Finite density aspects of leptogenesis

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Abstract. Leptogenesis takes place in the early universe at high temperatures and densities and a deviation from equilibrium in the decay of heavy Majorana neutrinos is a fundamental requirement for the generation of the asymmetry. The equations, commonly used for its description, are largely based on classical Boltzmann equations (BEs) while the source of CP-violation is a quantum interference phenomenon. In view of this clash, it is desirable to study such processes in terms of non-equilibrium quantum field theory. On the other hand, it is simpler to solve BEs rather than the corresponding quantum field theoretical ones. Therefore, we derive modified BEs from first principles in the Kadanoff-Baym (KB) formalism. The results, found for a simple toy model, can be applied to popular phenomenological scenarios by analogy. This approach uncovers structural differences of the corrected equations and leads to different results for the form of the finite density contributions to the CP-violating parameter. In the case of degenerate heavy neutrino masses, corresponding to the popular scenario of resonant leptogenesis, it allows to explicitly distinguish between regimes where BEs are applicable or inapplicable.

1. Introduction

The observation of the matter/anti-matter asymmetry of the universe (baryon to photon ratio $\eta_B \simeq 6 \cdot 10^{-10}$) represents a major puzzle of astro-particle physics. Within standard cosmology (including an inflationary phase) a dynamic generation mechanism is needed for its solution. From an early work by Sakharov it is evident that three conditions need to be fulfilled if this is to be achieved by a perturbative particle-physical process $i \to f$. In detail these conditions are: violation of baryon number $\langle B(i) \neq B(f) \rangle$, violation of $C$ and $CP$ ($\langle \Gamma(i \to f) \neq \Gamma(\bar{i} \to \bar{f}) \rangle$). Finally, as a consequence of CPT, these processes need to occur in a non-equilibrium situation. Assuming the amount of CP-violation present in the standard model, one finds baryon asymmetries much smaller than the observed value $\eta_B$. This discrepancy is often interpreted as a major hint for the existence of physics beyond the standard model.

1.1. Baryogenesis via Leptogenesis

Among the most popular baryogenesis scenarios, today, are variations in which a lepton asymmetry is produced first and which can therefore be classified as leptogenesis scenarios (see [1] for a recent review). The lepton asymmetry is subsequently converted (partly) into the observed baryon asymmetry by $B - L$ conserving sphaleron processes. The popularity of leptogenesis may be attributed to the comparatively moderate required overhead to the SM (in its simple versions). Moreover, this extension nicely fits into the picture of yet known beyond SM physics as it explains the small observed neutrino masses via the see-saw mechanism. In some scenarios observable predictions for collider and neutrino experiments are possible. We focus here on the lepton number violating decay of the heavy Majorana neutrinos (which act as counterweights on the see-saw), $N_i \to \ell + \phi$ and $N_i \to \bar{\ell} + \bar{\phi}$, as implied by their Yukawa interaction with lepton and Higgs fields. If these couplings are generically complex they can induce
the required CP-violation. If their magnitude is not too large the required deviation from equilibrium is
generated by the expansion of the universe. In the case of thermal leptogenesis with hierarchical heavy
neutrino masses ($M_1 \ll M_2, M_3$) the baryon asymmetry may be explained if $M_1 \gtrsim 10^9$ GeV which
can be derived from its relation to the light neutrino masses. Beyond this simplest phenomenological
case there is a multitude of more intricate scenarios such as resonant leptogenesis, flavoured leptogenesis
or leptogenesis in the MSSM. In general different processes with different initial and final states can be
relevant for the generation of the lepton asymmetry. The results presented here are mainly of a structural
kind and may be applied, in slightly modified form, to scenarios in which the CP-violation is due to the
interference of the tree-level, one-loop vertex and self-energy contributions to the (out-of-equilibrium)
two particle decay of a heavy species, such as in thermal leptogenesis:

\[
\begin{split}
\begin{array}{c}
\includegraphics[width=0.2\textwidth]{leptogenesis_vertex.png}
\end{array}
\end{split}
\]

The matrix elements squared may then be parametrized as

\[
|\varnothing\rightarrow\ell\phi\rangle|^2 = (1 + \epsilon_i) |M_i|^2, \quad |\varnothing\rightarrow\ell\bar{\phi}\rangle|^2 = (1 - \epsilon_i) |M_i|^2,
\]
in terms of CP-violating parameters $\epsilon_i = (\Gamma_{N_i\to\ell\phi} - \Gamma_{N_i\to\ell\bar{\phi}})/(\Gamma_{N_i\to\ell\phi} + \Gamma_{N_i\to\ell\bar{\phi}})$:

\[
\epsilon_i = -2 \sum_{j \neq i} \frac{3\{h^2(h)_i^j\}}{|h|^2} \left[ 3\{\bar{\phi}^*\rightarrow\ell\phi\} + 3\{\bar{\phi}^*\rightarrow\ell\bar{\phi}\} \right].
\]

