Adaptive resource management for multi-target tracking in co-located MIMO radar based on time-space joint allocation

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Abstract: Compared with the traditional phased array radar, the co-located multiple-input multiple-output (MIMO) radar is able to transmit orthogonal waveforms to form different illuminating modes, providing a larger freedom degree in radar resource management. In order to implement the effective resource management for the co-located MIMO radar in multi-target tracking, this paper proposes a resource management optimization model, where the system resource consumption and the tracking accuracy requirements are considered comprehensively. An adaptive resource management algorithm for the co-located MIMO radar is obtained based on the proposed model, where the sub-array number, sampling period, transmitting energy, beam direction and working mode are adaptively controlled to realize the time-space resource joint allocation. Simulation results demonstrate the superiority of the proposed algorithm. Furthermore, the co-located MIMO radar using the proposed algorithm can satisfy the predetermined tracking accuracy requirements with less comprehensive cost compared with the phased array radar.

Keywords: co-located multiple-input multiple-output (MIMO) radar, adaptive resource management, multi-target tracking, sub-array division, time-space joint allocation.

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1. Introduction

As a new type of radar, the multiple-input multiple-output (MIMO) radar has received significant attention recently [1–5]. Compared with the phased array radar, the MIMO radar has a low interception probability, strong anti-jamming ability, and can achieve better performance in weak targets detection and target parameters estimation [6–8].

In general, the MIMO radar can be divided into the co-located MIMO radar and the distributed MIMO radar [9]. In the distributed MIMO radar, the transmitting antennas are located far apart from one another relative to their distance to the targets [9]. However, actual difficulties including multistatic synchronization, and channel matrix estimation [10] still stop the distributed MIMO radar being used in application. As the extension of the conventional phased array radar, the co-located MIMO radar, whose transmitting and receiving antennas are located close to one another relative to the targets, has more practical values. Generally, the whole array in the co-located MIMO radar is divided into several sub-arrays, and different sub-arrays transmit mutually orthogonal waveforms. The width of the transmitting beam is much wider than that in the phased array radar. What is more, the beam width varies with the number of sub-arrays. Therefore, the co-located MIMO radar can illuminate multiple targets with a wide beam simultaneously or illuminate them in turns according to different sub-array numbers, which increases the freedom degree of resource management. In order to make full use of the limited system resource, the adaptive resource management for the co-located MIMO radar in multi-target tracking is especially necessary [11–14].

The essence of radar’s adaptive resource management is to control its working parameters [15–25]. As to the resource management for the co-located MIMO radar system, Yan et al. [15–17] put forth the resource management strategy called Minimax in the multi-target tracking scenario, whose aim is to improve the worst-case tracking accuracy through running out of all system resources. With the so-called Minimax strategy, Yan et al. [15] proposed a simultaneous multi-beam resource allocation (SMRA) strategy, where the transmitting beams and corresponding power are adaptively controlled. It is extended to the case which considers the clutter in [16]. In [17], the beam, power and waveform selection were jointly considered. The above works are Minimax-based, while a novel resource scheduling method based on the predefined tracking precision was put forward in [18], where the sub-array number, illuminated targets set and transmitting waveform
were adjusted effectively. Furthermore, an adaptive cost function (ACF) based on the predetermined tracking precision was put forward in [19–21]. For a given total power budget, through adaptively controlling the directions of beams and transmitting power, the ACF is optimized.

While lots of contributions have been made to the resource scheduling for the co-located MIMO radar by existing works, there are still some problems that need addressing.

Firstly, the co-located MIMO radar resource scheduling concentrates on space allocation of system resources [15–21], and the time allocation of system resources for the co-located MIMO radar is not considered.

Secondly, the system resources are run out to enhance the worst-case tracking precision [15–17]. However, in application, it is not necessary to use up the system resources to maximize the target tracking performance. To ensure the desired tracking performance, how to minimize system resource consumption has more practical values.

Thirdly, in [15–17,19–21], the mechanism of how to form the multi-beam was not given. One beam can only track one target, and the ability of simultaneous multiple targets illumination is not considered.

Finally, the posterior Cramér-Rao lower bound (PCRLB) was used as a metric of tracking performance in [15,16,19–21]. However, PCRLB is a theoretical lower bound and it is unable to measure the actual online tracking accuracy effectively.

Based on the above, a more practical resource management optimization model in the co-located MIMO radar for multi-target tracking is proposed here, where the system resource consumption is minimized to ensure the desired tracking accuracy. Based on the proposed optimization model, an adaptive algorithm for resource allocation is obtained, where the system sampling period, sub-array number, transmitting waveform energy, beam direction and illuminated targets set are controlled jointly to realize the time-space resource joint allocation.

(ii) The number of sub-arrays and the effect of it on the width of the transmitting beam are considered. The beam width can be changed through controlling the sub-array number. Specifically, the beam width of the co-located MIMO radar is wider than the one in the phased array radar when the sub-array number is larger. Thereby it has the capability of illuminating multiple targets simultaneously. Through the control of the sub-array number, the co-located MIMO radar can track multiple targets in turns or simultaneously, whereby the freedom degree of resource management increases.

