Natural Quintessence and the Brane World

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Abstract. Although quintessence models have many attractive cosmological features, they face two major difficulties. First, it has not yet been possible to find one which convincingly realizes the goal of explaining present-day cosmic acceleration generically using only attractor solutions. Second, quintessence has proven difficult to obtain within realistic microscopic theories, largely due to two major obstructions. Both of these difficulties are summarized in this article, together with a recent proposal for circumventing the second of them within a brane-world context. It is shown that this proposal leads to a broader class of dynamics for the quintessence field, in which its couplings slowly run (or: ‘walk’) over cosmological time scales. The walking of the quintessence couplings opens up new possibilities for solving the first problem: that of obtaining acceptable transitions between attractor solutions.

1 Introduction

The discovery by cosmologists that the Universe is currently dominated by two distinct types of unknown forms of matter is a development with truly Copernican implications for our picture of the Universe as a whole. We have known for some time that visible matter likes to cluster on large scales into galaxies and galaxy clusters, with 90% or more of the mass of these objects consisting of an unknown nonbaryonic ‘dark matter’ [1]. The more recent surprise was the discovery that this clustered dark matter itself makes up no more than 30% of the overall energy density of the Universe, with the remaining 70% apparently consisting of a different kind of unknown substance, sometimes called ‘dark energy’ [2].

It is absolutely breathtaking that so little is known about these two most abundant forms of matter. What is known is usually expressed in terms of their equations of state, through the ratio of pressure to energy density, \( w = p/\rho \). The formation of galaxies and galaxy clusters by gravitational attraction appears to require the dark matter to be ‘cold’, with \( w \) close to zero. Similarly, the current acceleration which the universal expansion is undergoing indicates \( w \lesssim -0.3 \) for the dark energy.

Perhaps the simplest explanation for the dark energy is that it is simply the energy density of the quantum vacuum, since this satisfies \( w = -1 \) and is generically nonzero in realistic quantum field theories. The problem with this explanation is that the predicted vacuum energy is typically at least \( 10^{56} \) times larger than what is observed. The Cosmological Constant Problem [3] is the
recognition that at present no way is known to naturally obtain a vacuum energy anywhere near the required size within a realistic microscopic theory.

Quintessence models [4] take a different tack to explain the dark energy. In these models the dark energy is attributed to the dynamics of a scalar field, \( \phi \), which is currently evolving in a cosmologically interesting way. Since \( w = (K - V)/(K + V) \), where \( K = \frac{1}{2} \dot{\phi}^2 \) and \( V \) are the scalar’s kinetic and potential energies, the condition that \( w \) be negative requires the scalar’s evolution at present must be slow in the sense that \( K \ll V \). (The vacuum energy is a special case of this kind of evolution, where \( K = 0 \).)

These models do not solve the Cosmological Constant Problem, since they do not provide natural reasons for \( V \) to be currently so small, but they can (potentially) explain why an evolving scalar field could naturally have an energy density which is now so similar to that of other forms of matter, like the dark matter. They can do so, firstly because their equations of motion often admit ‘tracking’ solutions, within which the scalar energy density closely follows (or tracks) the dominant energy density of the Universe as it evolves. Furthermore, the late-time evolution of the scalar field is often drawn to these ‘tracking’ solutions for wide choices of initial conditions, because these solutions are also ‘attractors’ for the scalar equations of motion.

Unfortunately, since these tracker solutions typically require the scalar itself not to be the dominant energy density, in order to become the dark energy the scalar must eventually leave the tracker solution. Although one might hope that this could also be naturally achieved – such as being perhaps due to a transient behaviour due to the crossover from radiation to matter domination – so far it has proven difficult to make a completely convincing cosmology along these lines.

The purpose of this article is to describe a new category of quintessence model, which could be called ‘Walking Quintessence’ [5,6], that may offer new ways to accomplish this crossover. They may do so because within these models the couplings of the scalar field slowly run (or walk) as the Universe evolves, and this walking may help facilitate the crossover between tracking solutions. What is remarkable is that these models were developed in an attempt to address a completely different set of (very serious) problems which arise once one tries to obtain quintessence from a realistic model of microscopic physics.

