Spin-Orbit Locking and Scissors Modes in rare earth crystals with uniaxial symmetry

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A recent experiment has questioned the standard relative value of spin-orbit and crystal-field strengths in rare-earth 4f electron systems, according to which the first should be one order of magnitude larger than the second. We find it difficult to reconcile the standard values of crystal field strength with the Single Ion Model of magnetic anisotropy. If in rare-earth systems the spin-orbit force is much larger than the crystal field, however, spin and orbit of 4f electrons should be locked to each other. For rare earths with non-vanishing spin, an applied magnetic field should rotate both spin and charge density profile. We suggest experiments to investigate the possible occurrence of such Spin-Orbit Locking, thus making a test of the standard picture, by studying the Scissors Modes in such systems.

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I. INTRODUCTION

On the basis of the standard values of spin-orbit and crystal-field strengths crystalline compounds fall into the following three categories [1]: "The relatively extended 3d electrons of the transition metal-ion series have crystal field energies of about 1 eV as compared to spin-orbit coupling of 0.05 eV. By contrast, rare earth 4f-electrons are close to the nucleus, largely screened from the crystal field, and characterized by a dominating spin-orbit coupling of about 0.2 eV as compared to a crystal-field interaction of the order of 0.01 eV. The magnetism caused by 4d, 5d and 5f electrons is intermediate, characterized by spin-orbit and crystal-field interactions that are both very strong". A recent paper, however, suggests that this intermediate situation might be common to many, or perhaps even all, 4f systems [2]. We will discuss this problem in the framework of the Single Ion Model of magnetic anisotropy of systems with uniaxial symmetry. We will regard the 4f electron system as a rigid rotor of ellipsoidal shape whose symmetry axis can precess around the symmetry axis of the cell. The action of the outer electrons is embodied in the crystal field caused by point charge ligands. We will find that, using the standard values of the parameters, the rotor is not polarized, due to large zero-point fluctuations which cause the average magnetism to vanish. Then assuming that the rotor is somehow polarized, we propose experiments to relate the spin-orbit to the crystal field strength. These experiments are based on the following property of a 4f electron system with nonvanishing spin if the spin-orbit force is sufficiently strong: "The charge cloud is rigidly coupled to the spin" so that they should rotate together under an applied magnetic field [1]. We call such a structure Spin-Orbit Locking. Spin-Orbit Locking can be tested by leaving a sample in a magnetic field which must be impulsively switched off. The 4f electron system will start oscillating and will go to the ground state emitting a photon.

![FIG. 1: The charge profile of the 4f electron system is rigidly coupled to the spin. In (a) the 4f-electron system is in the ground state in the presence of the magnetic field $\vec{B}$ set at an angle $\theta_B$ with the z-axis. It performs zero-point oscillations of amplitude $\theta_0$ around $\vec{B}$, the value of the angle at which the total potential (crystal field plus magnetic field) gets its minimum. In reality $\theta_B \geq \theta_0 >> \theta$. In (b) after the magnetic field is switched off the system starts oscillating (Scissors Mode) and goes to the ground state emitting a photon.]

\[a\] The name "rigid spin-orbit coupling" is often used in the literature on magnetism. The word "coupling", however, might suggest to readers of different fields a property of the spin-orbit force of a particle, rather than a structure of a many-body system.
a photon. The oscillating state is a Scissors Mode, which is a kind of collective excitation which has been predicted and observed in several many-body systems.

In Section 2 we will briefly summarize what is known about Scissors Modes and how they appear in different contexts. In particular it is interesting for us their observation in Bose-Einstein condensates by an experiment which gave the idea of the experiments we propose, and their prediction in crystals with axial symmetry, whose physics is (assumed by us to be) essentially the same as that of the 4f electron system. In Section 3 we will present and discuss the Rotor Model we will use to show how Spin-Orbit Locking can be investigated studying Scissors Modes. In Section 4 we will describe the experiments designed for such purpose, in Section 5 we will discuss the lifetime of the state which should be created in order to observe Scissors Modes and in Section 6 we will briefly summarize and discuss our results.

