Plasma graviton production in TeV-scale gravity

E Yu Melkumova
Department of Physics, Moscow State University, 119899, Moscow, Russia
E-mail: elenamelk@sr.d.sinp.msu.ru

Abstract. We develop the theory of interaction of classical plasma with Kaluza-Klein (KK) gravitons in the ADD model of TeV-scale gravity. Plasma is described within the kinetic approach as the system of charged particles and Maxwell field both confined on the brane. Interaction with multidimensional gravity living in the bulk with \( n \) compact extra dimensions is introduced within the linearized theory. The KK gravitons emission rates are computed taking into account plasma collective effects through the two-point correlation functions of the fluctuations of the plasma energy-momentum tensor. Apart from known mechanisms (such as bremsstrahlung and gravi-Primakoff effect) we find essentially collective channels such as the coalescence of plasma waves into gravitons which may be manifest in turbulent plasmas. Our results indicate that commonly used rates of the KK gravitons production in stars and supernovae may be underestimated.

1. Introduction

TeV-scale gravity proposal due to Arkani-Hamed, Dimopoulos and Dvali (ADD) [1] suggests that the standard model particles live in the four-dimensional subspace (the brane) of the D-dimensional bulk with \( n = D - 4 \) extra dimensions compactified on a torus which are inhabited only by gravity. The D-dimensional Planck mass \( M_* \) is supposed to be TeV-scale, and the large extra dimensions (LED) to have sub-millimeter size. In this scenario there is an infinite tower of massive KK gravitons [2] whose existence may be detected using table-top and collider experiments, astrophysical and cosmological observations. It has been pointed out that one of the strongest bounds on the parameters come from the supernova SN1987A data [1]-[10]. Processes contributing to energy loss due to graviton emission from SN1987A, the red giants and the Sun include photon-photon annihilation, electron-positron annihilation, nucleon-nucleon gravitational bremsstrahlung, gravi-Compton-Primakoff scattering [3]-[10].

Previous calculations of the KK graviton emission rates were performed using one-particle Feynman rules with subsequent averaging over Bose-Einstein and Maxwell-Boltzmann distributions to get the results reliable for finite temperature \( T \) [4]. These calculations, however, did not take into account plasma collective effects such as Debye screening, interaction of charged particles with plasma waves etc, which may be important in astrophysical conditions. This is the goal of the present contribution. We generalize kinetic theory of interaction of plasmas with gravitational waves developed
earlier in 3 + 1 dimensions [11] to the ADD model and present calculation of gravibremssstrahlung rate from non-relativistic thermal isotropic electron-ion plasma.

2. Kinetic approach

Consider collisionless plasma consisting of charged particles of types $\alpha$ with parameters $e_\alpha, m_\alpha$ described by microscopic distribution function [12, 13]

$$F_\alpha(x, p) = \sum_{i=1}^{N_\alpha} \delta(r - r_i(t))\delta(p - p_i(t)),$$

normalized as $\int F_\alpha(x, p)\, dp\, d^3x = N_\alpha$, satisfying the kinetic equation

$$\frac{\partial F_\alpha}{\partial t} + v \frac{\partial F_\alpha}{\partial r} + e_\alpha(E + [vB]) \frac{\partial F_\alpha}{\partial p} = 0.$$

They interact via the electromagnetic field $E$, $B$ satisfying Maxwell equations

$$\text{div}E = 4\pi\rho, \quad \text{rot}E = -\frac{\partial B}{\partial t}, \quad \text{div}B = 0, \quad \text{rot}B = 4\pi j + \frac{\partial E}{\partial t},$$

with the source terms

$$\rho(x) = \sum_\alpha e_\alpha \int F_\alpha(x, p)\, dp, \quad j(x) = \sum_\alpha e_\alpha \int \frac{p}{p_\alpha^0} F_\alpha(x, p)\, dp.$$

