Hadron Tomography

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Abstract. Generalized parton distributions describe the distribution of partons in the transverse plane. For transversely polarized quarks and/or nucleons, these impact parameter dependent parton distributions are not axially symmetric. These transverse distortions can be related to spin-orbit correlations as well as to (intuitively) to transverse single-spin asymmetries, allowing novel insights into quark orbital angular momentum from measurements of the Sivers and Boer-Mulders functions.

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INTRODUCTION

Generalized parton distributions (GPDs) [1] for \( \xi = 0 \) provide information about the distribution of partons in impact parameter space. The distribution of unpolarized quarks in unpolarized nucleons is given by the Fourier transform of the GPD

\[
q(x, b_\perp) = \int \frac{d^2 \Delta_\perp^2}{(2\pi)^2} H(x, 0, -\Delta_\perp^2) e^{-i\Delta_\perp \cdot b_\perp},
\]

where \( H \) is the GPD which appears in a decomposition of the Dirac form factor w.r.t. the momentum \( x \) of the active quark \( F_1(t) = \int dx H(x, \xi, t) \). The reference point for \( b_\perp \) is the \( \perp \) center of longitudinal momentum \( R_\perp = \sum_{i \in q,g} x_i r_{i,\perp} \) [4]. A similar relation exists for longitudinally polarized quarks \( \Delta q(x, b_\perp) \) with \( H(x, 0, -\Delta_\perp^2) \rightarrow \tilde{H}(x, 0, -\Delta_\perp^2) \). For a \( \perp \) polarized target the distribution of partons in impact parameter space is no longer axially symmetric and the deformation is described by the Fourier transform of the GPD \( E(x, 0, -\Delta_\perp^2) \). For example, for a nucleon polarized in the +\( \hat{x} \) direction, one finds [5]

\[
q(x, b_\perp) = \int \frac{d^2 \Delta_\perp^2}{(2\pi)^2} H(x, 0, -\Delta_\perp^2) e^{-i\Delta_\perp \cdot b_\perp} - \frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2 \Delta_\perp^2}{(2\pi)^2} E(x, 0, -\Delta_\perp^2) e^{-i\Delta_\perp \cdot b_\perp} \tag{2}
\]

(see Fig. [1]). Little is known about the GPD \( E \), beyond that it provides a \( x \) decomposition of the Pauli form factor as \( F_2(t) = \int dx E(x, \xi, t) \). Eq. (2) yields a \( \perp \) flavor dipole moment

\[
d_y = \int d^2 b_\perp q(x, b_\perp) b_\perp = \frac{\kappa_{q/p}}{2M}, \tag{3}
\]

where \( \kappa_{q/p} \) is the contribution from quark Flavor \( q \) to the anomalous magnetic moment of the nucleon (with the charge factored out). Neglecting the small contribution from strange quarks, the anomalous magnetic moments of the proton and neutron are sufficient to perform a flavor decomposition of the anomalous magnetic moment, yielding \( \kappa_{u/p} = 1.67 \) and \( \kappa_{d/p} = -2.03 \), i.e. significant \( \perp \) flavor dipole moments \( |d_q| \approx 0.2 fm \).
The physical origin for this deformations is the orbital motion of the quarks. When viewed in the Breit frame, quarks with orbital angular momentum in the transverse direction move towards the virtual photon on one side of the nucleon and away from it on the other side, i.e. they are shifted towards larger momentum fractions on the side of the nucleon where they move towards the virtual photon. As PDFs are rapidly falling functions of $x$, the resulting increase in $x$ for quarks on one side results in an enhancement when viewed at fixed $x$ and the PDFs appear shifted towards that side. Due to the opposite signs for $\kappa_{u/p}$ and $\kappa_{d/p}$, the shifts are into opposite directions for $u$ and $d$ quarks, indicating opposite signs for their orbital angular momenta.

SIVERS FUNCTION

The significant distortion of parton distributions in impact parameter space is expected to have observable consequences in other experiments. For example, in semi-inclusive DIS, when the virtual photon strikes a $u$ quark in a $\perp$ polarized proton, the $u$ quark distribution is enhanced on the left side of the target (for a proton with spin pointing up when viewed from the virtual photon perspective). Although in general the final state interaction (FSI) is very complicated, we expect it to be on average attractive thus translating a position space distortion to the left into a momentum space asymmetry to the right and vice versa (Fig. 2). Such a $\perp$ spin asymmetry is usually parameterized in terms of the Sivers function $f_{1T}^{\perp q}(x, k_{\perp}^2)$ entering the unintegrated parton density in a $\perp$ polarized target [7, 8, 9]

$$f_{q/p}^{\perp}(x, k_{\perp}) = f_{1}^{q}(x, k_{\perp}^{2}) - f_{1T}^{\perp q}(x, k_{\perp}^{2}) \frac{(\hat{P} \times k_{\perp}) \cdot S}{M}$$ (4)

