DEVIATIONS FROM LOW-ENERGY THEOREM FOR $V_L V_L$
SCATTERING DUE TO PSEUDO-GOLDSTONE BOSONS

Tomáš Bahnik
Department of Physics, Technical University Liberec,
Hákova 6, 461 17 Liberec, Czech Republic

Jiří Hořejší
Nuclear Centre, Faculty of Mathematics and Physics, Charles University,
V Holešovičkách 2, 180 00 Prague 8, Czech Republic

Possible deviations from a low-energy theorem for the scattering of strongly interacting longitudinally polarized $W$ and $Z$ bosons are discussed within a particular scheme of electroweak symmetry breaking. The scheme (suggested earlier by other authors in a slightly different context) is based on spontaneous breakdown of a $SU(4)$ symmetry to custodial $SU(2)$ subgroup. The physical spectrum of such a model contains a set of relatively light pseudo-Goldstone bosons whose interactions with vector bosons modify the low-energy theorem proven for a “minimal” symmetry-breaking sector. The Goldstone-boson manifold $SU(4)/SU(2)$ is not a symmetric space. In this context it is observed that, on the other hand, there is a large class of models of electroweak symmetry breaking, involving groups $G$ and $H$ such that the $G/H$ is a symmetric space and the corresponding rich multiplets of pseudo-Goldstone bosons do not influence the canonical low-energy theorem. For the scheme considered here, the relevant interactions are described in terms of an effective chiral Lagrangian and tree-level contributions of the pseudo-Goldstone boson exchanges to the vector boson scattering are computed explicitly. A comparison with the Standard Model is made.

1. Introduction

The nature of electroweak symmetry breaking (EWSB), i.e. the mechanism responsbile for generating the $W$ and $Z$ boson masses, constitutes perhaps the most important open issue of the present-day particle physics.¹ One of the few (indirect) experimental clues in this respect seems to be provided by the value of the famous parameter $\rho = m_W^2 / (m_Z^2 \cos^2 \theta_W)$, which is known to be close to unity. Such a value is understood quite naturally within spontaneously broken $SU(2) \times U(1)$ gauge theories if an unbroken “custodial” $SU(2)$ symmetry is present, giving automatically $\rho = 1$ in the lowest order.² Thus, the Goldstone realization of an appro-

¹ e-mail: tomas.bahnik@vslib.cz
² e-mail: jiri.horejsi@mff.cuni.cz
appropriate internal symmetry of a (largely unknown) “symmetry-breaking sector” and the ensuing Higgs mechanism (in a most general sense) seem to be highly plausible generic features of a realistic theory of electroweak interactions. Needless to say, the successful minimal standard model (SM) is built precisely along these lines, under an additional (technical) constraint of perturbative renormalizability and with the symmetry of the Higgs system being $O(4) \simeq SU(2) \times SU(2)$. On the other hand, given only the fairly weak phenomenological restrictions mentioned above, one may clearly envisage more general theoretical schemes for EWSB.

It is well known that the conceivable EWSB scenarios are basically of two distinct types. First, one may consider a (rather general) multiplet of elementary Higgs scalar fields and a corresponding “potential”, whose specific form triggers the spontaneous symmetry breakdown; one thus follows essentially the SM paradigm and perturbative renormalizability is maintained. Such a perturbative approach makes sense if the relevant couplings are sufficiently weak and this in turn means that a physical Higgs boson becomes relatively light, having mass far below 1 TeV or so.\cite{5,4} Second, there is a “strong-coupling scenario” (see e.g. Refs. 5, 6 for a review), in which EWSB is assumed to have a similar origin as e.g. the spontaneous breakdown of chiral symmetry within QCD. The corresponding new strong interaction responsible for EWSB may then be labelled generically e.g. as “technicolour”\cite{7} (though we will actually not invoke any particular dynamical model here). In a minimal variant of such a scheme there are just three (dynamically generated) Goldstone bosons that are used up in the Higgs mechanism, giving rise to the longitudinal polarizations (i.e. zero helicities) of massive vector bosons $W$ and $Z$; in the physical spectrum there is then no natural counterpart of the SM Higgs boson. Of course, one thus also abandons the concept of conventional renormalizability. There is a typical mass scale $\Lambda_{SB}$ of the symmetry-breaking (SB) sector and for sufficiently low energies ($E < \Lambda_{SB}$) an appropriate description of the dynamics is provided in terms of an effective “chiral” Lagrangian (cf. e.g. Refs. 8, 9 for review), in analogy with phenomenological pion Lagrangians in low-energy QCD.\cite{10,11} In this context, it should be noted that the current SM precision tests and analyses of electroweak radiative corrections suggest that the perturbative scenario with a light Higgs boson is favoured over the simplest technicolour models.\cite{15} Nevertheless, from a more general perspective, a strongly interacting SB sector within the $SU(2) \times U(1)$ electroweak gauge theory still represents a viable alternative – a clear-cut answer concerning the EWSB issue can only come from results of the future experiments at LHC or elsewhere.\cite{5,6}

For an electroweak theory with strongly interacting SB sector a remarkable general result is valid, namely a low-energy theorem (LET) concerning the scattering of longitudinally polarized vector bosons.\cite{14} This theorem specifies a universal leading behaviour of the $V_L V_L$ scattering amplitudes (the $V$ is a common symbol for $W$ or $Z$ ) for energies such that $m_V^2 \ll s \ll \Lambda_{SB}^2$. Roughly speaking, one observes linear growth with $s = E_{c.m.}^2$, characteristic for the $SU(2) \times U(1)$ gauge interactions alone (i.e. without the additional damping provided by a light Higgs boson within
SM). Note also that the LET formulated by Chanowitz et al.\cite{14} is analogous to the low-energy results known for pion-pion scattering\cite{10,11,12}; such a formal similarity is due to the famous equivalence theorem\cite{15} for Goldstone bosons and longitudinal vector bosons. The LET has been proved in Ref. 14 under the assumption that all would-be Goldstone bosons are “eaten” by $W$ and $Z$ resp. as a result of the Higgs mechanism, i.e. that the $W$ and $Z$ are the only light particles in the physical spectrum (“light” means a mass comparable with $m_W$ or less). In other words, the assumed symmetry pattern corresponds to $SU(2) \times SU(2)$ broken down to a (custodial) $SU(2)$, or simply $SU(2) \times U(1)$ down to the electromagnetic $U(1)$ (in the latter case the relation \(\rho = 1\) is not automatic and must eventually be imposed by hand).

