Tunable generator of radially and azimuthally polarized vortex Bessel beams based on the interference polarizer

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Abstract. The new optical system to generate azimuthally and radially polarized Bessel laser beams is proposed. The system is based on the transformation of the conical wave fronts when passing through interference polarizing plate. The polarization control characteristics occur when forming the beam by changing the divergence of the beam incident on the diffractive axicon. The radially and azimuthally polarized Bessel beams using binary phase axicon are experimentally obtained.

1. Introduction
Azimuthally and radially polarized laser beams have nowadays a variety of applications [1, 2], in particular for coupling in free space [3] and processing materials [4, 5]. Vortex beams with different polarization are also used in several problems [6]. Among the vortex beams, Bessel beams are important for practice [7]. In work [8] theoretical bases of non-paraxial propagation and transformation of non-homogeneously polarized Bessel beams are examined. Techniques for the formation of Bessel laser beams with inhomogeneous polarization differ both in the methods of forming the initial Bessel beam and in the properties of the optical systems forming the required polarization state. The universal means for solving both problems is presented by using liquid crystal light modulators [9], however the low resolution of these devices affected the quality of the experimental results obtained in [9]. In [10], the self-recovery property of not only the amplitude distribution but also the polarization state of Bessel-Gauss beams obtained using a spatial light modulator whose polarization state was formed with the aid of a sector plate (radial-polarization generator) was experimentally demonstrated. The transformation of circular polarization into inhomogeneous polarization by means of segmented polarization plates [11] leads to the appearance of a vortex phase in the beam. Unfortunately, the use of the sector plate [12] leads to a deterioration in the quality of the beams.

Higher quality beams can be obtained if the initial Bessel beam is obtained by using a refractive axicon, and further formation of the polarization state occurs in a biaxial birefringent crystal [13]. This method allowed to obtain experimentally both radial and azimuthal polarizations in the same optical system by rotating the direction of the polarization axis of the incident beam by 90\degree. A simpler solution is the use of a multilayer structure (one-dimensional photonic crystal) to form the polarization of the Bessel beam [14], besides, by using non-linear optical materials it is possible to control the properties of the crystal by additional irradiation with an auxiliary laser source. In such a variant of the optical system proposed in [14], the polarization state is restructured simply by controlling a non-
linear photonic crystal without any changes in the optical system. However, this principle has not been experimentally implemented yet. A simplified version of such a system [15], which was implemented in practice, contains a diffractive axicon instead of a refractive one, and instead of a multilayer structure it includes a Stoletov's pile, which also makes it possible to form a radial polarization. The use of the diffraction axicon increases the quality of the beam in addition to the advantages that diffraction optics provides. However, the diameter of the beam after passing throughout Stoletov's pile increases considerably due to the required large divergence angle, and collimation of the beam becomes a complex problem requiring the use of high-aperture optics of large diameter. In addition, the energy efficiency of Stoletov's pile is very low to achieve enough polarization contrast. No control of the polarization state of the beam in such a system is possible. In our work, we propose to take as the basis, the optical system proposed in [16], based on the transformation of conical beams in a multilayer interference structure. Such a system was effectively used to form a two-ring radially polarized beam [17]. The suitability of this system for the formation of radially polarized Bessel beams was experimentally shown in [18]. However, the control of the polarization state on it is impossible because of the large angles of incidence on the multilayer structure for obtaining radial polarization. A further increase in the angles for obtaining azimuthal polarization is not possible due to technological limitations in the production of diffractive optical elements [19] and a strong decrease in energy efficiency with increasing angles of incidence.

In this paper, it is proposed to form non-homogeneously polarized vortex Bessel beams with a controlled polarization state by changing the divergence of the beam, incident on the diffraction axicon. It has been shown theoretically and experimentally that when the operating angles of light incidence fall on an interference polarizer, radial and azimuthal polarizations can be obtained with the same diffraction axicon only by replacing the lenses in the illuminating beam.

2. Modeling of optical system
The basis of the proposed approach is the passage of a conic beam with circular polarization through a polarization plate [16]. It is known [17, 20] that circular polarization can be represented as a superposition of radial and azimuthal polarization with a vortex phase:

\[ e_x \pm ie_y = (\cos \phi e_r - \sin \phi e_\theta) \pm i(\sin \phi e_r + \cos \phi e_\theta) = \exp(\pm i\phi)(e_x + ie_y). \]  

(1)

As a result of the action of the polarization plate, one of the polarizations, radial or azimuthal, will be released depending on the angle of incidence, and the vortex phase will remain in the beam in both cases. A conic beam is proposed to be formed by using an axicon, which gives a zero-order Bessel beam with a maximum on the beam axis. After passing through the polarizing plate, the beam becomes non-uniformly polarized, but the presence of the vortex phase leads to the formation of a non-zero intensity on the optical axis with circular polarization. This fact will be further demonstrated by numerical modeling. Thus, a small central region with homogeneous polarization remains in the beam, but its energy contribution is negligibly small.

