Simulation of magnetization and levitation characteristics of HTS tape stacks

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Abstract. In this work it is presented a computational model of a magnetic levitation system based on stacks of high-temperature second generation superconducting tapes (HTS) GdBa2Cu3O7-x. Calculated magnetic field and the current distributions in the system for different stacks geometries in the zero-field cooling mode are also presented. The magnetization curves of the stacks in the external field of a permanent NdFeB magnet and the levitation force dependence on the gap between the magnet and the HTS tapes stack were obtained. A model of the magnetic system, oriented to levitation application, is given. Results of modeling were compared with the experimental data.

1. Introduction
Currently, high-temperature superconductors are widely applied in manufacturing current-carrying constructing elements. The superconducting ceramics are replaced by the thin second generation high-temperature superconductor tapes having high current-carrying characteristics, strength and simplicity of thermal stabilization. This led to the prospects of their use as the main material for manufacturing of trapped flux magnets, HTS levitation bearings, suspensions and other devices.

As a basic and simple element of a magnetic or levitation system, a stack of HTS tapes can be considered. The magnetic properties and levitation characteristics of the tapes stacks depend not only on critical characteristics of the used materials but its geometry (the number and shape of the tapes), conditions of magnetization and cooling. Therefore to design any devices based on the HTS tapes stacks it is necessary to determine the technical parameters of the elements taking into account the magnetic and transport characteristics of the tapes.

This work aims to develop simulation model that makes it possible to calculate magnetic and levitation properties of a system based on HTS tapes stacks of various configurations.

2. Computational model
The simulated magneto-levitation system is a stack of spatially separated planar superconducting layers. The geometry of the system is shown in the Figure 1. The measurements were performed on a stack of superconducting tapes produced by SuperOx (Russia) with the following characteristics. The thickness of the superconducting layer GdBa2Cu3O7-x is 1 micron. The thickness of the silver layer is 1.5 – 2 micron Width / diameter of tapes is 4 - 12 mm. The critical current (at 77 K, own field) - 150 A (criterion 1 μV / cm)
To study the behavior of a HTS tapes stack in an external magnetic field a finite element method was used. The model was developed on the base of the Partial Differential Equations (PDE) form of the Comsol Multiphysics software module.

To describe the superconducting magnetic system we used Faraday and Ampere laws equations. The resistance nonlinear dependence on the current is given by the power law (1) and the current-voltage characteristic can be written by formula (2):

\[ \rho = \frac{E_c}{I_c} \left( \frac{I}{I_c} \right)^{n-1} \]  
\[ E = \rho \cdot J \]  

where \( E_c \) is the electric field which equals 1 \( \mu \)V/cm, that corresponds to the critical current. The \( n \)-value is assumed to be 21, which is typical for the samples of HTS tapes. The critical current density \( J_c \) is determined on the basis of the transport measurements of the used HTS tapes.

Also, in this model it is taken into account the dependence of the critical current on the magnetic field perpendicular to the wide surface of the tapes. Calculations were performed within the two-exponential model [1]. Using it increases the convergence of the solution and also significantly reduces the computation time in comparison with Kim’s model.

In addition, the interaction force of the HTS tapes stack with the permanent magnet (the levitation force) is calculated by formula (3):

\[ F = \int_\Omega J \times B \, d\Omega \]  

where \( B \) is the magnetic field induction of the permanent magnet. The currents distribution \( J \) in the superconducting tapes of stack was calculated in result of the solutions of the PDE equations. Integration is performed by all entire volume of the superconductor.

In Comsol Multiphysics the general PDE standard equation view is (4):

\[ e_a \frac{\partial^2 u}{\partial t^2} + d_a \frac{\partial u}{\partial t} + \nabla \cdot \Gamma = F \]  

Here the dependent variable, magnetic field \( \mathbf{u} = H(x,y,z) \), is chosen in the entire space. All variables and parameters of equation (4) are given by (5)-(8):

\[ \Gamma = \rho \cdot J \cdot \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} \]
The current components are determined from the relations (9):

\[
\begin{align*}
\mathbf{J} &= \left\{ \begin{array}{l}
J_x = \frac{dH_x}{dy} - \frac{dH_y}{dz} \\
J_y = -\frac{dH_x}{dx} + \frac{dH_y}{dz} \\
J_z = \frac{dH_y}{dx} - \frac{dH_x}{dy}
\end{array} \right.
\end{align*}
\]  

To solve the problem it is also necessary to determine the boundary conditions. In our case it is convenient to use the Dirichlet conditions on the boundary of the external domain around the stack. The magnetic field is calculated as a superposition of external and internal contributions [2]. The internal contribution \( H_s = [H_{sx} \ H_{sy} \ H_{sz}] \) is calculated from the solution of the Ampere equation, the external contribution \( H_e = [H_{ex} \ H_{ey} \ H_{ez}] \) corresponds to the applied magnetic field. 

It should be noted that the singularity of the problem requires that the dimensions of the calculated domain outside the superconductor should exceed the sample size by at least 10 times (see Figure 1). One of the specific questions of developed model is the mesh construction question. In our case the ratio of the lateral dimensions of the stack to the thickness of the superconducting layer is \( \sim 10^3 \). Therefore, using a standard mesh leads to unsatisfactory results. We used multiscale structure: the mesh element’s size changes several times as we approach from the external domain boundary to the sample (Figure 2).

