The Capacity Region of the Cognitive Z-interference Channel with One Noiseless Component

Nan Liu*, Ivana Marić*, Andrea J. Goldsmith*, Shlomo Shamai (Shitz) †
*Dept. of Electrical Engineering, Stanford University, Stanford, CA 94305
†Department of Electrical Engineering, Technion, Technion City, Haifa 32000, Israel

Email: {nanliu@stanford.edu, ivanam@wsl.stanford.edu, andrea@wsl.stanford.edu, sshlomo@ee.technion.ac.il}

Abstract

We study the discrete memoryless Z-interference channel (ZIC) where the transmitter of the pair that suffers from interference is cognitive. We first provide upper and lower bounds on the capacity of this channel. We then show that, when the channel of the transmitter-receiver pair that does not face interference is noiseless, the two bounds coincide and therefore yield the capacity region. The obtained results imply that, unlike in the Gaussian cognitive ZIC, in the considered channel superposition encoding at the non-cognitive transmitter as well as Gel’fand-Pinsker encoding at the cognitive transmitter are needed in order to minimize the impact of interference. As a byproduct of the obtained capacity region, we obtain the capacity result for a generalized Gel’fand-Pinsker problem.

Index terms: Cognitive interference channel, capacity region, Z-interference channel, one-sided interference channel

The work of N. Liu, I. Marić and A. J. Goldsmith was supported in part from the DARPA ITMANET program under grant 1105741-1-TFIND, Stanford’s Clean Slate Design for the Internet Program and the ARO under MURI award W911NF-05-1-0246. The Work of S. Shamai was supported by the ISRC Consortium and by the European Commission in the framework of the FP7 Network of Excellence in Wireless COMMunications NEWCOM++.
I. INTRODUCTION

The interference channel (IC) [1] is a simple network consisting of two transmitter-receiver pairs. Each pair wishes to reliably communicate at a certain rate, however, the two communications interfere with each other. A key issue in such scenarios then, is how to handle the interference introduced by the simultaneous transmissions. This issue is not yet fully understood, and the problem of finding the capacity region of the IC remains open, except in special cases [2]–[12]. For a tutorial on the capacity results of the IC, see [13]. The Z-interference channel (ZIC) is an IC where one transmitter-receiver pair is interference-free. Although this is a simpler channel model than the IC, capacity results are still known only in special cases [6, Section IV], [14]–[16].

In certain communication scenarios, such as cognitive radio networks, some transmitters are cognitive, i.e., are able to sense the environment and thus obtain side information about transmissions in their vicinity. Perhaps due to the exciting promise of the cognitive radio technology to improve the bandwidth utilization and thus allow for new wireless services and a higher quality of service, the IC with one cognitive transmitter has been studied extensively [17]–[24]. Related channel models were also analyzed in [25], [26]. In the model considered in [17]–[25], it is assumed that due to the cognitive capabilities, the cognitive encoder noncausally obtains the full message of the non-cognitive transmitter. While this is a somewhat idealistic view of cognition in a wireless network, this model applies for example, to scenarios where the cognitive transmitter is a base station. Then, it can obtain side information via backhaul (high-capacity link such as an optical cable). This side information then enables interference reduction [27] by precoding at the cognitive encoder. Furthermore, it enables cooperation with the non-cognitive pair. In fact, one of the main difficulties in finding the capacity region of the traditional IC comes from distributed encoding. IC with one cognitive transmitter enables one-sided transmitter cooperation, and thus allows centralized encoding to some degree. This may be the reason why determining the capacity region of the cognitive IC is somewhat easier than the traditional IC. In particular, while the capacity region of the Gaussian IC in weak interference is not known (the sum capacity in certain weak interference regimes has recently been found in [28]–[30]), the capacity region of the cognitive Gaussian IC in weak interference has been determined [19], [20].

In this paper, we study a ZIC where the transmitter of the pair that suffers from interference is cognitive (see Fig. I). The capacity region of such a cognitive ZIC in the Gaussian case is straightforward to obtain, since by using dirty-paper coding [31] at the cognitive encoder, both communicating pairs can achieve the interference-free, single-user rates. However, limiting the study of the cognitive ZIC to the Gaussian case leaves some unsatisfaction to the understanding of the
problem. Firstly, it does not provide intuition as to how the interferer’s rate affects the rate of the
cognitive transmitter-receiver pair in a general channel. Secondly, it does not provide the insight into
the optimal codebook structure for the non-cognitive encoder, so that it minimizes interference caused
for the cognitive pair.

