ROTATIONAL EVOLUTION OF SOLAR-LIKE STARS IN CLUSTERS FROM PRE–MAIN SEQUENCE TO MAIN SEQUENCE: EMPIRICAL RESULTS

William Herbst
Astronomy Department, Wesleyan University, Middletown, CT 06459; wherbst@wesleyan.edu

AND

Reinhard Mundt
Max-Planck-Institut für Astronomie, Koenigstuhl 17, D-69117 Heidelberg, Germany

Received 2005 May 18; accepted 2005 July 14

ABSTRACT

Rotation periods are now available for ~500 pre–main-sequence (PMS) and recently arrived main-sequence stars of solar-like mass (0.4–1.2 $M_\odot$) in five nearby young clusters: the Orion Nebula cluster (ONC), NGC 2264, α Per, IC 2602, and the Pleiades. In combination with estimates of stellar radii these data allow us to construct distributions of surface angular momentum per unit mass at three different epochs: nominally, 1, 2, and 50 Myr. There are sufficient data that rotational evolution can now be discussed statistically on the basis of the evolution of these distributions, not just on the evolution of means or ranges, as has been necessary in the past. Our main result is illustrated in Figure 18 and may be summarized as follows: (1) 50%–60% of the stars on convective tracks in this mass range are released from any locking mechanism very early on and are free to conserve angular momentum throughout most of their PMS evolution, i.e., to spin up and account for the rapidly rotating young main-sequence stars; (2) the other 40%–50% lose substantial amounts of angular momentum during the first few million years and end up as slowly rotating main-sequence stars. The duration of the rapid angular momentum loss phase is ~5–6 Myr, which is roughly consistent with the lifetimes of disks estimated from infrared surveys of young clusters. The rapid rotators of Orion age lose less than 10% of their (surface) specific angular momentum during the next 50 Myr, while the slow rotators lose about two-thirds of theirs. A detectable part of this loss occurs even during the ~1 Myr interval between the ONC and NGC 2264. The data support the view that interaction between an accretion disk and star is the primary mechanism for evolving the broad, bimodal distribution of rotation rates seen for solar-like stars in the ONC into the even broader distributions seen in the young MS clusters.

Subject headings: stars: pre–main-sequence — stars: rotation

1. INTRODUCTION

The early evolution of rotation of solar-like stars (0.4–1.2 $M_\odot$ for the purposes of this paper) is a fundamental and surprisingly controversial subject that has recently attracted a good deal of theoretical and observational attention. For reviews of some recent conference discussions see Mathieu (2004) and Stassun & Terndrup (2003). A central issue has been to understand how the observed broad range of rotation rates, which is about a factor of 20 for pre–main-sequence (PMS) stars and larger for recently arrived main-sequence (MS) stars, comes to exist and how it evolves. The physical phenomena that have been proposed, modeled, and debated include the overall contraction of the star during its PMS phase with expected conservation of angular momentum, accretion that can either add or subtract angular momentum per unit mass, magnetically driven stellar winds that drain angular momentum, and internal redistribution of angular momentum. For comprehensive recent discussions and references to the earlier literature see, for example, Krishnamurthi et al. (1997), Sills et al. (2000), and Barnes (2003).

An area of particular importance (and controversy) is the putative role of “disk locking,” the theory that the angular velocity of the stellar surface may be locked to a location in the accretion disk several stellar radii above the photosphere. Originally proposed by Camenzind (1990) and Königl (1991) as an explanation for the slow rotation seen in many classical T Tauri stars (CTTSs), it has become an essential feature of most, if not all, models of angular momentum evolution of solar-like stars (e.g., Barnes et al. 2001; Tinker et al. 2002). At the same time, however, the concept has been criticized on observational and theoretical grounds, most recently by Matt & Pudritz (2004). As F. Shu discusses in the conference report by Stassun & Terndrup (2003), part of the problem is that there is no current “first principles” model of disk locking. The physical complexities of a stellar dynamo linked to an external disk are too much to contend with at present. Another problem is that the observed correlations between rotation and accretion disk indicators, although highly significant, are nonetheless weak in the sense of there being a good deal of scatter in the data (Herbst et al. 2002; Lamm et al. 2004). While there are good reasons for not expecting a tight correlation, including the difficulties of observing accretion disks with inner holes and the time lapse between the disappearance of active accretion and the substantial spin-up of the star due to contraction, doubts are raised on the observational side by some authors (Mathieu 2004).

Some recent analyses of the growing database on stellar rotation have also led to puzzling and contradictory conclusions with respect to disk locking and rotational evolution. Rebull et al. (2002) find that solar-like stars do not appear to conserve angular momentum as they contract during their first 3 Myr of existence, suggesting the importance of disk regulation. Yet, they also do not find the near-infrared excesses expected of disks for most stars. A similar puzzle is raised by Makidon et al. (2004), who find that while the mean size of stars (of similar mass) in the extremely young cluster NGC 2264 is smaller than in the Orion Nebula cluster (ONC), the period distribution in NGC 2264 is
indistinguishable from that in Orion. This, again, suggests that most stars contract without spinning up. Again, however, they cannot relate this to any observational evidence for the presence of disks around such a large fraction of the stars or any correlation between presence of a disk and rotation. Finally, in this vein, Rebull et al. (2004) find that “a significant fraction of all pre–main-sequence (PMS) stars may evolve at nearly constant angular velocity during the first ~3–5 Myr after they begin their evolution down the convective tracks.” In order to explain the rapid rotators (also known as “ultrafast rotators”) in older clusters such as α Per and IC 2602, these authors also argue, however, that at least 30%–40% of the PMS stars in their sample cannot actually be regulated for times exceeding 1 Myr. To summarize this body of work, Rebull and collaborators interpret the data on rotation periods of extremely young clusters to indicate that the majority of stars must be regulated for up to 4 Myr during their PMS contraction phase.

A very different picture has recently been proposed by Lamm et al. (2004, 2005). Based on their more extensive observational study of rotation in NGC 2264, they conclude that while, indeed, the NGC 2264 stars are smaller than their counterparts in the ONC, they also generally spin faster. A detailed description and account of the differences between the Makidon et al. (2004) and Lamm et al. (2005) results are given in the latter paper and need not be repeated here, but some aspects of the distinctively different interpretations are revisited in what follows.

The purpose of this paper is to reanalyze the existing data on stellar rotation in extremely young clusters in an attempt to clarify exactly what the observations say and do not say about the evolution of rotation of solar-like stars from the PMS to the MS. This new look is warranted, we believe, because there are finally enough rotation periods for stars in the relevant mass range at sufficiently different ages to allow a statistically valid comparison that employs the full distribution of rotation periods, not just a median or range. In §2 we describe and justify our approach to the subject, which is as empirical and as model independent as possible.

In §3 we present the distributions of the relevant quantities, rotation periods, stellar radii, and specific angular momentum at three different epochs (nominally 1, 2, and 50 Myr). In §4 we discuss the results in terms of other current work, and in §5 we summarize our conclusions.

2. ANALYSIS

2.1. Basic Issues and Assumptions

The magnitude of the angular momentum vector of an object rotating about a spin axis is

\[ J = I \omega, \]

where \( I \) is the moment of inertia and \( \omega \) is related to the rotation period \( (P) \) by

\[ \omega = \frac{2\pi}{P}. \]

The moment of inertia may be expressed in terms of the “radius of gyration,” \( R \), for a spherical body of radius \( R \). The radius of gyration is the distance from the spin axis that one would have to place a point mass, equal to the object’s mass \((M)\), to obtain the equivalent moment of inertia. Hence,

\[ I = M(kR)^2. \]

The value of \( k \) depends on the mass distribution within the object, as well as its shape. Since rotating stars become more and more distended at low latitudes with increasing spin rate, \( k \) is a function of \( P \). Combining the equations above, one can write that the specific angular momentum, \( j \), is given by

\[ j = \frac{J}{M} = 2\pi k^2 \frac{R^2}{P}. \]

It is the evolution of the specific angular momentum with time that we seek to constrain, and it is simply related to only three variable quantities, rotation period and radius, both of which may, at least in principle, be determined from the observations and radius of gyration, which can be approximated with good accuracy from a theory of rotating polytropes (Chandrasekhar 1935; James 1964).

The particular approach to studying the evolution of \( j \) adopted here, which differs in important ways from other approaches in the literature, is motivated by certain characteristics of the observations, as well as by the fact that \( R \) evolves rapidly during PMS contraction. We note that rotation periods have a range of 20 or more, that their distribution is highly mass dependent, but that they can be measured to a high degree of accuracy (~1%). Radii, on the other hand, are poorly determined for any one star but expected to have a vanishingly small range for stars of the same mass and age and a relatively weak mass dependence. Radii of 1 Myr old solar-like stars are expected to evolve quickly and, for angular momentum conservation, \( P \propto R^2 \), so \( P \) may be very sensitive to the precise age of the star. Therefore, to best constrain the evolution of \( j \) with time, one wishes to have a large sample of stars within a narrow mass range and with very nearly the same ages. This will allow one to accurately define the broad period distribution characteristic of a particular time and to allow the large scatter in measured radii to be dealt with by averaging. Clearly, what one requires is populous clusters, where there are a good number of stars of appropriate mass whose coevality is reasonably guaranteed by their cluster membership. The use of clusters in this way is nothing new, of course: it is simply the classical method used to investigate all aspects of stellar evolution, which has been employed by astronomers for more than a century.

