Generalized Rough Digraphs and Related Topologies

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Abstract. The primary objective of this paper, is to introduce eight types of topologies on a finite digraphs and state the implication between these topologies. Also we used supra open digraphs to introduce a new types for approximation rough digraphs.

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1. Introduction.

Rough set theory was introduced by Zdzislow Pawlak in 1982 [1]. He presented the conception of rough set inherently as a mathematical method to manipulate inexactness, uncertainty and vagueness in datagram analyses. This theory is an stretch of set theory for the studying of clearheaded systems diacritical by inadequate and incompletely information. The theory has found implementation in many domains, such as medicine, pharmacy, engineering and others. Furthermore, the prospered implementing of rough set theory in a diversity of problems has abundantly shown its benefit. A specific using of the theory is that property depreciating in databases. Giving a dataset with discretionary property weightings, it is tolerable to existing a subset of the original property that are the bulk informative. Rough set theory treated with the approximating of an arbitrarily subset of universe by depending on two observable or defined subsets, these subsets are named lower approximation and upper approximations, by utilization the terminology of these subsets in rough set theory knowing furtive in info regimes may be unraveled and manifested in the format of resolution norms [2]. We built on some of the results in [3], [4], [5], [6], [7], [8] and [9].

2. Preliminaries

In this part, we present some of essential notions in rough theory and peculiarities of lower approximation and upper approximation which are useful for our study.
Definition 2.1. [10] Let $X$ be non-empty set and $\tau$ be a collection of subsets of $P(X)$, the collection $\tau$ is said to be a topology on $X$ if $\tau$ satisfies:
(a) $X \in \tau, \emptyset \in \tau$.
(b) $\tau$ is closed within finite intersection.
(c) $\tau$ is closed within arbitrarily union.

If $\tau$ is a topology on $X$, then the pair $(X, \tau)$ is called a topological space. In this space, the subsets of $X$ which belong to $\tau$ are dubbed open sets, while the closed sets is represented by the supplement of the subsets of $X$ which belong to $\tau$ (that is the complement of open sets).

The approximation of lower and upper of a set is the basic conception of rough set theory, the approximation of space is the formalized categorization of acquaintance regarding the interesting domain. The partitioning represents a topological space, that topological space named approximation space and symbolized by $K = (X, R)$, so that $X$ is a set named space or universe while $R \subseteq X \times X$ is represented by an indescribable equivalence relation [2]. In the relation $R$, the equivalence classes are savvied blocks, grained or primary sets too. The equivalence class which includes $x \in X$ denoted by $R_x$.

Definition 2.2. [11] Let $K = (X, R)$ be an approximation space and $S$ is a subset of $X$, then the lower and the upper approximation of $S$ denoted consecutively by $L(S), U(S)$ and defined by

$$L(S) = \{x \in X; R_x \subseteq S\}, \quad U(S) = \{x \in X; R_x \cap S \neq \emptyset\}.$$  

According to the lower and upper approximations of a subset $S$ of $X$, $X$ can be dichotomizes in to three discrete areas, positive area (briefly $POSR(S)$), negative area (briefly $NEG_S(S)$) and boundary area (briefly $BR_S(S)$), where they are defined by

$$POSR(S) = L(S), \quad NEG_S(S) = X - U(S), \quad BR_S(S) = U(S) - L(S)$$

If $K = (X, R)$ be an approximation space, where $S$ and $F$ be two subsets of the universe $X$, the following properties of the Pawlak’s rough sets [1, 12],

\[
\begin{align*}
(L1) \quad & L(S) = [U(S)]^c \\
(L2) \quad & L(X) = X \\
(L3) \quad & L(S \cap F) = L(S) \cap L(F) \\
(L4) \quad & L(S \cup F) \supseteq L(S) \cup L(F) \\
(L5) \quad & \text{If } S \subseteq F \text{ then, } L(S) \subseteq L(F) \\
(L6) \quad & L(\emptyset) = \emptyset \\
(L7) \quad & L(S) \subseteq S \\
(L8) \quad & L(L(S)) = L(S) \\
(L9) \quad & L(U(S)) = L(S)
\end{align*}
\]

\[
\begin{align*}
(U1) \quad & U(S) = [L(S)]^c \\
(U2) \quad & U(X) = X \\
(U3) \quad & U(S \cup F) = U(S) \cup U(F) \\
(U4) \quad & U(S \cap F) \subseteq U(S) \cap U(F) \\
(U5) \quad & \text{If } S \subseteq F \text{ then, } U(S) \subseteq U(F) \\
(U6) \quad & U(\emptyset) = \emptyset \\
(U7) \quad & S \subseteq U(S) \\
(U8) \quad & U(U(S)) = U(S) \\
(U9) \quad & U(L(S)) = U(S)
\end{align*}
\]

Definition 2.3. [1] Let $K = (X, R)$ be an approximation space and $S \subseteq X$ then the accuracy measure of $E$ is symbolized by the symbol $A_S(E)$ and is predefined by

$$A_S(E) = 1 - \frac{|L(S)|}{|U(S)|}, \text{ wherein } |U(S)| \neq 0.$$
Also, the accuracy measure dubbed accuracy of approximation.

