NNLO Antenna Subtraction with One Hadronic Initial State

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HP2.3rd

The 3rd International Workshop on High Precision for Hard Processes at the LHC
**Motivation:**

- Tevatron and LHC: machines for QCD precision physics
  - new discovery potential related to how good we understand what we already know

- For precise predictions we need a precise determination of
  - coupling constants
  - parton distributions
  - quark masses
  - ...

- Need higher order calculations: NLO, NNLO ...
Subtraction at NLO

For an m-jet cross section, need to integrate \textit{numerically} over phase space:

\textbf{LO:}

\[ d\sigma_{\text{LO}} = \int d\Phi_m \ d\sigma_{\text{tree}} \]

\textbf{NLO:}

\[ d\sigma_{\text{NLO}} = \int d\Phi_{m+1} \ d\sigma_{\text{NLO}}^R + \int d\Phi_m \ d\sigma_{\text{NLO}}^V \]

\textbf{Problem:} same divergent structure as virtual part but summation occur only after phase space integration
Subtraction at NLO

For an m-jet cross section, need to integrate \textit{numerically} over phase space:

- LO:

\[
d\sigma_{\text{LO}} = \int d\Phi_m d\sigma_{\text{tree}}
\]

- NLO:

\[
d\sigma_{\text{NLO}} = \int d\Phi_{m+1} (d\sigma_{\text{NLO}}^{R} - d\sigma_{\text{NLO}}^{S}) + \left[ \int d\Phi_{m+1} d\sigma_{\text{NLO}}^{S} + \int d\Phi_m d\sigma_{\text{NLO}}^{V} \right]
\]

Solution: Introduce subtraction term which reproduces \(\sigma_{\text{NLO}}^{R}\) in all singular limits, and can be integrated analytically

\cite{Z. Kunszt, D. Soper}
Subtraction at NLO

For an m-jet cross section, need to integrate numerically over phase space:

LO:

\[ d\sigma_{LO} = \int d\Phi_m d\sigma_{\text{tree}} \]

NLO:

\[ d\sigma_{NLO} = \int d\Phi_{m+1} \left( d\sigma_{NLO}^R - d\sigma_{NLO}^S \right) + \left[ \int d\Phi_{m+1} d\sigma_{NLO}^S + \int d\Phi_m d\sigma_{NLO}^V \right] \]

Solution: Introduce subtraction term which reproduces \( \sigma_{NLO}^R \) in all singular limits, and can be integrated analytically.

Different subtraction methods exist: dipole, FKS, antenna,...

[Z. Kunszt, D. Soper]

[S. Catani, M. Seymour, S. Weinzierl, S. Frixione, Z. Kunszt, A. Signer, M. Grazzini, V. Del Duca, G. Sermonti, Z. Trocsanyi, D. Kosower, J. Campbell, M. Cullen, N. Glover, A. Daleo, A. Gehrmann-De Ridder, T. Gehrmann, D. Maitre]
NLO Antenna Subtraction

How is $d\sigma_{NLO}^{S}$ constructed within the antenna frame work? It must satisfy:

$$d\sigma_{NLO}^{R} \xrightarrow{\text{soft & collinear limit}} d\sigma_{NLO}^{S}$$

Real correction $d\sigma_{NLO}^{R}$ given by

$$d\sigma_{NLO}^{R} = \mathcal{N} \sum_{m+1} d\Phi_{m+1} \frac{1}{S_{m+1}} |\mathcal{M}_{m+1}^{0}|^2 J_{m+1}^{(m+1)} (k_1, \ldots, k_{m+1})$$

Exploit factorization of phase space and matrix element in soft and coll. limit:

$$d\Phi_{m+1} (\ldots, i, j, k, \ldots) \xrightarrow{j \text{ unresolved}} d\Phi_{m} (\ldots, I, K, \ldots) d\Phi_{X_{ijk}} (i, j, k, I, K)$$

$$|\mathcal{M}_{m+1}^{0} (\ldots, i, j, k, \ldots)|^2 \xrightarrow{j \text{ unresolved}} |\mathcal{M}_{m}^{0} (\ldots, I, K, \ldots)|^2 F (i, j, k) + \text{regular terms}$$

