Anatomy of $B_s \to PV$ decays and effects of next-to-leading order contributions in the perturbative QCD factorization approach

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In this paper, we will make systematic calculations for the branching ratios and the CP-violating asymmetries of the twenty one $B^0_s \to PV$ decays by employing the perturbative QCD (PQCD) factorization approach. Besides the full leading-order (LO) contributions, all currently known next-to-leading order (NLO) contributions are taken into account. We found numerically that: (a) the NLO contributions can provide $\sim 40\%$ enhancement to the LO PQCD predictions for $\mathcal{B}(B^0_s \to K^0 K^{*0})$ and $\mathcal{B}(B^0_s \to K^\pm K^{*\mp})$, or a $\sim 37\%$ reduction to $\mathcal{B}(B^0_s \to \pi^- K^{*+})$; and we confirmed that the inclusion of the known NLO contributions can improve significantly the agreement between the theory and those currently available experimental measurements; (b) the total effects on the PQCD predictions for the relevant $B \to P$ transition form factors after the inclusion of the NLO twist-2 and twist-3 contributions is generally small in magnitude: less than $10\%$ enhancement respect to the leading order result; (c) for the “tree” dominated decay $B^0_s \to K^+ \rho^-$ and the “color-suppressed-tree” decay $B^0_s \to \pi^0 K^{*0}$, the big difference between the PQCD predictions for their branching ratios are induced by different topological structure and by interference effects among the decay amplitude $A_{T,C}$ and $A_{P}$: constructive for the first decay but destructive for the second one; and (d) for $B^0_s \to V(\eta, \eta')$ decays, the complex pattern of the PQCD predictions for their branching ratios can be understood by rather different topological structures and the interference effects between the decay amplitude $A(V\eta_0)$ and $A(V\eta_s)$ due to the $\eta - \eta'$ mixing.

Key Words: $B_s$ meson decays; The PQCD factorization approach; Form factors; Branching ratios

I. INTRODUCTION

During the past two decades, the theoretical studies and experimental measurements for the two-body charmless hadronic decays of $B$ and $B_s$ mesons have played a very important role in testing the Standard Model (SM) and in searching for the possible signals of new physics (NP) beyond SM [1–7]. On the theory side, such decays have been studied systematically by employing rather different factorization approaches at the leading order (LO) or next-to-leading order (NLO), such as the generalized factorization approach [8–10], the QCD factorization (QCDF) approach [11–14] and the perturbative QCD (PQCD) factorization approach [15–19]. The resultant theoretical predictions from different approaches are generally consistent with each other within the errors.

On the experimental side, the early measurements for $B \to M_2 M_3$ decay modes (here $M_i$ stands for the light pseudo-scalar or vector mesons) mainly come from the BaBar and Belle collaboration in $B$ factory experiments [1, 6]. For $B_s \to M_2 M_3$ decays, however, LHCb Collaboration provide the dominant contribution [2–6]. Although some deviations or puzzles, such as the so-called $(R(D), R(D^*))$ and $(R_K, R_{K_s})$ anomalies, are observed so far, but there is no any solid flavor-related evidence for the existence of the new physics beyond the SM.

In the framework of the PQCD factorization approach, the charmless two-body hadronic decays $B^0_s \to M_2 M_3$ have been studied by some authors in recent years:

1. In 2004, Li et al. studied the pure annihilation $B_s \to \pi^+ \pi^-$ decay [20] and gave a leading order PQCD prediction for a large branching ratio $\mathcal{B}(B^0_s \to \pi^+ \pi^-) \sim 5 \times 10^{-7}$, which has been confirmed by recent CDF and LHCb measurements [21–24].

2. In 2007, Ali et al. completed the systematic study for the forty-nine $B^0_s \to PP, PV, VV$ decays at the LO level, presented their PQCD predictions for the CP-averaged branching ratios, the CP-violating asymmetries and some other physical observables [25]. For $B_s \to \pi^+ \pi^-$, for example, they also found a large theoretical prediction for its decay rate.

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(3) In 2014, Qin et al. studied the twenty $B_s \to PT$ decays (here $P$ and $T$ denote the light pseudo-scalar and tensor mesons) in the PQCD factorization approach at the LO level, and provided their predictions for the decay rates and CP-violating asymmetries of those considered decay modes [26].

(4) Very recently, we studied $B_s^0 \to (K\pi, KK)$ decays [27] and $B_s^0 \to (\pi\eta^{(0)}, \eta^{(0)}\eta^{(0)})$ decays [28] at the partial NLO level. We found that the currently known NLO contributions from different sources can interfere with the LO part constructively or destructively for different decay modes, while the agreement between the central values of the PQCD predictions for the decay rates and CP violating asymmetries and those currently available experimental measurements are indeed improved effectively after the inclusion of those NLO contributions [27, 28].

In this paper, by employing the PQCD factorization approach, we will make a systematic study for all two-body charmless hadronic decays $B_s \to PV$ (here $P = (\pi, K, \eta, \eta')$ and $V = (\rho, K^*, \phi, \omega)$), by extending the previous LO studies to the partial NLO level: including all currently known NLO contributions. We will focus on investigating the effects of the NLO contributions, specifically those newly known NLO twist-2 and twist-3 contributions to the form factors of $B \to P$ transitions [29, 30] under the approximation of $SU(3)$ flavor symmetry.

This paper is organized as follows. In Sec. II, we give a brief review about the PQCD factorization approach and we calculate analytically the relevant Feynman diagrams and present the various decay amplitudes for the considered decay modes in the LO and NLO level. We calculate and show the PQCD predictions for the branching ratios and CP violating asymmetries of all twenty-one $B_s \to PV$ decays in Sec III. The summary and some discussions are included in Sec. IV.

II. DECAY AMPLITUDES AT LO AND NLO LEVEL

As usual, we consider the $B_s$ meson at rest and treat it as a heavy-light system. Using the light-cone coordinates, we define the $B_s^0$ meson with momentum $P_1$, the emitted meson $M_2$ and the recoiled meson $M_3$ with momentum $P_2$ and $P_3$ respectively. We also use $x_i$ to denote the momentum fraction of anti-quark in each meson and set the momentum carried by the light anti-quark in $B_s$ and $M_{2,3}$ flavor symmetry) in the following forms:

$$P_1 = \frac{m_{B_s}}{\sqrt{2}}(1, 1, 0, T), \quad P_2 = \frac{M_2}{\sqrt{2}}(1, 0, 0, T), \quad P_3 = \frac{M_3}{\sqrt{2}}(0, 1, 0, T),$$

$$k_1 = (x_1 P_1^+, 0, k_{1T}), \quad k_2 = (x_2 P_2^+, 0, k_{2T}), \quad k_3 = (0, x_3 P_3^-, k_{3T}).$$

The integration over $k_{1T}^-$ and $k_{3T}^-$ will lead conceptually to the decay amplitude

$$A \sim \int \cdots \left[ C(t) \Phi_{B_s}(x_1, b_1) \Phi_{M_2}(x_2, b_2) \Phi_{M_3}(x_3, b_3) H(x_i, b_i, t) S_i(x_i) e^{-S(t)} \right] ,$$

where $b_i$ is the conjugate space coordinate of $k_{iT}$, $C(t)$ are the Wilson coefficients evaluated at the scale $t$, and $\Phi_{B_s}$ and $\Phi_{M_i}$ are wave functions of the $B_s$ meson and the final state mesons. The hard kernel $H(x_i, b_i, t)$ describes the four-quark operator and the spectator quark connected by a hard gluon. The Sudakov factor $e^{-S(t)}$ and $S_i(x_i)$ together suppress the soft dynamics effectively [15].

A. Wave functions and decay amplitudes

For the considered $B_s^0 \to PV$ decays with a quark level transition $b \to q$ with $q = (d, s)$, the weak effective Hamiltonian $H_{eff}$ can be written as [31]

$$H_{eff} \sim \frac{G_F}{\sqrt{2}} \left\{ V_{ub} V^*_{us} \left[ C_1(\mu) O_1^u(\mu) + C_2(\mu) O_2^u(\mu) \right] - V_{tb} V^*_{ts} \frac{10}{6} \sum_{i=3}^{10} C_i(\mu) O_i(\mu) \right\} + \text{h.c.}$$

where $G_F = 1.16639 \times 10^{-5}$ GeV$^{-2}$ is the Fermi constant, and $V_{ij}$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element, $C_i(\mu)$ are the Wilson coefficients and $O_i(\mu)$ are the four-fermion operators.

For $B_s$ meson, we consider only the contribution of Lorentz structure

$$\Phi_{B_s} = \frac{1}{\sqrt{6}} (\not{P}_{B_s} + m_{B_s}) \gamma_5 \phi_{B_s}(k_1),$$

and adopt the distribution amplitude $\phi_{B_s}$ as in Refs. [20, 25, 27].

$$\phi_{B_s}(x, b) = N_{B_s} x^2 (1 - x)^2 \exp \left[ - \frac{M_{B_s}^2 x^2}{2 \omega_{B_s}} - \frac{1}{2} (\omega_{B_s} b)^2 \right].$$
We also take $\omega_{B_s} = 0.50 \pm 0.05$ GeV in numerical calculations. The normalization factor $N_{B_s}$ will be determined through the normalization condition: $\int_0^1 dx \phi_{B_s}(x, b = 0) = f_{B_s}/(2\sqrt{2}N_c)$.

For $\eta$-$\eta'$ mixing, we also use the quark-flavor basis: $\eta_q = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\eta_s = s\bar{s}$ [32, 33]. The physical $\eta$ and $\eta'$ can then be written in the form of

$$
\begin{pmatrix}
\eta \\
\eta'
\end{pmatrix} = \begin{pmatrix}
\cos \phi & -\sin \phi \\
\sin \phi & \cos \phi
\end{pmatrix} \begin{pmatrix}
\eta_q \\
\eta_s
\end{pmatrix},
$$
(6)

where $\phi$ is the mixing angle. The relation between the decay constants $(f_{\eta q}^3, f_{\eta'}^3, f_{\eta q}^s, f_{\eta'}^s)$ and $(f_q, f_s)$ can be found for example in Ref. [33]. The chiral enhancements $m_{\eta q}^0$ and $m_{\eta q}^b$ have been defined in Ref. [34] by assuming the exact isospin symmetry $m_q = m_u = m_d$. The three input parameters $f_q, f_s$, and $\phi$ in Eq. (6) have been extracted from the data [32]

$$
f_q = (1.07 \pm 0.02)f_{\pi}, \quad f_s = (1.34 \pm 0.06)f_{\pi}, \quad \phi = 39.3^\circ \pm 1.0^\circ.
$$
(7)

With $f_{\pi} = 0.13$ GeV, the chiral enhancements $m_{\eta q}^0$ and $m_{\eta q}^b$ consequently take the values of $m_{\eta q}^0 = 1.07$ GeV and $m_{\eta q}^b = 1.92$ GeV [34].

