Correlation and numerical study of heat transfer for single row cross-flow heat exchangers with different fin thickness

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Abstract

Efficiency of heat exchanger and its dimensions are ones of the most important parameters to consider in engineering design activity. The size of heat exchanger can be more compact by introducing the fins to increase the heat transfer rate between the heat exchanger surface and the surroundings. Different engineering methods are used in heat exchanger design process. The proper correlations or modeling and simulation tools are often applied to receive the general recommendation at early stages of exchanger study. The paper investigates the accuracy of air-side correlations for a cross flow heat exchanger. Then, numerical model is applied for heat transfer investigation in modified fin designs. The performance of the fin-tube heat exchanger for different fin thickness is calculated. To give indications about the accuracy of numerical outcome, the most popular correlations are evaluated and results obtained from Ansys CFX program are verified. Analyzing the output, it seems that the implementation of the CFD model offers particular benefits especially when minor modification are applied to the fin surface for which the correlation equations are not defined.

1. Introduction

A heat exchanger is equipment that is used to transfer thermal energy among fluids or between a solid surface and a fluid at different temperatures. In order to intensify the heat transfer from the heat exchanger surface to fluid, it is possible to increase convection coefficient (by growing the fluid velocity), widen temperature difference between

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surface and fluid or increase the surface area across which convection occurs [1]. In a gas-to-liquid exchanger, the heat transfer coefficient on the liquid side is generally one order of magnitude higher than that on the gas side and to minimize the size of heat exchangers, fins are used on the gas side to increase the surface area. Individually finned tubes are more rugged and practical in large tube-fin exchangers. Tube-fin exchangers can also withstand pressures on the tube side. The highest temperature is limited by the type of bonding, materials employed, and material thickness [2]. All above causes that finned tube heat exchangers are used in different thermal systems for applications where heat energy is exchanged between different media. The heat transfer rate is often the primary interest of engineers [3].

Single row or double row heat exchangers are still common device in different industries due to the construction simplicity and the maintenance cost. Although different engineering methods are used in design processes, there is little experimental work and correlations in literature that can be used for air-side single row heat exchanger.

Jang et al. [4] study numerically and experimentally fluid flow and heat transfer over a multi-row (1-6 rows) plate-fin and tube heat exchanger. The effects of different geometrical parameters such as tube arrangement, tube row numbers and fin pitch are investigated in detail for different Reynolds number. They find that an average plate-fin and tube heat exchanger. The effects of different geometrical parameters such as tube arrangements, tube row numbers and fin pitch are investigated in detail for different Reynolds number. They find that an average plate-fin and tube heat exchanger focusing on different fin profiles [5] and different fluid velocities [6]. In both papers, the aerodynamic configuration of the fin is taken into account. The fin and tube geometry affects the flow direction and has the effect on the temperature changes and heat transfer conditions. Mass flow weighted average temperatures of air volume flow rate are calculated in the outlet section and compared for different fin/tube shapes. Yan and Sheen [7] carry out an experimental study to investigate the heat transfer and pressure drop characteristics of fin-and-tube exchangers with plate, wavy and louvered fin surfaces. Various comparison methods are adopted to evaluate the performance of the heat exchanger. Results are presented as plots of friction factor and Colburn factor against Reynolds number. The numerical results are compared with the existing correlations for four-row annular-finned tube bundles. Mon and Gross [8] investigate the effects of fin spacing on four-row annular-finned tube bundles in staggered and in-line arrangements by the three-dimensional numerical study. According to the flow visualization results, the boundary layer developments and horseshoe vortices between the fins are found to be substantially dependent on the fin spacing to height ratio and Reynolds number. Mendez et al. [9] examine the influence of fin spacing on the over-tube side of a single-row fin-tube heat exchanger through flow visualization and numerical computation. The investigation is done for single-row heat exchangers, then the conclusions are also applied to multi-row devices. Numerical studies of heat transfer are used by Wais [10]. Analyses are carried out to examine finned tube heat exchanger and to determine the performance of the radiator, when the tube-fin heat exchanger geometry is modified. The same author [11] investigates the heat transfer and thermal efficiency of finned tube heat exchanger with and without winglets at the fin surface. The analyses are carried out for different fin configurations. A deflector implementation can increase significantly the heat transfer for flows through the channel formed by circular fins. Erek et al. [12] examine numerically the influences of the changes in fin geometry on heat transfer and pressure drop of a plate fin and tube heat exchanger. The distance between fins is found to have a considerable effect on pressure drop. It is observed that placing the fin tube at downstream region affects heat transfer positively. Yoo et al. [13] analyze the cross-flow over tube banks and investigate the local and average heat transfer characteristics for staggered tube banks. They show that the variation of the local heat transfer coefficients is quite different from the first tube to the third tube, but they are similar afterwards. In comparison with that of the first tube, the average Nusselt number increases more than 30% and 65% on the second and third tubes, respectively. Taler [14] describes numerical methods for determining heat transfer coefficients in cross-flow compact heat exchangers. The methods base on the calculated and measured coolant outlet temperatures, selection the functional form for the Nusselt numbers, and the results of the CFD simulation of flow and heat transfer in the heat exchanger. In [15], Taler and Oclon present numerical and experimental methods for the determination of the average heat transfer coefficient for the air flow in a fin-and-tube heat exchanger. Results received numerically are almost 14% lower comparing to those received from tests. It can be explained by existence of thermal contact resistance between the fin and the tube as well as the non-uniform distribution of the air and water flows in a real device. They also propose a modified method for estimating the mean value of thermal contact resistance in a plate fin-and-tube heat exchanger [16]. CFD simulations allow predicting heat transfer correlations for the plate fin and tube heat exchangers after introducing the
thermal contact resistance into the algorithm. Oclon et al. [17] present the numerical study on the influence of inner tube surface fouling on the thermal performance of a high temperature fin-and-tube heat exchanger. Fouling influences the heat transfer characteristics and can reduce the performance of the heat exchanger.

