Homodyne detection and parametric down-conversion: a classical approach applied to proposed “loophole-free” Bell tests

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A classical model is presented for the features of parametric down-conversion and homodyne detection relevant to recent proposed “loophole-free” Bell tests. The Bell tests themselves are uncontroversial: there are no obvious loopholes that might cause bias and hence, if the world does, after all, obey local realism, no violation of a Bell inequality will be observed. Interest centres around the question of whether or not the proposed criterion for “non-classical” light is valid. If it is not, then the experiments will fail in their initial concept, since both quantum theorists and local realists will agree that we are seeing a purely classical effect. The Bell test, though, is not the only criterion by which the quantum-mechanical and local realist models can be judged. If the experiments are extended by including a range of parameter values and by analysing, in addition to the proposed digitised voltage differences, the raw voltages, the models can be compared in their overall performance and plausibility.

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I. INTRODUCTION

No test of Bell’s inequalities [1, 2] to date has been free of “loopholes”. This means that, despite the high levels of statistical significance frequently achieved, violations of the inequalities could be the effects of experimental bias of one kind or another, not evidence for the presence of quantum entanglement. Recent proposed experiments by García-Patrón Sánchez et al. and Nha and Carmichael [3, 4] show promise of being genuinely free from such problems. If the world in fact obeys local realism, they should not, therefore, infringe any Bell inequality.

The current article presents a classical (local realist) model that should be able, once all relevant details are known, to explain the results. It uses standard classical theory for homodyne detection, but for the behaviour of the “Optical Parametric Amplification” (“Parametric Down-Conversion”) source introduces new ideas.

As far as the “loophole-free” status of the proposed experiments is concerned, there would appear to be no problem. A difficulty that seems likely to arise, though, is that theorists may not agree that the test beams used were in fact “non-classical”, so the failure to infringe a Bell inequality will not in itself be interpreted as showing a failure of quantum mechanics [5]. The criterion to be used to establish the non-classical nature of the light is the observation of negative values of the Wigner density, and there is reason to think that, even if the standard method of estimation seems to show that these are achieved, this may not in fact be so.

In the quantum mechanical theory discussed in the proposals and in other recent papers [6, 7], the presence of negative Wigner densities is seen almost as a corollary of the observation of double peaks in the distribution of the phase-averaged voltage differences recorded by the homodyne detectors. Such double peaks, however, can readily be shown to occur naturally, under purely classical assumptions, so long as noise levels are low. They are a consequence of the fact that a sine function spends proportionately more time near the extremes than around the central, zero, value. It would seem that the theory that leads from their observation to the deduction of non-classicality may be in error. This, though, is a problem that is of no direct relevance to the classical model under consideration. Far from being, as suggested by Sánchez and others, the “hidden variable” needed, Wigner density plays no part whatsoever.

Regardless of the outcome of the Bell tests, and whether or not the light is declared to be non-classical, there are features of the experiments that can usefully
be exploited to compare the strengths of quantum mechanics versus (updated) classical theory as viable models. The two theories approach the situation from very different angles. Classical theory traces the causal relationships between phenomena, starting with the simplest assumption and building in random factors later where necessary. Quantum mechanics starts with models of complete ensembles, all random factors included. This, it is argued, is inappropriate, since two features of the proposed experiments demand that we consider the behaviour of individual events, not whole ensembles: the process of homodyne detection itself, and the Bell test.

II. THE PROPOSED EXPERIMENTS

The experimental set-up proposed by Sánchez et al. is shown in Fig. 1, that of Nha and Carmichael being similar. In the words of the Sánchez et al. proposal:

The source (controlled by Sophie) is based on a master laser beam, which is converted into second harmonic in a nonlinear crystal (SHG). After spectral filtering (F), the second harmonic beam pumps an optical parametric amplifier (OPA) which generates two-mode squeezed vacuum in modes A and B. Single photons are conditionally subtracted from modes A and B with the use of the beam splitters BS_A and BS_B and single-photon detectors PD_A and PD_B. Alice (Bob) measures a quadrature of mode A (B) using a balanced homodyne detector that consists of a balanced beam splitter BS_3 (BS_4) and a pair of highly-efficient photodiodes. The local oscillators LO_A and LO_B are extracted from the laser beam by means of two additional beam splitters BS_1 and BS_2.