Of course, one is allowed to consider more general SB schemes, without obviously violating the existing phenomenology. A question then immediately arises as to whether (and how) the LET may get modified. A general pattern of symmetry breakdown can be characterized by specifying the symmetry group $G$ of the SB sector and its unbroken subgroup $H$ (in particular, the $H$ may be conveniently chosen so as to contain a custodial $SU(2)$). Goldstone bosons associated with spontaneous breaking of the symmetry $G$ down to $H$ correspond to coordinates in the manifold (quotient space) $G/H$ and one can write a corresponding effective Lagrangian for the SB sector (involving a nonlinear realization of the symmetry $G$) in a model-independent way, employing the general CCWZ construction.\cite{16} When a subgroup $G_w = SU(2) \times U(1)$ of the $G$ is gauged, three Goldstone bosons disappear, but some other may survive (if $G/H$ is large enough) in the physical spectrum and acquire masses through gauge interactions. In general, such pseudo-Goldstone bosons (PGB) may be relatively light, i.e., have masses on the electroweak scale and thus they are expected to modify the LET for $V_L V_L$ scattering through the additional exchanges in the corresponding tree-level Feynman diagrams.\cite{14}

The purpose of the present paper is to examine such deviations from the LET quantitatively, in a pertinent model and using the corresponding effective Lagrangian. To this end, one might consider first e.g. the schemes discussed earlier by various authors in the context of explicit technicolour models.\cite{17,18,19} These examples include the SB patterns with $G/H = SU(n) \times SU(n) \times U(1)/SU(n) \times U(1)$, $SU(2n)/O(2n)$ and $SU(2n)/Sp(2n)$. Note that in all these cases, the quotient space $G/H$ is a symmetric space (i.e. there is a natural “parity” operation distinguishing the broken and unbroken generators of the $G$). Looking into the interactions of the corresponding physical PGB (see in particular Ref. 18), one finds that there are no modifications of the LET in question – interactions of the type $\pi VV$ (where $\pi$ stands for a PGB) become trivial within all these schemes.

It turns out that a minimal scheme suitable for our purpose is provided by the model suggested some years ago by Chivukula and Georgi.\cite{20} The corresponding SB pattern is $G/H = SU(4)/SU(2)$ and does lead to non-trivial $\pi VV$ interaction vertices; the study of deviations from the LET for $V_L V_L$ scattering within such a scheme is the main subject of our investigation. It should be stressed that through-
out our discussion we stay within the general framework of the CCWZ effective Lagrangian and do not attempt to find any particular dynamical model (technicolour or whatever) producing the SB pattern in question. As a side remark, let us also note that another interesting feature of the considered scheme is that it contains doubly charged PGB; in this respect, it has a similar signature as some particular weak-coupling models involving Higgs triplets (in addition to a standard doublet), discussed independently in the literature by several authors.\textsuperscript{21,22}

The present paper is organized as follows. In the next section the statement of the LET for $V_L V_L$ scattering is briefly summarized. In Sect.3 the $SU(4)/SU(2)$ EWSB scheme is described in some detail and Sect.4 is devoted to the calculation of the envisaged deviations from the LET due to the PGB exchange. Main results of the calculation are discussed briefly in Sect.5, while some further remarks of a more general character are left to Sect.6. Some uncomfortably long formulæ employed in our calculation are relegated to an Appendix.

2. Low-Energy Theorem for $V_L V_L$ Scattering

As noted in the Introduction, the LET due to Chanowitz et al.\textsuperscript{14} specifies the leading energy-dependence of the $V_L V_L$ scattering amplitudes within models involving a “minimal” strongly interacting SB sector, i.e. such that the only Goldstone bosons are those “eaten” by the $W^\pm$ and $Z$ via the Higgs mechanism. Here we will recapitulate briefly only the main results; for more details, the reader is referred to the original literature.

The theorem should hold for the energy domain $m_W^2 \ll s \ll \Lambda_{SB}^2$, with

$$\Lambda_{SB} = \min \{4\pi v, M_{SB}\} \sim 1\text{TeV},$$

where $v$ is the well-known electroweak scale, $v = (G_F \sqrt{2})^{-1/2} \approx 246$ GeV ($G_F$ being the Fermi constant) and $M_{SB}$ denotes a mass scale typical for the SB sector (e.g. the mass of the lightest resonance due to the strong EWSB force – a “technirho” or so). The essential statement of the LET (up to the order $O(g^2)$ in electroweak gauge coupling) is summarized in Table 1. Upon neglecting small constant terms, the LET expressions coincide with the high-energy ($E \gg m_W$) limit of the gauge part of the complete amplitude – for a detailed calculation see e.g. Ref. 23 (cf. also Ref. 24). The leading behaviour shown in Table 1 is universal, i.e. independent of any particular dynamical model reproducing the considered SB pattern.

