In this paper, we investigate a trade-off between the number of radar observations (or measurements) and their resolution in the context of radar range estimation. To this end, we introduce a novel estimation scheme that can deal with strongly quantized received signals, going as low as 1-bit per signal sample. We leverage for this a dithered quantized compressive sensing framework that can be applied to classic radar processing and hardware. This allows us to remove ambiguous scenarios prohibiting correct range estimation from (undithered) quantized base-band radar signal. Two range estimation algorithms are studied: Projected Back Projection (PBP) and Quantized Iterative Hard Thresholding (QIHT). The effectiveness of the reconstruction methods combined with the dithering strategy is shown through Monte Carlo simulations. Furthermore we show that: (i), in dithered quantization, the accuracy of target range estimation improves when the bit-rate (i.e., the total number of measured bits) increases, whereas the accuracy of other undithered schemes saturate in this case; and (ii), for fixed, low bit-rate scenarios, severely quantized dithered schemes exhibit better performances than their full resolution counterparts. These observations are confirmed using real measurements obtained in a controlled environment, demonstrating the feasibility of the method in real ranging applications.

**Keywords**

Radar, FMCW, Ranging, Compressive Sensing, Quantization, Dither, Projected Back Projection, Iterative Hard Thresholding, 1-bit.

**I. INTRODUCTION**

Civilian radar applications such as automotive radar design or the growing fields of smart cities are more and more in need of small form factor and affordable radars [1]–[3]. As these complex applications often requires the deployment of many radar sensors working in a collaborative mode, the increasing amount of data recorded by these systems challenges both data transmission and processing techniques.

In this paper, we focus on lightening the acquisition of radar signals; we strongly reduce the resolution (or bit-depth) of each samples collected by a radar sensor without sacrificing accurate depth estimation. More precisely, in order to be processed, the physical voltage signals coming from a radar must be digitized. This quantization process is often forgotten in the system model; high resolutions — and expensive — Analog to Digital Converters (ADCs) must then be used to mitigate the resulting quantization noise, inducing fairly large bit-rates. We propose to remove this limitation by integrating quantization directly in the signal model, using the framework of Quantized Compressive Sensing (QCS) [4]. We consider a digitization modeled by a scalar mid-rise uniform quantizer (or lower floor quantizer). Our aim is thus to minimize the impact of lowering the acquisition bit-rate on the quality of the estimated ranges, hence targeting possibly cheaper radar receiver implementations.
The Compressive Sensing (CS) theory leverages the low-complexity nature of structured signals (e.g., their sparsity, compressibility or low-rankness) to reduce the signal sampling rate at the acquisition [5], [6]. CS shows that, with high probability, one can stably and robustly estimate such signals by collecting a number of random linear measurements driven by the signal “information-rate”, e.g., its sparsity level. During the last ten years, many works have considered the association of the radar principles with CS theory: first, to increase a targets parameter resolution [1], and later to reduce the number of samples to be processed [2]. The survey [3] describes the reduced sampling rate of different compressive (or sub-Nyquist) radar systems, in comparison with traditional Nyquist sampling schemes, although digitization impact is not covered.

One-bit quantized compressive radar schemes have been studied in, e.g., [7]–[9]. One limiting effect is that, as the digitization becomes coarser, ambiguities might appear between different unquantized signals — and thus different target configurations — that are digitized to the same bits, rendering the estimation ambiguous. These works, however, failed to address these ambiguities. Our previous work in 1-bit quantization applied to Frequency-Modulated Continuous-Wave radar (FMCW) [10] showed that these ambiguities do happen in realistic settings and measurements and can be counteracted using a pre-quantization dither. Dithering amounts to adding a designed noise on the signal, before quantizer’s action, with the goal of attenuating quantization distortions [11], [12]. This procedure is also used in, e.g., LIDAR imaging [13] where dithering is implemented in a real set-up by physically varying a time-delay before the acquisition, and was studied for high sampling rate ADCs [14].

