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Fractal–fractional operator for COVID-19 (Omicron) variant outbreak with analysis and modeling

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Abstract

The fractal–fraction derivative is an advanced category of fractional derivative. It has several approaches to real-world issues. This work focuses on the investigation of the second wave of the Coronavirus in India. We develop a time-fractional order COVID-19 model with effects of disease which consists of system of fractional differential equations. Fractional order COVID-19 model is investigated with fractal–fractional technique. Also, the deterministic mathematical model for the Omicron effect is investigated with different fractional parameters. Fractional order system is analyzed qualitatively as well as verify sensitivity analysis. The existence and uniqueness of the fractional-order model are derived using fixed point theory. Also, the solutions are bounded for the new wave omicron. Solutions are derived to investigate the influence of fractional operator which shows the impact of the disease on society. Simulation has been made to understand the actual behavior of the OMICRON virus. Such kind of analysis will help to understand the behavior of the virus and control strategies to overcome the dissease in community.

Introduction

Currently, Atangana [1] has established a new plan for the fractal–fractional derivative of Fractional Calculus. The idea on this topic is very appropriate in many circumstances for dealing with some complicated issues. The operator has two orders, one is called fractional-order and the other is called fractal dimension. This new idea of deriving fractal fractions is preferable compared to the classic. This is because fractal operators and fractal dimensions can deal with simultaneous research can be done by dealing with fractal–fractional derivatives. The immense advantage of this operator is that you can create a model that better describes a system with a memory effect. In addition, there are many problems in the real world where you need to know how much information a system carries. Ali et al. [2] studied a SIR model to assess the dynamics of COVID 19 using fractals Fraction operator. By using power-law kernels and new applications, Akgul [3] described some advances in fractal–fractional differential equations.

Coronavirus is a virus. There are many species, some of which can cause disease. Coronavirus SARSCoV2, discovered in 2019, has caused a pandemic of a respiratory disease called COVID-19. It has caused millions of deaths worldwide. The biggest challenge in overcoming the ongoing COVID 19 pandemic is understanding its complex dynamics. There are several vaccines and medicines to control, but it is not yet known how these vaccines can entirely remove the virus. Further, in the second wave of COVID-19, the Omicron variant has been discovered and still, cases are increasing. So, proper and timely execution of precautionary measures can significantly reduce the number of newly infected cases worldwide [4–6].

From [7–11], we can deduce that these mathematical models were examined to be constructive in solving scientific and technological problems. In the above-mentioned works, the focal point of the researchers is on looking at coronavirus infections from several views and using fractional operators. Mathematical modeling of scientific and engineering problems using fractional operators also takes into account the attention of researchers around the world. Some months later, after the upsurge of the pandemic, several models for studying the
dynamics of this infection in different parts of the world can be found in the literature [12–16]. [17] and [18] examined mask and lockdown responses to COVID 19 disease dynamics, respectively individually. Few studies on COVID 19 dynamics from different stances can be found in [19–23]. A fractional order pandemic model is developed to examine the spread of COVID19 with and without Omicron variant and its relationship with heart attack using real data from the United Kingdom [24]. The interactions between COVID-19 and diabetes are investigated using real data from Turkey [25]. Implementation of a novel numerical method for solving differential equations with fractional variable-order in the Caputo sense to research the dynamics of a circulant halflifed system [26]. The fitting of parameters through least squares curve fitting technique is performed, and the average absolute relative error between COVID-19 actual cases and the model’s solution for the infectious class is tried to be reduced and the best fitted values of the relevant parameters are achieved [27].

In this effort, Introduction and historical background is discussed in section ‘Introduction’. Section ‘Primary notions’, some basic definition are provided regarding proposed techniques. Qualitative and sensitivity analysis has been made for fractional order system in Section ‘Fractional order COVID-19 model’. Existence and uniqueness are verified in Section ‘Existence and Uniqueness of the Model’. The advance technique Fractal–fractional is applied in Section ‘Fractal–fractional operator’ and simulation explained in Section ‘Simulation and discussion’. Conclude work in Section ‘Conclusion’ to analyze the actual behavior of omicron and control strategy for the model of fractional order by applying the fractal–fractional operator.

Primary notions

In this part, we present several primary notions of the fractal–fraction that are supportive to analyze the system.

Definition 1 ([5,28,29]). For power law kernel (FFP), let 0 ≤ ϕ, θ ≤ 1, thus \( B(t) \) for fractal–fractional operator is defined in the Riemann–Liouville as:

\[
\frac{d^\alpha}{dt^\alpha} B(t) = \frac{1}{\Gamma(\alpha - \varphi)} \int_0^t (t - \varphi)^{\alpha - \varphi - 1} B(\varphi) d\varphi, \tag{1}
\]

where \( \varphi > 0, \theta \leq n \in N, \) and \( N(0) = 1 = N(1). \)

Definition 2 ([5,28,29]). For exponentially decaying kernel (FFE), let 0 ≤ ϕ, θ ≤ 1, thus \( B(t) \) for fractal–fractional operator in the Riemann–Liouville is given as:

\[
\frac{d^\alpha}{dt^\alpha} B(t) = \frac{N(\varphi)}{\Gamma(\alpha - \varphi)} \int_0^t \exp[-\varphi/(1 - \varphi)] (t - \varphi)^{\alpha - \varphi - 1} B(\varphi) d\varphi, \tag{3}
\]

where \( \varphi > 0, \theta \leq n \in N, \) and \( N(0) = 1 = N(1). \)

Definition 3 ([5,28,29]). For generalized Mittag-Leffler kernel (FFM), let 0 ≤ ϕ, θ ≤ 1, thus \( B(t) \) for fractal–fractional operator in the Riemann–Liouville is defined as:

\[
\frac{d^\alpha}{dt^\alpha} B(t) = \frac{C(\alpha)}{\Gamma(\alpha - \varphi)} \int_0^t (t - \varphi)^{\alpha - \varphi - 1} \exp[-\varphi/(1 - \varphi)] B(\varphi) d\varphi, \tag{4}
\]

where \( 0 < \varphi, \theta \leq 1 \) and \( C(\alpha) = 1 - \alpha + \frac{\alpha}{\Gamma(\alpha)} \).

