Social Networking by Proxy: A Case Study of Catster, Dogster and Hamsterster

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ABSTRACT
The proliferation of online social networks in the last decade has not stopped short of pets, and many different online platforms now exist catering to owners of various pets such as cats and dogs. These online pet social networks provide a unique opportunity to study an online social network in which a single user manages multiple user profiles, i.e. one for each pet they own. These types of multi-profile networks allow us to investigate two questions: (1) What is the relationship between the pet-level and human-level network, and (2) what is the relationship between friendship links and family ties? Concretely, we study the online pet social networks Catster, Dogster and Hamsterster, the first two of which are the two largest online pet networks in existence. We show how the networks on the two levels interact, and perform experiments to find out whether knowledge about friendships on a profile-level alone can be used to predict which users are behind which profile. In order to do so, we introduce the concept of multi-profile social network, extend a previously defined spectral test of diagonality to multi-profile social networks, perform a two-level social network analysis, and present an algorithm for predicting whether two profiles were created by the same user. As a result, we are able to predict with very high precision whether two profiles were created by a same user. Our work is thus relevant for the analysis of other online communities in which users may use multiple profiles.

1. INTRODUCTION
Pet ownership is common in many countries. In the United States for instance, 47% of households owned at least one dog, and 46% at least one cat in 2012 [1]. It therefore comes as no surprise that specialized online social networking platforms exist specifically for pets. In general, online social networks may range from the very generic such as Facebook and Twitter, to the very specialized for dedicated communities related to hobbies, activities or professions. Nevertheless, the specific topic that unifies the community usually does not affect the basic mechanism of an online social network: A user creates an account to connect with other users. Online pet social networks are however different in this regard. In online pet networks, users can create any number of accounts, one for each pet they own. While individual persons cannot usually be stopped from creating multiple accounts in an ordinary online social network, this is usually proscribed, and when used for disruption is called sock puppetry [23]. For these reasons, information on the use of multiple accounts by users in online social networks is a rarely studied problem, and few datasets for its study exist. As an example, one study performs the task of predicting whether a given Wikipedia account is a sock puppet on less than a hundred accounts [18]. In contrast to this, we are able to perform a study on several hundreds of thousands of users in this paper.

In online pet social networks, a single user may (and is expected to) create one account for each owned pet. All social networking functionality such as entering personal information, creating friendship links to others, etc., are then performed on the pet level. Figure 1 illustrates how the multiple pet profiles created by a user form a family of pets. With their structure that allows multiple profiles per account, on-
Online pet social networks thus make it possible to investigate the following questions:

- How does the fact that individual users own multiple profiles influence the structure of the social network?
- Is it possible to predict that two accounts are managed by the same person?

These questions are analysed under multiple aspects in the remainder of the paper. In Section 2 we review related work and in Section 3 we describe our three datasets. In Section 4 we perform social network analysis, in order to determine crucial differences between both networks. In Section 5 we investigate the homophily on both levels, asking whether the account-level network is characterized by higher homophily values, and if yes, for which node properties this is true. In Section 6 we perform a spectral analysis of the networks, for which we introduce an extended spectral diagonality test in order to compare friendships with family ties. In Section 7 we analyse the problem of predicting that two profiles were set up by the same account, with the goal to find out whether this is possible at all, and if yes which structural and metadata properties are suitable for this task. Section 8 concludes the paper.

2. ONLINE PET NETWORKS

The analysis of social networks has its roots in the social sciences [17]. More recently, the use of social network datasets extracted from online social networking platforms have led to a large amount of research in computer science and network science. Online social networks allow people to connect via a platform in order to communicate, share content, or simply manage a list of connections for various purposes. In most such platforms, a single user account is used to manage a single user profile, to which users can add information such as their age, location, sex, favorite movies, songs, food, or any other metadata deemed interesting to the particular community. In only few cases can multiple profiles be created by a single user. An example is given by company or product pages on Facebook, of which one user can create many more than one. In that case however, there may be more than one user managing each profile, resulting in group-like semantics rather than profile-like semantics. In most online social networking platforms, the creation of multiple profiles by one user is not allowed, only possible by using multiple email addresses, or restricted to very specific users. On Wikipedia for instance, multiple accounts created by a single person are referred to as sock puppets, and are proscribed [23]. Therefore, few datasets are available and only little research has been conducted on the topic, an example being the detection of sock puppets on Wikipedia [15] using one hundred accounts. Text mining approaches to detect sock puppets in Wikipedia have been described, too [19]. Therefore, online pet social networks such as Catster, Dogster and Hamsterster present a unique opportunity to study a social network in which users manage multiple profiles. What is more, due to the fact that this is not proscribed by the sites, but instead represents the normal way of using them, information about identical users is openly available on these sites, making this study possible.

