Capturing Prospective Teachers' Beliefs about Mathematical Problem Solving

Muhtarom1*, Nizaruddin1, Sutrisno1, Pathuddin2

1Department of Mathematics Education, Universitas PGRI Semarang, Indonesia
2Department of Mathematics Education, Universitas Tadulako, Indonesia

Received March 4, 2020; Revised April 3, 2020; Accepted April 19, 2020

Copyright©2020 by authors, all rights reserved. Authors agree that this article remains permanently open access under the terms of the Creative Commons Attribution License 4.0 International License

Abstract Beliefs about problem solving are very important because they form the basis for learning mathematics. This research describes beliefs of prospective mathematics teachers toward mathematical problem solving. Participants of this research consisted of 157 prospective mathematics teachers at a private University in Semarang. Participants have sufficient background in the field of mathematics and pedagogy study in learning. Data were collected through semi-structured interviews and beliefs questionnaire on problem solving with answer choices. Questionnaire we used was to investigate the beliefs of prospective teachers about mathematical problem solving, while the interview process that we used was to obtain qualitative data. Based on the results of the questionnaire, each one of the prospective teacher students who had instrumentalist, platonic and constructivist beliefs was interviewed for their beliefs in mathematical problems, problem solving and problem solving processes. The results of the research show that most of prospective teacher tended to have platonist beliefs in every statement of mathematical problems and mathematical problem solving. But on the statement of the problem solving process, the most of prospective teachers tended to have constructivist beliefs. The belief perspective is built to help teachers understand and develop mathematical knowledge needed in solving mathematical problems.

Keywords Beliefs, Mathematical Problems Solving, Prospective Teachers

1. Introduction

Problem solving has become an important issue that is discussed deeply in mathematics education. NCTM (2000) recommends that problem solving can be the focus of mathematics instruction as the basis for developing thinking skills implemented in the 2013 curriculum. Simple problem solving is the process of accepting problems as a challenge to solve them. Polya (1973) defines problem solving as an attempt to find a way out of a difficulty, to achieve a goal that is not immediately achievable. A problem is a problem for someone at a time, but it is not a problem in the next time (Dossey, McCrone, O'Sullivan & Gonzales, 2006; Nizaruddin, Muhtarom, & Zuhri, 2019; Pathuddin, Budayasa, & Lukito, 2018; 2019), if one already knows the way or the process of getting the problem solving. Through mathematical problem solving, students are directed to develop their skills such as building new mathematical knowledge, solving problems in various contexts related to mathematics, applying the necessary strategies and reflecting the process of mathematical solution (Anderson, White and Sullivan, 2005; Chapman, 2015; Muhtarom, Juniati, & Siswono, 2017a; Nizaruddin, Muhtarom, & Murtianto, 2017). Greer, Verschaffel & Corte (2002), state that one of the most effective ways of generating beliefs in problem solving is through teachers, textbooks, learning strategies, and the main one is the use of problems around students for learning activities. During math lessons, students learn not only mathematical concepts and procedures, but also how to interact in the classroom, they learn about a set of beliefs, and they learn how to behave in math lessons. Students’ beliefs about mathematics, mathematics learning and their skills for solving mathematical problems can be understood through their beliefs in problem solving (Garofalo, 1989; Kloosterman & Stage, 1992; Mason, 2003). Garofalo (1989) shows different types of beliefs about mathematical problem solving that affect mathematical achievement, for example: the difficulty degree of the problem, the operations to be performed and the student's decision to check what has been done.

Obviously, beliefs to mathematical problems solving is an important thing that must be instilled in children since
the early days because belief can be the basis for disposition, the basis for action, the basis for change and the basis for learning (Chapman, 2015). Pehkonen & Pietila (2003) do not place beliefs in the human affective domain but somewhere between the cognitive domain and the affective domain, called the "twilight zone". Beliefs as subjective knowledge and perception can be interchangeable (Presmeg, 2002) in the context of mathematics. Beliefs as a cognitive and affective construct is essential for the problem-solving learning process (Bal, 2015; Callejo & Vila, 2009; Ozturk & Guven, 2015; Schoenfeld, 1992; Thompson, 1992). Prospective teachers can successfully solve the problem of belief in the importance of mathematics and the need to understand the problem; while students who believe in problem solving take a short time and they can handle it by memorizing the rules, believing that problem solving is difficult (Schoenfeld, 1992).