**Definition 2.4.** [13] A directed graph (briefly d.g.) express a pair \( D = (V(D), E(D)) \) such that \( V(D) \) named vertex set which is non-empty set and \( E(D) \) named edge set represented by ordered pairs of elements of \( V(D) \).

**Definition 2.5.** [14] A subdigraph \( Q = (V(Q), E(Q)) \) of a directed graph \( D = (V(D), E(D)) \) written \( Q \subseteq D \) if \( V(Q) \subseteq V(D) \) and \( E(Q) \subseteq E(D) \).

### 3. Generalized Rough Digraphs and Related Topologies

In this section, we present some of definitions and propositions anent a new types of topologies and the implication among them. Also we give many results, examples were provided.

**Definition 3.1.** Let \( D = (V(D), E(D)) \) is a finite digraph. The \( J \)-degree of \( r \), where \( r \in V(D) \), for all \( J \in \{ \emptyset, I, \cap, \cup, \langle O \rangle, \langle D \rangle, \langle \cap \rangle, \langle \cup \rangle \} \) defined by

\[
\begin{align*}
(a) \quad O-D(r) &= \{ u \in V(D); (r, u) \in E(D) \}, \\
(b) \quad I-D(r) &= \{ u \in V(D); (u, r) \in E(D) \}, \\
(c) \quad \cap-D(r) &= O-D(r) \cap I-D(r), \\
(d) \quad \cup-D(r) &= O-D(r) \cup I-D(r), \\
(e) \quad <\langle O \rangle\rangle-D(r) &= \cap_{r \in O-D(r)} O-D(r), \\
(f) \quad <\langle D \rangle\rangle-D(r) &= \cap_{r \in I-D(r)} I-D(r), \\
(g) \quad <\langle \cap \rangle\rangle-D(r) &= O-D(r) \cap I-D(r), \\
(h) \quad <\langle \cup \rangle\rangle-D(r) &= O-D(r) \cup I-D(r).
\end{align*}
\]

**Definition 3.2.** Let \( D = (V(D), E(D)) \) is a finite digraph and \( \theta_J : V(D) \rightarrow P(V(D)) \) be a mapping which assigns for all \( r \in V(D) \) its \( J \)-degree in \( P(V(D)) \). The pair \((D, \theta_J)\) is namable \( J \)-degree space (concisely \( J\)-DS).

**Theorem 3.3.** If \((D, \theta_J)\) is \( J\)-DS, then the a family

\[
\tau_J = \{ V(Q) \subseteq V(D); \text{ for each } r \in V(Q), J-D(r) \subseteq V(Q) \},
\]

for all \( J \in \{ \emptyset, I, \cap, \cup, \langle O \rangle, \langle D \rangle, \langle \cap \rangle, \langle \cup \rangle \} \) is a topology on \( D \).

**Proof.** For all \( J \in \{ \emptyset, I, \cap, \cup, \langle O \rangle, \langle D \rangle, \langle \cap \rangle, \langle \cup \rangle \} \). Clearly, \( V(D), \emptyset \in \tau_J \).

Let \( M, Q \in \tau_J \) and \( r \in V(M) \cap V(Q) \), then \( r \in V(M) \) and \( r \in V(Q) \), which implies that \( J-D(r) \subseteq V(M) \) and \( J-D(r) \subseteq V(Q) \), therefore \( J-D(r) \subseteq V(M) \cap V(Q) \) and then \( M \cap Q \in \tau_J \).

Let \( Q_i \in \tau_J \) for each \( i \in I \), and \( r \in \cup_i V(Q_i) \), which mean that there exists \( i_0 \in I \) where \( r \in V(Q_{i_0}) \subseteq \cup_i V(Q_i) \), therefore \( J-D(r) \subseteq V(Q_{i_0}) \subseteq \cup_i V(Q_i) \) this implies \( J-D(r) \subseteq \cup_i V(Q_i) \) and so \( \cup_i V(Q_i) \in \tau_J \).

**Example 3.4.** If \( D = (V(D), E(D)) \) is a finite digraph such that \( V(D) = \{ r_1, r_2, r_3, r_4 \} \), \( E(D) = \{ (r_1, r_2), (r_2, r_3), (r_2, r_4), (r_3, r_4), (r_4, r_3), (r_4, r_1) \} \).

![Diagram](image)

\[ \text{Diagram Image} \]
Figure 1: digraph given in Example 3.4.