The branch-cut singularities of the graphs (square brackets) can conveniently be computed by means of
the Cutkosky cutting rules which have an elegant representation as circling rules, e. g.

\[
3\{\bar{\phi}^*\rightarrow\ell\phi\} = \includegraphics[width=0.2\textwidth]{cutting_rule.png} + \includegraphics[width=0.2\textwidth]{cutting_rule.png}.
\]

The “circling rules” implied here are due to ’t Hooft and tell to sum all graphs with at least one circled
e external vertex. Circled vertices change their sign and the propagators connecting them to uncircled ones
are replaced by $D_0^\pm \propto \Theta(\pm p_0) \delta(p^2 - m_0^2)$ which may be interpreted as a cut.

1.2. Leptogenesis and Kinetic Theory

The first two Sakharov conditions, as stated above, refer to processes (and their $S$-matrix elements) in
vacuum. At the same time, it is evident from the third one that the finite density of the primordial plasma
plays a crucial role. Obviously some kind of kinetic equation is needed to study the temporal evolution
of baryon/lepton number. The conventional equations are based on kinetic theory which describes the
microscopic evolution of the system in terms of averaged classical quantities. The underlying physical
picture here is that the plasma is composed of freely propagating (in the context of cosmology there is an
additional external gravitational force) particles which occasionally undergo instantaneous interactions.
For each species there is a one-particle distribution function which represents the average probability to
find an on-shell particle of this kind in a given small space volume and momentum bin. To reflect this
picture in our notation we may write $\psi_i \rightarrow \ell$ for a general process, or $N_i \rightarrow \ell + \phi$, $N_i \rightarrow \ell + \bar{\phi}$
for the decays – into particles or antiparticles – crucial in thermal leptogenesis. The colored bullets make
explicit that a one-particle distribution function, e. g. $f_\ell(X, k) = f_\ell^K$, for on-shell particles is identified with
each of them. Their time dependence is given by generalized Boltzmann equations (BEs):

\[
k^{\mu}\mathcal{D}_\mu f^{\ell}_1(X, k) = \sum_{\text{interactions of } \ell} C^{\ell+\rightarrow+}_k[f^{\ell}_1](X, k).
\]
There is one equation for each species and a separate collision term on the r.h.s. for each of its interactions such as the important $2 \leftrightarrow 1$ collision term

\[ C_{\ell \phi}^{\text{gain}}(k) = \frac{1}{2} \int d\Pi_k \, d\Pi_p \, (2\pi)^4 \delta(k + p - q) \left[ \begin{array}{c} \text{particles} \\ \text{particles} \\ \text{particles} \end{array} \right] - \left[ \begin{array}{c} \text{antiparticles} \\ \text{antiparticles} \end{array} \right], \quad (3) \]

where a shortage for the gain- and loss-term has been introduced:

\[ \left[ \begin{array}{c} \text{particles} \\ \text{particles} \end{array} \right] = \left[ \begin{array}{c} \text{particles} \\ \text{antiparticles} \end{array} \right] \left[ 2(1 \pm f_k^\pm) f_q^\pm \right] \quad \text{and} \quad \left[ \begin{array}{c} \text{antiparticles} \\ \text{antiparticles} \end{array} \right] = \left[ \begin{array}{c} \text{particles} \\ \text{antiparticles} \end{array} \right] \left[ - f_k^\pm f_q^\pm \right]. \]

In this notation, the corresponding $2 \leftrightarrow 1$ collision term for antiparticles $C_{\ell \phi}^{\text{loss}}$ can be obtained by changing $(\ell, \phi) \rightarrow (-\ell, -\phi)$ in eqn. (3). The quantum statistical terms represent a first finite density effect (Pauli blocking or stimulated emission). The usual procedure continues with the derivation of a rate equation for $n_{B-L} = (n_p - n_\phi)$ by subtraction of the BEs for particles and antiparticles and integration over the free momentum $k$. In thermal equilibrium one finds for the r.h.s.

