In order to investigate the formation of relativistic jets at the center of a progenitor of a gamma-ray burst (GRB), we develop a two-dimensional general relativistic magnetohydrodynamic code. We show that the code passes many well-known test calculations, and confirm the reliability of the code. Then we perform a numerical simulation of a collapsar using a realistic progenitor model. It is shown that a jet is launched from the center of the progenitor. The structure of the jet is similar to the previous study: a Poynting flux jet is surrounded by the funnel-wall jet. Even at the final stage of the simulation, the bulk Lorentz factor of the jet is still low, and the total energy of the jet is still as small as $10^{48}$ erg. However, we find that the energy flux per unit rest-mass flux is as high as $10^{2}$ at the bottom of the jet. Thus, we conclude that the bulk Lorentz factor of the jet can be potentially high when it propagates outward. Also, as long as the duration of the activity of the central engine is long enough, the total energy of the jet can be large enough to explain the typical explosion energy of a GRB ($\sim10^{51}$ erg). It is shown that the outgoing Poynting flux exists at the horizon around the polar region, which proves that the Blandford–Znajek mechanism is really working. However, we conclude that the jet is mainly launched by the magnetic field amplified by the gravitational collapse and differential rotation around the black hole, rather than the Blandford–Znajek mechanism.

Key words: accretion, accretion disks – black hole physics – gamma rays: bursts – relativity – supernovae: general

1. INTRODUCTION

Gamma-ray bursts (GRBs) have been mysterious phenomena since their discovery in 1969 (Klebesadel et al. 1973). In the last decade, observational evidence for supernova (SN) and long GRB (in this study, we consider only long GRBs, so we refer to long GRBs as GRBs hereafter for simplicity) association has been reported (e.g., Woosley & Bloom 2006, and references therein).

Some of the SNe that are associated with GRBs were very energetic and blighted. The estimated explosion energies were of the order of $10^{52}$ ergs, and produced nickel mass was ($\sim0.5M_\odot$). Thus they are categorized as a new type of SNe (sometimes called hypernovae). That the explosion energy is so large is very important, because it cannot be explained by the standard core-collapse SN scenario; another mechanism should be working at the central engine of GRBs.

Two of the most promising scenarios are the collapsar scenario (Woosley 1993) and the magnetar scenario (U\v{s}ov 1992). In the collapsar scenario, a rapidly rotating black hole (BH) is formed at the center, while a rapidly rotating neutron star (NS) with strong magnetic fields ($\sim10^{15}$ G) is formed in the magnetar scenario. Many numerical simulations have been done for the collapsar scenario (MacFadyen & Woosley 1999; Proga et al. 2003; Proga & Begelman 2003; Mizuno et al. 2004a, 2004b; Proga 2005; Fujimoto et al. 2006; Shibata et al. 2006; Nagataki et al. 2007; Sekiguchi & Shibata 2007; Suwa et al. 2007; Barkov & Komissarov 2008) and the magnetar scenario (Takiwaki et al. 2004; Komissarov & Barkov 2007; Burrows et al. 2007; Bucciantini et al. 2008; Dessart et al. 2008; Takiwaki et al. 2009; Bucciantini et al. 2009). In this study, we investigate the collapsar scenario.

In the collapsar scenario, a black hole is formed as a result of gravitational collapse. Also, rotation of the progenitor plays an essential role. Due to the rotation, an accretion disk is formed around the equatorial plane. On the other hand, the matter around the rotation axis freely falls into the black hole. It is pointed out that the jet-induced explosion along the rotation axis may occur due to the heating through neutrino–anti-neutrino pair annihilations that are emitted from the accretion disk (Woosley 1993; MacFadyen & Woosley 1999; Fryer & Mészáros 2000). The effect of extraction of the angular momentum from the accretion disk by magnetic field lines that leave the disk surface (Blandford–Payne effect (Blandford & Payne 1982)) is also investigated by several authors (Proga et al. 2003; Proga & Begelman 2003; Mizuno et al. 2004a, 2004b; Proga 2005; Fujimoto et al. 2006; Nagataki et al. 2007; Suwa et al. 2007). Recently, the effect of extraction of the angular momentum from the black hole through outgoing Poynting flux (Blandford–Znajek effect (Blandford & Znajek 1977)) was investigated (Barkov & Komissarov 2008). In order to fully investigate the collapsar scenario, a high-quality numerical code that includes the effects of microphysics (neutrino physics, nuclear physics, and equation of state for dense matter) and macrophysics (magnetohydrodynamics, general relativity) has to be developed. Although many numerical studies have been reported, such a numerical code has not been developed yet. Thus we have to develop our numerical code step by step.

In this study, we investigate the dynamics of collapsars taking into account the general relativistic effects. Extraction of rotation energy from a rotating black hole is one of them. Also, even when the rotation energy is extracted from the accretion disk, the properties of the accretion disk should depend on the properties of the black hole: if the black hole is rotating, the innermost region of the accretion disk should be forced to corotate with the black hole. We investigate how a jet is launched at the center of a progenitor, and what the property of the jet is. Effects of rotation of the black hole on the formation of GRB jet have not been investigated so much. The study of Barkov & Komissarov (2008) is pioneering. However, only one case is investigated in their study, and the initial progenitor model they used is a simplified one-dimensional model without rotation...
and magnetic fields (Bethe 1990). Since there should be many initial conditions of progenitors (progenitor mass, metallicity, angular momentum, magnetic fields), it should be important to investigate the effects using a different initial condition from the previous study. In this study, we use a realistic initial condition for the progenitor model that is developed by Woosley & Heger (2006), in which rotation and magnetic fields are taken into account.

When we investigate the general relativistic effects, one has to develop a general relativistic magnetohydrodynamic (GRMHD) code. So far, there are many studies on GRMHD code for fixed background spacetimes using high-order conservative schemes based on either approximate or full wave-decomposition Riemann solvers (Gammie et al. 2003; Komissarov 2005; Anninos et al. 2005; Antón et al. 2006; Del Zanna et al. 2007; Tchekhovskoy et al. 2007) or non-conservative schemes (De Villiers & Hawley 2003; Anninos et al. 2005). Since the accepted mass onto the black hole is still less than the initial black hole mass in this study, we take the GRMHD code for the fixed background. Particularly, we develop our code using the formulation of Gammie et al. (2003) with the method of Noble et al. (2006) for transforming conserved variables to primitive variables.

The plan of the paper is as follows. In Section 2, we present the formulation of the GRMHD code. In Section 3, we show results of many well-known test calculations to confirm the reliability of the code. After showing the reliability, we present results of numerical simulations of collapsars in Section 4. Summary and discussion are presented in Section 5.

2. DEVELOPMENT OF THE GRMHD CODE

We have developed a two-dimensional GRMHD code following Gammie et al. (2003) and Noble et al. (2006). We have adopted a conservative, shock-capturing scheme with Harten, Lax, and van Leer (HLL) flux term (Harten et al. 1983) with flux-interpolated constrained transport technique (Toth 2000). We use a third-order total variation diminishing (TVD) Runge–Kutta method for evolution in time, while monotonized central slope-limited linear interpolation method is used for second-order accuracy in space (van Leer 1977). A two-dimensional (2D) scheme (2D Newton–Raphson method) is usually adopted for transforming conserved variables to primitive variables (Noble et al. 2006).

