The completed SDSS-IV extended Baryon Oscillation Spectroscopic Survey: a multitracer analysis in Fourier space for measuring the cosmic structure growth and expansion rate

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ABSTRACT

We perform a joint BAO and RSD analysis using the eBOSS DR16 LRG and ELG samples in the redshift range of $z \in [0.6, 1.1]$, and detect an RSD signal from the cross-power spectrum at a $\sim 4\sigma$ confidence level, i.e., $f_8 = 0.317 \pm 0.080$ at $z_{\text{eff}} = 0.77$. Based on the chained power spectrum, which is a new development in this work to mitigate the angular systematics, we measure the BAO distances and growth rate simultaneously at two effective redshifts, namely, $D_{\text{M}}r_d (z = 0.70) = 17.96 \pm 0.51$, $D_Hr_d (z = 0.70) = 21.22 \pm 1.20$, $f_8 (z = 0.70) = 0.43 \pm 0.05$, and $\sigma_8 (z = 0.845) = 0.30 \pm 0.08$. Combined with BAO measurements including those from the eBOSS DR16 QSO and Lyman-α sample, our measurement has raised the significance level of a non-zero $\Omega_\Lambda$ to $\sim 11\sigma$. The data product of this work is publicly available at https://github.com/icosmology/eBOSS_DR16_LRGxELG and https://www.sdss.org/science/final-bao-and-rsd-measurements/.

Key words: (cosmology:) cosmological parameters – (cosmology:) large-scale structure of Universe – cosmology: observations – (cosmology:) dark energy.

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1 INTRODUCTION

Large spectroscopic galaxy surveys are one of the key probes of both the expansion history and structure growth of the Universe, thus can in principle break the ‘dark degeneracy’ between scenarios of dark energy (DE; e.g. Weinberg et al. 2013) and modified gravity (MG; e.g. Koyama 2016), which are proposed as possible physical origins of the cosmic acceleration (Riess et al. 1998; Perlmutter et al. 1999).

Being almost not clustered, DE primarily affects the background expansion of the Universe, which can be probed by Baryonic acoustic oscillations (BAO), a special 3D clustering pattern of galaxies, formed in the early Universe due to interactions between photons and baryons. The BAO feature was first detected by both the Sloan Digital Sky Survey (SDSS) collaboration (Eisenstein et al. 2005) and the 2 degree Field Galaxy Redshift Survey (2dFGRS) collaboration in 2005 (Cole et al. 2005), and has been extensively investigated by a large number of studies since then.

MG, on the other hand, can dictate both the expansion and the structure formation of the Universe. After the required tuning for the cosmic acceleration, MG leaves imprints at the perturbation level, i.e., it can alter the history of the structure growth on linear and non-linear scales. On such scales where the peculiar motion of galaxies is relevant, the redshift space distortions (RSD) can be directly mapped by redshift surveys, as reported by its first detection in 2001 by the 2dFGRS collaboration (Peacock et al. 2001).

Measurements of BAO and RSD from redshift surveys and cosmological implications have been extensively performed (Percival et al. 2010; Beutler et al. 2011, 2012, 2017; Blake et al. 2013; Contreras et al. 2013; Kazin et al. 2014; Ross et al. 2015; Alam et al. 2017; Wang et al. 2017, 2018; Zhao et al. 2017b, 2019; Ata et al. 2018; Bautista et al. 2018; Gil-Marín et al. 2018; Zarrouk et al. 2018; Abbott et al. 2019; Zheng et al. 2019), but most of studies focus on the clustering of a single type of galaxies. This is, however, largely due to the fact that most finished galaxy surveys, including 2dFGRS and SDSS III-BOSS, only target at a single tracer in the same cosmic volume.

The statistical error budget of RSD measurements is dominated by the shot noise and the cosmic variance on small and large scales, respectively. While the former can be in principle reduced by increasing the number densities of the observed tracers, the latter is difficult to suppress, due to the fact that the number of large-scale modes is limited by the survey volume. One possible way to tackle the cosmic variance, however, is to combine multiple tracers with different biases covering the same footprint and redshift range (McDonald & Seljak 2009; Seljak 2009). The idea is that by contrasting different tracers of the same underlying density field, the uncertainty of statistics of the density field, which is dominated by the cosmic variance on large scales, can be cancelled out if the shot noise of all the concerning tracers is negligible, yielding a measurement of $R_b$, the ratio of effective biases, $R_b \equiv b_{	ext{eff}}^x/b_{	ext{eff}}^y$, between tracers $X$ and $Y$ without cosmic variance. The measured bias is effective because it includes the RSD term, namely, $b_{	ext{eff}} \geq b(1 + \mu \mu^t)$, where $b$, $\beta$ and $\mu$ are the linear bias, the RSD parameter and the cosine of the angle between the line-of-sight and the pair of tracers, respectively. The effective bias also receives a contribution from the primordial non-Gaussianity parametrized by $f_{NL}$ if $f_{NL} \neq 0$. By combining measurements of $R_b$ using various $\mu$ modes, parameters of $\beta$, or $f_{NL}$ can be determined to an arbitrary precision in the ideal case, where the shot noise is negligible.

It is challenging to run a multitracer survey, as different tracers may require different methods of target selection, different treatments of observational systematics, and different tracers have to be observed separately, making it expensive to build and perform. Alternative options include either creating ‘multitracer’ samples from a single-tracer survey by splitting the samples using luminosity or colour (Blake et al. 2013; Ross et al. 2014), or combining different tracers observed by different surveys (Beutler et al. 2016; Marín et al. 2016). These approaches may be subject to limits including a limited relative galaxy bias (samples in a single-tracer survey usually do not differ much in the galaxy bias), and a limited overlapping area (most galaxy surveys are designed to be complementary to each other, in terms of the sky coverage and/or redshift range; Wang & Zhao 2020).

Fortunately, the extended Baryon Oscillation Spectroscopic Survey (eBOSS) project has provided such an opportunity for a proper multitracer analysis. Targeted for both Luminous Red Galaxies (LRG) and Emission Line Galaxies (ELG) at $z \in [0.6, 1.1]$ in a large overlapping patch of sky, the eBOSS Data Release (DR) 16 provided a total of $\sim550,000$ spectra for the multitracer analysis, which is the largest sample for such an analysis to date. This is the natural motivation for this work.

In this analysis, we develop new methods for a joint BAO and RSD analysis using the DR16 LRG and ELG sample, and pay particular attention to the mitigation of possible systematics.

The paper is structured as follows. In Section 2, we describe the observational and simulated data sets used in this analysis, and in Section 3, we present the method, followed by mock tests and main result of this work in Section 4, before conclusion and discussion in Section 5.

This work is one of a series of papers presenting results based on the final eBOSS DR16 samples. The multitracer analysis of the same galaxy sample is performed in configuration space to complement this work (Wang et al. 2020). For the LRG sample, produced by Ross et al. (2020), the correlation function is used to measure BAO and RSD in Bautista et al. (2021), and the analyses of BAO and RSD from power spectrum are discussed in Gil-Marín et al. (2020). The LRG mock challenge for assessing the modelling systematics is described in Rossi et al. (2020). The ELG catalogues are presented in Ráichoor et al. (2021), and analysed in Fourier space (de Mattia et al. 2021) and in configuration space (Tamone et al. 2020), respectively. The clustering catalogue of quasar is generated by Lyke et al. (2020) and Ross et al. (2020). The quasar mock challenge for assessing the modelling systematics is described in Smith et al. (2020). The quasar clustering analysis in Fourier space is discussed in Neveux et al. (2020), and in configuration space in Hou et al. (2021). Finally, the cosmological implication from the clustering analyses is presented in eBOSS Collaboration (2020).

2 THE DATA SETS

In this section, we briefly describe the observational and simulated data sets used in this analysis.

2.1 The eBOSS DR16 LRG and ELG samples

Being part of the Sloan Digital Sky Survey-IV (SDSS-IV) project (Blanton et al. 2017), the eBOSS survey (Dawson et al. 2016; Zhao et al. 2016) started in 2014 using the 2.5-m Sloan telescope (Gunn et al. 2006) at the Apache Point Observatory in New Mexico.

The LRG targets are selected using optical and infrared imaging data over the entire SDSS imaging footprint. The optical imaging data are taken from the SDSS III/II (York et al. 2000) and III (Eisenstein et al. 2011) surveys in five passbands: $u$, $g$, $r$, $i$, $z$, while the infrared data are provided by the Wide Field Infrared Survey Explorer (WISE) survey (Wright et al. 2010). The ELG targets, however, are not...
A multitracer analysis of the eBOSS DR16 sample

Figure 1. The footprint of the DR16 LRG (larger red region) and ELG (smaller blue) samples in the NGC (left) and SGC (right), respectively, used in this analysis.

selected using the SDSS imaging observations. Instead, the g, r, z bands of the DECam Legacy Survey (Dey et al. 2019) photometric sample is used. After the eBOSS target selection, which is described in Ross et al. (2020) and Raichoor et al. (2017) for the LRG and ELG, respectively, the spectra are taken using the double-armed spectrographs (Smee et al. 2013), which were used for the Baryon Oscillation Spectroscopic Survey (BOSS) mission, as part of the SDSS-III project (Eisenstein et al. 2011).

The footprint of the LRG and ELG samples is shown in Fig. 1. The eBOSS LRG sample used in this work is a combination of the eBOSS LRG with those observed by the BOSS program at z > 0.6, and it is denoted as 'LRGpCMASS' in other companion papers. This sample covers the redshift range of $z \in [0.6, 1.0]$ with a sky coverage of $\sim 9500$ deg$^2$, and consists of approximately 255 K and 121 K galaxies in the northern galactic cap (NGC) and southern galactic cap (SGC), respectively. The ELG are selected to cover $z \in [0.6, 1.1]$, covering $\sim 730$ deg$^2$, with $\sim 174$ K redshifts in total.

