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Mode Tracking of Unidirectional Carbon-Based Composite Structures Using Modified Mode Shape Vectors

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Abstract: A comparison of mode shapes in isotropic structures can be efficiently performed using the modal assurance criterion (MAC) to determine the similarity between mode shape vectors. However, the unidirectional, carbon-based composite (UCBC) structure shows different dynamic characteristics according to the carbon fiber orientation, even for the same structural configuration. The MAC of a certain mode may result in a poor value for the CBC structures in the case of the existence of the distorted mode shape vector from reinforced carbon fibers. In this study, the mode tracking of the UCBC structure is proposed using the MAC value only under the modified mode shape vector to enhance the MAC value between relevant modes. Because the mode shape vectors of the UCBC structure are altered from those of the isotropic structure owing to the reinforced stiffness along the carbon fiber orientation, the modified mode shape vectors are calculated by multiplying the original vectors with the proposed modification window. The proposed method was verified for simple UCBC structures with five different carbon fiber orientations, from 0° to 90°. The UCBC structures were tracked for five modes, three bending and two torsional, and the results were discussed with reference to earlier study results.

Keywords: modified mode shape vector; modification window; unidirectional carbon-based composite structure; modal assurance criterion; experimental modal analysis

1. Introduction

Carbon-based composite (CBC) materials have been used in industries owing to their high specific stiffness and adequate damping qualities, and rigorous studies have been focused on the development of lightweight products or manufacturing processes. The Belleville spring was proposed, using the non-axisymmetric behavior [1], and both the reinforced concrete beam [2] and the external confinement by a carbon-fiber-reinforced plastic [3] were suggested to enhance the endurance capacity over given loading conditions. The embedment of ultra-thin electrospun nanofibers for the damage-tolerant composite structure was studied [4], and the application of laser fabrication of fiber-reinforced polymer was experimentally investigated to provide technical references [5]. The understanding of the anisotropic nature of the CBC structure or material itself is still a very important issue for advancing from current reinforced composites or development-related applications. In particular, some jobs were devoted to identifying the structural strength or damping properties; the bending resonance method [6] was proposed for the directional-dependent dynamic material property, and damping coefficients were measured for flax-epoxy composite plates [7], thermoplastic organic sheets [8]; the characterization of failure strain was conducted for three reinforcements, woven jute, carbon (3k), and glass fibers under the on- and off-axis [9]. A simple but reliable identification method is modal analysis from the prepared frequency response function (FRF) under the linearity of the target system. The modal analysis technique was used to identify the dynamic nature of CBC structures; the anisotropic panel constants were experimentally obtained.
through modal analysis [10], and the dynamic behavior of anisotropic simple structure modes, beam and arch, were simulated with the boundary element method [11]; a non-contact laser Doppler vibrometer was used for the experimental modal analysis of anisotropic composite laminates [12], and the complex laminate stiffness values were computed via modal analysis after introducing the extended version of the Resonalyser procedure of single-layer fiber-reinforced composites [13].

Modal Assurance Criterion (MAC) is one well-known scalar indicator used to validate modal parameters by calculating the similarity between two mode shape vectors between 0 and 1; the similarity between two vectors rates as high for a value of 1 and as low for a value of 0. For example, two distinct modes should have a very low MAC value owing to the orthogonality between two eigenvectors [14,15]. MAC was widely used for the validation of simulation modal in isotropic structures [16], model updating processes [17], or operational modal analyses [18]. The application of MAC was also extended to composite structures so that most previous studies were devoted to the validation of the identified modal parameters of composite structure [19–21]. In particular, a modified mode shape algorithm was applied for the polar–orthotropic circular composite structure via ZERNIKE polynomials and compensation of phase of eigenvectors [22]. The proposed modification of mode shape is effective for the circular-shape composite structure but not applicable for the general-shape composite structures.

