NLO corrections to the twist-3 amplitude in DVCS on a nucleon in the Wandzura-Wilczek approximation: quark case

N. Kivel\textsuperscript{a}\textsuperscript{*}, L. Mankiewicz\textsuperscript{b},

\textsuperscript{a} Institut für Theoretische Physik II, Ruhr-Universität Bochum, D-44780 Bochum, Germany
\textsuperscript{b} Center for Theoretical Physics, Polish Academy of Science, Al. Lotników 32/46, 02-668 Warsaw, Poland

Abstract

We computed the NLO corrections to twist-3, $L \rightarrow T$, flavor non-singlet amplitude in DVCS on a nucleon in the Wandzura-Wilczek approximation. Explicit calculation shows that factorization holds for NLO contribution to this amplitude, although the structure of the factorized amplitude at the NLO is more complicated than in the leading-order formula. Next-to-leading order coefficient functions for matrix elements of twist-3 vector and axial-vector quark string operators and their LO evolution equations are presented.

\textsuperscript{*}on leave of absence from St.Petersburg Nuclear Physics Institute, 188350, Gatchina, Russia
Introduction

Deeply virtual Compton scattering (DVCS) \([1, 2]\) on a nucleon, \(\gamma^* N \rightarrow \gamma N'\), is perhaps the cleanest hard reaction sensitive to the skewed parton distributions (SPD). According to celebrated factorization theorems \([3, 4, 5]\) the leading term in the \(1/Q^2\) expansion of the DVCS amplitude, where \(Q^2\) is large virtuality of the hard photon, can be expressed in terms of twist-2 skewed parton distributions. Twist-2 SPD’s are related to matrix elements of non-local twist-2 quark and gluon string operators. These matrix elements contain complete information about nucleon structure seen by a highly-virtual electromagnetic probe. For that reason in recent years DVCS has been the subject of extensive theoretical investigations. First experimental data have also became recently available (see e.g. \([6, 7, 8, 9]\)) and much more data are expected from JLAB, DESY, and CERN in the future.

The amplitude of \(\gamma^* N \rightarrow \gamma N'\) scattering receives contribution from both DVCS and Bethe-Heitler processes. Extraction of DVCS from the experimental cross-section is therefore not straightforward. The helicity analysis reveals that the interference term contains contributions of different photon helicity amplitudes weighted with sines and cosines of the angle between leptonic and hadronic scattering planes in the center-of-mass of the scattered photon and the nucleon \([10, 11]\). At present, the leading-twist DVCS amplitude has been studied to the NLO accuracy \([12, 13, 14]\). The analysis has revealed that the photon helicity is conserved by the tree-level contribution. Photon helicity-flip amplitude appears at the NLO level. The \(L \rightarrow T\) amplitude, corresponding to scattering of longitudinal virtual photon off nucleon, appears as twist-3, \(1/Q\) correction. As the typical experimentally accessible values of \(Q^2\) are by no means large, such leading power corrections may have significant effects on some of DVCS observables. In addition, twist-3 corrections typically scale as \(\sqrt{-t/Q}\), with \(t\) denoting the square of the momentum transfer, so the size of twist-3 corrections increases with \(t\). As it follows, taking into account these corrections is mandatory for understanding continuation of the twist-2 part of the DVCS amplitude to \(t = 0\).

The LO contribution to the DVCS amplitude on the nucleon to the twist-3 accuracy has been calculated in \([15, 16]\). Twist-3 corrections to the DVCS amplitude arise in a twofold way. First, as usual for a hard exclusive process, the hard amplitude with an additional parton, the gluon, taking part in the hard collision is suppressed by one power of \(1/Q\). Such a genuine twist-3 contribution involves matrix element of three-parton operator in a nucleon state. Second, there is a so-called Wandzura-Wilczek (WW) contribution which arises from a hard configuration with minimal number of hard partons involved in hard collision. Formally, the WW contribution arises because operators with external derivatives w.r.t. total translation in a transverse direction give nonzero contribution in the DVCS kinematics. As the current phenomenology of power corrections is consistent with an assumption that matrix elements of three-parton operators in a nucleon are small \([17]\), one can conjecture that the WW contribution can provide a rather accurate numerical description of twist-3 corrections.

In this paper we have computed the NLO contribution to the WW twist-3, \(L \rightarrow T\) flavor non-singlet DVCS amplitude. Besides obvious phenomenological applications, there is a broader, theoretical interest in such a calculation. A factorization theorem for twist-3 DVCS amplitude has neither been considered nor proven in the literature. As a consequence, although the direct calculation showed that the LO \(L \rightarrow T\) amplitude factorizes, there is no guarantee that factorization prevails in higher orders as well. At the same time there have
been no direct calculations supporting or disproving factorization of twist-3 contribution to DVCS beyond the leading order. The aim of this paper has been to provide an explicit example of NLO correction to twist-3 WW amplitude. Although computation of one-loop, flavor non-singlet amplitude may seem trivial, based on experience with NLO corrections to twist-2 amplitudes, calculation of NLO twist-3 contribution has turned out to be technically quite involved, warranting a more detailed discussion. Our calculation demonstrates that the amplitude does factorize at the NLO. It is tempting to interpret it as a hint that the factorization holds for this particular twist-3 DVCS helicity amplitude to all orders.

Our paper is organized in the following way: in the first section we introduce our definitions and notations. The next two sections are devoted to discussion of intricacies of the calculation: in the second section we derive convenient parametrization of matrix elements of vector and axial-vector quark string operators up to twist-3 accuracy in the WW approximation and in the third one we demonstrate technicalities of our approach by calculating LO twist-3 term in WW approximation. The fourth section contains discussion of our main result - the NLO correction to the WW $L \rightarrow T$ amplitude. Finally, we conclude.

