The Search for Quantum Gravity Signals

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Abstract

We give an overview of ongoing searches for effects motivated by the study of the quantum-gravity problem. We describe in greater detail approaches which have not been covered in recent “Quantum Gravity Phenomenology” reviews. In particular, we outline a new framework for describing Lorentz invariance violation in the Maxwell sector. We also discuss the general strategy on the experimental side as well as on the theoretical side for a search for quantum gravity effects. The role of test theories, kinematical and dynamical, in this general context is emphasized. The present status of controlled laboratory experiments is described, and we also summarize some key results obtained on the basis of astrophysical observations.

1 Introduction and preliminary remarks

1.1 The search for quantum gravity

Our present description of the laws of physics may be characterized as obtained from two types of constituents. The first type of constituent are theoretical frameworks which apply to all physical phenomena at any instant. These “universal” or “frame” theories are Quantum Theory (all matter is of microscopic origin), Special and General Relativity; SR and GR, (all kinds of matter locally have to obey the principles of Lorentz symmetry and behave in the same way in gravitational fields), and statistical mechanics which is a method to deal with all kinds of systems for a large number of degrees of freedom. The second type of constituent is nonuniversal and pertains to the description of the four presently-known interactions: the electromagnetic, the weak, the strong and the gravitational. The first three interactions are all described within a single formalism, in terms of a gauge theory. So far only gravity has not been successfully included into that scheme. One reason for that might be that gravity appears on both sides: it is an interaction but it is at the same time a universal theory. Universal theories like relativity and gravity are geometric in origin and do not rely on the particular physical system under consideration, whereas a description in terms of a particular interaction heavily makes use of the particular particle content. Therefore, gravity plays a distinguished role which may be the reason for the difficulty encountered in attempting to unify the other interactions with gravity and attempting to quantize gravity.
The most pressing problem for present-day theoretical physics is the unification of quantum theory with gravity, the so-called “quantum-gravity problem”. The standard scheme of quantization has been proven to lead to inconsistencies when applied to gravity. In particular, the emerging theory would not be perturbatively renormalizable. Various ideas have been explored in alternative to the standard quantization procedure, and some of the most popular approaches are based on string theory, canonical/loop quantum gravity, or non-commutative geometry. It is hoped that these quantum-gravity approaches may also provide a scheme for the unification of the four interactions, something that string theory already comes very close to doing.

A general feature of all these quantum-gravity scenarios is the appearance of new effects, often mediated by new fields, and in particular some of these effects appear to require a modification of some of the fundamental principles which SR and GR are based on.

1.2 Implications of a new quantum gravity theory

The possibility of solving the quantum-gravity problem is often described as intellectually exciting but of mere academic interest. However, the fact that a significant sample of quantum-gravity approaches appears to lead to modifications of some aspects of SR and GR suggests that the implications may go well beyond the academic interest. This point applies in particular to the area of modern metrology, the definition, preparation and transport of physical units. Since various atomic clocks on Earth are located at different height and geographical positions, it is clear that the uniqueness of the definition of time in terms of the TAI, the international atomic time, relies on the validity of SR and GR, see Fig.1.

Another key objective of modern metrology is to base all units on distinguished quantum effects. The reason for that is the precision achieved and the universal reproducibility which is based on the uniqueness of quantum mechanics. Beside the second and the meter, where this already has been done, one can base the unit of the electrical resistance, the Ohm, on the quantum Hall effect using \( R_H = K/n \) where \( n \in \mathbb{N} \) and \( K = h/e^2 \) is the von-Klitzing constant, and the unit of the electrical voltage on the Josephson effect via \( U_J = n\nu/K_J \) where \( K_J = 2e/h \) is the Josephson constant and \( \nu \) a given frequency, see Fig.2. These quantum definitions of units heavily rely on the validity of basic dynamical equations like the Maxwell, the Schrödinger, and the Dirac equation. If one of these equations was to be modified then some definitions of units would also be affected. Therefore, any high precision test of Lorentz invariance is also a test of the modern metrological scheme.

Moreover, if it turns out that the Maxwell and Dirac equations have to be modified then this may complicate future high precision navigation.

1.3 The magnitude of quantum gravity effects

Since the typical laboratory energies are of the order of 1 eV and the characteristic quantum-gravity energy scale is expected to be of the order of the Planck energy which is about \( 10^{28} \) eV, the
quantum-gravity effects in laboratory experiments are likely to come in at the order of $10^{-28}$ (or lower) which appears to be well beyond the reach of laboratory experiments, in spite of the many high-precision devices which are becoming available for searching for new effects. As stressed in several recent “quantum gravity phenomenology” reviews [2, 3, 4, 5] this suggests that the relevant phenomenology should rely on contexts of interest in astrophysics, rather than use controlled laboratory experiments. However, there may nevertheless be an important role for laboratory experiments in this phenomenology, especially considering the following observations:

- The arguments that suggest that the characteristic quantum-gravity energy scale should be of order $10^{28}$ eV must at present be viewed as inconclusive. One really needs the correct theory, which is still not established, in order to reliably estimate this scale. It may well be that the characteristic quantum-gravity energy scale is actually much lower than $10^{28}$ eV. For example, in scenarios with “large extra dimensions” the quantum-gravity effects, including deviations from Newton’s law, would be accessible at much lower energies (see e.g. [6]). Similarly, in some string-theory-motivated “dilaton scenarios” [7, 8] the Universality of Free Fall (UFF) would be violated already at the $10^{-13}$ level, and the PPN parameter $\gamma$, which in ordinary Einsteinian gravity is exactly 1, might be different from unity by up to $10^{-5}$.

- Even within the assumption that the quantum-gravity effects actually do originate at the Planck scale, one can find some (however rare) cases in which the small $10^{-28}$ effect is effectively “amplified” [2] by some large ordinary-physics number that characterizes the laboratory setup. For example, there has been considerable work on the possibility that quantum-gravity effects might significantly affect some properties of the neutral-kaon system [9, 10] and in those analyses the effects are amplified by the fact that the neutral-kaon system hosts the peculiarly small mass difference between long-lived and the short-lived kaons $M_{L,S}/|M_L - M_S| \sim 10^{14}$.
Validity of Maxwell and Schrödinger equations

Figure 2: Electrical units of the voltage $V$ (Volt), electrical resistance $\Omega$ (Ohm), and electrical current $A$ (Ampere) based on distinguished quantum effects. The experimental effects are interpreted in terms of the ordinary Maxwell and Schrödinger equation.

Similarly, in studies of quantum-gravity-induced interferometric noise [11] one can exploit the fact that some scenarios predict noise that increases as the frequency of analysis decreases, and therefore, searching for fundamental noises in high precision long–term stable devices (like optical resonators) may give new access to this domain of quantum gravity effects [12].

In an analogous way, the sensitivity of some clock–comparison experiments is amplified by a factor of $\frac{\hbar \nu}{m_p c^2} \sim 10^{18}$ where $\nu$ is the clock frequency used and $m_p$ the proton mass, see Sec.4.3.6.

- In some rare cases the laboratory experiment actually does have the required $10^{-28}$ sensitivity, as illustrated by the analysis reported in Ref. [13], which studies the implications of quantum-gravity-induced wave dispersion for the most advanced modern interferometers (the interferometers whose primary intended use is the search of classical gravity waves).

- The fact that low-energy data suggest that the electroweak and strong interactions unify at the GUT energy scale of $\sim 10^{16} \text{ GeV}$ may encourage us to conjecture that also the gravitational interaction would be of the same strength as the other interactions at that scale. This would mean that the characteristic energy scale of quantum gravity should be some three orders of magnitude smaller than the Planck energy (see e.g. [14]).

It appears therefore that we should complement the astrophysics searches with a wide program of laboratory/controlled experiments searching for the effects predicted by some quantum-gravity approaches. Although in most cases the potential sensitivity of astrophysics searches will be higher, the laboratory experiments have the advantage of providing access to a much larger variety of potential phenomena, and for the development of the relevant phenomenology it would be an extraordinary achievement to be able to analyze a new effect within a controlled/repeatable laboratory experiment with the possibility of a systematic modification of boundary and initial conditions, rather than relying on the “one chance” observations available in astrophysics.
1.4 Main quantum gravity schemes

As mentioned, the most popular approaches to the quantum-gravity problem are based on string theory, canonical/loop quantum gravity, or non-commutative geometry, see, e.g., [15]. For different reasons the development of phenomenology for each of these three ideas is still at a rather early stage of development. But several hints for experimentalists have already emerged.

- In canonical/loop quantum gravity the key difficulty is the fact that the techniques for obtaining the classical limit of the theory have not yet been developed. Since our phenomenology will usually be structured as a search of corrections to the classical effects, this is a very serious issue. However, several authors [16, 17, 18, 19], guided by the intuition from working with some candidate quasiclassical states, made analyses that led to the expectation that Lorentz symmetry should be broken in Loop Quantum Gravity, and as a result the Maxwell and Dirac equations should include extra terms of higher derivatives. But clearly these violations from Lorentz symmetry still cannot be viewed as a “prediction” of Loop Quantum Gravity because of the heuristic nature of the underlying arguments, and indeed there are some authors (see, e.g. Ref. [20]) who have presented arguments in favour of exact Lorentz symmetry for Loop Quantum Gravity.

- String theory schemes in any dimensions always predict new fields which couple in different ways to the various matter sectors. As a result a large variety of effects are allowed by string theory, including effects leading to violation of Lorentz invariance, violation of the UFF and violation of the Universality of the Gravitational Redshift (UGR) is encountered. From the viewpoint of the phenomenologist a disappointing aspect of string theory is that it appears that it cannot be falsified on the basis of these effects. String theory may predict many new low-energy effects, but it can also be easily tuned to avoid all of them. But of course it is still interesting to look for these new effects, and indeed a rich phenomenology is being developed. In particular, there is considerable work on a string-theory-motivated dilaton scenario [7] with deviations from the UFF at the order $10^{-13}$ and deviations of the PPN parameters $\gamma$ and $\beta$ at the order $10^{-5}$ and $10^{-9}$, respectively. Another large phenomenological effort is being devoted to a string-theory-motivated general framework, the Standard Model Extension (SME) [21], for the description of violations of Lorentz invariance that are codified in power-counting-renormalizable terms.

- Much of the quantum-gravity work based on noncommutative geometry has focused on the hypothesis that the correct quantum gravity should have as flat-spacetime limit a noncommutative version of Minkowski spacetime, with spacetime coordinate noncommutativity of the type $[x_\mu, x_\nu] = i\theta_{\mu\nu} + i\zeta_{\mu,\nu} x_\sigma$ for some appropriate choice of the coordinate-independent $\theta_{\mu\nu}$ and $\zeta_{\mu,\nu}$. In a noncommutative spacetime there is an absolute limitation on the localization of a spacetime point (event), and as a result the relevant theories are inevitably nonlocal. In some frameworks it is possible to give an effective commutative-spacetime description of this nonlocality, in which in particular – in terms of partial differential equations – higher order derivatives may occur. Another rather generic feature of these noncommutative versions of Minkowski spacetime is the emergence of anomalous dispersion relations. The simplest and most studied example of such an anomalous dispersion relation is

$$m^2 \simeq E^2 - \vec{p}^2 + \eta \vec{p}^2 \frac{E}{E_P} + O\left(\frac{E^4}{E_P^2}\right), \quad (1)$$

where $E_P$ is the Planck scale and $\eta$ is a numerical factor (expected to be of order 1) describing
the strength of the quantum gravity modification \( E_{QG} \equiv E_P/\eta \) is the characteristic scale of the modification of the dispersion relation.

| Geometrical characteristics | String Theory | Canonical/Loop quantum gravity | Non-commutative geometry |
|-----------------------------|---------------|--------------------------------|--------------------------|
| Particle characteristics    | starts from fixed (flat or deSitter) background depends on particle model | dynamical model independent | starts from fixed background model independent |

| Violations of               | Lorentz invariance | Lorentz invariance? (may depend on boundary conditions) | Lorentz invariance |
|-----------------------------|--------------------|-------------------------------------------------------|-------------------|
|                            | Universality of Free Fall | Universality of Free Fall? (not yet calculated) | Universality of Free Fall? (not yet calculated) |

1.5 Methods of phenomenological generalizations of dynamical equations

Since it was not covered extensively in recent quantum-gravity-phenomenology reviews [2, 3, 4, 5] we will here devote particular attention to the study of phenomenological generalizations of dynamical equations like the Dirac or the Maxwell equations. As always in quantum gravity phenomenology, these studies must be structured in terms of some test theories, bridging the gap between the rich (but often untreatable) formalisms used in the study of the full quantum gravity problem and the language which is appropriate to describe experiments. The structure of these test theories should be obtained by calculating some low energy approximation of the full quantum gravity scenario. This has been attempted for the “Liouville String” scenario [22, 23] as well as for the loop quantum gravity approach [16, 17, 18, 19]. But since various technical issues remain to be understood about the relevant approximation schemes, the phenomenology is being developed on the basis of rather general test theories describing modifications of the Dirac and the Maxwell equations.

One approach, which is being pursued mostly as starting point of the SME [24, 21, 25], adopts some general Lagrangian which is still quadratic in the field strengths or in the fermionic fields and requires further building principles like conservation of energy-momentum, Lorentz-covariance, conventional quantization, Hermiticity, microcausality, positivity of energy, gauge invariance, and power-counting renormalizability. The parameters of the SME are just additional interactions with constant fields\(^1\). The main advantages of this approach are the mathematical consistency and physical interpretability of the new theory in conventional terms and the fact that it provides the most conservative modification of established theories. But of course this may also turn into a disadvantage, if it eventually turns out that the correct quantum gravity requires more novel features, such as effects that are not described by power-counting-renormalizable terms. And indeed some authors have chosen to look beyond the SME setup, considering Planck-scale-suppressed effects which are in fact not described by power-counting-renormalizable terms (see, e.g., Refs. [28, 16, 29]).

Of course, it is also possible to renounce to the assumption of a Lagrangian generating the dynamical equations, and in fact there is a rich phenomenology being developed introducing the generalizations directly at the level of the dynamical equations. This is of course more general than the Lagrangian approach (see [30, 31] for examples dealing with generalized Dirac and Maxwell equation). In the case of the Dirac equation [30] one is led to effects like the violation of the UFF, of the UGR and of Local Lorentz Invariance (the effects violating Local Lorentz Invariance have been

\(^1\)This kind of generalizations in the photonic sector have been introduced and discussed earlier by Ni [26] and Haugan and Kauffmann [27].
also obtained later on in [25]). More effects than in the SME are encountered for the generalized Maxwell equation discussed in [31]. Charge conservation, which automatically comes out from the Lagragian approach, can be violated in models generalizing the field equations [31]. In addition, in [31] also more Lorentz Invariance violating parameters than in the SME have been found. However, particular care must be taken for the mathematical consistency of the formalism if one "by hand" generalizes the dynamical equations. This will be automatically secured if one employs a constructive axiomatic scheme in order to derive equations, as shown for example in Refs. [32, 33, 34] within a derivation of the generalized Dirac equation in terms of fundamental properties of the dynamics of fields.

