Shannon’s information entropies in position- and momentum-space and their sum $S$ are calculated for various $s$-$p$ and $s$-$d$ shell nuclei using a correlated one-body density matrix depending on the harmonic oscillator size $b_0$ and the short range correlation parameter $y$ which originates from a Jastrow correlation function. It is found that the information entropy sum for a nucleus depends only on the correlation parameter $y$ through the simple relation $S = s_0A + s_1A^2y - \lambda_sA$, where $s_0$, $s_1$, and $\lambda_s$ depend on the mass number $A$. A similar approximate expression is also valid for the root mean square radius of the nucleus as function of $y$ leading to an approximate expression which connects $S$ with the root mean square radius. Finally, we propose a method to determine the correlation parameter from the above property of $S$ as well as the linear dependence of $S$ on the logarithm of the number of nucleons.

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I. INTRODUCTION

Information theoretical methods have in recent years played an important role in the study of quantum mechanical systems in two cases: first in the clarification of fundamental concepts of quantum mechanics and second in the synthesis of probability densities in position and momentum space. An important step was the discovery of an entropic uncertainty relation which for a three-dimensional system has the form

$$S = S_r + S_k \geq 3(1 + \ln \pi) \simeq 6.434 \quad (\hbar = 1),$$

where

$$S_r = - \int \rho(r) \ln \rho(r) dr,$$

and

$$S_k = - \int n(k) \ln n(k) dk$$

are the Shannon’s information entropies in position- and momentum-space and $\rho(r)$, $n(k)$ are the density distribution (DD) and momentum distribution (MD), respectively, normalized to unity.

Inequality (1) is an information-theoretical uncertainty relation stronger than Heisenberg’s and does not depend on the unit of length in measuring $\rho(r)$ and $n(k)$, i.e. the sum $S = S_r + S_k$ is invariant to uniform scaling of coordinates, while the individual entropies $S_r$ and $S_k$ are not. The physical meaning of $S$ is that it is a measure of quantum-mechanical uncertainty and represents the information content of a probability distribution, in our case of the nuclear density and momentum distributions. Inequality (1) provides a lower bound for $S$ which is attained for Gaussian wave functions.

Information entropies were employed in the past to study quantum-mechanical systems. Recently we studied the position- and momentum-space information entropies $S_r$ and $S_k$, respectively, for the densities of various systems: the nucleon DD of nuclei, the valence electron DD of atomic clusters and the DD of trapped Bose alkali atoms. We found that the same functional form $S = a + b \ln N$ for the entropy sum as function of the number of particles $N$ holds approximately for the above systems in agreement with Refs. for atomic systems. In Ref. we found a link of $S$ with the kinetic energy of the system $T$ and a relationship of Shannon’s information entropies in position-space with an experimental quantity i.e. the root mean square (RMS) radius of nuclei and atomic clusters. In Ref. we used another definition of information entropy according to phase-space considerations and we derived an information-theoretic criterion for the quality of a nuclear DD, i.e. the larger $S$ the better the quality of nuclear model. In Ref. the DD, the MD and the Shannon’s information entropies have been calculated for nuclei using three different cluster expansions. The parameters of the various expressions have been determined by least-squares fit of the theoretical charge form factor to the experimental one. It was found that the larger the entropy sum the smaller the value of $\chi^2$, indicating that the maximal $S$ is a criterion of the quality of a given nuclear model according

1
to the maximum entropy principle. Only two exceptions to that rule were found out of many cases examined. Finally, in Ref. [18] we considered the single particle states of a nucleon in nuclei, a Λ in hypernuclei and a valence electron in atomic clusters. We proposed a connection of $S$ with the energy $E$ of the single particle states. Before proceeding, it is appropriate to mention that additional applications of entropy have attracted interest in recent years [6,7], but in a different spirit, in nuclear physics problems, such as in analysis of shell model eigenvectors. However, it is noted that in Ref. [7] the authors define a correlational entropy. This is a von Neumann entropy, which they apply in the framework of the nuclear shell model. In our case we use the definition of information entropy according to Shannon.

