Mathematical reasoning through the application of solid geometry

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Abstract. This study aims to find out how the process and results of students' reasoning abilities are faced with the problems of applying solid geometry. This research was conducted on 10 students of class VIII SMP in the even semester of the 2018/2019 school year. Data collection techniques using purposive sampling. Data triangulation process is carried out to confirm the results of the test and interview. The results of this study indicate that students' mathematical reasoning abilities measured by reasoning indicators have different results. Indicator of the ability to present mathematical statements verbally, in writing, drawings, diagrams, get good results because students have been able to write the information requested and find solutions to problems. Indicator of the ability to do mathematical manipulation to get enough results, because students in the process of working on mathematical manipulation are incomplete and inaccurate. Indicators compile evidence and give reasons to get bad results because students cannot give reasons precisely because the preparation of evidence is incomplete. The indicator draws conclusions from a statement getting bad results because students cannot deduce the questions given correctly due to a lack of understanding. Based on the results it was concluded that the student's reasoning ability was still lacking at the stage of giving reasons for the evidence compiled and inferring from a statement.

1. Introduction

Mathematics is a compulsory subject taught at all levels of education. The role of mathematics for life is very important, therefore teaching mathematics has been started from an early age. One of the goals of learning mathematics is to train mindsets and reasoning in deciding conclusions, developing the ability to solve problems, and developing the ability to convey information or communicate ideas through oral, written, drawing, graphic, map, diagram, etc. [1].

The National Council of Teachers of Mathematics (NCTM) mentions that there are five aspects of learning mathematics, one of which is a reasoning [2]. Reasoning skills are important in learning mathematics which can arouse students' thinking when solving mathematical problems. According to Shivakumar [3] reasoning skills are recognized as a key ability for humans to create, learn, and utilize knowledge. The reasoning is the act or process of thinking to conclude a conclusion or make a new statement based on the statement before and the truth has been proven [4]. NCTM revealed that
mathematical reasoning is any activity that exists in mathematics will not be separated from the process of reasoning [2]. In mathematical reasoning there are several activities as indicators, namely (1) the ability to present mathematical statements verbally, in writing, drawings, diagrams, (2) check the validity of an argument, (3) the ability to do mathematical manipulation, (4) compile evidence and provide evidence the reasons, (5) suggest, (6) draw conclusions from a statement, (7) find patterns or properties of mathematical symptoms to make generalizations [4]. The role of reasoning ability is very important in solving everyday problems [5]. For important reasons in learning mathematics, a change in approach to learning and teaching mathematics is needed [6].

According to the TIMSS 2011 data, Indonesia’s position is in the order of 38 out of 42 participants with an average score of 386 [7]. The data shows that Indonesia’s mathematical ability is still low compared to most TIMSS participating countries. PISA 2018 results show that Indonesia’s ranking is still low, where the average achievement score for mathematics is ranked 72 out of 78 countries evaluated [8]. Weak causes in mathematics are due to lack of interaction among students and also the lack of reasoning activities during the teaching and learning process of mathematics in the classroom and this results in unsatisfactory mathematical achievements in Indonesia [9]. One of the influential ways in the process of reasoning students is learning in the form of the application of mathematics in everyday life.

In learning mathematics, there is an approach related to the application of mathematics in everyday life. The approach is Realistic Mathematics Education (RME). RME is one approach that addresses problems that begin with traditional and abstract mathematics learning [10]. RME aims to make mathematics learning more interesting and meaningful for students by introducing teaching mathematics through contextual problems where the problem is in students’ knowledge and experience [11].

According to previous research, the RME approach has proven to be influential in mathematics learning. Like the theory of Realistic Mathematics Education as a theoretical framework for teaching low achievement in mathematics [12]. Besides that, how Realistic Mathematics Education (RME) improves students’ cognitive achievement in mathematics [13]. The results of previous studies indicate that RME can be used effectively to predict the achievement of mathematics learning. The study also revealed the relationship between mathematical reasoning abilities with various kinds of learning. Such as increasing the mathematical reasoning ability of middle school students by applying the Mind Mapping Strategy [14]. Then about solving problems based on metacognitive learning to improve students' mathematical reasoning skills [15]. The results of this study indicate that the treatment of learning can affect the ability of students' mathematical reasoning.

Based on the research above shows that students’ mathematical reasoning abilities can be influenced, measured, and the results are known. Therefore the researcher wants to examine how the process occurs and the results obtained from students' mathematical reasoning abilities when faced with the matter of the application of material to solid geometry.

