Heavy Baryon Masses in the $1/m_Q$ and $1/N_c$ Expansions

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The masses of baryons containing a single heavy quark are studied in a combined expansion in $1/m_Q$, $1/N_c$ and $SU(3)$ flavor symmetry breaking. Heavy quark baryon mass splittings are related to mass splittings of the octet and decuplet baryons. The $\Sigma^*_c$, $\Xi^*_c$ and $\Omega^*_c$ are predicted to the level of a few MeV. A number of bottom baryon mass splittings are predicted very accurately.

I. INTRODUCTION

The mass spectrum of baryons containing a single heavy quark is tightly constrained by the presence of approximate symmetries. In the heavy quark limit, hadrons containing a single heavy quark respect a heavy quark spin-flavor symmetry $[1]$. For finite $m_Q$, this symmetry is broken by effects of order $1/m_Q$. In the large $N_c$ limit, baryons with an approximate flavor symmetry possess a larger spin-flavor symmetry $[2]$. For finite $N_c$, this symmetry is broken by effects of order $1/N_c$. In the combined limit $m_b \to \infty$, $m_c \to \infty$, $N_c \to \infty$ for fixed $(m_c/m_b)$ and $(N_cA_{QCD}/m_b)$ results in a light quark and heavy quark spin-flavor symmetry $SU(6)_L \times SU(4)_h$ for baryons containing a single heavy quark. For finite $m_Q$ and $N_c$, this symmetry is violated by effects of order $1/N_c$ and $(1/N_c m_Q)$, and by $SU(3)$ flavor breaking. It is the purpose of this work to explore the implications of $SU(3)$ flavor breaking and the combined $1/m_Q$ and $1/N_c$ expansions for the masses of heavy quark baryons.

Section II begins with a brief review of the $1/m_Q$ expansion of heavy hadron masses in heavy quark effective theory. Section III presents the operator analysis of the $1/N_c$ expansion. Heavy baryon masses are analyzed in the combined $1/m_Q$ and $1/N_c$ expansion for isospin flavor symmetry in Section IV. The generalization to $SU(3)$ flavor symmetry is performed in Section V. The most precise predictions follow from the $SU(3)$ mass relations of Sect. V. Readers interested only in these results may go directly to subsection VA where they are presented. Isospin-violating mass splittings are considered in Section VI. Conclusions are presented in Section VII.

II. HEAVY QUARK BARYON MASSES IN THE $1/m_Q$ EXPANSION

The masses of hadrons containing a single heavy quark have been studied in the $1/m_Q$ expansion of heavy quark effective theory. A brief review of this expansion is presented in this section.

The HQET Lagrangian,

$$L_{HQET} = \overline{Q}_v (iv \cdot D) Q_v + \overline{Q}_v \left( \frac{iD^2}{2m_Q} \right) Q_v - Z_Q \overline{Q}_v g G_{\mu\nu} \sigma^{\mu\nu} Q_v + O \left( \frac{1}{m_Q^2} \right),$$

(2.1)

describes the interactions of a heavy quark $Q$ with fixed velocity $v$ inside a hadron containing a single heavy quark. The heavy quark mass $m_Q$ is removed from the QCD Lagrangian by the heavy quark field redefinition $[3]$. The residual mass term has been chosen so that there is no mass term in the HQET Lagrangian $[4]$.

The mass of a hadron containing a single heavy quark has an expansion in $1/m_Q$,

$$M(H_Q) = m_Q + \bar{\Lambda} - \frac{\lambda_1}{2m_Q} - d_H \frac{\lambda_2}{2m_Q} + O \left( \frac{1}{m_Q^2} \right),$$

(2.2)

where the order unity contribution $\bar{\Lambda}$ is the mass of the light degrees of freedom in the hadron, and the two $1/m_Q$ contributions are determined by the matrix elements

$$\lambda_1 = \left\langle H_Q(v) \left| \overline{Q}_v (iD)^2 Q_v \right| H_Q(v) \right\rangle$$

(2.3)

and

$$d_H \lambda_2 = \frac{1}{2} Z_Q \left\langle H_Q(v) \left| \overline{Q}_v g G_{\mu\nu} \sigma^{\mu\nu} Q_v \right| H_Q(v) \right\rangle.$$  (2.4)

In the above equation, $d_H$ is the Clebsch factor $d_H = -4(J_L \cdot J_Q)$, and $Z_Q$ is a renormalization factor with

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$Z_Q(\mu = m_Q) = 1$. Renormalization group scaling between the scales $m_b$ and $m_c$ yields $Z_b/Z_c = [\alpha_s(m_b)/\alpha_s(m_c)]^{9/25}$.

Neglecting $SU(3)$ flavor breaking, the masses of the lowest-lying pseudoscalar and vector mesons $D$ and $D^*$ for $Q = c$ and $B$ and $B^*$ for $Q = b$ are parametrized by

$$P_Q = m_Q + \bar{\Lambda}_{\text{meson}} \frac{\lambda_1}{2m_Q} - \frac{3\lambda_2}{2m_Q} + ..., \quad P_Q^* = m_Q + \bar{\Lambda}_{\text{meson}} \frac{\lambda_1}{2m_Q} + \frac{\lambda_2}{2m_Q} + ..., \quad (2.5)$$

whereas the masses of the lowest-lying spin-$\frac{1}{2}$ antitriplet (3) and spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ sextets (6) are parametrized by

$$T_Q = m_Q + \bar{\Lambda}_{\text{baryon}} \frac{\lambda_1}{2m_Q} + ..., \quad S_Q = m_Q + \bar{\Lambda}_{\text{baryon}} \frac{\lambda_1}{2m_Q} - \frac{4\lambda_2}{2m_Q} + ..., \quad (2.6)$$

to order $1/m_Q^2$ in the $1/m_Q$ expansion. Note that different parameters $\bar{\Lambda}$, $\lambda_1$, and $\lambda_2$ appear for mesons and baryons. When $SU(3)$ flavor breaking is not neglected, different parameters also appear for different members of the meson and baryon $SU(3)$ multiplets.

### III. OPERATOR ANALYSIS

The $1/N_c$ operator analysis for baryons containing light quarks was derived in detail in Ref. [4]. The operator analysis is generalized to baryons containing heavy quarks in this section. The approximate flavor symmetry of QCD with three light quarks $u, d, s$ and heavy quarks $c$ and $b$ is $SU(3) \times U(1)_c \times U(1)_b$. The corresponding spin-flavor symmetry for large-$N_c$ baryons is $SU(6) \times SU(2)_c \times SU(2)_b$. There is a separate spin symmetry for each flavor of heavy quark in the baryon in the large-$N_c$ limit. This heavy quark spin symmetry is operative for baryons containing multiple heavy quarks as well as baryons containing a single heavy quark. It is important to emphasize that large-$N_c$ heavy quark spin symmetry is present for baryons even if the quark flavor in question is not heavy enough to have a valid $1/m_Q$ expansion. For this reason, it is possible to study the consequences of large-$N_c$ heavy quark spin symmetry for baryons in the $1/N_c$ expansion without reference to the $1/m_Q$ expansion. We begin with this $1/N_c$ operator analysis and then generalize to the combined $1/m_Q$ and $1/N_c$ expansion.

The operator basis of the $1/N_c$ expansion for baryons containing heavy quarks is constructed out of the zero-body identity operator $\mathbb{1}$ and the generators of the spin-flavor algebra $SU(6) \times SU(2)_c \times SU(2)_b$. The $SU(6)$ generators are defined by

$$J^i_r = q^{\dagger} \frac{\sigma^r}{2} q, \quad T^a = q^{\dagger} \frac{\lambda^a}{2} q, \quad G^{ia} = q^{\dagger} \frac{i\lambda^a \sigma^r}{4} q,$$

where $J^i_r = J^i_u + J^i_d + J^i_s$, while the charm and bottom spin generators are defined by

$$J^i_c = c^{\dagger} \frac{\sigma^r}{2} c, \quad J^i_b = b^{\dagger} \frac{\sigma^r}{2} b. \quad (3.1)$$

The light quark spin-flavor algebra is given in Ref. [7]. The heavy quark spin symmetry commutation relations are simply

$$[J^i_Q, J^j_Q] = i\epsilon^{ijk} J^k_Q, \quad (3.2)$$

where $Q = c$ or $b$.

Operators in the $1/N_c$ expansion are polynomials in the 1-body operators. The set of polynomial operators is overcomplete. An operator basis which is complete and linearly independent is constructed by eliminating redundant operators using operator identities. The operator identities are derived in Ref. [7]. The structure of the operator identities is discussed in detail in Ref. [7], and will not be repeated here. For the case of baryons containing $N_\ell$ light quarks and $N_h$ heavy quarks such that $N_c = N_\ell + N_h$, the $SU(2)_{N_F}$ identities of Ref. [7] apply for $J^i$ replaced by $J^i_\ell$ and $N_c$ replaced by $N_\ell$ in Eq. (4.2) and Tables VII and VIII. Similarly, the $SU(2)_Q$ spin symmetry identities are the identities for one quark flavor with $N_c$ replaced by $N_Q$, namely

$$Q^\dagger Q = N_Q \mathbb{1}, \quad \{J^i_Q, J^j_Q\} = \frac{1}{2} N_Q (N_Q + 2) \mathbb{1}. \quad (3.3)$$

The presence of $N_\ell$ and $N_Q$ in the operator identities rather than $N_c$ follows from the group theory of the baryon representation for a fixed number of light quarks and a fixed number of heavy quarks of each flavor. Fig. 1 gives the $SU(6) \times SU(2)_c \times SU(2)_{Q'}$ spin-flavor representation for baryons containing $N_\ell$ light quarks and $N_h = N_Q + N_{Q'}$ heavy quarks, with $N_\ell + N_h = N_c$, where $Q$ and $Q'$ represent two different flavors of heavy quark $c$ and $b$. It is clear that the operator identities apply for the quark numbers $N_\ell$ and $N_{Q'}$. For example, the total number of quarks in the baryon is described by the 1-body $\rightarrow 0$-body identities

$$q^\dagger q + Q^\dagger Q + Q'^\dagger Q' = (N_\ell + N_Q + N_{Q'}) \mathbb{1} = (N_\ell + N_h) \mathbb{1} = N_c \mathbb{1}. \quad (3.4)$$
The $1/N_c$ expansion of a $l$-body QCD operator acting on baryons containing $N_\ell$ light quarks and $N_h$ heavy quarks has the form

$$O_{\text{QCD}}^{(l)} = N_\ell \sum_{m=0}^{N_\ell} \sum_{n=0}^{N_h} c^{(m+n)} \frac{1}{N_c^{m+n}} O^{(m)}_\ell O^{(n)}_h,$$

(3.5)

where $O^{(m)}$ denotes an $m$-body operator in the light quark spin-flavor generators $J^i_\ell$, $T^a$, and $G^{ia}$, and $O^{(n)}_h$ denotes a $n$-body operator in the heavy quark generators. The $1/N_c$ operator expansion only goes up to $N_c$-body operators since $N_c = N_\ell + N_h$. Each of the arbitrary coefficients of the $1/N_c$ expansion

$$c^{(m+n)} \left( \frac{1}{N_c} \right)$$

(3.7)

is order one at leading order in the $1/N_c$ expansion. The coefficients are only a function of $1/N_c$, since $N_\ell$, $N_Q$ and $N_{Q'}$ are to be regarded as fixed numbers, not operators, in the present expansion. The coefficients depend implicitly on the fixed ratios $N_\ell/N_c$, $N_Q/N_c$ and $N_{Q'}/N_c$. It is an important physical point that the operator expansion for baryons containing light and heavy quarks is suppressed by factors of $1/N_c$ since $N_\ell$, $N_Q$ and $N_{Q'}$ are all suppressed by $1/N_c$. This recasting of the operator expansion is independent of whether the quark flavor in question is heavy enough to have a valid $1/m_Q$ expansion. If the quark flavor of the heavy quark spin symmetry is independent of whether the quark flavor in question is heavy enough to have a valid $1/m_Q$ expansion. If the quark flavor is sufficiently heavy that it has a valid $1/m_Q$ expansion, then heavy quark spin symmetry is also a consequence of the heavy quark mass expansion. A second consistency requirement is that the large-$N_c$ power counting of the $1/N_c$-body operators $O^{(m+n)} \equiv O^{(m)}_\ell O^{(n)}_h$. This feature is present for a suppression factor of $1/N_c^{m+n}$, but not for $1/(N_c^m N_c^n)$. A second consistency requirement is that the operator expansion be invariant under a change of operator basis for the expansion. For example, it is possible to rewrite the operator basis using $J = J_\ell + J_h$, the total baryon spin, rather than $J_\ell$. This recasting of the operator expansion is obviously possible if $J$, $J_\ell$ and $J_h$ are all suppressed by $1/N_c$, but it is not consistent if $J_\ell$ is suppressed by $1/N_c$ and $J_h$ is suppressed by $1/N_c$. The conclusion that all the suppression factors are $1/N_c$ also follows from an analysis of the large-$N_c$ Feynman diagrams.

