G-Jitter Fully Developed Heat and Mass Transfer by Mixed Convection Flow between Two Parallel Plates with Constant Heat Flux

(Wan Nor Zaleha Amin, Ahmad Qushairi Mohammad, Sharidan Shafie* & Muhammad Qasim)

ABSTRACT

In this research, the problem of heat and mass transfer in mixed convection flow induced by g-jitter is studied analytically. The flow passes through an infinite length of vertical parallel plates. The left wall of the plate is prescribed with constant heat flux while the right wall is maintained at a constant temperature. The governing boundary layer equations are transformed into dimensionless equations by using the appropriate dimensionless variables. The behaviors of the velocity, temperature, and concentration profiles of the fluid are analyzed by Fourier method and illustrated graphically as well. Results show that due to an increase in value of wall temperature, the fluid temperature increases. It is found that the controlling parameters strongly affect the heat and mass transfer characteristics in g-jitter fluid flow.

Keywords: Constant heat flux; g-jitter; mixed convection; two vertical plates

INTRODUCTION

The mechanisms of convection include free, forced, and mixed. Among these three types, mixed convection is the most highlighted topic and is considered here. Scientists are still interested to disclose the characteristics of mixed convective fluid behavior due to its promising significance in various engineering and industrial procedures. Mixed convection concepts are useful in designing the cooling system with liquid metals, accelerators, nuclear reactor, pumps, and blood flow. These motivated numerous researchers such as Bakar et al. (2019), Chen (2004), Hammou et al. (2004), Kumari and Nath (2004), Kumar et al. (2009), and Sajid et al. (2010) explored the behavior of mixed convection flow in vertical plates through different methods to increase the concentration and temperature transfer performance of fluids.

Furthermore, space experiments have showed the unknown or nonexistent effect of g-jitter or residual acceleration associated with the microgravity environment. g-jitter is defined as the inertia effects due to quasi-steady, oscillatory or transient accelerations arising from crew motions and machinery vibrations in parabolic aircraft, space shuttles or other microgravity environments. It is important to understand the effects of g-jitter in design and maintaining the cooling board of a system to prevent such overheating cause problem like explosion in practical application. Recently, numerous attempt by Rawi et al. (2018, 2017a, 2017b) was conducted with the numerical research for the effect of g-jitter upon the mixed convection flow to overcome this situation.

Despite that, constant heat flux in an isothermal state is also desirable in many applications especially for an electric component in the circuit board. Therefore, a numerical study by Chen et al. (2000) of the fully developed mixed convection in a heated vertical channel filled with a porous medium with imposed uniform heat

ABSTRAK

Dalam kajian ini, masalah bagi aliran pemindahan haba dan jisim di dalam aliran olakan campuran yang disebabkan oleh ketar-g dikaji secara analitik. Aliran ini melepasi satu plat selari menegak yang panjanganya tidak terhingga. Dinding sebelah kiri plat ditetapkan dengan flaks haba tetap manakala dinding sebelah kanan pula dikekalkan pada suhu tetap. Persamaan menakluk bagi lapisan sempadan diibah ke dalam persamaan tak hermatra dengan menggunakan perboleh ubah tak hermatra yang berkesuatan. Telatah bagi profil halaju, suhu dan kepekatan bendalir dianalisis dengan kaedah Fourier dan juga diilustrasi secara bergraf. Keputusan menunjukkan bahawa dengan peningkatan nilai suhu dinding, suhu cecair juga meningkat. Didapati bahawa parameter pengawal sangat mempengaruhi ciri pemindahan haba dan jisim dalam aliran bentalir ketar-g.

Kata kunci: Dua plat menegak; flaks haba tetap; ketar-g; olakan campuran
flux at the plates was performed using the Brinkman
Forchheimer extended Darcy model. It is then being
solved using the Galerkin method. Next, Lee et al.
(2004) presented a research on a large eddy simulation
of heated vertical annular pipe flow in fully developed
turbulent mixed convection with heat flux. Navier
Stokes equations are solved using a second order of
finite volume method. The results showed that the strong
heating and buoyant force affect the flow structure by
reducing the turbulent intensities, shear stress, and
turbulent heat flux, particularly near the wall. The strong
heating near the inner wall causes an increased viscosity
and the consequential enhancement in the damping of
turbulence.

