Inclusive Pion Double-Charge-Exchange Above .5 GeV

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Abstract

A cascade model has been developed to study pion induced multichannel reactions (quasielastic, SCX, DCX, absorption and π-production) at pion energies above 0.5 GeV. The inclusive pion double-charge-exchange (DCX) reaction on $^{16}O, ^{40}Ca$ and $^{208}Pb$ nuclei in the energy range from 0.4 to 1.2 GeV is analyzed. Pion energy spectra and double differential cross section are calculated. The pion production is a determinant feature in the high energy pion nucleus reactions, and non pion production, DCX signal is sizeable only at forward angles and for high energy outgoing pions. It is shown that the contribution to inclusive DCX processes of the conventional mechanism, with two (or more) quasielastic SCX steps decreases very fast as a function of the energy and reaches very low values at energies above .6 GeV. This opens the opportunity of having sizeable contributions of exotic mechanisms that are negligible at the Δ-resonance energies.
1 Introduction

The pion induced Double Charge Exchange (DCX) reaction has been extensively studied [1, 2, 3, 4] at energies below and around the $\Delta$ resonance. However, many questions remain not sufficiently understood, like the angular dependence of the analog transitions at resonance energy [3], or the two-peaks structure of the spectra at forward angles that appear in the inclusive DCX process in light nuclei[5, 6]. There are many reasons for these difficulties. One is the existence of several mechanisms, some of them important, that interfere, and whose evaluation is not simple. Another one is the strong distortion of the pion waves at energies around the $\Delta$ resonance. Finally, for exclusive reactions, DCX processes are very sensitive to small details of the nuclear structure.

At higher energies the analysis of DCX reactions is much simpler, mainly, because the $\pi N$ cross section is much smaller, reducing the importance of distortion. Additionally, the contribution of some of the mechanisms, like DINT [9] and the absorption [10] mechanism, is expected to decrease rapidly with energy.

Furthermore, the angular dependence of the single charge exchange $\pi N$ reaction is strongly energy dependent. This has the consequence that at certain energies the sequential contribution to the DCX cross section results in deep minima. This result, found for the exclusive reaction in [11] will be shown here to be a general feature of inclusive DCX for all nuclei. The energies, at which the sequential mechanism is very small, and distortion is probably negligible, offer the best ground for the investigation of some exotic mechanisms, like those involving meson exchange currents [12], six-quark bags [13], and others involving more than two nucleons [14], which, although small around the $\Delta$ resonance, are not expected to decrease significantly with energy.

We know, from our experience at low energies, how strong is the dependence of the DCX cross section on the nuclear structure for exclusive processes. This dependence is smeared out in the inclusive case, when a sum over all possible final states is done. Additionally, inclusive DCX offers the advantage of a much higher yield than exclusive experiments and does not require such a good energy resolution.

At present there are no data on inclusive DCX in the energy range above .5 GeV, although some measurements are in progress now [15] at ITEP (Moscow). These experiments have stimulated the present work. There are, however, some data around .5 GeV which will be used to test our model.

The calculation presented here is a Monte Carlo simulation of the reaction. This is most appropriate for a multichannel situation (i.e. absorption, quasielastic, SCX, DCX, $\pi$-production) that renders a full quantum calculation infeasible.

Furthermore, similar calculations [7, 8] describe quite well all $\pi$-nucleus inclusive reactions around the $\Delta$-resonance: absorption, quasielastic scattering, single charge exchange, and double charge exchange. At higher energies the wavelength of the pion is shorter and therefore quantum interference effects should be less important.

This paper is organized as follows: Section 2 presents the basic elements of the variant of the cascade model considered here (details are given in Appendix
A). In Section 3 the results of calculations of pion spectra (energy and angular distributions) for the inclusive DCX are presented, and in Section 4 the obtained results are discussed.

2 Model

2.1 Basic considerations

A pion travelling inside a nucleus can be absorbed, can change direction, energy, charge, or even produce more pions. The basic inputs for our simulation will be the probabilities per unit length for each of these channels to happen. How these probabilities are obtained is presented below, in sections 2.2 - 2.4. Details on the simulation will be presented in Appendix A.

