Qualitative Aspects of Polarization Distributions In Excited Heavy Hadron Productions

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(September 16, 2018)

Abstract

Within the context of flux tube models, heavy quark fragmentation takes place through the breaking of flux tubes with the production of a (di)quark-anti(di)quark pair. It is found that the (di)quark produced are more likely to be found in an \( L_z = 0 \) state. This naturally leads to an suppression of the polarization distribution parameters \( w_{3/2} \) and \( \bar{w}_1 \) for \( (D_1, D_2^*) \) and \( (\Lambda_{c1}, \Lambda_{c1}^*) \) production respectively. The corresponding parameter \( w_1 \) for \( (\Sigma_c, \Sigma_c^*) \) production, however, is not suppressed, in agreement with the CLEO results but not the DELPHI one. Implications on the measurements of \( \Lambda_Q \) polarizations are discussed.
The production of excited heavy hadrons has been studied in Ref. [1]. It was observed that, due to heavy quark symmetry and parity conservation of the strong interaction, the relative production probabilities of the different helicity states of the light degrees of freedom can only depend on the absolute magnitude of \( j \), the helicity of the light degrees of freedom along the production axis, but not on its sign. As a result, the relative production probabilities of the \((D_1, D_{1}^*)\) system is controlled by a single parameter \( w_{3/2} \), which is defined to be the probability that \( j \) has its maximal value \( \frac{3}{2} \). A similarly defined parameter \( w_{1} \) controls the production of the \((\Sigma_c, \Sigma_{c}^*)\) system, while another parameter \( A \) describes the likelihood of the production of a spin-1 diquark instead of a spin-0 one, which translates into the probability of producing a \( \Sigma_c \) or \( \Sigma_{c}^* \) instead of a \( \Lambda_c \). The framework has been subsequently extended to describe the excited \( \Lambda \) resonances \((\Lambda_{c1}, \Lambda_{c1}^*)\) \[2\], with two more parameters \((B, \tilde{w}_1)\) defined in analogy to \((A, w_{1})\) for the \((\Sigma_c, \Sigma_{c}^*)\) system.

It must be emphasized that Ref. [1] and [2] are parametrizations rather than predictions of the fragmentation processes in the sense that they did not attempt to predict (or explain) the experimental values of the \( w \)’s. By measuring the angular distribution of the decay products, ARGUS [3,4] and CLEO [5,6] have measured \( w_{3/2} \). They found \( w_{3/2} \) to be small (best fit \( w_{3/2} \) is \(-0.30\), which is in the unphysical region; restricting to the physical region \( 0 \leq w_{3/2} \leq 1 \) gives \( w_{3/2} \approx 0 \)), meaning that transverse polarization is preferred to longitudinal. There is no theoretical understanding of why \( w_{3/2} \) is so small. In the case of excited \( B_c \) production, one can calculate \( w_{3/2} \) by perturbative QCD [7,8], and the result \( w_{3/2} = 29/114 \approx 0.24 \) is indeed on the small side. But it is dangerous to simply carry the result over to the \((D_1, D_{1}^*)\) where non-perturbative QCD effects are dominant. In the baryon sector, there is no measurement for \( \tilde{w}_1 \) yet, while the \( w_{1} \) measurements by CLEO and DELPHI yielded inconsistent results. DELPHI [10] obtained a small value for \( w_{1} \) (best fit \( w_{1} \) is \(-0.36\), which

\[1\] The DELPHI analysis use \( \Sigma_{b}^{(*)} \) production from \( Z^0 \) decays, not \( \Sigma_{c}^{(*)} \). By heavy quark symmetry, however, \( w_{1} \) should be the same for both cases.
is also unphysical; restricting to the physical region $0 \leq w_1 \leq 1$ again gives $w_1 \sim 0$ \[9,10\] while the CLEO result is consistent with an isotropic polarization ($w_1 = 0.71 \pm 0.13$ while an isotropic distribution gives $w_1 = \frac{2}{3}$) \[11\].