The mechanical properties of unidirectional carbon-based composites (UCBC) are dependent on both reinforcement and matrix; the structural stiffness is dominantly influenced by carbon fibers, and the equivalent damping coefficient is covered mainly by the binding matrix. Since the dynamics of UCBC structure were varied according to the carbon fiber orientation, previous studies investigated the sensitivity of the frequency-response function over several parameters, the service temperature, and the spectral loading pattern [23,24]. The sensitivity of modal parameters was discussed for a simple UCBC specimen over different five carbon fiber orientations in a previous study [25], and the relationship between the structural stiffness and the viscous damping coefficient was investigated for the same composite structure [26]. The tracking method of identified modes of interest is essential to provide reliable FRFs or modal information of the UCBC structure, so a direct investigation of the mode-tracking method was developed using multiple indicators, such as the MAC value, resonance frequency, and viscous damping coefficient in a recent study [27]. The reliable indicator of the MAC value was not sufficient to evaluate the similarity of mode shape vectors for the carbon-fiber-reinforced plastic structure because the mode shape vectors were distorted from the reinforced orientation of structural strength by the carbon fibers. Therefore, multiple indicators were proposed to tackle the limitation of mode-order tracking using only the MAC values. In that study, the variations in both the resonance frequency and the viscous damping coefficient were introduced as supplementary indicators because the two indicators increased or decreased according to their carbon fiber orientations. In this study, the feasibility of UCBC-structure mode tracking was studied using only the MAC value. The enhancement of MAC values can be expected by introducing a modified mode shape vector by multiplying the original mode shape vectors with a modification window. The proposed mode shape vector modification window was determined using the calculated distance between the measurement points and the normal line subjected to the carbon fiber orientation. The number of measurement locations was increased from seven to ten for the simple UCBC specimen, and the reference mode shape vectors were previously obtained from the experimental modal test of the isotropic SUS304 specimen. The mode shape vectors of the UCBC specimens were also obtained under the same experimental conditions as those used for the modal test. The proposed modification window was obtained from the distance of ten measurement locations, and the modified mode shape vectors were derived by multiplying the original mode shape vectors with the modification window. The evaluation of the proposed mode-tracking method was discussed for the two MAC value cases, using the original and modified mode shape vectors.
The highlight of this study was enhancing the reliability of tracking results of UCBC structures via the application of the modified MAC values in modes of interest, and its feasibility was verified experimentally with the comparison of the MAC values of the simple UCBC specimen. Based on a previous study [27], it is difficult to consider other indicators, both variations in the resonance frequency and the viscous damping coefficient as efficient mode-tracking tools because both indicators are susceptible to the selected mode case or configuration of composite structures. Because the modified mode shapes were derived from the modification window compensating for the distorted behavior from the carbon fibers, the proposed method is surely independent of the dynamics of the system and can be extended into general composite structures. The best study results may be successfully obtained using the tracking strategy with only the modified MAC values, and this study is the first step towards finding dynamic variations in composite structures using the modification of mode shapes in composite structures.

2. Mode Tracking Using the Modified Mode Shape Vector of the UCBC Structure

The linear time-invariant system generally consists of isotropic materials, and the dynamic behavior can be represented by the modal parameters, resonance frequency, and viscous damping coefficient. The n-degree-of-freedom governing equation of the linear system can be expressed by column vector \( X(t) = [x_1(t) \ldots x_n(t)]^T \) with three system matrices, mass \( (M) \), damping \( (C) \), and stiffness matrix \( (K) \), as shown in Equation (1). Here, \( [A]^T \) denotes the transposed matrix of matrix \( A \):

\[
M\ddot{X}(t) + CX(t) + KX(t) = [0 \ldots 0]^T
\]  

(1)

The time domain expression in Equation (1) can be transformed into the frequency domain by column vector \( R = [r_1(t) \ldots r_n(t)]^T \) with the relationship in Equation (2), and the transformed governing equation can be formulated as shown in Equation (3):

\[
X(t) = \Phi R
\]  

(2)

\[
\begin{bmatrix}
1 & \text{zeros} \\
\text{zeros} & 1
\end{bmatrix} \ddot{R} + \begin{bmatrix}
2\omega_{n,1}\xi_1 & \text{zeros} \\
\text{zeros} & 2\omega_{n,n}\xi_n
\end{bmatrix} \dot{R} + \begin{bmatrix}
\omega_1^2 & \text{zeros} \\
\text{zeros} & \omega_n^2
\end{bmatrix} R = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\]  

(3)

Here, the modal parameters \( \omega_{n,i} \) and \( \xi_i \) are defined as the resonance frequency and modal damping ratio in the i-th mode, respectively. In addition, \( \varphi_i \) denotes the mass-normalized mode shape vector in the i-th mode, and the matrix \( \Phi = [\varphi_1 \ldots \varphi_n] \) contains all mass-normalized mode shape vectors. The transformed damping coefficient is assumed to be the viscous damping coefficient proportional to the decoupled resonance frequency, and the viscous damping coefficient at the i-th mode \( (c_i) \) is expressed as in Equation (4). The structural stiffness at the i-th mode \( (k_i) \) is also defined in Equation (5):

\[
c_i = 2\omega_{n,i}\xi_i
\]  

(4)

\[
k_i = m_i(\omega_{n,i})^2
\]  

(5)

Here, \( m_i \) is modal mass at the i-th mode. The advantage of the linearized formulation shown in Equation (2) is the decoupled dynamics in each mode such that the dynamic behavior of the i-th mode can be expressed by the frequency response function between two locations, \( m \) and \( n \), as shown in Equation (6). Here, \( \omega_i, r_{mn}^i \), and \( j \) represent the frequency, residual, and imaginary units, respectively:

\[
H_{mn}^i(\omega) = \frac{r_{mn}^i}{(\omega_{n,i})^2 - \omega^2 + 2\omega_{n,i}\omega\xi_n^i}
\]  