Then, \(O-D(r_i) = \{r_1, r_e\}, \ O-D(r_2) = \{r_1, r_3\}, \ O-D(r_3) = \{r_3, r_e\}, \ O-D(r_e) = \{r_1\}.\)
\[I-D(r_i) = \{r_1, r_2, r_3\}, \ I-D(r_2) = \emptyset, \ I-D(r_3) = \{r_2, r_3\}, \ I-D(r_e) = \{r_1, r_3\}.\]
\[\cap-D(r_i) = \{r_1, r_3\}, \ \cap-D(r_2) = \emptyset, \ \cap-D(r_3) = \{r_1, r_3\}, \ \cap-D(r_e) = \{r_1\}.\]
\[\cup-D(r_i) = \{r_1, r_2, r_3\}, \ \cup-D(r_2) = \{r_2, r_3, r_e\}, \ \cup-D(r_3) = \{r_1, r_3\}, \ \cup-D(r_e) = \{r_1, r_3\}.\]
\[<O>-D(r_i) = \{r_1\}, \ <O>-D(r_2) = \emptyset, \ <O>-D(r_3) = \{r_3\}, \ <O>-D(r_e) = \{r_4\}.\]
\[<I>-D(r_i) = \{r_1\}, \ <I>-D(r_2) = \{r_2\}, \ <I>-D(r_3) = \{r_3\}, \ <I>-D(r_e) = \{r_1, r_2, r_3\}.\]
\[<\cap>-D(r_i) = \{r_1\}, \ <\cap>-D(r_2) = \emptyset, \ <\cap>-D(r_3) = \{r_3\}, \ <\cap>-D(r_e) = \{r_2\}.\]
\[<\cup>-D(r_i) = \{r_1\}, \ <\cup>-D(r_2) = \{r_2\}, \ <\cup>-D(r_3) = \{r_3\}, \ <\cup>-D(r_e) = \{r_1, r_2, r_3\}.\]

Remark 3.5. From the above results, the implication among different topologies \(\tau_J\) are explained in the following diagram (where \(\rightarrow\) implies \(\subseteq\))

Diagram 1

By using the above topologies, we present eight methods for approximation rough digraphs using interior and closure of the topologies \(\tau_J \) for all \(J \in \{O, I, \cap, \cup, <O>, <I>, <\cap>, <\cup>\}.\)

Definition 3.6. Let \((D, \theta_J)\) be J-DS. The subgraph \(Q \subseteq D\) is called J-open graph if \(V(Q) \in \tau_J\). While the complement of J-open graph is called J-closed graph. The family of every J-closed graphs of a J-DS is predefined by:
\[\Gamma_j = \{V(K) \subseteq V(D); [V(K)]^\# \in \tau_j\}.\]

Definition 3.7. Let \((D, \theta_J)\) be J-DS and \(Q \subseteq D\). The J-lower approximation of \(Q\) and J-upper approximation of \(Q\) are predefined consecutively by
\[L_J(Q) = \cup \{V(M) \in \tau_J: V(M) \subseteq V(Q)\} = J\text{-interior of } Q.\]
\[U_J(Q) = \cap \{V(M) \in \Gamma_j: V(Q) \subseteq V(M)\} = J\text{-closure of } Q.\]

Definition 3.8. Let \((D, \theta_J)\) be J-DS and \(Q \subseteq D\). The, J-positive, J-negative and J-boundary areas of \(Q\) are defined as
\[POS_J(V(Q)) = L_J(V(Q)), \ NEG_J(V(Q)) = V(D) - U_J(V(Q)),\]
\[ B_J(V(Q)) = U_J(V(Q)) - L_J(V(Q)) \]

**Definition 3.9.** Let \((D, \theta_J)\) be J-DS. The subgraph \(Q\) is dubbed \(J\)-exact (definable) graph if

\[ L_J(V(Q)) = U_J(V(Q)) = V(Q). \]

Otherwise is called \(J\)-rough graph.

**Definition 3.10.** Let \((D, \theta_J)\) is J-DS. The \(J\)-accuracy of the approximation of \(Q \subseteq D\) is predefined by

\[ A_J(V(Q)) = \left| \frac{|U_J(V(Q))|}{|U_J(V(Q))|} \right|, \text{ where } |U_J(V(Q))| \neq 0. \]

**Remark 3.11.** Clear that \(0 \leq A_J(V(Q)) \leq 1\) and \(Q\) is \(J\)-exact graph if \(B_J(V(Q)) = \emptyset\) and \(A_J(V(Q)) = 0\). Otherwise \(Q\) is \(J\)-rough.

**Remark 3.12.** From above results, we have a concluding that using of \(\tau \cap \) in construction the approximations of graphs is minutest than \(\tau_D \), \(\tau_I\) and \(\tau_U\). Also the using of \(\tau_<\) in construction the approximations of graphs is minutest than \(\tau_<D\), \(\tau_<I\) and \(\tau_<U\). Moreover, the topologies \(\tau_n\) and \(\tau_<\) are not necessarily comparable.

Now, some properties of the operators \(J\)-lower approximation and \(J\)-upper approximation, will be presented in the next proposition.

