Quasi-normal modes of a massless scalar field around the 5D Ricci-flat black string

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1. Introduction

Through an additional field or by perturbing the metric itself, the black hole suffers a damping oscillation phase. These are usually called quasi-normal modes (QNMs) or quasi-normal ringing. So, as the emission of a gravitational wave (GW), a normal model oscillation is replaced by the complex frequencies ‘quasi-normal’ where the real part represents the actual frequency and the imaginary part represents the damping of the oscillation. Those frequencies are directly connected to the black hole’s mass, charge, momentum and so on. In the view of QNMs’ evolution, there are three stages: the first one is the initial outburst from the source of perturbation, the second one is the damping (quasi-normal) oscillation and the last one is the asymptotic tails at very late time. The evolution significantly depends on the asymptotic behavior of the space. In perturbation theory, the linear perturbation of static black holes was first studied by Regge and Wheeler in 1957 [1]. Soon after that, Vishveshvara [2] presented the QNMs by calculating the scattering of gravitational waves (GW) around a Schwarzschild black hole. Then Press [3] gave the original term quasi-normal (QN) frequencies. These
perturbations have been studied extensively in the literature [4]. For in-depth reviews, one can refer to [5, 6]. People believe that the QN frequencies could be detected by GW observatories (LIGO, VIRGO, TAMT, GEO600, and so on) in future.

On the other hand, one kind of higher dimensional theory named induced matter theory was shown by Wesson and co-workers [7, 8] in the 1990s. They showed a non-compact fifth dimension and pointed out that the 4D source is induced from an empty 5D manifold. That is, the 5D manifold is Ricci-flat while the 4D hypersurface is curved by the 4D matters. So this theory is also called space-time-matter (STM) theory. Meanwhile, in the STM framework, there are many extensive papers discussing the quantum Dirac equation [9], the perihelion problem [10], Kaluza–Klein solitons [11, 12], black hole [13–16], solar system tests [18] and so on.

In the last decade, other robust extra dimensional models have appeared in gravitational-field theory such as the ADD model [19] and Randall–Sundrum I model [20]II model [21] with the additional spacelike dimensions. In those models, our world is a 3-brane which is embedded in the higher dimensional space (bulk). To avoid interactions beyond any acceptable phenomenological limits, standard model (SM) particles (such as fermions, gauge bosons, Higgs) are confined on a (3 + 1) dimensional hypersurface (3-brane) without accessing the transverse dimensions, except for the gravitons and scalar particles without charges under the SM gauge group. In this paper, we assume that the scalar field can freely propagate in the bulk.

If the matter trapped on the brane undergoes gravitational collapse, a black hole will form naturally and its horizon extends into the extra dimension transverse to the brane. Such a higher dimensional object which looks like a black hole on the brane is actually a black string in the higher dimensional braneworld [22]. One natural candidate is a Schwarzschild–de Sitter (SdS) black hole embedded into the 5D Ricci-flat space [13, 15–17]. It should be noted that the STM theory is equivalent to the braneworld [23–25].

Meanwhile, one of the exciting predictions in the large extra dimensional model [19] is that the CERN large hadron collider (LHC) will produce black holes by the collision of highly energetic particles when the scale of quantum gravity is near TeV [26]. Naturally, the detectable QNMs are studied widely in a higher dimensional background [27]. In this paper, we calculate the QN frequency of a massless scalar field around a 5D Ricci-flat black string space.

This paper is organized as follows: in section 2, the 5D Ricci-flat black string metric and the time-dependent radial equation about \( R_\omega (r, t) \) are represented. In section 3, by a tortoise coordinate transformation, the propagating master equation of the scalar field is obtained. In section 4, by using the third-order WKB method, the QN frequency is obtained in tables 1, 2 and 3. Section 5 is the conclusion. We adopt the signature \((+, −, −, −, −)\) and put \( \hbar, c \) and \( G \) equal to unity. Lowercase Greek indices \( \mu, \nu, \ldots \) will be taken to run over 0, 1, 2, 3 as usual, while capital indices \( A, B, C, \ldots \) run over all five coordinates \( (0, 1, 2, 3, 4) \).