In the present work we focus on the entropy sum $S$ of a nucleus using the analytical expressions of the DD and MD of Refs. [19,20,21]. The expressions of those distributions have been found for s-p and s-d shell nuclei using the factor cluster expansion of Clark and co-workers [22] and Jastrow correlation function which introduces short range correlations (SRC). Those expressions depend on the harmonic oscillator (HO) parameter and the correlation parameter. We studied the dependence of $S$ on those parameters and we found how this quantity is connected with a fundamental quantity, the RMS radius of the nucleus. Finally, we propose a way to determine the correlation parameter of the model using the dependence of $S$ on that parameter as well as the linear dependence on the logarithm of the number of nucleons. The HO parameter is determined equating the theoretical charge RMS radius of the nucleus with the experimental one.

The paper is organized as follows. In Sec. II, the general definitions related to the correlated DD, MD as well as the dependence of the entropy sum on the correlation parameter are given. In Sec. III, we study the dependence of the RMS radius on the correlation parameter and we give an approximate expression connecting the information entropy sum with the RMS radius of the nucleus. In Sec. IV, we present a method for the determination of the correlation parameter from the information entropy sum. Finally, in Sec. V, the summary of the present work is given.

II. CORRELATED ONE-BODY DENSITY OF S-P AND S-D SHELL NUCLEI AND THEIR ENTROPY

A general expression for the one-body density matrix of $N = Z$, s-p and s-d shell nuclei was derived in Refs. [20,21] using the factor cluster expansion of Ristig, Ter Low and Clark [22]. That expression depends on the HO parameter $b_0$ ($b_0 = (\hbar/(m\omega))^{1/2}$), the occupation probabilities of the various states and the correlation parameter $y$ that comes from the Jastrow type correlation function

$$f(r) = 1 - \exp[-y r_b^2], \quad r_b = r/b_0,$$

which introduces short range correlations. The correlation function $f(r)$ for large values of $r$ goes to 1 and goes to 0 for $r \to 0$. It is obvious that the effect of correlations introduced by the function $f(r)$ becomes large when the correlation parameter $y$ becomes small and vice versa.

The diagonal part of the one-body density matrix is the DD $\rho(r)$. The Fourier transform of the DD is the form factor

$$F(q) = \int \exp[iqr] \rho(r) dr,$$

while the MD $n(k)$ is given by the particular Fourier transform of the one-body density matrix

$$n(k) = \frac{1}{(2\pi)^3} \int \exp[ik(r-r')] \rho(r,r') d rdr'.$$

The expressions of $\rho(r)$, $n(k)$ and $F(q)$ (in the two body approximation for the cluster expansion) have the forms

$$\rho(r) = \frac{N_0}{\pi b_0^{3/2}} \left[ O_1(r_b) + O_2(r_b, y) \right],$$

$$n(k) = \frac{N_0 b_0^3}{\pi^{3/2}} \left[ \tilde{O}_1(k_b) + \tilde{O}_2(k_b, y) \right],$$

and

$$F(q) = N_0 \left[ \tilde{O}_1(q_b) + \tilde{O}_2(q_b, y) \right],$$
where $k_b = k q_0$ and $q_b = q b_0$. The terms $O_1$, $\tilde{O}_1$ and $\tilde{O}_2$ come from the one-body term of the cluster expansion of the one-body density matrix and the terms $O_2$, $\tilde{O}_2$ and $\tilde{O}_2$ come from the two-body term. Their expressions as well as the expression of the normalization factor $N_0$ are given in Refs. [10,21,22].

From the expressions of $\rho(x)$ and $n(k)$ the Shannon’s information entropies in position and momentum space and their sum $S = S_r + S_k$ can be calculated through Eqs. [3] and [4] for $\rho(x)$ and $n(k)$ normalized to 1.

For various values of the parameters $b_0$ and $y$ and for the $N = Z, s-p$ and $s-d$ shell nuclei: $^4$He, $^{12}$C, $^{16}$O, $^{24}$Mg, $^{28}$Si, $^{32}$S, $^{36}$Ar and $^{40}$Ca we calculated $S_r$, $S_k$ and $S = S_A$, treating $^{24}$Mg, $^{28}$Si, $^{32}$S, $^{36}$Ar as 1$d$ shell nuclei.

It is found that, for the above nuclei, $S_r$ and $S_k$ depend on both parameters, $b_0$ and $y$, while their sum $S_A$ depends only on the correlation parameter $y$. The calculated values of $S_A$ for the above mentioned nuclei versus $1/y$ are displayed by points in Fig. 1.