The operating principle of the interference polarizer leads to a high angular selectivity in the transformation of the polarized state of the transmitted light. The conic wave front has radial symmetry and ensures constant angles of light incidence on the polarizer and, thus, the polarization transformation over the entire beam area is the same for all rays and radially symmetric. The ordinary axicon (both refractive and diffractive) forms a conical wave front under illumination by a plane wave front. If it is required to change the propagation angles of the rays in the conical wavefront, then the exact solution is to replace the forming axicon by an axicon with another numerical aperture while maintaining the illumination by the plane wave front. However, both diffractive and refractive axicons are rather narrowly specialized and expensive elements, and it is often not possible to have a wide range of axicons with exact numerical aperture values. In the present paper, it is proposed to correct the angles of convergence of beams formed by an axicon by replacing a plane illuminating wave with a spherical wave. If the beam that illuminates an axicon is spherical (either diverging or converging), the wave front of the transmitted beam is not purely conical, but will have a quadratic "phase additive." This will lead to the fact that angles of propagation of rays of the beam will vary depending on the area of the radius and hence angles of incidence on the interference polarizer will also vary.
main issue when designing the optical system proposed is to estimate the impact of the quadratic "phase additives" in a beam with a conical wavefront at the "purity" of the polarization conversion. The numerical modeling presented below is aimed at estimating the variation of angles over the beam cross section as a function of various parameters in order to be able to compare it with the characteristics of the interference polarizer and to select the parameters for the optical system elements necessary to achieve the required quality indices of the generated beams.

The numerical aperture of the diffraction axicon is determined by the following formula:

\[ NA = \frac{\lambda}{d} \]  

(2)

where \( \lambda \) is the wavelength of the axicon lighting radiation, \( d \) is the period of the axicon rings.

Using the numerical aperture (2), we can determine the angle of convergence of the conical wave front:

\[ \theta = \arcsin \left( \frac{\lambda}{d} \right) \]  

(3)

In particular, when a diffraction axicon with a period of 2 \( \mu \text{m} \) is illuminated by laser radiation with a wavelength of 0.633 \( \mu \text{m} \) (helium-neon laser), we obtain a conic wave front converging at an angle of \( \theta \approx 18.45^\circ \). In the ray approximation (if one does not take into account the diffraction), all the rays will propagate at this angle throughout the optical axis. In this case, a zero-order Bessel beam is formed. Diffraction phenomena can be taken into account if we calculate numerically the angular spectrum of radiation transmitted through the diffraction axicon. The angular spectrum of radiation transmitted through a binary diffraction axicon with a period of 2 \( \mu \text{m} \) is shown in Fig. 1.

**Figure 1.** The results of calculating the angular spectrum of the radiation transmitted through the axicon (angles are indicated in degrees).

It is known [17, 21] that the binary diffraction axicon has two intensity peaks in the spectrum, corresponding to converging and diverging wave fronts. Figure 1 shows the angular spectrum for a converging conical wavefront. The position of the maximum of the angular spectrum for the Bessel beam depends on the numerical aperture (1), and its width from the radius of the aperture of the optical element. The width of the angular spectrum of the beam in Fig. 1b is about 0.01° in 0.5. If the size of the element's aperture increases, the angular spectrum will narrow and, in the limit, tend to the delta function at the angle (2).

In order to shift the maximum of the spatial frequencies to the zone of large convergence angles, it is possible to reduce the period of the axicon \( d \). In particular, in order to obtain \( \theta = 20^\circ \), is necessary to prepare an axicon with a period of \( d = 1.85 \mu \text{m} \), and for \( \theta = 25^\circ \) the period must be \( d = 1.5 \mu \text{m} \). However, as already mentioned, the manufacture of a set of axicons with different periods to ensure accurate angular values is very expensive. In addition, axicons with such small period values require alignment comparable with the period of diffractive axicon. Spatial light modulator (SLM) might help avoiding this problems, but it is not possible to experimentally implement axicons in the required period range using standard SLM, because its cell size is on the order of a few microns.

