A Flexible Integrated Scheduling Algorithm for Dynamic Determination Processing Equipment Considering the Root-Subtree Vertical and Horizontal Pre-Scheduling

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ABSTRACT Aiming at the existing flexible integrated scheduling algorithms that use unified rules to solve the conflicts between processes and equipment, and ignore the vertical and horizontal characteristics of the process relative positions in the process tree, a flexible integrated scheduling algorithm for dynamic determination processing equipment considering the Root-Subtree vertical and horizontal pre-scheduling (FISA-DEVH) is proposed. The process sorting strategy of the Root-Subtree vertical and horizontal pre-scheduling is proposed to sort the Root-Subtree processes of the same planned processing equipment in groups and load the Root-Subtree processes into the corresponding equipment queues, so as to fully explore the vertical and horizontal characteristics of the process. In order to give full play to the characteristics of the flexible equipment and realize the effective parallel processing between the processes, the flexible equipment optimal scheduling strategy of dynamically determining process is proposed to determine the final processing equipment of the process at each equipment driving moment and complete the production scheduling process from the Root-Subtree static pre-scheduling to dynamic fusion; The strategy uses the scheduling relationship between the unplanned processes and unplanned equipment to reduce the search scope of the combination scheme and improve the efficiency of the algorithm. The example tests show that FISA-DEVH has a better scheduling effect than the flexible integrated scheduling algorithm that processes the process tree with unified rules, and has universal significance for solving the flexible integrated scheduling problem.

INDEX TERMS Flexible integrated scheduling, root-subtree vertical and horizontal pre-scheduling, the same planned processing equipment, the unplanned processes, the unplanned equipment.

I. INTRODUCTION

Facing the trend of the market changes, the high-cost special equipment has been unable to meet the production needs of the enterprises, while the flexible equipment can meet the changes in market demand and allocate production capacity for multi-type products [1]. The scheduling system is the key technology of the industrial manufacturing system, compared with the traditional Job-shop scheduling problem, the flexible Job-shop scheduling problem (FJSP) [2], [3] is more in line...
with the actual needs of the current enterprise production environment, which is mainly reflected in that the flexible equipment can handle different types of processes. The FJSP not only needs to solve the process scheduling sequence problem, but also selects a reasonable processing equipment for the process, which increases the complexity of combinatorial optimization, and is a more complex NP-Hard problem. Some results have been achieved for the FJSP, which mainly focus on dynamic flexible Job-shop scheduling, distributed flexible Job-shop scheduling and multi-objective green flexible Job-shop scheduling [4], [5], [6], [7], [8], [9]. Although these algorithms solve the diversified and multi-objective FJSP, they are mainly aimed at the production scheduling problem of pure processing or assembly. For the diverse and small-batch products with tree-like constraints in the current actual production environment, Separation of processing and assembly could inevitably ignore the parallel relationship between the various processing stages of the product itself, the actual production process of the flexible workshop extends the entire product production cycle. Therefore, Xie et al. [10] proposed a flexible integrated scheduling that considers both product processing and assembly parallel relation, which is more suitable for the production needs of a small number or single complex products, the tree-like processing tree of the product is shown in Figure 1. So far, the integrated scheduling problem has been deeply studied in the single workshop, the special constraint relation and multi-workshops, and it is gradually developing to the direction of the flexible equipment manufacturing according to the actual production demand. The product model of the modular design idea [11], [12] is the same as the product process tree model of the flexible integrated scheduling production. Therefore, the flexible integrated scheduling problem has become a popular research direction. The existing flexible integrated scheduling algorithm uses strategies such as quasi-critical path [10], substantial path [13], [14] and reverse order layer priority [15] to complete all process sorting, and uses strategies such as shortest time, idle device driver and equipment preemption to determine the processing equipment of the process for completing the final scheduling scheme. It can be seen that these algorithms use the same scheduling rule for overall analysis and sorting of all processes in the product process tree, and ignores the vertical and horizontal characteristics of the process position on the process tree. Each scheduling is a large number of searches for all schedulable processes to determine the final processing equipment of the process.

In view of the above problems, this paper proposes a flexible integrated scheduling algorithm for dynamic determination processing equipment considering the Root-Subtree vertical and horizontal pre-scheduling (FISA-DEVH). Firstly, the scope of the process selection processing equipment is narrowed according to the shortest processing time method, and the product process tree model with the flexible integrated scheduling characteristics is converted into an improved process tree model with the same equipment characteristics, the decomposition granularity control principle is used to split the improved process tree into several Root-Subtrees to simplify the scale of the process tree and prepare for the analysis of the vertical and horizontal characteristics of the process in the internal constraint relationship of the flexible process tree; Then, all Root-Subtrees are pre-scheduled by the improved algorithm mainly based on vertical scheduling [18] and mainly on horizontal scheduling [19], it obtains the vertical and horizontal pre-scheduling completion time of each Root-Subtree, compares the vertical and horizontal pre-scheduling completion time of the Root-Subtree itself, excavates the vertical and horizontal advantages of each Root-Subtree, and determines the planned processing equipment of the process, the priority of all Root-Subtrees is set according to the selected pre-scheduling completion time from long to short, in combination with the equipment process pre-start time, all Root-Subtree processes are grouped and sorted with the planned processing equipment sequence, and the vertical and horizontal attributes of the process are used to enhance the process scheduling compactness in the improved process tree model; Next, according to the characteristics of the flexible processing equipment, the flexible equipment optimal scheduling strategy of dynamically determining process is designed to find more suitable processing equipment for the process, even if a longer processing time equipment is selected in the fusion scheduling process of some Root-Subtree processes, the effect of early processing and higher equipment utilization can be obtained; Finally, the non-Root-Subtree processes are retrieved and added to the Root-Subtree process scheduling sequence to form the final scheduling scheme. The main contributions of this paper are as follows:

- FISA-DEVH uses the static vertical and horizontal pre-scheduling method to excavate the urgency of the process itself, and truly achieves both vertical and horizontal scheduling process, that is, the process with high urgency is prioritized;
- The final processing equipment of the process is flexibly determined through dynamic fusion, which improves the substantial parallel processing capability of the process. The flexible equipment optimal scheduling strategy of dynamically determining process reduces the search and comparison scope of the
slicable process combination scheme and improves the algorithm solution efficiency under the premise of ensuring the completion time;
- The example verification shows that FISA-DEVH can effectively improve the equipment utilization rate, shorten the production cycle, and enhance the adaptability to product examples of different structures.