Another element of our approach is that we employ only rotation periods in establishing the angular momentum distribution, not \( v \sin i \) measurements. The reason is that rotation periods are known for a sufficient number of stars in the relevant mass range (0.4–1.2 \( M_\odot \)) and that there is no need to introduce the complications that arise from \( v \sin i \) studies. These include the inherent uncertainty due to the unknown inclination of the system and the problem of accuracy, especially for slowly rotating stars. The \( v \sin i \) measurements are very helpful at verifying rotation periods and resolving issues of harmonics or beat periods as discussed below. They are also useful for determining whether selection effects are present in the rotation properties determined from the periodically variable stars. They do not, however, contribute in an important way to the definition of the angular momentum distribution in a cluster when numerous direct and accurate measurements of \( \omega \) are available. Hence, we focus only on rotation periods. An implicit assumption in doing this is that the subset of cluster stars with known periods is unbiased with respect to rotation. In fact, this is unlikely to be true for all clusters, and we return to the issue in §4.

Finally, we mention a perhaps obvious point that we wish to make explicit. Our analysis initially assumes that there are not cluster-to-cluster differences of significance in the initial rotation
period distribution. In other words, we take all differences to be the result of evolution from a common starting distribution represented here by the ONC. No progress can be made empirically without this assumption since there is no theory or set of observations that can yet tell us how the $P$ distribution at 1 Myr is created or how much variance we should expect between clusters. Hence, we proceed with the assumption of a common starting $P$ distribution and, at the end of the analysis, consider whether any element of the results points to such cluster-to-cluster differences. Since every cluster must evolve under at least slightly different environmental conditions, it is not hard to imagine ways in which the initial angular momentum distributions could differ. Perhaps the surprising thing is that we ultimately find only weak evidence for some small differences in starting $P$ distributions.

2.2. Sample Selection: Clusters and Mass Range

The first step in our procedure is to identify clusters at a range of ages that have sufficient rotation results to be useful. There are two rich PMS clusters that are perfect for this work: the ONC and NGC 2264. Over 400 rotation periods are known in the ONC from the studies of Mandel & Herbst (1991), Attridge & Herbst (1992), Eaton et al. (1995), Choi & Herbst (1996), Stassun et al. (1999), Herbst et al. (2000), Carpenter et al. (2001), and Herbst et al. (2002). A similar number are known in NGC 2264 from work by Kearns et al. (1997), Kearns & Herbst (1998), Lamm et al. (2004), Makidon et al. (2004), and Lamm et al. (2005). Other small clusters such as IC 348 and NGC 1333 are not included because they add very few stars in the relevant mass range. Association and field stars are not included for the reasons discussed in the previous section. In particular, we emphasize the distinction that exists between the Orion association (Ori OB1) and the ONC. It has been known since the work of Blaauw (1964) and Warren & Hesser (1978) that the Orion star-forming region is a complex one with a variety of subassociations of different ages and one extremely populous, dense cluster (the ONC). The definition of the ONC that we employ is the one used by Hillenbrand (1997). In particular, it does not include the regions studied by Rebull (2001), which she calls the Orion "flanking fields," or the survey (apart from any ONC stars) of Carpenter et al. (2001), which covers a wide range of Orion association stars. This distinction between the ONC and the "Orion region" is central to our analysis and further sets it apart from the recent discussion by Makidon et al. (2004). For additional discussion of this critical point see Lamm et al. (2005).

The other three clusters that we employ are the Pleiades, α Per, and IC 2602. These were, again, selected because there are extensive photometric surveys that have found rotation periods for dozens of stars. Their ages are much greater than the ONC and NGC 2264 and usually quoted to be in the range 50–120 Myr. The solar-like stars in these clusters are on or close to the MS so the rapid phase of evolution of radius is completed, and we refer to them as the "MS clusters," to distinguish them from the ONC and NGC 2264. The period and angular momentum distributions of the MS clusters are much more similar to each other than any of them are to the ONC or to NGC 2264. As we show in what follows, the rather small differences in rotation properties that do exist among them can be understood in terms of the age range, wind losses, and perhaps selection effects. Since the primary focus of this paper is the much more dramatic angular momentum losses that accompany the evolution of some stars from PMS to MS, we combine the data on the MS clusters, correcting for the small differences probably caused by wind losses or selection effects, to create a substantial set of stars defining the rotation properties of recently arrived or soon to arrive MS stars. The authors express here their gratitude to J. Stauffer for maintaining the excellent database on young clusters assembled by the late Charles Prosser, on which the present results are based.

Our selection of the mass range to study is dictated by the availability of rotation periods within the MS clusters. Currently, there is good coverage available only for stars with effective temperatures between log $T_{\text{eff}}$ of about 3.80 and 3.55 corresponding to masses between about 1.2 and 0.4 $M_\odot$. It is increasingly difficult to get rotation periods for lower mass stars because they are faint and red, increasing the noise while reducing the signal as the spot-photosphere contrast decreases. In order that we can compare apples with apples, it is necessary to know what range of log $T_{\text{eff}}$ of PMS stars corresponds to this mass range on the MS. Unfortunately, there is no agreement yet among PMS modelers on precise evolutionary tracks; the situation is nicely exhibited in Figure 1 of Hillenbrand & White (2004). Fortunately, however, as one can see from that figure, regardless of the particular tracks followed, the models do at least agree on the range of log $T_{\text{eff}}$ values on the PMS that will map onto the range of MS log $T_{\text{eff}}$ values over which periodic stars are actually measured. This is approximately the range log $T_{\text{eff}} = 3.54–3.67$, corresponding to spectral classes of K4–M2 if one adopts the Cohen & Kuhi (1979) calibration (see below). Many stars with this $T_{\text{eff}}$ range in the ONC and NGC 2264 should end up as MS objects with masses between 0.4 and 1.2 $M_\odot$ regardless of the precise tracks followed. We are aware that on the high- and low-mass ends of the selected PMS star log $T_{\text{eff}}$ range some stars may end up on the MS outside of the considered mass range, but due to the weak dependence of $j$ on mass (see below), this will not influence our conclusions significantly.

Figure 1 shows the loci of stars selected for this study on an H-R diagram. The ONC and NGC 2264 are represented by mean relations based on the average radius employed (see below). Individual stars are plotted for the MS clusters. Overlaid on this are two sets of theoretical tracks that illustrate the range of results obtained by modelers. Tracks for a 1.0 and 0.5 $M_\odot$ star are shown from D’Antona & Mazzitelli (1997), and tracks for a 1.0 and 0.4 $M_\odot$ star from Palla & Stahler (1999). It is evident that, depending on whose results are adopted, one could predict an MS mass that differed by a factor of 2 for a star with given values of luminosity and effective temperature. The real problem is compounded, of course, by the difficulties of determining accurate values of luminosity and effective temperature for PMS stars (see below) and by the possibly important factors such as rotation and magnetic fields that are neglected in all of the models. For an evaluation of the potential importance of magnetic effects see D’Antona et al. (2000). However, it turns out that the models do at least agree on the point that the range of log $T_{\text{eff}}$ among PMS stars that will map onto the MS in the 0.4–1.2 $M_\odot$ range is the adopted range of 3.54–3.67, corresponding to a spectral class range of K4–M2.

Fortunately, neither the MS nor the PMS rotation data suggest that $j$ is a strong function of mass, so that if we have mismatched the mass ranges in the PMS and MS clusters to some degree, it should not have an important impact on the results. Herbst et al. (2001) reviewed the situation in the ONC and showed that, while rotation period, indeed, has a clear dependence on mass, $j$ varies little, if at all. In other words, while lower mass stars on the PMS do rotate significantly faster than their higher mass counterparts, they are also smaller by enough to leave $j$ nearly a constant across masses. Also, the differences among PMS stars do not become obvious until a mass lower than those considered here, namely,
0.25 $M_\odot$ on the D’Antona & Mazzitelli (1997) scale, is reached. We show in what follows that $j$ is also independent or nearly independent of mass for MS stars in the relevant mass range. This circumstance relieves some of the pressure to be certain that the range of PMS stars selected is precisely the one that will map onto the MS at 0.4–1.2 $M_\odot$. PMS stars of all masses, within the 0.08–2.0 $M_\odot$ range where rotation period data are available, have about the same range and distribution of $j$.

2.3. Rotation Periods

This work is based on the assumption that the photometric periods derived for G, K, and M stars in young clusters result from the rotation of a spotted surface and that the photometric period is, at least in most cases, an accurate reflection of the stellar surface rotation period. While there is little or no controversy on these points in the literature, it is, perhaps, worth briefly reviewing the evidence for this assertion. First, we note that photometric periods can be reliably and accurately determined. A 5 yr study of the young cluster IC 348 by Cohen et al. (2004) illustrates that while there are changes in the light-curve shapes from year to year and that in some seasons the light variations become incoherent, when a period is found it is always the same period to within the errors, which are typically 1%. Early reports of significant changes in periods for the same star (Bouvier et al. 1993) have not been confirmed nor have additional cases been reported. It is most likely a result of applying an inappropriate false alarm probability to the data that did not reflect the correlations that exist in the photometry (Herbst & Wittenmyer 1996; Rebull 2001).

The shapes of the light curves, the periods involved, the color behavior, the evolution of light-curve forms, the amplitudes, and the correlation of period with $v \sin i$ measurements all support the identification of the photometric period with the rotation period of the star and the source of the variations as spots (primarily cool, but sometimes hot) on the stellar photospheres. For the PMS stars, the photometric amplitudes indicate enormous spots covering substantial portions of the star’s surfaces and most likely situated at high latitudes. So far, there has been no definitive determination of any period change with time that would indicate spot migration and differential rotation as is seen, for example, on the Sun. The periods repeat to within their errors for all stars observed in multiple seasons and by multiple observers, except as noted already. This gives us confidence that the photometric period is a measurement of a fundamental stellar property, namely, the surface rotation rate. For the MS stars, the situation is similar except that the amplitudes of variation tend to be much smaller, only a few percent at most. Again, the fact that the periodicity is measuring the stellar rotation rate of stars in the MS clusters is affirmed by the excellent correlation of $P$ with $v \sin i$.

