Overview on the Trojan Horse Method in nuclear astrophysics

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Abstract. The use of the Trojan Horse Method (THM) appears as one of the most suitable tools for investigating nuclear processes of interest for astrophysics. THM has been demonstrated to be useful for exploring different nuclear reactions intervening both in stellar and primordial nucleosynthesis as well. Some recent results will be here discussed together with a brief discussion of the fundamental theoretical description. General details about the recently studied \(^{7}\text{Be}(n,\alpha)^{4}\text{He}\) reaction will be given.

1. Introduction

Experimental nuclear astrophysics aims at determining the reaction rates for astrophysically relevant reactions at their Gamow energies. For charged-particle induced reactions, the access to these energies is usually hindered, in direct measurements, by the presence of the Coulomb barrier between the interacting particles or by electron screening effects, which make hard the determination of the bare-nucleus S(E)-factor of interest for astrophysical codes. Among the goals of this charming research field, the understanding elemental abundances to energy production, as firstly pointed out in the seminal B\textsuperscript{2}FH paper [1], is here of interest. Indeed, the
curve of abundances reveals several features, confirming the action of both primordial and stellar nucleosynthesis. In this framework, the well established Big Bang Nucleosynthesis scenario allows us to understand light element production on the basis of an elegant formalism for which only the action of the baryon-to-photon ration $\eta$ parameter is on play (see [2] and ref. ther.). Beside the role played by the light element abundances Li-Be-B, the curve of the elements reveal additional features from which the action of precise processes driven the nucleosynthesis appear. This consideration involves the neutron capture reactions such as $s - \text{process}$ and $r - \text{process}$ that nowadays are considering as the best candidates for the understanding of heavy elements beyond iron. In this complex scenario, experimental nuclear astrophysics aims to measure the nuclear reaction cross section of interest right in correspondence of the Gamow peak. Because of the well know difficulties in accomplish this important mission, experimentalist has to use often extrapolation procedures to access the relevant Gamow energy peak, even if they are inevitably affected by several uncertainties such as the currently not-well-understood electron screening effects [3]. This effects, due to the electronic (atomic) cloud surrounding the positively charged nucleus, alter the cross-section at low-energies, thus shielding the pure Coulomb repulsion between the charged-interacting nuclei. This considerably inhibits the experimentalist to reach the ultra-low energy region, thus causing an additional problem for experimental nuclear astrophysicists. In addition, its understanding is far to be completely understood since the current available theoretical dynamics models (i.e. adiabatic approximation) largely underestimate the electron screening potential values with respect those measured in terrestrial laboratories.

In order to by-pass the difficulties connected with both Coulomb barrier penetration and electron screening effects, the Trojan Horse Method (THM) [4, 5, 6] has been largely applied for measuring the bare nucleus S(E)-factor for astrophysically relevant reactions, being its power the access to the bare-nucleus S(E)-factor without any kind of extrapolation.

THM allows one to extract the bare-nucleus cross-section of a charged-particle induced reaction $a + x \rightarrow c + C$ at astrophysical energies free of Coulomb suppression, by properly selecting the quasi-free (QF) contribution of a suitable $2 \rightarrow 3$ body reaction $a + A \rightarrow c + C + s$, performed at energies well above the Coulomb barrier, where the nucleus $A$ has a dominant $x \oplus s$ cluster configuration. As recent published results, THM has been used in studying several problems, ranging from BBN (see for instance Pizzone et al. [7]), light element burning reactions (Lamia et al. [8]; Tumino et al. [11]; Pizzone et al. [12, 13]; Spitaleri et al. [9, 10]), CNO reactions (Sergi et al. [14, 15], Palmerini et al. [16], Indelicato et al. [17]), removing/producing neutron reactions (Gulino et al. [19], Guardo et al. [20]), carbon-burning (Tumino et al. [21]). Recently, the extension to RIB’s charged particle induced reactions has been also provided, as discussed in [22, 23].

