Correction Factor for Analysis of MIMO Wireless Networks With Highly Directional Beamforming

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Abstract

Most theoretical studies for multiple input multiple output (MIMO) wireless networks with beamforming model the actual received signal power as a product of some fixed antenna gain and the nominal received signal power. The nominal received signal power being that from an omni-directional transmission on a single input single output (SISO) link. Since the power angular spectrum is often significantly different for line of sight (LOS) and non-LOS (NLOS) links, multiplying the same antenna gain for both LOS and NLOS links leads to erroneous estimates of the actual received signal power. We quantify the difference in effective antenna gain for NLOS and LOS links as a function of number of dominant paths in the channel using a popular narrowband geometric channel model, when the number of antennas at the transmitter and receiver is large. We call this difference the correction factor. Using the recent New Radio (NR) 3GPP channel model we confirm that the effective antenna gain for NLOS links can be about 10 dB lower than for LOS links.

I. INTRODUCTION

The power of a wireless channel is usually spread out in different physical angular directions. This is visualized in Fig. 1 using the power azimuth spectrum (PAS), which represents the received signal power at some receiver locations as a function of the azimuth angle of arrival. This phenomenon is usually captured through a geometric channel model which is a function of the angles of arrival (AOAs) and the angles of departure (AODs) of rays belonging to different clusters and the corresponding power per ray [1]. Although such models are popular in signal
processing and link level analytical studies \cite{2}, \cite{3}, as well as elaborate simulation studies \cite{3}, the inherent complexity of such models makes it extremely non-trivial to incorporate in network level theoretical analysis. Thus, usually the received signal power is modeled as a product of an angle of arrival/departure dependent antenna gain and the received signal power assuming omni-directional SISO transmissions, with at most modifying a unit mean small scale fading random variable to incorporate the effect of channel multipath \cite{4}, \cite{5}. Such a received signal power can be justified by employing beamforming on a keyhole channel matrix (which has rank 1) \cite{6}.

Traditionally, the channel was not distinguished as LOS or NLOS as the networks were not very dense and most links were NLOS. However, since next generation networks will keep getting denser, distinguishing between LOS and NLOS links has become of paramount importance to capture the different propagation conditions of these links. For example, \cite{7} and its several extensions incorporate a blockage dependent path loss model as well as unit mean small scale fading for coverage and rate analysis of mmWave cellular networks. These works still carry forward the classical assumption of the same fixed antenna gain on service links irrespective of whether the links are LOS or NLOS. However, since the power azimuth spectrum can be notably different for LOS and NLOS links, as seen in Fig. 1, the classical method to incorporate the same antenna gain for LOS and NLOS service links could be error prone, especially if the links employ highly directional beamforming. An explanation for this phenomenon is given as Remark 2 in Section III.

Since next generation networks are likely to exploit highly directional beamforming \cite{8}, it is necessary to re-consider the classical approach to incorporate same antenna gains for LOS and NLOS when using a key-hole channel model. Our proposal is to multiply a LOS or NLOS dependent correction factor $\Upsilon$ onto the desired received signal power (not interference power) using the key hole channel model. Since the received signal power is simply multiplied by a constant – different for LOS and NLOS – prior analyses done with a keyhole assumption (like \cite{7} and its extensions) are preserved. However, the numerical results will change, which could affect the engineering insights. We first undertake asymptotic analysis with number of antennas at the transmitter and receiver to estimate such a correction factor for desired signal power on LOS and NLOS links. Then we take on a similar analysis for interference power. Finally, we validate our results with Monte Carlo simulations using our simple narrowband channel as well as using the 3GPP NR channel in \cite{1} and discuss some key implications of including the
proposed correction factor values on system design.