When different investigators, using different telescopes, observing procedures, wavelengths, period search algorithms, and false alarm indicators, have studied the same clusters, they have found very compatible results. For the ONC, a comparison between the periods determined by Stassun et al. (1999) and by Herbst et al. (2002) has been given in the latter work, and it shows agreement to within the errors for 85% of the 113 stars in common between the studies. Cases of disagreement are almost always the result of harmonics (i.e., sometimes a star can develop spots on opposite hemispheres so that the rotation period is actually twice the photometric period) or beat periods with the observing frequency of once per night. Comparison of data sets obtained at different epochs and with different observing frequencies helps eliminate these cases, as does comparison with spectroscopic $v \sin i$ measurements (Rhode et al. 2001). Very similar results were obtained in comparing periods in NGC 2264.
reported by Makidon et al. (2004) with those found by Lamm et al. (2005), even though the epochs of observation were years apart. Again, there were 113 stars in common between the studies, and agreement to within the errors was found for all but 15 of them, i.e., 87% agreement. See Lamm et al. (2005) for further discussion of this comparison.

One final comment about rotation periods is, perhaps, in order. It is good to keep in mind that we measure only the surface rotation rate of the stars. Throughout this analysis, when we speak of rotation rate, that is what we mean. There is no guarantee and, indeed, no current way of knowing how the rotation rate varies with depth in the star. It is plausible that PMS stars rotate nearly as rigid bodies because they are believed to be fully convective. Hence, mass and angular momentum may be efficiently mixed from the surface to the core. On the MS, stars in this mass range have radiative cores, so it is unlikely that they are similarly mixed. From the observational perspective, it is actually impossible to say, and one may consider limiting cases such as conservation in shells or solid-body rotation (Wolff et al. 2004). It is certainly true that the angular momentum loss implied by our data is substantial for some stars, and it would clearly be much easier for a star to lose such angular momentum from a relatively narrow surface shell than from its entire mass. So, we caution the reader once more that the only quantity that can be observed is data is substantial for some stars, and it would clearly be much easier for a star to lose such angular momentum from a relatively narrow surface shell than from its entire mass. So, we caution the reader once more that the only quantity that can be observed is.

1. a profile of rotation with depth is in any star other than the Sun is a shell, not an average.

2. the angular momentum per unit mass that applies to the surface reader once more that the only quantity that can be observed is.

One difference between our analysis and others is that we do not rely on individual stellar radii to establish the age of a particular star or determine whether it belongs to a cluster. There is a huge scatter in the radii of stars in the ONC that, if interpreted literally, would mean that star formation has been ongoing there for about 10 Myr. While some authors have interpreted the scatter to indicate precisely that (Palla & Stahler 1999), we subscribe to the view argued for by Hartmann (2003) that, in fact, star formation in the ONC was a rapid process and that the large majority of the stars seen projected on the cluster have a single, common age of ~1 Myr. Hence, we use membership in the cluster (which is based almost exclusively on location on the sky) as the primary source of identifying a set of stars of homogeneous age and regard the large scatter in calculated radii as simply indicative of the difficulties of measuring the quantity. We show in what follows that, by using an average radius versus spectral class relationship in place of individual radii, we get a somewhat tighter distribution in the ONC.

2.4. Radii

Besides rotation periods, which are known to an accuracy of ~1%, one requires only stellar radii to determine \( j \) for a spherical star. Unfortunately, radii are very poorly known for individual PMS stars. The reason is that they can currently only be calculated from the fundamental relationship \( L = 4\pi R^2 \sigma T^4_{\text{eff}} \), which requires that luminosity \( (L) \) and \( T_{\text{eff}} \) be determined. \( L \), in turn, depends on the apparent brightness (which fluctuates nightly and even hourly for essentially all PMS stars), an extinction correction (which depends not only on establishing the intrinsic color and color excess but also on an assumed reddening law that could be abnormal in star-forming regions such as Orion), a bolometric correction, and a distance. Fortunately, the radius does only depend on the square root of the luminosity and is, therefore, linearly dependent on the distance.

Perhaps the largest uncertainty in radius determination for PMS stars is the translation required between an observable quantity, spectral type (or class, actually), and the theoretical quantity \( T_{\text{eff}} \). It is instructive to recall that the relationship that is still most commonly employed in the mass range of interest here is the one proposed by Cohen & Kuhi (1979). While one might hope that the longevity of this relationship is due to the rigor with which it was assembled, the authors’ own comments dispel any such notion. They clearly regarded the relationship they constructed in the 1970s as uncertain and subject to revision. They furthermore commented on the fundamental difficulty of ever obtaining an accurate \( T_{\text{eff}} \) measurement for a PMS star, which may be summarized in this way. The atmosphere of a T Tauri star is highly magnetized and, as a result, heterogeneous in terms of temperature. If it were not, we would not be able to detect rotation periods photometrically. On the other hand, a basic assumption of the expression \( L = 4\pi R^2 \sigma T^4_{\text{eff}} \) that the radiation from the star is isotropic, so that it can be determined by sampling what is received in the tiny solid angle defined by the Earth at the distance of the star. Clearly, this is not true, so fundamentally there is a difficulty in ever calculating accurate values of \( T_{\text{eff}} \) for such heterogeneous atmospheres. Besides this fundamental problem, there are a large number of practical difficulties in computing \( L, T_{\text{eff}} \) and therefore \( R \), which do not need to be reviewed here. The interested reader is referred to Hartmann (2003) and Rebull et al. (2004) for further discussion. Following Hillenbrand (1997), we adopt the attitude that there is, unfortunately, little more that can be done to improve the situation over what Cohen & Kuhi (1979) did, and we simply adopt the radii calculated by them for the ONC stars and by Rebull et al. (2002) for the NGC 2264 stars, which is based on the Hillenbrand (1997) approach.

One difference between our analysis and others is that we do not rely on individual stellar radii to establish the age of a particular star or determine whether it belongs to a cluster. There is a huge scatter in the radii of stars in the ONC that, if interpreted literally, would mean that star formation has been ongoing there for about 10 Myr. While some authors have interpreted the scatter to indicate precisely that (Palla & Stahler 1999), we subscribe to the view argued for by Hartmann (2003) that, in fact, star formation in the ONC was a rapid process and that the large majority of the stars seen projected on the cluster have a single, common common age of ~1 Myr. Hence, we use membership in the cluster (which is based almost exclusively on location on the sky) as the primary source of identifying a set of stars of homogeneous age and regard the large scatter in calculated radii as simply indicative of the difficulties of measuring the quantity. We show in what follows that, by using an average radius versus spectral class relationship in place of individual radii, we get a somewhat tighter distribution in the ONC.
example), and there is a relatively large sample of nearby stars to use as calibrators. The exact procedure used to assign radii is not critical for this study since the breadth of the period distribution is very large compared to the range of radii. It is the period distribution that dominates the form of the angular momentum distribution, not the radii. The procedure adopted is to use the $B - V$ photometry in Prosser’s online catalog along with the calibrations of $\log T_{\text{eff}}$ and bolometric correction of Bessell et al. (1998) to move to the theoretical plane. Uniform reddening, a standard extinction law with a ratio of total to selective extinction of 3.1, and the distance moduli recommended by Prosser were also adopted. For the Pleiades, these values are mean reddening $E(B - V) = 0.04$ mag and distance of 127 pc. For the $\alpha$ Per cluster, the values were $E(B - V) = 0.1$ mag and a distance of 165 pc. For IC 2602, we adopted $E(B - V) = 0.04$ mag and a distance of 150 pc.

### 2.5. Binary Stars

One issue in this analysis is how to handle binary stars. The effect of binaries is easily seen on the MS cluster H-R diagrams because the single-star locus is so well defined and the errors in the photometry and calibrations are relatively small compared to the sometimes large effect that a binary companion can have. A binary sequence parallel to and above the MS is readily apparent because the single-star locus is so well defined and the errors in the photometry and calibrations are relatively small compared to those that occur during the MS phase, presumably due to wind losses and internal angular momentum transport. Representative estimates from the literature, based on Li depletion, are 50 Myr for IC 2602 (Randich et al. 2001) and 70–75 Myr for $\alpha$ Per (Basri & Martin 1999). It is generally agreed that the age order from low to high for the MS clusters is IC 2602, $\alpha$ Per, and the Pleiades.

### 2.6. Cluster Ages

The precise ages of the five clusters employed here are less important than their ratios. The adopted age scale is based on two fiducial points, 1 Myr for the ONC and 120 Myr for the Pleiades. The ONC age comes from the analysis of Hillenbrand (1997), who actually derives 0.8 Myr as her best estimate, but given the uncertainties in the models and transformations from observational to theoretical plane already discussed, this is consistent with 1 Myr. The Pleiades age is based on discussions in the literature by Stauffer et al. (1998) and Terndrup et al. (2000). The age of NGC 2264 follows from the ONC by comparing the luminosity of PMS stars of the same spectral class. Two recent studies agree that the cluster is about a factor of 2 older than the ONC, which places its age at about 2 Myr (Makidon et al. 2004; Lamm et al. 2005).

### 2.7. Radii of Gyration

Finally, we need to consider the radius of gyration, $kr$, since some stars in our sample spin fast enough that they must be significantly distorted from a spherical shape. As noted above, we only consider in this empirical study the surface rotation and, therefore, need only be concerned with the surface shape. Fortunately, the problem of equilibrium shapes of rotating stars has been solved analytically for polytropes by Chandrasekhar (1935). Here we approximate PMS stars as polytropes of index $n = 1.5$ and zero-age main-sequence (ZAMS) stars as polytropes of index $n = 3.0$. Following Chandrasekhar (1935), we may then write that the surface of a rotating star is defined by

$$R(\theta) = R_o = a - bP_2(\theta),$$

where $\theta$ is the usual polar angle, $R_o$ is the radius of the nonrotating star, and $P_2(\theta)$ is the second-order Legendre polynomial. For a PMS star ($n = 1.5$) and ZAMS star ($n = 3$), respectively,

$$a = \begin{cases} 1.74225v + 1, & n = 1.5, \\ 1.99496v + 1, & n = 3, \end{cases}$$

and

$$b = \begin{cases} 3.86184v, & n = 1.5, \\ 27.8734v, & n = 3, \end{cases}$$

where

$$v = \frac{\omega^2}{2\pi G\rho_c}.$$
Fig. 3.—Effect of rotation on the surface shape and radius of gyration (plus sign) of an \(n = 1.5\) polytrope with a central density of \(0.8 \text{ g cm}^{-3}\), chosen to model an ONC star. Two rotation rates are compared with a nonrotating star. A 0.6 day rotation period corresponds to the maximum possible rotation rate (James 1964).

