Supersymmetric Lepton Flavour Violation and Neutrinos mass textures: an update.

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Abstract.
We revise the prospects for the observation of charged lepton flavour violation (LFV), in the light of the recent results from the LHC, MEG and neutrino experiments. We work in the context of the Minimal Supersymmetric Standard Model (MSSM) extended with massive neutrinos arising from a see-saw mechanism. The connection between the observed neutrino oscillations with flavour oscillations in the charged sector is established using a model for the Yukawa couplings arising from a SU(5) grand unified theory (GUT) with Abelian flavour symmetries.
We discuss in this scenario the possible observation on the radiative decays $\mu \to e\gamma$ and $\tau \to \mu\gamma$ and LFV processes that could be detectable at a linear collider (LC) with a centre-of-mass energy in the TeV range.

1. Introduction
The picture of neutrino masses and oscillations has recently improved by the evidence for a non-zero value of $\theta_{13}$ as found by several reactor experiments [1, 2, 3]. The explanation for neutrino observations has to arise from theories beyond the standard model (SM). These theories also predict charged LFV. The link between neutrino oscillations and the violations of the individual lepton numbers $L_{e,\mu,\tau}$ offers the possibility of observing processes such as $l_i \to l_j\gamma$ ($i \neq j$) [4]. The present experimental upper limits, summarised below, already constrain significantly the parameter space of theoretical models,

$$BR(\mu \to e\gamma) < 5.7 \times 10^{-12} \ [5],$$
$$BR(\tau \to \mu\gamma) < 4.4 \times 10^{-8} \ [6],$$
$$BR(\tau \to e\gamma) < 3.3 \times 10^{-8} \ [6].$$

The strongest constraint on radiative decays is the latest MEG upper limit on $BR(\mu \to e\gamma)$ [5].

Supersymmetric (SUSY) theories connect the problem of SM fermions' flavour with the physics of their scalar partners. Experimental evidence on the absence or smallness of charged LFV phenomena seems to indicate that SUSY partners acquire masses with a flavour independent SUSY breaking mechanism. Models like the constrained minimal supersymmetric standard model (CMSSM) with universal scalar, gaugino masses and trilinear terms at the GUT scale extend the SM with an economical amount of undetermined parameters and without introducing flavour conflicts. We use CMSSM framework supplemented with the two elements that we need to explain neutrino oscillations: a structure of the Yukawa interactions and a...
mechanism to explain small neutrino masses. As a model for the Yukawa textures we consider one inspired by SU(5) GUT models with Abelian flavour symmetries \([7, 8]\) while tiny neutrino masses can be explained through a see-saw mechanism.

In the context of SUSY, even if the soft scalar masses were universal at the unification scale, quantum corrections between the GUT scale and low energies would modify this structure via renormalization-group running that generates off-diagonal contributions giving rise to observable LFV signals. Examples are rare lepton decays \([4, 9, 10, 11, 12]\) and slepton production at a LC \([13, 14, 15, 16]\).

We focus our analysis to models with universality assumptions of the CMSSM and that are compatible with searches at the LHC, and that also respect cosmological relic density considerations \([17]\). As an output of our analysis, we discuss detection prospects at a Linear Collider (LC), taking into account the constraints that the recent LHC supersymmetry searches and the measurement of the Higgs mass \([18, 19]\) imposes on the observability of LFV processes.

In the following sections we make a brief survey of the Yukawa model we used and the details and a complete list of references are given in Ref \([20]\).