Since they play such an important part in the motivation of the models, the bulk of the article is devoted to these serious microphysical problems. The problems themselves are first summarized in the next section, followed by a description how they are addressed within the attractive brane-world picture which has emerged as a potential low-energy consequence of string theory. The results of a sample cosmology built on this model are then briefly presented, intended as first step towards a more systematic exploration of the cosmologies which are suggested by this class of models.
2 Naturalness Issues

From a microscopic perspective any viable quintessence model must have two very remarkable properties, which turn out to be quite difficult to arrange [7]. As is explained below, they must have:

- **Extremely Light Scalars**, whose mass must be at most $m_Q = 10^{-32}$ eV, and;

- **No New Long-Range Forces**, to the extent that these would ruin the agreement between General Relativity and observations on Earth, within the Solar System and beyond [8,9].

2.1 Light Scalars

The first requirement (very light scalars) is a very generic property of explanations of the dark energy in terms of a rolling scalar field. Its necessity may be seen from the scalar field equation of motion within a cosmological context:

$$\ddot{\phi} + 3H \dot{\phi} + \frac{\partial V}{\partial \phi} = 0, \quad \text{and} \quad H^2 = \frac{\rho}{3M_p^2},$$

where $M_p = 10^{18}$ GeV is the rationalized Planck mass, and $\rho$ is the total energy density (which is at present dominated by the dark energy).

To quantify how small this requires the scalar mass to be, it is instructive to consider the very broad class of models within which

$$V(\phi) = \mu^4 U(\phi/f).$$

Here $\mu$ and $f$ are arbitrary mass scales, whose size may be determined if the dimensionless function $U(x)$ and its derivatives are at present $O(1)$. In this case the present value of the scalar potential and its derivatives are $V \sim \mu^4$, $dV/d\phi \sim \mu^4/f$, etc., and the square of the scalar mass is of order $d^2V/d\phi^2 \sim \mu^4/f^2$. This turns out to be of order $(10^{-33}$ eV)$^2$ given the values which are required for $\mu$ and $f$.

The value $\mu \sim 10^{-3}$ eV is inferred by recognizing that these estimates also apply to the total scalar field energy, $\rho = K + V \sim V$, since present observations require the scalar field to be rolling with $K$ less than but of the same order of magnitude as $V$. In this way we see that $\mu$ controls the present dark energy density, $\rho \sim \mu^4 \sim (10^{-3}$ eV)$^4$, and so also $H \sim \mu^2/M_p$ and $\dot{\phi} \sim \sqrt{K} \sim \sqrt{V} \sim \mu^2$.

The value $f = M_p$ is determined from the scalar field equation, eq. (1), together with the above slow-roll conditions, since this implies the $\dot{\phi}$ term should be much smaller than the other two. Except for the case of a pure cosmological constant (for which only the last term is important), we therefore have $H \dot{\phi} \sim \partial V/\partial \phi$ and so $\mu^4/M_p \sim \mu^4/f$, from which we learn $f \sim M_p$ and hence $m_Q \sim \mu^2/f \sim 10^{-33}$ eV.

Such an extraordinarily small scalar mass is extremely difficult to achieve in a realistic microscopic theory. There are two separate aspects to this difficulty.
1. **Heirarchy Problem 1**: How does such a small nonzero mass arise as a combination of microscopic parameters?

2. **Heirarchy Problem 2**: Given that such a small mass is predicted by the theory of microscopic physics, how does it remain small as one integrates out all the physics between these microscopic scales and the cosmological scales at which it is measured? This is a problem because, for instance, a particle of mass $M$ and coupling $1/f$ shifts the scalar mass by an amount

$$\delta m \sim \frac{M^2}{4\pi f}$$

when it is integrated out.

Both of these problems are the direct analogs of two aspects of the famous heirarchy problem as applied to the Higgs field which breaks electroweak gauge symmetry within the Standard Model of particle physics. There the weak scale, $M_w \sim 100$ GeV is controlled by the scalar Higgs mass, and one asks how this can be so much smaller than, say, the Planck scale, $M_p \sim 10^{18}$ GeV.

The problem for quintessence models, however, is arguably much worse for two reasons. First, the quintessence scalar is many more orders of magnitude lighter than the basic microscopic physics scale $M_w$ than $M_w$ itself is from $M_p$. Second, the range of scales between $M_w$ and $m_Q$ is well studied by experiment, which presumably makes it harder to hide the other degrees of freedom which are typically invoked to alleviate Heirarchy Problem 2 (such as the superpartners which do the job if supersymmetry is the solution).