2. Basic features of the Trojan Horse Method

The Trojan Horse Method (THM) is an indirect technique allowing one to measure the cross section of a two-body reaction $A(x, c)C$ by properly selecting the quasi-free component of a suitable $2 \rightarrow 3$ body reaction $a(A, cC)s$ ([4, 5, 6]). Nucleus $a$ is chosen because of its large $a = x \oplus s$ configuration, its relatively low $x - s$ binding energy and its known radial wave function for the $x - s$ configuration. It represents the so-called “Trojan-horse nucleus”. The $2 \rightarrow 3$ reaction is induced at energies well above the Coulomb barrier of the $A + a$ interacting particles in order to induce the sub-process $A - x$ in the nuclear field. Thus, the breakup of $a$ is quasi-free since only cluster $x$ takes part to the binary process while the other counterpart $s$ acts as spectator, i.e. it maintains in the exit channel the same momentum distribution it had inside $a$ before its break-up. In addition, a specific role is played by the $x - s$ binding energy. In particular, it compensates for the projectile energy down to low, i.e. astrophysically relevant, energies thus appearing of immediate help for nuclear astrophysics purposes. In particular, since
the $A-x$ interaction occurs directly in the nuclear field, no Coulomb barrier penetration effects or screening phenomena affect the THM data in comparison with the direct ones where these two effects cause the well-known exponential decrease of the cross section values and the enhancing of the cross section values, respectively.

In the most simple theoretical description of THM by means of the plane wave impulse approximation (PWIA), the cross section of the quasi-free $a(A,cC)s$ reaction can be related to the one of the binary $A(x,c)C$ process via the formula ([6]):

$$\frac{d^3\sigma}{dE_c d\Omega_c d\Omega_C} \propto KF \cdot |\Phi(p_{xs})|^2 \cdot \frac{d\sigma}{d\Omega}^{H O E S}_{a-x}$$

where:

- $K F$ represents the kinematical factor, depending on masses, momenta, and angles of the outgoing particles, that takes into account the final state phase space factor;
- $|\Phi(p_{xs})|^2$ is the squared of the Fourier transform of the radial wave function describing the $x-s$ inter-cluster motion, usually in terms of Hänkel, Eckart or Hulthén functions depending on the $x-s$ system.
- $\frac{d\sigma}{d\Omega}^{H O E S}_{a-x}$ is the half-off-energy-shell (HOES) differential cross section for the two body reaction at the center of mass energy $E_{cm}=E_{cC}-Q$, where $Q$ represents the Q-value of the HOES $A(x,c)C$ reaction while $E_{cC}$ represents the relative $c-C$ energy measured in laboratory. The deduced cross section is HOES since, in the entrance channel, the transferred particle $x$ having mass $m_x$ is virtual, thus its energy and momentum are not related by the mass-shell equation $E_x=k_x^2/(2m_x)$. Under QF conditions, the relative $A-x$ energy is then determined by relation $E_{Ax}=p_{Ax}^2/(2\mu_{Ax})-\epsilon_{sx}$, being $\epsilon_{sx}$ the binding energy of the TH-nucleus. In the exit channel, the relation is restored since the emitted $c-C$ particles are real (see [6] for details).

3. Some detail on THM data analysis

As already discussed in the previous section, THM founds its theoretical formulation in the so-called “quasi-free reaction mechanisms”, widely used in the past for evaluating nuclear structure with particular regard to the cluster configurations (see for example the detailed discussion reported in [5]). By performing a devoted experiment for studying the three-body reaction $a+A\rightarrow c+C+s$, it is possible to connect the three-body cross section (measured in the laboratory) with the one of interest for astrophysics (extracted with the support of a dedicated theoretical formalism) through the relation [5]

$$\frac{d^3\sigma}{dE_c d\Omega_c d\Omega_C} \propto K F \cdot |\Phi(p_{xs})|^2 \cdot \left(\frac{d\sigma}{d\Omega}\right)^{H O E S}_{a-x}$$

where $K F$ represents the kinematical factor, $|\Phi(p_{xs})|^2$ is the square of the momentum distribution for the $x-s$ relative motion inside the TH-nucleus $A$, and $\frac{d\sigma}{d\Omega}^{H O E S}_{a-x}$ the half-off energy shell cross section. This last quantity represents the “bare-nucleus” cross section of interest for astrophysics, once it has been corrected for the penetrability through the Coulomb barrier and normalized to the available high-energy direct data.