The analysis of Ref. 14 shows that under the assumptions mentioned above all corrections to the scattering amplitudes from strongly interacting EWSB sector decouple, except for a renormalization of the $m_W$ and $\rho$. The LET has been derived there by means of three different methods, in particular via perturbative Feynman-diagram analysis, then with the help of an appropriate effective Lagrangian and also by using current-algebra techniques – the latter approach stresses an analogy with pion low-energy theorems.\textsuperscript{10,11,12}
Deviations from low-energy theorem ...

Table 1. Leading terms in the $V_L V_L$ scattering amplitudes predicted by the LET, $\rho = m^2_W / \left(m_Z^2 \cos^2 \theta_W \right)$

| Process                      | LET prediction       |
|------------------------------|----------------------|
| $W_L^+ W_L^- \rightarrow Z_L Z_L$ | $\frac{g^2 s}{4 m_W^2}$ |
| $W_L^+ Z_L \rightarrow Z_L W_L^+$ | $\frac{g^2 u}{4 m_W^2}$ |
| $W_L^+ W_L^+ \rightarrow W_L^+ W_L^+$ | $-\frac{g^2 s}{4 m_W^2} \left(4 - \frac{2}{\rho} \right)$ |
| $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$ | $-\frac{g^2 u}{4 m_W^2} \left(4 - \frac{2}{\rho} \right)$ |
| $Z_L Z_L \rightarrow Z_L Z_L$     | 0                    |

3. The $SU(4)/SU(2)$ Symmetry-Breaking Scheme

We will employ here the SB scheme proposed some years ago by Chivukula and Georgi\textsuperscript{20}, in which the original symmetry group $G$ of a strongly interacting SB sector is $SU(4)$ and its unbroken subgroup $H = SU(2)$ (playing the role of the custodial symmetry). No particular details of possible dynamics that might produce such a SB pattern are assumed and the relevant interactions are described in terms of a model-independent effective Lagrangian constructed according to the known general principles.\textsuperscript{16} The Lagrangian contains twelve Goldstone bosons; three of them are absorbed by the Higgs mechanism and the remaining ones become massive, with masses of order $\lesssim 1$ TeV depending on the parameters of the Lagrangian. In particular, the scheme involves doubly charged PGB that couple directly to two similarly charged $W$’s. These make interesting contributions to $W^+ W^+$ and $W^+ W^-$ scattering amplitudes.\textsuperscript{2} In the remainder of this section we summarize the essential properties of the relevant effective Lagrangian; some further details can be found in the original paper Ref. 20 and also in Ref. 23.

Let us denote by $SU(4)$ and $SU(2)$ resp. the Lie algebras of the initial global symmetry group $SU(4)$ and its unbroken subgroup $SU(2)$ resp. The generators of $SU(4)$ are decomposed as $\lambda_A = \{t_i, x_a\}$ where the $t_i$ form a basis in the unbroken $SU(2)$ and the $x_a$ are broken generators, i.e. correspond to a basis in the orthogonal subspace. Generators are normalized in such a way that $\text{Tr}\{\lambda_A \lambda_B^\dagger\} = \delta_{AB}$. According to general arguments\textsuperscript{16} the number of Goldstone bosons (GB) is equal to the number of broken generators, $n_{GB} = \text{dim} \, SU(4) - \text{dim} \, SU(2) = 12$. The GB, or broken generators, may in general be decomposed into irreducible multiplets with respect to the unbroken subgroup. In the considered case we have one GB quintuplet $\pi^{++,-,0,-,-}$, two triplets $\pi^3_{++,0,-}, \pi^3_{+,0,-}$ and one singlet $\pi^1$ under the $SU(2)$.

\textsuperscript{2}There is also a weak-coupling scheme\textsuperscript{21,22} with extended Higgs sector including multiplets classified under custodial $SU(2)$ as a quintuplet $H_5^{++,+,0,-,-}$, a triplet $H_3^{+,0,-}$ and two singlets $H_1^1$ and $H_1^1$.\textsuperscript{23}
The corresponding effective Lagrangian consists of three parts

\[ \mathcal{L} = \mathcal{L}_{IVB} + \mathcal{L}_{KE} + \mathcal{L}_m. \]  

(2)

The first term represents the usual gauge boson interactions (as well as their kinetic terms)

\[ \mathcal{L}_{IVB} = -\frac{1}{4} F_{\mu\nu}^i F^{i\mu\nu} - \frac{1}{4} G_{\mu\nu} G^{\mu\nu}. \]  

(3)

The \( \mathcal{L}_{KE} \) is the gauge invariant kinetic term of GB fields

\[ \mathcal{L}_{KE} = \frac{1}{2} f_j^2 J_{5\mu}^j \cdot J_{5\mu}^j + \frac{1}{2} f_3^2 J_{3\mu}^j \cdot J_{3\mu}^j 
+ \frac{1}{2} f_3' J_{3'}\mu \cdot J_{3'}\mu 
+ \alpha f_3 f_3' J_{3\mu}^j \cdot J_{3'}\mu 
+ \frac{1}{2} f_1^2 J_{1\mu}^j \cdot J_{1\mu}^j. \]  

(4)

The \( J_{\mu}^j \) are defined by a decomposition of the quantity \( e^{-i\Pi}(\partial_{\mu} + iW_{\alpha\mu}w_\alpha)e^{i\Pi} \) into broken and unbroken subspaces

\[ e^{-i\Pi}(\partial_{\mu} + iW_{\alpha\mu}w_\alpha)e^{i\Pi} = i \sum_{j=1}^{M} J_{\mu}^j \cdot x^j + T_{\mu} \]  

(5)

where the superscript \( j = 1, 3, 3', 5 \) labels irreducible multiplets and the dot denotes a “scalar” product inside given multiplet \( j \)

\[ J_{\mu}^j \cdot x^j = \sum_{a=1}^{n_j} J_{\mu a}^j x_a^j \]  

(6)

The GB fields are contained in

\[ \Pi = \sum_{j=1}^{M} \frac{\pi^j \cdot x^j}{f_j} \]  

(7)

where \( f_j \) are parameters with dimension of a mass (they are analogous to the pion decay constant in the low-energy chiral Lagrangian of QCD). The expression \( \partial_{\mu} + iW_{\alpha\mu}w_\alpha \) is ordinary gauge covariant derivative, i.e. \( W_{\mu\alpha} \) are gauge boson fields and the \( w_\alpha \) denote generators of a \( SU(2) \times U(1) \) electroweak gauge group including gauge coupling constants \( g \) and \( g' \).