II. SYSTEM MODEL

We will concentrate on a single transmitter-receiver pair in some wireless network. Let there be $N_t$ antennas at the transmitter and $N_r$ antennas at the receiver. If the link is NLOS, the narrowband channel between the transmitter-receiver pair is given by [1], [3], [10]

$$H_{\text{NLOS}} = \kappa \sqrt{\frac{\ell(d)}{\eta}} \sum_{i=1}^{\eta} \gamma_i a_r(\phi_i) a_t^*(\theta_i),$$

where $\ell(d)$ is the path gain (assumed deterministic function of $d$ for simplicity, although the analytical results will hold for a random $\ell(d)$), $\eta$ is the number of paths (assumed constant), $d$ is the transmission distance in meters and $\gamma_i$ is the small scale fading on path $i$ (random variable such that $\mathbb{E}[|\gamma_i|^2] = 1$) and $\kappa$ is a normalizing constant such that $\mathbb{E}[||H_{\text{NLOS}}||_F^2] = N_t N_r \ell(d)$. Note that $\eta = \kappa = 1$ implies a keyhole channel [6].

Assuming a uniform linear array at the receiver, the array response vector $a_r$ is given as follows [11].

$$a_r(\theta) = \left[1, e^{-j\theta}, e^{-2j\theta}, \ldots, e^{-(N_r-1)j\theta}\right]^T.$$  

Similarly, one can define $a_t$ by replacing $N_r$ with $N_t$. Note that $\phi_i$ and $\theta_i$ are spatial angles of arrival and departure (AOA/AOD). It is assumed that these AOAs and AODs are continuous.
**random variables.** They are related to the actual physical angles through the following equation
\[ \theta = 2\pi d \frac{\sin(\vartheta)}{\lambda}, \]
where \( \theta \) is the spatial angle, \( \vartheta \) is a physical angle, and \( d \) is inter-antenna spacing (usually chosen to be \( \lambda/2 \)). Note that we do not distinguish between intra-cluster or inter-cluster paths when we count \( \eta \). One can interpret \( \eta \) as the dominant paths, that is those contributing the most to the eigenvalues of the matrix \( H_{\text{NLOS}} \).

If the link is LOS, the narrowband channel is given by [1]
\[
H_{\text{LOS}} = \sqrt{\ell(d)} \left( \frac{K_R}{K_R + 1} a_r(\phi_0) a^*_t(\theta_0) + \kappa \sqrt{\frac{1}{\eta(K_R + 1)}} \sum_{i=1}^{\eta} \gamma_i a_r(\phi_i) a^*_t(\theta_i) \right),
\]
where \( K_R \) is the Rician K-factor. AOA and AOD given by \( \phi_0 \) and \( \theta_0 \) are constants corresponding to direct LOS path between the receiver and the transmitter. \( \eta \) and \( \kappa \) could have different LOS-specific values here, as compared to [1].

Assuming \( w \) is the combiner employed by the receiver and \( f \) is the precoder employed by the transmitter, the received signal power model is given as \( P_{\text{multi}} = ||w^* H f||^2 \), where \( H \) is the \( N_r \times N_t \) channel, which could be either \( H_{\text{LOS}} \) or \( H_{\text{NLOS}} \). The precoder and combiner are chosen so as to maximize \( ||w^* H f||^2 \), which is basically single stream beamforming.

**Goal:** We wish to compare \( P_{\text{multi}} \) with the mean received power obtained using a keyhole model (that is \( \eta = 1 \)) given by \( \mathbb{E} \left[ P_{\text{keyhole}} \right] = N_t N_r \ell(d) \), where \( \ell(.) \) could be different for LOS or NLOS links.

**Definition 1.** The proposed correction factor is defined as \( \Upsilon = \mathbb{E} \left[ P_{\text{multi}} \right] / \mathbb{E} \left[ P_{\text{keyhole}} \right] \).

Our proposal is that if one wants to use a keyhole model for simple system level analysis, then the corrected keyhole received signal power is given by \( P_{\text{keyhole}} \Upsilon \).

### III. Computing \( \Upsilon \) When \( N_t, N_r \) Are Large

Before we state the results, we make a couple of quick observations based on the results in [12].

**Observation 1:** As \( N_t, N_r \to \infty \), the array response vectors \( a_r \) in [1] and [2] converge to left singular vectors of \( H \) multiplied by \( \sqrt{N_r} \). Similarly, \( a_t(\cdot) \) in [1] and [2] converge to right singular vectors of \( H \) multiplied by \( \sqrt{N_t} \). Thus, the singular values converge to \( \sqrt{N_t N_r \gamma_i} \).

