Spontaneous CP violation in the Supersymmetric Higgs Sector

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Abstract

Spontaneous CP–violation in the minimal supersymmetric standard model with a gauge singlet and a cubic superpotential is examined. Although the tree–level Higgs potential conserves CP, it is shown that with the inclusion of the one–loop top–quark radiative effects, CP may be broken spontaneously. The CP–violating minimum requires two neutral ($h_1, h_2$) and one charged ($H^\pm$) Higgs–bosons to be relatively light with $m_{h_1} + m_{h_2} \lesssim 100 \text{ GeV}$ and $m_{H^\pm} \lesssim 110 \text{ GeV}$. The electric dipole moment of the electron is in the observable range of $(\frac{1}{3}$ to 3) $\times 10^{-27}$ e-cm.

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There are a number of reasons why it is interesting to study CP violation arising from the Higgs sector. Such CP violation could lead to large and soon–observable electric dipole moments for the electron and the neutron [1]. It may have a role to play in baryogenesis in the early universe [2]. It is even possible that this may be the origin of the CP violation observed in the Kaon system [3]. And such Higgs sector effects may be readily studied at present and future colliders.

In this Letter we explore CP violation phenomenology in the Higgs sector of low–energy supersymmetry, in the simplest model where such effects can appear [4]. That model is the minimal supersymmetric standard model (MSSM) to which a gauge–singlet Higgs superfield, $N$, has been added with the requirement that the superpotential contain only cubic terms [5]. There is no significant CP violation in the parameters of the Higgs effective potential (see below), so that any CP violation in the Higgs sector of the model must be spontaneous.

One of the main results we report is that spontaneous CP violation is indeed possible in this cubic singlet model. It has been shown by Ramao [6] that at tree–level spontaneous CP violation does not occur in this model. What allows spontaneous CP violation to occur in our case, in spite of Ramao’s theorem, is the radiative contribution to the effective potential due to a heavy top quark [7]. A striking feature of this model is that if CP is spontaneously broken then two of the neutral Higgs must be quite light. This is actually connected to Ramao’s theorem [6]: as the top–quark radiative effects are “turned off” the mass–squared of one (or more) of these light Higgs goes negative, so that there remains no CP–violating minimum. We show
that at best the masses of the two lightest neutral scalars add up to not much more than $M_Z$. In addition, the mass of the charged Higgs is found to be less than about 110 GeV. In this scenario the prospects for detecting these Higgs particles at LEP and LEP-200 are very good indeed. Moreover, since two of the neutral Higgs fields are light, the electron electric dipole moment (EDM) comes out in the experimentally accessible range of $(10^{-28}$ to $3 \times 10^{-27})$ e-cm (typically).

Only the radiative effects of the top–quark will be significant, so that the relevant terms in the superpotential are

$$W = \lambda H_1 H_2 N + \frac{1}{3} k N^3 + h_t Q H_2 T^c.$$  \hspace{1cm} (1)

Here $H_1 = \begin{pmatrix} H_1^0 \\ -H_1^- \end{pmatrix}$, $H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}$, so that $H_1 H_2 = \epsilon^{ab} H_a^1 H_b^2 = (H_1^0 H_2^0 + H_1^- H_2^+)$. The scalar potential for the fields $H_1$, $H_2$ and $N$ is then given by

$$V = V_0 + V_{\text{top}},$$

where

$$V_0 = \frac{1}{8} (g_1^2 + g_2^2)(|H_1|^2 - |H_2|^2)^2 + \frac{1}{2} g_2^2(|H_1|^2|H_2|^2 - |H_1 H_2|^2) + \lambda^2 \left( |H_1 H_2|^2 + |N|^2 (|H_1|^2 + |H_2|^2) \right) + k^2 |N|^4 + \lambda k (H_1 H_2 N^2 + H.c.) + \lambda A \lambda (H_1 H_2 N + H.c.) + \frac{1}{3} k A_k (N^3 + H.c.) + m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + m_3^2 |N|^2$$ \hspace{1cm} (2)

$$V_{\text{top}} = \frac{3}{16\pi^2} \left[ \left( h_t^2 |H_2|^2 + M_{sq}^2 \right) \ln \left( \frac{(h_t^2 |H_2|^2 + M_{sq}^2)}{Q^2} \right) - \left( h_t^2 |H_2|^2 \ln \left( \frac{h_t^2 |H_2|^2}{Q^2} \right) \right) \right].$$ \hspace{1cm} (3)

Here $|H_1|^2 \equiv H_1^\dagger H_1$ etc. $V_{\text{top}}$ denotes the leading–log top–quark one–loop radiative correction. We have assumed degenerate squarks, $M_{tiL}^2 = M_{tiR}^2 = M_{sq}^2 \gg (174 \text{ GeV})^2$. 