To describe gravitational radiation we will need the 3-spatial components of the plasma energy momentum tensor

$$T_{ij} = mT_{ij} + fT_{ij},$$

where

$$mT_{ij} = \sum_\alpha \int \frac{p_i p_j}{p_\alpha^0} F_\alpha(x, p)\, dp, \quad p_\alpha^0 = \sqrt{p^2 + m_\alpha^2}, \quad x \equiv \{t, r\},$$

$$fT_{ij} = -\frac{1}{4\pi} \left( E_i E_j + B_i B_j - \delta_{ij} \frac{E^2 + B^2}{2} \right).$$

To solve the system of equations (2-4) we use perturbation theory in terms of the electric charges. First we separate fluctuations from the average distribution using the approach of [11, 12]:

$$F(t, r, p) = f_\alpha^0 + \delta f_\alpha + \delta f_\alpha,$$

where $f_\alpha^0 \equiv <F_\alpha(x, p)>$ - the equilibrium Maxwell distribution function:

$$f_\alpha^0 = \frac{N_\alpha}{(2\pi v_{T_\alpha}^2)^{3/2}} e^{-\frac{1}{2} \frac{v^2}{v_{T_\alpha}^2}}, \quad v_{T_\alpha} = \sqrt{\frac{T_\alpha}{m_\alpha}}, \quad v_{T_\alpha} << 1,$$

$\delta f_\alpha^0$ - represents fluctuations due to chaotic particle motion ("zero" fluctuations), while $\delta f_\alpha(t, r, p)$ - stands for fluctuations due to electromagnetic interaction of the particles. Using the fact that $f_\alpha^0$ satisfies the free equation (for zero charges), we obtain:

$$\frac{\partial \delta f_\alpha}{\partial t} + v \frac{\partial \delta f_\alpha}{\partial r} + e_\alpha(E + [vB]) \frac{\partial (f_\alpha^0 + \delta f_\alpha + \delta f_\alpha)}{\partial p} = 0.$$
We then further expand the fluctuation $\delta f_\alpha$ in power series in terms of charges: $\delta f_\alpha = \delta f^{1}_\alpha + \delta f^{2}_\alpha + \ldots$, and use the Fourier-transformation

$$f(x) = \frac{1}{(2\pi)^4} \int f(k) e^{-ik_\mu x^\mu} d^4k$$

to get in the first two orders

$$\delta f^{1}_\alpha(k, p) = - \frac{ie_\alpha}{(\omega - kv)} F(k) \frac{\partial f^{0}_\alpha}{\partial p}, \quad F \equiv E + [vB];$$

$$\delta f^{2}_\alpha(k, p) = - \frac{ie_\alpha}{(2\pi)^4(\omega - kv)} \int F(k - k_1) \frac{\partial}{\partial p} (\delta f^{0}_\alpha(k_1, p) + \delta f^{1}_\alpha(k_1, p)) d^4k_1.$$  \hspace{1cm} (10)

The corresponding expansion of the charged particles energy-momentum tensor will read:

$$mT_{ik} = mT^{0}_{ik} + \delta mT^{0}_{ik} + \sum_{1}^{\infty} \sum_{\alpha} \int \frac{\eta_i \eta_j}{T^{\alpha}_{\alpha}} \delta f^{1}_\alpha(x, p) dp.$$  \hspace{1cm} (11)

First non-zero contribution to gravitational radiation comes from terms of the second order in the fields $E, B$, namely $\delta T_{ik} = \delta mT^{2}_{ik} + \delta \varphi T_{ik}$.

