Mathematical modeling of inclined accretion disks in cataclysmic variables

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Abstract. The work is devoted to synthetic light curves modeling for the observed binary stellar systems and an interpretation of their characteristic features including pre-eclipse humps, asymmetry in the vicinity of the eclipse and variability of light curves from revolution to revolution. The mathematical model of the accretion disk including the radiation cooling of the plasma, gravitational forces and incomplete plasma ionization has been studied. The results of the plasma flows calculation allow one to use the radiated energy for synthetic light curves construction. In a number of simulations the formation of an inclined accretion disk is noted. The inclination may be caused by the instability of the circumstellar plasma flowing around the disk at the initial stages of its formation. The disk obtained in the calculations retrogradely precesses with a period of about 40 orbital periods of the system. Due to this precession in different orbital periods the jet enters the disk in different places. This may explain the significant variability of the light curves of the binary star system as well as the presence of brightness humps at the eclipse.

1. Introduction
Eclipsing dwarf novae are probably the best objects for the study of accretion physics as the occultation of the accretion disc and white dwarf by the secondary, which can be used to constrain the geometry and parameters of the binary. Tomographic techniques such as eclipse mapping and Doppler tomography can be applied to probe the structure and dynamics of the accretion flow. On the other hand mathematical modelling of plasma flow in binary semidetached star systems accretion disks provides a way to analyze and interpret incoming observations data [1].

Many features of binaries light curves appear to be related both to the structure of the gas flow in the disk itself and to the behavior of the jet propagating from the Lagrange point \( L_1 \). Investigation of light curves especially for eclipsing systems gives information about system inclination, jet velocity at \( L_1 \) point, shock or shock-less behavior of jet-disk interaction [2].

We use the approach similar to one of [1] to construct the light curve of the PHL1445 system [3]. We consider three-dimensional hydrodynamic model of an accretion disk with a jet outflowing from the point \( L_1 \). This jet is a source of new matter for accretion disk. Additional model elements contributing to the luminosity of the system are a donor star (brown dwarf) and an accretor star (white dwarf). Because we are most interested to investigate the eclipse surroundings the using of the gas dynamic model [1] which assumes non-viscous partially ionized gas and pure hydrogen cooling function may be insufficient. So we change the inner part of the
calculated region of the gas dynamic model (radius \( \approx 4 R_{wd} \) with center in the accretor) for a thin disk with specified radiation flux on the surface. Such an approach allows us to delay the investigation of the accretor vicinity processes and at the same time allows to introduce into the model an additional quite physical tuning parameter.

The gas dynamic model of accretion disk is based on the following assumptions:

- partially ionized hydrogen plasma flow is described by compressible gas dynamic equations;
- not self-gravity nor magnetic fields are taken into account;
- plasma is treated as an ideal inviscid gas which heats solely due to conversion of the gravitational energy into the internal gas energy;
- energy losses are due to radiation cooling in optically thin collisional plasma.

2. Mathematical model

2.1. Governing equations

In accordance with the assumptions outlined above the mathematical model contains the set of hydrodynamic equations of ideal (inviscid) partially ionized hydrogen gas with account for the Roche gravitational potential and radiative gas cooling, written in non-inertial reference frame co-rotating with the binary orbital period:

\[
\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0, \tag{1}
\]

\[
\frac{\partial \rho \vec{v}}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v} \vec{v} + p \vec{I}) = -\rho \vec{v} \vec{\nabla} \Phi + 2 \rho \vec{v} \times \vec{\Omega}, \tag{2}
\]

\[
\frac{\partial e}{\partial t} + \vec{\nabla} \cdot (\vec{v} (e + p)) = -\rho \vec{v} \vec{\nabla} \cdot \vec{v} - \Lambda(\rho, T), \tag{3}
\]

where \( \rho \) is the density, \( \vec{v} \) is the velocity vector, \( p \) is the pressure, \( e \) is the total gas energy per unit volume, \( \vec{\Omega} \) is the angular rotational velocity of the binary system, \( \Lambda(\rho, T) \) is the radiative gas cooling function [1],

\[
\Phi = -\frac{GM_d}{\|\vec{r} - \vec{r}_d\|} - \frac{GM_a}{\|\vec{r} - \vec{r}_a\|} - \frac{1}{2} \|\vec{\Omega} \times (\vec{r} - \vec{r}_c)\|^2, \tag{4}
\]