1.6 Kinematical test theories for Lorentz invariance

Instead of modifying the dynamical equations, some authors have preferred to introduce new-physics effects at the level of kinematics. In particular, there is significant work on kinematical test theories for Lorentz invariance. Later on in these notes we will comment on some recent proposals of this type which have been motivated rather directly by some approaches to the quantum-gravity problem. We here want to comment on a predecessor, the Robertson–Mansouri–Sexl (RMS) theory, which was introduced by Robertson [35] and Mansouri and Sexl [36], where a modification of Lorentz–transformations are the basis for, e.g., anomalous effects of light propagation what can be tested. Compared with the dynamical approach the kinematical approach is more powerful since it is independent of the specific particle model under consideration: it discusses the transformation properties of observed quantities under transformations between inertial systems. One may however be uncomfortable with the fact that this RMS model assumes that in one frame light propagates isotropic. This requires to single out a preferred frame, which is usually taken to be the frame given by the cosmological radiation background. But this preferred role for the cosmological radiation background, which is after all a classical-physics feature, is not necessarily compelling. And one might ask how this programme should proceed if it turns out that, for example, there is also a stochastic gravitational wave background, which selects a different frame with respect to the one natural for the radiation background. Should one then make a choice? This choice would influence significantly the interpretation of the experiments. Some authors are also uncomfortable with the RMS scheme because of its lack of generality with respect to certain issues. In particular, it is not obvious that one should assume that in the preferred frame light propagates isotropically, something that might be violated in a Finslerian space–time, and the RMS scheme also does not allow a description of possible violations of Lorentz invariance leading to birefringence effects.

1.7 The scheme of exploring new physics with generalized Maxwell and Dirac equations

As mentioned in this notes we will devote particular attention to the possibility of exploring new physics with generalized Maxwell and Dirac equations. There is a long history in the discussion of violations of Lorentz invariance and of the UFF (also called Weak Equivalence Principle) due to some modifications of the Maxwell and Dirac equations.

Very early considerations are based on Mach’s principle and led to considerations of a hypothetical anomalous inertial mass term in the Schrödinger equation [37] which subsequently has been tested with very high precision by Hughes [38] and Drever [39], see also Sec.4.3.6. Later discussions have been performed by Liebscher and Bleyer [40, 41]. The most general Dirac equation based on basic properties like unique evolution, superposition principle, finite propagation speed and probability conservation has been discussed in [33]. Based on this, a generalized Pauli equation has been derived [30] which is most appropriate to confront this generalized theory with experiments. Since
the spin-$\frac{1}{2}$-sector of the SME is contained in this general scheme, also there a generalized Pauli equation can be derived [42].

Modifications of the Maxwell equations in view of a discussion of violations of Lorentz invariance and of the UFF has been first introduced by Ni [43, 26] and Haugan and Kauffmann [27]. The same sort of modifications can be found in the SME [25, 44]. More general modifications which also include charge non-conservation have been introduced in [31].

For exploring new physics in these frameworks, at first one discusses and explores isolated effects for the Dirac and the Maxwell equation. These are propagation phenomena of electromagnetic and matter waves and, in the case of electrodynamics, the static solutions for a point-like charge and magnetic moment. In a next step, combined effects which appear in electromagnetically bound systems have to be calculated, see, e.g., Sec.4.3.3. These combined effects govern the physics of atoms, molecules, and solids which serve as realizations of clocks, and of rulers.

1.8 What is a violation of Lorentz invariance?

A key focus for these notes are studies of possible departures from Lorentz symmetry. For this phenomenology a key question is how we may interpret or how we may proceed if we are detecting a signal which appears to violate Lorentz invariance. After all the attempts at a conventional-physics explanation have failed, and it is therefore certain that one is dealing with new physics, it would be nice to have some criteria for establishing what constitutes a robust basis for accepting or recognizing a violation of Lorentz invariance.

Of course in science one never establishes how things “are”, but one rather finds models that reproduce all observations. Therefore any interpretation will always be subject to further scrutiny. Still it is useful to keep in mind that some checks of consistency with the hypothesis of violation of Lorentz symmetry can be made. In particular, it is (at least in principle) relatively easy to distinguish between the hypothesis of a violation of Lorentz symmetry and the hypothesis of the discovery of a new interaction. If the effect signals a new interaction it should be possible to shield it (like the electromagnetic interaction), transform it away (like metrical gravity) and/or identify a cause of the effect. A violation of Lorentz invariance should instead be universal and it should not be possible to transform it away.

1.9 Main directions in searches for new physics

At present there are at least six main directions in the search for possible quantum gravity effects:

Search for orientation dependent effects. The effects one is looking for depend on the orientation of the laboratory. Early experiments of this kind are Michelson–Morley (see Sec.4.3.4) and Hughes–Drever experiments (see Sec.4.3.6). For power-counting-renormalizable effects the results of all experiments of this kind can be related to the SME, which gives the most general parametrization of power-counting-renormalizable anisotropy effects in various basic equations. Presently, for photons the anisotropy is limited by $10^{-15}$ and for nuclear matter by $10^{-30}$.

Search for a violation of the Universality of Free Fall. Within the string-theory-inspired dilaton scenarios there is a distinguished prediction of a violation of the UFF at the $10^{-13}$ level [7]. Presently, the UFF is confirmed at the level $5 \cdot 10^{-13}$.

Also anomalous spin couplings are considered (see, e.g., [45] for a short review), which are beyond the standard coupling to the gravitational field. Other works considering possible modifications of the UFF for various particles are [46, 47].
Figure 3: Schematics of the implications of modified Dirac and Maxwell equations.
Search for quantum gravity effects

Search for anisotropies
Search for violations of UFF
Search for violations of UGR
Search for anomalous dispersion
Search for space–time fluctuations

Figure 4: The main directions and characteristics of searches for quantum gravity induced effects: anisotropy effects, violation of UFF and UGR, modified dispersion and space–time fluctuations. A search for violations of isotropy, UFF and UGR amounts to a search of a violation of Einstein’s Equivalence Principle.

Search for a violation of the Universality of the Gravitational Redshift. String theory may also host deviations from the UGR [48]. In fact, it has been shown that violations and UFF and of UGR are strongly related [49, 50].

Presently, the UGR is confirmed by clock–clock comparison (atomic fountain clock vs. H–maser) at the $1.5 \cdot 10^{-5}$–level [51].

Search for a modified dispersion relation. Searches of modifications of the dispersion relation of the form (1), and its generalizations, constitute another direction of experimental quantum gravity efforts. This type of effect is beyond the SME, since it could not be described by power-counting-renormalizable terms. A possible manifestation of the modified dispersion relations is an energy dependent propagation velocity which should lead to different time–of–arrivals of the same event on a distant star when looked at it in different frequency channels, see e.g. [52]. Another effect is a frequency dependent position of interference fringes [13].

Search for space–time fluctuations. Search for fundamental space–time fluctuations [11] and associated fundamental decoherence effects in quantum systems [53, 54] have been also considered extensively in association with various classes of experiments [55, 12]. There has also been work on the implications of space–time fluctuations for the spreading of signals, which causes a sharp signal to turn into a fuzzy signal upon relatively long propagation times [2, 56].

Search for departures from CPT symmetry Since the CPT theorem is based on locality and Lorentz covariance, it is not surprising that the quantum-gravity approaches that involve some nonlocality and/or departures from Lorentz symmetry provide motivation for searches of departures from CPT symmetry. Noteworthy limits have been obtained using some analyses of the neutral-kaon system (see Ref. [57] and references therein) and other limits are now being pursued in the context of neutrino physics [10]. Indeed, it has been shown that CPT violation implies a violation of Lorentz invariance [58].
2 Quantum gravity predictions

2.1 String theory

The low-energy physics of String theory, as presently understood, could be very rich, with the presence of new fields and the possibility of a variety of new effects. This may provide the basis for a large phenomenological effort, even though one should keep in mind that the new effects are not genuine “predictions” of String theory: String theory could make room for these effects but it could equally well suppress them all. It appears at present not possible to falsify String theory on the basis of low-energy phenomenology, but it would nonetheless be very exciting if any of the new effects that String theory may host was actually found.

Lorentz invariance A possible violation of Lorentz invariance, due to spontaneous symmetry breaking, has been considered [59]. Though requirements like renormalizability and gauge invariance rule out spontaneous symmetry breaking of the Lorentz group in ordinary field theories it might happen for string theories. Lorentz symmetry breaking may arise from tensor–tensor–scalar couplings that are allowed because strings are extended objects which contain an infinite number of particle modes.

Universality of Free Fall The UFF is the most basic principle underlying GR. It is responsible for the possibility to geometrize the gravitational interaction and, thus, for the present understanding of space and time. However, in some string-inspired dilaton scenarios [60, 61, 7, 8] is has been claimed that the UFF might be violated, in terms of the Eötvös parameter, at the $10^{-13}$ level. Similar predictions are made by Wetterich [62] from a quintessence model where the dynamics of a cosmological field introduced to explain the dark energy, yields a violation of the UFF at the $10^{-14}$ level. Also in the “Liouville noncritical string” scenario [63] there are predictions for a violation of UFF due to the interaction of particles with the space–time foam which, however, are too small to be of today’s experimental interest.

Universality of Gravitational Redshift The same string-theory scalar field, which may couple to different matter fields in a different way thus leading to a violation of UFF, also leads to the effect that clocks based on different physical principles behave differently in the gravitational field.
and, thus, violate the UGR. This has been shown in [64, 48]. The relation between violations of UFF and UGR has been discussed in [49, 50]. Also quintessence models predict a violation of UGR [65].

**Anomalous dispersion** Since, in principle, the effective equations for the electromagnetic field and also for the Dirac field contain higher order derivatives, anomalous dispersion relations of the structure given in Eq.(1) could also come out from string theory [25]. However, in most cases they are neglected in the string-theory context since it is believed that they are less significant than the modifications, e.g., of the Dirac matrices in the case of the Dirac equation or of the constitutive relation in electrodynamics.

**Space–time fluctuations and decoherence** In the most popular “critical superstring” formulation of String Theory there has been so far no direct argument in favour of spacetime fluctuations and decoherence. However, in the “Liouville noncritical String” approach [66] space–time fluctuations and decoherence have been considered. For decoherence, possible implications for the neutral-kaon system have been investigated (see, e.g., Ref. [67] and references therein), while a direct investigation of spacetime fluctuations has been attempted by Percival and coworkers [53, 54].

### 2.2 Loop quantum gravity

As mentioned, in canonical/loop quantum gravity the key difficulty is the fact that the techniques for obtaining the classical limit of the theory have not yet been developed. In a certain sense one has an otherwise attractive candidate for quantum gravity, which however has not been shown to actually contain classical gravity as a limit. As a way to get some intuition for the type of effects that the theory might predict, when fully understood, several authors [16, 17, 18, 19] have proposed to start with the exploration of the properties of some candidate quasiclassical states.

**Lorentz invariance** By introducing the so–called weave states, which are candidate quasiclassical states of the space–time geometry, in the range between pure quantum geometry and classical geometry, one can motivate effective equations for the propagation of spinors and of electromagnetic fields [17, 18, 19, 68]. The resulting effective equations possess terms which violate Lorentz invariance, and also terms of higher order derivative. As mentioned there are some critiques on this approach, notably by Kozameh [20] who claimed that the breaking of LI came in by a particular choice of boundary conditions. Recently, it was also observed [69] that a plausible path toward the Loop-Quantum-Gravity classical limit should lead to a situation in which Lorentz symmetry is neither preserved nor broken, it would be deformed, in the sense we discuss more carefully later in our remarks on noncommutative spacetimes.

**Universality of Free Fall and Universality of Gravitational Redshift** To our knowledge there has been no analysis so far suggesting violations of the UFF and UGR in Loop Quantum Gravity. This is not surprising in light of the mentioned “classical limit” problem: one should first verify UFF and UGR in the classical limit and then look for possible quantum corrections.

**Anomalous dispersion** A modification of the energy-momentum dispersion relation is rather typical (though not inevitable) when Lorentz symmetry is broken. The arguments [17, 18, 19, 68] that suggest breaking of Lorentz symmetry in Loop Quantum Gravity do indeed provide motivation for modifications of the dispersion relation, since they lead to modified Maxwell and Dirac equations with higher order derivatives, suggesting a modified dispersion relation of the structure (1).
Space–time fluctuations and decoherence  There has not been much discussion of Space–
time fluctuations and decoherence in Loop Quantum Gravity. However, a recent proposal [70] for
a reformulation of Loop Quantum Gravity does lead to decoherence and the need to set up the
analysis within a density-matrix framework.

2.3 Non–commutative geometry

The formalism of noncommutative geometry appears to have rather wide (though not completely
general) applicability in the study of the quantum-gravity problem. In particular, a large number
of studies has been devoted to the hypothesis that the correct quantum gravity might admit a
regime which could be based on a non-commutative version of Minkowski spacetime. In some cases
non-commutative spacetimes prove useful at an effective-theory level (for example, in certain string
theory pictures [71] spacetime noncommutativity provides an effective theory description of the
physics of strings in presence of a corresponding external background), while other quantum-gravity
approaches (see, e.g., Ref. [72]) explore the possibility that a noncommutativity might be needed
for the correct fundamental description of spacetime.

The most studied noncommutative versions of Minkowski spacetime all fall within the parametriza-
tion \((\mu = 0, 1, 2, 3)\)

\[
[x_\mu, x_\nu] = i\theta_{\mu\nu} + i\zeta^\sigma_{\mu\nu} x_\sigma , \tag{2}
\]

with coordinate-independent \(\theta_{\mu\nu}\) and \(\zeta^\sigma_{\mu\nu}\). The choice of \(\theta_{\mu\nu}\) and \(\zeta^\sigma_{\mu\nu}\) is the key theoretical input and
the aspect to be determined experimentally.