(6)

The resonance frequencies are selected at the repeated peak points in the measured FRFs, except for the nodal point (zero response location), and the corresponding modal
damping ratio is obtained from the relationship between the half-power points and the resonance frequency. The i-th modal damping ratio ($\xi_{n,i}$) can be calculated using Equation (7). Here, $|a|$ denotes the absolute value of variable $a$, and $\omega_{n,i}^{(2)}$ and $\omega_{n,i}^{(1)}$ denote half-power points at resonance frequency $\omega_{n,i}$:

$$\xi_{n,i} = \frac{\omega_{n,i}}{2|\omega_{n,i}^{(2)} - \omega_{n,i}^{(1)}|}$$  \hspace{1cm} (7)

The modal parameter identification process is allowable for the UCBC structure if the composite structure can be assumed to be linear. However, the modal parameters of the UCBC structure can be altered under three conditions: temperature, spectral loading pattern, and carbon fiber orientation [23,24]. If the temperature is fixed at room temperature (22 °C), and the spectral loading pattern is only given by the impact force, the FRF of the UCBC structure in the i-th mode can be expressed as a function of the carbon fiber orientation, $\theta$, as shown in Equation (8). The FRFs of mechanical systems can be obtained via experimental modal analysis using an impact hammer or exciter over a fixed spectral loading pattern of external force, that is, impact, random, or harmonic, such that the FRF in Equation (8) can be a function of only the carbon fiber orientation, $\theta$:

$$H_{mn}^i(\omega, \theta) = \frac{r^i_{mn}}{\left(\omega_{n,i}(\theta)\right)^2 - \omega^2 + 2\omega\omega_{n,i}(\theta)\xi_{n,i}(\theta)j}$$  \hspace{1cm} (8)

The MAC value is a general indicator for validating the measured modal parameters using the orthogonal properties of eigenvectors and can be used to evaluate the similarity between mode shape vectors. If the mode shape vector in the i-th mode ($\varphi_i$) is compared with other vectors in the j-th mode ($\varphi_j$), the MAC formulation can be expressed as Equation (9):

$$MAC(i,j) = \frac{|(\varphi_i^T(\varphi_j^*)_j)|^2}{((\varphi_i^T(\varphi_j^*)_j) (\varphi_j^T(\varphi_j^*)_j)^*)}$$  \hspace{1cm} (9)

Here, $(\varphi_j^*)_j$ and $|\varphi_j|$ denote the complex conjugate and the absolute value of $\varphi_j$, respectively, and the maximum value of Equation (9) is 1 if $i=j$. If the number of measured locations is assumed to be $N$, several parallel sectional lines can be presented orthogonal to the carbon fiber orientation, as illustrated in Figure 1. A single projected line that passes through the center of the UCBC structure ($x_0$) can be introduced among all sectional lines, and all measured points can be projected on the single line at a distance from the normal line. Both the normal and reference lines pass through the center ($x_0$) of the UCBC structure with an angle difference, $\theta$. 
The distance of a certain measurement point from the normal line in Figure 1 can be explained by the graphical illustration in Figure 2. If one point is assumed to be located at \((z_x, z_y)\) and the normal line is given as \(Y = \cot(\theta)X\), distance \(d_x\) can be formulated as Equation (10) [28]. In addition, the row matrix \((D)\) of the distances of all \(N\) measured points on the projected line in Figure 1 can be obtained as shown in Equation (11).
\[
d_x(\theta) = \frac{|\cot(\theta)z_x - z_y|}{\sqrt{(\cot(\theta))^2 + 1}}
\]

\[
D(\theta) = [d_1(\theta) \quad d_2(\theta) \quad \ldots \quad d_{N-1}(\theta) \quad d_N(\theta)]
\]

The mode shape vector distortion from the isotropic material structure is caused by the reinforced stiffness of the carbon fiber, and the structural rigidity increases with respect to the carbon fiber orientation. Because the increased structural rigidity decreases the radius of curvature in the deflected structure, the decreased mode shape vector at the sectional line must be modified for comparison with other mode shape vectors with different carbon fiber orientations. Therefore, a modification window \( \hat{W}(\theta) \) at carbon fiber orientation \( \theta \) is proposed to modify the deflection value as proportional to the distance from the normal line (see Figure 1) in Equation (12). The maximum distance is one, and the other points become less than one with the distance ratio between the responsible and maximum points.

\[
\hat{W}(\theta) = \frac{D(\theta)}{\max(D(\theta))}
\]

Using the modification window, the modified mode shape vector \( \varphi^m \) can be derived as in Equation (13). Here, \( \text{diag}(\hat{W}) \) denotes the diagonal matrix from low matrix \( \hat{W} \).

\[
\varphi^m = \left( (\varphi) ^T \cdot \text{diag}(\hat{W}) \right) ^T
\]

Because the deflection becomes smaller as it approaches the center of the UCBC structure, the radius of the curvature in the deflected UCBC structure will be amplified. This concept can be explained by the \( N \) set of the distance row matrix, as shown in Figure 3. If the maximum distance is set as \( d_N \), the original mode shape decreases as it approaches the origin of the UCBC structure, \( x_0 \). Therefore, the radius of curvature of the modified mode shape is inversely amplified by applying the modification window to the original mode shape vector.