3
It is easy to see that all the parameters in eq. (2) can be made real by field redefinitions, except the ratio \( r \equiv A_\lambda / A_k \). We assume that parameter, too, is real. (It would be approximately so in most realistic scenarios [8].) To examine the CP–violating minimum, let us define

\[
D = \lambda k \\
E = \lambda A_\lambda / |N| = r \lambda A / |N| \\
F = \frac{1}{3} k A |N| / |H^0_1 H^0_2| .
\]

(4)

Here and throughout \( A \equiv A_k \). Let us further define the phases of the fields \( H^0_1, H^0_2 \) and \( N \) as follows:

\[
\text{arg}(H^*_1 H^*_2 N) = \eta \\
\text{arg}(N^3) = \zeta .
\]

(5)

There are three terms in eq. (2) that depend on \( \eta \) and \( \zeta \). Extremizing the potential with respect to these phases gives the CP–violating solution to be

\[
\cos \eta = \frac{1}{2} \left( \frac{DF}{E^2} - \frac{D}{F} - \frac{F}{D} \right) \\
\cos \zeta = \frac{1}{2} \left( \frac{DE}{F^2} - \frac{D}{E} - \frac{E}{D} \right) \\
\cos(\eta - \zeta) = \frac{1}{2} \left( \frac{EF}{D^2} - \frac{E}{F} - \frac{F}{E} \right) \\
E \sin \eta = -F \sin \zeta
\]

(6)

These equations can be represented graphically by the triangle in fig. 1. Equations (6) and (7) are respectively the law of cosines and the law of sines.

Denoting the expectation values (VEVs) of the neutral fields by \( \langle H_1 \rangle = \)
$v_1$, $\langle H_2 \rangle = v_2$ and $\langle N \rangle = x$, one can expand the fields around their minimum.

$$H_1 = v_1 + \frac{1}{\sqrt{2}} \frac{v_1}{|v_1|} \left[ \Phi_1 - i \frac{|v_2|}{v} \Phi_3 \right]$$

$$H_2 = v_2 + \frac{1}{\sqrt{2}} \frac{v_2}{|v_2|} \left[ \Phi_2 + i \frac{|v_1|}{v} \Phi_3 \right]$$

$$N = x + \frac{1}{\sqrt{2}} \frac{x}{|x|} [\Phi_4 + i \Phi_5]$$

with $v^2 = |v_1|^2 + |v_2|^2$. In the absence of CP-violation, $\Phi_{1,2,4}$ would be scalars and $\Phi_{3,5}$ pseudoscalars. One pseudoscalar gets eaten by the $Z$. (We are working in the unitary gauge.)

It is straightforward to expand the potential in eq. (2)-(3) about the CP-violating extremum to obtain the mass-squared matrix of $\Phi_i$, $i = 1, 2\ldots, 5$. In doing this it is convenient to adopt a common approach of choosing $Q^2$ in eq. (3) so that at the minimum of $V_0$ the relation $V'_\text{top} = 0$ is satisfied. (Then $V_\text{top}$ does not contribute explicitly to the equations for $v_1, v_2$ and $x$, it only corrects the (22) entry of the Higgs mass-squared matrix.) The mass-squared matrix elements are

$$M^2_{11} = \lambda_1 v_1^2 + 3r\lambda^2 v_2^2$$

$$M^2_{22} = (1 + \Delta)\lambda_1 v_2^2 + 3r\lambda^2 v_1^2$$

$$M^2_{33} = 3r\lambda^2 v^2$$

$$M^2_{12} = -\lambda_1 v_1 v_2 + (2 - 3r)\lambda^2 v_1 v_2$$

$$M^2_{13} = M^2_{23} = 0$$

$$M^2_{44} = r(4r - 1)A^2 + 36r^2\lambda^2 v_1^2 v_2^2/x^2 + 4r(6r - 1)\lambda A(v_1 v_2/x)\cos \eta$$

