Studying on the Optimum Frequency of Driving Voltage of Electro-osmotic

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Abstract. Electro-osmotic actuation is an effective method of mixing micro-scale fluids. In this paper, an annular micro-channel mixing model, in which electro-osmosis are used to mix fluids, is established to simulate fluid mixing condition. Simulation results show that for fixed flow parameters, there is a supply frequency that can optimize the mixing efficiency. As the frequency increases, the mixing efficiency increases firstly and then decreases. The optimal mixing efficiency is about 95\% at the frequency of 25 Hz. Furthermore, the influence of flow velocity, electrode position and voltage amplitude on the optimal frequency is proposed, and the physical mechanisms are discussed. Numerical analysis shows the electro-osmotic of electric field can produce a vortex, which can increase the mixing efficiency in micro-channels effectively. The mixing efficiency can be improved by increasing voltage amplitude and the electrode gap. When the velocity of fluids increases, the residence time of particles decreases, the frequency of optimization will increase accordingly.

1. Introduction

As an important driving method in micro-scale, electro-osmotic drive is widely used due to its advantages of low cost, reliable operation and high efficiency. In 1897, Helmholtz researched the liquid flow problem and the electricity phenomenon, and proposed the complete electrical double layer model theory. Furthermore, the fluid flow and heat transfer properties have been studied. For example, Yang et al [1] analyzed and solved the electro-osmotic/pressure mixing driving in rectangular micro-channels. Cummings et al [2] confirmed that the flow velocity of the fluid is steady and uniform in the electro-osmotic flow field.

Various micro-mixers have been studied. Feng et al [3] analytically studied electro-osmosis (EO) and induced charge electro-osmosis (ICEO) in the eccentric annulus, and proposed a chaotic micro-mixer by introducing Lagrangian chaos into the EO or the ICEO micro-mixers. He revealed that the created Lagrangian chaos performs a much better and faster mixing than either the pure EO or the pure ICEO. Zhang et al [4] designed and fabricated a new annular electro-kinetic chaotic mixer. Simulation results indicated that this mixer can achieve a good mixing of different kinds of fluids. Oddy et al [5] researched an electro-kinetic unstable micro-mixer with a sinusoidal electric field, and studied the mixing process by integrating the fluorescence intensity. The results indicated that the mixing effect is enhanced. Zhuang et al [6] studied the influence of electro-osmotic driving flow on the mixing of power-law fluid in T-shaped micro-channel channels, and focused on the influence of electric potential on the
enhancement mixing effect, where the instability phenomenon is utilized to enhance mixing in low Reynolds number flows of different micro-channel shapes. Park et al [7] found that cavity structures on the wall produce a repetitive vortex pattern yielding a higher mixing efficiency than that of a straight channel.

There are also many studies on the flow of micro-channels with isomeric potential on the surface. Using theoretical and numerical analyses, as well as experimental observations, Lin [8] explored the instability physics. The model indicated that the fluid forces associated with the thin dimension of the channel (transverse to both the conductivity gradient and the main flow direction) tends to stabilize the flow. Nayak et al [9] adopted the method of arranging heterogeneous potentials on the channel surface and found that the Newtonian fluid mixing in micro-channels can be enhanced by pressure gradient produced by eddy currents. Wang et al [10] presented a semi-analytical solution to study the flow behavior of periodical electro-osmosis in a rectangular micro-channel based on the Navier–Stokes equation and a nonlinear Poisson–Boltzmann equation. Chen et al [11] studied the flow mechanism of three kinds of power law fluids in micro-porous media with different structures. Yang et al [12] researched the fluid flow in plate micro-channel with heterogeneous zeta potential on its wall, and concluded that the apparent viscosity difference of power law fluid has different effects on the electro-kinetic phenomenon and has a significant effect on mixing efficiency. Besides, the shear thinning property of the fluid tends to increase the electromotive force phenomenon. The better mixing results can be obtained when the power law exponent decreases.