(iii) The tracking error covariance is used to construct the objective function in the resource management model. Different from the PCRLB, which is just a theoretical lower bound, the tracking error covariance can measure the actual online tracking accuracy effectively. The closer the tracking error covariance of a target is to the desired one, the better this target can achieve the desired tracking accuracy.

The remainder of the paper is organized in the following manner. In Section 2, the problem formulation is given firstly. Then, the co-located MIMO radar resource management optimization model is presented in Section 3. In Section 4, the resource management algorithm is given as the solution to the established model. Section 5 presents some numerical simulation results. Finally, in Section 6, the conclusions are given.

2. Problem formulation

Consider a co-located MIMO radar, where the total number of the elements is \( N \). When \( K \) sub-arrays are formed, each sub-array in the co-located MIMO radar contains \( L = N/K \) transmitting elements, as shown in Fig. 1.
be illuminated simultaneously with one wide beam in the co-located MIMO radar. At each probing moment, considering all the beams from $K$ different sub-arrays point at the same direction, the transmitting beam in Fig. 1 can be synthesized finally [26]. When receiving the echo information, the technique of digital beam-forming (DBF) is used to form multiple narrow receiving beams to cover the span that is illuminated by the transmitting beam, as shown in Fig. 1.

When tracking multiple targets, the co-located MIMO radar can use the formed wide beam to simultaneously illuminate multiple targets with a large sub-array number, or illuminate targets in turns by narrow beams with a small sub-array number as in the phased array radar. Therefore, the co-located MIMO radar has a larger flexibility in illuminating targets and a greater freedom degree of system resource management.

Assume that $Q$ targets are under tracking, the co-located MIMO radar resource scheduling aims to effectively save resource consumption while satisfying the desired tracking accuracy. The essence of the co-located MIMO radar resource scheduling is the adaptive control of system working parameters. Specifically, the resource management scheme needs to determine when and how to illuminate the targets, involving the following aspects. (i) When should the radar transmit the probing beam? It means the sampling period should be determined and indicates the resource allocation in the time domain. (ii) How many sub-arrays should the whole array be divided into? It determines the beam width and indicates the resource allocation in the space domain. (iii) How much energy should be consumed? (iv) Which targets should be illuminated?

Assume the latest system update time is $t_k$, the resource management for the co-located MIMO radar should determine the next system update time $t_{k+1}$, the sub-array number $K(t_{k+1})$, the transmitting waveform energy $e(t_{k+1})$, the beam direction $u_s(t_{k+1})$ and the illuminated targets set, where $t_{k+1} = t_k + T_{sys}$. Note that for brevity, the time index $t_{k+1}$ is often omitted below, unless doing so causes confusion. Here, the illuminated targets set is denoted by the working mode $M$. For example, three possible working modes can be obtained when two targets exist in the surveillance region, that is, $M \in \{\{1\},\{2\},\{1,2\}\}$. Specifically, Target 1 will be illuminated when the working mode $M = \{1\}$, Target 2 is illuminated when $M = \{2\}$, and Target 1 as well as Target 2 will be illuminated at the same time when $M = \{1,2\}$.

3. Co-located MIMO radar resource management model based on time-space joint allocation in multi-target tracking

In this section, the target dynamics and measurement model are firstly presented. Then the constraints and objective function of the proposed resource management optimization model are established.

3.1 Target dynamics and measurement model

In target tracking, we consider the target’s maneuverability, and adopt the interacting multiple model (IMM) algorithm to implement tracking [27 – 30]. Assume the total number of models in an IMM filter is $J$, and the dynamic motion of the $j$th model [31] is prescribed as

$$x_i^{(j)}(t_k) = F_i^{(j)}(T_{k-1})x_i^{(j)}(t_{k-1}) +$$

$$\Gamma_i^{(j)}(T_{k-1})w_i^{(j)}(t_{k-1}), \quad j = 1, 2, \ldots, J$$ (1)

where $x_i^{(j)}(t_k) = [x_i^{(j)}(t_k), \dot{x}_i^{(j)}(t_k), \ddot{x}_i^{(j)}(t_k), y_i^{(j)}(t_k), \dot{y}_i^{(j)}(t_k), \ddot{y}_i^{(j)}(t_k)]^T$ is the target state. Here, $[x_i^{(j)}(t_k), y_i^{(j)}(t_k)]$, $[\dot{x}_i^{(j)}(t_k), \dot{y}_i^{(j)}(t_k)]$ and $[\ddot{x}_i^{(j)}(t_k), \ddot{y}_i^{(j)}(t_k)]$ denote the position, velocity and acceleration of the $i$th target in the Cartesian coordinates, respectively. $F_i^{(j)}(T_{k-1})$ denotes the $j$th model’s transition matrix, and $T_{k-1} = t_k - t_{k-1}$. $w_i^{(j)}(t_{k-1})$ is the $j$th model’s process noise, and it obeys the zero-mean Gaussian distribution with a known covariance $Q_i^{(j)}(t_{k-1}),$ and $\Gamma_i^{(j)}(T_{k-1})$ denotes the $j$th model’s process noise input matrix.

