Optimal Ordering Policy with Stock-Dependent Demand Rate Under Retailer’s Two Stages Trade Credit Financing Using Discounted Cash Flow (DCF) Approach

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Abstract: Many researchers have assumed one stage trade credit financing. In this study, we considered two levels of trade credit policy using Discounted Cash Flow (DCF) approach. Demand rate is considered to be stock-dependent for the first level (credit demand) and constant for second level (cash demand). Mathematical models are derived under two different circumstances i.e., case I: The permissible delay period is less than or equal to the cycle time and case II: The permissible delay period is greater than or equal to the cycle time for settling the account. An algorithm is provided to determine the optimal order quantity and annual profit. In addition, numerical examples are presented to demonstrate the solution process. Finally, sensitivity analysis of the optimal solution is discussed with respect to different parameters.

Keywords: Discounted Cash Flow, Inventory, Two Level Credit Policy, Credit Linked Demand, Credit Period

Introduction

In classical Economic Order Quantity (EOQ) model, it is assumed that the supplier is paid for the items instantly after they are received. In practice, the supplier permits a certain fixed credit period to settle the account for invigorated retailer’s demand. The permissible delay in payment is helpful to attract new customer and increase sales. Inventory models with credit period were first developed by Goyal (1985) to push aside the difference between the selling price and purchase cost. Dave (1985) modified and extended Goyal (1985) model adding the fact that the selling price is necessarily higher than its purchase cost. Haley and Higgins (1973) established the first model to consider the economic order quantity under conditions of permissible delay in payment with deterministic demand. Shah (1993) considered a stochastic inventory model when delays in payments are permissible. Aggarwal and Jaggi (1995) modified Goyal (1985) model for deteriorating items. Jamal et al. (1997) further extended model (1995) allow for shortages. Chang et al. (2003) developed an EOQ model under supplier credits linked to ordering quantity for deteriorating items. Chung and Huang (2003) presented an Economic Production Quantity (EPQ) model for a retailer where the supplier offers a permissible delay in payments. Teng et al. (2012) presented an EOQ model under trade credit financing with increasing demand. Khanra et al. (2011) developed an EOQ model for time dependent demand when delay in payment is permissible. Many researchers like Chu et al. (1998; Chung et al., 2001; Davis and Gaither, 1985; Mandal and Phaujdar, 1989a; Chang et al., 2001; Chung and Liao, 2004; Saiedy and Moghadam, 2011) worked on inventory model by considering delay in payment. Ouyang et al. (2004) presented an inventory model with non instantaneous receipt under permissible delay in payments. Jaggi et al. (2007) developed the retailer’s optimal ordering policy under two stage trade credits financing using Discounted Cash Flow (DCF) approach.

In real world, the consumption rate is sometimes affected by the stock level. It is usually observed that a large pile of items on large rack in a supermarket will show the customer to purchase more and then generate demand. The consumption rate may fluctuate with the on hand inventory. Yang et al. (2010) presented an inventory model for deteriorating item with stock-dependent demand and partial backlogging. Soni and Shah (2008) established inventory model for retailer when demand is partially constant and partially...
dependent on the stock and the supplier offers progressive credit periods to settle the account. Teng et al. (2011) modified and extended the model (2008) for different situations. Mandal and Phaujdar (1989b) developed a production stock-dependent demand. Two closely related research papers/articles on stock-dependent rate published by Chang et al. (2010). Alfares (2007) established inventory models in which the demand rate depends on the inventory level and storage time-dependent holding cost. Pal et al. (1991) developed a deterministic inventory model assuming that the demand rate is stock-dependent for deteriorating items. Silver and Peterson (1985) observed that a sale at the retail level is proportional to the amount of inventory displayed. Gupla and Vrat (1986) established inventory model in which demand rate to be a function of initial stock level. Some of the related research in this area are by Wee (1995; Goh, 1994; Ray and Chaudhuri, 1997; Mandal and Maiti, 1999; Dye, 2002; Chung and Tsai, 2001; Yan and Cheng, 1998; Sarker et al., 1997) etc.

At present the effect of inflation plays an important role in any type of business. At present developing countries are facing large scale of inflation due to lock off, strike, natural calamities, political disturbances etc. Thus the effect of inflation cannot be disregarded in real word. Hou (2006) derived an inventory model for deteriorating items with stock-dependent consumption rate and shortages under inflation and time value of money discounting over a finite planning horizon. Ouyang et al. (2002) studied the thump of trade credit in the inventory system. Hou and Lin (2009) developed an inventory model to determine an optimal ordering policy for deteriorating item with delayed payment permitted by the supplier under-inflation and time discounting. Other related research papers/articles were considered by Chang (2004; Chung and Liao, 2006; Jaggi and Aggarwal, 1994; Chapman et al., 1985; Daellenbach, 1986; Haley and Higgins, 1973). Jaggi et al. (2007) determined the retailer’s optimal ordering policy under two stage trade credits financing using Discounted Cash Flow (DCF) approach.

Jaggi et al. (2007) developed an inventory model under two levels of trade credit policy by assuming the demand is a function of credit period offered by the retailer to the customer using Discounted Cash Flow (DCF) approach. In this study an attempt is made to formulate the mathematical model for stock-dependent credit demand and constant cash demand. The objective function to be maximized is appraised as the retailer’s net profit of the inventory system. The effect of parameters on the objective function is discussed numerically. An algorithm is provided to validate the proposed model.