II. RELATED WORKS

This section reviews the related works on the flexible integrated scheduling algorithms, aiming to solve the flexible integrated scheduling problem better. A new algorithm for complex product flexible scheduling with constraint between jobs (FISA-CPFS) in literature [10] uses a distributed method to select the processing equipment for the process according to the shortest time-consuming strategy, and then uses the quasi-critical path method to determine the processing sequence of the processes; The algorithm in literature [13] uses the equipment driving moment to determine the process equipment according to the shortest processing time, and combines the substantial short-path strategy to find the combination scheme that ends as soon as possible to determine the final processing equipment and processing sequence; Dynamic parallel integrated flexible scheduling algorithm based on device driver and essential path (FISA-DDEP) in literature [14] adopts the idle device driving strategy and the dynamic substantial path strategy to determine the processing equipment and the processing sequence of the parallel process, and realizes the substantial parallel processing between the processes; Flexible integrated scheduling algorithm based on reverse order layer priority (FISA-ROLP) in literature [15] uses the reverse order layer priority strategy and the dynamic quasi-length path strategy to determine the scheduling order of the process, and uses the equipment selection strategy and the device preemption strategy to determine the processing equipment of the target process; The literature [16], [17] proposed an intelligent algorithm based on the genetic algorithm framework to solve the flexible integrated scheduling problem. These algorithms are simply based on the vertical or horizontal strategy of the process tree to determine the processing equipment and the processing sequence, which cannot achieve both vertical and horizontal, and rely too much on the product structure attributes. When selecting the final processing equipment, the process needs to search a lot for all schedulable process combination schemes. Since the total time for manufacturing products is limited by the vertical and horizontal aspects of the process tree and the processing equipment, at the same time, the different product structures lead to different vertical and horizontal characteristics of the process tree internal partial order relationship. Therefore, it is necessary to study the internal characteristics of inside the product process tree in the flexible integrated scheduling problem, which adapts to the production forms of products with different structures. At present, there is no research result of the flexible integrated scheduling algorithm for vertical and horizontal characteristics analysis inside the product process tree.

III. PROBLEM DESCRIPTION AND ANALYSIS

A. PROBLEM DESCRIPTION AND RELATED DEFINITIONS

The flexible integrated scheduling problem is very different from the general integrated scheduling problem [20], which is mainly reflected in the multiple selection of the processing equipment in the flexible production process, which makes the process of obtaining the product production scheduling plan more complicated, and this complexity mainly comes from the parameters of various production equipment, product modules, and the variability of the combination between modules, etc., so a better flexible scheduling system is needed to exert the characteristics of the flexible integrated scheduling. The flexible integrated scheduling is mainly for scheduling the product flexible process tree with the decomposition of the product production tasks and the constrained structure, where the nodes of the process tree represent processing or assembly processes, and the directed edges represent the partial order relationship between processes, each process attribute is composed of process name, processable equipment set and corresponding processing time. Since the flexible processing equipment can handle different types of processes, and different equipment have basically different processing times for the same process, the flexible integrated scheduling needs to meet the following constraints: (i) the processes between different Root-Subtrees have parallel processing characteristics, but there is a competition relationship between processing equipment; (ii) the processing sequence of the process must meet the constraints of the process tree structure; (iii) there is a one-to-many relationship between process and flexible processing equipment, that is, the process can be selectively processed on multiple processing equipment; (iv) each processing equipment can only schedule one process in the same time period, and the processing process cannot be interrupted; (v) the same flexible processing equipment does not exist; (vi) the process is allowed to wait, and the processing equipment is allowed to be idle. In order to facilitate the description of FISA-DEVH, the relevant definitions are described as follows:

Definition 1: Improved process tree: The process tree model of the flexible integrated scheduling problem is simplified to an integrated scheduling process tree model with the same equipment characteristics according to the shortest processing time method.

Definition 2: Planned processing equipment: It selects the initial processing equipment for the process by the selected vertical or horizontal pre-scheduling scheme.

Definition 3: Planned process: In the pre-scheduling process, the process that has a binding relationship with the planned processing equipment.

Definition 4: Unplanned processing equipment: At equipment driving moment, the processing equipment is in an idle state, and the first process of the corresponding queue is empty or an unschedulable process.

Definition 5: Unplanned process: The first process of the queue is in a schedulable state, and the planned processing equipment corresponding to this process is in a busy state.
Definition 6: Vertical pre-scheduling completion time: The completion time of the Root-Subtree with the same processing equipment characteristics is pre-scheduled through the improved algorithm ① “dynamic critical path, short time” strategy in literature [18].

Definition 7: Horizontal pre-scheduling completion time: The completion time of the Root-Subtree with the same processing equipment characteristics is pre-scheduled through the improved algorithm ② “layer first, short time, long path” strategy in literature [19].

Definition 8: Root-Subtree priority: It sets the parameters of the Root-Subtree scheduling priority through the selected Root-Subtree vertical or horizontal pre-scheduling completion time.

Definition 9: Virtual root nodes: A node formed by merging all the processes between the root node process and its direct or indirect immediately preceding process, and the node’s in-degree is greater than 1 for the first time.

Definition 10: Equipment process pre-start time: It obtains the start processing time of the Root-Subtree processes on the corresponding planned processing equipment according to the selected Root-Subtree pre-scheduling scheme.

B. MATHEMATICAL MODEL ANALYSIS OF THE PROBLEM

It assumes that there is a product flexible process tree model, which consists of N processes, and can be split into G Root-Subtrees, the workshop includes M flexible processing equipment. The scheduling goal is to determine the scheduling sequence of the processes, and to find suitable processing equipment for each process under satisfying the constraint relation of the process tree, so as to obtain a production scheduling scheme with the shortest possible production time. The mathematical model of FISA-DEVH is as follows:

\[ T = \min \{ \max \{ F_m \} \} , \ 1 \leq m \leq M \]  \hspace{1cm} (1)

\[ s.t. \ \min \{ T_{n,1}, T_{n,2}, \ldots, T_{n,m} \} , \ 1 \leq n \leq N \]  \hspace{1cm} (2)

\[ s.t. \ \min \{ F_{n,m} \} , \ 1 \leq n \leq N \]  \hspace{1cm} (3)

\[ F_{n,m} = S_{n,m} + T_{n,m} \]  \hspace{1cm} (4)

\[ S_{n,m} \geq F_{n-1,m} \]  \hspace{1cm} (5)

\[ S_{n,m} \geq \max \{ F_{i,j}, \ldots, F_{x,y} \} \]  \hspace{1cm} (6)

\[ T^r_g / T^r_H = \begin{cases} > 1 & \text{if} \ 1 \leq g \leq G \\ = 1 & \text{if} \ < 1 \end{cases} \]  \hspace{1cm} (7)

In the above function, T represents the processing completion time of the product; \( F_m \) represents the processing completion time of the last process on the flexible processing equipment \( M_m \); \( T_{n,m} \) represents the processing time of the process \( P_n \) on the flexible processing equipment \( M_m \); \( S_{n,m} \) represents the start processing time of the process \( P_n \) on the flexible processing equipment \( M_m \); \( F_{n,m} \) represents the processing completion time of the process \( P_n \) on the flexible processing equipment \( M_m \); \( T^r_g \) represents the vertical pre-scheduling completion time of the Root-Subtree \( r^g \); \( T^r_H \) represents the horizontal pre-scheduling completion time of the Root-Subtree \( r^g \); \( r^g \) represents the Root-Subtree numbered \( g \) in the process tree. The function (1) indicates the objective function of FISA-DEVH, that is, the total processing time of the product is as short as possible; The function (2) indicates that a unique processing time and multiple processing equipment with the same function are determined for improved process tree; The function (3) indicates that the process is processed on the processing equipment that is finished as soon as possible; The function (4) indicates that each process is continuously processed on the processing equipment, and cannot be interrupted once processed; The function (5) indicates that the start processing time of the process must be greater than or equal to the processing completion time of the immediately preceding process on the same equipment; The function (6) indicates that the start processing time of each process must be greater than or equal to the processing completion time of all its immediately preceding processes; The function (7) indicates the ratio of the vertical pre-scheduling completion time to the horizontal pre-scheduling completion time of the Root-Subtree \( r^g \), and the vertical or horizontal advantage of the Root-Subtree is obtained according to different comparison results.

