There is no “first” quantization

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Abstract The introduction of spinor and other massive fields by “quantizing” particles (corpuscles) is conceptually misleading. Only spatial fields must be postulated to form the fundamental objects to be quantized (that is, to define a formal basis for all quantum states), while apparent “particles” are a mere consequence of decoherence. This conclusion is also supported by the nature of gauge fields.

1 Introduction

Decoherence theory [1] now allows us, in an appropriate sense, to derive classical concepts in terms of universal quantum theory. These classical concepts include clicks of a detector or spots on a plate, phenomena which represent outcomes of measurements, and which seem to indicate the presence of particles. Particle concepts are therefore usually presumed in a “quantization” procedure (which leads to the N-particle wave functions of quantum mechanics), and for an interpretation in terms of probabilities for the “occurrence of values” for particle properties (regarded as “observables”).

Relativistic theories, on the other hand, require quantum field theory, where single particle wave functions, together with classical fields, are used as arguments of field functionals, that now represent the general (pure) quantum states. The “occupation number” representation, resulting for free fields (coupled oscillators), then explains boson numbers by the numbers of nodes
in the wave functions for the amplitudes of all field modes. This definition of “particle” number may be extended beyond the harmonic approximation. Particle permutations are thereby reduced to identity operations – a consequence that is much deeper than a mere indistinguishability. In particular, it would explain the “new statistics” required for presumed quantum particles.

These arguments suggest to abandon a primordial particle concept entirely, and to replace it with fields only. While this is indeed what has always been done in the formalism of quantum field theory, particle concepts are still used in a fundamental way for its interpretation, for example when applied to scattering events. In a universal quantum field theory, spatial fields (rather than particle positions) do not only form the fundamental “configuration” space on which the wave function(al) is defined as a general superposition. Time-dependent quantum states may also describe apparently discontinuous “events” by means of a smooth but rapid process of decoherence.

So I agree with a consequence recently drawn by Ulfbeck and Bohr [2] that “no event takes place in the source itself as a precursor to the click in the counter”, while I disagree with their interpretation that the wave function “loses its significance” as soon as an event occurs “completely beyond law” in the counter. On the contrary, this event can be dynamically described in terms of a unitarily evolving (hence strongly entangled) universal wave function.

This conclusion must also affect the interpretation of the Wigner function, which is presently in vogue in non-relativistic quantum mechanics because of its (misleading) formal analogy to a classical phase space distribution.

2 General quantum systems

In order to set the stage for this comment, let me first define what I mean by a general (abstract) quantum system. This is the conceptual framework that remains when all specific aspects, such as those resulting from “quantizing” a certain classical system, are eliminated.

The kinematics of an abstract quantum system is defined by means of a “basis” of linearly independent states $|i\rangle$, with $i = 1, 2, \ldots, D$, subject to the superposition principle, which allows every state of the system to be written in the form

$$|\alpha\rangle = \sum c_i |i\rangle .$$  \hspace{1cm} (1)
This kinematical principle requires furthermore that any such (normalizable) superposition represents a possible physical state. For a certain system, the dimension $D$ may be finite, infinite, or the states $|i\rangle$ may even form a non-countable set. Although one can never strictly decide empirically whether $D$ is infinite or just very large, this difference is essential for the mathematical formulation of explicit models.

The dynamics of a quantum system by itself is assumed to be described by a Schrödinger equation,

$$i\frac{\partial}{\partial t} |\alpha\rangle = H |\alpha\rangle,$$

characterized by a hermitean matrix $h_{mn}$ in the basis (1). In quantum theories which contain gravity, the dynamics may degenerate to a static Wheeler-DeWitt equation, $H |\alpha\rangle = 0$.

On the one hand, this is very little, since there is no interpretation of these abstract states yet. On the other one, it is quite a bit, since the superposition principle is known to be very powerful, while the Hamiltonian may describe an enormous dynamical structure (dynamical locality, for example).

Fortunately, in general we have more.

