$\tau \to \eta(\eta')2\pi\nu, 3\pi\nu$ and WZW anomaly

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Abstract

The effects of the anomalous contact terms $\mathcal{L}_{\eta(\eta')\rho\pi\pi}$ are taken into account in calculating the decay rates of $\tau \to \eta(\eta')\pi\pi\nu$. The branching ratio of $\tau \to \eta\pi\pi\pi\nu$ is calculated. Theoretical result agrees with the data. It is the first time that the anomalous Wess-Zumino-Witten vertex $\eta\pi\pi$ is tested. A $a_1$ resonance is predicted in the final state of the three pions. The prediction of the branching ratio of $\tau \to \eta'\pi\pi\pi\nu$ is presented.
All the hadrons produced in $\tau$ hadronic decays are mesons made of the light quarks, therefore, the $\tau$ mesonic decays provide a test ground for all meson theories. Chiral symmetry plays an essential role in studying $\tau$ mesonic decays[1]. The Wess-Zumino-Witten (WZW) anomalous action[2] is general and model independent. It is important part of the meson theory. Test of the WZW action is very significant in the physics of strong interaction. Various anomalous WZW vertices have been used to calculate the decay widths of mesons[3,4]. The $\tau$ mesonic decays provide a comprehensive test ground for WZW anomaly. The abnormal vertices $\eta\rho\rho$ and $\omega\rho\pi$ derived from the WZW anomalous action have been studied[5] in $\tau$ mesonic decays. In Refs.[6] we have studied more abnormal $\tau$ mesonic decays. It is pointed out in Ref.[7] that the $\eta$ production in $\tau$ decay is associated with anomaly. Recently CLEO has reported a measurement of the branching ratio of $\tau \rightarrow \eta(3h^-)\nu[8]$

$$B(\tau^- \rightarrow \nu_\tau(3h^-)\eta) = (4.1 \pm 0.7 \pm 0.7) \times 10^{-4}. \quad (1)$$

In Ref.[7] an abnormal axial-vector current of $\eta$ and pions has been constructed to calculate the branching ratio of $\tau \rightarrow \eta3\pi\nu$. The theoretical prediction is

$$1.2 \times 10^{-6}$$

which is less than the experiment by more than two orders of magnitude. The decay $\tau \rightarrow \eta3\pi\nu$ is caused by the axial-vector current and, as pointed in Ref.[7], by the anomalous meson vertex. The question is what anomalous meson vertices are responsible for this decay mode?
As shown in the studies[1,5,6], resonances play essential role in \( \tau \) mesonic decays. In this letter we are going to find all anomalous WZW terms which contribute to this decay mode. In constructing the anomalous vertices, the resonances \( a_1 \) and \( \rho \) are needed to be taken into account. In doing so, an effective chiral theory of mesons is required.

In Ref.[4] an effective chiral theory of three nonets of pseudoscalar, vector, and axial-vector mesons is proposed. This theory has been used to study meson physics at low energies. So far, it is phenomenologically successful. The chiral symmetry breaking scale \( \Lambda \) is determined to be 1.6GeV[4], therefore, this theory is suitable to be applied to study \( \tau \) mesonic decays. Our studies of \( \tau \) mesonic decays based on this effective theory are presented in Ref.[6]. Theoretical results agree with the data reasonably well. The Wess-Zumino-Witten action is derived from the leading terms of the imaginary part of the effective Lagrangian[4]. In this letter the Lagrangian of Ref.[4] is used to derive all vertices needed. The expression of the axial-vector current(\( \Delta s = 0 \)) of mesons presented in Ref.[6] is taken. The calculations done in this letter are at the three level which is supported by the argument of large \( N_C \) expansion[4]. It is necessary to point out that in the studies of this letter there is no any new parameter.

The anomalous vertices can be found in the WZW action[2,3]. The formalism of the WZW action is model independent. However, the meson fields in the WZW action need to be normalized to physical fields by the normal part of the Lagrangian(see Refs.[3,4]). In
Ref. [4] a method is developed to derive the anomalous vertices which are found to have the same expressions as the one’s obtained from the WZW action. In this way the meson fields are normalized to physical mesons. In this letter we use the method presented in Ref. [4] to derive all anomalous vertices needed.