For the final state pseudo-scalar-meson $M = (\pi, K, \eta_q, \eta_s)$, their wave functions are the same ones as those in Refs. [35–38]:

$$
\Phi_M(P_t, x_t) = \frac{1}{\sqrt{6}}\gamma_5 [P_t\phi_M^A(x_t) + m_0\phi_M^P(x_t) + \zeta m_0(\eta - 1)\phi_M^T(x_t)],
$$
(8)

where $m_{0i}$ is the isospin mass of the meson $M_i$, $P_t$ and $x_t$ are the momentum and the fraction of the momentum of $M_i$s. The parameter $\zeta = 1$ or $-1$ when the momentum fraction of the quark (anti-quark) of the meson is set to be $x$. The distribution amplitudes (DA’s) of the meson $M$ can be found easily in Refs. [20, 38]:

$$
\phi_M^A(x) = \frac{3f_M}{\sqrt{6}}(1 - x) \left[ 1 + a_1^M C_1^{3/2}(t) + a_2^M C_2^{3/2}(t) + a_4^M C_4^{3/2}(t) \right],
$$
(9)

$$
\phi_M^P(x) = \frac{f_M}{\sqrt{2}6} \left[ 1 + \left( 30\eta_3 - \frac{5}{2}\rho_M^2 \right) C_2^{3/2}(t) - 3 \left[ \eta_3\omega_3 + \frac{9}{20}\rho_M^2 \left( 1 + 6a_2^M \right) C_4^{3/2}(t) \right] \right],
$$
(10)

$$
\phi_M^T(x) = \frac{f_M(1 - 2x)}{\sqrt{2}6} \left[ 1 + 6 \left[ 5\eta_3 - \frac{1}{2}\eta_3\omega_3 - \frac{7}{20}\rho_M^2 - \frac{3}{5}\rho_M^2 a_2^M \right] (1 - 10x + 10x^2) \right],
$$
(11)

where $t = 2x - 1$, $f_M$ and $\rho_M$ are the decay constant and the mass ratio with the definition of $\rho_M = (m_\pi/m_0^+, m_K/m_0^K, m_{\eta q}/m_0^{\eta_q}, m_{s s}/m_0^{s s})$. The parameter $m_{qq}$ and $m_{ss}$ have been defined in Ref. [34]:

$$
m_{qq}^2 = m_0^2 \cos^2 \phi + m_0^2 \sin^2 \phi - \frac{\sqrt{2}S}{f_q}(m_{\eta q}^2 - m_0^2) \cos \phi \sin \phi,
$$

$$
m_{ss}^2 = m_0^2 \sin^2 \phi + m_0^2 \cos^2 \phi - \frac{\sqrt{2}S}{f_s}(m_{\eta q}^2 - m_0^2) \cos \phi \sin \phi,
$$
(12)

with the assumption of exact isospin symmetry $m_q = m_u = m_d$. The explicit expressions of those Gegenbauer polynomials $C_1^{3/2}(t)$ and $C_2^{3/2}(t)$ can be found for example in Eq. (20) of Ref. [33]. The Gegenbauer moments $a_1^M$ and other input parameters are the same as those in Ref. [35]

$$
a_1^{\pi, \eta_q, \eta_s} = 0, \quad a_1^K = 0.06, \quad a_2^{\pi, K} = 0.25 \pm 0.15, \quad a_2^{\eta_q, \eta_s} = 0.115,
$$

$$
a_4^{\pi, K, \eta_q, \eta_s} = -0.015, \quad \eta_3 = 0.015, \quad \omega_3 = -3.0.
$$
(13)

For the $B_s^0 \to PV$ decays, only the longitudinal polarization component of the involved vector mesons contributes to the decay amplitude. Therefore we choose the wave functions of the vector mesons as in Ref. [25]:

$$
\Phi_V^L(P, \epsilon_L) = \frac{1}{\sqrt{6}} [g_L M_V \phi_V(x) + g_L P \phi_V(x) + M_V \phi_V^L(x)],
$$
(14)

where $P$ and $M_V$ are the momentum and the mass of the light vector mesons, and $\epsilon_L$ is the longitudinal polarization vector of the vector mesons. The twist-2 distribution amplitudes $\phi_V(x)$ in Eq. (14) can be written in the following form [25]

$$
\phi_\rho(x) = \frac{3f_\rho}{\sqrt{6}} x(1-x) \left[ 1 + a_1^{\rho, 3/2}(t) \right],
$$
(15)

$$
\phi_\omega(x) = \frac{3f_\omega}{\sqrt{6}} x(1-x) \left[ 1 + a_2^{\omega, 3/2}(t) \right],
$$
(16)

$$
\phi_K^\omega(x) = \frac{3f_K}{\sqrt{6}} x(1-x) \left[ 1 + a_1^{K, 3/2}(t) + a_2^{K, 3/2}(t) \right],
$$
(17)

$$
\phi_\phi(x) = \frac{3f_\phi}{\sqrt{6}} x(1-x) \left[ 1 + a_2^{\phi, 3/2}(t) \right],
$$
(18)
where $t = 2x - 1$, $f_V$ is the decay constant of the vector meson with longitudinal polarization. The Gegenbauer moments here are the same as those in Ref. [25]:

$$a_{1K^*}^{||} = 0.03 \pm 0.02, \quad a_{2\pi}^{||} = a_{2\pi}^{||} = 0.15 \pm 0.07, \quad a_{2K^*}^{||} = 0.11 \pm 0.09, \quad a_{2\rho}^{||} = 0.18 \pm 0.08.$$  (19)

While the twist-3 distribution amplitudes $\phi_V(x)$ and $\phi_T(x)$ are defined with the asymptotic form as in Ref. [25]:

$$\phi_V(x) = \frac{3f_V^T}{2\sqrt{6}} (2x - 1)^2, \quad \phi_T(x) = \frac{3f_V^T}{2\sqrt{6}} (1 - 2x),$$  (20)

where $f_V^T$ is the decay constant of the vector meson with transverse polarization.

### B. Example of the LO decay amplitudes

At the LO level, the twenty one $\bar{B}^0_s \to PV$ decays have been studied previously in Ref. [25], and the decay amplitudes as presented in Ref. [25] are confirmed by our independent recalculation. In this paper, we focus on the calculations of the effects of all currently known NLO contributions to these decay modes in the PQCD factorization approach. The relevant Feynman diagrams which may contribute to the considered $\bar{B}^0_s$ decays at the leading order are illustrated in Fig.1.

Based on the effective Hamiltonian $\mathcal{H}_{eff}$, each considered decay may receive contributions from one or more terms proportional to different Wilson coefficients $C_i(\mu)$ and/or their combinations $a_i$. According to the topological structure of the relevant Feynman diagrams for a given decay mode, i.e. which diagram provides the dominant contribution, one can classify the decays considered into the following four types:

1. The “color-allowed-tree” (“T”) decay: the dominant contribution comes from the terms proportional to $a_1$ and/or $C_2$;
2. The “color-suppressed tree” (“C”) decay: the terms with $a_2$ and/or $C_1$ provide the dominant contribution;
3. The “QCD penguin” (“P”) decay and the “Electroweak penguin” (“$P_{EW}$”) decays: the dominant terms are proportional to $C_{3-6}$ or $a_{3-6}$ and $C_{7-10}$ or $a_{7-10}$, respectively;
4. The “annihilation” (“Anni”) decays: if only the annihilation diagrams contribute.

At the leading order PQCD approach, as illustrated in Fig. 1, there are three types of diagrams contributing to the $\bar{B}^0_s \to PV$ decays considered in this paper: the factorizable emission diagrams (Fig. 1(a) and 1(b)); the hard-spectator diagrams (Fig. 1(c) and 1(d)); and the annihilation diagrams (Fig. 1(e)-1(h)). From the factorizable emission diagrams Fig. 1(a) and 1(b), the corresponding form factors of $B_s \to M_3$ transition can be extracted by perturbative calculations.

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1. For the sake of simplicity, one usually define the combinations of the Wilson coefficient in the form: $a_1 = C_2 + C_{1}/3$, $a_2 = C_1 + C_2/3$, $a_i = C_i + C_{i+1}/3$ and $a_j = C_j + C_{j-1}/3$ for $i = (3, 5, 7, 9)$ and $j = (4, 6, 8, 10)$. For given $\mu = [2, 5]$ GeV, one found numerically [25, 39]: $a_1 \approx C_2 \approx 1.1$ are large quantity, $C_1 \sim -0.2$ and $a_2 = 0.01 - 0.1$ are small ones, the QCD-penguins $|a_{3-6}| = 0.01 - 0.001$ are very small, and finally the electroweak-penguins $|a_{7-10}| = 10^{-3} - 10^{-4}$ are indeed tiny.
For the sake of completeness and the requirement for later discussions, we show here the total LO decay amplitudes for $B^0_s \rightarrow \pi^- K^{*+}$, $K^+ \rho^-$ and $\pi^0 K^{*0}$ decays. For other eighteen decay modes, one can found the expressions of their LO decay amplitudes easily in Ref. [25].

\[
A(B^0_s \rightarrow \pi^- K^{*+}) = V_{ub} V^*_{us} \left( f_{\pi} F_{\pi K^*} a_1 + M_{eK^*} C_1 \right) - V_{tb} V^*_{ts} \left( f_{\pi} F_{eK^*} [a_4 + a_{10}] \right. \\
\left. - f_{\pi} F_{\pi K^*} [a_6 + a_8] + M_{eK^*} [C_3 + C_9] \right) + f_{B_s} F_{eK^*} \left[ a_4 - \frac{1}{2} a_{10} \right] \\
- f_{B_s} F_{\pi K^*} \left[ a_6 - \frac{1}{2} a_8 \right] + M_{eK^*} \left[ C_3 - \frac{1}{2} C_9 \right] - M_{eK^*} \left[ C_5 - \frac{1}{2} C_7 \right],
\]

\[ (21) \]

\[
A(B^0_s \rightarrow K^+ \rho^-) = V_{ub} V^*_{us} \left( f_{\rho} F_{\rho K} a_1 + M_{eK} C_1 \right) - V_{tb} V^*_{ts} \left( f_{\rho} F_{eK} [a_4 + a_{10}] \right. \\
\left. + M_{eK} [C_3 + C_9] + M_{eK} F_{\rho K^*} [C_5 + C_7] \right) + f_{B_s} F_{eK} \left[ a_4 - \frac{1}{2} a_{10} \right] \\
+ f_{B_s} F_{\rho K^*} \left[ a_6 - \frac{1}{2} a_8 \right] + M_{eK} \left[ C_3 - \frac{1}{2} C_9 \right] + M_{eK} \left[ C_5 - \frac{1}{2} C_7 \right],
\]

\[ (22) \]

\[
\sqrt{2} A(B^0_s \rightarrow \pi^0 K^{*0}) = V_{ub} V^*_{us} \left( f_{\pi} F_{\pi K^*} a_2 + M_{eK^*} C_2 \right) - V_{tb} V^*_{ts} \left( f_{\pi} F_{eK^*} \left[ - a_4 - \frac{3}{2} a_{10} + \frac{3}{2} a_{10} \right] \right. \\
\left. - f_{\pi} F_{\pi K^*} \left[ a_6 + \frac{1}{2} a_8 \right] + M_{eK^*} \left[ - C_3 + \frac{3}{2} C_8 + \frac{1}{2} C_9 + \frac{3}{2} C_10 \right] \right) - f_{B_s} F_{\rho K^*} \left[ - a_6 + \frac{1}{2} a_8 \right] \\
+ f_{B_s} F_{eK^*} \left[ - a_4 + \frac{1}{2} a_{10} \right] + M_{eK^*} \left[ - C_3 + \frac{1}{2} C_9 \right] - M_{eK^*} \left[ - C_5 + \frac{1}{2} C_7 \right],
\]

\[ (23) \]

where $a_i$ are the combinations of the Wilson coefficients $C_i$ [25]. The individual decay amplitudes appeared in the above equations, such as $F_{eM_3}, F_{\rho K}^P$ and $F_{eK}^P$, are obtained by evaluating the Feynman diagrams in Fig. 1 analytically. The term $F_{eM_3}$ and $F_{\rho K}^P$, for example, comes from the factorizable emission diagrams with $(V-A)(V-A)$ and $(S-P)(S+P)$ current, respectively.