Lorentz invariance  An intense research effort has been devoted to the implications of noncom-
mutableity for the classical Poincaré symmetries of Minkowski spacetime. This is in fact one of the
few aspects of the relevant theories which can be analyzed rather exhaustively, and it leads to some
ideas for experimental testing. For the simplest noncommutative versions of Minkowski spacetime,
the canonical noncommutative spacetimes characterized by coordinate noncommutativity of type

\[
[x_\mu, x_\nu] = i\theta_{\mu\nu} \tag{3}
\]

with coordinate-independent \(\theta_{\mu\nu}\), a full understanding has been matured, and in particular it has
been established that the Lorentz-sector symmetries are broken [71, 73, 74] (in a sense analogous to
the popular mechanism of spontaneous symmetry breaking) by this type of noncommutativity.

At the next level of complexity [75], the one of Lie-algebra noncommutative spacetimes

\[
[x_\mu, x_\nu] = i\zeta^\sigma_{\mu\nu} x_\sigma , \tag{4}
\]

the description of symmetries is in some cases more demanding at the technical level. The most
studied of these Lie-algebra versions of Minkowski spacetime is “\(\kappa\)-Minkowski [76, 77] \((l, m = 1, 2, 3)\)

\[
[x_m, t] = \frac{i}{\kappa} x_m , \quad [x_m, x_l] = 0 . \tag{5}
\]

\(\kappa\)-Minkowski is a Lie-algebra spacetime that clearly enjoys classical space-rotation symmetry; moreover,
at least in a Hopf-algebra sense (see, e.g., Ref. [78, 79]), \(\kappa\)-Minkowski is invariant under
“noncommutative translations”. It is particularly noteworthy that in \(\kappa\)-Minkowski boost transfor-
mations are necessarily modified [78, 79, 80]. A first hint of this comes from the necessity of a
deformed law of composition of momenta, encoded in the so-called coproduct (a standard structure
for a Hopf algebra). As a result in this spacetime one expects departures from classical Lorentz
symmetry but of a type that does not lead to the emergence of a preferred class of inertial observers
(the “Relativity Principle” is still upheld). One usually refers to these cases as “deformations of Lorentz symmetry” (in alternative to the more familiar mechanisms that may lead to broken Lorentz symmetry).

**Universality of Free Fall and Universality of Gravitational Redshift**  Concerning the UFF and the UGR these approaches based on noncommutative versions of Minkowski spacetime are insufficient for the derivation of rigorous predictions, since indeed they focus on the Minkowski limit. They are motivated by the idea of some form of “quantization” of gravity, but at present they only incorporate this quantization in the noncommutativity of spacetime and gravitational effects are not yet described (although some attempts of generalization are under way).

**Anomalous dispersion**  The fact that noncommutative versions of Minkowski spacetime typically lead to departures from classical Lorentz symmetry (broken Lorentz symmetry in most cases, and, as mentioned, “deformed” Lorentz symmetry in some special cases like $\kappa$-Minkowski) in turn leads to the emergence of departures from the special-relativistic energy-momentum (“dispersion”) relation.

For the cases affected by IR/UV (infrared/ultraviolet) mixing the phenomenology based on modified dispersion relations must be conducted cautiously, since the IR/UV mixing may lead to infrared singularities, which are still not fully understood (or at least there is still a lively debate on how to handle them in phenomenology). We do not to comment on this possible phenomenology in these notes, but refer to Refs. [81, 74, 82].

When there is no singularity in the infrared, and assuming that the onset of the new effects is characterized by the Planck length, one will typically find (and indeed one finds in the examples which have been analyzed) dispersion relations that fall within the class

$$m^2 \simeq E^2 - \vec{p}^2 + \eta p^2 \left( \frac{E^n}{E_P^n} \right) + O \left( \frac{E^{n+3}}{E_P^{n+1}} \right),$$  \hspace{1cm} (6)

where $\eta$ and $n$ are model-dependent parameters which should be determined experimentally. Not only are $\eta$ and $n$ model dependent, but within a given model they may also take different values for different particles. In particular, in the analysis [73] of some noncommutative spacetimes one finds birefringence (different polarizations of light propagate at different speeds). The fact that some noncommutative spacetimes host this particle dependence suggests that they might also give rise to departures from the UFF and the UGR, but, as mentioned, the relevant formalisms are still at too early a stage of development for producing detailed predictions concerning the UFF and the UGR.

**Space–time fluctuations**  One of the original motivations for introducing the formalism of noncommutative geometry was the one of formalizing the limitations on the localization of spacetime points (events) at the quantum-gravity (Planck-scale) level, which are suggested by a variety of arguments combining general relativity and quantum mechanics (see, e.g., Refs. [83, 72, 84, 85, 86, 87, 88]). Unfortunately the analysis of the relevant formalisms has not yet been developed to the point of a phenomenologically useful description of this localization limits. The noncommutativity of the coordinates of course implies some associated limits on the combined measurement of pairs of coordinates, but phenomenology would need a more advanced level of description, one in which one could for example derive the implications of noncommutative geometry for the activities of an interferometer. Clearly if our interferometers are operating in a fuzzy (noncommutative) spacetime, rather than a sharp classical spacetime, one should expect that at some point the accuracy of the interferometer would be affected. This is indeed the expectation of those working on noncommutative spacetimes, but it has proven so far too hard to attempt to derive from the noncommutative geometry the relevant physical effects.
3 The test theories

The study of the quantum-gravity problem has provided motivation for the study of a rather large number of new effects. In light of the nature of the indications that are coming from the theory side it appears necessary to structure the phenomenological efforts according to a few alternative strategies.

In some cases a given quantum picture of spacetime or a given quantum-gravity scenario definitely predicts a specific effect, i.e. we can identify in the formalism an effect which, if not found, could falsify the theoretical picture. Then there is scope for a sharply aimed phenomenological effort, characterized only by the few parameters that the specific theoretical picture involves. A situation of this type is maturing in the study of certain noncommutative spacetimes, perhaps most notably the $\kappa$-Minkowski spacetime, whose single length parameter (here denoted by $\lambda$) could soon be constrained at a beyond-Planckian level.

In other cases, especially in ambitious attempts at a complete solution of the quantum-gravity problem, such as string theory and loop quantum gravity, one is dealing with an extremely complex and rich formalism which appears to have the potentiality of giving rise to a plethora of new effects, even though one is not able (at least at present) to establish that a certain new effect is definitely present in the relevant theoretical framework. This type of frameworks provide motivation for a general approach for the description of deviations from standard theories. Since even the most basic principles of present-day physics are not necessary “safe” in these new theories, one should be prepared for modifications of the fundamental equations describing physics. The most fundamental equations are the equations governing the standard model, that is, the gauge theory of the electroweak and strong interaction and the Einstein field equation, and the phenomenology should explore all candidate modifications of these equations.

Concerning the target sensitivity that this phenomenology should set for itself, obviously the Planck energy scale can provide some tentative guidance, but it appears that the phenomenology should be prepared for the possibility that another, possibly lower, scale might characterize at least some quantum-gravity effects. For those new effects, as in the example of some modified dispersion relations mentioned earlier, whose description automatically brings about a to-be-determined energy scale, it is of course of primary importance from a quantum-gravity perspective to find ways to test the hypothesis that the relevant energy scale is indeed the Planck scale. But even a lower sensitivity may turn out to be sufficient to uncover a quantum-gravity effect.

This large phenomenological effort, which as a whole should consider a large number of possible effects (and parametrizations of the effects) and a large range of possible magnitudes for these effects, must of course be structured in terms of some reference test theories. Reference to some widely-adopted test theories of course facilitates the comparison of sensitivities achieved by different experiments, and the test theories can also provide a language that bridges the gap between experiments and (usually rather formal) theory work on quantum gravity. But even in setting up such test theories a few alternative strategies should be explored simultaneously. A key issue is whether the new effects end up taking the form of new terms in the old type of formalisms or require even a new formalism. Usually, even when a new formalism is required for the full new theory, one can approximately incorporate the new effects in the old formalism. And on the basis of this experience it is natural to set up some test theories that indeed describe the new effects via some new terms in the old type of equations. But the phenomenology should also keep in mind that there have been cases in the history of physics where the description of the new physics really demanded a new formalism. For example, in order to introduce the new effects of special relativity in quantum mechanics one was tempted to formulate a “relativistic quantum mechanics” (introducing new structures in the old formalism) but it turned out to be necessary to invent relativistic quantum
field theory. We now understand that there is a limit of relativistic quantum field theory that can be described approximately in terms of a “relativistic quantum mechanics”, but the nature of the limiting procedure by which the old formalism emerges can only be established \textit{a posteriori}, when the full theory has been developed and understood. The study of the quantum-gravity problem has confronted us with new features like the mentioned IR/UV mixing, which for example should act as a warning against assuming \textit{a priori} that a naive low-energy limit of quantum gravity should give us back our present theories. Another example is the deformed law of composition of momenta in $\kappa$-Minkowski spacetime, which is incompatible with the standard field-theoretic setup and can only be switched off when all other $\kappa$-Minkowski modifications of Lorentz symmetry are switched off. Even at very low energies, if one keeps the $\kappa$-Minkowski dispersion relation (which could find room in the standard field-theory setup), but neglects the modification of the law of composition of momenta (which is not compatible with the standard field-theory setup), inconsistent results are obtained, including some significant \textit{violations} of Lorentz symmetry (whereas $\kappa$-Minkowski should only involve \textit{a deformation} of Lorentz symmetry).

So alongside the test theories that describe the new effects still in the old language, we should also develop some test theories that are prepared for the need of a new formalism. In practice the second type of test theories will need to simply avoid using too much of our present formalism. For example, as stressed here later in this Section, some phenomenology of Planck-scale departures from Lorentz symmetry focuses on pure kinematics only, in order to avoid the assumption that dynamics should be described within our present formalisms. An example of the opposite type is the SME framework, where one considers a plethora of new effects, but all still codified according our present rules, including power-counting-renormalizability.

In this Section we give a partial overview of test theories that can be used in quantum-gravity phenomenology. We describe in some detail, since it was not covered in detail in other recent quantum-gravity-phenomenology reviews [2, 3, 4, 5], a phenomenological scheme for the modification of the Maxwell and the Dirac equation. And we describe more briefly, since pedagogical descriptions are already available in Refs. [2, 3, 4, 5], some other test theories that are relevant for the search of anomalous dispersion and of quantum-spacetime fluctuations. In any case, since the present theories are well proven by all current experiments, it is reasonable to introduce tests theories through small modifications of the present theories.

A further important aspect of test theories is that, from an experimenter’s point of view, detailed test theories also allow ‘bookkeeping’ of the possible effects that can be bounded by different experiments: All, even very different, effects are related to the same set of parameters. Only by the use of test theories one is able to ‘compare’ different experiments.

### 3.1 The generalized Maxwell equation

#### 3.1.1 The model

A very general model for generalized Maxwell equations is based on the assumption that the homogeneous Maxwell equations $dF = 0$ are still true. This can be based on a definition of the electromagnetic field strength and the charge based on the Aharonov–Bohm effect [31]. In our approach the principles to formulate generalized inhomogenous Maxwell equations are:

- linear in the field strength,
- first order in the differentiation, and
- small deviation from the standard Maxwell equations.
Therefore, the generalized Maxwell equation which we are going to discuss and to confront with experiments is
\[ \lambda^{\mu\nu\rho\sigma} \partial_\nu F_{\rho\sigma} + \chi^{\mu\rho\sigma} F_{\rho\sigma} = 4\pi j^\mu. \]  \(7\)

The requirement of this equations to describe a small deviation from the standard theory leads to \( \lambda^{\mu\nu\rho\sigma} = \eta^{\mu[\rho} \eta^{\sigma]\nu] + \chi^{\mu\nu\rho\sigma} \) where \( \eta^{\mu\nu} \) is the Minkowski metric diag\((+ - - -)\) and, in that frame, all components of \( \chi^{\mu\nu\rho\sigma} \) and \( \chi^{\mu\nu\sigma} \) are small compared to unity. Therefore, all effects are calculated to first order in these quantities only.

In our approach, the constitutive tensor \( \chi^{\mu\nu\rho\sigma} \) and, thus, the tensor \( \chi^{\mu\nu\rho\sigma} \) are assumed to possess the symmetry \( \chi^{\mu\nu\rho\sigma} = \chi^{\nu\mu\rho\sigma} \) only (in the SME the constitutive tensor possesses the symmetry of the Riemann tensor \([44]\)). The decomposition of this constitutive tensor and of \( \chi^{\mu\nu\sigma} \) reads (see Ref. [31] for the definition of the various irreducible parts)

\[
\chi^{\alpha\beta\mu\nu} = \left(\right)^{(1)}W^{\alpha\beta\mu\nu} + \frac{1}{2} \epsilon^{\mu\nu}_{\rho\sigma} \eta^{\rho[\alpha} \Phi^{\beta]\sigma} + \frac{1}{12} \chi^{\epsilon\alpha\beta\mu\nu} - \eta^{\mu[\alpha} \Phi^{\beta]\nu] + \eta^{\nu[\alpha} \Phi^{\beta]\mu] + \nabla \cdot \left(\phi \delta^{\alpha\beta} - \frac{1}{6} W^{\alpha\beta\rho\sigma} \eta^{\mu[\rho} \eta^{\nu]\sigma] - \left(1\right)^{1/2} \chi^{\rho][\alpha \Phi^{\beta]} + \frac{1}{3} \left(2 \eta^{\mu(\alpha} \Delta^{\beta)\nu} - 2 \eta^{\nu(\alpha} \Delta^{\beta)\mu} - \eta^{\alpha\beta} \Delta^{\mu\nu}\right) + \chi^{\mu\rho\sigma} + \eta^{\rho}[\alpha \Phi^{\sigma)]\mu\right). \]

\(8\)

\(9\)

A 3+1 decomposition of the generalized Maxwell equations gives \(i,j = 1,2,3\)

\[4\pi \rho = \nabla \cdot E + \chi^{000i} \dot{E}_i + \chi^{0ij} \dot{B}_{ij} + \chi^{0\rho\sigma} \partial_\rho F_{\sigma\nu} + \chi^{0\nu\sigma} F_{\rho\sigma}\]

\[4\pi j^i = \dot{E}_i - \left(\nabla \times B\right)^i + \chi^{00ij} \dot{E}_j + \chi^{0i} \dot{B}_{jk} + \chi^{ij\rho\sigma} \partial_\rho F_{\sigma\nu} + \chi^{i\rho\sigma} F_{\rho\sigma}, \]

where \( E_i = F_{0i} \) and \( B_i = \frac{1}{2} \epsilon_{ijk} F_{jk}. \) Using the homogeneous Maxwell equations, the time derivative of \( B \) can be replaced by a spatial derivative of the electric field. The appearance of the term \( \chi^{000i} \) makes both equations to dynamical equations for the electric field rendering them to be an overdetermined system. Therefore we have to require the vanishing of the coefficient \( \chi^{000i}. \) Since this should be true for any chosen frame of reference, we have to require \( \chi^{(\mu\nu\rho)\sigma} = 0, \) what is identical to \( \left(1\right)Z_{(\alpha\beta\mu\nu)} = 0, \Xi_{\mu\nu} = 0, \) and \( \Delta_{\mu\nu} = -\frac{3}{4} Z_{\mu\nu}. \)