IV. DESIGN OF THE ALGORITHM SCHEDULING STRATEGY

A. DESIGN OF THE PROCESS SORTING STRATEGY OF THE ROOT-SUBTREE VERTICAL AND HORIZONTAL PRE-SCHEDULING

1) DESIGN OF THE FLEXIBLE PROCESS TREE

SIMPLIFICATION DECOMPOSITION STRATEGY

In order to mine the vertical and horizontal characteristics of the process in the flexible process tree, this paper needs to perform simplified preprocessing and decomposition preprocessing on the flexible process tree model. The process tree simplification preprocessing is to convert the product process tree with the flexible integrated scheduling characteristics into an improved process tree with the same equipment characteristics through the shortest processing time method, so as to reduce the complexity of the product process tree. The process tree decomposition preprocessing [21], [22] is to divide the improved process tree into several independent subtrees according to certain decomposition granularity rules. If the granularity is too coarse, the internal vertical and horizontal characteristics of the process tree cannot be analyzed; if the granularity is too fine, it will greatly increase the complexity of the algorithm. Therefore, the decomposition granularity control principle is to split the improved process tree into several Root-Subtrees. According to the structure of the process tree, all Root-Subtrees are named from left to right, that is, \( \{ r^1, r^2, \ldots, r^G \} \). Since different products have different structural characteristics, it is necessary to judge whether the root node of the improved process tree has more than 2 immediately preceding processes, if so, it directly loads the root node into the independent stack space and divides the improved process tree into several Root-Subtrees according to the decomposition granularity control principle; If not, then the root node of the process tree and its immediately
preceding process are merged into a virtual root node, and the loop judges until the virtual root node has two or more immediately preceding processes. Then, it loads all process nodes contained in the virtual root node into the independent stack space in the reverse order of the improved process tree structure, and the remaining processes are divided into several Root-Subtrees.

2) DESIGN OF THE ROOT-SUBTREE VERTICAL AND HORIZONTAL ADVANTAGE JUDGMENT STRATEGY

Since the algorithm ① and algorithm ② cannot be applied to the integrated scheduling problem with the same equipment characteristics, this paper needs to make appropriate modifications to the algorithm ① and algorithm ② under not changing the characteristics of the algorithm itself. On the basis of the original algorithm strategy, the early processing strategy and equipment balancing strategy [23], [24], [25] are introduced into the algorithm ① and algorithm ②. Among which, the early processing strategy is to load the process with multiple machinable equipment to the equipment with the earliest processing time; The equipment balancing strategy is that when a process with multiple machinable equipment has the same earliest start processing time on these equipment, the process is selected to be processed on the equipment with the smallest sum of the processing time. After the Root-Subtree set is formed, the improved algorithm ① and algorithm ② are used to pre-schedule each Root-Subtree respectively to obtain \(T_{Z}^{rt}\) and \(T_{H}^{rt}\). Next, it judges whether the Root-Subtree has a vertical advantage or a horizontal advantage according to the ratio results of \(T_{Z}^{rt}\) and \(T_{H}^{rt}\), and determines a unique planned processing equipment for each Root-Subtree process, the integrated scheduling problem with the same equipment characteristics is further simplified into a general integrated problem, and the function (7) is used as the objective function to analyze the vertical and horizontal advantages of the Root-Subtree.

3) DESIGN OF THE ROOT-SUBTREE PROCESS SORTING STRATEGY OF THE SAME PLANNED PROCESSING EQUIPMENT

In order to determine the scheduling order of all Root-Subtree processes, it will definitely produce the horizontal competition phenomenon of the planned processing equipment. Therefore, this paper designs the Root-Subtree process sorting strategy of the same planned processing equipment, which makes full use of the data in the selected Root-Subtree pre-scheduling scheme to seek a more compact processing scheduling sequence for the process. First, the strategy obtains the start processing time parameters of each Root-Subtree process on the corresponding planned processing equipment with the help of the selected Root-Subtree pre-scheduling scheme, that is, the equipment process pre-start time (denoted as \(S_{n,m}^{rt}\), which indicates the equipment process pre-start time of the process \(P_n\) on the processing equipment \(M_m\)); Then, the Root-Subtree processes are grouped by the planned processing equipment type, and the scheduling sequence of each group of process is determined according to the equipment process pre-start time from small to large, so as to complete the process cross-sorting operation between different Root-Subtrees and truly achieve both vertical and horizontal balance. When multiple Root-Subtree processes on the same planned processing equipment have the same equipment process pre-start time, this situation only occurs between different Root-Subtrees. The Root-Subtree process sorting strategy of the same planned processing equipment uses the Root-Subtree priority to determine the scheduling order of the corresponding process [26], [27]. Compared with the quasi-critical path of the process tree, as a branch of the improved process tree, the pre-scheduling completion time of the Root-Subtree can better illustrate the lower limit of the total processing time of the product, this paper sets the priority parameter of the Root-Subtree by comparing the pre-scheduling completion time, the longer the pre-scheduling completion time of the selected Root-Subtree is, the higher its own priority is, that is, the Root-Subtree process with the highest priority is preferentially scheduled, so that the immediately following processes of the process become schedulable process as soon as possible. It uses the function:

\[
Q_{rt}^{G} = \max\{\min\{T_{Z}^{rti}, T_{H}^{rti}\}, \cdots, \min\{T_{Z}^{rg}, T_{H}^{rg}\} - Q_{rt}^{G-1}, \cdots, -Q_{rt}\}
\]

to determine each Root-Subtree priority, \(Q_{rt}^{1}, Q_{rt}^{2}, \cdots, Q_{rt}^{G}\) indicates \(G\) Root-Subtree priority parameters from high to low. When there are multiple Root-Subtrees with the same pre-scheduling completion time, the Root-Subtree priority is set by comparing the number of processes contained in the Root-Subtree, that is, the more processes in the Root-Subtree, the higher the priority.

