Investigation on noise generation of open cavity flow using Lagrangian coherent structures

Shuaibin HAN, Yong LUO, Hu LI, Conghai WU and Shuhai ZHANG

1 State Key Laboratory of Aerodynamics, China Aerodynamics Research and Development Center, Mianyang Sichuan 621000, China.

Email: shuhai_zhang@163.com

Abstract. The noise generation mechanism of an open cavity flow is investigated using Lagrangian coherent structures (LCS) together with dynamic mode decomposition and Helmholtz decomposition methods. The flow field of an open cavity with the length-depth ratio \( L/D = 2 \), Mach number \( Ma = 0.8 \) and Reynolds number \( Re = 2500 \) is first obtained through direct numerical simulation with 5-th order weighted essentially non-oscillatory (WENO) scheme. LCSs are then obtained from flow field reconstructed by the mean velocity field and the decomposed modes. The interactions between LCSs inside the shear layer and the squeezing zone are found to be important sound sources. The method efficiently isolates the structures responsible for the noise generation.

1. Introduction
Flow over open cavity is a classic benchmark problem [1] which contains various acoustic and physical phenomena including multi-wave interaction, vortex-shear layer interaction and shear layer instabilities under acoustic excitation, et al. The noise generation of flow over cavity is of specific interest, and numerous literatures since Krishnamurty [2] have devoted to revealing the noise generation mechanism to provide insights into developing schemes to suppress the cavity noise. The basic flow processes of flow over cavity are as follows: the roll-up of coherent vortical structures in the shear layer are first caused by the Kelvin-Helmholtz instabilities, then the vortical structures impinge onto the trailing edge to generate the acoustic waves that propagate upwards, finally the acoustic waves reaching to the leading edge interact with the flow to further excite the hydrodynamic instabilities. The above processes form a closed feedback loop. The interaction of coherent vortical structures in the shear layer with the flow around the cavity trailing edge plays the main role in the noise generation. Revealing the coherent structure dynamics is the key to understanding the noise generation mechanism of flow over open cavity.

Various methods can be used to study the relation between coherent structures and acoustic waves. The methods can be largely divided into two categories, i.e., the Eulerian and Lagrangian methods. The dominant description and analysis of coherent structures have mainly based on the Eulerian approach, using the instantaneous velocity or other flow quantities, such as the Q-criterion [3] or \( \lambda_2 \) - criterion [4] to study vortical structures. Another Eulerian analysis method is to treat the coherent structures as wavepackets [5]. Through modelling the coherent flow as sets of wavepackets, the correlation between the far-field acoustic wave and the near field coherent structures can be revealed. There are also data-driven methods, such as Dynamic Mode Decomposition (DMD) [6], Proper Orthogonal Decomposition (POD) [7] and the most recent Spectral Proper Orthogonal Decomposition
(SPOD) [8], commonly used to extract coherent structures. However, the Eulerian approaches suffer the disadvantages of lacking temporal information and being subject, i.e. dependent on the reference frame.

Lagrangian approaches based on analyzing the flow variable trajectories in the temporal-spacial phase space can overcome these drawbacks. There are many lagrangian diagnostic tools, among which the Lagrangian Coherent Structures (LCS) is one of the broadly used and was coined by Haller & Yuan [9] to describe the most repelling, attracting and shearing material surfaces responsible for the organization of the bulk flow and as such has strong influence on the flow topology. The LCS has been broadly applied in various flows such as aerodynamics [10], biological flow [11], oceanic and atmospheric transport [12], flow separation [13] and vortex formation and shedding [14, 15] to name a few. Recently, the LCS is applied to the field of aeroacoustics as indicators of sound sources [16-18], and the works showed that LCS has the potential to reveal the physical acoustic sources in the flow. Gonzalez et al. [16] and Han S. et al. [19] showed that the FTLE is able to reveal both the acoustic waves and the coherent structures. For complex flow, the acoustic waves occupy a large range of frequency spectral with several dominant frequency modes. Nair et al. [17] and Premchand et al. [18] combined the LCS with DMD to isolate structures responsible for specific frequency acoustic waves. According to Goldstein [20], the acoustic wave is generated by the non-radiating flows, i.e., the hydrodynamic components. However, the DMD mode is actually a mixture of both hydrodynamic and acoustic components, the LCS extracted from DMD mode may still contain redundant structures not responsible for noise generation. The Helmholtz decomposition (HD) [21] decomposes the flow field into hydrodynamic and acoustic parts, therefore, extracting LCS from the hydrodynamic part of HD could possibly reveal the noise sources.