We first present the study of the new effect in $\tau \to \eta \pi \pi \nu$ which has been studied by many author[5] in terms of the anomalous vertex $\eta \rho \rho$. It is pointed out in this letter that indeed, this vertex $L_{\eta \rho \rho}$ plays major role in the decay $\tau \to \eta \pi \pi \nu$, however, there is another anomalous vertex which contributes about 20% of the decay rate. The study of $\tau \to \eta' \pi \pi \nu$ is presented. The anomalous WZW vertices derived in Ref. [4] are

$$L_{\eta \rho \rho} = \frac{N_C}{2\pi^2 g^2 f_\eta} \varepsilon^{\mu \nu \alpha \beta} \eta (-\sqrt{\frac{2}{3}} \sin \theta + \frac{1}{\sqrt{3}} \cos \theta) \partial_\mu \rho^i_\nu \partial_\alpha \rho^j_\beta,$$

$$L_{\eta' \rho \rho} = \frac{N_C}{2\pi^2 g^2 f_{\eta'}} \varepsilon^{\mu \nu \alpha \beta} \eta' (\sqrt{\frac{2}{3}} \cos \theta + \frac{1}{\sqrt{3}} \sin \theta) \partial_\mu \rho^i_\nu \partial_\alpha \rho^j_\beta, \quad (2)$$

where $\theta = -20^0$ and $f_\eta = f_{\eta'} = f_\pi$ are taken. $g$ is a universal coupling constant and is determined to be 0.39 in Ref. [6]. Besides the vertices (2), there are other vertices which contribute to these two decay modes. Using the same method deriving Eqs. (2)[4], we obtain

$$L_{\eta \rho \pi \pi} = \frac{2}{\pi^2 g f_\pi^3} (-\sqrt{\frac{2}{3}} \sin \theta + \frac{1}{\sqrt{3}} \cos \theta)(1 - \frac{4c}{g} - \frac{2c^2}{g^2}) \varepsilon^{\mu \nu \alpha \beta} \varepsilon_{ijk} \eta \partial_\mu \rho^i_\nu \partial_\alpha \pi^j \partial_\beta \pi^k,$$

$$L_{\eta' \rho \pi \pi} = \frac{2}{\pi^2 g f_\pi^3} (\sqrt{\frac{2}{3}} \cos \theta + \frac{1}{\sqrt{3}} \sin \theta)(1 - \frac{4c}{g} - \frac{2c^2}{g^2}) \varepsilon^{\mu \nu \alpha \beta} \varepsilon_{ijk} \eta' \partial_\mu \rho^i_\nu \partial_\alpha \pi^j \partial_\beta \pi^k. \quad (3)$$

where $c = \frac{f_\pi^2}{2gm_\pi^2}$. The difference between this study and previous calculations[5] is the
inclusion of the anomalous contact terms (3) in calculating the decay branching ratios. In the final states of the decay $\tau \to \eta(\eta')\pi\pi\nu$ caused by the vertices (2) the two pions have a $\rho$-resonance structure, while for the decay amplitudes determined by Eqs. (3) there are no $\rho$ resonance. Because of the cancellation in the factor $1 - \frac{4\alpha}{g} - \frac{2a^2}{g^2}$ the contributions of Eqs. (3) are smaller. This is similar to the decay $\omega \to \pi\pi\pi$ which is dominated by the vertex $\omega\rho\pi$ and the contribution of the contact term $\omega\pi\pi\pi$ is small [3,4]. Using the VMD and Eqs. (2,3), the decay branching ratios are calculated

$$B(\tau \to \eta\pi\pi\nu) = 1.9 \times 10^{-3},$$

$$B(\tau \to \eta'\pi\pi\nu) = 0.44 \times 10^{-5}. \quad (4)$$

The calculation shows that 28% of $B(\tau \to \eta\pi\pi\nu)$ is from the anomalous contact terms (3). The data are

$$B(\tau \to \eta\pi\pi\nu) = (1.71 \pm 0.28) \times 10^{-3} [9],$$

$$B(\tau \to \eta'\pi\pi\nu) < 0.8 \times 10^{-4} [8].$$

Theoretical results agree with the data.

