Fitting the exotic hadron spectrum with an additional quark

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Most of the exotic hadrons discovered over the last 20 years fit nicely into the quark model as normal mesons and baryons if the existence of a seventh flavor of quark is hypothesized. For the quark to reproduce the mass, spin, parity, production and decay modes of exotic hadrons, it would have to have a mass of $\sim$2.8 GeV, a charge of $-\frac{1}{3}$, and a W-boson-mediated interaction with the right-chiral component of the charm quark. The theory involving such a quark would ultimately have to show that it can reproduce all of the experimental data that are well explained by the Standard Model. As a first step toward that goal, this paper qualitatively argues how the 7-quark model could potentially reproduce a handful of experiments that would seemingly rule it out. It also makes many predictions that could be verified or falsified by current experiments.

INTRODUCTION

This paper assumes the following hypothesis: There is a fourth down-type quark (charge $-\frac{1}{3}$) whose mass is approximately 2.8 GeV. The W boson connects the right-chiral components of this new quark and the charm quark. Due to the theory having both left- and right-chiral weak interactions, the Z boson generates flavor changing neutral currents. Hadrons with the additional quark have W- and Z-induced “intrinsic charm” components to their eigenstates that facilitate the observed production and decay modes.

Section 1 shows how the measured masses, spin and parity of exotic hadrons map neatly as normal quark-model mesons and baryons involving the additional quark. Fig. 1 displays the mass spacings for $J^{PC}=1^{--}$ and shows that they are similar to the mass spacings seen for charmonium.

To discuss the production and decay processes of the proposed hadrons, it is first necessary to describe how the W and Z boson interactions of the hypothesized model differ from those of the Standard Model. For example, the seventh quark and right-chiral W boson interaction imply that this model generates an effective CKM matrix for the 6 known quarks that differs from that of the Standard Model. Section 2 shows how the model can nonetheless reproduce experimental CKM data.

Section 3 shows that the model’s mixture of right- and left-chiral interactions implies that the Z boson generates flavor-changing neutral currents (FCNCs), especially for interactions involving the seventh quark. It is argued that experimental upper limits on FCNC do not rule out the model. The section ends with a discussion about how gauge anomalies are cancelled in the model.

Generation of intrinsic charm along with the production and decay processes of the proposed hadrons are described in section 4. The following Quark Model (QM) mappings are proposed (denoting the hypothesized quark

| Name       | QM  | Quarks          |
|------------|-----|-----------------|
| $X^{\pm,0}(3250)$ | $1^3S_0$ | $f\bar{u}, u\bar{f}, \frac{\sqrt{2}}{2}(fd + df)$ |
| $X(3350)$    | $1^3S_1$ | $\frac{1}{\sqrt{2}}(f\bar{d} + df)$ |
| $\chi_{c0}(3860)$ | $1^3P_0$ | $\frac{1}{\sqrt{2}}(fd + df)$ |
| $\chi_{c1}(3872)$ | $1^3P_1$ | $\frac{1}{\sqrt{2}}(fd + df)$ |
| $T^{++}_{cc}(3875)$ | $1^3P_1$ | $u\bar{f}$ |
| $Z_{cs}^{+}(3900)$ | $1^3P_1$ | $f\bar{u}, u\bar{f}, \frac{1}{\sqrt{2}}(fd + df)$ |
| $Z_{cs}^{+}(3985)$ | $2^1S_0$ | $f\bar{u}, u\bar{f}, \frac{1}{\sqrt{2}}(fd + df)$ |
| $X^{\pm,0}(4020)$ | $2^3S_1$ | $f\bar{u}, u\bar{f}, \frac{1}{\sqrt{2}}(fd + df)$ |
| $X(4160)$    | $1^3D_2$ | $\frac{1}{\sqrt{2}}(fd + df)$ |
| $Y(4230)$    | $1^3D_1$ | $\frac{1}{\sqrt{2}}(fd + df)$ |
| $\chi_{c1}(4274)$ | $2^3P_1$ | $\frac{1}{\sqrt{2}}(fd + df)$ |
| $Z_{c}^{+}(4430)$ | $2^1P_1$ | $f\bar{u}, u\bar{f}$ |
| $Y(4500)$    | $3^3S_1$ | $\frac{1}{\sqrt{2}}(fd + df)$ |
| $\psi(4660)$ | $2^3D_1$ | $\frac{1}{\sqrt{2}}(fd + df)$ |
| $\chi_{c1}(4685)$ | $3^3P_1$ | $\frac{1}{\sqrt{2}}(fd + df)$ |
| $\chi_{c0}(4700)$ | $3^3P_0$ | $\frac{1}{\sqrt{2}}(fd + df)$ |
| $Y(4710)$    | $4^3S_1$ | $\frac{1}{\sqrt{2}}(fd + df)$ |
| $R(3760)$    | $1^3S_1$ | $f\bar{s} + sf$ |
| $X(3960)$    | $1^3P_0$ | $\frac{1}{\sqrt{2}}(f\bar{s} + sf)$ |
| $\chi_{c1}(4140)$ | $1^3P_1$ | $\frac{1}{\sqrt{2}}(f\bar{s} + sf)$ |
| $\psi(4360)$ | $2^3S_1$ | $\frac{1}{\sqrt{2}}(f\bar{s} + sf)$ |
| $R(4407)$    | $1^3D_1$ | $\frac{1}{\sqrt{2}}(f\bar{s} + sf)$ |
| $\chi_{c0}(4500)$ | $2^3P_0$ | $\frac{1}{\sqrt{2}}(f\bar{s} + sf)$ |
| $X^{\pm}(5568)$ | $2^3P_0$ | $ef, fe$ |
| $X(6600)$    | $1^3S_1$ | $f\bar{f}$ |
| $X(6900)$    | $1^3P_0$ | $f\bar{f}$ |
| $X(7200)$    | $2^3S_0$ | $f\bar{f}$ |
| $X(8322)$    | $1^3P_1$ | $\frac{1}{\sqrt{2}}(f\bar{b} + bf)$ |
| $P_{c}^{+}(4312)$ | $\Sigma_{f}^{+}$ | $fuu$ |
| $P_{c}^{0}(4459)$ | $\Sigma_{f}^{0}$ | $fud$ |
| $P_{c}^{0}(4338)$ | $A_{f}^{0}$ | $fud$ |

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Section 5 discusses a number of experimental results that seemingly rule out the possibility of the hypothesized additional quark and the model presented here (e.g. inclusive hadronic cross section data). Arguments are provided for how the model has the possibility of nonetheless reproducing the experimental data.

Section 6 provides alternative explanations for a few additional experiments, and section 7 presents a large number of predictions that are verifiable or falsifiable by current experiments.

1. HADRON MAPPINGS

This section elaborates on the summary table (1) to also show mass and \( J^{PC} \) measurements for each exotic hadron, and to compare mass spacings with those of \( s, c \) and \( b \) hadrons.

It will be proposed in section 4 that each \( \frac{1}{\sqrt{2}} (f \bar{d} + d \bar{f}) \) or \( \frac{1}{\sqrt{3}} (f \bar{s} + s \bar{f}) \) meson has a \( cc \) “intrinsic charm” component to its eigenstate. That component allows mesons of this type that have \( J^{PC} = 1^{--} \) to be created by the excited photon generated by \( e^+ e^- \) collisions.

With that in mind, figure 1 graphically portrays the proposed meson mapping of the several observed neutral \( J^{PC} = 1^{--} \) exotic hadrons (where \( f \bar{d} \) and \( f \bar{s} \) in the figure are shorthand for \( \frac{1}{\sqrt{2}} (f \bar{d} + d \bar{f}) \) and \( \frac{1}{\sqrt{3}} (f \bar{s} + s \bar{f}) \) CP eigenstates). The \( X(3350), R(3760), R(4407), \) and hadrons are from [5]. \( X(3350) \) and \( X^0(4020) \) have been observed, but not in \( e^+ e^- \rightarrow X \). The mass spacings of the these proposed \( f \bar{q} \) quark mesons generate patterns similar to those in the charmonium spectrum.

For the other proposed \( f \bar{d} \) mesons, mass differences are also consistent with analogous \( c \) and \( b \) mesons, as shown below.

Proposed \( \frac{1}{\sqrt{2}} (f \bar{d} + d \bar{f}) \) Mesons

| Name         | Mass | \( J^P \) | \( \Gamma \) | \( QM \) | \( \Delta m_c \) | \( \Delta m_b \) |
|--------------|------|-----------|-------------|---------|----------------|----------------|
| \( X^0(3250) \) | 3250 | \( < 45 \) | \( ? \)     | \( 1^1 S_0 \) | 1385           | 2030           |
| \( X(3350) \)   | 3350 | 70        | \( ? \)     | \( 1^3 S_1 \) | 1343           | 1975           |
| \( \chi_{c0}(3860) \) | 3862 | 201       | \( 0^+ \)   | \( 1^3 P_0 \) | 1519           | 1836           |
| \( \chi_{c1}(3872) \) | 3872 | 12        | \( 1^+ \)   | \( 1^3 P_1 \) | 1460           | 1854           |
| \( Z^0(3900) \)   | 3900 | 28        | \( 1^{--} \) | \( 1^1 P_1 \) | 1459           | 1839           |
| \( Z^0_{cs}(3985) \) | 3993 | 8         | \( ? \)     | \( 2^1 S_0 \) | 1444           | 1978           |
| \( X(4160) \)   | 4153 | 136       | \( 2^+ \)   | \( 1^1 D_2 \) | 1463           | 1836           |
| \( \psi(4230) \)  | 4223 | 49        | \( 1^{--} \) | \( 1^3 D_1 \) | 1397           | 1978           |
| \( \chi_{c1}(4274) \) | 4286 | 51        | \( 1^{--} \) | \( 2^1 P_1 \) | 1463           | 1836           |
| \( Y(4500) \)   | 4485 | 111       | \( 1^{--} \) | \( 3^3 S_1 \) | 1463           | 1836           |
| \( \psi(4660) \)  | 4670 | 72        | \( 1^{--} \) | \( 2^3 D_1 \) | 1397           | 1978           |
| \( \chi_{c1}(4685) \) | 4684 | 126       | \( 1^{--} \) | \( 3^3 P_1 \) | 1463           | 1836           |
| \( \epsilon(4700) \) | 4994 | 87        | \( 0^{++} \) | \( 3^1 P_0 \) | 1397           | 1978           |
| \( Y(4710) \)   | 4708 | 126       | \( 1^{--} \) | \( 4^3 S_1 \) | 1463           | 1836           |

The above masses and widths are in MeV. The \( Y(4500), Y(4710) \) and \( Z^0_{cs}(3985) \) data are from [4], [6] and [7], while the rest of the masses, widths and \( J^{PC} \) are from [8] (including the meson listing for Further States). The \( QM \) column shows the proposed Quark Model mapping of the above resonances as normal mesons; the \( \Delta m_c \) and \( \Delta m_b \) columns show the differences in mass compared to the \( c \bar{d} \) or \( b \bar{d} \) mesons with the same \( QM \) designations.

For each \( \frac{1}{\sqrt{2}} (f \bar{d} + d \bar{f}) \) meson, there should also be a \( \frac{1}{\sqrt{3}} \) \( (f \bar{s} - d \bar{s}) \) meson with similar mass but the opposite \( C \) quantum number.

