Complexity as the driving force for glassy transitions

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Abstract. The glass transition is considered within two toys models, a mean field spin glass and a directed polymer in a correlated random potential.

In the spin glass model there occurs a dynamical transition, where the system condenses in a state of lower entropy. The extensive entropy loss, called complexity or information entropy, is calculated by analysis of the metastable (TAP) states. This yields a well behaved thermodynamics of the dynamical transition. The multitude of glassy states also implies an extensive difference between the internal energy fluctuations and the specific heat.

In the directed polymer problem there occurs a thermodynamic phase transition in non-extensive terms of the free energy. At low temperature the polymer condenses in a set of highly degenerate metastable states.

1 Introduction

The structural glass transition is said to occur at the temperature $T_g$ where the viscosity equals $10^{14}$ Poise. The question why this transition occurs is often “answered” (more correctly: avoided) by saying that it is a dynamical transition. Surely, there is a continuum of time scales ranging from picoseconds to many years; at experimental time scales there is no equilibrium. Nevertheless, since some 20 decades in time are spanned, one would hope that equilibrium statistical mechanics can be applied in some modified way.

Crudely speaking, the observation time will set a scale. Processes with shorter timescales can be considered in equilibrium; processes with longer timescales are essentially frozen, as if they were random. To provide a (non-equilibrium) thermodynamic explanation of a model glassy transition will be the first subject of the present work.

Intuitively we expect that the resulting free energy is given by the logarithm of the partition sum, provided it has been restricted to those states that can be reached dynamically in the timespan considered. This non-equilibrium free energy will then differ from the standard case, and need not be a thermodynamic potential that determines the internal energy and entropy by its derivatives.
Experimentally one often determines the entropy $S_{\text{exp}}$, and thus the free energy $F_{\text{exp}} = U - TS_{\text{exp}}$, from the specific heat data by integrating $C/T$ from a reference temperature in the liquid phase down to $T$. As long as the cooling rate is finite there remains at zero temperature a residual entropy. In the limit of adiabatically slow cooling it vanishes.

Alternatively, a glass can be seen as a disordered solid. In this description the liquid undergoes a transition to a glass state with extensively smaller entropy. These states are sometimes called “states”, “metastable states”, “components” or, in spin glass theory, “TAP-states”. As the free energy then becomes much larger, it is not so evident from thermodynamic considerations why the system can get captured in such a state with much smaller Gibbs-weight $\sim \exp(-\text{volume})$. The explanation is that the condensed system then has lost part of its entropy, namely the entropy of selecting one out of the many equivalent states. This part, $I_{\hat{a}}$, is called the configurational entropy, complexity or information entropy [1] [2]. Its origin can be understood as follows. When the Gibbs free energy $F_{\hat{a}}$ of the relevant state $\hat{a}$ has a large degeneracy $N_{\hat{a}} \equiv \exp(I_{\hat{a}})$, the partition sum yields $Z = \sum_{a} \exp(-\beta F_{a}) \approx N_{\hat{a}} \exp(-\beta F_{\hat{a}})$, so $F = F_{\hat{a}} - T I_{\hat{a}}$ is the full free energy of the system. The entropy loss arises when the system chooses the state to condense into, since from then on only that single state is observed. [3] As the total entropy $S = S_{\hat{a}} + I_{\hat{a}}$ is continuous, so is the total free energy. For an adiabatic cooling experiment Jäckle has assumed that the weights $p_{a}$ of the states $a$ are fixed at the transition,[1] which implies that the free energy difference between the condensed phase and the liquid is positive and grows quadratically below $T_{c}$. This explains the well known discontinuity in quantities such as the specific heat. However, it may seem unsatisfactory that this higher free energy branch describes the physical state.

We shall first investigate these questions for the dynamical transition of a mean field spin glass model, and then for the static transition of a directed polymer in a correlated potential.

