Improving the Hydraulic Performance of Single Step Broad-Crested Weirs

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Abstract
In this study the hydraulic performance of single step broad-crested weir was improved. Through analyzing the parameters that have effect on the shape of the step and its influence on the flow characteristics, and energy dissipation percent ($E\%$) downstream (D/S) of the weir. The differential equation of gradually varied flow for the water surface profile over the weir was solved analytically. Furthermore, empirical relations for $E\%$ and discharge coefficient ($C_d$) due to the affecting factors were derived. The results showed that the weir model when the ratio of the length of D/S step to the length of the weir $L_2/L_1=0.5$ gives a higher $E\%$ in comparison with other weir models. Three types of flow regimes were observed, nappe flow below 350 cm$^3$/s/cm, transition flow 350-700 cm$^3$/s/cm and skimming flow upper than 700 cm$^3$/s/cm. The comparison between calculated values by the differential equation of gradually varied flow and experimental values gives a good agreement, the maximum difference is about 7%. Two empirical relations were obtained, the first to estimate $C_d$ in terms of the ratio for upstream U/S water head to U/S weir height $h/P_1$ and $L_2/L_1$. While the second relation to estimate $E\%$ in terms of the ratio for D/S water head to U/S weir height $h/P_1$, $L_2/L_1$ and the Froude number $F_{r2}$ with a high correlation coefficient.

Key Words: Hydraulics; weirs; performance; dissipation energy

تحسن الأداء الهيدرولوجي للهدرات ذات القمة العريضة مفردة الدرجة
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1. Introduction
The flow characteristics over broad-crested weirs were investigated by several investigators, they have mainly concentrated on the overflow above these weirs. Some of the published papers that studied the flow characteristics of broad-crested weirs include [1];[2]; [3]; [4], presented the results of an experimental study of supercritical flow over drops. The authors of [5] studied in experimental measurements the velocity and pressure distributions over a large broad-crested weir, their results showed the rapid flow distribution at the upstream end of the weir, while an overhanging crest design may affect the flow field. Also, they presented two empirical relations depend on the geometry of the weir. Dissipation of flow energy downstream (D/S) of hydraulic structures has been the interesting subject of many investigators. The authors of [6] focused on describing flows over small, stepped, homogeneous gabion spillways to quantify accurately the energy dissipation of standard designs and determine stilling-basin design parameters. The authors of [7] indicated that the stepped chute has been accepted to be the most powerful hydraulic structure to dissipate large flow energy downstream from steep hills. The authors of [8] proposed a relationship to express the relative energy dissipation in the presence and absence of boulders on the rock chutes. The authors of [9] investigated the relative energy dissipation in submerged flow conditions and proposed an exponential relationship which valid in different submerged conditions. The authors of [10] presented an experimental study on the hydraulics of flow through and over gabion-stepped weirs. They found that the energy loss in the gabion-stepped weirs were greater than those in the corresponding horizontal stepped weirs by approximately 7% to 14%. The authors of [11] changed the shape of traditional broad-crested weir by reducing D/S height of the weir to give it a new performance and make it as a water level controller and energy dissipater, the length of the D/S step was fixed, while the height of the D/S step was changed several times. The experimental results showed that the reduction in D/S height increased the energy dissipation percent up to 46%. Furthermore, the discharge coefficient was improved and gets higher values in comparison with traditional weirs.

This study is an extension to our previous study [11] by changing the D/S step length and fixing D/S step height. So that, the aim of this study is to investigate the effects of change the step geometry on the flow characteristics, and energy dissipation D/S of the stepped weir. Solve the differential equation of steady gradual flow for the water surface profiles over the weir. Also, derive empirical relations for the energy dissipation percent and discharge coefficient due to the affecting factors.

2. Theoretical Analysis
2.1 Gradually-Varied Flow Equation
The classic differential equation for steady gradual flow in open channels is ([12]; [13]):

\[
\frac{dy}{dx} = \frac{S_o - S_f}{1 - \alpha \frac{Q^2 T}{gA^3}}
\]

where:
- \( y \) = depth of flow, cm
- \( x \) = space coordinate in the flow direction, cm
- \( S_o \) = bed channel slope,
- \( S_f \) = energy slope,
- \( Q \) = discharge, cm\(^3\)/s
- \( T \) = top width of the flow, cm
- \( g \) = acceleration due to gravity, cm/s\(^2\)
\[ A = \text{cross sectional area, cm}^2 \]
\[ \alpha = \text{velocity distribution coefficient.} \]

