DOUBLE VOGAN SUPERDIAGRAMS OF LIE SUPERALGEBRAS AND SYMMETRIC SUPERSPACES

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ABSTRACT. A Vogan superdiagram is a set of involution and painting on a Dynkin diagram. It selects a real form, or equivalently an involution, from a complex simple Lie superalgebra. We introduce the double Vogan superdiagram, which is two sets of Vogan superdiagrams superimposed on an affine Dynkin diagram. They correspond to pairs of commuting involutions on complex simple Lie superalgebras, and therefore provide an independent classification of the simple locally symmetric or semisymmetric superpairs.

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1. Introduction

This article is supersymmetric conversion of the double Vogan diagrams of semisimple Lie algebras and symmetric spaces [2]. Homogeneous supermanifolds $G/\theta^0$, where $G$ is a real Lie superalgebra, $\theta$ is an automorphism of $G$ and involutive automorphism of $G_0$, can be viewed as analogs of symmetric spaces; called symmetric superspaces. Supermanifolds of this type carry additional interesting structures; are called semisymmetric superspaces. Since conjugate automorphisms create isomorphic superspaces. Supermanifolds (are special cases of noncommutative manifolds), are manifold with both bosonic and fermionic coordinates. Several string backgrounds which arise in the ADS (anti-de-sitter space)/CFT(conformal field theory) duality correspondence are described by integrable sigma-models with semisymmetric superspaces as target spaces. The inadequacy of the perturbation and gravity dual theory to solve the problem in analyzing the coupling constant from the Yang-Mill theory force to consider superstring in the appropriate $AdS_d \times S^b$ backgrounds [8].

Many common algebraic properties of different $AdS_d \times S^d$ backgrounds are quite interest in physics. The list of all semi-symmetric cosets with zero beta functions and central charge $\leq 26$ is focussed in [3]. A semi-symmetric superspaces is a supercoset
of a supergroup which possesses $\mathbb{Z}_4$ symmetry, results in a generalizations of $\mathbb{Z}_2$ invariant semisymmetric or symmetric superspace. The Anti-de-Sitter space and superspace backgrounds of $\mathbb{Z}_4$ coset for the worldsheet sigma-models is the Green-Schwarz string action on $AdS_5 \times S^5$ is $PSU(2,2|4)$ with $Sp(2,2) \times Sp(4)$ the invariant subgroup fixed by the $\mathbb{Z}_4$ automorphism. Quantization of strings in the $AdS_5 \times S^5$ background requires a sigma model based on a quotient supermanifold $PSU(1,1|2)/U(1) \times U(1)$. One of the ingredient of semisymmetric superspaces is Cartan domain which has analogs to phase space of a mechanical systems. In a different perspective and application point of view on supersymmetric conversion gives a class of deformed measures on the superdomain analogs to Euclidean field theory and statistical mechanics. Classification of symmetric superspaces means classification of string in the supersymmetric backgrounds. The polar purpose of this text is to use extended Dynkin diagrams of Lie superalgebras to produce associate degree combinatorial classification of the locally semisymmetric superpairs or equivalently the commuting involutions on complex Lie superalgebras. We shall call them double Vogan Superdiagrams. Suppose $G$ be a simple Lie superalgebra over a complex field with an automorphism $\theta$, that gives us $\theta^2 = \sigma_{\pm 1}$ ($\sigma$ a diagram involution; $\sigma_{\pm 1}$ is identical on the even part and act as the multiplication by $-1$ on the odd part) satisfies $\theta^2|G_0 = id$. From the preceding fact it is clear that $\sigma$ commutes with $\theta$ and it preserves the Cartan decomposition $G_0 = k_0 + p_0$.

In the following sections we extend the definitions of Double Vogan diagrams \([2]\) to Double Vogan superdiagrams. The details of Vogan diagrams and superdiagrams of Lie superalgebras in \([2, 5]\).

**Definition 1.1.** A double Vogan superdiagram is an Vogan superdiagram on $D$ (where $D$ is the Dynkin diagram ) with a diagram involution, such that the vertices fixed by the involution are uncircled or circled . We say that a double Vogan diagram represents a locally semisymmetric superpair $(G, G^\sigma)$ If

(a) the underlying affine Vogan superdiagram represents $G^\sigma$
(b) the Vogan superdiagram on the white vertices represents $\sigma \in \text{inv}(k_0)$
(c) the diagram involution and circling on the black vertices (black painted by fixed point due to Cartan involution $\theta$) represent $\sigma$ on $p_0^c$.

