Hybrid adaptive framework for coordinated control of distributed generators in cyber-physical energy systems

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Abstract: With the development of information and communications technology (ICT) and inundation of sensing devices, the control of smart grid is undergoing a paradigm shift from centralised/decentralised to a more distributed nature allowing each distributed generator to receive information from sensors at distant buses. In such systems, there is much interdependency between various power, control and communication parameters due to which the control of parameters from one domain gets affected by other. The central idea of this study is to develop a generic, hybrid and customised framework to jointly model the multi-disciplinary variables and their interactions present in the smart grid and to develop controllers in an adaptive manner to ensure better control of physical variables such as voltage irrespective of the changes in operating point brought about by changes in physical/cyber parameters. Hence, the different operating conditions of the power system have been modelled as multiple subsystems of a hybrid switching system and controller design is carried out by solving the optimisation formulations developed for delay-free and delay-existent cases using the theory of common Lyapunov function. The optimisation is carried out using the block coordinate descent methodology by converting the non-convex formulation into a series of convex problems with information from sensors on multiple buses providing scope for each DG to take a better control action. The central server would keep track of different parameters in the system that are prone to variations and make short-/long-term forecasts on the same. This information can be used in an online manner to adaptively update the distributed controllers to tackle different scenarios during the course of operation.

In practice, however, implementation of such CPES has to tackle issues pertaining individually to CCCP along with the ones that emerge from the combination of these domains. To promote flexibility in the CPES so as to accommodate different situations, the control framework needs to be more situation aware and adaptive, so that system does not become unstable due to unanticipated phenomena. The adaptive controllers shall depend not only on the structure of the physical system but also on the properties of communication and computation worlds. This calls for hybrid CPES modelling frameworks which contain inputs and outputs belonging to multiple fields including CCCP.

Moreover, these frameworks need to be generic to provide a direct pathway for many problems to be modelled into its structure. In CPES, there are many types of systems that need cyber-physical solutions ranging from transmission grids, distribution grids and microgrids each with different kinds of control problems. The CPES frameworks developed need to support all of these problems using a unified framework. The framework should also be customisable to the needs of a particular problem. For example, there might be some localities suffering from power deficit and are highly sensitive to changes in load. There might be certain other places such as hilly areas, where communication is a problem and the CPES becomes sensitive to delay. The erratic climatic conditions in some places may lead to damage in communication equipment. Similarly, the CPES may also experience different levels of sensitivities to different parameters depending on the time of the day. The load is generally high during the day and less in the morning. If the communication network of the grid is shared jointly for Internet browsing and downloading, the CPES may become sensitive to delay due to excessive data usage during evening times. Hence, to study these effects and provide relevant solutions, hybrid frameworks are necessary.
Augmenting the distribution system with communication for better voltage and frequency control has been under consideration for some time [14]. The usage of distributed [15–17] control frameworks with communication has gained importance over a centralised [18, 19] scheme as it enhances the reliability and scalability of the system. For example, Xin et al. [20] talk about a cooperative control strategy for multiple solar plants using minimal communications. Shafiee et al. [21] proposed a distributed secondary control technique which enhances the working of traditional droop control in a microgrid by adding a ubiquitous communication framework. Liang et al. [22] proposed a hybrid control scheme which works both in the presence and absence of a centralised communication scheme. There have also been certain works such as [23] which came up with communication routing algorithms and protocols to connect DGs and enhance power sharing in various microgrid configurations. Most of the works found in the literature are either specific to power domain or communication domain or computation domain even though they portray application to the smart grid. They do not model the inter-dependencies between parameters of various domains.

The work in [24] proposed a simple optimisation technique to design controller gains based on communication topology for voltage control in smart grids. A greedy algorithm was used to route the connections between sensors and DGs based on controller stability. Mishra et al. in [25] further improvised the routing strategy with a generalised algorithm to find more stable configurations with added constraints. Yet, in these works, both the constraints and parameters in the domains of communication and control remain fixed. In practise, various physical parameters such as load and communication parameters such as delay keep changing simultaneously over the course of the day. Lu et al. [26] tried to partially address this by modelling the smart grid system as a hybrid system but it could only capture the variation in delays. To our knowledge, no one has addressed the adaptive nature of a generic CPES framework having different types of dynamics.

To capture the inter-dependencies between communication and control and to design controllers adaptively for simultaneous variations in cyber-physical parameters, this paper proposes the following in fully connected smart grids:

1. A generic, hybrid and customisable framework to capture the dynamics of communication and control using the theory of hybrid switching systems.
2. Optimisation formulations using common Lyapunov function (CLF) to design controllers acknowledging variations in both physical and communication parameters in delay-free and delay-existent systems.
3. A block coordinate descent (BCD)-based technique for finding a solution to the above formulations which are non-convex in nature.

