Dark Radiation from Particle Decays during Big Bang Nucleosynthesis

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Cosmic microwave background (CMB) observations suggest the possibility of an extra dark radiation component, while the current evidence from big bang nucleosynthesis (BBN) is more ambiguous. Dark radiation from a decaying particle can affect these two processes differently. Early decays add an additional radiation component to both the CMB and BBN, while late decays can alter the radiation content seen in the CMB while having a negligible effect on BBN. Here we quantify this difference and explore the intermediate regime by examining particles decaying during BBN, i.e., particle lifetimes $\tau_X$ satisfying $0.1 \text{ sec} < \tau_X < 1000 \text{ sec}$. We calculate the change in the effective number of neutrino species, $N_{\text{eff}}$, as measured by the CMB, $\Delta N_{\text{CMB}}$, and the change in the effective number of neutrino species as measured by BBN, $\Delta N_{\text{BBN}}$, as a function of the decaying particle initial energy density and lifetime, where $\Delta N_{\text{BBN}}$ is defined in terms of the number of additional two-component neutrinos needed to produce the same change in the primordial $^4$He abundance as our decaying particle. As expected, for short lifetimes ($\tau_X \lesssim 0.1 \text{ sec}$), the particles decay before the onset of BBN, and $\Delta N_{\text{CMB}} = \Delta N_{\text{BBN}}$, while for long lifetimes ($\tau_X \gtrsim 1000 \text{ sec}$), $\Delta N_{\text{BBN}}$ is dominated by the energy density of the nonrelativistic particles before they decay, so that $\Delta N_{\text{BBN}}$ remains nonzero and becomes independent of the particle lifetime. By varying both the particle energy density and lifetime, one can obtain any desired combination of $\Delta N_{\text{BBN}}$ and $\Delta N_{\text{CMB}}$, subject to the constraint that $\Delta N_{\text{CMB}} \geq \Delta N_{\text{BBN}}$. We present limits on the decaying particle parameters derived from observational constraints on $\Delta N_{\text{CMB}}$ and $\Delta N_{\text{BBN}}$.

I. INTRODUCTION

Over the past decade, a “standard-model” cosmology has emerged, based, among other things, on precision measurements of the fluctuations in the cosmic microwave background (CMB) \[1, 2\], observations of type Ia supernovae \[3, 4\] and big bang nucleosynthesis (BBN) (see Ref. 5 for a recent review). In the standard cosmological model, the density of the universe is dominated at present by a cosmological constant ($\Lambda$) and cold dark matter (CDM), corresponding respectively to roughly 70% and 25% of the total density, with the remaining 5% in baryons. The present-day radiation content of the universe is negligible in comparison, but this radiation was the dominant component at early times.

While the cosmological observations are generally consistent with this standard model, there are a few unresolved issues. One of these involves the total radiation content of the universe. The energy density of the CMB is a simple function of the CMB temperature and is known to high accuracy. Similarly, given the observed number of light neutrinos ($N_\nu = 3$), one can calculate the neutrino energy density. (In fact, the “effective” number of neutrinos, $N_{\text{eff}}$ is slightly greater than 3 due to partial heating of the neutrinos in the early universe by electron-positron annihilation. Including these effects yields $N_{\text{eff}} = 3.046 \pm 0.006$ \[6\].) However, recent precision measurements of the CMB fluctuations are best fit by larger values of $N_{\text{eff}}$. (For a discussion of the effect of $N_{\text{eff}}$ on the CMB fluctuations, see Refs. 6, 7). The seven-year data from the Wilkinson Microwave Anisotropy Probe, combined with observations of baryon acoustic oscillations (BAO) and measurements of the Hubble parameter, $H_0$, give $N_{\text{eff}} = 4.34^{+0.86}_{-0.88} (68\% \text{ CL})$ \[2\]. Observations by the Atacama Cosmology Telescope combined with BAO and $H_0$ give $N_{\text{eff}} = 4.56 \pm 0.75 (68\% \text{ CL})$ \[10\]. Recent results from the South Pole Telescope combined with WMAP7, BAO, and $H_0$ give $N_{\text{eff}} = 3.86 \pm 0.42 (68\% \text{ CL})$ \[11\]. An analysis using combined datasets in Ref. 12 yields $N_{\text{eff}} = 4.08^{+0.77}_{-0.68} (95\% \text{ CL})$. Since this additional radiation component cannot interact electromagnetically, it has been dubbed “dark radiation”.

