SAR 3D sparse imaging based on CLA

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Abstract: A novel method for synthetic aperture radar (SAR) three-dimensional (3D) sparse imaging based on a coprime linear array (CLA) is proposed. To reduce the geometric resolution loss caused by under-sampled sparse SAR, a joint sparse imaging approach is used. In this scheme, the coprime sampled data of CLA is separately processed via the iterative reweight least-squares approach, and their recovered results are combined to cancel the false targets and grating-lobes to improve the imaging quality. Both simulated and experimented results demonstrate the effectiveness of the method. It shows that the exploiting of CLA and sparse recovery can significantly reduce the linear array antenna amount and the geometric resolution loss compared with the conventional uniform linear array SAR.

1 Introduction

A linear array synthetic aperture radar (LASAR) is a promising radar-imaging technique to producing a three-dimensional (3D) image of the observed scene [1, 2]. To achieving high geometric resolution, a long uniform linear array (ULA) is usually required for the LASAR system according to the traditional direction of arrival (DoA) theory. Commonly, a multi-input multi-output (MIMO) antenna can be used to reduce the transmitter/receiver inter-element spacing of units. Although the MIMO technique can reduce the real antenna elements of ULA, hundreds of virtual full-sampled elements are still needed to suppress the level of the sidelobe and grating-lobe, which bring a large amount of data to be stored and processed.

With advantages of super-resolution and sidelobe suppression, compressed sensing (CS)-based sparse imaging approaches have been successfully developed for the LASAR system in recent years [4, 5]. To guarantee the low coherence of the sensing matrix, random or jittered sparse under-sampling modes are usually used for the existing CS-based LASAR imaging approaches. However, the application of these random and jittered under-sampling modes is more difficult for the LASAR hardware system than that of uniform under-sampling.

Among a number of techniques that are available for the sparse array construction, the coprime linear array (CLA) is a very attractive due to its simplicity and effectiveness [6]. A prototype coprime array utilises a coprime pair of uniform linear subarrays, where one is of sensors with an inter-element spacing of units, whereas the other is of elements with another inter-element spacing of units. The numbers and are chosen to be coprime. In recent years, because of its superior performance, CLA has been widely discussed in the fields of DoA estimation and radar imaging. In [7], a novel coprime SAR (CopSAR) technique based on coprime slow-time acquisition was proposed for maritime surveillance. In [8], a novel adaptive beamforming algorithm for the coprime array was proposed by compressive sensing the virtual ULA signal. Moreover, an SAR sparse imaging method was presented based on coprime sampling and nested sparse sampling [9]. In [10], a novel coprime adjacent array SAR is proposed for azimuth location de-ambiguity for SAR ground moving targets. However, these methods mainly used CS-based recovery directly for CLA, which may result in some low false targets due to its high coherence of its sensing matrix. Hence, new methods should be developed to suppress these false targets for CLASAR 3D imaging.

In this paper, a novel data-acquisition mode and an imaging strategy for LASAR 3D sparse imaging are proposed by the joint CLA theory and CS-based sparse recovery method. To improve the 3D imaging quality of CLASAR, the coprime sampled data of CLA is separately processed via the iterative reweight least-squares (IRLS) approach, and the final results are combined to cancel the false targets and the grating-lobes of the direct CS-based method. Both simulated and experimented results demonstrate the effectiveness of the proposed approach.

2 CLASAR model and problem formation

2.1 Coprime linear array

The classical geometric model of a prototype CLA is illustrated in Fig. 1. Without loss of generality, we assume the two coprime integers and , and the unit inter-element spacing of CLA is set to . The sensors of CLA are positioned at

\[ S = \{nNd, 0 \leq n \leq N_2 - 1\} \cup \{mNd, 0 \leq m \leq N_1 - 1\} \]

(1)

Since the two subarrays of CLA share the first sensor at the zeroth position, the total number of the sensors used in CLA is only . The average space of the adjacent array in CLA is

\[ d_A = N_1(N_1 - 1)d/(N_1 + N_2 - 1) \]

(2)

Obviously, is usually much bigger than the conventional Nyquist space , so the direct beam-forming method for CLA may suffer from severe grating-lobes.
2.2 CLASAR imaging model

The model of CLASAR 3D imaging is shown in Fig. 2. The x-axis is the cross-range direction, the y-axis is the along-track direction, and the z-axis is the height direction. The position of the nth CLA element in the lth PRI and the rth range can be described as

$$q_{l,n} = (x_{l,n}, y_{l,n}, z_{l,n}), 0 \leq n \leq N_C, 0 \leq l \leq N_A$$

where $N_C$ denote the total number of CLA elements and $N_A$ is the total number of the PRIs. Assume that the underlying scene only consists of point-like scatterers. Then the scene can be approximated by several discrete cells, and let $\mathbf{P}_m = (x_m, y_m, z_m)$ and $\sigma_m$ denote the position and the scattering coefficient of the mth cell.