Almost none of the extant quintessence models address these issues, with the exception being those based on pseudo-Goldstone bosons, which address Heirarchy Problem 2. (Modifications of these models based on the brane world can also address Heirarchy Problem 1.) The models presented in the later sections are unique among those yet proposed in that both of these problems are related to the same microscopic quantities which explain why $M_w \ll M_p$.

### 2.2 Long-Range Forces

The incredibly small quintessence scalar mass also raises a related observational problem, since it implies that the exchange of this scalar must mediate a very long-range force. Furthermore, the strength with which this force couples to particles of energy $E$ is typically of order $E/f \sim E/M_p$, which makes it comparable to gravity. This is problematic, since many observations now strongly constrain the existence of gravitational-strength forces between macroscopic objects having a range longer than about 0.1 mm.

In the models which are described in subsequent sections this problem is evaded because the scalar couplings turn out to evolve over cosmological time scales. In the examples given it happens that these couplings evolve to become extremely small during the present epoch, which is when all of the very constraining observations have been made.
Another way to ensure acceptably small couplings is to arrange the scalar to couple to quantities, such as spin, which ordinary matter in bulk does not carry. This kind of coupling can occur in pseudo-Goldstone-boson models [10,12].

3 Large Extra Dimensions

New insights into naturalness problems, like the hierarchy between $M_w$ and $M_p$, have been made based on the recently much-discussed brane world picture, in which all observed elementary particles are confined to a domain-wall-like surface (or ‘brane’) which sits within a higher-dimensional ‘bulk’ spacetime. The simplest choice puts us on a 4-dimensional surface (or 3-brane) within a bulk space which has anywhere from 5 to 11 dimensions. By contrast, gravitational interactions in this picture are not confined to these branes. This kind of picture is very well motivated within string theory.

The fact that gravity and other interactions do not see the same number of dimensions lies at the root of the surprising realization that the fundamental string scale, $M_s$, can be much smaller than $M_p$ [13], and of the new perspective this has allowed for understanding low-energy naturalness problems. In particular, it can imply the relationship

$$\left(\frac{M_s}{M_p}\right)^2 \sim \frac{\alpha^2}{(M_s r)^n},$$

where $\alpha$ is the (open) string coupling and $r$ is the radius of the $n$ extra dimensions. For supersymmetric systems one generically has $M_w/M_p \sim (M_s/M_p)^2$ and so this remarkable formula allows the observed ratio $M_w/M_p \sim 10^{-16}$ (Hierarchy Problem 1, above) to be understood using parameters themselves no smaller than $\alpha \sim 1/(M_s r) \sim 0.01$ if $M_s \sim 10^{11}$ GeV and there are $n = 6$ extra dimensions [14]. (In this picture it is supersymmetry which accounts for Hierarchy Problem 2.)

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The brane-world variant which is of most interest for the present purposes, puts the string scale as low as is consistent with experiment, $M_s \sim M_w$ [15]. From eq. (4), one sees this is permitted (even if $\alpha$ is not small) provided $r$ is large enough. For instance, if $n = 2$ then $r \sim 0.1$ mm (or $1/r \sim 10^{-3}$ eV). In this picture $M_w/M_p \sim \alpha/(M_s r)$, so the hierarchy problem is not solved so much as translated into the problem of understanding the origin of the large hierarchy $M_s r/\alpha \sim 10^{16}$.

A remarkable feature of this scenario is that it provides a framework within which scalars can be naturally as light as $10^{-33}$ eV, and it is instructive to see how this works for specific examples. The success of the models of ref. [15] relies on choosing the extra dimensions to be torii and the quintessence field to be a component of the extra-dimensional metric – the radion, $r$. 
This construction directly solves Hierarchy Problems 1 as follows. First, because the radion field starts life as a component of the six-dimensional metric its kinetic term has the same origin as does the 4D graviton. Dimensionally reducing the 6D Einstein-Hilbert action to four dimensions gives

$$L_{\text{kin}} \sqrt{-g} = - \frac{M_p^2}{2} g^{\mu\nu} \left[ R_{\mu\nu} + \frac{4}{r^2} \partial_\mu r \partial_\nu r \right],$$

from which we see the canonically-normalized field is \( \phi \) where \( r = r_0 \exp(\phi/2M_p) \).