The rest of the paper organized as follows. In the next section, we provide the notations and assumptions for the proposed model. Mathematical formulation is established to manifest retailer’s net profit in section 3. Section 4, provides the optimal solution for finding optimal cycle time. In section 5, algorithm is developed for finding optimal solution. Numerical examples are provided to illustrate the solution algorithm in section 6. In section 7, sensitivity analysis of the optimal solution with respect to different parameters of the system is carried out. Finally, we draw the conclusion and future research in section 8.

Notations and Assumptions

The following notations are used through the manuscript:

\[ I(t) \]: The inventory level at time ‘t’
\[ Q \]: The order quantity
\[ S \]: The ordering cost per order at time zero
\[ c \]: The unit purchase cost of the item at time zero
\[ p \]: The unit selling price of the item at time zero
\[ t \]: Out-of-pocket inventory carrying charge per $ per year
\[ r \]: Discount rate per year
\[ I_e \]: The interest that can be earned per $ per year
\[ I_p \]: The interest charges payable per dollar per year \((I_p > I_e)\)
\[ m \]: Credit period granted by the retailer to his/her customers; \(T_1 \leq m\)
\[ T \]: The inventory cycle time in years
\[ T^* \]: Optimal inventory cycle time for case I in years
\[ T^{**} \]: Optimal inventory cycle time for case II in years
\[ Z(T) \]: Retailer’s annual net profit per cycle for case I
\[ Z(T) \]: Retailer’s annual net profit per cycle for case II
\[ Z^*(T^*) \]: Optimal retailer’s annual net profit per cycle for case I
\[ Z^{**}(T^{**}) \]: Optimal retailer’s annual net profit per cycle for case II
\[ Q \]: Order quantity
\[ Q^*_1 \]: Optimal order quantity for case I
\[ Q^{**}_2 \]: Optimal order quantity for case II

Assumptions:

In addition, the following assumption is being through manuscript:

- Replenishment rate is instantaneous
- Shortages are not allowed
- The annual demand rate consists of (a) regular cash demand and (b) credit demand. Thus demand function at time \(t\) is given by:
where, \( \alpha \) is known and constant cash-demand rate during the cycle \([0, T]\) and \( \beta \) is the credit demand rate during the customer’s credit demand rate during the customer’s credit period \( T_1 \):

- The model is considered for one item only
- The Discounted Cash Flow (DCF) approach is applied to consider the various the various cost at various times
- The supplier provides a credit period \( m \) to resolve the account to the retailer and retailer passes on a maximum credit \( T_1 \) to its customers to resolve the account. We assume \( T_1 \leq m \) and customer would resolve their account only on last day of the credit period \( T_1 \).

**Mathematical Formulation**

The inventory is depleted due to demand only. Thus, the rate of change of inventory at time \( t \) in \([0, T]\) is given by:

\[
\frac{dI(t)}{dt} = -[\alpha + \beta I(t)], \quad 0 \leq t \leq T_1
\]

\[
\frac{dI(t)}{dt} = -\alpha, \quad T_1 < t \leq T
\]

The rate of change of inventory can be easily seen in Fig. 1.

With the boundary condition \( I(0) = Q, \quad I(T) = 0 \), the solution of (1) and (2) is given by:

\[
I_1(t) = \left( Q + \frac{\alpha}{\beta} \right) e^{\beta t} - \frac{\alpha}{\beta}
\]

(3)

\[
I_2(t) = \alpha (T - t)
\]

(4)

and

\[
Q_1 = \left( Q + \frac{\alpha}{\beta} \right) \left( 1 - e^{\beta T_1} \right)
\]

(5)

\[
Q_2 = \alpha (T - T_1)
\]

(6)

Using (5) and (6) in (3), we get:

\[
I_1(t) = \alpha \left( \frac{e^{\beta (T_1-t)}}{\beta} - (T - T_1)e^{\beta t} \right), \quad 0 \leq t \leq T_1
\]

(7)

\[
Q = Q_1 + Q_2 = \left( Q + \frac{\alpha}{\beta} \right) \left( 1 - e^{\beta T_1} \right) + \alpha (T - T_1)
\]

or

\[
Q = \alpha \left( \frac{e^{\beta T_1} - 1}{\beta} + (T - T_1)e^{\beta T_1} \right)
\]

(8)

By using the discounted cash flow approach, the different components of the retailer’s net profit is calculated as follows:

The present value of the sales revenue is:

\[
= \frac{p}{T} \left\{ \alpha e^{-rt} dt + e^{-\beta T_1} \int_0^{T_1} \beta I_1(t) dt \right\}
\]

\[
= \frac{\alpha p}{T} \left[ 1 - e^{-rT} + e^{-\beta T_1} \left( \frac{e^{\beta T_1} - 1}{\beta} + (T - T_1)e^{\beta T_1} - T \right) \right]
\]

(9)

The present cost of placing order \( \frac{s}{T} \)

The ordering cost = \( \frac{cQ}{T} = \frac{c\alpha}{T} \left( \frac{e^{\beta T_1} - 1}{\beta} + (T - T_1)e^{\beta T_1} \right) \)

(10)

The present cost of out of pocket inventory carrying cost is:

\[
= \frac{i c}{T} \left( \int_0^{T_1} I_1(t) e^{-rt} dt + \int_{T_1}^{T_1} I_1(t) e^{-\beta t} dt \right)
\]

\[
= \frac{ic}{T} \left[ \left( \frac{1}{\beta} + T - T_1 \right) \left( \frac{e^{\beta T_1} - e^{\beta T_1}}{\beta + r} + e^{-\beta T_1} - 1 \right) + \frac{1}{\beta r} \right]
\]

(11)

\[
= \frac{ic}{T} \left[ \left( T - T_1 \right) e^{\beta T_1} + e^{-\beta T_1} - e^{-\beta T_1} - \frac{1}{\beta r} \right]
\]

(12)

The following two cases arise which is based on the value of \( T \) and \( m \).