2.2 $\pi N \rightarrow \pi N$

The probability per unit length of quasielastic scattering (or single charge exchange) is given by

$$P_{N(\pi^\lambda,\pi^\lambda')N'} = \sigma_{N(\pi^\lambda,\pi^\lambda')N'} \rho_N$$

(1)

where $N$ stands for neutron or proton, $\rho_N$ is the density of nucleons of type $N$, and $\sigma_{N(\pi^\lambda,\pi^\lambda')N'}$ is the elementary cross section for the reaction $\pi^\lambda + N \rightarrow \pi^\lambda' + N'$ obtained from Arndt’s phase shifts $[16]$. The density of protons is taken from experiment, and the density of neutrons is taken proportional to the density of protons in all results presented in this work.

When according with the probabilities of eq. (1), a quasielastic scattering took place, we executed the following algorithm. First, we chose randomly a nucleon, of the type $N$, from the fermi sea, then we boosted the $\pi$ and $N$ to their center of mass system. Finally, we selected the scattering angle (and therefore energy) of the outgoing particles using again the experimental cross sections $[16]$, and boosted the momenta to the lab. system. When the momentum of the outgoing nucleon in the lab. system is below the fermi level, we consider the event to be Pauli-blocked and therefore keep the pion initial charge and momentum unchanged.

2.3 Pion absorption

Even if pion absorption is a relatively small effect at high energies (which one could suggest from the rapid decrease of the pion-deuteron absorption cross section at the energy range from $0.3 - 1.0 GeV$ $[17, 18]$), there is a large number of pions at lower energies which are generated both by the quasielastic rescatterings and the pion production on the nuclear nucleons. The proportion of these pions that eventually comes out of the nucleus is essentially determined by the absorption strength.
Although pion absorption has been extensively studied at energies below 0.3 GeV, little theoretical work has been done about pion absorption by complex nuclei at high energies, and very little experimental information is available. In ref. [19] the effect of pion absorption on the pion-nucleus elastic and the SCX scattering has been studied in the energy range of 250 to 650 MeV. The absorptive part of the pion-nucleus optical potential was calculated within the framework of a many-body field theoretical approach. The model contains both two- and three-nucleon absorption mechanisms and it has been shown to agree quite well to the more complex microscopical model of ref. [20] in the $\Delta$ resonance region. Results show a quite weak absorption at high energies, as was expected. Another interesting result presented in ref. [19] is that, whereas in the resonance region three-body-absorption becomes comparable with the two-nucleon mechanism, as the pion energy increases the effects of three-body absorption decrease again, and the two-body mechanism becomes dominant, as is the case at low energy.

The probability per unit length of a pion to be absorbed is expressed in terms the imaginary part of the pion self-energy by the equation

$$P_{\text{abs}} = -\frac{\text{Im} \Pi_{\text{abs}}(k)}{k}.$$  

(1)

The imaginary part of the pion self-energy, related to two-nucleon pion absorption, which has been calculated in [19] is of the form

$$\text{Im} \Pi_{\text{abs}}^{(2)}(k) = -D_2 \frac{s \bar{\sigma}}{q} \rho^2$$  

(2)

Here, $D_2 = 0.0116 \, fm^5 \, mb^{-1}$, $k$ is the pion momentum in the lab system, $s$ is the square of the c.m. energy of a pion of momentum $k$ and a nucleon at rest, $\bar{\sigma} = (\sigma_{3/2} + \sigma_{1/2})/3$ is the spin-isospin averaged unpolarized $\pi N$ cross section, and the momentum of a virtual pion $q$ that appears in the model of ref. [19] is determined as

$$q = \left\{ \left[ \frac{(k_0^2 + 2m)^2 - \vec{k}^2}{2(k_0^2 + 2m)} \right]^2 - m^2 \right\}^{1/2}$$  

which in the nonrelativistic limit actually used in [19] goes to

$$q = \left[ m(k_0^2 - \vec{k}^2 / 2m) \right]^{1/2}$$

where $m$ is the mass of a nucleon.

The three-nucleon pion absorption has been calculated [19] in a similar fashion, and the contribution to the selfenergy is given by

$$\text{Im} \Pi_{\text{abs}}^{(3)}(k) = -D_3 \frac{s \bar{\sigma} s'}{q'} \frac{\bar{\sigma}}{q'} \rho^3$$  

(3)

with

$$q' = \left[ \frac{m}{2}(k_0^2 - \vec{k}^2 / 2m) \right]^{1/2}$$

The pion selfenergy $\Pi$ is related to the optical potential as $V = \Pi/2k_0$. 

