FLAVOR ASYMMETRY OF NUCLEON SEA FROM DETAILED BALANCE*

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In this study, the proton is taken as an ensemble of quark-gluon Fock states. Using the principle of detailed balance, we find $\bar{d} - \bar{u} \approx 0.124$, which is in surprisingly agreement with the experimental observation.

1. Hypothesis of Ensemble

In this talk, we introduce a new work on the flavor asymmetry of the nucleon sea as arising from the pure statistical effect without any parameter. It will be shown that we can reproduce the flavor asymmetry $\bar{d} - \bar{u} \approx 0.123$, which is in surprisingly agreement with the experimental observation, by just using the principle of detailed balance. We take the proton as an ensemble of a complete set of quark-gluon Fock states, and these different Fock states can be written as

$$|\psi^1\rangle = |uud\rangle = |\{uud\}, \{0, 0, 0\}\rangle$$
$$|\psi^2\rangle = |uudg\rangle = |\{uud\}, \{0, 0, 1\}\rangle$$
$$|\psi^3\rangle = |uud\bar{u}u\rangle = |\{uud\}, \{1, 0, 0\}\rangle$$
$$\cdots$$
$$|\psi^n\rangle = |uud\bar{u}u\cdots\bar{u}u \bar{d}d\cdots \bar{d}d g\cdots g\rangle = |\{uud\}, \{i, j, k\}\rangle,$$

where $\{uud\}$ represents the valence quarks of the proton, $i$ is the number of quark-antiquark $u\bar{u}$ pairs, $j$ is the number of quark-antiquark $d\bar{d}$ pairs, and $k$ is the number of gluons. Then the density operator of the ensemble is

$$\hat{\rho} = \rho_{0,0,0}|uud\rangle\langle uud| + \rho_{0,0,1}|uudg\rangle\langle uudg| + \cdots$$
$$= \sum_{i,j,k} \rho_{i,j,k}|uud\rangle\langle \{uud\}, \{i, j, k\}|\langle \{uud\}, \{i, j, k\}|,$$

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where $\rho_{i,j,k}$ is the probability of finding a proton in the state $|\{uud\}, \{i,j,k\}\rangle$ and should satisfy the normalization condition,

$$\sum_{i,j,k} \rho_{i,j,k} = 1. \quad (8)$$

It will be shown that $\rho_{i,j,k}$ can be calculated by using the principle of detailed balance without any parameter.

### 2. The Principle of Detailed Balance

Given an ensemble of quark-gluon Fock states that has $N_A(N_A = N\rho_A)$ of finding proton in state A and $N_B(N_A = N\rho_A)$ of finding proton in state B, then the states may change between each others from time to time. The principle of detailed balance means that the changing between any two states balance each other. So during a period of time, the number of events that changed from state A to state B ($n_{A\rightarrow B}$) equals the number of events that changed from state B to state A ($n_{B\rightarrow A}$),

$$n_{A\rightarrow B} = n_{B\rightarrow A}. \quad (9)$$

$n_{A\rightarrow B}$ is proportional to both $N_A$ and the transition probability of A to B ($R_{A\rightarrow B}$),

$$n_{A\rightarrow B} = N_A R_{A\rightarrow B} = N\rho_A R_{A\rightarrow B}. \quad (10)$$

We considered about B to A in the same way,

$$n_{B\rightarrow A} = N_B R_{B\rightarrow A} = N\rho_B R_{B\rightarrow A}. \quad (11)$$

Then combining (9), (10) and (11), we have

$$\rho_A = \frac{R_{B\rightarrow A}}{R_{A\rightarrow B}} \rho_B. \quad (12)$$

In order to know $\rho_{i,j,k}$, transition probabilities should be calculated out first.

The transition between two states has two ways: splitting and recombination.

In the splitting process, the transition probability is proportional to the number of partons that may split in a certain state,

$$R_{q\rightarrow qg} \propto N_q, \quad R_{g\rightarrow q\bar{q}} \propto N_g. \quad (13)$$

In the recombination process that involving two kinds of partons, the transition probability is proportional to both the number of those two kinds of partons that may recombine in a certain state,

$$R_{qq\rightarrow g} \propto N_q N_g, \quad R_{g\rightarrow q\bar{q}} \propto N_q N_g. \quad (14)$$

For a simple illustration, we neglect the $g \leftrightarrow gg$ processes because they give small contribution to the flavor asymmetry. We will use symbol

$$R_{A\rightarrow B}$$

for $|A\rangle \xrightarrow{\rho} |B\rangle$. \quad (15)
to present our calculations about transition probabilities.

(1) In the translation process involving the creation or annihilation of gluon, only $q \leftrightarrow qg$ will be considered while $g \leftrightarrow gg$ neglected. So we have

$$\{uud\}, \{i, j, k - 1\} \overset{3 + 2i + 2j}{\rightarrow} (3 + 2i + 2j)k \{uud\}, \{i, j, k\}.$$  \hfill (16)

Using formula (12), we obtain a recursion formula,

$$\frac{\rho_{i, j, k}}{\rho_{i, j, k - 1}} = \frac{1}{k!}.$$  \hfill (17)

We can further get a more general formula,

$$\frac{\rho_{i, j, k}}{\rho_{i, j, 0}} = \frac{1}{k!},$$  \hfill (18)

and we have $\rho_{i, j, 1} = \rho_{i, j, 0}$, which we will used later.

