Inverse Type II Seesaw Mechanism for Neutrino Masses

F. F. Freitas, C. A. de S. Pires, and P. S. Rodrigues da Silva

Departamento de Física, Universidade Federal da Paraíba,
Caixa Postal 5008, 58051-970, João Pessoa, PB, Brasil

(Dated: September 2, 2014)

Abstract

With the advent of the LHC, and the prospect of future colliders as the ILC, both programmed to run at TeV scale, it becomes imperative to investigate all possible manifestations of physics at that energy scale. This justify the recent effort in building seesaw mechanisms for small neutrino masses that work at TeV scale. In this work we propose an inverse of the type II seesaw mechanism. We show that, for such a seesaw to generate small neutrino masses, explicit lepton number violation must manifest itself at keV scale in conjunction with Higgs triplet components with masses around TeV. We obtain bounds on the mechanism coming from rare lepton decays and discuss the direct detection of the doubly charged component of the Higgs triplet at the ILC.

*Electronic address: felipefreitas@fisica.ufpb.br
I. INTRODUCTION

The typical energy scale involved in the canonical seesaw mechanisms\cite{1-2-3} is the GUT one ($10^{14}$ GeV). Consequently it cannot be probed neither at the present (LHC) nor at the future (ILC) colliders once they will run at TeV scale. Despite the lack of signature, seesaw mechanism is still seen as the most elegant way of explaining the smallness of the neutrino masses.

Remarkably, it is possible to have seesaw mechanisms working at TeV scale. Recently such mechanisms have received a lot of attention mainly because the LHC and future colliders as ILC will both dig physics into the TeV scale\cite{4}.

The most popular seesaw mechanism, whose signature lies at TeV scale, are the inverse seesaw ones (ISS)\cite{5}. It is called “inverse” because it requires lepton number symmetry to be explicitly violated around keV scale, contrary to the canonical seesaw where such violation happens at energy scales around $10^{14}$ GeV. The ISS mechanisms already developed in the literature are based on the type I\cite{5} and type III\cite{6} seesaw. It would be interesting to investigate the ISS mechanism based on the type II seesaw mechanism, aiming to find phenomenologically testable features that could make them more attractive\cite{7}.

In this work we develop the ISS based on the type II seesaw mechanism\cite{2}. We refer to it as the inverse type II seesaw (ISS2) mechanism. As in the case of the type II seesaw, it requires the existence of a triplet of scalars $\Delta$. In both cases the seesaw arises from the trilinear term, $\mu \Phi^T \Delta \Phi$, where $\Phi$ is the standard Higgs doublet. This term explicitly violates lepton number symmetry and $\mu$ is the energy scale where this happens. The ordinary type II seesaw requires $\mu = 10^{14}$ GeV and the mass spectrum of $\Delta$ lies at GUT scale. In the ISS2 mechanism, the explicit violation of the lepton number is supposed to occur at $\mu \simeq 10^{-1}$ keV with the mass spectrum of $\Delta$ lying at TeV scale. As these new scalars couple to leptons only, the ILC turns out to be the fairest place to probe this new physics. In view of this we study the behaviour of the process $e^+ + e^- \rightarrow \delta^{++} + \delta^{--}$ at the ILC. We also obtain bounds on the parameters involved in the ISS2 coming from rare lepton decays.

This work is organized as follows: in Sec. II we review the inverse seesaw in order to establish the framework for the ISS2 to be developed in Sec. III. We then work out the mass spectrum of the scalar sector of the model in Sec. IV. Next, in Sec. V we pursue the phenomenological bounds concerning the rare lepton decay $\mu \rightarrow e + \gamma$ and in Sec. VI we
study the collider signature of the ISS2, the production of a doubly charged scalar, $\delta^{++}$, at the ILC. We present our concluding remarks in Sec. VII.

II. THE INVERSE SEESAW MECHANISM

The canonical (type I) seesaw mechanism consists of adding to the standard model of electroweak interactions (SM) three right-handed singlet neutrinos, $\nu_R$, composing the following mass terms\[1\],

$$\mathcal{L} \supset -\bar{\nu}_R m_D \nu_L - \frac{1}{2} \bar{\nu}_R \mu \nu_L^c + \text{h.c.}$$

(1)

which result in the following mass matrix,

$$M_\nu = \begin{pmatrix} 0 & m_D^T \\ m_D & \mu \end{pmatrix}.$$  

(2)

On diagonalizing this mass matrix for the case of $\mu$ diagonal and degenerate, we get for the case $\mu >> m_D$,

$$m_\nu = \frac{m_D^T m_D}{\mu}.$$  

(3)

This mechanism easily provides neutrino masses at eV scale for $\mu$ at the GUT scale ($10^{14}$GeV) and $m_D$ at electroweak scale ($10^2$GeV).

Although elegant, the canonical seesaw is not phenomenologically attractive, once its signature is right-handed neutrinos with mass out of the reach of any terrestrial collider. Unfortunately, this also happens to other types of seesaw mechanisms since their essence is that lepton number should be explicitly violated at a very high energy scale.

On the opposite direction, there is the proposal of inverse seesaw (ISS) mechanism, which brings the lepton number violation to a very low energy scale, so arranged that its signature lies at TeV scale\[5, 7\]. Thus the ISS is a way of generating small active neutrinos mass while having its signature phenomenologically accessible at colliders.