If the above resonances are truly \( f \bar{d} \) mesons, then should also be \( f \bar{u} \) charged mesons with similar masses and widths. The following are proposed:

Proposed \( f \bar{u} \) and \( f \bar{d} \) Mesons

| Name         | Mass | \( J^P \) | \( \Gamma \) | \( QM \) | \( \Delta m_c \) | \( \Delta m_b \) |
|--------------|------|-----------|-------------|---------|----------------|----------------|
| \( X^\pm(3250) \) | 3250 | \( < 45 \) | \( ? \)     | \( 1^1 S_0 \) | 1385           | 2030           |
| \( T^+_{cc}(3875) \) | 3875 | 0.4       | \( 1^+ \)   | \( 1^3 P_1 \) | 1463           | 1839           |
| \( Z^{\pm}(3900) \) | 3900 | 28        | \( 1^+ \)   | \( 1^1 P_1 \) | 1465           | 1839           |
| \( Z^{\pm}_{cs}(3985) \) | 3985 | 13        | \( ? \)     | \( 2^1 S_0 \) | 1434           | 1958           |
| \( X^{\pm}(4020) \) | 4024 | 13        | \( ? \)     | \( 2^3 S_1 \) | 1397           | 1978           |
| \( Z^{\pm}(4430) \) | 4478 | 181       | \( 1^{--} \) | \( 2^1 P_1 \) | 1397           | 1978           |

Section 4 proposes a justification for the designation of \( T^+_{cc}(3875) \) as the isospin partner of \( \chi_{c1}(3872) \). It is assumed that the \( T^{0,0}_{\psi_1}(4000) \) [9] are the same as the \( Z^{\pm,0}_{cs}(3985) \).
The following $f\bar{s}$ mesons are also proposed:

Proposed $\frac{1}{\sqrt{2}}(f\bar{s} + s\bar{f})$ Mesons

| Name     | Mass   | $\Gamma$ | $J^{PC}$ | QM  | $\Delta m_c$ | $\Delta m_b$ |
|----------|--------|----------|----------|-----|---------------|---------------|
| R(3760) | 3766   | 22       | $1^-$    | $1^+S_1$ | 1648          | 1655          |
| X(3960) | 3956   | 43       | $0^+$    | $1^+P_0$ | 1639          |               |
| $\chi_{c1}(4140)$ | 4147 | 19       | $1^+$    | $1^+P_1$ | 1687          |               |
| $\psi(4360)$ | 4372 | 115      | $1^-$    | $2^+S_1$ | 1658          |               |
| R(4407) | 4407   | 128      | $1^-$    | $1^+D_1$ |               |               |
| $\chi_{c0}(4500)$ | 4474 | 77       | $0^+$    | $2^+P_0$ |               |               |

where $R(3760), X(3960)$ and $R(4407)$ data are from [2], [10] and [3]. Again, for each of the above mesons, there should be a $\frac{1}{\sqrt{2}}(f\bar{s} - s\bar{f})$ meson with similar mass but the opposite $C$ quantum number.

The following $f\bar{f}$ mesons are proposed:

Proposed $f\bar{f}$ Mesons

| Name     | Mass   | $\Gamma$ | $J^{PC}$ | QM  | $\Delta m_c$ | $\Delta m_b$ |
|----------|--------|----------|----------|-----|---------------|---------------|
| $X(6600)$ | 6638  | 440      | $?^?S$  | $1^+S_1$ | 1771          | 1411          |
| $X(6900)$ | 6847  | 191      | $0^+$    | $1^+P_0$ | 1716          | 1506          |
| $X(7200)$ | 7134  | 97       | $?^?S$  | $2^+S_0$ | 1748          | 1433          |

where $X(6900)$ was first observed by LHCb [11]. Both CMS [12] and ATLAS [13] confirmed the $X(6900)$ and suggested that fits to $J/\psi$ pair data were improved by also including $X(6600)$ and $X(7200)$ resonances [CMS values were used in the table]. To conserve $C$ and $P$ in this model, the $X(6600)$ decays by $X(6600) \rightarrow \gamma J/\psi J/\psi$, with a soft photon unobserved. The inclusive hadronic cross section data in this mass range appear to be consistent with a broad peak corresponding to the $X(6600)$ (see for example fig 10 in [14]). For the above table, $\Delta m_c$ and $\Delta m_b$ mean half the mass difference compared to $c\bar{c}$ and $b\bar{b}$ mesons with the same QM designation.

The final mapping is to hadrons currently categorized as pentaquarks. The following $fuu$ and $fud$ baryons are proposed in relation to their $suu$ and $sud$ counterparts:

Proposed $fuu$ Baryons

| Name     | Mass   | $\Gamma$ | $J^{PC}$ | $suu$ Name | $J^{P}$ | $\Delta m_s$ |
|----------|--------|----------|----------|------------|---------|---------------|
| $P_c^+(4312)$ | 4312  | 10       | $?^?S$  | $\Sigma(1620)$ | $\frac{1}{2}^+$ | 2692          |
| $P_c^+(4380)$ | 4380  | 200      | $?^?S$  | $\Sigma(1660)$ | $\frac{1}{2}^+$ | 2720          |
| $P_{\psi^+}^+(4377)$ | 4377 | 29       | $?^?S$  | $\Sigma(1670)$ | $\frac{1}{2}^+$ | 2675          |
| $P_c^+(4440)$ | 4440  | 21       | $?^?S$  | $\Sigma(1750)$ | $\frac{3}{2}^-$ | 2690          |
| $P_c^+(4457)$ | 4457  | 6        | $?^?S$  | $\Sigma(1775)$ | $\frac{3}{2}^-$ | 2682          |
| $P_c^+(4457)$ | 4457  | 6        | $?^?S$  | $\Sigma(1780)$ | $\frac{3}{2}^-$ | 2677          |

Proposed $fud$ Baryons

| Name     | Mass   | $\Gamma$ | $J^{PC}$ | $sud$ Name | $J^{P}$ | $\Delta m_s$ |
|----------|--------|----------|----------|------------|---------|---------------|
| $P_{\psi s}^0(4338)$ | 4338  | 7        | $\Lambda(1670)$ | $\frac{1}{2}^-$ | 2676          |
| $P_{\psi s}^+(4459)$ | 4459  | 17       | $?^?S$  | $\Sigma(1775)$ | $\frac{3}{2}^+$ | 2684          |
| $P_{\psi s}^+(4459)$ | 4459  | 17       | $?^?S$  | $\Sigma(1780)$ | $\frac{3}{2}^+$ | 2679          |

where the mass differences are relative to the masses of the $sud$ and $sud$ baryons in each line, the first $J^{P}$ column has measured values for the observed pentaquarks, and the second $J^{P}$ column has the quantum numbers of the analogous $sud$ and $sud$ baryons. The data for $P_c^+(4337)$ are from [15]. For the above table, it is assumed that the $P_c^+(4457)$ is a very close double peak analog of the $\Sigma(1775)$ and $\Sigma(1780)$. It appears that the data could support splitting $P_c^+(4457)$ into two resonances (see fig. 6 of [16]). It is also proposed that $P_{\psi s}^0(4459)$ from [17] is the $fud$ isospin partner of the $P_c^+(4457)$ resonance(s). Finally, it is proposed that $P_{\psi s}^+(4338)$ from [18] is an $fud$ isospin singlet.

Notably missing from the above tables are proposed mesons corresponding to the charged resonances $Z_c^+(4050), Z_c^+(4055)$, and $Z_c^+(4220)$. In the context of the model, it would be interesting to see if these resonances could be made consistent with other resonances listed above, threshold effects, feed-downs from heavier resonances, or possibly other Quark Model designations for $fu$.

Also missing are the $T_{c0}^{a^+}(2900)$ and its neutral isospin partner. Since mesons cannot have 2 units of electric charge, this model does not have a meson mapping for $T_{c0}^{a^+}(2900)$.

This theory does not rule out the possibility of actual 4- or 5-quark states such as tetraquarks, meson molecules, or pentaquarks. For example, it would not be in conflict with this model if the $Z_b(10610)$ and $Z_b(10650)$ were actually tetraquarks or meson molecules.

2. W BOSON INTERACTIONS

As mentioned above, the hypothesized quark has a right-chiral connection to the W boson. That implies a significant difference from the CKM matrix of the Standard Model. This section shows how the model can nonetheless reproduce existing CKM data.

In the Standard Model, there are three generations of down-type quarks and three generations of up-type quarks. The down-type quarks have a 3x3 mass matrix $M_D$ whose rows and columns connect left- and right-chiral quarks, respectively. Similarly, the up-type quarks have their own 3x3 mass matrix $M_U$. It is always possible to find four 3x3 unitary matrices $V$ that cause the following matrices to be diagonal: $V_D^T M_D V_D D^T$ and $V_U^T M_U V_U D^T$. The diagonal elements of these matrices are the mass eigenvalues ($m_d, m_s, m_b$) and ($m_u, m_c, m_t$).

According to the Standard Model, the W boson connects only the left-chiral up- and down-type quarks from the same generation. The Cabibbo-Kobayashi-Maskawa (CKM) matrix translates this same-generation connection into the mass eigenstates. Specifically, the Standard-
Model CKM matrix is given by:

\[
V_{\text{CKM}} = V_L^U \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix} V_{\text{CKM}}^{D\dagger} = 
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}.
\]

(8)

The unit matrix in the middle expression above has been inserted to emphasize the fact that in the Standard Model, the left-chiral components of all three generations are connected. Since the matrix in the middle is a unit matrix, \(V_{\text{CKM}}\) is just the product of two unitary matrices \((V_L^U\) and \(V_{\text{CKM}}^{D\dagger}\)), so it is also a unitary matrix.

This model proposes that the fourth down-type quark has a right-chiral connection with the second-generation up-type quark. In addition, the model proposes that the first-generation quarks also have a right-chiral (rather than left-chiral) connection. One reason for this second assumption is due to the fact that it facilitates gauge anomaly cancellation among quarks, as discussed at the end of the next section.

Given these differences, it is worth reviewing the direct experimental evidence pertaining to whether left-chiral or right-chiral components of the known quarks interact with the W boson. Recent top quark polarization measurements by ATLAS confirm that for the third generation, the data are consistent with the W boson connecting only left-chiral quarks [19]. In addition, measurements by Belle of tau lepton polarization in \(b \to c\) semi-leptonic decays imply that since the third generation connection is left-chiral, the second generation is also left-chiral [20].

On the other hand, there is no direct evidence that the W-boson connection for first-generation quarks is left-chiral. A complication is the fact that only about a third of the proton’s spin comes from the helicity of its quarks [21], so polarization of nucleons does not directly translate into polarization of constituent quarks. In other words, there is no direct experimental evidence that would contradict a model that had right-chiral (instead of left-chiral) first-generation quark interactions with the W boson, as long as the model was able to reproduce the observed CKM data.

With that in mind, this model assumes that the quark-W boson interaction terms in the Lagrangian take the following form:

\[
(\bar{u}_{L1} \bar{u}_{L2} \bar{u}_{L3}) \begin{pmatrix}
0 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix} \gamma^\mu \begin{pmatrix}
d_{L1} \\
d_{L2} \\
d_{L3}
\end{pmatrix} gW_\mu
+ (\bar{u}_{R1} \bar{u}_{R2} \bar{u}_{R3}) \begin{pmatrix}
1 & 0 & 0 \\
0 & 0 & 1 \\
0 & 0 & 0
\end{pmatrix} \gamma^\mu \begin{pmatrix}
d_{R1} \\
d_{R2} \\
d_{R3}
\end{pmatrix} gW_\mu,
\]

(9)

(plus the Hermitian conjugate) where the lower numerical indices denote quark generation. The model has two left-chiral and two right-chiral quark-boson interaction terms. In particular, the right-chiral (not left-chiral) first-generation quarks are connected by the W boson. Also, the right-chiral fourth-generation down-type and second-generation up-type quarks are connected by the W boson. That means that the second generation up-type quark (mostly \(c\)) has two W boson connections, one left-chiral (mostly \(s\)) and one right-chiral (mostly \(f\)).

With these interaction terms, the model has two versions of “CKM” matrices:

\[
V_{\text{CKM}}^+ = V_R^U \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix} V_R^{D\dagger}.
\]

(10)

\[
\pm V_L^U \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{pmatrix} V_L^{D\dagger}.
\]

Since this model has four down-type quarks \((d, s, b, f)\), the down-type quark mass matrix is a 4x4 matrix, as are the \(V_R^D\) and \(V_L^D\) unitary matrices that diagonalize it. Consequently the above “CKM” matrices are 3x4.

