Quenched QED on a momentum space lattice
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We discuss the advantages of using the momentum space lattice method for studying the phase diagram of quenched QED 4. Preliminary results of a numerical simulation are presented. They indicate that the method avoids the contamination by 4-fermi interactions which plagues the conventional position space non-compact formulation.

At last year’s conference, we introduced the momentum space lattice method and discussed its general features [1]. Here we would like to report on its application to quenched QED 4. Before going into the details of the simulation, however, it is worth explaining the advantages of the method for the problem at hand. We will therefore start with a brief reminder of earlier analytical and numerical results and motivate from there the introduction of the momentum space idea.

The analytical work of Miransky, Bardeen, Leung and Love (MBLL) [2] indicates that the phase diagram of quenched QED should be described in the (α, G) plane, where α is the usual QED coupling constant and G is the strength of a 4-fermi interaction. From the ladder Schwinger-Dyson equation, it is in fact possible to determine the shape of the critical line and to estimate the value of the critical exponents along this line [3]: They vary continuously from their mean-field value in the pure 4-fermi theory to an essential singularity at the MBLL point. Numerical simulations of quenched non-compact QED 4 appear to pick-up a particular point along this line (α = 0.44αc, G = 3.06), implying that non-compact QED actually has to be interpreted as a “mixture” of QED + 4-fermi. The values of the critical exponents at this point (δ = 2.2(1), β = 0.78(8), ν = 0.68(2)) were confirmed in more extensive simulations [4] and recent results strengthen the hypothesis of power-law scaling [5]. All of this however raises an important question: can we find a simulation technique that would allow us to move along the critical line and expose the physics of “pure” QED? We certainly expect that if such a technique can be found in the quenched case, it will play an important role in future simulations of the full theory.

A key point for carrying-out the above program would be to identify precisely the origin of 4-fermi interactions in non-compact QED with action:

\[ S = S_G + S_F = \frac{\beta}{4} \sum_{x,\mu,\nu} (\Delta^{\mu} \theta^\nu)^2 \]

\[ - \frac{1}{2} \sum_{x,\mu} \eta_\mu(x) \bar{\psi}_x [U_\mu(x) \psi_{x+\mu} - U_{\mu}^*(x-\mu) \psi_{x-\mu}] \]

where we have assumed

\[ D_{\mu\nu}(x-y) = \delta_{\mu\nu} V(x-y) \]

The first term represents a 4-fermi interaction, the second is a tadpole improved kinetic energy

\[ \exp[-V(0)/2\beta] S_F(U = 1) \]

An interesting candidate is revealed by using the quadratic nature of \( S_G \) and computing exactly the integration over the gauge fields. This operation leads to an effective action of the form:

\[ S_{\text{eff.}} = \frac{1}{4} \sum_{x,\mu} (\bar{\psi}_x \psi_{x+\mu}) \]

\[ + \exp[-V(0)/2\beta] S_F(U = 1) + \cdots \]

\[ \text{where we have assumed} \]

\[ D_{\mu\nu}(x-y) = \delta_{\mu\nu} V(x-y) \]
term, and the dots stand for the current-current interaction as well as 3-body and higher order inter-
actions. The form of (2) suggests that in order to get an effective action which is closer to “pure”
QED, we should try to get rid of the 4-fermi term. In fact, recent results on quenched models seem
to indicate that this term, when present, is very efficient in masking the rest of the dynamics: critical
exponents appear to be largely insensitive to the choice of the potential $V$ [6]. There is no
objection in principle to subtracting the 4-fermi interaction (multiplied by $\lambda^2$) by adding to the action:

$$S_+ = \frac{\lambda}{2} \sum_{x,\mu} \bar{\psi}_x [e^{i\phi_\mu(x)} \psi_{x+\mu} + e^{-i\phi_\mu(x-\mu)} \psi_{x-\mu}]$$

where $\phi_\mu$ is a new vector field which is freely integrated over between 0 and $2\pi$. Notice the relative
+ sign in (4) opposite to (1). This technique is however very slow numerically. In many respects, the problems are analogous to those encountered in simulations at finite chemical potential, where a term similar to (4) would appear at first order in an expansion of the lagrangian in powers of $\mu$ (in this analogy, $\lambda$ is identified with the chemical potential $\mu$). As in the chemical potential case, only small values of $\lambda$ can be considered here, so that the 4-fermi interaction could only be subtracted partially. A more radical modification of the model therefore seems necessary.

