Chook - A comprehensive suite for generating binary optimization problems with planted solutions

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We present Chook, an open-source Python-based tool to generate discrete optimization problems of tunable complexity with a priori known solutions. Chook provides a cross-platform unified environment for solution planting using a number of techniques, such as tile planting, Wishart planting, equation planting, and deceptive cluster loop planting. Chook also incorporates planted solutions for higher-order (beyond quadratic) binary optimization problems. The support for various planting schemes and the tunable hardness allows the user to generate problems with a wide range of complexity on different graph topologies ranging from hypercubic lattices to fully-connected graphs.

I. INTRODUCTION

The advent of quantum annealing devices [1, 2] has spawned a renewed interest in the development of heuristic approaches to solve discrete optimization problems. On the one hand, the quantum revolution has inspired remarkable classical algorithms (see, for example, Ref. [3]) running on conventional CMOS hardware that have raised the bar for emerging quantum annealing hardware. On the other hand, substantial progress has been made in recent years on the development of programmable devices based on alternative technologies such as, for example, the coherent Ising machine based on optical parametric oscillators [4, 5], digital MemComputing machines based on self-organizing logic gates [6, 7], and the ASIC-based Fujitsu Digital Annealer [8–10].

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The rapid progression of novel computing platforms and algorithms demands tunable hard optimization problems for benchmarking purposes. Much effort has been devoted to generate binary synthetic benchmark problems whose optimal configurations are known a priori [11–18]. These are frequently referred to as problems with planted solutions. In particular, benchmark problems that are easily scalable to large system sizes, and ideally with tunable hardness, facilitate systematic comparison of optimizers. Aside from their practical significance, theoretical investigation of such problem ensembles reveals intriguing insights into the nature of disordered systems, in particular, the interplay between frustration, thermodynamic behavior, and computational complexity [18, 19].

In this paper we introduce Chook (version 0.1.0), a comprehensive Python-based tool that integrates multiple solution planting schemes to provide a unified framework for generating benchmark problems for binary optimization problems. Chook currently supports tile planting [15, 19], Wishart planting [18], deceptive cluster loops [20], and equation planting [17]. In addition, the software allows for the construction of planted solutions for problems with higher-order \( (k > 2) \) interactions, by combining problems with lower-order \( (k \leq 2) \) interactions.

The paper is organized as follows. In Sec. [II] we present an overview of the solution planting schemes included in Chook. Section [III] provides instructions on installation and usage of the software, along with a detailed description of parameters and options, followed by concluding remarks.

II. SOLUTION PLANTING SCHEMES

In solution planting, the goal is to construct a binary cost function such that the minimizing configurations are known a priori. In the most general form, a cost function in Ising form with variables \( s = (s_1, \ldots, s_N) \), \( s_i \in \{\pm 1\} \) is given by

\[
H(s) = \sum_{j \in V} h_j s_j + \sum_{k=2}^{n} \sum_{(i_1, i_2, \ldots, i_k) \in E} J_{i_1 i_2 \ldots i_k} s_{i_1} s_{i_2} \cdots s_{i_k},
\]

where the hypergraph \( H = (V, E) \) with vertices \( V \) and hyperedges \( E \) captures the connectivity of the problem, and \( \{h_j\} \) are the local fields. A term containing a product of \( k \) spins, \( J_{i_1 i_2 \ldots i_k} s_{i_1} s_{i_2} \cdots s_{i_k} \), is referred to as a \( k \)-local interaction with \( J_{i_1 i_2 \ldots i_k} \) being the coupling constant. Equation (1) can be readily mapped onto a cost function of Boolean variables \( x = (x_1, x_2, \ldots, x_N) \), \( x_i \in \{0, 1\} \) in the form of a high-order polynomial unconstrained binary optimization (HOOBO) problem via the transformation \( s_i \to 1 - 2x_i \).

