Experimental modal analysis of an aluminum rectangular plate by use of the slope-assisted BOTDA method

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Abstract
We report an experimental modal analysis of an aluminum rectangular plate (50 cm × 30 cm × 0.3 cm), carried out by use of a Brillouin optical time-domain analysis (BOTDA) sensor operating in the slope-assisted configuration, i.e. at a fixed pump–probe frequency shift. Strain measurements were acquired along an optical fiber attached to the structure, at a maximum acquisition rate of 250 Hz, a spatial resolution of 30 cm and a sampling distance of 5 cm in both x- and y-directions. A sequence of dynamic tests, aimed to evaluate the resonant frequencies and strain modal shapes of the structure, were performed on the plate for various boundary conditions (plate clamped with four, three or two bolts). Comparison with finite element method (FEM) analysis and dynamic strain measurements with strain gauges shows that Brillouin based distributed sensors can be usefully employed to perform the modal analysis of a vibrating structure, even if the spatial resolution is comparable with the plate dimensions.

(Some figures may appear in colour only in the online journal)

1. Introduction

Modal analysis is a powerful technique capable of determining the dynamic response in structural dynamic problems. In general, applications of modal analysis cover a broad range of objectives: identification and evaluation of vibration phenomena, validation, structural integrity assessment, structural modification, and damage detection. It had been used on mechanical systems, transportation systems, and large civil engineering structures—anything that is subject to dynamic motion or vibration [1, 2]. Experimental modal analysis can be performed by use of a laser Doppler vibrometer (LDV), which allows remote, non-intrusive velocity measurements with a typical bandwidth of tens of MHz, and a velocity range up to a few m s⁻¹ [3]. However, the LDV is a single-point axial transducer, since it measures only the velocity at the point on which the laser spot is focused. The scanning laser Doppler vibrometer (SLDV) sensor, instead, measures the velocity in a grid, but requires a scanning system composed of two controlled orthogonal mirrors and needs a time-consuming scan. Accelerometers also suffer from the limitations of pointwise operation. Moreover, they may introduce a non-negligible mass loading on the tested structure, affecting modal parameter estimation especially when testing light or small structures [4, 5]. In [6], an electronic speckle pattern interferometry (ESPI) technology is proposed for experimental modal analysis on a plate. ESPI accesses a great number of measurement points simultaneously, providing high definition displacement fields in a short time. However, it needs preparation of the structure...
in the case of surfaces whose micro-roughness is comparable to the wavelength of the illumination source.

Optical fiber sensors may provide significant advantages over conventional vibration sensors, as they allow simultaneous multi-point strain measurements. Also, because the strain is directly related to the stiffness of the structure, it is more sensitive to structural defects with respect to displacement or velocity [7]. In previous work [8], a Brillouin optical time-domain analysis (BOTDA) configuration operated in the slope-assisted mode [9, 10] was employed in order to retrieve the natural frequencies and modal shapes of an aluminum cantilever beam. In this work, the same method is applied to measure the natural frequencies and modal shapes of a rectangular aluminum plate in different clamped conditions. The novelties of this work with respect to [8] can be summarized as follows: (i) the spatial resolution is reduced from 50 to 30 cm; (ii) the maximum acquisition rate is increased from 108 to 250 Hz; (iii) a method to calculate the absolute dynamic strain levels of the vibrating structure is applied, which also takes into account the different sensitivities to strain of the various fiber positions; (iv) a proper path for the fiber is used, allowing the modal shape to be reconstructed across a 2D structure.

In the following experiments, we compare the measured modal shape for each test case with the one provided by a numerical FEM analysis.

The remainder of this paper is organized as follows. In section 2 we discuss the employed technique. Section 3 reports the experimental and numerical results. Finally, section 4 concludes the paper.

