Monopole Chains in the Compact Abelian Higgs Model with doubly-charged Matter Field

M. N. Chernodub\textsuperscript{a,b}, R. Feldmann\textsuperscript{c}, E.-M. Ilgenfritz\textsuperscript{d}, A. Schiller\textsuperscript{c}

\textsuperscript{a} ITEP, B.Cheremushkinskaja 25, Moscow, 117218, Russia
\textsuperscript{b} Institute for Theoretical Physics, Kanazawa University, Kanazawa 920-1192, Japan
\textsuperscript{c} Institut für Theoretische Physik, Universität Leipzig, D-04109 Leipzig, Germany
\textsuperscript{d} Institut für Physik, Humboldt-Universität zu Berlin, Newtonstr. 15, D-12489 Berlin, Germany

Abstract

We study the properties of topological defects in the lattice compact Abelian Higgs Model with charge $Q = 2$ matter field. We find that monopoles and antimonopoles form chain-like structures which are dense in the confinement/symmetric phase. In this phase the mentioned structures explain both the confinement of single-charged and the breaking of strings spanned between doubly-charged test particles. This observation helps to understand how the non-diagonal gluons, once taken into consideration in the Abelian projection of gluodynamics, could reproduce in this framework the string breaking for adjoint charges.

1 Introduction

In a series of papers, we have studied the three-dimensional compact Abelian Higgs model (cAHM\textsubscript{3}) as a toy model \cite{1} for confinement and deconfinement in QCD. Starting from three-dimensional compact QED (cQED\textsubscript{3}), where monopoles and antimonopoles in the plasma phase are the agents of confinement at all couplings \cite{2}, we have studied deconfinement either induced by rising temperature (in cQED\textsubscript{2+1} \cite{3}) or occurring due to interaction with a single-charged ($Q = 1$) matter field in the framework of the cAHM\textsubscript{3}. In the latter case, the presence of matter fields eventually forces monopoles and antimonopoles to form magnetically neutral bound states \cite{1} connected by Abrikosov-Nielsen-Olesen (ANO) vortices \cite{4}. This mechanism of dipole formation is obviously different from that observed in the high temperature phase of cQED\textsubscript{2+1} \cite{5, 3}.

The role of monopoles is paralleled in four-dimensional Yang-Mills theory where, in the dual superconductor scenario \cite{6}, monopoles are also playing the role of agents of confinement (see, e.g. the reviews \cite{7}) and where the presence of a dynamical matter field changes the monopole dynamics in a way resulting in string breaking \cite{8}. Not only the similarities, also the differences to the four-dimensional non-Abelian theory are interesting in this context. One has to recognize that the dynamics of particle-like monopoles in four dimensions is richer and changes at the deconfinement transition in a way fundamentally different from the dynamics of instanton-like monopoles in three dimensions.
In 3 + 1 dimensional non-Abelian gauge theory, the Abelian monopoles are condensed at low temperatures and electric charges are confined [7]. However, as the temperature rises the monopoles become massive and static and the monopole condensate disappears. The static monopoles are unable to support confinement of timelike moving charges. However, such monopoles can still give rise to a non-zero spacelike string tension.

In 2 + 1 dimensional Abelian gauge theory, the monopoles form a plasma phase at low temperatures in which the electric charges are confined similarly to 3 + 1 dimensional theory. Increasing temperature leads to the formation of magnetic dipoles out of monopoles and antimonopoles. At sufficiently high temperatures the monopole plasma is destroyed and a dilute dipole plasma is formed. However, the dipole plasma is unable to create neither a “spatial” area law nor the confinement of static charges [5, 3].

In four-dimensional non-Abelian theory there is another, rivaling scenario of confinement based on center vortices [9]. A closer look reveals that it does not invalidate the monopole picture of confinement but puts it into another context, thus adding essential features to the monopole dynamics. In short, percolation or static behavior of the latter results from the respective properties of the vortices in the sense of monopoles running inside the vortex sheets. In the case of SU(2) this means that monopole worldlines are confined to vortex sheets, and magnetic flux between opposite monopoles is collimated inside the vortex sheet. This picture has been advocated by Greensite et al. [10] from the moment that they revived the vortex scenario of confinement. Later on, the picture helped to understand the different confinement properties (and scales) of fundamental and adjoint charges which were difficult to explain in the monopole picture [11].