If Target $i$ is illuminated at $t_k$, its range and bearing measurements can be obtained, which can be described by

$$z_i(t_k) = h(x_i(t_k)) + v_i(t_k)$$ (2)

where $h(x_i(t_k)) = [r_i(t_k), b_i(t_k)]^T$ is the nonlinear range and bearing measurement function. $v_i(t_k)$ denotes the measurement error, which is assumed to be zero-mean Gaussian distributed. The measurement error covariance is related to the range resolution and the echo signal-to-noise ratio (SNR), which will be given later.

3.2 Constraint conditions

In multi-target tracking, at first, the targets that need updating ought to be illuminated and detected. To be more specific, the illumination means that the transmitting beam can cover the targets. As shown in Fig. 2, in order to achieve successful illumination of Target $i$, the predicted position of the target $u_{i, pre}$ should fall into the range determined by the beam direction $u_s$ and the beam width $\phi(K)$. Therefore, the successful illumination constraint is described as

$$u_s - \phi(K)/2 < u_{i, pre} < u_s + \phi(K)/2, \quad \forall i \in M$$ (3)

where $i \in M$ means Target $i$ is under tracking. The transmitting beam width $\phi(K)$ can be calculated as follows [32]:

$$\phi(K) = \frac{1.76K}{N}.$$ (4)
Besides successful illumination, in order to detect the illuminated target, the detection probability of the target should exceed a given threshold. Thus, the successful detection constraint is

$$P_{d_i}(K, T_{sys}, e, M, u_s) \geq P_{d_{th}}$$  

where $P_{d_i}(K, T_{sys}, e, M, u_s)$ is the detection probability of Target $i$ when the sub-array number is $K$, the system sampling period is $T_{sys}$, the transmitting waveform energy is $e$, the working mode is $M$ and the beam direction is $u_s$. Besides, the term $P_{d_{th}}$ represents the detection probability threshold. The detection probability has a relationship with SNR, when the target’s radar cross section (RCS) is Swerling I distributed, and the detection probability of it [33] is

$$P_{d_i}(K, T_{sys}, e, M, u_s) = P_{In} \left[ \frac{1}{\ln(2)} (\frac{SNR_i(K, T_{sys}, e, M, u_s)}{1+SNR_i(K, T_{sys}, e, M, u_s)}) \right]$$  

where $SNR_i(K, T_{sys}, e, M, u_s)$ is the SNR of Target $i$ when the working parameters combination is $(K, T_{sys}, e, M, u_s)$, which can be calculated as follows [18,34]:

$$SNR_i(K, T_{sys}, e, M, u_s) = \frac{N^3 \varepsilon (\pi \eta_e)^2 \lambda^2 G_{i0}^{u_i,K}}{(4\pi)^2 K(R_i(T_{sys}))^2 N_0}$$  

where $\eta_e$ is the aperture efficiency, $\lambda$ is the wavelength, $N_0$ is the power spectrum density of radar receiver noise, $\sigma_i$ is the RCS of Target $i$, $R_i(T_{sys})$ that is related to the system sampling period is the range of Target $i$, $G_{i0}^{u_i,K}$ is the gain pattern of radar that can be calculated as follows:

$$G_{i0}^{u_i,K} = \left[ \exp \left( \frac{(u_{i,pre} - u_s)}{(\phi(K))^2} \right)^2 \right]$$  

where $c_0$ is a constant, namely $c_0 = -2 \ln 2$. As in (7), the RCS and range of the target are unknown, the RCS is estimated with an $\alpha$ filter and the predicted range is obtained according to the predicted target state.

$$P_i(K, T_{sys}, e, M, u_s) = \sum_{j=1}^{J} \mu_i^{(j)} (t^-_{k+1}) \{P_i^{pre}(t_{k+1}) + [\hat{x}_i(t^-_{k+1}) - \hat{x}_i(t^-_{k+1})][\hat{x}_i(t^-_{k+1}) - \hat{x}_i(t^-_{k+1})]^H \}$$  

### 3.3 Objective function

Assume that the desired tracking accuracy is described by the desired tracking error covariances of $Q$ targets, which is denoted by $P^\text{desire} = \{P_i^\text{desire}, i = 1, 2, \ldots, Q\}$, where $P_i^\text{desire}$ denotes the desired tracking accuracy of Target $i$. As mentioned in Section 2, the purpose of the co-located MIMO radar resource scheduling is how to effectively save the consumption of radar resource while satisfying the desired tracking accuracy. The system resource consumed by target tracking includes the time resource that can be described by $T_{sys}$ and the energy resource that can be reflected by $e$. The resource consumption and tracking accuracy requirements are considered comprehensively here, and the following objective function is formulated as

$$F(K, T_{sys}, e, M, u_s) = \alpha \frac{e}{e_{max}} + \beta \frac{1}{T_{sys}} + \gamma \frac{F(P^\text{desire}, P(K, T_{sys}, e, M, u_s))}{F(P^\text{desire}, 0)}$$  

