the Boltzmann approach [26, 29].

The general expectation is that there is no friction in local equilibrium [31]. Furthermore, it has been argued that the friction force saturates at leading order for high-velocities, such that near-luminal bubble propagation, or “runaway” behaviour is a generic possibility [20, 31]. This has been recently disputed [32], where it is argued that in local equilibrium the friction force per unit area follow the relationship

$$\left| F_{\text{friction}} \right| = \left( \gamma(v_w^2) - 1 \right) T |\Delta s|,$$

where $\gamma(v_w)$ is the Lorentz contraction factor of the asymptotic bubble wall velocity $v_w$, and $\Delta s$ the change in entropy density across the bubble. This force keeps growing with the velocity and prevents the bubbles from runaway behaviour.

The analysis of Ref. [32] was based on integrating the stress energy momentum tensor across the bubble wall and assuming a constant temperature and fluid velocity throughout. However, this does not exemplify how friction arises in the local dynamical equations for the scalar field and the plasma, or how to consistently compute both the bubble velocity and the associated entropy change. The goal of this paper is to confirm that indeed local equilibrium is compatible with subluminal bubble expansion, clarify the local origin of the friction forces, and provide consistent estimates of bubble velocities. Rather than arising from additional terms in the scalar’s equation of motion, the friction-like behaviour in the presence of local equilibrium is caused by the field-dependence of the local entropy and enthalpy density itself, which enters into the hydrodynamic equations of the plasma. As the scalar bubble expands it enforces local entropy and enthalpy changes in the plasma near the bubble wall, and conservation of stress-energy implies that the bubble must slow down. We will illustrate this effect quantitatively in an extension of the SM with additional scalars. We estimate bubble-wall velocities both from time-dependent solutions with radial symmetry, or by finding planar solutions to the static equations in the wall frame and matching them to consistent deflagration profiles away from the wall.

The paper is organized as follows. In section II we review the differential equations for the scalar field plus plasma, arising simply from imposing the conservation of the stress-energy tensor. Next, in section III we introduce the model used to illustrate the friction-like effects. Section IV presents the results for dynamical solutions with radial symmetry, while finally in section V we consider the asymptotic regime of constant velocity expansion and solve for static bubble profiles in the wall frame compatible with consistent deflagration solutions of the plasma equations away from the bubble. Conclusions are drawn in section VI.

II. DIFFERENTIAL EQUATIONS FOR BUBBLE PROPAGATION

We consider a system involving a real scalar field interacting with a thermal plasma. The stress-energy momentum tensor is given by the sum of contributions from both sectors

$$T^\mu_\nu = T^\mu_\nu_\phi + T^\mu_\nu_p,$$

where $\phi$ and $p$ denote respectively the scalar field and the plasma. We assume an ordinary scalar with a potential $V(\phi)$ plus a plasma modelled by a perfect fluid, which can be justified as the leading approximation in an expansion in terms of gradients of the plasma velocity. As such, we have

$$T^\mu_\nu_\phi = \partial^\mu \phi \partial^\nu \phi - \eta^{\mu\nu} \left( \frac{1}{2} \partial_\rho \phi \partial^\rho \phi - V(\phi) \right),$$

$$T^\mu_\nu_p = (\rho + p) u^\mu u^\nu - \eta^{\mu\nu} p = \omega \, u^\mu u^\nu - \eta^{\mu\nu} p.$$

In the above equations, $u^\mu$ with $\mu = 0, 1, 2, 3$ represents the fluid’s four-velocity, while $\rho$, $p$ and $\omega = \rho + p$ correspond to the pressure, energy density and enthalpy of the plasma. We assume the signature $(+, -, -, -)$ for the Minkowski metric and work in natural units with $c = 1$. In terms of the plasma velocity vector $v^i$ with $i = 1, 2, 3$, its magnitude $v = \sqrt{\sum_i (v^i)^2}$ and the Lorentz factor $\gamma(v) = 1/\sqrt{1 - v^2}$, the 4-velocity can be written as $u^\mu = \gamma(v)(1, v^1, v^2, v^3)$. Covariant conservation of the stress-energy momentum tensor in a cosmological background implies $\nabla_\mu T^\mu_\nu = 0$. Under the typical assumption of a phase transition that proceeds much faster than the Universe’s expansion, one may neglect the cosmological scale factor and replace covariant derivatives by ordinary ones. Doing so, the terms in $\nabla_\mu T^\mu_\nu$ involving $\partial^\nu \phi$ are proportional to the scalar field’s equation of motion in the plasma background and have to vanish separately. This yields

$$\Box \phi + \frac{\partial}{\partial \phi} (V(\phi) - p) = 0,$$

$$\partial_\mu (\omega u^\mu u^\nu - \eta^{\mu\nu} p) + \frac{\partial}{\partial \phi} \partial^\nu \phi = 0.$$  (4)

As initial time boundary conditions for the plasma, a fluid at rest with a temperature given by the nucleation temperature $T_{\text{nucl}}$ at which the bubble formation rate overcomes the Hubble expansion should be considered. For the scalar field, a perturbation of the critical bubble that extremizes the three-dimensional integral of the Lagrangian for static fields should be set as an initial condition.