Before discussing the beliefs of problem solving that becomes the focus of this article, we need to first describe the concept of beliefs (e.g. Anderson, White & Sullivan, 2005; Beswick, 2005; 2012; Boz, 2008; Cheng et al., 2009; Thompson, 1992), with categories: instrumentalist, platonist, and problem solving. Table 1 summarizes some of the expert views on these three categories of beliefs (Beswick, 2012).

The research of beliefs in problem solving has been carried out by several researchers (e.g. Duell, & Hutter, 2005; Kloosterman & Stage, 1992; Memnun, Hart & Akkaya, 2012; Schommer-Aikins., Mkomange & Ajagbe, 2012). For example, Schommer-Aikins., Duell, & Hutter (2005) find that beliefs in mathematics have a direct influence on the students' mathematical problem-solving performance. Mkomange & Ajagbe (2012) conclude that most mathematics prospective teachers have positive beliefs about the importance of understanding mathematical problems, problems with some completion ways and the type of mathematical learning emphasized by contemporary principles. Most of prospective teachers do not have enough beliefs in solving problems, especially about mathematical skills, place of mathematics and problem solving beliefs sub-dimensions (Memnun, Hart & Akkaya, 2012). Other researchers provide an overview of beliefs in problem solving by looking at aspects of the problem-solving process (Ozturk & Guven, 2015; Muhtarom, Juniati, & Siswono, 2017b), problem solving content knowledge and problem solving pedagogical knowledge (Chapman, 2015; Siswono, Kohar, & Hartono, 2017). Teachers’ beliefs have a strong relationship with the teacher's knowledge of problem solving. In particular, the instrumentalist's beliefs are consistent with the inadequate knowledge of problem solving, while platonist and constructivist teacher beliefs are consistent with their knowledge of problem solving (Muhtarom et al., 2017b; Siswono et al., 2017). This indicates that the development of prospective teachers’ beliefs in problem solving is a necessity. Based on the background, our research aimed to express the beliefs of prospective teachers toward mathematical problem solving.

2. Methods

2.1. Participants

Participants of this research consisted of 157 prospective mathematics teachers at a private University in Semarang. Participants have sufficient background in the field of mathematics and pedagogy study in learning.

2.2. Instruments and Procedures

The research data were collected through questionnaires of belief about problem solving compiled by researchers primarily by utilizing previously developed instruments (e.g. Chapman, 2015; Callejo & Vila, 2009; Kloosterman & Stage, 1992; Memnun, Hart & Akkaya, 2012; Siswono et al., 2017), with some modifications and additions to the choice of answers that cited the categories of beliefs: instrumentalist, platonist and constructivist. Before being used, a questionnaire of belief about problem solving has been performed expert validation and tested internal consistency with reliability coefficient 0.651. Questionnaire consisted of 14 statements divided into three parts, namely beliefs to math problems (4 statements), beliefs in problem solving (6 statements), and beliefs in problem-solving process (4 statements), as an example instrument presented in Table 2.

| Beliefs about nature mathematics | Beliefs about mathematics teaching | Beliefs about mathematics learning |
|---------------------------------|-----------------------------------|----------------------------------|
| Instrumentalist / as tool       | Content focused with an emphasis on performance | Skill Mastery, passive reception of knowledge |
| Platonist / body static         | Content focused with an emphasis on understanding | Active construction of understanding |
| Problem solving / constructivist| Learner focused                  | Autonomous exploration of own interest |
### 2.3. Data Analysis

Scores varied to show the belief level on mathematical problem solving from 1.00 (instrumentalist) to 3.00 (constructivist). Scores were given to each sample on each statement based on the following formula: average score = scores obtained: number of statements. Based on the average score the each participant’s beliefs are grouped into three categories: instrumentalist, platonist, and constructivist (see Table 3) (Siswono, Kohar, Kurniasari, & Astuti, 2016).

| Score                          | Categories of Beliefs        |
|--------------------------------|-----------------------------|
| Average Score <1.67            | Instrumentalist             |
| 1.67 ≤ Average Score ≤ 2.33   | Platonist                   |
| Average Score> 2.33            | Constructivist              |

These data were also supported by qualitative data to provide a description of prospective teachers’ beliefs in mathematical problem solving by using a set of interview guidelines. Prospective teachers were selected to be interviewed individually about their beliefs about problem solving, including problem knowledge, problem solving knowledge and knowledge of problem solving process. Data analysis was done by firstly reducing data, displaying data, and finally drawing conclusion and verification (Miles & Huberman, 1992). The conclusion was there was a description of the prospective teachers’ beliefs about mathematical problem solving.