The decay process $\tau \to \eta\pi\pi\pi\nu$ is more complicated than the one’s studied above. Only the axial-vector current contributes to these processes. The WZW anomalous interactions of mesons cause these decays. More vertices are involved. The Feynman diagrams of these decays are shown in Fig.1. The derivation of these vertices is lengthy. As done in Ref. [4],
by calculating $-\frac{2}{f_{\pi}} m_{\eta} < \bar{\psi}\gamma_{5}\psi >$ and $-\frac{2}{f_{\pi}} m_{\eta} < \bar{\psi}\lambda_{8}\gamma_{5}\psi >$, we derive all anomalous vertices contributing to these decays:

$$\mathcal{L}_{\eta a a} = f_{a}^{2} a^{2}_{\pi} f_{\eta} \varepsilon^{\mu\nu\alpha\beta} \eta \left( -\sqrt{\frac{2}{3}} \sin \theta + \frac{1}{\sqrt{3}} \cos \theta \right) \partial_{\mu}^{\prime} a_{\nu}^{i} \partial_{\alpha} a_{\beta}^{i},$$  \hspace{1cm} (5)

$$\mathcal{L}_{\eta \rho \pi} = \frac{1}{4\pi^{2} g} \left( \frac{2}{f_{\pi}} \right)^{3} (1 - \frac{4c}{g} - \frac{2c^{2}}{g^{2}}) \sqrt{\frac{2}{3}} \sin \theta + \frac{1}{\sqrt{3}} \cos \theta \varepsilon_{ijk} \varepsilon^{\mu\nu\alpha\beta} \eta \partial_{\mu}^{\prime} \rho_{j}^{i} \partial_{\alpha} \pi_{j} \partial_{\beta} \pi_{k},$$  \hspace{1cm} (6)

$$\mathcal{L}_{\eta a \pi \pi} = \frac{N_{C} f_{a}}{(4\pi)^{2}} \left( \frac{2}{f_{\pi}} \right)^{4} \sqrt{\frac{2}{3}} \sin \theta + \frac{1}{\sqrt{3}} \cos \theta (1 - \frac{10c}{3g} + \frac{8c^{2}}{3g^{2}}) \varepsilon^{\mu\nu\alpha\beta} \eta \partial_{\mu} a_{i}^{i} \partial_{\alpha} a^{2}_{\pi} \partial_{\beta} \pi_{i},$$  \hspace{1cm} (7)

where $a_{\mu}^{i}$ is the $a_{1}$ meson field and $f_{a}^{-1} = g \left( 1 - \frac{1}{2\pi^{2} g^{2}} \right)$. The vertices related to $\eta'$ meson are obtained by using the substitution

$$(-\sqrt{\frac{2}{3}} \sin \theta + \frac{1}{\sqrt{3}} \cos \theta) \eta \rightarrow (\sqrt{\frac{2}{3}} \cos \theta + \frac{1}{\sqrt{3}} \sin \theta) \eta'$$

in Eqs.(5-7).

The vertices $\mathcal{L}_{a \rho \pi}$ and $\mathcal{L}_{\rho \pi \pi}$ are involved in $\tau \rightarrow \eta \pi \pi \nu$. They are presented in Refs.[4,6]

$$\mathcal{L}_{a \rho \pi} = \varepsilon_{ijk} \{ A a^{i}_{\mu} \rho^{\mu} \pi^{k} - B a^{i}_{\mu} \rho_{\nu}^{j} \partial^{\nu} \pi^{k} \}$$  \hspace{1cm} (8)

$$A = \frac{2}{f_{\pi}} g f_{a} \left[ \frac{m_{a}^{2}}{g^{2} f_{a}^{2}} - m_{\rho}^{2} + p^{2} \left[ \frac{2c}{g} + \frac{3}{4\pi^{2} g^{2}} \left( 1 - \frac{2c}{g} \right) \right] \right]$$

$$+ q^{2} \left[ \frac{1}{2\pi^{2} g^{2}} - \frac{2c}{g} - \frac{3}{4\pi^{2} g^{2}} \left( 1 - \frac{2c}{g} \right) \right],$$ \hspace{1cm} (9)

$$B = -\frac{2}{f_{\pi}} g f_{a} \frac{1}{2\pi^{2} g^{2}} \left( 1 - \frac{2c}{g} \right),$$ \hspace{1cm} (10)

