Evaluating Link Prediction Accuracy on Dynamic Networks with Added and Removed Edges

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Abstract—The task of predicting future relationships in a social network, known as link prediction, has been studied extensively in the literature. Many link prediction methods have been proposed, ranging from common neighbors to probabilistic models. Recent work by Yang et al. [1] has highlighted several challenges in evaluating link prediction accuracy. In dynamic networks where edges are both added and removed over time, the link prediction problem is more complex and involves predicting both newly added and newly removed edges. This results in new challenges in the evaluation of dynamic link prediction methods, and the recommendations provided by Yang et al. [1] are no longer applicable, because they do not address edge removal. In this paper, we investigate several metrics currently used for evaluating accuracy of dynamic link prediction methods and demonstrate why they can be misleading in many cases. We provide several recommendations on evaluating dynamic link prediction accuracy, including separation into two categories of evaluation. Finally, we propose a unified metric to characterize link prediction accuracy effectively using a single number.

I. INTRODUCTION

The popularity of online social networking services has provided people with myriad new platforms for social interaction. Many social networking services also offer personalized suggestions of other people to follow or interact with, as well as websites or products that a user may be interested in. A key component in generating these personalized suggestions involves performing link prediction on social networks.

The traditional problem of link prediction on networks is typically defined as follows: given a set of vertices or nodes $V$ and a set of edges or links $E$ connecting pairs of nodes, output a list of scores for all pairs of nodes without edges, i.e. all pairs $(u, v) \notin E$, where a higher score for a pair $(u, v)$ denotes a higher predicted likelihood of an edge forming between nodes $u$ and $v$ at a future time.$^1$ Many link prediction methods have been proposed; see [2], [3] for surveys of the literature.

In this paper, we consider a more complex dynamic network setting where edges are both added and removed over time, which is often referred to as dynamic link prediction [4] or forecasting [5]. For instance, in a social network with timestamped edges denoting interactions between people (nodes), an edge may appear at several time instances where a pair of people are frequently interacting then disappear after interactions cease. Since existing edges between nodes may be removed at a future time, the dynamic link prediction problem is more complex and also involves computing a predicted score for existing edges, because they may disappear at a future time.

Evaluating link prediction accuracy involves comparing a binary label (whether or not an edge exists) with a real-valued predicted score. There are a variety of techniques for evaluation in this setting, including fixed-threshold methods such as F1-score and variable-threshold methods such as the area under the Receiver Operating Characteristic (ROC) curve, or AUC, and the area under the Precision-Recall (PR) curve, or PRAUC. Yang et al. [1] provide a comprehensive study of evaluation metrics for the traditional link prediction problem. Due to the severe class imbalance in link prediction (because only a small fraction of node pairs form edges), it was recommended to use PR curves and PRAUC for evaluating link predictors rather than ROC curves and AUC.

To the best of our knowledge, there has not been prior work on evaluating accuracy in the dynamic link prediction or forecasting setting we consider. Prior studies on dynamic link prediction have typically used AUC [4], [5], [7], [8], [10], log-likelihood [5], [10], [11], and maximum F1-score [10] as evaluation metrics.

The evaluation of several dynamic link prediction methods using current metrics is shown in Table I. We discuss these methods in further detail in Section II.C5. The table shows a clear disagreement between current metrics for dynamic link prediction accuracy. TS-Katz [8] has the highest AUC but a low PRAUC and maximum F1-score, while TS-Adj [6] has highest PRAUC and maximum F1-score, but lower AUC. The SBTM [9] ranks second in all three metrics. Which of these four methods is most accurate? We seek to answer this question in this paper. This type of disagreement among evaluation metrics has also been observed in prior studies, including [10], but has not been investigated further.

| Method | AUC | PRAUC | Max. F1-score |
|--------|-----|-------|---------------|
| TS-Adj [6] | 0.780 | 0.239 | 0.371 |
| TS-AA [7] | 0.777 | 0.065 | 0.144 |
| TS-Katz [8] | 0.879 | 0.077 | 0.149 |
| SBTM [9] | 0.799 | 0.138 | 0.337 |

$^1$Link prediction is also used to predict missing edges in partially observed networks, where the score denotes the predicted likelihood of an unobserved edge between $u$ and $v$. 