Many specialized online networking platforms exist, and online pet social networking platforms specifically have been studied before, although social network analyses have not been performed on them. Related work analysing online animal social networks has covered Catster, Dogster and Hamsterster, but only used small samples of the full networks for analysis: 2,000 dogs and 2,000 cats in [5], and 10,000 dogs and 10,000 cats in [24]. None of these works perform a network analysis. The latter paper asks the question whether knowledge about family ties can improve prediction of friendship ties; the question is answered positively.

A distinct topic is that of animal networks such as networks of sheep [7], dolphins [14] and macaques [21]. Those refer to social networks in which the actors are (usually wild) animals, whose social ties are not conditioned by humans. Another distinct concept is that of circles, as used for instance in Google+ [13]. Although families in pet networks have been called circles (e.g. in [24]), they are not the same concept as used on Google+. On Google+, a circle is a device to group one’s own friends. Hence, circles do not provide a new type of link beyond friendships, and cannot be compared to the families of online pet social networks.

3. DATASETS

We use datasets of Catster, Dogster and Hamsterster. Since Dogster and Catster share user accounts, we also report statistics on the union of these two. An overview of the datasets is given in Table 1. catster.com and dogster.com were both founded in 2004 [8]. Both sites are linked: A single user can create pet profiles on both sites, and individual cat and dog profile pages are interlinked via a family link when they were created by the same user. hamsterster.com is an independent site created in 2003 or 2004 [4]. Hamsterster appears to have been shut down as of October 2014 [1]. Other such “online social petworks” exist, such as

| Dataset        | #Pets | #Friendships | #Households | Pets per household |
|----------------|-------|--------------|-------------|--------------------|
| Catster        | 204,424 | 5,443,855    | 105,089     | 1.95               |
| Dogster        | 451,710 | 8,543,549    | 260,390     | 1.73               |
| Catster + Dogster | 623,766 | 13,991,746   | 333,111     | 1.87               |
| Hamsterster    | 2,950  | 12,531       | 1,575       | 1.87               |

Table 1: Datasets analysed.

1The exact creation date of Hamsterster is not known to us. The oldest accounts there date from 2003, but the domain hamsterster.com was registered in 2004 [4], and the phrase “after nearly ten years” written in October 2014 on Twitter suggests a creation date of 2004.

2As of October 2014, the Twitter account @HAMSTERsterTM states that Hamsterster had been closed “after nearly ten years”.
bunspace.com for rabbits, but are not studied in this paper. The suffix -ster in these names was likely chosen as a reference to friendster.com, created in 2002. We crawled Catster and Dogster from August 2011 to March 2012, and Hamsterster in February 2012.

On all three sites, a single user can create accounts for any number of pets. Catster and Dogster are connected, and thus a single user account can be used for both sites, although 90.3% of accounts across Catster and Dogster include only cats or only dogs. The group of pet profiles created by a single user makes up a household or family. Friendship links are allowed within a single household in Dogster and Catster, but are not allowed in Hamsterster. All friendship links are undirected.

Catster and Dogster allow only cats (*Felis catus*) and dogs (*Canis lupus familiaris* or *Canis familiaris*) respectively. Hamsterster allows multiple species of hamsters (subfamily Cricetinae) and gerbils (subfamily Gerbillinae), the most common species being the golden hamster (*Mesocricetus auratus*). The Hamsterster dataset contains at least one cat, a rat and five guinea pigs. We also found profiles in all three platforms apparently created for multiple pets (e.g., named “Hamster babies”). For each of the three sites, about two thirds of all users are located in the United States.

4. MULTI-PROFILE SOCIAL NETWORK ANALYSIS

The multi-profile social networks of Catster, Dogster and Hamsterster can be analysed using tools of social network analysis on two different levels: the profile level (pet level and the account level (family or household level). By performing social network analysis, we can derive several properties from a multi-profile social network. First, we can derive the differences and similarities between the two networks. Second, we can ask which of the two is more similar to a typical social network, in order to assess whether the network is better modeled as an account-level network to which profiles are attached, or a profile-level network in which the profiles are aggregated into groups.

4.1 Definitions

We now introduce a formal definition of a multi-profile social network, of which Catster, Dogster and Hamsterster are examples. A multi-profile social network is a social network in which each person is associated with one or more profiles, and in which the actual social relationships as well as the metadata such as age, sex and location are associated to individual profiles. In the online case, a multi-profile social network allows each user to manage one or more profiles. The set of profiles managed by a single account in a multi-profile social network may also be called a household or a family. The latter term in particular is used by the three studied online pet social networking sites.

