Origin of the Scaling Law in Human Mobility: Hierarchical Organization of Traffic Systems

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Uncovering the mechanism leading to the scaling law in human trajectories is of fundamental importance in understanding many spatiotemporal phenomena. We propose a hierarchical geographical model to mimic the real traffic system, upon which a random walker will generate a power-law travel displacement distribution with exponent -2. When considering the inhomogeneities of cities’ locations and attractions, this model reproduces a power-law displacement distribution with an exponential cutoff, as well as a scaling behavior in the probability density of having traveled a certain distance at a certain time. Our results agree very well with the empirical observations reported in [D. Brockmann et al., Nature 439, 462 (2006)].

Studies on the non-Poisson statistics of human behaviors have recently attracted much attention \cite{1,2,3,4}. Besides the inter-event or waiting time distribution, the spatial movements of human also exhibit non-Poisson statistics. Brockmann et al. \cite{4} investigated the bank note dispersal, as a proxy for human movements, and revealed indirectly a power-law distribution of human travel displacements. Gonzalez et al. \cite{5} studied the human travel patterns by measuring the distance of mobile phone users’ movements in different stations, and observed a similar scaling law. Actually, the mobility patterns of many animals also show power-law-like displacement distributions \cite{6,7,8}. The ubiquity of such kind of distributions attracts scientists to dig into the underlying mechanism. Some interpretations, such as optimal search strategy \cite{9,10}, olfactory-driven foraging \cite{11} and deterministic walks \cite{12}, have already been raised for the power-law displacement distribution in animals mobility patterns, however, they are based on the prey processes and thus cannot be used to explain the observed scaling law in human trajectories, which is still an open problem.

In this paper, we propose a model to mimic the human travel pattern, where the hierarchical organization of the real human traffic systems is taken into account. Our model can reproduce the power-law displacement distributions, as well as the scaling behavior in probability density of having traveled a certain distance at a certain time, agreeing very well with the empirical results reported in Ref. \cite{4}.

Let’s think about the real human traffic systems. Generally speaking, a district (e.g., a province or a state) usually has a core city, like its capital; around this core city, there are several big cities as the secondary centers (e.g., municipalities); then, each of these centers is rounded by some counties; and towns and villages will surround each of the counties. A hierarchical traffic system is built accordingly. Imaging people traveling from a town, \(a\), subordinating to the central city, \(A\), to another town \(b\) that is subordinated to the central city \(B\). There is usually no direct way connecting \(a\) and \(b\), and the typical route is \(a \rightarrow A \rightarrow B \rightarrow b\). This kind of hierarchical organization is not just inside a country or a district, but across the whole world. For example, if one wants to travel from the University of Science and Technology of China to the University of Fribourg, there is no direct way connecting Hefei and Fribourg, instead, one has to follow the route Hefei→Shanghai→Zürich→Fribourg although it is much longer than the geographical distance between Hefei and Fribourg. Such a hierarchical organization as well as the resulting scale invariance in road networks have already been demonstrated recently \cite{13}.

For simplicity, we call all the units cities. In our model, cities are organized in \(N\) layers. A uniform 3-layer system is shown in Fig. 1, in which, 81 cities locate on the centers of a \(9 \times 9\) lattice. The most central city is put in the first layer, and the whole region is divided into 9 sub-regions, each contains a \(3 \times 3\) lattice. Except the middle sub-region, all other eight sub-regions have their own central cities, namely the second layer cities, which locate at the centers of those sub-regions. Meanwhile, there are eight third layer cities around each of the second layer cities as well as the first layer city. An illustration is shown in Fig. 1. Denote \(N\) the number of layers and \(K\) the number of first layer cities. We assign \(M\) sub-regions to each of the \(K\) 1st-layer cities, with the 1st-layer city locating in the center, and \((M - 1)\) 2nd-layer cities are respectively put in the remain \((M - 1)\) sub-regions. Each of the \(KM\) sub-regions is further divided into \(M\) sub-sub-regions, with the 1st- or 2nd-layer cities locating in the center and \((M - 1)\) newly generated 3rd-layer cities put in the remain \((M - 1)\) sub-sub-regions. Repeating this process until the \(N\)th-layer cities are generated. For \(2 \leq n \leq N\), there are \(KM^{n-2}(M-1)\) \(n\)-th-layer cities. Note that, in this model, to make sure the lattice is fulfilled with cities, \(M\) must be equal to \((2q - 1)^2\) where \(q\) is a certain integer larger than 1.
The $K$ first layer cities are fully connected with each other, each of which is connected with the nearest $(M-1)$ $N$th-layer cities and the nearest $(M-1)$ second layer cities. Each of the second layer cities is connected with the nearest $(M-1)$ $N$th-layer cities, the nearest $(M-1)$ third layer cities, as well as the other $(M-2)$ second layer cities belonging to the same first layer city. Actually, for $1 \leq n < N$, each of the $n$th-layer cities is connected with the nearest $(M-1)$ $N$th-layer cities, the nearest $(M-1)$ $(n+1)$th-layer cities, as well as the other $(M-2)$ $n$th-layer cities belong to the same $(n-1)$th-layer city. Note that, all the connections are symmetry, and the direct move between two cities is allowed only if they are connected. Figure 1 illustrates an example with $N = 3$, $K = 1$ and $M = 9$. The modeled system can be viewed as a hierarchical network. Different from the real hierarchical networks [14] or the mathematical models [13, 10], it is hierarchical but not scale-free.

