Ultrasound elasticity imaging using physics-based models and learning-based plug-and-play priors

Narges Mohammadi\textsuperscript{1}, Marvin M. Doyley\textsuperscript{1}, Mujdat Cetin\textsuperscript{1,2}

\textsuperscript{1}Electrical and Computer Engineering Department, University of Rochester
\textsuperscript{2}Goergen Institute for Data Science, University of Rochester, Rochester, NY, USA

IEEE ICASSP 2021

This work has been partially supported by the National Science Foundation (NSF) under grants CCF-1934962 and DGE-1922591.
Goal:

• Improving FEM-based methods for elasticity imaging based on solving a constrained optimization problem.

• Existing methods typically assume fixed regularizers for various tissue types while advanced priors might be required.

• By integrating learning-based priors with physical forward models, a joint reconstruction framework is presented.

• The proposed PnP reconstruction approach guarantees data-driven reconstructions are consistent with the underlying physics.
Why Elasticity imaging?

• Ultrasound elasticity imaging for tissue stiffness quantification leading to reliable diagnostic decisions.

• Medical imaging concerns:
  • Fast elasticity reconstruction
  • Robust and accurate image reconstruction using limited noisy deformation measurements.
Why Elasticity imaging?

- Quasi-static
- Harmonic
- Transient

Sigrist RMS, Liau J, Kaffas AE, Chammas MC, Willmann JK. Ultrasound Elastography: Review of Techniques and Clinical Applications. Theranostics 2017
Quasi-static Elasticity modulus imaging

Step 1: applying force by ultrasound probe

Step 2: ultrasound imaging (multiple frames)

Step 3: displacement image acquisition

cross-correlation

\[ u(x, y) \]

Step 4: Young’s modulus reconstruction

Doyley, M.. “Model-based elastography: a survey of approaches to the inverse elasticity problem.” Physics in medicine and biology 57 3 (2012): R35-73.
Classical deterministic methods for solving inverse elasticity problem

- poor and unstable elasticity estimation in the presence of noise.

**Forward problem**

Equilibrium equation

\[ f_{true} = K(E)u \]

**Inverse problem**

Model-based/iterative modulus reconstruction

\[
\arg\min_{u,E} \left\| u - u^m \right\|^2_2 + \lambda \| \nabla E \|_1 \\
\text{s.t.} \quad K(E)u - f_{true} = 0
\]
Statistical Formulation for Elasticity Reconstruction

- integrating linear algebraic modeling of governing physical equation

\[
\begin{align*}
  f &= K(E)u + w \\
  w &\sim \mathcal{N}(0, \Sigma_w) \\
  f &= K(E)(u^m - n) + w \\
  n &\sim \mathcal{N}(0, \Sigma_n)
\end{align*}
\]

\[
D(u)E = K(E)u
\]

\[
\begin{align*}
  f &= D(u^m)E + \tilde{w} \\
  \tilde{w} &\sim \mathcal{N}(0, \Gamma) \\
  \Gamma &= \Sigma_w + K(E)\Sigma_n K(E)^T
\end{align*}
\]

\[
\hat{E} = \arg\min_E \quad \frac{1}{2} \left\| f - D(u^m)E \right\|_{\Gamma^{-1}}^2 + \lambda R(E)
\]

s.t. \quad E > 0

Signal-dependent colored noise
Fixed-point proximal gradient method (PGM)

- Decouple the iterative optimization steps into the data-fidelity and regularizer updates.
- Require proximal operator of regularizer
Proposed PnP approach

• Existing methods typically assume fixed regularizers for various tissue types while advanced priors might be required.

• Our approach: a joint PnP reconstruction framework by integrating learning-based priors with physical forward models.

• Each iteration of the elasticity image estimation process involves separate updates incorporating data fidelity and learning-based regularization.
Prior Learning

- DnCNN is used for exploiting the underlying prior information

\[ l(w) = \frac{1}{2N} \sum_{i=1}^{N} \left\| C_w(\tilde{E}) - (\tilde{E} - E) \right\|_F^2 \]
PnP for Elasticity reconstruction

1. Data-fidelity term
   - Displacement images
   - FEA procedure
     - Equilibrium eq., B.C., node2element conversion
   - $D(u^m)$
     - Forward linear operator
   - $g(E) = \frac{1}{2} \| f - D(u^m)E \|^2_{\Gamma^{-1}}$

2. Regularization term
   - $C_w$
     - DnCNN

3. Solving optimization problem
   - $E_{n+1} = \text{prox}_{E_n > 0}(C_w(E_n - \gamma_n \nabla g(E_n)))$

Reconstructed Elasticity image
Ultrasound Elasticity reconstruction results

Without regularizer | DnCNN Residual image | Post-processing

PnP approach | TV regularizer | True image
Conclusion:

- A PnP methodology for ultrasound elastography by combining statistical model-based representations and learning-based regularizers.

- The physical forward model incorporates statistical error modeling leading to a signal dependent correlated noise.

- Utilizing a fixed-point iterative approach and proximal gradient methods for solving the elasticity optimization problem.

- DnCNN is used for exploiting the underlying prior information which is plugged into the optimization function as the regularizer proximal operator.

- The PnP methodology ensures that the reconstructed images are consistent with the statistical physical model.
References:

- N. Mohammadi, M. M. Doyley, and M. Cetin, “A statistical framework for model-based inverse problems in ultrasound elastography,” ArXiv, vol. abs/2010.10729, 2020.

- K. Zhang, W. Zuo, S. Gu, and L. Zhang, “Learning deep CNN denoiser prior for image restoration,” CVPR, 2017, pp. 2808–2817.

- T. Meinhardt, M. Møller, C. Hazirbas, and D. Cremers, “Learning proximal operators: Using denoising networks for regularizing inverse imaging problems,” ICCV 2017, pp. 1799–1808.

- M. M. Doyley, “Model-based elastography: A survey of approaches to the inverse elasticity problem.”, Physics in Medicine and Biology, vol. 57, no. 3, R35–73, 2012.

- P. L. Combettes and J.-C. Pesquet, Proximal Splitting Methods in Signal Processing. 2009.