The flow chart of the process sorting strategy of the Root-Subtree vertical and horizontal pre-scheduling is shown in
B. DESIGN OF THE FLEXIBLE EQUIPMENT OPTIMAL SCHEDULING STRATEGY OF DYNAMICALLY DETERMINING PROCESS

The process sorting strategy of the Root-Subtree vertical and horizontal pre-scheduling is only a static virtual scheduling process to solve the horizontal conflict problem between processes, and does not really reflect the parallel processing capability of the flexible integrated scheduling. In order to give full play to the characteristics of the flexible processing equipment, the flexible equipment optimal scheduling strategy of dynamically determining process is proposed to determine the final processing equipment of the process. Under satisfying the constraints of the process tree, the strategy accurately utilizes the advantages of flexible processing equipment, that is, it can also obtain earlier completion time by selecting a longer processing time equipment for the process when the flexible processing equipment is idle, which can further improve the scheduling compactness of the process and the parallel efficiency, so as to complete the static pre-scheduling of the Root-Subtree process to the scheduling process of the dynamic fusion processing. First, a set of queues are established with the planned processing equipment sequence as the object, and the Root-Subtree processes on the same planned processing equipment are stored into the corresponding queue from front to back according to the scheduling order obtained by the process sorting strategy of the Root-Subtree vertical and horizontal pre-scheduling, and the temporary planning binding relationship between the Root-Subtree process and the planned processing equipment is established by setting the flag parameters $B^p_{n,m}$. When $B^p_{n,m} = 1$, the process $P_n$ can only be processed on its planned processing equipment $M_m$, and the equipment $M_m$ cannot process other processes. When $B^p_{n,m} = 0$, if the process $P_n$ is an unschedulable process, the equipment $M_m$ becomes an unplanned processing equipment, and the equipment $M_m$ can process other unplanned processes; If the equipment $M_m$ is in a busy state, the schedulable process $P_n$ becomes an unplanned processes, the process $P_n$ can be processed on other unplanned processing equipment. Next, it is necessary to judge whether to adjust $B^p_{n,m}$ according to the status of the first process in the queue and the status of the processing equipment, and the scheduling results of multiple processing equipment are compared to determine the final processing equipment of the process. In order to ensure the rationality of mutual selection between the processes and processing equipment, this paper compares the equipment driving moment [28] with the immediately preceding process completion time (the initial start processing time) of the first processes in the queues. Based on the comparison results, it determines whether the first processes in the queues can be queued. Thirdly, it uses the flexible integrated scheduling concept of the short usage time and early termination to determine the final processing equipment of the process, and generates a substantial scheduling plan for the Root-Subtree processes. Finally, the non-Root-Subtree processes stored in the independent stack are taken out and scheduled to form a final scheduling plan to complete the entire product.
Algorithm 1 The Process Sorting Strategy of the Root-Subtree Vertical and Horizontal Pre-scheduling

Input: The data information of the product processes

Output: The scheduling order of each group of Root-Subtree processes

1. Based on the simplified pre-processing principle of the flexible process tree, we compare the size of multiple processing times of each process itself and determine the unique processing time $T_n = \min\{T_{n,j}, T_{n,k}, \ldots\}$

2. Convert the flexible process tree into an improved process tree with the same equipment characteristics

3. if the in-degree of the root node > 1 then

4. Store the node process into $Stack_0$, which decomposes other remaining processes into several Root-Subtrees $\{rt_1, rt_2, rt_3, \ldots, rt_G\}$

5. else

6. Merge the root process and its predecessor process into the virtual root node "VR"

7. if there is an immediately preceding process before the virtual root node then

8. if the in-degree of the virtual root node > 1 then

9. Store the virtual root node into $Stack_0$, which decomposes other remaining processes into several Root-Subtrees $\{rt_1, rt_2, rt_3, \ldots, rt_G\}$

10. else

11. Jump to step 6

12. end

13. else

14. break

15. end

16. end

17. All Root-Subtrees are pre-scheduled by the improved algorithm ① and algorithm ② respectively to obtain $T^{rt}_Z$ and $T^{rt}_H$

18. if $T^{rt}_Z \leq T^{rt}_H$ then

19. The vertical pre-scheduling scheme

20. else

21. The horizontal pre-scheduling scheme

22. end

23. Obtain the equipment process pre-start time $S_{m,n}$ for each Root-Subtree process and determine the unique planned processing equipment of the Root-Subtree process

24. Sort the selected Root-Subtree pre-scheduling completion times in descending order using bubble sorting, and assign them to $Q^{1}_n, Q^{2}_n, \ldots, Q^{G}_n$ in turn to determine the priority parameters of each Root-Subtree

25. Group all Root-Subtree processes according to the corresponding planned processing equipment type, and determines the scheduling order of each group of Root-Subtree processes

26. return the scheduling order of each group of Root-Subtree processes
At the same time, if there are more than two processes with the same shortest processing completion time belonging to the same Root-Subtree, it compares the number of remaining processes in the corresponding queue, and selects the process with the largest number of processes in the queue at the current moment to be processed on the equipment $M_m$, which conforms to the flexible integrated scheduling concept of early termination and load balancing [31], [32].

Situation 4, at the equipment driving moment $T_{p+1}^D$, in addition to the unplanned processing equipment $M_m$, there is another unplanned processing equipment $M_{m+1}$, and there is only one unplanned process $P_{n+1}$, this situation needs to consider two forms: ① If $P_{n+1}$ can be processed on the equipment $M_m$ or $M_{m+1}$, $P_{n+1}$ is assigned to the equipment $M_m$ or $M_{m+1}$; ② If $P_{n+1}$ can be processed on the equipment $M_m$ and $M_{m+1}$ at the same time, $P_{n+1}$ is assigned to the equipment with a short processing completion time, that is, $\min \{ F_{n+1,m}, F_{n+1,m+1} \}$, if the processing completion time of $P_{n+1}$ on the two equipment is equal, that is, $F_{n+1,m} = F_{n+1,m+1}$, then $P_{n+1}$ is loaded to the equipment with the smaller equipment number for processing.

Situation 5, at the equipment driving moment $T_{p+2}^D$, in addition to the unplanned processing equipment $M_m$, there is another unplanned processing equipment $M_{m+1}$, and there are two unplanned processes $P_{n+1}$ and $P_{n+2}$ that can be processed by these two equipment, this situation needs to consider three forms: ① If $P_{n+1}$ can only be processed on the equipment $M_m$ and $P_{n+2}$ can only be processed on the equipment $M_{m+1}$, and there is no equipment competition problem, the two processes are directly loaded into their respective machinable equipment for processing; ② If $P_{n+1}$ and $P_{n+2}$ can be processed on both equipment $M_m$ and $M_{m+1}$ at the same time, there are two process allocation schemes, this paper selects the scheme with the shortest overall completion time to determine the final equipment, that is, $\min \{ \max \{ F_{n+1,m}, F_{n+2,m+1} \}, \max \{ F_{n+2,m}, F_{n+1,m+1} \} \}$ when the processing completion time of the two schemes are equal, then it arbitrarily selects a scheme; ③ When two equipment can simultaneously select $P_{n+1}$ for processing and $P_{n+2}$ with the second shortest processing completion time can be processed on the equipment $M_m$ or $M_{m+1}$, if $P_{n+2}$ is the second shortest process that can be processed on the equipment $M_m$, $M_{m+1}$ is the final processing equipment of $P_{n+1}$, otherwise, $M_m$ is the final processing equipment of $P_{n+1}$.