It is seen that $S_A$ is an increasing function both of $1/y$ and of the number of nucleons $A$ of the nucleus, while $S_A$ depends almost linearly on $1/y$. For that reason we fitted the numerical values of $S_A$ with the form

$$S \equiv S_A(y) = s_{0A} + s_{1A} \left( \frac{1}{y} \right)^{\lambda_{sA}},$$

(10)

separately for each nucleus, that is the parameters $s_{0A}$, $s_{1A}$, and $\lambda_{sA}$ depend on the mass number $A$ of the nucleus. The parameter $s_{0A}$ is determined from the values of the information entropy sum in the HO case, i.e.,

$$s_{0A} = S_A(\infty),$$

while the other two parameters are determined by least-squares fit of the values of $S_A$ calculated from Eq. (10) to the corresponding ones calculated from Eqs. [3] and [4]. The values of the parameter $s_{0A}$ and the best fit values of the parameters $s_{1A}$ and $\lambda_{sA}$ are displayed in Table I, while the values of $S = S_A(y)$ found from Eq. (10) using the above values of the parameters are displayed by lines in Fig. 1.

It is seen that the simple form of $S_A$, given by Eq. (10), reproduces very well the numerical values of $S_A$ for all nuclei considered. Also, there is a systematic trend of the values of the parameters $s_{0A}$, $s_{1A}$ and $\lambda_{sA}$. The parameter $s_{0A}$ depends linearly on the logarithm of $A$. That is expected, as $s_{0A}$ is equal to the information entropy sum in the HO case which depends linearly on the logarithm of the number of the nucleons. The parameter $\lambda_{sA}$ has smaller values in the closed shell nuclei $^4$He, $^{16}$O and $^{40}$Ca than in the corresponding neighboring open shell ones. Finally, the parameter $s_{1A}$ is almost a monotonic increasing function of $A$ with an exception for the nucleus $^{40}$Ca.

### III. THE DEPENDENCE OF THE INFORMATION ENTROPY ON THE RMS RADIUS

From the analytical expression of $\rho(x)$ given by Eq. (6) the analytical expression of the mean square radius of the nucleus can be found. It has, in units of the HO size parameter $b_0$, the form

$$r_0^2 \equiv \langle r^2 \rangle / b_0^2 = R_1 + R_2(y).$$

(11)

The term $R_1$ comes from the one-body term of the one-body density matrix and for the $N = Z, s-p$ and $s-d$ shell nuclei has the form

$$R_1 = \frac{1}{A} \left[ 6\eta_{1s} + 14\eta_{2s} + 30\eta_{1p} + 70\eta_{1d} \right],$$

(12)

where $\eta_{ml}$ are the occupation probabilities of the various states. The expression of the two-body term $R_2(y)$ is given in Ref. [4]. It depends on the parameter $y$ and the occupation probabilities of the various states.

The calculated values of the RMS radius $r_b$ in units of $b_0$ versus the parameter $1/y$ for the various $s-p$ and $s-d$ shell nuclei are displayed by points in Fig. 2. As in the case of the entropy sum $S_A$, $r_b$ is an increasing function of $1/y$ and of $A$. Although we know the analytical expression of $r_b$ we tried to find a simpler expression for it. We found that the simple approximate expression

$$r_b = r_0(y) = r_{0A} + r_{1A} \left( \frac{1}{y} \right)^{\lambda_{rA}},$$

(13)

reproduces well the values of $r_b$ calculated from Eq. (14). The parameter $r_{0A}$ is determined from the relation

$$r_{0A} = r_b(\infty) = R_1^{1/2},$$

3
while the parameters \( r_{1A} \) and \( \lambda_{r,A} \) are determined by least-squares fit of the values of \( r_b \) calculated from Eq. (13) to the values of \( r_b \) calculated from Eq. (11).

The values of the parameter \( \rho_{0A} \) and the best fit values of the parameters \( r_{1A} \) and \( \lambda_{r,A} \) for various nuclei are displayed in Table I, while the values of \( r_b \) calculated from the fitting expression (13) using the above values of the parameters are shown by lines in Fig. 2. It is seen that the simple approximate expression (13) reproduces quite well the numerical values of \( r_b \). As in the case of the parameters \( s_{0A}, s_{1A} \) and \( \lambda_{s,A} \) there is a systematic trend in the values of the parameters \( r_{0A}, r_{1A} \) and \( \lambda_{r,A} \). The parameters \( r_{0A}, r_{1A} \) and \( \lambda_{r,A} \) are increasing functions of \( A \) with exceptions in the nuclei \( ^{36}\text{Ar} \) and \( ^{40}\text{Ca} \) for the parameter \( \lambda_{r,A} \). Both of them are considered as \( 1d \) and \( 2s \) closed shell nuclei, respectively.

