**Focusing at the nanoscale by atomic holography?**

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**Abstract.** We propose a method to focus a cloud of condensed atoms by imprinting a phase on the atomic wave function. The main goal of our experiment under construction is to study the conditions of realization of atomic holography or deposition in the nanometer range. This experiment will also investigate the role of the atom-atom interactions in these processes.

1. Introduction

Over the last decade Bose-Einstein Condensate (BEC) becomes a commonly used object as starting point of many experiments. The ability to manipulate BEC with the help of magnetic- or electric-field allows a precise control on a macroscopic wavefunction, which opens the way to a wide range of applications such as atom laser and atomic lithography. In this later case, considering that the de Broglie wavelength $\lambda_{dB}$ of free falling $^{87}$Rb condensates at 5 m/s is 1nm, atomic lithography with BEC can be a practical way to pattern nano-objects in 2 or 3 dimensions, as a powerful alternative to the already demonstrated techniques using atomic beams or electron beams. Within this frame, our concerns are to focus a BEC and investigate the role of interatomic interactions in such processes. In this report we will first remind a method suggested earlier [1] to focus an atomic beam, and give numerical estimates of the resolution ability of our BEC setup. Finally we will briefly report on the progress in building the experiment, and conclude.

2. Focusing a condensate

One can understand the focusing of a wave function with a quadratic phase printing by the textbook example of free inflation of a quantum wave packet. Let us consider a 1D Gaussian wave packet having a width $a$ at time zero $\psi(x,0) \propto \exp \left( \frac{x^2}{2a^2} \right)$. After some time $t$ the wave packet expands and develops a quadratic phase

$$\psi(x,t) \propto \exp \left( -\frac{x^2}{2A^2} \right) \exp \left( i \left( \frac{m}{\hbar} \cdot \frac{A^2}{A^2 - a^2} \cdot \frac{x^2}{2} \right) \right)$$

(1)

where $A = a\sqrt{1 + \hbar^2 t^2 / a^4 m^2}$, $m$ is the mass of the particle (here the $^{87}$Rb atom), and $\hbar$ the Planck constant divided by $2\pi$. So if the initial condition of the wave packet is given by expression (1), one sees by time reversal $t \rightarrow -t$ symmetry that the wave packet of width $A$ will be focused to the minimal wave packet size $a$. Since one wants to reach significant focusing, the
Figure 1. Principle of the experiment. By illuminating a free falling condensate by a far red-detuned laser pulse, one can imprint a quadratic phase and focus this later. By changing the waist of the beam and the duration of the pulse, one can adjust the parameters of the atomic lens.

dimensionless term $\sqrt{1 + \frac{\hbar^2 t^2}{a^4 m^2}}$ has to be considerably larger than 1. If this condition is reached, one can make the substitution: $\frac{lt}{\hbar} = aA$. Once set the size of the object wave packet $A$ and its image $a$, the time interval is simply $t = m/\hbar \cdot Aa$ and $\phi(x) = x^2/2aA[1 + \frac{x^2}{2aA}] \approx \frac{x^2}{2aA}$ is the quadratic phase of the now starting wave packet. There is a parallel of this wave packet inflation and the Gaussian optics of electromagnetic waves where a small waist means a small confocal parameter. The equivalence with the wave packet inflation is in the time domain: a small size $a$ means a short time evolution from the starting point.

This time evolution is in the frame of the linear Schrödinger equation and is valid if the interaction term $g|\psi|^2$ with $g = \frac{4N\pi\hbar^2a}{m}$ is negligible with respect to the other energies. In other words, if the Gaussian wave packet is an eigenstate of the considered problem. This is not generally the case since the wavefunction in a BEC is a solution of the Gross-Pitaevskii equation and be written as $\psi(r, t) = \sqrt{\rho} \exp (iS(r, t))$ where $\rho$ is the density and $S$ is the phase.

In the Thomas-Fermi approximation (the kinetic energy term $\frac{\hbar^2 \nabla^2}{2m}$ is neglected) the density is an inverted parabola, provided an harmonic trapping potential for the atoms cooled down to the BEC state. Under these boundary conditions, the question of the time evolution of the BEC wavefunction has been adressed by [3]. It has been demonstrated that the Thomas-Fermi wave function keeps its original density form while the external harmonic potential varies in an arbitrary way. Moreover focusing occurs if a quadratic coordinate dependent phase is imprinted initially [6], [7] : the problem reduces to a scaling law.

