Vortex thermal fluctuations in underdoped Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ crystals

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Vortex thermal fluctuations in heavily underdoped Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ ($T_c=69.4$ K) are studied using Josephson plasma resonance (JPR). From the zero-field data, we obtain the $c$-axis penetration depth $\lambda_{c,0}(0)=230\pm10$ $\mu$m and the anisotropy ratio $\gamma(T)$. The low plasma frequency allows us to study phase correlations over the whole vortex solid state, and to extract a wandering length $r_w$ of vortex pancakes. The temperature dependence of $r_w$ as well as its increase with dc magnetic field is explained by the renormalization of the vortex line tension by the fluctuations, suggesting that this softening is responsible for the dissociation of the vortices at the first order transition.

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Vortex thermal fluctuations are considered essential in determining the $(H,T)$ phase diagram of layered high temperature superconductors (HTS), and notably the first order transition (FOT) in which the ordered vortex crystal transforms to a liquid state without long range phase coherence [1, 2]. Many scenarios, varying from vortex lattice melting described by a Lindemann criterion [2] to layer decoupling [3, 4, 5], all considering various degrees of coupling between the superconducting layers, have successfully been used to describe the position of the FOT in the $(H,T)$-plane. However, such fits to the FOT line have not been able to convincingly discriminate between the different models. Here, we aim to do just this, through a direct measurement of the amplitude, as well as the field and temperature dependence of vortex thermal excursions in the vortex solid phase (or “Bragg–glass” [2]) that lead to the FOT.

For this study, we use the layered Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ (BSCCO) compound, in which vortex excursions can conveniently be measured by the Josephson Plasma Resonance (JPR) technique [8, 9, 10]. Briefly, the inter-layer Josephson current $J^{\omega}_{m}(B)$ can be measured through the JPR frequency $\omega_{pl} \sim J^{\omega}_{m}^{(1/2)}$, at which the equality of charging and kinetic energy leads to a collective excitation of Cooper pairs across the layers. In turn, $\omega_{pl}^2(B,T) = \omega_{pl}^2(0,T)(\cos(\phi_{n,n+1}))$ intimately depends on the gauge-invariant phase difference $\phi_{n,n+1}$ between adjacent layers $n$ and $n+1$ [11]. Here, $\langle \ldots \rangle$ stands for thermal and disorder averaging. Thus, JPR is a probe of the interlayer phase coherence. The fluctuations of vortex lines created by a dc magnetic field applied perpendicularly to the layers modify the relative phase difference between adjacent layers and thus depress $\omega_{pl}$. In Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$, the ensemble of vortex lines should be described as stacks of two-dimensional pancake vortices. Thermal fluctuations shift the individual vortex pancakes with respect to their nearest neighbors in the $c$ direction, by a distance $r_{n,n+1} = u_{n+1} - u_n$. Here $u_n$ is the $ab$-plane displacement of the pancake vortex in layer $n$ with respect to the equilibrium position of the stack it belongs to (Fig. 1). The wandering length of vortex lines, which is related directly to the JPR frequency $\omega_{pl}^2$, can be then defined as $r_w = (r_{n,n+1}^2)^{1/2}$. Below, we shall only consider temperatures above $T = 42$ K, at which vortex pinning (quenched disorder) is unimportant [2, 12].

Underdoped BSCCO ($T_c = 69.4$ K) single crystals were grown by the traveling solvent floating zone method in 25 mbar O$_2$ partial pressure at the FOM-ALMOS center, the Netherlands [15]. The samples were post-annealed for one week at $700^\circ$C in flowing N$_2$. The advantage of using heavily underdoped BSCCO is that $\omega_{pl}(0,0) \approx 61$ GHz turns out to be very low, which allows us to measure the vortex meandering over the entire vortex phase diagram. Samples A and B (cut from the same crystal) have dimensions $1.35 \times 1 \times 0.04$ mm$^3$ and $0.7 \times 0.47 \times 0.04$ mm$^3$, respectively. Another sample from the same batch was used to determine the temperature of the FOT (Fig. 5b, inset), by measuring the paramagnetic peak at the FOT with a miniature Hall probe magnetometer [16].