In this paper, the flow field of a two-dimensional open cavity is obtained through direct numerical simulation (DNS). The sound sources are investigated using LCS together with the dynamic mode decomposition (DMD) method and the Helmholtz decomposition. The results for the two methods are compared. The main purpose of the paper is to address the issue that how the nearfield hydrodynamic coherent structures interacts with the flow around the trailing edge to generate the farfield acoustic waves in the Lagrangian view. The LCSs extracted from both DMD and HD modes reveal clearly the sound sources. Flow features such as shear layer roll-up and the vortical structures are identified to reveal the physical processes responsible for sound generation.

2. Computational details

The computation domain for the two-dimensional open cavity flow as Fig.1 consists of an open rectangular with a depth of $D = 1$ and a length of $L = 2$, and a large enough flow domain with $L_{l} = 7$ and $L_{r} = L_{y} = 20$ covering the radiating acoustic fields. A sponge zone with $L_{i} = 3$ and $L_{o} = L = 10$ is set to absorb and minimize reflections from computational boundaries, and thus avoid numerical pollution on the acoustic field. The mesh grid is $240 \times 120$ in the cavity and $810 \times 300$ out of the cavity with the grid near the wall intensified such that the minimal grid size is $5.0 \times 10^{-3}$ near the wall to resolve the boundary layer.

![Figure 1. Schematic of the open cavity flow.](image-url)
The numerical simulation is carried out through solving two-dimensional time-dependent Navier-Stokes equations. A fifth-order Weighted Essentially Non-Oscillating (WENO) [22] scheme, a fourth-order central finite difference scheme and a third-order Strong Stability Preserving (SSP) Runge-Kutta method are used to discretize the convection term, the viscous term and the time term of Navier-Stokes equations respectively. The flow parameters of Mach number, Reynolds number and Prandtl number are set to be $Ma = 0.8$, $Re = 2500$ and $Pr = 0.7$, respectively. A detailed description of the grid independence validation and comparison with experiments and other numerical results can be referred to Luo et al. [23] and are not shown here to avoid repetition.

3. Methods

3.1. Finite-time Lyapunov exponent

Lagrangian coherent structure (LCS) enables to unravel some invariant features of the flow, so in a way to show the flow ‘skeleton’, providing a new way to understand flow features such as flow separation, vortex dynamics, mixing and transportation et al. Various methods have been proposed to extract LCS in the flow [24], and the finite-time Lyapunov exponent (FTLE) is the most commonly used one. For particles advected in a non-autonomous system

$$\dot{x} = u(x,t)$$

(1)

the FTLE measures the separation rate of neighboring particles after a finite time interval. The calculation of FTLE is based on the flow map $F^t_{x_0}(x_0): x_0 \mapsto x(t, t_0, x_0)$ which maps the initial particle at $x_0$ at time $t_0$ to its final position $x(t, t_0, x_0)$ at time $t$. Then the FTLE is obtained as

$$\sigma^t_{x_0}(x_0) = \frac{1}{|t-t_0|} \ln \sqrt{\lambda_{\text{max}} \left( \nabla F^t_{x_0}(x_0)^\top \nabla F^t_{x_0}(x_0) \right)}$$

(2)

In practice, the FTLE can be calculated either in the backward time ($t < t_0$) or the forward time ($t > t_0$), and the ridges of FTLE reveals the attracting or repelling invariant manifolds of flow respectively. The backward FTLE generally shows resemblance to streaklines and reveals the flow structures more directly, therefore, in this paper we adopt backward FTLE to study the flow features.

For a complex flow consisting of multi-frequency perturbations, the FTLE calculated using the whole velocity field will not extract LCS at a particular frequency. The DMD method, however, can decompose the flow into dynamic modes consisting of only specific frequencies. Then we can reconstruct flow field using the average flow and the dynamic mode corresponding to a specific frequency. With the reconstructed flow field, the FTLE can be calculated and hence the LCS responsible for a particular frequency perturbation can be extracted.