**DVCS amplitude on a nucleon**

Let $p, p'$ and $q, q'$ denote momenta of the initial and final nucleons and photons, respectively. The amplitude of the virtual Compton scattering process

$$\gamma^*(q) + N(p) \rightarrow \gamma(q') + N(p') ,$$

is defined in terms of the nucleon matrix element of the $T$-product of two electromagnetic currents:

$$T^\mu\nu = i \int d^4x \ e^{-i(q+q')x/2} \langle p' | T [J_\mu^{(e.m.}(x/2)J_\nu^{(e.m.}(-x/2)] | p \rangle ,$$

where Lorentz indices $\mu$ and $\nu$ correspond to the virtual, respectively the real photon.

We shall consider the Bjorken limit, where $-q^2 = Q^2 \rightarrow \infty$, $2(p \cdot q) \rightarrow \infty$, with $x_B = Q^2/2(p \cdot q)$ constant, and $t \equiv (p - p')^2 \ll Q^2$. We introduce two light-like vectors $n, n^*$ such that

$$n \cdot n = 0, n^* \cdot n^* = 0, n \cdot n^* = 1.$$  \hspace{1cm} (3)

We shall work in a reference frame where the average nucleon momenta $P = \frac{1}{2}(p + p')$ and the virtual photon momentum $q$ are collinear along $z$-axis and have opposite directions. Such a choice of the frame results in the following decomposition of the momenta [10]:

$$P = n^* + \frac{m^2}{2} n$$

$$q = -2\xi n^* + \frac{Q^2}{4\xi} n$$

$$\Delta = p' - p = -2\xi n^* + m^2 \xi n + \Delta_\perp$$  \hspace{1cm} (4)
with \( \bar{m}^2 = m^2 - t/4 \), \( t = \Delta^2 \) being the squared momentum transfer, and

\[
2\xi = 2\xi' \frac{Q^2 - t}{Q^2 + 4\xi'^2\bar{m}^2}.
\]

Finally, \( \xi' \) is given by

\[
\xi' = \frac{2x_B^2}{x_B^2 + t/Q^2 + \sqrt{(2x_B^2 + t/Q^2)^2 + 16\xi n}} = \frac{x_B}{2 - x_B} + O(1/Q^2),
\]

with \( x_B = \frac{Q^2}{2p\cdot q} \).

We define the transverse metric and antisymmetric transverse epsilon tensors \( ^\perp \):

\[
g_{\mu\nu}^\perp = g_{\mu\nu} - n^\mu n^{*\nu} - n^{\nu} n^{*\mu}, \quad \epsilon_{\mu\nu}^\perp = \epsilon_{\mu\nu\alpha\beta} n^\alpha n^{*\beta}.
\]

In the following, we shall use the shorthand notation for

\[
a^+ \equiv a_\mu n^\mu, \quad a^- \equiv a_\mu n^{*\mu},
\]

where \( a_\mu \) is an arbitrary Lorentz vector.

In this paper we focus our attention on the \( L \rightarrow T \) DVCS amplitude where the virtual photon has longitudinal polarization. It appears first at the twist-3 level.\(^\dagger\) In order to calculate the leading contribution to this amplitude it is sufficient to expand (4) and (5) to twist-3 accuracy:

\[
P = \frac{1}{2}(p + p') = n^*, \quad \Delta = p' - p = -2\xi P + \Delta^\perp, \quad q = -2\xi P + \frac{Q^2}{4\xi} n, \quad q' = q - \Delta = \frac{Q^2}{4\xi} n - \Delta^\perp.
\]

The LO \( L \rightarrow T \) amplitude has the form [15, 16]:

\[
T_{0+}^{\mu\nu} = -\frac{(q + 4\xi P)^\mu}{(Pq)} \left[ g_{\perp}^{\nu\alpha} + \frac{P^\nu \Delta^\alpha_\perp}{(Pq)} \right] \frac{1}{2} \int_{-1}^{1} dx \left\{ F_\alpha(x, \xi) C^+(x, \xi) - i\epsilon^{\perp}_{\alpha\rho} \bar{F}\rho(x, \xi) C^-(x, \xi) \right\}.
\]

Note that the combination \((q + 4\xi P)^\mu\) is proportional to the longitudinal polarization vector of the initial photon. In order to have exact gauge invariance of the amplitude we keep in (10) terms of order \( \Delta^2_\perp/Q^2 \) applying prescription \( g_{\perp}^{\nu\alpha} \rightarrow g_{\perp}^{\nu\alpha} + \frac{P^\nu \Delta^\alpha_\perp}{(Pq)} \) for the twist-3 terms in the amplitude. The skewed parton distribution \( F_\mu(x, \xi) \) and \( \bar{F}_\mu(x, \xi) \) can be related to the twist-2 SPD’s \( H, E, \bar{H}, \bar{E} \) with help of WW relations. We shall give explicit expressions for them in the next section. Term proportional to \( \frac{P^\nu \Delta^\alpha_\perp}{(Pq)} \) is required to ensure electromagnetic gauge-invariance: the amplitude (10) is electromagnetically gauge invariant:

\[
q_\mu T_{0+}^{\mu\nu} = T_{0+}^{\mu\nu} q_\nu = 0
\]

\(^\dagger\)The Levi-Civita tensor \( \epsilon_{\mu\nu\alpha\beta} \) is defined as the totally antisymmetric tensor with \( \epsilon_{0123} = 1 \)
\(^\dagger\)We adopt here kinematical definition of twist i.e., terms suppressed by \( 1/Q \) are of twist-3.
formally to the accuracy $1/Q^2$. In the LO in QCD coupling coefficient functions are given by simple expression which have poles at $x = \pm \xi$:

$$C^\pm(x, \xi) = \frac{1}{x - \xi + i\varepsilon} \pm \frac{1}{x + \xi - i\varepsilon}.$$ 

It was demonstrated in [18] that despite presence of these poles the convolution integral in (10) is well-defined and therefore factorization is not violated at the LO.

