Investigation of the impact of steady-state VVER-1000 (1200) core characteristics on the reactor stability with respect to xenon oscillations*

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Abstract

The article presents a method for obtaining an analytical expression for the criterion of stability of a VVER-1000 (1200) reactor with respect to xenon oscillations of the local power in the core, containing an explicit dependence of the criterion ratio coefficients on the arbitrary axial neutron field distribution in steady states of the core. Based on the data of numerical experiments using a full-scale model of the Kalinin NPP power units, the authors present the results of checking the validity of this expression for the reactor stability criterion with respect to xenon oscillations for different NPPs with VVER-1000 (1200) reactors.

Keywords

Reactor stability, xenon oscillations, axial offset, VVER-1000 (1200), flexible (load tracing) operating modes

State of research

For the first time, analytical expressions were obtained for the criterion of stability of a VVER-1000 (1200) reactor with respect to xenon oscillations of the local power in the core, containing an explicit dependence of the criterion ratio coefficients on the arbitrary axial neutron field distribution in steady states of the core.

Under the conditions of maneuvering modes with a change in power during the day, non-stationary poisoning of the core with xenon occurs, which can lead to the occurrence of xenon oscillations of the local power throughout the VVER-1000 (1200) reactor core (Vygovsky et al. 2018, Averyanova et al. 2012). In this case, the task is to ensure the reactor stability with respect to xenon oscillations of the local power in the core. If the problem of ensuring stability is not solved, axial xenon oscillations can have a continuous character and, accordingly, be accompanied by an unlimited number of oscillation cycles of local power. This will lead to cyclic thermal loads on the fuel and the fuel cladding, which can change the thermomechanical properties of the fuel cell materials with a loss of characteristics necessary for the safe core operation (Averyanova et al. 2016).

For a symmetric neutron field distribution, it is not difficult to obtain an analytical expression for the reactor stability criterion. Similar expressions are found in many works (Bell and Glasstone 2001, Hitchcock 1963, Rudik...
1974, Semenov and Volman 2015, Ryabov and Semenov 2006, Vygovsky et al. 2011, Povarov et al. 2003). Under the condition of an asymmetric axial distribution of the neutron field, one more parameter appears that affects the reactor stability to xenon oscillations, i.e., the axial neutron power offset or the coefficient of nonuniformity of the axial distribution of energy releases. A mention of this and a description of the physical essence of this influence can be found in (Averyanova et al. 2008, Andrushechko et al. 2010). However, it is much more difficult to obtain final analytical expressions for the reactor stability criterion, taking into account an arbitrary axial neutron field distribution, and the authors are not aware of publications where such expressions could be presented. In our work, we set precisely this problem and obtained the final analytical expressions for the criterion ratio of the reactor stability to xenon oscillations at an arbitrary axial distribution.

In (Averyanova et al. 2008), simple calculations are presented that show that, when the maximum energy release shifts to the upper half of the core, while a constant reactor power is maintained, the average water temperature in the reactor decreases, and the temperature coefficient of reactivity decreases modulo, which reduces the reactor stability in comparison with the option of an symmetric axial neutron field distribution and, even more so, in comparison with the option of the shift of the maximum energy release to the lower half of the core. But this paper lacks analytical expressions that reflect these findings.

The analytical expressions obtained by us confirm the above findings and indicate explicitly the dependence of the temperature coefficient of reactivity on the coefficient of axial non-uniformity of energy releases (or the value of the axial neutron power offset), and thus allow us to quantitatively evaluate the effect of this dependence on the reactor stability to xenon oscillations.

In (Andrushechko et al. 2010), a statement is made that the more uniform the axial distribution of energy release in a nuclear reactor, the greater the probability of xenon oscillations is. This is not entirely correct, since the results of the performed numerical studies show that, when the maximum energy release is shifted to the upper half of the core, the reactor stability noticeably decreases in comparison with the state of the core at a uniform distribution.