![Figure 3. Original and amplified mode shapes of the UCBC structure; dashed: original, solid: modified.](image)

The modified mode shape relieves the distorted deflection in the carbon fiber orientation such that the similar result of the mode shape vector at the same mode order may reveal a high MAC value in UCBC structures. The MAC formulation can be expressed as Equation (14):

\[
MAC(i,j) = \frac{|(\varphi_i^m)\dagger(\varphi_j^m)|^2}{((\varphi_i^m)^\dagger(\varphi_i^m))(\varphi_j^m)^\dagger(\varphi_j^m)^\dagger)}
\]

(14)

3. Mode Shape Vector Identification of the UCBC Structure and MAC Value Calculation

The dynamic characteristics of the UCBC structure were determined using an experimental model test with an impact hammer, and the target UCBC structure was a simple specimen, which was the same as that used in previous studies [25–27]. The configuration of the simple rectangular specimen was 80 mm (W) x 150 mm (L) x 3 mm (H) and made with a 12-layer, stacked, unidirectional, pre-implemented (UD prepreg) USN 250A (SK Chemical, Seongnam, Republic of Korea), as illustrated in Figure 4 [29]. A large 12-layered UCBC plate was manufactured using an autoclave curing process (125 °C maximum temperature) and cut into simple specimens with five orientations: \( \theta_1 = 0^\circ \), \( \theta_2 = 30^\circ \), \( \theta_3 = 45^\circ \), \( \theta_4 = 60^\circ \), and \( \theta_5 = 90^\circ \).

![Figure 4](image)

Figure 4. Simple rectangular UCBC specimen with 12-layered UD prepreg and carbon fiber orientation \( \theta_j \).

Ten locations of uniaxial (+Z direction) accelerometers were selected so as to distribute them by maximizing the observability of FRFs in UCBC structures, as shown in Figure 5. All attached uniaxial accelerometers (model: 3225F2, Dytran, Chatsworth, CA, USA) had very small weights (each accelerometer: 1 g) compared to that of the UCBC specimen (56.5 g), such that the mass loading from the accelerometer could be neglected. The distance of ten measurement points (#1–#10) from the normal line is illustrated in Figure 6 for carbon fiber orientation \( \theta_j \).
Figure 5. Sensor attachment locations on the UCBC specimen: $\alpha$: 3 mm, $\beta$: 20 mm, $\gamma$: 37.5 mm.

Figure 6. Distance between measurement points (#1–#10) and the normal line for the UCBC specimen with carbon fiber orientation $\theta_j$.

The FRFs of the UCBC specimens were measured using Test.Lab equipment (Siemens, Munich, Germany), an impact hammer (Model: 5800B3, Dytran, Chatsworth, CA, USA), and ten accelerometers (#1–#10 in Figure 5). The free–free boundary condition was assigned to the UCBC specimen by hanging it with low-stiffness rubber bands, as shown in Figure 7a. The impact force was assigned at #3, FRFs were obtained by averaging ten times of the same data, and the frequency range was set between 10 Hz and 6400 Hz [25–27].
Figure 7. Free–free boundary condition for the experimental modal test: (a) UCBC specimen, (b) 304SS specimen.

The mode shape vectors of each UCBC specimen could be calculated for the candidate stable peak frequency points using the PolyMAX algorithm in the Test.Lab equipment. The reference mode shape vector was preliminarily obtained via the same condition as that of the modal test of the isotropic 304SS specimen, as shown in Figure 7b. The configuration of the SUS304 specimen was 80 mm (W) × 150 mm (L) × 3 mm (H). The force input was assigned from the impact hammer (Model: 5800B3) at #4, and ten accelerometers (model: 3225F2) were attached at the same location, as shown in Figure 5. The identified modal parameters of the 304SS specimen are listed in Table 1.

