Anomalous Transport in a Superfluid Fluctuation Regime

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Motivated by a recent experiment in ultracold atoms [S. Krinner et al., Proc. Natl. Acad. Sci. U.S.A. 113, 8144 (2016)], we analyze transport of attractively interacting fermions through a one-dimensional wire near the superfluid transition. We show that in a ballistic regime where the conductance is quantized, the absence of interaction, the conductance is renormalized by superfluid fluctuations in reservoirs. In particular, the particle conductance is strongly enhanced and the plateau is blurred by emergent bosonic pair transport. For spin transport, in addition to the contact resistance the wire itself is resistive, leading to a suppression of the measured spin conductance. Our results are qualitatively consistent with the experimental observations.

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Transport measurements often play crucial roles in revealing the fundamental nature of matter. In condensed matter physics, superconductivity, the Kondo effect, and the quantum Hall effect were all discovered with transport measurements. A two-terminal setup realized in ultracold atoms has opened up yet another avenue to explore strongly correlated systems through transport [1–7]. In ultracold atoms, quantum transport typically occurs in the clean limit. The bulk conductivity cannot distinguish between different quantum states, since the f-sum rule and the momentum conservation dictate that the conductivity involves the delta-function singularity at zero frequency whose weight does not depend on detailed states of matter [8]. However, transport through a constriction such as a quantum point contact allows one to distinguish between different states due to the breakdown of the momentum conservation at the constriction. Indeed, such a setup has unveiled different transport properties for non-interacting [5] and superfluid fermions [6].

Recently, particle conductance and spin conductance have been measured with a quantum point contact in ultracold fermions [7]. There, with increasing attractive interaction, both of them deviate significantly from quantized values [9] just above the superfluid critical temperature. More specifically, compared with the non-interacting limit, the particle conductance is enhanced, whereas the spin conductance is suppressed. These remarkable features stand in sharp contrast with the conventional wisdom that a conductance is not renormalized by an interaction in a one-dimensional wire [10,11]. Meanwhile, different from condensed matter situations, fluctuation and interaction effects in reservoirs may be significant in cold atom experiments.

On another front, to cope with effects of an interaction at reservoirs in ballistic transport presents a theoretical challenge, since Landauer’s approach does not operate with an interaction, and phenomenological approaches [11,15] used to explain the interaction effect in a one-dimensional wire cannot directly answer the question. When the conductance of the wire is small, a tunneling Hamiltonian approach is widely used to investigate the effect of interactions in reservoirs on transport [16,17,19]. However, to discuss the ballistic limit realized in Ref. [7], we must go beyond the linear response theory which has been widely used in tunneling experiments with correlated materials [19].

In this Letter, motivated by the ETH experiment [7] and the theoretical challenge mentioned above, we examine the effects of superfluid fluctuations in reservoirs on transport through a one-dimensional wire. To deal with ballistic transport, we apply the nonlinear response theory [21,25] to demonstrate that the breakdown of the quantization of conductance occurs by superfluid fluctuations. We show that transport of preformed pairs induced by superfluid fluctuations is essential to account for the breakdown. We also point out that in addition to the contact resistance, the resistance in the one-dimensional wire plays an important role in spin transport.

The Model.—We consider a system where two macroscopic reservoirs with superfluid fluctuations are connected by a quantum point contact (a one-dimensional channel). In the ETH experiment, the constriction has potential variations that take place over length scales larger than $1/k_F$ with the Fermi momentum $k_F$. This implies that when transport near the Fermi energy is concerned, the adiabatic approximation is justified in which the detailed shape in the constriction is irrelevant [9,20]. Thus, for the single channel case, we can start with the following Hamiltonian ($\hbar = k_B = 1$):

$$H = \sum_{j=L,R} \sum_{\sigma=\uparrow,\downarrow} \sum_{p} g c_{j,p,\sigma}^\dagger c_{j,p,\sigma} + V_j \right) + H_T, \quad (1)$$

$$V_j = -g \sum_{p,p',q} c_{j,p+q,\uparrow}^\dagger c_{j,-p',\downarrow}^\dagger c_{j,-p',\downarrow} c_{j,p+q,\uparrow}, \quad (2)$$

$$H_T = \int dxdy \sum_{\sigma} J(x,y) \psi_{L,\sigma}(x) \psi_{R,\sigma}(y) + h.c., \quad (3)$$