Although this picture is very intuitive, a few words of caution are in order. First of all the above mechanism is strictly true only in mean field models for the FSI as well as in simple spectator models [10]. Furthermore, even in such mean field models there is no one-to-one correspondence between quark distributions in impact parameter space and unintegrated parton densities (e.g. Sivers function). Although both are connected by a Wigner distribution [11], they are not Fourier transforms of each other. Nevertheless, since the primordial momentum distribution of the quarks (without FSI) must
FIGURE 2. The transverse distortion of the parton cloud for a proton that is polarized into the plane, in combination with attractive FSI, gives rise to a Sivers effect for $u$ ($d$) quarks with a $\perp$ momentum that is on the average up (down).

be symmetric we find a qualitative connection between the primordial position space asymmetry and the momentum space asymmetry (with FSI). Another issue concerns the $x$-dependence of the Sivers function. The $x$-dependence of the position space asymmetry is described by the GPD $E(x, 0, -\Delta^2_\perp)$. Therefore, within the above mechanism, the $x$-dependence of the Sivers function should be related to the $x$ dependence of $E(x, 0, -\Delta^2_\perp)$, for example, in a mean field model for the FSI, the second moment of the Sivers function is obtained as

$$\int d^2k_\perp f^{lT}_{1T}(x, k_\perp^2) k_\perp^2 \propto \int d^2b_\perp \delta(x, b_\perp) \nabla \cdot I(b_\perp),$$

where $I(b_\perp)$ describes the FSI [6]. However, the $x$ dependence of $E$ is not known yet and we only know the Pauli form factor $F_2 = \int dxE$. Nevertheless, if one makes the additional assumption that $E$ does not fluctuate as a function of $x$ then the contribution from each quark flavor $q$ to the anomalous magnetic moment $\kappa$ determines the sign of $E^q(x, 0, 0)$ and hence of the Sivers function. Making these assumptions, as well as the very plausible assumption that the FSI is on average attractive ($\nabla \cdot I(b_\perp) \leq 0$), one finds that $f^{lT}_{1T} < 0$, while $f^{lT}_{1T} > 0$. Both signs have been confirmed by the HERMES experiment [12].

For the flavor analysis of the Sivers function, there exists a useful sum-rule, which states that the average $k_\perp$ summed over all $x$, and all quarks and gluons vanishes [13]

$$\sum_{i=q,g} \int dx \int d^2k_\perp f^{lT}_{1T}(x, k_\perp^2) k_\perp^2 = 0. \quad (5)$$

However, Eq. (5) is not a trivial consequence of $\perp$ momentum conservation in the SIDIS process, as the different $f^{lT}_{1T}$ are extracted from different experiments. To illustrate this point, we consider a gedanken experiment using the scalar diquark model as a target. Imagine first a probe that only couples to the fermion $f$ in the model. The 2nd moment of $f^{lT}_{1T}$ would be evaluated by measuring the $\perp$ momentum of the fermions produced in a DIS experiment with a probe that couples only to the fermion. Now consider a second probe which couples only to the scalar in this model. This time only the $\perp$ momenta of the scalars are measured in the final state. The sum rule (5) states that the 2nd moment of the fermion $\perp$ momentum distribution in the first experiment plus the 2nd moment of the scalar $\perp$ momentum distribution in the second experiment should add up to zero. In contradistinction, mere momentum conservation yields that the $\perp$ momenta of all constituents in the final state (both active and spectator) in one single experiment should add up to zero. Eq. (5) is a different statement than that. Present experiments [12, 14] seem to indicate that the sum rule is already close to being satisfied by summing over $u$ and $d$ quarks only, suggesting possibly a small Sivers effect for the glue [15].
CHIRALLY ODD GPDS AND THE BOER-MULDERS FUNCTION

Transversity Distribution in Impact Parameter Space

Even in an unpolarized target, the distribution of quarks with a certain transverse spin may not be axially symmetric either as the transverse quark polarization singles out a direction as well. The details of this distortion are described by the chirally odd GPD $\bar{E}_T(x,0,-\Delta_{\perp}^2)$ [16]. Physically, what happens here is that although all orientations for the quark orbital motion are equally likely, there may be a certain preferred correlation between the quark orbital motion and the quark spin (e.g. due to spin-orbit effects). Therefore, by looking at distributions of quarks with a certain transversity one does at the same time single out (at least preferentially) quarks with a certain orientation of the orbital motion (Fig. 3). Here the sign of the correlation depends on the sign of $\bar{E}_T$, which is one of the interesting pieces of information that we are trying to find out. Once again, there will be an “enhancement” of PDFs on the side of the nucleon where the quarks move towards the virtual photon. This enhancement manifests itself in a distortion sideways relative to the (transverse) quark spin, which is described by the GPD $\bar{E}_T$. The distortion is $\perp$ to the quark spin and its sign is determined by the sign of $\bar{E}_T$. When quark with a certain $\perp$ polarization are knocked out in a DIS experiment, the momentum distribution of these quarks in the final state thus exhibits a left-right asymmetry analogous to the asymmetry described by the Sivers function, except that the $\perp$ spin of the nucleon is replaced by the $\perp$ spin of the struck quark [17]

$$f_{q^+/p}(x,k_{\perp}) = \frac{1}{2} \left[ f_1^q(x,k_{\perp}^2) - h_1^{\perp-q}(x,k_{\perp}^2) \frac{(\hat{P} \times k_{\perp}) \cdot S_q}{M} \right].$$

