Dual-induced multifractality of human online activity

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Abstract. Recent discoveries of human activity reveal the existence of long-term correlation and its relation with the fat-tailed distribution of inter-event times, which imply that there exists the fractality of human activity. However, works further analyzing the type of fractality and its origin still lack. Herein, DFA and MFDFA methods are applied in the analysis of time series of online reviewing activity from Movielens and Netflix. Results show the long-term correlation at individual and whole community level, while the strength of such correlation at individual level is restricted to activity level. Such long-term correlation reveals the fractality of online reviewing activity. In our further investigation of this fractality, we first demonstrate it is multifractality, which results from the dual effect of broad probability density function and long-term correlation of time series in online reviewing activity. This result is also verified by three synthesized series. Therefore, we conclude that the combining impact of both broad probability density function and long-term correlation is the origin of multifractality behaving in human online activity.

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1. Introduction

Difficulty usually emerges from the characterization and understanding of complex systems, since we cannot split a complex system into several simpler subsystems without tampering its dynamical properties \[1\]. To address this difficulty, researchers have to turn to analyze their macroscopic properties. Time series analysis is a good case in point because the behavioral evolution of a complex system is suggested by its output records restricted to time scale.

Human activity is deemed as a typical complex system, of which several macroscopic properties have been unveiled recently via time series analysis (e.g., detrended fluctuation analysis, DFA \[2, 3, 4\]). For examples, the periodicity is found in diverse human activities, such as web surfing \[5\], online game log-in \[6\], task submission to linux server \[7\] and purchasing in E-commerce \[8\]; the log-term correlation, commonly behaving in many physical, biotic, economic and ecological systems \[9, 10, 11, 12, 13, 14, 15, 16, 17\], is also discovered in human interactive activity and positively increased with activity level \[18\].

Following that, Rybski et al. \[19\] investigate human communication activities in a social network to show the relation between long-term correlation and inter-event clustering. They present that at individual level the long-term correlation in time series of events is strongly dominated by the power-law distribution of inter-event times (named ‘Levy type correlation’ in Ref \[19\]), while at whole community level, it is a generic property of system since it arises from interdependencies in the activities. Meanwhile, Zhao et al. \[20\] analyze the time series of inter-event times obtained from online reviewing activities to unveil long-term correlation (i.e., memory property) restricted to activity level. They find that there is an abnormal scaling behavior associated with long-term anticorrelation, which is brought by the bimodal distribution of inter-event times.

These long-term correlations also imply the existence of fractality in time series of human online activity. However, two unanswered questions limits our much deeper recognition of human activity: (1) what type of fractality in time series of human activity and (2) what is the origin of such fractality. To address them, facilitated by the Internet technology, we investigate the time series of human online activity from two movie reviewing websites, Movielends and Netflix. The analysis of long-term correlation is first presented to reveal the fractality in time series of the online reviewing activity. At the individual level, we apply DFA on time series of events composed by users with same activity level and the corresponding shuffled ones in which each user’s distribution of inter-event times is reserved. The long-term correlations positively increase with activity levels. Moreover, because the the distributions of inter-event intervals at different activity levels don’t strictly follow a power law, there is trivial difference between the Hurst exponents \[21, 22\] of original and shuffled time series. The empirical result is a little different from the finding in human communication activity. A detailed discussion will be presented in Sec.4. At whole community level, the similar analysis on time series
of events aggregated from all users’ activities show the stronger long-term correlation with Hurst exponents roughly close to 0.9 and 1.0 for Movielens and Netflix, respectively.

To further reveal the type of such fractality and understand its origin, we use multifractal detrended fluctuation analysis (MF DFA) \[23, 24, 25\] to probe the singularity spectrum. Dependence between generalized Hurst exponent \(H(q)\) and \(q\)-order statistical moments \(\langle q \rangle\) exhibits the multifractality in time series of events at whole community level. Though multifractality still keeps after these time series are randomly shuffled, manifestly changes happen in the value of the generalized Hurst exponent \(H(q)\). A legible result is suggested by the singularity spectrum. We hypothesize that such multifractality forms for the dual effect of broad probability density function (PDF) and long-range correlation \[23\]. This hypothesis is also demonstrated by our synthesized series. Therefore, we conclude that multifractality exists in human online activity series and the combination impact of broad probability density function and long-term correlation is at the root of such multifractality.