Situation 6, at the equipment driving moment $T_{p+3}^D$, in addition to the unplanned processing equipment $M_m$, there is another unplanned processing equipment $M_{m+1}$, and two equipment have three or more unplanned processes that can be processed, that is, an unplanned process set is formed. According to the strategy with the shortest processing completion time, two equipment select the processing processes from the unplanned process set, this situation needs to consider two forms: ① If the unplanned processes selected by two equipment are different processes, that is, there is no process competition problem, the process with the shortest processing completion time will be loaded into the corresponding equipment for processing; ② If the unplanned process selected by two equipment is the same process, it is recorded as the unplanned process $P_{n+1}$, at the same time, the process $P_{n+2}$ and $P_{n+3}$ are unplanned processes with the second shortest processing completion time that can be processed on the equipment $M_m$ and $M_{m+1}$, there are at most $C_2^3 = 3$ kinds of allocation combination schemes. Because it cannot meet the integrated scheduling concept of early termination by simultaneously selecting the unplanned process with the second shortest processing completion time, the combination with the shortest overall completion time is selected from the two effective combination schemes, that is, $\min \{ \max \{ F_{n+1,m}, F_{n+3,m+1} \}, \max \{ F_{n+2,m}, F_{n+1,m+1} \} \}$.

In order to simplify the complexity of the algorithm, when the processing completion time of two combined schemes is equal, one scheme is arbitrarily selected.

Through the above situation analysis, if there is no competition of the processing equipment, it can find the process with the shortest processing completion time for each processing according to the situation 1, situation 2 and situation 3; If there is competition in the processing equipment, it can find the process allocation combination scheme with the shortest overall processing completion time for the competing equipment according to the situation 4, situation 5 and situation 6. Since the above situation analysis is limited to two processing equipment, the complex situation of multi-equipment and multi-process competition can be solved by using this strategy in a circular manner. The flow chart of the flexible equipment optimal scheduling strategy of dynamically determining process is shown in Figure 3. The specific implementation steps of the strategy are as follows:

V. ALGORITHM DETAILED DESIGN AND COMPLEXITY ANALYSIS

A. ALGORITHM DETAILED DESIGN

In order to meet the needs of different product structures, it deeply excavates the vertical and horizontal characteristics of the internal structure of the product process tree under the flexible integrated scheduling environment to enhance the compactness and load balance of the process. The flow chart of FISA-DEVH is shown in Figure 4. The specific implementation steps of FISA-DEVH are as follows:

B. ALGORITHM COMPLEXITY ANALYSIS

It assumes that the number of the process contained in the complex product is $N$, the number of the flexible processing equipment is $M$, the number of the Root-Subtree is $G$, and the number of non-Root-Subtree processes contained in the virtual root node is $R$.

(1) The process sorting strategy of the Root-Subtree vertical and horizontal pre-scheduling. In the process of the Root-Subtree process sorting, according to the simplified preprocessing principle of the process tree, the flexible processing equipment of $N$ processes and the corresponding processing time are traversed, since each process can be processed on $M$ equipment at most, it is necessary to compare $M - 1$ times.
to determine the shortest processing time equipment of the process, so the complexity of forming the improved process tree is $O(N \times (M - 1))$, on this basis, the complexity of splitting the improved process tree and forming the Root-Subtree set is $O(N)$; Each Root-Subtree is pre-scheduled through the improved algorithm ① and algorithm ②, the vertical and horizontal pre-scheduling completion times are obtained and compared, and the complexity of selecting the Root-Subtree pre-scheduling scheme and determining the processing equipment is $G \times O((N-R/G)^2)$; The complexity of determining the Root-Subtree priority according to the pre-scheduling completion time is $O(G^2)$; According to the equipment process pre-start time of the Root-Subtree process and the Root-Subtree priority parameters, the complexity of the Root-Subtree process sorting on the planned processing equipment is $M \times O((N-R/M)^2)$.

(2) The flexible equipment optimal scheduling strategy of dynamically determining process. In the process of determining the final processing equipment of the process, the Root-Subtree processes of the same planned processing equipment need to be loaded into the corresponding equipment queue, and the complexity of obtaining the equipment driving moment is $O(N \times (M - 1))$; The complexity of establishing and updating the schedulable process set and the idle equipment set is $O(N^2)$, in the worst case, there are at most $M$ idle equipment and $N - 1$ schedulable processes, the complexity of judging whether the first process of the queue is the schedulable processes and the corresponding processing equipment is the idle equipment is $O(N)$; Under satisfying the constraint relationship between the vertical process tree and the horizontal processing equipment, the complexity of the planned process being processed on the planned processing equipment is $O(1)$; When there are unplanned processes with the same shortest processing time on multiple unplanned processing equipment, it is necessary to search for the second shortest processing processes for these equipment from the unplanned processes that can be processed, in the worst case, there are $M/2$ unplanned processing equipment and $M/2$ second-shortest unplanned processes at the same time, so according to the different combinations between unplanned processes and unplanned processing equipment, the combination scheme with the shortest overall processing.
completion time is selected, the complexity of this stage is $O(M \log M)$; The complexity of adding the non-Root-Subtree processes in the virtual root node to the Root-Subtree process scheduling sequence is $O(N)$.

In the actual production process, $N \gg M$, $N \gg G$ and $N \gg R$, so the complexity of FISA-DEVH is a quadratic polynomial $O(N^2)$.

VI. ALGORITHM EXAMPLE ANALYSIS AND COMPARISON

A. ALGORITHM EXAMPLE ANALYSIS

FISA-DEVH is a theoretical analysis process of the flexible integrated scheduling problem, which does not depend on the product process tree structure, and has universal significance. In order to better describe the execution process of the algorithm, the algorithm is used to perform scheduling analysis on a product example, it assumes that the complex product H consists of 23 processes and is processed in a workshop with 4 flexible processing equipment. The flexible process tree model of the product H is shown in Figure 5, each node consists of three layers of data information, the first layer represents the process name, the second layer represents the processing equipment name and the third layer represents the processing time.

Firstly, according to the structural characteristics of the flexible process tree, the process tree is transformed into an improved process tree with the same equipment characteristics through the simplified decomposition strategy of the flexible process tree, at the same time, it uses decomposition preprocessing for the improved process tree to take out the root node “Root” and store it into Stack$0$, splits the improved process tree to form a Root-Subtree set, and names $\{rt_1, rt_2, rt_3\}$. The schematic diagram of the improved process tree of the product H and its splitting Root-Subtree is shown in Figure 6.