### 3 Interpretation through measurements

Measurements are interactions of the system with an appropriate device. We know that there are specific system states $|x\rangle$, say, which cause the “pointer” of a certain device to move into a position that depends on the state $|x\rangle$. For general states $|\alpha\rangle$, this happens with Born probability $|\langle x|\alpha\rangle|^2$. For this purpose, an inner product has to be added to the kinematics defined in Sect. 2. Since different pointer positions exclude each other, we have to require $\langle x|x'\rangle = 0$ for $x \neq x'$.

There are various ways to describe such measurements.

(a) Traditional (Bohr): The pointer is described in classical terms. Its “position” may be the actual position of a spot on the photographic plate, or, for a different device, the impulse on a macroscopic (Brownian) particle that we can observe under the microscope. In the first case, we usually presume (more or less tacitly) that the measurement interaction is local, such that the “quantum object” must have been at this position, too. Similarly, in the second case, we presume momentum conservation in order to conclude that the quantum object must have changed its momentum correspondingly, or must have lost it in the case of absorption. (Bohr emphasized
that conservation laws are essential for the Copenhagen interpretation.) So one concludes that the “quantum object” exhibits properties of a particle (position or momentum) when being measured, even though we are forced to conclude that it cannot possess both properties at the same time or when not being observed. However, position and momentum may be used to define two different bases for the corresponding quantum states, with coefficients $c_i$ becoming wave functions $\psi(r)$ in the position representation, where (1) assumes the form $|\alpha\rangle = \int d^3r \, \psi(r)|r\rangle$.

The particle concept had proven useful earlier – though not with perfect results – in statistical mechanics (for molecules) and in Bohr-Sommerfeld quantum mechanics (for atomic electrons). By means of formal considerations (based on the Hamiltonian form of mechanics) this historical root led to a general “quantization” procedure, applicable to classical dynamical systems. These quantization rules and their consequences form the subject of this comment. For a dynamical system that can be brought into Hamiltonian form, any configuration space defines a basis for all quantum states, while canonical momenta form another one (usually related to the former by a Fourier transform – only at this point an operator algebra based on classical Poisson brackets becomes relevant). In this way one obtains wave functions on configuration space, and, in particular, the well established non-local many-particle wave functions $\psi(r_1, \ldots, r_N, t)$. Note, however, that the concepts of spin and permutation symmetry were added for empirically reasons.

(b) Quantum pointers (von Neumann [3]): Because of the generality of quantization rules, it appeared natural to describe the pointer position “x” by a quantum state, $|P_x\rangle$, too. If understood as a narrow wave packet of a massive pointer, it may approximately define both position and momentum (in accordance with the uncertainty relations or Fourier theorem). For the specific states $|x\rangle$, a measurement can then be written as a unitary evolution in the tensor product space,

$$|x\rangle |P_0\rangle \rightarrow |x\rangle |P_x\rangle \quad .$$

(3)

Apparently, Bohr was never ready to accept this extension of the application of quantum theory to macroscopic objects, even though he applied the uncertainty relations to them. Equation (3) defines an effective interaction Hamiltonian between system and pointer (neglecting all details), but inevitably leads into the well-known measurement problem, which seems to require either the existence of macroscopic superpositions (Schrödinger cats) or a “second dynamics” (the collapse of the wave function).
(c) Universal quantum theory (Everett [4]): In the next “natural step”, quantum theory was not only applied to the system and its measurement device, but also to their environment (the rest of the universe). Quantitative dynamical considerations then require that all systems in the universe are strongly entangled [5]. Subsystems can possess quantum states by themselves only “relative” to states of the rest, in particular relative to states of measurement devices or observers. The relation to observers is not merely formal: it implies a radically novel definition of “separate observers” in terms of the wave function – required as a consequence of entanglement. These relative states are factor states in dynamically autonomous “branches” of the global wave function. Restricting consideration to subsystems of the universe (as is realistic for local interactions affecting local observers) leads to the concept of decoherence and the formation of effective ensembles of subsystem states [1]. Phase relations defining (macroscopic) Schrödinger cats are almost immediately dislocalized, and thus become irrelevant to local observers. Since the universe is closed, there are no external measurement devices to be used for an operational interpretation of global (Everett) quantum states, and all observable properties must now in principle be derived from the invariant structure of the universal Hamiltonian.