From the functional forms of \( S_A(y) \) and of \( r_b(y) \) given by Eqs. (10) and (13), respectively, the dependence of \( S_A \) on the RMS radius of the nucleus can be found. That dependence has the form

\[
S_A(r_b) = s_{0A} + s_{1A} \left( \frac{r_b - r_{0A}}{r_{1A}} \right)^{\lambda_{s,A}/\lambda_{r,A}} \tag{14}
\]

Thus, the information entropy sum of a nucleus has been connected with a fundamental quantity, the RMS radius of the nucleus measured in units of the HO size \( b_0 \).

### IV. Determination of the Correlation Parameter from the Information Entropy

In recent works, it has been shown that the information entropy sum of a quantum system (electrons in atoms \([2,3]\), nucleons in nuclei and valence electrons in atomic clusters \([2]\) and correlated Bose atoms in a harmonic trap \([14]\)) depends approximately linearly on the logarithm of the number of particles, given by the form

\[
S = S(A) = a + b \ln A, \tag{15}
\]

where \( a \) and \( b \) depend on the considered system.

The question that arises is how that property can be used in practice. A possible way is to determine \( S_A \) for two nuclei (such that \( ^4\text{He} \) and \( ^{40}\text{Ca} \) for which there are enough experimental data and then to find \( a \) and \( b \) of Eq. (15) from the relations

\[
a = \frac{S(4) \ln 40 - S(40) \ln 4}{\ln 40 - \ln 4}, \tag{16}
\]

\[
b = \frac{S(40) - S(4)}{\ln 40 - \ln 4}. \tag{17}
\]

\( S(4) \) and \( S(40) \) can be found calculating first the charge form factors of \( ^4\text{He} \) and \( ^{40}\text{Ca} \) from the relation

\[
F_{ch}(q) = F(q)f_{cm}(q)f_p(q)\,F_{DF}(q),
\]

where \( F(q) \) is the point form factor of the nucleus calculated from Eq. (3), \( f_{cm}(q) \) is the Tassie-Barker center-of-mass correction and \( f_p(q) \) and \( F_{DF}(q) \) are the correction for the finite proton size and the Darwin-Foldy relativistic correction, respectively \([2]\).

The parameters \( b_0 \) and \( y \) for \( ^4\text{He} \) and \( ^{40}\text{Ca} \) are determined by least-squares fit of the theoretical \( F_{ch}(q) \) to the experimental, with the constraint that the experimental charge RMS radius is to be reproduced. The values of the parameters \( b_0 \) and \( y \) as well as the values of \( \chi^2 \) are displayed in Table II. In the same table the values of \( b_0 \) and \( \chi^2 \) in the HO case \((y = \infty)\) are also shown. In that case \( b_0 \) is determined from the above mentioned constraint. The experimental and the theoretical \( F_{ch}(q) \), calculated with and without SRC, for the two nuclei are shown in Fig. 3.

With the values of \( b_0 \) and \( y \) determined in the above described way, for the two nuclei \( ^4\text{He} \) and \( ^{40}\text{Ca} \), we calculated the point \( \rho(r) \) and \( n(k) \) from Eqs. (3) and (8) and the Shannon’s information entropies \( S_r \) and \( S_k \) from Eqs. (3) and (8) and their sum, \( S(4) \) and \( S(40) \). Substituting the values of \( S(4) \) and \( S(40) \) into Eqs. (16) and (17), the parameters \( a \) and \( b \) are determined. The calculated values

\[
a = 5.4029 \quad \text{and} \quad b = 0.9360,
\]

are quite close to the values \( a = 5.325 \) and \( b = 0.858 \) which have been found in Ref. [19] with SkIII interaction.
Rearranging Eq. (10) and replacing \( S_A(y) \) by \( S(A) \) from Eq. (15), we may write

\[
y = \left( \frac{s_{1A}}{S(A) - s_{0A}} \right)^{1/\lambda_{sA}}.
\]