$$M^2_{55} = 3rA^2 + 12r\lambda^2 v_1^2 v_2^2/x^2 + 12r\lambda A(v_1 v_2/x)\cos \eta$$

$$M^2_{45} = 4r\lambda A(v_1 v_2/x)\sin \eta$$
\begin{align*}
\mathcal{M}_{14}^2 &= -r \lambda A v_2 \cos \eta + 2 \lambda^2 x v_1 - 6 r \lambda^2 v_2^2 v_1/x \\
\mathcal{M}_{24}^2 &= -r \lambda A v_1 \cos \eta + 2 \lambda^2 x v_2 - 6 r \lambda^2 v_2^2 v_2/x \\
\mathcal{M}_{34}^2 &= r \lambda A \sin \eta \\
\mathcal{M}_{15}^2 &= -3 r \lambda A v_2 \sin \eta \\
\mathcal{M}_{25}^2 &= -3 r \lambda A v_1 \sin \eta \\
\mathcal{M}_{35}^2 &= -3 r \lambda A \cos \eta - 6 r \lambda^2 v_1 v_2 v/x
\end{align*}

where $\lambda_1 = \frac{1}{2}(g_1^2 + g_2^2) = M_Z^2/v^2$, $A \equiv A_k$ and $r A \equiv A_\lambda$. Here and henceforth

$v_1, v_2$ and $x$ stand for the absolute values of the VEVs. $\Delta$ in the second line of eq. (9) is the top–quark radiative correction defined to be

$$
\Delta = \frac{1}{\lambda_1} \frac{3 h_4^2}{4 \pi^2} \left( \ln \left( \frac{M_{sq}^2}{m_t^2} \right) + p \right). \tag{10}
$$

The parameter $p$ represents possible non–logarithmic corrections which may be significant if $\tilde{t}_L - \tilde{t}_R$ mixing is not negligible [9]. The physical charged Higgs boson mass is given by

$$
m_{H^\pm}^2 = M_W^2 + (3 r - 1) \lambda^2 v^2. \tag{11}
$$

Since $\lambda_1$ and $v = \sqrt{v_1^2 + v_2^2} = 174 \text{ GeV}$ are known, our parameter set is $
\{\tan \beta, x, A, r, \lambda, \cos \eta, \Delta\}$ where $\tan \beta = v_2/v_1$. (We have used eqs. (4) and (6) to eliminate $k$ in favor of the angle $\cos \eta$.)

Since two of the neutral Higgs bosons ($h_1$ and $h_2$) are relatively light in this model, their masses and mixings are constrained by the LEP data on Higgs searches. In fig. 2 is displayed a plot of the experimentally allowed region in the $\lambda - \cos \eta$ plane for fixed values of the other parameters. The first constraint is that $h_1$ and $h_2$ not have been produced in the decay of a
real $Z$ [10]. Since there is no significant suppression in the $Zh_1h_2$ coupling,
$Z \rightarrow h_1 + h_2$ should be kinematically forbidden. The bottom (small $\lambda$)
boundary of the allowed region in fig. 2 corresponds to $m_{h_1} + m_{h_2} = M_Z$. The
second constraint is that a light Higgs particle not have been produced in the
decay $Z \rightarrow Z^*h$. If $h = \sum_{i=1}^{5} \alpha_i \Phi_i$, where $\sum_i |\alpha_i|^2 = 1$, then the cross section
for the process is approximately proportional to $|\alpha_1 \sin \beta + \alpha_2 \cos \beta|^2 m_h^{-1}$. The
condition we have imposed, that gives the upper (large $\lambda$) boundary of the
allowed region is [10] $m_h > (60 \text{ GeV})|\alpha_1 \sin \beta + \alpha_2 \cos \beta|^2$. In addition, the
‘pseudoscalar’ should be heavier than 20 GeV [11], which is always satisfied
in the fit of fig. 2 (see Table 1).