Except for a few platforms like Azure Quantum, most of the software for binary optimization problems mainly targets up to 2-local interactions, in which case Eq. (1) reduces to

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H(s) = \sum_{j \in V} h_j s_j + \sum_{(i,j) \in E} J_{ij} s_i s_j,
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where \( G = (V,E) \) is the underlying problem graph. Among the planting schemes supported by Chook, tile planting, Wishart planting, and deceptive cluster loops methods construct cost functions with 2-local interactions. Equation planting and \( k \)-local planting methods are capable of generating problems with higher-order \((k > 2)\) local interactions.

In what follows, we provide a summary of each solution planting scheme supported by Chook. For a detailed description, the reader is referred to the the original references introducing the methods.

A. Tile planting

In tile planting [15] one seeks to decompose the problem graph into edge-disjoint vertex-sharing subgraphs and embed elementary Ising subproblems that share a common ground state over the subgraphs. The method produces scalable problems with highly-tunable complexity on cubic and square lattice topologies [15, 19]. It also extends to arbitrary graph structures, e.g., via lattice animals.

Consider a decomposition of the problem graph \( G = (V,E) \) into subgraphs \( \{G_i = (V_i,E_i)\} \) that ensures no edges are shared among the subgraphs (i.e., edge-disjoint). For each subgraph, we define an Ising cost function of the form

\[
\mathcal{H}_i = \sum_{(i,j) \in E_i} J_{ij} s_i s_j. \tag{3}
\]

The subproblem Hamiltonians \( \{\mathcal{H}_i\} \) are added to obtain the complete Hamiltonian \( \mathcal{H}_{TP} \) over \( G \) as

\[
\mathcal{H}_{TP} = \sum_i \mathcal{H}_i. \tag{4}
\]

The subproblems are constructed such that they share a common ground state configuration \( t \), therefore the entire problem has a ground state characterized by the same local configuration \( t \) occupying the constituent subgraphs. For simplicity, \( t \) is taken to be the ferromagnetic ground state, i.e., \(+1,+1, \ldots,+1\) (modulo \( \mathbb{Z}_2 \) symmetry). Once the problem is constructed, the planted ferromagnetic ground state can be concealed by a gauge randomization in which the couplers \( \{J_{ij}\} \) are transformed as \( J'_{ij} \leftarrow J_{ij} q_i q_j \), where \( q \) is an arbitrary ground state.

Chook supports the generation of tile-planted problems on square and cubic lattices with periodic boundary conditions. The regular structure of these lattices allows for a problem-graph decomposition that naturally renders a subset of the unit cells as the subgraphs. Figure 1 illustrates this decomposition for square lattices, in which the resulting unit-cell subgraphs (dark color) form a checkerboard pattern.

For square lattices we define four subproblem classes \( \{C_i\}, i \in \{1,2,3,4\} \) that correspond to unit cycles (plaquettes) with different levels of frustration [see Fig. 2(a)] for an illustration]. A subproblem from the class \( C_i \) is constructed by assigning the antiferromagnetic value \(+1\) to a chosen coupler, the ferromagnetic value \(-1\) to randomly selected \( i - 1 \) couplers, and \(-2\) to the remaining couplers. Planted instances are generated by first assigning a subproblem type for each subgraph in the lattice, followed by a random rotation of the plaquette to allow for more disorder. We define instance classes based on the probability distribution over classes \( \{C_i\} \) according to which subproblem types are assigned to subgraphs. Specifically, we denote \( p_i \) to be the probability of choosing subproblems from class \( C_i \), and uniquely define each instance class based on the three probability parameters \( \{p_1,p_2,p_3\} \), where \( p_1 + p_2 + p_3 \leq 1 \). Multiple complexity transitions have been observed in the multidimensional phase space defined by these parameters [19].