When we perform simulations of GRMHD, the modified Kerr–Schild coordinate is basically adopted with mass of the black hole (M) fixed, where the Kerr–Schild radius r is replaced by the logarithmic radial coordinate x1 = ln r. When we show the result, the coordinates are transferred from the modified Kerr–Schild coordinates to Kerr–Schild ones for convenience. In the following, we use G = M = c = 1 unit. G is the gravitational constant, c is the speed of light, and M is the gravitational mass of the black hole at the center. Throughout this paper, we follow the standard notation (Misner et al. 1970).

2.1. Formalism

There are 13 variables that appear in the equations of GRMHD: rest-mass density (\(\rho\)); internal energy density (u); pressure (p); four-velocity of fluid (\(u^\mu\)); and Faraday tensor (\(F^{\mu\nu}\)). Note that the Faraday tensor has only six independent components due to the relation \(F^{\mu\nu} = -F^{\nu\mu}\). We can reduce the number of independent variables to 8 using the MHD condition \((u_\mu F^{\mu\nu} = 0)\), equation of state \((p = (\Gamma - 1)u)\): \(\Gamma\)-law gas is assumed), and the unit length of the four-velocity \((u_\mu u^\mu = -1)\). Note that there are three independent equations of the MHD condition. We choose \((\rho, u, u^\nu, B^\nu)\) as the eight independent variables, where \(u^\nu\) is the space component of the four velocity. \(B^\nu\) can be written as \(\Psi_B^\nu\), where \(\alpha\) is the lapse function \((\alpha = \sqrt{-1/g^00})\) and \(\Psi_B^\nu\) is the magnetic field measured by the fiducial observer (FIDO) whose four velocity is \(u^\nu = (-\alpha, 0, 0, 0)\). We call these independent variables as the primitive variables. Later, we introduce the conserved variables. Of course, the number of the conserved variables is also 8. Thus, we require eight basic equations to follow the time evolution of the system.

The basic equations of GRMHD represent the rest-mass conservation, the energy–momentum conservation, and space component of the induction equation that determines the time evolution of the magnetic fields. These are

\[
\partial_t (\sqrt{-g} \rho u^\nu) = -\partial_i (\sqrt{-g} \rho u^\nu) \tag{1}
\]

\[
\partial_t (\sqrt{-g} T^\nu_{\;\;\nu}) = -\partial_i (\sqrt{-g} T^\nu_{\;\;\nu}) + \sqrt{-g} T^\nu_{\;\;\nu} \Gamma^\nu_{\;\;\nu} \tag{2}
\]

\[
\partial_t (\sqrt{-g} B^\nu) = -\partial_j [\sqrt{-g} (b^j u^\nu - b^j u^\nu)] \tag{3}
\]

where \(T^{\mu\nu}\) is the stress-energy tensor that is composed of the sum of the matter part \(T^{\mu\nu}_{\text{Matter}} = (\rho + p)u^\mu u^\nu + pg^{\mu\nu}\) and electromagnetic part \(T^{\mu\nu}_{\text{EM}} = F^{\mu\alpha} F_{\alpha\nu} - g^{\mu\alpha} F_{\alpha\beta} F_{\beta\nu}/4\). The factor of \(\sqrt{4\pi}\) is absorbed into the definition of the Faraday tensor (Gammie et al. 2003), \(b^\nu\) is introduced so that Equation (3) looks simple, and it is defined as \(b^\nu = e^{\mu\nu\lambda\kappa} u_\lambda F_{\nu\kappa}\), where \(\epsilon^{\mu\nu\lambda\kappa} = (-1/\sqrt{-g})[\mu\nu\lambda\kappa]\). [\(\mu\nu\lambda\kappa\)] is the completely antisymmetric symbol.

In this study, we adopt the conservative scheme for integration of the GRMHD equations. In this case, the left terms of Equations (1)–(3) are considered to be fundamental variables and called the conserved variables. The right terms of Equations (1)–(3) are flux terms with a source term (the second right term of Equation (2)).

Since we have to estimate pressure of the fluid, we have to estimate the primitive variables from the conserved variables at each time step. The problem is that the primitive variables cannot be expressed analytically by the conserved variables. Thus, we have to use the Newton–Raphson method to obtain the primitive variables from the conserved ones (Noble et al. 2006).

Basically, we adopt the 2D scheme introduced by Noble et al. (2006) to calculate the primitive variables. However, it sometimes happens that the 2D scheme fails to converge well, and the primitive variables cannot be obtained precisely. In such a case, we first adopt the 1Dw scheme introduced by Noble et al. (2006) and see whether the 1Dw scheme converges. If it converges well, we adopt the primitive variables obtained by the 1Dw scheme for the next time step. If not, we adopt the second choice explained in the following subsection.

2.2. Supplemental Method to Calculate Primitive Variables

Following Noble et al. (2006), we introduce convenient variables \(v^2, W, Q^\mu, \text{and } \tilde{Q}^\mu\). These variables, of course, depend on the primitive variables. The definitions of these variables are:

\(v^2 = v_i v^i\); \(W = \omega\gamma\); \(Q^\mu = \alpha T^\mu_{\;\;0}\); and \(\tilde{Q}^\mu = j^\mu j^\nu Q_{\nu}\).

where \(v^i\) is the fluid velocity relative to the FIDO, \(\omega = \rho + u + p\), and \(\gamma = 1/\sqrt{1 - v^2}\). It is apparent that \(Q^\mu\)
and $\bar{Q}_{\mu}$ can be written analytically by the conserved variables. On the other hand, $v^2$ and $W$ cannot be expressed analytically by the conserved variables. Thus, we have to solve $v^2$ and $W$ numerically in order to determine the proper, corresponding primitive variables.

Here we show that an upper limit and lower limit for $W$ can be obtained before searching for a solution of $W$ and $v^2$ numerically. Thanks to this fact, all we have to do is to seek the solution with the condition $W_{\text{min}} \leq W \leq W_{\text{max}}$. From Equations (28) and (29) in Noble et al. (2006), $W$ and $v^2$ satisfy the following equations:

$$v^2_{\text{eq29}} = \frac{\bar{Q}^2 W^2 + (\bar{Q}_{\mu} n_{\mu})^2 (9/2 + (3/2) W)}{(9/2 + W)^2 W^2}$$

$$v^2_{\text{eq29}} = \frac{2}{3} \left[ \frac{(\bar{Q}_{\mu} n_{\mu})^2}{2 W^2} - W + p - (\bar{Q}_{\mu} n_{\mu}) \right] - 1.$$  

From these equations and the relation $0 \leq v^2 \leq 1$, $v^2$ and $W$ satisfy the following relations:

$$f(W) = W^4 + 2(3/2) W^3 + (9/4 - \bar{Q}^2) W^2 - 2(\bar{Q}_{\mu} n_{\mu})^2 W$$

$$g(W) = W^3 + \left[ \frac{2}{3} \right] (9/2 + (\bar{Q}_{\mu} n_{\mu}) - p) W^2 - \frac{1}{2} (\bar{Q}_{\mu} n_{\mu})^2 \leq 0$$

$$h(W) = W^3 + \left\{ (9/2 + (\bar{Q}_{\mu} n_{\mu}) - p) W^2 - \frac{1}{2} (\bar{Q}_{\mu} n_{\mu})^2 \right\} \geq 0.$$  

Since $f(W = 0) \leq 0$, $f'(W) = 0$ and at least one of the solutions for $f'(W) = 0$ is less than 0, there is only one positive solution $W_{a}$ that satisfies $f(W_a) = 0$. Thus, from Equation (6), $W$ has to be greater than $W_a$.

