Modeling and Simulation of Non-ideal Buck Converter in CCM

Huifang Xu\textsuperscript{a}, Ming Cheng\textsuperscript{b} and Fan Zhang\textsuperscript{*}

School of Electronic Communication Engineering Anhui Xinhua University, Hefei, China

\textsuperscript{*}Corresponding author e-mail: 704762735@qq.com, \textsuperscript{a}xhftea@163.com, \textsuperscript{b}276202054@qq.com

Abstract. The circuit average equivalent model of non-ideal Buck converters in CCM was established on the basis of energy conservation average method and time averaging equivalent circuit method, which is considering the influence of the every element’s equivalent parasitic parameters and inductance current ripples existing in actual converters, and the influence of the output current oscillation for the circuit performance. Analysis and simulation by Matlab were implemented. It could be seen that the new model is intuitive and has clear physical meaning. Therefore, it can reflect the actual operation of the system more accurately.

1. Introduction

With the extensive application of DC-DC converter system to portable electronic devices, there is a growing deviation between the ideal model and the actual circuit. Therefore, it is necessary to establish a non-ideal small signal model that is closer to the actual circuit to analyze the stability and dynamic small signal characteristics of the silicon, so as to ensure the stable and reliable operation of the system.

The method of circuit average equivalent modeling is based on the circuit structure of dc-dc converter, which is using time average technique to average and linearize the nonlinear elements in the circuit and could be obtained linearized average equivalent circuit model. The biggest advantage of this method are that the equivalent circuit model is consistent with the original circuit topology, and the physical significance is clear. but when the number of circuit components increases, it takes a lot of computation to get the average topology. The circuit average method mainly includes time averaging equivalent circuit method in references[1-4] and energy conservation average method in references[5-6].

Based on the references [1-10] which are modeling achievements of dc-de converter, large signal average equivalent circuit model of non-ideal Buck converters in the continuous conduction mode (CCM) was established. The model considered the influence of the every element’s equivalent parasitic parameters and inductance current ripples existing in actual converters, and combined with energy conservation average method and time averaging equivalent circuit method. On this basis, dc equivalent circuit model and small-signal equivalent circuit model of the Buck converter are established. The model is universality to the non-ideal Buck converter in continuous conduction mode (CCM). It could be seen that the new model is intuitive and has clear physical meaning. Therefore, it can reflect the actual operation of the system more accurately.
2. Equivalent Circuit of Non-IDEAL Buck Converter

The power level of Buck converter which is considering non-ideal parasitic parameters is showed in Fig.1. The active switch MOSFET is equivalent to the series of the ideal switch $S$ and the on-resistance $R_S$. The diode $D$ is equivalent to the series of the ideal switch $D$, the forward voltage $V_D$ and the on-resistance $R_D$. $R_L$ and $R_C$ are equivalent series resistors of filter inductance $L$ and filter capacitor $C$ respectively. Assuming that the switching period of the switching element $S$ is $T_S$, and the on-off time is $T_{on}$, then the duty cycle is $D = T_{on} / T_S$.

![Figure 1. Non-ideal Buck converter considered parasitic parameters](image)

For the non-ideal Buck converter considering the influence of inductance current ripples in CCM, supposing the maximum value, minimum value and average value of inductance current in a switching cycle are $I_{\text{max}}$, $I_{\text{min}}$ and $I_L$, respectively, the current waveform $i_L(t)$, $i_S(t)$ and $i_D(t)$ flowing through the inductance $L$ and on the two switching tubes $S$ and $D$ are shown in Fig.2.