$$\mathcal{L}_{\rho \pi \pi} = \frac{2}{g} \varepsilon_{ijk} \rho_{\mu}^{i} \partial^{\mu} \pi^{k} - \frac{2}{\pi^{2} f_{\pi}^{2} g} \left[ (1 - \frac{2c}{g})^{2} - 4\pi^{2} c^{2} \right] \varepsilon_{ijk} \rho_{\nu}^{i} \partial_{\nu} \pi^{j} \partial^{\mu} \pi^{k},$$ \hspace{1cm} (11)

where $p$ is the momentum of $\rho$ meson and $q$ is the momentum of $a_{1}$. These vertices have been
used to calculate the decay widths of $a_1 \rightarrow \rho \pi$, $3\pi$, the ratio of $d/s$, the width of $\rho \rightarrow \pi\pi$ and the width of $\tau \rightarrow 3\pi\nu$[4,6]. The theoretical results agree with the data.

The axial-vector current is needed to calculate the decay rates of $\tau \rightarrow \eta\pi\pi\pi\nu$. In terms of chiral symmetry and dynamical chiral symmetry breaking, the expression of the axial-vector current of mesons is obtained in Ref.[6] (Eqs.(10,23,24,25) of Ref.[6]). We use this axial-vector current to calculate some $\tau$ mesonic decay rates in Ref.[6]. Theoretical results agree with data. Using the axial-vector current and the vertices(Eqs.(5-7,8-11)), the decay rate of $\tau \rightarrow \eta\pi\pi\pi\nu$ is calculated. In the subprocesses shown in Fig.1 there are anomalous vertices and normal vertices. The anomalous vertices are at the forth order in low energy expansion, hence the strengths of anomalous vertices are weaker. This argument is supported by the narrow decay widths of the $\omega$ and $f_1(1280)$ mesons whose decays are resulted by anomalous vertices[4]. Because the vertices(8,11) are normal and at the second order in low energy expansion, the strengths of the vertices(8,11) are stronger. Therefore, the $\rho$ and the $a_1$ mesons have broader widths. Only the anomalous vertex $\mathcal{L}_{\eta\pi\pi\pi}$ contributes to the subprocess shown in Fig.1(a), therefore, the contribution of this process to the decay is small. For the subprocess(Fig.1(b)) there are normal vertices and anomalous ones. For the anomalous vertex $\mathcal{L}_{\eta\rho\pi\pi}(6)$, because of the cancellation in the factor $(1 - \frac{4c}{g} - \frac{2g^2}{g^2})$, the anomalous vertex $\mathcal{L}_{\eta\rho\pi\pi}(6)$ is very weak. The contribution of this subprocess is small. In the third subprocess(Fig.1(c)) there is a $\rho$ resonance. The minimum of $q^2$ of this $\rho$ is
$m_\rho + m_\eta = 1.32 \text{GeV}$ which is much greater than $m_\rho$. This effect suppresses the contribution of this subprocess. As a matter of fact, the axial-vector current derived from the vertex (7) is similar with the one presented in Ref.[7] in which it is shown that the contribution of this vertex is very small. Numerical calculation supports these arguments. The contribution of these subprocesses is about few percents. Finally, only the fourth subprocess(Fig.1(d)) survives. There is no cancellation in the anomalous vertex $L_{\eta aa}$ and the strength of the normal vertex $L_{a\rho\pi}$ is strong. Therefore, we expect that the diagram Fig.1(d) explains the decay rate of $\tau \to \eta\pi\pi\pi\nu$.

The discussion above leads to that the process $\tau \to \eta\rho\pi\nu$ is dominant the decay $\tau \to \eta\pi\pi\pi\nu$. The Feynman diagram, Fig.1(d), shows that there are two subprocesses in this decay: $\tau \to \eta a_1\nu$ and $a_1 \to \rho\pi$. As mentioned above, the former is caused by the anomalous vertex $L_{\eta aa}(5)$ which is never tested before and the later is resulted by the vertex(8) which has been tested. It is necessary to point out that the anomalous vertices $L_{\eta vv}(v = \rho, \omega, \phi)$ have been tested by $\eta \to \gamma\gamma$, $\rho \to \eta\gamma$, $\omega \to \eta\gamma$, $\phi \to \eta\gamma$, and $\tau \to \eta\pi\pi\nu$. However, it is the first time that the anomalous WZW vertex $L_{\eta aa}$ is tested.