As an illustration of the effect of rotational distortion, we show in Figure 3 the surface shapes predicted by this theory for two rotation periods (1 and 0.6 days) of a PMS star. We have adopted a value of \(\rho_c = 0.8\) in cgs units, which is representative of an ONC star according to the models. The smaller period value is very close to the maximum rotation rate \((P = 0.602\) days\) allowed for such an object (James 1964). It is also quite close to the maximum observed rotation rate \((P = 0.66\) days\) for ONC stars in the mass range considered here. For our purposes, it is the effect of the distortion on the calculation of the radius of gyration that is relevant. Assuming that the surface shell is thin compared to the radius of the star and of uniform density, one can easily integrate over the surface shape to calculate a value of \(k\). For a perfect sphere, \(k = (2/3)^{1/2}\). In general,

\[
k^2 = \left(\frac{4}{3} a^4 + \frac{16}{15} a^3 b + \frac{8}{7} a^2 b^2 + \frac{16}{105} a b^3 + \frac{52}{1155} b^4\right) \times \left(2a^2 + \frac{2}{5}b^2\right)^{-1}.
\]

Values of \(k\) for the three shape solutions are shown by plus signs in Figure 3. The shapes for ZAMS stars \((n = 3)\) are essentially the same, but because of higher central densities, significant distortion occurs only at rotation periods significantly shorter than 1 day.

Clearly rotational distortion is not important for most of our sample, but it is important for the most rapidly rotating stars. As noted, the relevant quantity is \(k\), which enters as the second power in the calculation of \(j\). To assess and correct for the rotational flattening, we have calculated \(k\) by the above formulation. Stellar models suggest \(\rho_c = 0.8\) for ONC stars, \(\rho_c = 1.5\) for NGC 2264 stars, and \(\rho_c = 80\) for ZAMS stars, which we adopt. Figure 4 shows how \(k\) varies with rotation period for models representing the ONC, NGC 2264, and the ZAMS stars, respectively. In each case, we have terminated the calculations at the location represented by the most rapidly rotating star actually observed in each of these clusters (within the mass range considered here). It is, perhaps, worth noting that in each case the shortest period star observed is well matched to the shortest period expected based on James (1964) calculations. Stars with shorter periods would have surface gravities at their equators that were less than zero. Clearly, neglecting rotational flattening in calculating \(j\) for the most rapidly rotating stars in our sample would lead to errors as large as \(30\%\). It is also clear, however, that for most stars the correction for flattening will be trivially small.

3. RESULTS

Employing the principles outlined in § 2, we now derive and discuss the distributions of \(P\), \(R\), and \(j\) for solar-like stars at three different characteristic ages: 1, 2, and 50 Myr. This is followed by a discussion, in § 4, of the evolution of \(j\).

3.1. Rotation Period Distributions

It is clear from the discussion above that the form of the \(j\) distribution for young clusters is primarily determined by the distribution of rotation periods because radii vary little in comparison to periods. It is, therefore, instructive to look first at the rotation period distributions of the clusters. Figure 5 shows the rotation period distribution for the 150 stars within the selected \(T_{\text{eff}}\) range in the ONC, which corresponds to mass between 0.4 and 1.2 \(M_\odot\). It has the familiar bimodal character first reported by Attridge & Herbst (1992), with peaks near 2 and 8 days. We emphasize that this figure contains every published rotation period for members of the ONC, regardless of source, but does not include periods for stars in Orion outside of the ONC. The reason, again, is that in our view such stars are not likely to be of the same age, and therefore radius, as the ONC members. “Orion” stars that are members of the flanking fields or the greater Orion association are likely to be a heterogeneous mix of stars of different age, mostly older than the ONC, in our view. Therefore, one would not expect them to exhibit as much structure (i.e., bimodality) in their period distributions, and, in general, one would expect more rapid rotators. This is, in fact, exactly what is reported by Rebull (2001) for the “flanking fields” and by Carpenter et al. (2001) for the greater Orion association.
The rotation period distribution for the 173 stars in NGC 2264 that lie within the specified color range appropriate to a spectral class range of K4–M2 is shown in Figure 6. The 142 stars with periods detected by Lamm et al. (2005) and appropriate values of color were supplemented by 31 stars of quality 1 from Makidon et al. (2004). Reasons for not using quality 2 stars from Makidon et al. (2004) are given by Lamm et al. (2005). A double-sided Kolmogorov-Smirnov (K-S) test shows that there is no significant difference between the distribution shown in Figure 6 and the period distribution for stars in the same color range chosen only from the sample of Makidon et al. (2004). It is also quite clear that the period distributions of the ONC and NGC 2264 stars in this mass range are not drawn from the same parent population. A K-S test indicates that they are different at the 99.7% confidence limit. While this contradicts the statement in Makidon et al. (2004) that there is no significant difference between “Orion” and NGC 2264, it should be kept in mind that by “Orion” those authors are generally not referring to the ONC but to the greater Orion association. In fact, as Figure 7 of Makidon et al. (2004) shows, their period distribution in NGC 2264 does differ from that in the ONC at the 99% confidence limit when a K-S test is applied. Other features of this distribution have been discussed by Lamm et al. (2005) and include its bimodality, with peaks near 1 and 4 days and the extended tail of slowly rotating stars.

Rotation periods for the three MS clusters are shown in Figure 7. Combined, there are 148 stars, enough to reasonably define the distribution in a statistical sense. However, it is not entirely valid to simply combine these three clusters because they do not have the same period distributions, as is evident from the figures and confirmed by a K-S test. Clearly, α Per has a higher proportion of rapid rotators than do the other two clusters. From a strictly empirical point of view, this could be caused by an age difference between the clusters and a general slowdown of rotation with age expected from wind losses, by mass-dependent rotation properties and a difference in the mass distributions between the clusters, or by some other selection effect. It is easier to explore these issues in the j plane than in the P plane, so we postpone the task of combining the data until after a discussion of radii. The combined period distribution shown in the bottom right panel of Figure 7 is not strictly valid given the real differences between the clusters. However, because these differences are relatively small and a main feature of the combined plot, namely, its evident bimodal nature, is worth noting, we show the distribution as a didactic exercise.

To summarize, even without correcting for the effects of radius, a few things are clear about the rotation distributions of PMS and recently arrived MS stars of solar-like mass. First, there are indications in Figure 7 that the rotational bimodality observed for the ONC and NGC 2264 continues into the early MS phase; this becomes more evident when the j distributions are discussed below. Second, the period distributions are significantly different from one another at each age step. And third, the trend is for most stars to spin faster as they age, exactly as one would expect if angular momentum conservation were involved, at least to some degree. To assess things more physically and quantitatively, we need to examine j rather than P. This, in turn, requires that we take account of the stellar radii, a task to which we now turn.

3.2. Stellar Radii

Figure 8 shows the distribution of radii as a function of log $T_{\text{eff}}$ for ONC stars in our periodic sample. As expected, there is a wide scatter but no clearly evident trend with temperature. All data were taken directly from Hillenbrand (1997). Taken at face value, the wide range of radii would indicate stellar ages that range from about 0.1 to 10 Myr (Palla & Stahler 1999). As noted previously, our position is that this scatter is dominated by errors and that the actual age (and therefore radius) spread in the ONC is probably quite small. Since there is no clear trend of $R$ with $T_{\text{eff}}$ visible in the data, we adopt the median radius of $R = 2.09 \, R_\odot$ for all stars. This is a more robust value than the mean ($2.3 \pm 0.1$) because of the outliers at large radius. We show below that adopting a single value of $R = 2.09 \, R_\odot$, as opposed to individual radii, has no effect on the calculated j distribution other than to tighten it. Since rotation periods are very accurately determined and have a large range, while radii are evidently poorly determined but expected to have a very small range, we argue that this is the most appropriate procedure if the intention is to obtain the best estimate of the j distribution of a cluster population.

Fig. 5.— Rotation periods for stars in the ONC with spectral types (K4–M2) appropriate to the log $T_{\text{eff}}$ range of 3.54–3.67. Periods are based on the work of Stassun et al. (1999) and Herbst et al. (2002) as summarized in the latter paper. Effective temperatures are assessed by spectral type, as reported by Hillenbrand (1997), and employ the calibration of Cohen & Kuhi (1979). One star, with a period of 35 days, lies outside the boundaries of this figure.