The explicit expressions of $F_{eM_3}$ and other decay amplitudes at the leading order in QCD approach can be found, for example, in Ref. [25]. For the sake of the conveniences of the reader, we show $F_{eP}, F_{eV}$ and $F_{\rho K}^P$ here explicitly:

\[
F_{eP} = 8\pi C_F M^4_{B_s} f_P \int_0^1 dx_1 dx_3 \int_0^{\infty} b_1 db_1 db_3 \phi_{B_s}(x_1, b_1) \\
\times \left\{ \left[ (1 + x_3) \phi^A_p(x_3) + r_p (1 - 2x_3) (\phi^p_p(x_3) + \phi^L_p(x_3)) \right] \cdot \alpha_s(t_a) E_c(t_a) h_c(x_1, x_3, b_1, b_3) \\
+ 2r_p \phi^p_p(x_3) \cdot \alpha_s(t_b) E_c(t_b) h_c(x_3, x_1, b_3, b_1) \right\},
\]

\[ (24) \]

\[
F_{eV} = 8\pi C_F M^4_{B_s} f_P \int_0^1 dx_1 dx_3 \int_0^{\infty} b_1 db_1 db_3 \phi_{B_s}(x_1, b_1) \\
\times \left\{ \left[ (1 + x_3) \phi_v(x_3) + r_v (1 - 2x_3) \left[ \phi_v^A(x_3) + \phi_v^L(x_3) \right] \right] \cdot \alpha_s(t_a) E_c(t_a) h_c(x_1, x_3, b_1, b_3) \\
+ 2r_v \phi_v^L(x_3) \cdot \alpha_s(t_b) E_c(t_b) h_c(x_3, x_1, b_3, b_1) \right\},
\]

\[ (25) \]

\[
F_{\rho K}^P = 16\pi r_p C_F M^4_{B_s} f_P \int_0^1 dx_1 dx_3 \int_0^{\infty} b_1 db_1 db_3 \phi_{B_s}(x_1, b_1) \\
\times \left\{ \left[ (1 + x_3) \phi_v(x_3) + r_v (2 + x_3) \phi_v^A(x_3) - r_v x_3 \phi_v^L(x_3) \right] \cdot \alpha_s(t_a) E_c(t_a) h_c(x_1, x_3, b_1, b_3) \\
+ 2r_v \phi_v^L(x_3) \cdot \alpha_s(t_b) E_c(t_b) h_c(x_3, x_1, b_3, b_1) \right\},
\]

\[ (26) \]

where $C_F = 4/3$ and $\alpha_s(t_i)$ is the strong coupling constant. In the above functions, $r_v = M_v/M_{B_s}$ and $r_p = M_p^0/M_{B_s}$, with $M_p^0$ the chiral mass of the pseudoscalar meson. The explicit expression of the functions $E_i(t_i)$, the hard scales $t_i$, the hard functions $h_i(x_j, b_j)$ and more details about the LO decay amplitudes can also be found in Ref. [25].
FIG. 2. Typical Feynman diagrams for NLO contributions: the vertex corrections (a-d); the quark-loops (e-f), the chromo-magnetic penguin contributions (g-h), and the NLO twist-2 and twist-3 contributions to $B_s \to (K, \eta_s)$ transition form factors (i-l).

C. NLO contributions

During the past two decades, many authors have made great efforts to calculate the NLO contributions to the two-body charmless decays $B/B_s \to M_2 M_3$ in the framework of the PQCD factorization approach. At present, almost all such NLO contributions become available now:

1. The NLO Wilson Coefficients (NLO-WC): which means that the NLO Wilson coefficients $C_i(m_W)$, the renormalization group running matrix $U(m_1, m_2, \alpha)$ at NLO level (for details see Eq. (7.22) of Ref. [31]) and the strong coupling constant $\alpha_s(\mu)$ at two-loop level will be used in the numerical calculations [31], instead of the ones at the LO level.

2. The NLO vertex corrections (VC) as given in Refs. [12, 16], and as illustrated in Fig. 2(a)-2(d).

3. The NLO contributions from the quark-loops (QL) as described in Ref. [16], with the relevant Feynman diagrams as shown in Fig. 2(e) and 2(f).

4. The NLO contributions from the chromo-magnetic penguin (MP) operator $O_{8g}$ [40], as illustrated in Fig. 2(g)-2(h).

5. The NLO twist-2 and twist-3 contributions to the form factors of $B \to \pi$ transitions have been completed very recently in Refs. [29, 30], the typical Feynman diagrams are those as shown in Fig. 2(i)-2(l). Based on the $SU(3)$ flavor symmetry, we could extend directly the formulas for the NLO contributions to the form factor $F_{0,1}^{B \to \pi}(0)$ as given in Refs. [29, 30] to the cases for $B_s \to (K, \eta_s)$ transitions after making some proper modifications for the relevant masses or decay constants of the mesons involved.

6. In Ref. [41], we made the first calculation for the scalar pion form factors $F_{0,1}^{(1)}(s)$ up to the NLO level, which describes the LO and NLO ($O(\alpha_s^2)$) contributions to the factorizable annihilation diagrams of the considered $B \to \pi \pi$ decays. We found numerically that (a) the NLO part of the form factor $F_{0,1}^{(1)}$, i.e., the NLO annihilation correction, is very small in size, but has a large strong phase around $-55^\circ$, and therefore may play an important role in producing large CP violation for the relevant decay modes; and (b) the NLO annihilation correction can produce only a very small enhancement (less than 3% in magnitude) to their branching ratios for $B \to \pi^+ \pi^-$ and $\pi^0 \pi^0$ decays [41].

In this paper, we adopt directly the formulas for all currently known NLO contributions from Refs. [12, 16, 27–30, 38, 40, 41] without further discussions about the details. At present, the calculations for the NLO corrections to the LO hard spectator (Fig. 1(c)-1(d)) and the non-factorizable annihilation diagrams (Fig. 1(g),1(h)) have not been completed yet. For most $B^0_s \to PP$ with $P = (\pi, K, \eta_s)$ as studied in Refs. [27, 28, 38, 42], furthermore, we have made the comparative studies for the magnitude of all relevant LO and NLO contributions from different kinds of Feynman diagrams in great details and found that the LO contributions from the hard spectator and annihilation diagrams are always much smaller than the corresponding dominant LO contribution from the emission diagrams (Fig. 1(a)-1(b)), those still unknown NLO contributions in the PQCD
approach are in fact the higher order corrections to the small LO pieces, and consequently should be much smaller than their LO counterparts in magnitude, say less than 5% of the dominant LO contribution, and could be neglected safely. On the other hand, it is worth of mentioning that the uncertainty of current theoretical predictions for the decay rates or CP violating asymmetries of those charmless hadronic two-body decays of \( B/B_s \) mesons is generally around 20 to 30 percent.

According to Refs. [12, 16, 43], the vertex corrections can be absorbed into the redefinition of the Wilson coefficients \( a_i(\mu) \) by adding a vertex-function \( V_i(M) \) to them.

\[
a_{1,2}(\mu) \to a_{1,2}(\mu) + \frac{\alpha_s(\mu)}{9\pi} C_{1,2}(\mu) V_{1,2}(M),
\]

\[
a_i(\mu) \to a_i(\mu) + \frac{\alpha_s(\mu)}{9\pi} C_i(\mu) V_i(M), \quad \text{for} \ i = 3, 5, 7, 9,
\]

\[
a_j(\mu) \to a_j(\mu) + \frac{\alpha_s(\mu)}{9\pi} C_{j-1}(\mu) V_j(M), \quad \text{for} \ j = 4, 6, 8, 10,
\]

(27)

where \( M \) denotes the meson emitted from the weak vertex (i.e. the \( M_2 \) in Fig. 2(a)-2(d)). For a pseudo-scalar meson \( M \), the explicit expressions of the functions \( V_i(M) \) have been given in Eq. (6) of Ref. [43]. For the case of a vector meson \( V \) one can obtain \( V_i(V) \) from \( V_i(P) \) by some appropriate replacements: \( \phi^A \to \phi^V \), \( \phi^P \to -\phi^V \), and the decay constant \( f_P \to f_V, f_V^P \) [43].

The NLO “Quark-Loop” and “Magnetic-Penguin” contributions are in fact a kind of penguin corrections with the insertion of the four-quark operators and the chromo-magnetic operator \( \mathcal{O}_{9g} \), respectively, as shown in Figs. 2(e,f) and 2(g,h). For the \( b \to s \) transition, for example, the corresponding effective Hamiltonian \( H^{qg}_{eff} \) and \( H^{mp}_{eff} \) can be written in the following form:

\[
H^{qg}_{eff} = - \sum_{q=u,c,t} \sum_{q' = q} \frac{G_F}{\sqrt{2}} V_{qb} V_{q'b} \frac{\alpha_s(\mu)}{2\pi} C^{qg}(\mu, l^2) \left[ b \gamma_\rho (1 - \gamma_5) T^{a}_5 \right] (q' \gamma^\nu T^{a}_5 q'),
\]

(28)

\[
H^{mp}_{eff} = - \frac{G_F}{\sqrt{2}} g_s \frac{3m_b}{8\pi^2} V_{1b} V_{ts} C^{eff}_{9g} \delta_{1s} \sigma^{\mu\nu} (1 + \gamma_5) T^{a}_5 G^{a}_{\mu\nu} b_j,
\]

(29)

where \( l^2 \) is the invariant mass of the gluon which attaches the quark loops in Figs. 2(e,f), and the functions \( C^{qg}(\mu, l^2) \) can be found in Refs. [16, 33]. The \( C^{eff}_{9g} \) in Eq. (29) is the effective Wilson coefficient with the definition of \( C^{eff}_{9g} = C_{9g} + C_5 \) [16].

