Green modified function of the equation of the internal gravity waves in the stratum of the stratified medium with constant average flow

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Abstract.
In the present paper construction of the modified function of Green equation for internal gravity waves in the stratum of the stratified medium at presence of constant average flows is considered, properties of the corresponding spectral problems, the modified eigenfunctions and eigenvalues are investigated. Usage of the modified function of Green equation can give in some physically interesting events more friendly representations of the solutions for the fields of the internal gravity waves, including the wave fields disturbed by the non-local disturbing bodies.

Green equation for the internal gravity waves at presence of the average shift flows satisfies the equation

\[
\left(L, \frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) = \delta(t)\delta(x)\delta(y)\delta(z-z'),
\]

\[
L = \frac{D^2}{Dt^2} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) - \frac{D}{Dt} \left( \frac{\partial^2 V_1}{\partial z^2} \frac{\partial}{\partial x} + \frac{\partial^2 V_2}{\partial z^2} \frac{\partial}{\partial y} \right) + N^2(z) \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right).
\]

L - the operator of the internal waves in Boissinesq approximation, N(z) is Brunt-Väisälä frequency; V_1, V_2 - the components of the current velocity V = \{V_1, V_2, 0\} on some horizon z (\sqrt{V_1^2 + V_2^2} = V);

\[
\frac{D}{Dt} = \frac{\partial}{\partial t} + V_1 \frac{\partial}{\partial x} + V_2 \frac{\partial}{\partial y}.
\]

Boundary and the initial conditions are taken in the form of

\[
\Gamma = 0 \quad (z = 0, H); \quad \Gamma \equiv 0, \quad t < 0.
\]

By means of \( \Gamma \) we construct the solution U of the equation (B1) for the arbitrary right part of (B1): Q = Q(t, x, y, z), depending on the particular statements of the problem

\[
U = \int dt' \int dx' \int dy' \int dz' \Gamma(t-t', x-x', y-y', z-z') Q(t', x', y', z')
\]

Green function for the internal waves by the Fourier transform on the variables t, x, y is reduced to the following view

\[
\Gamma = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} d\lambda \int_{-\infty}^{\infty} d\mu \int_{-\infty}^{\infty} d\omega e^{i\lambda x + i\mu y - i\omega t} G(\omega, \lambda, \mu, z, z')
\]
where \( G \) is the solution of the boundary problem
\[
L_0 G = -\delta(z - z'); \quad G = 0, \ z = 0, H
\] (B5)
and \( L_0 \) is Taylor-Goldstein operator
\[
L_0 = (\omega - f)^2 \frac{\partial^2}{\partial z^2} + \left\{ k^2 \left[ N^2 - (\omega - f)^2 \right] + \frac{\partial^2 f}{\partial z^2} (\omega - f) \right\},
\]
\[
f \equiv \lambda V_1(z) + \mu V_2(z), \quad k^2 = \lambda^2 + \mu^2
\] (B6)

The initial condition determines the bypass of the poles and cuts of \( G \) at integration of (B4) \((\varepsilon > 0, \varepsilon \to 0)\). The expression for \( G \) looks like
\[
\Gamma = G(t, x, y, z, z') + \sum_{n=1}^{\infty} \frac{1}{2\pi^2} \int_{-\infty}^{\infty} d\lambda \int_{-\infty}^{\infty} d\mu \ e^{i\lambda x + i\mu y - i\omega_n (\lambda, \mu)t} \ \frac{\varphi_n(z)\varphi_n(z')}{d_n (\omega_n - f(z'))^2}
\] (B7)
where \( \varphi_n(z) \) and \( \omega_n(\lambda, \mu) \) is the eigenfunctions and the eigenfrequencies of the operator \( L_0 \)

\( G_m \) is the contribution of the continuous spectrum of the operator \( L_0 \).
The wave zone defined by (B7) at the great values of \( |x|, |y|, |t| \) is limited by two curves - by the forward wave front and the rear wave front. At \( V = \text{const} \) the contribution of the continuous spectrum and the rear wave front fade away.

Later we shall consider the Green function representations at \( V = \text{const} \) in the modified form allowing more completely to determine the spatial structure of (B7) both at the great distances and in the immediate proximity to the sources of the internal gravity waves. It will be shown, that each mode of Green function consists of the sum of three terms, the first of which describes the internal waves propagating from the source, the second term describes the effects of the non-stationarity of the source localized in some vicinity around it, the third term describes the effects of the pushed away liquid (the internal jump) caused by the source. The analysis of the gained expressions for the constant and oscillating with the frequency \( \Omega \) source \( Q(t, x, y, z) \) will be conducted below, at that each of Green function items will be represented in the form of the single integral suitable for the analytical and numerical analysis.

The routine method of a solution of the non-uniform boundary problem of the (B5) type is expansion of \( G \) into the full set of the linearly-independent functions, being the solutions of the corresponding uniform problem.

At the same time it seems useful looking for the required solution to use the other full sets of the linearly-independent functions expansion, which in the certain sense meet the required statements of the problem and allows to gain some additional information about behavior of Green function. In the capacity of such set we shall take the solution of the following boundary problem
\[
\frac{\partial^2 \psi_n(z, \chi)}{\partial z^2} + \chi \left[ \frac{N^2(z)}{\nu_n(\chi)} - 1 \right] \psi_n(z, \chi) = 0
\]
\[
\psi_n = 0, \ \ z = 0, H
\]
\[
\chi \in (-\infty, \infty), \quad \nu_n \in (-\infty, \infty)
\] (B8)

At the positive values of \( \chi \) the solution (B8) describes the vertical modes of oscillations of the particles in the stratified medium at lack of the flows with the values of the wave number
square $\chi$ and the square of the frequency $\nu_n(\chi)$. At the negative $\chi$ as it will be shown below, (B8) describes the oscillations of the liquid in areas of the internal jumps. To distinguish the boundary problem (B8) from the normally used (in which $\chi$ is always $\chi > 0$), below we shall call (B8) as the modified boundary problem.

