Resonant Landau–Zener transitions in a helical magnetic field

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Abstract
Spin-dependent electron transport has been studied in magnetic semiconductor waveguides (nanowires) in the helical magnetic field. We have shown that—apart from the well-known conductance dip located at the magnetic field equal to the helical-field amplitude $B_h$—the additional conductance dips (with zero conductance) appear at a magnetic field different from $B_h$. This effect occurring in the non-adiabatic regime is explained as resulting from the resonant Landau–Zener transitions between the spin-split subbands.

Keywords: spin transistor, spintronic, helical magnetic field

(Some figures may appear in colour only in the online journal)

1. Introduction
The experimental realization of an effective spin transistor remains a challenge facing spintronics since the pioneering concept proposed by Datta and Das [1]. According to the original idea [1], the operation of the spin transistor is based on the gate-controlled spin–orbit interaction (SOI) of the Rashba form [2]. The current of spin-polarized electrons is injected from the ferromagnetic source into the conduction channel formed in a two-dimensional electron gas and is ballistically transported to the ferromagnetic drain. The state of the transistor depends on the electron spin orientation modulated via the Rashba SOI by the voltage applied to the gate attached close to the channel. The operation of a spin transistor has been studied in many theoretical papers [3–7]. However, the experiments [8, 9] indicate that the signals obtained in the up-to-date realized spin transistors based on the SOI are rather low, which results from the low efficiency of the spin injection from the ferromagnet into the semiconductor [10] and the spin relaxation. The SOI causes the scattering processes to affect the spin states of electrons, e.g., by the Elliott–Yafet or Dyakonov–Perel mechanism [11]. Although the spin relaxation is proposed to be suppressed by equating the Rashba and Dresselhaus term [12, 13], the concept of the non-ballistic spin transistor proposed by Schliemann et al in reference [12] is still waiting for experimental realizations.

An alternative spin transistor design with the spin signal observed over the distance 50 μm has been recently described by Žutić and Lee [14] and experimentally demonstrated by Betthausern et al in reference [15]. In this approach, the spin control is realized by combining the homogeneous and helical magnetic fields. The latter is generated by ferromagnetic stripes located above the conduction channel. The spin state of electrons flowing through the channel is protected against a possible decay by keeping the transport in the adiabatic regime [16]. The transistor action is driven by the diabatic Landau–Zener transitions [17, 18] induced by the appropriate tuning of the homogeneous magnetic field. For these suitably chosen conditions, the backscattering of spin-polarized electrons appears, which gives rise to the large increase of the resistance; i.e., the transistor goes over into the ‘off’ state. In contrast to the SOI-based spin transistor, the proposed design is robust against the scattering processes [19].

Motivated by the experiment [15], we have performed computer simulations of the spin-dependent transport in the helical magnetic field for the nanostructure similar to that of reference [15]. The calculations of the conductance as a function of the Fermi energy reveal additional conductance dips, which cannot be understood in terms of the ordinary Landau–Zener theory. This effect, not reported in previous studies [15, 19], is explained as resulting from the resonant inter-subband transition between the spatially modulated spin-
dependent subbands and can considerably change the conductance of the nanowire.

2. Model

We consider the two-dimensional waveguide (nanowire) in the $x - y$ plane made of (Cd,Mn)Te in the presence of the magnetic field $B(r)$, which is the superposition of the homogeneous magnetic field applied along the $z$-axis $B_{\text{ext}} = (0, 0, -B_{\text{ext}})$ and the helical magnetic field $B_h(r)$ taken on in the form [19]

$$B_h(r) = B_h \left( \sin \frac{2\pi x}{a}, 0, \cos \frac{2\pi x}{a} \right),$$

where $B_h$ is the amplitude of the helical field and $a$ is the period of the magnetic field modulation. The Hamiltonian of the electron is given by

$$\hat{H} = \frac{1}{2m_{\text{eff}}} \left[ \hat{p} + eA(r) \right]^2 + \frac{1}{2} g_{\text{eff}} \mu_B B(r) \cdot \sigma,$$

where $m_{\text{eff}}$ is the conduction-band effective mass, $A(r) = (B \times r)/2$ is the vector potential, $g_{\text{eff}}$ is the effective $g$-factor, $\mu_B$ is the Bohr magneton, and $\sigma$ is the vector of Pauli matrices. In the presence of the magnetic field, the $s$-$d$ exchange interaction between the conduction band electrons and Mn ions in (Cd,Mn)Te leads to the giant Zeeman splitting of the conduction bands, with the effective $g$-factor ranging from 200–500 [20].