Fig. 6.— Rotation periods for stars in NGC 2264 with $R - 1$ values appropriate to the log $T_{\text{eff}}$ range of 3.54–3.67. The periods come from Lamm et al. (2004) for 142 stars and Makidon et al. (2004) for 31 additional stars that were not detected as periodic by Lamm et al. (2004). Only quality 1 stars from Makidon et al. (2004) were used. This distribution differs from the one shown in Fig. 5 for Orion at the 99.7% confidence limit according to a K-S test.
In Figure 9 we show the radii of NGC 2264 stars in our sample. Only 60 of the 173 periodic stars have radius estimates because spectral types are not available for the rest. Radii are based on the data and procedures of Rebull et al. (2002), which are identical to what Hillenbrand (1997) has employed in the ONC. There should be no systematic errors introduced by this procedure, therefore. Again, we adopt the median radius of $1.70 R_\odot$ rather than the mean ($1.81 \pm 0.03$) to minimize the effect of outliers at large radius. It is clear that, within the adopted range, stars of the same spectral class in the ONC are generally larger than those in NGC 2264 by about 25%. During the Hayashi phase $R$ depends on age ($t$) as $R \propto t^{-1/3}$, so this implies that the ONC is about one-half the age of NGC 2264. A fiducial age of 1 Myr for the ONC implies an age of 2 Myr for NGC 2264. The fact that NGC 2264 is somewhat older than the ONC, based on the observation that stars of the same effective temperature are somewhat less luminous, is now well documented in the literature (Makidon et al. 2004;...
Lamm et al. 2004). As in the ONC, we find no evidence for a dependence of \( R \) on \( \log T_{\text{eff}} \) over the limited range of interest in this study.

Radii for stars in the MS clusters can be determined much more accurately than for PMS stars, as discussed above. Hence, we use individual values of radius in calculating \( j \) for these stars. The distribution of radii for each cluster is shown in Figure 10. In general, the stars describe a very tight (main) sequence with a parallel binary sequence above it. There are only two widely discrepant points. One, in the Pleiades, is the star HII 1280, a K7 star with one of the shortest rotation periods measured (7.25 hr). Its radius is well below the MS because its measured color is too blue for its brightness. The cause of this discrepancy is unknown but could be related to its extreme rotation. The one discrepant star in IC 2602 is B1 34, with a radius clearly too large for its effective temperature. It is interesting that it has one of the longest rotation periods measured in the cluster (7.25 hr) and one obviously discrepant star in IC 2602 (B1 34, which has one of the longest rotation periods in the cluster).

A visual summary of this section is given in the right panel of Figure 1. The solid lines indicate the median radii for the PMS clusters, and individual radii are plotted for the MS cluster members. It is interesting to note that although the age difference between the ONC and NGC 2264 is quite small compared to the age difference between either cluster and the MS stars, the radius difference is relatively more substantial. In other words, the rapid difference between either cluster and the MS stars, the radius difference between the ONC and NGC 2264 is quite small compared to the age difference.

3.3. \( j \) Distributions

The calculation of \( j \) follows directly from \( P \) and \( R \). For convenience we normalize the results to \( j \) for the Sun (\( j_0 \)), which is based on an adopted solar radius of \( 6.96 \times 10^{10} \) cm and a mean surface rotation period of 25 days. For a spherical shell, \( k \) has the value \((2/3)^{0.5} \) so \( j_0 = 9.4 \times 10^{15} \text{ cm}^2 \text{ s}^{-1} \). To begin, we computed \( j \) for the ONC stars in two ways, using the individual radii and using the median radius of 2.09 \( R_\odot \). A comparison of the resulting \( j \) distributions is shown in Figure 11. As expected, there is no systematic difference between these, but the distribution based on the median radius is tighter. As argued above, we believe that the large scatter in the radii in the PMS clusters is primarily due to errors in their determination, not to real variation, so we are not surprised that the distribution based on individual radii is broader. It simply reflects an additional source of scatter, in our view. In what follows we use only the distribution based on the mean radii, for both the ONC and NGC 2264.

Herbst et al. (2001) showed that although the \( P \) distribution is a function of mass in the ONC, the \( j \) distribution is nearly independent of mass over the range 0.1–1.5 \( M_\odot \). The more rapid rotation characteristic of lower mass stars (outside the mass range considered here) is compensated for by their smaller radii. For the more limited mass range considered here it is not surprising, therefore, to find no evidence for a dependence of \( j \) on \( T_{\text{eff}} \), as illustrated in Figure 12. Similarly, there is no evidence for...
a dependence of $j$ on effective temperature in NGC 2264, although only about $\frac{1}{3}$ of the sample has known spectral type.

Figure 13 shows the values of $j$ calculated for stars in the MS clusters, again as a function of $T_{\text{eff}}$. Overall it is clear that there is a wide distribution of $j$ at all temperatures (masses) and that little or no trend of $j$ with mass is apparent. There are small differences between the clusters that may or may not be significant, as discussed in the text. In particular, $\alpha$ Per has a set of eight low-mass stars that are all rapidly rotating and no slow rotators of comparable mass. It also has a greater proportion of rapid rotators at all masses than the Pleiades.

It is evident from the H-R diagram (Fig. 1) that these are also not fully contracted to the MS. There is no corresponding set of more slowly rotating stars. It is hard to say whether this is a real, significant difference given the small number of stars involved.

A K-S test does show that, like the rotation period distribution, the $j$ distribution of $\alpha$ Per is significantly different from the Pleiades (and IC 2602). This is shown clearly in Figure 14, where frequency distributions for all three clusters are displayed. The Pleiades distribution differs from $\alpha$ Per at the 99% confidence level, containing more low-$j$ stars. IC 2602 is intermediate in its properties (and contains less stars), differing from each of the other clusters at only the 1–2 $\sigma$ level. The combined $j$ distribution is shown in the bottom right panel for illustrative purposes only. It is clearly bimodal, reflecting the bimodal period distributions of both the Pleiades and $\alpha$ Per.

From a purely empirical view it is not entirely appropriate to combine the $j$ distributions of the three MS clusters since there is evidence that they were not drawn from the same parent population. One interpretation is that the ages of the three clusters are sufficiently different that the action of normal stellar winds over the time interval between them is sufficient to have measurably slowed the Pleiades stars with respect to the $\alpha$ Per and IC 2602 stars. Another is that there are selection effects that are biasing the distributions. A third is that the clusters simply did not begin their lives with the same initial $j$ distributions. Unfortunately, there is no way of knowing for sure which of these possible effects is, indeed, important, but fortunately the differences among the MS clusters are small compared to the differences between the PMS and MS clusters.

We proceed empirically by asking whether there is a simple transformation of the MS cluster $j$ distributions that leads to...
statistically acceptable agreement among them. The Pleiades distribution is taken as the fiducial point, and we seek to transform its $j$ distribution to each of the others by applying a constant scale factor (the "$j$-factor"). The results are shown in Figure 15. It is clear from this exercise that if the Pleiades stars with known rotation periods all had about 1.75 times more angular momentum per unit mass than their counterparts in the IC 2602 cluster, the distributions would be statistically indistinguishable from one another. The corresponding factor for best transforming the Pleiades $j$ distribution to the IC 2602 distribution is about 1.35. In both cases, we find that the demonstrably older cluster, the Pleiades, has lower $j$-values than the younger MS clusters, in agreement with the common supposition that wind losses are draining some angular momentum from these young stars on a timescale of tens of millions of years.

We can assess the situation a bit more quantitatively by assuming that a Skumanich-type (Skumanich 1972) wind-loss relation ($\omega \propto t^{-1/2}$) applies to all stars. In that case, a loss by a factor of 1.75 in $j$ would imply an aging by a factor of 3, indicating a current age for $\alpha$ Per of 40 Myr if the Pleiades is 120 Myr old. Similarly, we would compute an age of 65 Myr for IC 2602 by this process. These are reasonably consistent with the quoted ages of the clusters given above although the order by age is not correct, and there is little doubt that $\alpha$ Per is older than IC 2602. We attribute this small inconsistency in the rotation properties of these clusters to the relatively small number of stars with known rotation properties and to possible selection effects in the data discussed below. At the $2 \sigma$ level (K-S factor $>0.1$) we find agreement in the $j$ distributions of $\alpha$ Per and the Pleiades for $j$-factors of $1.3-2.2$, corresponding to “Skumanich” ages of 25–70 Myr for that cluster. For IC 2602, the corresponding numbers are a $j$-factor of 0.9–1.5 and an inferred age of 50–150 Myr. Since this simple empirical scaling process has neglected complications such as saturated winds that are probably of importance for the more rapid rotators in our sample, it is actually remarkable that we get as good an agreement as we do with ages inferred by more accurate methods.

In Figure 16 we show the transformed $j$ distributions of the MS clusters corrected for angular momentum (presumably wind) losses and adjusted to a common age, namely, the age of IC 2602 (~50 Myr). These $j$ distributions are now sufficiently similar to have been drawn from the same parent population and can be combined. The bimodal nature of the combined $j$ distribution continues to be quite clear. We take this combination of the adjusted rotation distributions to be representative of stars in this mass range at about the time they arrive on the MS. Note that we have not corrected the distribution for the presence of binary stars. For illustrative purposes only, we show the effect of such a correction in Figure 17. There would be a minor shift to lower values of $j$. The reason for not including this correction when discussing the evolution of $j$ distributions is that it is impossible to make it for the PMS clusters. There, any binary sequence is lost in the observational scatter. If a correction were made to the MS clusters but not to the PMS clusters, we would clearly not be making a valid comparison, so we adopt the procedure of ignoring the (relatively small) correction in both cases.

4. THE EVOLUTION OF ROTATIONAL DISTRIBUTIONS OF SOLAR-LIKE STARS

We have formed three distributions of $j$ representative of a cluster population of solar-like stars at three different times,
Fig. 16.—The $j$ distributions of the three clusters with MS stars shifted to a common age of ~50 Myr (i.e., the age of IC 2602) using the $j$-factors based on Fig. 15. No correction is made for binary stars because binaries cannot be identified or corrected for in the clusters containing PMS stars. They have been treated as if they were single stars.