For toroidal compactifications, direct dimensional reduction of the higher-dimensional Einstein-Hilbert action gives no radion potential at all (provided that the cosmological constant is chosen to vanish, as usual). This is an artifact of the classical approximation, however, and a potential for \( r \) is generated once quantum effects are included, such as through the Casimir effect which predicts (for large \( r \)) a potential of the form \( V \sim 1/r^4 \). If this potential can be stabilized to have a minimum for some \( r = r_0 \) (more about this later), then it predicts \( \mu \sim 1/r_0 \).

More remarkably, this model also addresses Hierarchy Problem 2, since these predictions are protected from being ruined as physics between the weak scale and the quintessence mass is integrated out. Although the stability of the prediction for \( f \) against quantum corrections is fairly trivial, it is the stability of the potential which bears closer examination.

There turn out to be two reasons for this success. The Casimir effect predicts \( \mu \sim 1/r \) largely because the potential is generated when modes having energies of order \( 1/r \) are integrated out. Now consider the effect of integrating out modes with energies lower than \( 1/r \). For scales \( M \lesssim 1/r \) the effective theory is four-dimensional, and the naive corrections to \( V(r) \) are correct, since no symmetries preclude generating a radion potential. Integrating out a mode of mass \( M \) then contributes to \( V(r) \) terms of order \( \delta \mu^4 \sim M^4/(4\pi)^2 \), which is not dangerous since it predicts \( \delta \mu \sim M/\sqrt{4\pi} \lesssim 1/r \).

The key is at energies above the scale \( 1/r \), where the effective theory is six-dimensional and so is constrained by additional symmetries like 6D general covariance. For these scales the result of integrating out modes with \( M \gg 1/r \) may be expanded in powers of the 6D curvature \( R_{mnpq} \). If the extra dimensions were to have spherical geometry then \( R_{mnpq} \propto 1/r^2 \), and these curvature terms constitute dangerously large contributions to the radion potential. For flat spaces like torii, however, \( R_{mnpq} \) is independent of \( r \) and these terms are not dangerous.

The question for torii becomes whether it is possible to keep the internal dimensions flat despite the existence of large vacuum energies on the various branes on which our observed particles live. It is a special feature of two dimensions that this is so, since Einstein’s equations predict flat geometries around point sources.

We see that in this picture the quintessence scales \( \mu \) and \( f \) are predicted to be respectively given by \( 1/r \) and \( M_p \) because the quintessence field has its microscopic origin as part of the higher-dimensional geometry. The success of
the predicted dark energy density, \( V \sim 1/r^4 \), and quintessence mass, \( m_Q \sim 1/(M_p r^2) \), then follow from the choice \( 1/r \sim 10^{-3} \) eV which is required in any case to solve the electroweak hierarchy, since \( M_w/M_p \sim 1/(M_w r) \). This success does not crucially hinge on identifying the quintessence field as a mode of the metric, since variant brane-world models with similar properties may also be built wherein the quintessence field is a pseudo-Goldstone mode in the bulk \([12]\).

4 Radius Stabilization and ‘Walking’ Quintessence

In order to more precisely pin down the cosmology the explicit form for the quintessence potential is required, and within the brane world picture being presented this amounts to providing a mechanism for stabilizing the radion at large values. This section relates a concrete proposal for doing so, as made in ref. \([5]\).

Besides providing an interesting cosmology in its own right, there is another reason for exploring this specific proposal in more detail. This other reason is to illustrate why worrying about naturalness issues is useful when exploring phenomenological applications (such as quintessence cosmology). Naturalness issues are useful precisely because the cosmology of quintessence models depends in such a detailed way on the precise form of the low-energy potential. On one hand, the naturalness problem states that only very specific kinds of potentials are likely to arise as the low-energy limit of realistic microphysics. On the other hand, it has proven difficult to build a completely convincing quintessence cosmology purely by guessing different kinds of potentials. It may be true that it is only the very few potentials which can arise from real microphysics that can also provide a realistic description of cosmology as well. If so, it should be invaluable to be able to explore in detail any potential which does arise as the low-energy limit of real microphysics.

In the present instance the stabilization mechanism proposed suggests a qualitative new feature of the low-energy theory: it predicts that the effective low-energy couplings and masses depend logarithmically on the quintessence field, and so these all slowly run (i.e. walk) over cosmological time scales. This leads in a natural way to extended quintessence models \([18]\), but with a specific and cosmologically-interesting type of field-dependence. Indeed the proposal of ref. \([5]\) was initially motivated by the earlier discovery of the attractive cosmologies which can result from quintessence models having exponential potentials with logarithmic corrections \([19]\).