For wide rectangular channel sections:
\[ q = \frac{Q}{T}; \quad R = y; \quad A = yT; \quad \text{assume } \alpha = 1; \quad \text{then equation (1) becomes:} \]
\[ \frac{dy}{dx} = \frac{S_o - S_f}{1 - \frac{q^2}{gy^3}} \quad (2) \]

since \( y_c = \sqrt[3]{\frac{q^2}{g}} \)

where \( y_c = \text{critical depth of flow} \)
\[ \frac{dy}{dx} = \frac{S_o - S_f}{1 - \left(\frac{y_c}{y}\right)^3} \quad (3) \]

For horizontal channel; \( S_o = 0; \quad \text{then} \)
\[ \frac{dy}{dx} = -\frac{S_f}{1 - \left(\frac{y_c}{y}\right)^3} \quad (4) \]

From Manning’s equation
\[ S_f = \frac{q^2n^2}{y^{10/3}} \quad (5) \]
\[ \frac{dy}{dx} = -\frac{q^2n^2}{y^{10/3}} \quad (6) \]

Let: \( k = -q^2n^2; \quad \lambda = \frac{y}{y_c}; \quad d\lambda/dy = 1/y_c; \quad dy = y_c d\lambda \)

Substituting in equation (6)
\[ \frac{y_c d\lambda}{dx} = \frac{k}{\left(\lambda y_c\right)^{10/3}} \quad (7) \]
\[ \frac{d\lambda}{dx} = \frac{k}{\left(\frac{y_c^{13/3} \lambda^{10/3}}{1 - \frac{1}{\lambda^3}}\right)} \quad (8) \]

Let: \( k_1 = k/y_c^{13/3}; \quad \text{then} \)
\[ \frac{d\lambda}{dx} = \frac{k_1}{\lambda^{10/3} \left(1 - \frac{1}{\lambda^3}\right)} \]

By indefinite integration;
\[ k_1 dx = \left(\lambda^{10/3} - \lambda^{13/3}\right) d\lambda \]
\[ x = \frac{1}{k_1} \left(\frac{3}{13} \lambda^{13/3} - \frac{3}{4} \lambda^{4/3} + c\right) \quad (9) \]

The value of “c” can be found from the boundary conditions, at \( y = y_c; \quad \lambda = 1. \)

For the weir model of \( L_2/L_1 = 0.5 \) and run of \( q = 397 \text{ cm}^2/\text{s} ; \quad n = 0.011, \) at \( y_c, \quad x = -17.25 \text{ cm} \) then the value of “c” was found to be equal to 0.735; then equation (9) becomes
\[ x = \frac{1}{k_1} \left(\frac{3}{13} \lambda^{13/3} - \frac{3}{4} \lambda^{4/3} + 0.735\right) \quad (10) \]
2.2 Dimensional Analysis

A- Weir discharge coefficient, \(C_d\)

The selected parameters that have an influence on the weir coefficient of discharge can be functionally expressed as follows

\[ f_1(q, H, P_1, P_2, L_1, L_2, R, g) = 0 \]  

(11)

where:

- \(q\) = discharge over the weir per unit width, \(\text{cm}^3/\text{s/cm}\)
- \(H\) = U/S water head above the weir crest, \(\text{cm}\)
- \(P_1\) = U/S weir height, \(\text{cm}\)
- \(P_2\) = D/S weir height, \(\text{cm}\)
- \(L_1\) = length of the weir, \(\text{cm}\)
- \(L_2\) = length of D/S step, \(\text{cm}\)
- \(R\) = radius of U/S corner of the weir, \(\text{cm}\)
- \(g\) = acceleration due to gravity, \(\text{cm/s}^2\)

The parameters in equation (11) may be expressed in nondimensional form as

\[ C_d = \frac{q}{\frac{2}{3} H \sqrt{\frac{2}{3} g H}} = f_2\left(\frac{H}{P_1}, \frac{P_2}{P_1}, \frac{L_1}{L_1}, \frac{L_2}{P_1}, R\right) \]  

(12)

Since, \(P_1;\ P_2;\ L_1\) and \(R\) are fixed in this study then equation (12) can be rewritten as

\[ C_d = f_3\left(\frac{H}{P_1}, \frac{L_2}{P_1}\right) \]  

(13)

B- Energy Dissipation Percent

The energy dissipation percent (\(E\%\)) due to the change in D/S step geometry can be expressed by the following functional relationship

\[ E\% = f(q, P_1, H, h, P_2, L_1, L_2, g) \]  

(14)

where:

- \(E\%\) = energy dissipation percent,
- \(q\) = discharge over the weir per unit width, \(\text{cm}^3/\text{s/cm}\)
- \(P_1\) = U/S weir height, \(\text{cm}\)
- \(H\) = U/S water head above the crest, \(\text{cm}\)
- \(h\) = D/S water head, \(\text{cm}\)
- \(P_2\) = D/S weir height, \(\text{cm}\)
- \(L_1\) = length of the weir, \(\text{cm}\)
- \(L_2\) = length of D/S step, \(\text{cm}\)
- \(g\) = acceleration due to gravity, \(\text{cm/s}^2\)

