Towards an Accurate Mathematical Model of Generic Nominally-Typed OOP

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Abstract

The construction of GNOOP as a domain-theoretic model of generic nominally-typed OOP is currently underway. This extended abstract presents the concepts of ‘nominal intervals’ and ‘full genericification’ that are likely to help in building GNOOP as an accurate mathematical model of generic nominally-typed OOP. The abstract also presents few related category-theoretic suggestions. The presented concepts and suggestions are particularly geared towards enabling GNOOP to offer a precise and simple view of so-far-hard-to-analyze features of generic OOP such as variance annotations (e.g., Java wildcard types) and erased generics (e.g., Java type erasure).

1 Extended Abstract

Nominally-typed OO languages are among the top most-used programming languages today. Examples of nominally-typed OO languages include industrial-strength mainstream OO programming languages such as Java [25], C# [4], C++ [2], and Scala [33]. Recently, we presented NOOP as a domain-theoretic model of (non-generic) nominally-typed OOP [5, 6, 7] and compared it to well-known structural models of OOP [12, 9], proving—contrary to the mantra “inheritance is not subtyping”—that type inheritance and OO subtyping are in full one-to-one correspondence in nominally-typed OOP [21]. To support the development of NOOP, we also illustrated the technical and semantic value of nominal typing and nominal subtyping to mainstream OO developers and language designers [11].

Generic types add to the expressiveness of type systems of nominally-typed OO programming languages [15, 17, 23, 24, 1, 16, 3, 25, 30, 41]. Generics provide OO class designers the ability to abstract their classes over some types, and thus define them “generically,” independent of particular instantiations that class clients may later use. Generics move the decision as to what actual types to be used for some of the types used
inside the class to the usage-sites of a class (i.e., are decided by the clients of the class) rather than be declared and fixed at declaration-sites (i.e., decided by class designers).

Generics also offer OO programmers more flexibility, given that different type parameters of a generic class can be used at different usage sites, even in the same program. Without generics, such a capability could only be simulated by a cooperation between class designers and their clients, depending on OO subtyping and using the so-called “generic idiom.” Using the generic idiom is not a type-safe alternative to generics, given that using it involves requiring clients to insert downcasts by hand. Because they circumvent the type system, programs with downcasts can be type unsafe [27].

We believe building a domain-theoretic model of generic nominally-typed OOP along the lines of NOOP (e.g., as in [10]) will offer better chances for having a deeper and more solid understanding of features of generic mainstream OO languages, such as Java erasure, variance annotations (including Java wildcards), polymorphic methods, generic type inference and so on. It will also demonstrate the utility and importance of including nominal type information in mathematical models of OOP.

The plenty of research done on OO generics ([15, 17, 23, 24, 25, 1, 16, 3, 30, 41]) has proven that generics is a complex feature to analyze. Features such as Java wildcards [40], in particular, while designed to ameliorate the conceptual mismatch between parametric polymorphism and OO subtyping polymorphism, have proven to be difficult to accurately model [39, 19, 18, 38].

Based on our ongoing effort to build a mathematical model of OO generics, we believe a crucial reason for PL researchers not accurately modeling generics is their use of structural models of OOP. We demonstrate this here by summarizing our explorations towards constructing a simple yet precise mathematical model of OO generics that is based on a nominal rather than structural view of OOP.

Along the lines of our construction of NOOP, we are currently constructing GNOOP to be a domain-theoretic model of OOP that includes full nominal type information found in generic nominally-typed OOP.

The main nominal construct in NOOP are class signatures (see [5, 6, 7]). Class signatures in NOOP include full type name information (a.k.a., nominal information) found in non-generic nominally-typed OOP. For GNOOP, the main nominal construct are class signature constructors. When applied to type arguments, class signature constructors get instantiated to (ground) class signatures. (Appendix A presents the formal definition of generic class signatures.) Based on the definition of ground signature names and ground signatures, the construction of GNOOP goes along similar lines of constructing NOOP. (See [10] for more details on generic class signatures and on the construction of GNOOP.)

As constructed, GNOOP however does not (yet) model some important features of generic OOP, such as polymorphic methods, bounded type variables and variance annotations (e.g., Java wildcards).

We discuss how to model polymorphic methods in [10]. For the accurate modeling of bounded type variables and wildcards/variance annotations, we are considering adopting a unified approach to both features, using the notions of ‘nominal (or, named) intervals’ and ‘full generification.’ We discuss both notions below.

