Fully developed triplet proximity effect

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We present a model for fully developed triplet proximity effect in superconductor-ferromagnet heterostructures. Within the circuit-theory approximation, we evaluate the Green’s functions, the density of states, and the Josephson current that depend essentially on the magnetization configuration.

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Research of heterostructures that combine superconducting (S) and ferromagnetic (F) elements gives insight to the problem of mutual influence of superconductivity and ferromagnetism, allows realization of exotic superconducting states such as the L-O-F-F state and triplet ordering. Promises applications that utilize the spin degree of freedom. While this research has started more than three decades ago, interest to the above topics has yet stipulated recent important developments, both theoretical and experimental. Those concern Josephson $\pi$-junctions, triplet superconductivity, and Josephson spin valves.

The superconducting proximity effect is characteristic for most S/F heterostructures and is distinct from the effect in non-magnetic S/N systems. In the normal part of a S/N structure, the superconducting correlations persist at distances of the order of normal-metal coherence length $\xi_N$. This length scale can grow large at sufficiently small temperatures $T$. In a diffusive material, $\xi_N = \sqrt{\hbar D/2\pi k_B T}$, $D$ being the diffusivity. In contrast to this, superconducting correlations in a ferromagnet, where an exchange field $h$ is present, are quenched at much shorter scale $\xi_h = \sqrt{\hbar D/\hbar}$. Hence one might conclude that the proximity and Josephson effects are strongly suppressed in S/F heterostructures. However, some experiments seem to contradict this statement, indicating proximity correlations at much larger scale. Though these experiments may be explained by interface effects, they have motivated a proposal of an interesting mechanism for long-range proximity effect in ferromagnets. It was shown that inhomogeneity in the direction of exchange field generates superconducting correlations of two electrons with the same spin, i.e. triplet correlations. Such triplet proximity effect (TPE) is not suppressed by an exchange field and penetrates the ferromagnet at the scale of $\xi_N$. Recently, a substantial Josephson current has been reported for a fully polarized ferromagnet. The experiment can only be explained by TPE.

An immediate problem is that the theoretical predictions so far have been elaborated in assumption that TPE is weak and can be treated perturbatively. This makes it difficult to determine an unambiguous experimental signature of TPE to distinguish it from the conventional effect. Experimentally, the Josephson current due to TPE does not seem to be smaller than that due to a fully developed conventional proximity effect.

In this work, we address a fully developed TPE that significantly changes the density of states (DOS) at the Fermi level. We show that the DOS increases. This is in contrast with complete suppression of DOS by a fully developed conventional proximity effect. Similar to the conventional effect, the change in the DOS is restricted to the energy window $\simeq E_{Th}$, the Thouless energy of the structure, provided $E_{Th} \ll \Delta$, $\Delta$ being the energy gap in the superconductor. Therefore the corresponding Josephson current is of the same order as for the conventional proximity effect, though its magnitude essentially depends on the magnetization configuration in the structure. For magnets where both spin directions are present at the Fermi surface (ferromagnetic metals), we find both $\pi$-junctions (with negative supercurrents) and common 0-junctions (with positive supercurrents), depending on the magnetization directions. For fully polarized magnetic materials (half-metals), we find a continuous dependence of the equilibrium superconducting phase difference on magnetization directions. Finite current may be induced thereby at fixed zero phase difference.

We concentrate on a S/F/S heterostructure fabricated by deposition of two superconducting electrodes onto a conducting ferromagnetic film (Fig. 1a). We proceed with the so-called circuit theory that is a finite-element technique for semiclassical Green’s function method, which has been applied to S/F structures in Ref. 9. Circuit theory is convenient since we aim at presenting an idealized TPE without consideration and subsequent optimization of concrete geometry of the structure. Besides, it allows simple analytical presentation of the results.

We build a minimal circuit-theory model for a ferromagnetic metal, later adjusting it to a half-metal. The left and right parts of the structure contain regions where superconducting and magnetic correlations meet (points 1 and 3 in Fig 1.a). Following, we represent each region by a normal-metal node connected to a superconducting reservoir and a ferromagnetic insulating reservoir (FIR). The role of FIR is to represent the exchange field $h_k (k = 1, 2, 3)$ induced in the node. The middle of
ducting reservoir a unit vector \( n \) in FIRs. The magnetization direction in FIR equations is the matrix current into the corresponding \( \sigma \) are vectors composed of Pauli matrices in the Nambu relevant energies are much smaller than the superconducting TPE \([7]\). b) The corresponding circuit consists of three nodes representing the parts 1−3 of the structure. c) The nodes 1 and 3 are replaced by effective S/F reservoirs.

The two energy scales of the model are the typical exchange field \( h \) and the Thouless energy \( E_{Th} \approx g^0/\delta G_Q \). The condition \( h \leq E_{Th} \) corresponds to a short S/F structure with dimensions shorter or of the order of \( \xi_h \). In this case, the conventional proximity effect overshadows TPE; singlet anomalous components of \( G \) in all nodes either exceed or are of the same order as the triplet ones. Since we want to single out the TPE, we turn to the opposite limit \( E_{Th} \ll h \) of longer structures.