2 The $p$-spin glass

We first analyze these thermodynamic questions within a relatively well understood spin glass model, the mean field $p$-spin interaction spin glass. For a system with $N$ spins we consider the Hamiltonian

$$\mathcal{H} = - \sum_{i_1 \prec i_2 \prec \cdots \prec i_p} J_{i_1 i_2 \cdots i_p} S_{i_1} S_{i_2} \cdots S_{i_p}$$

with independent Gaussian random couplings, that have average zero and variance $J^2 p!/(2N^{p-1})$.

Kirkpatrick and Thirumalai[4] pointed out for the case of Ising spins that for $p = 3$ there is a close analogy with models for the structural glass transition, and that its properties are quite insensitive of the value of $p$ as long as $p > 2$. 


The spherical limit of this model, where the spins are real valued but subject to the spherical condition $\sum_i S_i^2 = N$, is very instructive. It has received quite some attention recently. The static problem was solved by Crisanti and Sommers. [5] There occurs a static first order transition to a state with one step replica symmetry breaking (1RSB) at a temperature $T_g$.

The dynamics of this spherical model was studied by Crisanti, Horner and Sommers (CHS) [6] and Cugliandolo and Kurchan [7]. Both groups find a sharp dynamical transition at a temperature $T_c > T_g$, which can be interpreted on a quasi-static level as a 1RSB transition. This dynamical transition is sharp since in mean field the metastable states have infinite lifetime. For $T < T_c$ one of the fluctuation modes is massless (“marginal”), not unexpected for a glassy state. At $T_c$ there is a lower specific heat.

These dynamical approaches are the equivalent for the spin glass of the mode coupling equations for the liquid-glass transitions. [8] At a critical temperature $T_c > T_g$ a dynamic phase transition has been reported. The presence of a sharp transition has been questioned, however. [9]

CHS integrate $C/T$ to define the “experimental” entropy $S_{exp}$ and the resulting free energy $F_{exp} = U - T S_{exp}$ exceeds the paramagnetic one quadratically. The interpretation of metastable states (“TAP-states”) in this system is discussed in ref. [10] The statistics of those states was considered by Crisanti and Sommers (CS). [11] Assuming that the result of long time dynamics follows through being stuck in the metastable state of highest complexity, they reproduced the “experimental” free energy obtained of CHS. This confirms Jäckle’s prediction of a quadratically higher free energy in the glassy state.

The long-time dynamics of 1RSB transitions fixes $q_0 (= 0$ in zero field), $q_1$ and $x$, which are just the plateau values and the breakpoint of a related Parisi order parameter function, respectively. In $p$-spin models they can simply be derived from a 1RSB replica calculation provided one fixes $x$ by a marginality criterion for fluctuations on the $q_1$ plateau. [4] [6] [7] The present author recently assumed that this is a very general phenomenon. [12] This was motivated by the expectation that a dynamical transition will automatically get trapped in a state with diverging time scale, if present. In a Potts model this then predicts a dynamical transition with marginal $q_0$ plateau and stable $q_1$ plateau.

As the replica free energy is minimized in this procedure, it lies near $T_c^-$ below the paramagnetic value and has a larger slope. Though this is exactly what one expects at a first order phase transition, it is a new result for dynamical glassy transitions. It is the purpose of the present work to discuss the physical meaning of the mentioned free energies.
2.1 The replica free energy

At zero field the 1RSB replica calculation involves the plateau value \( q_1 \) and the breakpoint \( x \). It yields the free energy \[ F_{\text{rep}} = -\frac{\beta J^2}{4} + \frac{\beta J^2}{4} \xi q_1^p \]

\[-\frac{T}{2x} \log(1 - \xi q_1) + \frac{T \xi}{2x} \log(1 - q_1) \]

where \( \xi = 1 - x \). The first term describes the paramagnetic free energy. Here and in the sequel, we omit the \( T = \infty \) entropy. It is a constant, only fixed after quantizing the spherical model, [13] that plays no role in the present discussion. For the marginal solution \( q_1 \) is fixed by equating the lowest fluctuation eigenvalue to zero, which gives

\[
\frac{1}{2} p(p - 1) \beta J q_1^p (1 - q_1)^2 = 1.
\]

The condition \( \partial F / \partial q_1 = 0 \) then yields \( x = x(T) \equiv (p - 2)(1 - q_1) / q_1 \). This dynamical transition sets in at temperature \( T_c = J \{ p(p - 2)q^p / 2(p - 1)q^{p-1} \}^{1/2} \) where \( x \) comes below unity. The same transition temperature follows from dynamics.

