Neutrino signatures of near-critical supernova outflows

Alexander Friedland\textsuperscript{1} and Payel Mukhopadhyay\textsuperscript{1,2}
\textsuperscript{1}SLAC National Accelerator Laboratory, Stanford University, Menlo Park, CA 94025, USA
\textsuperscript{2}Physics Department, Stanford University, Stanford, CA 94305, USA

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In a core-collapse supernova, after the explosion is launched, neutrino heating above the protoneutron star creates an outflow of matter. This outflow has been extensively investigated as a nucleosynthesis site. Here, we revisit this problem motivated by the modeling of neutrino flavor transformations. In this case, it is crucial to understand whether the outflow has a termination shock: its existence observably alters neutrino oscillations a few seconds into the explosion. We derive physical criteria for the formation of this shock, in terms of neutrino luminosity, average energy, protoneutron star radius and mass, and the postshock density. For realistic physical conditions, the system is found to be on the edge of shock formation, thus reconciling seemingly disparate numerical results in the literature. Our findings imply that neutrino signatures of modulated matter effects are a sensitive probe of the inner workings of the supernova.

Introduction.— The mechanism of core-collapse supernova explosions has been intensely investigated for over six decades \cite{1–6}. Simulations agree that the shock front at first stalls at the radius of several hundred kilometers. If the explosion is to form a neutron star, the shock must be revived within a second, cutting off accretion, which would otherwise continue, forming a black hole. It is believed that the revival is driven by neutrinos streaming out of the nascent protoneutron star (PNS) \cite{7,8}. Some of these neutrinos deposit energy just above the PNS surface, in the so-called gain region \cite{9}, from where it is then convected throughout the post-shock region \cite{10}. The full problem at this stage involves three-dimensional hydrodynamics and neutrino transport, giving rise to some of the most sophisticated and resource-intensive supercomputing calculations in the world \cite{11–15}.

To test this picture, it is essential to obtain direct experimental information on the evolution of the matter profile around the PNS during the crucial first several seconds after the shock is revived. Since the region of interest is shrouded by the stellar envelope, it cannot be directly observed in photons. The task thus falls to neutrinos (and also, potentially, gravitational waves). In the era of large detectors, such as DUNE and Hyper-Kamiokande, it becomes very important to identify the relevant signatures in the neutrino signal and to establish a quantitative connection between them and the physical conditions inside the exploding star.

It has been known for several decades that the density profile in the envelope of the star might leave imprints in the neutrino signal by flavor oscillations. Specifically of interest here is the time-dependent modulation of the MSW effect \cite{10,17} by the explosion \cite{18}.

The so-called MSW resonance operates at densities where the matter potential is comparable to the vacuum splitting between a pair of mass eigenstates. In a supernova, there are two resonant densities, of order $10^3$ g/cm$^3$ and $10^4$ g/cm$^3$ \cite{19}, corresponding to the two splittings in the neutrino mass-squared spectrum, $\Delta m^2$. The outcome of the transformations depends on the rate with which the density changes in the resonant layers. When the density scale height is much larger than the relevant neutrino oscillation length, the evolution is adiabatic. This is observed to happen for solar neutrinos \cite{20,22}, and also expected to be the case in a typical iron-core-collapse supernova at the onset of the explosion.

A few seconds into the explosion, however, the situation changes. First, when the expanding front shock rolls through the layer with densities $\sim 10^4$ g/cm$^3$, the profile there becomes effectively discontinuous \cite{18} and the flavor evolution nonadiabatic. Just as important, however, is the density distribution behind the shock front \cite{23,24}. There, a low-density region is formed, known as the hot bubble \cite{18,24}. Density features in and around the hot bubble, such as shocks and turbulence can have a great impact on the evolution of neutrino flavor \cite{23,25}.

The hot bubble is created by a high-entropy material that emanates from the PNS surface due to neutrino heating there. The existence of this outflow at the late stages of the explosion is investigated in the seminal 1986 work by Duncan, Shapiro and Wasserman \cite{26} and has been elaborated on in \cite{27,30}. It has been actively studied in the context of the nucleosynthesis calculations \cite{27,37}, including possible sterile neutrino effects \cite{38,41}.

So far, all this is well known. Yet, in connecting the physical properties of the outflow to the neutrino signal, a number of questions, requiring multi-disciplinary analysis, remain open. In this letter, we will derive the condition for the existence of the outflow termination shock in the hot bubble and explore some of its consequences.

