Pions near condensation under compact star conditions

Cristián Villavicencio*1 | Marcelo Loewe2 | Alfredo Raya3

1Departamento de Ciencias Básicas - Facultad de Ciencias, Universidad del Bío-Bío, Chillán, Chile
2Instituto de Física, Pontificia Universidad Católica de Chile, Santiago, Chile
3Instituto de Física y Matemáticas, Universidad Michoacana de San Nicolás de Hidalgo, Morelia, Michoacán, México

Correspondence
*Avda. Andrés Bello 720, Casilla 447 - CP: 3800708, Chillán, Chile. Email: cvillavi@uc.cl

Funding Information
FONDECYT, 1130056, 1150471 and 1150847. ConicytPIA/Basal, FB0821. CIC-UMSNH, 4.22. CONACyT, 256494.

The behavior of pions is studied in systems where their normal leptonic decay is forbidden. When thermal fluctuations are present, a low decay rate is generated, and as a consequence of lepton recombination, the amount of pions remains almost unaltered. Compact stars conditions are favorable for the formation of such intermediate state of charged pions: near condensation and almost stable, leading to a continuum source of anti-neutrinos. In particular, protoneutron stars could be an scenario where this state of matter is relevant.

KEYWORDS:
neutron star, hadrons, thermal field theory

1 | INTRODUCTION

Motivated by the possibility of new hadronic phases under extreme conditions, interesting phenomena can occur in dense media. In particular, charged pions in a degenerated leptonic system, as in the case in neutron stars, can experience interesting changes in their behavior.

The influence of pion or kaon condensate in the decay rate of neutron stars is still an open question. In particular, the formation of a pion Bose-Einstein condensation in chemical equilibrium with leptons needs that the electron chemical potential reaches the pion mass. However, it is not clear if repulsive s-wave or attractive p-wave dominates, with the consequences that in the first case pion mass increases, and in the second case pion mass diminishes (Ohnishi, Jido, Sekihara, & Tsubakihara 2009). Also the existence of hyperons reduces the electron chemical potential (Heiselberg & Hjorth-Jensen 2000).

The population of pions in the normal phase depends strongly on temperature, so, for cold neutron stars, their influence is negligible. However, for protoneutron stars the amount of pions in normal phase can increase considerably. Nevertheless, because of their fast decay, pions in normal phase are usually ignored in the literature and rarely considered in the description of relative population of particles inside neutron stars or protoneutron stars (Nakazato, Sumiyoshi, & Yamada 2008).

Charged pions decay mainly into muons (the 99.99% of them) and eventually into electrons (< 0.01%). Because of Pauli blocking, if the system is degenerated with respects of leptons, the leptonic Fermi levels are occupied, forbidding the pion decay since there is no room in the phase space to locate the emerging lepton. This scenario provides an interesting question about the role of pions in terms of stability and the emissivity of neutrinos when thermal effects are significant.

Here we will explore the possibility of finding charged pions in this low decay process, which we denote it conveniently pions in metastable state, in the language of nuclear physics for denoting long-lived isotopes. For a more detailed description of this work, see (Loewe, Raya, & Villavicencio, 2017) and also references therein. Here we will present the main ideas and results.
2 | PION DECAY RATE

The decay rate of pions into leptons is originated from electroweak process, where the Lagrangian term that governs this interaction is given by

$$\mathcal{L}_{\pi\ell} = f_{\pi\ell} G_F \bar{\psi}_\ell \not{D} \not{\Sigma} - (1 - \gamma_5) \psi_\ell + \bar{\psi}_\ell \not{D} \not{\Sigma} - (1 - \gamma_5) \psi_\ell,$$  \hspace{1cm} (1)

where \(\pi^\pm\) are the charged pion fields, \(\psi_\ell\) describes the lepton field (muons or electrons) and \(\psi_\nu\) the associated neutrino field. The derivative

$$D_\alpha \pi^\pm = (\partial_\alpha \pm i \mu_\alpha \delta_{60}) \pi^\pm\hspace{1cm} \text{(2)}$$

includes the pion chemical potential, and \(G_F = 1.17 \times 10^{-5} \text{ GeV}^{-2}\) is the Fermi constant.