As far as we know the only ISS mechanism proposed until now is based on the canonical type I seesaw\[1\]. Basically, it requires the addition of six new SM singlet right-handed neutrinos ($N_i, S_i$) with $i = 1, 2, 3$. The mechanism can be engendered by requiring the following mass terms\[5, 7\],

$$\mathcal{L} = -\bar{\nu}_L m_D N_R - \bar{N}_R M_N S_R^c - \frac{1}{2} \bar{S}_R \mu S_R^c + \text{h.c.},$$

(4)
written in the matrix form where the family structure is implicit. This leads to the mass matrix,

$$M_\nu = \begin{pmatrix} 0 & m_D^T & 0 \\ m_D & 0 & M_N^T \\ 0 & M_N & \mu \end{pmatrix}, \quad (5)$$

where $m_D$, $M_N$ and $\mu$ are $3 \times 3$ mass matrices. Without loss of generality, we consider that $\mu$ is diagonal, and suppose the following hierarchy , $\mu << m_D << M_N$. What makes the texture in Eq. (5) interesting from the phenomenological point of view is that after block diagonalization of $M_\nu$, we obtain, in a first approximation, the following effective neutrino mass matrix for the standard neutrinos:

$$m_\nu = m_D^T M_N^{-1} \mu (M_N^T)^{-1} m_D. \quad (6)$$

There are two aspects in the ISS that makes it profoundly distinct from the canonical seesaw mechanisms: the double suppression by $M_N$, which is an additional mass scale related to the mixing among the six new right-handed neutral fermions, and the fact that the Majorana mass scale $\mu$ in this scheme does not have to be at GUT scale, by the contrary, it can be very low. The sub-eV active neutrinos masses are then obtained by keeping $M_D$ at electroweak scale, $M_N$ at TeV scale and $\mu$ at keV scale. Also, the new neutral fermions get masses at TeV scale and their mixing with the standard neutrinos are modulated by the ratio $M_D M_N^{-1}$. The core of ISS is the fact that no huge mass scale has to be called for to explain the observed smallness of neutrino mass, while the heavy states can be brought to an accessible scale, which has to be at TeV scale due to nonunitarity effects. Thus the ISS mechanism can be probed at the LHC collider.

**III. THE INVERSE TYPE II SEESAW MECHANISM.**

It is appropriate to first recall some important features of the type II seesaw mechanism. As we mentioned earlier, a triplet of scalars, $\Delta$, is added to the SM content and transforms under $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ as $(1, 3, 2)$. Its components are bileptons since they carry two units of lepton number, which is explicitly violated at the scalar potential through the trilinear coupling $\mu \Phi^T i \sigma_2 \Delta \Phi$, where $\Phi$ is the SM scalar doublet and $\mu$ is a mass dimensional coupling. It happens that constraint equations from the minimum of the scalar potential lead
to \( v_\Delta = \frac{m_D^2}{\mu} \), with \( m_D \) the Dirac mass scale for neutrinos, associated to the SM electroweak breaking. However, \( v_\Delta \) sets up the mass scale of active neutrinos, a small \( v_\Delta \) implies small neutrino masses. Again, to obtain \( v_\Delta \) around eV, we need \( \mu \) around GUT scale \((10^{14} \text{ GeV})\) for \( m_D \) at electroweak scale. As a consequence, the collider signature of this mechanism, which are the new scalars that compose the triplet, are heavy and cannot be probed at the LHC or in any other collider running at TeV scale. In this section we apply the essence of the ISS2 mechanism which is to impose that the parameter \( \mu \), which modulates the explicit breaking of the lepton number, lies at eV scale with the mass of the new particles, here the scalars composing the triplet \( \Delta \), lying at TeV scale.

In order to implement the ISS2 mechanism, we need to add a triplet to the scalar content of SM,

\[
\Delta \equiv \begin{pmatrix} \delta^+ & \sqrt{\frac{2}{\sqrt{2}}} \\ \sqrt{\frac{2}{\sqrt{2}}} & \delta^+ & \delta^0 \end{pmatrix},
\]

(7)

which, together with the SM scalar doublet, \( \Phi = (\phi^+ \phi^0)^T \), compose the following potential,

\[
V(\Phi, \Delta) = -m_H^2 \Phi^\dagger \Phi + \frac{\lambda}{4} (\Phi^\dagger \Phi)^2 + M_\Delta^2 Tr[(\Delta^\dagger \Delta)] + [\mu (\Phi^T i \sigma^2 \Delta^\dagger \Phi) + H.c]
\]

\[
+ \lambda_1 (\Phi^T \Phi)Tr[(\Delta^\dagger \Delta)] + \lambda_2 (Tr[(\Delta^\dagger \Delta)])^2 + \lambda_3 Tr[(\Delta^\dagger \Delta)^2]
\]

\[
+ \lambda_4 \Phi^\dagger \Delta^\dagger \Delta \Phi + \lambda_5 \Phi^\dagger \Delta \Delta^\dagger \Phi.
\]

(8)

Recalling that \( \Delta \) is a bilepton, we remark that the trilinear term in the above potential explicitly violates the lepton number.

When considered in the framework of the SM, the doublet \( \Phi \) gets responsible for the charged fermion masses after spontaneous symmetry breaking. The addition of the triplet \( \Delta \), with its neutral component developing a nonzero vacuum expectation value (VEV), opens the possibility of generating neutrino masses, as we are going to see further on.