Definition 4 ([5,28,29]). The FFP of \( B(t) \) with order \( (\alpha, \theta) \) and having power law type kernel is described as:

\[
\frac{d^\alpha}{dt^\alpha} B(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t - \varphi)^{\alpha - 1} |\varphi^\theta B(\varphi) d\varphi. \tag{5}
\]

Definition 5 ([5,28,29]). The FFP of \( B(t) \) with order \( (\alpha, \theta) \) and having exponentially decaying type kernel is described as:

\[
\frac{d^\alpha}{dt^\alpha} B(t) = \frac{\Theta(1 - \varphi^\theta - 1) B(t)}{N(\varphi)} + \frac{\alpha \Theta}{\Gamma(\alpha)} \int_0^t \varphi^\theta B(\varphi) d\varphi. \tag{6}
\]

Fractional order COVID-19 model

The analysis of different techniques for COVID-19 infectious dynamics is crucial in the presence of several intervention schemes. Given the vital role of intervention techniques, numerous analysts have achieved new epidemic models with several intervention techniques for COVID-19 in a homogeneous host population. The proposed model [30] has a total population of \( N \) that is split into six compartments.

Fractional order COVID-19 model

The involved variables are \( A_K, A_I, A_L, A_J, A_R, A_M \) can be illustrated as susceptible, exposed, infected, asymptotically infected, recovered and reservoir persons. In case of parameters, \( \lambda, \psi \) show birth and death rates. Further, \( \zeta, \zeta_1, \zeta_2 \) represent the coefficient of transmission, multiple, and disease transmission. \( \zeta_1 \) is an infection of asymptotic, parameter \( \zeta_6 \) show incubation, while \( \zeta \) is infected transmission. Moreover, we express the rate of recovery as \( \zeta_5, \zeta_7, \zeta_8 \) persons as \( \zeta_9 \), while asymptotic and reservoir transmission viruses are \( \zeta_{10}, \zeta_{11} \) respectively. For the better interpretation of (8), we rearrange it with the addition of some constants.

\[
\begin{align*}
\frac{dA_K}{dt} &= \lambda A_K - \zeta_1 A_K - \zeta_1 A_K (A_I + \zeta_1 A_I - \zeta_1 A_I), \\
\frac{dA_I}{dt} &= \zeta_0 A_K - \zeta_1 A_K (A_I + \zeta_1 A_I) + \zeta_1 A_I A_J - \zeta_1 A_I A_J, \\
\frac{dA_R}{dt} &= \zeta_0 A_I (A_J + \zeta_1 A_I) - \zeta_1 A_I A_J, \\
\frac{dA_J}{dt} &= \zeta_0 A_I (A_I + \zeta_1 A_I), \\
\frac{dA_M}{dt} &= -\frac{A_I}{\zeta_1 + \zeta_1 A_J} - \zeta_1 A_M.
\end{align*}
\]

with initial condition

\[
\begin{align*}
A_K(0) &= A_K, \quad A_I(0) = A_I, \quad A_J(0) = A_J, \\
A_R(0) &= A_R, \quad A_M(0) = A_M.
\end{align*}
\]

where \( \zeta_{10} = \frac{\zeta_{10}}{\psi}, \zeta_{11} = (1 - \zeta_{15}) \zeta_{15}, \zeta_{15} = \lambda \zeta_{17}, \zeta_{16} = \zeta_{16} + \zeta_1 \) and \( \zeta_{17} = \zeta_9 + \zeta_1 \)

Positivity analysis and equilibria

To illustrate the equilibrium points, put the left-hand side of (9) equal to zero.

\[
\begin{align*}
Q_1(A_K, A_I, A_J, A_R, A_M) &= \frac{\omega}{\zeta_9} 0, 0, 0, 0, 0,
\end{align*}
\]

and

\[
\begin{align*}
Q_2(A_K, A_I, A_J, A_R, A_M) &= 0, 0, 0, 0, 0.
\end{align*}
\]

So we have,

\[
\begin{align*}
A_K^* &= \frac{\zeta_{13} \zeta_{14} \zeta_{16} (\zeta_{14} + \zeta_{15} + \zeta_{11})}{\zeta_{14} \zeta_{15} (\zeta_{14} + \zeta_{15} + \zeta_{11})}, \\
A_I^* &= \frac{\zeta_{13} \zeta_{14} \zeta_{16} (\zeta_{14} + \zeta_{15} + \zeta_{11})}{\zeta_{14} \zeta_{15} (\zeta_{14} + \zeta_{15} + \zeta_{11})},
\end{align*}
\]
In this work,\( R^*_R \) is a positive invariant set.