Due to the vanishing of these irreducible parts we get \( \partial_\mu \left(\chi^{\mu\nu\rho\sigma} \partial_\nu F_{\rho\sigma}\right) = 0 \) so that only \( \chi^{\mu\nu\sigma} \) may lead to charge non–conservation:

\[4\pi \partial_\alpha j^\alpha = \chi^{\alpha\mu\nu} \partial_\alpha F_{\mu\nu} = \left(1\right) \chi^{\alpha\mu\nu} \partial_\alpha F_{\mu\nu} + \eta^{\alpha\mu} \nu^{\rho\sigma} \partial_\rho F_{\nu\sigma}. \]

\(12\)

The validity of the homogenous Maxwell equations allows to require the vanishing of \( \chi^{\mu[\nu\rho\sigma]} \) without any physical consequences. This leads to \( X = 0, Z^{\mu\nu} = W^{\mu\nu}_\alpha, \) and \( \Upsilon^{\mu\nu} = \frac{1}{2} \Phi^{\mu\nu}. \)

### 3.1.2 Radiation effects

In order to discuss the effects due to the anomalous terms in radiation phenomena, we derive the wave equation

\[0 = \dot{E}_i - \Delta E_i + \left(\nabla \left(\nabla \cdot E\right)\right)^i + \chi^{ij\rho\sigma} \partial_\rho \partial_\nu E_j + 2 \chi^{i0j} \dot{E}_i + \chi^{ij} \partial_k E_j \]

\(13\)
and make a plane wave ansatz $E = E^0 e^{-i(k \cdot x - \omega t)}$ which results in equations for the amplitude and its derivative:

$$
0 = (\omega^2 - k^2)\delta_{ij} + k_i k_j + 2\chi^{i\mu j\nu} k_\mu k_\nu E_j^0. 
$$

$$
0 = -2\omega \dot{E}_i^0 - 2k_i \nabla E_i^0 + k_j \partial_j E_j^0 + k_\mu \partial_\mu E_j^0 + 2\chi^{i\mu j} (k_\mu \partial_\mu E_j^0 + k_\rho \partial_\rho E_j^0) + 2\omega \chi^{0j} E_j^0 - k_k \chi^{ikj} E_j^0, 
$$

$$
0 = \ddot{E}_i^0 - \Delta E_i^0 + \partial_j \partial_j E_j^0 + 2\chi^{i\mu j\nu} \partial_\mu \partial_\nu E_j^0. 
$$

The first equation gives the dispersion relation

$$
\omega = \left(1 + \rho(n) \pm \sqrt{\sigma^2(n) - \rho^2(n)}\right) |k| 
$$

with $\rho(n) = -\frac{1}{2} \Phi^{\mu\nu} n_\mu n_\nu$ and $\sigma^2(n) = \frac{1}{2} \eta_{\alpha\gamma} \eta_{\beta\delta} \chi^{\alpha\mu\beta\gamma} n_\mu n_\nu n_\rho n_\sigma$, where $n_\mu = k_\mu / \omega = (1, k/|k|)$. This generalizes results in Ref. [89, 44].

The $\pm$ sign in front of the square root indicates a hypothetical birefringence. If no birefringence is observed, then $(1) W^{\mu\rho\sigma} = 0$ and $\Psi^{\mu\nu} = 0$. For the interpretation of this result, see [90]. It has been shown [44] that from astrophysical observations birefringence can be excluded at the level of $10^{-32}$. The remaining $\rho(n)$ leads to an anisotropic speed of light which has been excluded in laboratory experiments [91] at the $10^{-15}$ level. Vanishing anisotropy implies $\Phi^{\mu\nu} = 0$. Furthermore, from (15) a propagation equation for the amplitude can be derived

$$
v^\mu \partial_\mu E_i^0 = - \left(2\omega \chi^{0j} - k_k \chi^{ikj}\right) E_j^0, 
$$

where $v^\mu$ is the group velocity of the light ray. This is directly related to the charge non-conservation parameter $\chi^{\mu\rho\sigma}$. Since no precession of the polarization has been inferred from astrophysical observation [92] the tensor $\chi^{\mu\rho\sigma}$ vanishes at the order $10^{-42}$ GeV.

From all these requirements, the remaining generalized Maxwell equations are (we let aside the factor $W$ since this can be absorbed into a redefinition of the electric charge and current)

$$
4\pi j^\alpha = \partial_\mu F^{\mu\alpha} - \frac{1}{2} \eta^{\alpha\mu\nu} Z^{\nu\beta} \partial_\beta F_{\mu\nu} + \frac{3}{2} \eta^{\beta\mu} Z^{\nu\alpha} \partial_\beta F_{\mu\nu} + \frac{1}{2} \eta^{\alpha\beta} Z^{\mu\nu} \partial_\beta F_{\mu\nu}. 
$$

In the SME approach the $Z^{\mu\nu}$ are absent. Therefore, in our approach it is not possible to establish Lorentz invariance by radiation experiments only. Further experiments are needed.

### 3.1.3 Electromagnetostatics

The 3+1 decomposition of the above equations gives

$$
\frac{\rho}{\epsilon_0} = -\nabla \cdot E - \dot{\zeta} \cdot (\nabla \times E) - \epsilon \zeta \cdot (\nabla \times B), 
$$

$$
\mu_0 j = \frac{1}{c^2} \dot{E} - \frac{1}{c^2} \dot{\zeta} \times \dot{E} - (\dot{\zeta} \cdot \nabla) B + \nabla \times B - \frac{1}{c} \nabla (\zeta \cdot E) + \frac{1}{c} \zeta (\nabla \cdot E), 
$$

where we used SI units and defined $\zeta^i := \frac{3}{2} Z^{0i}$ and $\dot{\zeta}^i := \frac{3}{4} \epsilon_{ijk} Z^{jk}$.

The generalized Maxwell equations for a point charge at the origin are given by (20,21) with $\rho = q \delta(r)$ and $j = 0$. With $E = \nabla \phi$ and $B = \nabla \times A$ and the gauge $\nabla \cdot A = 0$ we get to first order in the perturbations

$$
\phi = \frac{1}{4\pi \epsilon_0} \frac{q}{r}, \quad A = \frac{q \zeta}{4\pi \epsilon_0 c r}. 
$$
This gives a magnetic field

\[ B = \frac{q}{4\pi \varepsilon_0 c} \frac{\zeta \times r}{r^3}. \] (23)

Therefore, our model includes the feature that a point charge also creates a magnetic field.

If the source of the Maxwell equations is a magnetic moment \( m \) localized at the origin, then the Maxwell equations are (20,21) with \( \rho = 0 \) and \( \mathbf{j} = m \times \nabla \delta(r) \). We assume again a static situation and get to first order

\[ \mathbf{A} = \frac{\mu_0 m \times r}{4\pi r^3}, \quad \phi = \frac{\mu_0 c (\zeta \times m) \cdot r}{4\pi r^3}. \] (24)

A magnetic moment also creates an electric field of an electrical dipole with dipole moment \( d = \mu_0 c \varepsilon_0 \zeta \times m \). This feature is "dual" to the previous case.

For the new Lorentz invariance violating parameters \( \zeta \) and \( \hat{\zeta} \) there seem to exist no experimental results. However, we expect strong bounds on the \( \zeta^i \) components from measurements with SQUIDs and from atomic spectroscopy. With SQUIDs weak magnetic fields of down to \( 10^{-14} \) T can be measured. If we assume that a measurement with SQUIDs of a magnetic field from a point charge does not lead to any magnetic field larger than the SQUID sensitivity, then, from \( |\lambda \zeta/(2\pi \varepsilon_0 c \rho)| \leq 10^{-14} \) T for a line charge density \( \lambda = 0.01 \) C/m at a distance of 1 cm, we get the estimate \( |\zeta| \leq 2.7 \cdot 10^{-17} \). We strongly encourage experimentalists to carry out such an experiment.

The \( \zeta^i \) will also lead to a hyperfine splitting, additional to the usual one. We get for the interaction Hamiltonian of an electron in the magnetic field (23) of the nucleus

\[ H_\zeta = \mu_{el} \cdot \frac{q \zeta \times r}{4\pi \varepsilon_0 c r^3}. \] (25)

If we choose the \( z \)-axis in direction of \( \zeta \), then the corresponding energy shift \( \Delta E_{nlm} = \langle \psi_{nlm} | H_\zeta | \psi_{nlm} \rangle \) leads, e.g., to

\[ \Delta E_{210} = -\frac{q \zeta_x \mu_x}{48 \pi \varepsilon_0 c a^2}. \] (26)

With \( \mu_x = e \hbar / m_e \) this yields \( \Delta E_{210} = \zeta_x 1.8 \cdot 10^{-2} \) eV. The state of the art of high precision measurements of energy levels is of the order \( \Delta E/E \approx 10^{-15} \). Since the measured energy levels are still well described within the standard theory one gets for energies of about 10 eV at best an estimate \( |\zeta_x| \leq 10^{-14} \) which, however, is not as good as a direct measurement discussed above might yield.

The parameter \( \hat{\zeta} \) gives rise to small deviations from the unperturbed quantities only and, thus, cannot be measured such precisely.

### 3.2 The generalized Dirac equation

#### 3.2.1 The model

The Dirac equation can be derived from the principles of (i) unique evolution, (ii) superposition principle, (iii) locality, (iv) conservation of probability, and (iv) Lorentz invariance. The requirements (i) – (iii) leads to a linear system of partial differential equations of first order, requirement (iv) ensures the hyperbolicity of these equation. Using (i) to (iv) one gets

\[ 0 = i \gamma^\mu \partial_\mu \psi - M \psi \] (27)

where the matrices \( \gamma^\mu \) in general do not fulfill a Clifford algebra, that is, there is no underlying Riemannian metric. As a consequence, the characteristic surface of this equation is given, as in the
case of the Maxwell equation, by a forth order equation \(0 = \det(\gamma^\mu k_\mu)\) and shows birefringence and anisotropy. The same phenomenon we encounter with the mass shell equation \(0 = \det(\gamma^\mu p_\mu + M)\).

Also models with higher order derivatives as they appear in, e.g., in the effective equations for spin \(\frac{1}{2}\) particles within loop quantum gravity, have been considered [93].

### 3.2.2 Propagation of Dirac particles

There are two quasi–classical phenomena which one can derive from this generalized Dirac equation [30]: the equation of motion for the position in a nonrelativistic WKB aproximation

\[
\dot{a}^i = -\left(\delta^{ij} + \alpha^{ij} \bar{\alpha}^{ij} S^k\right) \partial_j U + \beta_{kl} \delta^{ij} \partial_j U^{kl}
\]

where \(U\) is the Newtonian potential, \(U^{ij}\) the Newtonian gravitational potential tensor [94], \(S^i\) the spin and \(\alpha^{ij}\), \(\bar{\alpha}^{ij}\), and \(\beta_{ij}\) are anomalous terms connected with the \(X^{\mu\nu}\) and an anomalous coupling to \(U^{ij}\), see [30]. The corresponding equation for the precession of the spin is

\[
\dot{S}^i = \epsilon^{ijk} \Omega_j S_k \quad \text{with} \quad \Omega_i = \tau_{ik}^p p_k p_l + \tau_{ij}^p p_j + \tau_i^n,
\]

where again the anomalous parameters \(\tau_{ik}^p\), \(\tau_{ij}^p\), and \(\tau_i^n\) are connected with the \(X^{\mu\nu}\) and the anomalous coupling to \(U^{ij}\). In both cases the dynamics is influenced by anomalous parameters. The acceleration can be probed, e.g., by atom interferometry, the spin precession by the corresponding spin precession experiments performed, e.g., in high energy experiments.

### 3.2.3 Spectroscopy

For quantum particles also spectroscopy can give valuable information about the underlying dynamics of the fields. The anomalous parameters give rise to additional energy shifts and splitting of spectral lines. As an example, we mention the energy shifts occuring the Hughes–Drever like experiments (see Sec.4.3.6) where a valence particle (a valence electron or a valence proton in the atomic’s nucleus) leads to a splitting of the Zeeman lines [30] which, among others, yields the best estimates on an hypothetical anomalous inertial mass of the proton and on a hypothetical space-time torsion [95].

Further effects like a modification of the spreading of a wave packet have been discussed in [96].

### 3.3 Generalized gravitational field

In physics, geometry cannot be separated from the motion of physical objects. That something can be ”geometrized” means that the dynamics of a certain object is independent from properties characterizing the object. In GR, the geometrization comes from the independence of the path of structureless particles from the mass and decomposition, the only characteristics of a structureless particle. The path is determined by its initial position and velocity only and, thus, given by an equation of the form \(\ddot{x}^\mu + H^\mu(x, \dot{x}) = \alpha \dot{x}^\mu\). Then the motion is not related to the particular particles and, thus, can be assigned to an underlying geometry. The geometry underlying this geometrization of the path dynamics is called ”path structure”. This very general structure becomes more familiar when we assume that at each space-time point there is a frame so that the equation of motion for all particles reduces to \(\ddot{x}^\mu = 0\) (Einstein’s elevator). Then the equation of motion reads \(\ddot{x}^\mu + H^{\mu\sigma}(x)\dot{x}^\sigma = \alpha \dot{x}^\mu\). The compatibility of this structure with the light cone structure then leads to a Weylian space–time which can be reduced to the ordinary Riemannian space-time by imposing the additional requiremet of the non–occurrence of the second clock effect [97, 98].
The next step in establishing a theory for gravity then is to determine the equations which give the metric in terms of the matter content in the universe. In a particular case we get Einstein’s field equations. A general parametrizations for a wide range of possible field equations for the metric is given by the PPN formalism, see e.g. [94]. With the most important PPN parameters $\beta$ and $\gamma$, the metric is given by

\begin{align}
  g_{00} &= 1 - 2U + 2\beta U^2 \\
  g_{0i} &= V_i \\
  g_{ij} &= -(1 + 2\gamma U) \delta_{ij}
\end{align}

(30) (31) (32)

where $U$ is the Newtonian potential and $V_i$ is a gravitational field connected with a matter current. In this representation the metric is very well adapted for a confrontation with experimental data, e.g., from light deflection, redshift, gravitational time delay, perihelion shift, and data from binary systems. Einstein’s field equations are characterized by $\beta = \gamma = 1$. Recently, it has been proposed in a string theory inspired dilaton model that $\beta$ and $\gamma$ may differ from unity by $10^{-9}$ and $10^{-5}$, respectively. The latter one is very close to the recent measurement by the Cassini mission [99] which gave $|\gamma - 1| \leq 2 \cdot 10^{-5}$.