In the superposition of the magnetic fields, the subbands corresponding to the electron with the parallel $q^+$ and antiparallel $q^-$ spin orientation (with respect to the magnetic field) depend on postion $x$. The spatially dependent subband energy minima $E^\pm$ are given by (see figure 1)

$$E^\pm(x) = \pm \frac{1}{2} g_{\text{eff}} \mu_B B_h \sqrt{1 + \gamma^2 + 2\gamma \cos(2\pi x/a)},$$

where $\gamma = B_{\text{ext}}/B_h$. Parameter $\gamma$, which can be tuned by changing the homogeneous magnetic field $B_{\text{ext}}$, determines the spatial modulation of the spin-split subbands, as depicted in figure 1.

We have performed the numerical calculations of the conductance by the tight-binding method on the square lattice with $\Delta x = \Delta y = 1$ nm using the Kwant package [21]. In the calculations, the following values of the parameters have been used: the effective mass of the electron in CdTe, $m_{\text{eff}} = 0.1m_e$, where $m_e$ is the free electron mass, and $g_{\text{eff}} = 200$. We adopt the hard-wall boundary conditions in the $y$ direction, assuming the width of the conduction channel $W = 30$ nm. The value of the helical magnetic field amplitude $B_h$ has been taken on the basis of the experimental report [15] and is equal to $B_h = 50$ mT.

3. Results and discussion

The previous studies [15, 19] of the spin control in the helical magnetic field have been performed in the adiabatic regime, in which the spin orientation of electrons flowing through the nanostructure follows the spatial modulation of the magnetic field. In the considered nanostucture, the adiabaticity [16] can be defined by the parameter $Q = \omega_L/\omega_{\text{mod}}$, where $\omega_L = g_{\text{eff}} \mu_B B/h$ is the frequency of the spin Larmor precession and $\omega_{\text{mod}} = 2\pi v_F/a$ is the frequency of the magnetic field modulation measured in the electron rest frame, where $v_F$ is the Fermi velocity. In the adiabatic regime, $Q \gg 1$.

In this paper, we study the spin transport in the regime, in which the adiabaticity condition is weakened. In order to achieve this, we change the period $a$ of the helical-field modulation. Figure 2 presents the conductance as a function of the homogeneous magnetic field $B_{\text{ext}}$ and the Fermi energy $E_F$ for the different values of $a$ corresponding to different $Q$.

In the calculations, we have considered only the two lowest-energy spin-split subbands. The Fermi energy is varied within the range, which ensures that the electrons are injected into the conduction channel from the lowest-energy subband. The conductance as a function of the magnetic field and Fermi energy exhibits the wide minimum centered around the value of the helical magnetic field amplitude $B_h$ (figures 2 and 3).

The conductance minimum in the range of the magnetic field ($B_h - \Delta B_{\text{ext}}, B_h + \Delta B_{\text{ext}}$) can be easily understood in terms of the Landau–Zener transitions between the spatially modulated Zeeman splitted energy subbands. The value of $\Delta B_{\text{ext}}$ depends on the position of the Fermi energy with respect to the spatially dependent energy subbands. The mechanism leading to the central wide dip has been explained in detail in reference [15, 19].

