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FPC: A New Approach to Firewall Policies Compression

Yuzhu Cheng, Weiping Wang*, Jianxin Wang, and Haodong Wang

Abstract: Firewalls are crucial elements that enhance network security by examining the field values of every packet and deciding whether to accept or discard a packet according to the firewall policies. With the development of networks, the number of rules in firewalls has rapidly increased, consequently degrading network performance. In addition, because most real-life firewalls have been plagued with policy conflicts, malicious traffics can be allowed or legitimate traffics can be blocked. Moreover, because of the complexity of the firewall policies, it is very important to reduce the number of rules in a firewall while keeping the rule semantics unchanged and the target firewall rules conflict-free. In this study, we make three major contributions. First, we present a new approach in which a geometric model, multidimensional rectilinear polygon, is constructed for the firewall rules compression problem. Second, we propose a new scheme, Firewall Policies Compression (FPC), to compress the multidimensional firewall rules based on this geometric model. Third, we conducted extensive experiments to evaluate the performance of the proposed method. The experimental results demonstrate that the FPC method outperforms the existing approaches, in terms of compression ratio and efficiency while maintaining conflict-free firewall rules.

Key words: firewall; firewall policy; network security; firewall rules compression

1 Introduction

Firewalls are critical components of network security and are deployed at all the entrances between a private network and the Internet to monitor all incoming and outgoing packets. The function of a firewall is to examine the field values of every packet and decide whether to accept or discard a packet according to the firewall policies. The policy is specified as a sequence of rules, each of which has a predicate over some packet header fields and a decision to be performed upon the packets that match the predicate. A packet can be viewed as a tuple with a finite number of the fields such as the source Internet Protocol (IP) address, destination IP address, source port number, destination port number, and protocol type. When a packet arrives at a firewall, the firewall searches for the first (i.e., the highest priority) rule with which the packet matches, and executes the decision of the rule. In general, the decision of a rule is accept or discard. Two firewalls are semantically equivalent to each other if and only if they have the same decision for every possible packet. In this paper, we consider the enhanced Firewall Policies Compression (FPC) problem: Given a firewall $f$, generate another firewall $f'$ that is semantically equivalent to $f$ but has fewer conflict-free rules.

In high-speed firewalls, filtering is performed by special hardware (i.e., Ternary Content Addressable Memory (TCAM)[1]), which is not only expensive but also constraints on the size of firewall rule sets. Thus, a natural optimization criterion for firewall
rule sets is to minimize their sizes while maintaining the semantic equivalence. This will also reduce the maximum delay in the sequential rules evaluation by the firewalls. Another optimization criterion is to ensure the rules are conflict-free based on the following two points [2]: First, because of the conflicts among rules and the resulting order sensitivity, firewall rules are logically entangled, and the correct ordering is critical but difficult. The semantics of any rule in a firewall cannot be correctly understood without examining previously listed rules. Second, if a firewall administrator unintentionally swaps any two conflicting rules, wrong actions may be performed, resulting in irreparable or tragic consequences.

In Ref. [2], Cheng et al. presented an approach for designing firewalls based on a multidimensional matrix. In their proposed method, a Firewall Design Matrix (FDM) is first designed. Then, a construction algorithm and a generation algorithm are applied in the FDM to generate and compress the target firewall rules, which are disjoint and non-conflicting. However, we find that the rules can be further compressed when rules with the same decision are allowed to overlap; here, the overlapping does not affect the semantics of the rules. Figure 1 illustrates an example of this instance. Assuming there are three firewall rules $r_1: F_1 \in [1, 2] \land F_2 \in [4, 7] \rightarrow \text{accept}$, $r_2: F_1 \in [3, 4] \land F_2 \in [2, 8] \rightarrow \text{accept}$, and $r_3: F_1 \in [5, 7] \land F_2 \in [4, 7] \rightarrow \text{accept}$, as shown in Fig. 1a, since these rules have the same decision, and are allowed to overlap, they can be further compressed to two rules: $r_1: F_1 \in [1, 7] \land F_2 \in [4, 7] \rightarrow \text{accept}$, and $r_2: F_1 \in [3, 4] \land F_2 \in [2, 8] \rightarrow \text{accept}$, as shown in Fig. 1b.

On the other hand, rectilinear polygons frequently arise in Very Large Scale Integration (VLSI) layout and artwork analyses as well as computer graphics [3]. Often, functions are more easily performed on a rectilinear polygon by considering the polygon as being composed of several rectangles. A set $T$ of rectangles is said to cover the polygon $P$ if $P$ is the union of $T$. If the rectangles in $T$ are disjoint, then we call the set $T$ to be a partition of $P$.