### 3.4 Anomalous dispersion

As mentioned various quantum–gravity scenarios lead us to considering Planck-scale modified dispersion relations of the general type (6). The derivation of the modified dispersion relation can take a rather different path in different approaches. For example in $\kappa$-Minkowski spacetime, where one has the support of the deformed (Hopf) algebra, the dispersion relation is obtained rather simply, through a Casimir of the spacetime-symmetry algebra, just like the classical dispersion relation is obtained through the mass Casimir of the Poincaré (Lie) algebra. But in some scenarios the argument may be rather involved, as shown by the case of Loop Quantum Gravity, in which we are still unable to perform a full rigorous derivation of the dispersion relation. This would require a sort of “Minkowski vacuum” for Loop Quantum Gravity, which is still unknown. One therefore relies on some states which are not the vacuum, but may reproduce some of the characteristics of the vacuum. For example, some authors [16, 17] have performed the analysis using the so-called Loop-Quantum-Gravity “weave states”.

A key point for the development of test theories based on this modified dispersion relations concerns the possibility of assuming that a field-theoretic formulation be admissible. Some authors [29, 100] have relied on the field-theoretic setup, in spite of the fact that it is inevitably nonrenormalizable (within the field-theoretic setup Eq. (6) corresponds to nonrenormalizable dimension-5 operators). Other authors, perhaps the majority, have been concerned not only with the nonrenormalizability of the corresponding field theory, but also with the possibility that the relevant quantum-gravity scenarios might have to be based rather significantly on decoherence, something that field theory by construction cannot accommodate. These other authors have preferred avoiding to commit to a formulation of dynamics and are basing their phenomenology exclusively on kinematics.

The pure-kinematics test theories only rely on the form of the dispersion relation and the form of the law of energy-momentum conservation, which is usually assumed to be unmodified.

A formal description of a field-theory-based test theory hosting the modified dispersion relations (6) is discussed in Refs. [29, 101]. For example, for a pure abelian gauge theory one introduces the extra term

\[ \frac{1}{E_P} n^a F_{ad} n \cdot \nabla (n_b \tilde{F}^{bd}) \]

(33)
where $n_a$ is a background four-vector that codified the breakup of Lorentz symmetry.

### 3.5 Space–time fuzziness and decoherence

The quantum-gravity literature on spacetime fuzziness (or “spacetime foam”) originates from a rich collection of arguments [83, 72, 84, 85, 86, 87, 88], combining general relativity and quantum mechanics, which suggest that in the Planck-scale regime there should be some absolute limitations on the measurability of distances. It turns out to be most convenient [5] to characterize operatively this spacetime fuzziness as an irreducible (fundamental) Planck-scale contribution to the noise levels in the readout of interferometers. Interferometer noise can in principle be reduced to zero in classical physics. Ordinary quantum properties of matter already introduce an irreducible noise contribution. Spacetime fuzziness would introduce another irreducible source of noise, reflecting the fact that the distances involved in the experiment would be inherently unsharp in a foamy spacetime picture.

While, as mentioned in the preceding sections, nearly all approaches to the quantum-gravity problem predict some limitations on the accuracy of localization, and therefore predict some spacetime fuzziness, one is usually unable to rigorously derive from first principles a detailed description of the physical consequences of this fuzziness, such as the mentioned interferometric noise. A phenomenology is being developed nonetheless, exploiting [5, 102, 103, 104] the fact that the role of the Planck scale in the needed formulas has only a limited number of options, as a result of the constraints introduced by dimensional analysis.

The key input needed for this phenomenology turns out to be the power spectrum $\rho(f)$ of the Planck-scale-induced strain noise [102]. Combining some intuition about the stochastic-like features of spacetime fuzziness and the dependence on the Planck length one can rather easily reach a model of $\rho(f)$. For example if the effects depend linearly on the Planck length $L_p \equiv 1/E_p$ and the underlying phenomena are of random-walk type one is inevitably led to

$$\rho_h \sim L_p f^{-2} \Lambda^{-2} \equiv \zeta L_p f^{-2} L^{-2}, \quad (34)$$

The proportionality to the square of the inverse of the frequency is a direct result of the assumption of random-walk-type processes. The length scale $\Lambda$ is needed on the basis of the dimensional analysis of the equation, and is to be treated as a free parameter to be constrained experimentally. One may choose to make reference to a dimensionless parameter, $\zeta$, which may be used to express the ratio of $\Lambda$ with the length $L$ of the arms of an interferometer or of an optical resonator.

Other hypothesis [5, 102, 103, 104] about the stochastic-like features of the underlying Planck-scale processes lead to other forms of $f$ dependence (and $L_p$ dependence) of the strain noise power spectrum. In general one should find ($\alpha > -1$)

$$\rho_h \sim \zeta_{\alpha\beta} L_p^{1+\alpha} f^{-2-\beta} L^{-2-\alpha-\beta}. \quad (35)$$

There is therefore some interest in attempting [5, 102, 103, 104, 12] to improve limits on the parameters $\zeta_{\alpha\beta}$.

The idea of spacetime fuzziness also motivates some research work on Planck-scale-induced decoherence. It is in fact rather plausible that spacetime fuzziness might affect the time evolution of quantum-mechanical states in such a way that, for example, a pure state might evolve into a mixed state. A Planck-scale decoherence scenario which has been extensively studied, for what concerns the phenomenological aspects, is the one of Ref. [57] (and references therein) which is motivated by the “Liouville Strings” approach [66, 67] and describes the decoherence effects within a density-matrix formalism.
4 Controlled laboratory experiments

The realization that, as a result of the large astrophysical propagation distances and (in some cases) the large energies of the particles involved, in a few astrophysics contexts the analysis of quantum-gravity effects can achieve Planck-scale sensitivity has generated a large interest over the last few years, as documented in Refs. [2, 3, 4, 5] and references therein. However, the realm of astrophysics is also affected by some key limitations, which encourage us to complement this phenomenology with controlled/laboratory experiments. In astrophysics we are just limited to the role of “observers” rather than having the opportunity to devise, set up and control a measurement procedure. In interpreting astrophysical observations one is faced with the problem that the source as well as the space in between (that might contain gasses, other electromagnetic fields etc.) are not under control of the experimenter, and this will, in the majority of cases, lead only to model-dependent results. Moreover, as mentioned above, the study of the quantum-gravity problem can motivate the search of a variety of new effects, which we would like to study one by one, whereas in astrophysics we must take what Nature offers, and this is not going to cover all aspects of the phenomenology of interest.

In this Section we discuss a few significant plans for a quantum-gravity-phenomenology with controlled/laboratory experiments. The description of examples of astrophysics studies is postponed to the next section, and will be structured more succinctly, since several recent reviews (e.g., Refs. [2, 3, 4, 5]) have already covered in detail the approach based on astrophysics.

4.1 Universality of Free Fall

One of the first physical laws stated and subject to tests is the UFF. It began with the free fall tests of Galileo and its tests using a tilted plane. The best tests today use torsion balances and confirm the UFF at the level of $5 \times 10^{-13}$ [105]. The best free fall tests reach the level of $10^{-10}$ [106, 107, 108]. Future tests to be performed in space on dedicated satellite missions, namely MICROSCOPE and STEP, should be able to tests the UFF to a level of $10^{-15}$ and $10^{-18}$ respectively [109, 110]. Therefore, MICROSCOPE as well as STEP are able to cover predictions from some quantum gravity motivated predictions.

4.2 Universality of the Gravitational Redshift

In these tests the ticking rates of two clocks is compared while the clocks change their position in a gravitational field. As in tests of the UFF, all pairs of clocks have to be considered. Presently, the best tests has been reported in [51] where two atomic clocks, an atomic fountain clock and a hydrogen maser, are subject to the changing gravitational field of the Sun. The two clocks show the same redshift to an accuracy of $1.5 \times 10^{-5}$. Other tests compared atomic clocks and clocks based on microwave and optical resonators and gave in both cases the same redshift within $2 \times 10^{-2}$ [111, 112].

4.3 Lorentz invariance

4.3.1 Accurate, clean, and comprehensive tests

Test of the principle of (local) Lorentz invariance (LI) are an extremely rich field for the search of fundamental physics. One reason is the universal validity of LI as a framework for all theories of nature. A violation of LI would thus manifest itself in virtually all branches of physics, and this allows to exploit the highest precision measurements as tests of LI. In test theories, the possible violations of LI are encoded into a set of parameters. The effects of these parameters in various experiments can be calculated (at least in principle), so that results on the parameters can thus be
obtained from experiments. These theories have been advanced to a level where the outcome of all these experiments can be compared. This also answers the question whether two experiments test different aspects of relativity or whether they are different methods for observing the same effect.

Ideally, the experimental verification of LI should be accurate, clean, and comprehensive. Accurate refers to the ability to detect very small violations and clean refers to how well the hypothetical effect sought in experiments is understood and referred to a set of parameters in the test theory. This is a requirement to both the experiment and the underlying theory. It ensures that a violation of relativity does not accidentally cancel. Comprehensiveness denotes the degree of completeness to which the theoretically conceivable violations of LI are excluded.

This summary wants to present the most important experimental techniques that have been used for bounding violations of LI. The variety of methods makes this an interesting and challenging subject for experimental physicists. As a general observation, tight limits on spin-dependent parameters can be obtained by comparing similar particles having different polarizations. The spin-independent effects have to be measured by comparing to independent standards, e.g., a macroscopic body as a length standard. Another common feature of most experiments is that they seek for a Lorentz-violating dependence of an effect as a function of the orientation of the apparatus or the velocity of the laboratory frame in space. For a laboratory located on Earth, the relevant rotations and velocities can be given by Earth's orbit as well as rotation.

4.3.2 Applications and every day tests

LI has found many applications in fundamental and applied research, in technology and, thus, even in every day life. One striking example on how every day these may be are the accelerated electrons in cathode ray tubes used in color television sets: Their mass is increased by about 5% relative to their rest mass. For a less mundane example, the accurate quantum field theoretical prediction of the electron's anomalous magnetic moment $g = 2.002 \ldots$ to twelve significant digits [113] (maybe the most accurate prediction of a 'complicated' number in any science) also is a confirmation of LI.

In the global positioning system, a receiver determines its position by comparing timing signals received from accurate reference clocks located in satellites. The positions of the satellites are known, and the difference in the timing signals gives the relative distances that allows the receiver to determine its spatial coordinates and the local time. However, since the reference clocks are moving with respect to the receiver, their rates are subject to time dilation (and also to the redshift due to Earth’s gravity). Not taking into account these effects would lead to a position error of many kilometers [114].

There are also technologies whose very operating principle is a relativistic effect. Certain microwave oscillators ('gyrotrons') [115] and the free electron laser [116] use the relativistic Doppler effect to generate short-wavelength radiation. In a free electron laser, a beam of electrons is directed along an arrangement of periodically poled magnets, where the period $p$ is of the order of a centimeter. The moving electrons have a kinetic energy $E \gg m_e c^2$, and thus see a Lorentz-contracted period $pm_c^2/E$. The corresponding oscillating magnetic field in the electron’s rest frame thus has a frequency $\nu_e = cE/(pm_c^2)$ and forces the electrons to an oscillatory motion that leads to the emission of electromagnetic waves with a frequency $\nu_e$. The Lorentz transformations back to the laboratory frame concentrate these waves into a tight cone in the forward direction of the electron beam, with a blue-shifted frequency $\nu_{lab} = \nu_e E/(m_e c^2) = cE^2/(pm_e^2c^4)$. Thus, highly concentrated radiation and a small wavelength $\lambda = p(m_e c^2/E)^2$ are achieved. The two Lorentz transformations give the quadratic dependence on $1/E$.

Particle accelerators are maybe the most prominent example of a technology where relativistic effects are built in. The velocity-dependence of the relativistic mass, for example, is taken into
account in the construction of the machines, and, most importantly, the understanding of the high-energy particle reactions provides maybe the richest field for the application of relativistic (quantum field) theories. These have not only given rise to an enormous output of fundamental physics discoveries, but the apparent correctness of relativistic quantum field theory is also a confirmation of relativity itself.

While the success of relativistic physics in fundamental and applied research gives us confidence into the theory, the ‘every-day tests’ provide no high-precision confirmation: On the one hand, machines like particle accelerators, microwave oscillators etc. are constructed in such a way that small tolerances do not lead to malfunction, and this also means that small errors in the underlying theory may not affect functionality. Also, the ‘every day tests’ are usually not clean: In accelerator experiments, for example, an incomplete understanding of the hadronic processes might cover a tiny violation of relativity. For these reasons (and for achieving comprehensiveness), LI has been tested in dedicated experiments since its inception.

4.3.3 Macroscopic matter effects

Lorentz violation affects the properties of macroscopic bodies through a modification of its microscopic constituents. These modifications, in particular the change of the geometry, have been studied for the interpretation of experiments. As an important result, within the SME matter effects do not cancel the sensitivity in interferometer or cavity tests of Lorentz invariance. For a model which may lead to a cancellation see [117]. Instead, they enhance the sensitivity for Lorentz violation in electrodynamics, but only slightly for cavity materials presently in use. Moreover, the theory concerning the Dirac sector [118] allowed to derive the first experimental limits on some of the electron coefficients $c_{\mu\nu}$, at a level of $10^{-14}$. The theories summarized here constitute a complete description of all SME effects that influence experiment using vacuum-filled cavities. These experiments are thus particularly clean tests of Lorentz invariance.