4 Quantization

The “canonical quantization rules” require that classical configuration variables $q$ define a basis of quantum states, $|q\rangle$, for the “corresponding” Hilbert space. Similarly, the quantum Hamiltonian is obtained from the classical Hamiltonian $H(p, q)$ by replacing the canonical variables $p$ and $q$ with operators $P = \int dp |p\rangle p\langle p|$ and $Q = \int dq |q\rangle q\langle q|$, respectively, (a procedure that cannot be unique because of the factor ordering problem). We will here essentially be concerned with the construction of the basis only, which is then often used for a fundamental probability interpretation, while the operators are defined to act on the quantum states spanned by the basis in the form (1).

We now understand in principle how classical properties emerge from the quantum system by means of decoherence (for example as narrow Gauss packets in the canonical basis $|q\rangle$): their superpositions would immediately decohere. However, since decoherence depends on the environment, the question arises whether the basis obtained by quantizing a classical theory is always a fundamental one for the quantum system of interest. If effec-
tive classical variables, \( q(t) \), are known for a certain system, we have to conclude according to the superposition principle that all their superpositions \( \int dq \psi(q, t) |q\rangle \) must in principle exist as physical states, but this formal “quantization” procedure, based on classical concepts, does often not lead to a fundamental basis (understood in a hierarchical sense if required). Let me give three examples:

(1) The rigid rotator is classically described by means of the Euler angles \( \phi, \theta, \chi \), say. Their symplectic structure defines the geometry of this configuration space. Canonical quantization then leads to wave functions \( D(\phi, \theta, \chi) \). They represent an effective approximation for certain states of a many-body system (forming a rotational band), which have more fundamentally to be described by a wave function \( \psi(r_1, \ldots, r_N) \). The stability of a rigid body, required for this approximation, is itself based on quantum properties of the many-body system. In general, no eigenstates for the Euler angles exist in terms of the many-body states that form the rotational band [6], since the approximation of a rigid body breaks down at high values of angular momentum.

(2) \( N \)-particle systems would upon quantization lead to wave functions depending on \( 3N \) position variables not restricted by permutation symmetries. However, states of zero-mass bosons, for example, can also be derived by quantizing a lattice or a continuum of coupled oscillators (a “field”). Its quantization leads to equidistant energy levels (oscillator quanta), which can be interpreted as boson numbers. It is for this reason that field amplitudes appear as boson creation and annihilation operators. Since photon number eigenstates are not robust against decoherence,\(^1\) their observed classical states are indeed fields, which upon canonical quantization give rise to field functionals rather than particle wave functions. For massive and, in partic-

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\(^1\)There is a popular misundstanding of decoherence, found particularly in the context of welcher Weg experiments. It assumes that decoherence is defined by the disappearance of spatial interference fringes, observable only in the statistics of events. However, entanglement with an inaccessible environment destroys phase relations between coefficients \( c_i \) in (local) individual quantum states (1). These superpositions define spatial waves only in the special (though important) cases of (effective) quantized single mass points or single oscillator quanta on a spatial lattice. Apparent events, such as those appearing in measurements and giving rise to statistical aspects, “occur” according to the Schrödinger equation in another (later) process of decoherence. The latter affects superpositions of different measurement outcomes, such as spots on a plate. In other situations, depending on the relevant environment, other (individual) quasi-classical states may be produced by decoherence, for example precisely those coherent states of coupled oscillators that define “field” modes [9]. These two extremes of decoherence, caused by one or the other measurement device, are conventionally interpreted as a “wave particle dualism”.