A Minimal Overlapping Cover (MOC) of a rectilinear polygon $P$ is defined as the cover of $P$ that has the fewest number of rectangles. A Minimal Non-overlapping Cover (MNC) of a rectilinear polygon $P$ is defined as a partition of $P$ with the minimum number of rectangles [4]. Figures 2a and 2b show a rectilinear polygon $P$ with $|\text{MNC}(P)| = 3$ and $|\text{MOC}(P)| = 2$, respectively.

As defined in Ref. [2], the FDM method can be seen as the MNC problem of a multidimensional rectilinear polygon, and the target firewall rules are disjoint and conflict-free. However, if overlapping is allowed, it can be regarded as an MOC problem in a multidimensional space. In general, $|\text{MOC}(P)|$ is less than $|\text{MNC}(P)|$ [4], as shown in Fig. 2. This means if the firewall rules compression problem is transformed into an MOC problem, the number of rules can be further reduced with a higher degree compared with the FDM method. In this paper, we propose a new method to transform the firewall rules compression and a novel algorithm that solves...
the corresponding MOC problem. This is our main innovation compared to prior researches\cite{2,5–9} on the firewall rules compression. However, to the best of our knowledge, current studies of the MOC problem are concentrated on a two-dimensional space\cite{9–13}. Therefore, we need to consider how to deal with this problem in a multidimensional case, which has been proved to be NP-hard\cite{5}.

The main contributions of this study can be summarized as follows:

(1) We present a new approach in which a geometric model is first constructed to transform the firewall rules compression problem into a multidimensional MOC problem.

(2) We propose a heuristic algorithm, FPC, to solve the multidimensional MOC problem, which is totally different from prior methods.

(3) We evaluated the performance of our approach by conducting extensive experiments on synthetic firewalls with different sizes. The experimental results demonstrate that FPC outperforms the existing approaches, in terms of compression ratio and efficiency, while maintaining conflict-free firewall rules.

The rest of this paper is organized as follows. Related works are presented in Section 2. In Section 3, we define terms related to our approach and present the algorithm for compressing the original firewall rules. In Section 4, we give the compression results of synthetic firewall rules. Finally, conclusion is drawn in Section 5.

2 Related Work

In this study, we convert the FPC problem to an MOC problem in a multidimensional space. To our knowledge, except several special cases, this problem in the general setting has never been studied in prior works. The optimal polynomial time algorithms for a one-dimensional case have been developed in Ref. \cite{14}. In addition, a two-dimensional rules compression problem has been proved to be NP-hard\cite{6}. Applegate et al.\cite{5} proposed an optimal polynomial time algorithm for a two-dimensional problem when there are only two decisions and all rules must be strip rules, and then with this result, they created an approximation algorithm for the general two-dimensional problem. However, it is not clear how to apply their ideas to a space with more than two dimensions.

Daly et al.\cite{7} presented Diplomat to address the multidimensional access control rules compression problem, where, the key idea is to transform higher-dimensional target patterns to lower-dimensional patterns by dividing the original pattern into a series of hyper-planes. Two adjacent planes are then selected, and their differences are resolved by adding rules to specify where the two planes differ. After resolution, the two planes are compatible and can be merged into a single plane. In this approach, each difference between two adjacent planes requires specification by additional rules. Therefore, the compression ratio is largely affected by the differences among the rules.

Cheng et al.\cite{2} presented the FDM approach for designing firewalls. In their proposed method, the original firewall rules are mapped into a set of unit spaces in a multidimensional matrix. These unit spaces are disjoint from each other, and this approach can be seen as an MNC problem\cite{4} of a multidimensional space. Based on this research, we transform the task of firewall rules compression to a multidimensional MOC problem. Given a rectilinear polygon $P$, the MOC problem requires determining the minimum number of axis-parallel rectangles whose union covers $P$. For the MOC problem on a two-dimensional space, researchers have proposed several distinguished schemes\cite{5,9–13}. However, only little has been done for cases with over two dimensions, and real-life firewalls are typically five-dimensional. In this paper, we propose a heuristic approach for the first time to cover the multidimensional rectilinear polygon with the minimum number of rectilinear hyper-rectangles.

3 Proposed Approach

In this section, we first define the relevant concepts of our approach, and then we present our FPC algorithm. Table 1 lists the notations used in this paper.

| Notation | Description |
|----------|-------------|
| $f$      | Firewall    |
| FDM      | Firewall design matrix model |
| MOC      | Minimal overlapping cover |
| MNC      | Minimal non-overlapping cover |
| $F_i$    | $i$-th dimension |
| $D(F_i)$ | Domain of $F_i$ |
| $M_k$    | A $k$-dimensional matrix |
| $u,v$    | Unit space |
| $R(u,v,F_i)$ | Spatial relation between $u$ and $v$ in $F_i$ |
| $c_i$    | Grid cell |
| REC      | Rectangle |
| $P$      | Polygon |
| $L_k$    | Linked list |
3.1 Preliminaries

A firewall rule generally has the form \( (\text{predicate}) \rightarrow (\text{decision}) \), and any (predicate) of the firewall rules has the form “\( F_i \in D(F_1) \land \cdots \land F_k \in D(F_k) \)”. A field \( F_i \) is a variable whose value is taken from a predefined interval of non-negative integers, called the \( F_i \) domain and denoted by \( D(F_i) \). For example, the domain of the source address in an IP packet is \([0, 2^{32} - 1]\).

