The magnetic diagnostics subsystem of the LISA Technology Package

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Abstract. The Magnetic Diagnostics Subsystem of the LISA Technology Package (LTP) on board the LISA Pathfinder (LPF) spacecraft includes a set of four tri-axial fluxgate magnetometers, intended to measure with high precision the magnetic field at the positions they occupy. However, their readouts do not provide a direct measurement of the magnetic field at the positions of the test masses. Therefore, an interpolation method must be implemented to obtain this information. However, such interpolation process faces serious difficulties. Indeed, the size of the interpolation region is excessive for a linear interpolation to be reliable, and the number of magnetometer channels does not provide sufficient data to go beyond that poor approximation. Recent research points to a possible alternative to address the magnetic interpolation problem by means of neural network algorithms. The key point of this approach is the ability neural networks have to learn from suitable training data representing the magnetic field behaviour. Despite the large distance to the test masses and the insufficient magnetic readings, artificial neural networks are able to significantly reduce the estimation error to acceptable levels. The learning efficiency can be best improved by making use of data obtained from on-ground measurements prior to mission launch in all relevant satellite locations and under real operation conditions. Reliable information on that appears to be essential for a meaningful assessment of magnetic noise in the LTP.

1. Introduction
LISA Pathfinder (LPF) is a science and technology demonstrator mission, led by programmed by the European Space Agency (ESA), within its LISA (Laser Interferometer Space Antenna) mission activities [1]. LISA (Laser Interferometer Space Antenna) is a joint ESA-NASA mission which will be the first low frequency (milli-Hz) gravitational wave detector, and also the first space-borne gravitational wave observatory. LPF’s payload includes the LISA Technology Package (LTP), and this will be the highest sensitivity geodesic explorer flown to date. The LTP is designed to measure relative accelerations between two test masses (TMs) in nominal free fall (geodesic motion) with a noise budget of

\[ S_{\delta a,LPF}(\omega) \leq 3 \times 10^{-14} \left[ 1 + \left( \frac{\omega}{2\pi} \right)^2 \frac{3}{3 \text{ mHz}} \right]^2 \text{ m s}^{-2} \sqrt{\text{Hz}} \quad (1) \]
Figure 1. View of the LPF science-craft. The LCA is in the center, surrounded by a double cylindrical shield. Outside it, a number of electronic boxes are represented, most of which are sources of magnetic field. The four magnetometers are the grey little boxes labelled with the 3-axis Magnetometer label and are mounted on the outer cylindrical shell.

in the frequency band between 1 mHz and 30 mHz [2].

Noise in the LTP arises as a consequence of various disturbances, mainly generated within the spacecraft itself, which limit the performance of the instrument. A number of these disturbances are monitored and dealt with by means of suitable devices, which form the so-called Diagnostics Subsystem [3]. In LPF, this includes thermal and magnetic diagnostics, plus the radiation monitor, which provides counting and spectral information on ionizing particles hitting the spacecraft. The magnetic diagnostics system will be the subject of our attention here.

One of the most important functions of the LTP magnetic diagnostics is the determination of the magnetic field and its gradient at the positions of the TMs. For this, it includes a set of four tri-axial fluxgate magnetometers, intended to measure with high precision the magnetic field at the positions they occupy in the spacecraft — see figure 1. However, their readouts do not provide a direct measurement of the magnetic field at the positions where the TMs are, and an interpolation method must therefore be implemented to calculate it. In the circumstances we face, this is a difficult problem, mostly because the magnetometer layout is such that they are too distant from the locations of the TMs compared with the typical distances between magnetic sources on the satellite. It is important to solve this problem as magnetic noise can be as high as 40% of the total budget [2] given in Eq. (1), and hence it must be properly quantified.

2. Description of the problem

Magnetic noise in the LTP is allowed to be a significant fraction of the total mission acceleration noise because 40% of the total noise (3 \times 10^{-14} \text{ m s}^{-2} \text{ Hz}^{-1/2}) can be apportioned to magnetism, see Eq. (1). This noise occurs because the residual magnetisation and susceptibility of the TMs couple to the surrounding magnetic field. The magnetic field and its gradient randomly fluctuate in the regions occupied by the TMs, thus resulting in a randomly fluctuating force:

\[ \delta F = \left\langle \left[ \left( \frac{\chi}{\mu_0} B \right) \cdot \delta \nabla \right] B + \frac{\chi}{\mu_0} [\delta B \cdot \nabla] B \right\rangle V \]  

(2)
In this expression $\mathbf{B}$ is the magnetic field in the TM, $\nabla \mathbf{B}$ stands for the magnetic field gradients, $\chi$ its magnetic susceptibility ($\chi = 2.5 \times 10^{-5}$), $\mathbf{M}$ its density of magnetic moment (magnetisation, with a nominal value of $2 \times 10^{-4}$ A/m), $V$ is the volume of the TMs ($V = 9.7336 \cdot 10^{-5}$ m$^3$), $\mu_0$ is the vacuum magnetic constant, $4\pi \times 10^{-7}$ m kg s$^{-2}$ A$^{-2}$, and $\langle \cdots \rangle$ indicates TM volume average of the enclosed quantity. Finally, $\delta \mathbf{B}$ represents the fluctuation of the magnetic field, and $\delta \nabla$ stands for the fluctuation of the gradients.