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ular, charged boson fields (where “mass” is defined by a specific term in the field equations), occupation numbers and quasi-local states for each oscillator quantum are robust and may therefore appear classical in many cases – though not in the non-environment-entangled state of their Bose-Einstein condensate (cf. [7]). Permutation of bosons thus becomes a \textit{redundancy}, since this concept of bosons does not depend on a primordial \textit{particle} concept any more. For example, the permutation symmetry of many-particle wave functions does \textit{not} represent any physical entanglement: it disappears in terms of wave modes (cf. [8]).

(3) The most important example (though not the subject of this Letter) is the \textit{dressing} of elementary fields. It may simply lead to a \textit{renormalization} of parameters characterizing effective fields, or even require quite new fields – hopefully in the form of a unification. While not restricted to \textit{quantum} field theory (mass renormalization was known in classical electron theory, for example), the results of dressing must be expected to depend essentially on quantum theory.

Fermions (but also massive bosons) are usually regarded as particles on a fundamental level. This is particularly evident in Bohm’s theory [10], where unobservable trajectories in classical configuration space are postulated for particles \textit{and} for electromagnetic and other fields. (Variants of Bohm’s theory with photon trajectories instead of time-dependent Maxwell fields have recently been claimed to be in conflict with quantum theory and experiments [11]. While the analysis of these experiments appears doubtful, this modified Bohm theory – in contrast to the original one – seems to have never been proven equivalent to quantum theory.)

In the Heisenberg picture, “observables” corresponding to classical particle variables are often assumed to “exist” but not to “possess values”. However, since many-particle \textit{quantum} states are represented by wave functions (on a space of $3N$ dimensions), while particle aspects, such as spots on a plate or clicks in a counter, \textit{emerge} by means of decoherence in accordance with a universal Schrödinger equation, I concluded ten years ago [12] that “there are no particles” in quantum mechanics any more. Their rôle in the quantization procedure (for defining the corresponding configuration space as a stage for the wave function) is nonetheless widely used as an argument for a probability interpretation in terms of particles.

In canonical quantum electrodynamics, wave functionals $\Psi[\psi(\mathbf{r}), \mathbf{A}(\mathbf{r}), t]$ or $\Psi[\psi(\mathbf{r}), \psi^*(\mathbf{r}), \mathbf{A}(\mathbf{r}), t]$ describe general quantum states. They represent entangled superpositions of different values of all field amplitudes, thus leading to field \textit{operators} and their canonical momenta for $\mathbf{A}(\mathbf{r})$ and $\psi(\mathbf{r})$. Since
\( \psi(r, t) \) was itself obtained from particle quantization, this procedure is often called a “second” quantization. This interpretation is obviously wrong, since a true second quantization would lead to wave functionals defined on \textit{many}-particle wave functions. The “one-particle wave function” \( \psi(r, t) \) is a perfectly local \textit{field}, that would not allow one to describe EPR type non-locality, for example. While a quantized spinor field was \textit{historically} a second step, we must now simply conclude that spinor fields (rather than particle positions) define a correct \textit{basis} for electron and other fermion quantum states, even though they hardly ever appear as quasi-classical objects. Position \( r \) never represents a dynamical variable; it occurs as an \textit{index} of the true variables (\( \psi \) and other dynamical fields).

In other words: there are not even particles “before” quantization, that is, characterizing in any way the “configuration” space on which a fundamental wave function(al) is defined. According to the present state of the art, the “second” quantization in terms of fields is the first and only one, while particles represent a derived and \textit{effective} concept. Their appearance is no more than the result of decoherence by means of local interactions: it leads to robust (quasi-classical) local effects in the cloud chamber or detector, representing droplets or clicks, respectively. The occupation number (rather than particle) basis for electron states has recently been experimentally confirmed by anti-bunching [13], while the non-invariance of neutrons under \( 2\pi \)-rotations was directly observed long ago [14]. This double-valuedness of spinor fields under full spatial rotations may explain the restriction of their occupation numbers to 0 and 1.

