Some properties of meta-stable supersymmetry-breaking vacua in Wess-Zumino models

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Abstract

As a contribution to the current efforts to understand supersymmetry-breaking by meta-stable vacua, we study general properties of supersymmetry-breaking vacua in Wess-Zumino models: we show that tree-level degeneracy is generic, explore some constraints on the couplings and present a simple model with a long-lived meta-stable vacuum, ending with some generalizations to non-renormalizable models.

Introduction

In the search for a natural model of dynamical supersymmetry breaking, it was suggested by Intriligator et al. [1] that supersymmetry need not be broken by a stable vacuum and that the non-supersymmetric vacuum could be a long-lived meta-stable vacuum, with possible but slow tunnelling towards a stable, supersymmetric vacuum. This idea has recently attracted much attention [2,4] since it gives more freedom to dynamical supersymmetry breaking, removing for instance the Witten index constraint. It could also be of some interest with respect to possible embeddings in string theory [3,5,6] or M theory [7].

This note presents some simple remarks on the possibility of meta-stable supersymmetry-breaking vacua in Ó Raifeartaigh-like models. Although some of these may be known to experts, they have not, to our knowledge, appeared in literature, and they could be useful in coming efforts to build realistic models.

We first present some properties of supersymmetry-breaking vacua in renormalizable Wess-Zumino models: they are necessarily degenerate at tree level

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and this degeneracy is lifted by a one-loop pseudomodulus stabilization. We then study meta-stability due to tunnelling towards a neighbouring supersymmetric vacuum and show that the lifetime can be parametrically long while leaving the couplings finite. We then try to generalize these results to non-renormalizable models.

1 Renormalizable models

We consider several chiral superfields $\phi^a$ with canonical Kähler potential $K = \phi^*_a \phi^a$ and superpotential $W$, third-order polynomial of the $\phi^a$.

Suppose there exists a non-supersymmetric vacuum: the potential $V = |\partial W|^2$ for scalar fields admits a local non-zero minimum. The fourth-order expansion of $V$ around the vacuum, exact for a third-order superpotential, is then:

$$V = |\partial W|^2 + 2\Re \left( \partial^c W^\dagger \partial_{abc} W \delta \phi^a \right) + \left| \partial_{ab} W \delta \phi^b \right|^2 + \Re \left( \partial^c \partial^d W^\dagger \partial_{abc} \delta \phi^a \delta \phi^b \delta \phi^c \right) + \Re \left( \partial^c \partial^d W^\dagger \partial_{abc} \delta \phi^a \delta \phi^b \delta \phi^c \delta \phi^d \right) + O(\delta \phi^4).$$

(1)

so that obvious necessary conditions for such a vacuum are:

$$\begin{cases} 
\partial W \neq 0 \\
\partial W^\dagger \partial^2 W = 0.
\end{cases}$$

(2)

We shall now try to find some consequences of those conditions in the form of constraints on the superpotential.

Degeneracy

In this paragraph we show that the potential is necessarily exactly degenerate at tree level. Using expansion (1) of the potential at vacuum point, we find:

$$\delta V = \left| \partial_{ab} W \delta \phi^b \right|^2 + \Re \left( \partial^c \partial^d W^\dagger \partial_{abc} \delta \phi^a \delta \phi^b \right) + \Re \left( \partial^c \partial^d W^\dagger \partial_{abc} \delta \phi^a \delta \phi^b \delta \phi^c \delta \phi^d \right) + O(\delta \phi^4).$$

(3)
This must be positive in order for the vacuum to be (meta)stable. But taking \( \delta \phi^a = \delta z \partial^a W^\dagger \), with some complex \( \delta z \), we find, using formula (2):

\[
\delta V = \Re \left( (\partial W^\dagger)^3 \partial^3 W \delta z^2 \right) + O(\delta z^3).
\]

(4)

The first term, if non-zero, is negative for some phase of \( \delta z \). As the form is supposed to be positive, this yields:

\[
\partial_{abc} W \partial^a W^\dagger \partial^b W^\dagger \partial^c W^\dagger = 0.
\]

