Engineering analysis of foundations in the chasm zone, taking into account the soil subgrade nonlinear behavior

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Abstract. The article solves the problem of stress and strain state of the subgrade waned by cylindrical neckline, modeling the chasm zone. The solution was obtained taking into account the soil subgrade nonlinear behavior based on using the A.I.Bonkin bilinear subjection. The visual results of calculations in the form of stresses and strains isofields are presented in the article. The obtained solution made it possible to offer an updated analytical model of the buildings and constructions karst-resisting foundations for consideration. A variable coefficient of subgrade reaction, varying according to the bilinear law was adopted in the proposed analytical model.

Introduction

The stress and strain state of the soil subgrade waned by karst chasm is advisable to predict using the analytical models describing the subgrade reaction coefficient changing [1,2,3]. Two analytical models are presented on the Fig. 1. On the first one (Fig. 1 (a)), the coefficient of subgrade reaction \( K \) changes with stepwise from zero value to the calculation value beyond its borders. On the second one (Figure 1 (b)), the decreasing coefficient \( K_1' \) not regulated by the distance from the pothole area and its quantitative value enters. This article is devoted to this coefficient definition for using it in the buildings and constructions karst-resisting foundation calculations.

Figure 1. Existing analytical models

The Authors of the article have already considered [1] the problem of the uniformly loaded layer stress and strain state of soil with limited thickness, waned by cylindrical neckline (Figure 2), the diameter of which is regulated by the regulatory documents [2].
As an output of solving the problem in the linear formulation, the expressions for stresses and strains in a cylindrical coordinate system are obtained (Eq. 1):

\[
\sigma_z = p_0; \sigma_r = \frac{v p_0}{(1-\nu)} \left[ 1 - \left( \frac{b}{r} \right)^2 \right] ; \sigma_\theta = \frac{v p_0}{(1-\nu)} \left[ 1 + \left( \frac{b}{r} \right)^2 \right] \\
w = \frac{\beta p_0 (H-z)}{E} ; u = -\frac{v p_0 b^2}{2rG(1-\nu)}.
\]

(1)

It is important to note that the formula for vertical deformations \( w \) does not depend on the distance from the cylindrical area to the chasm; therefore, the waning effect of the subgrade in the zone of contact with chasm is not realized. The calculation model, describing this foundation behavior in approximate way is shown on the Fig. 1 (a).

This work proposes to give a quantitative subgrade waning estimate in the zone of contact with a chasm and to develop a refined analytical model for calculating karst-resisting foundations, based on the nonlinear formulation assigned problem.

**Pre-conditions for using nonlinear calculations**

Instead of the usual deformation characteristics of the soil - \( E \) and \( \nu \), there are two alternative deformation characteristics in the formula: the Bulk Modulus Elasticity-\( K \) and the shear modulus- \( G \). Then, Hooke’s law takes the following form:

\[
\varepsilon_i = \frac{\sigma_i - \sigma_m}{2G} + \frac{\sigma_m}{K}, (i = r, \varphi, z) ; \sigma_m = \frac{\sigma_z + \sigma_\theta + \sigma_r}{3}.
\]

(2)

The soil deformation characteristics separation into shear and volumetric is very important for the soil strain nonlinear theory, because the soil differently resists to volumetric deformations and to the shape changings strains.

Thus, as the average stresses \( \sigma_m \) increase, the volume strain \( \varepsilon_v \) increases too, but asymptotically it tends to a certain value, that is, the volume strain \( K \) \( (\sigma_m) \) module is a function of the average stresses \( \sigma_m \) and increases with their growth [4, 5, 6]. With shape changings strains, the opposite is true: with increasing tangential stresses \( \tau \), the angular strain increases hyperbolically and tends to infinity with a certain limiting intensity of shearing stresses \( \tau^* \), so the shear modulus \( G \) is a function of shear stresses \( \tau \) and decreases with their growth [5,6,7,8].

It is important to note, that shear modulus- \( G \) depends on the average stresses \( \sigma_m \), because the shearing stresses intensity limited value \( \tau^* \) also depends on them (Fig. 3).
There are a lot of theories for describing nonlinear deformability of soils. The authors apply A.I. Botkin bilinear subjection in the article [4,5,6] (Eq. 3):

$$G_{pl} = G_0 \frac{\tau^*_i - \tau^*_i}{\tau^*_i}; \quad \tau^*_i = \sigma^*_m \cdot \tan \varphi + c. \quad (3)$$

In addition to the soil deformation characteristics, this dependence also allows to introduce the stress parameters: the angle of internal friction $\varphi$ and the coefficient of adhesion $c$. Thus, having the soil mechanical characteristics “standard” set ($E$, $\nu$, $\varphi$ and $c$), it is possible to get a solution to a nonlinear problem, approximate to the subgrade actual behavior.