If a modified Bohm theory with photon trajectories is indeed in conflict with quantum theory, I would expect this to apply to Bohm’s original theory in the \textit{relativistic} case, too. One may instead need Bohm trajectories for fields only [15] in order to remain consistent with relativistic quantum theory and with experiments (while even these “consistent” Bohm trajectories remain unobservable and in this sense meaningless [16]).

The conclusion that there is only a quantum theory of \textit{fields} does not, of course, contain any novel consequences for the \textit{formalism} of conventional quantum field theory. However, it undermines the usual interpretation of quantum states as probability amplitudes for (conceptually primordial) particles. This consequence may affect also other aspects of the Heisenberg picture. One may even argue whether a functional of \textit{fields} will survive in a \textit{future} quantum theory. Its form as a functional of many effective “particle” fields is certainly the most successful theory yet, but unified quantum field theories (supersymmetry or M-theory, for example) are no more than
promising proposals for a more fundamental one. Quantum field theories have the important advantage, however, to allow the formulation of local dynamics by means of a Hamiltonian density, defined as a function of field operators and their derivatives.

5 Constraints

Gauge theories are using constraints, which may be understood as a means to eliminate “unphysical” degrees of freedom, or redundancies (“gauges”), in order to define the proper stage for the wave function. For example, the permuted positions of two “identical particles”, or two magnetic potentials which lead to the same magnetic field and loop integrals \( \int A(r) \cdot ds \), have to be physically identified. While this should be done before quantization, that is, when defining the Hilbert space basis in (1), it is often more convenient, or the only feasible way, to apply “quantum constraints” in the form

\[
C |\alpha\rangle = 0 \tag{4}
\]

to an unphysical (too large) Hilbert space based on unconstrained variables. The latter is thus restricted to states being symmetric (invariant) under the group of all unphysical transformations, such as \( \exp(iC\phi) \) (see [17] and Giulini’s Sect. 6.3 of [1] for a relation to superselection rules). These two procedures are expected to be equivalent, while the enlarged Hilbert space remains irrelevant for any physical interpretation.

In quantum gravity, for example, “momentum constraints” \( P_i |\alpha\rangle = 0 \), applied to the Hilbert space spanned by all spatial metrics \( h_{kl}(r) \), with \( i, k, l = 1, 2, 3 \), are known to symmetrize the physical states under all transformations which connect different spatial metrics that represent the same abstract spatial geometry (such as those related by a mere coordinate transformation). A coordinate-free description of three-geometry is not explicitly known, in general.

There is a catch, however. Invariance under the gauge proper (the local choice of a basis of gauge group generators) is related to the existence of a physical “gauge field” (see the note added in proof in [18]). The parallel transport of these generators (a connection on the corresponding fibre bundle), which is required for the meaningful definition of relative gauge transformations at different points, must itself represent gauge-independent (abstract) geometry, and may thus give rise to active (physical) oscillations (“bosons” after their quantization). It is for this physical reason that the
gauge field “should be varied in the Lagrangean” [18]. Invariance then holds
trivially under local basis transformations, but (if defined) also under active
global ones (Mach’s principle understood as a redundancy).

This picture supports the view, entertained in this comment, that even
fermions have to be fundamentally described by quantum fields rather than
quantum particles. These fields may carry properties (“charges”) that are
affected by transformations under the gauge group – for example a “clas-
sical” phase characterizing a complex spinor field. This phase does not
describe a quantum superposition (as it would for quantized charged par-
ticles). In Weyl’s classical gauge theories, their later application (in 1929) to
the “quantum” phase (see [19]) appears as a deus ex machina, while it would
be entirely natural for complex pre-quantum (“classical”) spinor fields.

In the same sense as the observation of a radiation reaction in the ab-
sence of any absorbers [20] has been regarded, in classical context, as evi-
dence for the reality of fields (in contrast to time-symmetric action at a
distance), decoherence of the source by its own radiation in the absence of
any events in absorbers would demonstrate the reality of the corresponding
wave functional (which here describes entanglement between the radiation
and its source).

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