where $c_{j,p,\sigma}$ ($c_{j,p,\sigma}^\dagger$) is the fermionic annihilation (creation) operator with momentum $p$ and spin $\sigma$ in reservoir $j$, and the energy $\xi_{j,p,\sigma} = p^2/2m - \mu_{j,\sigma}$ is measured.
ETH experiment \([7]\). Below, we focus on the BCS regime
the system with a broad-Feshbach resonance used in the
single-particle Hamiltonian and the interaction with an
and second terms on the right-hand side of Eq. (1) are the
\(\mu\) from the chemical potential
fluctuation pair tunneling. (d) Higher-order diagram of the
operator in the real space with its argument
on the right with a renormalized tunneling amplitude \(\tilde{t}\)
can be replaced by the direct pair exchange diagram shown
inside the circle with a cross mark). (d) Higher-order diagram of the
fluctuation pair tunneling. Each circle with a cross mark represents the tunneling amplitude \(t\). This process can be replaced by the direct pair exchange diagram shown on the right with a renormalized tunneling amplitude \(t\) (double circle with a cross mark). (d) Higher-order diagram of the
fluctuation pair tunneling.

from the chemical potential \(\mu_{\sigma}\). In addition, \(\psi_{i,\sigma}\) is the
operator in the real space with its argument \(x\) (y) representing a position in the left (right) reservoir. The first and second terms on the right-hand side of Eq. (1) are the single-particle Hamiltonian and the interaction with an attractive coupling \(-g\) \((g > 0)\), respectively, and describe the system with a broad-Feshbach resonance used in the ETH experiment \([7]\). Below, we focus on the BCS regime
\((1/k_{F}a) < 0\) with the s-wave scattering length \(a\) \([26–29]\) above the superfluid transition temperature \(T_{c}\). To discuss the case of a single conducting channel, we set \(t(x, y) = \delta(x - x_{0})\delta(y - y_{0})\) \([26]\) \([30]\) \([31]\), where near the Fermi energy the tunneling amplitude \(t\) can be chosen to be a real constant without loss of generality, and \(x_{0} (y_{0})\) is the entrance (exit) point in the quantum point contact. In fact, this tunneling Hamiltonian can precisely reproduce the known universal conduction properties in the quantum point contact including ballistic transport with superfluid reservoirs \([6]\) \([30]\).

In terms of Eq. (3), the mass and spin current operators are given by
\[ I_{\text{mass}} = -\sum_{x, \sigma} \bar{N}_{x, \sigma} = -\sum_{x, \sigma} i[H_{T}, N_{x, \sigma}], \quad I_{\text{spin}} = -i[H_{T}, N_{x, \uparrow} + i[H_{T}, N_{x, \downarrow}], \quad \text{where} \quad N_{x, \sigma} = \int dx \psi_{x, \sigma} \bar{\psi}_{x, \sigma}(x) \text{is the number operator with spin} \ \sigma \ \text{in the reservoir} \ \text{L}. \quad \text{We note that the number operator in each reservoir commutes with the Hamiltonian except for} \ H_{T}. \quad \text{In the presence of a chemical-potential difference between the reservoirs,} \ \nu = \mu_{\uparrow} - \mu_{\downarrow} = \mu_{L, \uparrow} - \mu_{R, \uparrow} \neq 0 \ (\nu = \mu_{\downarrow} - \mu_{\uparrow} = -\mu_{L, \downarrow} + \mu_{R, \downarrow}), \text{the mass (spin) current is induced. Then,}\]

the averages of the mass and spin currents at time \(\tau\) are given by
\[
I_{\text{mass/spin}}(x_{0}, y_{0}, \tau) = 2\text{Im}\{e^{-i\nu \tau}(A_{\tau}(x_{0}, y_{0}, \tau))_{H} + e^{i\nu \tau}(A_{\tau}(x_{0}, y_{0}, \tau))_{H}\}, \quad (4)
\]
where \(A_{\tau}(x_{0}, y_{0}, \tau) = t\psi_{x_{0}, \sigma}^{\dagger}(y_{0}, \tau)\psi_{x_{0}, \sigma}(x_{0}, \tau), \text{and} \langle \cdot \cdot \rangle_{H} \text{means the thermal average for the Hamiltonian} (1).