\[ \int \frac{d^3k}{(2\pi)^3 E_k} \left[ C_{\ell \phi}^{\text{gain}}(k) - C_{\ell \phi}^{\text{loss}}(k) \right] \neq 0, \quad \text{using} \quad \left[ \begin{array}{c} \text{particles} \\ \text{antiparticles} \end{array} \right] = \left[ \begin{array}{c} \text{antiparticles} \\ \text{antiparticles} \end{array} \right]. \]

This means that, in contradiction to the third Sakharov condition, a non-zero asymmetry is produced even in thermal equilibrium. The physical reason for this discrepancy is a miscounting in eqn. (3) and the corresponding term for antiparticles which fail to take into account the finite probability that a heavy neutrino produced – within a small time interval – in the inverse decay of lepton and Higgs may decay again in the same small time interval. The well-known technical solution to this “double-counting problem” is to subtract the real intermediate state contribution to the $2 \leftrightarrow 2$ collision term:

\[ \text{in } C_{\ell \phi}^{\text{gain}} \text{ and } C_{\ell \phi}^{\text{loss}}. \]

This works because of the generalized optical theorem, but only if stimulated emission and blocking factors are dropped. How can consistent equations be obtained which include the quantum statistical factors? One may also be tempted to ask: Why are no distribution functions identified with the internal lines in eqn. (1) which can also be on-shell, as indicated by the cuts. Given the importance of the conclusions often drawn from the requirement of successful baryogenesis (in multiple different models) it seems surprising that comparatively few effort has been made to gain a better understanding of the underlying kinetic description.

2. Systematic Kadanoff–Baym Approach

The weaknesses of the bottom-up approach described above suggest to consider leptogenesis within a more fundamental framework. Our approach was to pursue a consistent top-down derivation of BEs from non-equilibrium QFT for a toy-model in curved space-time [3, 4, 5, 6, 7]. Other recent interesting approaches have been followed in [8, 9, 10, 11]. Our model includes a complex scalar field (toy-baryons) and two mixing real scalars (toy-neutrinos) with Yukawa interaction:

\[ S = \int \sqrt{-g} \, d^4x \, \mathcal{L}, \quad \text{with} \quad \mathcal{L} \supset -\frac{\lambda}{2!} (bb)^2 - \frac{g_i}{2!} \psi_i bb - \frac{g_i}{2!} \psi_i \bar{b}b, \quad i = 1, 2. \]

1 A redefinition of the widths can also lead to consistent RIS-subtracted equations in this case, see e.g. [2].
Despite its simplicity the model captures important features of baryogenesis/leptogenesis which are common to many out-of-equilibrium decay scenarios. In particular, what has been said above remains valid if we make the replacement (\(\llangle\rightarrow \llparenthesis\llangle\), \(\llangle\rightarrow \llparenthesis\llangle\)). This yields quantum-corrected BEs of the form (2) where we identify as different symmetrization factors may be needed.

The first step is the derivation of self-consistent Schwinger–Dyson equations for the full propagator of the toy-baryons from the (covariant) generating functional for Greens functions with local and bi-local external sources:

\[
D^{-1}(x, y) = \mathcal{G}^{-1}(x, y) - \Sigma(x, y),
\]

where the self-energy \(\Sigma(x, y) = 2i\delta \Gamma_2[D, \Phi]/\delta D(y, x)\) is obtained from the 2PI functional at 3-loop order

\[
\Gamma_2 = \llparenthesis + \llangle + \llangle + \llangle + \cdots \cdot .
\]

There is a similar (matrix-)equation for the full propagators of the toy-neutrinos \(G^{ij}\). Together they lead to a coupled system of Kadanoff–Baym equations (KBEs) for quantum correlation functions, i.e. the statistical propagator and the spectral function (here only for the toy-baryons):

\[
[\Box + m^2(x)] D_F(x, y) = \int_0^\infty \sqrt{-g} d^4 z \Sigma_F(x, z) D_F(z, y) - \int_0^\infty \sqrt{-g} d^4 z \Sigma_\rho(x, z) D_F(z, y) ,
\]

\[
[\Box + m^2(x)] D_\rho(x, y) = \int_0^\infty \sqrt{-g} d^4 z \Sigma_\rho(x, z) D_\rho(z, y) .
\]