Table 1. Modal parameters of 304SS specimen.

| Resonance Frequency (Hz) | Modal Damping Ratio (%) | Mode Shape       |
|--------------------------|--------------------------|------------------|
| 647.2                    | 0.6                      | Bending (first)  |
| 734.5                    | 0.6                      | Twisting (first) |
| 1601                     | 0.8                      | Twisting (second)|
| 1773.2                   | 0.6                      | Bending (second) |
| 2341.2                   | 0.6                      | Bending (third)  |

The dynamic characteristics of five UCBC specimens were also calculated for two modal parameters, the resonance frequency and the modal damping ratio, for the frequency range between 10 Hz and 6400 Hz. To investigate the five interesting modes of the UCBC specimens, three bending and two torsional modes, the reference mode shape vectors specified in Table 1 were used to compare their similarity with those from five UCBC specimens. To overcome the low MAC values of the UCBC specimens obtained in previous studies [27], the distorted mode shape vectors, owing to the carbon fiber orientation, were relieved by the modified mode shape vectors of the UCBC specimens in Equation (12). MAC values were calculated between the reference mode shape vectors and the UCBC specimen vectors using Equation (13), and the information from the first five modes of the UCBC specimens were derived from the MAC results, as shown in Table 2. The detailed mode shape vectors of first five modes are listed in Appendix A.
Table 2. Modal parameters and MAC values of UCBC specimens.

| Specimen | Resonance Frequency (Hz) | Modal Damping Ratio (%) | MAC  | Mode Type       |
|----------|--------------------------|-------------------------|------|-----------------|
| UCBC #1  | 1961.6                   | 5.4                     | 82.7 | Bending (first) |
|          | 318                      | 3.8                     | 97.2 | Torsional (first) |
|          | 1064.5                   | 2.7                     | 86.1 | Torsional (second) |
|          | 3103.6                   | 5.8                     | 51.7 | Bending (second) |
|          | 884.1                    | 2.3                     | 77.3 | Bending (third)  |
| UCBC #2  | 2539.4                   | 7.4                     | 55.5 | Bending (first) |
|          | 306.5                    | 2.5                     | 42   | Torsional (first) |
|          | 749.3                    | 2.5                     | 56.2 | Torsional (second) |
|          | 2162.1                   | 4.4                     | 52.9 | Bending (second) |
|          | 1199.8                   | 2.1                     | 44.2 | Bending (third)  |
| UCBC #3  | 285.7                    | 2.6                     | 49.4 | Bending (first) |
|          | 520                      | 1.9                     | 80.3 | Torsional (first) |
|          | 721.1                    | 1.9                     | 53.4 | Torsional (second) |
|          | 2069.3                   | 3.4                     | 65.4 | Bending (second) |
|          | 1688                     | 2.3                     | 83.8 | Bending (third)  |
| UCBC #4  | 267.6                    | 3                       | 64.8 | Bending (first) |
|          | 387.1                    | 2.3                     | 91.8 | Torsional (first) |
|          | 881.6                    | 2.2                     | 79.4 | Torsional (second) |
|          | 680.8                    | 3.4                     | 39.6 | Bending (second) |
|          | 2504.9                   | 4.8                     | 72.2 | Bending (third)  |
| UCBC #5  | 158.3                    | 3.5                     | 64.1 | Bending (first) |
|          | 2480.5                   | 5.2                     | 83.5 | Torsional (first) |
|          | 336                      | 6                       | 72.8 | Torsional (second) |
|          | 425.6                    | 4                       | 87.6 | Bending (second) |
|          | 2581.1                   | 0.7                     | 35.9 | Bending (third)  |

The efficiency of the proposed modified mode shape vector was verified by comparing the MAC values with those from the original vectors, as summarized in Table 3. The average MAC value of the UCBC specimens was higher for the modified mode shape vector than for the original mode shape cases, except for UCBC #2.

Table 3. Modal parameters and MAC values of UCBC specimens.

| Specimen | Modified Mode Shape | Original Mode Shape | Mode Type       |
|----------|---------------------|---------------------|-----------------|
|          | Each MAC | Averaged MAC | Each MAC | Averaged MAC |          |
| UCBC #1  | 82.7      | 97.2      | 86.1      | 51.7      | 77.3      | 73.7 | 81.6 | 69.4 | Bending (first) |
|          | 55.5      | 42        | 56.2      | 52.9      | 44.2      | 46.7 | 66.1 | 65.7 | Torsional (first) |
| UCBC #3  | 49.4      | 80.3      | 53.4      | 65.9      | 65        | Bending (first) |

The efficiency of the proposed modified mode shape vector was verified by comparing the MAC values with those from the original vectors, as summarized in Table 3. The average MAC value of the UCBC specimens was higher for the modified mode shape vector than for the original mode shape cases, except for UCBC #2.
The results in Table 3 revealed that the proposed method could enhance the MAC values for most mode types of UCBC specimens but did not always guarantee increased MAC values with the modified mode shape vector. In particular, the average MAC value for UCBC specimen #2 decreased from 65.7 to 50.2. This might be caused by the mismatch of mode shape vectors with reference vectors after applying the modification window in Equation (11). Indeed, the proposed modification window is proportional to the distance of the measurement point from the normal line only and does not evaluate the effectiveness of the MAC values in each mode type or carbon fiber orientation. Rather, this study focuses on the feasibility of the modified mode shape vector with a modification window to enhance the MAC values of the UCBC structure.