Wind termination shock.— To set up the problem, let us consider the density profile a few seconds into the explosion. Fig. 1 shows, schematically, two physical possibilities for this profile. The Figure is inspired by the simulation results in \cite{22,22}, but with the relevant physical features intentionally emphasized. The blue curves depict density while the red ones show the corresponding entropy per baryon.

In both cases, the front shock is seen at $3 \times 10^4$ km, beyond which the profile is still that of the progenitor star, parameterized as a power law of the radius.
diabatic compression by the passing shock causes a jump in entropy, seen at the front shock location. The material in the density “bump” is moving outward.

The focus of this discussion is on the post-shock region, where behind the “bump” we see the high-entropy bubble occupying $r \lesssim 2 \times 10^4$ km. This entropy is generated by neutrino heating close to the PNS and carried by the out-flowing material into the bulk of the bubble. The outflow then runs into the “bump” and the panels illustrate two possible regimes how this can happen. In the top panel, the outflow first reaches supersonic speeds and then experiences a termination shock, causing a density jump and additional entropy generation. In the bottom panel, the outflow remains subsonic and smoothly flows into the back of the “bump”. The supersonic outflow is traditionally called a “neutrino-driven wind”, while the subsonic case is often referred to as a “breeze”.

Neutrino flavor transformations occur primarily at the MSW resonant matter densities, indicated in Fig. 1 by the horizontal colored bands. The shock fronts are discontinuities where flavor evolution is completely non-

1 The density change between the low-entropy and high-entropy regions (labeled $\rho_{PS}$ and $\rho_f$ in the top panel), the so-called contact discontinuity, is also of interest for neutrino transformations. Detailed analysis shows that this feature is physically distinct from the shocks, thanks to hydrodynamic instabilities that wash out its surface [23]. The resulting turbulent region can have distinct neutrino signatures [25].
to neutrino emission in $e^-e^+$ annihilation; and, finally, $(\nu N)$ heating due to neutrino/antineutrino absorption on nucleons in the medium. Once the temperature drops below $T \sim 0.5$ MeV, nucleons combine into Helium-4, which shuts off the heating. Beyond that radius, entropy per baryon $S$ is conserved along the outflow (cf. Fig. 1).

While the equations are simple, the physical context and the resulting numerical treatment involve some subtlety. The key question is which boundary conditions (b.c.) should be imposed? The system of three equations requires three b.c. Two of them are the initial (small-$r$) $T$ and $S$, but what is the third one? Sometimes, it is stated that the third b.c. is the mass loss rate $\dot{M}$. Yet, physically, $\dot{M}$ is a derived quantity. There must be something in the problem that dictates when one obtains a breeze or a wind and the resulting value of $\dot{M}$.

The subject starts in Ref. 26, which solves the problem following the standard approach for stellar winds (cf. 44). Namely, 26 iterates the location and physical conditions at the sonic point (where the outflow speed equals the speed of sound). The equations are solved first inward, to match the conditions at the surface of the PNS, and then from the sonic point to infinity. This framework thus presupposes that the outflow reaches supersonic speeds. A conceptually similar approach, with different numerical implementation, is followed in 28. Other studies, such as 27 and 29, in addition to the method of 26, also considered imposing boundary pressure ($T \sim 0.1$ MeV) at large radii ($\sim 10^4$ km). With this, they obtained subsonic outflows, and $\dot{M}$ values that were somewhat lower than those of the wind solution.

The consistency of the different approaches has been debated in 28,30. For the purposes of the r-process nucleosynthesis, however, the practical applications of the two approaches were not that different. The reason is that the early parts of the outflow, where the proton-to-neutron ratio is set, turn out to be sufficiently similar in the two cases. Thus, the question of the eventual termination shock could be sidestepped. However, the issue of which method to use, when one does not a priori know if the solution is a wind or a breeze, remains open. As we have already discussed, the problem of neutrino flavor evolution brings it back into sharp focus.

The answer is that the third physical b.c. is always the pressure at large radii, in the post-front-shock region. But how the system is to be solved depends on the value of this pressure. When this pressure is sufficiently high, the system can be solved by “shooting” the initial velocity of the outflow, or equivalently, $\dot{M}$. For sufficiently small far pressures, however, the algorithm is different: one first builds the wind solution that expands into empty space at infinity. One then shoots the location of the termination shock, $R_{ts}$, to match the far pressure b.c. The density (velocity) jump at $R_{ts}$ is given by

$$v_2/v_1 = \rho_1/\rho_2 = (2T_1S_1/m_N + v_1^2)/(7v_1^2),$$

where $v_1$, $T_1$, and $S_1$ are pre-shock quantities and therefore are taken from the wind solution at point $R_{ts}$.