**Observation 2:** As \( N_r \to \infty \), \( a^*_r(\phi_i) a_r(\phi_j) = N_r \mathbb{I}(i = j) \). Similarly \( a^*_t(\theta_i) a_t(\theta_j) = N_t \mathbb{I}(i = j) \) as \( N_t \to \infty \).
Note that Observation 1 implies that for large number of antennas, the Frobenius norm 
\[ \|H_{\text{NLOS}}\|_F^2 = \kappa^2 \ell(d) N_t N_r \sum_{i=1}^\eta |\gamma_i|^2 / \eta. \]
Thus \( \mathbb{E} [\|H_{\text{NLOS}}\|_F^2] = N_t N_r \ell(d) \kappa^2 \), which implies that
the normalizing constant \( \kappa = 1 \). Similarly one can conclude that \( \kappa = 1 \) in (2).

**Theorem 1.** For large \( N_t \) and \( N_r \), if the link is NLOS, then
\[ P_{\text{multi}} \approx N_t N_r \ell(d) \max_{i=1, \ldots, \eta} |\gamma_i|^2 / \eta. \]
Irrespective of the number of antennas, if the link is LOS and \( K_R \gg 1 \) then
\[ P_{\text{multi}} \approx N_t N_r \ell(d) \frac{K_R}{K_R + 1}. \]

**Proof.** Optimal combiner and precoder correspond to the singular vectors corresponding to
the maximum singular value norm of \( H \). Using the asymptotic Observations 1 and 2 for finite but
large number of antennas, we can arrive at the desired result. For NLOS channel, \( w = \frac{1}{\sqrt{N_r}} a_r(\phi_1) \)
and \( f = \frac{1}{\sqrt{N_t}} a_t(\theta_1) \) assuming \( |\gamma_1| = \max_i |\gamma_i| \), without loss of generality. Thus,
\[ P_{\text{multi}} = \|w^* H f\|^2 \]
\[ \approx \|\kappa \sqrt{\ell(d)} N_t N_r / \eta \gamma_1\|^2 = N_t N_r \ell(d) \frac{|\gamma_1|^2}{\eta}. \]

For LOS, since \( \mathbb{E} [|\gamma_i|^2] = 1 \), by Markov inequality \( \mathbb{P} (|\gamma_i|^2 > \eta K_R) < 1/\eta K_R \). Thus, owing
to \( K_R \gg 1 \) one can conclude that with high probability \( w = a_r(\phi_0) \) and \( f = a_t(\theta_0) \), which
corresponds to the AOA and AOD for direct LOS path. Using Observation 2, it can be concluded
that \( P_{\text{multi}} \approx N_t N_r \ell(d) \frac{K_R}{K_R + 1} \).

**Corollary 1.** If \( \gamma_i \) are complex normal random variables and independent of each other, \( \Upsilon \approx \frac{1}{\eta} \sum_{k=1}^\eta (1/k) \) for NLOS links. If \( \gamma_i \) are all identically equally to complex normal \( \gamma_1 \), \( \Upsilon \approx \frac{1}{\eta} \) for
NLOS links. For LOS links and \( K_R \gg 1 \), \( \Upsilon \approx \frac{K_R}{1+K_R} \approx 1 \).

**Proof.** In this case, \( |\gamma_i|^2 \) are exponentially distributed random variables with unit mean. Maximum of \( \eta \) independent exponential random variables is given by \( \sum_{k=1}^\eta (1/k) \). By Theorem 1, \( \Upsilon \approx \frac{1}{\eta} \sum_{k=1}^\eta (1/k) \) if \( \gamma_i \) are complex normal random variables and independent of each other.
Similarly, the other two results in the corollary can be derived.

From Theorem 1, NLOS received signal power can be significantly overestimated with the
keyhole model for \( \eta = 10 \), which translates to \( \Upsilon = -4.6 \)dB if \( \gamma_i \) are identically equal to \( \gamma \), and
to \( \Upsilon = -10 \)dB if \( \gamma_i \) are independently but identically distributed. Note that this is an analytical
result and that well accepted wideband models (like in [1]) will have unequal distribution of powers amongst paths within and across different clusters. Estimating $\Upsilon$ in these settings is an avenue for further research. In the next section we simulate the new 3GPP channel model to highlight that $\Upsilon \ll 1$ even after considering a more realistic NLOS channel. For LOS signals, [1] indicates choosing mean $K_R = 9$dB and [13] indicates mean $K_R = 10$dB. Thus, the correction factor is about 0.9 on average for LOS and so can be neglected without much loss in accuracy.

