Determination of the turbulent parameter in accretion disks: effects of self-irradiation in 4U 1543–47 during the 2002 outburst

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ABSTRACT
We investigate the viscous evolution of the accretion disk in 4U 1543–47, a black hole binary system, during the first 30 days after the peak of the 2002 burst by comparing the observed and theoretical accretion rate evolution \( M(t) \). The observed \( M(t) \) is obtained from spectral modelling of the archival RXTE/PCA data. Different scenarios of disk decay evolution are possible depending on a degree of self-irradiation of the disk by the emission from its centre. If the self-irradiation, which is parametrized by factor \( C_{irr} \), had been as high as \( \sim 5 \times 10^{-3} \), then the disk would have been completely ionized up to the tidal radius and the short time of the decay would have required the turbulent parameter \( \alpha \sim 3 \). We find that the shape of the \( M(t) \) curve is much better explained in a model with a shrinking high-viscosity zone. If \( C_{irr} \approx (2–3) \times 10^{-4} \), the resulting \( \alpha \) lie in the interval 0.5–1.5 for the black hole masses in the range 6–10 M\(_{\odot}\), while the radius of the ionized disk is variable and controlled by irradiation. For very weak irradiation, \( C_{irr} < 1.5 \times 10^{-4} \), the burst decline develops as in normal outbursts of dwarf novae with \( \alpha \sim 0.08–0.32 \). The optical data indicate that \( C_{irr} \) in 4U 1543–47 (2002) was not greater than approximately \( (3–6) \times 10^{-4} \). Generally, modelling of an X-ray nova burst allows one to estimate \( \alpha \) that depends on the black hole parameters. We present the public 1-D code freddi to model the viscous evolution of an accretion disk. Analytic approximations are derived to estimate \( \alpha \) in X-ray novae using \( M(t) \).

Key words: accretion – accretion disks – binaries: general – methods: numerical – X-rays: individual: 4U 1543–47

1 INTRODUCTION

The viscosity in the accretion disks is explained by the magneto-hydrodynamic (MHD) turbulence, developing due to the magneto-rotational instability (Hawley & Balbus 1991; Velikhov 1959; Chandrasekhar 1961). MHD simulations of the turbulence have been generally providing the turbulent parameter \( \alpha \sim 0.01 \). More recently, numerical simulations have achieved a value of \( \alpha \sim 0.1 \) for the regions with the partial ionization and convection (Hirotse et al. 2014). Yet observations sometime indicate higher values: \( \alpha \sim 0.1–0.4 \) (King et al. 2007), 0.5–0.6 (Suleimanov et al. 2008; Malanchev & Shakura 2015). In this work, we study in detail an outburst of one X-ray nova and derive constraints on \( \alpha \).

The thermal instability in the zone of partial ionization of hydrogen in an accretion disk around a compact star is believed to be a trigger mechanism of outbursts in X-ray novae. The Disk Instability Model (DIM) was originally put forward to explain dwarf novae outbursts (see reviews by Smak 1984b; Lasota 2001). The accumulation of mass in the cold disk (where the effective temperature is \( \lesssim 10^4 \) K and the matter is not ionized) between bursts leads to the situation when the surface density exceeds a critical value, and the transition to the hot state occurs on the thermal timescale. As the model predicts, this happens first at some radius, proceeding then to other disk rings outwards and inwards as an ‘avalanche’ and converting the disk to the hot state. The viscous evolution of the disk leads to an increase of the central accretion rate, which is observed as an X-ray outburst if the central body is a compact object. To reproduce outburst cycles of dwarf and X-ray novae, the \( \alpha \)-parameter should be higher in the hot state by 1–2 orders of magnitude than in the cold state (see, e.g., Lasota 2001; King et al. 2007).

The central accretion rate during a burst is determined by the viscous evolution in the high-viscosity hot part of the disk because the cold part evolves at a relatively low rate. The analysis of the viscous evolution of X-ray novae during outbursts provides an estimate of the turbulent parameter \( \alpha \) in the hot state. The size of the hot part of the disk depends on the radial temperature distribution, meanwhile the disk temperature depends on the viscous heating rate and the incident flux of central X-rays. Previously, Suleimanov et al. (2008) have analysed X-ray and optical light-curves of A 0620-00 (1975) and GS 1124-68 (1991), which have...
fast-rise exponential-decay (FRED) X-ray light curves. They have considered the completely ionized disks with the fixed outer radius and obtained values of $\alpha$ depending on the black hole parameters (see also Malanchev & Shakura (2015)).

In the present work, we study details of the viscous evolution of the disk in X-ray transient 4U 1543–47 during the first ~30 days of the decay of its FRED-like outburst in 2002. The archival X-ray spectral observations by RXTE/PCA are used to derive the evolution of the central accretion rate $\dot{M}(t)$. System 4U 1543–47 (V* IL Lup) is a low mass X-ray binary (LMXB) that shows outbursts about every ten years. The compact accreting object is a reliable black hole (BH) candidate. The binary has an orbital period of 1.116 day (Orosz 2003), which is longer than those of most LMXBs with known orbital periods (Ritter & Kolb 2003). For example, the X-ray novae with BH candidates A 0620–00, GS 1124–68, GS 2000+25, and GRO 0422+32 have orbital periods less than 10 hours. The optical companion to 4U 1543–47 is an A2V star with a mass of $M_{\text{opt}} \sim 2.5 M_\odot$. The mass of the black hole has been estimated as $M_\bullet = 2.7 \pm 0.5 M_\odot$ or $M_\bullet = 9.4 \pm 2 M_\odot$, and the distance $9.1 \pm 1.1$ kpc (Orosz et al. 1998a, 2002). The latest observational result is a BH mass of $M_\bullet = 8.4 \pm 10.4 M_\odot$, and an inclination of $20.7 \pm 1.5^\circ$ for the binary system (Orosz 2003).

Different scenarios or stages of the disk evolution during an outburst of an X-ray Nova are possible, which generally follow each other. During some time, the size of the hot zone can be constant, being approximately equal to the tidal truncation radius in the binary system. In case of 4U 1543–47, this requires a strongly irradiated accretion disk. For less irradiated disk, the hot zone should be shrinking, although the irradiation may play an important role in the disk evolution. Finally, a stage of virtually negligible irradiation follows, resembling the disk decay in dwarf novae.

Using the observational data for 4U 1543–47, we study all these options. For a grid of the parameters $m_\alpha$ and $a_{Kerr}$, we compare the accretion rate $\dot{M}(t)$ obtained from the RXTE data with the theoretical results. The optical light curves in $V$ and $J$ bands obtained by Buxton & Bailyn (2004) are used to check the degree of self-irradiation. We assess the turbulent $\alpha$-parameter in the hot state, as well as other disk properties of 4U 1543–47, and generalize our modelling results.

To model the evolution of a hot irradiated disk, 1-D code 
1Dcode has been developed and used in this work. The main input parameters of the code are as follows: the turbulent parameter $\alpha$, the BH mass $m_\bullet$\footnote{We use two notations for the black hole mass: $M_\bullet$ is the mass in grams and $m_\bullet = M_\bullet / M_\odot$ is the mass in solar masses.}, and Kerr parameter $a_{Kerr}$. For a non-irradiated disk, we expect that a cooling front, which resembles that in a dwarf-nova disk, controls $\dot{M}(t)$ and the X-ray light curve. To model such evolution, we follow the approach used by Kotko & Lasota (2012) for normal outbursts of dwarf novae.

The structure of the paper is as follows. The spectral mod-
The modelled peak accretion rate of the 2002 outburst of 4U 1543–47 (at MJD 52446) in different spectral models with \( i = 20^\circ \) and \( J_{\text{gal}} = 1.7 \). The list of these and other parameters can be found in Table A1 versus the BH mass. The results for both spectral combinations are plotted, although the difference of their peak accretion rates is rather imperceptible: The symbols indicate the different values of \( a_{\text{Kerr}} \): 0 (circles), 0.1 (triangle), 0.4 (square), 0.6 (diamonds), 0.9 (crosses), and 0.998 (asterisks). Two straight lines show the accretion rates corresponding to the Eddington limit \( L_{\text{Edd}} \) and different efficiencies of the accretion onto the black hole: for \( a_{\text{Kerr}} = 0.0 \) (solid) and \( a_{\text{Kerr}} = 0.998 \) (dashed). See Appendix A for details.

### 2 SPECTRAL MODELLING OF THE ACCRETION RATE EVOLUTION OF 4U 1543–47 IN 2002

The outburst of 4U 1543–47 in 2002 had a FRED-like X-ray light curve with the first minor X-ray/IR re-flare at \( \sim 15 \) day and the second bigger IR/optical re-flare at \( \sim 40 \) day from the peak (Park et al. 2004; Buxton & Bailyn 2004). We analyse the same data obtained with the Proportional Counter Array aboard the RXTE observatory as in Park et al. (2004).