Fig. 17.—The $j$ distributions of the three older clusters corrected for the presence of binary stars. This was done by fitting a line to the single star sequence of radius vs. log $T_{\text{eff}}$ and applying that radius to all stars. The dashed lines show the $j$ distributions for the binary-corrected sample, while the solid lines show the observed $j$ distributions without a binary correction. As may be seen, the presence of binaries makes a small difference. We have not attempted to use this correction in the analysis because there is no way to correct the PMS stars for this effect. We assume, therefore, that the binaries have a roughly equal (small) effect on the $j$ distributions at all ages and may safely be ignored.
nominally 1 (ONC), 2 (NGC 2264), and 50 Myr (combined MS clusters). Figure 18 compares these distributions in pairs and then with all three shown for clarity. K-S tests reveal that each distribution is significantly different from the others at more than a 99% significance level (see Table 1). While the distributions are shown as fractions of the total for easy comparison with one another, it should be recalled that there are 150–175 stars in each so they are reasonably well defined.

A new and, we believe, significant feature of angular momentum evolution emerges from this comparison. As seen clearly in Figure 18, the high-$j$ sides of the distributions are rather similar in all three data sets, while the low-$j$ sides evolve dramatically as the population ages. In other words, rapidly rotating PMS stars appear to evolve with very little additional loss of angular momentum to the MS, while slowly rotating stars in the ONC must lose substantial additional amounts as they progress to the MS. Although quite evident in the figure, we can quantify the result by dividing the sample into a rapidly rotating half and a slowly rotating half. Applying the K-S test to each half independently yields the significance values given in Table 1. The rapid rotator side of the distribution shows only small, if any, indications for evolution with time, while the slow rotator side evolves dramatically.

In our opinion, this feature of the evolution of $j$ distributions provides dramatic new support for the disk-locking theory of angular momentum evolution, as we now discuss. An overview of the argument is as follows. According to the disk-locking theory, the slower rotators in the ONC should be those still interacting with their disks, while the rapid rotators should have lost most or all such interaction at an earlier stage. Assuming that once the influence of a disk on a star’s rotation has waned it does not tend to reappear, one would expect rapid rotators at the ONC age to show only small angular momentum losses as they age further. The high-$j$ side of the distribution should not evolve much with time, therefore, precisely as is observed. If large angular momentum losses are to occur in any stars, it should be the slow rotators, since these are the ones that still have disks. Again, this is precisely what Figure 18 shows.

A second aspect of rotational evolution revealed in Figure 18 that provides additional new support for the disk-locking theory is the clear difference in $j$ distributions seen between ONC and NGC 2264 ages. By its nature, the disk-locking theory predicts that the most rapid evolution of $j$ (recall that $j$ is the angular momentum per unit mass at the surface of the star) will occur during the most rapid contraction phases. The amount of $j$ loss should scale with radius of the star, not time elapsed. Since evolution of radius is most rapid during the initial stages of PMS evolution (see Fig. 1), the disk-locking theory would predict a detectable evolution of the $j$ distributions even on the very short timescale (∼1 Myr) represented by the difference in ages between the ONC and NGC 2264. Other theories of angular momentum loss (e.g., by winds) would predict an evolution that would be more steady with time and would not lead to detectable differences among PMS clusters with such similar ages. We now explore these arguments in more detail, considering first the evolution of the rapid rotators.

4.1. Rapid Rotators: Near Conservation of Angular Momentum

Conservation of angular momentum on Figure 18 would be indicated, of course, by no change in the distributions with time.
The good agreement evident among all distributions on the rapid side, therefore, means that rapid rotators are evolving in a manner essentially indistinguishable from conservation of angular momentum: wind losses or other angular momentum losses are small or negligible. This means that there is no need to posit any additional physics other than PMS contraction and conservation of angular momentum to account for the “ultrafast rotators” in young clusters, i.e., the stars populating the high-j side of the distribution in the MS clusters. To quantify this result, we note from Table 1 the K-S probability that the high-j sides were drawn from the same parent populations. The K-S probability is less than $10^{-7}$ from the K-S analysis (or just from the appearance of the distributions in Fig. 18) that it is the slowly rotating stars that are evolving in j so dramatically. Clearly, slow rotators are losing substantial amounts of angular momentum as they age. To quantify the significance level of the effect, we compare the low-j halves of the distributions using the K-S test (see Table 1). Even in the case of the comparison between NGC 2264 and the ONC, where the age difference is only ~1 Myr, there is a highly significant difference in their j distributions on the low-j side. A K-S test indicates that there is only a $9 \times 10^{-7}$ chance that the distributions have the same parent populations. The K-S probability is less than $10^{-16}$ when the ONC or NGC 2264 is compared to the MS clusters. It is clear from Figure 18 that slowly rotating stars lose substantial amounts of angular momentum during their contraction to the MS.

It may be seen that no overall scaling of the distributions can transform one into another. The reason, of course, is that the j distributions evolve with time not only by shifting their medians toward lower j but also by broadening dramatically. There is simply no way to understand the evolution of these distributions with time without considering the slow rotators separately from the rapid rotators. This, of course, is what one would expect from a disk-locking theory of angular momentum evolution. Slow rotators should be the ones still locked to their disks and, therefore, the ones that should continue to lose additional amounts of angular momentum. This is precisely what our data suggest is happening. Lacking a quantitative theory of disk locking, it is hard to make a more compelling comparison of the data and theory. However, it is possible to estimate empirically what sort of evolution is required under the disk-locking paradigm to account for the observations. That is done in the next section.

### 4.3. A Disk-locking Model for the Data

A clear indication from this study is that stars with $j > 10 j_\odot$ in the ONC must evolve with very little angular momentum loss during the next 50 Myr if the j distribution is to transform into that seen for young MS clusters. At the same time, a significant fraction of stars with $j \lesssim 10 j_\odot$ must lose substantial amounts of angular momentum, typically a factor of 3, in 50 Myr to populate the low-j side of the distribution exhibited by the MS clusters. This angular momentum loss must, furthermore, be initially rapid to account for the significant evolution toward lower j already seen in NGC 2264. Clearly, these aspects of the evolution point to some kind of locking as the physical mechanism, and the correlations suggest disk locking. Lacking a predictive theory of disk locking, it is difficult to go much further with a quantitative comparison, especially since the locking is likely to be imperfect, as has been discussed by Lamm et al. (2005). We can, however, make some qualitative comparisons between the theory and data, a task to which we now turn.

To begin, we inquire whether there is a simple transformation of the data between the ONC, NGC 2264, and the MS that can account for the evolution of the j distribution. From an empirical viewpoint, it is clear that this transformation must be applied only to the low angular momentum side; otherwise, the reasonably good fit that already exists on the high-j side (Table 1)
would be lost. To explore the simplest possible transformation that might work, we divided the samples into two sets, a low angular momentum group comprising a fraction \( f \) of the whole sample and a high angular momentum group comprising the rest of the sample. The low angular momentum group was then multiplied by a \( j \)-factor (obviously less than 1) and compared them with the MS sample using the K-S test. This simulates continued loss of angular momentum for the already low-\( j \) stars, as expected in the case of disk locking. Exploring \((f, j\)-factor\)-space in this manner led to the discovery that there is a fairly narrow range in these parameters, which does, in fact, allow one to match distributions across time in a statistically acceptable way. Our best fits are listed in Table 2 and shown in Figure 19. The parameters adopted are given in the figures. For the bottom right panel we combined the PMS data from the ONC and NGC 2264 by first transforming the ONC to NGC 2264 age, using the results shown in the top left panel, and then combining these two clusters. Obviously there is not much difference between doing this and transforming the individual clusters.

Our conclusion from this exercise is that it is possible to adequately model, at least in a statistical sense, the evolution of all of the \( j \) distributions in terms of a simple scaling of a fraction of the already slowly rotating population. Quantitatively, the fraction affected is 40%–50% and the scale factors required are given in Table 2 and are quite substantial. How consistent are these numbers, which are derived entirely from an empirical assessment of the data, with the predictions of disk-locking theory?

As noted above, there is no quantitative disk-locking theory with which to compare due to the theoretical difficulties mentioned above, so we take the simple, first-order assumption that rotation period remains constant during the disk-locked phase. Assuming a starting radius of \( 2.09 \, R_\odot \), appropriate to the ONC, one can estimate the radius of the stars at the time the disk locking must cease, again assuming that the rotation period remains fixed. These radii are given in Table 2. In the case of the comparison between the ONC and NGC 2264 it is not necessary, of course, that the disk-locked phase has, indeed, ceased. The radius given is simply the radius to which the NGC 2264 stars must have contracted with constant period to match the \( j \) distributions.

Looking first at NGC 2264, we see that the derived value of \( R = 1.8 \, R_\odot \), based simply on the rotational period distributions and the simplest possible assumptions consistent with a disk-locking interpretation, is remarkably close to the median value adopted for the cluster \( R = 1.7 \, R_\odot \) from measurements of the luminosity and effective temperatures of the stars. We take this excellent agreement to be an indication that, to first order, the idea of disk locking (for 40%–50% of the stars) provides a good

| Cluster Comparison        | \( f \) | \( j \)-Factor | \( R \) (\( R_\odot \)) | DLT (Myr) |
|---------------------------|--------|---------------|------------------------|-----------|
| ONC and NGC 2264............ | 0.4    | 0.74          | 1.80                   | ...       |
| ONC and MS.................. | 0.45   | 0.32          | 1.18                   | 5.5       |
| NGC 2264 and MS............. | 0.5    | 0.44          | 1.19                   | 5.4       |
| (ONC + 2264) and MS......... | 0.5    | 0.42          | 1.17                   | 5.8       |

- \( f \) The fraction of the whole sample that must be disk locked.
- \( j \)-Factor The factor by which the \( j \)-values of that fraction are scaled.
- \( R \) The radius to which the stars have contracted while remaining disk locked, assuming a constant period and starting radius of \( 2.09 \, R_\odot \).
- DLT The (disk locking) time required for contraction to the radius in the fourth column, assuming that \( R \propto t^{-1/2} \), as is appropriate to the Hayashi-phase contraction.

