Method censored samples consideration in composite fatigue testing

I V Gadolina¹, I L Serebrjakova² and S I Smelov³

¹ IMASH RAN, Moscow, Russia, gadolina@mail.ru
² MGTU named after Bauman, docent, Moscow, Russia
³ ORPE “Technologiya”, Obninsk, Russia

Summary. Taking into consideration the huge scatter of fatigue performance of composite, as well as the experimentally proved absence of unconditional fatigue limit, the method for statistical improvement was developed which allows to improve the statistical quality of fatigue curve. Some preliminary considerations about mechanical testing tool in quality control are shown. The source of data scatter in fatigue testing is discussed. For building of the method, the statistical approach from the group of intensive use of computer, namely, bootstrap was employed. The model example on the base of real testing was investigated. Applying the method will help to include the information of censored samples into the result of testing.

Key words. Polymer Composite Material, quality, fatigue, censored sets, bootstrap

1. Introduction

Studies of the mechanical properties of Polymer Composite Materials (PCM) show that they have serious advantages over traditional structural metals and alloys, mainly related to high specific characteristics of static strength. A polymer composite is a multi-phase material in which reinforcing fillers are integrated with a polymer matrix, resulting in synergistic mechanical properties that cannot be achieved from either component alone. Nowadays, the development of science and technology leads to the fact that the price of production of composites of various types is constantly decreasing, while the quality is increasing. Thus, this type of material is gaining new areas of application. Due to the increasingly widespread use of composites [1], nowadays priority is given to quality requirements. Some advanced control methods are successfully applied not only for diagnostic of technical state but also for the quality control [2, 3].

In [4, 5] the advanced methods of non-destructive (NDI) and destructive testing were applied. As one of possibilities, the composite specimens were subjected to through-transmission ultrasonic (TTU) NDI that generated C-scans to quantify the planar-damaged areas using image analysis software. Additional inspections techniques, e.g., microscopy and thermal imaging, are also used for the damage tolerance investigation.

Along with the use of modern physical methods in the problem of analyzing the quality of composites, the mechanical characteristics of products will never lose their value: such as strength characteristics obtained during static tests (tension, articles, shear), rigidity (elastic modulus). In [6] the shear strength of carbon-reinforced composites was investigated with the aim of the analysis of the stability of the production process. In Figure 1 the diagrams of box-and-whiskers [6,7] for several batches are shown. One can see a significant scatter here even inside the single batch. The aim of the investigation was to develop the method for outliers’ discharge. In this particular example, the reading $\tau = 83.9$ MPa was accepted to be an outlier, that meant this value was withdrawn and wasn't used for future consideration.

To improve quality production and to control the stability of the technical process the Control Charts present a good tool [8]. In Figure 2 an example of Control Chart for ultimate stress of some batches of composite samples. is shown. Although this investigation is not finished by now, this example one can see how the process “says for itself” in the interactive procedure – some improving changes the chart’s
limits. Batches #8 and #10 are beyond the limits, this means that the process is not stable and therefore is difficult to be controlled.

![Scattering fields of some batches of composites](image1)

**Figure 1.** Scattering fields of some batches of composites

![Control chars for compressive ultimate stress [MPa] of some batches: above – sample means, below – sample ranges](image2)

**Figure 2.** Control chars for compressive ultimate stress [MPa] of some batches: above – sample means, below – sample ranges

The characteristics of fatigue resistance stand apart. Despite the fact that composites are increasingly used in aircraft and spacecraft and other products subject to repeated cyclic loads, there is still an opinion that the static strength of composites is a sufficient strength indicator. This is partly due to the fact that, the slope of the fatigue curve of composite specimens is large compared to metals one. If for metals the value \( m = 5 \ldots 9 \) (the fatigue exponent) is typical, then for composites it is not uncommon to notice \( m = 15 \) (and more). Such a large value in the first place means a significant scatter of the fatigue properties, which is not surprising. As it was stated earlier [9], two important factors increase the scatter of fatigue characteristics of composites. First, Composites strength demonstrates greater scatter comparing with metals. Second, scatter in fatigue is greater, comparing with static tests. For composites, large scatter is
also true for static characteristics (see Figures 1, 2), and as it is well known, in fatigue, the scatter is always greater.

2. Fatigue investigation of composites (preliminary notes)

Fatigue investigation of composites of varied types as well as material behaviour under cyclic loads has been drawn the attention of many scientists [10 -13]. As it was stated before and following the common sense, one can conclude, that the information about censoring should be included somehow into the statistical conclusions. It would improve the statistical data of fatigue characteristics. Indeed, we know that the particular specimen (non-broken until the cycling base of testing, run-out, censored) withstood some number of cycles (large number!) without breaking. It might be not have broken until the testing base due to 1) positive traits combination of the individual specimen; or 2) due to unreasonable low stress amplitude $\sigma_a$ chosen. It is usually recommended to take stress amplitude for the first level of fatigue testing as $S=2 \times (0.75 \ldots 0.85) \sigma_{ul}$ where $\sigma_{ul}$ is the ultimate stress mean estimated by 6 or 5 specimens tested. It can be stated, that in $\sigma_{ul}$ estimation the error is very probable.