Hence, in this paper, we study a discrete memoryless cognitive ZIC. We first derive an upper bound
on the capacity region. The technique that we use in obtaining the converse was introduced by Korner
and Marton in [32], and was proven to be useful in the solution of several problems in multi-user
information theory [15], [16], [32], [33], including the Gel’fand-Pinsker problem [27]. We apply this
technique twice to obtain the upper bound on the capacity region. Next, we derive a lower bound on
the capacity region where the non-cognitive pair uses superposition encoding to control the amount
of interference it causes for the cognitive pair. Unlike in the IC, this encoding approach has not been
applied in the cognitive IC literature, with the exception of concurrent and independent work [23].
Finally, we show that the lower and upper bounds meet when the channel between the non-cognitive
pair is noiseless. We denote this channel model as the cognitive ZIC with one noiseless component.
From the capacity results, we conclude that it is optimal for the interference-causing (non-cognitive)
pair to use superposition encoding; the inner codeword is decoded by the receiver of the cognitive pair
while Gel’fand-Pinsker coding is performed against the outer codeword at the cognitive transmitter.

The capacity region of the discrete memoryless cognitive IC is known in some special cases [18],
[19], [23]. The tight result we derive in this paper does not fall into these special cases, as explained
in more details in Section V. Furthermore, the cognitive ZIC with one noiseless component is the first
channel model for which superposition encoding at the non-cognitive transmitter is not only required
but also optimal.

Note that, in general, the capacity region of the traditional ZIC in which the interference-free
transmitter-receiver pair is noiseless, is unknown. The most we know about this scenario is the sum
capacity [15]. Thus, the results in this paper provide yet another example where finding the capacity
the possibility of centralized (joint) encoding by the cognitive transmitter.

The considered problem is also intimately related to the Gel’fand-Pinsker (GP) problem [27] where a transmitter-receiver pair communicates in the presence of interference noncausally known at the encoder (see Fig. 2). By viewing the non-cognitive encoder in the cognitive ZIC as a source of this interference, we arrive to a generalized GP problem. Instead of the state being i.i.d. as in the GP problem, in the generalized GP model considered in this paper, the state is uniformly distributed on a set of size $2^{nR_2}$, where $R_2$ is a number between 0 and the logarithm of the cardinality of the state space. The further generalization is that, unlike in [27], in our model one can optimize the set, i.e., the structure of the interference. The solution of this paper shows that the optimal interference has a superposition structure.

II. SYSTEM MODEL

Consider a ZIC with two transition probabilities $p(y_1|x_1, x_2)$ and $p(y_2|x_2)$. The input and output alphabets are $X_1$, $X_2$, $Y_1$ and $Y_2$.

Let $W_1$ and $W_2$ be two independent messages uniformly distributed on $\{1, 2, \cdots, M_1\}$ and $\{1, 2, \cdots, M_2\}$, respectively. Transmitter $i$ wishes to send message $W_i$ to Receiver $i$, $i = 1, 2$. Transmitter 1 is cognitive in the sense that, in addition to knowing $W_1$, it knows the message $W_2$. An $(M_1, M_2, n, \epsilon_n)$ code for this channel consists of a sequence of two encoding functions

$$f_1^n : \{1, 2, \cdots, M_1\} \times \{1, 2, \cdots, M_2\} \to X_1^n,$$

$$f_2^n : \{1, 2, \cdots, M_2\} \to X_2^n,$$

and two decoding functions

$$g_i^n : Y_i^n \to \{1, 2, \cdots, M_i\}, \quad i = 1, 2$$

with probability of error

$$\epsilon_n = \max_{i=1,2} \frac{1}{M_1M_2} \sum_{w_1,w_2} \Pr [g_i^n(Y_i^n) \neq w_i | W_1 = w_1, W_2 = w_2].$$

A rate pair $(R_1, R_2)$ is said to be achievable if there exists a sequence of $(2^{nR_1}, 2^{nR_2}, n, \epsilon_n)$ codes such that $\epsilon_n \to 0$ as $n \to \infty$. The capacity region of the cognitive ZIC is the closure of the set of all achievable rate pairs.