Quantitative assessment of magnetic noise in the LTP clearly requires real-time monitoring of the magnetic field, which in LPF is done by means of a set of four tri-axial fluxgate magnetometers (12 magnetic channels) [4]. These devices have a high-permeability magnetic core, so they must be kept somewhat far from the TMs. The TMs are located at $x = \pm 0.1880$ m ($y = 0$), and the magnetometers are located at the following positions: magnetometer $M_1$ is located at $x = -0.0758$ m and $y = 0.3694$ m, magnetometer $M_2$ is at $x = 0.0758$ m and $y = -0.3694$ m, whereas magnetometers $M_2$ and $M_4$ are along the x-axis but at positions $x = \pm 0.3765$ m ($y = 0$). Therefore, the minimum distance of a magnetometer to the position of the TM is $\sim 20$ cm (all TMs and magnetometers are place within the same $z-$plane). The consequence of this is that the measured field does not directly give that at the positions of the TMs, as is needed. Hence, a procedure to estimate the field at these positions, based on the data from the magnetometers, must be established.

As previously mentioned, the sources of magnetic field are essentially electronics boxes plus a few genuinely magnetic components inside the spacecraft. The interplanetary magnetic field is orders of magnitude weaker, hence of little relevance to the effects considered here, and solar panel effects will not be considered. There are no sources of magnetic field inside the LTP Core Assembly (LCA), all being placed outside its walls but within the spacecraft. Therefore, the magnetic field is smaller towards the centre of the LCA than it is in its periphery, where the magnetometers take measurements. The number of sources identified by Astrium (prime contractor of the mission) is around 40, and these can be modelled as point magnetic dipoles [5]. In the following sections two different interpolation schemes for the Magnetic Diagnostics System of LISA Pathfinder will be presented and their performance compared.

3. Classical interpolation theory

The LCA region is treated as a vacuum, with a magnetic field of zero divergence and curl, $\nabla \cdot \mathbf{B}(x, t) = 0$ and $\nabla \times \mathbf{B}(x, t) = 0$. Hence, a scalar function $\Psi(x, t)$ exists such that

$$\mathbf{B}(x, t) = \nabla \Psi(x, t) \quad \text{with} \quad \nabla^2 \Psi(x, t) = 0$$

The solution to this equation can be expressed as an orthogonal series of the form

$$\Psi(x, t) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} M_{lm}(t) r^l Y_{lm}(\mathbf{n})$$

where $r \equiv |\mathbf{x}|$ and $\mathbf{n} \equiv \mathbf{x}/r$ are the spherical coordinates of the field point $\mathbf{x}$. Finally, the multipole coefficients, $M_{lm}(t)$, depend on the sources of magnetic field, and boundary conditions. The interpolated magnetic field is given by cutting off the series after the terms with $l = l_{\text{max}} \equiv L$, or

$$\mathbf{B}_{\text{estim}}(x, t) = \sum_{l=1}^{L} \sum_{m=-l}^{l} M_{lm}(t) \nabla [r^l Y_{lm}(\mathbf{n})]$$

The number of terms in this sum is $N(L) = \sum_{l=1}^{L} (2l + 1) = L(L + 2)$ which obviously equals the number of multipole coefficients needed to evaluate the sum. For example, we have $N(2) = 8$.
Table 1. Averaged estimation errors in the components of the magnetic field and of its modulus. They are reported in relative percentage.

| TM  | \(\varepsilon_{B_x} \) [%] | \(\varepsilon_{B_y} \) [%] | \(\varepsilon_{B_z} \) [%] | \(\varepsilon_{|B|} \) [%] |
|-----|----------------|----------------|----------------|----------------|
| TM1 | 496            | 323            | 349            | 79             |
| TM2 | 645            | 539            | 352            | 68             |

and \(N(3) = 15\). On the other hand, the number of magnetometer data channels is 12 — three channels per magnetometer, as the devices are tri-axial. This means we cannot extend push the series beyond the quadrupole (\(l = 2\)) terms. Since we have 12 data channels we have some redundancy in determining the first eight \(M_{lm}(t)\) coefficients up to \(l = 2\), but we do not have sufficient information to evaluate the next seven octupole terms.

4. Results of the classical interpolation method

The most salient features of the numerical analysis of the above mentioned method can be briefly summarised. First, we find that magnetic field estimation errors are very variable, ranging from a few percent to more than 1000% with only slight changes in the dipoles’ configuration and, secondly, these huge uncertainties happen in an utterly random and unpredictable way. A summary of the numerical results is presented in table 1. The conclusion is quite simple: the intrinsic linear character of the interpolation scheme is not capable of reliably reproducing the field structure inside the LCA and hence at the positions of the TMs. In addition to this chaotic behaviour not being predictable, the average error is also unacceptably large. The ultimate reason for such poor performances is the small number of magnetometers as well as the positions they occupy: four magnetometers only allow for a field multipole expansion up to quadrupole terms, which means that the field values at the TMs are just linearly interpolated between magnetometer readouts at the boundary of the LCA. Further, the magnetometers are closer to the magnetic field sources than they are to the TMs, which prevents resolution of the spatial field structure details inside the LCA using only linear terms in the space coordinates.

