Beams destruction by explosion of non-contact charges of condensed explosives in water

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Abstract. The analysis of effects of the hydrodynamic field created by the explosion of condensed explosive charge on the simplest construction element in the form of beams in the water is performed. The features of the load due to the location of the beam in the water are taken into account by the attached mass introduction. The classical condition of the element destruction is used, according to which it is necessary that the stress in the element is greater than the ultimate resistance of the material and refined of O.E. Vlasov’s formula on the proportionality of the bending moment to the acting explosive load pulse is used.

1. Introduction

When blasting charges in the air at close distances from construction elements, the main load carriers are explosion products, at farther distances – an air shock wave, i.e. in both cases the load is transmitted through a medium whose density is significantly less than the density of the material of the construction element.

In case of the non-contact charges explosion in the water, the load on construction elements is transmitted through the medium, whose density is comparable to the density of the construction material.

With the beginning of the element deformation in the water, not only the element itself comes into motion, but also a significant part of the medium particles in direct contact with it, as a result of this phenomenon, the movement of the element is inhibited. This effect of the medium resistance to the displacement of a solid body in it in hydrodynamics is taken into account by the introduction of attached mass [1, 2].

2. Physical analog (main accepted assumptions)

1) The force of the liquid resistance to the body motion is replaced by the force of inertia, which occurs when the hypothetical attached mass is moved together with the body [1, 2].
2) We use refined O.E. Vlasov’s formula [2].
3) Two cases are considered when the beam is fully or partially immersed into the water.
4) We assume that the beam is freely supported by non-deformable (non-destructible) supports.

2.1. Mathematical model

Consider a beam with freely supported ends located in the water. For such beam in the air the deflection from explosive load is determined by the formula [1]:
\[ Z = \frac{2l}{\pi^2 \beta m_0} \sum_{j=1}^{\infty} \frac{1}{j^2} \sin j \pi \frac{x}{l} \sin \frac{j^2 \pi^2}{l^2} \beta t \int_0^1 i(\xi) \sin \frac{j \pi \xi}{l} d \xi, \]  

where \( l \) is the length of the beam, \( m_0 \) is the linear mass of the beam, \( x \) is the coordinate of its section, \( t \) is the time, \( i \) is the linear impulse of the explosive load, \( \beta \) is a parameter characterizing the mechanical properties of the beam material

\[ \beta = \sqrt{\frac{EJ}{m_0}}, \]

\( E \) is the modulus of the beam material elasticity, \( J \) is the moment of inertia of the cross section relative to the corresponding axis.

In case of the beam location in the water when it is freely supported on the fixed non-emptying supports, the attached mass \([2]\) should be introduced, in this case the value \( \beta \) is calculated by the formula

\[ \beta = \sqrt{\frac{EJ}{m_0 + m_*}}, \]

where \( m_* \) is the linear attached mass \([2]\).

For bending moment, the relation \([3]\) is known

\[ M = -EJ \frac{\partial^2 Z}{\partial x^2}. \]

Formula (1) for beam in the water takes the form

\[ Z = \frac{2l}{\pi^2 \beta(m_0 + m_*)} \sum_{j=1}^{\infty} \frac{1}{j^2} \sin j \pi \frac{x}{l} \sin \frac{j^2 \pi^2}{l^2} \beta t \int_0^1 i(\xi) \sin \frac{j \pi \xi}{l} d \xi, \]

therefore,

\[ M = \frac{2\beta}{l} \sum_{j=1}^{\infty} \sin j \pi \frac{x}{l} \sin \frac{j^2 \pi^2}{l^2} \beta t \int_0^1 i(\xi) \sin \frac{j \pi \xi}{l} d \xi. \]

Differentiating function (5) by time, we get the formula for speed

\[ u = \frac{\partial z}{\partial t} = \frac{2}{(m_0 + m_*)l} \sum_{j=1}^{\infty} \sin j \pi \frac{x}{l} \cos \frac{j^2 \pi^2}{l^2} \beta t \int_0^1 i(\xi) \sin \frac{j \pi \xi}{l} d \xi. \]