**Existence and uniqueness of the model**

In this portion, we aim to discover the existence and uniqueness of the model by adopting the technique named, linear growth. Consider the model

\[
\begin{align*}
F_{\tilde{t}}^2A_S &= \epsilon_0 - \epsilon_1 A_S - \epsilon_3 A_S(A_1 + \epsilon_5 A_{L1}) - \epsilon_4 A_S A_M, \\
F_{\tilde{t}}^2A_E &= \epsilon_0 A_S - \epsilon_3 A_S(A_1 + \epsilon_5 A_{L1}) + \epsilon_4 A_S A_M \\
& \quad + \epsilon_1 A_S - \epsilon_3 A_S - \epsilon_4 A_S E, \\
F_{\tilde{t}}^2A_I &= \epsilon_1 A_S - \epsilon_3 A_S A_M - \epsilon_4 A_S A_M, \\
F_{\tilde{t}}^2A_R &= \epsilon_0 A_S \epsilon_1 A_S \epsilon_2 A_S \epsilon_3 A_S \epsilon_4 A_S A_M, \\
F_{\tilde{t}}^2A_M &= \epsilon_0 A_S + \epsilon_1 A_S - \epsilon_3 A_S - \epsilon_4 A_S A_M.
\end{align*}
\]

(11)

For this, we validate the following two conditions:

(1) \(|Z(t, A_S, A_E, A_I, A_R, A_M)|^2 \leq (1 + |A_M|^2)\)

(2) \(|\|Z(t, A_S, A_E, A_I, A_R, A_M)\|_2 \leq \|A_S - A_M\|_{\infty}^2\)

Initially,

\[|Z(t, A_S, A_E, A_I, A_R, A_M)|^2 = |\epsilon_0 - \epsilon_1 A_S - \epsilon_3 A_S(A_1 + \epsilon_5 A_{L1}) - \epsilon_4 A_S A_M|^2 \leq 2\epsilon_0^2 + 2|\epsilon_1 A_S - \epsilon_3 A_S(A_1 + \epsilon_5 A_{L1}) - \epsilon_4 A_S A_M|^2 \leq 2\epsilon_0^2 + 6\epsilon_1^2|A_S|^2 + 6\epsilon_3^2|A_S(A_1 + \epsilon_5 A_{L1}) + \epsilon_4 A_S A_M|^2 \leq 2\epsilon_0^2 + 6\epsilon_1^2|A_S|^2 + 6\epsilon_3^2|A_S|^2 + 6\epsilon_4^2|A_S|^2 + 6\epsilon_4^2|A_M|^2
\]

If \(6\epsilon_3^2|A_S|^2 + 6\epsilon_4^2|A_M|^2 < 1\), we get

\[|Z(t, A_S, A_E, A_I, A_R, A_M)|^2 \leq \mathcal{V}_1(1 + |A_M|^2), \quad \text{where} \quad \mathcal{V}_1 = 2\epsilon_0^2\]

\[|Z(t, A_S, A_E, A_I, A_R, A_M)|^2 \leq \mathcal{V}_2(1 + |A_M|^2), \quad \text{where} \quad \mathcal{V}_2 = 6(\epsilon_0^2 + \epsilon_1^2 + \epsilon_3^2).\]

**Theorem 7.** The outcome of the presented system (9) along with initial conditions is unique and bounded in \(R^*_R\).

**Proof.** For the time interval \((0, \infty)\), we can get the existence and uniqueness of (9). Later, we express \(R^*_R\) is a positive invariant region.

From (9), we get

\[
\begin{align*}
F_{\tilde{t}}^2A_S &= \epsilon_0 \geq 0, \\
F_{\tilde{t}}^2A_E |_{A_M = 0} &= \epsilon_1 A_S - \epsilon_3 A_S(A_1 + \epsilon_5 A_{L1}) \geq 0, \\
F_{\tilde{t}}^2A_I |_{A_M = 0} &= \epsilon_1 A_S - \epsilon_3 A_S \geq 0, \\
F_{\tilde{t}}^2A_R |_{A_M = 0} &= \epsilon_1 A_S - \epsilon_3 A_S \geq 0, \\
F_{\tilde{t}}^2A_M |_{A_S = 0} &= \epsilon_1 A_S - \epsilon_3 A_S \geq 0.
\end{align*}
\]

(10)

If \(A_S(0), A_E(0), A_I(0), A_R(0), A_M(0) \in R^*_R\), then according to (10), the result cannot escape from the hyperplanes. Consequently, the domain \(R^*_R\) is a positive invariant set.
\[ \frac{\gamma_5^2}{\gamma_4^2(A_E^2)} < 1, \text{ we get} \]
\[ |Z_2(t, A_S, A_E, A_I, A_{L_1}, A_{R_1}, A_{R_2}, A_{M_1})|^2 \leq \gamma_3(1 + |A_I|^2), \]
where \( \gamma_3 = 2\gamma_4^2(A_E^2) \).

\[ |Z_3(t, A_S, A_E, A_I, A_{L_1}, A_{R_1}, A_{R_2}, A_{M_1})|^2 = |\mathcal{C}_{15}A_E - \mathcal{C}_{14}A_{L_2}|^2 \]
\[ \leq 2|\mathcal{C}_{14}A_E|^2 + 2|\mathcal{C}_{15}A_{L_2}|^2 \]
\[ \leq 2\mathcal{C}_{14}^2A_E^2 + 2\mathcal{C}_{15}^2A_{L_2}^2 \]
\[ \leq 2\mathcal{C}_{14}^2A_E^2 \left[ 1 + \frac{2\mathcal{C}_{15}^2A_{L_2}^2}{2\mathcal{C}_{14}^2A_E^2} \right] \]
If \( \frac{\gamma_5^2}{\gamma_4^2(A_E^2)} < 1, \text{ we get} \)
\[ |Z_4(t, A_S, A_E, A_I, A_{L_1}, A_{R_1}, A_{R_2}, A_{M_1})|^2 \leq \gamma_4(1 + |A_I|^2), \]
where \( \gamma_4 = 2\gamma_5^2(A_E^2) \).

