Implementation of controlled multi-qudit operations for a solid-state quantum computer based on charge qudits

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(Dated: April 1, 2022)

We consider a mechanism to generate controllable qudit-qudit interactions in a charge-position paradigm for a quantum computer, through the use of auxiliary states. By controlling the tunneling rates onto these auxiliaries from the qudits proper, we can controllably switch the entangling operations. We consider a practical architecture in which to realize such a computer and examine the associated Hilbert space dimension.

PACS numbers: 03.67.Lx,85.35.Be

Quantum computing has been identified as an important field recently, and significant work is being undertaken to find suitable systems in which to observe scalable, coherent interactions. Recent advances in semi-conductor technology have opened up new possibilities for silicon-based solid-state realizations of quantum computers, which are highly attractive due to their compatibility with conventional Silicon metal-oxide-semiconductor technology. All scalable quantum computing proposals require some particle that can be placed into a superposition of several distinguishable quantum states, and for which particle-particle entanglement can be achieved. The Hilbert space dimension of an $N$ particle quantum register obtained by entangling all the particles simultaneously has been shown to be a good measure for the power of a quantum computer.

One model for a scalable quantum computer is based on localization of an electronic charge in potential wells, the so-called charge-qubit approach. This has been discussed by Ekert and appears in various flavors in the literature, see for example Refs. and. A generic problem with charge-based quantum computing schemes is that, although it is relatively easy to generate single particle operations, i.e., to realize the Hadamard transform gate, it is usually quite difficult to obtain controllable particle-particle interactions. The mechanism normally suggested is the Coulomb interaction, which has the problem of being difficult to turn off. It may be possible to perform universal operations with an “always on” interaction, and schemes for realizing global operations in this setting have been considered by Benjamin and Pachos and Vedral. However, for practical purposes it seems highly desirable to have controllable multi-particle interactions. Here we suggest ways to achieve this by making use of local tunneling to auxiliary states, which has the effect of switching the Coulomb action by changing the effective distance between components of the electronic wavefunction.

The basic schemes we propose are in principle not limited to a particular charge-based implementation, and may in fact be useful for a variety of charge-based proposals including, for instance, Cooper-pair box schemes. However, for concreteness we consider a generalization of a specific recent proposal by Hollenberg et al. where the confining potentials are obtained from individual Phosphorus implants in a Silicon matrix. We envisage this being fabricated via a “bottom up” approach to nanofabrication such as has been recently realized. Hollenberg’s charge proposal deserves particular consideration in our opinion since it takes advantage of existing fabrication technologies and medium-scale realizations of such an architecture appear to be within experimental reach in the near future. Although the original proposal involved charge qubits, we shall consider a generalization to qudits (i.e., systems whose single particle Hilbert space has dimension $D \geq 2$), and concentrate on qutrit structures ($D = 3$) as they have been shown to optimize the total Hilbert space dimension of the composite system. We propose concrete scalable architectures that permit efficient, controlled multi-particle gates in this setting, which is essential for the operation of quantum error-correction algorithms.

The donor impurities in the Hollenberg et al. scheme can be regarded as quantum dots that create an electric potential with $D$ local minima located at the sites of the donor impurities, which confines the shared electron as shown in Fig. 1. The $D$ ground states $|d\rangle$ for $d = 1, \ldots, D$, corresponding to the various localizations of the electron at the donor impurities, form a basis for the Hilbert space $H$ of a single charge qudit. The height of the potential barrier between a pair of adjacent quantum dots belonging to the same qudit can be manipulated by applying an external electric potential via surface electrodes located between the two sites. By adjusting the voltages applied we can manipulate the height of the potential barrier and therefore the rate of tunneling between.
FIG. 1: Schematic of potential created by three donor impurities forming a charge qutrit with ground state energy levels of the shared electron, shown here localized in the left well, as well as a S-gate above the center dot and two B-gates between dots 1, 2 and 2, 3, respectively.

FIG. 2: Scalable arrangement of quantum dots partitioned into qutrits with fixed, permanent Coulomb interaction between $|1\rangle$-states of all adjacent qudits.

adjacent sites. We follow the notation of Ref. [12] and refer to these surface electrodes as barrier or B-gates. Quantum tunneling through the potential barriers leads to the creation of coherent superposition states of the localized charge qudit states $|d\rangle$. Furthermore, by applying electric potentials to surface electrodes located directly above the donor impurities we can create asymmetries in the potential, and thus change the ground state energy $E_d$ of state $|d\rangle$, introducing local energy shifts. We shall call these electrodes (energy) shift gates, or S-gates. It can be shown that by combining S-gates and B-gates, arbitrary single qudit operations (local unitary operations) can be performed.

One of the main advantages of the proposed charge qudit architecture compared to the original Kane proposal [11] using the nuclear spins of the donor atoms as qubits is the possibility of easy readout of quantum information via single electron transistors [14], the suitability of which has already been shown for quantum computing applications when run in RF mode [15]. Moreover, charge qudit quantum logic operations can be implemented using voltage gates only, without the need for additional radio frequency control pulses. Besides reducing the operational complexity, this should largely avoid problems of non-selective excitation of multiple sites, which may complicate the implementation of selective single and multi-qubit quantum logic operations in the original Kane proposal.

One of the main disadvantages of charge qudits compared to nuclear spin qubits is their much shorter coherence lifetime. However, local operations can be performed much faster for charge qudits than for nuclear spin qubits, on the order of $10^{-11}$ seconds, and estimates suggest that the decoherence lifetime of a charge qudit should be much longer than that, at least on the order of 10 ns [9, 16]. It also seems likely that advances in technology and control will lead to further reductions of the gate operation time, and that the coherence lifetime might be increased though decoherence control measures. For instance, continuous quantum error correction by means of quantum feedback control has recently been shown to be able to reduce spontaneous emission errors [17], and similar control techniques might be able to reduce decoherence.