2. Data

Data sets from Movielens and Netflix both record individuals’ reviews and markings on movies at a certain time, and are filtered according to the criteria of activity level, \(M \geq 55\) (see definition in Sec. 4). We thus finally obtain 26,884 users (38.4% of total users) and 10,000,054 records in a long duration of 4,703 days (nearly 13 years) for Movielens, and 17,703 users (99.6% of total users) and 100,477,917 records in a long duration of 2,243 days (nearly 6 years) for Netflix. As the records in Movielens are almost sampled from its creation date, there are so many noise users. Nevertheless, both the sizes of the filtered users in these two data sets are more than \(10^5\).

To convert these records into time series of events, we introduce two variables \(x(t)\) and \(x_{tot}(t)\) which means the events per day of a single user and whole community respectively. These time series serve as the base of our subsequently analysis. A visual illustration is shown in Fig. 1 where (a) and (b) indicate the activity records of two typical users for Movielens and Netflix, respectively, (c) and (d) show corresponding time series of events at individual level, while (e) and (f) represent the time series of events at whole community level. In Fig. 1 we also can obverse the clusters of records or events which suggests the existence of burstyness in human online activity.

3. Method

3.1. Detrended fluctuation analysis

The method of DFA has been proven for usefully revealing the extent of long-term correlation \[2, 3, 4\], and is less sensitive to the scaling range for the additional degrend process \[26\]. To keep our description self-contained, we briefly introduce the steps of this method as follows:
i) Calculate the profiles $Y(t')$ of time series $x(t)$,

$$Y(t') = \sum_{t=1}^{t'} x(t) - \langle x(t) \rangle, t' = 1, ..., N$$  \hspace{1cm} (1)

ii) Divide $Y(t')$ into $s$ non-overlapping segments with length $N_s$ in an increasing order. Generally, $N$ doesn’t exactly equal to the product of $s$ and $N_s$ (i.e., $(N_s = \lfloor N_s \rfloor)$), suggesting that the last part of $Y(t')$ are missed. Therefore we once divide $Y(t')$ from the opposite direction in order to incorporate whole $Y(t')$. Thus, there are $2N_s$ different segments. Moreover, the value of $s$ is sampled from the logarithmic space, $s = \frac{N}{2^{\text{int}(\log_{\frac{N}{2}} s) - 2}}, ..., \frac{N}{2^{s}}, \frac{N}{2^{2s}}$, which can keep smoothness of curve between $F(s)$ and $s$.

iii) Given $s$, the profile $Y(t')$ in each segment is detrended separately. Least-square fit is applied for determining $\chi^2$-functions for each segment, such as for $v = 1, 2, ..., N_s$,

$$F^2(v, s) = \frac{1}{s} \sum_{j=1}^{s} [Y((v - 1)s + j) - \omega_v^s]^2;$$  \hspace{1cm} (2)

and for $v = N_s + 1, ..., 2N_s$,

$$F^2(v, s) = \frac{1}{s} \sum_{j=1}^{s} [Y((N - v - N_s)s + j) - \omega_v^s]^2$$  \hspace{1cm} (3)
where $w_n^v$ is the $n$-order polynomial fitting for segment $v$.

iv) Calculate the fluctuation function,

$$F(s) = \left[ \frac{1}{2N_s} \sum_{v=1}^{2N_s} F^2(v, s) \right]^{\frac{1}{2}} \sim s^H,$$

where $H$ is the Hurst exponent. The value of $H$ reveals the extent of long-term correlation in time series. It indicates the long-term anticorrelation for $0 < H < 0.5$, no correlation for $H = 0.5$, and long-term correlation for $H > 0.5$.