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and \( s', \bar{\rho} \) have the same meaning as \( s \) and \( \bar{\rho} \), but are evaluated at a kinetic energy of the pion equal to two thirds of the real one \( (T'_{\pi} = \frac{2}{3}T_{\pi}) \), and \( D_3 = 1.15 \times 10^{-7} \text{fm}^8 \text{mb}^{-2} \text{MeV}^{-1} \).

The pion selfenergy pieces of eqs. (2) and (3) can readily be translated to a probability per unit length by the relation \( P_{abs} = \frac{-\text{Im} \Pi_{abs}(k)}{k} \).

In [21] (see, also Erratum in [22]), the pion absorption effect on the pion-nucleus scattering at 800 MeV has been estimated using the quasi-deuteron (qd) model. Taking into account that at energies well above the resonance region the two-body absorption becomes dominant [19], it is interesting to compare the outlined above model (Eq.(2)) with the quasi-deuteron model.

The imaginary part of the pion selfenergy \( \text{Im} \Pi(k) \) within the framework of the quasi-deuteron model is given by

\[
\text{Im} \Pi_{abs}(q_{qd})(k) = -8\pi \Gamma \text{Im} B_{\pi 2N}(E) \rho_n \rho_p
\]

where \( \rho_p \) and \( \rho_n \) are the densities of protons and neutrons. The kinematical factor \( \Gamma \) is determined as

\[
\Gamma = \frac{\mathcal{M}_{\pi A}}{\mu_{\pi d}}
\]

where, \( \mathcal{M}_{\pi A} \) and \( \mu_{\pi d} \) are the \( \pi \)-nucleus and \( \pi \)-deuteron reduced masses correspondingly, and \( \gamma \) is the relativistic transformation factor of the \( \pi - 2N \) scattering amplitude from the \( \pi \)-nucleus to the \( \pi - 2N \) c.m.s.

\[
\gamma = \frac{\omega_{\pi}(k)\omega_{\pi}(k')E_{2N}(\kappa)E_{2N}(\kappa')}{\omega_{\pi}(q)\omega_{\pi}(q')E_{2N}(P)E_{2N}(P')}\bigg[\frac{1}{T_{\pi}}\bigg]^{1/2}
\]

Here, \( \tilde{q} \) and \( \tilde{q}' \) are the pion momenta before and after the collision in the \( \pi \)-nucleus c.m.s.; \( \tilde{\kappa} \) and \( \tilde{\kappa}' \) are the pion momenta before and after the collision in the \( \pi - 2N \) c.m.s., and \( \tilde{P} \) and \( \tilde{P}' \) are the total momenta of the 2N subsystem before and after the collision in the \( \pi - 2N \) c.m.s. We use here the “frozen” approximation which means that \( \tilde{P} = -2\tilde{q}/A \) and \( \tilde{P}' = \tilde{P} - \tilde{\Delta} \), where \( \tilde{\Delta} = \tilde{q}' - \tilde{q} \) is the momentum transfer.

The imaginary part of the absorption parameter \( B \) is given by

\[
\text{Im} B = (1/4\pi)W(T_{\pi})/(2\rho_d(0))
\]

where \( \rho_d(0) \) is the deuteron density at \( r = 0 \). So as in [21] this quantity is calculated using the square-well potential model of a deuteron. The parameter \( W \) is related to the pion-deuteron total cross section as \( W \equiv \sigma(\pi^+ d \rightarrow pp) \). The total pion-deuteron absorption cross section at the energy range from 0.3 GeV to 1 GeV can be obtained from the differential cross section data of [17, 18]. The energy dependence of \( W(T) \) can be approximated as

\[
W(T_{\pi}) = \frac{\alpha_1}{T_{\pi}} + \frac{\alpha_2}{T_{\pi}^2} + \frac{\alpha_3}{T_{\pi}^3}
\]

where \( T_{\pi} \) is the pion kinetic energy in the laboratory system (measured in fm), and the parameters \( \alpha \) are

\[
\alpha_1 = 0.171, \quad \alpha_2 = -0.612 \text{ fm}^{-1}, \quad \alpha_3 = 1.780 \text{ fm}^{-2}
\]
2.4 Pion production

Pion production is a determinant feature in the high energy pion nucleus reactions. The inelastic channels have a cross section comparable, or even larger than the elastic channels at energies above 0.6 GeV. Although two (or more) pion production channels are possible at the energies considered, the inelastic cross section is clearly dominated by the single pion production \[23\]. In this work the multipion production channels have been ignored \[2\].