### 2. Klein–Gordon equation in the 5D Ricci-flat black string space

A class of 5D black hole solutions has been presented by Mashhoon, Wesson and Liu [7, 13, 15] under the STM scenario. Briefly, the static, three-dimensional spherically symmetric line element takes the form

\[
dS^2 = \frac{\Lambda \xi^2}{3} \left[ f(r) \, dt^2 - \frac{1}{f(r)} \, dr^2 - r^2 (d\theta^2 + \sin^2 \theta \, d\phi^2) \right] - d\xi^2,
\]

where \( \xi \) is the open non-compact extra dimension coordinate. The part of this metric inside the square bracket is exactly the same line element as the 4D Schwarzschild–de Sitter solution,
which is bounded by two horizons—an inner horizon (black hole horizon) and an outer horizon (one may call this the cosmological horizon).

The radial-dependent metric function \( f(r) \) takes the form

\[
f(r) = 1 - \frac{2M}{r} - \frac{\Lambda}{3} r^2,
\]

where \( \Lambda \) is the induced cosmological constant and \( M \) is the central mass. Metric (1) satisfies the 5D vacuum equation \( R_{AB} = 0 \). Therefore, there is no cosmological constant when viewed from 5D. But when viewed from 4D, there is an effective cosmological constant \( \Lambda \). So one can treat this \( \Lambda \) as a parameter which comes from the fifth dimension. This solution has been studied in many works [28–31], focusing mainly on the induced constant \( \Lambda \), the extra force and so on.

We redefine the fifth dimension \( \xi = \sqrt{\frac{3}{\Lambda}} e^{\sqrt{\frac{\Lambda}{3}} y} \). With this redefinition, the metric (1) can be rewritten as

\[
\text{d}S^2 = e^{2\sqrt{\frac{\Lambda}{3}} y} \left[ f(r) \, \text{d}t^2 - \frac{1}{f(r)} \, \text{d}r^2 - r^2 (\text{d}\theta^2 + \sin^2 \theta \, \text{d}\phi^2) - \text{d}y^2 \right].
\]

Using the line element (1), the metric function (2) and the above new extra dimension, a Randall–Sundrum (RS) type brane model is built up. Now, let us show the configuration in detail. There are two branes in this model: one brane is at \( y = 0 \) and the other brane is at \( y = y_1 \). So the fifth dimension becomes finite. It could be very large as the RS II model [20] or very small as the RS I model [21]. The 4D line element represents exactly the Schwarzschild–de Sitter black hole on a hypersurface (\( \xi \) or \( y = \) constant). However, viewing from the 5D space, the horizon does not form a 4D sphere—it looks like a black string lying along the extra dimension. So, we call solution (1) a 5D Ricci-flat black string solution.

The metric function (2) can be expressed by the horizons as follows:

\[
f(r) = \frac{\Lambda}{3r} (r - r_c)(r_e - r)(r - r_o).
\]

The null hypersurface of this black string space is determined by its singularity \( f(r) = 0 \). Obviously, the solutions to this equation are the inner horizon \( r_c \), outer horizon \( r_e \), and a negative solution \( r_o = -(r_e + r_c) \). The last one has no physical significance. Here we only consider the real solutions. \( r_c \) and \( r_e \) are given as

\[
\begin{cases}
  r_c = \frac{2}{\sqrt{\Lambda}} \cos \eta, \\
  r_e = \frac{2}{\sqrt{\Lambda}} \cos(120^\circ - \eta),
\end{cases}
\]

where \( \eta = \frac{1}{3} \arccos(-3M \sqrt{\Lambda}) \) with \( 30^\circ \leq \eta \leq 60^\circ \). The real physical solutions are accepted only if \( \Lambda \) satisfy \( \Lambda M^2 < \frac{1}{9} \) [13].