$F (i, j, k)$: soft eikonal factor or collinear splitting function,

$I, K$: remapped on-shell momenta: $i + j + k = I + K$. 
And thus $d\sigma_{\text{NLO}}^S$ can be constructed as:

$$d\sigma_{\text{NLO}}^S = N \sum_{m+1} d\Phi_{m+1} \frac{1}{S_{m+1}} \sum_j X^0_{ijk} |\mathcal{M}_m|^2 J^{(m)}_m (k_1, \ldots, k_{m+1})$$

where $X^0_{ijk} \xrightarrow{j \text{ unresolved}} F (i, j, k)$. 

Pictorially:

$$\sum_{m+1} d\Phi_{m+1} |\mathcal{M}_{m+1}|^2 J^{(m+1)}_m \rightarrow \sum_{m+1} d\Phi_{m} |\mathcal{M}_{m}|^2 J^{(m)}_m \sum_j d\Phi X^0_{ijk} X^0_{ijk}$$
NLO antenna subtraction

- NLO antenna function $X_{ijk}^0$ contains all soft and collinear configuration of parton $j$ emitted between two hard color-connected partons $i$ and $k$

$$X_{ijk}^0 = S_{ijk,IK} \frac{|M_{ijk}^0|^2}{|M_{IK}^0|^2}$$

- Antennae computed from matrix elements of physical processes

- Integrated subtraction term can be computed analytically

$$|M_m|^2 J_m^{(m)} d\Phi_m \int d\Phi X_{ijk}^0 X_{ijk}^0 \propto |M_m|^2 J_m^{(m)} d\Phi_m \int d\Phi_3 |M_{ijk}^0|^2$$
Cross section for hadronic initial state: $(pp, p\bar{p})$

\[
\frac{d\sigma}{d\hat{\sigma}_{ab}} = \sum_{h_1, h_2, a, b} \int_0^1 \frac{d\xi_1}{\xi_1} \frac{d\xi_2}{\xi_2} f_a^{h_1} (\xi_1, \mu_F^2) f_b^{h_2} (\xi_2, \mu_F^2) \, d\hat{\sigma}_{ab} \left( \xi_1 P_1, \xi_2 P_2, \mu_F^2 \right)
\]
Cross section for hadronic initial state: \((ep)\)

\[
d\sigma = \sum_{h_1,a,b} \int_0^1 \frac{d\xi_1}{\xi_1} \frac{d\xi_2}{\xi_2} f_{h_1}^{a,b} (\xi_1, \mu_F^2) \delta (1 - \xi_2) \, d\hat{\sigma}_{ab} (\xi_1 P_1, \xi_2 P_2, \mu_F^2)
\]
Hadronic initial state

- **final-final:**
  Applied to $e^+e^- \rightarrow 3$ jets at NNLO [A. Gehrmann De-Ridder, T. Gehrmann, N. Glover, G. Heinrich; S. Weinzierl]

- **initial-final:**
  Sufficient for DIS (2+1)-jet [A. Daleo, T. Gehrmann, D. Maître; A. Daleo, A. Gehrmann-De Ridder, T. Gehrmann, G. L]

- **initial-initial:**
  Needed for vector boson plus jet production [A. Daleo, T. Gehrmann, D. Maître]
  [R. Boughezal, A. Gehrmann-De Ridder, M. Ritzmann]
m-jet cross section

n-parton contribution to the m-jet cross section \((p = \xi_1 P_1, r = \xi_2 P_2)\):

\[
d\hat{\sigma}^{i}_{ab} (p, r) = \mathcal{N} \sum_{n} d\Phi_n (k_1, \ldots, k_n; p, r) \frac{1}{S_n} |\mathcal{M}_n (k_1, \ldots, k_n; p, r)|^2 J_m^{(n)} (k_1, \ldots, k_n)
\]

