GAPLESS CFL AND ITS COMPETITION WITH MIXED PHASES

MARK ALFORD
Physics Department, Washington University, St. Louis, MO 63130, USA

CHRIS KOUVARIS AND KRISHNA RAJAGOPAL
Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

We recently argued that as the density of quark matter decreases, there is a continuous transition from the color-flavor-locked (CFL) phase to a gapless “gCFL” phase. The reason is the growing importance of the strange quark mass, and the constraint of electric/color neutrality. In this paper we discuss mixed phases that achieve neutrality by charge separation, and might offer an alternative to the gCFL phase. We find that none of the obvious mixtures is favored relative to gCFL.

1. Introduction

There is a strong possibility that quark matter may occur in the core of compact (“neutron”) stars, and at sufficiently high density it is expected to be color superconducting: the attractive QCD interaction between the quarks at the Fermi surface causes them to condense in Cooper pairs. At asymptotic densities, where the up, down and strange quarks can be treated on an equal footing and the potentially disruptive strange quark mass can be neglected, quark matter is in the color-flavor locked (CFL) phase 1, in which quarks of all three colors and all three flavors form Cooper pairs 2.

However a very interesting question is what happens at lower densities. In dense matter with quark chemical potential $\mu$ and pairing gap $\Delta$, the strange quark mass has negligible impact on CFL pairing when $M_s^2 \ll \mu \Delta$. At compact-star densities $\mu \sim 350$ to $500$ MeV, $M_s \sim 80$ to $500$ MeV, and $\Delta \sim 10$ to $100$ MeV, so it is quite possible that the strange quark has a strong influence on the pairing.

*Talk given by C. Kouvaris.
Starting at asymptotic density and reducing the density by reducing $\mu$, we expect CFL pairing to be disrupted, at first by kaon condensation\(^3\) (which we do not discuss), and then by the transition to gapless CFL (gCFL)\(^4,5\) with the appearance of gapless modes. In this paper we assume that this happens before the transition from quark matter to hadronic matter. We first review some of the features of gCFL, and then compare it with various charge-separated mixed phases, finding that they are not energetically favored even before we include their extra energy costs arising from Coulomb (color-)electric fields and the surface tension.

### 2. The Gapless CFL Phase

Stable bulk matter has to be neutral under all the gauged symmetries (color and electromagnetic), and equilibrated under all interactions including the weak interaction. These conditions are imposed by introducing chemical potentials coupled to the gauged charges: $\mu_\text{e}$ couples to negative electric charge, and $\mu_3, \mu_8$ couple to the color generators $T_3$ and $T_8$, generators of the Cartan subalgebra of the color group. $\mu_\text{e}, \mu_3, \mu_8$ are determined by requiring that the corresponding charge densities vanish. The pairing ansatz we use is

$$\langle \psi_\alpha^a C_\gamma 5\psi_\beta^b \rangle \sim \Delta_1 \epsilon^{\alpha \beta \gamma} \epsilon_{a b 1} + \Delta_2 \epsilon^{\alpha \beta \gamma} \epsilon_{a b 2} + \Delta_3 \epsilon^{\alpha \beta \gamma} \epsilon_{a b 3} \quad (1)$$

The gap parameters $\Delta_1, \Delta_2$ and $\Delta_3$ describe down-strange, up-strange and up-down Cooper pairs, respectively. Above a critical $M_s^2/\mu = 2\Delta$, the CFL phase is replaced by a new gapless CFL (gCFL) phase\(^4\). The defining properties of the gapless CFL phase arise in its dispersion relations, not in its pattern of gap parameters. However, it is useful for orientation to list the patterns of gap parameters for all the phases we shall discuss:

| Phase       | $\Delta_2 = \Delta_1 = \Delta_{\text{CFL}}$ | gCFL: $\Delta_3 > \Delta_2 > \Delta_1 > 0$ |
|-------------|------------------------------------------|-----------------------------------------|
| CFL         | $\Delta_3 \approx 0$                     | $\Delta_1 = \Delta_2 = 0$              |
| g2SC        | $\Delta_3 > 0$, $\Delta_1 = \Delta_2 = 0$ | 2SCus: $\Delta_2 > 0$, $\Delta_1 = \Delta_3 = 0$ |

The 2SCus phase has the same free energy as 2SC at $M_s = 0$, and to leading order in $M_s$ if their respective nonzero gap parameters have the same value\(^6\). However, an NJL model calculation indicates that $\Delta_2$ in 2SCus is always less than $\Delta_3$ in 2SC, so 2SCus is never favored\(^5\).

### 3. Mixed phases

The gCFL phase can be viewed as a distortion of the CFL phase, induced by the stress of a non-zero strange quark mass, combined with the constraints of color and electromagnetic neutrality. However, it is possible for
Figure 1. Schematic illustration of the dependence of the free energy $\Omega$ on a gauged chemical potential $\mu_i$, showing conditions for the occurrence of mixed phases. Charge $Q_i = -\partial \Omega / \partial \mu_i$ is given by the slope. Squares mark the neutral points. Panel (a): at the neutral value of $\mu_i$ for each phase, the other phase has lower free energy, so there is a coexistence point (black dot) for oppositely-charged phases with lower free energy than either neutral phase. Depending on Coulomb and surface energy costs, a mixed phase may exist there. Panel (b): phase $B$ has higher free energy than phase $A$ at the value of $\mu_i$ where $A$ is neutral. The two phases never coexist with opposite charge, so no mixed phase is possible.