II. SCISSORS MODES

Scissors Modes are collective excitations in which two particle systems move with respect to each other conserving their shape. It was first predicted to occur in deformed atomic nuclei [3] by a semiclassical Two Rotor Model in which protons and neutrons were assumed to form two interacting rotors to be identified with the blades of scissors. Their relative motion (Fig. 2) generates a magnetic dipole moment whose coupling with the electromagnetic field provides the signature of the mode. After its discovery [4] in a rare earth nucleus, $^{156}$Gd, and its systematic experimental and theoretical investigation [5] in all deformed atomic nuclei, it was predicted to occur in several other systems including metal clusters [6], quantum dots [7], Bose-Einstein [8] and Fermi [9] condensates and crystals [10,11] (but clearly observed till now only in Bose-Einstein condensates [12]). In all these systems one of the blades of the scissors must be identified with a moving cloud of particles (electrons in metal clusters and quantum dots, atoms in Bose-Einstein and Fermi condensates, individual atoms in crystal cells) and the other one with a structure at rest (the trap in Bose-Einstein and Fermi condensates, the lattice in metal clusters, quantum dots and crystals). These systems can be described by a One Rotor Model. Scissors Modes in crystals have been studied only in the framework of semiclassical models in which an atom is regarded as a rigid body which can rotate around the axes of its cell under the electrostatic force generated by the ligands. We considered crystals with uniaxial and cubic symmetry. In the first case the precessing ion was treated as one rotor, in the second case as the body obtained by superimposing three ellipsoids at right angles. In the presence of uniaxial symmetry the photoabsorption cross section is characterized by a linear dichroism [10] (Fig.3). A numerical estimate for LaMnO$_3$ gives a M1 transition amplitude of the order of 0.8˚A, an excitation energy of about 4 eV or 9 eV and a value of the zero point oscillation amplitude $\theta_0^2 \approx 0.3$ or 1 depending on how the atomic moment of inertia is evaluated [10]. The uncertainty in the evaluation of the moment of inertia in a semiclassical model will be shortly discussed below. We remark that the above values of $\theta_0$ imply a substantial polarization. With cubic symmetry the dichroism disappears, but the values of excitation energy and M1 transition amplitude are only
slightly changed.

In the present work we suggest that in a crystal in which Spin-Orbit Locking occurs Scissors Modes can be excited also by applying a magnetic field which has an appropriate time-dependence. The mechanism of excitation would be similar to that used with Bose-Einstein Condensates in magnetic traps [12]. In these systems one gives a sudden twist to the trap inducing oscillations of the atomic cloud. In crystals with Spin-orbit Locking the combined effect of the crystalline electrostatic field and of an external magnetic field is to create a potential well (corresponding to the magnetic trap of Bose-Einstein Condensates) which aligns atomic spin and density profile at some angle with the direction of easy magnetization. If we perform a sudden variation of the magnetic field (which corresponds to twist the trap of Bose-Einstein Condensates) the atom will start oscillating around the axes of the cell and will go to the new minimum of the potential emitting a photon (Fig.1). As one can a priori guess the angle by which we can rotate the atom is very small compared with the proper amplitude of Scissors oscillations, so that the amplitude of the Scissors Mode wave function in the initial state prepared by applying a magnetic field is also very small. But because of the huge number of cells in a macroscopic sample an observable number of photons of the energy of the Scissors Mode might be produced.

If Scissors Modes exist they will affect the dispersive effects in the channels with their quantum numbers, which are $J^{π} = 1^+$. The knowledge of their properties should then be of some importance in the study of crystals with strong spin-orbit coupling, because the magnetic anisotropy of these systems is at the origin of many interesting technological applications including magnetic storage devices and sensors, spin-torque nano-oscillators for high-speed spintronics and spin-optics [13]. In this connection we emphasize that in each of the systems studied so far, Scissors Modes provide specific pieces of information. In nuclear physics they are related to the superfluidity of deformed nuclei, in Bose-Einstein Condensates provide a signature of superfluidity, in metal clusters they are predicted to be responsible for paramagnetism. It is thus per se interesting, apart from being a test for Spin-Orbit Locking, to know whether they exist also in crystals, which would add support to the idea that they are a universal feature of many-body systems.