### 3. Results and Discussion

There were fourteen statements to measure the beliefs of prospective teachers on mathematical problem solving. Table 4 obviously shows that the distribution of the number of prospective teachers choosing each answer option for each statement. For example, the statement of ‘mathematical problem maker’ indicated that 25.4% of prospective teachers believed that mathematical problems should be made by teachers ‘spontaneously’ in accordance with textbooks. 51% of prospective teachers believed that mathematical problems should be specially designed by teachers in accordance with material needs lessons that were appropriate to textbooks, and 23.6% of prospective teachers believed that mathematical problems should be specially designed by the teachers according to the material needs and problems that were made by the students themselves. Furthermore, the prospective teachers believed that the purposes of the problems were: the students were able to complete according to the procedure according to the teacher or book (10.8%), the students were able to understand the relation among mathematical ideas, concepts, and mathematical procedures to be able to solve the problems (67.5%) and
students were actively involved in performing mathematical tasks in exploring and formulating ideas to be able to complete the problems (21.7%). Furthermore, 31 prospective teachers improved the misunderstanding of students by providing a more detailed explanation, especially in the wrong part, 74 prospective teachers improved the misunderstanding of students by involving students more actively and more carefully in carrying out the learning activities and 52 prospective teachers improved the misunderstanding of students by allowing students to discuss by comparing ideas and deciding the best way to improve their misconceptions.

As described in Table 4, we can note that the most of prospective teachers tended to have platonist beliefs about every statement of mathematical problems and mathematical problem solving. However, in the statement of the problem-solving process, the most of prospective teachers tended to have constructivist beliefs. Specifically, the results of the research showed that 55.4% of prospective teachers had platonist beliefs about mathematical problem solving, 40.1% of prospective teachers had constructivist-oriented beliefs about mathematical problem solving, and the rest had beliefs in the solving of instrumentalist-oriented mathematical problem solving.

### Table 4. Results Description of Each Statement

| Category               | Statement                                                        | The answer choice of Instrumentalist | The answer Choice of Platonist | The answer Choice of Constructivist |
|------------------------|------------------------------------------------------------------|-------------------------------------|-------------------------------|-------------------------------------|
|                        |                                                                  | n n                                 | % %                           | n n                                 | % % | n n                                 | % % |
| Mathematical problems  | The type of problem that must be solved by the students         | 23 23                              | 14.6 14.6                      | 79 79                              | 50.4 50.4                      | 55 55                             | 35.0 35.0 |
|                        | Mathematical problems maker                                     | 40 40                              | 25.4 25.4                      | 80 80                              | 51.0 51.0                      | 37 37                             | 23.6 23.6 |
|                        | The purpose of giving problems in learning                      | 17 17                              | 10.8 10.8                      | 106 106                           | 67.5 67.5                      | 34 34                             | 21.7 21.7 |
|                        | The way of students in learning problem-solving                 | 31 31                              | 19.7 19.7                      | 72 72                              | 45.9 45.9                      | 54 54                             | 34.4 34.4 |
| Problem Solving        | Problem solving as a learning objective of mathematics          | 16 16                              | 10.2 10.2                      | 91 91                              | 58.0 58.0                      | 50 50                             | 31.8 31.8 |
|                        | Problem solving as a process                                    | 24 24                              | 15.3 15.3                      | 76 76                              | 48.4 48.4                      | 57 57                             | 36.3 36.3 |
|                        | Problem solving as a basic skill                                 | 43 43                              | 27.4 27.4                      | 45 45                              | 28.7 28.7                      | 69 69                             | 43.9 43.9 |
|                        | Improving students' misunderstanding                            | 31 31                              | 19.7 19.7                      | 74 74                              | 47.2 47.2                      | 52 52                             | 33.1 33.1 |
|                        | Students who have difficulty                                    | 13 13                              | 8.3 8.3                        | 82 82                              | 52.2 52.2                      | 62 62                             | 39.5 39.5 |
|                        | How to train students to solve math problems                    | 43 43                              | 27.4 27.4                      | 48 48                              | 30.6 30.6                      | 66 66                             | 42.0 42.0 |
| Problem Solving Process| Understanding math problems                                     | 20 20                              | 12.7 12.7                      | 40 40                              | 25.5 25.5                      | 97 97                             | 61.8 61.8 |
|                        | Planning for solving math problems                              | 18 18                              | 11.5 11.5                      | 84 84                              | 53.5 53.5                      | 55 55                             | 35.0 35.0 |
|                        | Implementing the planning in a math problem solving              | 30 30                              | 19.1 19.1                      | 27 27                              | 17.2 17.2                      | 100 100                           | 63.7 63.7 |
|                        | Looking back the completion                                     | 27 27                              | 17.2 17.2                      | 86 86                              | 54.8 54.8                      | 44 44                             | 28.0 28.0 |
Furthermore, we presented the result of interview with Mayya (woman and pseudo name) as based on the result of belief questionnaire in solving problem, the participant seemed to illustrate her belief in the instrumentalist view. Mayya had belief that to be able to solve the problem, one had to remember the way ever taught by the teacher then imitate the way of completion. Mayya had sufficient understanding in the process of solving the problem. Her idea was guiding students to solve math problems such as, asking students to read the problem, understanding the concepts of the given problem, and understanding that in planning the problem solving, one should make completion procedures, then use the procedures that had been made to get the solution of the problem. The truth of the answer was believed to be based on the process of completion stage and correctness of the calculation process undertaken. We can see the interview quote as follows.