A different version of CKM matrix should be used depending on whether a vector current or an axial vector current process is being considered. Specifically:

\[
\text{Vector current decays: } V_{\text{CKM}}^+ \\
\text{Axial vector current decays: } V_{\text{CKM}}^-.
\]

Consider the following quark mass matrices:

\[
M_D = \begin{pmatrix}
-0.0338 & 0.0137 & 0.0246 & -0.0527 \\
-0.0252 & -0.0725 & -0.0636 & -0.0804 \\
0.1054 & 1.4894 & -3.8554 & 0.1329 \\
1.4717 & -0.0800 & 0.2408 & 2.3052
\end{pmatrix}
\]

(12)

\[
M_U = \begin{pmatrix}
0.00131 & -0.00191 & 0.0122 \\
0.6493 & 1.1126 & -5.3477 \\
1.7379 & -1.0843 & 171.94
\end{pmatrix}.
\]

where each matrix element is in GeV. The four unitary matrices that diagonalize these mass matrices can be found by calculating the eigenvectors of the matrices multiplied by their transposes. Using those unitary matrices, the diagonalized mass matrices reproduce the 7 quark masses (including \(m_f = 2.74\) GeV). The magnitudes of CKM matrices defined by eqs (10) and (11) are

\[
|V_{\text{CKM}}^+| = \begin{pmatrix}
0.9745 & 0.2239 & 0.0043 & 0.0094 \\
0.2323 & 0.9661 & 0.0388 & 0.9720 \\
0.0088 & 0.0402 & 0.9973 & 0.0619 \\
0.9742 & 0.2252 & 0.0042 & 0.0093
\end{pmatrix}.
\]

(13)

\[
|V_{\text{CKM}}^-| = \begin{pmatrix}
0.2165 & 0.9805 & 0.0394 & 1.0278 \\
0.0144 & 0.0349 & 0.9974 & 0.0621
\end{pmatrix}.
\]
Comparing the first 3 columns (excluding the $f$ quark column) to data from [22] (and references therein), it can be seen that the above matrices do a good job of reproducing measured magnitudes of CKM matrix elements. The model could also reproduce CP-violating phases, but that has been left out of the present discussion for simplicity.

Many variations of the mass matrices can generate CKM matrices similar to the ones seen above. The particular mass matrices in eq (12) were chosen to give an example for how CKM matrices could be reproduced and FCNC could be generated (as discussed in the next section). A better pair of mass matrices could be obtained by a more rigorous global fit to experimental data.

3. Z BOSON INTERACTIONS

In the proposed model, the couplings to the Z boson by quark generation are:

\[
\begin{align*}
\bar{u}_i \gamma^\mu u_i Z_\mu & \quad g_L & \quad g_R \\
i = 1 & \quad -\frac{2}{3}x & \quad \frac{1}{2} - \frac{2}{3}x \\
2 & \quad \frac{1}{2} - \frac{2}{3}x & \quad \frac{1}{2} - \frac{2}{3}x \\
3 & \quad -\frac{2}{3}x & \quad -\frac{2}{3}x \\
SM & \quad \frac{1}{2} - \frac{2}{3}x & \quad -\frac{2}{3}x
\end{align*}
\]

\[
\begin{align*}
\bar{d}_i \gamma^\mu d_i Z_\mu & \quad g_L & \quad g_R \\
i = 1 & \quad \frac{1}{3}x & \quad -\frac{1}{2} + \frac{1}{3}x \\
2 & \quad -\frac{1}{2} + \frac{1}{3}x & \quad \frac{1}{3}x \\
3 & \quad -\frac{1}{2} + \frac{1}{3}x & \quad \frac{1}{3}x \\
4 & \quad \frac{1}{3}x & \quad -\frac{1}{2} + \frac{1}{3}x \\
SM & \quad -\frac{1}{2} + \frac{1}{3}x & \quad \frac{1}{3}x
\end{align*}
\]

where $x = \sin^2 \theta_W$, $i$ is an index for a generation, and the “SM” rows show the Standard Model couplings for up- and down-type quarks.

Since the couplings of this model differ by generation, Z boson connections mix mass eigenstates. For example, the Z mixing matrix for left-chiral down-type quarks is:

\[
V_{ZD}^L = V_L^D \begin{pmatrix} \frac{1}{3}x & 0 & 0 & 0 \\ -\frac{1}{2} + \frac{1}{3}x & 0 & 0 & 0 \\ 0 & -\frac{1}{2} + \frac{1}{3}x & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3}x \end{pmatrix} V_L^{D\dagger}
\]

\[
= \begin{pmatrix} 0.0520 & -0.1093 & -0.0030 & 0.0032 \\ -0.1093 & -0.3972 & -0.0003 & 0.0147 \\ -0.0030 & -0.0003 & -0.4210 & -0.0304 \\ 0.0032 & 0.0147 & -0.0304 & 0.0749 \end{pmatrix},
\]

where the diagonalizing matrix $V_L^D$ was determined from $M_D$ of eq (12) and $x = \sin^2 \theta_W$ was taken from [23]. There are also Z mixing matrices for right-chiral down-type quarks as well as for right- and left-chiral up-type quarks.

Most of the off-diagonal FCNC elements of these matrices are small, but a few are more significant. The only Z mixing matrix elements with magnitudes greater than 0.003 are $V_{Zu3}, V_{Zd1}, V_{Zf1}, V_{Zf3}, V_{Zf4}, V_{Zb1}, V_{Zc1},$ and $V_{Zb2}$. Given the sample mass matrices presented above, the last two elements imply FCNC that is smaller than the limits set by [24]. More detailed work (including modeling of leptons in the theory) would be needed to determine whether the $V_{Zu3}$ and $V_{Zd1}$ mixing of the current mass matrices are consistent with experimental data.

With the mass matrices of eq (12), the model predicts significant FCNC mixing of the $f$ quark with other down-type quark flavors. Of particular note is the fact that the $b \rightarrow f + Z$ coupling of this model has a magnitude of 0.03. Comparing to the 0.04 magnitude of the $b \rightarrow c + W$ coupling, it can be seen that the former FCNC decays of this model can generate a non-negligible production mechanism for $f$-quark hadrons.

Now that the W and Z boson interactions have been specified, it is possible to discuss gauge anomalies. In the Standard Model quark gauge anomalies are cancelled by lepton gauge anomalies. In this model, quark gauge anomalies cancel amongst themselves, independently of leptons. This quark anomaly cancellation can be seen by noting that for every right-chiral interaction in eqs (9), (14) and (15), there is a corresponding left-chiral interaction. Another way to see this cancellation is to create Dirac fermions out of the following R+L pairings of up- and down-type quarks, where the numerical subscripts denote quark generation: $U_1 = u_{R1} + u_{L3}, U_2 = u_{R2} + u_{L2}, U_3 = u_{R3} + u_{L1}, D_1 = d_{R1} + d_{L3}, D_2 = d_{R2} + d_{L4}, D_3 = d_{R3} + d_{L1}, D_4 = d_{R4} + d_{L2}$.

With these pairings, Dirac quarks only have vector current interactions (no axial vector current) with the W and Z bosons. So there are no gauge anomalies generated by quarks.

A complete model would have to include additional leptons so that gauge anomalies of leptons also separately cancel amongst themselves. Such a model has been published [25, 26].

4. PRODUCTION AND DECAY PROCESSES

A. W-induced Intrinsic Charm

A meson can be modelled as having two parts: the first part is comprised of a valence quark and antiquark; the second part is a “sea” of quark-antiquark pairs created by gluon interactions. In the context of this picture, W boson interactions can create components of the meson eigenstate that have different valence quarks but the same “sea”. Some of these components could be considered “intrinsic charm”.

Consider a $df$ meson, where $d$ and $f$ are the valence
quarks. Surrounding these is a sea of $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ pairs (as well as other flavors of quarks). The $f$ quark can exchange a W boson with one of the $d$ sea quarks, enabling $df + d\bar{d} \rightarrow d\bar{c} + u\bar{d}$. Alternatively, with Cabibbo suppression, the W boson could enable $df + u\bar{u} \rightarrow u\bar{c} + d\bar{d}$ or $df + d\bar{d} \rightarrow c\bar{c} + d\bar{d}$. After this rearrangement, the sea is the same, but the valence quarks are different; in addition to $df$, the valence quark eigenstate includes small components of $uc$ and $c\bar{c}$.

An analogous interaction could take place with $u\bar{u}$ sea quark pairs. The $f$ valence quark can exchange a W boson with one of the $u$ sea quarks, enabling $df + u\bar{u} \rightarrow u\bar{c} + d\bar{d}$ or with Cabibbo suppression $df + u\bar{u} \rightarrow u\bar{c} + d\bar{s}$. This matrix element has the opposite sign since the valence quark can exchange a W boson with one of the $\bar{u}$ quarks, as opposed to the $d\bar{u}$ case where W is exchanged between two antiquarks, as opposed to the $dd$ case where W is exchanged between an antiquark and a quark. If it is assumed that a sea with one more or one fewer $q\bar{q}$ pair is indistinguishable from the original sea, then W interaction with a $u\bar{u}$ sea pair effectively adds small $u\bar{c}$ and $u\bar{c}d\bar{s}$ components to the $df$ valence quark eigenstate.

Similarly, W interaction with an $s\bar{s}$ sea pair adds small $c\bar{c}d\bar{s}$ and $-ucd\bar{s}$ components to the $df$ valence quark eigenstate (where the second has Cabibbo suppression).

These same interactions could take place with all of the $dd$, $u\bar{u}$ and $s\bar{s}$ sea quark pairs. Assuming all of these diagrams lead to indistinguishable final states, they should all be added, being weighted by $\rho(q\bar{q})_{sea}$, the sea-quark pair density of the quark flavor $q$ that interacts with the W boson. Despite the fact that each W-boson interaction has the mass squared of the W boson in the denominator, it is proposed that the sum of all of the indistinguishable interactions with every quark pair in the sea can lead to measureable additional contributions to the $df$ meson eigenstate.

In a similar way, a Z boson can induce the valence quark transition $f \rightarrow b, f \rightarrow s$ or $f \rightarrow d$, interacting with a sea quark on the other side. For the decays discussed below, only the first of these will be taken into account. Again, it is proposed that the sum of all of the indistinguishable interactions with every quark pair in the sea can lead to measureable additional contributions to the $df$ meson eigenstate.

The full “$df$ meson” valence quark eigenstate can then be approximated by

$$df \rightarrow df + \epsilon(c\bar{c} + uc(1 - R_u)/\sin \theta_C)$$

$$+ \epsilon' d\bar{b} + \epsilon(u\bar{c}d\bar{s}(R_u - R_s) - c\bar{c}d\bar{s}R_s/\sin \theta_C) + \ldots.$$  \hspace{1cm} (17)

where $\epsilon$ and $\epsilon'$ are some small constants, $\theta_C$ is the Cabibbo angle, and

$$R_q = \rho(q\bar{q})_{sea}/\rho(d\bar{d})_{sea}.$$  \hspace{1cm} (18)

The same mechanism modifies an $f\bar{d}$ meson to have an eigenstate that is the charge conjugate of the above state. This implies that the C-parity eigenstate $(f\bar{d})_+ = \frac{1}{\sqrt{2}}(d\bar{f} + f\bar{d})$ has a $c\bar{c}$ component, but the $(f\bar{d})_- = \frac{1}{\sqrt{2}}(d\bar{f} - f\bar{d})$ eigenstate does not due to cancellation. In addition, both of these eigenstates have $c\bar{c}(d\bar{s} \pm s\bar{d})$ components, where the sign depends on the particular hadron as described below.

The $R_q$ ratios can be different for each hadron, but could presumably be determined by measuring relative decay rates in different channels. Most of the decays of exotic hadrons discussed below involve the $c\bar{c}$ component, but some of them involve other components. Based on observed decays of $\psi(4360)$ and $Z_{c \bar{c}}^{+,0}(3985)$, the factor $R_q/\sin \theta_C$ may not be negligible in those mesons. Based on the observed decay of $X^{0}(3530)$, its factor of $(1 - R_u)/\sin \theta_C$ may not be negligible. Also, since these factors vary by hadron, it could be that the $(1 - R_u)/\sin \theta_C$ component is negligible in some hadrons (where $\rho(q\bar{q})_{sea} \approx \rho(d\bar{d})_{sea}$) but not in others.