From this set of KBEs, which includes all kinds of quantum effects, we obtain “quantum-corrected BEs” following a sequence of systematic approximations. This includes the introduction of a center coordinate \(X\) and relative coordinates (covariantly), sending the initial time to \(-\infty\), a Wigner transformation wrt. to the relative coordinate, a gradient expansion wrt. to the center coordinate, neglecting the Poisson brackets (i.e. memory effects) as well as the so-called Kadanoff-Baym ansatz and a quasi-particle approximation. Assuming that the full propagator of the toy-neutrinos \(G^{ij}\) can be approximated by a diagonal one, the first six steps lead to

\[
[k^{\mu} D_\mu f^{ij}(X, k)] D_\rho(X, k) = -\frac{1}{2} \left[ D_{\gamma\gamma}(X, k) \Sigma_\gamma(X, k) - \Sigma_{\gamma\gamma}(X, k) D_\gamma(X, k) \right],
\]

where \(D_{\gamma\gamma}(X, p) \equiv D_F(X, p) \pm \frac{1}{2} D_\rho(X, p) = \{ (1 + f^{ij}(X, p)) D_\rho(X, p), f^{ij}(X, p) D_\rho(X, p) \}\) (Kadanoff-Baym ansatz) which defines distribution functions in terms of quantum correlation functions. The relevant contribution to the self-energy is

\[
\Sigma_{\gamma\gamma}(X, k) = -\frac{1}{2} \int d\Pi_{\gamma} d\Pi_{\gamma} (2\pi)^4 \delta^4(k + p - q) \left[ 1 + \epsilon_i(X, q, p) \right] G^{ij}_{\gamma\gamma}(X, q) D_{\gamma\gamma}(X, p)
\]

If the spectral functions are sharply peaked on-shell they can be approximated by Dirac-deltas (quasi-particle approximation). This yields quantum-corrected BEs of the form (2) where we identify as collision terms

\[
G^{ij}_{\gamma \gamma}(k) = \frac{1}{2} \int d\Pi_{\gamma} d\Pi_{\gamma} (2\pi)^4 \delta(k + p - q) \left( 1 + \epsilon_l^\gamma \right) \llangle \llangle - \llangle \llangle ,
\]

Surely the toy-model may fail to capture important features of a given phenomenological scenario such as gauge-interactions or processes which involve SM fermions in the case of thermal leptogenesis. However, often minor modifications such as different thermal masses or decay widths will be sufficient. In section (1) different signs in quantum statistical factors and different symmetrization factors may be needed.
and the corresponding equation for antiparticles, obtained by $(\Phi, \emptyset) \rightarrow (\emptyset, \Phi)$, and $(1 + \epsilon_i^{th})$ is to be exchanged by $(1 - \epsilon_i^{th})$. The CP-violating parameter in eqn. (5) appears as overall prefactor. By detailed balance, this guarantees that no asymmetry is produced in thermal equilibrium, since the square brackets vanish in this case. As before $\epsilon_i^{th}$ receives vertex and self-energy contributions, $\epsilon_i^{th} = \epsilon_i^{V,th} + \epsilon_i^{S,th}$. Both of them depend on the distribution functions of the toy-baryons:

$$
\epsilon_i^{V,th} = -\frac{1}{8\pi M_i^2} \sum \left\{ \frac{|g_j|^2}{g_i^* g_j} \right\} \int \frac{d\Omega_i}{4\pi} \frac{1 + f_{E_1} \Phi + f_{E_2} \Phi}{M_j^2 / M_i^2 + \frac{1}{2} (1 + \cos \theta_i)}, \quad E_{1,2} = \frac{1}{2} (E_q^0 \mp |q| \cos \theta_q t).
$$

and an analogous result for $\epsilon_i^{S,th}$.

3. Application to Phenomenological Theories

The form of eqn. (6) is reminiscent of older results for the finite density contributions to CP-violating parameters in the phenomenological scenario of thermal leptogenesis, see e.g. [1]. However, an important difference is that the established results exhibit an additional term quadratic in $\epsilon_i^{th}$.

The thermal circling rules employed here differ from their vacuum version used in eqn. (1). In particular the (cut) propagators are to be replaced by their thermal versions. These circling rules lead to the correct results for the toy model, such as eqn. (6). For the phenomenological scenario of thermal leptogenesis it leads to

$$
\epsilon_i^{V,th} = \frac{1}{16\pi} \sum_{j \neq i} \left\{ \frac{(h h)^2_{ij}}{(h h)_{ii}} \right\} M_j \int \frac{d\Omega_i}{4\pi} \frac{1 - \cos \theta_i}{M_j^2 / M_i^2 + \frac{1}{2} (1 + \cos \theta_i)} \left\{ 1 - f_{E_1}^{\Phi,eq} + f_{E_2}^{\Phi,eq} \right\}.
$$

The thermal approach reveals the nature of the finite density effects represented by the distribution functions as absorption and subsequent re-emission of the intermediate on-shell particles by the plasma. Vertex and self-energy contribution exhibit the same $f^{eq}$ dependence and yield the vacuum result for $f^{eq} \rightarrow 0$. $\epsilon_i^{th}$ is momentum dependent and always enhanced by medium corrections. The enhancement can lead to a correction of the final asymmetry by about 20%. These results are in agreement with [9].

