Effect of Endometriosis to fallopian tube of the peristaltic-ciliary flow of third grade fluid in a finite narrow tube

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Abstract. The present prospective theoretical investigation deals with analysis of the peristaltic-ciliary transport of a developing embryo within the fallopian tubal fluid in the human fallopian tube under the effect of Endometriosis. This disease make the peristalsis ciliary flow become to peristalsis flow. A mathematical model induced flow of viscoelastic fluid characterized by the third grade fluid in a finite two dimensional narrow tube. That research is study the effect of couple stress to peristaltic–ciliary flow to Non-Newtonian fluids. Non-linear partial differential equations are solved by perturbation method. Flow variables like axial and radial velocities, appropriate residue time over tube length, pressure difference over have been derived under the assumption of long wavelength and low Reynolds number approximation and the expression for pressure rise is obtained by using wavelength and stream function are analysed for embedded parameter. This study is done through by the “MATHEMATICA”

Keywords: peristaltic flow, perturbation method

1. Introduction
In this paper we describe the adverse impact of various pathological conditions, such as endometriosis, infection ciliary motility. Cause altered or impaired cilia activity. Is one of the tubal disorders, which are related to infertility that result in partial or total fallopian tube obstruction. Tubalendometriosis is identified in fallopian tubes, most commonly involving the distal end [49]. Endometriosis of the tube can be found within the lumen by focal replacement of tubal epithelium by uterine mucosa [27]; or myosalpinx or on the serosa resulting in lower ciliary beat frequency, lower ciliate cells percentage, weaker muscular contractile activity, and lower contraction frequency [38]. We will study the effect of couple stress to peristaltic-ciliary flow with this disease under the assumptions of low Reynolds number and long wavelength we solved the governing equation by the perturbation method when Deborah number is small and study the effect of the parameters on the axial and radius velocity [3,22] pressure gradient and stream function

2. Mathematical Formulation
We assumethesurface has cyclic peristaltic contraction it’s generated asinusoidal wave while the swaying motions of the cilia tips generate metachronal wave and both of them generate a travelling wave [1]
\[
H_{\theta}(\bar{z}, \bar{t}) = r_{t} + b \sin \left( \frac{2\pi}{\lambda} \bar{z} \right)
\]

\[
f_{\theta}(\bar{z}, \bar{t}) = r_{t} + b \sin \left( \frac{2\pi}{\lambda} (\bar{z}, \bar{t}) \right)
\]

Where amplitude of sinusoidal wave

\[
\bar{Z} = \bar{z} + c_{t}, \quad H_{\theta}(\bar{Z}, \bar{t}) = \bar{f}(\bar{z})
\]

\[
h(\bar{Z}) = r_{t} + b \sin \left( \frac{2\pi}{\lambda} (\bar{z} - c_{t}) \right)
\]

We assume fallopian tubal fluid as an incompressible third grade fluid and consider it within fallopian tubal fluid fill the tube.

Let \( (r, \Theta, z) \) denoted the cylindrical coordinates in the radial, tangential and axial direction.

Let \( v = (u, v, w) \) the velocity components in those directions [1], and \( t \) is the time.

The extra stress tensor for incompressible third grade fluid is given by

\[
\bar{\tau} = \mu \bar{A}_{1} + \kappa_{1} \bar{A}_{2} + \kappa_{2} \bar{A}_{3} + B_{1} \bar{A}_{3} + B_{2} (\bar{A}_{1} \bar{A}_{2} + \bar{A}_{2} \bar{A}_{1}) + B_{3} (\text{tr} \bar{A}_{1}^{2}) \bar{A}_{3}
\]

Where \( \mu \) is the coefficient of shear viscosity, \( \kappa_{1}, \kappa_{2}, B_{1}, B_{2} \) and \( B_{3} \) material constant, \( \bar{A}_{1}, \bar{A}_{2}, \text{ and } \bar{A}_{3} \) respectively are the first, second and third Rivli-Ericksen tensors

\( \mu \geq 0, \kappa_{1} \geq 0, \quad B_{1} = B_{2} = 0, \quad B_{3} \geq 0 \) and \( |\kappa_{1} + \kappa_{2}| \leq \sqrt{\mu B_{3}} \)

\[
\bar{A}_{\alpha} = \bar{B}_{\alpha} \bar{A}_{\alpha-1} + \bar{A}_{\alpha-1} (\nabla \bar{V}) + (\nabla \bar{V})^{T} \bar{A}_{\alpha-1} = 2, 3
\]

