Anti–de Sitter/ boundary conformal field theory correspondence in the non-relativistic limit

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Abstract

Boundary conformal field theory (BCFT) is the study of conformal field theory (CFT) in semi-infinite space-time. In non-relativistic limit ($x \to \epsilon x, t \to t, \epsilon \to 0$), boundary conformal algebra changes to boundary Galilean conformal algebra (BGCA). In this work, some aspects of AdS/BCFT in non-relativistic limit were explored. We constrain correlation functions of Galilean conformal invariant fields with BGCA generators. For a situation with a boundary condition at surface $x = 0$ ($z = \bar{z}$), our result is agree with non-relativistic limit of BCFT two-point function. We also, introduce holographic dual of boundary Galilean conformal field theory.

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1 Introduction

Recently, there has been some interest in extending the AdS/CFT correspondence to non-relativistic field theories [1, 2], where the non-relativistic conformal symmetry was obtained by a parametric contraction of the relativistic conformal group. Galilean conformal algebra (GCA) arises as a contraction relativistic conformal algebras [1, 3, 4], where in $d = 4$ the Galilean conformal group is a fifteen parameter group which contains the ten parameter Galilean subgroup. Beside Galilean conformal algebra, there is another non-relativistic algebra, the twelve parameter Schrödinger algebra [5, 6]. The dilatation generator in the Schrödinger group scales space and time differently, $x_i \to \lambda x_i$, $t \to \lambda^2 t$, but in contrast the corresponding generator in GCA scales space and time in the same way, $x_i \to \lambda x_i$, $t \to \lambda t$.

Infinite dimensional Galilean conformal group has been reported in [3]. The generators of this group are: $L^n = -(n+1)t^n x_i \partial_i - t^{n+1} \partial_n$, $M_i^n = t^{n+1} \partial_i$ and $J_{ij}^n = -t^n (x_i \partial_j - x_j \partial_i)$ for an arbitrary integer $n$, where $i$ and $j$ are specified by the spatial directions which obey commutation relation of the Virasoro-Kac-Moody algebra [3]. There is a finite dimensional subgroup of the infinite dimensional Galilean conformal group which is generated by $(J_{ij}^0, L_\pm^1, L_0, M_i^\pm_1, M_i^0)$. These generators are obtained by contraction ($t \to t$, $x_i \to \epsilon x_i$, $\epsilon \to 0$, $v_i \sim \epsilon$) of the relativistic conformal generators. The gravity dual of finite GCA was considered in [3, 4, 7] and the metric with finite 2d GCA isometry was obtained in [8].

The presence of free surfaces or walls in macroscopic systems which are at the critical point, lead to the large variety of physical effects. Since, using boundary condition effect is shown to be very helpful in various branch in physics, the systems with boundary conditions have been considered by both theorists [9] and experimentalists [10]. The situation with walls or free surfaces opens a new area in condensed matter physics [11]. In reference [12], the research on semi-infinite systems which exhibits a non-equilibrium bulk phase transitions was initiated and the effects of boundary condition on direct percolation were considered.

Holographic dual of a conformal field theory with a boundary (BCFT) was proposed in [13]. The main idea of AdS/BCFT correspondence was started with asymptotically AdS geometry with Neumann boundary condition on the metric as one approaches to the boundary [13, 14]. The geometry is modified by imposing two different boundary conditions on the metric. The boundary is divided into two parts $\partial M = N \cup Q$ where $\partial Q = \partial N$ [13]. The metric has Neumann boundary condition on $Q$ and Dirichlet boundary condition on $N$. With this boundary condition the AdS geometry is divided into...
two parts and the gravitational theory lives in one part of this space. This modified geometry could provide a holographic dual for BCFT. Boundary conformal field theory (BCFT) defined in domains with a boundary [15]. In this work we extend AdS/BCFT correspondence to non-relativistic version. When non-relativistic CFT lives in semi-infinite space, one sector of Galilean conformal group is removed. For example, if we have a boundary condition on surface $z = \bar{z}$ ($t - x = t + x$) or $x = 0$, translation, Boost and spatial-spacial conformal transformation are removed. So, two-point function in this situation is completely different from situation without boundary condition (free space). Two-point function of BCFT in the situation with a boundary condition at surface $z = \bar{z}$ was calculated in [16, 17]. In this paper we calculate two-point and three point functions of BGCA from gravity dual [7] and quantum field theory method in the boundary [18, 19]. Our results agree with results [16, 17] in non-relativistic limit. We also, introduce holographic dual of non-relativistic limit of BCFT (BGCA). The paper organized as follow: In section 2 we give a brief review of 2d GCA. In section 3 we calculate two-point and three point correlation functions of Galilean conformal invariant fields in semi-infinite space. In section 4 we introduce holographic dual of non-relativistic BCFT, then we calculate two-point function from gravity dual. Finally, in section 5, we close by some concluding remarks.