There are two main spectral components in the 4U 1543–47 burst spectra, as found by Park et al. (2004): the multicolour blackbody-disk thermal emission with a maximum around 1 keV and the non-thermal component at higher energies. To describe the thermal emission of the disk, Park et al. (2004) used the diskbb model as a spectral component in XSPEC (Arnaud 1996). In this model, parameter \( T_{\text{in}} \), the ‘inner’ disk temperature, is an indicator of the accretion rate through the inner edge of the disk. An actual value of the accretion rate is related to \( T_{\text{in}} \) in a complex way because of the general relativity effects.

We use the kerrbb spectral model (Li et al. 2005) in XSPEC that takes into account general relativity effects on flux production and propagation of photons in the vicinity of a black hole, with \( M \) as a free parameter. Here we outline the principle features of our modelling, and the details can be found in Appendix A.

The kerrbb parameters, which we set free, are the accretion rate \( \dot{M} \) and the distance \( d \). For each fit, we fix the BH mass \( M_b \) and the dimensionless Kerr parameter \( a_{\text{Kerr}} \). The disk inclination is the binary orbit inclination, \( 20^\circ/7 \) (Orosz 2003), or, alternatively, \( 32^\circ \) (a value suggested by spectral modelling; Morningstar & Miller 2014). The list of these and other parameters can be found in Table A1 in Appendix A.

To describe the non-thermal component as the thermal emission comptt (Steiner et al. 2009) or the ad-ditive model simpl (Titarchuk 1994). Thus, two combinations of spectral components are used, including either simpl or comptt. As we have found, both spectral combinations give very similar peak magnitudes of \( M \).

For each spectral combination, we have obtained at least 20 accretion rate curves: for each pair of values of the BH mass and the Kerr parameter. An example of the evolution of some spectral parameters is shown in Fig. 1, where we adopt observationally suggested (hereafter ‘central’) disk parameters: \( m_x = 9.4, a = 0.4, i = 20.7^\circ \) (Orosz 2003).

The peak accretion rates versus the BH parameters are shown in Fig. 2. The higher \( a_{\text{Kerr}} \), the less the accretion rate is needed to generate the observed X-ray flux. The obtained accretion peak rates for \( a_{\text{Kerr}} = 0 \) are very close to the critical Eddington rate calculated as \( L_{\text{Edd}}/\left(\tau_0 a_{\text{Kerr}} \right)^2 \). We made additional spectral fits, fixing the distance at its average value \( \rho_{\text{gal}} \). The additional fits demonstrate that, apparently, the accretion rate decays smoothly (empty circles in Fig. 1). The bump on the \( M(t) \) curve (solid circles around MJD 52460) is probably a result of strong variations of the non-thermal component, or a manifestation of an extra spectral component, which cannot be properly described by the presumed spectral model.

### 3 DETERMINATION OF THE VALUE OF \( \alpha \) FROM X-RAY NOVA OUTBURSTS

The viscous evolution of an accretion disk involves the radial redistribution of the viscous torque and the mass density. This process is described by the equation of diffusion type:

\[
\frac{\partial \Sigma}{\partial t} = \frac{1}{4\pi} \frac{(GM_b)^2}{h^3} \frac{\partial^2 F}{\partial h^2},
\]

where \( h = \sqrt{GM_b \sigma} \) is the specific angular momentum, \( \Sigma \) is the surface density, \( F = 2 \pi W_{\nu} r^2 \) is the viscous torque expressed using the height-integrated viscous stress tensor \( W_{\nu} \) (see, e.g., Lyubarskij & Shakura 1987). The viscous torque and surface density in the Keplerian disk are related as follows:

\[
F = 3 \pi h \nu_i \Sigma,
\]

where \( \nu_i \) is the kinematic coefficient of turbulent viscosity.

Using \( F(h) \) as a radial characteristic is very effective. First, boundary conditions are usually set on \( F \) or its first derivative \( \partial F/\partial h = M \). Second, the viscous heating in the disk is explicitly related to the viscous torque, \( Q_{\nu} = 3 \left( GM_b \sigma \right)^2 F/(8 \pi h^3) \), and this relation holds for cases with \( M = 0 \) too.

To illustrate how the viscous torque behaves, we plot schematically the \( F(h) \)-distribution in the disk with the hot inner and cold outer parts (Fig. 3). At the inner boundary of the disk around the black hole, the viscous torque is zero: \( F_{\nu} = 0 \). A positive slope corresponds to mass inflow and vice versa. In the outer, cold neutral disk, the viscous torque is depressed. Between the cold and hot
Figure 3. A schematic representation of the viscous torque distribution in the disk, which has the hot inner and cold outer zone. The grey arrows show the direction of the mass flow, and their heights indicate the representative moduli of the accretion rate. The slope of the curve $F(h)$ equals the accretion rate. In the outer cold disk, the accretion rate is suppressed due to a lower value of the kinematic coefficient of turbulent viscosity. There are two locations where the accretion rate $M = \partial F/\partial h$ becomes zero.

zone, where the hydrogen undergoes recombination/ionization, the opacity coefficient varies strongly. If the disk in a binary system is fully ionized during some period of time, the outer radius of the disk may be considered stationary, corresponding to the dashed line (the corresponding quasi-stationary distribution of $F$ is derived by Lipunova & Shkurova 2000). The mass is lost from the disk through its inner boundary. When the accretion rate drops to such level that the surface density in the hot disk becomes less than a critical value, the outside-in transition to the cold state begins (Smak 1984b).

At the outer boundary of the hot zone (the dashed line in Fig. 3), the viscous torque $F$ reaches an extremum. Subsequently, we assume that the mass flow becomes zero at the boundary: $\partial F/\partial h = 0$. There is a mass outflow in the transition zone, and the mass accumulates in the cold zone while the transition region moves towards the centre. In the cold zone, it takes a longer time for a quasi-stationary distribution to develop, hence, the slope of $F(h)$ is not constant there.

There are three successive stages of an outburst decay in X-ray novae as suggested by Dubus et al. (2001): (1) the hot zone has the constant radius $R_{\text{out}}$ due to the strong irradiation (this stage can be absent if the disk is too large); (2) the cooling front begins to move, its position being determined by the irradiation; (3) the cooling front propagates with the speed $\sim \alpha_{\text{hot}} u_{\text{sound}}$, when the irradiation is not important. Two first stages are possible only if the tidal interactions with the companion truncate the disk at $R_{\text{out}}$.

3.1 The hot disk with a fixed outer radius

If the outer radius is constant, the $\alpha$-folding time of the accretion rate viscous in the disk around a stellar-mass black hole in a binary system is

$$\tau_{\text{exp}} \approx 0.45 R_{\text{out}}^2/v_i(R_{\text{out}})$$

(3)

The above relation is obtained as an approximation of the exact solution for evolution of the disk whose kinematic viscosity $v_i(r)$ does not change with time (Lipunova 2015). Such a viscous disk with the fixed outer radius yields a precisely exponential decay of $M(t)$.

In $\alpha$-disks (Shakura & Sunyaev 1973), the kinematic coefficient of turbulent viscosity depends on the surface density and, respectively, on time: $v_i = v_i(r, t)$. The solution for the decay of a hot $\alpha$-disk with a fixed outer radius has been found by Lipunova & Shakurova (2000). The accretion rate decays as $M \propto t^{-10/3}$ in the regime of the Kramers opacity. Such a steep decay can be observed in X-rays as an exponential decay.

The analytic solution may be applied for a time interval when the whole disk is in the hot state. This can happen due to a high viscous heating or a strong irradiating flux, keeping the whole disk in the completely ionized state. (King & Ritter 1998; Shabbaz et al. 1998; Esin et al. 2000; Dubus et al. 2001, also Meyer & Meyer-Hofmeister (1984, 1990)). Ertan & Alpar (2002) have suggested that the radius $R_{\text{hot}}$ may be constant, although smaller than the disk size, if a specific height profile is developed with a stationary boundary between the hot and cold zone, beyond which the disk is shadowed from the central X-rays.

Let us estimate $\alpha$ using (3). Let us parametrize the local viscous stress tensor in the equatorial plane of the disk as $w_{\text{visc}} = \alpha P_c$, where $P_c$ is the total pressure (Shakurova & Sunyaev 1973). For the outer parts of the hot disk, the gas pressure dominates. On the other hand, $w_{\text{visc}} = 3/2 \omega K v_i P_c$ (c.f. Eq. (2)). Substituting $P_c/\rho_c = K T_c/\mu$ yields a relation between $\nu$ and $v_i = 1/2 (\nu/\omega K)(K T_c/\mu)$. Following the work of Ketsaris & Shakurova (1998), we use the relation between $T_c$ and the half-thickness $z_0$, which is the depth where the optical thickness is $\tau = 2/3$ and $T = T_{\text{disk}}: K T_c/\mu = \omega K z_0^2/\Pi_1$; $\Pi_1$ is a dimensionless parameter depending on the total optical thickness of the disk in the vertical direction; it has been calculated by Ketsaris & Shakurova (1998) and Malanchev et al. (2017). We arrive at the relation $v_i(r) = \frac{1}{3} (\nu/\omega K) (K T_c z_0^2/\Pi_1)$ and, hence,

$$\alpha \sim 0.15 \left( \frac{R_{\text{out}}}{2 R_c} \right)^{3/2} \left( \frac{z_0}{0.05} \right)^{-3/5} \left( \frac{M_c}{10^4 M_\odot} \right)^{-1/2} \left( \frac{\tau_{\text{exp}}}{30^2} \right)^{-1} \times \Pi_1 ,$$

(4)

twhere $\tau_{\text{exp}}$ is the $\alpha$-folding of the accretion rate decay, $z_0$ is the disk half-thickness near the outer radius, and $\Pi_1 = 5.5 - 6$. To estimate $\alpha$, one should substitute $z_0$ corresponding to the peak of an X-ray nova outburst.

