Gravitation in the solar system and metric extensions of General Relativity

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Outline

- The evidence for General Relativity
- The Pioneer anomaly
- Metric extensions of General Relativity
- Phenomenological consequences
- Conclusion
General Relativity as a metric theory

The gravitational field identifies with the metric in a Riemannian space-time:

- ideal clocks measure the proper time along their trajectory

\[ ds^2 \equiv g_{\mu\nu} \, dx^\mu \, dx^\nu \]

- freely falling probes (masses and light rays) follow geodesics

\[ \delta \int ds = 0 \]

One of the most accurately tested principles of physics.

The gravitation field couples to its sources through a relation between space-time curvature and energy-momentum tensors:

- one curvature tensor has a null divergence (Bianchi identities) like the energy-momentum tensor (conservation laws)

\[ E_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \]

\[ E^\mu_{\nu;\mu} \equiv 0 \]

\[ T^\mu_{\nu;\mu} \equiv 0 \]

- in General Relativity, the two tensors are simply proportional to each other (Einstein-Hilbert equation)

\[ E_{\mu\nu} = \frac{8\pi G_N}{c^4} T_{\mu\nu} \]

Tests amount to compare geodesics predicted by a metric solution of this equation with observations.
**Parametrized post-Newtonian metrics**

Solution of GR for the metric in the solar system (using spatially isotropic coordinates; the Sun is treated as a point-like motionless source):

\[
\begin{align*}
g_{00} & = (1 + 2\phi + 2\phi^2 + \ldots) \\
g_{jk} & = -(1 - 2\phi + \ldots) \delta_{jk}
\end{align*}
\]

with Newton potential:

\[
\phi \equiv -\frac{G_N M}{r c^2}, \quad |\phi| \ll 1
\]

GR is usually tested through confrontation with the family of PPN metrics:

\[
\begin{align*}
g_{00} & = (1 + 2\alpha \phi + 2\beta \phi^2 + \ldots) & \alpha = 1 \text{ fixes } G_N \\
g_{jk} & = -(1 - 2\gamma \phi + \ldots) \delta_{jk} & \beta = \gamma = 1 \text{ in GR}
\end{align*}
\]

- Motions for light and matter are predicted as the geodesics of this metric
- Comparison between observations and predictions is expressed in terms of anomalies of the PPN parameters: 
  \[\beta - 1, \gamma - 1\]
Tests of General Relativity

Tests in the solar system confirm General Relativity:

- Ranging on planets
- Astrometry and VLBI
- LLR = Lunar Laser Ranging (1969 – ongoing)
- Doppler velocimetry on artificial probes

Tests are consistent with GR and give bounds for potential deviations

\[ |\gamma - 1| \lesssim 3 \times 10^{-5} \]
\[ |\beta - 1| \lesssim 1 \times 10^{-4} \]

Living Reviews in Relativity, C.F. Will (2001)
Tests of Newton law

The Newtonian dependence of the potential is also well tested:

Search for a Yukawa correction

\[ \phi(r) = -\frac{G_N M}{c^2 r} \left(1 + \alpha e^{-\frac{r}{\lambda}}\right) \]

Experiments performed in the submillimeter range test the presence of a "fifth force"

Potential deviations from the Newtonian behavior remain possible at long ranges

\[ \lambda \gg 10^6 \text{m} \]

J. Coy, E. Fischbach, R. Hellings, C. Talmadge, and E. M. Standish (2003)
The Pioneer anomaly

Pioneer 10 Launch: 2 March 1972

Trajectories of Pioneer 10/11:
Elliptical (bound) orbits before the last fly-by;
Hyperbolic (escape) orbits after the last fly-by.
The velocity of the probe relatively to the station on Earth is deduced from the Doppler observable:

\[ \frac{f}{f_0} = 1 - \frac{2v}{c} \]