With this procedure, we obtain a family of solutions for a range of physically plausible values for the final density $\rho_f$, imposed at $10^4$ km. We fix neutrino luminosity $L = 8 \times 10^{51}$ erg/s, average energy $\epsilon = 20$ MeV, the mass of the PNS, $M = 1.4M_\odot$, and the gain (starting) radius $R = 20$ km [42]. The results are plotted in Fig. 2. The curves for $r < 10^4$ km are outputs of the calculation. They are matched to profiles for $r > 10^4$ km of the front shock moving through the progenitor profile. At the contact discontinuity, pressure equality is imposed.

We see that gradual adjustment of $\rho_f$ can trigger a qualitative change in the nature of the outflow. For large far densities, we obtain a family of smooth curves. These curves are completely subsonic and the entire flow remains causally connected. As the far density is reduced, the velocity curve develops a cusp at $r \sim 500$ km, where the derivative changes discontinuously. This mathematically singular behavior occurs at the sonic point and has a physical origin: the information about the b.c. at large radius now travels inward only up to the sonic point.

As $\rho_f$ is decreased further, the jump in the derivative becomes the jump in the function itself: a shock front develops. The causal information about the large-$r$ b.c. now travels only up to the shock location; the pre-shock part of the outflow “believes” it is expanding into empty space and is thus universal. For the lowest far density considered in Fig. 2 800 g/cm$^3$, the shock is located at 4000 km, and the density jump is $\rho_2/\rho_1 \simeq 4.4$. For vanishingly small far pressures, the shock radius grows with-

![FIG. 2. Formation of the shock as a function of density, $\rho_f$, at large radius, $10^5$ cm. The panels show the profiles of speed and density of the outflow as $\rho_f$ is varied. Termination shocks start to form as $\rho_f$ falls below $\rho_{f, crit}$.](image-url)
numerical results in the literature. Ref. [32], which uses high heating rates, obtains very robust shocks. Ref. [18] does not have termination shocks because the simulations of the Livermore group used a heavy, $20M_{\odot}$ progenitor (see Fig. 5 in [24]), with concomitantly high density in the post-shock region. Ref. [42] also has no termination shock for a $18M_{\odot}$ progenitor, while, for a $10.8M_{\odot}$ progenitor, a termination shock appears for $t = 3$ s. Using our results in Fig. 2, we estimate that for heating rates similar to Ref. [42], the boundary between progenitors that develop termination shocks at $t = 2 - 3$ s and those that do not lies around $12M_{\odot}$. This estimate needs to be investigated with detailed numerical simulations.

We also stress that the shock properties can be time-dependent [32]. This is explained by the interplay of the evolving neutrino luminosities and post-shock density. As the critical condition is crossed, the shock appears at the sonic point and moves out, or travels backwards towards the sonic point and disappears. This phenomenon is physically distinct from a reverse shock, which is caused by the front shock hitting a density feature in the progenitor profile [43] and a detailed analysis is required [49, 50]. Thus, it is essential to extend numerical simulations [11, 13, 15, 51–53], which are often cut off at 1-2 s, to the entire duration of the neutrino signal.

**Neutrino signatures.**— The overall conclusion of this analysis is that the existence and properties of the termination shock offer a sensitive probe of the physical conditions inside the exploding star. What are the corresponding neutrino signatures of this physics? Fig. 3 shows two spectra that could be observed in the DUNE-like 40 kt liquid argon detector, with and without the termination shock, for profiles from [32]. The oscillation calculation includes 3-flavor matter effects as well as collective neutrino oscillations computed in multi-energy, multi-angle, spherically symmetric approach [54]. Normal mass hierarchy is assumed, as seems to be favored by the latest oscillation data from T2K [56, 57] and NOVA experiments [58, 59], as reflected in global fits [60, 61]. The signal is simulated using the SNOwGLoBES [62] package and integrated from 2.2 to 3.2 seconds. The Figure shows that the impact of the shock on the detectable signal could have 5-sigma statistical significance for a source at 3 kpc. Details of the calculations and the analysis strategies will be discussed elsewhere.