The cAHM is of interest to mimic this particular aspect due to its non-perturbative features which arise from the presence of two types of topological defects: monopoles and Abrikosov-Nielsen-Olesen vortices [4]. In the single-charged cAHM tight monopole pairs are bound by ANO vortices. In three dimensions monopoles and anti-monopoles experience a logarithmically rising attractive potential due to an anomalous dimension of the photon propagator induced by the matter fields [12]. The interaction guarantees the formation of monopole-antimonopole pairs. When such pairs are forming, asymptotic confinement is simultaneously lost for all test charges $q$. This is not the case we are now interested in.

In this paper we therefore consider the cAHM with doubly-charged ($Q = 2$) Higgs fields. Besides of its importance in condensed matter physics [13] this model is attractive in our context because it is clear [14] that the dynamical Higgs field can only screen external particles with even charge $q$. Before the transition to the Higgs phase sets in, the $q = 1$ test charges (the analogue of fundamental charges) should be confined at large separations, while $q = 2$ test charges (the analogue of adjoint charges) should suffer string breaking. The assumed formation of only magnetic dipole states is not able to explain this selective string breaking. Finally, in the Higgs phase, none of the test charges will be confined, which corresponds with the formation of monopole and vortex clusters with a size of the order of the lattice spacing. Such clusters may consist of pure vortex rings or also they may contain monopoles.

2 Some analytic considerations

To enable analytical considerations and to simplify the numerical simulations, we study the model in the London limit where the Higgs field $\Phi$ at site $x$ is represented only by its phase
\( \Phi_x = \exp (i \varphi_x) \). The Wilson-type action of the model with \( Q \)-charged Higgs field is

\[
S_{W}[\theta, \varphi] = -\beta \sum_P \cos(d\theta)_P - \kappa \sum_l \cos(d\varphi - Q \theta)_l ,
\]

where \( \theta_l \) is the link angle representing the compact gauge field and \((d\theta)_P\) is the plaquette angle representing its curl. \( \beta \) denotes the gauge coupling and \( \kappa \) is the hopping parameter. Here and below we use the compact notations of differential forms on the lattice (see the second of Refs. [7] for a review).

The monopoles and the ANO-vortices appear due to compactness of the phase angles \( \theta_l \) and \( \varphi_x \), respectively. For the sake of our arguments it is instructive to perform the Berezinsky-Kosterliz-Thouless [15] (BKT) transformation in order to rewrite the partition function of the cAHM3 in terms of those defects. We use the Villain-type action (with couplings \( \tilde{\beta} \) and \( \tilde{\kappa} \)) instead of (1):

\[
Z = \pi \int_{-\pi}^{\pi} \! D\theta \int_{-\pi}^{\pi} \! D\varphi \sum_n \left( c_2^2 \right)^n \sum_l \left( c_1^1 \right)^l e^{-\tilde{\beta} ||d\theta + 2\pi n||^2 - \tilde{\kappa} ||d\varphi - Q \theta + 2\pi l||^2} ,
\]

where \( n \) and \( l \) are integer-valued forms living on plaquettes \( c_2 \) and links \( c_1 \), respectively.

Applying the BKT transformation [15] (see also the second of Refs. [7]) with respect to the gauge and the Higgs fields and integrating them out we rewrite the partition function (2) as

\[
Z \propto Z_d = \sum \sum e^{-S_d(*m, *j_m)}
\]

with the defect action\(^1\)

\[
S_d(*m, *j_m) = 4\pi^2 \tilde{\beta} (*m, \frac{1}{\Delta + M^2} *m) + 4\pi^2 \tilde{\kappa} (*j_m, \frac{1}{\Delta + M^2} *j_m) .
\]

Here \( M = Q(\tilde{\kappa} / \tilde{\beta})^{1/2} \) is the tree-level mass of the gauge boson, \( \Delta \) is the lattice Laplacian. The integer-valued forms \(*m\) and \(*j_m\) represent monopoles and vortices, respectively, living on sites and links of the dual lattice. A site \(*c_3\) (link \(*c_2\)) of the dual lattice is dual to a cube \(c_3\) (plaquette \(c_2\)) of the original lattice. The constraint in the second sum of Eq. (3) requires that the vortices begin (and end) at the monopole (anti-monopole) positions, \( \delta *j_m = Q *m \). This indicates that a vortex carries a fraction \( 1/Q \) of the total magnetic flux emanating from a monopole.

