Heavy quark diffusion in pre-equilibrium stage of heavy ion collisions

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The drag and diffusion coefficients of the heavy quarks have been evaluated in the pre-equilibrium phase which is expected to be formed in the early stages of the evolving fireball produced in heavy ion collisions at RHIC and LHC energies. The interaction of the probe with the gluon in the pre-equilibrium phase has been treated within the framework of perturbative QCD. For the pre-equilibrium gluon distribution function we have used the KLN and Classical Yang Mills (CYM) models. It is observed that the magnitude of both the transport coefficients have significant values in the pre-equilibrium phase and comparable to the magnitudes obtained for a system of kinetically equilibrated gluonic system. However, these values are larger than the value estimated for a chemically equilibrated quark gluon plasma. The results may have significant impact on the experimental observable like the suppression and elliptic flow of single electron spectra originating from the decays of heavy mesons produced in heavy ion collisions at RHIC and LHC energies.

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I. INTRODUCTION

The primary intent of the ongoing nuclear collision programmes at the Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC) energies is to create a new state of matter known as Quark Gluon Plasma (QGP), the bulk properties of which are governed by the light quarks and gluons. Heavy quarks (HQs \(\equiv\) charm and beauty) are produced at very early stage of Heavy Ion Collisions (HIC) due to the initial hard scatterings. Therefore, they experience the entire evolution of the medium created as an after-effect of HIC. Due to their large masses, heavy quarks can be considered as executing Brownian motion in the gluonic fluid, even if some significant correction may occur in the case of charm quarks \(1\). HQs has generated a significant interest in the recent past as a probe of the QGP medium (see \(2\) for a review). The suppression of the momentum distributions \(R_{AA}(p_{T})\) \(\equiv\) of charm quark at large momentum in the thermal medium and their elliptic flow \(v_{2}\) \(\equiv\) have been used as effective tools to characterize the system formed in HIC at RHIC and LHC collisions.

Several attempts have been made to study these factors within the framework of Fokker Plank equation \(\equiv\) and Boltzmann equation \(\equiv\) \(\equiv\) \(\equiv\). However, the roles of pre-equilibrium phase have been ignored in these works. The evolution of heavy flavors before the formation of QGP are currently approximated by free streaming. The final heavy meson spectra and elliptic flow may be affected by the presence of the pre-equilibrium phase because the HQs are produced in the early stage of the initial hard scatterings due to their large masses. The effect of the pre-equilibrium phase could be more significant at the low-energy nuclear collisions where the bulk will take a larger time to reach equilibrium.

The present work addresses the relevance of the pre-equilibrium phase produced in heavy ion collisions at RHIC and LHC energies in the context of the heavy flavour as a probe for characterizing QGP. Our motivation in this paper is to calculate the drag and diffusion coefficients when the HQs are interacting elastically with the non-equilibrated gluons constituting the medium formed in HIC.

The paper is organized as follows. In the next section we discuss the formalism used to evaluate the drag and diffusion coefficients of the heavy quarks in the pre-equilibrium era. Section III is devoted to present the results. Section IV contains summary and discussions.

II. FORMALISM

The Boltzmann Transport Equation (BTE) describing the evolution of the HQs can be written as:

\[
\left[ \frac{\partial}{\partial t} + \frac{\vec{p}}{E} \frac{\partial}{\partial \vec{x}} + \vec{F} \cdot \frac{\partial}{\partial \vec{p}} \right] f(\vec{x}, \vec{p}, t) = \left[ \frac{\partial f}{\partial t} \right]_{\text{collisions}}
\]

For two-body collisions, the right hand side of BTE, which is called the collision integral is given by:

\[
\left[ \frac{\partial f}{\partial t} \right]_{\text{collisions}} = \int d^3k [w(\vec{p}+\vec{k}, \vec{k}) f(\vec{p}+\vec{k}) - w(\vec{p}, \vec{k}) f(\vec{p})].
\]

Here, \(w(\vec{p}, \vec{k})\) is the rate of collision of HQ which changes its momentum from \(\vec{p}\) to \(\vec{p} - \vec{k}\), can be expressed as:

\[
\omega(p, k) = gC \int \frac{d^3q}{(2\pi)^3} f'(q) v_{\sigma, p,q \rightarrow k,q+k}
\]

where \(f'\) is the phase space distribution of the particles in the bulk, \(v\) is the relative velocity between the two collision partners, \(\sigma\) denotes the cross section and \(gC\) is the statistical degeneracy.