Let's introduce the following notation $\Lambda_n(\chi) = -\chi/\nu_n(\chi)$. At that (B8) shall to take the form of Sturm-Liouville boundary problem creating for any substantial $\chi$ the full set of the eigenfunctions $\psi_n(\chi, z)$ being inter-orthogonal in the space with the scalar product $(a, b)_{N^2} = \int_0^H N^2(z) a b^* dz$. The spectrum of the eigennumbers $\Lambda_n(\chi)$, $n = 1, 2, K$ is substantial, discrete and contains no more than finite number of the negative values. The number $n$ of the eigenfunction $\psi_n(\chi, z)$ is determined by the number of its zero. To show it for any $\chi$ it is suitable to take advantage of the method of the phase functions. Let's represent the solution of the problem (B8) in the form of

$$
\psi_n(\chi, z) = A_n(\chi, z) \left[ \sin\left(\chi^{1/2} z\right) \cos(B_n(\chi, z)) + \cosh\left(\chi^{1/2} z\right) \sinh(B_n(\chi, z)) \right]
$$

(B9)

By analogy to the method of variation of parameters we shall superimpose on the function $A_n, B_n$ the following requirement

$$
\frac{d}{dz} \psi_n = A_n \left[ \cos B_n \frac{d}{dz} \sinh(\chi^{1/2} z) + \sin B_n \frac{d}{dz} \cosh(\chi^{1/2} z) \right]
$$

(B10)

Substitution of (B9) in (B8) with the allowance for (B10) gives the equations for the functions $A_n, B_n$

$$
A_n(\chi, z) = A_n(\chi, 0) \exp\left[ -\frac{\chi^{1/2}}{\nu_n(\chi)} \right] \times
\int_0^z N^2(z) \cos B_n \sinh(\chi^{1/2} z) + \sin B_n \cosh(\chi^{1/2} z) \times \left[ \cos B_n \cosh(\chi^{1/2} z) - \sin B_n \sinh(\chi^{1/2} z) \right]
$$

(B11)

$$
\frac{dB_n}{dz} = \chi^{1/2} \frac{N^2(z)}{\nu_n(\chi)} \left[ \cos B_n \sinh(\chi^{1/2} z) + \sin B_n \cosh(\chi^{1/2} z) \right]^2
$$

(B12)

Now we shall write (B9) in the form of

$$
\psi_n = A_n \sinh(\chi^{1/2} z) \sin \left[ \text{mod}_{2\pi}(B_n) + \arctg \text{th}(\chi^{1/2} z) \right]
$$

(B13)

In view of the monotonicity $B_n$ (the sequent of (B12)) and the boundary conditions we come to the conclusion, that at $\chi > 0$ the number $n$ is determined by the quantity of zeros of the function $\psi_n$. At $\chi < 0$ it is enough in (B9) - (B13) to substitute $B_n$ with $iB_n$ and $\psi_n$ with $i\psi_n$. Then similarly to (B9) - (B13) we shall have

$$
\psi_n = A_n \sinh(\chi^{1/2} z) \cos(\chi^{1/2} z)
$$

(B14)

$$
\frac{dB_n}{dz} = -\chi^{1/2} \frac{N^2(z)}{\nu_n(\chi)} \left[ \sinh(\chi^{1/2} z) \cosh(\chi^{1/2} z) - \sin(\chi^{1/2} z) \right]^2
$$

(B15)

$$
\psi_n = A_n \sinh(\chi^{1/2} z) \sin \left[ \text{mod}_{2\pi}(\chi^{1/2} z) + \arctg B_n \right]
$$

(B16)
From (B15), (B16) follows, that at \( \chi < 0 \) the number \( n \) is determined by the quantity of zeros of the function \( \psi_n \).

The behavior of \( \psi_n(\chi, z) \) at variation of \( z \) (the distribution of zeros of the function \( \psi_n \) on the axis \( z \)) is determined by the sign of the value \( \chi \left[ \frac{N^2(z)}{\nu_n(\chi)} - 1 \right] \). At \( \chi \geq 0 \), \( \nu_n \geq 0 \) and \( \chi \leq 0 \), \( \chi \leq 0 \) zeros of the function \( \psi_n \) are concentrated in the area of the axis \( z \) with the maximum value of the Brunt-Väisälä frequency \( N(z) \). At \( \chi < 0 \), \( \nu_n > 0 \) zeros of the function \( \psi_n \) are concentrated in the area of with the minimum value of \( N(z) \). At \( \chi > 0 \), \( \nu_n < 0 \) there is no oscillating solutions.

Let’ consider the dependence of \( \nu_n(\chi) \) from \( \chi \). From (B8) follow the ratios

\[
\chi \int_0^H dz \left[ \frac{N^2(z)}{\nu_n(\chi)} - 1 \right] |\psi_n(z, \chi)|^2 = \int_0^H dz \left| \frac{d\nu_n(\chi, z)}{dz} \right|^2 \quad (B17)
\]

\[
\frac{d\nu_n(\chi)}{d\chi} = \frac{\nu_n^2(\chi)}{\chi} \frac{\int_0^H dz \left[ \frac{N^2(z)}{\nu_n(\chi)} - 1 \right] |\psi_n(\chi, z)|^2}{\int_0^H dz N^2(z)|\psi_n(z, \chi)|^2} \quad (B18)
\]

From which follows the monotonicity \( \nu_n(\chi) \), or the non-negativity \( \frac{d\nu_n(\chi)}{d\chi} \) for any \( \chi \). In view of correspondence of the quantity of zeros of the function \( \psi_n \) to its number \( n \) we using (B8) determined five special points of the dispersion curves \( \nu_n(\chi) \)

1) \( \nu_n(\chi) \rightarrow \max N^2(z) \)
   \( \chi \rightarrow +\infty \)
2) \( \nu_n(0) = 0 \)
3) \( \nu_n(\chi) \rightarrow -\infty \)
   \( \chi \rightarrow -\left( \frac{\pi n}{H} \right)^2 + 0 \) \quad (B19)
4) \( \nu_n(\chi) \rightarrow +\infty \)
   \( \chi \rightarrow -\left( \frac{\pi n}{H} \right)^2 - 0 \)
5) \( \nu_n(\chi) \rightarrow \min N(z) \)
   \( \chi \rightarrow -\infty \)

In particular, at \( N^2(z) = \frac{\chi N^2}{\chi + \left( \frac{\pi n}{H} \right)^2} \) \quad (B20)
Comparing (B20) and (B17) we come to the conclusion, that for particular number \( n \) it is possible to introduce the effective thickness of the stratum of the stratified liquid, in which the internal wave is propagating, and effective Brunt-Väisälä frequency of this stratum

\[
\bar{H}_n(\chi) = \left[ \frac{H}{\int_0^H \frac{1}{\pi n} \frac{d\psi_n}{dz} \, dz} \right]^{1/2}
\]  
(B21)

\[
\bar{N}_n(\chi) = \left[ \frac{H}{\int_0^H N^2(z) \psi_n^2 \, dz} \right]^{1/2}
\]  
(B22)