In this paper, based on the parameter of mixing efficiency, the optimal supply frequency is obtained for the micro-fluid mixture with an annular structure. Furthermore, the influence of fluid velocity, electrode position and power amplitude on the optimal supply frequency is analyzed.

2. Model and simulation results

2.1. Physical model

The structure of annular micro-mixer is shown in Fig. 1. Two liquids flow in the mixer through the inlets $A_1$ and $A_2$ respectively. The inlets of $A_1$ and $A_2$ are both 5μm in width. The two fluids pass through a rectangular channel, whose length $L_1=26\ \mu m$ and then into a ring-type mixing chamber with an inner radius $R_1=5\ \mu m$ and an outer radius $R_2=15\ \mu m$. There are four electrodes on the outer wall (1, 3 for the positive electrode, 2, 4 for the negative electrode), the angle between each electrode and the horizontal direction (acute angle) $\theta=45^\circ$. The electrode is loaded with sinusoidal voltage to enhance the mixing of two fluids. After passing through the annular mixer, the two fluids flow out through a passage, whose length $L_2=26\ \mu m$ and the section length of outlet $A_3$ is 10 μm.

![Figure 1. Micro-mixer geometry](image)

The Navier-Stokes equation for incompressible flow is used to describe the flow field in the micro-channel:

$$\rho \frac{\partial u}{\partial t} - \nabla \cdot (\nabla u + (\nabla u)^T) + \rho u \cdot \nabla u + \nabla p = 0$$

(1)

The convection-diffusion equation is applied to account for the mixing of fluids:
\[
\frac{\partial c}{\partial t} + \nabla \cdot (-D \nabla c) + u \cdot \nabla c = 0 \tag{2}
\]

Where the dynamic viscosity coefficient \(\eta = 10^{-3}\) Pa\cdot s; the fluid velocity of the entrance \(u = 0.1\) mm/s; the fluid density \(\rho = 1000\) kg/m\(^3\); \(c\) is the fluid concentration, the concentration of inlet \(A_1\) is 1 mol/m\(^3\); the concentration of inlet \(A_2\) is 0 mol/m\(^3\); the fluid diffusion coefficient \(D = 10^{-11}\) m\(^2\)/s.

Helmholtz-Smoluchowski equation is used to describe the relationship between the magnitude of electro-osmotic velocity and the tangential component of the applied electric field [4]:

\[
u = \frac{\varepsilon_w \zeta_0}{\eta} \nabla_T V \tag{3}
\]

Where \(\varepsilon_w = \varepsilon_0 \varepsilon_r\) denotes the fluid’s electric permittivity (F/m), \(\varepsilon_r\) represents relative electric permittivity of the fluid, \(\varepsilon_r = 80.2\); \(\zeta_0\) represents the zeta potential at the channel wall, \(\zeta_0 = -0.1\) V; the applied voltage \(V = V_0 \sin(\omega t)\), amplitude of the AC voltage \(V_0 = 0.1\) V; the value of the angular frequency \(\omega\) is 50 Hz.

2.2. Numerical Simulation Results

The parameter of mixing efficiency is introduced to evaluate the degree of mixing [13]:

\[
\eta = 1 - \frac{\int_{A_3} |C - C_\infty|}{\int_{A_3} |C_0 - C_\infty|} \tag{4}
\]

Where \(C\) is the micro-channel exit \(A_3\) fluid concentration; \(C_\infty\) is the concentration of perfectly mixed fluid, and \(C_0\) is the concentration of completely unmixed fluid (at the initial stage). Thus, the mixing efficiency \(\eta = 1\) indicates a perfectly mixed state, while a mixing efficiency \(\eta = 0\) indicates a completely unmixed state.

![Figure 2. Relationship between efficiency and angular frequency](image)

The relationship between mixing efficiency and angular velocity is shown in Fig. 2. As the frequency increases, the mixing efficiency increases firstly and then decreases, achieving the optimal mixing efficiency at 31 Hz with a mixing ratio of 95%. When the angular frequency is bigger than 132 Hz, the mixing efficiency remains constant.