We can understand the behavior of $g(W)$ from its first derivative for $W$:

$$g'(W) = W \left[ 3W + 2 \left\{ \frac{1}{2} (9/2 + (\bar{Q}_{\mu} n_{\mu}) - p) \right\} \right].$$

It is apparent that $W = 0$ is a solution for $g'(W) = 0$. As for the other solution(s), it is not so obvious because the pressure $p$ depends on $W$ and $v^2$. However, it will be natural to consider that the monotonic relation holds between $W$ and $p$. It means that the pressure rises when $W$ becomes larger. If this assumption is adopted, as long as $g(W) = 0$ has another solution, it is a positive one $W = W_{b} \geq 0$. This is because when $W = 0$, $p$ should also be 0, and $[3W + 2(1/2)W^2 + (\bar{Q}_{\mu} n_{\mu}) - p]$ is a positive value. Thus, $g(W) = 0$ has only one positive solution $W_b$. This holds even if $g(W) = 0$ has only one solution $W = 0$. Also, same conclusion can be derived for $h(W)$: there is only one positive solution $W_c$ that satisfies $h(W_c) = 0$.

Since $h(W) \geq g(W)$, the relation $W_b \leq W_c$ holds. In conclusion, $W$ has to be in the range $W_{\text{min}} = \max(W_a, W_c) \leq W \leq W_b = W_{\text{max}}$. Thus, all we have to do is to find a solution of $W$ that satisfies $v^2_{\text{eq29}} = v^2_{\text{eq39}}$ in this range. This procedure is more expensive than the 2D and 1DW schemes, but the solution for $W$ and $v^2$ is more likely to be found because the range for the solution of $W$ is determined a priori. Thus, we use this method as a supplementary one to obtain the primitive variables.

3. TEST CALCULATIONS

Using the GRMHD code that is developed in this study, we check whether it can pass many well-known test calculations.

![Figure 1. Simulation of one-dimensional shock tube test (Komissarov 1999). The state at $t = 1.0$ is shown in the figure. The number of grid points is 600. The calculation region is set to be $-2 \leq x \leq 2$. The upper left panel shows density, the upper right panel shows pressure, the lower left panel shows the velocity in the $x$-direction, and the lower right panel shows the bulk Lorentz factor.](image)

The first three tests are Special relativistic hydrodynamic/magnetohydrodynamic (SRHD/SMHD) calculations, while the rest of three tests are GRMHD ones.

### 3.1. Shock Tube Problems

One-dimensional (1D) shock tube tests are the most basic test problems for SRHD/SMHD. We have carried out a number of the test simulations introduced in Komissarov (1999) and Balsara (2001). Here we describe only two of them. One is the shock tube test1 and the other is the collision test (Komissarov 1999; Mizuno et al. 2006). The initial left and right states are summarized in Table 1. Number of grid points is 600 for both simulations. The results are shown in Figures 1 and 2, which show that the test calculations are well solved as in the previous studies.

### 3.2. Double Shock Problems

Here the 2D shock tube problem is done to confirm whether the shock dynamics in the multidimensional flow can be solved safely. This problem includes the interactions of shocks, rarefactions, and contact discontinuities. Initially, a square
computational domain is prepared in the $x$–$y$ plane and divided into four quarter boxes. The initial condition in each box is summarized in Table 2. This condition is same with previous study (Del Zanna & Bucciantini 2001; Zhang & MacFadyen 2006; Mizuta et al. 2006). We use $400\times400$ uniform grid points in a square computational box. The boundary conditions are open ones. The density contour at the final stage of the simulation is shown in Figure 3, which shows that the GRMHD code reproduces the previous studies

### 3.3. Cylindrical Explosion Test

Here we go to a relativistic MHD test. A famous cylindrical blast explosion test is done (Komissarov 1999; Del Zanna et al. 2003; Leismann et al. 2005). We use the $[0,1]\times[0,1]$ Cartesian grid with a resolution of $N_x = N_y = 250$ grid points. We define an initially static background with $\rho = 1.0$, $p = 0.01$, and $B_z = 4.0$. The relativistic flow comes out by setting a much higher pressure, $p = 10^5$ within a circle of radius $r = 0.08$ placed at the center of the domain. $\Gamma$ for the equation of state is set to be $4/3$. Final time is set to be 0.4. The result is shown in Figure 4. The upper left panel shows the density contour in logarithmic scale. The upper right panel shows the pressure contour in logarithmic scale. The lower left panel shows bulk Lorentz factor. The lower right panel shows the divergence of the magnetic fields with magnetic field lines. These results are consistent with the previous studies. Especially, the divergence of the magnetic fields is kept as small as $10^{-14}$.

### 3.4. Gammie’s Flow

Next we consider a test of general relativistic MHD. A steady, magnetized inflow solution on the equatorial plane around a Kerr black hole is considered (Takahashi et al. 1990; Gammie 1999). Initially, the steady inflow solution for a Kerr parameter $a = 0.5$ is set, and time evolution of the system is followed by the GRMHD code. In this calculation, the Boyer–Lindquist coordinate is used. The calculation region is set to be $(2.0 - r \leq 4.04)$ and $(0.5 - 10^{-3} \leq \theta/\pi \leq 0.5 + 10^{-3})$. The model is run for $t = 1.5$. The physical values at boundaries are fixed throughout the simulation. Results are shown in Figure 5: density; radial component of the four-velocity; the $\phi$ component of the four-velocity; and $\Psi^0$ at the final stage of the simulation. When the initial state is written in the same figure, we can see that the final state coincides with the initial state. To show it more quantitatively, we introduce the norms of the errors for these values as a function of the number (N) of grid points in the radial coordinate. The definition of the norm of the error is $\Sigma_{i=1}^{N} |a_{(\text{final})} - a_{(\text{initial})}| / \Sigma_{i=1}^{N} |a_{(\text{initial})}|$. In Figure 6, the norms of errors are shown. We can see that these values converge roughly proportional to $N^{-2}$, as expected.