We denote a multi-profile social network by $G = (V, W, E, m)$, where $V$ is the set of profiles, $W$ is the set of accounts, $E \subseteq V \times V$ is the set of friendship edges connecting profiles, and $m : V \rightarrow W$ is a mapping from profiles to accounts. Individual profiles will be denoted by the letters $u, v$, etc., while accounts will be denoted by the letters $i, j$, etc. As in other social networks, additional metadata for profiles, accounts and friendships may be defined. The online pet social networks we study include extensive profile metadata (described in Section 3.1), but do not include account metadata, because they present everything from the point of view of the pet. The graph $G_0 = (V, E)$ then represents the profile-level social network, while $G_a = (W, m(E))$ represents the account-level social network, using the definition

$$m(E) = \{\{i, j\} \mid i \neq j \land \exists \{u, v\} \in E : m(u) = i \land m(v) = j\},$$

that is, $G_a$ is the result of identifying vertices in $G_0$ that are in the same household, not including loops in the result. An overview of the differences between the two levels of net-
works is shown in Table 2 in terms of numerical statistics.

### 4.2 Demographic Characteristics

The distribution of sexes and ages of pets is shown in Figure 2 (b-c). Both sexes are equally distributed in Catster and Dogster, and the age distribution reflects the pet’s life spans. On average, there are two pets to one household. The average number of pets per household is consistent over all three pet types; it is 1.95 for cats, 1.73 for dogs and 1.87 for hamsters (see Table 1). The distribution of pets per household (shown in Figure 3(a)) is power-law-like, with similar power law exponents for all three sites. The fitted power law exponents using the method described in [16, Eq. (5)-(6)] are 3.62 for Hamsterster ($p_{\text{min}} = 5$), 3.63 for Catster ($p_{\text{min}} = 6$), 3.90 for Dogster ($p_{\text{min}} = 4$) and 3.79 for Catster and Dogster combined ($p_{\text{min}} = 5$). The fitted parameter $p_{\text{min}}$ denotes the starting point of the fit.

The fact that the number of pets per household follows a power-law distribution closely is interesting. In usual social networks, this is explained through a process of preferential attachment, i.e., persons with many friends are more likely to make new friends. In the case of profiles, it would mean that accounts with many profiles are more likely to create new profiles. Whether this is the correct explanation cannot be explained by the data however. Nonetheless, the distributions of pets per household follow power laws much more closely than the number of friends per profile.

Thus, the account-level networks have about half the number of nodes as the profile-level networks. In terms of the number of edges (the volume of the network), the account-level networks are smaller by a factor of ten (Catster), four (Dogster) and three (Hamsterster). The lower value for Hamsterster can be explained by the fact that Hamsterster does not allow friendship edges within families, but also by the fact that in Hamsterster, the average number of friendships is lower (8.5) than in Catster (53.3) and Dogster (37.8).

### 4.3 Are Pet Networks Scale-free?

The distribution of the node degrees in a network is an important characteristic of the network. Many network models such as the preferential attachment model [2] predict the degree distribution to be scale-free, i.e., the number of nodes with degree $d$ to be proportional to the power $d^{-\gamma}$ for some constant $\gamma$. Along with estimating $\gamma$, we also used the Gini coefficient to measure the equality of the friendship distribution [12].

The degree distributions of the profile-level networks as well as the account-level networks are plotted in Figure 2 (d) and the values of the fitted power-law exponent $\gamma$ and the Gini coefficient are given in Table 2. The power law exponent is computed using a minimum degree $d_{\text{min}}$, using the robust method given in [16, Eq. (5)-(6)].

Beyond the fact that the average degree is lower in the account-level networks than in the profile-level networks, we observe that in Catster and Dogster, the power-law exponent $\gamma$ is larger for the account-level network than for the profile-level network, while the Gini coefficient is smaller in the account-level network than in the profile-level network. Both observations are consistent with each other, as a large Gini coefficient and a small power-law exponent both denote a more equal degree distribution [12]. This indicates that the account-level networks have a more equal distribution of degrees than the profile-level network, i.e., the account-level networks are more regular. Both statistics are however in the range usual for social networks; $\gamma$ is in the range 2.1–2.5 and the Gini coefficient is in the range 60–70%.

### 5. HOMOPHILY IN PET NETWORKS

The term homophily refers to the tendency of people connected through social ties to be similar to each other. More precisely, homophily can be measured by a network’s assortativity with respect to a given node property. A network then displays positive homophily (assortativity) when two randomly chosen connected persons are more similar than two randomly chosen persons without regard to connections [15]. Inversely, a network displays negative homophily (disassortativity) when the opposite is the case. By analysing the homophily in online pet social networks, we want to answer the following questions:

- Which is higher, the homophily between friends, or within families? If the homophily between friends is...
Pets per household (p)

\[ P(x > p) \sim p^{-3.5} \]

Catster
Dogster
Catster + Dogster
Hamsterster

Number of friends (d)

\[ P(x > d) \sim d^{-2.2} \]