- **LO**: \(n = m\)
- **NLO**: \(n = m + 1\)
- **NNLO**: \(n = m + 2\)

Subtraction term for initial-final singularity:

\[
d\hat{\sigma}^{S_{(if)}} = \mathcal{N} \sum_{m+1} d\Phi_{m+1} (k_1, \ldots, k_{m+1}; p, r) \frac{1}{S_{m+1}}
\]

\[
\times \sum_{j} X_{i,jk}^{0} |\mathcal{M}_m (k_1, \ldots, k_{m+1}; xp, r)|^2 J_{m}^{(m)} (k_1, \ldots, k_{m+1})
\]
I-F NLO phase space factorization

- Kinematics is now: \( q + p \rightarrow k_j + k_k \Rightarrow q + xp \rightarrow K_K \)

- Limits:
  - \( xp \rightarrow p \quad K_K \rightarrow k_k \) when \( j \) soft
  - \( xp \rightarrow p \quad K_K \rightarrow k_j + k_k \) when \( j \parallel k \)
  - \( xp \rightarrow p - k_j \quad K_K \rightarrow k_k \) when \( j \parallel i \)

- Phase space factorization for \( m + 1 \) particles:

\[
d\Phi_{m+1}(k_1, \ldots, k_{m+1}; p, r) = d\Phi_m(k_1, \ldots, K_K, \ldots, k_{m+1}; xp, r) \times \frac{Q^2}{2\pi} d\Phi_2(k_j, k_k; p, q) \frac{dx}{x}
\]
Obtain antennae functions by crossing final-final NLO antennae

\[\sum_{m+1} d\Phi_{m+1} |M_{m+1}|^2 J^{(m+1)}_m \rightarrow \sum_{m+1} d\Phi_m |M_m|^2 J^{(m)}_m \sum_j \frac{Q^2}{2\pi} d\Phi_2 \frac{dx}{x} X^0_{i,j,k}\]

Again integrated subtraction term can be computed **analytically**:

\[X^0_{i,j,k}(x) = \frac{1}{C(\epsilon)} \int d\Phi_2 \frac{Q^2}{2\pi} X^0_{i,j,k}, \quad C(\epsilon) = (4\pi)^\epsilon \frac{e^{-\epsilon\gamma_E}}{8\pi^2}\]

[A. Daleo, T. Gehrmann, R. Maitre]
Integrated subtraction term has to be convoluted with PDFs

Make change of variable and obtain

\[ d\sigma^{S(if)}(p, r) = \sum_{m+1} \sum_{j} \frac{S_m}{S_{m+1}} \int \frac{d\xi_1}{\xi_1} \int \frac{d\xi_2}{\xi_2} \int_{\xi_1}^{1} \frac{dx}{x} f_a^h(\frac{\xi_1}{x}) f_b^h(\xi_2) \times C(\epsilon) \mathcal{X}_{i,j,k}^0(x) d\hat{\sigma}^B(\xi_1 P_1, \xi_2 P_2) \]

Mass factorization can be carried out

Phase space integration in \( d\hat{\sigma}^B \) and convolutions can be done numerically
Subtraction at NNLO

Structure of NNLO m-jet cross section

\[ d\sigma_{\text{NNLO}} = \int d\Phi_{m+2} \left( d\sigma_{\text{R,NNLO}}^{\text{R}} - d\sigma_{\text{NNLO}}^{\text{S}} \right) + \int d\Phi_{m+2} d\sigma_{\text{NNLO}}^{\text{S}} \]
\[ + \int d\Phi_{m+1} \left( d\sigma_{\text{V,1,NNLO}}^{\text{V,1}} - d\sigma_{\text{NNLO}}^{\text{VS,1}} \right) + \int d\Phi_{m+1} \left( d\sigma_{\text{NNLO}}^{\text{VS,1}} + d\sigma_{\text{NNLO}}^{\text{MF,1}} \right) \]
\[ + \int d\Phi_{m} \left( d\sigma_{\text{NNLO}}^{\text{V,2}} + d\sigma_{\text{NNLO}}^{\text{MF,2}} \right). \]