The organisation of this paper is as follows. Section 2 gives an idea of modelling various aspects in a generic CPES system. Section 3 describes the proposed CLF-based optimisation frameworks capable of designing controllers for multi-parametric variations in these systems, whereas Section 4 explains the BCD technique for solving the proposed optimisation problems. Section 5 describes the application of the developed techniques to the context of voltage control in a smart distribution grid and the results obtained on application of these procedures on certain scenarios such as load fluctuations, delay fluctuations and communication failure in a 4-bus distribution feeder have been presented in Section 6. Appropriate conclusions with possible future work have been indicated in Section 7.

### 2 Cyber-physical model

The structure in Fig. 1 consists of both physical components such as sensors, DGs and controllers which deal with physical parameters such as voltage and frequency and communication components placed at the physical components which deal with parameters such as bandwidth, speed, data loss etc. Hence, the model adopted for representing such a system must be able to represent parameters both from the physical and communication worlds. This section describes various assumptions and models pertaining to physical and communication domains of the generalised CPES along with the distributed control structure and the switching system representation of the overall system. The detailed application of this modelling framework is described in Section 5 for a multi-input-multi-output (MIMO) voltage control problem of an n-bus distribution feeder.

#### 2.1 Physical model

Let the smart grid contain Ns number of buses at which sensors are placed. Furthermore, let there be No number of controllers which are controlling a specific physical variable at these points. The distributed controllers present at the DGs sense the variables and come up with a strategy to vary their outputs for the bus variables to reach their reference values. The MIMO system model of the grid can be considered in the form as below:

$$\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) \\
y(t) &= Cx(t)
\end{align*}$$

where $x(t)$ is the state vector of length $n$, $u(t)$ is the control vector of length $n_c$, and $y(t)$ represents the observation vector from sensors of length $n_s$. Thus

$$A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n_c \times n}, C \in \mathbb{R}^{n_s \times n}$$

Most of the dynamic control problems in various types of smart grid systems including transmission, distribution and microgrids can be modelled into state-space format. Even, if the system may be non-linear, it can be linearised around its equilibrium point. The observations in the system are considered to be noiseless.

#### 2.2 Communication structure

Most of the equipment in the smart grid are assumed to be equipped with wired/wireless communication interfaces. These interfaces can be clubbed into three types of nodes: sensor nodes, controller nodes and the server nodes.

There exist two types of communication networks:

- **Network-1** consists of a central server which has receiver nodes to get the data from sensors and a transmitter node to send data to the DGs. This network functions mostly when there is a need to update the existing distributed controllers based on large parameter variations. The central server tracks the data of various parameters of the smart grid and forecasts their future values to design situation-aware distributed controllers adaptively.
- **Network-2** consists of interconnections between the sensors and controllers which will be operational on a regular basis. The $n_s$ sensor nodes placed at individual buses collect the data coming out of the sensors and send them to $n_c$ controller nodes placed at
DGs which compute the control references in a distributed manner.

The communication structure is fully connected which means that every controller receives information from all the sensors. The data flow among these nodes is assumed to be continuous, i.e. the sampling rate is high, the quantisation error is low and there is no issue of packet loss.

2.3 Distributed control scheme

The following distributed state-feedback control structure:

$$u(t) = Kx(t)$$  \hspace{1cm} (2)$$

is used, where $K \in \mathbb{R}^{n \times n}$ shows the connection structure between controllers and sensors. This control structure allows inputs from multiple sensors to be available to individual distributed controllers. If $K_{ij}$ is non-zero, it represents the existence of a connection between controller $i$ and sensor $j$. For full communication structure subscribed in this paper, all the elements of $K$ matrix would be non-zero. Substituting (2) into (1), results in

$$\dot{x}(t) = \hat{A}x(t)$$  \hspace{1cm} (3)$$

where $\hat{A} = A + BK$ represents closed-loop system matrix. Without the inclusion of the controller, the system stability is defined by the eigenvalues of the matrix $A$, whereas once the controller is added, the system's stability gets defined through eigenvalues of the matrix $\hat{A}$. If the real parts of all the eigenvalues are negative, then the system matrix $\hat{A}$ is stable.