In (9), since three terms have different physical meanings, the normalized resource consumption and the tracking accuracy offset are adopted here to formulate the objective function, where the first term denotes the normalized energy resource consumption, the second term represents the normalized time resource consumption and the last term describes the tracking accuracy offset. In the objective function, $0 = \{0, i = 1, 2, \ldots, Q\}$, where $0_i$ is the zero matrix with the same dimension of $P^\text{desire}$. $F(X, Y)$ is a function that measures the difference between two groups of matrices $X$ and $Y$, which can be calculated as

$$F(X, Y) = \frac{1}{Q} \sum_{i=1}^{Q} f(X_i, Y_i)$$  

where the term $f(X_i, Y_i)$ represents the matrix difference between $X_i$ and $Y_i$ [21]. In the objective function (9), the adjustable coefficients $\alpha$, $\beta$ and $\gamma$ represent the emphasis of the system on the energy consumption, time resource consumption and tracking accuracy offset respectively, which meet $\alpha + \beta + \gamma = 1$.

In order to calculate the objective function in (9), the predicted tracking error covariances of $Q$ targets are represented by
\( \mathbf{P}(K, T_{\text{sys}}, e, M, u_s) = \{ \mathbf{P}_i(K, T_{\text{sys}}, e, M, u_s), i = 1, 2, \ldots, Q \} \) should be obtained, where the element \( \mathbf{P}_i(K, T_{\text{sys}}, e, M, u_s) \) denotes the predicted tracking error covariance of Target \( i \). In the framework of IMM, the predicted tracking error covariance of Target \( i \) can be obtained in (11), where \( J \) is the model number in the IMM filter, the symbol \([\cdot]^{\dagger}\) denotes conjugate transpose operation, and \( \mu_i^{(j)}(t_{k+1}) \) denotes the \( j \)th model's predicted probability. The term \( \bar{\mathbf{x}}_i(t_{k+1}) \) in (11) denotes the predicted state vector of Target \( i \) from the model \( j \), and \( \bar{\mathbf{x}}_i(t_{k+1}) \) denotes the combined predicted state vector of Target \( i \), such that

\[
\bar{\mathbf{x}}_i(t_{k+1}) = \sum_{j=1}^{J} \mu_i^{(j)}(t_{k+1}) \mathbf{x}_i^{(j)}(t_{k+1}),
\]

(12)

\[
P_i^{(j)}(t_{k+1}^{\text{pre}}) = \begin{cases} 
I - K_i(K, T_{\text{sys}}, e, M, u_s) \mathbf{H} & , i \in M, \\
\mathbf{P}_i^{(j)}(t_{k+1}), & i \notin M
\end{cases}
\]

(13)

For the targets in the working mode \( M \), namely the targets that need updating, the term \( \mathbf{P}_i^{(j)}(t_{k+1}^{\text{pre}}) \) in (11) represents the predicted tracking error covariance of Target \( i \) from the model \( j \). For other targets, which are not to be illuminated, \( \mathbf{P}_i^{(j)}(t_{k+1}) \) denotes the predicted error covariance of Target \( i \) from the model \( j \). Therefore, \( \mathbf{P}_i^{(j)}(t_{k+1}) \) can be calculated as (13). In (13), \( \mathbf{I} \) denotes the unit matrix, and \( \mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \). The term \( K_i(K, T_{\text{sys}}, e, M, u_s) \) in (13) is the Kalman gain matrix of Target \( i \), which can be expressed as

\[
K_i(K, T_{\text{sys}}, e, M, u_s) = \mathbf{P}_i^{(j)}(t_{k+1})^{\dagger} (\mathbf{H} \mathbf{P}_i^{(j)}(t_{k+1}))^{\dagger} + \mathbf{R}_i(K, T_{\text{sys}}, e, M, u_s)^{-1}
\]

(14)

where the symbol \([\cdot]^{\dagger}\) denotes transpose operation, and \( \mathbf{R}_i(K, T_{\text{sys}}, e, M, u_s) \) is the measurement error covariance matrix that can be calculated as

\[
\mathbf{R}_i(K, T_{\text{sys}}, e, M, u_s) = \mathbf{J}_i \begin{bmatrix} \sigma_{r,i}^2(K, T_{\text{sys}}, e, M, u_s) & 0 \\
0 & \sigma_{b,i}^2(K, T_{\text{sys}}, e, M, u_s) \end{bmatrix} \mathbf{J}_i^{\dagger}
\]