III. THE ROTOR MODEL

In the Single-Ion Model of magnetic anisotropy each rare earth ion is assumed to be independent from the others. In the study of the magnetic properties only the motion of the $4f$ electron system as a whole is considered, disregarding its excitations. In other words the $4f$ electron system is treated as a rigid rotor with spin, but, as far as we understand, the kinetic energy of this rotor is neglected. For the application we want to do, however, it is necessary to look at this point closely in order to understand under which conditions such an approximation can be justified.

We consider the dynamics of the $4f$ electron system with respect to a frame of reference fixed with the cell and $x, y, z$-axes parallel to the cell axes. We introduce the principal frame of inertia of this system, with axes $ξ, η, ζ$. We thus introduce 6 collective degrees of freedom, namely the position of the origin (which coincides with the centre of mass) and the Euler angles $α, β, γ$ of the principal frame. There remain $3Z_{4f} - 6$ intrinsic position coordinates, $q_i$, say, where $Z_{4f}$ is the number of $4f$ electrons. The explicit use of such coordinates is terribly cumbersome for antisymmetric wave functions, but they can be introduced in implicit form [14-15] if necessary.

The microscopic Hamiltonian of the $4f$ electron system can always be written in the following way

$$H_{4f} = \frac{P^2}{2Zm_e} + H_{\text{rotational}}(α, β, γ) + H_{\text{intr}}(q, s) + H_{\text{coupl}}(α, β, γ, q, s) + V \quad (1)$$

where $s_i$ are the spins of the electrons. The first term is the kinetic energy associated with the center of mass motion ($\vec{P}$ being the total momentum and $m_e$ the electron mass). The second term is the rotational energy of the system as a whole, the third term the energy of the electrons in their principal frame, the fourth an interaction between rotational and intrinsic degrees of freedom, the last the crystal field potential. There is no term coupling the centre of mass coordinates with the intrinsic coordinates, because according to Galilean invariance the intrinsic motion does not depend on the centre of mass motion. On the contrary, intrinsic and rotational motion are coupled, because the moment of inertia depends on the intrinsic motion, and because of the centrifugal and Coriolis forces. If the term $H_{\text{coupl}}$ is large, intrinsic excitations will disrupt the collective rotational term, and the collective Euler angles will not correspond to physical degrees of freedom. In other words, the above form of the hamiltonian is always valid but of no practical use in such a case. If instead $H_{\text{coupl}}$ is small and we can disregard the intrinsic excitations in the energy range of interest, we get the Hamiltonian of a rigid rotor. The terms $H_{\text{intr}}, H_{\text{coupl}}$ might be studied in principle with the methods of [14-15], but without dwelling into such complicate analysis, we can come to a generally sound conclusion looking at the shape of the system: if it has a well defined charge distribution which is its intrinsic property, namely not determined by external fields, the collective approximation is generally acceptable for the lowest lying states. This seems to be the case for the $4f$ electron system of most rare earths, because they have a well pronounced quadrupole moment $I$. In any case this is the approximation at the basis of the Single Ion Model, in which the intrinsic motion is altogether ignored.

For spherical rare earths, as $Gd^{3+}$ and $La^{3+}$, $H_{\text{rotational}} = H_{\text{coupl}} = 0$ in Eq. (1) (the number of intrinsic variables becomes $Z_{4f} - 3$). This is due to the fact
that a spherical body cannot rotate in quantum mechanics. Its spin can instead rotate, but there is no kinetic energy associated with its motion. For deformed ions, instead, the dynamics is determined by the Hamiltonian

$$H_{4f} \approx \frac{P^2}{2Zm_e} + H_{\text{rotational}}(\alpha, \beta, \gamma) + \nabla(\alpha, \beta, \gamma, \Sigma)$$  \hspace{1cm} (2)

where \(\Sigma\) is the total spin of the system, and \(\nabla\) results from the microscopic potential acting on the single electrons. It is important to note that such an approximation is generally acceptable only for the first collective excited state or at most the first few ones. This observation will become of consequence in the discussion in Section 5 of the lifetime of the state prepared by applying a magnetic field to the crystal.