2.4 Delay representation in the MIMO model

Delay existing between various sensors and controllers has been incorporated in the MIMO model as follows [24]:

$$\dot{x}(t) = Ax(t) + \sum_{i,j \in \mathbb{N}} b^{i,j}(\cdot, \cdot)k(j, \cdot)x(t - d_{ij})$$  \hspace{1cm} (4)$$

where $(j, i) \in \mathbb{R}$ means that the connection between sensor $j$ and controller $i$ is established, $b^{i,j}(\cdot, \cdot)$ is the $i$th column of the matrix $B$, $k(j, \cdot)$ is the $j$th row of the matrix $K$ and $d_{ij}$ is the delay between sensor $j$ and controller $i$. A particular case of delay has been considered in this work where the delay is small. The wireless networks can deliver the data in order of milliseconds, whereas the system matrix $\hat{A}$ is stable.

$$\hat{A} = A + BK$$

where $\hat{A} = (I - BDK)(A + BK)$ and $D$ is the delay matrix formulated as

$$D_{ij} = \begin{cases} d_{ij} & \text{if } j \text{ and } i \text{ are connected} \\ 0 & \text{if } j \text{ and } i \text{ are not connected} \end{cases}$$  \hspace{1cm} (7)$$

2.5 Switching system representation

The model shown in (1) represents the working of the smart grid only for a particular value/small range of parameters. However, the variation of the parameters during the course of operation is quite large. The overall system can alternatively be modelled as a hybrid system consisting of $n$ switching subsystems

$$\dot{x} = A_i x + B_i u; \quad t_k \leq t < t_{k+1}$$

where each subsystem captures the set of physical and communication parameters operational for a particular period of time during the day. For example, $A_i$ can represent the power system load from 5 a.m. to 7 a.m., whereas $A_i$ can represent the load between 7 a.m. and 9 a.m. and so on.

3 CLF-based controller design

Many physical variables in the power system such as bus voltages get affected by multiple parameters both from the communication and the physical worlds. These parameters might change either in the natural course of system operation or sometimes as a matter of choice of the operator. Hence, the set of distributed controllers as a whole must be capable of stabilising the system over a wide range of parameters.

For this purpose, the switching representation of the system as presented in (8) has been adopted which can consider multiple scenarios as multiple subsystems. A CLF [27–29] can exist in such a way that it stabilises all the subsystems considered in the switching representation simultaneously. Thus, the controller devised keeping the CLF in perspective can stabilise all the scenarios considered. The conditions for the availability of such a function can be shaped into a linear matrix inequalities (LMI) formulation, where different electrical loading conditions or different delay conditions are represented as various switching subsystems.

It is to be noted that for our purpose, we shall consider only two subsystems at a time since the increase in the number of subsystems reduces the likelihood of existence of the CLF. The forecast tool running in the central server will be of much use in deciding the two subsystems to be considered for a particular time of the day. The optimisation frameworks which have been derived for no-delay systems and delay systems basing on this concept have been presented in this section.

3.1 Systems without delay

For designing controllers on the basis of CLF, we assume two closed-loop switching subsystems $\dot{x} = A_{k}x$ and $\dot{x} = A_{k}x$, where $A_{k} = A_{i} + BK$ and $A_{k} = A_{i} + BK$ satisfy the common Lyapunov stability criterion.

Considering a Lyapunov function $V_{1} = x^{T}P_{1}x$ for $A_{k}$ and applying $V_{1} < 0$ which means

$$A_{k}^{T}P_{1} + P_{1}A_{k} = - P_{1}, \quad P_{1} > 0, \quad P_{1} > 0$$  \hspace{1cm} (9)$$

Similarly, considering the Lyapunov function $V_{2} = x^{T}P_{2}x$ for the system $\dot{x} = A_{2}x$ and applying $V_{2} < 0$ for stability

$$A_{2}^{T}P_{2} + P_{2}A_{2} = - P_{2}, \quad P_{2} > 0, \quad P_{2} > 0$$  \hspace{1cm} (10)$$

Now, consider the time derivative of the second Lyapunov function $V_{2}$

$$V_{2} = x^{T}A_{k}^{T}P_{2}x + x^{T}P_{1}A_{k}x = - x^{T}P_{1}x$$

Substituting (10) into (9) results in

$$A_{k}^{T}P_{1} + P_{1}A_{k} = - P_{1}, \quad P_{1} > 0, \quad P_{1} > 0$$  \hspace{1cm} (12)$$

On applying the assumption...
\[ A_K A_{K_2} = A_{K_2} A_K \]  
\[ \text{in (12) and realigning, it can be seen that} \]
\[ A_K^T (K_2 P_2 + P_2 A_K) + (K_2 P_2 + P_2 A_K) A_K = P_0 \]  
\[ \text{which means} \]
\[ A_K^T \hat{P} + \hat{P} A_K = P_0 \]  
\[ \text{where} \hat{P} = A_K^T P_2 + P_2 A_K. \]