- \( d\sigma_{\text{NNLO}}^{\text{S}} \): real radiation subtraction term for \( d\sigma_{\text{NNLO}}^{\text{R}} \),
- \( d\sigma_{\text{NNLO}}^{\text{VS,1}} \): one loop real subtraction term for \( d\sigma_{\text{NNLO}}^{\text{V,1}} \),
- \( d\sigma_{\text{NNLO}}^{\text{V,2}} \): two loop virtual corrections,
- \( d\sigma_{\text{NNLO}}^{\text{MF,i}} \): mass factorization counter terms (i=1,2).

Each column is numerically finite and free of IR \( \epsilon \)-poles.
I-F NNLO: double real radiation

- Obtain antennae functions by crossing final-final NNLO antennae

Phase space factorization similar to NLO, with one particle more

\[ d\Phi_{m+2} (k_1, \ldots, k_j, k_k, k_l, \ldots, k_{m+2}; p, r) = \]

\[ d\Phi_m (k_1, \ldots, K_L, \ldots, k_{m+2}; xp, r) \frac{Q^2}{2\pi} d\Phi_{X_{i,j,k,l}} (k_j, k_k, k_l, p, q) \frac{dx}{x} \]

- Again integrated subtraction term can be computed analytically

- 2 → 3 particle phase space
Single unresolved limit of 1-loop amplitude:

\[ \text{Loop}_{m+1}^{j \, \text{unresolved}} \rightarrow \text{Split}_{\text{tree}} \times \text{Loop}_m + \text{Split}_{\text{loop}} \times \text{Tree}_m \]

Thus:

\[ X^1_{i,jk} = S_{i,jk;I,K} \left( \frac{M^1_{i,jk}}{M^0_{I,K}} \right)^2 - X^0_{i,jk} \left( \frac{M^1_{I,K}}{M^0_{I,K}} \right)^2 \]
## Initial-final antenna functions

| Initial | Final | Tree Level | One Loop |
|---------|-------|------------|----------|
| **Quark initiated** | **tree level** | **one loop** |
| **quark-quark** | | |
| $q \rightarrow gg$ | $A^0_{q,gg}$ | $A^1_{q,gg}$, $\tilde{A}^1_{q,gg}$, $\hat{A}^1_{q,gg}$ |
| $q \rightarrow ggg$ | $A^0_{q,ggg}$, $\tilde{A}^0_{q,ggg}$ |
| $q \rightarrow q'q'$ | $B^0_{q,q'q'}$ |
| $q' \rightarrow q\bar{q}q'$ | $B^0_{q',q\bar{q}'}$ |
| $q \rightarrow q\bar{q}$ | $C^0_{q,q\bar{q}}$, $C^0_{q,q\bar{q}q}$, $C^0_{q,q\bar{q}q}$ |
| **quark-gluon** | | |
| $q \rightarrow gg$ | $D^0_{q,gg}$ | $D^1_{q,gg}$, $\hat{D}^1_{q,gg}$ |
| $q \rightarrow ggg$ | $D^0_{q,ggg}$ |
| $q \rightarrow q'q'$ | $E^0_{q,q'q'}$ | $E^1_{q,q'q'}$, $\tilde{E}^1_{q,q'q'}$, $\hat{E}^1_{q,q'q'}$ |
| $q \rightarrow q'q'g$ | $E^0_{q,q'q'g}$, $\tilde{E}^0_{q,q'q'g}$ |
| $q' \rightarrow q'q$ | $E^0_{q',q'q}$ |
| $q' \rightarrow q'qq$ | $E^0_{q',q'qq}$, $\tilde{E}^0_{q',q'qq}$ |
| **gluon-gluon** | | |
| $q \rightarrow qg$ | $G^0_{q,gg}$ | $G^1_{q,gg}$, $\tilde{G}^1_{q,gg}$, $\hat{G}^1_{q,gg}$ |
| $q \rightarrow ggg$ | $G^0_{q,ggg}$, $\tilde{G}^0_{q,ggg}$ |
| $q \rightarrow q\bar{q}'\bar{q}'$ | $H^0_{q,qq\bar{q}'\bar{q}'}$ |
Integrated antenna computation