Secondly, it uses the improved algorithm 1 and algorithm 2 to perform pre-scheduling operations on each Root-Subtree, and obtains $T^Z_{rt}$ and $T^H_{rt}$, the vertical and horizontal comparison of $\{rt_1, rt_2, rt_3\}$ pre-scheduling Gantt chart is

![Diagram](image-url)
Algorithm 2 The flexible equipment optimal scheduling strategy of dynamically determining process

Input: The data information of the product processes
Output: The overall scheduling scheme of the product and outputs the Gantt chart

1. Initialize \( i = 1 \); Initialize the number of devices as \( M \); Define the number of the unplanned processing equipment as \( K_{upe} \); Define the number of the unplanned processes as \( K_{app} \)
2. While \( i \leq M \) do
   3. If at a certain equipment driving moment \( T_{P}^{D} \), \( M_{m} \) is in an idle state according to the increasing order of the equipment number then
      4. If the first process \( P_{n} \) in the equipment queue is a schedulable process then
         5. \( P_{n} \) is processed according to the first form of the situation 2
      6. Else
         7. If the first process \( P_{n} \) in the equipment queue is a schedulable process then
            8. \( B_{n,m}^{p} = 0 \), Change \( P_{n} \) to an unplanned process according to the first form of the situation 1
         9. Else \( B_{n,m}^{p} \) remains unchanged, \( P_{n} \) and \( M_{m} \) do not perform any operations until the next equipment driving moment
      10. End
   11. End
   12. Change \( M_{m} \) to unplanned processing equipment, \( i = i + 1 \)
13. End
14. If \( K_{upe} = 2 \) then
   15. It operates according to the situation 3
16. Else
   17. If \( K_{app} = 1 \) then
      18. It operates according to the situation 4
   19. Else
      20. It operates according to the situation 5
   21. End
   22. If \( K_{app} = 2 \) then
      23. It operates according to the situation 6
   24. Else
      25. It operates according to the situation 6
   26. End
   27. End
28. End
29. Return the overall scheduling scheme of the product and outputs the Gantt chart

Algorithm 3 The flexible integrated scheduling algorithm for dynamic determination processing equipment considering the Root-Subtree vertical and horizontal pre-scheduling

Input: The data information of the product processes
Output: The overall scheduling scheme of the product and outputs the Gantt chart

1. Organize the data information of the product processes into a standardized and flexible integrated scheduling process tree model
2. Use the process sorting strategy of the Root-Subtree vertical and horizontal pre-scheduling to determine the Root-Subtree process scheduling order of the same planned processing equipment.
3. According to the scheduling processing sequence, the Root-Subtree processes of the same planned processing equipment are sequentially loaded into \( Queue_{1}, Queue_{2}, \ldots, Queue_{M} \)
4. Initialize processing time of the product is set to 0
5. Initialize \( B_{n,m}^{D} = 1 \)
6. Initialize equipment driving moment \( T_{0}^{D} = 0 \)
7. Initialize a schedulable process set \( S_{sp} \) and an idle equipment set \( S_{ie} \)
8. Adds the schedulable processes to \( S_{sp} \), and adds the idle equipment to the \( S_{ie} \)
9. While \( Queue_{1}, Queue_{2}, \ldots, Queue_{M} \) is not empty do
   10. Determine the final processing equipment of the Root-Subtree process by using the flexible equipment optimal scheduling strategy of dynamically determining process
   11. At the equipment driving moment \( T_{k-1}^{D} \), it calculates \( F_{m} \) of all busy equipment, and determines the earliest processing completion time as the next equipment driving moment \( T_{k}^{D} \)
   12. Compares \( T_{k}^{D} \) and \( F_{m} \), updates \( S_{sp} \) and \( S_{ie} \)
13. End
14. While \( Stack_{0} \) is not empty do
   15. Pops up the top process and adds it to the Root-Subtree process scheduling sequence according to the information stored in the stack
16. End
17. Return the overall scheduling scheme of the product and outputs the Gantt chart

shown in Figure 7, Figure 8 and Figure 9. Next, the pre-scheduling scheme is selected according to the comparison results of \( T_{Z}^{rt} \) and \( T_{H}^{rt} \), and \( Q_{1}, Q_{2}, Q_{3}^{Z}, Q_{3}^{H} \) are set. The specific analysis process of the Root-Subtree vertical and horizontal advantages and the Root-Subtree priority parameters is shown in Table 1.

Thirdly, according to the selected Root-Subtree prescheduling scheme in Table 1, it obtains \( S_{n,m}^{D} \) of all Root-Subtree processes and determines the corresponding planned processing equipment, at the same time, it compares the size of \( S_{n,m}^{D} \), and determines the Root-Subtree process scheduling sequence of the same planned processing equipment in combination with the priority parameters of the Root-Subtree. The specific description process of all Root-Subtree process sorting is shown in Table 2. Then, it establishes the
TABLE 1. The specific analysis process of the Root-Subtree vertical and horizontal advantages and the Root-Subtree priority parameters.

| Root-Subtree name | Vertical pre-scheduling completion time | Horizontal pre-scheduling completion time | Comparison results | Vertical and horizontal advantage analysis results | Root-Subtree priority |
|-------------------|----------------------------------------|-------------------------------------------|---------------------|-----------------------------------------------|----------------------|
| rt₁               | 100                                    | 90                                        | T_r^v₁ / T_r^h₁ >1  | Horizontal                                     | Q_r³                 |
| rt₂               | 110                                    | 130                                       | T_r^v₂ / T_r^h₂ <1  | Vertical                                       | Q_r¹                 |
| rt₃               | 95                                     | 95                                        | T_r^v₃ / T_r^h₃ =1  | Vertical                                       | Q_r²                 |

TABLE 2. The specific description process of all Root-Subtree process sorting.

| Planned processing equipment sequence | Root-Subtree process name: equipment processes start time | Root Subtree Priority Parameters | Scheduling sequence of the same planned processing equipment |
|---------------------------------------|----------------------------------------------------------|---------------------------------|----------------------------------------------------------|
| A1:70                                 | Q_n¹                                                   | 5                              |                                                         |
| B2:10                                 | Q_n²                                                   | 2                              |                                                         |
| M1                                    |                                                        |                                |                                                         |
| C5:0                                  | Q_n²                                                   | 1                              |                                                         |
| C3:10                                 | Q_n²                                                   | 3                              |                                                         |
| C4:40                                 | Q_n²                                                   | 4                              |                                                         |
| A2:40                                 | Q_n²                                                   | 3                              |                                                         |
| B8:0                                  | Q_n¹                                                   | 1                              |                                                         |
| M2                                    |                                                        |                                |                                                         |
| B3:55                                 | Q_n¹                                                   | 4                              |                                                         |
| B1:90                                 | Q_n¹                                                   | 5                              |                                                         |
| C2:10                                 | Q_n²                                                   | 2                              |                                                         |
| A6:0                                  | Q_n³                                                   | 3                              |                                                         |
| A4:10                                 | Q_n³                                                   | 5                              |                                                         |
| A3:40                                 | Q_n²                                                   | 7                              |                                                         |
| M3                                    |                                                        |                                |                                                         |
| B5:0                                  | Q_n¹                                                   | 1                              |                                                         |
| B4:10                                 | Q_n²                                                   | 4                              |                                                         |
| C6:0                                  | Q_n²                                                   | 2                              |                                                         |
| C7:20                                 | Q_n²                                                   | 6                              |                                                         |
| A5:10                                 | Q_n³                                                   | 3                              |                                                         |
| B7:0                                  | Q_n¹                                                   | 1                              |                                                         |
| M4                                    |                                                        |                                |                                                         |
| B6:30                                 | Q_n²                                                   | 4                              |                                                         |
| C8:0                                  | Q_n³                                                   | 2                              |                                                         |
| C1:75                                 | Q_n²                                                   | 5                              |                                                         |

FIGURE 7. The vertical and horizontal comparison of the Root-Subtree rt₁ pre-scheduling Gantt chart.