From (15), if \( P_0 > 0 \) and \( A_K \) is stabilising, then \( \hat{P} < 0 \) which means that \( P_0 \) is also stabilising for \( A_K \). Adding a \( \gamma \) parameter, this condition can be rewritten as
\[ A_K^T P_2 + P_2 A_K + \gamma I < 0 \]  
\[ \text{Thus, under the conditions mentioned above,} \ V_2 \text{becomes the CLF for both} \ A_{K_2} \text{and} A_K. \]

\[ \| K \|_2 \leq \rho \]  
\[ \text{Thus, if a controller} \ K \text{and matrix} P_2 \text{can be obtained while satisfying assumptions (10), (12), (13), (16) and (17), it ensures that the controller can stabilise the system when switched from} A_{K_2} \text{to} A_K \text{or vice versa. Formulating these assumptions as constraints, the optimisation problem can be written in the following way:} \]
\[ \max_{\gamma, K} \gamma \]  
\[ \text{s.t.} \ A_K^T P_2 + P_2 A_K + \gamma I < 0, \]  
\[ \text{condition (12) holds,} \]
\[ A_K A_{K_2} = A_{K_2} A_K, \]  
\[ A_K^T P_2 + P_2 A_K < 0, \]  
\[ \| K \|_2 \leq \rho. \]

### 3.2 Systems with delay

For this case, let the two switching system dynamics under consideration be
\[ \dot{x} = A_K x + B f_i(x), \quad \| f_i(x) \|_2 \leq \alpha \| x \|_2 \]
\[ \dot{x} = A_K x + B f_i(x), \quad \| f_i(x) \|_2 \leq \alpha \| x \|_2 \]  

Considering a common controller \( u = K x \) for both the system dynamics in (19) and (20) results in the following switched system representation:
\[ \dot{x} = A_{K_1} x + f_i(x) \]
\[ \dot{x} = A_{K_1} x + f_i(x) \]  

Here, \( A_{K_1} = A_1 + B K \) and \( A_{K_2} = A_1 + B K \) show the system design for two different physical parameters, whereas \( f_i(x) = -B D_K A_{K_2} x \) and \( f_j(x) = -B D_K A_{K_2} x \) signify different delays present in the two subsystems.

It is assumed that the dynamics of the two subsystems mentioned in (19) and (20) are stabilised by Lyapunov functions \( V_i(x) = x^T P_i x \) and \( V_j(x) = x^T P_j x \), respectively, where \( P_i, P_j > 0 \). The time derivative of \( V_i \) can be written as
\[ \dot{V}_i(x) = x^T P_i \dot{x} + x^T f_i(x) \]
\[ = x^T P_i (A_{K_1} x + f_i(x)) + x^T A_{K_1}^T f_i(x) P_i x \]  

This equation can be rewritten as
\[ \dot{V}_i(x) = y^T F_i y \]

where
\[ F_i = \begin{bmatrix} A_{K_1}^T P_2 + P_2 A_{K_1} & P_1 \\ P_2 & -P_0 \end{bmatrix} \text{ and } y = [x^T \ f_i(x)]^T \]

It is also known that \( \| f_i(x) \|_2 \leq \alpha \| x \|_2 \). This can be easily rewritten as
\[ y^T G_i y \leq 0 \]  
\[ \text{where} \]
\[ G_i = \begin{bmatrix} -\alpha^2 I & 0 \\ 0 & I \end{bmatrix} \]  

Now, for stability, \( \dot{V}_i = y^T F_i y < 0 \) must exist and it will happen if there exists a \( y^T G_i y \leq 0 \) such that \( F_i - \gamma G_i < 0 \) which results in
\[ \begin{bmatrix} A_{K_1}^T P_2 + P_2 A_{K_1} + \gamma \alpha^2 I & P_1 \\ P_2 & -\gamma I \end{bmatrix} < 0 \]  

Following a similar procedure for \( V_i \) gives rise to:
\[ \begin{bmatrix} A_{K_1}^T P_2 + P_2 A_{K_1} + \gamma \alpha^2 I & P_1 \\ P_2 & -\gamma I \end{bmatrix} < 0 \]  