In order to develop the scalar sector and obtain its spectrum, we shift the neutral components of \( \Phi \) and \( \Delta \) in the usual way,

\[
\phi^0, \delta^0 \rightarrow \frac{1}{\sqrt{2}} (v_{\phi,\Delta} + R_{\phi,\Delta} + iI_{\phi,\Delta}),
\]

(9)

After this shift, a set of constraint equations over the parameters has to be imposed to find
the minimum of the scalar potential,

\[-m^2_H + \frac{1}{4}(v_\phi^2 \lambda + 2v_\Delta(v_\phi(\lambda_1 + \lambda_4) - 2\sqrt{2}\mu)) = 0,\]

\[M^2_\Delta v_\Delta + v_\Delta^2(\lambda_2 + \lambda_3) + \frac{v_\phi^2v_\Delta}{2}(\lambda_1 + \lambda_4) - \frac{1}{\sqrt{2}}v_\phi^2\mu = 0. \tag{10}\]

These constraint equations are crucial for implementing the ISS2 mechanism to be developed in this work. As in the canonical ISS mechanism\(^5\), where the bare mass terms for the right-handed neutrinos are assumed to lie around keV scale, here we assume that the parameter \(\mu\) in the potential above, Eq. (8), lies around keV scale too. As we will show, this assumption implies a small VEV for \(\delta^0\).

To see this perceive that, on assuming that \(\mu\) and \(v_\Delta\) are smaller than any other energy parameter of the potential, the first constraint in Eq. (10) implies \(m^2_H \simeq \frac{1}{4}v_\phi^2\lambda\), while the second one leads to,

\[v_\Delta \simeq \frac{1}{\sqrt{2}}v_\phi M_\Delta^{-1}\mu v_\phi M^{-1}. \tag{11}\]

This expression is at the heart of ISS2 mechanism. It means that \(v_\Delta\) gets suppressed due the tiny scale associated to lepton number violation, \(\mu\). Notice that, similarly to the case of canonical ISS mechanism, \(v_\Delta\) at the eV scale requires \(M_\Delta\) at TeV scale and \(\mu\) at keV scale (while \(v_\phi\) is the electroweak scale).

The Yukawa interactions involving the triplet \(\Delta\) and the standard leptonic doublet \(L = (\nu, e)^T_L\) are given by,

\[\mathcal{L}_Y = Y_{ij}\bar{L}i\sigma_2\Delta L_j + H.c. \tag{12}\]

When \(\Delta\) develops a VEV, these Yukawa interactions lead to the following neutrino mass matrix,

\[m^{\nu}_{ij} = Y_{ij}v_\Delta = \frac{Y_{ij}(v_\phi M_\Delta^{-1})\mu(v_\phi M_{\Delta}^{-1})}{\sqrt{2}}. \tag{13}\]

Observe that, concerning the magnitude, the above expression for the neutrino masses recovers the one that appears in the canonical ISS mechanism, with the advantage that the matrix structure in ISS2 becomes much more simple to handle. Moreover, we need only to introduce a scalar triplet to SM instead of six neutral fermions, at least, as in canonical ISS mechanism.
IV. SPECTRUM OF SCALARS

Motivated by the running of the LHC at TeV scale, the spectrum of scalars composing the triplet $\Delta$ has been extensively investigated in the last years\cite{8}. It is important to stress that in order to pursue such investigations the focus had to be on the parameter space appropriate to leave some track of the new scalars in LHC. This means that, if the aim is to look for these new scalars through enhanced couplings to SM particles, suitable for LHC searches, $v_\Delta$ has invariably to be taken far away from the eV scale. The price to be paid rests on the loss of any natural explanation for the smallness of neutrino masses. In the opposite direction, in this paper we give emphasis on the ISS2 mechanism, which means to obtain the scalar spectrum for a scenario where $v_\Delta$ and $\mu$ are kept small enough lying in the range from eV to keV. Here we consider the consequence of this choice for the parameters in the scalar spectrum of ISS2 so as to explore its implications in the next section.

From the scalar potential, Eq. (8), together with the constraint equations, Eq. (10), we obtain the following mass matrix for the CP-even neutral scalars in the basis $(R\phi, R\Delta)$,

$$m^2_{h_0} = \begin{pmatrix} A & B \\ B & C \end{pmatrix},$$

(14)

where the terms $A$, $B$ and $C$ are,

$$A = \frac{v_\phi^2 \lambda}{2},$$

$$B = \frac{v_\phi (v_\Delta (\lambda_1 + \lambda_4) - \sqrt{2} \mu)}{2v_\Delta},$$

(15)

$$C = \frac{4v_\Delta^3 (\lambda_2 + \lambda_3) + \sqrt{2} v_\phi^2 \mu}{2v_\Delta}.$$  

In the limit $v_\phi \gg \mu, v_\Delta$, we obtain the following eigenvalues,

$$m^2_{h_0} \simeq \frac{v_\phi^2 \lambda}{4},$$

$$m^2_{H_0} \simeq m^2_{h_0} + \left( \frac{1}{\sqrt{2} v_\phi} \frac{\mu}{v_\Delta} \right) v_\phi^2.$$  

(16)

Regarding the eigenvectors, we obtain,

$$\begin{pmatrix} h^0 \\ H^0 \end{pmatrix} \simeq \begin{pmatrix} 1 & \sqrt{\frac{v_\phi}{v_\Delta}} \\ -\sqrt{\frac{v_\Delta}{v_\phi}} & 1 \end{pmatrix} \begin{pmatrix} R_\phi \\ R_\Delta \end{pmatrix}.$$  

(17)
We recognize that \( h^0 \) is the standard Higgs, while \( H^0 \) is a second Higgs that survives in the model. For \( v_\Delta \approx 1 \text{eV} \) and \( v_\phi \approx 10^2 \text{GeV} \), we get \( \frac{v_\Delta}{v_\phi} \approx 10^{-11} \). In this case we see that \( h^0 \) decouples from \( H^0 \).

