Adaptive Control and Multi-variables Projective Synchronization of Hyperchaotic Finance System

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Abstract. This paper introduces a new hyperchaotic finance system and show that it is a self-excited hyperchaotic attractor with the coexistence of double attractor with difference initial conditions for the same set of parameter values. Nonlinear feedback control function $u_i(t)$, $(i = 1,2,3,4)$ via adaptive control laws are design for the control and synchronization of the 4D-hyperchaotic finance system with the multiple values of the economics scaling factor $\alpha_i$, $(i = 1,2,3,4)$, a special case of projective synchronization. The proposed schemes are able to stabilize as well as globally synchronize the two identical finance systems evolving from different initial conditions with uncertain parameter for the different values of $\alpha_i$, $(i = 1,2,3,4)$. The analytical results were verified through numerical simulation.

1. Introduction

Chaos phenomenon is an interesting subject of discuss among the researchers of different fields today due to it wide applications in biological, physical, chemical, and financial systems as well as electronic engineering, secure communication, signal processing, fibre optical communication scheme [1-5].

For economics systems, the chaotic behavior was found in 1985 [6], since then researches on the complicated economics system via nonlinear method have been successful [7-8].

Lyapunov exponent is a critical prerequisite use to distinguish the 3D chaotic system from a hyperchaotic system. The 3D chaotic system has only one positive Lyapunov exponent; thus it is difficult to meet some certain requirement in practical application such as in secure communication [9-10]. But hyperchaotic system has at least two positive Lyapunov exponents thus has more prominent advantage over the 3D chaotic system due to its higher dimensions and more unpredictable behaviors [11-12].
In order to achieve financial control and synchronization of both developed and developing countries and different areas, we need to solve more problems of high order finance chaotic systems. Different approaches have been reported in the literature for chaos control and synchronization such as; active control [13], adaptive control [14], backstepping technique [15]-[16], sliding mode control [17-18]. The methods can be used for systems with either known or unknown parameters.

Multistability is a system property which refers to systems that neither stable nor totally instable, but that alternate between two or more mutually exclusive states (attractors) over time [19]. Multistability, namely, the coexistence of different attractors is a feature of many nonlinear dynamical models, ranging from the geophysical fluid dynamics to model of finance in economics, together with engineering applications. Multistability depends on the choice of initial conditions, as well as small changes in parameter, so that a sudden transition can occur to a different attractor [20].

Since the finance chaotic and hyperchaotic systems were proposed, many works has been done on them (see the references therein). However, there are some limitations in the existing results. For instance, in the stabilization problem of such systems, the designed control functions \( u_i(t) \) are too complex to use in practical application. On the other hand, the coexistence (multistability) attractors with different initial conditions for the same set of parameter values have not been investigated.

Motivating by the above limitations, the present work address the problems by investigated the multistability of the system, reducing the controller complexity using the scaling factor \( \alpha \), which could be varies in order to achieve a stable economic growth. The numerical simulation results are presented to validate effectiveness of the designed controller.

2. Mathematical Formulation

In constructing the hyperchaotic finance system model, four factors are involves the: money, production, labor force and stock, and are expressed by four-order nonlinear ordinary differential equations shown in system (1).

The mathematical model considered in this paper is 4D hyperchaotic finance system as described in equation (1).

The system describes the time variations of the state variables: the interest rate \( x \), the investment demand \( y \), the price exponent \( z \) and the average profit margin \( w \).

\[
\begin{align*}
\dot{x} &= -a(x + y) + w \\
\dot{y} &= -y - axz \\
\dot{z} &= b + axy \\
\dot{w} &= -c xz - d w
\end{align*}
\]

Where \( a \) is the saving, \( b \) is the per investment cost, \( c \) is the elasticity of demands of commercials and \( d \) is the governing parameter of the system.

System (1) exhibit complex hyperchaotic behavior with the real constant parameter values \( a = 3.0 \), \( b = 15.0 \), \( c = 0.5 \) and \( d = 0.12 \) with the portrait of the phase attractor displayed in Figure 1.
3. Adaptive Control Technique

3.1 Design of the Adaptive control function $u_i(t)$

\[
\begin{align*}
\dot{x} &= -a(x + y) + w + u_1 \\
\dot{y} &= -y - axz + u_2 \\
\dot{z} &= b + axy + u_3 \\
\dot{w} &= -cxz - dw + u_4
\end{align*}
\]

Where $u_i(t)$, $(i = 1, 2, 3, 4)$ is the control input to design.