**Proposition 3.13.** If \((D, \theta_J)\) is J-DS and \(M, Q \subseteq D\). Then

- (L1) \(L_J(V(Q)) = [U_J(V(Q'))]^\dagger\)
- (L2) \(L_J(V(D)) = V(D), L_J(\emptyset) = \emptyset\)
- (L3) If \(V(M) \subseteq V(Q)\) then, \(L_J(V(M)) \subseteq L_J(V(Q))\)
- (L4) \(L_J(V(M)) \cap V(Q) = L_J(V(M) \cap V(Q))\)
- (L5) \(L_J(V(M)) \cup V(Q) = L_J(V(M) \cup V(Q))\)
- (L6) \(L_J(V(Q)) \subseteq V(Q)\)
- (L7) \(L_J(L_J(V(Q))) = L_J(V(Q))\)

- (U1) \(U_J(V(Q)) = [L_J(V(Q'))]^\dagger\)
- (U2) \(U_J(V(D)) = V(D), U_J(\emptyset) = \emptyset\)
- (U3) If \(V(M) \subseteq V(Q)\) then, \(U_J(V(M)) \subseteq U_J(V(Q))\)
- (U4) \(U_J(V(M) \cap V(Q)) = U_J(V(M) \cap U_J(V(Q)))\)
- (U5) \(U_J(V(M) \cup V(Q)) = U_J(V(M) \cup U_J(V(Q)))\)
- (U6) \(V(Q) \subseteq U_J(V(Q))\)
- (U7) \(U_J(U_J(V(Q))) = U_J(V(Q))\)

Proof. The proof is evident, by employing peculiarities of closure and interior.

The next example explains the comparison between our approach and approach in Yousif and Sara approach [15, 16].

**Example 3.14.** Let \((D, \theta_J)\) be J-DS where \(D = (V(D), E(D))\), \(V(D) = \{r_1, r_2, r_3, r_4\}\) and \(E(D) = \{(r_1, r_3), (r_2, r_3), (r_1, r_2), (r_2, r_1)\}\)
From Yousif and Sara approach [15, 16], we have

\[
\begin{align*}
\Omega^D(r_1) &= \{r_2\}, \quad \Omega^D(r_2) = \{r_3\}, \\
\Omega^D(r_3) &= \{r_1\}, \quad \Omega^D(r_4) = \emptyset.
\end{align*}
\]

\[
\begin{align*}
U^D(r_1) &= \{r_2, r_4\}, \quad U^D(r_2) = \{r_3\}, \\
U^D(r_3) &= \{r_1\}, \quad U^D(r_4) = \emptyset.
\end{align*}
\]

\[
\begin{align*}
\langle \langle \rangle^D(r_1) &= \{r_1\}, \quad \langle \langle \rangle^D(r_2) = \{r_2\}, \\
\langle \langle \rangle^D(r_3) &= \{r_1\}, \quad \langle \langle \rangle^D(r_4) = \emptyset.
\end{align*}
\]

\[
\begin{align*}
\langle \rangle^D(r_1) &= \{r_2, r_3\}, \quad \langle \rangle^D(r_2) = \{r_2\}, \\
\langle \rangle^D(r_3) &= \{r_2\}, \quad \langle \rangle^D(r_4) = \emptyset.
\end{align*}
\]

\[
\begin{align*}
\langle \rangle^D(r_1) &= \{r_2, r_3\}, \quad \langle \rangle^D(r_2) = \{r_2\}, \\
\langle \rangle^D(r_3) &= \{r_2\}, \quad \langle \rangle^D(r_4) = \emptyset.
\end{align*}
\]

\[
\begin{align*}
\tau_{\langle \rangle} &= P(V(D)), \quad \tau_{\langle \rangle} = P(V(D)).
\end{align*}
\]

\[
\begin{align*}
\tau_{\langle \rangle} &= \{V(D), \emptyset, \{r_1\}, \{r_2\}, \{r_3\}, \{r_4\}, \{r_1, r_2\}, \{r_1, r_3\}, \{r_1, r_4\}, \{r_2, r_3\}, \{r_2, r_4\}, \{r_3, r_4\}\}.
\end{align*}
\]

\[
\begin{align*}
\mathcal{F}_{\xi} &= \{V(D), \emptyset, \{r_1\}, \{r_2\}, \{r_3\}, \{r_4\}, \{r_1, r_2\}, \{r_1, r_3\}, \{r_1, r_4\}, \{r_2, r_3\}, \{r_2, r_4\}, \{r_3, r_4\}\}.
\end{align*}
\]

From Yousif and Sara approach [15, 16], we have

\[
\begin{align*}
\mathcal{F}_{\xi} &= \{V(D), \emptyset, \{r_1\}, \{r_2\}, \{r_3\}, \{r_4\}, \{r_1, r_2\}, \{r_1, r_3\}, \{r_1, r_4\}, \{r_2, r_3\}, \{r_2, r_4\}, \{r_3, r_4\}\}.
\end{align*}
\]

\[
\begin{align*}
\Omega_{\xi}(V(D)) &= \{V(D), \emptyset, \{r_1\}, \{r_2\}, \{r_3\}, \{r_4\}, \{r_1, r_2\}, \{r_1, r_3\}, \{r_1, r_4\}, \{r_2, r_3\}, \{r_2, r_4\}, \{r_3, r_4\}\}.
\end{align*}
\]