The following assumptions have been made in similar terms to that of (9) and (10) after adding terms to represent uncertainty:
\[ A_{K_1}^T P_2 + P_2 A_{K_1} + \gamma \alpha^2 I = -P_0, \quad P_0 > 0, \quad P_i > 0 \]
\[ A_{K_1}^T P_2 + P_2 A_{K_1} + \gamma \alpha^2 I = -P_i, \quad P_i > 0, \quad P_0 > 0 \]  

It is seen that (28) and (29) when applied to (26) and (27) give rise to the following equation:
\[ \begin{bmatrix} -P_{i-1} & P_i \\ P_i & -\gamma I \end{bmatrix} < 0 \quad i = 1, 2. \]  

Also, substituting (29) into (28)
\[ \gamma \alpha \dot{I} + A_{K_1}^T (A_{K_1}^T P_2 + P_2 A_{K_1} + \gamma \alpha^2 I) \]
\[ + (A_{K_1}^T P_2 + P_2 A_{K_1} + \gamma \alpha^2 I) A_{K_1} = P_0 \]  

Substituting another assumption
\[ A_{K_1} A_{K_2} = A_{K_2} A_{K_1} \]  
\[ \text{into (31) and rearranging we obtain} \]
\[ A_{K_1}^T P + P A_{K_2} - \hat{P} = 0 \]  
\[ \text{where} \]
\[ \hat{P} = P_i + \gamma \alpha \dot{I} (A_{K_1}^T + A_{K_2}) \]  
\[ P = A_{K_1}^T P_2 + P_2 A_{K_1} + \gamma \alpha^2 I \]  
\[ \text{Now, if} \]
\[ \hat{P} > 0 \]  
\[ \text{then from (33) we find that} \]
\[ A_{K_1}^T P + P A_{K_2} > 0 \]  

Since \( A_{K_1} \) is stabilising, \( P < 0 \) which means
that is, the Lyapunov function $V_i$ stabilises the system $A_{K_i}$ also becoming the CLF for the two systems mentioned in (19) and (20). After converting assumptions (17), (28)–(30), (32) and (34) into constraints, the optimisation problem for finding the value of $K$ is formulated as below:

$$\begin{align*}
\max & \quad \gamma_i + \gamma_s \\
s.t. & \quad (30)
\end{align*}$$

Solving the optimisation in (37) would result in a controller $K$ that can stabilise the systems with two different parametric conditions as mentioned in (19) and (20).

4 BCD technique

The optimisation problems in (18) and (37) are non-convex and difficult to solve within a reasonable amount of time. An approximate solution may, however, be obtained by utilising the BCD [30] philosophy. The idea here is to carry out the optimisation with respect to smaller blocks of variables, in an iterative fashion. Interestingly, for the problems at hand, it will be shown that these smaller subproblems are convex and easy to solve. The following algorithms delineate the procedure adopted for the same.

Algorithm 1: Finding $K$ for delay-free systems

1. Initialise $A_{K_i} := A_i - \beta I$, $n = 1, 2$, $P_0 := I$, $\rho = 5$.
2. Find $P^{(0)}_i$ and $P^{(0)}$ by solving (18) using constraints mentioned in (10), (12) and (16).
3. do
4. Initialise $P^*_i = P^{(0)}_i$, $\gamma^{(0)} = \gamma^{(0-1)}$.
5. Replace $A_{K_i}$ with $A_{K_i}^{(0)}$ in (12).
6. Solve for $K^{(0)}$ using (17), (28)–(30), (32) and (34).
7. Find $P^{(0)}_i$ and $P^{(0)}$ by solving (18) using constraints mentioned in (10), (12) and (16).
8. while $\gamma^{(i-1)} - \gamma^{(i)} > 10^{-6}$
9. end while.

For instance, the optimisation problem in (18) is divided into two parts. First, stable $A_{K_i}$ matrices are assumed in the initialisation process and the optimisation process is carried out only with constraints pertaining to $\gamma$ and $P$. With these values fixed, the controller $K$ is obtained by solving (18) only with respect to constraints pertaining to $K$ as shown in step 6. Furthermore, $\gamma$ and $P$ are obtained which is further utilised to find $K$. Thus, the iterations continue, till the value of $\gamma$ converges.