In our study we have found that if the Fermi energy exceeds $E_{\text{min}}^+$, defined as the minima of the band $q^+$ (see figure 1), the additional conductance dips appear for $B_{\text{ext}} \neq B_h$ (see figures 2(a), (b) and 3(b)). This effect has not
been reported in the previous studies [15, 19] and cannot be explained as resulting from the ordinary Landau–Zener transitions, for which the conductance is suppressed only for $B_{\text{ext}} \approx B_h$. The analysis of the conductance maps in figures 2(a) and (b) allows us to extract two characteristic features of this effect: (i) The period of the occurrence of the additional dips (measured as a function of the Fermi energy) decreases with the increasing period $a$ of the magnetic field modulation (see figures 2(a) and (b)), and (ii) The additional conductance dips disappear in the adiabatic regime (we see that for $a = 8 \mu$m ($Q \approx 25$) the additional conductance dips are strongly suppressed (see figure 2(c))). In order to find the physical interpretation of these additional minima of conductance, we have analyzed the electron properties for six characteristic values of the magnetic field marked by labels (I)–(VI) on the red curve in figure 3(b). The chosen values of the magnetic field correspond to the subsequent maxima and minima of the conductance. Figure 4 displays the electron density in the nanostructure calculated for points (I)–(VI). In case (I), we observe the transmission of electrons through the waveguide with the slight increase of the charge density in the vicinity of the point $x = a/2$. At this point the electron transmission is completely blocked in cases (II), (IV), and (VI). The partial transmission with the non-zero electron density for $x > a/2$ has been obtained in cases (III) and (V). Figure 4 shows that the three conductance dips (II, IV, VI) correspond to the backscattering of electrons in the vicinity of $x = a/2$. We note that the conductance dips for $B_{\text{ext}} \neq B_h$ appear only in the non-adiabatic regime for the short period $a$, which corresponds to $Q < 5$. In this regime, the modulated magnetic field, acting on the electron in its reference frame, changes so fast that the electron spin cannot adapt to changes of the magnetic field. This means that the electron, initially injected into the lowest subband $\varphi^+$, does not remain in this subband when flowing through the nanostructure. Therefore the quantum state of the electron can be described by the linear combination of the eigenstates with the spin orientation parallel and antiparallel to the magnetic field. The wave functions of these eigenstates can be approximately expressed as $\varphi^\pm (x, y) = c_\pm(x) \varphi^\pm (y; x)$, where $\varphi^\pm (y; x)$ are the wave functions obtained from the diagonalization of Hamiltonian \ref{eq:2} for coordinate $x$, treated as a parameter. Then,

\begin{equation}
\Psi(x, y) = c_+(x) \varphi^+(y) + c_-(x) \varphi^-(y),
\end{equation}
where the space-dependent coefficients are given by

$$c_\phi(x) = \int \left[ \phi^\pm(y) \right]^\# \Psi(x, y) \, dy. \quad (5)$$

The contribution of each subband $|c_\phi(x)|^2$ is presented in figure 5. We see that for the magnetic fields (II), (IV), (VI), i.e., for the conductance dips, the contribution to the wave function in the vicinity of $x = a/2$ originates only from the subband $\phi^+$. Since the electrons are injected into the nanowire in state $\phi^-$, related to the lowest-energy subband, one can conclude that the conductance dips correspond to the inter-subband transitions that occur with the probability close to one. On the contrary, for the magnetic fields (III) and (V), the probability of the inter-subband transition is less than 1. Although the main conductance dip for $B_{ext} = B_h$ (point (IV)) can be easily interpreted in terms of the ordinary Landau–Zener transition [15, 17–19], the occurrence of the dips for $B_{ext} \neq B_h$ cannot be explained by the existing theory.

The physics behind this effect can be explained as follows. If the period of the spatial magnetic field modulation is short, the adiabaticity condition for the spin transport through the nanostructure is violated. In this non-adiabatic regime, the state of the electron, initially injected into the lowest-energy subband $\phi^-$, is a linear combination of the eigenstates with the spin orientation parallel and antiparallel to the magnetic field (equation 4). This results in the probability of the inter-subband transition being non-zero. If the Fermi energy $\mu_F$, i.e., the energy of the injected electrons, exceeds $E_{min}$, the electrons flowing through the nanostructure experience the effective quantum well created in the spatially varying subband $\phi^+$. In this quantum well, the quasi-bound electron states are created. If the Fermi energy becomes equal to the energy of one of the quasi-bound states formed in subband $\phi^+$, the probability of the inter-subband transition approaches 1. This effect, being analogous to resonant tunneling, is called the resonant Landau–Zener transition. As a result of these resonant transitions, all electrons injected into the nanowire are transmitted to the subband $\phi^+$. However, since the energy of the subband $\phi^+$ in the right contact is higher than the Fermi energy, the electrons are backscattered, which gives rise to the conductance dip. This mechanism is schematically illustrated in figure 6.