The DMD method has been adopted by several literatures [17, 18] to investigate the noise source. However, the decomposed mode consists both hydrodynamic and acoustic perturbation. As the true noise source would contain only hydrodynamic components, the noise source extracted this way possibly contains redundant structures not responsible for noise generation. In this paper, we propose to adopt the Helmholtz decomposition to extract the true noise sources as comparison. The Helmholtz decomposition isolates the irrotational (longitudinal process, contains mainly acoustic components) and the solenoidal (transverse process, contains mainly hydrodynamic components) parts. So it is an aerodynamic/acoustic splitting technique. Since the two-dimensional open cavity flow is actually a single frequency system, the LCS extracted from hydrodynamic component can also be treated as responsible for the single dominant frequency mode of the cavity flow. And we expect the HD-LCS to isolate the structures responsible for noise generation.

3.2. Dynamic mode decomposition

The dynamic mode decomposition (DMD) is a data-driven method widely used to extract coherent structures in complex flows. The method is based on Koopman analysis [6] and decomposes the flow
into dynamic modes that are temporal orthogonal. Each DMD mode represents a basis function corresponding to a particular frequency. To obtain the DMD modes, the velocity fields are arranged into the snapshot matrix form as:

\[ U_1^{N-1} = \{u_1, u_2, u_3, \ldots, u_{N-1}\} \]

\[ U_2^N = \{u_2, u_3, u_4, \ldots, u_N\} \]

where \( u_i \) denotes the \( i^{th} \) velocity snapshot. A linear tangent approximation could be applied to obtain

\[ U_2^N = AU_1^{N-1} \]

Then the DMD frequencies and corresponding DMD modes are the eigenvalues and eigenvectors of the matrix A. It would be numerically expensive to solve the eigenvalues of A directly. Adopting the single value decomposition (SVD) would improve the efficiency. Rearrange the velocity snapshot matrix \( U_1^{N-1} \) into the form

\[ U_1^{N-1} = V\Sigma W^T \]

then a companion matrix having the same eigenvalues and eigenvectors as A can be expressed as \( S = V^T AV \), and the dynamic modes are obtained as

\[ \Psi = U_1^{N-1}\Phi \]

where \( \Phi \) is the right eigenvector matrix of S.

The velocity field at time \( t_i \) can be reconstructed using the several dominant dynamic modes through a linear combination

\[ u(x, t_i) = \sum_{j=0}^{m} a_j(t_i) \Psi_j \]

In general, the time coefficients \( a_j(t_i) \) can be expressed by the eigenvalue corresponding to the \( j^{th} \) dynamic mode as \( \lambda_j^{-1} \). However, for a complex flow, the amplitude of dynamic modes can vary with time, and hence the time coefficients would be slightly different. Alenius [25] proposed a modified approach to obtain the time coefficients to track the true time evolution of the dynamic modes. The equation (8) is an over determined system and can be solved using the least squares method. Then the several dominant DMD modes span the velocity and the time evolution of the dynamic modes can be studied.

**Helmholtz decomposition**

Helmholtz decomposition decompose the compressible velocity field into a scalar potential and a vector potential:

\[ u = \nabla \phi + \nabla \times A \]

The gradient of scalar potential \( \nabla \phi \) represents the irrotational acoustic part, and \( \nabla \times A \) represents the solenoidal hydrodynamic part. For a periodic flow with a steady base flow satisfying \( \nabla \cdot u_0 = 0 \), assuming that \( \nabla \cdot A = 0 \), a set of Poisson equation can be obtained by taking divergence and curl of equation (9), respectively,

\[ \nabla^2 \phi' = \nabla \cdot u' \]

\[ \nabla^2 A' = -\nabla \times u' \]

where \( u' = u - u_0 \). In this paper, we solve the scalar Poisson equation (10) to isolate the acoustic component \( u'_a = \nabla \phi' \) of the velocity perturbation. The boundary condition is set as follows: the
potential $\phi'$ is set to be zero at inlet and outlet, for the top boundary and wall boundary, a Neuman condition is set [26]. Then the hydrodynamic component is obtained by $u' = u - u_0 - \nabla \phi'$. As pointed out by P. Jordan [27], the generalised acoustic analogy can be written as

$$L_{\tau_\omega}(q_A) = s(\overline{q}_D)$$

(12)

where $L_{\tau_\omega}$ is a linear operator describing the evolution of $q_A$, a disturbance generated and carried by $\overline{q}_D$. In this paper, we reconstruct flow field using the $u_0$ and $u'$ to represent the $\overline{q}_D$. The LCS is then extracted from the reconstructed flow field. The LCS extracted this way is expected to be responsible for the cavity noise generation.