The current paper investigates the air-side correlations for a cross flow heat exchanger presented in the literature and verifies the numerical results for heat transfer received from Ansys CFX program. The numerical outcome of heat transfer coefficient form 3D model is compared to the results received from the empirical equation for the fin-tube heat exchanger of different fin thicknesses. The model allows considering the heat transfer in three directions that is advantage, comparing to other methods, for which the temperature and velocity profile is two-dimensional.

2. Fin-tube cross-flow heat exchanger geometry

The analysis of heat transfer from finned surfaces involves solving second-order differential equations and is often a subject of researches including also the variable heat transfer coefficient as a function of temperature or the fin geometrical dimensions. In general, the study of the extended surface heat transfer compromises the movement of the heat within the fin by conduction and the process of the heat exchange between the fin and the surroundings by convection [18]. For the ideal case, the optimized profile of the symmetrical radial fin of least material can be found from the generalized differential equation [19]. It leads to the parabolic fin shape for which the heat flux is less sensitive to the variation of the tip temperature than in the case of rectangular and trapezoidal fin profiles.

In practice, flow maldistribution is common during the air flow and influences the performance of heat exchangers. The analysis and design of heat exchangers consider problems in which the temperature of the fluid changes as it flows through a passage as a result of heat transfer between the wall and the fluid. For heat transfer analyses, at least the following heat transfer surface geometrical properties are needed on each side of a two-fluid exchanger:

- minimum free-flow area,
- core frontal area,
- heat transfer surface area which includes both primary and fin area,
- hydraulic diameter, and
- flow length.

Typical fin-tube geometry, with minimum cross-sectional area, is presented in Figure 1.

![Fig. 1. Fin–tube geometry.](image)

Surface area of one sector (consists of fin and tube) are defined as:

- surface area of fins: \( A_f = \frac{1}{2} \pi (D_f^2 - D^2) \) + \( \pi D_f \delta \)
- surface area of tube between fins: \( A_f = \pi \, D \, s \)
- total surface area: \( A_t = \pi \, D \, (s + \delta) \)

Reynolds number, maximum fluid velocity and Nusselt number is defined as:

\[
Re_D = \frac{\rho \, v_{\text{max}} \, D}{\eta} \tag{1}
\]

\[
v_{\text{max}} = \frac{m_f}{A_0 \, \rho} \tag{2}
\]

\[
N_u = \frac{h \, D}{k_f} \tag{3}
\]

The heat exchanger characteristic dimensions are written for different fin thickness in Table 1:

| Fin version | \( R_f = D_{f}/2 \) mm | \( R = D/2 \) mm | \( \delta_t \) mm | \( \rho_{f} \) mm | \( \rho_{t} \) mm | \( \delta \) mm |
|-------------|-------------------------|-----------------|------------------|------------------|------------------|--------|
| (a)         | 20.5                    | 12.5            | 2.0              | 3.0              | 46               | 1.2    |
| (b)         | 20.5                    | 12.5            | 2.0              | 3.0              | 46               | 1.0    |
| (c)         | 20.5                    | 12.5            | 2.0              | 3.0              | 46               | 0.8    |

3. Correlation for external heat transfer in fin-tube crossflow heat exchanger

The value of heat transfer depends on local fluid velocity, fluid properties and details of the tube bank geometry. Correlations that allow calculating average heat transfer coefficient, \( \overline{h} \), are derived from experimental data and take into account geometrical features.