By analytical evaluations, we find the following two points:

1. The four pure annihilation type decays \( B^0_s \to \pi\rho \) and \( B^0 \to \pi\omega \) do not receive the NLO contributions from the vertex corrections, the quark-loop and the magnetic-penguin diagrams. The only NLO contributions are included by using the NLO-WCs, instead of the LO ones.

2. For the remaining seventeen decay channels, besides the LO decay amplitudes, one should take those NLO contributions into account:

\[
\begin{align*}
A^{(u)}_{K^0\phi} & \to A^{(u)}_{K^0\phi} + M^{(u,c)}_{K^0\phi}, & A^{(t)}_{K^0\phi} & \to A^{(t)}_{K^0\phi} - M^{(t)}_{K^0\phi} + M^{(g)}_{K^0\phi},
A^{(u)}_{\rho^- K^+} & \to A^{(u)}_{\rho^- K^+} + M^{(u,c)}_{\rho^- K^+}, & A^{(t)}_{\rho^- K^+} & \to A^{(t)}_{\rho^- K^+} - M^{(t)}_{\rho^- K^+} - M^{(g)}_{\rho^- K^+},
A^{(u)}_{\pi^- K^{*+}} & \to A^{(u)}_{\pi^- K^{*+}} + M^{(u,c)}_{\pi^- K^{*+}}, & A^{(t)}_{\pi^- K^{*+}} & \to A^{(t)}_{\pi^- K^{*+}} - M^{(t)}_{\pi^- K^{*+}} - M^{(g)}_{\pi^- K^{*+}},
A^{(u)}_{\eta K^{0\pi}} & \to A^{(u)}_{\eta K^{0\pi}} + \frac{1}{2} M^{(u,c)}_{\eta K^{0\pi}}, & A^{(t)}_{\eta K^{0\pi}} & \to A^{(t)}_{\eta K^{0\pi}} - \frac{1}{2} M^{(t)}_{\eta K^{0\pi}} - \frac{1}{2} M^{(g)}_{\eta K^{0\pi}},
A^{(u)}_{K^{*0}\eta_s} & \to A^{(u)}_{K^{*0}\eta_s} + M^{(u,c)}_{K^{*0}\eta_s}, & A^{(t)}_{K^{*0}\eta_s} & \to A^{(t)}_{K^{*0}\eta_s} - M^{(t)}_{K^{*0}\eta_s} - M^{(g)}_{K^{*0}\eta_s},
A^{(u)}_{\phi \eta_s} & \to A^{(u)}_{\phi \eta_s} + M^{(u,c)}_{\phi \eta_s}, & A^{(t)}_{\phi \eta_s} & \to A^{(t)}_{\phi \eta_s} - M^{(t)}_{\phi \eta_s} - M^{(g)}_{\phi \eta_s},
A^{(u)}_{\phi^0 K^0} & \to A^{(u)}_{\phi^0 K^0} + \frac{1}{2} M^{(u,c)}_{\phi^0 K^0}, & A^{(t)}_{\phi^0 K^0} & \to A^{(t)}_{\phi^0 K^0} - \frac{1}{2} M^{(t)}_{\phi^0 K^0} - \frac{1}{2} M^{(g)}_{\phi^0 K^0},
A^{(u)}_{\omega K^0} & \to A^{(u)}_{\omega K^0} + \frac{1}{2} M^{(u,c)}_{\omega K^0}, & A^{(t)}_{\omega K^0} & \to A^{(t)}_{\omega K^0} - \frac{1}{2} M^{(t)}_{\omega K^0} - \frac{1}{2} M^{(g)}_{\omega K^0},
\end{align*}
\]

(30)

where the terms \( A^{(u,t)}_{\cdot} \) refer to the LO amplitudes, while \( M^{(u,c,t)}_{\cdot} \) and \( M^{(g)}_{\cdot} \) are the NLO ones, which describe the NLO contributions from the up-loop, charm-loop, QCD-penguin-loop, and magnetic-penguin diagrams, respectively.
It is straightforward to calculate the decay amplitudes $\mathcal{M}_{B^0M_1M_3}^{(q)}$ and $\mathcal{M}_{B^0M_1M_3}^{(mp)}$. As mentioned in the previous section, since the Lorentz structure of wave functions for vector mesons is different from those for pseudoscalar mesons, there are also two different kinds of decay amplitudes $\mathcal{M}_{B^0M_1M_3}^{(q)}$ and $\mathcal{M}_{B^0M_1M_3}^{(mp)}$. First, when the $M_2$ is a pseudoscalar meson and $M_3$ is a vector meson, the NLO decay amplitudes $\mathcal{M}_{B^0M_1M_3}^{(q)}$ and $\mathcal{M}_{B^0M_1M_3}^{(mp)}$ can be written in the form:

$$\mathcal{M}_{B^0}^{(q)} = -8m_{B_s}^4 \frac{g_F^2}{2\sqrt{2\pi c}} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 b_2 b_3 b_4 b_5 b_6 b_7 b_8 b_9 \phi_{B_s}(x_1) \left\{ [(1 + x_3)\phi^A_{p}(x_2)\phi_v(x_3)] - 2r_p\phi^F_{p}(x_2)\phi_v(x_3) + r_v(1 - 2x_3)\phi^A_{p}(x_2)(\phi^F_v(x_3) + \phi^D_v(x_3)) - 2r_p\phi^F_{p}(x_2)(2 - x_3)\phi^D_v(x_3) - 2r_p\phi^A_{p}(x_2)(2 - x_3)\phi^F_v(x_3) - 2r_p\phi^A_{p}(x_2)\phi^F_v(x_3) + 2r_p\phi^A_{p}(x_2)\phi^F_v(x_3) \right\},$$

$$\mathcal{M}_{B^0}^{(mp)} = 16m_{B_s}^6 \frac{g_F^2}{2\sqrt{2\pi c}} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 b_2 b_3 b_4 b_5 b_6 b_7 b_8 b_9 \phi_{B_s}(x_1) \times \left\{ [(1 - x_3)\phi_v(x_3) - r_v(3\phi^A_v(x_3) + \phi^F_v(x_3))] - r_v(3\phi^A_v(x_3) + \phi^F_v(x_3)) - r_v(3\phi^A_v(x_3) + \phi^F_v(x_3)) - r_v(3\phi^A_v(x_3) + \phi^F_v(x_3)) + 2r_p\phi^F_{p}(x_2)(2 - x_3)\phi^D_v(x_3) - 2r_p\phi^A_{p}(x_2)(2 - x_3)\phi^F_v(x_3) + 2r_p\phi^A_{p}(x_2)\phi^F_v(x_3) + 2r_p\phi^A_{p}(x_2)\phi^F_v(x_3) \right\},$$

When $M_2 = V$ and $M_3 = P$, however, the corresponding decay amplitudes can be written as:

$$\mathcal{M}_{B^0}^{(q)} = -8m_{B_s}^4 \frac{g_F^2}{2\sqrt{2\pi c}} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 b_2 b_3 b_4 b_5 b_6 b_7 b_8 b_9 \phi_{B_s}(x_1) \left\{ [(1 + x_3)\phi_v(x_3)] - 2r_p\phi^F_{p}(x_2)\phi_v(x_3) + r_v(1 - 2x_3)\phi^A_{p}(x_2)(\phi^F_v(x_3) + \phi^D_v(x_3)) - 2r_p\phi^F_{p}(x_2)(2 - x_3)\phi^D_v(x_3) - 2r_p\phi^A_{p}(x_2)(2 - x_3)\phi^F_v(x_3) - 2r_p\phi^A_{p}(x_2)\phi^F_v(x_3) + 2r_p\phi^A_{p}(x_2)\phi^F_v(x_3) \right\},$$

$$\mathcal{M}_{B^0}^{(mp)} = 16m_{B_s}^6 \frac{g_F^2}{2\sqrt{2\pi c}} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 b_2 b_3 b_4 b_5 b_6 b_7 b_8 b_9 \phi_{B_s}(x_1) \times \left\{ [(1 - x_3)\phi_v(x_3) - r_v(3\phi^A_v(x_3) + \phi^F_v(x_3))] - r_v(3\phi^A_v(x_3) + \phi^F_v(x_3)) - r_v(3\phi^A_v(x_3) + \phi^F_v(x_3)) - r_v(3\phi^A_v(x_3) + \phi^F_v(x_3)) + 2r_p\phi^F_{p}(x_2)(2 - x_3)\phi^D_v(x_3) - 2r_p\phi^A_{p}(x_2)(2 - x_3)\phi^F_v(x_3) + 2r_p\phi^A_{p}(x_2)\phi^F_v(x_3) + 2r_p\phi^A_{p}(x_2)\phi^F_v(x_3) \right\}.$$
be written in the form:

\[
f^+(q^2)|_{\text{NLO}} = 8\pi m_{B_s}^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_{B_s}(x_1, b_1) \times \left\{ F_K \left[ \phi_K^P(x_2) - \phi_K^T(x_2) \right] \cdot \alpha_s(t_1) \cdot e^{-S_{b_s, \kappa(t_1)}} \cdot S_{t}(x_2) \cdot h(x_1, x_2, b_1, b_2) + \left( 1 + x_2 \eta \right) \left[ 1 + F_{T_2}^{(1)}(x_1, \mu_f, q^2) \right] \phi_K^A(x_2) + 2r_K \left( \frac{1}{\eta} - x_2 \right) \phi_K^P(x_2) - 2r_2 r_K \phi_K^P(x_2) \right\} \cdot \alpha_s(t_2) \cdot e^{-S_{b_s, \kappa(t_2)}} \cdot S_{t}(x_2) \cdot h(x_1, x_2, b_1, b_2) + 2r_K \phi_K^P(x_2) \left( 1 + F_{T_3}^{(1)}(x_1, \mu_f, q^2) \right) \alpha_s(t_2) \cdot e^{-S_{b_s, \kappa(t_2)}} \cdot S_{t}(x_2) \cdot h(x_1, x_2, b_1, b_2) \right\},
\]

(35)

where \( \eta = 1 - q^2/m_{B_s}^2 \), with \( q^2 = (P_{B_s} - P_3)^2 \) and \( P_3 \) is the momentum of the meson \( M_3 \) which absorbed the spectator \( \bar{s} \) quark of the \( B_s^0 \) meson, \( \mu \) (\( \mu_f \)) is the renormalization (factorization) scale, the hard scale \( t_{1,2} \) are chosen as the largest scale of the propagators in the hard \( b \)-quark decay diagrams [29, 30]. The explicit expressions of the threshold Sudakov function \( S_t(x) \) and the hard function \( h(x_i, b_j) \) can be found in Refs. [29, 30]. The NLO correction factor \( F_{T_2}^{(1)}(x_1, \mu_f, q^2) \) and \( F_{T_3}^{(1)}(x_1, \mu_f, q^2) \) appeared in Eq. (35) describe the NLO twist-2 and twist-3 contributions to the form factor \( f^+(q^2) \) of the \( B_s \to K \) transition respectively, and can be written in the following form [29, 30]:

\[
F_{T_2}^{(1)} = \frac{\alpha_s(\mu_f) C_F}{4\pi} \left[ \frac{21}{4} \ln \frac{\mu_f^2}{m_{B_s}^2} - \left( \frac{13}{2} + \ln r_1 \right) \ln \frac{\mu_f^2}{m_{B_s}^2} + \frac{7}{16} \ln^2 (x_1 x_2) + \frac{1}{8} \ln^2 x_1 \right.
\]

\[
+ \left. \frac{1}{4} \ln x_1 \ln x_2 + \left( -\frac{1}{4} + 2 \ln r_1 + \frac{7}{8} \ln \eta \right) \ln x_1 + \left( \frac{3}{2} + \frac{7}{8} \ln \eta \right) \ln x_2 \right. \]

\[
+ \left. \frac{15}{16} \ln \eta - \frac{7}{16} \ln^2 \eta + \frac{3}{2} \ln^2 r_1 - \ln r_1 + \frac{101\pi^2}{48} + \frac{219}{16} \right],
\]