**Conclusions.**— We demonstrated that the formation and properties of the termination shock at the end of the neutrino-driven outflow inside an exploding supernova are sensitive to the physical conditions that exist inside a core-collapse supernova several seconds into the explosion. While the shockwave and the central engine are shrouded from the outside by the stellar envelope, pronounced signatures of the termination shock are imprinted in the neutrino signal. These signatures open up additional physics opportunities for the upcoming large detectors, such as DUNE. We hope that our semi-analytical results on the shock physics can be supported and improved with detailed numerical simulations, and also applied to other environments, such as binary mergers [63, 65].
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[1] M. E. Burbidge, G. R. Burbidge, W. A. Fowler, and F. Hoyle, Synthesis of the elements in stars, Rev. Mod. Phys. 29, 547 (1957)
[2] S. A. Colgate and M. H. Johnson, Hydrodynamic Origin of Cosmic Rays, Phys. Rev. Lett. 5, 235 (1960)
[3] S. A. Colgate and R. H. White, The Hydrodynamic Behavior of Supernovae Explosions, Astrophys. J. 143, 626 (1966)
[4] H. A. Bethe and J. R. Wilson, Revival of a stalled supernova shock by neutrino heating, The Astrophysical Journal 295, 14 (1985)
[5] H. A. Bethe, Supernova mechanisms, Rev. Mod. Phys. 62, 801 (1990)
[6] S. Colgate, M. Herant, and W. Benz, Neutron star accretion and the neutrino fireball, Phys. Rept. 227, 157 (1993)
[7] S. Woosley and T. Janka, The physics of core-collapse supernovae, Nature Phys. 1, 147 (2005), arXiv:astro-ph/0601261
[8] H.-T. Janka, Neutrino-Driven Explosions, in Handbook of Supernovae edited by A. W. Alasia and P. Murdin (2017) p. 1095, 1702.08825 [astro-ph.HE]
[9] H. A. Bethe, Supernova 1987A: An Empirical and analytic approach, Astrophys. J. 412, 192 (1993)
[10] M. Herant, W. Benz, W. Hix, C. L. Fryer, and S. A. Colgate, Inside the supernova: A Powerful convective engine, Astrophys. J. 435, 339 (1994) arXiv:astro-ph/9404024
[11] E. J. Lentz, S. W. Bruenn, W. R. Hix, A. Mezzacappa, O. E. B. Messer, E. Endeve, J. M. Blondin, J. A. Harris, P. Marronetti, and K. N. Yakunin, Three-dimensional Core-collapse Supernova Simulated Using a 15 M⊙ progenitor, Astrophys. J. 807, L31 (2015) arXiv:1505.05110 astro-ph.HE
[12] B. Müller, T. Melson, A. Heger, and H. T. Janka, Supernova simulations from a 3D progenitor model – Impact of perturbations and evolution of explosion properties, Mon. Not. Roy. Astron. Soc. 472, 491 (2017), arXiv:1705.00620 [astro-ph.SR]
[13] D. Radice, A. Burrows, D. Vartanyan, M. A. Skinner, and J. C. Dolence, Electron-Capture and Low-Mass Iron-Core-Collapse Supernovae: New Neutrino-Radiation-Hydrodynamics Simulations, Astrophys. J. 850, 43 (2017) arXiv:1702.03927 [astro-ph.HE]
[14] D. Vartanyan, A. Burrows, D. Radice, M. A. Skinner, and J. Dolence, Revival of the Fittest: Exploding Core-Collapse Supernovae from 12 to 25 M⊙, Mon. Not. Roy. Astron. Soc. 477, 3091 (2018) arXiv:1801.08148 [astro-ph.HE]
[15] A. Burrows, D. Radice, and D. Vartanyan, Three-dimensional supernova explosion simulations of 9-, 10-, 11-, 12-, and 13-M⊙ stars, Mon. Not. Roy. Astron. Soc. 485, 3153 (2019) arXiv:1902.00547 [astro-ph.SR]
[16] L. Wolfenstein, Neutrino Oscillations in Matter, Phys. Rev. D17, 2369 (1978)
[17] S. P. Mikheev and A. Yu. Smirnov, Resonant amplification of neutrino oscillations in matter and solar neutrino spectroscopy, Nuovo Cim. C9, 17 (1986)
[18] R. C. Schirato and G. M. Fuller, Connection between supernova shocks, flavor transformation, and the neutrino signal, arXiv:astro-ph/0205390 (2002), arXiv:astro-ph/0205390 [astro-ph]
[19] A. S. Dighe and A. Yu. Smirnov, Identifying the neutrino mass spectrum from the neutrino burst from a supernova, Phys. Rev. D62, 033007 (2000) arXiv:hep-ph/9907423
[20] Q. Ahmad et al. (SNO), Direct evidence for neutrino flavor transformation from neutral current interactions in the Sudbury Neutrino Observatory, Phys. Rev. Lett. 89, 011301 (2002) arXiv:nuc-ex/0204008
[21] J. Hosaka et al. (Super-Kamiokande), Solar neutrino measurements in Super-Kamiokande-I, Phys. Rev. D 75, 112001 (2006) arXiv:hep-ex/0508055
[22] M. Agostini et al. (BOREXINO), Comprehensive measurement of pp-chain solar neutrinos, Nature 562, 505 (2018)
[23] R. Tomas, M. Kachelriess, G. Raffelt, A. Dighe, H. T. Janka, and L. Scheit, Neutrino signatures of supernova shock and reverse shock propagation, JCAP 0409, 015 arXiv:astro-ph/0407132 [astro-ph]
[24] S. E. Woosley and R. D. Hoffman, The alpha-process and the r process, Astrophys. J. 395, 202 (1992)
[25] A. Friedland and A. Gruzinov, Neutrino signatures of supernova turbulence, arXiv:astro-ph/0607244 (2006), arXiv:astro-ph/0607244 [astro-ph]
[26] R. C. Duncan, S. L. Shapiro, and I. Wasserman, Neutrino-driven winds from young, hot neutron stars, Astrophys. J. 309, 141 (1986)
[27] Y. Z. Qian and S. E. Woosley, Nucleosynthesis in neutrino driven winds: 1. The Physical conditions, Astrophys. J. 471, 331 (1996) arXiv:astro-ph/9611094
[28] T. A. Thompson, A. Burrows, and B. S. Meyer, The Physics of protoneutron star winds: implications for r-process nucleosynthesis, Astrophys. J. 562, 887 (2001) arXiv:astro-ph/0105004 [astro-ph]
[29] K. Otsuki, H. Tagoshi, T. Kajino, and S.-y. Wanajo, General relativistic effects on neutrino driven wind from young, hot neutron star and the r process nucleosynthesis, Astrophys. J. 533, 424 (2000) arXiv:astro-ph/9911164 [astro-ph]
[30] S. Wanajo, T. Kajino, G. J. Mathews, and K. Otsuki, The r-Process in Neutrino-driven Winds from Nascent, “Compact” Neutron Stars of Core-Collapse Supernovae, Astrophys. J. 554, 578 (2001) arXiv:astro-ph/0102261
[31] S. Wanajo, The rp-process in neutrino-driven winds, Astrophys. J. 647, 1323 (2006) [arXiv:astro-ph/0602488]