![Figure 2. Each current waveform of Buck converter in CCM](image)

3. Modeling of non-ideal Buck converter in CCM

In CCM mode, considering the influence of inductance current ripples and according to energy conservation average method, the parasitic parameters of switching elements in Fig.2 can be equivalent to the inductance branch. The effective value $I_{L,\text{rms}}$, $I_{S,\text{rms}}$ and $I_{D,\text{rms}}$ of the inductance current and two switching tubes in a switching cycle are obtained as follows.

\[ I_{L,\text{rms}} = \sqrt{\frac{1}{T_S} \int_0^{T_S} i_L^2(t) \, dt} = I_L \cdot \sqrt{1 + \frac{\Delta I_L^2}{3I_L^2}} \quad (1) \]

\[ I_{S,\text{rms}} = \sqrt{\frac{1}{T_S} \int_0^{T_S} i_S^2(t) \, dt} = I_L \cdot \sqrt{D \left(1 + \frac{\Delta I_L^2}{3I_L^2}\right)} \quad (2) \]

\[ I_{D,\text{rms}} = \sqrt{\frac{1}{T_S} \int_0^{T_S} i_D^2(t) \, dt} = I_L \cdot \sqrt{(1-D) \left(1 + \frac{\Delta I_L^2}{3I_L^2}\right)} \quad (3) \]
Therefore, According to the energy conservation method, the equivalent average voltage $V_E$ converted by parasitic voltage $V_D$ is as follow.

$$V_E = (1 - D)V_D$$  \hfill (4)

$R_S$, $R_D$ and $R_L$ are converted and combined to obtain the total equivalent average resistance $R_E$ as follows.

$$R_E = [R_L + DR_S + (1 - D)R_D]\left(1 + \frac{\Delta i_L^2}{3i_L^2}\right)$$  \hfill (5)

Therefore, the non-ideal buck converter can be equivalent to the circuit shown in Fig.3.

![Figure 3. Non-ideal Buck converter after equivalent conversion](image)

According to the time averaging equivalent circuit method, if the disturbance amount of duty cycle is $\hat{d}$, then the instantaneous value of duty cycle is $d = D + \hat{d}$. Combined with Fig.2, the average variable of current $i_S(t)$ which is flowing through the power switch $S$ in a switching cycle can be expressed as follow.

$$\bar{i}_S = \frac{1}{T_s} \int_{0}^{T_s} i_S(t)dt = \frac{1}{T_s} \int_{0}^{T_s} i_L(t)dt = di_L$$  \hfill (6)

In the average equivalent circuit, it is considered that the average value of the input voltage source is equal to the instantaneous value, that is $v_{gs} = v_g(t)$, then the average variable of voltage $v_D(t)$ at both ends of the switch $D$ can be expressed as follow.

$$\bar{v}_D = \frac{1}{T_s} \int_{0}^{T_s} v_D(t)dt = \frac{1}{T_s} \int_{0}^{T_s} v_g(t)dt = dv_g$$  \hfill (7)

Therefore, Considering the input voltage source directly connected in series with active power switch $S$, the power switch $S$ can be replaced by a controlled current source $di_L$, and the passive switch $D$ can be replaced by a controlled voltage source $dv_g$. And then the large signal average equivalent circuit model of non-ideal Buck converter under CCM can be obtained, as shown in Fig. 4.

![Figure 4. Large signal average equivalent circuit model of non-ideal Buck converter in CCM](image)
Ignore the small signal perturbations, in Fig. 4, letting \( v_g = V_g \), \( v_o = V_o \), \( d = D \), \( d_i = DI_L \), \( dv_g = DV_g \), and make the inductance \( L \) short circuit, capacitor \( C \) open circuit. At the same time, it is assumed that under ideal conditions, the ideal transformer can transform dc, then the controlled voltage source and current source can be replaced by the ideal transformer. The DC equivalent circuit model of the non-ideal Buck converter in CCM as shown in Fig. 5 is obtained.