- Reduce phase space integrals to master integrals
- Integration over inclusive 2- or 3-particle phase space using differential equations in $q^2$ and $x = -\frac{q^2}{2 p \cdot q}$
- Boundary condition from explicit computation at $x = 1$
- 9 real and 6 virtual masters:
Computation of master integrals

- Masters computed using differential equations
- Example: \( (d = 4 - 2\epsilon) \)

\[
\begin{align*}
    x \frac{\partial I[2]}{\partial x} &= -\frac{d-4}{2} I[2] + \frac{3d-8}{2} \left(1 + \frac{1}{x-1}\right) \frac{I[0]}{Q^2} \\
    Q^2 \frac{\partial I[2]}{\partial Q^2} &= (d - 4) I[2] \quad \Rightarrow \quad I[2] \propto (Q^2)^{-2\epsilon}
\end{align*}
\]

- Boundary condition from explicit computation at \( x = 1 \)
- Putting all together:

\[
I[2] = \frac{2^{-7+4\epsilon}}{\pi^{3-2\epsilon}} \frac{\Gamma(1 - \epsilon)^3}{\Gamma(3 - 3\epsilon) \Gamma(2 - 2\epsilon)} \frac{3\epsilon - 2}{1 - 2\epsilon} (1 - x)^{1-2\epsilon} x^\epsilon (Q^2)^{-2\epsilon} \text{ hypergeometric function}
\]

- For simple masters exact result in \( \epsilon \to \) expanded with HypExp
  [T. Huber, D. Maître]
- For the others expansion up to needed power of \( \epsilon \)
Check with DIS structure functions

- Completed full set of integrated $2 \to 3$ tree-level and $2 \to 2$ one-loop antennae
- Cross check with NNLO DIS structure functions
  - DIS cross section for photon exchange
    \[
    \frac{d^2 \sigma}{dx \, dy} = \frac{2\pi \alpha^2}{Q^4} s \left[ (1 + (1 - y)^2) F_2(x, Q^2) - y^2 F_L(x, Q^2) \right]
    \]
- Checks $A$, $B$ and $C$ type antenna functions
- At NLO (before mass factorization)
  \[
  \frac{1}{C_f} \left( F_{2,q}^{(1)} - \frac{d - 1}{d - 2} F_{L,q}^{(1)} \right) = 4A_{q,qg}^0 + 8\delta (1 - z) F_q^{(1)}
  \]
  \[
  \frac{1}{d - 2} \left( F_{2,g}^{(1)} - \frac{d - 1}{d - 2} F_{L,g}^{(1)} \right) = -4A_{g,qg}^0
  \]
Check with DIS structure functions

- Completed full set of integrated $2 \to 3$ tree-level and $2 \to 2$ one-loop antennae
- Cross check with NNLO DIS structure functions
  - DIS cross section for photon exchange

$$\frac{d^2\sigma}{dx\,dy} = \frac{2\pi\alpha^2}{Q^4} s \left[ (1 + (1 - y)^2) F_2(x, Q^2) - y^2 F_L(x, Q^2) \right]$$

- Checks $A$, $B$ and $C$ type antenna functions
- At NLO (before mass factorization)

\[ \frac{1}{C_f} \left( F_{2,q}^{(1)} - \frac{d-1}{d-2} F_{L,q}^{(1)} \right) = - \frac{1}{d-2} \left( F_{2,g}^{(1)} - \frac{d-1}{d-2} F_{L,g}^{(1)} \right) = -4 A_{g,gq}^{0} \]

[Full agreement!]

