Constraints on the Distortion of Area Objects in the Process of Map Objects Displacement

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Abstract  The displacement operator is an important and still a hot topic in map generalization. In the generalization product, symbols must be unambiguous and easily perceived and readily understood, which makes space competition among features an important obstacle in the process of map objects displacement. Space conflict between objects, through propagation process, may be spread and more objects may drift into it. In order to maintain symbols equilibrium and spatial relationship between objects, some unimportant symbols or parts of symbols should be distorted under constraints according to visual graphic resolution thresholds to figure out space competition among map features. Three constraints including position, legibility and characteristics are important for the maintenance of symbols equilibrium and spatial relationship, which are discussed in this paper. The skeleton is introduced to represent area objects figure characteristic, in which an area object can be separated to parts hierarchically according to their importance in the construction. Then, the finite element method is applied to the map objects’ displacement and distortion, in which a strategy for the parameters of finite element method is discussed.

Keywords  map objects displacement; constraints; object distortion; finite element method

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Introduction

In map generalization, the magnification and simplification of symbols are usually followed by the change of symbols volume and spatial position, which is the process of space competition and reasonable redistribution among map symbols. In order to maintain legible symbols and spatial relationship between objects, some map objects should be displaced. The purpose of map objects displacement is to avoid space conflict based on some displacement rules and constraints\\[1]. However, in the area of dense objects and severe space competition, some objects need to be distorted and rotated under constraints in the process of objects displacement\\[2-5]. Many scholars have researched area object distortion and rules in map generalization\\[6], and have discussed area object distortion and rotation\\[7-9]. Generally speaking, object distortion is a kind of dynamic pressurized distortion, and the spatial structural characteristics of objects need to be preserved in this process of distortion. In this paper, we analyze the constraints of map objects displacement and area object distortion first, then use the finite element method to
solve the map objects displacement and distortion, while considering preserving the object spatial structural characteristics at the same time.

1 The finite element method of map objects displacement

The finite element method is a kind of numerical analysis method combining elastic theory and computational mathematics. This method disperses the object region into finite elements connecting each other by node, and simplifies the complicated geometrical shape, material characteristics and boundary condition, then uses simple functions to represent approximately the distribution rules of displacement. Based on variational principles or other mechanics theories, the mechanical characteristics relationship between force and displacement can be built, and the approximation can be found through some computational models. In the process of dispersing the area using triangle elements, we supposed that one element has three nodes as $i$, $j$, $m$, and every node has two node force components and two displacement components in $X$ and $Y$ coordinates, respectively, as $F_{ix}$, $F_{iy}$, $u$ and $v$, then the relationship between force and displacement is as follows:

$$
\begin{bmatrix}
F_{ix} & F_{iy} & F_{jx} & F_{jy} & F_{mx} & F_{my}
\end{bmatrix}^T = \Delta \begin{bmatrix} B \end{bmatrix}^T \begin{bmatrix} D \end{bmatrix} \begin{bmatrix} B \end{bmatrix} \begin{bmatrix} u_i & v_i & u_j & v_j & u_m & v_m \end{bmatrix}
$$

(1)

Where $\Delta$ is the area of triangle cell, $B$ is the geometric matrix of triangle cell, and $D$ is physics function,

$$
D = \frac{E}{1-\mu} \begin{bmatrix}
1 & \nu & 0 \\
\nu & 1 & 0 \\
0 & 0 & (1-\mu)/2
\end{bmatrix}
$$

(2)

where $E$ is the elastic modulus of the cell which can control the spread degree, and $\mu$ is the Poisson's ratio of the cell that represents cell's distorted relationship in the vertical direction.

Let $[k]^T$ represents $\Delta[B]^T[D][B]$, then $[k]^T$ is namely the rigidity matrix of the cell that represents the conversion relationship between node force and node displacement. Based on the node displacement calculation principle in the finite element method, the relationship between force and displacement in the whole structure is decided by total rigidity matrix. When the cells' rigidity matrix conditions are known, the total rigidity matrix can be formed by these cells based on a certain rule. This rule is that total rigidity matrix is evaluated to zero first, sub-matrix $[K_n]^T$ of each cell's rigidity matrix is placed into the corresponding position in the total rigidity matrix according to row and column from the subscript. If there are many sub-matrices in the same position, they will be united. From the forming rule of the total rigidity matrix, the matrix is only related to material physics characteristics and structural geometrical size. That is, elastic modulus and Poisson's ratio control the relationship between node force and node displacement after dispersing map using a certain triangle network.

