Levelized Cost of Electricity Oriented Modular String Inverter Design Optimization for PV Generation System Using Geometric Programming

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ABSTRACT  Levelized cost of energy (LCOE) is a commonly used metric to assess the cost-to-benefit ratio over the lifetime of an energy resource such as photovoltaics (PV). However, power electronics engineers tend to rely on metrics such as efficiency and power density which do not guarantee lifetime cost optimality. Recent work has shown that an LCOE-focused optimization approach can yield improved system designs, leading to improved lifetime performance with balanced lifetime cost and energy generation. This paper outlines an LCOE optimization framework for PV power electronics that uses geometric programming. The large number of circuit parameters and nonlinear nature of the system equations pose significant barriers. Our approach allows for decoupling of design variables which in turn enables superior computational efficiency and a near-optimal solution. By incorporating the power electronics design process and magnetic loss mechanism into the convex design framework, the optimization engine yields practically implementable parameters for a PV converter that minimizes LCOE. An optimization example for a cascaded modular PV inverter architecture is presented that suggests 3.35\% LCOE improvement can be achieved by the new power electronics and the advanced optimization. The proposed optimization framework can be applied to other power generation systems to evaluate the effect of the power electronics design on system lifetime costs and efficiency.

INDEX TERMS  Convex optimization, geometric programming, levelized cost of energy, lifetime energy cost, magnetic loss modeling, solar inverters, modular multilevel converter.

I. INTRODUCTION

Demand and the importance of renewable energy have been rapidly increasing because of recent technology advancements that are leading to significant improvements in cost-effectiveness, comparable to conventional generation [1]. Among renewable energy resources, photovoltaics (PV) are becoming increasingly common in all sectors, i.e., at the utility, commercial, and residential scales. Recent reports show that new installations of PV have increased by an order of magnitude during the last decade, and the generation cost of a PV system is even lower than conventional energy sources in some regions [2].

To improve PV system performance and displace retiring conventional generation, recent research and development...
Efforts have focused on cost reduction with balanced performance [3]. The Levelized cost of energy (LCOE) is a widely accepted metric used to evaluate a technology’s performance [4]. The LCOE measures the energy cost required to generate a unit amount of energy for a system’s lifetime, e.g., $/kWh. To estimate the lifetime cost and energy yield, it factors in comprehensive system-level characteristics, such as cost for procurement and installation, PV panel efficiency and degradation, cost for maintenance and downtime, and financial factors for investment. By reflecting and balancing multiple aspects, the LCOE-oriented system design can be used to maximize system performance.

Prior LCOE-oriented works on PV generation systems have analyzed the cost share of the system components and their impacts on the LCOE in the scope of the entire system [5], [6]. Moving the scope inside the system, several studies have been conducted to compare the technologies or different conditions in a region [7]–[10]. Most prior work using the LCOE has not considered the PV inverter, which is an essential element for dc-to-ac power conversion and the grid interface: in general, parameters in the power conversion (e.g., efficiency and cost) are assumed to be given or negligible. In power electronics designs, on the other hand, power conversion efficiency and component cost are common design metrics, but, in general, their impact on the system-level performance metrics has not been a high-priority. The inverter performance, however, has been found to be not negligible; moreover, inverter performance can significantly impact the entire system’s performance, including its LCOE [11]. An LCOE optimization methodology for a PV inverter system incorporating reliability of the components, reported in [12], demonstrated how components design optimization leads to LCOE reduction. In [13], Saridakis et al. presented a design optimization approach to evaluate the impact of using Silicon Carbide (SiC) power switches in PV inverters. By balancing the switching frequency and output filter structure, they showed the potential of the SiC PV inverters with device cost reduction, compared to Silicon-based ones. A recent work reported in [14] studied the impact of different PV power conversion system architectures on LCOE, including conventional central and string inverters and multiport dc transformers, with battery storage at utility-scale. Reference [15] discusses PV inverter LCOE improvement by employing absolute active power control to contribute to extending the inverter lifetime with thermal stress on power switches controlled. Since the LCOE of a generation system can be designed to represent the impact of design parameters of interest in detail, an LCOE-oriented design methodology can derive improved design by balancing both efficiency and cost with proper system modeling [11], [12], [14]. In addition, it can be used to evaluate the relative value of a new technology compared to the baseline state-of-the-art solutions [13]–[15].

This paper presents an LCOE-oriented PV inverter design optimization. Distinguished from prior works that used exhaustive search [11], [15] or genetic algorithm [12], [13], geometric programming (GP) is employed in this optimization work for its superior computation efficiency. In addition, GP can guarantee the global optimal point obtained with detailed models, if the optimization problem is well constructed to have a specific form that is compatible with the method [16]. Section II presents an introduction to the GP optimization. Component-level modeling for the power conversion loss and the cost to model the LCOE improvement from a new technology are introduced in Section III. Section IV presents an optimization case study of a cascaded modular PV inverter system design with the GP algorithm, rated for a 200-kW commercial-scale PV generation. The proposed LCOE-oriented design optimization is validated by comparing its results with ones from the physics model-based method. Section V concludes the paper with final remarks.

II. GEOMETRIC PROGRAMMING

Geometric programming (GP) is an optimization problem-solving algorithm that has a special form of objective and constraint functions, called posynomial. The posynomial function can be defined as:

$$f_{posy}(x) = \sum_{k=1}^{K} (c_k x_1^{a_{k1}} x_2^{a_{k2}} \ldots x_n^{a_{kn}}),$$

where $x_1, x_2, \ldots, x_n$ are optimization variables; and coefficients $c_k$ and exponents $a_{k1}, a_{k2}, \ldots, a_{kn}$ are the model parameters, which are determined by the model characteristics. Here, the optimization variables and the coefficients are positive real, and the exponents are real values. This is why this form is called “posynomial,” which is “positive,” and “polynomial.” $K$ is the number of terms in the posynomial function, and it is in monomial form when $K = 1$. The standard form of GP is expressed as:

$$\text{minimize } f_o(x) = \sum_{k=1}^{K} (c_k x_1^{a_{k1}} x_2^{a_{k2}} \ldots x_n^{a_{kn}})$$

subject to

$$f_i(x) = \sum_{l=1}^{L} (c_{il} x_1^{a_{i1}} x_2^{a_{i2}} \ldots x_n^{a_{in}}) \leq 1, i = 1, \ldots, m,$$

$$h_j(x) = c_j x_1^{a_{j1}} x_2^{a_{j2}} \ldots x_n^{a_{jn}} = 1, j = 1, \ldots, p,$$

where $f_o$ is the objective function, which should be minimized in the optimization problem; and $f_i$ and $h_j$ are the inequality and the equality constraint functions, respectively, which should be satisfied in the optimal point.

GP retains benefits in the convex optimization since is a special expansion of the convex [16]. The convex optimization is mathematically proven to find the global optimal point if it exists. It is also known to have polynomial time complexity [17]; it can solve an optimization problem in a computationally-efficient manner. The computation performance of the convex optimization (and GP) has been improved attributed to the advancements in solvers, e.g., MOSEK [18]. Moreover, it has been recently found that...
many practical optimization problems, especially in electrical circuit design and system optimization problems, have model functions that are equivalent or well approximated to the posynomial form; therefore, GP has been widely applied in practical electrical circuit/system design optimization problems [19]–[24].