The result of the system (2) is validated using Lyapunov stability theory (LST) by construct a positive Lyapunov function $V(x, y, z, w)$ as follows:

\[
V = 0.5 \left( x^2 + y^2 + z^2 + w^2 + \tilde{a}\tilde{a} + \tilde{b}\tilde{b} + \tilde{c}\tilde{c} + \tilde{d}\tilde{d} \right)
\]

Where: $\tilde{a} = a - \bar{a}$, $\tilde{b} = b - \bar{b}$, $\tilde{c} = c - \bar{c}$ and $\tilde{d} = d - \bar{d}$ are the estimated values of the unknown parameter respectively.

The time derivative of equation (3) along the trajectories of the system (1) is:

\[
\dot{V} = x\dot{x} + y\dot{y} + z\dot{z} + w\dot{w} + \tilde{a}\tilde{a}\dot{\tilde{a}} + \tilde{b}\tilde{b}\dot{\tilde{b}} + \tilde{c}\tilde{c}\dot{\tilde{c}} + \tilde{d}\tilde{d}\dot{\tilde{d}}
\]

To ensure the convergences of the controller $u_i(t)$ $(i = 1, 2, 3, 4)$ to the origin asymptotically, the control function $u_i(t)$ is choose as follows;
\[ u_1 = a(x + y) - w - x \]
\[ u_2 = y + axz - y \]
\[ u_3 = -b - axy - z \]
\[ u_4 = cxz + dw - w \]

The substitution of equation (2) into the equation (3) yields the equation below.
\[
\dot{V} = x[-a(x + y) + w + u_1] + y[-y - axz + u_2] + z[b + axy + u_3] + \\
w[-cxz - dw + u_4] + \tilde{a}[-\tilde{a} - x^2 - xy - xyz] + \tilde{b}[-\tilde{b} + z] + \tilde{c}[-\tilde{c} - xzw] + \\
\tilde{d}[-\tilde{d} - w^2] 
\] (6)

From equation (6), the parameter update law is estimated in equation (7).
\[
\dot{\tilde{a}} = -(x^2 + xy + xyz) + xyz \\
\dot{\tilde{b}} = z \\
\dot{\tilde{c}} = -xzw \\
\dot{\tilde{d}} = -w^2
\] (7)

The substitution of equations (5) and (7) respectively into equation (4) result to equation (8)
\[
\dot{V} = -x^2 - y^2 - z^2 - w^2 - \tilde{a}^2 - \tilde{b}^2 - \tilde{c}^2 - \tilde{d}^2 < 0
\] (8)

Remarkably, \( V \) is a quadratic positive definite Lyapunov function and it time derivative \( \dot{V} \) is a quadratic negative definite function.

According to the Lyapunov stability theory (LST), the system (2) can converge to the unstable equilibrium \( E_0 \) with the control function \( u_i(t) \), \( i = 1,2,3,4 \) in equation (5) and the parameter update law in equation (7).

### 3.2 Numerical Simulation Results

In this section, fourth-order Runge-kutta algorithm is adopted with the following initial conditions of the state variables \( x(0) = 0.8 \), \( y(0) = 0.6 \), \( z(0) = -0.9 \) and \( w(0) = -0.8 \) with time grid of 0.001. Fixing the parameters values of \( a, b, c \) and \( d \) of the system as in figure 1 above, the system (2) is solved with the control input \( u_i(t) \), \( i = 1,2,3,4 \) as defined in equation (5).

The results display in figure 2 shows that the state variables move hyperchaotically when the control function \( u_i(t) \) are switched off and when the activated at \( t = 50 \), the state variables are converges asymptotically at the unstable equilibrium point \( E_0(0,0,0,0) \) according to the Lyapunov stability theory (LST).
Figure 2. Time responses of the state variables \((x, y, z, w)\) for hyperchaotic finance system when the controller is activated at \(t = 50\) to stabilize at the origin.
4. Multi-variable Projective Synchronization

4.1 Adaptive Synchronization with multiple values of $\alpha_i$

From system (1), we let: $x = x_1$, $y = x_2$, $z = x_3$ and $w = x_4$.