Although factors of $1/N_\ell$ and $1/N_h$ do not appear as explicit suppression factors in the $1/N_c$ operator expansion, $N_\ell$ and $N_h$ do appear implicitly through the matrix elements of $O_\ell$ and $O_h$. Matrix elements of the operator $O^{(m)}_\ell$ are at most order $(N_\ell)^m$, whereas matrix elements of $O^{(n)}_h$ are at most $(N_h)^n$, so that

$$\frac{1}{N_c^{m+n}} \langle O^{(m)}_\ell O^{(n)}_h \rangle \lesssim \left( \frac{N_\ell}{N_c} \right)^m \left( \frac{N_h}{N_c} \right)^n.$$

(3.8)

Both $N_\ell/N_c$ and $N_h/N_c$ are suppression factors so long as neither $N_\ell$ nor $N_h$ is equal to $N_c$. Thus, for baryons containing both light and heavy quarks, higher-body operators in the $1/N_c$ expansion are suppressed relative to lower-body operators if the operator basis is written in terms of light-quark and heavy-quark 1-body operators. Alternative operator bases are less ideal since they will not have this feature. For example, the highest spin baryon states have $J \sim N_c$, so that the use of the 1-body operator $J$ instead of $J_\ell$ will not lead to an operator basis with this feature. For the physical situation of interest in this work, namely baryons containing a single heavy quark, the $N_\ell/N_c$ and $N_h/N_c$ suppression factors will be $2/3$ and $1/3$, respectively.

The large-$N_c$ heavy quark spin symmetry is independent of whether the quark flavor in question is heavy enough to have a valid $1/m_Q$ expansion. If the quark flavor is sufficiently heavy that it has a valid $1/m_Q$ expansion, then heavy quark spin symmetry is also a consequence of the $m_Q \to \infty$ limit, independent of whether the large-$N_c$ limit is taken. It thus follows that heavy quark spin symmetry violation is suppressed by $(1/N_c m_Q)$ for very heavy quarks.

For the special case of baryons containing a single heavy quark, there is a heavy quark flavor symmetry in the limit that each of the heavy quark flavors is regarded as infinitely heavy. More specifically, for baryons with $N_h = 1$ in the limit $m_h \to \infty$ and $m_c \to \infty$ with $m_c/m_h$ held fixed, heavy quark spin symmetry for each of the heavy flavors is promoted to a heavy quark spin-flavor symmetry $SU(4)_h$ $^1$. In the presence of heavy quark flavor symmetry (regardless of the origin of the heavy quark flavor symmetry), heavy quark spin-flavor symmetry for baryons is a consequence of the large $N_c$ limit. Thus, when the large-$N_c$ limit, $N_c \to \infty$ with $N_c A_{\text{QCD}}/m_Q$ fixed, is added to the heavy quark limit, there is a $SU(6)_\ell \times SU(4)_h$ spin-flavor symmetry for baryons with one heavy quark $^1$. The heavy quark spin-flavor algebra is generated by the 1-body operators

$$J^i_h = Q^i \frac{a_i}{2} Q, \quad J^a_h = Q^a \frac{T^a}{2} Q,$$

(3.9)

$^1$For example, if there were two moderately heavy quarks of similar mass in QCD, there would be a flavor symmetry amongst these quarks and a spin-flavor symmetry for baryons containing these quarks in the large $N_c$ limit. There would not be a spin-flavor symmetry for mesons in this situation.
where \( a = 1, 2, 3 \) is the heavy flavor quark index. The charm and bottom quark spin and number operators are related to the heavy quark operators and \( N_h \) through

\[
J_h^i = J_h^i + J_h^i,
\]

\[
I_h^i = \frac{1}{2} (N_{\text{charm}} - N_h),
\]

\[
G_h^{3a} = \frac{1}{2} \left( J_h^i - J_h^i \right),
\]

and \( N_h = N_{\text{charm}} + N_b \). The heavy quark flavor symmetry only applies to baryons with \( N_h = 1 \), so operators in the \( 1/N_c \) expansion for these baryons will contain no more than one heavy quark 1-body operator \( J_h^i, I_h^i \) and \( G_h^{3a} \).

Finally, the expansion (3.3) can easily be generalized to encompass baryons with differing numbers of light quarks and heavy quarks \( Q \) and \( Q' \) by including all operators up to \( N_c \)-body in the light quark 1-body operators and the heavy quark 1-body operators, which now consist of \( N_Q, J_Q, N_{Q'} \) and \( J_{Q'} \). Notice that the heavy quark numbers of operators \( N_Q \) and \( N_{Q'} \) are now to be regarded as heavy quark 1-body operators since one is not restricting to baryons with fixed numbers of \( Q \) and \( Q' \) quarks. (Because of the constraint \( N_c = N_e + N_Q + N_{Q'} \), it is not necessary to introduce \( N_e \) as an additional 1-body operator as well.) The coefficients of the expansion (3.3) contain implicit \( N_Q/N_c \) and \( N_{Q'}/N_c \) dependence which appear as operators in the generalized expansion.

### IV. Masses of Heavy Baryons with Fixed Strangeness

In this section, the masses of baryons containing heavy quarks are analyzed in the combined \( 1/m_Q \) and \( 1/N_c \) expansion for the special case of fixed strangeness. It is useful to study this special case because it illustrates the structure of the combined expansion without the complication of flavor \( SU(3) \) breaking. The analysis here depends only on the presence of isospin flavor symmetry. The general \( SU(3) \) mass analysis is given in Secs. V and VI.

The baryons containing a single heavy quark \( Q \) with zero strangeness are the \( \Lambda_Q, \Sigma_Q \) and \( \Sigma_Q^* \). The HQET \( 1/m_Q \) expansion of these masses is given by

\[
\Lambda_Q = m_Q + \Lambda_T - \frac{\lambda_{1T}}{2m_Q} + ...
\]

\[
\Sigma_Q = m_Q + \Lambda_S - \frac{\lambda_{1S}}{2m_Q} - \frac{4\lambda_2}{2m_Q} + ...
\]

\[
\Sigma_Q^* = m_Q + \Lambda_S - \frac{\lambda_{1S}}{2m_Q} + \frac{2\lambda_2}{2m_Q} + ...
\]

(4.1)

to order \( 1/m_Q^2 \) in the \( 1/m_Q \) expansion. The leading term in this expansion is exactly \( m_Q \) to all orders in the \( 1/N_c \) expansion. Each of the subleading parameters \( \Lambda, \lambda_1 \) and \( \lambda_2 \) has an expansion in \( 1/N_c \),

\[
\Lambda = c_0 N_c\mathbb{1} + c_1 \frac{1}{N_c} J_T^2,
\]

\[
\frac{\lambda_1}{2m_Q} = c_0' \frac{1}{2m_Q} N_Q + c_2' \frac{1}{N_c^2} \frac{1}{2m_Q} N_Q J_T^2,
\]

(4.2)

\[
- d_H \frac{\lambda_2}{2m_Q} = c_2' \frac{2}{N_c m_Q} (J_T \cdot J_Q),
\]

where the factor of 2 in the \( \lambda_2 \) expansion follows from the factor of 4 in the definition of \( d_H = -4 (J_T \cdot J_Q) \). The unknown coefficients in Eq. (4.2) are a function of \( 1/N_c \) and the QCD scale \( \Lambda_{QCD} \). Each coefficient has an expansion in \( 1/N_c \) beginning at order unity in the \( 1/N_c \) expansion. (The coefficients do not depend on \( 1/m_Q \) since the HQET parameters are defined to occur at a definite order in the \( 1/m_Q \) expansion.) Without loss of generality, it is possible to set \( c_0 \equiv \Lambda \), and use this relation as the definition of a QCD scale. The remaining coefficients are now given by a dimensional power of \( \Lambda \) times a dimensionless function of \( 1/N_c \) beginning at order unity in the \( 1/N_c \) expansion. For example, the coefficient \( c_2 \) is proportional to \( \Lambda \), whereas the \( 1/m_Q \)-suppressed operators have coefficients proportional to \( \Lambda^2 \).

The numbers of operators of the \( 1/N_c \) expansions for \( \Lambda, \lambda_1 \) and \( \lambda_2 \) are in one-to-one correspondence with the numbers of parameters \( \Lambda_T, \lambda_S, \lambda_{1T}, \lambda_{1S} \) and \( \lambda_2 \) appearing in Eq. (4.1). The \( 1/N_c \) expansion predicts that the \( \Lambda \) parameters of the 3 and 6 are equal to \( N_e, \Lambda \) with a splitting of relative order \( 1/N_c^2 \) compared to the leading contribution. Similarly, the \( \lambda_1 \) parameters of the 3 and 6 are order unity in the \( 1/N_c \) expansion with a splitting of relative order \( 1/N_c^2 \). Finally, the \( \lambda_2 \) parameter for baryons is order \( 1/N_c \). The \( 1/N_c \) scaling of \( \lambda_2 \) can be tested by comparing the meson and baryon hyperfine splittings resulting from the heavy quark spin symmetry-violating chromomagnetic operator. The baryon mass splitting is only measured in the charm system at present. The charm meson hyperfine mass splitting is given by

\[
(D^* - D) = \frac{2\lambda_2^{\text{meson}}}{m_c} + ..., \tag{4.3}
\]

whereas the analogous charm baryon mass splitting is

\[
(\Sigma_c^* - \Sigma_c) = \frac{3\lambda_2^{\text{baryon}}}{m_c} + ... \tag{4.4}
\]

The naive \( 1/N_c \) scaling predicted by the baryon \( 1/N_c \) expansion implies

\[
\lambda_2^{\text{baryon}} \sim \frac{1}{N_c} \lambda_2^{\text{meson}}. \tag{4.5}
\]

\(^1\)Recent measurements of the \( \Sigma_c^* \) masses by DELPHI are probably unreliable and will be ignored.
The measured charm meson and baryon mass differences $(D^* - D) = 141$ MeV and $(\Sigma^*_c - \Sigma_c) = 77.1 \pm 5 \pm 5$ MeV are in remarkable agreement with this scaling. The $(D^* - D)$ mass difference determines the canonical heavy quark symmetry suppression factor $\Lambda_{QCD}^2/m_c$.

$$\frac{\Lambda_{QCD}^2}{m_c} \sim \frac{\Lambda_{QCD}^2}{m_c} = \frac{1}{2}(D^* - D).$$

(4.6)

The $\lambda_1$ parameters for baryons and mesons also can be related. The $1/N_c$ expansion implies

$$\lambda_1^{\text{baryon}} \sim \lambda_1^{\text{meson}}.$$  

(4.7)

Eq. (4.3) could be used to estimate the magnitude of $\lambda_1^{\text{baryon}}$ if $\lambda_1^{\text{meson}}$ is known accurately. The most recent extraction of $\lambda_1^{\text{meson}}$ is too uncertain, however, so the naive estimate

$$\lambda_1^{\text{baryon}} \sim \frac{\Lambda_{QCD}^2}{2m_Q},$$

(4.8)

is used in Eq. (4.3).

We now proceed to analyze the masses of baryons containing heavy quarks using the combined $1/m_Q$ and $1/N_c$ operator expansion developed in the previous section. We consider several different mass expansions.

Let us first consider the operator expansion for strangeness zero baryons containing a single flavor of heavy quark $Q$. The three baryons masses $\Lambda_Q, \Sigma_Q$ and $\Sigma'_Q$ are parametrized by the three operators $\lambda_3^Q$:

$$M = (m_Q + N_c \Lambda + \ldots) 1 + \frac{1}{N_c} J^2 + \frac{2}{N_c m_Q} (J_z \cdot J)$$

(4.9)

where the leading $N_c$ and $m_Q$ dependence of each operator appears explicitly. The leading operator $1$ has a coefficient given exactly by $m_Q + N_c \Lambda$ up to a correction of order $1/m_Q$ in the combined $1/m_Q$ and $1/N_c$ expansion. It is to be understood that every other operator is accompanied by a coefficient

$$c(\Lambda, \frac{1}{m_Q}, \frac{1}{N_c})$$

(4.10)

with an expansion in $1/m_Q$ and $1/N_c$ beginning at order unity. For example, expanding to order $1/m_Q^2$ in the $1/m_Q$ expansion, there is an order $(1/2m_Q)$ contribution to the operator $1$ from the $\lambda_1$ contribution proportional to $\lambda_1^Q$ in Eq. (4.2) and an order $(1/N_c^2)(1/2m_Q)$ contribution to the operator $J^2$ from the $\lambda_2$ contribution proportional to $\lambda_2^Q$. Each of the operators in Eq. (4.3) contributes to one specific linear combination of the three baryon masses. The operators and their corresponding mass combinations are tabulated in Table I, together with the operator matrix element and the leading $1/m_Q$ and $1/N_c$ dependence for each mass splitting. The operator matrix elements depend implicitly on $N_t = 2$ and $N_Q = 1$. Up to unknown coefficients of order unity, the combined $1/m_Q$ and $1/N_c$ expansion predicts that the mass combinations $\Lambda_Q, \frac{1}{2}(\Sigma_Q + 2\Sigma'_Q) - \Lambda_Q$, and $(\Sigma_Q - \Sigma'_Q)$ satisfy a hierarchy given by $(m_Q + N_c \Lambda, 2\Lambda/N_c$ and $\frac{1}{2}(2\Lambda^2/N_c m_Q)$. For $Q = c$, the experimental values of the mass splittings are $2285 \pm 0.6$ MeV, $219 \pm 3 \pm 3$ MeV and $77 \pm 5 \pm 5$ MeV, which compares very favorably with the theoretical hierarchy for canonical values of $m_c$ and $\Lambda$. (Note that the experimental values and errors of the hyperfine splittings are dominated by the uncertainty in the mass measurement $\Sigma = 2530 \pm 5 \pm 5$ MeV.)