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TABLE I

ILLUSTRATION OF DISAGREEMENT AMONG CURRENT METRICS USED TO EVALUATE DYNAMIC LINK PREDICTION ACCURACY
Inspired by the work of Yang et al. [11] in the traditional link prediction setting, we provide a thorough investigation of evaluation metrics for the dynamic link prediction problem. Our aim is not to identify the most accurate link prediction algorithm, but rather to establish a set of recommendations for fair and effective evaluation of the accuracy of dynamic link prediction algorithms.

Our main contributions are the following:

- We discuss why currently used metrics for dynamic link prediction can be misleading (Section IV).
- We illustrate the importance of geodesic distance for the dynamic link prediction task and the dominance of edges at distance 1 (Section V).
- We separate the dynamic link prediction problem into two different link prediction problems based on geodesic distance and suggest metrics for fair and effective evaluation for each of the two problems (Section VI).
- We propose a unified metric that characterizes link prediction accuracy using a single number and demonstrate that it avoids the shortcomings of currently used metrics for dynamic link prediction (Section VII).

II. BACKGROUND

A. Problem Definition

The dynamic link prediction or forecasting problem is defined as follows. Given a set of nodes $\mathcal{V}$ and a set of edges $\mathcal{E}$ connecting pairs of nodes, output a list of scores for all pairs of nodes, where a higher score for a pair $(u, v)$ denotes a higher predicted likelihood of an edge between $u$ and $v$ at a future time. Again, the main difference in the dynamic link prediction task compared to traditional link prediction is the need to output scores for node pairs where an edge is already present, because the edge may be removed in the future.

We consider dynamic networks observed at discrete time steps $1, 2, \ldots, T$. A common prediction setting used in time series forecasting is the rolling 1-step forward prediction: for each $t = 1, \ldots, T - 1$, one trains a model using times $1$ to $t$ then predicts time $t + 1$. In this paper, we perform dynamic link prediction in the rolling 1-step forward prediction setting. The output of the link predictor contains $T - 1$ sets of predicted scores for times $2$ to $T$ (trained using times $1$ to $T - 1$, respectively), which are then compared against $T - 1$ sets of binary outputs denoting the actual states (edge or no edge) of all node pairs at times $2$ to $T$.

To evaluate accuracy, we concatenate all of the predicted scores into a single vector and all of the binary outputs into a second vector. This setting has been adopted in many past studies including [4], [5], [10], [11]. As noted in [11], we exclude node pairs corresponding to newly appearing nodes at any particular time step, since the identities of these new nodes are unknown at the time the prediction is computed.

B. Data Sets

We use two data sets as running examples throughout this paper. The first is the NIPS co-authorship data collected by Globerson et al. [12], consisting of papers from the NIPS conferences from 1988 to 2003. Nodes in the NIPS data denote authors, and undirected edges denote collaborations between authors. Each year is used as a time step, and an edge between two nodes at a particular time step denotes that the authors co-wrote a paper together in the NIPS conference that year. The data set contains 2,865 authors; we remove all authors who never collaborated with any other authors in the data set, leaving 2,715 authors (nodes).

The second data set is the Facebook data collected by Viswanath et al. [13]. Nodes denote users, and directed edges represent interactions between users via posts from one user to another user’s Facebook wall. All interactions are timestamped, and we use 90-day time steps (similar to the analyses in [9], [13]) from the start of the data trace in June 2006, with the final complete 90-day interval ending in November 2008, resulting in 9 total time steps. Viswanath et al. collected data on over 60,000 nodes. To make the dynamic link prediction problem more computationally tractable, we filter out nodes that have both in- and out-degree less than 30 in the aggregated network over all time steps, leaving 1,330 nodes.

Summary statistics for the two data sets are shown in Table II. The edge probability at each time step is given by the number of actual edges divided by the number of possible edges, i.e. the number of node pairs. We define a new edge at time $t$ as an edge that did not appear in any time step $t' < t$. We define a previously observed edge at time $t$ as an edge that appeared in at least one time step $t' < t$. Notice the large disparity between the new and previously observed edge probabilities—we will re-visit this point in Section IV.

C. Methods for Dynamic Link Prediction

Most methods for dynamic link prediction in the literature fall into one of three classes.