Let us now consider the contribution of vortices and monopoles to the potential \( V_q(R) \) between a pair of external test particles with charges \( \pm q \), separated by a distance \( R \). The potential is given in terms of the average of the Wilson loop \( W_q(R, T) = \exp \{ iq(J, \theta) \} \) in the form \( V_q(R) = -T^{-1} \log \langle W_q(R, T) \rangle + \text{const} \). The current \( J \) runs around a contour of rectangular shape, \( R \times T \) with \( R \ll T \). Using the same transformations which led us from Eq. (2) to Eq. (3), the vacuum expectation value of the Wilson loop will emerge factorized

\(^1\)In the journal version of this article the \((\Delta + M^2)^{-1}\) operator in the first term of the r.h.s. is inadvertently written as \( \Delta^{-1} \).
as \( \langle W_q \rangle = \langle W_q \rangle_{\text{ph}} \cdot \langle W_q \rangle_{\text{d}} \). The first factor is simply the perturbative self-interaction of the external loop

\[
\langle W_q \rangle_{\text{ph}} \propto \exp \left\{ - \frac{q^2}{4\beta} \left( J, \frac{1}{\Delta + M^2 J} \right) \right\}
\]

due to massive photon exchange. The non-perturbative factor is due to topological defects:

\[
\langle W_q \rangle_{\text{d}} = \mathbb{Z}_d^{-1} \sum_{m} \sum_{j_m} e^{-S_{\text{int}}(m,j_m)} Z_{j_m=Q^* m}
\]

with

\[
S_{\text{int}}(m,j_m; J) = -2\pi i \frac{q}{Q} (d^* j_m, \frac{1}{\Delta + M^2 J}) + 2\pi i \frac{q}{Q} \mathbb{L}(j_0, J).
\]

The first term in the interaction \( S_{\text{int}} \) is a Yukawa-type interaction (which reduces in the limit \( M \to 0 \) to a usual cQED-like interaction) between a monopole and the charged test particle. The second term is given by the linking number \( \mathbb{L}(j_0, J) = (d^* j_0, \Delta^{-1} J) \in \mathbb{Z} \) between the external particle trajectory \( J \) and the closed part \( \cdot j_0 \) of the vortex ensemble \( \cdot j_m \). The linking number describes the Aharonov-Bohm (AB) topological interaction \[16\].

The contribution of vortices to the potential \( V_q \) is twofold. At first, and most importantly, the closed vortices interact with the electrically charged external particles via the AB effect if \( q/Q \notin \mathbb{Z} \). Secondly, the vortices influence the monopole dynamics via the constraint, contributing in this way to the potential indirectly, via the monopoles and their interaction with the test particles. Since there are \( Q \) vortices attached to each (anti-)monopole, the vortices force the monopoles to form magnetically neutral states which are dipoles for \( Q = 1 \). If \( Q = 2 \), however, the monopoles may form not only dipoles but also extended loop-like structures depicted in Figure 1. A typical loop structure contains vortex lines which, in general, are non-orientable closed loops consisting of vortex segments with definite orientations of magnetic fluxes. A junction of segments with different flux orientations must contain a monopole. Thus non-orientable vortex loops should necessarily contain magnetic monopoles.

A separation of a general vortex ensemble, \( \cdot j_m = \cdot j_0 + \cdot j_m' \), into closed vortices, \( \delta \cdot j_0 = 0 \), and open ones, \( \delta \cdot j_m' = Q \cdot m \), is ambiguous. However, in the sum (6) the ambiguity disappears.