2 GCA in 2d

Galilean conformal algebra (GCA) in 2d is obtained by contracting 2d conformal symmetry [18]. Two-dimensional Conformal algebra is described by two copies of Virasoro algebra. In quantum field theory (QFT) level, two-dimensional ($z = t + x$, $\bar{z} = t - x$) CFT generators

\[
\mathcal{L}_n = z^{n+1} \partial_z, \quad \bar{\mathcal{L}}_n = \bar{z}^{n+1} \partial_{\bar{z}},
\]

obey the Virasoro algebra

\[
[\mathcal{L}_m, \mathcal{L}_n] = (n - m) \mathcal{L}_{m+n} + \frac{c_R}{12} m(1 - m^2) \delta_{m+n,0},
\]

\[
[\bar{\mathcal{L}}_m, \bar{\mathcal{L}}_n] = (n - m) \bar{\mathcal{L}}_{m+n} + \frac{c_L}{12} m(1 - m^2) \delta_{m+n,0}.
\]

In non-relativistic limit ($t \to t$, $x \to \epsilon x$ with $\epsilon \to 0$), the GCA generators $L_n$ and $M_n$ are constructed by Virasoro generators $L_n$ and $M_n$

\[
L_n = \lim_{\epsilon \to 0} (\mathcal{L}_n + \bar{\mathcal{L}}_n) = (n + 1) t^n \partial_x + t^{n+1} \partial_t,
\]

\[
M_n = - \lim_{\epsilon \to 0} \epsilon (\mathcal{L}_n - \bar{\mathcal{L}}_n) = -t^{n+1} \partial_x.
\]
From Eqs. (2) and (3), one obtains centrally extended 2d GCA
\[
[L_m, L_n] = (n - m)L_{m+n} + C_1 m(1 - m^2)\delta_{m+n,0},
\]
\[
[L_m, M_n] = (n - m)M_{m+n} + C_2 m(1 - m^2)\delta_{m+n,0},
\]
\[
[M_n, M_m] = 0.
\]
The GCA central charges \((C_1, C_2)\) are related to CFT central charges \((c_L, c_R)\) as:
\[
C_1 = \lim_{\epsilon \to 0} \frac{c_L + c_R}{12}, \quad C_2 = \lim_{\epsilon \to 0} \frac{c_L - c_R}{12}.
\]
From above equations, for a non-zero and finite \((C_2, C_1)\) in the limit \(\epsilon \to 0\), it can be seen that we need \(c_L - c_R \propto O(\frac{1}{\epsilon})\) and \(c_L + c_R \propto O(1)\). Similarly, rapidity \(\xi\) and scaling dimensions \(\Delta\), which are the eigenvalues of \(M_0\) and \(L_0\) respectively, are given by
\[
\Delta = \lim_{\epsilon \to 0} (h + \overline{h}), \quad \xi = -\lim_{\epsilon \to 0} \epsilon (h - \overline{h}),
\]
where \(h\) and \(\overline{h}\) are eigenvalues of \(L_0\) and \(\overline{L}_0\) respectively. Equation (5) tells us that, \(h + \overline{h}\) is of order \(O(1)\) while \(h - \overline{h}\) must be order \(O(\frac{1}{\epsilon})\), for the finite \(\Delta, \xi\).

