Distributed filtering stochastic parameter learning algorithm for discrete-time random nonlinear systems with delay

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Abstract. Distributed filtering algorithm for a discrete time random nonlinear stochastic systems associated with state delay for the distributed wireless communication sensors are discussed in this paper. Stochastic Parameter Learning Algorithm (SPLA) is tend to aim at obtaining a collection of stochastic filter parameters in a finite limited time horizon, that minimize the traces of the upper limits which is permitted to reduce the error variance matrices of the concerned stochastic filter system's states and delay measurements. Filter gain values of the filter derived by the determination of Riccati type difference equations, estimates systems states with delay. Two different filter rules are taken into account for the SPLA discrete time random nonlinear systems with steady state space equations model. Zero mean distinct covariance matrix along with the constructive state values and constant time delay are focused in compatible dimensions. The variance of the projected systems predicted noise and the actual estimated noise are validated through numerical examples.

1. Introduction
In the recent days stochastic parameter systems along with difference equations [13,14, 18] have been found to be a vital mechanism for estimating various applications of various fields in the real world [1, 6]. Formal logic has long been at the center of analysis for the purpose of extending every theoretical consequences and significant applications. This theoretical nonlinguistic variable victimization in communication technology [11], further requires accurate information from the system and achieves preferred performance once systems with inaccurate parameters are eliminated [11]. The modeling strategies [15] that supported the feasibility constraints applied to wireless device networks [17].

Compared with different models, the Stochastic Parameter Learning Algorithm (SPLA) model is basically a nonlinear model with sensitive estimate the capability and, compatibility of nonlinear systems with [16] gains and manage filtering approaches in an adaptive manner. Although there is a lot of new random stochastic parameter controller attribute-based research, contributes the Stochastic Parameter Learning Algorithm (SPLA) model and its performance of various estimation of signal processors [19].

In reality, huge data sets of real time problems were tackled with the help of the stochastic parameters representations, such as wave propagation, demonstrating the wide application of the estimation of the signals and systems [20] and so on. Furthermore, the SPLA model compensates for the lack of typical Kalman and extended Kalman filtering. Primarily, discarding the higher order [5] Taylor's series growth term in the extended Kalman filter, one in each addition device (coordinates) of the Kalman filter to operate nonlinearities, affects the accurateness of the estimate, particularly when a nonlinearity exists within the system and the formation of the stability analysis [7] and exponential stability analysis [12] in a periodical manner [3, 8].
Provoked by the previous deliberations, it is of theoretical and reasonable significance to handle the discrete [9] and distributed discrete time filtering problem for a collection of SPLA discrete time random nonlinear systems with state space equations \([4, 5]\). There is a tendency to aim at designing a successful and effective filter parameter that minimizes the trace of an allowed limit for the variation in the filtering error. The main segment of this article is summarized below: (i) A completely unique distributed filter is built on to determine the delays, of wireless communication topology. (ii) An adaptive repetitive approach is envisaged with Riccati-type difference equations to promise non-optimal filtering performances; and (iii) Gain of the filter is resulting and allowed to identify the variance matrices.

The paper is organized as follows. In section II, discrete time random nonlinear systems were considered and applied various validation rule to enrich our SPLA model. Further two different filter rules are taken into account for the SPLA discrete-time random nonlinear systems with state space equations model. In section III, Stochastic Parameter Learning Algorithm (SPLA) for various acceptable parameters are taken into the consideration. Section IV validates the result with the numerical example and section V concludes the paper.

2. Projected and designed problem formulation

In general, consider the subsequent discrete time random nonlinear systems with respect to the time domain is:

\[
x_{k+1} = f(x_k) + g(x_{k-\tau}) + B(w_k),
\]
\[
y_k^a = h^a(x_k) + D^a v_k^a
\]
\[
x_k = x_{i_k},\quad k = -\tau, -\tau + 1, \ldots 0 \text{ and } i = 1, 2, 3, \ldots n
\]

where \(x_k \in \mathbb{R}^{n_x}\) represents the state of the system; \(y_k^a\) is the output measured by the \(a\)th sensor; \(f, g\) and \(h^a\) are proverbial functions of \(x_k/x_{k-\tau}\) analyzed at the given horizon; \(w_k \in \mathbb{R}^w\) and \(v_k^a \in \mathbb{R}^v\); \(\tau\) represents the constant time delay; \(x_{i_k}\) be the initial state; and constant matrices \(B\) and \(D^a\) with \(n \times n\) dimensions.