Figure 1: A schematic view of simplest vortex-monopole configurations and a pure ANO vortex configuration in the cAHM\(3 \) with \( Q = 2 \) dynamical Higgs field.
Orientable vortices with magnetic flux cannot lead to confinement of electric charges \cite{17}. Indeed, the long-range interaction between such vortices and the Wilson loop may be provided only by the AB effect in a form visible in Eq. (6) (here we implicitly lean upon that we are working in a model with a mass gap in which any interactions – except for the topological ones like via the linking number – must be suppressed). However, numerical simulations of the non-compact version of the AHM$_3$ show \cite{17} that a confinement phase is absent while the AB-interaction does exist. The vortices in this model can only be orientable since monopoles are absent.

On the other hand, in the Maximal Center Gauge of the four-dimensional Yang-Mills theory (YM$_4$) the so-called center vortices are known to lead to an area law of the fundamental Wilson loop via the linking number representing the AB effect \cite{10, 18}. The center vortices are non-orientable and therefore there is no contradiction with the conclusions of Ref. \cite{17}. Moreover, considered in an Abelian sense the non-orientable center vortices can be represented as segments of orientable vortices with monopoles separating the segments with different flux orientations from each other, similarly to what we will find in the doubly-charged cAHM$_3$.

We conclude that in the doubly-charged cAHM$_3$ both monopoles and vortices are responsible for the confinement of $q = 1$ charges and the breaking of the string between $q = 2$ charges. Note that in the single-charged cAHM$_3$ non-orientable vortices do not exist, the AB effect is absent and the vortices do not feature in producing the confining forces. In this case monopoles alone are responsible for confinement while the vortices can only create magnetic dipoles and, in this way, destroy confinement.

3 Numerical simulations

To check these general ideas, we have performed a numerical study of the confining or non-confining properties of electrically charged test particles in the doubly-charged cAHM$_3$. The phase structure of the model was investigated in Refs. \cite{14, 13}. Before the Higgs phase sets in, external particles with charge $q = 1$ must be linearly confined in this model whereas particles with charge $q = 2$ show a flattening of the potential at a certain distance. In order to observe this effect we have simulated the model (11) on a $32^2 \times 8$ lattice ($L_t = 8$). The choice of the asymmetric lattice is dictated by the fact that in the case of symmetric lattices the potential can be measured only using Wilson loops which are not suitable for an observation of the string breaking effect. We have performed simulations at fixed gauge coupling constant, $\beta = 1.2$, the choice of which was motivated by visualization reasons: to see clearly the monopole structures the density of the monopoles must be neither too high nor too low. We have used about $10^4$ independent measurements for each value of the hopping parameter $\kappa$.

Defining a Polyakov loop of charge $q$ at position $(x_1, x_2)$ via

$$P_q(x_1, x_2) = \exp \left\{ iq \sum_{x_t=1}^{L_t} \theta_3(x_1, x_2, x_t) \right\}$$

with the obvious notation $\theta_1 = \theta_{\mu}(x_1, x_2, x_t)$ we show the v.e.v. of the position average $P_q$ in Figure 2(a) as a function of the hopping parameter $\kappa$. At small (large) $\kappa$ the v.e.v’s of both loops are low (high) which corresponds to the confinement (Higgs) phase. One can clearly see

\footnote{Thus, one can ascribe the leading role in the YM$_4$ confinement phenomenon solely to the vortices (as in}
that the average Polyakov loop for the $q = 1$ external charges, which are really confined on the left of the transition, is more sensitive to the transition than $P_2$. The much less sensitivity of $P_2$ is consistent with the observation of the $q = 2$ string breaking (due to $Q = 2$ particles popping up in the vacuum) discussed below.