The JPR measurements were carried out using the cavity perturbation technique in the Laboratoire des Solides Irradiés (on sample A) and the bolometric method in...
the Institute for Solid State Physics at the University of Tokyo (on samples A and B). For the cavity perturbation technique, the sample was glued in the center of the top cover of a cylindrical Cu cavity used in the different TM_{01i} (i = 0, . . . , 4) modes. These provide the correct configuration of the microwave field at the sample location, in which $E_{rf} \parallel c$-axis and $H_{c, f} \approx 0$ [13]. The unloaded quality factor $Q_0$ is measured as function of temperature and field to obtain the power absorbed by the sample (Fig. 2). The bolometric method [15] consists in measuring the heating of the sample induced by the absorption of the incident microwave power when the JPR is excited [3] [17]. Furthermore, reversible magnetization measurements were carried out on sample A using a commercial superconducting quantum interference device magnetometer in order to extract the London penetration depth $\lambda_{L,ab}(T)$ for currents in the $ab$-plane [13].

Figure 3 shows the JPR frequency $f_{JPR} = \omega_{pl}/2\pi$ in zero field obtained by the above-mentioned methods on samples A and B. $\omega^2_{pl}$ is proportional to the maximum interlayer Josephson current along the c-axis [11].

$$\omega^2_{pl}(H,T) = \omega^2_{pl}(0,T) \cos(\phi_{n,n+1}) = \frac{2\pi\mu_0 c^2 s}{\epsilon_r \Phi_0} J_m^{(c)}(B,T)$$

(1)

where $J_m^{(c)}(B,T) = J_m^{(c)}(0,T) \cos(\phi_{n,n+1})$ is the maximum Josephson current, $s = 1.5$ nm is the interlayer spacing, $\epsilon_r$ the high-frequency relative dielectric constant and $\Phi_0$ the flux quantum. Using $\omega_{pl}(0,T) = c/\lambda_{L,ab}(T)\sqrt{\epsilon_r}$ and $\epsilon_r=11.5$ [17], we obtain the London penetration depth for currents along the c-axis, $\lambda_{L,c}(T)$. When divided by the $\lambda_{L,ab}(T)$—data from reversible magnetization, this yields, without any model assumptions, the anisotropy parameter $\gamma(T) \equiv \lambda_{L,c}(T)/\lambda_{L,ab}(T)$, shown in the Inset to Fig. [3]. Typically, at $T = 0.5 T_c$, $\lambda_{L,c} \approx 240$ $\mu$m and $\lambda_{L,ab} \approx 400$ nm, so that $\gamma \approx 600$.

FIG. 2: Field sweep experiment on sample A at $T = 66$ K in the TM_{012} mode of the cavity ($f = 22.9$ GHz). The power absorbed (●, left axis) in the sample shows a maximum at $B_{JPR} = 5.3$ G at which $\omega = \omega_{pl}$ (open arrow). At the same field, the resonance frequency of the cavity (○, right axis) displays a double-peak structure, indicated by the closed arrows, and a jump (arrow between dashed lines).

FIG. 3: The JPR frequency in zero magnetic field for samples A (data below 40 GHz) and B (data above 40 GHz) vs the reduced temperature $T/T_c$. The bolometric method (●) and the cavity perturbation technique (○) have been used. We use a spline fit (solid line) in the extraction of the wandering length (see text). Inset: experimental temperature dependence of $\gamma$, obtained by the division of the experimental $\lambda_{L,c}(T)$ data by the $\lambda_{L,ab}(T)$—data from reversible magnetization. Consistent with other data for the same material [8]. Note that $\gamma$ decreases as function of temperature.

To analyze our JPR data in non-zero magnetic fields, we should divide $\omega_{pl}(B,T)$ by the zero-field result depicted in Fig. 3. In the absence of a model that satisfactorily describes $\omega_{pl}(0,T)$ over the temperature range, we resort to a spline fit to the experimental data. Next, we extract the vortex wandering length $r_w$ as follows. In the single vortex regime, at very low fields $B < B_J = \Phi_0/\lambda_J^2$, $B < B_\lambda = \Phi_0/4\pi\lambda_{L,ab}^2$, Bulaevskii and Koshelev derived [3, 14]

$$1 - \frac{\omega^2_{pl}(B,T)}{\omega^2_{pl}(0,T)} \approx \frac{\pi B r_w^2}{2\Phi_0} \ln \frac{\lambda_J}{r_w}$$