It can be seen from the formulae obtained that the maximum values of the velocity corresponding to the load application moment \( t = 0 \) and the maximum values of the bending moment corresponding to the moment of the first maximum deflection \( t_1 = \frac{l^2}{2\pi \beta} \)

\[ M_{\text{max}}(x) = \frac{2\beta}{l} \sum_{j=1}^{\infty} \sin j \pi \frac{x}{l} i(\xi) \sin \frac{j \pi \xi}{l} d \xi, \]

are related by the relation:

\[ M_{\text{max}}(x) = \beta(m_0 + m_*)u_{\text{max}}(x). \]

Since according to the initial condition

\[ u_{\text{max}}(x) = \frac{\partial z}{\partial t} \bigg|_{t=0} = \frac{i(x)}{(m_0 + m_*)}, \]

then we will finally get
\[ M_{\text{max}}(x) = \beta 
abla_i(x) \cdot \Phi(x) \,, \]  

where \( \Phi(x) \) is a function represented by the relations:

\[ \Phi(x) = \frac{1}{\left[ 1 + \frac{\partial^2 z}{\partial x_{\text{max}}^2} \right]^{\frac{3}{2}}} \left( 1 + \Omega(x) \right) \]

\[ \Omega(x) = \frac{\sum_{k=0}^{\infty} \sin(2k \pi x^2) S_{2k}}{\sum_{k=0}^{\infty} \sin \left( \frac{(2k+1)\pi x}{l} \right) S_{2k+1}} \]

\[ \left( \frac{\partial z}{\partial x} \right)_{\text{max}} = \frac{2}{\pi \beta (m_0 + m_s)} \sum_{k=0}^{\infty} \frac{1}{2k+1} \cos \left( \frac{(2k+1)\pi x}{l} \right) S_{2k+1} \]

\[ S_{2k} = \int_0^l i_i(\xi) \sin \left( \frac{2k \pi x}{l} \right) d\xi \]

\[ S_{2k+1} = \int_0^l i_i(\xi) \sin \left( \frac{(2k+1)\pi x}{l} \right) d\xi \]

The ratios (9) were found as a result of refinement of the O.E. Vlasov’s formula [4].

2.2. Special cases of structure location
When studying the effect of an explosion on elements of construction elements located in the water, two cases that are significantly different from each other should be distinguished [1, 5]:

1. The construction element is completely in the water;
2. Only a portion of the element surface contacts with the water and the remainder contacts with the air.

In the first case, the element during its movement drives the entire attached mass, and in the second – only a part of this mass. At the same time, the amount of load created by the explosion under other similar conditions for these cases will be different, namely: in the first case it will be less than in the second [1].

As an example, explaining this difference, consider the blasting effect of the same spherical charge weighing \( C \) on two identical beams of the square cross section with the side of square \( h \). It is assumed that the charge is removed from the beams by the same distance \( a \). Let the first beam be completely surrounded by water, and the second beam be submerged only half.

Then the attached linear mass for the first beam has the form [2]

\[ m_1 = \frac{\pi}{4} \rho h^2 \],

but for the second beam

\[ m_2 = \frac{\pi}{8} \rho h^2 \].

Herewith correspondingly, coefficients \( \beta \) are calculated by formulas.
\[
\beta_1 = \sqrt{\frac{EJ}{m_0 + m_1}} = \frac{h}{\sqrt{12}} \left( \frac{E}{\rho_0 + \frac{\pi}{4} \rho_s} \right),
\]
\[
\beta_2 = \sqrt{\frac{EJ}{m_0 + m_2}} = \frac{h}{\sqrt{12}} \left( \frac{E}{\rho_0 + \frac{\pi}{8} \rho_s} \right),
\]
\(\text{(10)}\)

where \(\rho_0\) is the density of the beam material, \(\rho_s\) is the density of water.