In this paper we investigate a trade-off between the number of radar observations (or measurements) and their resolutions in the case of an FMCW radar with one transmitting and one receiving antenna. While this setting might seem restrictive as it only considers target range recovery, its setup allows us to perform thorough tests using both simulations and actual radar measurements. To this end, two range estimation algorithms, adapted to quantized radar signal, are used: Projected Back Projection (PBP) [11] and Quantized Iterative Hard Thresholding (QIHT) [15]. Compared to [10], this work deeply investigates the comparisons between severely quantized and high-resolution measurements constrained to the same bit-rate, i.e., between quantity and quality.

Let us summarize the main contributions of this work: (i) we show that ambiguities due to the combination of the intrinsic radar Fourier domain with harsh quantization exist and are removed using dithered quantization; (ii) we observe that as the number of measurements $M$ grows, non-dithered quantization yields range estimation error (using either PBP or QIHT) that saturates whereas the dithered schemes reach a decaying error when $M$ increases; (iii) we show that QIHT provides the best performances at low resolution for harsh bit-rate condition; and (iv) we confirm all the above observations on a controlled laboratory set-up using an FMCW radar.

The rest of the paper is structured as follows. In Sec. II, the complete FMCW radar model (i.e., its transmission and reception principles) is introduced, as well as a linear inverse problem formulation focused on a Fourier sensing model of the range profile. Sec. III defines the quantization procedure applied on the received radar signal. We then prove that unavoidable ambiguities are induced by this scheme, i.e., the existence of distinct received signals (and thus distinct range profiles) whose quantized measurements are identical. A dithered quantizer is then proposed to cancel out these ambiguous situations. Sec. IV describes two algorithms capable to estimate sparse range profiles from quantized observations, namely PBP and QIHT. Finally, we demonstrate the efficiency of our approach through intensive Monte Carlo simulations in Sec. V, and via real radar measurements in Sec. VI, before concluding in Sec. VII.
Notations: Vectors and matrices are denoted with bold symbols. The imaginary unit is \( i = \sqrt{-1} \), \([D] := \{1, \ldots, D\}\) for \( D \in \mathbb{N} \), \( \mathbf{Id} \) is the identity matrix, \( \text{supp} \, u = \{i : u_i \neq 0\} \) is the support of \( u \), \( \lfloor \cdot \rfloor \) is the flooring operator, and \( |S| \) is the cardinality of a set \( S \). \((\cdot)^*\) denotes the complex conjugate and the adjoint operator for scalar and matrices, respectively. For \( p \geq 1 \), the \( \ell_p \)-norm of a complex vector \( u \) reads
\[
\| u \|_p := \left( \sum_k |u_k|^p \right)^{1/p},
\]
with \( \| u \| := \| u \|_2 \) and \( \| u \|_\infty = \max_k |u_k| \).

II. Radar System Model

We study here an FMCW radar with one transmitting and one receiving antenna. The radar’s transmitting antenna emits a signal \( s(t) \) modeled as
\[
s(t) = \sqrt{P_t} \exp\left( i 2\pi \left( \int_{0}^{t} f_c(\xi) d\xi \right) + i \phi_0 \right),
\]
where \( P_t \) is the transmitted power, \( f_c(t) \) the transmitted frequency pattern, and \( \phi_0 \in [0, 2\pi] \) is the initial phase of the oscillator.

The carrier frequency pattern \( f_c(t) \) of an FMCW radar can be characterized as a saw-tooth function (see Fig. 1):
\[
f_c(t) = f_0 + B \left( \frac{t}{T} \mod 1 \right),
\]
with \( f_0 \) the central frequency, \( \mod \) the modulo operator, \( T \) the duration of one ramp, and \( B \) the spanned bandwidth. Note that, in practical applications, \( B \) is not a design parameter but a constraint imposed by government regulations.

Now considering the received signal model, let us first focus on one static target located at a range \( R > 0 \) from the receiving antenna. In a noiseless setting, the received signal \( r(t) \) is
\[
r(t) = A s(t - \tau_0),
\]
where \( A \) is the complex received amplitude, \( \tau_0 \) is the round-trip delay between the radar and the target and is defined as \( \tau_0 = 2R/c \), and \( c \) is the speed of light. For the sake of simplicity, in the rest of our presentation, the complex value \( A \) will refer to a global constant amplitude that may change from one line to the other in the description of the reception and demodulation processes.