**Case I: \( m \leq T \)**

In this case, the retailer deposits the assembled revenue from cash sales in the period \([0, m]\) and also from credit sales in time period \([T_1, m]\) in to an account that earns interest rate \( I_e \). At credit period ‘\( m \)’ credit period, the account have to be resolved, it is assumed that account will be fixed by proceeds of sells produced up to credit period \( m \) and by taking a short term credit at an interest rate of \( I_p \) in between \((T - m)\) for financing the remaining stock. Therefore, the present interest earned is:
Case II: $m \geq T$

In this case the credit period $m$ is longer than or equal to cycle time $T$, therefore the retailer gets interest on each sales during the period $[0, m]$ and also on credit sales in between $[T, m]$ and pay no interest for the raw material in stock.

The present interest payable is:

$$\frac{pl}{T} \left[ \frac{1-e^{-rt}}{r} + \frac{r}{T} \left( e^{-rt} - e^{-m} \right) \right] = \frac{ap}{T} \left[ \frac{1-e^{-rt}}{r} + \frac{r}{T} \left( e^{-rt} - e^{-m} \right) \right]$$

The retailer’s net profit $Z(T)$ can be expressed as

$$Z(T) = \text{Sales revenue} + \text{interest earned} - \text{purchase cost} - \text{ordering cost} - \text{cost of out of pocket inventory carrying cost}$$

Therefore, the retailer’s annual profit, $Z(T)$ is given by $Z(T) = \text{Sales revenue} + \text{interest earned-purchase cost-ordering cost-carrying cost-interest payable}$:

$$= \frac{cl}{T} \int_{0}^{r} l(t) e^{-rt} dt = \frac{acl}{T} \int_{0}^{r} (T-t) e^{-rt} dt$$

$$= \frac{acl}{rT} \left( T-m \right) e^{-m} + \frac{e^{-rt} - e^{-m}}{r}$$

(14)

(15)

$$= \frac{cl}{T} \int_{0}^{r} l(t) e^{-rt} dt = \frac{acl}{T} \int_{0}^{r} (T-t) e^{-rt} dt$$

(13)

(16)

(17)

(18)
Determination of Optimal Solution

To determine optimal value of $T$, taking the first derivative of $Z_i(T)$ and $Z_2(T)$ with respect to $T$, we obtain:

$$
\frac{dZ_i(T)}{dT} = -\alpha p \left[ \frac{1-e^{-rt}}{T} - Te^{-rt} + \left\{ e^{-rt} + \frac{I}{r} (e^{-rt} - e^{-m}) \right\} \left\{ \frac{e^{\beta t} - 1}{\beta} - T e^{\beta t} \right\} \right] + \frac{L}{r} \left[ \frac{1-(1+rm)e^{-m}}{r} \right] + \frac{s}{T^2} + \frac{ca}{T^2} \left\{ \frac{1 - T e^{\beta t}}{\beta T} \right\} + \frac{ic\alpha}{T^2} \left\{ \frac{1}{\beta + r} \right\} + \frac{1}{r} \left\{ \left( T + \frac{1}{r} \right) e^{-rt} - \left( m + \frac{1}{r} \right) e^{-m} \right\}
$$

(19)

and

$$
\frac{dZ_i(T)}{dT} = -\alpha p \left[ \frac{1-e^{-rt}}{T} - Te^{-rt} + \left\{ e^{-rt} + \frac{I}{r} (e^{-rt} - e^{-m}) \right\} \left\{ \frac{e^{\beta t} - 1}{\beta} - T e^{\beta t} \right\} \right] + \frac{L}{r} \left[ \frac{1-(1+rm)e^{-m}}{r} \right] + \frac{s}{T^2} + \frac{ca}{T^2} \left\{ \frac{1 - T e^{\beta t}}{\beta T} \right\} + \frac{ic\alpha}{T^2} \left\{ \frac{1}{\beta + r} \right\} + \frac{1}{r} \left\{ \left( T + \frac{1}{r} \right) e^{-rt} - \left( m + \frac{1}{r} \right) e^{-m} \right\}
$$

(20)

Our aim is to find maximum retailer’s annual profit. The necessary and sufficient condition to maximize $Z_i(T)$; $i = 1, 2$, for a given value $T$ are respectively $\frac{dZ_1(T)}{dT} = 0$ and $\frac{d^2Z_2(T)}{dT^2} > 0$; $i = 1,2$. (Appendix).