\( \vec{r} \) is the radius-vector, \( M_d, M_a \) are masses of the components, \( \vec{r}_d, \vec{r}_a \) are their radius-vectors, \( \vec{r}_c \) is the system barycentre radius-vector, \( G \) is the Newton gravity constant. The system of hydrodynamic equations is completed by the equation of state of partially ionized gas

\[
e = \frac{3}{2} p + \rho \frac{R_y}{m_p} i + \frac{\rho \|\vec{v}\|^2}{2}, \quad p = \rho kT \frac{1 + i}{m_p}, \tag{5}
\]

\[
i = \left[ 1 + p \left( \frac{2\pi \hbar^2}{m_e} \right)^{3/2} \left( kT \right)^{-5/2} e^{R_y/kT} \right]^{-1/2}, \tag{6}
\]

where \( i \) is the degree of ionization, \( T \) is the temperature, \( \varepsilon \) is the specific internal energy of gas, \( k \) is the Boltzmann constant, \( R_y \) is the Rydberg unit of energy, \( m_p \) and \( m_e \) are the masses of a proton and an electron.

The integral cooling function for pure hydrogen was calculated as a function of gas density and temperature assuming local thermodynamical equilibrium (i.e. the Saha-Boltzmann distribution of atomic level populations) [1].
2.2. Gas dynamics simulation setup

Let us introduce the scales for the main parameters of the problem:

- the space scale equals the distance between centers of the system components \( L_0 = a \);
- the time scale is the system’s orbiting period \( t_0 = 76.3 \text{ min.} \);
- the mass scale is the sum of masses of components: \( M_0 = M_d + M_a \), where the donor mass \( M_d = 0.1 M_\odot \), and the accretor mass \( M_a = 0.73 M_\odot \), \( M_\odot \) is the solar mass;
- the temperature scale equals donor’s surface temperature \( T_0 = 2100 \text{ K} \);
- the density scale equals \( \rho_0 = 10^{-8} \text{ g cm}^{-3} \).

Other scales can be constructed from the basic units in the standard way. The time scale is in units of the orbital period of the binary system, \( t_0 = 2\pi \sqrt{L_0^3/GM_0} \). The angular velocity scale is \( \Omega_0 = 1/t_0 \), and the dimensionless orbital angular velocity of the binary is \( \Omega = 2\pi \). The cooling function scale is \( \Lambda_0 = \rho_0 L_0^2/t_0^3 \).

We took a rotational ellipsoid as a computational domain, center of this ellipsoid coincides with the center of the accretor; axis of rotation of ellipsoid is co-directed with axis \( Oz \), its center coincides with the accretor center, the major half axis equals 0.692 and the smaller one is 0.510. The major axis is chosen so that \( L_1 \) point lays on the border of the computational region. The spherical cavity (\( R = 0.08 \)) which includes the accretor and its vicinity is placed in the center of the calculated volume. The example of tetrahedral grid used in calculations is presented in Fig. 1.

Initial conditions are modeling unperturbed state of plasma fixed in the rotating coordinate system. The background density and temperature values are \( \rho = 10^{-5} \) and \( T = 10^{-4} \) correspondingly. Non-reflective conditions are selected for the inner boundary side; we set the plasma stream (jet) with density \( \rho = 3 \) for outer boundary in the vicinity of the point \( L_1 \). The mass inflow rate into calculated area was chosen to ensure the mass flow at \( \dot{M} \approx 10^{-11} M_\odot \text{ yr}^{-1} \).

Changing the density of the jet allows to vary the mass flow through the point \( L_1 \), without leading to a fundamental change in the flow pattern, while the speed of the jet has a significant influence on the location of the disk structure elements in the space, and, consequently, on the final light curve.

The three-dimensional solver described by [1] was used for the numerical solution of plasma flow simulation. It assumes the use of the finite volume type method for unstructured tetrahedral grids with the integration of flows on the faces of the cells using a modified the HLLC solver for the Riemann gas dynamics problem. Radiation cooling accounting at each step of integrating the system of equations (1)–(3) is done by additional substep and solving the equation for energy with the cooling function in the right side of the equation using the Dorman-Prince 4(5) [4]. We used partition of the computation domain into subdomains and MPI technologies. Calculations were performed on the cluster K-60 named after M.V. Keldysh of the Russian Academy of Science (RAS) using up to 500 processing cores.
2.3. Calculating the light curve
The total energy flux $F$ observed from the model system is calculated by integrating the visible structure along observer’s line of sight:

$$F = \int_S I(x, y) \frac{dS}{d^2},$$

(7)

where $I(x, y)$ is the intensity in the plane perpendicular to the line of sight (picture plane), $x$ and $y$ are the coordinates on this plane, $d$ is the distance to the source.