A cognitive ZIC with one noiseless component is a cognitive ZIC where the channel between $X_2$ to $Y_2$ is noiseless, i.e., $p(y_2|x_2)$ is a deterministic one-to-one function.
Throughout the paper, we use the following shorthand for random vectors: $K^i \triangleq K_1, K_2, \ldots, K_i$ and $K^i_{i+1} \triangleq K_{i+1}, K_{i+2}, \ldots, K_n$.

### III. Converse

In this section, we provide an upper bound on the capacity region of the cognitive ZIC.

**Theorem 1:** Achievable rate pairs $(R_1, R_2)$ belong to a union of rate regions given by

$$R_1 \leq I(U; Y_1|V) - I(U; X_2|V)$$  \hspace{1cm} (5)

$$R_2 \leq I(X_2; Y_2|V) + \min \{I(V; Y_1), I(V; Y_2)\}$$  \hspace{1cm} (6)

where the union is over all probability distributions $p(v, u, x_2)p(x_1|u, x_2)$ and the mutual informations are calculated according to the distribution

$$p(v, u, x_1, x_2, y_1, y_2) = p(v, u, x_2)p(x_1|u, x_2)p(y_1|x_1, x_2)p(y_2|x_2).$$  \hspace{1cm} (7)

**Proof:** The proof is provided in Section VII-A.

This converse result is obtained by using the converse technique of Korner/Marton [34, page 314] two times, resulting in two auxiliary random variables.

### IV. Achievability

The achievability scheme uses a combination of superposition encoding at the non-cognitive encoder and GP encoding of the outer codeword of interference at the cognitive encoder. The performance is given in the following theorem.

**Theorem 2:** The union of rate regions given by

$$R_1 \leq I(U; Y_1|V) - I(U; X_2|V)$$  \hspace{1cm} (8)

$$R_2 \leq I(X_2; Y_2|V) + \min \{I(V; Y_1), I(V; Y_2)\}$$  \hspace{1cm} (9)

is achievable, where the union is over all probability distributions $p(v, u, x_2)p(x_1|u, x_2)$ and the mutual informations are calculated according to the distribution in (7).
Proof: The proof is provided in Section VII-B.

Remark: The proposed achievability scheme is a special case of the independent and concurrent work [23, Theorem 2] by setting $U_{10} = V$, $(U_{11}, U_{10}) = (V_{11}, U_{10}) = X_1$, $V_{20} = \phi$, $V_{22} = U$, $L_{20} = R_{20} = 0$, $L_{11} = R_{11}$ and then swapping the indices of 1 and 2 because in [23], the second transmitter-receiver pair is cognitive.

V. CAPACITY REGION OF THE COGNITIVE ZIC WITH ONE NOISLESS COMPONENT

In general, the achievability results in (8)-(9) and the converse results in (5)-(6) do not meet, due to the fact that

$$I(U; X_2|V) \geq I(U; Y_2|V)$$

(10)

because the random variables satisfy (7) which implies that Markov chain $U \rightarrow (V, X_2) \rightarrow Y_2$ holds.

However, in the case where the channel output $Y_2 = X_2$, the achievability results and the converse results meet, yielding the capacity region. More specifically, we have the following capacity results for the cognitive ZIC.

Theorem 3: For cognitive ZIC with one noiseless component, i.e., $p(y_2|x_2)$ is a deterministic one-to-one function, the capacity region is given by the union of rate regions:

$$R_1 \leq I(U; Y_1|V) - I(U; X_2|V)$$

(11)

$$R_2 \leq H(X_2|V) + \min \{I(V; Y_1), I(V; X_2)\}$$

(12)

where the union is over all probability distributions $p(v, u, x_2)p(x_1|u, x_2)$ and the mutual informations are calculated according to the distribution in (7).

Remark: Similar to the solution of the GP problem, one may restrict $p(x_1|u, x_2)$ to be a deterministic function, i.e., $p(x_1|u, x_2)$ only takes the values of 0 and 1, in the union in Theorem 3. To see this, observe that for a fixed $p(v, u, x_2)$, only (11) and the term $I(V; Y_1)$ in (12) depend on $p(x_1|u, x_2)$. The right-hand side of (11) can be written as

$$I(U; Y_1|V) - I(U; X_2|V) = \sum_v p(v) (I(U; Y_1|V = v) - I(U; X_2|V = v))$$

(13)

