Crossover from photon to exciton-polariton lasing

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\textbf{Abstract.} We report on a real-time observation of the crossover between photon and exciton-polariton lasing in a semiconductor microcavity. Both lasing phases are observed at different times after a high-power excitation pulse. Energy-, time- and angle-resolved measurements allow for the transient characterization of carrier distribution and effective temperature. We find signatures of Bose-Einstein condensation, namely macroscoping occupation of the ground state and narrowing of the linewidth in both lasing regimes. The Bernard-Douraffourgh condition for inversion was tested and the polariton laser as well as the photon laser under continuous wave excitation were found to operate below the theoretically predicted inversion threshold.
Bose-Einstein condensation (BEC) of exciton-polaritons in semiconductor microcavities and photons in dye-filled microcavities have been reported in recent years. Unlike atomic condensates in harmonic traps, where a coherent state is achieved by cooling down of the bosonic thermal distribution, the condensates (coherent states) of polaritons and photons can be formed without thermal equilibrium for example by parametric amplification or under nonresonant excitation at negative detunings. Moreover, the spectra of vertical cavity surface emitting semiconductor lasers (VCSELs) frequently show thermal tails coexisting with the lasing mode that suggest thermal equilibrium of photons. Essentially the experimental observations of polariton BEC in the strong coupling regime, to photon BEC in the weak coupling regime and weak coupling lasing or VCSEL operation have very similar signatures. Carriers are distributed according to the Bose-Einstein distribution, the emission narrows in energy and the first-order spatial coherence builds up. Recently we showed that spontaneous symmetry breaking, which is the Landau criterion for the phase transition can also be observed in polariton and photon lasers.

However, the physical processes by which condensation and conventional lasing occur are fundamentally different. Condensation is a purely thermodynamic phase transition during which the total free energy of the system is minimized, whereas conventional lasing is a balance between the gain from inversion and the loss in the system. In a conventional semiconductor laser, lasing occurs by the stimulated emission of the cavity photons from the e–h plasma. Above a threshold density, the stimulated emission becomes faster than the thermalization rate. As the result, a dip is formed in the carrier distribution, which is called kinetic hole burning. This is because the thermalization process can no longer supply the lost carriers at sufficient speed. In condensation, however, the system remains thermalized while lasing. The question whether the term BEC or lasing is appropriate for degenerate condensates of exciton polaritons and photons is still a subject of great debate in the scientific community.

The crossover from strong to weak coupling according to the coupled oscillator model takes place when the exciton-photon coupling strength equals half of the difference between the decay rates of cavity photons and excitons. This may be achieved by changing the optical pumping strength. The exciton linewidth increases and the oscillator strength decreases with the increase of pumping intensity, which brings the system from strong to weak coupling. This transition is not to be confused with the Mott transition from an exciton gas to an electron hole plasma. Whilst the distinction between strong and weak coupling in a microcavity is straightforward, as the dispersion relations exhibit specific differences, it is very hard to identify the exact point of the Mott transition by standard spectroscopic means. A transition to the weak-coupling regime with increasing pumping strength in steady state has been observed by several groups and the carrier densities at the onset of photon lasing compare well with the Mott density.
paper completes this series as it investigates the dynamical transition from the weak to the strong exciton-light coupling regime in a planar semiconductor microcavity excited by a short high-power excitation pulse. We particularly investigate the distributions of carriers during this crossover and discuss the possibility of a BEC of photons. We observe clear features of polariton and photon lasing and find quasi-thermal distributions of quasiparticles in the weak- and in the strong-coupling regime, which could imply BEC of photons and polaritons. A closer look at the temporal dynamics and the change of the effective temperatures during the transition provide insight into the nature of the observed lasing modes and the thermodynamic state of the system. We further investigate the build-up of photon lasing under continuous wave excitation and investigate the question whether the system is inverted by means of the Bernard-Dourauffough condition for lasing.