4.1 Stabilization via Six-Dimensional Logarithms

The models now described are based on the observation that \( V(r) \) would naturally be minimized at large values for \( r \) if it were to depend logarithmically on \( r \):

\[
V(r) = V_0 \left( \frac{r}{\ell} \right)^p \left[ 1 + \epsilon \log \left( \frac{r}{\ell} \right) \right] + \ldots ,
\]

(6)
where $a$ and $\epsilon$ are constants and $\ell$ is a microscopic length scale, such as $\ell \sim 1/M_s$. The ellipses indicate other terms which fall off with a higher power of $\ell/r$ relative to those shown. A potential of this form has a minimum at $r_0 \sim \ell \exp(1/\epsilon)$, which is exponentially larger than $\ell$ if $\epsilon$ is moderately smaller than one. (Values $\epsilon \sim 1/50$ are sufficient to generate the desired hierarchy in the models explored in [5].)

The key point is that a potential of precisely this form is expected if the effective six-dimensional theory at scales $E > 1/r$ contains a renormalizable (i.e. marginal) coupling, $g$. If so, then this coupling runs logarithmically with $r$ and loop corrections to the radion potential involving only this coupling have the logarithmic form of eq. (6), with a coefficient $\epsilon \sim g^2/(4\pi)^3$ which is naturally small. In six dimensions most interactions are not renormalizable, but such couplings can exist, such as a cubic coupling amongst six-dimensional scalar fields.

Given this generic mechanism it is clear that logarithms in $r$ are not restricted to appear in the low-energy theory only within the radion potential. Rather, they arise generically as radiative corrections to all couplings, and it is this logarithmic dependence which is responsible for the walking of these couplings as $r$ evolves over cosmological time scales.

### 4.2 A Sample Quintessence Cosmology

Logarithmically evolving couplings are very attractive for quintessence cosmology for two reasons. First, they introduce non-minimal couplings between the quintessence field and ordinary matter, and these turn out to provide new kinds of scaling attractor solutions. Which attractor is the endpoint for a given choice of initial conditions is determined by the parameters of the quintessence potential, and this observation is at the root of the second attractive cosmological feature of having walking couplings. A given cosmological solution can cross over from the basin of attraction of one attractor to another since the walking of the couplings moves the boundaries of these domains of attraction within field space.

Ref. [6] provides a preliminary exploration of the cosmology of models built along these lines. An example of a viable cosmology which is obtained in this way is illustrated in Figure 1, which shows the cosmological evolution of the energy density in radiation, matter and the radion kinetic and potential energies. Figure 2 shows, for the same model, the coupling, $\eta$, of the radion to ordinary matter as a function of the universal scale factor, from which it is clear that the model evades current constraints on long-range forces (which require $\eta < 0.03$).

A constraint which is specific to models where it is the radion which is the quintessence field is the requirement that the radion not roll too far since the epoch of nucleosynthesis, since such a roll would predict an unacceptably large change in the ratio of weak and gravitational couplings, $M_w/M_p \sim 1/(M_s r)$, between then and now. Figure 3 shows how $M_s r$ evolves for the cosmology shown in Figure 1, and so demonstrates that its evolution can be acceptably small.

Although a promising start, this cosmology still leaves much to be desired. In particular, the constancy of $r$ is not generic and must be partially arranged by adjusting the parameters of the potential and the initial energy in the scalar
However the broader framework of walking quintessence models seems to provide a fruitful place to search for a quintessence model which is truly natural in all senses of the word.

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Fig. 1. The logarithm of the energy density of radiation (solid blue), matter (solid red), scalar kinetic (green dashed) and radion potential (cyan dashed), plotted against the logarithm of the universal scale factor (normalized to unity at present).

Fig. 2. The quintessence-matter coupling, $\eta$, plotted against the logarithm of the scale factor for the same cosmology as Figure 1. The observational constraint is $\eta < 0.03$ during the present epoch.
Fig. 3. The logarithm of the ratio $M_p/M_w \sim M_\pi r$ against scale factor for the same cosmology as for Figure 1. This ratio must be within roughly 10% of its present value $10^{16}$ at nucleosynthesis ($a \sim 10^{10}$).