(2) In the translation process involving the creation or annihilation of a pair of $\bar{uu}$, we have

$$\{\bar{uud}\}, \{i - 1, j, 1\} \overset{1}{\rightarrow} i(i + 2) \{\bar{uud}\}, \{i, j, 0\}.$$  \hfill (19)

Using formula (12), we obtain a recursion formula,

$$\frac{\rho_{i, j, 0}}{\rho_{i - 1, j, 1}} = \frac{1}{i(i + 2)}.$$  \hfill (20)

Using relation $\rho_{i, j, 1} = \rho_{i, j, 0}$, we get

$$\frac{\rho_{i, j, 0}}{\rho_{i - 1, j, 0}} = \frac{1}{i(i + 2)},$$  \hfill (21)

$$\frac{\rho_{i, j, 0}}{\rho_{i - 0, j, 0}} = \frac{2}{i!(i + 2)!}.$$  \hfill (22)

(3) In the translation process involving the creation or annihilation of a pair of $\bar{dd}$, we have

$$\{\bar{uud}\}, \{i, j - 1, 1\} \overset{1}{\rightarrow} j(j + 1) \{\bar{uud}\}, \{i, j, 0\}.$$  \hfill (23)

Using formula (12), we obtain a recursion formula,

$$\frac{\rho_{i, j, 0}}{\rho_{i, j - 1, 1}} = \frac{1}{j(j + 1)}..$$  \hfill (24)

Using relation $\rho_{i, j, 1} = \rho_{i, j, 0}$, we get

$$\frac{\rho_{i, j, 0}}{\rho_{i, j - 1, 0}} = \frac{1}{j(j + 1)},$$  \hfill (25)

$$\frac{\rho_{i, j, 0}}{\rho_{i, 0, 0}} = \frac{1}{j!(j + 1)!}.$$  \hfill (26)
Combining (18), (22), and (26), we obtain the general formula
\[ \frac{\rho_{i,j,k}}{\rho_{0,0,0}} = \frac{2}{i!(i+2)!j!(j+1)!k!}. \]  
(27)

From this result and the normalization condition (8), all \( \rho_{i,j,k} \) can be calculated out as shown in Table 1. From Table 1, we get the numbers of intrinsic gluons and sea quarks of the proton,
\[ \bar{u} = \sum_{i,j,k} i \rho_{i,j,k} = 0.308, \]  
(28)
\[ \bar{d} = \sum_{i,j,k} j \rho_{i,j,k} = 0.432, \]  
(29)
\[ g = \sum_{i,j,k} k \rho_{i,j,k} = 0.997, \]  
(30)
\[ \bar{d} - \bar{u} = 0.124. \]  
(31)

The flavor sea-quark asymmetry \( \bar{d} - \bar{u} \) can be checked by experiments directly because its \( Q^2 \) dependence is small. It is a surprise that our result is in excellent agreement with the recent experimental result \( \bar{d} - \bar{u} = 0.118 \pm 0.012 \). This good agreement indicates that the principle of detailed balance plays an essential role in the structure of proton. We also give a complete set of Fock states for the proton, with the probability of finding each Fock state calculated without any parameter, as shown in Table 1.

Table 1. The probabilities, \( \rho_{i,j,k} \), of finding the quark-gluon Fock states of the proton.

| i | j | \( \{|uud\}, \{i,j,0\} \) | \( \rho_{i,j,0} \) | \( \rho_{i,j,1} \) | \( \rho_{i,j,2} \) | \( \rho_{i,j,3} \) | \( \rho_{i,j,4} \) | ... |
|---|---|----------------------|--------|--------|--------|--------|--------|---|
| 0 | 0 | \( |uud\rangle \) | 0.167849 | 0.167849 | 0.083924 | 0.027975 | 0.006994 | ... |
| 1 | 0 | \( |uud\bar{u}\rangle \) | 0.055950 | 0.055950 | 0.027975 | 0.009325 | 0.002331 | ... |
| 0 | 1 | \( |uudd\rangle \) | 0.083924 | 0.083924 | 0.041962 | 0.013987 | 0.034979 | ... |
| 1 | 1 | \( |uudd\bar{u}\rangle \) | 0.027975 | 0.027975 | 0.013987 | 0.004662 | 0.001166 | ... |
| 0 | 2 | \( |uudddd\rangle \) | 0.013987 | 0.013987 | 0.006994 | 0.002331 | 0.000583 | ... |
| 2 | 0 | \( |uudd\bar{u}\rangle \) | 0.006994 | 0.006994 | 0.003497 | 0.001166 | 0.000291 | ... |
| 1 | 2 | \( |uudd\bar{u}\rangle \) | 0.004662 | 0.004662 | 0.002331 | 0.000777 | 0.000194 | ... |
| 2 | 1 | \( |uudd\bar{u}\rangle \) | 0.003497 | 0.003497 | 0.001748 | 0.000583 | 0.000146 | ... |
| 0 | 3 | \( |uudddd\rangle \) | 0.001166 | 0.001166 | 0.000583 | 0.000194 | 0.000049 | ... |
| 3 | 0 | \( |uudd\bar{u}\rangle \) | 0.000466 | 0.000466 | 0.000233 | 0.000078 | 0.000019 | ... |

References
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