At that

\[
\min_{z} N(z) \leq \bar{N}_n(\chi) \leq \max_{z} N(z)
\]

\[
\bar{H}_n\left(\frac{\pi n}{H}\right) = H
\]

After that \( \nu_n(\chi) \) will look like

\[
\nu_n(\chi) = \frac{\chi \bar{N}_n^2(\chi)}{\chi + \left(\frac{\pi n}{\bar{H}_n(\chi)}\right)^2}
\]  
(B23)

And for \( \frac{d\nu_n(\chi)}{d\chi} \) we shall have

\[
\frac{d\nu_n(\chi)}{d\chi} = \frac{\nu_n(\chi)}{\chi} \left( 1 - \frac{\nu_n(\chi)}{\bar{N}_n^2(\chi)} \right) = \frac{\left(\frac{\pi n}{\bar{H}_n(\chi)}\right)^2 \bar{N}_n^2(\chi)}{\chi + \left(\frac{\pi n}{\bar{H}_n(\chi)}\right)^2}
\]  
(B24)
The corresponding (B19)-(B24) graphs of the function \( \nu_n(\chi) \) have the typical appearance shown on the Fig.B1

\[
\Gamma = \sum_{n=1}^{\infty} \Gamma_n \quad (B26)
\]

\[
\Gamma_n = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} d\mu \int_{-\infty}^{\infty} d\nu e^{i(\lambda + \mu - \nu)z} k^2 \frac{\nu_n(k^2)}{[\omega + i\epsilon - f]^2 - \nu_n(k^2)} \psi_n(k^2, z) \psi_n^*(k^2, z) \int_{0}^{H} dz N^2(z) \left| \psi_n(k^2, z) \right|^2
\]

Then representing the solution of the boundary problem (B5) in the form of the set \( \psi_n(\chi, z) \) for \( \chi = k^2 \) expansion

\[
G = \sum_{n=1}^{\infty} p_n \psi_n(k^2, z) \quad (B25)
\]

and solving \( p_n \) with the help of convolution (5) with \( \psi_n(k^2, z) \) we come to the following expression for Green function

\[
\Gamma = \sum_{n=1}^{\infty} \Gamma_n \quad (B26)
\]

\[
\Gamma_n = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} d\mu \int_{-\infty}^{\infty} d\nu e^{i(\lambda + \mu - \nu)z} k^2 \frac{\nu_n(k^2)}{[\omega + i\epsilon - f]^2 - \nu_n(k^2)} \psi_n(k^2, z) \psi_n^*(k^2, z) \int_{0}^{H} dz N^2(z) \left| \psi_n(k^2, z) \right|^2
\]

\[
k^2 = \lambda^2 + \mu^2
\]

Considering, that the axis \( x \) is always directed against the current velocity \( f = -\lambda V \), we gain the equation for the poles of the integrand (B27)

\[
\nu_n = (\omega + i\epsilon - f)^2 \quad (B28)
\]

Or in view of the formula (B23)

\[
N_n^2(k^2)(\lambda^2 + \mu^2) = (\omega + i\epsilon + \lambda V)^2 \left[ \lambda^2 + \mu^2 + \frac{\pi n}{H_n(k^2)} \right]^2 \quad (B29)
\]

The integration is most simply fulfilled with respect to \( \omega \), for what close the contour of integration in the negative half-plane of the variable \( \omega \) (complex) and gain after application of the residue theorem (for \( t > 0 \), which allows to take one of the integrals in (B26) - (B27).

\[
\Gamma_n = -\frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} d\lambda \int_{-\infty}^{\infty} d\nu e^{i(\lambda(x+tf) + \nu)z} \sin(t V_n(k^2)) \frac{V_n(k^2)}{k^2} \frac{\nu_n(k^2, z) \psi_n^*(k^2, z')}{\int_{0}^{H} dz N^2(z) \left| \psi_n(k^2, z) \right|^2} =
\]

\[
= -\frac{1}{2\pi} \int_{-\infty}^{\infty} dk J_0(kR) \sin(t \sqrt{V_n(k^2)}) \frac{V_n(k^2)}{k^2} \frac{\nu_n(k^2, z) \psi_n^*(k^2, z')}{\int_{0}^{H} dz N^2(z) \left| \psi_n(k^2, z) \right|^2}
\]

\[
= -\frac{1}{2\pi} \int_{-\infty}^{\infty} dk J_0(kR) \sin(t \sqrt{V_n(k^2)}) \frac{V_n(k^2)}{k^2} \frac{\nu_n(k^2, z) \psi_n^*(k^2, z')}{\int_{0}^{H} dz N^2(z) \left| \psi_n(k^2, z) \right|^2}
\]
\[ R \equiv \left[ (x + Vt)^2 + y^2 \right]^{\frac{1}{2}} \]

where \( J_0(kR) \) is Bessel function of the zero-order.

For \( t < 0 \) the contour of integration is closed in the upper half plane, that result in \( \Gamma_n = 0 \) according to the initial condition of (B2). In the coordinate system moving together with the flow \( (x \to x - Vt) \) the expression (B30) possesses the circular symmetry and coincides with the expression for Green function gained in [47-69]. It is evident, that if values of \( t, |x|, |y| \) are small, \( \Gamma_n \) may be gained only by the numerical integration (B30).

In the practical applications, for example, in the problem about generation of the internal gravity waves by the pulsing sources, often it is suitable to use the spectral density \( \tilde{\Gamma}_n \) determined by the formula

\[ \Gamma_n = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{\Gamma}_n d\omega \]

In this case in (B27) it is necessary to conduct \( \lambda \) or \( \mu \) integration.

The \( \lambda \) integration as applied to the problem about generation of the internal gravity waves by the source of the constant intensity \( (\omega = 0) \) was used in [47-69]. At that Green function was represented in the form of the eigenfunctions row of the general boundary problem (the modified problem (B8) coincides with the general problem at \( \chi > 0 \). As the result only the real poles \( \lambda = \pm \lambda_n (\mu^2) \) are taken into consideration, that nevertheless does not change the asymptotics of Green function at the great values of \( t, |x|, |y| \) determined only by the real poles.