2.2 Components

A state, called a component by Palmer, [2] is labeled by \( a = 1, 2, \ldots, N \), and has a local magnetization profile \( m^a_i = \langle S_i \rangle^a \). Its free energy \( F_a \) is a thermodynamic potential that determines the internal energy and the entropy by its derivatives. In the present model \( F_a = F_{\text{TAP}}(m^a) \) is know explicitly. It is a minimum of the "TAP" free energy functional [14] [10] [11]

\[
F_{\text{TAP}}(m_i) = -\sum_{i_1 < \cdots < i_p} J_{i_1 \cdots i_p} m_{i_1} \cdots m_{i_p} - H \sum_i m_i
\]

\[-\frac{NT}{2} \log(1 - q) - \frac{N \beta J^2}{4} (1 + (p - 1)q^p - pq^{p-1}) \]

where \( q = (1/N) \sum_i m_i^2 \) is the self-overlap. The state \( a \) occurs with weight \( p_a \) that is set by the type of experiment one describes. (In practice these weights are usually unknown.) Given the \( p_a \)'s one can define the "component averages" such as \( \overline{F} = \sum_a p_a F_a \), \( \overline{C} = \sum_a p_a C_a \) and even the complexity [1] [2] \( I = -\sum_a p_a \ln p_a \). For any observable, the component overage is the object one obtains when measuring over repeated runs and averaging over the outcomes. According to the Gibbs weight the probability of occurrence is \( p_a = \exp(-\beta F_a(T))/Z \), with \( Z = \sum_a \exp(-\beta F_a) \).

The nice thing of the present model is that many questions can be answered directly. After setting \( \partial F_{\text{TAP}} / \partial m_i = 0 \), we can use this equation to
express $F_a$ in terms of $q_a$ alone. This gives the simple relation $F_a = N f(q_a)$ where
\[ f(q) = \frac{\beta J^2}{4} [-1 + (p - 1)q^p - (p - 2)q^{p-1}] \]
\[ - \frac{Tq}{p(1 - q)} - \frac{T}{2}\log(1 - q) \] (5)

The resulting saddle point equation for $q_a$ coincides with the marginality for $q_1$ given below eq. (2). Since $F_a$ only depends on the selfoverlap $q_a$, it is self-averaging. In the paramagnet one has $m_i = q = 0$, so both eqs. (4) and (5) reproduce the replica free energy $F = -N\beta J^2/4$. From the replica analysis we know that at $T_c^-$ the value of $q_a \approx q_1$ is $q_1 = q_c = (p - 2)/(p - 1)$. The component free energy $F_a = N f(q_c)$ exceeds the free energy of the paramagnet by an extensive amount. As expected from experimental knowledge on glasses, the internal energy is found to be continuous. At $T_c^-$ the free energy difference is solely due to the lower entropy, $S_a = -N\beta J^2/4 - I_c$, where
\[ I_c = N \left( \frac{1}{2}\log(p - 1) + \frac{2}{p} - 1 \right) \] (6)

is the value of complexity at the transition point.

