Comment on the stability of the Yukawa couplings and the cosmological problems of intersecting brane models

Tomohiro Matsuda

Laboratory of Physics, Saitama Institute of Technology, Fusaiji, Okabe-machi, Saitama 369-0293, Japan

Abstract

In string theory, stabilization of moduli fields and their cosmological implications have been discussed by many authors. In this paper we do not consider conventional modulus, nor relative distance between two branes. We focus our attention to a relative position of three intersecting branes. Surprisingly, there had been no phenomenological argument on the stabilization of such moduli. We will show that the area of the corresponding triangle is not a free parameter, but an effective potential is generated from conventional loop corrections in the low energy effective theory. Of course, the stabilization does not induce any serious problem, because one is allowed to modify other parameters of the model to adjust the Yukawa couplings. Then the stabilization puts a constraint that is a different nature from the ones that have been discussed before. We also discuss cosmological problems and show a simple idea that can solve the problem.

\textsuperscript{1}matsuda@sit.ac.jp
1 Stability and cosmological problems of intersecting brane models

In spite of the great success of quantum field theory and classical Einstein gravity, there is still no consistent unification scenario in which quantum gravity is successfully included. Perhaps the most promising scenario in this direction is string theory, in which consistency of the quantum gravity is ensured by a requirement of additional dimensions. Originally the size of extra dimensions was assumed to be as small as $M_p^{-1}$. However, later observations showed that there is no reason to require such a tiny compactification radius. In this respect, what we had seen in the old string theory was a tiny part of the whole story. In the new scenario, the compactification radius (or the fundamental scale) is an unknown parameter that should be determined by observations. In models with large extra dimensions, the observed Planck mass is obtained by the relation $M^2_p = M^2_\ast + 2V_n$, where $M_\ast$ and $V_n$ denote the fundamental scale of gravity and the volume of the $n$-dimensional compact space. In this scenario the standard model fields are expected to be localized on a wall-like structure and the graviton propagates in the bulk. The most natural embedding of this picture in the string theory context is realized by a brane construction. Thus it is quite important to construct models of the brane world where the observed fermion spectrum of the standard model is included in the low energy effective theory. In this respect, chirality of the fermions and the family replication are two of the most important characteristics of the standard model, which must be included in the fundamental theory. Possiblities for fermion chirality in brane construction are already discussed by many authors. One of the examples is to locate D3-branes on a orbifold singularity. An alternative is discussed in, where Dp-branes are put intersecting at non-vanishing angles. Considering open strings stretched between them, strings living at the intersecting points become chiral fermions in four-dimensional effective Lagrangian. Phenomenological aspects of intersecting brane world are discussed in ref. and many authors.

In this paper we discuss stabilization and cosmological aspects of the Yukawa couplings in the intersecting brane models. Here we do not pretend to make it clear the whole story of the cosmological evolution, nor to discuss all the possible problems, but simply focus on
some cosmological criteria that will become important in generic situations. Of course, we know that sometimes cosmology of the models for the braneworld (or models with large extra dimensions) seems quite peculiar.\textsuperscript{2} In any case, we know historically that the characteristic features of phenomenological models are revealed by discussing their cosmological problems. Then it will be quite natural to consider a question, “What is the characteristic problem of the intersecting brane models that would be induced by cosmology?” In this paper we will discuss one of the possible answers for the above question.

In the models of the intersecting branes, several kinds of moduli fields are expected to exist at low energy effective theory. Some of them might be fixed by the natural mechanisms and integrated away from the effective theory, while others might not.\textsuperscript{3} In this paper we focus our attention to the Yukawa couplings, which come from the instantons of triangles bounded by three branes. It was noticed in ref.\textsuperscript{[4, 12]} that Yukawa couplings among three fields living at brane intersections will arise from the calculation of worldsheet instantons involving three boundary conditions.