[E. Zijlstra, W. van Nerveen; S. Moch, J. Vermaseren, A. Vogt]
Check with $\phi$-DIS structure functions

- Structure functions for a scalar particle coupling only to gluons
- Permits to check integrated $\mathcal{F}$, $\mathcal{G}$ and $\mathcal{H}$-type antenna functions
- DIS cross section for scalar exchange has only one structure function: $T_{\phi,i}$, for $i = q, g$

Some example

\[
T^{(1)}_{\phi,g} = 2N F^0_{g,gg} + 2n_f G_{g,q\bar{q}} + 4 \delta (1 - z) F^{(1)}_g
\]

\[
\frac{1}{C_f (1 - \epsilon)} T^{(1)}_{\phi,q} = -4N G^0_{q,qg}
\]

\[
T^{(2)}_{\phi,g} \bigg|_{N^2} = F^0_{g,ggg} + 4F^1_{g,ggg} + \delta (1 - z) \left( 8F^{(2)}_g + 4F^{(1)}_g \right)
\]

[S. Moch, O. Savai, J. Vermaseren, A. Vogt]
Check with $\phi$-DIS structure functions

- Structure functions for a scalar particle coupling only to gluons
- Permits to check integrated $F$, $G$ and $H$-type antenna functions
- DIS cross section for scalar exchange has only one structure function: $T_{\phi,i}$, for $i = q, g$

Some example

$$T_{\phi,g}^{(1)} = 2N F_{g,gg}^0 + F_{g,qq} + 4 \delta (1 - z) F_g^{(1)}$$
$$\frac{1}{C_f (1 - \epsilon)} T_{\phi,q}^{(1)} = -4N G$$
$$T_{\phi,g}^{(2)}\bigg|_{N^2} = 4 F_{g,gg}^{(2)} + 4 F_g^{(1)}$$

[3. Moch, G.Sabbir, J. Vermaseren, A. Vogt]
Conclusions

- Antenna subtraction scheme
  - subtraction method based on collecting all IR and collinear radiation between two pair of color connected hard partons
  - final-final case applied successfully at NNLO for $e^+e^- \rightarrow 3$-jet
  - all ingredient for initial-final subtraction now available
  - cross check of initial-final antennae with DIS structure functions is completed

- Potential applications:
  - NNLO DIS (2+1)-jet production
  - contribution to hadron-collider jet production
**Outlook:**

**DIS (2+1)-jet production @ NNLO**

Needed for several reasons:

- Determination of $\alpha_s$

---

**Dijet cross sections: constraints on pPDFs**

- Gluon fraction and theoretical uncertainties in the phase-space region of the measurements:
  - Predicted Gluon fraction:
    - 75% at low $Q^2$
    - > 60% at $Q^2 \sim 500$ GeV$^2$

→ PDF uncertainty large in regions of phase space where the gluon fraction is still sizeable

→ high precision dijet data have the potential to constrain further the proton PDFs when included in the global fits

- Simultaneous fit to all 62 measurements of inclusive, 2- and 3-jet cross sections
- Result dominated by theoretical uncertainty, missing higher orders

**NNLO calculations needed**

**Gluon pdfs**

**H1 data**
- $\alpha_s$ fit to jets
- Theory PDF

**Figure:**

- $\alpha_s$ from Jet Cross Sections

**Legend:**
- H1 data
- $\alpha_s$ fit to jets
- Theory PDF

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C Glasman (Universidad Autónoma de Madrid)
Outlook:

**DIS (2+1)-jet production @ NNLO**

- **All ingredients present**
  - real matrix elements,
    - [K. Hagiwara, D. Zeppenfeld, F. A. Berends, W. Giele, H. Kuft, N. K. Falck, D. Graudenz, G. Kramer]
  - mixed real-virtual matrix elements,
    - [Z. Bern, L. J. Dixon, D. A. Kosower, S. Weinzierl, E. W. N. Glover, D. J. Miller, J. M. Campbell]
  - two loop matrix elements,
    - [L. W. Garland, T. Gehrmann, E. W. N. Glover, A. Kourkoutsa, E. Remiddi]
  - subtraction terms.
    - [A. Daleo, A. Gehrmann De-Ridder, T. Gehrmann, D. Maître, G. L.]