Using the vertices(5,8,11) and the axial-vector current of Ref.[6] in the chiral limit the amplitude of the decay $\tau \to \eta\rho\pi\nu$ is derived as

$$< \eta(p)\rho^0(k')\pi^-(k)|\bar{\psi}_\tau + \gamma_{\mu}\gamma_5\psi|0> = \frac{i}{\sqrt{8\omega\omega'}E} \frac{f_\pi^2}{\pi^2f_\pi} (-\sqrt{\frac{2}{3}}\sin\theta + \frac{1}{\sqrt{3}}\cos\theta)$$
\[
g^2 f_a m_\rho^2 - i f_a^{-1}\sqrt{q^2} \Gamma_a(q^2) \frac{1}{q^2 - m_\rho^2 + i\sqrt{q^2} \Gamma_a(q^2)} \{A(q^2_1) g_{\beta\lambda} + B k_\beta k_\lambda\} \epsilon^{\mu\alpha\beta\gamma} \epsilon^\gamma \]

\[
\text{where } q = p + k + k', q_1 = q - p, \text{ and } k, k', \text{ and } p \text{ are momentum of pion, } \rho, \text{ and } \eta \text{ respectively.}
\]

The width of the \(a_1\) meson is derived from the vertex (8)

\[
\Gamma_a(q^2) = \frac{k_a}{12\pi} \frac{1}{\sqrt{q^2 m_a}} \left\{ (3 + \frac{k_a^2}{m_\rho^2}) A^2(q^2) - 2 A(q^2) B(q^2 + m_\rho^2) \frac{k_a^2}{m_\rho^2} + k_a^4 \frac{q^2}{m_\rho^2} B^2 \right\},
\]

(13)

where \(k_a^2 = \frac{1}{4q^2}(q^2 + m_\rho^2 - m_\pi^2)^2 - m_\rho^2\). Making the substitution \(q^2 \to q_1^2\) in Eq. (13), the \(\Gamma_a(q_1^2)\) is obtained.

This matrix element has the strength of the vertex \(L_{a_1\rho\pi}\). In the matrix element (13) there is second \(a_1\) resonance

\[
\frac{1}{q_1^2 - m_\rho^2 + i\sqrt{q_1^2 \Gamma_a(q_1^2)}}.
\]

The minimum of the \(q_1^2\) is \((m_\rho + m_\pi)\) which is less than \(m_a\). Therefore, the matrix element is enhanced by the second \(a_1\) resonance. These two factors make the subprocess (Fig. 1(d)) the main contributor of the decay \(\tau \to \eta\pi\pi\pi\nu\). The term \(g^2 f_a m_\rho^2\) comes from dynamical chiral symmetry breaking which is the origin of the mass difference of the \(a_1\) and the \(\rho\) mesons [6].

Using the matrix element, in the chiral limit the decay rate of \(\tau \to \eta\rho\pi\pi\nu\) is found to be

\[
d\Gamma(\tau^- \to \eta\rho^0 \pi^- \nu) = \frac{G^2}{(2\pi)^5} \cos^2\theta_c \frac{1}{1152} \frac{1}{m_\pi^2 q^2} (m_\tau^2 - q^2)^2 (m_\tau^2 + 2q^2) \]

\[
\frac{(f_a^2 a_\rho g^2 f_\pi^2 + f_a^{-2} q^2 \Gamma_a^2(q^2))}{\pi^2 f_\pi} \frac{1}{(q^2 - m_\rho^2)^2 + q^2 \Gamma_a^2(q^2)} \frac{1}{(q_1^2 - m_\rho^2)^2 + q_1^2 \Gamma_a^2(q_1^2)}
\]

\[
\{3A^2(q_1^2) \frac{q^2}{q^2} (q_\max^2 - q_\min^2) - \frac{1}{6} A(q_1^2) B(q \cdot p)^2 \frac{1}{q^4} [(q^2 - q_\min^2)^3 - (q^2 - q_\max^2)^3]\}.
\]