The offset in practice for the second fuel loads of VVER-1000 (1200) always has a noticeable positive value, since the neutron distribution shifts to the top of the core due to significantly less the burnout of fuel in the upper part of the core as compared to the lower one for the first fuel load. Fuel burnup in the upper part of the core occurs to a lesser extent as compared to the lower part for the first fuel load due to a shift in the axial neutron field distribution for the first load to the lower part of the core. For every second load, it is useful to have an analytical description of the reactor stability, taking into account the influence of the axial distribution on it. This is what the practice of applying the obtained analytical expressions for the operation of NPP power units with VVER-1000 (1200) can be. This is especially important for making a decision on testing maneuvering modes for the second fuel load of NvNPP-7 and LNPP-II-2.

Undoubtedly, the obtained criterion is useful for clarifying the idea of the nature of the occurrence of xenon oscillations in thermal neutron reactors. It is advisable to use the information on the obtained criterion in the process of training future specialists for the nuclear industry and for retraining personnel of nuclear power plants. Moreover, these materials are already used in the educational process at the Departments of Automation of NRNU MEPhI and Equipment and Operation of Nuclear Power Plants of OINPE NRNU MEPhI.

### Stability of the spatial neutron field distribution

Xenon transients in the reactor core are caused by a violation of the equilibrium state, i.e., the dynamic equilibrium between the neutron flux density and the concentration of $^{135}\text{Xe}$ and $^{133}\text{Xe}$ nuclei. Xenon stability is understood as the ability of the core to restore the equilibrium state of spatial xenon distribution and spatial local power distribution throughout the core.

The changes in reactivity caused by xenon processes (integral xenon processes) as well as changes in the spatial distribution of energy releases in the core (spatial xenon processes) are of practical importance for the reactor operation.

The reactor stability to xenon oscillations is characterized by the stability index ($\alpha$) and the oscillation period ($T_{\text{osc}}$), which are determined in the analysis of free xenon oscillations obtained experimentally or by means of computational modeling. In this case, the time variation of a certain scalar quantity characterizing the energy release distribution in the core is considered. Such a scalar value is the axial offset ($AO$), i.e., the percentage ratio of the difference between the powers of the upper and lower halves of the core to the total power. Free spatial xenon oscillations have a sinusoidal character with periodic alternation of the ascending and descending phases, corresponding to an increase and decrease in the values of the parameter characterizing xenon oscillations. The deviation of the $AO$ value from its equilibrium value ($AO^*$), corresponding to the equilibrium distribution of xenon, is represented as (Averyanova and Filimonov 2009, Kosourova et al. 2010)

$$A(t) = A(t_0)\exp(\alpha t)\cos(\omega t),$$  

where $A = AO - AO^*$; $\omega = 2\pi/T$; $\tau = t - t_0$ ($t_0$ is the moment of reaching the first extremum); $T$ is the period of free xenon oscillations; $\alpha = T^{-1}\ln(A/A_1)$, $A_1$ is the amplitude of the first maximum, $A_2$ is the amplitude of the second maximum.

The behavior of $AO$ in time controls the stability of the reactor. The operational efficiency of a power unit with a VVER-1000 (1200) reactor is determined by minimizing
the deviation of $AO$ from its stationary values (Maksimov et al. 2015).

A characteristic of the stability of stationary states is such a parameter as the reactor stability index ($\alpha$) with respect to free xenon oscillations of the local power in the core (Kosourov et al. 2010). At $\alpha < 0$, the reactor is stable, and the oscillations damp; at $\alpha > 0$, the reactor is unstable, and the oscillations do not damp.

**Analytical expression procedure**

To obtain an expression that determines the criterion of reactor stability to xenon oscillations, the two-point approximation of the neutron kinetics model is used.