The allowed region in fig. 2 corresponds to the fixed parameters having
the values $\tan \beta = 1, r = 0.8, A = 3v, x = 3.8v$ and $\Delta = 3.8$. This is
a relatively large value for $\Delta$, but is realized if the top Yukawa is large,
near its renormalization-group fixed point. For example, if $M_{sq} = 3 \text{ TeV}$,
$h_t = 1.05$ (at $Q^2 = (3 \text{ TeV})^2$) and $p = 1$ in eq. (10), we obtain $\Delta = 3.8$ at
$Q^2 = m_t^2$. (This involves a further running from $Q^2 = M_{sq}^2$ to $m_t^2$ assuming
non–SUSY two–doublet spectrum.) A larger value of $h_t$ will be inconsistent
with perturbative unification (in the desert scenario), so a larger $\Delta$ seems
to be unlikely, unless the squark masses are heavier than 3 TeV. If $\Delta$ is
taken to be smaller (with the other parameters fixed at the above values),
the allowed region rapidly shrinks and vanishes at $\Delta \simeq 3.1$ corresponding to
$M_{sq} \simeq 1.5 \text{ TeV}$ (with the same $h_t$). Similarly, if $\tan \beta$ is increased above 1, the
allowed region shrinks and disappears at $\tan \beta \simeq 1.3$. In this scenario of CP–
violation, then, the parameters $\Delta(M_{sq}, h_t)$ and $\tan \beta$ are tightly constrained.
The same is seen from fig. 2 to be true of the parameter $\lambda$ which must lie
in a rather narrow range (around 0.2 for \( r = 0.8 \)). Numerically it is found that as \( r \) decreases the allowed range of \( \lambda \) shifts upward, and vice versa. On the other hand, the parameters \( r, A, x \) and \( \cos \eta \) can vary over a rather wide range. For example, we find solutions for \( 0.5 \leq r \leq 2.7 \). Much of the above behavior can be understood fairly well analytically.

Consider certain subdeterminants of the 5×5 mass–squared matrix given in eq. (9). If the CP–violating extremum is to be a local minimum, this matrix must have no negative eigenvalues and its subdeterminants must be all non–negative.

\[
\det_{1235} = 9\lambda^2 r^2 A^2 \sin^2 2\beta \sin^2 \eta \sqrt{v^6} \left\{ \frac{1}{2} \lambda_1 \left[ \frac{1}{2} \lambda_1 \Delta - 2(3r - 1)\lambda^2 \right] - \lambda^4 (3r - 1)^2 \right\} \\
\det_{345} = 3\lambda^2 r^3 A^2 x^{-2} (3r - 1) \sin^2 \eta \left\{ 9\lambda^2 \sin^2 2\beta v^6 + 4A^2 x^2 v^2 + 12\lambda A \sin 2\beta \cos \eta x v^4 \right\}
\]

where \( \det_{i,j..k} \) is the determinant of the submatrix corresponding to the \( ij..k \)th rows and columns. Since \( r \geq 0 \) from \( M^2_{33} \), eq. (13) implies that \( (3r - 1) \geq 0 \). If one sets \( \Delta = 0 \), then the right hand side of eq. (12) with \( (3r - 1) \geq 0 \) is negative. This is the result of Ramao [6], that at tree–level spontaneous CP violation does not occur in the model.

Eqs. (12)-(13) allow us to place bounds on \( \lambda^2 \) if CP is spontaneously broken in the Higgs sector.

\[
0 \leq (3r - 1)\lambda^2 \leq \frac{1}{2} \lambda_1 \left( \sqrt{1 + \Delta} - 1 \right) .
\]

One also sees from eq. (12) and (13) that the light Higgs masses are made largest by making \( \sin 2\beta \) large, that is, \( \tan \beta \simeq 1 \) is preferred. As a consequence of eq. (14), the charged Higgs boson mass is predicted to be less than
about 110 GeV (see eq. 11).

More insight can be obtained by considering the small \( \lambda \) limit, which is justified by eq. (14) and by our numerical results. As \( \lambda \to 0 \), the mixing between the \( \Phi_{1,2,3} \) block and \( \Phi_{4,5} \) block goes away. If one block diagonalizes to obtain the \( 3 \times 3 \) block to order \( \lambda^2 \) one finds

\[
\mathcal{M}^2(\Phi_1, \Phi_2, \Phi_3) = \begin{pmatrix}
\lambda_1 v_1^2 & -v_1 v_2 (\lambda_1 + 2(3r - 1)\lambda^2) & 0 \\
0 & \lambda_1 v_2^2 (1 + \Delta) & 0 \\
0 & 0 & 0
\end{pmatrix}
+ C \lambda_1 \begin{pmatrix}
v_2^2 \cos^2 \eta & v_1 v_2 \cos^2 \eta & -v v_2 \sin \eta \cos \eta \\
v_1 v_2 \cos^2 \eta & v_1^2 \cos^2 \eta & -v v_1 \sin \eta \cos \eta \\
-v v_2 \sin \eta \cos \eta & -v v_1 \sin \eta \cos \eta & v^2 \sin^2 \eta
\end{pmatrix}
\]