It is easy to demonstrate, that usage of the modified boundary problem determines the presence besides the couple of the real poles also a couple of clearly imaginary poles \( \lambda_n (\mu^2) \). Really, from (B29) follows (for the poles not shifted because of the component \( i\varepsilon \) presence)

\[ \lambda_n^2 (\mu^2) = \frac{1}{2} \left[ \frac{\tilde{N}_n (k^2)}{V^2} - \mu^2 - \left( \frac{\pi n}{\tilde{H}(k^2)} \right)^2 + \frac{\tilde{N}_n (k^2)}{V^2} - \mu^2 - \left( \frac{\pi n}{\tilde{H}_n (k^2)} \right)^2 + 4\mu^2 \frac{\tilde{N}_n (k^2)}{V^2} \right] \]

As the values \( \tilde{N}_n (k^2) \) and \( \tilde{H}_n (k^2) \) by definition are real, then from (B31) follows the reality of \( \lambda_n^2 (\mu^2) \). We shall note, that if \( N(z) \approx \text{const} \), then \( \tilde{N}_k \) and \( \tilde{H}_n \) practically do not depend on \( k \) and (B31) directly determines the poles \( \lambda_n (\mu^2) \). The shift of the real poles in the complex plane caused by the component \( i\varepsilon \) is determined from expansion (B28) with respect to \( \varepsilon \)

\[ \lambda_n (\mu^2)_e = \lambda_n (\mu^2)_{e=0} + i\varepsilon \frac{\partial \lambda_n (\mu^2)}{\partial \varepsilon} + \Lambda \]

where
\[
\frac{\partial \varphi_n(\mu^2)}{\partial \varepsilon} = \frac{V}{\frac{\partial v_n(k^2)}{\partial k^2}} - V^2
\]  

(B32)

The spectral density \( \bar{\Gamma}_n \) at \( \varepsilon = 0 \) is represented in the form of

\[
\bar{\Gamma}_n = \frac{1}{(2\pi)^2} \int d\mu \int_{-\infty}^{\infty} d\lambda e^{i(\lambda \varepsilon + \mu |\lambda|)} \frac{v_n(k^2)}{k^2[(\lambda V + i\varepsilon)^2 - v_n(k^2)]} \psi_n(k^2, z) \psi_n^*(k^2, z) \int_0^\infty dz N^2(z) |\psi_n(k^2, z)|^2 + K^* \quad (B33)
\]

Here and below \( K^* \) is the corresponding complex conjugate item. Later, considering the function \( v_n(k^2) \) and the function

\[
b_n(k^2) = \frac{\psi_n(k^2, z) \psi_n^*(k^2, z)}{\int_0^\infty dz N^2(z) |\psi_n(k^2, z)|^2}
\]

analytically prolonged into the area of the complex expression \( \lambda (k^2 = \lambda^2 + \mu^2) \), we shall calculate the integral with respect to the variable \( \lambda \) by means of the residue theorem. As a result, at the values of \( x > 0 \) we shall gain

\[
\bar{\Gamma}_n = \frac{1}{2\pi} \int_{0}^{\infty} d\mu e^{i\mu |x|} \frac{\sin(\lambda_n(\mu^2)x)}{\lambda_n(\mu^2)} \frac{v_n(k^2)b_n(k^2)}{k^2[V^2 - \partial v_n(k^2)]} + \frac{1}{2\pi} \int_{0}^{\infty} d\mu e^{i\mu |x|} \frac{v_n(k^2)b_n(k^2)}{2\lambda_n(\mu^2)} \frac{v_n(k^2)b_n(k^2)}{k^2[V^2 - \partial v_n(k^2)]} + K^* \quad (B34)
\]

at the value of \( x < 0 \)

\[
\bar{\Gamma}_n = \frac{1}{2\pi} \int_{0}^{\infty} d\mu e^{i\mu |x|} \frac{\sin(\lambda_n(\mu^2)x)}{\lambda_n(\mu^2)} \frac{v_n(k^2)b_n(k^2)}{k^2[V^2 - \partial v_n(k^2)]} + \frac{1}{2\pi} \int_{0}^{\infty} d\mu e^{i\mu |x|} \frac{v_n(k^2)b_n(k^2)}{2\lambda_n(\mu^2)} \frac{v_n(k^2)b_n(k^2)}{k^2[V^2 - \partial v_n(k^2)]} + K^* \quad (B35)
\]

where \( k^2 = \mu^2 + \lambda_n^2(\mu^2) \) and \( v_n(k^2) = V^2\lambda_n^2(\mu^2) \).

From \( B34 \), \( B35 \) follows, that the integral with the positive values of \( \lambda_n^2(\mu^2) \) determines the wave component of the source generated field

Allowing, that at \( \lambda_n^2(\mu^2) \geq 0 \) and \( k^2 = \mu^2 + \lambda_n^2(\mu^2) \geq 0 \) the values \( v_n(k^2) \) and \( \frac{\partial v_n(k^2)}{\partial k^2} \) are limited.

\[
v_n(k^2) < \text{max } N^2(z)
\]
\[
\frac{\partial v_n(k^2)}{\partial k^2} \leq \frac{\partial v_n(k^2)}{\partial k^2} \bigg|_{k^2=0} = \frac{N_n^2(0)\tilde{H}_n^2(0)}{\pi^2n^2}
\]

\[
\lim_{\mu \to \infty} \frac{\partial v_n(k^2)}{\partial k^2} = 0
\]

We gain, that the wave field always exists in the field of \( x < 0 \). In the field of \( x > 0 \) there is only the final quantity of the modes of the internal waves, which numbers \( n \) do not exceed the value \( \frac{N_n(0)\tilde{H}_n(0)}{\pi V} \).

Later, for ease we shall consider the requirement \( \frac{N_n(0)\tilde{H}_n(0)}{\pi V} < 1 \) as always fulfilled. The asymptotics of the wave component \( \Gamma_n(\omega = 0) \) at the great values of \( |x|, |y| \) is determined by the standard methods and corresponds to the results gained in [47-69]. The integral with the negative \( \lambda_n^2(\mu^2) \) in (B34), (B35) because of the presence of \( e^{-|\lambda_n(\mu^2)||k|} \) describes the fast fading at the increase of \( |x| \) component \( \Gamma_n \). If the wave field disappears at absence of the stratification - \( N(z) \equiv 0 \) (according to \{ B31 \} \( \lambda_n^2(\mu^2) \) is the non-negative component only in the point \( \mu = 0 \)), then as it will be shown later, the fast fading term of (B34), (B35) is different from zero, and due to that, describes the effects of the liquid expulsion by the source. We shall also note, that unlike the wave term, the integrand of the second integral in (B34), (B35) has the singularities in some areas of integration. Below, in the more general event of \( \omega \neq 0 \) it will be shown, that these singularities are removed at the calculation with the help of \( \Gamma_n \) for the field of the internal waves from the lengthened along the y axis source – the well known problem of the energy infinity of the internal gravity waves generated by the point source.