Fig. 1 shows that almost all the ELG are in the footprint of the LRG, but the overlapping region only covers about 8 per cent of the LRG coverage. As we show in a later section (Section 4), this makes the autopower spectrum of the LRG sample, which is largely dominated by the LRG that do not overlap with the ELG, not closely related to the cross-power between the LRG and ELG samples (a quantitative discussion is in Section 4). The number density distribution in redshift is displayed in Fig. 2. Apparently, the overlap between these two samples in redshift is significant, and the densities of both samples are sufficiently high, which enables a multitracer exercise.

2.2 The simulated mock samples

A large number of mock samples, each of which has the same clustering property of the eBOSS DR16 sample, are required to estimate the data covariance matrix. In this analysis, we use the Extended Zel’dovich (EZ) mocks, which consist of 1000 realizations, produced following the prescription in Zhao et al. (2021) and Chuang et al. (2015). The number of total realizations of the EZmocks we have, which is 2000 for the LRG and ELG (1000 for each), is sufficient given the total number of data points (including those for the cross-power spectrum multipoles) we used, which is 208, for a joint LRG and ELG analysis.\(^1\) To reflect the actual situation of the eBOSS observations, observational systematics, including the depth-dependent radial density, angular photometric systematics, fibre collision, redshift failure, etc., is implemented in the pipeline for producing these mocks (see Zhao et al. 2021 for more details).

The cosmological parameters used for the EZ mocks are listed in equation (1), where the parameters are: the physical energy density of cold dark matter and baryons, the sum of neutrino masses, the amplitude of the linear matter power spectrum within 8 $h^{-1}$ Mpc, the power index of the primordial power spectrum, and the (derived) scale of the sound horizon at recombination, respectively.

\[
\Theta \equiv \left\{ \Omega_\Lambda h^2, \Omega_b h^2, \sum M_i/eV, \sigma_8, n_s, r_s/\text{Mpc} \right\} = [0.1190, 0.022, 0, 0.8288, 0.96, 147.74]|_f = [0.1190, 0.022, 0, 0.8225, 0.96, 147.66]|_{\text{EZ}}. \tag{1}
\]

\(^1\)The Hartlap factor is 0.895 in our case, which does not significantly deviate from unity, and is included in the likelihood analysis to correct for the data covariance matrix (Hartlap, Simon & Schneider 2007).
Throughout the paper, the subscript or superscript ‘f’ denotes the fiducial value.

### Table 1. Abbreviations used in this work with meanings.

| Abbreviation | Meaning                              |
|--------------|--------------------------------------|
| LRG          | Luminous Red Galaxies               |
| ELG          | Emission Line Galaxies              |
| LRG (P)      | $P_l(k)$ for LRG                    |
| LRG (Q)      | $Q_l(k)$ for LRG                    |
| ELG (P)      | $P_l(k)$ for ELG                    |
| ELG (Q)      | $Q_l(k)$ for ELG                    |
| X            | The cross power between LRG and ELG |
| QQP          | LRG (Q) + ELG (Q) + X (P)           |
| POP          | LRG (P) + ELG (Q) + X (P)           |
| PPP          | LRG (P) + ELG (P) + X (P)           |
| $z_L$        | $z_{\text{eff}}$ (LRG) = 0.70       |
| $z_E$        | $z_{\text{eff}}$ (ELG) = 0.845      |
| $z_X$        | $z_{\text{eff}}$ (LRG × ELG) = 0.77 |
| FoM          | Figure of Merit                     |
| NGC          | Northern Galactic Cap               |
| SGC          | Southern Galactic Cap               |
| LoS          | Line of Sight                       |

We list another set of parameters in equation (1), which is the fiducial cosmology we adopt for this analysis.

Note that the EZmocks for different tracers are produced using the same set of random seeds, thus the clustering of different tracers are intrinsically correlated. This is crucial for the multitracer analysis in this work.

### 3 Methodology

We describe the method used in this work, including a brief review of the multitracer method, a development of the chained power spectrum to mitigate the angular systematics, and prescriptions of creating the power spectrum template, measuring the power spectrum multipoles with the survey window function, handling the mismatch of $z_{\text{eff}}$ between different tracers, and performing parameter estimations. For the ease of presentation, we include a mini-dictionary in Table 1 for abbreviations used in this paper.

#### 3.1 The multitracer method

The clustering of galaxies, as biased tracers of the underlying dark matter field, is subject to the cosmic variance on large scales. The cosmic variance is an intrinsic source of uncertainty for surveys probing a single type of galaxies, but can be significantly suppressed by contrasting the clustering of multiple types of galaxies covering the same range of redshifts and footprints, if the number density of the overlapping tracers is sufficiently high so that the shot noise is negligible on large scales (McDonald & Seljak 2009; Seljak 2009).

As described previously, the eBOSS DR16 sample consists of two types of tracers partially overlapping in cosmic volume at $z < 1.1$, allowing for a multitracer analysis to probe the BAO and RSD jointly.

Under the assumption of Gaussianity, the covariance matrix for power spectrum multipoles of DR16 tracers for a given $k$ mode can be modelled as (White, Song & Percival 2009)

$$C = \begin{bmatrix} LLLL & LLEE & LLLE \\ SYM & EEEE & EEEL \\ LELE & & & \end{bmatrix}.$$  

#### 3.2 The effective redshifts

The measured galaxy cross-power spectrum between tracers A and B in a redshift slice is actually a combination of power spectra at multiple redshifts (Zhao et al. 2019), i.e.,

$$P = \frac{\sum P(z_i) w_A^i w_B^i}{\sum w_A^i w_B^i},$$  \hspace{1cm} (4)

where $z_i$ is the average redshift for the $i$th galaxy pair made of galaxies $w_A^i$ and $w_B^i$, and the summation is over all galaxy pairs in the catalogue. Traditionally, the clustering analysis is performed at a single effective redshift, $z_{\text{eff}}$, for simplicity. This is an approximation, which can be understood from the following Taylor expansion,

$$P(z) = P(z_{\text{eff}}) + P'(z - z_{\text{eff}}) + \frac{1}{2} P''(z - z_{\text{eff}})^2 + O(P''')$$  \hspace{1cm} (5)

where

$$\begin{align*}
\text{AAAA} &= \left(P_A + \frac{1}{n_A}\right)^2; \\
\text{ABAB} &= \frac{1}{2} \left[ P_{AB}^2 + \left(P_A + \frac{1}{n_A}\right) \left(P_B + \frac{1}{n_B}\right) \right]; \\
\text{AABB} &= P_{AB} \left(P_A + \frac{1}{n_A}\right),
\end{align*}$$

for $\{A, B\} \in \{L, E\}$. The autopower spectrum for tracers A and B are expressed as $P_A$ and $P_B$, respectively, and $P_{AB}$ denotes the corresponding cross-power. The shot noise of each tracer are shown as $n_A$ and $n_B$, respectively.

It is worth noting that using C as the data matrix for the likelihood analysis, or equivalently, using both the auto- and cross-power spectra in the analysis, one essentially measures a ratio between the autopower spectra of two biased tracers. In the low-noise limit, i.e., $n_A \to \infty, n_B \to \infty$, this ratio can be determined to an infinite accuracy, since the power spectrum for the matter field, which is subject to the cosmic variance, is cancelled out. Interestingly, the RSD parameter, $\beta = f/b$ where $f$ and $b$ are the logarithmic growth rate and the linear bias, respectively, is involved in the measured ratio, thus the marginalized uncertainty of the RSD parameter is proportional to the shot noise, i.e., $\beta$ is measured without the cosmic variance (McDonald & Seljak 2009). Admittedly, in a realistic situation, the gain from the multitracer method can be degraded by a few factors even in the low-noise limit, including the non-Gaussian correction to the distribution of the matter field, but this effect is sub-dominant on large scales, on which the modes are more relevant for measuring the RSD.
Combining equations (4) and (5) yields,
\[ P = P(z_{\text{eff}}) + P^\Delta_1 + \frac{1}{2} P^\Delta_2 + \mathcal{O}(P^{\Delta_1}) \]
where
\[ \Delta_1 = \sum \frac{z_i w_i^A w_i^B}{w_i^A w_i^B} - z_{\text{eff}}, \]
\[ \Delta_2 = \sum \frac{z_i^2 w_i^A w_i^B}{w_i^A w_i^B} - 2z_{\text{eff}} \sum \frac{z_i w_i^A w_i^B}{w_i^A w_i^B} + z_{\text{eff}}^2. \]

Diminishing \( \Delta_1 \) by properly defining \( z_{\text{eff}} \) as
\[ z_{\text{eff}} = \sum \frac{z_i w_i^A w_i^B}{w_i^A w_i^B}, \]
where \( w_i \) is the total weight of each sample, leaves a residual \( \Delta_2 \) term,
\[ \Delta_2 = \sum \frac{z_i^2 w_i^A w_i^B}{w_i^A w_i^B} - \left( \sum \frac{z_i w_i^A w_i^B}{w_i^A w_i^B} \right)^2. \]
Thus, one has to make sure that \( \Delta_2 \) (and higher order residuals) is sufficiently small to be ignored for the redshift distribution of the concerning galaxy sample, when using a fixed power spectrum template, otherwise the analysis may be subject to systematics.