### 3.5. Blandford–Znajek Monopole Solution

Further we continue to test the GRMHD code. We consider the Blandford–Znajek monopole solution (Blandford & Znajek 1977). This analytic solution has also been numerically investigated in previous studies (Komissarov 2004b; McKinney & Gammie 2004; Tanabe & Nagataki 2008).

The computational domain is axisymmetric, with a grid that extends from $r_{in} = 0.98r_s$ to $r_{out} = 230$ and from $\theta = 0$ to $\theta = \pi$, where $r_s$ is the outer event horizon. The numerical resolution is $300\times300$. As an initial condition, we put the zeroth-order terms of the monopole solution around the black hole (Komissarov 2004b). That is, $\Psi^0 = -n_s^*F^{0\nu} = (0, \alpha \sin \theta / g, \sqrt{-g} g, 0)$ in the Kerr–Schild coordinate where $^*F^{0\nu}$ and $g$ are the dual field tensor and determinant of the Kerr–Schild metric. The plasma velocity relative to the FIDO is set to zero initially, and its pressure and density are set to small

![Figure 2](image-url)  
Figure 2. Simulation of one-dimensional collision test (Komissarov 1999). The state at $t = 1.5$ is shown in the figure. The number of grid points is 600. The calculation region is set to be $-2 \leq x \leq 2$. The left panels show density, velocity in the $x$-direction, and bulk Lorentz factor (from top to bottom), while the right panels show pressure, velocity in the $x$-direction, and $y$-component of the magnetic field (from top to bottom).

![Figure 3](image-url)  
Figure 3. Simulation of the 2D shock tube problem. The density contour at $t = 0.4$ is shown in the figure. Numbers of grid points are $400 \times 400$. The calculation region is set to be $0 \leq x \leq 1$ and $0 \leq y \leq 1$.

![Figure 4](image-url)  
Table 2
Initial Conditions for 2D Shock Tube Problem

| Region | $x$ | $y$ | $\rho$ | $\rho$ | $v_x$ | $v_y$ |
|--------|----|----|-------|-------|------|------|
| A      | $0 \leq x \leq 0.5$ | $0.5 \leq y \leq 1$ | 0.1    | 1     | 0.99 | 0    |
| B      | $0.5 \leq x \leq 1$ | $0.5 \leq y \leq 1$ | 0.1    | 0.01  | 0    | 0    |
| C      | $0 \leq x \leq 0.5$ | $0 \leq y \leq 0.5$ | 0.5    | 1     | 0    | 0    |
| D      | $0.5 \leq x \leq 1$ | $0 \leq y \leq 0.5$ | 0.5    | 1     | 0    | 0.99 |

**Notes.** $\gamma$ for the equation of state is set to be 5/3. Final time is set to be 0.4.
value \( P = \rho = 9 \gamma^2 / 100 \) so that the system becomes force-free like. Also, to keep the magnetization reasonably low, when the critical condition \( 0.01 B^2 \geq \gamma^2 \rho + (\Gamma \gamma^2 - (\Gamma - 1)) u \) is satisfied, density and internal energy are increased by the same factor so that the critical condition holds (Komissarov 2004b). \( \Gamma \) is set to be \( 4/3 \). We have performed numerical simulations with the Kerr parameters \( 0, 0.01, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, 0.99, \) and \( 0.995 \) until time \( t = 200 \).

The total energy flux, which is the integrated outgoing Poynting flux over the zenith angle, can be written as

\[
\dot{E} = 2\pi \int_0^\pi d\theta \sqrt{-g} (-T^\theta_\theta) = 2\pi \int_0^1 dx_2 \left( \frac{d\theta}{dx_2} \right) \\
\times \sqrt{-g} (-T^\theta_\theta) = 2\pi \int_0^1 dx_2 F_E, \tag{10}
\]

where \( x_2 = \theta / \pi \) is introduced as a convenient variable (Gammie et al. 2003).

In Figure 7, the outgoing Poynting fluxes \( F_E \) as a function of the zenith angle for the Blandford–Znajek monopole solution are shown. The fluxes are measured at \( r = 20 \) at the final stage of the simulations. We would like to note that the outgoing Poynting flux hardly depends on the radius where it is evaluated. This means that the conservation of the outgoing Poynting flux is confirmed numerically.

In Figure 8(a), we plot the total energy flux \( \dot{E} \) at the final stage for small Kerr parameter \( (0 \leq a \leq 0.2) \) by rectangular points. The dashed line is just the interpolation of the calculated values. For comparison, the second-order analytical solution is shown by the dotted line, and the forth-order analytical solution is shown by the solid line. From this comparison, we can see that all of them coincide with each other. Thus, the result of the numerical simulation by the GRMHD code is confirmed by analytical solutions.

The situation becomes different for a large Kerr parameter. In Figure 8(b), we plot the same values with Figure 8(a), but with wide range of the Kerr parameter \( (0 \leq a \leq 1) \). We can see clearly the difference among three cases. This is because the analytical solution is obtained by the perturbation method in the Kerr parameter, and it is applicable only for a small Kerr parameter. Of course, there is no such limitation for the numerical simulations. Thus, the total energy flux obtained by the numerical simulation is more reliable than the analytical estimation (see Tanabe & Nagataki 2008 for detailed discussion).
Figure 5. Gammie’s equatorial inflow solution in the Kerr metric with $a = 0.5$ and magnetization parameter $F_{\theta \phi} = 0.5$. The number of grid point is 1024. The state at $t = 1.5$ is shown in the figure. The panels show density, radial component of the four-velocity, the $\phi$ component of the four-velocity, and $\Re \phi$ at the final stage of the simulation. Boyer–Lindquist coordinates are used for the simulation.

Figure 6. Convergence results for Gammie’s equatorial inflow solution in the Kerr metric with $a = 0.5$ and magnetization parameter $F_{\theta \phi} = 0.5$. Norms of the error for $\rho$, $u^r$, $u^\phi$, and $\Re \phi$ at the final stage of the simulation are shown in the figure. The straight line represents the slope expected for second-order convergence. The definition of the norm of the error is written in the text.

Figure 7. Outgoing Poynting fluxes as a function of the zenith angle for the Blandford–Znajek monopole solution with the Kerr parameter $a = 0.01, 0.1, 0.15, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$, and $0.95$. The fluxes are measured at $r = 20$ and $t = 200$. Numbers of grid point are $300 \times 300$.

3.6. Fishbone and Moncrief’s Test

Here we present a final test of the GRMHD code. A steady and stationary torus (Fishbone & Moncrief 1976; Abramowicz et al. 1978) around a Kerr black hole that is supported by both centrifugal force and pressure is solved numerically. Of course, it should be solved as a steady and stationary state.

We have integrated a Fishbone–Moncrief solution around a Kerr black hole with $a = 0.9$. We set $u^r u^\phi = 4.45$ and $r_{in} = 6.0$. The grid extends radially from $R_{in} = 1.40$ to $R_{out} = 20$. The
same floors with Gammie et al. (2003) are used for \( \rho \) and \( u \). The numerical resolution is \( N \times N \) and the solution is integrated for \( t = 10 \). The resulting norm of the error, which converges roughly proportional to \( N^{-2} \), is shown in Figure 9.