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1 An introduction to domain theory is presented in [22].
Variance annotations in generic OOP are designed to combine the benefits and abstraction power of parametric polymorphism (i.e., provided by plain generics, with no variance) with the benefits and abstraction power of OO subtyping polymorphism. Variance annotations allow different instantiations of generic types to be included in the subtyping relation. (See [28] for a more detailed discussion of variance annotations.) Variance annotations come in two main flavors in mainstream generic OOP: usage-site variance annotations (used e.g., in Scala and C#) and declaration-site variance annotations (used e.g., in Java; more well-known as Java wildcards).

A critical observation to be made early on when analyzing variance annotations is that infinite chains of supertypes could now occur in the subtyping relation. For example, given the declaration of class Enum in Java, and with a typical class declaration such as class C extends Enum<C>, consider the chain of Java types C, which has Enum<C> as a supertype (given the declaration of class C), which, by subtyping rules for wildcard types, has supertype Enum<? extends C>, which in turn using the same rules has Enum<? extends Enum<C>> as a supertype, which has Enum<? extends Enum<? extends Enum<C>>> as a supertype, which has Enum<? extends Enum<? extends Enum<C>>> as a supertype, ... and so on. (Did you note the repeating pattern?)

In fact, building on this observation we further made the observation that (the graph of) the subtyping relation in Java (or in generic nominally-typed OOP, more generally)—when few artificial restrictions on wildcards are relaxed, e.g., them having both lower and upper bounds—is a fractal. Fractals, as graphs, are characterized by having ‘self-similarity’. Due to invariant subtyping, covariant subtyping and contravariant subtyping rules, we noted that three “transformed” copies of the subtyping relation are embedded inside the relation (See [8] for more details and illustrations.) While this fractal observation may not be surprising, given the inductive definition of subtyping, the intricacy in generic OO subtyping comes when noting the three transformations of the subtyping relation resulting from having three forms of subtyping rules between generic types.

As mentioned earlier, to model bounded type variables and variance annotations we consider using what we call ‘nominal intervals’ and ‘full generification’, where

- a nominal interval is a type variable name with both upper and lower bounds where the lower bound is guaranteed to be a subtype of the upper bound (nominal intervals will model bounded type parameters), and
- full generification means that all types inside a class (e.g., as the type of a field, the type of a method parameter/return value, or passed as a type argument inside a parameterized type—including wildcard types and nested type arguments) get replaced by (a.k.a., are “captured” into) new additional synthetic type parameters of the class. (Appendix B presents the formal definition of fully-generified generic class signatures. Appendix C presents illustrating code examples).

Existential types, originally from mathematical logic, are used to model wildcards in virtually all research on generic (a.k.a., polymorphic) OOP. We believe nominal intervals are simpler than existential types as models of wildcards. The notion of intervals we use is a simple generalization of the notion of intervals over total orders (e.g., over real numbers or inte-
gers) to intervals over partial orders (namely, over the subtyping relation). Naming (i.e., giving names to) intervals regains (and also nicely conceals) the existential nature of wildcards as intervals.

As to full generification, because of their capturing in synthetic type variables, a type argument inside a fully generified class will always be a type variable (i.e., the name of a nominal interval, like any other type inside the class) not a parameterized type. As such, full generification results in what can be called ‘single-nested generics’, where in the code of a fully-generified class there will be no explicit multi-level nesting of type arguments (e.g., as in the type `List<List<String>>`, or in the supertype `EQUATABLE<List<EQUATABLE<E>>>` where E is a type parameter of class `EQUATABLE` [26]). Multi-level nesting of type arguments in fully-generified code will only be implicit and indirect, expressed always via (original or synthetic) type variables.

In addition to our approach being simple and intuitive, based on preliminary analysis we made we expect this approach—based on nominal intervals and full generification—to particularly allow better, more accurate modeling of Java wildcards and Java erasure—two features of generics that, as we hinted above, have not been modeled accurately so far.

For wildcards, it should be noted that full generification results in the “disappearance” of wildcard types. Due to capturing them as (synthetic) type variables/parameters—which are modeled as nominal intervals—full generification results in a uniform treatment of wildcard types and type variables.

For type erasure, full generification makes all (non-type-variable) types inside a class to be explicitly expressed in the type parameters clause of the class. Doing a syntactic comparison on the class code, namely between its legacy non-generic version and its generic version, a raw type (the erased version of a generic type) can then be modeled by some instantiation of the fully generified generic class. The code comparison will reveal the type arguments to be used for the synthetic type parameters, which were not explicitly included in the original generic version of the class.