The subsequent SPLA structure is inserted to form a state estimate intended for the predicted and designed digital system:

**Rule i**: IF \(\beta_{k,1}^a\) is \(M_{i,1}^a\), and \(\beta_{k,2}^a\) is \(M_{i,2}^a\), ..., and \(\beta_{k,m}^a\) is \(M_{i,m}^a\) is chosen randomly then,

\[
\begin{cases}
  x_{k+1} = A_i x_k + F_i x_{k-\tau} + B w_k, \\
  y_k^i = C_i^a x_k + D^a v_k^a, \quad i = \{1, 2, \ldots, n\} \\
  x_k = x_{i_k},\quad k = -\tau, -\tau + 1, \ldots 0 \text{ and } i = 1, 2, \ldots, n
\end{cases}
\]

where \(M_{i,j}^a (j = 1, 2, \ldots, m)\) is the random set (choosing any random element from \(M_{i,j}^a\)): \(A_i, F_i\) and \(C_i^a\) are arrays of proverbial constants with compatible dimensions. \(\{\beta_{k,1}^a, \beta_{k,2}^a, \ldots, \beta_{k,m}^a\}\) are the fundamental variable and denote

\[
\beta_k^a = \{\beta_{k,1}^a, \beta_{k,2}^a, \ldots, \beta_{k,m}^a\}.
\]

Further (2) can be extended like,

\[
\begin{cases}
  x_{k+1} = \sum_{l=1}^n p_i^l (\beta_k^a) A_i x_k + B w_k + \sum_{l=1}^n p_i^l (\beta_{k-\tau}^a) F_i x_{k-\tau} \\
  y_k^a = \sum_{l=1}^n p_i^l (\beta_k^a) C_i^a x_k + D^a v_k^a
\end{cases}
\]

(3)
here $p^a_i (\beta^{a}_k) = \frac{\psi^a_i (\beta^{a}_k)}{\sum^n_{j=1} \psi^a_j (\beta^{a}_{kj})}$ and $\psi^a_i (\beta^{a}_k) = \Pi^n_{j=1} M^a_{ij} (\beta^{a}_{kj}) \geq 0$, and $p^a_i (\beta^{a}_k)$ is the normalized weight and deduce it as

$$\sum^n_{i=1} p^a_i (\beta^{a}_k) = 1, \quad p^a_i (\beta^{a}_k) \geq 0$$

To shorten the notation, to delineate $p^a_i \Leftrightarrow p^a_i (\beta^{a}_k)$. Now, rewrite (3) as

$$\begin{cases} x_{k+1} = A_{i,k}x_k + F_{i,k-\tau}x_{k-\tau} + Bw_k, \\ y^a_k = C^a_{i,k}x_k + D^a v^a_k, \\ x_k = x_{i,k}, \quad k = -\tau, -\tau + 1, \ldots 0 \text{ and } i = 1, 2, \ldots, n \end{cases}$$

(4)

here

$$A_{i,k} = \sum^n_{i=1} p^a_i A_i$$

$$F_{i,k-\tau} = \sum^n_{i=1} p^a_i F_i$$

$$C^a_{i,k} = \sum^n_{i=1} p^a_i C^a_i$$

(5)

Subsequently, the communication topology is applied in distributed wireless sensors network and that is delineated $l^a_{in} \mid M_a$ subject to the degree of input of the node $a$.