**FDM.** An FDM over fields \((F_1, F_2, \ldots, F_k)\) is a \( k \)-dimensional matrix \( M_k \). Each dimensional coordinate in \( M_k \) is represented by \( F_i \), and the corresponding value is within a range of \([0, D(F_i)]\), \(1 \leq i \leq k\). Any of the firewall rules with the form \( (\text{predicate}) \rightarrow (\text{decision}) \) can be formally represented as an \( M_k \). In other words, the predicate of each rule can be expressed as a multidimensional rectangular region in \( M_k \), and the region value reflects the rule’s decision: value \( = 0 \) (if \( \text{decision} = \text{discard} \)), or value \( = 1 \) (if \( \text{decision} = \text{accept} \)).

**Unit space.** An \( M_k \) may have a set of multidimensional rectangular regions with a value of “1”, each of which can be represented by its low vertex coordinates and the corresponding distance, denoted as \([l_1, l_2, \ldots, l_k](d_1, d_2, \ldots, d_k]\), where \( l_i \) is the vertex coordinate representing the lower bound of \( D(F_i) \), and \( d_i \) is the distance representing the size of \( D(F_i) \). These regions are referred as the unit spaces of \( M_k \).

The procedure of mapping a firewall rule \( r_i \) to a \( k \)-dimensional matrix \( M_k \) is presented in Ref. [4]. Figures 3a and 3b illustrate this process. Assuming there are four firewall rules \( r_1: F_1 \in [0, 2] \Rightarrow \text{discard}, \ r_2: F_1 \in [3, 5] \land F_2 \in [0, 1] \Rightarrow \text{accept}, \ r_3: F_1 \in [5, 5] \land F_2 \in [3, 5] \Rightarrow \text{discard}, \ r_4: F_1 \in [0, 5] \land F_2 \in [1, 5] \Rightarrow \text{accept} \), and we map rule \( r_i \) to \( M_k \) in reverse order, while assigning the value of mapping region as “0” or “1” based on if \( \text{decision} = \text{discard} \) or \( \text{accept} \). Figure 3a shows the FDM to which rule \( r_4 \) is mapped. There is only one unit space in this figure, denoted as \([0, 1), (6, 5)]. \) Then after \( r_3, r_2 \) and \( r_1 \) are mapped, the resulting FDM is shown in Fig. 3b, and the final three unit spaces are \([0, 2)(5, 4)], [(3, 0)(3, 2)], \) and \([5, 2)(1, 1)]\), corresponding to \( u_1, u_2, \) and \( u_3 \), respectively.

For ease of representation, we define the spatial relation between two unit spaces in a certain dimension. Given any two unit spaces \( u \) and \( v \), the **spatial relation** between them in a certain dimension \( F_i \) can be represented as \( R(u, v, F_i) \), where the value is an element of \( \text{adjacent}, \text{disjunct}, \text{crossed}, \text{covered}, \text{included}, \) and \( \text{equivalent} \), as shown in Fig. 4.

Let \( u = ([t_1^{(u)}, \ldots, t_k^{(u)}], [d_1^{(u)}, \ldots, d_k^{(u)}]) \), \( v = ([t_1^{(v)}, \ldots, t_k^{(v)}], [d_1^{(v)}, \ldots, d_k^{(v)}]) \),

1. If \( t_i^{(u)} + d_i^{(u)} < t_i^{(v)} \), \( R(u, v, F_i) = \text{disjunct} \);
2. If \( t_i^{(u)} + d_i^{(u)} = t_i^{(v)} \), \( R(u, v, F_i) = \text{adjacent} \);
3. If \( t_i^{(u)} < t_i^{(v)} < t_i^{(u)} + d_i^{(u)} \), \( R(u, v, F_i) = \text{crossed} \);
4. If \( t_i^{(v)} < t_i^{(u)} < t_i^{(u)} + d_i^{(u)} \), \( R(u, v, F_i) = \text{included} \);
5. If \( t_i^{(u)} < t_i^{(v)} < t_i^{(u)} + d_i^{(u)} < t_i^{(v)} \), \( R(u, v, F_i) = \text{covered} \);
6. If \( t_i^{(u)} = t_i^{(v)} \), \( R(u, v, F_i) = \text{equivalent} \).

