Timing evolution of accreting strange stars

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Abstract

It has been suggested that the QPO phenomenon in LMXB’s could be explained when the central compact object is a strange star. In this work we investigate within a standard model for disk accretion whether the observed clustering of spin frequencies in a narrow band is in accordance with this hypothesis. We show that frequency clustering occurs for accreting strange stars when typical values of the parameters of magnetic field initial strength and decay time, accretion rate are chosen. In contrast to hybrid star accretion no mass clustering effect is found.

PACS number(s): 04.40.Dg, 12.38.Mh, 26.60.+c, 97.60.Gb

Key words: accretion, accretion discs - stars: interiors - stars: magnetic fields - pulsars: general - X-rays: binaries

\footnotesize{1 Supported by DFG grant No 436 ARM 17/7/00
2 Supported by DAAD}
1 Introduction

The recent discovery of new phenomena in the physics of X-ray binaries is giving a new and powerful tool to probe dense hadronic matter (Miller et al., 1998) and possibly to infer the existence of a deconfined phase of quark matter in the cores of the central accretors in these systems. Various signals of a phase transition to deconfined quark matter have been suggested in the form of peculiar changes of observables, such as the pulse timing (Glendenning & Weber, 2001; Poghosyan et al., 2001), and the thermal evolution of isolated pulsars (Schaab et al., 1997; Page et al., 2000b; Blaschke et al., 2001). Even more intriguing than the existence of a quark core in a neutron star, is the possible existence of a new family of compact stars consisting completely of a deconfined mixture of up (u), down (d), and strange (s) quarks, together with an appropriate number of electrons to guarantee electrical neutrality. In the literature, such compact stars have been referred to as strange quark stars or shortly strange stars (SS), and their constituent matter as strange quark matter (SQM). In a series of recent papers (Dey et al., 1998; Li et al., 1999) it has been argued that the compact objects in some X-ray sources are likely strange star candidates. Particularly interesting candidates are the compact stars in the newly discovered millisecond X-ray pulsar SAX J1808.4–3658, and in the atoll source 4U 1728–34.

In the present paper, we model the spin evolution of an accreting strange star in a binary stellar system, from the so called “death line” up to the millisecond pulsar phase. We explore the dependence of the SS spin evolution upon the mass accretion rate and upon those physical quantities which regulate the temporal evolution of the torque acting on the spinning star. We try to constrain these quantities to have a population clustering of strange stars in agreement with the spin frequency distribution for observed Z sources in LMXBs with kHz QPOs. Possible signatures to distinguish strange stars from ordinary neutron stars are also briefly discussed.

2 Accretion model and magnetic field evolution

We consider the spin evolution of a strange star under mass accretion from a low-mass companion star as a sequence of stationary states of configurations (points) in the phase diagram spanned by the angular velocity $\Omega$ and the baryon number $N$. The process is governed by the change in angular momentum $J(N, \Omega) = I(N, \Omega) \Omega$

$$\frac{d}{dt}(J(N, \Omega)) = K_{\text{ext}}, \quad (1)$$
where $I(N, \Omega)$ is the moment of inertia of the star and

$$K_{\text{ext}} = \sqrt{GM\dot{M}r_0} - N_{\text{out}}$$

is the external torque due to both the specific angular momentum transferred by the accreting plasma and the magnetic plus viscous stress given by $N_{\text{out}} = \kappa \mu^2 r_c^{-3}$, $\kappa = 1/3$ [Lipunov, 1992]. For a star with radius $R$ and magnetic field strength $B$, the magnetic moment is given by $\mu = R^3 B$. The co-rotating radius $r_c = (GM/\Omega^2)^{1/3}$ is very large ($r_c \gg r_0$) for slow rotators. The inner radius of the accretion disc is

$$r_0 \approx \left\{ \begin{array}{ll}
R, & \mu < \mu_c \\
0.52 r_A, & \mu \geq \mu_c
\end{array} \right.$$  

where $\mu_c$ is that value of the magnetic moment of the star for which the disc would touch the star surface. The characteristic Alfvén radius for spherical accretion with the rate $\dot{M} = mN$ is $r_A = (2\mu^{-4}GM\dot{M})^{-1/7}$ [Bhattacharya & van den Heuvel, 1991]. Since we are interested in the case of fast rotation for which the spin-up torque due to the accreting plasma in Eq. (2) is partly compensated by $N_{\text{out}}$, eventually leading to a saturation of the spin-up, we neglect the spin-up torque in $N_{\text{out}}$ which can be important only for slow rotators [Ghosh & Lamb, 1979].