Considering only the soft scattering between the two collision partners \(\equiv\), the integro-differential Eq. \(\equiv\) re-
duced to the Fokker Planck equation:
\[
\frac{\partial f}{\partial t} = \frac{\partial}{\partial p_i} \left[ A_i(p) f + \frac{\partial}{\partial p_j} [B_{ij}(p) f] \right],
\]
(4)
where the kernels are defined as
\[
A_i = \int d^3k w(\vec{p}, \vec{k}) k_i,
\]
(5)
and
\[
B_{ij} = \frac{1}{2} \int d^3k w(\vec{p}, \vec{k}) k_i k_j.
\]
(6)
for \( p \rightarrow 0 \), \( A_i \rightarrow \gamma p_i \) and \( B_{ij} \rightarrow D \delta_{ij} \) where \( \gamma \) is the drag coefficient and \( D \) is the diffusion coefficient. We notice that while the drag and diffusion coefficients are evaluated with \( f'(q) \) in Eq. (3) that is in thermal equilibrium, the derivation of the Fokker-Planck equation is more general and valid for a generic distribution function for the bulk matter.

Our motivation in this paper is to calculate these coefficients for heavy quarks having elastic collisions with the non-equilibrated gluons in the pre-equilibrium stage of the heavy ion collisions. The drag and diffusion coefficients of the heavy quarks propagating through the pre-equilibrium phase consisting of gluons have been evaluated using pQCD. For the elastic scattering of the HQ of momentum \( p_1 \) with a non-equilibrated gluon, \( g \) of momentum \( p_2 \) i.e. for the process, \( HQ(p_1) + g(p_2) \rightarrow HQ(p_3) + g(p_4) \), the drag coefficient, \( \gamma \) can be expressed in terms of \( A_i \) as \( \gamma = p_i A_i / p^2 \) (see also \( \gamma \)).

\[
\gamma = p_i A_i / p^2
\]
(7)
where \( A_i \) takes the form
\[
A_i = \frac{1}{2 E_{p_1}} \int \frac{d^3p_2}{(2\pi)^3 E_{p_2}} \int \frac{d^3p_4}{(2\pi)^3 E_{p_4}} \int \frac{d^3p_3}{(2\pi)^3 E_{p_3}} \frac{1}{g_{HQ}} \sum |M|^2 (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) f(p_2) \left\{ 1 + f(p_4) \right\} \langle (p_1 - p_3) \rangle \equiv \langle \langle (p_1 - p_3) \rangle \rangle.
\]
(8)
The \( g_{HQ} \) denotes the statistical degeneracy of HQ, \( f(p_2) \) is the momentum distribution of the of the gluons in the initial channel (bulk). Similarly, \( 1 + f(p_4) \) represents momentum distribution of the gluons with Bose enhancement effects in the final channel. \( |M|^2 \) represents the modulus square of the spin averaged invation amplitudes for \( HQ + g \) elastic scattering process. The drag coefficient (in Eq. (3)) is the measure of the average of the momentum transfer, \( p_1 - p_3 \) weighted by the interaction through \( |M|^2 \).

Similarly the diffusion coefficient, \( D \), can be defined as:
\[
D = \frac{1}{4} \left[ \langle \langle p_i^2 \rangle \rangle - \langle \langle p_1 \cdot p_3 \rangle \rangle \right] / p_1^2
\]
(9)
The following expression can be used to evaluate the drag and diffusion coefficients by appropriately choosing, \( Z(p_3) \),
\[
\ll Z(p_1) \gg = \frac{1}{512 \pi^4} \frac{1}{E_{p_1}} \int_0^\infty \frac{p_3^3 dp_3 d(cos \chi)}{E_{p_2}}
\]
\[
\hat{f}(p_2) \{ 1 + f(p_4) \} \frac{\lambda^4(s, m_{p_1^2}, m_{p_2}^2)}{\sqrt{8}} \int_1^\infty d(cos \theta_{c.m.})
\]
\[
\frac{1}{g_{HQ}} \sum |M|^2 2\pi d\phi_{c.m.} Z(p_3)
\]
(10)
where \( \lambda(s, m_{p_1^2}, m_{p_2}^2) = \{ s - (m_{p_1} + m_{p_2})^2 \} \{ s - (m_{p_1} - m_{p_2})^2 \} \).