Now $\frac{dZ_1(T)}{dT} = 0$; $i = 1,2$. Give the following equation in T:

$$
\alpha p \left[ \frac{1-e^{-rt}}{T} - Te^{-rt} + \left\{ e^{-rt} + \frac{I}{r} (e^{-rt} - e^{-m}) \right\} \left\{ \frac{e^{\beta t} - 1}{\beta} - T e^{\beta t} \right\} \right] - s - ca \left\{ \frac{e^{\beta t} - 1}{\beta} - T e^{\beta t} \right\} - ic\alpha \left\{ \frac{1}{\beta + r} \right\} + \frac{I}{r} \left\{ \frac{1-(1+rm)e^{-m}}{r} \right\} + \frac{1}{r} \left\{ \left( T + \frac{1}{r} \right) e^{-rt} - \left( m + \frac{1}{r} \right) e^{-m} \right\} = 0
$$

(21)

and
To get the optimal cycle time \( T = T^* \) for case I and \( T = T^{**} \) for case II, we have to solve Equations (21) and (22), for which \( \frac{d^2 Z_i(T)}{dT^2} < 0 \). for \( i = 1, 2 \) (Appendix).

Since it is difficult to solve above Equations (21) and (22), for finding the exact value of \( T \), therefore, we make use of the second order approximations for exponential terms, i.e., \( e^{-rT} = 1 - rT + \frac{r^2T^2}{2} \), \( e^{\beta_{i1}} = 1 - \beta_{i1} + \frac{\beta_{i1}^2T^2}{2} \) and \( e^{-rT_i} = 1 - rT_i + \frac{r^2T_i^2}{2} \) etc.

Hence Equations (21) and (22) reduces to:

\[
\alpha p \left[ \frac{1-e^{-rT}}{r} - T e^{-rT} \right] \left[ 1 + I(T_i - m) \left( \frac{rT_i + rm}{2} - 1 \right) \right] (1 + \beta_{i1}) + \frac{I T_i^2}{2} (1 - rm) \]

\[ -s + c_{\text{c}} \alpha \beta_{i1} \left( 1 + \beta_{i1} + \frac{ic\alpha}{2} \left( T_i^2 + T T_i^2 - T \right) \right) = 0 \]

\[
\alpha p \left[ \frac{1-e^{-rT}}{r} - T e^{-rT} \right] \left[ 1 + I(T_i - m) \left( \frac{rT_i + rm}{2} - 1 \right) \right] (1 + \beta_{i1}) + \frac{I T_i^2}{2} (1 - rm) \]

\[ -s + c_{\text{c}} \alpha \beta_{i1} \left( 1 + \beta_{i1} + \frac{ic\alpha}{2} \left( T_i^2 + T T_i^2 - T \right) \right) = 0 \]

Again, we make use of the second order approximations for exponential terms, i.e., \( e^{-rT} = 1 - rT + \frac{r^2T^2}{2} \), \( e^{\beta_{i1}} = 1 - \beta_{i1} + \frac{\beta_{i1}^2T^2}{2} \) and \( e^{-rT_i} = 1 - rT_i + \frac{r^2T_i^2}{2} \) etc.

Hence Equations (8) (15) and (17) reduce to:

\[
Q = \alpha \left[ \left( 1 + \beta_{i1} + \frac{\beta_{i1}^2T^2}{2} \right) - \frac{\beta_{i1}^2}{2} (1 + \beta_{i1}) \right] \]

\[
Z_i(T) = \frac{\alpha p}{T} \left[ \left( 1 - \frac{rT_i}{2} \right) + \left( 1 + \beta_{i1} + \frac{\beta_{i1}^2T^2}{2} \right) - 1 + I(T_i - m) \left( \frac{rT_i + rm}{2} - 1 \right) \right] \left( 1 + \beta_{i1} \right) + \frac{I T_i^2}{2} (2T - T_i + \beta T T_i - \beta T_i^2) + \frac{I T_i^2}{2} (1 - rm) \]

\[
\frac{s}{T} \left[ \left( 1 + \beta_{i1} \right) + \frac{ic\alpha}{2T} (T^2 + T T_i^2 - T) \right] - \frac{\alpha c I(T_i - m)}{2T} (T - m(1 - rm)) \]

and

\[
Z_i(T) = \frac{\alpha p}{T} \left[ \left( 1 - \frac{rT_i}{2} \right) + \left( 1 + \beta_{i1} + \frac{\beta_{i1}^2T^2}{2} \right) - 1 + I(T_i - m) \left( \frac{rT_i + rm}{2} - 1 \right) \right] \left( 1 + \beta_{i1} \right) + \frac{I T_i^2}{2} (2T - T_i + \beta T T_i - \beta T_i^2) + \frac{I T_i^2}{2} (1 - rm + \frac{r^2m^2}{2}) \]

\[
\frac{s}{T} \left[ \left( 1 + \beta_{i1} \right) + \frac{ic\alpha}{2T} (T^2 + T T_i^2 - T) \right] - \frac{\alpha c I(T_i - m)}{2T} (T - m(1 - rm)) \]

\[
\text{(23)}
\]

\[
\text{(24)}
\]
Now, we summarize the above results and establish the following algorithm to find the optimal solution.