The main uncertainty in the above formula is the radius of the disk. In addition, the evolution of the half-height of the $\alpha$-disk leads to a variation of the numerical factor in (4). However, the modeling of the disk evolution can provide a self-consistent value of $\alpha$. This was done for outbursts of A 0620–00 (1975) and GS 1124–68 (1991) by Suleimanov et al. (2008). Using our code freddi, we find that for any fully-ionized viscously-accreting thin disk

$$\alpha \approx 0.21 \left( \frac{R_{\text{out}}}{R_c} \right)^{25/16} \left( \frac{\tau_{\text{exp}}}{30^2} \right)^{-5/4} \left( \frac{M_{\text{max}}}{10^3 M_\odot} \right)^{-3/8} m_5^{16} ,$$

(5)

for the Kramers opacity, or

$$\alpha \approx 0.20 \left( \frac{R_{\text{out}}}{R_c} \right)^{17/12} \left( \frac{\tau_{\text{exp}}}{30^2} \right)^{-9/7} \left( \frac{M_{\text{max}}}{10^3 M_\odot} \right)^{-3/7} m_5^{27} ,$$

(6)

for the OPAL approximation. Power indexes in the above expressions are obtained when substituting the thickness of the disk in (4) by its analytic expression from Suleimanov et al. (2007). An accuracy of the numerical factors is 5%. Details of the freddi code can be found in Appendix B.

Let us estimate $\alpha$ for 4U 1543–47, supposing that the fast viscous evolution has swept the whole disk. The Roche lobe effective radius of the primary in 4U 1543–47 is $R_{\text{in}} \approx 4.2 - 5.3 R_\odot$ (Eggleton 1983) for the BH mass lying in the interval $M_t = 6 - 10 M_\odot$. The maximum disk radius in 4U 1543–47 can be as large as $\sim 4 R_\odot$, if the tidal interactions with the companion truncate the disk at

2 Freddi can be freely downloaded from the authors’ web page http://xray.sai.msu.ru/~malanchev/freddi/.
\( R_{\text{hid}} \approx 0.8 R_{\text{Edd}} \) (Paczynski 1977; Suleimanov et al. 2007). When substituting the radius and other parameters for 4U 1543–47 in (5) or (6), namely, \( t_{\text{exp}} = 15^4 \), \( m_{\text{h}} = 9.4 \), \( M_{\text{max}} = 1.4 \times 10^{10} \text{ g/s} \) (from Fig. 1, the upper left panel), we obtain \( \alpha \approx 3.2 \), \( \sim 3.3 \), an unpalatable value. This result indicates that the disk evolved in a different manner. Most probably, during the burst of 2002 the outer parts of the disk in 4U 1543–47 remained cold.

### 3.2 Disks with \( R_{\text{hot}} \) controlled by irradiation

After the recombination starts, the radius of the hot disk decreases. If the X-ray heating is strong, the radius of the hot zone \( R_{\text{hot}} \) is determined by the incident flux, that is, by the central accretion rate. Dubus et al. (2001) argue that the transition between the hot and cold portions of the disk in X-ray transients is determined by the position where \( T_{\text{irr}} = T_{\text{Edd}} \). This is explained by the fact that the cold state exists only for lower \( T_{\text{irr}} \) (Meyer & Meyer-Hofmeister 1984; Tuchman et al. 1990). Some evidence for such behaviour was found by Hynes et al. (2002) in the 1999–2000 outburst of XTE J1555+226, although they suggested that the condition at the hot zone radius \( R_{\text{hot}} \) may be more complex.

We adopt the following parametrization for the flux, \( Q_{\text{irr}} \), heating the outer disk (Lyutyi & Sunyaev 1976; Cunningham 1976):

\[
Q_{\text{irr}} = \sigma_{\text{SB}} T_{\text{irr}}^4 = C_{\text{irr}} \frac{L_{\text{bol}}}{4 \pi R_{\text{phot}}^2},
\]

where \( L_{\text{bol}} = \eta(\alpha_{\text{Kerr}}) \dot{M} c^2 \) is the bolometric flux, \( \eta(\alpha_{\text{Kerr}}) \) is the accretion efficiency, \( \sigma_{\text{SB}} \) is the Stephan–Boltzmann constant, and \( C_{\text{irr}} \) is the irradiation parameter. We assume in the present study that \( C_{\text{irr}} = \text{const} \) (see discussion in §6.1).

We calculate the radius of the hot zone \( R_{\text{hot}}(t) \) from

\[
R_{\text{hot}} = \frac{\eta(\alpha_{\text{Kerr}}) M(t)}{4 \pi R_{\text{irr}}^2},
\]

where \( T_{\text{irr}} = 10^4 \text{ K} \) and \( M(t) \) is the accretion rate through the inner edge of the disk.

In Fig. 4, we show the hot zone size versus the BH mass for 4U 1543–47, calculated for two different irradiation factors: \( C_{\text{irr}} = 5 \times 10^{-3} \) (the value suggested for X-ray novae (Dubus et al. 2001)) and \( C_{\text{irr}} = 5 \times 10^{-1} \). The black symbols for \( R_{\text{hot}} \) correspond to the peak accretion rate (MJD 25446) and the green symbols online represent the accretion rate at the end of the studied interval (MJD 52474).

When the accretion rate is high and \( R_{\text{hot}}(M) > R_{\text{hid}} \), the hot zone extends through the entire disk and, hence, the viscously evolving region has a constant radius. As we can see from Fig. 4 for \( C_{\text{irr}} = 5 \times 10^{-1} \) (top), the whole disk would have been hot during almost entire investigated time interval. For lower \( C_{\text{irr}} \), the hot zone radius becomes smaller than \( R_{\text{hid}} \) (see Fig. 4, the lower panel) and cannot be constant during the investigated time interval.

The relative importance of the irradiation compared to the viscous heating can be expressed as follows:

\[
Q_{\text{irr}} = \frac{4}{3} \tau_{\text{Kerr}} C_{\text{irr}} \frac{G M(t)}{c^2} \frac{1}{R_{\text{phot}}(t)}
\]

(e.g. Suleimanov et al. 2007), where \( \tau_{\text{phot}} = 2 G M/c^2 \) and the viscous heating

\[
Q_{\text{vis}} = \frac{3}{8} \frac{G M(t)}{\pi} \frac{F}{r_{\text{phot}}^2}, \quad F = M(t) h f(r). \]

For the quasi-stationary solution of Eq. (1), the dimensionless factor \( f(r) \approx 0.7 \) at the radius where \( M = 0 \) (Lipunova & Shakura 2000). The ratio (9) does not depend on the accretion rate in the approximation of constant \( C_{\text{irr}} \). The illumination is more important for bigger disks. If \( Q_{\text{irr}} > Q_{\text{vis}} \) at \( R_{\text{hot}}(t) \), a change in \( R_{\text{hot}}(t) \) is determined by a variation of the irradiating flux. As the latter depends on \( M(t) \), the radius of the hot zone shifts at a rate determined by the viscous timescale at \( R_{\text{hot}} \).

Fig. 5 presents values of \( C_{\text{irr}} \), critical in the context of explaining the decay of 2002 burst of 4U 1543–47. The higher sequence of symbols represents the minimum values of \( C_{\text{irr}} \) above which the whole modelled light curve should be explained by the irradiation-controlled decay.
controlled hot zone. These values are found from the condition \( Q_{\text{in}}/Q_{\text{vis}} = 1 \) at \( R_{\text{hot}} \) at the end of the studied time interval, taking into account Eq. (8).

To model evolution of the irradiation-controlled hot disk, we solve the viscous diffusion equation (1) on the interval from \( R_{\text{in}} \) to \( R_{\text{hot}} \) using our \textsc{freddi} code. The condition \( M = 0 \) is set at \( R_{\text{hot}} \). The outer boundary thus corresponds to the vertical dashed line in Fig. 3. Physically, the transition to the cold disk starts there, so we determine \( R_{\text{hot}} \) at each step using (8).