2. Method
This type of research is a qualitative descriptive study. Qualitative research is a method used to analyze the condition of natural objects, inductive data analysis, and qualitative research results that emphasize the meaning of generalization [16]. The purpose of this study is to uncover naturally the mathematical reasoning ability of students through the application of solid geometry. This research was conducted on 10 students of class VIII SMPN 1TawangmanguKaranganyar in the even semester of the 2018/2019 academic year. Data collection techniques in this study used purposive sampling. In this study, students were given questions with material to build solid geometry that have been modified according to their application in daily life. Student work outcomes are measured using reasoning indicators that have been determined by researchers. Indicators of reasoning used in this study are: (1) The ability to present mathematical statements verbally, in writing, pictures, diagrams; (2) Ability to carry out mathematical
manipulations; (3) Prepare evidence and provide reasons; (4) Draw conclusions from a statement. Then the results of student work are analyzed every indicator according to the categories obtained. The next stage is interviews with students to find out more patterns from their reasoning process when completing the questions given. A description of the reasoning indicators used in this study can be seen in Table 1.

Table 1. Description of reasoning indicators

| Indicator |
|-----------|
| 1 The ability to present mathematical statements verbally, in writing, pictures, diagrams. |
| 2 The ability to do mathematical manipulation. |
| 3 Compile evidence and give reasons. |
| 4 Take conclusions from a statement. |

| Indicator | Description |
|-----------|-------------|
| 1         | Students write the requested data information. Students do the process of finding solutions. |
| 2         | Students write mathematical models. Students do mathematical manipulation and do calculations. |
| 3         | Students carry out the process of finding evidence from the requested data. Students give reasons for the evidence compiled. |
| 4         | Students understand the statement given. Students give conclusions about the statement given. |

Data triangulation was carried out to confirm the findings. The data confirmation process is carried out to compare test and interview data. The stages of data analysis in this study involved: (1) giving students the test questions to solid geometry; (2) analyzing test results; (3) conducting interviews with several students; and (4) analyze the results of the interview. The final step is to conclude. In this step, researchers conclude from the results of data analysis that has been done before.

3. Results and Discussion

3.1 Results

The results of students' reasoning tests using the application questions of solid geometry measured using reasoning indicators can be seen paste in the following Table 2.

Table 2. Scoring the results of students' reasoning tests

| Indicator | Student score | Maximum score | Percentage | Qualification |
|-----------|---------------|---------------|------------|--------------|
| The ability to present mathematical statements verbally, in writing, pictures, diagrams. | 90 | 120 | 75% | Good |
| The ability to do mathematical manipulation. | 74 | 120 | 61.6% | Enough |
| Compile evidence and give reasons. | 39 | 120 | 32.5% | Bad |
| Take conclusions from a statement. | 48 | 120 | 40% | Bad |

The data above shows that there are differences in results between the measurement indicators. Therefore we need an analysis of the results of student work to find out the stages of students' reasoning and problems encountered in working on the given problem. The first indicator shows good results, from a maximum score of 120 students get a score of 90 or 75% can answer the question well. This shows that in the first indicator students can use their reasoning well. The first indicator is given the following problem. Draw a scout tent that resembles an isosceles triangular prism and determine the volume if the tilted side
of the tent picture is 13 cm, the width of the tent picture is 10 cm, and the length of the tent picture is 15 cm. The following is one of the results of student work for the first indicator.

Figure 1. SA answer to problem 3

From the students' answers (SA), it was found that students had written down the requested data information, but students in writing the information did not match what the question wanted. This can be seen in the picture that students did not make like a tent along with the size that was not made original. In the next stage, students can find the desired solution to the problem, but students are less sequential in their work.

For the second indicator, the results are obtained with enough categories, from a maximum score of 120 students get a score of 74 or 61.6% can answer the questions given. This shows that students are quite capable of using their reasoning power to answer the questions given. The second indicator is given the following problem. A block-shaped music box has a length equal to two times its width and a height of half its width. The size of the music box is changed so that the length is three times the original and the width is twice the original, while the height is fixed. If the surface area of the music box was originally 448 cm$^2$, then the volume after enlarging is? The following is one of the results of student work for the second indicator.