Let us consider the proposed way of adjusting the angles of convergence of beams formed by an axicon by replacing a plane illuminating wave with a spherical wave [22].

It is easy to see that in principle there are two options for this adjustment: either reducing the convergence angles for an axicon with a higher numerical aperture due to illumination by a diverging wave front, or increasing the angles for an axicon with a lower numerical aperture due to illumination.
by a converging wave front. Obviously, the second option is preferable, because it is technologically easier (more the period of the axicon rings) and, in addition, this option wins in the distance of saving the Bessel beam.

The maximum distance to which the axle-generated Bessel beam will be preserved is determined by the formula [23]:

$$z_{\text{max}} \approx R/NA$$  \hspace{1cm} (4)

where $R$ is the axicon radius.

It follows from expression (4) that the higher the numerical aperture of the optical element, the shorter the working distance. In particular, for an axicon with a period of 2 μm and a radius $R = 8$ mm, the maximum distance $z_{\text{max}} \approx 25$ mm. To increase this distance, it is possible to increase the size of the element, but in this case it will also be necessary to expand the illuminating beam, which ultimately leads to a decrease in the useful energy. Therefore, it is preferable to use axicons with a smaller numerical aperture.

Illumination of an axicon by a converging spherical beam will theoretically result in a broadening of the angular spectrum and a large-scale change in the Bessel beam at various distances from the optical element. This behavior is characteristic of the beams formed by lensacons [24] and fracticons (fractional axicons) [25].

Figure 2 shows the results of calculating the angular spectrum under illumination of a binary diffraction axicon with a radius $R = 8$ mm with a period of 2 μm with a converging spherical beam with a focal length of 80 mm.

Figure 2. Results of calculating the angular spectrum under illumination by a converging spherical beam (angles are indicated in degrees).

The width of the angular spectrum of the beam in Fig. 2 is about 0.7° in 0.5

Thus, illumination of an axicon by a convergent spherical beam makes it possible to increase the numerical aperture of the beam being formed. However, the angular spectrum of the transmitted radiation also expands. But, as will be shown later, this expansion agrees with the operating angles range of the interference polarizer, which allows to preserve the "purity" of the polarization of the non-uniformly polarized Bessel beam formed. We also note that in this case the Bessel beam decreases scales with increasing distance from the optical element, which leads to a decrease in the conservation distance of the Bessel beam in comparison with a separate axicon [25].

3. Experimental study

It is known from the previous results [18] that the angular difference in the positions of maximum transmission of p- and s-polarized radiation for an interference polarizer is about 30%. Taking into account the simulation results, the maximum transmission of the interference polarizer for p-polarization was chosen at a 20° incidence angle. In this case, the transmission maximum for s-polarized radiation should be obtained at an angle of incidence of about 25°. It was supposed to use in the experiment a binary diffraction axicon with a period of 2 μm, and to obtain a beam for the passage of p-polarization under illumination by a plane wave (in this case, radially polarized radiation at the output is obtained), and for passing s-polarized radiation to illuminate an axicon by a spherical wave with the same parameters, as in the simulation in Figure 2. The results of modeling for such an axicon when illuminated by a plane wave (see Figure 1) give a slightly different position of the maximum of
the angular spectrum, about \( \theta \approx 18.45^\circ \). However, because of the small width of the spectrum (see Figure 1b), the value of this angle need not be \( 20^\circ \), but may be different within the bandwidth of the interference polarizer. As will be shown below, the shift of the maximum of the angular spectrum of the axicon toward smaller angles makes it possible to improve the "purity" of the resulting polarization state and to equalize the transmission for p and s-polarized radiation.