A $d\bar{b}$ meson should also have $c\bar{c}$ and other components in its eigenstate. However, these contributions are much smaller for two reasons: (i) the W boson coupling of $b$ to $c$ is much smaller than $f$ to $c$, and (ii) since $b$ quarks are more massive, the density of sea quarks for $b$-quark mesons is smaller. As a result, it is proposed that decay modes analogous to the ones discussed below for $f$-quark mesons are mostly negligible for $b$-quark mesons.

Due to similar W-boson interactions, an $s\bar{f}$ meson should have the following eigenstate:

$$s\bar{f} \rightarrow s\bar{f} + \epsilon(c\bar{c}d\bar{s} + uc\bar{s}(1 - R_u)/\sin \theta_C)$$

$$+ \epsilon' d\bar{b} + \epsilon(u\bar{c}d\bar{s}(R_u - R_s) - c\bar{c}d\bar{s}R_s/\sin \theta_C) + \ldots.$$  \hspace{1cm} (19)

Again, the $s\bar{f}$ meson’s eigenstate is the charge conjugate of the above state, so an $(f\bar{s})_+$ meson has a $c\bar{c}$ component, but an $(f\bar{s})_-$ meson does not.

Using similar reasoning, notable eigenstate components of $b\bar{f}, c\bar{f}$ and $s\bar{b}$ mesons include:

$$b\bar{f} \rightarrow b\bar{f} + \epsilon'(bb + ff) + \ldots$$

$$c\bar{f} \rightarrow c\bar{f} + \epsilon(c\bar{c}u\bar{d}(1 - R_u)/\sin \theta_C) + \ldots$$

$$s\bar{b} \rightarrow s\bar{b} + \epsilon cc0.04R_s/\sin \theta_C + \ldots.$$  \hspace{1cm} (20)

The first of these allows $(b\bar{f})_+$ mesons to be created in $e^+e^-$ collisions. The second and third facilitate possible evidence for an $f\bar{c}$ meson discussed below.

A double W-induced interaction with sea quarks could generate the following:

$$f\bar{f} \rightarrow f\bar{f} + \epsilon'^2(1 + (R_u/\sin \theta_C)^2)c\bar{c}c\bar{c} + \ldots.$$  \hspace{1cm} (21)

This can allow $f\bar{f}$ mesons to decay to two $J/\psi$.

By similar reasoning, an $fqq'$ baryon should have the following eigenstate:

$$fqq' \rightarrow fqq' + \epsilon(dqq' c\bar{c} + dqq' c\bar{c}u(1 - R_u)/\sin \theta_C)$$

$$+ \epsilon' dqqq' + \epsilon(sqq' c\bar{c}u(R_u - R_s) - sqqq' c\bar{c}R_s/\sin \theta_C) + \ldots.$$  \hspace{1cm} (22)

The same logic can be applied to a $duu$ proton. In this case, $2duc + duuc + (1 - R_u)ducuc$ components are created with one factor of Cabibbo suppression and $ducuc$
with 2 factors of Cabibbo suppression. In other words, in this model W-mediated interactions with sea quarks create the “intrinsic charm” of the proton. An interesting feature of this model is that it predicts that the charm quark content of the proton eigenstate is approximately twice as large as the anticharm content. The reverse is true of the neutron. It has been estimated that intrinsic charm+anticharm contributes \( \sim 0.6\% \) to proton momentum [27]. That means that a single Cabibbo-suppressed component like \( du\bar{u}c \) contributes \( \sim 0.2\% \).

If the density of sea quarks in an \( fuu \) meson was similar to that in a proton, the \( du\bar{u}c \) part of the \( fuu \) baryon eigenstate would contribute \( \sim 0.2\% \) to the \( fuu \) baryon momentum. This gives a rough idea of the order of magnitude of the amount of intrinsic charm expected in \( fuu \) baryons. The \( \bar{c}c \) contributions to \( f \)-quark meson states could potentially be significantly larger.

If \( (f\bar{d})_\pm \) are mass eigenstates for a particular quark-model state, then there is no CP violation in decays of those mesons. However, if the mass eigenstates are linear combinations of \( (f\bar{d})_\pm \), then their decays will generate CP violation, much like the \( K^0_L \) and \( K^0_S \).

**B. Vector mesons**

In this model the \( X(3350) \), \( X^0(4020) \), \( \psi(4230) \), \( Y(4500) \) and \( \psi(4660) \) are all meson mass eigenstates that are mostly the \( J^{PC} = 1^{--} \) CP eigenstates of \( (f\bar{d})_+ \). The \( R(3760) \), \( \psi(4360) \) and \( R(4407) \) are mostly \( 1^{-+} \) CP eigenstates of \( (f\bar{s})_+ \). Since the CP eigenstates all have \( J^{PC} = 1^{-+} \), a photon from an \( e^+e^- \) collision can create the \( \bar{c}c \) component within these and thereby create these mesons.

The model predicts another set of neutral \( (f\bar{d})_- \) and \( (f\bar{s})_- \) vector mesons with similar masses but which are mostly CP eigenstates with the opposite C-parity. These \( 1^{++} \) CP eigenstates are not created in \( e^+e^- \) collisions, since they do not have a \( \bar{c}c \) component or the requisite quantum numbers. However, if there is CP violation, the mass eigenstates could be created in \( e^+e^- \) collisions by connection to their \( 1^{-+} \) CP eigenstate components. Except where specified otherwise below, it will be assumed that CP violation is small, so that the mass eigenstates are mostly the same as the CP eigenstates.

The strongest decay for all of the \( (f\bar{d})_+ \) vector mesons (mass permitting) is to emit a pion and decay to a \( 1^{++} \) \( f\bar{d} \) axial vector meson. Since \( \psi(4230) \rightarrow \pi^0 Z^0_2(3900) \) has been observed, the \( Z^0_2(3900) \) is mapped as the \( 1^3P_1 \) meson of \( (f\bar{d})_+ \). Since \( \psi(4230) \rightarrow \pi^+ Z^+_2(3900) \) has also been observed, the \( Z^+_2(3900) \) are mapped to the \( 1^3P_1 \) mesons of \( u\bar{f} \) and \( f\bar{u} \). A similar decay is expected for \( \psi(4660) \rightarrow \pi^+ Z^+_2(4430) \), where the latter are mapped as \( 2^1P_1 \) mesons of \( u\bar{f} \) and \( f\bar{u} \).

The next strongest decay is to emit a pion and decay to a \( 1^{--} \) \( f\bar{d} \) vector meson. For example, the following decays are predicted: \( \psi(4660) \rightarrow \pi^0 \psi(4230), \psi(4230) \rightarrow \pi^0 X^0(4020) \) and \( \psi(4230) \rightarrow \pi^+ X^+(4020) \). The last 2 decays may have contributed to observed \( e^+e^- \rightarrow \pi^\pm X^\mp(4020) \) processes.

An \( f \)-quark vector meson can also decay by gamma emission to a \( 1^{++} \) \( f \)-quark axial vector meson. For example, the decay \( \psi(4230) \rightarrow \gamma \chi_{c0}(3872) \) has been observed.

All of the above decays are from the primary valence quark eigenstate component inside an \( f \)-quark meson. But \( f \)-quark mesons can also decay when one of the smaller components in its eigenstate decays (e.g. \( \bar{c}c \)). For example, an \( (f\bar{d})_+ \) vector meson can decay radiatively to \( \bar{c}c \) plus a photon or strongly to \( \bar{c}c \) plus a gluon and/or a meson.

With regard to the latter, \( \psi(4230) \) has been observed decaying to \( f_0 \psi, \eta J/\psi, \eta' J/\psi \) and \( \omega \chi_{c0} \). Also the charged and neutral versions of \((4020)\) have been observed decaying to \( \pi h_c \) and \( \rho_0 h_c \). From eq (19), it can be seen that with relative strength \( R_\psi / \sin \theta_C \), the \( \bar{c}c \) component within the \( (f\bar{s})_+ \) vector mesons can decay in a similar fashion. Since the decay \( \psi(4360) \rightarrow \eta J/\psi \) has been observed, it suggests that the factor \( R_\psi / \sin \theta_C \) may not be extremely small.

The \( \psi(4230) \), the \( Y(4500) \) [4], and the \( Y(4710) \) [6] have all been observed decaying to \( K^+K^- \) \( J/\psi \). One contribution to these decays may be the \( \bar{c}c \) component of these mesons decaying to \( f_0(980) \) \( J/\psi \) or \( f_0(1370) \) \( J/\psi \) followed by \( f_0 \rightarrow K^+K^- \).

Similarly, \( f_0 \rightarrow \pi^+\pi^- \) processes could be contributing to observed \( \psi(4360) \rightarrow \pi^+\pi^- J/\psi \) and \( \psi(4360) \rightarrow \pi^+\pi^- h_c \) decays. If so, one would expect to see a small resonance for \( \psi(4360) \rightarrow K^+K^- J/\psi \) in the data from [4]. The data appear to be able to accommodate a small feature at that mass. The \( \psi(4360) \rightarrow f_0 J/\psi \) decays come from its \( \bar{c}c \) component which from eq (19) is proportional to \( R_\psi / \sin \theta_C \). In this model, observation of these decays implies that this factor is non-negligible.

Vector mesons can also decay by their \( \bar{c}c \) component emitting a gluon which then creates one or more \( qq \) pairs. Since gluon emission leaves the \( \bar{c}c \) component colored, to get back to colorless hadrons, the \( c \) goes with one decay product and the \( \bar{c} \) goes with the other. This is the mechanism for the observed decays \( \psi(4660) \rightarrow \Lambda^+_c \Lambda^-_c \) and \( \rightarrow D^+_c D^0_c \)(2536), where the second decay involves the gluon creating an \( s\bar{s} \) pair.

As for radiative decays from a \( \bar{c}c \) component, \( \psi(4230) \rightarrow \gamma \eta_c \) has possibly been seen.

All of the decays up to this point have involved decays either of the primary component or of the \( \bar{c}c \) component of an \( f \)-meson’s valence quark eigenstate. The observed decay of the \( X^0(3350) \rightarrow \Lambda^+_c \bar{p} \) [1] is different in that it involves decay of the \( c\bar{u} \) component of the eigenstate. For example, the \( c\bar{u} \) component can emit a gluon and become colored. The gluon splits into two gluons that create \( u\bar{u} \) and \( d\bar{d} \), then the \( c \) joins with \( u\bar{u} \) and the \( \bar{u} \) joins with \( u\bar{u} \) to make colorless combinations.

In this model, the observed \( X^0(3350) \) resonance is mapped to the \( 1^3S_1 \) meson of \( (f\bar{d})_+ \). The \( c\bar{u} \) component in \( X^0(3350) \) is proportional to the factor \( (1 - R_\psi) / \sin \theta_C \). Since the decay to \( \Lambda^+_c \bar{p} \) is observed, that factor is assumed
to be non-negligible. In the observed resonance, there is a hint of a small shoulder around 3550 MeV; it would be consistent with the mass spacings if the main peak was the $^1S_1$ meson of $(f\bar{d})_+$ and this shoulder was the $^1S_0$ meson of $(f\bar{s})_+$. Those same data have evidence of another 2.8σ resonance with a mass of around 3.8 GeV. That resonance could potentially be mapped to a combination of the $^1P_1$ meson of $(f\bar{d})_+$ (the $\chi_{c1}(3872)$) and the $^1S_1$ meson of $(f\bar{s})_+$ (the $R(3760)$). The main point is that in the context of this model, these data provide evidence for the existence of $c\bar{u}$ components of valence quark eigenstates in $f$-quark mesons.

