Simulation of flows with breaking internal waves generated by an obstacle

S N Yakovenko\(^1\) and N V Gavrilov\(^2\)

\(^1\)Khristianovich Institute of Theoretical and Applied Mechanics SB RAS, Novosibirsk, Russia
\(^2\)Lavrentyev Institute of Hydrodynamics SB RAS, Novosibirsk, Russia

E-mail: yakovenk@itam.nsc.ru

Abstract. Numerical study of a stably stratified flow with obstacle of different shapes is performed by means of the Navier–Stokes and scalar equations for incompressible fluid with the Boussinesq approximation of buoyancy effects. The results of computations agree well with the concurrent experiment study with the towed body in the water tank with pycnocline.

1. Introduction

Instability and turbulence development in breaking lee waves generated by the obstacle of height \(h\) in a stably stratified flow with constant inflow velocity \(U\) (figure 1a) is studied by DNS and LES at wide ranges of Reynolds number, \(50 \leq \text{Re}(= \frac{Uh}{\nu}) \leq 4 \times 10^4\), relating to tank measurements, and Prandtl or Schmidt number \((1 \leq \text{Pr} \leq 2000)\) for natural and laboratory study conditions. The case of \(\text{Re} = 4000,\ \text{Pr} = 1\) was tested in \([1, 2]\). The density field instability arising after wave overturning reveals a range of spanwise spectra modes. The smallest mode \(\lambda_y \sim 0.5h\) represents oscillations of Rayleigh–Taylor instability (RTI), growing and resulting in periodic convective structures with associated Kelvin–Helmholtz instability rolls (figure 1b). At late transition times the smaller vortices transform into the larger ones with the dominant mode \(\lambda_y \sim 2.5h\) at \(t > 30\) when the large-scale toroidal vortex structures become evident as observed in both \([1]\) and \([3]\).

![Figure 1](image-url)

Figure 1. Pathlines at \(y = 0, t = 37.5\) (a), density contours at \(x = 2.5h, t = 23, 24, 25\) (b) for \(\text{Re} = 4000,\ \text{Pr} = 1\).
A clear molecular diffusion effect is seen: lower Re or Pr give smaller density gradients and drops in the unstable area viewed in figure 1, delay the RTI growth start, decrease its speed at linear stage, and increase the perturbation wavelength [4]. For Pr = 1, Re = 200 spanwise instability is weak, so not visible in density contours and does not lead to turbulence. The results show transition to turbulence at Re ≥ 200 for Pr = 7 and Re ≥ 100 for Pr = 700.

The intention of the present study is to numerically simulate the lee wave overturning region for various obstacle shapes and two cases of initial stable density distributions (constant-gradient one and step-like profile with pycnocline) and to examine the influence of these conditions on the details of instability development and transition to turbulence.

2. Numerical model
Numerical simulation of a stratified flow is based on the three-dimensional unsteady continuity, Navier–Stokes and scalar (density) equations in Cartesian coordinates (x, y, z) for incompressible fluid with the Boussinesq approximation of buoyancy effects. As in [1], discretization of equations is carried out on a staggered grid to prevent mismatch between the velocity and pressure fields. All details of the governing equations non-dimensionalized by inflow velocity $U$ and obstacle height $h$, and their numerical realization are given in [1, 2].

To overcome the problems of insufficient resolution, SGS models of Smagorinsky type for velocity and scalar equations are used for cases of high Reynolds or Prandtl/Schmidt numbers, with the standard value of Smagorinsky constant $C_s = 0.1$, and the same boundary conditions, sponge layers, refined uniform grid as in [1, 2]. The SGS Prandtl/Schmidt number is chosen to be $Pr_{sgs} = 0.3$ which is enough to efficiently remove the excessive numerical noise distorting the scalar field plots and spectra both in ILES with $C_s = 0$ and in LES with $C_s = 0.1$ and $Pr_{sgs} = 1.0$ [5].

**Figure 2.** Streamlines for non-dimensional times $t = 40$ (a), 45 (b), 50 (c), 55 (d), 60 (e), 75 (f), 90 (g), 100 (h), 115 (i) at $y = 0$ in a flow with “hill” located at $|x| \leq 3.56h$ and $0 \leq z \leq h$. 