4. Numerical results and analysis

4.1. Basic flow features of open cavity flow

The velocity and pressure perturbation at the point $y = 1$ and $x = 0.995$ along the cavity mouth is shown in Fig.2. The periodic perturbation is evident and the time period of the flow is $T = 3.75$. Performing fast Fourier transform (FFT) to 468 snapshots (5 time periods, time interval between snapshots is $\Delta t = 0.04$) of the computational data to get the frequency spectrum (Fig.3), it’s found that at the sample point the frequency of the dominant mode is $St = fL/U = 0.668$, which is close to the Rossiter 2-nd mode $St_2 = 0.686$ according to the semi-empirical formula

$$St = \frac{fL}{U} = \frac{n - \gamma}{M + 1/k}, n = 1, 2, \ldots$$

(13)

and agrees well with the experiment result $St_e = 0.656$ of Krishnamurty [2]. The following frequency peaks are the multiplication of the dominant frequency.

Figure 2. Velocity and pressure at the point $y = 1$ and $x = 0.995$ along the cavity mouth.

Figure 3. Frequency spectrum of velocity and pressure at the point $y = 1$ and $x = 0.995$ along the cavity mouth.
The streamline and vorticity contour at several typical times during a period is shown in Fig.4. There are mainly four circulations in the cavity, which form the main topology structures of the flow in the cavity. Notice that the vorticity contour splits up at the trailing edge in Fig.4(b) and a closed streamline forms at the leading edge in Fig.4(e), which could be thought as the vortex breakdown and the vortex formation in the Eulerian frame and important sound generation processes. The shear layer along the cavity mouth is evident during the whole period. However, the interactions in the shear layer are not revealed by the Eulerian method.

![Streamline patterns with vorticity during a period of the open cavity flow.](image)

**Figure 4.** Streamline patterns with vorticity during a period of the open cavity flow.

The time evolution of the pressure at trailing edge is shown in Fig.5. The pressure experiences a sharp drop from \( t = 0 \) to \( t = 0.2T \) when the shear layer impinges onto the trailing edge. The root mean square of pressure is calculated as

\[
p_{rms} = \sqrt{\frac{\int (p')^2 dt}{T}}
\]

and is shown in Fig.6. From the distribution of \( p_{rms} \), the directivity of the noise is clearly revealed. The \( p_{rms} \) has the strongest level at the trailing edge, where the vortex impinges on the edge and generate the noise. The \( p_{rms} \) also has a local maximum at the middle of the shear layer, which indicates that the interaction of flows in the shear layer also contributes to the noise radiation.

![Time evolution of pressure at trailing edge.](image)

**Figure 5.** Time evolution of pressure at trailing edge.
4.2. Dynamic Mode Decomposition, Helmholtz Decomposition and corresponding Lagrangian Coherent Structures

The flow dynamics are further studied using LCS together with DMD and HD. The first and second DMD modes for pressure and velocity are shown in Fig.7 and Fig.8, respectively. The acoustic, hydrodynamic and mean velocity components of Helmholtz decomposition are shown in Fig.9. As we described previously, the DMD mode is actually a mixture of acoustic and hydrodynamic components at a specific frequency, therefore, the far field perturbation and near field coherent wavepacket structures in the cavity are simultaneously revealed in both the two DMD modes. For the Helmholtz decomposition, the acoustic and hydrodynamic components shows distinct features. The acoustic component propagates in and out the cavity, while the hydrodynamic component mainly locates around the cavity. And the acoustic component is an order of magnitude smaller than that of the hydrodynamic component.

Figure 6. Distribution of $p_{rms}$

Figure 7. The first DMD mode with St=0.68.

(a) Pressure   (b) U   (c) V
Figure 8. The second DMD mode with St=1.36

(a) Pressure  (b) U  (c) V

Figure 9. Acoustic, hydrodynamic components of Helmholtz decomposition and the mean velocity field.