Figure 2. DPS answer to problem 2
Based on the results of student work (DPS) above it can be seen that students can write mathematical models according to the problems even though they are simple. In working on the problems, students do mathematical manipulations imperfectly. There are several steps to pass and some symbols that are not written. The results of student work above have not been completed and the solutions asked by the questions have not been answered. Nevertheless, students have been able to do the stages of writing models and manipulating.

In indicators three and four the results are categorized as poor. For the third indicator, the results obtained a score of 39 or 32.5% of the maximum score of 120 by entering into the bad category. From these results, it can be estimated that students have not been able to do the reasoning process properly. The third indicator is given the following problem. There is a cube-shaped glass box and in it there is a pyramid miniature that is congruent with the glass box. Investigate whether the difference in the volume of the glass box and the volume of the pyramid miniature is equal to 2 times the volume of the pyramid miniature if it is known that the height of the miniature is the same as the height of the glass box! Why?
The following is one of the results of student work for the third indicator.

![Figure 3. KPLP answer to problem 3](image)

From students' answers (KPLP) it is known that students have done the process of looking for evidence well, but students do not do the process of finding differences. Resulting in at a time when students should give reasons for the evidence that was obtained correctly. But the student makes the mistake by directly deducing what he got from the first step, so in giving reasons it becomes wrong.

In the fourth indicator, the results are obtained with a bad category wherein the maximum score of 120 students gets a score of 48 or 40% of the maximum score. In the fourth indicator, students are given the problem as follows. "Limas is a solid geometry structure whose blanket consists of triangular buildings with one common point" Is that statement correct? What is the common point called? And what is the connection with the height of the pyramid? The following is one of the results of student work for the fourth indicator.
The answers of students (FR) show that in the first step students can understand the purpose given problems. But in the second step where students are asked to provide conclusions about the questions given, students give conclusions incorrectly.

To confirm the results of student work that have been obtained it is necessary to interview the subjects of the SA, DPS, KPLP, and FR. The following interviews with KPLP subjects can be seen in table 3.

Table 3. Results of the first interview

| Code | Interview result |
|------|------------------|
| Q1   | Can you understand the meaning of the first problem? |
| A1   | Yes, I can understand that. |
| Q2   | Why don't you draw according to what is asked about the problem? |
| A2   | Because I draw it easier with that shape. And easier for me to understand. |
| Q3   | Can you find the volume of the tent? How can you do it? |
| A3   | Yes, I can. By finding the height of the tent first, use Pythagoras, then use the prism volume formula to find the tent volume. |

The following results of interviews with DPS subjects can be seen in table 4.

Table 4. Second interview results

| Code | Interview result |
|------|------------------|
| Q4   | Have you read second problem carefully? |
| A4   | Yes, I have read it several times. |
| Q5   | Do you understand the purpose of the problem? |
| A5   | Yes, I understand it. |
| Q6   | How do you solve this problem? |
| A6   | First, write the known length and height using a mathematical model according to the problem, then enter the length, height, and width into the beam surface area formula, then do the mathematical manipulation process so that it can operate. |
| Q7   | From this method can you find the length, width, and height? |
| A7   | Yes, it can be found, after that just search for the volume of the beam as a requested problem. |
| Q8   | In your work there are no results from the volume of the beam, why? |
A8: That's because I forgot, I guess I'm just looking for the size of the sides of the enlarged beam.

The following results of interviews with KPLP subjects can be seen in table 5.

Table 5. Third interview results

| Code | Interview result |
|------|------------------|
| Q9   | Have you read the third problem carefully? |
| A9   | Yes, I have read it in earnest. |
| Q10  | Do you understand the purpose of the problem? |
| A10  | Yes, I already understand the matter of wanting. |
| Q11  | Are you able to look for evidence as to the desired problem? |
| A11  | Yes, I can look for it. |
| Q12  | How do you look for it? |
| A12  | The first is by considering the size of the sides of the glass case and pyramid, then look for both volumes. |
| Q13  | By getting the two volumes is it enough to compile the evidence? |
| A13  | Yes, it seems enough because there is nothing else to look for. |
| Q14  | After you compile the evidence can you give a reason as asked about? |
| A14  | Yes, so the answer to that problem is incorrect because it is not appropriate between the statement in the problem with the evidence that I'm looking for. |

The following results of interviews with FR subjects can be seen in table 6.