Catster [p]
Dogster [p]
Hamsterster [p]
Catster [a]
Dogster [a]
Hamsterster [a]

Figure 3: Power law-like distributions in pet networks. (a) Complementary cumulative distributions of pets per household for the three sites, as well as for Catster and Dogster combined (as single accounts may create profiles on both sites). (b) Complementary cumulative degree distribution in the profile-level [p] and account-level [a] networks.

higher, this would indicate that the pets are the primary actors in the networks, and that families are merely organizational structures, but that a proper social network analysis would have to consider the pet-level network. On the other hand, a higher homophily within families would indicate that the family (or household) is the primary social structure in the network, and that a social network analysis would have to consider the household-level structure to accurately reflect the social structure.

• Which profile properties correlate with two pets being friends, and with two pets being in the same household? The features indicative of a shared household will give insight about the behavior of the users’ choice of pets, while the features indicative of friendship links will be indicative of the social networking behavior of users.

In order to answer these questions, we propose two complementary assortativity coefficients that apply to multi-profile social networks, whose ratio is measure of the relative strength of intra-household homophily as compared to across-friendship homophily.

5.1 Methodology

Many different node properties can be subject to homophily analysis, and the exact method used for measuring it depends on the data type considered. In the online pet social networks we analyse, the data that can be added to a pet’s profile fall into three categories:

• Categorical variables
  - The sex of a pet (male / female). The sex is a mandatory field for all pets.
  - The race of a pet. For cats and dogs, the race corresponds to the breed. For hamsters, the race corresponds to one of multiple species of hamsters and gerbils. The race is a mandatory field for all pets.
  - The pet’s coloration. The coloration is mandatory for all hamsters and optional for cats (69% of profiles include it). It is not used on Dogster.

• Numerical variables
  - The profile creation date. It is known for all pets on all three sites.
  - The birth date. The birth date is mandatory for all hamsters, and optional on Catster and Dogster. It is known for 76% of cats and 80% of dogs.
  - The weight. On Catster, the weight can be specified as an exact number in pounds, and is known for 58% of cats. On Dogster, one out of five ranges can be chosen (1–10 lbs, 11–25 lbs, 26–50 lbs, 51–100 lbs, 100+ lbs). The weight is not used on Hamsterster.
  - The number of friends of a pet.
  - The location (“home”) of a pet can be specified on all three sites. We converted the location strings to latitude-longitude pairs using the Google Geocoding API [6]. The geolocation is known for 68% of cats, 78% of dogs and 99% of hamsters.

We additionally use as a feature the join age, defined as the age of the pet at the time of profile creation.

We define two measures of assortativity for multi-profile networks: one that measures homophily on the profile friendship level \( r_p \) and one that measures homophily on the account level \( r_a \). For the friendship level, we consider the
friendship edges between pets in the networks. For the account level, we consider all pairs of pets that are in the same household. As in most social networks, we expect to observe a certain amount of homophily in the pet friendship network. We further hypothesize that the homophily between pets within a single household is larger than the homophily for pets connected by friendship links. Therefore, we compute measures of homophily for both levels, based on the available pet characteristics.

For categorical variables, we base the assortativity coefficients on [15, Eq. (2)]. Let \( C \) be the set of possible values of the categorical variable, \( P_x(i, j) \) the probability that a randomly chosen connected pair of profiles (either via a friendship edge for \( x = p \), or in the same household for \( x = a \)) are in the categories \( i \in C \) and \( j \in C \) respectively, and \( P_x(i) = \sum_j P_x(i, j) \). Then, we define the friendship assortativity coefficient \( r_p \) and the household assortativity coefficient \( r_a \) using

\[
 r_x = \frac{\sum_i P_x(i, i) - \sum_i P_x(i)^2}{1 - \sum_i P_x(i)^2}.
\] (2)

The assortativity coefficients defined in this way equal one for perfect positive homophily, and lie between negative one and zero for negative homophily.

For numerical variables, we use the Pearson correlation coefficient between the numerical properties of connected pets, as defined in [15, Eq. (20)]. Let \( \text{var}_x(X) \) be the variance of the numerical profile characteristic weighted by the number of neighbors of the profile in the friendship graph, and \( \text{cov}_x(X, Y) \) the covariance between the characteristics of pairwise connected profiles, using again \( x \) as a for friendship connections and \( x = a \) for pairs of profiles of the same account. Then the assortativity coefficients \( r_p \) and \( r_a \) are given by

\[
 r_x = \frac{\text{cov}_x(X, Y)}{\text{var}_x(X)}.
\] (3)

Note that this expression is simplified from the usual Pearson correlation coefficient because the relationships are symmetric. The values of \( r_x \) range from \(-1\) to \(+1\) and are one for perfect positive homophily and \(-1\) for perfect negative homophily.