The massless scalar field \( \Phi \) in the 5D black string space satisfies the Klein–Gordon equation \( \Box \Phi = 0 \), where \( \Box = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^A} \left( \sqrt{g} g^{AB} \frac{\partial}{\partial x^B} \right) \) is the 5D d’Alembertian operator. We assume that the separable solutions are of the form

\[
\Phi = \frac{1}{\sqrt{4\pi \omega r}} R_{\omega}(r, t) L(y) Y_{lm}(\theta, \phi),
\]

where \( R_{\omega}(r, t) \) is the radial time-dependent function, \( Y_{lm}(\theta, \phi) \) is the usual spherical harmonic function. The dependent equation about \( R_{\omega}(r, t) \) in the QNMs aspect is

\[
- \frac{1}{f(r)} \frac{1}{r^2} \frac{\partial^2}{\partial t^2} \left( \frac{R_{\omega}}{r} \right) + \frac{\partial}{\partial r} \left( r^2 f(r) \frac{\partial}{\partial r} \left( \frac{R_{\omega}}{r} \right) \right) - [\Omega r^2 + l(l + 1)] \frac{R_{\omega}}{r} = 0.
\]

\( 3 \)
where $\Omega$ is the eigenvalue of function $L(y)$. The fifth dimensional equation about $L(y)$ is

$$\frac{d^2 L(y)}{dy^2} + \Lambda \sqrt{\frac{\Lambda}{3}} \frac{dL(y)}{dy} + \Omega L(y) = 0,$$

(8)

which is discussed carefully in [16]. In the Randall–Sundrum double branes system, the modes along the extra dimension are quantized by means of stable standing waves, and then the eigenvalue is naturally discretized. The discrete spectra of $L(y)$ are

$$L_n(y) = C e^{-\sqrt{\frac{\Lambda}{3}} y} \cos \left( n\pi \frac{y}{y_1} \right),$$

(9)

and the quantum parameter $\Omega_n$ is

$$\Omega_n = \frac{n^2 \pi^2}{y_1^2} + \frac{3}{4} \Lambda,$$

(10)

where $n = 1, 2, 3 \ldots$ and $y_1$ is the thickness of the bulk.

3. The master equation for propagation of the scalar field in the bulk

It is known that the radial direction determines the evolution of black hole radiation. The time variable of equation (7) can be removed by the Fourier component $e^{-i\omega t}$ via

$$R_{\omega}(r, t) \rightarrow \Psi_{\omega n}(r) e^{-i\omega t},$$

(11)

where the subscript $n$ presents a new wavefunction unlike the usual 4D case $\Psi_{\omega n}$ [33]. Equation (7) can be rewritten as

$$\left[ -f(r) \frac{d}{dr} \left( f(r) \frac{d}{dr} \right) + V(r) \right] \Psi_{\omega n}(r) = \omega^2 \Psi_{\omega n}(r),$$

(12)

whose potential function is given by

$$V(r) = f(r) \left[ \frac{1}{r} \frac{df}{dr} + \frac{l(l+1)}{r^2} + \Omega \right].$$

(13)

Now we introduce the tortoise coordinate

$$x = \int \frac{dr}{f(r)}.$$  

(14)

The tortoise coordinate can be expressed with the surface gravity as follows:

$$x = \frac{1}{2M} \left[ \frac{1}{2K_c} \ln \left( \frac{r - 1}{r_c - 1} \right) - \frac{1}{2K_c} \ln \left( \frac{1 - r}{r_c} \right) + \frac{1}{2K_c} \ln \left( \frac{1 - r}{r_o} \right) \right],$$

(15)

where

$$K_i = \frac{1}{2} \left| \frac{df}{dr} \right|_{r = r_i}.$$  

(16)

So under the tortoise coordinate transformation (14), the radial perturbation equation is obtained as

$$\left[ -\frac{d^2}{dx^2} + V(r) \right] \Psi_{\omega n}(x) = \omega^2 \Psi_{\omega n}(x).$$