(5)

Then if we choose \( \delta \phi^a = \varphi^a \delta z^2 + \partial^a W^\dagger \delta z \), with any \( \varphi^a \) such as \( \varphi^a \partial_a W = 0 \) and make the same calculation, we find:

\[
\delta V = 2 \Re \left( \partial^b W^\dagger \partial^c W^\dagger \partial_{abc} W \varphi^a \delta z^3 \right) + O(\delta z^4).
\]

(6)

At leading order in \( \delta z \), positivity implies:

\[
\partial^b W^\dagger \partial^c W^\dagger \partial_{abc} W \varphi^a = 0.
\]

(7)

As this is true for any \( \varphi^a \) orthogonal to \( \partial^a W^\dagger \) and for \( \partial^a W^\dagger \) itself, this gives:

\[
\partial^b W^\dagger \partial^c W^\dagger \partial_{abc} W = 0.
\]

(8)

From this we infer that, for a finite shift in the \( \partial W \) direction, \( \Delta \phi^a = \delta \partial^a W^\dagger \),

\[
\Delta V = 0.
\]

(9)

In other words, the potential is degenerate in the \( \partial W \) direction.

**Coupling conditions**

If we choose the considered supersymmetry-breaking vacuum to be at \( \phi^a = 0 \) and the direction \( \phi^0 \equiv X \) to be the direction of \( \partial W^\dagger \), the orthogonal directions being labelled by indices \( i, j, ... \), the superpotential, given the previous result, can be written as follows:

\[
W = \xi X + \frac{1}{2} \left( \mu_{ij} + \lambda_{ij} X + \frac{1}{3} \lambda_{ijk} \phi^k \right) \phi^i \phi^j.
\]

(10)

with \( \xi \) a real positive number parametrizing the amount of supersymmetry breaking. The vacuum extends on the complex line \( \phi^i = 0 \), with \( X \) taking any
value. Instead of keeping the background $\langle X \rangle$ as a free parameter, we shall shift it to zero by a change of $\mu$.

The masses of the bosonic and fermionic fields around that vacuum are generically given by the eigenvalues of the following mass matrices:

\[
\begin{align*}
M_0^2 &= \begin{pmatrix}
\partial^2 W^\dagger \partial^2 W & \partial^2 W^\dagger \partial \phi \\
\partial W^\dagger \partial^2 W & \partial^2 W \partial^2 W^\dagger
\end{pmatrix}, \\
M_{1/2}^2 &= \begin{pmatrix}
\partial^2 W^\dagger \partial^2 W & 0 \\
0 & \partial^2 W \partial^2 W^\dagger
\end{pmatrix}.
\end{align*}
\]

(11)

In this case:

\[
\begin{align*}
M_0^2 &= \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & \mu^\dagger \mu & 0 & \xi \lambda^\dagger \\
0 & 0 & 0 & 0 \\
0 & \xi \lambda & 0 & \mu \mu^\dagger
\end{pmatrix}, \\
M_{1/2}^2 &= \begin{pmatrix}
0 & 0 & 0 \\
0 & \mu^\dagger \mu & 0 \\
0 & 0 & 0 \\
0 & 0 & \mu \mu^\dagger
\end{pmatrix}.
\end{align*}
\]

(12)

The zero lines and columns correspond, in the bosonic part, to the complex, classically massless, $X$ direction and, in the fermionic part, to its superpartner the goldstino. They can be left out of the matrices, thus giving:

\[
\begin{align*}
M_0^2 &= \begin{pmatrix}
\mu^\dagger \mu & \xi \lambda^\dagger \\
\xi \lambda & \mu \mu^\dagger
\end{pmatrix}, \\
M_{1/2}^2 &= \begin{pmatrix}
\mu^\dagger \mu & 0 \\
0 & \mu \mu^\dagger
\end{pmatrix}.
\end{align*}
\]

(13)