Whereas at low energies a considerable amount of data is available for most isospin channels, including differential cross sections, our knowledge is more fragmentary in the energy regime addressed here. Data has been taken from compilation \[23\] and \[24, 25\], to obtain parametrizations of the \(\pi N \rightarrow \pi \pi N\) total cross sections. Then, for each channel, the probability per unit length is given by the equation

\[
P_{N(\pi,2\pi)N'} = \sigma_{N(\pi,2\pi)N'} \rho_N
\]

When, according to this probability, a pion production event of a certain isospin channel has taken place, we proceed in the following way. First, a nucleon of the type \(N\) is randomly chosen from the fermi sea. Then the scattering angles and energies of the outgoing particles are selected, using the 3-body phase space distribution. When the momentum of the outgoing nucleon in the lab system is below the fermi sea level, the event is considered to be Pauli-blocked and therefore the pion initial charge and momentum are unchanged, and no any new pions are produced.

There are five independent pion production channels induced by charged pions, namely,

1. \(\pi^+ + p \rightarrow \pi^+ \pi^+ n\)
2. \(\pi^+ + p \rightarrow \pi^+ \pi^0 p\)
3. \(\pi^+ + n \rightarrow \pi^+ \pi^0 n\)
4. \(\pi^+ + n \rightarrow \pi^0 \pi^0 p\)
5. \(\pi^+ + n \rightarrow \pi^+ \pi^- p\)

The cross sections of (1) and (2) are parametrized as

\[
\sigma_{(1)} = \sigma_{in} \cdot 0.2 \left[1 - 0.05 \left(T_\pi - T_{\pi\text{th}}\right)\right],
\]

\[
\sigma_{(2)} = \sigma_{in} \cdot 0.8 \left[1 - 0.05 \left(T_\pi - T_{\pi\text{th}}\right)\right],
\]

where \(\sigma_{in}\) is the inelastic \(\pi^+ + p\) cross section obtained from Arndt’s phase shifts, \(T_\pi\) is the kinetic energy of a pion in units of \(fm^{-1}\) and \(T_{\pi\text{th}}\) is the kinetic energy of the pion production threshold in the same units.

\[2\] Note that this omission will not affect practically the higher energy part of the pion spectra, because for these channels some energy, at least two pion masses, has necessarily been spent.
In a similar fashion the other three channels have been parametrized. The formulas are

\[ \sigma(3) = (\sigma_{in} - \sigma(4) - \sigma(5)) \left[1 - 0.075(T_\pi - T_{\pi\text{th}})\right], \]

\[ \sigma(4) = \sigma_{in} \left[0.30 - 5.069 \times 10^{-2}T_\pi + 5.004 \times 10^{-3}T_\pi^2\right] \]

\[ \sigma(5) = \sigma_{in} \left[-2.78 \times 10^{-2} + 0.315T_\pi - 4.154 \times 10^{-2}T_\pi^2\right] \]

where \( \sigma_{in} \) is now the inelastic \( \pi^+ + n \) cross section, and \( T_\pi, T_{\pi\text{th}} \) have the same meaning as above. In all these cases the reproduction of data is quite satisfactory.

Apart from the five additional channels induced by a \( \pi^- \) that will be obtained by charge symmetry, there are three channels induced by a \( \pi^0 \):

\( (a) \) \( \pi^0 + N \rightarrow \pi^0\pi^0N \)
\( (b) \) \( \pi^0 + N \rightarrow \pi^+\pi^-N \)
\( (c) \) \( \pi^0 + N \rightarrow \pi^0\pi^cN' \)

where \( N \) is a nucleon, \( \pi^c \) is a charged pion, and the charge of the nucleon \( N' \) is determined by the charge balance in the channel (c). We use \( \sigma(a) = \sigma(b) = \sigma(c) = \sigma_{in}/3 \), where \( \sigma_{in} \) is the \( \pi^0N \) inelastic cross section. These channels are less important, because the experiment will begin with a charged pion. Therefore the neutral pions are secondary pions. That means that there are fewer of them, and also that in the average they have less energy and will not affect much the higher energy part of the spectra of interest here.

### 3 Results

There are some data of pion-nucleus scattering at energies around 0.5 GeV. We compare the results of our program with these data, because data at higher energies are yet very preliminary. We should remark however that we expect our results to be in better agreement with experiment at higher energies, where the use of a semiclassical approximation like this one is more justifiable, and where the pion interaction with nucleons is weaker.