Suppose that the CLA sensors transmit chirp signals. After range compression and azimuth focusing, the CLASAR echoes are expressed as

$$s(r, l, n) = \sum_m \sigma_m g(k, l) \exp(-j2kR_{l,m}), 0 \leq r \leq N_R$$

where $r, l,$ and $n$ are the range, the along-track, and the cross-track direction domain, respectively, $k$ is the wave number, $R_{l,m}$ is the range between $\mathbf{P}_m$ and $q_{l,m}$, $N_R$ denotes the total number of the cell range, and $\chi(r, l)$ is the range-azimuth ambiguity function of CLASAR. If the range-azimuth migration of $s(r, l, n)$ is corrected exactly, the echoes of (4) in the lth PRI and the rth range can be expressed as a linear measurement model:

$$y_{l,i} = A_{l,i} f_{l,i} + n_{l,i}, 0 \leq r \leq N_R, 0 \leq l \leq N_A$$

where $y_{l,i} = [s(r, l, 1), \ldots, s(r, l, N_C)]^T$ is the echo vector of CLASAR after echo realignment in the lth PRI and the rth range, $f_{l,i}$ is the scattering coefficient of targets after range-azimuth compression, $A = [\varphi_1, \ldots, \varphi_{N_C}]^T$ is the sensing matrix of $y_{l,i}$ with $\varphi_m = [\exp(-j2kR_{l,1,m}), \ldots, \exp(-j2kR_{l,N_C,m})]^T$ being the $m$th PRI and the rth range, $n_{l,i}$ denote the noises in $y_{l,i}$, and $M$ is the total number of image cells in the $l$th PRI and the $r$th range.

Since the underlying scene is usually predominantly space sparse, CS-based algorithms can be used for CLASAR sparse 3D imaging. The vector $f_{l,i}$ can be reconstructed by solving the following $\ell_1$ norm optimisation:

$$\tilde{f}_{l,i} = \arg \min_f \| f_{l,i} \|_{1, s.t.} \| y_{l,i} - A_{l,i} f_{l,i} \|_2 \leq \varepsilon$$

where $\varepsilon$ is the estimated noise level of $y_{l,i}$. The problem of (6) can be solved by different types of CS-based algorithms, such as basis pursuit and orthogonal match-pursuit, and IRLS.

2.3 Problem formation

According to CS theory, the exact recovery of the sparse vector $f_{l,i}$ required that the mutual correlation of the sensing matrix $A_{l,i}$ should satisfy

$$\mu(A_{l,i}) \leq 1/(2K - 1)$$

where $\mu(A_{l,i})$ is the mutual correlation of matrix $A_{l,i}$ and $K$ is the sparsity of $f_{l,i}$.

Obviously, the value of $\mu(A_{l,i})$ should be small enough for CS-based imaging. To satisfy (7), a random-sampling non-uniform linear array antenna is usually used for LASAR CS-based imaging. However, unlike the random-sampled linear array antenna, the sampling space of CLASAR is larger, so the value of $\mu(A_{l,i})$ is much higher, which may result in the false targets for CLASAR CS-based imaging. Hence, some new methods should be developed to suppress these false targets of CLASAR.

3 CLASAR 3D sparse imaging

3.1 Sparse imaging via joint recovery

Due to CS-based algorithms, the ghosts of the recovered result in (6) will be occurred in the cells with a high correlation value of $A_{l,i}$. Then, the ghosts of LASAR CS-based imaging with the $N_1$ times and $N_2$ times under-sampled ULA will be presented in these positions, respectively,

$$Z_{N_1} = \{iW/N_1, i = 1, \ldots, [W/N_1]\}$$

$$Z_{N_2} = \{iW/N_2, i = 1, \ldots, [W/N_2]\}$$

where $W$ is the efficient observed width of the standard Nyquist-sampled ULA. Obviously, as $N_1$ and $N_2$ are coprime integers, $Z_{N_1}$ and $Z_{N_2}$ are located in different places of the recovered image. Hence, the ghosts of the CLA can be suppressed by combining its two under-sampled ULA as

$$Z = Z_{N_1} \cap Z_{N_2}$$

Inspired by this idea, we present a joint sparse imaging method of CLASAR that is able to suppress the ghost result from the sparse CLA. For simplicity, the subscript $r$ and $l$ of (6) are ignored. The result of suppressed ghosts will be estimated by joining the results of the three types of linear arrays as

$$\tilde{f} = \min_{\tilde{f}} \{ \tilde{f}_{N_1}, \tilde{f}_{N_2}, \tilde{f}_{All} \}$$

$$\tilde{f}_{N_1} = \arg \min_f \| f \|_{1, s.t.} \| y_{N_1} - A_{N_1} f \|_2 \leq \varepsilon_i$$

$$\tilde{f}_{N_2} = \arg \min_f \| f \|_{1, s.t.} \| y_{N_2} - A_{N_2} f \|_2 \leq \varepsilon_i$$

$$\tilde{f}_{All} = \arg \min_f \| f \|_{1, s.t.} \| y_{All} - A_{All} f \|_2 \leq \varepsilon_{All}$$

where $\tilde{f}_{N_1}, \tilde{f}_{N_2}$, and $\tilde{f}_{All}$ are the estimated results of the $N_1$ ULA, $N_2$ ULA, and all elements of CLA. In this paper, IRLS is used to solve the problem of (12). Moreover, loop over all the subscript $r$ and $l$ of the echoes, and then the final recovered 3D result of CLASAR is obtained.