Note that the couplings $\lambda_{ijk}$ play no role in the mass terms around this line of vacua. The background $\phi = 0$ is a vacuum only if the matrix $M_0^2$ is positive. This condition can be written:

\[
\forall \psi_1, \psi_2, \quad \|\mu \psi_1\|^2 + \|\mu^\dagger \psi_2\|^2 + 2 \Re (\psi_2^\dagger \lambda \psi_1) \geq 0.
\]

(14)

Suppose $\det \mu = 0$: as a symmetrical matrix, $\mu$ can be written in a certain basis of the fields:

\[
\mu = \begin{pmatrix}
\bar{\mu} & 0 \\
0 & 0
\end{pmatrix},
\]

(15)
with \( \det \tilde{\mu} \neq 0 \). Then, with
\[
\lambda = \begin{pmatrix} \tilde{\lambda} & \Lambda \\ \bar{t} \Lambda & \tilde{\lambda}^\prime \end{pmatrix}, \quad \psi = \begin{pmatrix} \tilde{\psi} \\ \tilde{\psi}^\prime \end{pmatrix},
\]
the positivity condition becomes:
\[
2\xi \Re \left( \tilde{\psi}^\dagger_2 \tilde{\lambda} \tilde{\psi}_1 + \tilde{\psi}^\dagger_2 \bar{t} \Lambda \tilde{\psi}_1 + \tilde{\psi}^\dagger_2 \Lambda \tilde{\psi}_1^\prime + \tilde{\psi}^\dagger_2 \tilde{\lambda}^\prime \tilde{\psi}_1^\prime \right) + \| \tilde{\psi}_1 \|^2 + \| \tilde{\psi}_2 \|^2 \geq 0. \tag{17}
\]
This implies \( \tilde{\lambda}^\prime = 0 = \Lambda \): the primed directions are massless and can be removed from the calculation. We shall then consider that \( \det \mu \neq 0 \). The positivity condition is then equivalent to:
\[
\forall \psi_1, \psi_2, \| \psi_1 \|^2 + \| \psi_2 \|^2 + 2\xi \Re \left( \psi^\dagger_2 U \psi_1 \right) \geq 0, \tag{18}
\]
with \( U \equiv \mu^{-1} \lambda \mu^{-1} \). Taking an eigenvalue \( u \) of \( U \) and a corresponding eigenvector \( U \psi_1 = u \psi_1 \) with \( \| \psi_1 \| = 1 \), then choosing \( u^* \psi_2 = -|u| \psi_1 \), we find:
\[
1 - \xi |u| \geq 0. \tag{19}
\]
To put it in words, the positivity condition implies that all eigenvalues of \( U \) have modules inferior to \( \xi^{-1} \):
\[
\det \left( \mu^{-1} \lambda \mu^{-1} - u \right) = 0 \Rightarrow |u| \leq \xi^{-1}, \tag{20}
\]
or equivalently
\[
0 < |v| < \xi \Rightarrow \det \left( \mu^2 - v \lambda \right) \neq 0. \tag{21}
\]

**Tree-level stability**

In order for the vacuum to be effectively stable, the matrix \( M_0^2 \) has to be positive on the whole complex line \( X \). The coupling \( \mu(X) \) is equal to \( \mu + \lambda X \), so that, using condition (21) for stability, we find:
\[
0 < |v| < \xi \Rightarrow \det \left[ (\mu + X \lambda)^2 - v \lambda \right] \neq 0 \quad \forall X. \tag{22}
\]
This is a strong condition since the determinant, being a holomorphic function of \( X \), has no complex root and is thus a constant:

\[
0 < |v| < \xi \Rightarrow \partial \det \left[ (\mu + X\lambda)^2 - v\lambda \right] = 0
\]  

(23)

for all \( X \) once again, where \( \partial \) stands for the derivative with respect to \( X \). As this is a holomorphic function of \( v \), it must be zero for all \( v \), so that \( \det((\mu + X\lambda)^2 - v\lambda) \) is only a function of \( v \), from which we deduce that the eigenvalues of \( \mu^{-1}\lambda\mu^{-1} \) are the same all along the complex line \( X \).