(15)

where \( C \equiv [4r(3r - 1)/(4r - 1)] (\lambda^2/\lambda_1) \). From the eigenvalues of \( \mathcal{M}^2(\Phi_1, \Phi_2, \Phi_3) \) a non–trivial lower bound on \( \lambda^2 \) can be derived as follows. Label the eigenvalues to be \( M_{I}^2 \leq M_{II}^2 \leq M_{III}^2 \). It is possible by considering the eigenvalues as a function of \( C \) (defined above) to show that \( M_{I}^2 \leq CM_{Z}^2 = C \lambda_1 v^2 \). (This can be done by plotting the eigenvalues of \( \mathcal{M}^2(C)/M_{Z}^2 \) versus \( C \) and considering how often they cross the line \( y = C \) which can be determined from the equation \( \det \{ \mathcal{M}^2(C)/M_{Z}^2 - CI \} = 0 \). This is easily seen to be quadratic (not cubic) in \( C \) and to have two real roots, one of which is \( C = 0 \).) From eq. (15) one sees that for small \( \lambda^2 \) and \( \Delta \geq \cot^2 \beta - 1 \) the largest entry of \( \mathcal{M}^2 \) is greater than \( (1 + \Delta)\lambda_1 v_2^2 = (1 + \Delta)M_{Z}^2 \sin^2 \beta \). Therefore \( M_{III}^2 \geq (1 + \Delta)M_{Z}^2 \sin^2 \beta \).

The trace of \( \mathcal{M}^2 \) implies that

\[
M_{I}^2 + M_{II}^2 + M_{III}^2 = M_{Z}^2 (1 + \Delta \sin^2 \beta + C).
\]

Thus \( M_{I}^2 + M_{II}^2 \leq M_{Z}^2 (C + \cos^2 \beta) \). This together with \( M_{I}^2 \leq CM_{Z}^2 \) imply that if \( C \leq \cos^2 \beta \), then \( (M_{I} + M_{II})/M_{Z} \leq \sqrt{C + \cos \beta} \), while if \( C \geq \cos^2 \beta \), then \( (M_{I} + M_{II})/M_{Z} \leq \sqrt{2C + 2\cos^2 \beta} \). In order to have \( M_{I} + M_{II} \geq M_{Z} \) it then must be that \( C \geq (1 - \cos \beta)^2 \). Using the definition of \( C \) one gets a lower
bound on $\lambda^2$, which combining with eq. (14) gives

$$\left(\frac{4r - 1}{4r}\right) \lambda_1 (1 - \cos \beta)^2 \leq (3r - 1) \lambda^2 \leq \frac{1}{2} \lambda_1 (\sqrt{1 + \Delta} - 1). \quad (16)$$

For the values of $r = 0.8, \Delta = 3.8, \tan \beta = 1$ that are used in fig. 2 one has $0.13 \leq \lambda \leq 0.41$ which is seen to be satisfied well by the numerical results. One can also derive from eq. (16) a better lower limit on $r$ than $r \geq 1/3$.

A remark about the small $\lambda$ limit is in order. From eq. (4) and fig.1 it can be seen that as $\lambda \to 0$, $x/A \to r/k$. ($k$ in fig. 2 is about 0.63, the fixed point value.) Terms have been dropped in deriving eq. (15) that are down by $O(\lambda x/A)$. For large $r$, these terms become important, which tend to decrease the light eigenvalues of $M^2$. This is why no realistic solution exists for large $r$ ($r \gtrsim 2.7$).

In fig. 2 we have also plotted contours of the electric dipole moment of the electron. These have been computed using the results of Leigh, Paban and Xu (LPX) [1]. Only the graphs with the same topology as the top quark graphs (figs. 1 and 2 of LPX), and only the contributions from the lightest two neutral Higgs have been included. This should be good to about 20%. The typical values of $d_e$ are in the range $(10^{-28}$ to $3 \times 10^{-27}$) e-cm, interesting for future atomic experiments. In Table 1 are presented the masses and mixings of the lightest Higgs for various $\lambda$ and $\cos \eta$ in fig. 2 from which we have computed $d_e$.