This discussion supports the picture of the glass as a disordered solid, where the entropy of the component the system condenses in, and thus the component average $\overline{S}$, is much smaller than the entropy of the paramagnet. In real glasses this loss of entropy is due to the reduced phase space that arises by trapping of the atoms in a glassy configuration. In the quantized system $\overline{S}$ will vanish at $T = 0$. [13]

2.3 Value of the complexity

Kirkpatrick, Thirumalai and Wolynes [15] were the first to study the role of the complexity for Potts glasses in static situations in the temperature range $T_g < T < T_c$, see also [4] for Ising spin glasses. Statically (that is to say, on timescales $\sim \exp(N)$) the system condenses into a state with higher free energy but with complexity such that the total free energy is exactly equal to the paramagnetic free energy. Here we will investigate the role of the complexity on timescales $\sim N^\gamma$, relevant for the dynamical transition at $T_c$.

The free energies discussed for this problem are plotted in Figure 1.

A simple calculation shows that the ‘experimental’ free energy of CHS and CS, and the marginal replica free energy obtained from eq. (2) [12] have the following connection with the component average free energy $\overline{F} = N f(q_1)$:
\[ F_{exp} = \overline{F} - T I_c \] (7)
\[ F_{repl} = \overline{F} - TI = \overline{F} - \frac{T I_c}{x(T)} \] (8)
Fig. 1. Free energies of a spherical spin glass with random quartet couplings, after subtraction of the paramagnetic value. a) Marginal replica free energy b) Static replica free energy c) “Experimental” free energy, obtained by integrating $C/T$ and by analysis of the degeneracy of the TAP states.

Since $x(T_c) = 1$ both expressions are at $T_c$ equal to the paramagnetic free energy.

In order to trace back the difference between (7) and (8) we have decided to redo the analysis of the TAP equations. Hereto we consider the generalized partition sum

$$Z_u = \sum_a e^{-u\beta F_a(T)} \equiv e^{-\beta F_u}$$

(9)

For $u = 0$ we thus calculate the total number $N$ of TAP-states, while for $u = 1$ we consider their partition sum. The sum over the TAP states can be calculated using standard approaches. A 1RSB pattern is assumed for the 6 order parameters. For instance, $q_{\alpha \beta} = (1/N) \sum_i m_i^\alpha m_i^\beta$ takes the values $q_d$ for $\alpha = \beta$ and $q_1$ for $\alpha \neq \beta$ both inside a $\tilde{x} \ast \tilde{x}$ diagonal block of the 1RSB Parisi matrix, while vanishing outside these blocks. At fixed breakpoint $\tilde{x}$ the 12-dimensional saddle point can be found explicitly. For the long time limit of the dynamical approach the marginality condition should be taken, [12] in the form given in eq. (3). As expected, the above replica expression for $q_1$ is found back as solution of $\partial f(q_d)/\partial q_d = 0$ at $q_d = q_1$. The result $q_1 = q_d$ asserts that the mutual overlap between different states in the same cluster is equal to the selfoverlap. Like in the replica calculation of the ordinary partition sum, $\tilde{x}$ can still take any value. In analogy with the marginal replica calculation of
The complexity as the driving force for glassy transitions

eq (2), we expect \( \tilde{x} \) to be fixed by the vanishing of a fluctuation eigenvalue. We have therefore analyzed that \( 12 \times 12 \) longitudinal fluctuation matrix at marginality. For any value of \( \tilde{x} \) it automatically has 3 zero eigenvalues, proving the marginality. Another eigenvalue vanishes for \( \tilde{x} = 0 \), \( \tilde{x} = 1 \) (twice) and for \( \tilde{x} = x(T)/u \).

From this we infer that \( \tilde{x} = x(T)/u \), and thus \( I = uI_c/x(T) \). In our case \( u = 1 \) it just implies that the calculated complexity is the replica value \( I \), not \( I_c \), the one of CS. This leads to the conclusion that nothing went wrong in the replica calculation of the dynamical phase transition: the replica free energy is a generating function for the mean field equations, and its saddle point value is the logarithm of the partition sum.