1) Univariate Time Series Models: Perhaps the most straightforward approach to dynamic link prediction is to apply standard univariate time series models to each node pair. Autoregressive Integrated Moving Average (ARIMA) models were used for dynamic link prediction in studies [2], [8]. A special case, the ARIMA($p, 1, 0$) model, is an exponentially-weighted moving average (EWMA) model, which has been used in studies [4], [6], [14], [15]. Another approach is to model the probability of an edge between a pair of nodes to be proportional to the previous number of occurrences of that edge [5], [10], [11], [14], i.e. a cumulative or growing window.
average, rather than an exponentially-weighted one. Dunlavy et al. \cite{Dunlavy2008} referred to the EWMA as the collapsed weighted tensor and the cumulative average as the collapsed tensor.

Univariate time series approaches treat each node pair separately by ignoring the rest of the network altogether. In doing so, the predictors based on univariate time series models are limited in their predictive ability; for instance, they only predict future occurrences of previously observed edges and cannot predict new edges. Thus these predictors are often used as baselines for comparison purposes. In many cases, however, these baselines have proven to be surprisingly competitive in accuracy as evaluated by existing metrics such as AUC \cite{Handbook2001}, \cite{Handbook2008}, which can be quite deceiving as we discuss in Section \ref{sec:baseline}.

2) Similarity-Based Methods: Node similarity-based methods have been among the earliest proposed methods for the traditional link prediction problem. These methods exploit the large number of triangles that are observed empirically in networks such as friendship networks to predict new edges. Typically used methods include common neighbors, Adamic-Adar, Jaccard coefficient, preferential attachment, and Katz \cite{Katz1957}. These methods are often used in a static setting, where only a single snapshot of a network is available.

In the case of dynamic networks, these similarity-based methods have been used in several different manners. Huang and Lin \cite{Huang2013} aggregated the dynamic network over time to form a static network then apply similarity-based methods. G"unes \cite{Gunes2013} et al. computed node similarities at each time step then model these similarities using ARIMA models. Dunlavy et al. \cite{Dunlavy2008} proposed a truncated version of the Katz predictor based on a low-rank approximation of a weighted average of past adjacency matrices. From these studies, it appears that the Adamic-Adar and Katz predictors have been the most accurate among the similarity-based predictors.

Similarity-based methods have the opposite weakness of link predictors based on univariate time series models; that is, they ignore whether an edge has occurred in the past between a pair of nodes. These methods are sometimes used together with univariate time series models in practice \cite{Huang2013, Lin2011}.

3) Probabilistic Generative Models: An alternative approach for dynamic link prediction is to fit a probabilistic generative model to the sequence of observed networks. A generative model for a dynamic network represents the network (up to time $t$) by a set of unobserved parameters $\Theta_t$. Given the values of the parameters, it then provides a model for the probability of an edge between any pair of nodes $(u, v)$ at time $t + 1$, which is used as the link prediction score for $(u, v)$. Since the parameters $\Theta_t$ are unobserved, one typically estimates them from the sequence of networks then uses the estimated parameters to compute the link prediction score. The link prediction or forecasting accuracy is often used as a measure of goodness-of-fit for the generative model.

Several classes of generative models for dynamic networks have been proposed, including dynamic latent feature models and dynamic stochastic block models. In a latent feature model, every node in a network has an unobserved (typically binary) feature vector. An edge between two nodes is then formed conditionally independently of all other node pairs given their feature vectors. These models have been adapted to dynamic networks by allowing the latent features to change over time \cite{De2011}, \cite{De2012}, \cite{De2013}. Such models have tremendous flexibility; however, fitting these models typically requires Markov chain Monte Carlo (MCMC) methods that scale up to only a few hundred nodes.

Stochastic block models (SBMs) divide nodes into classes, where all nodes within a class are assumed to have identical statistical properties. An edge between two nodes is formed independently of all other node pairs with probability dependent only on the classes of the two nodes, giving the adjacency matrix a block structure where blocks correspond to pairs of classes. SBMs have been extended to the dynamic network setting by allowing the edge probabilities and class memberships to change over time \cite{Decelle2011}, \cite{Lin2011}, \cite{Tylenda2013}. The models proposed in \cite{Lin2011, Tylenda2013} can be fit using an extended Kalman filter and local search procedure that scales to a few thousand nodes, an order of magnitude larger than methods for fitting dynamic latent feature models.

