Computation of optical waveguide interaction for quantum gates implementation

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Abstract. The system of two coupled optical dual-mode waveguides is considered. The coupling of the system is studied to find a circuit for building a control switch for two qubit gates. The classical coupled mode theory is applied and the exact expressions for coupling coefficients are derived. The parameters of the system for performing the desired operations are numerically computed and analysed. The system describing the influence of intermodal interactions is solved numerically. The distortions are analysed.

1. Introduction

The physical implementation of quantum computer is a problem of vital importance since the idea was suggested by Richard Feynman [1]. One of the main obstacles on the way to build a multiple qubit quantum computer is a quickness of the system decoherence. The optical quantum computing is an approach, that allows to reduce the effect of decoherence due to the low effectiveness of photon-photon and photon-environment interaction. The conventional linear optical model is based on the usage of the bulk optics devices such as beamsplitters, polarizing beamsplitters, mirrors, phase shifters etc. However, the practical implementation would require the usage of integrated optics devices, where the bulk optics devices are replaced with waveguides. The universal set of quantum gates based on waveguides interaction is described in [2].

Nevertheless, the existing models of linear optical two-qubit gates are probabilistic with 1/9 probability of success, that can be increased only with a large number of ancilla qubits, thus, making them unscalable for building a complex computational scheme [3]. The alternative is to use nonlinear effects for quantum gate implementation. However, the low effectiveness of Kerr-like nonlinearities would require long optical paths, that could potentially accumulate the computational error due to the waveguide border imperfections [4].

One of the possible solutions was suggested in [5], where it is suggested to encode quantum bits with two mode optical waveguides. This work was devoted to the development of this approach by studying the system of two similar two-mode waveguide that can be used as a control switch for nonlinear CNOT gate.

2. Two mode waveguide optical model

The idea to perform quantum computing using two mode optical waveguides was suggested in [5]. |0⟩ qubit state can be encoded with a TE₀ waveguide mode, while |1⟩ with TE₁. The quantum theory of rays allows one to note the analogy between the transverse field distribution in the waveguide and the quantum mechanical wavefunction, so the system behaves as a quantum bit [6]. One of the main benefits of this approach is the high optical density of the light within the core region of the waveguide that increases the intensity of nonlinear interaction, so it can be used to perform qubit operations, thus
making the model closer to possible practical implementations. Note that coding of qubit by two modes is used not only in waveguides but also in free space channels where the Gaussian beam modes play the role of waveguide modes (see, e.g., [7]).

The model of CNOT gate for two mode waveguide qubits is described in [5] (see figure 1). The main part of the circuit is the waveguide Mach-Zehnder interferometer with nonlinearities in its arms with intensity dependent refractive index. The waveguide coupling regions perform the waveguide state dependent mode separation and combine operations. Thus, the closer is the state of the control qubit to |1⟩, the higher is intensity of light, that is transferred to the upper arm of Mach-Zehnder interferometer and the larger is the phase shift the target qubit is subjected to. The necessary condition for the correct operation of the system is, that the coupling region transfers TE1 mode of the control waveguide to the upper MZI arm, while keeping TE0 within the control waveguide. The adjustment of the coupling region parameters (the coupling length L and the distance R between the waveguides) is needed to make the circuit perform operations in the desired way. It is our goal.

3. The coupled mode theory
The interaction between two mode waveguides can be described using the coupled mode theory [8]. This approach is approximate because the field in the system is presented as the fields of the individual waveguides with amplitude coefficients.

\[
E = \sum_{v,m=0,1} A_{vm}(z) E_{vm}
\]

\[
H = \sum_{v,m=0,1} A_{vm}(z) H_{vm}
\]

The fields are substituted into the Maxwell equations.

\[
\nabla \times H = i\omega \varepsilon_0 \hbar^2 (x, z) E
\]

\[
\nabla \times E = -i\omega \mu_0 H
\]