In Figure 2(b) we show the Polyakov loop susceptibilities vs. $\kappa$. The susceptibility $\chi_{P_1}$ related to $q = 2$ external particles changes between roughly constant levels in the two phases at the transition point. At the same time, $\chi_{P_1}$ shows a clear maximum signaling the transition from confinement to deconfinement. We have fitted the susceptibility $\chi_{P_1}$ in the vicinity of the maximum by the function

$$\chi_{P_1}^{\text{fit}} = \frac{C_1}{C_2 + (\kappa - \kappa_c)^2}$$

with $C_{1,2}$, $\gamma$ and $\kappa_c$ being fit parameters and we localize the transition at $\kappa_c = 1.300(1)$.

The potentials

$$V_q(|\vec{R}|) = -\frac{1}{L_t} \log \left\langle P_q(\vec{0})P_q^*(\vec{R}) \right\rangle,$$  \hspace{1cm} (9)$$

extracted from the Polyakov loop correlators are presented in Figure 3 for both values of the electric test charges $q = 1, 2$. We show the potentials both in the confinement (at $\kappa = 1.275$) and the Higgs phase (at $\kappa = 1.325$). In the confinement phase the potential for the $q = 1$ external charges is approximately linearly rising at large distances (the flattening for $R \to 16$ is a result of lattice periodicity). The $q = 2$ potential shows a rapid flattening corresponding to the dynamical $Q = 2$ particle creation from the vacuum and, eventually, to the string breaking already in the confinement phase. In the Higgs phase all potentials show flattening due to deconfining nature of this phase.

Let us now focus on the topological defects which should explain this. The simplest characteristic of a topological defect is its density. The monopole and the vortex densities are

$$\rho_{\text{mon}} = \frac{1}{V} \sum_{c=3} |^*m|, \quad \rho_{\text{vort}} = \frac{1}{3V} \sum_{c=2} |^*j_m|,$$  \hspace{1cm} (10)$$

Ref. [10, 15] but in this case the vortices must be non-orientable which implies the existence of monopoles living on the vortex sheets.
respectively. The monopole charge is defined in the standard way,
\[
m = \frac{1}{2\pi} [d\theta]_{2\pi},
\]
where \([\cdots]_{2\pi}\) denotes the integer part modulo \(2\pi\). The vortex current is, following Ref. [19], defined as
\[
j = \frac{1}{2\pi} (d\varphi - Q\theta)_{2\pi} + Q [d\theta]_{2\pi}.
\]

According to Figure 3, the densities (shown in lattice units) of the monopoles and vortices are gradually decreasing functions of \(\kappa\) already in the confinement phase. Towards the Higgs phase, the density of monopoles and vortices drops faster, a fact which agrees with the expectation that the confining properties of the cAHM\(_3\) are due to the topological defects.
One has to realize that in the confinement phase the monopoles cannot be in a plasma state because in this case the potentials for both external charges \( q = 1, 2 \) would have to be linearly rising at large separations, contrary to our observation (Figure 3). On the other hand, the monopoles cannot form magnetically neutral monopole-antimonopole bound states (dipoles) exclusively because in this case both \( q = 1, 2 \) potentials \( V_q \) would have to show flattening – essentially for distances larger than the dipole size – again in contradiction to the observed behavior (Figure 3). Thus the only possible kind of monopole configurations – which could explain both the linearly rising potential for the \( q = 1 \) electric charges and the string breaking for the \( q = 2 \) charges – is a monopole chain schematically plotted in Figure 1.

If a vortex loop contains monopoles, the monopoles and antimonopoles are mutually alternating along the chain. Thus the magnetic flux coming from a monopole inside the chain is separating into two parts, gradually squeezing into vortices of finite thickness and, as a consequence, forming a non-orientable closed magnetic flux. Each piece of such a vortex carries in average a half flux, \( \Phi = 2\pi/Q \equiv \pi \). If such a flux pierces the \( q \)-charged Wilson loop it provides a contribution to the loop close to \((-1)^q\). This leads to the necessary disorder for odd-charged external particles (eventually leading to a linearly rising potential) whereas even-charged particles are not confined. In Figure 5 we visualize a typical monopole/vortex configuration observed in our numerical simulations. This example shows the presence of the monopole chains confirming the physical picture described above.