In general, given any two unit spaces \( u \) and \( v \), if they are disjunct in a certain dimension or are adjacent in two or more dimensions, they are independent of each other. Accordingly, any unit space in an independent unit space set is independent of that in another independent unit space sets.

**Grid cell.** In a certain dimension \( F_i \), if two unit spaces \( u \) and \( v \) satisfy \( R(u, v, F_i) = \text{equivalent} \), then we cut \( v \) into two or three sub-unit spaces, in which there exists a sub-unit space \( v' \) that satisfies \( R(u, v', F_i) = \text{equivalent} \). A cutting operation is iteratively conducted on all the unit spaces, until there are only disjunct or adjacent spatial relations between any two sub-unit spaces in all the \( k \) dimensions. We call these sub-unit...
spaces grid cells.

For any grid cell, the following two properties are satisfied:

**Property 1.** Given any two grid cells in a \( k \)-dimensional space, if they are adjacent in a certain dimension, then their coordinate values are identical in the rest \( k - 1 \) dimensions.

**Proof:** Proof by contradiction. Given two grid cells \( c_1 \) and \( c_2 \) in a \( k \)-dimensional space, they are adjacent in the first dimension. Suppose their coordinate values are not completely identical in the rest \( k - 1 \) dimensions; for example, the coordinate value of \( c_1 \) is less than that of \( c_2 \) in dimension \( F_i \) \((1 \leq i \leq k)\). According to the definition of grid cell, we will let the coordinate value of \( c_1 \) be the secant to partition \( c_2 \) in dimension \( F_i \). Then, at this time, \( c_2 \) is equal to \( c_1 \) in dimension \( F_i \) after the partition. This contradicts the assumption that the coordinate value of \( c_1 \) is less than that of \( c_2 \) in dimension \( F_i \). Therefore, given any two grid cells in \( k \)-dimensional space, if they are adjacent in a certain dimension, then their coordinate values are identical in the rest \( k - 1 \) dimensions.

**Property 2.** Given any grid cell \( c_j \) in a \( k \)-dimensional space, if it has an adjacent grid cell \( c_j \) in a certain dimension, then \( c_j \) can be expanded to \( c_j \) to form a hyper-rectangle.

**Proof:** Given two grid cells \( c_1 \) and \( c_2 \) in a \( k \)-dimensional space, if they are adjacent in the dimension \( F_i \), then their coordinate values are identical in the rest \( k - 1 \) dimensions based on Property 1. Since the upper coordinate value of \( c_1 \) is equal to the lower coordinate value of \( c_2 \) in dimension \( F_i \), these two grid cells can be combined in dimension \( F_i \) to form a hyper-rectangle.

For example, there are seven grid cells from \( c_1 \) to \( c_7 \) in Fig. 3c, which are partitioned from the unit spaces \( u_1 \), \( u_2 \), and \( u_3 \) (Fig. 3b). The dimension of these grid cells is equivalent to that of the unit spaces, and each grid cell is either disjunct or adjacent to each other. Taking the grid cell \( c_1 \) as an example, \( R(c_1, c_2, F_2) = \text{adjacent} \) and \( R(c_1, c_2, F_3) = \text{equivalent} \); therefore, these two grid cells can be combined to form a rectangle \( c_1 - c_2 \).

### 3.2 Methodologies

In this section, we formally describe our FPC method.

#### 3.2.1 Problem description

First, a geometric model is first constructed to transform the FPC problem into an MOC problem in a multidimensional space.

Specifically, using the FDM method, we map the firewall rules into a multidimensional matrix to form a set of unit spaces, which constitute several multidimensional rectilinear polygons. In the multidimensional space, any firewall rule can be described by a multidimensional rectangle. Therefore,
the firewall rules compression problem can be transformed to use the minimum number of hyperrectangles to cover these rectilinear polygons (MOC).

As mentioned in Section 1, the MOC problem is NP-hard in two or more dimensions[5], and to the best of our knowledge, there are no heuristic algorithms on MOC problems in a multidimensional case.

3.2.2 FPC heuristic algorithm

Considering that the solution of MOC problem in multidimensional space is NP-hard, we propose a heuristic method — FPC. The main idea of this method is as follows:

(1) Use the FDM algorithm to map the firewall rules into one or more multidimensional rectilinear polygons.

(2) Divide each multidimensional polygon into a set of grid cells, so that these grid cells can be merged more conveniently in the heuristic algorithm.

(3) Propose a heuristic algorithm based on a “greedy” strategy. The core idea of the algorithm is to define a multidimensional rectangle which covers as many grid cells as possible in each step.

Step 1. Mark all grid cells white, and determine a certain dimension $F_i$ of the multidimensional polygon.