Also, Sivasamy et al. (2010) conducted a research
of jet impingement cooling of constant heat flux on the
mixed convection flow of a porous channel numerically
using Darcy model and found that the increasing values
of Grashof and Reynolds numbers will increase the
Nusselt number. Later, Roşca and Pop (2013) studied
numerically the mixed stagnation point past vertical
plate with second-order slip using bvp4c from Matlab
with constant heat flux. The paper performed a stability
analysis to verify the stability of the results and found
out that the second order slip considerably affect the heat
and flow characteristics.

Moreover, Altunkaya et al. (2017) conducted a
research to investigate analytically the open-ended
vertical microchannel using a perturbation technique with
constant heat flux. The author considered the analysis for
Nusselt number against the mixed convection parameter,
rarefaction, and viscous dissipation. The results showed
that as the mixed convection and rarefaction increases,
Nusselt number is decreasing.

Recently, Rosas et al. (2017) conducted an
experimental study of mixed convection heat transfer
which is carried out in a vertical channel with one-

sided semi-cylindrical constriction with prescribed heat
flux while the other bounding wall is insulated and
adiabatic. The results showed that variation of the local
temperature distributions with angular position and
spanwise location become evident and their relation to
the presence of a complex 3D vortex structure that
devotes close to the semi-cylindrical constriction.
Moreover, empirical correlations for the overall Nusselt
number are obtained using both Reynolds and Grashof
numbers as controlling parameters.

These review of literature suggests that little
attention has been paid to evaluate the mixed convection
in discretely heated surfaces. Clearly, from the foregoing
discussion, there are no prior results available for mixed
convection heat and mass transfer between two vertical
plates with the effect of g-jitter by a prescribed heat flux
boundary condition. Heat flux is relevant to influence
heat and mass transfer rates. In particular, this research
aims to employ Fourier method and also to expand
the following works by Sharidan et al. (2005) which
investigate analytically the effect of g-jitter on mixed
convection flow in two vertical parallel plates with constant wall temperature boundary condition. Moreover,
this study is very crucial to explore the effect of g-jitter
and heat flux for understanding the fluid flow behavior in
microgravity environment.

### FORMULATION AND METHODS

Consider an incompressible fluid which unsteadily
flows between two vertical parallel plates of width \( h \),
uniform temperature at the right wall and heat flux
at the left wall. A stationary Cartesian coordinate is
chosen such that the \( x \)-coordinate along the surface and \( y \)-coordinate is normal to it. Pressure gradient
of oscillatory type is applied in the flow direction.
The presence of g-jitter fully developed effect is also
considered. Assuming time \( t = 0 \), the flow is at constant
temperature \( T_0 \), concentration \( C_0 \) with the mean velocity
\( u_0 \) and pressure \( p_0 \). The assumption of fully developed
flow means that the axial (\( x \)-direction) velocity depends
only on the transverse coordinate \( y \). Then, from the
continuity equation, the transverse velocity \( v \), must be
zero. The axial velocity \( u \), and the fluid temperature \( T \),
and concentration \( C \), are assumed to be functions of \( y 
plus function of time \( t \). The pressure variation is found
to be a linear function of \( x \) and \( t \), and it is assumed that the
gravity acceleration is given by \( g' (t) = g_0 \sin(\omega t) \) where
\( g_0 \) is the magnitude and \( \omega \) is the frequency of the g-jitter
field.

In the view of these assumptions, the initial
and boundary layer conditions governing the mixed
convection flow of the fluid past a vertical parallel
plate containing continuity (Kamal et al. 2019,
momentum, energy, and mass equations (Naser et al.
2013; Rajvanshi & Saini 2010) can be cast into the
following forms:

\[
\mathbf{v} \cdot \mathbf{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}
\]

\[
\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} + g'(t)\rho |\beta_\gamma| (T - T_0) + \beta_c (C - C_0). \tag{2}
\]

\[
\frac{\partial T}{\partial t} = \frac{k}{(\rho c_p)} \frac{\partial^2 T}{\partial y^2}. \tag{3}
\]

\[
\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial y^2}. \tag{4}
\]

The appropriate initial and boundary conditions are given by

\[
t \leq 0: \quad u = u_0, T = T_0, \quad C = C_0 \quad \text{for} \quad 0 < y < h
\]

\[
t > 0: \quad u = 0, \frac{\partial T}{\partial y} = -\frac{q}{k}, \quad C = C_1 \quad \text{at} \quad y = 0 \tag{5}
\]

\[
u = 0, T = T_2, \quad C = C_2 \quad \text{at} \quad y = h
\]
where $D$ is the mass diffusivity, $\mu$ is the dynamic viscosity of the fluid, $\rho$ is the fluid density, $\beta_\tau$ and $\beta_c$ are the volumetric coefficient of thermal expansion and concentration of the fluid respectively, $\nu$ is the kinematic viscosity, $p$ is the fluid pressure, $k$ is the thermal conductivity of the fluid, $c_p$ is the specific heat of the fluid, of the constant pressure, $q$ is the heat flux, $h$ is the width between two plates, $C_1$ is the concentration of the left wall and $T_2$ and $C_2$ is the temperature and concentration of the right wall respectively. The flow configurations and coordinate system are shown in Figure 1.