Expanding equation (23) in powers of \( v \), we find, for all \( n \geq 0 \) and for all \( X \):

\[
\text{tr}\{[(\mu + X\lambda)^{-2}\lambda]^n(\mu + X\lambda)^{-1}\lambda}\} = 0.
\]  

(24)

This is not an obviously solvable condition although, for \( n = 0 \), it is equivalent to say that \( \mu^{-1}\lambda \) is nilpotent. We shall solve it in the following simple case.

*Renormalizable three-field model*

If there are only three superfields fields \( X, \phi^1, \phi^2 \), the matrices in question are \( 2 \times 2 \) and the solutions to the nilpotence condition can be written, in a certain basis:

\[
\mu = \begin{pmatrix} \mu' & \mu \\ \mu & 0 \end{pmatrix}, \quad \lambda = \begin{pmatrix} \lambda & 0 \\ 0 & 0 \end{pmatrix}.
\]  

(25)

\( \mu' \) can even be set to zero by a shift of \( X \), and a phase shift of \( \phi_1 \) and \( \phi_2 \) can be used to make \( \mu \) and \( \lambda \) real positive. Condition (22) then gives:

\[
0 < |v| < \xi \Rightarrow \mu^2(\mu^2 - v\lambda) \neq 0,
\]  

(26)

i.e. \( \mu^2 \geq \xi\lambda \). The general three-field tree-level stable renormalizable superpotential with a supersymmetry-breaking vacuum is then:

\[
W = \xi X + \mu\phi^1\phi^2 + \frac{1}{2}\lambda X(\phi^1)^2 + \frac{1}{6}\lambda_{ijk}\phi^i\phi^j\phi^k,
\]  

(27)

with \( i, j, k = 1, 2 \). This is none other than the usual Ó Raifeartaigh model with an additional \( \lambda_{ijk} \) interaction term that is irrelevant for mass calculation. The masses in the background \( \phi^i = 0 \), apart from the zero-mass complex particle corresponding to the flat \( X \) direction and from the goldstino, can then be calculated from the mass matrix:
\[ m_B^2 = \frac{1}{2} \left[ 2\mu^2 + \lambda^2 |X|^2 + \epsilon\xi \lambda \right] \]

\[ \pm \sqrt{\left( \lambda^2 |X|^2 + \epsilon\xi \lambda \right)^2 + 4\mu^2 \lambda^2 |X|^2}, \] \hspace{1cm} (28)

\[ m_F^2 = \frac{1}{2} \left[ 2\mu^2 + \lambda^2 |X|^2 \right] \]

\[ \pm \sqrt{\lambda^4 |X|^4 + 4\mu^2 \lambda^2 |X|^2}, \] \hspace{1cm} (29)

where \( \epsilon^2 = 1 \). These masses are all positive given the condition \( \mu^2 \geq \xi\lambda \).

**One-loop stability**

The stabilization of the pseudo-modulus by one-loop potential lifting is a well-known result, which can be found for instance in appendix A of [1]. We recall here the main lines of the calculation, with an additional check that the potential is non-tachyonic at infinity. The one-loop correction to the vacuum energy is given in model (27) by:

\[ V_{\text{eff}} = \frac{1}{64\pi^2} \text{Str} \left( M^4 \ln \frac{M^2}{\Lambda^2} \right). \] \hspace{1cm} (30)

That energy, once again, does not depend on the \( \lambda_{ijk} \) couplings. It is not \textit{a priori} independent from the modulus \( X \) and thus generates a correction potential along that direction. This could make the vacuum instable if the potential develops tachyonic directions. But if the correction is positive for \( |X| \to \infty \), then there exists at least one potential minimum that will be a (meta)stable vacuum.