**Table 1:** $L_d(V(Q))$, $U_d(V(Q))$, $L_{\langle \rangle}(V(Q))$, $U_{\langle \rangle}(V(Q))$, $L_{\langle \rangle}(V(Q))$ and $U_{\langle \rangle}(V(Q))$

| $P(V(D))$ | $L_d(V(Q))$ | $U_d(V(Q))$ | $L_{\langle \rangle}(V(Q))$ | $U_{\langle \rangle}(V(Q))$ | $L_{\langle \rangle}(V(Q))$ | $U_{\langle \rangle}(V(Q))$ |
|-----------|------------|-------------|-----------------|-----------------|-----------------|-----------------|
| $\{r_1\}$ | $\emptyset$ | $\{r_2, r_3\}$ | $\{r_1\}$ | $\{r_1\}$ | $\{r_1\}$ | $\{r_1\}$ |
| $\{r_2\}$ | $\emptyset$ | $\{r_2\}$ | $\{r_2\}$ | $\{r_2\}$ | $\{r_2\}$ | $\{r_2\}$ |
| $\{r_3\}$ | $\emptyset$ | $\{r_3\}$ | $\{r_3\}$ | $\emptyset$ | $\{r_3, r_4\}$ |
| $\{r_4\}$ | $\emptyset$ | $\{r_4\}$ | $\emptyset$ | $\{r_4\}$ | $\{r_3, r_4\}$ |
| $\{r_1, r_2\}$ | $\emptyset$ | $\{r_1, r_2, r_3\}$ | $\{r_1, r_2\}$ | $\{r_1, r_2\}$ | $\{r_1, r_2\}$ | $\{r_1, r_2\}$ |
| $\{r_1, r_3\}$ | $\emptyset$ | $\{r_1, r_3\}$ | $\{r_1, r_3\}$ | $\{r_1, r_3\}$ | $\{r_1, r_3\}$ | $\{r_1, r_3\}$ |
| $\{r_1, r_4\}$ | $\emptyset$ | $\{r_1, r_4\}$ | $\{r_1, r_4\}$ | $\{r_1, r_4\}$ | $\{r_1, r_4\}$ | $\{r_1, r_4\}$ |
| $\{r_2, r_3\}$ | $\emptyset$ | $\{r_2, r_3\}$ | $\{r_2, r_3\}$ | $\{r_2, r_3\}$ | $\{r_2, r_3\}$ | $\{r_2, r_3\}$ |
| $\{r_2, r_4\}$ | $\emptyset$ | $\{r_2, r_4\}$ | $\{r_2, r_4\}$ | $\{r_2, r_4\}$ | $\{r_2, r_4\}$ | $\{r_2, r_4\}$ |
| $\{r_3, r_4\}$ | $\emptyset$ | $\{r_3, r_4\}$ | $\{r_3, r_4\}$ | $\{r_3, r_4\}$ | $\{r_3, r_4\}$ | $\{r_3, r_4\}$ |
Remark 3.14. The above proposition and example can be considered as one of the difference between our approaches and Yousif and Sara approach [15]. So, we can say that our approach is the actual circularization of Yousif and Sara approach because the numbers of exact graph in our approach more than Yousif and Sara approach.

Definition 3.15. Let \((D, \theta_J)\) be \(J\)-DS. Then for each \(J \in \{O, I, \cap, \cup, <O>, <I>, <\cap>, <U>\}\), the subgraph \(Q \subseteq D\) is named:

(a) \(J\)-regular open (shortly \(R_J\)-open) if \(V(Q) = Int_J(\text{Cl}_J(V(Q)))\)
(b) \(J\)-pre-open (shortly \(P_J\)-open) if \(V(Q) \subseteq Int_J(\text{Cl}_J(V(Q)))\)
(c) \(J\)-semi-open (shortly \(S_J\)-open) if \(V(Q) \subseteq Cl_J(Int_J(V(Q)))\)
(d) \(\omega_J\)-open if \(V(Q) \subseteq Int_J(\text{Cl}_J(Int_J(V(Q))))\)
(e) \(b_J\)-open if \(V(Q) \subseteq Int_J(\text{Cl}_J(V(Q))) \cup Cl_J(Int_J(V(Q)))\)
(f) \(\beta_J\)-open if \(V(Q) \subseteq Cl_J(Int_J(\text{Cl}_J(V(Q))))\)

Remark 3.16.