It is easy to see that the fluctuations of the center of mass are confined within such a small region that they can be ignored, and the center of the rotor can be assumed standing at a fixed position. The situation is in general different for the quantum fluctuations of the rotor axes. We restrict ourselves to a rotor with axial symmetry, and assume its symmetry axis along the \(\zeta\)-axis. Its rotational Hamiltonian is

$$H_{\text{rotational}} = \frac{\hbar^2}{2I} \left( -\frac{\partial^2}{\partial \theta^2} - \cot \theta \frac{\partial}{\partial \theta} + \frac{1}{\sin^2 \theta} L_z^2 \right)$$  \hspace{1cm} (3)

where \(\theta\) is the angle between the \(z, \zeta\)-axes, \(L_z = -i\hbar \frac{\partial}{\partial \theta}\) is the \(z\)-component of the orbital angular momentum, and \(I\) the moment of inertia with respect to the \(\xi\) and \(\eta\)-axes. For the potential we assume

$$\nabla = \frac{1}{2} \mu \sin^2 \theta + \mu \cdot \vec{B},$$  \hspace{1cm} (4)

where \(\mu\) is the total magnetic moment, \(\vec{B}\) the total magnetic field acting on the system and \(C\) a restoring force constant. In the absence of magnetic field the dynamics is determined by the parameter

$$\theta_0^2 = \frac{\hbar}{\sqrt{2C\mu}}.$$  \hspace{1cm} (5)

When \(\theta_0 \to 0\) the axis of the rotor can be assumed to lie along the \(z\)-axis, the direction of easy magnetization, and its zero-point fluctuations can be ignored and. For \(\theta_0 \sim 1\) the zero-point fluctuations cannot be neglected, but the rotor is still polarized within an angle of order \(\theta_0\). For \(\theta_0 \gg 1\) there is no polarization at all.

Let us make an estimate of \(\theta_0\) according to the standard values of the parameters reported in [1]. The restoring force constant is

$$C = 2K_1V_c$$  \hspace{1cm} (6)

where \(V_c\) is the volume of the cell and \(K_1\) the lowest order uniaxial anisotropy constant. This latter can be expressed in terms of the second-order uniaxial crystal field parameter \(A_0^2\) and of the quadrupole moment of the atom \(Q_2\)

$$V_cK_1 = \frac{3}{2}Q_2A_0^2.$$  \hspace{1cm} (7)

For typical rare earth compounds such as \(R_2Fe_{14}B\) and \(R_2Fe_{17}N_3\), \(A_0^2 = 30\text{ meV}/a_0^2\) and \(-36\text{ meV}/a_0^2\) respectively, while for most rare earth ions \(|Q_2| \approx 0.5a_0^2\) \((a_0 \approx 0.5\text{ Å})\), so that \(C \approx 45\text{ meV}\). For the present estimate of an order of magnitude we assume for the moment of inertia the expression appropriate to a rigid body

$$I_{\text{rigid}} = \frac{2}{5}m_0Z_{4f}\langle r_{4f}^2 \rangle$$  \hspace{1cm} (8)

where \(\langle r_{4f}^2 \rangle\) is the mean square radius of the \(4f\)-electrons. Then setting \(\langle r_{4f}^2 \rangle \approx a_0^2\) as appropriate to all \(4f\)-rare earth electron systems and \(Z_{4f} \approx 10\) as appropriate, for instance, to \(Dy, Ho, Er\) we get \(\theta_0^2 \approx 10\). By comparison we remind that the values we quoted for \(La Mn O_3\) are one order of magnitude smaller \([10, 11]\), and that for the atomic nuclei \(\Sigma\) of the rare earths \(\theta_0^2 \approx 10^{-2}\). The value of \(\theta_0\) is so large because of the small value of the restoring force constant \(C\) and of the moment of inertia of the \(4f\) electrons system (due to their small mean square radius). We find it difficult to reconcile the Single Ion Model with the standard values of the parameters, a difficulty which might be related with the observation of Ref. [2]. Notice that increasing the crystal field strength by one order of magnitude would not alter our conclusion.

