Quantum friction imprints on the geometric phase of a moving atom in front of a dielectric plate

Fernando C. Lombardo and Paula I. Villar
Departamento de Física Juan José Giambiagi, FCEyN UBA and IFIBA CONICET-UBA, Facultad de Ciencias Exactas y Naturales, Ciudad Universitaria, Pabellón I, 1428 Buenos Aires, Argentina

Abstract. We compute the non-unitary geometric phase for the moving atom under the presence of the vacuum field and a dielectric mirror, analytically and numerically. We consider the atom (represented by a two-level system) moving in front of a dielectric plate, and study how decoherence of the particle’s internal degrees of freedom can be found in the corrections to the geometric phase accumulated by the atom. We consider the particle to follow a classical, macroscopically-fixed trajectory and by integrating over the vacuum field and the microscopic degrees of freedom of the plate we may calculate friction effects. We find a velocity dependence in the correction to the unitary geometric phase due to quantum frictional effects. We also show in which cases decoherence effects could, in principle, be controlled in order to perform a measurement of the geometric phase using standard interferometry procedures.

1. Introduction
A quantum system can retain the information of its motion when it undergoes a cyclic evolution in the form of a geometric phase (GP), which was first put forward by Pancharatman in optics [1] and later studied explicitly by Berry in a general quantal system [2]. Since the work of Berry, the notion of geometric phases (GPs) was shown to have important consequences for quantum systems. Berry demonstrated that quantum systems could acquire phases that are geometric in nature. He showed that, besides the usual dynamical phase, an additional phase that was related to the geometry of the space state was generated during an adiabatic evolution. Since then, great progress has been achieved in this field. As an important evolvement, the application of the geometric phase has been proposed in many fields, such as the geometric quantum computation. Due to its global properties, the geometric phase is propitious to construct fault tolerant quantum gates. In this line of work, many physical systems have been investigated to realise geometric quantum computation, such as NMR (Nuclear Magnetic Resonance) [3], Josephson junction [4], Ion trap [5] and semiconductor quantum dots [6]. The quantum computation scheme for the GP has been proposed based on the Abelian or non-Abelian geometric concepts, and the GP has been shown to be robust against faults in the presence of some kind of external noise due to the geometric nature of Berry phase [7, 8, 9]. It was therefore seen that the interactions play an important role for the realisation of some specific operations. As the gates operate slowly compared to the dynamical time scale, they become vulnerable to open system effects and parameters fluctuations that may lead to a loss of coherence. Consequently, study of the GP was soon extended to open quantum systems. Following this idea, many authors have
analysed the correction to the GP under the influence of an external environment using different approaches (see [10, 11, 12, 13] and references therein).

In this framework, we propose to track evidence of vacuum fluctuations in the geometric phase acquired by a neutral particle moving in front of an imperfect mirror. By measuring the interference pattern of the particle, it could be possible to find a dependence of the correction to the unitary geometric phase upon the velocity of the particle. The pattern obtained can be an indirect proof of the existence of a quantum frictional force [14]. We shall consider a neutral particle coupled to a vacuum field, which is also in interaction with the internal degrees of freedom of a dielectric plate. The particle’s trajectory will be, along this paper, kept as an externally-fixed variable. We shall assume the particle moving at a constant velocity $v$, as is the most popular scenario in the literature [15, 16]. As we are interested in the dynamics of the internal degree of freedom of the particle, we will consider the neutral particle as a two-level quantum (a qubit) system (regarding it as a simple model for the atom) coupled to the vacuum field. We shall use a simple model for the microscopic degrees of freedom of the mirror, as we have done in a previous work [17]: a set of uncoupled harmonic oscillators, each of them also interacting locally with the vacuum field. In Fig.1 there is a picture of the global system and the interactions among the subsystems.

In order to consider how the relative motion between the particle and the plate affects the geometric phase acquired (and therefore consider the effects of quantum friction in the GP), we shall follow the procedure presented in previous works [17, 18], where we have used the in-out effective action approach, to account for the dissipative effects. Decoherence effects have been studied in Ref. [18], evaluating the influence action of composite environment (vacuum field + plate) over the particle [19].