The first step to solving the practical optimization problem with GP is to model the target system in a posynomial form. In case the system cannot be modeled, by nature, in posynomial form, the original non-posynomial model is fitted functions to minimize the deviation of the relative errors: 

$$\sum_{i=1}^{N} r_i^2 = \sum_{i=1}^{N} \left( f_i(x) - f_{fist}(x) \right)^2$$  \hspace{1cm} (3)

or the maximum relative error

$$\max \left( \left| \frac{f_i(x) - f_{fist}(x)}{f_i(x)} \right| \right), \quad i = 1, \ldots, N$$  \hspace{1cm} (4)

is commonly used as the error term, where \(N\) is the number of data points of \((x, f(x))\). The fitting process is a nonlinear optimization problem, and general nonlinear least-square methods—such as Gauss-Newton [25] or Levenberg-Marquardt [26]—can be used to find the unknowns that minimize the error. These methods are not guaranteed to converge to the global optimum, and the solution depends on the initial values of unknowns. So the accuracy of the solution can be enhanced by repeated trials with changing initial values. There are several techniques to reduce the error in the posynomial fitting [27] [28]. In these works, the fitting error is reduced by applying a larger number of monomial terms or by breaking the data set into clusters and performing the fitting with each cluster.

The GP problem with optimization variables that are contained within a certain discrete set, such as an integer, is called mixed-integer geometric programming (MIGP) [16] [29]. It cannot be solved with the same method as the ordinary GP, and methods for solving the MIGP have some drawbacks compared to the GP solver. The methods to solve the MIGP can be classified into two types. The first is the heuristic method. To obtain a solution, the heuristic method first relaxes the integer constraints and solves the relaxed noninteger GP problem. After solving, the relaxed values of the variables are rounded to integer variables, and the optimization process is completed by solving the reduced-order optimization problem, with integer variables fixed to the rounded values. Once formulated, it can calculate the optimum in a computationally efficient manner; however, as the process of relaxation and rounding is approximation of the original, it may find a design point that deviates from the global optimum. The second is the global method. In this method, the GP optimization runs multiple cases with different sets of fixed integer variables to find the global optimal. Since it explores the entire design space, it can find the global optimum at the cost of additional computation. Considering the trade-offs (computation and accuracy), one should choose the method for an MIGP problem.

### III. OPTIMIZATION MODEL

LCOE presents the lifetime energy cost of a generation system. This value incorporates system performance metrics, including the system efficiency, cost, degradation of the equipment, and interest rate; therefore, an LCOE-oriented optimization can identify the optimal system design for the minimum lifetime cost. LCOE for a generation system can be defined:

$$LCOE = \frac{C_{life}}{E_{life}} = \frac{C_0 + \sum_{t=1}^{T} C_t}{8760 \cdot P_{rated} \cdot \bar{\eta} \cdot \left( \sum_{t=1}^{T} (1 - \delta)^{t-1} \right)}$$  \hspace{1cm} (5)

where \(C_{life}\) and \(E_{life}\) are the lifetime cost and energy generated, respectively. \(C_0\) is the initial investment cost including installation cost; \(C_t\) is the yearly dispensed cost; and \(P_{rated}\) is the rated power of the system; \(\gamma = \bar{\eta} \cdot \delta\) is the ratio of actual power per year considering the system efficiency \((\eta)\) and capacity factor \((c_t)\), considering irradiance and temperature for a geographic region; \(\delta\) is the degradation factor, such as PV module degradation; and \(i\) is the interest rate.

By comparing the LCOE value to baselines, the effectiveness and performance of a new technology can be evaluated. In case of evaluating a new power electronics technology, i.e., replacement of the PV-to-grid power conversion architecture, the parameters related to the power conversion would vary, including the installation cost (inverter and transformer cost), \(C_0\), and the power conversion efficiency, \(\eta\). On the other hand, the rest of the parameters in (5) would remain unchanged, such as cost and degradation factor for PV modules. Based on this rationale, the LCOE of the system with a new topology can be re-written and simplified, focusing on the changes, as:

$$LCOE_{new} = \frac{(\bar{C}_0 - \Delta C_0) + \sum_{t=1}^{T} C_t}{8760 \cdot P_{rated} \cdot \bar{\eta} + \Delta \eta \cdot \left( \sum_{t=1}^{T} (1 - \delta)^{t-1} \right)}$$  \hspace{1cm} (6)

where \(\Delta C_0\) and \(\Delta \eta\) are the cost and efficiency change by applying the new topology, respectively. \(\bar{C}_0, \bar{\eta}, LCOE, E_{life}\) are the initial cost, efficiency, LCOE, and lifetime energy generation of the baseline topology, respectively. The LCOE improvement resulting from replacing the power conversion architecture in the new topology can be expressed as follows:

$$\frac{\Delta LCOE}{LCOE} = \frac{LCOE - LCOE_{new}}{LCOE} = \frac{\Delta C_0}{E_{life} \cdot LCOE} + \frac{\Delta \eta}{\bar{\eta}}.$$  \hspace{1cm} (7)

Since lower LCOE translates into lower cost required to generate the same amount of energy, the greater LCOE improvement factor (7) is, the better (lower) LCOE is expected.
Based on this, (7) is used as the objective function for the optimization in the study.

To compute the improvement factor for a new power conversion technology, its cost and loss model are required to derive $\Delta C_0$ and $\Delta \eta$, as shown in (7). To abstract the two differentials, the major components in the power conversion that determine the two key metrics should be properly modeled. Since the power semiconductor devices and magnetic components, i.e., transformers and inductors, are the major drivers for the power electronics cost and loss, modeling of the two is explained in the subsections.

A. SWITCH LOSS MODELING

The loss of the power semiconductor switch is represented by the conduction loss and the switching loss, which can be expressed as:

$$P_{\text{cond}} = i_{\text{rms}}^2 R_{\text{ds(on)}},$$  
(8)

$$P_{\text{sw}} = 2C_{\text{oss}} v_{\text{sw}}^2 f_{\text{sw}},$$  
(9)

respectively, where $R_{\text{ds(on)}}, C_{\text{oss}}$ are the on-state resistance and the output capacitance of a switch device. $i_{\text{rms}}, v_{\text{sw}}, f_{\text{sw}}$ are the root-mean-square (RMS) current, drain-source voltage, and switching frequency of the switch, respectively. Here, the relationship between $R_{\text{ds(on)}}$ and $C_{\text{oss}}$ can be modeled considering the bare die area and height of the switch. The output capacitance, $C_{\text{oss}}$, can be modeled as the capacitance of the parallel plate capacitor as:

$$C_{\text{oss}} = A_{\text{die}} \frac{W}{d} + C_{\text{fixed}},$$  
(10)

where $W, A_{\text{die}}$ are the width and height of the bare switch die, and $C_{\text{fixed}}$ is the offset capacitance. In case of a vertical SiC MOSFET, $W$ and $A_{\text{die}}$ can be modeled as:

$$W = \frac{2v_{BD}^{\text{die}}}{E_{\text{C,SiC}}},$$

$$A_{\text{die}} = K_{\text{IC,SiC}} \frac{v_{BD}^{\text{die}}}{R_{\text{ds(on)}},}$$  
(11)

where $v_{BD}$ is the blocking voltage of the switch. Coefficients of the switch characteristics, $C_{\text{fixed}}, E_{\text{C,SiC}}, K_{\text{IC,SiC}}$ and $\kappa$, can be extracted from the device datasheet. Substituting (10) and (11) into (9), the switching loss can be expressed in the posynomial form of $R_{\text{ds(on)}}$ and $f_{\text{sw}}$ as:

$$P_{\text{sw}} = 2 \left( \frac{K_{\text{IC,SiC}} E_{\text{C,SiC}} v_{\text{BD}}^{\kappa-1}}{2R_{\text{ds(on)}},} + C_{\text{fixed}} \right) v_{\text{sw}}^2 f_{\text{sw}}.$$  
(12)

B. TRANSFORMER AND INDUCTOR LOSS MODELING

The loss of magnetic components can be modeled by the core loss and the copper loss. The core loss of the unit volume core can be expressed by the iGSE method [30] as:

$$P_{\text{core}} = \frac{1}{T_s} \frac{K_{fe}}{(2\pi)^{\alpha-1}} \left[ \int_{0}^{2\pi} |\cos \theta|^{\alpha-2-\beta-\alpha} d\theta \right] \left( \int_{0}^{T_s} (B_{pk-pk})^{\beta-\alpha} \left| \frac{dB}{dt} \right|^{\alpha} dt \right)$$  
(13)

where $\alpha, \beta, K_{fe}$ are parameters related to the core material and geometry and can be found in the core datasheet. $T_s$ and $B_{pk-pk}$ are the unit period of the AC excitation and the peak-to-peak value of the flux density of the core section $B(t)$ within a single excitation period, respectively. In this model equation, the core loss can be expressed as a posynomial if $B(t)$ is properly modeled. Since $B(t)$ is based on the voltage profile applied to the component, its modeling and approximation process needed would depend on the circuit topology and operation. Section IV will provide an example modeling process of the core loss to formulate a posynomial form for the case study.

For the magnetic copper loss, Dowell’s equation can be used [31]. This model function is derived by considering the dc resistance, skin effect, and proximity effect of the winding. Fig. 1 shows the design concept of the winding. In this figure, the shape of the winding is assumed to be rectangular, and $t_{cu}, t_w$ are the thickness and width of a winding. $W_{\text{core}}$ and $Th_{\text{core}}$ are the width and thickness of the winding area in the core, respectively. Assuming that the current waveform at the winding is sinusoidal, the copper loss of a single layer winding in Fig. 1 can be expressed as:

$$P_{\text{layer}} = \frac{\phi R_{\text{dc}}}{(N_t/m)^2} \left( \frac{f_h^2 + f_0^2}{f_h^2} \right) \left( \frac{\sin (2\phi) + \sin (2\phi)}{\cos (2\phi) - \cos (2\phi)} \right)$$

$$- 4f_h f_0 \sinh (\phi) \cos (\phi) + \cosh (\phi) \sin (\phi) \cos (\phi)$$

$$\cosh (2\phi) - \cos (2\phi)$$

$$= \frac{\phi R_{\text{dc}}}{(N_t/m)^2} \left( \frac{f_h^2 + f_0^2}{f_h^2} \right) G_1(\phi) - 4f_h f_0 G_2(\phi),$$  
(14)

where $N_t, m, f_h, f_0$ are the total winding turns, the number

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**FIGURE 1.** Winding geometry for copper loss modeling of magnetic components.
of layers, and the magneto-motive forces (MMF) on both sides of the winding, respectively. \( R_{dc} \) is the dc resistance of the single-layer winding, expressed as \( R_{dc} = \rho \cdot \frac{N_{t,MLT}}{m \cdot \delta} \), where \( MLT \) is the mean length per single turn, \( \phi = \frac{l_{cu}}{\delta} \)

with \( \delta = \sqrt{2\pi \mu_0 f_{ac}} \) is the skin depth, where \( f_{ac} \) is the frequency of the AC excitation.

The total copper loss of the component is calculated by adding the copper loss of every layer, considering \( f_h \) and \( f_0 \) of each layer. In the case of the transformer, the total copper loss is dependent on the winding structure of the primary and the secondary winding; previous work [32] showed that the copper loss is minimized if the primary and the secondary winding are alternately placed layer by layer. This placement is called a fully interleaved placement. The MMFs, \( f_0 \) and \( f_h \), for the fully interleaved winded transformer can be expressed as shown in Fig. 2a.

In Fig. 2a, the MMF values of the primary winding and the secondary winding in a certain layer can be expressed as:

\[
\begin{align*}
\phi_{0, pri} &= f_{h, sec} = 0, \\
\phi_{h, pri} &= f_{0, sec} = \frac{N_{t, pri} i_{rms,pri}}{m},
\end{align*}
\]

where \( N_{t,pri}, i_{rms,pri} \) are the total turns and the RMS current value of the primary-side winding, respectively. Equation (15) is satisfied if the MMF from the primary winding and the secondary winding are in opposite directions and therefore cancel each other. By substituting (15) into (14), the copper loss of the fully interleaved transformer by a single AC frequency can be presented as:

\[
P_{layer,XF} = \frac{\phi R_{dc,pri}}{(N_{t,pri}/m)^2} \left( \frac{N_{t,pri} i_{rms,pri}}{m} \right)^2 G_1(\phi)
\]

where \( R_{dc,pri} = \left( \frac{N_{t,pri}}{\sqrt{N_{t,sec}}} \right)^2 \) is assumed. \( N_{t,sec}, i_{rms,sec} \) are the total turns and the RMS current value of the secondary-side winding, respectively. The total copper loss of the transformer considering up to \( n_{th} \) harmonic current components can be calculated as:

\[
P_{copper,XF} = \sum_{j=1}^{m} \sum_{k=1}^{n} P_{layer,XF}
\]

\[
= 2m R_{dc,pri} \sum_{k=1}^{n} \sqrt{\phi k_{rms,kth}^2} \frac{G_1(\sqrt{\phi k_{rms,kth}})}{m}
\]

where \( i_{rms,kth} \) is the RMS current value of the \( k^{th} \) harmonics component.