The last term parametrizes electroweak contributions to the GB scattering of the order \( g^2 \) and/or \( g'^2 \). It contains mass terms and nonderivative self-interactions of the GB. Using this part of the Lagrangian we can identify fictitious GB, eaten in the Higgs mechanism, real GB, which remain massless and the PGB with masses induced by explicit symmetry breaking given by electroweak interaction. The determination of the \( \mathcal{L}_m \) is known as the vacuum alignment problem.\(^{17,18,19} \) Within the

\(^{b}\)We adopt here the terminology of Ref. 25.
scheme under consideration we have just three massless GB (namely the fictitious ones) and nine massive PGB.

The final form of the $L_m$ reads

$$-L_m = V(\Pi) = (a + 2|d|)f^4 Tr\{w_\alpha(\Pi)x^5\} \cdot Tr\{w_\alpha(\Pi)x^5\} + (b + 2|d|)f^4 Tr\{w_\alpha(\Pi)x^3\} \cdot Tr\{w_\alpha x^3\} + (c + 2|d|)f^4 Tr\{w_\alpha(\Pi)x^1\} \cdot Tr\{w_\alpha(\Pi)x^1\} + df^4 Tr\{w_\alpha(\Pi)\sigma_a \times 1\} \cdot Tr\{w_\alpha(\Pi)1 \times \tau_a\}$$

where the $x^j$ denote multiplets of broken generators, $\sigma$ and $\tau$ are Pauli matrices and $w_\alpha(\Pi) = e^{-i\Pi}w_\alpha e^{i\Pi}$. Parameters $a, b, c$ are positive dimensionless numbers of the order of unity and $f$ has dimension of a mass. It is expected that different $f$'s, i.e. the $f_j$ in $L_{KE}$ and the $f$ in $L_m$ are of the same order of magnitude. Only the value of the $f_3$ is fixed by the relation

$$f_3 = \frac{v}{\sqrt{2}} \approx 174 \text{ GeV}$$

following from the structure of the gauge boson mass matrix.

4. Deviations from LET Due to PGB Exchanges

4.1. General discussion

Table 2 gives an overview of general types of PGB exchange contributions to the processes in question. In the considered model there are six types of $\pi VV$ vertices, as one can see from the relevant part of the interaction Lagrangian

$$\mathcal{L}_{VV\pi} = -agm_W \frac{f_3}{\sqrt{2} f_5} W^+ W^+ \pi^- \pi^- + c.c. + agm_W \sqrt{\frac{8 f_3}{3 f_1}} W^+ W^- \pi^1 - agm_W \frac{f_3}{\sqrt{3} f_5} W^+ W^- \pi^0 - \frac{agm_Z f_3}{\cos \theta_W f_5} W^+ Z \pi^0 + c.c.$$
where
\[ \tan \theta_W = g'/g, \quad m_W = gf_3/\sqrt{2}, \quad m_Z = f_3\sqrt{g'^2 + g^2}/\sqrt{2} \quad \text{and} \quad e = g\sin \theta_W. \]

The fictitious GB $G^\pm$ are eliminated in the $U$-gauge. Unfortunately, only the doubly charged field $\pi_5^{++}$ is a mass eigenstate while the other $\pi$ fields, in general, are not. As an example, we have performed numerical diagonalization of the GB mass matrix for the values of parameters $\alpha = 0.5, 0.9, a, b, c, d = 1$ and with all the $f'$s taken to be equal.\(^{23}\) The result has been expressed in multiples of $f$. Using the only fixed value of $f = f_3$ in (8), we can get an estimate of the PGB masses. The mass eigenstate fields $\varphi$ and fields $\pi$ are related by
\[
\begin{align*}
\pi_1 &= 0.107 \varphi_7 + 0.997 \varphi_8 \\
\pi_5^+ &= -i 0.997 \varphi_9 + i 0.076 \varphi_{11} \quad \pi_5^- = (\pi_5^+)^\dagger 
\end{align*}
\]

The $\varphi_{7,8}$ and $\varphi_{9,11}$ are real and complex scalar fields resp. In what follows we will neglect this fact and assume that the $\pi$ fields are themselves mass eigenstates. To plot the relevant graphs we have chosen masses of $\pi_1$ and $\pi_5$ according to eigenvalues calculated\(^{23}\)
\[
m_{\pi_1} \approx 3.1 f \approx 500 \text{ GeV} \quad m_{\pi_5} \approx 1.9 f \approx 350 \text{ GeV}
\]

with $f = f_3 = 174$ GeV.