BBN is also quite sensitive to the total radiation content in the universe, but here the evidence is more ambiguous. Recent calculations of the relic helium abundance by Izotov and Thuan \[12\] and by Aver, Olive, and Skillman \[13\] have reached opposite conclusions, with the former arguing for an additional dark radiation component and the latter concluding that the standard number of neutrinos suffices. A recent analysis by Mangano and Serpico \[13\] gives $\Delta N_{\text{eff}} \leq 1 (95\% \text{ CL})$. (See also the discussion in Ref. 16). An overview combining CMB and BBN constraints on dark radiation can be found in Ref. 17, while constraints on the physical properties of the dark radiation are discussed in Refs. 12, 18.

Given these hints of new physics, a number of models have been proposed to account for additional dark radiation. The simplest way to achieve this is to add additional relativistic relic particles, as suggested by, e.g., Refs. \[18, 22\]. In this case, the value for $N_{\text{eff}}$ determined by BBN and the CMB should be the same. Additional relativistic energy density can also be provided by a neutrino chemical potential, as in the models discussed in Refs. 18, 21.

Alternately, if one wishes to produce an increase in $N_{\text{eff}}$ in the CMB, but retain the standard-model value for $N_{\text{eff}}$ in BBN, then an obvious possibility is the production of relativistic, non-electromagnetically-interacting particles from the decay of a massive relic particle after BBN. Such a scenario for the dark radia-
recently considered by Ichikawa et al. [25], and further elaborated by Fischler and Meyers [26] and Hasenkamp [27]. If we let $N_{BBN}$ denote the value of $N_{eff}$ as measured by BBN, and $N_{CMB}$ be the value determined from the CMB and other low-redshift measurements, then these decaying particle models predict $N_{BBN} \ll N_{CMB}$, while additional stable relativistic degrees of freedom give $N_{BBN} = N_{CMB}$.

In this paper, we fill in the gap between these two regimes by examining decaying particle scenarios in which the particle decays during BBN. Such models produce a relation between $N_{BBN}$ and $N_{CMB}$ that varies from $N_{BBN} = N_{CMB}$ at short particle lifetimes (when the particle decays before the onset of BBN) to $N_{BBN} \ll N_{CMB}$ when the particle decays after the conclusion of BBN. In the next section, we give a detailed discussion of our calculation and present our results for $N_{BBN}$ and $N_{CMB}$ as a function of the decaying particle abundance and lifetime. Our conclusions, including observational limits, are discussed in Sec. III.

II. DARK RADIATION FROM A DECAYING PARTICLE

We assume a standard flat Friedman-Robertson-Walker model with the expansion rate given by:

$$H = \frac{\dot{R}}{R} = \left(\frac{8 \pi G \rho}{3} \right)^{1/2},$$

where $R$ is the scale factor and $\rho$ is the energy density. To this standard cosmological model we add a nonrelativistic particle $X$, which is unstable and decays with lifetime $\tau_X$. By assumption, $X$ decays only into "invisible" relativistic decay products, which do not interact electromagnetically and can thus form the dark radiation (see Refs. [23, 27] for examples of such models). The effects of such decays during BBN were previously considered by the authors of Ref. [28] and our treatment closely follows theirs. (For another early discussion of BBN with such decays, see Ref. [29].) The main focus of Ref. [28] was the constraint from BBN that could be placed on these decaying particles, while in this paper, we will be interested in using such models to provide dark radiation, and to determine the relation between $N_{BBN}$ and $N_{CMB}$ as a function of the model parameters.