4. Resonant Case

The self-energy contribution $\epsilon_i^{S} \propto M_i M_j / (M_i^2 - M_j^2)$ can be strongly enhanced in the case of degenerate heavy neutrino masses $M_i \simeq M_j$. This enhancement corresponds to the popular scenario of resonant leptogenesis. According to the classical computation it allows to lower the temperature scale of leptogenesis significantly, thus (in supersymmetric scenarios) it allows to circumvent the gravitino problem. In this case the finite width needs to be included in the heavy neutrino propagator. For the toy-model the established computation leads to:

$$
\epsilon_i^{S} = -\frac{|g_j|^2}{16\pi} \sum \left\{ \frac{g_i^* g_j}{g_i^* g_j} \right\} \frac{M_j^2 - M_i^2}{(M_j^2 - M_i^2)^2 + M_j^2 F_j^2}.
$$

Moreover, the results for causal products are known to agree with those obtained in the imaginary time formalism for the self-energy loop and the three-point function. The vertex which is always circles has the smallest/largest time component which agrees with the Boltzmann picture.
The derivation of BEs, sketched in section (2), requires sharply peaked spectral functions $G^{ij}_\rho$. In the non-resonant case the on-shell peaks are well separated by the mass splitting, but for $M_i \approx M_j$ the mass splitting is much smaller than the masses themselves ("resonant" case) and may in addition be comparable to the width of the spectral functions ("maximal resonant" case). In the latter regime the peaks of the spectral functions may overlap significantly which prevents the unambiguous definition of quasi-particles and their one-particle distribution functions. Also the behaviour of the off-diagonal components of the correlation functions can be important so that the effects cannot be described in terms of on-shell CP-violating parameters. In the resonant regime these concepts can still be employed. However, it is important to take into account that the mass spectrum of the quasi-particle excitations is modified by the finite density of the medium since the enhancement depends sensitively on the mass splitting. An improved analysis [5] leads to modified expressions for the CP-violating parameters:

$$c_i^S \equiv \frac{|g_{ij}|^2}{16\pi} \sum \left\{ \frac{g_j g_i^*}{g_i^* g_j} \right\} \frac{\Delta_{ij} - \delta_{ij}}{c^2 |\Delta_{ij} - \delta_{ij}|^2 + s^2 \left[ \Delta_{ij}^2 + (M_j \Gamma_j)^2 \right]^2 L_\rho},$$

where $c = \cos(\delta_{CP})$, $s = \sin(\delta_{CP})$, $L_\rho(X, p) = \int \frac{d^4k}{(2\pi)^4} [1 + j_{E_1} + j_{E_2}]$ encodes medium corrections discussed above for the hierarchical case and $\Delta_{ij}$, $\delta_{ij}$ describe finite density corrections to the masses. On the other hand, there is also a significant change of the in-medium decay width $\Gamma_i^{med}$ which can have an opposite effect on final baryon asymmetry. These corrections may already lead to significant quantitative modifications in the resonant case, while there is even more uncertainty in the maximal resonant case where the derivation of BEs fails.

5. Conclusions

In the case of hierarchical heavy neutrino masses quantum-corrected BEs can be derived systematically for leptogenesis from the corresponding set of KBEs. These BEs are free of the double-counting problem and include a different result for the CP-violating parameter. This can be reconciled with thermal QFT if causal products are used in the computation. The corrections can be significant but for a given scenario further thermal effects must be included to obtain quantitative results. The application of the Boltzmann picture can be extended to a regime of intermediate degeneracy but fails in the maximal resonant case where the spectral functions overlap due to their finite width. It is highly important to test the different steps in our systematic derivation in order to find out when BEs are applicable. For example memory effects could be important in the resonant case. Also the next-to-leading order in the gradient expansion leads to a qualitatively different contribution to the rate equations [6]. These can be neglected in the case of the standard scenario of thermal leptogenesis, however. An alternative is to compare directly to solutions of the corresponding set of KBEs. In [13, 8] an approach to solve them explicitly in the special case of leptogenesis has been sketched.

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