All models, calculated by \textsc{freddi} for the present study, are obtained with the quasi-stationary distribution as the initial one. This corresponds to the calculation of the decaying part of the burst, although \textsc{freddi} can simulate the light curve from the early rise (see Appendix B).

### 3.3 Cooling front without irradiation

If X-ray heating is very low, a so-called cooling front, surrounding the hot zone, propagates towards the centre while the accretion rate decreases. The heating and cooling fronts without irradiation have been studied to explain dwarf nova bursts (Hoshi 1979; Meyer & Meyer-Hofmeister 1981; Smak 1984a; Meyer 1984; Lin et al. 1985; Cannizzo 1994; Vishniac & Wheeler 1996; Menou et al. 1999; Smak 2000).

Numerical modelling of DIM in dwarf novae shows that, if \( \alpha_{\text{hot}} \) is constant, the speed of the cooling front approaches a characteristic constant velocity of order of \( \alpha_{\text{hot}} u_{\text{sound}} \) (Meyer 1984; Cannizzo 1994; Vishniac & Wheeler 1996), with the hot inner part of the disk evolving in a self-similar way (Menou et al. 1999).

Using analytic approximations to numerical results of DIM, Kotko & Lasota (2012) considered about 20 outbursts of dwarf novae and AM CVn stars to determine the hot disk viscosity parameter \( \alpha_{\text{hot}} \) from the measured decay times. This method relies on the implied velocity of the cooling front propagation. Kotko & Lasota (2012) have also used the DIM, which is able to reproduce the normal outbursts of dwarf novae, in order to relate the amplitudes of the outbursts and the recurrence times. Both methods yield \( \alpha_{\text{hot}} \sim 0.1 \sim 0.2 \).

Similarly, to reproduce the light curve of 4U 1543–47 in the model of the cooling front propagation without irradiation, we adopt the approximations to numerical modelling by Menou et al. (1999). The accretion rate decays as

\[
M(t) = M_{\text{peak}} (R_{\text{cool}}(t)/R_{\text{hot, peak}})^{\frac{2}{3}},
\]

where the radius of the hot zone

\[
R_{\text{cool}}(t) = R_{\text{hot, peak}} - u_{\text{cool}} t
\]

can be found using the front velocity

\[
u_{\text{cool}} = k \alpha u_{\text{sound}}, \quad u_{\text{sound}} = \sqrt{R T_{\text{cool}}/\mu}, \quad k \approx 1/14.
\]

At the front, the disk temperature at the central plane is \( T_{\text{cool}} = 4.7 \times 10^4 \) K (Kotko & Lasota 2012) and the molecular weight \( \mu = 0.6 \). Since irradiation is not important, the maximum radius of the hot disk is found from the rate of the viscous heating (10) at the peak:

\[
Q_{\text{in}}(R_{\text{hot, peak}}) = \sigma_{\text{SH}} T_{\text{hot}}^4, \quad T_{\text{hot}} = 10^4 \text{ K}.
\]

As can be seen from Fig. 5, the above approach is appropriate for 4U 1543–47 (2002) if \( C_{\text{in}} \leq 1.5 \times 10^{-4} \). The low set of symbols in Fig. 5 represents the critical values of the irradiation parameter \( C_{\text{in}} \) for 4U 1543–47 in 2002, below which the self-irradiation could not control the disk evolution. The values are obtained from the condition that \( Q_{\text{in}}/Q_{\text{vis}}(R_{\text{hot}}) = 1 \) at the burst peak, taking into account Eq. (8). Since ratio (9) decreases with the radius, the effect of irradiation becomes only less during the decay as \( R_{\text{hot}} \) decreases.

By applying the approximation of the constant-speed cooling front, \( \alpha \) may be expressed as follows:

\[
\alpha_{\text{hot}} \approx 0.16 \frac{R_{\text{hot, peak}}}{R_{\text{in}}} \left( \frac{\tau_{\text{cool}}}{10^4} \right)^{-1} \left( \frac{k}{1/14} \right)^{-1} \left( \frac{u_{\text{sound}}}{25 \text{ km s}^{-1}} \right)^{-1}.
\]

Expressing the hot-zone size at the peak \( R_{\text{hot, max}} \) from the accretion rate \( M_{\text{max}} \) using (13), we rewrite the last relation as:

\[
\alpha_{\text{hot}} \approx 0.07 \left( \frac{M_{\text{max}}}{10^{11} \text{ g s}^{-1}} \right)^{1/3} \left( \frac{\tau_{\text{cool}}}{10^4} \right)^{1/3} \left( \frac{u_{\text{sound}}}{25 \text{ km s}^{-1}} \right)^{-1} \left( \frac{R_{\text{hot, max}}}{10^{13} \text{ cm}} \right)^{1/3}.
\]

The numerical factor in (15) depends on the power-law index of \( M(R_{\text{hot}}) \) in relationship (11).

### 4 RESULTS OF FITTING OF THE ACCRETION-RATE EVOLUTION FOR 4U 1543–47 (2002)

#### 4.1 Mild irradiation

The viscous evolution of the hot part of the disk whose size is controlled by irradiation (§3.2) can explain the shape of the observed curve \( M(t) \). For characteristic values of \( C_{\text{in}} \) shown in Fig. 5 (the upper boundary between the empty and shaded areas, \( C_{\text{in}} = (2 - 3) \times 10^{-4} \)), we solve equation (1) numerically using code \textsc{freddi} to find \( M(t) \). Comparing it with \( M(t) \) derived from the spectral modelling (see §2), we find the best-fit values of \( \alpha \) as a grid of BH parameters (Fig. 6).

Higher values of \( C_{\text{in}} \) would correspond to larger hot-zone size \( R_{\text{hot}} \), according to (8). Since the observed \( \tau_{\text{cool}} \) is fixed, this would require bigger values of \( \alpha \).

An example of a model with the minimal \( \chi^2 \) statistic is presented in Fig. 7. The top panel shows the evolution of the rate of accretion through the inner radius for the parameters indicated in the plot, along with the spectral-modelling results from Fig. 1. In the lower panel of Fig. 7, the evolution of the irradiation-controlled hot-zone size is shown for the same parameters by black. By green and red (online), we show the hot-zone radius evolution for other sets of parameters, representing extreme cases within the parameters’ limits considered. Values of the accretion rate and distance to the source are unique for each set.

Values of \( \alpha \), found by \textsc{freddi} for such scenario, lie in the interval ~ 0.5 – 1.5 (see Fig. 6). It is possible that \textsc{freddi} does not fully account for complex effects appearing when the outer boundary \( R_{\text{hot}} \) is moving. This can bias the resulting \( \alpha \) towards bigger values. We discuss this further in §6.2. The results, presented in Fig. 6, can be approximated in a way similar to (5) or (6). The approximation has the same power indexes from the disk parameters, while the dimensionless numerical factor is less: 0.17, for Kramers opacity, and 0.15, for OPAL approximation, with an accuracy of ~ 12%.

If \( C_{\text{in}} \) lied in the interval \((1.5 - 3) \times 10^{-4} \), it would follow from the picture described in §3.2 and 3.3, that the cooling front of ‘dwarf-nova-disk’ type had started sometime during the investigated time interval. A visible change in the evolution of \( M(t) \) is expected, when irradiation fails to support the outer hot disk (Dubus et al. 2001), but further modelling is needed to study this in detail.

Notice that there are no suspicious turns in the \( M(t) \) curve during MJD 25446–25474 (see the note at the end of §2).
Determination of the turbulent parameter $\alpha$ in 4U 1543−47 (2002) had, if its size was controlled by irradiation ($\S$3.2). Values of $\alpha$ are obtained for values of $C_{irr}$ shown in Fig. 5 by bigger symbols. Each value of $\alpha$ is obtained using $\chi^2$ minimization of the $\dot{M}(t)$ curve. The radius of the hot zone corresponds to $T_{irr}=10^4$ K. As before, the symbols indicate different values of $a_{Kerr}$: 0 (circles), 0.1 (triangle), 0.4 (square), 0.6 (diamonds), 0.9 (crosses), and 0.998 (asterisks). Each pair of parameters $(m_x, a_{Kerr})$ has two resulting $\alpha$ for two XSPEC models, with either simple or comptt component.

4.2 No irradiation

For $C_{irr} \lesssim 1.5 \times 10^{-4}$, we can describe the burst decay using the same approach as used for normal outbursts of dwarf novae (see $\S$3.3). For the cooling front moving with a constant speed and the peak radius of the hot zone corresponding to $T_{hot} = 10^4$ K, we obtain $\alpha$ shown in Fig. 8 for the same grid of the values $m_x$ and $a_{Kerr}$. All values of $\alpha$ are in the interval 0.08 – 0.32.

Fig. 9 shows the modelled accretion rate for the specific BH parameters: $m_x = 9.4$ and $a_{Kerr} = 0.4$ (the top panel). The lower panel of Fig. 9 shows the hot-zone radius evolution for three sets of the parameters. As can be seen from the comparison of Figs. 7 and 9, the hot-zone radius is less for the no-irradiation scenario for the same disk parameters. The smaller size of the hot disk is one of the reasons for lesser values of $\alpha$. Notice that the optical observations could be used to constrain the radius of the hot disk.