We say a diagram represents a Lie superalgebra involution $\sigma$ if there are canonical root vectors $X_\alpha$ such that $\sigma X_\alpha = X_{\sigma \alpha}$ if $\alpha \neq \sigma \alpha$ and $\sigma X_\alpha = \pm X_\alpha$ if $\alpha = \sigma \alpha$

**Definition 1.2.** An almost double Vogan superdiagram is an Vogan superdiagram with a diagram involution, such that the vertices fixed by the involution are uncircled or circled.

**Remark 1.3.** An almost double Vogan superdiagram is a weaker than a double Vogan superdiagram only because $\sum a_{\alpha}$ may not be even.

**Theorem 1.4 (\([1]\)).** Let $G_\mathbb{R}$ be a real Lie superalgebra such that $G$ is a Lie superalgebra of the series $W, J$ and $H$, $\sigma$ be an involutive automorphism of $G$ and the pair $(G, \sigma)$ be Hermitian, i.e., a Hermitian symmetric superspace corresponds to it. Then $G \cong H(n)$ or $H^+(n)$. 
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Proposition 1.5. Let $\sigma \in \text{inv}(\mathfrak{t}_0)$ on $D_l$. If $\sigma$ extend to an involution on $\mathcal{G}^c$, then the Vogan diagram on $D_l$ extends to an almost double super Vogan diagram. Furthermore if $\mathcal{G}$ is of Hermitian type then

(a) the diagram involution preserves the black vertices implies $\sigma = 1$ on $\mathfrak{g}_0$ and $\sigma = 1$ on $\mathfrak{g}_1^c$

(b) the diagram involution interchanges the black vertices implies $\sigma = 1$ on $\mathfrak{g}_0$ and $\sigma = -1$ on $\mathfrak{g}_1^c$

Proof. Let $\sigma \in \text{inv}(\mathfrak{t}_0)$ extend to an involution on $\mathcal{G}$. If $\sigma$ extend to $\text{aut}_{2,4}\mathcal{G}$, then $\sigma$ permutes the extreme weight spaces of $\mathcal{G}_1$. But since $\sigma|\mathcal{G}_0$ is represented by $(c, d)$ where $c$ coloring of vertex and $d$ is a diagram involution, on $\mathcal{G}_0$, it permutes the simple root spaces of $\mathcal{G}_0$. Hence $\sigma$ permutes the lowest weight of $\mathcal{G}_1$ and $d$ extends to $\text{inv}\mathcal{G}$.

\[ \gamma(Z_0) = B(Z_0, [X_\gamma, X_{-\gamma}]) = B(\sigma Z_0, [\sigma X_\gamma, \sigma X_{-\gamma}]) = \gamma(\sigma Z_0) \]

\[ \gamma(Z_1) = B(Z_1, [X_\gamma, X_{-\gamma}]) = B(\sigma Z_1, [\sigma X_\gamma, \sigma X_{-\gamma}]) = B(\sigma Z_1, [X_\gamma, X_{-\gamma}]) = -\gamma(\sigma Z_1) \]

For part (b), suppose that the diagram involution interchanges two black vertices $\delta$ and $\gamma$.

\[ \delta(Z_0) = B(Z_0, [X_\delta, X_{-\delta}]) = B(\sigma Z_0, [\sigma X_\delta, \sigma X_{-\delta}]) = B(\sigma Z_0, [X_\delta, X_{-\delta}]) = \gamma(\sigma Z_0) \]

\[ \delta(Z_1) = B(Z_1, [X_\delta, X_{-\delta}]) = B(\sigma Z_1, [\sigma X_\delta, \sigma X_{-\delta}]) = B(\sigma Z_1, [X_\delta, X_{-\delta}]) = \gamma(\sigma Z_1) \]

\[ \gamma(\sigma Z_0) + \gamma(\sigma Z_0) = \gamma(-Z_0) + \gamma(\sigma Z_0) = 0 \quad \text{and} \quad \gamma(\sigma Z_1) + \gamma(\sigma Z_1) = \gamma(-Z_1) + \gamma(\sigma Z_1) = 0 \]

We Know $\sum a_\alpha = 0$ that proved the proposition. \hfill $\square$

Theorem 1.6. \[9\] Given $\theta \in \text{aut}_{2,4}(\mathcal{G})$, there exists a unique real form $\mathcal{G}_R$ such that $\theta$ restricts to a Cartan automorphism on $\mathcal{G}$. Conversely every real form $\mathcal{G}_R$ has a unique Cartan automorphism $\theta \in \text{aut}_{2,4}(\mathcal{G})$.