**Fig. 19.**—The \( j \) distributions of the ONC and NGC 2264 (corrected for 10% wind losses) have been adjusted (on the low angular momentum side) by the \( j \)-factors indicated on each panel, which is applied to the fraction \( f \) having the lowest \( j \)-values. These are the simplest transformations that give adequate fits to the data. Clearly, one requires shifts by large factors applied to 40%–50% of the stars on the slow rotating side of the distributions.
way of describing the evolution of rotation from ONC age to NGC 2264 age. Going further, we can ask how this might continue to the MS clusters. Here we find that the typical radius at which disk locking must end is about 1.2 \( R_\odot \), again under the assumption of a constant rotation period. If disk locking continued beyond that point, the stars in the MS clusters would rotate too slowly to have evolved in this way from the PMS distributions. It may be seen in the right panel of Figure 1 that the “release point” of \( \sim 1.2 \ R_\odot \) happens to correspond roughly with the end of the Hayashi phase for the more massive stars in our sample. The time to contract to such a radius can, therefore, be estimated using the fact that \( R \propto t^{-1/3} \) during the Hayashi phase, and this leads to an estimate for disk-locking times of about 5–6 Myr (see Table 1). To summarize, a quantitative evaluation of the evolution of \( j \) with time indicates that simple transformations of the data consistent with the first-order ideas of disk-locking theory provide a wholly adequate description of the data. The timescale required for the process of, at most, 5–6 Myr is in good agreement with estimates of disk lifetimes based on near-infrared studies (Haisch et al. 2001).

### 4.4. Selection Bias and Caveats

It has been assumed that, in all clusters, the set of stars with detected rotation periods is a representative sample in terms of their rotation properties of the cluster as a whole. There are two ways in which this assumption could be (and probably is, to some extent) wrong. First, in the PMS clusters there is a likely bias against finding rotation periods for slow rotators for the following reason. Slow rotators are statistically more likely to be actively accreting (i.e., classical) TTSs. The irregular variability that accompanies accretion makes it more difficult to detect rotational signals in the light curves of such objects. Cohen et al. (2004), for example, have recently discussed this issue in some detail. Hence, the \( j \) distributions of the ONC (and NGC 2264) may have a bias against slow rotators. In the ONC, where this comparison has been made, we have found no significant difference between the \( v \sin i \) distributions of stars with and without rotation periods discovered by spot modulation (Rhode et al. 2001). However, this study is not entirely definitive because of the limited sample. A more detailed analysis by Herbst et al. (2002) based on a number of considerations concluded that there was a likely bias against slow rotators in the ONC sample but probably only at about the 15% level.

In the MS sample there is also a likely bias against slow rotators, but for a different reason. Clusters such as the Pleiades are so spread out on the sky that they must be photometrically monitored on an individual star basis, as opposed to including the entire cluster on a single or few CCD images. The selection of which stars to monitor for rotational variability may be biased if it is made with reference to known \( v \sin i \) measurements. J. Stauffer (2005, private communication) indicates that such a bias does indeed affect the Pleiades sample used here since the observers (primarily he and C. Prosser) preferentially selected known rapid rotators for study assuming that they would more likely yield measurable rotational periods with the least investment of observational time. The extent of the bias can be estimated by the fact that in the full sample of stars with known \( v \sin i \), 49 out of 102 (48%) have \( v \sin i < 10 \text{ km s}^{-1} \), while among the stars with known rotation periods, only 14 out of 44 (32%) fall in that category. This same bias probably does not affect the other MS clusters to as great an extent (J. Stauffer 2005, private communication).

Our conclusion is that both the PMS and MS samples are probably biased to some extent against slow rotators, but that the degree of biasing is probably only of the order of 15%. This is a relatively small effect that might act to increase somewhat the estimated fractions of disk-locked stars if we had a more representative rotational sample. In the future, it might be possible to evaluate this effect more definitively by increasing the numbers of stars with \( v \sin i \) measurements and to lessen the effect by obtaining more rotation periods for the slower rotators in the Pleiades. For now, it is hard to see how this bias could affect our principal results. Only in the case that we were missing a substantial number of very slow rotators in the ONC would much change in Figure 18. Since PMS monitoring programs often extend over several months, they would have no difficulty detecting very slow rotators if they existed, so apparently they do not. Therefore, just to populate the slowly rotating star bins of the current MS distribution requires a good deal of loss of angular momentum from a sizable fraction of stars. If there are even more slow rotators in these MS clusters than is represented by the distributions shown here, then disk locking must be even more common than our current analysis indicates.

Finally, we should explicitly address the underlying assumption of this analysis that the initial \( j \) distributions of the five clusters considered were enough alike that the differences we measure today reflect evolutionary effects and not initial conditions. The main reason for such an assumption is that no progress can be made without it. If some or all measured differences are assigned to initial conditions, then we can say nothing about evolution. If theory or observations are someday able to establish that the Pleiades had a much different \( j \) distribution when it was 1 Myr old than the ONC has today, our analysis and interpretation are obviously invalid. Given the current state of the field, we suspect that this will not happen for a long time, if ever. This paper shows that the current \( j \) distributions can be understood in terms of an evolutionary sequence from a common initial distribution represented by the ONC if one simply allows angular momentum to be conserved for about half the sample and disk locking to affect the other half for about 5 Myr. At present, we find no inconsistencies in the data that would seem to require that we abandon the simplifying assumption of a common initial \( j \) distribution among the clusters included in this analysis.

### 4.5. Comparison with Results Obtained by Other Authors

The question of angular momentum evolution of solar-like stars from PMS to MS has been addressed frequently over the past few years by a number of authors, as cited throughout this paper. To some extent there has been disagreement on the following issues: (1) the bimodal nature of the period (or \( j \)) distribution in the ONC, (2) the bimodal nature of the period (or \( j \)) distribution in NGC 2264, (3) whether there is evidence for spin-up of stars between the ONC and NGC 2264 ages, and (4) whether slowly rotating stars in the ONC or NGC 2264 actually have active accretion disks. See the review by Mathieu (2004) for a concise summary of much of the debate. On the other hand, there is substantial agreement on the main features of the rotational evolution of solar-like stars as summarized in the Abstract of this paper and in § 5, which follows.

Here we would like to address areas of disagreement in the light of the new results presented in this study and emphasize the agreement that exists on some major points. On the bimodal nature of the period distribution in the ONC, we think that it is fair to say that the issue is entirely resolved now to everyone’s satisfaction. The source of the controversy was that several studies of “Orion” did not find a clearly bimodal period distribution similar to what was first reported by Attridge & Herbst...
(1992) and Choi & Herbst (1996). Two of these studies (Rebull et al. 2001; Carpenter et al. 2001) were, in fact, not focused on the ONC but on the greater Orion association. The third (Stassun et al. 1999) did not cover the same mass range. As additional data have accumulated from a variety of sources, the original result has been strengthened. Figure 5 contains all of the currently available data on rotation periods in the ONC for stars in the relevant mass range. There is no question that the solar-like stars in the ONC have a bimodal period distribution.

In retrospect, it is not surprising in the least, but indeed expected, that the period distributions for other samples of stars that were limited neither by mass nor by position on the sky should show a different period distribution. We now know, for example, that less massive stars in the ONC spin faster (Herbst et al. 2001) so mixing masses within a period distribution tends to wipe out structure such as bimodality. This is probably the main reason why Stassun et al. (1999) did not find a bimodal period distribution. Their sample contained many more low-mass stars than the samples analyzed by Attridge & Herbst (1992) and Choi & Herbst (1996). In fact, when one limits the Stassun et al. (1999) sample to stars in the mass range considered here, it is distinctly bimodal. These points have been raised and expounded upon in several papers, including most recently by Herbst et al. (2002). Hopefully, it will now be clearly recognized that the bimodal nature of the ONC is not actually a controversial subject any longer.

It is also not controversial that the greater Orion association (the “flanking fields”) of Rebull (2001) and the survey of Carpenter et al. (2001) have a greater preponderance of rapidly rotating stars than the ONC and do not show a clearly bimodal distribution. Our interpretation of this fact is that these samples are likely to contain a good mixture of older stars associated with earlier star-forming episodes in the greater Orion association. Being older, these stars would have had more time to contract and spin up, just as about half of the NGC 2264 stars have spun up with respect to the ONC. In fact, we would argue that the average age of this more heterogeneous (than the ONC) population is probably close to NGC 2264’s average age of 2 Myr, since the rotation period distributions of what Makidon et al. (2004) call “Orion” and NGC 2264 are not significantly different according to them.

We also note that previous studies of the Orion region, as listed above, have found the ONC to be the youngest portion of the cluster and that estimating ages by radii is difficult. Furthermore, when Lamm et al. (2005) attempted to divide their NGC 2264 sample into a younger and older half by location on an H-R diagram, they found no significant difference in the rotation properties. Taken at face value, this would mean that older, more contracted stars do not spin faster than their younger counterparts, contradicting the results of this study. In fact, we believe, it is simply another indication of the fact that determining ages of PMS stars by determining their radii is fraught with difficulty.

That brings us to the conclusions of Rebull et al. (2004), who found no evidence for spin-up of stars due to contraction during the PMS phase but did conclude that 30%–40% of the stars on convective tracks in the relevant mass range could have been released by the time they are 1 Myr old. Apparently their PMS sample was a little too small at any given age to identify the subtle change in the broad $j$ distributions that occurs between 1 and 2 Myr. On the other hand, their main conclusion, based on comparing the PMS to the recently arrived MS clusters, is not too different from ours. We simply find 10%–30% more stars populating the rapid rotator portion of the sample. Further discussion of these points has been included in Lamm et al. (2005) and need not be pursued here. The distinction between 30%–40% of the stars conserving angular momentum and 50%–60% is probably small enough that we should be more impressed with the similarity of these numbers than their difference. The studies agree in pointing to disk locking as the most likely source of the angular momentum drain.