In the much constrained model considered above, the potential for \( |X| \to \infty \) is easily calculated since we have the expressions (28) and (29) for the masses. These expressions, for the plus sign of \( \pm \), give:

\[ m^4 \ln \frac{m^2}{\Lambda^2} \simeq \left[ \lambda^4 |X|^4 + 2\lambda^2 |X|^2 (2\mu^2 + \epsilon\xi \lambda) + 2\mu^4 + 2\epsilon \mu^2 \xi \lambda + \epsilon^2 \xi^2 \lambda^2 \right] \]

\[ \times \left[ \ln \frac{\lambda^2 |X|^2}{\Lambda^2} + \frac{2\mu^2 + \epsilon\xi \lambda}{\lambda^2 |X|^2} - \frac{6\mu^4 + 6\epsilon \mu^2 \xi \lambda + \epsilon^2 \xi^2 \lambda^2}{2\lambda^4 |X|^4} \right], \] \hspace{1cm} (31)

where \( \epsilon = \pm 1 \) for bosons and \( \epsilon = 0 \) for fermions. Thus only \( \epsilon^2 \) and upper terms contribute (as 2) to the supertrace. The dominant term for the plus contributions to the supertrace is therefore \( 2\xi^2 \lambda^2 \ln |X|^2 \). As for the minus
terms, they are of order \( |X|^2 \) and therefore negligible, so that:

\[
V_{\text{eff}} \sim \frac{2\xi^2\lambda^2}{64\pi^2} \ln |X|^2.
\] (32)

As this is positive for \( |X| \to \infty \) and as the potential is everywhere well defined, it must admit one or several minima, so that there exists (meta)stable vacua with no tachyon at one loop\(^1\).

In fact, \( X = 0 \) is such a minimum: expanding the masses around \( X = 0 \), the potential reads:

\[
V_{\text{eff}} \simeq V_0(\Lambda) + \frac{\lambda^2\mu^2|X|^2}{32\pi^2} F(x) + O(|X|^4),
\] (33)

\[
F(x) \equiv \frac{1 + x^2}{x} \ln \left( \frac{1 + x}{1 - x} \right) + 2 \ln(1 - x^2) - 2,
\] (34)

with \( x \equiv \lambda \xi / \mu^2 \leq 1 \). As the bracketed function of \( x \) —call it \( F(x) \)— is always positive, with \( F(0) = 0 \) and \( F(1) = 4 \ln 2 \), the model has a (meta)stable vacuum at \( X = 0 \). The bosonic masses at that point are \( \mu^2 \) (complex), \( \mu^2(1 \pm x) \) (real) and \( \mu^2\lambda^2 F(x) / 32\pi^2 \) (complex, one-loop light mass); the fermionic masses are \( \mu^2 \) (two Weyl spinors) and of course exactly zero for the Goldstino.

**Meta-stability and supersymmetric vacua**

After studying the local behaviour of the system around the supersymmetry-breaking vacuum, we shall now give some hints of possible non-perturbative effects due to other vacua. In the general model (10) as well as in the three-field model (27), the vacuum is not generically unique, and in particular, there exists generically a supersymmetric vacuum, except, for instance, if we impose some global symmetry on the model \[8\]. The vacuum will then be meta-stable, tunnelling towards a more stable, generally supersymmetric, vacuum.

For simplicity, let us study the Ó Raifeartaigh-like model. Besides the studied vacuum, there will generically be four supersymmetric vacua; in special cases there can be less of them (three or two) or even a whole complex line of degenerate supersymmetric vacua. In other special cases, as the original

\(^1\) Note that such a vacuum might exist without need of tree-level stability on the *whole* complex plane of the pseudo-modulus—a sufficient condition would be tree-level stability in a region around the vacuum; however, after fruitlessly searching for a counter-example, we conjecture that, in renormalizable models, local one-loop stability is only achieved in models with global tree-level stability.
Ó Raifeartaigh model, there will not be any possible supersymmetric vacuum, but several supersymmetry-breaking vacua.