In treating the metal fatigue testing results, the problem of censored samples also exists. In [14 15] the problem of run-outs is considered for metal welded specimens. The authors believe, that there is an infinitive fatigue limit $\sigma_\infty$ in their objects. Their task became even more complicated due to this fact. The authors of [14] try to estimate the fatigue limit $\sigma_\infty$ from the information of specimens unbroken until the testing base $N_b$. To solve this problem, they apply the Maximum likelihood method (MLM) (method of estimating parameters of a population, e.g. regression coefficients) to estimate values for which the likelihood of obtaining the observed data is maximized.

There is an underlying linear relationship between $\lg S$ and $\lg N$ of the Basquin equation form:

$$\lg N = \lg A - m \log S \quad (1),$$

where $m$ is the fatigue exponent (slope) and $\log A$ is the intercept. (1) can be re-written in a form that is commonly used to describe S-N curves in design rules:

$$S^m N = A \quad (1a),$$

here $S$ is the stress amplitude, usually in [MPa] and $N$ is the cycle number until failure (or some other criterion of failure in case of composite testing).

The most detailed investigation of the fatigue life of composite used in Aircraft Structure can be found in [4]. Following at some extend the earlier work [16] the authors of [4] employ the Weibull distribution not only to analyze the scatter but also for supplementing the information of the broken during cyclic loading specimens with the information of 1) static tests; 2) censored specimens. The conception of Load-Enhancement is widely used there. In Figure 3 the fatigue curves from [4] are shown. The information in Figure 3 will be further used during the approbation of the developed method (see below). The other example of Weibull-Gnedenko distribution application can be found in [17].
There are no many investigations aimed to direct $\sigma_\infty$ estimation in composites (following, for example, stair-case method). Luckily, the times have changed, and the researchers started to investigate the giga-cycle domain in fatigue, that is more than $10^8$ cycles [18]. To speed up the testing, the special ultrasound machine with the testing technique was developed [19]. This new in science the testing approach allowed (although not without some doubts) to shed some light on whether $\sigma_\infty$ exists in composites.

In Figure 4 the selected from [19] data of testing Carbon fibre reinforced plastics (CFRP) which are widely used in the aerospace field are shown. The testing was performed under (attention!) ultrasound frequency of 965 Hz. The special cooling system was developed for this testing. In Figure 4 one can see that S–N curve decrease when the number of loading cycles exceeds $10^6$. The Plato between $10^7$ and $10^8$ might be noticed in Figure 4. When the number of cycles is greater than $10^8$, the fatigue strength continues to decrease. The authors conclude, that for composites the S–N curve has no fatigue limit similar to traditional metal materials.
3. Method

The method is developed for improving the quality of linear regression of fatigue curve (1) by the special consideration of censoring. The assumptions behind the method are: 1) there is no infinitive fatigue limit for composites [19]; 2) the equation (1) is true with considering \( \lg(S) \) as an independent variable (factor) and \( \lg(N) \) as a function (dependent variable, normally distributed). It is worth reminding, that historically the fatigue curve is always drawn in an alternative way: horizontal axe for \( \lg(N) \) and vertical axe for \( \lg(S) \). It this way the fatigue curves also are drawn in this paper. While evaluating the regression coefficients by linear regression tool the justice is restored: \( y \rightarrow \lg(N) \) and \( x \rightarrow \lg(S) \).

Let’s accept, that there are \( p \) pairs of \( S, N \), which are registered failures during fatigue test. Also, we have \( q \) censored tests on the lower level \( S^* \), for which only the number of specimens tested is known as well as the ultimate longevity without breaking until the testing was finished at the base, say \( N_b=106 \). This type of data limitation is called the single time right censoring. Right censoring occurs when a subject leaves the study before an event occurs, or the study ends before the event has occurred. Opposite to the reliability test during service, where the times of censoring differ [20] in fatigue testing all the censored items have the same life, namely \( N_b \).