(17)

It is evident that equation (17) is exactly the same as the Regge–Wheeler equation in QNMs. The incoming or outgoing particle flowing between inner horizon $r_c$ and outer horizon $r_o$ is
reflected and transmitted by the potential $V(r)$. Substituting the quantum parameters $\Omega_n$ (10) into equation (13), the quantum potentials are obtained as follows:

$$V_n(r) = f(r) \left[ \frac{1}{r} \frac{df(r)}{dr} + \frac{l(l+1)}{r^2} + \frac{n^2 \pi^2}{y_1^2} + \frac{3}{4} \Lambda \right],$$

(18)

which are illustrated in figures 1 and 2. The 5D potential contains the quantum number $n$ which is higher and thicker than the 4D’s when $\Omega = 0$. Also, the height and the thickness of the former increase with bigger $n$. Meanwhile, for the increasing cosmological constant $\Lambda$, the potential also becomes higher and thicker, and the interval between $r_e$ and $r_c$ is larger, too.

According to the quantum potential (18), the QNMs for massless scalar particles propagating in the black string space satisfy the boundary conditions [5, 6]

$$\Psi_{obs}(x) \approx C_{\pm} \exp(\pm i \omega x) \quad \text{as} \quad x \longrightarrow \pm \infty,$$

(19)

denoting pure ingoing waves at the event horizon $r_e$ and pure outgoing waves at the cosmological horizon $r_c$.

4. The QN frequency of the massless scalar field with the third-order WKB method

The numerical WKB approximation is an effective method to obtain the complex QN frequencies by using the well-known Bohr–Sommerfeld rule. It was originally shown by Schutz et al [34] and was later developed to the third order by Iyer et al [35, 36] and to the sixth order by Konoplya [37]. Then after that this method was extensively used in various spaces [38]. The third-order WKB formula for QN frequencies has the form [35, 36],

$$\omega^2 = [V_0 + (-2V_0')^{1/2} \Lambda] - i \left( p + \frac{1}{2} \right) (-2V_0'')^{1/2}(1 + \Omega),$$

(20)

$$p = \begin{cases} 
0, 1, 2, \ldots, \text{Re } \omega > 0, \\
-1, -2, -3, \ldots, \text{Re } \omega < 0,
\end{cases}$$

(21)
where

$$\tilde{\lambda} = \frac{1}{(-2V_0')^{1/2}} \left[ \frac{1}{8} V_0^{(4)} V_0' \left[ \frac{1}{4} + \alpha^2 \right] - \frac{1}{288} \left[ \frac{V_0'' V_0^{(4)}}{V_0'^2} \right] (7 + 60\alpha^2) \right], \quad (22)$$

$$\tilde{\Omega} = \frac{1}{(-2V_0')^{1/2}} \left[ \frac{5}{6912} \left[ \frac{V_0'' V_0^{(4)}}{V_0'^2} \right] (77 + 188\alpha^2) \right.
- \frac{1}{384} \left[ \frac{V_0'' V_0^{(4)}}{V_0'^2} \right] (51 + 100\alpha^2) + \frac{1}{2304} \left[ \frac{V_0'' V_0^{(4)}}{V_0'^2} \right] (67 + 68\alpha^2)
+ \frac{1}{288} \left[ \frac{V_0'' V_0^{(5)}}{V_0'^2} \right] (19 + 28\alpha^2) - \frac{1}{288} \left[ \frac{V_0'' V_0^{(5)}}{V_0'^2} \right] (5 + 4\alpha^2) \right], \quad (23)$$

where $\alpha = p + 1/2$ and symbol $p$ is the various overtones. The primes and superscript $(n)$ denote differentiation with respect to the tortoise coordinate $x$. The subscript 0 on a variable denotes the value at $x_0$, which is the position of maximum $V(x)$, namely, $V_0^{(n)} = \frac{d^n V}{dx^n}(x=x_0)$.