5. A novel approach: neural network algorithms

Search for an alternative approach to the classical interpolation scheme is imperative, otherwise the information provided by the magnetometers will not be useful for the main goal of the LTP magnetic diagnostics system, i.e., to quantify the contribution of the magnetic noise to the total system noise. Here we present some promising results obtained using a completely different methodology, neural networks [6]. Artificial neural networks are made up of interconnecting artificial neurons (programming constructs that mimic the properties of biological neurons) that have the capacity to learn from processing data. Neural networks are often used in solving nonlinear classification and regression tasks by learning from data, hence are worth investigation for the present problem [7]. There are four sets of tasks which need to be implemented when solving a problem using artificial neural networks: (i) Neuron model selection, (ii) model and architecture selection, (iii) learning paradigm and learning algorithm selection and (iv) performance assessment. In this paper we will focus on tasks (i), (ii) and (iv), the third one is beyond the scope of this paper.

The neuron is the basic unit of any neural network. It collects the inputs from all other neurons connected to it and computes a weighted sum of all the signals injected into it, generally adding a bias as well. If we represent the inputs by a vector \(\mathbf{x} \equiv (x_1, \ldots, x_n)\), and the weights
Figure 2. Field estimation performance for $B_x$, $B_y$, $B_z$ and $|B|$ using artificial neural networks. Note that the estimation errors are now very small and that the distributions of errors have zero mean errors and standard deviations below 10%, which represents an increase of one order of magnitude in the estimation performance.

by a $w \equiv (x_1, \ldots, w_n)$ then this operation consists of calculating the sum

\[ \Sigma = w_0 + \sum_{k=1}^{n} w_k x_k \equiv w_0 + w^T x \quad (3) \]

where the superindex $T$ stands for transpose matrix. In this case, $w^T$ is a row vector while $x$ is a column vector, so that $w^T x$ is the scalar product of $w$ and $x$. A term $w_0$ is added to form the most general linear function of the vector argument $x$; it is called the bias.

Artificial neural networks are software or hardware models inspired by the structure and behaviour of biological systems, and they are created by a set of neurons distributed in layers. There are many different types of neural networks in use today, but the architecture of a so-called feed-forward network, where each layer of neurons is linked with the next by means of a set of weights, is the most commonly used, and will also be used here. The data streams coming from the magnetometers will be considered the system inputs, while magnetic field results and their gradients at the positions of the TMs will be the system outputs.

6. Implementation and results

The implementation of the neural network algorithm for this specific application has been performed according to the following characteristics. The implemented architecture is a feed-forward network implemented with only one hidden layer (15 neurons in the hidden layer). The input vector is characterized by the 12 onboard magnetometers channels and the output vector by the field and gradient values at the positions of the TMs.

Training and testing have been done based on different field realisations. Training has been performed using a backpropagation algorithm on batches of 1 000 examples, i.e., each example will consist in the magnetic field at the magnetometers’ positions, plus the magnetic field and gradient at the TMs positions, all of them corresponding to a given configuration of the 37 dipole sources [5]. Two different batches of examples, each one including 1 000 realisations of a possible magnetic environment, have been generated. The first batch has been used as the training set for a neural network with 12 inputs (3 inputs for each of the 4 vector magnetometers) and 16 outputs representing the field information at the position of the two TMs (3 field plus 5 gradient components per TM). Note that only 5 of the 9 gradient components $\partial B_i/\partial x_j$ are independent.
This is because \( \partial B_i / \partial x_j \) is a traceless and symmetric matrix. The second batch has been used for validation to assess the performance of the network in front of magnetometers readings not used during the training procedure.

Figure 2 shows the distribution of relative percentage errors of the estimated components of the magnetic field at the positions of each TM. The plots are based on the results of the 1000 validation runs previously described. As can be observed, the order of magnitude of the errors of the estimated fields and gradients are now below \( \sim 15\% \). This represents a reduction of estimation errors of more than one order of magnitude in comparison with the multipole expansion method, which is a substantial improvement for the magnetic diagnostics for LISA Pathfinder.

7. Conclusions
The research presented here allows to draw the following conclusions. First, neural network algorithms represent a significant improvement of the quality of the magnetic field and gradient interpolation in LISA Pathfinder. We find that where classical interpolation methods produce very erratic results with very large (over 500\%) average estimation errors, neural network interpolation algorithms achieve zero mean errors with 10\% standard deviation. However, we stress that these results require a careful choice of the training data set and of the training algorithm. Hence, we foresee that real measured data will be required to perform a proper training on the actual spacecraft magnetic configuration. Nevertheless, the results obtained so far are promising, although they have been obtained for DC field and gradient estimation. Current research is underway for estimation in the presence of magnetic field drifts and fluctuations and will be presented elsewhere.

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