5 OPTICAL EMISSION FROM THE DISK IN 4U 1543−47 (2002)

Last decade studies have provided evidence that the optical emission during an X-ray Nova outburst could be produced not only by the outer parts of the disk illuminated with the central radiation. The low-frequency portion of the non-thermal emission from the BH vicinity can contribute to optical flux. Studies of the correlation between X-ray, optical, infrared, and radio emission of some X-ray transients suggest that jets are responsible for at least a part of the emission (e.g., Russell et al. 2006; Rahoui et al. 2011).
thermal electrons in the central disk corona can produce power-law OIR spectra (the hybrid hot-flow model; see Poutanen & Veledina (2014)).

Here we consider the optical emission only from the accretion disk. Figs. 10 and 11 present a comparison of the modelled optical light curves with the data of Buxton & Bailyn (2004). If not stated otherwise, the optical flux is integrated only from the hot part of the multi-colour disk using $M(t)$ derived from the spectral modelling. The effective temperature of a disk ring is calculated as $T_{eff} = (Q_{opt} + Q_{vis})/Q_{vis}$.

The modelled optical flux depends on the irradiation parameter $C_{irr}$, BH mass and Kerr parameter (because the mass and accretion efficiency affect derived $M$), and hot-zone radius. The hot-zone radius itself depends on $C_{irr}$ via (8) or on $a$ via (12). It is important to note that a value of the turbulent parameter $\alpha$, derived in the previous section, is a dynamical characteristic of the disk and depends on $C_{irr}$ but does not affect directly the observed optical flux in the case of the irradiation-controlled hot zone.

The $V$ and $J$ optical bands are chosen because there are data in these bands just before the onset of the burst. Spectral flux densities at $\lambda = 5500 \, \AA$ and $\lambda = 12600 \, \AA$ are converted to the optical $V$ and $J$ magnitudes using the zero-fluxes $F_{V}^{0} = 3.750 \times 10^{-9} \, \text{erg} \, \text{s}^{-1} \, \text{cm}^{-2} \, \text{Å}^{-1}$ and $F_{J}^{0} = 3.021 \times 10^{-10} \, \text{erg} \, \text{s}^{-1} \, \text{cm}^{-2} \, \text{Å}^{-1}$ (Cox 2015). The optical interstellar extinction is accounted for in the plotted light curves as $A_{V} = 1.6 \, \text{mag}$ (Orosz et al. 1998b) and $A_{J} = 0.282 \, A_{V}$ (Buxton & Bailyn 2004).

To plot the model curves, we add the modelled optical fluxes to the pre-burst flux, namely, 16.43 mag in $V$ and 15.13 mag in $J$. The modelled magnitudes do not include a possible optical flux from other sources: due to reprocessed X-rays by the outer cold disk, companion star, jet, or corona. There is an exception though: we show the upper limit on the flux from the disk, calculated as if the accretion rate is uniform up to $R_{tid}$: the curves with the notation $R_{opt} = R_{tid}$ in Figs. 10 and 11 (blue online).

Emission in the $J$ band is much more variable than in $V$. Buxton & Bailyn (2004) have shown that the optical/IR spectrum at the secondary maximum arising ~ 40 days after the peak is explained by a jet (low-frequency power-law component in the spectrum). Other large $J$ variations, visible before the 40th day, may also be associated with emission of this type.

Figures 10 and 11 demonstrate that the irradiation parameter $C_{irr}$, BH mass and Kerr parameter are the most important for the optical emission. From the $V$ data alone, $C_{irr} \approx (3 - 6) \times 10^{-4}$ is suggested, taking into account possible extra emission from the irradiated outer cold disk. A value of $C_{irr}$ deduced from the $J$ data is approximately the same, although there is some uncertainty due to large flux variations in the $J$ band. Very weak irradiation, $C_{irr} \approx 1.5 \times 10^{-4}$, needed to allow the dwarf-nova type evolution (§ 3.3), is less preferred from the point of view of optical observations but could not be excluded altogether in view of other possible sources of the optical photons.

6 DISCUSSION

6.1 Values of $C_{irr}$

There are several similar definitions of $C_{irr}$ in the literature depending on how the central luminosity is expressed. Esin et al. (2000) have used the total X-ray luminosity obtained from analysing the X-ray data, and this $C_{irr}$ is greater by a factor of $L_{X}/L_{X}$ than that defined by (7); Dubus et al. (1999) have expressed the irradiation
Determination of the turbulent parameter $\alpha$ in 4U 1543−47 (2002) 9

The characteristic values of $C_{ir}$ (Fig. 5), obtained for 4U 1543−47 (2002), are smaller compared to those usually derived on the theoretical and observational grounds. The irradiation factor $C_{ir}$ should be $< 6 \times 10^{-3}$ in order for the hot-zone radius to be variable. On the other hand, Dubus et al. (2001) adopted $C_{ir} \sim 5 \times 10^{-3}$ to reproduce the light curves of X-ray novae. When analysing Swift optical/UV/X-ray broad-band data of XTE J1817−330 (the outburst of 2006), Gierliński et al. (2009) have found out that the spectra are consistent with a model of reprocessing a constant fraction of $10^{-3}$ of the bolometric X-ray luminosity (disk plus nonthermal tail), arguing in favour of the direct illumination of the black hole disk. For the 1999–2000 outburst of the BH transient XTE J1859+226, Hynes et al. (2002) have estimated the irradiation parameter $C_{ir} \sim 7 \times 10^{-3}$, assuming the accretion efficiency to be $\eta_{acc} = 0.1$.

Estimate (17) agrees with the value of $C_{ir}$ for 4U 1543−47, assuming a presence of some relativistic focusing and the fact that the relative half-thickness $z_0/r$ near $R_{tid}$ at the peak reaches $\sim 0.07$. There is no need to contrive ways for increasing the value given by (17). Instead, one has to answer a question what is the reason for $C_{ir}$ being lower during the particular burst of this X-ray transient.

Figure 2 demonstrates that the disk was close to the Eddington limit in the 2002 outburst. Above the Eddington limit, outflows from the disk are expected (Shakura & Sunyaev 1973). There was possibly a weak outflow in 4U 1543−47 (2002), which was able to effectively attenuate X-rays. In principle, this may constrain $C_{ir}$ in X-ray transients with about-Eddington accretion rates.

It is also possible that the factor $C_{ir}$ depends on the proximity of $R_{hot}$ to the tidal truncation radius of the disk. In short-period X-ray novae, $R_{hot}$ at the peak is close to $R_{tid}$, while its peak value in 4U 1543−47 $R_{hot}$ is apparently only about $(0.5 – 0.7)R_{tid}$.

6.2 Models with irradiation-controlled size of the hot zone

Figure 12 presents two attempts of modelling the decay of the 4U 1543−47 burst in 2002 in the assumption that the radius of the evolving disk remained constant. The plot illustrates that the assumption of constant size for the hot zone cannot in principle yield the observed shape of $M(t)$, regardless of the values of $R_{hot}$ and $\alpha$.

The stage of irradiation-controlled hot zone with changing size was considered by King & Ritter (1998). The equation for the shrinking hot-zone mass,

$$M_{hot} = \alpha M_{in} + \frac{d}{dt} \left( \Sigma(R_{hot}) \pi R_{hot}^2 \right),$$

(18)

can be solved if one assumes a quasi-stationary evolution of the disk. In this case, $M_{hot} \propto \Sigma(R_{hot}) R_{hot}^2$. The relationship (B1) between $\Sigma$ and $F$ and condition (8) for determining the radius $R_{hot}$ of the hot zone yield the following power-law relationship between the mass of the hot part of the disk $M_{hot}$ and the inner accretion rate $M_{in}$:

$$M_{hot} \propto M_{in}^{\alpha_{55}}$$

(19)

(Kramers opacity). Expression (19) is fully confirmed by the FREDDI calculations. It can be deduced from (19) that $M \propto (t_{end} – t)^{\alpha_{55}}$, which evaluates the moment in time when the formal solution gives the zero disk mass. Notice that the consideration of this stage for the case, when the kinematic turbulent viscosity $v_t$ is independent of time and radius, yields $M \propto (t_{end} – t)^{\alpha}$. (‘linear decay’; by King & Ritter 1998). However, variable $v_t$ is inherent for $\alpha$-disks.

More involved numerical models used for calculating thermal stability and evolution of a disk with the irradiation-controlled cooling front (see, e.g., Dubus et al. 2001) suggest that the simple model...
expressed by (18) may not provide fully reliable $\alpha$ values. The quasi-stationary approximation may prove to be too crude to accurately assess the velocity of the boundary of the uniform-viscosity zone.