It would provide support for this model to observe both $c\bar{u}$ and $c\bar{c}$ component decays from the same hadron. One way to do this would be to observe the decay $X(3350) \rightarrow \pi^0 J/\psi$. In the data for $\psi(4230) \rightarrow \pi^0 \pi^0 J/\psi$, it was mentioned that including a resonance around 3.4 GeV in these data could potentially help the fit [28]. It would be interesting to study those data with more statistics.

It is possible that evidence for an $(f\bar{b})_+$ vector meson was seen in a past experiment. According to the arguments in the intrinsic charm section above, an $(f\bar{b})_+$ meson should have $Z$-induced $ff$ and $bb$ components in its eigenstate, so it could be created as a resonance in $e^+e^-$ collisions. How big would such a resonance be? It appears that the $e^+e^-$ cross section data in the 8.6 to 9.4 GeV range could potentially be better fitted by including a resonance (see for example fig 10 in [14]). Given the mass of the $f$ quark in this model, such a resonance could correspond to the $^3D_1$ and/or $^3D_2$ meson of $(f\bar{b})_+$.

In that case, the $^3D_1$ or $^3D_2$ meson of $(f\bar{b})_+$ could have a mass just slightly larger than that of $\Upsilon(1S)$. Just like in the $\psi(4230) \rightarrow \gamma \chi_{c1}(3872)$ decay discussed above, this $(f\bar{b})_+$ meson could decay by gamma emission to the $^1P_1$ meson of $(f\bar{b})_+$. Given the mass of the $f$ quark in this model, the latter meson could have a mass near 8.3 GeV.

In 1984, the Crystal Ball collaboration reported evidence from their 1983 data that the $\Upsilon(1S)$ was radiatively decaying to a narrow resonance at 8322 MeV [29]. The resonance was not seen in 1984 Crystal Ball data or in those of other experiments with $e^+e^-$ centered on the $\Upsilon(1S)$ resonance [30]. However it was noticed that $R_{hadron}$ for the original dataset was smaller than $R_{hadron}$ for the other $\Upsilon(1S)$ datasets. This suggested that it was possible that the mean CM energy of the 1983 Crystal Ball data may have been a bit higher than the $\Upsilon(1S)$ resonance. If so, and if there was another $1^-\bar{f}$ resonance just a little bit more massive than the $\Upsilon(1S)$, then the 1983 resonance could have been real, not an experimental fluke. It was stated that “the 1984 result rules out the 1983 result to the 90% C.L. level unless some state exists between about 16-26 MeV above the Upsilon(1S)” [31]. That possibility was thought to be very unlikely, so it was assumed that the 1983 evidence of a resonance and smaller $R_{hadron}$ value were both experimental flukes that should be ignored.

This model opens the door to the possibility that the 8322 MeV resonance in 1983 data was not a fluke. The $^4S_3$ or $^3D_1$ meson of $(f\bar{b})_+$ could have a mass 16-26 MeV above that of the $\Upsilon(1S)$, and it could decay radiatively to the $^1P_1$ meson of $(f\bar{b})_+$, producing the resonance seen in the 1983 data. The other experiments would not have seen that resonance since their CM energies were centered on the $\Upsilon(1S)$ resonance rather than a little above it.

### C. Axial vector mesons

In this model the $Z_c^+(3900)$ and $\chi_{c1}^0(3872)$ are the $^1P_1$ and $^3P_1$ mesons of $(f\bar{b})_+$ with $J^{PC}=1^{-+}$ and $1^{++}$. As mentioned above, the $Z_c^\pm,0(3900)$ are typically produced in the strong decays $\psi(4230) \rightarrow \pi^\pm 0 Z_c^\pm,0(3900)$. The $\chi_{c1}^0(3872)$ is predicted to have charged isospin partners $\chi_{c1}^\pm(3872)$ ($uf$ and $f\bar{u}$ mesons). Both neutral and charged $^1P_1$ mesons can be produced in decays of a B meson to a $\chi_{c1}^0(3872)$ and a kaon or pion. These weak decays can be mediated by $b \rightarrow cW \rightarrow c\bar{c}s$, where the $c\bar{c}$ generates the $\chi_{c1}^0(3872)$. Alternatively, they could be mediated by the flavor changing neutral current process $b \rightarrow fZ \rightarrow f\bar{d}s$, where the $f\bar{d}$ generates the $\chi_{c1}(3872)$. Patterns in the data seem to indicate that the $W$-mediated process may be larger. This implies that the $c\bar{c}$ component of the $\chi_{c1}(3872)$ may be substantial, possibly carrying 1-2% (or more) of that meson’s momentum.

Like the vector mesons, the axial vector mesons can decay by the $c\bar{c}$ component (or another component) of their valence quark eigenstate emitting a photon, gluon, and/or a meson. To this last point, $Z_c^\pm(3900)$ and $\chi_{c1}^0(3872)$ have been observed decaying by pion emission to $\pi^\pm J/\psi$ and $\pi^0\chi_{c1}$.

Since pions have isospin $I=1$ and the strong interaction is blind to electric charge, the $Z_c^\pm(3900)$ and $\chi_{c1}^0(3872)$ can analogously decay to $\pi^\pm J/\psi$ and $\pi^\pm \chi_{c1}$. The first of these has been observed. It would be interesting to search for the second. In this model, the $Z_c^\pm(4430)$ are the $^2P_1$ mesons of $uf$ and $f\bar{u}$; they have been observed decaying to $\pi^\pm J/\psi$ and $\pi^\pm \chi_{c1}$.

Decay by rho emission enables $Z_c^{\pm,0}(3900) \rightarrow \rho^{\pm,0} \eta_c$. The charged version of this has been observed. Direct rho emission should also contribute to $\chi_{c1}^0(3872) \rightarrow \rho^{\pm,0} J/\psi$, but it is assumed that this is highly suppressed due to the larger mass of $J/\psi$ and the smaller mass and width of the $\chi_{c1}^{\pm,0}(3872)$. In particular, it is assumed that direct rho emission generates a signal less than 5% of that of the electromagnetic decay of the neutral $\chi_{c1}^0(3872)$ to $(\gamma \rightarrow \rho^0) J/\psi$ that is described below. If true, then the magnitude of a signal from $\chi_{c1}^0(3872) \rightarrow \rho^{\pm,0} J/\psi$ would be expected to be less than 5% that of $\chi_{c1}^0(3872) \rightarrow \pi^+\pi^- J/\psi$. The hint of a signal of that magnitude at a mass of 3873 mentioned in [32] could in fact be evidence for the $\chi_{c1}(3872)$. It would be interesting to study that decay with more statistics.
The neutral $\chi^0_{c1}(3872)$ has another decay channel not shared by its charged partners. It can decay to a $c\bar{c}$ meson by emitting a photon. That photon can then create a $1^-\gamma q\bar{q}$ meson like a $\rho^0$ or an $\omega$. It is proposed that this mechanism is not only responsible for the observed decays $\chi^0_{c1}(3872) \to \gamma J/\psi$ and $\chi^0_{c1}(3872) \to \gamma\psi(2S)$, it is also responsible for the vast majority of $\chi^0_{c1}(3872) \to \pi^+\pi^- J/\psi$ decays. In the latter, the emitted photon creates a $\rho^0$ (or $\omega$ [33]) that quickly decays to $\pi^+\pi^-$. As mentioned above, it is proposed that this kind of radiative decay to $\pi^+\pi^- J/\psi$ contributes at least 20 times as much as the strong decay to $\rho^0 J/\psi$.

The $\chi_{c1}(4140)$, $\chi_{c1}(4274)$, and $\chi_{c1}(4685)$ are $1^{++}$ mesons of $(f\bar{d})_+$ and $(f\bar{s})_+$ that can decay by their $c\bar{c}$ component emitting a phi meson directly or emitting photon that produces a phi meson. All of these mesons have been observed decaying to $\phi J/\psi$. It would be interesting to see if they could also be observed decaying to $\gamma J/\psi$, $\omega J/\psi$ or $\rho^0 J/\psi$.

Axial vector mesons can also decay by gluon emission. In gluon-mediated $(f\bar{d})_+$ decays, the $c\bar{c}$ component emits a gluon which then creates a $u\bar{u}$, $d\bar{d}$, or $s\bar{s}$ pair. Since gluon emission leaves $c\bar{c}$ colored, the quarks have to rearrange to become colorless again, forming $u\bar{u} + c\bar{c}$, $c\bar{d} + d\bar{c}$, or $s\bar{s} + s\bar{c}$.

The $1^{+-}$ and $1^{++}$ mesons $Z^0_{c1}(3900)$ and $\chi^0_{c1}(3872)$ can decay by this method to $(D^*D + D^*\bar{D})^0$ and $(D^*D - D^*\bar{D})^0$, respectively. Since strong decays are blind to electric charge, this picture also implies that $Z^+_{c1}(3900)$ and $\chi^+_{c1}(3872)$ can decay to $(D^*D)^0$ and $(D^*\bar{D})^0$, when there is enough mass. If the $\chi^+_{c1}(3872)$ is the $T^+_{c1}(3775)$, then its mass is less than that of both $D^0\bar{D}^0$ and $D^{*+}\bar{D}^0$, so it cannot decay by this channel.

It is next interesting to consider the case where a $\chi_{c1}(3872)$ decays to a $\pi^0$ and a $0^{++}$ configuration of $c\bar{c}$. The $c\bar{c}$ then decays by emitting a gluon to produce D mesons. But what are the CP-allowed combinations of D mesons for the decay of a $0^{++}$ configuration of $c\bar{c}$?

It is well known from the $\psi(3770)$ that a $1^{--} c\bar{c}$ configuration decays to $(D\bar{D})^0$. Looking at the $D^0\bar{D}^0$ part of this, the daughter mesons can be expressed in terms of the C-parity eigenstates $D^0_\pm = \frac{1}{\sqrt{2}} (D^0 \pm \bar{D}^0)$. In terms of these, a $1^{--} c\bar{c}$ configuration decays to

\[
\frac{1}{\sqrt{2}} (D^0 (\bar{p}) D^0_+ (\bar{p}) - \bar{D}^0 (\bar{p}) D^0_+ (\bar{p}))
= \frac{1}{\sqrt{2}} (D^0_+ (\bar{p}) D^0 (-\bar{p}) - D^0_+ (\bar{p}) D^0 (-\bar{p})).
\]

The above configuration changes sign under C. It also changes sign under P, giving the decay products the correct $1^{--}$ quantum numbers.

A $0^{++}$ configuration can be constructed from either $D^0_+ (\bar{p}) D^0_+ (\bar{p})$ or $D^0_+ (\bar{p}) D^0 (\bar{p})$. Each configuration has terms in it that can be generated by the $c\bar{c}$ component of the meson. When each configuration separates to decay, it can generate terms with $D^0 D^0$, $D^0 \bar{D}^0$, or $D^0 \bar{D}^0$. But when the decay diagrams are added up, the terms with the latter cancel. The average over diagrams generates the following:

\[
\frac{1}{\sqrt{2}} (D^0_+ (\bar{p}) D^0 (-\bar{p}) + \bar{D}^0 (\bar{p}) D^0 (-\bar{p}))
= \frac{1}{\sqrt{2}} (D^0 (\bar{p}) D^0_+ (-\bar{p}) + \bar{D}^0 (\bar{p}) D^0_+ (-\bar{p})).
\]

In other words, this model predicts that $\chi^0_{c1}(3872)$ will predominantly decay to $\pi^0 D^0\bar{D}^0$ or $\pi^0 D^0 D^0$, but not to $\pi^0 D^0 D^0$ (separately from its decays to $D^{*0} D^0$ and $D^0 D^{*0}$).