From Eqs. (1), (2) one can obtain a first order differential equation for the evolution of angular velocity

$$\frac{d\Omega}{dt} = \frac{K_{\text{ext}}(N, \Omega) - K_{\text{int}}(N, \Omega)}{I(N, \Omega) + \Omega(\partial I(N, \Omega)/\partial \Omega)_N}.$$  

where the internal torque term defined as

$$K_{\text{int}}(N, \Omega) = \Omega \dot{N} \left( \frac{\partial I(N, \Omega)}{\partial N} \right)_\Omega.$$  

Solutions of (3) are trajectories in the $\Omega - N$ plane describing the spin evolution of accreting compact stars. Since for the hybrid stars $I(N, \Omega)$ exhibits characteristic functional dependences [Blaschke et al., 2001] at the deconfinement phase transition line $N_{\text{crit}}(\Omega)$ we expect observable consequences in the $\dot{P} - P$ plane when this line is crossed. In the case of SS we have no expectation of the mass clustering due to absence of phase transition effects in strange matter.
In our model calculations we assume that both the mass accretion and the angular momentum transfer processes are slow enough to justify the assumption of quasistationary rigid rotation without convection. For a more detailed description of the method and analytic results we refer to Chubarian et al. (2000) and the works of Hartle & Thorne (1968), as well as Sedrakian & Chubarian (1968).

The time dependence of the baryon number for the constant accretion rate $\dot{\mathcal{N}}$ is given by

$$N(t) = N(t_0) + (t - t_0)\dot{\mathcal{N}}. \quad (5)$$

For the magnetic field of the accretors we consider the exponential decay (Bhattacharya & van den Heuvel, 1991)

$$B(t) = [B(0) - B_\infty] \exp(-t/\tau_B) + B_\infty. \quad (6)$$

We solve the equation for the spin-up evolution (3) of the accreting star for decay times $10^7 \leq \tau_B[\text{yr}] \leq 10^9$ and initial magnetic fields in the range $0.2 \leq B(0)[\text{TG}] \leq 4.0$. The remnant magnetic field is chosen to be $B_\infty = 10^{-4}\text{TG}$ (Page et al., 2000a).

At high rotation frequency, both the angular momentum transfer from accreting matter and the influence of magnetic fields can be small enough to let the evolution of angular velocity be determined by the dependence of the moment of inertia on the baryon number, i.e. on the total mass. This case is similar to the one with negligible magnetic field considered in (Shapiro & Teukolsky, 1983; Burderi et al., 1999; Chubarian et al., 2000), where $\mu \leq \mu_c$ in Eq. (3), so that only the so called internal torque term (4) remains.

### 3 Equation of state for strange quark matter

To describe the properties of strange quark matter, we used a recent model for the equation of state (EoS) derived by Dey et al. (1998). This model is based on a dynamical density-dependent approach to confinement. This EoS has asymptotic freedom built in, shows confinement at zero baryon density, and deconfinement at high density. In this model the quark interactions is described by a color-Debye-screened interquark potential originating from gluon exchange, and by a density-dependent scalar potential which restores chiral symmetry at high density (in the limit of massless quarks). This density-dependent scalar

$1 \ \text{TG} = 10^{12} \ \text{G}$
Table 1

Ground state properties of SQM for the equations of state used in the present work. $(E/A)_{gs}$ (in MeV) is the energy per baryon, $\rho_{gs}$ ($\times 10^{14}$ g/cm$^3$) the mass density, and $n_{gs}$ (fm$^{-3}$) the baryon number density. The two equations of state SS1 and SS2 differ for the choice of the parameter $\nu$ entering in the expression of the in-medium quark masses.