For two flavors of heavy quark $Q = c$ and $Q' = b$, the previous analysis of the baryon mass spectrum holds for each flavor of heavy quark separately. In the presence of heavy quark flavor symmetry, a combined analysis of the six heavy baryon masses using heavy quark spin-flavor operators is possible. This mass expansion for baryons containing a single heavy quark of flavor $Q$ or $Q'$ is given by

$$M = \left[\frac{1}{2}(m_Q + m_{Q'}) + N_c \Lambda + \ldots\right] 1 + P^b_{\frac{3}{2}}(m_Q - m_{Q'}) + \frac{1}{N_c} J_c^2 + \frac{1}{N_c^2} J^2_{h_b}(J_z) \left(\frac{1}{m_Q} + \frac{1}{m_{Q'}}\right)$$

(4.11)

where leading $1/m_Q$ and $1/N_c$ dependences are given explicitly and unknown coefficients of the form Eq. (4.10) are understood to accompany each operator [with the exception of the order $m_Q$ and $N_c \Lambda$ terms given explicitly on the first line of Eq. (4.11)]. The important new operators of the heavy quark spin-flavor analysis are the heavy quark flavor-violating operators which do not violate heavy quark spin symmetry: $I^3_{\frac{3}{2}}$ and $I^3_{\frac{1}{2}}(J_z)^2$. The most interesting operator is $I^3_{\frac{3}{2}}(J_z)^2$. The leading order $J^2$ hyperfine splitting for heavy baryons is a heavy quark flavor-independent splitting of order $1/N_c$. The $I^3_{\frac{3}{2}}(J_z)^2$ operator represents heavy quark flavor symmetry-breaking in the $J^2$ hyperfine splittings. This heavy flavor symmetry-breaking hyperfine mass splitting is order $1/N_c^2$ times $(1/2m_Q - 1/2m_{Q'})$, and is sensitive to the subleading $(1/N_c^2)(1/2m_Q)$ contribution to the $J^2$ hyperfine splitting. Similar remarks hold for the operator $I^3_{\frac{3}{2}}$, which represents heavy quark flavor symmetry breaking in the singlet $1$ mass, except that this case is

\[ \text{The parameter values } m_c = 1450 \text{ MeV, } \Lambda_{QCD} = \Lambda = 310 \text{ MeV, } \Lambda_{QCD}^2/m_c = 66 \text{ MeV, } \Lambda_{QCD}/m_c = 0.21, \text{ and } m_b = 4757 \text{ MeV.} \text{ These parameter values follow from the charm baryon mass analysis of this work, and are consistent with canonical values used in the heavy quark literature.} \]
less interesting since there is a leading contribution equal to \((m_Q - m_{Q'})\) which dominates the subleading contribution proportional to \((1/2m_Q - 1/2m_{Q'})\).

The mass combinations corresponding to the operators \(I_h^3\) and \(J_f^3 (J_f)^2\) are tabulated in Table II. Both operators probe heavy flavor violation in \(\Lambda_1\). The latter operator is sensitive to the the \(\lambda_1\) contribution proportional to \(c_2^\prime\), while the former operator depends on the \(\Lambda_1\) operator proportional to \(c_0^\prime\) only at subleading order. The \(\Sigma_{Q}^\pm\) masses are not reliably measured at present, so the \(I_h^3 (J_f)^2\) splitting cannot be evaluated. Instead, the mass relation

\[
\frac{1}{3}(\Sigma_b + 2\Sigma_b^\mp) - \Lambda_b = \frac{1}{3}(\Sigma_c + 2\Sigma_c^\mp) - \Lambda_c \tag{4.12}
\]

can be used to predict the bottom baryon \(J_f^2\) hyperfine splitting to a theoretical accuracy of order \(2/N_c^2\) times \(\Lambda^3 (1/2m_c - 1/2m_b)\), which is about 5 MeV. Thus, the mass splitting \(\frac{1}{3}(\Sigma_b + 2\Sigma_b^\mp) - \Lambda_b\) is predicted to be equal to \(219 \pm 7\) MeV using Eq. (4.12). The measured \((\Lambda^+_1 - \Lambda_0^3)\) splitting of \(-3338 \pm 5 \pm 4\) MeV gives a measure of \((m_c - m_b)\), up to corrections of order \(\Lambda^3 (1/2m_c - 1/2m_b)\) which is about 23 MeV.

The remaining operators in the expansion (4.11) correspond to sum and difference combinations of \(Q\) and \(Q'\) splittings. There is no advantage to studying these combinations rather than the \(Q\) and \(Q'\) splittings separately, since there is no cancellation of leading order contributions in these terms. For example, there is no reason to study \(J_f \cdot J_h\) and \(J_f G_{h}^{\alpha\beta}\) rather than \(J_f \cdot J_Q\) and \(J_f \cdot J_{Q'}\), which are order \(2/N_c m_Q\) and \(2/N_c m_{Q'}\), respectively.

Additional information about the heavy quark mass spectrum can be obtained by generalizing the expansion to encompass baryons with differing number of heavy quarks. The strangeness zero baryons for one flavor of heavy quark \(Q\) consist of the \(N_Q = 0\) baryons \(N\) and \(\Delta\); the \(N_Q = 1\) baryons \(\Lambda_Q, \Sigma_Q, \Sigma_Q'\); and the \(N_Q = 2\) baryons \(\Xi_{QQ}\) and \(\Xi_{QQ}^\mp\); and the \(N_Q = 3\) baryon \(\Omega_{QQQ}\). The \(1/N_c\) operator basis for strangeness zero baryons with \(N_Q = 0, 1, 2, 3\) heavy quarks consists of the eight operators \(1, N_Q, J_f^2, (J_f \cdot J_Q), N_Q J_f^2, N_Q^2, N_Q (J_f \cdot J_Q), \) and \(N_Q^3\). Notice that \(N_Q\) is to be regarded as a 1-body heavy quark operator along with \(J_Q\) when considering baryons with differing numbers of heavy quarks. Although it is interesting to study this expansion for \(N_Q = 0, 1, 2, 3\) baryons, the principal interest of this work will be to relate mass splittings of baryons containing a single heavy quark to the well-measured mass splittings of baryons containing no heavy quarks, namely baryons in the spin-\(\frac{1}{2}\) octet and spin-\(\frac{3}{2}\) decuplet. Thus, we will restrict the operator expansion to baryons with \(N_Q = 0\) and 1. The operator expansion for strangeness zero baryons containing no heavy quark and baryons containing a single heavy quark of flavor \(Q\) is given by

\[
M = N_Q (m_Q + \ldots) + N_c \Lambda_1 + \frac{1}{N_c} J_f^2 + \frac{2}{N_c m_Q} (J_f \cdot J_Q) + \frac{1}{N_c^2} N_Q J_f^2, \tag{4.13}
\]

where leading \(1/m_Q\) and \(1/N_c\) dependences are given explicitly and unknown coefficients of the form Eq. (4.10) are understood to accompany each operator with the exception of the order \(m_Q\) and \(N_c\Delta\) terms. Since one is only considering baryons containing up to one heavy quark, each operator in the expansion has at most one heavy quark 1-body operator \(N_Q J_f^2\). Comparing with Eq. (4.9), there are two new operators in the present expansion: \(N_Q J_f^2\). These operators reduce to \(\Pi\) and \(J_f^2\) for baryons with fixed \(N_Q = 1\), reproducing expansion (4.9). The most interesting operator is \(N_Q J_f^2\) which represents the heavy quark number-dependent contribution to the \(J_f^2\) hyperfine splittings. This heavy quark number-dependent mass splitting is order \((1/N_c^2)\). The \(N_Q\) operator is less interesting since it corresponds to the leading \(N_Q\)-dependent mass splitting which is equal to \(m_Q\). There is a subleading order unity contribution to this splitting.

The mass combinations corresponding to \(N_Q\) and \(N_Q J_f^2\) are tabulated in Table III. The \([\Lambda_Q - \frac{1}{3}(5N - \Delta)]\) splitting of \(1419.2 \pm 0.7\) and \(4757.2 \pm 6.4\) for \(Q = c\) and \(b\), respectively, give a measure of \(m_c\) and \(m_b\), up to corrections of order \(\Lambda\). The \(N_Q J_f^2\) mass relation,

\[
\frac{1}{3}(\Sigma_Q + 2\Sigma_Q') - \Lambda_Q = \frac{2}{3} (\Delta - N), \tag{4.14}
\]

is predicted to be satisfied to order \((1/N_c^2)\). Relation (4.14) has been obtained previously in the context of the Skyrme model where heavy baryons appear as bound states of Skyrmions and heavy mesons \([12,13]\). The present derivation is model-independent, and provides a prediction for the accuracy to which the relation is satisfied. The theoretically predicted accuracy can be tested in the charm baryon sector where the relevant masses are measured. The \((\Delta - N)\) splitting of \(292 \pm 1\) MeV and the \([\frac{1}{3}(\Sigma_c + 2\Sigma_c') - \Lambda_c]\) splitting of \(219 \pm 4.6\) MeV are both \(J_f^2/N_c\) splittings with \(J_f^2\) matrix elements of 3 and 2, respectively. The mass relation (4.14) is satisfied to \(24 \pm 5\) MeV, to be compared with the predicted violation of \((1/N_c^2)\Lambda\) times the matrix element \(\langle N_Q J_f^2 \rangle = 2\), which is about \(69\) MeV. (Alternatively, the violation should be suppressed by a factor of \((1/N_c)\) compared to the charm baryon \(J_f^2\) splitting of \(219\) MeV.) Relation (4.14) can be used to predict the bottom baryon \(J_f^2\) hyperfine splitting \([\frac{1}{3}(\Sigma_b + 2\Sigma_b^\mp) - \Lambda_b]\) to an accuracy of \((1/N_c^2)\) as well. The theoretical prediction is

\[
\frac{1}{3}(\Sigma_b + 2\Sigma_b^\mp) - \Lambda_b = 195 \pm 69\ MeV, \tag{4.15}
\]

which is consistent with, but much less accurate than, the prediction made from the previous expansion using the \(I_h^3 (J_f)^2\) mass combination.

It is instructive to generalize the expansion (4.13) to two heavy quark flavors \(Q\) and \(Q'\). The mass expansion
for strangeness zero baryons containing zero or one heavy quark of flavor $Q$ or $Q'$ is given by

$$M = N_Q (m_Q + ...) + N_{Q'} (m_{Q'} + ...) + N_c \Lambda \text{I}$$

$$+ \frac{1}{N_c} J^2 + \frac{1}{N_{Q'}} N_Q J^2_\ell + \frac{1}{N_c} N_{Q'} J^2_\ell$$

$$+ \frac{2}{N_c m_Q} (J_\ell \cdot J_Q) + \frac{2}{N_{Q'} m_{Q'}} (J_\ell \cdot J_{Q'}) ,$$

(4.16)

where leading $1/m_Q$ and $1/N_c$ dependences are given explicitly and unknown coefficients are understood to accompany all operators except the $m_Q$ and $N_c \Lambda$ terms. This expansion can be rewritten in terms of heavy quark spin-flavor operators as follows,

$$M = \frac{1}{2} N_h [(m_Q + m_{Q'}) + ...] + I_h^3 (m_Q - m_{Q'}) + N_c \Lambda \text{I}$$

$$+ \frac{1}{N_c} J^2 + \frac{1}{N_{Q'}} N_h J^2_\ell + \frac{1}{N_c} J^2_\ell$$

$$+ \frac{1}{N_c} I_h J^2 (J_\ell) + \frac{2}{m_Q} \left( \frac{1}{2 m_Q} - \frac{1}{2 m_{Q'}} \right)$$

$$+ \frac{1}{N_c} (J_\ell \cdot J_h) \left( \frac{m_Q}{m_{Q'}} + \frac{1}{m_Q} \right)$$

$$+ \frac{1}{N_c} J_\ell G^3_h$$

(4.17)

where $N_h = N_Q + N_{Q'}$. The interesting suppressions of the heavy quark flavor symmetry-violating and heavy quark number-dependent $J^2_\ell$ splittings discussed in the preceding paragraphs are manifest.