Assuming the material of the beam is linearly resilient and considering that in this case
\[
M = \beta_i = w\sigma,
\]
\(\text{(11)}\)

where \(w\) is the moment of the beam resistance, \(\sigma\) — stress, and also considering that in order to break the element, it is necessary that the stress in the element \(\sigma\) is greater than the limit resistance of the material, we obtain:
\[
\sigma \geq k_0^* k_1 \cdot R_z,
\]
\(\text{(12)}\)

where \(k_0^*\) is the coefficient of homogeneity for guaranteed destruction, \(k_1\) is the ratio of yield limits in dynamic loading of the element to the yield strength in static loading, \(R_z\) is the normative resistance of the material during bending. Note that according to the studies presented in the work [1]
\[
k_0^* = \frac{R_1^{\max}}{R_2},
\]
\(\text{(13)}\)

where \(R_1^{\max}\) is the maximum resistance of the material.

From the ratios (11), (12) we obtain
\[
i = \frac{w}{\beta} k_0^* k_1 R_2.
\]

Impulse intensity
\[
i = k_s \cdot bi,
\]
\(\text{(14)}\)

where \(k_s\) is the shape coefficient [1], \(b\) is the beam width, \(i\) is the specific impulse.

Value of specific impulse acting in the water can be determined by formula [6, 7]:
\[
i = A_r \frac{C_a^2}{r}.
\]
\(\text{(15)}\)

The design impulse for the considered cases of beam location in the water is different [8]. For the first beam that is completely in the water, it is equal to the difference in specific impulses acting on the opposite faces of the beam
\[
i = A_r \frac{C_a^2}{r} \left[ \frac{1}{r - \frac{h}{2}} - \frac{1}{2 \left( r + \frac{h}{2} \right)} \right] = A_r \frac{C_a^2}{r} \frac{1 + 2 \frac{h}{a}}{2a \left( 1 + \frac{h}{a} \right)}.
\]
\(\text{(16)}\)

For the second beam, due to the fact that nothing opposes from the opposite face, the specific impulse is calculated using the formula [9]
\[
i_2 = \frac{A_r C_a^2}{a}.
\]

Thus, for the first beam
\[ i_i = k_i b A C^3 \frac{1 + 2 \frac{b}{a}}{2a(1 + \frac{h}{a})} = \frac{w}{2} k_i k_i R_i. \]  

(17)

Taking into account the expression for \( \beta_i \):

\[ \beta_i = \frac{h}{\sqrt{12}} \sqrt{\frac{E}{\rho_0 + \frac{\pi}{4} \rho}}, \]

as well as the ratio \( w = \frac{bh^2}{6} \) we obtain from (17)

\[ C = \left[ \frac{2k_i k_0 \cdot R_i ah \left(1 + \frac{h}{a}\right)}{k_i A \left(1 + 2 \frac{h}{a}\right) \left(\frac{3E}{\rho_0 + \frac{\pi}{8} \rho}\right)} \right]^{\frac{3}{2}}. \]  

(18)

For a beam half placed in water, accordingly, we obtain

\[ C = \left[ \frac{k_i k_0 \cdot R_i ah}{k_i A \left(\frac{3E}{\rho_0 + \frac{\pi}{8} \rho}\right)} \right]^{\frac{3}{2}}. \]  

(19)

Since \( \frac{2 \left(1 + \frac{h}{a}\right)}{1 + 2 \frac{h}{a}} \geq 1 \) the amount of charge required to destroy a beam completely submerged into the water will be substantially greater than to destroy a beam half submerged into the water.

3. A special case of explosive loading

3.1. Impulse evenly distributed over the entire span of the beam.

Let the beam, freely supported at the ends, be acted upon by an impulse evenly distributed over the entire span of the beam, that is, at all points of the surface receiving the load, the specific impulse is the same, \( i = \text{const} \). Such a load will be, for example, during the explosion at a great distance from the beam of a large charge; during the explosion of an elongated charge located parallel to the beam; during the explosion of a flat charge and in other similar cases [10].

Since the specific impulse is at all points, so is the intensity of the impulse along the length of the beam (linear impulse) is \( i_i = \text{const} \).