(a) The above graphs are dubbed \(J\)-supra open graphs and the collection of \(J\)-supra open graphs of \(D\) symbolized by the symbol \(K_JO(D)\) for every \(K = R, P, S, \omega, \alpha, \beta\).
(b) The \(J\)-supra closed graphs is the complement of the \(J\)-supra open graphs where the families of \(J\)-supra closed graphs of \(D\) symbolized by the symbol \(K_JC(D)\) for every \(K = R, P, S, \omega, \alpha, \beta\).
(c) The family \(\omega_JO(D)\) idealizes a topology on \(D\), furthermore, the \(J\)-supra interior and the \(J\)-supra closure idealizes the \(J\)-interior and the \(J\)-closure respectively.

Remark 3.17. The implication between the topologies \(\tau_J(f)(\text{consecutively } \Gamma_J)\) and the precedent collection of \(J\)-supra open graphs (consecutively \(J\)-supra closed graphs) are explained the next diagram (where \(\rightarrow\) implies \(\subseteq\))

Diagram 2
By usage the $J$-supra open graph, we can present new causeways for approximation rough graphs using the $J$-supra interior and the $J$-supra closure for all topology of $\tau$, as the next definitions

**Definition 3.18.** Let $(D, \theta, J)$ be $J$-DS and $Q \subseteq D$. Then for all $J \in \{O, I, \cap, \cup, <O>, \langle J, \langle \cap, \langle \cup, \rangle \rangle \rangle \}$ and $K \in \{R, P, b, S, \alpha, \beta\}$, the $J$-supra lower approximation of $Q$ and $J$-supra upper approximation of $Q$ are predefined consecutively by

$$L^K_J(V(Q)) = \bigcup \{V(M) \in K, O(D); V(M) \subseteq V(Q)\} = J\text{-supra interior of } Q,$$

$$U^K_J(V(Q)) = \bigcap \{V(M) \in K, C(D); V(Q) \subseteq V(M)\} = J\text{-supra closure of } Q.$$

**Definition 3.19.** Let $(D, \theta, J)$ be $J$-DS and $Q \subseteq D$. Then for all $J \in \{O, I, \cap, \cup, <O>, \langle J, \langle \cap, \langle \cup, \rangle \rangle \rangle \}$ and $K \in \{R, P, b, S, \alpha, \beta\}$, the $J$-supra positive, $J$-supra negative and $J$-supra boundary areas of $Q$ are predefined consecutively by

$$POS^K_J(V(Q)) = L^K_J(V(Q)),\quad NEG^K_J(V(Q)) = V(D) - U^K_J(V(Q)),$$

$$B^K_J(V(Q)) = U^K_J(V(Q)) - L^K_J(V(Q))$$

**Definition 3.20.** Let $(D, \theta, J)$ be $J$-DS and $Q \subseteq D$. Then for all $J \in \{O, I, \cap, \cup, <O>, \langle J, \langle \cap, \langle \cup, \rangle \rangle \rangle \}$ and $K \in \{R, P, b, S, \alpha, \beta\}$, the $J$-supra accuracy of the $J$-supra approximations of $Q \subseteq D$ is predefined by

$$A^K_J(V(Q)) = \frac{|L^K_J(V(Q))|}{|U^K_J(V(Q))|}, \text{ where } |U^K_J(V(Q))| \neq 0.$$  

It is clear that $0 \leq A^K_J(V(Q)) \leq 1$.

The essential properties of the $J$-supra approximations are mentioned in the next proposition.

**Proposition 3.21.** Let $(D, \theta, J)$ be $J$-DS and $Q, M \subseteq D$. Then, for every $J \in \{O, I, \cap, \cup, <O>, \langle J, \langle \cap, \langle \cup, \rangle \rangle \rangle \}$ and $K = R, P, b, S, \alpha, \beta$.

(L1) $L^K_J(V(Q)) = [U^K_J(V(Q))]^c$,

(L2) $L^K_J(V(D)) = V(D), L^K_J(\emptyset) = \emptyset$,

(L3) If $V(Q) \subseteq V(M)$ then,

(L4) $L^K_J(V(Q)) \subseteq L^K_J(V(M))$,

(L5) $L^K_J(V(Q) \cup V(M)) = L^K_J(V(Q)) \cup L^K_J(V(M))$,

(U1) $U^K_J(V(Q)) = [U^K_J(V(Q))]^c$,

(U2) $U^K_J(V(D)) = V(D), U^K_J(V(\emptyset)) = \emptyset$,

(U3) If $V(Q) \subseteq V(M)$ then,

(U4) $U^K_J(V(Q) \cap V(M)) \subseteq U^K_J(V(Q)) \cap U^K_J(V(M))$,

(U5) $U^K_J(V(Q) \cup V(M)) = U^K_J(V(Q)) \cup U^K_J(V(M))$.
\[
(L6) \quad L^J_f (V(Q)) \subseteq V(Q), \quad (U6) \quad V(Q) \subseteq U^J_f (V(Q)),
\]
\[
(L7) \quad L^J_f (U^J_f (V(Q))) = L^J_f (V(Q)), \quad (U7) \quad U^J_f (U^J_f (V(Q))) = U^J_f (V(Q)).
\]

**Remark 3.21.** The collections of all regular open graphs of \( D, R_JO(D), \) are smaller than the topologies \( \tau_J, \) (that is \( R_JO(D) \) idealized a special case of the topologies \( \tau_J \)) hence we will not using it in our approaches.

