Reconstruction of Deceleration Parameters from Recent Cosmic Observations

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In this paper, three kinds of simple parameterized deceleration parameters $q(z) = a + \frac{b}{1+z}$, $q(z) = a + \frac{bz}{1+z}$ and $q(z) = \frac{a}{b} \ln(1+z)$ are reconstructed from the latest SNe Ia Gold dataset, observational Hubble data and their combination. It is found that the transition redshift from decelerated expansion to accelerated expansion $z_T$ and current decelerated parameter values $q_0$ are consistent with each other in $1\sigma$ region by only using SNe Ia Gold dataset and observational Hubble data in three parameterizations respectively. By combining the SNe Ia Gold dataset and observational Hubble data together, a tight constraint is obtained. With this combined constraints, $z_T$ is $0.505^{+0.080}_{-0.052}$, $0.368^{+0.059}_{-0.036}$, $0.767^{+0.121}_{-0.126}$ with $1\sigma$ error in three parameterizations respectively. And, it is easy to see that $z_T$ separates from each other in $1\sigma$ region clearly.

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I. INTRODUCTION

At the end of the last decade, the observations of High redshift Type Ia Supernova from two teams [1, 2] indicated that our universe is undergoing accelerated expansion. Meanwhile, this suggestion was strongly confirmed by the observations from WMAP [3, 4, 5, 6] and Large Scale Structure survey [7]. To understand the late-time accelerated expansion of the universe, a large part of models are proposed by assuming the existence of an extra energy component, dubbed dark energy, which has negative pressure and dominates at late time to push the universe from decelerated expansion to accelerated expansion. In principle, a natural candidate to dark energy could be a small cosmological constant $\Lambda$ which has the constant equation of state (EOS) $w_\Lambda = -1$. However, there exist serious theoretical problems: fine tuning and coincidence problems. To overcome the coincidence problem, dynamical dark energy models, such as quintessence [8], phantom [9], k-essence [11], Chaplygin gas [12], holographic dark energy [13], etc., are proposed.

Another approach to study the dark energy is by an almost model-independent way, i.e., by a parameterized EOS of dark energy which is implemented by giving a concrete form of the EOS of dark energy directly, such as $w(z) = w_0 + w_1 z$ [14], $w(z) = w_0 + w_1 \frac{1}{1+z}$ [15, 16], $w(z) = w_0 + w_1 \ln(1+z)$ [17], etc.. Through this method, the evolution of dark energy with respect to the redshift $z$ is explored, and it is found that the current constraints favor a dynamical dark energy, though the cosmological constant is not ruled out in $1\sigma$ region [18]. Also, the dark energy favors a quintom-like dark energy, i.e. a crossing of the cosmological constant boundary $w = -1$. In all, it is an effective method to rule out the dark energy models. As known, the universe is dominated by dark energy and is undergoing accelerated expansion at present and was dominated by dark matter and underwent a decelerated epoch in the past. In another words, the universe underwent a transition from decelerated expansion to accelerated expansion. So, to realize the transition, the parameterized decelerated parameter is presented in a model independent way by giving a concrete form of decelerated parameter which is positive in the past and negative at present [15, 20, 21, 22]. Moreover, it is interesting and important to know what is the transition time $z_T$ from decelerated expansion to accelerated expansion. This is the main point of this paper to be explored in the model-independent way. In this paper, the SNe Ia Gold dataset and observational Hubble data are used to constrain the transition redshift $z_T$ and current value of decelerated parameter.

This paper is structured as follows. In section II three kinds of parameterized decelerated parameters are constrained by latest 182 SNe Ia Gold data points compiled by Riess [18] and observational Hubble data [23]. Section III is the conclusion.