(36)

\[
F_{T_3}^{(1)} = \frac{\alpha_s(\mu_f) C_F}{4\pi} \left[ \frac{21}{4} \ln \frac{\mu_f^2}{m_{B_s}^2} - \frac{1}{2} \left( 6 + \ln r_1 \right) \ln \frac{\mu_f^2}{m_{B_s}^2} + \frac{7}{16} \ln^2 x_1 - \frac{3}{8} \ln^2 x_2 \right.
\]

\[
+ \left. \frac{9}{8} \ln x_1 \ln x_2 + \left( -\frac{29}{8} + \ln r_1 + \frac{15}{8} \ln \eta \right) \ln x_1 + \left( -\frac{25}{16} + \ln r_2 + \frac{9}{8} \ln \eta \right) \ln x_2 \right.
\]

\[
+ \left. \frac{1}{2} \ln r_1 - \frac{1}{4} \ln^2 r_1 + \ln r_2 - \frac{9}{8} \ln \eta - \frac{1}{8} \ln^2 \eta + \frac{37\pi^2}{32} + \frac{91}{32} \right],
\]

(37)

where \( r_i = m_{B_s}^2/\xi_i^2 \) with the choice of \( \xi_1 = 25 m_{B_s} \) and \( \xi_2 = m_{B_s} \). For the \( B_s \to (K, \eta_s)V \) decays, the large recoil region corresponds to the energy fraction \( \eta \sim O(1) \). The factorization scale \( \mu_f \) is set to be the hard scales

\[
t^a = \max(\sqrt{x_3 \eta} m_B, 1/b_1, 1/b_3), \quad \text{or} \quad t^b = \max(\sqrt{x_1 \eta} m_B, 1/b_1, 1/b_3),
\]

(38)

corresponding to the largest energy scales in Fig. 1(a) and 1(b), respectively. The renormalization scale \( \mu \) is defined as [30, 38, 44]

\[
\mu = t_s(\mu_f) = \left\{ \exp \left[ c_1 + \left( \ln \frac{m_B^2}{\xi_1^2} + \frac{5}{4} \right) \ln \frac{m_B^2}{m_{B_s}^2} \right] \right\}^{2/21} \mu_f,
\]

(39)

with the coefficients

\[
c_1 = - \left( \frac{15}{4} - \frac{7}{16} \ln \eta \right) \ln \eta + \frac{1}{2} \ln \frac{m_B^2}{\xi_1^2} \left( 3 \ln \frac{m_B^2}{\xi_1^2} + 2 \right) - \frac{101}{48} \pi^2 - \frac{219}{16},
\]

\[
c_2 = - \left( 2 \ln \frac{m_B^2}{\xi_1^2} + \frac{7}{8} \ln \eta - \frac{1}{4} \right),
\]

\[
c_3 = \frac{7}{8} \ln \eta + \frac{3}{2}
\]

(40)
III. NUMERICAL RESULTS

In the numerical calculations, the following input parameters will be used implicitly. The masses, decay constants and QCD scales are in units of GeV [5]:

\[
\begin{align*}
\Lambda_{\text{MS}}^{(f=5)} &= 0.225, \quad f_{B_s} = 0.23 \pm 0.02, \quad f_K = 0.16, \quad f_\pi = 0.13, \quad f_\rho = 0.209, \quad f_\rho^T = 0.165, \\
M_{B_s} &= 5.37, \quad m_K = 0.494, \quad m_\pi^0 = 1.4, \quad m_0^K = 1.9, \quad f_\omega = 0.195, \quad f_\rho^T = 0.145, \\
|f_K^*| &= 0.217, \quad |f_\rho^T| = 0.185, \quad f_\phi = 0.231, \quad f_\Delta^T = 0.20, \quad m_\rho = 0.77, \quad m_\omega = 0.78, \\
m_K^* &= 0.89, \quad m_\rho = 1.02, \quad \tau_{B_s^0} = 1.497\text{ps}, \quad m_b = 4.8, \quad M_W = 80.42.
\end{align*}
\]

(41)

For the CKM matrix elements, we also take the same values as being used in Ref. [25], and neglect the small errors on \(V_{ud}, V_{us}, V_{ts}\) and \(V_{tb}\)

\[
|V_{ud}| = 0.974, \quad |V_{us}| = 0.226, \quad |V_{ub}| = (3.68^{+0.11}_{-0.08}) \times 10^{-3}, \quad |V_{td}| = (8.20^{+0.59}_{-0.27}) \times 10^{-3}, \\
|V_{ts}| = 40.96 \times 10^{-3}, \quad |V_{tb}| = 1.0, \quad \alpha = (99^{+4}_{-9.4})^\circ, \quad \gamma = (59.0^{+9.7}_{-3.7})^\circ.
\]

(42)

For the considered \(B_s^0\) decays, the decay amplitude for a given decay mode with \(b \to q\) transitions can be generally written as

\[
A(B_s^0 \to f) = V_{ub} V_{us}^{*} T - V_{tb} V_{ts}^{*} P = V_{ub} V_{us}^{*} T \left[ 1 + z e^{i(-\theta + \delta)} \right],
\]

(43)

where \(q = (d, s)\), \(\theta\) is the weak phase (the CKM angles), \(\delta = \text{arg}[P/T]\) are the relative strong phase between the tree \((T)\) and penguin \((P)\) diagrams, and the parameter \(z\) is the ratio of penguin to tree contributions with the definition

\[
z = \left| \frac{V_{ub} V_{us}^{*}}{V_{ub} V_{us}^{*}} \right| \left| P \right| \left| T \right|,
\]

(44)

the ratio \(z\) and the strong phase \(\delta\) can be calculated in the PQCD approach. Therefore the CP-averaged branching ratio, consequently, can be defined as

\[
B(B_s^0 \to f) \propto \frac{1}{2} \left[ |A|^2 + |\bar{A}|^2 \right] = |V_{ub} V_{us}^{*} T|^2 \left[ 1 + 2z \cos \theta \cos \delta + z^2 \right],
\]

(45)

where the ratio \(z\) and the strong phase \(\delta\) have been defined in the above equations.

In Table I, we list the PQCD predictions for the CP-averaged branching ratios of the considered \(B_s^0\) decays. The label “LO” means the full leading order PQCD predictions. For other four cases with the label “+VC”, “+QL”, “+MP” and “NLO”, the NLO Wilson coefficients \(C_i(\mu)\) and \(C_\alpha(\mu)\) at two-loop level are used implicitly. The label “+VC” means the additional NLO “Vertex correction” is included. The label “+QL” (“+MP”) means both “VC” and “QL” (“QL” and “MP”) NLO contributions are taken into account simultaneously. And finally the label “NLO” means that all currently known NLO contributions are taken into account: the newly known NLO corrections to the form factor \(F_0^{B_s \to \bar{K}^0}(0)\) and \(F_0^{B_s \to \eta}(0)\) also be included here. In Table I, for the sake of comparison, we also list the LO PQCD predictions (in the seventh column) as given in Ref. [25], the QCDF predictions as given in Ref. [12] (the eighth column) and in Ref. [14] (the ninth column) respectively. The main theoretical errors come from the uncertainties of the various input parameters: dominant ones from \(\omega_{B_s} = 0.50 \pm 0.05\) GeV, \(f_{B_s} = 0.23 \pm 0.02\) GeV and the Gegenbauer moments like \(a_2^\pm K = 0.25 \pm 0.15\). The total errors of the NLO PQCD predictions as listed in Table I are obtained by adding the individual errors in quadrature.

Among the twenty one \(B_s^0 \to PV\) decays considered in this paper, only three of them, say \(B_s^0 \to \pi^- K^{*+}, K^{*+} K^{*-}\) and \(B_s^0 \to K^0 \bar{K}^{0*}\), have been measured recently by LHCb experiments [45]. For \(B_s^0 \to \eta \phi\) decay, the LHCb Collaboration put an upper limit at 95% C.L. on its decay rate very recently [46]. We list those measured values and upper limit in the last column of Table I and will compare those theoretical predictions with them.

From our PQCD predictions for the branching ratios, the previous theoretical predictions as given in Refs. [12, 14, 25] and the data [45, 46], as listed in Table I, we have the following observations:

1. The LO PQCD predictions for branching ratios of \(B_s^0 \to (\pi, K, \eta(1'))V\) decays as given in Ref. [25] ten years ago are confirmed by our independent calculations. Some little differences between the central values of the LO predictions are induced by the different choices or upgrade of some input parameters, such as the Gegenbauer moments and the CKM matrix elements.