The \( J \)-supra approximations are extremely interesting in rough context because the it can assists in the detecting of unobserved information in datagram collected from real life implementations. Furthermore, the utilization of the \( J \)-supra formats can assists for more developments in the notional and implementations of rough graphs, because the boundary area will decreased or abolished by increasing the lower approximation and decreasing the upper approximation, as the following results explained.

**Proposition 3.22.** Let \((D, \theta_J)\) be \( J-DS \) and \( Q \subseteq D. \) Then, for every \( J \in \{ O, I, \cap, \cup, <O>, <I>, <\cap>, <\cup> \} \) and \( K \in \{ R, P, b, S, \alpha, \beta \} \) such that \( K \neq R,\)

\[
L_J(V(Q)) \subseteq L^J_f (V(Q)) \subseteq V(Q) \subseteq U^J_f (V(Q)) \subseteq U_J(V(Q))
\]

Proof. For each \( J \in \{ O, I, \cap, \cup, <O>, <I>, <\cap>, <\cup> \} \) and \( K \in \{ R, P, b, S, \alpha, \beta \} \) such that \( K \neq R, \)

\[
L_J(V(Q)) = \bigcup \{ V(M) \in K_JO(D); V(M) \subseteq V(Q) \}
\]

By Proposition (2)

\[
L^J_f (V(Q)) = \bigcap \{ V(F) \in K_JC(D); V(Q) \subseteq V(F) \}
\]

\[
U^J_f (V(Q)) = \bigcap \{ V(F) \in \Gamma_J; V(Q) \subseteq V(F) \} \quad \text{since} \quad K_JC(D) \subseteq \Gamma_J,
\]

\[
= U_J(V(Q))
\]

From (1), (2) and (3) we get

\[
L_J(V(Q)) \subseteq L^J_f (V(Q)) \subseteq V(Q) \subseteq U^J_f (V(Q)) \subseteq U_J(V(Q))
\]

**Corollary 3.23.** Let \((D, \theta_J)\) be \( J-DS \) and \( Q \subseteq D. \) Then, for each \( J \in \{ O, I, \cap, \cup, <O>, <I>, <\cap>, <\cup> \} \) and \( K \in \{ P, b, S, \alpha, \beta \} \) such that \( K \neq R, \)

\[
(a) \quad B_J(V(Q)) \subseteq B^J_f (V(Q)), \quad (b) \quad A_J(V(Q)) \subseteq A^J_f (V(Q))
\]

We will presenting the next example to explain the prominence of using \( J \)-supra conception in rough context and to expressing the precedent results.

**Example 3.24.** Let \((D, \theta_J)\) be \( J-DS \) where \( D = (V(D), E(D)), V(D) = \{ r_1, r_2, r_3, r_4 \} \) and \( E(D) = \{(r_1, r_1), (r_1, r_2), (r_2, r_1), (r_2, r_2), (r_3, r_1), (r_3, r_2), (r_3, r_3), (r_3, r_4), (r_4, r_2)\}.\)
Figure 3: digraph given in Example 3.24.

\[ O-D(r_1) = \{r_1, r_2\}, O-D(r_2) = \{r_1, r_2, r_3\}, O-D(r_3) = V(D), O-D(r_4) = \{r_4\}. \]

\[ \tau_O = \{V(D), \emptyset, \{r_1\}, \{r_2\}, \{r_1, r_2\}, \{r_1, r_2, r_3\}\}, \text{ and } I_O = \{V(D), \emptyset, \{r_1\}, \{r_2, r_3\}, \{r_1, r_2, r_3\}\}. \]

We shall calculate the \( J \)-supra approximations for \( J = O \) and \( K = P, b, \beta \).

\[ P_O O(D) = \{V(D), \emptyset, \{r_1\}, \{r_2\}, \{r_1, r_2\}, \{r_1, r_2, r_3\}, \{r_1, r_2, r_4\}, \{r_1, r_2, r_3, r_4\}\}. \]

\[ P_O C(D) = \{V(D), \emptyset, \{r_1\}, \{r_2\}, \{r_1, r_2\}, \{r_1, r_2, r_3\}, \{r_1, r_2, r_4\}, \{r_1, r_2, r_3, r_4\}\}. \]

\[ b_O O(D) = \{V(D), \emptyset, \{r_1\}, \{r_2\}, \{r_1, r_2\}, \{r_1, r_2, r_3\}, \{r_1, r_2, r_4\}, \{r_1, r_2, r_3, r_4\}\}. \]