3.3 The time dependence of the BAO and RSD parameters

Care must be taken when cross-correlating galaxy samples, because different samples may have different effective redshifts, even if they perfectly overlap. One could, in principle, use different \( z_{\text{eff}} \) to generate templates for autocorrelation of each tracer, and for their cross-correlation respectively, but this inevitably requires additional parameters for BAO and RSD, which may degrade the efficiency of the multitracer technique. One way out is to relate the BAO and RSD parameters at different redshifts by a general parametrization. For this purpose, we follow Zhao et al. (2019) to use the parametrization for evaluating the optimal redshift weights, when necessary.

\[ \alpha_\perp(z) = \alpha_\perp(z_p) + \left[ \alpha_\parallel(z_p) - \alpha_\perp(z_p) \right] x, \]
\[ \alpha_\parallel(z) = \alpha_\parallel(z_p) + 2 \left[ \alpha_\parallel(z_p) - \alpha_\perp(z_p) \right] x, \]
\[ f(z) = f(z_p) \left( \frac{1 + z}{1 + z_p} \right)^{3\gamma} \left( \frac{\alpha_\parallel(z)}{\alpha_\parallel(z_p)} H(z) \right)^{2\gamma}, \]
where \( \gamma \) is the growth index introduced in Linder (2005), \( z_p \) is the pivot redshift, \( x \equiv \chi(z)/\chi(z_p) - 1 \) and \( \chi(z) \) and \( H(z) \) are the co-moving distance and the Hubble function at redshift \( z \), respectively. This set of parametrization has been proven to be sufficiently general to cover a broad class of cosmologies in a wide redshift range (Zhu, Padmanabhan & White 2015; Wang et al. 2019). In this work, we use this framework to relate BAO and RSD parameters at \( z = 0.77 \) and \( z = 0.845 \), which is well within the validity of this parametrization, given the uncertainty of the eBOSS DR16 sample.

3.4 Measuring the auto and cross-power spectrum multipoles

The measurement of the power spectrum multipoles can be performed efficiently using the Fast Fourier Transformation (FFT; Bianchi et al. 2015; Scoccimarro 2015), based on the Yamamoto estimator (Yamamoto et al. 2006),
\[ \hat{P}_I(k) = \frac{2}{I + 1} \int \frac{d\ell}{4\pi} \left[ \int \frac{d\mathbf{r}_1}{\mathbf{r}_1^2} F(r_1) e^{i k \cdot r_1} \right. \]
\[ \times \left. \int \frac{d\mathbf{r}_2}{\mathbf{r}_2^2} F(r_2) e^{-i k \cdot r_2} \mathcal{L}_\ell(\hat{k} \cdot \hat{r}_2) - P_{\text{shot}} \right], \]
where \( P_{\text{shot}} \) is the shot noise component, and the integral is over the entire volume of the survey. The line of sight (LoS) of pairs is approximated as the LOS of one of the galaxies in the pair, i.e., \( \mathcal{L}_\ell(\hat{k} \cdot \hat{r}_2) \approx \mathcal{L}_\ell(\hat{k} \cdot \hat{r}_2) \), and the overdensity field is estimated as Feldman, Kaiser & Peacock (1994),
\[ F(r) = \frac{w(r)}{r^{1/2}} \left[ n(r) - c_m(r) \right], \]
where \( w \) is the total weight of each galaxy, and \( n, n_s \) denotes the number density of the data and random samples, respectively. The quantity \( c \) is the ratio of the weighted numbers of the data and random, and the normalization \( I \) is evaluated as,
\[ I \equiv \int dr w^n(r)^2 \approx \alpha \sum_i w_i^2 n_i \alpha. \]
Note that the above approximation using sums over the randoms is only valid for the autopower. For the cross power, one has to take the overlapping geometry into account. A practical way is to assign random galaxies of both tracers on to a grid, and for each tracer, compute \( w_i^n \) for each grid cell, and compute the product \( \sqrt{w_i^n} \sqrt{w_j^n} \) for each grid cell, and sum over the cells. The final result for \( I \) and \( P_{\text{shot}} \) for each tracer is summarized in Table 3.

We use a 1024^3 grid for evaluating \( F \) and \( w^n \), use a fourth-order B-spline for interpolation, and correct for the aliasing effect following Jing (2005). We use the following estimator to measure the cross-power between tracers A and B, which makes use of the spherical

\[ \text{Table 3. Statistics of the galaxy sample used in this work. Quantities } P_{\text{shot}} \text{ and } I, \text{ as defined in equations (11) and (13), are the shot noise subtracted from the measured monopole, and the normalization factor for the power spectrum measurement, respectively.} \]

| Area (deg^2) | LRG(N) | LRG(S) | ELG(N) | ELG(S) | X(N) | X(S) |
|-------------|--------|--------|--------|--------|------|------|
| Leven | 6,934 | 2,560 | 370 | 358 | 370 | 358 |
| N | 255,741 | 121,717 | 83,769 | 89,967 | – | – |
| P_{\text{shot}} (h^{-1}\text{Mpc})^3 | 12,641 | 11,995 | 5,318 | 4,498 | – | – |
| I | 6.18 | 3.00 | 5.42 | 5.93 | 0.88 | 1.54 |
where the addition Theorem (Arfken & Weber 1995) to factorize the Legendre polynomial into a product of spherical harmonics,

\[ P_\ell(k) = \frac{2\ell + 1}{2I} \int \frac{d\Omega_k}{4\pi} \left[ F_{0,A}(k) F_{\ell,B}(-k) + F_{0,B}(k) F_{\ell,A}(-k) \right], \]

where

\[ F_\ell(k) = \int dr F(r)e^{ikr} \mathcal{L}_\ell(k \cdot \hat{r}) \]

\[ = \frac{4\pi}{2\ell + 1} \sum_{m=-\ell}^{\ell} Y_{\ell m}(\hat{k}) \int dr F(r) Y_{\ell m}^*(\hat{r}) e^{ikr}. \]

### 3.5 The chained power spectrum multipoles

To minimize the impact from unknown systematics, we propose a new observable to use, which is the ‘chained power spectrum multipoles’, as defined below, which is immune to any angular systematics, i.e., any contaminant coupling to the transverse mode.

The observed power spectra \( P_{\ell}^{\text{obs}}(k, \mu) \) can be understood as follows. If the angular systematics \( X(k) \) only contaminates the transverse mode, i.e., the \( \mu = 0 \) mode, then it can be modelled in the following way, as discussed in (Hand et al. 2017).

\[
P_{\ell}^{\text{obs}}(k, \mu) = P_{\ell}^{\text{true}}(k, \mu) + X(k) \delta_0(\mu),
\]

where \( \delta_0 \) is the Dirac-\( \delta \) function. A multipole expansion of equation (14) shows,

\[
P_{\ell}^{\text{obs}}(k) = P_{\ell}^{\text{true}}(k) + \frac{2\ell + 1}{2} X(k) \mathcal{L}_\ell(0),
\]

with \( \mathcal{L}_\ell \) being the Legendre polynomial of order \( \ell \). Proceed equation (15) to the next non-vanishing order, we get,

\[
P_{\ell+2}^{\text{obs}}(k) = P_{\ell+2}^{\text{true}}(k) + \frac{2\ell + 5}{2} X(k) \mathcal{L}_{\ell+2}(0).
\]

Eliminating \( X(k) \) from equations (15) and (16), we obtain the following relation,

\[
Q_{\ell}^{\text{obs}} = Q_{\ell}^{\text{true}},
\]

where \( Q_{\ell} \) is the chained power spectrum multipoles,

\[
Q_{\ell} = P_{\ell} - A_{\ell} P_{\ell+2},
\]

and

\[
A_{\ell} \equiv \frac{(2\ell + 1) \mathcal{L}_\ell(0)}{(2\ell + 5) \mathcal{L}_{\ell+2}(0)}.
\]

Unlike the observed \( P_{\ell} \), the observed \( Q_{\ell} \) is immune to the angular systematics, as demonstrated by equation (17), thus is a better quantity to use for data analysis.

For the first three multipoles of \( Q \), equation (18) means,

\[
\begin{pmatrix}
Q_0 \\
Q_2 \\
Q_4
\end{pmatrix} = \begin{pmatrix}
1 & -A_0 & 0 & 0 \\
0 & 1 & -A_2 & 0 \\
0 & 0 & 1 & -A_4
\end{pmatrix} \begin{pmatrix}
P_0 \\
P_2 \\
P_4 \\
P_6
\end{pmatrix}.
\]

To reconstruct \( P \) from \( Q \), a truncation in \( P_{\ell} \) is necessary, otherwise the above matrix equation is not invertible. As \( P_{\ell} = 0(\ell > 4) \) in linear theory, we show an example in which \( P_6 \) is set to zero after finding \( Q_4 \) from data. An matrix inversion of the first \( 3 \times 3 \) block of the transformation matrix in equation (20) yields the cleaned \( P \), denoted as \( P^c \).

\[
\begin{pmatrix}
P_0^c \\
P_2^c \\
P_4^c
\end{pmatrix} = \begin{pmatrix}
1 & A_0 & A_2 \\
0 & 1 & A_2 \\
0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
Q_0 \\
Q_2 \\
Q_4
\end{pmatrix} = \begin{pmatrix}
1 & 0 & 0 & -A_0 & A_2 & A_4 \\
0 & 1 & 0 & -A_2 & A_4 \\
0 & 0 & 1 & -A_4
\end{pmatrix} \begin{pmatrix}
P_0 \\
P_2 \\
P_4 \\
P_6
\end{pmatrix}.
\]

This equation is physically transparent: the role of measured \( P_6 \), which is supposed to be zero as a theoretical prior chosen in this example, is to provide an estimate of the transverse contamination, \( X(k) \).

We caution that the window function of galaxy surveys can complicate the above formalism, because \( P_{\ell}^{\text{true}} \) receives contributions from not only \( P_{\ell+2}^{\text{true}} \), but also \( P_{\ell+2}^{\text{true}} \), which are the true \( P(k) \) multipoles with similar orders, due to the convolution with the anisotropic survey window function. A complete prescription for mitigating the angular systematics with the window function effect is beyond the scope of this paper, but we argue that the chained power spectrum method developed here can remove the primary angular systematics, because \( P_{\ell}^{\text{true}} \) dominates \( P_{\ell}^{\text{obs}} \), even with the window function effect.