(a) $|u'_r|$  (b) $|u'_i|$  (c) $|u_0|$

Figure 10. FTLE of mean velocity field.

The FTLE calculated using the mean velocity field is first presented in Fig.10. There is no evident LCS for the mean velocity. The FTLE of mean velocity just highlights the convection effect in the cavity. To extract the Lagrangian coherent structures, we calculate the backward FTLE from
reconstructed velocity field as described in section 3. The reason for using the mean velocity to reconstruct the velocity field is to include the convection effect. The time interval for calculating the backward FTLE is $T = 3.75$, the time period of the flow, which is long enough to detect the LCSs.

![Figure 11. Spatial-temporal evolution of the dynamic mode-1 LCS for the open cavity flow.](image)

The FTLE corresponding to the dynamic mode-1 highlights the interactions inside the shear layer. The two LCSs along the cavity mouth and a squeezing zone (Fig.11) are evident during the whole time period. The LCS-1 begins to form at the leading edge at $t = 0$. Then the LCS-1 extends and interacts with LCS-2 to squeeze the flow between them, and a squeezing zone forms inside the shear layer. The squeezing zone convects with the two LCSs to the downstream and becomes long and narrow due to the shear and compression of the two LCSs from $t = 0$ to $t = 0.8T$. Then the squeezing zone impinges onto the trailing edge of the cavity and begins to split into two parts at $t = T$. Focusing on the flow around the trailing edge, the squeezing zone impinges onto the trailing edge and finally split into two parts from $t = 0$ to $t = 0.2T$. It should be noted that a sharp pressure drop (Fig.5), which indicates the release of acoustic energy, is associated with the process. Since the reconstructed flow field consists of only the mean velocity and the dominant frequency perturbation ($St = 0.668$), the impingement of the squeezing zone and the interactions between the two LCSs could be thought as the sources of noise. The FTLE corresponding to mode-2 (Fig.12) reveals mainly the shear layer. However, only the convection of LCS is revealed, no interaction is found. Therefore, the dynamic mode-2 contains no sound sources.

![Figure 12. Spatial-temporal evolution of the dynamic mode-2 LCS for the open cavity flow.](image)
For the hydrodynamic components of Helmholtz decomposition, the FTLE distribution (Fig.13) is almost the same as that of DMD mode-1 FTLE and is only slightly different from the shape of the squeezing zone. The flow dynamics of LCS interaction inside the shear layer and the impingement onto the trailing edge of the squeezing zone is the same. Therefore, both the HD-LCS and the DMD-LCS reveal the structures responsible for noise generation.

5. Conclusion
The noise generation of the open cavity flow is investigated using LCS together with the DMD and HD methods. By reconstructing flow field using the mean velocity field and the dynamic mode from DMD or hydrodynamic component from HD, the corresponding LCSs are obtained. It’s found that the interactions between the two LCSs inside the shear layer and the squeezing zone are important sound sources. The LCS together with DMD and HD is an efficient method to identify sound sources in complex flow and reveal the mechanism of noise generation.

6. Acknowledgement
This research is partially supported by the National Numerical Windtunnel project. The authors acknowledge support from State Key Laboratory of Aerodynamics (no. JBKYC190105), National Natural Science Foundation of China (no. 11732016) and Sichuan Science and Technology Program (no. 2018JZ0076).

References
[1] Rowley, C.W., Colonius, T. and Basu, A.J. On self-sustained oscillations in two-dimensional compressible flow over rectangular cavities. J. Fluid Mech., 455, 315–346, 2002.
[2] Krishnamurty, K. Sound radiation from surface cutouts in high speed flow. PhD thesis, California Institute of Technology, 1956.
[3] Hunt, J.C.R., Wray, A. and Moin, P. Eddies, stream and convergence zones in turbulent flows. Proceedings of the Summer Program, Center for Turbulence Research CTR-S88, 193–208, 1988.
[4] Jeong, J. and Hussain, F. On the identification of a vortex. J. Fluid Mech., 285, 69–94, 1995.
[5] Jordan, P. and Colonius, T. Wavepackets and turbulent jet noise. Annu. Rev. Fluid Mech. 45, 173-195, 2013.
[6] Schmid, P.J. Dynamic mode decomposition of numerical and experimental data. J. Fluid Mech., 656, 5–28, 2010.
[7] Sirovich, L. Turbulence and dynamics of coherent structures. part I: coherent structures.
Quarterly of Applied Mathematics. 45(3), 561-571, 1987.