Højholt[7] researched the application of finite element method to buildings displacement of map. He dispersed a map using triangulation, and used the different parameters of building inner cells and outer cells to control building displacement and distortion. This method can effectively consider the relationship between force and displacement, and combine the cell's rigidity matrix to form the total rigidity matrix. It can also synthetically assess the influences on building displacement and distortion raised from local force and boundary conditions, thereby receiving the approximation using the transmitting character of force. However, this method is restricted in the dense objects area because the limited displacement space can cause a spatial relationship conflict in the process of compressing objects. As Fig.1 shows, there are

Fig.1  Overlapped by roads and buildings after the buildings displacement
overlapped areas by roads and buildings after the building displacement. At the same time, due to the absence of transmitting controls of force, large-scale objects displacement might be formed.

2 Constraints on the distortion of area objects

Generally, single object displacement may cause a group of objects displacement due to the influence of object spatial conflict transmission raised from space competition in the process of displacement. However, because of the constraints of the whole map characteristic and boundary condition, the object displacement is a constrained process which is restricted in limited spatial range. When the intense space competition causes illogical object displacement or object overlap, the distortion or deleting part of secondary objects needs to be considered in order to support more space to satisfy the object displacement. Usually, the different objects or the different parts of one object have different influences on map or object structural characteristic. In the process of displacement, the appropriately distorting secondary object under constraints is accepted if it has no influence on the main structural characteristic.

The aim of constraints on the objects displacement is to maintain the legible map content, beautiful layout and consistent spatial relationship. Constraint conditions contain: (1) absolute constraints and optional constraints based on different constraint intensity and aims, (2) global constraints and local constraints with definite scope, (3) inner constraints such as legibility and topological relation and outer constraints such as map equilibrium and spatial characteristic maintenance, and (4) independent constraints, which mean the area of signs must be larger than the given value in order to keep legibility, and dependent constraints, which mean two objects cannot be in the same position, considering the element relationship. Therefore, we consider mainly the following constraints in the process of map object displacement.

2.1 Position constraints

Spatial position contains two parts: the absolute position which is an important attribute of map object, and opposite position which represents the spatial relationship between object and object group. For some map objects such as boundary lines and control points with a special position, the absolute position must be preserved strictly. The opposite position describes the spatial relationship among objects such as topology, direction, distance, clustering and so on. It reflects the spatial composing relationship of map objects. Also, it is a basis for constituting the corresponding constraints conditions.

2.2 Characteristic constraints

The object's characteristic is one of the important components of map signs. The boundary of the area object is a closed curve, which can be described by area, perimeter, and dimension and so on. However, in spatial shapes, the object's extension and concave-convex condition are more important characteristics. The object's extension can be described by trend or skeleton line, and concave-convex condition by the complex degree of object boundary. The minimal convex hull is the set of convex mark points. In the process of area objects displacement and distortion, area objects skeleton maintenance is one of the main constraints conditions and an important approach to keep mark points.

2.3 Legibility constraints

When we process the spatial relationship of map objects, one of the important tasks is maintaining map legibility without sign overlap, intersection and tangency. Therefore, when we process the area object, we should not only keep a distance between objects to show clear boundary, but also prevent sharp angles in object boundaries.

3 Process of area objects distortion

Based on the constraints conditions and the finite element method, we can use the following methods to constrain area objects distortion in the process of map displacement: (1) extracting area object skeleton line according to the weight of every component in the area object, (2) defining the influencing scope and constraints conditions of object displacement, (3) us-
ing triangulation to form finite elements, (4) setting boundary condition according to constraints and evaluating the parameters, (5) calculating the displacement of area object and post processing. The process is described as shown in Fig. 2.