The classical description, working from the same figure, is just a little different. Quantum theoretical terms such as “squeezed vacuum” and are not used since they are not appropriate to the model and would cause confusion. The description might run as follows:

The master laser beam (which is, incidentally, pulsed) is frequency-doubled in the crystal SHG. After filtering to remove the original frequency, the beam is used to pump the crystal OPA, which outputs pairs of classical wave pulses at half the input frequency, i.e. at the original laser frequency. The selection of pairs for analysis is done by splitting each output at an unbalanced beamsplitter (BS_A or BS_B), the smaller parts going to sensitive detectors PD_A or PD_B. Only if there are detections at both PD_A and PD_B is the corresponding homodyne detection included in the analysis. The larger parts proceed to balanced homodyne detectors, i.e. ones in which the intensities of local oscillator and test inputs are approximately equal. The source of the local oscillators LO_A and LO_B is the same laser that stimulated, after frequency doubling, the production of the test beams.

III. HOMODYNE DETECTION

In (balanced) homodyne detection, the test beam is mixed at a beamsplitter with a local oscillator beam of the same frequency and the two outputs sent to photodetectors that produce voltage readings for every input pulse. In the proposed “loophole-free” Bell test the difference between the two voltages will be converted into a digital signal by counting all positive values as +1, all negative as –1.

![Fig. 2: Inputs and outputs at the beamsplitter in a homodyne detector. E_L is the local oscillator beam, E the test beam, E_r and E_t the transmitted and reflected beams respectively.](image)

A. Classical derivation of the predicted voltage difference

Assume the test and local oscillator signals have the same frequency, \( \omega \), the time-dependent part of the test signal being modelled by \( e^{i\omega t} \), where (ignoring a constant phase offset \( \phi = \omega t \) is the phase angle, and the local oscillator phase and test beam phases differ by \( \theta \). [Note that although complex notation is used here, only the real part has meaning: this is an ordinary wave equation, not a quantum-mechanical "wave function". To allay any doubts on this score, the derivation is partially repeated with no complex notation in the Appendix.]
Let the electric fields of the test signal, local oscillator and reflected and transmitted signals from the beamsplitter have amplitudes $E$, $E_L$, $E_r$ and $E_t$ respectively, as shown in Fig. 2. Then, after allowance for phase delays of $\pi/2$ at each reflection and assuming no losses, we have

$$E_r = \frac{1}{\sqrt{2}}(Ee^{i(\phi+\pi/2)} + E_Le^{i(\phi+\theta)}) \quad (1)$$

and

$$E_t = \frac{1}{\sqrt{2}}(Ee^{i\phi} + E_Le^{i(\phi+\theta+\pi/2)}). \quad (2)$$

The intensity of the reflected beam is therefore

$$E_rE_r^* = \frac{1}{2}(E^2 + E_L^2 + 2EE_Le^{i(\pi/2-\theta)} + 2EE_Le^{-i(\pi/2-\theta)})$$

$$= \frac{1}{2}(E^2 + E_L^2 + 2EE_L\cos(\pi/2 - \theta))$$

$$= \frac{1}{2}(E^2 + E_L^2 + 2EE_L\sin\theta). \quad (3)$$

Similarly, it can be shown that the intensity of the transmitted beam is

$$E_tE_t^* = \frac{1}{2}(E^2 + E_L^2 - 2EE_L\sin\theta). \quad (4)$$

If the voltages registered by the photodetectors are proportional to the intensities, it follows that the difference in voltage is proportional to $2EE_L\sin\theta$. When digitised, this translates to the step function mentioned above. The probabilities for the two possible outcomes are, as shown in Fig. 3

$$p_- = \begin{cases} 1 & \text{for } -\pi < \theta < 0 \\ 0 & \text{for } 0 < \theta < \pi \end{cases} \quad (5a)$$

and

$$p_+ = \begin{cases} 0 & \text{for } -\pi < \theta < 0 \\ 1 & \text{for } 0 < \theta < \pi \end{cases} \quad (5b)$$