**Algorithm**

The following steps are to be followed to find optimal annual profit and order quantity:

**Step 1:** Determine $T^*$ from (23), if $T \geq m$, evaluate $Z_i(T^*)$, from (26)

**Step 2:** Determine $T^{**}$ from (24), if $T < m$, evaluate $Z_i(T^{**})$, from (27)

**Step 3:** If the condition $T^* \geq m$ and $T^{**} < m$ is satisfied, go to step 4, otherwise go to step 5

**Step 4:** Compare $Z_i(T^*)$ and $Z_i(T^{**})$ and find the maximum profit

**Step 5:** If $T^* > m$ is satisfied but $T^{**} > m$, then $Z_i(T^*)$ the maximum profit, else if $T^* < m$, but $T^{**} < m$, then $Z_i(T^{**})$ is the maximum profit

**Numerical Examples**

**Example 1:** (Case I & II) Maximum Retailer’s Annual Profit $Z_i(T^{**})$

The following data is considered for inventory system:

- $\alpha = 1000$ units per year, $\beta = 0.1$, $r = 13\%$, $m = 5.0$ year, $T_I = 0.5$ year, $c = $20/ unit, $i = 0.15$, $I_c = 9\%$, $I_p = 14\%$, $s = $700/ unit, $p = $ 60 / unit. Solving Equation (23), we get $T = T^* = 7.67797$ years, the corresponding values of $Q = Q_i^* = 12087.5$ units and maximum retailer’s annual profit $Z_i(T) = Z_i^*(T^*) = $ 8058.34

- Again solving Equation (24), we have $T = T^{**} = 0.531856$ year, the corresponding values of $Q = Q_i^* = 545.989$ units and maximum retailer’s annual profit $Z_i(T) = Z_i^*(T^{**}) = $ 53980.6

Here $T^* > m$ and $T^{**} < m$ and $Z_i^*(T^*) < Z_i^*(T^{**})$. Hence the maximum average profit in this case is $Z_i^*(T^{**}) = $ 53980.6. Where optimal cycle time is $T = T^* = 0.531856$ year

- The economic order quantity is $Q = Q_i^* = 545.989$ units

**Example 2:** (Case I) Maximum Retailer’s Annual Profit $Z_i^*(T^*)$

The following data is considered for inventory system:

- $\alpha = 1000$ units per year, $\beta = 0.1$, $r = 13\%$, $m = 0.0822$ year, $T_I = 0.0274$ year, $c = $50/ unit, $i = 0.15$, $I_c = 9\%$, $I_p = 14\%$, $s = $500/ unit, $p = $ 60 / unit. Solving Equation (23), we get $T = T^* = 0.225374$ year, the corresponding values of $Q = Q_i^* = 225.955$ units and maximum retailer’s annual profit $Z_i(T) = Z_i^*(T^*) = $ 5843.36

- Again solving Equation (24), we have $T = T^{**} = 0.431152$, the corresponding values of $Q = Q_i^* = 432.297$ units and maximum retailer’s annual profit $Z_i(T) = Z_i^*(T^{**}) = $ 4837.67

Here $T^{**} > m$ which contradicts case II, only case I holds as $T^* > m$. Hence the maximum average profit in this case is $Z_i^*(T^*) = $ 5843.36. Where optimal cycle time is $T = T^* = 0.225374$ year

- The economic order quantity is $Q = Q_i^* = 225.955$ units

**Example 3:** (Case II) Maximum Retailer’s Annual Profit $Z_i^*(T^{**})$

The following data is considered for inventory system:

- $\alpha = 1000$ units per year, $\beta = 0.1$, $r = 13\%$, $m = 0.8$ year, $T_I = 0.4$ year, $c = $50/ unit, $i = 0.15$, $I_c = 9\%$, $I_p = 14\%$, $s = $500/ unit, $p = $ 60 / unit. Solving Equation (23), we get $T = T^* = 0.371669$ year, the corresponding values of $Q = Q_i^* = 378.513$ units and maximum retailer’s annual profit $Z_i(T) = Z_i^*(T^*) = $ 8439.39

- Again solving Equation (24), we have $T = T^{**} = 0.437502$, the corresponding values of $Q = Q_i^* = 447.032$ units and maximum retailer’s annual profit $Z_i(T) = Z_i^*(T^{**}) = $ 8624.52

Here $T^* < m$ which contradicts case I, only case II holds as $T^{**} < m$. Hence the maximum retailer’s annual profit in this case is $Z_i^*(T^{**}) = $ 8624.52, where optimal cycle time is $T = T^{**} = 0.437502$ year

- The economic order quantity is given by $Q = Q_i^* = 447.032$ units

**Sensitivity Analysis**

By using the same data as in example 1, we study the effect of the changes in a single parameter keeping other parameters same on the optimal solution as shown in following Tables 1-8.