Finally, to possibly further simplify and strengthen our model, we are considering using some category theoretic tools in our construction of a mathematical model of generic nominally-typed OOP. While category theory has been recently getting applied, at an increasing rate, in other scientific disciplines [31, 13, 37], the use of category theory in computer science, and in defining the semantics of programming languages in particular, is well known. It is well known, for example, that cartesian-closed categories (CCCs) provide accurate models of typed lambda calculus [36, 22]. Further, a fruitful point of view takes the objects of a category to be the types of a programming language [34]. In such a view, type constructors (i.e., generic classes, in an OOP language) are seen as functors between categories. F-bounded generics (where type variables of generic classes are used in their own bounds [14]) can then be interpreted using F-coalgebras [20, 29, 35]. As such, building on these earlier successes in the use of category theory in programming languages research, we intend to consider using operads [37] to model the self-similarity (see page 3 above) of the subtyping relation in generic nominally-typed OOP.

We believe the inclusion of nominal information, the use of notions such as nominal intervals and full generification, and the possible use of category theory, may hold the key towards constructing a mathematical model of generic nominally-typed OOP that, unlike prior mathematical models of OOP, is simple yet accurate enough to provide a solid under-
standing of the main features of generic nominally-typed OOP.

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A Formal Definition of Generic Class Signatures

Formally, akin to NOOP class signatures, GNOOP class signature constructors are defined as

\[ SC = N \times X^* \times GN^* \times (L \times GNX^*)^* \times (L \times GNX \times GNX^*)^* \]  

(1)

where \( SC \) is the set of class signature constructors, \( N \) is the set of class/interface/trait names, \( X \) is the set of type variable names, \( L \) is the set of member (i.e., field/method) names, and \( \times \) and \( ^* \) are the cross-product and finite sequences set constructors.\(^2\)

For ‘generic signature names’ \( GN \), we have

\[ GN = N \times GNX^* \]

and for ‘generic signature names or type variables’ \( GNX \), we have

\[ GNX = GN + X. \]

(Note the mutual dependency between \( GN \) and \( GNX \), and that only members of \( N \), not \( X \), can be paired with members of \( GNX^* \).)

B Fully-Generified Generic Class Signatures

Formally, in an approach based on nominal intervals and full generification, the equations for generic signatures will be as follows (in contrast to

\(^2\)As such, \( X^* \) is the set of (finite) sequences of type variables. As a component inside a class signature constructor, a member of \( X^* \) is the sequence of type variables whose members can be used inside this class signature constructor. Ordering of elements in a sequence of type variables (i.e., an element of \( X^* \)) does matter (type arguments are matched with type variables based on the order of each in their respective sequences). Repetition is not allowed allowed in elements of \( X^* \).
using $X$ in the equations above, $Y$ in the following equations is the set of both synthetic and original type variable names, where we have

$$\text{GN} = N \times Y^*$$
$$\text{GNY} = \text{GN} + Y$$

(Note the single-nesting of generic types. There is no mutual dependency between GN and GNY, as that between GN and GNX above.)

For nominal intervals (modeling bounded type variables, and wildcard types captured in synthetic type variables), we have

$$\text{YB} = Y \times \text{GNY} \times \text{GNY}.$$ (All type variables have lower and upper bounds, in addition to a name. For a triple in YB to be a nominal interval the lowerbound (the second component of the triple) must be a subtype of the upperbound (the third component of the triple). Since they may stand for different (unknown) types, nominal intervals with the same lower and upper bounds but with different names are different intervals—hence nominality. Also, a non-circularity convention—as in Scala, and more recently in Java$^3$—will enforce that a type variable does not occur by itself directly or indirectly in its own bounds.)

Then, for class signature constructors, we have

$$\text{SC} = N \times Y^* \times \text{GN}^* \times (L \times Y)^* \times (L \times Y^* \times Y)^*$$

(Note: In comparison with Equation (1), all types of fields and methods are now references to (original or synthetic) type variables/nominal intervals. A class signature constructor is thus now fully generified.)

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$^3$Based on the announced Java 7.0 language enhancements (at http://docs.oracle.com/javase/7/docs/technotes/guides/language/enhancements.html#javase7), making Java follow the footsteps of other nominally-typed OO languages (such as Scala) in allowing naked type variables occur as bounds of earlier-declared type variables (i.e., having naked forward references) seems to be a (surprising) unannounced language enhancement in Java 7.0 (released in 2011), even though the relaxation of this restriction is mentioned in [25, Sec. 6.3] where it is mentioned that ‘the scope of a class’s type parameter is the type parameter section of the class declaration ...’.