To summarize, the model of the irradiation-controlled shrinking hot zone implemented in $\text{raaon}$ describes the shape of $M(t)$ in 4U 1543–47 quite satisfactorily. On the other hand, its current version probably overestimates $\alpha$. Notice that the model of a completely ionized disk, also calculated by $\text{raaon}$, does not suffer from this shortcoming.

### 6.3 Determination of $M$ from observations

It is important to consider the general relativity effects when analysing the disk evolution in real systems. For given X-ray data, the peak accretion rate depends strongly on the Kerr parameter $a_{\text{Kerr}}$. $M$ changes by a factor of $\sim 25$ for $a_{\text{Kerr}} = 0 - 0.998$. This factor incorporates the variation of accretion efficiency of rotating black holes with different $a_{\text{Kerr}}$. Also, in the vicinity of a black hole, the outgoing X-ray spectrum is disturbed by the effects of Doppler boosting, gravitational focusing, and the gravitational redshift (Cunningham 1975).

In the present study, we use the spectral model of the relativistic disk $\text{kerrbh}$ developed by Li et al. (2005). As mentioned by the authors, $M$ in $\text{kerrbh}$ is the ‘effective’ accretion rate, while an actual one should be greater by a factor of $(1 + \eta_{\text{sat}})$, where $\eta_{\text{sat}}$ is the ratio of the disk heating due to a non-zero inner torque to the heating caused by the fall of accreting matter. For a non-rotating black hole, the magnetic torque at the inner edge of a thin magnetized accretion disk is only $\sim 2\%$ of the inward flux of the angular momentum (Shafee et al. 2008). In the present work, we assume $\eta_{\text{sat}} = 0$.

If the innermost stable circular orbit is $0.5 G M_i/c^2$ closer to the centre than a canonical value of $6 G M_i/c^2$ for a non-rotating black hole, as argued by Reynolds & Fabian (2008), then the related disk-power increase corresponds to a formal variation of $a_{\text{Kerr}}$ from 0 to 0.15. As can be seen from Fig. 2, a higher Kerr parameter for the black hole translates into a smaller accretion rate. This uncertainty of relativistic disk models tends to be smaller for higher $a_{\text{Kerr}}$ (Miller et al. 2009). In view of such uncertainties, we varied the Kerr parameters within some range to see how the results depended on them.

There are other modern XSPEC models for emission of the relativistic disk, for example, $\text{kerrbb}$ and $\text{slimbh}$, which are more sophisticated in some respects. However, these have the limitations that hinder us from using them: the $\text{kerrbb}$ model covers limited ranges for the accretion rate and spectral energy; $\text{slimbh}$ has the luminosity as an input parameter, instead of the accretion rate.

### 6.4 Variation of the model assumptions

Results of spectral fitting by Morningstar & Miller (2014) suggest that the inclination of the inner disk in 4U 1543–47 differs from the binary inclination. When taking as the inclination the central value from the range derived by Morningstar & Miller (2014), $i = 32\%$, we obtain lower resulting accretion rates. The decrease depends on $a_{\text{Kerr}}$: it is $\sim 86\%$ for $a_{\text{Kerr}} = 0.998$ and $\sim 24\%$ for $a_{\text{Kerr}} = 0$ (Fig. 13). This inclination makes the resulting $a$ smaller by a factor of 1.3 for $a = 0.998$ and by 1.1 for $a_{\text{Kerr}} = 0.0$ (for the Kramers opacity), as can be found from relationship (5).

Spectral modelling with the colour factor $f_{\text{col}} = 1$ gives systematically higher peak accretion rates and, thus, higher estimates for $\alpha$.

In Fig. 14, we compare models with three different opacity implementations. The parameters of the disk are set as follows: $m_{\text{a}} = 10$, $\alpha = 0.5$, $R_{\text{sat}} = 10^{11}$ cm $\equiv$ const. We see that the difference in the opacity does not affect the results significantly in contrast to other uncertainties involved in the modelling.

**Figure 12.** The evolution of a strongly-irradiated disk, $C_{\text{irr}} = 5 \times 10^{-3}$ (solid line), and evolution of a disk with the fixed radius of the hot zone (dashed line). Top panel: the accretion rate found by minimizing $\chi^2$; lower panel: the radius of the hot zone.

**Figure 13.** The ratio of the peak accretion rates of the 4U 1543–47 (2002) outburst for $i = 21\%$ and $i = 32\%$. The designations are the same as in Fig. 2. The dependence on $a_{\text{Kerr}}$ is due to the relativistic focusing of X-rays.

**Figure 14.** The modelled evolution of $M(t)$ with different opacity implementations for the solar abundances: the dashed line is for Kramers law $\kappa = 5 \times 10^{24} \rho / T_{5/2}^3$ cm$^{-2}$ g$^{-1}$; the dotted line is for the analytic approximation to the OPAL tables $\kappa = 1.5 \times 10^{20} \rho / T_{5/2}^2$ cm$^{-2}$ g$^{-1}$ (Bell & Lin 1994); the solid line is calculated using the code of Malanchev & Shakura (2015) for the OPAL tables (Iglesias & Rogers 1996). Disk radius is constant, $C_{\text{irr}} = 0$. The designations are the same as in Fig. 2.
7 SUMMARY

Bursts of X-ray novae are crucial laboratories to probe models of the non-stationary disk accretion. It is important therefore that a model of the viscous disk evolution takes into account the self-irradiation of the disk in a self-consistent way.

We assume the concept of the high turbulent parameter $\alpha$ in the hot ionized part of the disk and low $\alpha$ in the cold neutral zone. Coleman et al. (2016) have considered a dwarf-nova disk with $\alpha \sim 0.01$ in the ionized and neutral zones and with $\alpha$ one order higher in the zone with the partial ionization, following some recent results of numerical MHD simulations that have achieved a value of $\alpha \sim 0.1$ for the regions with the partial ionization and convection (Hirose et al. 2014). However, such low-$\alpha$ disks may face a difficulty at explaining the characteristic times of the exponential decays (several tens of days) and the time delays between the optical and X-ray band during outburst rises (several days), observed in some X-ray novae.

If an X-ray outburst is produced due to the variations of the central accretion rate in a completely ionized disk, a nearly exponential decay is expected for the disk with a constant outer radius. For such decays, which may take place in short-periodic X-ray novae, the turbulent parameter $\alpha$ can be estimated using analytic relationships (5)–(6) and the disk evolution can be calculated using the MHD code. Slower decays happen in the accreting viscous flows, which are radially expanding (ordinarily, this is not a case of a close binary system), or if a companion star provides additional matter to the disk. Faster decays are due to the progressive shrinking of the viscously-evolving zone (the zone with high $\alpha$). Detailed modelling is needed to discriminate between these cases because the apparent similarity to an exponential decay may mask different scenarios.

The difference between the disk instability models proposed for explaining the bursts in dwarf and X-ray novae is thought to be mainly due to the major role of X-ray irradiation in the latter group of objects (e.g., van Paradijs & Verbunt 1984). The typical irradiation parameter $C_{in} \sim 5 \times 10^{-3}$ was suggested (Dubus et al. 1999; Lasota et al. 2008).

We derived and analysed the accretion rate evolution $\dot{M}(t)$ during the first 30 days after the peak of 2002 outburst of 4U 1543–47, when it was in the high/soft state. We found that the observed evolution requires $\alpha \sim 3.2 - 3.3$ for the irradiation parameter $C_{in} \sim 5 \times 10^{-3}$. This happens because the strong illumination leads to a large size of the hot part of the disk, and the short decay time can be obtained only with a high viscosity parameter. The shape of the $\dot{M}(t)$ curve of 4U 1543–47 (2002) cannot be produced by a hot zone with constant size; rather, it is consistent with the model of shrinking high-viscosity zone.

Putting aside mixed scenarios, there were two options depending on whether or not the central irradiating flux controlled the hot zone size. These corresponded to $C_{in} \approx (3 - 6) \times 10^{-4}$ (`mild irradiation’ case) and $C_{in} \lessapprox 1.5 \times 10^{-4}$ (`no irradiation’ case) for 4U 1543–47 (2002). In the quasi-stationary model of the irradiation-controlled hot zone, we obtained $\alpha \sim 0.5 - 1.5$ for the range of the BH masses $6 - 10 M_\odot$. The second case would resemble a decay in a dwarf nova disk and required $C_{in} \sim 0.08 - 0.32$.

