The transfer of the intensity of edge cracks during their propagation in elastoviscousplastic medium in plane stress

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Abstract In this paper take investigation dynamic stress-strain field in material near δ-neighbor edge of crack: longitudinal cheer, cut and antiplane deformation which propaganda with velocity $C_1$ and $C_2$ of elasticity longitudinal ware and cheer wave. For the plane stress material in front of edge crack build ordinary differential equation for transfer intense the forerunner edge crack with calculation the field of stress in front of crack. Exactly solution for intense of edge crack build in this paper and chon graphics for a distance when edge of crack stop.

1. Introduction
The phenomenon of brittle fracture of solids is so deep in its physical nature, so widely in mathematical and experimental approaches to its study and mathematical models that interest in it does not fade to the present time. Physical experiments at the macro level allow us to develop a mathematical model of spatial deformation of an elastic continuous environment and to solve the problem of limited static equilibrium of rigid elastic bodies with cracks based on it [1-12].

Special attention should be paid to the work [13-18], where the problems of the limited equilibrium of elastic bodies were considered based on of various physical representations:
1) the tensile strength of the material in the vicinity of the crack tip [4-7];
2) the equality of the mechanical energy flowing from the region in the vicinity of the crack tip to the surface energy of the free boundary in the vicinity of the vertex [3-4].

A meticulous analysis of both approaches showed their equivalence with the corresponding dependence of the selected model parameters.

Macro-and microanalysis of the physical state of the material in the vicinity of the front edge of the crack showed [13-15, 1] that the model of elastic behavior of the material, that allows unlimited stress values at the top, does not describe the stress-strain state of the material in the nearest vicinity of the top reliably.

The research of the material’s state at the nanoscale [13-15, 16-17] using electron microscopy revealed complex processes of chemical transformations of solids near the top of the crack and the formation of the films in sliding nanoasperity.

The plastic behavior of the material near the singular point was taken into account by a number of researchers [21-35] in the framework of the problems of limiting equilibrium of elastic solids.
The analysis of the dynamic behavior of cracks [1-10] required the determination of the propagation velocity of the vertices (front edges) of cracks. In [1] it is shown that the plane problem of crack propagation contains three speeds as the internal parameters – the speed of propagation of elastic longitudinal waves $C_1$, the velocity of propagation of shear waves $C_2$ and the speed of propagation of surface waves of Rayleigh $C_R$. The experimental determination of the crack tip propagation velocity is difficult due to its large size, small size of the experimental samples and the creation of boundary conditions on the sample. It is found that the velocity of the vertex moving fluctuates in the region of shear wave propagation velocity $C_2$ [9-10].

In view of the transience of the crack tip propagation process and the delay in the process of stress state changing in the solid state, it is natural to study the crack tip propagation under the conditions of quasi-stationarity of the stress state of the material before the crack tip.

In the study the dynamics of cracks in [1] it was proposed to allocate a small $\delta$-neighborhood summit and to consider its inelastic behavior in the framework of continuum mechanics without isolation of the free boundaries of the nascent cracks.

The joint consideration of the kinematics of possible static states of cracks of various types near their front edges and the kinematics of deformation of the elastic medium behind the fronts of longitudinal and shear waves made it possible to establish the following correspondence [11].

1) Behind the front of the longitudinal wave of plastic loading, a discontinuous distribution of the longitudinal deformation velocity is possible, which corresponds to displacements in the vicinity of the crack edge of the longitudinal shear [11] (figure 1).

![Image of the front $\Sigma_1$ of the longitudinal plastic loading wave.](image)

On the figure 1 discontinuous velocities behind $\hat{V}_n - \hat{\nu}_n = [V_n]_{\delta}$ the front $\Sigma_1$ when passing through the line $L_\delta$. At a distance $\delta = C_1 \cdot \Delta t$, a local velocity $[V]_{\delta}$ gap will lead to a discontinuity in the displacements of the longitudinal shear behind the leading edge $L$ $[U_n]_t = [V_n]_{\delta} \cdot \Delta t = \delta [V]_{\delta} / C_1$ and the formation of a cracked longitudinal shear.