### 2. Neutrino Mass Textures and Predictions for Neutrino Observables

We choose a model inspired by a SU(5) GUT combined with family symmetries \([7, 8]\) such that the Yukawa textures take the form:

\[
Y_\ell \propto \begin{pmatrix}
\varepsilon^4 & \varepsilon^3 & \varepsilon \\
\varepsilon^3 & \varepsilon^2 & 1 \\
\varepsilon^3 & \varepsilon^2 & 1
\end{pmatrix}, \quad Y_\nu \propto \begin{pmatrix}
|\varepsilon|^{n_1} & |\varepsilon|^{n_2} & |\varepsilon|^{n_3} \\
|\varepsilon|^{n_1} & |\varepsilon|^{n_2} & |\varepsilon|^{n_3} \\
|\varepsilon|^{n_1} & |\varepsilon|^{n_2} & |\varepsilon|^{n_3}
\end{pmatrix}
\]

(4)

where \(Y_{\ell,\nu}\) stand for the Yukawa couplings of charged leptons and neutrinos respectively. \(n_i\) denote the U(1) charges and the heavy Majorana mass matrix is given by

\[
M_N \propto \begin{pmatrix}
|\varepsilon|^{2+n_1} & |\varepsilon|^{n_1+n_2} & |\varepsilon|^{n_1+n_3} \\
|\varepsilon|^{n_1+n_2} & |\varepsilon|^{2+n_2} & |\varepsilon|^{n_2+n_3} \\
|\varepsilon|^{n_1+n_3} & |\varepsilon|^{n_2+n_3} & |\varepsilon|^{2+n_3}
\end{pmatrix}.
\]

(5)

The SU(5) structure of the model implies that the charged-lepton mass matrix is the transpose of the down-quark mass matrix, which relates the mixing of the left-handed leptons to that of the right-handed down-type quarks. The latest property implies that a large mixing can take place in the lepton sector while having a small mixing in the down quark sector, as suggested by a natural explanation of the Cabibbo-Kobayashi-Maskawa (CKM) matrix.

The light neutrinos mass matrix is, thanks to the see-saw mechanism,

\[
m_{\nu} \approx m_{\nu} \frac{1}{M_N} m_{\nu}^{DT} \propto \begin{pmatrix}
|\varepsilon|^2 & |\varepsilon| & |\varepsilon|
\end{pmatrix}.
\]

(6)

Note that \(Y_\ell Y^{\dagger}_\ell \sim m_{\nu}\) at the lowest order, thus, given the following diagonalizations of the Dirac and Majorana mass matrices,

\[
V_\ell^T (Y_\ell Y^{\dagger}_\ell) V_\ell^* \propto \text{diag}(y_\ell^2, y_\ell^2, y_\ell^2),
\]

(7)

\[
V_D^T (Y_\nu Y^{\dagger}_\nu) V_D^* \propto \text{diag}(y_\nu^2, y_\nu^2, y_\nu^2),
\]

(8)

\[
U_N^T M_N U_N \propto \text{diag}(M_1, M_2, M_3),
\]

(9)

\[
U_{\nu}^T m_{\nu} U_{\nu} \propto \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}),
\]

(10)
Figure 1. Correlations between the neutrino mixing angles before and after imposing the constraints on the model coefficients as discussed in the text. The solid lines indicate the experimental bounds, and the small black crosses represent models satisfying all the constraints. The large magenta cross corresponds to model that is discussed in the text and used for numerical calculations.

$V_\ell$ and $U_\nu$ diagonalize matrices with a similar structure. The Maki-Nakagawa-Sakata (MNS) matrix is given by

$$U_{\text{MNS}} \equiv U = V_\ell^\dagger U_\nu,$$

(11)

The texture is determined up to coefficients $O(1)$ in every entry of the matrices. We take the expansion parameter to be $\varepsilon = 0.2$ and multiply the entries of $Y_\ell$, $Y_\nu$ and $M_N$ given by Eqs. (4) and (5) by coefficients in the range $\pm [0.5, 2]$. We require the ranges for the mixing angles of Ref. [21] and assume a neutrino mass hierarchy with a mass splitting of the order of the neutrino masses and $m_{\nu_3} \sim 0.05$ eV. In addition we make a further selection by requiring:

- The coefficients of $Y_\nu$ and $M_N$ are matched to a light neutrino mass matrix $m_{\text{eff}}$ of the form (6), with entries that deviate by a factor between 1/2 and 2 from those in Eq. (6).