The system under study is a GaAs microcavity grown by molecular beam epitaxy. Previous works have shown that lasing in the weak coupling can be observed in this sample under continuous wave (CW) excitation [10], whilst nonlinearities in the strong-coupling regime are accessible under pulsed excitation, due to the sample overheating [25]. At higher excitation power the emission switches to the weak-coupling regime, similar to the observations in reference [24]. We show that the photon and polariton lasing occur at different times after the excitation pulse and that above threshold photon lasing is followed by polariton lasing. Experiments were carried out using a liquid helium cooled wide-field view cold finger cryostat. The exciton-cavity mode detuning was set to $-0.5\text{meV}$. Transform limited pulses from a femtosecond Ti:Sapph oscillator tuned to a reflection minimum of the Bragg mirror were focused to a $30\text{\mu m}$ spot through an objective with a high numerical aperture ($\text{NA} = 0.7$). The dispersion relation was imaged through the same objective onto the slit of a monochromator equipped with a water cooled CCD and a streak camera with ps resolution. For temporal resolution the momentum space was scanned across the crossed slits of the streak camera and the monochromator.

Figure 1a shows a snapshot of a bi-linearly interpolated image of the microcavity dispersion 3 ps after optical excitation at the excitation density $P = 4P_{th}$ (where $P_{th} = 7\text{mW}$ is the power threshold for lasing). The photoluminescence intensity is displayed in false-color logarithmic scale. The inset of figure 1a shows the same microcavity dispersion in false-color linear scale. Solid white lines indicate the exciton-polariton branches in the linear regime and the dashed lines show the bare cavity and exciton modes. Observation of the bare cavity photon dispersion confirms that the excitation pulse brings the microcavity to the weak-coupling regime. Figure 1b shows the same as figure 1a but at 55 ps after optical excitation. White circles indicate the intensity maxima of the recorded spectrum at each detection angle, following the cavity (a) and the polariton mode (b). The lower exciton-polariton dispersion is uniformly blue-shifted due to the repulsive interaction with the exciton reservoir. It is instructive to compare these results with the lower excitation power sufficient to excite a polariton condensate. Figure 1c shows a snapshot of the microcavity dispersion 32 ps after optical
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Figure 1. Dispersion images at different times and excitation powers. (a), (b), (c), (d) Dispersions at excitation powers of $P = 4P_{th}$ (a, b) and $P = 1.1P_{th}$ (c, d) at 3 ps (a), 55 ps (b), 32 ps (c) and 192 ps (d). The insets in (a), (b), (c) show the same dispersions but in linear color scale. At $P = 4P_{th}$ a transition from the weak to the strong-coupling regime is clearly observed between (a) and (b), whilst at lower excitation powers a blue shifted exciton-polariton laser at early times relaxes towards the modes of the linear strong-coupling regime. Images are bilinearly interpolated. The solid lines indicate the shape of the modes at low excitation powers and the dashed lines are the corresponding bare exciton and cavity modes. The white circles follow the location of maxima corresponding to the measured photon and exciton-polariton modes. (e) Typical energy resolved evolution of the groundstate emission intensity for $P = 8P_{th}$. (f) Linewidth as a function of time.

excitation at the excitation density $P = 1.1P_{th}$. Similarly to figure [1], a near uniformly blue-shifted lower exciton-polariton dispersion is observed. Figure [1] shows a snapshot of the microcavity dispersion at 192 ps under the same optical excitation, when the exciton-polariton dispersion in the linear regime is fully recovered as a result of the depletion of the exciton reservoir. Therefore, using time-resolved dispersion imaging we observe the dynamics of the transition of the microcavity eigenstates through three distinctively different regimes: from the weak-coupling regime where we observe a bare cavity mode, to the non-linear strong-coupling regime featuring a blue-shifted lower exciton-polariton branch, through to the linear strong-coupling regime where the exciton-polariton dispersion is not altered spectrally. The temporal evolution of the groundstate energy is depicted in figure [1], showing the redshift of the emission with time. This reflects the transition from the weak to the strong-coupling regime and can be understood as an effect of the depletion of the carrier reservoir. The emission linewidth reflects the coherence properties in the three regimes. Starting from a linewidth of ∼1 meV in the photon lasing regime the linewidth initially increases when the system enters the transitory regime and then narrows down to the linewidth of the polariton laser [26] (figure [1]). The time-resolved spectra and linewidth evolutions in the linear and nonlinear strong coupling regime are given in the supplemental material 1. The time axis was rescaled to account for the temporal distortion caused by the use of the
Figure 2. Energy distribution and Temperatures. (a) the occupation measured at different energies are given for several times after the optical excitation for $P = 4P_{th}$. At 3 ps a photon laser at the energy of 1.610 eV coexists with thermalized photons at $\sim 30$ K populating the cavity dispersion. At 55 ps and 76 ps a Bose-Einstein distribution of exciton-polaritons with a degenerate ground state is observed. After 116 ps the occupancy of the ground state reaches one. (b) Analysis of the spectra for $P = 1.1P_{th}$ at different angles and times show the coexistence of an exciton-polariton lasing mode at the ground state with a thermalized population at early times. We map the depletion of the condensate until the linear exciton-polariton regime, where the distribution exhibits a bottleneck. (c) The temporal evolution of temperature for $P = 1.1P_{th}$ (red triangles) and $P = 4P_{th}$ (green circles). The dashed lines in (a) and (b) are BE fits with a spectral range of $> 3k_BT$.