Substituting potential (18) into $\tilde{\lambda}$ and $\tilde{\Omega}$, we can obtain the vital QN frequencies for the massless scalar field in the 5D black string space. Meanwhile, it is known that the WKB approximation is accurate for the low-lying QNM modes, but it fails to calculate the higher order modes. Therefore, the condition $l > p$ is employed and the QN frequencies of fundamental key cases: $(l = 1, p = 0)$, $(l = 2, p = 0)$ and $(l = 2, p = 1)$ are listed in tables 1, 2 and 3, respectively. Meanwhile, potential (13) clearly illustrates that when $\Omega = 0$ the Regge–Wheeler equation (17) is naturally reduced to the 4D SdS case. So these tables also include a comparison with the results of the 4D case. Here we should note that the notation $\Omega(n)$ does not indicate $n = 0$. To avoid confusion about parameters $\Omega$ and $\Omega(n)$, we provide some explanation in the conclusion. Here, we adopt $M = 1$ and $y_1 = 10$ and analyze the QN frequencies from two aspects: cosmological constant $\Lambda$ and quantum number $n$. 

\[ \text{Figure 2. The potentials } V_1(r) \text{ versus radial coordinate } r \text{ with } \Lambda = 0.02 \text{ (plus), } \Lambda = 0.04 \text{ (dotted), } \Lambda = 0.06 \text{ (dash-dot), } \Lambda = 0.08 \text{ (dashed) and } \Lambda = 0.10 \text{ (solid). Here, we adopt } M = 1, n = 1, l = 1 \text{ and } y_1 = 10 \text{ (a very large extra dimension).} \]
Firstly, for a given cosmological constant $\Lambda$, it is shown that the real parts (Re $\omega$) increase with a bigger quantum number $n$. But the absolute value of the imaginary parts (|Im $\omega$|) decreases for bigger $n$. In general, the actual frequencies in 5D are larger than 4D’s, and the scalar field in 5D decays more slowly than that in 4D. With increasing $n$, the QN frequencies become larger and the scalar field decays more slowly.

Secondly, for a given $n$ we can also read that Re $\omega$ and |Im $\omega$| decrease with larger $\Lambda$. This means that with the increasing cosmological constant $\Lambda$ the actual frequency becomes smaller and the scalar field decays more slowly. These results are in agreement with the results of the 4D SdS case with the sixth-order WKB method [39]. As mentioned above, the two circumstances for given $n$ and $\Lambda$ can be manifested in figures 3 and 4 which are obtained by table 1.

In order to study the relationship between the actual frequency and the damping of the oscillation, we also plot the Im $\omega$ versus Re $\omega$ graph in figure 5. Obviously, the absolute value of the imaginary parts increases entirely with the larger real parts. However, when the quantum number $n$ becomes larger, there is a break point in the curve. In other words, |Im $\omega$| does not monotonously increase with Re $\omega$ for bigger $n$, especially in the case of $n = 2, 3$.

To explain the reason, we should refer to this black string’s reflection (or transmission) [17]. In the square barrier model [17], the reflection or transmission coefficients have analytic forms. As one knows that the reflection should be stronger with higher barriers. The reflection coefficients of $n = 2, 3$ would be larger than the cases of $n = 1$ or $\Omega = 0$ from the usual viewpoint. In contrast, the values of reflection coefficients of $n = 2, 3$ are smaller than the cases of $n = 1$ or $\Omega = 0$. If the resonant effect of quantum mechanics exists in the barriers, the peculiar behavior is easily observed. Mathematically, an oscillatory cosine function $\cos(2kxd)$
Figure 3. The real parts of QN frequencies ($\text{Re}\omega$) of quasi-normal of the scalar field in the 5D Ricci-flat black string space with $l = 1$, $p = 0$, $M = 1$ and $\gamma_1 = 10$. We denote pentagrams with $n = 1$, squares with $n = 2$, diamonds with $n = 3$ and triangles with $\Omega = 0$.