Where \( \nabla \bar{V} \) represents the velocity gradient in the cylindrical coordinates, \( (\nabla \bar{V})^{T} \) is the transpose of the velocity gradient [1]

\[
Z = \begin{bmatrix}
\bar{Z}_{R\bar{R}} & \bar{Z}_{R\bar{\Theta}} & \bar{Z}_{R\bar{Z}} \\
\bar{Z}_{\Theta\bar{R}} & \bar{Z}_{\Theta\Theta} & \bar{Z}_{\Theta\bar{Z}} \\
\bar{Z}_{Z\bar{R}} & \bar{Z}_{Z\Theta} & \bar{Z}_{Z\bar{Z}} \\
\end{bmatrix}
\]

\[
\bar{Z}_{R\bar{R}} = \bar{Z}_{\Theta\Theta} = \bar{Z}_{Z\bar{Z}} = \bar{Z}_{Z\Theta} = 0
\]

3. The Governing Equation

The equation of motion for incompressible couple-stress fluid in fallopian tube and continuity equation [8]

\[
\frac{1}{R} \frac{\partial}{\partial R} (\bar{q}R) + \frac{\partial \bar{w}}{\partial Z} = 0
\]

In R-direction

\[
\rho \left( \frac{\partial \bar{u}}{\partial R} + \bar{w} \frac{\partial \bar{u}}{\partial Z} \right) = -\frac{\partial \bar{p}}{\partial R} + \frac{1}{R} \frac{\partial}{\partial R} (R \bar{Z}_{RR}) + \frac{\partial}{\partial Z} (\bar{Z}_{RZ}) - \frac{1}{R} \bar{Z}_{R\Theta}
\]

In Z-direction

\[
\rho \left( \frac{\partial \bar{v}}{\partial R} + \bar{w} \frac{\partial \bar{v}}{\partial Z} \right) = -\frac{\partial \bar{p}}{\partial R} + \frac{1}{R} \frac{\partial}{\partial R} (R \bar{Z}_{ZR}) + \frac{\partial}{\partial Z} (\bar{Z}_{Z\bar{Z}}) - n \nabla^{4} \bar{w}
\]

Where \( \bar{u}, \bar{w} \) component of the velocity in \( R, Z \) direction, \( \rho \) constant density of the fluid. \( P \) is pressure, \( \bar{Z}_{RR}, \bar{Z}_{RZ}, \bar{Z}_{ZR} \) and \( \bar{Z}_{Z\bar{Z}} \) component of extra stress tensor and the velocity gradient the velocity gradient

\[
\nabla^{2} = \frac{\partial^{2}}{\partial R^{2}} + \frac{1}{R} \frac{\partial}{\partial R} + \frac{\partial^{2}}{\partial Z^{2}}, \nabla^{4} = \nabla^{2} (\nabla^{2})
\]
4. Dimension analysis of governing equation

The fluid in fallopian tube became independent of time \( \bar{t} \), because we introduce a wave frame \((\bar{R}, \bar{Z})\) moving at a speed from the fixed \((r, z)\), by the following.

\[
\bar{Z} = \bar{z} + \bar{c} \bar{t}, \quad \bar{w} = \bar{w}(\bar{r}, \bar{z}) + \bar{c}, \quad \bar{Z}(\bar{R}, \bar{Z}, \bar{t}) = \bar{Z}(\bar{r}, \bar{z}), \quad \bar{R}(\bar{R}, \bar{Z}, \bar{t}) = \bar{R}(\bar{r}, \bar{z})
\]

And by using dimensionless parameters:

\[
\begin{align*}
\bar{r} &= \frac{r}{\bar{r}},
\bar{z} &= \bar{z},
\bar{w} &= \bar{w}
\end{align*}
\]

continuity equation and motion equation can be written as

\[
\frac{\partial \bar{u}}{\partial \bar{r}} + \frac{\partial \bar{w}}{\partial \bar{z}} = 0
\]

By the low Reynolds number and long wavelength we get:

\[
\frac{\partial}{\partial \bar{r}} (\bar{r} \bar{w}) = \frac{\partial p}{\partial \bar{r}}
\]