Next we follow the time evolution of the Fishbone–Moncrief solution with magnetic fields. The vector potential \( A_\theta \propto \max(\rho/\rho_{\text{max}} - 0.2, 0) \), where \( \rho_{\text{max}} \) is the peak density in the torus, is introduced (Gammie et al. 2003). The field is normalized so that the minimum value of \( p_{\text{gas}}/p_{\text{mag}} \) becomes 10. The time integration extends for \( t = 2000 \). The number of grid points is \( 256 \times 256 \), and the grid extends radially from \( R_{\infty} = 1.40 \) to \( R_{\infty} = 300 \), while it extends in the zenith angle from \( \theta = 0 \) to \( \theta = \pi \).

The results, which are projected on the \((r \sin \theta, r \cos \theta)\)-plane, are shown in Figure 10. The upper left panel shows the initial state. The lower left panel shows the final state of the simulation with magnetic fields. The lower right panel is same with the lower left one, but it shows the wider region. The upper right panel shows the result of the simulation without magnetic fields. The lower right panel is same with the upper right one, but it shows the wider region. The upper right panel shows the result of the simulation with magnetic fields. The vector potential \( A_\phi \) is introduced (Gammie et al. 2003). The field is normalized so that the minimum value of \( p_{\text{gas}}/p_{\text{mag}} \) becomes 10. The time integration extends for \( t = 2000 \). The number of grid points is \( 256 \times 256 \), and the grid extends radially from \( R_{\infty} = 1.40 \) to \( R_{\infty} = 300 \), while it extends in the zenith angle from \( \theta = 0 \) to \( \theta = \pi \).

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4. SIMULATION OF A LONG GRB

Since our code has passed the many test calculations shown in the previous section, we now simulate the dynamics of a collapsar using the code. However, we have to say beforehand that no microphysics are included in the code such as nuclear reactions, neutrino processes, and equation of state for dense matter. So this is the first step of our project to simulate the dynamics of a collapsar and formation of a relativistic jet of a long GRB.
Effects of magnetic fields are taken into account in the model 12TJ. However, again, since 1D calculation is done, we do not know the configuration of the magnetic fields. It is difficult to extrapolate magnetic fields that satisfy the condition \( \text{div} \mathbf{B} = 0 \) everywhere. Also, there are much uncertainty on the amplitude of the magnetic fields in a progenitor. Thus, we do not use the information on magnetic fields of the model 12TJ. Rather, we adopt the same treatment in Section 3.6. That is, the vector potential \( A_\phi \propto \max(\rho/\rho_{\max} - 0.2, 0) \sin^4 \theta \), where \( \rho_{\max} \) is the peak density in the progenitor (after extracting the central part of the progenitor that has collapsed and formed a black hole). The field is normalized so that the minimum value of \( p_{\text{gas}}/p_{\text{mag}} = 10^2 \). The definition of \( p_{\text{mag}} \) is \( p_{\text{mag}} = b^2/2 \). The reason why we adopt the strong dependence on the zenith angle for \( A_\phi \) is so as not to suffer from discontinuity of magnetic fields at the polar axis. The resulting biggest amplitude of the magnetic fields is \( 7.4 \times 10^8 \) G at \( r = 950 \) (2.8 \( \times \) 10^3 cm).

We use a simple equation of state \( p_{\text{gas}} = (\Gamma - 1)u \), and set \( \Gamma = 4/3 \) so that the equation of state roughly represents radiation gas.

As for the boundary condition in the radial direction, we adopt the outflow boundary condition for the inner and outer boundaries (Gammie et al. 2003). As for the boundary condition in the zenith angle direction, the axis of symmetry boundary condition is adopted for the rotation axis, while the reflecting boundary condition is adopted for the equatorial plane. As for the magnetic fields, the equatorial symmetry boundary condition, in which the normal component is continuous and the tangential component is reflected, is adopted.

4.2. Results

In Figure 11, contours of rest-mass density at the central region are shown. Color represents the density in units of g cm\(^{-3}\) in logarithmic scale. The upper panel (Figure 11(a)) represents the contours of rest-mass density at \( t = 110,000 \), while the lower panel (Figure 11(b)) shows the contours at \( t = 180,000 \). The time unit corresponds to \( 9.85 \times 10^{-6} \) s. These results are projected on the \((r \sin \theta, r \cos \theta)\)-plane. The length unit in the vertical/horizontal axes corresponds to \( 2.95 \times 10^5 \) cm. Arrows represent the velocity fields \( u_r, u_\theta \). At \( t = 110,000 \) (Figure 11(a)), the rest mass accretes into the black hole almost steadily (see also the mass accretion rate onto the black hole, Figure 13 later). After that, the jet is launched from the black hole (Figure 11(b)). Figure 12 is the same figure as Figure 11(b), but for a wider region (upper panel, Figure 12(a)), and a narrower region (lower panel, Figure 12(b)). A jet is clearly seen along the rotation axis in Figure 12(a), while the horizon of the black hole is seen at the center in Figure 12(b). In Figure 13, mass accretion rate history on the horizon is shown. The definition of the mass accretion rate is
Figure 11. Contours of rest-mass density at the central region in logarithmic scale, in which cgs units are used assuming that the gravitational mass of the black hole is $2 M_\odot$. The length unit in the vertical/horizontal axes corresponds to $2.95 \times 10^5$ cm. The upper panel (a) shows the state at $t = 110,000$ (that corresponds to 1.0835 s), while the lower panel (b) shows the one at $t = 180,000$ (that corresponds to 1.773 s). These results are projected on the $(r \sin \theta, r \cos \theta)$-plane. Arrows represent the velocity fields $(u^r, u^\theta)$.

\[ \dot{M} = 2 \times 2\pi \int_0^\theta d\theta \sqrt{-g} \rho u^\theta / u^r. \]  

(11)

It takes about 0.15 s for the inner edge of the matter to reach the horizon. When the matter reaches there, there is an initial spike of the mass accretion rate. After that, there is a quasi-steady state like Figure 11(a) is realized. Then, the jet launches at $\sim 1.1$ s.

After that, the mass accretion rate varies rapidly with time.

We show color contours of the plasma beta ($p_{\text{gas}} / p_{\text{mag}}$) in logarithmic scale at $t = 180,000$ in Figure 14. As expected, the plasma beta is low in the jet region while it is high in the accretion disk region. We show color contours of bulk Lorentz factor around the central region at $t = 180,000$ in Figure 15(a) (upper panel, in logarithmic scale). Contours of the energy flux per unit rest-mass flux ($E = -T^r / (\rho u^r)$), which is conserved for an inviscid fluid flow of magnetized plasma, are also shown in Figure 15(b) (lower panel, in logarithmic scale). This value represents the bulk Lorentz factor ($\Gamma_\infty$) of the inviscid fluid element when all of the internal and magnetic energies are converted into kinetic energy (McKinney 2006a).