\[ |Z_5(t, A_S, A_E, A_I, A_{L_1}, A_{R_1}, A_{R_2}, A_{M_1})|^2 = |\mathcal{C}_{13}A_I + \mathcal{C}_{15}A_{L_2} - \mathcal{C}_{14}A_{R_1}|^2 \]
\[ \leq 2|\mathcal{C}_{13}A_I|^2 + 2|\mathcal{C}_{15}A_{L_2}|^2 + 2|\mathcal{C}_{14}A_{R_1}|^2 \]
\[ \leq 4\mathcal{C}_{13}^2A_I^2 + 4\mathcal{C}_{15}^2A_{L_2}^2 + 2\mathcal{C}_{14}^2A_{R_1}^2 \]
\[ \leq 4\mathcal{C}_{13}^2A_I^2 + \mathcal{C}_{15}^2A_{L_2}^2 \]
\[ \times \left[ 1 + \frac{2\mathcal{C}_{14}^2A_{R_1}^2}{4\mathcal{C}_{13}^2A_I^2 + \mathcal{C}_{15}^2A_{L_2}^2} \right] \]
If \( \frac{\gamma_5^2}{\gamma_4^2(A_E^2)} < 1, \text{ we get} \)
\[ |Z_6(t, A_S, A_E, A_I, A_{L_1}, A_{R_1}, A_{R_2}, A_{M_1})|^2 \leq \gamma_6(1 + |A_{R_1}|^2), \]
where \( \gamma_6 = 4\mathcal{C}_{13}^2A_I^2 + \mathcal{C}_{15}^2A_{L_2}^2 \).

\[ |Z_7(t, A_S, A_E, A_I, A_{L_1}, A_{R_1}, A_{R_2}, A_{M_1})|^2 = |\mathcal{C}_{10}A_I + \mathcal{C}_{11}A_{L_2} - \mathcal{C}_{14}A_{R_1}|^2 \]
\[ \leq 2|\mathcal{C}_{10}A_I|^2 + 2|\mathcal{C}_{11}A_{L_2}|^2 + 2|\mathcal{C}_{14}A_{R_1}|^2 \]
\[ \leq 4\mathcal{C}_{10}^2A_I^2 + 4\mathcal{C}_{11}^2A_{L_2}^2 + 2\mathcal{C}_{14}^2A_{R_1}^2 \]
\[ \leq 4\mathcal{C}_{10}^2A_I^2 + \mathcal{C}_{11}^2A_{L_2}^2 \]
\[ \times \left[ 1 + \frac{2\mathcal{C}_{14}^2A_{R_1}^2}{4\mathcal{C}_{10}^2A_I^2 + \mathcal{C}_{11}^2A_{L_2}^2} \right] \]
If \( \frac{\gamma_5^2}{\gamma_4^2(A_E^2)} < 1, \text{ we get} \)
\[ |Z_8(t, A_S, A_E, A_I, A_{L_1}, A_{R_1}, A_{R_2}, A_{M_1})|^2 \leq \gamma_8(1 + |A_{R_1}|^2), \]
where \( \gamma_8 = 4\mathcal{C}_{10}^2A_I^2 + \mathcal{C}_{11}^2A_{L_2}^2 \).

That shows, function fulfill the growth condition.

Further, we validate the Lipschitz condition. If
\[ |Z_9(t, A_S, A_E, A_I, A_{L_1}, A_{R_1}, A_{R_2}, A_{M_1}) - Z_9(t, A_S, A_E, A_I, A_{R_1}, A_{M_1})|^2 \]
\[ = |(\mathcal{C}_{14} + \mathcal{C}_{15})(A_E - A_{R_1})|^2 \]
then
\[ \sup_{t \in [0, T]} |Z_9(t, A_S, A_E, A_I, A_{L_1}, A_{R_1}, A_{R_2}, A_{M_1}) - Z_9(t, A_S, A_E, A_I, A_{L_1}, A_{R_1}, A_{M_2})|^2 \]
\[ = \mathcal{C}_{14}^2 + \mathcal{C}_{15}^2 \sup_{t \in [0, T]} |A_E - A_{R_1}|^2 \]
\[ \|Z_9(t, A_S, A_E, A_I, A_{L_1}, A_{R_1}, A_{R_2}, A_{M_1}) - Z_9(t, A_S, A_E, A_I, A_{L_1}, A_{R_1}, A_{M_2})\|_0^2 \]
\[ = \mathcal{V}_3\|A_E - A_{R_1}\|_0^2 \]
If
\[ |Z_9(t, A_S, A_E, A_I, A_{L_1}, A_{R_1}, A_{R_2}, A_{M_1}) - Z_9(t, A_S, A_E, A_I, A_{L_1}, A_{R_1}, A_{M_2})|^2 \]
then
\[ \sup_{t \in [0, T]} |Z_9(t, A_S, A_E, A_I, A_{L_1}, A_{R_1}, A_{R_2}, A_{M_1}) - Z_9(t, A_S, A_E, A_I, A_{L_1}, A_{R_1}, A_{M_2})|^2 \]
\[ = \mathcal{C}_{14}^2 + \mathcal{C}_{15}^2 \sup_{t \in [0, T]} |A_E - A_{R_1}|^2 \]
\[ \|Z_9(t, A_S, A_E, A_I, A_{L_1}, A_{R_1}, A_{R_2}, A_{M_1}) - Z_9(t, A_S, A_E, A_I, A_{L_1}, A_{R_1}, A_{M_2})\|_0^2 \]
\[ = \mathcal{V}_3\|A_E - A_{R_1}\|_0^2 \]
If
Hence,
\[
\max \left\{ 6(\epsilon_R^2 + \epsilon_M^2)(A_1 + \epsilon_3 A_{1s})^2 + \epsilon_M^2 |A_M|^2 \right\} 
\]
\[
\frac{2\epsilon_R^2}{2\epsilon_R^2} + \frac{2\epsilon_M^2}{2\epsilon_R^2} \left( 4\epsilon_R^2 |A_1|^2 + \epsilon_M^2 |A_M|^2 \right)
\]
\[
(4\epsilon_R^2 |A_1|^2 + \epsilon_M^2 |A_M|^2) < 1.
\]
(9) has particular solution.