The subsequent filter rule is taken into account for the SPLA discrete time random state space equations model (2)

**Filter Rule** : IF $\beta^{a}_{k,1}$ is $M^a_{j,1}$, and $\beta^{a}_{k,2}$ is $M^a_{j,2}$, and, ..., and $\beta^{a}_{k,m}$ is $M^a_{j,m}$ THEN

$$\tilde{x}^a_{k+1|k} = A_j \tilde{x}^a_{k|k} + F_j \tilde{x}^a_{k-\tau|k-\tau}$$

$$\hat{x}^a_{k+1|k} = \hat{x}^a_{k+1|k} + K^a_{k+1} (y^a_{k+1} - C^a_{k+1} \hat{x}^a_{k+1|k}) + H^a_{k+1} \sum_{b \in N_a} l_{ab} (y^a_{k+1} - C^a_{k+1} \hat{x}^a_{k+1|k})$$

(6)

where $\tilde{x}^a_{k|k} \in \mathbb{R}^n_x$ means that the estimation of $x_k$ at time instant $k$, $\hat{x}^a_{k+1|k} \in \mathbb{R}^n_x$ depicts the prediction related to error, $y^a_{k+b} (b = 1, 2, \ldots, a)$ is that the measurement of the filter device, $A_j$, $F_j$ and $C_j$ are constant matrices in system (4) and $K^a_{k+1}$ and $H^a_{k+1}$ are desired filter gains.

$$\tilde{x}^a_{k+1|k} = A_{j,k} \tilde{x}^a_{k|k} + F_{j,k-\tau} \tilde{x}^a_{k-\tau|k-\tau}$$

$$\hat{x}^a_{k+1|k} = \hat{x}^a_{k+1|k} + K^a_{k+1} (y^a_{k+1} - C^a_{j,k+1} \hat{x}^a_{k+1|k}) + H^a_{k+1} \sum_{b \in N_a} l_{ab} (y^a_{k+1} - C^a_{j,k+1} \hat{x}^a_{k+1|k})$$

(7)

were
\[
A_{j,k} = \sum_{j=1}^{n} p_j^a A_j,
\]
\[
F_{j,k-\tau} = \sum_{j=1}^{n} p_j^a F_j,
\]
\[
C_{j,k}^a = \sum_{j=1}^{n} p_j^a C_j^a
\]

As a result, the estimation error is calculated from \(\tilde{x}_{k+1}^a := x_k - \tilde{x}_{k+1}^a\) and filter prediction error is calculated \(\tilde{x}_{k+1}^a := x_{k+1} - \tilde{x}_{k+1}^a\). Furthermore \(P_{k+1}^a := \mathbb{E}\{\tilde{x}_{k+1}^a (\tilde{x}_{k+1}^a)^T\}\) represent the variance of the estimation error and \(P_{k+1}^a := \mathbb{E}\{\tilde{x}_{k+1}^a (\tilde{x}_{k+1}^a)^T\}\) represents the variance of filter.

3. Stochastic Parameter Learning Algorithm (SPLA) for various acceptable parameters

In this paper, we applied two acceptable parameter sequences \(K_{k+1}^a\) and \(H_{k+1}^a\) for algorithmic filter for (7) as:

(i) for SPLA system (4) subject to state delays, associated with the variance and
(ii) the distributed discrete filter parameters \(K_{k+1}^a\) and \(H_{k+1}^a\) errors has to be calculated through SPLA.

Lemma 3.1. Consider the matrix \(A = A^T > 0\), and the matrix results \(\xi_k(A) = \xi_k^T(A) \in \mathbb{R}^{m \times m}\) and \(\phi_k(A) = \phi_k^T(A) \in \mathbb{R}^{m \times m}\) and beyond the finite horizon \([0,N]\) based on time satisfies \(\xi_k(B) \geq \xi_k(A), \forall A \leq B = B^T\) and \(\phi_k(B) = \xi_k(A)\). Then, \(M_{k+1} = \xi_k(M_k), N_{k+1} = \phi_k(M_k), M_0 = N_0 > 0\) satisfying \(M_{k+1} \leq N_{k+1}\).