From (1) and (3), the received signal is thus:
\[
r(t) = A \exp \left( i 2\pi \left( \int_{0}^{t} f_c(\xi) d\xi \right) + i \phi_0 \right).
\]

After coherent base-band demodulation with the transmitted signal (1), \( i.e., \) by replacing \( r(t) \) by its multiplication with \( s^*(t) \), the expression (4) reduces to
\[
r(t) = A \exp \left( -i 2\pi \int_{0}^{t} f_c(\xi) d\xi \right).
\]

If \( f_c \) follows the saw-tooth model (2), the integral in (5) becomes
\[
\int_{0}^{t} f_c(\xi) d\xi = \int_{0}^{t} f_0 + B \frac{\xi}{T} d\xi
\]
\[
= \tau_0 f_c(t) - B \frac{\tau_0^2}{2T}.
\]
Combining (5) with (6) allows us to express the received signal $r(t)$ in base-band, i.e., we have

$$r(t) = A \exp \left( -i2\pi\tau_0 f_c(t) \right),$$  \hspace{1cm} (7)$$

where $A$ also encompasses the static phase-shift $-\frac{B}{\pi} \tau_0^2$ in (6). In words, (7) shows that the coherent demodulation expresses the time difference coming from the target as a carrier frequency difference between the transmitted and received signals. This frequency shift linked to the range is represented in Fig. 1. Sampling $r(t)$ at the receiver at a rate $T/N$ for some integer $N$, i.e., at time samples $t_m := m(T/N)$, $m \in \mathbb{Z}$, gives

$$r[m] = A \exp \left( -i2\pi f_m \frac{2B}{c} \right),$$  \hspace{1cm} (8)$$

with $f_m := f_c(t_m) = f_0 + B \left( \frac{m}{N} \mod 1 \right)$. A single ramp can thus be sampled over at most $N$ time samples, which implicitly determines both the resolution $c/(2B)$ and the maximum range $R_{\text{max}} := cN/(2B)$ at which $R$ can be estimated.

Let us now turn to a multi-target scenario restricted to a purely additive model; all the targets are in a direct line of sight from the radar, without any possible multipath propagation. Taking into account the radar range resolution $(c/2B)$ and $R_{\text{max}}$, we discretize the range domain $(0, R_{\text{max}})$ with $N$ ranges $R := \{R_n := n(c/2B), 1 \leq n \leq N\}$. A range profile resulting from $K$ targets with ranges in $R$ is expressed as a $K$-sparse vector $\alpha = (a_1, \cdots, a_N)^T$, i.e., the amplitude $a_n \neq 0$ if there is a target at the $n^{th}$ range bin $R_n$, and $\|\alpha\|_0 := |\text{supp} \alpha| \leq K$. Then, the single target case (8) generalizes to the multi-target sensing model

$$r[m] = \sum_{n=1}^{N} a_n e^{-i2\pi f_m \frac{2B}{c}} = \sum_{n=1}^{N} a_n' e^{-i2\pi \frac{m}{N}},$$  \hspace{1cm} (9)$$

where $a_n' = a_n e^{-i2\pi f_0 B}$. In words, each observation $r[m]$ at time $t_m$ amounts to probing the $m^{th}$ frequency of the discrete Fourier transform the range profile $\alpha' = (a_1', \cdots, a_N')^T$. Hereafter, since $\alpha'$ encodes the same range profile than $\alpha$ (up to a modulation), we drop the prime symbol for the sake of simplicity. Classically, in a Nyquist sensing scenario, if we collect $N$ samples $r = (r[1], \cdots, r[N])^T$, (9) is equivalent to $r = F^* \alpha$, with $F$ the Fourier matrix (i.e., $F_{mn} := \exp(i2\pi \frac{m}{N})$), and an inverse Fourier transform recovers $\alpha$. For noisy observations, a sampling over multiple ramps — hence reaching an oversampled sensing model — yields a robust estimate of $\alpha$.

In this work, we leverage the sparsity assumption made on $\alpha$ to allow this estimation through severely quantized, possibly oversampled, received signal samples. Without quantization, Compressive Sensing (CS) theory from partial random Fourier sensing matrices shows that, with high probability, we can recover any $K$-sparse vector $\alpha$ from only $M = O(K \log^4 N)$ random
samples of $r$ [5]. However, as made clear in Sec. III, QCS aims to reduce the impact of signal measurement quantization in signal estimation by possibly increasing the number of measurements beyond $N$; what truly matters in QCS is indeed the total bit-rate $B$ (i.e., $M \times b$, the bit depth $b$) used to encode the observations [4], [11].