### III. Initial Conditions

In most of the earlier works the distribution functions, appearing in Eq. (10) are taken to be an equilibrium distribution for either quarks, anti-quarks (Fermi-Dirac) or gluons (Bose-Einstein); the transport coefficients are then calculated when the HQ is travelling through the thermalised medium, the latter assumed to be formed within the time scale 0.3 - 1 fm/c after the collisions, depending on the center of mass energy of the HIC. In this article instead we will focus on HQ propagation in the pre-thermalized medium which, according to the general picture of the pre-equilibrium stage, is usually assumed to be formed mainly by gluons; in particular we evaluate the transport coefficients in the pre-equilibrium stage. This can be achieved by substituting the distribution functions in Eq. (10) by pre-equilibrium distributions computed according to a model of the pre-equilibrium stage which we specify shortly after; we chose the normalization of the several distributions used in this work in order to obtain the same total gluon number of thermal distribution at the temperature of the thermal medium, which we assume to be equal to \( T = 0.34 \text{ GeV} \) for the case of collisions at RHIC energy and \( T = 0.51 \text{ GeV} \) for collisions at LHC energy. We then compare the drag and diffusion coefficients with those obtained in the thermalized phase at the aforementioned initial temperature.

We now specify the models we consider in this article for the pre-equilibrium stage. According to the general understanding of the dynamics of pre-equilibrium stage the initial strong gluon fields (the glasma) shatters into gluon quanta in a time scale which is of the order of the inverse of the saturation scale \( Q_s \); therefore any model of the pre-equilibrium stage has to include in some way the saturation scale. The first one we consider corresponds to an initial distribution computed from the classical Yang-Mills (CYM) gluon spectrum \( [24] \), which assumes the initial gluon fields can be expanded in terms of massless gluonic excitations. Since we do not perform CYM calculations instead using results obtained in \( [24] \), we refer to this article for further details.

Beside CYM we consider the model known as factorized KLN model \( [30, 31] \) which includes the saturation
FIG. 1: Variation of drag coefficient as a function of momentum for charm quark at RHIC energy. The result that corresponds to thermal gluons (kinetic equilibrium) and thermal quarks and gluons (chemical equilibrium) are evaluated at a temperature 340 MeV.

FIG. 2: Variation of diffusion coefficient as a function of momentum for charm quark at RHIC energy. The result that corresponds to thermal gluons (kinetic equilibrium) and thermal quarks and gluons (chemical equilibrium) are evaluated at a temperature 340 MeV.

FIG. 3: Variation of $D/A$ as a function of momentum for charm quark at RHIC energy. The result that corresponds to equilibrium cases are evaluated at a temperature 340 MeV.

FIG. 4: Same as Fig. 1 using Eq. 13.
phase transport coefficients with the equilibrated phase transport coefficients (both kinetic and chemical equilibrium) keeping the number of particles fixed in both the phases. In this work the temperature dependence of the strong coupling $\alpha_s$ is taken from Ref. [32] corresponding to temperature $T=0.34$ GeV for the QGP phase for the sake of comparison. We have used the same coupling for both the equilibrium and pre-equilibrium phase. The Debye screening mass, $m_D^2 = 8\alpha_s(N_c + N_f)T^2/\pi$, needed to shield the infra-red divergence associated with the t-channel scattering amplitude is estimated at $T = 0.34$ GeV for RHIC energy for the kinetic equilibrium. Here $N_c$ and $N_f$ are the number of color and flavors respectively. For both the pre-equilibrium and equilibrium phase we are using the same Debye screening mass.