3 Two-point function in semi-infinite space

In this section we find the correlation functions in semi-infinite space with a boundary condition at surface \(z = \overline{z}\). We now turn to derive the consequences of Galilean conformal invariance for the correlation. In general, we expect a quasi-primary field \(\mathcal{O}\) to be characterized by its Galilean conformal dimension \(\Delta\) and rapidity \(\xi\) (These fields are invariant under finite sub-group that is generated by sub-algebra \(\{L_{-1}, M_{-1}, L_0, M_0, L_1, M_1\}\)). We would like to find the form of two-point and three-point functions of the Galilean conformal invariant operators in semi-infinite space. Firstly, we find the form of the commutators \([\mathcal{L}_n, \mathcal{O}]\) and \([\overline{\mathcal{L}}_n, \mathcal{O}]\), then we obtain the form of \([\mathcal{L}_n, \mathcal{O}]\) and \([M_n, \mathcal{O}]\) as following
\[
[\mathcal{L}_n, \mathcal{O}(z, \overline{z})] = [\mathcal{L}_n, U\mathcal{O}(0)U^{-1}] = [\mathcal{L}_n, U]\mathcal{O}(0)U^{-1} + U\mathcal{O}(0)[\mathcal{L}_n, U^{-1}]
\]
\[
+ U[\mathcal{L}_n, \mathcal{O}(0)]U^{-1} = U\{U^{-1}\mathcal{L}_nU - \mathcal{L}_n\}\mathcal{O}(0)U^{-1}
\]
\[
+ U\mathcal{O}(0)\{\mathcal{L}_n - U^{-1}\mathcal{L}_nU\}U^{-1} + \delta_{n,0}h\mathcal{O}(z, \overline{z})
\]
$U$ and $O(z, \overline{z})$ are defined as
\[ O(z, \overline{z}) = UO(0)U^{-1} \quad \text{where} \quad U = e^{z\mathcal{L}^{-1} + \overline{z}\mathcal{\overline{L}}^{-1}} \quad (8) \]

By using the Hausdorff formula we get
\[ U^{-1} \mathcal{L}_n U = e^{-z\mathcal{L}^{-1} - \overline{z}\mathcal{\overline{L}}^{-1}} \mathcal{L}_n e^{z\mathcal{L}^{-1} + \overline{z}\mathcal{\overline{L}}^{-1}} = e^{-z\mathcal{L}^{-1} \mathcal{L}_n e^{z\mathcal{L}^{-1}}} \]
\[ = \mathcal{L}_n + [\mathcal{L}_n, z\mathcal{L}^{-1}] + \frac{1}{2!}[[\mathcal{L}_n, z\mathcal{L}^{-1}], z\mathcal{L}^{-1}] + \ldots \]
\[ = \sum_{k=0}^{n+1} \frac{(n+1)!}{(n+1-k)!k!} (z)^k \mathcal{L}_{n-k} \]

and
\[ \mathcal{L}'_n = U^{-1} \mathcal{L}_n U - \mathcal{L}_n = \sum_{k=1}^{n+1} \frac{(n+1)!}{(n+1-k)!k!} (z)^k \mathcal{L}_{n-k} \quad (9) \]

The Eq. (7) gives us
\[ [\mathcal{L}_n, O(z, \overline{z})] = U \{[\mathcal{L}'_n, O(0)] + \delta_{n,0} hO(0)\} U^{-1} \]
\[ = z^{n+1} [\mathcal{L}^{-1}, O(z, \overline{z})] + z^n (n+1)U[\mathcal{L}_0, O(0)]U^{-1} \quad (10) \]