3. Gravitational radiation

In ADD model one considers [2] the linearized $D = 4 + n$ dimensional Einstein equations

$$\Box \psi_{MN} = \kappa^2 D T_{MN},$$

where $\psi_{MN} = h_{MN} - \eta_{MN} h^P_P/2$, $h_{MN} = g_{MN} - \eta_{MN}$, the indices $M, N$ run over the brane $\mu, \nu = 0, 1, 2, 3$ and $n$ directions on the torus, and the harmonic gauge $\partial_N \psi^{MN} = 0$ is understood. Formula for the total (integrated over space and time) energy loss on gravitational radiation in ADD model was recently derived in [14]:

$$E = \frac{\kappa^2 D}{16\pi^3 V_d} \sum_{N \in \mathbb{Z}^d} \int d^k T_{SN}(k) T_{LR}(k) \tilde{\Lambda}^{SNLR}_{\alpha\beta\gamma\delta} \left|_{k^0 = \sqrt{|k|^2 + (2\pi N/\Lambda)^2}} \right|, \quad V_d = (2\pi L)^D - D,$$  \hspace{1cm} (12)

where $T_{SN}(k)$ is the four-dimensional Fourier-transform, and

$$\tilde{\Lambda}^{SNLR}_{\alpha\beta\gamma\delta} = \frac{1}{2} \left[ \eta^{SL}_{\alpha} \eta^{NR}_{\beta} + \eta^{SR}_{\alpha} \eta^{NL}_{\beta} \right] - \frac{1}{D - 2} \eta^{SN}_{\alpha} \eta^{LR}_{\beta}.$$  \hspace{1cm} (13)

is the polarization projection operator. Since the stress tensor has only brane indices $\mu = 0, i$ and it satisfies the conservation equation $k_\mu T^{\mu
u}(k) = 0$, one can eliminate its components $T^{\mu
u}$ in favor of $T^{\nu\rho}$. The resulting expression will contain the sum of various order terms from which the lowest non-zero contribution comes from the product of the second order terms. We also have to normalize the energy loss of the stationary plasma per unit volume and per unit time, which is achieved by introducing the two-point correlation function according to the relation

$$< T_{\alpha\beta}(k) T^{*}_{\alpha'\beta'}(k + \Delta k) > = < T_{\alpha\beta} T^{*}_{\alpha'\beta'} > k^\alpha(\Delta k).$$
The final expression for the gravitational emission rate to all graviton modes per unit plasma volume per unit time will read

\[
P = \frac{x^2}{16\pi^3 V_d} \sum_{\mathbf{k} \in \mathbb{Z}^d} \int d\mathbf{k} \text{<} \delta T_{ik} \delta T_{ik'}^* \text{>}_k \tilde{\Lambda}^{ik'k} \bigg|_{k^0 = \sqrt{|k|^2 + (2\pi N/L)^2}}, \tag{15}\]

\[
\tilde{\Lambda}^{ik'k} = \frac{1}{2} \left[ \Delta_{ik} \Delta_{kk'} + \Delta_{ik'} \Delta_{kk'} \right] - \frac{1}{D - 2} \Delta_{ik} \Delta_{kk'}, \quad \Delta_{ik} = \delta_{ik} - n_in_k, \quad n = \frac{k}{k^0}. \tag{16}\]

In our approach we did not use explicit decomposition of the full multidimensional metric perturbation \(h_{MN}\) in massless and massive modes, these correspond to \(N = 0\) and \(N \neq 0\) terms in the sum respectively. For massless modes the three-dimensional vector \(n\) is the unit vector, but not for massive modes. Note that \(h_{MN}\) has bulk components due to the trace term in the wave equation.

To calculate gravitational bremsstrahlung from the non-relativistic plasma we need the longitudinal field and the related fluctuation function. Keeping as the source in Maxwell equations terms of the zero and the first order

\[
\text{div} \mathbf{E} = 4\pi e \int d\mathbf{v} (\delta f^0 + \delta f^1), \quad \delta f^1 = -i \frac{e}{m\omega - \mathbf{k}v} \mathbf{E}_k \frac{\partial f^0}{\partial \mathbf{v}}, \tag{17}\]

one can express the first order electric field through zero fluctuations:

\[
\mathbf{E}_k = -4\pi i \frac{k}{|k|^2 \varepsilon_L(k)} \rho^0_k, \quad \rho^0_k = \sum e \int d\mathbf{v} \delta f^0_k(v), \tag{18}\]