Algorithm 2: Finding $K$ for systems with delay

1. Initialise $A_{K_i}^{(0)} := A_i - \beta I$, $n = 1, 2$, $P_0 := I$, $\rho = 5$.
2. Find $\gamma^{(0)}_i$, $\gamma^{(0)}_s$, $P^{(0)}_i$ and $P^{(0)}$ by solving (37) using constraints mentioned in (28)–(30).
3. do
4. Initialise $P^*_i = P^{(0)}_i$, $P^*_s = P^{(0)}$, $\gamma^{(0)}_i = \gamma^{(0-1)}_i$, $\gamma^{(0)}_s = \gamma^{(0)}_s$.
5. Replace $A_{K_i}$ with $A_{K_i}^{(0)}$ in (12).
6. Solve for $K^{(0)}$ using (17), (28)–(30), (32) and (34).
7. Using the value of $K^{(0)}$ obtained in the previous step, find $\gamma^{(1)}_i$, $\gamma^{(1)}_s$, $P^{(1)}_i$ and $P^{(1)}$ by solving (37) using constraints mentioned in (28)–(30).
8. while $\gamma^{(i-1)} - \gamma^{(i)} > 10^{-6}$
9. end while.

The first two steps of the algorithms are meant to improve the feasibility of solution for the next optimisation in the process. The controller achieved will be able to stabilise the intended physical variable in the smart grid for two zones simultaneously due to CLF-based formulation. However, the two zones should be chosen with proper discretion depending on the zones in which the system will operate in a particular period of time.

5 Application to voltage control in smart grids

This section describes the application of the general system model and control design previously developed for the purpose of controlling the bus voltages in a smart grid scenario.

5.1 Description of the smart grid

Fig. 2 shows an $n$-bus distribution feeder, where bus voltages $V_i = [V_{1i}, \ldots, V_{ni}]$ are controlled with the help of DGs, which are modelled as voltage sources $V_c = [V_{1c}, \ldots, V_{nc}]$. Each DG consists of a power electronic inverter with a DC side capacitor. The DG is connected to the power system with the help of an inductor. The buses present in the grid may be connected to a substation whose effect is ignored in this paper to effectively portray the contribution of DGs. The control response of DGs is much faster than conventional sources due to low inertia and fast acting power electronic controllers.

On applying Kirchoff’s current law at each bus in Laplace domain and converting them to time domain, it is possible to arrive at the state-space representation of the system represented in (1). Applying nodal analysis at bus-$i$

$$\frac{V_{hi} - V_{ci}}{sL_{ci}} + \frac{V_{hi} - V_{bi}}{R_c + sL_{ci}} = 0$$

On converting this to time domain

$$\left(\frac{L_c}{sC_{ci}} + 1\right)V_{hi} - sV_{ci} + \frac{R_c}{sC_{ci}}V_{hi} - \frac{R_c}{sC_{ci}}V_{ci} - \frac{L_c}{sC_{ci}}V_{ci} = 0$$

Applying similar procedure at all the buses, the final time-domain equations can be written as follows:

$$TV(t) = TV(t) + TV_c(t) + TV_e(t)$$

where

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Tables 1 Grid parameters

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| $R_1$     | 0.175 | $L_1$     | 0.0005|
| $R_2$     | 0.1667| $L_2$     | 0.0004|
| $R_3$     | 0.2187| $L_3$     | 0.0006|

Equation (42) can be further rewritten as:

$$\Delta \hat{V}_i(t) = A' \Delta V_i(t) + B' \Delta V_i(t) + M' \Delta V_i(t)$$

where

$$A' = T'^{-1} T, \quad B' = T'^{-1} B, \quad M' = T'^{-1} M$$

Equation (47) can be further rewritten as:

$$\Delta V_i(t) = V_i - V_{\text{ref}}$$

and applied onto (47) to arrive at the state-space representation of the system represented in (1), where $A = A'$ and $B = A'M + B'$. Sensors placed at various buses collect the information on respective bus voltages $V_{i,j} = 1, \ldots, n$. As full communication architecture has been assumed, each sensor relays its information to all the DGs present in the smart grid. Using this information, each DG needs to generate a proper voltage $V_i = 1, \ldots, n$ such that the bus voltages reach their reference values $V_{\text{ref}}$. The CLF-based hybrid distributed controller design methodology designs a $K$ matrix that assures quick convergence of bus voltages when different parameters such as load and communication delay change or communication link are lost.

5.2 Functioning of the central server

With the tools adopted and developed in the previous sections, the central server of the smart grid can be utilised to design adaptive controllers in an online manner in the presence of various forecast data on load, climate etc. The central server keeps track of forecast data, computes the most appropriate controllers in an online manner and updates the $K$ values in the local DG controllers for every time slot consisting of 2 to 3 h. The following procedure is adopted for adaptively designing the CLF-based controllers:

1. Plot the maximum eigenvalues of the system matrix for the known range of parameters.
2. Segregate the plot into uniform smaller zones $z_1, z_2, \ldots, z_N$ on the basis of maximum eigenvalue as shown in Fig. 3.
3. Find the system matrices with worst eigenvalues for all $N$ individual zones $A_1, A_2, \ldots, A_N$. These are obtained by plugging in the worst-case parameter values of individual zones.
4. Select the particular zone in which the grid is operating during the current time slot and a neighbour zone, where the grid will operate in the next time slot as per the forecast data, say $z_i$ and $z_2$.
5. Solve the appropriate CLF formulation as presented in Section 4 and obtain the corresponding $K$ matrix.