For the CP-odd neutral scalars we get the mass matrix in the basis \((I_\Delta, I_\phi)\),

\[
m^2_A = \sqrt{2} \mu \begin{pmatrix} 2v_\Delta & -v_\phi \\ -v_\phi & \frac{v_\phi^2}{2v_\Delta} \end{pmatrix}.
\]  

In the limit \( v_\phi \gg \mu > v_\Delta \), we obtain the following eigenvalues,

\[
m^2_{C^0} = 0, \\
m^2_{A^0} \approx \frac{1}{\sqrt{2}} \frac{v_\phi^2 \mu}{v_\Delta},
\]

with their respective eigenvectors,

\[
\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \begin{pmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} I_\phi \\ I_\Delta \end{pmatrix},
\]

where,

\[
\sin \beta = \frac{2v_\Delta}{\sqrt{v_\phi^2 + 2v_\Delta^2}}, \quad \cos \beta = \frac{v_\phi}{\sqrt{v_\phi^2 + 2v_\Delta^2}}.
\]  

As we are assuming \( v_\phi \gg v_\Delta \), we have that \( \sin \beta \to 0 \) and \( \cos \beta \to 1 \) which means that \( G^0 \) decouples from \( A^0 \). Also, \( G^0 \) is a Goldstone boson that will be eaten by the SM neutral gauge boson \( Z \), and \( A^0 \) is a massive CP-odd scalar that survives in the particle spectrum.

The mass matrix for the singly charged scalars in the basis \((\delta^+, \phi^+)\) is given by,

\[
m^2_+ = (\sqrt{2} \mu - \frac{v_\Delta \lambda_4}{2}) \begin{pmatrix} v_\Delta & -v_\phi \\ \frac{v_\phi}{\sqrt{2}} & \frac{v_\phi^2}{2\sqrt{2}} \end{pmatrix}
\]

In the limit \( v_\phi \gg v_\Delta, \mu \), we obtain the following eigenvalues,

\[
m^2_{G^+} = 0, \\
m^2_{H^+} \approx \frac{\sqrt{2}}{2} \left( \frac{\mu}{v_\Delta} - \lambda_4 \right) v_\phi^2,
\]

where \( G^+ \) is the Goldstone boson eaten by the SM charged gauge bosons, \( W^\pm \), while \( H^\pm \) are massive scalars remaining in the spectrum. The mixing mass matrix for \( \delta^\pm \) and \( \phi^\pm \) is the same one that appears in Eq. (20). Thus, the singly charged scalars decouple too.
In regard to the doubly charged scalars, $\delta^{\pm\pm}$, we obtain the following expression for its mass in the limit $v_\phi \gg \mu, v_\Delta$,

$$m^{2}_{\delta^{++}} \simeq \frac{\sqrt{2}}{2} \left( \frac{\mu}{v_\Delta} - \lambda_4 \right) v_\phi^2.$$  

(24)

It is noticeable that in both, the type II seesaw and the ISS2 mechanisms, the new scalars composing the triplet $\Delta$ decouple from the SM scalars. In other words, we have the Englert-Brout-Higgs boson, $h^0$, and the new massive scalars, $H^0, A^0, H^\pm$ and $\delta^{\pm\pm}$, all decoupled in the particle spectrum. The difference is that in the ISS2 mechanism, $v_\Delta$ around eV requires $M_\Delta$ at TeV scale. The degeneracy among $H^+$ and $\delta^{++}$ is a consequence of taking $v_\Delta$ at eV scale. With such mass scale, the new scalars may be probed at LHC or ILC, but before discussing this we first delve into the constraints coming from the rare decay $\mu \rightarrow e\gamma$, which must impose some restriction to the parameter space we are interested in.

V. NEUTRINO MASSES AND THE RARE LEPTON DECAY $\mu \rightarrow e\gamma$

In the flavor basis the Yukawa couplings in Eq. (12) provide the following expression for neutrino masses,

$$m_{ij}^{\nu} = Y_{ij} v_\Delta$$  

(25)

This is a $3 \times 3$ mass matrix which is diagonalized by a $3 \times 3$ mixing matrix $U$,

$$m^D = U m_{ij} U^\dagger$$  

(26)

where $m^D = \text{diag}(m_1, m_2, m_3)$ is the diagonal neutrino mass matrix and $U$ is the PMNS mixing matrix, which is usually parametrized in the following way,

$$U = \begin{pmatrix}
    c_{12}c_{13} & s_{12}c_{13} & s_{13} \\
    -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\
    s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13}
\end{pmatrix},$$  

(27)

where $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$, while in this work we neglect CP violation phases.