The respective free Lagrangian for the fields involved are

$$\mathcal{L}_\pi = D_\alpha \pi^+ D^\alpha \pi^- - m_\pi^2 \pi^+ \pi^-,\hspace{1cm} \text{(3)}$$

$$\mathcal{L}_\ell = \bar{\psi}_\ell [i \not{D} + \mu_\ell \gamma_0 - m_\ell] \psi_\ell,\hspace{1cm} \text{(4)}$$

$$\mathcal{L}_\nu = \bar{\psi}_\nu [i \not{D} + \mu_\nu \gamma_0] \psi_\nu,\hspace{1cm} \text{(5)}$$

where their associated chemical potentials are included. This is an effective model in the sense that we consider all the hadronic parameters as density dependent. This affects directly the pion mass and pion decay constant.

The decay rate can be obtained from the retarded \(\pi^-\) propagator

$$D_{\pi^-(p)} = \frac{1}{(p_0 + m_\pi)^2 - E_{\pi^-}^2 - \Pi(p) \mid_{p_0 = p_0 + i\epsilon}}.\hspace{1cm} \text{(6)}$$

The denominator of the dressed propagator is expanded around the physical real pole, obtaining a Breit-Wigner propagator with a momentum dependent decay rate defined as

$$\Gamma_{\pi^-} = \frac{1}{E_{\pi^-}} \text{Im} \Pi(E_{\pi^-} - \mu_\ell + i\epsilon, p).\hspace{1cm} \text{(7)}$$

where \(\Pi(p_0, p)\) is the charged pion self-energy and where \(E_{\pi^-} = \sqrt{p^2 + m_\pi^2}\) is the pion energy. The imaginary part of the pion comes from the one-loop correction diagram from the weak interaction term described in Eq. (1) and thermal effects are considered using the Matsubara formalism.

We consider that the leptons involved are in chemical equilibrium from the process \(\mu^- \rightarrow e^+ \bar{\nu}_e + \nu_\mu\), therefore their chemical potentials follow the relation \(\mu_\mu - \mu_\nu = \mu_e - \mu_\nu \equiv \mu_e - \mu_\nu\). Also negative charged pions will be in chemical equilibrium from the process \(\pi^- \rightarrow \ell^- + \bar{\nu}_\ell\), so pion chemical potential follows the relation \(\mu_\mu = \mu_e - \mu_\nu\). Under such considerations, the decay rate including thermal effects is

$$\Gamma_{\pi^-}(p) = \Gamma_{\pi^-} m_{\pi^-} E_{\pi^-} \left[1 + \frac{T}{2a_\ell |p|} \ln \left(\frac{1 + e^{-E_{\pi^-}/T}}{1 + e^{-E_{\pi^-}/T}}\right) \right],\hspace{1cm} \text{(8)}$$

were \(\Gamma_{\pi^-}\) is the pion decay width at zero temperature and chemical potential, the lepton and neutrino energy terms are defined as \(E_\ell^\pm = (1 - a_\ell) E_\ell^\pm \pm a_\ell |p|\), and with \(a_\ell = (m_e^2 - m_\ell^2)/2m_\ell^2\). It is assumed, of course, that \(m_\pi > m_\ell\).

A more enlightening expression can be obtained in the rest frame:

$$\Gamma_{\pi^-}(0) = \Gamma_{\pi^-} \left[1 - n_F(M_\ell - \mu_\ell) - n_F(M_\nu + \mu_\nu)\right],\hspace{1cm} \text{(9)}$$

where the mass terms are defined as \(M_\ell = (m_\ell^2 + m_\nu^2)/2m_\pi\), and \(M_\nu = (m_e^2 - m_\ell^2)/2m_\pi\). From this expression one can see that the thermal corrections can be expressed as \(1 - n_\ell - n_\nu = (1 - n_\ell)(1 - n_\nu) - n_\ell n_\nu\), which is interpreted as the availability to create a lepton and an antineutrino in the thermal bath minus the probability of finding a lepton and an antineutrino in the thermal bath. This is often referred as the direct and inverse process \(\Gamma_d - \Gamma_i\). The macroscopic interpretation of the decay rate obtained in this way is explained in Weldon [1983]: the rate at which the slightly out-of-equilibrium system can reach thermal equilibrium. Although it is not well understood, this interpretation is the most accepted one.

A direct consequence in Eq. (9) can be observed at zero temperature. The decay rate vanishes if \(\mu_\ell > M_\ell\). This is the phenomenon we want to study in more detail.