Figure 2a shows the occupancy as a function of energy at different times for $P = 4P_{th}$. At early times (3 ps), whilst still in the weak-coupling regime, we observe a massively-occupied cavity mode ground state on top of a thermalized tail of excited states. In the transitory regime ($10 - 54$ ps) the dispersion cannot be mapped because the linewidth at higher angles is strongly broadened and therefore a distribution of population is unattainable. At later times, 55 ps and 76 ps after optical excitation whilst in the strong-coupling regime, we observe a largely occupied exciton-polariton ground state coexisting with a thermalized exciton-polariton gas. After $\sim 116$ ps the ground state is no longer degenerate and the particle distribution is close to a Boltzmann distribution. At even later times, the occupation of the ground state cannot be resolved as it is four orders of magnitude lower than at the peak emission intensity. Figure 2b shows successive snapshots of the emission intensity of exciton-polaritons as a function of energy for low excitation powers ($P = 1.1P_{th}$). We observe a largely occupied ground exciton-polariton state on top of a thermalized exciton-polariton gas, in low-excitation nonlinear regime. The depletion of the ground state and the loss of thermalization occur around the same time ($\sim 86$ ps) as a bottleneck builds out (140 ps and 192 ps). Figure 2c shows the temporal evolution of temperature in the transition from photon to exciton-polariton condensate (green circles), and from polariton condensate to a thermalized exciton-polariton gas (red triangles). Effective temperatures were extracted by fitting a Bose-Einstein distribution to the measured angular distribution of the
emission intensity \[27\] (dashed grey lines in figure 2a and (b)). This analysis provides insight into the thermodynamics of the system and how far how far away from thermal equilibrium the quasiparticles are. The effective temperature in the weak-coupling regime (\(\sim 32\) K) is higher than in the strong-coupling regime (\(\sim 16\) K), while in both cases the quasiparticles remain warmer than the lattice temperature (\(\sim 6\) K). The lower effective temperature of the polariton gas reflects the longer timescale on which they thermalize with respect to photons. At the formation stage of the photon laser, the effective photon temperature is higher (\(\sim 32\) K) than the subsequent exciton-polariton gas (\(\sim 16\) K).

The photon gas thermalizes via absorption and re-emission processes in the intracavity quantum wells, similar to the mechanism of photon thermalization in a dye-filled microcavity \[9\]. This thermalization mechanism is analogous to the exciton-polariton thermalization in the strong-coupling regime if the system is below the Mott transition and the excitons are still present in the weak-coupling regime. In this case each photon state has a finite exciton fraction even in the weak-coupling regime, which allows for efficient interaction with phonons and other dressed photons. On the other hand, the observed photon lasing mode occurs at much shorter times than the usual exciton formation rates of tens of picoseconds \[28\] in GaAs. In this case the thermalized distribution originates from the ultrafast self-thermalization of an electron hole plasma (tens of femtoseconds) \[29\]. Thermalization and BEC in an ionized plasma is in principle possible \[30\], through Compton scattering.