3.2. Multifractal detrended fluctuation analysis

DFA method helps us to acquire the long-term correlation of time series, thereby ensure its fractality. To further analyze such fractality and its origin, we modify DFA method to introduce MFDFA method [23, 24, 25], where equation (4) is modified as follows:

$$F(s) = \left[ \frac{1}{2N_s} \sum_{v=1}^{2N_s} [F^2(v, s)]^{q/2} \right]^q \sim s^{H(q)}.$$  \hspace{1cm} (5)

$H(q)$ is the generalized Hurst exponent. For monofractal time series, $H(q)$ is independent from $q$, while for multifractal one, $H(q)$ depends on $q$. In addition, the multifractality of time series may be brought by several key factors, such as long-term correlation and broad probability density function (PDF). To figure out the origin of multifractality, we randomly shuffle the time series to reduce the long-term correlation but preserve the same PDF, and applied the MFDFA method once again. If the multifractality only results from the PDF, it will be reserved in the shuffled one, while if the multifractality only comes from long-term correlation, it will disappears. If the long-term correlation and PDF dually affect time series, we can expect that the multifractality still keeps but the value of generalized Hurst exponent ($H(q)$) changes.

A much more legible way to characterize a multifractal time series is the singularity spectrum $f(\alpha)$. The horizontal span of $f(\alpha)$ represents the strength of multifractality. Specifically, a very narrow $f(\alpha)$ approximately indicates a monofractal time series, while a wide one suggests a multifractal time series. To acquire its analytical relation with $\alpha$, we introduce Renyi exponent $\tau(q)$ [27, 28] with the equation given by follows:

$$\tau(q) = qH(q) + 1.$$  \hspace{1cm} (6)

Applying the Legendre transformation [29], we finally get the relation between $f(\alpha)$ and $\alpha$

$$\alpha = \tau'(q) \text{ and } f(\alpha) = q\alpha - \tau(q)$$  \hspace{1cm} (7)

or equivalently (by using (6))

$$\alpha = H(q) + qH'(q) \text{ and } f(\alpha) = q[\alpha - H(q)] + 1$$  \hspace{1cm} (8)
4. RESULTS

4.1. Individual level

To effectively categorize users into different activity levels, we define $M_i$ as the total records of a single user $i$ ($M_i = \sum_{i=1}^{N} x_i(t)$, where $N$ is the length of the series), and convert it into a logarithmic scale, $L_i = int(lnM_i)$. And, $L$ ranges from 4 to 8 in Movielens while it increases from 4 to 12 in Netflix. According to logarithmic activity levels, we firstly present the distributions of inter-event times in Fig. 2 where the left and right panels respectively indicate Movielens and Netflix. As shown in Fig. 2, both of them show a fat tail that suggests the burstyness of human online activity [30]. More concretely, for these users with lower activity levels (e.g. $L < 6$), their inter-event times aren’t exactly power law distributed. For example, in Fig. 3, the PDFs of inter-event times of users with activity levels $L = 4$ and $L = 5$ in Movielens apparently follow an exponential cut-off power-law distribution via least squared estimating method. Thus, the power-law distribution isn’t the only type to characterize the fat tail of inter-event clustering (i.e., burstyness), which is a little different from the empirical result found in human communication activity [19].

![Figure 2](image_url)

**Figure 2.** (Color online) Inter-event time distribution at different activity levels. The left and right panels respectively indicate Movielens and Netflix. The dash lines are guide for power-law distributions. The fat tail is both found in inter-event time distributions for Movielens and Netflix, which suggests the bustyness of human online activity.

The bustyness of human online activity (or fat tailed inter-event time distribution) potentially suggests that the time series of events behave long-term correlation [31] [19]. Thus, we apply DFA method to calculate the Hurst exponent in each time series of events obtained from a single user. Note that the least square estimating method is applied for fitting trend, and the F-statistic test confirms the significant of fitting results (see more in Supplementary Materials). Then, these Hurst exponents are further scaled according
Figure 3. Inter-event time distributions of users in Movielens whose activity levels are respectively $L = 4$ and $L = 5$. Compared with exact power-law distributions, they can be fitted by exponential cut-off power-law distributions via least squared estimating method.

Figure 4. Inter-event time distributions of Netflix whose activity levels are respectively $L = 4$ and $L = 5$. Compared with exact power-law distributions, they can be fitted by exponential cut-off power-law distributions via least squared estimating method.

Figure 5. Inter-event time distributions of users in Movielens whose activity levels are respectively $L = 4$ and $L = 5$. Compared with exact power-law distributions, they can be fitted by exponential cut-off power-law distributions via least squared estimating method.

to the same activity levels of users and averaged. As shown Fig. 4, the average of Hurst exponents as a function of activity level is presented, whose values are much more than 0.5 that indicates the uncorrelated behavior. Thus, it can be claimed that the long-term correlation exists in these time series of events at individual level. It also worthy to be noted that there is an approximately positive relation between Hurst exponent and activity level both for Movielens and Netflix, similar to those results found in the traded values of stocks and communication activity [32, 33, 19].