- **Next steps:**
  - implementation of a parton level Monte Carlo event generator.
Backup Slides
**NNLO double real subtraction**

\[ d\sigma^S_{\text{NNLO}} : \text{double real subtraction} \rightarrow \text{different configurations} \]

- (a) one unresolved parton
- (b) two color-connected unresolved partons
- (c) two almost color-connected unresolved partons
- (d) two color-unconnected unresolved partons

(a): one unresolved parton:

- one unresolved parton but the experimental observable selects only \( m \) jets,
- three parton antenna function \( X^0_{i,j,k} \) can be used (like at NLO)
NNLO double real subtraction

\[ d\sigma_{\text{NNLO}}^S \]: double real subtraction → different configurations

(a) one unresolved parton
(b) two color-connected unresolved partons
(c) two almost color-connected unresolved partons
(d) two color-unconnected unresolved partons

(b): two color-connected unresolved partons:

\[ X_{i,j,k,l}^0 \]

four parton antenna function \( X_{i,j,k,l}^0 \)

complete set of four parton antennae for i-f configuration is now available
NNLO double real subtraction

\[ d\sigma^{S}_{\text{NNLO}}: \text{double real subtraction} \rightarrow \text{different configurations} \]

- (a) one unresolved parton
- (b) two color-connected unresolved partons
- (c) two almost color-connected unresolved partons
- (d) two color-unconnected unresolved partons

(c): two almost color-connected unresolved partons:

\[
\text{share a common radiator}
\]

\[
\text{accounted for by products of two tree-level three-parton antennae functions}
\]

\[
\text{distinguish cases where common radiator is in the initial or final configuration}
\]
NNLO double real subtraction

\[ d\sigma_{\text{NNLO}}^S \]: double real subtraction → different configurations

(a) one unresolved parton
(b) two color-connected unresolved partons
(c) two almost color-connected unresolved partons
(d) two color-unconnected unresolved partons

(d): two color-unconnected unresolved partons:

two well separated partons in the colour chain

product of independent three-parton antenna functions
NNLO Antenna subtraction

\[ d\sigma_{VS,1}^{\text{NNLO}} \]: one loop real subtraction \(\rightarrow\) several requirements

(a) remove explicit IR poles from loop

(b) subtract single unresolved limits

(c) remove oversubtracted terms

(a): remove poles from loop integral:

- virtual correction has IR poles which have to be removed by means of the real counterpart

- subtraction term contains integrated antenna \(\chi_{i,j,k}^0\)
**NNLO Antenna subtraction**

*d\sigma_{NNLO}^{VS,1}*: one loop real subtraction $\rightarrow$ several requirements

1. (a) remove explicit IR poles from loop
2. (b) subtract single unresolved limits
3. (c) remove oversubtracted terms

(b): subtraction of single unresolved limits:

- subtraction of singular configurations originating when the real radiation correction to the one loop amplitude becomes soft or collinear.
- subtraction term is a combination of three-parton tree-level $X_{i,j,k}^0$ and three parton one-loop $X_{i,j,k}^1$ antenna functions.
NNLO Antenna subtraction

\[ d\sigma_{VV,1}^{NNLO} \]: one loop real subtraction \( \rightarrow \) several requirements

(a) remove explicit IR poles from loop
(b) subtract single unresolved limits
(c) remove oversubtracted terms

(c): remove oversubtracted terms:

- remove terms which are common to both previous contributions and are oversubtracted
- subtraction term contains initial-final and final-final antenna