For cubic lattices, Chook uses three subproblem types \( F_{22}, F_{42}, \) and \( F_6 \), each having two, four, and six frustrated facets, respectively [see Fig. 2(b)]. Each subproblem class consists of 48 members that are equivalent by octahedral symmetry under the operations of rotation and reflection [21]. Similar to the square lattice case, problem construction begins by assigning subproblem types for the subgraphs according to the chosen probability distribution over classes \( \{F_{ij}\} \). Then for each subproblem type, one of the 48 members are selected randomly. Instance classes are defined based on the two probability parameters \( \{p_{F_{22}}, p_{F_{42}}\} \), \( p_{F_{22}} + p_{F_{42}} \leq 1 \), where \( p_{F_{22}} \) and \( p_{F_{42}} \) are the probabilities of choosing subproblems from the \( F_{22} \) class and \( F_{42} \) class, respectively. By varying these parameters one can achieve drastic changes in typical complexity, with higher concentrations of \( F_6 \) subproblems resulting in problems that are many orders of magnitude harder than random problems with bimodal and Gaussian disorder [15].

FIG. 1. Decomposition of a square lattice with periodic boundary conditions into edge-disjoint, unit-cell subgraphs (shaded cells) on which Ising subproblems are embedded. Under periodic boundary conditions, each vertex is shared by exactly two subgraphs.
can be shown that the problem Hamiltonian and zero the diagonal to obtain
in terms of a parameter is varied.

\[ \chi \]

\( b \)

Wishart planting \[ 18 \] generates Ising Hamiltonians on complete graphs of the form
in which the coupler matrix \( J \) follows a Wishart distribution, a matrix generalization of the \( \chi^2 \) distribution. The model exhibits a first-order phase transition in temperature, and allows for dramatic variations in typical hardness over many orders of magnitude as a control parameter is varied.

In Wishart planting, one defines the coupler matrix \( J \) in terms of a \( N \times M \) real-valued matrix \( W \), where \( N \) is the number of spins and \( M (M \geq 1) \) is a tunable parameter which regulates the typical hardness and the thermodynamic behavior of the problem ensemble. Specifically, we define \( J \) as
and zero the diagonal to obtain

\[ J = \frac{1}{N} WW^T, \]

and can be shown that the problem Hamiltonian \( \mathcal{H}_{WP} \) then becomes

\[ \mathcal{H}_{WP}(s) = \frac{1}{N} \|W^T s\|^2 - \frac{1}{2} \text{Tr}(J). \]

Because the first term is in positive semidefinite quadratic form, \( \mathcal{H}(s) \) is minimized when \( s = t \) at which \( W^T t = 0 \). Thus, the goal is to construct the matrix \( W \) for a given planted solution \( t \) such that the condition \( W^T t = 0 \) is satisfied. We achieve this by drawing the \( M \) columns \( \{\omega^\mu\} \) of \( W \) from a correlated Gaussian distribution \( \omega^\mu \sim \mathcal{N}(0, \Sigma) \), where \( \Sigma = N[I_N - tt^T/N]/(N-1) \) is the covariance matrix. More precisely, we successively sample uncorrelated Gaussian variates \( z^{\mu} \sim \mathcal{N}(0, I_N) \) and then multiply by the square root of \( \Sigma \) to obtain \( \omega^\mu \), i.e., \( \omega^\mu = \Sigma^{\frac{1}{2}} z^{\mu} \). It can be shown that \( \langle \omega^\mu, t \rangle = 0 \) for all \( \mu \) generated in this manner. The matrix \( WW^T \) is known to follow a Wishart distribution. Analogous to the clause-to-variable ratio in Boolean satisfiability problems, we define an equation-to-variable ratio \( \alpha = M/N \) for modulating the typical computational hardness of the problem ensemble. The class exhibits a pronounced easy-hard complexity transition as \( \alpha \) is varied \[ 15 \].