Figure 4. The imaginary parts of QN frequencies ($\text{Im}\omega$) of quasi-normal of the scalar field in the 5D Ricci-flat black string space with $l = 1$, $p = 0$, $M = 1$ and $\gamma_1 = 10$. We denote pentagrams with $n = 1$, squares with $n = 2$, diamonds with $n = 3$ and triangles with $\Omega = 0$.

is contained in the expression of reflection coefficients $R$ or transmission coefficients $T$ [17]. (For a detailed discussion on those behaviors, see [17].) The scattering potential of $n = 2, 3$ also has the peculiar QN frequencies by the same resonant effect. From figure 1, we can see that with larger $n$ the peak of potential slips the cosmological horizon and the potential becomes higher and wider, especially in the case of $n = 2, 3$. Hence with the enhancing of the resonant effect the QN frequencies decay more slowly, even though the QN frequencies
Figure 5. The imaginary parts (Im $\omega$) versus the real parts (Re $\omega$) in the 5D Ricci-flat black string space with $l = 1, p = 0, M = 1$ and $y_1 = 10$. We denote pentagrams with $n = 1$, squares with $n = 2$, diamonds with $n = 3$ and triangles with $\Omega = 0$.

become larger. So the anomalous values of $n = 2, 3$ are the result of the resonant effect of quantum mechanics in the barriers.

5. Conclusion

In this paper, we have used the third-order WKB approximation to calculate the quasi-normal frequencies of the massless scalar field outside a 5D black string. We summarize what has been achieved.

(1) In this 5D Ricci-flat black string space, the QNMs are studied by fixing either the cosmological constant $\Lambda$ or the quantum number $n$. From the result we find that the scalar field decays more slowly with the increasing $\Lambda$ or $n$. For a given cosmological constant $\Lambda$, the 5D actual frequency becomes bigger with increasing $n$. While for a given $n$, the frequency becomes smaller with the bigger cosmological constant $\Lambda$. In other words, the 5D QN frequencies are larger than 4D’s ($\Omega = 0$).

(2) As one candidate of the higher dimensional black hole, the 5D Ricci-flat black string implies something interesting to us. The quantum number $n$ depicts a new wave solution $\Psi_{n}^{\text{dtn}}$ of the Schrödinger wavelike equation. The spectrum of original potential is discrete for the existence of quantum number $n$. The non-trivial radiation can reveal much valuable information that characterizes the higher dimensional background, such as the dimensionality of space, the topological structure and so on. Here, the QN frequencies of the 5D Ricci-flat black string are determined by the black hole mass $M$, the effective cosmological constant $\Lambda$, the quantum number $n$ and the thickness of the bulk $y_1$. The information about an extra dimension is encoded in these QN frequencies such as the magnitude of extra dimension and the thickness of the bulk. It is known that the best method to probe the black hole is the detectable QN spectrum. If an extra dimension does exist and is visible near the black hole, maybe these QN frequencies can prove its existence.
(3) To ensure the validity of results, tables and figures, we use Mathematica software to design a program and calculate carefully these QN frequencies. The induced four dimensional results ($\Omega = 0$) are exactly identical to the 4D SdS black hole’s [39]. Otherwise, just as the 4D case [39], the QN frequencies decrease with the increasing cosmological constant $\Lambda$. Of course, this program is tested on some other black holes such as a 4D Schwarzschild black hole [36], and the same results are obtained, which are not presented in this paper. This method gets the desired effects and is hence believable.

(4) The reason why we discuss not the gravitational perturbation but a test scalar field is that this paper is a continuation of the previous work [16] to a certain extent. As we know from the spirit of string theory, the standard model fields are confined on a 3-brane except for gravitons and scalar particles. The original goal to introduce the scalar field is to examine the effect of an extra dimension on black hole radiation. Surprisingly, the radial component of the 5D Klein–Gordon equation can be rewritten exactly as the Regge–Wheeler form. Considering the QNMs boundary condition, we have studied the spectrum of QN frequencies by the usual third-order WKB method. After all, the QNMs of higher dimensional black hole are very attractive and interesting. Certainly, the basic gravitation field is detected more easily by the gravitational wave than other fields. Anyway, it is interesting to study this case and further work is needed.