4) Other Methods: Dunlavy et al. \cite{Dunlavy2008} proposed to use matrix and tensor factorizations, namely truncated singular value decomposition (TSVD) and canonical decomposition/parallel factors (CANDECOMP/PARAFAC or CP) tensor models, respectively. Tylenda et al. \cite{Tylenda2013} proposed a “time-aware” version of a local probabilistic model based on the maximum-entropy principle. The approach involves weighted constraints based on the times at which edges occurred.

5) Methods Considered in This Paper: In this paper, we consider methods from each of the first three categories:

- TS-Adj \cite{Dunlavy2008}: a univariate time series model applied to each node pair.
- TS-AA \cite{Lin2011}: a similarity-based method that extends the Adamic-Adar link predictor to the dynamic setting by applying a time series model to the Adamic-Adar scores over time for a node pair.
- TS-Katz \cite{Tylenda2013}: a similarity-based method that extends the Katz predictor to the dynamic setting by applying a time series model to the Katz scores over time for a node pair.
- SBTM \cite{Lin2011}: a probabilistic generative model based on stochastic block models.

We emphasize again that the objective of this paper is not to identify the best prediction algorithm, thus this list is not exhaustive. For simplicity, we use the EWMA, which corresponds to ARIMA(0, 1, 0) with forgetting or decay factor of 0.5 as the time series model for each of the methods with prefix TS. Higher accuracy is likely attainable by better model selection for the ARIMA model parameters, but it is outside the scope of this paper.

\footnote{Adamic-Adar is not applicable to directed networks so we first convert the Facebook network to an undirected network before applying TS-AA.}

\footnote{The approach is slightly different from what was proposed in \cite{Tylenda2013} and is similar to the approach used in \cite{Tylenda2013} for TS-AA; we find this approach to be almost universally more accurate than the approach in \cite{Lin2011}.}
### TABLE III

**CONFUSION MATRIX FOR BINARY PREDICTION**

| Actual 1’s (P) | Predicted 1’s (TP) | False Negatives (FN) |
|---------------|--------------------|----------------------|
| Actual 0’s (N) | False Positives (FP) | True Negatives (TN) |

III. **EXISTING EVALUATION METRICS**

The currently employed evaluation methods discussed in the introduction and shown in Table I indicate the lack of a principled metric, which makes it difficult to evaluate the accuracies of dynamic link prediction methods. Most of the evaluation metrics used in link prediction have been borrowed from other applications such as information retrieval and classification. Hence these metrics are naturally biased to favor certain aspects over others, which may result in either over- or under-representing the accuracy of a particular method.

The output of a link predictor is usually a set of real-valued scores, which are compared against a set of binary labels, where each label denotes the presence (1) or absence (0) of an edge. One technique for comparison is to threshold the scores at a fixed value, transforming the real-valued scores into binary predictions. These binary predictions can then be compared against the binary labels by computing the confusion matrix shown in Table III then using metrics based on the confusion matrix. A second technique involves sweeping the threshold over the entire range of predicted scores and plotting a threshold curve displaying the variation of one metric against another. A third technique, applicable only to probabilistic models, is to evaluate the likelihood of the model given the set of binary labels.

#### A. Information Retrieval-Based Metrics

In information retrieval, one is typically concerned with two metrics calculated from the confusion matrix in Table III: precision (\( \frac{TP}{TP+FP} \)) and recall (\( \frac{TP}{TP+FN} \)). Precision and recall are often combined into a single measure using their harmonic mean, known as the F1-score (\( 2 \cdot \frac{\text{precision} \cdot \text{recall}}{\text{precision} + \text{recall}} \)).

The precision, recall, and F1-score all vary with the choice of threshold applied to the real-valued scores. As an alternative to choosing a threshold, one sometimes computes the precision at \( k \), also known as the top \( k \) predictive rate, which denotes the number of correctly predicted links from the top \( k \) scores. In the traditional link prediction setting, \( k \) is typically chosen to be equal to the number of actual new edges \( P \) [2]. Relative metrics are also used, such as the improvement in top \( k \) predictive rate as compared to expected rate of a random predictor [2]. Yang et al. [1] discussed and empirically demonstrated several shortcomings of using fixed-threshold metrics in the traditional link prediction setting, which led to unstable results and disagreements as the threshold was varied. We observe these shortcomings also in the dynamic link prediction setting.