This approximation is described by a system of ordinary nonlinear differential equations for the balance of xenon and iodine, supplemented by an algebraic neutron balance equation in one-group in neutron energy and one-dimensional in geometry approximations, and simplified expressions for feedbacks in reactivity with small deviations of the average power. To analyze the reactor stability to xenon oscillations, the original system of ordinary nonlinear differential equations is reduced to a consolidated system of linearized equations. Further, the Laplace transform is applied to this system. Using this transform, we can obtain the final form of the third order characteristic equation. Based on the Hurwitz criterion, a criterion of the reactor stability with respect to xenon oscillations is derived in the case of an asymmetric axial neutron field distribution:

\[
\frac{\langle \Sigma_f \rangle V_0}{\sigma_{av} N_0} \left[ 2\lambda_{x_e} + \lambda \lambda_{x_e} \right] + \frac{\langle \Sigma_f \rangle V_0}{\sigma_{av} N_0} \left[ 6\lambda_{x_e}^2 + 2\lambda \lambda_{x_e} \right] + \frac{4\lambda_{x_e} + 16\lambda_{x_e}}{8\sigma_{av} N_0} + 4\lambda_{x_e} C \right] \times (2)
\]

where $b$ is the conversion factor from MW to fis/(s×cm$^3$); $\gamma = \gamma_{x_e} + \gamma_{x_e}^2 + \gamma_{x_e} \lambda_{x_e}$ is the total fraction of the yield of $^{135}$I per fission of the heavy isotope ($^{235}$U, $^{239}$Pu, $^{241}$Pu); $\gamma_{x_e}$ is the total fraction of the yield of $^{135}$Xe per fission of the heavy isotope ($^{235}$U, $^{239}$Pu, $^{241}$Pu); $\gamma_{x_e}^2$ is the total fraction of the yield of $^{135}$Te per fission of the heavy isotope ($^{235}$U, $^{239}$Pu, $^{241}$Pu); $D_i$ is the diffusion coefficient throughout the lower part of the core; $D_f$ is the diffusion coefficient throughout the upper part of the core; $a N_0$ is the coolant heating, deg/$(\Sigma_f) V_0$ is the average macroscopic uranium fission cross section; $\sigma_{av}$ is the average macroscopic cross section of thermal neutron absorption by $^{135}$Xe; $H$ is the height of the reactor core; $V_0$ is the volume of the reactor core; $\lambda_{x_e}$ is the xenon radioactive decay constant; $\lambda$ is the iodine radioactive decay constant; $k$ is the axial core non-uniformity factor, $k = N_{av} / \langle 0.5 N_0 \rangle$; $N_{av}$ is the average number of fissions (or average power); $N_{av} = N_{av} + N_{av}$; $N_{av}$ is the average number of fissions in the lower part of the core (or average power); $N_{av}$ is the average number of fissions in the upper part of the core (or average power); $d \rho / dN$ is the power coefficient of reactivity; $|d \rho / dT|$ is the temperature coefficient of reactivity in the lower part of the core; $|d \rho / dT|$ is the temperature coefficient of reactivity in the upper part of the core; $\nu s$ is the number of secondary neutrons.

Note that the coolant temperature in the lower part of the core is lower than in the upper part. It is known that the temperature coefficient of reactivity consists of two parts. One of them is determined by the density effect of reactivity and depends on the derivative of the water density with respect to its temperature, and this part of the coefficient is directly proportional to the value of this derivative. The value of the derivative of the water density with respect to temperature (modulo) decreases as the water temperature decreases, which follows from the thermodynamic properties of water and steam, and this leads to a decrease (modulo) in the temperature coefficient of reactivity. For this reason, the temperature coefficient of reactivity in the lower half of the core is smaller modulo than in the upper one.

The left side of (2) can be represented as a function $F$ of the parameters $k$ and $\sigma_{av} N_0 / \langle 0.5 N_0 \rangle V_0): F(k, \sigma_{av} N_0 / \langle 0.5 N_0 \rangle V_0)$. Provided that $\sigma_{av} N_0 / \langle 0.5 N_0 \rangle V_0 > 1$, the derivative $F$ with respect to the parameter $k$ turns out to be positive, i.e., $\delta F / \delta k > 0$. For VVER-1000 and VVER-1200 reactors at rated power, the parameter $\sigma_{av} N_0 / \langle 0.5 N_0 \rangle V_0)$ is noticeably greater than unity, which suggests that an increase in the parameter $k$ leads to an increase in the value of the left side of inequality (2) and contributes to an increase in the reactor stability to xenon vibrations. Otherwise, with a decrease in $k$ and a shift in the axial neutron field distribution to the upper part of the core, the reactor stability to xenon oscillations will decrease.