The existing studies have shown that after introducing an electric field, the tangential velocity of the liquid on the wall increases, and a vortex forms in the mixing section of the channel, which enhances the mixing of the liquids. The properties of vortex field are affected by the frequency of the applied electric field, so there is an optimized angular frequency at which the mixing efficiency reaches its maximum.
Figure 3 shows the trajectories of the particles in liquid 1. After entering the mixing section of the channel, particles that originally moving in parallel start to disperse due to the vortex field and finally flow out from the outlet. Compared to the distribution near the inlet of the channel, particles are more evenly distributed at the outlet. The color of the trajectory represents the moving velocity of particles. It can be seen that the velocities of particles will increase when they are close to the wall, which is a result of the influence of electric field. The difference of velocities between the wall and the middle part of the channel leads to the occurrence of vortex, which scatter particles to different locations of the channel, leading to the mixing of the liquids.

![Particle tracking diagram](image)

**Figure 3.** Particle tracking diagram

When the electric field frequency is less than the optimal frequency, the electric field changes slowly relative to the time it takes for the particles to pass through the mixing section. And there is not enough driving force for the particles to move in vortex and the mixing efficiency is not satisfactory. On the other hand, if the electric field frequency is far greater than the optimal frequency, the movement of particles can’t keep up with the changes of electric field force, which will also have a detrimental effect on the mixing efficiency.

### 3. Effect of parameters on mixing efficiency

The influence of flow and electric field parameters on mixing efficiency will be studied in the following.

#### 3.1. Effect of fluid velocity

![Graph](image)

**Figure 4.** Relationship between mixing efficiency and angular frequency at different fluid velocities
The effect of fluid velocity on mixing efficiency is studied by changing the inlet velocity of the fluid (Fig. 4). Numerical simulation results show that with the increase of fluid velocity, the maximum mixing efficiency decreases while the frequency of maximum mixing efficiency increases. As the residence time of the fluid in the mixing zone decreases, in order to complete the mixing in a shorter time, the optimized angular frequency needed to form the vortex increases.

3.2. **Effect of electrode position**

By changing the angle of the electrode position, the influence of the angular frequency on the mixing efficiency is analyzed (Fig. 5). As the angle decreases, the distance between electrode 1 and electrode 2 increases, and the same is true for electrode 3 and electrode 4. As the distance between the electrodes increases in the direction of x-axis, the effect extent of the electric field expands toward the inlet and outlet sections of the passage. Therefore, the maximum mixing efficiency is increased.

3.3. **Effect of voltage amplitude**

By changing the angle of the electrode position, the influence of the angular frequency on the mixing efficiency is analyzed (Fig. 5). As the angle decreases, the distance between electrode 1 and electrode 2 increases, and the same is true for electrode 3 and electrode 4. As the distance between the electrodes increases in the direction of x-axis, the effect extent of the electric field expands toward the inlet and outlet sections of the passage. Therefore, the maximum mixing efficiency is increased.
The initial voltage amplitude is set as 0.08V, 0.09V, 0.10V, 0.11V and 0.12V. The effect of the electric field angular velocity on mixing efficiency is shown in Fig. 6. It can be seen that with the increase of voltage amplitude, the optimal mixing efficiency increases, while the frequency of maximum mixing efficiency remains unchanged. The increase in the amplitude of the electric field enhances the intensity of the vortex and thus promotes the mixing of liquid.

4. Conclusion
The simulation results of the fluid mixing in the annular micro-channel show that the tangential velocity of the fluid close to the wall can be increased by the addition of alternating electric field, thus generating the vortex and improving the mixing efficiency. For the fixed-size micro-channel, the mixing efficiency exhibits the following patterns:

1. As the power frequency increases, the mixing efficiency increases firstly and then decreases. There is an optimum frequency to maximize the mixing efficiency.
2. Increasing the distance between the electrodes in the direction of x-axis is conducive to improve the mixing efficiency.
3. For the applied voltage, the optimized angular frequency increases with the fluid velocity, and the mixing efficiency can be improved by increasing the voltage amplitude.

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