There is one PDG-listed observation that is characterized as $\chi^0_{c1}(3872) \to \rho^0 D^0\bar{D}^0$. The referenced Belle paper notes that “it is not possible to separate the contributions of $D^0 D^0\pi^0$ and $D^0 D^0\pi^0$ to the peak” [34]. More recently, BESIII found no evidence of decays of $\chi^0_{c1}(3872) \to \rho^0 D^0 D^0$ separate from decays to $D^{*0} D^0$ and $D^0 D^{*0}$ [35]. The simplest version of this model is consistent with $\chi^0_{c1}(3872) \to \rho^0 D^0 D^0$ only coming from $\chi^0_{c1}(3872)$ decays to $D^{*0} D^0$ and $D^0 D^{*0}$. However, from the arguments given above, the model also predicts that $\chi^0_{c1}(3872)$ has 3-body decays to $\pi^0 D^0 D^0$ and $\pi^0 D^0 D^0$. It would be interesting to search for these decays.

If $\chi_{c1}(3872)$ does decay this way, then the charged versions should have the decays: $\chi_{c1}(3872) \to \pi^+ D^0 D^0$ and $\chi_{c1}(3872) \to \pi^- D^0 D^0$. The positively charged version of this has been observed, motivating the mapping of $T^+_{c1}(3775)$ to $\chi_{c1}(3872)$.

It should also be pointed out that the strongest decays expected for $1^{+-}$ f-quark mesons like $Z_c(3900)$ and $Z_c(4430)$ are S-wave decays to a pion and a lower-mass $1^{--}$ f-quark meson. The $\chi_{c1}(3872)$ does not have analogous decays. This is the primary reason that the $Z_c(3900)$ width is much larger than the $\chi_{c1}(3872)$ width. It would be interesting to search for the strong decays $Z^+_c(4430) \to \pi^+\psi(2320)$ or $Z^+_c(3900) \to \pi^0\phi X(0200)$, $\pi^0 X^\pm(4020)$ or $\pi^\pm X^+(4020)$.

### D. Other hadrons

In this model, the $1^+ S_0$ meson of $(f\bar{d})_+$ is the $X^0(3250)$ with $J^{PC} = 0^{-+}$. The $c\bar{c}$ component of its eigenstate has the same quantum numbers as $\eta_c$, which is known to decay to $K^+\bar{p}\Lambda + c.c.$ Therefore the decay $X^0(3250) \to K^+\bar{p}\Lambda + c.c.$ is enabled by this model, and those decays have been observed [8] (Mesons: Further States). The $X^+_c(3250)$ $uf$ and $fu$ mesons that decay by emitting a $\pi^+$ before the above $c\bar{c}$ decays have also been observed.

The $\chi_{c0}(3860)$ and $\chi_{c0}(3960)$ are the $1^+ P_0$ $0^{++}$ mesons of $(f\bar{d})_+$ and $(f\bar{s})_+$ that have been observed decaying to $DD$ and $D^+_s D^+_s$, respectively. In this model, these decays involve the $c\bar{c}$ components of those mesons emitting a gluon that creates a colored $q\bar{q}$ pair.

The $\chi_{c0}(4500)$ and $\chi_{c0}(4700)$ are $0^{++}$ mesons of $(f\bar{d})_+$ and $(f\bar{s})_+$ that can decay by their $c\bar{c}$ component either directly emitting a phi meson or emitting a photon that produces a phi meson. Both of these $\chi_{c0}$ resonances have been observed decaying to $\phi J/\psi$. It would be interesting...
to see if these as well as $\chi_{c0}(3960)$ could also be observed decaying to $\gamma J/\psi$, $\omega J/\psi$ or $\pi^+\pi^- J/\psi$.

In [36], the $Z_{c}^+(3985)$ was observed as a resonance of the D mesons in $e^+e^- \rightarrow K^+D^+_cD^0$ and $e^+e^- \rightarrow K^0D^+_cD^0$. The resonance was largest at a CM energy of 4680 MeV. In this model, the first part of this is $e^+e^- \rightarrow \psi(4680)$, where $\psi(4680)$ is a predicted $3^3S_1$ 1$^-$ meson of $(f\bar{s})^+$. The second part is the strong decay $\psi(4680) \rightarrow K^\pm Z_c^\mp$, where $Z_{c}^\pm(3985)$ are the 0$^-$ and 2$^-$ $S_0$ mesons of $fu$ and $uf$. From eq (17), replacing a $D$ quark with a $u$ quark and taking the charge conjugate of the whole thing, it can be seen that the $cc\bar{s}\bar{u}$ component of $Z_{c}^-$ can decay to the observed D mesons. This is another indication that the factor $R_s/sin\theta_C$ in these mesons may not be extremely small.

As noted in eq (21), an $f\bar{f}$ meson has a $cc\bar{c}\bar{c}$ component to its eigenstate. For the $X(6900)$ which is mapped to the $1^3P_0$ meson of $f\bar{f}$, this component can simply separate and generate the observed $X(6900) \rightarrow J/\psi J/\psi$ decay. Since the $X(6600)$ is mapped to the $1^3S_1$ meson of $f\bar{f}$, it can decay by $X(6600) \rightarrow J/\psi J/\psi$, with the photon unobserved. That unobserved photon could contribute to the very large apparent width of the $X(6600)$. As the $2^3S_0$ meson of $f\bar{f}$, the $X(7200)$ does not need to emit a photon in its decay $X(7200) \rightarrow J/\psi J/\psi$, so it is much narrower.

As seen in eq (22), the $f\bar{c}$ baryons of this model have 5-quark components to their eigenstates. The pentaquark resonances observed so far are from decays of one of these 5-quark components separating into a $cc$ meson and a proton or $\Lambda$.

It is interesting to speculate whether evidence for an $f\bar{c}$ meson has been seen. From the charge conjugate of eq (20), such a meson has a $c\bar{c}\bar{d}\bar{u}$ component to its eigenstate. That component could decay to a $\pi^-$ and a $cc$ configuration. As can also be seen from eq (20), that $cc$ configuration could generate an $sb$ meson like a $B_{u}^{s}$. Evidence for a narrow resonance in $\pi^- B_{u}^{s}$ at 5568 MeV was reported in [37] but was not reproduced in other experiments that had different CM energies or kinematics. The $X^- (5568)$ has a mass that would make it an attractive candidate for the $2^3P_0$ state of $f\bar{c}$.

**E. Select Predictions**

Most of the predictions of this model are made in section 7 of the paper, but a few are described in more detail here to provide more explanation about how the predictions arise from the model.

According to this model, an $f\bar{d}$ meson like $\chi_{c1}^-(3872)$ has a $c\bar{c}\bar{s}\bar{u}$ component in its eigenstate. This component can emit a $K^-$ and then a gluon, predicting the decay $\chi_{c1}^-(3872) \rightarrow K^-D^+\pi^-$. Since the magnitude of the $c\bar{c}\bar{s}\bar{u}$ component is $(R_u - R_s)$ times that of the $c\bar{c}\bar{d}\bar{u}$ component but the former component has more phase space, the branching ratio for this predicted decay should be at least as large as that of $\chi_{c1}^+(3872) \rightarrow \pi^+D^0D^0$ if $R_s \geq 2R_u$. Through a similar mechanism but possibly slightly smaller branching ratio, the model predicts $\chi_{c1}^0(3872) \rightarrow K^-D^0\pi^+$. These decays would provide evidence for the $c\bar{c}s\bar{u}$ eigenstate component predicted by this model.

If the factor $(1 - R_u)/sin\theta_C$ is not negligible, then similar decays could be seen where the $K^-$ in the above decays is replaced by a $\pi^-$. Taking the charge conjugate, the model predicts the decays $\chi_{c1}^+(3872) \rightarrow D^-\pi^+\pi^+$ and $\chi_{c1}^0(3872) \rightarrow D^-\pi^0\pi^+ or \rightarrow D^-\pi^+\pi^-$.

From eq (22), if $(R_u - R_s)$ and $R_s/sin\theta_C$ have similar magnitudes in the eigenstate of a $\Lambda_f f\bar{u}d$ baryon, then the $c\bar{u}\bar{s}u$ and $sd\bar{u}c$ components have similar magnitudes. In that case, the predicted decay $\Lambda_f \rightarrow \Lambda^-K^-\pi^+$ should have a branching ratio similar to that of $P_{\Lambda_{0}}^{\Lambda_0} \rightarrow \Lambda J/\psi$. This kind of decay could potentially be observed by measuring the invariant mass of $\Lambda_f^-K^-\pi^+$ in the decays $\Lambda_f^0 \rightarrow \Lambda_f^-K^-K^-\pi^-$. These studies could reveal new narrow $\Lambda_f$ baryons predicted by this model, with mass, $J^P$ of $\sim$3795, $\frac{1}{2}^+$ and $\sim$4200, $\frac{3}{2}^-$. 

**5. REPRODUCING SELECT EXPERIMENTAL RESULTS**

This section addresses a number of experimental results that would seemingly rule out the model proposed here. Qualitative arguments are given for how the model could potentially still reproduce data from those experiments.

**A. Inclusive Hadronic Cross Sections**

This model would appear to have a problem reproducing inclusive hadronic cross section data. The model introduces an additional quark and increases the magnitude of the right-chiral coupling of the charm quark with the Z boson. As a result, it would seem that this model’s prediction for inclusive hadronic cross section data should be significantly larger than that of the Standard Model. Since “R Values” over a wide range of energies appear to be well reproduced by the Standard Model (see [38]), they should not be well reproduced by this model.

However, it will be argued that an important interference effect has been left out of previous calculations of inclusive hadronic cross sections.

Calculating an inclusive cross section involves taking the sum of exclusive cross sections over a complete set of orthogonal final states. It has been argued that when calculating the inclusive cross section, any complete set of orthogonal final states can be used; they will all generate the same result. The argument further posits that there is no requirement that a final state must be observable. In that case, three of the possible definitions of a complete set of “final” states for $e^+e^-$ collisions are (a) an off-shell photon and a $Z$ boson, (b) all flavors of lepton-antilepton and quark-antiquark pairs, or (c) all
flavors of lepton-antilepton pairs and all states involving hadrons. According to the above argument, calculation of the inclusive cross section for $e^+e^- \to \pi^+\pi^-$ collisions generates the same result no matter which of the three sets is used.

But this is not correct. If set (a) is used, the Z boson and the photon are treated as orthogonal final states, so there is no interference between them. If set (b) is used, then for $\sqrt{s}$ near the mass of the Z boson, interference between the Z boson and the photon is included (see for example eq (1.34) of [39]). The inclusion of interference is justified by the fact that there are $ll$ final states that could have been generated either by $e^+e^- \to Z \to ll$ or by $e^+e^- \to \gamma^* \to ll$ (and the same for $q\bar{q}$). Due to this difference as to whether or not interference is included, calculation of the inclusive cross section for $e^+e^-$ collisions generates different results, depending on whether complete orthogonal set (a) or (b) is used.

Despite this known effect, it has been argued that there would be no difference in inclusive cross section calculations using complete set (b) vs. set (c). Such an equality could only arise if there was no possibility of interference between different $qq$ states in the set (c) calculation. If set (c) included a final hadronic state $X$ that could have been created either by $e^+e^- \to q\bar{q} \to X$ or by $e^+e^- \to q'\bar{q}' \to X$, where $q \neq q'$, then “flavor interference” would need to be taken into account for that state when doing a set (c) calculation. The argument that calculations using set (b) and set (c) generate the same result relies on there being no hadronic states $X$ like this in set (c).

But that is clearly not the case. The hadronic state $X = \pi^+\pi^-$ is one that can be created either by $e^+e^- \to u\bar{u} \to \pi^+\pi^-$ or by $e^+e^- \to d\bar{d} \to \pi^+\pi^-$. More generally, the class of states comprised of two jets, where each jet has no net beauty, charm, strangeness or baryon number also has those two creation modes. There are many more hadronic states $X$ that have both of these creation modes. When using set (c) for calculation of the inclusive cross section, the flavor interference between $u\bar{u}$ and $d\bar{d}$ intermediate states should be included. After including it, there is no reason to expect that inclusive cross section calculations using set (b) and set (c) should give the same result.