Different from the transformer, the MMF of a common inductor accumulates as additional layers are stacked, as illustrated in Fig. 2b. The MMF values of the inductor by the \( j_{th} \) layer are expressed as:

\[
\begin{align*}
\phi_0(j) &= (j - 1) N_i t_{rms}/m \\
\phi_h(j) &= \phi_0(j + 1) = j N_i t_{rms}/m.
\end{align*}
\]

By substituting (18) into (14), the copper loss of the \( j^{th} \) layer by the fundamental frequency current can be expressed as follows:

\[
P_{layer,IND}(j) = \frac{\phi R_{dc}}{(N_i/m)^2} \left( f_h^2 + f_0^2 \right) G_1(\phi) - 4 f_h f_0 G_2(\phi)
\]

\[
= \phi R_{dc} \frac{i_{rms}^2}{m} \left( \left( j^2 + (j - 1)^2 \right) G_1(\phi) - 4 j (j - 1) G_2(\phi) \right).
\]

Similar to the transformer modeling, the total copper loss of the inductor, considering up to \( n_{th} \) harmonics, can be
modeled from (19):

\[ P_{\text{copper,IND}} = \sum_{j=1}^{m} \sum_{k=1}^{n} \sum_{m=1}^{n} \sqrt{k}\phi R_{dc} i_{\text{rms},kth}}^2 \times \left( \left( j^2 + (j - 1)^2 \right) G_1 \left( \sqrt{k}\phi \right) - 4j (j - 1) G_2 \left( \sqrt{k}\phi \right) \right) \]

\[ = m R_{dc} \sum_{k=1}^{n} \sqrt{k}\phi i_{\text{rms},kth}}^2 \times \left( G_1 \left( \sqrt{k}\phi \right) + \frac{2}{3} (m^2 - 1) \left( G_1 \left( \sqrt{k}\phi \right) - 2G_2 \left( \sqrt{k}\phi \right) \right) \right) \]

\[ = \left( \frac{2}{3} (m^2 - 1) \right) \left( G_1 \left( \sqrt{k}\phi \right) - 2G_2 \left( \sqrt{k}\phi \right) \right) \]

Note, for GP optimization, since \( G_1 \) and \( G_2 \) in the copper loss models, (17) and (20), are not posynomials as of function of \( \phi \), they should be approximated. As the RMS harmonic current, \( i_{\text{rms},kth} \), depends on the circuit topology and operation, an example fitting process is presented in Section IV.

\section*{C. COST MODELING}

The component costs for a power conversion architecture can be abstracted by developing scalable models as functions of key performance metrics. For the power switches, by assuming that the semiconductor switch cost is proportional to the die area, the switch cost can be modeled as:

\[ C_{\text{semi}} = \frac{k_{\text{semi, cost}}}{K_{\text{SiC}}} A_{\text{die}} = k_{\text{semi, cost}} \frac{V_{\text{BD}}^2}{R_{\text{ds(on)}}}. \quad (21) \]

Using the market data of the switch under consideration, the cost model coefficient, \( k_{\text{semi, cost}} \), can be calculated.

For the magnetic components, in this study, it is assumed that their cost is determined by the core cost, and, based on this, the cost for transformers and inductors is modeled to be proportional to core volume:

\[ C_{\text{core}} = k_{\text{core}} V_{\text{core}} \quad (22) \]

where \( V_{\text{core}} \) is the relative core volume of the magnetic component, and \( k_{\text{core}} \) is the cost of the unit volume core.

\section*{IV. CASE STUDY: OPTIMIZATION ENGINE DEVELOPMENT USING GEOMETRIC PROGRAMMING FOR DIRECT LOW-VOLTAGE DC TO MEDIUM-VOLTAGE AC PV POWER CONVERSION SYSTEM}

In this section, an optimization case study for a cascaded modular PV inverter architecture is presented. Fig. 3 shows the system block diagram under this study using circuit + control, (C^2), modules. The C^2 architecture features cascaded modular power conversion with stacked converter modules to directly generate medium-voltage AC output without grid frequency (50/60 Hz) step-up transformers from low-voltage DC input, i.e., <1500 V. Each C^2 module is connected to a PV string, and the PV DC power is transformed to the three-phase AC driven by a quadruple active bridge (QAB) with high-frequency transformers followed by three full-bridge inverters for individual phases, as illustrated in Fig. 3. The QAB stage operates as a DC transformer (DCX) with a fixed conversion ratio to improve the efficiency. The three phase outputs of the modules are connected in series with interleaved pulse-width modulation (PWM) signals to synthesize the multilevel medium-voltage output. Further details of the power conversion can be found in [33].

\section*{A. SYSTEM MODELING}

Based on the circuit and system operation, the LCOE improvement the C^2 system can achieve, compared to a conventional PV generation architecture, can be modeled as follows. The modeling approach considers major components in power conversion, including the QAB primary switches (4ea/module), QAB secondary switches (12ea/module), inverter switches (12ea/module), high-frequency transformers (3ea/module), and QAB leakage inductor (3ea/module). To represent the system characteristics for the LCOE-oriented optimization, the system modeling should incorporate the loss and cost mechanism of the individual components. As presented in Section III, the loss functions, \( P \), and the cost functions, \( C \), for the C^2 system can be defined as:

\[ P_{\text{QAB},C} = f_{1} \left( N_{\text{module}}, f_{\text{on,pri}}, f_{\text{on,sec}}, f_{\text{sw,QAB}} \right) \]

\[ P_{\text{INV},C} = f_{2} \left( N_{\text{module}}, f_{\text{on,sec}}, f_{\text{sw,INV}}, f_{\text{sw,QAB}} \right) \]

\[ P_{\text{IND},C} = f_{3} \left( N_{\text{module}}, f_{\text{on,ind}}, f_{\text{sw,INV}}, f_{\text{sw,QAB}} \right) \]

\[ C_{\text{semi,QAB}} = f_{4} \left( N_{\text{module}}, f_{\text{on,semi}}, f_{\text{sw,INV}}, f_{\text{sw,QAB}} \right) \]

\[ C_{\text{semi,INV}} = f_{5} \left( N_{\text{module}}, f_{\text{on,semi}}, f_{\text{sw,INV}}, f_{\text{sw,QAB}} \right) \]

\[ C_{\text{core,IND}} = f_{6} \left( N_{\text{module}}, f_{\text{on,core}}, f_{\text{sw,IND}}, f_{\text{sw,INV}} \right) \]

\[ (23) \]

where \( N_{\text{module}}, f_{\text{on}}, f_{\text{sw}}, \) and \( f_{\text{on}} \) are the number of modules, the on-state resistance of the primary-side switch, the secondary-side switch, the inverter switch, the switching frequency of the QAB, the switching frequency of the inverter, and the core volume of the transformer and inductor, respectively. The magnetic component variables, \( N_{\text{t}}, m_{\text{t}}, t_{\text{w}}, \) and \( t_{\text{cu}} \) are the geometric parameters of the transformer and inductor, representing the number of turns, number of layers, and the winding width and winding thickness of the transformer and inductor, respectively. Here, in the modeling process, the power and voltage ratings of the cascaded modules in the target system are assumed to be identical. This assumption is based on the common practice found in the field, mostly for controllability, scalability with standardized modules, and cost-effectiveness in commercialization and mass production. Similar approaches in modular power conversion systems are found in [34], [35].

The function models, \( f_{1}, ..., f_{7} \), in (23) will be represented in posynomial form, with respect to the design variables, to perform the GP optimization.