Table 6. Fourth interview results

| Code | Interview result |
|------|------------------|
| Q15  | Have you read the fourth problem carefully? |
| A15  | Yes, I have. |
| Q16  | Do you understand the purpose of the problem? |
| A16  | Yes, I understand the meaning of the problem. |
| Q17  | What do you think of the statement of the problem? Why? |
| A17  | In my opinion, the question in question number 4 is correct. Because if I picture, the statement is appropriate. |
| Q18  | Can you find the link between the statement and the question in question 4? |
| A18  | Yes I can find it, in my opinion, the point of this alliance is the top of the pyramid where it is also high in the pyramid. |

3.2. Discussion

Based on research that has been done shows that each indicator gets results with different categories. The first indicator gets results in the good category, the second indicator gets enough results, the third indicator gets bad results, and the fourth indicator gets bad results.

In the first indicator with a good category, students can solve the problems given by writing the requested information and can find solutions. In writing information, students can do well, but in the process, students are still less than perfect. In finding solutions students are able to do it correctly but are still less sequential in writing and less careful in writing symbols.
In the second indicator with enough categories, students can complete the given problem, but there are still many improper works in each of the steps. In the section write mathematical models students do not write information that is known in advance. Do not write down information related to the mathematical model created. In the mathematical manipulation section several steps are skipped, still not careful enough in changing variables and writing symbols. The work done is also incomplete as desired problems. Nevertheless, students have been able to do the stages of writing models and manipulating.

In the third indicator with a bad category, students can understand the purpose of the problem and can answer the first few steps. Students can arrange evidence even though there are still deficiencies in the process. Students do a lot of work in order and there is still a lot of information that is not included. In the section giving reasons for evidence that has been compiled, students cannot give exact reasons about the evidence that was compiled because there is still information or evidence that is not sought. Imperfect students in preparing evidence make mistakes in understanding in giving reasons.

In the fourth indicator with a bad category, students are able to understand the statements given in the questions. But in the step of giving conclusions from the problem students make many mistakes. The error occurred because of the lack of students in understanding the intentions that the questions wanted.

Based on the results of the analysis that has been done about student answers found that the process of reasoning students can not be achieved in full. This is in line with one study that students' mathematical reasoning abilities have not been reached optimally on certain indicators [17]. This shows that the students' mathematical reasoning ability is not optimal. The students' mathematical reasoning ability is not yet maximized due to various factors. One of the factors that influence is the habits of students who only focus on the aspect of calculation. This can be seen in the results of research where most students are capable of mathematical calculations but are unable to take a reason or conclusion from what they are doing. Other research also states that students cannot provide arguments and make conclusions correctly to make reasoning abilities not optimal [18].

Not being able to fully understand is also a factor in the attainment of reasoning indicators. Students are not able to meet the indicators of measured mathematical reasoning ability because they do not understand what is given the problem and the lack of understanding of the concept [19]. This is seen in the results of students working on the third and fourth reasoning indicator questions and is strengthened by the results of their interviews. Students are wrong in understanding the purpose of the questions given to make the results wrong. Students have difficulty understanding the purpose of the problem, formulating what is known from the problem, the student's completion plan is not directed, and the student's calculation process is wrong [20].

Increasing students' reasoning abilities to be perfect is an important thing. To do this, appropriate learning strategies are needed in the teaching and learning process following the characteristics of students [14]. For this reason, teachers are expected to choose the right strategy in the learning process. Using ICT in teaching and learning can attract students to do mathematics learning [21, 22]. The use of ICT in learning geometry can help students understand abstract concepts [23]. The teacher is also expected to be able to provide more understanding of the material being taught rather than just focusing on the calculation process itself.

4. Conclusion
The use of application problems in the material of flat side space has been proven to affect the students' mathematical reasoning abilities. This is indicated by the existence of different results on several indicators of reasoning that are used. Based on a maximum score of 120 the ability to present mathematical statements verbally, in writing, drawings, diagrams get 75% results included in either category, where students have been able to write the information requested and find solutions to problems. The ability to do mathematical manipulation got 61.6% results included in the sufficient category, where
students in the process of doing mathematical manipulation are incomplete and inaccurate. Compiling evidence and giving reasons to get 32.5% results included in the category of bad, where students can not give reasons precisely because the preparation of evidence is incomplete. Taking conclusions from a statement getting 40% results included in the bad category, where students can not deduce the questions given correctly due to lack of understanding. The ability of students to stage the stages of compiling evidence and giving reasons, as well as concluding a statement has fewer results. Various factors can affect the results, one of the factors is the mathematical ability and understanding of the material that is different for each student. This research is conducted with a sample limited to one class and at a certain time, so any generalization taken from this study must be done carefully.

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