### Stability analysis

Here, we utilize the fractal–fraction approach. For this, consider

\[
\begin{align*}
A(t) &= \epsilon_0 - \epsilon_1 A_s - \epsilon_3 A_{1s} + \epsilon_3 A_{1s} - \epsilon_4 A_2 A_M, \\
A_e(t) &= \epsilon_0 - \epsilon_1 A_s - \epsilon_3 A_{1e} + \epsilon_3 A_{1e} - \epsilon_4 A_2 A_M, \\
A_i(t) &= \epsilon_0 - \epsilon_1 A_s - \epsilon_3 A_{1i} + \epsilon_3 A_{1i} - \epsilon_4 A_2 A_M, \\
A_h(t) &= \epsilon_0 - \epsilon_1 A_s - \epsilon_3 A_{1h} + \epsilon_3 A_{1h} - \epsilon_4 A_2 A_M, \\
A_m(t) &= \epsilon_0 - \epsilon_1 A_s - \epsilon_3 A_{1m} + \epsilon_3 A_{1m} - \epsilon_4 A_2 A_M.
\end{align*}
\]

Here, we use fixed point results to achieve our required result, so we can express the model as

\[
\begin{align*}
A(t) &= \epsilon_0 - \epsilon_1 A_s - \epsilon_3 A_{1s} + \epsilon_3 A_{1s} - \epsilon_4 A_2 A_M, \\
A_e(t) &= \epsilon_0 - \epsilon_1 A_s - \epsilon_3 A_{1e} + \epsilon_3 A_{1e} - \epsilon_4 A_2 A_M, \\
A_i(t) &= \epsilon_0 - \epsilon_1 A_s - \epsilon_3 A_{1i} + \epsilon_3 A_{1i} - \epsilon_4 A_2 A_M, \\
A_h(t) &= \epsilon_0 - \epsilon_1 A_s - \epsilon_3 A_{1h} + \epsilon_3 A_{1h} - \epsilon_4 A_2 A_M, \\
A_m(t) &= \epsilon_0 - \epsilon_1 A_s - \epsilon_3 A_{1m} + \epsilon_3 A_{1m} - \epsilon_4 A_2 A_M.
\end{align*}
\]

We can rewrite the system (12) as:

\[
\begin{align*}
D^\alpha_0 A(t) &= \frac{h^\alpha}{h^\alpha} V(t, \xi(t)), \\
D^\beta_0 \xi(t) &= \xi(t).
\end{align*}
\]

By replacing \( D^\alpha_0 \) by \( D^\alpha_0 \) and applying fractional integral, we have

\[
\int_0^t \frac{h^\alpha}{h^\alpha} \left( t - \beta \right) V(t, \xi(t)) \, dt 
\]

where

\[
\begin{align*}
\begin{pmatrix}
A_s(t) \\
A_e(t) \\
A_i(t) \\
A_h(t) \\
A_m(t)
\end{pmatrix} &= \begin{pmatrix}
A_s(0) \\
A_e(0) \\
A_i(0) \\
A_h(0) \\
A_m(0)
\end{pmatrix} \\
\xi(t) &= \begin{pmatrix}
A_s(t) \\
A_e(t) \\
A_i(t) \\
A_h(t) \\
A_m(t)
\end{pmatrix} = \begin{pmatrix}
A_s(0) \\
A_e(0) \\
A_i(0) \\
A_h(0) \\
A_m(0)
\end{pmatrix}.
\end{align*}
\]

If \( Q = [0, T] \), and we apply a Banach space \( \mathcal{X} = Q \times Q \times Q \times Q \), then the norm is defined as:

\[
\| \xi(t) \| = \max \{ |A_s(t)| + |A_e(t)| + |A_i(t)| + |A_h(t)| + |A_m(t)| \}.
\]

An operator \( \mathcal{S} : \mathcal{X} \rightarrow \mathcal{X} \) can be express as:

\[
\mathcal{S}(\xi(t)) = \xi(t) + \frac{\beta^{h\alpha-1}(1-\beta)}{C_D(\xi)} \eta(t, \xi(t)) \sum_{t \in T} \int_0^t \beta^{h\alpha-1}(t-\beta) \sigma(\xi(t)) \, d\beta
\]

For a function \( \sigma(\xi(t)) \) attains the extension with Lipschitz condition, then for \( \mathcal{C} \subseteq \mathcal{B} \) there exists some positive constants \( \xi, \eta \) such that

\[
\sigma(\xi(t)) \leq \mathcal{S}_�(\xi(t)) + \xi \eta
\]

and for \( \mathcal{C} \subseteq \mathcal{B} \) there exists a constant \( \xi \eta > 0 \) such that

\[
\| \sigma(\xi(t)) - \sigma(\xi(t)) \| \leq \xi \eta \| \xi(t) - \xi(t) \|.
\]

### Definition 8.

For all \( \mathcal{C} \subseteq (\mathcal{W}[0, T], \mathcal{F}) \), and for \( Y \) an operator \( \mathcal{S}_{X, h} \) then given model is Ulam–Hyres stable

\[
\left\| \mathcal{S}_{X, h} \mathcal{F}(\xi(t)) - \mathcal{F}(\xi(t)) \right\| \leq Y, \quad t \in [0, T],
\]

so, we get \( \sigma \in (\mathcal{W}[0, T], \mathcal{F}) \) such that

\[
\| \sigma(t) \| \leq \mathcal{S}_{X, h} (Y, \xi(t)) \leq [0, T].
\]

Let a perturbation \( e \in W[0, T] \), then \( e(t) = 0 \).