which is a linear combination of convex functions of $p(x_1|u, x_2)$ [27, Proposition 1 (ii)]. Thus, the right-hand side of (11) is a convex function of $p(x_1|u, x_2)$ and the maximum is achieved by a deterministic function. For the fixed $p(v)$, $I(V; Y_1)$ is convex in $p(y_1|v)$ [35, Theorem 2.7.4], which is a linear function of $p(x_1|u, x_2)$ for fixed $p(u, x_2|v)$ and $p(y_1|x_1, x_2)$. Thus, $I(V; Y_1)$ is a convex function of
\( p(x_1|u, x_2) \) and the maximum is achieved by a deterministic function. Hence, both (11) and \( I(V; Y_1) \) in (12) are maximized by a deterministic \( p(x_1|u, x_2) \).

We conclude from Theorem 3 that, in the special case of noiseless channel between the interference-free transmitter-receiver pair, to minimize the effect of interference caused to the cognitive transmitter-receiver pair, the non-cognitive pair uses superposition encoding, allowing the cognitive pair to decode the inner codeword. In contrast to the Han-Kobayashi scheme [9] for the traditional ZIC, where the outer codeword of the interferer is treated as noise, here, due to the cognitive capability of the transmitter that faces interference, GP encoding is performed on the outer codeword to further reduce the effect of interference.

The capacity region of the discrete memoryless cognitive IC is known in some special cases [18], [19], [23]. The cognitive ZIC with one noiseless component is not a special case of [18, Theorem 3] as it does not satisfy either of the two conditions of strong interference. It satisfies Assumption 3.1 but not Assumption 3.2 in [19], and therefore its capacity region is not characterized by [19, Theorem 3.4]. The capacity results in Theorem 3 is not a special case of [23, Theorem 5] as the received signal of the cognitive pair is not a deterministic function of the two channel inputs. Rather, in the cognitive ZIC with one noiseless component, the received signal of the non-cognitive pair is a deterministic function. Furthermore, it does not satisfy the mutual information inequality required in [23, Theorem 5].

VI. DISCUSSION

In the case where \( Y_2 = X_2 \), the cognitive ZIC problem can be seen as a form of generalized GP problem, where \( X_2 \) is the channel state that affects the communication between transmitter-receiver pair 1. This formulation generalizes the GP problem in the sense that, instead of the state (random parameters of the channel) being i.i.d., the state is uniformly distributed on a set of size \( 2^{nR_2} \). Furthermore, we are allowed the freedom, not only to design the codebook of the cognitive transmitter, but also the structure of the set where the states lie, in order to maximize the number of bits transmitted between the cognitive pair. We are then interested in the capacity of the cognitive transmitter-receiver pair, denoted as \( C(R_2) \), which is a function of \( R_2 \).

Using the capacity region for the cognitive ZIC with one noiseless component in Theorem 3, we see that the capacity of the cognitive pair when the state uniformly takes a value from a set of \( 2^{nR_2} \) sequences is

\[
C(R_2) = \max_{p(v,u,x_2),p(x_1|u,x_2)} I(U; Y_1|V) - I(U; X_2|V) 
\] (14)
where the maximum is over all distributions \( p(v, u, x_2), p(x_1|u, x_2) \) that satisfy

\[
H(X_2|V) + \min \{ I(V; Y_1), I(V; X_2) \} \geq R_2
\]  \hspace{1cm} (15)

Thus, in the generalized GP problem, when given the rate of the possible channel states \( R_2 \), the optimal interference has a superposition structure.

**Remark:** When \( R_2 = \log |X_2| \), \( C(R_2) \) reduces to the GP rate where the state is i.i.d. and uniformly distributed on set \( X_2 \). This can be seen as follows: first, by choosing \( V = \phi \) and \( p(x_2) \) to be the uniform distribution on \( X_2 \) in the maximization of (14), we obtain the GP rate. Hence, we conclude that \( C(\log |X_2|) \) is no smaller than the GP rate. On the other hand, when \( R_2 = \log |X_2| \), according to (15), the distribution we are allowed to maximize over in (14) has to satisfy

1) \( p(x_2) \) is the uniform distribution on \( X_2 \)