![Figure 2. Multiscale mesh structure for five HTS tapes stack. Insets: enlarged mesh near the edge of the tape.](image)

In general, the developed model is three-dimensional, however, in order to decrease the calculation time we additionally used the fact that the outer layers of the stack shield the applied magnetic field. Consequently, the inner layers of the HTS stack of tapes can be considered as two-dimensional layers of zero thickness. Our research has shown that with this approach the calculation time is reduced by more than 60 times, the maximum discrepancy between the 2D and the 3D models does not exceed 10% (Figure 3).
3. Results and discussion

The magnetic and levitation characteristics of the stacks were calculated for square and round shape of tapes, width/diameter is 12 mm, number of tapes in a stack was varied from 1 to 5.

Gradient magnetic field was produced by the permanent Nd-Fe-B magnet, 30 mm diameter, 10 mm thickness, class n42 (induction on the surface in the range 0.30 - 0.35 T). During the magnetization the HTS stack slowly approaches to the magnet (at a constant speed of 2 mm/s), and then moves away from it. The stack fall into an inhomogeneous magnetic field whose magnitude varies with time. Thus we realized zero field cooling mode magnetization.

Shown in the Figure 3 dependence $B_z(r)$ is for the moment of complete removal of the magnetic field for 2D (the HTS tape thickness is equal zero) and 3D (superconductor thickness is 1 μm) models and experimental data for the one tape in a stack.

![Figure 3](image)

**Figure 3.** Profile of the trapped magnetic flux $B_z(r)$ by the HTS tapes stack. The edge of the sample is at $r = 6$ mm, $r = 0$ mm - center of the sample.

The calculated magnetic field and the current distributions for the tapes stacks of different geometries were obtained. We performed comparison of the calculated distributions of the magnetic flux trapped by the stacks with the experimental dependences of Hall magnetometry measurements. Scanning height was 0.7 mm [3]. The calculated data are in consistence with the experimental data for the samples of circular shape (Figure 4 (a)) and the square shape (Figure 4 (b)).

![Figure 4](image)

**Figure 4.** Dependence of $B_z(r)$ for the round tape HTS (a) and for the square HTS tape (b) The edges of the sample: $r = 6$ mm and $r = -6$ mm.
The magnetization curves of the tapes stacks were calculated in the external gradient field of the permanent magnet. Comparison of the calculated hysteresis loops for the one and five HTS tapes in the stack with the experimental dependences shows qualitative and quantitative coincidence (Figure 5).

![Figure 5](image)

**Figure 5.** Theoretical and experimental magnetization curves for round-shaped tapes stacks in an external field of a permanent magnet for the single tape (a) and for 5 tapes in the stack (b).

The dependence of the levitation force $F$ on the levitation gap $z$ between the magnet and the tapes stacks was obtained (Figure 6). The maximum calculated levitation force increases in proportion to the number of tapes in the stack from 1 to 5 which corresponds to the experimental data. However, in general, for a large number of tapes (more than 50-60) the dependences of the magnetization and the levitation force on the number of tapes in the stack are not linear [5].

![Figure 6](image)

**Figure 6.** Dependence of the levitation force on the levitation gap (a stack of 5 tapes).

The differences in the dependences can be explained by the presence of external factors (in particular, the thermal processes, the angular dependences of the critical current on the magnetic field) that were not taken into account in the modeling, model simplifications and by the features of an experimental method (for more details see [4]). But that discrepancies are minor and including into the model for additional parameters negatively affects on the calculation time. Nevertheless, the analytical model demonstrates a qualitative coincidence with the experimental results.
Obtained results indicate the existence of optimal parameters and system configurations to achieve the maximum performance of devices based on HTS tapes stacks. Thus, the developed model allows to predict the characteristics and optimize the parameters of the levitation and magnetic systems for various applications.

4. Conclusions
Based on the use of the finite element method the physical model of the magnetic levitation system based on the HTS stacks tapes was developed. In the model it was used the features of the stacks properties, which on the one hand necessitated the construction of a multiscale mesh and from the other hand helped to significantly reduce the computational time in comparison with the standard mesh of elements. Using the model, the loops of the magnetization of the tapes stacks with various shapes in a gradient magnetic field of a permanent magnet were calculated. Moreover the profiles of the trapped magnetic flux were obtained.

In addition, the dependence of the levitation force on the levitation gap was calculated for the system of HTS tapes stack - permanent magnet.

We carried out comparison of the computational and the experimental data, good coincidence of the results was shown.

The advantage of developed model is the possibility of changing all input parameters of the system, as well as changes in the geometry and properties of HTS samples, which makes the model applicable to the calculation and optimization of magnetic and levitation systems of various configurations (in the fields of building of new transport systems of various scales, electrical rotating machines, energy and wind generators).

Acknowledgments
This work was supported by the National Research Nuclear University MEPhI Academic Excellence Project (contract 02.A03.21.0005).

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