In Fig.1 we show the results of the present model (solid line) for the quasielastic \( \pi^+ \) scattering in \( ^{12}\text{C} \), compared with the experimental data from ref. [26].

The quasielastic peak is well reproduced, in both size and width, at all angles. The size, being absorption of very little importance at this energy in our model, is governed by the elementary \( \pi N \) cross sections and by the fermi motion of the nucleons. The calculation we present has been done assuming a fermi momentum of 250 MeV all over the nucleus. In fact, the use of a local fermi momentum, obtained from the local nucleon density, results in a worse agreement with the data, producing narrower peaks and overestimating clearly the cross section at forward angles.
Our results underestimate the cross section at pion energies below the quasielastic peak. This seems to be a fact common to other cascade codes (see, discussion in ref. [24] and Sect. 4)

Of course we cannot reproduce the elastic peak, important at low angles, given that we do not have collisions of the pions with the nucleus as a whole. Only pure quasielastic scattering is our aim here.

In Fig. 2 we show the results of the present model (solid line) for single charge exchange scattering at 500 MeV. This is a very important channel for us, because double charge exchange requires two single charge exchange scatterings. Again, the quasielastic peak size and width are in a quite good agreement with data. This should be expected, as this peak is dominated by a single pion-nucleon collision, as it was the case for quasielastic scattering. The only additional information is that the ratio between different scattering channels is the same as in the elementary \( \pi N \) collisions. Note also that below the peak, one can observe the same behavior found in quasielastic scattering. There are some pions missing in that region. In this figure, we show separately the pions coming from \( \pi \) production, which are not enough to agree with the data from [27]. Even the total suppression of pion absorption would not be enough to improve significantly the agreement with data.

In Fig.2 we also compare the data with the cascade-exiton model (CEM). This model of nuclear reactions [28] was proposed initially to describe nucleon induced reactions at bombarding energies below or at \( \sim 100 \) MeV and was later developed for a larger interval of bombarding energies and for the analysis of pion-nucleus reactions (see, e.g. [29] and references therein.) The results of CEM, shown by the short-dashed curves, agree with the results of the present model. The CEM calculations also underestimate the low energy part of the pion spectra.

Finally, before concentrating in the DCX channel, we will present some results for pion-nucleus reactions at higher energies. Our purpose is to identify the main features of these processes. In Figs.3-5 we analyze the reaction \( \pi^+ + ^{40}Ca \rightarrow \pi + X \), and Figs.6,7 deal with the same reaction in lead. In all figures we split the total cross section into two pieces: a quasifree piece, given by those pions that come out of the nucleus after having only quasifree scatterings, and a pion production piece, given by the pions coming from events in which at least a pion production took place. Note that in this latter case, quasifree scatterings, prior or subsequent to the pion production itself, could have occurred.

Fig.3 shows the energy spectra of the outgoing pions. Although the energy spectra begin at 0 MeV, we should remember that the results at very low energies are not meaningful, because the cascade method is not appropriate there. In ref. [4] it has been shown that quantum calculations begin to differ appreciably from the present kind of approach, and for inclusive processes at energies around 100 MeV.

Let us begin discussing the quasielastic channel. In it, we can separate a region of high energies, where only pions coming from one (or several) quasifree scatterings contribute, and a second region dominated by pions coming from \( \pi \) production events. Note the little dip around 150 MeV in the "quasifree" part, and also the change of curvature in the same region of the "production" part. Both are due
to the strong absorption in the $\Delta$ resonance region. At energies above 300 MeV absorption effects are small for both the two models discussed before.

The situation is similar for the SCX channel, although the quasifree peak is smaller. As expected, because DCX requires at least two scatterings, the quasifree peak is much smaller in its case.

The importance of pion production channels contribution to DCX has already been shown at lower beam energies. At 600 MeV and above, DCX is totally dominated by $\pi$ production. Of course, this is true except for the small region of phase space where $\pi$ production is forbidden. Therefore, if we want to learn something about alternative DCX mechanisms, we should concentrate our attention in this region. Otherwise, any signal will be blurred by the large number of pions coming from production channels. And these production channels are not well known, neither in size, nor in the angle-energy dependence of its cross sections.

The angular behavior of the reaction is shown in Fig. 4. All channels are forward peaked, essentially the "quasifree" part. Note that one of the uncertainties in our model is, as mentioned above, the angle-energy structure of the $N(\pi,2\pi)N'$ amplitudes, that has been included as being proportional to phase space. Thus, the "production" part reflects mostly the boost of the isotropic process from the CM to the lab system.