![Figure 5. DC equivalent circuit model of non-ideal Buck converter in CCM](image)

Therefore, the steady-state value \( D \) of duty cycle \( d \) can be expressed as:

\[
V_O - V_D + (R_L + R_D) I_L \left( 1 + \frac{\Delta T^2}{3 L^2} \right) = V_o + (R_D + R_S) I_L \left( 1 + \frac{\Delta T^2}{3 L^2} \right)
\]

(8)

Among them \( I_L = \frac{V_O}{R} \), and

\[
V_O = \frac{DV_g - V_E}{1 + \frac{R_E}{R}}
\]

(9)

Each parameter in Fig. 4 are separated the perturbation and made to equal the sum of dc component and ac component, that is \( v_g = V_g + \hat{v}_g \), \( i_L = I_L + \hat{i}_L \), \( v_o = V_o + \hat{v}_o \), \( d = D + \hat{d} \), and the parameters of the two nonlinear controlled sources are separated as follows:

\[
\begin{align*}
&d_i = (D + \hat{d}) (I_L + \hat{i}_L) = DI_L + \hat{D} I_L + I_L \hat{d} + \hat{D} \hat{i}_L \\
dv_g = (D + \hat{d}) (V_g + \hat{v}_g) = DV_g + \hat{D} V_g + \hat{V}_g \hat{d} + \hat{D} \hat{v}_g
\end{align*}
\]

(10)

It is Assumed that the converter is satisfied with the small signal hypothesis, that is \( \hat{v}_g \ll V_g \), \( \hat{i}_L \ll I_L \), \( \hat{v}_o \ll V_o \), \( \hat{d} \ll D \). The quadratic product terms \( \hat{D} \hat{i}_L \) and \( \hat{D} \hat{v}_g \) of the ac small signal are negligible small quantities of the second order. To remove the dc components of the above kinds, the linear small-signal equivalent circuit of non-ideal Buck converter in CCM as shown in Fig. 6 can be obtained.

![Figure 6. Small-signal equivalent circuit model of non-ideal Buck converter in CCM](image)
According to Fig.6, the expressions for the power level transfer functions of non-ideal Buck converter are described as follows.

\[
G_{ob}(s) = \frac{\hat{v}_o(s)}{v_o(s)} = \frac{DR}{R + R_E} \cdot \frac{1 + \frac{s}{\omega_1}}{1 + \frac{s}{Q\omega_b} + \left(\frac{s}{\omega_b}\right)^2} \tag{11}
\]

\[
G_{od}(s) = \frac{\hat{v}_d(s)}{d(s)} = \frac{V_g R}{R + R_E} \cdot \frac{1 + \frac{s}{\omega_2}}{1 + \frac{s}{Q\omega_b} + \left(\frac{s}{\omega_b}\right)^2} \tag{12}
\]

\[
G_{ig}(s) = \frac{\hat{i}_g(s)}{i_o(s)} = \frac{D}{R + R_E} \cdot \frac{1 + \frac{s}{\omega_3}}{1 + \frac{s}{Q\omega_b} + \left(\frac{s}{\omega_b}\right)^2} \tag{13}
\]

\[
G_{id}(s) = \frac{\hat{i}_d(s)}{d(s)} = \frac{V_g}{R + R_E} \cdot \frac{1 + \frac{s}{\omega_2}}{1 + \frac{s}{Q\omega_b} + \left(\frac{s}{\omega_b}\right)^2} \tag{14}
\]

\[
Z_o(s) = \frac{\hat{v}_o(s)}{i_o(s)} = \frac{R R_E}{R + R_E} \left(1 + \frac{s}{\omega_1} \right) \left(1 + \frac{s}{\omega_3} \right) \frac{1 + \frac{s}{\omega_2}}{1 + \frac{s}{Q\omega_b} + \left(\frac{s}{\omega_b}\right)^2} \tag{15}
\]

\[
G_{E}(s) = \frac{\hat{i}_E(s)}{i_o(s)} = \frac{R}{R + R_E} \cdot \frac{1 + \frac{s}{\omega_1}}{1 + \frac{s}{Q\omega_b} + \left(\frac{s}{\omega_b}\right)^2} \tag{16}
\]

Among the above categories, the free resonance frequency is \(\omega_b = \sqrt{\frac{R + R_E}{(R + R_E)C}}\), zero points are \(\omega_1 = \frac{1}{R C}\), \(\omega_2 = \frac{1}{(R + R_E)C}\), \(\omega_3 = \frac{R_E}{L}\), quality factor is \(Q = \frac{\sqrt{(R + R_E)R + R_E}}{RR_E C + RR_E C + R_E C + L}\).