For the geolocation, we use the distance correlation [20] as a measure of homophily, based on the great circle distance between pairs of locations. Since locations are two-dimensional, the distance correlation is able to represent the orientation of the correlation as does the Pearson correlation, but cannot represent the direction of the correlation. Therefore the distance correlation ranges from zero to one, with one denoting perfect correlation and zero denoting no correlation. The location is always the same for pet profiles created by a single user and therefore the family-level homophily for the location is always trivially one.

All three types of assortativity measures are zero when neither positive nor negative homophily is observed. To compare the both the assortativity coefficients on the friendship level and on the account level, we define the multi-profile assortativity ratio of a profile characteristic as

\[
 r_{rel} = \frac{r_p}{r_a}.
\] (4)

By construction \( r_{rel} \) is larger than one if the assortativity is higher within profiles of one account than across friendships, and smaller than one if it is the assortativity across friendships that is higher.

5.2 Discussion

Table 3 shows the complete homophily analysis. For all features, the homophily within households is larger than the homophily between friends, and thus all multi-profile assortativity ratios are larger than one. This indicates, as we would expect from pets, that the underlying social network is primarily one of humans and not one of pets. However, the pet friendship network is not completely unassortative, as it displays positive assortativity \((r > 0.5)\) by join date for all three sites.

For the intra-household homophily, high values \((r > 0.5)\) can be observed for the join date and the number of friends. Small positive assortativity \((r > 0.1)\) can be observed for the race, the birth date, the join age, and the pet’s weight. The largest multi-profile assortativity ratio \((r_{rel} > 10)\) can be observed for the breed in Catster, the number of friends in Hamsterster, the join age in Catster and Hamsterster, and the pet weight in Catster.

In terms of race, Dogster has a particularly high intra-household homophily, indicating that owners of several dogs tend to prefer dogs of the same breed, while this is only true to a small extent for cats and hamsters. The sex and coloration of pets show no homophilic tendencies. The number of friends of a pet show negative assortativity on the friendship, and positive assortativity within households. This indicates that while the friendship ties display the usual degree dissassortativity of real social networks, the numbers of friends of pets within one household are similar, and therefore the degree of a pet is a function of the owner, not of the pet. The homophily with respect to he join date and birth date is higher in Hamsterster. This is consistent with the fact that hamsters have shorter lives.

In conclusion, we find that the intra-household homophily is higher than the friendship homophily. Thus, with respect to profile features, these pet social networks largely follow the underlying human social networks. This conclusion is however only based on profile properties, and does not take into account the network structures. Therefore, we investigate the pet and human-level network structures in the next section.

6. RELATIONSHIP BETWEEN FRIENDSHIPS AND FAMILY TIES

So far, we have analysed the friendship and family ties on an individual level. We now perform several experiments to analyse the available networks as a whole, and to determine the relationships between the friendship network and family tie network at the structural level. In order to do so we extend the spectral diagonal test described in [10], which was originally used to analyse the temporal evolution of a
network, to the comparison of the friendship network with the ownership structure in the multi-profile network. The result is a test that allows us to directly observe relationships between both structures, and a measure of the consistency between friendships and family ties.

6.1 Definitions

The graphs \( G_p \) and \( G_a \) can be represented by the adjacency matrices \( A_p \in \{0,1\}^{|V| \times |V|} \) and \( A_a \in \{0,1\}^{|W| \times |W|} \), defined as follows:

\[
(A_p)_{uv} = \begin{cases} 
1 & \text{when } \{u, v\} \in V \\
0 & \text{when } \{u, v\} \notin V
\end{cases},
\]

\[
(A_a)_{ij} = \begin{cases} 
1 & \text{when } \{i, j\} \in W \\
0 & \text{when } \{i, j\} \notin W
\end{cases}.
\]

Both matrices are symmetric. We also define a matrix giving the relationship between profiles and accounts. Let \( R \in \{0,1\}^{|V| \times |W|} \) be the matrix defined by

\[
R_{ui} = \begin{cases} 
1 & \text{when } m(u) = i \\
0 & \text{when } m(u) \neq i
\end{cases}.
\]

\( R \) is rectangular, and by definition each row has a single entry equaling one. By construction, the following relationship holds:

\[
A_a = [R^T A_p R],
\]

where the matrix operator \([X]\) rounds all nonzero entries of \( X \) to one, and all diagonal entries to zero. We also define the family matrix \( F \in \{0,1\}^{|V| \times |V|} \) whose entries equal one when two profiles are managed by the same account and zero otherwise:

\[
F_{uv} = \begin{cases} 
1 & \text{when } m(u) = m(v) \\
0 & \text{when } m(u) \neq m(v)
\end{cases}
\]

The following relationship can then be established:

\[
F = RR^T.
\]

Note that the diagonal elements of \( F \) are all one, since every profile is in the same account as itself.