3.1. Recommended correlation to calculate the average Nusselt number for staggered tube banks by Engineering Sciences Data Unit [21]

The correlation can be applied for Reynolds number range \( 2 \times 10^3 \leq \text{Re} \leq 4 \times 10^4 \) and \( 0.13 < \frac{s}{l} < 0.57 \), \( 1.15 < \frac{X_i}{X_l} < 1.72 \):

\[
\overline{N_u} = 0.242 \, \text{Re}^{0.658} \left( \frac{s}{l} \right)^{0.297} \left( \frac{X_i}{X_l} \right)^{-0.091} \cdot \text{Pr}^{1/3} \cdot F_1 \cdot F_2 \tag{4}
\]

where
- \( X_i \) — transverse tube pitch,
- \( X_l \) — longitudinal tube pitch,
$F_1$ – factor for fluid property variation (significant only at high temperatures)
$F_2$ – factor for number of fin-tube rows ($F_2 = 0.76$ is applied for all correlations).
  1.0 for four or more rows,
  0.92 for three rows
  0.84 for two rows
  0.76 for one row

3.2. Correlation of Briggs and Young [19], [22], [23]

$$
\overline{N_u} = 0.134 \, \text{Re}^{0.681} \, \text{Pr}^{1/3} \left( \frac{s}{l} \right)^{0.200} \left( \frac{s}{\delta} \right)^{0.1134}
$$

(5)

The correlation is based on experimental data for six row tube banks laid out on equilateral triangular pitch and $1 \cdot 10^3 \leq \text{Re} \leq 1.8 \cdot 10^4$, $11.13 \, \text{mm} < D < 40.89 \, \text{mm}$, $1.42 \, \text{mm} < l < 16.57 \, \text{mm}$, $0.33 \, \text{mm} < \delta < 2.02 \, \text{mm}$, $0.89 \, \text{mm} < s < 2.97 \, \text{mm}$, $24.38 \, \text{mm} < X_i < 111.00 \, \text{mm}$.

3.3. Effective heat transfer coefficient

Effective heat transfer coefficient, for the air flowing outside and at right angles to the axes of a bank of finned pipes, can be represented approximately by the dimensional equation [24]:

$$
\overline{h'} = \frac{A_f - A_i}{A_T} \overline{h}
$$

(6)

$$
\overline{h} = 5.29 \frac{V_{IN}^{0.6}}{D^{0.4}} \left( \frac{X_i}{X_i - D} \right)^{0.6}
$$

(7)

4. Mean temperature coefficient and heat transfer in heat exchanger

Total heat transfer can be calculated taking into consideration fin efficiency:

$$
Q = \overline{h} \Delta T \left( \eta_f A_f + A_i \right) = \overline{h'} \Delta T \, A
$$

(8)

where

- $\eta_f$ - fin efficiency
- $\overline{h'}$ - effective heat transfer coefficient

To evaluate the heat transfer, it is necessary to find the effective mean temperature difference, $\Delta T$. Since the fluid temperature changes in fluid flow through the tube bank, the fluid temperature difference $\Delta T_{\text{Fluid}}$ can be calculated from energy exchanged as:

$$
\dot{Q} = \overline{h} \Delta T \left( \eta_f A_f + A_i \right) = m_f c_f \Delta T_{\text{Fluid}}
$$

(9)

where

\( m_f \) and \( c_f \) are the mass flow rate and the specific heat of the fluid.
\[ \Delta T = \frac{(T_0 - T_{OUT}) - (T_0 - T_{IN})}{\ln \frac{T_0 - T_{OUT}}{T_0 - T_{IN}}} \]  \hspace{1cm} (10)

and for \( T_{IN} > T_{OUT} \)

\[ \Delta T_{\text{fluid}} = T_{IN} - T_{OUT} \]  \hspace{1cm} (11)

After transformation

\[ \Delta T_{\text{fluid}} = \frac{\tilde{h} \left( \eta_f A_f + A_t \right)}{m_f c_f} \Delta T \]  \hspace{1cm} (12)

\[ T_{OUT} = T_{IN} - \frac{\tilde{h} \left( \eta_f A_f + A_t \right)}{m_f c_f} \Delta T \]  \hspace{1cm} (13)

\[ \Delta T = (T_{IN} - T_0) \frac{1 - \exp \left( -\frac{\tilde{h} \left( \eta_f A_f + A_t \right)}{m_f c_f} \Delta T \right)}{\tilde{h} \left( \eta_f A_f + A_t \right)} \]  \hspace{1cm} (14)