Step 2. Choose a grid cell $c$ with the minimum coordinate value in $F_i$. When there is a grid cell with the right adjacent spatial relation with $c$, then merge as many grid cells as possible in this dimension to form a RECtangle (REC). Record REC (which corresponds to a rule) and mark it gray. Then the convex part of the gray area in the polygon is determined to be marked as black.

Step 3. Remove the black area from the multidimensional polygon, and then continue with Step 2 in the remaining multidimensional polygons. When all the polygons are marked in gray, output the rectangles and corresponding rules.

The heuristic algorithm FPC possesses the following two properties:

Property 3. In Step 2, REC always covers the region starting from the convex, and the region contains the most grid cells.

Property 4. The previous REC coverage process does not result in a situation whereby the region in the remaining multidimensional polygons, which can be covered by a REC, has to be covered by multiple RECs.

Property 3 is obvious. We prove Property 4 as follows:

Proof: Suppose $k$-dimensional rectangles are needed to cover the original multidimensional polygon $P$. For example, $P$ comprises five grid cells: $c_1$, $c_2$, $c_3$, $c_4$, and $c_5$, as shown in Fig. 5. According to the algorithm implementation process, each time the REC covers, the area that contains the initial grid cell and covers the most grid cells is selected. Then, in the multidimensional rectangles $c_1$, $c_3$, and $c_5$ which are formed in the REC coverage, we remove the convex region ($c_1$ and $c_5$, which are represented by the gradient-filled area in Fig. 5). At this point, the problem is equivalent to proving that after the convex region is removed, the remaining polygon $P'$ can be covered by a maximum of $k - 1$ multidimensional rectangles. In a multi-dimensional polygon $P$, at least one multidimensional rectangle is needed to cover the convex region. Moreover, based on the property of the MNC problem, the gray region in Fig. 5 allows repeated coverage; therefore, the rest regions of a multidimensional polygon can be covered by up to $k - 1$ multidimensional rectangles.

According to the properties of FPC algorithm, in the algorithm implementation process, the REC always covers the region that contains the most grid cells. The algorithm embodies the greedy idea and achieves the local optimal coverage.

3.2.3 Steps and analyses of FPC algorithm

Our algorithm comprises the following four steps:

(1) Map the firewall rules $f$ to an FDM;
(2) Divide the rectilinear polygon into grid cells;
(3) Perform FPC on the multidimensional polygon;
(4) Generate the resulting firewall rules $f_0$ from the derived rectangles.

Step 1: Map the firewall rules $f$ to an FDM. Given a firewall $f$, we map the rules into a multidimensional matrix to form a set of unit spaces. The procedure of mapping a firewall rule $r_i$ to a $k$-dimensional matrix $M_k$ is presented in Ref. [2]. Then, we derive the independent unit space sets, each of which can be used to constitute a multidimensional rectilinear

![Fig. 5 An example of rectangle covering process.](image)
complexity is \( O(kn) \), where \( k \) is the dimension.

Let us consider an intuitive example to show the process of this approach, supposing there are five original user specified rules.

Given the firewall \( f \) shown in Fig. 6 as input, we can obtain three unit spaces \([(0, 4)(3, 4)], [(2, 2)(1, 2)], \) and \([(3, 2)(1, 3)]\) after the mapping procedure, which can constitute a rectilinear polygon, as shown in Fig. 7a.

Step 2: Divide the rectilinear polygon into grid cells.

In this step, the multidimensional rectilinear polygon (which consists of unit spaces) is divided into grid cells. For any two unit spaces \( u \) and \( v \), if they satisfy these two conditions: (1) in dimension \( F_x \), \( R(u, v, F_x) = \text{adjacent} \), and (2) in any other dimension \( F_y \), \( R(u, v, F_y) = \text{included} \) or \( R(u, v, F_y) = \text{crossed} \), we let the coordinate value of \( u \) in dimension \( F_y \) to be the secant, then cut \( u \) into two or three sub-unit spaces. This operation on unit spaces is iteratively conducted in a certain dimension until all the \( k \) dimensions are completed.

In the worst case, \( n \) unit spaces need to be cut for \( (k - 1)n(n - 1) \) times in a \( k \)-dimensional space. The time complexity is \( O(kn(n - 1)) \), and the number of grid cells is \( n + (k - 1)n(n - 1) \). Here \( n \) is the number of unit spaces. However, when all the unit spaces are independent of each other in \( k \) dimensions, the number of unit spaces is the same as the number of grid cells.