The above analysis which applies to $S = 0$ baryons in the isospin limit also can be performed in the $S = -1$ sector. These mass combinations are provided in Tables I, II and III as well. The charm baryon masses $\Xi_c$ and $\Xi^*_c$ are well-measured, but the lone $\Xi' _c$ mass measurement is unpublished and possibly unreliable. A precise measurement of the $\Xi'_c$ mass is likely at CLEO in the not too distant future. A prediction of the $\Xi'_c$ mass using the $N_Q J^2_\ell$ mass relation

$$\frac{1}{3} (\Xi'_Q + 2 \Xi'_c) - \Xi_Q = \frac{2}{3} \left[ \Sigma^* - \frac{1}{4} (3 \Sigma + \Lambda) \right]$$

(4.18)

is possible, but not terribly accurate. Relation [4.18] is predicted to be satisfied to an accuracy of order $(2 \Lambda/N_c^2)$, which is about 69 MeV. The $\left[ \Sigma^* - \frac{1}{4} (3 \Sigma + \Lambda) \right]$ mass splitting is 210.8 ± 0.4 MeV, so Eq. [4.18] predicts $\left[ \frac{1}{3} (\Xi'_Q + 2 \Xi'_c) - \Xi_c \right] = 140.5 ± 69$ MeV. The $\Xi_c \equiv \frac{1}{2} (\Xi^*_c + \Xi^*_c)$ mass is 2467.1 ± 1.2 MeV and the analogous $\Xi'_c$ mass is 2644.0 ± 1.6 MeV, so the $\Xi'_c$ prediction is not very useful. A much more precise prediction of the $\Xi'_c$ mass is made in Sect. V. The bottom baryon $S = -1 J^2_\ell$ splitting also is predicted by relation [1.18] to the same theoretical accuracy. Once the $\Xi'_c$ mass is measured, the bottom baryon $J^2_\ell$ splitting can be accurately predicted from the charm baryon mass splitting.

The results of this section follow from the combined $1/m_Q$ and $1/N_c$ expansion using only isospin symmetry of the $u$ and $d$ quarks. The $SU(3)$ analysis including $SU(3)$ flavor breaking is provided in the next section. The $SU(3)$ analysis leads to more precise predictions than the isospin analysis. The isospin symmetry predictions of this section are consistent with the mass predictions of Sec. V. The isospin-breaking analysis is given in Sec. VI.

V. $SU(3)$ ANALYSIS OF HEAVY BARYON MASSES

The $SU(3)$ analysis of the masses of baryons containing a single heavy quark in the combined $1/m_Q$ and $1/N_c$ expansion is a straightforward generalization of the isospin analysis of the previous section. The lowest-lying baryons containing a single heavy quark are the spin-$\frac{1}{2}$ 3 which consists of the isosinglet $\Lambda_Q$ and the isodoublet $\Xi_Q$; the spin-$\frac{1}{2}$ 6 which consists of the isotriplet $\Sigma_Q$, the isodoublet $\Xi'_Q$, and the isosinglet $\Omega_Q$; and the spin-$\frac{3}{2}$ 6 which consists of the isotriplet $\Sigma'_Q$, the isodoublet $\Xi^*_Q$, and the isosinglet $\Omega^*_Q$. Heavy baryon masses will be denoted by their particle labels. Each of the $I = 0$ masses refers to the average mass of the isomultiplet. For example, $\Xi = \frac{1}{2} (\Xi^*_c + \Xi^*_c)$, etc. In addition to the eight $I = 0$ heavy baryon masses, there is an $I = 0$ off-diagonal mass $\Xi_Q \Xi_Q \Xi_Q = \Xi'_Q \Xi_Q$ between the spin-$\frac{1}{2}$ 3 isodoublets, which will be referred to as $\Xi_Q \Xi_Q \Xi_Q$ throughout the rest of the paper. The $I = 0$-off diagonal mass is defined to be the average off-diagonal mass of the spin-$\frac{1}{2}$ 3 $S = -1$ isodoublets,

$$\Xi_c \Xi_c' = \frac{1}{2} \left( \Xi^*_c \Xi^*_c + \Xi^*_c \Xi^*_c \right),$$

$$\Xi_c \Xi_c' = \frac{1}{2} \left( \Xi^*_c \Xi^*_c + \Xi^*_c \Xi^*_c \right).$$

Many of the charm baryon masses are now measured. The measured $I = 0$ masses are

$$\Lambda_c = 2285.0 ± 0.6 \text{ MeV } [14],$$

$$\Xi_c = 2467.7 ± 1.2 \text{ MeV } [14],$$

$$\Sigma_c = 2452.9 ± 0.6 \text{ MeV } [14, 15, 16],$$

$$\Omega_c = 2704 ± 4 \text{ MeV } [14],$$

$$\Xi^*_c = 2644.0 ± 1.6 \text{ MeV } [17].$$

The masses $\Xi_c' \sim 2560 \text{ MeV } [18]$ and $\Sigma^*_c = 2530 ± 5 ± 5 \text{ MeV } [14]$ are measured, but are less reliable or precise. The $\Omega_c^*$ mass and the off-diagonal mass $\Xi_c \Xi_c'$ are unmeasured. At present, only the bottom baryon

$$\Lambda_b = 5623 ± 5 ± 4 \text{ MeV } [19]$$

(5.3)

is accurately measured. As stated earlier, the reported $\Sigma_b^{(*)}$ measurements by DELPHI are not used.
The operator analysis including $SU(3)$ flavor breaking decomposes into $SU(3)$ flavor singlet, octet and 27 expansions:

$$M = M^{(1)} + \epsilon M^{(8)} + \epsilon^2 M^{(27)}, \quad (5.4)$$

where $\epsilon$ is an $SU(3)$-violating parameter whose magnitude is governed by the quark mass difference ($m_s - \bar{m}$) divided by the chiral symmetry breaking scale $\Lambda_\chi$. The symmetry breaking parameter $\epsilon \sim 0.25$. The flavor singlet, octet and 27 mass expansions are given by

$$M^{(1)} = (m_Q + N_c \Lambda + ...) \mathbb{I} + \frac{1}{N_c} J^2 + \frac{2}{N_c m_Q} (J_t \cdot J_Q),$$

$$M^{(8)} = T^8 + \frac{1}{N_c} J^8 G^8 + \frac{1}{N_c m_Q} J^8 G^8$$

$$+ \frac{2}{N_c^2 m_Q} (J_t \cdot J^8 T^8),$$

$$M^{(27)} = \frac{1}{N_c} [T^8, T^8] + \frac{2}{N_c^2 m_Q} J^8 G^8 [T^8, G^8], \quad (5.5)$$

where leading $1/m_Q$ and $1/N_c$ dependences are given explicitly and unknown coefficients are understood to accompany all operators (except for the leading terms of the $\mathbb{I}$ operator). The operators appearing in the flavor-27 expansion require flavor singlet and octet subtractions: these subtractions are implied, but not given explicitly throughout this paper. The relevant flavor-27 projection operator for an operator $O^{ab}$ with two symmetric flavor indices $(ab)$ is

$$\delta^{a8} \delta^{b8} - \frac{1}{8} \delta^{ab} - \frac{3}{5} \delta^{a8} \delta^{b8}. \quad (5.6)$$

The operators and their corresponding mass combinations are tabulated in Table IV, together with the $SU(3)$ and $J_Q$ representations of the operator, the operator matrix element, and the leading $1/m_Q$, $1/N_c$ and $SU(3)$ flavor-breaking factors. Table V combines these factors to predict a hierarchy of mass combinations. The numerical values of the theoretical hierarchy are evaluated for canonical values of parameters $m_Q$, $\Lambda$, $(\epsilon^2 \Lambda_\chi)$ and $(\epsilon^2 \Lambda_\chi) \mathbb{I}$ in the columns labeled $Q = c$ and $Q = b$. It is not possible to evaluate seven of the nine mass combinations using the measured charm baryon masses without knowledge of $\Omega_c^*$ and $\Xi_c^*$. The measured mass combinations $\frac{1}{3} (\Lambda_c + 2 \Xi_c)$ and $(\Lambda_c - \Xi_c)$ which appear in column four were used to determine the parameters $(m_\pi, N_c \Lambda)$ and $(\epsilon \Lambda_\chi)$, and so do not test the theoretical hierarchy.

Table V predicts a hierarchy of mass relations obtained by successively neglecting operators in the mass expansion $(5.4)$. The most accurate mass relation is the flavor-27 heavy quark spin-violating relation

$$\frac{1}{4} [(\Sigma^*_Q - \Sigma_Q) - 2 (\Xi^*_Q - \Xi_Q) + (\Omega^*_Q - \Omega_Q)] = 0, \quad (5.7)$$

which is satisfied up to a correction of order $\frac{2}{3}(1/N_c^2)(\Lambda_c^2/m_Q)(\epsilon^2 \Lambda_\chi)$, which is about 0.23 MeV for $Q = c$ and 0.07 for $Q = b$. Thus, Eq. $(5.7)$ is essentially an exact relation for both the charm and the bottom baryons. The physical content of this relation is that the three chromodynamic mass splittings of the sextet, $(\Sigma^*_Q - \Sigma_Q), (\Xi^*_Q - \Xi_Q)$ and $(\Omega^*_Q - \Omega_Q)$, contain only a singlet contribution and a contribution which is linear in strangeness. Differences of the splittings are equal,

$$(\Sigma^*_Q - \Sigma_Q) - (\Xi^*_Q - \Xi_Q) = (\Xi^*_Q - \Xi_Q) - (\Omega^*_Q - \Omega_Q). \quad (5.8)$$

Eq. $(5.8)$ is the equal spacing rule of Savage $[22]$ for the chromodynamic mass splittings of the sextet.

The next most accurate relation is the flavor-27 mass relation,

$$\frac{1}{6} [(\Sigma_Q + 2 \Sigma^*_Q) - 2 (\Xi_Q + 2 \Xi^*_Q) + (\Omega_Q + 2 \Omega^*_Q)] = 0, \quad (5.9)$$

which is expected to be satisfied to order $\frac{2}{3}(1/N_c)(\epsilon^2 \Lambda_\chi)$, or about 4.4 MeV for $Q = c, b$. This mass relation respects heavy quark spin symmetry. It implies that the three spin-averaged masses of the sextet, $\frac{1}{3} (\Sigma_Q + 2 \Sigma^*_Q)$, $\frac{2}{3} (\Xi_Q + 2 \Xi^*_Q)$ and $\frac{1}{3} (\Omega_Q + 2 \Omega^*_Q)$, receive a singlet contribution and a contribution linear in strangeness. Differences of the splittings are predicted to be equal

$$\frac{1}{3} (\Xi_Q + 2 \Xi^*_Q) - \frac{1}{3} (\Sigma_Q + 2 \Sigma^*_Q)$$

$$= \frac{1}{3} (\Omega_Q + 2 \Omega^*_Q) - \frac{1}{3} (\Xi_Q + 2 \Xi^*_Q), \quad (5.10)$$

at the 8.8 MeV level. Eq. $(5.10)$ is an equal spacing rule for the spin-averaged masses of the sextet.

The two flavor-27 mass relations Eqs. $(5.7)$ and $(5.9)$ together imply the vanishing of the spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ sextet flavor-27 combinations

$$\Sigma_Q - 2 \Sigma^*_Q + \Omega_Q = 0,$$

$$\Sigma^*_Q - 2 \Xi^*_Q + \Omega^*_Q = 0, \quad (5.11)$$

to a precision of about 8.8 MeV. The first of these equations was obtained previously by Savage $[22]$. 

---

**The analytic flavor dependence of the flavor-27 expansion is given here. The non-analytic contribution of order $\epsilon^{3/2}$ is proportional to $\frac{1}{2} (m_s^2 - m_b^2 + m_\pi^2)$ which is numerically very small, and does not dominate the $\epsilon^2$ counterterm $[21]$. The order $\epsilon^{3/2}$ contribution is computed in chiral perturbation theory in Ref. $[22]$.

††Canonical values for the flavor octet and flavor-27 mass splittings are $\epsilon \Lambda_\chi = 225$ MeV and $(\epsilon^2 \Lambda_\chi) = 8.8$ MeV. These values are obtained from the octet and decuplet baryon masses. Note that the flavor-27 mass parameter is considerably smaller than $\epsilon$ times the octet mass parameter, and may be underestimated.
There are two additional mass relations

\[
\frac{1}{6} \left[ 3 (\Sigma_Q^* - \Sigma_Q) - (\Xi_Q^* - \Xi_Q) - 2 (\Omega_Q - \Omega_Q) \right] \]

\[
- \frac{5}{2\sqrt{3}} \Sigma_Q \Xi_Q = 0, \quad (5.12)
\]

\[
\Xi_Q \Xi_Q = 0,
\]

satisfied to orders \( \frac{1}{2N_c} \) and to \( \frac{1}{N_c} \), respectively, which are numerically comparable: about 11.5 MeV for \( Q = c \) and 3.5 for \( Q = b \). The second relation gives an order of magnitude estimate of the \( I = 0 \) off-diagonal mass as \( \Xi_Q \Xi_Q \). While the first relation implies a determination of this off-diagonal mass in terms of the sextet masses. The two relations together yield the mass relation

\[
\frac{1}{6} \left[ 3 (\Sigma_Q^* - \Sigma_Q) - (\Xi_Q^* - \Xi_Q) - 2 (\Omega_Q - \Omega_Q) \right] = 0,
\]

which is predicted to hold to 16.6 \( \pm \) 11.5 MeV for \( Q = c \) and to 5.1 \( \pm \) 3.5 MeV for \( Q = b \). This relation combined with the flavor-27 relation \((5.7)\) implies that the chromomagnetic mass splittings of the sextet are equal,

\[
(I = \\Sigma_Q^* - \Sigma_Q) = (\Xi_Q^* - \Xi_Q) = (\Omega_Q - \Omega_Q).
\]

\[(5.14)\]

Eq. \((5.14)\) was obtained previously by Savage \[22\]. The combined \(1/m_Q, 1/N_c\) and \(SU(3)\) flavor-breaking expansion predicts that this equation need not be very accurate. The two equalities in Eq. \((5.14)\) are predicted to be satisfied only to 19.9 \( \pm \) 13.8 MeV for \( Q = c \) and to 6.1 \( \pm \) 4.2 MeV for \( Q = b \). The equality \((\Sigma_Q^* - \Sigma_Q) = (\Omega_Q - \Omega_Q)\) is predicted to be satisfied only to 39.8 \( \pm \) 27.6 MeV for \( Q = c \) and 12.2 \( \pm \) 8.4 MeV for \( Q = b \).

