RESIDUAL CONFINEMENT
IN HIGH-TEMPERATURE YANG-MILLS THEORY*

A. MAAS, J. WAMBACH
Darmstadt University of Technology, Schloßgartenstraße 9, D-64289 Darmstadt, Germany

B. GRÜTER, R. ALKOFER
Tübingen University, Auf der Morgenstelle 14, D-72076 Tübingen, Germany

The infrared behavior of Landau gauge gluon and ghost propagators are investigated in Yang-Mills theory at non-vanishing temperatures. Self-consistent solutions are presented for temperatures below the presumed phase transition and in the infinite temperature limit. Gluon confinement is manifest in the infrared behavior of these propagators. As expected confinement prevails below the phase transition. In the infinite-temperature limit a qualitative change is observed: the chromoelectric sector exhibits a near-perturbative behavior while long-range chromomagnetic interactions, mediated by soft ghost modes, are still present. The latter behavior is in agreement with corresponding lattice results. It furthermore implies that part of the gluons are still confined.

1. Introduction

It is by now well established that QCD undergoes a phase transition or at least a rapid crossover at some critical temperature. The naive expectation is that the novel kind of matter formed above the phase transition is a plasma of deconfined gluons and quarks reaching ideal gas behavior at very high temperatures. Recent lattice calculations (see e.g. Cucchieri et al.1) have raised doubts about this simple picture. In addition, it is known that the dimensionally reduced theory, to which the equilibrium state evolves in the infinite temperature limit, is confining. The present work further investigates the properties of finite- and high-temperature QCD within the Dyson-Schwinger approach2.

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The following investigations are restricted to pure Yang-Mills theory as substantial evidence exists that the non-perturbative features of QCD are generated in the gauge sector. Hence, the theory studied here is an equilibrium Yang-Mills theory governed by the Euclidean Lagrangian
\[ \mathcal{L} = \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \bar{c}^a \partial_\mu D^{ab}_\mu c^b \]
with the field strength tensors \( F_{\mu\nu}^a \) and the covariant derivative \( D^{ab}_\mu \). \( A^a_\mu \) denotes the gluon field and \( \bar{c}^a \) and \( c^a \) are the ghost fields describing part of the quantum fluctuations of the gluon field. The gauge chosen is the Landau gauge which is best suited for the purpose at hand.

The gluon and ghost fields are described by their respective two-point Green’s functions or propagators. Their infrared properties are linked to the presence of confinement. Especially, a particle is absent from the physical spectrum if its propagator \( D(q^2) \) vanishes in the infrared
\[ \lim_{q \to 0} D(q^2) = 0. \tag{1} \]

### 2. Results

To present results we first define dressing functions from the propagators:
\[ D_G(q) = -\frac{G(a_0^2, q^2)}{q^2}, \]
\[ D_{\mu\nu}(q) = P^{T}_{\mu\nu}(q) \frac{Z(a_0^2, q^2)}{q^2} + P^{L}_{\mu\nu}(q) \frac{H(a_0^2, q^2)}{q^2}. \]

Here \( G \) denotes the ghost dressing function, \( Z \) and \( H \) the ones for gluons being transverse or longitudinal with respect to the heat bath. To obtain these functions the truncated set of Dyson-Schwinger equations displayed in Fig. 1 have been solved\(^3,4,5\). The necessity to truncate the infinite set of coupled Dyson-Schwinger equations generates several problems with gauge invariance. These have been dealt with and the errors are, at least qualitatively, under control.

Calculations have been performed at \( T = 0^3, T < T_c^4 \) and \( T \to \infty^5 \). Those at \( T < T_c \) have used a toroidal discretized space while the others were done in the continuum. Note that, for \( T \to \infty \), the theory becomes dimensionally reduced to a three-dimensional (3d) Yang-Mills theory plus an adjoint Higgs field, where \( Z \) corresponds to the 3d gluon (the chromomagnetic sector of the original 4d theory) and \( H \) to the Higgs (the chromoelectric sector of the original 4d theory).
Figure 1. The truncated Dyson-Schwinger equations at finite temperature. The dotted lines denote ghosts, the dashed lines longitudinal gluons and the wiggly lines transverse gluons. Lines with a full dot represent self-consistent propagators and small dots indicate bare vertices. The open circled vertices are full and must be constructed in a given truncation scheme. A bare ghost-gluon vertex and slightly modified bare gluon vertices have been used. Note that for soft modes the ghost-longitudinal and the 3-point coupling of three longitudinal and of one longitudinal and two transverse gluons vanish.

Solutions for the ghost dressing function are shown in Fig 2. It is found that they are not affected by temperature qualitatively. The divergence at zero momentum indicates the mediation of long-range forces which are still present at $T \to \infty$. In addition, this divergence is connected to the confinement mechanism$^2$ and thus indicates the presence of some residual gluon confinement in the high-temperature phase.

Figure 2. The ghost dressing function compared to lattice results$^6$: The left panel is for $T < T_c$ while the right panel shows $T \to \infty$. $g_3$ is the dimensionful coupling constant in the dimensionally reduced theory.

The results for the transverse gluon are shown in Fig. 3. Below the phase transition, the propagator exhibits confinement by virtue of condition (1). It becomes steeper in the infrared with increasing temperature.
In the high-temperature phase there is no qualitative difference. Hence the transverse gluon propagator also shows explicitly the presence of residual confinement which can be interpreted as over-screening. In fact, the temperature dependence is very slight. The lattice data at $T = 0$ show strong finite-volume effects in the infrared and are expected to bend over for larger volumes as is the case in the dimensionally reduced theory, where significantly larger lattices can be employed.

For the longitudinal gluon propagator the results are shown in Fig. 4. At $T = 0$, $H = Z$ and thus longitudinal gluons are confined as well. At non-zero temperature, $H$ becomes shallower in the infrared, and is thus severely affected by finite-volume effects. It, however, still exhibits confinement according to volume-scaling studies. In the high-temperature phase, the situation is much different. In contrast to the transverse sector, it shows screening and behaves similar to a massive particle. Nevertheless a comparison to lattice results and perturbation theory indicates that the Higgs propagator contains sizeable non-perturbative effects.

3. Conclusions
We have analyzed the infrared behavior of gluon and ghost propagators employing Dyson-Schwinger equations. At vanishing and small temperatures the results show manifestly gluon confinement. It is found that, even in the $T \to \infty$ limit, strong long-range correlations are present, leading to a non-perturbative behavior of the soft modes. Part of the gluons are
still confined: The Gribov-Zwanziger scenario (see e.g. Zwanziger\textsuperscript{9}) applies at all temperatures. These results, together with the one of lattice calculations, demonstrate the presence of strong non-perturbative effects even at high temperatures. In the infrared sector of a Yang-Mills theory soft quantum fluctuations win over thermal fluctuations at all temperatures.

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