2) Behind the front of the shear wave of plastic deformation $\Sigma_2$ possible discontinuous distribution of shear strains tangent to the precursor $L_\delta$ crack edge $L$ (figure 2), which at a distance $\delta$ behind the front $\Sigma_2$ leads to the origin of the edge $L$ of the antiplane shear crack.
Figure 2. Image of the front $\Sigma_2$ of the shear plastic-loading wave.

On the figure 2 a discontinuity of the tangent to $L_0$ the velocity $[V_t]_{L_0} = \hat{V}_x - \hat{V}_z$. Distance $\delta$ local gap tangent to $L_0$ speed will break the tangents to $L$ displacements $[U_x] = \delta[V_t]_0/C_2$. Relative to the plane $P_1$ orthogonal to $L$, this shift leads to the formation of cracks antiblockage shift.

3) Behind the front of shear waves $\Sigma_2$ possible to distribute the discontinuities orthogonal $[V_{t_2}]$ edge $L_0$ harbinger of cracks $L$ which is at a distance of $\delta$ behind the front $\Sigma_2$ leads to the crack nucleation of separation.

Figure 3. The image of the front $\Sigma_2$ of the shear plastic-loading wave.

On the figure 3 a discontinuity of the tangential velocities orthogonal to the leading edge of the precursor $L_0$ of the tearing fracture $L$ $[V_{t_2}] = \hat{V}_{t_2} - \hat{V}_{t_2}$. At a distance $\delta$ behind the front $\Sigma_2$, such a discontinuity of the velocity $[V_{t_2}]$ orthogonal to the edge $L_0$ of the precursor of the crack leads to the initiation of the edge $L$ of the separation crack.

The propagation of plastic loading waves has been studied by many authors [11]. It is established that at the front longitudinal $\Sigma_1$ and shear $\Sigma_2$ waves of plastic deformation is continuous, the relationship of stress with elastic deformation obeys the law, and in the near zone behind the front $\Sigma$ waves takes the form [11, 35]

$$[\sigma_{ij} n_j = \rho C_1 V_{t_j} n_j - \text{for } \Sigma_1]$$

$$[\sigma_{ij} t_j = \rho C_2 [V_{t_j} t_j] - \text{for } \Sigma_2]$$

Here: $[V_{t_j} n_j = [V_{t_j}]; [V_{t_j} t_j = [V_{t_j}]]$; $n_j r_i = 0$ ($i, j = 1, 2, 3$).
The dependences of stresses on the velocity of motion of the medium behind the front longitudinal \( \Sigma_1 \) and shear \( \Sigma_2 \) waves allow you to enter the characteristic values for the velocity spatial gaps on the lines \( L_\delta \) as the harbingers of the front edges of the cracks \( L \)

\[
\begin{align*}
[\hat{V}_n]_a - [\hat{V}_n]_0 &= [V_n]_a = \bar{\omega}_n; \\
[\hat{V}_\tau]_a - [\hat{V}_\tau]_0 &= [V_\tau]_a = \bar{\omega}_\tau.
\end{align*}
\]

2. Propagation of the front edges of cracks in the stressed material

The distribution of the front edges of cracks, their intensity and attenuation during propagation, is significantly influenced by the type and magnitude of the stress state ahead of the edge, as well as the direction of the normal \( \bar{n} \) edge of the crack relative to the stress state in front of it.

The stress state of the medium before the edge of the creeping crack is given by the values of the main stresses \( \sigma_1, \sigma_2, \sigma_3 \) and their directions \( \bar{m}_1, \bar{m}_2, \bar{m}_3 \), which are determined by the Euler angles (figure 4)

\[
\begin{align*}
\bar{m}_1 &= (\cos\phi \cos\psi - \sin\phi \sin\psi \cos\theta; \cos\phi \sin\psi + \sin\phi \cos\psi \cos\theta; \sin\phi \sin\theta), \\
\bar{m}_2 &= (-\sin\phi \cos\psi - \cos\phi \sin\psi \cos\theta; \sin\phi \sin\psi - \cos\phi \cos\psi \cos\theta; \cos\phi \sin\theta), \\
\bar{m}_3 &= (\sin\phi \sin\theta; -\cos\psi \sin\theta; \cos\theta).
\end{align*}
\]

Figure 4. a) picture of the main \( \bar{m}_1, \bar{m}_2, \bar{m}_3 \) stress directions \( \sigma_1, \sigma_2, \sigma_3 \); b) the image of the normal section in the vicinity of the crack tip \( L \).