In order to assess the method, it is customary to underline the role of some of the most important sources of uncertainties in a typical THM experiment and/or data analysis:

- **Role of the experimental momentum distribution.** The determination of the experimental momentum distribution is one of the most important steps of a typical THM
analysis, this reflecting the presence of the quasi-free reaction mechanism. This is usually performed in terms of both Plane Wave Impulse Approximation (PWIA) and Distorted Wave Born Approximation (DWBA). By referring to the $^{11}$B+p case, its has been found that the two approach, PW and DWBA, nicely agree within lower momentum of the exiting neutron, being the same result confirmed in a variety of THM experiment discussed in the literature (i.e. [14]). Other THM nuclei have been used up to now, such as $^6$Li($\alpha$$\otimes$$d$), $^9$He($d$$\otimes$$p$) and, very recently, $^{14}$N($^{12}$C$\otimes$$d$).

In addition, the role of the momentum distribution and, in particular, of its experimental FWHM on THM data has been firstly investigated in the work of [24]. Additionally, in [25] we have investigate the role of the $d$-state when describing the deuteron ground-state wave function. In particular, both the $s$ and $d$ state wave functions have been calculated by using the exact form and the asymptotic one. In order to evaluate the impact on THM data, we have introduced the $d$ state contribution in the THM analysis of [26], resulting in an 0.5% variation on the absolute value of the $S(0)$ for the $^{11}$B+p reaction.

- **Introduction of the penetration factor trough the Coulomb barrier.** THM results are naturally bared from Coulomb penetration effects. However, in order to get THM results in absolute units, a normalization procedure is required by using the available direct data. Thus, THM data need to be corrected for analytical function describing the penetration through the Coulomb barrier, as given in several text book (see for example [3]). To such purposes, ones has to fix a cut-off radius in terms of the standard formula $r=1.2(A_1^{1/3}+A_2^{1/3})$ fm [3], even if such choice could introduce some uncertainties in the final results. This further source of uncertainty has been investigated in [26], leading to an overall $\sim$14% of uncertainty on the final evaluation of the zero-energy $S(E)$-factor;

- **Energy resolution effects.** THM data are expressed in terms of the relative energy between two-out-of three $c$ and $C$ detected particles, being this experimental solution sufficient to reconstruct completely the kinematic for a reaction having three-particles in the exit channel by properly applying energy and momentum conservation rules. Thus, energy resolution effects on the relative $E_{cc}$ energy are measured by means of standard errors propagation theory, taking into account both energy and angular resolution due to the adopted experimental setup. In the case of the $^{11}$B+p reaction discussed in [26], this has been evaluated to be $\sim$40 keV. Improved relative-energy resolution have been obtained in recent works such as [20, 17].

4. Recent THM studies on the cosmologically relevant $^7$Be($n,\alpha$)$^4$He reaction

In the case of the cosmologically relevant $^7$Be($n,\alpha$)$^4$He reaction, the corresponding cross section has been subject of different studies on which direct and indirect methods have been used [27, 28, 29, 30, 31].

In [30], the $^7$Be($n,\alpha$)$^4$He cross section has been deduced by applying the charge-symmetry hypothesis (CSH) to already existing $^7$Li($p,\alpha$)$^4$He THM data for which a validation for the $^7$Be($n,\alpha$)$^4$He case derive from the agreement between the direct measurements performed by [29] and the one of [27], being these last ones based on CSH. Taking advantage of such evidence, for the purpose of our THM work, two data sets have been considered for applying CSH to the already existing THM $^7$Li($p,\alpha$)$^4$He data. In particular, we adopted the data discussed in [12, 32]. These data allowed for the extraction of the $^7$Li($p,\alpha$)$^4$He via a deuteron and $^4$He breakup THM experiments, separately. In addition, because we are interested in using the experimental data useful for the $^7$Be($n,\alpha$)$^4$He investigation, only part of available data have been considered. In particular, because of the difference in mass of the two entrance channels $^7$Li+$p$ and $^7$Be+$n$, a difference of 1.644 MeV is present between the center-of-mass energies covered in the two cases. For such a reason, only the $^7$Li($p,\alpha$)$^4$He THM cross section data, $\sigma_{p\alpha}$, covering a center-of-mass
energy $E_{Li-p}>1.644$ MeV have been taken into account. These data have been then converted to the $\sigma_{na}$ ones of the $^7$Be($n,\alpha$)$^4$He channel by using the identity:

$$\sigma_{na} \cdot \frac{E_{Li-p} - 1.644}{P_{l=1}^n(E_{Li-p} - 1.644)} = \sigma_{pa} \cdot \frac{E_{Li-p}}{P_{l=1}^p(E_{Li-p})}$$

where $P_{l=1}^n$ represent the penetrability for the neutron and proton channel, respectively [30].