FIGURE 8. The vertical and horizontal comparison of the Root-Subtree rt₂ pre-scheduling Gantt chart.

equipment queue Queue₁, Queue₂, Queue₃, Queue₄ according to the planned processing equipment sequence M1, M2, M3 and M4, and loads all Root-Subtree processes into the corresponding queue in turn according to the scheduling sequence of the same planned processing equipment in Table 2. The sequence of loading the Root-Subtree
The parallel process into Queue1: \{A1→C4→C3→B2→C5→Header\}; The sequence of loading the Root-Subtree process into Queue1: \{B1→B3→A2→C2→B8→Header\}; The sequence of loading the Root-Subtree process into Queue3: \{A3→C7→A4→B4→A6→C6→B5→Header\}; The sequence of loading the Root-Subtree process into Queue5: \{C1→B6→A→C8→B7→Header\}, so as to complete the planned binding relationship between the Root-Subtree process and the planned processing equipment, and \(B_{n,m}\) is initialized to 1.

Fourth, according to the flexible equipment optimal scheduling strategy of dynamically determining process, it finds more reasonable processing equipment for the Root-Subtree process, and forms the substantial scheduling sequence of the Root-Subtree processes. The starting processing time of the entire product processing task is 0, that is, the initial equipment driving moment \(T_D^0 = 0\), all equipment are in an idle state, and the first processes of all queues are schedulable processes, it maintains the original planning binding relationship between the process and the equipment, and the first processes of the queues are dequeued and loaded into the corresponding planned processing equipment. After all equipment and the first process of the queue are processed at the equipment driving moment \(T_D^0 = 0\), it calculates the next equipment driving moment \(T_D^1 = \min \{F_{C5,1}, F_{B8,2}, F_{B5,3}, F_{B7,4}\}\), that is, \(T_D^1 = 10\), and loops in turn until all Root-Subtree processes scheduling is completed. The specific analysis process of the final processing equipment for determining the Root-Subtree process is shown in Table 3 and Table 4.

Finally, the non-Root-Subtree process “Root” in the Stack0 is popped out and loaded into the corresponding processing equipment according to the shortest processing time information stored in the stack to form the final scheduling plan of the product H. FISA-DEVH is used to schedule the product H, and the output Gantt chart is shown in Figure 10.

The following uses FISA-CPFS, FISA-DDEP and FISA-ROLP to schedule the product H in Figure 5 respectively, and the superiority of FISA-DEVH is analyzed by the sequence of process scheduling. The scheduling Gantt chart of FISA-CPFS, FISA-DDEP and FISA-ROLP is shown in Figure 11, Figure 12 and Figure 13.

It can be seen from these four figures, the total processing time of the product H scheduled by FISA-DEVH, FISA-CPFS, FISA-DDEP and FISA-ROLP are respectively 175, 200, 180 and 185. Comparing the scheduling results of four algorithms, FISA-DEVH deeply excavates the vertical and horizontal advantages of the Root-Subtree itself to complete the process cross-sorting operation between different Root-Subtrees without increasing the complexity of the algorithm, it truly achieves both vertical and horizontal balance, narrows the search scope of the schedulable process.
TABLE 3. The specific analysis process of the final processing equipment for determining the Root-Subtree process at the equipment driving moment.

| Equipment driving moment | Processing equipment/whether it is idle | The first process of the queue/whether it is the schedulable process | Whether it is unplanned processing equipment | Machinable unplanned process | The selected process |
|--------------------------|----------------------------------------|---------------------------------------------------------------|---------------------------------------------|-----------------------------|---------------------|
| $T_2^D = 0$              | M1/Yes                                  | C5/ Yes                                                       | No                                          | --                          | C5                  |
|                         | M2/Yes                                  | B8/ Yes                                                       | No                                          | --                          | B8                  |
|                         | M3/Yes                                  | B5/ Yes                                                       | No                                          | --                          | B5                  |
|                         | M4/Yes                                  | B7/ Yes                                                       | No                                          | --                          | B7                  |
|                         | M1/No                                   | B2/ Yes                                                       | No                                          | --                          | B2                  |
| $T_1^D = 10$             | M2/No                                   | C2/ Yes                                                       | No                                          | --                          | --                  |
|                         | M3/Yes                                  | C6/ Yes                                                       | No                                          | --                          | C6                  |
|                         | M4/ No                                  | C8/ Yes                                                       | No                                          | --                          | --                  |
|                         | M1/ No                                  | C3/ Yes                                                       | No                                          | --                          | --                  |
| $T_2^D = 20$             | M2/ Yes                                 | C2/ Yes                                                       | No                                          | --                          | C2                  |
|                         | M3/ No                                  | A6/ Yes                                                       | No                                          | --                          | --                  |
|                         | M4/ No                                  | C8/ Yes                                                       | No                                          | --                          | --                  |
|                         | M1/ No                                  | C3/ Yes                                                       | No                                          | --                          | --                  |
| $T_3^D = 30$             | M2/ No                                  | A2/ No                                                        | No                                          | --                          | --                  |
|                         | M3/ Yes                                  | A6/ Yes                                                       | No                                          | --                          | A6                  |
|                         | M4/ Yes                                  | C8/ Yes                                                       | No                                          | --                          | C8                  |

FIGURE 14. The comparison of the total processing time by FISA-CPFS, FISA-DDEP, FISA-ROLP and FISA-DEVH scheduling 50 product instances.

combination scheme, and makes full use of the particularity of the flexible processing equipment to obtain a production scheduling plan with a shorter total processing time. In addition, FISA-CPFS, FISA-DDEP and FISA-ROLP use unified vertical or horizontal rules to solve the process conflict problem, which leads to the lack of flexibility in the scheduling of the process and the selection of the processing equipment, so that the processes choose unreasonable processing equipment, which causes other processes related to it to delay processing and affects the product scheduling effect. As can be seen from Figure 11, since FISA-CPFS adopts the fixed sequence and the principle of early processing, the process A6 is allocated to the equipment M1, which causes it to lose the flexibility of the independent process and increase the processing time, so that the subsequent associated process of the process A6 is delayed; As can be seen from Figure 12, FISA-DDEP uses the dynamic substantive path strategy of the unified shortest processing time to solve all process conflicts, at the equipment driving moment $T_2^D = 20$, since the dynamic substantial path length of the planned process B6 is 110, and the dynamic substantial path length of the planned process B7 is 115, the planned process B6 is scheduled preferentially, it makes the process B6 and the process B7 lose the parallel processing effect, so that the process B3 cannot be processed as soon as possible, which affects the overall scheduling effect; As can be seen from Figure 13, FISA-ROLP uses the reverse order layer first strategy and the dynamic long path strategy to sort the processes, this kind of the horizontal sorting strategy may make the process schedule on the equipment with long processing time, the dynamic
TABLE 4. The specific analysis process of the final processing equipment for determining the Root-Subtree process at the equipment driving moment (Continued Table 3).