3 | METASTABILITY CONDITIONS

The effect of Pauli blocking inhibits the the particle decay, and only thermal fluctuations can overcome this situation. We can write the decay rate separated as the contribution at zero temperature and the thermal fluctuations

$$\Gamma_{\pi^-(0)} = \Gamma_{\ell 0} + \delta \Gamma_T,\hspace{1cm} \text{(10)}$$

where \(\Gamma_0\) is the decay rate in the \(T = 0\) limit and \(\delta \Gamma_T\) the thermal contribution. We will denote the pion system as metastable if \(\Gamma_0 = 0\). This definition, as was pointed in the introduction, is in analogy of nuclear physics definition of particles with finite lifetime but considerably longer than usual. From Eq. (8), we have

$$\Gamma_0 = \frac{\Gamma_{\pi^-} m_{\pi^-}}{E_{\pi^-}} \left[1 + \frac{\mu_\ell - E_{\ell^+}}{2a_\ell |p|} \theta(\mu_\ell - E_{\ell^+}) - \mu_\nu - \frac{E_\ell^-}{2a_\ell |p|} \theta(\mu_\ell - E_{\ell^-})\right].\hspace{1cm} \text{(11)}$$

From the above equation is easy to see that \(\Gamma_0\) vanishes for \(\mu_\ell > E_{\ell^+}\). This condition involves two direct restrictions. One is that the pion momentum must be less than some critical momentum,

$$|p| < \frac{m_\ell^2 + m_\nu^2}{2m_\ell^2} q_F - \frac{m_\ell^2 - m_\nu^2}{2m_\ell^2} \mu_\ell \equiv p_\ell,\hspace{1cm} \text{(12)}$$
where \( q_F = \sqrt{\mu_F^2 - m_F^2} \) is the Fermi momentum of the leptons at zero temperature. The other relation is the one related with the lepton chemical potential, and it must be greater that a critical chemical potential,

\[
\mu_\pi > \frac{m_\pi^2 + m_e^2}{2 m_\pi} \equiv \mu_c.
\]

If we consider the vacuum pion mass \( m_\pi = 139.5 \text{ MeV} \), the critical lepton chemical potential for muons \( (m_\mu = 105.6 \text{ MeV}) \) is \( \mu_c = 109.74 \text{ MeV} \).

### 4 | THERMAL EQUILIBRIUM

As was pointed previously, the interpretation of the decay as the imaginary pole of the retarded green function is related with the time that a system slightly out of equilibrium takes to reach the thermal equilibrium. In addition to the pions, we also need to consider another process that involves the same participants [Kuznetsova, Habs, & Rafelski (2008): leptons recombined with antineutrinos. This process of virtual neutrinos from the heat bath captured by leptons increases the pion population. We will explore which are the necessary conditions for a system of pions and leptons in order to reach thermal equilibrium simultaneously. This happens if the decay rates of pions and the rate of lepton recombination are of similar magnitude \( \Gamma_\pi \sim \Gamma_\pi^e \).

The lepton rate is obtained also from the retarded propagator of leptons through weak interaction corrections described by the interaction Lagrangian defined in Eq. (1). The dressed tor of leptons through weak interaction corrections described by the interaction Lagrangian defined in Eq. (1).

\[
S^{dr}_\ell = \frac{Q + m - \Sigma}{Q^2 - m_\ell^2 - \Pi_+} P_+ + \frac{Q + m - \Sigma}{Q^2 - m_\ell^2 - \Pi_-} P_-,
\]

where \( Q = (q_0 + \mu_\ell, \mathbf{q}) \) is the lepton self-energy through weak interactions, \( P_\pm = \frac{1}{2} (1 \pm \gamma_5 \mathbf{q} \gamma_\mu) \), are the helicity projectors and \( \Pi_\pm(q) = 2 Q \cdot A \pm (Q_0 A_0 - |\mathbf{q}| A_0) \) the mass corrections for each helicity. Now the procedure is the same as in Sec. 3 by setting \( q_0 \to q_0 + i \epsilon \) in the retarded propagator. The recombination rates for each lepton helicity is

\[
\Gamma_\ell = \frac{1}{E_\ell} \text{Im} \Pi_\pm(E_\ell - \mu_\ell + i \epsilon, \mathbf{q}),
\]

where \( E_\ell = \sqrt{q^2 + m_\ell^2} \) is the lepton energy. In order to simplify the analysis, we calculate the average of the rates for the different helicities. Defining \( \Gamma_\ell = (\Gamma_+ + \Gamma_-)/2 \), after Matsubara summation, the average lepton recombination rate is