Now we have $[\mathcal{L}^{-1}, O] = \partial_z O$ and $[\mathcal{L}_0, O] = hO$ ($\mathcal{L}_0$ and $\mathcal{L}^{-1}$ generate $z$-dilatation and $z$-translation, respectively). Hence we obtain (for $n \geq -1$)
\[ [\mathcal{L}_n, O(z, \overline{z})] = (z^{n+1}\partial_z + (n+1)hz^n)O \quad (11) \]

We can exchange $\mathcal{L}_n$ with $\mathcal{\overline{L}}_n$ and using the above steps (7)-(11). We get
\[ [\mathcal{\overline{L}}_n, O(z, \overline{z})] = (\overline{z}^{n+1}\partial_{\overline{z}} + (n+1)h\overline{z}^n)O \quad (12) \]

From Eqs. (12), (13) and by using the definitions of $L_n$ and $M_n$ (3), we can find the form of commutators $[L_n, O]$ and $[M_n, O]$
\[ [L_n, O] = (t^{n+1}\partial_t + (n+1)t^n x\partial_x + (n+1)(t^n \Delta - n x t^{n-1} \xi))O \]
\[ [M_n, O] = (t^{n+1}\partial_x - (n+1)t^n \xi)O \quad (14) \]

Correlation functions of GCA are constrained by the above equations in free space [19]. If we have a boundary in $x$ direction, symmetries in this direction is removed obviously. So, in the situation with a boundary condition at
surface \( x = 0 \) (\( z = \tau \)), Galilean symmetry group reduces to one copy of non-relativistic version of Virasoro group which is generated by \( L_n \) \cite{10, 11}. We can use this subgroup to calculate two-point function. Firstly, we consider the invariance under time translation which is generated by \( L^{-1}_1 \)<

\[
\begin{align*}
<0 | [L^{-1}_1, G] | 0 > = 0 & \Rightarrow G = G(x_1, x_2, \tau) \quad \tau = t_1 - t_2
\end{align*}
\]

(15)

where \( G = < \mathcal{O}_1 \mathcal{O}_2 > \) is two-point function of two quasi-primary operators \( \mathcal{O}_1 \) and \( \mathcal{O}_2 \). Invariance under dilatation constrains two-point function as

\[
\begin{align*}
<0 | [L_0, G] | 0 > = 0
\end{align*}
\]

(16)

\[
\begin{align*}
\Rightarrow \sum_{i=1}^{2} (t_i \partial_{t_i} + x_i \partial_{x_i} + \Delta_i)G = 0
\end{align*}
\]

\[
(\tau \partial_{\tau} + x_1 \partial_{x_1} + x_2 \partial_{x_2} + \Delta)G = 0 \quad \Delta = \Delta_1 + \Delta_2
\]

Invariance under spatial component of special conformal transformation is

\[
\begin{align*}
<0 | [L_1, G] | 0 > = 0
\end{align*}
\]

(17)

\[
\begin{align*}
\Rightarrow \sum_{i=1}^{2} (t_i^2 \partial_{t_i} + 2t_i x_i \partial_{x_i} + 2t_i \Delta_i - x_i \xi_i)G
\end{align*}
\]

\[
= ((t_1^2 - t_2^2) \partial_{\tau} + 2(t_1 x_1 \partial_{x_1} + t_2 x_2 \partial_{x_2})
+ 2(t_1 \Delta_1 + t_2 \Delta_2 - x_1 \xi_1 - x_2 \xi_2))G
\]

\[
= (\tau^2 \partial_{\tau} + 2t_2 (\tau \partial_{\tau} + x_1 \partial_{x_1} + x_2 \partial_{x_2})
- 2(x_1 \xi_1 + x_2 \xi_2) + 2(\tau x_1 \partial_{x_2} + 2(t_1 \Delta_1 + t_2 \Delta_2))G
\]

\[
= (\tau^2 \partial_{\tau} - 2(x_1 \xi_1 + x_2 \xi_1) + 2\tau x_1 \partial_{x_2} + 2\tau \Delta_1)G = 0
\]

where in the last step, Eq. (16) was used. We make the following ansatz

\[
G(x_1, x_2, \tau) = \tau^{-2\Delta_1} G'(u, v), \quad u = \frac{x_1}{\tau}, \quad v = \frac{x_2}{\tau}
\]