Fig. 4 shows the relationship of the optimization variables and the model functions of the C^2 system. The system
efficiency and the total installation cost are calculated from (23) as:

\[ \eta_{\text{new}} = 1 - \frac{P_{\text{loss,}C2}}{P_{\text{rated}}} = 1 - \frac{N_{\text{module}}}{P_{\text{rated}}} \left( P_{QAB,C2} + P_{INV,C2} + P_{XF,C2} + P_{IND,C2} \right), \]

and

\[ C_{0,\text{new}} = N_{\text{module}} \left( C_{\text{semi,}QAB} + C_{\text{semi,}INV} + C_{XF,C2} + C_{IND,C2} \right). \]

The LCOE improvement factor of the \( C^2 \) system can be calculated by substituting the calculated values in (24) and (25) and the LCOE, \( E_{\text{life}}, \eta, C_0 \), values for the conventional one into (7). The modeling process of each loss and the cost model (23) is explained in the following.

1) Switch Loss Modeling

First, the switch loss model for the \( C^2 \) system, \( P_{QAB,C2} \) and \( P_{INV,C2} \), are presented. The power loss of the QAB switches, \( P_{QAB,C2} \), can be expressed as:

\[ P_{QAB,C2} = P_{\text{cond,}QAB} + P_{\text{sw,}QAB} = (4i_{\text{rms,}pri}^2R_{\text{ds(on),}pri} + 12i_{\text{rms,}sec}^2R_{\text{ds(on),}sec}) \]

\[ + 12 \left( \frac{k^2}{2} C_{\text{oss,}sec} i_{\text{sw,}QAB} \right), \]

where \( i_{\text{rms,}pri} \) and \( i_{\text{rms,}sec} \) are the RMS current of the primary-side switches and secondary-side switches of the QAB, respectively. Fig. 5 shows the voltage and current waveform of the QAB in the \( C^2 \) topology. Variables, \( v_{pri}, v_{sec}, i_L, i_{\text{sw,}pri}, \Delta t, T_{sw} \) are the primary-side voltage, secondary-side voltage, inductor current, primary-side switch current, time delay between the primary and secondary voltage, and the switching period of the QAB, respectively. In
the QAB, a phase difference between \( v_{pri} \) and \( v_{sec} \) allows the electrical power to be delivered to the secondary side. The RMS value of \( i_{sw,pri} \) is represented as follows [33]:

\[
i_{rms,pri} \approx \frac{i_{pv}}{1 - \frac{\phi_{max}}{\pi}} \sqrt{1 - \frac{\phi_{max}}{3\pi}} = \frac{P_{rated}}{N_{module}v_{pv}} \frac{1}{1 - \frac{\phi_{max}}{\pi}} \sqrt{1 - \frac{\phi_{max}}{2}} (27)
\]

where \( \phi_{max} = \frac{\Delta t}{T_{sw}} \) is the phase shift between the primary-side voltage and the secondary-side voltage at the rated power, which is set as \( \pi/6 \) for practicality. \( v_{pv}, i_{pv} \) are the voltage and current from the PV panel connected to the QAB primary side. As discussed, the QAB has three secondary-side bridges to output three-phase AC, and the secondary-side current, \( i_{sw,sec} \), contains the line frequency component. The RMS value of \( i_{sw,sec} \) considering the line frequency component can be expressed as:

\[
i_{rms,sec} \approx \frac{v_{dc}\phi_{max}}{2\pi f_{sw,QAB}L_{lk}} \sqrt{1 - \frac{4\phi_{max}}{9\pi^2}}, (28)
\]

where \( L_{lk} \) is the designed leakage inductance of the QAB for the rated power at \( \phi = \phi_{max} \), \( v_{dc} \) is the dc-link voltage at the QAB secondary side and the inverter input, which can be calculated as:

\[
v_{dc} = m_{INV}v_{g,pk} = n_{TR}v_{pv} \tag{29}
\]

where \( m_{INV} \) is the output voltage margin of the inverter, \( v_{g,pk} \) is the peak phase voltage of the grid, and \( n_{TR} = \frac{N_{i, pri}/N_{i, sec}}{N_{module}} \) is the turns ratio of the QAB transformer. Substituting (27), (28), and (29) into (26), the QAB switch loss model can be represented as the posynomial function form with respect to variables \( N_{module}, R_{ds(on), pri}, R_{ds(on), sec}, f_{sw,QAB} \).

The switching loss of the QAB switches can be modeled by incorporating the degree of soft switching achieved over an AC line cycle and the other switching losses. Using a current source(s), e.g., leakage inductance or magnetizing inductance, the QAB can be designed to achieve zero-voltage switching (ZVS) to mitigate the turn-on switching loss, which is the major loss component of major carrier switches, such as SiC MOSFETs under this study. Since the QAB delivers AC power, fluctuating at double-line frequency (100 or 120 Hz), the ZVS condition may depend on the line cycle and the source of soft switching. The QAB primary-side switches can straightforwardly obtain the full ZVS, independent of the line cycle, since the sum of the three individual AC phase currents flowing through the primary side is kept constant. The secondary-side switches can also achieve the ZVS by securing the enough inductive current and PWM dead time over the entire line cycle, e.g., having moderate magnetizing inductance current that remains constant to discharge parasitic device capacitances, independent of line cycle. Further discussion on the soft switching techniques for the QAB can be found in [36].

With the QAB designed to achieve the full ZVS, the other switching losses remaining should be incorporated into the switch model to result in a reasonable design optimization. To represent the inevitable switching losses, including MOSFET gate drive loss and turn-off loss, the switching-loss factor, \( k \), in (26), is set to 0.1 in this study.

The loss model of the inverters is expressed as:

\[
P_{INV,C2} = P_{cond,INV} + P_{sw,INV}
\]

\[
= 12i_{rms,INV}^2R_{ds(on),INV} + 12(2v_{dc}^2C_{oss,sec}f_{sw,INV}) \tag{30}
\]

Here, the single-phase inverters of the \( C^2 \) architecture are implemented with full-bridge inverters operating in the bipolar modulation and hard switching, no soft-switching technique employed. In this case, the RMS current of the inverter switches, \( i_{rms,INV} \), considering the switching ripple can be expressed as follows [37]:

\[
i_{rms,INV} = \sqrt{\frac{i_{rms,g}^2 + i_{rms,ripple}^2}{12}} = \sqrt{\frac{v_{dc}^2}{2(N_{module}/f_{sw,INV})}} \tag{31}
\]

As in the QAB case, the inverter switch loss model can also be expressed in posynomial form with respect to \( N_{module}, f_{sw,INV}, \) and \( R_{ds(on),INV} \) by substituting (29) and (31) into (30). Here, an additional constraint is required to ensure that the switching ripple current of the inverter stays below the limit. Equation (32) shows the inequality constraint regarding this limit. It is a posynomial form with respect to variables \( N_{module} \) and \( f_{sw,INV} \).

\[
i_{ripple, pk} = \frac{v_{dc}}{2(N_{module}/f_{sw,INV})(L_{INV})} \leq i_{ripple, lim} \tag{32}
\]