In the presence of superfluid fluctuations, we should consider contributions arising from fermionic quasiparticles and fluctuation pairs \([17]\). Below, such fluctuations are considered up to the Gaussian level in each propagator, which is reasonable in a regime \(10^{-3} \lesssim (T - T_{c})/T_{c} \lesssim 10^{-1}\) for the case of three-dimensional reservoirs \([17]\) relevant to the ETH experiment.

**Fermionic quasiparticle current.** We now examine a steady current induced by fermionic quasiparticles. By the assumption of the steady state, we can put \(\tau = 0\) without loss of generality. Then, the mass and spin currents can be expressed as
\[
I_{\text{mass/spin}} = \frac{\nu}{2\pi} \int d\omega \text{Re}[G_{x_{0}, y_{0}, \omega}^{R}] = G_{x_{0}, y_{0}, \omega}^{R}, \quad G_{x_{0}, y_{0}, \omega}^{R} = -i \int d\tau e^{i\nu \tau} \langle \psi_{x_{0}, \sigma}^{\dagger}(x_{0}, \tau)\psi_{x_{0}, \sigma}(y_{0}, 0) \rangle_{H}
\]
the Keldysh Green’s function \([25]\), and we use
\[
\int d\omega G_{x_{0}, y_{0}, \omega}^{R} = 0 \quad \text{for the retarded Green’s function} \ G_{x_{0}, y_{0}, \omega}^{R} \ [25] \ [31].
\]

As shown in Fig. 1(b), the single-particle Green’s function is renormalized by the fluctuation pair propagator (Fig. 1(a)) up to the Gaussian level. As pointed out in Ref. \([30]\), the effect of \(t\) must be incorporated to all orders in the ballistic limit. By using an analysis similar to the case of noninteracting fermions \([22]\) \([30]\) \([32]\), the fermionic quasiparticle contribution to the conductance per spin is obtained as \([26]\)
\[
G_{\eta} \approx \frac{1}{\hbar} \int_{-\infty}^{\infty} d\omega \frac{4\pi^{2}t^{2} p^{2}(\omega)}{[1 + \pi^{2}t^{2} p^{2}(\omega)]^{2}} \left( -\frac{\partial n_{F}(\omega)}{\partial \omega} \right), \quad (5)
\]
where \(n_{F}(\omega) = (e^{\omega/\nu} + 1)^{-1}\) is the Fermi distribution function at temperature \(T, g(R)(\omega) = \sum_{p} g(p, \omega)\) with the retarded Green’s function \(g^{R}(p, \omega)\) in the reservoirs, and \(\rho(\omega) = -\text{Im}[g^{R}(\omega)]/\pi\) is the density of state (DOS). We note that the conductance depends neither on \(x_{0}\) nor \(y_{0}\) \([25]\). To obtain the above result, an expansion up to linear order in \(V\) is considered, since \(V/\mu_{L(R)} \lesssim 0.1\) and no significant deviation from the linear order is found in the ETH experiment. We note that the same expression holds for the mass and spin currents. In the case of small transmittance where \(1 + \pi^{2}t^{2} p^{2}(\omega)^{2} \approx 1\) in the denominator, Eq. (5) essentially reduces to the Ambegaokar-Baratoff formula \([17]\) \([33]\). On the other hand, in the absence of the fluctuations, the conductance is reduced to \(G_{\eta} = \frac{T}{\hbar}\), and is equivalent to Landauer’s formula with transmittance \(T_{0} = 4\pi^{2}t^{2} p_{0}^{2}(0) / (1 + \pi^{2}t^{2} p_{0}^{2}(0))^{2}\), where \(p_{0}\) is the DOS for noninteracting fermions and we use the fact that the change of \(p_{0}\) around the Fermi level
is much smaller than that of $\frac{\partial n}{\partial T}$ \cite{22,30,32}. In this limit, the quantized conductance, $1/h$, is obtained for $t = 1/(\pi \rho_0(0))$.

The superfluid fluctuations renormalize the conductance of fermionic quasiparticles and generate that of preformed pairs. The fermionic quasiparticle conductance, in general, tends to be suppressed due to pseudogap effect \cite{34,35}; however, in the case of three-dimensional reservoirs, such suppression is found to be negligible in the experimentally relevant regime ($T - T_c)/T_c \sim 10^{-1}$ \cite{26}.