[32] A. Arcones, H.-T. Janka, and L. Scheck, Nucleosynthesis-relevant conditions in neutrino-driven supernova outflows. 1. Spherically symmetric hydrodynamic simulations, Astron. Astrophys. 467, 1227 (2007) [arXiv:astro-ph/0612582 [astro-ph]]

[33] S. Wanajo, H.-T. Janka, and S. Kubono, Uncertainties in the nu-p-process: supernova dynamics versus nuclear physics, Astrophys. J. 729, 46 (2011) [arXiv:1004.4487 [astro-ph.SR]]

[34] A. Arcones and F. K. Thielemann, Neutrino-driven wind simulations and nucleosynthesis of heavy elements, J. Phys. G 40, 013201 (2013) [arXiv:1207.2527 [astro-ph.SR]]

[35] L. F. Roberts, S. E. Woosley, and R. D. Hoffman, Integrated Nucleosynthesis in Neutrino Driven Winds, Astrophys. J. 722, 954 (2010) [arXiv:1004.4916 [astro-ph.HE]]

[36] A. Arcones, C. Frohlich, and G. Martinez-Pinedo, Impact of supernova dynamics on the vp-process, Astrophys. J. 750, 18 (2012) [arXiv:1112.4651 [astro-ph.HE]]

[37] J. Bliss, M. Witt, A. Arcones, F. Montes, and J. Pereira, Survey of astrophysical conditions in neutrino-driven supernova ejecta nucleosynthesis, Astrophys. J. 855, 135 (2018) [arXiv:1802.00737 [astro-ph.HE]]

[38] I. Tamborra, G. G. Raffelt, L. Hudepohl, and H.-T. Janka, Impact of eV-mass sterile neutrinos on neutrino-driven supernova outflows, JCAP 01, 013 (2015) [arXiv:1110.2104 [astro-ph.SR]]

[39] M.-R. Wu, T. Fischer, L. Huther, G. Martinez-Pinedo, and Y.-Z. Qian, Impact of active-sterile neutrino mixing on supernova explosion nucleosynthesis, Phys. Rev. D 89, 061303 (2014) [arXiv:1305.2382 [astro-ph.HE]]