On the figure 4 \( L \) – the front edge of the crack; \( L_\delta \) – the harbinger of a front edge of the crack; \( \Sigma_\delta \) – surface \( \delta \)-neighborhood of the edges of the crack \( L \), restricting the viscoplastic behavior of the material; \( \Sigma \) is the front line of the disturbances generated by the crack at the time of its occurrence.

In equations (16, 17) the right part contains the state of stress ahead \( \Sigma_\delta \) in the form \( \sigma_{ij}^\Sigma n_i n_j \) and \( \sigma_{ij}^\Sigma n_i \tau_j \), where \( \bar{n} \) is normal to \( L_\delta \) is the only to crack of; \( \bar{\tau} \) is the vector tangent to \( \Sigma_\delta \) has different directions: in the case of a crack \( \bar{\tau} \) of transverse shear is tangential to \( L_\delta \), and for the case of cracks \( \bar{\tau} \) perpendicular separation \( L_\delta \) (figure 3).
Select the local coordinate system \( (\overrightarrow{n}, \overrightarrow{r}, \overrightarrow{t}) \) that coincides with the \( x_1x_2x_3 \) axis directions (figure 4).

\[
\overrightarrow{n} = (1,0,0); \quad \overrightarrow{r} = (0,1,0); \quad \overrightarrow{t} = (0,0,1).
\] (6)

Calculate \( \sigma_{ij}n_in_j = I_{nn}^+ \)

\[
I_{nn}^+ = \sigma_{ij}m_km_jn_in_j = \sigma_1(\cos \varphi \cos \psi - \sin \varphi \sin \psi \cos \theta)^2
+ \sigma_2(\sin \varphi \cos \psi + \cos \varphi \sin \psi \cos \theta)^2 + \sigma_3(\sin \psi \sin \theta)^2.
\] (7)

Calculate \( \sigma_{ij}n_in_i^j = I_{n\tau_1}^+ \)

\[
I_{n\tau_1}^+ = \sigma_{ij}m_km_jn_in_i^j = \sigma_1(\cos \varphi \cos \psi - \sin \varphi \sin \psi \cos \theta)(\cos \varphi \sin \psi + \sin \varphi \cos \psi \cos \theta)
+ \sigma_2(-\sin \varphi \cos \psi - \cos \varphi \sin \psi \cos \theta)(\sin \varphi \sin \psi - \cos \varphi \cos \psi \cos \theta)
+ \sigma_3(\sin \psi \sin \theta)(-\cos \psi \sin \theta).
\] (8)

Calculate \( I_{n\tau_2}^+ \)

\[
I_{n\tau_2}^+ = \sigma_{ij}m_km_jn_in_i^j = \sigma_1(\cos \varphi \cos \psi - \sin \varphi \sin \psi \cos \theta)\sin \varphi \sin \theta
+ \sigma_2(-\sin \varphi \cos \psi - \cos \varphi \sin \psi \cos \theta)\cos \varphi \sin \theta
+ \sigma_3(\sin \psi \sin \theta)(\cos \varphi \sin \theta).
\] (9)

In the case when \( \theta = 0 \), the third principal voltage \( \sigma_3 \) is collinearly straightening \( L_{\delta} \) in the plane \( \Pi \), we get

\[
I_{nn}^+ = \sigma_1(\cos \varphi \cos \psi - \sin \varphi \sin \psi \sin \theta)^2 + \sigma_2(\sin \varphi \cos \psi + \cos \varphi \sin \psi \sin \theta)^2;
\] (10)

\[
I_{n\tau_1}^+ = \sigma_1(\cos \varphi \cos \psi - \sin \varphi \sin \psi \sin \theta)(\cos \varphi \sin \psi + \sin \varphi \cos \psi \sin \theta)
+ \sigma_2(-\sin \varphi \cos \psi - \cos \varphi \sin \psi \sin \theta)(\sin \varphi \sin \psi - \cos \varphi \cos \psi \sin \theta);
\] (11)

\[
I_{n\tau_2}^+ = 0.
\] (12)