2) Magnetic Component Loss Modeling

The loss of the magnetic components in the \( C^2 \) system occurs at the inductors and transformers in the QAB. In general, the magnetic loss can be divided into core loss and copper loss. The core loss can be presented in posynomial forms as functions of the design variables, \( V_{core, XF} \), \( V_{core, IND} \), \( N_{i, XF} \), \( N_{i, IND} \), \( f_{sw,QAB} \), and \( T_{sw,QAB} \). Fig. 6 shows the flux density waveforms, \( B(t) \), for the transformers and the inductors in the QAB, which can be expressed as:

\[
B_{XF}(t) = \frac{v_{XF}}{N_{t, A_{up}} v_{pu, t}} \frac{N_{t, XF}A_{Bus}V_{core, XF}}{v_{pu} (T_{sw, QAB} - t)} \tag{33}
\]

\[
(0 \leq t \leq 0.5T_{sw, QAB}) \quad \text{and} \quad (0.5T_{sw, QAB} \leq t \leq T_{sw, QAB})
\]
Here, the peak value of the flux density should be kept under the limit, \( B_{\text{max}} \), to avoid the core saturation. Constraints related to the saturation can be expressed as:

\[
B_{XF,pk} = \frac{0.5v_{pu}T_{sw,QAB}}{N_{t, XF}A_{\text{Base}}V_{\text{core, XF}}^{2/3}} \leq B_{\text{max}}
\]

\[
B_{1\text{ND},pk} = \frac{v_{pu}\Delta t}{N_{t, 1\text{ND}}A_{\text{Base}}V_{\text{core, 1\text{ND}}}^{2/3}} \leq B_{\text{max}}
\]

(36)

where \( B_{XF,pk} \) and \( B_{1\text{ND},pk} \) are the peak values of flux density, \( B_{XF}(t) \) and \( B_{1\text{ND}}(t) \), respectively. As shown, these constraints are in monomial form to the design variables; they can be applied to the GP optimization without modification.

In contrast to the core loss model, the native copper loss model is not in posynomial form to variables, \( V_{\text{core, XF}}, V_{\text{core, 1\text{ND}}}, \) and \( f_{sw,QAB} \); therefore, the posynomial fitting to the total copper loss is required so that the loss model can be applied to the GP optimization.

To derive the copper losses of the QAB inductors and transformers, the RMS values of the harmonic current components can be expressed, referring to illustration in Fig. 5, by using the Taylor series:

\[
i_{\text{rms, pri}} = i_{L,pk}\sum_{k=1}^{n} \frac{2\sqrt{2}\sin(\kappa\phi\pi)}{\alpha(\kappa\pi)^{2}}.
\]

(37)

Substituting (37) into (17), the total copper loss of the transformer can be expressed as:

\[
P_{\text{copper, XF}} = 2mR_{dc, pri}i_{L,pk}^{2}\sum_{k=1}^{n} \sqrt{\phi} \left( \frac{2\sqrt{2}\sin(\kappa\phi\pi)}{\alpha(\kappa\pi)^{2}} \right)^{2}
\]

\[
\times \left( \sinh(2\sqrt{2}\phi) + \sin(2\sqrt{2}\phi) \right)
\]

\[
+ \left( \cosh(2\sqrt{2}\phi) - \cos(2\sqrt{2}\phi) \right)
\]

\[= 2mR_{dc, pri}i_{L,pk}^{2}F_{ac, XF}(\phi).
\]

(38)

By fitting \( F_{ac, XF} \) to posynomial form with respect to \( \phi = t_{cu}/\delta \), the total copper loss can be modeled as posynomial with respect to \( t_{cu} \) and \( f_{sw,QAB} \). In the same manner, the total copper loss of the inductor is:

\[
P_{\text{copper, 1\text{ND}}}
\]

\[
= R_{dc,i_{L,pk}}^{2}\sum_{k=1}^{n} m\sqrt{\phi} \left( \frac{2\sqrt{2}\sin(\kappa\phi\pi)}{\alpha(\kappa\pi)^{2}} \right)^{2}
\]

\[
\times \left( G_{1}\sqrt{\phi} + \frac{2}{3}(m-1) \left( G_{1}(\sqrt{\phi}) - 2G_{2}(\sqrt{\phi}) \right) \right)
\]

\[= R_{dc,i_{L,pk}}^{2}F_{ac, 1\text{ND}}(m, \phi),
\]

(39)

Fig. 7a and 7b show the plots of the non-posynomial terms, \( F_{ac, XF} \) and \( F_{ac, 1\text{ND}} \), with respect to \( \phi \).

Next, the posynomial fitting process of \( F_{ac, XF} \) and \( F_{ac, 1\text{ND}} \) is presented. Posynomial functions should have convex or affine characteristics; however, as shown in Figs. 7a and 7b, \( F_{ac, XF} \) and \( F_{ac, 1\text{ND}} \) have a concave shape starting from \( \phi \approx 2.2 \), and this area aggravates the fitting

![Diagram of Flux Density](image-url)
error. In this case, higher fitting accuracy can be obtained by dividing the original function into several clusters and performing fitting to each cluster. In this case study, $F_{ac, XF}$ is divided into two clusters at $\phi = 2.2$, and $F_{ac, IND}$ is divided into three clusters at $\phi = 1$ and $\phi = 2$. Figs. 8 and 9 show the fitting result to $F_{ac, XF}$ and $F_{ac, IND}$, with and without clustering. The posynomial functions for the copper loss factors from the fitting are:

$$F_{ac, XF}(\phi) = \begin{cases} 
0.2698\phi^{0.3268} + 0.9901\phi^{-0.0403} + 0.5965\phi^{0.3398} + 0.6940\phi^{2.486} + 0.3732\phi^{-0.0293} + 1.2633 
& (0 < \phi \leq 2.2) \\
0.2658\phi^{0.3268} + 0.7054\phi^{-0.6689} + 0.9176\phi^{0.2666} + 1.1906\phi^{0.9569} + 1.2674\phi^{2.1699} + 0.0591 
& (\phi \geq 2.2) 
\end{cases}$$

(40)

This clustering technique is distinguished from the commonly used max-monomial fitting method [27] or the softmax-posynomial fitting method [38]. Fig. 10 illustrates the difference between the methods. The conventional clustering methods reported in [27] and [38] combines the fitting results of each cluster into a single fitting function. It is proven to work well when fitting into a convex function but not in a concave function case. The proposed clustering separates the original functions into several convex functions and solves each optimization problem with separate functions. This technique, as a result, has an optimization performance trade-off between model accuracy and computation time since more clusters require additional computation for solving subproblems.