(5) It should be noted that the parameter $\Omega$ is not the same as $\Omega_n$. In fact, parameter $\Omega$ is introduced to separate the variables $R_\omega(r, t)$, $L(y)$ and $Y_{lm}(\theta, \phi)$ in this paper. But parameter $\Omega_n$ is a particular eigenvalue which is deduced from the original $\Omega$ under the standing wave condition [16],

$$y_1 \sqrt{\Omega - \frac{3}{4} \Lambda} = n \pi.$$  

In other words, $\Omega$ is a free parameter, while $\Omega_n = \frac{n^2 \pi^2}{y_1^2} + \frac{3}{4} \Lambda$ is constrained by $n$, $y_1$ and $\Lambda$. Meanwhile, the quantum number $n$ is a positive integer i.e. $n > 0$ according to the condition (24) [16]. Furthermore, the cosmological constant $\Lambda$ is nonzero in the de Sitter universe. So considering the conditions $n > 0$ and $\Lambda > 0$, we can get the eigenvalue $\Omega_n > 0$. By the way, when $n = 0$ the steady stand wave (9) cannot be formed and this case must be abandoned. On the other hand, the potential function (13) indicates that the case $\Omega = 0$ (not $n = 0$) corresponds to usual 4D SdS. Hence, the notations $\Omega = 0$ in the figures and the tables just show that of 4D SdS. Of course, some other notation could be used in principle.