We can see that the bulk Lorentz factor of the jet is still low (Figure 15(a)), but it can be potentially as high as $10^5$ at large radius (Figure 15(b)). At $t = 180,000$, the strength of the magnetic field ($\sqrt{4\pi b^2}$) at the bottom of the jet is found to be $\sim 10^{15}$ G, and $u^\phi / u^\theta$ is 0.1 at $r_{\text{ms}}$ on the equatorial plane. Here $r_{\text{ms}}$ is the marginally stable orbit. For the Kerr black hole with $a = 0.5$, $r_{\text{ms}}$ is 4.23. As stated in Section 4.1, the initial biggest amplitude of the magnetic fields is $7.4 \times 10^8$ G at $r = 950$ where the initial density is $\sim 10^6$ g cm$^{-3}$, the expected amplification factor of the magnetic fields due to the gravitational collapse and differential rotation around the black

Figure 12. Same as Figure 11(b), but for a wider region (upper panel (a)), and a narrower region (lower panel (b)). The horizon of the black hole is seen at the center of the lower panel.

Figure 13. Mass accretion rate history on the horizon. The unit $M_\odot$ s$^{-1}$ is used assuming that the gravitational mass of the black hole is $2 M_\odot$ throughout the calculation.
hole is \((\rho/\rho_0)^{2/3} \times (d\Phi/dt/2\pi) \times \Delta t \sim 100 \times 0.016 \times 180,000 \sim 3 \times 10^5\). Thus, the initial magnetic field can be amplified as large as \(10^{15}\) G, which is roughly consistent with the amplitude of the magnetic fields at the bottom of the jet. At the late phase, the magneto-rotational instability (MRI) may be also working, which is discussed in the following section.

In Figure 16, the contour of the \(\phi\) component of the vector potential \((A_\phi)\) at \(t = 180,000\) is shown. Level surfaces coincide with poloidal magnetic field lines, and field line density corresponds to poloidal field strength. As expected, the magnetic fields are strong in the jet region, which makes the plasma beta very low, which suggests that the magnetic fields are related to the jet formation. From Figure 16, the opening angle of the jet is estimated as \(5^\circ-6^\circ\). From Figure 14, 15(b), and 16, this jet should correspond to the Poynting flux jet (Hawley & Krolik 2006). This jet is surrounded by the funnel-wall jet region (Hawley & Krolik 2006), which is shown in Figure 17 and 18 later.

In Tables 3 and 4, the integrated energies of matter and electromagnetic field at \(t = 180,000\) are shown. The integrated region is between the horizon and \(r = 200\) (for Table 3) or \(r = 40\) (for Table 4), and within the zenith angle measured from
the polar axis. As for the matter component, the contribution of the rest-mass energy is subtracted. That is,

\[
E_{\text{Matter}} = 2 \times 2\pi \int_{r_s}^{200} d\rho \int_{0}^{\theta} d\theta \sqrt{-g} (T_{0,\text{Matter}}^0 - \rho \mu^0 u^0_0). \tag{12}
\]

Factor 2 is coming from the symmetry of the system with respect to the equatorial plane. The field part can be written as

\[
E_{\text{EM}} = 4\pi \int_{r_s}^{200} d\rho \int_{0}^{\theta} d\theta \sqrt{-g} T_{0,\text{EM}}^0. \tag{13}
\]

It can be seen that the energy in the electromagnetic field dominates that in matter within \( r \leq 40 \), while they become comparable within \( r \leq 200 \). Also, the integrated energy is still less than the typical explosion energy of a GRB (\( \sim 10^{51} \) erg; Frail et al. 2001).

Next, we show the rest-mass density, outgoing mass flux, plasma beta, and outgoing Poynting flux in Figure 17. The top panel shows the rest-mass density (g cm\(^{-2}\)) as a function of the zenith angle at \( r = 10r_{\text{ms}} = 42.3 \) and \( t = 180,000 \). It is seen that the low-density region is realized around the polar axis, which corresponds to the Poynting flux jet region (0.1 radian corresponds to 5.7). Here, we define the Poynting flux jet region as the region where (1) \( \mu^0 \) is positive and (2) plasma beta is less than unity. Thus, as explained below, the Poynting flux jet region can be defined by the second top panel and second bottom panel. The second top panel shows the outgoing mass flux \( \sqrt{-g} \rho \mu^0 \) (g cm\(^{-2}\) s\(^{-1}\)) at \( r = 10r_{\text{ms}} \) and \( t = 180,000 \). It is seen from this panel that the mass outflow is not mainly in the Poynting flux jet region, but in the surrounding region. This is because the matter suffers from the centrifugal force and cannot be confined within the narrow, Poynting flux jet region (De Villiers et al. 2003; Hirose et al. 2004; McKinney & Gammie 2004; Kato et al. 2004; De Villiers et al. 2005; Hawley & Krolik 2006). Here, we define the funnel-wall region as the region where (1) \( \mu^0 \) is positive and (2) plasma beta is greater than unity. We discuss this point below with Figure 18. The second bottom panel shows the plasma beta (\( \rho_{\text{gas}} / \rho_{\text{mag}} \)) at \( r = 10r_{\text{ms}} \) and \( t = 180,000 \). From this panel, the boundary between the Poynting flux jet and the funnel-wall jet is located at \( \theta \sim 0.2 \) rad. The bottom panel shows the outgoing Poynting flux (in units of \( 10^{50} \) erg s\(^{-1}\) rad\(^{-1}\)) at \( r = r_s \) and \( t = 180,000 \). The definition of the outgoing Poynting flux here is

\[
F_{\text{BZ}} = 2 \times 2\pi \sqrt{-g} T_{0,\text{EM}}^0. \tag{14}
\]

Factor 2 is coming from the assumption of the symmetry of the system with respect to the equatorial plane. We can see that the positive outgoing Poynting flux exists at the jet region (\( 0 \leq \theta \leq 0.23 \) in radian). The integrated energy of the outgoing Poynting flux at \( r = r_s \) and \( t = 180,000 \) is \( 4.6 \times 10^{46} \) erg s\(^{-1}\). Since the duration of the jet in this study is \( \sim 0.7 \) s, this outgoing Poynting flux seems to be too weak to explain the energy of the jet listed in Tables 3 and 4. Thus, we conclude that the jet is mainly launched by the magnetic field amplified by the gravitational collapse and differential rotation around the black hole, rather than the Blandford–Znajek mechanism in this study.

Finally, we show the contour of the outgoing mass flux \( \sqrt{-g} \rho \mu^0 \) (g cm\(^{-2}\) s\(^{-1}\)) at \( t = 180,000 \) in logarithmic scale in Figure 18. It is seen that the mass outflow is not mainly around the polar region, but in the surrounding region. This is similar to the funnel-wall jet in the previous study (e.g., Figure 7 of Hawley & Krolik 2006). The reason why the mass outflow is not in the polar region is, as stated above, matter suffers from the centrifugal force and cannot be confined within the narrow, Poynting flux jet region. The dark blue region corresponds to the region where \( \mu^0 \) is negative.