If \( \omega \neq 0 \), then the poles \( \lambda_n(\omega,\mu^2) \) should be determined from the equation (B29), which already is impossible to represent in the form of (B31) (the equation (B31) is gained by the solution of (B29), in which the values \( \tilde{N}_n(k^2) \) and \( \tilde{H}_n(k^2) \) formally are considered as not dependent on \( k^2 \). If by analogy with the conclusion from (B31) to consider \( \tilde{N}_n(k^2) \) and \( \tilde{H}_n(k^2) \) formally nondependent on \( k^2 \), then it is possible to gain the equations \( \lambda_n(\omega,\mu^2) \) in the form of the solution of the equation of the fourth order (B26) with the help of Descartes-Euler method or Ferrari method allowing by virtue of the reality of \( \tilde{N}_n(k^2) \) and \( \tilde{H}_n(k^2) \) to discuss the behavior of the real and imaginary parts of the poles \( \lambda_n(\omega,\mu^2) \). It is apparent, that at \( \omega \neq 0 \) the poles become complex, however because of the bulkiness of the gained expressions the analytical analysis of \( \Gamma_n \) behavior is hampered.

The more acceptable expression of \( \Gamma_n \) from the point of view of the analysis and calculation at any values of \( x, y \) allows to gain the integration in (B27) with respect to the variable \( \mu \). Before this it is suitable to make the substitution of the variables \( \lambda \rightarrow \lambda - \omega/V \) and to represent \( \Gamma_n \) in the form of
\[
\tilde{\Gamma}_n = \frac{1}{(2\pi)^2} \int_0^\infty d\lambda \int d\mu e^{i(\lambda + \mu)\frac{z}{V}} \psi_n(k^2) \frac{\psi_n(k^2, z')}{k^2 \left( (\lambda V + i \epsilon)^2 - \nu_n(k^2) \right)} + K^*
\]

(B36)

\[
k^2 = \left( \lambda - \frac{\omega}{V} \right)^2 + \mu^2
\]

The poles \( \mu_n(\lambda, \omega) \) of the integrand (B36) satisfy the equation

\[
\nu_n(k^2) = (\lambda V + i \epsilon)^2
\]

(B37)

Or with allowance for (B23)

\[
\mu_n^2(\lambda, \omega) = -\left( \lambda - \frac{\omega}{V} \right)^2 + \left( \frac{\pi n}{H_n(k^2)} \right) \left[ \frac{N_n(k^2)}{N_n(k^2) - (\lambda V + i \epsilon)^2} - 1 \right]
\]

(B38)

\[
k^2 = \left( \lambda - \frac{\omega}{V} \right)^2 + \mu_n^2(\lambda, \omega)
\]

From (B38) the follows the reality of squares of the non-shifted \((\epsilon = 0)\) poles \( \mu_n^2(\lambda, \omega) \) for any values of \( \lambda \) follows and \( \omega \).

Owing to limitlessness of the function \( \nu_n \) (one of the differences of the modified boundary problem from the normal problem) the equation (B37) is always solvable for \( \mu_n^2(\lambda, \omega) \). At that in compliance with the behavior of \( \nu_n \) (Fig. B1) the following situations can take place:

a) \( \lambda < -\frac{z}{V} \) - there is one positive value of the parameter \( \nu_n \) in (B37) and as the result of it - one pair of the poles \( \mu_n(\lambda, \omega) \),

\[
\min N(z) \quad \max N(z)
\]

b) \( \frac{z}{V} < \lambda < \frac{z}{V} \) - there is one positive and may be one negative value of the parameter \( \nu_n \) in (B37) and accordingly two pairs (pure real and pure imaginary) poles \( \mu_n(\lambda, \omega) \) (and both pairs may be pure imaginary \( \mu_n \)),

\[
\min N(z) \quad \max N(z)
\]

c) \( \lambda < \frac{z}{V} \) - there is one negative value of the parameter \( \nu_n \) in (B37), that is one pair of the pure imaginary poles \( \mu_n(\lambda, \omega) \).

We should note, that if Brunt-Väisälä frequency \( N(z) \) weakly depends on \( z \), then the values of \( \tilde{N}_n(k^2) \) and \( H_n(k^2) \) as the first approximation do not depend on \( k^2 \), and (B38) directly determines \( \mu_n(\lambda, \omega) \).

The Fig. B2, B3, B4 show the graphs of \( \mu_n^2(\lambda, \omega) \) at \( \omega = 0 \), \( \omega > 0 \) and \( \omega < 0 \). Digits I, II, III mark the sections of the curves \( \mu_n^2(\lambda, \omega) \) of the definite sign of \( \mu_n^2 \). At that any \( \lambda \) and \( \omega \) meets

\[
\left| \mu_n^{II}(\lambda, \omega) \right| < \frac{\omega}{V} ; \quad \left| \mu_n^{III}(\lambda, \omega) \right| > \frac{\pi n}{H_n(\infty)}
\]

(B39)
Considering the functions $\psi_n(k^2)$ and $\tilde{b}_n(k^2) = \frac{\psi_n(k^2, z)\psi_n^*(k^2, z')}{\int_0^{|\psi_n(k^2, z)|^2}}$

as analytically prolonged in the area of the complex $\mu$ we shall take the integral with respect to $\mu$ in (B36) by means of the residue theorem. Allowing for the shift of the poles from the real axis, we close the contour of integration in the upper half plane. Using the formulas (B23), (B24), and also (B37), (B38) we shall gain for $\tilde{\Gamma}_n$

$$\tilde{\Gamma}_n = (\tilde{\Gamma}_n^I + \tilde{\Gamma}_n^{II} + \tilde{\Gamma}_n^{III}) e^{-i\sigma(t+\frac{x}{\sqrt{v}})}$$  \hspace{1cm} (B40)

where

I) $\mu_n^2 = (\mu_n^I)^2 \geq 0$

$$\tilde{\Gamma}_n^I = \frac{1}{4\pi} \int_0^{\max N(z)/V} d\lambda e^{i\Delta x} e^{-i\mu_n(\lambda, \omega)\|\beta\|+\frac{\pi}{2}} \tilde{b}_n(k^2) \left[ \mu_n^I(\lambda, \omega)\left|\lambda^2 V^2 - \tilde{N}_n^2(k^2)\right| + K^* \right] \hspace{1cm} (B41)$$