The resulting equations for amplitude coefficients are multiplied with conjugate fields and integrated over the cross sections.
\[ \sum_{v, m = 0, 1}^{+\infty} \int dx \left( \frac{\partial A_{vm}}{\partial z} \mathbf{e}_z (\mathbf{H}_{vm} \times \mathbf{E}_{v'm'}) + \frac{\partial A_{vm}}{\partial z} \mathbf{e}_z (\mathbf{E}_{vm} \times \mathbf{H}_{v'm'}) \right) \\
- i\omega \epsilon_0 (n_{core}^2 - n_{clad}^2)_{(v+1)\text{mod}2} A_{vm} E_{vm} E_{v'm'} = 0, \quad v', m' = 0, 1 \] (3)

One cannot find the general solution in an analytic form. It can be made numerically. However, it is possible to simplify the analysis by neglecting terms, that describe weak interactions of high orders. The introduction of coupling coefficients \( c_m \) and \( D_m' \) yields to the following system of four differential equations.

\[ \begin{align*}
\frac{\partial A_{00}}{\partial z} c_0 + A_{10} iD_0^0 + A_{11} iD_1^0 e^{i\Delta \beta z} &= 0 \\
\frac{\partial A_{01}}{\partial z} c_1 + A_{10} iD_0^1 e^{-i\Delta \beta z} + A_{11} iD_1^1 &= 0 \\
\frac{\partial A_{10}}{\partial z} c_0 + A_{00} iD_0^0 + A_{01} iD_0^1 e^{i\Delta \beta z} &= 0 \\
\frac{\partial A_{11}}{\partial z} c_1 + A_{00} iD_0^1 e^{-i\Delta \beta z} + A_{01} iD_1^1 &= 0
\end{align*} \] (4)

It is possible to simplify the system further by neglecting the terms of intermodal interaction with \( e^{i\Delta \beta z} \) as well. The analytical solution can be found for the simplified system.

\[ \begin{align*}
A_{0m}(z) &= A_{0m}(0) \cos \left( \frac{D_m^m}{c_m} z \right) - iA_{1m}(0) \sin \left( \frac{D_m^m}{c_m} z \right) \\
A_{1m}(z) &= A_{1m}(0) \cos \left( \frac{D_m^m}{c_m} z \right) - iA_{0m}(0) \sin \left( \frac{D_m^m}{c_m} z \right)
\end{align*} \] (5)

In this approximation the system transfers \( TE_1 \) mode to another waveguide, while keeping \( TE_0 \) to remain in the same, when the following condition in integers is satisfied.

\[ \frac{4v_1 + 1}{4v_0} = \frac{D_1^1}{D_0^0} / c_0 \quad v_0, \ v_1 \in \mathbb{N} \] (6)

The analytical expressions for \( D_m^m / c_m \) were obtained by directly evaluating the integral expressions with TE mode fields given in [8].

\[ \begin{align*}
D_0^0 / c_0 &= 4\pi^2 (n_{core}^2 - n_{clad}^2) \cos(\chi_0 d) \exp[-\gamma_0 (R-d)] \beta_0^{-1} (\lambda n_{clad})^{-2} (\gamma_0^2 + \chi_0^2)^{-1} \\
&\times \left[ \gamma_0^2 \cos(\chi_0 d) \sin(\gamma_0 d) + \chi_0 \sin(\chi_0 d) \cos(\gamma_0 d) \right] \left[ d + \frac{\sin(2\gamma_0 d)}{2\gamma_0} + \frac{\cos(2\chi_0 d)}{2\chi_0} \right]^{-1} \\
D_1^1 / c_1 &= 4\pi^2 (n_{core}^2 - n_{clad}^2) \sin(\chi_1 d) \exp[-\gamma_1 (R-d)] \chi_1 \beta_1^{-1} (\lambda n_{clad})^{-2} (\gamma_1^2 + \chi_1^2)^{-1} \\
&\times \left[ \chi_1 \cos(\chi_1 d) - \gamma_1 \sin(\chi_1 d) \right] \left[ d - \frac{\sin(2\chi_1 d)}{2\chi_1} + \frac{\sin(2\gamma_1 d)}{2\gamma_1} \right]^{-1}
\end{align*} \] (7)