Note that the probabilities are undefined for integral multiples of $\pi$. In practice it would be reasonable to assume that, due to the presence of noise, all the values were 0.5, but for the present purposes the integral values will simply be ignored.

### IV. FRESH IDEAS ON PARAMETRIC DOWN-CONVERSION

If the frequencies and phases of both test beams and both local oscillators were all identical apart from the applied phase shifts, the experiment would be expected to produce step function relationships between counts and applied shifts both for the individual (singles) counts and for the coincidences.

It may safely be assumed that this is not what is observed. It would have shown up in the preliminary trials on the singles counts (see ref. [7]), which would have followed something suggestive of the basic predicted step function as the local oscillator phase shift was varied. What is observed in practice is more likely to be similar to the results obtained by Schiller et al. [9]. Their Fig. 2a, reproduced here as Fig. 4, shows a distribution of photocurrents that is clustered around zero, for $\theta$ taking integer multiples of $\pi$, but is scattered fairly equally among positive and negative values in between.

![Fig. 4: A typical scatter of “noise current” (related to voltage difference) for varying local oscillator phase. (Reproduced from S. Schiller et al., Phys. Rev. Lett. 87 (14), 2933 (1996). Permission requested, June 12, 2005.)](image-url)
would be expected if we had full "rotational invariance" \[10\]. If the ideas presented here are correct, we have instead, in the language of an article by the present author \[11\], only binary rotational invariance. Schiller's scatter of photocurrent differences is seen as evidence that the relative phase can (under perfect conditions) take just one of two values, 0 or \(\pi\). The scatter is formed from a superposition of two sets of points, corresponding to two sine curves that are out of phase, together with a considerable amount of noise.

It is suggested that the two "phase sets" arise from the way in which the pulses are produced, which involves, after the frequency doubling, the degenerate case of parametric down-conversion, the latter producing pulses that are (in contrast to the quantum-mechanical assumption of conjugate frequencies) of exactly equal frequency. Consider an initial pump laser of frequency \(\omega\). In the proposed experiment, this will be doubled in the crystal SHG to 2\(\omega\) then halved in OPA back to \(\omega\). At the frequency doubling stage, one laser input wave peak gives rise to two output ones. Assuming that there are causal mechanisms involved, it seems inevitable that every other wave peak of the output will be exactly in phase with the input. When we now use this input to produce a downconversion, the outputs will be in phase either with the even or with the odd peaks of the input, which will make them either in phase or exactly out of phase with the original laser.

We thus have two classes of output, differing in phase by \(\pi\). If we define the random variable \(\alpha\) to be 0 for one class, \(\pi\) for the other, this will clearly be an important "hidden variable" of the system.

The theory of (degenerate) parametric down-conversion assumed here differs in several respects from the accepted quantum-mechanical one, and also from Stochastic Electrodynamics. No attempt is made to give a full explanation of the physics involved. The theory is rather of the nature of an empirical result, effectively forced on us by a number of different experiments in quantum optics (to be discussed in later papers). Accepted theory says (if, indeed, it allows at all for the existence of "photons" with definite frequencies \[12\]) that though the sum of the frequencies of the two outputs is equal to that of the pump laser, even in the degenerate case the two will differ slightly \[12\]. The exact frequency of one will be \(\frac{1}{2}(\omega_0 + \delta \omega)\), that of the other \(\frac{1}{2}(\omega_0 - \delta \omega)\), where \(\omega_0\) is the pump frequency. If this is the case in the proposed experiment, though, it will severely reduce the visibility of any coincidence curve observed when the experimental beam is mixed back with the source laser in the homodyne detector.