Where

\[
\bar{Z}_{\bar{R}Z} = \mu \bar{u}_R + \mu \bar{w}_R + \kappa_1 \bar{w} \bar{u}_R + \kappa_1 \bar{w}_R \bar{u} + \kappa_1 \bar{u} \bar{u}_R + \kappa_1 \bar{w}_R \bar{u} + \kappa_1 \bar{w} \bar{w}_R + 3 \kappa_1 \bar{u}_R \bar{u}_R + \kappa_1 \bar{w}_R \bar{w}_R + 2 \kappa_2 (\bar{u}_R \bar{w}_R + \bar{w}_R \bar{u}_R) + 2 \kappa_2 (\bar{u}_R \bar{u}_R + \bar{w}_R \bar{w}_R)
\]

5. The Dimensionless Form of Boundary

\[
h(\bar{x}) = 1 + \phi \sin 2\pi(x)
\]

Where

\[
\phi = b \frac{\bar{r}}{\bar{r}_t}, \quad e = \frac{A2\pi}{\lambda}
\]

6. Solution of the Governing Equation

\[
\frac{\partial}{\partial \bar{r}} \left( \bar{r} \frac{\partial \bar{w}}{\partial \bar{r}} + 2\text{De} \bar{r} \left( \frac{\partial \bar{w}}{\partial \bar{r}} \right)^3 \right) - \kappa_3 \bar{r} \nabla^4 \bar{w} = \frac{\partial p}{\partial \bar{z}} \bar{r}
\]

Where \( \text{De} \) is the Deborah number and \( p(z) = \frac{\partial p}{\partial \bar{z}} \) is the pressure gradient, the equation was solved by the perturbation method[1] assuming \( \text{De} \ll 1 \) small parameter, let

\[
\begin{align*}
\bar{w} &= \bar{w}_0 + \kappa_3 \bar{w}_1 \\
u &= \bar{u}_0 + \kappa_3 \bar{u}_1 \\
p &= \bar{p}_0 + \kappa_3 \bar{p}_1
\end{align*}
\]
The series of \( w(r, \bar{z}) \), \( \psi(r, \bar{z}) \) and \( p(\bar{z}) \) was expanded:

\[
w(r, \bar{z}) = w_{01}(r, \bar{z}) + Dw_{02}(r, \bar{z}) + D_{r}^{2}w_{03}(r, \bar{z}) + \kappa_{3}(w_{11}(r, \bar{z}) + De w_{12}(r, \bar{z}) + D_{r}^{2}w_{13}(r, \bar{z})) \tag{22}
\]

\[
u(r, \bar{z}) = u_{01}(r, \bar{z}) + Du_{02}(r, \bar{z}) + D_{r}^{2}u_{03}(r, \bar{z}) + \kappa_{3}[u_{11}(r, \bar{z}) + De u_{12}(r, \bar{z}) + D_{r}^{2}u_{13}(r, \bar{z})] \tag{23}
\]

\[
p(\bar{z}) = p_{01}(\bar{z}) + De p_{02}(\bar{z}) + D_{r}^{2}p_{03}(\bar{z}) + \kappa_{3}[p_{11}(\bar{z}) + De p_{12}(\bar{z}) + D_{r}^{2}p_{13}(\bar{z})] \tag{24}
\]

Using the above expression, we get

\[
\frac{\partial}{\partial r}[\frac{\partial w_{0}}{\partial r} + \kappa_{3} r \frac{\partial w_{1}}{\partial r} + 2 De r \left(\frac{\partial w_{0}}{\partial r}\right)^{2} + 6 \kappa_{3} De r \left(\frac{\partial w_{1}}{\partial r}\right)] - \kappa_{3} r \Psi \frac{\partial w_{0}}{\partial r} = \frac{\partial}{\partial z}\left[\frac{\partial p_{0} + \kappa_{3} p_{1}}{\partial z}\right] \tag{25}
\]

