RENORMALIZATION AND RESUMMATION IN FIELD THEORIES

A. JAKOVÁC*
HAS Research Group “Theory of Condensed Matter” and TU Budapest, H-1521 Budapest, Hungary

ZS. SZÉP†
Research Group for Statistical Physics of the Hungarian Academy of Sciences, H-1117 Budapest, Hungary

Using resummation in perturbation theories at finite temperature or in non-equilibrium is unavoidable to obtain consistent results. Resummation, however, is often in conflict with renormalization. In this talk we give two possible solutions to problems of this type in static resummation. One is the use of auxiliary finite temperature UV divergent counterterms. The other is to use a renormalization scheme fitted to the environment. We also suggest a way how one could proceed in case of dynamic resummations.

1. Introduction

In perturbation theories at finite temperature and non-equilibrium environment we are often faced with infrared (IR) divergent diagrams. Although this divergence can be of physical origin, but more often it is just an artefact of a not properly organized perturbation series, and it disappears after summing up a certain subset of diagrams.

A zero temperature example is the on-shell singularity of the perturbation series of a propagator. In this case one uses 1PI resummation (Schwinger-Dyson equations) that makes possible to shift the inverse propagator. At finite temperature a similar phenomenon occurs since the environment modifies the properties of the propagating particles. As a consequence we have to count with temperature dependent mass terms\(^1\), or self-energies\(^2\). In these cases the Schwinger-Dyson resummation is again

\*e-mail: jakovac@planck.phy.bme.hu
\†e-mail: szepzs@achilles.elte.hu
necessary to avoid divergent perturbation series. Coupling constant resum-
mation is needed near a second order phase transition point or in case of non-Abelian gauge theories.

In many cases it happens, however, that new, unexpected, temperature (or, more generally, environment) dependent ultraviolet (UV) divergences appear. An illustrative example is the $\Phi^4$ model. Here the one-loop correction to the self-energy at finite temperature and in dimensional regularization reads as

$$m^2 = \frac{m^2}{16\pi^2} \left[ \frac{1}{\varepsilon} + \gamma_E - 1 + \ln \frac{m^2}{4\pi\mu^2} \right] + \frac{1}{2\pi^2} \int_{m}^{\infty} d\omega \sqrt{\omega^2 - m^2} n(\omega). \quad (1)$$

This diagram is UV divergent, so we need a mass counterterm $\delta m^2 = \frac{m^2}{16\pi^2} \frac{1}{\varepsilon}$. If we resum the mass we effectively substitute $m^2 \to M^2_T$, a temperature-dependent mass term, and the same diagram now requires $\delta m^2 = \frac{M^2_T}{16\pi^2} \frac{1}{\varepsilon}$. This is $T$ and $\varepsilon$ dependent in the same time, in contradiction with the physical picture that short distance physics cannot be affected by the physics in the IR.

In order to understand the problem deeper let us try to understand, why, in general, resummation and renormalization can be in conflict. The theory that defines the system is, in fact, the bare theory. The renormalized theory is already a resummation in the bare theory: the bare parameters are divided into sum of infinite parts (counterterms) and these parts are taken into account at different levels of perturbation theory. In this subdivision the organizing principle is the UV relevance of the diagrams. There is a subtle balance between the diagrams and counterterms, and a series of theorems ensure that this method finally works consistently for a certain set of theories (the perturbatively renormalizable ones).

If we encounter other type of divergencies at finite energy scales we are forced to do another resummation. In the course of this IR resummation we order the diagrams according to their IR relevance. “Normal” and counterterm diagrams, however, have different IR relevance in general, and so IR resummation will segregate them. So UV divergences do not drop out, the resummed theory will be UV inconsistent.

The above analysis suggest immediately a way out of this problem: we should resum the appropriate counterterm diagrams together with the most IR relevant “normal” diagrams. Here we describe how to do that.
2. Counterterm resummation by hand

We demonstrate the method for the mass resummation, but it will be applicable for other cases (coupling constant resummations) as well. We start from the generic procedure of resummation: we add to and subtract from the Lagrangian the same term and treat the two parts in different order in perturbation theory:

\[ -L_{\text{mass}} = \frac{m^2 + \Delta M^2}{2} \varphi^2 + \left( \frac{\delta m^2}{2} \varphi^2 - \frac{\Delta M^2}{2} \varphi^2 \right), \]

where the subscripts under the brace denote the order where the given term starts to contribute. If \( m^2 = m^2_{\text{bare}} \), \( \delta m^2_{\text{bare}} = 0 \) and \( m^2_{\text{bare}} + \Delta M^2 = m^2_R \), then we describe renormalization. If \( \Delta M^2 \) is temperature dependent, then we have the “thermal counterterm” method. In this case, however, as we have seen, \( \delta m^2 \) must be temperature dependent to make the theory finite at one loop level.