An alternative to fixed-threshold metrics is to use threshold curves, which work by shifting the threshold, computing the confusion matrix for each threshold, and finally computing metrics based on the confusion matrices. Threshold curves for different predictors are often compared using a single scalar measure, typically the area under the curve. In information retrieval, the commonly used threshold curve is the Precision-Recall (PR) curve. We denote the area under the PR curve by PRAUC. Simply linearly interpolating between points on the PR curve has been shown to be inappropriate for calculating PRAUC; we use the proper interpolation approach as discussed in [13].

PR curves consider only prediction of the positives and are generally used for needle-in-haystack problems common in information retrieval, where negatives dominate and are not interesting. For link prediction, PR curves give credit for correctly predicting edges but do not give credit for correctly predicting non-edges. Due to the sparsity of most types of networks including social networks, the number of non-edges is much greater than the number of edges, so Yang et al. [1] recommend the use of PRAUC for evaluation in the traditional link prediction setting.

1) **Uses in Dynamic Link Prediction:** In the dynamic link prediction setting, Kim et al. [10] proposed to use the maximum F1-score over all possible threshold values, i.e., identifying the point on the PR curve that maximizes F1-score. In this manner, it utilizes a single threshold that is determined by sweeping the PR curve rather than choosing a threshold a priori. This metric displays similar evaluation properties as PRAUC due to its dependence on the PR curve.

The normalized discounted cumulative gain (NDCG) over the top \( k \) link prediction scores [17] is another information retrieval-based metric that has been used for evaluating dynamic link prediction accuracy. It is a fixed-threshold metric that suffers from the same drawbacks as other fixed-threshold metrics as discussed by Yang et al. [1].

2) **Shortcomings for Dynamic Link Prediction:** We argue that the PR curve is inappropriate for dynamic link prediction because it only considers the edges (positives). Accurate prediction of existing edges that do not appear at a future time (negatives), is an important aspect of dynamic link prediction and is not captured by the PR curve! Thus the PR curve and metrics derived from the PR curve, such as PRAUC and maximum F1-score, may be highly deceiving in the dynamic link prediction setting. Notice from Table I that the most accurate link predictor according to PRAUC and maximum F1-score is the TS-Adj baseline predictor that does not predict any new edges! We expand on this discussion in Section V-B.

#### B. Classification-Based Metrics

In classification, the commonly used metric is classification accuracy (\( \frac{TP+TN}{P+N} \) for binary classification) over all data points, which are node pairs in the case of link prediction. Classification accuracy is often deceiving in the case of highly imbalanced data, where high accuracy can be obtained even by a random predictor.

In binary classification, one is often concerned with the true positive rate (\( TPR = \frac{TP}{TP+FP} \)) and false positive rate (\( FPR = \frac{FP}{FP+TN} \)), which can be calculated from the confusion matrix in Table III for a fixed threshold. By sweeping the
Given a probabilistic model for observed data, the likelihood of a set of parameters is given by the probability of the observations given those parameter values. Since the actual parameter values are unknown, one typically calculates the likelihood using optimal parameter estimates or the estimated posterior distribution of the parameters given the observed data. It is often easier and more numerically stable to work with the log-likelihood rather than the likelihood itself, so the log-likelihood of a model is usually reported in practice.

1) Uses in Dynamic Link Prediction: Likelihood-based metrics are often used for evaluating link prediction accuracy for generative models and are a natural fit given their probabilistic nature. In the dynamic link prediction setting, the log-likelihood has been used in studies [5], [10], [14] as a metric for dynamic link prediction accuracy. Researchers often also calculate the log-likelihood of a baseline model, which is then used to measure relative improvement of a proposed model in terms of log-likelihood. For instance, studies [5], [11] use a Bayesian interpretation of a cumulative average as a baseline model.

2) Shortcomings for Dynamic Link Prediction: Yang et al. [1] claimed that AUC is deceiving for evaluation of accuracy in the traditional link prediction setting due to the locality of edge formation. They found empirically that the probability of forming a new edge between a pair of nodes decreases as the geodesic (shortest path) distance between the node pair increases. We demonstrate in Section IV that this problem is even greater in the dynamic link prediction setting, where edges at distance 1, i.e. edges that have been previously observed, are also considered in the evaluation.