The vertices $\alpha$ are equipped with canonical positive integer called labelling of vertices $a_\alpha$ without the nontrivial common factor such that $\sum a_\alpha \alpha = 0$.

Theorem 1.7 (Main Theorem). Every double Vogan diagram represents a locally symmetric superpair $(\mathcal{G}, \mathcal{G}^c)$ unique up to conjugation. Conversely, every locally symmetric pair is represented by a double Vogan superdiagram.

The double Vogan superdiagrams leads the way to explain of $\mathfrak{t}_0^c$ involutions to $\mathcal{G}^c$ involutions.

2. Preliminaries on Basic Lie Superalgebras

In current section, we recall some background on Lie superalgebras and set up the notation for subsequent sections. We will modify the Borel and de Siebenthal Theorem for Lie superalgebra.

Theorem 2.1. Let $\mathcal{G}$ be a non complex real Lie superalgebra and Let the Vogan diagram of $\mathcal{G}$ be given that corresponding to the triple $(\mathcal{G}, \mathfrak{h}_0, \Delta^+)$. Then $\exists$ a simple system $\prod'$ for $\Delta = \Delta(\mathcal{G}^c, \mathfrak{h})$, with corresponding positive system $\Delta^+$, such that $(\mathcal{G}, \mathfrak{h}_0, \Delta^+)$ is a triple and there is at most two painted simple root in its Vogan diagrams of $\text{sl}(m,n), D(m,n)$ and at most three painted vertices in $D(2,1;\alpha)$. Furthermore suppose the automorphism associated with the Vogan diagram is the identity, that $\prod' = \alpha_1, \cdots, \alpha_l$ and that
\( \omega_1, \cdots, \omega_l \) is the dual basis for each even part such that \( \langle \omega_j, \alpha_k \rangle = \delta_{jk}/\epsilon_{kk} \), where \( \epsilon_{kk} \) is the diagonal entries to make cartan matrix symmetric. The the each painted simple root of even parts may be chosen so that there is no \( i^{'} \) with \( \langle \omega_i - \omega_{i^{'}}, \omega_{i^{''}} \rangle > 0 \) for each even part.

**Proof.** We have proved the theorem in [5].

**Theorem 2.2.** [5] If an abstract Vogan diagram is given, then there exist a real Lie superalgebra \( G \), a Cartan involution \( \theta \), a maximally compact \( \theta \) stable Cartan subalgebra and a positive system \( \Delta^+_{\theta} \) for \( \Delta = \Delta(\mathfrak{g}, \mathfrak{h}) \) that takes \( i_{\mathfrak{n}} \) before \( \sigma_{\mathfrak{n}} \) such that the given diagram is the Vogan diagram of \( (G, \mathfrak{h}_{\mathfrak{n}}, \Delta^+_{\theta}) \).

**Corollary 2.3.** Let \( \sigma \) be an involution on a complex Lie superalgebra \( G \). Then \( \sigma \) can be represented by a Vogan superdiagram with at most two black vertex \( \alpha \), where \( a_\alpha = 1 \) or \( a_\alpha = 2 \).

**Proof.** Suppose \( \sigma \) is an involution on a complex contragradient Lie superalgebras. We always find a real form \( G_R \) of \( G \) such that \( \sigma \) restricts to a Cartan Involution on \( G \). It commutes with \( \theta \). By choosing a \( \sigma \) stable maximally compact Cartan subalgebra \( \mathfrak{h} \). Let \( G_0 = \mathfrak{t}_0 + \mathfrak{p}_0 \) be the Cartan decomposition arising from \( \sigma \). The above theorems 2.1, 2.2 say the Vogan superdiagram can be chosen to have at most two painted vertex \( \alpha \), where \( a_\alpha = 1 \) or \( a_\alpha = 2 \). This complete the proof of corollary.