One can suggest that in the confinement phase the monopole chains are percolating as in Figure 5 (so that the monopoles are relevant to infrared physics similarly to the monopoles in QCD [7]) whereas in the Higgs phase the monopole chains are relatively short. To check this idea we have analyzed the cluster structure and show in Figures 6(a,b) the normalized distributions of the observed mutually disconnected clusters with respect to the number of monopoles and antimonopoles \( N \) they contain \( (D_M) \) and with respect to the number of vortices links (the vortex length \( L \)) \( (D_V) \). One can see that in the confinement phase, \( \kappa < \kappa_c \), the distributions \( D_M(N) \) and \( D_V(L) \) have a two-peak structure. The first peak corresponds to the ultraviolet clusters and is centered at small monopole numbers \( N \) or small length \( L \), respectively. 4 The

---

4Vortex loops without monopoles obviously are not shown in the histogram of \( D_M \).
second peak is due to large (infrared) chain-like clusters which extend all over the volume of the lattice. An example from the confining phase near to the transition is shown in Figure 5. In the confinement phase the infrared peak in both distributions is clearly present whereas in the Higgs phase the infrared peak is absent. This is the signal of deconfinement.

4 Discussion and outlook

Summarizing, we have found that in the presence of a doubly-charged Higgs field the monopoles must form chain-like structures. This offers an explanation of both confinement of single-charged electric particles and string breaking for doubly-charged test particles. This physical picture – observed in our study in the compact Abelian gauge theory with a $Q = 2$ dynamical Higgs field – has a close analogy with non-Abelian gluodynamics where tight correlations between Abelian monopoles and center vortices (each in the respective Abelian projection) have been found \[20\]. Our observation also suggests a natural way for the formation of monopole sheets (instead of chains) in gluodynamics. For example, in the pure SU(2) gauge model (chosen here for simplicity of discussion) the Abelian monopoles are defined with the help of an Abelian gauge, in which the off-diagonal gluons (originally ignored in the Abelian projection) play the role of the doubly-charged matter fields coupled minimally to the leading diagonal gluons. These matter fields may cause the monopole trajectories to be confined inside sheets, a mechanism which in turn should be responsible for the simultaneous occurrence (in the pure gauge model) of confinement for fundamental charges (quarks) and flattening of the potential between adjoint charges (gluons). A more detailed study of monopole chain formation and confining properties of the $Q = 2$ model will be presented elsewhere \[21\].
Acknowledgements

We are grateful to J. Greensite for useful comments to an earlier version of the manuscript. This work is supported by grants RFBR 01-02-17456, DFG 436 RUS 113/73910, RFBR-DFG 03-02-04016, JSPS S04045 and MK-4019.2004.2. E.-M. I. is supported by DFG through the DFG-Forschergruppe “Lattice Hadron Phenomenology” (FOR 465).

References

[1] M. N. Chernodub, E.-M. Ilgenfritz and A. Schiller, Phys. Lett. B 547, 269 (2002); ibid. B 555, 206 (2003); M. N. Chernodub, R. Feldmann, E.-M. Ilgenfritz and A. Schiller, Phys. Rev. D 70, 074501 (2004).

[2] A. M. Polyakov, Nucl. Phys. B 120, 429 (1977).

[3] M. N. Chernodub, E.-M. Ilgenfritz and A. Schiller, Phys. Rev. D 64, 054507 (2001); ibid. D 64, 114502 (2001); Phys. Rev. Lett. 88, 231601 (2002).

[4] A. A. Abrikosov, Sov. Phys. JETP, 32, 1442 (1957); H. B. Nielsen and P. Olesen, Nucl. Phys. B 61, 45 (1973).

[5] N. Parga, Phys. Lett. B 107, 442 (1982); N. O. Agasyan, K. Zarembo, Phys. Rev. D 57, 2475 (1998).

[6] G. ’t Hooft, in Proceedings of the International Conference on High Energy Physics, edited by A. Zichichi (Editrice Compositori, 1976), p. 1225; S. Mandelstam, Phys. Rept. 23 C, 45 (1976); G. ’t Hooft, Nucl. Phys. B190, 455 (1981).