| EoS     | $(E/A)_{gs}$ | $\rho_{gs}$ | $n_{gs}$  |
|---------|--------------|-------------|-----------|
| SS1     | 888          | 12.3        | 0.779     |
| SS2     | 926          | 14.1        | 0.858     |
| B60$_{150}$ | 836        | 4.6         | 0.295     |

Potential arises from the density dependence of the in-medium effective quark masses $M_q$, which in the model of Dey et al. (1998) are taken to depend on the baryon number density $n_B$ according to

$$M_q(n_B) = m_q + 310 \text{ (MeV) sech} \left( \frac{\nu n_B}{n_0} \right),$$

(7)

where $q(= u, d, s)$ is the quark flavor index, $n_0 = 0.16$ fm$^{-3}$ is the normal nuclear matter density, and $\nu$ is a parameter. The effective quark mass $M_q(n_B)$ goes from its constituent mass value at zero density, to its current mass $m_q$ as $n_B$ goes to infinity. Here we consider two different parameterizations of the EoS, which correspond to a different choice for the parameter $\nu$. The equation of state SS1 (SS2) corresponds to $\nu = 0.333$ ($\nu = 0.286$). These two models for the EoS give absolutely stable SQM according to the strange matter hypothesis (Bodmer, 1971; Witten, 1984), see Tab. 1.

Medium dependent mechanisms for confinement and their consequences for the EoS of quark matter, have been explored by many authors using different QCD motivated phenomenological models (Blaschke et al., 1990, 1999; Blaschke & Tandy, 2000; Drago et al., 1999). In addition to the previous model for the EoS, we make use of the MIT bag model EoS (Farhi & Jaffe, 1984) for strange quark matter for non-interacting quarks, with strange quark mass $m_s = 150$ MeV, massless $u$ and $d$ quarks, and with bag constant $B = 60$ MeV/fm$^3$ (hereafter the B60$_{150}$ EoS).

4 Results and discussion

In Fig. 1 we show evolutionary paths of accreting strange stars in the mass-radius (MR) plane (top panels) corresponding phase diagrams (lower panels) for two different EoS (SS1 right panels and B60$_{150}$ left panels). In each panel we show two trajectories of a strange star initially rotating with frequency
\( \Omega(0) = 0.001 \text{ Hz}; \) for which the initial magnetic field is \( B(0) = 7 \text{ TG} \) and its decay time is \( \tau_B = 10^8 \text{ yr} \). The mass accretion rate onto the star is \( \dot{M} = 10^{-9} M_\odot/\text{yr} \). The solid lines show the evolution of strange star configurations with initial gravitational mass \( M(0) = 1.4M_\odot \); the dashed lines show that of configurations with initial baryon mass \( N(0) = 1.4N_\odot \). In the insets of Fig. 1 we show the stable branches for configurations rotating with maximal frequency (dash-dotted lines) in compare to static ones (dotted lines).

![Graphs showing evolutionary paths for strange stars](image)

**Fig. 1.** Evolutionary paths for strange stars in the mass-radius plane (top panels) and in the frequency-baryon number plane (lower panels) for the equations of state SS1 (left panels) and B60\textsubscript{150} (right panels), see text.

From the evolution paths \( \Omega(N) \) one can see that in all cases of initial masses the magnetic braking force is strong enough to stop fast spin-up of the star and saturate the frequency of rotation. This can lead to an effect of frequency clustering. However such effect for strange stars is correlated with the accretion rate.
In Fig. 2, we show the result of our calculation for the spin evolution of the accreting strange star with EoS SS1 in the $\dot{P} - P$ diagram, where $P = 2\pi/\Omega$ is the period of rotation. The parameters of the accretion model are chosen such as to correspond to values extracted from observations made on LMXBs, which are divided into Z sources with $\dot{M} \sim 10^{-8} M_{\odot}/yr$ and A(toll) sources with $\dot{M} \sim 10^{-10} M_{\odot}/yr$ (Bhattacharya & van den Heuvel, 1991; van der Klis, 2000; Glendenning & Weber, 2001).

In Fig. 2, $\dot{P} - P$ diagrams for accreting strange stars with equation of state SS1. We show the dependence on the initial magnetic field $B(0)$ (upper panel), mass accretion rate $\dot{M}$ (lower panel), and magnetic field decay time $\tau_B$ (middle panel).