Intuitively the helicity of the light degrees of freedom can be viewed as the sum of two different contributions. One is the spin of the “brown muck” $\vec{S}$, which is 0 for $\Lambda$ type baryons, $\frac{1}{2}$ for mesons and 1 for $\Sigma$ type baryons. Then this “brown muck” may orbit around the heavy quark with orbital angular momentum $\vec{L}$, giving an additional contribution to the helicity. We will see that, if heavy quark fragmentation can be understood as breaking of color flux tubes as suggested by the Lund models \[12\], the Artru–Mennessier model \[13\] and the UCLA model \[14\], then the orbital angular momentum naturally prefers a transverse polarization, explaining the smallness of $w_{3/2}$ and predicts a small $\tilde{w}_1$; $w_1$, on the other hand, is not required to be small, \textit{i.e.}, the CLEO numbers are preferred.

Due to the non-abelian nature of QCD, the color field is expected to be confined into tube-like regions (flux tubes) by the tri-gluon coupling. The flux tubes have constant tension and ends at colored objects like quarks or diquarks. Such a picture is supported by Regge phenomenology, quarkonium spectroscopy, bag models and lattice QCD calculations. For concreteness, let’s study the process $Z^0 \rightarrow c\bar{c}$ and the subsequent fragmentation and hadronization. Just after the $Z^0$ decay, both quarks are in general very off-shell and will fragment by the emission of hard gluons, which is governed by perturbative QCD. Eventually such gluon bremsstrahlung will bring the off-shell energy down to $\Lambda_{QCD}$ scale, and non-perturbative QCD will be important. This is when the flux tube model become a reasonable description of the dynamics. In a coordinate system in which the $c$ quark travel along the positive $z$-axis in the $c\bar{c}$ center of mass frame ($Z^0$ rest frame if the momenta carried away by the hard gluons are negligible), there will be a flux tube lying along the $z$-axis joining the two quarks. The flux tube will be characterized by a constant linear energy density (tension) $\kappa \sim 0.2(\text{GeV})^2$, which leads to a linear potential between the quarks. For a long flux tube, it will be energetically favorable to break the flux tube by the production of a quark-antiquark pair (or a diquark-antidiquark pair) to shorten the flux tube and hence
reduce the energy stored in the flux tube. This is the QCD analog of pair creation in a strong electric field and is completely non-perturbative in nature.

If we ignored the finite thickness of the flux tube, the pair-created quark-antiquark pair will be produced right on the $z$-axis, where the flux tube is. Moreover, due to the tension of the flux tube, the antiquark will be linked by the flux tube to the $c$ quark and move towards the $c$ quark, \textit{i.e.,} along the positive $z$ direction. In general there will also be transverse momenta, which has a gaussian distribution centered at zero, but let us ignore that for a moment. Then all colored objects (quarks and flux tubes) are on the $z$-axis, and the system has a rotational symmetry about the $z$-axis. As a result, the $z$-component of the orbital angular momentum $L_z$ is conserved. Since the system starts out with vanishing $L_z$ (nothing is orbiting), the final hadron must have $L_z = 0$. Notice that $L^2$ of the final hadron is not necessarily equal to zero, as the system does not have a spherical symmetry and hence $L^2$ is not conserved.