We follow Ref. [30] and parametrize the density of the decaying particle in terms of its number density relative to the entropy density, $s$, prior to decay ($t \ll \tau_X$):

$$Y_X = \frac{n_X}{s},$$

where $s$ is given by

$$s = \frac{2\pi^2}{45} g_{ss} T_\gamma^3,$$

In Eq. (3), $T_\gamma$ is the photon temperature, and $g_{ss}$ is the effective number of "entropy" degrees of freedom, defined as

$$g_{ss} = \sum_{\text{bosons}} g_B(T_i/T_\gamma)^3 + (7/8) \sum_{\text{fermions}} g_F(T_i/T_\gamma)^3,$$

where the sum is over all relativistic bosons and fermions.

To this standard cosmological model we add a nonrelativistic particle $X$, decaying only into invisible relativistic decay products at $t = \tau_X$:

$$\rho_X = \rho_{X0} \left(\frac{R}{R_0}\right)^{-3} e^{-t/\tau_X},$$

while equation (5) must be integrated numerically (although an analytic solution can be derived for $t \ll \tau_X$). It is clear from Eqs. (4)-(6) that $\rho_{dec}$, and thus, the increase in $N_{eff}$, depends only on $Y_X m_X$ and $\tau_X$.

Evolving these equations to calculate $\rho_X$ and $\rho_{dec}$, and converting $\rho_{dec}$ into an effective number of neutrinos, we derive $\Delta N_{CMB}$, the change in $N_{eff}$ as measured by the CMB, as a function of $Y_X m_X$ and $\tau_X$, where we confine our attention to the case where the $X$ particle fully decays before last scattering. A contour plot of $\Delta N_{CMB}$ as a function of $Y_X m_X$ and $\tau_X$ is given in Fig. 1.

In the limit where the decaying particles themselves never dominate the expansion, one can calculate $\Delta N_{CMB}$ as a function of $Y_X m_X$ and $\tau_X$, as in Ref. [28]. Rewriting the results of Ref. [28] in terms of our parameters, we find

$$\Delta N_{CMB} = 8.3 \left(\frac{Y_X m_X}{\text{MeV}}\right) (\tau_X / \text{sec})^{1/2}.$$

A comparison of Eq. (8) with the results displayed in Fig. 1 shows that Eq. (8) is accurate to within $\sim 10\%$ for the curves displayed in Fig. 1. Note that our analytic expression for $\Delta N_{CMB}$ differs significantly from that derived in Ref. [24], as the latter used the "sudden decay approximation," in which all of the energy density of the decaying particle is taken to be converted into relativistic decay products at $t = \tau_X$.

Adding additional energy density during BBN affects all of the element abundances, but the effect is largest
FIG. 1: Contour plot of $\Delta N_{CMB}$, the change in the effective number of neutrinos determined by CMB observations due to a decaying particle with lifetime $\tau_X$ and energy density prior to decay parametrized in terms of $Y_X m_X$, where $Y_X$ is the initial number density of the particle relative to entropy density, and $m_X$ is the particle mass. Curves correspond to, from bottom to top, $\Delta N_{CMB} = 0.1, 0.2, 0.3, 0.4, 0.7, 1.0, 1.5, 2.0$.

We use the Kawano \cite{33} version of the Wagoner \cite{34,35} Big Bang nucleosynthesis code to calculate the change in the primordial $^4$He abundance with the addition of $\rho_X$ and $\rho_{\text{dec}}$ as given by Eqs. (5) and (6), taking a baryon-photon ratio of $\eta = 6.1 \times 10^{-10}$. (Note that since we are presenting the change in the $^4$He abundance produced by the decaying particle, rather than the absolute helium abundance itself, our results are quite insensitive to the assumed value of $\eta$; see, e.g., Fig. 6 of Ref. \cite{3}). We determine the change in the number of relativistic neutrinos that gives exactly the same change in the $^4$He abundance as a decaying particle with a given abundance and lifetime. Thus, for any pair of values for $Y_X m_X$ and $\tau_X$, we have a corresponding change $\Delta N_{BBN}$ that produces the same effect on BBN. (The change in the other element abundances can be ignored here). In Fig. 2, we give a contour plot of $\Delta N_{BBN}$ as a function of $Y_X m_X$ and $\tau_X$.