### 5. SUMMARY AND DISCUSSION

In order to investigate the formation of relativistic jets at the center of a progenitor of a GRB, we have developed a 2D GRMHD code. In order to confirm the reliability of the code, we have shown that the code passes many well-known test calculations. Although we believe that the GRMHD code presented in this study is sufficiently reliable, further test calculations will be still helpful to confirm the reliability (e.g., Magnetized tori around Kerr black holes (Komissarov 2006), and Blandford Znajek Paraboloidal solution (McKinney 2006b)), which will be shown in the forthcoming paper.

Then, we have performed a numerical simulation of a collapsar using a realistic progenitor model. We have followed the

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### Table 3

| \( \theta \) | 0.714 | 1.43 | 2.14 | 2.86 | 3.57 |
|---|---|---|---|---|---|
| Matter | 1.44E+46 | 7.09E+46 | 1.70E+47 | 3.14E+47 | 5.10E+47 |
| Field | 2.96E+46 | 1.11E+47 | 2.39E+47 | 4.12E+47 | 6.30E+47 |
| \( \theta \) | 4.29 | 5.00 | 5.71 | 6.42 | 7.14 |
| Matter | 7.66E+47 | 1.10E+48 | 1.52E+48 | 2.05E+48 | 2.69E+48 |
| Field | 8.91E+47 | 1.19E+48 | 1.52E+48 | 1.88E+48 | 2.26E+48 |

**Notes.** The energy is written in units of erg. As for the matter component, the contribution of the rest-mass energy is subtracted.

### Table 4

| \( \theta \) | 0.714 | 1.43 | 2.14 | 2.86 | 3.57 |
|---|---|---|---|---|---|
| Matter | 6.89E+43 | 3.15E+44 | 8.96E+44 | 2.03E+45 | 3.95E+45 |
| Field | 8.60E+45 | 3.44E+46 | 7.73E+46 | 1.37E+47 | 2.14E+47 |
| \( \theta \) | 4.29 | 5.00 | 5.71 | 6.42 | 7.14 |
| Matter | 6.76E+45 | 1.04E+46 | 1.47E+46 | 1.96E+46 | 2.50E+46 |
| Field | 3.08E+47 | 4.19E+47 | 5.46E+47 | 6.91E+47 | 8.52E+47 |

**Notes.** Same as Table 3, but integration is done for \( R \leq 40 \).
time evolution of the system for 1.77 s, and it is shown that a jet is launched from the center of the progenitor. The structure of the jet is similar to the previous study; a Poynting flux jet is surrounded by the funnel-wall jet. Even at the final stage of the simulation, the bulk Lorentz factor of the jet is still low, and the total energy of the jet is still as small as \(10^{48}\) erg. However, we found that the energy flux per unit rest-mass flux \(\mathcal{E} = -T^t_t / (p u^t)\) is as high as \(10^2\) at the bottom of the jet. Thus, we conclude that the bulk Lorentz factor of the jet can be potentially high when it propagates outward. Also, as long as the duration of the activity of the central engine is long enough, the total energy of the jet can be large enough to explain the typical explosion energy of a GRB (\(\sim 10^{51}\) erg). It is shown that the outgoing Poynting flux exists at the horizon around the polar region, which proves that the Blandford–Znajek mechanism is really working. However, we conclude that the jet is mainly launched by the magnetic field amplified by the gravitational collapse and differential rotation around the black hole, rather than the Blandford–Znajek mechanism in this study.

As for the efficiency of converting the released gravitational energy to the jet’s energy, it can be estimated as follows: the mass accretion rate is \(\sim 0.05 \dot{M}_\odot\) s\(^{-1}\) (Figure 13), the total energy of the jet at the final stage is \(\sim 10^{48}\) erg (Table 3), and the duration of the jet is \(\sim 0.7\) s (Figure 13). Thus, the efficiency can be estimated as \(\sim 2 \times 10^{-5}\). When we use the outgoing Poynting flux at the horizon (\(4.6 \times 10^{46}\) erg s\(^{-1}\)), the efficiency is as low as \(\sim 6 \times 10^{-7}\). These values seem to be very small compared with the previous study (De Villiers et al. 2005; McKinney & Narayan 2007); one of the reasons is that they used an almost steady disk model. On the other hand, we used a realistic progenitor model that collapses gravitationally whose mass accretion rate is fairly high. Second reason may be that the efficiency becomes higher with time: the mass accretion rate will become smaller, and the jet energy might be larger due to the amplification of the magnetic fields due to winding-up (and MRI) effects. Also, when the initial amplitude of the magnetic field is set to be larger (as in Barkov & Komissarov 2008), the efficiency may be enhanced. Further, we should investigate the dependence of the dynamics on progenitor models as well as the Kerr parameter of the black hole. We are planning to investigate this point systematically in the forthcoming paper.

It is well known that the system is unstable against MRI when there is a strong negative shear profile \((dQ/d ln r)\) (Balbus & Hawley 1991, 1994). The saturation toroidal magnetic field strength is roughly expected to be \(B_\phi \sim (4\pi \rho)^{1/2} r_\Omega\) (Akiyama et al. 2003; Akiyama & Wheeler 2005), which is confirmed by semi-global simulations (Obergaulinger et al. 2009). The saturation poloidal magnetic field strength is roughly an order of magnitude smaller (Obergaulinger et al. 2006). Thus \(B_\phi\) may be amplified by MRI as strong as \(10^{15}\) G at \(r_{\text{ms}}\). The characteristic timescale for saturating the MRI is the Alfvén crossing time: \(t_A \sim 0.1\) ms \(R_\phi \rho_\phi^{1/2} / B_{15}\), where \(R_\phi\) is the radius in units of \(10^6\) cm. Thus, this characteristic timescale can be shorter than the winding-up timescale for strong magnetic fields. However, the length scale of the mode with the largest MRI growth rate is approximately \(\lambda_{\text{MRI}} \sim 50 P_{0.5} B_{10}^{1/2} / \rho_0^{1/2}\) cm, where \(P_{0.5}\) is the rotation period in units of 0.5 ms, which is too short to be resolved numerically. At least, it is not resolved in the beginning of the simulations. After the magnetic field is amplified to a certain value due to gravitational collapse and winding-up effect, MRI may be working in this study (Obergaulinger et al. 2006b; Ott et al. 2006; Burrows et al. 2007; Dessart et al. 2008). It will be necessary to develop a sophisticated code that takes into account the effect of MRI effectively with help of semi-global simulations (Obergaulinger et al. 2009) in order to evaluate the influence of MRI on the dynamics of a collapsar.

It is well known that it becomes difficult to precisely obtain the matter part of the primitive variables \((\rho, u, u^t)\) by the Newton–Raphson method (Noble et al. 2006) due to numerical truncation errors (Komissarov 2002, 2004a, 2004b; McKinney & Gammie 2004; Komissarov 2005; McKinney 2006b) when the electromagnetic part of the stress-energy tensor \(T^\mu_\nu\) greatly exceeds the matter part \(T^\mu_\nu_{\text{Matter}}\). The problem is that the time integration of the electromagnetic part does not become so reliable. This is because the Blandford–Znajek mechanism is really working. However, we conclude that the jet is mainly launched by the magnetic field amplified by the gravitational collapse and differential rotation around the black hole, rather than the Blandford–Znajek mechanism in this study.