Mayya: What students should do first is that students must understand concepts in mathematics. Then students should be more active reading, learning then completing some math questions. When they understand the exact concept, the students can solve the problem in various ways. Mathematics does not have only one way of solving, and there are many ways and answers that are true and appropriate.

Mayya: To plan the problem solving it means we understand the problem first. After that we make the completion procedures. Later we know how the process to get the answer.

Mayya: We know the stages then continue to count so that the answer is met. So do it based on the problems and the stages.

Mayya: We are back to problem solving. If our procedures counting is appropriate, the answer is correct.

Subjek Insiatun (nama perempuan dan pseudo) tampak memiliki keyakinan terhadap pemecahan masalah dalam pandangan platonis. Participant Insiatun (woman and pseudo name) seemed to have belief in solving problems in platonist view. Insiatun understood that the purpose of giving problems in learning activities was that students were able to understand the relationship between mathematical concepts and procedures. Therefore, students should have tried to master the mathematical concepts associated with the problem first, then solved the math problem. Participant also stated that the role of teachers was to train students many times to master the concept and make connections between mathematical concepts. Mathematical problems had to be made by the teacher in accordance with the needs of the subject matter in accordance with the textbook and they had some solutions. To be able to solve mathematical problems, the first step that was believed by Insiatun was to know the available information, what was asked and to find out enough information available to answer the problem. The next was making a connection between the mathematical concepts used and how it related to other material concepts and then made the completion steps, solved the problem according to plan by checking every step of completion, calculation and concepts that had been applied so that they understood why an answer was true according to the context of the problem.

Septin (nama perempuan dan pseudo) tampak memiliki keyakinan terhadap pemecahan masalah dalam pandangan konstruktivis. Participant Septin (woman and pseudo name) seemed to have belief in solving problems in the constructivist view. Subject understood that the purpose of giving problem in learning activities was to make students actively involved in carrying out mathematical tasks in exploring ideas. Therefore, they tried to solve the problem in their own way based on the knowledge and experience they had. Septin also stated that the role of teachers was to guide students to try to solve problems in their own way based on knowledge and experience. Mathematical problems had to be made by the teacher in accordance with the material needs and problems created by the students themselves and they had some solutions. To be able to solve math problems, the first step that Septin believed was to know whether the information was available enough to answer the problem, separate the main information from the problem and represent the problem. The next step was making connections between the mathematical concepts used and how they related to other material concepts and then arranging the completion steps, completing the problems based on the strategy-oriented plan and checking every step at a time and whether the steps were correct so they understood why an answer was true according to the context of the problem. Here is a brief excerpt of the interview.