3.2 Flowchart of the algorithm

According to the principle of CLA joint recovery, the main flows of the proposed algorithm are demonstrated in Fig. 3. The core of the algorithm is the sparse recovery by using three different types
of arrays independently and joining the obtained results using the minimisation method.

4 Results

4.1 Simulation data

To demonstrate the effectiveness of the proposed algorithm for CLASAR 3D sparse imaging, simulation results of point targets are presented in this section. Furthermore, the conventional IRLS CS-based algorithm is used as the compared sparse recovery method to analyse the performance of target reconstruction. The main parameters of the CLASAR simulation are displayed in Table 1. Also, the echoes after range-azimuth compression are added noises with SNR = 20 dB.

Fig. 4 shows the 1D imaging results of five points in the cross-track direction in the case of full ULA elements, 25% random ULA elements, \( M = 6 \) under-sampled ULA elements, \( N = 7 \) under-sampled ULA elements, and the \( M = 6 \) and \( N = 7 \) CLA elements, using the conventional CS-based IRLS method and the proposed method, respectively. Obviously, we can see that the false targets obtained by the proposed joint method are lower than that of the direct IRLS method on random or coprime linear array.

Fig. 5 displays the 3D imaging results of the six points in the case of full ULA elements, \( M = 6 \) under-sampled ULA elements, \( N = 7 \) under-sampled ULA elements, and the \( M = 6 \) and \( N = 7 \) CLA elements, using the conventional CS-based IRLS method and the proposed method, respectively, where the range migration in the echoes is not corrected. As can be seen from Fig. 5, the conventional CS-based IRLS method suffers serious grating-lobes and false targets with coprime samples. However, the proposed joint sparse imaging algorithm can effectively improve the quality of image and reduce the level of grating-lobes with CLA.

4.2 Experimental data

Some experimental data of a ground-based LASAR system is implemented for performance verification in this section. The main parameters of the ground-based LASAR system are as follows: the carrier frequency is \( f_c = 9.62 \) GHz, the bandwidth is \( B = 120 \) MHz, the ULA length is \( L = 1.0 \) m, and the range from the scene centre to the radar platform is \( R_0 = 40 \) m. In the experiment, a virtual ULA with 108 elements is synthesised by the movement of an antenna.

Fig. 6 shows the tested scene of ground-based LASAR imaging and the virtual ULA, respectively. The scene mainly contains three balls on the ground. The imaging results by the conventional CS-based IRLS method with the whole ULA elements and the proposed sparse imaging approach with \( M = 3 \) and \( N = 4 \) CLA are displayed in Fig. 7. Clearly, the 3D image of the three balls obtained by the proposed imaging algorithm is almost the same as with full ULA elements. The ghosts of the CLA are suppressed by joint sparse recovery. The experimental results demonstrate the effectiveness of the proposed algorithm.

5 Conclusion

In this paper, a novel imaging approach for LASAR 3D sparse imaging is proposed by the joint CLA theory and CS-based sparse recovery method. To improve the 3D imaging quality of CLASAR, the coprime-sampled data of CLA is separately processed via the IRLS approach, and the final results are combined to cancel the false targets and the grating-lobes of the directly CS-based method. Both simulated and experimented results demonstrate the effectiveness of the proposed approach. The results demonstrate that the CLA can significantly reduce the real antenna elements of LASAR with the CS-based sparse imaging method.

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Fig. 3. Main flows of the proposed algorithm

Table 1. Simulation parameters

| Parameters       | Value     | Parameters       | Value     |
|------------------|-----------|------------------|-----------|
| centre frequency | 37.5 GHz  | airplane speed   | 100 m/s   |
| bandwidth        | 150 MHz   | PRF              | 2000      |
| APC number       | 4096      | antenna aperture | 3 m       |
| platform altitude| 1000 m    | incidence angle  | 90°       |
| sampling rate    | 200 MHz   | ULA length       | 5 m       |

Fig. 4. Imaging results of five points target in the cross-track direction
(a) IRLS, full ULA elements, (b) IRLS, 25% random elements, (c) IRLS, \( M = 6 \) under-sampled ULA elements, (d) IRLS, \( N = 7 \) under-sampled ULA elements, (e) IRLS, \( N = 7 \) under-sampled ULA elements, (f) The proposed method, CLA elements

Fig. 5. 3D imaging results of the six points target
(a) IRLS, full ULA elements, (b) IRLS, \( M = 6 \) under-sampled ULA elements, (c) IRLS, \( N = 7 \) under-sampled ULA elements, (d) The proposed method, CLA elements

Fig. 6. Tested scene and the virtual ULA of the experiment data
(a) The tested scene, (b) The virtual ULA
Fig. 7. Imaging results of the tested three balls
(a) IRLS method with all ULA elements, (b) The proposed approach with M = 3 and N = 4 CLA

7 References

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