In Fig.5 we have shown the variation of drag coefficient of the charm quark as a function of momentum in the pre-equilibrium phase for two different (initial) gluon distributions and compared the results with the equilibrated phase (both kinetic and chemical) at $T=0.34$ GeV. It has been found that the magnitude of the drag coefficient in the pre-equilibrium phase is significant and is the same order of magnitude of the kinetic equilibrated phase, indicating substantial amount of interaction of the charm quarks with the pre-equilibrated gluons. We observed that the drag for chemically equilibrated QGP is smaller (dotted line) than the (purely) gluonic system(solid line). To keep the total number of particles fixed some of the gluons in the purely gluonic system has to be replaced by quarks in the (chemically equilibrated) QGP and the cross section for charm quark-light quarks interaction is smaller than charm quark-gluon interaction which results in lower drag for QGP. Later in this section we will display results where the Debye screening mass is calculated in a more consistent way. Between CYM and KLN distributions, the CYM gives rise to a larger value of the drag coefficient in comparison with the KLN case. The KLN provides harder momentum gluon distribution, hence having smaller momentum difference with the charm quark than the CYM case; as the drag coefficient is a measure of the momentum transfer weighted by the interaction strength, the KLN gives rise to lower drag compared to the drag obtained by the CYM distribution.

The variation of the diffusion coefficient of charm quarks with momentum is depicted in Fig.6 for both the
We observed that the KLN has harder momentum distributions than the CYM case, hence having larger width. Through interaction the probe will acquire the average width of the bulk; since width is the measure of the diffusion coefficient, heavy flavors diffuse faster (larger diffusion) in case of KLN medium. This is reflected in the results displayed in Fig. 2.

In Fig. 5 we have depicted the diffusion to drag ratio, $D/\gamma$ as function of momentum. $D/\gamma$ can be used to understand the deviation of the calculated values from the value obtained by using Fluctuation-Dissipation theorem (FDT). Since the KLN has a harder momentum distribution, results obtained from KLN input deviates more from FDT.

From here after we will discuss results using Debye screening mass that has been calculated self consistently by from the expression below:

$$m_D^2 = \pi \alpha_s g_G \int \frac{d^3 p}{(2\pi)^3} \frac{1}{p^2} (N_c f_g + N_f f_q)$$  \hspace{1cm} (13)

where $g_G$ is the gluon degeneracy factor.

In Fig. 8 and 9 we have shown the variation of the drag and diffusion coefficients, respectively, using the Debye screening mass evaluated using Eq. 13 appropriate gluon (and quark for the QGP case) distributions. It is interesting to note that when this done then we obtain similar values of the drag coefficients for all the three different distributions (KLN, CYM and thermal (g)).

The drag and diffusion coefficients for bottom quarks in the pre-equilibrium phase are displayed in Fig. 10 and 11, respectively, showing behavior qualitatively similar to that of charm quarks.

In Fig. 11 we have shown the variation of drag coefficient of the charm quark as a function of momentum in the pre-equilibrium phase for LHC energy. The drag coefficient in the pre-equilibrium era for LHC collision condition has also been compared with the equilibrium drag coefficient evaluated at $T=0.51$ GeV. We have used the same coupling for both the equilibrium and pre-equilibrium phase. The value of $m_D$ is taken from Eq. 13 for all the cases. We find that the magnitude of the drag coefficient in pre-equilibrium phase is similar to that obtained for the kinetic equilibrium case. The variation of the diffusion coefficient of charm quarks with momentum is plotted in Fig. 9 at LHC energy. The drag and diffusion coefficients for bottom quarks at the LHC energy in the pre-equilibrium phase are displayed in Fig. 10 and 11, respectively, showing behavior qualitatively similar to that of charm quarks.

V. SUMMARY AND DISCUSSIONS

In this article we have studied the drag and diffusion coefficients of heavy quarks in the pre-equilibrium phase...
of relativistic heavy ion collisions at the RHIC and LHC energies, where the medium mainly consists of gluons. For the pre-equilibrium distribution we have used two different initial conditions i.e. CYM and KLN.

We have compared the pre-equilibrium phase transport coefficients with the equilibrated phase transport coefficients keeping the number of particles fixed in both the phases. We have found that the magnitude of the transport coefficients in the pre-equilibrium phase is comparable to (or in some cases even more than) the values obtained with a thermal gluonic system. The pre-equilibrium drag and diffusion coefficients are much larger in magnitude than the thermal QGP case. This is due to the smaller cross section of the heavy quark + light quark interaction than heavy quark + gluon interaction for fixed number of particle.

The results may have significant impact on the experimental observable like the suppression of single electron spectra originating from the decays of heavy mesons as well as on their elliptic flow. We will address these aspects in future works.

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