(15)

where the elements \( \sigma_{r,i}^2(K, T_{\text{sys}}, e, M, u_s) \) and \( \sigma_{b,i}^2(K, T_{\text{sys}}, e, M, u_s) \) are the variances of the range measurement error and the azimuth measurement error of Target \( i \), respectively [35]. To be more specific, \( \sigma_{r,i}(K, T_{\text{sys}}, e, M, u_s) \) and \( \sigma_{b,i}(K, T_{\text{sys}}, e, M, u_s) \) can be calculated as

\[
\sigma_{r,i}(K, T_{\text{sys}}, e, M, u_s) = \frac{\Delta r(K, T_{\text{sys}}, e, M, u_s)}{\sqrt{12}},
\]

(16)

\[
\sigma_{b,i}(K, T_{\text{sys}}, e, M, u_s) = \frac{B_w}{c \cdot \sqrt{2\text{SNR}_i(K, T_{\text{sys}}, e, M, u_s)}}
\]

(17)

where the term \( \Delta r(K, T_{\text{sys}}, e, M, u_s) \) is the transmitting waveform's range resolution, \( B_w \) denotes the round-trip beam width, \( \text{SNR}_i(K, T_{\text{sys}}, e, M, u_s) \) can be obtained according to (7), and \( c \) denotes a constant. The term \( \mathbf{J}_i \) in (15) denotes the Jacobian transform matrix from polar coordinates to Cartesian coordinates [31]:

\[
\mathbf{J}_i = \begin{bmatrix} \cos b_i(t_{k+1}^-) & -r_i(t_{k+1}^-) \sin b_i(t_{k+1}^-) \\
\sin b_i(t_{k+1}^-) & r_i(t_{k+1}^-) \cos b_i(t_{k+1}^-) \end{bmatrix}
\]

(18)

where \( r_i(t_{k+1}^-) \) and \( b_i(t_{k+1}^-) \) are the predicted range and the bearing of Target \( i \). The term \( \mathbf{P}_i^{(j)}(t_{k+1}) \) in (13) and (14) denotes the predicted error covariance of Target \( i \) from the model \( j \), which can be calculated as

\[
\mathbf{P}_i^{(j)}(t_{k+1}) = \mathbf{F}_i^{(j)}(T_{i,k}) \mathbf{P}_i^{(j)}(t_{k}) (\mathbf{F}_i^{(j)}(T_{i,k}))^{\dagger} + \mathbf{G}_i^{(j)}(T_{i,k}) \mathbf{Q}^{(j)}(t_{k}) (\mathbf{P}_i^{(j)}(T_{i,k}))^{\dagger}
\]

(19)

where \( \mathbf{F}_i^{(j)}(t_{k}) \) is the tracking error covariance of Target \( i \) from the model \( j \) at \( t_k \).

In summary, combining the proposed objective function in (9) and the constraints in (3) and (5), the co-located MIMO radar resource management optimization model can be established as

\[
\min_{K, T_{\text{sys}}, e, M, u_s} \mathbb{E}(K, T_{\text{sys}}, e, M, u_s)
\]

s.t. \( \{ u_s - \phi(K)/2 < \mathbf{u}_{i,\text{pre}} < u_s + \phi(K)/2, \quad \mathbf{P}_{\mathbf{d}_{i}}(K, T_{\text{sys}}, e, M, u_s) \geq \mathbf{P}_{\mathbf{d}_{\text{th}}}, \quad \forall i \in M \} \)

(20)

Through solving the optimization model in (20), the optimal working parameters combination \( (K_{\text{opt}}, T_{\text{sysopt}}, e_{\text{opt}}, M_{\text{opt}}, u_{s}) \) will be obtained in the sense of minimizing the proposed objective function. In (20), the parameters \( K \) and \( u_s \) reflect the resource allocation in the space domain, and the parameter \( T_{\text{sys}} \) reflects the resource allocation in the time domain. It can be seen that the time-space joint allocation of system resources is realized by controlling the working parameters of the co-located MIMO radar.

4. Adaptive resource management algorithm based on time-space joint allocation

As shown in (20), the resource management optimiza-
tion problem involves five working parameters of the co-
located MIMO radar, namely the sub-array number, the system sampling period, the transmitting waveform energy, the beam direction and the working mode.

For the array that has \( N \) elements, which is always the integer power of 2, the possible sub-array number can form the set \( K_{\text{set}} = \{1, 2, \ldots, 2^{i-1}, \ldots, N\} \) \((i = 1, 2, \ldots, \log_2 N + 1)\), whose size is denoted as \( N_K \). For the sampling period, there is usually a maximum value \( T_{\text{max}} \) and a minimum value \( T_{\text{min}} \), so the possible set \( (T_{\text{sys}})_{\text{set}} \), whose size is \( N_{T_{\text{sys}}} \), can be obtained by discretization of the interval \([T_{\text{min}}, T_{\text{max}}]\). When \( Q \) targets are under tracking, the possible working mode can be chosen from the set \( \{\{1\}, \{2\}, \ldots, \{Q\}, \{1, 2\}, \ldots, \{Q-1, Q\}, \ldots, \{1, 2, \ldots, Q\}\} \), whose size is \( N_M = C_Q^1 + C_Q^2 + \cdots + C_Q^{Q-1} + C_Q^Q \). Consider there are different waveforms in the predefined waveform library with total \( N_C \) kinds of energies. At each update time, the corresponding energy in \( c_{\text{set}} \) should be selected to meet the desired tracking accuracy requirements.