**Accuracy of calculations of xenon processes**

To check the validity of the expression given above, we carried out computational studies of the VVER-1000
xenon stability using the PROSTOR software package (Vygovsky et al. 2004) with a nuclear data library prepared using the UNK program (Anokhin et al. 2001). PROSTOR is the software core of the reactor and primary circuit equipment models as part of the full-scale simulators (PMT) of Kalinin NPP-2,-3,-4 and Rostov NPP-3,-4. PROSTOR is also used as part of the analyzer of operating modes of a reactor plant with a VVER reactor in the nuclear physics laboratory (NPhL) of the Kalinin NPP and in the training center (TC) of the Novovoronezh NPP.

This software package has been verified on many experimental and calculated NPP data based on acceptance test programs for the above-mentioned full-scale simulators and analyzers (Vygovsky et al. 2014). The comprehensive results of the verification of the reactor plant model are presented in the certification materials of Rostekhnadzr. As for the errors in calculating the main neutronic characteristics of the core, it can be stated that they do not exceed the errors in the calculations using the BIPR-7A, NOSTRA and IR programs. Figures 1, 2 show the results of verification of the PROSTOR software package based on comparing the calculated data obtained for this package with the experimental data obtained for various NPP power units with VVER-1000 reactors (Vygovsky et al. 2014). In addition to the above figures with the graphs of the behavior of the axial neutron power off-set as a function of time, the calculated and experimental integral parameters of xenon oscillations are given, i.e., the stability index and the oscillation period, which confirm the satisfactory accuracy of the calculations using the PROSTOR software package. The integral parameters of the oscillations are given in Tab. 1.

### Calculations of the dynamics of xenon processes

Oscillations were excited by immersion of the working (tenth) group of control elements of the reactor control and protection system (CPS) from the initial position of 90% extraction along the core height at the initial steady state of the reactor. Throughout the entire process, the critical state of the reactor was maintained at a constant power level by changing the concentration of boric acid in the coolant.

Tables 2, 3 show the results of PROSTOR-based numerical calculations of the stability indices and periods of free axial xenon oscillations at the beginning and end of the 22nd campaign of Kalinin NPP-2.

It follows from the analysis of these tables that the change in the stability index occurs in accordance with the changes in the parameters that determine the condition

| Oscillation parameters | Stability index α, h⁻¹ | Oscillation period T, h |
|------------------------|-------------------------|------------------------|
| Calculation for Kalinin NPP-3 | -34×10⁻⁷ | 27.7 |
| Experiment at Kalinin NPP-3 | -(33.4±0.7)×10⁻⁷ | 27.9±0.5 |
| Calculation for Kalinin NPP-4 | -1.91×10⁻³ | 28.0 |
| Experiment at Kalinin NPP-4 | -(1.93±0.04)×10⁻³ | 27.7±0.5 |

**Figure 1.** Results of modeling free xenon oscillations at a power equal to 75% of the rated value for the first campaign of Kalinin NPP-3. Axial offset behavior.

**Figure 2.** Results of modeling free xenon oscillations at a power equal to 75% of the rated value for the first campaign of Kalinin NPP-4. Axial offset behavior.
for the occurrence of xenon oscillations of the axial offset in the core. These parameters are the reactor power, the average macroscopic neutron fission cross section in the fuel, the number of secondary neutrons per fission, and the total power coefficient of reactivity.

Tables 4, 5 show the stability indices and periods of free axial xenon oscillations at the beginning of the first campaign of Kalinin NPP-3 and at the beginning of the second campaign of the same unit.