There are constraints for the transformer/inductor practical design. First, the number of layers, $m_{XF}$ and $m_{IND}$, and the number of turns per layer, $N_{t, XF}/m_{XF}$ and $N_{t, IND}/m_{IND}$, should be integer. The entire width and thickness of the winding should also be less than the width and thickness of the winding area of the core so that all windings can be placed within the core. (42) shows the expressions of the geometric constraints:

$$W_{winding, XF} = \frac{N_{t, XF}}{m_{XF}} (T_{w, XF} + T_{w, sp, XF})$$

$$W_{winding, IND} = \frac{N_{t, IND}}{m_{IND}} (T_{w, IND} + T_{w, sp, IND})$$

(42)

$$Th_{winding, XF} = m_{XF} (T_{cu, XF} + T_{cu, sp, XF})$$

$$Th_{winding, IND} = m_{IND} (T_{cu, IND} + T_{cu, sp, IND})$$

$$W_{core, XF} = W_{Base} V_{core, XF}^{1/3}$$

$$W_{core, IND} = W_{Base} V_{core, IND}^{1/3}$$

where $W_{winding, XF}$ and $W_{winding, IND}$ are the width of the transformer and inductor winding; $Th_{winding, XF}$ and $Th_{winding, IND}$ are the thickness of the transformer and inductor winding; and $W_{Base}$ and $Th_{Base}$ are the width and thickness of the unit volume core, respectively. The space between the windings, $T_{w, sp}$ and $T_{cu, sp}$, are predetermined by considering the isolation voltage level and size limit. In this case study, It is assumed that $T_{w, sp} = 1$ mm and $T_{cu, sp} = 0.254$ mm.
3) Cost Modeling

The cost models of the $C^2$ system components, $C_{semi,QAB}$, $C_{semi,INV}$, $C_{core,XF}$, and $C_{core,IND}$ can be expressed as:

$$C_{semi,QAB} = 4C_{semi, pri} + 12C_{semi, sec} = k_{semi,cost} \left( \frac{v_{BD, pri}}{R_{ds(on), pri}} + 12 \frac{v_{BD, sec}}{R_{ds(on), sec}} \right)$$

$$C_{semi,INV} = 12C_{semi, INV} = 12k_{semi,cost} \left( \frac{v_{BD, sec}}{R_{ds(on), INV}} \right)$$

$$C_{core,XF} = 3k_{core} V_{core,XF}$$

$$C_{core,IND} = 3k_{core} V_{core,IND}.$$  \hfill (43)

As shown, the cost model functions have posynomial forms with respect to design variables, $R_{ds(on)}$ and $V_{core}$, directly applicable for the GP optimization. The model equations are based on two assumptions: i) the same SiC switch technology (latest CREE SiC devices in this study) is used for the QAB and the inverters, scalable with the key parameters, i.e., voltage rating and on-resistance, and ii) the same magnetic core material in the same geometry (EE cores from TDK in this study) are used in the transformers and the inductors, scalable with core volume. It makes the optimization problem more tractable and provide system-level insights. In case different types of semiconductor switches or cores are used, they can be reflected by modifying coefficients $k_{semi,cost}$ and $k_{core}$ in the cost model.

4) GP Optimization Model

The final GP optimization form for the $C^2$ system design is expressed as follows:

$$\text{maximize } \frac{\Delta LCOE}{LCOE}$$

subject to

$$B_{XF,pk} \leq B_{max}$$

$$i_{ripple,INV} \leq i_{ripple, max}$$

$$W_{winding,XF} \leq W_{core,XF}$$

$$W_{winding,IND} \leq W_{core,IND}$$

$$T_{h,winding,XF} \leq T_{h,core,XF}$$

$$T_{h,winding,IND} \leq T_{h,core,IND}.$$  \hfill (44)
In the optimization model (44), five design variables, $N_{\text{module}}, N_{t,\text{XF}}, m_{XF}, N_{t,\text{IND}},$ and $m_{\text{IND}}$ are integer values in practice. As a result, it is an MIGP problem, requiring a certain algorithm for optimization. In this case study, a heuristic method is chosen to obtain lower computation time.

Fig. 11 shows the flowchart of the heuristic algorithm used. In Step I, the five integer variables in (44) are relaxed as noninteger, and we obtain the temporary optimal points by solving the relaxed GP problems. As shown in (40) and (41), $\phi$ is clustered in a copper loss model fitting. The number of optimization problems generated is equal to the number of clusters in the copper loss fitting, and the solution can be obtained by solving each problem and selecting the solution that maximizes the LCOE improvement factor among them. In this case study, there are six subproblems from the clustering, two by three, two in the transformer loss fitting, three in the inductor fitting.

In the next steps, as shown in Fig. 11, relaxed variables are fixed to certain integer values to satisfy the practical design limitation. In Step II, the number of modules, $N_{\text{module}}$, and the number of turns per layer, $N_{t,\text{XF}}/m_{XF}$, are rounded to the integer values. First, two candidates of integer $N_{\text{module}}$ are selected, which are the floor and ceiling values of the value from Step I. Next, $N_{t,\text{XF}}/m_{XF}$ is rounded to maintain the turns ratio of the transformer $n_{TR} = N_{t,\text{sec}}/N_{t,\text{XF}}$ at its optimal value from Step I. As shown in (29), the transformer turns ratio determines the dc-link voltage and thus the switch...
Pre-optimization tasks:
- Posynomial system modeling as in (40)
- Determination of system ratings and parameters

Relaxation of integer free variables in (40)
\[ (N_{\text{module}}, N_{\text{XF}}, m_{\text{XF}}, N_{\text{t,IND}}, m_{\text{IND}}) \]

Solving relaxed optimization problem using Geometric Programming solver

Rounding variables \( N_{\text{module}}, N_{\text{XF}} \) into integer:
- \( N_{\text{module}} \): Floor / Ceiling values of solution in Step I
- \( N_{\text{XF}} \): Value selection for nearest \( n_{TR} \) (Fig. 12)

Solving reduced optimization problem:
Same problem, but values of \( N_{\text{module}} \) and \( N_{\text{XF}} \) fixed

Rounding rest of relaxed variables \( (m_{\text{XF}}, N_{\text{t,IND}}, m_{\text{IND}}) \) into integer

Solving final optimization problem w/ fixed integer variables

End

FIGURE 11. Flowchart of the proposed optimization method with two integer variable iteration processes to identify practical optimal design values.

FIGURE 12. Rounding of \( N_{t, XF} \) into integer: selection of candidates.

voltage ratings required. Consequently, it would determine the switching loss and cost of the switches. According to (21), the cost of the switches exponentially scale with the dc-link voltage, so the design with a low dc-link voltage in a certain range tends to outperform in the LCOE. Therefore, with the deviation of \( n_{TR} \) from the value in Step I limited, the degradation in the iterated LCOE improvement in Step II can be minimized. In addition, the number of turns per layer of the transformer should be integer values—both the primary side, \( N_{t, XF}/m_{XF} \), and the secondary side, \( N_{t, sec}/m_{XF} \).

Fig. 12 shows the potential design choices of the number of primary and secondary turns per layer of the transformer. As shown, \( N_{t, XF}/m_{XF} \) and \( N_{t, sec}/m_{XF} \) are selected under the three design criteria:
- \( n_{TR, min} \leq N_{t, sec}/N_{t, pri} \leq n_{TR, max} \)
- \( |N_{t, sec}/N_{t, pri} - n_{TR, Step I}| \) should be minimized.
- Among the \( (N_{t, XF}/m_{XF}, N_{t, sec}/m_{XF}) \) pairs with same \( N_{t, sec}/N_{t, pri} \), the pairs with a smaller number of turns should be selected for less copper loss.