\[
\varepsilon_L(k) = 1 + \frac{4\pi e^2}{m\omega |k|^2} \int dp \frac{(k\mathbf{v})}{(\omega - k\mathbf{v})} k \frac{\partial f^0_k(v)}{\partial \mathbf{v}}, \tag{19}\]

the last line being the longitudinal permittivity of the isotropic plasma. Using this, one then finds the two-point correlation functions defined above \[13\]. Correlation function for zero-order particle density \(\delta n^2_0\) is calculable for the Maxwell distribution (8) in the finite form, while the field correlation functions can be then expressed through this quantity. Using the equations (17) and (18) one find the following relations between \(\delta \rho^2_k, \delta E^2_k\) and \(\rho^2_k\):

\[
< \delta \rho^2_k >_k = < \rho^2_k >_k, \quad < E^2 >_k = \frac{16\pi^2 |\rho^2_k >_k|}{|\varepsilon_L(k)|^2}, \quad < \delta \rho^2 >_k = \rho^2_k = \sqrt{\frac{2\pi^3 n_0 e^2}{|k|v_T^2}} e^{-\frac{|k|^2}{2v_T^2}}. \tag{20}\]

Substituting this to (fluctuations of) the energy-momentum tensors and making use of the formula (15), one obtains after some rearrangements:

\[
< \delta T_{ik} \delta T_{ik'}^* \text{>}_k \tilde{\Lambda}^{ik'k'} = \int d^4k \left\{ A(k_1, k) \frac{T}{|\varepsilon_L(k_2)|^2} < \delta n^2 >_{k_1} + B(k_1, k) < \delta n^2 >_{k_1} < \delta n^2 >_{k_2} \right\}. \tag{21}\]

According to the relations (20), the first term here is the product of the field and particle density correlation functions, while the second is quadratic in the field correlators. The following frequency regions has to be distinguished:

\[\text{Density fluctuations can be characterized by correlation functions whose Fourier-transforms exhibits homogeneity and isotropy: } < \delta n(r_1, t_1) \delta n(r_2, t_2) > \equiv < \delta n^2 >_{r,t}, \quad r = r_1 - r_2, t = t_1 - t_2. \quad \text{The space-time Fourier transformation will give the spectral fluctuation density: } < \delta n^2 >_{k} = \int d\mathbf{r} \int dt e^{-ikr + i\omega t} < \delta n^2 >_{r,t}, < \delta n(k)\delta n(k') > = (2\pi)^4 < \delta n^2 >_k \delta^4(k - k').\]
Debye shielding becomes manifest, then
\[ \omega < \omega_L, \quad \omega_L = \sqrt{\frac{4\pi^2 e^2 N_0}{m}} \]
and the long wavelength: \( |\mathbf{k}| < \frac{1}{r_D^*} \), where \( r_D^* = \frac{r_d}{\omega_L} \) is the Debye shielding radius.

ii) For \( \omega > \omega_L \), \( |\mathbf{k}|v_T \gg 1 \), then \( \varepsilon_L \simeq 1 \) which corresponds to electromagnetic interaction switched off.

iii) For \( \omega < \omega_L \) one has \( \varepsilon_L(k) \simeq 1 + (|\mathbf{k}|r_D)^{-2} \) which corresponds to the Debye shielding.

Correspondingly, we have to distinguish radiation losses due to gravitational bremsstrahlung (denominators are non-zero, both terms being taken into account), and the coalescence of two Langmuir plasmons into graviton (the second term with both denominators at Langmuir frequencies).