Let us present a simple model of meta-stable supersymmetry-breaking vacuum tunnelling to a supersymmetric vacuum. We shall change variables for simplicity and write it:

\[
W = h \left[ \Phi^2 \phi_1 - m \Phi (\phi_1 + \alpha \phi_2) \right],
\]

(35)

with \(\alpha^2 < 1/8\). This model admits a degenerate supersymmetric vacuum at \(\Phi = 0, \phi_1 + \alpha \phi_2\), and a meta-stable supersymmetry-breaking vacuum at \(\Phi = (3 + \sqrt{1 - 8 \alpha^2})/4 \times m, (m - 2 \Phi) \phi_1 + \alpha m \phi_2 = 0\). The latter is stabilized at one loop for \(\phi_1 = \phi_2 = 0\).

The lifetime of the meta-stable vacuum can be easily evaluated: as the model has a \(U(1)_R\) symmetry under which \(\Phi\) is neutral and \(\phi_1\) and \(\phi_2\) have charge 2, the plane \(\phi_1 = \phi_2 = 0\) is stable under the equations of motion and the least potential barrier path will involve only \(\Phi\) changes. Moreover, for \(\phi_1 = \phi_2 = 0\), the potential is invariant under \(\Phi \rightarrow \Phi^*\), so that, as the meta-stable value of the field is real, it remains so during the bounce. One can then write the potential as a function of a real \(\Phi\):

\[
V(\Phi) = h^2 \left[ \Phi^4 - 2m \Phi^3 + m^2 (1 + \alpha^2) \Phi^2 \right].
\]

(36)

We can then, for small \(\alpha\), use the known results on the behaviour of false vacua [9] in the thin-wall approximation, for which the energy density difference between the two vacua, here \(h^2 m^4 \alpha^2 + O(\alpha^4)\), is small. At leading order in \(\alpha^2\), the probability of tunnelling per unit time per unit volume then reads:

\[
\Gamma/V \propto \exp \left[ -\frac{\pi^2}{24 h^2 \alpha^6} \right],
\]

(37)

with an average radius of the tunnelling region:

\[
\bar{\rho} = \frac{1}{\sqrt{2hm\alpha^2}}.
\]

(38)

The lifetime of the meta-stable, supersymmetry-breaking vacuum is thus parametrically great in the limit where field \(\phi_2\) decouples and the supersymmetry-breaking scale is small.
2 Non-renormalizable generalizations

We shall now try to extend the previous results to non-renormalizable models, i.e. higher-order superpotentials and non-canonical Kähler potentials.

Degeneracy for canonical Kähler

The degeneracy theorem for renormalizable models is easily extended to general superpotentials, provided we keep a canonical Kähler potential: it can be shown that a non-zero minimum of a potential of the form $V = |\partial W|^2$ is always perturbatively degenerate.

We shall then use a recurrence to show that the potential at supersymmetry-breaking vacuum point is flat at all orders in the $\partial W^\dagger$ direction. Let us use the convention $A_k \equiv (\partial W^\dagger)^k \partial^{k+1} W$, where $k$ of the indices of the multiple derivative are contracted with the $k$ simple derivatives. The vacuum conditions can then be written $A_0 \neq 0, A_1 = 0$.

Let us now suppose, as a recurrence condition, that for some non-zero integer $n$, $A_k = 0 \forall 1 \leq k \leq n$. Let us then consider a variation of the fields $\phi^i$ around the vacuum $\delta \phi^i = \partial^i W^\dagger \delta z + \varphi^i \delta z^{n+1}$, with $\varphi^i \partial_i W = 0$. The leading term of the variation of $V$ for small $\delta z$ must be positive whatever the choice of the direction $\varphi^i$.