The best separation between the islands is achieved in the limit \( \delta_2 \gg \delta_{1,3} \), where the regions 1 and 3 adjacent to the superconductors are much longer than the middle part of the system. We will see that in this limit the superconducting correlations in the nodes 1 and 3 are between electrons of opposite spin. As to the node 2, the correlations are between electrons of the same spin only: there TPE is present in its purest form.

Under these conditions, the last term in the first Eq. \([1]\) can be neglected and \( G_{1,3} \) can be determined separately from \( G_2 \). Owing to their big size, the nodes 1 and 3 act as effective reservoirs for the node 2 (Fig. 1c). The Green’s functions at these nodes are determined by the competition between the corresponding superconducting reservoir and FIR and read \([3]\) \((k = 1,3; h_k > \Delta_k; \tau_\pm = [\tau_1 \pm i\tau_2]/2)\):

\[
G_k = \frac{h_k \tau_3 - i(\Delta_k^* \tau_+ + \Delta_k \tau_-)(n_k \cdot \sigma)}{\sqrt{h_k^2 - |\Delta_k|^2}}
\]

where we assume \( \epsilon \ll h_k, \Delta_k \). As seen from the structure of \([2]\), the effective S/F reservoirs supply superconducting correlations that are different for opposite spin directions. These correlations are most pronounced if \( h_k \approx \Delta_k \). We will see that this condition is optimal for TPE. At this stage of research it is difficult to immediately relate this condition to specifics of the structures in hand such as geometry, film thickness, transparency of S/F interface etc. Since the observations of \([2]\) suggest that TPE is close to the optimal one, we are convinced that this condition is realizable.

Now the node 2 is connected to reservoirs only. Its Green’s function is determined by the balance of the matrix currents into these reservoirs. The two S/F reservoirs are connected in parallel, so that their net effect is additive and can be represented by a matrix \( M = (g_1^0 G_1 + g_3^0 G_3)/(g_1^0 + g_3^0) \equiv M_0 \tau_3 - i(M^* \tau_+ + M \tau_-) \cdot \sigma \). It is important that the resulting \( G_2 \) splits into two independent blocks corresponding to two spin projections.
on \( n_2 \). The separation into blocks allows us to treat half-metals on equal footing with ferromagnetic metals. While for a ferromagnetic metal both blocks contribute to physical quantities, only a single block does so in a half-metal. It is implied that in any case the S/F reservoirs support both spin directions, otherwise they would not be superconducting.

We write the block structure as follows:

\[
G_2 = S \left[ \begin{array}{cc} G_\downarrow & 0 \\ 0 & G_\uparrow \end{array} \right] S^{-1},
\]

\[ S \equiv (1 - \tau_3)/2 - i \sigma_y (n_2 \cdot \sigma)(1 + \tau_3)/2. \]

In these notations,

\[
G_\uparrow = \frac{1}{\sqrt{A_1}} \left[ \begin{array}{cc} M_0 - i\epsilon/E_{Th} & -i(M_x^* - iM_y^*) \\ -i(M_x + iM_y) & -M_0 + i\epsilon/E_{Th} \end{array} \right],
\]

where \( A_1 \equiv (M_0 - i\epsilon/E_{Th})^2 - |M_x + iM_y|^2 \) and the \( z \) axis is chosen in the direction of \( n_2 \). The result for \( G_\downarrow, A_\downarrow \) is obtained by replacing \( M_x \pm iM_y \) with \( M_x \mp iM_y \). The advantage of such notations is that the block structure is made explicit. The non-diagonal elements of \( G_\uparrow \) correspond to triplet anomalous averages \( \langle \psi_\uparrow \psi_\downarrow \rangle, \langle \psi_\downarrow \psi_\downarrow \rangle \), whereas the singlet-pairing averages \( \langle \psi_\uparrow \psi_\uparrow \rangle, \langle \psi_\downarrow \psi_\downarrow \rangle \) vanish. This manifests a pure TPE. The fully developed effect is characterized by \( h_k \gtrsim |\Delta_k| \), in which case the non-diagonal and diagonal elements of \( G_\uparrow, G_\downarrow \) are of the same order of magnitude.

The triplet anomalous averages \( \langle \psi_\uparrow \psi_\downarrow \rangle \) acquire a phase factor \( e^{i\chi} \) upon rotation by an angle \( \chi \) about the \( z \) axis. This leads to an interesting interplay between the superconducting phase difference \( \phi_3 - \phi_1 \equiv \phi \) and the relative longitude angle between \( n_1 \) and \( n_3 \), \( \chi_3 - \chi_1 \equiv \chi \). To see this explicitly, we express the magnetization direction vectors \( n_1, n_3 \) in the spherical coordinates (\( \chi, \theta \) standing for the longitude and latitude respectively).

In these notations,

\[
M_x + iM_y = a_1 e^{i(\phi_1 + \chi_1)} + a_3 e^{i(\phi_3 + \chi_3)},
\]

where \( a_k \equiv g^0_k |\Delta_k| \sin \theta_k / (g^0_k + g^3_k) \sqrt{h_k^2 - |\Delta_k|^2} \), so that the superconducting phase and the longitude always come together.