**Nonlinear settlement calculation**

To obtain the convenient for analytical work expression for $\tau^*_i$, we substituted the tensions tensor components obtained expressions $\sigma_z$, $\sigma_i$ and $\sigma_\varphi$ and carried out the simple transformations (Eq. 4):

$$\tau^*_i = p_0 \frac{(1 + \nu)}{3(1 - \nu)} \cdot \sigma^*_m \cdot \tan \varphi + c. \quad (4)$$

As a result of the expression analysis for the tension components, taking into account the tension equality, we obtain the following expression for the tension intensity (Eq. 5):

$$\tau_i = \frac{p_0}{\sqrt{3(1 - \nu)}} \sqrt{3\nu^2 \left( \frac{b}{r} \right)^4 + (1 - 2\nu)^2}. \quad (5)$$

We obtain the expression for vertical deformations $w$ by substituting the obtained expressions for $\tau^*_i$ and $\tau_i$ in (Eq. 3) and carrying out the simple transformations. On the base of this expression we build the vertical displacements isofields (Figure 4).
Figure 4. Isofields of vertical strains $w$, taking into account the subgrade nonlinear behavior

This pattern of the array vertical displacements isofields is fundamentally different from the solution obtained in the linear formulation — there is a significant increase of displacements in the chasm zone, which decays at some distance from the chasm.

Comparing the obtained results with the linear calculation data in the article [1] it is unequivocally traced that the foundation draft, obtained taking into account that nonlinear soil deformability, is greater than the precipitation obtained by the linear theory. It is important to note that as the load $P_0$ increases, this difference increases as the intensity of the tangential stresses $\tau_i$ with a tendency toward $\tau_i^*$. The shear modulus $G$ value, in turn, tends to zero $G \to 0$, thus increasing the first term (shape changing) in the expression for the deformation.

On the obtained data basis the graph of subgrade reaction $C_z$ coefficient varying in the chasm zone under the base slab was built (Figure 5).

Figure 5. Graph of subgrade reaction $C_z$ coefficient varying in the chasm zone under the base slab in the in the zone of contact with the chasm.
The obtained expressions and graphs for the variable coefficient of subgrade reaction \( C_z \) are nonlinear in nature and are difficult to use in engineering practice.

**Formation of the analytical of the subgrade.**

In the article, for the simplified description of the dependence of the variable coefficient of subgrade reaction \( C_z \) from the radial distance to the axis of the pothole \( r \), the bilinear dependence, which is divided into two sections at the radial distance \( r = R \) is accepted.

- when \( r \geq R \), the coefficient of subgrade reaction \( C_z \) takes on a value without taking into account the effect of waning of the subgrade by the cylindrical neckline \( C_{z,\text{max}} \);
- when \( r \leq R \), the coefficient of subgrade reaction \( C_z \) increases with the increasing of radial distance \( r \) be linear dependence, it increases from \( C_{z,\text{min}}(b) \) to \( C_{z,\text{max}}(R) \).

A graphic representation to the description of the bilinear dependence is given below on Figure 6.

**Figure 6. Subgrade reaction \( C_z \) coefficient bilinear diagram**

However, in engineering calculations it is more convenient to use not two stiffness coefficients \( C_{z,0} \) and \( C_{z,\text{red}}(b) \), but the ratio of them - the attenuation coefficient \( k_{\text{red}} \) (Eq. 6):.

\[
k_{\text{red}} = \frac{C_{z,\text{red}}(b)}{C_{z,0}}.
\]

The waning area \( R \) radius is taken from the assumption that in this point the subgrade reaction \( C_z \) coefficient value taking into account the wane, \( C_{z,\text{red}}(R) \) differs by no more than 5% from the subgrade reaction coefficient without taking into account the wane in \( C_{z,0}(R) \). In this case, 2 (two) main parameters of this dependence are determined unambiguously, and it is much more convenient to use...
them in engineering practice than the complex non-linear dependencies. Formulas for determining these parameters are given below (Eq. 7):

$$k_{red} = \left( \frac{1 + \nu}{3(1 - \nu)} \tan \phi + \frac{c}{p_0} - \frac{\sqrt{7\nu^2 - 4\nu + 1}}{\sqrt{3}(1 - \nu)} \right) \left( \frac{1 + \nu}{(1 - \nu)} \tan \phi + 3\frac{c}{p_0} - \frac{(1 - 2\nu)}{\sqrt{3}(1 - \nu)} \right)$$

$$R = \frac{b}{4 \sqrt{0.05 \frac{1 + \nu}{\sqrt{3}} \tan \phi + 0.05 \frac{c\sqrt{3}(1 - \nu)}{p_0} + 0.95(1 - 2\nu)}^2 - (1 - 2\nu)^2} \frac{3\nu^2}{3\nu^2}.$$  (7)

The waning area R radius is taken from the assumption that in this point the value of coefficient of subgrade reaction $C_z$, taking into account the wane, $C_{z,red}(R)$ differs by no more than 5% from coefficient of subgrade reaction without taking into account the wane in $C_{z,0}(R)$. In this case, two main parameters of this dependence are determined unambiguously, and it is much more convenient to use them in engineering practice than the complex non-linear dependencies. Formulas for determining these parameters are given below (Eq. 7):

**Summary**

The obtained parameters for the wane of the foundation subgrade ($k_{red}$ and R) have a wide range of changes depending on the mechanical characteristics of the soil subgrade and structural elements of the underground part of buildings and structures. Even with the presence of "weak" soils and in the case of load transfer of a conditionally low load on the subgrade, the attenuation coefficient $k_{red}$ is in the range from 0.3 to 0.7, and the zone of waning of the subgrade R expands beyond the limits of the chasm by a value from b to 2b.

**References**

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