Fluctuation pair current.—We now consider a current carried by the fluctuating (preformed) pairs that makes a dominant contribution to the conductivity in dirty superconductors \cite{17}. As shown on the left of Fig. 1 (c), the lowest-order diagram already contains a factor $t^4$. In usual tunneling experiments where $\pi t \rho_0(0) \ll 1$ \cite{10,20,40}, this contribution is negligible compared with the fermionic quasiparticle current, and has not been considered for realistic situations. However, in the ballistic regime in which $\pi t \rho_0(0) \approx 1$, one needs to consider it seriously. To evaluate the fourth-order diagram, we calculate the third-order response function, which is related to an imaginary-time correlation function through analytic continuation \cite{10,21}. Up to linear order in $V$, the fluctuation pair current at the fourth order in $t$ behaves as $I_p^F(t) \approx t^4(2V)/(T - T_c)$. Here, the factor $2V$ originates from the pair exchange between the reservoirs, and the factor $1/(T - T_c)$ reflects the superfluid fluctuations. We note that as in the case of spin conductivity \cite{38}, the fluctuation pair contribution in the spin current vanishes. This reflects the fact that the pair exchange is not caused by a spin bias. Thus, the enhancement of the current by fluctuation pairs only occurs for mass transport.

We also note that the left-hand side of Fig. 1 (c) can be replaced by the right-hand side of Fig. 1 (c) up to linear order in $V$. Namely, the fluctuation pair contribution can be expressed in terms of an effective hopping amplitude $t = (\pi t \rho_0(0))^2/(2T)$ \cite{26} and the retarded pair-fluctuation propagator whose expression in the vicinity of $T_c$ is given by \cite{17}

$$L^R(q, \omega) = \frac{8T}{\pi \rho_0(0)} \frac{1}{\omega - (\tau^{-1}_{GL} + \frac{2T \xi^2}{\pi^2 q^2})},$$

where $\tau^{-1}_{GL} = 8(T - T_c)/\pi$ and $\xi^2 = 7\zeta(3)v_F^2/(16d\pi^2 T^2)$ with the Fermi velocity $v_F$ and the dimension of the system $d$. Thus, this contribution can be calculated as tunneling of the preformed pairs for a given effective hopping amplitude and bias $2V$, and therefore the multiple tunneling processes of the preformed pairs represented by power series in $t$ can be systematically evaluated as depicted in Fig. 1 (d). By using the nonlinear response theory, we obtain the conductance contributed from the fluctuation pair per spin up to linear order in $V$ as \cite{26}

$$G_p \approx \frac{1}{\hbar} \int_{-\infty}^{\infty} \frac{d\omega}{\sinh^2(\frac{\omega}{2T})} \frac{2T^2}{\pi^2} \left\{1 - \frac{2}{\pi T} \left(\sum_q \text{Im}[L^R(q, \omega)]\right)^2\right\}.$$

We note that the above formula indeed reflects bosonic transport, since the term $1/\sinh^2(\omega/(2T))$ in the integrand is the derivative of the Bose distribution function with respect to $\omega$ (note that such a term is absent in Eq. (5)). Thus, fluctuation pairs make a positive contribution to the mass conductance. Such a contribution is already visible in the regime $(T - T_c)/T_c \sim 10^{-1}$ \cite{26}.

Conductances in the single-channel regime.—We now compare our theory with the ETH experiment in the ballistic single-channel regime. To this end, one may also consider an interaction effect inside the wire.

For mass transport, the mass current operator commutes with the bulk Hamiltonian containing an interaction in the wire \cite{8,20} (except, of course, for the tunneling term), the wire resistance is expected to be negligible, and the conductance calculation obtained above is directly applicable to the ETH experiment. An essential input parameter in the theory is the ratio $T/T_F$ with the Fermi temperature $T_F$, which is extracted from the experiment \cite{7,26}. By assigning the ballistic limit

![FIG. 2. Particle conductance $G_{mass}$ (a) and the spin conductance $G_{spin}$ (b) as a function of $T/T_c$ in the single-mode regime. Circles with error bars and solid curves represent the ETH experimental data \cite{7} and our theoretical calculations, respectively. The blue and red colors show the experimental results with $T/T_F = 0.075$ and 0.1, respectively. In (b) the wire resistance is estimated so as to be compatible with the experiment by the relation $R_s/h \sim e^{2}\pi^2/T$.](image-url)
$\pi \rho_0(0) = 1$, we compare the theory with the experiment and find excellent agreement as shown in Fig. 2 (a). A crucial point here is that the conductance is enhanced due to the bosonic fluctuation-pair contribution. Since our theory is based on an expansion from $T_c$, some deviation is expected at $T/T_c \gtrsim 2$.