This conclusion has been supported by a calculation for the spherical p-spin glass in a transverse field \( \Gamma \). [17] In that extension of the model there again occurs a dynamical transition from the paramagnet to a 1RSB spin glass state, at transition temperature \( T_c(\Gamma) \). The paramagnet of this model is non-trivial. There is a first order transition line (that we call pre-freezing line) separating regions with large and small ordering in the z-direction. [18] This line intersects the dynamical PM-1RSB transition line at a point \( (T^*, \Gamma^*) \). Beyond this point there occurs a first order PM-SG transition with a finite latent heat. We expect the location of the transition line to follow from matching of free energies. The replica free energy is indeed suited for that, while the ‘experimental’ free energy does not lead to a meaningful match.

The free energy is \( F_{\text{repl}} \) is the physical one, in the sense that it takes into account the correct value of the complexity. Nevertheless the increase of complexity, \( I \sim 1/T \) for low \( T \), remains to be explained.

### 2.4 Specific heat versus energy fluctuations

It would be nice to have a measurable quantity that probes the multitude of states. One object that should be accessible, at least numerically, is the specific heat. The standard expression \( C = dU/dT = \sum_a d(p_a U_a)/dT \) is likely to differ from the component average fluctuations of the internal energy:

\[
\overline{C} = \sum_a C_a = \sum_a p_a dU_a/dT = \beta^2 \sum_a p_a \langle \Delta U_a^2 \rangle. 
\]

The interesting question is whether their difference is extensive. Based on experience in a toy model, [19] we think it generally is in systems with 1RSB. Since in the present model the energy fluctuations are too small at \( H = 0 \), [20] it can only occur in a field.

From the internal energy in a small field we obtain

\[
\frac{1}{N} C(T, H) = \frac{1}{2} \beta^2 J^2 (1 + (p - 1)q^p - pq^{p-1}) \\
- \beta^2 H^2 \frac{(p - 1)^2(p - 2)(1 - q)^2}{p(pq + 2 - p)}
\]

(10)

On the other hand, a short calculation shows that \( C_a = -T d^2 F_a/dT^2 \) remains only a function of \( q_a \) at the marginal point, which in the present
model takes the field-independent value \( q_1 = q_d \). This implies that \( C \) is field-independent as well, thus satisfying the Parisi-Toulouse hypothesis [21]. Interestingly enough, we find \( C < \bar{C} \), whereas Palmer derives the opposite at equilibrium. Our reversed “dynamical” inequality is a new result that is due to the marginality.

The reversed dynamical inequality occurs due to non-equilibrium effects. We conjectured that it generally takes place outside equilibrium, for instance in cooling experiments above \( T_c \) in the three dimensional Edwards-Anderson spin glass. Some numerical support for this behavior was found, see [22].

3 Directed polymer in a correlated random potential

We introduce a new, simple model with a static glassy transition. Consider a directed polymer (or an interface without overhangs) \( z(x) \) in the section \( 1 \leq x \leq L \) and \( 1 \leq z \leq W \) of the square lattice with unit lattice constant. In the Restricted Solid-on-Solid approximation the interface can locally be flat \( (z(x+1) = z(x)) \) or make a single step \( (\Delta z = \pm 1) \); larger steps are not allowed. The polymer is subject to periodic boundary conditions \( z(0) = z(L) \) and we allow all values of \( z(0) \).

Further there is a random energy cost \( V(z) \) per element of the polymer at height \( z \). Note that this is a correlated random potential, with energy barriers parallel to the \( x \)-axis.

3.1 The partition sum

The partition sum of this system can be expressed in the eigenvalues of the tridiagonal transfer matrix \( T \) that has diagonal elements \( \exp(-\beta V(z)) \) and off-diagonal elements \( \exp(-\beta J) \)

\[
Z = \text{tr} e^{-\beta \mathbf{H}} = \text{tr} T^L = \sum_{w=1}^{W} (\Lambda_w)^L \tag{11}
\]

For a pure system \( (V(z) = 0 \text{ for all } z) \) at temperature \( T = 1/\beta \) Fourier analysis tells that for small momentum \( \Lambda(k) = \Re(1 + e^{-\beta J + i k})/(1 - e^{-\beta J + i k}) \approx \exp(-\beta \mathbf{f}_B - D k^2/(2\pi^2)) \) with bulk free energy density \( \mathbf{f}_B \) and diffusion coefficient \( D \) that can be simply read off and are temperature dependent.