Consequently, our sensing scheme is determined by sampling the received signal $r(t)$ over a set of $M$ (discrete) time samples $\mathcal{T} = \{t'_m : 1 \leq m \leq M\}$ determined as follows. If $M < N$, then $(t'_1, \cdots, t'_M)$ is picked uniformly at random among all possible subset of $M$ time samples of $\{t_m : 1 \leq m \leq N\}$. If $M > N$, in an effort to obtain an acquisition time as short as possible, we take then $t'_m = t_m$ for $1 \leq m \leq N\lfloor M/N \rfloor$, i.e., the first $\lfloor M/N \rfloor$ ramps are fully sampled, and the set of $M' = M - N\lfloor M/N \rfloor$ remaining samples is picked uniformly at random among all possible subset of $M'$ time samples of $\{t_m : N\lfloor M/N \rfloor + 1 \leq m \leq N(\lfloor M/N \rfloor + 1)\}$, i.e., the last ramp is randomly sub-sampled.

Correspondingly, these time samples are associated with $M$, possibly non-distinct, frequencies $\{f'_m = f_c(t'_m) : 1 \leq j \leq M\}$. Finally, if $\Omega$ is a multiset (i.e., a set with repeated elements) representing the indices of these frequencies in $[N]$, the final CS model, before quantization, reads

$$r = \Phi a = F_{\Omega}^* a,$$

where $\Phi := F_{\Omega}^*$, $F_{\Omega}$ gathers the (possibly repeated) columns of $F$ indexed in $\Omega$, and $r$ follows the sampling of $r(t)$ over $\mathcal{T}$. Note that for $M > N$, the addition of a dither ensures that the observations of $r$ over repeated frequencies carry additional information (see Sec. III).

### III. Quantization: Model & Ambiguity

We select in this work on a uniform $b$-bit scalar quantizer applied componentwise onto complex vectors, separately on the real and the imaginary domains, i.e.,

$$Q_b^c(r) = Q_b(\Re(r)) + iQ_b(\Im(r)),$$

where $b$ is the number of bits per vector component (i.e., the I and Q channels), or bit depth. This quantization takes place on the received base-band signal $r$ using ADCs with a resolution of $b$ bits. In (11), $Q_b(\cdot)$ is the standard mid-riser quantizer of quantization step size $\delta > 0$ [11], [12]

$$Q_b(\lambda) := \delta\lfloor \frac{\lambda}{\delta} \rfloor + \frac{\delta}{2}, \quad \forall \lambda \in \mathbb{R}.$$

The step size is set to $\delta = \alpha_b \Delta$, where $\Delta$ is the dynamic range of the ADC, i.e., its voltage range $[-\Delta, \Delta]$, and $\alpha_b = 2^{1-b}$ ensures that the bit-depth of each sample is $b$. For example, for $b = 1$, the ADC is then a simple voltage comparator over its domain, i.e., $2Q_1(\cdot)/\Delta \equiv \text{sign}(\cdot)$. This definition assumes that the quantizer is adjusted to the variations of $r$, i.e., we must have $\Delta \geq ||r||_\infty$, with $\Delta$ as small as possible to minimize the quantization distortion which scales like $O(\delta)$. Note that one can also decide to set $\Delta \geq |r[m]|$ only for a significant fraction of indices $m$, e.g., if $||r||_\infty$ is not bounded. Hereafter, we just assume that $\Delta$ is given.