Given the choice between calculations using sets (b) and (c), it is proposed that the set (c) calculation is more consistent with other quantum mechanical interference calculations, since all of the final states in the set are observable. The problem with using set (c) is a practical one. Hadronization is a highly nonperturbative process, making calculation of the inclusive cross section impossible analytically. Some new numerical algorithms would be needed to calculate the cross section in the presence of flavor interference. Despite these calculational difficulties, inclusive hadronic cross section calculations using set (b) (the usual choice) should be treated with skepticism.

A different justification for using set (b) in calculations would be if there was reason to believe that the effects of flavor interference were insignificant. To qualitatively address that, it is instructive to revisit the hadronic state $X = \pi^+\pi^-$. As mentioned above, $X$ can be created either by $e^+e^- \to u\bar{u} \to \pi^+\pi^-$ or by $e^+e^- \to d\bar{d} \to \pi^+\pi^-$. The matrix elements for these two processes should be added before being squared. Even though hadronization is nonperturbative, isospin symmetry means that the $u\bar{u} \to \pi^+\pi^-$ and $d\bar{d} \to \pi^+\pi^-$ parts of the two matrix elements should be approximately equal. Therefore, the sum of matrix elements should have a partial cancellation due to the opposite signs of the electric (or Z boson) charges in the $e^+e^- \to u\bar{u}$ and $e^+e^- \to d\bar{d}$ parts of each diagram. For high CM energies below the Z pole (e.g. $20 < \sqrt{s} < 45$ GeV), comparing $(\frac{2}{3} - \frac{1}{3})^2$ to $(\frac{2}{3})^2 + (\frac{1}{3})^2$, one can see that the flavor interference of this sum is significant and destructive. Repeating the analysis for other hadronic states $X$ with increasing complexity, significant interference is seen, sometimes destructive, sometimes constructive.

The situation is further complicated by the fact that interference can also be generated from $s\bar{s}$ intermediate states. That means that the net interference effect may be different for low CM energies (e.g. below 3 GeV) than for high energies (e.g. above 10 GeV). The reason is because fewer $s\bar{s}$ pairs (relative to $u\bar{u}$ and $d\bar{d}$ pairs) are created for the former than for the latter. This difference in effect is incorporated into the hypothesis below.

What about interference involving the heavier quark flavors? Consider the exclusive process $e^+e^- \to B^+B^-$. The final hadronic state could be generated by either $e^+e^- \to B^+B^-$ or by $e^+e^- \to u\bar{u} \to B^+B^-$. But the latter would require nonperturbative generation of a $b\bar{b}$ pair. Such pair creation should have a very small contribution since $2m_b \gg \Lambda_{QCD}$. On the other hand, since $2m_u \ll \Lambda_{QCD}$, a $u\bar{u}$ pair can easily be generated, so $e^+e^- \to B^+B^-$ should be generated almost exclusively from the $e^+e^- \to b\bar{b}$ subprocess. That same argument would hold for any hadronic final state in which at least one jet carries a beauty quantum number. This suggests that the usual approach of ignoring flavor interference should be a good approximation for hadronic states involving beauty or charm quarks, since the masses of those quarks are much larger than $\Lambda_{QCD}$.

Similarly, it should be a good approximation to ignore flavor interference when calculating the contribution to the inclusive cross section generated by the additional $f$ quark. If so, then below the Z pole, $e^+e^- \to ff$ events contribute $\sim \frac{1}{3}$ to the R value.

In summary, in the absence of a calculation that fully takes into account $u/d/s$ flavor interference in inclusive cross sections, it is hypothesized that such a calculation with the 6 known quarks would approximately agree with the non-interference calculation for CM energies below 3 GeV, but would generate an R value $\sim 1/3$ smaller for CM energies above 10 GeV. If the hypothesis is correct, then above 10 GeV, something else would be needed to add $1/3$ back to the calculated R value and reproduce the data. The $f$ quark of this paper could perform that
needed role.

Most of the above arguments were made for inclusive cross sections below the Z pole. But similar arguments could be made at the Z pole using Z boson couplings to the quarks. It is similarly hypothesized that including u/d/s interference in calculations would reduce the calculated hadronic partial width of the Z boson so that the calculation would underpredict the data. The contribution of the \( f \) quark (and the larger coupling of the charm quark) to the hadronic partial width would have to be added to reproduce the data.

The above hypothesis could be proven or disproven by a calculation that fully takes into account u/d/s flavor interference in inclusive hadronic cross sections. The goal of this section was to show that there is reason to doubt the usual interference-free calculation and that it is plausible that a full calculation could require something like the \( f \) quark to reproduce the data.

\[ \text{B. Cross Section Spikes} \]

If the \( f \) quark exists, then why haven’t experiments seen telltale narrow resonance spikes in lepton pair cross sections, such as those seen for the \( J/\psi \) and \( \Upsilon \)? The reason that the \( J/\psi \) and \( \Upsilon \) resonances are narrow is because their masses are below the thresholds for decay to \( DD \) or \( BB \), respectively. The allowed decay channels are suppressed by the OZI rule. In the allowed OZI-suppressed channels, decay to a pair of leptons is competitive with decay to hadrons, so the resonant spikes can be clearly seen for example in \( e^+e^- \rightarrow \mu^+\mu^- \) cross sections [40].

In the hypothesized model, the \( 1^3S_1 \) \( f \bar{f} \) meson’s mass is larger than twice the mass of the \( 1^1S_0 \) \( f\bar{u} \) and \( f\bar{d} \) mesons. Specifically, it was proposed above that the \( 1^1S_0 \) mesons have masses of around 3250 MeV, while the \( 1^3S_1 \) \( f\bar{f} \) meson is the observed \( X(6600) \) resonance with a mass of around 6638 MeV [13]. In that case, the \( X(6600) \) would decay strongly to a \( 1^1S_0 \) \( f\bar{u} \) (or \( f\bar{d} \)) meson and its conjugate by a gluon creating a \( u\bar{u} \) (or \( d\bar{d} \)) pair. Decay of the \( X(6600) \) to a pair of leptons would not be competitive with this preferred strong decay to hadrons. That would explain why no 5\( \sigma \) resonance in \( e^+e^- \rightarrow \mu^+\mu^- \) has been observed between the \( \psi(2S) \) and \( \Upsilon(1S) \) spikes [40].

That being said, the 3.5\( \sigma \) hint of a resonance in \( e^+e^- \rightarrow \mu^+\mu^- \) seen at 7250 MeV [40] could correspond to the \( 2^3S_1 \) \( f\bar{f} \) meson of \( f \). That mapping would be consistent with the section 1 mapping of the \( X(7200) \) resonance observed in \( J/\psi \) pair production to the \( 2^3S_1 \) meson of \( f\bar{f} \).

Why haven’t resonant spikes been seen in inclusive hadronic cross sections in this mass range? It is because the strong decay of the \( X(6600) \) generates a wide resonance (440 MeV), that gets flattened further by initial state radiation, similar to what is seen for the \( \Upsilon(4S) \). The inclusive hadronic cross section datasets in the \( \sqrt{s} = 6.5 \) to 8 GeV range are in significant disagreement with each other (see for example fig 10 in [14]). However, both sets of data appear to be able to accommodate a hadronic resonance that coincides with the \( X(6600) \) seen in \( J/\psi \) pair data.

\[ \text{C. Z-charm coupling} \]

The precision experiments at LEP and SLD measured partial width and asymmetry data for \( e^+e^- \rightarrow Z \rightarrow bb \) and \( e^+e^- \rightarrow Z \rightarrow c\bar{c} \) events. Based on those data, the experiments inferred the left-chiral and right-chiral couplings of the \( Z \) boson to the \( b \) and \( c \) quarks. The inferred couplings matched those of the Standard Model.

To a first approximation (neglecting FCNC), the \( b \) quark in this model has the same coupling to the \( Z \) boson as in the Standard Model, but the \( c \) quark has a different coupling (as can be seen in eq (14)). This model must be able to reconcile how the different \( Zc\bar{c} \) coupling could still reproduce the data.

The first step to that reconciliation is to calculate the lifetime of the new proposed \( f \) quark. An upper-limit estimate of the \( f \) quark lifetime relative to that of the \( b \) quark can be found by assuming that it only decays weakly and using the zeroth order term in Heavy Quark Effective Theory [41]. Treating the phase space of vector and axial vector couplings to the \( W \) boson similarly to how they are treated in neutron decay, one has:

\[ \frac{\tau_f}{\tau_b} < \frac{m_b^5 (|V_{cb}|^2 + 3|V_{cb}|^2)}{m_f^5 (|V_{cf}|^2 + 3|V_{cf}|^2)} \sim 1\%, \]  

where \( V^\pm \) are the CKM matrices defined in eq (10). To get the numerical result, a mass for the \( f \) quark of 2.8 GeV was used (relative to 1.28 for \( c \) and 4.18 for \( b \)), along with the CKM matrices of eq (13).

The above estimate is appropriate for a free \( f \) quark, but after hadronization, \( f \) quark hadrons decay even more quickly, since their primary decay is via the strong interaction to intrinsic charm and other components in their eigenstate, as described in section 4. Nonetheless, the upper-limit estimate of eq (25) is sufficient for the present discussion.

One of the main ways that \( b \) and \( c \) quarks were identified at LEP and SLD was from the secondary vertex seen when the \( b \) quark decayed to a \( c \) quark (or the \( c \) quark decayed to an \( s \) quark). The \( b \) quark lifetime translated into an “impact parameter” distance for its secondary vertex of around 300 microns. With experimental resolution of 20-70 microns, the \( b \) secondary vertex was reliably identified [39].

According to the lifetime calculation above, the impact parameter of a secondary vertex for the \( f \) quark should then have been less than 3 microns. This means that the secondary \( f \rightarrow c \) vertex would not have been seen. For this reason, \( Z \rightarrow f\bar{f} \) events would have been mostly grouped together with \( Z \rightarrow c\bar{c} \) events in \( c \) quark partial width and asymmetry measurements.

This grouping would lead to effective vector current and axial vector current couplings of \( c \)-quark pairs with...
where the above $V$ couplings are elements of $V$-mixing matrices like the one in eq (16). Each $P_q$ specifies the probability that a term would have survived the various tags and cuts imposed on $Z \to c\bar{c}$ partial width and asymmetry data.

If the following choices are made:

For partial width: $P_f = P_b = 1$, $P_s = P_d = 0.5$

For asymmetry: $P_f = P_b = 0.46$, $P_s = P_d = 0.23$, \( (27) \)

then the effective couplings $V_{Zc\bar{c},eff}^{V,A}$ of this model are very similar to the couplings of the $Z$ boson to the charm quark in the Standard Model. Since the data are fit by the Standard Model, they would also be fit by this model with the above choices of probabilities.

Those choices qualitatively make sense for the following reasons: In the LEP and SLD experiments, $Z \to b\bar{b}$ events were mostly double tagged, so $Z \to f\bar{b}$ events would mostly not have been included in $Z \to b\bar{b}$ measurements. On the other hand, $Z \to c\bar{c}$ events were mostly single tagged, and the best tags were ones that counted $c$ quarks (or $D$ mesons). Since each $b$ or $f$ quark would produce a $c$ quark, that justifies $P_f = P_b = 1$. Since $s$ and $d$ quarks would not have produced a $c$ quark, that justifies $P_s = P_d = 0.5$ (e.g. only the $f$ quark in an $f\bar{s}$ event would produce a $c$ quark).

For asymmetry measurements, additional tags were used, including measurements of opposite hemisphere charge. When interpreting $f$-quark events as $c$-quark events, a little more than half of the $f$-quark events would have had the wrong sign for their opposite hemisphere charge (the “little more” is due to jets with nonzero baryon number). This qualitatively justifies the choices for the asymmetry probabilities.

The main point of this section is that despite the additional quark and different $Zc\bar{c}$ coupling, it is possible that the model could have reproduced the experimental data.