We shall consider the situation of randomly located potential barriers parallel to the \( x \)-axis. Here we assume binary disorder, so \( V(z) = 0 \) with probability \( p = \exp(-\mu) \) or \( V(z) = V_1 > 0 \) with probability \( 1 - p \). Eq. (11) is dominated by the largest eigenvalues. It is well known that they occur due to Lifshitz-Griffiths singularities. These are due to lanes of width \( \ell \gg 1 \) in which all \( V(z) = 0 \), bordered by regions with \( V(z) \neq 0 \). These dominant configurations are the “components”, “TAP states” or “metastable states” of our previous discussion. The eigenfunction centered around \( z_0 \) has inside
the lane the approximate form $\cos[\pi(z - z_0)/\ell]$ while it decays essentially exponentially outside due to the disorder. These states can thus be labeled by $a = (z_a, \ell_a)$. Since $k \to \pi/\ell$ the free energy of this state follows as

$$
\beta F_\ell \equiv -L \ln \Lambda_\ell \approx \beta f_B L + \frac{DL}{2\ell^2} \quad (12)
$$

The typical number of regions with $\ell$ successive sites with $V = 0$ is $N_\ell = W(1-p)^2 p^\ell$. We now choose $W = \exp(\lambda L^{1/3})$ so the states with width $\ell$ have a configurational entropy or complexity $I_\ell \equiv \ln N_\ell \approx \lambda L^{1/3} - \mu \ell$.

### 3.2 The TAP-partition sum

For large $L$ we may restrict the partition sum to these dominant states. We thus evaluate, instead of eq. (11), the ‘TAP’ partition sum

$$
Z = \sum_\ell N_\ell e^{-\beta F_\ell} \quad (13)
$$

Note that it is obtained by simply omitting the contributions of states with low eigenvalue (high free energy). The total free energy

$$
\beta F = -\ln Z = \beta f_B L - \lambda L^{1/3} + \mu \ell + \frac{DL}{2\ell^2} \quad (14)
$$

has to be optimized in $\ell$. The largest $\ell$ which occurs in the system can be estimated by setting $N_\ell \approx 1$, yielding

$$
\ell_{\text{max}} = \frac{\lambda L^{1/3}}{\mu} \quad (15)
$$

It is a geometrical length, independent of $T$. Let us introduce $\tilde{D} = D\mu^2/\lambda$. The free energy of this state reads

$$
\beta F = \beta f_B L + \frac{1}{2} \lambda L^{1/3} \tilde{D} \quad (16)
$$

At low enough $T$ the optimal length is smaller than $\ell_{\text{max}}$,

$$
\ell = \left( \frac{DL}{\mu} \right)^{1/3} = \tilde{D}^{1/3} \ell_{\text{max}} \quad (17)
$$

The free energy of this phase is

$$
\beta F = \beta f_B L + \frac{1}{2} \lambda L^{1/3}(3\tilde{D}^{1/3} - 2) \quad (18)
$$

For $\tilde{D} > 1$ ($T > T_g$) the interface is in an essentially non-degenerate state. For $\tilde{D} < 1$ it lies in one of the $N_\ell \gg 1$ relevant states, which is reminiscent to a glass. So the model has a glassy transition at $\tilde{D} = 1$. 
The internal energy of a state of width $\ell$ is

$$U_\ell = u_B L + \frac{L}{2\ell^2} \frac{\partial D}{\partial \beta} = u_B L + \left(\frac{\lambda L}{D^2}\right)^{1/3} \frac{1}{2} \frac{\partial \tilde{D}}{\partial \beta}$$  \hfill (19)