One final comment is in order here. Note that using symmetry of the GPD’s, see (33) below, the LO amplitude can be written in a shorter form as

$$T^\mu_{0+} = \frac{(q + 4\xi P)^\mu}{2(Pq)} \left[ g_{\perp}^{\mu\alpha} + \frac{P_\perp^\mu \Delta_1}{(Pq)} \right] \int_{-1}^{1} dx \frac{-2}{x + \xi} \left( F_{\perp\alpha}(x, \xi) + i\varepsilon_{\alpha\rho}^\perp \tilde{F}_{\perp\rho}^\alpha(x, \xi) \right).$$

In the following we shall, for notational simplicity, always rewrite lengthy amplitudes in a similar way, making use of symmetries of corresponding GPD’s. One can easily make use of these symmetries to transform expressions into the original form.

**Light-cone expansions of the matrix element**

Calculation of NLO corrections to the $L \rightarrow T$ amplitude requires knowledge of the nucleon matrix element of vector and axial-vector non-local quark string operators

$$\langle p' | \bar{\psi}(x) \gamma^\sigma \psi(y) | p \rangle$$

and

$$\langle p' | \bar{\psi}(x) \gamma^\sigma \gamma_5 \psi(y) | p \rangle$$

to the twist-3 accuracy but for arbitrary $x$ and $y$. We shall restrict ourselves to the Wandzura-Wilczek approximation i.e., we neglect quark-gluon-quark operators arising in the twist expansion. As we shall show below, the result is given in terms of distribution functions which parametrize matrix elements of string operators restricted to the light-cone:

$$F_{\mu}(x, \xi) = \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} e^{-ix\lambda} \langle p' | \bar{\psi}(\frac{1}{2} \lambda n) \gamma_\mu \psi(-\frac{1}{2} \lambda n) | p \rangle,$$

$$\tilde{F}_{\mu}(x, \xi) = \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} e^{-ix\lambda} \langle p' | \bar{\psi}(\frac{1}{2} \lambda n) \gamma_\mu \gamma_5 \psi(-\frac{1}{2} \lambda n) | p \rangle.$$  

To simplify notation, we have omitted in the above formulae the path-ordered exponential connecting the quark fields.

As discussed in [16, 19], to the twist-3 accuracy the above matrix elements can be parametrized as:

$$F^W_{\mu}(x, \xi) = P_{\mu} F(x, \xi) + F_{\perp\mu}(x, \xi).$$

$$\tilde{F}^W_{\mu}(x, \xi) = P_{\mu} \tilde{F}(x, \xi) + F_{\perp\mu}(x, \xi).$$
Here we have explicitly separated longitudinal, twist-2:

\[ F(x, \xi) = n^\rho F_\rho^{WW}(x, \xi) = \langle \gamma^+ \rangle (H + E)(x, \xi) - \langle \frac{1}{m} \rangle E(x, \xi) , \]  
(19)

\[ \tilde{F}(x, \xi) = n^\rho \tilde{F}_\rho^{WW}(x, \xi) = \langle \gamma^+ \gamma_5 \rangle \tilde{H}(x, \xi) - \langle \frac{\gamma_5}{m} \rangle \xi \tilde{E}(x, \xi) , \]  
(20)

and transverse, twist-3 components:

\[ F_{\perp \mu}(x, \xi) = -\frac{\Delta_{\perp \mu}}{2\xi} F(x, \xi) + \int_{-1}^{1} du \left( G_\mu(u, \xi) W_+(x, u, \xi) + i\epsilon_{\perp \mu} G_\alpha^\alpha(u, \xi) W_-(x, u, \xi) \right) , \]  
(21)

\[ \tilde{F}_{\perp \mu}(x, \xi) = -\frac{\Delta_{\perp \mu}}{2\xi} \tilde{F}(x, \xi) + \int_{-1}^{1} du \left( \tilde{G}_\mu(u, \xi) W_+(x, u, \xi) + i\epsilon_{\perp \mu} G_\alpha^\alpha(u, \xi) W_-(x, u, \xi) \right) . \]  
(22)

A shorthand notation \( \langle \ldots \rangle \) denotes \( \bar{U}(p') \ldots U(p) \) and \( m \) is the nucleon mass.

Functions \( G^\mu \) and \( \tilde{G}^\mu \) and the Wandzura-Wilczek kernels \( W_\pm(x, u, \xi) \) have been introduced in Refs. [16, 19]. They are defined as :

\[ G^\mu(u, \xi) = \langle \gamma_\perp^\mu \rangle (H + E)(u, \xi) - \frac{\Delta_{\perp \mu}}{2\xi} \left[ u \frac{\partial}{\partial u} + \xi \frac{\partial}{\partial \xi} \right] F(u, \xi) , \]  
(23)

\[ \tilde{G}^\mu(u, \xi) = \langle \gamma_\perp^\mu \gamma_5 \rangle \tilde{H}(u, \xi) - \frac{\Delta_{\perp \mu}}{2\xi} \left[ u \frac{\partial}{\partial u} + \xi \frac{\partial}{\partial \xi} \right] \tilde{F}(u, \xi) . \]  
(24)

\[ W_\pm(x, u, \xi) = \frac{1}{2} \left\{ \theta(x > \xi) \theta(u > x) - \theta(x < \xi) \theta(u < x) \right\} \frac{1}{u - \xi} \]  

\[ \pm \frac{1}{2} \left\{ \theta(x > -\xi) \theta(u > x) - \theta(x < -\xi) \theta(u < x) \right\} \frac{1}{u + \xi} . \]  
(25)

The flavor dependence in the amplitude can easily be restored by a substitution :

\[ F_\mu(\tilde{F}_\mu) \rightarrow \sum_{q=u,d,s,...} e_q^2 F_\mu^q(\tilde{F}_\mu^q) . \]  
(26)