6.2 Methodology

We seek to compare the friendship-level network and the family tie network using a spectral diagonality test, a technique that was initially introduced to study time-evolving networks under the spectral evolution hypothesis, i.e., the hypothesis that under time evolution, the eigenvalues of a network’s adjacency matrix change while its eigenvector stays nearly constant [10]. Two matrices with the same eigenvectors are related by spectral transformations [11], and if they are adjacency matrices their relationship indicates how the one type of edge is related to the other type of edge. If \( A_1 \) and \( A_2 \) are the adjacency matrix of a single network at two different timepoints and defined on the same node set, then the spectral diagonality test first computes the rank-\( k \) eigenvalue decomposition

\[
A_1 = U \Lambda U^T,
\]

and then sets out the write an eigenvalue decomposition-like expression for \( A_2 \), using the same eigenvalue matrix \( U \) as for the first matrix:

\[
A_2 = U \Delta U^T.
\]

If both \( A_1 \) and \( A_2 \) have the same set of eigenvectors, then the last equation is a proper rank-\( k \) eigenvalue decomposition of \( A_2 \), and \( \Delta \) gives its eigenvalues. Solving for \( \Delta \) gives

\[
\Delta = U^T A_2 U.
\]

If the \( k \)-by-\( k \) matrix \( \Delta \) is diagonal, then the spectral evolution hypothesis is true, and if \( \Delta \) is nearly diagonal, then the hypothesis is nearly true. Furthermore, comparing the diagonal entries of \( \Lambda \) and \( \Delta \) gives an indication as to the actual algebraic function connecting the two matrices, such as matrix powers or exponentials [11].

In the context of multi-profile networks, our goal is to learn the relationship between the friendship network and the family relationships. Thus, we apply the spectral diagonality test to the matrices \( A_p \) and \( F \). First, we compute the rank-\( k \) diagonalization of the adjacency ma-

|                  | Catster | Dogster | Hamsterster |
|------------------|---------|---------|-------------|
| Race             | 0.0138+ | 0.1372+ | 0.1556+     |
| Sex              | 0.0048+ | 0.0472+ | 0.0075+     |
| Coloration       | 0.0076+ | 0.0599+ | 0.0075+     |
| Weight range     |        |         |             |
| #Friends         |        |         |             |
| Birth date       |        |         |             |
| Join date        |        |         |             |
| Join age         |        |         |             |
| Weight           |        |         |             |
| Location         | 0.0888 |         |             |

R and + denote an estimate on the error of less than 0.1% and 1%, respectively [15, Eq. (5)]

a Categorical variable; numbers denote the assortativity coefficient [15, Eq. (2)]
b Numerical variable; numbers denote the Pearson correlation coefficient [15, Eq. (21)]
c In Dogster, the weight can only be chosen from a predefined set of ranges

d In Catster, the exact pet weight can be specified

Not computed for households as all pets in one household share their location.
\[ A_p = U \Lambda U^T \]  

We then compute \( \Delta \):

\[ \Delta = U^T F U = U^T R R^T U \]

Testing the \( k \)-by-\( k \) matrix \( \Delta \) for diagonality then gives an indication whether both matrices are related, and the relationship between the matrices \( \Lambda \) and \( \Delta \) gives an indication of the path relationships between friend and family relations. We use the value \( k = 250 \) in all calculations. We additionally also define the coefficient of diagonality, which measures what proportion of the matrix \( F \) is explained by a spectral transformation of \( A_p \). We define the coefficient of diagonality as the proportion of square entry weights in \( \Lambda \) that lie on the diagonal:

\[ \delta = \frac{\sum_{i,j} \Delta_{ij}^2}{\sum_{i,j} \Delta_{ij}^2} \]

The coefficient ranges from zero to one, and attains one when the two matrices have the exact same eigenvectors. The denominator is the squared Frobenius norm of \( \Delta \), and since the Frobenius norm is invariant under orthogonal transformations, it follows that \( \delta \) is the largest number such that \( F \) can be written as a sum of a spectral transformation of \( A_p \) and another matrix. Thus, \( \delta \) denotes to what extent the family relationships are represented by friendships. Note that this coefficient works in an opposite way to well-known co-spectrality measures [9], which aim to measure how similar the eigenvalues of two matrices are, while \( \delta \) aims to measure to what extent they share the same eigenvectors.

6.3 Experiments

We compute the matrix \( \Delta \) as described above for the three sites, and show the result in Figure 4 (a-c). Furthermore, Table 4 shows the diagonality coefficient \( \delta \) of the tests. The results show that all three datasets display a partial diagonality for the matrix \( \Delta \). The diagonality coefficient \( \delta \) is 20% for Dogster, 28% for Catster, and 55% for Hamsterster. We may conclude from this that friendship links and family ties are the most consistent with each other on Hamsterster. All three results are consistent with temporal network evolution results given in [10].