Combining Eqs. (25), (26) and (27), we can write the Yukawa couplings in terms of the
phenomenological parameters (mixing angles and neutrino masses) by the following way,

\[
Y_{11} = \frac{1}{v_\Delta}(c_{12}^2(m_1 c_{13}^2 + m_2 s_{12}^2) + m_3 s_{13}^2),
\]

\[
Y_{22} = \frac{1}{v_\Delta}(m_3 c_{13}^2 s_{23} + m_1 (c_2 s_{12} + c_{12} s_{13} s_{23})^2 + m_2 (c_{12} c_{23} - s_{12} s_{13} s_{23})^2),
\]

\[
Y_{33} = \frac{1}{v_\Delta}(m_3 c_{13}^2 s_{23} + m_2 (c_{23} s_{12} s_{13} - c_{12} s_{23})^2 + m_1 (c_{12} c_{23} s_{13} - s_{12} s_{23})^2),
\]

\[
Y_{12} = \frac{1}{v_\Delta}(c_{12}(m_2 c_{12} - m_1 c_{13}) c_{23} s_{12} + ((m_3 - m_1 c_{12}^2) c_{13} - m_2 c_{12} s_{12}^2) s_{13} s_{23}),
\]

\[
Y_{13} = \frac{1}{v_\Delta}(c_{23} s_{13}((m_3 - m_1 c_{12}^2) c_{13} - m_2 c_{12} s_{12}^2) + c_{12} s_{12} s_{23}(m_2 c_{12} + m_1 c_{13})),
\]

\[
Y_{23} = \frac{1}{v_\Delta}(-\frac{1}{2}(m_2 - m_1(1 - 2 s_{23}^2)) (2. s_{12} c_{12}) 2. s_{12} c_{12} s_{13}
+ c_{23} s_{13}(m_3 c_{13}^2 + c_{12}^2(m_2 + m_1 s_{13}^2) + s_{12}^2(-m_1 + m_2 s_{13}^2))).
\] (28)

According to recent data on neutrino physics, the values of the angles involved in the above mixing matrix are\([9]\),

\[
\theta_{12} \simeq \frac{\pi}{5.4}, \quad \theta_{23} \simeq \frac{\pi}{4}, \quad \theta_{13} \simeq \frac{\pi}{20},
\] (29)

while for the masses of the neutrinos, for the normal hierarchy (NH) and inverted hierarchy (IH) cases, we have

\[
m_1, m_2 = \sqrt{m_1^2 + \Delta m^2_{\odot}}, m_3 = \sqrt{m_1^2 + \Delta m^2_{\text{atm}}} \text{ (NH)},
\]

\[
m_3, m_1 = \sqrt{m_3^2 + \Delta m^2_{\text{atm}} - \Delta m^2_{\odot}}, m_2 = \sqrt{m_3^2 + \Delta m^2_{\text{atm}}} \text{ (IH)},
\] (30)

with \(\Delta m^2_{\odot} \simeq 0.0086 \text{ eV}^2\) and \(\Delta m^2_{\text{atm}} \simeq 0.048 \text{ eV}^2\).

Together with the above constraints we also take advantage of the cosmological bound on neutrino masses\([10]\),

\[
m_1 + m_2 + m_3 = 0.35 \text{ eV}.
\] (31)

The ISS2 mechanism relates the three parameters, \(v_\Delta, \mu\) and \(M_\Delta\) through Eq. (11). The essence of the mechanism lies in the fact that lepton number is explicitly violated at low energy scale. In our work this happens in the scalar potential of the model through the trilinear term characterized by the \(\mu\) parameter. The whole scheme will work if for a small \(\mu\), neutrino masses at eV scale are generated. However, a small \(\mu\) requires a small \(M_\Delta\) in order to have \(v_\Delta\) at eV scale. Nevertheless, the existence of the triplet \(\Delta\) in a lepton number violating scenario leads to rare leptonic decays mediated by the charged scalars \(\delta^+\),
and \(\delta^{++}\) engendered by the Eq. (12). The more stringent of the rare decays is \(\mu^- \rightarrow e^- + \gamma\) which occurs through a loop mediated by the charged scalars \(\delta^{\pm\pm}\) and \(H^+\). For the case of degeneracy among \(H^+\) and \(\delta^{\pm\pm}\), the branching ratio (BR) for this process is given by [11],

\[
BR(\mu \rightarrow e\gamma) \simeq \frac{27\alpha|Y_{11}Y_{12} + Y_{12}Y_{22} + Y_{13}Y_{32}|^2}{64\pi G_F^2 m_{\delta^{++}}^3},
\]

where \(\alpha\) is the fine structure constant, \(Y_{ij}\) are the Yukawa coupling constants given in Eq. (28), \(G_F\) is the fermi constant and \(m_{\delta^{\pm\pm}}\) is the mass of doubly charged scalar. The present experimental bounds on this process is \(BR(\mu \rightarrow e\gamma) < 5.7 \times 10^{-13}\) [12]. Our proposal here is to check all possible values for \(\mu\) and \(M_\Delta\) that lead to a \(v_\Delta\) at the eV scale and concomitantly obey the bound \(BR(\mu \rightarrow e\gamma) < 5.7 \times 10^{-13}\). The results are summarized into the curves presented in FIG. 1.