The remaining mass combinations in Table V are not suppressed enough to yield useful mass relations.

The two flavor-27 mass relations can be used to predict the poorly measured \( \Xi_Q^* \) mass and the unmeasured \( \Omega_Q \) mass. \[1\] The mass \( \Sigma_Q^* = 2530 \pm 7 \) MeV is known to greater precision than the mass relation \((5.13)\) which would be needed to extract a third charm baryon mass. The masses \( \Xi_Q^* = 2578.5 \pm 4.8 \) MeV and \( \Omega_Q^* = 2758.0 \pm 11.7 \) MeV are obtained using the flavor-27 sextet mass relations \((5.11)\). A different, more precise, extraction of these masses will be obtained shortly.

The analysis of Sec. IV showed that additional constraints on the heavy baryon hierarchy follow from studying heavy quark flavor symmetry violation and heavy quark number dependence in the heavy baryon mass splittings which do not violate heavy quark spin symmetry. For the \(SU(3)\) analysis, these mass splittings correspond to the operators \( I_5, J_2, T^8, J_2G^8 \), and \( \{T^8, T^8\} \) times \( I_5 \) and \( N_Q \), respectively. It is useful to study the mass combinations corresponding to these operators since the leading order contribution of each \( J_Q = 0 \) \(SU(3)\) splitting cancels out of the corresponding heavy quark flavor-violating and heavy quark number-dependent splitting.

The \( I_5 = 1 \) heavy quark flavor symmetry-violating mass splittings are tabulated in Table VI. These mass combinations correspond to splittings between heavy charm and bottom baryons, and are suppressed by a relative factor of \(1/N_c\) times \((1/2m_c - 1/2m_b)\) in comparison to the leading heavy quark flavor symmetric contribution to the heavy baryon mass splitting except for the \(I_5^4\) splitting which is proportional to \((m_c - m_b)\). The most accurate \( I_5 = 1 \) mass relations in Table VI can be used to predict the bottom baryon mass splittings from the corresponding charm baryon mass splittings.

Heavy quark number-violating mass splittings between baryons containing a single heavy quark \( Q \) and baryons containing no heavy quarks are given in Table VII. These mass combinations relate heavy baryon mass splittings to splittings amongst the spin-\( \frac{1}{2} \) octet and spin-\( \frac{3}{2} \) decuplet baryons. The mass combinations are suppressed by a relative factor of \((1/N_c)\) in comparison to the leading contribution to the heavy baryon mass splitting (except for the \(N_Q\) mass splitting which is proportional to \(m_Q\)). The \( N_Q \) and \( N_QJ^2_2 \) mass combinations are the \(SU(3)\) generalizations of the \(SU(2)\) mass combinations studied in Sec. IV. The linear combination of octet and decuplet masses appearing for these operators are the \(I^4_1\) and \(J^2_2\) mass combinations for \(N_Q = 0\) baryons derived in Ref. \[3\]. The derivation of the flavor octet and flavor-27 mass combinations is more subtle. There are five flavor octet \((J_Q = 0)\) operators amongst the \(N_Q = 0\) and \(N_Q = 1\) baryons: \(T^8, J_1G^8, (J_2)^2T^8, N_QT^8\) and \(N_QJ_1G^8\). It is important to note that the 4-body operator \(N_Q(J_1)^2T^8\) is a redundant operator. The mass combination corresponding to \(N_QJ_1G^8\) is the mass combination with vanishing matrix elements of the remaining four operators. Since the matrix element of \((J_1)^2T^8\) must vanish on this combination, the linear combination of octet and decuplet masses appearing in the \(N_QJ_1G^8\) mass combination is not simply the \(J_1G^8\) mass combination for \(N_Q = 0\) baryons. \[3\] Similar remarks apply for the flavor 27 mass combinations. There are three flavor-27 \((J_Q = 0)\) operators amongst the \(N_Q = 0\) and \(N_Q = 1\) baryons: \(\{T^8, T^8\}, \{T^8, J_1G^8\}\) and \(N_Q\{T^8, T^8\}\). The 4-body operator \(N_Q\{T^8, J_1G^8\}\) is a redundant operator. The mass combination corresponding to the \(N_Q\{T^8, T^8\}\) operator is the mass combination with vanishing matrix elements of the two other operators.

There are two accurate mass relations obtained by neglecting heavy quark number-violating operators. Neglect of the flavor-27 operator \(N_Q\{T^8, T^8\}\) yields the mass relation

\[
\frac{1}{6} \left[ (\Sigma_Q + 2\Sigma_Q^*) - 2 (\Xi_Q + 2\Xi_Q^*) + (\Omega_Q + 2\Omega_Q^*) \right] =
\frac{1}{3} \left[ \frac{1}{4} (2N - 3\Sigma - 2\Xi + \frac{1}{7} (4\Delta - 5\Sigma^* - 2\Xi^* + 3\Omega) \right],
\]

\[(5.15)\]
Eq. (5.15) is a factor of \( \frac{1}{N_c} \) with the estimated accuracy of Eq. (5.9). Neglect of the relation Eq. (5.9), which sets the heavy baryon mass combination equal to zero. The linear combination of octet and decuplet masses on the right-hand side of the equation equals \(-4.43 \text{ MeV} \) with negligible error, which agrees with the estimated accuracy of Eq. (5.9) and (5.16) yields a prediction for the heavy baryon mass splitting \( \Xi \) mass is significantly larger than the unpublished WA-89 measurement of \(~2560 \text{ MeV} \). All of the other charm baryon masses are accurately measured, so the predicted masses for \( \Sigma_c^* \), \( \Xi_c^* \) and \( \Omega_c^* \) together with the measured charm baryon masses can be used to evaluate the mass combinations of the combined \( 1/m_Q \) and \( 1/N_c \) expansion.

The numerical values of the mass combinations then can be compared with the theoretical hierarchy predicted by \( 1/m_Q \), \( 1/N_c \) and \( SU(3) \) flavor-breaking suppressions.

Using the derived masses and the measured masses of all the other charmed baryons, it is possible to evaluate the mass hierarchy of Table V. These numbers are listed in the column labeled Expt in Table V. Good agreement with the theoretical hierarchy is found. Only the predictions for the \( J_Q^2 \) and \( J_Q^0 \) mass combinations and the two mass combinations involving \( \Xi, \Xi' \) can be regarded as pure predictions since the choice of parameters \( m_c, \Lambda, (\epsilon \Delta \chi \epsilon) \) and \( (\epsilon^2 \Delta \chi \epsilon) \) affects four of the mass combinations, and relation (5.7) was imposed to extract the unknown and inaccurately measured charm baryon masses.

It also is worthwhile to study the implications of the predictions Eq. (5.13) for the charm baryon mass spectrum. The chromomagnetic mass splittings are evaluated to be

\[
\begin{align*}
(\Sigma_c^* - \Sigma_c) &= 79.6 \pm 5.3 \text{ MeV}, \\
(\Xi_c^* - \Xi_c) &= 61.1 \pm 3.0 \text{ MeV}, \\
(\Omega_c^* - \Omega_c) &= 42.6 \pm 7.3 \text{ MeV},
\end{align*}
\]

which implies that the linear in strangeness number contribution to the chromomagnetic splittings is negative, so that \( (\Sigma_c^* - \Sigma_c) > (\Xi_c^* - \Xi_c) > (\Omega_c^* - \Omega_c) \). This ordering seems reasonable based on intuition from the quark model: in the quark model, the hyperfine operator \( J_q \cdot J_Q \) is suppressed by \( 1/m_Q m_Q \), which is a greater suppression for \( q = s \) than for \( q = u, d \). The differences of these splittings are equal as dictated by Eq. (5.14), and given by \( 18.5 \pm 4.6 \text{ MeV} \), so Eq. (5.14) which holds at this level is not very accurate. The spin-averaged sextet masses are evaluated to be

\[
\begin{align*}
\frac{1}{3} (\Sigma_c + 2 \Sigma_c^*) &= 2506.0 \pm 3.5 \text{ MeV}, \\
\frac{1}{3} (\Xi_c + 2 \Xi_c^*) &= 2623.6 \pm 1.4 \text{ MeV}, \\
\frac{1}{3} (\Omega_c + 2 \Omega_c^*) &= 2732.4 \pm 4.3 \text{ MeV}.
\end{align*}
\]

The sextet mass differences

\[
\frac{1}{3} (\Sigma_c + 2 \Sigma_c^*) - \frac{1}{3} (\Sigma_c + 2 \Sigma_c^*) = 117.6 \pm 3.8 \text{ MeV},
\frac{1}{3} (\Omega_c + 2 \Omega_c^*) - \frac{1}{3} (\Xi_c + 2 \Xi_c^*) = 108.8 \pm 4.5 \text{ MeV},
\]

are equal at the 8.8 MeV level. The \( J_Q^2 \) hyperfine splittings in each strangeness sector are

\[
\frac{1}{3} (\Sigma_c + 2 \Sigma_c^*) - \Lambda_c = 221.0 \pm 3.6 \text{ MeV},
\frac{1}{3} (\Xi_c + 2 \Xi_c^*) - \Xi_c = 155.9 \pm 1.8 \text{ MeV}.
\]
The difference of these splittings is predicted to be large,
\[
\left[ \frac{1}{3} (\Sigma_c + 2\Sigma_c^*) - \Lambda_c \right] - \left[ \frac{1}{3} (\Xi_c' + 2\Xi_c') - \Xi_c \right] = 65.1 \pm 4.0 \text{ MeV.} \tag{5.24}
\]

Finally, now that all the charm baryon masses have been determined, it is possible to return to the problem of predicting the bottom baryon masses from the charm baryon masses. The heavy quark spin-violating splittings of the bottom baryons can be obtained by rescaling the charm baryon mass splittings by a factor \((m_c/m_b) \sim 0.3\). Thus, the chromomagnetic mass splittings of the bottom baryons are predicted to be
\[
(\Sigma_b^* - \Sigma_b) \sim 23.8 \pm 1.6 \text{ MeV},
\]
\[
(\Xi_b^* - \Xi_b) \sim 18.3 \pm 0.9 \text{ MeV},
\]
\[
(\Omega_b^* - \Omega_b) \sim 12.8 \pm 2.2 \text{ MeV},
\]
where the precise numerical values in Eq. (5.25) will change outside errors if a different scale factor is used since the error on the scale factor has not been taken into account. For example, using the more rigorous scale factor \((Z_b/Z_c)(m_c/m_b) \sim 0.24\) reduces the central values by about 5 MeV. The mass relation Eq. (5.15) and the second flavor octet mass relation of Table VI yield precise predictions for two additional bottom baryon mass combinations
\[
\frac{1}{6} \left[ (\Sigma_b + 2\Sigma_b^*) - 2(\Xi_b + 2\Xi_b^*) + (\Omega_b + 2\Omega_b^*) \right]
\]
\[
= -4.43 \pm 1.5 \text{ MeV},
\]
\[
\left\{ -\frac{5}{8} (\Lambda_b - \Xi_b) \right. \left. \right\}
\]
\[
+ \frac{1}{24} \left[ 3(\Sigma_b + 2\Sigma_b^*) - (\Xi_b + 2\Xi_b^*) - 2(\Omega_b + 2\Omega_b^*) \right]
\]
\[
= 42.89 \pm 2.1 \text{ MeV},
\]
where the errors represent the combined theoretical and experimental accuracy of each relation. There are two additional mass relations which are less accurate, namely the second and third mass relations in Table VI, with theoretical accuracies of about 4.8 and 5.1 MeV, respectively:
\[
(\Lambda_b - \Xi_b) = -182.7 \pm 4.9 \text{ MeV},
\]
\[
\frac{1}{3} (\Lambda_b + 2\Xi_b) + \frac{1}{18} \left[ 3(\Sigma_b + 2\Sigma_b^*) + 2(\Xi_b + 2\Xi_b^*) + (\Omega_b + 2\Omega_b^*) \right]
\]
\[
= 176.1 \pm 5.4 \text{ MeV.}
\]

Combined with the measured \(\Lambda_b\) mass, these seven constraints determine the seven masses \(\Xi_b, \Sigma_b, \Xi_b^*, \Omega_b, \Sigma_b^*, \Xi_b^*, \text{ and } \Omega_b^*.\) The three chromomagnetic splittings were given already in Eq. (5.23). The four mass relations in Eq. (5.26) and (5.27) together with the \(\Lambda_b\) measurement determine the four spin-averaged mass combinations
\[
\frac{1}{3} (\Lambda_b + 2\Xi_b) = 5744.8 \pm 5.8 \text{ MeV},
\]
\[
\frac{1}{3} (\Sigma_b + 2\Sigma_b^*) = 5844.0 \pm 8.9 \text{ MeV},
\]
\[
\frac{1}{3} (\Xi_b' + 2\Xi_b) = 5961.6 \pm 10.8 \text{ MeV},
\]
\[
\frac{1}{3} (\Omega_b + 2\Omega_b^*) = 6070.3 \pm 23.6 \text{ MeV},
\]
where the precision of the extraction is limited by the theoretical accuracy of the least accurate mass relations. Once two additional bottom baryon masses are measured, a more precise extraction of the remaining unmeasured masses will be possible using only the most accurate mass relations in Eq. (5.23). Some linear combinations of the spin-averaged mass combinations are determined accurately; however. For example, the two mass relations in Eq. (5.26) imply
\[
\left[ \frac{1}{3} (\Sigma_b + 2\Sigma_b^*) - \Lambda_b \right] - \left[ \frac{1}{3} (\Xi_b' + 2\Xi_b) - \Xi_b \right]
\]
\[
= 65.1 \pm 3.6 \text{ MeV.} \tag{5.29}
\]

VI. ISOSPIN-VIOLATING MASS SPLITTINGS

Isospin-violating mass splittings are analyzed in this section for completeness. Almost all isospin splittings will be at the sub-MeV level, so isospin symmetry is a very good symmetry for the heavy baryon masses. Isospin breaking arises due to differences in the \(u\) and \(d\) quark masses and electromagnetic charges. These two sources of isospin breaking will be denoted by the parameters \(\epsilon\) and \(\epsilon'\). Isospin breaking due to quark mass differences is purely \(I = 1\), whereas the electromagnetic mass splittings are second order in the quark charge matrix and can be \(I = 1, 2\). Both sources of isospin breaking produce comparable mass splittings in QCD, so \(\epsilon' \sim \epsilon''\). All \(I = 2\) mass splittings are electromagnetic mass splittings suppressed by \(\alpha_{EM}/4\pi\), and will be proportional to the parameter \((\epsilon''\Lambda_x)\). The \(I = 1\) mass splittings can arise from either source of isospin breaking. Since these effects are comparable, the splittings will be written in terms of the parameter \((\epsilon''\Lambda_x)\).