Hence:
\[
\begin{align*}
\dot{x}_1 &= -a(x_1 + x_2) + x_4 \\
\dot{x}_2 &= -x_2 - ax_1 x_3 \\
\dot{x}_3 &= b + ax_1 x_2 \\
\dot{x}_4 &= -cx_1 x_3 - dx_4 \\
\end{align*}
\]

Equation (9) is designated as the drive or master which will transmit the signal to the system (10) the error dynamical variables is obtained in equation (12).

\[
\begin{align*}
\dot{y}_1 &= -a(y_1 + y_2) + y_4 + \alpha_1 u_1 \\
\dot{y}_2 &= -y_2 - ay_1 y_3 + \alpha_2 u_1 \\
\dot{y}_3 &= b + ay_1 y_2 + \alpha_3 u_3 \\
\dot{y}_4 &= -cy_1 y_3 - dy_4 + \alpha_4 u_4 \\
\end{align*}
\]

Where $\alpha_i$ $(i = 1, 2, 3, 4)$ is the economic scaling factor to be estimated appropriately and $u_i(t)$ $(i = 1, 2, 3, 4)$ is the control function to be derives.

The error vector between the drive (master) in equation (9) and the response (slave) in equation (10) is defined as;
\[
e_t = y_t - x_t
\]

By subtracting equation (9) from equation (10) using the definition of the error vector in equation (10) the error dynamical variables is obtained in equation (12).

\[
\begin{align*}
\dot{e}_t &= -a(e_t + e_2) + e_4 + \alpha_1 u_1 \\
\dot{e}_2 &= -e_2 - a(x_t e_3 + x_3 e_1 + e_1 e_3) + \alpha_2 u_2 \\
\dot{e}_3 &= a(x_t e_2 + x_2 e_1 + e_1 e_2) + \alpha_3 u_3 \\
\dot{e}_4 &= -c(x_t e_3 + x_3 e_1 + e_1 e_3) - de_4 + \alpha_4 u_4 \\
\end{align*}
\]

We select the positive Lyapunov function $V(t)$ as;
\[
V = 0.5(e_1^2 + e_2^2 + e_3^2 + e_4^2 + \tilde{a}^2 + \tilde{b}^2 + \tilde{c}^2 + \tilde{d}^2)
\]

Where $\tilde{a} = a - \tilde{a}$, $\tilde{b} = b - \tilde{b}$, $\tilde{c} = c - \tilde{c}$ and $\tilde{d} = d - \tilde{d}$ are the estimate values of the unknown parameter $a$, $b$, $c$ and $d$ respectively.

The time derivative of equation (13) along the trajectories of equation (12) is given in equation (14).
\[
\dot{V} = e_1 \dot{e}_1 + e_2 \dot{e}_2 + e_3 \dot{e}_3 + e_4 \dot{e}_4 + \tilde{a} \dot{\tilde{a}} + \tilde{b} \dot{\tilde{b}} + \tilde{c} \dot{\tilde{c}} + \dot{\tilde{d}}
\]

The substitution of equation (12) into equation (14) is shown in equation (15).
\[
\begin{align*}
\dot{V} &= e_1[-a(e_1 + e_2) + e_4 + \alpha_1 u_1] + e_2[-\dot{e}_2 - a(x_t e_3 + x_3 e_1 + e_1 e_3) + \alpha_2 u_2] \\
&+ e_3[a(x_t e_2 + x_2 e_1 + e_1 e_2) + \alpha_3 u_3] + e_4[-c(x_t e_3 + x_3 e_1 + e_1 e_3) - de_4 + \alpha_4 u_4] \\
&+ \tilde{a}[\tilde{a} - e_1^2 - e_4] - e_2(x_t e_3 + x_3 e_1 + e_1 e_3) + e_4(x_t e_2 + x_2 e_1 + e_1 e_2)] \\
&+ \tilde{b}[\tilde{b}] + \tilde{c}[\tilde{c} - e_4(x_t e_3 + x_3 e_1 + e_1 e_3)] + \tilde{d}[\tilde{d} - e_4^2]
\end{align*}
\]
The control input \( u(t) \) together with the scaling factor and the parameter update law are respectively estimated from equation (15) as shown in equations (16) and (17) in that order.