In the top panel of FIG. 1, we see that the smaller the \(\mu\) the lighter the \(M_\Delta\). Considering the two curves shown in the figure we can infer that for \(\mu\) in the range \((1 - 10)\) eV, we must have \(M_\Delta\) spanning \(10^{2-3}\) GeV in order to have \(v_\Delta\) at eV scale. It has to be remarked that we have some freedom in choosing the \(\mu\) value, but it is desirable that its values do not vary much beyond those presented in the top panel of FIG. 1, otherwise we would push \(M_\Delta\) to scales higher than TeV, escaping from current collider searches. In the bottom panel, we have plotted the values of \(M_\Delta\) and \(v_\Delta\) constrained to obey the upper bound on the rare \(\mu \rightarrow e\gamma\) decay, \(BR(\mu \rightarrow e\gamma) < 5.7 \times 10^{-13}\), representing the shaded area in the plot. Observe that there is a considerable overlapping between the curves and the shaded region in these plots. The important outcome in this analysis is that there is plenty of space to obtain light neutrino masses through the ISS2 mechanism while obeying the bounds coming from small neutrino masses and rare leptonic decays.

We call the attention to the fact that, in the regime of the ISS2 mechanism that can be testable at TeV scale, the values of the mass parameters are such that \(\delta^{++}\) and \(\delta^+\) are degenerated in mass, Eqs. (23) and (24). Consequently, \(\delta^{++}\) can decay into a pair of charged leptons and charged gauge bosons, \(W^{\pm}\). However, the decay into \(W^{\pm}\) is strongly suppressed in our model due to the smallness of \(v_\Delta\). In what concerns the hierarchy of neutrino masses, there are two situations to be considered. In the case of NH, \(\delta^{++}\) will decay preferentially into a \(\mu^+\tau^+\) pair, with BR around 46% and into pairs of \(\mu^+\mu^+\) and \(\tau^+\tau^+\), with BR\approx 23% each. For the case of IH, \(\delta^{++}\) will decay preferentially, with \(BR \approx 46\%\), into a \(e^+e^+\) pair, while it decays 27% of the times into the \(e^+\tau^+\) pair and 15% into the \(\mu^+\mu^+\) pair. Thus,
although the analysis made above, concerning the smallness of the neutrinos masses, as well as the bounds from rare leptonic decays are not affected by the neutrino mass hierarchy, we see that the decays of $\delta^{++}$, may offer a way of establish such hierarchy if this ISS2 mechanism rules. We explore this signature next.

VI. SIGNATURE OF THE ISS2 MECHANISM AT THE ILC

The phenomenology of $\delta^{++}$ has been intensively investigated at the LHC\cite{8} and a direct search in the CMS and ATLAS colliders has been already performed and the result was the limit $m_{\delta^{++}} > 459$ GeV for $\delta^{++}$ decaying 100% into $\mu^+\mu^+$ pairs \cite{13}. Investigation of direct detection of $\delta^{++}$ at ILC through diboson decay scenario ($W^+W^+$) has also been considered in \cite{14}. As we have discussed at the end of the last section, in the regime of validity of the ISS2 mechanism, the detection of $\delta^{++}$ must occur through dilepton decay scenarios: $e^+e^- \rightarrow \delta^{++} + \delta^{--} \rightarrow l^+l^+l^-l^-$, where $l = e, \mu, \tau$ with BR of the same order of magnitude for the most important channels, but surely not 100% into $\mu^+\mu^-$ pairs, recalling that diboson decays are suppressed in ISS2. Thus, the LHC bound and the results of the ILC investigation cannot be applied for the case of ISS2 mechanism.

In this section we discuss the direct detection of the $\delta^{++}$ in the framework of the ISS2 mechanism at the ILC for the cases of NH and IH of neutrino masses. We consider the ILC with a luminosity of 1000 fb$^{-1}$ and center of mass energy of 1000 GeV. The main contributions for $\delta^{++}$ production and subsequent decay are presented in FIG. 2 \cite{15}. In FIG. 3 we plot the cross section (top panel) and signal (bottom panel), as a function of the mass parameter $M_\Delta$, for the process $e^+e^- \rightarrow \delta^{++} + \delta^{--} \rightarrow \mu^+\mu^-\mu^+\mu^-$ when we have NH, and $e^+e^- \rightarrow \delta^{++} + \delta^{--} \rightarrow e^+e^-e^+e^-$ for the IH case. We also included the expected backgrounds from SM. In order to obtain these results we made use of the packages MadGraph5 \cite{16} and CalcHep \cite{17} and considered the following kinematic cuts,

$$p_T^l \geq 10\text{GeV, } |\eta^l| \leq 2.5,$$

where $p_T^l$ and $|\eta^l|$ are the transverse momentum and pseudo rapidity for the first most energetic lepton.

In FIG. 4 we show the same processes but now as function of Center of Mass energy, $E_{CM}$. In these plots we have considered only the lowest scalar mass for which the cross section
and signal are higher, \( m_{\delta^{++}} = 200 \text{ GeV} \) (see the plots in FIG. 3 recalling that \( m_{\delta^{++}} \approx M_\Delta \)), although the pattern would be qualitatively the same for the mass range in the hundreds of GeV.

As we can see from these plots, the ILC will be very sensitive to the detection of a doubly charged scalar as predicted by the ISS2 mechanism, while its mass is below 500 GeV. This seems to complement the search at LHC for a doubly charged scalar in the usual ISS mechanism [13], which bounds its mass to be just higher than about 460 GeV. Besides, the very distinct outcomes for the cross section and/or signal when the different hierarchies of neutrino mass are taken into account, allows ILC to be able of distinguishing between normal or inverted hierarchy.