One recognizes the first equation in Eq. (4) as the equation of motion of the scalar field at finite temperature.
Indeed, under the assumption of local thermal equilibrium with temperature \( T \), the pressure is related to the free energy, which itself is related to the thermal corrections \( V_T \) to the effective potential \( p = -V_T \). Hence, we may denote \( V(\phi) - p = V(\phi, T) \) and recover the standard equation of motion at finite temperature. Equations equivalent to (4) were obtained in Ref. [23], where the authors expressed the total pressure as a radiative contribution proportional to \( T^4 \) and the additional field dependent terms. We make no such distinction here, thus the simpler notation. Furthermore, the authors of Ref. [23] added a phenomenological friction term without spoiling stress-energy conservation. This corresponds to substituting the r.h.s. of the two equations in (4) by \( -\eta \omega^{\mu} \partial_{\mu} \phi \) and \( \eta \omega^{\mu} \partial_{\mu} \phi \partial_{\mu} \), respectively, where \( \eta \) is a friction parameter.

In the second equation of (4), it should be noted that the terms involving field derivatives of the pressure cancel, but the terms proportional to \( \partial \omega / \partial \phi \) survive. Under local thermal equilibrium, one can relate \( \omega \) to the entropy density \( s = \omega / T \), so that the terms proportional to \( \partial \omega / \partial \phi \) account for local entropy changes across the bubble wall. It is precisely these terms which give rise to friction-like effects and subluminal bubble propagation. In fact, this connection to entropy changes across the bubble wall matches the result shown in Ref. [32]. The former approach directly assumed a steady state expansion, planarity and a common temperature on both sides of the bubble. Our treatment goes beyond the former simplifications by incorporating the friction-like effects at the level of the local field and plasma equations.

We note that standard thermodynamic identities allow the computation of the entropy density in terms of the pressure or equivalently \( V_T \), whose one-loop expression for a general model is a standard result of thermal field theory:

\[
\omega(\phi, T) = T s = T \frac{\partial p}{\partial T} = -T \frac{\partial V_T(\phi, T)}{\partial T}. \tag{5}
\]

This considerably simplifies the calculation of backreaction effects under the assumption of local equilibrium, and allows a quick recovery of the lengthier derivations of entropy in e.g. Ref. [32].

It is worth mentioning that the usual friction terms parameterized by \( \eta \) lead to a violation of the conservation of the total entropy of the universe, and thus correspond to out-of-equilibrium, irreversible processes. Indeed, adding the friction term to the second equation in (4), contracting with \( u_\nu \) and using the thermodynamic identities of Eq. (5) leads to

\[
\partial_{\mu} (su^{\mu}) = \frac{\eta}{T} (u^{\mu} \partial_{\mu} \phi)^2. \tag{6}
\]

Integrating the former equation over a region of spacetime between times \( t_i \) and \( t_f \), applying the divergence theorem and assuming a fluid at rest at the boundary gives \( S(t = t_f) - S(t = t_i) = \int d^3x \frac{1}{2} (u^{\mu} \partial_{\mu} \phi)^2 \), where \( S \) is the total entropy in the spatial volume. In local equilibrium one expects conservation of \( S \), and thus it is consistent to take \( \eta = 0 \). Nevertheless, as we will show in the following sections, friction-like behaviour persists. As the expansion is reversible due to the conservation of entropy, the effective force slowing down the bubble is non-dissipative, and we will refer to it as a backreaction as opposed to a friction force. Its effect will be shown in two ways: by solving the dynamical equations (4), and by directly looking for solutions of their static limit so as to constrain the possible wall velocities [23]. Indeed, a large bubble propagating with constant speed has a steady profile up to subleading curvature effects. As such, static solutions to (4) capture the field and fluid near the wall can directly be searched for. For a bubble propagating in the \( z \) direction with \( v^z \equiv v \), the static equations can be written as [23]