**Photon sector** Within the electrodynamic sector of the SME, Lorentz violation leads to a modified velocity of light $c = c_0 + \delta c$ and also to a modified Coulomb potential of a point charge $e$, $\Phi(\vec{x}) = e^2/(4\pi|\vec{x}|) + V$, where [44]

$$V = \frac{e^2}{8\pi} \frac{\vec{x} \cdot \kappa_{DE} \cdot \vec{x}}{|\vec{x}|^3}.$$  \hspace{1cm} (36)

The influence of this on solids can be treated for ionic crystals [119], which in the simplest case (e.g., NaCl) consists of a lattice of ions with opposite charges. The lattice is formed by the balance between attractive Coulomb forces and a quantum mechanical repulsion due to the overlap of the ionic orbitals. Perturbative calculations show that the change in the repulsive potential due to Lorentz violation is negligible. For estimating the influence of the modification of the Coulomb potential $V$, the force generated by the modified Coulomb potential is summed over all ions and equated to the elastic force associated with a distortion of the lattice. This leads to the relative change of the length

$$\frac{\delta L_z}{L} = A \left[ (2\sigma - 3\tau_\parallel)(\kappa_{DE})_\parallel - 3\tau_\perp(\kappa_{DE})_\perp \right]$$ \hspace{1cm} (37)

with $\sigma$ and $\tau$ being constants obtained from summing the Coulomb potential in analogy to the Madelung constants. $(\kappa_{DE})_\parallel = (\kappa_{DE})_{zz}$, $(\kappa_{DE})_\perp = (\kappa_{DE})_{xx} + (\kappa_{DE})_{yy}$ and

$$A = -\frac{1}{2} \frac{e^2 v_1 v_2 N_m N_A \rho}{8\pi E_Y \Ma}.$$ \hspace{1cm} (38)
Table 1: Length change coefficients.

| Material | $a_\parallel$ | $a_\perp$ | Material | $a_\parallel$ | $a_\perp$ |
|----------|--------------|-----------|----------|--------------|-----------|
| NaCl     | -0.28        | 0.10      | LiF      | -1.06        | 0.37      |
| sapphire | -0.03        | 0.01      | quartz   | -0.11        | 0.04      |

$v_1$ and $v_2$ are the number of valence charges for the atoms, and $E_Y$ is the elastic modulus. The contributions of the length change and the change in the speed of light give the total frequency change of a cavity filled with vacuum due to Lorentz violation in electrodynamics

$$\frac{\delta \nu_{\text{cav}}}{\nu_{\text{cav}}} = -a_\parallel \hat{N} \kappa_{DE} \hat{N} - \left(\frac{1}{2} + a_\perp\right) \left[ \hat{E}^* \kappa_{DE} \hat{E} + (\hat{N} \times \hat{E}^*) \kappa_{DE} (\hat{N} \times \hat{E}) \right].$$

Here, $a_\parallel = A (2 \sigma - 3 \tau_\parallel)$ and $a_\perp = -3 A \tau_\perp$. This has been simplified by noting that astrophysical tests lead to $(\kappa_{HB}) = - (\kappa_{DE})$. For practical materials sapphire and quartz the length change effect is negligible (see Table 1). For future experiments using resonators made of other materials, however, the influence might be stronger and enhances the sensitivity.

The model can be extended to include the additional non-Lagrangian coefficients of the most general Maxwell equations that are linear and first order in the derivatives [31, 120]. It is found that the non-SME terms do not additionally modify the geometry of crystals.

**Fermionic sector** Here, the starting point is the nonrelativistic hamiltonian $h = h_0 + \delta h$ of a free electron in the SME, as described in [42]. Most of the terms contained therein (Tab. 5) have zero expectation value within the rest frame of a solid. The only term that doesn’t drop out for non-spin-polarized materials (spin-polarized materials can also be treated[118]) is the modification of the kinetic energy of the electron, $\delta h = E'_{jk} p_j p_k$. Since the electrons inside crystals have a nonzero expectation value $\langle p_i p_j \rangle$, which is a function of the geometry of the lattice, Lorentz violation will cause a geometry change (‘strain’) of the crystal.

Strain is conventionally expressed by the strain tensor $e_{ij}$. For $i = j$, it represents the relative change of length in $x_i$-direction, and for $i \neq j$, it represents the change of the right angle between lines originally pointing in $x_i$ and $x_j$ direction. A general linear relationship between the Lorentz violating quantity $E'_{jk}$ and strain is given by

$$e_{dc} = B_{dpj} E'_{pj}.$$  

with a 'sensitivity tensor' $B_{dpj}$ that has to be determined from a model of the crystal. $B_{dpj}$ can be taken as symmetric in the first and last index pair; symmetry under exchange of these pairs will hold only for some simple crystal geometries, like cubic. Thus, the tensor has at most 36 independent elements. To calculate the sensitivity tensor, the electronic states are described by Bloch wave functions to determine $\langle p_i p_j \rangle$; the corresponding strain is calculated using elasticity theory. As a result, the sensitivity tensor $\mathcal{B}_{dcjp} = \mu_{dcmp} \kappa_{mj} + \mu_{dcmj} \kappa_{mp}$, where

$$\kappa_{mj} = \frac{N_{e,u} h^2}{m |\det(l_{ij})|} k_{ml} k_{jk} \xi_{lk},$$

(41)

can be calculated. $N_{e,u}$ is the number of valence electrons per unit cell, $|\det(l_{ij})|$ is the volume of the unit cell expressed by the determinant of the matrix of the primitive direct lattice vectors, $k_{ml}$ is the matrix containing the primitive reciprocal lattice vectors, and $\mu_{dcmp}$ the elastic compliance constants. The symmetric $3 \times 3$ matrix $\xi_{lk}$ is given by the Fourier coefficients of the Bloch wave
Table 2: Sensitivity coefficients.

| Mat.         | $B_{11}$ | $B_{12}$ | $B_{13}$ | $B_{14}$ | $B_{31}$ | $B_{33}$ | $B_{41}$ | $B_{44}$ |
|--------------|----------|----------|----------|----------|----------|----------|----------|----------|
| Au           | 24.13    | -11.06   |          |          |          |          |          | 12.34    |
| Al₂O₃        | 3.58     | -1.05    | -0.53    | 0.014    | -0.57    | 3.14     | 0.004    | 5.08     |
| Nb           | 6.80     | -2.40    |          |          |          |          |          | 17.9     |
| fused quartz | 2.64     | -0.32    |          |          |          |          |          | 3.95     |

functions. Its six parameters are unknown at this stage and can, e.g., be determined from a simple model that leads to $\bar{\xi}_{lk} \sim \delta_{lk}$. To eliminate these unknowns, an alternative method is used to calculate the strain for the simple case of isotropic Lorentz violation $E'_{jk} \sim \delta_{jk}$, and the result is compared to Eq. (40) [118]. This yields six equations from which $\kappa_{ab}$ (that depends solely on material properties) can be determined and re-inserted into Eq. (40):

$$B_{dcpj} = \mu_{dcmp}\lambda_{aamj} + \mu_{dcmj}\lambda_{aamp}.$$ (42)

Note that the theory now needs no assumptions that go beyond the use of Bloch states. Thus, it is very generally applicable and accurate.

For convenience, we arrange the independent elements of $e_{ab}$ into a six-vectors $e = (e_{xx}, e_{yy}, e_{zz}, e_{yz}, e_{zx}, e_{xy})$ and express Eq. (40) as $e = B \cdot E'$, where $B$ is a 6 × 6 'sensitivity matrix'. Tab. 2 gives values for the materials presently used for cavities. Gold is included as an example for a material with exceptional sensitivity.

The influence of the electron terms $c_{\mu\nu}$ on hydrogen molecules have also been calculated [121]. Here, an explicit wave-function can be obtained from first principles, and Lorentz-violating changes in the frequencies of electronic and (ro-) vibrational transitions, as well as the bond length, have been obtained. This allows new tests of Lorentz symmetry that use molecules. A change in the index of refraction of dielectric media is also connected to violation of LI [122]. It has to be taken into account for certain cavity experiments.

4.3.4 Cavity experiments

Polarization experiments, see Sec.5, comparing light with light, cannot measure all 19 coefficients of the ordinary constitutive tensor $(k_F)_{\kappa\lambda\mu\nu}$, whereas cavity and interferometer experiments, comparing light with matter, can. Moreover, cavity experiments can measure coefficients for Lorentz violation in the electron’s equation of motion that have not been measured by other methods. Cavity experiments have been developed out of the well known interferometer experiments originally performed by Michelson, Morley [123, 124] as well as Kennedy, Thorndike [125, 126], and others.

The basic principle is to measure the time of flight of light rays that transverse a path, which is usually defined by a pair of mirrors and a spacer. A change connected to the orientation or the velocity of the apparatus in space would indicate Lorentz violation. Interferometers indicate such a change by a shift in fringes formed by the interference of two beams. In cavities, however, interferometry between multiple beams is used. Each transverses the cavity a large number of times that is limited by losses such as imperfect mirrors. Today, about $10^5$ reflections are used, and $\sim 10^6$ seem technically possible. Thus, a large effective distance of propagation is reached in a compact apparatus, that can be much better shielded from temperature fluctuations and (seismic) vibrations, and is less sensitive to gravitational bending.

2In the large interferometers dedicated to the search for gravitational waves, shielding against seismic vibrations with frequencies above a couple of Hz is effected by pendulum-like mounting of the interferometer end mirrors. Thus, however, the dc position of the mirrors is much less stable than in the rigid cavities described here.
Table 3: Overview of recent cavity experiments (WGR = whispering-gallery resonator). The notation 1−2(4) in the column for $\tilde{\kappa}_{e^-}$ indicates that the experiment provides 4 independent limits on parameters of $\tilde{\kappa}_{e^-}$, the lowest individual being about 1, the highest about 2 parts in $10^{15}$ (absolute mean plus standard error). If no number in brackets is given, all matrix elements have been limited. The notation $\sim 5$ indicates that, although a detailed analysis within the SME was not performed, the experiment should limit at least one of the parameters at the indicated level.

| Ref.  | type                      | $\tilde{\kappa}_{e^-} \times 10^{15}$ | $\tilde{\kappa}_{\phi^+} \times 10^{11}$ | $c_{AB} \times 10^{15}$ |
|-------|---------------------------|--------------------------------------|-----------------------------------------|-------------------------|
| [127] | (1979) rotat. quartz cavity/CH$_4$ | $\sim 6$                           | $\sim 10$                              |                         |
| [128] | (1980) quartz cavity/CH$_4$    |                                      |                                        |                         |
| [112] | (2000) sapphire cavity/iodine |                                      |                                        |                         |
| [129] | (2003) Nb microwave cavities  | 140-430 (4)                         | 200 (1)                                |                         |
| [91]  | (2003) sapphire cavities     | 4-18 (4)                            | 3-28                                   |                         |
| [118] | (2003) reanalysis of [127, 91]| 4-18 (4)                            | 3-28                                   | 2-100 (3)               |
| [130] | (2004) sapphire WGR/Cs clock | 2-8 (4)                             | 3-5                                    |                         |

In cavity experiments, one measures the resonance frequencies

$$\omega = 2\pi \frac{mc}{2nL}$$

($m$ is a constant mode number, $c$ is velocity of light parallel to the cavity axis, $n$ the index of refraction of the medium, and $L$ the cavity length) of a cavity, defined by the boundary conditions for standing waves. A Lorentz-violating shift in $c$ and/or $L$ and $n$ connected to a rotation or boost of the apparatus would lead to a measurable shift in $\omega$. The shift in $c$ is, of course, due to Lorentz violation in the photonic sector. The shift in $L$ has been determined above.

The first experiment to make full use of the potential that reference cavities offer for precision experiments was performed by Brillet and Hall [127] (see Tab. 3). A laser was stabilized to a resonance of a fused quartz cavity (actually, ultra-low expansion (ULE) glass ceramics). Both the cavity as well as the laser were rotating on a platform. Their frequency was compared to a stationary methane (CH$_4$) frequency standard. The data showed frequency variations of a few parts in $10^{13}$ that where ascribed to a slight tilt of the rotation axis with respect to Earth’s gravity. After a transformation of the signal into the sidereal frame, however, this effects becomes oscillatory and can thus be separated from any signals of cosmic origin. The experiment was performed before the developement of an elaborate theoretical framework. A detailed re-analysis of the published result within the SME is, unfortunately, not possible, since only a single signal component is given. For deriving detailed SME results, it would be desirable to have at least 10 independent components.

In a similar setup, a non-rotating quartz cavity (ULE) was compared against a CH$_4$ standard by Hils and Hall [128]. The signal period was given by Earth’s rotation. On that time-scale, however, the ULE cavity showed a significant drift that limited the accuracy of that experiment. The experiment was interpreted in the RMS test theory (Sec.1.6) and provided the best limits of $6.6 \times 10^{-5}$ on the velocity-dependence parameter at the time. As above, a detailed analysis within the experiment from the published data is not feasible. This experiment was improved by a factor of about three by Braxmaier et al. [112], who used a cavity made from crystalline sapphire, operated at the temperature of liquid helium (4K). The crystalline material showed a remarkable absence of
to give separate limits on four components of $\tilde{\kappa}$ published in 2003 [129]. Two superconducting microwave cavities made of Niobium (Nb) were used about $2^{-15}$ limits on several parameters, thus representing progress towards a comprehensive verification of LI.

The accuracy of this experiment was lower than that of older experiments, it determined simultaneous parameter space. It determined four elements of $\tilde{\kappa}$ limits on the elements of $\tilde{\kappa}$ internal reflection. Also this experiment was operated for a sufficiently long time to state separate creep, i.e., no systematic long term drifts of the cavity length. Thus, it was possible to search for a signal with 1 year period, given by Earth’s orbit.

The first experiment to determine simultaneous limits on the photonic SME coefficients was published in 2003 [129]. Two superconducting microwave cavities made of Niobium (Nb) were used to give separate limits on four components of $\tilde{\kappa}_{e-}$ and one limit on $\tilde{\kappa}_{o+}$ (Tab. 3). Although the accuracy of this experiment was lower than that of older experiments, it determined simultaneous limits on several parameters, thus representing progress towards a comprehensive verification of LI.

The experiment of Müller *et al.* [91, 131], improved both the accuracy as well as the coverage of parameter space. It determined four elements of $\tilde{\kappa}_{e-}$ to a level of about $10^{-15}$. The three elements of $\tilde{\kappa}_{o+}$ enter the signal suppressed by the velocity of Earth’s orbit, $\beta \sim 10^{-4}$, and where thus limited to about $10^{-11}$ (Tab. 4). That one of these seven parameters was different from zero at $1.8\sigma$ does not mean that Lorentz violation was detected. Rather, the probability of a mean value to be larger than the standard error is 23% even if the true value is zero, so one or two such instances are to be expected if the error bars are realistic. The experiment compared two cryogenic sapphire resonators pointing in orthogonal directions over a period of 399 days. This made it possible to independently measure Fourier signal components separated by as little as 1/1 year in frequency space, and thus allowed the experiment to give separate limits on the elements of $\tilde{\kappa}_{o+}$.