2. For the “QCD-Penguin” decays \(B_s^0 \to K^0 \bar{K}^{0*}\) and \(B_s^0 \to K^{*+} K^{*-}\), the NLO contributions can provide \(\sim 30\%\) to \(45\%\) enhancements to the LO PQCD predictions of their branching ratios. For the “tree” dominated decay \(B_s^0 \to \pi^- K^{*+}\), however, the NLO contribution will result in a \(37\%\) reduction of the LO PQCD prediction for its branching ratio. The
TABLE I. The PQCD predictions for the CP-averaged branching ratios (in units of $10^{-6}$) of the considered $B^0_s$ decays. As a comparison, we also list the theoretical predictions as given in Refs. [12, 14, 25], and those currently available measured values [45] or upper limit at 95% C.L. [46].

| Mode          | Class | LO + V + C + QL + MP | NLO | PQCQD [25] | QCD [12] | QCDF 2 [14] | Data [45, 46] |
|---------------|-------|-----------------------|-----|------------|----------|-------------|---------------|
| $B^0_s \to \pi^- K^{*0}$ | T     | 6.32                  | 5.12| 4.01       | 3.96     | 3.96±0.01   | 7.6±0.0     |
| $B^0_s \to K^+ K^{*-}$ | P     | 9.03                  | 10.75| 12.30| 12.24| 12.32±0.01 | 10.7±0.0    |
| $B^0_s \to K^0 \bar{K}^{*-}$ | P     | 9.95                  | 12.41| 14.46| 14.38| 14.39±0.01 | 11.6±0.0    |
| $B^0_s \to K^0 \rho^0$ | T     | 18.6                  | 16.3| 16.6| 16.4| 15.9±0.01 | 17.8±0.0    |
| $B^0_s \to \pi^0 K^{*-0}$ | C     | 0.08                  | 0.20| 0.21| 0.21| 0.21±0.01 | 0.07±0.0    |
| $B^0_s \to \eta \phi$ | P     | 3.3                   | 0.89| 1.40| 1.21| 1.26±0.01 | 3.6±0.0     |
| $B^0_s \to \eta' \phi$ | P     | 0.25                  | 0.63| 0.76| 0.51| 0.59±0.01 | 0.19±0.0    |
| $B^0_s \to K^0 \phi$ | P     | 0.20                  | 0.22| 0.23| 0.23| 0.24±0.01 | 0.16±0.0    |
| $B^0_s \to K^+ \phi$ | C     | 0.10                  | 0.39| 0.36| 0.34| 0.34±0.01 | 0.08±0.0    |
| $B^0_s \to K^0 \omega$ | C     | 0.14                  | 0.51| 0.56| 0.62| 0.62±0.01 | 0.15±0.0    |
| $B^0_s \to \eta K^{*0}$ | C     | 0.20                  | 0.22| 0.22| 0.22| 0.20±0.01 | 0.17±0.0    |
| $B^0_s \to \eta' K^{*-0}$ | C     | 0.11                  | 0.26| 0.33| 0.37| 0.35±0.01 | 0.09±0.0    |
| $B^0_s \to \phi' K^{*-0}$ | C     | 0.14                  | 0.51| 0.56| 0.62| 0.62±0.01 | 0.15±0.0    |
| $B^0_s \to \phi K^{*-0}$ | C     | 0.20                  | 0.22| 0.22| 0.22| 0.20±0.01 | 0.17±0.0    |
| $B^0_s \to \eta K^{*-0}$ | C     | 0.11                  | 0.26| 0.33| 0.37| 0.35±0.01 | 0.09±0.0    |
| $B^0_s \to \eta' K^{*-0}$ | C     | 0.14                  | 0.51| 0.56| 0.62| 0.62±0.01 | 0.15±0.0    |
| $B^0_s \to \omega K^{*-0}$ | C     | 0.07                  | 0.11| 0.11| 0.11| 0.11±0.01 | 0.04±0.0    |
| $B^0_s \to \omega' K^{*-0}$ | C     | 0.30                  | 0.35| 0.35| 0.35| 0.35±0.01 | 0.44±0.0    |

resultant enhancements or the reduction, fortunately, are all in the right direction. After the inclusion of the NLO corrections, the NLO PQCD predictions for these three decays become well consistent with those currently available data within one standard deviation. In order to show numerically the improvements due to inclusion of the NLO corrections, we define the ratios $R_{1,2,3}$ of the measured values and the PQCD predictions for those three measured decay modes:

$$R_1 = \frac{B(B_s^0 \to \pi^- K^{*-0})_{\text{exp}}}{B(B_s^0 \to \pi^- K^{*-0})_{\text{PQCQD}}} \approx \left\{ \begin{array}{l} 0.52, \text{ LO}, \\
0.83, \text{ NLO}, \end{array} \right. $$

$$R_2 = \frac{B(B_s^0 \to K^+ K^{*-0})_{\text{exp}}}{B(B_s^0 \to K^+ K^{*-0})_{\text{PQCQD}}} \approx \left\{ \begin{array}{l} 1.38, \text{ LO}, \\
1.02, \text{ NLO}, \end{array} \right. $$

$$R_3 = \frac{B(B_s^0 \to K^0 \bar{K}^{*-0})_{\text{exp}}}{B(B_s^0 \to K^0 \bar{K}^{*-0})_{\text{PQCQD}}} \approx \left\{ \begin{array}{l} 1.65, \text{ LO}, \\
1.14, \text{ NLO}, \end{array} \right. $$

It is easy to see that the agreements between the PQCD predictions and the three measured values are indeed improved significantly due to the inclusion of the NLO contributions. This is a clear indication for the important role of the NLO contributions in order to understand the experimental measurements.

(3) For the “tree” dominated decay $B_s^0 \to K^+ \rho^-$, the NLO contribution results in a ~ 15% reduction against the LO result, but its branching ratio is still at 1.6 x 10^{-5} level, the largest one of all decays considered in this paper. We believe that this decay mode could be measured by LHCb soon. For the “color-suppressed-tree” decay $B_s^0 \to \pi^0 K^{*-0}$, however, although the NLO contribution can provide a large ~ 150% enhancement, but the theoretical predictions for its branching ratio in the LO and NLO PQCD or in the QCDF approaches[12, 14] are always at the level of 10^{-5}, much smaller than that for $B_s^0 \to K^+ \rho^-$ decay. At the LO and NLO level, one can read out the ratio of the branching ratios of these two decays from Table I

$$R_4 = \frac{B(B_s^0 \to K^+ \rho^-)}{B(B_s^0 \to \pi^0 K^{*-0})} \approx \left\{ \begin{array}{l} 232, \text{ LO}, \\
76, \text{ NLO}, \end{array} \right. $$

(49)
In order to understand so large difference, we made careful examinations for the LO decay amplitudes of these two decay modes and found the two reasons. Firstly, as shown explicitly in Eqs. (22,23), the dominant part of the decay amplitudes for these two decays are very different in magnitude (in units of $10^{-4}$):

\[
A_T(\bar{B}_s^0 \to K^+ \rho^-) = V_{ub} V_{us}^{\ast} [f_\rho F_{K^\ast} a_1 + M_{eK} C_1] = 17.45 - 49.38 i, \quad (50)
\]

\[
A_C(\bar{B}_s^0 \to \pi^0 K^{\ast0}) = V_{ub} V_{us}^{\ast} [f_\rho F_{K^\ast} a_2 + M_{eK} C_2] / \sqrt{2} = -2.15 + 0.42 i, \quad (51)
\]

where $a_1 = C2 + C1/3 \approx C2 \approx 1.1$ is a large quantity, while $|a_2| \approx |C1 + C2/3| \approx 0.1$ a small one. The ratio of these two magnitudes $|A_T|/|A_C| \approx 33$ is therefore very large. This is the main reason of the large difference of these two branching ratios. Secondly, there is a strong constructive interference among the large $A_T$ and $A_P$ for $\bar{B}_s^0 \to K^+ \rho^-$, but a destructive one between the small $A_C$ and $A_P$ for $\bar{B}_s^0 \to \pi^0 K^{\ast0}$ decay. Numerically, one finds (in units of $10^{-4}$) that

\[
A(\bar{B}_s^0 \to K^+ \rho^-)_{LO} = (17.45 - 49.38 i) + (6.39 - 2.00 i) = 23.83 - 51.38 i, \quad (52)
\]

\[
A(\bar{B}_s^0 \to \pi^0 K^{\ast0})_{LO} = (-2.15 + 0.42 i) + (1.19 - 2.34 i) = -0.96 - 1.92 i. \quad (53)
\]

For the corresponding CP-conjugated decay modes, we also find similar behaviour

\[
A(\bar{B}_s^0 \to K^- \rho^+)_{LO} = (15.5 + 50.0 i) + (3.24 - 5.84 i) = 18.7 + 44.2 i, \quad (54)
\]

\[
A(\bar{B}_s^0 \to \pi^- K^{\ast0})_{LO} = (1.473 - 1.62 i) + (-0.75 - 2.51 i) = 0.72 - 4.13 i. \quad (55)
\]

From above four decay amplitudes, it is simple to define the ratio $R_4(\langle |A|^2 \rangle)$ of the square of the decay amplitudes:

\[
R_4(\langle |A|^2 \rangle) = \frac{|A(\bar{B}_s^0 \to K^+ \rho^-)_{LO}|^2 + |A(\bar{B}_s^0 \to K^- \rho^+)_{LO}|^2}{|A(\bar{B}_s^0 \to \pi^0 K^{\ast0})_{LO}|^2 + |A(\bar{B}_s^0 \to \pi^- K^{\ast0})_{LO}|^2} \approx 248, \quad (56)
\]

which is indeed close to the ratio of the CP-averaged branching ratios: $R_4^{LO} \approx 232$ as defined in Eq. (49). From above numerical results, it is straightforward to understand the large difference between the LO PQCD predictions for $B(\bar{B}_s^0 \to K^+ \rho^-)$ and $B(\bar{B}_s^0 \to \pi^0 K^{\ast0})$. At the NLO level, the ratio $R_4^{NLO} \approx 76$ can be interpreted in a similar way.

(4) The two $\bar{B}_s^0 \to \phi \eta, \phi \eta'$ decays are very similar in nature, the difference between the PQCD predictions for $B(\bar{B}_s^0 \to \phi \eta)$ and $B(\bar{B}_s^0 \to \phi \eta')$ is rather large at LO level: $R_4^{LO}(B) = 3.3/0.25 \approx 13.2$, but become smaller at NLO level: $R_4^{NLO}(B) = 1.28/0.59 \approx 2.14$, after the inclusion of the NLO contributions. For $\bar{B}_s^0 \to \phi \eta$ decays, the NLO contribution results in a 62% reduction for its branching ratio. For $\bar{B}_s^0 \to \phi \eta'$ decay, however, the inclusion of the NLO contribution leads to a 136% enhancement to its LO result. How to understand these special features for these two decay modes? The major reason is the unique $\eta - \eta'$ mixing pattern. We know that the decay amplitude for $\bar{B}_s^0 \to V(\eta, \eta')$ with $V = (\phi, \omega, \rho^0, K^0)$ can be written as

\[
A(\bar{B}_s^0 \to V \eta) = A(V \eta) \cos(\phi) - A(V \eta) \sin(\phi), \quad (57)
\]

\[
A(\bar{B}_s^0 \to V \eta') = A(V \eta) \sin(\phi) + A(V \eta) \cos(\phi), \quad (58)
\]

where $\phi = 39.3^\circ$ is the mixing angle of $\eta - \eta'$ system [32]. Since $\sin(\phi) = 0.63$ has the same sign with $\cos(\phi) = 0.77$ and are similar in magnitude, the interference between the two parts, consequently, may be constructive for one channel but destructive for another, or vise versa.