\[
\Gamma_\ell = \frac{m_\pi}{2 m_\ell} \frac{m_\ell}{E_\ell} \frac{T}{2 b_\ell |q|} \left[ \ln \left( \frac{1 - e^{(E_\ell - \mu_\ell)/T}}{1 - e^{-(E_\ell - \mu_\ell)/T}} \right) - \ln \left( \frac{1 + e^{(E_\ell + \mu_\ell)/T}}{1 + e^{-(E_\ell + \mu_\ell)/T}} \right) \right],
\]

where again the pion decay constant in vacuum \( \Gamma_\pi^e \) is present. The pion and neutrino energy terms are defined as \( E_\pi^e = (1 + b_\pi) E_\ell \pm b_\ell |q| \) and \( E_\nu^e = b_\nu E_\ell \pm b_\ell |q| \), respectively, with the constant \( b_\ell = (m_\ell^2 - m_\pi^2)/2 m_\ell^2 \).

Now, we need to find a window in the parameter space where pions and leptons reach thermal equilibrium satisfying \( \Gamma_\pi \sim \Gamma_\pi^e \). The parameters involved are the neutrino chemical potential, the lepton chemical potential, the temperature, the pion energy and the lepton energy. To reduce the number of parameters we consider pions in the rest frame, where the decay rate is given in Eq. (1). In the same way, near the Fermi surface, fluctuations of leptons in the degenerated environment are produced. So we take the lepton energy as the Fermi energy in the decay rate, which leads to

\[
\Gamma_\ell = \frac{m_\pi}{2 m_\ell} \frac{m_\ell}{E_\ell} \frac{T}{2 b_\ell |q|} \left[ \frac{\sinh((b_\ell \mu_\ell + \mu_\nu)/T)}{\sinh((b_\ell \mu_\ell + \mu_\nu)/T)} + \frac{\sinh(b_\ell q_\ell / T)}{\sinh(b_\ell q_\ell / T)} - \frac{\sinh((b_\ell \mu_\ell + \mu_\nu)/T)}{\sinh((b_\ell \mu_\ell + \mu_\nu)/T)} - \frac{\sinh(b_\ell q_\ell / T)}{\sinh(b_\ell q_\ell / T)} \right].
\]

Figure 1 shows the leptonic chemical potential as a function of temperature necessary to have simultaneous thermal equilibrium for lepton pion system, considering pions in rest frame and leptons at the Fermi surface. The band considers the possibility to fluctuate a bit, with \( \frac{1}{2} \Gamma_\ell < \Gamma_\pi < \frac{3}{2} \Gamma_\ell \). For simplicity we consider \( \mu_\nu = 0 \), which means that all real neutrinos escape from the star once created. This is a usual approximation in neutron stars, but not necessarily valid for protoneutron stars [Pons, Miralles, Prakash, & Lattimer (2001)].

From the two plots, for the pion muon process (upper plot) and for pion electron process (lower plot), at least the region for \( T > 1 \text{ MeV} \) considers values of the lepton chemical potential in the metastability region, \( \mu_\pi > \mu_\ell \). For lower temperatures this is still valid for temperatures less than 1 keV, but we are interested in this region because it will be the region of higher emissivity as we will show next.

### 5 | EMISSIVITY

One important consequence of charged pion metastability is the neutrino emission in this soft decay process. The most efficient mechanisms in neutron stars are the URCA and modified URCA process. The emissivity is related with the cooling process, which is the main insight to speculate of different
amplitude for pions going into leptons and antineutrinos is defined as
\[
\langle \ell \bar{\nu} \rangle \int d^4x \mathcal{L}_{\pi\ell} | \pi^- \rangle = i \mathcal{M}(2\pi)^4 \delta^4(p - q - k).
\] (20)

At chemical equilibrium, the neutrino emissivity from charged pions decay is then
\[
e_\pi = \frac{\tilde{\Gamma}_{\pi\ell} m_\pi T^2}{2\pi^2 a_\ell} \int_{m_\pi}^\infty dE_\pi n_B(E_\pi - \mu_\pi) \left[ \frac{E_\pi - E_\ell}{T} \ln(1 + e^{(E_\ell - \mu_\ell)/T}) \right]^{E_\ell}_{E_\pi},
\] (21)

where the integral variable \(E_\ell\) is evaluated in \(E_\pi^\pm\), defined after Eq. (8). This result is compared with the URCA emissivity by setting \(\mu_\pi = 0\). For the pion decay constant and also for nucleon masses in the URCA emissivity, we use the Brown-Rho scaling \(\text{Brown & Rho} [1991]\), with a scaling factor 0.8. Here we show the region where the emissivity from pion decay reaches its maximum value compared with the URCA emissivity. Note that for low pion mass and temperatures of the order of 1 MeV, the neutrino emissivity from pions decaying into muons can be of the same order of magnitude than the URCA emissivity. This is the region where, in the case of emissivity, consequences of metastable pions are more appreciable.