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References

[1] Regge T and Wheeler J 1957 Phys. Rev. 108 1063
[2] Vishveshwara C V 1970 Nature 227 936
[3] Press W H 1971 Astrophys. J. 170 L105
[4] Chandrasekhar S 1975 Proc. R. Soc. A 343 289
Blome H-J and Mashhoon B 1984 Phys. Lett. A 100 231
Ferrari V and Mashhoon B 1984 Phys. Rev. D 30 295
Leaver E W 1985 Proc. R. Soc. A 402 285
Nollert H-P and Schmidt B G 1992 Phys. Rev. D 45 2617
Arkani-Hamed N, Dimopoulos S and Dvali G R 1998 *Astrophys. J.* 16 R159
[6] Kokkotas K D and Schmidt B G 1999 *Living Rel. Rev.* 2 2 (Preprint gr-qc/9909058)
[7] Wesson P S 1999 *Space-Time-Matter* (Singapore: World Scientific)
[8] Overduin J M and Wesson P S 1997 *Phys. Rep.* 283 303 (Preprint gr-qc/9805018)
[9] Macias A, Fuentes y Martinez G J and Obregon O 1993 *Gen. Rel. Grav.* 25 549
[10] Lim P H and Wesson P S 1992 *Astrophys. J.* 397 L91
[11] Billyard A, Wesson P S and Kalligas D 1995 *Int. J. Mod. Phys. D* 4 639
[12] Liu H Y and Wesson P S 1996 *Phys. Lett. B* 381 420
[13] Liu H Y 1991 *Gen. Rel. Grav.* 23 759
[14] Liu H Y and Wesson P S 1992 *J. Math. Phys.* 33 3888
[15] Mashhoon B, Liu H Y and Wesson P S 1994 *Phys. Lett. B* 331 305
[16] Liu M L, Liu H Y, Xu L X and Wesson P S 2006 *Mod. Phys. Lett. A* 21 2937 (Preprint gr-qc/0611137)
[17] Liu M L, Liu H Y, Luo F and Xu L X 2007 *Gen. Rel. Grav.* 39 1389 (Preprint 0705.2465)
[18] Liu H Y and Wesson P S 2000 *Astrophys. J.* 538 386 (Preprint gr-qc/0003034)
[19] Arkani-Hamed N, Dimopoulos S and Dvali G R 1999 *Phys. Lett. B* 429 263 (Preprint hep-ph/9803315)
Arkani-Hamed N, Dimopoulos S and Dvali G R 1999 *Phys. Rev. D* 59 086004 (Preprint hep-ph/9807344)
Antoniadis I, Arkani-Hamed N, Dimopoulos S and Dvali G R 1998 *Phys. Lett. B* 436 257 (Preprint hep-ph/9803315)
[20] Randall L and Sundrum R 1999 *Phys. Rev. Lett.* 83 4690 (Preprint hep-th/9906064)
[21] Randall L and Sundrum R 1999 *Phys. Rev. Lett.* 83 3370 (Preprint hep-th/9905221)
[22] Chamblin A, Hawking S W and Reall H S 2000 *Phys. Rev. D* 61 065007
[23] Dadhich N, Maartens R, Papadopoulos P and Rezzolla V 2000 *Phys. Lett. B* 487 1 (Preprint hep-th/0003061)
[24] Ponce de Leon J 2001 *Mod. Phys. Lett. A* 16 2291 (Preprint gr-qc/0111011)
[25] Seahra S S 2003 *Phys. Rev. D* 68 104027 (Preprint hep-th/0309081)
[26] Liu H Y 2003 *Phys. Lett. B* 560 149 (Preprint hep-th/0206198)
[27] Emparan R G, Horowitz T and Myers R C 2000 *Phys. Rev. Lett.* 85 499 (Preprint hep-th/0003118)
Dimopoulos S and Landsberg G 2001 *Phys. Rev. Lett.* 87 161602 (Preprint hep-ph/0106295)
Giddings S B and Thomas S 2002 *Phys. Rev. D* 65 056010 (Preprint hep-ph/0106219)
[28] Cardoso V, Lemos J P S and Yoshida S 2003 *J. High Energy Phys.* JHEP12(2003)041 (Preprint hep-th/0311260)
Cardoso V, Lemos J P S and Yoshida S 2004 *Phys. Rev. D* 69 044004
Kanti P and Konoplya R A 2006 *Phys. Rev. D* 73 044002 (Preprint hep-th/0512257)
Konoplya R A 2003 *Phys. Rev. D* 68 124017 (Preprint hep-th/0309330)
Kanti P, Konoplya R A and Zhidenko A 2006 *Phys. Rev. D* 74 064008 (Preprint gr-qc/0607048)
[29] Mashhoon B, Wesson P S and Liu H Y 1998 *Gen. Rel. Grav.* 30 555
[30] Wesson P S, Mashhoon B, Liu H Y and Saiko W N 1999 *Phys. Lett. B* 456 34
[31] Liu H Y and Mashhoon B 2000 *Phys. Lett. A* 272 26 (Preprint gr-qc/0005079)
[32] Mashhoon B and Wesson P S 2004 *Class. Quantum Grav.* 21 3611 (Preprint gr-qc/0401002)
[33] Ping Y L, Liu H Y and Xu L X 2007 *Int. J. Mod. Phys. A* 22 985 (Preprint gr-qc/0610094)
[34] Breivik I and Simonsen B 2001 *Gen. Rel. Grav.* 33 1839
[35] Iyer S and Will C M 1987 *Phys. Rev. D* 35 3621
[36] Iyer S 1987 *Phys. Rev. D* 35 3632
[37] Konoplya R A 2003 *Phys. Rev. D* 68 024018 (Preprint gr-qc/0303052)
[38] Konoplya R A 2002 *Gen. Rel. Grav.* 34 329 (Preprint gr-qc/0109096)
Zaslavskii O B 1991 *Phys. Rev. D* 43 605
Simone L E and Will C M 1992 *Class. Quantum Grav.* 9 963
Kokkotas K and Schutz B F 1988 *Phys. Rev. D* 37 3378
Berti E and Kokkotas K 2003 *Phys. Rev. D* 67 064020 (Preprint gr-qc/0301052)
Zhidenko A 2004 *Class. Quantum Grav.* 21 273 (Preprint gr-qc/0307012)