In equations (15) and (16), the output current \(\hat{i}_o(s)\) is the test current, and the direction is from right to left in Fig.6. \(Z_o(s)\) is the output impedance of the open loop and \(G_{E}(s)\) is the transfer function of the inductive current to the output current. The two equations reflect the influence of the output current fluctuation on the circuit performance.

Among the above equations (11) to (16), letting \(s \to 0\), the low-frequency asymptotes of small signal characteristics can be obtained as follows. \(G_{ob}(0) = \frac{DR}{R + R_E}\), \(G_{od}(0) = \frac{V_g R}{R + R_E}\), \(G_{ig}(0) = \frac{D}{R + R_E}\).
$$G_{id}(0) = \frac{V_g}{R + R_E}, \quad Z_o(0) = \frac{RR_E}{R + R_E}, \quad G_{iz}(0) = -\frac{R}{R + R_E}.$$ It can be seen that they are related to the total equivalent series resistance $R_E$ in the inductance branch, that is, the low-frequency asymptotes of small signal characteristics is related to the two switching tubes and the equivalent parasitic parameters of the inductance.

If letting $s \to \infty$, the high-frequency asymptotes of small signal characteristics can be obtained as follows. $G_{vg}(\omega) = 0$, $G_{id}(\omega) = 0$, $G_{ig}(\omega) = 0$, $G_{id}(\omega) = 0$, $Z_o(\omega) = \frac{RR_C}{R + R_C}$, $G_{iz}(\omega) = 0$. As can be seen, $Z_o(\omega)$ is related to the parasitic parameters $R_C$ of capacitance.

It is worth to point out that the dc gain of the output voltage to the input voltage obtained from equation (11) is:

$$\frac{V_o}{V_g} = G_{vg}(0) = \frac{D_R}{R + R_E}$$

(17)

It can be seen that the equations (17) and (9) can be said the dc gain of the output voltage to the input voltage, but the value is different, equation (17) is obtained by the small-signal equivalent circuit model, and the equation (9) is obtained by dc equivalent model, which contains a diode positive pressure drop $V_D$ and does not contain ac small signal component. In the subsequent derivation process, equation (17) is used for ac small signal model, otherwise, equation (9) is used if it refers to dc gain.

Therefore, the disturbance quantity of the average inductance current $\dot{i}_L(t)$ and the disturbance quantity of the average output voltage $\dot{v}_o(t)$ can be expressed as follows.

$$\dot{i}_L(s) = G_{id}(s)\dot{d}(s) + G_{ig}(s)\dot{v}_g(s) + G_{iz}(s)\dot{v}_o(s)$$

(18)

$$\dot{v}_o(s) = G_{id}(s)\dot{d}(s) + G_{ig}(s)\dot{v}_g(s) + Z_o(s)\dot{v}_o(s)$$

(19)

4. Simulation

In this paper, Matlab is used to simulate the model [11-12], and the simulation parameters are: $V_g = 15V$, $V_o = 10V$, $R = 10\Omega$, $L = 127\mu H$, $R_L = 0.72\Omega$, $C = 247\mu F$, $R_C = 20m\Omega$, $R_S = 10m\Omega$, $R_D = 30m\Omega$, $V_D = 0.45V$, Inductive current ripple $\Delta i_L = 0.064$, and the frequency of the switch $f_s = 200kHz$.