The tool to produce this quadratic phase is an impulse of dipole potential $U(x, t)$ produced by a red detuned laser beam of intensity $I(x, t)$:

$$U(x, t) = \frac{\hbar \Gamma^2 I(x, t)}{8 \Delta I_{\text{sat}}}$$

(2)

The laser pulse is generally of Gaussian shape $I(x, t) = I_0 \exp(-2x^2/w_0^2)$ which can be approximated in a quadratic profile $\exp(-2x^2/w_0^2) \approx \left[1 - \frac{2x^2}{w_0^2}\right]$ as long as $A \ll w_0$. Eventually the printed phase by the laser pulse is $\phi_L(x, (T)) = \frac{U(x)}{\hbar}T$, where $T$ is pulse duration (Fig.1).

Fig. 2 presents the results of a calculation of the focal length with this method. These calcu-
Figure 2. Calculated dependence of the focal length from the size of the focused condensate within the limits of the free propagating Gaussian model. Calculation parameters: peak intensity $I_0 = 3 \text{W/cm}^2$, detuning $\Delta = -1 \text{GHz}$, and waist of the beam $w_0 = 1 \text{mm}$. To each point corresponds its own duration of laser pulse $T$.

lations were performed for an initial wave packet size $A = 50 \mu \text{m}$, typical for our QUIC-trap parameter, and illuminated by a 50 mW diode laser pulse, 1GHz red detuned from $F = 2 \rightarrow F' = 3$ cycling transition of $^{87}\text{Rb}$, within a 2-level approximation. Assuming an atomic condensate with a zero vertical velocity at the start of the free evolution, it is shown that the focal length is shorter than 0.2mm when $\alpha < 100 \text{nm}$, and hence an even shorter depth-of-field. This difficulty could be overcome for atomic lithography applications by accelerating atoms along the vertical direction, with the help of an external field gradient.

If one neglects the aberrations and astigmatism effects the size of the spot is ultimately bounded by the diffraction limit which is $R_{\text{diff}} = 0.61 \lambda_{\text{dB}}/\alpha$, where $\alpha = A/f$, and $f$ is the focal length. Assuming a focal length $f = 80 \mu \text{m}$, an initial wave packet size $A = 50 \mu \text{m}$, and $\lambda_{\text{dB}} = 1 \text{nm}$, one find an ultimate spatial resolution of such a set-up close to 1nm, slightly smaller than the de Broglie wavelength of the wave packet.

3. Experimental setup

The chart in figure 1 is to be implemented in an experimental set-up under construction. We are working with $^{87}\text{Rb}$ atoms in the QUIC-trap configuration [4]. The atoms are initially collected in a magneto-optical trap (goal pressure $10^{-9} \text{mbar}$) where Rb atoms are expelled sequentially from a Rb-dispenser. The laser sources are Sanyo laser diodes, actively locked on D2 Rb-line by saturation spectroscopy techniques. We tune precisely the cooling and repumping frequency with Acousto-Optic Modulators (AOM). In addition to AOMs, complete extinction is achieved with mechanical shutters. In the preliminary experiment we were able to collect $2 \cdot 10^7$ atoms and where MOT lifetime was 5-6s.

These atoms will then be magnetically transported [5] over a distance of 60cm to a second Ultra-High Vacuum glass cell (goal pressure $10^{-11} \text{mbar}$) using differential pumping method, aiming at long BEC lifetime. Quantum degeneracy will be achieved by evaporative techniques [2]. We choose a 16mm-wide cell in the QUIC region to achieve high phase-space density inside the QUIC trap. The parameters of the QUIC-trap are: $B_{\text{min}} = 1.8 \text{G}$, $B' = 378 \text{G/cm}$, $B'' = 508 \text{G/cm}^2$, $v_{\text{trans}} = 352 \text{Hz}$, $v_{\text{long}} = 29 \text{Hz}$. 
4. Conclusion
We have presented a method for focusing a cloud of condensed atoms by using a dipole force interaction. The effect of the quadratic phase imprinting on the atomic condensate wavefunction is considered. Preliminary calculations in the frame of a noninteracting atomic cloud exhibit the possibility to image an atomic condensate in the nanometric range.

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