We have selected the DCX channel to show a double differential cross section in Fig. 5. Given the angular behavior observed previously it is only logical to find that the best place to isolate a clean, non-production, DCX signal occurs at forward angles and for high energy pions. The rest of the spectra are totally dominated by the $\pi$-production channels. Observe again the dip produced by the absorption at energies around the $\Delta$ resonance.

Fig. 6 shows the pion spectra for the reaction $\pi^+ + ^{208}$Pb $\rightarrow \pi + X$ at 1200 MeV. Many of the main features found in Calcium at 600 MeV are also relevant for this case. Let us mention that there is an even stronger dominance of the production channels, except for the quasielastic channel at the very high energy region. The reason is the smallness of the $\pi N$ charge exchange cross section at this energy. Obviously, this implies a small "quasifree" SCX cross section and an even smaller "quasifree" DCX cross section.

The angular distribution, not presented here, is similar to that of Calcium, although a bit more forward peaked, essentially in the quasielastic channel. Fig. 7 shows the DCX double differential cross section for the reaction $\pi^+ + ^{208}$Pb $\rightarrow \pi + X$ at 1200 MeV at the same angles presented before for Calcium. Quasifree DCX is hardly visible with the scale used in the figure, as it could be expected from Fig. 6.

Finally, we have selected as observable the integrated DCX cross section, putting as a cut that the energy of the final pion is, at most, 150 MeV below the beam energy. This eliminates practically all cases in which there is a pion production. The results, in Calcium, and as a function of the beam kinetic energy, are presented in Fig. 8. Very similar results are obtained putting an additional cut in angles, given that most pions fulfilling the previous energy condition go forward. For comparison we also present the quasielastic and SCX case. Note that whereas the quasielastic
channel changes by around a factor two (from 500 to 1300 MeV), SCX loses one order of magnitude and DCX decreases almost 3 orders of magnitude.

We do not consider this as a prediction of an extremely low DCX cross section at high energies. We do not expect, nor claim that. First, because of a "technical" reason, DCX is so tiny because quasifree SCX is quite small. A change of the SCX $\pi N$ cross section by a 2% of a typical scale, say the elastic $\pi N$ cross section, would mean a factor 4 for our result. In other words, the error bars are only statistical and do not include the errors coming from the elementary cross sections used as input. A second, more important point is that the curve corresponds only to the ingredients of the code, namely, to two consecutive quasifree single charge exchange $\pi N$ collisions. Let us them state our result in a meaningful way: The contribution to inclusive DCX processes of the conventional mechanism, with two (or more) quasieelastic SCX steps decreases very fast as a function of the energy and reaches very low values, compared with the quasieelastic channel, at energies above 600 MeV.

4 Discussion

In the present paper a cascade model has been developed to study pion induced multichannel reactions (quasieelastic, SCX, DCX, absorption and $\pi$-production) at pion energies above .5 GeV.

The model has been checked by comparing with the experimental data [26, 27] for quasielastic pion scattering and single charge exchange $\pi^-$ scattering in $^{12}$C at 0.5 GeV. We also compared the results of the present model with the CEM [28, 29] calculations.

It has been shown that for quasielastic scattering the quasielastic peak is well reproduced, in both size and width, at all angles, but our results clearly underestimate the cross section at pion energies below the quasielastic peak. This seems to be a fact common to other cascade codes as it is remarked in ref [26], indicating possibly, some interesting piece of physics missing in our description of the pion-nucleus reactions [30]. The CEM calculations also underestimate strongly the cross section at this energy range.

For the single charge exchange scattering we also observe some missing of pions below the quasielastic peak.

One could ascribe the missing cross sections to several causes that should certainly be investigated further. For instance, at lower energies, a sizeable enhancement of the $(\pi, 2\pi)$ cross sections, when comparing to quasifree calculations, had been predicted in ref [31] and was later found in ref [32]. The effect was related to the change of the dispersion relation of the pions in the medium. Speaking in simple terms, the pions are attracted by the medium. Unfortunately we do not know so well the pion propagation properties at higher energies, and the results of ref. [31] cannot be easily extrapolated. In ref. [26] Zumbo et al. suggest the formation of a narrow $\sigma$ meson, with little interaction with the medium, that would leave the nucleus prior to its decay into two pions. One should mention that the
obvious candidate, a weaker pion absorption, it is difficult to reconcile with the agreement obtained in the quasielastic peak, that for different angles is situated at the same energy, and also with the pion absorption data in the resonance energy region.