The velocity is found to have the form:

\[
w(r, \bar{z}) = -1 - \frac{1}{4} P_{01}(z)(-r^{2} + h(z)^{2}) + \frac{1}{16} De(-4 P_{02}(z)(-r^{2} + h(z)^{2}) + P_{01}(z)^{3}(-r^{4} + h(z)^{4}) + \frac{1}{16} De^{2}(-4 P_{03}(z)(-r^{2} + h(z)^{2}) + 3 P_{01}(z)^{2} P_{02}(z)(-r^{4} + h(z)^{4}) - P_{01}(z)^{5}(-r^{6} + h(z)^{6}) + \frac{1}{4}(P_{11}(z)r^{2} - P_{11}(z)h(z)^{2}) + De(-h(z)P_{12}(z) + P_{12}(z)r + 4h(z)P_{13}(z)^{3} - 4r P_{01}(z)^{3} + h(z)^{3} P_{01}(z)^{3} P_{11}(z) - r^{3} P_{01}(z)^{2} P_{11}(z)) + \frac{1}{40} De^{2}\left[\frac{1}{40}(560 h^{3} P_{01}(z)^{3} - 560 h^{3} P_{01}(z)^{3} P_{02}(z) + 480 h(z) P_{01}(z)^{3} P_{02}(z) + 36 r^{5} P_{01}(z)^{3} P_{11}(z) + 5 r^{6} P_{01}(z)^{3} P_{11}(z) - 36 h^{3} P_{01}(z)^{3} P_{11}(z) - 5 h^{3} P_{01}(z)^{3} P_{11}(z) - 15 r^{6} P_{01}(z)^{3} P_{11}(z) + 15 h(z)^{3} P_{01}(z)^{3} P_{12}(z) + 20 r^{3} P_{01}(z)^{3} P_{12}(z) + 3 h(z)^{3} P_{01}(z)^{3} P_{11}(z) + 40 h(z) P_{13}(z) - 40 h(z) P_{13}(z) \kappa_{3}\right] \tag{26}
\]

Similarly the second component velocity is

\[
u = -\frac{1}{16} r(-4 h(z) P_{01}(z) h'(z) + r^{2} P_{01}'(z) - 2 h(z)^{2} P_{01}'(z)) - \frac{1}{128} De^{2} r(-2 h(z)^{5} P_{01}(z)^{5} h'(z) + 48 h(z)^{3} P_{01}(z)^{2} P_{02}(z) h'(z) - 32 h(z) P_{03}(z) h'(z) + 5 r^{6} P_{01}(z)^{4} P_{01}'(z) - 20 h(z)^{4} P_{01}(z)^{4} P_{11}(z) - 8 r^{4} P_{01}(z) P_{02}(z) P_{11}'(z)) + \frac{1}{32} De r(-4 h(z)^{3} P_{01}(z)^{3} h'(z) + 8 h(z) P_{02}(z) h'(z) + r^{4} P_{01}(z)^{2} P_{01}'(z) - 3 h(z)^{4} P_{01}(z)^{2} P_{01}'(z) - 2 r^{2} P_{02}'(z) + 4 h(z)^{2} P_{02}'(z) + a 2 (De^{2} + \frac{1}{16} (4 rh(z) P_{11}(z) h(z) - 3 h(z)^{4} P_{01}(z)^{2} P_{01}'(z) - 2 r^{2} P_{02}'(z) + 6 h(z)^{2} P_{11}'(z) + 4 r^{2} P_{01}(z)^{4} P_{11}'(z) - 45 r h(z)^{2} P_{11}(z) h'(z) + 15 r P_{12}(z) h'(z) + 120 r^{2} P_{01}(z)^{2} P_{01}'(z) - 180 r h(z) P_{01}(z) P_{01}'(z) - 12 r^{4} P_{01}(z) P_{01}'(z) - 30 h(z)^{3} P_{01}(z) P_{01}'(z) P_{01}'(z) + 6 r^{4} P_{01}(z)^{3} P_{11}'(z) - 15 r h(z)^{3} P_{01}(z)^{3} P_{11}'(z) - 10 r^{2} P_{12}'(z) + 15 r h(z)P_{12}'(z)) -
\[
\frac{1}{6720} r(80640r^3 P_{01} ' (z) P_{01} (z^2) + 40320 P_{01} (z)^2 h(z)') - 1260 h(z)^3 P_{01} (z) h(z)'
\]
\[
120960 h(z)^3 P_{01} (z)^5 h(z) + 15120 h(z)^4 P_{01} (z)^4 P_{11} (z) h(z)' - 2520 h(z)^5 P_{01} (z)^4 P_{11} (z) h(z) + 5040 h(z)^3 P_{01} (z) P_{02} (z) P_{11} (z) h(z) + 3360 P_{33} (z) h(z)'
\]
\[
- 201600 h(z)^3 P_{01} (z)^4 P_{01} (z) - 3456 r P_{01} (z)^3 P_{11} (z) P_{01} ' (z) + 420 r^6 P_{01} (z)^3 P_{11} (z) P_{01} (z) + 120960 h(z)^5 P_{01} (z)^3 P_{11} (z) P_{01} ' (z) - 1680 h(z)^6 P_{01} (z)^3 P_{11} (z) P_{01} ' (z) - 53760 P_{01} (z) P_{01} ' (z) + 420 r^4 P_{01} (z)^3 P_{01} (z) - 1260 h(z)^4 P_{01} (z)^3 P_{01} ' (z) - 420 r^4 P_{01} (z)^3 P_{01} (z) + 1260 h(z)^4 P_{02} (z) P_{11} (z) P_{01} ' (z) - 864 r^5 P_{01} (z)^3 P_{11} (z) + 105 r^8 P_{01} (z)^4 P_{11} (z) + 3024 h(z)^5 P_{01} (z)^4 P_{11} (z) - 420 h(z)^6 P_{01} (z)^3 P_{11} (z) - 420 r^4 P_{01} (z)^3 P_{02} (z) P_{11} (z) + 1260 h(z)^4 P_{01} (z) P_{02} (z) P_{11} (z) - 2240 r P_{13} (z) + 3360 h(z) P_{13} (z)) \]