At $\tilde{D} = 1$ this coincides with the paramagnetic value, simply because $\ell \to \ell_{\text{max}}$. It is easily checked that the free energy (18) is a thermodynamic potential, and yields the same value for $U$. At the transition it branches off quadratically from (16). In the glassy phase the specific heat

$$C = \frac{dU}{dT} = c_B L + \frac{L}{2\ell^2} \partial_T \partial_\beta D + \frac{1}{3} (\lambda L D^{-5})^{1/3} (T \partial_T \tilde{D})^2$$  \hfill (20)

exceeds the component averaged specific heat $\overline{C} = Lc_B + (L/2\ell^2)\partial_T \partial_\beta D$. In contrast to previous model, the specific heat is larger in the glassy phase than in the paramagnet. This is because the free energy is lower.

### 3.3 On overlaps and hierarchy of phase space

In a given realization of disorder we define the ‘overlap’ of two states $a$ and $b$, centered around $z_a$ and $z_b$, respectively, as

$$q_{ab} = \lim_{t \to \infty} \langle \delta_{z(0),z_a} \delta_{z(t),z_b} \rangle$$  \hfill (21)

In the high temperature phase there is one non-degenerate state, so $P(q) = \delta(q - q_1)$. In the glassy phase we expect that $q_{ab} = q_1$ for all optimal states $(a, b)$ at temperature $T$. The reason is that at thermodynamic equilibrium the whole phase space can be traversed, and negligible time is spent in non-optimal states. If so, then though there are many states, one still has $P(q) = \delta(q - q_1)$ and there is no replica symmetry breaking. This is standard for equilibrium situations without frustration.

This puts forward the picture of replica symmetry breaking and hierarchy of phase space being a dynamical effect. At given timescale only some nearby states can be reached, “states within the same cluster”. At larger times other clusters can be reached, and for times larger than the ergodic time of a large but finite system, all states are within reach. Only in the thermodynamic limit phase space splits up in truly disjoint sets. To investigate the validity of this picture in detail, one should solve the dynamics of the polymer.

### 3.4 The polymer model at $\tilde{T} = 1/T$

The comparison to the $p$-spin model is most direct when we compare the $p$-spin model at temperature $T$ with the polymer model at temperature $\tilde{T} = 1/T$. In this interpretation, coming from high $\tilde{T}$, the polymer undergoes a gradual freezing into TAP states. This truly becomes relevant when the domain size is of order $\ell \sim L^{1/3}$, where the complexity starts to be smaller.
than \( \log W = \lambda L^{1/3} \). This gradual freezing shows explicitly that the dynamical transition, as found in the mean field \( p \)-spin glass, is smeared in finite dimensions.

For \( \tilde{T} \) going down to \( \tilde{T}_c \), the polymer gets captured in states with free energy closer and closer to the lowest free energy state available at that temperature.

As in 1RSB spin glasses, the complexity also vanishes to leading order for \( \tilde{T} \downarrow \tilde{T}_c \). In the low \( \tilde{T} \) phase the complexity is no longer of order \( L^{1/3} \). This is similar to the low \( T \) phase of the static \( p \)-spin model, where the complexity is non-extensive in the glassy phase.

When considered as function of \( \tilde{T} \), the specific heat makes a downward jump when cooling the system from large \( \tilde{T} \) below \( \tilde{T}_c \). The absence of a sharp dynamical transition and the vanishing of the complexity that occurs in this polymer model as function of \( \tilde{T} \) are very analogous to the expected behavior of realistic glasses.

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This happens since at $H = 0$ its (free) energy fluctuations are not $O(\sqrt{N})$ but $O(1)$, as can be seen by expanding the result for $\ln[Z^n]_\text{av}$ to order $n^2$. For $H = 0$ there appear no terms of order $n^2 N$; for $H \neq 0$ they do appear.

Such behavior has been observed for $T > T_g$ in a numerical cooling experiment in the 3d Edwards-Anderson model. (H. Rieger, private communication, April 1995)