In Fig. 2 we explore the sensitivity of the model to changes of the parameters with respect to the set used in Fig. 1 ($B(0) = 7$ TG, $\tau_B = 10^8$ yr, $\dot{M} = 10^{-8} M_{\odot}/yr$) for accretors with initial baryon mass $N(0) = 1.4 N_{\odot}$. In the upper panel we vary the initial magnetic field $B(0) = 5, 7, 8$ TG, in the middle one the magnetic field decay time $\tau_B = 10^7, 10^8, 10^9$ yr and in the lower panel the accretion rate $\dot{M} = 10^{-11}, 10^{-9}, 10^{-8} M_{\odot}/yr$.

We see from this figure that changes of the accretion parameters not only shift (choice of the initial magnetic field) and deform (choice of the magnetic field decay time) the interval where the spin-up is saturated (dip in $\dot{P}$), but for some cases this effect can be washed out by a variation of the accretion
rate. This phenomenon means that the existence of a frequency clustering for strange stars requires a strong limitation of possible values of the accretion parameters.

In Fig. 3 we plot the 'Waiting time' \( \tau = |P/\dot{P}| \) (Poghosyan et al., 2001) of the strange star as a function of the spin frequency, for the EsoS SS1 and B60\(_{150}\). In order to obtain an enhanced waiting time (frequency clustering) in the interval \( 220 \text{ Hz} < \nu < 380 \text{ Hz} \) which corresponds to recent observations (Glendenning & Weber, 2001), we have to choose the following parameters: \( B(0) = 2.5 \text{TG} \), \( \tau_B = 10^8 \text{ yr} \), \( \dot{M} = 10^{-9} M_\odot/\text{yr} \) for SS1 model and \( B(0) = 3 \text{TG} \), \( \tau_B = 10^8 \text{ yr} \), \( \dot{M} = 10^{-8} M_\odot/\text{yr} \) for B60\(_{150}\) model. For both cases the initial gravitational mass is the same \( 1.4 M_\odot \), which corresponds to \( 1.83 N_\odot \) initial baryon mass for SS1 and \( 1.71 N_\odot \) for B60\(_{150}\) model. The main difference between both of these scenarios for the frequency clustering is the initial baryon mass and the mass accretion rate. An independent determination of these quantities to a sufficient accuracy could thus rule out one of the compact star models. At present, this could be done only for a minority of objects (Lamb & Miller, 2001) and bears some model dependence. (Poghosyan et al., 2001).

Fig. 3. Waiting times as a function of spin frequency for spin-up evolution of strange stars with equations of state B60\(_{150}\) and SS1.
5 Conclusions

We have investigated the question whether the clustering of spin frequencies which has been observed for the compact objects in LMXBs is consistent with the hypothesis that at least some of these objects are strange stars. We have applied a standard model of magnetic disc accretion and find that population clustering in a narrow band of spin frequencies between 220 ≤ ν[Hz] ≤ 380 can occur for typical parameter values of the model. Inspection of the response to parameter variations shows that the lowering of the magnetic field decay time can wash out the effect as well as a change in the accretion rate. The changes in the initial magnetic field leave the waiting time distribution rather unchanged but shift the interval of period clustering.

On the other hand, the observation of frequency clustering alone is no indication for the presence of a strange star. A standard bag model EoS shows a similar waiting time pattern as the strange quark matter EoS does. For the EoS investigated in this paper no clustering of masses has been obtained. This is a striking difference to hybrid star configurations [Poghosyan et al., 2001], which could be used in order to constrain further our approaches to the EoS of superdense matter provided a sufficiently large sample of LMXBs could be observed and their frequencies and masses could both be extracted with sufficient accuracy.

Acknowledgement

I.B., H.G. and G.P. acknowledge the hospitality of Rostock University. This work was supported in part by the DEUTSCHE FORSCHUNGSGEMEINSCHAFT (DFG) under Grants No. 436 ARM 17/7/00, 436 ARM 17/5/01, by the GRADUIERTENKOLLEG “Stark korrelierte Vielteilchensysteme” and by the DEUTSCHER AKADEMISCHER AUSTAUSCHDIENST (DAAD). We thank D. Aguilera for discussions and careful reading of the manuscript.

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