One of the appealing properties of AUC is its interpretation as the probability of a randomly selected positive instance appearing above a randomly selected negative instance [19]. In the traditional classification setting, where instances are assumed to be independent and identically distributed (iid), this interpretation can be very useful. However, as we demonstrate in Section IV, node pairs are certainly not iid, and edge formation probabilities vary greatly based on whether an edge has previously existed. Using only this information, one can construct a predictor that achieves high AUC, as evidenced by the TS-Adj predictor in Table I. Hence pooling together all node pairs to evaluate AUC can be highly deceiving.

C. Likelihood-Based Metrics

Given a probabilistic model for observed data, the likelihood of a set of parameters is given by the probability of the observations given those parameter values. Since the actual parameter values are unknown, one typically calculates the likelihood using optimal parameter estimates or the estimated posterior distribution of the parameters given the observed data. It is often easier and more numerically stable to work with the log-likelihood rather than the likelihood itself, so the log-likelihood of a model is usually reported in practice.

1) Uses in Dynamic Link Prediction: The AUC gives a single value that can be used to compare accuracy against other models and is the most commonly used metric for evaluating dynamic link prediction accuracy [4], [8], [7], [3], [10]. The main difference compared to the traditional link prediction task is that the AUC is computed over all possible node pairs, not only node pairs without edges.

Güneş et al. [7] also evaluated AUC over smaller subsets of node pairs, such as node pairs with no edges over the past 3 time steps. Splitting up the evaluation into different subsets is a step in the right direction; however, Güneş et al. [7] chose the subsets in a somewhat ad-hoc fashion and still rely on AUC over all node pairs as an evaluation metric, which is problematic as we discuss in the following. We present a principled approach for splitting up the evaluation of dynamic link prediction accuracy in Section IV.

2) Shortcomings for Dynamic Link Prediction: In general, log-likelihoods may be very complex to calculate due to the effects of constant terms that are usually ignored when maximizing the log-likelihood. Additionally it is not possible to obtain likelihood values for link predictors that are not based on probabilistic models. Thus the scope of this metric is limited both by its complexity and applicability to only a small subset of link prediction techniques.

IV. THE EFFECT OF GEODESIC DISTANCE ON DYNAMIC LINK PREDICTION

One of the main differences between the typical machine learning setting and the link prediction setting is that node pairs are not independent and identically distributed (iid). It has been shown that the probability of forming an edge between two nodes is highly dependent on the length of the shortest path between them, often called the geodesic distance or just the distance. In the traditional link prediction problem, most edges are formed at geodesic distance 2, and the probability of edge formation generally decreases monotonically with increasing geodesic distance [1].

In the dynamic link prediction problem, we also need to consider node pairs at geodesic distance 1, i.e. pairs of nodes for which an edge has previously been formed, because these edges may or may not re-occur in the future. In the Facebook data set, we find that the majority (almost 80%) of edges are formed at distance 1, as shown in Fig. 1a. Additionally Fig. 1b shows the empirical probability that an edge is formed between two nodes as a function of geodesic distance. Notice that the
edge probability is over 30 times higher at distance 1 compared to distance 2 and over 300 times higher than at distances 3 and above! Thus it does not make sense to pool over all node pairs when evaluating dynamic link prediction accuracy (e.g. using AUC or PRAUC), because the overwhelming majority of positive instances occur at distance 1!

In the traditional link prediction problem, Yang et al. [1] suggested to evaluate link prediction accuracy separately at each distance. However this is a cumbersome approach, so they proposed also to use the PRAUC as a single measure of accuracy over all distances. As we have discussed in Section III, PRAUC is problematic in the dynamic link prediction setting because it ignores the negative class, so we cannot use the same approach as in [1]. Instead, recognizing that most edges are formed between node pairs with a previously observed edge, we propose to separate the dynamic link prediction problem into two problems.

V. SEPARATION INTO TWO LINK PREDICTION PROBLEMS

Part of the difficulty in evaluating accuracy in the dynamic link prediction setting is related to the problem itself. Dynamic link prediction combines two problems: prediction of new links (distance $\geq 2$) and prediction of previously observed links (distance = 1). These two problems have very different properties in terms of difficulty, which primarily relates to the level of class imbalance in the two problems.