Fig. 3 shows the difference in the power spectrum multipoles measured from ELG mocks with and without an injected angular systematics for the normal (denoted as \( P \), left panels) and the chained power spectrum (\( Q \), right). It demonstrates that the chained power spectrum is immune to the injected systematics, namely, the
difference is consistent with zero for $Q$, while an offset on large scales is visible in $P$.

For the eBOSS DR16 sample, we find that $Q$ is rather noisy, as it involves the $P_8$ component, which is barely informative on linear scales. We thus choose not to use $Q$, for this work. Admittedly, we learn less of the galaxy clustering from $Q$, than from $P_0$, $P_2$, and $P_4$, but the information loss can be largely compensated by adding $P_4^X$, multipoles of cross-power spectrum between LRG and ELG, to the analysis. As LRG and ELG are selected using different photometry, we assume that the angular systematics of these tracers are uncorrelated, i.e., $P_4^X$ is immune to angular systematics.

In principle, we can use the following data vectors for analysis,

$$
\begin{align*}
\text{PPP} & = (P_0^1, P_0^4, P_0^5, P_1^1, P_1^4, P_1^5, P_2^0, P_2^4, P_2^5) \; ; \\
\text{PQP} & = (P_0^1, P_1^1, Q_1^0, Q_1^4, Q_1^5, P_2^0, P_2^4, P_2^5) \; ; \\
\text{QQP} & = (Q_1^0, Q_1^4, Q_1^5, P_2^0, P_2^4, P_2^5) \; .
\end{align*}
\tag{22}
$$

where ’L’, ’E’, and ’X’ denote observables for the LRG, ELG, and their cross correlation, respectively. Apparently, PPP and QQP are the most aggressive and most conservative combinations, respectively, and PQP is in between. We shall make the choice in Section 4, after validating our pipeline by performing analyses on the mocks using all these combinations.

### 3.6 The power spectrum template

The TNS model (Taruya, Nishimichi & Saito 2010) has been widely used as a theoretical template for analyses using the autopower spectrum with the linear and non-local bias terms included (McDonald & Roy 2009; Butler et al. 2017). For multiple tracers, the TNS model can be generalized as follows,

$$
P_{g}^{AB}(k, \mu) = D_{g;g}(k, \mu) \left[ P_{g}^{AB}(k) + 2f \mu^2 P_{g;g}^{AB}(k) \right] + f^2 \mu^4 P_{g;g}^{AB}(k) + A^{AB}(k, \mu) + B^{AB}(k, \mu) ,
\tag{23}
$$

where

$$
P_{g;g}^{AB}(k) = b_1^A b_1^B P_{g;g}(k) + \left( b_2^A b_2^B + b_4^A b_4^B \right) P_{g;g}(k)
\quad + \left( b_4^A b_2^B + b_4^B b_2^A \right) P_{g;g}(k)
\quad + \left( b_4^A b_4^B + b_4^B b_4^A \right) P_{g;g}(k)
\quad + \left( b_{2a}^A b_{2a}^B + b_{2a}^B b_{2a}^A \right) \sigma_2^2 P_{g;g}(k)
\quad + b_{2b}^A b_{2b}^B P_{g;g}(k) + b_{2b}^A b_{2b}^B P_{g;g}(k) + N_{AB},
\tag{24}
$$

$$
P_{g;g}(k, \mu) = \frac{1}{2} \left[ (b_1^A + b_1^B) P_{g;g}(k) + (b_2^A + b_2^B) P_{g;g}(k) + (b_4^A + b_4^B) P_{g;g}(k) + \left( b_{2a}^A + b_{2a}^B \right) \sigma_2^2 P_{g;g}(k) + \left( b_{2b}^A + b_{2b}^B \right) P_{g;g}(k) \right] ,
\tag{25}
$$

$$
P_{g;g}(k) = P_{g;g}(k) ,
\tag{26}
$$

$$
D_{g;g}(k, \mu) = \left\{ 1 + [k \mu \sigma_z]^2 / 2 \right\} ^{-1} ,
\tag{27}
$$

with a full derivation of the $A^{AB}$ and $B^{AB}$ terms for the multitracer case included in Appendix A. This template restores the form for the autopower if $A = B$.

The subscripts $\delta$ and $\theta$ denote the overdensity and velocity divergence fields, respectively, and $P_{g;g}$, $P_{g;g}$, and $P_{g;g}$ are the corresponding non-linear auto- or cross-power spectrum, evaluated using the regularized perturbation theory (RegPT) up to second order (Taruya et al. 2012). The linear matter power spectrum $P_m(k)$ is calculated using CAMB (Lewis, Challinor & Lasenby 2000). Terms $b_1$ and $b_2$ stand for the linear bias and the second-order local bias, respectively. We have eliminated the second-order non-local bias $b_3$ and the third-order non-local bias $b_4$ using the following relation (Baldauf et al. 2012; Chan, Scoccimarro & Sheth 2012; Saito et al. 2014),

$$
\begin{align*}
b_{12} & = -\frac{4}{7} (b_1 - 1) , \\
b_{34} & = \frac{32}{315} (b_1 - 1) .
\end{align*}
\tag{28}
$$

Note that the template of the cross power cannot be represented using that for the autopower by redefining a new set of bias parameters in the framework of the TNS model, as explicitly shown in Appendix B, therefore we choose not to introduce an additional set of bias parameters for the cross power for theoretical consistency, although this approach is taken for the analysis in the configuration space (Ross et al. 2014; Wang et al. 2020).

### 3.7 The Alcock–Paczynski effect

The Alcock–Paczynski (AP) effect (Alcock & Paczynski 1979) distorts the observed power spectrum due to a possible mismatch between the input cosmology, which is used to convert redshifts to distances, and the true cosmology hidden in the observations. This effect creates anisotropy at the background level, via the following dilation parameters,

$$
\begin{align*}
\alpha_\perp & = \frac{D_{\delta}(z) r_d^\perp}{D_{\delta}(z) r_d^\perp} , \\
\alpha_\parallel & = \frac{D_{\delta}(z) r_d^\parallel}{D_{\delta}(z) r_d^\parallel} ,
\end{align*}
\tag{29}
$$

with

$$
D_{\delta}(z) = (1 + z) D_{\delta}(z) ; \quad D_{\delta}(z) = c / H(z) ,
\tag{30}
$$

where $D_{\delta}(z)$, $H(z)$ are the angular diameter distance and the Hubble function at redshift $z$, respectively, and $c$ is the speed of light. The $\alpha$ parameters then distort the wavenumber $k$ and $\mu$, which is the cosine of the angle between the LoS and the galaxy pair, in the following way,

$$
\begin{align*}
k' & = \frac{k}{\alpha_\perp} \left[ 1 + \mu^2 \left( \frac{1}{F^2} - 1 \right) \right] ^{1/2} , \\
\mu' & = \frac{\mu}{F} \left[ 1 + \mu^2 \left( \frac{1}{F^2} - 1 \right) \right] ^{-1/2} ,
\end{align*}
\tag{29}
$$

where $F = \alpha_\parallel / \alpha_\perp$ (Ballinger, Peacock & Heavens 1996), and the resultant power spectrum multipole with order $\ell$ reads,

$$
P_{\ell}^{AB}(k) = \frac{2(\ell + 1)}{2\alpha_\parallel^2 \alpha_\perp} \int_{-1}^{1} d\mu P_{g}^{AB} \left( k'(k, \mu), \mu'(\mu) \right) L_{\ell}(\mu) .
\tag{30}
$$

7The numeric code for evaluating the $A^{AB}$, $B^{AB}$ terms for the cross-power is available at http://www2.yukawa.kyoto-u.ac.jp/~atsushi.taruya/cpt_pack.html.
3.8 The survey window function

To account for the geometry of the survey, we follow Wilson et al. (2017) to compute the survey window functions for the autopower spectrum of all tracers, and the cross-power spectrum between LRG and ELG, using the pair-count approach,

$$W_{l}^{AB}(s) = \frac{(2l + 1)}{4\pi s^3} \sum_{i,j} w_{i,m}^A(s) w_{j,m}^B(s) \Delta l \left( S_{i,m} \cdot S_{j,m} \right),$$

where superscripts $A, B$ denote different types of tracers, and again, $A = B$ is the limit for the autocorrelation. This is a multitracer generalization of the formalism in Gil-Marín et al. (2020), and note that, the factor $I$ appears in the denominator, as suggested by de Mattia & Ruhlmann-Kleider (2019), to match the normalization in the measurement of power spectrum, so that the final BAO and RSD measurement does not depend on how exactly the power spectrum is normalized.

3.9 The radial integral constraint

Due to the ignorance of the true selection function of the galaxy survey, which is needed for a clustering analysis, the redshift distribution of actual observations, $n(z)$, is used instead as the selection function for analysis. This can in principle bias the final measurement of BAO and RSD parameters if not accounted for. The resultant bias, quantified as the radial integral constraint (RIC), is recently investigated in de Mattia & Ruhlmann-Kleider (2019), and corrected for in the theoretical model for the power spectrum in the eBOSS DR16 ELG analysis (de Mattia et al. 2021). In this work, we take a different approach, namely, subtracting the RIC component from the data directly. In practice, we analyse two sets of EZmocks, with different treatments of the randoms so that one has the RIC effect, while the other does not. Then a comparison of the measured power spectrum from these two sets provides an estimate of the RIC component of the power spectrum. The two approaches are indistinguishable if the RIC barely depends on cosmology, which is proven to be true for the DR16 sample using mock.