### Table 4. Stability index and period of axial xenon oscillations in the core and in the beginning of the first campaign of Kalinin NPP-3.

| Reactor power, % | α, h⁻¹ | T₀, h | H₀ | k | σ_x | (Σ_x)', cm⁻¹ | δp/δN×10⁻¹ | %/MW |
|-----------------|--------|------|----|---|-----|-------------|-------------|-------|
| 105             | -0.027 | 27.09| 70 | 1.072 | 209455 | 0.00888 | 2.55 | -0.3059 |
| 95              | -0.033 | 27.98| 70 | 1.064 | 209964 | 0.00891 | 2.55 | -0.3245 |
| 85              | -0.039 | 29.18| 70 | 1.056 | 210436 | 0.00894 | 2.55 | -0.3455 |
| 75              | -0.045 | 30.55| 70 | 1.049 | 210868 | 0.00897 | 2.55 | -0.3692 |
| 65              | -0.049 | 32.49| 70 | 1.043 | 211240 | 0.00900 | 2.55 | -0.3978 |
| 55              | -0.054 | 34.82| 70 | 1.036 | 211543 | 0.00902 | 2.55 | -0.4262 |
| 45              | -0.058 | 37.80| 70 | 1.031 | 211744 | 0.00902 | 2.55 | -0.4462 |

### Table 5. Stability index and period of axial xenon oscillations in the core and in the beginning of the second campaign of Kalinin NPP-3.

| Reactor power, % | α, h⁻¹ | T₀, h | H₀ | k | σ_x | (Σ_x)', cm⁻¹ | δp/δN×10⁻¹ | %/MW |
|-----------------|--------|------|----|---|-----|-------------|-------------|-------|
| 105             | -0.009 | 27.38| 70 | 0.983 | 187955 | 0.00908 | 2.56 | -0.3618 |
| 95              | -0.001 | 27.94| 70 | 0.970 | 188697 | 0.00911 | 2.56 | -0.3736 |
| 85              | -0.012 | 28.88| 70 | 0.957 | 189416 | 0.00913 | 2.56 | -0.3882 |
| 75              | -0.023 | 29.82| 70 | 0.946 | 190113 | 0.00916 | 2.56 | -0.3930 |
| 65              | -0.032 | 31.15| 70 | 0.935 | 190776 | 0.00918 | 2.56 | -0.4042 |
| 55              | -0.039 | 32.56| 70 | 0.924 | 191392 | 0.00921 | 2.56 | -0.4510 |
| 45              | -0.044 | 34.59| 70 | 0.912 | 191957 | 0.00923 | 2.56 | -0.4773 |

Despite the fact that the fuel enrichment and the power coefficient of reactivity increase, and the microscopic cross-section of xenon poisoning decreases for the second fuel load, the first load at the beginning of the campaign turns out to be more stable than the second load at the same power values. This occurs due to the change in the axial profile of the neutron power for the second campaign as compared to the initial load. An analysis of expression (2) proves this seemingly paradoxical result. As we can see, the values of \( k_\alpha (k_\alpha = N_{\text{ref}} / (0.5N_{\nu})) \), indicated in Tab. 4 and 5 for the first load, \( k_\alpha > 1 \) and the neutron field are shifted towards the lower part of the core; for the second load, \( k_\alpha < 1 \) and the field are shifted to the upper part of the core. A decrease in \( k_\alpha \), as follows from (2), decreases the left-hand side of this inequality and, at least, decreases modulo the stability index of free xenon oscillations of the axial neutron power offset. All this weakens the reactor ability in this state of the core to ensure the stability of xenon processes in the core.

### Conclusion

An analytical form of the criterion of stability of VVER-1000 (1200) with respect to xenon processes in the core is obtained, taking into account the arbitrary axial distribution of the neutron field in the reactor stationary states. The quality of this criterion was checked on the basis of the results of numerical experiments using the PROSTOR software package based on the data obtained from a number of power units of Russian NPPs with VVER-1000 reactors.

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