Spatial Mode Diversity for Robust Free-Space Optical Communications

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Free-space communication links are severely affected by atmospheric turbulence, which causes degradation in the transmitted signal. One of the most common solutions to overcome this is to exploit diversity. In this approach, information is sent in parallel using two or more transmitters that are spatially separated, with each beam therefore experiencing different atmospheric turbulence, lowering the probability of a receive error. In this work we propose and experimentally demonstrate a generalization of diversity based on spatial modes of light, which we have termed modal diversity. We remove the need for a physical separation of the transmitters by exploiting the fact that spatial modes of light experience different perturbations, even when travelling along the same path. For this proof-of-principle we selected modes from the Hermite-Gaussian and Laguerre-Gaussian basis sets and demonstrate an improvement in Bit Error Rate by up to 54%. We outline that modal diversity enables physically compact and longer distance free space optical links without increasing the total transmit power.

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of FSO communications when compared to radio.

In a typical diversity model, multiple transmitters (lasers) transmit identical signals, \( x_i(t) \), at the same time with intensity \( g_i \). These signals propagate through separate channels represented by channel impulse responses, \( h_i(t) \). The signal is then detected by a single receiver (photodiode) with receiver sensitivity, \( r \). The resulting received signal, \( y(t) \) is then found from

\[
y(t) = r \sum_i g_i x_i(t) + h_i(t) + n(t),
\]

where an Additive White Gaussian Noise (AWGN) component, \( n(t) \), may be incorporated. Traditionally, “separate channels” means separate paths, which in turbulence means a physical separation of at least \( r_0 \). Now we will alter this paradigm to interpret “separate channels” as distinct modes with differing behaviour in turbulence.

If the signal is strong then the contribution of noise to errors is insignificant and the dominant cause of errors is the channel gain itself. If the channel gains are statistically independent then we can write the overall probability of an error in the diversity case as

\[
\text{Pr}[E_{\text{diversity}}] = \prod_i \text{Pr}[E_i],
\]

where the probability of an error occurring for the \( i \)th channel is defined as \( \text{Pr}[E_i] \). This always results in a lower probability of error than a single channel case (since \( \text{Pr}[E_i] \leq 1 \)). This standard diversity scheme is similar to Equal Gain Combining (EGC) and will only show a diversity gain if the individual channels are statistically independent [19].

To implement the above using modal diversity, identical or- der yet orthogonal HG\(_p\) and LG\(_p\) beams were chosen, motivated by the fact that HG modes are robust to tip/tilt which are the primary aberrations of atmospheric turbulence [18]. The order of the beams is given by \( N = n + m = 2p + |\ell| \) for HG and LG beams respectively. The completeness property of both bases allow us to express any element of one basis as a linear combination of elements from the other basis using the transformation relations below [20]:

\[
LG_{n,m}(x,y,z) = \sum_{k=0}^{N} b(n,m,k) HG_{N-k,k}(x,y,z)
\]

\[
b(n,m,k) = \frac{(N-k)!k!}{2^N n! m!} \frac{1}{k!} \frac{d^k}{dt^k} [(1-t)^n(1+t)^m]_{t=0}
\]

The LG modes have been written in terms of \( n \) and \( m \), which are indices typically used for HG modes. Traditionally, LG modes are given in terms of azimuthal index \( \ell \) and radial index \( p \), which can be recovered as \( \ell = n - m \) and \( p = \min(n,m) \). For example, the LG\(_2\) mode may be written as

\[
LG_2 = \frac{1}{2} HG_0^2 - \frac{i}{2} HG_1^2 + 0 \times HG_2^2 - \frac{i}{2} HC_1^2 - \frac{1}{2} HG_0^2.
\]

Notice that the HG\(_2\) component has a zero weighting, making the HG\(_2\) mode orthogonal to the LG\(_1\) mode. Figure 1 shows an experimental test of this orthogonality. Conveniently, both of these modes have an order of \( N = 4 \), however, not all LG modes contain an orthogonal HG mode with the same mode order, and vice versa. Similarly, HG\(_4\) is orthogonal to LG\(_3\) with \( N = 8 \). Both of these mode sets were tested in the experiment.
visible. This is expected because the effect of turbulence has become negligible. The diversity gain is still present because of the independent nature of the effect of turbulence on the HG and LG modes.

These results can be put into context by calculating the effective propagation distance gain at a specific BER [21]. If Kolmogorov turbulence is assumed, then we can write $r_0$ as a function of $C_n^2$, the refractive index structure parameter, the wavelength, $\lambda$, and the propagation distance, $z$:

$$r_0 = 0.185 \left( \frac{\lambda^2}{C_n^2} \right)^{3/5}$$

This equation can then be solved for $z$, resulting in:

$$z \approx \frac{0.0600647 \lambda^2}{C_n^2 r_0^{5/3}}$$

In this experiment, $\lambda = 660$ nm and a typical value for $C_n^2$ is $10^{-14}$ m$^{-2/3}$ in strong turbulence [21]. Three arbitrary BERs were selected and in Table 1 the corresponding $r_0$ values are provided. From Eq. 7, the corresponding theoretical propagation distances are also shown. It is clear that significant improvements in propagation distance are possible using modal diversity.

Traditionally, diversity has been theoretically and experimentally demonstrated with a $r_0$ (or greater) separation between transmit and/or receive apertures. In this work we have outlined the concept of modal diversity, and shown that a significant diversity gain can in fact be achieved in a far more compact system, without $r_0$ separation between apertures or beams. While we have used orthogonal HG and LG modes of the same order for the demonstration, we expect modal diversity to be observed in any system with judiciously selected mode sets, whether differing in mode order, size or mode type. We speculate that it should be possible to formalise the concept of “separate” in terms of a newly defined modal distance, $r_M$, akin to the previous definition in terms of a physical distance, $r_0$. That is, how far would two modes have to be separated by in mode space, $r_M$, in order to ensure the same gain as a physical separation of $r_0$?

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**Fig. 2.** Experimental setup showing two transmitters and a single receiver for transmit diversity. The insets to the right are holograms for (a) generating an LG$_{12}$ beam, (b) generating an HG$_{22}$ beam, (c) superposition of the aforementioned holograms and (d) example turbulence with a grating with SR=0.7. The grey area on the holograms is due to complex amplitude modulation.

**Fig. 3.** Bit Error Rate of modes with order $N = 4$ with varying turbulence strength showing a clear improvement for the diversity case (solid lines). The vertical line indicates the approximate size of the beams.

**Fig. 4.** Bit Error Rate of modes with order $N = 8$ with varying turbulence strength showing a clear improvement for the diversity case (solid lines).
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Importantly, the measured diversity gain is shown to increase the allowable propagation distance (determined by an acceptable error rate) by a significant margin at both strong and weak turbulence strengths. This finding can be used to significantly improve future FSO communication links in terms of range and reliability.

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| Distance Gain (%) | LG1距离 (mm) | Diversity r0 (mm) | LG1 Distance (km) | Diversity Distance (km) |
|-------------------|---------------|-------------------|-------------------|-------------------------|
| 54                | 16.6          | 12.8              | 0.24              | 0.37                    |
| 71                | 10.2          | 7.4               | 0.55              | 0.93                    |
| 137               | 4.5           | 2.68              | 2.13              | 5.06                    |