3.10 Parameter estimation

With a modified version of cosmomc (Lewis & Bridle 2002) that supports sampling the BAO and RSD parameters using a TNS template, we use the Markov chain Monte Carlo algorithm to sample the following general parameter space,

$$P = \{ \alpha_L, \alpha_j, f, \sigma_8, \{ b_1 \sigma_8 \}, \{ b_2 \sigma_8 \}, \{ \sigma_v \}, \{ N \} \},$$

where quantities in the inner bracket denote a collection of parameters for each tracer in each galactic cap, e.g.,

$$\{ b_1 \sigma_8 \} = \{ b_{1,N}^L \sigma_8, b_{1,L}^E \sigma_8, b_{1,N}^E \sigma_8, b_{1,L}^S \sigma_8 \},$$

and $N$ is fixed to zero for the cross power. We list wide flat priors for these parameters in Table 4.

Note that we use separate sets of bias parameters for the NGC and SGC, to account for the fact that unknown systematics may yield slightly different amplitudes of power spectra in different galactic caps. This treatment is consistent with BOSS DR12 analyses (Alam et al. 2017), and with other eBOSS DR16 analyses (Gil-Marín et al. 2020; de Mattia et al. 2021).

By default, we assign a full set of the above parameters for a joint analysis at three effective redshift, resulting in joint BAO and RSD measurements at $z_L = 0.70$, $z_X = 0.77$, and $z_E = 0.845$, dubbed as the ‘$3z$’ measurement. Alternatively, we also perform a ‘$2z$’ measurement, by relating parameters at $z_X$ with those at $z_E$ using the parametrization introduced in equation (10) with $z_p = z_E = 0.845$. This essentially spends the information of the cross-power for measuring parameters at the effective redshift of the ELG sample, which yields a joint measurement at $z_L$ and $z_E$. The reason for combining the autopower of the ELG with the cross-power is the following:

(i) As we shall present in Section 4, the power spectrum of the ELG, or the cross-power on their own, struggles to constrain the BAO parameter well due to the low signal-to-noise ratio, which results in loose and highly non-Gaussian constraints without combining with each other:

(ii) The ELG sample is known to be much more contaminated by systematics than the LRG sample (Tamone et al. 2020; de Mattia et al. 2021), thus combining with the cross-power is an efficient way to mitigate the systematics, in addition to using the $Q_i$’s as observables;

(iii) The LRG sample, on the other hand, is much less subjective to systematics (Gil-Marín et al. 2020; Bautista et al. 2021), and it can provide a decent measurement on its own, making it unnecessary to combine it with the cross-power;

(iv) Tomographic information on the lightcone is key for probing physics including the nature of DE (Zhao et al. 2017a), thus we choose not to compress all the power spectra into a measurement at a single redshift.

All the above arguments support for performing the ‘$2z$’ measurement, which will be presented in Section 4 as the primary result of this paper.

In this work, we use data points in the range of $k \in [0.02, 0.15] h$ Mpc$^{-1}$ for all spectra, as motivated by the LRG analysis (Gil-Marín et al. 2020), and have confirmed that this is an appropriate choice for the multitracer analysis, based on analyses using the mocks. In all cases, we combine the likelihoods for the NGC and SGC using a direct sum, and properly correct for the (inverse) data covariance matrix with relevant correction factors suggested by Hartlap et al. (2007) and Percival et al. (2014).

We analyse the chains using GetDist (Lewis 2019), after the chains are fully converged, namely, the Gelman and Rubin statistics $R - 1 < 0.01$ in all cases (An, Brooks & Gelman 1998; Lewis & Bridle 2002).

4 RESULTS

This section is devoted to the main result of this work. We show our measurement of power spectrum multipoles from the EZmocks and from the DR16 galaxy sample, respectively, from which we derive a joint constraint on BAO and RSD parameters at multiple effective redshifts, after validating our pipeline using the EZmocks.

| Table 4. Parameters sampled with flat priors used in this analysis. |
|---------------------------------------------------------------|
| Parameter | Flat prior |
| $\alpha_L$ | [0.5, 1.5] |
| $\alpha_j$ | [0.5, 1.5] |
| $f \sigma_8$ | [0, 3] |
| $b_1 \sigma_8$ | [0, 10] |
| $b_2 \sigma_8$ | [-10, 10] |
| $\sigma_v$ | [0, 20] |
| $N$ | $[-5, 5] \times P_{shot}$ |
A multitracer analysis of the eBOSS DR16 sample

4.1 The power spectrum multipoles

Figs 4 and 5 show the measurement of power spectrum multipoles $P_\ell$ and $Q_\ell$ for the LRG and ELG samples in the NGC and SGC, respectively. The shaded bands illustrate the measurements (68 per cent CL uncertainty around the averaged power spectra) from 1000 realizations of the EZmocks, and the data points with error bars are from the DR16 galaxy sample. Although measurements of the auto power spectrum in $P_\ell$ are presented and extensively discussed in Gil-Marín et al. (2020) and de Mattia et al. (2021) for the LRG and ELG samples, respectively, they are included here for completeness, which is helpful for presenting and discussing the measurement of $Q_\ell$ and the cross-power spectrum.

As expected, we see that $Q_\ell$ generally has larger uncertainties compared to $P_\ell$, because $P_{\ell+2}$, which is less well determined than $P_\ell$, is involved in $Q_\ell$. However, as claimed earlier, the unknown systematics in the data, if exists and couples to the $\mu = 0$ mode, should be largely suppressed by using $Q_\ell$ instead. Interestingly, for $P_\ell$ measured from the ELG (NGC) sample, which is believed to be contaminated more by systematics than the SGC (de Mattia et al. 2021),
an offset between the DR16 sample and the EZmock in the quadrupole is clearly visible on scales at $k \leq 0.06 \, h \, \text{Mpc}^{-1}$, which might signal a component of unknown systematics. However, this glitch vanishes completely in the corresponding $Q_\ell$.

The cross-power spectra in both galactic caps are successfully detected and well measured, although the signal-to-noise ratio is less than that of the auto-power spectra. We find that there is almost no qualitative difference in $P_\ell$ and $Q_\ell$ (up to $\ell = 2$) for the cross power, reinforcing that the cross power is less affected by the systematics, as systematics for different tracers should be uncorrelated. Fig. 6 shows the anisotropic cross-power spectrum, which is reconstructed from the measured $P_0$, $P_2$, and $P_4$. An RSD pattern, which is the elongation of the clustering along $k_\parallel$, is clearly visible in both galactic caps.

The correlation matrix for PQP is shown in Fig. 7, from which we see that $P^X$ strongly correlates with $Q^E$, but less with $Q^L$. This is due to the fact that the ELG sample almost entirely overlaps with the LRG sample, so that the majority of the ELG contributes to the cross power. On the other hand, the LRG sample covers a much larger volume than the ELG, thus only a small fraction of the LRG is counted in the cross correlation. The correlation coefficient between $Q^L$ and $Q^E$ is relatively less (around +0.3), for the same reason.

Fig. 8 presents the window function multipoles measured from the random catalogues of the DR16 sample. As mentioned previously, the normalization is performed in a way to match that for the power spectrum measurement, thus $W_0$ on small scales does not necessarily go to unity. These window function multipoles are used to convolve the theoretical power spectrum prediction to account for the survey geometry following Wilson et al. (2017), before a proper comparison between theory and data can be performed.
4.2 Demonstration using the EZmocks

We perform a joint ‘3z’ fit on the averaged power spectra of 1000 realizations of the contaminated EZmocks using data vectors of PPP, PQP, and QQP, respectively, for a validation and demonstration, and present the result in the upper triangle part of Fig. 9.

To start with, we notice that PPP and QQP provide the tightest and weakest constraint, respectively, and PQP is in between, as expected. The constraint from QQP and PQP are in excellent agreement with the expected values for all parameters, but the constraint from PPP can deviate by a noticeable amount, e.g., the constraints on \( \alpha_\parallel(z) \) and \( f_8|_{\parallel}(z_E) \) are higher or lower than the expected value by \( \sim 1\sigma \), due to the systematics in the ELG mock sample. This makes us decide not to use PPP for this work, although it provides the tighest constraint.

QQP, on the other hand, unnecessarily trashes information of the LRG sample, which significantly dilutes the constraint at \( z = 0.70 \). Due to these reasons, we choose to use PQP for presenting the primary result of this paper, as it is a reasonable compromise between retaining the constraining power of the data, and mitigating the systematics in the ELG sample. One point worth noting is that, the cross power, almost on its own, is able to provide a decent measurement at \( z_{\text{eff}} = 0.77 \) with nearly no bias at all, which once again shows the robustness of the cross power against the systematics.

Mock tests with other data combinations and parametrizations are performed, and e.g., the case of PQP with ‘2z’ is shown in red solid contours in the upper triangle. We also run tests with different cutoff scales for the power spectrum, and different widths of \( k \) bins, and find that the choice adopted in this work is reasonably optimal. As illustrated, our pipeline is well validated, i.e., the constraint derived in all cases are consistent with the expected ones well within the uncertainty, from all the tests. These mock tests also demonstrate that the cross-power spectrum is informative, and more robust against systematics than the autopower spectrum.

4.3 Measurements from the DR16 sample

The ‘3z’ and ‘2z’ measurements from the DR16 galaxy sample (using PQP) are presented in Table 5 and in the lower triangle and the diagonal part of Fig. 9.