[40] E. Plumbi, I. Tamborra, S. Wanajo, H.-T. Janka, and L. Hudepohl, Impact of neutrino flavor oscillations on the neutrino-driven wind nucleosynthesis of an electron-capture supernova, Astrophys. J. 808, 188 (2015) [arXiv:1406.2596 [astro-ph.SR]]

[41] Z. Xiong, M.-R. Wu, and Y.-Z. Qian, Active-sterile Neutrino Oscillations in Neutrino-Driven Winds: Implications for Nucleosynthesis, Astrophys. J. 880, 10.5847/1538-4357/ab2870 (2019), arXiv:1904.09371 [astro-ph.HE]

[42] T. Fischer, S. C. Whitehouse, A. Mezzacappa, F. K. Thielemann, and M. Liebendorfer, Protonneutron star evolution and the neutrino driven wind in general relativistic neutrino radiation hydrodynamics simulations, Astron. Astrophys. 517, A80 (2010) [arXiv:0908.1871 [astro-ph.HE]]

[43] J. Wittler, H. T. Janka, and K. Takahashi, Nucleosynthesis in neutrino-driven winds from protoneutron stars I. The α-process, A&A 286, 841 (1994).

[44] E. N. Parker, Interplanetary dynamical processes. (Interscience Publishers, 1963).

[45] L. Landau and E. Lifshitz, Fluid Mechanics, Course of Theoretical Physics, Vol. 6 (Butterworth-Heinemann, Oxford, 1987).

[46] M. C. Potter, D. C. Wiggert, B. Ramadan, and T. I.-P. Shih, Mechanics of Fluids (Cengage Learning, Stamford, CT, 2012).

[47] P. Slane, Pulsar Wind Nebulae, in Handbook of Supernovae edited by A. W. Al sabiti and P. Murdin (2017) p. 2159, arXiv:1703.09311.

[48] H. T. Janka and E. Mueller, The first second of a type-II supernova: convection, accretion, and shock propagation, Astrophys. J. 448, L109 (1995) [astro-ph/9503015].

[49] R. Buras, H.-T. Janka, M. Rampp, and K. Kifonidis, Two-dimensional hydrodynamic core-collapse supernova simulations with spectral neutrino transport. 2. models for different progenitor stars, Astron. Astrophys. 457, 281 (2006) [arXiv:astro-ph/0512189].

[50] I. Panov and H.-T. Janka, On the Dynamics of Protoneutron Star Winds and r-Process Nucleosynthesis, Astron. Astrophys. 494, 829 (2009) [astro-ph/0805.1848].

[51] B. Müller, T. Melson, A. Heger, and H. T. Janka, Supernova simulations from a 3D progenitor model – Impact of perturbations and evolution of explosion properties, Mon. Not. Roy. Astron. Soc. 472, 491 (2017) [arXiv:1705.00620 [astro-ph.SR]]

[52] D. Vartanyan, A. Burrows, D. Radice, A. M. Skinner, and J. Dolence, A Successful 3D Core-Collapse Supernova Explosion Model, Mon. Not. Roy. Astron. Soc. 482, 351 (2019) [arXiv:1809.05106 [astro-ph.HE]]

[53] E. O’Connor et al., Global Comparison of Core-Collapse Supernova Simulations in Spherical Symmetry, J. Phys. G 45, 104001 (2018) [arXiv:1806.04175 [astro-ph.HE]]

[54] H. Duan, G. M. Fuller, and Y.-Z. Qian, Collective neutrino oscillations, Annual Review of Nuclear and Particle Science 60, 569594 (2010).

[55] H. Duan and A. Friedland, Self-induced suppression of collective neutrino oscillations in a supernova, Phys. Rev. Lett. 106, 091101 (2011) [arXiv:1006.2359 [hep-ph]]

[56] K. Abe et al. (T2K), Measurement of neutrino and antineutrino oscillations by the T2K experiment including a new additional sample of νμ interactions at the far detector Phys. Rev. D 96, 092006 (2017) [Erratum: Phys.Rev.D 98, 019902 (2018)], arXiv:1707.01048 [hep-ex]

[57] P. Dunne, Latest neutrino oscillation results from t2k (talk at NEUTRINO 2020), 10.5281/zenodo.3959558 (2020).