The difference in difficulties of the two problems can be seen in Table II. Notice that the probability of a new edge being formed is tiny compared to a previously observed edge! Thus the new link prediction problem involves much more severe class imbalance (i.e. difficulty) compared to the previously observed link prediction problem. By pooling together all node pairs when calculating AUC or PRAUC, the evaluation is heavily biased towards the previously observed link prediction problem. As a result, all of the metrics shown in Table II are biased in this manner. Instead, node pairs corresponding to possible new edges should be separated from node pairs corresponding to possible re-occurring edges, and accuracy metrics should be computed separately.

A. Prediction of New Edges

We begin by considering the prediction of new edges that have not been observed at any previous time. Actually this is simply the traditional link prediction problem, and the recommendations in [1] apply here as well. The main recommendation is to use PR curves rather than ROC curves due to the abundance of true negatives as indicated by the extreme class imbalance shown in Table II. By using PR curves, the overwhelming number of true negatives generated by link prediction algorithms are excluded from the evaluation.

TS-Adj is capable only of predicting previously observed edges, as discussed in Section II.C Thus its predictions for new links are random guesses, so it achieves the random baseline AUC of 0.5 and PRAUC of $\frac{1}{M}$. The similarity-based methods TS-AA and TS-Katz are extensions of the Adamic-Adar and Katz predictors for traditional link prediction, and hence, they should be expected to perform better than the SBTM for new link prediction, especially because the SBTM does not consider geodesic distance. We see from Table IV that this is indeed the case in both data sets, although the difference is much more pronounced in terms of PRAUC.

Thus we support the recommendation in [1] to use PRAUC to evaluate accuracy of new link prediction.

B. Prediction of Previously Observed Edges

The second problem in dynamic link prediction involves predicting edges that are currently present or were present at a previous time. As shown in Table III the class imbalance is several orders of magnitude less severe than in the case of predicting new edges.

Another major difference from new link prediction is the relevance of negatives (non-edges). Accurate prediction of negatives is highly relevant because the removal of edges over time contributes a significant portion of the network dynamics. For example, in the NIPS co-authorship network, we find that over 85% of edges observed at any time step are deleted at the following time step.

A good evaluation metric for the task of predicting previously observed links must provide a balanced evaluation between the positive and negative classes. The metrics based on the PR curve are biased towards the positive class. We hence propose to use AUC, which is based on the ROC curve and does account for negatives. Many of the shortcomings of AUC pointed out by Yang et al. [1] for the new link prediction task are not present in the previously observed link prediction task because the class imbalance is not nearly as significant.

From the AUC and PRAUC values for previously observed link prediction in Table IV, TS-Adj is the most accurate according to both metrics on both data sets. This is not surprising because TS-Adj can only predict previously observed edges. However AUC and PRAUC do not necessarily agree in general; for example, consider TS-AA and TS-Katz on the NIPS data. TS-AA has higher PRAUC but lower AUC, and the

| Method | New Link AUC | New Link PRAUC $\times 10^{-3}$ | Prev. Observed AUC | Prev. Observed PRAUC |
|--------|--------------|-------------------------------|-------------------|----------------------|
| TS-Adj [6] | 0.500 | 0.033 | 0.855 | 0.099 |
| TS-AA [7] | 0.534 | 0.882 | 0.646 | 0.057 |
| TS-Katz [8] | 0.535 | 0.735 | 0.694 | 0.049 |
| SBTM [9] | 0.531 | 0.055 | 0.713 | 0.066 |

| Method | New Link AUC | New Link PRAUC $\times 10^{-3}$ | Prev. Observed AUC | Prev. Observed PRAUC |
|--------|--------------|-------------------------------|-------------------|----------------------|
| TS-Adj [6] | 0.500 | 1.19 | 0.705 | 0.417 |
| TS-AA [7] | 0.712 | 14.4 | 0.560 | 0.293 |
| TS-Katz [8] | 0.768 | 14.8 | 0.579 | 0.297 |
| SBTM [9] | 0.700 | 4.41 | 0.649 | 0.326 |
Fig. 2. Comparison of (a) ROC and (b) PR curves for previously observed link prediction on NIPS data. TS-AA performs better at low recall (TPR) and worse at high recall, resulting in lower AUC but higher PRAUC.

Fig. 3. Link prediction scores of (a) TS-AA and (b) TS-Katz sorted in descending order (blue lines) corresponding to all node pairs for which an edge was previously observed. Red vertical lines denote node pairs that form an edge at the following time step. TS-AA correctly predicts more edges at high scores but misses many edges at low scores compared to TS-Katz.