\[
\alpha_i u_i = a(e_i + e_2) - e_4 - k_i e_i
\]

\[
\alpha_i u_2 = e_2 + a(x_i e_3 + x_3 e_1 + e_i e_3) - k_2 e_2
\]

\[
\alpha_i u_3 = -a(x_i e_2 + x_2 e_1 + e_i e_2) - k_3 e_3
\]

\[
\alpha_i u_4 = c(x_i e_3 + x_3 e_1 + e_i e_3) + d e_4 - k_4 e_4
\]

(16)

Where \( k_i (i = 1, 2, 3, 4) \) is the arbitrary feedback control gain to be determine

\[
\hat{a} = -e_1^2 - e_2 e_3 - e_2 (x_i e_1 + x_2 e_3 + e_i e_1) + e_3 (x_i e_2 + x_2 e_1 + e_i e_2) - a
\]

\[
\hat{b} = -b
\]

\[
\hat{c} = -e_4 (x_i e_3 + x_3 e_1 + e_i e_3) - c
\]

\[
\hat{d} = -e_4^2 - d
\]

For the error dynamics system (12) to converge to the equilibrium point asymptotically, the condition \( V = -\sum k_i e_i^2 \) must be satisfied.

**Theorem:** For a given nonzero economic scaling factor \( \alpha \), the multi variable projective synchronization between two systems (9) and (10) will occur by the adaptive control law (19) and the parameter update law (17).

By substitute equations (16) and (17) respectively into equation (14) gives the result in equation (18) which satisfied the above condition.

\[
\dot{V} = -k_i e_i^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 - a^2 - b^2 - c^2 - d^2 < 0
\]

(18)

The Lyapunov function \( V \) is a positive definite and its derivative \( \dot{V} \) is negative definite in the neighborhood of the zero solution for system (12). Based on the Lyapunov stability theory (LST), the error dynamical system (12) can converge to the origin asymptotically. Thus, by Barbalat’s lemma in Lyapunov stability theory [21], one can conclude that \( e_i(t) (i = 1, 2, 3, 4) \) (equation (12)) is globally and exponentially stable. Consequently, the drive (master) and response (slave) in equations (9) and (10) respectively are globally and exponentially synchronized for all initial conditions \( x_i(0), y_i(0) \) \( \in R^4 \) and update law (17).

In order to vary the value of the scaling factor \( \alpha_i \ (i = 1, 2, 3, 4) \), the control function \( u_i(t) \) in equation (16) is reduce to equation (19) as illustrated below.

\[
u_i = \frac{1}{\alpha_i} [a(e_i + e_2) - e_4 - k_i e_i]
\]

\[
u_2 = \frac{1}{\alpha_2} [e_2 + a(x_i e_3 + x_3 e_1 + e_i e_3) - k_2 e_2]
\]

\[
u_3 = \frac{1}{\alpha_3} [-a(x_i e_2 + x_2 e_1 + e_i e_2) - k_3 e_3]
\]

\[
u_4 = \frac{1}{\alpha_4} [c(x_i e_3 + x_3 e_1 + e_i e_3) + d e_4 - k_4 e_4]
\]

(19)
4.2 Numerical Simulation Results

In order to demonstrate the effective of the proposed techniques, the classical fourth-order Runge-Kutta routine is employed to solve the control law (19) and update law (17). The initial conditions of the drive (master) equation (9) are: 

\[ x_1(0) = -0.5, \quad x_2(0) = 0.6, \quad x_3(0) = 0.6 \quad \text{and} \quad x_4(0) = 0.8 \]

while that of the response (slave) equation (10) are:

\[ y_1(0) = -10.0, \quad y_2(0) = 1.0, \quad y_3(0) = 0.2 \quad \text{and} \quad y_4(0) = 10.0 \]

A time step of 0.001 is used with the values of the constant parameters \(a, b, c\) and \(d\) as in Figure 1 and the control feedback gain \(k_i = 1\) to ensure complex hyperchaotic dynamics of the state variables in systems (9) and (10).

Figure 3. Multi-stability with double coexistence of the hyperchaotic attractors for the different initial conditions.