\[
-\phi''(z) + \frac{\partial}{\partial \phi} (V(\phi, T)) = 0,
\]

\[
\omega \gamma^2 v^2 + \frac{1}{2} (\phi'(z)^2) - V(\phi, T) = c_1, \quad \omega \gamma^2 v = c_2,
\]

where \( c_1, c_2 \) are constants which can be traded for the temperature \( T_+ \), and velocity \( v_+ \), in front of the bubble wall. We assume a bubble propagating towards positive \( z \), so that in the wall frame the fluid velocity \( v_+ \) is negative. The last two equations in (7) can be used to express the temperature and velocity in terms of the Higgs field and its derivatives, which then leaves a single equation for the scalar field with a non-standard potential that depends on \( \phi'(z) \). The boundary conditions are \( \phi'(z) = 0, z \to \pm \infty \), and \( \phi \to 0, z \to \infty \). For given \( v_+ < 0 \) and \( T_+ \) this can be satisfied only for a specific choice of the value \( \phi_-(v_+, T_+) \) of the field behind the wall, leading to a prediction of the fluid velocity \( v_-(v_+, T_+) \) behind the bubble.

A further constraint is that very far from the wall the temperature should coincide with \( T_{\text{nuc}} \), which enforces a constraint on the pairs of values \( v_+, T_+ \) and leads to a one-parameter family of solutions. The constraint however is not simply \( T_+ = T_{\text{nuc}} \), as far from the wall the fluid is actually not expected to remain static, but follow a self-similar solution as a function of \( |x|/t \) [33]. For relative velocities between the fluid in front of the wall and the latter below the speed of sound in the plasma, \( c_s^2 = \partial_T p / \partial_T \rho \), one expects deflagration solutions in which the fluid heats up and compresses in front of the bubble and is left at rest in its wake [25] [33]. In the planar case this leads to a region of constant temperature and velocity in front of the wall, which ends up in a shock front in front of which the velocity drops abruptly to zero and the temperature drops to the nucleation temperature. In order that the solutions of (7) are compatible with such a shock front, stress-energy conservation can be used at the front to find the desired constraint on \( v_+, T_+ \). Considering the the frame in which the fluid is unperturbed far behind and in front of the wall, and equating the fluid velocity between wall and shock front deduced from the solutions of (7) and from the shock
Once solutions of (7) satisfying the constraint (8) are found, the bubble wall velocity corresponds to $v_\text{w}$. In this way one ends up with a one-parameter family of solutions. Ref. [23] asserted that the solutions for the equations including the friction parameter were unique; we suggest that the discrepancy may come from the fact that the scalar equation with non-standard potential equivalent to (7) has solutions with $\phi'(z) \to 0$, $z \to \pm \infty$ even when the modified potential has non-degenerate minima. This is a consequence of the fact the the former potential depends on $\phi'(z)$ and thus the standard energy conservation arguments don’t apply.

To end this section, let us point out that the friction force (11) can be derived directly from the second identity in Eq. (7) evaluated at both sides of the wall (where $\phi'(z) = 0$), once one identifies the backreaction pressure $|\vec{F}_{\text{back}}|/A$ with $|\Delta V(\phi, T)|$, and under the approximation of a constant temperature and fluid velocity, the latter identified with $-v_w$. In reality, the situation is more complicated as the temperature and velocity change across the bubble, a more complete result is

$$
\frac{|\vec{F}_{\text{back}}|}{A} = |\Delta (\gamma^2 v^2 \omega)| = |\Delta ((\gamma^2 - 1)T \delta)|.
$$

III. EXAMPLE MODEL

To illustrate the friction effects, we consider an extension of the SM by an $N$-dimensional multiplet $\chi$ of complex scalar singlets with $U(N)$-preserving couplings, including interactions with the Higgs $\Phi$:

$$
\mathcal{L} \supset -m_H^2 \Phi^\dagger \Phi - \frac{\lambda}{2} (\Phi^\dagger \Phi)^2 - m_\chi^2 \chi^\dagger \chi - \frac{\lambda_\chi}{2} (\chi^\dagger \chi)^2 - \lambda_{H \chi} \Phi^\dagger \Phi \chi^\dagger \chi.
$$

In the above equation, we have assumed a background for the neutral component of the Higgs $h$. $g_{*SM} \sim 106.75$ denotes the number of effective relativistic degrees of freedom in the SM plasma, while $g_1$ and $g_2$ are the hypercharge and weak gauge couplings in the normalization compatible with Grand Unification, and $y_t, y_b$ are the bottom and top quark Yukawa couplings respectively.