2) \( I(V; Y_1) \geq I(V; X_2) \)

which means

\[
C(\log |X_2|) = \max_{p(v, u|x_2)p(x_1|u, x_2): I(V; Y_1) \geq I(V; X_2)} I(U; Y_1|V) - I(U; X_2|V) \]  \hspace{1cm} (16)

\[
\leq \max_{p(v, u|x_2)p(x_1|u, x_2): I(V; Y_1) \geq I(V; X_2)} I(V; Y_1) + I(U; Y_1|V) - I(V; X_2) - I(U; X_2|V) \]  \hspace{1cm} (17)

\[
= \max_{p(v, u|x_2)p(x_1|u, x_2): I(V; Y_1) \geq I(V; X_2)} I(U, V; Y_1) - I(U, V; X_2) \]  \hspace{1cm} (18)

\[
\leq \max_{p(v, u|x_2)p(x_1|u, v, x_2)} I(U, V; Y_1) - I(U, V; X_2) \]  \hspace{1cm} (19)

where in (16)-(19), we have implicitly assumed that \( p(x_2) \) is the uniform distribution. By setting \( (U, V) = \bar{U} \) in (19), we see that (19) is the GP rate, which means that \( C(\log |X_2|) \) is no larger than the GP rate. Thus, we conclude that \( C(\log |X_2|) \) is equal to the GP rate where the state is i.i.d. and uniformly distributed on set \( X_2 \).

**VII. PROOFS**

**A. Proof of Theorem 7**

Following from Fano’s inequality [35], we have

\[
nR_1 \leq H(Y_1^n) - H(Y_1^n|W_1) + n\epsilon_n \]  \hspace{1cm} (20)
and
\[ nR_2 \leq H(Y_2^n) - H(Y_2^n|W_2) + n\epsilon_n \]  \hspace{1cm} (21)
\[ \leq H(Y_2^n) - H(Y_2^n|W_2, X_2^n) + n\epsilon_n \]  \hspace{1cm} (22)
\[ = H(Y_2^n) - H(Y_2^n|X_2^n) + n\epsilon_n \]  \hspace{1cm} (23)
\[ = H(Y_2^n) - \sum_{i=1}^{n} H(Y_2|X_{2i}) + n\epsilon_n \]  \hspace{1cm} (24)

where (23) follows from the Markov Chain \( W_2 \rightarrow X_2^n \rightarrow Y_2^n \), and (24) follows from the memoryless property of the channel \( p(y_2|x_2) \).

Applying the technique [34, page 314, eqn (3.34)] twice, we obtain
\[ H(Y_1^n) - H(Y_2^n) = \sum_{i=1}^{n} H(Y_1|Y_1^{i-1}, Y_2^n) - H(Y_2^n|Y_1^{i-1}, Y_2^n) \]  \hspace{1cm} (25)
\[ H(Y_1^n|W_1) - H(Y_2^n|W_1) = \sum_{i=1}^{n} H(Y_1|Y_1^{i-1}, Y_2^n, W_1) - H(Y_2^n|Y_1^{i-1}, Y_2^n, W_1). \]  \hspace{1cm} (26)

Define auxiliary random variables as
\[ V_i = Y_1^{i-1}, Y_2^n, \quad i = 1, 2, \ldots, n. \]  \hspace{1cm} (27)

Further define \( Q \) to be an auxiliary random variable that is independent of everything else and uniform on the set \( \{1, 2, \ldots, n\} \), and
\[ V = (V_Q, Q), \quad U = (V, W_1), \quad X_1 = X_1Q, \quad X_2 = X_2Q, \quad Y_1 = Y_1Q, \quad Y_2 = Y_2Q. \]  \hspace{1cm} (28)

It is straightforward to check that the random variables thus defined satisfy (7).

Following from (25) and (26), we have
\[ \frac{1}{n} (H(Y_1^n) - H(Y_2^n)) = H(Y_1|V) - H(Y_2|V) \]  \hspace{1cm} (29)
\[ \frac{1}{n} (H(Y_1^n|W_1) - H(Y_2^n|W_1)) = H(Y_1|U) - H(Y_2|U). \]  \hspace{1cm} (30)