It is interesting to ask if the Higgs sector CP–violation alone can account for $\epsilon$ and $\epsilon'$ in the K meson system. Pomarol [3] has shown that this may indeed be possible in a more generalized version of the SUSY singlet model. Since the effects of spontaneous CP violation felt in the fermion sector in our
model is identical, we expect the results of Ref. 3 to hold in our case as well.

As for the neutron edm ($d_n$), although it is difficult to predict its value precisely due to strong interaction uncertainties, we expect $d_n$ to be near the experimental limit, $d_n \sim 10^{-26}$ e-cm. Indeed, if the only source of CP-violation is in the Higgs sector, then the usual fine-tuning of the phase of the gluino mass term can be avoided. This phase will be zero at tree-level, and will arise only at one-loop, giving experimentally consistent value for $d_n$.

So far we have said nothing about whether the CP-violating minimum that we have been studying is also the absolute minimum. For fixed $|H_1|, |H_2|$ and $|N|$ it is easy to see that $V$ at the CP-violating extremal angles (eq. 6) is lower than at the CP conserving values $\eta, \zeta = 0, \pi$. Of course the true CP-violating and CP-conserving minima will not be at the same values of $H_1, H_2$ and $N$. However, it can be shown analytically that for small $\lambda$ the CP-violating minimum is the lowest [9]. We have verified this result numerically for some reasonable values of the parameters.

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Figure Captions

Fig. 1. This triangle shows the relation between the extremal values of the phase angles $\eta$ and $\zeta$ and the magnitudes of the fields and the parameters of the Higgs potential. If $D, E$ and $F$ are not in such a relation that this triangle can be drawn, then there is no CP–violating extremum.

Fig. 2. A plot of the allowed region (inside the closed curves) of the $(\lambda - \cos \eta)$ plane, for $\tan \beta = 1, r = 0.8, A = 3v, x = 3.8v$ and $\Delta = 3.8$. The lower boundary of the region corresponds to $m_{h_1} + m_{h_2} = M_Z$. The upper boundary is the constraint from non–observation of $Z \rightarrow Z^* + h$. The contours inside the region represent the electron EDM in units of $10^{-27}$ e-cm.

Table 1. Some selected points in the $(\lambda - \cos \eta)$ plane for the same parameter values as in fig. 2. $m_{h_1}$ and $m_{h_2}$ are the masses (in GeV) of the lightest two neutral Higgs $h_1$ and $h_2$. The $R_{ij}$ are defined by $h_i = \sum_{i=1}^{5} R_{ij} \Phi_j$. The electron EDM $d_e$ is in units of $10^{-27}$ e-cm.

| $\cos\eta$ | $\lambda$ | $m_{h_1}$ | $m_{h_2}$ | $R_{11}$ | $R_{12}$ | $R_{13}$ | $R_{21}$ | $R_{22}$ | $R_{23}$ | $d_e$ |
|------------|-----------|-----------|-----------|---------|---------|---------|---------|---------|---------|------|
| -0.5       | 0.235     | 34.3      | 56.9      | -0.78   | -0.29   | 0.55    | 0.17    | 0.83    | -10.0   |      |
| 0.0        | 0.17      | 49.5      | 41.6      | -0.92   | -0.29   | 0.25    | 0.08    | 0.97    | 1.75    |      |
| 0.0        | 0.21      | 42.5      | 53.3      | 0.88    | 0.31    | 0.35    | -0.33   | -0.11   | 0.94    | 3.0   |
| 0.5        | 0.195     | 59.2      | 32.0      | -0.82   | -0.24   | 0.52    | 0.49    | 0.18    | 0.85    | 11.5  |
| 0.5        | 0.225     | 63.2      | 30.6      | -0.73   | -0.21   | 0.65    | 0.61    | 0.23    | 0.76    | 15.2  |
| 0.5        | 0.253     | 68.5      | 26.2      | -0.67   | -0.19   | 0.74    | 0.68    | 0.27    | 0.68    | 21.5  |
| 0.67       | 0.223     | 66.7      | 24.5      | -0.81   | -0.23   | 0.55    | 0.51    | 0.20    | 0.84    | 19.8  |