If nevertheless the magnetic anisotropy of the rare earths is due to a substantial polarization of the single ions, we might have a direct information about the relative values of spin-orbit and crystal field strength. Indeed if the spin-orbit force remains larger than the crystal field Spin-Orbit Locking should occur for the rare earth ions with non-vanishing spin. In the following we suggest experiments to investigate the occurrence of Spin-Orbit Locking, thus making a test of the standard picture, by studying the Scissors Modes in such systems.

A. Eigenstates and eigenvalues of the Rotor Model

We set a magnetic field in the \(y-z\)-plane at an angle \(\theta_B\) with the \(z\)-axis, so that its components are \(B_x = 0\), \(B_y = B\sin \theta_B\), \(B_z = B\cos \theta_B\), where \(B\) is its strength. Since the uniaxial symmetry is broken by the external magnetic field, it is convenient to introduce the cartesian coordinates \(x = \sin \theta \cos \phi, y = \sin \theta \sin \phi, z = \cos \theta\), which are the direction cosines of the axes of the atom. In the presence of a strong polarization the angle \(\theta\) is very small, so that we can make the approximations

$$x \approx \theta \cos \phi, \quad y \approx \theta \sin \phi, \quad z \approx 1 - \frac{1}{2}(x^2 + y^2).$$  \hspace{1cm} (9)

The interaction with the magnetic field becomes

$$-\vec{\mu} \cdot \vec{B} \approx -\mu B(y \sin \theta_B + z \cos \theta_B)$$  \hspace{1cm} (10)
where $\mu = gJ \mu_B$, $\mu_B$ being the magnetic moment of the electron, $g$ the Landé’s factor and $J$ the total angular momentum of the ion. The values of $\theta_B$ can be restricted to the interval $(0, \frac{\pi}{4})$. Then the total potential takes its minimum at $x = 0, y = \frac{\mu_B \sin \theta_B}{C_B}$ and the harmonic approximation to the hamiltonian is

$$H_B \approx \frac{\hbar^2}{2I} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) + \frac{1}{2} C x^2 + \frac{1}{2} C_B (y - \bar{y})^2$$

where $C_B = C + \mu B \cos \theta_B$ and we neglected the constant $-\mu B \cos \theta_B$. It describes the motion of the projection of the end points of the symmetry axis of the ion on the $x - y$ plane. The eigenfunction are

$$\psi_{n_1, n_2}(\phi) = \left( \frac{\pi}{2n_1 + 1} \right)^{\frac{1}{2}} \frac{1}{\theta_0} H_{n_2} \left( \frac{x}{\theta_0} \right)$$

$$\times H_{n_2} \left( \frac{y - \bar{y}}{\theta_0} \right) \exp \left( -\frac{x^2}{2 \theta_0^2} \right)$$

where $H_n$ are Hermite polynomials and $\theta_0 = \frac{\hbar}{\sqrt{4I}} \approx \frac{\hbar}{\sqrt{C}}$.

The first excited states at zero magnetic field are $\psi_{1,0}^{(0)}, \psi_{0,1}^{(0)}$. They are analogous to the Scissors Modes of condensed atoms in magnetic traps, which can be depicted as opening and closing scissors. Because at zero magnetic field the crystal field has axial symmetry, however, the eigenfunctions must be eigenfunctions of angular momentum. They are combinations of $\psi_{0,1}^{(0)}$ and $\psi_{1,0}^{(0)}$

$$\psi_\pm^{(0)} = \frac{1}{\sqrt{2}} (\psi_{1,0}^{(0)} \pm i \psi_{0,1}^{(0)})$$

All these states have excitation energy $E_S \approx \hbar \sqrt{\frac{\mu B}{I}}$.