VII. CONCLUSIONS

In this work we proposed and developed the inverse type II seesaw mechanism and showed that it may provide neutrino mass at eV scale with new physics just around the corner (the electroweak scale). The essence of the mechanism lies in the fact that lepton number is explicitly violated at eV to keV scale. To naturally obtain such a low scale, we take advantage of a type II seesaw structure resembling the ordinary inverse seesaw scheme. The gain in doing this is somewhat impressive: while canonical ISS demands at least six new extra neutral fermions to be added to SM and a resulting neutrino mass matrix very combersome to deal with, the ISS2 only needs a scalar triplet to be added, leading to a very simple neutrino mass matrix.

The implementation of the mechanism relies on the addition of a scalar triplet (\( \Delta \)) to the standard model content. We have shown that the model accommodates small neutrino mass and oscillation while being consistent with upper bounds on the rare leptonic decay \( \mu \rightarrow e\gamma \). Also, the existence of a doubly charged scalar, \( \delta^{++} \), composing the Higgs triplet \( \Delta \), is the best signature of the mechanism that can be probed at TeV scale. We developed the phenomenology of the ISS2 mechanism at the ILC analysing the cross section and signal for the case of NH and IH neutrino masses by searching for the doubly charged scalar, \( \delta^{++} \), which in the framework of the ISS2 mechanism, decay preferentially into leptons through the process \( e^+e^-\rightarrow\delta^{++} + \delta^{--}\rightarrow l^+l^+l^-l^- \). We obtained that the ILC with luminosity of \( 1000 fb^{-1} \) and \( E_{CM} \) in the range \( 500 - 1000 \text{ GeV} \) may detect the \( \delta^{++} \) with mass in the range
200 – 500GeV. We checked that a δ++ in this range of mass is compatible with neutrino mass and oscillation parameters besides the non-observed lepton violation process μ → eγ. Finally, we also obtained that the ILC may distinguish among normal or inverted hierarchy, another feature which makes this ISS2 mechanism more attractive and simple than the usual ISS.

Acknowledgments

FFF is supported by Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES). CASP and PSRS are supported by Conselho Nacional de Pesquisa e Desenvolvimento Científico - CNPq. The authors thanks Alexandre Alves for useful discussions.

[1] M. Gell-Mann, P. Ramond, and R. Slansky, in supergravity, edited by P. van Nieuwenhuizen and D. Z. Freedman (North-Holland, amsterdam, 1979); T. Yanagida, in proceedings of the Workshop on the Unified Theory and the Baryon number in the Universe, edited by O. Sawada and A. Sugamoto (KEK Report No. 79-18, Tsukuba, Japan, 1979); R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980).
[2] M. Magg, C. Wetterich, Phys. Lett. B94, 61 (1980); R. N. Mohapatra, G. Senjanovic, Phys. Rev. D23, 165 (1981); E. Ma, U. Sarkar, Phys. Rev. Lett. 80, 5716 (1998).
[3] R. Foot, H. Lew, X. G. He, G. C. Joshi, Z. Phys. C44, 441 (1989).
[4] Seesaw mechanisms at TeV scale and their effects have being investigated in various contexts, see Refs.: A. Abada, M. E. Krauss, W. Porod, F. Staub, A. Vicente and C. Weiland, arXiv:1408.0138; A. Abada, A.M. Teixeira, A. Vicente and C. Weiland, JHEP 1402 (2014), 091; A. Das, P. S. Bhupal Dev and N. Okada, Phys. Lett. B 735 (2014), 364; C-H Lee, P. S. Bhupal Dev, R. N. Mohapatra, Phys. Rev. D88 (2013) 9, 093010 ; A. Das and N.Okada, Phys. Rev. D 88 (2013), 11, 113001; A. Abada, D. Das, A.M. Teixeira, A. Vicente and C. Weiland, JHEP 1302 (2013), 048; K. Kumericki, I. Picek, B. Radovic, Phys. Rev. D86 (2012), 013006 ; S. Kanemura, T. Nabeshima, H. Sugiyama, Phys. Rev. D85 (2012), 033004; P. S. Bhupal Dev, R. N. Mohapatra, Phys. Rev. D81 (2010), 013001; W. Grimus, L. Lavoura, Phys. Lett. B687 (2010), 188; He Zhang, Shun Zhou, Phys. Lett. B685 (2010), 297; I. Gogoladze, N.
Okada, Q. Shafi, Phys. Lett. B 6bf 72 (2009), 235; F. Bonnet, D. Hernandez, T. Ota, W. Winter, JHEP 0910 (2009), 076; Zhi-zhong Xing, Shun Zhou, Phys. Lett. B679 (2009), 249; K. Huitu, S. khalil, H. Okada, S. K. Rai, Phys. Rev. Lett. 101 (2008), 181802; J. Kersten, A. Y. Smirnov, Phys. Rev. D76 (2007), 073005; D. Atwood, S. Bar-Shalom, A. Soni, Phys. Rev. D76 (2007), 033004; F. del Aguila, J. A. Aguilar-Saavedra, R. Pittau, JHEP 0710 (2007), 047; F. del Aguila, J. A. Aguilar-Saavedra, R. Pittau, J. Phys. Conf. Ser. 53 (2006), 506; J. C. Montero, C. A. de S. Pires, V. Pleitez, Phys. Lett. B502 (2001), 167; E. Ma, Phys. Rev. Lett. 86 (2001), 2502.