3. Computation results  

The combined effects of obstacle shape, flow depth $D$, lower/upper boundaries, inflow density profile, Froude number $F_h = U/(Nh)$ where $N$ is Brunt–Väisälä frequency are explored here. For example, for a flow above the two-dimensional cosine-shaped hill with length $L = 3.56h$ at obstacle half-height and with $D = 10h$, $F_h = 0.6$, $Re = 100$, $Pr = 700$, pathlines reveal (figure 2) the wave overturning at $t > 28$, $x \approx 2.3h$, $z \approx 3.2h$, recirculation zone at $t > 42$, its weak turbulization at $t \sim 60$, intermittent behavior without recirculation/overturning at $t \sim 75$ or 100. Along the span, organized periodic structures exist [4] without significant changes for long periods as those in the laboratory experiments [3] at $Re = 150$.

For a flow with the thin vertical barrier at the same $Re$, $Pr$ as in the case above and for different values of $D = 5h$, $F_h = 1.33$, the processes of lee wave overturning and recirculation appearance are observed at larger times and positions (figure 3). Moreover, the laminar-like recirculation region is now developed into stable configuration survived persistently as in early computational studies [6], so intermittent behavior of the wave-breaking region is absent for this case, unlikely with the features shown in figure 2.

![Figure 3](image_url)

**Figure 3.** Streamlines at $t = 45$ (a), 50 (b), 60 (c), 70 (d), 300 (e) and $y = 0$ in a flow past “fence” placed at $x = 100h$, $0 \leq z \leq h$.

The third flow case (figure 4) has the complicated triangle-like obstacle shape which consists of two straight sides of different angles to the horizontal plane and the central fragment related to semicircle. Moreover, the initial density distribution has multiple layers of constant density values and gradients (see the scalar $f(z)$ profile) like the pycnocline in oceans. The flow blockage by the obstacle
is here much larger since $D = 2h$ only, and other parameters are $Re = 783$, $Pr = 2000$ (sugar solution in water), $F_h = 0.19$ (based on the $f(z)$ drop shown in figure 4).

Figure 4. The experiment scheme with sharp-density drop and towed body of quasi-triangle shape.

The physical experiment (figure 5) corresponding to the third case (figure 4) is performed in water tanks in Lavrentyev Institute of Hydrodynamics SB RAS (see e.g. [7, 8] for similar earlier studies).

The preliminary results of comparison between the laboratory experiments (figure 5) and numerical simulations (figures 6 and 7) show agreement for this situation where instability growth due to internal wave overturning is observed too and needs further exploration.

Figure 5. Experimental flow visualization with internal wave breaking at $y = 0$ and non-dimensional time $t(u_0/h) = 2.18$ (a), 4.37 (b), 6.00 (c), 8.73 (d), 12.00 (e), 16.38 (f).
Figure 6. Density contours at $t(u_0/h) = 2$ (a), 3 (b), 3.5 (c), 4 (d), 4.5 (e), 5 (f), 6 (g), 9 (h) for $y = 0$ in a flow past triangular hill.
Figure 7. Streamlines at $t(u_0/h) = 2$ (a), 3 (b), 3.5 (c), 4 (d), 4.5 (e), 5 (f), 6 (g), 9 (h) for $y = 0$ in a flow past triangular hill.
Acknowledgments
The author thanks Prof I.P. Castro and Dr T.G. Thomas for help in breaking lee wave study. The work was done with use of Iridis computing resources at the University of Southampton and NUSC services at the Novosibirsk State University. The study is made partly within the framework of the Program of Fundamental Scientific Research of the state academies of sciences in 2013-2020 (Project No. AAAA-A17-117030610128-8) and Integrated Basic Research Program II.1 of SB RAS (Project No. 2).

References
[1] Yakovenko S N, Thomas T G and Castro I P 2014 J. Fluid Mech. 760 466
[2] Yakovenko S N, Thomas T G and Castro I P 2011 J. Fluid Mech. 677 103
[3] Eiff O F and Bonneton P 2000 Phys. Fluids 12 1073
[4] Yakovenko S N 2017 J. Phys.: Conf. Ser. 894 012112
[5] Yakovenko S N, Thomas T G and Castro I P 2014 Progress in Turbulence V: Proceedings of the iTi Conference in Turbulence 2012 ed A Talamelli et al. Springer Proceedings in Physics vol 149 (Springer) p 233
[6] Paisley M F, Castro I P and Rockliff N J 1994 Stably Stratified Flows: Flow and Dispersion over Topography (Clarendon Press, Oxford) p 39
[7] Gavrilov N V and Liapidevskii V Yu 2010 J. Applied Mechanics and Technical Physics 51 471
[8] Gavrilov N, Liapidevskii V and Gavrilova K 2012 Nonlin. Processes Geophys. 19 265