Measurement at \( z_{\text{eff}} = 0.70 \) is well performed, thanks to the robust LRG observations. However, the ‘3z’ measurement at \( z_{\text{eff}} = 0.77 \) and 0.845 are rather weak for some parameters, including all parameters for the ELG and \( f_8|_{\parallel}(z_E) \), compared to those measured from the mean of mocks. This is largely because the ELG sample is subject to systematics including the redshift failures, and unfortunately this kind of systematics affect both auto- and cross-correlations, so that the BAO feature gets distorted in the ELG auto- and cross-correlation functions (Tamone et al. 2020; Wang et al. 2020). However, the cross power can constrain \( f_8|_{\parallel}(z_X) \) fairly well, namely, \( f_8|_{\parallel}(z_X) = \ldots \).
Figure 9. The 1D posterior distribution (panels on the diagonal) and 68 and 95 per cent CL contour plots for BAO and RSD parameters derived from the eBOSS DR16 (panels in the lower triangle) and EZmock catalogues (upper triangle), respectively. For measurements from the DR16 catalogue, results are derived at three (blue) and two (red) effective redshifts, respectively, denoted as DR16 (3z) and DR16 (2z) in the legend. For results derived from the mocks, measurements are performed at three effective redshifts, from three different data combinations: QQP3z (grey solid contours); PQP3z (filled), and PPP3z (grey dashed). We also perform the mock test at two effective redshifts, denoted as PQP2z (red solid). The dashed horizontal and vertical lines illustrate the fiducial model used to produce the EZmocks, which is identical to that used for this work.

$0.317 \pm 0.080$, which is a $\sim 4\sigma$ detection of the RSD signal, as visually illustrated in Fig. 6.

Due to the large correlation between $Q^E$ and $P^X$ as shown in Fig. 7, the BAO and RSD parameters measured at $z_X = 0.77$ and $z_E = 0.845$ are correlated. For example, $\text{corr}[\alpha_{\parallel}(z_X), \alpha_{\parallel}(z_E)] = 0.50$, $\text{corr}[\alpha_{\parallel}(z_X), \alpha_{\perp}(z_E)] = 0.45$ and $\text{corr}[\sigma_8(z_X), \alpha_{\parallel}(z_E)] = -0.33$. This means that the weak constraints at $z_E$ can be improved, if the cross-power spectrum is used to constrain parameters at $z_E$, which is designed as the ‘2z’ measurement, as described in Section 3.10.

Comparing the ‘2z’ (top red layer in Fig. 9) with ‘3z’ (bottom blue) measurements, we see that the constraint on all parameters at $z_E$ is significantly improved, primarily due to the contribution of the cross-power spectrum. Specifically, $\alpha_{\perp}(z_E)$, which is almost unconstrained in ‘3z’ (it has no upper bound given the wide flat prior), is now measured at a precision of 4 per cent with a perfectly Gaussian
distribution with the cross-power combined in ‘2z’. The constraint on $f_{\text{rs}}(z_E)$ is also improved by a significant amount, namely, the error bar is reduced by a factor of 2.5. We notice that parameters at $z_L$ and $z_E$ are more correlated in the ‘2z’ measurement, due to the cross-power spectrum, as shown in Fig. 10.

Constraints on BAO and RSD parameters at $z_L$ and $z_E$ are extensively studied in companion papers of Gil-Marín et al. (2020) and de Mattia et al. (2021), respectively, using $P_l$ of the LRG and ELG samples separately. As an independent analysis using different methods in various aspects, we find that our results are fully consistent with these analyses within statistical uncertainties, as explicitly compared in Fig. 11. One noticeable difference, though, is seen for the uncertainty of parameters at $z_E$. The error bars derived in de Mattia et al. (2021) are highly asymmetric, because of the non-Gaussian likelihood distribution. However, the posterior measured in this work is much closer to Gaussian, due to the contribution from the cross power.

Final data product of this work is summarized in equation (34) and Fig. 12, which are data vectors and covariance matrices for the chained power spectrum and the cross power.

\[
\text{Table 6. The constraints on } \Omega_M, \Omega_A \text{ from BAO data sets, with } H_0 r_d \text{ marginalized over.}
\]

|            | BOSS                      | BOSS + this work | Full BAO       |
|------------|---------------------------|------------------|----------------|
| $\Omega_A$ | 0.706 ± 0.239             | 0.864 ± 0.186    | 0.752 ± 0.069  |
| $\Omega_M$ | 0.443 ± 0.204             | 0.480 ± 0.172    | 0.302 ± 0.021  |
| corr[$\Omega_M, \Omega_A$] | 0.85              | 0.75            | 0.55           |
| S/N        | 2.95                      | 4.65            | 10.95          |

BAO distances and $f_{\text{RS}}$ at two redshifts,
\[
V(0.70) = [17.954 \pm 0.509, 21.221 \pm 1.198, 0.434 \pm 0.050]
\]
\[
V(0.845) = [18.897 \pm 0.776, 20.910 \pm 2.862, 0.297 \pm 0.081]
\]

where $V \equiv \{D_M r_d, D_H r_d, f_{\text{RS}}\}$. These measurements are overplotted with external measurements published in recent years, including one from the Planck 2018 observations (Planck Collaboration VI 2020), based on a Lambda cold dark matter ($\Lambda$CDM) model. Compared with the Planck result, our measurement at $z_E$ shows a roughly 2$\sigma$ difference, especially on $D_M r_d$ and $f_{\text{RS}}$. The same trend is independently found in de Mattia et al. (2021) in the RSD measurement, which used a completely different scheme to mitigate the angular systematics. This may suggest interesting new physics beyond $\Lambda$CDM, although it may be subject to unknown residue systematics in the ELG sample, even after the mitigation by using the chained power spectrum and the cross power.

Projecting our BAO measurement on to the $\Omega_M, \Omega_A$ plane with $H_0 r_d$ marginalized over as performed in Zhao et al. (2019), we find that the constraint is largely improved by combining our measurement with the that derived from the BOSS DR12 sample, namely, the error on $\Omega_A$ is reduced by 22 per cent, and the correlation with $\Omega_M$ is lowered from 0.85 to 0.75, which raises the significance of $\Omega_A > 0$ from 2.95$\sigma$ to 4.65$\sigma$, as shown in Table 6. Combining other BAO data to date as illustrated in Fig. 13, including the DR16 QSO and Lyman-$\alpha$ measurements, the non-zero $\Omega_A$ is now favoured at a $\sim 11\sigma$ confidence level, which is consistent with the multitracer analysis in the configuration space in a complementary paper (Wang et al. 2020).

5 CONCLUSION AND DISCUSSIONS

eBOSS is a first galaxy survey to observe multiple tracers with a large overlap in the cosmic volume, which naturally motivated this work, as a study of BAO and RSD using multiple tracers in Fourier space.

This work is based on the eBOSS DR16 LRG and ELG samples in redshift range of $z \in [0.6, 1.1]$, with more than 550 000 galaxies in total. Being a first ELG sample for cosmological analysis in history, the DR16 ELG sample is analysed with particular care, to mitigate the systematics in the observations. For this purpose, we develop a new method using the chained power spectrum multipoles ($Q_l$), and have demonstrated using EZmocks that it can efficiently remove angular systematics. Being simply the algebraic difference between the normal power spectrum multipoles ($P_l$) with different orders, $Q_l$ is less well measured. Fortunately, the information loss in using $Q_l$ can be compensated by the cross-power spectrum between the LRG and ELG samples, which itself is least affected by angular systematics.

We measure both $P_l$ and $Q_l$ for each tracer, as well as their cross-power spectrum, and perform a joint BAO and RSD analysis at multiple redshifts after validating our pipeline using the EZmocks.
Figure 12. A compilation of BAO (left) and RSD measurements in recent years (right), including this work (filled red circle) and others derived from catalogues of DR16 Lyman-α forest (DR16 Ly α; du Mas des Bourboux et al. 2020), DR16 quasar (DR16 QSO; Neveux et al. 2020; Hou et al. 2021), DR14 quasar (tomographic BAO and RSD measurements at 4 effective redshifts; Zhao et al. 2019), BOSS DR12 (consensus BAO and RSD measurements at 3 effective redshifts; Alam et al. 2017), DR12 (tomographic BAO and RSD measurements at 9 effective redshifts; Zhao et al. 2017b; Zheng et al. 2019). The shaded bands illustrate the 68 per cent CL constraint derived from Planck 2018 observations (Planck Collaboration VI 2020), in the framework of a ΛCDM model. In the BAO figure, the upper and lower curves (and associated data points) are \( D_M/c/\sqrt{z} + 2 \) and \( \sqrt{z}D_H/r_d - 2 \), respectively.

Figure 13. Constraints on \( \Omega_M \) and \( \Omega_\Lambda \) using BAO observations alone. The left-hand panel shows the 68 and 95 per cent CL constraints derived from three data sets: (I) the BOSS DR12 BAO consensus result at three effective redshifts (‘BOSS gal.’; blue dashed) (Alam et al. 2017); (II) BOSS DR12 combined with this work (red dash–dotted; Note that we eliminated the BOSS BAO measurement for \( z \in [0.6, 0.8] \)) in this combination, because BOSS galaxies in this redshift range are included in the DR16 LRG sample; (III) a further combination with BAO measurements using samples of MGS (Ross et al. 2015), 6dFGS (Beutler et al. 2011), eBOSS DR16 QSO (Hou et al. 2021), and DR16 Lyman-α forest (du Mas des Bourboux et al. 2020) (‘Full BAO’; grey filled). The right-hand panel shows the 1D posterior distribution of \( \Omega_\Lambda \) using three data sets. The posteriors are normalized so that the area under each curve is unity.

Methods developed in this work are directly applicable to forthcoming multitracer surveys including Dark Energy Spectroscopic Instrument (DESI; DESI Collaboration 2016). Given the higher S/N of DESI, we expect the information loss to be reduced when using the chained power spectrum, with the cross-power spectrum between different tracers included in the analysis. This makes it possible for mitigating angular systematics without degrading the statistical precision.
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We used the FFTW library and the jobfork tool for numerical calculation.

DATA AVAILABILITY

The data product of this work is publicly available at https://github.com/icosmology/eBOSS_DR16_1RGxELG and https://www.sdss.org/science/final-bao-and-rsd-measurements/.