II) $\mu_n^2 = (\mu_n^I)^2 \geq 0$

$$\tilde{\Gamma}_n^{II} = \frac{1}{4\pi} \int_{\min N(z)/V}^{\infty} d\lambda e^{i\Delta x} e^{-i\mu_n(\lambda, \omega)\|\beta\|} \tilde{b}_n(k^2) \left[ \mu_n^{II}(\lambda, \omega)\left|\lambda^2 V^2 - \tilde{N}_n^2(k^2)\right| + K^* \right] \hspace{1cm} (B42)$$

III) $\mu_n^2 = (\mu_n^I)^2 \geq 0$

$$\tilde{\Gamma}_n^{III} = \frac{1}{4\pi} \int_0^{\max N(z)/V} d\lambda e^{i\Delta x} e^{-i\mu_n(\lambda, \omega)\|\beta\|} \tilde{b}_n(k^2) \left[ \mu_n^{III}(\lambda, \omega)\left|\lambda^2 V^2 - \tilde{N}_n^2(k^2)\right| + K^* \right] \hspace{1cm} (B43)$$

Here $k^2 = \lambda^2 + \mu_n^2(\lambda, \omega)$

The integrands in (B41) - (B43) have a singularity in the points of $\lambda$, where $|\mu_n(\lambda, \omega)| = 0$

also the essential singularity in the points $\lambda^2 V^2 = \tilde{N}_n(k^2)$. If the source $Q$ in (B3) is punctual, then (B40) describes the field of the waves formed by it and the essential singularity in integrals remains. If $Q$ is the source lengthened along axis $y$, then at the calculation (B3) may be recorded as

$$\int_{-\infty}^{\infty} dy' \Gamma_n(y - y')Q(y') = \sum_{\lambda} Q(y_{\lambda}) \Gamma_n(y - y_{\lambda}, \Delta y_{\lambda})$$

where $\Gamma_n(y, \Delta y_{\lambda}) = \frac{1}{2\Delta y_{\lambda}} \int_{y_{\lambda}-\Delta y_{\lambda}}^{y_{\lambda}+\Delta y_{\lambda}} dy' \Gamma_n(y)$  \hspace{1cm} (B44)

$$2\Delta y_{\lambda} = y_{\lambda+1} - y_{\lambda}$$  \hspace{1cm} (B45)
Expression for the "averaged" function $\Gamma_n$ differs by introduction in the integrands of the normalizing multipliers of the following type

$$\frac{\sin(\mu_n(\lambda, \omega)\Delta y)}{\mu_n(\lambda, \omega)}$$

(B46)

As a result in the denominator of (B41) - (B43) there is the product

$$\frac{\sin(\mu_n(\lambda, \omega)\Delta y)}{\mu_n(\lambda, \omega)}$$

of (B46), that with allowance for (B38) removes the essential singularity.

In the case of the source non-lengthened along axis $y$ such regularization is not possible. Another singularity arising at $|\mu_n(\lambda, \omega)| \to 0$ is removed with the help of the requirement of generation, according to which Green function should to convert into zero at $|y| \to \infty$. The contribution of this singularity to the integral apparently does not depend on $y$ and for its removal it is enough to subtract from the Internal its value in any other point. As for the calculation of $\Gamma_n$ it should be $\Gamma_n(|y| \to \infty) = 0$, then it is possible to use the step by step calculation (beginning from $y >> 1$)

$$\Gamma_n(|y| - \Delta y) = \Gamma_n(|y| + \Delta y) + \bar{\tilde{\Gamma}}_n(y, \Delta y)$$

(B47)

The value of $\bar{\tilde{\Gamma}}_n$ is determined by the ratio of (B47) identically and the formula for it differs from $\bar{\tilde{\Gamma}}_n$ by introduction in the integrand of the multiplier $-2i\sin(\mu_n|\Delta y)$ removing the singularity at $\mu_n = 0$

The described procedure of the regularization also improves convergence of the partial sums at calculation of Green function as the sum with respect to $n$ (because at $n \to \infty$ the value of $\mu_n$ is proportional to $n$). The regularization of expressions (B34), (B35) for $\bar{\tilde{\Gamma}}_n$ is similarly conducted at $\omega = 0$. For this purpose it is necessary to transfer from the variable $\mu$ in the integral to other variable $T$ determined for the corresponding sections of the monotonicity of the curve $|\lambda_n(\mu^2)|$ by the formula $\mu = \mu_n(T^2)$ from which we gain

$$\lambda_n^2(\mu_n^2(S^2)) = T^2$$

(B48)

$$d\mu \left[ \frac{\nu_n(k)}{k^2} \right] = dT \left[ \frac{\tilde{N}_n^2(k)}{\mu_n(T^2)} \right]$$

(B49)

$$\frac{d|\lambda_n(\mu^2)|}{d\mu} = \left[ \frac{\mu \frac{\partial \nu_n(k^2)}{\partial k^2}}{\lambda_n(\mu^2) \left[ V^2 - \frac{\partial \nu_n(k^2)}{\partial k^2} \right]} \right]$$

(B50)
That reduces singularities of the integrands (B34), (B35) to the singularities of the integrands (B41) - (B43). The monotonicity \( |\lambda_n(\mu^2)| \) on the sections of the integration of (B34), (B35) is proved by differentiation of (B28) with respect to \( \mu \)

\[
\frac{d|\lambda_n(\mu^2)|}{d\mu} = \frac{\mu \frac{\partial \nu_n(k^2)}{\partial k^2}}{|\lambda_n(\mu^2)| \left[ \nu^2 - \frac{\partial \nu_n(k^2)}{\partial k^2} \right]}
\]  

(B50)

Whence in view of the formulas (B23), (B24) we have the signdefiniteness of \( d\lambda_n / d\mu \) on the given intervals. Hereinafter the presence of the indicated regularization everywhere is meant.