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II. RECONSTRUCTION OF DECELERATION PARAMETER

We consider a flat FRW cosmological model containing dark matter and dark energy with the metric
\[ ds^2 = -dt^2 + a^2(t)dx^2. \]  
(1)

The Friedmann equation of the flat universe is written as
\[ H^2 = \frac{8\pi G}{3} (\rho_m + \rho_{de}), \]  
(2)

where, \( H \equiv \dot{a}/a \) is the Hubble parameter, and its derivative with respect to \( t \) is
\[ \ddot{H} = \frac{\ddot{a}}{a} - \left( \frac{\dot{a}}{a} \right)^2, \]  
(3)

which combined with the definition of the deceleration parameter
\[ q(t) = -\frac{\ddot{a}}{aH^2}, \]  
(4)

gives
\[ \dot{H} = -(1 + q) H^2. \]  
(5)

By using the relation \( a_0/a = 1 + z \), the relation of \( H \) and \( q \), i.e., Eq. (5) can be written in its integration form
\[ H(z) = H_0 \exp \left[ \int_0^z [1 + q(u)] d\ln(1 + u) \right], \]  
(6)

where the subscript "0" denotes the current values of the variables. If the function of \( q(z) \) is given, the evolution of the Hubble parameter is obtained. In this paper, we consider three kinds of parameterized deceleration parameters:

- **A.** \( q(z) = a + \frac{b}{1+z} \), \( H(z) = H_0 (1 + z)^{1+a+b} \exp \left( -\frac{b}{1+z} \right) \)

- **B.** \( q(z) = a + \frac{b}{2(1+z)^2} \), \( H(z) = H_0 (1 + z)^{1+a} \exp \left( \frac{b}{2(1+z)^2} \right) \)

- **C.** \( q(z) = \frac{1}{2} + \frac{a+b}{2(1+z)^2} \) \[21\], \( H(z) = H_0 (1 + z)^{3/2} \exp \left[ \frac{b}{2} + \frac{z^2 - b}{2(1+z)^2} \right] \)

where, \( a, b \) are constants which can be determined from the cosmic observations, such as Sne Ia and observational Hubble data. From the explicit expressions of Hubble parameters, this mechanisms can also be treated as parameterizations of Hubble parameters which can been constrained from observational Hubble data directly.

Before constraining the parameterizations of decelerated parameters, we give brief discussion on the parameterized equations. It is easy to obtain the current values of decelerated parameters which determined at the redshift \( z = 0 \) are \( q_0.A = a \), \( q_{0B} = a \) and \( q_{0C} = 1/2 + b \) in three parameterizations A, B and C respectively. Also, the transition redshift \( z_T \) from decelerated expansion to accelerated expansion can also be obtained by solving the equation of \( q(z = z_T) = 0 \). Then, they can be written as a function \( z_T = z_T(a, b) \) in terms of parameters \( a \) and \( b \) uniformly, if the equation has real root.

Now, these models can be constrained by the Sne Ia Gold dataset and observational Hubble data. The Sne Ia Gold dataset contains 182 Sne Ia data \[18\] by discarding all Sne Ia with \( z < 0.0233 \) and all Sne Ia with quality='Silver'. Constraints from Sne Ia can be obtained by fitting the distance modulus \( \mu(z) \)
\[ \mu_{th}(z) = 5 \log_{10}(D_L(z)) + \mathcal{M}, \]  
(7)

where, \( D_L(z) \) is the Hubble free luminosity distance \( H_0 d_L(z) \) and
\[ d_L(z) = (1 + z) \int_0^z \frac{dz'}{H(z')} \]  
(8)

\[ \mathcal{M} = M + 5 \log_{10} \left( \frac{H_0}{Mpc} \right) + 25, \]  
(9)

\[ = M - 5 \log_{10} h + 42.38, \]  
(9)
where, $M$ is the absolute magnitude of the object (Sne Ia here). With Sne Ia dataset, the best fit values of parameters in dark energy models can be determined by minimizing

$$\chi^2_{SneIa}(p_s) = \sum_{i=1}^{N} \frac{(\mu_{obs}(z_i) - \mu_{th}(p_s,z_i))^2}{\sigma_i^2}, \quad (10)$$

where $N = 182$ for Sne Ia Gold dataset, $\mu_{obs}(z_i)$ is the moduli obtained from observations, $\sigma_i$ is the total uncertainty of the Sne Ia data, and $p_s$ denotes the parameters contained in the model. The Sne Ia datasets are used as cosmic constraints can also be found in [24].