The intensity $I(x, y)$ is obtained from the radiation transfer equation:

$$I(x, y) = \int_0^{+\infty} \frac{\bar{\Lambda}(x, y, z)}{4 \pi} e^{-\tau(x, y, z)} dz,$$

(8)

$$\tau(x, y, z) = \int_0^z \alpha_{Pl}(\rho(x, y, z), T(x, y, z)) ds'',$$

(9)

where $z$ is the distance along the line of sight, $\bar{\Lambda}$ is the cooling function $\Lambda$ averaged over the hydrodynamical time step, $\tau$ is the optical depth between the observer and the volume element, $\alpha_{Pl}$ is the Planck mean absorption coefficient.

The absorption coefficient $\alpha_{Pl}$ is set to infinity in the donor star and accretor surface, as well as on the surface of the thin disk around the accretor. In all other points, like in [1], this value is obtained by averaging over the Planck function using data of gas dynamic calculation of radiated energy flux:

$$\alpha_{Pl}(\rho, T) = \frac{\Lambda(\rho, T)}{4\sigma_{SB}T^4},$$

(10)

where $\sigma_{SB}$ is the Stefan–Boltzmann constant.

With the passing of the beam through the surface of the donor (or accretor) or through the thin disk close to the accretor we stopped integrating of the transfer equation. The intensity was set for each point of the picture plane. This intensity corresponds to the object on the line of sight. The surface of the donor star does not coincide with the surface of its Roche lobe. Donor radius is taken equal to 0.9 of the Roche lobe size in the direction of the system rotation axis.

3. Results
We performed 3D gas dynamic modeling of the gas outflow from the vicinity of point $L_1$, to the Roche lobe of accretor of the system PHL1445. The accretion disk is formed over 5 orbital periods. Figure 2 demonstrates the standard three dimensional accretion disk image with donor, accretor and thin disk at the moment $t = 5$. It is located in the equatorial plane. Average disk thickness is 0.06, ratio of the disk thickness to the disk diameter is 1/10. A detached shock wave forms and compaction of the jet as it enters the disk. The gas in the disk is cold (its temperature varies slightly in the disk and does not exceed $5 \cdot 10^3$ K) and rather weakly ionized (disk ionization coefficient does not exceed 0.2).

The simulations using rather fine mesh shown another regime of accretion disk formation. During few orbital periods from jet flow start the protodisk is light and unstable. Instability of disk edge which is flowed around by interstellar gas leads to disk inclination, see Fig.3. The disk inclination leads to relocation of hot domain in disk-jet interaction region from the disk edge to its surface. The hot domain location affects the light curve details such as the eclipse hollow and humps.

Applying technique described in [1] the synthetic light curves for system inclination 87° are calculated. The donor star surface is modelled to be opaque with temperature $T = 2100$K. Inside the computational domain the accretor is modelled as white dwarf with thin and bright
Figure 2. Three-dimentional image of the accretion disk with the donor, the accretor and the thin disk. By color for the accretion disk is shown the averaged for the time step energy release $\bar{\Lambda}$

Figure 3. Three-dimentional image of the inclined accretion disk formation. From left to right from top to bottom time moments $t = 1.1; 1.6; 2.1; 2.6; 3.1; 3.6$
circumstellar disk. Figure 4 shows the modelled and averaged (with error bars) observational light curves. The inclined disk light curve has the hump inside eclipse similar to observational one (near phase $\phi = 0.99$). Such hump cannot be explained using planar model of the accretion disk.

4. Conclusions
A three-dimensional mathematical model of the accretion disk in a semidetached binary star system PHL1445 with a small donor is considered. The model includes the equations of gas dynamics taking into account gravitation and radiation cooling of gas. A numerical solution was obtained using a parallel CFD solver for unstructured tetrahedral meshes. Modeling shows that accretion disk can be unstable during first 5 orbital periods after jet flow arises. The instability of disk edge leads to formation of inclined disk. Disk inclination relocates the region of disk-jet interaction to the disk surface. It leads to registration of the hump in the light curve during the eclipse.

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