One can construct problems with discrete coupler values via a simple modification of the sampling procedure. When generating \( \{\omega^\mu\} \), instead of using Gaussian variates \( z^{\mu} \sim \mathcal{N}(0, I_N) \), one can sample from a Rademacher distribution, i.e., draw samples independently and uniformly from \( \{-1, 1\} \). It can be shown that the scaled couplers \( J'_{ij} = N^2(N-1)J_{ij} \) assume values in the set of equally-spaced integers given by

\[ J'_{ij} \in \{-4M(N-1)^2, \ldots, -4, 0, 4, \ldots, 4M(N-1)^2\}. \]

C. Deceptive cluster loops

Deceptive cluster loops (DCL) \[ 20 \] are a class of benchmark problems designed for the Chimera topology of the D-Wave 2000Q and D-Wave 2X quantum annealers, although the ideas can be generalized to other topologies. DCL problems are derived from the conventional frustrated cluster loops (FCL) problems \[ 22 \]. They have an additional control parameter that conceals the logical structure of the problem for a particular range of values. Such a feature makes these problems hard to solve using algorithms that exploit the underlying logical structure.

Both DCL and FCL problems are based on the traditional frustrated loop problems introduced in Ref. \[ 12 \]. Frustrated loop problems are constructed by generating \( M = \alpha N \) loops on a graph \( G = (V, E) \), where \( N \) is the number of nodes and \( \alpha \) is the loop-to-node ratio. The loops are generated by placing random walkers on random nodes. A random walk is terminated when it crosses its own path, and the trailing tail segment is discarded to form a closed loop. On each loop \( k \), the coupler values \( J_{ij}^k \) are set to the ferromagnetic value \(-1\) except for a single, randomly-chosen coupler for which we assign the antiferromagnetic value \(+1\). The total Hamiltonian is formed by adding up the couplers belonging to all the loops

\[ \mathcal{H}_{\text{DCL}} = \sum_{(i,j) \in E} \sum_{k=1}^M J_{ij}^k s_i s_j. \]
The problem is discarded if \( \sum_{k=1}^{M} J_{ij}^{k} > R \) for any edge \((i, j) \in E\), where \( R \) is referred to as the “ruggedness.” In frustrated cluster loop (FCL) problems, frustrated loops are generated on a two-dimensional logical lattice embedded on a Chimera graph. Here, all couplers within each Chimera unit cell are set to the ferromagnetic value \(-1\), forcing all physical spins within the unit cell to behave as a logical spin. The frustrated loops are then generated on the resulting \( L_x \times L_y \) two-dimensional logical lattice, where \( L_x \) and \( L_y \) are the linear dimensions of the parent Chimera graph.

DCL problems are an extension of FCL problems. Here, all inter-cell couplers between Chimera unit cells are multiplied by a scaling factor \( \lambda \), while leaving all intra-cell couplers intact. For small values of \( \lambda \) (\( \lambda \approx 1 \)), DCL problems can be described by a virtual planer model with each Chimera unit-cell behaving as a single virtual variable. For \( \lambda \gg 1 \), the model behaves as a virtual fully-connected bipartite problem. For intermediate values of \( \lambda \), the problem behavior is nontrivial and no logical structures (as it happens for either small or large values of \( \lambda \)) can be determined to simplify the optimization of the problem; see Ref. [20] for details.

### D. Equation planting

Systems of linear equations modulo 2, also known as “exclusive or satisfiability” (XORSAT) equations, are solvable in polynomial time by Gaussian elimination, but when formulated as optimization problems, they are often challenging for heuristic solvers [23–25]. Equation planting [17] casts XORSAT problems as \( k \)-local \((k > 1)\) Ising cost functions.

Consider a linear system of equations modulo 2 with \( N \) Boolean variables \( \{x_1, x_2, \ldots, x_N\} \) and \( M \) equations

\[
\sum_{j=1}^{N} a_{ij} x_j = b_i, \quad (10)
\]

for \( i \in \{1, \ldots, M\} \), where the coefficients \( \{a_{ij}, b_i\} \in \{0, 1\} \). Alternatively, each equation can be written in terms of the bit-wise XOR operation as \( a_{ij} x_1 \oplus \cdots \oplus a_{ij} x_N = b_i \). When expressed in the Ising form, the above equation becomes

\[
\prod_{j: a_{ij}=1} s_j = (-1)^{b_i}, \quad s_j \in \{\pm 1\}, \quad (11)
\]

in which only the variables with non-zero coefficients are included in the product. The linear system can be cast as an optimization problem with an Ising cost function, i.e.,

\[
\mathcal{H}' = \sum_{i=1}^{M} \left[ \prod_{j: a_{ij}=1} s_j - (-1)^{b_i} \right]^2, \quad (12)
\]

which, after dropping irrelevant constants, reduces to

\[
\mathcal{H}_{\text{XORSAT}} = -\sum_{i=1}^{M} (-1)^{b_i} \prod_{j: a_{ij}=1} s_j. \quad (13)
\]