\[ b_O C(D) = \{V(D), \emptyset, \{r_1\}, \{r_2\}, \{r_1, r_2\}, \{r_1, r_2, r_3\}, \{r_1, r_2, r_4\}, \{r_1, r_2, r_3, r_4\}\}. \]

| \( P(V(D)) \) | \( \tau_O \) | \( P_O \) | \( b_O \) | \( \beta_O \) |
|-----------------|-------------|-------------|-------------|-------------|
| \( \{r_1\} \)   | \( \emptyset \) | \( r_1 \)    | \( r_1 \)    | \( r_1 \)    |
| \( \{r_2\} \)   | \( \emptyset \) | \( r_2 \)    | \( r_1 \)    | \( r_1 \)    |
| \( \{r_3\} \)   | \( \emptyset \) | \( r_3 \)    | \( r_2 \)    | \( r_2 \)    |
| \( \{r_4\} \)   | \( \{r_4\} \) | \( \emptyset \) | \( r_1 \)    | \( r_1 \)    |
| \( \{r_1, r_2\} \) | \( \{r_1, r_2\} \) | \( \emptyset \) | \( r_1 \)    | \( r_1 \)    |
| \( \{r_1, r_3\} \) | \( \{r_1, r_3\} \) | \( \emptyset \) | \( r_1 \)    | \( r_1 \)    |
| \( \{r_1, r_4\} \) | \( \{r_1, r_4\} \) | \( \emptyset \) | \( r_1 \)    | \( r_1 \)    |
| \( \{r_2, r_3\} \) | \( \{r_2, r_3\} \) | \( \emptyset \) | \( r_1 \)    | \( r_1 \)    |
| \( \{r_2, r_4\} \) | \( \{r_2, r_4\} \) | \( \emptyset \) | \( r_1 \)    | \( r_1 \)    |
| \( \{r_3, r_4\} \) | \( \{r_3, r_4\} \) | \( \emptyset \) | \( r_1 \)    | \( r_1 \)    |
| \( \{r_1, r_2, r_3\} \) | \( \{r_1, r_2, r_3\} \) | \( \emptyset \) | \( r_1 \)    | \( r_1 \)    |
| \( \{r_1, r_2, r_4\} \) | \( \{r_1, r_2, r_4\} \) | \( \emptyset \) | \( r_1 \)    | \( r_1 \)    |
| \( \{r_1, r_3, r_4\} \) | \( \{r_1, r_3, r_4\} \) | \( \emptyset \) | \( r_1 \)    | \( r_1 \)    |
| \( \{r_2, r_3, r_4\} \) | \( \{r_2, r_3, r_4\} \) | \( \emptyset \) | \( r_1 \)    | \( r_1 \)    |
Example (3.24) show that there are many rough graphs in extracts of unobserved information in datagram collected from real-life implementations.

| $[r_0, r_1]$ | $\emptyset$ | $[r_0, r_2, r_3]$ | $[r_2]$ | $[r_0, r_3]$ | $[r_2, r_3]$ | $[r_0, r_2]$ | $[r_0, r_1]$ |
|--------------|-------------|------------------|---------|-------------|-------------|-------------|-------------|
| $[r_2, r_3]$ | $[r_2]$     | $V(D)$           | $[r_2, r_3]$ | $[r_2, r_0, r_3]$ | $[r_2, r_0]$ | $[r_2, r_3]$ | $[r_2, r_0]$ |
| $[r_0, r_1]$ | $[r_2]$     | $[r_0, r_1]$    | $[r_2]$   | $[r_2, r_0]$ | $[r_2, r_0]$ | $[r_2, r_1]$ | $[r_2, r_0]$ |
| $[r_0, r_2]$ | $[r_2, r_1]$ | $V(D)$           | $V(D)$   | $V(D)$      | $V(D)$      | $V(D)$      | $V(D)$      |
| $[r_0, r_3]$ | $[r_2, r_2]$ | $V(D)$           | $V(D)$   | $V(D)$      | $V(D)$      | $V(D)$      | $V(D)$      |
| $V(D)$       | $V(D)$      | $V(D)$           | $V(D)$   | $V(D)$      | $V(D)$      | $V(D)$      | $V(D)$      |

From the above table we can notice that:

(a) Implementing the $J$-supra approximations is extremely interesting for obliterating the abstruseness of rough graphs, and this would help to extract and detecting of furtive information in statements aggregated from real-life applications.

(b) The best $J$-supra approach is $\beta_j$, (since $\beta_j$ is minutest than the other kinds of $J$-supra open graphs.

(c) There are many rough graphs in $\tau_O$, but it is $J$-supra exact such as the shadowed graphs.

**Conclusion.**

By employing the $J$-supra open graph, a newfound ways for approximation rough graphs for each topology of $\tau_J$ are presented. Applying $J$-supra approximations helps to extract of unobserved information in datagram collected from real-life implementations. Example (3.24) show that there are many rough graphs in $\tau_O$ it is $J$-supra exact, $\beta_j$ is the best $J$-supra approach since it is more accurate than the other types.

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