Lemma 3.2. In the projected system (4), we tend to construct \(X_{k+1} = \mathbb{E}\{x_{k+1} x_{k+1}^T\}\) because the variance matrix of state \(y\) is delineated as:

\[
X_{k+1} \leq (1 - \gamma_k) A_i X_k A_i^T + BB^T + (1 - \gamma_k^{-1}) F_i X_{k-\tau} F_i^T
\]

According to the idea of Lemma 2, the variance of the predicted noise is specified in the succeeding theorem.

Theorem 3.3. For the system (4) with algorithmic filter (7), the variance of the prediction error \(P_{k+1}^a\) is also the variances of the estimation error \(P_{k+1}^a\) are with subsequent ratio

\[
P_{k+1}^a = \mathbb{E}\{\tilde{x}_{k+1}^a (\tilde{x}_{k+1}^a)^T\}
\]

\[
= A_{i,k} P_{k-\tau} A_{i,k}^T + F_{i,k-\tau} P_{k-\tau} F_{i,k-\tau}^T + BB^T + (A_{i,k} - A_{j,k}) \mathbb{E}\{\tilde{x}_{k-\tau}^a (\tilde{x}_{k-\tau}^a)^T\} (A_{i,k} - A_{j,k})^T +
\]

\[
(\Phi_{i,k-\tau} - F_{i,k-\tau}) \mathbb{E}\{\tilde{x}_{k-\tau}^a (\tilde{x}_{k-\tau}^a)^T\} (\Phi_{i,k-\tau} - F_{i,k-\tau})^T + (A_{i,k} - A_{j,k}) \mathbb{E}\{\tilde{x}_{k-\tau}^a (\tilde{x}_{k-\tau}^a)^T\} (A_{i,k} - A_{j,k})^T +
\]

\[
A_{i,k} \mathbb{E}\{\tilde{x}_{k-\tau}^a (\tilde{x}_{k-\tau}^a)^T\} (A_{i,k} - A_{j,k})^T + (A_{i,k} - A_{j,k}) \mathbb{E}\{\tilde{x}_{k-\tau}^a (\tilde{x}_{k-\tau}^a)^T\} (A_{i,k} - A_{j,k})^T +
\]

\[
F_{i,k-\tau} \mathbb{E}\{\tilde{x}_{k-\tau}^a (\tilde{x}_{k-\tau}^a)^T\} (F_{i,k-\tau} - F_{j,k-\tau})^T + (F_{i,k-\tau} - F_{j,k-\tau}) \mathbb{E}\{\tilde{x}_{k-\tau}^a (\tilde{x}_{k-\tau}^a)^T\} (F_{i,k-\tau} - F_{j,k-\tau})^T
\]

(10)
\[ p_{k+1|k+1} = E \left\{ \hat{x}_{k+1|k+1}^a \left( \hat{x}_{k+1|k+1}^a \right)^T \right\} \]

\[ = \Gamma_{k+1|k}^{1,a} \, p_{k+1|k}^a \left( \Gamma_{k+1|k}^{1,a} \right)^T + K_{k+1}^a \, \Gamma_{k+1|k}^{2,a} (K_{k+1}^a)^T + K_{k+1}^a \, \Gamma_{k+1|k}^{3,a} (K_{k+1}^a)^T + H_{k+1}^a \, \sum_{b \in \mathcal{N}_a} l_{ab} \Gamma_{k+1|k}^{3,b} (H_{k+1}^a \sum_{b \in \mathcal{N}_a} l_{ab})^T + H_{k+1}^a \, \sum_{b \in \mathcal{N}_a} l_{ab} (C_{i,k+1}^b - C_{j,k+1}^b) X_{k+1} \times \]

\[ \left[ H_{k+1}^a \, \sum_{b \in \mathcal{N}_a} l_{ab} (C_{i,k+1}^b - C_{j,k+1}^b) \right] \left[ H_{k+1}^a \, \sum_{b \in \mathcal{N}_a} l_{ab} (C_{i,k+1}^b - C_{j,k+1}^b) \right]^T + H_{k+1}^a \, \sum_{b \in \mathcal{N}_a} l_{ab} (C_{i,k+1}^b - C_{j,k+1}^b) X_{k+1} \times \]