Having calculated effective mean temperature difference, \( \Delta T \), average heat transfer coefficient, \( \tilde{h} \), and fin efficiency, \( \eta_f \), the rate of heat transfer can be found from Eq. (11)

The fin efficiency value \( \eta_f \) can be achieved from equation [20]:

\[ \eta_f = \frac{\tanh \left( \sqrt{\frac{2 \tilde{h}}{\kappa_f}} \cdot \psi \right)}{\sqrt{\frac{2 \tilde{h}}{\kappa_f}} \cdot \psi} \]  \hspace{1cm} (15)

where

\[ \psi = \frac{D}{2} \left( \frac{D_f}{D} - 1 \right) \left( 1 + 0.35 \ln \frac{D_f}{D} \right) \]  \hspace{1cm} (16)

5. Results of heat transfer calculations

Calculations are done for circular fin-tube heat exchanger. Three-dimensional models are performed to find heat transfer characteristics between a finned tube and the air for different fin shapes in order to find the heat transfer rate between the air and the fin material during the air flow in the cross flow heat exchanger. The model allows
considering the heat transfer in three directions. The output is compared with the results received from the correlation formula.

Using the described correlation, the heat transfer is determined based on defined the mass flow rate (inlet velocity 4.0 m/s), inlet temperature of the fluid (300 °C) and the internal tube surface temperature (70 °C). Values of $v_{\text{max}}$, $N_u$ for one row, effective heat transfer coefficient and fluid outlet temperature, received from correlation functions for each fin version, are written in Table 2.

|            | Correlation Eq. (4) | Correlation Eq. (5) | Correlation Eq. (7) |
|------------|---------------------|---------------------|---------------------|
| ($v_{\text{max}}$, m/s) | (a) 12.60 (b) 11.74 (c) 11.00 | (a) 12.60 (b) 11.74 (c) 11.00 | (a) 12.60 (b) 11.74 (c) 11.00 |
| ($N_u$, (one row)) | 71.06 70.00 68.95 | 60.58 60.93 61.56 | --- --- --- |
| $h$, W/(m² K) | 74.19 73.08 71.98 | 63.25 63.61 64.27 | 64.66 64.66 64.66 |
| $T_{\text{OUT}}$, °C | 254.5 255.3 256.1 | 260.6 260.6 260.4 | 259.8 260.0 260.2 |

Numerical analysis are also carried out to examine modified finned tube heat exchangers and the influence of the fin thickness on the heat transfer. The numerical outcome of heat transfer coefficient form 3D model is compared to the results received from the correlations for the fin-tube heat exchanger of uniform fin thickness. Correlations are used to check the numerical calculation of the heat transfer and its accuracy in relation to fin shape modifications. Results are presented in Table 3, where $\Delta = \frac{\Delta T_{\text{correlat}} - \Delta T_{\text{correlat}}}{\Delta T_{\text{correlat}}} \cdot 100\%$.

|            | Correlation Eq. (4) | Correlation Eq. (5) | Correlation Eq. (7) |
|------------|---------------------|---------------------|---------------------|
| $\Delta T_{\text{correlat}}$, °C | (a) 45.5 (b) 44.7 (c) 43.9 | (a) 39.4 (b) 39.4 (c) 39.6 | (a) 40.2 (b) 40.0 (c) 39.2 |
| $\Delta T_{\text{model}}$, °C | (a) 45.7 (b) 43.3 (c) 41.4 | (a) 45.7 (b) 43.3 (c) 41.4 | (a) 45.7 (b) 43.3 (c) 41.4 |
| $\Delta$ | -0.4% 3.1% 5.7% | -16.0% -9.9% -4.5% | -13.7% -8.2% -5.6% |

6. Conclusions

The main objective of this research is to determine numerically the performance of the heat transfer process in a single row fin-tube cross flow heat exchanger for different fin configurations. The most popular correlations are applied for heat transfer evaluation. For Briggs and Young correlation, the heat transfer decreases with the fin thickness increase. The opposite results are seen for the other correlations. The heat transfer is also analyzed by means of numerical computation. The results are verified with the known correlations for circular fins of constant thickness.
Analyzing the output received from numerical calculations with those gathered from correlations, it seems that the differences are within the standard deviation and numerical techniques can predict heat transfer coefficients with acceptable accuracy. The use of the CFD model offers particular benefits especially when minor modification are applied to the fin surface for which the correlation equations are not defined, for instance fin thickness modification. However, comparative analysis are still required and the numerical model should be examined, verified with proper correlations or experimental values.