The preliminary experiments (see ref. \[1\]) using just one output beam may already be sufficient to show that the interference is stronger than would be the case if there were any difference between local oscillator and test beam frequencies. It is known that the source laser has quite a broad band width, i.e. that \(\omega_0\) is not constant. Though it is likely that it is only part of the pump spectrum that induces a down-conversion, so that the band width of the test beam may be considerably narrower than that of the pump, it too is non zero. It follows that agreement of frequency between this and the test beam must be because we are always dealing, in the degenerate case, with exact frequency doubling and halving.

V. A CLASSICAL MODEL OF THE PROPOSED BELL TEST

In the proposed Bell test of Sánchez et al., positive voltage differences will be treated as +1, negative as –1. Applying this version of homodyne detection to both branches of the experiment, the CHSH test \((-2 \leq S \leq 2)\) will then be applied to the coincidence counts. Under quantum theory it is expected that, so long as "non-classical" beams are employed, the test will be violated. However, since there are no obvious loopholes in the actual Bell test (see later), local realism should win: the test should not be violated. In the classical view, this prediction is unrelated to any supposed non-classical nature of the light.

A. The basic local realist model

If we take the simplest possible case, in which to all intents and purposes all the frequencies involved are the same, the hidden variable in the local realist model is clearly going to be the phase difference (\(\alpha = 0\) or \(\pi\)) between the test signal and the local oscillator. If high visibility coincidence curves are seen, it must be because the values of \(\alpha\) are identical for the A and B beams. Assuming no noise, the basic model is easily written down.

From equation \(5\), the probability of a –1 outcome on side A is

\[
p_-(\theta_A, \alpha) = \begin{cases} 
1 & \text{for } \pi < \theta_A - \alpha < 0 \\
0 & \text{for } 0 < \theta_A - \alpha < \pi,
\end{cases}
\]

where \(\theta_A\) is the phase shift applied to the local oscillator A, \(\alpha\) is the hidden variable and all angles are reduced modulo \(2\pi\). Similarly, the probability of a +1 outcome is

\[
p_+(\theta_A, \alpha) = \begin{cases} 
0 & \text{for } \pi < \theta_A - \alpha < 0 \\
1 & \text{for } 0 < \theta_A - \alpha < \pi,
\end{cases}
\]

Assuming equal probability \(\frac{1}{2}\) for each of the two possible values of \(\alpha\) \[14\], the standard "local realist" assumption that independent probabilities can be multiplied to give coincidence ones leads to a predicted coincidence rate of

\[
P_{++}(\theta_A, \theta_B) = \frac{1}{2}p_+(\theta_A, 0)p_+(\theta_B, 0) + \frac{1}{2}p_+(\theta_A, \pi)p_+(\theta_B, \pi),
\]

\[8\]
with similar expressions for $P_{+-}$, $P_{-+}$ and $P_{--}$.

The result for $\theta_A = \pi/2$, for example, is

$$P_{++}(\pi/2, \theta_B) = \begin{cases} 0 & \text{for } -\pi < \theta_B < 0 \\ 1/2 & \text{for } 0 < \theta_B < \pi. \end{cases}$$

(9)

For $\theta_A = -\pi/2$ it is

$$P_{++}(-\pi/2, \theta_B) = \begin{cases} 1/2 & \text{for } -\pi < \theta_B < 0 \\ 0 & \text{for } 0 < \theta_B < \pi. \end{cases}$$

(10)

Note that, as illustrated in Fig. 5, the coincidence probabilities cannot, in this basic model, be expressed as functions of the difference in detector settings, $\theta_B - \theta_A$. This failure, marking a significant deviation from the quantum mechanical prediction, is an inevitable consequence of the fact that we have (as mentioned earlier) only binary, not full, rotational invariance.