(2)

where the Josephson length $\lambda_J = \gamma s$. We stress that this relation is meaningful only for small excursions $r_w \leq 0.6\lambda_J$, i.e. for $\langle \cos(\phi_{n,n+1}) \rangle = \omega^2_{pl}(B,T)/\omega^2_{pl}(0,T) \lesssim 1$. More generally, one expects an increase of $1 - \langle \cos(\phi_{n,n+1}) \rangle$ with $r_w$ up to a plateau for large $r_w$, as was found in recent simulations of the evolution of $1 - \langle \cos(\phi_{n,n+1}) \rangle$ versus $(u)/a_0 \sim r_w/a_0$ for a pancake gas ($a_0 = \sqrt{\Phi_0/B}$ is the intervortex spacing) [29]. The numerical data show that $1 - \langle \cos(\phi_{n,n+1}) \rangle$ is almost quadratic in $r_w$ for $0 \lesssim 1 - \langle \cos(\phi_{n,n+1}) \rangle \lesssim 0.7 - 0.8$, in agreement with Eq. (2), if the weak logarithmic dependence on $\lambda_J/r_w$ is disregarded. Thus, we use

$$r_w^2 = \frac{2\Phi_0}{\pi B}(1 - \langle \cos(\phi_{n,n+1}) \rangle)$$

(3)

to obtain an approximation of the wandering length. Since $r_w = \langle (u_{n+1} - u_n)^2 \rangle^{1/2} = \langle (u^2 - (u_n - u_{n+1}))^2 \rangle^{1/2}$, one has, in the case of completely uncorrelated layers (e.g. for a pancake gas), $r_w = \langle 2u_n^2 \rangle^{1/2} = \sqrt{2}u_n$. Disregarding the “anticorrelated” situation with $u_n \cdot u_{n+1} < 0$,
correlations between pancake positions in different layers yield $r_w$ in the vortex solid. The RMS thermal vortex displacement $u$ can be obtained by equipartition of the associated elastic energy with the thermal energy, $U = \frac{1}{2} k_B T$. The vortex lattice tilt modulus

$$c_{44}(k) \approx \frac{B^2/\mu_0}{1 + \lambda_0^2 k_0^2 + \lambda_0^2 Q_s^2} + \frac{\varepsilon_0}{2\gamma^2 a_0^2} \ln \left( \frac{k_{\text{max}}^2}{k_0^2 + (Q_s/\gamma)^2} \right)$$

$$+ \frac{\varepsilon_0}{2\lambda_0 Q_s a_0^2} \ln \left( 1 + \frac{a_0^2}{21.3 r_w^2} \right),$$

(4)

calculated by Koshelev and Vinokur [21] and Goldin and Horowitz [22], consists of three terms: the nonlocal collective (lattice) term, the vortex line tension term, determined by Josephson coupling between layers, and a third term due to the electromagnetic dipole interaction between pancakes. Of particular interest here is the logarithmic correction to the temperature dependence of the second term, introduced by the cutoff $k_{\text{max}} = \pi/r_w$, which corresponds to the smallest meaningful deformation [21, 22]. To proceed, we evaluate $U_{el}$ at the typical vortex line deformation wavevectors parallel and perpendicular to the layers, $k_\parallel = \pi/r_w$ and $Q_z = \pi \gamma/2a_0 \ll 2\pi/s$. Writing $K_0 = \sqrt{4\pi/a_0}$, $r_w^2 = \alpha a^2$ and $x = a_0/r_w$, equipartition yields

$$r_w^2 \approx \frac{\alpha s^2 k_B T \gamma^2}{\varepsilon_0} \left[ \frac{4}{\pi (x^2 + \frac{1}{4})} + \frac{1}{2} \ln \left( \frac{0.66x^2}{21.3} \right) \right]^\frac{1}{2} \left( \frac{a_0}{\lambda_{L,ab}} \right)^2 \ln \left( 1 + \frac{x^4}{21.3} \right)^\frac{1}{2},$$

(5)

All parameters in Eq. (5), and notably $\varepsilon_0(T)/\gamma^2(T) = \Phi_0^2/4\pi \mu_0 \lambda_{L,c}^2$, are known from experiment, which allows a direct comparison to the $r_w(T)$ -data. Very good agreement is obtained for the magnitude, the temperature, as well as the field dependence of $r_w$ for the lowest three fields, using the single free parameter $\alpha = 0.95$. For

FIG. 4: $1 - \langle \cos(\phi_{n,n+1}) \rangle$ vs temperature for different magnetic fields. We extract $r_w$ from these data using Eq. (4).

For $T < 0.96T_{\text{FOT}}$, $r_w(T/T_{\text{FOT}})$ roughly overlaps for all fields, whereas for $T > 0.96T_{\text{FOT}}$ the curves deviate from each other.