We consider the simplest case where a random walker is consequently moving from one city to a random neighboring city (two cities are said to be neighboring if they are connected). Figure 1(d) shows a typical trajectory of a walker moving from a lower-layer city to a higher-layer city in other sub-region. The displacement of the walker in one step is defined as the geometric distance $L$, and the distribution of $L$ is what we mainly concern in this paper. As shown in Fig. 2, burstiness of long-range travels is clearly observed and the distribution of travel displacement, $P(L)$, approximately obeys a power-law form with exponent $-2$.

The essential physics of this model is a random-walk process in a geographical network where edges are of different geometric distances. For a random walker in a connected symmetry (undirected) network, in the long time limit, each edge has the same chance to be visited (this proposition is hold even for a very heterogenous network, since for an arbitrary node, the number of times being visited is proportional to its degree while the probability that a specific adjacent edge of this node is consecutively visited is inversely proportional to the degree. Details can be found in Ref. [17]). Therefore, the displacement distribution of a random walker is equivalent to the distribution of edges’ geometric distances. Let $d_n$ ($n > 1$) denote the average geometric distance of edges connecting two $n$th-layer cities (they must belong to the same $(n-1)$th-layer city) and an $n$th-layer city and an $(n-1)$th-layer city (the former belongs to the latter), and $m_n$ denote the total number of edges contributed to $d_n$. Obviously, for $1 < n \leq N$, $d_n = d_N \sqrt{M^{N-n}}$. 

FIG. 1: (Color online) Illustration of the hierarchical structure and moving rules in the present model. (a)-(c) show the connections among the different-layer cities (black lines) and the same-layer cities (blue lines). Where the green, red, and black circles denote the cities in 3rd, 2nd and 1st layers, respectively. (d) gives a typical trajectory of a walker moving from a lower-layer city to a higher-layer city in other sub-region. A fractal network model with similar motivation can be found in Ref. [13].

FIG. 2: (Color online) Burstiness of long-range travels. Parameters are set as $N = 5$, $M = 9$, and $K = 9$. We choose $N = 5$ because it is typical in the real world, such as province-city-county-town-village in China and region-department-arrondissement-canton-commune in France. (a) The sequence of the displacement, $L$, of a random walker in $10^4$ consecutive steps. (b) The walker’s trajectory in 2000 consecutive steps. (c) The distribution $P(L)$, where the dash line, as a guide for eyes, is of slope -2. The data in panel (c) is obtained by averaging over 100 independent runs, and each lasts $10^5$ steps.
M, N and \( d_N \) can be considered as constants, and thus we have \( d_n \sim M^{-n/2} \). On the other hand, for \( n > 1 \), \( m_n = K C_3^2 M^{n-2} \) where \( C_3^2 = M/(M-1)/2 \). That is, \( m_n \sim M^n \). Roughly speaking, \( d_n \) and \( m_n \) play the roles of geometric distance and the number of edges associated with such a distance. As we have already obtained the scaling \( m_n \sim d_n^{-2} \), we can deductive that the displacement distribution for a system with sufficiently large \( N \) and in the long time limit obeys the scaling \( P(L) \sim L^{-2} \), which is in accordance with the simulation result shown in Fig. 2(c). The observed result implies that the scaling law in human trajectories may results from the inherent hierarchical organization in traffic systems.