The \(I = 1\) flavor octet, \((10 + \overline{10})\) and 27 mass expansions are given by
\[
M^{(10+\overline{10})} = T^3 + \frac{1}{N_c} J^i G^{i3} + \frac{1}{N_c m_Q} \frac{2}{m^2} J^i Q^i G^{i3} + \frac{1}{N_c^2 m_Q^2} (J_i \cdot J_Q) T^3,
\]
\[
M^{(27)} = \frac{1}{2} \left[ T^3, T^3 \right] + \frac{1}{N_c} \left[ T^3, G^{i8} \right] + \frac{1}{N_c^2 m_Q} J^i Q^i \left[ \{T^3, G^{i8} \} + \{T^8, G^{i3} \} \right],
\]
whereas the \(I = 2\) flavor-27 mass expansion is given by
\[
M^{(27)} = \frac{1}{N_c} \left[ T^3, T^3 \right] + \frac{1}{N_c^2 m_Q} J^i Q^i \left[ T^3, G^{i3} \right].
\]
The mass combinations corresponding to these operators are tabulated in Table VIII for $Q = c$ and Table IX for $Q = b$. Table X combines the operator matrix element, $1/m_Q$, $1/N_c$ and flavor-breaking suppression factors to predict a hierarchy of mass relations. The numerical values of the theoretical hierarchy are evaluated for canonical values of the parameters in the columns labeled $Q = c$ and $Q = b$. The largest isospin mass splitting is $(\Xi^0_5 - \Xi^0_8)$ or $(\Xi^0_6 - \Xi^0_3)$, which are equal and of order a few MeV in magnitude. These splittings can be used to determine the isospin breaking parameter $(\epsilon'\Lambda_{\chi})$ which sets the scale of the mass hierarchy. Experimentally, $(\Xi^0_5 - \Xi^0_8) = 5.2 \pm 2.2$ MeV [14]. The next largest mass splitting is the $I = 2$ mass combination

$$\frac{1}{10} (\Sigma_c^{++} - 2\Sigma_c^+ + \Sigma_c^0) + \frac{2}{5} (\Sigma_c^{++} - 2\Sigma_c^+ + \Sigma_c^0) \quad (6.3)$$

or the analogous splitting for $Q = b$, which will be about $3.5 \pm 1.5$ MeV. The $I = 1$ mass splitting

$$-\frac{5}{8} (\Xi_c^+ - \Xi_c^0) + \frac{1}{24} [2(\Sigma_c^{++} - \Sigma_c^0) + (\Xi_c' - \Xi_c^0)]$$

$$+ \frac{1}{12} [2(\Sigma_c^{++} - \Sigma_c^0) + (\Xi_c' - \Xi_c^0)] \quad (6.4)$$

or the $Q = b$ analogue will both be about $1.0 \pm 0.5$ MeV. All remaining isospin mass combinations are predicted to be sub-MeV. The mass relations obtained by neglect of these operators are given in Tables VIII and IX.

**VII. CONCLUSIONS**

It has been shown that there are light quark and heavy quark spin-flavor symmetries for baryons containing a single heavy quark in the combined heavy quark and large $N_c$ limits. A (spin⊗flavor) operator expansion in $1/N_c$ and $1/m_Q$ has been constructed for heavy quark baryons at finite $m_Q$ and $N_c$. Heavy quark spin symmetry is present for $N_c \to \infty$ or for $m_Q \to \infty$, so any violation of heavy quark spin symmetry is suppressed by $(1/N_c m_Q)$. In the presence of heavy quark flavor symmetry, heavy quark spin-flavor symmetry is present for $N_c \to \infty$. For the physical situation of two heavy flavors $Q = c$ and $Q = b$, heavy quark flavor symmetry is justified by the heavy quark limit [1]. Heavy quark spin-flavor symmetry $SU(4)_h$ is a better symmetry for baryons than for mesons because violation of the symmetry is suppressed by $(1/N_c m_Q)$, rather than $1/m_Q$. In addition to heavy quark spin-flavor symmetry, there is a light quark $SU(6)$ spin-flavor symmetry for heavy quark baryons in the large $N_c$ limit [3].

The masses of baryons containing a single heavy quark have been analyzed in a combined expansion in $1/m_Q$, $1/N_c$, and $SU(3)$ flavor symmetry breaking. The naive $1/N_c$ scalings work beautifully for the baryon masses. A mass hierarchy is predicted by the $1/m_Q$, $1/N_c$ and flavor-breaking expansion. The most suppressed operators yield mass relations which are well-satisfied experimentally. The precision of the mass relations and the magnitude of mass combinations are predicted by the expansion. Heavy baryon mass splittings have been related to octet and decuplet mass splittings. The most accurate mass relations, Eq. (7.7), (5.13), and (7.16), have been used to predict the unmeasured or poorly measured charm baryon masses $\Sigma^c_9$, $\Omega^c_9$ and $\Omega^c_8$ to a precision of several MeV. A number of interesting features of the charm baryon mass spectrum are found. These include: (i) the mass splitting

$$\left[\frac{1}{3} (\Sigma_c + 2\Sigma_c') - \Lambda_c\right] - \left[\frac{1}{3} (\Xi_c' + 2\Xi_c) - \Xi_c\right] \quad (7.2)$$

is predicted to be large, (ii) the chromomagnetic hyperfine mass splittings satisfy a reverse hierarchy,

$$(\Sigma_c^* - \Sigma_c) > (\Xi_c^* - \Xi_c) > (\Omega_c^* - \Omega_c), \quad (7.3)$$

and (iii) the equal spacing rule [22]

$$(\Sigma_c^* - \Sigma_c) - (\Xi_c^* - \Xi_c) = (\Xi_c^* - \Xi_c) - (\Omega_c^* - \Omega_c) \quad (7.4)$$

is almost exact. Bottom baryon mass splittings are predicted in terms of charm baryon mass splittings and mass splittings of the octet and decuplet baryons. Some bottom baryon mass splittings are predicted very accurately.

As this work neared completion, manuscript [23] appeared. The present analysis shows that Eq. (2d) of [23] is not well satisfied.

Noted added in proof: There are no renormalon ambiguities in the operator coefficients which correspond to physical mass splittings. The renormalon ambiguity of the heavy quark mass $m_Q$ and $\Lambda_{\chi}^{baryon} \sim N_c A_{QCD}$ is of order $A_{QCD}$, and formally appears as a term $N_Q A_{QCD}$ in the expansion (4.13).

The two very accurate mass relations Eqs. (5.7) and (5.15) yield very precise predictions for $\Xi_c^*$ and $(\Sigma_c^* + \Omega_c^*)$. A new CLEO measurement of $\Sigma_c^* = 2518.6 \pm 2.2$ MeV is significantly lower than previous measurements. In addition, there is a somewhat more accurate value for $\Omega_c = 2699.9 \pm 2.9$ MeV from the E687 Collaboration. The more accurate $\Omega_c$ value changes the $\Xi_c^*$ prediction slightly to

$$\Xi_c^* = 2580.8 \pm 2.1 \text{MeV}.$$
The lower $\Sigma^*$ mass implies a larger value for the predicted $\Omega^*$ mass,
\[
\Omega^* = 2760.6 \pm 6.4 \text{MeV}.
\]
The mass combination quoted in Eqs. (5.20)–(5.29) will be modified accordingly. The only significant modification is that the differences of the chromomagnetic mass splittings $(\Sigma_Q^* - \Sigma_Q)$, $(\Xi_Q^* - \Xi_Q')$ and $(\Omega_Q^* - \Omega_Q)$ are much smaller than before, which implies that relation (5.13) is as small as possible. This occurs if the magnitude of the first mass combination in Eq. (5.12) is dominated by the $\Xi_Q\Xi_Q'$ mixing term.

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Figure Captions

FIG. 1. Baryon representation of $SU(2N_F) \times SU(2)_Q \times SU(2)_{Q'}$ spin-flavor symmetry for large finite $N_c$. The total number of boxes equals $N_c = N_{\ell} + N_h$ where $N_h = N_Q + N_{Q'}$.
TABLE I. Mass splittings of baryons containing a single heavy quark $Q$ for strangeness $S = 0$ and $S = -1$ baryons. Each operator is an isospin singlet; the $J_t \cdot J_Q$ operator violates heavy quark spin symmetry. The operator matrix element and $1/m_Q$, $1/N_c$ and isospin flavor-breaking suppressions of each mass combination are tabulated. The singlet mass combination has a contribution $m_Q$ and a contribution of order $N_c$ at leading order, i.e. $m_Q + N_c \Lambda$.

| Operator | $(SU(2), J_Q)$ | Mass Combination | $\langle O \rangle$ | $1/m_Q$ | $1/N_c$ | Flavor |
|----------|----------------|------------------|---------------------|---------|---------|--------|
| $I^b_1$  | (1, 0)         | $\Lambda_Q$     | 1                   | $*$     | 1       | 1      |
| $J^2_t$  | (1, 0)         | $\frac{1}{3} (\Sigma_Q + 2\Sigma_Q^*) - \Lambda_Q$ | 2 | 1 | $1/N_c$ | 1 |
| $J_t \cdot J_Q$ | (1, 1) | $\Sigma_Q - \Sigma_Q^*$ | $\frac{3}{2}$ | $2/m_Q$ | $1/N_c$ | 1 |
| $I^b_1$  | (1, 0)         | $\Xi_Q^*$       | 1                   | $*$     | 1       | 1      |
| $J^2_t$  | (1, 0)         | $\frac{1}{3} (\Xi_Q^* + 2\Xi_Q^*) - \Xi_Q$ | 2 | 1 | $1/N_c$ | 1 |
| $J_t \cdot J_Q$ | (1, 1) | $\Xi_Q^* - \Xi_Q$ | $\frac{3}{2}$ | $2/m_Q$ | $1/N_c$ | 1 |

TABLE II. $I_b = 1$ heavy baryon mass splittings which violate heavy quark flavor symmetry, but preserve heavy quark spin symmetry for strangeness $S = 0$ and $S = -1$ baryons. The mass splittings are suppressed by $(1/2m_c - 1/m_b)$ and one additional factor of $1/N_c$. The $I^2_b$ mass combination has a leading contribution of $(m_c - m_b)$.

| Operator | $(SU(2), J_Q)$ | Mass Combination | $\langle O \rangle$ | $1/m_Q$ | $1/N_c$ | Flavor |
|----------|----------------|------------------|---------------------|---------|---------|--------|
| $I^2_b$  | (1, 0)         | $(\Lambda_c - \Lambda_b)$ | 1 | $(m_c - m_b)$ | 1 | 1 |
| $J^2_t$  | (1, 0)         | $\left[\frac{1}{3} (\Sigma_c + 2\Sigma_c^*) - \Lambda_c\right] - \left[\frac{1}{3} (\Sigma_b + 2\Sigma_b^*) - \Lambda_b\right]$ | 2 | $\left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)$ | $1/N_c^2$ | 1 |
| $I^2_b$  | (1, 0)         | $(\Xi_c - \Xi_b)$ | 1 | $(m_c - m_b)$ | 1 | 1 |
| $J^2_t$  | (1, 0)         | $\left[\frac{1}{3} (\Xi_c^* + 2\Xi_c^*) - \Xi_c\right] - \left[\frac{1}{3} (\Xi_b^* + 2\Xi_b^*) - \Xi_b\right]$ | 2 | $\left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)$ | $1/N_c^2$ | 1 |