To restore the lost data of the potential longevities non-broken \( q \) specimens the bootstrap samples are generated [20, 21]. Each bootstrap sample consists of \( p \) items but chosen in special random order. Consider that each of \( p \) pair \((\lg S, \lg N)\) has its unique number like shown in Table 1:

| \( j \) | \( 1 \) | \( 2 \) | \( 3 \) | \( 4 \) | \( 5 \) | \( 6 \) |
|-------|-------|-------|-------|-------|-------|-------|
| 1     | 6     | 7     | 7     | 8     | 4     | 7     |
| 2     | 4     | 2     | 4     | 4     | 2     | 8     |
| 3     | 1     | 8     | 7     | 3     | 5     | 3     |
| 4     | 6     | 1     | 2     | 7     | 9     | 2     |
| 5     | 7     | 10    | 8     | 10    | 6     | 2     |
| 6     | 3     | 6     | 3     | 7     | 3     | 5     |

The formal description of bootstrap procedure is as follows [21]. Let the random sample with the dimension \( n \) be pooled out of totally unknown probability distribution \( F \)

\[ X_i=x_i, F\sim F, i=1,2, \ldots, n, \quad (2), \]

Which are independently and equally distributed. In our situation it will be the flat distribution, because there are two coordinates: \( \lg N \) and \( \lg S \). Bootstrap procedure for single-set task will be as follows:

1) Draw the sample probability distribution \( \hat{F} \), assuming in each points \( x_i \) the mass \( 1/n \);
2) Under fixed \( \hat{F} \) draw out it the random sample with dimension \( n \), like this: \( X^*=(x^*_1, x^*_i \sim \hat{F}, i=1,2, \ldots, n \) and it will be called the bootstrap-sample, \( X^*=(X^*_1, X^*_2, \ldots, X^*_n) \), \( x^*= (x^*_1, x^*_2, \ldots, x^*_n) \). Note, that it wouldn’t be the permutation distribution, because the values are chosen with the replacement;
3) Approximate the sample distribution \( R(X, \hat{F}) \) by bootstrap distribution \( R^*=R(X^*, \hat{F}) \) by the random procedure (2) under \( \hat{F} \), fixed on its’ registered value.

Indexes in Table 1 address to information about \( \lg S \) and \( \lg N \) (Figure 3). For each of \( j \)-th sample the regression equation is built. In this example we have 6 equations with unique coefficients \( \lg A \) and \( m \) in the equation (1). Using these 6 equations we can get 6 extrapolated values \( N_i^* \), \( i=1 \ldots 6 \) for \( S^* \) and they be different. Considering those values are valid, at the next step we include them into refined fatigue curve estimation and get the better statistical parameters.
Further explanation is given by example from [4], \( p = 13 \). In Figure 3. Here some points are shown black (broken) and some are shown red (censored), \( q = 6 \). They were accepted to be unknown, under assumption that the testing was performed only until \( 10^5 \). On the base of data concerning the breaking event, the fatigue curve equation was estimated by the above described procedure (bootstrap) by 13 points (Figure 3, black points):

\[
\lg N = 39.49 - 16.94 \lg S
\]  

(3).

After performing the bootstrap imitation, we got 6 extra points of \( \lg N \) for the lower value of \( S^* = 112 \) [MPa], \( \lg (S^*) = 2.05 \).

4. Results

In Figure 5 in normal probability plot (function “qqplot” in [22]) two distributions of the random value of longevities for the specimens, tested at the level \( S^* = 112 \) MPa are shown. With green in figure 3, the points are shown, which were not in reality censored, but were accepted to be censored with the aim of the method approbation. One distribution (shown blue) is for the real values, obtained in the experiment, which in reality was performed until the base \( 10^7 \) cycles. The values obtained by the method, described above, which are distribution of bootstrap approximation, are shown red. As one could see, the distributions coincide well. The distributions characteristics, shown in Table also demonstrate good agreement.

![Normal Q-Q Plot](image)

**Figure 5.** Distribution of random value of \( \lg (N) \). blue- real: red - modelled

| Table 2. Parameters of experimental and modelled samples for the random durabilities for value \( S = 112 \) [MPa] |
|---------------------------------------------------------------|
| **Experimental sample** | **Modelled sample** |
| mean | 6.78244 | 6.801667 |
| RMS | 0.129617 | 0.122052 |

After obtaining 6 extra points, the fatigue curve equation was recalculated:

\[
\lg N = 39.23 - 15.82 \lg S
\]  

(4)

Having more points for estimation (19 instead to 13 initially) the quality of equation improves. The quality of regression equation might be characterized by the determination coefficient \( R^2 \):

\[
R^2 = D(\bar{y}) / D(y) = 1 - D(e) / D(y)
\]  

(5)

where \( D(y) \) is the total sum of squares; \( D(e) \) is variance of the model’s predictions.
Basing on (5) determination coefficient for (3) equals: $R^2 = 0.909$ and for (4) equals: $R^2 = 0.941$, so the improvement was achieved.

5. Conclusions

Among important for composite quality control characteristics, the mechanical ones play an important role. In fatigue resistance studies censoring (runouts) often take place. On the base of investigations made during past years as in the field of investigation of metal fatigue as well in the composite fatigue study, the method for improving the regression equation was proposed. The method considers the censored results by means of developed on the base of a bootstrap model approach. This type of statistical model belongs to the group of the methods of intensive computers use. On the example of real testing, taken from the reference data, the workability of the proposed method is shown.

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