\[ \left[ H_{k+1}^a \, \sum_{b \in \mathcal{N}_a} l_{ab} (C_{i,k+1}^b - C_{j,k+1}^b) \right] \left[ H_{k+1}^a \, \sum_{b \in \mathcal{N}_a} l_{ab} (C_{i,k+1}^b - C_{j,k+1}^b) \right]^T + H_{k+1}^a \, \sum_{b \in \mathcal{N}_a} l_{ab} (C_{i,k+1}^b - C_{j,k+1}^b) X_{k+1} \times \]

\[ \left[ H_{k+1}^a \, \sum_{b \in \mathcal{N}_a} l_{ab} (C_{i,k+1}^b - C_{j,k+1}^b) \right] \left[ H_{k+1}^a \, \sum_{b \in \mathcal{N}_a} l_{ab} (C_{i,k+1}^b - C_{j,k+1}^b) \right]^T \]

\[ \text{where,} \]

\[ \Gamma_{k+1|k}^{1,a} = I - K_{k+1}^a C_{j,k+1}^b - H_{k+1}^a \sum_{b \in \mathcal{N}_a} l_{ab} C_{j,k+1}^b, \]

\[ \Gamma_{k+1|k}^{2,a} = (C_{i,k+1}^b - C_{j,k+1}^b) X_{k+1} (C_{i,k+1}^b - C_{j,k+1}^b)^T \]

\[ \Gamma_{k+1|k}^{3,a} = D^a (D^a)^T, \]

\[ \Gamma_{k+1|k}^{3,b} = D^b (D^b)^T \]

Subtract (6) from (4) and we have

\[ \hat{x}_{k+1|k}^a = x_{k+1} - \hat{x}_{k+1|k}^a \]

\[ = A_{k} \hat{x}_{k}^a + F_{i,k} \hat{x}_{k-\tau|k-\tau}^a + B_{w,k} + (A_{i,k} - A_{j,k}) \hat{x}_{k|k}^a + (F_{i,k-\tau} - F_{j,k-\tau}) \hat{x}_{k-\tau|k-\tau}^a \]

\[ \text{where } F_{i,k-\tau} \text{ is stands for filter gain. (10) are often obtained from the actual fact that } \hat{x}_{k|k}^a \text{ and } \hat{x}_{k-\tau|k-\tau}^a \]

are independent of the noise.

Subsequently, according to (4) and (7), we can derive

\[ \hat{x}_{k+1|k+1}^a = x_{k+1} - \hat{x}_{k+1|k+1}^a \]

\[ = x_{k+1} - K_{k+1}^a (y_{k+1} - C_{j,k+1}^a \hat{x}_{k+1|k+1}^a) - H_{k+1}^a \sum_{b \in \mathcal{N}_a} l_{ab} (y_{k+1} - C_{j,k+1}^a \hat{x}_{k+1|k+1}^a) \]

\[ \text{Note that } \hat{x}_{k+1|k+1}^a = x_{k+1} - \hat{x}_{k+1|k+1}^a \text{ we rewrite (13) as} \]

\[ \hat{x}_{k+1|k+1}^a = \left( 1 - K_{k+1}^a C_{j,k+1}^a - H_{k+1}^a \sum_{b \in \mathcal{N}_a} l_{ab} C_{j,k+1}^a \right) \hat{x}_{k+1|k+1}^a - K_{k+1}^a \left( C_{i,k+1}^a - C_{j,k+1}^a \right) X_{k+1} + D_{k+1}^a \]
By multiplying (14) by its regularity, we immediately obtain (12) by the fact that \( \mathbb{E}\{v_{k+1}\} = 0 \) and has no relation to the states of the system.