(18)

so, Eq. (17) gives

\[
(u \partial_u - v \partial_v - 2(u \xi_1 + v \xi_2))G'(u, v) = 0,
\]

(19)

Solution of this equation is \cite{9, 20}.

\[
G'(u, v) = \chi(uv) \exp(2(u \xi_1 - v \xi_2))
\]

(20)

where \( \xi \) is an arbitrary function. The final result for two-point function is

\[
G(x_1, x_2, \tau) = \delta_{\Delta_1, \Delta_2} \tau^{-2\Delta} \chi \left( \frac{x_1 x_2}{\tau^2} \right) \exp \left( \frac{2}{\tau} (x_1 \xi_1 - x_2 \xi_2) \right)
\]

(21)
where $\Delta = \Delta_1 = \Delta_2$. It is clear that, two-point function near the boundary is different from other places \cite{19}. Two-point function of BCF T for scaler fields was calculated in \cite{16, 17}
\begin{equation}
G(z_1, z_2, \bar{z}_1, \bar{z}_2) = \frac{1}{4} \left( \frac{1}{|z_1 - z_2|^2\Delta} + \frac{1}{|\bar{z}_1 - \bar{z}_2|^2\Delta} + \frac{1}{|z_1 - \bar{z}_2|^2\Delta} + \frac{1}{|\bar{z}_1 - z_2|^2\Delta} \right)
\end{equation}

In non-relativistic limit ($t \to t, x \to \epsilon x$) we have
\begin{align}
\lim_{\epsilon \to 0} (z_1 - z_2) &= \lim_{\epsilon \to 0} (t_1 + \epsilon x_1 - t_2 - \epsilon x_2) = t_1 - t_2 \\
\lim_{\epsilon \to 0} (\bar{z}_1 - \bar{z}_2) &= \lim_{\epsilon \to 0} (t_1 - \epsilon x_1 - t_2 + \epsilon x_2) = t_1 - t_2 \\
\lim_{\epsilon \to 0} (z_1 - \bar{z}_2) &= \lim_{\epsilon \to 0} (t_1 + \epsilon x_1 - t_2 + \epsilon x_2) = t_1 - t_2 \\
\lim_{\epsilon \to 0} (\bar{z}_1 - z_2) &= \lim_{\epsilon \to 0} (t_1 - \epsilon x_1 - t_2 - \epsilon x_2) = t_1 - t_2
\end{align}

From above equations we obtain
\begin{equation}
\lim_{\epsilon \to 0} G(z_1, z_2, \bar{z}_1, \bar{z}_2) = \delta_{\Delta_1, \Delta_2} \tau^{-2\Delta}
\end{equation}

which is agree with our result \cite{21}. (For scalar field $\xi_i$ is equal to zero.)

Now using above method, we calculate three-point correlation function in semi-infinite space-time with a boundary condition at surface $x = 0$.

Consider the three-point function as
\begin{equation}
G(x_1, x_2, x_3, t_1, t_2, t_3) = \langle \phi_1(x_1, t_1) \phi_2(x_2, t_2) \phi_3(x_3, t_3) \rangle
\end{equation}

where $\phi_1, \phi_2$ and $\phi_3$ are Galilean conformal invariant fields. Invariance under the time translation symmetry implies $G = G(x_1, x_2, x_3, \tau, \sigma)$ where $\tau = t_1 - t_3$ and $\sigma = t_2 - t_3$. We constrain $G$ by scale invariance as
\begin{align}
&\langle 0 \mid [L_0, \phi_1\phi_2\phi_3] \mid 0 \rangle = 0 \\
\Rightarrow &\sum_{i=1}^{3} \left( t_i \partial_{t_i} + x_i \partial_i + \Delta_i \right) G \\
= &\left( \tau \partial_\tau + \sigma \partial_\sigma + x_1 \partial_{x_1} + r_2 \partial_{x_2} + x_3 \partial_{x_3} + \Delta_1 + \Delta_2 + \Delta_3 \right) G = 0
\end{align}
From the invariance under the time component of non-relativistic special conformal transformation we get