- That the hierarchy of eigenvalues of $Y_\ell Y_\ell^\dagger$ (which, as discussed above, has a similar structure to $m_{\text{eff}}$ and $Y_\nu Y_\nu^\dagger$) preserves the order of the gauge eigenstates.

After a search of about $10^9$ fits we display in Fig. 1 the predictions for the neutrino mixing angles corresponding to the above criteria. We can see that, within this class of models, most of solutions that reproduce the correct range of $\theta_{12}$ and $\theta_{23}$ also predict a neutrino mixing angle $\theta_{13}$ that is compatible with the data from [1, 2] and also with the global analysis of neutrino data [22]. We find values for $\theta_{12}$ mostly on the lower part of its experimental range, while values with maximal $\theta_{23} = \pi/4$ are not frequent. This would have been the case when $U_{\text{MNS}}$ arose from a $m_{\text{eff}}$ with the structure of Eq. (6) and moderate 2-3 mixing from the charged lepton sector. However, with our conditions, both $V_\ell$ and $U_\nu$ in Eq. (11) arise from diagonalization of matrices with a large 2-3 mixing and our solutions do not present a distribution of $\sin \theta_{23}$ around 1/2.
We selected the marked point as reference for further analysis, this corresponds to:

\[
Y_\ell \propto \begin{pmatrix}
\varepsilon^4 & 2\varepsilon^3 & -1.75\varepsilon \\
-0.5\varepsilon^3 & 1.9\varepsilon^2 & 0.5 \\
-0.5\varepsilon^3 & -0.7\varepsilon^2 & 1.25
\end{pmatrix}, \quad Y_\nu \propto \begin{pmatrix}
\varepsilon^{[1\pm n_1]} & \varepsilon^{[1\pm n_2]} & 2\varepsilon^{[1\pm n_3]} \\
0.75\varepsilon^{[n_1]} & \varepsilon^{[n_2]} & -0.5\varepsilon^{[n_3]} \\
\varepsilon^{[n_1]} & \varepsilon^{[n_2]} & 1.25\varepsilon^{[n_3]}
\end{pmatrix}.
\]

\[
M_N \propto \begin{pmatrix}
\varepsilon^{[2\pm n_1]} & \varepsilon^{[n_1+n_2]} & -\varepsilon^{[n_1+n_3]} \\
\varepsilon^{[n_1+n_2]} & \varepsilon^{[2\pm n_2]} & -\varepsilon^{[n_2+n_3]} \\
-\varepsilon^{[n_1+n_3]} & -\varepsilon^{[n_2+n_3]} & \varepsilon^{[2\pm n_3]}
\end{pmatrix}.
\]

(12)

The predictions for the neutrino angels are, \(\sin^2 \theta_{13} = 0.019\), \(\sin^2 \theta_{12} = 0.28\), \(\sin^2 \theta_{23} = 0.40\). We observe no correlation between LFV and particular arrangements of neutrino parameters. However, LFV is maximized by large out-diagonal elements on \(V_\ell\) and by the choice of charges \(n_i\). Neutrino fits are independent of these charges because despite they affect \(Y_\nu\) and \(M_N\) but not their combination in \(m_{eff}\).

3. Scenarios for Charged-Lepton-Flavour Violation

A complete GUT model with an Abelian symmetry that determines the Yukawa textures also implies a flavour dependence of the soft mass matrices on the family charges. The determination of the resulting model at \(M_{GUT}\) depends on the details of the SUSY breaking and RGE evolution of the soft terms [23]. In this work we assume that this model is the MSSM extended by a seesaw with soft terms that do not deviate significantly from the universality conditions of the CMSSM.