[58] M. Acero et al. (NOVA), First Measurement of Neutrino Oscillation Parameters using Neutrinos and Antineutrinos by NOVA, Phys. Rev. Lett. 123, 151803 (2019) [arXiv:1906.04907 [hep-ex]]

[59] A. Himmel, New oscillation results from the nova experiment (talk at NEUTRINO 2020), 10.5281/zenodo.3959581 (2020).

[60] I. Esteban, M. Gonzalez-Garcia, A. Hernandez-Cabezudo, M. Maltoni, and T. Schwetz, Global analysis of three-flavour neutrino oscillations: synergies and tensions in the determination of θ23, δCP, and the mass ordering, JHEP 01, 106 (2019) [arXiv:1811.05487 [hep-ph]]

[61] P. F. de Salas, D. V. Forero, S. Garaiazzo, P. Martinez-Mirav, O. Mena, C. A. Ternes, M. Trícola, and J. W. F. Valle, 2020 global reassessment of the neutrino oscillation picture 2020, arXiv:2006.11237 [hep-ph]

[62] J. Albert et al., Snowglobes, https://webhome.phy.duke.edu/~schol/snowglobes/ (2020).

[63] B. Metzger, A. Piro, and E. Quataert, Time-Dependent Models of Accretion Disks Formed from Compact Object Mergers, Mon. Not. Roy. Astron. Soc. 390, 781 (2008) [arXiv:0805.4415 [astro-ph]]

[64] S.-Y. Wanajo and H.-T. Janka, The r-process in the neutrino-driven wind from a black-hole torus, Astrophys. J. 746, 180 (2012) [arXiv:1106.6142 [astro-ph.SR]]

[65] A. Perego, S. Rosswoog, R. M. Cabezón, O. Korobkin, R. Käppeli, A. Arcones, and M. Liebendörfer, Neutrino-
driven winds from neutron star merger remnants. Mon. Not. Roy. Astron. Soc. 443, 3134 (2014) arXiv:1405.6730 [astro-ph.HE].
Appendix 1. —

Table I shows the values of neutrino outflow parameters as the far boundary condition, $T_f$, is varied and $L$, $\epsilon$, $M$, and $R$ are held fixed to their values in the caption. Values of the final density, $\rho_f$, entropy $S_f$ and density jump across the shock, $\rho_2/\rho_1$ are also presented. It can be seen that the first six rows do not have any entries for the termination shock radius ($R_{ts}$) and the Shock jump factor ($\rho_2/\rho_1$). This is because, in all these cases, the flow is subsonic. The critical solution has density $\rho_f \simeq 3.2 \times 10^3$ gm/cm$^3$; temperature $T \simeq 0.0665$ MeV and entropy $S_f \simeq 48.13$. Note that this density $\rho_f$ is the density at the far boundary radius $10^4$ km before the contact discontinuity in the high entropy region.

Note that, strictly speaking, the critical parameter is temperature, which is directly related to the far pressure boundary condition. It can be converted to critical density assuming the typical values of $S \sim 50$, as done for brevity in the main text. The critical temperature depends on the parameters of the problem as follows (cf. Eq. 5):

$$T_{crit} \simeq 0.0665 \text{ MeV} \left( \frac{L}{8 \times 10^{51} \text{ erg/s}} \right)^{0.9} \left( \frac{\epsilon}{20 \text{ MeV}} \right)^{1.7} \left( \frac{M}{1.4 M_\odot} \right)^{-1.33} \left( \frac{R}{20 \text{ km}} \right)^{0.3}. \quad (7)$$

| $T_f$ (MeV) | $R_{ts}$ (10$^4$ km) | $\rho_f$ (10$^3$ gm/cm$^3$) | $S_f$ | Shock jump factor ($\rho_2/\rho_1$) |
|-------------|-----------------------|--------------------------|-----|-------------------------------|
| 1           | 0.18                  | 58.54                    | 52.88 | -                            |
| 2           | 0.16                  | 39.9                     | 50.59 | -                            |
| 3           | 0.11                  | 12.10                    | 48.61 | -                            |
| 4           | 0.068                 | 3.40                     | 48.22 | -                            |
| 5           | 0.068                 | 3.23                     | 48.21 | -                            |
| 6           | 0.0665                | 3.20                     | 48.19 | -                            |
| 7           | 0.0664                | 5.5                      | 48.13 | 1.13                         |
| 8           | 0.0661                | 6.5                      | 48.13 | 1.24                         |
| 9           | 0.0658                | 7.5                      | 48.37 | 1.46                         |
| 10          | 0.0653                | 8.5                      | 48.56 | 1.67                         |
| 11          | 0.0648                | 10.5                     | 49.44 | 2.04                         |
| 12          | 0.062                 | 13.5                     | 50.98 | 2.50                         |
| 13          | 0.057                 | 20.0                     | 55.57 | 3.25                         |
| 14          | 0.051                 | 30.0                     | 61.85 | 3.93                         |
| 15          | 0.047                 | 40.0                     | 67.87 | 4.38                         |