An alternative way of seeing the same result is to consider the wave function of the antiquark in the relative momentum space\footnote{An similar argument in the relative position space also holds analogously.}. Since the light antiquark is moving towards to $c$ quark, it means that the relative momentum $\vec{p}$ of the light antiquark with respect to the $c$ quark is in the positive $z$ direction. In other words, the wave function of the antiquark in the momentum space is a wave packet peaked at some point $\vec{p}_0$ on the positive $z$ axis. We do not know the exact shape of the wave function, but the rotational symmetry about the $z$ axis mandates that the wave function can depend only on $p_z$ and $\sqrt{p_x^2 + p_y^2}$, but not the azimuthal angle $\theta$. As a result, the expectation value of $L_z = i \partial_\theta$ vanishes. Notice that our argument holds even if the spread of the wave function is large with respect to $|\vec{p}_0|$ and the wave function is non-vanishing even for points off the $z$-axis. As long as the wave function is azimuthally symmetric, $L_z$ has to vanish. Hence we see that $L_z$ in heavy meson production vanishes if transverse momenta are negligible. The polarization of the intrinsic
spin of the antiquark $\vec{S}$, on the other hand, is not constrained in any way as long as it is cancelled by that of the quark produced at the same time. Since the flux tube models do not have a preference over the orientation of $\vec{S}$, we naturally assume it to be isotropic. The total helicity of the light degrees of freedom, then, is the sum of $\vec{L}$, under the constraint $L_z = 0$, and $\vec{S}$, with an isotropic distribution.

For a heavy meson with orbital angular momentum $L^2$, the possible helicity states for the light degrees of freedom range over $j = -L - \frac{1}{2}, -L + \frac{1}{2}, \ldots, L - \frac{1}{2}, L + \frac{1}{2}$. By the conservation of parity, the probability of finding the light degrees of freedom with helicity $j$ is the same as that with helicity $-j$. We will define $W_j$ the probability of finding the light degrees of freedom with helicity either $j$ or $-j$. The sum of all these probabilities equals to unity, i.e., $\sum W_j = 1$. In particular, for $L = 1$, $W_{3/2} = w_{3/2}$ defined in Ref. [1], and $W_{1/2} = 1 - w_{3/2}$. The analysis above suggests that $W_{1/2} = 1$ and all other $W$’s vanish, for all values of $L$. In other words, the light degrees of freedom will be in the lowest helicity state, and be as transversely polarized as allowed by quantum mechanics. In particular, for the $(D_1, D_2^*)$ system, $w_{3/2} = W_{3/2} = 0$ seen by ARGUS and CLEO [3–6].

Our formalism can be extend to describe the production of heavy baryons as well. Different flux tube models have different descriptions of the production of the diquark-antidiquark pairs. In the simplest models, the diquark appears as a single entity, and the analysis above can be adopted in a straightforward manner. In some other models, like the “popcorn model” [15], the two quarks in the diquark are produced in stages. The analysis above will be invalidated if the first quark-antiquark pair moves off the $z$-axis before the second pair is created, as such off-axis configuration will break the azimuthal symmetry. It turns out that, however, the first quark-antiquark pair will instead slide along the flux tube as “curtain quarks” but not wander off the flux tube. Hence, even in these models, all colored objects still lie on the $z$-axis, azimuthal symmetry is preserved, and hence the resultant heavy baryon will also have $L_z = 0$.

Since $S = 0$ for a $\Lambda$ type diquark, the light degrees of freedom of a $\Lambda$ type heavy baryon
with orbital angular momentum \( L^2 \) can have helicity \( j = -L, \ldots, L \). Define as before \( W_j \) as the probability of finding the light degrees of freedom with helicity either \( j \) or \(-j\). Then the analysis above suggests that \( W_0 = 1 \) and all other \( W \)'s vanish. In particular, for the \((\Lambda_{c1}, \Lambda_{c1}^*)\) system, \( W_1 = \tilde{w}_1 \) as defined in Ref. [2], and \( W_0 = 1 - \tilde{w}_1 \). Our analysis then predicts \( \tilde{w}_1 \) to be small. On the other hand, \( S = 1 \) for a \( \Sigma \) type diquark, and the light degrees of freedom of a \( \Sigma \) type heavy baryon with orbital angular momentum \( L^2 \) can have helicity \( j = -L-1, -L, \ldots, L, L+1 \). Since \( \vec{S} \) is supposed to have an isotropic distribution, \( S_z \) is equally likely to be found in the +1, 0, or −1 states. This gives \( W_0 = \frac{1}{3}, \ W_1 = \frac{2}{3} \) and all the other \( W \)'s vanish. For the \((\Sigma_c, \Sigma_c^*)\) system, which is not orbitally excited, our analysis suggests an isotropic distribution, i.e., \( w_1 = \frac{2}{3} \) for \( w_1 \) defined as in Ref. [1]. This is in agreement with the CLEO result [11] but not the DELPHI one [9,10].