For finite number of antennas we would expect $\Upsilon$ to be closer to 1 than in the results in Corollary 1. This is because we make use of Observation 2 to neglect the inner product of different array response vectors while arriving at the approximation in Theorem 1.

**Remark 1.** Corollary 1 will hold even if the channels (1) and (2) include a patch antenna gains multiplied to array steering vectors for uniform planar arrays (like in [2]), in which case both the single and multipath signal power will be multiplied by the maximum per-element patch antenna gain.

**Remark 2** (Intuitive explanation for $\Upsilon < 1$). Usually an omni-directional path loss model is used for analysis to enable its wider applicability to different antenna array designs at the transmitters and receivers [3]. Such a path loss model is computed as follows. First, the powers corresponding to different measured angles to generate the PAS, which are spaced such that the beam patterns corresponding to different angles of arrival have minimal overlap, are added up at the receiver. This is the aggregate received signal power. Next, path loss is computed by dividing the aggregate received power by the transmit power and the maximum antenna gains at the transmitter and the receiver [3]. Thus, if highly directional beamforming is employed for actual data transmission such that only a fraction of angular space corresponds to some maximum antenna gain, then the average received signal power estimate using the omnidirectional received signal power multiplied by the maximum antenna gain is likely to overestimate the actual average received power.

For a NLOS interfering link, the received signal power assuming a keyhole model is given by $P_{r_{\text{keyhole,int}}} = \ell(d)|\gamma|^2G_r(\phi, \phi')G_t(\theta', \theta)$, where $G_r(\phi, \phi') = \frac{1}{\sqrt{N_r}}|a_r^*(\phi) a_r(\phi')|^2$ with $\phi$ and $\phi'$ denoting the beamsteering direction at the receiver and the keyhole channel AOA. Similarly $G_t$ can be written replacing subscript $r$ with $t$ and $\phi$ with $\theta$. Unlike the desired signal power case, here $\phi$ and $\theta$ are not chosen to maximize $||w^*Hf||^2$ but can be random angles distributed
according to some continuous distribution.

\[ P_{r,\text{multi, int}} = \frac{\ell(d)}{\eta} \left| \sum_{i=1}^{\eta} \frac{\gamma_i}{\sqrt{N_tN_r}} a_r^*(\phi) a_r(\phi_i) a_t^*(\theta_i) a_t(\theta) \right|^2. \]  

(3)

The LOS interfering received power can be written similarly.

**Theorem 2.** If \( \gamma_i \) are independent zero mean random variables with unit variance, \( \{\theta_i\} \) are identically distributed, \( \{\phi_i\} \) are identically distributed and \( \{\gamma_i\} \) are independent of \( \{\theta_i, \phi_i\} \), then \( \mathbb{E}[P_{r,\text{multi, int}}] = \mathbb{E}[P_{r,\text{keyhole, int}}] \) for NLOS interfering links.

**Proof.** Using independence of \( \gamma_i \) and that these are zero mean random variables, the cross terms while expanding the norm squared in (3) become zero and thus, \( \mathbb{E}[P_{r,\text{multi, int}}] \) is equal to

\[
\frac{\ell(d)}{\eta} \sum_{i=1}^{\eta} \mathbb{E}[|\gamma_i|^2] \mathbb{E}[G_r(\phi, \phi_i)G_t(\theta_i, \theta)] = \frac{\ell(d)\eta\mathbb{E}[G_r(\phi, \phi_1)G_t(\theta_1, \theta)]}{\eta} = \mathbb{E}[P_{r,\text{keyhole, int}}].
\]

The above theorem indicates that a correction factor is not necessary for NLOS interfering links, if the assumptions in the theorem hold true. However, depending on the structure of the arrays, the per-element antenna gains and joint distribution of \( \{\gamma_i, \phi_i, \theta_i\} \) a \( < 1 \) or \( > 1 \) correction factor may be necessary for interfering links. We do not delve into computing a correction factor for NLOS interfering links, since unlike the desired signal power computations in this section, this is highly dependent on the implementation and also the mathematical models used to incorporate antenna patterns for computing interference. For example the model in [7], [14] uses a step pattern, wherein the value of back lobe gain can be modified differently for LOS and NLOS to incorporate a correction factor, if any. In the next section, we will develop some numerical intuition for the result in Theorem 1. Also we will validate our intuition by making a similar observation with the 3GPP NR channel model [1].