2. Non-unitary geometric phase

We consider the moving particle to be a two-level quantum system (spin-$1/2$) coupled to a composite environment formed by the vacuum field and the dielectric mirror. Therefore, the main system bare Hamiltonian is given by

$$H_{\text{sys}} = \Delta \sigma_z,$$

which simply represents a cyclic evolution with period $\tau = 2\pi/\Delta$ if isolated.

For simplicity, we are only considering a dephasing spin–bath interaction, neglecting relaxation. We take a product initial state for the complete spin-bath system as

$$\rho(0) = |\psi_0\rangle \langle \psi_0| \otimes |\varepsilon(0)\rangle \langle \varepsilon(0)|,$$

where $|\psi_0\rangle = \cos(\theta/2)|0\rangle + \sin(\theta/2)|1\rangle$ is a superposition initial state for the two–level system, and $|\varepsilon(0)\rangle$ is a general initial state of the composite environment. To compute the global phase gained during the evolution, one can use the Pancharatnam’s definition [1], which has a gauge dependent part (i.e a dynamical phase $\Phi_d = \pi \cos \theta$) and a gauge independent part, commonly known as geometric phase $\Phi_g = \pi (1 + \cos \theta)$.

It is widely known that, when coupled to a bath, the reduced density matrix of the particle system satisfies a mater equation of the form (see for example [11] and references therein):

$$\dot{\rho}_r = -i[H_{\text{sys}}, \rho_r] - D(t)[\sigma_z, [\sigma_z, \rho_r]].$$

This model describes a purely decoherent mechanism, solely containing a diffusion term $D(t)$, where no energy exchange between system and the composite bath is present. The diffusion coefficient $D(t)$ can be evaluated following Ref.[18], and its explicit form is not relevant at this point. Therefore, the coupling to the bath affects the system such that its reduced density matrix at a time $t$ (after integrating out all the bath degrees of freedom) is
Figure 1. (Color online) We present a simple diagram of the system under consideration. That vacuum field is a massless scalar field $\phi(x)$ and the internal degrees of freedom of the plate are $\psi(x)$. The internal degrees of freedom of the particle will be considered as a two-level system $\sigma_z$.

\[
\rho_r(t) = \cos^2(\theta_t)|00\rangle + \sin^2(\theta_t)|11\rangle
+ \sin(\theta_t)\cos(\theta_t)e^{-i\Omega t}|01\rangle
+ \sin(\theta_t)\cos(\theta_t)e^{i\Omega t}|10\rangle,
\]

where we have used

\[
\sin(\theta_t) = \frac{2(\epsilon_+ - \cos^2(\theta_t))}{\sqrt{|r(t)|^2\sin^2(\theta) + 4(\epsilon_+ - \cos^2(\theta_t))^2}},
\]

\[
\cos(\theta_t) = \frac{|r(t)|\sin(\theta)}{\sqrt{|r(t)|^2\sin^2(\theta) + 4(\epsilon_+ - \cos^2(\theta_t))^2}}.
\]

Here, $r(t)$ is the decoherence factor induced by the composite environment (vacuum field and dielectric mirror). The eigenvalues of the above reduced density matrix are easily calculated, yielding:

\[
\epsilon_{\pm}(t) = \frac{1}{2} \pm \frac{1}{2}\sqrt{\cos^2(\theta) + |r(t)|^2\sin^2(\theta)}.
\]

The phase $\Phi$ acquired by the open system after a period $\tau$ is defined as [10]:

\[
\Phi = \text{arg}\left[\sum_k \sqrt{\epsilon_k(\tau)\epsilon_k(0)}\langle\Psi_k(0)|\Psi_k(\tau)\rangle e^{-\int_0^\tau dt\langle\Psi_k(t)|H_0|\Psi_k(t)\rangle}\right],
\]

where $|\Psi_k(t)\rangle$ and $\epsilon_k(\tau)$ are respectively the instantaneous eigenvectors and eigenvalues of $\rho_\tau(t)$. Here, $k$ refers to the two modes (+ and −) of the qubit model we are dealing with. The central result of Eq. (7) is to extract the purification independent part of the phase – which can be defined as a geometric phase because it is gauge invariant and reduces to the known results in the limit of a unitary evolution. In order to estimate the geometric phase, we only need to
consider the eigenvector $|\Psi_+(t)\rangle$ since $\varepsilon_-(0) = 0$. Therefore, in this case, only the ‘+ mode’ contributes to the GP. By the use of the several expressions obtained above, one can reach the final formula for the GP