Neutral neutrality to be achieved by a mixture of oppositely charged phases at the same pressure. This happens in the two-flavor case, where a gapless 2SC phase exists, but a mixed phase is free-energetically favored. We now ask whether such a mixed phase might be preferred over the gCFL phase.

The two possible situations are schematically illustrated in Fig. 1, which shows generic free energy curves $\Omega(\mu_i)$ for two phases $A$ and $B$. In Fig. 1b there is no coexistence point and hence no mixed phase is possible. In Fig. 1a there is a coexistence point of oppositely-charged phases, and its free energy is lower than that of either neutral phase, so if the energy of the electric fields induced by the charge separation and the surface tension of the interface is small enough, a neutral mixed phase will be free-energetically preferred over either homogeneous neutral phase.

In quark matter we have gauge charges associated with color as well as electromagnetism. However, it seems unlikely that mixed phases involving color-charged components could form. The color gauge coupling is strong, so separating color charges into different domains will produce color-electric fields with very high energy cost. It seems likely, therefore, that mixed phases will consist of components that are individually color-neutral, with
different values of the color chemical potentials $\mu_3$ and $\mu_8$, but which are electrically charged, with a common electric chemical potential $\mu_e$. In this case there are still color-electric energy costs, associated with the color electric field induced at the phase boundary by the gradient in $\mu_3$ and $\mu_8$. We will not try to calculate these interface costs, since our argument is that most of the mixed phases are excluded even before we include the electrostatic and interface energy costs.

In the region $(M_s^2/\mu) > 2\Delta$ where gCFL is the free-energetically favored homogeneous neutral phase, the other less-favored quark matter phases are 2SC, 2SCus, and unpaired quark matter (2). Note that there is no CFL solution, charged or neutral, in this region. We have compared the mixed phases with neutral gCFL. We performed calculations in an NJL model at $\mu = 500$ MeV, with coupling chosen so that $\Delta_{\text{CFL}} = 25$ MeV at $M_s = 0.45$. When we vary $M_s$ at fixed $\mu$ we find the gCFL region is $47$ MeV $< M_s^2/\mu < 130$ MeV. At the upper limit there is a convergence of the free energies of g2SC, gCFL, and unpaired quark matter, so there is a first-order transition from gCFL to unpaired, with a possible window of g2SC. We have performed calculations of the various possible mixtures. We now discuss the results.

(1) unpaired+gCFL. For color neutral unpaired and gCFL phases, the situation is typically that of Fig. 1(b) so mixed phases are ruled out. This is true for all values of $M_s^2/\mu$ except for a range of a few MeV just below $M_s^2/\mu = 130$ MeV, where the neutral gCFL and neutral unpaired free energies cross. There, a mixed phase may arise, although it may be superseded by other more favorable possibilities such as the crystalline phase.

(2) 2SC+2SCus. We find the situation of Fig 1a: a neutral mixed phase exists. However, its free energy is higher than that of gCFL even before electrostatic and surface energy costs are included. At $M_s^2/\mu = 80$ MeV, it has $\Omega = -14.93 \times 10^6$ MeV$^4$, vs. $\Omega_{\text{gCFL}} = -18.01 \times 10^6$ MeV$^4$. (These free energies are both measured relative to that of neutral unpaired quark matter.) We have checked that a neutral 2SC+2SCus mixed phase is also free-energetically unfavored relative to homogeneous gCFL at $M_s^2/\mu = 51.2$ MeV, which is just above the CFL $\rightarrow$ gCFL transition.

(3) gCFL+2SCus. The $\mu_e$ dependence of the free energies of these two phases is as in Fig 1b, so a mixed phase is not possible. We have verified this at $M_s^2/\mu = 51.2$ MeV and 80 MeV.

(4) gCFL+2SC. At $M_s^2/\mu = 51.2$ MeV we find the situation of Fig 1b, so no mixed phase is possible. At $M_s^2/\mu = 80$ MeV, the free-energy dependence is of the type shown in Fig. 1a, so a mixed phase is possible, and since
gCFL itself is one of the components its free energy is necessarily lower than that of neutral gCFL. However, \( \Omega_{gCFL}(\mu_e) \) depends very weakly on \( \mu_e \): the \( \Omega_{gCFL}(\mu) \) parabola is very shallow. This suppresses the mixed phase in two ways. (i) the free energy of the mixed phase is only lower than that of neutral gCFL by a very small margin (0.0012 × 10^6 MeV^4 at \( M_s^2/\mu = 80 \) MeV), so there is very little chance that it will survive once electrostatic and surface costs are included; (ii) The charge density of gCFL, proportional to \( \partial \Omega/\partial \mu_e \), is very small so the mixed phase must be dominantly gCFL, with a tiny admixture of 2SC, to achieve neutrality. It is known that such highly asymmetric mixed phases have the highest electrostatic energy costs\(^9\).

We conclude that the gCFL phase is a very strong candidate for the “next phase down in density” after CFL. None of the mixed phases that we investigated was able to compete with it. It remains to be seen what role kaon condensation (in CFL and gCFL) plays, and whether a completely different phase such as the crystalline (LOFF) phase\(^{11}\) can compete with gCFL at sufficiently low density.

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