If the stratification of the liquid is absent ( \( N(z) \equiv 0 \) ), then according to (B41) - (B43) the terms of \( \Gamma^I_n \) and \( \Gamma^III_n \) are fading, and \( \Gamma^II_n \) becomes

\[
\tilde{\Gamma}^II_n = \frac{1}{4\pi} \int_0^\infty d\lambda e^{-i\lambda z} e^{-\frac{1}{2\lambda} \left( \lambda - \omega - \frac{m}{\nu} \right)^2 + \frac{\partial}{\partial k} b_n(k^2)} \frac{\partial}{\partial k} + K^* 
\]  

(B51)

At the calculation of the field \( W \) of the vertical component of the velocity from the point source, as it is known, it is necessary to differentiate Green function with respect to \( z \) and twice to apply the operator \( \frac{D}{Dt} = \frac{\partial}{\partial t} - \nu \frac{\partial}{\partial x} \). As a result from the denominator of (B51) the value \( \lambda^2 V^2 \) will disappear, and with consideration of the regularizing procedure of (B47) the expression for the field \( W \) will look like

\[
W = \frac{1}{2\pi} \int_0^\infty d\lambda e^{i\lambda z} e^{-\frac{1}{2\lambda} \left( \lambda - \omega - \frac{m}{\nu} \right)^2 + \frac{\partial}{\partial k} b_n(k^2)} \frac{\partial}{\partial k} + K^* 
\]  

(B52)

For the greater obviousness and convenience of the analysis of the formulas (B41) - (B43) we shall conduct in integrals (B42), (B43) the substitution of the variable \( \lambda \) with the variable \( k \) by the formula

\[
\lambda = \lambda(k) = \sqrt{\frac{\nu_n(k^2)}{V}} 
\]  

(B53)

Realization of such a substitution follows from properties of the function \( \nu_n(\chi) \). At that we shall gain

\[
\mu_n^2(\lambda(k),\omega) = k^2 - \left( \omega - \sqrt{\nu_n(k^2)} \right)^2 \frac{1}{V^2} 
\]  

(B54)

And accordingly for \( \Gamma^I_n \) and \( \Gamma^III_n \) we shall gain
\[ \tilde{\Gamma}_n^I = -\frac{1}{4\pi} \int_0^\infty dk \ e^{i\sqrt{V_n(k^2)^2 - (\omega - \sqrt{V_n(k^2)})^2}} \frac{\sqrt{V_n(k^2)}}{k} \frac{\tilde{b}_n(k^2)}{\sqrt{k^2 V^2 - (\omega - \sqrt{V_n(k^2)})^2}} + K^* \]  

At \( k^2 > \frac{(\omega - \sqrt{V_n(k^2)})^2}{V^2} \), and

\[ \tilde{\Gamma}_n^III = -\frac{1}{4\pi} \int_0^\infty dk e^{i\sqrt{V_n(k^2)^2 - (\omega - \sqrt{V_n(k^2)})^2}} \frac{\sqrt{V_n(k^2)}}{k} \frac{\tilde{b}_n(k^2)}{\sqrt{(\omega - \sqrt{V_n(k^2)})^2 - k^2 V^2}} + K^* \]  

At \( k^2 > \frac{(\omega - \sqrt{V_n(k^2)})^2}{V^2} \)

We shall note, that \( \Gamma_n^I + \Gamma_n^III \) may be recorded as one integral with respect to \( k \) from 0 up to \( \infty \). By analogy with (B53) - (B56) we shall conduct in the integral (B41) substitution of the variable \( \lambda \) with the variable \( k \) by the formula

\[ \lambda = \tilde{\lambda}(k) = \frac{\sqrt{V_n(-k^2)}}{V} \]  

That also ensures variation \( \lambda \) in the necessary limits. At that we shall gain

\[ \mu^2_n(\tilde{\lambda}(k), \omega) = \left[ k^2 - \left( \omega - \sqrt{V_n(-k^2)} \right)^2 \frac{1}{V^2} \right] \]  

And accordingly for \( \tilde{\Gamma}_n^II \)

\[ \tilde{\Gamma}_n^II = \frac{1}{4\pi} \int_{\infty}^{\infty} \int_{\infty}^{\infty} \ e^{i\sqrt{V_n(-k^2)^2 - (\omega - \sqrt{V_n(-k^2)})^2}} \frac{\sqrt{V_n(-k^2)}}{k} \frac{\tilde{b}_n(-k^2)}{\sqrt{(\omega - \sqrt{V_n(-k^2)})^2 + k^2 V^2}} + K^* \]  

Formulas (B55), (B56), (B59) and (B40) evidently demonstrate dependence of \( \tilde{\Gamma}_n \) on coordinates \( t, x, y, z \) and characteristics of stratification \( v_n(\chi) \) and \( \psi_n(\chi, z) \).

Value \( \Gamma_n^I \) determines the wave component of Green function, which asymptotics is determined at the great values of \( |x|, |y| \) by the points of the stationary phase satisfying to the equation
\[
\begin{align*}
\frac{x}{V} \frac{\partial}{\partial k^2} \sqrt{v_n(k^2)} + \frac{y}{V} \frac{kV^2}{\partial k^2} + \left( -\sqrt{v_n(k^2)} + \omega \right) \frac{\partial}{\partial k^2} \sqrt{v_n(k^2)} = 0
\end{align*}
\] (B60)

The lines of the constant phase describing the wave fronts are set at that by the equation

\[
\sqrt{v_n(k^2)} \frac{x}{V} + \frac{y}{V} k^2 V^2 - \left( \omega - \sqrt{v_n(k^2)} \right)^2 - \omega \left( t + \frac{x}{V} \right) = \Phi = \text{const}
\] (B61)

In (B61) it is necessary to substitute the value of \( k \) gained from (B60), however the more convenient method of receiving the lines of the constant phase consists in the representation of these lines in the parametric form \( x = x(k), y = y(k) \). At that the functions \( x(k), y(k) \) are gained by the solution of the system of two equations (B60), (B61), where \( k \) it is considered as the parameter.