Then, Equations (2), (3) and (4) are transformed into the following dimensionless equations (Sharidan et al. 2005)

\[
\frac{\partial U}{\partial \tau} = - \frac{\partial P}{\partial X} + \frac{\partial^2 U}{\partial Y^2} + \frac{Gr}{Re} g(\tau) \cdot \left[ \theta + N \phi \right].
\]

(6)

\[
\frac{\partial \theta}{\partial \tau} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial Y^2}
\]

(7)

\[
\frac{\partial \phi}{\partial \tau} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial Y^2}
\]

(8)

where the dimensionless variables are defined as

\[
U = \frac{u}{u_0}, \tau = \frac{rt}{h}, \rho = \frac{\rho \nu u_0}{\rho}, X = \frac{x}{h}, Y = \frac{y}{h}, \theta = \frac{T - T_0}{\frac{h q}{k}}, \phi = \frac{C - c_0}{c_2 - c_1}, g(\tau) = \frac{g^*}{g_0}
\]

(9)

with Grashof number $Gr$, Reynolds number $Re$, Prandtl number $Pr$ and Schmidt number $Sc$ are given by

\[
Gr = \frac{g \beta_\tau \rho h^4}{\nu^2}, Re = \frac{u_0 h}{\nu}, Pr = \frac{v c_p \mu}{k}, Sc = \frac{v}{D}
\]

(10)

Dimensionless boundary conditions are:

\[
\tau \leq 0 \quad U = 1, \theta = 0, \quad \phi = \phi_0 \quad \text{for} \quad 0 < Y < 1
\]

\[
\tau > 0 \quad U = 0, \frac{\partial \theta}{\partial Y} = -1, \quad \phi = \phi_c \quad \text{at} \quad Y = 0
\]

\[
U = 0, \theta = \tau, \quad \phi = 1 \quad \text{at} \quad Y = 1
\]

(11)

Parameter of dimensionless wall temperature and concentration $r_\tau$ and $r_c$ are defined as

\[
r_\tau = \frac{T_2 - T_0}{hq/k}, \quad r_c = \frac{C_1 - C_0}{C_2 - C_1}
\]

(12)

Using Fourier method, (7) and (8) have the solution of form

\[
\theta(Y, \tau) = 1 + r_\tau - Y,
\]

(13)

\[
\phi(Y, \tau) = r_c + (1 - r_c) Y.
\]

(14)

Assuming $U = e^{\alpha \tau} \Phi(Y), P = e^{\alpha \tau} F(X)$ and $g(\tau) = e^{\alpha \tau}$ (Sharidan et al. 2005) along with overall mass conservation equation $\int_0^1 \Delta F(Y) dY = 1$, then the solution of $\Phi(Y)$ is given by

\[
\Phi(Y) = \frac{1}{\beta^2} \left[ \frac{\partial F}{\partial X} \right]_{Y=0}^{Y=1} + \left[ \frac{\sinh \beta (1-Y)}{\sinh \beta} \right] + \left[ \frac{\sinh \beta Y \frac{\partial F}{\partial X} \frac{Gr}{Re} (r_c + N)}{\sinh \beta} \right]
\]

(15)

where $\beta^2 = i \beta$ with $\partial F / \partial X$ is given by

\[
\frac{\partial F}{\partial X} = \frac{\beta^3 \sinh \beta}{2 \cosh \beta - \beta \sinh \beta} - \frac{Gr [1 + 2 \cdot r_\tau + N (r_c + 1)]}{2 Re}
\]

(16)
VALIDATION

For validation purposes, the solutions of (13), (14) and (15) are then being substitute back to (6), (7), and (8) and found that the left-hand side, LHS of (6), (7) and (8) are equal to right-hand side, RHS, respectively. Solutions (13), (14) and (15) are also observed fully satisfied the boundary conditions (11). Therefore, it is verified that the solutions of concentration (14), temperature (13) and velocity profiles (15) are corrects.