[7] T. Suzuki, Nucl. Phys. B Proc. Suppl. 30, 176 (1993); M. N. Chernodub and M. I. Polikarpov, in Confinement, duality, and nonperturbative aspects of QCD, edited by P. van Baal (Plenum Press, New York, 1998), p. 387 (hep-th/9710205); R.W. Haymaker, Phys. Rept. 315, 153 (1999).

[8] C. DeTar, O. Kaczmarek, F. Karsch and E. Laermann, Phys. Rev. D 59, 031501 (1999); K. Schilling, Nucl. Phys. B Proc. Suppl. 83, 140 (2000); V. G. Bornyakov, M. N. Chernodub, H. Ichie, Y. Koma, Y. Mori, M. I. Polikarpov, G. Schierholz, H. Stüben and T. Suzuki, Prog. Theor. Phys. 112, 307 (2004); G. S. Bali, Th. Duessel, Th. Lippert, H. Neff, Z. Prkacin and K. Schilling, hep-lat/0409137.

[9] G. ’t Hooft, Nucl. Phys. B 153, 141 (1979); G. Mack, in Recent Progress in Gauge Theories, edited by G. ’t Hooft et al. (Plenum Press, New York, 1980), p. ??; H. B. Nielsen and P. Olesen, Nucl. Phys. B 160, 380 (1979).

[10] L. Del Debbio, M. Faber, J. Greensite and S. Olejník, in New Developments in Quantum Field Theory, edited by P. Damgaard and J. Jurkiewicz (Plenum Press, New York, 1998), p. 47 (hep-lat/9708023).

[11] J. Greensite, Prog. Part. Nucl. Phys. 51, 1 (2003) (hep-lat/0301023).
[12] H. Kleinert, F. S. Nogueira, A. Sudbø, Phys. Rev. Lett. **88**, 232001 (2002); A. Sudbø et al., *ibid.*, **89** 226403 (2002); H. Kleinert, F. S. Nogueira, A. Sudbo, Nucl. Phys. **B 666**, 361 (2003).

[13] J. Smiseth, E. Smorgrav, F. S. Nogueira, J. Hove and A. Sudbo, Phys. Rev. **B 67**, 205104 (2003).

[14] E. Fradkin and S. H. Shenker, Phys. Rev. **D 19** (1979) 3682; M. B. Einhorn and R. Savit, Phys. Rev. **D 17** (1978) 2583; *ibid.* **D 19** (1979) 1198.

[15] V. L. Berezinsky, Sov. Phys. JETP **32**, 493 (1971); J. M. Kosterlitz, D. Thouless, J. Phys. **C 6**, 1181 (1973).

[16] M. I. Polikarpov, U. J. Wiese and M. A. Zubkov, Phys. Lett. **B 309**, 133 (1993).

[17] M. N. Chernodub, F. V. Gubarev, M. I. Polikarpov, Phys. Lett. **B 416**, 379 (1998).

[18] L. Del Debbio, M. Faber, J. Greensite and S. Olejnik, Phys. Rev. **D 55**, 2298 (1997); R. Bertle, M. Faber, J. Greensite and S. Olejnik, JHEP **9903**, 019 (1999).

[19] M. N. Chernodub, M. I. Polikarpov, M. A. Zubkov, Nucl. Phys. **B Proc. Suppl. 34**, 256 (1994).

[20] J. Ambjorn, J. Giedt and J. Greensite, JHEP **0002**, 033 (2000); A. V. Kovalenko, M. I. Polikarpov, S. N. Syritsyn and V. I. Zakharov, [hep-lat/0402017](http://arxiv.org/abs/hep-lat/0402017); V. I. Zakharov, in: H. Suganuma, N. Ishii, M. Oka, H. Enyo, T. Hatsuda, T. Kunihiro and K. Yazaki (Eds.), Color Confinement and Hadrons in Quantum Chromodynamics, World Scientific, Singapore, 2004, p. 112, [hep-ph/0309301](http://arxiv.org/abs/hep-ph/0309301).

[21] M. N. Chernodub, R. Feldmann, E.-M. Ilgenfritz and A. Schiller, in preparation.