Fig.7 shows the comparison between the control-output transfer function Bode diagram of the ideal model and the non-ideal model, in which the real line is the non-ideal model and the dotted line is the ideal model. It can be seen that the ideal model is the non-ideal model are slightly different in the low and middle-frequency band, but after $10^3Hz$, their amplitude-frequency characteristic or phase-frequency characteristics have bigger difference, and , and the amplitude of the non-ideal model decreases more slowly, the phase is relatively advanced. This is also can see from Equation (10), the ideal model relative to the ideal model adds a zero, so that the amplitude-frequency characteristic curve near the zero point relative to the ideal model of slope, and the leading phase. The non-ideal model relative to the ideal model increases a zero point $\omega_z$, thus, the slope of the amplitude-frequency characteristic curve increases by 20dB/dec near the zero point and produces a leading phase of $90^\circ$.

It can be seen that when modeling DC-DC converter, if only ideal assumptions are made, the equivalent model of DC-DC converter will have a large deviation from the actual circuit, which will affect the design of control circuit. Therefore, it is necessary to consider the non-ideal factors of DC-DC converter in the modeling process.
5. Conclusion
The circuit average equivalent model of non-ideal Buck converters in CCM was established in this paper. The model considered the influence of every element’s equivalent parasitic parameters and inductance current ripples existing in actual converters. The expressions of the power level transfer functions of non-ideal Buck converter are obtained and are proved that the transfer functions are affected by non-ideal factors. The necessity of the non-ideal model is verified by the simulation of the control-output transfer function. The model was intuitive, had clear physical meaning and can reflect the actual operation of the system more accurately. The modeling method is universal and suitable for other types of converters, such as Boost, Buck-Boost, etc.

Acknowledgments
This work is supported by the University Outstanding Young Talents Foundation of Anhui under Grant No.gxyq2017127.

References
[1] Robert W. Erickson, Dragan Maksimovic, “Fundamentals of Power Electronics (Second Edition)”, Kluwer Academic Publishers, 2001.
[2] Xu Jianping, Yu Juebang, “Time averaging equivalent circuit analysis of a resonant switching converter”, International Journal of Electronics, vol.67, no.6, pp.937-948, 1989.
[3] A. AMMOS, “An advanced PWM-Switch Model Including Semiconductor Device Nonlinearities”, IEEE Transactions on Power Electronics, vol.18, no.5, pp.1230-1237, 2003.
[4] O A1-Naseem’ R Erickson, “Prediction of Switching Loss Variations by Averaged Switch Modeling”, IEEE Applied Power Electronics Conference, pp.726-731, 2000.
[5] Davoudi A, Jatskevich J., “Realization of Parasitics in State-Space Average-Value Modeling of PWM DC-DC Converters”, IEEE Trans. Power Electronics, vol.21, no.4, pp.1142 - 1147, 2006.
[6] Yuri Tanovitski, Gennady Kobzhev,Danil Savin, “General Solutions of Nonlinear Equations for the Buck Converter”, International Conference on Nonlinear Dynamics of Electronic Systems
(CCIS), volume 438, pp.142-147, 2014.

[7] Qiu Y, Xu M, Sun J J, “A generic high-frequency model for the nonlinearities in Buck converters”, IEEE Trans. Power Electron, vol.22, no.5, pp.1970-1977, 2007.

[8] Robert Sheehan, “A New Way to Model Current-Mode Control, Part 2”, Power Electronics Technology Magazine, 2007.

[9] Yie-Tone Chen, Cing-Hong Chen, “A DC-DC Buck Converter Chip with Integrated PWM/PFM Hybrid-Mode Control Circuit”, Power electronics and Drive Systems, International Conference on Digital Object Identifier, pp.181-186, 2009.

[10] Mahesh Patil, Pankaj Rodey, “Buck Converter in Open Loop”, Control Systems for Power Electronics, pp.21-27, 2015.

[11] Zhi-bang Xu, Xiao-ying Shi, “Education Platform for Simulation of Buck Converter in Matlab GUI”, Technology and Management, AISC165, PP.617-626, 2012.

[12] Adel A. Elbaset, M. S. Hassan, “Small-Signal MATLAB/Simulink Model of DC–DC Buck Converter”, Design and Power Quality Improvement of Photovoltaic Power System, pp.97-114, 2016.