2. Method

In order to acquire time-varying strain changes, a BOTDA set-up operated in the slope-assisted (SA) configuration [8–10], i.e. at a fixed pump–probe frequency shift, was employed. In brief, the method relies on the measurement of the probe signal amplification induced by stimulated Brillouin scattering (SBS), following the launch of a counter-propagating pump pulse. Provided that the frequency shift between the two beams is tuned within the rising or falling slope of the Brillouin gain spectrum (BGS) of the fiber, the probe wave temporal amplification at each position will be modulated by the dynamic strain acting on the fiber, as a result of the BGS spectral shift (see figure 1). Therefore, for each injected optical pulse, a full picture of the strain acting on the fiber is obtained, at a spatial resolution related to the pump pulse duration. Theoretically, the acquisition rate only depends on the pump pulse repetition rate, which in turn is limited by the fiber length. For example, for a fiber length of 10 m, a pulse repetition rate as large as 10 MHz could be adopted, owing to a pulse roundtrip time of 100 ns. In practice, some averaging of the probe signal is necessary in order to improve the signal-to-noise ratio, especially when using a single-mode fiber and a polarization scrambler to average out the state-of-polarization related Brillouin gain fluctuations. Therefore, the actual acquisition rate will be rather limited by the number of averages, and therefore by the required signal-to-noise ratio. Another limitation of the method is that the Brillouin frequency shift oscillations should lie within the BGS linear slope range in order to ensure linearity [8]. This puts a limit on the dynamic strain range. However, for short pump pulses, the BGS linewidth is so large that this condition is easily satisfied [10]. For example, in our experiments a pulse duration of 3 ns gave rise to a BGS linear slope range of ∼150 MHz, resulting in a dynamic strain range as large as ∼3000 με. Finally, it should be considered that bonding the fiber to the structure generally induces some strain along the fiber itself, resulting in a non-uniform Brillouin frequency shift baseline. If the spatial Brillouin shift variations are not negligibly small compared to the BGS bandwidth, the use of a single pump–probe frequency shift to monitor the whole fiber simultaneously results in different strain sensitivities for different fiber positions. In our experiments, a proper correction procedure was applied to the dynamic measurements, in order to compensate for these sensitivity changes [11]. The procedure consists in acquiring the BGS at each fiber position statically, by using the standard BOTDA technique. The sensitivity to dynamic strain is then calculated for each position, based on the slope of the local BGS at the chosen working point. The retrieved sensitivity is then applied to the dynamic measurements, so as to convert the signal retrieved from each position to a strain waveform.

3. Experimental and numerical results

The experiments were performed on a 50 cm × 30 cm × 0.3 cm (L × W × H) aluminum plate (see figure 2). A single-mode G.657.B3 optical fiber, with an outer jacket of 900 μm and a Brillouin frequency shift of 10710 MHz at room temperature, was glued to the upper surface of the plate by means of a cyanoacrylate adhesive. The fiber was deployed in order to measure the longitudinal strain of the plate along the x direction. Moreover, the same fiber was run seven times along the plate, with a 5 cm step in the y-direction, in order to retrieve the dynamic strain at several points across the plate.
Figure 2. A schematic top view of the aluminum plate employed for the experiments. The red line represents the path of the optical fiber attached to the plate. The white circles represent the clamps.

Figure 3. Brillouin frequency shift static profile, acquired with the plate in rest conditions at 30 cm spatial resolution. The insets show the BGSs at two sample positions ($z = 7.55$ m and $z = 10.76$ m), with the corresponding slopes at the working point frequency.

Figure 4. The vibration amplitude measured at a fiber position close to the plate center, when exciting the structure in the spectral range 25–85 Hz.

with a single measurement. About one meter of fiber was left between the various bonded segments, in order to avoid overlap of the Brillouin signals associated with the various fiber strands.

The input excitation was provided by a magnetic shaker (TIRAvib 50009), attached to the plate center, and acting so as to apply a vertical force to the plate. The shaker was characterized by a rated peak force of 9 N, a maximum travel of 3 mm, a rated current of 2.7 A and a 2–13 000 Hz frequency range.

After gluing the fiber to the plate, a static acquisition of the Brillouin frequency shift profile was performed with the plate in rest conditions, in order to determine the optimal pump–probe frequency shift to be employed in the dynamic tests. For this measurement, the BOTDA set-up was operated in standard mode, i.e. by sweeping the pump–probe frequency shift over a proper range. A 3 ns duration of the pump pulse was set, corresponding to a spatial resolution of 30 cm. The Brillouin shift profile retrieved by curve-fitting is shown in figure 3. Besides the Brillouin shift peak at the beginning of the fiber, associated with an SMF-28 pigtail, each fiber strand glued to the plate is clearly recognizable, owing to the strain induced by the bonding procedure.

In particular, a Brillouin frequency shift variation as large as 50 MHz is estimated along the glued fiber. Note that, due to the limited spatial resolution, the actual Brillouin shift changes can be even larger than those estimated by curve-fitting the various BGSs. However, as far as modal shape measurement is concerned, the accuracy of the static Brillouin shift profile reconstruction does not come into play. Rather, it is essential to estimate accurately the sensitivity of the sensor to dynamic strain at the various positions, from the slope of each BGS. Therefore, the slopes of the various BGSs acquired statically were evaluated at the chosen set point frequency of 10 860 MHz, located along the falling edge of each BGS. As an example, we report in figure 3 the BGSs at two fiber positions, which correspond to the middle positions of strands #1 and #3, respectively, together with the computed sensitivity at the working point. At the position $z = 7.55$ m, a BGS slope of $113.1 \mu V MHz^{-1}$ was found, corresponding to a dynamic strain sensitivity of $\sim 5.7 \mu V \mu e^{-1}$. The sensitivity at each position was employed in the dynamic tests in order to calculate the strain amplitude at each position [11].