The observational Hubble data are based on differential ages of the galaxies [25]. In [26], Jimenez et al. obtained an independent estimate for the Hubble parameter using the method developed in [25], and used it to constrain the EOS of dark energy. The Hubble parameter depending on the differential ages as a function of redshift $z$ can be written in the form of

$$H(z) = -\frac{1}{1+z} \frac{dz}{dt}. \quad (11)$$

So, once $dz/dt$ is known, $H(z)$ is obtained directly [23]. By using the differential ages of passively-evolving galaxies from the Gemini Deep Deep Survey (GDDS) [27] and archival data [28], Simon et al. obtained $H(z)$ in the range of $0 \lesssim z \lesssim 1.8$ [23]. The observational Hubble data from [23] are list in Table I.

| $z$ | 0.09 | 0.17 | 0.27 | 0.40 | 0.88 | 1.30 | 1.43 | 1.53 | 1.75 |
|-----|------|------|------|------|------|------|------|------|------|
| $H(z)$ (km s$^{-1}$ Mpc$^{-1}$) | 69 | 83 | 70 | 87 | 117 | 168 | 177 | 140 | 202 |
| 1σ uncertainty | ±12 | ±8.3 | ±14 | ±17.4 | ±23.4 | ±13.4 | ±14.2 | ±14 | ±40.4 |

**TABLE I:** The observational $H(z)$ data [23] (see [29, 30] also).

The best fit values of the model parameters from observational Hubble data [23] are determined by minimizing

$$\chi^2_{H_{obs}}(p_s) = \sum_{i=1}^{9} \frac{(H_{th}(p_s,z_i) - H_{obs}(z_i))^2}{\sigma^2(z_i)}, \quad (12)$$

where $p_s$ denotes the parameters contained in the model, $H_{th}$ is the predicted value for the Hubble parameter, $H_{obs}$ is the observed value, $\sigma(z_i)$ is the standard deviation measurement uncertainty, and the summation is over the 9 observational Hubble data points at redshifts $z_i$. In our three cases, the derived $H_{th}$ contains parameter $H_0$ which is current value of Hubble parameter, and $H_0 = 72 \pm 2$ are taken as a prior.

Fitting the 182 Sne Ia Gold data only, the minimum $\chi^2$ and the best fit parameters $a$, $b$ and the transition times (or redshift) $z_T$ in these three kinds of parameterizations are listed in Table II. The evolutions of the decelerated parameters $q(z)$ with 1σ errors are plotted in Fig. 11.

| Parameters | $\chi^2_{\text{min}}$ | $a$ | $b$ | $z_T$ | $q_0$ |
|-------------|-----------------|-----|-----|------|------|
| A. $q(z) = a + \frac{b}{1 + z}$ | 156.44 | $-0.84^{+0.22}_{-0.22}$ | $1.05^{+0.25}_{-0.25}$ | $0.39^{+0.10}_{-0.05}$ | $-0.84^{+0.22}_{-0.22}$ |
| B. $q(z) = a + \frac{b}{1 + z}$ | 156.71 | $-1.07^{+0.30}_{-0.30}$ | $5.68^{+2.00}_{-2.00}$ | $0.34^{+0.11}_{-0.05}$ | $-1.07^{+0.30}_{-0.30}$ |
| C. $q(z) = \frac{1}{1 + z}$ | 156.54 | $1.46^{+1.22}_{-1.22}$ | $-1.46^{+0.28}_{-0.28}$ | $0.36^{+0.12}_{-0.05}$ | $-0.94^{+0.28}_{-0.28}$ |

**TABLE II:** The best fit results with 1σ error of constraints from 182 Sne Ia Gold dataset.

By fitting the dataset from 9 points of observational Hubble data alone, we obtain the results of the minimum $\chi^2$ and the best fit parameters $a$, $b$ and the transition times (redshift) $z_T$ which are listed in Table III. The evolutions of the decelerated parameters are plotted in Fig. 2.

In the above, three kinds of parameterized decelerated parameters have been constrained by the Sne Ia Gold dataset and observational Hubble data respectively. However, the cosmological parameters have degeneracies in almost all cosmological observables. So, it is necessary to combine all available probes or observations to break the degeneracies to obtain tight constraints. In the remained parts of this section, the Sne Ia Gold dataset and observational Hubble data are combined together to give a tight constraint to parameterized decelerated parameters.
The best fit parameter values of $a$, $b$ and $z_T$ can be obtained by minimizing the summation of $\chi^2$ of Sne Ia Gold dataset and Hubble parameter data in these three kinds of parameterizations

$$\chi^2_{\text{total}}(P_s) = \chi^2_{\text{Sne Ia}}(P_s) + \chi^2_{\text{Hubble}}(P_s).$$

The results of combined constraints are listed in Table III. The evolutions of the decelerated parameters $q(z)$ with $1\sigma$ error are plotted in Fig. 3.