6 Results

This section describes various results obtained while designing CLF-based controllers for voltage control in a 4-bus system. Cases such as the simultaneous variation in load, delay and loss of communication link have been explored for demonstrating the efficacy of the proposed method.

6.1 Simulation configuration

A sample 4-bus system is used for demonstrating voltage control whose parameters have been given in Table 1. The value of $R_i$ and $L_i$ depend on the amount of load resistance and reactance at a particular point of time. For all the cases, $L_i$ has been taken to be 0.0148. Also, the value of $L_{dc} = 0.001$ where $n = 1, 2, 3, 4$.

The different system matrices used for different loading conditions $R_i = 0.001$ and 0.5 are given as follows:

$$A_1 = \begin{bmatrix} -150.438 & -63.001 & -21.476 & -0.000 \\ -50.657 & -94.501 & -32.214 & -0.000 \\ 35.604 & 9.197 & 53.691 & -0.001 \\ 155.010 & 138.913 & 100.579 & -0.003 \end{bmatrix}$$

Voltage control, $\Delta V_i$ is the control input $u$ and the following state is chosen:

$$x(t) = \Delta V_i(t) - M' \Delta V_i(t)$$

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is modelled as a resistance and reactance. As described previously, these cases have been carried out using the CVX optimisation tool \[31\]. These scenarios including a change in load resistance, change in delay predicted for the upcoming period of the day. Along with this, the load variation is also an inevitable change. The controller-2 designed using optimisation formulation for delay case has demonstrated that it can stabilise the bus voltages for simultaneous change in load and delay profile. The results indicate that the controller-2 designed with delay considerations settles quickly after the disturbance compared with the controller-1 designed without delay considerations.

### 6.2 Numerical results

The system described previously has been tested in various scenarios including a change in load resistance, change in delay and failure in the communication link. All of these studies have been carried out using the CVX optimisation tool \[31\]. These cases can further be combined for describing practical scenarios in the cyber-physical framework.

#### 6.2.1 Change in load resistance: The load on the power system is modelled as a resistance and reactance. As described previously, two zones of operations \(A_1\) and \(A_2\) with different load resistances have been selected and the controller has been designed. The \(K\) matrix designated as controller-1 can comfortably take care of voltage control within these two neighbouring zones as well as maintain system stability while switching between the two zones of operation. When, the system is predicted to go beyond the bound of the selected two zones, the controller gets updated by the central server. The result for this case is shown in (52) and (53)

\[
K = \begin{bmatrix}
1.5063 & 0.4404 & 0.5500 & -1.5126 \\
0.5038 & 0.7212 & 0.5915 & -1.4257 \\
-0.0774 & 0.2346 & 1.0336 & -1.3883 \\
-1.6643 & -1.7800 & -0.7057 & -1.7950 \\
\end{bmatrix}
\]  

maximum eigenvalue of \(A_1 = -0.0039\)  
maximum eigenvalue of \(A_{K_1} = -2.3859\)  
maximum eigenvalue of \(A_2 = -18.8524\)  
maximum eigenvalue of \(A_{K_2} = -21.5665\)  
\[\| K \|_2 = 3.1688\]  

Fig. 4a shows the bus voltages when the load is switched from \(R = 0.5\) to 0.01 at time \(t = 500\) ms. The results indicate that voltage at bus-4 reduces due to switching in the absence of controller which is taken care by addition of controller into the system:

i. Comparison of bus voltages between grid with controller-1 and uncompensated grid.

ii. The magnitude of DG controller voltages in the presence of controller-1.