$$\Phi = \Delta \int_0^T dt \cos^2(\theta_t). \quad (8)$$

The GP can be straightforwardly computed if we have the definition of $r(t)$. The decoherence factor $r(t)$ induced by this kind of composite environment has been already calculated in Ref.[18] but for the case of a harmonic oscillator internal degree of freedom for the moving particle. In the present case, we obtain the the decoherence factor induced upon the two-level system as:

$$r(t) = \exp \left\{ -\frac{\gamma_0 t}{2} \left( 1 + \frac{2}{3} v^2 + \lambda^2 v^2 e^{-\frac{2\Omega}{\sqrt{1-v^2}}} \right) \right\}, \quad (9)$$

by taking the limit $\Omega \gg \omega_0$ in [18], which means that the bare frequency of the harmonic oscillator is negligible compared with the natural frequency $\Omega$ of the dielectric harmonic oscillator degrees of freedom in the mirror. In the process, we have considered the internal degrees of freedom of the dielectric plate to be an infinite set of uncoupled harmonic oscillators of frequency $\Omega$. Each of these oscillators is interacting locally in position with the vacuum field through a coupling constant $\lambda$. The two-level system (modeled by $\sigma_z$ operator), also interacts linearly and locally with the vacuum field, but through a coupling constant named $g$. We have set dimensionless coupling constant to the dielectric plate $\tilde{\lambda}^2 = \lambda^2 / \Omega^2$ and $\gamma_0 = g^2$.

It is important to note that the decoherence factor contains a term proportional to the coupling between the vacuum field and the dielectric mirror ($\sim \gamma_0 \tilde{\lambda}^2$), in addition to the expected term corresponding to the coupling to the vacuum field, proportional to $\gamma_0$. The dependence of the decoherence factor upon the velocity $v$ is also of remarkable importance. The contribution coming from the presence of the dielectric plate becomes less important when $v \to 0$ and grows for large values of $v$.

In Fig.2 we plot the ratio between the total geometric phase from Eq.(8) and the unitary phase $\Phi_u = \pi (1 + \cos \theta)$, as a function of the initial angle $\theta$ and the tangential velocity $v$ for fixed parameters of $\tilde{\lambda}$, $\tilde{\Omega}$ and $\gamma_0$. Therein, we can see that for small values of the initial angle (i.e. a spin very similar to $|\downarrow\rangle$) and very low values of the velocity, the GP obtained for this system is very similar to the one obtained for an isolated quantum system (i.e. a spin 1/2 particle evolving freely). The bigger difference in the open GP and the isolated one is obtained for bigger angles and bigger values of $v$.

The correction to the unitary GP is relevant for big values of $v$. What is important to remark, is that the mere presence of a velocity contribution to the phase, is an indication of the friction effect over the quantum degree of freedom of the atom. In this sense, the measurement of the geometric phase could, in principle, be an alternative way to find out quantum friction in a laboratory, even though the considered velocities in experiments are still far away from a relativistic case with $v \to 1$.

Finally, we can perform a series expansion in $\gamma_0$ and $\tilde{\lambda}$ (up to first non-trivial orders) in Eq.(8) in order to obtain an analytical expression for the GP acquired by the particle:

$$\Phi_{\text{approx}} \approx \pi \sin^2 \theta + \frac{\pi^2}{3} \frac{\gamma_0}{(1 - v^2)} \cos \theta \sin^2 \theta \times \left[ (3 - v^2 - 2v^4) + 3v\tilde{\lambda}^2 e^{-\frac{2\Omega}{\sqrt{1-v^2}}} \right]. \quad (10)$$

In the particular case in which the coupling between the atom and the dielectric plate is switched off, $\tilde{\lambda} = 0$, the correction to the unitary phase is given by $\delta \Phi \sim \pi^2 \gamma_0 (1 +$
Figure 2. (Color online). We present the geometric phase normalised \((\Phi/\Phi_g)\) by the unitary geometric phase \(\phi_g = \pi(1 + \cos(\theta))\) as function of \(\theta\) and \(v\). Parameters used: \(\tilde{\lambda} = 10\), \(\gamma_0 = 0.05\), \(\tau = 4\pi/\Delta\) and \(\tilde{\Omega} = 0.03\).

\[ \left(\frac{2}{3v^2}\right) \cos \theta \sin^2 \theta \] which agrees with the result obtained for a two-level system coupled to an environment composed by an infinite set of harmonic oscillators at equilibrium with \(T = 0\) \cite{11, 12, 20, 21, 22, 23, 24}. However, we can see that in this case the result is enhanced by a factor \(1 + 2/3v^2\). This situation corresponds to the atom only coupled to the vacuum field. Up to the lowest perturbative order, the same result can be obtained in the limiting case of \(v \to 0\).