Now, we find the long-term correlation and the fat-tailed inter-event time distribution in the Netflix and Movielens. To ensure the existence of the relation between long-term correlation and inter-event time distribution, we shuffle the original Movielens and Netflix series but preserve individuals’ inter-event time distribution. The step is shown as follows: i) extract inter-event time of each individuals; ii) shuffle the extracted data; iii) rebuild the new series via the shuffled data (keep the first time stamp of each individuals unchange); iv) apply 1-order and 2-order DFA on new series. Blue line in figure 4 represents the results of DFA applied on shuffled series. Comparing with the green line and the blue line in fig. 4, we only find trivial difference between them, which shows the possible relation between the long-term correlation and inter-event time distribution. Since a strict power-law distribution does not show in the inter-event time distribution of movielens and netflix, the exact analytical relation which is same as Rbyski cannot be derived [19].

4.2. Whole community level

Although the inter-event interval distributions in respect to activity levels show fat tail, we still want to probe whether this property is reserved in whole community (i.e, system). Figure 5 shows that the inter-event time distributions of Movielens and Netflix at whole community level. It can be seen that it is fitted by an exponential cut-off power law distribution for Movielens and approximate power law one for Netflix, which suggests
that the fat tail is generic. Furthermore, the events are aggregated from all users in whole community, and the resulting time series is investigated to unveil the long-term correlation. As shown in Fig 6, the Hurst exponents obtained from 1-order and 2-order DFA are robust and approximately close to 0.9 and 1 for Movielens and Netflix, respectively, which shows very strong long-term correlation. These Hurst exponents also associate with the spectrum of time series obeying $1/f$ scaling, suggesting the self-organized criticality of system.

To further verify such property, we randomly shuffle these two series and use the same DFA method. We find their Hurst exponents change into 0.5. Therefore, we assume that long-term correlation is a universal property showing in different time series when combining with the conclusion drawn from the analysis of individual level. Long-term correlation shown at individual and whole community level means the fractality of human activity, but few works have further analyzed the type of the fractal and its origin. Inspired by [23], we introduce MFDFA method to analyze these data sets. When analyzing Movielens and Netflix series via MFDFA method, we firstly keep the value of $q$ at 0.1 and fit $F_q(\Delta t)$ and $\Delta t$ on double logarithmic coordination with least square estimating method to get the value of generalized Hurst exponent $H(q)$. Then, we update $q$ by adding 0.1 to $q$ in each step and apply the same fitting method.

**Figure 4.** (Color online) Hurst exponents as a function of activity level for Movielens and Netflix. The results obtained from original time series of events and shuffled ones are respectively plotted with green circles and blue square. With the increase of activity levels, the long-term correlation becomes stronger Moreover, the trivial difference between them reveals the long-term correlation having a potential relation with fat-tailed inter-event time distribution.
Figure 5. (Color online) Inter-event time distribution at whole community level. The power-law with exponential cut-off relation behaves in Movielens, while power-law relation behaves in Netflix. This result shows the burstyness is generic to system.

Finally, when $q$ equals to 10, we stop our update and plot the relation between $H(q)$ and $q$. Figure 7(a) and (b) show the results of 1-order MFDFA for Movielens and Netflix. The dependence between $H(q)$ and $q$ shows the multifractality of these two time series.

To figure out the origin of the multifractality in Movielens and Netflix series, we randomly shuffled these two time series and applied 1-order MFDFA method on the shuffled data. Though the multifractality still keeps, a decreasing trend happens to the value of $H(q)$ (see in Fig. 7(c) and (d)). As shown in Fig. 7(e) and (f), we also shows the singularity spectrum of the corresponding data. It also can be found that the horizon span of the spectrum has a little change, but changes happen to the value of $\alpha$. Therefore, we conclude that the multifractality results from the dual effect of broad probability density function and long-term correlation.
Figure 7. (Color online) Relation between $H(q)$ and $q$ deriving from 1-order MFDFA (a)-(d) and the corresponding singularity spectrum (e) and (f), where panels (a) and (b) are obtained from the original time series and panels (c) and (d) are obtained from the shuffled ones. Though the multifractality keeps, there are significant changes happened to the values of $H$ and $\alpha$. This results reveals the multifractality for Netflix and Movielens and the formation results from the duel effect of broad probability density function and long-term correlation.