\[ <0 \mid [L_1, \phi_1 \phi_2 \phi_3] \mid 0 \rangle = 0 \]  
\[ \Rightarrow \sum_{i=1}^{3} (t_i^2 \partial_t + 2t_i x_i \partial_t + 2t_i \Delta_i - x_i \xi_i) G \]
\[ = ((t_1^2 - t_3^2) \partial_t + (t_2^2 - t_3^2) \partial_t + 2(t_1 x_1 \partial_{x_1} + t_2 x_2 \partial_{x_2} + t_3 x_3 \partial_3) \]
\[ + 2(t_1 \Delta_1 + t_2 \Delta_2 + t_3 \Delta_3) - x_1 \xi_1 - x_2 \xi_2 - x_3 \xi_3) G \]
\[ = (\tau^2 \partial_t + \sigma^2 \partial_t + 2t_3(\tau \partial_t + \sigma \partial _t + x_1 \partial_{x_1} + x_2 \partial_{x_2} + x_3 \partial_3) \]
\[ - x_1 \xi_1 - x_2 \xi_2 - x_3 \xi_3 + 2\tau x_1 \partial_{x_1} + 2\sigma x_2 \partial_{x_2} + 2(t_1 \Delta_1 + t_2 \Delta_2 + t_3 \Delta_3) G \]
\[ = (\tau^2 \partial_t - x_1 \xi_1 - x_2 \xi_2 - x_3 \xi_3 + 2\tau x_1 \partial_{x_1} + 2\sigma x_2 \partial_{x_2} + 2(t_1 \Delta_1 + t_2 \Delta_2 + t_3 \Delta_3)) G \]
\[ + \sigma^2 \partial_t + 2\sigma x_2 \partial_{x_2} + 2\tau \Delta_1 + 2\sigma \Delta_2) G = 0 \]

where in the last equation we have used Eq. (26). We make the ansatz

\[ G = \delta_{\Delta_1 + \Delta_2, \Delta_3} \tau^{-2\Delta_1} \sigma^{-2\Delta_2} G' \]  
(28)

and simplify the above equations as

\[ (\tau^2 \partial_t + 2\tau x_1 \partial_{x_1} - x_1 \xi_1 - x_3 \xi_3) G_1' = 0 \]  
(29)
\[ (\tau \partial_t + x_1 \partial_{x_1} + x_3 \partial_{x_3}) G_1' = 0 \]
\[ (\sigma^2 + 2\sigma x_2 \partial_{x_2} - x_2 \xi_2) G_2' = 0 \]
\[ (\sigma \partial_\sigma + x_2 \partial_{x_2}) G_2' = 0 \]

where \( G' = G_1'(x_1, x_3, \tau) G_2'(x_2, \sigma) \), or

\[ (\sigma^2 \partial_\sigma + 2\sigma x_2 \partial_{x_2} - x_2 \xi_2 - x_3 \xi_3) G_1' = 0 \]  
(30)
\[ (\sigma \partial_\sigma + x_2 \partial_{x_2} + x_3 \partial_{x_3}) G_1' = 0 \]
\[ (\tau^2 + 2\tau x_1 \partial_{x_1} - x_1 \xi_1) G_2' = 0 \]
\[ (\tau \partial_\tau + x_1 \partial_{x_1}) G_2' = 0 \]

where \( G' = G_1'(x_2, x_3, \sigma) G_2'(x_1, \tau) \). By using the method of characteristic [20], we may found the general solution of these equations