For $1 \leq k \leq n$, the $k$-th order of variation of $V$ in $\delta z$ reads:

$$\delta^k V = \sum_{i=0}^k \frac{\delta z^i \delta z^{k-i}}{i! (k-i)!} A_{k-i}^\dagger A_i = 0$$  \hfill (39)

by recurrence condition. Furthermore, for $0 \leq k \leq n$, the $(n + k + 1)$-th order reads:

$$\delta^{n+k+1} V = \sum_{i=0}^{n+k+1} \frac{\delta z^i \delta z^{n+k-i+1}}{i! (n+k-i+1)!} A_{n+k-i+1}^\dagger A_i$$

$$+ 2 \Re \left\{ \sum_{i=0}^k \frac{\delta z^{n+i+1} \delta z^{k-i}}{i! (k-i)!} A_k^\dagger \left[ \varphi (\partial W^\dagger)^i \partial^{i+2} W \right] \right\}$$

$$= 2 \Re \left\{ \delta z^{n+k+1} \left[ \frac{1}{(n+k+1)!} A_0^\dagger A_{n+k+1} + \frac{1}{k!} \varphi A_{k+1} \right] \right\}. \hfill (41)$$

These terms must all be zero since, if one of them were not, the leading order in $\delta z$ would be of the form $\Re (\delta z^{n+k+1})$, which takes negative values for some
\[ \frac{1}{(n + k + 1)!} A_0^\dagger A_{n+k+1} = -\frac{1}{k!} \varphi A_{k+1}. \]  

(42)

For \( k = 0 \), this gives us \( A_0^\dagger A_{n+1} = 0 \) and, since the equation must hold for every initial choice of the direction \( \varphi \), it yields, for \( k = n \), \( \varphi A_{n+1} = 0 \). From these two results we finally conclude that \( A_{n+1} = 0 \): the recurrence condition is verified one step further. An additional result, if we take \( \varphi = 0 \), is that the potential in the \( \partial W^\dagger \) direction is flat up to order \( 2n + 1 \). As the recurrence condition is true for \( n = 1 \), it is true for all \( n \), and the potential is flat at all orders.

Thus for a canonical Kähler potential and an analytic superpotential, a supersymmetry-breaking vacuum is always degenerate since for any complex \( z \), \( V(\phi_0 + z\partial W^\dagger) = V(\phi_0) \).

**Non-canonical Kähler potentials**

That theorem only holds for a canonical Kähler potential: for a generic Kähler potential, the vacuum need not be degenerate at all, as is obvious from the following one-superfield counter-example:

\[ K = \phi^\dagger \phi - \frac{1}{4m^2} (\phi^\dagger \phi)^2, \]

\[ W = h \left[ \frac{\alpha^2 (3 - \alpha^2)}{2} m^2 \phi^2 - \alpha \phi^3 + \frac{1}{4m} \phi^4 \right], \]

(43)

(44)

where \( \alpha < 1 \). The lagrangian density for the scalar part of this theory is:

\[ L = \left( 1 - \frac{1}{m^2} |\phi|^2 \right) \partial_\mu \phi^\dagger \partial^\mu \phi - \frac{h^2 |\phi|^2}{m^2 - |\phi|^2} \left( |\phi|^2 - 3 \alpha m \phi + \alpha^2 (3 - \alpha^2) m^2 \right)^2. \]

(45)

There is a supersymmetric vacuum at \( \phi = 0 \) and a meta-stable, non-denegerate supersymmetry-breaking vacuum at \( \phi = \alpha m \), the mass of the scalar (complex) particle around that vacuum being \( h\alpha^3 m / (1 - \alpha^2) \): there is no pseudo-modulus even at tree level. As \( \alpha \to 0 \), the meta-stable vacuum becomes long-lived, according to the thin-wall approximation model.

Non-renormalizable models are therefore far less constrained as regards the properties of their vacua and it seems difficult to characterize them by generic
features. Long-lived meta-stable vacua are still easily found, as the previous example shows, but still require some fine tuning in the couplings.

Conclusion

This paper aimed to be a modest exploration of the properties of meta-stable supersymmetry breaking in non-gauged Wess-Zumino-like theories—degeneracy and modulus stabilization. In order for this type of $F$-term breaking to be transplanted in a realistic theory, the essential problem would be the one-loop light mass of the pseudo-modulus, yielding an unobserved light scalar in addition to the generic massless fermion.

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