Although the present model can reproduce the power-law displacement distribution, the absolute value of exponent is higher than the empirical ones [4, 5], and the model is obviously oversimplified. Firstly, real cities are not located in a completely uniform matter, but with some irregularity. Secondly, the model assumes that each city has the same attraction for the walker, however, in the real world, a central city is generally much more attractive than a small town. We next propose a modified model taking into consideration the inhomogeneous locations and attractions of cities. In this inhomogeneous model, all cities are randomly distributed in an \( S \times S \) square (we keep the number of cities in each layer the same as the original model), each of the \( n \)th-layer cities (\( 2 \leq n \leq N \)) is connected to the nearest higher-layer city, and two \( n \)th-layer cities are connected if they are connected to the same higher-layer city. All the \( K \) 1st-layer cities are fully connected to each other.

As mentioned above, the center city should have greater attraction, which is represented by a layer-dependent weight, \( w_n = r^{N-n} \), where \( n \) denotes the layer and \( r \geq 1 \) is a free parameter. The probability that the walker will move along an edge is proportional to its weight (A similar weighted random walk model has previously been proposed to explain the nonlinear dependence of the airport throughput on the connectivity [13]). Clearly, the larger \( r \) indicates higher heterogeneity. In Fig. 3, we show an illustration of a typical trajectory in a 3-layer inhomogeneous model, and in Fig. 4 we report the trajectories for \( r = 1 \) and \( r = 2 \) in a 5-layer inhomogeneous model. As shown in Fig. 5, for the inhomogeneous model, the displacement distribution, \( P(L) \), is still heavy-tailed and can be well fitted by a power-law function with an exponential cutoff, as \( P(L) = c(a + L)^{-\beta} e^{-L/\epsilon_c} \). In addition, as shown in the inset of Fig. 5, when \( r \) increases from 1.0 to 2.0,
the probability ing the trajectory of a random walker, one can obtain spatiotemporal statistics of real human mobility. Providing the power-law exponent, $\beta$, monotonically decreases from 2.70 to 1.51, covering the range of empirical observations $[4, 5]$. This result suggests that the inhomogeneity in cities’ locations enlarges the absolute value of such exponent $\beta$. The former enlarges the exponent $\beta$, while the latter depresses it (essentially, the inhomogeneity of attractions results from the inhomogeneous population density and economic development). Actually, as shown in Fig. 5, with tunable strength of inhomogeneity, the exponent $\beta$ is also tunable. When $r = 2$, meaning the topper-layer cities having greater attractions, the statistical features produced by our model are very close to the empirical ones reported in Ref. [4], not only the displacement distribution, but also the spatiotemporal statistics of mobility.

Finally, we check whether our model can reproduce the spatiotemporal statistics of real human mobility. Providing the trajectory of a random walker, one can obtain the probability $W(d, t)$ of having traveled a distance $d$ at time $t$ (the same technique has been adopted in preparing Fig. 2a in Ref. [4], please see details there). As shown in Fig. 6, a scaling behavior $r(t) \sim t^\alpha$ with $\alpha \approx 1.0$ is clearly observed, which agrees well with the empirical result, $\alpha \approx 0.95$, reported in Ref. [4]. Similar scaling behavior can also be observed for $r = 1.0$, however, the exponent, $\alpha \approx 0.5$, is far less than the empirical value. In addition, providing the travel displacement distribution, this scaling behavior with $\alpha$ around 1.0 can not be reproduced by a Lévy flight $[4]$.

Uncovering the human traveling pattern is of fundamental importance in the understanding of various spatiotemporal phenomena $[4, 5]$, and may find applications in the design of traffic systems $[19]$, the control of human infectious disease $[20]$, the military service planning $[21]$, and so on. Although empirical results about the scaling law of long-range human travels have been reported for years, it lacks the understanding of the underlying mechanism. This work gives raise to a very possible reason causing the heavy-tailed displacement distribution, $P(L)$, that is, the hierarchical organization of traffic systems. The secondary ingredient, also playing appreciable role in determining the traveling pattern, is the inhomogeneities of the locations and attractions of cities: The former enlarges the exponent $\beta$, while the latter depresses it (essentially, the inhomogeneity of attractions results from the inhomogeneous population density and economic development). Actually, as shown in Fig. 5, with tunable strength of inhomogeneity, the exponent $\beta$ is also tunable. When $r = 2$, meaning the topper-layer cities having greater attractions, the statistical features produced by our model are very close to the empirical ones reported in Ref. [4], not only the displacement distribution, but also the spatiotemporal statistics of mobility.

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