The experiment of Wolf *et al.* used a slightly different setup, in which resonances of a cryogenically cooled microwave whispering gallery resonator (WGR) are used. The WGR is a cylinder made from crystalline sapphire, in which the microwave mode travels along the perimeter, guided by total internal reflection. Also this experiment was operated for a sufficiently long time to state separate limits on the elements of $\tilde{\kappa}_{o+}$. In this experiment, the results for the three parameters are significant at about $2\sigma$. As explained above, however, this is to be expected from standard statistics even if all parameters are zero, and thus not an indication of Lorentz violation.

The theory of the influence of fermionic LI violation in the cavity geometry [118] made it possible to state the first limits on three electron parameters contained in $c_{\mu\nu}$. Since this influence is material dependent, it can be separately measured by performing experiments with different cavity materials, here the experiments of Brillet and Hall [127] and Müller *et al.* (Tab. 4). Theoretically, all components of $c_{\mu\nu}$ can be limited by this method, if sufficiently detailed experimental results are available. As explained above, this is unfortunately not the case for the older experiments.

| Parameter | [91, 118] | [91, 130, 122] |
|-----------|-----------|----------------|
| $(\tilde{\kappa}_{e-})^{XY}/10^{-15}$ | $1.7 \pm 2.6$ | $-1.7 \pm 1.6$ |
| $(\tilde{\kappa}_{e-})^{XZ}/10^{-15}$ | $-6.3 \pm 12.4$ | $-4.0 \pm 3.3$ |
| $(\tilde{\kappa}_{e-})^{YZ}/10^{-15}$ | $3.6 \pm 9.0$ | $0.5 \pm 2.52$ |
| $((\tilde{\kappa}_{e-})^{XX} - (\tilde{\kappa}_{e-})^{YY})/10^{-15}$ | $8.9 \pm 4.9$ | $2.8 \pm 3.3$ |
| $(\tilde{\kappa}_{o+})^{XY}/10^{-11}$ | $14 \pm 14$ | $-1.8 \pm 1.5$ |
| $(\tilde{\kappa}_{o+})^{XZ}/10^{-11}$ | $-1.2 \pm 2.6$ | $-1.4 \pm 2.3$ |
| $(\tilde{\kappa}_{o+})^{YZ}/10^{-11}$ | $0.1 \pm 2.7$ | $2.7 \pm 2.2$ |
| $c^{e}_{(YZ)}/10^{-15}$ | $0.21 \pm 0.46$ | $0.21 \pm 0.46$ |
| $c^{e}_{(XZ)}/10^{-15}$ | $-0.16 \pm 0.63$ | $-0.16 \pm 0.63$ |
| $c^{e}_{(XY)}/10^{-15}$ | $\lesssim 1$ | $0.76 \pm 0.35$ |
| $(c^{e}_{XX} - c^{e}_{YY})/10^{-15}$ | $\lesssim 8$ | $1.2 \pm 0.64$ |
| $|c^{e}_{XX} + c^{e}_{YY} - 2c^{e}_{ZZ} - 0.25(\tilde{\kappa}_{e-})^{ZZ}|/10^{-12}$ | $\lesssim 1$ | $\lesssim 1$ |
For a complete determination of the photon and electron coefficients, the experiment of Müller et al. has to be compared to an experiment that gives as many signal components, but uses dissimilar cavities. The experiment by Wolf et al. [130] could be used for this purpose. However, in the whispering-gallery type cavity, 98% of the electromagnetic field energy are confined within a refractive material. Thus, a shift in the index of refraction also connected to Lorentz violation alters the Lorentz-violation signal. This has been taken into account in [121], leading to more comprehensive and more accurate limits (Tab. 4). The additional hypothetical shift in the index of refraction of sapphire contributes to the higher accuracy.

4.3.5 Doppler shift experiments

To measure the relativistic Doppler shift, the frequency of a moving oscillator is measured as a function of the velocity with respect to the detector. Such measurements can be easily described within the RMS framework, where a parameter $\alpha$ describes the strength of the second order Doppler shift. $\alpha = -1/2$ is predicted by SR. In the SME and other dynamical test theories, the analysis is more complicated, since it must contain the nature of the moving clock. The most precise test yet gives $|\alpha_{MS} + 1/2| \lesssim 1.8 \times 10^{-7}$ [132] by spectroscopy on lithium ions moving within a storage ring. The accuracy of this result is limited by the knowledge of the rest-frame transition frequency of lithium.

Other methods to measure the Doppler shift include $\gamma$-ray spectroscopy using a rotating Mössbauer absorber [133]. While the resolution of the frequency measurement is much higher, the velocity is limited to maybe 100 m/s due to centrifugal forces in the rotor, making these experiment less accurate.

In the SME and other dynamical test theories, the interpretation of such experiments is different: Here, the physics of the clock and the detector is described by dynamical equations, from which the received frequency can be calculated. A departure from the relativistic Doppler shift would arise from the extra terms in the dynamical equations. Thus, the analysis must contain the nature of the moving clock. Although this was already discussed [134], at the time of this writing no such analysis has been performed yet.

4.3.6 Clock-comparison experiments

The basic idea is to compare the oscillation frequencies of two physical systems (“clocks”) and look for any change with the orientation or velocity of the clocks in space. The highest precision limits are achieved by comparing the frequencies of dissimilar clocks as they rotate with the Earth.

If the influence of the Lorentz-violating parameters on the frequencies of the clock can be calculated (by use of perturbation theory), definite bounds can be obtained by a comparison to the outcome of the experiment. However, the parameters of the SME are so manifold that a calculation of the individual influences is generally difficult, especially if nuclear energy levels are involved. In these cases, the problem has been simplified by use of simplified models for nuclear matter and assuming that all but a few parameters for Lorentz violation are zero [42]. This gives insight into how many of the parameters can be bounded at which level of accuracy. Further assuming no cancellations between the parameters, limits on many parameters for Lorentz violation that lead to spin-dependent effects for the electron, the proton, and the neutron could be obtained (Tab. 5).

In the tests originally performed by Hughes [38] and Drever [39] at the beginning of the sixties, the clocks are based on transitions in atoms or ions between states characterized by the orientation of an electronic or nuclear spin (hyperfine or Zeeman transitions). If the rotation–invariance of the energy $E = mc^2$ of the nucleus was violated, the energy levels of the states involved would depend on the orientation of the clock in space. This would cause a modulation of the clock frequency $\nu_1$ with
the influence of the parameters due to Lorentz violation could be calculated. Because of the relatively small numerical factors, the near future [137, 138].

A vapour cell is filled with two species of noble gas (129Xe and 3He). Population inversion for both nuclei, the experiment obtains limits that do not rely on the above assumptions that have been made in order to simplify the theory. The frequency change of the He maser $\delta \nu_J = -3.5b_j^n + 0.012d_j^n - 0.012g_{D, J}^n$ due to Lorentz violation could be calculated. Because of the relatively small numerical factors, the influence of the parameters $b_j^n$, $d_j^n$, and $g_{D, J}^n$ can be neglected, and the experiment results in a limit $b_j^n = (6.4 \pm 5.4) \times 10^{-32} \text{ GeV}$ [135, 136]. Space–based experiments of this type are also planned for the near future [137, 138].

A similar trick is used in 'co-magnetometer' experiments [143]. Since the spin-dependent Lorentz violation fields couple to matter in a similar way as magnetic fields, magnetometers (based on the precession frequency of a spin) are sensitive to these terms. The relative sensitivity to Lorentz violation and magnetic background fields may be different for different types of atoms, so simultaneous operation of two spatially overlapped magnetometers can be used to eliminate the sensitivity to the magnetic field.
Table 6: Overview of clock–comparison experiments with sensitivity to neutron and proton mass anisotropy $\delta m$. The quoted accuracy applies for the measurement of $\nu_1$.

| Reference | clock 1 | $\nu_1$ | clock 2 | $\nu_2$ | $\delta m/m_{n,p}$ |
|-----------|---------|---------|---------|---------|-------------------|
| [139]     | $^9$Be  | 303 MHz | $^1$H   | 23.91401 GHz | 10$^{-26}$        |
| [140]     | $^{201}$Hg | 5.5 Hz  | $^{199}$Hg | 15 Hz    | 10$^{-28}$        |
| [141]     | $^{21}$Ne | 1.000 kHz | $^3$He   | 9.650 kHz | 10$^{-28}$        |
| [142]     | $^{199}$Hg | 4.0321 Hz | $^{133}$Cs | 1.858 kHz | 10$^{-29}$        |
| [135]$^a$ | $^3$He  | 1.7 kHz | $^{129}$Xe | 4.9 kHz   | 5 $\times$ 10$^{-31}$ |

$^a$ see also [136]

4.3.7 Other tests

Limits on spin dependent electron coefficients have been obtained by torsion balances with spin polarized solids (i.e., permanent magnets). One experiment yielded $\tilde{b}_Z \simeq (2.7 \pm 1.6) \times 10^{-25} m_e$ [144, 145]; in a similar experiment [146], $(|\tilde{b}_X|^2 + |\tilde{b}_Y|^2)^{1/2} \leq 6.0 \times 10^{-26} m_e$ and $|\tilde{b}_Z| \leq 1.4 \times 10^{-25} m_e$ have been found.

Hydrogen spectroscopy can prospectively limit linear combinations of $\tilde{b}_f$, $\tilde{b}_p$, to about $10^{-27}$ GeV [147]. Due to the simplicity of Hydrogen, the sensitivity of these tests is accurately calculable, making this a clean test. Comparing the frequencies of hydrogen masers, [148] find $|\tilde{b}_f + \tilde{b}_p| \lesssim 2 \times 10^{-28}$ GeV.

4.4 Anomalous dispersion in interferometry

The study of anomalous dispersion relations, although usually motivated on the basis of a break up of Lorentz symmetry, deserves to be discussed separately, if nothing else because of the huge interest that has been attracted by this phenomenology in the recent literature (see, e.g., Refs. [2, 3, 4, 5]). Anomalous dispersion is of course a prototypical propagation effect, and its study would immediately suggest resorting to astrophysics contexts, where the propagation distances can be gigantic. We here want to stress that however there is at least one class of controlled/laboratory experiments where extremely high sensitivity to anomalous dispersion can be achieved. This has been stressed in Ref. [13] on the basis of the observation that conceivably, if we manage to operate them at high sensitivity using two different-wavelength beams, modern interferometers (of the type used for gravity-wave searches) might achieve [13] sensitivity to $\eta \sim 1$, for the $n = 1$ case in Eq. (6).

4.5 Space–time fluctuations and decoherence

Concerning the models of Planck-scale-induced strain noise in interferometry described in Section 3.5 the quality of experimental limits is improving quickly. For example, already available data constrain $\zeta$ to be much smaller than 1 in the random-walk scenario of Eq. 34. This is achieved both using large “free-mirror” laser-light interferometers [102, 103, 104, 12], such as TAMA (soon improving with LIGO and VIRGO), and using small-size laser-light interferometers whose mirrors are rigidly connected [12].

The fact that we are still unable to derive a quantitative description of spacetime fuzziness from the various quantum-gravity proposals does not allow us to have a good intuition for the significance of these limits. And, while theory work attempts to clarify the situation, it appears necessary to attempt to push the limits as far as possible.

For what concerns the density-matrix Planck-scale decoherence formalism mentioned in Section 3.5 the best limits were obtained using data from the CPLEAR neutral-kaon experiment. The
neutral-kaon system is very sensitive to new physics because of the delicate balance of conventional-physics scales that governs its peculiar features. The density matrix formalism of Ref. [57] involves several dimensionless parameters which one may guess to be all obtained as roughly the ratio between the kaon mass and the Planck scale. For some of the parameters, CPLEAR data already exclude this possibility experimentally.

5 Observations in astrophysics

As mentioned, the realization that tests of some quantum-gravity effects with extremely high sensitivity could be based on observations in astrophysics has generated over these past few years a rather large interest. In turn this has resulted in the production of quite a few general reviews of the subject, of which Refs. [2, 3, 4, 5] are just a representative small sample. We refer the reader to these reviews (and references therein) for a detailed discussion of the relevant phenomenological proposal. Still, in order to render our own review somewhat self-contained, in this Section we comment briefly on some of the most studied methods.

5.1 SME astrophysics

There has been a sizeable effort of constraining SME parameters using astrophysical observations of radiation. Of course the interpretation of the data on radiation emitted by distant sources as tests of LI is faced with the problem that the source as well as the space in between (that might contain gasses, other electromagnetic fields etc.) are not under control of the experimenter, and the analysis inevitably acquires a certain level of model dependence.

The distance of the source can only be inferred from studying other radiation, and therefore the analysis must always compare radiation with radiation, which in some cases renders the analysis insensitive to spin-independent effects. But of course the extremely long propagation distance is very valuable for constraints on other types of effects.

The SME 4-vector \( (k_{AF})^e \) from the photon sector (Tab. 7) (that is also expected to vanish for theoretical reasons [149]) has been ruled out by birefringence measurements on radiation emitted by distant radio galaxies [92]. The idea is that \( k_{AF} \neq 0 \) leads to a splitting of photons in two circularly polarized modes with different phase velocities, according to a dispersion relation

\[
|k| = \omega \mp \frac{1}{2} \left( |k_{AF}| - |\mathbf{k}_{AF}| \cos \theta \right). \tag{44}
\]

Here, \( k = (\omega, \mathbf{k}) \) denotes the wave vector and \( \mathbf{k}_{AF} \) the spatial components of \( k_{AF} \). The angle \( \theta \) is between the direction of \( \mathbf{k}_{AF} \) and \( \mathbf{k} \). The + and - signs correspond to right- and left-handed circularly polarized waves, respectively. As in the Faraday effect, this causes a rotation of the polarization of a linearly polarized plane wave by an angle \( \delta \phi = ((k_{AF})_0 - |k_{AF}| \cos \theta) L / 2 \), where \( L \) denotes the distance travelled and \( \theta \) is the angle between the direction of propagation and \( \mathbf{k} \). The radio galaxies are expected to emit radiation that is polarized either parallel or perpendicular to their observed elongation. A rotation of the polarization would cause a difference between the observed polarization and elongation; if no difference is observed, an upper limit on \( k_{AF} \) can be derived. Studying 160 sources, [92] one obtains \( |k_{AF}| = \sqrt{(k_{AF})^a(k_{AF})^a} \lesssim 10^{-42} \text{GeV} \).