3.1 Area object skeleton and hierarchy

The skeleton line is comprised of some points which have the same distance to two-side boundaries in a 2D graph. It is also a mid-axes line that represents the shape character of the area object and is used to converse the area object into a line object. The area object with tree structure represents the skeleton with a network structure in which weightiness is hierarchical too and the basic skeleton is the most important one. In the process of sampling the area object shape, the secondary skeletons are usually processed earlier than the basic skeletons. The local hierarchy of the area object can be formed based on the hierarchy of skeletons.

3.2 Boundary condition of object displacement

Considering map equilibrium and spatial character, object displacement needs to be restricted in a limited scope whose boundary is defined by a displacement safe region[11]. Then, the boundary conditions of FEM can be defined based on the boundary of object displacement. When the node is converted into a boundary point, the displacement equals to zero no matter what force. Therefore, the boundaries which affect scope and nodes or which keep absolute spatial position can be seen as the boundary conditions.

3.3 Parameters evaluation and adjustment

Generally, elastic modulus and Poisson's ratio are invariable in the same cell, but they are different in different cells. The elastic modulus is inversely proportional to node displacement, and the Poisson's ratio is proportional to cell distortion in the vertical direction under a certain force. Based on the hierarchy of the area object weightiness and relationship between elastic modulus, Poisson's ratio and object displacement, the evaluation strategy of elastic modulus and Poisson's ratio can be developed as follows: (1) the basic skeleton should be evaluated with maximal elastic modulus and minimal Poisson's ratio because the basic skeleton can compose the area object skeleton and its shape must be maintained strictly; (2) the secondary skeleton can be evaluated with less elastic modulus and bigger Poisson's ratio due to the lower shape weightiness of the secondary skeleton; (3) the elastic modulus is given as minimum and the Poisson's ratio is maximum in order to obtain enough space for object displacement, if the region between objects can satisfy least distance. However, when object displacement reaches a critical distance, further compressing object distance may produce spatial relationship conflict. Here, the elastic modulus and Poisson's ratio need to be adjusted to keep minimal distance based on the evaluation strategy of the basic skeleton.
3.4 Post processing

The shape similarity of the triangle cell has an influence on the solution precision. After interpolating the object boundary to form triangle cells with similar size and shape, the linear boundary can usually become the curve one. Therefore, post processing is necessary to keep similar object shape, for example orderly reconnecting the original nodes.

3.5 Computation and analysis

As a case study, Fig.3(a) shows the original objects including roads, building pools and so on. Fig.3(b) is the extracted objects skeleton based on longest skeleton line, and Fig.3(c) represents displacing points, loading directions and boundaries. Loading size can be realized by constraining displacement length. In this case, the displacement length of all displacing points is 1 m and the direction is vertical. Fig.3(d) describes objects' classification based on object weightiness. There are three classes: skeleton components, non-skeleton components, and the others between objects. Among these, the skeleton components are the most important. Due to the effect of road expansion and boundary condition, the cell weightiness inside road is equal to that between objects. Based on object weightiness, the evaluation strategy of elastic modulus and Poisson’s ratio can be defined as shown in Table 1. When the object distance reaches the threshold in the process of object displacement, the elastic modulus and the Poisson’s ratio will be adjusted to the value of skeleton cell. Fig.3(e) shows the result of triangulation in FEM, and Fig.3(f) is the result of displacement. The total displacing distance is 10 m with 1 m one time. The result shows that the object skeleton is maintained preferably and the non-skeleton is more distorted under extrusion produced by object displacement conditions. The above computing process is achieved in Ansys 8.

Fig.3  Map objects displacement and distortion
Table 1  Main parameter of finite elements

| Node loading          | Boundary node          | Elements properties |   |
|-----------------------|------------------------|---------------------|---|
| 1m displacement in the | No displacement in the  | E       | μ  | E  | μ  | E  | μ  |
| direction of Y        | direction of X and Y   | 20      | 0.2| 2  | 0.6| 0.2| 0.9|

4  Conclusion

Object distortion is one of the important problems in map object displacement. Usually, gradually deleting local parts can achieve simplification of objects. However, because object distortion is dynamic and pressurized, the finite element method can better consider the constraints condition on displacement and distortion and maintain the spatial structural characteristics and spatial relationship. The automatism and entirety of this method are better than other local processing methods in many aspects such as local selecting and computing process.

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