Using the values of the parameters \( s_{0A}, s_{1A} \) and \( \lambda_{sA} \) given in Table I and the values of \( S(A) \) calculated from Eq. (13), the correlation parameter \( y \) is determined for the other nuclei without any fit to experimental data. The HO parameter \( b_0 \), can be determined now for each nucleus from the relation

\[
r_{exp}^2 = \langle r^2 \rangle - \frac{b_0^2}{A} + r_p^2 + \frac{h^2}{2m^2c^2} - \frac{1}{A} \left( \frac{\lambda^2}{2} + \frac{\eta}{A} \right),
\]

where \( r_{exp}^2 \) is the experimental mean square charge radius of the nucleus and \( \langle r^2 \rangle \) is the point mean square radius calculated from Eq. (11). The last three terms of Eq. (19) are the corrections due to the spurious center-of-mass motion, the finite proton size and the Darwin-Foldy relativistic correction, respectively.

The values of \( b_0 \) and \( y \) for the various \( s-p \) and \( s-d \) shell nuclei determined in the way above described, as well as the values of the least-squares errors, in the comparison of the theoretical \( F_{\text{ch}}(q) \) to the experimental and the entropy sum \( S \) are displayed in Table II. In the same table the values of \( b_0, \chi^2 \) and \( S \) when SRC are not included (HO case) are also shown. From Table II we can see that there is a systematic behavior of the parameter \( y \). The values of \( y \) are always smaller (that is large correlations) in the closed shell nuclei, \( ^4\text{He}, ^{16}\text{O}, ^{36}\text{Ar} \) and \( ^{40}\text{Ca} \), than in the neighboring open shell ones. It is mentioned that \( ^{36}\text{Ar} \) is treated as 1d closed shell nucleus. The above behavior indicates that there should be a shell effect in the case of closed shell nuclei for the values of the correlation parameter \( y \). Similar behavior was found in Ref. [29] where the determination of the parameters \( b_0 \) and \( y \) were made by individual fit of the theoretical \( F_{\text{ch}}(q) \) to the experimental one. From the same table we can see that \( y \) is a monotonically increasing function of the number of nucleons of the closed shell nuclei.

The theoretical \( F_{\text{ch}}(q) \) with and without SRC, as well as the experimental ones for various nuclei have been plotted versus the momentum transfer \( q \) in Figs. 4 and 5. It is remarkable that without fit to the experimental charge form factors, the present method gives good form factors, reproducing the diffraction minima and maxima in the correct place. In nearly all cases, the \( \chi^2 \) values found with SRC are better than the corresponding values in the HO case. However, the assessment of the quality of the calculated form factors should not be based solely on the values of the least-squares errors but also on the fulfillment of the requirement that all the diffraction minima are reproduced in the correct place. Thus, comparing the quality of the form factors calculated in the present method with the ones calculated with the harmonic oscillator model, we can say that the quality of the form factors are considerably better in the former case. All the diffraction minima (even the third one which seems to exist in the experimental data of \( ^{24}\text{Mg}, ^{28}\text{Si} \) and \( ^{32}\text{S} \)) are reproduced in the present method while in the harmonic oscillator model they are not. We note also that, in the case of the nucleus \( ^{36}\text{Ar} \) there are not experimental data for the form factor. The exception which appears in \( ^{16}\text{O} \), where the value of \( \chi^2 \) with SRC is worse compared with the value of \( \chi^2 \) without SRC, should not be taken as a drawback of the present method. The reason is that there exist many experimental points at low momentum transfer where the HO model gives good form factor, while there are a few experimental points at high momentum transfers where the present method reproduces these points very well, as well as all the diffraction minima and maxima. That can be seen in Fig. 4b. If the experimental points were distributed uniformly, then the values of \( \chi^2 \) calculated within the present method would be smaller than the ones calculated within the HO model. Thus, we should conclude that even in \( ^{16}\text{O} \) the theoretical \( F_{\text{ch}}(q) \) calculated within the present method is better than that calculated within the HO model.

It should be noted that in the above analysis, the open shell nuclei, except \( ^{12}\text{C} \), have been treated as 1d shell nuclei. However, the same analysis could be made if they were considered as 1d-2s shell nuclei, provided that the corresponding occupation probabilities of the various states are known. That analysis was made, as an example, for \( ^{36}\text{Ar} \), assuming that the occupation probabilities of the various states are: \( \eta_{1s} = \eta_{1p} = \eta_{2s} = 1 \) and \( \eta_{1d} = 0.8 \). The values of the parameters \( b_0 \) and \( y \) which were found are: \( b_0 = 1.7861 \, \text{fm} \) and \( y = 7.7658 \). Thus, the value of the parameter \( y \) now becomes larger (less correlations) than that of the closed shell nucleus \( ^{40}\text{Ca} \) and closer to the values of the open shell nuclei \( ^{24}\text{Mg} \) and \( ^{28}\text{Si} \) (see Table II).