The flat problem of crack propagation is understood to be the case when the edge of a crack is orthogonal to the plane of the stress state \( (\sigma_1, \sigma_3) \) (figure 4). The principal stress \( \sigma_3 \) is constant and directed along the edge of the crack and it is logical to consider not the curved edge \( L_{\delta} \), but the straight line. The angle \( \theta \) in this case is equal to zero, and the form of the invariants \( I_{nn}, I_{n\tau_1}, I_{n\tau_2} \) is simplified when \( \theta = 0 \) and \( \psi = 0 \).

\[
I_{nn}^+ = \sigma_1 \cos^2 \varphi + \sigma_2 \sin^2 \varphi;
\] (13)

\[
I_{n\tau_2}^+ = 0.
\]

The invariants \( I_{nn}^+, I_{n\tau_1}^+, I_{n\tau_2}^+ \) in the case \( \sigma_3 = 0 \) are defined

\[
I_{nn}^+ = \frac{2\sigma_1 - \sigma_2}{3} \cos^2 \varphi + \frac{2\sigma_2 - \sigma_1}{3} \sin^2 \varphi = \frac{\sigma_1}{3}(\cos^2 \varphi + \cos 2\varphi) + \frac{\sigma_2}{3}(\sin^2 \varphi - \cos 2\varphi)
= \frac{\sigma_1 - \sigma_2}{3} \cos 2\varphi + \frac{\sigma_1}{3} \cos^2 \psi + \frac{\sigma_2}{3} \sin^2 \varphi;
\] (14)
\[ I_{n_1}^+ = \left( \frac{2\sigma_1 - \sigma_2}{3} - \frac{2\sigma_2 - \sigma_1}{3} \right) \frac{1}{2} \sin 2\varphi = \frac{\sigma_1 - \sigma_2}{2} \sin 2\varphi; \]

\[ I_{n_2}^+ = 0. \]

Deviatoric complexes \( I_{n_1}^+, I_{n_2}^+, I_{n_3}^+ \) differ from those calculated by replacing the main stresses \( \sigma_1, \sigma_2, \sigma_3 \) with deviatoric ones

\begin{align*}
\sigma_1' &= \sigma_1 - \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) = \frac{2\sigma_1 - (\sigma_2 + \sigma_3)}{3} \\
\sigma_2' &= \sigma_2 - \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) = \frac{2\sigma_2 - (\sigma_1 + \sigma_3)}{3} \\
\sigma_3' &= \sigma_3 - \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) = \frac{2\sigma_3 - (\sigma_1 + \sigma_2)}{3} \\
I_1' &= \sigma_y' - \sigma_y = \frac{2}{3}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_1\sigma_3 - \sigma_2\sigma_3)
\end{align*}

3. Propagation of the front edges of cracks in highly stressed material with high elastic parameters

Consider the transport equations for the intensities of the front edges:

1) longitudinal shear cracks \( \left[ \overline{F}_{\text{ns}} \right] = \overline{\omega}_n \)

\[ \frac{\delta \overline{\omega}_n}{\delta t} - \Omega \overline{\omega}_n = \left( c_2/c_1 \right)^2 \left[ \overline{F}_{\text{ns}} \right] \]

Here:

\[ \left[ \overline{F}_{\text{ns}} \right] = -\frac{1}{\eta} \left( \frac{4}{3} \frac{\overline{\omega}_n}{\mu} + I_{n_1}^+ \right) \left( 1 - \frac{k\sqrt{2}}{\sqrt{I_2' + 4 \mu \frac{\overline{\omega}_n}{c_1} I_n + \frac{8}{3} \mu^2 \left( \frac{\overline{\omega}_n}{c_1} \right)^2}} \right) \]

2) tear-off and shear cracks \( \left[ \overline{F}_{\text{tr}} \right] = \overline{\omega}_t \)

\[ \frac{\delta \overline{\omega}_t}{\delta t} - \Omega \overline{\omega}_n = \left[ \overline{F}_{\text{tr}} \right] \]

Here:

\[ \left[ \overline{F}_{\text{tr}} \right] = -\frac{1}{\eta} \left( I_{n_1}^+ + \frac{\overline{\omega}_t}{c_2} \right) \left( 1 - \frac{k\sqrt{2}}{\sqrt{I_2'}} \right) \]

where \( I_2' = I_{n_1}^+ + 2\mu^2 \left( \frac{\overline{\omega}_t}{c_2} \right)^2 \).