By dimensional analysis

\[ E\% = f\left(\frac{H}{P_1}, \frac{h}{P_1}, \frac{P_2}{P_1}, \frac{L_2}{L_1}, \frac{q^2}{gh^3}\right) \]  

(15)

Since \(P_1\) and \(P_2\) are fixed and \(h\) value related to \(H\) and \(q^2/gh^3\) is Froude number at Sec. (2) then equation (15) becomes
\[
E = f_2 \left( \frac{h}{P_1}, \frac{L_2}{L_1}, Fr \right)
\]

(16)

3. Experimental Setup and Procedure

The stepped broad-crested weirs were installed in a horizontal, glass-walled rectangular channel, 0.50 m wide, 10 m long, and with 0.45 m height. Water was pumped from a laboratory sump to the V-notched tank from which water dropped to a dissipation tank with three dissipating baffles then water enters the horizontal channel (Plate 1). The discharge per unit width \((q)\) varied from 100 to 800 \(\text{cm}^3/\text{s/cm}\), and it was measured by the V-notched weir tank. Upstream (U/S) water surface heads were started to measure at a location when \(X/P_1 > 2\) U/S of the weir models as recommended by [11]. Seven weir models were made from thermo-stone and painted to decrease the surface roughness, the models were fixed at a distance of 3.5 m from the channel inlet. One of the weir models was a traditional round-nosed broad-crested weir, its length was 48 cm, 50 cm wide and 12 cm height with 6 cm radius of the round-nose. The other four models involved the above dimensions of the weir with D/S stepped height \(P_2= 6\) cm and D/S lengths \(L_2= 12, 18, 24\) and \(30\) cms. While in the last two models the length of the weirs were increased to 54 and 60 cms with D/S lengths 30 and 36 cms respectively, as shown in Table 1. Each test included the measurements (under free flow conditions) for water surface levels, U/S flow depth, D/S depth over the step, and water discharge.

Table 1. Details of the tested weir models.

| Model No. | \(L_1\) (cm) | \(L_2/L_1\) | Run No. | \(H\) (m) | \(q\) (cm^3/sec) | \(C_d\) | \(Fr\) |
|-----------|---------------|--------------|---------|-----------|-----------------|--------|-------|
| 1         | \(L_1=48\) \(L_2=48\) | 1            | 1-7     | 0.157-0.215 | 270 -595       | 0.968-0.988 | 4.3-4.6 |
| 2         | \(L_1=48\) \(L_2=30\) | 0.625        | 8-14    | 0.138-0.241 | 196 -794       | 0.897-0.952 | 5.5-5.6 |
| 3         | \(L_1=48\) \(L_2=24\) | 0.5          | 15-21   | 0.12-0.22  | 139-631        | 0.954-0.988 | 4.7-6.1 |
| 4         | \(L_1=48\) \(L_2=18\) | 0.375        | 22-28   | 0.142-0.228 | 211-690       | 0.845-0.879 | 5.2-7.2 |
| 5         | \(L_1=48\) \(L_2=12\) | 0.25         | 29-35   | 0.105-0.234 | 100-737       | 0.853-0.923 | 7.6-8.2 |
| 6         | \(L_1=54\) \(L_2=30\) | 0.56         | 36-42   | 0.115-0.232 | 125-721       | 0.907-0.954 | 5.3-6.3 |
| 7         | \(L_1=60\) \(L_2=36\) | 0.6          | 43-49   | 0.159-0.233 | 279-729       | 0.869-0.915 | 1.8-2.1 |
Plate 1. Photograph showing the laboratory horizontal channel with V-notched tank and dissipation tank.

4. Results and Discussion

Three types of flow regimes were observed in all tests, that was, nappe flow, transition flow, and skimming flow. A free-falling nappe was found at small discharge (< 350 cm³/s/cm) when the flow depth is smaller than step height. For transition flow at intermediate discharge (350-700 cm³/s/cm), the free-falling disappears and the water surface was wavy, and flow depth a little pit less than step height. While for skimming flow at large discharge (> 700 cm³/s/cm) the water surface was smooth and no nappe was visible. Furthermore, the step was submerged beneath a strong current (Plates 2, 3 and 4).