Comparison with the modelized velocity (using GR) shows a discrepancy which varies linearly with time:

\[ v_{\text{obs}} - v_{\text{model}} \sim -a_P \left(t - t_{\text{in}}\right) \]

\[ a_P \sim 0.9 \text{ nm s}^{-2} \]

J. Anderson et al, Phys. Rev. D 65 (2002) 082004
Can the Pioneer anomaly be compatible with other gravity tests?

• The equivalence principle (EP) cannot be violated at the level of the Pioneer anomaly:

\[ a_N \sim 1 \mu m \ s^{-2} \quad , \quad a_P \sim 1 \text{nm} \ s^{-2} \]

→ the geometrical interpretation of Einstein theory is preserved:
  ✓ gravitation is described as a Riemannian metric theory
  ✓ motions are identified with geodesics

• But Einstein-Hilbert equation can be modified:
  → modifications emerge naturally from radiative corrections to GR due to coupling between the graviton and other fields
  → gravitation coupling is modified and depends on scale
  → this leads to modifications of the metric around a gravitational source and then of geodesic motions
  → this results in phenomenological consequences
Metric extensions of GR

Two gravitation sectors must be considered:

- Einstein curvature contains two independent components, one related to the trace (sector 1), the other one to the traceless part (sector 0).
- Einstein-Hilbert relation is replaced by a general coupling involving two running constants. In the linearized theory (in Fourier space) and for a pointlike motionless stress tensor, it becomes:

\[
E_{\mu\nu} = E_{\mu\nu}^{(0)} + E_{\mu\nu}^{(1)} \quad T_{\mu\nu} = \delta_{\mu0}\delta_{\nu0}T_{00}
\]

\[
E_{\mu\nu}^{(0)} = \left\{ \pi_{\mu}^{0}\pi_{\nu}^{0} - \frac{\pi_{\mu\nu}\pi^{00}}{3} \right\} \frac{8\pi\tilde{G}^{(0)}}{c^4}T_{00} \quad E_{\mu\nu}^{(1)} = \frac{\pi_{\mu\nu}\pi^{00}}{3} \frac{8\pi\tilde{G}^{(1)}}{c^4}T_{00}
\]

\[
\pi_{\mu\nu} = \eta_{\mu\nu} - \frac{k_{\mu}k_{\nu}}{k^2}
\]

\[
\tilde{G}^{(0)} = G_N + \delta\tilde{G}^{(0)} \quad \tilde{G}^{(1)} = G_N + \delta\tilde{G}^{(1)}
\]

The solution remains in the vicinity of GR:

\[
E = [E]_{GR} + \delta E
\]

- the Einstein curvature of GR vanishes outside gravitational sources:
  \[
  [E]_{GR} = 0 \quad \text{where} \quad T \equiv 0
  \]
- the Einstein curvature of a general metric extension contains two non vanishing components in empty space:
  \[
  E = \delta E^{(0)} + \delta E^{(1)}
  \]
Two gravitation potentials are generated:
the general isotropic and stationary metric is written in terms of two potentials which parametrize the new phenomenological freedom:

\[
g_{00} = [g_{00}]_{GR} + \delta g_{00} \\
g_{rr} = [g_{rr}]_{GR} + \delta g_{rr}
\]

and are related to the modification of Einstein curvatures:

\[
\frac{\delta g_{00}}{[g_{00}]_{GR}} = 2 \int \frac{\delta \Phi_N' + ([g_{00}]_{GR} - 1)\delta \Phi_P'}{[g_{00}]_{GR}^2} dr \\
\frac{\delta g_{rr}}{[g_{rr}]_{GR}} = \frac{2r(\delta \Phi_N - \delta \Phi_P)'}{[g_{00}]_{GR}}
\]

\[
\delta E_0^0 \equiv 2\Delta(\delta \Phi_N - \delta \Phi_P) \\
\delta E_r^r \equiv -\frac{2}{r}\delta \Phi_P'
\]