Consider

1. \( f \) or \( Y > 0 \), we have \( |\sigma(t)| \leq Y \)

2. \( \mathcal{F}_{X, h} \mathcal{S}_{X, h} \mathcal{F}(\xi(t)) = \mathcal{F}(\xi(t)) + \sigma(t) \).

Lemma 9. If

\[
\left\| \mathcal{F}_{X, h} \mathcal{F}(\xi(t)) \right\| \leq \mathcal{F}(\xi(t)) + \sigma(t), \quad \mathcal{F}(0) = \xi_0,
\]

is solution of a perturbed model then following relation holds:

\[
\| \sigma(t) - \mathcal{F}(\xi(t)) \| \leq \xi_{X, h} T \| \| \xi(t) - \xi(t) \| \| \mathcal{F}(\xi(t)) \|
\]

\[
\xi_{X, h}(t) = \frac{\beta^{h\alpha-1}(1-\beta)}{C_D(\xi)} T \| \| \xi(t) - \xi(t) \| \| \mathcal{F}(\xi(t)) \|.
\]

Lemma 10. If \( \theta < 1 \), then model (9) has Ulam–Hyres stable solution under the state of Eq (16) with Lemma 9.

Proof. Let \( e \in \mathcal{A} \) be a remarkable consequence, and \( \mathcal{C} \subseteq \mathcal{A} \) be any consequence of the model, then

\[
\| \xi(t) - \xi(t) \| = \| \xi(t) - \xi(t) \| + \frac{\beta^{h\alpha-1}(1-\beta)}{C_D(\xi)} \eta(t, \xi(t)) \sum_{t \in T} \int_0^t \beta^{h\alpha-1}(t-\beta) \sigma(\xi(t)) \, d\beta
\]

\[
\leq \| \xi(t) - \xi(t) \| + \frac{\beta^{h\alpha-1}(1-\beta)}{C_D(\xi)} \eta(t, \xi(t)) \sum_{t \in T} \int_0^t \beta^{h\alpha-1}(t-\beta) \sigma(\xi(t)) \, d\beta
\]


\[
\begin{align*}
&+ \frac{\zeta h}{CD\xi \xi} \int_0^t \zeta h^{-1}(t - \zeta h^{-1}v(\zeta, \xi(t))d\zeta) \\
&+ |0(0) + \frac{h}{CD\xi \xi} \int_0^t \zeta h^{-1}(t - \zeta h^{-1}v(\zeta, \xi(t))d\zeta) \\
&- |t(0) + \frac{h}{CD\xi \xi} \int_0^t \zeta h^{-1}(t - \zeta h^{-1}v(\zeta, \xi(t))d\zeta) \\
&+ \frac{\zeta h}{CD\xi \xi} \int_0^t \zeta h^{-1}(t - \zeta h^{-1}v(\zeta, \xi(t))d\zeta) \\
&\leq \xi_{t, h} Y + (\frac{h}{CD\xi \xi}) \mid E(\xi, h) |L_c |\mid t - t(0), \xi(t) \mid.
\end{align*}
\]

Consequently,
\[
\|C - \chi\| \leq \xi_{t, h} Y + \|C - \chi\|.
\]

Thus, it can be
\[
\|C - \chi\| \leq \xi_{t, h} Y,
\]
where \(\xi_{t, h} = \frac{\zeta h}{\|\|}\) and that is required result.

**Fractal-fraction operator**

Here, we use piece-wise interpolation of Lagrange with fractal-fraction to set up operator for system (9). We proceed Eq. (13) through the fractal-fraction integral. Thus
\[
A_S(t) = A_S(0) + \frac{h}{\int (\mathcal{X}^2)} \int_0^t \phi^{(1)}(t - \beta)A_S(t, \mathcal{X}; \beta, \mathcal{X}; \beta) d\beta,
\]

\[
A_E(t) = A_E(0) + \frac{h}{\int (\mathcal{X}^2)} \int_0^t \phi^{(1)}(t - \beta)A_E(t, \mathcal{X}; \beta, \mathcal{X}; \beta) d\beta,
\]

\[
A_f(t) = A_f(0) + \frac{h}{\int (\mathcal{X}^2)} \int_0^t \phi^{(1)}(t - \beta)A_f(t, \mathcal{X}; \beta, \mathcal{X}; \beta) d\beta,
\]

\[
A_L(t) = A_L(0) + \frac{h}{\int (\mathcal{X}^2)} \int_0^t \phi^{(1)}(t - \beta)A_L(t, \mathcal{X}; \beta, \mathcal{X}; \beta) d\beta,
\]

\[
A_R(t) = A_R(0) + \frac{h}{\int (\mathcal{X}^2)} \int_0^t \phi^{(1)}(t - \beta)A_R(t, \mathcal{X}; \beta, \mathcal{X}; \beta) d\beta,
\]

\[
A_A(t) = A_A(0) + \frac{h}{\int (\mathcal{X}^2)} \int_0^t \phi^{(1)}(t - \beta)A_A(t, \mathcal{X}; \beta, \mathcal{X}; \beta) d\beta,
\]