It is obvious that the state $\psi_{0,0}^{(B)}$ is an excited state (but not an eigenstate) of the rotor hamiltonian at zero magnetic field. It has a life time $\tau(\psi_{0,0}^{(B)})$ which will be estimated in Section 5. The probability to find in $\psi_{0,0}^{(B)}$ Scissors states is

$$P(B) = |\langle \psi_{0,0}^{(B)} | \psi_\pm^{(0)} \rangle|^2 \approx \frac{1}{2} \left( \frac{\mu B \theta_0^3 \sin \theta_B}{\hbar^2} \right)^2$$

We note that the above expression has been obtained by setting the magnetic field in the $y - z$ plane at a small angle with the $z$-axis. The same result would be obtained if it were set at a small angle with the $z$-axis in the $x, z$ plane, in which case the minimum of the potential would occur at $\pi \neq 0, \pi = \bar{y} = 0$. On the contrary we would get $P(B) = 0$ if the magnetic field were at a small angle with the $x$- or $y$-axes, giving $\pi = y = 0, \pi \neq 0$, because a state containing a quantum of oscillation along the $z$-axis is orthogonal to $\psi_\pm^{(0)}$.

In view of the difficulty with the rotor description of the Single Ion Model, discussing the experiments we will not use the restoring force constant $C$. We will express all the quantities in terms of the zero point oscillation amplitude $\theta_B$, which is unknown but must be of the order or smaller than 1 if the atom is polarized, and of the moment of inertia, for which we have a reasonable estimate. Therefore we will parametrize the excitation energy according to

$$E_S = \hbar \sqrt{\frac{C}{I}} = \frac{1}{\theta_0^2} \frac{h^2}{I}$$

We note that in the framework of a semiclassical model there is no unique prescription, in general, for the evaluation of the moment of inertia. For instance one might include or exclude that part of the constituents which has a spherical shape [1118]. In the presence of Locking such ambiguity is reduced because, qualitatively, the $4f$ electrons which contribute a spherical charge distribution have a nonvanishing spin and therefore rotate with the electrons which determine the charge deformation.

### IV. THE PROPOSED EXPERIMENTS

We consider 3 experiments, which are essentially 3 ways of performing one and the same experiment

i) we set a sample of the crystal in a magnetic field which remains constant for a time sufficient for the ion in the cell to go to the state $\psi_{0,0}^{(B)}$. Then we switch off the magnetic field in a time not longer than $\tau(\psi_{0,0}^{(B)})$, Fig.1

ii) the magnetic field is switched on in a time not longer than $\tau(\psi_{0,0}^{(B)})$ and then kept constant for a much longer time

iii) the magnetic field is pulsed with a cycle not longer than $\tau(\psi_{0,0}^{(B)})$.

Let us now discuss the first experiment. After we switch off the magnetic field the state $\psi_{0,0}^{(B)}$ will decay to the ground state $\psi_{0,0}^{(0)}$ through various processes. In order to investigate Scissors Modes we select the decay accompanied by emission of a photon of energy $E_S$. The probability of this process is $P(B)$. In conclusion the expected number of photons of energy $E_S$ is

$$N_{\text{photons}} = N_{\text{atoms}} P(B)$$

where $N_{\text{atoms}}$ is the number of effective atoms in the sample, namely the atoms whose decay photons are not absorbed in the sample itself. They are contained in a volume equal to the surface $S$ of the sample times the photon radiation length $\lambda$

$$N_{\text{atoms}} = \rho \lambda S$$

where $\rho$ is the number of atoms per unit volume. We can get a lower bound to the number of radiated photons by assuming a lower bound for $\lambda$ equal to the interatomic distance which is of the order of $3\AA$, and an upper bound assuming $\lambda = \frac{1}{\sigma \rho}$, where $\sigma$ is the photoabsorption cross.
section. \(\sigma\) has been evaluated in \[11\]

\[
\sigma = 6\pi^3 \frac{\alpha \hbar^2}{m_e^2 c^2} \frac{1}{\theta_0^2}
\]

(18)

where \(c\) the velocity of light and \(\alpha\) the fine structure constant. We then get for an experiment of the first type

\[
3 S \rho P(B) < N_{\text{photons}} < \frac{1}{\sigma} S \rho P(B).
\]