### 3.1. Gravitational bremsstrahlung

This corresponds to \( Re\varepsilon_L \neq 0 \). The main contribution to the integral comes from the region \( \omega < |\mathbf{k}|v_T \), so for \( k_0 > \omega_L \), \( |\mathbf{k}|r_D > 1 \), then \( \varepsilon_L \simeq 1 \). In opposite limit \( k_0 < \omega_L \), Debye shielding becomes manifest, then \( \varepsilon_L(k_1) \simeq 1 + (|\mathbf{k}|r_D)^{-2} \). Therefore we obtain in the two cases:

\[
P \simeq \frac{128}{\sqrt{\pi}}(1 + \sqrt{2})e^4 N_0^2 v_T \frac{T\ln n}{M^2} I(n) \ln \Lambda + \frac{8}{3} I_2(n), \quad k_0 > \omega_L, \quad (22)
\]

\[
P \simeq \frac{128}{\sqrt{\pi}}(1 + \sqrt{2})e^4 N_0^2 v_T \frac{\omega^4}{M^4} I(n) \ln \Lambda, \quad k_0 < \omega_L, \quad (23)
\]

where \( \ln \Lambda \) is the Coulomb logarithm, \( \Lambda = r_D^*/\hat{r} \), \( r_D^* = \frac{r_d}{\omega_L} \) - is the electron Debye radius, \( \hat{r} \) is determined from the condition \( e^2/\hat{r} = T \) and \( M_* = \frac{1}{\pi_D^*} \) is the D-dimensional Planck mass. The coefficients \( I_1(n), I_2(n) \) depend on the number of extra dimensions \( n \), \( 2 \leq n \leq 7 \) as follows: \( I_1(n) = 1.1, 1.07, 0.97, 0.83, 0.67, \quad I_2(n) = 4.8, 6.35, 7.3, 7.5, 7.1 \). If \( n = 0 \) \( I_1(n) = 1, I_2(n) = 1 \) these results coincide with those in 3+1 dimensions [11]. Without collective effects, the result in 3+1 dimensions corresponds to the quadruple formula calculation with consequent thermal averaging [15].

### 3.2. Coalescence of two Langmuir plasmons into graviton

For coalescence of two Langmuir plasmons into the graviton the field correlation function can be presented as

\[
\langle E^2 \rangle = 4\pi^2 T(\delta(\omega - \omega_L) + \delta(\omega + \omega_L)) \quad (24)
\]

The graviton frequency will be twice the Langmuir frequency and the rate of emission will be

\[
P^0 = 2e^4 N_0^4 v_T \frac{(2\omega_L)^{n+1}}{M_*^{n+2}} I_n, \quad (25)
\]
with \( I_n = 1.4, 1.9, 2.2, 2.3, 2.2, \) for \( n = 2, \ldots, 6. \)

Compare with the result of the ref. [4] for the bremsstrahlung losses which was obtained using the one-particle approach:

\[
P \simeq \sum_j \frac{\Gamma\left(\frac{3}{2} + n\right)n_en_jZ_j^2\alpha^2I_{GB}(n)}{\Gamma\left(\frac{3}{2}\right)} T^{n+1}M_s^{n+2}, \tag{26}
\]

where \( n_e \) and \( n_j \) are the electrons and ions densities, \( Z_j \) is the ions charge, \( \alpha = e^2 \), and the numerical integral \( I_{GB}(n) \) is presented for \( n = 2, 3, 4, 5, 6 \).

Their results are obtained using the one-particle approach (26). Being applied to red giants with the electron temperature \( T \sim 8.6 \text{ keV} \) and the electron density \( n_e = 3.0 \cdot 10^{29} \text{ cm}^{-3} \), assuming the conservative upper limit on the energy-loss rates of red giants \( \dot{\varepsilon} \sim 100 \text{ erg g}^{-1} \text{ sec}^{-1} \), the authors of [4] obtained the upper limit for Planck mass \( M_s \) to be \( 10^6 \) for \( n = 2 \) and \( 1.7 \cdot 10^{-5} \) for \( n = 3 \). Our result (23) for the bremsstrahlung losses is about three time greater than (26). Moreover, an additional enhancement comes from the in coalescence process (25).

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