The core-halo mass relation of ultra-light axion dark matter from merger history

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In the context of structure formation with ultra-light axion dark matter, we offer an alternative explanation for the mass relation of solitonic cores and their host halos observed in numerical simulations. Our argument is based entirely on the mass gain that occurs during major mergers of binary cores and largely independent of the initial core-halo mass relation assigned to hosts that have just collapsed. We find a relation between the halo mass $M_h$ and corresponding core mass $M_c$, $M_c \propto M_h^{2\beta-1}$, where $(1 - \beta)$ is the core mass loss fraction. Following the evolution of core masses in stochastic merger trees, we find empirical evidence for our model. Our results are useful for statistically modeling the effects of dark matter cores on the properties of galaxies and their substructures in axion dark matter cosmologies.

I. INTRODUCTION

Ultra-light scalar field dark matter with mass $\sim 10^{-22}\text{eV}$ has recently been the subject of intense research (e.g., [1]). Possible constituents are ultra-light axions (ULAs) that are produced non-thermally via the misalignment mechanism [10–12]. If self-interactions can be neglected, this type of dark matter candidate is often referred to as fuzzy dark matter (FDM) [13, 14]. On scales smaller than the de Broglie wavelength of particles with the halo’s virial velocity, the so-called quantum Jeans length [14, 15], quantum effects suppress gravitational collapse and produce flat halo cores. Simulations of cosmological structure formation [2] and merging solitonic solutions [3, 16] based on the Schrödinger-Poisson (SP) equations indicate that FDM halos contain distinct cores surrounded by NFW-like profiles. [2] find that the mass of these cores, $M_c$, is related to the halo mass, $M_h$, by a power law relation, $M_c \propto M_h^{1/3}$. They propose an explanation based on the relation $M_c = \alpha (|E|/M)^{1/2}$, where $E$ is the total energy, $M$ is the total mass, and $\alpha$ is a constant of order unity, which they motivate heuristically with non-local consequences of the Heisenberg uncertainty relation. Identifying $E$ and $M$ with the energy of the halo $E_h$ and its virial mass $M_h$, they arrive at the numerically measured core-halo mass relation [3].

Note that while $M_c \sim |E|^{1/2}M^{-1/2}$ is consistent with the intrinsic scaling properties of the SP equations (see, e.g., [17]), it is not unique (i.e., it can be multiplied by any scale invariant combination of $|E|$ and $M$). Removing any residual effects of the scaling symmetry by constructing and analysing scale invariant quantities, [10] were unable to reproduce this relation in simulations of solitonic core mergers. Furthermore, the model of [3] does not account for the combined evolution of $M_c$ and $M_h$ by halo mergers after the initial collapse of density perturbations which is known to be an important ingredient in hierarchical structure formation.

Comparing the initial and final masses of merging cores, [10] find a universal behavior of the core mass loss in mergers that depends nearly entirely on the mass ratio. Implementing this relation in a semi-analytic code for galaxy formation, [18] studies the effects of the core on the substructure of Milky way-sized FDM halos. Fig. [10] obtained with the method described in [18], shows the core-halo mass relation from [3].

FIG. 1. Core mass with respect to halo mass for Milky way-sized FDM halos at $z=0$ from stochastic merger trees (the definition of $N_{\text{major}}$ will be shown later). The solid line shows the core-halo mass relation from [3].

\[ \log_{10} \left( \frac{M_c}{M_{\odot}} \right) = 8.65 \pm 0.05 \]

\[ \log_{10} \left( \frac{M_h}{M_{\odot}} \right) = 12.2 \pm 0.1 \]

\[ N_{\text{major}} = 1.5 \pm 0.2 \]

\[ \beta = 0.3 \pm 0.1 \]

\[ \alpha = 20 \pm 5 \]

\[ E_h = 10^{58} \text{erg} \]

\[ M_h = 10^{12.5} M_{\odot} \]

\[ M_c = 10^{8.65} \text{M}_{\odot} \]

\[ \rho_c = 10^{-22} \text{eV} \]

\[ h = 0.73 \]

\[ \Omega_m = 0.27 \]

\[ \Omega_{\Lambda} = 0.73 \]

\[ \Omega_{\text{cdm}} = 0.27 \]

\[ \Omega_{\text{b}} = 0.045 \]

\[ \Omega_{\text{cold}} = 0.75 \]

\[ \Omega_{\text{w}} = 0.25 \]

\[ \Omega_{\text{r}} = 0.002 \]

\[ \Omega_{\text{cdm}} = 0.27 \]

\[ \Omega_{\text{b}} = 0.045 \]

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are necessary. In particular, our model is independent of the dynamics of halo formation by gravitational collapse and hence insensitive to the initial core-halo mass relation of newly formed halos. We find a simple relation between the core and halo mass whose slope is a function of the core mass loss fraction. We provide numerical evidence for this dependence using stochastic merger trees.