Upon the formation of excitons the effective temperature approaches the lattice temperature through carrier phonon scattering in the picosecond timescale. Exciton-polaritons have larger exciton fraction than the photons, which is why they interact stronger with acoustic phonons and between themselves. The cooling of exciton-polaritons occurs on a longer timescale providing a different temperature for the exciton-polariton gas with respect to that of the exciton reservoir and the host lattice. In the intermediate regime of the temporal transition from photon to exciton-polariton laser, the distinction between the weak and strong-coupling regime in momentum space vanishes and the energy appears more and more red shifted, indicating broadband emission similar to the kind observed in reference \[31\] or the coexistence of polariton and photon lasing \[32\].

The buildup of the photon laser is below the temporal resolution of our detection apparatus. Therefore we show time-integrated photoluminescence spectra under CW excitation at different temperatures and excitation densities. Figure 3a shows pump power dependent data at 60 K. The distribution thermalizes at a temperature close to the lattice temperature (solid line) and upon saturation the ground state becomes macroscopically occupied, as observed in reference \[4\]. Next we induce the transition from a thermalized distribution to a photon laser by lowering the temperature, to demonstrate further similarities to atom \[5, 6\] and polariton condensates \[2\] (figure 3b). We use CW excitation at a constant excitation power and study the emission pattern of the cavity mode as a function of temperature. At a critical temperature of about 90 K the
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Figure 3. Photon lasing in the steady state. (a) At 60 K the distribution of photons thermalizes at a temperature close to the lattice temperature and the ground state becomes macroscopically occupied at a certain excitation power. (b) Photon distributions for different cryostat temperatures. The transition can equally be induced by lowering of the temperature. Photon distributions follow the lattice temperature and the ground state becomes degenerate around 60 K. While the lattice temperature can be varied down to \( \sim 6 \) K the photon distribution does not go below 60 K. Straight lines indicate Boltzmann distributions for 60 K (solid), 95 K (dashed) and 120 K (dotted). (c) Difference between the Fermi levels of electrons and holes as a function of carrier density. For three different temperatures, corresponding to the cryostat temperature (6 K), the temperature in the weak coupling measured for pulsed excitation (35 K) and under CW excitation (60 K). The arrows indicate the experimental conditions.

thermal distribution achieves the degeneracy threshold and photons start condensing at the cavity ground state. Although such behavior is characteristic of a thermodynamic phase transition, it is more likely due to the change of the cavity mode energy with respect to the electron hole transition energy in a similar fashion to VCSELs \[33\]. The temperature of the photon gas follows the lattice temperature between 60 K and 120 K, but does not go below 60 K which is slightly above the exciton binding energy in bulk GaAs \[19\]. This might be an indication that the electron-hole pairs are unbound in this case. In the time-resolved experiments we detect a lower temperature for the photon condensate (figure 2c), due to lesser heating of the sample under pulsed excitation.

Next we test the Bernard-Douraffourg condition for lasing \[34\] by comparing the emission energy in the weak coupling regime \( E_{\text{weak}} \) with the difference between the Fermi energies \( \Delta E_F \) of electrons in the conduction band and holes in the valence band for the weak-coupling regime. Conventional lasing occurs when \( \Delta E_F > E_{\text{weak}} \). Calculations of the Fermi-energies and carrier densities are provided in the supplemental material 3. Figure 3c shows \( \Delta E_F \) as a function of the density of electron hole pairs. Arrows indicate the experimental conditions. The system is close to the theoretically estimated inversion-threshold, at the onset of photon lasing under pulsed excitation. However, the density in the CW case remains an order of magnitude below the inversion density predicted for 60 K, which is the lowest photon temperature measured under CW excitation suggesting that photons are coalescing to a condensed state.
In conclusion, we have studied the dynamic transition from a photon to a polariton laser after a high-power excitation pulse. Dispersion images clearly show the transition from the weak to the strong coupling. Both regimes exhibit the same signatures of Bose-Einstein condensation: A macroscopic occupation of the ground state on top of a thermalized tail, narrowing of the linewidth and narrowing of the distribution in momentum space. We have shown that the transition to the photon laser can be induced by decreasing the temperature. The carrier densities remain below the inversion threshold in the CW excitation regime as well as in the strong coupling regime. The effective temperatures show how far from thermal equilibrium the system is at different times after the excitation pulse and the evolution of the linewidth maps the transition between two coherent states by a passage through an incoherent state. The results presented here as well as the recently reported observation of spontaneous symmetry breaking and long-range order \[11\] calls for further studies of condensation in the weak-coupling regime.

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