First we give general expressions of tree-level scattering amplitudes of longitudinally polarized gauge bosons for each row of the Table 2. Then we take into account the particle content of the $SU(4)/SU(2)$ model. We will write the amplitude in a form
\[
\mathcal{M}^{(n)}_{\pi\pi} = -g_1g_2A^{(n)}_{k\pi}, \quad k = s, t, u .
\]

The couplings $g_1, g_2$ can be read off from (9), e.g.
\[
g_{WW\pi_5^+} = -2\alpha g m_W \frac{f_3'}{\sqrt{2} f_5'}, \quad g_{WW\pi_5^-} = -\alpha g m_W \frac{f_3'}{\sqrt{3} f_5'}, \quad \ldots 
\]

Note the factor 2 in the coupling involving two identical particles. We denote the contributions to the scattering amplitude coming from PGB exchange as $M_{SB}^{(n)}$. Then the complete amplitude of the process $\#n$ is given by $\mathcal{M}^{(n)} = M_{\text{gauge}}^{(n)} + M_{SB}^{(n)}$. Exact formulae for $M_{\text{gauge}}^{(n)}$ are given in Ref. 23 and also in the earlier papers Ref. 26.

Figures 1-4 compare the tree-level amplitudes corresponding to SM and the considered $SU(4)/SU(2)$ scheme. The Higgs boson mass is taken to be $m_H = 500$ GeV. The masses of PGB $\pi^5$ and $\pi^1$ are $m_{\pi^5} = 350$ GeV and $m_{\pi^1} = 500$ GeV.
Deviations from low-energy theorem

![Diagram](image)

Fig. 1. Tree-level amplitude of the process $WW \rightarrow ZZ$ as a function of $\sqrt{s}$ in SM (solid), $SU(4)/SU(2)$ model (dashed), and pure gauge amplitude $M_{\text{gauge}}^{(1)}$ (dotted). The Higgs boson mass $m_H = 500$ GeV, width $\Gamma_H = 0$, $\cos \theta_{CM} = 0.5$, $\rho = 1$, $m_Z = 91.2$ GeV, $\sin^2 \theta_W = 0.231$, $\rho^2 = 4 \times 10^{-4} \approx 0.42$, $m_{\pi} = 350$ GeV, $m_{\pi^3} = 500$ GeV.

resp. as discussed above, $\alpha = 0.5$ and $f_3 = f_5 = f_1$. The parameters are the same for all the plots. Pure gauge amplitudes $M_{\text{gauge}}^{(n)}$ (dotted line) are also plotted. As mentioned in Section 2, the high-energy limit of the $M_{\text{gauge}}^{(n)}$ practically coincides with the amplitude given by the canonical LET.

### 4.2. Results for Scattering Amplitudes

Using values of coupling constant from (9) and high-energy limits of $A$'s given in (A.2) we get for the process #1

$$M_{SB}^{(1)} = -\frac{\alpha^2 g^2 s}{m_W^2} \left[ \frac{f_3}{f_1} \right]^2 \frac{2s \sqrt{\rho}}{3} + \left( \frac{f_3}{f_5} \right)^2 \left( \frac{m_Z^2 \rho}{4m_W^2} (t + u) - \frac{s \sqrt{\rho}}{6} \right) + O(s^0)$$  \hspace{1cm} (14)$$

where the contributions from individual terms (see (A.3)) are clearly distinguished. Of course, in the considered model $\rho = m_W^2/m_Z^2 \cos^2 \theta_W = 1$. For the simplest case $f_3 = f_5 = f_1$ and using $s + t + u = 0$ we get

$$M_{SB}^{(1)} = -\frac{\alpha^2 g^2 s}{m_W^2} \left[ \frac{1}{2} - \frac{m_Z^2}{4m_W^2} \right] + O(s^0)$$

In the case of the process #2 the symmetry-breaking amplitude differs from $M_{SB}^{(1)}$ by replacement $s \leftrightarrow u$. Taking into account only the leading terms in $s$ we get an
The relevant formulae for processes \#3 and \#4 are easily obtained by setting $m_Z = m_W$ in (A.1) or (A.4).

\[
\mathcal{M}^{(3)}_{SB} = \frac{\alpha^2 g^2 s}{4m_W^2} \left[ 1/12 \left(f_{3}\over f_{5}\right)^2 (6u + t + s) + \frac{2}{3} \left(f_{3}\over f_{1}\right)^2 (s + t) \right] + O(s^0) \quad (16)
\]

which for our choice of the $f$'s simplifies to

\[
\mathcal{M}^{(3)}_{SB} = \frac{\alpha^2 g^2 s}{4m_W^2} + O(s^0) .
\]

\[
\mathcal{M}^{(4)}_{SB} = \frac{\alpha^2 g^2 u}{4m_W^2} \left[ 1/12 \left(f_{3}\over f_{5}\right)^2 (6u + t + s) + \frac{2}{3} \left(f_{3}\over f_{1}\right)^2 (s + t) \right] + O(s^0) \quad (17)
\]

and for $f_{3v} = f_{5} = f_{1}$ this becomes

\[
\mathcal{M}^{(4)}_{SB} = \frac{\alpha^2 g^2 u}{4m_W^2} + O(s^0)
\]
Deviations from low-energy theorem.

![Graph showing the process W^+ W^+ \rightarrow W^+ W^+ as a function of \sqrt{s} in SM (solid), SU(4)/SU(2) model (dashed), and pure gauge amplitude M^{(3)}_{gauge} (dotted).]

5. Discussion of Results

Using the formulae (14), (15), (16) and (17) we can discuss the modifications of the low-energy theorems caused by PGB exchanges. Table 3 displays the high-energy limit (s ≫ m_W^2, m_{Z_k}^2, m_H^2) of the symmetry breaking amplitudes in the extended model (up to the factor \(-\frac{\alpha_s^3 f_0^2}{4m_W^2}\)) and in the SM with a light Higgs boson (up to the factor \(-\frac{g^2}{4m_W^2}\)). The approximate relation s + t + u ≡ 0 is used.