As we will see in the next section, surface area of a triangle depends also on the volume and the structure of the compactification, which could be fixed by some known mechanisms. Thus we consider in this paper the most optimistic scenario in which the geometry of the compactified space is already fixed and does not induce another problem. In this case the ambiguity of the area of the triangle appears only through the shift of the relative position of the three branes, which cannot be fixed by the conventional mechanism for the moduli stabilization.

For the distance between two branes, it is known that an effective potential is generated when supersymmetry is broken. This potential is a simple perturbative effect of the string, but cannot stabilize the relative position of intersecting branes, because the distance is already 0 for such branes. On the other hand, it was noted in ref.\textsuperscript{[12]}, that in the context of the intersecting brane world, Yukawa couplings are generated by the

\textsuperscript{2}Constructing successful models for inflation with a low fundamental scale is still an interesting problem\textsuperscript{[6, 7]}. Baryogenesis and inflation in models with a low fundamental scale are discussed in \textsuperscript{[8, 9, 10]}. We think constructing models of particle cosmology with large extra dimensions is very important since future cosmological observations would determine the fundamental scale of the underlying theory.

\textsuperscript{3}Shift of a gauge coupling during inflation is discussed in \textsuperscript{[11]}. 
worldsheet instantons involving three different boundary conditions and three different intersections. In this paper we will discuss that the effective potentials generated through conventional fermion loop corrections stabilize the Yukawa couplings. The stabilization of the Yukawa couplings is good news for the intersecting brane models, although it forces some modifications to the previous models. In this paper, however, we also point out that the cosmological problems might remain even if one considers the above mechanism for the stabilization. One reason is that generic models of intersecting brane world contain at least three triangles of intersecting branes that correspond to three generations, which might lead to three degenerated vacua where each triangle shrinks. Then it is natural to worry about the problem of cosmological domain walls that interpolate between two of the three degenerated vacua. Even if the fluctuations of the corresponding massless modes are suppressed, the allowed initial condition for the brane configuration is quite restricted. One way to avoid these problems is to include at least one “large” correction to the effective potential that involves the areas of the triangles. Such a potential will stabilizes the brane configuration during cosmological evolution of the early Universe. Unfortunately, in this case one should have to worry about the significant correction that might affect the effective Yukawa couplings in the standard model. At present, it seems impossible to include such corrections within the setups of the conventional intersecting brane models. Another way is to assume that the translational invariance is explicitly broken at the beginnings and there is no freedom for the brane positions to be shifted. In this case, one must answer why branes are fixed at the present positions. Explicit breaking of the reparametrization invariance would be possible, but it requires significant modification of the original scenario. From the discussions above, it seems rather difficult to construct a model where the present brane configuration is ensured by introducing another mechanisms for the stabilization. Alternatively, one can assume that the three generations are not equal but slightly different in their geometrical settings, so that the cosmological domain walls become unstable even if they are produced. The last example seems to be the most realistic. For example, one may assume “tiny warping” that resolves the degeneracy of the three vacua and destabilizes the domain wall. In this case, our criteria will merely put a lower bound for the warping factor. Let us explain the situation

\[4\] Of course the situation is much better than that without any mechanism for the stabilization.
in more detail. The areas of the triangles depend on the parameter $A$ in eq. (2.4), which represents the Kähler structure of the torus. If the parameter $A$ is slightly warped so that it depends on the generation, it induces energy difference in (2.6). In this case, our criterion for the unstable domain walls puts a lower bound on the magnitude of the energy difference between different vacua. For $\sigma \geq (10^5 \text{GeV})^3$, the required energy difference $\epsilon$ is

$$\epsilon > \frac{\sigma^2}{M_p^2}$$

where $\sigma$ is the tension of the domain wall. When the tension of the domain wall is smaller than $(10^5 \text{GeV})^3$, one should consider another bound from the nucleosynthesis,