Let us stress an important limitation of a too direct quantization of the radar sensing model (10): the existence of distinct vectors whose quantized Fourier observations are sent to the same quantized vector, rendering the estimation process ambiguous. This bears similarities with known ambiguities in 1-bit CS with binary matrices [16] and for QCS for multiple antennas and a single target [10]. We show here that the same effect exists for multiple targets and one receiving antenna.
This ambiguity is explained by the following construction. Given two distinct \( n_0, n_1 \in [N] \), we build \( a_0 = b_{n_0} e^{-i\psi_{n_0}} \) and \( a_1 = a_0 + \gamma b_{n_1} e^{-i\psi_{n_1}} \), with \( \psi_{n_0} \) and \( \psi_{n_1} \) two arbitrary phases in \([-\pi, \pi]\), \( 0 < \gamma < 1 \), and \( b_i \in \{0, 1\}^N \) the (canonical) vector whose components are all 0 but the \( i^{th} \) \((i \in [N])\). The signal \( a_0 \) can be seen as one unit-amplitude target at location \( R_{n_0} \), while \( a_1 \) contains an additional target at \( R_{n_1} \) with amplitude \( \gamma \). According to the CS model (10), the acquired received signals are \( r_0 = \Phi a_0 \) and \( r_1 = \Phi a_1 \), with

\[
\begin{align*}
    r_0[m] &= e^{-i\psi_{n_0}} e^{-i2\pi \frac{m n_0}{N}}, \\
    r_1[m] &= e^{-i\psi_{n_0}} e^{-i2\pi \frac{m n_0}{N}} + \gamma e^{-i\psi_{n_1}} e^{-i2\pi \frac{m n_1}{N}},
\end{align*}
\]

(12)

Interestingly, there exist parameter values where the quantizer (11) sends the two signals to the same quantized vector, \( i.e. \), for which the ambiguity condition (AC) holds:

\[
Q_b^C(r_0) = Q_b^C(r_1). \tag{AC}
\]

Consequently, in these cases, while the \( \ell_2 \)-distance \( \|a_1 - a_0\| = \gamma \) is non-zero, recovering both \( a_1 \) and \( a_0 \) from their identical quantized observations is impossible. Let us study when (AC) occurs for 1-bit quantization \( (b = 1) \), \( i.e. \), \( Q_1^C(\cdot) \propto \text{sign}(\Re(\cdot)) + i \text{sign}(\Im(\cdot)) \). In this case, (AC) involves that \( r_0[m] \) and \( r_1[m] \) are always in the same quadrant of the complex plane \( \mathbb{C} \) for all \( m \). Since from (12) \( r_1[m] \) lies on a circle of center \( r_0[m] \) and radius \( \gamma \) in \( \mathbb{C} \), regardless of the values of \( \psi_{n_1} \) or \( R_{n_1} \) (see Fig. 2a), (AC) holds if

\[
\min_m \min(|\Re(r_0[m])|, |\Im(r_0[m])|) > \gamma.
\]

(13)

As \( r_0[m] = e^{-i(\psi_0 + 2\pi \frac{2m}{N})} \), (13) shows a clear dependency between the parameters \( \psi_0 \), \( N \), \( M \), and \( n_0 \) for two quantized vectors to be indistinguishable. For instance, if \( n_0 = N/4 \), then we just need \( \gamma < \min(|\sin \psi_0|, |\cos \psi_0|) \) for (AC) to hold for any values of \( \psi_1 \) and \( n_1 \) (see Fig. 2a). Similar examples can be constructed for other values of \( n_0 \), as well as with multiple targets, with then more restriction on the amplitudes of the additional targets as suggested in Fig. 2b. For \( b > 1 \), there also exist vectors satisfying (AC), but their \( \ell_2 \)-distance must decay if \( b \) increases since \( Q_b \) splits \( \mathbb{C} \) into square cells of size \( 2^{1-b} \Delta \). Therefore, if an algorithm wrongly estimates \( r_1 \) with the value of \( r_0 \), its error decays as \( 2^{-b} \) if \( b \) increases, but this error is not ensured to decay if \( M \) increases.

In this work, we stress that the previous ambiguities can be removed by voluntary introducing randomness in the quantization, \( i.e. \), by inserting a random dither in the quantizer input. Consequently, one can design algorithms whose estimation error of range
profile decay as $M$ increases. While dithered quantization is a well-known strategy to improve signal estimation techniques (see, e.g., [12], [13], [17]), its use in quantized compressive sensing is recent and we follow here the approach of [11].