The existence of compact cores in the halo substructure has many potentially observable direct signatures in, for instance, rotation curves of dwarf galaxies [19, 21], gravitational lensing [21–24], globular cluster streams in the Milky Way [25–28] or the thickening of the thin galactic disk [27–33]. Indirectly, the effects of compact cores on star formation at high redshifts may be probed by reionization history and the high-z galaxy luminosity function.

In addition to providing a simple explanation for the core-halo mass relation, it is straightforward to produce realizations of our stochastic model from modified EPS merger trees [18]. Since - as we will show - the core mass is determined by the individual accretion history, it can be modeled more realistically using individual mass accretion histories that recover not only the mean core-halo mass relation but also its scatter.

II. CORE-HALO MASS RELATION

Given the mass loss fraction of cores during each merger, we can calculate the evolution of core mass along the merger history. Since cores merge and relax to their final state on a dynamical time scale once they begin to overlap [16], we only need to consider isolated binary mergers.

To calculate the evolution of core masses, we first need to know the initial core masses for halos without progenitors, i.e., those that form from direct collapse. Their mass is determined by the cutoff mass in the halo mass function. As shown in [18], the cutoff mass depends only mildly on redshift, so the directly collapsed halos have approximately equal masses, \( M_{c,\text{min}} \), independent of their collapse redshift. The initial core masses are therefore also expected to have roughly equal values, \( M_{c,\text{min}} \).

Halo mergers change both core and halo masses. [10] find that only mergers with mass ratio \( \mu < 7/3 \) yield an increased core mass \( M_c = \beta(M_{c,1} + M_{c,2}) \). Here, \( M_{c,1} \) and \( M_{c,2} \) are the masses of the initial cores and \((1 - \beta)\) is the core mass loss fraction, where \( \beta \sim 0.7 \) independent of the initial core masses. We refer to mergers with \( \mu < 7/3 \) as major mergers. Larger mass ratios (minor mergers) leave the core mass of the more massive halo unchanged and result in the total disruption of the smaller halo. Smooth accretion corresponds to accretion with very high mass ratios and is treated in the same way.

In summary, in our model three types of physical interactions can change the core and halo masses: smooth accretion (see [18] for more details), minor mergers, and major mergers. The first two increase the mass of the halo but leave the core mass unchanged. Major mergers increase both halo and core masses.

Our model is based on a simplified description of the merging process. Suppose that \( N \) halos with halo mass \( M_{h,\text{min}} \) and core mass \( M_{c,\text{min}} \) merge to form one halo with mass \( M_h \) whose contribution from mergers is \( N M_{h,\text{min}} \). Assuming that the mass contributed by smooth accretion is also proportional to \( N M_{h,\text{min}} \), we have

\[
M_h = \alpha N M_{h,\text{min}}. \quad (1)
\]

If the final halo encounters \( N_{\text{major}} \) major mergers and \( N_{\text{minor}} \) minor mergers, then

\[
N = N_{\text{major}} + N_{\text{minor}} + 1. \quad (2)
\]

The more major mergers the final halo encounters, the more minor mergers it also tends to have. Hence, we can assume

\[
N_{\text{minor}} = b(\beta) N_{\text{major}}. \quad (3)
\]

We will show below that the assumptions Eqs. (1) and (3) are reasonable. Given the mass of the final halo, Eqs. (1), (2) and (3) allow us to estimate the number of major mergers it has encountered:

\[
N_{\text{major}} = \frac{1}{1 + b(\beta) \left( \frac{M_h}{\alpha M_{h,\text{min}}} - 1 \right)} \approx \frac{1}{1 + b(\beta) \frac{M_h}{\alpha M_{h,\text{min}}}}. \quad (4)
\]

Since minor mergers do not change the core mass, we only need to consider major mergers when estimating the final core mass \( M_c \). Suppose that during every major merger, both progenitors have the same core mass, i.e. the \((N_{\text{major}} + 1)\) first-formed halos with core mass \( M_{c,\text{min}} \) merge pairwise and form \( N_{\text{major}} + 1 \) halos with core mass \( 2\beta M_{c,\text{min}} \). This process continues until the formation of the final halo with mass \( M_h \). The other \((N - N_{\text{major}} - 1)\) first-formed halos are assumed to be accreted by minor mergers, thus they do not affect the core mass.