Table 3. A comparison of high-energy limit of the individual PGB and Higgs contributions to the scattering amplitudes of W_L and Z_L. \(\rho = 1\), s + t + u = 0.

| Process                  | \(\pi^5_{++}\) | \(\pi^5_{+}\) | \(\pi^5_0\) | \(\pi^1\) | SM Higgs |
|--------------------------|----------------|--------------|-------------|-----------|----------|
| \(W_1^+ W_2^- \rightarrow Z_3 Z_4\) | \(\frac{s}{\cos^2 \theta_W f_5}\) | \(\frac{s}{f_5}\) | \(\frac{8s}{3f_1}\) | s         |          |
| \(W_1^+ Z_2 \rightarrow Z_3 W_4^+\)   | \(\frac{u}{\cos^2 \theta_W f_5}\) | \(\frac{u}{f_5}\) | \(\frac{8u}{3f_1}\) | u         |          |
| \(W_1^+ W_2^+ \rightarrow W_3^+ W_4^+\) | \(\frac{2s}{f_5}\) | \(\frac{s}{3f_5}\) | \(\frac{8s}{3f_1}\) | s         |          |
| \(W_1^+ W_2^- \rightarrow W_3^+ W_4^-\) | \(\frac{2u}{f_5}\) | \(\frac{u}{3f_5}\) | \(\frac{-8u}{3f_1}\) | u         |          |

As regards the process ZZ \rightarrow ZZ, neutral PGB exchanges in s, t and u channels...
Fig. 4. Tree-level amplitude of the process $W^+W^- \to W^+W^-$ as a function of $\sqrt{s}$ in SM (solid), $SU(4)/SU(2)$ model (dashed), and pure gauge amplitude $M_{\text{gauge}}^{(4)}$ (dotted).

Contribute there. In the limit $s,t,u \gg m_W, m_Z, m_\pi$ one thus gets an amplitude proportional to $s + t + u \sim 0$ (see (A.2)). The signs of neutral PGB contributions to the processes #3 and #4 are the same as the Higgs’ ones. It is interesting that no modifications occur for $\alpha = 0$. The value of $\alpha$ is restricted by the GB kinetic term to $|\alpha| < 1$. The presence of a term proportional to $\alpha$ is due to the existence of two different triplets $3$ and $3'$ (see (4)). The fact that there can be more than one $H$-irreducible multiplet was pointed out in Refs. 27, 28, where a most general Lagrangian is written in the form

$$L_{BB} = \sum_{i=1}^{n} f_i^2 j_{\mu}^i \cdot j^{i\mu}$$

(18)

with $n$ being the number of $H$-irreducible multiplets. Thus the fact that it is possible to combine different multiplets was neglected. A detailed discussion of the structure of an effective chiral Lagrangian containing combinations of different multiplets along with the classification of GB and the corresponding definition of the $U$-gauge can be found in Ref. 23. Table 4 gives a more compact view on the PGB contribution in the case $f_3 = f_5 = f_1$. Comparing Fig. 1 ($WW \to ZZ$) and Fig. 4 ($W^+W^- \to W^+W^-$) we can see a manifestation of the opposite signs of the couplings

$$g_{ZZ\pi_0} = 2 \frac{\alpha m_Z}{\cos \theta_W} \frac{f_3}{\sqrt{3} f_5}$$

and

$$g_{WW\pi_0} = -\frac{\alpha m_Z}{\sqrt{3}} \frac{f_3}{f_5}$$

(19)

(see (9)) around the pole at $\sqrt{s} = m_{\pi^0} = 350$ GeV. In the region $s > m_{\pi^0}, m_H = \ldots$
Deviations from low-energy theorem

Table 4. A comparison of the LET, the high-energy limit of Higgs-boson and the PGB contributions with $f'_3 = f_5 = f_1$.

| Process | LET | SM Higgs | $SU(4)/SU(2)$ |
|---------|-----|----------|--------------|
| $W^+_1 W^-_2 \to Z_3 Z_4$ | $\frac{g^2 s}{4 m_W}$ | $\frac{g^2 s \sqrt{s}}{4 m_W}$ | $-\frac{\alpha^2 g^2 s \sqrt{s}}{4 m_W} \left[ \frac{1}{2} - \frac{m_H^2 \sqrt{s}}{4 m_W} \right]$ |
| $W^+_1 Z_2 \to Z_3 W^+_4$ | $\frac{g^2 u}{4 m_W}$ | $\frac{g^2 u \sqrt{s}}{4 m_W}$ | $-\frac{\alpha^2 g^2 u \sqrt{s}}{4 m_W} \left[ \frac{1}{2} - \frac{m_H^2 \sqrt{s}}{4 m_W} \right]$ |
| $W^+_1 W^+_2 \to W^+_3 W^+_4$ | $-\frac{g^2 s}{4 m_W} \left( 4 - \frac{3}{\rho} \right)$ | $\frac{g^2 s}{4 m_W}$ | $\frac{\alpha^2 g^2 s}{4 m_W}$ |
| $W^+_1 W^-_2 \to W^+_3 W^-_4$ | $-\frac{g^2 u}{4 m_W} \left( 4 - \frac{3}{\rho} \right)$ | $\frac{g^2 u}{4 m_W}$ | $\frac{\alpha^2 g^2 u}{4 m_W}$ |

500 GeV the effect of the PGB exchange points in the same direction as that of the SM Higgs boson. However, the growth of the pure gauge amplitudes is not damped by the PGB exchange, at least for our simple choice of the $f$’s.