The result of such investigation show a marked agreement with the trend of the cross section data of [27] and [29], with the advantage of producing a cross section measurement right in the energy region of BBN. The good agreement once again showed the goodness of our assumption as previously done in [27]. The THM reaction rate has been then used for running the evolutionary code described in [7] to derive the primordial $D,^3He,^7Li$ abundances. Besides the already mentioned agreement for deuteron and helium isotopes, a marked disagreement appears for lithium, thus leaving still open the $Li$-problem in cosmology.

The devoted THM experiment for determining the $^7$Be($n,\alpha$)$^4$He cross section is discussed in [31]. The $^2$H($^7$Be,$\alpha\alpha$) experiment was performed at the EXOTIC facility [33] of Laboratori Nazionali di Legnaro (INFN-LNL) using a 20.4 MeV $^7$Be beam impinging on a CD$_2$ target with a thickness of 400 $\mu$g/cm$^2$. In the quasi-free (QF) $^2$H($^7$Be,$\alpha\alpha$) process, deuteron undergoes its break-up in neutron (participant) and proton (spectator). The detection setup was thought with the aim of detecting the two emerging alpha particles while the kinematical quantities of the undetected proton were reconstructed via momentum-energy conservation laws. In addition, since only quasi-free (QF) events were considered for the THM analysis, the setup covered the kinematical region corresponding to the QF-angular pairs, i.e. the angular pairs at which the spectator maintains the same momentum distribution it had inside the deuteron before its break-up ([6]).

To assess the proper selection of the exit channel the experimental Q-value spectrum has been deduced for the selected events, leading to an experimental value of about 16.76 MeV, in agreement with the theoretical one of 16.765 MeV. A Gaussian fit of such a peak leads to a FWHM of about 2 MeV. The trend of the momentum distribution for the p-n intercluster motion inside deuteron has been used in order to select the QF-reaction mechanism. The agreement shown in [31] marks unambiguously the presence of the QF-reaction mechanism thus allowing us to further proceed in the extraction of the $^7$Be($n,\alpha$)$^4$He cross section. Thus, the two body reaction cross section was properly evaluated taking into account HOES (half-off energy shell effects) as well as normalization to the available direct data of [27, 28, 29].

The HOES differential cross section expressed was then extracted and converted to the on-energy-shell (OES) cross section by correcting the angular distribution and the centrifugal-barrier penetrability based on the orbital angular momentum of $l = 1$ which arises from the broad $p - wave$ $^7$Be-n resonances at $^8$Be excitation energies around 20 MeV [31]. Then the OES cross section was normalized to the data of [27]. The present data nicely overlap with those of the previous experiments [27, 28, 29, 30]. The derived reaction rate shows a fair agreement with that of [27], with improved uncertainty at BBN energies. Although the primordial $^7$Li abundance evaluated via the code of [7] still remains far from the observed one, the THM investigation allowed for reducing the uncertainty on the corresponding reaction rate.

A third experimental THM investigation expanded the idea of the EXOTIC experiment aiming at measuring the $^7$Be($n,p$)$^7$Li reaction at the CRIB (Center-for-Nuclear-Study RI beam separator) facility, as briefly introduced in [34]. For such a purpose, the THM experiment has been tough in order to detect both the product of the $^2$H($^7$Be,$\alpha\alpha$) process as well as those coming from the $^2$H($^7$Be,$p^7$Li)$p$ one. For such a purpose, two more $\Delta E$-E telescopes have been placed in a symmetrical configuration with respect the beam direction at the most forward angles with 20-$\mu$m-thick silicon detectors. The coincidence measurement were simultaneously available as well with the same setup in given telescope pairs. The present setup allows better angular
and energy resolutions by installing Parallel Plate Avalanche Counters (PPACs [22]) enabling event-by-event beam tracking, and a thinner CD2 target of 64 µg/cm², and silicon detectors with a better position resolution, and with a higher incident beam energy with smaller energy spread of 22.1±0.1 MeV. The preliminary normalized excitation functions for the ⁷Be(n, p₀)⁷Li and the ⁷Be(n, α)⁴He channels are roughly consistent with the previous studies present in the literature. Thanks to the improved experimental setup, our preliminary results suggest the possibility of extracting information also on the (n, p₁) channel, i.e. the one for which ⁷Li is left in its first excited state (≈478 keV). Data analysis is still in progress.

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