IV. SOLVING FOR TIME-DEPENDENT SOLUTIONS WITH A NEURAL NETWORK

In this section we focus on solving the time-dependent equations (1). We assume radial symmetry, with the velocity field having a radial component $v^r \equiv v$, and with $v, h, T$ being functions of $r, t$. As initial conditions we use $T(r, t = 0) = T_{\text{nuc}}(r)$, $v(r, t = 0) = 0$, while for the Higgs we use the critical bubble perturbed with a non-relativistic boost (as otherwise the bubble would remain static): $h(r, t = 0) = h_{\text{crit}}(r)$, $\partial_t h(r, t)|_{t=0} = -\dot{h}_{\text{crit}}(r)$, with $\delta = 0.2$.

A. Setup

In order to find time-dependent solutions to Eqs. (4) we follow the technique pioneered in Ref. [23] and implement an artificial neural network (NN). The method relies on recasting the partial differential equations (PDEs) as an optimization procedure –for which NN are uniquely suited– of the form $\mathcal{L} = 0$, where $\mathcal{L}$ is a positive loss
function to be minimized by the NN. The network is constructed by considering an initial layer of 2 inputs \( \xi_n = (r, t) \) that are to be mapped to a final layer with 3 outputs \( N_m \) which are to be approximations of the solutions \( \varphi_m = (v, h, T) \) to the differential equations. The inputs are mapped to successive hidden layers of \( k \) elements, from the combined action of linear transformations between each layer and the action of a real activation functions; a final linear mapping gives the final outputs. For example, for one hidden layer one has

\[
N_m(\xi; \{w, b\}) = \sum_{k,n} w_{mk}^h g(w_{kn}^b \xi_n + b_n^b) + b_m^b,
\]

where \( g \) is the activation function, \( \omega_{mk}^{h}, \omega_{mk}^{b} \) are known as “weights”, and \( b^h, b^b \) are the “biases”. A set of weights and biases which minimize the loss function associated with the system of differential equations are searched for. Writing the latter in the form

\[
F_m(\xi, \varphi_m(\xi), \partial_q \phi_n(\xi)) = 0,
\]

with \( m, n \in \{1, 2, 3\} \), \( p, q \in \{1, 2\} \), and assuming boundary conditions (BCs) for boundary points \( \xi_b \) of the form

\[
B_a(\xi_b, \varphi_n(\xi_b), \partial_q \phi_n(\xi_b)) = 0,
\]

the loss function is constructed from considering a discrete set of “training points” \( \xi \) including boundary points \( \xi_{b,j} \), and evaluating \( F_m \) and \( B_a \) on them:

\[
\hat{L}(\{w, b\}) = \sum_{i,m} c_m F_m(\xi_i, N_m(\xi_i), \partial_j^q N_m(x_i))^2 + \sum_{j,a} d_a B_a(\xi_{b,j}, N_m(\xi_{b,j}), \partial_j^q N_m(\xi_{b,j}))^2.
\]

Above, the derivatives of the network outputs can be obtained analytically from \( \frac{\partial}{\partial \phi} \). The coefficients \( c_m \) and \( d_a \) represent relative weightings for each PDE and BC, required to ensure that all PDEs and BCs contribute comparably to the loss function. We implement the NN with 13 hidden layers with 10 nodes each, with tanh activation functions. We choose the training examples from an evenly spaced 80 \times 80 grid. We use the `pytorch` package along with the Adam optimizer for the NN gradient descent. To avoid getting trapped in sub-optimal local minima of the smooth loss function, we take care to reduce the learning rate through cosine annealing with warm restarts. For fast convergence of our solution, we first pretrain the NN with a template solution implemented as a boundary condition for low \( t \). This is obtained using Wolfram Mathematica’s PDE solver, which is only able to provide reliable solutions for a small time interval. After the NN is in the correct vicinity of solution, we remove the pretrained template from the loss function and train according to \( \frac{\partial}{\partial \phi} \). This allows reliable solutions for time intervals that cannot be reached with the Mathematica solver.

\[
B. Dynamic transition results
\]

From the previous NN setup we were able to obtain solutions in which the individual loss functions \( F_m \) in dimensionless units (obtained by rescaling quantities with appropriate powers of the W mass \( m_W \approx 80 \text{ GeV} \)) take values \( \lesssim 5 \times 10^{-3} \). We show the resulting dynamical profiles of \( h, T, v \) in Fig. 1 as a function of \( r \) in dimensionless units for 5 equally spaced timestamps between \( t = 0 \) and \( t = 50/m_W \). We note that the Mathematica solver was only able to compute accurate solutions for \( t \lesssim 15/m_W \). The scalar profile settles to a slow expansion, while the velocity and temperature profiles show the formation of a faster propagating front, in accordance with the expectations of a deflagration solution with self-similar fluid behaviour. Confirming the latter would require extending the solutions to even later times; a more efficient means is to directly look for static wall solutions with consistent hydrodynamic profiles as we show in the next section.