Hence, the ratio (6) can be satisfied by adjusting the coupling distance \( R \) to match almost any given pair of \( v_0 \) and \( v_1 \). The coupling length \( L \), that makes the system perform the desired control operation can be expressed as

\[ L = (2D_1^1)^{-1} c_1 (4v_1 + 1) \] (8)
4. Numerical results
The numerical evaluation of the coupling coefficients and various propagation constants as a solution of transcendental equation was implemented for a symmetrical system of coupled waveguides. The system parameters are the light wavelength $\lambda$, the waveguide width $2d$ and the refractive indices $L$, $n_{\text{core}}$ and $n_{\text{clad}}$ of the core and cladding layers respectively. The algorithm for determination of the distance between waveguides $R$, so that the coupling coefficient are adjusted to satisfy the condition (6), was implemented. The secondary goal, that is achieved by the algorithm is the minimization of the coupling length $L$, so the compactness of the device is increased and the potential error due the waveguide boundaries imperfections is reduced. In order to perform that, the algorithm tests a number of parameters $R$, the one that yields the minimum value of $L$ is selected.

The analysis above was performed for the system of equations (4), that was simplified by neglecting of weak intermodal interactions. In addition, the system (4) without simplifications was solved numerically with the finite difference method in order to estimate the influence of intermodal interaction and the potential error due to them.

It was obtained, that including of intermodal interactions distorts the performance of the device notably. For the system with parameters $\lambda = 1.018 \, \mu m$, $n_{\text{core}} = 1.57$, $n_{\text{clad}} = 1.55$, $d = 1.83 \, \mu m$, $R = 3.96 \, \mu m$ (figure 2, 3), that would perform ideally in the theoretical model governed by equations (5), it was obtained, that the loss in the signal due to the intermodal interaction is about 0.5%. The loss is due to the undesired guidance of modes of the different order and the related phase shift in the signal mode. The error of that order should be taken into account, as it would multiply in the complex scheme of several consecutive CNOT gates, however it could be coped with by the application of various quantum error correction algorithms.

Figure 2. Propagation of waveguide modes for $|0\rangle$ as input parameter in the control qubit. (a) TE$_0$ in the control waveguide. (b) TE$_0$ in the target waveguide. (c) TE$_1$ in the control waveguide. (d) TE$_1$ in the target waveguide.
Figure 3. Propagation of waveguide modes for [1] as input parameter in the control qubit. (a) TE₀ in the control waveguide. (b) TE₀ in the target waveguide. (c) TE₁ in the control waveguide. (d) TE₁ in the target waveguide.

5. Conclusion
The system of two slab symmetrical two mode coupled optical waveguides was considered with the aim to be used as a switcher for nonlinear CNOT gate. The coupled mode theory approximation was applied to simulate the behaviour of the system. The approximate analytical model was studied in order to obtain the parameters, that would make the coupling perform in the desired way. The extended differential equations system was solved numerically with the finite difference method to consider the influence of the intermodal interactions. It was obtained, that the intermodal interaction could cause the computational error of 0.5 %. However, the possible distortions due to the waveguide bends and boundary imperfections were not considered.

References
[1] Feynman R 1986 Found Phys. 16 507
[2] Gavrilo M, Gortinskaya L, Pestov A, Popov I and Tesovskaya E 2007 Phys. Part. Nuclei Lett. 4 137
[3] Knill E, Laflamme R and Milburn J 1997 Nature 409 46
[4] Milburn G 1989 Phys. Rev. Lett. 62 2124
[5] Fu J and Tang S 2003 Chinese Phys. Lett. 20 1426
[6] Gloge D and Marcuse D 1969 J. Opt. Soc. Am. 59 1629
[7] Faleeva M and Popov I 2020 Nanosyst. Phys. Chem. Math. 11 651
[8] Marcuse D 1982 Light Transmission Optics ed S Mitra (New York: van Nostrand Reinhold) chapter 10 pp 519–531