Direct calculation gives for the lines of the constant phase the pattern of the waves, close to the standard Kelvin wave wedge. The wave pattern is symmetrical with respect to the axis \( x \) and the pattern boundary can be determined from (B60)

\[
\max \left| \frac{y}{x} \right| = \max_k \left[ \frac{\sqrt{v_n(k^2)}}{\partial k^2} \right] \left( V^2 - \left( \frac{\sqrt{v_n(k^2)}}{k} - \frac{\omega}{V} \right)^2 \right) \left( \frac{\sqrt{v_n(k^2)}}{k} \right)
\] (B62)

If to consider, that \( \frac{\partial \sqrt{v_n(k^2)}}{\partial k} = C_n^e(k), \frac{\sqrt{v_n(k^2)}}{k} = C_n^e(k) \) is the group and phase velocities of the plane internal wave with a wave number \( k \) it is receivable (46) in the form of

\[
\max \left| \frac{y}{x} \right| = \max_k \left[ \frac{C_n^e(k)}{V^2 - C_n^e(k) \left( \frac{\omega}{k} \right)^2} \right] \left( V^2 - C_n^e(k) C_n^e(k) + \frac{\omega}{k} C_n^e(k) \right)
\] (B63)

In the system of coordinates connected with the source, the velocity of relocation of the wave zone \( S \) in the tangential direction is determined by the expression (\( \theta \) - the angle of the wave wedge opening)
\[ S(\omega) = V \sin \theta_{\text{max}} = \frac{V}{\sqrt{1 + \tan^2 \theta_{\text{max}}} = \frac{V}{\sqrt{1 + \frac{1}{(\text{max}(y/x))^2}}} = \]
\[ = \max_k \left\{ \frac{V}{\sqrt{1 + \frac{x^2}{y^2}}} \right\} = \max_k \left\{ \frac{C_n^f(k) \sqrt{V^2 - \left(\frac{C_n^f - \omega}{k}\right)^2}}{\sqrt{(C_n^g)^2 + V^2 + 2 \left(\frac{\omega}{k} - C_n^f(k)\right) C_n^g(k)}} \right\} \] (B64)

From (B64) follows, that at \( \omega = 0 \) the velocity of relocation of the wave-front is equal to
\[ S(0) = \frac{C_n^g(0) \sqrt{V^2 - \left(\frac{C_n^f(0)}{k}\right)^2}}{\sqrt{V^2 + C_n^g(0) (C_n^g(0) - C_n^f(0))}} = \frac{C_n^g(0) \sqrt{V^2 - \left(\frac{C_n^f(0)}{k}\right)^2}}{\sqrt{V^2 - \left(\frac{C_n^f(0)}{k}\right)^2 + (C_n^f(0) - C_n^g(0))^2 + C_n^g(0) C_n^f(0)}} \] (B65)

Or considering, that \( C_n^g(0) = C_n^f(0) \)
\[ S(0) = C_n^g(0) \sqrt{1 - \frac{(C_n^f(0))^2}{V^2}} \] (B66)

that is the velocity of relocation of the wave-front set does not exceed the maximum possible velocity of the internal gravity waves. If \( \omega \neq 0 \), then \( S \) apparently will be different for \( \omega = +|\omega_0| \) and \( \omega = -|\omega_0| \), the situation met at the construction of the field of the internal waves from oscillating sources, and, for example, for an event of the oscillating source it will lead to formation of two wave zones superimposed on each other, which boundaries do not coincide. At that we shall mark, that the wave in the last case is modulated according to \( \omega_0 \) frequency of oscillation.

The item \( \bar{\Gamma}_{n}^{\Pi} \) (B59) is quickly fading at the increase of \( |y| \). Its singularity is the behavior in the vertical direction as the function \( \psi_n(\chi, z) \) at \( \chi = -k^2 \) is localized in the stratum with the minimal values of Brunt-Väisälä frequency. Considering, that \( \bar{\Gamma}_{n}^{\Pi} \) at the absence of stratification \( (N(z) \equiv 0) \) does not transform in zero, it is possible to draw the conclusion, that this item determines the effects of the flow streamlining of the source (the internal jumps).

The item \( \bar{\Gamma}_{n}^{\III} \) (B56) is different from zero only in the case of \( \omega \neq 0 \) (because as it has been noted, we suppose \( V^2 > (C_n^f(k))^2 = \frac{C_n^f(k^2)}{k^2} \)) and thus determines the effects of the nonstationarity of the sources of the internal waves. The asymptotics \( \bar{\Gamma}_{n}^{\III} \) at the great values of \( |x|, |y| \) wanes less fast, than \( \bar{\Gamma}_{n}^{\II} \), that is stipulated by behavior of the index of the exponential curve in the integrand (B56) in the
complex plane of the variable $k$. In this case the asymptotics can be evaluated by the saddle point approximation.

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Fig. B1 Qualitative behavior of the dispersive curves $v_n(\chi)$ for the first two modes (the curve 1 - the first mode, 2 - the second mode,), $N_1$ - the minimal value of Brunt-Väisälä frequency $N(z)$, $N_2$ - the maximum value of $N(z)$, $m_n = -\frac{\pi^2 n^2}{H^2}$ $(n=1,2)$. 
Fig. B2 Qualitative behavior of the dispersive curves $\mu_n^2(\lambda, \omega)$ for the first three modes at $\omega > 0$ (the curve 1 - the first mode, 2 - the second mode, 3 - the third mode), the curve A – function $(-\lambda^2)$, the curve B – function $-(\lambda - \omega/V)^2$, $N_1$ - the minimal value of Brunt-Väisälä frequency $N(z)$, $N_2$ - the maximum value of $N(z)$, $M = \omega^2 V^2$, $M_n = -\left(\frac{m}{H(\infty)}\right)^2$. 
Fig. B3 Qualitative behavior of the dispersive curves $\mu_n^2(\lambda, \omega)$ for the first three modes at $\omega = 0$ (the curve 1 - the first mode, 2 - the second mode, 3 - the third mode), the curve A – function $(-\lambda^2)$, the curve B – function $-(\lambda - \omega/V)^2$, $N_1$ - the minimal value of Brunt-Väisälä frequency $N(z)$, $N_2$ - the maximum value of $N(z)$, $M = \omega V^2$, $M_n = -\left(\frac{m}{H(\infty)}\right)^2$. 
Fig. B4  Qualitative behavior of the dispersive curves \( \mu_n^2(\lambda, \omega) \) for the first three modes at \( \omega < 0 \) (the curve 1 - the first mode, 2 - the second mode, 3 - the third mode), the curve A – function \( -\lambda^2 \), the curve B – function \( -\left(\lambda - \omega / V\right)^2 \), \( N_1 \) - the minimal value of Brunt-Väisälä frequency \( N(z) \), \( N_2 \) - the maximum value of \( N(z) \), \( M = \omega^2 V^{-2} \), \( M_n = -\left(\frac{m}{H(x)}\right)^2 \).