Figures 8 and 9 show the variations in the structural stiffness and viscous damping coefficient with the increase in carbon fiber orientation, respectively, and both variation trends showed different results as compared to those from previous studies [25–27]. The simple but direct analysis will be to check it again with other two other multiple indicators, variations in the resonance frequency and viscous damping coefficient, as was performed in the reference [27]. The other approach is focused on updating the proposed mode shape by applying for a different modification window rather than the current one, which is just proportional to the distance between the measurement location and the normal line. The optimal curvature or line of the modification window may enhance the MAC values of UCBC structure as future research.

|     | Norm. Stiffness (kN/kg) |     | Norm. Stiffness (kN/kg) |
|-----|-------------------------|-----|-------------------------|
|     | 65.4                    | 64.3| Bending (second)        |
|     | 83.8                    | 53.2| Bending (third)         |
| UCBC #4 | (θ = 60°)              |     |                         |
|     | 64.8                    | 88.8| Bending (first)         |
|     | 91.8                    | 93.3| Torsional (first)       |
|     | 79.4                    | 69.6| Bending (second)        |
|     | 39.6                    | 47.9| Bending (second)        |
|     | 72.2                    | 42.9| Bending (third)         |
| UCBC #5 | (θ = 90°)              |     |                         |
|     | 64.1                    | 81.8| Bending (first)         |
|     | 83.5                    | 53.3| Torsional (first)       |
|     | 72.8                    | 68.8| Bending (second)        |
|     | 87.6                    | 82.4| Bending (second)        |
|     | 35.9                    | 9.8 | Bending (third)         |

**Figure 8.** Variations in the structural stiffness considering only MAC values: (a) bending modes, –○–: first bending mode; –□–: second bending mode; –×–: third bending mode; (b) twisting modes, –○–: first twisting mode; –□–: second twisting mode.
Figure 9. Variations in the viscous damping coefficient considering only MAC values: (a) bending modes, ––: first bending mode; ––: second bending mode; ––: third mode; (b) twisting modes, ––: first twisting mode; ––: second twisting mode.

4. Conclusions

The usefulness of the modified mode shape vector of the UCBC specimens was determined by multiplying the original vector with the modification window. The modification window was proposed as the normalized distance row matrix after calculating the distance between the measurement point and the normal line along the carbon fiber orientation. The similarity of the mode shapes of the UCBC structure was distorted by the decreased deflection by the reinforced carbon fibers with respect to the carbon fiber orientation; thus, the amplification of the radius of curvature on the mode shape is expected to enhance the MAC value of the UCBC structure. The proposed modification of the mode shape vector was verified using the MAC values of simple UCBC specimens with five different carbon fiber orientations, from 0° to 90°. Reference mode shapes were previously obtained via an experimental modal test of an isotropic SUS304 specimen. The enhancement of MAC values can be attained for five interesting modes, three bending and two torsional, using a modified mode shape vector instead of the original vector. However, the MAC value with the modified mode shape cannot validate for some cases, such as in UCBC specimen #2. Therefore, this study focused on the feasibility of the modified mode shape vector for MAC value calculation. Further studies should be conducted to determine the optimal modification window considering the carbon fiber orientation or the mode type.

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Appendix A
Table A1. Mode shape vector of SUS304 specimen. (i: imaginary unit).

| Mode 1       | Mode 2       | Mode 3       | Mode 4       | Mode 5       |
|--------------|--------------|--------------|--------------|--------------|
| 4.51 - 0.09i | -3.11 + 0.05i| 5.19 + 0.07i | 2.43 - 0.00i | -7.02 + 0.11i|
| 4.17 - 0.08i | 3.00 - 0.09i | -4.88 - 0.06i| 2.36 - 0.03i | -6.83 + 0.15i|
| -1.11 + 0.01i| -0.97 + 0.00i| -0.86 + 0.00i| -2.27 + 0.01i| 0.61 + 0.01i  |
| -1.09 + 0.05i| 1.00 - 0.00i | 0.77 + 0.00i | -2.40 + 0.03i| 0.78 - 0.06i  |
| -0.89 + 0.09i| 1.04 + 0.04i | -0.61 + 0.03i| 2.30 - 0.00i | 0.51 - 0.10i  |
| -0.91 + 0.02i| -1.03 + 0.04i| 0.65 + 0.01i | 2.27 - 0.01i | 0.93 - 0.06i  |
| 4.21 - 0.13i | 3.04 - 0.15i | 4.86 - 0.00i | -2.31 + 0.02i| -6.58 + 0.16i|
| 4.05 - 0.14i | -2.94 + 0.02i| -4.75 - 0.05i| -1.99 - 0.01i| -6.78 + 0.24i|
| -3.48 + 0.07i| -0.02 - 0.02i| -4.30 - 0.05i| 0.02 - 0.03i | -5.39 + 0.19i|
| -3.49 + 0.07i| -0.05 - 0.00i| 4.23 - 0.00i | 0.08 + 0.00i | -4.93 + 0.14i|