$$\epsilon > \lambda \sigma M_{EW}^2 / M_p,$$

where $\lambda \geq 10^{-7}$ is required, and $M_{EW}$ is the electroweak scale. In the present model, the tension of the domain wall is about $\sigma \simeq M_{EW}^3$ and the energy difference $\epsilon$ is induced by the shift of the Kähler structure $A$, if $A$ is warped.\(^5\) Then it is straightforward to calculate the required bound for the warp factor from eq. (1.2) and (2.6).\(^6\) At present one cannot remove the theoretical uncertainties concerning the prefactors and numerical factors in the exponents, but the result suggests that the tiny shift in the Kähler structure $A$ is enough to remove the unwanted domain walls. Since the required warping is tiny, one can expect that the small perturbation induced by the additional flavor-violating components might solve the problem, which is ignored so far. Of course, the problem is not confined to the issue of the stability of the Yukawa couplings, but should be solved including the stabilization of the whole moduli fields, which should be discussed in the forthcoming papers.

One might also think that the relative position of the intersecting three branes could be fixed without introducing any mechanism for the stabilization, if the homogeneous initial condition is achieved during inflation. In this case, however, it is difficult to con-

\(^5\)To be more precise, here the Kähler structure $A$ is supposed to distinguish the generation. This can be realized by introducing additional breaking of the flavor symmetry, or by assuming that the positions of the triangles are weakly fixed on the warped manifold.

\(^6\)Here the energy difference is induced by the light fermions whose Yukawa couplings depend explicitly on $A$. Since the area of the heaviest fermion vanishes in each vacuum, the Yukawa coupling (2.5) of the heaviest fermion can not depend on $A$.\)
struct models where the fluctuations of the positions of the branes are safely suppressed throughout inflation and reheating. In models where the electroweak symmetry is spontaneously broken by radiative corrections, there is a lower bound for the largest Yukawa coupling, which suggests that the initial condition (and its fluctuation) must be finely tuned to be within the restricted area. Obviously, phenomenological bound is so tight that the unnatural fine tuning is required, even if our mechanism for the stabilization works. Otherwise, the electroweak symmetry breakdown does not start because none of the fermions develops $O(1)$ Yukawa coupling.

In any case, changes are required for the present models of the intersecting brane world. Of course, the situation is much better than the models where stabilization of the Yukawa couplings is ignored. As we have discussed above, once the effect of the stabilization is included, a small warp factor may solve the problem of the initial brane condition, because it can destabilize the unwanted domain walls.

2 1-loop potential and vacuum degeneracy

In this section we consider the simplest example in ref.[4, 12]. What we would like to see is the Yukawa couplings in the quark sector. The Yukawa coupling among two chiral fermions and one Higgs boson cannot appear from perturbative effects of the string theory, but induced by worldsheet instanton corrections of the corresponding triangle that has three boundaries of the intersecting branes and three vertices where matter fields live.

Here we consider the simplest case and derive the expression for Yukawa couplings. When computing a sum of worldsheet instantons, the simplest example comes from D-branes wrapping 1-cycles in a $T^2$, where branes are intersecting at one angle. Here we associate each brane to complex number $z_\alpha$, $(\alpha = a, b, c)$,

\[
\begin{align*}
  z_a &= R \times (n_a + \tau m_a) \times x_a \\
  z_b &= R \times (n_b + \tau m_b) \times x_b \\
  z_c &= R \times (n_c + \tau m_c) \times x_c.
\end{align*}
\]

Here $(n_\alpha, m_\alpha) \in \mathbb{Z}^2$ denote the 1-cycle the brane $\alpha$ wraps on $T^2$ and $x_\alpha \in \mathbb{R}$ is an arbitrary
number. \( \tau \) is the complex structure of the torus. These branes are given by a straight line in \( \mathbb{C} \). The triangle corresponding to a Yukawa coupling must involve three branes, which has the form \((z_a, z_b, z_c)\) with \(z_a + z_b + z_c = 0\). The solution is

\[
x_\alpha = I_{\beta \gamma} x / d,
\]

where \( x = x_0 + l, x_0 \in \mathbb{R}, l \in \mathbb{Z} \) and \( d = \text{g.c.d.}(I_{ab}, I_{bc}, I_{ca}) \). Here \( I_{\beta \gamma} \) stands for the intersection number of branes \( \beta \) and \( \gamma \). Indexing the intersection points, one can obtain a simple expression for \( x_0 \),