TABLE III. Mass splittings between baryons containing a single heavy quark $Q$ and baryons containing no heavy quarks for strangeness $S = 0$ and $S = -1$ baryons. The mass splittings are suppressed by an additional factor of $1/N_c$. The $N_Q$ mass combination has a leading order contribution of $m_Q$.

| Operator | $(SU(2), J_Q)$ | Mass Combination | $\langle O \rangle$ | $1/m_Q$ | $1/N_c$ | Flavor |
|----------|----------------|------------------|---------------------|---------|---------|--------|
| $N_Q$    | (1, 0)         | $\Lambda_Q - \frac{1}{4} (5N - \Delta)$ | 1 | $m_Q$ | 1 | 1 |
| $N_QJ^2_t$ | (1, 0) | $\left[\frac{1}{3} (\Sigma_Q + 2\Sigma_Q^*) - \Lambda_Q\right] - \frac{2}{3} (\Delta - N)$ | 2 | 1 | $1/N_c^2$ | 1 |
| $N_Q$    | (1, 0)         | $\Xi_Q - \frac{1}{4} \left[\frac{5}{3} (3\Sigma + \Lambda) - 2\Sigma^*\right]$ | 1 | $m_Q$ | 1 | 1 |
| $N_QJ^2_t$ | (1, 0) | $\left[\frac{1}{3} (\Xi_Q^* + 2\Xi_Q^*) - \Xi_Q\right] - \frac{2}{3} \left[\Sigma^* - \frac{1}{4} (3\Sigma + \Lambda)\right]$ | 2 | 1 | $1/N_c^2$ | 1 |
TABLE IV. Mass splittings of baryons containing a single heavy quark $Q$.  $SU(3)$ flavor and heavy quark spin $J_Q$ symmetry quantum numbers of each operator and mass combination are given explicitly. The operator matrix element for each mass combination and suppression factors of $1/m_Q$, $1/N_c$ and $SU(3)$ flavor-breaking $\epsilon$ are tabulated. The singlet operator is $m_Q + N_c \Lambda$ at leading order.

| Operator | $(SU(3), J_Q)$ | Mass Combination | $\langle Q \rangle$ | $1/m_Q$ | $1/N_c$ | Flavor |
|----------|----------------|-----------------|----------------|----------|--------|--------|
| $1$      | (1, 0)         | $\frac{1}{3} (\Lambda_Q + 2\Xi_Q)$ | 1             | *        | *      | 1      |
| $J_1^2$  | (1, 0)         | $-\frac{3}{8} (\Lambda_Q + 2\Xi_Q) \pm \frac{1}{2i} [3 (\Sigma_Q + 2 \Sigma_Q^*) + 2 (\Xi_Q + 2 \Xi_Q^*) + (\Omega_Q + 2 \Omega_Q^*)]$ | 2             | 1        | $1/N_c$ | 1      |
| $J_3 \cdot J_Q$ | (1, 1) | $\frac{1}{6} [3 (\Sigma_Q - \Sigma_Q) + 2 (\Xi_Q - \Xi_Q) + (\Omega_Q - \Omega_Q)]$ | $\frac{3}{4}$ | $2/m_Q$ | $1/N_c$ | 1      |
| $T^8$    | (8, 0)         | $(\Lambda_Q - \Xi_Q)$ | $\frac{3}{4}$ | 1        | 1      | $\epsilon$ |
| $J_5^* G^{\text{18}}$ | (8, 0) | $-\frac{\Xi}{8} (\Lambda_Q - \Xi_Q) \pm \frac{1}{2\sqrt{3}} [3 (\Sigma_Q + 2 \Sigma_Q^*) - (\Xi_Q + 2 \Xi_Q^*) - 2 (\Omega_Q + 2 \Omega_Q^*)]$ | $\frac{1}{2} \frac{15}{\sqrt{3} N_c}$ | 1        | $1/N_c$ | $\epsilon$ |
| $J_3^* G^{\text{G18}}$ | (8, 1) | $\Xi_Q \Xi_Q$ | $\frac{3}{8}$ | $2/m_Q$ | $1/N_c$ | $\epsilon$ |
| $(J_3 \cdot J_Q) T^8$ | (8, 1) | $\frac{1}{6} [3 (\Sigma_Q^* - \Sigma_Q) - (\Xi_Q^* - \Xi_Q^*) - 2 (\Omega_Q - \Omega_Q)] - \frac{\Xi}{2\sqrt{3}} = \Xi_Q^\prime$ | $\frac{1}{2} \frac{15}{\sqrt{3} N_c}$ | $2/m_Q$ | $1/N_c^2$ | $\epsilon$ |
| $\{ T^8, T^8 \}$ | (27, 0) | $\frac{1}{6} \left[ (\Sigma_Q + 2 \Sigma_Q^*) - 2 (\Xi_Q + 2 \Xi_Q^*) + (\Omega_Q + 2 \Omega_Q^*) \right]$ | $\frac{9}{16}$ | $2/m_Q$ | $1/N_c^2$ | $\epsilon$ |
| $J_5^* \{ T^8, G^{\text{G18}} \}$ | (27, 1) | $\frac{1}{15} \left[ (\Sigma_Q^* - \Sigma_Q) - 2 (\Xi_Q^* - \Xi_Q) + (\Omega_Q^* - \Omega_Q) \right]$ | $\frac{9}{16}$ | $2/m_Q$ | $1/N_c^2$ | $\epsilon$ |

TABLE V. Heavy baryon mass hierarchy for $Q = c$ and $Q = b$. The experimental charm baryon mass splittings are evaluated using theoretically extracted values for the three masses $\Sigma'_c = 2532.5 \pm 5.3$ MeV, $\Xi'_c = 2582.9 \pm 2.5$ MeV and $\Omega'_c = 2746.6 \pm 6.1$ MeV. Agreement between the theory values appearing with an asterisk and experiment is imposed by the determination of the parameter values $m_c$, $\Lambda$, $(\epsilon \Lambda_c)$ and $(\epsilon^2 \Lambda_c)$. Comparison of the remaining theory and experimental values provides support for the theoretical hierarchy.

| Mass Combination | Theory | $Q = c$ | Expt $Q = c$ | $Q = b$ |
|------------------|--------|---------|-------------|---------|
| $\frac{1}{3} (\Lambda_Q + 2\Xi_Q)$ | $m_Q + N_c \Lambda$ | 2380* | 2406.8 ± 0.8 | 5687 |
| $-\frac{1}{15} (\Lambda_Q + 2\Xi_Q) \pm \frac{1}{15} [3 (\Sigma_Q + 2 \Sigma_Q^*) + 2 (\Xi_Q + 2 \Xi_Q^*) + (\Omega_Q + 2 \Omega_Q^*)]$ | $2\frac{1}{15} \Lambda$ | 207 | 176.1 ± 1.9 | 207 |
| $\frac{1}{6} [3 (\Sigma_Q - \Sigma_Q) + 2 (\Xi_Q - \Xi_Q) + (\Omega_Q - \Omega_Q)]$ | $3\frac{1}{6} \frac{\Lambda^2}{m_Q}$ | 66* | 67.3 ± 3.1 | 20 |
| $\Lambda_Q - \Xi_Q$ | $\frac{3}{2\sqrt{3}} \frac{\epsilon \Lambda_c}{m_Q}$ | -195* | -182.7 ± 1.3 | -195 |
| $\Xi_Q \Xi_Q^\prime$ | $\frac{1}{2\sqrt{3}} \frac{15}{8} \frac{1}{N_c} \frac{\Lambda}{m_Q} (\epsilon \Lambda_c)$ | 40.6 | 42.9 ± 1.9 | 40.6 |
| $\frac{1}{6} \left[ (\Sigma_Q + 2 \Sigma_Q^*) - 2 (\Xi_Q + 2 \Xi_Q^*) + (\Omega_Q + 2 \Omega_Q^*) \right]$ | $\frac{9}{16} \frac{\Lambda}{m_Q}$ | 11.8 | -- | 3.5 |
| $\Xi_Q^\prime$ | $\frac{3}{2\sqrt{3}} \frac{15}{8} \frac{1}{N_c} \frac{\Lambda}{m_Q} (\epsilon \Lambda_c)$ | 11.3 | (15.4 ± 3.6) - $\frac{5}{2\sqrt{3}} \Xi_c$ | 3.4 |
| $-\frac{\Xi}{2\sqrt{3}} = \Xi_Q^\prime$ | $\frac{9}{16} \frac{\Lambda}{m_Q} (\epsilon^2 \Lambda_c)$ | -4.4* | -4.4 ± 3.1 | -4.4 |
| $\frac{1}{6} \left[ (\Sigma_Q^* - \Sigma_Q) - 2 (\Xi_Q^* - \Xi_Q) + (\Omega_Q^* - \Omega_Q) \right]$ | 0.23 | 0 ± 2.7 | 0.07 |
| Operator | $(SU(3), J_{Q})$ | Mass Combination | $\langle O \rangle$ | $1/m$ | $1/N_c$ | Flavor |
|----------|------------------|------------------|-----------------|--------|----------|-------|
| $I_h^3$  | (1, 0)           | $\frac{1}{3} (\Lambda_c + 2\Xi_c) - \frac{1}{3} (\Lambda_b + 2\Xi_b)$ | 1               | $m_c - m_b$ | 1        | 1     |
| $I_h^3(J_5)^2$ | (1, 0)          | $\left[ -\frac{1}{3} (\Lambda_c + 2\Xi_c) + \frac{1}{12} (3\Sigma_c + 2\Xi'_c + \Omega_c) + \frac{1}{9} (3\Sigma'_c + 2\Xi'_c + \Omega'_c) \right]$ $- \left[ -\frac{1}{3} (\Lambda_b + 2\Xi_b) + \frac{1}{12} (3\Sigma_b + 2\Xi'_b + \Omega_b) + \frac{1}{9} (3\Sigma'_b + 2\Xi'_b + \Omega'_b) \right]$ | 2               | $\frac{1}{2m_c} - \frac{1}{2m_b}$ | $1/N_c^2$ | 1     |
| $I_h^3T^8$ | (8, 0)           | $\left( \Lambda_c - \Xi_c \right) - \left( \Lambda_b - \Xi_b \right)$ | $\frac{3}{2\sqrt{3}}$ $\left( \frac{1}{2m_c} - \frac{1}{2m_b} \right)$ | $1/N_c$ | $\epsilon$ |       |
| $I_h^3J_i^\nu G^i$ | (8, 0)           | $\left[ -\frac{5}{8} (\Lambda_c - \Xi_c) + \frac{1}{12} (3\Sigma_c - \Xi'_c - 2\Omega_c) + \frac{1}{12} (3\Sigma'_c - \Xi'_c - 2\Omega'_c) \right]$ $- \left[ -\frac{5}{8} (\Lambda_b - \Xi_b) + \frac{1}{12} (3\Sigma_b - \Xi'_b - 2\Omega_b) + \frac{1}{12} (3\Sigma'_b - \Xi'_b - 2\Omega'_b) \right]$ | $\frac{1}{2\sqrt{3}} \frac{15}{8}$ $\left( \frac{1}{2m_c} - \frac{1}{2m_b} \right)$ | $1/N_c^2$ | $\epsilon$ |       |
| $I_h^3\{T^8, T^8\}$ | (27, 0)          | $\left[ \frac{1}{3} (\Sigma_c - 2\Xi'_c + \Omega_c) + \frac{1}{3} (\Sigma'_c - 2\Xi'_c + \Omega'_c) \right]$ $- \left[ \frac{1}{3} (\Sigma_b - 2\Xi'_b + \Omega_b) + \frac{1}{3} (\Sigma'_b - 2\Xi'_b + \Omega'_b) \right]$ | $\frac{3}{2}$ $\left( \frac{1}{2m_c} - \frac{1}{2m_b} \right)$ | $1/N_c^2$ | $\epsilon^2$ |       |
| Operator | \((SU(3), J_Q)\) | Mass Combination | \(|O|\) | \(1/m_Q\) | \(1/N_c\) | Flavor |
|----------|-----------------|------------------|------|--------|--------|-------|
| \(N_Q\) | (1, 0) | \(\frac{1}{3} (\Lambda_Q + 2\Xi_Q) - \frac{1}{3} \left[ \frac{5}{3} (2N + 3\Sigma + \Lambda + 2\Xi) - \frac{1}{10} (4\Delta + 3\Sigma^* + 2\Xi^* + \Omega) \right] \) | 1 | \(m_Q\) | 1 | 1 |
| \(N_Q J^2\) | (1, 0) | \(-\frac{5}{3} (\Lambda_Q - \Xi_Q) + \frac{1}{15} \left[ 3 (\Sigma_Q + 2\Sigma_Q^*) - (\Xi'_Q + 2\Xi'_Q) - 2 (\Omega_Q + 2\Omega_Q^*) \right] \) | 2 | 1 | \(1/N^2\) | 1 |
| \(N_Q T^{8}\) | (8, 0) | \((\Lambda_Q - \Xi_Q) - \frac{1}{8} (6N - 3\Sigma + \Lambda - 4\Xi) + \frac{1}{20} (2\Delta - 3\Xi - \Omega) \) | \(\frac{3}{2\sqrt{3}}\) | 1 | \(1/N_c\) | \(\epsilon\) |
| \(N_Q J^2 G^{18}\) | (8, 0) | \(-\frac{5}{8} (\Lambda_Q - \Xi_Q) + \frac{1}{27} \left[ 3 (\Sigma_Q + 2\Sigma_Q^*) - (\Xi'_Q + 2\Xi'_Q) - 2 (\Omega_Q + 2\Omega_Q^*) \right] \) | \(\frac{1}{2\sqrt{3}}\) | 1 | \(1/N^2\) | \(\epsilon\) |
| \(N_Q \{T^{8}, T^{8}\}\) | (27, 0) | \(\frac{1}{9} \left[ (\Sigma_Q + 2\Sigma_Q^*) - 2 (\Xi'_Q + 2\Xi'_Q) + (\Omega_Q + 2\Omega_Q^*) \right] + \frac{1}{9} \left[ -\frac{1}{9} (2N - 3\Sigma - 2\Xi) - \frac{1}{9} (4\Delta - 9\Sigma^* - 2\Xi^* + 3\Omega) \right] \) | \(\frac{3}{2}\) | 1 | \(1/N^2\) | \(\epsilon^2\) |
TABLE VIII. Isospin-violating mass splittings of baryons containing a single charm quark.