5. Discussion

So far, we have analyzed the Netflix and Movielens at individual and the whole community level. At individual level, we find the fat-tailed inter-event time distribution and its relation with the discovered long-term correlation, but we cannot get the exact analytical relation like in [19] for the fact that inter-event time distributions of movielens and netflix do not strictly follow power-law relation. At the whole community level, we also find the same properties as what we find on individual level. Then, we verify the fractality of real time series. To further analyze such fractality and its origination, we have introduced MFDFA method and finally find the multifractality of human online activity. We hypothesize such multifractality derives from the combination impact of broad probability density function and long-term correlation.

To verify such hypothesis, we create three synthetic time series. The first one is a random series which follows a power-law distribution ($y \sim x^{-2}$). The second
Figure 8. Relation between $H(q)$ and $q$ obtained from 1-order MFDFA (a)-(c) and the corresponding singularity spectrum (d)-(f) of three synthetic time series, where pink square and blue circle respectively indicates the results of original and shuffled time series. Note that the first column shows a synthetic time series that follows a power law distribution $y \sim x^{-2}$, the second one describes a synthetic time series whose Hurst exponent is 0.9, the third one represents a synthetic time series that combines of the former two time series. Only the third time series behaves a similar result to real one.

one is a monofractal series with long-term correlation ($H = 0.9$) created by FFM method [34, 35]. The third one is the combination of the first and second series (see more in Appendix). Then, we apply 1-order MFDFA for these time series and their corresponding randomly shuffled ones. Specifically, as shown in Fig. 8 in the first time series, we find the existence of multifractality both in original and shuffled ones, reflected by overlaps between the curve of $H(q)$ and $q$; in the second time series, we only find the monofractality and change only happening in the value of $H(q)$ between original and shuffled ones; in the third time series, we find the multifractality and change happening in the value of $H(q)$ between original and shuffled ones. The corresponding singularity spectrums of these three synthetic time series are shown in the second row of Fig. 8 which presents a much visualized result. Especially, in the third time series, the significant horizon span and gaps simultaneously appear in original and shuffled ones, in consistency with the empirical result.

Synthesizing the results from our created time series, we are able to verify the multifractality of human online activity and conclude that it is induced by the combining effect of broad probability density function and long-term correlation. Nevertheless, lacking in appropriate models to explain the mechanism of producing such time series, questions still exist on the recognition of this dual effect. For example, what causes the broad density probability distribution of real time series? Which preference hidden in human activity lead to the long-term correlation? We hope future work will show the answers of these questions.
Appendix

Generation of synthetic time series

According to limit central theorem, time series \( x(t) \) that follows power-law distribution 
\[ p(x) = \beta x^{-(1+\beta)} \]  
can be created as follows:
\[ x(t) = (r(t)/\beta)^{-\frac{1}{1+\beta}}, \]  \hspace{1cm} (9)

where \( r(t) \) is a time series sampled from a uniform distribution \((U(0,1))\). In our analysis, we set \( \beta = 1 \), which leads to \( x(t) \) obeying a power-law distribution, \( p(x) = x^{-2} \).

Applying the FFM method \([34, 35]\), we create a time series with long-term correlation and not following any typical distribution. The procedures are shortly introduced as follows:

i) Generate a 1-dimension random series \( \{U_i\} \) which follows a Gaussian distribution, and derive its fourier transform coefficients \( \{U_q\} \).

ii) Obtain \( \{S_q\} \) from the Fourier transformation of \( \{C(l)\} \), where
\[ C(l) = \langle \mu_i \mu_{i+l} \rangle = (1 + l^2)^{-\gamma/2}. \]

iii) Calculate \( N_q = [S(q)]^{1/2} U_q \).

iv) Derive our desired time series \( N_r \) via the inverse fourier transformation of \( N_q \).

At last, we provide a method to combine these two series for synthesizing the third time series,
\[ X(t) = (N_r(t)/\beta)^{-\frac{1}{1+\beta}} \]  \hspace{1cm} (10)
where \( N_r \) is the time series with long-term correlation.

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