As explained above, the first-formed halos have nearly identical core masses, hence the assumption that all major merger have core mass ratio \( \mu = 1 \) is reasonable for the first generation of merging events. As halos continue to merge, we overestimate the core mass because there will be major mergers with \( \mu > 1 \) and correspondingly smaller core mass growth.

Finally, after \( \log_2(N_{\text{major}} + 1) \) generations of major merger events and \( N_{\text{minor}} \) minor merger events, the final halo has a core mass of

\[
M_c = (2\beta)^{\log_2(N_{\text{major}} + 1)} M_{c,\text{min}} = (N_{\text{major}} + 1)^{\log_2(2\beta)} M_{c,\text{min}} 
\approx (N_{\text{major}})^{\log_2(2\beta)} M_{c,\text{min}}. \quad (5)
\]
In order to test the core-halo mass relation on initial conditions, we implemented a power-law initial relation $M_{c, ini} \propto M_{h, ini}^n$ for halos that have no progenitors in the modified Galacticus code. Fig. 4 shows the results for $n = 1/3$, $n = 1$, and $n = 2$. Clearly, the core-halo mass relation at $z = 0$ depends only very weakly on the initial mass distribution. Hereafter, we will use $n = 1/3$ to set the initial core mass.

Next, we verify the two assumptions made in deriving Eq. (1) and Eq. (3). The left panel of Fig. 4 shows the halo mass $M_h$ with respect to the number of first-formed halos $N$ obtained from merger trees. Despite large scatter at small $N$ representing halos that have only encountered few mergers and are thus more strongly affected by the uncertainty of individual events, the assumed linear dependence Eq. (4) fits well.

The center panel of Fig. 4 shows the number of minor mergers $N_{minor}$ with respect to the number of major mergers $N_{major}$. Again, at small $N_{major}$ the data points have large scatter, but in general the assumption Eq. (3) gives a reasonable fit. Finally, the right panel of Fig. 4 shows the halo mass $M_h$ with respect to the number of major mergers $N_{major}$. This plot is a combination of the first two and is just meant to give a more relevant comparison between Eq. (1) inferred from the two assumptions and the results from merger trees.

To study the impact of the core mass loss fraction, we varied the value of parameter $\beta$ between 0.5 and 1. Correspondingly, we must also modify the definitions of minor and major merger: if the core mass ratio is larger (smaller) than $\beta/(1 - \beta)$, the merger is defined as minor (major) merger. For $\beta = 0.7$, we obtain the former definition.

Before showing the results from merger trees, we consider two extreme cases. In the case of $\beta = 0.5$, the core mass does not change during any of the three possible interactions. The final core mass is solely determined by the initial core mass and independent of the final halo mass. On the contrary, for $\beta = 1$, all mergers will be major mergers and the final core mass is given by $M_c = N M_{c, ini}$. Since the halo mass is also proportional to $N$ (Eq. (11)), in this case the core-halo mass relation is linear. Expressed in the form $M_c \propto M_h^{\gamma(\beta)}$, we thus have $\gamma(0.5) = 0$ and $\gamma(1) = 1$. A simple linear parameterization for $\gamma(\beta)$ is $2\beta - 1$ which yields the core-halo mass relation

$$M_c \propto M_h^{2\beta - 1}. \quad (8)$$

Note that it is very similar to Eq. (4) obtained from the merger history.

Fig. 5 shows the core-halo mass relation at present time for different $\beta$ and compares them with the predictions from Eq. (4), Eq. (4), and the linear parameterization Eq. (5). Despite the simplifications in deriving Eq. (4), we find reasonable agreement for the core-halo mass relation for different core mass loss fractions $(1 - \beta)$. At larger halo masses (implying more major mergers), the prediction of our model Eq. (4) tends to overestimate the

Substituting Eq. (4) into Eq. (5), we have

$$M_c = \left[\frac{1}{1 + b(\beta)} \frac{M_h}{a M_{h, min}}\right]^{\log_2(2\beta)} M_{c, min} = A M_h^{\log_2(2\beta)}. \quad (6)$$