**IV. Numerical Results**

For Monte Carlo simulations in Fig. 2 and Fig. 3 we use the following setup. The receiver is assumed to be at the origin and potential transmitters are randomly distributed in an area of \( 1 \times 1 \) km\(^2\). All links are assumed to be NLOS. We plot the effective antenna gain on the X axis. This is defined as received signal power divided by path loss \( \ell(d) \), which eliminates the dependence on path loss function. AOA and AOD are distributed around the direct LOS path as a Gaussian random variable with standard deviation of 20°.
Fig. 2 plots the effective antenna gain cumulative distribution function (CDF) for single and multi-path channel scenarios with \( \gamma_i = \gamma \) for all \( i \). As can be seen, a keyhole model (\( \eta = 1 \)) indeed overestimates the signal power, and including a correction factor gives much better estimates. Fig. 2 shows that for a \( 64 \times 64 \) system (that is \( N_t = N_r = 64 \)), \( \Upsilon = -4 \)dB for \( \eta = 5 \) but it decreases to \(-5\)dB for \( \eta = 10 \). As \( \eta \) approaches the finite number of antennas in the simulation, the asymptotic approximation with \( N_t \) and \( N_r \) becomes weaker. See [15] for seeing how virtual channel approximation could be used for tighter analytical estimates of \( \Upsilon \). For a \( 256 \times 256 \) system, \( \Upsilon \) decreases to \(-7\)dB for \( \eta = 10 \) as compared to \(-5\) dB for a \( 64 \times 64 \) system.

Fig. 3 considers the scenario when \( \gamma_i \) are all independent with \( \eta = 10 \) and a \( 64 \times 64 \) system. In this case, \( \Upsilon \approx -1.7\)dB when unit mean small scale fading is used to compute received signal power. This will approach to \(-4.6\)dB as per Theorem 1 when \( N_t, N_r \to \infty \). Note the distribution of \( \eta = 10 \) varies significantly from “\( \eta = 1 \), unit mean exp ss”. A better match is obtained if for the keyhole model, small scale fading equivalent to \( \max_i |\gamma_i|^2 \) is incorporated, but it needs a correction factor of \(-8.4\)dB to be incorporated for a better match with the multipath channel scenario.

Finally, we validate our insight for the need to incorporate a correction factor by comparing the effective gain (averaged over small scale fading) for LOS and NLOS received signal power with 3GPP channel model at 73 GHz and a \( 64 \times 64 \) uniform planar array system. As seen from Fig. 4 there is a drop of about 12 dB in NLOS median gain compared to LOS for a \( 64 \times 64 \) MIMO system, which is very significant.

V. IMPLICATIONS

For dense outdoor-to-outdoor cellular networks, a user would likely associate with a LOS base station (BS) and thus the signal to noise ratio (SNR) coverage estimates wouldn’t vary significantly, except for the tail probability when a user associates with a NLOS BS that affects the cell edge rates. Otherwise, we expect such a correction factor to be significant since there is significant probability of connecting to a NLOS BS since the SNR distribution itself will shift by \( \Upsilon \). We expect the significance of such a correction factor to also be significant in analysis of multi-hop mmWave cellular networks wherein the fiber site deployment will be relatively sparse and thus there will be a question as to whether a relay should go for a NLOS direct hop to fiber base station or whether it should relay over multiple LOS hops. Given that the correction
Fig. 2: $\gamma_i = \gamma$ for all $i = 1, \ldots, \eta$.

Fig. 3: $\{\gamma_i\}$ are all independent and 64 x 64 system. Here, “exp ss” implies small scale fading is exponential and “max exp ss” implies small scale fading is the maximum of $\eta$ exponentials.

factor introduced in this letter doesn’t affect LOS links but strongly affects NLOS links, LOS hops will be even more strongly favoured over NLOS hops.

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