\[ G = \delta_{\Delta_1 + \Delta_2, \Delta_3} (t_1 - t_3)^{-2\Delta_1} (t_2 - t_3)^{-2\Delta_2} \exp\left(\frac{x_1 \xi_1}{t_1 - t_3} + \frac{x_2 \xi_2}{t_2 - t_3}\right) \]  
(31)
\[ \times \left(\chi_1 \left(\frac{x_1 x_3}{(t_1 - t_3)^2}\right) \exp\left(-\frac{x_3 \xi_3}{t_1 - t_3}\right) + \chi_2 \left(\frac{x_2 x_3}{(t_2 - t_3)^2}\right) \exp\left(-\frac{x_3 \xi_3}{t_2 - t_3}\right)\right) \]
\[ + \Sigma (exchanging \quad 2 \leftrightarrow 3 \quad \text{or} \quad 1 \leftrightarrow 3) \]

where \( \chi_1 \) and \( \chi_2 \) are arbitrary functions. Three-point function near the boundary is obviously different from other places.
4 Holographic dual of non-relativistic BCFT

Recently, holographic dual of BCFT was considered [13, 14, 16]. $AdS_3$ with Neumann boundary condition at surface $z = \bar{z}$ is holographic dual of $BCFT_2$. In this situation the symmetry group of boundary conformal field theory is generated by one copy of Virasoro algebra [16]. We introduce non-relativistic version of this gravity dual as a holographic dual of non-relativistic BCFT. The $AdS_3$ metric in Poincare coordinates is

$$ds^2 = \frac{1}{r^2}(-dt^2 + dr^2 + dx^2)$$  \hspace{1cm} (32)

where $r$ is a radial coordinate and $(x,t)$ are boundary coordinate. In the Eddington-Finkelstein coordinates which define by $r = r'$ and $t = t' + r'$ the $AdS_3$ metric is given by

$$ds^2 = \frac{1}{r^2}(dt^2 - 2dtdr + dx^2)$$  \hspace{1cm} (33)

The Killing vectors of $AdS_3$ read as

$$P = \partial_x \quad B = (t - r)\partial_x - x\partial_t$$  \hspace{1cm} (34)

$$K_x = (t^2 - 2tr - x^2)\partial_x + 2tx\partial_t + 2rx\partial_r + 2x^2\partial_x$$

$$H = -\partial_t \quad D = -t\partial_t - r\partial_r - x\partial_x$$

$$K = -(t^2 + x^2)\partial_t - 2r(t - r)\partial_r - 2(t - r)x\partial_x$$

In non-relativistic limit

$$t \rightarrow t \quad r \rightarrow r \quad x \rightarrow \epsilon x$$  \hspace{1cm} (35)

we obtain the contracted Killing vectors

$$P = \partial_x \quad B = (t - r)\partial_x \quad K_x = (t^2 - 2tr)\partial_x \quad H = -\partial_t$$  \hspace{1cm} (36)

$$D = -t\partial_t - r\partial_r - x\partial_x \quad K = -t^2\partial_t - 2(t - r)(r\partial_r + x\partial_x)$$

We can define an infinite extension of these vectors field in the bulk [3]

$$M^{(n)} = (t^{n+1} - (n + 1)rtn)\partial_x$$  \hspace{1cm} (37)

$$L^{(n)} = -t^{n+1}\partial_t - (n + 1)(t^n - nrt^{n-1})(x\partial_x + r\partial_r)$$

These vector fields obey the commutation relation (4) (without central charges). We can see that, these vector fields at the boundary $r = 0$ reduce to Killing vectors of contracted $CFT_2$ [3]. The vector fields $M^{(n)}$ only
act on the spatial coordinate $x$, so if we have a boundary condition at surface $x = 0$ ($z = \bar{z}$), these vector fields are removed from all Killing vectors in the bulk. Now we consider the action of the Virasoro generators $L^{(n)}$ (remanded Killing vectors) on $AdS_3$ metric in non-relativistic limit. We introduce non-relativistic limit of $AdS_3$ metric which is given by $AdS_2 \times R$ metric [3]

$$ds^2 = \frac{1}{r^2}(-2dt dr + dt^2 + dx^2) \rightarrow \frac{1}{r^2}(-2dt dr + dt^2)$$ (38)