The $V$ and $J$ light curves favour the value $C_{in}$ in the range $\sim (3 - 6) \times 10^{-4}$, making the model of irradiation-controlled hot zone in 4U 1543–47 more plausible. Nevertheless, there remains the possibility that the X-ray illumination can be of low importance for this and some other X-ray nova outbursts.

| Model component | Parameter | Value |
|-----------------|-----------|-------|
| $T_{Babs}$      | $n_H$     | $4 \times 10^{23}$ cm$^{-2}$ |
| $kerrbb$        | $\eta_{tot} = F_{in} \omega_{rad}/\eta_{acc} M c^2$ | 0 |
|                 | $\gamma_{Kerr}$ | 0, 0.6, 0.9, 0.998 |
|                 | $m_*$     | 6, 7, 8, 9, 10 |
|                 | $M$       | $0 \sim 10^{21}$ g s$^{-1}$ |
|                 | $d$       | $0 \sim 10^4$ kpc |
|                 | $f_{col}$ | $1.7 \sim (1.0)$ |
|                 | irradiation flag | 1 |
|                 | limb-darkening flag | 1 |
|                 | normalization | 1 |
| $simpl$         | $\Gamma$  | $0 - 4.5$ |
|                 | $f_{sc}$  | $10^{-3} - 0.3$ |
|                 | only up-scattering flag | 1 |
| $compTT$        | redshift  | 0 |
|                 | $T_0$     | $10^{-4} - 4$ keV |
|                 | $kT$      | $10 - 10^4$ keV |
|                 | $\tau_{plasma}$ | $10^{-2} - 200$ |
| $laor$          | $E_{line}$ | $5 - 7$ keV |
|                 | Index     | 3 |
|                 | $R_0$     | $1.235 - 400 G M_*/c^2$ |
|                 | $R_{out}$ | $400 G M_*/c^2$ |
|                 | $i$       | $20 - 73$ (32°) |
| $smedge$        | normalization | $0 \sim 10^{24}$ photons cm$^{-2}$s$^{-1}$ |
|                 | $E_{edge}$ | $6 - 10$ keV |
|                 | $\tau_{max}$ | $0 - 10$ |
|                 | Index     | $-2.67$ |
|                 | Width     | 7 keV |

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APPENDIX A: SPECTRAL MODELLING

We analyse the archival data for the 2002 outburst of 4U 1543–47 obtained with the Proportional Counter Array aboard the RXTE observatory. We have selected the same data as in Park et al. (2004). In particular, long observations of 17, 19, 20, and 28 June 2002 were divided in two equal intervals. The “Standard-2” data from all Xe layers of PCU-2 have been used.

The basic reduction is made with the help of the $\text{SAS}$ script of HEASoft 6.18. The background PCA model for bright sources is used. The good time intervals are selected according to the conservative options: the source elevation should have been greater than 10 degrees; the pointing offset, less than 0.02 degrees; 30 min should have elapsed after the observatory passage through the South Atlantic Anomaly. We calculate dead time corrections for the source and background to adjust the exposures, make re-
response matrices, and extract spectra using tools pcadeadcalc2 and pcaextspect2.

The spectral fitting is done using XSPEC 12.9.0. Systematic errors of 1% are added via the command system. We use the absorption model tbabs with the hydrogen column $4 \times 10^{21}$ cm$^{-2}$ (according to the LAB (Kalberla et al. 2005) and GASS (Kalberla & Haud 2015) surveys; see Alfa Hi Surveys tool at Argelander-Institut für Astronomie) together with $\text{wlin}$ abundances. The spectral fitting is carried out in the 2.9–25 keV interval.

To describe the thermal emission from the disk, the kerrbb spectral model is used. The spectral parameters can be found in Table A1. The accretion rate $\dot{M}$ and the distance $d$ are set free. We fix the BH mass $M_\bullet$ and the dimensionless Kerr parameter $a_\ast$. The disk inclination is fixed and equal to $20^\circ$. The accretion rate $\dot{M}$ and the zero-torque condition at the inner edge ($q_{\text{in}} = 0$).

To describe the non-thermal component as the emission component in the high-temperature plasma near the disk, we use the convolution model simpl (Steiner et al. 2009). This empirical model, in which a fraction of seed photons produces the power-law component, can be used for any spectrum of the seed photons. The model has two free parameters: the photon power-law index and the fraction of scattered photons. Since simpl redistributes input photons to higher and lower energies, the energy interval should be extended to adequately cover the relevant band. Following Steiner et al. (2009), we compute the model over 1000 logarithmically spaced energy bins between 0.05 and 50 keV. We hold the power-law index $\Gamma$ between 0 and 4 and choose the 'up-scattering' mode.

As an alternative, we also model the power-law tail with comptt (Titarchuk 1994), which calculates the Compton scattering as a convolution using the scattering Green’s function. In comptt, we set free the seed photons’ and plasma temperatures, the optical depth, and the normalization parameter, choosing the ‘disk’ geometry.

Following Park et al. (2004), we include a spectral component to fit a broad edge-like absorption spectral feature. It is suggested that the feature near 7 keV is produced by the K-shell absorption of iron, smeared due to reflection or partial absorption of X-rays by the optically thick accretion disk (Ebisawa et al. 1994). The smedge spectral model reproduces the broadened Fe-absorption line (K-absorption structure) and should be multiplied with the continuum spectrum. We conﬁne the parameter $E_{\text{edge}}$, which approximately corresponds to the energy of the iron K-edge (Ebisawa 1991), to values within 6 to 10 keV, and the smearing width parameter is fixed to be 7 keV, as in Park et al. (2004).

The spectral model $\text{tbabs+simpl+kerrbb}_n + \text{smedge}$ gives acceptable ﬁts (the reduced $\chi^2 < 1.5$) for the most spectra with chosen masses and Kerr parameters.

Park et al. (2004) have found an evidence of the Fe Ka-line in the spectra of the source during the outburst. They have concluded that the XSPEC laor model that takes into account the relativistic effects leading to line broadening (Laor 1991) suits the line component better than the Gaussian model. We conﬁrm this conclusion. In laor, the energy of the line centre is varied freely between 6.4 and 7 keV, following Park et al. (2004), while the inner radius is thawed. The other parameters, frozen by default, remain unchanged.

Morningstar & Miller (2014) have used the spectral component reconv=frelionx to describe the feature near 7 keV. We could not use it because the spectral component is not able to provide satisfactory results at times close to the peak, apparently due to a limitation on the photon index in reflionx.

APPENDIX B: THE FREDDI CODE OVERVIEW

The freddi$^3$ code is designed to solve the differential equation (1) with two boundary conditions on the viscous torque $F: F_{\text{in}} = 0$ and $(\partial F/\partial h)_{\text{out}} = M_{\text{out}} = 0$.

The code uses the analytic relationship between the surface density $\Sigma$ and the viscous torque $F$:

$$\Sigma = \frac{(GM_\bullet)^2 F^{|-n|}}{4\pi(1-m)D_h^{m-1}},$$  

where $m$ and $n$ are the dimensionless constants that equal 3/10 and 4/5, respectively, for the Kramers opacity law, and 1/3 and 1, for $n = \rho/T^{3/2}$ (Bell & Lin 1994), $D$ is a diffusion coefficient (Lyubarskij & Shakura 1987; Suleimanov et al. 2008). The dimensional diffusion coefficient $D$ in (B1) is approximately constant for specific disk parameters, as it depends on the parameter $\tau_0$ (comparable to the disk optical thickness), and the dependence is rather weak for $\tau_0 \gg 1$ (Suleimanov et al. 2007). We use a constant value of $D$ corresponding to $\tau_0 = 10^3$.

In freddi, the outer radius $R_{\text{out}}$ of the evolving disk can be varied so that either the effective temperature $T_{\text{eff}}$ or the irradiation temperature $T_{\text{irr}}$ would be constant. Thus, the shift of $R_{\text{out}}$ tracks the temperature variation. The outer boundary condition $M_{\text{out}} = 0$ means that the distribution of the viscous torque in the outer cold disk is ignored.

To solve diffusion equation (1), it is necessary to know an initial distribution of either the function $F(h)$ or the function $\Sigma(h)$, where $h = \sqrt{GM_\bullet/T}$ is the specific angular momentum. freddi provides a possibility to choose one of the following initial distributions:

- Power law for the surface density: $\Sigma \propto (h/h_{\text{ref}})^{m-1}$.  
- Power law for the viscous torque: $F \propto (h/h_{\text{ref}})^{\tau}$.  
- Sinus law for the viscous torque: $F \propto \sin \left(\frac{h-h_{\text{ref}}}{h_{\text{in}}-h_{\text{ref}}}\right)$.  
- Two-parametric Gaussian distribution for the viscous torque.  
- Quasi-stationary distribution: $F \propto f_r \left(\frac{h-h_{\text{ref}}}{h_{\text{in}}-h_{\text{ref}}}\right)^{m-1}$, where $f_r \left(\frac{h}{h_{\text{ref}}}\right)$ is a coordinate part of the self-similar analytic solution of the diffusion equation in the assumption of the fixed outer radius and zero inner radius (Lipunova & Shakura 2000).

All results presented in § 3.2 are obtained using the quasi-stationary distribution as the initial one.