The dynamic tests were carried out by still setting a pulse duration of 3 ns, while the number of averages was varied each time, in order to find the best compromise between signal-to-noise ratio and acquisition frequency. The sampling rate was 2 GS s$^{-1}$, corresponding to a sampling distance of 5 cm along the fiber.

In the first test, the plate was clamped to an optical bench with four M6 bolts, each one placed at a distance of 3 cm from the plate corner in both $x$- and $y$-directions (see figure 2). The first natural frequency of the plate was identified by driving the shaker with a chirped current signal, having an amplitude of 2 A and a frequency ranging from 25 to 85 Hz in 12 s. The number of averages for the Brillouin signals was set to 32, resulting in an acquisition rate of $\sim 250$ S s$^{-1}$ (one updated strain profile every 4 ms). The squared modulus of the fast Fourier transform (FFT) of the signal measured at a fiber position close to the plate center (strand #4) is shown in figure 4. A natural frequency of 55.2 Hz is identified within the considered range.
As a next step, the strain modal shape of the identified mode was measured. To this aim, a sinusoidal current with an amplitude of 2 A was applied to the shaker, so as to excite the plate at the measured resonance frequency. The modal shape was retrieved by performing the FFT of the Brillouin gain signal acquired at each fiber position, and taking the real part of the resonance frequency component. Note that taking the real part allowed us to extract both the amplitude and the phase of the vibration. The number of averages was set to 64, resulting in an acquisition rate of 173 S s$^{-1}$. Up to 12 000 strain profile acquisitions were performed, giving rise to an overall acquisition time of $\sim$70 s. In figure 5(a) we report the dynamic strain profile acquired for each of the seven fiber strands attached to the plate. For validation proposes, we show in figure 5(b) the corresponding FEM result achieved by the commercial software Comsol Multiphysics$^{\text{R}}$, in which the computed strains were normalized in order to assume unitary magnitude at the center of the plate. Numerical simulations were performed by modeling each clamp as a fixed point constraint, and assuming for the plate a Young modulus of 69 GPa. The experimental profiles are in relatively good agreement with the numerical simulation. In particular, in both cases we observe that the strain becomes compressive in the proximity of the four plate corners (leftmost and rightmost regions of fibers #1 and #7). The modal assurance criterion (MAC), which indicates the correlation between the two mode shapes [12], is 0.87, revealing good consistency between the experimental and numerical shapes.

While the discrepancy between the measured and numerical profiles can be mostly attributed to the limited spatial resolution of the sensor, the natural frequency of the first resonance, computed by FEM analysis, is 51.8 Hz. The difference between the numerical and experimental values of the natural frequency may be attributed to the effect of the fiber attached to the plate, which was not considered in the simulations due to memory space limitations of the PC running the FEM software. Actually, the diameter of the silica core of the optical fiber is 125 $\mu$m, which is not negligible with respect to the thickness of the plate. Further study is needed to analyze the influence of the attached fiber on the modal parameters of the structure under test.

Figure 5. The strain modal shape of the first resonance of the plate fixed with four clamps, as measured using the fiber optic sensor (a) or calculated using the FEM analysis (b).

In order to facilitate the comparison between experimental and numerical results, we report in figure 6 the measured and computed strain modal shapes across the structure. Note that, in the case of the experimental map, a distinct value of the strain amplitude was available each 5 cm along both $x$- and $y$-directions, providing 77 data collection points.

In order to further assess the accuracy of the sensor, the dynamic strain measured by the fiber was compared with a similar measurement carried out by the use of two identical strain gauges (6/120LY11 by HBM). One strain gauge (SG1) was glued to the plate surface in the proximity of the plate center, while the second one (SG2) was bonded near the plate end (see figure 2). The gauge factor of the strain gauges was 2.05. In figure 7, we compare the strain amplitudes measured by SG1 with those acquired by the fiber at the middle of strand #4, while varying the current amplitude applied to the shaker. Still, the plate was excited at its resonance. It is seen that the strain amplitudes provided by the fiber are always smaller (about 30%) than those provided by SG1. The difference should be mainly attributed to the limited spatial resolution of the fiber sensor (30 cm), compared to the gauge length of the strain gauge (6 mm). Actually, while the gauge length of the strain gauge is so small that the strain amplitude of the vibrating plate is relatively constant along its length (see the numerical modal shape in figure 5(b)), the opposite is true for the fiber, i.e. the smoothing effect owing to the 30 cm spatial resolution results in a weaker strain amplitude. The same circumstance may explain, partially, the non-null strain amplitudes measured by the fiber at the plate ends (see figure 5(a)). In particular, we note that fiber strands #4 and #5 indicate a positive strain at the plate ends, while the corresponding numerical strain is null. While BOTDA measurements near the plate ends are surely affected by the limited spatial resolution, we also note from figure 7 that strain gauge SG2 reveals a positive strain (i.e., in phase with the dynamic strain from SG1), although much weaker than the strain measured by the fiber (the amplitude of the strain measured by SG2 was $\sim$0.5 $\mu$e at 3 A driving current). Therefore, it should be concluded that the discrepancy between BOTDA measurements and FEM data...
near the plate ends is not entirely due to measurement error, rather some strain is actually produced around the plate ends by the shaker excitation, in opposition to what is indicated by the FEM analysis.