Given a range profile $\mathbf{a} \in \mathbb{C}^N$, our dithered QCS sensing model is thus defined by

$$y = A_\delta(\mathbf{a}) := Q^C_\delta(\Phi \mathbf{a} + \mathbf{\xi}),$$

where $\mathbf{\xi} \in \mathbb{C}^M$ is a complex dither defined as $\xi_i = \xi_i^R + i\xi_i^I$, with $\xi_i^R, \xi_i^I \sim \mathcal{U}(-\frac{\delta}{2}, \frac{\delta}{2})$. This dither induces more diversity in the quantized measurements, especially for $M > N$. Moreover, $E_{\mathbf{\xi}} A_\delta(\mathbf{a}) = \Phi \mathbf{a},$ i.e., the dither cancels out the quantization error in expectation, or, equivalently, if $M$ is large [11]. Note that this also changes the dynamic range of the signal before quantization, i.e., we must adapt the range $\Delta \geq \|r\|_{\infty} + \frac{\delta}{2}$.

IV. RECONSTRUCTION ALGORITHM

To reconstruct the range profile $\mathbf{a}$ from the quantized measurements $y$, two algorithms are studied. The first is Projected Back Projection (PBP) and is defined as follow:

$$\hat{\mathbf{a}} = \mathcal{H}_K \left( \frac{1}{M} \Phi^* y \right),$$

where $K$ is the range profile sparsity, assumed known a priori, $\mathcal{H}_K$ is the hard-thresholding operator setting all the components of its vector input to zero but those with the $K$ largest amplitudes.

The advantages of PBP are threefold. First, its complexity is $O(N \log N)$ since $\Phi^*$ only requires the computation of an inverse FFT applied on a zero-padding\footnote{In this sense, PBP is similar to a Maximum Likelihood Estimator.} of $y$ from $\Omega$ to $[N]$ (or $[\rho N]$ for $\rho = O(1)$ ramps) and $\mathcal{H}_K$ involves a vector component ordering of $O(N \log N)$ computations. Second, as a function of $y$, PBP does not explicitly invoke the dither $\mathbf{\xi}$; its implementation only requires the knowledge of $\Phi$, i.e., of $\Omega$. Finally, in the context of dithered QCS, PBP enjoys of a reconstruction error that decays when $M$ increases for all sensing matrices $\Phi$ respecting with high probability the restricted isometry property (RIP) [11], such as for the random partial Fourier matrix in (10). For a sparse range profile $\mathbf{a}$, the reconstruction guarantees is

$$\|\mathbf{a} - \hat{\mathbf{a}}\| = O(M^{-\frac{1}{2}}).$$

In other words, compared to the undithered context, no counterexamples exist that would make this error stagnate when $M$ is increased.

Note that (15) is a root-mean-square error bound for the estimation of $\mathbf{a}$. In this work, our interest is, however, to characterize the range recovery of target, i.e., the support of $\mathbf{a}$. Interestingly, since $\|\mathbf{a} - \hat{\mathbf{a}}\|_{\infty} \leq \|\mathbf{a} - \hat{\mathbf{a}}\|$, if $\mathbf{a}$ is $K$-sparse, with $K$ given, and if we know that $\min\{|a_i| : i \in \text{supp } \mathbf{a}\} > \eta$ for some $\eta > 0$, then, one can expect that

$$M \geq C/\eta^2 \quad \Rightarrow \quad \text{supp } \hat{\mathbf{a}} = \text{supp } \mathbf{a},$$

for some $C > 0$. Indeed, support recovery is ensured if $|\hat{a}_i| > |\hat{a}_j|$ for all $i \in \text{supp } \mathbf{a}$ and all $j \in [N] \setminus \{\text{supp } \mathbf{a}\}$, which is achieved if $|a_i| - |\hat{a}_i - a_i| > |\hat{a}_j - a_j|$. This holds if $|a_i| > \eta > 2\|\mathbf{a} - \hat{\mathbf{a}}\|_{\infty} = O(M^{-1/2})$, or if $M \geq C/\eta^2$.