The ground state of the Ising cost function corresponds to the solution of the linear system, given that the linear system is solvable. Here we limit our attention to \( k \)-regular \( k \)-XORSAT problems, where each equation contains exactly \( k \) randomly selected variables out of the \( N \) variables (hence \( k \)-XORSAT), and each variable appears in exactly \( k \) equations (hence \( k \)-regular). Such a linear system consists of \( N \) equations (i.e., \( M = N \)), and the resultant Ising Hamiltonian contains \( N \) \( k \)-local terms. If the linear system has solutions, the ground-state energy of the Ising cost function is \( -M \), and the ground-state degeneracy (i.e., the number of minimizing configurations) is given by \( g = 2^{N-r} \), where \( r \) is the number of linearly independent (in arithmetic modulo 2) rows of the matrix \( A = (a_{ij}) \). For random \( k \)-regular \( k \)-XORSAT instances, \( r \) is typically \( \mathcal{O}(N) \) and hence the number of ground states grows sub-exponentially [26–28].

### E. \( k \)-local planting

Here we introduce a method of planting solutions for Hamiltonians with higher-order \((k > 2)\) interactions by combining Hamiltonians with lower-order \((k \leq 2)\) interactions and known ground states.

Consider a set of \( n \) problems described by the Hamiltonians \( \{\mathcal{H}^{(1)}(s^{(1)}), \mathcal{H}^{(2)}(s^{(2)}), \ldots, \mathcal{H}^{(n)}(s^{(n)})\} \) whose ground state energies are \( \{E^{(1)}_0, E^{(2)}_0, \ldots, E^{(n)}_0\} \). Note that the Hamiltonians are completely independent of each other in that the underlying problem graphs do not share any vertices or edges. The composite Ising cost function defined by the product

\[
\mathcal{H}_{\text{comp}}(\{s^{(1)}, s^{(2)}, \ldots, s^{(n)}\}) = \prod_{i=1}^{n} [\mathcal{H}^{(i)}(s^{(i)}) - E^{(i)}_0] \quad (14)
\]

is minimized for \( s^{(i)} = t^{(i)}, \ i \in \{1, \ldots, n\} \), where \( t^{(i)} \) is a ground state of \( \mathcal{H}^{(i)} \). The composite problem contains \( N = \sum_{i=1}^{n} N^{(i)} \) spins, where \( N^{(i)} \) is the number of spins in the \( i \)th problem. The locality of the highest-order term in the composite Hamiltonian \( \mathcal{H}_{\text{comp}} \) is \( k_{\text{max}} = \sum_{i=1}^{n} k_{\text{max}}^{(i)} \), with \( k_{\text{max}}^{(i)} \) being the locality of the highest order term in \( \mathcal{H}^{(i)} \).

Chook constructs higher-order \((k > 2)\) cost functions with even \( k_{\text{max}} \) by using tile-planted problems and Wishart-planted problems as constituent problems [29]. Because these problem types consist of only 2-local interactions, one cannot construct composite problems with odd \( k_{\text{max}} \) using these two problem types alone. Therefore when generating problems with odd \( k_{\text{max}} \), we also use a trivial Hamiltonian with 1-local interactions, namely, Ising spins coupled to a bimodal random field, given by
\[ H_{\text{1-local}} = \sum_{i=1}^{N_l} h_i s_i, \] where \( N_l \) is the number of spins and \( \{h_i\} \) are the random fields drawn independently and uniformly from \{+1, −1\}. The ground state of \( H_{\text{1-local}} \) is trivially obtained by aligning all spins with their random fields.