An interference polarizer with an operating angle of \( 20^\circ \) for a wavelength of 632.8 nm was designed and manufactured by OAO IZOVAK (Republic of Belarus, Minsk). The polarizer consisted of 43 alternating layers of Nb2O5 / SiO2 of different thickness, deposited on a quartz substrate with a diameter of 25.4 mm and a thickness of 3 mm. Polarization characteristics of the transmission of an element were investigated on a spectral ellipsometer J.A. Wollam V-VASE in the range of angles 0-40° in 0.2° increments. The wavelength of the probing radiation was set to 632.8 nm, the spectral width was no more than 0.5 nm. For greater accuracy, before each measurement, the emission level of the spectral block of the ellipsometer (baseline) was determined, and the results of the measurements were averaged over time in 10 points. The dependence of the transmission of p- and s-polarized radiation on the angle of incidence is shown in Fig. 3. It follows from the measured characteristics that the interference polarizer provides a ratio of the transmission coefficients of the radial and azimuthal components \( T_p / T_s \) of at least 80: 1 to 90: 1 for an operating angle of about \( 20^\circ \). The maximum transmission for \( T_p \) is 72%, and the transmission for \( T_s \) is 62%. The width of the passband of the interference polarizer was about 4° in the level of 0.5 for both p- and s-polarized radiation. Thus, taking into account the width of the angular spectra of the radiation transmitted through the axicon obtained in modeling, it can be concluded that there is some freedom in choosing the position of the maxima of the angular emission spectrum. In this case, since the transmission for s-polarized radiation is smaller, it is desirable to accurately match the maxima of the angular emission spectrum transmitted through the axicon and the transmission of the polarizer, and for p-polarized radiation, it is possible to achieve a transmission equalization of the components by shifting the maximum of the angular spectrum of the incident radiation. As can be seen from Fig. 3, the maximum shift to a smaller side by approximately 1.5 ° allows both to equalize the transmission of the components and to reduce the parasitic transmission of the other polarization, thereby improving the "purity" of the resulting radial polarization. It is these angles for the angular spectrum of the axicon that we sought when conducting the experiment.

![Figure 3. Transmission of radial \( T_p \) and azimuthal \( T_s \) components by an interference polarizer.](image)

To create a zero-order Bessel beam, a phase diffraction axicon with a period of 2 μm and a diameter of 20 mm, made on a quartz substrate with the help of the CLWS-200S photewriter, was used in the work. An electronic photograph of the fragment of the microrelief of the diffraction axicon is shown in Fig. 4.

The study of the formation of radial and azimuthally polarized vortex Bessel beams was carried out on an optical apparatus, the scheme of which is shown in Fig. 5. It included: a linearly polarized helium-neon laser, a spatial beam-beam expander, a quarter-wave plate (\( \lambda/4 \)), diffraction axicon (Axicon), polarizer (PI), magnifying lens (40×), an analyzer (A), a video matrix DCM310 (CCD). The beam expander consisted of a 20× objective, a 15 μm point diaphragm, and a collimating lens L1 with a focal length of 200 mm. The radius of the illuminating beam at the level of 0.5 after the expander is about 10 mm.
Figure 4. Electronic photograph of the fragment of the microrelief of the diffraction axicon.

Figure 5. Schemes of the experimental setup for obtaining radially (a) and azimuthally (b) polarized vortex Bessel beams.

Figure 6 shows the intensity distributions obtained for different positions of the transmission axis of the analyzer. The azimuthal angle of 0° corresponds to the vertical orientation of the analyzer axis. It follows from the obtained data that the generated Bessel vortex beam has a radial polarization.

Figure 6. Formed vortex Bessel beam with radial polarization at various positions of the analyzer: (a) 0°, (b) 45°, (c) 90°, (d) 135°, (e) without analyzer.

To form an azimuthally polarized vortex Bessel beam, in accordance with the foregoing, it is necessary to illuminate the diffraction axicon by a spherical beam with a focus of 80 mm. For this purpose, the collimating lens L1 with a focal length of 200 mm was replaced by a lens with a focal length of 50 mm, located at a distance of 150 mm from the point diaphragm (see Fig. 5b). The diffraction axicon was placed close to this lens, and the focus of the resulting convergent spherical beam was obtained just about 80 mm from it. The radius of the illuminating beam decreases to approximately 8 mm due to the approach of the lens to the point diaphragm. Figure 7 shows the intensity distributions obtained for different positions of the analyzer axis. From the obtained data it follows that the generated vortex Bessel beam has an azimuthal polarization.

Figure 7. Formed vortex Bessel beams with azimuthal polarization at various positions of the analyzer: (a) 0°, (b) 45°, (c) 90°, (d) 135°, (e) without analyzer.

4. Conclusion
In this way, a tunable optical system has been proposed for the formation of both radially and azimuthally polarized vortex Bessel beams, which includes an illuminating optical system, a binary-
phase axicon and an interference polarizer. The reorganization of the system is based on the shift of the maximum of the angular spectrum of radiation transmitted through the diffraction axicon due to its illumination by a spherical wave instead of a plane one. The formation of radial and azimuthal polarizations was carried out by an interference polarizer at mean angles of incidence 18.45° and 25°, respectively. Images of non-homogeneously polarized beams were obtained with the aid of a high-aperture microscope. Radial- and azimuthally polarized vortex Bessel beams can be used for optical communication and material processing.

5. References

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