### 6. ALTERNATIVE EXPLANATIONS OF SELECT DATA

#### A. CKM 3 sigma deviations

Currently there are no CKM measurements that disagree with the Standard Model by $5\sigma$. There are, however, some $3\sigma$ hints of disagreement. For example, measurements of CKM matrix elements lead to a first-row unitarity calculation of $0.9985 \pm 0.0005$ [22], which is a $3\sigma$ variation from the unitary value of 1 predicted by the Standard Model. In addition, the following $2.9\sigma$ difference is measured:

Vector Current: $|V_{us}| = 0.2231 \pm 0.0006$

Axial Vector Current: $|V_{us}| = 0.2254 \pm 0.0006$, \( (28) \)

where the first result above is from semi-leptonic kaon decay and the second is from leptonic [42]. Also, $V_{cb}$ measured via exclusive decays differs by $2-3\sigma$ from $V_{cb}$ measured via inclusive decays [43].

Since the Standard Model has only left-chiral $W$ boson interactions and six quarks, its CKM matrix should be unitary and there should be no difference between vector-current and axial-vector-current CKM matrices. If increased precision causes the above discrepancies to exceed $5\sigma$, it will therefore be problematic for the Standard Model.

The model presented here has the flexibility to reproduce the above $3\sigma$ differences. In fact, the mass parameters of eq (12) were chosen to reproduce them, as can be seen from the $V_{us}$ elements generated by those matrices. Also, the FCNC of this model enables $b \to f \to c$ decays that will contribute to inclusive measurements of $V_{us}$. That could provide a qualitative explanation for the difference between inclusive and exclusive measurements of that CKM matrix element.

### B. Weak Radiative Decay of Hyperons

For several decades, there has been a debate about how to reproduce data from weak radiative decays of hyperons [44]. Hara’s theorem states on general grounds that in the limit of flavor SU(3) symmetry, these decays should have very small parity violation [45]. The large asymmetry observed in these decays can be interpreted in one of two ways: (a) flavor SU(3) symmetry is significantly broken for weak interactions or (b) Hara’s theorem is violated. Baryon magnetic moments show that flavor SU(3) symmetry provides a good approximation for electromagnetic interactions, so if Hara’s theorem is not violated, a reason must be provided for why flavor SU(3) is significantly broken just for weak interactions. On the other hand, violation of Hara’s theorem would require violation of electromagnetic gauge invariance, locality (at the hadron level), or $CP$ symmetry.

This model assumes Hara’s theorem is not violated and provides an alternative explanation for the breaking of flavor SU(3). As in the Standard Model, the $d$ and $s$ quarks have the same electric charge, so SU(3) flavor symmetry should be a good approximation for electromagnetic interactions. Unlike the Standard Model, they have different weak interaction couplings, as can be seen from eqs (10) and (15). Therefore, even in the limit that the $d$ and $s$ quark masses are the same, flavor SU(3) is significantly broken for the allowed parity-violating hadronic matrix elements.

At the quark level, in addition to the Standard-Model process involving exchange of a $W$ boson between va-
lence and sea quarks, this model includes a process involving a Z boson. For example in \( \Sigma^+ \to p + \gamma \), the latter involves the FCNC process \( s \to d + Z \), with the Z then having a flavor-conserving interaction with one of the \( \Sigma^+ \) sea quarks. If this FCNC effect is occurring, it could play a proportionally larger role in decays such as \( \Xi^- \to \Sigma^- + e^+e^- \) that have fewer W boson mediated radiative decay modes. If the decay \( \Xi^- \to \Sigma^- + e^+e^- \) was observed with a branching ratio significantly larger than predicted by the Standard Model, that would provide support for the proposed existence of FCNC interactions involving \( s \) and \( d \) quarks.

C. Forward Z+c data

The LHCb collaboration recently measured from pp collisions the number of events emerging at very forward rapidities that involved a Z boson and a charm jet [46]. The number of measured events was significantly larger than the number predicted by Standard-Model event generators that incorporate Parton Distribution Functions (PDFs) that assume that the only charm quarks inside of a proton are those generated perturbatively (by gluons). On the other hand, the data were well reproduced by PDFs that assume that a proton has some “intrinsic charm” [27].

As described in section 4, this model can accommodate intrinsic charm in an amount that is compatible with the above measurements.

7. Predictions

Based on the hypothesis, a number of predictions are made. A few were described in section 4, but there are many more below:

A. Decays to the primary eigenstate component

1. The decays \( Z_c^{\pm,0}(3900) \) to \( \pi^{\pm,0}X^0(4020) \), \( \pi^0X^{\pm}(4020) \) and \( \pi^\mp X^{\pm}(4020) \) will be observed. They will have branching ratios at least ten times larger than \( Z_c^{\pm,0}(3900) \) to \( \pi^{\pm,0}\psi(4230) \).

2. Similar decays with similar branching ratios will be observed for \( Z_c^{\pm,0}(3900) \) to \( \pi^{\pm,0}X^0(3350) \), \( \pi^0X^{\pm}(3350) \) and \( \pi^\mp X^{\pm}(3350) \), where \( X^\pm(3350) \) are the predicted charged isospin partners of the observed \( X^0(3350) \).

3. The decay \( \psi(4660) \to \pi^0\pi^0\psi(4230) \) will be observed with a branching ratio at least five times that of \( \psi(4660) \) to \( \pi^0\pi^0\psi(4230) \).

4. In the above decay, most of the branching ratio will be through a resonance in the invariant mass of \( \pi^0\psi(4230) \) at around 4430 MeV that will be observed. This is the predicted \( Z_c^0(4430) \) isospin partner of the observed \( Z_c^0(4430) \).

B. Decays to c\bar{c} components

8. In the invariant mass of both \( \psi(2S)J/\psi \) and \( \psi(2S)\psi(2S) \) a resonance at 7500 MeV almost as prominent as the 7200 MeV resonance will be observed. It is the \( \Xi_{c}^{++}(7540) \) resonance of J/\psi.

9. The decays \( \chi_{c0}^+(3872) \to \pi^0D^0D^0 \) and \( \chi_{c1}^+(3872) \to \pi^0D^0D^0 \) will be observed. They will have branching ratios similar to that of \( T_{c0}^+(3875) \) to \( \pi^+D^0D^0 \).

10. The decays \( \chi_{c2}^+(3872) \to \pi^{\pm}\chi_{c1} \) will be observed, where \( \chi_{c2}^+(3872) \) in this model is the same as the observed \( T_{c2}^+(3875) \). The branching ratios for these will be similar to that of \( \chi_{c1}^+(3872) \to \pi^0\chi_{c1} \).

11. The decay \( X^0(3250) \to (K\bar{K}\pi)^0 \) will be observed with a branching ratio at least ten times that of the observed decay to \( K^+\Lambda\bar{p} \).

C. Decays to c\bar{u} components

12. The decay \( \chi_{c1}^+(3872) \to K^-D^+\pi^- \) and its charge conjugate will be observed. These will have branching ratios at least as large as that of \( T_{c1}^+(3875) \) to \( \pi^+D^0D^0 \).

13. \( \chi_{c1}^{0}(3872) \to K^-D^0\pi^+ \) will be observed and have a branching ratio a little smaller than the above decays.

14. The decays \( \chi_{c1}^{0}(3872) \to D^-\pi^+\pi^- \) and \( \chi_{c1}^{0}(3872) \to D^-\pi^0\pi^- \) will be observed.

15. In decays \( \Lambda_b \to \Lambda_c^+K^-\bar{K}^+\pi^- \), neutral isospin-0 “\( A_f \)” resonances will be observed in \( \Lambda_c^+K^- \) that have masses and \( J^P \) of \( \sim 3795, \frac{3}{2}^+ \) and \( \sim 4200, \frac{1}{2}^- \).

16. In decays \( \Lambda_b \to \Lambda_c^+K^-\bar{K}^+\pi^- \), charged isospin-1 “\( \Sigma_f^- \)” resonances will be observed in \( \Lambda_c^+K^- \) that have masses and \( J^P \) of \( \sim 3890, \frac{1}{2}^+ \) and \( \sim 4280, \frac{3}{2}^- \).

17. The decays \( Z_c^+(3900) \to K^+D^0 \) and its charge conjugate will be observed with branching ratios similar to that of \( Z_c^+(3900) \) to \( \pi^+J/\psi \).

18. The decays \( \psi(4660) \to \Lambda_c^+\Sigma^- \) and its charge conjugate will be observed with branching ratios similar
to that of $\psi(4660) \rightarrow \Lambda_c^+ \Lambda_c^-$.  
19. The decay $\psi(4360) \rightarrow \Lambda_c^+ \bar{p}$ will be observed with a branching ratio around 10-20% that of $X(3350) \rightarrow \Lambda_c^+ \bar{p}$. 
20. In $e^+e^- \rightarrow \Lambda_c^+ \bar{p}$, a resonance with the mass and width of $X(3350)$ will be observed. 
21. In $e^+e^- \rightarrow \pi^0 J/\psi$, a resonance with the mass and width of the $X(3350)$ will be observed.

D. Other predictions

22. More detailed measurement of the inclusive cross section for $e^+e^- \rightarrow$ hadrons in the 6-8 GeV range will reveal resonances whose masses and widths correspond to the $X(6600)$ and the evidence for a resonance at 7250 MeV mentioned in [40]. 
23. The inclusive cross section for $e^+e^- \rightarrow$ hadrons in the 8.6 to 9.4 GeV range will be better fit by including a resonance at around 8.9-9.0 GeV. This is the $3^3S_1$ and/or $2^3D_1$ resonance of $(f\bar{b})_+$. 
24. In invariant mass data for $(X(3250)B_1(5721))^0$ a resonance matching the one above will be observed. 
25. In invariant mass data for $(Z_c(3900)B^0)$ a resonance with mass slightly larger than $Y(1S)$ will be observed. This is the $4^3S_1$ or $3^3D_1$ resonance of $(f\bar{b})_+$. 
26. In invariant mass data for $(Z_c(3900)D^\pm)$ resonances with mass around 5860 MeV will be observed. These are the $2^3D_1$ resonances of $f\bar{c}$ and $c\bar{f}$. 
27. An $f\bar{u}$ isospin partner will be found for each $f\bar{d}$ meson listed in section 1 (and vice versa). 
28. The undetermined $J^{PC}$ values of the hadrons listed in section 1 will be determined to be consistent with the quark-model mappings in those sections. 
29. More precision will cause the first-row unitarity calculation of the Standard-Model CKM matrix to differ from 1 by more than 5σ. 
30. More precision will cause the difference between vector-current and axial-vector-current measurements of $|V_{ub}|$ and ultimately $|V_{ub}|$, $|V_{cd}|$ and $|V_{cs}|$ to exceed 5σ. 
31. More precision will cause the difference between inclusive and exclusive measurements of $|V_{ub}|$ to exceed 5σ. 
32. The decays $\Xi^- \rightarrow \Sigma^- e^+ e^-$ and $\Sigma^+ \rightarrow p e^+ e^-$ will be observed. Their branching ratios will exceed the Standard-Model predictions by 5σ.

CONCLUSION

The Standard Model has been extraordinarily successful in reproducing a wide range of precision experiments performed over the last 50 years. Due to this unprecedented success, it is difficult to believe that a theory significantly different from the Standard Model could also reproduce those same data. Nonetheless, it is possible.

The most problematic data for a theory involving an additional quark are those from inclusive hadronic cross section measurements. These data appear to leave no room for an additional quark. However, calculations of the inclusive cross section have always left out interference between $e^+ e^- \rightarrow u\bar{u} \rightarrow X$ and $e^+ e^- \rightarrow d\bar{d} \rightarrow X$ processes where the unobserved $q\bar{q}$ intermediate states generate the same observable hadronic state $X$. It has been argued here that if this interference is taken into account, Standard-Model calculations of inclusive hadronic cross sections may underpredict the data. The hypothesis implies that reproducing the data would require an additional quark like the one proposed here.

Once the additional quark has been hypothesized, most of the exotic hadrons observed over the last 20 years can be nicely mapped to normal mesons and baryons involving the additional quark. With this structure in place, a large number of falsifiable predictions have been made regarding quark-model consequences of the additional quark, evidence of right-chiral W boson interactions, and evidence of flavor changing Z boson interactions.

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