Even if we start with universal soft-terms at \(M_{GUT}\), at the intermediate scale where the seesaw takes place, \(M_3\) (taken as the mass of the heaviest Majorana neutrino), the slepton mass matrices and \(Y_\ell\) cannot be diagonalized with a single superfield rotation. Thus, the interactions lepton-slepton-gaugino can mix flavours. To understand this mismatch of the leptons and sleptons rotations we can consider the soft masses evolution from \(M_{GUT}\) to \(M_3\) in a basis such that \(Y_\nu\) is diagonal, at \(M_3\) the right handed neutrinos decouple and the REG can be rewritten in terms of \(Y_\nu\) diagonal. In this basis the soft terms involving left slepton are not diagonal and can be written in terms of the matrix \(V_{LFV} = V_D^T V_\ell\):

\[
m_{LL}^2 = V_{LFV}^T (m_{LL}^2) d V_{LFV},
\]

while the \(A\)-terms become:

\[
A_\ell = V_{LFV}^T (A_\ell) d.
\]

(14)

Here \((m_{LL}^2) d\) and \((A_\ell) d\) are the soft terms resulting from the RGE running of the universal soft terms at the GUT scale to \(M_3\) with the fields written in a basis such that \(Y_\nu\) is diagonal. The successive run of the MSSM RGEs to \(M_{SUSY}\) do not have a significant effect on the off-diagonal elements.

The matrix \(V_{LFV}\) can be evaluated from Yukawa textures. We use the fit of Eq. (12) obtained by adjusting the matrices to the neutrino parameters at a low energy scale. RGE effects are taken into account by considering the global scale evolution of the neutrino mass set initially by imposing \(m_{\nu_3} = 0.05\) eV and the seesaw condition at a common scale \(M_3\) to determine the strength of the largest Dirac Yukawa coupling. Since we assume hierarchical neutrinos, we neglect the evolution of the non-diagonal matrix elements of effective neutrino mass and the lightest generations. These effects are small [24] and can be included in the uncertainty of the coefficients without affecting the stability of the solutions. Furthermore, no sizeable effect is expected from including lower right handed neutrino scales since the Dirac Yukawa decreases
with them becoming tiny when $M_1$ and $M_2$ are much lower than $M_3$. In our computation we use the matrices given in Eq. (12), $V_\ell$ has large out-diagonal elements and the $n_\ell$ charges that are not relevant to fit neutrino data become important in the determination of LFV effects as can be seen in the values of $V_{LFV}$ of table 1. We need now to evaluate the SUSY spectrum and couplings using the CMSSM as general framework. The recent LHC measurement of the Higgs mass seems to point towards large SUSY masses in these scenarios. However, the global analysis of the CMSSM parameter space of Ref. [17] yielded two almost equally good fits to the available data, one with relatively light sparticle masses and $\tan \beta \sim 16$, and the other with heavier sparticle masses and $\tan \beta \sim 45$, corresponding to the following CMSSM sets of parameters:

\begin{align}
(a) \quad & \tan \beta = 16, \quad m_0 = 300 \text{ GeV}, \quad M_{1/2} = 910 \text{ GeV}, \quad A_0 = 1320 \text{ GeV}, \\
(b) \quad & \tan \beta = 45, \quad m_0 = 1070 \text{ GeV}, \quad M_{1/2} = 1890 \text{ GeV}, \quad A_0 = 1020 \text{ GeV}.
\end{align}

Point (a) belongs to the region where the WMAP-favoured range of $\Omega h^2$ is achieved via $\chi - \tilde{\tau}$ coannihilation. Point (b) lies in the funnel region where the neutralino LSP annihilates rapidly via direct-channel $H/A$ poles.

4. LFV in radiative decays.

The branching ratios (BRs) of the decays $\ell_j \rightarrow \ell_i + \gamma$ are calculated using full mass matrices diagonalization and the formulae of Ref. [9]. In Fig. 2 we show numerical predictions for the LFV branching ratios. We can see the effect of varying $M_N$ from $6 \times 10^{14} \text{ GeV}$ down to $10^{12}$.
GeV for the choices of right-handed neutrino charges of Table 1. The branching ratios are larger
for the lower-mass scenario with \( \tan \beta = 16 \), due to the lighter spectrum. The new MEG bound
on \( BR(\tau \rightarrow \mu \gamma) \) imposes constraints on the ”see-saw” scale for all for all the charge choices
of Table 1 at point (a) while the predictions of point (b) are in the range of the experimental
searches.