For $\bar{B}_s^0 \to \phi \eta$ and $\phi \eta'$ decays, for example, we find the LO PQCD predictions for their decay amplitudes (in units of $10^{-4}$)

\[
A(\bar{B}_s^0 \to \phi \eta)_{LO} = (13.25 + 4.23 i) \cdot \cos(39.3^\circ) - (-17.16 - 8.61 i) \cdot \sin(39.3^\circ) = 21.01 + 8.68 i, \quad (59)
\]

\[
A(\bar{B}_s^0 \to \phi \eta')_{LO} = (13.25 + 4.23 i) \cdot \sin(39.3^\circ) + (-17.16 - 8.61 i) \cdot \cos(39.3^\circ) = -4.86 - 3.96 i. \quad (60)
\]
And it is easy to see that $A(\phi \eta_b)$ interfere constructively with $A(\phi \eta_s)$ for $\bar{B}_s^0 \rightarrow \phi \eta$ decay, but destructively with $A(\phi \eta_b)$ for $\bar{B}_s^0 \rightarrow \phi \eta'$ decay. Such pattern of interference leads to the large ratio of $|A|^2$

$$R_5^{\text{LO}}(|A|^2) = \frac{|A(\bar{B}_s^0 \rightarrow \phi \eta)|_{\text{LO}}^2}{|A(\bar{B}_s^0 \rightarrow \phi \eta')|_{\text{LO}}^2} = 13.14 \approx R_5^{\text{LO}}(B).$$  \hspace{1cm} (61)

When the NLO contributions are taken into account, however, we find numerically

$$A(\bar{B}_s^0 \rightarrow \phi \eta)^{\text{NLO}} = (0.47 + 9.47 i) \cdot \cos(39.3\)°) - (-14.76 - 5.76 i) \cdot \sin(39.3\)°) = 9.66 + 10.88 i, \hspace{1cm} (62)$$

$$A(\bar{B}_s^0 \rightarrow \phi \eta')^{\text{NLO}} = (0.47 + 9.47 i) \cdot \sin(39.3\)°) + (-14.76 - 5.76 i) \cdot \cos(39.3\)°) = -11.02 + 1.57 i, \hspace{1cm} (63)$$

and similar numerical results for $A(\bar{B}_s^0 \rightarrow \phi \eta|^{\text{NLO}}$ and $A(\bar{B}_s^0 \rightarrow \phi \eta')^{\text{NLO}}$. It is then simple to define the ratio $R_5^{\text{NLO}}$ in the following form

$$R_5^{\text{NLO}}(|A|^2) = \frac{|A(\bar{B}_s^0 \rightarrow \phi \eta)|^{\text{NLO}}^2 + |A(\bar{B}_s^0 \rightarrow \phi \eta')^{\text{NLO}}^2}{|A(\bar{B}_s^0 \rightarrow \phi \eta)^{\text{NLO}}|^2 + |A(\bar{B}_s^0 \rightarrow \phi \eta')^{\text{NLO}}|^2} = 2.2 \approx R_5^{\text{NLO}}(B).$$  \hspace{1cm} (64)

One can see that the strength of the interference at the NLO level become a little weaker than that at the LO level, the value of the ratio consequently changed its value from a large 13 to a relatively small one 2.2. For $\bar{B}_s^0 \rightarrow \phi \eta'$ decay, based on the data collected in the period of RUN-I, the LHCb Collaboration put an upper limit on its branching ratio very recently [46]: $B(\bar{B}_s^0 \rightarrow \phi \eta') < 0.82 \times 10^{-6}$ at 90% and 1.01 $\times 10^{-6}$ at 95% confidence level (CL). The PQCD predictions and the QCDF prediction as given in Ref. [12] agree with this limit, while the central value of the QCDF prediction as given in Ref. [14] is likely too large. The future LHCb and/or Belle-II measurements for this kind of decays may be helpful for us to examine the mixing pattern between $\eta - \eta'$ system.

For $\bar{B}_s^0 \rightarrow \omega \eta$ and $\omega \eta'$ decays, the difference between the PQCD predictions for their branching ratios can be understood by a similar mechanism: the interference effects between the decay amplitude $A(\omega \eta)$ and $A(\omega \eta_b)$.

(5) For the “color-suppressed-tree” decay $\bar{B}_s^0 \rightarrow \eta K^{*0}$, the total NLO contribution is negligibly small. For other three same kind decays $\bar{B}_s^0 \rightarrow K^0(\rho^0, \omega)$ and $\bar{B}_s^0 \rightarrow \eta K^{*0}$, however, the NLO contributions can provide a factor of $2 \rightarrow 4$ enhancement to their branching ratios. The central values of the NLO PQCD predictions for the branching ratios of above four decays agree well with those QCDF predictions as given in Ref. [12] within one standard deviation, but smaller than those QCDF predictions as given in Ref. [14] by a factor of $3 \rightarrow 5$. Such model differences will be examined by the LHCb (RUN-II) and/or Belle-II experiments.

(6) For the three “Electroweak-Penguin” $\bar{B}_s^0 \rightarrow \eta(\rho^0) \rho^0$ and $\eta(\rho^0) \phi$ decays, the NLO corrections comes only from the usage of the NLO Wilson coefficients $C_i(\mu)$, the $\alpha_s(\mu)$ at two-loop level and the so-called “Vertex corrections”. The enhancement or reduction due to the inclusion of the NLO contributions are always not large: less than 45% in magnitude. The PQCD predictions for their decay rates agree well with those in QCDF approach [12, 14].

(7) For the four pure “annihilation” decays, the only NLO correction comes from the usage of the NLO Wilson coefficients $C_i(\mu)$ and the $\alpha_s(\mu)$ at two-loop level. For $B_s \rightarrow \pi^+ \rho^-$ and $\pi^0 \rho^0$ decays, the NLO corrections will lead to $\sim 50\%$ reduction on their LO PQCD predictions for branching ratios, but the NLO PQCD predictions are still at the $10^{-7}$ level, much larger than those QCDF predictions as given in Refs. [12, 14] by roughly one to two orders of magnitude. The forthcoming LHCb and Belle II experimental measurements can help us to examine such large theoretical difference.

The $\bar{B}_s \rightarrow \pi^0 \omega$ decay is also a pure “annihilation” decay, but the theoretical predictions for its branching ratios in both the PQCD and QCDF approaches are always tiny in size: less than $10^{-8}$ and be hardly measured even in the future LHCb experiments.

(8) By comparing the numerical results as listed in the sixth column (“+MP”) and seventh column (“NLO”), one can see easily that, the effects due to the inclusion of the NLO pieces of the $B_s \rightarrow K$ or $B_s \rightarrow \eta_s$ transition form factors are always small: $\sim 10\%$ for the first seventeen decays. For the remaining four pure “annihilation” decays, in fact, they do not receiver such kinds of NLO corrections.

(9) The still missing NLO contributions in the pQCD approach are the ones to the LO hard spectator and the non-factorizable annihilation diagrams. But from the comparative studies for the LO and NLO contributions from different sources in Refs. [24, 41, 42], we do believe that those still missing NLO contributions are most possibly the higher order corrections to the small LO quantities, and therefore can be safely neglected.
TABLE II. The LO and NLO PQCD predictions for the direct CP asymmetries $A_f^{\text{dir}}$ (in units of $10^{-2}$) of the considered $B_s^0 \to PV$ decays. As comparisons, the LO PQCD predictions as given in Ref. [25] and the central values of the NLO QCDF predictions as given in Ref. [12] are listed in last two columns.

| Mode | Class | LO | NLO | PQCD[25] | QCDF[12] |
|------|-------|----|-----|----------|----------|
| $B_s^0 \to K^+ \rho^-$ | T     | 16.5 | 11.3$^{+2.9}_{-2.8}$ | 14.2$^{+3.5}_{-5.6}$ | -1.5 |
| $B_s^0 \to \pi^0 K^{*0}$ | C     | -49.4 | 19.7$^{+3.7}_{-4.9}$ | -47.1$^{+36.4}_{-31.8}$ | -45.7 |
| $B_s^0 \to K^0 \rho^+$ | C     | 72.1 | 69.4$^{+6.2}_{-5.5}$ | 73.4$^{+17.5}_{-17.2}$ | 24.7 |
| $B_s^0 \to K^0 \omega$ | C     | -59.3 | -84.7$^{+1.1}_{-4.5}$ | -52.1$^{+23.1}_{-15.2}$ | -43.9 |
| $B_s^0 \to \pi^0 \phi$ | PEW   | 15.1 | 49.2$^{+0.4}_{-0.5}$ | 13.3$^{+2.6}_{-1.8}$ | 27.2 |
| $B_s^0 \to K^0 \phi$ | P     | - | -2.9$^{+1.2}_{-1.4}$ | 0 | -10.3 |
| $B_s^0 \to \pi^0 \omega$ | ann   | 4.7 | 3.8$^{+0.5}_{-0.7}$ | 6.0$^{+0.9}_{-0.6}$ | - |
| $B_s^0 \to \pi^- \rho^+$ | ann   | 1.6 | -1.2$^{+3.3}_{-2.2}$ | 4.6$^{+3.6}_{-1.6}$ | - |
| $B_s^0 \to \pi^+ \rho^-$ | ann   | -4.3 | -8.5$^{+5.7}_{-4.8}$ | -1.3$^{+4.9}_{-3.5}$ | - |
| $B_s^0 \to \pi^0 \rho^0$ | ann   | 1.0 | 4.6$^{+2.5}_{-3.6}$ | 1.7$^{+3.9}_{-3.4}$ | - |
| $B_s^0 \to \pi^- K^{*0}$ | T     | -17.2 | -12.1$^{+1.2}_{-3.5}$ | -19.0$^{+3.7}_{-5.6}$ | 0.6 |
| $B_s^0 \to K^+ K^{*-}$ | P     | -34.1 | -21.6$^{+4.9}_{-4.3}$ | -36.6$^{+4.3}_{-3.6}$ | 2.2 |
| $B_s^0 \to K^- K^{*-}$ | P     | 50.1 | 46.6$^{+7.4}_{-6.7}$ | 55.3$^{+10.8}_{-11.2}$ | -3.1 |
| $B_s^0 \to K^0 K^{*0}$ | P     | - | 0.8$^{+1.0}_{-0.1}$ | 0 | 1.7 |
| $B_s^0 \to K^0 K^{*0}$ | P     | - | 0.1$^{+1.0}_{-0.05}$ | 0 | 0.2 |
| $B_s^0 \to \eta K^{*0}$ | C     | 38.5 | 30.6$^{+10.9}_{-8.5}$ | 51.2$^{+15.6}_{-14.4}$ | 40.2 |
| $B_s^0 \to \eta' K^{*0}$ | C     | -37.2 | -63.4$^{+3.3}_{-2.6}$ | -51.1$^{+15.6}_{-19.8}$ | -58.6 |
| $B_s^0 \to \eta \rho^0$ | PEW   | -14.3 | 37.7$^{+2.3}_{-2.3}$ | -9.2$^{+2.5}_{-2.8}$ | 27.8 |
| $B_s^0 \to \eta' \rho^0$ | PEW   | 23.9 | 54.1$^{+2.3}_{-1.3}$ | 25.8$^{+4.6}_{-4.4}$ | 28.9 |
| $B_s^0 \to \eta \omega$ | P, C  | -9.4 | -35.8$^{+2.3}_{-3.6}$ | -16.7$^{+16.5}_{-19.4}$ | - |
| $B_s^0 \to \eta' \omega$ | P, C  | 11.3 | -23.5$^{+5.4}_{-4.6}$ | 7.7$^{+10.4}_{-4.2}$ | - |
| $B_s^0 \to \eta \phi$ | P     | -1.2 | -4.1$^{+0.3}_{-0.2}$ | -1.8$^{+0.6}_{-0.6}$ | -8.4 |
| $B_s^0 \to \eta' \phi$ | P     | 5.1 | 14.2$^{+2.3}_{-1.2}$ | 7.8$^{+1.9}_{-1.6}$ | -62.2 |

Now we turn to the evaluations of the CP-violating asymmetries for the considered decay modes. In the $B_s$ system, we expect a much larger decay width difference: $\Delta \Gamma_s/(2 \Gamma_s) \sim -10\%$ [5]. Besides the direct CP violation $A_f^{\text{dir}}$, the CP-violating asymmetry $S_f$ and $H_f$ are defined as usual [25]

$$A_f^{\text{dir}} = \frac{\lambda}{1 + |\lambda|^2}, \quad S_f = \frac{2 \text{ Im}[\lambda]}{1 + |\lambda|^2}, \quad H_f = \frac{2 \text{ Re}[\lambda]}{1 + |\lambda|^2}. \quad (65)$$

They satisfy the normalization relation $|A_f|^2 + |S_f|^2 + |H_f|^2 = 1$, while the parameter $\lambda$ is of the form

$$\lambda = \eta f e^{2i\beta} \frac{A(B_s^0 \to f)}{A(B_s^0 \to \bar{f})}, \quad (66)$$

where $\eta_f$ is $+1(-1)$ for a CP-even(CP-odd) final state $f$ and $\beta_f = \arg(-V_{ts}V_{tb}^*)$ is very small in size.