The integral in the formula (1) is then equal to [11, 12]

\[ S = \int_0^l i_i(\xi) \sin \frac{j\pi \xi}{l} d\xi = i_i \left( \frac{1}{j\pi} \cos \frac{j\pi \xi}{l} \right) \bigg|_0^l = \frac{i_i l}{j\pi} \left[1 - (-1)^j \right]. \]

Substituting this value into the formula (1), we obtain a formula for deflection determining
In this case, the bending moment is expressed as

\[
M = -EJ \frac{\partial^2 z}{\partial x^2} = \frac{2\beta_i}{\pi} \sum_{j=1}^{\infty} \sum_{l=1}^{\infty} \frac{1 - (-1)^j}{j^3} \sin j\pi x l \sin j^2\pi^2 l^2 \beta t.
\]  

(21)

The maximum deflections are achieved at moments \( t_i \) corresponding to the ratio

\[
\sin \frac{j^2\pi^2 l^2 \beta t_i}{l^2} = \sin \frac{2n-1}{2} \pi
\]

that is

\[
\frac{j^2\pi^2 l^2 \beta t_i}{l^2} = \frac{2n-1}{2} \pi
\]

where \( n = 1, 2, 3, \ldots \).

From here we find

\[
t_i = \frac{2n-1}{2} j^2 \pi \beta
\]

(22)

Consequently, the maximum deflections in any section

\[
z = \frac{2iL^2}{\pi^3 \beta m_0} \sum_{j=1}^{\infty} \frac{1 - (-1)^j}{j^3} \sin j\pi x l \sin j^2\pi^2 l^2 \beta
\]

(23)

and the maximum of the maximum deflections of the beam will be reached in its middle, while it is equal to

\[
z_{\text{max}} \left( \frac{l}{2} \right) = \frac{4iL^2}{\pi^3 \beta m_0} \left[ 1 - \frac{1}{3^2} - \frac{1}{5^2} - \frac{1}{7^2} + \ldots \right]
\]

(24)

Maximum bending moments

\[
M_{\text{max}} = \frac{2\beta_i}{\pi} \sum_{j=1}^{\infty} \sum_{l=1}^{\infty} \frac{1 - (-1)^j}{j} \sin j\pi x l \sin j^2\pi^2 l^2 \beta
\]

(25)

and the maximum of the maximum bending moments is reached in the middle of the span and is equal to

\[
M_{\text{max}} \left( \frac{l}{2} \right) = \frac{2\beta_i}{\pi} \left[ 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots \right]
\]

(26)

3.2. Determination of the destructive impulse intensity.

Let’s us write down the condition of guaranteed destruction of the beam in accordance with the properties of the material of the beam material in the form [1]

\[
\sigma_{\text{max}} = \frac{M_{\text{max}}}{W} \geq k_0 \cdot k_1 \cdot R_2,
\]

(27)

where \( W \) is the moment of resistance of the beam.

The condition (27) means that for guaranteed destruction of the beam, the stress of its most stressed section must be greater than the maximum possible dynamic ultimate strength of the material or, in extreme cases, equal to it.

Using the formula (26), we obtain for \( i_i \) the relation

\[
i_i \geq \frac{\pi k_0 \cdot k_1 \cdot R_2 \cdot W}{2\beta \left[ 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots \right]}
\]

(28)

The formula (28) determines the intensity of the destructive impulse. Knowing it, the value of the charge can be found, whose explosion at a given distance from the beam in the water ensures its guaranteed destruction.
4. Conclusions
In the work under consideration, the value of the intensity of a running destructive impulse obtained using one of the classical dynamic criteria of destruction was found [1]. A comparative analysis of the values of the charges required for the destruction of beams completely submerged in the water and submerged only half has been carried out. It was found that the amount of the charge required to destroy a beam completely submerged in the water will be significantly greater than to destroy a beam that is half submerged in the water.

This effect must be taken into account when calculating load-bearing construction elements that are only partially in contact with the water. In addition, in the calculations of beams for guaranteed destruction, the refined O.E. Vlasov’s formula, obtained in [4], since the unspecified formula is valid only for the middle section of the beam.

Note in the work [8] estimates the magnitude of the pulse acting on a stationary barrier in the water, depending on the energy characteristics of the explosive charge and its location in the medium. In this paper, we found a condition for guaranteed destruction of the beam from the action of the explosion of such a charge in the water.

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