The following inferences can be made from the results obtained from Tables 1-8:

- When the cash demand ‘$\alpha$’ increases, the order quantity ($Q_i$) and net profit $Z_i(T)$ will also increase. Similarly if the credit demand ‘$\beta$’ increases, the order quantity ($Q_i$) slightly increases and net profit $Z_i(T)$ increases. That is, change in ‘$\alpha$’ will lead the positive change in ($Q_i$) and change in $Z_i(T)$

- When the cash demand ‘$\alpha$’ increases, order quantity ($Q_i$) and net profit $Z_i(T)$ will also increase. Similarly in purchase cost ‘$c$’ increases, the order quantity ($Q_i$) and net profit $Z_i(T)$ will also increase. That is, change in ‘$c$’ will lead the positive change in ($Q_i$) and $Z_i(T)$
Table 1. Variation of cash demand ‘α’ and credit demand ‘β’

| α | T | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 |
|---|---|-----|-----|-----|-----|-----|-----|
| 1000 | T | 0.225457 | 0.225541 | 0.225625 | 0.225709 | 0.225794 | 0.22588 |
|   | Z(T) | 5869.550000 | 5895.810000 | 5922.130000 | 5948.520000 | 5974.980000 | 6001.500000 |

Table 2. Variation of cash demand ‘α’ and unit purchase cost ‘c’

| α | c | 45 | 50 | 60 | 75 | 90 |
|---|---|----|----|----|----|----|
| 1000 | T | 0.232056 | 0.239482 | 0.247801 | 0.257201 | 0.267939 | 0.280359 |
|   | Z(T) | 232.655000 | 240.101000 | 248.443000 | 257.840000 | 268.637000 | 281.091000 |

Table 3. Variation of cash demand ‘α’ and unit selling price ‘p’

| α | p | 65 | 70 | 75 | 80 | 90 |
|---|---|----|----|----|----|----|
| 1000 | T | 0.221683 | 0.218147 | 0.214755 | 0.211500 | 0.208369 | 0.205356 |
|   | Z(T) | 222.254000 | 218.708000 | 215.308000 | 212.043000 | 208.903000 | 205.883000 |

When the cash demand ‘α’ increases, order quantity (Q) and net profit Z(T) will also increase. Similarly if selling price ‘p’ increases, order quantity (Q) decreases while net profit Z(T) increases. That is, change in ‘α’ leads positive change in (Q) and Z(T) and the change in ‘β’ causes negative change in (Q) and positive change in Z(T).

When the cash demand ‘α’ increases, order quantity (Q) and net profit Z(T) will also increase. Similarly if ordering cost ‘s’ increases, order quantity (Q) increases while net profit Z(T) decreases. That is, change in ‘α’ leads positive change in (Q) and negative change in Z(T) and the change in ‘s’ causes negative change in both (Q) and Z(T).
### Table 3. Continue

| Q               | 233.4710000 | 229.7140000 | 226.1100000 | 222.6490000 | 219.3210000 | 216.1180000 |
|-----------------|-------------|-------------|-------------|-------------|-------------|-------------|
| Z_{r(T)}       | 1211.8400000 | 17571.0000000 | 2302.8200000 | 28447.6700000 | 33931.7000000 | 39387.7000000 |
| T               | 0.2030120    | 0.1997170   | 0.1965570   | 0.1935210   | 0.1906020   | 0.1879720   |
| Q               | 224.2380000  | 224.2730000  | 236.4700000  | 232.8170000  | 229.3050000  | 225.9240000  |
| Z_{r(T)}       | 13452.3000000 | 19403.1000000 | 25355.2000000 | 31308.4000000 | 37262.9000000 | 43218.4000000 |
| T               | 0.1953540    | 0.1921570   | 0.1890890   | 0.1861420   | 0.1833080   | 0.1805800   |
| Q               | 254.6080000  | 250.4410000  | 246.4410000  | 242.6000000  | 238.9050000  | 235.3940000  |
| Z_{r(T)}       | 14795.1000000 | 21245.3000000 | 27696.9000000 | 34149.6000000 | 40603.7000000 | 47058.9000000 |
| T               | 0.1885410    | 0.1854290   | 0.1824420   | 0.1795740   | 0.1768140   | 0.1741580   |
| Q               | 264.6290000  | 260.2600000  | 256.0760000  | 252.0410000  | 248.1660000  | 244.4370000  |
| Z_{r(T)}       | 16145.9000000 | 23095.7000000 | 30046.8000000 | 36999.8000000 | 43953.1000000 | 50908.1000000 |
| T               | 0.1824280    | 0.1793910   | 0.1764770   | 0.1736780   | 0.1709850   | 0.1683920   |
| Q               | 274.3360000  | 269.7680000  | 265.3850000  | 261.1750000  | 257.1250000  | 253.2250000  |
| Z_{r(T)}       | 17503.8000000 | 24953.2000000 | 32404.1000000 | 39856.4000000 | 47310.1000000 | 54765.1000000 |

### Table 4. Variation of cash demand ‘α’ and ordering cost’s

| α   | ε→  | 550  | 600  | 650  | 700  | 750  | 800  |
|-----|-----|------|------|------|------|------|------|
| 1000 | T   | 0.236001 | 0.246173 | 0.259546 | 0.265364 | 0.274464 | 0.283279 |
| 1100 | T   | 0.225374 | 0.235055 | 0.243355 | 0.253318 | 0.261978 | 0.270656 |
| 1200 | T   | 0.216120 | 0.225374 | 0.232462 | 0.242830 | 0.251107 | 0.259123 |
| 1300 | T   | 0.207968 | 0.216846 | 0.225374 | 0.233591 | 0.241531 | 0.249221 |
| 1400 | T   | 0.200716 | 0.209259 | 0.217466 | 0.225374 | 0.233014 | 0.240413 |
| 1500 | T   | 0.194210 | 0.202453 | 0.210372 | 0.218002 | 0.225374 | 0.232512 |