| Equipment driving moment | Processing equipment/whether it is idle | The first process of the queue/whether it is the schedulable process | Whether it is unplanned processing equipment | Machinable unplanned process | The selected process |
|--------------------------|----------------------------------------|--------------------------------------------------|--------------------------------------------|----------------------------|---------------------|
|                         | M1/Yes                                 | C3/Yes                                           | No                                         | —                          | C3                  |
|                         | M2/No                                  | A2/No                                            | No                                         | —                          | —                   |
|                         | M3/Yes                                 | B4/Yes                                           | No                                         | —                          | B4                  |
|                         | M4/No                                  | A5/Yes                                           | No                                         | —                          | —                   |
|                         | M1/No                                  | C4/No                                            | No                                         | —                          | —                   |
|                         | M4/Yes                                 | A5/Yes                                           | No                                         | —                          | A5                  |
|                         | M1/Yes                                 | C4/No                                            | Yes                                        | B6                         | B6                  |
|                         | M2/No                                  | A2/No                                            | No                                         | —                          | —                   |
|                         | M3/Yes                                 | C7/Yes                                           | No                                         | —                          | C7                  |
|                         | M4/No                                  | B6/Yes                                           | No                                         | —                          | —                   |
|                         | M1/No                                  | C4/No                                            | No                                         | —                          | —                   |
|                         | M2/No                                  | A2/No                                            | No                                         | —                          | —                   |
|                         | M3/No                                  | A3/Yes                                           | No                                         | —                          | —                   |
|                         | M4/Yes                                 | C1/No                                            | Yes                                        | A3                         | A3                  |
|                         | M1/No                                  | C4/Yes                                           | No                                         | —                          | —                   |
|                         | M2/No                                  | A2/No                                            | No                                         | —                          | —                   |
|                         | M3/Yes                                 | —                                                | Yes                                        | C4                         | C4                  |
|                         | M4/No                                  | C1/No                                            | No                                         | —                          | —                   |
|                         | M1/No                                  | A1/No                                            | No                                         | —                          | —                   |
|                         | M2/Yes                                 | A2/Yes                                           | No                                         | —                          | —                   |
|                         | M3/No                                  | —                                                | No                                         | —                          | —                   |
|                         | M4/No                                  | C1/No                                            | Yes                                        | B3/40                      | B3                  |
|                         | M1/Yes                                 | A1/No                                            | Yes                                        | B3/40                      | B3                  |
|                         | M2/No                                  | B3/Yes                                           | No                                         | —                          | —                   |
|                         | M3/No                                  | —                                                | No                                         | —                          | —                   |
|                         | M4/No                                  | C1/No                                            | No                                         | —                          | —                   |
|                         | M1/No                                  | A1/No                                            | No                                         | —                          | —                   |
|                         | M2/Yes                                 | A1/Yes                                           | Yes                                        | A1/35                      | A1                  |
|                         | M3/No                                  | —                                                | No                                         | —                          | —                   |
|                         | M4/Yes                                 | C1/No                                            | Yes                                        | A1/25                      | A1                  |
|                         | M1/No                                  | —                                                | No                                         | —                          | —                   |
|                         | M2/Yes                                 | B1/No                                            | Yes                                        | C1/35                      | C1                  |
|                         | M3/Yes                                 | —                                                | Yes                                        | C1/25                      | C1                  |
|                         | M4/No                                  | C1/Yes                                           | No                                         | —                          | —                   |
|                         | M1/Yes                                 | —                                                | Yes                                        | —                          | —                   |
|                         | M2/Yes                                 | B1/Yes                                           | No                                         | —                          | B1                  |
|                         | M3/No                                  | —                                                | No                                         | —                          | —                   |
|                         | M4/No                                  | —                                                | No                                         | —                          | —                   |

long path of the process B2 and A2 on the same layer is respectively 85 and 100, so that the process A2 is preferentially scheduled to process on equipment M1, resulting in the processing of B2 on equipment M3, which affects the overall scheduling effect.

B. ALGORITHM COMPARISON

In order to verify the performance of FISA-DEVH in the flexible integrated scheduling problem and its adaptability to products with different structures, 50 product processing trees were randomly generated with different structures. The following parameters were randomly generated: the number of the processes is [20] and [60], the number of the tight front processes of each process is [0, 5], the number of the processing equipment is [3] and [6], the number of the Root-Subtree is [3] and [6]. Figure 14 depicts the comparison of the total processing time by FISA-CPFS, FISA-DDEP, FISA-ROLP and FISA-DEVH scheduling 50 product instances; Figure 15 depicts the comparison of the average processing time by above four algorithms scheduling 50 product instances.

The above description further confirms the necessity of analyzing the internal vertical and horizontal characteristics of the product process tree in the flexible integrated scheduling problem, FISA-DEVH determines the reasonable process scheduling sequence of the same planned processing equipment through the vertical and horizontal static pre-scheduling
method, and excavates the urgency of the process scheduling and apply to the product instances with different structures, at the same time, it uses the characteristics of the flexible processing equipment to dynamically and reasonably determine the final processing equipment of the process, improves the parallel processing ability between processes, realizes the overall optimization of the product scheduling effect, and reflects the universality and superiority of the algorithm in actual production scheduling.

VII. CONCLUSION

In the field of the flexible integrated scheduling, this paper firstly proposes a flexible integrated scheduling algorithm for dynamic determination processing equipment considering the Root-Subtree vertical and horizontal pre-scheduling. On the premise of not increasing the complexity of the algorithm, FISA-DEVH improves equipment utilization, and optimizes the product production goals. The conclusions are as follows:

(1) The vertical and horizontal characteristics of the Root-Subtree processes are analyzed through pre-scheduling method, and it designs the Root-Subtree process sorting strategy of the same planned processing equipment to group the Root-Subtree processes and determine their scheduling order, so as to enhance the process compactness on the basis of ensuring the flexibility of the process scheduling.

(2) According to the grouping and ordering of the Root-Subtree processes, the processes are sequentially loaded into the corresponding equipment queues, and at each equipment driving moment, the first process of the queue with high urgency is given priority to be processed, which narrows the search range of the schedulable process combination scheme and improves the algorithm solution efficiency.

(3) The flexible equipment optimal scheduling strategy of dynamically determining process determines the processing equipment of the process on the basis of considering the characteristics of the flexible processing equipment and the constraint relationship of the process tree in all aspects, so as to improve the parallel processing efficiency of the processes.

(4) Compared with the flexible integrated scheduling algorithm that uses unified rules to solve the conflict between processes and equipment, FISA-DEVH has better flexibility and superiority, which pays attention to the vertical and horizontal characteristics of the process in the associated position of the process tree, and can be applied to different product structure requirements.

In summary, FISA-DEVH seeks a reasonable product scheduling scheme on the basis of analyzing the internal structure of the process tree, and the scheduling effect is better and more practical. It has certain theoretical value and practical significance, and expands the new idea of two-way mutual selection between process and equipment in the flexible integrated scheduling problem.

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