Note that although the relation Eq. (6) does not explicitly depend on redshift, the prefactor $A$ does since $\alpha$ and $b$ change with redshift. On the other hand, the exponent of $M_h$ only depends on the core mass loss fraction. Treating $(2\beta - 1)$ as a small number, Eq. (6) yields

$$M_c \propto M_h^{\log_2(2\beta)} \approx M_h^{(2\beta - 1)/\ln 2} \approx M_h^{1.44(2\beta - 1)} \quad (7)$$

to leading order. This relation overestimates the core mass when binary mergers with $\mu > 1$ are involved. We will account for this effect below when presenting our numerical results.

In order to test the core-halo mass relation given in Eq. (5), we use the modifications to the SAM code Galacticus [34, 35] for FDM and build 2000 merger trees for halos with $4 \times 10^{11} < M_h < 4 \times 10^{13} M_\odot$. The mass resolution is set to $2 \times 10^8 M_\odot$. The parameter $\beta$ is set to 0.7 unless otherwise specified.

Fig. 2 shows an example of 15 trajectories of the core-halo mass relation chosen from the merger trees, i.e. each line shows the evolution of the core and halo mass along the main branch of one tree. The effects of smooth accretion, minor mergers, and major mergers can clearly be seen in this figure. Smooth accretion and minor mergers increase the halo mass while the core mass remains constant. Major mergers increase both core and halo mass.

Eq. (6) predicts that while the proportionality factor $A$ may depend on the initial core mass $M_{c, min}$, the exponent is independent of it.
core masses. Eq. (8) gives a slightly better fit, implying that we can use it as a correction to Eq. (6). For $\beta = 0.7$ [10], Eq. (6) yields $M_c \propto M_h^{3/2}$. It is close to the relation $M_c \propto M_h^{3/3}$ and fits the cosmological simulations [2] equally well.

It should be noted that the merger trees constructed using the method described in [12] are not very accurate at redshifts $z > 3$. Consequently, the core-halo mass relation obtained from these merger trees may contain some uncertainties from mergers at higher redshifts. To estimate the possible effects, we ran the merger trees for CDM with a mass resolution of $5 \times 10^5 M_\odot$ to mimic the cutoff of the HMF for FDM (the halos below the mass resolution are assumed to be smoothly accreted). Then we assigned the core masses to the halos in the same way as for FDM. We find similar results for the core-halo mass relation, implying that the uncertainties in merger trees at higher redshifts do not affect our conclusions.

In order to compare the core mass predicted for FDM halos with observations, the prefactor $A$ in Eq. (6) is also important. According to our results, we can replace $\log_2(2\beta)$ in Eq. (6) with $2\beta - 1$ to give a better estimate of the core-halo mass relation. If we further assume that at the beginning, i.e. prior to any mergers, there were only pure solitons (instead of virialized halos produced by mergers of solitons), the initial core mass is $M_{c, \text{min}} = \frac{1}{4}M_{h, \text{min}}$ by definition [2]. Then we have

$$M_c = \frac{1}{4}B \left( \frac{M_h}{M_{h, \text{min}}} \right)^{2\beta - 1} M_{h, \text{min}},$$

(Eq. 9)

where $B \equiv 1/[\alpha(1+b(\beta))]^{2\beta-1}$. The redshift dependence is implicitly contained in the function $B$. If $\beta = 2/3$, Eq. (9) coincides exactly with the core-halo mass relation (Eq. (6)) in [3].

III. CONCLUSIONS

By considering the merger history of dark matter halos in scenarios with ultra-light bosonic dark matter, we offer an alternative explanation for the core-halo mass relation observed in cosmological simulations. We provide evidence for our model using stochastic merger trees and show that the core-halo mass relation depends only on the mass loss fraction of cores during binary mergers, $M_c \propto M_h^{2\beta - 1}$. We find that for $\beta = 0.7$ [10], this relation fits numerical data from cosmological simulations very well [3].
FIG. 5. The core-halo mass relation at merger may still allow the construction of a stochastic though a unique core-halo mass relation does not exist in side of individual halos and during halo mergers. Although a single solitonic core, each dark matter halo hosts a large number of these objects and mergers take place both inside of individual halos and during halo mergers. Although a unique core-halo mass relation does not exist in this case, the universal mass gain for each substructure merger may still allow the construction of a stochastic model similar to ours. We will explore this possibility in future work.

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