Assume at instant \( t_k \), after updating, the states information of \( Q \) targets can be denoted as \( \{t_{k(i)}, \tilde{x}_i(t_{k(i)}), P(t_{k(i)})\} \) \((i = 1, 2, \ldots, Q)\), where \( t_{k(i)} \) is the latest update time for Target \( i \). The next system update time \( t_{k+1} \) and the corresponding working parameters \( (K(t_{k+1}), T_{\text{sys}}, e(t_{k+1}), M(t_{k+1}), u_s(t_{k+1})) \) are determined by the adaptive resource management algorithm based on time-space joint allocation as follows:

\[
F(P_{\text{desire}}, P(K, T_{\text{sys}}, e, M, u_s)) = \\
\frac{1}{Q} \sum_{i=1}^{Q} (\text{abs}(P_{i,11}^{\text{desire}} - P_{i,11}(K, T_{\text{sys}}, e, M, u_s)) + \text{abs}(P_{4,44}^{\text{desire}} - P_{4,44}(K, T_{\text{sys}}, e, M, u_s))). \tag{21}
\]

In the objective function in (9), the third term is calculated as (21), where \( \text{abs}(\cdot) \) denotes the operation of calculating the absolute value, \( P_{i,11}^{\text{desire}} \) and \( P_{i,11}(K, T_{\text{sys}}, e, M, u_s) \) are the first diagonal elements in the matrix \( P_{\text{desire}} \) and \( P_i(K, T_{\text{sys}}, e, M, u_s) \) respectively, \( P_{4,44}^{\text{desire}} \) and \( P_{4,44}(K, T_{\text{sys}}, e, M, u_s) \) are the fourth diagonal elements in the matrices \( P_{\text{desire}} \) and \( P_i(K, T_{\text{sys}}, e, M, u_s) \) respectively.

**Step 1** For each possible \((K_j, M_j) \) \((j = 1, 2, \ldots, N_K \times N_M)\), initialize the population in the GA, where the size of it is \( N_P \) and the generation number is \( G \), and the coding process is as follows:

(i) For \( T_{\text{sys}} \), take a random value that is denoted as \( \lambda_t \) on the interval \([0,1]\), then divide the interval into \( N_{T_{\text{sys}}} \) parts. In these \( N_{T_{\text{sys}}} \) parts, find out the index \( \varsigma \) of the sub-interval where \( \lambda_t \) is, and finally code \( T_{\text{sys}} = (T_{\text{sys}})_{\text{set}}(\varsigma) \).

(ii) For \( e \), a similar coding method as the one of \( T_{\text{sys}} \) is used to obtain the index \( \eta \), and code \( e \) as \( c_{\text{set}}(\eta) \).

(iii) To determine \( u_s \), at first, \( u_{\text{set}} \) should be determined based on the obtained \( T_{\text{sys}} \). Specifically, calculate the predicted positions of all targets in \( M_j \) according to \( T_{\text{sys}} \), and the minimum azimuth and maximum azimuth of these predicted positions are formed into an interval \( \mathcal{I} \). Thus, \( u_{\text{set}} \) is obtained by discretization of \( \mathcal{I} \). Then, a similar coding method as the one of \( T_{\text{sys}} \) is used to obtain the index \( \varphi \), and code \( u_s \) as \( u_{\text{set}}(\varphi) \).

**Step 2** Perform the selection and crossover operation according to the elite strategy [36,37] based on the fitness, where the fitness is calculated as follows.

(i) For each individual that meets the constraints in (20), calculate its fitness \( F = -F(K, T_{\text{sys}}, e, M, u_s) \) according to (9).

(ii) For each individual that does not meet the constraints in (20), add the penalty factor \( \xi_p \) to its fitness \( F = -F(K, T_{\text{sys}}, e, M, u_s) \) that is calculated according to (9), where \( \xi_p \) is a large negative constant.

**Step 3** Perform the mutation operation, where the real mutation is chosen as the mutation operator. Calculate the fitness value of each individual in the sub-population with the similar method in Step 2. Then the sub-population and the parent population are merged. Rank the individuals in the merged population according to their values of fitness from high to low, and select the first \( N_P \) individuals as the new population.

**Step 4** Repeat Step 2 and Step 3 till the number of iterations reaches the threshold \( G \) or the change of optimal fitness between generations is smaller than \( \vartheta_d \), where \( \vartheta_d \) is a very small positive constant. The searching process ends, and the optimal \((e, T_{\text{sys}}, u_s)_{\text{opt}}\) under the \((K_j, M_j) \) \((j = 1, 2, \ldots, N_K \times N_M)\) combination is obtained.