3. Conclusions

We have considered the effects of quantum vacuum fluctuations on a particle moving parallel to an imperfect mirror. We have presented a simple model in which the vacuum field is a massless, real, scalar field coupled to the microscopic degrees of freedom of the mirror and the internal degree of freedom of the particle (following analysis done previously in Refs.\cite{17, 18}). The plate is formed by uncoupled unidimensional harmonic oscillators, each of them interacting locally in position with the vacuum field. The macroscopic trajectory of the particle is externally fixed, and its internal degree of freedom is considered as a simple two-level system, also coupled to the scalar field. Using previous results for decoherence effects found in Refs.\cite{17, 18}, we have estimated the decoherence factor when the internal degree of freedom of the particle moving with parallel velocity \(v\) is a two-state system, as a toy model for an atom travelling over the dielectric plate. With the expression of the decoherence factor, it was possible to calculate the corrections to the geometric phase of the quantum states of the atom, induced by the interaction with the composite environment.

We have found that the phase acquired by the moving particle is different to the one the particle would have acquired if it has evolved freely. By measuring the correction to the unitary geometric phase, one can get an insight of the dependance of the phase on the parameters. In this way, we have seen that the bigger the velocity of the particle, the more correction to the phase. It is also noticeable that the effect of noise is bigger for initial states near the equator of the Bloch sphere.

Finally, we have obtained an approximate expression for the phase acquired and compared this result to the exact geometric phase. In this case, it is possible to see that the expression gives an accurate result for small values of the coupling constants (as expected), as well as small angles. It is also possible to note that in the case in which the coupling with the mirror can be neglected, the result obtained for the correction to the geometric phase would correspond to the one obtained in the case a qubit is solely in interaction with an infinite set of harmonic oscillators at thermal equilibrium with \(T = 0\). Finally, it is remarkable that, in the case of the...
moving atom, there is a factor proportional to $v^2$ which enhances the correction to the phase, even though we are in the small velocity case $v < 1$.

We expect that in an interference experiment, the parameters of our model could be chosen in a way that would maximise the decoherence effects. By increasing the decoherence effect, the unitary geometric phase results in a major correction. In this sense, as quantum friction has not been measured in labs yet, we expect that an indirect evidence could be obtained from measuring the environmental induced corrections to the geometric phase. The dependence of the correction on the velocity $v$ would be an indirect way to finally measure quantum friction.

Acknowledgments
This work was supported by UBA, CONICET and ANPCyT - Argentina. We would like to H. Thomas Elze for the organization of DICE 2016. FCL acknowledges to B. Lok Hu by the very interesting conversations about quantum friction, decoherence, and geometric phases.