From the equations for the transfer, it follows that in the case when the crack plane coincides with the main directions of the stress tensor in front of the floor \( L_n \), all three \( \overline{\omega} \) main (7, 8, 9) stresses \( \sigma_1, \sigma_2, \sigma_3 \) influence the change due to the work of friction forces on spatial tangential deformations.
Analytical solution of nonlinear multiparametric ordinary differential equations (16, 17) is practically impossible for intensities $\bar{\sigma}_n$ and $\bar{\sigma}_t$ leading edges of separation and shear fractures, but for the case of large $\sigma_j^+$ and $\mu$ we can construct a limiting solution.

The considered limiting case corresponds to a small value of the plasticity limit $k$ which allows us to relate the obtained results to the further propagation of the leading edges of the test in the environment of the viscoelastic Maxwell model.

4. The transfer intensity of the front edges of the fracture in the plane problem in the environment of Maxwell

Equations (16, 17) are simplified $k \sqrt{I_2}$:

$$\frac{\delta \bar{\sigma}_n}{\delta s} - \Omega \bar{\sigma}_n = -\left(\frac{c_2}{c_1}\right)^2 \frac{1}{\eta} \left(\frac{4}{3} \mu \frac{\bar{\sigma}}{c_1} + I_{nn}^+ \right).$$

(18)

$$\frac{\delta \bar{\sigma}_t}{\delta s} - \Omega \bar{\sigma}_t = -\frac{1}{\eta} \left(\frac{\bar{\sigma}}{c_2} + I_{nt}^+ \right).$$

(19)

Calculate the expressions for $I_{nn}^+$ and $I_{nt}^+$ for the case of plane location cracks in the plane $(x_1, x_2)$, i.e., the third main voltage $\sigma_3$, i.e. $\theta=0$, and the orientation of the first principal stress $\sigma_1$ define an angle $\varphi$ when $\psi=0$. Next, consider the case of a plane stress state by putting $\sigma_3^+=0$.

$$I_{nn}^+ = \frac{\sigma_1^+ - \sigma_2^+}{2} \cos^2 \varphi + \frac{\sigma_2^+ - \sigma_1^+}{2} \sin^2 \varphi = \frac{\sigma_1^+ - \sigma_2^+}{2} \cos 2\varphi .$$

$$I_{nt}^+ = \frac{\sigma_1^+ - \sigma_2^+}{2} \cos \varphi \sin \varphi - \frac{\sigma_2^+ - \sigma_1^+}{2} \sin \varphi \cos \varphi = \frac{\sigma_1^+ - \sigma_2^+}{2} \sin 2\varphi .$$

(20)

$$I_{nt}^+ = 0 .$$

Curvature $\Omega_3$ in the case of a rectilinear front edge $L_3$ cracks will be constant

$$\Omega_3 = -\frac{1}{2\delta}$$

(21)

The transport equations (18, 19) are linear ordinary differential equations

$$\frac{\delta \omega_j}{\delta s} + a_j \omega_j = I_j^+ , \quad (j=1, 2, 3)$$

(22)

where: for $j=1$; $w_1=w_\omega$; $a_1 = \frac{1}{\eta} \left(\frac{c_2}{c_1}\right)^2 \mu + \frac{1}{2\delta} ; \quad I_1^+ = I_{nn}^+ = \frac{\sigma_1^+ - \sigma_2^+}{2} \cos 2\varphi ;$

for $j=2$; $w_2 = w_{nt}$; $a_2 = \frac{1}{\eta} \frac{\mu}{c_2} + \frac{1}{2\delta} ; \quad I_2^+ = I_{nt}^+ = \frac{\sigma_1^+ - \sigma_2^+}{2} \sin 2\varphi ;$

(23)

for $j=3$; $w_3 = w_{nt}$; $a_3 = \frac{1}{2\delta} + \frac{\mu}{c_2 \eta} ; \quad I_3^+ = I_{nt}^+ = 0 .$

The solution of equation (23) is known [11]

$$\omega_j = (\omega_j^0 - I_j/a_j) \exp(-a_j(s-s_0)) + I_j/a_j ,$$

(24)
where $s$ is counted from the leading edge of $L$ crack.