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APPENDIX A: THE EXTENDED TNS MODEL FOR THE CROSS-POWER SPECTRUM

A1 Preliminaries

Throughout the report, we work with the distant-observer limit, and assume that the line-of-sight direction is parallel to the z-axis. Then the observed redshift space may be written as

\[ s = r - f u_z(r) \hat{z}, \]  

(A1)

where the quantity \( u_z \) is the normalized velocity field along the LoS, defined by \( u_z \equiv -v_z/(aH f) \). The density field in observed redshift space, \( \delta^{(S)} \), is expressed in Fourier space as

\[ \delta^{(S)}(k) = \int d^3r \left[ \delta(r) + f \nabla u_z(r) \right] e^{ikr}u_z(r) \]  

(A2)

with \( \mu \equiv k/f \).

We are particularly interested in the cross-correlation between the different samples (with different bias parameter). We denote the number density fluctuation of the objects \( A \) and \( B \) by \( \delta_A \) and \( \delta_B \). Also, we consider that the velocity for each object do not simply trace the underlying mass density field, i.e., we generically allow for velocity biases for each tracer, and is labelled as \( u_{A,B} \). Then, the cross-power spectrum is expressed as

\[ P^{(S)}(k) = \int d^3x e^{i k \cdot x} \left\langle \chi^{j_1 A_1} A_2 A_3 \right\rangle \]  

(A3)

with \( x = r - r' \). Here we define

\[ \Delta u_z \equiv u_{A,B}(r) - u_{B,z}(r'). \]  

(A4)

A2 Modelling redshift-space cross-power spectrum at weakly non-linear regime

To derive the expression relevant in the weakly non-linear regime, we follow Taruya et al. (2010), and rewrite equation (A3) with

\[ P^{(S)}(k) = \int d^3x e^{i k \cdot x} \left\langle \chi^{j_1 A_1} A_2 A_3 \right\rangle \]  

(A5)

with the quantities \( j_1, A_i \) given by

\[ j_1 = -i k \mu, \]

\[ A_1 = f \Delta u_z, \]

\[ A_2 = \delta_A(r) + f \nabla_z u_{A,z}(r), \]

\[ A_3 = \delta_B(r') + f \nabla_z u_{B,z}(r'). \]

Then, with a help of cumulant expansion theorem, we obtain

\[ P^{(S)}(k) = \int d^3x e^{i k \cdot x} \exp \{ \left\langle j_1^{(A_1)} \right\rangle \} \left[ \left\langle \chi^{j_1 A_1} A_2 A_3 \right\rangle + \left\langle \chi^{j_1 A_1} A_2 \right\rangle \left\langle \chi^{j_1 A_1} A_3 \right\rangle \right]. \]  

(A6)

Here, \( \left\langle \cdot \right\rangle \) indicates the cumulant.

As it is clear from the expression, the exponential pre-factor \( \exp \{ \left\langle j_1^{(A_1)} \right\rangle \} \) can be non-perturbative, and it leads to a strong damping even at large scales. We thus keep it untouched. But, at weakly non-linear scales, we may expand the rest of the terms regarding \( j_1 \) as a small expansion...
Here, the term $\frac{1}{2}j_i^2 \langle A_i^2 \rangle_{\text{c},x}$ is ignored according to Taruya et al. (2010). For more simplification, we shall assume that $\exp\{\langle h A_i \rangle_{\text{c}}\}$ is independent of separation $x$, and is expressed as (even) function of $k \mu$. With this assumption/ansatz, the model of redshift-space cross-power spectrum, $P_{\text{AB}}^{(S)}$, is given by

$$P_{\text{AB}}^{(S)}(k) = D_{\text{POS}}(k \mu ; c_{\text{AB}}) \left[ \tilde{P}_{\text{mm}}(k) + \tilde{A}(k) + \tilde{B}(k) \right]$$

with

$$\tilde{P}_{\text{mm}}(k) = \int d^3 x \ e^{k \cdot x} \langle A_1 A_2 \rangle_{\text{c},x},$$

$$\tilde{A}(k) = j_1 \int d^3 x \ e^{k \cdot x} \langle A_1 A_2 A_3 \rangle_{\text{c},x},$$

$$\tilde{B}(k) = j_1^2 \int d^3 x \ e^{k \cdot x} \langle A_1 A_2 \rangle \langle A_1 A_3 \rangle_{\text{c},x}.$$
where $\tilde{B}_s^{(\text{sym})}$ and $\tilde{B}_s^{(\text{non-sym})}$ are defined below:

$$\tilde{B}_s^{(\text{sym})}(k_1, k_2, k_3) = b_A b_B B_{211}(k_1, k_2, k_3) + c_A c_B f^2 \left( \frac{k_2}{k_2} \right)^2 \left( \frac{k_3}{k_3} \right)^2 B_{222}(k_1, k_2, k_3),$$

$$\tilde{B}_s^{(\text{non-sym})}(k_1, k_2, k_3) = b_A c_B f \left( \frac{k_3}{k_3} \right)^2 B_{212}(k_1, k_2, k_3) + b_B c_A f \left( \frac{k_2}{k_2} \right)^2 B_{222}(k_1, k_2, k_3).$$

(A17)

With the form given above, the $\tilde{A}$ is expanded as

$$\tilde{A}(k, \mu) = \frac{k^3}{(2\pi)^2} \sum_{n=1}^{3} \sum_{a,b} \mu^{2n} f^{a+b-1} \int_0^\infty dr \int_{-1}^1 dx
\times \left\{ A_{ab}^n(r, x) B_{2ab}(p, k - p, -k) + \tilde{A}_{ab}^n(r, x) B_{2ab}(k - p, p, -k) + \hat{A}_{ab}^n(r, x) B_{2ab}(k - p, -k, p) \right\},$$

(A18)

where we define $r = p/k$ and $x = (k \cdot x)/(kp)$. Then, according to appendix B of Taruya et al. (2010), the coefficients $A_{ab}^n$, $\tilde{A}_{ab}^n$, and $\hat{A}_{ab}^n$ are derived, and the non-vanishing coefficients are expressed as follows:

$$A_{11}^1 = r x b_A b_B c_A, \quad A_{12}^1 = -\frac{r^2(-2 + 3x)(x^2 - 1)(k_2)}{2(1 + r^2 - 2x)} b_A c_B^2, \quad A_{11}^2 = r x b_A c_A c_B,$$

$$A_{21}^2 = \frac{r(2x + r - 2x^2 + 3x) + r^2(-3 + 5x^2)}{2(1 + r^2 - 2x)} b_A c_B^2, \quad A_{22}^2 = -\frac{r^2(-2 + 3x)(x^2 - 1)}{2(1 + r^2 - 2x)} c_A c_B,$$

$$A_{31}^2 = \frac{r(2x + r - 2x^2 + r x(-3 + 5x))}{2(1 + r^2 - 2x)} c_A c_B,$$

$$\tilde{A}_{11}^1 = -\frac{r^2(-1 + r x)}{(1 + r^2 - 2x)} b_A b_B c_B, \quad \tilde{A}_{22}^1 = \frac{r^2(-1 + 3x)(x^2 - 1)}{2(1 + r^2 - 2x)} c_A c_B^2, \quad \tilde{A}_{11}^2 = \frac{r^2(-1 + 3x + 3x^2 - 5x^3)}{2(1 + r^2 - 2x)} c_A c_B^2,$$

$$\tilde{A}_{12}^2 = \frac{r^2(-1 + 3x + 3x^2 - 5x^3)}{2(1 + r^2 - 2x)} c_A c_B^2, \quad \tilde{A}_{11}^3 = -\frac{r^2(-1 + r x)(x^2 - 1)}{2(1 + r^2 - 2x)} b_A c_B^2, \quad \tilde{A}_{12}^3 = -\frac{r^2(-1 + r x)}{1 + r^2 - 2x} b_A c_B.$$

(A19)

The contributions coming from the symmetric bispectrum $\tilde{B}_s^{(\text{sym})}$, i.e. $A_{11}^1$, $A_{22}^1$, $\tilde{A}_{11}^1$, and $\tilde{A}_{22}^1$, coincide with those obtained in the autopower spectrum case (Taruya, Nishimichi & Bernardeau 2013), but others do not necessarily reproduce the previous results.

**A2.2 $B$ term**

We first rewrite equation (A12) with

$$\tilde{B}(k, \mu) = \frac{(k \mu f)^2}{2} c_A c_B \int \frac{d^3 p}{(2\pi)^3} \left[ \tilde{F}_A(p) \tilde{F}_B(k - p) + \tilde{F}_A(k - p) \tilde{F}_B(p) \right].$$

(A20)

The integrand of this expression is symmetric under $p \leftrightarrow k - p$. Then, as similarly done in the autopower spectrum case (Taruya et al. 2010), we expand the $\tilde{B}$ term in powers of $f$ and $\mu$:

$$\tilde{B}(k, \mu) = \frac{k^3}{(2\pi)^2} \sum_{n=1}^{4} \sum_{a,b=1}^{2} \mu^{2n} (-f)^{a+b} \int_0^\infty dr \int_{-1}^1 dx \tilde{B}_{ab}(r, x) \frac{P_a(k \sqrt{1 + r^2 - 2x}) P_b(k r)}{(1 + r^2 - 2x)^{a+b}}.$$

(A21)

Note again that $r = p/k$ and $x = (p \cdot x)/(kp)$. With the symmetric form of equation (A20), the integral over $r$ and $x$ can be replaced with

$$\int_0^\infty dr \int_{-1}^1 dx \rightarrow 2 \int_0^\infty dr \int_{-1}^{\text{Min}[1, 1/(2r)]} dx.$$ 