For spin transport, the spin current operator and the Hamiltonian do not commute even in the absence of the tunneling term, giving rise to the wire resistance [Fig. 11]. In the presence of an attractive interaction, a spin gap, $\Delta_s$, shows up. A typical estimation suggests $10\text{nK} \lesssim \Delta_s \lesssim 50\text{mK}$, where the lower bound is estimated with the Yang-Gaudin model at the density $n \sim 10^6/m$ and the upper bound is determined from the binding energy of the confinement-induced resonance $\sim 0.6\hbar\omega_\perp$, where $\omega_\perp$ is the transverse confinement frequency [42, 43]. In Fig. 2 (b), we show the spin conductance $G_{\text{spin}}$ whose resistance is the sum of the contact resistance and wire resistance. The wire resistance is estimated so as to be compatible with the experiment by assuming $R_w/h \sim e^{\Delta_s/T}$ [44]. Our result shows that the wire resistance for spin transport is of the order of the contact resistance, implying that even in the ballistic limit a nonnegligible chemical potential drop occurs inside the wire due to the interaction between $\uparrow$ and $\downarrow$ spin components.

We also comment on an effect of the spin gap near the contacts. The spin gap in the wire originates from the strong nesting effect allowed in a one-dimension system [44]. On the other hand, near the contact, multiple channels that render the spin gap smeared out through the dimensional crossover are present. Thus, the effect of the spin gap near the contacts is expected to make negligible contributions to the contact resistance.

**Effects of the gate potential, trapping, and interaction on particle conductance**—Now, we discuss how the particle conductance is affected by the gate potential, trap potential, and interaction. Since the gate and trap potentials shift the energy levels of the conducting channels, these effects can be incorporated by introducing multiple tunneling amplitudes, each of which depends on the gate and trap potentials [7, 26, 45, 46]. The tunneling amplitudes as the input parameters are determined so as to reproduce the weakest-interaction data in the experiment based on Landauer’s formula for noninteracting fermions, since there, $(T - T_c)/T_c > 1$ and the superfluid fluctuations are expected to be minuscule [26]. On the other hand, the interaction strength $1/(k_F a)$ is directly related to how close the system is to $T_c$, since increasing $1/(k_F a)$ towards the unitarity leads to an enhancement of $T_c$ [7, 26].

Figure 3 compares the results of our theory with the ETH experiments for different interaction strengths $(T/T_F = 0.1$ in Fig. 3 (a) and $T/T_F = 0.075$ in Fig. 3 (b) [7]). For the weakest interaction $1/(k_F a) = -2.1$, the grey curves are obtained by fittings of the experimental data by assuming Landauer’s formula. For stronger interaction strengths, where $(T - T_c)/T_c < 1$, we use our theory by incorporating the superfluid fluctuations to calculate the particle conductance by assigning tunneling amplitudes estimated from the data at $1/(k_F a) = -2.1$. As shown in Fig. 3, the particle conductance is enhanced and deviates from Landauer’s formula, which is consistent with the experiment [7]. As in the case of the single-channel regime, enhancement is caused by the preformed pairs. The discrepancy between the theory and the experiment occurring at the larger gate potential in Fig. 3 (b) may be due to an effect of the higher transverse channels that are not treated in the theory.

**Summary**—We have shown that superfluid fluctuations cause two competing effects in two-terminal transport through a quantum point contact: suppression of the conductance of fermionic quasiparticles and enhancement due to bosonic preformed pairs. The former is negligible in the ETH experiment, since the depletion of the DOS near the Fermi level is negligible. The latter in the ballistic regime is shown to be significant due to the absence of the Pauli exclusion principle. Thus, the net conductance can exceed the upper bound of the quantized conduc-
tance $1/h$ for noninteracting fermions through multiple tunneling processes that are captured with the nonlinear response theory. Such transport is ideally realized in an impurity-free system with perfect transmittance such as cold atoms and high-mobility semiconductors in which diffusive properties in a one-dimensional wire, which tends to suppress the bosonic transport, can be ignored. We have also shown that spin transport is affected by the wire resistance originating from the spin gap whose determination with no ambiguity requires the more precise knowledge of the particle density in the wire.

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Note added.—Recently, we became aware of works by M. Kanász-Nagy et al. [47] and B. Liu et al. [48], both of which discuss the anomalous conductances from different perspectives.

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