The stream function is defined

\[ w(r, z) = \frac{1}{r} \frac{\partial}{\partial r} \psi \]

from which, we get

\[ \psi (h, z) = \]

\[ \frac{1}{80} (-\frac{80}{3} De(4 P_{01} + 12 De P_{01}^2 P_{02} - P_{12} - De P_{13}) r^3 \alpha_2 + 80 De P_{01}^2 (28 De P_{01}^3 - 2 P_{11} - De P_{12}) r^5 \alpha_2 + \frac{72}{5} De^2 P_{01}^4 P_{11} r^7 \alpha_2 + 5 r^4 (P_{01} + 3 De P_{01}) P_{11} \alpha_2) + \frac{5}{6} De^2 P_{01}^4 r^8 P_{01} + 2 P_{11} \alpha_2) - \frac{5}{6} De P_{01} r^6 (P_{01} (P_{01} + 3 De P_{02}) + 6 De P_{02} P_{11} \alpha_2) - \frac{1}{2} r^2 \left( -80 - 80 De (4 P_{01} + 12 De P_{01}^2 P_{02} - P_{12} - De P_{13}) h[z] \alpha_2 + 40 De P_{01}^2 (28 De P_{13}^3 - 2 P_{11} - De P_{12}) h[z] \alpha_2 + 72 De P_{01}^4 P_{11} h[z] \alpha_2 + 20 h[2z] P_{01} + De P_{02} + De P_{03} + P_{11} \alpha_2) + 5 De^2 P_{13}^4 h[2z] P_{01} + 2 P_{11} \alpha_2 - 5 De P_{01} h[z]^4 (P_{01} + 3 De P_{02}) + 6 De P_{02} P_{11} \alpha_2) \]  

\[ \text{... (28)} \]

7. Results and Discussions

Number (De) and pressure gradient (ε) and couple stress in axial (w) and radial (u) velocities, pressure gradient and stream function all result in this section made by plotting graphs using Mathematica.

7.1. Velocity distribution (w)

We will study the effect of pressure gradient at the tube entrance (ε), amplitude ratio (Ø) and Deborah number (De), couplestress parameter (κ_3).

7.1.1. Effect of Deborah number (De):

To study the effect of De of velocity distribution (w) keep (κ_3, ε, Ø) fixed at (0.01, 0.0001, 0.13, 9, 1.5) respectively and give De number the values (0.020, 0.024, 0.026). As the previous result makes De increase, the velocity increasing as well. Because the decreasing in dynamic viscosity
7.1.2. Effect of couple stress parameter ($\kappa_3$):
Refer to study the effect of $\alpha_3$ on the velocity distribution, we keep the $(De, \epsilon, \phi)$ fixed $(0.024, 0.001, 9, 1.5, 0.13)$ respectively and the value of $\alpha_3$ is $(0.001, 0.01, 0.1)$ respectively. The increasing in $\alpha_3$ makes dynamic viscosity decreasing that makes the velocity range increases as well.