References

[1] P. Wais, One row fin heat exchanger numerical optimization, Proceedings of International Congress on Thermodynamics, 4-7 Sept Poznan, (2011) 709-716.
[2] R. K. Shah, D. P. Sekulic, Fundamentals of Heat Exchanger Design, Wiley, 2003.
[3] P. Wais, Extended Surfaces (Fins and Pins), in: R.B. Hetnarski, Encyclopedia of Thermal Stresses, Vol 3, Springer, Dordrecht, 2014, 1536-1550.
[4] J. Y. Jang, M. C Wu., W. J. Chan, Numerical and experimental studies of three dimensional plate-fin and tube heat exchangers. International Journal of Heat and Mass Transfer 14, (1996) 3057-3066.
[5] P. Wais, J. Taler, Fin shape optimization in tube heat exchangers by means of CFD program, 2nd International Conference on Engineering Optimization, Sept. 6 - 9, Lisbon, Portugal, (2010) 1-10.
[6] P. Wais, Fluid flow consideration in fin-tube heat exchanger optimization, Archives of Thermodynamics, 31, (2010) 87-104.
[7] W. M. Yan, P. J. Sheen, Heat transfer and friction characteristics of fin-and-tube heat exchangers, International Journal of Heat and Mass Transfer 43, (2000) 1651-1659.
[8] M. S. Mon, U. Gross, Numerical study of fin-spacing effects in annular-finned tube heat exchangers. International Journal of Heat and Mass Transfer 47, (2004) 1953–1964.
[9] R. Romero-Mendez, M. Sen, K. T. Yang, R. McClain, Effect of fin spacing on convection in a plate fin and tube heat exchanger, International Journal of Heat and Mass Transfer 43, (2000) 39-51.
[10] P. Wais, Fin-tube heat exchanger performance for different louver angles, Zeszyty Naukowe Politechniki Rzeszowskiej Mechanika 86, (2014) 115-122.
[11] P. Wais, Influence of fin thickness and winglet orientation on mass and thermal efficiency of cross-flow heat exchanger, Applied Thermal Engineering 102, (2016) 184-195.
[12] A. Erek, B. Ozerdem, L. Bilir, Z. Ilken, Effect of geometrical parameters on heat transfer and pressure drop characteristics of plate fin and tube heat exchangers. Applied Thermal Engineering 25, (2005) 2421–2431.
[13] S. Y. Yoo, H. K. Kwonb, J. H. Kima, A study on heat transfer characteristics for staggered tube banks in cross-flow. Journal of Mechanical Science and Technology 21, (2007) 505-512.
[14] D. Taler, Methods for obtaining heat transfer correlations for plate finned heat exchangers using experimental and CFD simulated data, Archives of Thermodynamics 25, (2004) 31-54.
[15] D. Taler, P. Ocłoń, Determination of heat transfer formulas for gas flow in fin-and-tube heat exchanger with oval tubes using CFD simulations, Chemical Engineering and Processing 83, (2014) 1–11.
[16] D. Taler, P. Ocłoń, Thermal contact resistance in plate fin-and-tube heat exchangers, determined by experimental data and CFD simulations, International Journal of Thermal Sciences 84, (2014) 309–322.
[17] P. Ocłoń, S. Lopata, M. Nowak, A. C. Benim, Numerical study on the effect of inner tube fouling on the thermal performance of high-temperature fin-and-tube heat exchanger, Progress in Computational Fluid Dynamics, An International Journal, 5, (2015) 290–306.
[18] P. Wais, Fin-tube heat exchanger optimization, in: J. Mitrovic, Heat Exchangers – Basics design applications, In-Tech Rijeka, (2012) 343-366.
[19] A. Kraus, A. Aziz, J. Welty, Extended surface heat transfer, A Willey Interscience Publication, 2001.
[20] F. C. McQuiston, D. R. Tree, Optimum space envelopes of the finned tube heat transfer surface, ASHRAE Transactions, Vol. 78, Part 2, (1972) 144-152.
[21] G. H. Hewitt, G. L. Shires, T. R. Bott, Process Heat Transfer, CRC Press, 1994.
[22] R. W. Serth, Process heat transfer: principles and applications, Elsevier USA, 2007.
[23] T. Kuppan, Heat exchanger design handbook, Marcel Dekker USA, 2000.
[24] O. James, J. O. Maloney, Perry’s chemical engineers’ handbook. Mc Graw-Hill, USA, 2008.