To resum counterterms we can follow the same strategy, but now at one loop level. We write

\[ -L_{\text{mass}} = \frac{m^2_R + \Delta M^2}{2} \varphi^2 + \left( \frac{\delta m^2}{2} \varphi^2 - \frac{\Delta M^2}{2} \varphi^2 \right) + \left( -\frac{\delta m^2_T}{2} \varphi^2 \right), \]

\( \delta m^2_T \), referred as “compensating counterterm” should be temperature dependent and divergent (in \( \Phi^4 \) model \( \delta m^2_T = \frac{\Delta M^2}{16\pi^2} \frac{1}{z} \)); but its value is irrelevant from the point of view of the complete theory, being subtracted one loop later. Otherwise it is a normal counterterm, its value should be determined order by order. In mass-independent schemes (where \( \delta m^2 = zm^2 \), where \( z \) is mass-independent), however, we expect that the shift of the tree level mass \( m^2 \rightarrow m^2 + \Delta M^2 \) will imply the mass counterterm shift \( \delta m^2_T = zm^2 \rightarrow zm^2 + z\Delta M^2 \), so we get \( \delta m^2_T = z\Delta M^2 \). This is not the most general case (any finite term could be added), but an especially comfortable one. In this case, namely, we arrive at

\[ \Pi_{\text{resum, MS}}^{m^2 + \Delta M^2} = \Pi_{\text{MS}}^{m^2}\bigg|_{m^2 \rightarrow m^2 + \Delta M^2}, \]

ie. we do the renormalization first, and then substitute the resummed mass into the finite expression. It can be proven that this choice is in fact consistent to all orders in perturbation theory.
3. Resummation scheme

We can make the procedure of the previous subsection more systematic by noting that the value of the tree level mass in renormalized perturbation theory is not fixed, it is in fact the consequence of the scheme chosen. The previous subsection was, from this point of view, the description of a scheme where the tree level mass is \( m_{\text{MS}}^2 + \Delta M^2 \). We will call this scheme resummation scheme. In a more generic framework resummation scheme is a scheme that suits well the perturbation theory in an external environment\(^7\). We define it that the infinite pieces of the counterterms cancel the UV divergences, while the finite pieces should diminish IR sensitivity of the theory.

As this definition shows, counterterms in a resummation scheme will depend on the environment. Therefore changing the environment will imply a change in the scheme; but results in different schemes are not comparable\(^a\). We should project the results onto a fixed reference scheme (we chose \( \overline{\text{MS}} \) for concreteness). Since we change from one scheme to another, the projections can be done by adapting the renormalized parameters of the Lagrangian\(^4\). In case of \( \Phi^4 \) theory there exist functions

\[
m = m(m_{\text{MS}}^2, \lambda_{\text{MS}}, T), \quad \lambda = \lambda(m_{\text{MS}}^2, \lambda_{\text{MS}}, T), \quad \zeta = \zeta(m_{\text{MS}}^2, \lambda_{\text{MS}}, T),
\]\n
that for any \( n \)-point function

\[
G_{\text{MS}}^{(n)}(p_i; m_{\text{MS}}, \lambda_{\text{MS}}) = \zeta^n G_{\text{resum}}^{(n)}(p_i; m, \lambda).
\]

In perturbation theory this equation holds up to a given order: the difference is the effect of the resummation performed by the resummation scheme.

Technically we determine (5) from the condition that the bare parameters should be the same\(^4\): \( m_{\text{MS}}^2 + \delta m_{\text{MS}}^2 = m^2 + \delta m^2 \) in case of mass resummation. In the on-shell scheme we obtain from this condition

\[
m^2 = m_{\text{MS}}^2 + \Pi_{\text{MS}}(m^2).
\]

This matching equation is just the gap equation in the language of the resummation.

At one loop level an equivalent form could have been reached if we had wrote \( m_{\text{MS}}^2 \) as the argument of \( \Pi_{\text{MS}} \). The two cases correspond to different resummations: one with \( \Pi_{\text{MS}}(m^2) \) yields the super-daisy, while the other with \( \Pi_{\text{MS}}(m_{\text{MS}}^2) \) the daisy resummation.

\(^a\)For example the mass in the finite temperature on shell scheme is always \( m^2 \); there is no explicit dependence on the temperature!
Resummation scheme can be used to resum any parameters of the Lagrangian in a momentum-independent way (or at a fixed momentum). For example we can resum the coupling constant in $\Phi^4$ theory. The details are in Ref.\textsuperscript{5}. The results for the matching equations (5) read in high temperature expansion

$$
\frac{1}{\lambda} - \frac{1}{\lambda_{\text{MS}}} = \frac{T}{16\pi M} - \frac{3}{32\pi^2} \ln \frac{T^2}{\tilde{\mu}^2}
$$

$$
m^2 - m_{\text{MS}}^2 = \frac{\lambda_{\text{MS}} T^2}{24} - \frac{\lambda_{\text{MS}} T m}{8\pi} + \frac{\lambda_{\text{MS}} m^2}{32\pi^2} \ln \frac{T^2}{\tilde{\mu}^2}.
$$

(8)

At high temperature $m^2 = m_{\text{MS}}^2(T) + \frac{1}{2\pi} \lambda_{\text{MS}}(T) T^2$, where the arguments refer to the renormalization scale in $\text{MS}$ scheme. Near the critical point $m \sim T - T_c$, signaling a second order phase transition. The coupling constant at high temperatures behaves as $\lambda \sim \sqrt{\lambda_{\text{MS}}(T)}$, characteristic to a 3D, dimensionally reduced theory. Near the critical point $\lambda \sim m \to 0$.

4. Outlook

So far we have spoken about momentum-independent resummations. The whole procedure goes through, however, if momentum dependence is concentrated only to the IR regime: for example when we have $\lambda(p \to \infty) = \lambda_{\text{resum}}$. In fact the UV behavior is much more under control, and it is not affected by the IR problems of the theory. So a possibility for momentum dependent cases would be to make a partial resummation of the IR momentum dependence, and leave the UV part for perturbation theory. This work is in progress.

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