One line of argument that is sometimes raised against disk locking is that it does not work in detail, i.e., that slowly rotating stars in the PMS clusters do not appear preferentially to have disks. Because this argument is often repeated, we reiterate here that it is not true. A statistically significant correlation between rotation and various disk indicators such as near-infrared excess and H$\alpha$ emission strength has been shown to exist in both the ONC and NGC 2264 (Herbst et al. 2002; Lamm et al. 2005; Dahm & Simon 2005). Apparently, some investigators feel that these correlations should be tighter than they are to be convincing, even though they have a high statistical significance. Our opinion is that there is a lot of scatter introduced into the relationships by the inherent variability of TTSs, the difficulty of detecting disks, and the timescales for disk dissipation and subsequent spin-up. While rotation rates can be accurately measured to 1% and respond only slowly (i.e., on timescales of $10^5$–$10^6$ yr) to external influences, indications of the presence or absence of disks are notoriously difficult to measure and some, including H$\alpha$ equivalent width, ultraviolet excess, and even near-infrared excess, can vary on timescales as short as hours or days (Herbst et al. 1994; Carpenter et al. 2001). In our opinion, it is that variability and observational difficulties that make the scatter in relations between rotation and disk properties so large. The fact that we can, in spite of this scatter, find statistically significant correlations between rotation and disk indicators is hard to understand if there was no physical link. Obviously, this is an area in which additional monitoring and observations of selected stars should prove fruitful.

5. SUMMARY

We have formed $j$ distributions for stars of solar-like mass at three different ages: nominally 1, 2, and 50 Myr. The distributions at all times are broad and bimodal. The rapidly rotating side evolves with only a small or negligible loss of angular momentum while the slowly rotating side shows much greater angular momentum losses as expected from disk locking (see Fig. 18). The data indicate that a broad range of rotation rates is established within 1 Myr, presumably by magnetic interactions between the stars and their accretion disks. The subsequent rotational evolution can be characterized as spin-up during PMS contraction with conservation of angular momentum (plus, perhaps, a small amount of angular momentum loss through a stellar wind) for at least half of the stars. About 40%–50%, however, all of which are already slow rotators at 1 Myr, must lose substantial additional amounts of angular momentum ($\sim 70\%$) by the time they reach the MS. Furthermore, they must lose a good deal of this angular momentum quickly. In the short time interval ($\sim 1$ Myr) between ONC and NGC 2264 ages, while most stars spin up conserving angular momentum, 40%–50% do not. The observed size of the angular momentum loss, the fact that it affects preferentially the slowly rotating side of the $j$ distribution, and its rapid action over times as brief as 1 Myr all support an interpretation of disk locking as the physical cause. We argue that all of the currently available observational evidence on PMS rotation and disks is consistent with this picture: one-half, or more, of solar-like stars in clusters are no longer locked to their disks by 1 Myr while 40%–50% maintain such a locking for times of order 5 Myr. These results are in reasonable agreement with what others have found for
disk-locking times, percentage of stars affected, and disk survival times based on infrared excess measurements (Haisch et al. 2001; Tinker et al. 2002; Wolff et al. 2004).

We note that our interpretation is based on two necessary assumptions that can be tested by further observation. First, we assume that the \( j \) distributions derived from rotation period determinations are representative of the full cluster \( j \) distributions. If rotation period determinations are significantly biased against slow rotators, as is likely at some level, we may have underestimated the percentage of disk-locked stars. Such tests as can be done at present suggest that the effect is only at the \( \sim 15\% \) level at most and will, therefore, not have a major impact on our results. Second, we assume that the other four clusters in our sample had \( j \) distributions similar to that displayed by the ONC when they were at a similar age. If that is not true, then differences between their current \( j \) distributions that we attribute to evolution might, in fact, be due to differences in initial conditions. The only way to check on this possibility is to increase the number of clusters at all ages that have sufficient rotation periods to define distributions. This will not be easy because appropriate clusters are farther away and will require extended periods of observation on larger telescopes than have been used heretofore. Finally, we note that our results apply only to stars of solar-like mass (0.4–1.2 \( M_\odot \)). It would be interesting to know if things were different for lower mass stars, but that will require many more rotation periods for low-mass stars in MS clusters, an observationally challenging task.

We thank John Stauffer for information and advice regarding the periods and radii of the MS stars and for a critical reading of a first draft of this manuscript that led to substantial revisions. We thank our close and long-time collaborators Coryn Baier-Jones, Markus Lamm, Catrina Hamilton, and Eric Williams for help with various aspects of this study. We thank the referees, Sidney Wolff and Steve Strom, for their constructive suggestions on the original submitted manuscript. We are indebted to the many Wesleyan students who, for more than two decades, have manned the telescope to obtain data for this project. Finally, we thank NASA for their support of this work over the years through its Origins of Solar Systems program, most recently grant NAG5-12502 to W. H.

REFERENCES

Attridge, J. M., & Herbst, W. 1992, ApJ, 398, L61
Barnes, S., Sofia, S., & Pinsonneault, M. 2001, ApJ, 548, 1071
Barnes, S. A. 2003, ApJ, 586, 464
Basi, G., & Martin, E. L. 1999, ApJ, 510, 266
Bessell, M. S., Castelli, F., & Plez, B. 1998, A&A, 337, 321
Blaauw, A. 1964, ARA&A, 2, 213
Bouvier, J., Cabrit, S., Fernandez, M., Martin, E. L., & Matthews, J. M. 1993, A&A, 272, 176
Camenzind, M. 1990, Rev. Mod. Astron., 3, 234
Carpenter, J. M., Hillenbrand, L. A., & Skrutskie, M. F. 2001, AJ, 121, 3160
Chandrasekhar, S. 1935, MNRAS, 95, 207
Cohen, M., & Kuhi, L. V. 1979, ApJS, 41, 743
Cohen, R. E., Herbst, W., & Williams, E. C. 2004, AJ, 127, 1602
Dahm, S. E., & Simon, T. 2005, AJ, 129, 829
D'Antona, F., & Mazzitelli, I. 1997, Mem. Soc. Astron. Italiana, 68, 807
D'Antona, F., Ventura, P., & Mazzitelli, I. 2000, ApJ, 543, L77
Eatmon, N. L., Herbst, W., & Hillenbrand, L. A. 1995, AJ, 110, 1735
Haisch, K. E., Lada, E. A., & Lada, C. J. 2001, AJ, 121, 2065
Hartmann, L. 2003, ApJ, 585, 398
Herbst, W., Bailey-Jones, C. A. L., & Mundt, R. 2001, ApJ, 554, L199
Herbst, W., & Bailer-Jones, C. A. L., Mundt, R., Meisenheimer, K., & Wackerman, R. 2002, A&A, 396, 513
Herbst, W., Herbst, D. K., Grossman, E. J., & Weinstein, D. 1994, AJ, 108, 1906
Herbst, W., Rhode, K. L., Hillenbrand, L. A., & Curran, G. 2000, AJ, 119, 261
Herbst, W., & Wittenmyer, R. 1996, BAAS, 28, 1338
Hillenbrand, L. A. 1997, AJ, 113, 1733
Hillenbrand, L. A., & White, R. J. 2004, ApJ, 604, 741
James, R. A. 1964, ApJ, 140, 552
Kearns, K. E., Eaton, N. L., Herbst, W., & Mazzurco, C. J. 1997, AJ, 114, 1098
Kearns, K. E., & Herbst, W. 1998, AJ, 116, 261
Königl, A. 1991, ApJ, 370, L39
Krishnamurthi, A., Pinsonneault, M. H., Barnes, S., & Sofia, S. 1997, ApJ, 480, 303
Lamm, M. H., Baier-Jones, C. A. L., Mundt, R., Herbst, W., & Scholz, A. 2004, A&A, 417, 557
Lamm, M. H., Mundt, R., Baier-Jones, C. A. L., & Herbst, W. 2005, A&A, 430, 1005
Makidon, R. B., Rebull, L. M., Strom, S. E., Adams, M. T., & Patten, B. M. 2004, AJ, 127, 2228
Mandel, G. N., & Herbst, W. 1991, ApJ, 383, L75
Mathieu, R. D. 2004, in IAU Symp. 215, Stellar Rotation, ed. A. Maeder & P. Eunens (Dordrecht: Kluwer), 113
Matt, S., & Pudritz, R. E. 2004, ApJ, 607, L43
Palla, F., & Stahler, S. W. 1999, ApJ, 525, 772
Randich, S., Pallavicini, R., Mioia, G., Stauffer, J. R., & Balachandran, S. C. 2001, A&A, 372, 862
Rebull, L. M. 2001, AJ, 121, 1676
Rebull, L. M., Wolf, S. C., & Strom, S. E. 2004, AJ, 127, 1029
Rebull, L. M., Wolf, S. C., Strom, S. E., & Makidon, R. B. 2002, AJ, 124, 546
Rhode, K. L., Herbst, W., & Mathieu, R. D. 2001, AJ, 122, 3258
Sills, A., Pinsonneault, M. H., & Terndrup, D. M. 2000, ApJ, 534, 335
Skumanich, A. 1972, ApJ, 171, 565
Stassun, K. G., Mathieu, R. D., Mazeh, T., & Vrba, F. J. 1999, AJ, 117, 2941
Stassun, K. G., & Terndrup, D. 2003, PASP, 115, 505
Stauffer, J. R., Schultz, G., & Kirkpatrick, J. D. 1998, ApJ, 499, L199
Terndrup, D. M., Stauffer, J. R., Pinsonneault, M. H., Sills, A., Yuan, Y., Jones, B. F., Fischer, D., & Krishnamurthi, A. 2000, AJ, 119, 1303
Tinker, J., Pinsonneault, M., & Terndrup, D. 2002, ApJ, 564, 877
Warren, W. H., & Hessell, J. E. 1978, ApJS, 36, 497
Wolff, S. C., Strom, S. E., & Hillenbrand, L. A. 2004, ApJ, 601, 979