Figure 7 illustrates an example of this step. Given unit spaces \( u_1, u_2, \) and \( u_3 \) as the input, since \( R(u_3,\ u_1,\ F_1) = \text{adjacent} \), and \( R(u_3,\ u_1,\ F_2) = \text{included} \), we use the upper coordinate of \( u_3 \) in dimension \( F_2 \) as the secant to cut \( u_1 \) into two sub-unit spaces \( u_{11} \) and \( u_{12} \), which is represented by the gradient-filled area in Fig. 7b. Then this cutting operation is iteratively conducted on all the grid cells, and finally, we obtain seven grid cells from \( c_1 \) to \( c_7 \) (as shown in Fig. 7c). Here all the grid cells have the same degree of dimensions.

Step 3: Perform FPC on the multidimensional polygon.

(1) To conveniently cover the grid cells, we designed a directed graph structure on the spatial relations among these grid cells. To describe the spatial relations, we consider each grid cell as a vertex in the digraph. If two grid cells are adjacent in a certain dimension, we use a directed line segment which connects these two vertices to describe their adjacency relationship.

Taking the grid cells in Fig. 7c as an example, \( R(c_1, c_4, F_1) = \text{adjacent} \), \( R(c_4, c_7,F_1) = \text{adjacent} \), and \( R(c_1, c_2, F_2) = \text{adjacent} \), their spatial relations can be represented as the digraph as shown in Fig. 8. Here, Dim \( F_1 \) and Dim \( F_2 \) are the abbreviations of dimensions \( F_1 \) and \( F_2 \), respectively. For a \( k \)-dimensional case, we can
construct \(k\) digraphs, and in each digraph, each vertex only appears only once.

(2) Based on this digraph, we first choose an initial grid cell. Suppose there are \(k\) dimensions from \(F_1\) to \(F_k\), we list all the grid cells which has the minimum coordinate value in the first dimension \(F_1\). Then, we search these grid cells for the one that has the minimum coordinate value in the second dimension \(F_2\). The search operation is iteratively conducted in all the \(k\) dimensions until the initial grid cell \(c\) is found. Let \(N\) be the number of grid cells; then, the time complexity of choosing the initial grid cell is \(O(N \log N)\).

For simplicity, we illustrate this method using a two-dimensional case. As shown in Fig. 8a, \(c_1\) and \(c_2\) have the smallest coordinate value in \(F_1\) among all the grid cells. In addition, the coordinate value of \(c_1\) is smaller than that of \(c_2\) in \(F_2\), as shown in Fig. 8a. Therefore, \(c_1\) is the initial grid cell.

(3) First, we search for the linked list \(l_k\) whose first vertex is \(c\) in the \(k\)-th digraph; suppose there are \(l_k\) vertex \((c_1 - c_{l_k})\) in the list, then in the \((k - 1)\)-th digraph, we traverse all the \(l_k\) linked lists whose starting points are from \(c_1\) to \(c_{l_k}\). Let \(l_k\) be the length, and let the shortest chain length of the traversed \(l_k\) linked lists be the width of the rectangle, we can obtain a two-dimensional rectangle. The operation is iteratively conducted from the \(k\)-th digraph to the first digraph until a \(k\)-dimensional hyper-rectangle is obtained. Then, we remove all the convex vertexes \(c_v\) from the \(k\) digraphs. In a certain dimension \(F_i\), we can identify that the grid cells constitute convex vertexes in the \(k\)-dimensional space, and remove them from the \(k\) digraphs if they satisfy the following two conditions: (1) Each grid cell is the starting or the ending vertex of the linked list in this dimension \(F_i\); (2) One or more \((k - 1)\)-dimensional convex polygons in the other dimensions are exclusively composed of these grid cells. The above operation is iteratively conducted until all the vertices are removed.

Let \(N\) be the number of grid cells, to compute the time complexity. We first give the following observations:

(1) In the \(k\) dimensions, all grid cells can form \(k\) digraphs, and in each digraph, each vertex appears only once.

(2) In each round of covering process, we start from the initial grid cell. Then, we separately perform the traversal operation on the \(k\) dimensions to find the largest multidimensional rectangle. Because each grid cell is covered at most once in each round, the traversal operation does not exceed the number of grid cells that needs to be covered in this round.

(3) In each round of covering process, we remove the convex vertices after the multidimensional rectangles are formed. Because in each round, more than one convex vertex will be removed. The number of grid cells required to be covered in the \(t\) round is not more than \(N - t + 1\), where \(1 \leq t \leq N\).

According to the above consideration, the time complexity of grid cells covering in the worst case is \(\sum_{t=1}^{N} (N - t + 1)\), that is \(O(N^2)\), while in the best case, the grid cells are independent of each other, and the time complexity is \(O(N)\).