Figure 3 showed the multi-stability of the system (1) with double coexistence of the hyperchaotic attractors for the different initial conditions of the drive (in red) \((x_1, x_2, x_3, x_4) = (-0.5, 0.6, 0.6, 0.8)\) and the response (in green) \((y_1, y_2, y_3, y_4) = (-10.0, 1.0, 0.2, 10.0)\) for the same parameter values \((a, b, c, d) = (3.0, 15.0, 0.5, 0.12)\). This is a special property of a system which refers to neither stable nor totally unstable, but alternate between two or more mutually exclusive states (attractors) over time. However, for complete synchronization \(\alpha_i = 1\).

The results of the simulation displayed the states trajectories of the drive (master) system (9) and the response (slave) system (10) in figure 4 ((a), (b), (c) and (d)). The states of the response (slave) (10) track the dynamics of the drive (master) (9) when the controller is activated at \(t = 50\). The error vectors \(e_i(t)\) \((i = 1, 2, 3, 4)\) (equation (12)) converges to the origin at \(t = 50\) as shown in figure 5 and the synchronization norm \(e = \sqrt{e_1^2 + e_2^2 + e_3^2 + e_4^2}\) is depicted in in Figure 6.
Figure 4. The time response of the state variables drive (master) and response (slave) for complete synchronization ($\alpha_i = 1$)

Figure 5. The error vectors between the drive (master) and the response (slave) for complete synchronization ($\alpha_i = 1$)

Figure 6. Synchronization norm for complete synchronization ($\alpha = 1$)
Figure 7. The time response of the state variables drive (master) and response (slave) for multi-variables projective synchronization ($\alpha_1 = 1, \alpha_2 = 2, \alpha_3 = 3, \alpha = -4$).

Figure 8. The error vectors between the drive (master) and the response (slave) for multi-variables projective synchronization ($\alpha_1 = 1, \alpha_2 = 2, \alpha_3 = 3, \alpha = -4$).

Figure 9. Synchronization norm for anti-synchronization for multi-variables projective synchronization ($\alpha_1 = 1, \alpha_2 = 2, \alpha_3 = 3, \alpha = -4$).
For multi-variable projective synchronization, $\alpha_1 = 1$, $\alpha_2 = 2$, $\alpha_3 = 3$ and $\alpha_4 = -4$. The states of the drive (master) and the response (slave) are depicted in Figure 7 ((a), (b), (c) and (d)). The states of the response (slave) (10) track the dynamics of the drive (master) (9) when the controller is switched on at $t = 50$. The error vectors $e_i(t)$ ($i = 1, 2, 3, 4$) (equation (12)) converges to the origin at $t = 50$ as shown in Figure 8 and the synchronization norm $e = \sqrt{e_1^2 + e_2^2 + e_3^2 + e_4^2}$ is depicted in Figure 9.

Figure 10 present the dynamics of the parameter estimation errors. The parameter $a, b, c$ and $d$ converges from $(3.0, 15.0, 0.5, 0.12)$ to the origin $(0.0, 0.0, 0.0)$. However, figure 11 display the time response of the parameter update law (17) updated from the initial values $a_1(0) = 3.0$, $b_1(0) = 15.0$, $c_1(0) = 0.5$ and $d_1 = 0.12$ to the estimated values $a = 5.0$, $b = 10.0$, $c = 2.5$ and $d = 1.5$ respectively.
5. Conclusions

This work studied the 4D hyperchaotic finance system. The multistability property of the system was discussed. The adaptive control method with parameter update laws has been applied to control the finance hyperchaotic system while the multi-variables projective technique with the update is used to synchronize between the two identical financial systems.

The novel adaptive controller $u_i(t)$ stabilizes as well as globally synchronized the two identical systems at different initial values of $x_i(t)$ and $y_i(t)$. The orbits of the error vector (12) converges to the origin asymptotically when the controller is activated at $t = 50$ as $t \to \infty$, that is the stabilization of the error dynamic (12) is achieved by the controller (19).

The stabilization of the error vector (12) indicates an effective way to regulate and control the interest rate ($x$ variable) when the inflation occurs and chaos arises in nonlinear finance system. When the saving (parameter $a$) is small, the inflation will increase hence chaos will appear. Therefore, there is a need for parameter updating in finance system.

In various economic systems, efforts are made towards reducing the investment cost ($b$), thus practical implementation of the update laws shall be very useful.

We established that the proposed method is an effective measure to; regulate, revive as well as track the economy of the nations in particular during the global crisis of COVID-19 pandemic that paralyzes the world economy.

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