7.1.3. Effect of amplitude ratio ($\phi$):
To study the effect of $\phi$ on the velocity distribution we keep the $(De, \alpha_3, \epsilon)$ fixed as $(0.024, 0.001, 0.0001, 9, 1.5)$ respectively and the value of $\phi$ is $(0.11, 0.13, 0.15)$ respectively. As the previous result makes $\phi$ increase, the velocity very decreasing, when sinusoidal wave of larger amplitude and the metachronal wave of larger amplitude merge together to form a travelling wave of larger amplitude, then slowness in flow of the third grade fluid in the direction of propagation of the travelling wave of tube surface happens. Velocity range is equal. $\phi$
7.1.4. Effect of pressure gradient at the tube entrance $\varepsilon$:
To study the effect of $\psi$ on the velocity distribution we keep the $(De, \alpha_3, \epsilon, \varepsilon)$ fixed as $(0.024, 0.001, 0.0001, 9, 1.5)$ respectively and the value of $\psi$ is $(0.11, 0.13, 0.15)$ respectively. As the previous result makes $\psi$ increase, the velocity very decreasing, when sinusoidal wave of larger amplitude and the metachronal wave of larger amplitude merge together to form travelling wave of larger amplitude, then slowness in flow of the third grade fluid in the direction of propagation of the travelling wave of tube surface happens to

![Figure (4)](image)

**Figure (4)** vertical velocity for different values $\psi$, when $De = 0.020, \kappa_3 = 0.01, \psi = 0.13 \varepsilon = 8, \varepsilon = 8.5, \varepsilon = 9$

7.2. Velocity distribution
We will study the effect of the pressure gradient at the tube entrance ($\varepsilon$), amplitude ratio ($\psi$) and Deborah number ($De$), couple stress parameter ($\kappa_3$).

7.2.1. Effect of pressure gradient at the tube entrance $\varepsilon$:
To study the effect of $\varepsilon$ on the velocity distribution ($\upsilon$) we keep $(De, \alpha_3, \epsilon, \varepsilon, \kappa_3, K)$ fixed $(2.5, 0.1, 0.1, 0.13, 1.5)$ respectively and the value of $\epsilon$ is $(8, 8.5, 9)$ respectively. As the previous result makes $\varepsilon$ increase, the velocity range decrease as well. And the velocity is still fixed in all state after $z>1$.

![Figure (5)](image)

**Figure (5)** velocity for different values $\psi$, when $De = 2.5, \kappa_3 = 0.1, \psi = 0.13 \varepsilon = 8, \varepsilon = 8.5, \varepsilon = 9$

7.2.2. Effect of Deborah number $De$:
To study the effect of $(De)$ on the velocity distribution $(\upsilon)$ we keep $(\varepsilon, \alpha_3, \epsilon, \psi, K)$ fixed $(9, 0.1, 0.1, 0.13, 1.5)$ respectively and the value of $De$ is $(2.5, 3.5, 4.5)$ respectively the following result is made
as $De$ when $(z \leq -1.5)$ in the range of the velocity increasing with the increasing of $De$ but after $(z \geq 1.5)$ the velocity decreasing with the increasing of $De$ and became fixed in $(z \geq 1)$.

\[ \text{Figure (6)} \text{ velocity for different values } De, \text{ when } \kappa_3 = 0.1, \emptyset = 0.13, \varepsilon = 9 \]

\[ \text{De}=2.5, \text{De}=3.5, \text{De}=4.5 \]

7.2.3. Effect of couple stress parameter ($\kappa_3$):

To study the effect of ($\kappa_3$) on the velocity distribution ($u$) we keep ($De, \emptyset, \varepsilon$) fixed (2.5, 0.13, 9) and the value of $\kappa_3$ is (0.1, 0.2, 0.3) respectively the increasing in $\kappa_3$ made decreasing in velocity when ($r \leq -0.3$) and when ($r \geq -0.3$) the velocity increasing and still fixed when ($r \geq 0.1$).

\[ \text{Figure (7)} \text{ velocity for different values } (\kappa_3), \text{ when } \varepsilon = 9, \kappa_3 \]

\[ =2.5, \kappa_3=3.5, \kappa_3=4.5 \]

7.2.4. Effect of amplitude ratio ($\emptyset$):

To study the effect of ($\emptyset$) in the velocity distribution ($u$) we keep ($De, \kappa_3, \varepsilon$) fixed (2.5, 0.1, 9) and the value of $\emptyset$ is (0.13, 0.15, 0.17) respectively the increasing in $\emptyset$ there is decreasing when ($r \leq -0.3$) but when ($r \geq -0.3$) the velocity became decreasing and still fixed when ($r \geq 0$) equal to zero.
7.3. **Stream function:**

We study the effect of the pressure gradient at the tube entrance ($\varepsilon$), amplitucle ratio ($\phi$) and Deborah number ($De$), couple stress parameter ($\kappa_3$) to the stream function.