Finally, comparing the values of the information entropy sum, which were calculated with and without SRC and are displayed in Table II, it is seen that the introduction of SRC increases the information entropy sum by 3% to 5% in agreement with the simple model of SRC used in Ref. [10].
V. SUMMARY

In the present work a systematic study of Shannon’s information entropy sum $S$ has been made for various $N = Z$, $s$-$p$ and $s$-$d$ shell nuclei using correlated one-body density matrix which depends on the HO size $b_0$ and the correlation parameter $y$.

It is found that, for all the nuclei we have examined, $S$ depends only on $y$ through a simple two-parameter relation. A similar approximate expression holds for the RMS radius of the nucleus $r_b$, measured in units of $b_0$. It is found also that a simple relation connects $S$ with $r_b$.

From the dependence of $S$ on $y$ and its linear dependence on the logarithm of the number of nucleons of the nucleus, the correlation parameter $y$ for a nucleus can be determined, provided that there are enough experimental data for two neighboring nuclei. It is mentioned that, usually, the two parameters of the correlated one-body density matrix are determined for each nucleus by least-squares fit of the theoretical $F_{ch}(q)$ to the experimental. Within the present method, those parameters are determined even in those cases where there are not any experimental data for the charge form factor as this has been made for the nucleus $^{36}$Ar. The only experimental data which are used are the experimental charge RMS radius of the nucleus, as well as the experimental charge form factors and RMS radii of only two nuclei, those of $^4$He and $^{40}$Ca. The procedure which is followed to determine the parameters of the model is the following: First we determine the parameters $a$ and $b$ of Eq. (15) from the experimental data of $^4$He and $^{40}$Ca. Thus, the values of $S(A)$ for each nucleus are known. From the value of $S(A)$ for a particular nucleus and from Eq. (10) we determine the correlation parameter $y$ corresponding to each nucleus. From that value of $y$ and from Eq. (11) the parameter $b_0$ is determined so that the theoretical RMS charge radius to be the experimental one. It is noted also that, using the sum of the information entropies $S_r$ and $S_k$, the short range correlation parameter has been determined indirectly, from the density distribution, as well as from the momentum distribution. This appears to be an interesting feature of the present method, since that parameter is usually determined only from the density distribution. We would like to mention that ”experimental data” for the momentum distribution are not directly measured but are obtained by means of $y$-scaling analysis [31] and are only known for $^4$He and $^{12}$C.

It would be interesting if the dependence of the information entropy sum on the various parameters could be examined with more sophisticated models, such as the density dependent interactions, and if the present method could be applied for the determination of a parameter of those models.

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TABLE I. The values of the parameters $s_{0A}$, $s_{1A}$ and $\lambda_{sA}$ of the information entropy sum $S_A$ of the relation (10), and of the parameters $r_{0A}$, $r_{1A}$ and $\lambda_{rA}$ of the RMS radius of the relation (13) for various $s$-$p$ and $s$-$d$ shell nuclei.

| Nucleus | $s_{0A}$ | $s_{1A}$ | $\lambda_{sA}$ | $r_{0A}$ | $r_{1A}$ | $\lambda_{rA}$ |
|---------|---------|---------|-------------|---------|---------|-------------|
| $^4$He  | 6.4342  | 1.0410  | 1.0064      | 1.2247  | 0.4740  | 1.5221      |
| $^{12}$C| 7.5086  | 2.1885  | 1.1548      | 1.4720  | 1.5579  | 1.8282      |
| $^{16}$O| 7.6069  | 2.6464  | 1.1529      | 1.5     | 2.5069  | 1.9549      |
| $^{24}$Mg| 8.0933 | 3.7445  | 1.2390      | 1.6330  | 4.1480  | 2.1115      |
| $^{28}$Si| 8.2096 | 4.1641  | 1.2548      | 1.6691  | 5.2681  | 2.1959      |
| $^{32}$S| 8.2901  | 4.5837  | 1.2659      | 1.6956  | 6.2356  | 2.2275      |
| $^{36}$Ar| 8.3490 | 4.9578  | 1.2681      | 1.7159  | 6.2862  | 2.1758      |
| $^{40}$Ca| 8.4347 | 4.7275  | 1.2208      | 1.7321  | 6.5364  | 2.0809      |