Notice that (29) implies that there exists a number \( \gamma \) where
\[ \frac{1}{n} H(Y_1^n) = H(Y_1|V) + \gamma \]  \hspace{1cm} (31)
\[ \frac{1}{n} H(Y_2^n) = H(Y_2|V) + \gamma \]  \hspace{1cm} (32)
\[ 0 \leq \gamma \leq \min \{I(V;Y_1), I(V;Y_2)\} \]  \hspace{1cm} (33)
where (33) follows because \( H(Y_1^n) \leq nH(Y_1) \) and \( H(Y_2^n) \leq nH(Y_2) \) and
\[
H(Y_2^n) = \sum_{i=1}^{n} H(Y_{1i}|Y_{1i-1}^i) \geq \sum_{i=1}^{n} H(Y_{1i}|Y_{1i-1}^i, Y_{2i+1}^n) = nH(Y_1|V) \tag{34}
\]

Following from (24), we have
\[
R_2 = \frac{1}{n}H(Y_2^n) - \frac{1}{n} \sum_{i=1}^{n} H(Y_{2i}|X_{2i}) + \epsilon_n
= H(Y_2|V) + \gamma - H(Y_2|X_2, Q) + \epsilon_n \tag{35}
\]
\[
= H(Y_2|V) + \gamma - H(Y_2|X_2) + \epsilon_n \tag{36}
\]
\[
\leq H(Y_2|V) + \min \{ I(V; Y_1), I(V; Y_2) \} - H(Y_2|X_2) + \epsilon_n \tag{37}
\]
\[
= I(X_2; Y_2|V) + \min \{ I(V; Y_1), I(V; Y_2) \} + \epsilon_n \tag{38}
\]

where (35) follows from (32) and the definition of the random variables in (28); (36) follows by
the memoryless nature of the channel \( p(y_2|x_2) \); (37) follows from (35); and (38) follows because
the random variables satisfy (7) which implies that Markov chain \( V \rightarrow X_2 \rightarrow Y_2 \) holds.

Following from (20), we have
\[
R_1 \leq \frac{1}{n}H(Y_1^n) - \frac{1}{n}H(Y_1^n|W_1) + \epsilon_n
= \frac{1}{n}H(Y_2^n) + H(Y_1|V) - H(Y_2|V) - \frac{1}{n}H(Y_2^n|W_1) - H(Y_1|U) + H(Y_2|U) + \epsilon_n \tag{39}
\]
\[
= H(Y_1|V) - H(Y_2|V) - H(Y_1|U) + H(Y_2|U) + \epsilon_n \tag{40}
\]
\[
= I(U; Y_1|V) - I(U; Y_2|V) + \epsilon_n \tag{41}
\]

where (39) follows from (29) and (30); (40) follows from the fact that \( Y_2^n \) only depends on \( X_2^n \)
and the channel noise induced by \( p(y_2^n|x_2^n) \), and is therefore independent of \( W_1 \); and (41) follows because
the random variables satisfy (7) which implies that Markov chain \( V \rightarrow U \rightarrow (Y_1, Y_2) \) holds.

We obtain the desired upper bound on the capacity region from (38) and (41).

B. Proof of Theorem 2

Since the encoding/decoding procedure follows the standard steps, the detailed calculation of the
probability of error is omitted.

\textit{Codebook generation:}

Fix a distribution \( p(v, u, x_2)p(x_1|u, x_2) \).

The codebook at Transmitter 2 is generated as follows: generate \( 2^{n\gamma} \) sequences \( v^n \) in an i.i.d.
fashion using \( p(v) \). These \( v^n \) sequences constitute the inner codebook. For each \( v^n \), generate \( 2^{n(R_2 - \gamma)} \)
sequences $x^n_2$ in an i.i.d. fashion using $p(x_2|v)$. These $x^n_2$ constitute the outer codebook of Transmitter 2 associated with $v^n$.

The codebook at Transmitter 1 (the cognitive transmitter) uses the same inner codebook as Transmitter 2 and the outer codebook of Transmitter 1 is generated as follows: for each $v^n$ sequence, generate $2^{n(R_1+R_0)}$ sequences $u^n$ in an i.i.d. fashion using $p(u|v)$. These $u^n$ constitute the outer codebook of Transmitter 1 associated with $v^n$. Randomly distribute them into $2^{nR_1}$ many bins. Each bin will contain approximately $2^{nR_0}$ many $u^n$ sequences.

**Encoding:**

Transmitter 2 splits its message $W_2$ into two independent parts $W_{2a}$ and $W_{2b}$, with rates $\gamma$ and $R_2 - \gamma$, respectively. For $W_{2a} = w_{2a}$ and $W_{2b} = w_{2b}$, it finds the $w_{2a}$-th codeword in the inner codebook, denoted as $\bar{v}^n$, and transmits the $w_{2b}$-th codeword in the outer codebook (denoted as $\bar{x}^n_2$) of Transmitter 2 associated with $\bar{v}^n$.