**Proposition 2.4.** We obtain a Lie superalgebra superinvolution \( \sigma_{\mathfrak{t}_0} \oplus \sigma_{\mathfrak{p}_0} \oplus \sigma_1 \in \text{aut}_{2,4}(G) \), where \( \sigma = \sigma_0 + \sigma_1 \) such that \( r \sum a_\alpha = \text{even} \) for \( \text{sl}(m,n), \text{osp}(2m+1,2n) \), except \( D(2,1;\alpha) \) and \( F(4) \).

**Proof.** \( \mathfrak{t}_{0ss} = [\mathfrak{g}_0, \mathfrak{g}_0] \mathfrak{g} = \mathfrak{t}_{0ss} + \zeta_0 \), for \( \mathfrak{g} = \mathfrak{sp}(n) \) or \( \mathfrak{psl}(n|n) \), \( \sigma(\mathfrak{t}_{0ss}) = \mathfrak{t}_{0ss} \)

\[
\sigma([\mathfrak{g}_0, \mathfrak{g}_0]) = [\sigma(\mathfrak{g}_0), \sigma(\mathfrak{g}_0)] \\
\sigma([\mathfrak{g}_0, \mathfrak{g}_1]) = [\mathfrak{g}_0, \sigma(\mathfrak{g}_1)] \\
\Rightarrow \sigma(\mathfrak{t}_{0ss}) = \mathfrak{t}_{0ss}
\]

For \( X, Y \in \mathfrak{p}_0 \) and \( Z \in \mathfrak{t}_0 \)

\[
\sigma_{\mathfrak{t}_0}[X, Y] = [\sigma_{\mathfrak{p}_0}X, \sigma_{\mathfrak{p}_0}Y] \\
f[X, Y] = [\sigma_{\mathfrak{p}_0}X, \sigma_{\mathfrak{p}_0}Y]
\]

\[
\sigma_{\mathfrak{t}_0}[Z, [X, Y]] = [\sigma_{\mathfrak{t}_0}Z, \sigma_{\mathfrak{t}_0}[X, Y]] \\
= [\sigma_{\mathfrak{t}_0}Z, [\sigma_{\mathfrak{p}_0}X, \sigma_{\mathfrak{p}_0}Y]] \\
= [[\sigma_{\mathfrak{t}_0}Z, \sigma_{\mathfrak{p}_0}X], \sigma_{\mathfrak{p}_0}Y] + [\sigma_{\mathfrak{p}_0}X, [\sigma_{\mathfrak{t}_0}Z, \sigma_{\mathfrak{p}_0}Y]] \\
= [\sigma_{\mathfrak{p}_0}[Z, X], \sigma_{\mathfrak{p}_0}Y] + [\sigma_{\mathfrak{p}_0}X, \sigma_{\mathfrak{p}_0}[Z, Y]] \\
= f[[Z, X], Y] + f[X, [Z, Y]] \\
= f[Z, [X, Y]]
\]

For \( X, Y \in \mathfrak{p}_0 \) and \( Z \in \mathfrak{g}_1 \) From Lemma 10.1.3 we get \( \sigma_1^2 = \pm \text{id} \) for \( \mathfrak{g}_1 \) be an irreducible \( \mathfrak{g}_0 \) module \( \sigma \in \text{Aut}(\mathfrak{g}) \) and \( \sigma_0^2 = \text{id} \).
\[ \sigma_1[Z, [X, Y]] = [\sigma_1 Z, \sigma_{\bar{Y}} [X, Y]] \\
= [\sigma_1 Z, [\sigma_{\bar{Y}} X, \sigma_{\bar{Y}} Y]] \\
= [[\sigma_1 Z, \sigma_{\bar{Y}} X], \sigma_{\bar{Y}} Y] + [\sigma_{\bar{Y}} X, \sigma_1 Z, \sigma_{\bar{Y}} Y] \\
= [\sigma_{\bar{Y}} [Z, X], \sigma_{\bar{Y}} Y] + [\sigma_{\bar{Y}} X, \sigma_{\bar{Y}} [Z, Y]] \\
= f[[Z, X], Y] + f[X, [Z, Y]] \\
= f[Z, [X, Y]] \]

The vertices \( \alpha \) are equipped with canonical positive integers \( a_\alpha \) without nontrivial common factor such that \( \sum a_\alpha \alpha = 0 \). By expressing \( \beta \) with all white and grey vertices we obtain \( c_\gamma = 0 \) and

\[ \beta = \sum_O c_\alpha \alpha \]