The modal analysis was repeated under different boundary conditions. In particular, the natural frequency identification and strain modal shape measurement were repeated after removing, each time, one of the four bolts fixing the structure to the optical bench.

In all cases, the natural frequency of the tested structure, identified in the range 25–85 Hz, was 30.5 Hz (against the value of 27.8 Hz provided by the FEM analysis). The strain modal shapes were measured by setting a number of averages equal to 128, giving rise to an acquisition rate of 107 S s\(^{-1}\). The number of acquired strain profiles was 6000, resulting in an overall acquisition time of \(\sim\)56 s. In figure 8 we summarize the results of the four measurements.

In analogy to the previous test, we computed for each test case the corresponding FEM mode shape (see figure 9). Still, the numerically calculated strain shapes were normalized in order to have unitary magnitude at the plate center. Both the experimental and numerical maps reveal a strain peak in correspondence with the \(y\)-position of the removed clamp, and at an \(x\)-position in the proximity of the opposite clamp.

The MACs computed for the four cases shown in figures 8 and 9 are 0.13, 0.18, 0.32 and 0.34, respectively. The large discrepancy between experimental and numerical profiles, as witnessed by the reduced MACs, may be explained by observing that, in this case, the theoretical profiles have more rapid changes across the plate compared to the previous test, thus leading to a major influence of the relatively coarse spatial resolution of our sensor on the strain shape reconstruction accuracy.

Finally, we studied the case in which the plate is clamped with only two bolts (\(C_1\) and \(C_3\)). For this test, the spectral range chosen for natural frequency identification was extended from 5 to 85 Hz. The experimentally retrieved natural frequencies in the investigated range were 10.7 and 54.5 Hz, while the corresponding values provided by the FEM analysis were 6.7 Hz and 54.6 Hz, respectively. The experimental and numerical modal shapes for the two identified modes are reported in figures 10 and 11, respectively (note that, to improve visibility, the \(x\)- and \(y\)-axes have been inverted with respect to the previous figures). The numbers of averages for the two mode shape measurements were set to 512 and 64, resulting in acquisition rates of 33 S s\(^{-1}\) and 173 S s\(^{-1}\), respectively.

From the experimental data, we observe that the strain levels are larger compared to the previous tests, due to the wider oscillation of the plate with fewer clamps. In both cases, a reasonably good agreement exists between the numerical and experimental profiles. In particular, the MACs between experimental and numerical shapes were 0.73 and 0.92 for the first and second resonances, respectively. The greater consistency between experimental and theoretical profiles is justified, in this case, by the smoother shapes of the plate modes in the latter configuration, turning the spatial resolution of the sensor into a less critical factor.
Figure 8. Experimental strain modal shapes of the plate fixed with three clamps, excited at its first resonance (30.5 Hz). The removed clamp was (a) $C_1$, (b) $C_2$, (c) $C_3$ or (d) $C_4$.

Figure 9. Numerical strain modal shapes of the plate fixed with three clamps. The removed clamp was (a) $C_1$, (b) $C_2$, (c) $C_3$ or (d) $C_4$. 
4. Conclusions

A slope-assisted BOTDA sensor has been employed to perform a modal analysis of an aluminum rectangular plate. The dynamic strain was measured simultaneously at various positions on the structure, with a sampling distance of 5 cm in both x- and y-directions, a spatial resolution of 30 cm and a maximum acquisition rate of 250 Hz. The system allows strain measurements on a two-dimensional contour map for planar surface samples. The accuracy of the sensor was assessed by both FEM analysis and comparison with strain measurements performed by the use of strain gauges. The presented set-up can be used to perform vibration tests on different flexible structures, supporting the design optimization and control process of the structures. Future developments of this technology relate to the possibility to enhance the system performance, i.e. signal-to-noise ratio, spatial resolution and maximum detectable vibration frequency.

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