#### 6.2.2 Change in communication delay and load: If, it is possible to know the region of maximum delay bound, the controllers can be designed for the maximum delay predicted during a particular period of the day along with the maximum delay predicted for the upcoming period of the day. Along with this, the load variation is also an inevitable change. The controller-2 designed using optimisation formulation for delay case has demonstrated that it can stabilise the bus voltages for simultaneous change in load and delay profile. The results obtained with controller-2 when two subsystems have been considered: one with a maximum delay of 0.5 ms operating in the region \(A_1\) and the other with a maximum delay of 1 ms operating in the region \(A_2\):

\[
D_1 = \begin{bmatrix}
0.2500 & 0.5000 & 0.4375 & 0.3125 \\
0.3750 & 0.3125 & 0.2500 & 0.4375 \\
0.5000 & 0.4375 & 0.3125 & 0.2500 \\
0.3750 & 0.2500 & 0.4375 & 0.3125 \\
\end{bmatrix} \times 10^{-3}
\]

\[
D_2 = \begin{bmatrix}
0.5000 & 1.0000 & 0.8750 & 0.6250 \\
0.7500 & 0.6250 & 0.5000 & 0.8750 \\
1.0000 & 0.8750 & 0.6250 & 0.5000 \\
0.7500 & 0.5000 & 0.8750 & 0.6250 \\
\end{bmatrix} \times 10^{-3}
\]

\[
K = \begin{bmatrix}
0.9661 & -0.0401 & -0.6552 & -1.7199 \\
-0.1347 & 0.4414 & -0.4786 & -1.7313 \\
-0.6294 & -0.4689 & -0.0069 & -1.6075 \\
-2.2338 & -2.1026 & -2.1243 & -2.0097 \\
\end{bmatrix}
\]  

maximum eigenvalue of \(A_1 = -0.0039\)  
maximum eigenvalue of \(A_{K_1} = -2.3938\)  
maximum eigenvalue of \(A_2 = -18.8524\)  
maximum eigenvalue of \(A_{K_2} = -21.9100\)  
\[\| K \|_2 = 4.6872\]  

Fig. 5a shows the bus voltages when both loads are switched from \(R = 0.5\) to 0.01 and delay of 1 ms is added at time \(t = 200\) ms. Fig. 5b shows the DG controller voltages in the presence of controller designed for both load and delay variations for the same. The results indicate that the controller-2 designed with delay considerations settles quickly after the disturbance compared with the controller-1 designed without delay considerations:

i. Comparison of bus voltages between the situation with controller-1 designed only for load variation and the situation with controller-2 designed for both load and delay variations.

ii. The magnitude of DG controller voltages for a system with controller-2 designed for both load and delay variations.

#### 6.2.3 Failure of a communication link: The following are the details of the controller-3 designed for a case, where there has been a communication failure between sensor 3 and DG 3. Moreover, the system parameters have also been changed due to change in load resistance from \(A_1\) to \(A_2\):
Comparison of bus voltages between the situation with controller-1 designed only for load variation and the situation with controller-2 designed for both load and delay variations.

Results for change in load and delay at $t = 200\,\text{ms}$

Fig. 5

(a) Comparison of bus voltages between the grid with controller-1 designed only for load variation and the situation with controller-2 designed for both load and delay variations, (b) Magnitude of DG controller voltages for system with controller-2 designed for both load and delay variations.

Maximum eigenvalue of $A_1 = -0.0039$

Maximum eigenvalue of $A_K = -3.0426$

Maximum eigenvalue of $A_3 = -18.8524$

Maximum eigenvalue of $A_K = -22.1455$

$||K||_1 = 3.8840$

(59)

Fig. 6a shows the bus voltages when the load is switched from $R = 0.5$ to $0.01$ at time $t = 500\,\text{ms}$ for two controllers after experiencing a loss in the communication link between sensor 3 and controller 3. The controller-1 designed without considerations of link loss makes the bus voltages unstable, whereas the controller-3 designed with link loss consideration stabilises the system after the disturbance. Fig. 6b shows the DG controller voltages in the systems with controller-3 designed with communication link loss considerations when the load is changed at $t = 500\,\text{ms}$.

7 Conclusion and future work

This paper presents a generic hybrid and adaptive framework for enhancing the distributed control in the smart grid using communication under simultaneous variation of various electrical and communication parameters. The strength of this paper has been the novel control frameworks from the cyber-physical perspectives. It has been shown that the variation of parameters for a sample coordinated voltage control problem in a distribution feeder can be modelled as a hybrid system and the associated controllers can be designed using convex optimisation. Delay and link loss can play a crucial role in CPES and these issues have also been addressed. Results show that the controllers developed using this technique have been effective in situations of a load change, delay change and communication link failure. The overall control procedure not only performs well in situations with variation in multiple parameters but also increases the range of operation for these controllers.

A more detailed model of the smart grids with dynamic loads will be pursued as the future scope of this work. The authors are working toward introducing a distributed routing framework so as to increase the reliability of the overall system. The framework could also contain a provision for considering the dispatchability of the DGs. The system could take into account network uncertainties such as packet loss and computational constraints as well as non-linearity of grid dynamics.

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