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It is very important to evaluate the terminal bulk Lorentz factor, because GRBs are considered to be emissions from relativistic jets with bulk Lorentz factor greater than 10\(^2\) (e.g., Lithwick & Sari 2001). Although an ad hoc thermal (and kinetic) energy deposition into the polar region seems to lead to relativistic jets with bulk Lorentz factors \(\sim 100\) (Aloy et al. 2000, 2002; Zhang et al. 2003, 2004; Mizuta et al. 2006; Mizuta & Aloy 2009; Morsony et al. 2007; Wang et al. 2008), it is still controversial whether such ad hoc energy deposition is justified by numerical simulations with proper neutrino physics (Nagataki et al. 2007). On the other hand, numerical study on the acceleration of electromagnetically powered jet requires quite high resolution (Komissarov et al. 2007; Narayan et al. 2007; Tchekhovskoy et al. 2008; Komissarov et al. 2009). Due to the reason, a simplified jet model with an idealized boundary condition is used at present in order to investigate whether the initial Poynting flux can be effectively converted into kinetic energy (Tchekhovskoy et al. 2008; Komissarov et al. 2009). According to their results, as long as confinement of the jet is realized, acceleration operates over several decades in radius, and considerable fraction of the Poynting flux can be converted into the kinetic energy. Thus, from the high ratio of the Poynting flux relative to rest-mass density seen in our study (Figure 15(b)), a relativistic jet with a high bulk Lorentz factor may be realized at large radius. This will be also investigated in our future study. At present, in our simulation, the head of the jet reaches \(R \approx 5000\) at the final stage of the simulation \((t = 180.000, \text{see Figure 19})\). The length unit of 5000 corresponds to \(1.48 \times 10^9\) cm for the black hole with 2 \(M_\odot\), which is still inside of the progenitor. We would like to follow the propagation of the jet to evaluate its terminal bulk Lorentz factor, although this is impossible at present. One can see from Figure 19 that the numerical resolution at large radii is not very good. This is because the modified Kerr–Schild coordinate is used in our simulation, and much of the radial grids are used for the inner-most region. In the future, we would like to study the propagation of the jet using an adequate numerical grids for the purpose. In such a simulation, we will be able to study whether the star itself explodes as a SN (or hypernova) explosion (note that the blast wave, including the jet, has not reached the surface of the progenitor yet in our simulation).

It is true that the 2D restriction can be a significant limitation. First, anti-dynamo theorem (Moffat 1978) prevents the
Thus, we are planning to develop the more generic MHD turbulence. Hydrodynamic instabilities which produce coherent internal magnetized flows rather than indefinite maintenance of the poloidal magnetic field in the face of dissipation. Second, axisymmetric simulations tend to overemphasize the channel mode (Hawley & Balbus 1992), which produces coherent internal magnetized flows rather than the more generic MHD turbulence. Hydrodynamic instability in the azimuthal direction may also be very important (Nagakura & Yamada 2008, 2009). Thus, we are planning to develop a three-dimensional GRMHD code (De Villiers et al. 2003; Hirose et al. 2004; De Villiers et al. 2005; Hawley & Krolik 2006; Beckwith et al. 2008; Shafee et al. 2008; McKinney & Blandford 2009) and investigate the difference between 2D simulations of collapsars with three-dimensional ones.

In this study, we assumed that the central region of the progenitor has collapsed and a black hole is formed at the center with surrounding envelope unchanged. Thus, we solved the GRMHD equation on a fixed background. But the final goal of our project is to study how a GRB is formed from the gravitational collapse of a massive star. Thus, we are planning to develop a GRMHD code on a dynamical background, which makes the study on the gravitational collapse and black hole formation at the center of a massive star possible (Shibata 2003; Sekiguchi & Shibata 2004, 2005; Baiotti et al. 2005; Duez et al. 2006; Sekiguchi & Shibata 2007).

In this study, photodisintegration of nuclei and neutrino processes are not taken into account. Photodisintegration absorbs considerable amount of thermal energy, and cooling/heating due to neutrino processes may have great influence on the dynamics of a collapsar (Di Matteo et al. 2002; Kohri & Mineshige 2002; Nagataki et al. 2003a; Surman & McLaughlin 2004; Kohri et al. 2005; Lee et al. 2005; Gu et al. 2006; Nagataki et al. 2007; Kawana & Mineshige 2007; Rossi et al. 2008; Zhang & Dai 2009; Cannizzo & Gehrels 2009). As for the photodisintegration of nuclei and neutrino cooling, the thermal pressure should be decreased when such microphysics are taken into account, which should be negative effects for the successful explosion of the progenitor. On the other hand, pair annihilation of electron-type neutrinos may be a key process to drive a GRB jet (Woosley 1993; MacFadyen & Woosley 1999; Asano & Fukuyama 2000, 2001; Miller et al. 2003; Surman & McLaughlin 2005; Kneller et al. 2006; Shibata et al. 2007; Birk et al. 2007). Thus, we are planning to include these microphysics in our code, and perform more realistic simulations of collapsars.

The SNe associated with GRBs often show peculiar properties. Some are very energetic and blighted (Galama et al. 1998; Iwamoto et al. 1998; Hjorth et al. 2003; Malesani et al. 2004), but others prohibit such blighted SNe from being accompanied (Fynbo et al. 2006; Della Valle et al. 2006; Gal-Yam et al. 2006). Since the brightness of SNe depends on the mass of produced 56Ni (Woosley et al. 1999; Nakamura et al. 2001), it is suggested that there is a huge variety of the amount of 56Ni in a SN that associates with a GRB. It is still controversial where and when 56Ni is produced in a SN accompanied by a GRB (Nagataki 2000; Nagataki et al. 2003b, 2006). It may be produced in a GRB jet (Maeda et al. 2002; Maeda & Nomoto 2003; Tanaka et al. 2007; Maeda et al. 2008; Tominaga 2009; Maeda & Tominaga 2009; Bucciantini et al. 2009), or it may be produced in (or outflow from) the accretion disk around the black hole (MacFadyen & Woosley 1999; Pruet et al. 2004; Fujimoto et al. 2004; Surman et al. 2006; Hu & Peng 2008), or it may be synthesized around a proto-neutron star (Uzdensky & MacFadyen 2007). At present, it is impossible to investigate the explosive nucleosynthesis in a collapsar in our code because nuclear reactions are not taken into account. We are planning to include this effect, and study the site of 56Ni production. Also, study of a GRB as a possible site where very heavy elements and light elements are synthesized is very important (Lemoine 2002; Beloborodov 2003; Suzuki & Nagataki 2005).

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Figure 19. Same as Figure 11(b), but the drawn region is 10000 × 10000. The length unit of 10000 corresponds to 2.95 × 10^7 cm for the black hole with 2 M_☉.

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