Looking at Fig. 2 ($WZ \to ZW$) and Fig. 3 ($W^+ W^+ \to W^+ W^+$) one may notice that the PGB act in an opposite way than the Higgs boson, in contradiction with the last row of the Table 4. But the energies covered by the figures are not in the region $\sqrt{s} \gg m_\pi, m_H$. There is a point near 1200 GeV where $M_{SB}^{(2)}$ and $M_{SB}^{(3)}$ change the sign. Nevertheless they cannot cancel the “bad” high-energy behaviour of the pure gauge amplitude.

6. Concluding Remarks

The analysis performed in the present paper serves mainly an illustrative purpose. We have shown, by means of an explicit calculation, that a “canonical” low-energy theorem for strongly interacting $W_L$ and $Z_L$, proven for a “minimal” symmetry-breaking scenario, may indeed be violated within an extended scheme involving physical PGB with masses on the electroweak scale.

The considered scheme (introduced earlier in a slightly different context), corresponding to spontaneous breaking of the symmetry $SU(4)$ down to (custodial) $SU(2)$ subgroup, is somewhat unusual in that the Goldstone-boson manifold $G/H = SU(4)/SU(2)$ is not a symmetric space. For reader’s convenience let us recall that a $G/H$ (the quotient space for a group $G$ broken down to $H$) is symmetric space if there is a “parity” operation (an involutive automorphism) $P$ on the Lie algebra of the $G$, distinguishing the unbroken and broken generators $t_i$ and $x_a$ in such a way that e.g.

$$P t_i P = + t_i, \quad P x_a P = - x_a$$

A useful criterion (a necessary condition) for the existence of a $P$ satisfying (20) is that any commutator of the broken generators can be expressed as a linear combination of the unbroken ones. For the SB scheme discussed in the present paper one can show that this criterion is indeed violated (in particular, a commutator of the generators belonging to the two different triplets does not have the required
property). In this connection it is quite instructive to realize that several rather broad classes of EWSB schemes involving rich spectra of PGB have been studied previously\textsuperscript{17,18,19}, mostly within the framework of technicolour models. As noticed in the Introduction, in any of these models the corresponding Goldstone-boson manifold $G/H$ is a symmetric space and the PGB interactions do not influence the canonical low-energy theorem.

One should also notice that the scheme examined here has similar signatures as some weak-coupling models with additional Higgs triplets investigated by other authors\textsuperscript{21,22}, in that doubly charged scalar bosons appear in the physical spectrum. However, at present it is not clear to us how to construct e.g. a Higgs potential that would reproduce precisely the SB pattern considered in this paper. A discussion of such a problem, as well as a comparison of scattering amplitudes calculated here with those obtained e.g. in the model of Ref. 22 would deserve a separate treatment.

**Acknowledgements**

This work has been partially supported by the grant GACR–202/98/0506. One of us (J.H.) is indebted to Dr. M.Stöhr for technical assistance.

**References**

1. M.Peskin, preprint SLAC-PUB-7497, hep-ph/9705479.
2. P.Sikivie et al., Nucl. Phys. **B173**, 189 (1980).
3. J.Gunion et al., *The Higgs Hunter’s Guide* (Addison-Wesley, Redwood City, CA, 1990).
4. P.Langacker and H.Weldon, Phys. Rev. Lett. **52**, 1377 (1984); H.Weldon, Phys. Lett. **146B**, 59 (1984); D.Comelli, J.Espinosa, Phys. Lett. **B388**, 793 (1996).
5. M.Chanowitz, in *Perspectives on Higgs Physics*, ed. G.Kane (World Scientific, Singapore, 1993), p.343.
6. D.Dominici, Riv. Nuovo Cim. **20**, Ser.4, No.11 (1997).
7. E.Farhi and L.Susskind, Phys. Rep. **74**, 277 (1981).
8. F.Feruglio, Int. J. Mod. Phys. **A8**, 4937 (1993).
9. J.Wudka, Int. J. Mod. Phys. **A9**, 2301 (1994).
10. S.Weinberg, Physica A **96**, 327 (1979).
11. J.Gasser and H.Leutwyler, Ann. Phys. **158**, 142 (1984).
12. J.Donoghue, E.Golowich and B.Holstein, *Dynamics of the Standard Model* (Cambridge Univ. Press, Cambridge, UK, 1992).
13. G.Altarelli, R.Barbieri and F.Caravaglios, Int. J. Mod. Phys. **A13**, 1031 (1998).
14. M.Chanowitz, M.Golden and H.Georgi, Phys. Rev. Lett. **57**, 2344 (1986); Phys. Rev. **D36**, 1490 (1987).
15. J.Cornwall, D.Levin and G.Tiktopoulos, Phys. Rev. **D10**, 1145 (1974); C.Vayonakis, Lett. Nuovo Cim. **17**, 383 (1976); M.Chanowitz and M.Gaillard, Nucl. Phys. **B261**, 379 (1985); G.Gounaris, R.Koegerler and H.Neufeld, Phys. Rev. **D34**, 3257 (1986); H.-J.He, Y.P.Kuang and X.Li, Phys. Rev. Lett. **69**, 2619 (1992).
16. S.Coleman, J.Wess and B.Zumino, Phys. Rev. **177**, 2239 (1968); C.Callan, S.Coleman and B.Zumino, ibid., 2247.
17. M.Peskin, Nucl. Phys. **B175**, 197 (1980).
18. S.Chadha and M.Peskin, Nucl. Phys. **B185**, 61 (1981).
19. J.Preskill, Nucl. Phys. **B177**, 21 (1981).
Deviations from low-energy theorem