The cognitive encoder knows $W_2$ and therefore knows $\bar{x}^n_2$ and $\bar{v}^n$. For $W_1 = w_1$, it look into the $w_1$-th bin in the outer codebook of Transmitter 1 associated with $\bar{v}^n$, and find the $u^n$ (denoted as $\bar{u}^n$) that is jointly typical with $\bar{x}^n_2$ conditioned on $\bar{v}^n$. This can be done almost always as long as

$$R_0 \geq I(U; X_2|V) \tag{42}$$

is satisfied. The cognitive encoder then transmit an $x^n_1$ sequence generated i.i.d. conditioned on $\bar{u}^n$ and $\bar{x}^n_2$ using $p(x_1|u, x_2)$.

**Decoding:** Receiver 2 first finds the unique $v^n$ sequence in the inner codebook that is jointly typical with received sequence $y^n_2$ while treating everything else as noise. This can be done if

$$\gamma \leq I(V; Y_2) \tag{43}$$

Based on the $v^n$ sequence it decoded, Receiver 2 then finds the unique $x^n_2$ sequence that is jointly typical with $y^n_2$ conditioned on $v^n$ in the outer codebook of Transmitter 2 associated with $v^n$. This can be done if

$$R_2 - \gamma \leq I(X_2; Y_2|V) \tag{44}$$

Receiver 1 first finds the unique $v^n$ sequence in the inner codebook that is jointly typical with received sequence $y^n_1$ while treating everything else as noise. This can be done if

$$\gamma \leq I(V; Y_1) \tag{45}$$
Based on the $v^n$ it decoded, Receiver 1 then finds the unique $u^n$ sequence that is jointly typical with $y^n_2$ conditioned on $v^n$ in the outer codebook of Transmitter 1 associated with $v^n$. This can be done if

$$R + R_0 \leq I(U; Y_1 | V)$$

(46)

Based on (42)-(46), using Fourier-Motzkin elimination, we obtain the desired result.

**ACKNOWLEDGEMENT**

The authors would like to thank Dr. Wei Kang for the helpful discussions.

**REFERENCES**

[1] C. E. Shannon. Two-way communication channels. In *Proc. 4th Berkeley Symp. Math. Stat. Prob.*, volume 1, pages 611–644, Berkeley, CA, 1961.

[2] R. Ahlswede. Multi-way communication channels. In *Proc. 2nd Int. Symp. Inform. Theory*, pages 23–52, Tsahkadsor, Armenian S.S.R., 1971.

[3] H. Sato. The two-user communication channels. *IEEE Trans. on Information Theory*, 23(3):295–304, May 1977.

[4] A. B. Carleial. Interference channels. *IEEE Trans. on Information Theory*, 24(1):60–70, January 1978.

[5] R. Benzel. The capacity region of a class of discrete additive degraded interference channels. *IEEE Trans. on Information Theory*, 25(2):228–231, March 1979.

[6] A. El Gamal and M. Costa. The capacity region of a class of deterministic interference channels. *IEEE Trans. on Information Theory*, 28(2):343–346, March 1982.

[7] A. B. Carleial. A case where interference does not reduce capacity. *IEEE Trans. on Information Theory*, 21:569–570, September 1975.

[8] H. Sato. On the capacity region of a discrete two-user channel for strong interference. *IEEE Trans. on Information Theory*, 24(3):377 – 379, May 1978.

[9] T. Han and K. Kobayashi. A new achievable rate region for the interference channel. *IEEE Trans. on Information Theory*, 27(1):49–60, January 1981.

[10] H. Sato. The capacity of the Gaussian interference channel under strong interference. *IEEE Trans. on Information Theory*, 27(6):786–788, November 1981.

[11] M. Costa and A. El Gamal. The capacity region of the discrete memoryless interference channel with strong interference. *IEEE Trans. on Information Theory*, 33(5):710–711, September 1987.

[12] N. Liu and S. Ulukus. The capacity region of a class of discrete degraded interference channels. In *44th Annual Allerton Conference on Communications, Control and Computing*, Monticello, IL, September 2006.

[13] G. Kramer. Review of rate regions for interference channels. In *2006 International Zurich Seminar on Communications*, ETH Zurich, Switzerland, February 2006.

