The wavelet transform has been used for numerous studies in astrophysics, including signal–noise periodicity and decomposition as well as the signature of differential rotation in stellar light curves. In the present work, we apply the Morlet wavelet with an adjustable parameter $a$, which can be fine-tuned to produce optimal resolutions of time and frequency, and the Haar wavelet for decomposition at levels of light curves. We use the WaveLab–package (library of Matlab routines for wavelet analysis) for the decomposition and a modified version of Colorado–package for the wavelet maps of synthetic and observed light curve. From different applications, including Virgo/SoHO, NSO/Kitt Peak, Voyager 1 and Sunspot data and synthetic light curve produced by different simulators, we show that this technique is a solid procedure to extract the stellar rotation period and possible variations due to active regions evolution. In this paper we show the Morlet Wavelet Amplitude Maps, respectively corresponding to oscillations in the photospheric magnetic field of the Sun (NSO/Kitt Peak data), the daily averages of the magnetic field strength $B$ versus time measured by Voyager 1 (V1) during 1978, and synthetic light curve produced by A. F. Lanza. We can also identify the noise level, as well as the contribution for the light curves produced by intensity, variability and mean lifetime of spots. Thus, we can identify clearly the temporal evolution of the rotation period in relation to other periodicity phenomena affecting stellar light curves. In this context, because the wavelet technique is a powerful tool to solve, in particular, not trivial cases of light curves, we are confident that such a procedure will play an important role on the CoRoT data analysis.

Keywords: stars: stellar rotation; computational astrophysics: wavelets transform; stars: CoRoT space telescope

1. Introduction

Signal processing plays a central role of a truly enormous range of the Astrophysical problems. These include, for example, rotational periodicity, behavior of stellar magnetic activity (flares, spots, faculae and plages), oscillations and noise. Much of traditional signal processing has relied upon a relatively small class of problems, for example, stationary and cyclostationary signal characterized by a form of translational invariance. It is not surprising that the Fourier Transform (FT) is considered...
the key tool in the analysis and manipulation of these problems. But, there are many signals whose defining characteristic is their invariance not to translation but rather to scale, i.e., the process exhibit a dependence on different time scales. In addition, while the FT plays a central role in the analysis and manipulation of both statistically and deterministically translation-invariant signal, the Wavelet Transform (WT) plays an analogous role for the kinds of scale-invariant signal.

Wavelet analysis is becoming a common tool for analyzing localized variations of power within a time series. By decomposing a time series into time–frequency plane, it is possible to determine both the dominant modes of variability and how those modes vary in time (see Torrence & Compo 1998 for further details)\(^1\). Contrary to classical Fourier analysis that decomposes a signal into different sines and cosines which are not bounded in time, the wavelet transform uses functions characterized by scale (period) and position in time. The wavelet transform has been used for numerous studies in astrophysics, including signal–noise periodicity and decomposition as well as the signature of differential rotation in stellar light curves. In the present work, we apply the Morlet wavelet with an adjustable parameter \(a\) (see Section 2), which can be fine-tuned to produce optimal resolutions of time and frequency, where for large values of \(a\) give better time resolution. The use of the Morlet wavelet allow the best trade–off between time and frequency resolution, as the gaussian function is its own Fourier transform (Oliver et al. 1998)\(^2\). In addition, we apply the Haar wavelet for decomposition at levels of light curves. We use the WaveLab–package (library of Matlab routines for wavelet analysis) for the decomposition and a modified version of Colorado–package for the wavelet maps of synthetic and observed light curve. We also apply the wavelet analysis of signals with gaps and of unevenly spaced data (Frick et al. 1998)\(^3\).

2. Procedures and data analysis

Many wavelet families can be proposed, depending on the nature of problem. The most commonly used in astrophysical applications is the Morlet Wavelet, that can be defined as being the generalization of the windowed Fourier Transform. The wavelet transform uses a window whose width is a function of the frequency. Several types of wavelets can be used. However, if the signal is sinusoidal, the wavelet should also be chosen to be sinusoidal. On the other hand, the Morlet wavelet (Grossmann & Morlet 1984)\(^4\) represents a sinusoidal oscillation contained within a Gaussian envelope. Then the wavelet transform can be written as

\[
W(t) = e^{-a[\nu(t-\tau)]^2}e^{-i2\pi\nu(t-\tau)},
\]

represents a sinusoidal oscillation contained within a Gaussian package. Then the wavelet transform is given by

\[
WT(\nu, \tau) = \sqrt{2\pi \nu} \int S(t)W^*[\nu(t - \tau)]dt,
\]
where $W^*$ is the complex conjugate of $W$ and $S(t)$ is the signal. Where an element of the discrete wavelet transform is

$$WT(\nu, \tau)_j = \sqrt{2\pi \nu} S_j (t_{j+1} - t_j) e^{-a[\nu(t_j - \tau)]^2} e^{-i2\pi \nu (t_j - \tau)}. \quad (3)$$