15

20. R.S.Chivukula and H.Georgi, Phys. Lett. B182, 181 (1986).
21. H.Georgi and M.Machacek, Nucl. Phys. B262, 463 (1985).
22. R.Vega and D.Dicus, Nucl. Phys. B329, 533 (1990).
23. T.Bahnik, PhD thesis, Charles University, Prague, 1997.
24. J.Horejsi, Introduction to Electroweak Unification (World Scientific, Singapore, 1994).
25. S.Weinberg, Phys. Rev. D13, 974 (1976).
26. B.Dutta and S.Nandi, Mod. Phys. Lett. A9, 1025 (1994); C.H.Llewellyn Smith and M.Bento, Nucl. Phys. B289, 36 (1987); T.Han et al., Phys. Rev. D42, 3052 (1990); G.Kane, M.Duncan and W.Repko, Nucl. Phys B272, 517 (1986).
27. D.Boulware and L.S.Brown, Ann. Phys. 138, 392 (1982).
28. M. Bando, T. Kugo and K. Yamawaki, Phys. Rep. 164, 218 (1988).

Appendix A
This appendix summarizes intermediate formulae used in our calculations.

Process \( W^+(k_1) + W^-(k_2) \to Z(k_3) + Z(k_4) \)

\[
A^{(1)}_{k\pi} (s) = \frac{\langle \varepsilon_1 \cdot \varepsilon_2 \rangle \langle \varepsilon_3 \cdot \varepsilon_4 \rangle_{\text{long}}}{s - m^2_{\pi}} = \frac{(s - 2m^2_W)(s - 2m^2_Z)}{4m^4_W m^2_Z (s - m^2_{\pi})} \\
A^{(1)}_{t\pi} (s, \cos \theta_{cm}) = \frac{\langle \varepsilon_1 \cdot \varepsilon_3 \rangle \langle \varepsilon_2 \cdot \varepsilon_4 \rangle_{\text{long}}}{t - m^2_t} = \frac{(s\beta^2_Z - 4m^2_W \beta^2_Z - s\beta_W \beta_Z \cos \theta_{cm})^2}{16m^4_W m^2_Z \beta^2_W \beta^2_Z (t - m^2_t)} \\
A^{(1)}_{u\pi} (s, \cos \theta_{cm}) = \frac{\langle \varepsilon_1 \cdot \varepsilon_4 \rangle \langle \varepsilon_2 \cdot \varepsilon_3 \rangle_{\text{long}}}{u - m^2_u} = A^{(1)}_{t\pi} (s, -\cos \theta_{cm})
\]

where

\[
t = m^2_W + m^2_Z - \frac{s}{2} + \frac{s}{2} \beta_W \beta_Z \cos \theta_{cm} \\
u = m^2_W + m^2_Z - \frac{s}{2} - \frac{s}{2} \beta_W \beta_Z \cos \theta_{cm} \\
\beta_W = \sqrt{1 - \frac{4m^2_W}{s}}, \quad \beta_Z = \sqrt{1 - \frac{4m^2_Z}{s}}.
\]

\( \theta_{cm} \) is the angle between \( k_1 \) and \( k_3 \) in the c.m. system. Since we assume that the PGB are light, i.e. \( m_{\pi} \approx m_{W} \), the high-energy \( (s \gg m^2_W) \) expansion of the amplitudes takes a simple form

\[
A^{(1)}_{k\pi} = \frac{k}{4m^2_W m^2_Z} + O(s^0), \quad k = s, t, u
\]

\[
t = -\frac{s}{2}(1 - \cos \theta_{cm}), \quad u = -\frac{s}{2}(1 + \cos \theta_{cm})
\]

The symmetry-breaking part of the complete amplitude is

\[
\mathcal{M}^{(1)}_{SB} = \mathcal{M}^{(1)}_{s\pi} + \mathcal{M}^{(1)}_{t\pi} + \mathcal{M}^{(1)}_{u\pi}.
\]
The kinematical variables are related by
\[
t = m_W^2 + m_Z^2 - \frac{s}{2} + \frac{(m_Z - m_W^2)^2}{2s} + 2k^2 \cos \theta_{cm}
\]
\[
u = -2k^2(1 + \cos \theta_{cm})
\]
where in the c.m. system
\[
k^2 = \frac{1}{4s} \left[ s^2 + (m_W^2 - m_Z^2)^2 - 2s(m_W^2 + m_Z^2) \right] = |k_i|^2 \quad i = 1, 2, 3, 4.
\]

High-energy expansions are the same as in (A.2).

**Process** \( W^+(k_1) + Z(k_2) \to Z(k_3) + W^+(k_4) \)

The relevant formulae for processes #3 and #4 are easily obtained by setting \( m_Z = m_W \) in (A.1) or (A.4).

\[
A^{(3,4)}_{SS} = \frac{(2m_W^2 - s)^2}{4m_W^2(s - m_Z^2)}
\]
\[
A^{(3,4)}_{SS} = \frac{(2m_W^2 - s)^2}{4m_W^2(s - m_Z^2)}
\]

\[
M_{SS}^{(3)} = M_{\pi^+\pi^+}^{(3)} + M_{\pi^-\pi^0}^{(3)} + M_{\pi^0\pi^0}^{(3)} + M_{\pi^0\pi^0}^{(3)} + M_{u\pi^1}^{(3)} + M_{u\pi^1}^{(3)}.
\]

**Process** \( W^+(k_1) + W^-(k_2) \to W^+(k_3) + W^-(k_4) \)

For the sake of completeness we give also the corresponding results for the process #4

\[
M_{SS}^{(4)} = M_{u\pi^1}^{(4)} + M_{\pi^0\pi^0}^{(4)} + M_{s\pi^0}^{(4)} + M_{s\pi^0}^{(4)} + M_{u\pi^1}^{(4)} + M_{u\pi^1}^{(4)}.
\]