(19)

For a rare earth ion with \(Z_{4f} = 10\)

\[
E_S \approx 8 \frac{\mu}{\theta_0^3} \text{eV}.
\]

(20)

If the \(4f\) electrons have \(gJ \approx 10\), and are in a crystal of surface \(S = 1 \text{nm}^2\) and density \(\rho \approx 0.03 \text{A}^{-3}\), set in a magnetic field \(B = 10T\), we get

\[
3 \times 10^4 \theta_0^6 < N_{\text{photons}} < 1.5 \times 10^8 \theta_0^6.
\]

(21)

In an experiment of the second type the initial state of the atom is \(\psi_{0,0}\). Then we switch on the magnetic field according to the schematic low:

\[
B(t) = B \frac{t}{t_1}, \quad 0 < t < t_1; \quad B(t) = B, \quad t_1 < t.
\]

(22)

Applying standard perturbation theory to first order we get that the amplitude for the magnetic field to excite the Scissors Mode \(\psi_{0,1}\) is

\[
-\hbar^{-1} \int_0^t dt' e^{i E_{t'}/\hbar} (\psi_{0,1} | \vec{B} | t') \psi_{0,0} = \theta_B.
\]

(23)

The value of the matrix element \[11\] is:

\[
\langle \psi_{0,1} | \vec{B} | t' \psi_{0,0} \rangle \approx \frac{1}{2\theta_0} \mu B(t) \sin \theta_B,
\]

(24)

and the integral in Eq.(23) yields

\[
\int_0^{t_1} dt e^{i E_{t}/\hbar} B(t) \approx \frac{B t_1}{\hbar}.
\]

(25)

In conclusion the probability per atom of exciting the Scissors Mode is

\[
\left( \frac{B \mu}{E_S} \right)^2 \left( \frac{\sin \theta_B}{\theta_0} \right)^2 = \frac{2}{\theta_0^4} P(B).
\]

(26)

Since all the excited atoms will eventually decay the number of photons to be expected in the second type of experiment is

\[
N_{\text{photons}} \approx N_{\text{atoms}} \frac{2}{\theta_0^4} P(B),
\]

(27)

which is \(2 \theta_0^4\) times the number of photons in an experiment of the first type. Finally an experiment of the third type can be analyzed in similar way. Such an experiment offers the advantage that the number of observed photons is proportional to the number of pulses of the magnetic field.

The photons emitted by the decay of the state \(\psi^{(B)}_{00}\) have a signature which should help identifying them. As observed after Eq.(14) photons with energy \(E_S\) and momentum parallel to the cell axis should be produced provided the magnetic field is set in the \(x, z\)-plane at a small angle with the \(x\)-axis, but no photons should be produced if the magnetic field were at a small angle with the \(x\) or \(y\)-axes.

The experiments can be performed looking for photons of energy given by \[15\] varying \(\theta_0\) in the range \(0 < \theta_0 < 1\). If the result is positive, we learn that both Scissors Modes and Spin-Orbit Locking exist. If the result is negative, we need to do a photoabsorption experiment to establish if Scissors Modes exist, and then Locking does not, or Scissors modes do not exist, in which case we do not learn anything about Locking.

Concerning the choice of the sample we must exclude the ion \(Eu^{3+}\) because its angular momentum is zero \[1\]. Moreover our analysis might apply only qualitatively to the ions \(Gd^{3+}\) and \(La^{3+}\) whose \(4f\) electron system is spherical. The hamiltonian of these ions does not have a kinetic term and the restoring force is of pure magnetic nature. All compounds with these rare earths, however, could be used by comparison, in order to exclude spurious effects. Concrete examples of suitable compounds can be \(R_2Fe_{14}B\) and \(R_2Fe_{17}N_3\) where the rare earth ions can be \(Dy, Ho\) and \(Er\) which all have \(Z_{4f} \approx 10\) and \(gJ \approx 10\).