[14] I. Sason. On achievable rate regions for the Gaussian interference channel. *IEEE Trans. on Information Theory*, 50(6):1345–1356, June 2004.

[15] R. Ahlswede and N. Cai. *General Theory of Information Transfer and Combinatorics, Lecture Notes in Computer Science*, Vol. 4123, chapter Codes with the identifiable parent property and the multiple-access channel, pages 249–257. Springer Verlag, 2006.
[16] N. Liu and A. Goldsmith. Superposition encoding and partial decoding is optimal for a class of Z-interference channels. In *IEEE International Symposium on Information Theory*, Toronto, CA, July 2008.

[17] N. Devroye, P. Mitran, and V. Tarokh. Achievable rates in cognitive radio channels. *IEEE Trans. on Information Theory*, 52(5):1813–1827, May 2006.

[18] I. Marić, R. D. Yates, and G. Kramer. Capacity of interference channels with partial transmitter cooperation. *IEEE Trans. on Information Theory*, 53(10):3536–3548, October 2007.

[19] W. Wu, S. Vishwanath, and A. Arapostathis. Capacity of a class of cognitive radio channels: Interference channels with degraded message sets. *IEEE Trans. on Information Theory*, 53(11):4391–4399, November 2007.

[20] A. Jovičić and P. Viswanath. Cognitive radio: An information-theoretic perspective. *Submitted to IEEE Trans. on Information Theory*, http://www.arxiv.org/pdf/cs.IT/0604107.pdf, 2006.

[21] I. Marić, A. Goldsmith, G. Kramer, and S. Shamai(Shitz). On the capacity of interference channels with a cognitive transmitter. *European Trans. on Telecommunications, invited*, 19:405–420, April 2008.

[22] J. Jiang and Y. Xin. On the achievable rate regions for interference channels with degraded message sets. *IEEE Trans. on Information Theory*, 54(10):4707–4712, October 2008.

[23] Y. Cao and B. Chen. Interference channel with one cognitive transmitter. In *Asilomar Conference on Signals, Systems, and Computers*, Pacific Grove, CA, October 2008.

[24] S. Sridharan and S. Vishwanath. On the capacity of a class of MIMO cognitive radios. In *IEEE Information Theory Workshop (ITW 2007)*, Lake Tahoe, CA, September 2007.

[25] Y. Liang, A. Somekh-Baruch, V. Poor, S. Shamai(Shitz), and S. Verdú. Cognitive interference channels with confidential messages. In *45th Annual Allerton Conference on Communication, Control and Computing*, Allerton House, Monticello, IL, September 2007.

[26] Y. Cao, B. Chen, and J. Zhang. A new achievable rate region for interference channels with common information. In *Proc. IEEE Wireless Comm. and Networking Conf (WCNC 2007)*, Hong Kong, China, March 2007.

[27] S. I. Gelfand and M. S. Pinsker. Coding for channel with random parameters. *Probl. Contr. and Inform. Theory*, 9(I):19–31, 1980.

[28] X. Shang, G. Kramer, and B. Chen. New outer bounds on the capacity region of Gaussian interference channels. In *IEEE International Symposium on Information Theory*, Toronto, Canada, July 2008.

[29] V.S. Annapureddy and V.V. Veeravalli. Gaussian interference networks: Sum capacity in the low interference regime. In *IEEE International Symposium on Information Theory*, Toronto, Canada, July 2008.

[30] A.S. Motahari and A.K. Khandani. Capacity bounds for the Gaussian interference channel. In *IEEE International Symposium on Information Theory*, Toronto, Canada, July 2008.

[31] M. Costa. Writing on dirty paper. *IEEE Trans. on Information Theory*, 29(3):439 – 441, May 1983.

[32] J. Korner and K. Marton. Images of a set via two channels and their role in multi-user communication. *IEEE Trans. on Information Theory*, 23(6):751–761, Nov. 1977.

[33] J. Korner and K. Marton. General broadcast channels with degraded message sets. *IEEE Trans. on Information Theory*, 23(1):60–64, Jan. 1977.

[34] I. Csiszar and J. Korner. *Information Theory: Coding Theorems for Discrete Memoryless Systems*. Academic Press, 1981.

[35] T. M. Cover and J. A. Thomas. *Elements of Information Theory*. Wiley-Interscience, 1991.