\[9\]
\[ + [A^2(q_1^2) - 2A(q_1^2)Bk \cdot k' + B^2(k \cdot k')^2] \frac{(q \cdot p)^2}{12m_\rho^2q_1^4} [(q^2 - q_{\min}^2)^3 - (q^2 - q_{\max}^2)^3] \]
\[ dq_2^2 dq_1^2, \quad (14) \]

where

\[ q_{\max}^2 = m_\rho^2 + \frac{1}{\sqrt{q_1^2}}(q^2 - q_1^2)(\sqrt{m_\rho^2 + l^2} + l), \quad q_{\min}^2 = m_\rho^2 + \frac{1}{\sqrt{q_1^2}}(q^2 - q_1^2)(\sqrt{m_\rho^2 + l^2} - l), \]

where \( l = \frac{1}{2\sqrt{q_1^2}}(q_1^2 - m_\rho^2) \) and \( a_\eta = -\sqrt{\frac{2}{3}}\sin\theta + \frac{1}{\sqrt{3}}\cos\theta \). The branching ratio is computed to be

\[ B(\tau^- \to \eta\rho^0\pi^-\nu) = 2.93 \times 10^{-4}. \quad (15) \]

Theoretical result agrees with the data reasonably well. This theory predicts a \( \rho \) resonance structure in the two pion state and a \( a_1 \) resonance in the three pion state of the decay \( \tau \to \eta\pi\pi\pi\nu \).

Because of the value of \( m_{\eta'} \) the decay \( \tau \to \eta'\rho\pi\nu \) is forbidden. Only the diagrams, Fig.1(a,b), contribute to the decay. Because of kinematic reason the \( \rho \) resonance amplitude in the amplitude derived from the vertex \( \mathcal{L}_{\eta'\rho\pi\pi} \) provides a very strong suppression. The contribution of this diagram is completely negligible. The decay rate derived from the anomalous vertex \( \mathcal{L}_{\eta'a\pi\pi\pi} \) is

\[ d\Gamma(\tau^- \to \eta'\pi^+\pi^-\pi^-\nu) = \frac{G^2\cos^2\theta}{9216(2\pi)^7} \frac{p^2}{48m_\tau^2q^4} F^2(m_\tau^2 - q^2)^2(m_\tau^2 + 2q^2) \]
\[ [(q^2 + p^2 - m_{\eta'}^2)^2 - 4q_2^2p^2] \frac{1}{4q_2^2} (q^2 + m_{\eta'}^2 - p^2)(q^2 - m_{\eta'}^2) \]
\[ [(q^2 + p^2 - m_{\eta'}^2)^2 - q^2p^2] \]

10
\[
\frac{m_{\rho}g^4f_a^4 + q^2\Gamma_a^2(q^2)}{(q^2 - m_a^2)^2 + q^2\Gamma_a^2(q^2)} dq^2 dp^2,
\]
\[
F = \frac{2N_C}{(4\pi)^2}\frac{2}{f_\pi^2}(\sqrt{\frac{2}{3}}\cos\theta + \frac{1}{\sqrt{3}}\sin\theta)(1 - \frac{10}{3}c - \frac{8}{3}c^2),
\]
where \(q^2 = (p_\tau - p_\nu)^2\) and \(p^2 = (q - p_{\eta'})^2\),
\[
B(\tau \to \eta'\pi\pi\nu) = 0.64 \times 10^{-8}.
\]

To conclude, it is predicted that the anomalous contact terms of \(L_{\rho\eta\pi\pi}\) contributes about 28% of the decay rate of \(\tau \to \eta\pi\pi\nu\). The decay mode \(\tau \to \eta\pi\pi\pi\nu\) provides the first test on the anomalous vertex \(L_{\eta_{aa}}\). Theoretical results agree with data. A \(a_1\) resonance structure in the final state of \(\tau \to \eta\pi\pi\pi\nu\) is predicted. The decay rate of \(\tau \to \eta'\pi\pi\pi\nu\) is predicted too. The smallness of the decay rate of \(\tau \to \eta'\pi\pi\pi\nu\) originates in the large value of \(m_{\eta'}\) which is not zero in the chiral limit.

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Fig.1 CAPTION

Diagrams of the decay $\tau \rightarrow \eta\pi\pi\nu$