The SME matrices \( (\tilde{\kappa}_{e^+})^{jk} \) and \( (\tilde{\kappa}_{\nu^+})^{jk} \) from the photon sector (Tab. 7) have also been restricted by polarization measurements. A difference of the phase velocity \( \Delta v_p \) of the two polarization modes travelling over a distance \( L \) would lead to a change in the relative phase

\[
\Delta \phi \approx 2\pi \Delta v_p \frac{L}{\lambda}, \tag{45}
\]
Table 7: Effects of the SME parameters of the photonic sector on the phase velocity of light. # denotes the number of degrees of freedom contained in each symbol. $\beta = v/c$ is the velocity of the experiment relative to the frame of reference in which the parameters are defined.

| Parameters | # makes $c$ dependent on | limit | Ref. |
|------------|--------------------------|-------|------|
| $\vec{k}_{AF}$ | circular polarization | $10^{-42}$ GeV | |
| $(\tilde{\kappa}_{e+})^{j k} = (\tilde{\kappa}_{e+})^{k j}$ | 6 linear polarization | $10^{-32}$ | [150, 44] |
| $(\tilde{\kappa}_{e-})^{j k} = -(\tilde{\kappa}_{o-})^{k j}$ | 3 linear polarization | $10^{-32}$ | [150, 44] |
| $(\tilde{\kappa}_{e+})^{j k} = (\tilde{\kappa}_{o+})^{k j}$ | 6 direction of propagation | $10^{-15}$ | [91, 130] |
| $(\tilde{\kappa}_{o+})^{j k} = -(\tilde{\kappa}_{o+})^{k j}$ | 3 direction of propagation, $O(\beta)$ | $10^{-11}$ | [91, 130] |
| $\tilde{\kappa}_{tr}$ | 1 motion of experiment $O(\beta^2)$ | | |

where $\lambda$ is the wavelength. This modifies the polarization, e.g., light that is initially linearly polarized would in general become elliptically polarized. The large factor $L/\lambda$ makes the experiment very sensitive. A change of the polarization state at the receiver occurs as a consequence of either a change in $L$ or $\lambda$. For an astrophysical source, $L$ is fixed, but one can measure the dependence on $\lambda$ (making the assumption that the emitted polarization is relatively constant over a range of wavelengths). From data on 16 sources at $0.04 \text{Gpc} \leq L \leq 3.53 \text{Gpc}$, a limit of $2 \times 10^{-32}$ (90% confidence level) on all elements of $(\tilde{\kappa}_{e+})^{j k}$ and $(\tilde{\kappa}_{o-})^{j k}$ is obtained [150, 44].

Similar analyses of synchrotron radiation from the Crab nebula and other sources are reported by [151]. As stressed in the next subsection, they interpret the result in terms of a modified dispersion relation of the form (6). Since their analysis uses birefringence methods, it should be possible to interpret the same result in terms of the SME: The wavelength of the $\sim 0.3 \text{MeV}$ radiation is $\sim 4 \times 10^{-12}$ m, and the distance is assumed to be $L = 0.5 \text{Gpc}=1.5 \times 10^{25}$ m or about $10^{37} \lambda$. Since the upper limit on polarization changes with $\lambda$ is of the order of one radian, an analysis of these results in terms of the SME will likely lead to a limit of the order of $10^{-37}$ on at least one component of either $(\tilde{\kappa}_{e+})^{j k}$ or $(\tilde{\kappa}_{o-})^{j k}$.

Another kind of astrophysical limit is derived from the following idea: If Lorentz symmetry is not exact, the limiting velocity for different particles does not need to be equal. A 'fast' particle with a higher limiting velocity may then emit Cherenkov radiation that consists of particles with lower limiting velocities. Thus, its velocity would eventually be reduced to below the threshold for Cherenkov radiation in a time that is short compared to astrophysical timescales, so particles above a certain energy would not exist in cosmic radiation. From observations of cosmic radiation with energies of up to $3 \times 10^{11} \text{GeV}$, the maximum velocity of several particles can be constrained to deviate no more than a few parts in $10^{-20} \ldots 10^{-24}$ from $c$ [152].

### 5.2 Anomalous dispersion

A special role within the phenomenology inspired by the possibility of Lorentz-symmetry violations is played by effects of anomalous dispersion. The fact that (as also stressed earlier in these notes) several approaches to the quantum-gravity problem motivate the study of Planck-scale-modified dispersion relations, has led, over the last few years, to a very large effort on the phenomenology side.

The first popular idea for searches of Planck-scale anomalous dispersion was based [28] on properties of gamma-ray bursts. These bursts travel cosmological distances and contain a rich time structure, and as a result they can be used to set rather significant limits on the wavelength dependence of the speed of photons which would follow from a dispersion relation of the type (6).
The present best limits obtained using this strategy constrain (see, e.g., Refs. [52, 153, 154]) the parameter $\eta < 300$ for the case $n = 1$ in (6). And planned telescopes should provide an improvement of a few orders of magnitude within a few years [155, 156].

For the case in which the modified dispersion relation is assumed to hold within a field-theoretic setup very stringent (beyond-Plankian) limits can be obtained from an analysis of synchrotron radiation from the Crab nebula [100]. However, the field-theoretic setup in the photon sector imposes birefringence and invites one to consider a particle/helicity dependence of $\eta$, leading to many parameters. Only one of these parameters can be constrained using the Crab-nebula synchrotron-radiation analysis [157, 100].

The case of the Crab-nebula synchrotron radiation is interesting because, as stressed already in the previous subsection, this same set of data can be valuable both for the SME and for the phenomenology based on (6). Clearly the physics described by the SME matrices is different from the energy-dependent effects [28] associated with (6). In principle, simultaneous limits on both effects could be obtained by studying the energy-dependence of the phase-shift. This illustrates that some observation is sometimes interpreted in two theoretical frameworks to produce results that appear to be unrelated, while the actual observation is the same.

Still concerning the hypothesis of a description of birefringence based on (6) a limit of order $|\eta| < 2 \cdot 10^{-4}$, for $n = 1$, can be inferred [158] using observations of polarized light from distant galaxies.

Perhaps the most exciting opportunity for this phenomenology based on (6) is provided by the study of ultrahigh-energy cosmic rays. Before reaching our observatories ultrahigh-energy cosmic rays travel gigantic (cosmological) distances and our expectations concerning the structure of the cosmic-ray spectrum depend strongly on the interactions of cosmic rays with photons in the cosmic microwave background, which in particular should produce a cut off in the spectrum around $5 \cdot 10^{19}$ eV. A modification of the dispersion relation could lead [159, 160, 161] to a modification of this cut off prediction. The Auger cosmic-ray observatory, now starting to take data, has the sensitivity needed to probe very small values of $\eta$ for $n = 1$, and even in the case $n = 2$ Auger could still be sensitive to $\eta$ of order 1.

### 5.3 Spacetime fuzziness and decoherence

Some proposals concerning the use of observations in astrophysics to explore the quantum-gravity ideas of spacetime fuzziness and decoherence have also been made. In Ref. [162] it is argued that evidence of a good phase coherence of light from extragalactic sources could be used to constrain spacetime-fuzziness scenarios. This proposal opens a valuable phenomenological window, but it might be necessary to seek improved analyses, taking into account the comments made in Ref. [163] concerning the quantum-gravity aspects, and in Ref. [164] concerning the astrophysics aspects.

Concerning decoherence certain aspects of neutrino astrophysics may play a role [10] somewhat similar to the one played by laboratory studies of the neutral-kaon system.

### 6 Summary and outlook

We have given a rather general overview of the phenomenology work which finds motivation in the study of the quantum-gravity problem. The field has grown so wide that we shall not here argue that we gave a complete overview, but we did set as one of our primary goals the one of illustrating some alternative strategies which are being pursued for this phenomenology. Within this spectrum of research programmes one finds on one side a phenomenology of the type illustrated by studies of the modified dispersion relation (6), with only a minimal parametrization, a nearly exclusive focus
on the hypothesis that the onset of the new effects should be characterized by a scale which is rather close to the Planck scale, and the expectation that the new effects should not be describable in terms of renormalizable field-theory operators. And on the opposite side the strategy involves very general parametrizations, a phenomenology which looks for the new effects even at scales much different from the Planck scale, but assuming throughout that the present formalisms can accommodate the new effects, so that one is led for example to the SME field theory and/or to the type of modified Maxwell and Dirac equations which we discussed. To our knowledge there are no other reviews covering this wide spectrum of approaches, and we thought it might be beneficial to provide one. In spite of the differences in strategy there are a number of common challenges for all these approaches, besides the obvious common goal of contributing to the search of a solution for the quantum-gravity problem.

Another key goal for our work was to stress that this candidate effects that have been discussed in the quantum-gravity literature are not merely of academic interest, as it is sometimes assumed. We believe that the example of metrology, which we discussed in Subsection 1.2, should act as a clear warning of the potential (even technological) implications of these studies.

We expect that this field will prove to be among the most exciting of the next 10 or 20 years, since over this period we can clearly see several opportunities for sharp improvement of the present sensitivities toward the relevant effects. A good example is provided by the Auger cosmic-ray data which will surely amuse us for the next couple of years, and are going to provide a much clearer picture of the high-energy cosmic-ray spectrum. Similarly, the next generation of gamma-ray telescopes, such as GLAST, will provide a huge improvement for the relevant gamma-ray studies.

While many Lorentz-violating effects have been bounded, some to impressive levels of precision, improvements in terms of accuracy, cleanliness and comprehensiveness are expected from improved versions of the old experiments and from experiments of new kinds. New turntable versions of the cavity tests are performed at several locations. The shorter timescale of the rotation (minutes compared to 24 h for Earth’s rotation) allows to collect much more data in a given time and to exploit the time scale of the optimum sensitivity of the experiment. This should allow two to three orders of magnitude improvements, if the systematic effects associated with active rotation (like the bending of the resonators caused by a slight tilt of the rotation axis relative to Earth’s gravity) can be controlled. As an alternative idea, cavity experiments could also search for birefringence in isotropic materials induced by Lorentz violation. Such experiments could perform the same tests as conventional Michelson-Morley type experiments, but would be immune against a broad range of systematic effects, including length changes of the cavity [122].

A very interesting challenge is to put some of these experiments into space. This would have several advantages. The absence of seismic vibrations is important for experiments involving macroscopic matter. Experiments in space also offer the possibility to choose the time scale of rotation and orbit of the satellite which provide the necessary modulation of the Lorentz violation in the frame of the experiment. On Earth, both time scales (24 h and 365 d) are relatively long. In space, these would be reduced to minutes and hours without introducing the systematic effects of turntable experiments on Earth. Several proposals [138, 165, 166] have been made for experiments on the International Space Station ISS and dedicated satellites.

New types of terrestrial experiments are suggested by new theoretical work. For example, the relatively low accuracy of the bounds on the photon parameters \( \hat{\kappa}_{0+} \) can be improved by exploring the static limit of Lorentz-violating Maxwell equations [31, 167]. Such experiments can also bound the Lorentz-violating non-SME terms of the most general Maxwell equations [31].

Concerning the UFF, we expect improvements by one order of magnitude within the next few years for laboratory experiments and up to five orders with dedicated satellite missions. These experiments, when carried through successfully, will definitely for the first time decide on the viability
of certain quantum gravity predictions.

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A Standard Model Extension definitions and conventions

Rich and extensive literature exists on the SME, so a full description is neither necessary nor possible here. However, we introduce briefly the definition and standard notation for Lorentz violating quantities in the SME.

Lagrangian The SME starts from a Lagrangian formulation of the Standard Model, adding all possible observer Lorentz scalars that can be formed from the known particles and Lorentz tensors. Taken from the full SME that includes all known particles, the Lagrangian involving the Dirac field $\psi$ of one fermion and the electromagnetic field $F^{\mu\nu}$ can be written as

$$L = \frac{i}{2} \bar{\psi} \Gamma_{\nu} D^{\nu} \psi - \frac{1}{2} \bar{\psi} M \psi + \text{h.c}$$

$$- \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{4} (k_F)^{\kappa\lambda\mu\nu} F^{\kappa\lambda} F^{\mu\nu} + \frac{1}{2} (k_{AF})^{\kappa} \epsilon_{\kappa\lambda\mu\nu} A^\lambda F^{\mu\nu} ,$$

where h.c. denotes the Hermitian conjugate of the previous terms, and $A^\lambda$ is the vector potential. The symbols $\Gamma_{\nu}$ and $M$ are given by

$$\Gamma_{\nu} = \gamma_{\nu} + c_{\mu\nu} \gamma^\mu + d_{\mu\nu} \gamma_5 \gamma^\mu + e_{\nu} + i f_{\nu} \gamma_5 + \frac{1}{2} g_{\lambda\mu\nu} \sigma^{\lambda\mu} ;$$

$$M = m + a_{\mu} \gamma^\mu + b_\mu \gamma_5 \gamma^\mu + \frac{1}{2} H_{\mu\nu} \sigma^{\mu\nu} .$$

The SME introduces such parameters into the Lagrangian of each type of fermion, as denoted by superscripts added to the field $\psi$ as well as to the symbols $a_{\mu}, b_\mu, c_{\mu\nu}, d_{\mu\nu}, e_\nu, f_\nu, g_{\lambda\mu\nu},$ and $H_{\mu\nu}$. The $\gamma_{\nu}, \gamma_5$ and $\sigma^{\mu\nu}$ are the conventional Dirac matrices, and $D^{\nu}$ is the usual covariant derivative. The tensors entering $M$ have the dimension mass, the others are dimensionless. $H_{\mu\nu}$ is antisymmetric; $g_{\lambda\mu\nu}$ is antisymmetric in its first two indices. $c_{\mu\nu}$ and $d_{\mu\nu}$ are traceless. Gauge invariance and renormalizability excludes $e_\nu, f_\nu,$ and $g_{\lambda\mu\nu},$ so these may be assumed to be either zero or suppressed relative to the other terms. Analogous terms are obtained in the framework of the generalized Dirac equation described in Sec.3.2.

Lorentz violation for the photons is encoded in the tensors $(k_F)^{\kappa}$ and $(k_{AF})^{\kappa}$ of the non-Lagrangian framework described in Sec.3.1.

Coordinate and field definitions Some of the Lorentz violating parameters contained in one sector of the SME can be absorbed into the other sectors by coordinate and field redefinitions. For example, in a hypothetical world containing only photons and electrons, nine components of $(k_F)^{\kappa}$ could be moved into the nine symmetric components of $c_{\mu\nu}$. By definition, either the photon or the electron sector could be taken as conventional with respect to these parameters, with
the Lorentz violation in the other sector. The presence of other particles, of course, changes this picture. We can still assume that one of the sectors is conventional, but then in general the other sectors are Lorentz-violating. Loosely speaking, in experiments where one compares the sectors against each other, only differential effects are meaningful.

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