**Step 5** For each working parameters combination \((K_j, M_j, (e, T_{\text{sys}}, u_s)_{\text{opt}}) \) \((j = 1, 2, \ldots, N_K \times N_M)\), calculate its value of objective function according to (9). Choose the working parameters combination, whose objective function value is the minimum, as the optimal combination, and denote it as \((K_{\text{opt}}, M_{\text{opt}}, e_{\text{opt}}, (T_{\text{sys}})_{\text{opt}}, (u_s)_{\text{opt}})\). According to the obtained optimal working parameters combination, we can obtain the next system update time \( t_{k+1} = t_k + (T_{\text{sys}})_{\text{opt}} \) and the corresponding working parameters \((K(t_{k+1}), M(t_{k+1}), e(t_{k+1}), T_{\text{sys}}, u(t_{k+1}))\)
\( u_s(t_{k+1}) = (K_{opt}, M_{opt}, e_{opt}, (T_{sys})_{opt}, (u_s)_{opt}) \).

**Step 6** Illuminate with the optimal working parameters, that is, the working parameters \((K(t_{k+1}), M(t_{k+1}), e(t_{k+1}), T_{sys}, u_s(t_{k+1})) \) at \( t_{k+1} \). Update the state of Target \( i \) \((i \in M(t_{k+1}))\), whereby the states information \((\hat{x}_i(t_{k+1}), P_i(t_{k+1})) \) \((i \in M(t_{k+1}))\) is obtained.

**Step 7** Repeat Steps 1–6 above till the simulation time is over.

The corresponding flow chart of the proposed algorithm is shown in Fig. 3.

5. Numerical simulation results

Consider the scenario with three targets. Target 1, whose initial location is \((122.5, 122) \) km, moves from 0 s to 150 s and maneuvers from 90 s to 110 s. Target 2, whose initial location is \((123, 127) \) km, moves from 20 s to 150 s with the speed of \([110, 0]\) m/s. Target 3, whose initial location is \((123, 122.1) \) km, moves with the constant velocity of \([30, 30]\) m/s from 20 s to 150 s. They are assumed to be Swerling I targets with the average RCS of \(1 \) m².

The co-located MIMO radar that is located at \((0, 0)\) km has a linear array with totally 4096 elements and the distance between adjacent elements is half wavelength, where the wavelength is \(\lambda = 0.0545 \) m. The range resolution of the tracking waveforms is \(\Delta r = 22.5 \) m, the working frequency is 10 GHz, \(P_{th} = 10^{-9}\), \(P_{sh} = 0.95\). In the optimization model, \(\alpha = 0.05\), \(\beta = 0.05\) and \(\gamma = 0.9\). In the GA, \(N_P = 100\), \(G = 100\), the crossover probability is \(P_c = 0.9\), and the mutation probability is \(P_m = 0.05\).

The desired tracking accuracy is described by the tracking error variances in \(x\) and \(y\) positions. For all the three targets, the desired tracking accuracy in \(x\) and \(y\) positions are assumed to be \(30 \) m², namely \(P_{desire} = 30\) \((i = 1, 2, 3)\). The possible working mode can be chosen from the set \(M_{set} = \{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}\), where the meanings of the elements in \(M_{set}\) are shown in Table 1. Note that the element in \(M_{set}\) is simplified as the corresponding number for convenience, as shown in Table 1. In addition, the possible system sampling period set is \((T_{sys})_{set} = \{0.1, 0.3, 0.5, 1, 1.3, 1.7, 2, 2.3, 2.5, 3\} \) s, the possible set of the transmitting waveform energy is \(e_{set} = \{1.35, 2.25, 4.05, 5.85, 11.7, 23.4, 2\}\) J, and the possible

Fig. 3 Flow chart of the proposed algorithm
The adaptive tracking of these three targets is realized by the proposed resource management algorithm for the co-located MIMO radar based on time-space joint allocation, which determines the optimal working mode, the transmitting waveform energy, the sub-array number, the system sampling period and the beam direction. The changing of them in one Monte Carlo run are shown in Figs. 4 – 8, respectively.

It can be seen that the co-located MIMO radar can adaptively control its working parameters. Specifically, before 20 s, the working mode keeps to be one, the transmitting waveform energy remains low, the sub-array number is small, the system sampling period is large, and the transmitting beam illuminates Target 1. The reason is that

| Number | Element | Physical meaning |
|--------|---------|------------------|
| 1      | {1}     | Track Target 1   |
| 2      | {2}     | Track Target 2   |
| 3      | {3}     | Track Target 3   |
| 4      | {1,2}   | Track Target 1 and Target 2 |
| 5      | {1,3}   | Track Target 1 and Target 3 |
| 6      | {2,3}   | Track Target 2 and Target 3 |
| 7      | {1,2,3} | Track Target 1, Target 2 and Target 3 |

The sub-array number set is $K_{set} = \{1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096\}$.