At $.6 \text{GeV}$ and above, DCX is almost totally dominated by $\pi$ production. To learn something about exotic DCX mechanisms, we should concentrate our attention in the region of phase space where $\pi$ production is forbidden. Otherwise, any signal will be blurred by the large number of pions coming from production channels. Furthermore, these production channels are not well known, neither in size, nor in the angle-energy dependence of its cross sections.

The calculations for the inclusive DCX at energy $1.2 \text{GeV}$ show that many of the main features found at $600 \text{ MeV}$ are also relevant at these higher energies. There is an even stronger dominance of the production channels, except for the quasielastic channel at the very high energy region. The reason is the smallness of the $\pi N$ charge exchange cross section at these energies.

There are some interesting results in the literature concerning exclusive DCX processes at high energies. In particular the reactions $^{18}O(\pi^+, \pi^-)^{18}Ne$ and also $^{14}C(\pi^+, \pi^-)^{14}O$ have been studied in ref. [11] at energies up to $1400 \text{ MeV}$. Their resulting cross sections present two deep minima at energies around 700 and 1300 MeV. That result does not depend on nuclear structure or the specific nuclei chosen. It simply reflects the energy dependence of the $\pi N$ SCX amplitude. Thus one expect to get a similar result for the inclusive DCX process and, possibly, gaining in yield and requiring a less precise energy measurement of the final pion, because there is no need to separate clearly a given final state of the target nucleus. To study this point, we have selected as observable the integrated DCX cross section (Fig. 8), putting as a cut that the energy of the final pion is, at most, $150 \text{ MeV}$ below the beam energy. This eliminates practically all cases in which there is a pion production. Very similar results are obtained putting an additional cut in angles, given that most pions fulfilling the previous energy condition go forward. Whereas the quasielastic channel loses less than one order of magnitude (from $.5 \text{ GeV}$ to $1.3 \text{ GeV}$), SCX changes by a factor $1/40$, and DCX decreases 3 orders of magnitude.

It should be noted that, within the present approach, the contribution to inclusive DCX processes is generated by the conventional mechanism, with two (or more) quasielastic SCX steps. The results presented in Fig.8 show that the sequential mechanism decreases very fast as a function of the energy and reaches very low values, compared with the quasielastic channel, at energies above $600 \text{ MeV}$. In this situation, it is very important to have a possibility to compare these results with the experimental data and to consider other mechanisms of the DCX (like MEC) which might do not decrease so fast at high energies.
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A Details on the simulation

We generate pions, of a given momentum and charge, which travel in the \( z \) direction, with a random impact parameter \( \vec{b} \), obeying \( |\vec{b}| \leq R \), where \( R \) is an upper bound for the nuclear radius. We choose \( R \) such that \( \rho(R) \approx 10^{-3} \rho_0 \), with \( \rho_0 \) the normal nuclear matter density. At the beginning, the pions are placed at the point \((\vec{b}, z_{in})\), with \( z_{in} = -\sqrt{R^2 - |\vec{b}|^2} \), and then, we proceed to move them along the \( z \) direction, in small steps, until either the pions get out of the nucleus or interact.

Let us assume that \( P(q, r, \lambda) \) is the probability of interaction per unit length, at the point \( r \), of a pion of momentum \( \vec{q} \) and charge \( \lambda \). We choose an interval \( \delta l \), such that \( P(q, r, \lambda)\delta l \) is small compared to unity. Then we generate a random number \( x \in [0, 1[. \) We have two possibilities:

(a) \( x > P\delta l \). In this case there is no interaction, and the pion travels a distance \( \delta l \) along the direction of its momentum \( \vec{q} \).

(b) \( x < P\delta l \). In this case there is interaction. According to its respective weights, we decide whether it has been absorption, quasielastic scattering, charge exchange, or pion production.

When it has been quasielastic, or charge exchange, we use the procedure defined in section 2.2 to find the new energy, and direction of the pion, and continue to propagate it along its new direction, checking at every step if new interactions take place.

When it has been a pion production case, we choose the channel, according with the respective weights given by their cross sections, and select the energy and direction of the two final pions using the algorithm described in section 2.4. Then we store the data of one of the pions and keep moving the other one.