We now discuss the temperature and field dependence of $r_w$ in the vortex solid. The temperature dependence of the magnetic energy, yield $r_w < \sqrt{2u}$, i.e., $r_w$ is a lower limit for the root mean squared (RMS) displacement $u$ of the vortex line. Figure 4 shows $1 - \omega_3^2(B,T)/\omega_3^2(0,T) = 1 - \langle \cos(\phi_{n,n+1}) \rangle$ as function of temperature in different dc fields. The temperature dependence of the wandering $\langle r \rangle^2$ in the vortex solid. The RMS thermal vortex displacement $\langle r \rangle$ vs $T/T_{\text{FOT}}$ for different magnetic fields in strongly underdoped BSCCO. For $B = 27.5, 22.4, 15.3, 12.4$ and $9.9$ G, arrows show the temperature of the FOT. The dotted line shows the evolution of $0.9s(k_B T\gamma^2/\varepsilon_0 s)^{1/2}$. Solid lines are fits to Eq. (4) with the single parameter $\alpha = 0.95$, omitting the term $4/\pi (x^2 + \frac{1}{4})$ for $B > 15$ G. Inset: phase diagram of a sample cut from the same crystal. Full and open triangles stand for the FOT and the irreversibility fields, respectively.

FIG. 5: (a) : Experimental $r_w$ vs $T/T_{\text{FOT}}$ for different magnetic fields. The dotted line shows the evolution of $0.9s(k_B T\gamma^2/\varepsilon_0 s)^{1/2}$. Solid lines are fits to Eq. (4) with the single parameter $\alpha = 0.95$, omitting the term $4/\pi (x^2 + \frac{1}{4})$ for $B > 15$ G. (b) : $r_w$ vs $T_T/T_{\text{FOT}}$. Solid lines are guides to the eye. Inset : phase diagram of a sample cut from the same crystal. Full and open triangles stand for the FOT and the irreversibility fields, respectively.
higher fields, Eq. (6) gives the correct magnitude of \( r_w \), but too weak a temperature dependence. However, excellent fits of both the temperature and field dependence can be obtained for all fields, with the same \( a = 0.95 \), by omitting the nonlocal collective term, \([i.e. \ 4/\pi (x^2 + 1) \] in Eq. (6)], see Fig. 5a. While the main temperature dependence of \( r_w \) comes from the prefactor \( \gamma^2 T/\varepsilon_0 \) in Eq. (6) (dotted line in Fig. 5), the behavior of \( r_w \) in the vortex solid can only be understood as the result of the logarithmic correction arising from the softening of the line tension term by thermal fluctuations [21, 22]. The field dependence, originating from \( Q_x \), explicitly indicates that vortex lines are correlated (line-like) on distances that well exceed the layer spacing \( s \).

The experimental data can also be used to compare the terms entering Eq. (6). Deep inside the vortex solid, the line tension always dominates over the magnetic coupling and the nonlocal collective contribution. At very low fields (\( B < 10 \ G \)), the line tension term is largest all the way to the FOT. Eq. (6) then reduces to Eq. (40) of Ref. [22] with \( Q_x \approx \pi \gamma/2a_0 \) instead of \( 2\pi/s \). At higher fields, the nonlocal collective contribution is expected to increase, eventually exceeding the Josephson coupling (line tension) term close to the FOT (for \( B > 20 \ G \)). Nevertheless, the very good fits obtained when only the line tension term is taken into account in Eq. (6) suggest that the line tension term always dominates \( c_{44} \) near the FOT. Moreover, we find that the electromagnetic coupling as well as the shear contribution to \( U_{el} \) are, under all circumstances, negligible. This renders Lindenmann-like [22] or dislocation-mediated (Kosterlitz-Thouless like) melting, as well as the vortex–line evaporation [5] scenarios very unlikely. Rather, the large thermal excursions of pancake vortices bring about the softening of the line tension contribution to \( c_{44} \) for the large-wavevector modes that lead to the FOT. This would comply with recent measurements showing that vortex lattice order is not a prerequisite for the FOT [23]. For deformations with smaller wavevectors, Josephson coupling still contributes to the line tension even in the vortex liquid, leading to, \( e.g. \), the anisotropic vortex response to columnar defects in heavy-ion irradiated samples.

Summarizing, we have carried out JPR measurements on heavily underdoped \( \text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8 \) crystals. These data yield the \( c \)-axis penetration depth, the anisotropy parameter \( \gamma(T) \), and the wandering length of vortex lines \( r_w \). The observed temperature and field dependences of \( r_w \) suggest that thermal fluctuations soften the Josephson coupling contribution to the tilt modulus for short wavelengths [22], which leads us to believe that this softening drives the FOT.

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