The maximum and the minimum of the turns ratio, \( n_{TR, min} \) and \( n_{TR, max} \), can be expressed as:

\[
\begin{align*}
n_{TR, max} &= \frac{v_{BV, max}}{m_{BV} v_{pV}}, \\
n_{TR, min} &= \frac{v_{g, pk}}{N_{\text{module}} m_{\text{INV}} v_{pV}},
\end{align*}
\]

where \( v_{BV, max}, m_{BV}, \) and \( m_{INV} \) are the maximum switch voltage rating available, the boundary voltage margin for
the semiconductor switch, and the output voltage margin of the inverter. Having boundaries is to avoid significant variation of the turns ratio iterated in Step II that may result from rounding of $N_{\text{t},XF}/m_{XF}$ and $N_{\text{t},sec}/m_{XF}$. Five design candidates selected from the criteria, as an example, are illustrated in Fig. 12. At the final stage of Step II, the optimization engine reruns for the selected cases; as a result, it runs multiple times but with reduced free variables, the two design variables, $N_{\text{module}}$ and $N_{\text{t},XF}/m_{XF}$ predetermined. Among multiple optimal designs identified in Step II, one with maximum LCOE improvement is selected.

In Step III, first, the other integer variables are finalized; $m_{XF}$, $N_{\text{t,IND}}$, and $m_{IND}$ are rounded from the non-integer values identified in Step II. These variables are rounded to the nearest integer values. The last integer variable left, $N_{XF}$, is determined as $m_{XF}$ rounded accordingly. With optimal design values practical for the integer variables identified, the final optimal design is derived from the GP optimization with them fixed that refines the other design variables for the maximum LCOE improvement.

The system parameter values given for the optimization are listed in Table 1. The core characteristics for the magnetic components loss and cost in this case study are based on EELP 102 set with N87 from TDK [39]. For the semiconductor power switches, electrical characteristics and market data for commercial SiC power devices from Cree are used.

### B. OPTIMIZATION RESULTS

In this section, the performance of the GP optimization is validated with the design values derived for the example system. To serve as a reference reliable for the performance evaluation, results from an exhaustive search algorithm using the original system models, without approximations for convexity, are used. The algorithm searches the entire design space formed with a range of free variables. The reference data sets generated from the algorithm are reliable since the original models are based on common approaches that are widely accepted in the literature, and also they have been validated for the modular multilevel architecture for the case study in the prior works. Reference [33] validated topological feasibility, [40] substantiated control, and [36] provided loss and efficiency models. They include experimental results for the architecture as well as experimental validation of the loss models.

Based on the discussion, the performance of the GP optimization is evaluated by comparing with the exhaustive search. Fig. 13 presents two performance metrics of the brute-force method, the computation time and LCOE improvement achieved, as a function of the number of data points per variable. As expected, the computation time exponentially increases with increasing data points since the total number of cases searched is proportional to $N_{\text{d}}^{n_{f}}$, where $n_{d}$ and $n_{f}$ are the number of data points per variable and the number of free variables. On the other hand, notably, the LCOE improvement factor saturates approaching 3.3%, following the initial steady increase beyond a certain point, which implies it approaches a design point near the optimal, not guaranteed to be the definite solution. Fig. 13 also clearly shows heavy computation required to derive a near-optimal point. In this case, it takes 2000 seconds to find a design to approach 3.3% of LCOE improvement.

Table 2 compares the computation time taken for each step of the proposed GP optimization and the LCOE improvement estimates to the exhaustive search. The reference data from the exhaustive search is taken from the case with 20 samples per variable. For the proposed GP optimization, the result of Step I, direct from the convex optimization, indicates the LCOE improvement of 3.46% with design values impractical. Following, Step II iterates to a new optimization point with two variables adjusted to practical values, $N_{\text{module}}$ and $N_{\text{t},XF}$, and based on that Step III achieves the final design with the other three integer variables rounded (refer to Figs. 11 and 12 for details). Fig. 14 locates the initial

---

**TABLE 1.** System parameters used in optimization.

| Parameter   | Value   |
|-------------|---------|
| $f_{\text{rated}}$ | 200 kW  |
| $V_{\text{grid}}$ | 13.2 kV  |
| $V_{\text{pv}}$ | 1000 V  |

**Parameters of benchmark system**

| Parameter   | Value   |
|-------------|---------|
| LCOE        | 6 Cents/kWh |
| $\eta_{\text{bench}}$ | 0.97  |
| $e_{t}$     | 0.24   |
| $C_{0}$     | 16,536 US Dollars |

**FIGURE 13.** Computation time and maximum LCOE improvement factor found as a function of the number of samples per variable by using the exhaustive algorithm. Computation exponentially increases with improvement saturating.

**TABLE 2.** Performance comparison of GP optimization compared to exhaustive search.

| Test case  | Time [s] | LCOE [%] |
|------------|----------|----------|
| Step I     | 6.01     | 3.46     |
| Step II    | 9.76     | 3.36     |
| Step III   | 13.23    | 3.35     |
| Exhaustive | 70,448   | 3.30     |
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### TABLE 3. Key designs values from the GP optimization and exhaustive search. Variables adjusted to practical values are highlighted in bold at the step of iteration.

| Methods       | \( N_{\text{module}} \) | \( R_{ds,\text{pri}} \) [\( \text{m}\Omega \)] | \( R_{ds,\text{sec}} \) [\( \text{m}\Omega \)] | \( f_{sw,\text{QAB}} \) [kHz] | \( f_{sw,\text{INV}} \) [kHz] | \( V_{core,\text{XF}} \) [\( \text{cm}^2 \)] | \( V_{core,\text{IND}} \) [\( \text{cm}^2 \)] | \( N_{t,\text{XF, pri}} \) [turns] | \( N_{t,\text{XF, sec}} \) [turns] | \( N_{t,\text{IND}} \) [turns] |
|---------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Step I        | 18.7           | 104            | 149            | 106            | 237            | 7.31           | 195            | 91.4           | 38.3           | 28.7           | 13.5           |
| Step II       | 19             | 106            | 146            | 105            | 126            | 7.07           | 135            | 92.3           | 51.5           | 38.9           | 19.6           |
| Step III      | 19             | 106            | 146            | 105            | 125            | 7.07           | 136            | 91.2           | 52             | 39             | 20             |
| Exhaustive    | 19             | 110            | 140            | 110            | 89             | 8.86           | 155            | 89.4           | 64             | 48             | 22             |

**FIGURE 14.** Final design point from the GP optimization overlaid on the design surface generated from the original models. The initial result (Step I) is iterated to the final design (Step III) with practical design values.

### TABLE 4. LCOE improvement factor error from curve fitting of transformer and inductor copper loss models.

| Methods       | Copper loss, XF [W] | Copper loss, IND [W] | \( \frac{\Delta \text{LCOE}}{\text{LCOE}} \) [%] |
|---------------|---------------------|----------------------|-------------------------------------|
| Step III (w/ fitting) | 32.48               | 16.37                | 3.33                               |
| Step III (w/o fitting) | 32.17               | 15.29                | 3.35                               |
| Error (%)     | 0.96                | 7.05                 | 0.41                               |

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