### III. DISCUSSION AND CONCLUSION

In this paper, by a model-independent way, we have used three kinds of parameterized decelerated parameters to obtain the transition time or redshift $z_T$ from decelerated expansion to accelerated expansion and the current value of decelerated parameter $q_0$. To obtain the best fit values of the transition redshift and decelerated parameters, Sne...
Ia Gold dataset, observational Hubble data and their combination are used as cosmic observational constraints. With these cosmic observations, three parameterized decelerated parameters are reconstructed. In Fig. 1 where only Sne Ia Gold dataset is used, it can be seen that three kinds of parameterizations all have transitions from decelerated expansion to accelerated expansion. Also, it is found that the transition redshift $z_T$ and current value of decelerated parameter $q_0$ are consistent with each other in 1σ region. The results may be caused by two possible reasons: a. the Sne Ia Gold data points are not enough to discriminate them from each other; b. three parameterizations just coincide with each other and have the same transition redshift and current value of decelerated parameter. In Fig. 2 where observational Hubble data is used alone, the similar results as shown in Fig. 1 can also be obtained. However, the current values of decelerated parameter have larger 1σ intervals which could come from the relative lack of observational Hubble data points. Noticeably, in Fig. 2 (the right panel) the transition from decelerated expansion to accelerated expansion is not clear in 1σ region in parameterized C case where only observational Hubble data is used. But, the best fit curve across the boundary of $q = 0$. This means that a different result will be obtained by choosing different datasets as cosmic constraints. To give a tight constraint to the transition redshifts and current values of decelerated parameter of three kinds of parameterizations, a combined constraint of Gold Sne Ia and observational data is introduced. The results are shown in Table. IV and Fig. 3 where the combined constraint is used, it can be seen that the transition redshifts $z_T$ and current values of decelerated parameter $q_0$ separate from each other clearly in 1σ region, i.e., three kinds of parameterizations do not overlap with each other in 1σ regions. This means that the results rely on the concrete forms of the parameterized equations strongly just as pointed out in Ref. [31, 32], namely parameterization dependence. The parameterization dependence is the potential drawback of parameterized decelerated parameter and parameterized EOS of dark energy. Now, a new question would be asked, which one parameterized decelerated parameters describe our universe evolution or no one? To answer this question completely, only the Sne Ia Gold dataset and observational Hubble data are far from enough and another cosmic observational constraints would be added. At last, it is worth to remind that the chosen of cosmic observational datasets has a strong impact on the results, as shown in Fig. 1 is not only in shrinking the error regions but also in different best fit values, this is so-called dataset dependence. In this paper, about the datasets of Sne Ia, the Gold dataset is used only. It would be interesting to use SNLS dataset as another constraint. And, we leave the comparison of Sne Ia Gold and SNLS datasets as cosmic constraints in the future work.

| Parameters | $\chi^2_{\text{min}}$ | $a$ | $b$ | $z_T$ | $q_0$ |
|------------|----------------|-----|-----|-------|------|
| A. $q(z) = a + \frac{b}{1+z}$ | 162.19 | $-0.657^{+0.153}_{-0.153}$ | $1.956^{+0.535}_{-0.535}$ | $0.505^{+0.089}_{-0.052}$ | $-0.697^{+0.153}_{-0.153}$ |
| B. $q(z) = a + \frac{bx}{1+z}$ | 161.01 | $-0.982^{+0.242}_{-0.242}$ | $4.992^{+1.319}_{-1.319}$ | $0.368^{+0.059}_{-0.036}$ | $-0.982^{+0.232}_{-0.232}$ |
| C. $q(z) = \frac{1}{2} + \frac{a}{1+z}$ | 165.89 | $-0.849^{+0.069}_{-0.069}$ | $-0.910^{+0.089}_{-0.089}$ | $0.767^{+0.121}_{-0.126}$ | $-0.410^{+0.089}_{-0.089}$ |

TABLE IV: The best fit results of the combined constraints from 182 Sne Ia Gold dataset and 9 observational Hubble data.
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