When generics were initially introduced in Java 5.0 (in 2004), it was required that a type variable bound not be a naked type variable that is declared later in the type parameters clause of a generic class/interface (possibly due to an indirect influence from [14], which has the same restriction). The original restriction was reflected, for example, in question Type-Parameters.FAQ302 of (earlier versions of) the Java Generics FAQ [30], where it is mentioned that the code

```java
<S extends T, T extends Comparable<S>> T create(S arg) { ... }
```

which now successfully compiles in Java, is in error, and that the forward reference (to type variable T) in the code is illegal.

The non-circularity convention of Scala—i.e., that a type parameter may not be bound directly or indirectly by itself—is explicitly mentioned in the Scala language specification, at least as of Scala 2.7 [32, Sec. 4.4] (in 2009). Java now seemingly (silently) follows suit and makes the scope of a type parameter include the whole type parameter clause. Therefore, in Scala and in the latest versions of Java, it is possible for a type parameter to appear as part of its own bounds or the bounds of any of the other type parameters in the same clause. (We thank Martin Odersky for the brief discussion on this point.)
C Full-Generification Code Examples

For an illustration of full generification, consider the simple generic class declaration

```java
class C<T> {
    private Integer count;
    private T t;
    C(T t) { this.t = t; count = 0; }
}
```

Using the notation \([VN:LB-UB]\) for a nominal interval that denotes a bounded type variable with name \(VN\), a lower bound \(LB\) and an upper bound \(UB\) (and where \(O\) and \(N\) are shorthands for a type \(Object\), at the top of the subtyping hierarchy, and a type \(Null\) at its bottom), the declaration of class \(C\), when fully generified, gets translated to

```java
// T is original, T1 to T3 are synthetic
class C<[T:N-0],[T1:Integer-Integer],
[T2:T-T],[T3:T-T]> {
    private T1 count;
    private T2 t;
    C(T3 t) { this.t = t; count = 0; }
}
```

Using the same notation, when fully generified, the (non-generic) class declaration

```java
class DecorCanvas extends Canvas {
    void drawShapes(List<? extends Shape> ss){
        for(s: ss){
            s.draw(this);
            drawDecor(s);
        }
    }
}
```

gets translated to the (generic) class declaration

```java
// T1 and T2 are synthetic
class DecorCanvas<[T1:N-Shape],
[T2:List<T1>-List<T1>]> extends Canvas {
    void drawShapes(T2 ss){
        for(s: ss){
            s.draw(this);
            drawDecor(s);
        }
    }
}
```
and the class declaration

```java
class ListCopier<T> {
    void copy(List<? extends T> src,
               List<? super T> dest){
        for(int i=0; i < src.size(); i++)
            dest.set(i, src.get(i));
    }
}
```

gets translated to

```java
// T is original, T1 to T4 are synthetic
class ListCopier<[T:N-O],
                [T1:N-T],[T2:List<T1>-List<T1>],
                [T3:T-O],[T4:List<T3>-List<T3>]> {
    void copy(T2 src, T4 dest){
        for(int i=0; i < src.size(); i++)
            dest.set(i, src.get(i));
    }
}
```

For a more intricate example of full generification, consider the declaration

```java
class Box<T> {
    private T t;
    Box(T t) { this.t = t; }
    void put(T t) { this.t = t; }
    T take() { return t; }
    boolean equalTo(Box<?> other) {
        return this.t.equals(other.t); }
    Box<T> copy() { return new Box<T>(t); }
}
```

which, when fully generified, gets translated to

```java
// T is original, T1 to T11 are synthetic
class Box<[T:N-O],[T1:T-T],[T2:T-T],[T3:T-T],
         [T4:T-T],[T5:Boolean-Boolean],
         [T6:T-T], /* Not [T6:N-O], due to
         * "wildcard capturing" */
         [T7:Box<T6>-Box<T6>],[T8:T-T],
         [T9:Box<T8>-Box<T8>],
         [T10:T-T],[T11:Box<T10>-Box<T10>]> {
    private T1 t;
    Box(T2 t) { this.t = t; }
    void put(T3 t) { this.t = t; }
    T4 take() { return t; }
    T5 equalTo(T7 other) {
        return this.t.equals(other.t); }
    T9 copy() { return new T11(t); }
}
```