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only Target 1 exists in the airspace before 20 s, the narrow transmitting beam can cover Target 1, so the sub-array number is small. When the sub-array number is small, the transmitting gain is high, so the transmitting energy keeps low. Moreover, Target 1 does not maneuver before 20 s, and therefore the system sampling period can be relatively large.

Once multiple targets appear after 20 s, the sub-array number and the transmitting waveform energy increase, the system sampling period decreases, and the transmitting beam tends to illuminate three or two targets simultaneously. Specifically, combine Fig. 4, Fig. 6 and Fig. 8, we can see the sub-array number becomes larger than one at most of the time after 20 s, which makes the probing beam wide enough to simultaneously illuminate multiple targets. Combine Fig. 5 and Fig. 7, it can be seen that a large transmitting waveform energy and a small system sampling period are chosen during the time period of 20 – 150 s. This is because more system resources are consumed to obtain enough SNR in order to meet the desired tracking accuracy.

In order to investigate the performance of the proposed algorithm, the average results of 100 Monte Carlo runs are also given. The average sampling period of the system is given in Fig. 9, it can be seen that the system sampling period decreases at 20 s, which means that more time resource is consumed. The reason is that the overall tracking performance is poor when Target 2 and Target 3 begin to exist in the surveillance region. Herein, the algorithm will order the radar to allocate more system resources to Target 2 and Target 3, namely Target 2 and Target 3 will be illuminated frequently by the radar to improve the overall tracking performance.

In addition, the system sampling period also decreases during the time period of 90 – 110 s, the reason is that Target 1 maneuvers during this period. In order to compensate for the loss of tracking performance due to the maneuvering motion of Target 1, the algorithm will drive the radar to allocate more system resources to Target 1. Specifically, Target 1 is illuminated by the co-located MIMO radar more frequently, leading to the decrease of system sampling period during this time period. Fig. 10 shows the average transmitting waveform energy, it can be seen that the waveform energy increases during the time period of 25 – 150 s. This is because multiple targets are tracked at the same time during this time period. In order to satisfy the tracking accuracy requirements, more energy is required to obtain enough SNR.

The estimated tracking accuracy in the $x$ position of Target 1 is shown in Fig. 11.
the actual tracking accuracy of Target 2 is also close to the desired tracking accuracy, that is, the desired tracking accuracy of Target 2 is ensured by the proposed algorithm too. The estimated tracking accuracy in the $x$ position of Target 3 is similar with the one of Target 2, whereby the figure is not given repeatedly. Besides, the estimated tracking accuracies of three targets in the $y$ position are similar with the ones in the $x$ position, which are not shown repeatedly either.

As this paper is an extension of [18], the proposed resource management algorithm is compared with the one in [18]. Furthermore, to show the advantage of MIMO compared with the phased array radar, the resource management algorithm for the phased array radar in [38] is also compared, where the simulation scene remains the same. The comparison of the comprehensive cost in (9) among the proposed resource management algorithm based on time-space joint allocation, the algorithm in [18] and the algorithm in [38] are given in Fig. 13. It can be seen that the comprehensive cost of the proposed algorithm is lower than that of the other two existing algorithms during the target tracking process. The reason is that the system resource allocation in the time domain was not considered both in [18] and [38], and the sub-array number was fixed at 1 in [38] at the same time. However, the time-space joint allocation of system resource is considered comprehensively in the proposed algorithm. In addition, the sub-array number can be changed flexibly, which also contributes to the improvement of the comprehensive cost.

The average running time of the proposed algorithm is also investigated, which can be calculated as

$$\tau_{\text{sim}} = \frac{1}{N_{MC}} \sum_{n=1}^{N_{MC}} \frac{1}{M_n} t^n$$

where $N_{MC}$ is the number of Monte Carlo trials, $M_n$ and $t^n$ are the sampling times and the running time of the $n$th Monte Carlo trial, respectively. When the GPU memory is 8 G, the test result of running time is 4.011 0 s. In our future work, the optimization of the proposed algorithm will be further considered to improve its efficiency.

![Fig. 12 Tracking accuracy in $x$ position of Target 2](image)

![Fig. 13 Cost comparison among the proposed algorithm, the one in [18] and the one in [38]](image)

**6. Conclusions**

For the co-located MIMO radar in multi-target tracking, how to minimize the total resource consumption while satisfying the tracking accuracy requirements is of great significance. In this paper, the co-located MIMO radar resource management optimization model in multi-target tracking is established. Through solving the optimization problem, an adaptive resource management algorithm based on time-space joint allocation for the co-located MIMO radar is proposed. The sub-array number, the system sampling period, the transmitting waveform energy, the beam direction and the working mode are controlled jointly. Numerical simulation results demonstrate the effectiveness of the proposed algorithm. Using the proposed algorithm, the co-located MIMO radar can satisfy the predetermined tracking accuracy requirements with less comprehensive cost compared with existing algorithms.

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