Septin: The purpose of giving problems to students is of course to entice the student to think creatively, so that they can solve the problem not only from one point of view, but from another point of view. Thus, it was enticing the students to think creatively.

Septin: So teachers and students should be able to make math problems. So later the students do the questions from the books of which the problems were made by the students themselves. By doing so the students could be more developed because they are able to make questions-based problems.

Septin: Mathematical problem solving is a strategy used by students to discover why the answer can be true.

Most mathematics prospective teachers had platonist and constructivist beliefs about mathematical problem solving. It means their beliefs were in line with the current reformation movement in mathematics education in Indonesia. For example, to improve students' misconceptions, they would involve students more actively and more carefully in performing learning activities (platonist), and allow students to discuss by comparing their ideas to determine the best way to correct misconceptions (constructivist). The prospective teachers
believed that the most important thing a person had to do to make the students able to solve the math problem was to ask the students to try to solve the problem in their own ways based on their knowledge and experience. Basically they chose problems requiring mathematical thinking and reasoning. However, at the same time they preferred a problem that did not spend too much time. It was clear that they still held some instrumentalist beliefs toward mathematical problems solving. For example, some prospective teachers tended to make mathematical problems which were appropriate to textbooks or student worksheets that had one way of completion, students tended to see the problem solving as a reason for practicing computing skills, following a predetermined sequence of steps while solving problems and only oriented on the correct answer. Understanding the beliefs of prospective teachers on mathematical problem solving is very important. Further research is needed to understand the practice of learning-oriented development of beliefs about problem solving and how to involve teachers in learning it (Chapman, 2015, Muhtarom et al., 2017b; 2018; 2019; Siswono et al., 2017).

In this research, we obtained the data using a set of questionnaires and interview guidelines that the reliability had been customized and tested. This is different from the commonly used method of exploring belief in mathematical problem solving (e.g. Duell, & Hutter, 2005; Kloosterman & Stage, 1992; Memnun, Hart & Akkaya, 2012; Schommer-Aikins., Mkomange & Ajagbe, 2012). The research could also be the basis for studying specifically about the role of belief in mathematical problem solving. We proposed extending this category to include belief in mathematical problem solving (see Table 5), so the research on belief not only focused on beliefs about: mathematics nature, mathematics teaching and mathematics learning (Anderson, White & Sullivan, 2005; Beswick, 2005; 2012; Boz, 2008; Cheng et al., 2009, Thompson, 1992). However, another thing we need to be aware of is that one's belief is not necessarily consistent in all categories.

| Beliefs about nature mathematics | Beliefs about mathematics teaching | Beliefs about mathematics learning | Beliefs about mathematics problem solving |
|---------------------------------|-----------------------------------|-----------------------------------|------------------------------------------|
| Instrumentalist / as tool       | Content focused with an emphasis on performance | Skill Mastery, passive reception of knowledge | Remembering the way of completion that has been taught and emphasizing the step sequence performance and orienting to the right answer |
| Platonist / body static         | Content focused with an emphasis on understanding | Active construction of understanding | Making a connection between mathematical concepts and understanding why an answer is true |
| Problem solving / constructivist| Learner focused                    | Autonomous exploration of own interest | Orienting to the development of problem-solving strategy and understanding why an answer is true |

Table 5. Extension of Beliefs Categories
4. Conclusions

Our results show that the most of prospective teachers tend to have platonist beliefs about every statement of mathematical problems and mathematical problem solving. However, in the statement of the problem-solving process, the most of prospective teachers tend to have constructivist beliefs. Prospective teacher students must realize the importance of problem solving beliefs because belief can be the basis for disposition, the basis for action, the basis for change and the basis for learning. The perspective beliefs presented in this article can be constructed to provide a framework of beliefs to help teachers understand and develop the mathematical knowledge needed in mathematical problem solving.

Acknowledgements

We would like to thank Universitas PGRI Semarang for supporting this research.

REFERENCES

[1] Anderson, J., White, P., & Sullivan, P. (2005). Using a Schematic Model to Represent Influences on, and Relationships Between, Teachers’ Problem-Solving Beliefs and Practices. Mathematics Education Research Journal, 17(2), 9-38.