Taking Fig. 8 as an example, since \(c_1\) is the initial grid cell, we first search with the initial grid cell \(c_1\) for the linked list and find \(c_1 - c_2\) (Fig. 8b). Then, we use \(c_1 - c_2\) as the edge of rectangle and extend the edge to the maximum in \(F_1\); then, we can obtain the rectangle \(c_1 - c_2 - c_4 - c_5\), as shown in Fig. 8a. Since \(c_1\) and \(c_2\) are the starting points of simultaneously linked lists in \(F_1\), and they constitute a linked list which does not contain any other vertex in \(F_2\), it means \(c_1\) and \(c_2\) are convex grid cells in \(F_1\). Similarly, \(c_5\) is a convex grid cell in \(F_2\). After all the convex grid cells have been removed, we find the initial grid cell \(c_3\) among the rest grid cells. The covering operation is recursively conducted until we get the second rectangle \(c_3 - c_4 - c_6 - c_7\). At this time, these four grid cells \(c_3, c_4, c_6,\) and \(c_7\) are all convex. When we removed them, no any grid cell is left, and the algorithm ends. According to the above implementation process, we can finally obtain two rectangles \(c_1 - c_2 - c_4 - c_5\) and \(c_3 - c_4 - c_6 - c_7\) after two rounds of covering operation.

Figure 9 shows the corresponding rectilinear polygon covering process. Given the polygon \(P\) shown in Fig. 7c as input, it has seven grid cells from \(c_1\) to \(c_7\). In the first round of covering, \(c_1\) is the initial grid cell, and the maximal rectangle \(c_1 - c_2 - c_4 - c_5\) is added to the cover \(T\). Here, the gradient-filled area represents the already covered area, as shown in Fig. 9a. Next, we remove the convex grid cells \(c_1, c_2,\) and \(c_5\), which are represented by the shaded area shown in Fig. 9b. Now the contracted polygon in Fig. 9b has the initial grid cell \(c_3\), and the rectangle \(c_3 - c_4 - c_6 - c_7\) is added to \(T\) in this iteration, as shown in Fig. 9c. Finally, following the contraction, all the grid cells are removed and the algorithm terminates normally, as shown in Fig. 9d.

Step 4: Generate the resulting firewall rules \(f'\) from the derived rectangles.
In this step, the resulting firewall rules are generated from the derived rectangles. We first calculate the unit spaces which are formed by those rectangles. Then, we sort them in a descending order according to their regional sizes. The corresponding ordered firewall rules are generated from these ordered unit spaces. Finally, since all the unmarked regions denote the discard condition, a default firewall rule “\( F_1 \in D(F_1) \land F_2 \in D(F_2) \land \cdots \land F_k \in D(F_k) \rightarrow \text{discard} \)” is fixed for the last rule to ensure the completeness of the firewall rules.

For \( \text{REC}_1 = \{c_1, c_2, c_4, c_5\} \) and \( \text{REC}_2 = \{c_3, c_4, c_6, c_7\} \), we can obtain the corresponding unit spaces: \( \{(0, 4)(3, 4), (2, 2)(3, 3)\} \). Based on our rules-generating algorithm, the resulting firewall rules are \( r_1: F_1 \in [0, 2] \land F_2 \in [4, 7] \rightarrow \text{accept} \), \( r_2: F_1 \in [2, 4] \land F_2 \in [2, 4] \rightarrow \text{accept} \), and the default rule \( r_3: F_1 \in [0, 5] \land F_2 \in [0, 5] \rightarrow \text{discard} \).

In this step, we calculate the regional sizes of those hyper-rectangles, and then sort them in a descending order. Let \( N \) be the number of grid cells; then, in the worst case, the number of hyper-rectangles is \( N \), and the time complexity is \( O((k + \log N)N) \).

Through comprehensive analysis and evaluation, we can derive that the time complexity of our algorithm as \( O(T_{\text{best}}) = O(ckn + kn(n - 1) + kn^2 + (k + \log n)n) = O(kn^2) \) in the best case and \( O(T_{\text{worst}}) = O(ckn + kn(n - 1) + kn^2 + (k + 2\log n)n^2) = O(kn^4) \) in the worst case.

4 Experimental Results

In this section, we evaluate the effectiveness and efficiency of the FPC algorithm.

4.1 Effectiveness

Theorem 1: The proposed FPC heuristics approach can certainly reduce the number of rules with a higher degree compared to the FDM method.

Proof: As defined in Ref. [2], the FDM method can be seen as the MNC problem of a multidimensional rectilinear polygon, while the FPC approach can be regarded as an MOC problem in a multidimensional space. For FDM, the resulting number of compressed rules is \( |MOC| \), while the resluting number of rules in FPC is \( |MOC| \). As proved in Ref. [4], \( |MOC| \leq 2|MOC| - 1 \). This result demonstrated that by using an MOC versus an MNC, we can reduce the number of rectangles in the cover by a factor of at most two.