4. Numerical validation
To validate the efficiency of the state estimation technique designed for different SPLA systems, consider the following:

\[
\dot{x}_0(t) = -\frac{gsin(x_1(t)) + (2b/2m)x_2(t)}{4l/3 - am\cos^2(x_1(t))} + \frac{(am/2)x_2(t)\sin(2x_1(t))}{4l/3 - am\cos^2(x_1(t))}
\]

where \( x_1 \) and \( x_2 \) represent the initial and final state of SPLA systems and \( a = 1/(M + m) \) depends oscillation behaviors of the input and output; \( g \) describes the gravity constant (if the communicative signal is in the space network); \( 2l \) signifies that the duration of the signal; \( b \) represents the damping constant, \( v(t) \) denotes the disturbance; \( \tau \) express the delay in the steady state.

Suppose there are five different sensors with a direct communication then as a result within the limitation, the simulation parameters and results of nodes are:

\[
x_{k+1} = \sum_{i=1}^{3} h_i(x_{1,k}) A_i x_k + B w_k + \sum_{i=1}^{3} h_i(x_{1,k}) F_i, k - \tau x_{k-\tau},
\]

\[
y_k = \alpha_k \sum_{i=1}^{3} h_i(x_{1,k}) C_i x_k + D v_k
\]

were

\[
A_1 = \begin{bmatrix} 0.892 & 0.088 \\ -0.170 & 0.974 \end{bmatrix}, \quad F_1 = \begin{bmatrix} 0.892 & 0.088 \\ -0.170 & 0.974 \end{bmatrix}
\]

\[
A_2 = \begin{bmatrix} 0.897 & 0.298 \\ -0.357 & 0.973 \end{bmatrix}, \quad F_2 = \begin{bmatrix} -0.023 & 0.007 \\ 0.015 & -0.045 \end{bmatrix}
\]

\[
A_3 = \begin{bmatrix} 0.856 & 0.21 \\ 0.25 & -0.87 \end{bmatrix}, \quad F_3 = \begin{bmatrix} -0.005 & 0.005 \\ 0.05 & 0.08 \end{bmatrix}
\]

\[
C_1 = \begin{bmatrix} -0.32 \\ -0.2 \end{bmatrix}, \quad C_2 = \begin{bmatrix} -0.47 \\ 0.24 \end{bmatrix}
\]

\[
C_3 = \begin{bmatrix} 0.264 \\ -0.42 \end{bmatrix}, \quad B = \begin{bmatrix} 0.05 \\ 0.08 \end{bmatrix}
\]

\[
D = \begin{bmatrix} 0.05 \\ 0.08 \end{bmatrix}
\]

The distributed recursive filter [10] along with recursive sequence is depicted as [2]:

\[
\hat{x}_{k+1|k} = \sum_{i=1}^{3} h_j(\hat{x}_{1,k|k}) A_j \hat{x}_{k|k} + \sum_{i=1}^{3} h_j F_j(\hat{x}_{1,k-\tau|k-\tau}) \hat{x}_{k-\tau|k-\tau}
\]

\[
\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1} \times \left( y_{k+1} - \alpha \sum_{i=1}^{3} h_j(\hat{x}_{1,k+1|k}) C_j \hat{x}_{k+1|k} \right)
\]

Hence the system states and their estimates, confirms that the projected algorithmic rule works well in the SPLA systems' states.
5. Conclusion
In this paper, the advantage of discrete type distributed algorithmic filtering for a collection of discrete time random nonlinear systems subject to steady state and delays of the distributed wireless sensors was investigated. The SPLA model was introduced to get an exact the sequences of the signals and system. The distributed algorithmic filter was designed on the finite horizon to confirm a non optimal margin for the variance of the filtering error using two difference equations of Riccati type. Further discrete time random nonlinear systems were considered and applied various validation rule to enrich our SPLA model. Two different filter rules are taken into account for the SPLA discrete-time random nonlinear systems with state space equations model and Stochastic Parameter Learning Algorithm (SPLA) model. Predicted error and the estimated error are evaluated and validated through numerical examples.

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