[2] Bal, A. P. (2015). Examination of the Mathematical Problem-Solving Beliefs and Success Levels of Primary School Teacher Candidates Through the Variables of Mathematical Success and Gender. Educational Sciences: Theory & Practice, 15(5), 1373-1390.

[3] Beswick, K. (2005). The Beliefs/Practice Connection in Broadly Defined Contexts. Mathematics Education Research Journal, 17(2), 39-68.

[4] Beswick, K. (2012). Teachers’ Beliefs about School Mathematics and Mathematicians’ Mathematics and Their Relationship to Practice. Educational Studies in Mathematics, 79, 127-147.

[5] Boz, N. (2008). Turkish Pre-Service Mathematics Teachers’ Beliefs about Mathematics Teaching. Australian Journal of Teacher Education, 33 (5): 66-80.

[6] Callejo, M. L., & Vila, A. (2009). Approach to Mathematical Problem Solving and Students’ Belief Systems: Two Case Studies. Educational Studies in Mathematics, 72(1), 111-126.

[7] Chapman, O. (2015). Mathematics Teachers’ Knowledge for Teaching Problem Solving. LUMAT, 3(1), 19-36.

[8] Cheng, M. M., Chan, K.W., Tang, S. Y., & Cheng, A. Y. (2009). Pre-Service Teacher Education Students’ Epistemological Beliefs and Their Conceptions of Teaching. Teaching and Teacher Education, 25(2), 319–327.

[9] Dossey, J. A., McCrone, S. S., O’Sullivan, C., & Gonzales, P. (2006). Problem Solving in the PISA and TIMSS 2003 Assessment. Technical Report, US Department of Education.

[10] Garofalo, J. (1989). Beliefs and Their Influence on Mathematical Performance. Mathematics Teacher, 82, 502-505.

[11] Greer, B., Verschaffel, L., & Corte, E.D. (2002). The Answer is Really 4.5: Beliefs about Word Problems. In Gilah, L. C. Erkki, P & Gunter, T. (ed). Belief: A Hidden Variable in Mathematics Education?. Dordrecht: Kluwer Academic Publishers.

[12] Kloosterman, P., & Stage, K. F. (1992). Measuring Beliefs about Mathematical Problem Solving. School Science and Mathematics, 92(3), 109-115.

[13] Mason, L. (2003). High School Students’ Beliefs about Maths, Mathematical Problem Solving, and Their Achievement in Maths: A Cross-Sectional Study. Educational Psychology, 23(1), 73-86.

[14] Memmun, D. S., Hart, L. C., & Akkaya, R. (2012). A Research on the Mathematical Problem Solving Beliefs of Mathematics, Science and Elementary Pre-Service Teachers in Turkey in terms of Different Variables. International Journal of Humanities and Social Science, 2(24), 172-184.

[15] Miles, M., & Huberman, A. M. (1992). Analisis Data Kualitatif: Buku Sumber tentang Metode-Metode Baru. Jakarta: UI Press.

[16] Mkomange, W. C & Ajegbe, M. A. (2012). Prospective Secondary Teachers’ Beliefs about Mathematical Problem Solving. International Journal of Research in Management & Technology, 2(2), 154-163.

[17] Muhtarom., Juniati, D., & Siswono, T. Y. E. (2017a). Exploring Beliefs in a Problem-Solving Process of Prospective Teachers’ with High Mathematical Ability. Global Journal of Engineering Education, 19(2), 130-136.

[18] Muhtarom, Juniati, D., & Siswono, T. Y. E. (2017b). Consistency and Inconsistency of Prospective Teachers’ Beliefs in Mathematics, Teaching, Learning and Problem Solving. AIP Conference Proceedings (Vol. 1868, No. 1, p. 050014). AIP Publishing. Retrieved from http://aip.scitation.org/doi/abs/10.1063/1.4995141.

[19] Muhtarom., Juniati, D., & Siswono, T. Y. E. (2018). Exploring Prospective Teachers’ Beliefs about Nature of Mathematics. Journal of Engineering and Applied Sciences, 13(10), 3547-3554. https://doi.org/10.36478/jeasci.2018.35 47.3554.