\[
A_M(t) = A_M(0) + \frac{h}{\int (\mathcal{X}^2)} \int_0^t \phi^{(1)}(t - \beta)A_M(t, \mathcal{X}; \beta, \mathcal{X}; \beta) d\beta,
\]

\[
A^{(s+1)} = A^{(s)} + \frac{h}{\int (\mathcal{X}^2)} \int_0^t \phi^{(1)}(t - \beta)A_S(t, \mathcal{X}; \beta, \mathcal{X}; \beta) d\beta,
\]

\[
A^{(s+1)} = A^{(s)} + \frac{h}{\int (\mathcal{X}^2)} \int_0^t \phi^{(1)}(t - \beta)A_E(t, \mathcal{X}; \beta, \mathcal{X}; \beta) d\beta,
\]

\[
A^{(s+1)} = A^{(s)} + \frac{h}{\int (\mathcal{X}^2)} \int_0^t \phi^{(1)}(t - \beta)A_f(t, \mathcal{X}; \beta, \mathcal{X}; \beta) d\beta,
\]

\[
A^{(s+1)} = A^{(s)} + \frac{h}{\int (\mathcal{X}^2)} \int_0^t \phi^{(1)}(t - \beta)A_L(t, \mathcal{X}; \beta, \mathcal{X}; \beta) d\beta,
\]

\[
A^{(s+1)} = A^{(s)} + \frac{h}{\int (\mathcal{X}^2)} \int_0^t \phi^{(1)}(t - \beta)A_R(t, \mathcal{X}; \beta, \mathcal{X}; \beta) d\beta,
\]

\[
A^{(s+1)} = A^{(s)} + \frac{h}{\int (\mathcal{X}^2)} \int_0^t \phi^{(1)}(t - \beta)A_A(t, \mathcal{X}; \beta, \mathcal{X}; \beta) d\beta,
\]

\[
A^{(s+1)} = A^{(s)} + \frac{h}{\int (\mathcal{X}^2)} \int_0^t \phi^{(1)}(t - \beta)A_M(t, \mathcal{X}; \beta, \mathcal{X}; \beta) d\beta.
\]
Fig. 1. Simulation of $A_S(t)$ under Fractal–fractional with Mittag-Leffler kernel.

$$A_S(p^{-1}, A_S^{p-1}, A_E^{p-1}, A_I^{p-1}, A_L^{p-1}, A_R^{p-1}, A_M^{p-1}),$$

$$A_I(p) = \frac{\beta - t_{p-1}}{(t_p - t_{p-1})} A_S(p, A_S^p, A_E^p, A_I^p, A_L^p, A_R^p, A_M^p) - \frac{\beta - t_p}{(t_p - t_{p-1})} A_S(p, A_S^{p-1}, A_E^{p-1}, A_I^{p-1}, A_L^{p-1}, A_R^{p-1}, A_M^{p-1}).$$

$$A_{Ia}(p) = \frac{\beta - t_{p-1}}{(t_p - t_{p-1})} A_S(p, A_S^p, A_E^p, A_I^p, A_L^p, A_R^p, A_M^p) - \frac{\beta - t_p}{(t_p - t_{p-1})} A_S(p, A_S^{p-1}, A_E^{p-1}, A_I^{p-1}, A_L^{p-1}, A_R^{p-1}, A_M^{p-1}).$$

$$A_{R}(p) = \frac{\beta - t_{p-1}}{(t_p - t_{p-1})} A_S(p, A_S^p, A_E^p, A_I^p, A_L^p, A_R^p, A_M^p) - \frac{\beta - t_p}{(t_p - t_{p-1})} A_S(p, A_S^{p-1}, A_E^{p-1}, A_I^{p-1}, A_L^{p-1}, A_R^{p-1}, A_M^{p-1}).$$

$$A_{M}(p) = \frac{\beta - t_{p-1}}{(t_p - t_{p-1})} A_S(p, A_S^p, A_E^p, A_I^p, A_L^p, A_R^p, A_M^p) - \frac{\beta - t_p}{(t_p - t_{p-1})} A_S(p, A_S^{p-1}, A_E^{p-1}, A_I^{p-1}, A_L^{p-1}, A_R^{p-1}, A_M^{p-1}).$$

(21)

Fig. 2. Simulation of $A_E(t)$ under Fractal–fractional with Mittag-Leffler kernel.

$$A_S(p^{-1}, A_S^{p-1}, A_E^{p-1}, A_I^{p-1}, A_L^{p-1}, A_R^{p-1}, A_M^{p-1}),$$

Thus, we have

$$A_S^{(k+1)} = A_S^{(0)} + \frac{\hbar}{\mathcal{D} \mathcal{B}(Z)} A_S(t_0, A_S^0, A_E^0, A_I^0, A_L^0, A_R^0, A_M^0)$$

$$+ \frac{\gamma \tau \hbar}{\mathcal{D} \mathcal{B}(Z) \Gamma(k+2)} \sum_{p=1}^{\infty} \left[ (p^{\beta} A_S(t_p, A_S^p, A_E^p, A_I^p, A_L^p, A_R^p, A_M^p) - \frac{\beta - t_p}{(t_p - t_{p-1})} A_S(t_p, A_S^{p-1}, A_E^{p-1}, A_I^{p-1}, A_L^{p-1}, A_R^{p-1}, A_M^{p-1})) \right]$$

$$\times ((k + 1 - p)\tau^2 (i - p + 2 + p) - (k - p)\tau(2 + 2 \chi + \kappa)) - p^{-1}$$

$$A_S(t_{p-1}, A_S^{p-1}, A_E^{p-1}, A_I^{p-1}, A_L^{p-1}, A_R^{p-1}, A_M^{p-1})$$