The $a$–parameter controls the resolution in both frequency and time (Baudin et al. 1994):

$$\Delta \tau = \frac{1}{\nu} \sqrt{\frac{\ln 2}{a}} \quad (4)$$

$$\Delta \nu = \frac{\nu}{\pi} \sqrt{a \ln 2} \quad (5)$$

and therefore, we have the following uncertainty relationship:

$$\Delta \tau \cdot \Delta \nu = \frac{\ln 2}{\pi}. \quad (6)$$

The relative frequency resolution is uniquely determined by $a$–parameter, whereas the time resolution depends on the frequency itself to hold the number of oscillations inside the wavelet constant. We choose $a = 0.005$ in our analysis in order to balance the time and frequency resolution.

Fig. 1. Morlet Wavelet Amplitude Map of the NSO/Kitt Peak data. The local wavelet power spectrum of the record of sunspot areas for periods for between about 22 days and 1.5 years. We also can to visual other periodocities, such as, $\sim 30$–days (differential rotational period), 158–days, 1–year (seasonal period) and 1.3 years ($\sim 474.5$ days) periodicities. The maximum variance corresponds to 1.3 years.

3. Results and Conclusions

Figures 1, 2 and 3 show the Morlet Wavelet Amplitude Maps, respectively corresponding to oscillations in the photospheric magnetic field of the Sun (NSO/Kitt...
Fig. 2. Morlet Wavelet Amplitude Map of the daily averages of the magnetic field strength $B$ versus time measured by Voyager 1 (V1) during 1978, corresponding from 2 to 3 AU. In this figure we can see that the average differential rotation is less than of the Fig. 1. This effect can be caused by dispersion of solar magnetic field on the distance.

Fig. 3. Morlet Wavelet Amplitude Map of the synthesized light curve in one CoRoT passbands for a main-sequence star with $T_{\text{eff}} = 6000$ K, log $g = 4.5$ (cgs units), a rotation period of 3 days and a facular behaviour of type F. The period of 1.5 days corresponds to a spot $180^\circ$ shifted longitude.

Peak data), the daily averages of the magnetic field strength $B$ versus time measured by Voyager 1 (V1) during 1978, and synthetic light curve produced by A. F. Lanza. The NSO/Kitt Peak data, as well as Voyager 1 and synthetic light curve data, are up to date the best proxies for the stellar light curves that will be obtained by CoRoT Space Mission.

Several time series will be used as example of wavelet analysis. These series include the NSO/Kitt Peak data used to measure the 1.3–year (rotation of the Sun near the base of its convection zone) and 158–day (high–energy solar flares related to
a periodic emergence of magnetic flux that appears near the maxima of some solar cycles) periodicities as well as rotational period of $\sim 30$ days\cite{hasler2002} (see Fig. 1). We have used a Wavelet analysis (Hempelmann & Donahue 1997; Hempelmann 2002; Lanza et al. 2003\cite{lanza2003}) to recover information on the solar rotation rate from its stellar-like light curve. We also apply the wavelet analysis in Sunspot data, reproducing results found in the literature (Oliver et al. 1998; Krivova & Solanki 2002\cite{krivova2002}). We also analyze synthetic light curve produced by a theoretical simulator, developed by the Group of Stellar Astronomy at Natal, obtaining well–defined periodicities compared with the real values.

We confirm the results obtained by analysis based on the Lomb-Scargle periodogram and by the Phase Dispersion Minimization method (PDM) for period search using the PERANSO program\cite{strassmeier2003}. But, in contrast with these methods, the wavelet procedure makes possible a global and local analysis of the periodicities\cite{polygiannakis2003}.

The period of rotational modulation changes during the solar cycle because the variability of: first, the latitude of the activity belts and the mean lifetime of the surface features. The rotational modulation signal can be masked by the active region evolution due to its variability.

The use of wavelet applied in a signal, demonstrated to be a powerful tool for analysis of signals with non–stationary features. On the other hand, we can through the wavelet method establish clearly the presence of a persistent signal. But, through the decomposition at level (or frequency)\cite{jpolygiannakis2003} we can obtain other periodicities that are not visible when we treat simultaneously all–frequencies.

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