In order to show that the resonant Landau–Zener transition leads to the conductance dips for $B_{ext} \neq B_h$, we display in figure 7 the conductance as a function of the Fermi energy. The triangles mark the numerically calculated (we use the finite difference method) energies of the quasi-bound states in the effective quantum well created in the subband $\phi^+$. We see that these energy levels agree very well with the positions of the conductance dips. The results of the calculations of the quasi-bound state energy levels as functions of the magnetic field $B_{ext}$ and the Fermi energy $\mu_F$ are presented in figure 2(b) by white dashed curves. The good agreement between the positions of the conductance dips and the energies of the quasi-bound states allows us to conclude that the conductance dips result from the resonant Landau–Zener transition between the corresponding subbands. We note that the necessary condition for this effect is the non-adiabaticity of the transport (in our case resulting from the sufficiently short period $a$ of the magnetic field spatial modulation), because only in this regime the state of the electron is the linear combination of the two eigenstates with the opposite spins. In the adiabatic regime, the electron injected into the lowest-energy subband $\phi^-$ remains in this state when flowing through the entire nanostructure. Therefore, the probability of the Landau–Zener transitions is zero. This explains why the effect presented in this paper disappears for the long period of the magnetic field modulation (see figure 2(c)), for which the transport can be treated as adiabatic. This also explains the lack of the experimental observation of this effect in [15]. We believe that after applying the helical magnetic field with a sufficiently short modulation period, the additional conductance dips can be observed.

In the present calculations, we have assumed a narrow width of the conduction channel and considered only the two lowest-energy spin-split subbands. The increase of the conduction channel width—under the assumption of the constant Fermi energy—leads to the increase of the number of subbands (transport modes) participating in the electron transport. The influence of the many subbands on the adiabatic
Landau–Zener transition in the helical magnetic field has been studied in reference [19]. It has been shown [19] that the increasing number of the transport modes causes the decrease of the conductance dip for $B_h = B_{\text{ext}}$. However, in order to observe the conductance dips for the system with many sub-bands, we need to ensure that the system is partially spin-polarized, which means that the Fermi energy has to take the value between the energies of two spin-split subbands. Then, all the bands with the spin-split subbands lying below the Fermi level transmit the electrons, while the bands with one spin-split subband below the Fermi energy and the second subband above the Fermi energy scatter the electrons back, giving rise to the conductance dip. Since many bands can transmit the electrons without the backscattering, the conductance dip becomes less pronounced. In the non-adiabatic regime, we have performed additional test calculations with many bands included and found that the results of reference [19] for the central conductance dip are also correct for the additional dips studied in the present paper. The only difference is that in the non-adiabatic regime, more additional dips for $B_{\text{ext}} \neq B_h$ appear if the number of the transport modes, which backscatter, increases. This also explains why

Figure 5. Spatially varying minima $E^\pm(x)$ of the conduction subband energy (upper panels) and the contribution of each subband to the wave function $|c_1|^2$ (red line) and $|c_2|^2$ (blue line), calculated for the magnetic fields (I)–(VI) marked in figure 3(b).
the conduction changes observed in experiment [15] are very small.

The additional conduction dips result from the non-adiabatic transitions between the spin-split subbands. In the experimental setup, the non-adiabaticity can be controlled by changing the distance between the ferromagnetic stripes, which changes the helical-field period. In order to observe the additional conductance dips, this period should be sufficiently small. The effect is also more pronounced for the small number of transport modes. Therefore, the conduction channel should be as narrow as possible.

4. Summary

In summary, we have demonstrated that the current flowing in the helical magnetic field through the waveguide (nanowire) made of the magnetic semiconductor exhibits the additional conductance dips for $B_{\text{ext}} \neq B_0$. This effect has been explained as resulting from the resonant Landau–Zener transitions between the spin-split subbands that lead to the spin backscattering. In our opinion, our finding can be important for the spin transistor design based on the helical magnetic field. Until now the transport in the new design is restricted to the adiabatic regime. We have shown that in the non-adiabatic regime, i.e., for the sufficiently small distance between the ferromagnetic stripes, the spin transistor action can also be driven by the resonant Landau–Zener transitions.

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