[8] Towne, A. Schmidt, O.T. and Colonius, T. Spectral proper orthogonal decomposition and its relationship to dynamic mode decomposition and resolvent analysis. J. Fluid Mech., 825, 1113–1152, 2017.

[9] Haller, G. and Yuan, G. Lagrangian coherent structures and mixing in two-dimensional turbulence. Physica D, 147, 352–370, 2000.

[10] Green, M., Rowley, C. and Smits, A. The unsteady three-dimensional wake produced by a trapezoidal pitching panel. J. Fluid Mech. 685, 117-145, 2011.

[11] Peng, J. and Dabiri, J. Transport of inertial particles by Lagrangian coherent structures: application to predator-prey interaction in jellyfish feeding. J. Fluid Mech. 623, 75-84, 2009.

[12] Beron-vero, F., Olascoaga, M. and Goni, G. Oceanic mesoscale eddies as revealed by Lagrangian coherent structures. Geophys Res. Lett. 35, L12603, 2008.

[13] Shuaibin HAN, Shuhai ZHANG and Hanxin ZHANG. A Lagrangian criterion of unsteady flow separation for two-dimensional periodic flows. Applied Mathematics and Mechanics, 39(7), 1007-1018, 2018.

[14] Rockwood, M. P., Taira, K. and Green, M. A. Detecting vortex formation and shedding in cylinder wakes using lagrangian coherent structures. AIAA Journal, 55(1), 15-23, 2016.

[15] HAN Shuaibin, LUO Yong, ZHANG Shuhai. Lagrangian vortex dynamics in the open cavity flow. Advances in Aeronautical Science and Engineering. 10(5), 691-697,734, 2019. (in Chinese)

[16] Gonzalez, D. R., Speth, R., and Gaitonde, D. V., et al . Finite-time Lyapunov exponent-based analysis for compressible flows. Chaos, 26(8), 083112, 2016.

[17] Nair, V., Alenius, E. and Boji, S., et al . Inspecting sound sources in an orifice-jet flow using Lagrangian coherent structures. Computers & Fluids, 140, 397–405, 2016.

[18] Premchand, C. P., George, N. B., Raghunathan M., et al . Lagrangian analysis of intermittent sound sources in the flow-field of a bluff-body stabilized combustor. Physics of Fluids, 31, 025115, 2019.

[19] HAN Shuaibin, LUO Yong, ZHANG Shuhai. Relation between the Finite-time Lyapunov exponent and acoustic wave. AIAA Journal. 57(12), 5114-5125, 2019.

[20] Goldstein, M.E. On identifying the true sources of aerodynamic sound. J. Fluid Mech. 526, 337–347, 2005.

[21] Helmholtz, H. On integrals of the hydrodynamical equations which express vortex motion. J. Reine Angew. Math. 55, 25-55, 1858.

[22] Jiang G.S. and Shu, C.W. Efficient implementation of weighted ENO schemes. Journal of Computational Physics. 126(1), 202–228, 1996.

[23] LUO Yong, LI Hu, HAN Shuaibin and ZHANG Shuhai. Direct numerical simulations of self-sustained oscillations in two-dimensional open cavity for subsonic and supersonic flow. Advances in Applied Mathematics and Mechanics. (2020) (in press)

[24] Hadjighasem, A., Farazmand, M., Blazevski, D. et al. A critical comparison of Lagrangian methods for coherent structure detection. Chaos, 27, 053104, 2017.

[25] Alenius E. Mode switching in a thick orifice jet, and LES and dynamic mode decomposition approach. Computers & Fluids, 90, 101-112, 2014.

[26] Stefan Schoder, Klaus Roppert and Manfred Kaltenbacher. Postprocessing of direct aeroacoustic simulation using Helmholtz decomposition. AIAA Journal. 58(7), 3019-3027, 2020.

[27] Jordan, P. Analysis techniques for aeroacoustics: noise source identification. In Noise Sources in Turbulent Shear Flows: Fundamentals and Applications. Springer, Vienna , 197-287, 2013.