TABLE II. The values of the parameters $b_0$ (in fm) and $y$, the $\chi^2$, the RMS charge radius $\langle r_{ch}^2 \rangle^{1/2}$ (in fm) and the information entropy sum $S$ for various $s$-$p$ and $s$-$d$ shell nuclei. The theoretical RMS charge radii are equal to the experimental of Ref. [25].

| Nucleus | Case | $b_0$ | $y$ | $\chi^2$ | $\langle r_{ch}^2 \rangle^{1/2}$ | $S$ |
|---------|------|------|----|---------|---------------------------------|----|
| $^4$He  | SRC  | 1.2497 | 3.7857 | 9.40   | 1.676                             | 6.7068 |
|         | HO   | 1.3335 | ∞    | 53.46  | 1.676                             | 6.4342 |
| $^{12}$C| SRC  | 1.5617 | 7.1294 | 153.46 | 2.471                             | 7.7351 |
|         | HO   | 1.6108 | ∞    | 181.31 | 2.471                             | 7.5086 |
| $^{16}$O| SRC  | 1.6451 | 5.1782 | 417.05 | 2.730                             | 8.0044 |
|         | HO   | 1.7554 | ∞    | 202.09 | 2.730                             | 7.6069 |
| $^{24}$Mg| SRC | 1.7609 | 7.8711 | 221.07 | 3.075                             | 8.3839 |
|         | HO   | 1.8222 | ∞    | 226.41 | 3.075                             | 8.0933 |
| $^{28}$Si| SRC | 1.7226 | 7.8711 | 322.63 | 3.086                             | 8.5282 |
|         | HO   | 1.7860 | ∞    | 472.93 | 3.086                             | 8.2096 |
| $^{32}$S| SRC  | 1.7781 | 7.4140 | 669.72 | 3.248                             | 8.6531 |
|         | HO   | 1.8559 | ∞    | 850.03 | 3.248                             | 8.2901 |
| $^{36}$Ar| SRC | 1.7885 | 7.0790 | 3.327  | 8.7634                             |
|         | HO   | 1.8801 | ∞    | 3.327  | 8.3490                             |
| $^{40}$Ca| SRC | 1.8397 | 7.1632 | 168.44 | 3.479                             | 8.8620 |
|         | HO   | 1.9526 | ∞    | 230.60 | 3.479                             | 8.4347 |
FIG. 1. The information entropy sum $S_A$ versus the correlation parameter $1/\gamma$ for various $s$-$p$ and $s$-$d$ shell nuclei. The points correspond to the numerical values of $S_A(\gamma)$ and the lines come from the fitting expression (10).

FIG. 2. The RMS radius $r_b$, in units of the HO size $b_0$, versus the correlation parameter $1/\gamma$ for various $s$-$p$ and $s$-$d$ shell nuclei. The points correspond to the values of $r_b$ calculating from Eq. (11) and the lines come from the fitting expression (13).

FIG. 3. The charge form factors of nuclei $^4$He (a) and $^{40}$Ca (b). The solid lines correspond to the case when SRC are included and the parameters $\gamma$ and $b_0$ are determined by least squares fit of the theoretical charge form to the experimental with the constraint the calculated RMS charge radius is to be the experimental one. The dot lines correspond to the HO case when $b_0$ is determined from the experimental RMS charge radius. The experimental points for $^4$He are from Ref. [26] and for $^{40}$Ca from Ref. [27].
FIG. 4. The charge form factors of nuclei $^{12}$C (a), $^{16}$O (b) and $^{24}$Mg (c). The solid lines correspond to the case when SRC are included and the parameters $y$ and $b_0$ are determined from Eq. (18) and the experimental RMS charge radius, respectively. The dot lines correspond to the HO case when the parameter $b_0$ is determined from the experimental RMS charge radius. The experimental points for $^{12}$C and $^{16}$O are from Ref. [28] and for $^{24}$Mg from Ref. [29].

FIG. 5. The charge form factors of nuclei $^{28}$Si (a), $^{32}$S (b) and $^{36}$Ar (c). The various cases are as in Fig. 4. The experimental points for $^{28}$Si and $^{32}$S are from Ref. [29].