| Operator | \((SU(3), J_c)\) | Mass Combination | \(\langle O \rangle\) | \(1/m_c\) | \(1/N_c\) | Flavor |
|----------|-----------------|-----------------|----------------|------------|----------|--------|
| \(T^3\) | (8, 0)          | \((\Xi^+_c - \Xi^0_c)\) | 1              | 1          | 1        | \(\epsilon'\) |
| \(J^c_i G^{i3}\) | (8, 0)          | \(-\frac{5}{8} (\Xi^+ - \Xi^0) + \frac{1}{24} \left[ 2 (\Sigma^{+*} + \Sigma^0) + (\Xi^{+*} - \Xi^0) \right] + \frac{1}{24} \left[ 2 (\Sigma^{+*} + \Sigma^0) + (\Xi^{+*} - \Xi^0) \right]\) | \(\frac{5}{8}\) | 1 | \(1/N_c\) | \(\epsilon'\) |
| \(J^c_i G^{i3}\) | (8, 1)          | \(\frac{1}{2} \left[ A^+_c \Sigma^+_c + \frac{1}{2} (\Xi^+_c \Xi^+_c + \Xi^0_c \Xi^0_c) \right] \) | \(\frac{3 \sqrt{3}}{8}\) | 2/m_c | 1/N_c | \(\epsilon'\) |
| \((J_c \cdot J_c) T^3\) | (8, 1)          | \(-\frac{1}{6} \left[ 2 (\Sigma^{+*} + \Sigma^0) + (\Xi^{+*} - \Xi^0) \right] \) | \(\frac{1}{6}\) | 2/m_c | 1/N_c^2 | \(\epsilon'\) |
| \(\{T^3, T^8\}\) | (27, 0)         | \(\frac{1}{3} \left[ (\Sigma^{++} - \Sigma^{00}) - 2 (\Xi^+_c - \Xi^0_c) \right] \) | \(\frac{1}{\sqrt{3}}\) | 1 | 1/N_c | \(\epsilon'\) |
| \(J^c_i \{T^3, G^{i8}\} + \{T^8, G^{i3}\}\) | (27, 1)         | \(-\frac{1}{6} \left[ 2 (\Sigma^{+*} + \Sigma^0) + (\Xi^{+*} - \Xi^0) \right] \) | \(\frac{\sqrt{5}}{2}\) | 2/m_c | 1/N_c^2 | \(\epsilon'\) |
| \(J^c_i \{T^3, G^{i8}\} - \{T^8, G^{i3}\}\) | (10 + \(\overline{10}, 1\)) | \(-\frac{1}{6} \left[ 2 (\Sigma^{+*} + \Sigma^0) + (\Xi^{+*} - \Xi^0) \right] \) | \(\frac{2}{3}\) | 2/m_c | 1/N_c^2 | \(\epsilon'\) |
| \(\{T^3, T^3\}\) | (27, 0)         | \(\frac{1}{10} (\Sigma^{++} - 2 \Sigma^{+} + \Sigma^{0}) + \frac{2}{5} (\Sigma^{++} - 2 \Sigma^{+} + \Sigma^{0})\) | 2 | 1 | 1/N_c | \(\epsilon''\) |
| \(J^c_i \{T^3, G^{i3}\}\) | (27, 1)         | \(-\frac{1}{4} (\Sigma^{++} - 2 \Sigma^{+} + \Sigma^{0}) + \frac{1}{4} (\Sigma^{++} - 2 \Sigma^{+} + \Sigma^{0})\) | \(\frac{5}{8}\) | 2/m_c | 1/N_c^2 | \(\epsilon''\) |
TABLE IX. Isospin-violating mass splittings of baryons containing a single bottom quark.

| Operator                        | (SU(3), J₀) | Mass Combination | (O) | 1/mb | 1/N₀ | Flavor |
|--------------------------------|-------------|-----------------|-----|------|------|--------|
| I = 1                          |             |                 |     |      |      |        |
| \( T^3 \)                      | (8, 0)      | (\( \Xi^0_b - \Xi^-_b \)) | \( \frac{3\sqrt{2}}{8} \) | 1/mb | 1/N₀ | \( \epsilon' \) |
| \( J^i G^{i3} \)              | (8, 0)      | \(-\frac{3}{8} (\Xi^0_b - \Xi^-_b) + \frac{1}{2} (\Sigma^+_b - \Sigma^-_b) + (\Xi^0_b - \Xi^-_b)\) | \( \frac{3\sqrt{2}}{8} \) | 1/mb | 1/N₀ | \( \epsilon' \) |
| \( J^i G^{i3} \)              | (8, 1)      | \( \frac{1}{8} [\Lambda^0_0 \Sigma^0_0 + \frac{1}{2} (\Xi^0_b \Xi^0_b - \Xi^-_b \Xi^-_b)] \) | \( \frac{3\sqrt{2}}{8} \) | 2/mb | 1/N₀ | \( \epsilon' \) |
| \( (J_t \cdot J_b) T^3 \)     | (8, 1)      | \(-\frac{1}{8} [2 (\Sigma^+_b - \Sigma^-_b) + (\Xi^0_b - \Xi^-_b)] \) | \( \frac{3\sqrt{2}}{8} \) | 2/mb | 1/N₀ | \( \epsilon' \) |
| \( \{T^3, T^8\} \)            | (27, 0)     | \( \frac{1}{8} [(\Sigma^+_b - \Sigma^-_b) - 2 (\Xi^0_b - \Xi^-_b)] \) | \( \frac{1}{8} \sqrt{3} \) | 1/mb | 1/N₀ | \( \epsilon' \) |
| \( J^i_b \{\{T^3, G^{i8}\} + \{T^8, G^{i3}\}\} \) | (27, 1)     | \(-\frac{1}{8} [(\Sigma^+_b - \Sigma^-_b) - 2 (\Xi^0_b - \Xi^-_b)] \) | \( \frac{1}{8} \sqrt{3} \) | 2/mb | 1/N₀ | \( \epsilon' \) |
| \( J^i_b \{\{T^3, G^{i8}\} - \{T^8, G^{i3}\}\} \) | (10 + \( \bar{10} \), 1) | \(-\frac{1}{8} [\Lambda^0_0 \Sigma^0_0 - (\Xi^0_b \Xi^0_b - \Xi^-_b \Xi^-_b)] \) | \( \frac{3}{8} \) | 2/mb | 1/N₀ | \( \epsilon' \) |
| I = 2                          |             |                 |     |      |      |        |
| \( \{T^3, T^3\} \)            | (27, 0)     | \( \frac{1}{8} (\Sigma^+_b - 2 \Sigma^0_b + \Sigma^-_b) + \frac{2}{3} (\Sigma^+_b - 2 \Sigma^0_b + \Sigma^-_b) \) | \( \frac{3}{8} \sqrt{2} \) | 2/mb | 1/N₀ | \( \epsilon'' \) |
| \( J^i_b \{T^3, G^{i3}\} \)  | (27, 1)     | \(-\frac{1}{8} (\Sigma^+_b - 2 \Sigma^0_b + \Sigma^-_b) + \frac{4}{3} (\Sigma^+_b - 2 \Sigma^0_b + \Sigma^-_b) \) | \( \frac{3}{8} \sqrt{2} \) | 2/mb | 1/N₀ | \( \epsilon'' \) |
TABLE X. Hierarchy of heavy baryon mass splittings in decreasing order of magnitude for \( Q = c \) and \( Q = b \). Superscripts refer to the isospin \( I_3 \) of the baryon rather than the electromagnetic charge. The parameters \((\epsilon'\Lambda_\chi)\) and \((\epsilon''\Lambda_\chi)\) are expected to be comparable and of order a few MeV.

| Mass Combination | \( Q = c \) | \( Q = b \) |
|------------------|-------------|-------------|
| \( \Xi^+_Q - \Xi^-_Q \) | \((\epsilon'\Lambda_\chi)\) | \(1.0(\epsilon'\Lambda_\chi)\) |
| \( \frac{1}{2\pi} \left( \Sigma^{+1}_Q - 2\Sigma^0_Q + \Sigma^{-1}_Q \right) + \frac{2}{\pi} \left( \Sigma^{+1}_Q - 2\Sigma^0_Q + \Sigma^{-1}_Q \right) \) | \(2\frac{1}{\sqrt{20}}(\epsilon''\Lambda_\chi)\) | \(0.67(\epsilon''\Lambda_\chi)\) |
| \( \frac{5}{3} \left( \Xi^+_Q - \Xi^-_Q \right) + \frac{1}{3\sqrt{3}} \left[ \left( \Sigma^{+1}_Q - \Sigma^{-1}_Q \right) + \left( \Xi^+_Q - \Xi^-_Q \right) \right] \) | \(\frac{5}{3} \frac{1}{\sqrt{20}}(\epsilon'\Lambda_\chi)\) | \(0.21(\epsilon'\Lambda_\chi)\) |
| \( \frac{1}{8} \left[ \left( \Sigma^{+1}_Q - \Sigma^{-1}_Q \right) - 2 \left( \Xi^+_Q - \Xi^-_Q \right) \right] \) | \(\frac{1}{\sqrt{20}} \frac{1}{\sqrt{3}}(\epsilon'\Lambda_\chi)\) | \(0.05(\epsilon'\Lambda_\chi)\) |
| \( \frac{1}{8} \left[ \left( \Sigma^{+1}_Q - \Sigma^{-1}_Q \right) - 2 \left( \Xi^+_Q - \Xi^-_Q \right) \right] \) | \(\frac{3}{8} \frac{1}{\sqrt{20}} \frac{2\Lambda}{m_Q}(\epsilon'\Lambda_\chi)\) | \(0.09(\epsilon'\Lambda_\chi)\) |
| \( \frac{1}{8} \left[ \left( \Sigma^{+1}_Q - \Sigma^{-1}_Q \right) - 2 \left( \Xi^+_Q - \Xi^-_Q \right) \right] \) | \(\frac{5}{8} \frac{1}{\sqrt{20}} \frac{2\Lambda}{m_Q}(\epsilon'\Lambda_\chi)\) | \(0.06(\epsilon'\Lambda_\chi)\) |
| \( \frac{1}{8} \left[ \left( \Sigma^{+1}_Q - 2\Sigma^0_Q + \Sigma^{-1}_Q \right) + \frac{1}{\sqrt{20}}(\epsilon''\Lambda_\chi)\) | \(0.03(\epsilon''\Lambda_\chi)\) | \(0.009(\epsilon''\Lambda_\chi)\) |
| \( \frac{1}{8} \left[ \left( \Sigma^{+1}_Q - \Sigma^{-1}_Q \right) - 2 \left( \Xi^+_Q - \Xi^-_Q \right) \right] \) | \(\frac{3}{8} \frac{1}{\sqrt{20}} \frac{2\Lambda}{m_Q}(\epsilon'\Lambda_\chi)\) | \(0.01(\epsilon'\Lambda_\chi)\) |
| \( \frac{1}{8} \left[ \left( \Sigma^{+1}_Q - \Sigma^{-1}_Q \right) - 2 \left( \Xi^+_Q - \Xi^-_Q \right) \right] \) | \(\frac{3}{8} \frac{1}{\sqrt{20}} \frac{2\Lambda}{m_Q}(\epsilon'\Lambda_\chi)\) | \(0.003(\epsilon'\Lambda_\chi)\) |
| \( \frac{1}{8} \left[ \left( \Sigma^{+1}_Q - \Sigma^{-1}_Q \right) - 2 \left( \Xi^+_Q - \Xi^-_Q \right) \right] \) | \(\frac{3}{8} \frac{1}{\sqrt{20}} \frac{2\Lambda}{m_Q}(\epsilon'\Lambda_\chi)\) | \(0.009(\epsilon'\Lambda_\chi)\) |
| \( \frac{1}{8} \left[ \left( \Sigma^{+1}_Q - \Sigma^{-1}_Q \right) - 2 \left( \Xi^+_Q - \Xi^-_Q \right) \right] \) | \(\frac{3}{8} \frac{1}{\sqrt{20}} \frac{2\Lambda}{m_Q}(\epsilon'\Lambda_\chi)\) | \(0.0026(\epsilon'\Lambda_\chi)\) |