TABLE I. Table of values for the termination shock radius $R_{ts}$ vs. far-end temperature ($T_f$) boundary conditions. Final density ($\rho_f$), entropy per baryon ($S_f$) and the shock jump factor ($\rho_2/\rho_1$) are also shown. The following parameters are held fixed: $L_{\nu e} = 8 \times 10^{51}$ erg/s, $\epsilon = 20$ MeV, $R = 20$ km, $M = 1.4 M_\odot$. Empty entries imply that for those sets of boundary conditions, there is no termination shock formation.

Appendix 2. —

Fig. 4 shows an extended version of Fig. 2 in the main text, with the four panels depicting the profiles of the speed ($v$), entropy ($S$), temperature ($T$) and density ($\rho$) of the neutrino-driven outflows as the far boundary condition $T_f$ is varied. For completeness, the input parameters used for the simulation are $L_{\nu e} = 8 \times 10^{51}$ erg/s, $\epsilon = 20$ MeV, $M = 1.4 M_\odot$ and $R = 20$ km.

The top panel and the bottom panels are the same ones as in Fig. 2. The second panel shows the evolution of $S$. Additional entropy generation at the termination shock is clearly visible. This entropy generation is expected at termination shocks. Its possible impact on the r-process nucleosynthesis has been investigated in [32].

The third panel shows the evolution of temperature ($T$) for both subsonic flows and flows with termination shocks. Since the pressure is dominated by radiation, the temperature is also a proxy for pressure, $P \propto T^4$. This panel thus illustrates pressure matching at our prescribed far boundary of $10^4$ km to the pressure of the “bump” indicated with the vertical dashed line. The existence of the critical temperature ($T_{crit}$), separating breeze and wind solutions is clearly visible.

Comparison of the third and fourth panels shows that the density profile has additional features compared to the temperature profile, indicating the presence of layers with different entropies per baryon. The boundary condition for density $\rho_f$ can be obtained from far boundary pressure ($\propto T^4$) by assuming $S \sim 50$.

The mass of the progenitor for which the critical curve is achieved is roughly estimated to be $\sim 12 M_\odot$ in this calculation. The exact value of the progenitor mass above which termination shocks will not form will depend on $L$,
\( \epsilon, M \) and \( R \). If \( L \) is extremely high, termination shocks can form in higher mass progenitors of \( M \sim 15M_\odot \) as has already been seen in numerical simulations \([32]\).

We emphasize that the outflows in the supernova are found to be near critical, that is, on the edge of forming the shocks for typical conditions that are present several seconds into the explosion. We hope that the novel scaling law derived in this paper can find applications to other astrophysical environments, such as binary mergers where the conditions are inherently anisotropic.

![Graph](image_url)

**FIG. 4.** Formation of the shock as a function of temperature at large radius. The four panels show the evolution of speed, entropy, temperature and density of the outflow as the far temperature b.c is varied. It is clear that there exists a \( T_{f, \text{crit}} \) below which termination shocks start to form.
As described in the main text, by numerically solving Eqs. (1), (2) and (3) on a grid of points, we established the dependence of the critical far density on the physical parameters of the problem: $L$, $\epsilon$, $M$, and $R$. Here, we give a illustration of this process. Fixing the starting radius to $R = 20$ km and the average neutrino energy to $\epsilon = 20$ MeV, we explore the relationship between luminosity and density at criticality. The results are shown in Fig. 5 where the points show numerically determined critical parameters and the lines show the inferred power law fit. The quality of the fit given in Eq. (5) in the main text and Eq. (7) in Appendix 1 is apparent. The two sets of points correspond to the values of $M = 1.4M_\odot$ and $M = 2.0M_\odot$.

For luminosity values below the critical points, the solution is subsonic, while above it is supersonic, with a termination shock.

The power law can also be understood analytically, as will be discussed in detail in a followup paper.

**FIG. 5.** Critical luminosity vs. far end density ($\rho_{PS}$). Below the critical values, the outflow solutions are subsonic while above it, the solutions are supersonic and forms termination shocks. The points show numerically determined critical parameters and the lines show the inferred power law fit. The two sets of points illustrate the dependence on the PNS mass values.