The light-cone expansion of the matrix elements (13,14) can be derived in a similar manner. To the twist-3 accuracy needed for the present calculation the result is

\[ \langle P + \Delta/2 | \bar{\psi}(x) \gamma^\rho \gamma_5 \psi(y) | P - \Delta/2 \rangle = \int_{-1}^{1} du \epsilon^{i(P_x)(u-\xi)-i(P_y)(u+\xi)} \left\{ P^\sigma F(u, \xi) + F_\perp^\sigma(u, \xi) + \frac{1}{2} i P^\sigma (x + y)_{\rho} \Delta_\perp^\rho F(u, \xi) + i P^\sigma (x - y)_{\rho} G^\rho_\perp(u, \xi) \right\} \]  
(27)

\[ \langle P + \Delta/2 | \bar{\psi}(x) \gamma^\rho \gamma_5 \psi(y) | P - \Delta/2 \rangle = \int_{-1}^{1} du \epsilon^{i(P_x)(u-\xi)-i(P_y)(u+\xi)} \left\{ P^\sigma \tilde{F}(u, \xi) + \tilde{F}_\perp^\sigma(u, \xi) + \frac{1}{2} i P^\sigma (x + y)_{\rho} \Delta_\perp^\rho \tilde{F}(u, \xi) + i P^\sigma (x - y)_{\rho} \tilde{G}^\rho_\perp(u, \xi) \right\} . \]  
(28)
where now both $x$ and $y$ are off the light-cone. New distributions $G_1^\sigma(u, \xi)$ and $\tilde{G}_1^\sigma(u, \xi)$ can be expressed through the skewed distributions (17), (18):

$$G_1^\sigma(u, \xi) = u F_\perp^\sigma(u, \xi) - \xi \epsilon_\perp^{\sigma k} \tilde{F}_\perp k(u, \xi) - \frac{1}{2} i \epsilon_\perp^{\sigma k} \Delta_{\perp k} \tilde{F}(u, \xi)$$

(29)

$$\tilde{G}_1^\sigma(u, \xi) = u \tilde{F}_\perp^\sigma(u, \xi) - \xi \epsilon_\perp^{\sigma k} F_\perp k(u, \xi) - \frac{1}{2} i \epsilon_\perp^{\sigma k} \Delta_{\perp k} F(u, \xi)$$

(30)

Keeping in mind calculation of the $L \rightarrow T$ amplitude it is useful to recall symmetry properties of skewed parton distributions. Symmetry properties of twist-2 distributions $H(u, \xi)$, $E(u, \xi)$, $\tilde{H}(u, \xi)$, $\tilde{E}(u, \xi)$ with respect to interchange $u \leftrightarrow -u$ are given by:

$$H(-u, \xi) = -H(u, \xi), \quad E(-u, \xi) = -E(u, \xi),$$

(31)

$$\tilde{H}(-u, \xi) = \tilde{H}(u, \xi), \quad \tilde{E}(-u, \xi) = \tilde{E}(u, \xi).$$

(32)

As it follows,

$$F(-u, \xi) = -F(u, \xi), \quad F_{\perp \mu}(-u, \xi) = F_{\perp \mu}(u, \xi)$$

(33)

$$\tilde{F}(-u, \xi) = \tilde{F}(u, \xi), \quad \tilde{F}^\mu_{\perp}(-u, \xi) = \tilde{F}^\mu_{\perp}(u, \xi)$$

(34)

In the following we shall often need Fourier transforms of the matrix elements (27) and (28). For example, for the vector matrix element one finds:

$$\int \frac{d^4 k'}{(2\pi)^4} e^{-i(k'y)} \int \frac{d^4 k}{(2\pi)^4} e^{-i(kx)} \langle P + \Delta/2 | \bar{\psi}(y + \frac{1}{2} x) \gamma^\sigma \psi(y - \frac{1}{2} x) | P - \Delta/2 \rangle =$$

$$\int_{-1}^{1} du \left\{ P^\sigma F(u, \xi) + F_{\perp}^\sigma(u, \xi) - P^\sigma \left( F(u, \xi) \Delta_{\perp k}^\sigma \frac{\partial}{\partial k_\rho} + G_1^\sigma(u, \xi) \frac{\partial}{\partial k_\rho} \right) \right\} \delta^{(4)}(2 \xi P + k') \delta^{(4)}(u P - k)$$

(35)

Similar result holds for the Fourier transform of the axial matrix element.

**Covariant calculation of the $L \rightarrow T$ amplitude in the WW approximation**

The aim of this section is to present a method of calculation of the $L \rightarrow T$ DVCS amplitude in the WW approximation which can be efficiently applied at the NLO.

The crucial simplification arises due to the fact that according to (10) in the present kinematics the Lorentz indices $\mu$ and $\nu$ of the $L \rightarrow T$ amplitude point in the longitudinal, respectively transverse directions. As it follows, $\mu$ can be carried only by longitudinal vectors $P$ and $q$ and $\nu$ has to be carried by transverse components of other vectors present in the problem. Symbolically, one can classify possible contributions as:

$$(q \text{ or } P)^\mu \times \left\{ \Delta_{\perp}^\nu, i \epsilon_\alpha^{\perp \perp} \Delta_{\perp}^\alpha, F_{\perp}^\nu, G_1^\nu(u, \xi), \tilde{F}_{\perp}^\nu, \tilde{G}_1^\nu(u, \xi) \right\}$$

(36)

Hence, it is necessary to compute only those terms which can be casted in the above form.
The starting point for calculation is the standard definition of DVCS amplitude (2) which can be rewritten as:

\[
T_{\mu\nu} = i \int d^4x \ e^{-i(q+q')x/2} \langle p' | T[J_{\text{e.m.}}^\mu(x/2)J_{\text{e.m.}}^\nu(-x/2)] | p \rangle = \\
i \int d^4x \ e^{-i(q+q')x/2} \int d^4y d^4z \langle p' | \bar{\psi}(y)H(x, y, z)\psi(z)| p \rangle
\]

(37)