Additionally, we show in Figure 4 (d-f) the relationship between the diagonal elements of the matrix \( \Lambda \) (the eigenvalues of \( A_p \)) and the diagonal elements of \( \Delta \). This type of plot serves to find out which matrix functions best maps one matrix to another [11]. The three mappings seen in the plot allow us to draw two conclusions. First, the plots are nearly symmetrical around the Y axis, indicating that the best mapping matrix function is an even function, i.e., paths of even lengths of friendships should be used to predict family ties. Secondly, for Catster and Hamsterster, the distribution of eigenvalues follows a nearly linear trend, indicating that a linear spectral graph transformation may be used, i.e., only short paths are relevant, and longer even paths (of length four, six, etc.) are not relevant. This is however not observed for Dogster.

7. PREDICTING FAMILY TIES

A family tie can be thought to exist between two pets that are in the same family, i.e., whose profiles were created by...
Given a multi-profile social network $G = (V, W, E, m)$, we want to predict whether two profiles are managed by the same account, i.e., information contained in $W$ and $m$, using only the profile-level network $G_p = (V, E)$, including the metadata associated with it. In the case of pet social networks, we use the available pet profile information along with the pet-level friendship links for learning. We investigate the following indicators (i.e., features), each of which applies to a pair of profiles $\{u, v\}$:

- **Degree difference**: The difference of degrees.
- **Friend**: This feature is one if there is a friendship between the two profiles and zero otherwise.
- **Common friends**: The number of common friends between the two profiles.
- **Jaccard index**: The Jaccard index between the sets of friends of the two profiles \(\text{Jaccard} = \frac{|u \cap v|}{|u \cup v|}\). This is related to the number of common friends, being normalized by the number of friends of either profile.
- **Same X**: 1 when \(X(u) = X(v)\), 0 otherwise.
- **Difference in X**: \(-|X(u) - X(v)|\)

We also perform a logistic regression prediction, combining all features given above. Let \(f_i(u, v)\) be the values for all features \(i\) enumerated above. Then, a logistic regression model takes the form

\[
 f_{\text{reg}}(u, v) = \frac{1}{1 + \exp\left(-a - \sum_i b_i f_i(u, v)\right)} .
\]  

The regression parameters \(b_i\) as well as \(a\) are learned using a training set of profile pairs. The training profile pairs are sampled from each dataset such that it contains \(e\) pairs of profiles that are in the same household and \(e\) pairs of profiles that are not in the same profile. This training set is disjoint from the test set defined in a similar way below.

### 7.2 Experimental Setup

In order to measure the accuracy of each prediction method, we use a test set defined in the same manner as the training set, i.e., we randomly sample pet pairs known to be in the same family, and pet pairs known not to be in the same family. This test set is disjoint from the training set used for learning the regression parameters. The accuracy of the prediction methods is measured using the area under the curve (AUC) \(3\), which measures the probability that our prediction gives the correct ordering when applied to two randomly chosen pairs of profiles. Thus the AUC is 1/2 for a random prediction, and one for a perfectly accurate prediction. It is less than 1/2 for inverted predictions, i.e. predictions methods that become better when their values are negated. A perfectly inaccurate prediction has an AUC of zero. Table 6 gives the AUC values for each method separately and for the regression predictions, as well as the learned regression weights for each of the three sites.

### 7.3 Discussion

We observe that in all three sites, pets in the same household can be detected with an AUC of over 99% using the regression predictor. This means that given two pairs of pets, one of which from the same household and one of which from two different households, our algorithm will detect which is which in over 99% of cases. This high value can be explained by the fact that certain individual indicators are better distributed on a logarithmic scale. The additive term of one is used to take into account degrees of zero.

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**Table 5**: Definitions of the features used for family tie prediction. Each feature is given as a function of an unordered profile pair $\{u, v\}$.

| Feature | Definition |
|---------|------------|
| Degree difference | \(\log(1 + d(u)) - \log(1 + d(v))\) |
| Friend | \(1\) when $\{u, v\} \in E$ \(0\) otherwise |
| Common friends | \(|\{w \in V \mid \{u, w\}, \{v, w\} \in E\}|\) \(|\{w \in V \mid \{u, w\}, \{v, w\} \in E\}|\) |
| Jaccard index | \(|\{w \in V \mid \{u, w\}, \{v, w\} \in E\}|\) \(1\) when $X(u) = X(v)$ \(0\) otherwise |
| Same X | \(-|X(u) - X(v)|\) |