In the case that after moving the step \( \delta l \) the pion gets out of the nucleus, and when the pion is absorbed, we check whether there are some other pions left inside the nucleus (these pions would have been produced previously at some step). If there are some other pions, we select one of them and begin to propagate it from its current position.

When there are no pions left, we store the data -energy and angles- of all pions that got out of the nucleus, if any, and begin again the full procedure by generating a new initial pion.

Note that between interactions pions follow a straight trajectory. Thus, even some classical effects like the change in direction due to the real part of the potential, strong and coulombian, are neglected.

A.1 Integrated cross sections

Let \( N \) be the total number of incident pions, and let \( N_c \) be the total number of events of a given channel, i.e. number of cases in which there is a \( \pi^- \) in the final state, then, the integrated cross section for that channel is given by

\[
\sigma_c = \pi R^2 \frac{N_c}{N}
\]
A.2 Differential cross sections

As an example, we will show the way in which angular cross sections are calculated. Energy distributions, or double differential cross sections are obtained in a similar way. To calculate differential angular cross sections we divide the cosine of the polar angle in $N_\mu$ equal angular intervals. If $\mu$ is the cosine of the polar angle with which a pion leaves the nucleus, we associate to it the discrete value of the angular bin in which it falls. Thus

$$k = 1 + \left[ \frac{\mu + 1}{\delta \mu} \right]$$

where, $[.]$ means that we take only the integer part, and

$$\delta \mu = \frac{2}{N_\mu}.$$ 

If we get a total number of $n_k$ pions in the $k$ bin, we have

$$\frac{d\sigma_k}{d\Omega} = \pi R^2 \left( \frac{1}{2\pi \delta \mu} \right) \frac{n_k}{N}$$
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Figure Captions

**Fig. 1.** Double differential cross section for the $^{12}\text{C}(\pi^+, \pi^+)_X$ reaction at $T_\pi=500$ MeV. Continuous line: full model. The data, taken from ref. [26], also include the $^{12}\text{C}(\pi^-, \pi^-)_X$ reaction.

**Fig. 2.** Double differential cross section for the $^{12}\text{C}(\pi^-, \pi^0)_X$ reaction at $T_\pi=500$ MeV. Continuous line: full model, long-dashed line: $\pi$-production channels, short-dashed line CEM model [29]. Data from ref. [27].

**Fig. 3.** Calculated spectra of the $^{40}\text{Ca}(\pi, \pi')_X$ reactions at $T_\pi=600$ MeV. Continuous line: full model, long-dashed line: $\pi$-production channels, short-dashed line: quasielastic channels.

**Fig. 4.** Calculated angular distributions of the $^{40}\text{Ca}(\pi, \pi')_X$ reactions at $T_\pi=600$ MeV. Continuous line: full model, long-dashed line: $\pi$-production channels, short-dashed line: quasielastic channels.

**Fig. 5.** Calculated angle-energy distributions of the $^{40}\text{Ca}(\pi^+, \pi^-)_X$ reaction at $T_\pi=600$ MeV. Continuous line: full model, dashed line: quasielastic channels.

**Fig. 6.** Calculated spectra of the $^{208}\text{Pb}(\pi, \pi')_X$ reactions at $T_\pi=1200$ MeV. Continuous line: full model, long-dashed line: $\pi$-production channels, short-dashed line: quasielastic channels.

**Fig. 7.** Calculated angle-energy distributions of the $^{208}\text{Pb}(\pi^+, \pi^-)_X$ reaction at $T_\pi=1200$ MeV. Continuous line: full model, dashed line: quasielastic channels.

**Fig. 8.** Calculated cross section of the $^{40}\text{Ca}(\pi, \pi')_X$ reactions as a function of the energy. Error bars represent statistical uncertainty.
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fig. 1
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$C(\pi^-,\pi^0)$

$\theta=50^\circ$
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$^{40}\text{Ca}(\pi^+\pi^+)X$

$E_{\pi^-} - E_{\pi} > 150 \text{ MeV}$

$^{40}\text{Ca}(\pi^+\pi^0)X$

$E_{\pi^-} - E_{\pi} > 150 \text{ MeV}$

$^{40}\text{Ca}(\pi^+\pi^-)X$

$E_{\pi^-} - E_{\pi} > 150 \text{ MeV}$

$\sigma \text{ (mb)}$

$E_{\pi} \text{ (MeV)}$

Fig. 8