1) Compression ratios. To evaluate the effectiveness of the firewall rules compression algorithm, we first define the compression ratio metric. Given a firewall \( f \), we use FPC to represent our proposed FPC approach. Let \( \text{FPC}(f) \) be the firewall produced by applying algorithm FPC, and \( |f| \) be the number of rules in firewall \( f \). The compression ratio of FPC over \( f \) is \( \frac{|\text{FPC}(f)|}{|f|} \).

2) Experimental data. Firewall rules are considered confidential because of various security concerns, thus, it is not easy to get many real-life firewall rules for evaluation. To circumvent this issue and effectively evaluate the performance of the proposed approach, we generated a set of 50 synthetic firewall rules, comprising 20 to 5000 rules. We divided this set into three smaller sets based on the number of rules. The small set contains the 20 smallest classifiers, and the middle set contains the next 15 larger classifiers, while, the classifiers in the large set all have at least 500 rules, with the largest having 5000 rules. The predicate of each rule has five fields: source IP, destination IP, source port, destination port, and protocol type. We generated these rules by ClassBench\textsuperscript{[15]}, which is a well-known benchmark that provides classifiers similar
to real classifiers used in Internet routers and inputs traces corresponding to the classifiers.

(3) Methodology. In our experiments, the rules set was divided into five groups based on sizes, and each group contained eight firewall rules with different rule numbers. We addressed each rule in the five groups by using the FDM, Diplomat, and FPC schemes. The average value was obtained and considered as a performance metric.

As shown in Fig. 10, the compression ratios of FDM, Diplomat, and FPC are all slightly around the average values, which indicates that the compression results are fairly stable. Note that the time complexity of Firewall Decision Diagram (FDD) construction is $O(n^k)$ \cite{16}. For the typical five-dimensional firewall rules, $k$ is 5. As a result, the time complexity of Diplomat, which includes the steps of FDD construction, difference resolution, etc., is not less than $O(n^k)$. When the number of rules were more than 1000, the algorithm execution time was over 200 h. Therefore, in this study, no more than 1000 rules were tested for Diplomat.

The mean compression ratios for FDM, FPC, and Diplomat on each set of classifiers (Table 2) show that FDM offers little improvement over Diplomat, and in many cases they provide equally good solutions. However, FPC outperformed them on most of the 50 classifiers. Specifically, on the small and medium sets, FPC resulted in improvements of 8.06% and 6.17% over Diplomat. On the large set, FPC moderately outperformed FDM and Diplomat.

In general, if intersection conditions exist as shown in Fig. 11, the compression ratio of FPC would be better than those of FDM and Diplomat, and the resulting compressed rules would be conflict-free.

### 4.2 Efficiency

The proposed algorithms were implemented in Java JDK1.6, and we carried out experiments on a server running Linux with Intel six CPU 2.0 GHz. The running

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### Table 2 Compression ratios of firewall rules.

| Rules set         | FDM (%) | Diplomat (%) | FPC (%) |
|-------------------|---------|--------------|---------|
| Small (20–80)     | 51.13   | 54.94        | 46.88   |
| Medium (100–400)  | 53.83   | 55.5         | 49.33   |
| Large (500–5000)  | 63.47   | 63.0         | 61.15   |
| Mean              | 56.14   | 57.81        | 52.45   |

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![Fig. 10 Performance comparison of FDM, Diplomat, and FPC.](image-url)
times of FPC and Diplomat (Fig. 12) show that FPC executes more efficiently than Diplomat. Its execution time is within a second on small rules sets, which contain dozens of rules, and up to a few hours for some large sets containing thousands of rules. As stated above, the time complexity of FPC in the worst case is $O(n^4)$. In fact, the execution time is close to the best case of $O(n^2)$; this is because most of the unit spaces maintain disjunct spatial relations in the multidimensional space. For Diplomat, the time complexity is $O(n^5)$; when the number of rules increases, its execution time increases dramatically.

5 Conclusion

In this paper, we propose a novel scheme FPC to compress firewall rules into conflict-free rules semantically identical to the original rules. First, a geometric model is constructed to transform the firewall rules compression problem into an MOC problem in a multidimensional space, and then, the multidimensional MOC problem is solved using the proposed FPC scheme. We conducted extensive experiments to evaluate the performance of the proposed method. The experimental results show that FPC outperforms FDM and Diplomat in terms of compression ratios. Most of the time, FPC resulted in a high compression ratio while maintaining conflict-free firewall rules. In our future work, we will develop software-defined networking applications\textsuperscript{[17]} that utilize our compression method, and evaluate the feasibility of our method with corresponding experiments. In addition, we will apply powerful techniques studied in computer science, such as parameterized method\textsuperscript{[18–21]}, to our method.

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include network security, privacy protection, and related areas.

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