| Eqn. | Variables | LHS | RHS |
|------|-----------|-----|-----|
| (8)  | $S_c = 0.3$ | 0   | 0   |
| (7)  | $N = 1, Gr/Re = 0$ | 0   | 0   |

RESULTS AND DISCUSSION

The results are then computed into mathematical software named Matlab and the various values of the desired parameters are then generated and presented graphically. The quantities plotted along the vertical axis are temperature, concentration and velocity profiles which the equations of solutions are

\[
\theta(Y, \tau) = 1 + rT - Y, \quad (17)
\]

\[
\varphi(Y, \tau) = rC + (1 - rC)Y, \quad (18)
\]

and it represents the value of driving force

\[
g^* (t) = \text{Imag} \left( g_0 e^{i\omega t} \right) \quad \text{and} \quad u_0 (t) = u_0 \sin(\omega t). \quad (20)
\]

Figure 2 shows the effect of dimensionless concentration $r_C$ for the concentration profile. Values of ratio for $r_T$ and $r_C$ are taken in the range of 0 to 1. It is shown that as $r_C$ increased, the concentration also increased. Meanwhile, Figure 3 demonstrates the effect of dimensionless wall temperature $r_T$ on temperature distribution. From the graph, as $r_T$ increases, $\theta$ also increases. This situation is attributable as $r_T$ and $r_C$ increases, more hot and less concentrated fluid are carried through the vertical plates faster due to the increasing fluid temperature which consequently results in higher wall temperature gradient. These imply that concentration and temperature ratio is directly proportional to the concentration and temperature distribution, respectively.

![Concentration Profile](image)
From Figures 4 to 7, graphical results for the velocity profiles are shown. The parameters incorporated in the problem are mixed convection $Gr/Re$, buoyancy ratio $N$, oscillating $\tau$, temperature and concentration ratio parameter $r_T$ and $r_C$.

The analysis on the effect of mixed convection parameter $Gr/Re$ on the fluid flow is shown in Figure 4. It is found that the velocity of the fluid increases when $Gr/Re$ increases. The velocity profile is always symmetric about the centerline of the channel at $Y = 0.5$.

The presence of mixed convection enables us to seek the consequence of the relaxation time on the fluid velocity. This is because the mixed convection raised the wall temperature which drives the fluid to become further from the surface resulting the high speed of flow. It enhances the heat transfer when they assist the forced flow. This results indicates a good agreement with the past scheme (Mahat et al. 2017).

Furthermore, Figure 5 illustrates the velocity profile for different values of buoyancy ratio $N$. It can be seen that the fluid flow accelerated to the right wall as $N$ increases. This imply that the buoyancy aiding the fluid to move faster to upwards direction.
Figure 6 analyzes the effect of oscillating parameter $\Omega \tau$ towards the behaviour of the fluid velocity $U$. Values of $\Omega \tau$ is taken between 0 and $2\pi$ because the same behaviors are shown for the increasing values of $2\pi$. It is observed that there is an increasing reversed flow close to the left wall and only a small reversed flow closed to the right wall, respectively. This is due to the prescribed heat flux at the left wall where the fluid flow rises as the temperature increased. Moreover, the fluid velocity fluctuated as $\Omega \tau$ increases. It has a good correlation with the past results (Rawi et al. 2017c). The oscillating velocity has passed a quasi-steady state in a specific time about $2\pi$ and subsequently it fluctuates with fundamentally the same frequency of the g-jitter. This is due to the velocity become nonlinear, following around the identical scheme as g-jitter modulation.

Additionally, very interesting situations can be seen in Figure 7(a), where when $r_T > r_c$, the velocity profile showed a full reversed flow when the oscillating parameter $\Omega \tau$ is increased while in Figure 7(b) it is partial reversed flow when $\Omega \tau$ increased as $r_T > r_c$. These flow behaviour are the same from those case where the constant wall temperature is implied by Sharidan et al. (2005). Physically, the oscillating parameter will affect the velocity profiles.
This paper focused on heat and mass transfer by mixed convection flow between two parallel plates with constant heat flux, in the existence of g-jitter effect. Therefore, the following conclusions are made based on present research. Due to the increasing value of the wall temperature $r_T$ and concentration ratio $r_C$, the temperature $\theta$ and concentration of the fluid increased. These two parameters attributed to improvement of heat transfer efficiency. It enhanced the fluid to flow faster as it is increases. For velocity distribution, the velocity $U$ of the fluid increased as mixed convection parameter $Gr/Re$ and buoyancy ratio $N$ rised. Meanwhile, the velocity fluctuated with an increase in oscillating parameter $\Omega\tau$.

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