### Table 5. Variation of cash demand ‘α’ and credit demand ‘β’ (at s = 15)

| α   | β   | 0.2  | 0.3  | 0.4  | 0.5  | 0.6  | 0.7  |
|-----|-----|------|------|------|------|------|------|
| 1000 | T   | 0.0733754 | 0.0736225 | 0.0738738 | 0.0741293 | 0.0743888 | 0.0746524 |
| 1100 | T   | 0.0699907 | 0.0702495 | 0.0705127 | 0.0707801 | 0.0710517 | 0.0713257 |
| 1200 | T   | 0.0670415 | 0.0673116 | 0.0675860 | 0.0678648 | 0.0681479 | 0.0684352 |
| 1300 | T   | 0.0644432 | 0.0647230 | 0.0650882 | 0.0652979 | 0.0655919 | 0.0658902 |
| 1400 | T   | 0.0621289 | 0.0624200 | 0.0627155 | 0.0630156 | 0.0633200 | 0.0636288 |
| 1500 | T   | 0.0600530 | 0.0603539 | 0.0606594 | 0.0609694 | 0.0612839 | 0.0616028 |

Sensitivity Analysis (Case II)

### Table 5. Variation of cash demand ‘α’ and credit demand ‘β’ (at s = 15)

| α   | β   | 0.2  | 0.3  | 0.4  | 0.5  | 0.6  | 0.7  |
|-----|-----|------|------|------|------|------|------|
| 1000 | T   | 0.0733754 | 0.0736225 | 0.0738738 | 0.0741293 | 0.0743888 | 0.0746524 |
| 1100 | T   | 0.0699907 | 0.0702495 | 0.0705127 | 0.0707801 | 0.0710517 | 0.0713257 |
| 1200 | T   | 0.0670415 | 0.0673116 | 0.0675860 | 0.0678648 | 0.0681479 | 0.0684352 |
| 1300 | T   | 0.0644432 | 0.0647230 | 0.0650882 | 0.0652979 | 0.0655919 | 0.0658902 |
| 1400 | T   | 0.0621289 | 0.0624200 | 0.0627155 | 0.0630156 | 0.0633200 | 0.0636288 |
| 1500 | T   | 0.0600530 | 0.0603539 | 0.0606594 | 0.0609694 | 0.0612839 | 0.0616028 |

DOI: 10.3844/jmssp.2015.75.87

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### Table 7. Variation of cash demand \( 'c' \) and unit purchase cost \( 'c' \) (at \( s = 15 \))

| \( \alpha \) | \( p \) | 65   | 70   | 75   | 80   | 85   | 90   |
|-------------|-------|------|------|------|------|------|------|
| 1000 \( T \) | 0.0673476 | 0.062846 | 0.0592261 | 0.056232 | 0.0537072 | 0.0515423 |
| 1000 \( Q \) | 67.4947000 | 62.9843000 | 59.3510000 | 56.3498000 | 53.8169000 | 51.6416000 |
| 1100 \( Z(T) \) | 14554.6000000 | 19600.1000000 | 24641.0000000 | 29678.8000000 | 34714.5000000 | 39748.6000000 |
| 1100 \( Z(T) \) | 10535.2000000 | 10570.3000000 | 10606.8000000 | 10665.2000000 | 10684.6000000 | 10921.6000000 |
| 1500 \( T \) | 0.0567626 | 0.0542767 | 0.0521738 | 0.0503678 | 0.0487973 | 0.0474171 |
| 1500 \( Q \) | 85.3211000 | 81.5820000 | 78.4190000 | 75.7025000 | 73.3403000 | 71.2643000 |
| 1500 \( Z(T) \) | 22096.0000000 | 29666.2000000 | 37229.0000000 | 44786.0000000 | 52338.4000000 | 59887.1000000 |

### Table 8. Variation of cash demand \( 'a' \) and ordering cost \( 's' \)

| \( \alpha \) | \( s \) | 14   | 13   | 12   | 11   | 10   | 5    |
|-------------|-------|------|------|------|------|------|------|
| 1000 \( T \) | 0.0706576 | 0.0680941 | 0.0654316 | 0.0626576 | 0.0595767 | 0.0424105 |
| 1000 \( Q \) | 70.8138000 | 68.2433000 | 65.5735000 | 62.7919000 | 59.8830000 | 42.4892000 |
| 1100 \( Z(T) \) | 9534.5200000 | 9568.0800000 | 9602.9200000 | 9639.2100000 | 9677.1400000 | 59887.1000000 |
| 1100 \( Z(T) \) | 77.5931000 | 75.3770000 | 72.1586000 | 68.8173000 | 65.3600000 | 46.1900000 |
| 1500 \( T \) | 0.0597567 | 0.0553797 | 0.0515922 | 0.0493305 | 0.0473934 | 0.0378840 |
| 1500 \( Q \) | 83.8362000 | 83.6510000 | 80.3990000 | 77.0025000 | 73.4563000 | 52.2516000 |
| 1500 \( Z(T) \) | 14532.4000000 | 14596.1000000 | 14638.5000000 | 14682.7000000 | 14728.9000000 | 15003.8000000 |
Conclusion and Future Research

In this study, the retailer’s optimal ordering policy under two stage trade credit financing is developed using Discounted Cash Flow (DCF) approach. An algorithm is established to obtain the optimal solution. The sensitivity analysis of the optimal solution with respect to the parameters is also discussed. The results show some phenomena as follows: (i) A higher value of cash demand ‘a’ caused higher value of retailer’s annual profit, (ii) A higher value of credit demand ‘β’ causes higher value of retailer’s annual profit, (iii). A higher value of unit purchase cost ‘c’ causes lower value of retailer’s annual profit, (iv). A higher value of selling price ‘p’ causes higher value of net profit. That is, the retailer should increase the net profit per transfer from the increase of cash demand, credit demand, purchase cost and selling price. Second order approximation is used for exponential terms to find exact values of cycle time ‘T*, order quantity ‘Q’ and retailer’s annual profit Z(T).