Figures 1, 2, and 3 present the variation of nondimensional water surface profiles (w.s.ps) for different discharges over three models of stepped weirs, where \( L_2/L_1 = 0.25, 0.375 \) and 0.625 respectively. In all of these figures w.s.ps. tends to reduce gradually when the water approach the weirs. Once the water across the crest of the weir the water passed rapidly and the flow changed from gradually varied flow to rapidly varied flow. The measured values of w.s.ps. and the calculated values by equation (10) over different stepped weir models ,for different discharges, were tested but the nearest with a good matching when \( L_2/L_1 = 0.5 \), and used to plot in Figure 4. The plotted area for the measured and calculated values were shown in the reach of gradually varied flow. In this reach the comparison between both w.s.ps. gives a good agreement and the maximum difference is about 7% .
Plate 2. Flow over stepped weirs.

Plate 3. Nappe flow.

Plate 4. Skimming flow.
Figure 1. Dimensionless water surface profile for weir model of \( \frac{L_2}{L_1}=0.25 \).

Figure 2. Dimensionless water surface profile for weir model of \( \frac{L_2}{L_1}=0.375 \).

Figure 3. Dimensionless water surface profile for weir model of \( \frac{L_2}{L_1}=0.625 \).
Figure 4. Comparison between calculated values with measured ones for the water surface profile over stepped weir for $L_2/L_1=0.5$.

Figure 5 depicts the variation of the energy dissipation percent $E\%$ with the Froude number $Fr_2$ for different stepped weirs. It was shown that $E\%$ increases as $Fr_2$ increases. On the other hand, the weir model when $L_2/L_1=0.5$ gives a higher $E\%$ in comparison with other weir models, for example when $Fr_2=6$ the values of $E\%$ for weir models 0.25, 0.625 and 0.5 were 19%, 22% and 34% respectively.

Figure 5. Variation of $E\%$ with $Fr_2$ for different step weirs.

Effect of the nondimensional parameter $h/P_1$ on $E\%$ for different weir models were shown in Figure 6, it was observed that $E\%$ decreased gradually with the increase of $h/P_1$. Also, it was observed that $E\%$ for the traditional weir model (i.e., $L_2/L_1=0$) almost remains constant with the increase of $h/P_1$. While $E\%$ at other weir models was decreased clearly, for example, when $h/P_1=0.4$ the values of $E\%$ for weir models 0.375, 0.25, 0.625 and 0.5 were 15%, 21%, 24% and 30% respectively.
Figure 6. Dimensionless water surface profile for weir model of $L_2/L_1=0.25$.

The variation of $C_d$ with $H/P_1$ was shown in Figure 7, where $C_d$ was found to increase with the increase of $L_2/L_1$, as observed in the figure the highest values of $C_d$ when $L_2/L_1=0.625$.

Figure 7. Relation between $C_d$ and $H/P_1$ for weirs.

5. Empirical Relations

Based on equation (13), multiple nonlinear regression analysis was used to correlate $C_d$ with both $(H/P_1)$ and $(L_2/L_1)$ in an empirical power relation

$$C_d = 1.02 \left( \frac{H}{P_1} \right)^{0.05} \left( \frac{L_2}{L_1} \right)^{0.41}$$

with a correlation coefficient of 0.89.

The relation between $C_d$ values predicted by equation (17) and experimental values of $C_d$ is plotted in Figure 8 showing a good agreement.
Another empirical power relation based on equation (16) was obtained for the variation of $E\%$ with $(h/P_1)$, $(L_2/L_1)$ and $Fr_2$

$$E\% = 24.4 \left( \frac{h}{P_1} \right)^{-0.57} \left( \frac{L_2}{L_1} \right)^{0.02} Fr_2^{-1.24}$$  \hspace{1cm} (18)

with a correlation coefficient of 0.96.

A comparison between $E\%$ values predicted by equation (18) and observed values experimentally is shown in Figure 9 and showing a good agreement.

**Figure 8. Variation of predicted values of $C_d$ with the observed ones for all models.**

**Figure 9. Variation of predicted values of $E\%$ with the observed ones for all models.**
6. Conclusions
Depending upon the results and analysis of this study the following conclusions are summarized below:

1. The weir model when $L_2/L_1 = 0.5$ gives a higher $E\%$ in comparison with other weir models. So, it can be recommended to use this weir in the design of prototype weirs.

2. Three types of flow regimes were observed, nappe flow below 350 cm$^2$/s, transition flow 350-700 cm$^2$/s and skimming flow upper than 700 cm$^2$/s.

3. Over single step weirs there are two parts of flow, in the first part the flow is Gradually-varied flow while in the second part the flow is rapidly- varied flow.

4. A direct solution for the equation of gradually-varied flow at horizontal channel was derived, the comparison between calculated and experimental values gives a good agreement and the maximum difference is about 7%.

5. Two empirical relations were obtained, the first to estimate $C_d$ in terms of $H/P_1$ and $L_2/L_1$ as shown in Equation (17). While the second relation to estimate $E\%$ in terms of $h/P_1$, $L_2/L_1$ and $Fr_2$ as shown in Equation (18).

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