The DOS is determined from \( G_2 \) and generally is different for the opposite spin directions.

\[
\nu_{\uparrow,\downarrow}(\varepsilon) = \nu_0^{\uparrow,\downarrow} \text{Re} \left( 1 - \frac{a_1^2 + a_3^2 + 2a_1 a_3 \cos(\phi \pm \chi)}{(M_0 - i\epsilon/E_{Th})^2} \right)^{-1/2},
\]

where \( \nu_0^{\uparrow,\downarrow} \) is the normal-state DOS. As shown in Fig. 2a), \( \nu \) always exceeds the normal-state value at \( \varepsilon \ll E_{Th} \). This is in contrast with a suppression of \( \nu \) manifesting the conventional proximity effect. Such an enhanced DOS is therefore a signature of TPE. The peak at small energies is followed by a wider dip at energies of the order of the Thouless energy \( E_{Th} \) so that the total number of states remains unchanged [Fig. 2b)]. The dependence of \( \nu \) on the superconducting phase/longitude is also characteristic of TPE. As seen from Eq. 6, for a given spin direction the effect of superconducting phase can be always compensated by a rotation of one of the magnetization directions \( n_1, n_3 \) about \( n_2 \). While for a ferromagnetic metal the total DOS \( \nu = \nu^\uparrow + \nu^\downarrow \) is an even function of \( \phi \) and \( \chi \) [Fig. 2b)] this generally not so for a half-metal. In the latter case, for a symmetric setup \( a_1 = a_3 \), the DOS \( \nu(0) \) can be modulated by either phase from 1 to its maximum value \( 1\sqrt{1 - 4a_1^2/M_0^2} \).

In the model under consideration, the Josephson current is given by the Keldysh component of the matrix current through either junction \( g_{1,3}^0 \) integrated over energy,

\[
I = -\frac{e \hbar}{4}\int_{-\infty}^{\infty} d\varepsilon \text{Re} \text{Tr} \tau_3 [G_1, G_2] \tanh \frac{\varepsilon}{2T}.
\]
The block structure of $G_2$ gives rise to two contributions to the current that correspond to opposite spin directions. The integral in Eq. (4) is logarithmic, converging at energies $E_{Th} \ll \epsilon \ll \Delta_{cut}$, $\Delta_{cut} \simeq \min(\Delta, h, h - \Delta)$. We assume $T \ll E_{Th}$. With logarithmic accuracy,

$$I_{r, \downarrow} = -\frac{2a_1a_3(g^u_n + g^d_n)}{e} \sin(\phi \pm \chi)E_{Th} \ln \left(\frac{\Delta_{cut}}{E_{Th}}\right) . \quad (8)$$

Both the scale of the current ($I \simeq gE_{Th}/e$) and the logarithmic structure are similar to the common proximity effect. The difference is the dependence of the current on the magnetization directions. For a ferromagnetic metal, the total current $I_I + I_\downarrow$ is odd in $\phi$. Interestingly, the sign of the Josephson current is opposite to that in a common Josephson contact provided $|\chi| < \pi/2$. This signals a $\pi$-junction [14], which can be changed to a common 0-junction by changing $\chi$. Since the accuracy of the logarithmic approximation is always questionable, we evaluate the integral numerically assuming $\Delta_{cut}/E_{Th} = 50$. The resulting dependence of the current on both phases is not precisely harmonic, though close to it. (Figs. 3). As seen in Fig. 3(b), the second harmonic of the current becomes dominant in the vicinity of $\chi = \pi/2$. This implies that the transition between ”$\pi$” and ”0” states follows a scenario of Ref. 16 where both states are stable in this vicinity.

For a half-metal, the situation is very different. In this case, the superconducting phase corresponding to the energy minimum just follows $\chi$ and changes continuously instead of ”jumping” between the values of 0 and $\pi$. There is a finite supercurrent at zero phase difference. Therefore, rotation of the magnetizations $n_{1,3}$ about $n_2$ is equivalent to the effect of extra magnetic flux $\chi\Phi_0/2\pi$ enclosed in a large loop that includes the junction. In our opinion, this facilitates an unambiguous experimental verification of TPE.

In both cases, not only the supercurrent is zero at the energetically favorable phase difference, but also TPE is reduced, vanishing completely for an ideally symmetric setup. This is because TPE increases the electron energy in the central node, as seen from the fact that the DOS is enhanced at low energies.

In conclusion, we have proposed a simple model for fully developed triplet proximity effect in S/F/S structures. In contrast to the common proximity effect, TPE enhances the DOS at low energies. The Josephson current exhibits a peculiar dependence on the magnetization configuration that is essentially different for a ferromagnetic metal and a half-metal. Those are signatures of TPE to be observed experimentally.

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FIG. 4: The Josephson current for a half-metal versus $\phi$ for $\chi$ varying from 0 to $\pi/2$ with a step $\pi/8$. The $I - \phi$ curves shift to the left with increasing $\chi$. 