While requiring a single iteration, PBP does not ensure that its estimate $\hat{\mathbf{a}}$ is consistent with $y$, i.e., $A_\delta(\hat{\mathbf{a}}) \neq y = A_\delta(\mathbf{a})$;
the quantized sensing model is thus not fully exploited while estimating $a$ from $y$. To solve this situation, [15] has proposed the Quantized Iterative Hard Thresholding (QIHT) algorithm, i.e., a variant of the Iterative Hard Thresholding (IHT) [18] and of the Binary IHT [19], iteratively enforcing both consistency and sparsity of a signal estimate. QIHT is defined by

$$\hat{a}^{j+1} = H_K[\hat{a}^j + \frac{\mu}{M} \Phi^*(y - A_b(\hat{a}^j))],$$

where $j$ is the iteration index, $\mu$ is a step size parameter, and $\hat{a}^0$ is the PBP estimate. Compared to PBP, this algorithm is not ensured to converge. However, numerically, QIHT often provides a sparse and consistent estimate. If this happens at the $J$th iteration, i.e., $y - A_b(\hat{a}^J) = 0$, and if $\Phi$ is a random Gaussian matrix, the QIHT estimate $\hat{a} = \hat{a}^J$ reaches an error $\|a - \hat{a}\| = O(1/M)$ [20]. Consequently, we decide to also investigate the efficiency of QIHT for the radar sensing model (14).

While QIHT has more to offer in terms of reconstruction by enforcing the consistency, one must also note that knowing the dither at the reconstruction, as imposed by the computation of $A_b$ in (17), will impact the physical implementation of the system. Indeed PBP could use analogical random noise source such as a noise diode [14], whereas QIHT would require a more advanced implementation.

V. Numerical Results

![Numerical Results](image)

Fig. 3: [best viewed in color] (a) and (b): TPR vs $\log_2 B$ for PBP; (c) and (d): Comparison between PBP (disks) and QIHT (triangles) in function of $\log_2 B$. In all figures, solid, dashed and dotted curves stand for dithered, undithered and unquantized schemes, respectively. The first (second) gray vertical line represents a bit-rate of $2^B (2^{13})$ bits corresponding to $M = 256 (M = 8192)$ for 1-bit and $M = 16 (M = 256)$ for no quantization. In (a) and (b), the resolution is represented by colors, orange for 1-bit, green for 2-bits and gray in absence of quantization. In (c) and (d) blue stands for 1-bit PBP, red for 1-bit QIHT and gray for no quantization. Figures (a,c) and (b,d) are for $K = 2$ and $K = 10$, respectively.

We here challenge the possibility of recovering sparse range profiles from quantized radar observations, i.e., from measurements associated with the dithered QCS model (14). To this aim, we present the result of extensive Monte Carlo (MC) simulations for various parameters of our setup: we have set the sparsity level $K$ — the number of targets — in $[2, 10]$, a total bit-rate $B = bM$ in $[2^B, 2^{13}]$ with measurement number $M$ in $[2^B, 2^{13}]$ and a bit depth $b \in [1, 32]$, $N = 256$. Concerning QIHT, we have set $\mu = 1$ and a total number of iterations between 20 and $100K$, with an early stop if, either, the consistency
The PBP performances are rather poor for larger values of $K$. Furthermore, for large values of $K$, the drop in performances in Fig. 3d between the non-dithered and dithered schemes for the 1-bit PBP is reduced for 1-bit QIHT. In Fig. 3d, the dithered 1-bit QIHT is markedly better than any other methods for $B = 2^9$ bits and above, reducing the bit-rate by as much as 93.75% compared to the classic high resolution Nyquist sampling scheme. This bit-rate corresponds to $M \geq 2N = 512$ for 1-bit and $M \geq N/2^4 = 16$ in absence of quantization; at harsh bit-rates quantity outweighs quality.

Finally, we study in Fig. 4a the TPR of PBP and QIHT vs $K$ for a fixed bit-rate $B = 2^9$. Here also, the gain offered by the explicit knowledge of the dither in QIHT is quite obvious. For low $K$, both of the dithered schemes have better TPR than their non dithered counterparts. However, as $K$ increases above 4, the performances of the 1-bit dithered PBP plummets quickly.
Fig. 4: [best viewed in color] (a) TPR vs number of targets for 1-bit PBP and 1-bit QIHT with $B = 2^9$ bits; (b) TPR vs bit-rate using real FMCW radar measurements for $K = 2$. In all figures, PBP is represented by disks and QIHT by triangles, blue stands for 1-bit PBP, red for 1-bit QIHT, and gray for no quantization.