### III. CHOOK

CHOOK is a platform-independent, Python-based tool distributed under the Apache License 2.0. The code is openly available on the GitHub software sharing platform (https://github.com/dilinanp/chook), as well as being included in this submission (see ancillary files). It is also hosted on the Python Package Index (PyPI) for easy installation via the package management system pip. We ask you to please cite this work if you choose to use CHOOK.

#### A. Installation

CHOOK requires Python version 3.4 or above for installation. We recommend installing CHOOK in a Python virtual environment to avoid potential conflicts with packages installed system-wide. If the Python package management system pip is available, CHOOK can be installed from the command line by running the command:

```bash
$ pip install chook
```

Alternatively, chook can be directly downloaded from PyPI or the GitHub repository and can be installed with the command:

```bash
$ python setup.py install
```

During installation, the prerequisite Python libraries numpy, scipy, and more-itertools will be automatically installed if they are not already available.

#### B. Usage

CHOOK can be executed from the command line providing two required positional arguments `<problem_type>` and `<config_file>`, followed by zero or more optional arguments. The basic usage ignoring the optional arguments is

```bash
$ chook <problem_type> <config_file>
```

where the different types of `<problem_type>` are listed in Table I and `<config_file>` is a configuration file in INI format that contains the problem-type specific parameters (see Sec. IID). Table I shows the complete list of optional arguments that can be appended to change the default behavior of CHOOK.

| Problem type            | INI section header |
|-------------------------|--------------------|
| Tile planting           | [TP]               |
| Wishart planting        | [WP]               |
| Deceptive cluster loops | [DCL]              |
| Equation planting       | [XORSAT]           |
| k-local planting        | [K_LOCAL]          |

#### C. Output

CHOOK stores the generated problem instances in a subdirectory under the current working directory, which is named according to the problem type and major problem-specific parameters. For example, tile-planted problems on a square lattice with linear lattice size \( L = 32 \) and tuning parameters \( p_1 = 0.2, p_2 = 0.5, \) and \( p_3 = 0.1 \) will be stored in a subdirectory named `tile_planting_2D_L_32_p1_0.2_p2_0.5_p3_0.1`. Each problem instance will be stored in a separate file within the said subdirectory. Except for deceptive cluster loop problems for which the ground states cannot be decoded (except in specific limits), the ground-state energies will be stored in a separate file `gs_energies.txt` with two columns: the instance file name and the ground state energy. For the case of XORSAT problems, `gs_energies.txt` will contain a third column, the ground state degeneracy.

The generated instances are expressed in the Ising form by default, but can be cast in the binary HOBO form by setting the optional argument `-o hobo`. By default, the instance files are in plain text format. Each line in the instance file corresponds to a single \( k \)-local interaction term following the format

\[ <i_1> <i_2> \ldots <i_k> <J> \]

where the first \( k \) elements are the indices of the spin (Boolean) variables, and the last element \(<J>\) is the coupling constant. For the case of field terms (i.e., 1-local interactions), the entry consists of two terms: the index of the spin (Boolean) variable followed by the field value. The user has the option to save the instances in the JavaScript Object Notation (JSON) format by setting the optional argument `-f json`.

#### D. Configuration file

As the second command-line argument, CHOOK requires a configuration file in INI format that specifies the parameters associated with the selected problem type. CHOOK is distributed with a sample configuration file.
The code supports the construction of cost functions with problems (or “subproblems”) of three supported types, namely, tile planting, Wishart planting, and bimodal random field terms. For odd values of \( k_{\text{max}} \), one should use \( (k_{\text{max}} - 1)/2 \) subproblems with 2-local interactions, and a single subproblem with a 1-local term (i.e., a bimodal random field term).

IV. SUMMARY

We have presented a Python-based suite, Chook, for generating discrete optimization problems with known solutions using multiple popular solution planting schemes. Chook is distributed freely via Python Package Index (PyPI) and GitHub software sharing platform, and allows for fast and easy installation on any platform. The code supports the construction of cost functions with tunable hardness and local interactions spanning 2-local to arbitrary higher-order. We believe that Chook will be highly beneficial for generating benchmark problem sets for current and future generations of programmable devices for discrete optimization.