V. LIFE TIME OF THE STATE \(\psi^{(B)}_{00}\)

A crucial requirement for the proposed experiments to be feasible is that the time required to switch on or off the magnetic field must not be large with respect to the life time of the state \(\psi^{(B)}_{00}\), \(\tau\left(\psi^{(B)}_{00}\right)\), otherwise this state will go adiabatically to the state \(\psi^{(0)}_{00}\) without photon emission. We need therefore an estimate of this life time.

We notice that to order \(\theta_0\) the state \(\psi^{(B)}_{00}\) has a nonvanishing component only on the Scissors Mode, so that

\[
\tau\left(\psi^{(B)}_{00}\right) = P(B)^{-1} \tau\left(\psi^{(0)}_{00}\right).
\]

(28)

Using the expression of the life time of the Scissors Modes evaluated in Ref.\[10\] we get

\[
\tau\left(\psi^{(B)}_{00}\right) = \frac{1}{3} \frac{m^2 c^4}{\alpha} \frac{\hbar}{\mu B} \theta_0^4 \hbar.
\]

(29)

For the values of the parameters used in the estimate of the number of photons we get \(\tau\left(\psi^{(B)}_{00}\right) \approx 3 \left(\theta_0 \sin \theta_B \right)^2 \text{sec.}\)
There is a problem, however. As already said all the excited collective eigenstates of the hamiltonian (2), apart from the Scissors states, are not physically realized, with the possible exception of a few. In order to respect unitarity these states must be replaced by other states, which could contribute to the width of the state \( \psi^{(B)}_{00} \) through single particle cascade processes. As soon as these processes will start, they will begin to disrupt the collectivity of \( \psi^{(B)}_{00} \), making the decay through emission of photons of energy \( E_S \) impossible.

The missing states can only be determined using the microscopic hamiltonian (1), which is completely outside the scope of the present paper. In this connection, however, we can make some considerations of general character. Before the state \( \psi^{(B)}_{00} \) is substantially altered by single particle processes, a sufficient number of steps must occur. Each step will involve one power of the relevant coupling constant. Therefore we can expect that the contribution to the width of cascade processes will not exceed the collective contribution, provided the coupling constants are small enough and the number of steps large enough to alter significantly the structure of the state. We should feel reasonably justified in neglecting electromagnetic single particle transitions because of the smallness of the fine structure constant. Concerning phonons, since their energies can at most be 50 meV, the number of steps involved should be sufficiently large.

Anyhow the estimate (29) provides an upper bound to the lifetime of the state \( \psi^{(B)}_{00} \).

VI. SUMMARY

Our original motivation was to propose an experiment to excite and detect Scissors Modes in crystals, alternative to photoabsorption, in order to see if such states exist. Scissors Modes were predicted by a Two Rotor Model, which describes the motion of two interacting deformed bodies. The physics of the Two Rotor Model has been applied by many authors to systems in which only one deformed body is moving in a nonspherical potential, which we can call a One Rotor Model. Then we thought of using a One Rotor Model for the 4f electron system of the rare earths, to design an experiment by which we might investigate Scissors Modes in crystals.

In the course of this work, we read Ref.[2], which questions the standard values of spin-orbit and crystal field strength in the rare earths, and we thought that the experiments we were designing might also provide direct information about this issue, namely about the existence of Spin-Orbit Locking. We stress, however, that even if the existence of Spin-Orbit Locking (or rigid spin-orbit coupling) were regarded as firmly established, the experiments we suggest would have their validity: in this case they would give unambiguous information about the existence of Scissors Modes.

In the design of the experiments we adopted the Single Ion Model of magnetic anisotropy. We then realized that, apparently, in such a model the kinetic energy of the ion representing collectively the 4f electron system is disregarded, but if taken into proper account with current values of the relevant parameters for nonspherical systems, it would destroy the polarization and therefore the magnetism of the ion. This is at variance with spherical ions, because the rotation of the spins of the latter is not associated to any kinetic energy.

In conclusion in its present form our work contains two issues. First we raise the question of how to derive the Single Ion Model from a microscopic hamiltonian. Secondly, assuming that magnetism in the rare earths is however associated with single ion polarization, we propose experiments to detect Scissors Modes and investigate Spin-Orbit Locking.

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