III. CONCLUSIONS

In comparing Figs. 1 and 2, we see that for $\tau_X < \sim 0.1$ sec, $\Delta N_{BBN} = \Delta N_{CMB}$. In this short-lifetime limit, all of the decaying particle energy density is converted into dark radiation before BBN begins, so both BBN and the CMB “see” the same $N_{\text{eff}}$.

In the opposite limit, $\tau_X > \sim 1000$ sec, the contours in Fig. 2 become horizontal lines. In this long-lifetime limit, all of the $X$ particles decay after BBN, and the increase in the expansion rate that alters the $^4$He abundance is due entirely to the energy density of the nonrelativistic particles before they decay. Thus, in this limit, $\Delta N_{BBN}$ becomes a function only of $Y_X m_X$ and is independent of $\tau_X$. Note that $\Delta N_{BBN}$ never goes to zero in the long lifetime limit precisely because of this contribution to the expansion rate from the nonrelativistic particles. However, by increasing $\tau_X$, one can make $\Delta N_{BBN}/\Delta N_{CMB}$
FIG. 2: Contour plot of $\Delta N_{BBN}$, the change in the effective number of neutrinos giving the same change in the primordial $^4$He abundance as a decaying particle with lifetime $\tau_X$ and energy density prior to decay parametrized in terms of $Y_X m_X$, where $Y_X$ is the initial number density of the particle relative to entropy density, and $m_X$ is the particle mass. Curves correspond to, from bottom to top, $\Delta N_{BBN} = 0, 0.1, 0.2, 0.3, 0.4, 0.7, 1, 1.5, 2$. The interesting transitional regime, then, is precisely the one we have explored: $0.1 \text{ sec} \lesssim \tau_X \lesssim 1000 \text{ sec}$. This would be the regime of interest if more precise measurements of $N_{eff}$ from the CMB and BBN yielded nonzero values for both $\Delta N_{CMB}$ and $\Delta N_{BBN}$ with $\Delta N_{BBN} \neq \Delta N_{CMB}$. It this case, it is possible to simply read off, from Figs. 1-2, values of $Y_X m_X$ and $\tau_X$ that give the desired values for $\Delta N_{BBN}$ and $\Delta N_{CMB}$. Note, however, that one always has $\Delta N_{CMB} \geq \Delta N_{BBN}$ in this scenario, so this model can be falsified by observations contradicting this inequality.

As an example, we show, in Fig. 3, the limits on $Y_X m_X$ and $\tau_X$ using the upper and lower bounds on $\Delta N_{CMB}$ from Ref. [12] and the upper bound on $\Delta N_{BBN}$ from Ref. [13]. As expected, the upper bound on $\Delta N_{BBN}$ cuts into the region favored by $\Delta N_{CMB}$ at short lifetimes, but current bounds are not sufficiently restrictive for this to be a major effect. Tighter bounds from future observational data will, of course, shrink this allowed region.

Our results can be generalized to more complicated scenarios, such as a particle that decays into both dark radiation and electromagnetically-interacting particles. Even a small branching ratio into the latter can produce a markedly different effect on BBN if the decay products are energetic enough to photofission the primordial nuclei (see, e.g., Refs. [36–38] and references therein). These effects, however, tend to be minimal for the lifetimes ($\tau_X \lesssim 10^3 \text{ sec}$) considered here, since the electromagnetically-interacting particles thermalize rapidly at high temperatures. Even in this case, however, the electromagnetically-interacting decay products can heat the photons relative to the neutrino background, potentially decreasing $N_{eff}$ instead of increasing it [39].

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FIG. 3: Observational constraints from the CMB and BBN in the $\tau_X, Y_X m_X$ plane, where $\tau_X$ is the decaying particle lifetime, $Y_X$ is the initial number density of the particle relative to entropy density, and $m_X$ is the particle mass. The region between the two solid curves is allowed by the CMB bounds on $N_{\text{eff}}$ from Ref. [12], while the area below the dashed curve is the allowed region from BBN limits on $N_{\text{eff}}$ given by Ref. [15]. The shaded (yellow) region satisfies both constraints.

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