Table A2. Mode shape vector of UCBC #1 specimen. (i: imaginary unit).

| Mode 1       | Mode 2       | Mode 3       | Mode 4       | Mode 5       |
|--------------|--------------|--------------|--------------|--------------|
| -8.57 - 2.65i| 5.62 + 0.92i | -10.15 - 0.40i| -6.39 + 3.48i| 12.38 - 0.17i|
| -13.34 - 4.28i| -4.91 - 3.69i| 5.01 - 1.20i | -0.11 + 1.33i| 11.14 + 0.32i|
| -1.49 + 0.87i| 1.54 + 0.07i | 2.38 + 0.15i | 6.92 + 0.63i | -2.80 + 0.08i|
| -4.28 + 2.52i| -1.20 - 1.86i| 0.32 - 0.42i | 0.36 + 1.34i | -3.13 + 0.22i|
| -2.93 + 1.94i| -1.26 - 0.32i| 1.24 - 0.41i | -0.31 - 2.31i| -2.79 + 0.23i|
| 2.94 - 2.74i | 0.85 + 4.32i | -5.38 + 0.73i| -7.66 - 0.64i| -1.94 - 0.24i|
| -9.69 + 2.88i| -5.62 + 0.06i| -2.55 - 0.52i| 2.44 + 1.54i | 10.73 - 0.00i|
| -10.58 + 0.79i| 5.70 - 0.31i| 9.60 - 1.21i | 0.58 - 2.45i | 10.47 + 0.25i|
| 16.66 - 4.28i| -0.10 + 0.34i| 5.92 - 0.84i | -3.25 - 1.50i| -0.11 + 0.17i|
| 16.53 - 0.83i| 0.13 - 0.49i | -5.81 - 0.19i| 4.33 - 0.42i | 0.16 - 0.06i |

Table A3. Mode shape vector of UCBC #2 specimen. (i: imaginary unit).

| Mode 1       | Mode 2       | Mode 3       | Mode 4       | Mode 5       |
|--------------|--------------|--------------|--------------|--------------|
| -0.83 + 2.82i| -5.59 + 2.67i| -10.56 + 1.78i| -1.42 + 1.77i| -6.10 - 0.43i|
| -0.50 + 3.11i| 2.32 - 0.04i | 1.91 + 1.15i | -8.47 + 2.94i| -6.70 - 0.12i|
| 2.44 + 2.50i | 1.71 + 0.90i | 2.66 + 0.59i | 2.51 + 0.61i | 3.22 + 0.14i |
| -1.16 - 0.41i| 1.08 - 1.13i | 1.50 - 0.82i | 6.96 + 2.15i | -1.45 + 0.05i|
| -2.30 - 1.34i| 1.37 - 1.02i | -0.75 - 0.04i| -6.76 + 1.08i| -1.40 - 0.02i|
| 1.26 + 0.93i | 0.65 + 0.57i | -4.95 + 1.71i| -2.35 + 1.66i| 3.60 + 0.06i |
| 1.99 + 2.67i | 2.69 - 0.01i | -3.34 + 1.85i| 8.82 - 0.64i | -6.70 - 0.43i|
| -0.38 + 1.28i| -5.67 + 2.50i| 9.82 - 2.07i | 1.06 - 1.66i | -5.79 - 0.15i|
| 1.60 - 5.80i | 0.89 - 1.25i | 7.30 - 2.41i | 1.44 - 1.09i | -0.32 + 0.06i|
| 3.03 - 3.82i | 1.04 - 1.07i | -7.28 + 0.23i| -1.71 - 0.31i| -0.38 - 0.40i|

Table A4. Mode shape vector of UCBC #3 specimen. (i: imaginary unit).