\[
x_0(i, j, k) = \frac{i}{I_{ab}} + \frac{j}{I_{cb}} + \frac{k}{I_{ca}} + \frac{I_{ab} \epsilon_c + I_{ca} \epsilon_b + I_{bc} \epsilon_a}{I_{ab} I_{bc} I_{ca}},
\]

where the parameter \( \epsilon_\alpha \) correspond to shifting the positions of the three branes. Using this solution, one can compute the areas of the triangles whose vertices lie on the triplet of intersections \((i, j, k)\),

\[
A_{ijk}(l) = \frac{1}{2} (2\pi)^2 |I_{ab} I_{bc} I_{ca}| \left( x_0(i, j, k) + l \right)^2
\]

where \( A \) represents the Kähler structure of the torus. The corresponding Yukawa coupling is given by

\[
Y_{ijk} \sim \sigma_{abc} \sum_{l \in \mathbb{Z}} \exp \left( -A_{ijk}(l) \right),
\]

where \( \sigma_{ijk} = \text{sign}(I_{ab} I_{bc} I_{ca}) \) is a real phase.

Now our question is how one can determine the areas of the triangles. A perturbative force between branes can produce potential for the distance between two branes. However, it is obvious that this force cannot affect the area of a triangle when branes are intersecting. On the other hand, one can see from eq. (2.5) that almost all the parameters are determined if the windings of the branes and the structure of the manifold are fixed by some mechanisms. The only ambiguity that might remain at low energy effective theory is one parameter of three \( \epsilon_\alpha \), which corresponds to shifting the relative brane position. For the area of a triangle, only one of the three parameters \( \epsilon_\alpha \) is independent.

An effective potential for the area of a triangle is obtained by considering a well-known 1-loop correction from fermion loops of the form\[14\]

\[
\Delta V(\phi_c) = -\frac{3}{64\pi^2} V_{ijk}^4 \phi_c^4 \ln \left( \frac{\phi_c^2}{\mu^2} \right),
\]

\[\text{See ref.}[12] \text{ for more detail.}\]
where \( \phi_c \) denotes the classical field. From eq. (2.6) and (2.3), one can easily see that the 1-loop correction stabilizes the area of the triangle.

Because of the exponential form of the potential, intersecting branes are stabilized when one of the areas of the three triangles vanishes. In general, models for the intersecting branes are designed so that the three triangles cannot shrink simultaneously to a point, so that they satisfy the phenomenological requirements. Thus it is straightforward to construct the models in which one of the three Yukawa couplings becomes large, while others remain (hierarchically) small.

### 3 Conclusions and Discussions

Stabilization of the moduli fields and their cosmological implications are quite important in any phenomenological models of string theory. In this paper we examined a peculiar cosmological problem of the moduli field in models of the intersecting brane world. We focused our attention to a relative position of three branes and discussed its cosmological problems. The relative position of the intersecting three branes is not determined by conventional perturbative effects. To obtain an effective potential for the corresponding moduli, one should consider string effects that have more than two boundaries. In models of the intersecting brane world, instanton effects that have three boundaries play an important role in determining Yukawa couplings. We considered a simple model and discussed how the relative position among three intersecting branes appears in the effective potential. We showed that the effective potential that contains the area of the triangle is generated by the 1-loop corrections of fermions, which involve the Yukawa couplings. Cosmological aspects of the model are also discussed in this context. The constraints on the models become slightly stronger if one includes the stabilization of the Yukawa couplings, since the area of the triangle is no longer a free parameter of the model. However, once the stabilization is included, there appears a chance to solve the cosmological problem of the initial brane configuration. As we have discussed, a tiny

\textsuperscript{9}Note that the triangle of the largest Yukawa coupling always shrinks to a point. As a result, the areas of the triangles are no longer the free parameters, but are always determined once the geometrical settings are fixed.
warp factor can induce the energy difference between degenerated vacua, which removes the cosmological domain walls.

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