### B. Fine-tuning the model

Many practical considerations mean that the final local realist prediction will probably not look much like the above step function. It may not even be quite periodic. The main logical difference is that, despite all that has been said so far, the actual variable that is set for the local oscillators is not directly the phase shift but the path length, and, since the frequency is likely to vary slightly from one signal to the next (though always keeping the same as that of the pump laser), the actual phase difference between test and local oscillator beams will depend on the path length difference and on the frequency. In a complete model, therefore, the important parameters will be path length and frequency, with phase derived from these.

If frequency variations are sufficiently large, the situation may approach one of rotational invariance (RI), but it seems on the face of it unlikely that this can be achieved without loss of correlation. If we do have RI, the model becomes the standard realist one in which the predicted quantum correlation varies linearly with difference in phase settings, but it is more likely that what will be found is curves that are not independent of the choice of individual phase setting. They will be basically the predicted step functions but converted to curves as the result of the addition of noise.

### C. The role of the “event-ready” detectors

In the quantum-mechanical theory, the expectation of violation of the Bell test all hinges on the production of “non-classical” light. The light directly output from the crystal OPA is assumed to be Gaussian, i.e. it takes the form of pulses of light that have a Gaussian intensity profile and also, as a result of Fourier theory, a Gaussian spectrum. When this is passed through an unbalanced beamsplitter (BS$_A$ or BS$_B$) and a “single photon” detected by an “event-ready” detector, the theory says that the subtraction of one photon leaves the remaining beam “non-Gaussian”. Although there is mention here of single photons, the theory is concerned with the ensemble properties of the complete beams, not with the individual properties of its constituent pulses.

In the local realist (classical) model, the shapes of the spectra are not relevant except insofar as a narrow band width is desirable for the demonstration of dramatic correlations. The event-ready detectors play, instead, the
important role of selecting for analysis only the strongest down-converted output pairs, it being assumed that the properties of the transmitted and reflected light at the unbalanced beamsplitters are identical apart from their amplitudes. It is likely that those detected signals that are coincident with each other will be genuine “degenerate” ones, i.e. of exactly equal frequency, quasi-resonant with the pump laser. The unbalanced beamsplitters and the detectors PD_A and PD_B need to be set so that the intensity of the detected part is sufficient to be above the minimum for detection but low enough to ensure that all but the strongest pulses are ignored.

In neither theory are the event-ready detectors really needed in their “Bell test” role of ensuring a fair test (see below), since the homodyne detectors are almost 100% efficient.

VI. VALIDITY OF THE PROPOSED BELL TEST

Coincidences between the digitised voltage differences will be used in the CHSH Bell test \cite{13,14}, but avoiding the “post-selection” that has, since Aspect’s second experiment \cite{17}, become customary. The Sánchez et al. proposal is to use “event-ready” detectors, as recommended by Bell himself for use in real experiments \cite{18}. None of the usual loopholes \cite{19} are expected to be applicable:

1. With the use of the event-ready detectors, non-detections are of little concern. The detectors (PD_A and PD_B in Fig. 1) act to define the sample to be analysed, and the fact that they do so quite independently of whether or not any member of the sample is then also detected in coincidence at the homodyne detectors ensures that no bias is introduced here. The estimate of “quantum correlation” \cite{20} to be used in calculating the CHSH test statistic is \[ E = \frac{N_{++AB} + N_{--AB} - N_{+AB} - N_{-AB}}{N_{AB}} \], where the \( N \)'s are coincidence counts and the subscripts are self-explanatory. This contrasts with the usual method, in which the denominator used is not \( N_{AB} \) but the sum of observed coincidences, \( N_{++AB} + N_{--AB} + N_{+AB} + N_{-AB} \). The use of the latter can readily be shown to introduce bias unless it can be assumed that the sample of detected pairs is a fair one. That such an assumption fails in some plausible local realist models has been known since 1970 or earlier \cite{17,21}.