Fig. 14 shows a comparison of the diffusion equation solutions obtained with freddi for different analytic opacity laws and the solution for tabulated opacity values (OPAL tables; Iglesias & Rogers 1996). We alternatively use the Kramers opacity $\kappa = 5 \times 10^5 \rho/T^{7/2}$ cm$^{-2}$ g$^{-1}$ or $\kappa = 1.5 \times 10^5 \rho/T^{3/2}$ cm$^{-2}$ g$^{-1}$ Bell & Lin (1994). The initial power-law distribution of the viscous torque with an index of $k_F = 6$ is set. The third solution (the solid line) is obtained with a code described in Malanichev & Shakura (2015) using tabulated OPAL opacities and a quasi-stationary initial distribution. The values of the disk parameters used in all of the three calculations are the same: $a = 0.5$, $m_\bullet = 10$, $q_{\text{disk}} = 0$, and

$^3$ FREDDI – Fast Rise Exponential Decay: accretion Disk model Implementation. The code can be downloaded from http://xray.sai.msu.ru/~malanchev/freddi/
The high speed of \( \text{freddi} \) and a specifically developed interface make it very useful for fitting the Shakura-Sunyaev \( \alpha \)-parameter and the disk radius \( R_{\text{in}} \). One can set the distance \( d \), the irradiation factor \( C_{\text{irr}} \), and the inclination \( i \) of the system to obtain the X-ray flux and the spectral flux density at any wavelength. It should be noted that \( \text{freddi} \) uses an analytical vertical structure and thus works orders of magnitude faster than any other code, which uses tabulated opacity values and numerically solving the vertical structure equations. \( \text{freddi} \) has several tuning parameters: a time step, a number of coordinate steps and a type of coordinate grid (linear or logarithmic in terms of \( h \)). \( \text{freddi} \) is written on C++ and has a user-friendly command-line interface.

REFERENCES

Arnaud K. A., 1996, in G. H. Jacoby & J. Barnes ed., Astronomical Society of the Pacific Conference Series Vol. 101, Astronomical Data Analysis Software and Systems V. pp 17–20
Bell K. R., Lin D. N. C., 1994, ApJ, 427, 987
Buxton M. M., Bailyn C. D., 2004, ApJ, 615, 880
Camizzo J. K., 1994, ApJ, 435, 389
Chandrasekhar S., 1961, Hydrodynamic and hydromagnetic stability. International Series of Monographs on Physics. Oxford: Clarendon, 1961
Coleman M. S. B., Kotko I., Blaes O., Lasota J.-P., Hirose S., 2016, MNRAS, 462, 3710
Cox A., 2015, Allen’s Astrophysical Quantities. Springer New York, https://books.google.ru/books?id=TjDtCAAAQBAJ
Cunningham C. T., 1975, ApJ, 202, 788
Cunningham C., 1976, ApJ, 208, 534
Dubus G., Lasota J.-P., Hameury J.-M., Charles P., 1999, MNRAS, 303, 139
Dubus G., Hameury J.-M., Lasota J.-P., 2001, A&A, 373, 251
Ebisawa K., 1991, PhD thesis, Institute of Space and Astronautical Science/Japan Aerospace Exploration Agency
Ebisawa K., et al., 1994, PASJ, 46, 375
Esin A. A., Kuulkers E., McClintock J. E., Narayan R., 2000, ApJ, 532, 105
Eggleton P. P., 1983, ApJ, 268, 368
Ertan U., Alpar M. A., 2002, A&A, 393, 205
Esin A. A., Kuulkers E., McClintock J. E., Narayan R., 2000, ApJ, 532, 105
Gierliński M., Done C., Page K., 2009, MNRAS, 392, 1106
Höflich P., 1997, Progress of Theoretical Physics, 61, 1307
Hawley J. F., Balbus A. A., 1991, ApJ, 376, 223
Hirose S., Blaes O., Kroluk J. H., Coleman M. S. B., Sano T., 2014, ApJ, 787, 1
Hynes R. I., Haswell C. A., Chaty S., Shrirer C. R., Cui W., 2002, MNRAS, 331, 169
Iglesias C. A., Rogers F. J., 1996, ApJ, 464, 943
Kalberla P. M. W., Haar U., 2015, A&A, 578, 478
Kalberla P. M. W., Burton W. B., Hartmann D., Arnael E. M., Bajaja E., Morras R., Pöppel W. G. L., 2005, A&A, 440, 775
Ketsaris N. A., Shakura N. I., 1998, Astronomical and Astrophysical Transactions, 15, 193
King A. R., Ritter H., 1998, MNRAS, 293, L42
King A. R., Pringle J. E., Livio M., 2007, MNRAS, 376, 1740
Kotko I., Lasota J.-P., 2012, A&A, 545, A112
Laor A., 1991, ApJ, 376, 90
Lasota J.-P., 2001, New Astronomy Review, 45, 449
Lasota J.-P., Dubus G., Kruk K., 2008, A&A, 486, 523
Li L.-X., Zimmerman E. R., Narayan R., McClintock J. E., 2005, ApJS, 157, 335
Lin D. N. C., Faulkner J., Papaloizou J., 1985, MNRAS, 212, 105
Lipunova G. V., 2015, ApJ, 804, 87
Lipunova G. V., Shakura N. I., 2000, A&A, 356, 363
Lyubarskij Y. E., Shakura N. I., 1987, Soviet Astronomy Letters, 13, 386
Lyutyi V. M., Sunyaev R. A., 1976, Soviet Astr., 20, 290
Malanchev K. L., Postnov K. A., Shakura N. I., 2017, MNRAS, 464, 410
Menou K., Hameury J.-M., Stehle R., 1999, MNRAS, 305, 79
Mescheryakov A. V., Revnivtsev M. G., Filipponi E. V., 2011, Astronomy Letters, 37, 826
Meyer F., 1984, A&A, 131, 303
Meyer F., Meyer-Hofmeister E., 1981, A&A, 104, L10
Meyer F., Meyer-Hofmeister E., 1984, A&A, 140, L35
Meyer F., Meyer-Hofmeister E., 1990, A&A, 239, 214
Miller J. M., Reynolds C. S., Fabian A. C., Minniti G., Gallo L. C., 2009, ApJ, 697, 900
Morrison W. R., Miller J. M., 2014, ApJ, 793, L33
Orosz J. A., 2003, in van der Hucht K., Herrero A., Esteban C., eds, IAU Symposium Vol. 212, A Massive Star Odyssey: From Main Sequence to Supernova. p. 365 (arXiv:astro-ph/0209841)
Orosz J. A., Jain R. K., Bailyn C. D., McClintock J. E., Remillard R. A., 1998b, ApJ, 499, 375
Orosz J. A., Jain R. K., Bailyn C. D., McClintock J. E., Remillard R. A., 1998, ApJ, 499, 375
Orosz J. A., Polisensky E. J., Bailyn C. D., Toutellotte S. W., McClintock J. E., Remillard R. A., 2002, Bulletin of the American Astronomical Society, 34, 1124
Paczynski B., 1977, ApJ, 216, 822
Park S. Q., et al., 2004, ApJ, 610, 378
Poutanen J., Veledina A., 2014, Space Sci. Rev., 183, 61
Rahoui F., Lee J. C., Heinz S., Hines D. C., Pottschiadt K., Wilms J., Grinberg V., 2011, ApJ, 736, 63
Reynolds C. S., Fabian A. C., 2008, ApJ, 675, 1048
Ritter H., Kolb U., 2003, A&A, 404, 301
Russell D. M., Fender R. P., Hynes R. I., Brocklepp C., Homan J., Jonker P. G., Buxton M. M., 2006, MNRAS, 371, 1334
Shafee R., McKinney J. C., Narayan R., Tchekhovskoy A., Gammie C. F., McClintock J. E., 2008, ApJ, 687, L25
Shahbaz T., Charles P. A., King A. R., 1998, MNRAS, 301, 382
Shakura N. I., Sunyaev R. A., 1973, A&A, 24, 337
Smak J., 1984a, Acta Astronomica, 34, 161
Smak J., 1984b, PASP, 96, 5
Smak J., 2000, New A Rev., 44, 171
Steiner J. F., Narayan R., McClintock J. E., Ebisawa K., 2009, PASP, 121, 1279
Suleimanov V., Meyer F., Meyer-Hofmeister E., 1999, A&A, 350, 63
Suleimanov V., Meyer F., Meyer-Hofmeister E., 2003, A&A, 401, 1009
Suleimanov V. F., Lipunova G. V., Shakura N. I., 2007, Astronomy Reports, 51, 549
Suleimanov V. F., Lipunova G. V., Shakura N. I., 2008, A&A, 491, 267
Titarchuk L., 1994, ApJ, 434, 570
Tuchman Y., Mineshige S., Wheeler J. C., 1990, ApJ, 359, 164
Velikhov E. P., 1959, Soviet Journal of Experimental and Theoretical Physics, 9, 995
Vishniac E. T., Wheeler J. C., 1996, ApJ, 471, 921
van Paradis J., Verbunt F., 1984, in Woosley S. E., ed., American Institute of Physics Conference Series Vol. 115, American Institute of Physics Conference Series. pp 49–62, doi:10.1063/1.345565