Assuming \( g \) of non-Hermitian type, its double Vogan superdiagram has two black vertex \( \{ \delta, \gamma \} \), each black vertex for each even part with

\[ \beta = \sum 2\gamma + 2\delta + \sum_{D_{\bar{t}_0}} a_\alpha \]

Since \( \sigma \beta = \beta \), so we have \( b_\alpha = b_{\sigma \alpha} \). By taking \( b_\gamma = 2 \) and \( b_\delta = 2 \), we can rewrite the \( \beta = \sum_O b_\alpha \alpha \).

\[ \sum 2\gamma + 2\delta + \sum_{D_{\bar{t}_0}} a_\alpha \alpha - \sum_O b_\alpha \alpha = 0 \]

\[ \Rightarrow a_\gamma + a_\delta = 4 \]

\[ fX = (-1)^{\deg(X)} bX \]

\[ \sigma_{t_0} X = f(-1)^{\deg X} X \]

For \( g \) of Hermitian type

\[ \sum \gamma + \delta + \sum_{D_{\bar{t}_0}} a_\alpha - \sum_O b_\alpha = 0 \]

\[ \Rightarrow \sum a_\gamma + a_\delta = \text{even} \]

**Proof of Main Theorem 1.7.** The proof of the main theorem follows from [2, 9]. From Theorem 2.1 and Proposition 2.4, it follows that every double Vogan superdiagram represents a locally semisymmetric super pair. From a given semisupersymmetric pair \((G, G^\sigma)\), we obtain an affine Vogan superdiagram. There exists a Cartan involution \( \theta \) of \( G \). Let \( \mathfrak{s}(N, F) \) denote the skew symmetric and \( \mathfrak{s}(N, F) \) be symmetric \( N \times N \) matrices over \( F \). By Choosing a \( \theta \) stable real form \( \mathfrak{g}_0 \) (\( \theta \) restricts to a Cartan involution on \( \mathfrak{g}_0 \)). We get \( \mathfrak{t}_0 \subset \mathfrak{s}(2n, \mathbb{C}) \) and \( \mathfrak{p}_0 \subset \mathfrak{s}(2n, \mathbb{C}) \). The adjoint \( \mathfrak{t}_0 \) on \( \mathfrak{s}(2n, \mathbb{C}) \) preserves the real form \( \mathfrak{s}(2n, \mathbb{R}) \). Let the Cartan decomposition of \( \mathfrak{g}_0 = \mathfrak{t}_0 + \mathfrak{p}_0 \). \( \mathfrak{t}_0 = \mathfrak{t}_{0ss} + \zeta_0 \). \( \sigma \) preserves the semisimple part. By proposition 2.4 we get \( r \sum a_\alpha = \text{even} \). This proved the Theorem.

Extended Dynkin diagram of \( A(m, n) \): \( A^{(1)}(m, n) \)
Extended Dynkin diagram of $B(m, n)$: $B^{(1)}(m, n)$

Extended Dynkin diagram of $B(0, n)$: $B^{(1)}(0, n)$

Extended Dynkin diagram of $D(m, n)$: $D^{(1)}(m, n)$

Extended Dynkin diagram of $D(2, 1; \alpha)$: $D^{(1)}(2, 1; \alpha)$

Extended Dynkin diagram of $C(n)$: $C^{(1)}n$
Extended Dynkin diagram of $F(4)$: $F^{(1)}(4)$

\[ \circ \leftrightarrow \otimes \rightarrow \circ \leftrightarrow \circ \]

Extended Dynkin diagram of $G(3)$: $G^{(1)}(3)$

\[ \circ \leftrightarrow \otimes \rightarrow \circ \leftrightarrow \circ \]

3. Double Vogan diagram and classification of symmetric superspaces

We shall reproduce the list of all the irreducible Hermitian symmetric superspaces with base equal to a product of classical domains and with a simple group of motion as in [1].

(i) $SU(m|n)/(P)S(U(m - r|n - s)) \times U(r|s)$
(ii) $PSUQ(m)/PS(UQ(r) \times UQ(m - r))$
(iii) $SU\Pi(m)/S(U\Pi(r) \times U\Pi(m - r))$
(iv) $PQ(m)/P_{\text{Sim}}GL(r|m - r)$
(v) $Osp_r(m|2n)/O_r(2) \times Osp_r(m - 2|2n)$
(vi) $Osp_r(2m|2n)/U(m|n)$
(vii) $Osp^*(2m|2n)/U(m|n)$

The one-to-one corresponding of the above classical domain of symmetric superspaces and the real forms of Lie superalgebras produce below the double Vogan superdiagram and semisymmetric superspaces.