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### Table II. Optional command-line arguments supported by Chook.

| Argument flag    | Description                                                                 |
|------------------|-----------------------------------------------------------------------------|
| -n `<num_instances>` | `<num_instances>` is the number of instances to be generated (default value: 10). |
| -o `<output_format>` | `<output_format>` can either be `ising` or `hobo`, and specifies whether the problem is expressed in the Ising form or the binary HOBO form (default value: `ising`). |
| -f `<file_format>` | `<file_format>` can either be `txt` or `json`, and specifies whether the output files are in text format or the JSON format (default value: `txt`). |
| -h, --help       | Show a help message and exit.                                               |

With the provided params.in file which includes sample parameter specifications for all problem types. Each parameter is specified as an INI property with a name and a value, separated by an equal sign, i.e., name = value. Properties are grouped into sections according to the problem types they are associated with, and the sections begin with section headers of the form `[@<problem_type>]`. Table II shows the section headers associated with the supported problem types. A section ends when the next section header is encountered, or when the end of file is reached. When Chook is executed, only the section associated with the chosen problem type is read from the configuration file, and the rest of the file is ignored. Table III shows a comprehensive list of supported problem-specific properties. We recommend the users to modify the provided params.in file to meet their needs rather than scripting their own configuration file to avoid potential errors.

We now describe the parameter specification for the \( k \)-local planting method. In Chook, \( k \)-local problems are constructed by combining a sequence of constituent problems (or “subproblems”) of three supported types, namely, tile planting, Wishart planting, and bimodal random field terms. Parameter specifications for the subproblems should be grouped into separate INI sections with user-defined headers and should be listed below the main section `[K_LOCAL]`. The section `[K_LOCAL]` contains two properties: \( k_{\text{max}} \) which represents the locality \( k_{\text{max}} \) of the highest-order term in the composite Hamiltonian, and `subproblem_id_list` that accepts a list of identifiers representing the subproblems. Each subproblem identifier `<subproblem_id>`, enclosed within square brackets, i.e., `[<subproblem_id>]`, is used as the header of the section under which the subproblem properties are listed. Subproblem identifiers should begin with a letter, and may contain a combination of letters and numbers. One can reuse the same subproblem (with the same parameter specifications) multiple times in the construction procedure, in which case the corresponding subproblem identifier should be repeated accordingly in the `subproblem_id_list`. In addition to the usual problem-specific properties, each section defining a subproblem should contain an additional property `subproblem_type`, which is used to identify the type of the subproblem being defined. The allowed values are, “RF” for the bimodal random field terms, “TP” for tile planting, and “WP” for Wishart planting. As the bimodal random field terms can be trivially optimized, Chook enforces the user to minimize its usage, and it is only allowed when constructing problems with odd values of \( k_{\text{max}} \). Therefore, for even values of \( k_{\text{max}} \), one should use \( k_{\text{max}}/2 \) subproblems with 2-local interactions, which can be tile-planted problems and/or Wishart problems. For odd values of \( k_{\text{max}} \), one should use \( (k_{\text{max}} - 1)/2 \) subproblems with 2-local interactions, and a single subproblem with a 1-local term (i.e., a bimodal random field term).
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One can define two additional subproblem classes, $F_{21}$ and $F_{41}$, having the same number of frustrated facets as the $F_{32}$ and $F_{42}$ classes, respectively. Each of these classes also consists of 48 members that are equivalent under octahedral symmetry.