Due to heavy quark symmetry, our analysis is obviously also applicable to \( b \) quark fragmentation as well. It can also be easily generalized to other excited heavy hadrons. By the \( L_z = 0 \) rule, the light degrees of freedom of excited heavy mesons will have \(|j| = \frac{1}{2}\), excited \( \Lambda \) type baryons \(|j| = 0\), and for excited \( \Sigma \) type baryons, \(|j| \) can either be 0 or 1, with probabilities \( \frac{1}{3} \) and \( \frac{2}{3} \) respectively. One exception is the \( P \)-wave \( \Sigma \) type baryon, with the spin of light degrees of freedom \( \vec{s}_L = \vec{L} + \vec{S} = \vec{0} \). Then there will be only one helicity state and \( W_0 = 1 \) trivially. In general, however, care must be taken to apply our analysis to very excited heavy hadrons, as the flux tube models may cease to be good descriptions with high excitation energies.

It is interesting to see the implication of our analysis on the measurements of heavy quark polarization in \( Z^0 \to c\bar{c}, b\bar{b} \) processes. As discussed in Ref. [1], in general (except one special case) all polarization information of the heavy quark is lost in the meson sector. On the other hand, since the “brown muck” of \( \Lambda_Q \) is spinless, the polarization of the heavy quark should be retained in the baryon sector. This effect, however, is modified by the presence of secondary \( \Lambda_Q \)’s, i.e., those produced in decays of excited heavy baryons. Since the polarizations of these secondary \( \Lambda_Q \)’s do not necessarily align with the initial heavy quark, the overall polarization is diluted. It has been shown [2] that, if one only includes
secondary \( \Lambda_Q \)'s from the \((\Sigma_Q, \Sigma_Q^*)\) and \((\Lambda_{c1}, \Lambda_{c1}^*)\) doublets, the polarization is diluted by the factor

\[
P = \frac{1 + \frac{A}{9}(1 + 4w_1) + \frac{B}{9}(1 + 4\tilde{w}_1)}{1 + A + B}.
\]  

(1)

The first, second and third terms in both the numerator and the denominator correspond to \( \Lambda_Q \) produced directly, from \((\Sigma_Q, \Sigma_Q^*)\) decays and and from \((\Lambda_{c1}, \Lambda_{c1}^*)\) decays respectively.

With \( w_1 = \frac{2}{3}, \tilde{w}_1 = 0 \) and both \( A \) and \( B \) assuming the default Lund value 0.45, it is found that \( P = 0.65 \). For comparison, \( P = 0.58, 0.72 \) and 0.79 for \( w_1 = \tilde{w}_1 = 0, \frac{2}{3} \) and 1 respectively. In the standard model, one find that the \( b \) and \( c \) quarks produced by \( Z^0 \) decay is partially polarized with \( P_b = -0.94 \) and \( P_c = -0.67 \). Hence our analysis predicts \( P_{\Lambda_b} = P_b P = -0.61 \) and \( P_{\Lambda_c} = P_c P = -0.44 \). Note that this is two standard deviations away from both the ALEPH result \( \langle P_{\Lambda_b} \rangle = -0.23^{+0.24}_{-0.20} \text{ (stat.)}^{+0.08}_{-0.07} \text{ (syst.)} \) and the recent DELPHI preliminary result \( \langle P_{\Lambda_b} \rangle = -0.08^{+0.35}_{-0.29} \text{ (stat.)}^{+0.18}_{-0.16} \text{ (syst.)} \). This should not be interpreted as the failure of our analysis, since the ALEPH and DELPHI central values \(-0.23 \) and \(-0.08 \) correspond to \( P = 0.24 \) and 0.08 respectively, which are not achievable for any choice of \( w_1 \) and \( \tilde{w}_1 \) anyway. Instead, this probably means that effects of other resonances, like the \( P \)-wave \( \Sigma_Q \)'s and \( D \)-wave \( \Lambda_Q \)'s, are not negligible. Also the hypothesis that \( A = B = 0.45 \) has not yet been tested. In fact, DELPHI preliminary results \( 1 < A < 2 \) with large uncertainty \( \langle P_{\Lambda_b} \rangle = -0.08^{+0.35}_{-0.29} \text{ (stat.)}^{+0.18}_{-0.16} \text{ (syst.)} \) suggests that more \( \Sigma_Q^* \) are produced than the Lund model expects, and hence the depolarization is more severe. Evidently more accurate measurements on the parameters \( A, B \) and the \( w \)'s are necessary to clarify the situation.