The difference in the comparison of neutron emissivity from metastable pions, instead of normal pions, can be more appreciable by approximating EC. (21) in a low temperature expansion. This is possible since the temperature values we are considering are much smaller than the pion mass. When the critical momentum defined in Eq (12) is much larger than the temperature \(p_c \gg T\), which is the case in the region of temperature we show in Fig. 2, the emissivity is proportional to \(T^2\) times an exponential suppression term
\[
e_\pi \approx \tilde{\Gamma}_{\pi\ell} m_\pi^4 g(\mu_\ell) \left( \frac{T}{2\pi m_\pi} \right)^2 e^{-(E_\ell - \mu_\ell)/T},
\] (22)

with
\[
g(\mu_\ell) = \frac{2\alpha_F}{b_\ell m_\pi} + \frac{2\alpha_e m_\pi}{p_\ell} + \frac{2p_\ell}{a_\ell m_\pi} q_F + p_c,
\] (23)

and all the other terms defined after EQ. (8), (12) and (16). The result in

\[\text{FIGURE 1} \quad \text{Chemical potential and temperature values where pion-lepton chemical equilibrium is favorable. The band widths correspond to a half order of magnitude difference between the widths: } 0.5 < \Gamma_{\pi\ell} / \Gamma_\pi < 1.5. \quad 
\]

The colors and lines refer to \(m_\pi = 140 \text{ MeV (solid blue)}, m_\pi = 115 \text{ MeV (dashed red)} \) and \(m_\pi = 200 \text{ MeV (dot-dashed green)} \).
low decaying process inside compact stars. This condition has a direct impact in the emissivity of neutrinos from the pion decay process with a behavior of $\epsilon \sim T^2 e^{-(E_{\mu} - \mu_L)/T}$ instead of the usual pion gas emissivity $\epsilon \sim T^{3/2} e^{-(m_\pi - \mu_L)/T}$ which is commonly used.

The conditions for thermal equilibrium are basically satisfied in the range of interest, where the lepton chemical potential is higher than the critical chemical potential needed for metastability, and the temperature where emissivity is maximal, between $T \approx 1 - 10$ MeV.

Although the emissivity is low compared with URCA process, and even much less than the modified URCA process, there are many other factors that can change this estimations. For the moment, the possibility of this source of neutrino emission be of the same order than the URCA neutrino emission, for low pion mass case at $T \sim 1$ MeV, could be a good signal.

For now, this state of matter doesn’t seem to be a very significant contribution, other than being another source of neutrinos. However, many estimations that considers pion in unstable state can be revisited. The effects of considering metastable pions and not unstable pions in particle populations may have a direct impact.

ACKNOWLEDGMENTS

The authors acknowledge the support of FONDECYT (Chile) under Grants No. 1130056, 1150471 and 1150847; Conicyt-PIA/Basal (Chile) Grant No. FB0821; CIC-UMSNH (México) Grant No. 4.22; CONACyT (México) Grant No. 256494.

REFERENCES

Brown, G. E., & Rho, M. 1991, Phys. Rev. Lett., 66, 2720-2723.
Heiselberg, H., & Hjorth-Jensen, M. 2000, Phys. Rept., 328, 237-327.
Jaikumar, P., Prakash, M., & Sch"{a}ffer, T. 2002, Phys. Rev., D66, 063003.
Kuznetsova, I., Habs, D., & Rafelski, J. 2008, Phys. Rev., D78, 014027.
Loewe, M., Raya, A., & Villavicencio, C. 2017, Phys. Rev., D95(9), 096013.
Nakazato, K., Sumiyoshi, K., & Yamada, S. 2008, Phys. Rev., D77, 103006.
Ohnishi, A., Jido, D., Sekihara, T., & Tsubaki, K. 2009, Phys. Rev., C80, 038202.
Pons, J. A., Miralles, J. A., Prakash, M., & Lattimer, J. M. 2001, Astrophys. J., 553, 382-393.
Weldon, H. A. 1983, Phys. Rev., D28, 2007.
Yakovlev, D. G., Kaminker, A. D., Gnedin, O. Y., & Haensel, P. 2001, Phys. Rept., 354, 1.