Specific patterns of fracture intensity transfer take the form:

For a longitudinal shear crack in $j=1$

$$
\omega_{\mu\nu} = \left( \omega_{\mu\nu}^0 - \frac{\sigma_1 - \sigma_2}{a_1} \cos 2\phi \right) \exp(-a_1(s-s_0)) + \frac{\sigma_1 - \sigma_2}{2a_1} \cos 2\phi.
$$

(25)

for crack separation in $j=2$, we obtain

$$
\omega_{nt_1} = \left( \omega_{nt_1}^0 - \frac{\sigma_1 - \sigma_2}{a_2} \sin 2\phi \right) \exp(-a_2(s-s_0)) + \frac{\sigma_1 - \sigma_2}{2a_2} \sin 2\phi;
$$

(26)

for the antiplane shear crack in $j=3$, we obtain

$$
\omega_{nt_2} = \omega_{nt_2}^0 \exp(-a_3(s-s_0)).
$$

(27)

From the expressions (25-27) for intensities of cracks as a function of the traversed edge distance $s > \delta$ it follows that in materials with large dynamic viscosity $\eta \to \infty$ the intensity of the edges $k$ of the cracks is stored.

The depth of $s^*$ penetration of cracks in the material depends on the viscosity, the magnitude of the stress state and the direction of the crack

$$
s_{mn}^* - \delta = \frac{1}{a_1} \ln \left( \frac{2a_1 \omega_{\mu\nu}^0 / (\sigma_1 - \sigma_2) + \cos 2\phi}{\cos 2\phi} \right),
$$

(28)

$$
s_{nt_1}^* - \delta = \frac{1}{a_2} \ln \left( \frac{2a_2 \omega_{nt_1}^0 / (\sigma_1 - \sigma_2) + \sin 2\phi}{\sin 2\phi} \right),
$$

(29)

$$
\left. s_{nt_2}^* - \delta \to \infty. \right)
$$

(30)

Expressions (28-30) for the intensity of the front edges of cracks are more convenient to present in dimensionless form

$$
\Lambda_{\mu\nu} = \omega_{\mu\nu} / \omega_{\mu\nu}^0; \quad \Lambda_{nt_1} = \omega_{nt_1} / \omega_{nt_1}^0; \quad \Lambda_{nt_2} = \omega_{nt_2} / \omega_{nt_2}^0.
$$

For longitudinal shear crack

$$
\Lambda_{\mu\nu} = e^{-a_1(s^* - \delta)} + \frac{\sigma_1 - \sigma_2}{2a_1 \omega_{\mu\nu}^0} \cos 2\phi \left( 1 - e^{-a_1(s^* - \delta)} \right).
$$

(31)

For tear-off crack

$$
\Lambda_{nt_1} = e^{-a_2(s^* - \delta)} + \frac{\sigma_1 - \sigma_2}{2a_2 \omega_{nt_1}^0} \sin 2\phi \left( 1 - e^{-a_2(s^* - \delta)} \right).
$$

(32)

For antiplane shear crack

$$
\Lambda_{nt_2} = e^{-a_3(s^* - \delta)}.
$$

(33)

For figure 5 graphs of the relative intensity $\Lambda_{\mu\nu}$ of the crack edges depending on the distance traveled and the direction $\phi$ of crack propagation are presented.
Figure 5. Image of the qualitative behavior of the relative intensity \( \Lambda_{\text{nn}} \) of the leading edge of the longitudinal shear crack depending on the distance \( S \) and orientation of the main stresses \( \phi \) relative to the direction \( \bar{n} \) of the edge of the crack until it stops \( \Lambda_{\text{nn}} (S^*) = 0 \) at the depth \( S = S^*(\phi) \).

5. Conclusion
The wave model for propagation of an edge crack show that in plane case their intensive depend from orientation crack and field of stress in front of an edge in the process propagation.

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