By following points with constant \( h(r, t) = 0.5 \) we can estimate the bubble’s position and velocity; the latter is plotted with a solid line in Fig. 2 which shows that the bubble’s velocity settles to \( \lesssim 0.25 \). This is in contrast to the result, illustrated with a dashed line, when the terms proportional to \( \partial \omega / \partial \phi \) are omitted in Eq. (1). In this case the bubble velocity quickly approaches the speed of light. This confirms our observation that the field-dependence of the enthalpy is responsible for the friction-like behaviour.

V. STATIC PLANAR BUBBLE PROFILES AND CONSISTENT DEFLAGRATIONS

In this section we report the results of looking for the static solutions of (7) satisfying the hydrodynamic constraints of (8). By choosing values of \( v_w, T_+ \) and substituting the first two identities in (7), the potential in the scalar equation becomes a function \( V_W(h(z), h'(z)) \). We have found a one-parameter branch of solutions satisfying all constraints with \( T_+ \leq T_+^{\text{max}} = 115.476 \text{ GeV} \); for temperatures in the vicinity of the upper bound the constraints cannot be satisfied. The wall velocity is restricted to be \( v_w \leq 0.33 \), compatible with the dynamical results of the previous section, in which temperatures remained below \( T_+^{\text{max}} \) (see Fig. 1) and the wall velocity approached 0.25. The wall velocity \( v_w \), the exact back-action force of Eq. (9) and the approximation of Eq. (1) found in Ref. [32] are illustrated in Fig. 3. We find qualitative agreement with Eq. (1) up to deviations below 25%, which are due to the changes of \( T \) and \( v \) across the bubble.

The static profiles for the scalar have a typical width as in Fig. 1 \( L \sim 20/m_W \sim 30/T_+ \). The local equilibrium approximation is expected to hold if \( L/\gamma(v_w) \) is above the mean free path \( \lambda_{\text{mfp}} \) of particles in the plasma. With \( v_w \lesssim 0.3 \) the Lorentz contraction factor is of order one, while Ref. [13] estimated \( \lambda_{\text{mfp}} \lesssim \).
Figure 1: Dynamical evolution of $h$, $T$, and $v$ in dimensionless units. The curves from left to right correspond to time steps from $t m_W = [0, 50]$ with $m_W \Delta t = 10$.

Figure 2: Bubble velocity versus time in dimensionless units, including the effect of the field dependence of the enthalpy (solid line) or without it (dashed line).

$\hat{m}_W^2(T)/(10\pi \alpha_w^2 T^3)$, where $\hat{m}_W^2(T)$ is the temperature-dependent $W$ mass, and $\alpha_w = g_2^2/(4\pi)$. In our bubbles, we have $h \lesssim 1.5 m_W \sim T$, giving $\hat{m}_W^2(T) \lesssim T^2/9$ and $\lambda_{mfp} \lesssim 3/T$. Hence the local equilibrium approximation is indeed justified.

VI. CONCLUSION

We have shown that in a first-order phase transition in which equilibrium is maintained locally and the total entropy is conserved, the bubbles of the energetically favoured phase are slowed down by hydrodynamic effects related to the dependence of the enthalpy on the scalar field background. This is in contrast to the common view that links bubble friction with out-of-equilibrium effects. The backreaction effects of the plasma can be accounted for by using conservation of the stress-energy momentum tensor and incorporating the background-field-dependence of the plasma’s pressure and enthalpy, which can be derived straightforwardly from the thermal corrections to the effective potential. We have illustrated these effects by using a neural network to solve the differential equations for bubble propagation in an SM extension with additional scalars. We also constrained the possible asymptotic bubble velocities by considering the equations in the static limit, while accounting for consistent hydrodynamic deflagration profiles. The resulting backreaction force, computed from Eq. (9), is consistent with the results of Ref. [32], which exclude runaway bubbles, up to $O(20\%)$ effects related to the change of velocity and temperature across the wall. We have considered a scenario in which the hypothesis of local equilibrium is justified. However, we expect that even in cases where some particles are out of equilibrium, a similar reduction in bubble velocity will still result due to a backreaction force from the equilibrated plasma. This effect should be properly accounted for.
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