[20] Muhtarom, Juniati, D., & Siswono, T. Y. E. (2019). Examining Prospective Teacher Beliefs and Pedagogical Content Knowledge Towards Teaching Practice in Mathematics Class: A Case Study. Journal on Mathematics Education, 10(2), 185-202. https://doi.org/10.22342/jme.10 .2.7326.185-202.

[21] Muhtarom., Murtianto, Y. H & Sutrisno. (2017). Thinking Process of Students with High-Mathematics Ability (A Study on QSR NVivo 11-Assisted Data Analysis). International Journal of Applied Engineering Research, 12(17), 6934-6940.
[22] NCTM. (2000). Principles and Standards for School Mathematics. NCTM: Reston, VA.

[23] Nizaruddin, Muhtarom., & Sugiyanti. (2017). Improving Students’ Problem-Solving Ability in Mathematics through Game-Based Learning Activities. World Transactions on Engineering and Technology Education, 15(2), 102-107.

[24] Nizaruddin, N., Muhtarom, M., & Zuhri, M. S. (2019). Improving Mechanical Engineering Students’ Achievement in Calculus through Problem-Based Learning. Universal Journal of Educational Research 7(12), 2729-2733, DOI: 10.13189/ujer.2019.071221.

[25] Ozturk, T., & Guven, B. (2016). Evaluating Students’ Beliefs in Problem Solving Process: A Case Study. Eurasia Journal of Mathematics, Science & Technology Education, 12(3), 411-429.

[26] Pathuddin, P., Budayasa, I. K., & Lukito, A. (2018). Metacognitive Knowledge of A Student in Planning the Solution of Limit Problems. Journal of Physics: Conference Series (Vol. 1108, No. 1, p. 012032). IOP Publishing.

[27] Pathuddin, P., Budayasa, I. K., & Lukito, A. (2019). Metacognitive Activity of Male Students: Difference Field Independent-Dependent Cognitive Style. Journal of Physics: Conference Series (Vol. 1218, No. 1. p. 012025). IOP Publishing.

[28] Pehkonen, E., & Pietilä, A. (2003). On Relationships Between Beliefs and Knowledge in Mathematics Education. Proceedings of the CERME-3 (Bellaria) meeting. http://www.dm.unipi.it/~didattica/CERME3/draft/proceedings_draft/TG2

[29] Polya, G. (1973). How To Solve It, Second Edition. Princeton University Press, Princeton, New Jersey.

[30] Preiss, N. (2002). Beliefs about the Nature of Mathematics in the Bridging of Everyday and School Mathematics Practices. In G. C. Leider, E. Pehkonen, dan G. Torner (ed). Beliefs: A Hidden Variable in Mathematics Education?, Dordrecht: Kluwer Academic Publisher.

[31] Thompson, A. G. (1992). Teachers’ Beliefs and Conceptions: A Synthesis of the Research. in D. A. Grouws (Ed.), Handbook of Research on Mathematics Teaching and Learning (pp. 127–146). Reston, VA: National Council of Teachers of Mathematics. Retrieved from http://psycnet.apa.org/psycinfo/1992-97586-007.

[32] Schoenfeld, A.H. (1992). Learning to Think Mathematically, in A.D. Grouws (Ed). Handbook of research on Mathematics Learning and Teaching.

[33] Schommer-Aikins, M., Duell, O.K., & Hutter, R. (2005). Epistemological Beliefs, Mathematical Problem-Solving Beliefs and Academic Performance of Middle School Students. The Elementary School Journal, 105(3), 289-304.

[34] Siswono, T. Y. E., Kohar, A. W., Kurniasari, I., & Astuti, Y. P. (2016). An Investigation of Secondary Teachers’ Understanding and Belief on Mathematical Problem Solving. Journal of Physics: Conference Series (Vol. 693, No. 1. p. 012015). IOP Publishing. Retrieved from http://iopscience.iop.org/article/10.1088/1742-6596/693/1/012015/meta.

[35] Siswono, T. Y. E., Kohar, A. W., & Hartono, S. (2017). Secondary Teachers’ Mathematics-related Beliefs and Knowledge about Mathematical Problem-solving. Journal of Physics: Conference Series (Vol. 812, No. 1. p. 012046). IOP Publishing. Retrieved from http://iopscience.iop.org/article/10.1088/1742-6596/812/1/012046/meta.