The components of the metric in the $(t, r)$ directions survive and we receive to $AdS_2$ metric. The spatial direction $x$ is fiber over this $AdS_2$. Virasoro generators $L^{(n)}$ act non-trivially on all coordinate

$$r \rightarrow r' = r(1 + a_n(n + 1)(t^n - nrt^{n-1}))$$

$$t \rightarrow t' = t(1 + a_nt^n)$$ (39)

where $a_n$ is infinitesimal parameter. From above equation we have

$$dr \rightarrow dr' = dr(1 + a_n(n + 1)(t^n - nrt^{n-1}))$$

$$+ r a_n n(n + 1)t^{n-2}((t - (n - 1)r)dt - tdr)$$

$$dt \rightarrow dt' = dt(1 + (n + 1)a_nt^n)$$ (40)

So in non-relativistic limit (35) we get

$$ds^2 = \frac{1}{r^2}(-2dt dr + dt^2) \rightarrow \frac{1}{r^2}(-2dt dr + dt^2 + 2n(n^2 - 1)a_n r^n t^{n-2}dt^2)$$ (41)

The $SL(2, R)$ subgroup which is generated by $L^0, L^\pm$ are exact isometries of non-relativistic version of boundary $AdS_3$. Near the boundary $r = 0$ the diffeomorphisms of above metric has a fall-off like $r^2$, so other $L^n$ are asymptotic isometries of non-relativistic $AdS_3$. One copy of Virasoro algebra is asymptotic symmetry of non-relativistic $AdS_3$ metric with a boundary condition. Following [7], we calculate two-point function from gravity dual. Equation of motion for massive scalar field on the $AdS_3$ background (32) is given by

$$\frac{1}{\sqrt{G}}\partial_M(\sqrt{G}G^{MN}\partial_N \phi(t, r, x)) - m^2 \phi(t, r, x) = 0$$ (42)

In non-relativistic limit [35] we have

$$\frac{1}{\sqrt{G}}\partial_a(\sqrt{G}g^{ab}\partial_b \phi) - m^2 \phi = 0 \quad \partial_x^2 \phi = 0$$ (43)
The first equation can be obtained from the following action

\[ I = \int dt dr \sqrt{G^{1/2}} (g^{ab} \partial_a \phi \partial_b \phi + m^2 \phi^2) \]  

(44)

General solution of the equation of motion of the above action is

\[ \phi(t, r) = r e^{-i\omega t} (AI_\alpha(\omega r) + BK_\alpha(\omega r)) \]  

(45)

where \( \alpha = \sqrt{m^2 + 1} \). From AdS\(_3\)/CFT\(_2\) correspondence, we can find the bulk solution as

\[ \phi(t, r) = c_\delta \Delta^{-2} \int dt' \phi_\delta(t')(\frac{r}{r^2 + |t - t'|^2})^\Delta \]  

(46)

where \( \phi_\delta \) is a Dirichlet boundary value at \( r = \delta \) and \( \Delta = \alpha + \frac{1}{2} \). The above equation can be used to read two-point function of GCA\(_2\)

\[ <\phi(t_1)\phi(t_2)> \sim (t_1 - t_2)^{-2\Delta} \]  

(47)

which is agree with results (21) and (24).

5 Conclusion

Galilean conformal algebra (GCA) arises as a contraction of conformal algebra. We can use 2d GCA to constrain correlation functions. Correlation functions of Galilean conformal invariant fields in 2d for space-time without boundary condition were found in [19]. We calculated two-point and three-point functions in semi-infinite space with a boundary condition at surface \( z = \bar{z} \) \( (x = 0) \), by using some methods in quantum field theory [21] and from gravity dual [47]. Our results [21] and [47] are agree with two-point function of BCFT in non-relativistic limit [24]. We also, introduce holographic dual of BCFT in non-relativistic limit (BGCA). AdS\(_3\) with boundary condition and in non-relativistic limit has asymptotic isometries which are generated by one copy of non-relativistic version of Virasoro algebra.

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