5. LFV Observation at a Linear Collider

In Ref. [13] it was shown that if the flavour mixing is introduced in the \( LL \) slepton sector, as is
the case for the models under consideration here, slepton-pair production and LFV decays such as:

\[
e^+ e^- \rightarrow \tilde{\ell}_i^- \tilde{\ell}_j^+ \rightarrow \tau^\pm \tilde{\mu}_i^0 \tilde{\mu}_j^0, \\
e^+ e^- \rightarrow \tilde{\nu}_i \tilde{\nu}_j \rightarrow \tau^\pm \tilde{\chi}_1^+ \tilde{\chi}_1^-,
\]

lead to a cross section of the order of 1 fb, the reference value used in [13], for a LFV signal
of \( \mu^\pm \tau^\pm \) pairs that can be distinguished from the background, according to the considerations
in [15].

Here, we extend our previous results by considering the full structure of the Yukawa couplings,
thus comparing the LFV production of charged leptons of all generations and use the updated
supersymmetric benchmark points compatible with the LHC measurement of \( m_{h} \).

We present in Fig. 3 the expected cross sections as functions of \( \sqrt{s} \) for the same sets of charges
as those in Fig. 2. For definiteness, the right-handed neutrino mass scale is set to \( M_N = 10^{14} \)
GeV for point (b) and the restrictive value of \( M_N = 2 \times 10^{13} \) such that the new MEG bound
on \( BR(\mu \rightarrow e\gamma) \) is preserved. We observe that, by varying this scale in a similar range as in
Fig. 2, the cross sections also change by approximately an order of magnitude: according to the
see-saw conditions, larger values of \( M_N \) imply larger Dirac neutrino Yukawa couplings, and thus
stronger RGE effect. The results depend sensitively on the sparticle spectrum. If we restrict
ourselves to the benchmark CMSSM parameter choice discussed above, we would require \( \sqrt{s} \) at
TeV scale in order to have significant LFV signals in slepton mixing.

6. Conclusions

In our presentation, we revisited the signatures of charged LFV in theoretically-motivated
scenarios, studying the correlations arising in CMSSM models with parameter values that are
favoured by the LHC and cosmological considerations. We have explored these issues using
updated experimental input from neutrino data, particularly recent measurements of \( \theta_{13}, \) MEG
and the LHC.

Models based in SU(5) with Abelian flavour symmetries provide very interesting possibilities
for understanding the hierarchy of fermion masses and mixings. We performed a big scan of
fits to the neutrino data paying attention to the the naturalness of the fit and avoiding artificial
cancellations arising from specific choices of coefficients. We get a pattern of neutrino predictions
and correlations compatible with the global analysis of neutrino data [22]. In general, we found
that fittings with similar predictions for the neutrino parameters may lead to very different LFV
predictions. This can provide information on the heavy Majorana neutrino matrix.

The new MEG bound on \( \mu \rightarrow e\gamma \) is still compatible with the LHC data. At the point
with lighter SUSY spectrum it determines an upper limit for the see-saw scale while for point
with large SUSY spectrum, LFV radiative decays are still interesting for experimental searches.
Regarding LC prospects, it was possible to establish correlations between the expected rates
for radiative LFV decays and LFV \( \tau\mu \) pair production at a future LC. Within the CMSSM
framework studied, the absence of a supersymmetry signal in the LHC data and the discovery
Figure 3. Values of the cross sections $\sigma(e^+e^- \rightarrow \tilde{\ell}_i^- \tilde{\ell}_j^+ \rightarrow \ell_a^\pm \ell_b^\mp + 2 \chi^0)$ ($\ell_a \neq \ell_b$ as indicated for each panel) as functions of $\sqrt{s}$. The line styles are the same as those in Fig. 2. For the point (a) we use $M_N = 2 \times 10^{13}$ GeV. While for the point (b) we use $M_N = 10^{14}$ GeV.

of a neutral Higgs weighing $\sim 125$ GeV imply that observation of slepton flavour violation at a LC will be possible for only energies beyond 1 TeV.

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