VI. A Unified Evaluation Metric

By separating the dynamic link prediction problem into two problems with separate evaluation metrics, we are able to fairly evaluate different methods for dynamic link prediction. However, one often desires a single metric to capture the “overall” accuracy rather than two metrics, analogous to the role of F1-score combining precision and recall.

In the dynamic link prediction setting, any such metric should capture both the predictive power in new link and previously observed link prediction. In Section V we concluded that PRAUC is the better evaluation metric for new link prediction and that AUC is the better evaluation metric for previously observed link prediction. A unified evaluation metric could thus consist of the mean of the two quantities. Notice, however, from Table IV that the two quantities have very large differences in magnitude, despite both being in the same range [0, 1]. Thus the arithmetic mean is inappropriate because it would be dominated by the AUC value for previously observed link prediction. The harmonic mean is also inappropriate because it would be dominated by the PRAUC for new link prediction, which has a much larger reciprocal.

We recommend instead to use the geometric mean of the two quantities after a baseline correction, which we denote by

\[ \text{GMAUC} = \sqrt{\frac{\text{PRAUC}_{\text{new}} - \frac{P}{P+N}}{1 - \frac{P}{P+N}} \cdot 2(\text{AUC}_{\text{prev}} - 0.5)}, \]

where \( P \) and \( N \) denote the number of actual edges and non-edges over the set of node pairs considered for new link prediction. The baseline correction subtracts the PRAUC and AUC values that would be obtained by a random predictor. The use of the geometric mean is motivated by the GMean metric proposed by Kubat et al. [20] for evaluating classification accuracy in highly imbalanced data sets. The geometric mean has several nice properties in this setting:

- It is based on threshold curves and avoids the pitfalls of fixed-threshold metrics as discussed in [1].
- It accounts for the different scales of the PRAUC for new edges and AUC for previously observed edges without being dominated by either quantity.
- It is 0 for any predictor that can only predict new edges or can only predict previously observed edges.

The final point addresses an observation from several previous papers [4], [5], [10] on generative models for dynamic networks: baseline methods (e.g. TS-Adj) that predict only previous observed edges tend to perform quite competitively in terms of AUC when evaluated on the entire network. The GMAUC for a baseline predictor of this sort would be 0 due to its inability to predict any new edges at all.

The accuracies of several dynamic link predictors using the evaluation metrics proposed in this paper are shown in Table VI According to the proposed GMAUC metric, TS-Katz is the best predictor for both data sets due to its ability to accurately predict both previously observed and new edges. Notice that, for the NIPS data, TS-Katz has the highest GMAUC despite
not being the most accurate in either task. This is due to the balanced evaluation of new and previously observed link prediction used in the proposed GMAUC metric.

The data set used to compute the metrics shown in Table I is actually the same Facebook data used in Table V. Notice that the least accurate method according to all three metrics in Table IV—TS-AA, actually becomes the second most accurate once the evaluation is properly split up into new and previously observed links. This is primarily due to its strength in new link prediction compared to the SBTM, which does not consider geodesic distance for new link prediction, and to TS-Adj, which does not consider new edges at all.

VII. Conclusions

In this paper, we thoroughly examined the problem of evaluating accuracy in the dynamic link prediction setting where edges are both added and removed over time. We find that the overwhelming majority of edges formed at any given time are edges that have previously been observed. These edges should be evaluated separately from new edges, i.e., edges that have not formed in the past between a pair of nodes. The new and previously observed link prediction problems have very different levels of difficulty, with new link prediction being orders of magnitude more difficult. None of the currently used metrics for dynamic link prediction perform this separation and are thus dominated by the accuracy on the easier problem of predicting previously observed edges.

Our main recommendations are as follows:

1) Separate node pairs for which edges have previously been observed from the remaining node pairs, and evaluate link prediction accuracy on these two sets separately.
2) For node pairs without previous edges, i.e., the new link prediction problem, evaluate prediction accuracy using PRAUC due to the tremendous class imbalance.
3) For node pairs with previous edges, evaluate prediction accuracy using AUC due to the importance of predicting negatives (non-edges).

4) If a single metric of accuracy is desired, evaluate new and previously observed link prediction using separate metrics then combine the metrics rather than computing a single metric over all node pairs.
5) Use the proposed GMAUC metric as the single accuracy metric to provide a balanced evaluation between new and previously observed link prediction.

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