(6)

Using the Collins effect [18], i.e. the $\perp$ asymmetry in the distribution of hadrons in a jet originating from a transversely polarized quark, one can ‘tag’ the spin of the active quark: relative to the lepton scattering plane the pion distribution in the final state exhibits a $\cos(2\phi)$ asymmetry proportional to the Collins as well as the ‘Boer-Mulders function’ $h_1^{\perp-q}(x,k_{\perp}^2)$, which can thus be extracted.
Are All Boer-Mulders Functions Alike?

While the signs of the chirally even GPD $E^q$ are known from the flavor decomposition of the anomalous magnetic moment, no such information is available for the chirally odd GPD $\bar{E}^q$. Therefore, the only information on $\bar{E}^q$ comes from models and lattice studies. We considered a bag model for both the nucleon as well as the pion, as well as the NJL model for the pion [22]. In all of these calculations we find that $\kappa_T = \int dx \bar{E}^q > 0$ for both $q = u, d$. Moreover, a constituent quark model [20], using a Melosh rotation to generate the spin-orbit structure of the light-cone wave functions, also yields $\kappa_T^u > 0$ and $\kappa_T^d > 0$ for the nucleon. All of these models also yielded numerical values for $\kappa_T$ that were larger than for the (chirally even) $\kappa$. Of course, all of these are just model results and one may question their validity. Fortunately, moments of $\int dx \bar{E}^q$ have been calculated in lattice gauge theory and were also found to be positive and numerically larger than $\kappa$.

Similar to the chirally even GPD $E$, $\bar{E}_T$ also requires the interference between wave function components that differ by one unit of quark orbital angular momentum. In the bag model, the ground state wave function only contains nonzero orbital angular momentum in its lower component, which is related to the upper component through the free Dirac equation. In the constituent quark model the situation is similar. At least in those models the sign of $\bar{E}_T$ can thus be understood from the relative phase between upper and lower components in $s$-wave solutions to the free Dirac equation [22]. Amazingly, lattice QCD yields the same signs [19, 21]. The same sign for $\kappa_T^u$ and $\kappa_T^d$ can also be understood on the basis of large $N_C$, as for $N_C \to \infty$ one finds $\bar{E}_T^u = \bar{E}_T^d$.

The fact that $\kappa_T$ is larger than $\kappa$ can also be understood in these models (bag model and constituent quark model), which have in common that the nucleon wave function is constructed from a product of independent quark wave functions, which one would also do in a mean field approximation appropriate for $N_C \to \infty$. What these models/approximations have in common is that the nucleon wave function is constructed from independent quark wave functions. However, regardless whether they which although these independent quark wave functions have $J_z = +\frac{1}{2}$ or $J_z = -\frac{1}{2}$, both yield the same correlation between the quark spin and the quark orbital motion as both the spin and the orbital motion are reversed in the $J_z = -\frac{1}{2}$ wave function. When one constructs the nucleon state with total angular momentum an spin equal to $\frac{1}{2}$, for both $u$ and $d$ quarks this involves both quark wave functions with $J_z = +\frac{1}{2}$ as well as $J_z = -\frac{1}{2}$. In the computation of the GPD $E$, which is sensitive to the correlation between quark orbital angular momentum and the nucleon spin, the involvement of quark wave functions with both $J_z = \pm\frac{1}{2}$, results in a certain cancellation between different terms in the familiar $SU(6)$ wave functions used to construct the nucleon state. However, in the case of $\bar{E}_T$, no such cancellation occurs, as each quark wave function has the same correlation between its orbital angular momentum and its spin, and all $SU(6)$ components contribute coherently to $\bar{E}_T$. As a result, there is no cancellation and these models all yield $\kappa_T > \kappa$, and in fact all models where the nucleon state is constructed from products of identical (up to $J_z$) quark wave functions, e.g. mean field models, should share this property. Again, lattice calculations give results that are consistent with these models.
SUMMARY

Parton distributions in impact parameter space are deformed when either the nucleon is polarized or when one considers the distribution or polarized quarks. These deformations provide a simple mechanism for SSAs, based on the assumption that the FSI is on average attractive. The predicted signs for the Sivers functions for $u$ and $d$ quarks were confirmed in a recent experiment \cite{12}. Various models as well as lattice QCD calculations predict that the lowest moment of $\bar{E}_T$ is positive for both $u$ and $d$ quarks and numerically larger than the lowest moment of its chirally even counterpart $E$. As a result, the Boer-Mulders function for $u$ as well as $d$ quarks is expected to be larger than the Sivers function for the same flavor and with the same sign as the Sivers function for $u$ quarks, i.e. negative.

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