Function \( H(x, y, z) \) is given by connected Feynman diagrams with amputated external fermion legs, see Fig. 1. Note that equation (37) is valid only in the WW approximation i.e. if twist-3 contributions involving three points quark-gluon-quark matrix elements are neglected. Using Fiertz identity we can further transform (37) into:

\[
T_{\mu\nu} = i \int d^4x \ e^{-i(q+q')x/2} \int d^4y \int d^4z \left\{ \tilde{H}_V^\sigma(x, y, z)\langle p' | \bar{\psi}(y)\gamma_\sigma\psi(z)| p \rangle \\
- \tilde{H}_A^\sigma(x, y, z)\langle p' | \bar{\psi}(y)\gamma_\sigma\gamma_5\psi(z)| p \rangle \right\}.
\]

(38)

Here we have introduced notation

\[
\tilde{H}_V^\sigma(x, y, z) = \frac{1}{4} \text{tr} \left\{ \gamma_{\sigma} (\gamma_{\sigma}\gamma_5) H(x, y, z) \right\}
\]

(39)

Using translational invariance and equations (27),(28) and (35) one easily obtains corresponding expression in momentum space. The final result for the twist-3 part of \( T_{\mu\nu} \) which gives rise to the \( L \rightarrow T \) amplitude reads:

\[
T_{\mu\nu} = \int_1^{-1} du \int d^4k \delta^4(uP - k) \int d^4k' \delta^4(2\xi P + k') \int d^4k'' \delta^4(q + \xi P - k''')
\left\{ \begin{array}{l}
F_\perp^\sigma(u, \xi) + \frac{1}{4} \text{tr} \left\{ \gamma_{\sigma} (\gamma_{\sigma}\gamma_5) H(x, y, z) \right\} \\
F_\perp^\sigma(u, \xi) + \frac{1}{4} \text{tr} \left\{ \gamma_{\sigma} (\gamma_{\sigma}\gamma_5) H(x, y, z) \right\}
\end{array} \right\}
\]

(40)

The starting point for calculation is the standard definition of DVCS amplitude (2) which can be rewritten as:

\[
T_{\mu\nu} = i \int d^4x \ e^{-i(q+q')x/2} \langle p' | T[J_{\text{e.m.}}^\mu(x/2)J_{\text{e.m.}}^\nu(-x/2)] | p \rangle = \\
i \int d^4x \ e^{-i(q+q')x/2} \int d^4y d^4z \langle p' | \bar{\psi}(y)H(x, y, z)\psi(z)| p \rangle
\]

(37)

Figure 1: Graphical representation of the function \( H(x, y, z) \)
Functions $H_{V(A)}(k, k', k'')$ are Fourier transforms of functions $\tilde{H}_{V(A)}(x, y, z)$:

$$H_{V(A)}^\sigma(k, k', k'') = i \int d^4x e^{-ik''x} \int d^4ye^{iy(k+k'/2)} \int d^4ze^{-iz(k-k'/2)} \tilde{H}_{V(A)}^\sigma(x, y, z). \tag{41}$$

Resulting flow of momenta is shown in Fig. 2. Due to $\delta$-functions in (40) one can put $k = uP$, $k' = -2\xi P$ and $k'' = q + \xi P$ after differentiation. Thus, calculation of the twist-3 DVCS amplitude in the WW approximation has been reduced to the calculation of hard parton diagrams and their derivatives with respect to external momenta.

Above considerations are quite general and valid at any order of perturbation theory if the factorization is not broken. Let us now illustrate them with the example of calculation of the LO contribution to the twist-3 $L \to T$ amplitude (10). In this case the hard amplitudes $H_{V(A)}^\sigma(k, k', k'')$ are given by diagrams shown in Fig. 3. Simple calculation gives:

$$H_{V(A)}^\sigma(k, k', k'') = \frac{1}{4} \text{tr} \left\{ \gamma_\sigma(\gamma_\sigma\gamma_5)\gamma^\mu(k+k'')\gamma_\nu \right\} (k+k'')^2 + i0, \tag{42}$$

i.e. in the Born approximation $H_{V(A)}^\sigma(k, k', k'')$ does not depend on $k'$. Note that one need to evaluate only one diagram as contribution arising from the crossed one can be obtained using symmetry considerations and (33). Explicit calculation according to (40) yields:

$$T_{\mu\nu}^{\delta+} = \frac{1}{2(Pq)} \int_{-1}^{1} du \frac{1}{u + \xi - i0} \left\{ \frac{1}{2} \Delta_+^k P^\mu F(u, \xi) + [(u + \xi)P^\mu - q^\mu]F_+^\nu(u, \xi) + P^\mu G_1^\nu(u, \xi) \right. \right.$$  

$$+ \frac{1}{2} i\epsilon^\nu_{\perp k} \Delta_+^k P^\mu \tilde{F}(u, \xi) - [(u + \xi)P^\mu + q^\mu]i\epsilon^\nu_{\perp k} \tilde{F}_+^k(u, \xi) + P^\mu i\epsilon_\perp \tilde{G}_1^k(u, \xi) \right\}. \tag{43}$$
Figure 4: Feynmann diagrams for the NLO correction in WW approximation for $H(k, k', k'')$

Using the identity which follows from (29), (30):

$$G^\nu_1(u, \xi) + \frac{1}{2} \Delta^\nu \bar{F}(u, \xi) + i \epsilon^\nu_{\perp k} G^k_1(u, \xi) - \frac{1}{2} i \epsilon^\nu_{\perp k} \Delta^k \bar{F}(u, \xi) = (u - \xi) \left[ F^\nu_1(u, \xi) + i \epsilon^\nu_{\perp k} \bar{F}^k_1(u, \xi) \right]$$

one can easily cast (43) into the form

$$T_{0+}^{\mu \nu} = -\frac{1}{2(Pq)} (4\xi P^\mu + q^\mu) \int_{-1}^{1} du \frac{1}{u + \xi - i0} \left\{ F^\nu_{\perp 1}(u, \xi) + i \epsilon^\nu_{\perp k} \bar{F}^k_{\perp 1}(u, \xi) \right\} + \frac{1}{2(Pq)} P^\mu \int_{-1}^{1} du F^\nu_{\perp 1}(u, \xi).$$

Taking into account the contribution from the crossed diagram one finds that the term proportional to the integral $\int_{-1}^{1} du F^\nu_{\perp 1}(u, \xi)$ cancels out and the final expression reads:

$$T_{0+}^{\mu \nu} = -\frac{1}{2(Pq)} (4\xi P^\mu + q^\mu) \int_{-1}^{1} du \left\{ \frac{1}{u + \xi - i0} \left[ F^\nu_{\perp 1}(u, \xi) + i \epsilon^\nu_{\perp k} \bar{F}^k_{\perp 1}(u, \xi) \right] - \frac{1}{\xi - u - i0} \left[ F^\nu_{\perp 1}(u, \xi) - i \epsilon^\nu_{\perp k} \bar{F}^k_{\perp 1}(u, \xi) \right] \right\},$$

which is precisely the expression given in (10).