Fig. 5: (a) Experimental setup: radar in front of the simulator. (b) Block representation of the 2 targets simulator by AMG

below its non dithered version. On the other hand, QIHT with dithering always outperforms its performances with non dithered quantization. We thus conclude that, provided a uniform dithering can be implemented efficiently and later reproduced in QIHT, dithered quantization has always a positive impact on the range estimation.

VI. MEASUREMENT IN LABORATORY

Sec. V has focused on the study of range estimation performances from noiseless and synthetic simulations, under a perfect linear sensing model (before quantization) where an idealized radar interacts with point-like targets. We thus present here different tests of resilience of both our model and algorithms by confronting them with real data acquired in a controlled laboratory setting.

The radar used for this experiment is the KMD2 radar [21], i.e., an FMCW radar with one transmitting antenna and 3 receiving antennas. The radar lies in the “K”-band and its bandwidth can be extended up to 770 Mhz. The AMG43-007 [22] is a target simulator distributed by AMG-microwave which is able to simulate a target with varying velocity, range, and power. In the context of this work, two simulators are used with the velocity set to zero and with a power changing according to a logarithmic uniform distribution. This setup allows to simulate target ranges up to 64 m by 1 m step. We thus set the bandwidth of the FMCW radar to 150 Mhz to match this spatial resolution.

The radar is placed in front of these two simulators and emits the frequency pattern (1), the signal received by the two simulators is then delayed and attenuated according to user defined parameters and then re-emitted towards the radar (see Fig. 5a and Fig. 5b). This process allows the simulation of a specific support while adding the concrete effect associated with the radar that are not taken into account in the developed model. These effects range from the inherent noise in RF and electronics hardware, IQ imbalance, non linearities in the coherent demodulation and all other non idealities related to radar applications.
This experimental setup has thus the ability to combine the rigor and completeness of Monte Carlo simulations with the possibility to program and repeat specific scenarios \((i.e., \text{specific } \alpha)\), and to test them against a real acquisition system.

We recorded 196 runs with different sparse range profiles using the same parametrization as in Fig. 3c. We observed that the SNR of the configuration in Fig. 5a is sufficiently high to neglect the impact of the noise on the quantization. Note that this effect was briefly addressed in [10] by experimentally adjusting the dither to the noise amplitude, and its thorough theoretical study is ongoing.

The curves in Fig. 4b exhibit the same tendencies than in Fig. 3c. The only difference is the TPR at which the non dithered schemes saturate; an effect most probably due to some discrepancies in the range profile amplitudes between this setup and the previous simulations. Once again, 1-bit dithered QIHT is the algorithm with the highest TPR from \(B = 2^6\) bits to \(2^{13}\), \(i.e.,\) the bit-rate of a full resolution Nyquist sensing. These results from real measurements are fully consistent with the previously developed theory and simulations; this paves the way to more complete and practical realizations of the proposed quantized architecture.

\section*{VII. Conclusion & Future work}

In this article, we demonstrated that a pre-quantization dither removes unavoidable range estimation ambiguities when one quantizes the received radar signal. Moreover, in this dithered scheme, we proved that severe quantization, as low as 1-bit per received signal sample, still allows for an accurate range profile estimation as soon as the total bit-rate is large enough; a tradeoff between the number of radar observations (or measurements) and their resolution (or bit-depth) must be respected. Moreover, we showed that for low bit-rate scenarios, low bit-depth exhibits better performances than an unquantized scheme. These results are achieved thanks to two QCS reconstruction algorithms, PBP and QIHT, that leverage the sparsity of the range profile. Moreover, when the number of targets – and thus the sparsity level of the range profile – increases, Monte Carlo simulations proved that QIHT still provides high range estimation performances by promoting consistency with the quantized radar observations. As a proof of concept, we obtained similar range estimation performances from quantized observations of an actual radar in a controlled environment; hence showing that this QCS radar framework could apply in radar applications with limited bit-rate, \(e.g.,\) for radar signal reception with cheap ADC. Future work will address the interplay between the dither and the background noise, with a practical realization of the proposed highly quantized and dithered architecture.

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