The PPN metric is recovered as a particular case:

\[
\delta \Phi_N = (\beta - 1)\Phi^2 + O(\Phi^3), \quad \delta \Phi_P = -(\gamma - 1)\Phi + O(\Phi^2)
\]

The anomalous potentials, \(\delta \Phi_N\), \(\delta \Phi_P\) promote Eddington parameters \(\beta - 1\), \(\gamma - 1\) to the status of space dependent functions.
Modified Newton potential

- The first anomalous potential $\delta \Phi_N$ corresponds to a modification of Newton law and is strongly constrained by planetary tests:
  - the third Kepler law is verified on the orbital period of Mars compared to the radius of its orbit (measured independently)
  - the perihelion precessions of the planets agree with GR

- Deviations $\delta \Phi_N$ needed to explain the Pioneer anomaly are too large to remain unnoticed on planetary tests, if they are assumed to have a Yukawa or linear form.
- Deviations could in principle appear only after Saturn:
  - this possibility must be confronted to the ephemeris of outer planets (or other objects there)
Anomalous Pioneer acceleration

Modified solutions for light-like propagation (time delay function) and massive probe geodesics are obtained when using a metric extension of GR:

- a Pioneer-like anomaly results for probes with escape trajectories.
- We evaluate this effect by:
  - calculating the Doppler velocity, taking into account the perturbation of light-like propagation to and from the probes, as well as the perturbation of the motion of the probes
  - writing the derivative of this velocity as an acceleration:
  - subtracting the result of the standard calculation for GR:

\[ \delta a \equiv a - [a]_{GR} \]

- A constant anomalous acceleration is found:
  - for a metric anomaly linear in \( r \) in the first sector:
    \[ \delta \Phi_N \simeq \frac{r}{\ell_H} \]
  - or a metric anomaly quadratic in \( r \) in the second sector:
    \[ \delta \Phi_P \simeq -\frac{c^2}{3G_N M} \frac{r^2}{\ell_H} \]
  - or a superposition of these two anomalies.
Effects in the inner solar system

The two potentials affect the perihelion precessions of planets:

\[
\frac{\delta \Delta \varpi}{2\pi} \simeq u (u \delta \Phi_P)'' - \frac{c^2 u}{2G_N M} \delta \Phi_N'' \\
+ \frac{e^2 u^2}{8} \left( (u^2 \delta \Phi_P'' + u \delta \Phi_P')'' - \frac{c^2 u}{2G_N M} \delta \Phi_N''' \right) + \ldots
\]

Planetary perihelion precessions may be used to obtain constraints in the range

\[ r \sim \text{UA} \]

The two potentials affect the propagation of electromagnetic waves:

\[
\delta \Delta \theta \sim - \frac{G_N M}{c^2} \frac{\partial}{\partial \rho} \left( \delta \gamma(\rho) \ln \frac{4r_1 r_2}{\rho^2} \right)
\]

the deflection anomaly may be seen as an anomaly of Eddington parameter, increasing with the impact parameter:

\[
\delta \gamma(\rho) = - \frac{G_0}{G_N} + \frac{\zeta_0(\rho) \rho^2}{2G_N} \quad 2\delta \Phi_N - \delta \Phi_P \equiv - \frac{G_0 M}{c^2 r} + \frac{M}{c^2} r \zeta_0(r)
\]

Eddington tests may reveal the modification of GR through a space dependence of \( \gamma \) (GAIA).
Conclusion

• A larger phenomenological framework is available:
  – metric extensions of GR preserve its geometrical interpretation but change the coupling of gravity to energy-momentum tensors
  – the obtained generalizations of PPN metric promote Eddington parameters to the status of scale-dependent functions

• Motions in the solar system must be reanalysed in the new framework, taking into account the two potentials.

• Further tests can be performed:
  – check predictions against the recently recovered Pioneer data
  – search for Pioneer-related anomalies:
    • in the motions of planets or other objects, with dedicated probes
    • in Eddington / Shapiro tests
  – look for a scale dependence of Pioneer-related anomalies