One comment is in order here. In the present calculation of the Wilson coefficient one does not need to apply QCD equations of motion (EOM) like it was done in Refs. [15] and [20]. EOM had been used at the previous stage to derive the light-cone expansion of the matrix elements (27) and (28). After it is done, the appropriate convolution of matrix elements with the hard amplitude according to (40) leads directly to the twist-3 $L \rightarrow T$ Compton amplitude.

**NLO corrections to the WW twist-3 amplitude**

Technique described in the previous section can be applied without any modifications to the calculation of the NLO flavor non-singlet, quark contribution to the $L \rightarrow T$ Compton amplitude. The corresponding Feynman diagrams shown in Fig. 4 are the same as for the NLO contributions to twist-2 amplitude. Nevertheless, besides trivial technical complications due to derivatives of corresponding diagrams w.r.t. external momenta, some new subtleties appear here which deserve a detailed discussion before final presentation of NLO results.

A convenient method for regularizing loop integrals arising at the NLO is provided by dimensional regularization. We kept space-time dimension $d = 4 - 2\epsilon$ and applied the
MS subtraction scheme. For the $\gamma_5$ matrix in $D$-dimension we used the t'Hooft-Veltman definition in terms of four antisymmetric gamma-matrices $\mathbb{G}$. Powerful check of the resulting calculation is provided by the electromagnetic gauge-invariance of the final answer. We computed all possible contributions according to (36) and check that in the sum of all diagrams only such combination of momenta appears, $q^{\mu} + 4\xi P^{\mu}$, which is consistent with electromagnetic gauge-invariance.

So far, it is a standard discussion familiar from calculations of higher-order coefficients in twist-2 amplitudes. New and interesting element appears when one considers a possible structure of the NLO answer. Obviously, proportionality of the $L \to T$ amplitude in (10) to $q^{\mu} + 4\xi P^{\mu}$ is fixed by gauge-invariance arguments and does not change when one goes from LO to NLO accuracy. On the other hand, the LO amplitude depends only on combinations of $F^a_0$ and $\tilde F^a_0$. Contributions involving $F$ and $\tilde F$, which might in principle be present according to (40), cancel. This cancellation is, however, specific to the Born approximation and does not hold at higher orders. We therefore expect that the NLO expression for the $L \to T$ amplitude has the form:

$$T^\mu_0 = \frac{(q + 4\xi P)^\mu}{2(Pq)} \left[ g^{\mu\alpha} + \frac{P^\nu \Delta^\alpha_1}{(Pq)} \right] \int_1^1 dx C_1(x, \xi; \mu^2/Q^2) \left( F_\perp(x, \xi) + i\epsilon_{\alpha\rho} \tilde F_\rho(x, \xi) \right),$$

(47)

Here we used symmetry properties of skewed parton distributions (33) in order to rewrite the contribution from crossed diagrams in terms of direct ones.

Note now that term containing $\Delta_1^\alpha F(x, \xi)$ may interfere with a corresponding piece of the $L \to L$ amplitude $T^\mu_0$ which appears at the NLO and describes the twist-2 transition between longitudinally polarized initial and final photons. In the massless QCD the corresponding contribution to the hadronic tensor $T^\mu_0$ has logarithmic singularity $\sim \ln [q^2/q^2]$ as the virtuality of the final photon tends to zero. Of course, the $L \to L$ scattering amplitude obtained by contraction of $T^\mu_0$ with polarization vectors of initial and final photons vanishes in this limit. In the Feynman gauge the contribution to $T^\mu_0$ arises from the box diagram [14]. From the general parametrization, valid for non-zero $q^2$:

$$T^\mu_0 = \frac{1}{(Pq)^2} \left( q^\mu (Pq) - P^\mu q^2 \right) \left( q^\nu (Pq) - P^\nu q^2 \right) A_{LL},$$

(48)

one finds that formally $T^\mu_0$ has a twist-3 part which can indeed contribute to $T^\mu_0$ with index $\mu$ transverse and $\nu$ longitudinal. Retaining only the contribution singular as $q^2$ goes to zero, one finds:

$$T^\mu_0 \mid_{tw-3} = (q + 4\xi P)^\mu (-\Delta^\mu_0) A_{LL},$$

$$A_{LL} = \frac{1}{2(Pq)} \frac{\alpha_s(\mu^2)}{4\pi} C_F \int_1^1 dx \frac{F_1(x, \xi)}{(\xi + x)^2} \ln \left[ \frac{q^2/q^2}{(\xi + x)(\xi - x)} \right].$$

(49)

Self-consistency of such requires to perform additional finite renormalization of the local axial current operator [21].
Note that expression for $A_{LL}$ is finite as the space-time dimension $d = 4$ but instead it has singularity when $q'^2 \to 0$. We regulated it by a small non-zero value of $q'^2$. In order to obtain the NLO $L \to T$ amplitude the contribution (49) must be subtracted from result of calculation of diagrams in Fig. 4 in $d = 4 - 2\epsilon$ dimensions and with $q'^2 \neq 0$. In practice, in the Feynman gauge only the box graph gives contribution singular as $q'^2 \to 0$.