We present our proposal for a scalable charge-qudit quantum computer in Fig. 3. As mentioned above, we differentiate it from previous models by the use of (a) qutrits to optimise the Hilbert-space dimensionality, and (b) auxiliary dots to effectively switch the Coulomb interaction to mediate particle-particle interactions. The qudits are arranged vertically, alternatingly above and below a center row of auxiliary quantum dots which mediate interactions between multiple adjacent qudits. The alternating arrangement of the qudits above and below the row of auxiliary quantum dots reduces unwanted Coulomb interactions between quantum dots belonging to different qudits. The efficiency of this shielding effect can be further enhanced by adding “trenches” filled with a conducting material between adjacent qudits as shown in the figure. Since the donor impurities are buried, realizing such shielding trenches would require a 3D structure. One way to realize such a structure would be to metalize a sheet of delta-doped Silicon, which could be achieved with a bottom-up strategy [12]. The staggering of the qudits further reduces unwanted interactions between diagonally facing qudits. The efficiency of multi-qudit operations might be enhanced by embedding the auxiliary quantum dots into a high-permittivity material to facilitate the Coulomb interaction. This would require that the silicon substrate be replaced by another mate-
FIG. 3: Scalable arrangement of quantum dots partitioned into qutrits with auxiliary quantum dots to mediate the Coulomb interaction.

Material in the region occupied by the auxiliary quantum dots. This would be difficult to realize given current technology, however it may well become feasible with further advances in fabrication technology. The high permittivity material would reduce the time needed to implement multi-qudit gates, increasing the number of such operations that can be performed within the coherence lifetime of the qudit register, potentially improving the performance of a working quantum device.

To activate the Coulomb interaction between two or more adjacent qudits, we coherently transfer the populations of state $|1\rangle$ of the corresponding qudits to the auxiliary state $|0\rangle$ by simultaneously lowering the height of the potential barriers between the corresponding quantum dots by applying suitable gate voltages to the auxiliary tunneling gates. Then we restore the potential barriers and let the Coulomb interaction between the auxiliary quantum dots create the necessary controlled phase gate. After a certain predetermined time, the populations of the auxiliary quantum dots are coherently transferred back to the original qudit states by simultaneously lowering the corresponding auxiliary potential barriers, thus switching off the Coulomb interaction. The details of how to implement a sufficient set of elementary gates required for universal quantum computation and efficient qudit error correction will be discussed in a forthcoming, more detailed paper.

Note that this arrangement of the qudits is scalable and permits not only the implementation of arbitrary two-qudit gates between any pair of adjacent qudits by combining controlled phase gates with single qudit operations, but, in principle, the implementation of arbitrary gates involving a set of $k \leq N$ adjacent qudits. Furthermore, since the effects of the control fields (voltage gates) are strictly spatially confined, unlike for radio-frequency control fields for instance, it should be easily possible to implement multi-qudit interactions on disjoint subsets of qudits simultaneously.

One drawback of the proposed architecture is that it requires a comparatively large number of auxiliary sites to mediate the interactions. In principle, it is possible to reduce the number of auxiliary quantum dots required by modifying the architecture as shown in Fig. 4 for instance. In this modified arrangement the auxiliary sites are horizontally offset such that each auxiliary site is shared by four qudits. This reduces the number of auxiliary sites required by half. However, a potential drawback of this architecture is that, while it should be easy to implement four-qudit gates, it would appear to be difficult to implement controlled two-qudit interactions since the Coulomb interaction will always affect all four qudits surrounding an auxiliary quantum dot.

Finally, we consider the effect of the auxiliary quantum dots on the Hilbert space dimension of the register. As mentioned earlier, in absence of auxiliary quantum dots, partitioning into qutrits will always optimize the Hilbert space dimension of the quantum register. Clearly, given
any fixed number of quantum dots, the need for auxiliary quantum dots reduces the dimension of the Hilbert space of the quantum register. If one auxiliary site is needed for each qudit, as in the architecture shown in Fig. 3 for instance, then the dimension of the Hilbert space of a register of $K$ quantum dots partitioned into $D$-qudits will be $D^K/(D+1)$ instead of $D^K/D$. If only a single auxiliary site for every two qudits is needed as in Fig. 4 then the Hilbert space dimension of the register will be $D^K/(D+0.5)$ instead. Fig. 5 shows the Hilbert space dimension of a qudit register comprised of a fixed number of quantum dots as a function of qudit size for the three architectures discussed. The graph clearly shows that partitioning into qutrits maximizes the Hilbert space dimension of the quantum register if either no auxiliary quantum dots or only a single dot per pair of qudits is required. If one auxiliary dot for each qudit is used then partitioning into four-qudits increases the Hilbert space dimension slightly. However, this increase is rather small compared to the increase from qubits to qutrits and may be offset by other considerations such as the increased complexity of single qudit gates or the number of measurements required to extract a comparable amount of quantum information for $D > 3$.

In summary, we have proposed concrete scalable architectures for a charge qudit quantum computer that allow the efficient implementation of controlled multi-qudit gates by making use of auxiliary sites to mediate multi-qudit interactions. Our emphasis has been on the implementation of such schemes for a practical realization of a charge qudit quantum computer based on donor impurities embedded in a Silicon matrix, but they are not fundamentally limited to this specific case.

SGS and DKLO are supported by the Cambridge-MIT Institute project on quantum information. ADG is supported by the Australian Research Council.

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