The proposed model can be further extended in several ways. For example, we may add pricing strategy into consideration. We may also extend the model to allow for constant deterioration rate or a two-parameter Weibull distribution. In addition, we can consider the demand as a function of time, price as well as quality.

We may extend the model to allow for constant deterioration rate or a two-parameter Weibull distribution. In addition, we can consider the demand as a function of time, price as well as quality. Finally, we could generalize the model to allow for shortages, quantity discount or others.

Acknowledgment

The authors are thankful to anonymous reviewers for valuable comments and to the Editor-in-chief for handling of the manuscript.

Funding Information

The authors have no support or funding to report.

Author’s Contributions

Rakesh Prakash Tripathi: Paper formation, Mathematical formulation, discussion of data analysis, contributed to the writing of the manuscript and publication.

Harenrda Singh: Coordination of research work and publication of the manuscript.

Neha Sang: Design the research plan, organization, development and publication of the manuscript.

Ethics

In this paper second order approximation have been used for exponential terms to find closed form optimal solution. With the help of differential calculus the author’s have obtained the retailer’s annual net profit per unit time.

Appendix A

To prove this appendix, we first prove the following lemma.

Lemma 1: If a function \( G(T) = \frac{F(T)}{T} \), where \( F(T) \) is a differential function of \( T \) two times, then the maximum value of \( G(T) \) exist at \( T = T^* \) if \( \frac{d^2(F(T))}{dT^2} < 0 \), at \( T = T^* \).

Proof: It is given that \( G(T) = \frac{F(T)}{T} \). For extremum, the necessary condition is \( \frac{d(G(T))}{dT} = 0 \). But
\[
\frac{d(G(T))}{dT} = -\frac{F(T)}{T^2} + \frac{1}{T} \frac{d(F(T))}{dT} = \frac{1}{T} \left( T \frac{d(F(T))}{dT} - F(T) \right) \]
\[
\frac{d(G(T))}{dT} = 0, \quad \text{gives } T \frac{d(F(T))}{dT} = F(T) = 0. \quad (i)
\]

Let Equation (i) be satisfied for \( T = T^* \).

Again
\[
\frac{d^2(G(T))}{dT^2} = -\frac{2}{T} \left( T \frac{d(F(T))}{dT} - F(T) \right) + \frac{1}{T} \frac{d^2(F(T))}{dT^2}
\]
Or
\[
\frac{d^2(G(T))}{dT^2} = \frac{1}{T} \frac{d^2(F(T))}{dT^2}
\]
from (i)

We know that the sufficient condition for existence of a maximum value of \( G(T) \) is \( \frac{d^2(G(T))}{dT^2} < 0 \). Hence the Lemma 1.

Here \( Z(T) = \frac{1}{T} \left[ SR + IE - PC - OC - IC - IP \right] \). For extremum value at \( T = T^* \)

Where Sales revenue = \( SR/T \), interest earned = \( IE/T \), Purchase cost = \( PC/T \), Ordering cost = \( OC/T \), Inventory carrying cost = \( IC/T \), Interest payable = \( IP/T \).

At \( T = T^* \), the necessary condition is \( \frac{dZ(T)}{dT} = 0 \), which gives Equation (18).

If \( T = T^* \) be a maximum value of \( Z(T) \), then at \( T = T^* \), we have
\[
\frac{d^2Z(T)}{dT^2} = -\frac{1}{T} \left[ \left( \frac{d^2(SR)}{dT^2} + \frac{d^2(IE)}{dT^2} + \frac{d^2(PC)}{dT^2} - \frac{d^2(OC)}{dT^2} - \frac{d^2(IC)}{dT^2} - \frac{d^2(IP)}{dT^2} \right) \right] \quad \text{By Lemma 1.}
\]

Now at \( T = T^* \)
\[
\frac{d^2Z(T)}{dT^2} = -\frac{re^{ct}}{T} (rp + ic + cl_s) < 0.
\]

Hence the proof of Appendix A.
Appendix B:

We have \( Z_0(T) = \frac{1}{T}\left[ SR + IE_r - PC - OC - IC \right] \).

Where Sales revenue = SR/T, interest earned = IE_r/T.

Purchase cost = PC/T, Ordering cost = OC/T, Inventory carrying cost = IC/T.

For extremum value of \( Z_r(T) \), the necessary condition is

\[ \frac{dZ_0(T)}{dT} = 0, \]

which gives Equation (19). If \( T = T^{**} \) be a maximum value of \( Z_0(T) \), then at \( T = T^{**} \)

\[ \frac{d^2Z_0(T)}{dT^2} = -\frac{r}{T} \left( \frac{I}{r} + ic \right) < 0. \]

Hence the proof of Appendix B.

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