Explicit calculation according to the discussion above gives in $d = 4 - 2\epsilon$ dimensions the following result for the coefficient functions $C^{(1)}_{\perp}(x, \xi; \mu^2/Q^2)$, $C(x, \xi; \mu^2/Q^2)$ and $\tilde{C}(x, \xi; \mu^2/Q^2)$ in (47):

$$C^{(1)}_{\perp}(x, \xi; \mu^2/Q^2) = \frac{1}{x + \xi} \left\{ -\frac{1}{\epsilon} \left[ 1 + 2 \ln \left( \frac{\xi + x}{2\xi} \right) \right] (1 + \epsilon \ln[\mu^2/Q^2]) - 5 - 2 \ln \left( \frac{\xi + x}{2\xi} \right) + 4 \ln \left( \frac{\xi - x}{2\xi} \right) - \frac{\xi - x}{\xi + x} \ln \left( \frac{\xi - x}{2\xi} \right) + \ln \left( \frac{\xi + x}{2\xi} \right)^2 \right\} ,$$

$$C(x, \xi; \mu^2/Q^2) = \frac{1}{\epsilon \xi (\xi + x)} (1 + \epsilon \ln[\mu^2/Q^2]) + \frac{1}{\xi^2 - x^2} \left( \frac{2x}{\xi^2} - \ln \left( \frac{\xi + x}{2\xi} \right) + \frac{2x}{\xi - x} \ln \left( \frac{\xi + x}{2\xi} \right) \right) ,$$

$$\tilde{C}(x, \xi; \mu^2/Q^2) = \frac{1}{\epsilon \xi (\xi + x)} (1 + \epsilon \ln[\mu^2/Q^2]) + \frac{1}{\xi^2 - x^2} \left( 2 - \ln \left( \frac{\xi + x}{2\xi} \right) - \frac{2x}{\xi - x} \ln \left( \frac{\xi + x}{2\xi} \right) \right) .$$

Two important comments are of order here. First, in the above equations it is understood that $\xi$ has infinitesimally small imaginary part $\xi \to \xi - i\epsilon$. Moreover, as the combination $F_{\perp\alpha} + i\epsilon_{\alpha\rho}^\perp \tilde{F}_\perp^\rho$ is continuous at the point $x = -\xi$, the resulting integrals in (47) are well defined, in complete analogy with the situation in the LO amplitude. However, in order to complete proof of factorization of the $L \to T$ amplitude at the NLO one has to demonstrate more, namely that the structure of singular terms is indeed such that they can be absorbed, following the standard procedure, into renormalization of the LO $L \to T$ amplitude.

In order to show this one has to derive one-loop evolution equations for the combination $F_{\perp\alpha} + i\epsilon_{\alpha\rho}^\perp \tilde{F}_\perp^\rho$ which appears at the LO in (12). One possibility is to use definitions (21),(22) and the well-known evolution equations of the twist-2 SPD’s $F$ and $\tilde{F}$, see, for instance, [3]. Another way is to compute directly the one-loop evolution of the transverse components of the light-cone matrix elements (15) and (16). The result is:

$$\mu^2 \frac{d}{d\mu^2} (F_{\perp\alpha} + i\epsilon_{\alpha\rho}^\perp \tilde{F}_\perp^\rho)(x, \xi) = \frac{\alpha_s(\mu^2)}{2\pi} C_F \int_{-1}^{1} du \left[ V_+(u, x, \xi)(F_{\perp\alpha} + i\epsilon_{\alpha\rho}^\perp \tilde{F}_\perp^\rho)(u, \xi) + U_+(u, x, \xi)(\Delta_\perp F + i\epsilon_{\alpha\rho}^\perp \Delta_\perp \tilde{F})(u, \xi) \right] .$$

The evolution kernels are defined as:

$$V_+(u, x, \xi) = \left[ \theta(\xi - x < u) - \theta(u < x < \xi) \right] \left( \frac{1}{u - x} \right) ,$$

$$+ \left[ \theta(-\xi < x < u) - \theta(u < x < -\xi) \right] \left( \frac{1}{u - x} \frac{(\xi + x)^2}{(\xi + u)^2} \right) .$$
\[ U_+(u, x, \xi) = \left[ \theta(\xi < x < u) - \theta(u < x < \xi) \right] \frac{\xi + x}{4\xi^2(u - \xi)} + \theta(\xi < x < u) - \theta(u < x < -\xi) \left( \frac{\xi + x(3\xi + u)}{4\xi^2(u + \xi)^2} \right) \quad (56) \]

\[ -\left[ \theta(-\xi < x < u) - \theta(u < x < -\xi) \right] \left( \frac{\xi + x}{4\xi^2(u - \xi)} + \frac{\xi + x}{4\xi^2(u + \xi)^2} \right). \quad (57) \]

Here the plus prescription is to be understood as
\[ (X(u, x, \xi))_+ = X(u, x, \xi) - \delta(x - u) \int dz X(u, z, \xi) \quad (58) \]

As one can see the evolution equation of the WW contribution is not homogenous, i.e. the RHS of (53) receives contributions which are not given in terms of the combination of GPD’s on the LHS. Precisely these terms generate contributions to the one-loop renormalization of the LO amplitude which exactly match singular terms arising from “bare” coefficients \( C \) and \( \tilde{C} \). Indeed, with the help of equations (54) and (56) one finds that convolution integrals of the tree level coefficient function with kernels \( V_+ \) and \( U_+ \) do reproduce the pole contributions in (50), (51) and (52) as it should be:

\[ \frac{\alpha_s(\mu^2)}{4\pi} C_F \int_{-1}^1 dx \frac{-2}{(\xi + x)} V_+(u, x, \xi) = -\frac{\alpha_s(\mu^2)}{2\pi} C_F \frac{1}{\xi + u} \left( 1 + 2 \ln \frac{\xi + u}{2\xi} \right), \quad (59) \]

\[ \frac{\alpha_s(\mu^2)}{2\pi} C_F \int_{-1}^1 dx \frac{-2}{(\xi + x)} U_+(u, x, \xi) = \frac{\alpha_s(\mu^2)}{2\pi} C_F \frac{1}{\xi} \frac{1}{\xi + u}. \quad (60) \]