Every real simple Lie superalgebra is either a realified complex simple Lie superalgebra $G^\sigma$, such that $G^\sigma \otimes R \subset G$. Every real form of a Lie superalgebra $G$ equals $G^\theta$, where $\theta$ is an involutive, antilinear automorphism of $G$. In semisimple Lie algebras each irreducible hermitian symmetric space of noncompact type is equivalent to a Cartan domain is a sympletic manifold. The natural $Z_2$ graded or super generalizations of Cartan domains which arises in Hermitian symmetric spaces are called Cartan superdomains. The invariant metric on $G/G^\theta$ is obtained from the invariant metric on $G$ whenever the latter exists. In the category of symmetric superspaces we have only a weak analog of the notion of compactness. We shall call the symmetric superspaces with compact base compact.

Furthermore a symmetric superspaces will be said to be Hermitian if it is associated with a pair $(G, \theta)$ such that on $G/G^\theta$ there is a $G^\theta$ invariant complex structure. Such a symmetric superbase with Hermitian base is Hermitian. Conjugate automorphisms yields isomorphic superspaces.

Double Vogan diagram and semisymmetric superspaces of $A(m, n)$
\[(p)sl_r(m|n)/sl_r(p) \oplus sl_r(m - p) \oplus u(1) \oplus \mathbb{R}\]

\[su^*(2m|2n)/sl(n) \oplus u^*(2m - 2p) \oplus u^*(2p) \oplus u(1) \oplus \mathbb{R}\]

\[su^*(2m|2n)/o^*(2m) \oplus o^*(2n)\]

\[su^*(2m|2n)/sp(2m, 2p) \oplus sp(2n, 2q)\]

\[(p)su(m, p|n, q)/su(p, m - p) \oplus su(r, n - r) \oplus i\mathbb{R}\]

\[(p)su(m, p|n, q)/osp(m, p|n)\]
Double Vogan diagram of and semisymmetric superspaces $B(m, n)$

The below first Vogan diagram which contains the extreme right black painted root is from the original Dynkin diagram color.

Double Vogan diagram of $D(m, n)$

$osp(m, p|2n)/(sp(m, \mathbb{R}))$

$osp^*(2m|2n)/(sp(r, s) \oplus so^*(2p))$
Double Vogan diagram of and semisymmetric superspaces $D(2, 1; \alpha)$

\[ D(\alpha)/ (\text{su}(2) \oplus \text{su}(2) \oplus \mathfrak{sl}(2, \mathbb{R})) \]

\[ D(\alpha)/ (\mathfrak{sl}(2, \mathbb{R}) \oplus \mathfrak{sl}(2, \mathbb{R}) \oplus \mathfrak{sl}(2, \mathbb{R})) \]

Double Vogan diagram of and semisymmetric superspaces $C(n)$

\[ osp^{*}(2m|2n)/(\mathfrak{sp}(m, \mathbb{R}) \oplus \mathfrak{so}(p, q)) \]

\[ osp(2|2n)/(\mathfrak{sp}(n, \mathbb{R}) \oplus \mathfrak{so}(2)) \]

\[ osp(2|2n)/(\mathfrak{sp}(r, s) \oplus \mathfrak{so}(2)) \]

Double Vogan diagram of $F(4)$

\[ F(4)/(\text{su}(2, \mathbb{R}) \oplus \mathfrak{so}(1, 6)) \]

\[ F(4)/(\text{su}(2, \mathbb{R}) \oplus \mathfrak{so}(2, 5)) \]
DOUBLE VOGAN SUPERDIAGRAMS OF LIE SUPERALGEBRAS AND SYMMETRIC SUPERSPACES

\[ F(4)/(\mathfrak{sl}(2, \mathbb{R}) \oplus \mathfrak{so}(3, 4)) \]
\[ F(4)/(\mathfrak{sl}(2, \mathbb{R}) \oplus \mathfrak{so}(7)) \]
Double Vogan diagram of and semisymmetric superspaces \( G(3) \)

\[ G(3)/(\mathfrak{sl}(2, \mathbb{R}) \oplus g_c) \]
\[ G(3)/(\mathfrak{sl}(2, \mathbb{R}) \oplus g_s) \]

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