(A22)
This would help to improve the convergence of numerical integration, avoiding poles. The coefficient $\tilde{B}_{ii}^b$, is derived based on appendix B of Taruya et al. (2010), and the results are summarized below:

\[ \tilde{B}_{11}^1 = \frac{r^2}{2} (x^2 - 1) b_{\lambda} b_{\mu} c_{\lambda} c_{\mu}, \quad \tilde{B}_{12}^1 = \frac{3r^2}{16} (x^2 - 1)^2 c_{\lambda} c_{\mu}(b_{\lambda} c_{\mu} + b_{\mu} c_{\lambda}), \quad \tilde{B}_{21}^1 = \frac{3r^4}{16} (x^2 - 1)^2 c_{\lambda} c_{\mu}(b_{\lambda} c_{\mu} + b_{\mu} c_{\lambda}), \]

\[ \tilde{B}_{22}^2 = \frac{3r^4}{16} (x^2 - 1)^3 c_{\lambda}^2 c_{\mu}^2, \quad \tilde{B}_{21}^2 = \frac{r}{8} (r + 2x - 3r x^2)c_{\lambda} c_{\mu} b_{\lambda} b_{\mu}, \quad \tilde{B}_{22}^2 = \frac{3r^4}{16} (x^2 - 1)^2 r (r + 2x - 5r x^2)c_{\lambda} c_{\mu} (b_{\lambda} c_{\mu} + b_{\mu} c_{\lambda}). \]

\[ \tilde{B}_{21}^3 = \frac{3r^2}{8} (x^2 - 1)(-2r + 5r + 30r x - 35r x^2)^2 c_{\lambda}^2 c_{\mu}^2, \quad \tilde{B}_{22}^3 = \frac{3r^4}{16} (x^2 - 1)^2 r (-6 + 5r^2 + 30r x - 35r x^2)^2 c_{\lambda}^2 c_{\mu}^2. \]

Setting $b_{\lambda}, b_{\mu}, c_{\lambda}$, and $c_{\mu}$ to unity, the above expressions exactly coincide with those presented in Taruya et al. (2010).

**APPENDIX B: THE RELATION BETWEEN AUTO AND CROSS-POWER SPECTRUM TEMPLATES**

To show the relation and difference between the auto- and cross-power spectrum templates in an explicit way, here we rewrite equation (23) by introducing two sets of bias parameters

\[ b_1 = \frac{b_1^0 + b_1^b}{2}; \quad \Delta b_1 = \frac{b_1^0 - b_1^b}{2}, \]  

(1.1)

and

\[ b_2 = \frac{b_2^0 + b_2^b}{2}; \quad \Delta b_2 = \frac{b_2^0 - b_2^b}{2}. \]  

(1.2)

With equation (28), $b_{1i}^0$ and $b_{2i}^0$ can be written as

\[ b_{1i}^0 = -\frac{4}{7} (b_{1i}^0 - 1) = b_{2i} + \Delta b_1; \quad b_{2i}^0 = \frac{4}{7} (b_{1i}^0 - 1) = b_{2i} - \Delta b_1, \]  

(1.3)

\[ b_{3i} = \frac{32}{315} (b_{1i}^0 - 1) = b_{3i} - \frac{32}{315} \Delta b_1, \quad b_{3i}^0 = \frac{32}{315} (b_{1i}^0 - 1) = b_{3i} + \frac{32}{315} \Delta b_1. \]  

(1.4)

Substituting these new parameters into equation (24) gives

\[ P_{g,ij}^{AB} (k) = P_{g,ij}^g (k) + \Delta P_{g,ij}^g (k), \]  

(1.5)

where $P_{g,ij}^g$ takes the form of the auto-power, i.e.,

\[ P_{g,ij}^g (k) = b_{1i}^0 P_{g,ij}^g (k) + 2 b_{1i} b_{2j} P_{g,ij}^g (k) + 2 b_{2i} b_{1j} P_{g,ij}^g (k) + 2 b_{2i} b_{2j} P_{g,ij}^g (k) + 2 b_{2i} b_{2j} \sigma_0^2 (k) P_{M}^g (k) + b_{2i}^2 P_{g,ij}^g (k) + b_{2i}^2 P_{g,ij}^g (k) + N_{AB}. \]  

(1.6)

and

\[ \Delta P_{g,ij}^g (k) = - (\Delta b_1)^2 P_{g,ij}^g (k) - 2 \Delta b_1 \Delta b_2 P_{g,ij}^g (k) + \frac{8}{7} (\Delta b_1)^2 P_{g,ij}^g (k) + \frac{8}{7} \Delta b_1 \Delta b_2 P_{g,ij}^g (k) - \frac{64}{315} \Delta b_1^2 \sigma_0^2 (k) P_{M}^g (k) - (\Delta b_2)^2 P_{g,ij}^g (k) - \frac{16}{49} \Delta b_1 \Delta b_2 P_{g,ij}^g (k). \]  

(1.7)

For $P_{g,ij}^{AB}$, we find that both $\Delta b_1$ and $\Delta b_2$ vanish, so

\[ P_{g,ij}^{AB} (k) = P_{g,ij}^g (k). \]  

(1.8)

To see how the A and B terms change under transformation of bias parameters, we first rewrite them in the following form,
\[ A^{AB}(k, \mu) = \mu^2 f \left[ A_{11a}(k) b_A b_B c_A + A_{11b}(k) b_A b_B c_B \right] + \mu^2 f^2 \left[ A_{12a}(k) b_B c_A^2 + A_{12b}(k) b_A c_B^2 \right] \\
+ \mu^4 f^2 \left[ A_{22a}(k) b_B c_A^2 + A_{22b}(k) b_A c_B^2 \right] + \mu^4 f^2 \left[ A_{23a}(k) c_A^2 c_b + A_{23b}(k) c_A c_B^2 \right] \\
+ \mu^6 f^2 \left[ A_{33a}(k) c_A^2 c_B + A_{33b}(k) c_A c_B^2 \right] , \]  
(B9)

\[ B^{AB}(k, \mu) = \mu^2 \left[ f^2 B_{13}(k) b_A b_B c_A + f^2 B_{14}(k) c_A c_B (b_A c_B + b_B c_A) + f^2 B_{15}(k) c_A^2 c_B \right] \\
+ \mu^4 \left[ f^2 B_{22}(k) b_A b_B c_A c_B + f^2 B_{23}(k) c_A c_B (b_A c_B + b_B c_A) + f^2 B_{24}(k) c_A^2 c_B \right] \\
+ \mu^6 \left[ f^2 B_{33}(k) c_A c_B (b_A c_B + b_B c_A) + f^2 B_{34}(k) c_A^2 c_B \right] \\
+ \mu^8 f^2 B_{44}(k) c_A^2 c_B^2 . \]  
(B10)

Setting \( c_A = c_B = 1 \) as assumed in this paper, and eliminating \( b_A, b_B \) using equation (B1), we obtain,
\[ A^{AB}(k, \mu) = A(k, \mu) + \Delta A(k, \mu) , \]  
(B11)

with
\[ A(k, \mu) = \mu^2 f^2 \left[ A_{11a}(k) + A_{11b}(k) \right] b_1^2 \\
+ f^2 \left[ \mu^2 A_{12a}(k) + \mu^2 A_{12b}(k) + \mu^4 A_{22a}(k) + \mu^4 A_{22b}(k) + \mu^4 A_{23a}(k) + \mu^4 A_{23b}(k) \right] b_1 \\
+ f^3 \left[ \mu^4 A_{23a}(k) + \mu^4 A_{23b}(k) + \mu^6 A_{33a}(k) + \mu^6 A_{33b}(k) \right] , \]  
(B12)

\[ \Delta A(k, \mu) = - \mu^2 f \left[ A_{11a}(k) + A_{11b}(k) \right] (\Delta b_1)^2 \\
- f^2 \left[ \mu^2 A_{12a}(k) + \mu^2 A_{12b}(k) + \mu^4 A_{22a}(k) + \mu^4 A_{22b}(k) + \mu^4 A_{23a}(k) + \mu^4 A_{23b}(k) \right] \Delta b_1 , \]  
(B13)

\[ B^{AB}(k, \mu) = B(k, \mu) + \Delta B(k, \mu) , \]  
(B14)

\[ B(k, \mu) = f^2 \left[ \mu^2 B_{13}(k) + \mu^4 B_{22}(k) \right] b_1^2 + f^3 \left[ \mu^2 B_{13}(k) + \mu^4 B_{23}(k) + \mu^6 B_{33}(k) \right] \times 2b_1 \\
+ f^4 \left[ \mu^2 B_{23}(k) + \mu^4 B_{34}(k) + \mu^6 B_{44}(k) + f^4 \mu^8 B_{44}(k) \right] , \]  
(B15)

\[ \Delta B(k, \mu) = - f^2 \left[ \mu^2 B_{13}(k) + \mu^4 B_{22}(k) \right] (\Delta b_1)^2 . \]  
(B16)

Finally, the relation between the auto- and cross-power spectrum templates is,
\[ P^{AB}_g(k, \mu) = P_g(k, \mu) + \Delta P_g(k, \mu) , \]  
(B17)

where
\[ P_g(k, \mu) = D_{FgG}(k, \mu) \left[ P_{g,44}(k) + 2 f \mu^2 P_{g,60}(k) + f^2 \mu^4 P_{g,80}(k) + A(k, \mu) + B(k, \mu) \right] \]  
(B18)

is the auto-power spectrum, and
\[ \Delta P_g(k, \mu) = D_{FgG}(k, \mu) \left[ \Delta P_{g,44}(k) + \Delta A(k, \mu) + \Delta B(k, \mu) \right] \]  
(B19)

gives the difference.

The above calculation explicitly shows that the template of the cross power cannot be represented using that for the auto-power by redefining a single set of bias parameters.

This paper has been typeset from a \TeX/\LaTeX\ file prepared by the author.

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