Now, combining equations (50), (51), (52) with (59) one obtains the NLO Wilson coefficients in their final form:

\[ C^{(1)}_-(x, \xi; \mu^2/Q^2) = \frac{1}{x + \xi} \left\{ -\left[ 1 + 2 \ln \frac{\xi + x}{2\xi} \right] \ln[\mu^2/Q^2] \right. \]

\[ -5 - 2 \ln \left[ \frac{\xi + x}{2\xi} \right] + 4 \ln \left[ \frac{\xi - x}{2\xi} \right] - \frac{\xi - x}{\xi + x} \ln \left[ \frac{\xi - x}{2\xi} \right] + \ln \left[ \frac{\xi + x}{2\xi} \right]^2 \right\}, \quad (61) \]

\[ C(x, \xi; \mu^2/Q^2) = \frac{1}{\xi(x + x)} \ln[\mu^2/Q^2] + \frac{1}{\xi^2 - x^2} \left( \frac{2x}{\xi} - \ln \left[ \frac{\xi + x}{2\xi} \right] + \frac{2x}{\xi - x} \ln \left[ \frac{\xi + x}{2\xi} \right] \right), \quad (62) \]

\[ \tilde{C}(x, \xi; \mu^2/Q^2) = \frac{1}{\xi(x + x)} \ln[\mu^2/Q^2] + \frac{1}{\xi^2 - x^2} \left( 2 - \ln \left[ \frac{\xi + x}{2\xi} \right] - \frac{2x}{\xi - x} \ln \left[ \frac{\xi + x}{2\xi} \right] \right). \quad (63) \]

The above equations are our main result. They explicitly show that the NLO correction to the \( L \to T \) amplitude can be put into a factorizable form in a self-consistent way.
Conclusions

In this paper we have demonstrated factorizability of the NLO correction to the twist-3, Wandzura-Wilczek, flavor non-singlet DVCS amplitude corresponding to scattering of longitudinal photon off nucleon. We have shown that, similarly to the LO, the singularity structure of the corresponding Wilson coefficients is such that the convolution integrals with SPD’s are well defined. After the interference with the twist-2 LL amplitude is properly taken into account all IR singular terms arising in the calculation of the $L \to T$ amplitude can be absorbed into renormalization of the LO result in a usual manner. Besides the phenomenological applications of new Wilson coefficients found in this paper our result strongly suggests that factorization holds for the twist-3, $L \to T$ DVCS amplitude beyond LO and possibly to all orders in the $\alpha_S$ expansion.

Acknowledgments

LM wants to acknowledge kind hospitality extended to him during numerous visits to Regensburg and Bochum Universities. Research reported in this paper has been supported in part by Polish-German PAN-DFG grant, Kovalevskaja Program of Alexander von Humboldt Foundation, by the BMBF and by the COSY-Julich project.

References

[1] D. Müller, D. Robaschik, B. Geyer, F.M. Dittes, J. Horejsi, Fortschr. Phys. 42 (1994) 101.
[2] X. Ji, Phys. Rev. D55 (1997) 7114.
[3] A. V. Radyushkin, Phys. Rev. D56 (1997), 5524.
[4] X. Ji and J. Osborne, Phys. Rev. D58 (1998) 094018.
[5] J. C. Collins and A. Freund, Phys. Rev. D59 (1999) 074009.
[6] P. R. Saull [ZEUS Collaboration], “Prompt photon production and observation of deeply virtual Compton scattering,” hep-ex/0003030.
[7] Rainer Stamen [H1-Collaboration], “Measurement of the Deeply Virtual Compton Scattering at Hera”, H1prelim-00-17, DIS 2000 and IHEP 2000.
[8] A. Airapetian [HERMES Collaboration], hep-ex/0106068.
[9] M. Amarian [HERMES collaboration], “DVCS and exclusive meson production measured by HERMES”, talk at workshop “Skewed Parton Distributions and Lepton - Nucleon Scattering”, DESY, Sept. 2000, http://hermes.desy.de/workshop/TALKS/talks.html
[10] K. Goeke, . V. Polyakov and M. Vanderhaeghen, Prog. Part. Nucl. Phys. 47 (2001) 401 [arXiv:hep-ph/0106012].
[11] A. V. Belitsky, D. Muller and A. Kirchner, Nucl. Phys. B 629 (2002) 323 [arXiv:hep-ph/0112108].
[12] X. D. Ji and J. Osborne, Phys. Rev. D 58 (1998) 094018 [arXiv:hep-ph/9801260].
[13] A. V. Belitsky and D. Muller, Phys. Lett. B 417 (1998) 129 [arXiv:hep-ph/9709379].
[14] L. Mankiewicz, G. Piller, E. Stein, M. Vanttinen and T. Weigl, Phys. Lett. B 425 (1998) 186 [arXiv:hep-ph/9712251].
[15] M. Penttinen, M. V. Polyakov, A. G. Shuvaev and M. Strikman, Phys. Lett. B 491 (2000) 96 [arXiv:hep-ph/0006321].
[16] A. V. Belitsky and D. Muller, Nucl. Phys. B 589 (2000) 611 [hep-ph/0007031].
[17] D. V. Kiptily and M. V. Polyakov, arXiv:hep-ph/0212372.
[18] N. Kivel, M. V. Polyakov, A. Schafer and O. V. Teryaev, Phys. Lett. B 497 (2001) 73 [arXiv:hep-ph/0007315].
[19] N. Kivel and M. V. Polyakov, Nucl. Phys. B 600 (2001) 334 [hep-ph/0010150].
[20] I. V. Anikin, B. Pire and O. V. Teryaev, Phys. Rev. D 62 (2000) 071501 [arXiv:hep-ph/0003203].
[21] S. A. Larin, Phys. Lett. B 303 (1993) 113 [arXiv:hep-ph/9302240].