[5] R. N. Mohapatra, Phys. Rev. Lett. 56 (1986), 561; R. N. Mohapatra, J. W. F. Valle, Phys. Rev. D34 (1986), 1642.

[6] Inverse type III seesaw mechanism was developed in the Refs.: F. -X. Josse-Michaux and E. Molinaro, Phys. Rev. D 87 (2013), 036007; S. Morisi, E. Peinado and A. Vicente, J. Phys. G 40 (2013), 085004; E. Ma, Mod. Phys. Lett. A 24 (2009), 2491; D. Ibanez, S. Morisi and J. W. F. Valle, Phys. Rev. D 80 (2009), 053015.

[7] A. Abada, M. Lucente, Nucl. Phys. B885 (2014), 651; S. S. C. Law, K. L. McDonald, Phys. Rev. D87 (2013) 11, 113003; A. G. Dias, C. A. de S . Pires, P. S. Rodrigues da Silva, A. Sampieri, Phys. Rev. D86 (2012), 035007; A. G. Dias, C. A. de S. Pires, P. S. Rodrigues da Silva, Phys. Rev. D84 (2011), 053011; S. C. Park, Kai Wang, Phys. Lett. B701 (2011), 107; M. Hirsch, T. Kernreiter, J. C. Romao, A. Villanova del Moral, JHEP 1001 (2010), 103; P. S.Bhupal Dev, R. N. Mohapatra, Phys. Rev. D81 (2010), 013001; S. Khalil, Phys. Rev. D82 (2010), 077702; T. Fukuyama, A. Ilakovac, T. Kikuchi, K. Matsuda, J. High Energy Phys. 06 (2005), 016.

[8] P. F. Perez, Tao Han, Gui-yu Huang, Tong Li, Kai Wang, Phys. Rev. D78 (2008), 015018; A. Arhrib, R. Benbrik, M. Chabab, G. Moultaka, M. C. Peyranere, L. Rahili, J. Ramadan, Phys. Rev. D84 (2011), 095005; A. G. Akeroyd, S. Moretti, Phys. Rev. D84 (2011), 035028; A. Melf, M. Nemevsek, F. Nesti, G. Senjanovic, Y. Zhang, Phys. Rev. D85 (2012), 055018; E. J. Chun, P. Sharma, Phys. Lett. B728 (2014), 256.

[9] G. L. Fogli, E. Lisi, A. Marrone, D. Montanino, A. Palazzo, and A. M. Rotunno, Phys. Rev. D86 (2012), 013012; M. C. Gonzalez-Garcia, M. Maltoni, J. Salvado, T. Schwetz, JHEP 1212 (2012), 123.

[10] P. A. R. Ade et al. [Planck Collaboration], arXiv:1303.5076 [astro-ph.CO]; K. N. Abazajian, E.
[11] A. G. Akeroyd, Mayumi Aoki, and Hiroaki Sugiyama, Phys. Rev. D 79 (2009), 113010; For a general formulae for $f_1 \rightarrow f_2 \gamma$, see: L. Lavoura, Eur. Phys. J. C 29 (2003), 191.

[12] MEG Collaboration, Phys. Rev. Lett. 110 (2013), 201801.

[13] G. Aad et al. [ATLAS Collaboration], Phys. Rev. D 85, 032004 (2012); S. Chatrchyan et al. [CMS Collaboration], Eur. Phys. J. C 72, 2189 (2012).

[14] S. Kanemura, M. Kikuchi, K. Yagyu, H. Yokoya, arXiv:1407.6547.

[15] We have to remark that, as we saw in the end of Sec. V, there is a $\delta^{++}$ decay channel involving the $\tau$ lepton which is important too. However, since the hadronically decaying $\tau$ is harder to measure and is not as clean as the lighter leptonic decays, we leave it outside this analysis.

[16] J. Alwall, M. Herquet, F. Maltoni, O. Mattelaer and T. Stelzer, JHEP 1106, 128 (2011).

[17] A. Belyaev, N. D. Christensen, A. Pukhov, Comput. Phys. Commun. 184 (2013), 1729.
FIG. 1: Allowed values for $v_\Delta$(GeV) and $M_\Delta$(GeV) considering the seesaw scheme and the rare lepton decay $\mu \rightarrow e\gamma$. The top panel shows the points for $\mu = 10$ eV (Thick) and $\mu = 1$ eV (Dashed) taking into account only the constraint from the seesaw Eq. (11). The bottom panel gives us the region in the parameter space (with varying $\mu$) that conforms to the bound $BR(\mu \rightarrow e\gamma) < 5.7 \times 10^{-13}$. The shaded area is the experimentally allowed region.
FIG. 2: Main contributions for the process $e^+ + e^- \rightarrow \delta^{++} + \delta^{--} \rightarrow l^+ l^- l^+ l^-$ for the case of $l = e$ and $l = \mu$ considering both hierarchies for neutrino masses separately.
FIG. 3: Cross section (in logarithmic scale) and signal as function of $M_\Delta$, at the ILC for NH and IH cases. BG is background and L is luminosity.
FIG. 4: The same as FIG. 3 but as a function of $E_{CM}$ instead of $M_{\Delta}$, for $m_{\delta^{++}} = 200$ GeV.