2. There is no possibility of synchronisation problems, since a pulsed source is used.

3. No “accidentals” will be subtracted.

4. The “locality” loophole can be closed by using long paths and a random system for choosing the “detector settings” (local oscillator phase shifts) during the propagation of the signals.

The system is almost certainly not going to be “rotationally invariant” (not all phase differences will be equally likely), but this will not invalidate the Bell test. It may, however, be important in another way. It is likely that high visibilities will be observed in the coincidence curves (i.e. high values of \((\text{max} - \text{min})/(\text{max} + \text{min})\) in plots of coincidence rate against difference in phase shift), leading to the impression that the Bell test ought to be violated. These visibilities, though, will depend on the absolute values of the individual settings. High ones will be balanced by low, with the net effect that violation does not in fact happen.

VII. VALIDITY AND SIGNIFICANCE OF NEGATIVE ESTIMATES FOR WIGNER DENSITIES

Part of the evidence that is put forward as indicating that negative Wigner densities are likely to be obtained consists in the observation that, when \( \theta \) is varied randomly, the distribution of observed voltage differences shows a double peak (see Fig. 7). There is a tendency to observe roughly equal numbers of + and – results but relatively few near zero. The fact that the relationship depends on the sine of \( \theta \) is, however, sufficient to explain why this should be so.

To illustrate, let us consider the following. The sine of an angle is between 0 and 0.5 whenever the angle is between 0 and \( \pi/6 \). It is between 0.5 and 1.0 when the angle is between \( \pi/6 \) and \( \pi/2 \). Since the second range of angles is twice the first yet the range of the values of the sine is the same, it follows that if all angles in the range 0 to \( \pi \) are selected equally often there will be twice as many sine values seen above 0.5 as below.

When allowance is made for the addition of noise, the production of a distribution such as that of Fig. 7 for the average when angles are sampled uniformly comes...
as no surprise. Clearly, as the experimenters themselves recognise, the dip is not in itself sufficient to prove the non-classical state of the light. For this, direct measurement of the Wigner density is required, but there is a problem here. No actual direct measurement is possible, so it has to be estimated, and the method proposed is the Radon transformation \[22\]. It is claimed \[23\] that in other experiments Wigner densities calculated by this procedure have shown negative regions, but perhaps the method should be checked for validity?

In any event, as already explained, the natural hidden variable relevant to the proposed experiment is the phase of the individual pulse, not any statistical property of the whole ensemble.

VIII. SUGGESTIONS FOR EXTENDING THE EXPERIMENT

The basic set-up would seem to present an ideal opportunity for investigation of some key aspects of the quantum and classical models, as well as the operation of the Bell test “detection loophole”.

The operation of the detection loophole could be illustrated if, instead of using the digitised difference voltages of the homodyne detectors, the two separate voltages are passed through discriminators. The latter operate by applying a threshold voltage that can be set by the experimenter and counting those pulses that exceed it. These can be used in a conventional CHSH Bell test, i.e. using total observed coincidence count as denominator in the estimated quantum correlations \(E\). The model that has been known since Pearle’s time (1970) predicts that, as the threshold voltage used in the discriminators is increased and hence the number of registered events decreased (interpreted in quantum theory as the detection efficiency being decreased), the CHSH test statistic \(S\), if calculated using estimates \(E = (N_{++} + N_{--} - N_{+-} - N_{-+})/(N_{++} + N_{--} + N_{+-} + N_{-+})\), will increase. If noise levels are low, it may well exceed the Bell limit of 2.

The existence of the two phase sets could also be investigated if either the raw voltages or the undigitised difference voltages are analysed. So long as the noise level is low, the existence of the two superposed curves, one for \(\alpha = 0\) and the other for \(\alpha = \pi\), should be apparent. It would be interesting to investigate how the pattern changed as optical path lengths were varied. Schiller’s pattern might be hard to reproduce using long path lengths, where exact equality is needed unless the light is monochromatic.