An important aspect of the 4-fermi interaction in $S_{\text{eff}}$, is that it is directly related to the compact nature of the coupling of the fermions to the gauge field in the original model (as can be seen through the steps leading to (2)). On the other hand, the momentum space lattice method [4] has a non-compact coupling and therefore avoids the generation of 4-fermi interactions. In fact, the method simply uses the continuum lagrangian of QED. The model is investigated on a torus of length $L$ and the momentum expansion is truncated. After gauge fixing (to the Feynman gauge) and Fourier expansion, the action reads:

$$S = -\frac{1}{2} \beta \sum_q \theta_\mu^*(q)(2\pi)^2 \theta_\mu(q) + \sum_k \chi(k)[\gamma_\mu(2\pi k_\mu) + \rho]\chi(k)$$

$$- \sum_{k,k'} \chi(k) \gamma_\mu(k - k') \chi(k')$$

where, instead of the usual fields $\psi$ and $A_\mu$, we have used the following dimensionless variables:

$$\chi(k) = L^{5/2} \psi(k) \quad \theta_\mu(q) = eL^3 A_\mu(q)$$

and $\beta = 1/e^2$, $\rho = mL$. In the above formula, the $k_\mu$'s are integers or half-integers depending on whether the boundary conditions for the fermions are periodic or antiperiodic. The $q_\mu$'s are integers and take all the possible values of the momentum transfer between $k$ and $k'$. The gauge fields therefore live on a lattice which is twice as big (in each direction) as the fermionic lattice. The method has several advantages among which is the fact that it does not suffer from the fermion doubling problem. From the numerical point of view, the procedure is however more involved than conventional position space lattice methods, the reason being the non-locality of the coupling of the fermions to the gauge field. This is partially taken care of by repeated use of a fast Fourier transform which reduces the computer time requirements to $O(N \log N)$. When quoting our results below, we give the size of the “photon lattice” since it is the one used in the FFT and therefore determines the computer time requirements. This size in turn is constrained by the specifics of the FFT routine and the choice of the boundary conditions for the fermions. Up to now, we have considered lattices of size (6)$^4$, (10)$^4$ and (15)$^4$. We have measured the mass gap and $<\bar{\psi}\psi>$ by respectively using point and volume sources on the momentum space lattice (both methods make use of the fact that the propagator should be diagonal in momentum after statistical average). Identifying the mass gap $M$ with the inverse of the correlation length, we expect at the critical coupling:

$$M \propto m^{\nu/\beta\delta}$$

From our (10)$^4$ measurements, we obtain the estimates $\beta_c = 0.8$ and $\beta\delta/\nu \approx 2$. We can go one
step further and write the equation of state in the form:

\[ \frac{m}{M^{\beta \delta/\nu}} = F \left( \frac{\Delta \beta}{M^{1/\nu}} \right) \]

\[ \Delta \beta = \beta - \beta_c \quad (8) \]

Fixing \( \beta \delta/\nu = 2 \), and varying \( \beta_c \) and \( \nu \), we obtain a best fit to our full data sample for \( \beta_c = 0.79 \) and \( \nu = 1.22 \). This value of \( \nu \) is anomalously large and indicates that we are indeed probing the physics of a non-trivial theory (By comparison, the value of \( \nu \) obtained on a position space lattice was \( \nu = 0.68 \) and it would be \( \nu = 0.5 \) in mean field theory). Similarly, the value \( \beta \delta/\nu \approx 2 \) is consistent with Miransky scaling. It is important to complete this picture by independent measurements of the critical indices \( \beta \) and \( \delta \). We measured \( \langle \bar{\psi} \psi \rangle \) on lattices of size \((10)^4\) and \((15)^4\) and found respectively \( \delta \approx 2.0 \) and \( \delta \approx 1.7 \). The trend here is interesting and indicates that as the size of the lattice is increased, it becomes possible to detect exponents which are further away from their mean-field value (see fig. 1). Since, on a position space lattice, distinguishing between the critical behaviour of QED and a Nambu Jona-Lasinio model is a difficult task, it is interesting to see how the momentum space lattice behaves in this respect. For that purpose, we ran simulations in which we replaced \((2\pi q)^2\) in equation (5) by a constant, so that the long range QED potential becomes a point-like interaction. The estimates for \( \delta \) are now \( \delta = 3.5 \pm 0.2 \) on a \((10)^4\) lattice and \( \delta = 3.2 \pm 0.1 \) on a \((15)^4\) lattice. These can very easily be discriminated from the QED values given above and we reach the conclusion that on a momentum space lattice, the shape of the potential really matters (at the difference of the position space situation). The results presented above are obviously very preliminary and more detailed analyses are clearly needed. However, they all indicate that the momentum space lattice method is successful in removing 4-fermi interactions from QED simulations. The trends that we have established as a function of the lattice size, open the way for more detailed simulations of QED on larger lattices and the extraction of “infinite volume” critical exponents. The method of course also has its limitations: it is computationally very expensive due to the non-locality of the interaction and a careful analysis of its gauge invariance properties is needed. It remains however, so far, the only method we have with the potential to map out the entire critical line of the quenched model.

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