| Mode 1       | Mode 2       | Mode 3       | Mode 4       | Mode 5       |
|--------------|--------------|--------------|--------------|--------------|
| -21.43 + 3.14i| 3.96 + 0.05i | -8.47 + 0.01i| 4.73 + 1.42i | 1.30 + 1.43i |
| -2.26 + 0.01i| -6.28 - 0.00i| 1.32 - 0.06i | 18.42 - 0.57i| 0.69 + 13.51i|
| 0.92 + 0.10i | 1.97 + 0.11i | 4.19 + 0.03i | -1.44 + 0.05i| -0.18 + 0.86i|
| 6.69 - 1.20i | -1.08 - 0.02i| 1.98 + 0.15i | -8.05 + 0.43i| -0.024 + 1.69i|
| 6.36 - 1.43i | -1.24 - 0.11i| -1.69 - 0.17i| 5.16 + 1.88i | -0.68 + 2.13i|
| 0.47 - 1.09i | 1.88 - 0.09i | 4.33 - 0.20i | 3.15 - 0.53i | -0.61 + 1.31i|
| -1.92 + 2.32i| -6.42 + 0.12i| -1.07 + 0.43i| -15.42 + 2.44i| 1.84 + 12.35i|
| -21.66 + 2.14i| 4.13 + 0.07i| 8.55 + 0.35i | -6.28 + 1.55i| 1.13 + 3.72i |
| 9.45 - 0.89i | 3.82 - 0.09i | 5.27 - 0.10i | 3.04 - 2.61i | 0.85 - 0.52i |
| 9.70 + 0.88i | 3.61 + 0.29i | -5.62 + 0.18i| 1.61 - 0.46i | 0.27 - 1.72i |
Table A5. Mode shape vector of UCBC #4 specimen. (i: imaginary unit).

| Mode 1          | Mode 2          | Mode 3          | Mode 4          | Mode 5          |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| −11.51 + 2.76i  | 4.85 + 0.20i    | −2.80 + 0.39i   | −9.01 + 0.19i   | −5.48 + 4.63i   |
| −5.03 + 1.56i   | −6.20 + 0.52i   | 6.07 + 0.48i    | −0.15 + 0.06i   | −8.51 + 6.11i   |
| 1.92 + 0.26i    | 1.79 + 0.13i    | −0.79 + 0.00i   | 4.81 + 0.07i    | 1.68 + 1.90i    |
| 3.86 + 0.51i    | −1.38 + 0.06i   | −4.95 + 0.54i   | 2.43 + 0.05i    | 1.56 − 1.68i    |
| 3.48 + 0.17i    | −1.64 + 0.28i   | 5.11 − 0.40i    | −2.87 + 0.17i   | 0.32 + 2.50i    |
| 1.36 − 0.50i    | 1.95 − 0.12i    | 1.26 + 0.08i    | −4.63 − 0.06i   | 1.02 + 1.13i    |
| −5.05 + 0.12i   | −6.04 + 0.02i   | −5.59 + 0.27i   | −0.11 + 0.06i   | −0.71 − 3.82i   |
| −10.87 + 0.58i  | 5.31 − 0.49i    | 3.33 − 0.42i    | 8.37 + 0.10i    | −1.19 − 1.66i   |
| 6.92 − 1.37i    | 2.17 − 0.19i    | 3.78 − 0.24i    | 4.89 − 0.22i    | −4.59 − 1.68i   |
| 6.81 + 0.51i    | 1.73 − 0.45i    | −2.98 − 0.57i   | −5.35 + 0.40i   | −3.42 + 2.78i   |

Table A6. Mode shape vector of UCBC #5 specimen. (i: imaginary unit).

| Mode 1          | Mode 2          | Mode 3          | Mode 4          | Mode 5          |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| −3.33 − 2.35i   | 8.46 + 1.89i    | −8.15 + 0.35i   | 3.39 + 0.79i    | 0.79 − 0.65i    |
| −9.64 + 0.96i   | −3.73 − 0.34i   | 7.41 − 1.10i    | 2.98 − 0.21i    | 0.87 + 1.19i    |
| 2.89 + 0.38i    | 6.10 + 0.29i    | 2.09 + 0.27i    | −2.89 + 0.19i   | −0.02 + 0.92i   |
| 0.93 + 1.53i    | −6.99 + 1.87i   | −2.82 + 1.28i   | −2.87 − 0.67i   | −0.57 − 0.05i   |
| 1.15 − 1.07i    | −5.89 − 0.13i   | 1.86 + 0.35i    | 2.21 − 1.90i    | −0.78 + 0.14i   |
| 3.59 + 0.83i    | 9.01 + 0.81i    | −2.40 + 0.42i   | 2.15 − 0.41i    | 1.32 + 2.1i     |
| −10.18 + 1.63i  | −4.25 + 1.01i   | −7.30 + 1.39i   | −2.71 + 0.08i   | −0.35 + 0.44i   |
| −4.35 − 0.08i   | 2.63 + 2.74i    | 6.46 + 0.12i    | −1.49 − 1.33i   | 0.03 + 1.75i    |
| 6.12 + 1.53i    | −1.09 − 2.67i   | 8.09 − 0.48i    | 0.04 − 1.73i    | −2.33 + 0.04i   |
| 7.02 − 0.80i    | −0.84 + 3.58i   | −8.91 + 0.71i   | −0.20 + 0.70i   | 0.69 + 0.40i    |

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