If the primary goal of the experimenter is clearly set out to be the comparison of the performance of the two rival models, rather than merely the conduct of a Bell test, further ideas for modifying the set-up will doubtless emerge when the first experiments have been done.

IX. CONCLUSION

The proposed experiments would, if the “non-classicality” of the light could be demonstrated satisfactorily, provide a definite answer one way or the other regarding the reality of quantum entanglement. They could usefully be extended to include empirical investigations into the operation of the Bell test detection loophole. Perhaps more importantly, though, they present valuable opportunities to compare the performance of the two theories in both their total predictive power and their comprehensibility. Are parameters such as “Wigner density” and “degree of squeezing” really the relevant ones, or would we gain more insight into the situation by talking only of frequencies, phases and intensities? Parameters such as the detection efficiency and the transmittance of the beamsplitters will undoubtedly affect the results, but do they do this in the way the quantum theoretical model suggests? It will take considerably more than just the minimum runs needed for the Bell test if we are to find the answers.

The detailed predictions of the local realist model cannot be given until the full facts of the experimental set-up and the performance of the various parts are known, but it gives, in any event, a simple explanation of the double-peaked nature of the distribution of voltage differences. The peaks arise naturally from the way in which homodyne detection works, and the quantum theoretical idea that they are one of the indications of a non-classical beam or of negative Wigner density would not appear to be justifiable. The idea that a classical beam can become non-classical by the act of “subtracting a photon” is, equally, of doubtful validity. In the classical model, the only effect of the subtraction and detection of part of each beam is to aid the selection for coincidence analysis of those pulses that are likely to be most strongly correlated.

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APPENDIX A: ALTERNATIVE CLASSICAL DERIVATION OF THE HOMODYNE DETECTION FORMULA

A derivation is given here that does not involve complex numbers and hence confirms that the equations in the text are ordinary wave equations, not quantum-mechanical “wave functions”.

The relationship between intensity difference and the local oscillator phase can be checked as follows:

Assume the two input beams are
Experimental beam: \( E \cos \phi \), where, as before, \( \phi = \omega t \)

Local oscillator: \( E_L \cos(\phi + \theta) \)

Then the output beams, assuming a 50-50 beamsplitter and no losses, can be written

Reflected beam:

\[
E_r = \frac{1}{\sqrt{2}} (E \cos(\phi + \pi/2) + E_L \cos(\phi + \theta))
= \frac{1}{\sqrt{2}} (-E \sin \phi + E_L \cos(\phi + \theta)) \quad \text{(A1)}
\]

Transmitted beam:

\[
E_t = \frac{1}{\sqrt{2}} (E \cos \phi + E_L \cos(\phi + \pi/2))
= \frac{1}{\sqrt{2}} (E \cos \phi - E_L \sin(\phi + \theta)). \quad \text{(A2)}
\]

Let us define a (constant) angle \( \psi \) such that \( \tan \psi = E/E_L \), making \( E \cos \psi = E_L \sin \psi \).

Consider the case when \( \theta = 0 \). We have

\[
E_r = \frac{1}{\sqrt{2}} (E/\sin \psi)(-\sin \psi \sin \phi + \cos \psi \cos \phi)
= \frac{1}{\sqrt{2}} (E/\sin \psi) \cos(\psi + \phi), \quad \text{(A3)}
\]

so that amplitude is proportional to \( \frac{1}{\sqrt{2}} E/\sin \psi \) and intensity to \( \frac{1}{2} E^2/\sin^2 \psi \).

Similarly, the intensity of the transmitted beam is also proportional to \( \frac{1}{2} E^2/\sin^2 \psi \). The voltage difference from the homodyne detector is therefore expected to be zero. A zero difference is also found for \( \theta = \pi \), but other values of \( \theta \) produce more interesting results.