* We use the logarithm because the distribution of degrees is better distributed on a logarithmic scale. The additive term of one is used to take into account degrees of zero.
8. SUMMARY AND CONCLUSIONS

We have analysed the three online pet social networks Catster, Dogster and Hamsterster under the aspect of them being multi-profile networks, as they allow individual users to create any number of profiles, for each of their pets. We have shown that multi-profile networks can be analysed on two levels: the profile level and the account level. Our experiments showed that the two networks are related, but not identical, as the profile-level network is smaller, has smaller degrees, has a more equal degree distribution, less clustering and lower average path lengths. We also showed that a multi-profile network implicitly contains household links, and therefore a comparison between friendship and household links can be performed. We confirmed through a homophily analysis that intra-household homophily is higher than across-friendship homophily, and defined the multi-profile assortativity ratio in order to measure that difference.

In experiments, we found that the pet breed, join age and weight display the highest differences. Through extended spectral tests of diagonality, we were able to discover the relationship between friendships links and family ties in the network. Finally, we showed that it is possible to predict whether two profiles were created by the same user with a very high precision. In regards to this high precision, we conclude that it should be possible in principle to analyse the behavior of users creating multiple accounts on social networking platforms where this is not allowed. While corresponding datasets are inherently difficult to come by, a corresponding analysis would shed light on user behavior in terms of whether the profiles they create can be considered individual actors in the social network, or whether the person-level network should rather be considered. Although the methods developed in this paper can be applied to such datasets, we do not expect the individual numerical results to hold for the individual features, as users knowing that the creation of multiple accounts is not allowed can be expected to behave in a largely different way than users who are allowed to do this.

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Table 6: Results of family tie prediction.

| Feature          | Cat | Dog | Ham. | Regression weights |
|------------------|-----|-----|------|--------------------|
| Degree difference| 82.3% | 75.7% | 72.3% | 0.00 – 0.27 – 0.22 |
| Frienda          | 50.3% | 50.6% | — | 4.83 – 3.76 |
| Common friends   | 79.6% | 91.5% | 71.7% | – 0.46 – 0.71 – 4.98 |
| Jaccard index    | 82.8% | 92.2% | 76.2% | 5.78 – 9.73 – 1.25 |
| Same race        | 66.4% | 66.2% | 76.4% | 1.32 – 3.08 – 0.92 |
| Same sex         | 51.9% | 50.3% | 54.2% | 0.07 – 0.02 – 0.09 |
| Same colorationb | 57.2% | — | 59.4% | 0.95 – 5.59 |
| Same location    | 87.2% | 90.3% | 99.6% | 11.02 – 8.92 – 21.21 |
| Birth date difference | 53.7% | 50.1% | 73.5% | – 0.41 – 0.30 – 0.42 |
| Same join date   | 79.7% | 74.6% | 78.2% | 6.08 – 5.44 – 6.21 |
| Join date difference | 90.8% | 87.6% | 91.9% | 1.19 – 0.87 – 0.24 |
| Join age difference | 52.7% | 48.7% | 66.2% | 0.42 – 0.30 – 0.88 |
| Weight differencec | 41.6% | — | — | – 0.01 – — |
| Same weightc     | — | 61.9% | — | 0.52 – — |

The best individual predictor, the join date difference, achieves an AUC near to 90% for all three sites, indicating that users often create multiple pet accounts in quick succession. This may be explained by the fact that the sites have only been in operation for a decade. After a longer time period of observation, we may expect this number to go down. In contrast to this, the birth date of a pet is not a good indicator for being in the same household (AUC near to 50% for Catster and Dogster), indicating that users of the pet social networks do not have pets all born in quick succession; this is consistent with the behavior of many people acquiring new pets only after old ones die.

The location is a good individual indicator too, as by construction pets of the same household must have the same location.

Properties of pets such as the sex, the race, the color and the weight are not good indicators, with most AUC values not differing much from 1/2. The highest AUC values among these is achieved by the species of hamsters (76%), the breed of cats and dogs (66%) and the weight ranges on Dogster (62%). This indicates that there is a slight tendency for owners to own pets of the same breed, and dogs of comparable weight. The failure of cat weight’s to predict anything can be explained by the low variance in cat weights in general, as compared to the high variance of dog weights.

The indicators based on the friendship network achieve AUC values from 70% to 90%, also indicating good prediction performance. The only exception is the existence of a friendship link itself, whose AUC is very near to 1/2. We may interpret this as users not being sure what to make of the possibility to connect two of their own pets with a friendship link; some users do it and some do not. This result is consistent with the symmetric shape of the plots in Figure 4 (d-f), which indicate that paths of even length of friendship links should be used.

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Hamsterster does not allow friendship links within one household.
Dogster does not allow to specify a dog’s coloration.
Catster allows exact weights and Dogster has weight ranges.
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