The leading correction to our analysis comes from the transverse momenta acquired by the (di)quark-anti(di)quark pair. These transverse momenta have a random (gaussian) distribution with \( \langle p_{\perp} \rangle = 0 \) and \( \langle p_{\perp}^2 \rangle \sim (0.3\text{GeV})^2 \). Our analysis is a good description only if \( \langle p_{\perp}^2 \rangle \gg \langle p_{\perp}^2 \rangle, \) i.e., the produced (di)quark-anti(di)quark pair move essentially in the \( z \)

\(^3\)The definition of polarization used here differs from that in Ref. \( \| \) by a negative sign to facilitate comparison with the ALEPH results.
direction to preserve the azimuthal symmetry. Since both of these are governed by non-perturbative QCD, they should both be of order \( \Lambda_{\text{QCD}} \), and it is not obvious that \( \langle p_z^2 \rangle \) should be much larger than \( \langle p_\perp^2 \rangle \). Physically, however, we do expect \( \langle p_z^2 \rangle = \frac{1}{2} \langle p_\perp^2 \rangle \) in the absence of the flux tube, and hence, under the tension of the flux tube, \( \langle p_z^2 \rangle \) to be at least as large as \( \frac{1}{2} \langle p_\perp^2 \rangle \). Consequently, we expect the probability of having orbitally excited hadrons with non-vanishing \( L_z \) to be suppressed by \( O(\frac{1}{2} \langle p_\perp^2 \rangle / \langle p_z^2 \rangle) \).

We conclude that, in flux tube models, orbitally excited heavy hadrons tend to have \( L_z = 0 \). This gives a natural explanation of the small observed value of \( w_{3/2} \), and a small value is predicted for \( \tilde{w}_1 \) but not \( w_1 \). A large value of \( \tilde{w}_1 \) (for an isotropic distribution \( \tilde{w}_1 = \frac{2}{3} \)) would be fatal to our scheme, while a small \( w_1 \) will mean that there are physics not captured by the flux tube model to make the diquark spin \( \vec{S} \) anisotropic. It is expected that the next round of experiments will resolve the controversy on \( w_1 \) and possibly measure \( \tilde{w}_1 \) as well, putting our prediction to test. Our analysis is qualitative in nature; our ignorance of the \( \langle p_z^2 \rangle \) prevents us from making quantitative predictions. On the other hand, it is important to note that our analysis is entirely non-perturbative in nature. This complements the perturbative calculations in Ref. \[7,8\] and suggests that the suppression of high helicity states is a genuine consequence of QCD, not that of the perturbative approximation.

**ACKNOWLEDGMENTS**

This paper is inspired by Adam Falk’s talk at Cornell \[15\]. I am grateful to David Cassel, John Elwood, Adam Falk and John Yelton for valuable discussions. The work is supported in part by the National Science Foundation.
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