References

[1] Pancharatnam S 1956 Generalized Theory of Interference, and its Applications. Part I. Coherent Pencils Proc. Indian Acad. Sci. A 44, 247
[2] Berry M Y 1984 Quantal Phase Factors Accompanying Adiabatic Changes Proc. R. Soc. Lond. A 392 45
[3] Jones J A, Vedral V, Ekert A and Castagnoli G 2000 Geometric quantum computation using nuclear magnetic resonance Nature 403 869
[4] Faoro L, Siewert J and Fazio R 2003 Non-Abelian Holonomies, Charge Pumping, and Quantum Computation with Josephson Junctions Phys. Rev. Lett. 90 028301
[5] Duan L M et. al. 2001 Geometric Manipulation of Trapped Ions for Quantum Computation Science 292 1695
[6] Solinas P, Zanardi P, Zangh N and Rossi F 2003 Semiconductor-based geometrical quantum gates Phys. Rev. B 67 121307
[7] Zanardi P, Rasetti M 1999 Holonomic Quantum Computation Phys. Lett. A 264, 94
[8] Xiang-Bin W and Keiji M 2001 Nonadiabatic Conditional Geometric Phase Shift with NMR Phys. Rev. Lett. 87, 097901
[9] Sjoqvist E, Tong D M, Andersson M L, Hessmo B, Johansson M, and Singh K 2012 Non-adiabatic holonomic quantum computation New J. Phys. 14, 103035
[10] Tong D M, Sjoqvist E, Kwek L C, and Oh C H 2004 Kinematic Approach to the Mixed State Geometric Phase in Nonunitary Evolution Phys. Rev. Lett. 93, 080405; see also Tong D M, Sjoqvist E, Kwek L C, and Oh C H 2005 Erratum: Kinematic Approach to the Mixed State Geometric Phase in Nonunitary Evolution [Phys. Rev. Lett. 93, 080405 (2004)] Phys. Rev. Lett. 95, 249902
[11] Lombardo F C and Villar P I 2006 Geometric phases in open systems: A model to study how they are corrected by decoherence Phys. Rev. A 74, 042311
[12] Lombardo F C and Villar P I 2008 Environmentally induced corrections to the geometric phase in a two-level system Int. J. of Quantum Information (IJQI) 6, 707713
[13] Villar P I 2009 Spin bath interaction effects on the geometric phase Phys. Lett. A 373, 206
[14] Pendry J B 1997 Shearing the vacuum - quantum friction J. Phys. Condens. Matter 9, 10301; Pendry J B 2010 Quantum frictionfact or fiction? New J. Phys. 12, 033028; Pendry J B 2010 Reply to comment on 'Quantum frictionfact or fiction? New J. Phys. 12, 068002; Philbin T G and Leonhardt U 2009 No quantum friction between uniformly moving plates New J. Phys. 11, 033035; Leonhardt U 2010 Comment on 'Quantum FrictionFact or Fiction?' New J. Phys. 12, 068001; Volokitin A I and Persson B N J 2007 Near-field radiative heat transfer and noncontact friction Rev. Mod. Phys. 79, 1291
[15] Hoye J S and Brevik I 2010 Casimir friction force and energy dissipation for moving harmonic oscillators Europhys. Lett. 91, 60003; Barton G 2010 On van der Waals friction. I: Between two atoms New J. Phys. 12, 113044; Barton G 2010 On van der Waals friction. II: Between atom and half-space New J. Phys. 12, 113045
[16] Intravaia F, Behunin R O, and Dalvit D A R 2014 Quantum friction and fluctuation theorems Phys. Rev. A 89 050101
[17] Farías M B, Fosco C D, Lombardo F C, Mazzitelli F D, Rubio López A E 2015 Functional approach to quantum friction: Effective action and dissipative force Phys. Rev. D 91, 105020
[18] Farías M B and Lombardo F C 2016 Dissipation and decoherence effects on a moving particle in front of a dielectric plate Phys. Rev. D 93, 065035

[19] Feynman R and Vernon F 1963 The Theory of a General Quantum System Interacting with a Linear Dissipative System Ann. Phys. (N.Y.) 24, 118; Lombardo F C, Mazzitelli F D, and Rivers R J 2003 Decoherence in Field Theory: General Couplings and Slow Quenches Nucl. Phys. B 672, 462; Lombardo F C, Mazzitelli F D, and Rivers R J 2001 Classical behaviour after a phase transition Phys. Lett. B 523, 317; Lombardo F C, Rivers R J, and Villar P I 2007 Decoherence of domains and defects at phase transitions Phys. Lett. B 648, 64

[20] Cucchietti F M, Zhang J F, Lombardo F C, Villar P I and Laflamme R 2010 Geometric Phase with Nonunitary Evolution in the Presence of a Quantum Critical Bath Phys. Rev. Lett. 105 240406

[21] Villar P I and Lombardo F C 2011 Geometric phases in the presence of a composite environment Phys. Rev. A 83, 052121

[22] Lombardo F C and Villar P I 2013 Nonunitary geometric phases: A qubit coupled to an environment with random noise Phys. Rev. A 87, 032338

[23] Lombardo F C and Villar P I 2015 Correction to the geometric phase by structured environments: The onset of non-Markovian effects Phys. Rev. A 91, 042111

[24] Lombardo F C and Villar P I 2015 Geometric phase and quantum correlations for a bipartite two-level system Journal of Physics: Conference Series 626 012043.