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Note that when multiplying Wishart problems to generate higher-order Hamiltonians the range of the couplers grows, thus making the problems considerably harder to solve. We recommend using sub-problem classes with integer coupler values to avoid this problem.
TABLE III. Problem-specific properties defined in the configuration file. Note that the term “variables” refers to either spins (in Ising form) or Boolean variables (in HOBO form).

| Section header | Property name | Description |
|----------------|---------------|-------------|
| dimension      | L             | Spatial dimension $D$ of the periodic lattice on which problems are constructed. Should be set to 2 for a square-lattice and 3 for a cubic-lattice. |
|                | p1, p2, p3    | Probabilities $\{p_1, p_2, p_3\}$ for constructing problems on square-lattice topology. $p_i$ is the probability of choosing subproblems from class $C_i$. Should satisfy the conditions $p_1 + p_2 + p_3 = 1.0$ and $0 \leq p_i \leq 1$, $i \in \{1, 2, 3\}$. These parameters are ignored when `dimension = 3`. |
|                | gauge_transform | If set to `yes`, the planted ferromagnetic ground state will be concealed via a gauge randomization. Allowed values: `{yes, no}` |
|                | N             | The number of variables $N$ in the planted problem. |
|                | alpha         | Equation-to-variable ratio $\alpha = M/N$, where $M$ is the number of columns in the matrix $W$. Note that $M$ is determined from $\alpha$ as $M = \alpha N$, and will be rounded to the nearest non-zero integer. Thus the value of $\alpha$ internally represented by Chook can be different from the user-provided value. |
|                | discretize_couplers | If set to `yes`, the code generates problems with discrete couplers by sampling from a Rademacher distribution instead of a Gaussian distribution when constructing the matrix $W$. Allowed values: `{yes, no}` |
|                | gauge_transform | If set to `yes`, the planted ferromagnetic ground state will be concealed via a gauge randomization. Allowed values: `{yes, no}` |
|                | Lx, Ly        | Linear dimensions $L_x$ and $L_y$ of the Chimera graph on which problems are constructed ($1 \leq L_x, L_y \leq 16$). Frustrated loops are generated on the $L_x \times L_y$ logical lattice which treats Chimera unit cells as logical variables. The number of (physical) variables is $N = 8L_xL_y$. |
|                | alpha         | Loop-to-node ratio $\alpha$ defined by $\alpha = M/N_l$, where $N_l = L_x \times L_y$ is the number of logical variables, and $M$ is the number of loops generated on the logical lattice. Note that $M$ is determined from $\alpha$ as $M = \alpha N_l$, and will be rounded to the nearest non-zero integer. Thus the value of $\alpha$ internally represented by Chook can be different from the user-provided value. |
|                | R             | Ruggedness $R$ that limits the range of coupler strength as $|\sum_{k=1}^{M} J_{ij}| > R$. Must be an integer greater than zero. |
|                | lambda        | Scaling factor $\lambda \geq 1$ by which the inter-cell couplers between Chimera unit cells are scaled. For $\lambda = 1$, DCL problems are equivalent to FCL problems. |
|                | k             | Locality $k$ of the terms in the resultant Ising cost function (equivalent to the number of variables per equation in $k$-regular $k$-XORSAT). |
|                | N             | The total number of variables $N$ in the problem ($N \geq k$). This is equivalent to the number of equations in the $k$-regular $k$-XORSAT representation. |
|                | k_max         | Locality $k_{\text{max}} > 2$ of the highest-order term in the composite Hamiltonian. |
|                | subproblem_id_list | A list of user-defined, comma-delimited identifiers representing the subproblems. Subproblem identifiers should begin with a letter and can include alphanumeric characters. Each identifier enclosed by square brackets, i.e., `[subproblem_id]`, is used as the section header under which the properties of the subproblem are grouped. A subproblem identifier can be repeated multiple times in `subproblem_id_list`, if one chooses to reuse the corresponding subproblem specification multiple times. |
|                | subproblem_type | A numeric code used to identify the type of subproblem being defined. Should be included with every INI section defining a subproblem, in addition to the subproblem-specific properties. Allowed values: `RF` - Bimodal random field, `TP` - Tile planting, `WP` - Wishart planting |
|                | <subproblem-specific properties> | Define all the properties associated with the selected subproblem type. If the subproblem type is a bimodal random field, define a single property $R$, which specifies the number of variables in the subproblem. |