For example, for \( \theta = \pi/2 \) we find

\[
E_r = \frac{1}{\sqrt{2}} (E/\sin \psi)(-\sin \psi \sin \phi - \cos \psi \sin \phi)
= \frac{1}{\sqrt{2}} (E/\sin \psi) \sin \phi(-\sin \psi - \cos \psi), \quad \text{(A4)}
\]

\[
E_t = \frac{1}{\sqrt{2}} (E/\sin \psi)(\sin \psi \cos \phi - \cos \psi \cos \phi)
= \frac{1}{\sqrt{2}} (E/\sin \psi) \cos \phi(\sin \psi - \cos \psi). \quad \text{(A5)}
\]

The difference in intensities is therefore proportional to

\[
\text{Difference} = \frac{1}{2} (E^2/\sin^2 \psi) [(-\sin \psi - \cos \psi)^2
- (\sin \psi - \cos \psi)^2]
= \frac{1}{2} (E^2/\sin^2 \psi) 4 \sin \psi \cos \psi
= \frac{1}{2} E^2 \cos \psi/\sin \psi
= 2E^2E_L/E
= 2EE_L. \quad \text{(A6)}
\]

This is consistent with the result obtained by the method in the main text, so it seems safe to accept that as being correct.

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\[\text{[5] The predicted violation is in any case small, so failure may be put down to other “experimental imperfections”.} \]
\[\text{[6] A. I. Lvovsky, H. Hansen, T. Aichele, O. Benson, J. Mylyne and S. Schiller, “Quantum state reconstruction of the single-photon Fock state”, Phys. Rev. Lett. 87, 050402 (2001), http://qis.ucalgary.ca/quantech/FockPRL.pdf.} \]
\[\text{[7] J. Wenger, R. Tualle-Brouri and Ph. Grangier, “Non-gaussian statistics from individual pulses of squeezed light”, Phys. Rev. Lett. 92, 153601 (2004), http://arXiv.org/abs/quant-ph/0402192.} \]
\[\text{[8] The phase offset depends on the difference in optical path lengths, which will not in practice be exactly constant due to thermal oscillations. If a complete model is ever constructed, this should therefore be a parameter.} \]
\[\text{[9] S. Schiller, G. Breitenbach, S. F. Pereira, T. Müller, and J. Mylyne, “Quantum statistics of the squeezed vacuum by measurement of the density matrix in the number state representation”, Phys. Rev. Lett. 87 (14), 2933 (1996).} \]
\[\text{[10] “Rotational invariance” means the hidden variable takes all possible values with equal probability. In the current context, if the experiment does indeed produce high visibility coincidence curves, the hidden variable responsible will be the common phase difference between test beams and local oscillators before application of the phase shifts (“detector settings”) \( \theta_A \) and \( \theta_B \). It is argued that in the proposed experiment there will be at best approximate rotational invariance, if there is appreciable variation in} \]
the (again common) frequency.

[11] C. H. Thompson, “Rotational invariance, phase relationships and the quantum entanglement illusion”, http://arxiv.org/abs/quant-ph/9912082.

[12] See, for example, P. G. Kwiat and R. Y. Chiao, who write: “In the light of observed violations of Bell’s inequalities, it is incorrect to interpret these results in terms of an ensemble of conjugate signal and idler photons which possess definite, but unknown, conjugate energies before filtering and detection. Any observable, e.g. energy or momentum, should not be viewed as a local, realistic property carried by the photon before it is actually measured.” Phys. Rev. Lett. 66, 588 (1991).

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[14] It is possible that there will not be quite perfect symmetry between the two values of $\alpha$, if the frequency doubling process does not quite eliminate the initial laser frequency.

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[20] In the derivation of Bell inequalities such as the CHSH inequality, the statistic required is simply the expectation value of the product of the outcomes. This is commonly referred to in this context as the “quantum correlation”. Only when there are no null outcomes, possible outcomes being restricted to just +1 or −1, does it coincide with ordinary statistical correlation.

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