The PQCD predictions for the direct CP asymmetries $A_f^{\text{dir}}$, the mixing-induced CP asymmetries $S_f$ and $H_f$ of the considered decay modes are listed in Table II and Table III. In these two tables, the label “LO” means the LO PQCD predictions, the label “NLO” means that all currently known NLO contributions are taken into account, the same definition as for the NLO PQCD predictions for the branching ratios as in Table I. The errors here are defined in the same way as for the branching ratios. As a comparison, the LO PQCD predictions as given in Ref. [25] and the central values of the NLO QCDF predictions as given in Ref. [12] are also listed in Table II and III. Since the mechanism and the sources of the CP asymmetries for the considered decay modes are very different in the PQCD approach and the QCDF approach, we here listed the central values of the NLO QCDF predictions only. Unfortunately, no experimental measurements for the CP asymmetries of the $B_s^0$ decays considered here are available at present.

From the PQCD predictions for the CP violating asymmetries of the considered $B_s^0$ decays as listed in the Table II and III, one can see the following points:

1. For all $\bar{B}_s^0 \to PV$ decays, the LO PQCD predictions for their CP asymmetries obtained in this paper do agree well with those as given in Ref. [25].
TABLE III. The LO and NLO PQCD predictions for the mixing-induced CP asymmetries (in units of $10^{-2}$) $S_f$ (the first row) and $H_f$ (the second row). The meaning of the labels are the same as those in Table II.

| Mode            | Class | LO      | NLO     | PQCD [25] |
|-----------------|-------|---------|---------|-----------|
| $B^0 \to K_S \rho^0$ | C     | $-54.1 \pm 15.2$ | $-7.9 \pm 16.2$ | $-57 \pm 56$ |
|                 |       | $-41.2 \pm 8.9$   | $-71.5 \pm 5.4$ | $-36 \pm 20$  |
| $B^0 \to K_S \omega$  | C     | $-55.5 \pm 3.2$   | $30.1 \pm 4.4$  | $-63 \pm 29$  |
|                 |       | $-58.3 \pm 5.6$   | $43.8 \pm 3.2$  | $-57 \pm 40$  |
| $B^0 \to K_S \phi$   | P     | $-72.1 \pm 0.2$   | $95.6 \pm 0.4$  | $-72$         |
|                 |       | $-69.3 \pm 2.2$   | $33.4 \pm 1.0$  | $-69$         |
| $B^0 \to \pi^0 \phi$ | $P_{EW}$ | $-16.1 \pm 4.3$   | $-8.7 \pm 4.3$  | $-7 \pm 10$   |
|                 |       | $97.2 \pm 0.6$    | $86.6 \pm 0.8$  | $98 \pm 1$    |
| $B^0 \to \pi^0 \rho^0$ | Anni | $-20.1 \pm 1.4$   | $24.5 \pm 1.2$  | $-19 \pm 2$   |
|                 |       | $97.3 \pm 0.3$    | $96.8 \pm 0.7$  | $99$         |
| $B^0 \to \pi^0 \omega$ | Anni | $-97.2 \pm 0.2$   | $97.6 \pm 0.1$  | $-97 \pm 11$  |
|                 |       | $-24.6 \pm 1.3$   | $21.2 \pm 1.1$  | $-22 \pm 29$  |
| $B^0 \to \eta\omega$ | P,C   | $-4.5 \pm 2.3$    | $11.1 \pm 2.1$  | $-2 \pm 2$    |
|                 |       | $97.5 \pm 0.3$    | $93.3 \pm 0.3$  | $99 \pm 1$    |
| $B^0 \to \eta^\prime\omega$ | P,C  | $-19.3 \pm 5.2$   | $35.4 \pm 7.9$  | $-11 \pm 5$   |
|                 |       | $96.6 \pm 3.3$    | $91.8 \pm 4.1$  | $99$         |
| $B^0 \to \eta\rho^0$ | $P_{EW}$ | $20.8 \pm 2.3$    | $10.3 \pm 2.6$  | $15 \pm 15$   |
|                 |       | $97.1 \pm 0.4$    | $92.6 \pm 0.1$  | $98 \pm 1$    |
| $B^0 \to \eta^\prime\rho^0$ | $P_{EW}$ | $-29.4 \pm 1.3$   | $11.1 \pm 1.4$  | $-16 \pm 11$  |
|                 |       | $92.2 \pm 3.6$    | $82.3 \pm 3.7$  | $95 \pm 1$    |
| $B^0 \to \eta\phi$  | P     | $-3.2 \pm 0.5$    | $-4.2 \pm 0.5$  | $-3 \pm 2$    |
|                 |       | $99.9 \pm 0.1$    | $99.4 \pm 0.1$  | $100 \pm 1$   |
| $B^0 \to \eta^\prime\phi$ | P   | $-8.6 \pm 0.4$    | $6.1 \pm 0.5$   | $0 \pm 2$     |
|                 |       | $99.9 \pm 0.1$    | $99.1 \pm 0.1$  | $100 \pm 2$   |

(2) For most $B^0 \to PV$ decays, the changes of the PQCD predictions for the CP asymmetries induced by the inclusion of the NLO corrections are basically not large in size. For $B^0 \to \pi^0 \eta^\prime$, $\eta \rho^0$ and $\eta^\prime \omega$ decays, however, the PQCD predictions for their $A^{dir}_{\phi}$ can change sign after the inclusion of the NLO corrections. For $B^0 \to \pi^0 \phi$, $\eta \rho^0$, $\eta \omega$ and $\eta^\prime \phi$ decays, on the other hand, the NLO enhancements on their $A^{dir}_{\phi}$ can be larger than a factor of two.

(3) By comparing the numerical results as listed in Table II, one can see that the PQCD and QCDF predictions for the CP-asymmetries of the considered decays are indeed quite different, due to the very large difference in the mechanism to induce the CP asymmetries in the pQCD approach and the QCDF approach. In the PQCD approach, fortunately, one can calculate the CP asymmetries for the pure annihilation decays. From Table II one can see that the PQCD predictions for the $A^{dir}_{\phi}$ of the four pure annihilation decays $B^0_s \to \pi(\omega, \rho)$ are small: less than 10% in magnitude.

(4) Since the currently measured $B_s \to \pi^- K^{*+}$ and $\bar{B}_s \to K^+ K^{*-} + K^- K^{*+}$ decays have a large decay rates at the level of $10^{-5} - 10^{-6}$, their relatively large direct CP asymmetries from $-30\%$ to around $50\%$ could be measured in the near future LHCb or Belle-II experiments. For $B_s \to K^0(\rho^0, \omega)$ and $\eta'(K^{*0}, \rho^0)$, however, it might be very difficult to measure their large direct CP asymmetries (around $50\%$ in magnitude), due to their very small branching ratios at the level of $10^{-7} - 10^{-8}$.

(5) The mixing-induced CP asymmetries $S_f$ and $H_f$ for the considered twelve decay modes are shown in Table III. For $B_s \to K_s(\omega, \phi)$ and $\omega(\pi^0, \eta')$ decays, although their $S_f$ are large in size, it is still very difficult to measure them due to their very small decay rates.

IV. SUMMARY

In summary, we calculated the CP-averaged branching ratios and CP-violating asymmetries for all twenty one $B^0 \to PV$ decays with $P = (\pi, K, \eta, \eta')$ and $V = (\rho, K^*, \phi, \omega)$ by employing the PQCD factorization approach. All currently known
NLO contributions, specifically those newly known NLO twist-2 and twist-3 contributions to the relevant form factor $F_0^{\eta\to K}(0)$ and $F_0^{\eta\to \mu\mu}(0)$, are taken into account.

From our analytical evaluations and numerical calculations, we found the following points:

1. The LO PQCD predictions for the branching ratios and CP-violating asymmetries of $B_s \to PV$ decays as presented in Ref.[25] are confirmed by our independent calculations. The effects of the NLO contributions on the PQCD predictions for the branching ratios and CP asymmetries of the considered decay modes are channel dependent and will be tested by future experiments.

2. For the three measured decays $\bar{B}_s^0 \to K^0\bar{K}^{*0}$, $K^\pm K^{*\mp}$ and $\pi^-K^{*+}$, the NLO contributions can provide a large enhancement (about 30% - 45%) or a reduction ($\sim 37\%$) to the LO PQCD predictions for their branching ratios, respectively. From the variations of the ratios $R_{1,2,3}$, one can see that the agreements between the PQCD predictions and the measured values are improved significantly due to the inclusion of the NLO contributions. This is the major reason why we have made great efforts to calculate the NLO contributions in the PQCD factorization approach.

3. For the considered $B_s \to PV$ decays, the effects from the inclusion of the NLO twist-2 and twist-3 contributions to the form factor $F_0^{\eta\to K}$ and $F_0^{\eta\to \mu\mu}$ are always small: less than 10% in magnitude.

4. For the “tree” dominated decay $\bar{B}_s^0 \to K^{0}\rho^-$ and the “color-suppressed-tree” decay $\bar{B}_s^0 \to \pi^0K^{*0}$ decay, the different topological structure and the strong interference effects (constructive or destructive) between decay amplitude $A_{T,C}$ and $A_P$ together leads to the very large difference in their decay rates.

5. For $\bar{B}_s^0 \to V(\eta, \eta')$ decays, the complex pattern of the PQCD predictions for their branching ratios can be understood by the difference of the major contributing Feynman diagrams, and the interference effects (constructive or destructive) between the decay amplitude $A(V\eta)$ and $A(V\eta_b)$ due to the $\eta - \eta'$ mixing.

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