$$\times ((k + 1 - p)\tau^{k+1} - (k - p)\tau(1 + \chi + \kappa)).$$

$$A_E^{(k+1)} = A_E^{(0)} + \frac{\hbar}{\mathcal{D} \mathcal{B}(Z)} A_E(t_0, A_S^0, A_E^0, A_I^0, A_L^0, A_R^0, A_M^0)$$

$$+ \frac{\gamma \tau \hbar}{\mathcal{D} \mathcal{B}(Z) \Gamma(k+2)} \sum_{p=1}^{\infty} \left[ (p^{\beta} A_E(t_p, A_S^p, A_E^p, A_I^p, A_L^p, A_R^p, A_M^p) - \frac{\beta - t_p}{(t_p - t_{p-1})} A_E(t_p, A_S^{p-1}, A_E^{p-1}, A_I^{p-1}, A_L^{p-1}, A_R^{p-1}, A_M^{p-1})) \right]$$

$$\times ((k + 1 - p)\tau^2 (i - p + 2 + p) - (k - p)\tau(2 + 2 \chi + \kappa)) - p^{-1}$$

$$A_E(t_{p-1}, A_E^{p-1}, A_I^{p-1}, A_L^{p-1}, A_R^{p-1}, A_M^{p-1})$$

$$\times ((k + 1 - p)\tau^{k+1} - (k - p)\tau(1 + \chi + \kappa))$$

Fig. 3. Simulation of $A_E(t)$ under Fractal–fractional with Mittag-Leffler kernel.

Fig. 4. Simulation of $A_M(t)$ under Fractal–fractional with Mittag-Leffler kernel.
Fig. 5. Simulation of $A_k(t)$ under fractal–fractional with Mittag-Leffler kernel.

Fig. 6. Simulation of $A_k(t)$ under fractal–fractional with Mittag-Leffler kernel.

\[
A_{E_1}(t_{p-1}, A_{S_1}, A_E^{−1}, A_{I_1}^{−1}, A_{R_1}^{−1}, A_{M_1}^{−1}) = A_0(0) + \frac{\Phi^{\alpha}h}{\mathcal{H}(\chi)(\chi + 2)} \sum_{p=0}^{\infty} \left[ h(t_{p-1}, A_{S_1}, A_E^{−1}, A_{I_1}^{−1}, A_{R_1}^{−1}, A_{M_1}^{−1}) \right]
\]

Thus, (22) is the generalized form of (9).

Simulation and discussion

In this section, consider the numerical simulations of the proposed scheme using fractal–fractional technique for fractional order COVID-19 model for newly infected case of Omicron. To identify the potential effectiveness of Corona virus disease transmission in the Community, the COVID-19 fractional-order model in the case of new wave omicron in India is presented to analyze with simulations. That is why; we used fractal-fractional derivative with Mittag-Leffler law of the COVID-19 in case of new wave omicron in India with the initial conditions are provided. The various numerical methods identify the mechanical features of the fractional-order model with the time-fractional parameters. The results of the nonlinear system memory were also detected with the help of fractional values. Figs. 1–6 represents the simulations obtained by fractal-fractional method. It is easily observed that in Figs. 1 and 5, all compartments starts decreasing by decreasing the fractional values which converge to steady state while in Figs. 2–4 and 6 starts increasing by decreasing fractional values. It has been shown that physical processes are better explained using the fractional-order derivatives, which are the most notable and reliable component compared to the classical-order case. Existing non-integer-order models are less profitable compared to those operators. The behaviors of the dynamics found in the various fractional orders are shown in the form of numerical results that have been reported. From pictorial depictions, we can see the fluctuation of different variables. Initially, the susceptible population goes very high, but it decreases in a few days due to the expeditious growth in exposed, asymptotically infected, recovered, and reservoir peoples. The chart of the exposed population climbs up then gradually trends from top to bottom. As time passes, the strength of infection reduces the effects of the virus on the community. Thus, the population of infected persons drops to the ground. In the same way, the number of asymptotically infected people increases within a few days as can be seen in Figs. 3 and 4, but later, they also fall off. As a result, recovered people shoot up in the community as can be seen in Fig. 5. This method provide us better understanding of Omicron and its control position after certain time using fractional operator.

Conclusion

The dangerous corona virus and the deadly epidemic of Omicron disease in today’s pandemic have caused millions of deaths to date.
This article shows the second wave dynamics of COVID-19 called Omicron. The pictorial results show the behavior of the virus that helps to understand the spread of the disease. We consider the COVID-19 model of second wave with fractional operator for this work to check the dynamical behavior of infection of disease in society. In this regard, fractal–fractional derivative gave a realistic approach to analyze the effect of disease which will be helpful for such type of epidemic. Sensitivity analysis has been made to see the rate of change every parameter for threshold hold condition as well as analyze qualitatively. The existence and unique solution of the fractional order model were made with the help of fixed point theory. Numerical simulation has been made to check the actual behavior of COVID-19 effect for the second wave infected case by Omicron which shows that infected individuals start decreasing after few day. Such kind of results are very helpful for planning, decision-making, and developing control strategies to overcome the effect of the second wave of COVID-19 in the form of Omicron in society. Now Omicron version is slowly growing throughout the globe and it is feasible to put into effective lockdown for mid-2022 (June) in India and we finish that the 1/3 wave which can be both excessive unfold or much less on the end of May 2022.

CRediT authorship contribution statement

Muhammad Farman: Conceptualization. Maryam Amin: Methodology. Ali Akgül: Investigation. Aqeel Ahmad: Supervision. Muhammad Bilal Riaz: Supervision. Sheraz Ahmad: Methodology.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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