7.3.1. **Effect of Deborah number ($De$):**

We have studied the effect of ($De$) in stream function we keep ($\kappa_3, \phi, \varepsilon$) fixed as (0.001, 0.3, 9) respectively and the value of ($De$) as (0.020, 0.024, 0.026) respectively the increasing in the value of ($De$) made increasing in stream function

\[ 0.3, 0.2, 0.1, 0.0, 0.1, 0.2, 0.3 \]

\[ 1, 10, 13 \]

\[ 5, 10, 12 \]

\[ 0, 5, 10, 12 \]

\[ 1, 10, 13 \]

\[ 0.020, De = 0.024, De = 0.026 \]

7.3.2. **Effect of couple stress parameter ($\kappa_3$):**

We have studied the effect of ($\kappa_3$) in stream function we keep ($De, \phi, \varepsilon$) fixed as (0.20, 0.3, 9) and the value of $\kappa_3$ (0.001, 0.002, 0.003) respectively and the increasing in $\kappa_3$ made the stream function increasing.

**Figure (8)** velocity for different values $\phi$, when $De = 2.5$, $\kappa_3 = 0.1$, $\varepsilon = 9$ $\phi = 0.13$, $\phi = 0.15$, $\phi = 0.17$

**Figure (9)** stream function for different values $De$, when $\phi = 0.3$, $\kappa_3 = .001$, $\varepsilon = 9$ $De = 0.020, De = 0.024, De = 0.026$
7.3.3. Effect of amplitude ratio ($\Phi$):
We study the effect of $\Phi$ in stream function keep $(De, \kappa_3, \varepsilon)$ fixed as $(0.20, 0.001, 9)$ respectively and the value of $\Phi$ $(0.3, 0.4, 0.5)$ respectively the increasing of $\Phi$ made the stream function increasing.

**Figure (11)** stream function for different values $\Phi$, when $\Phi = 0.3, De = 0.020, \varepsilon = 9, \kappa_3 = 0.001, \kappa_3 = 0.002, \kappa_3 = 0.003$

7.3.4. Effect of pressure gradient at the tube entrance ($\varepsilon$):
We study the effect of $\varepsilon$ in stream function keep $(De, \kappa_3, \Phi)$ fixed $(0.20, 0.001, 0.3)$ respectively and the value of $\varepsilon$ is $(8, 8.5, 9)$ respectively the increasing in $\varepsilon$ made the stream function increasing.

**Figure (12)** stream function for different values $\varepsilon$, when $\kappa_3 = 0.001, De = 0.20, \Phi = 0.3, \varepsilon = 8, \varepsilon = 8.5, \varepsilon = 9$
7.4. Pressure distribution:
In this section we have studied the effect the parameters ($\epsilon, De, \kappa_3$) in pressure distribution.

7.4.1. Effect of Deborah number ($De$):
To study the effect ($De$) we keep ($\kappa_3, \varnothing, \epsilon$) fixed as $(0.01, 0.3, 9)$ respectively and the value of $De$ $(0.020, 0.024, 0.026)$ the increasing in $De$ made the increasing of pressure.

Figure (13) pressure gradient for different values of $De$, when, $\kappa_3 = 0.01, \varnothing = 0.3, \epsilon = 9, De = 0.020, De = 0.024, De = 0.026$

7.4.2. Effect of couple stress parameter ($\kappa_3$):
To study the effect of ($\kappa_3$) we keep ($De, \varnothing, \epsilon$) fixed as $(0.20, 0.3, 9)$ respectively and value of ($\kappa_3$) $(0.01, 0.02, 0.03)$ respectively the increasing in $\kappa_3$ made the pressure increasing.

Figure (14) pressure gradient for different values of $\kappa_3$, when, $De = 0.20, \varnothing = 0.3, \epsilon = 9, \kappa_3 = 0.01, \kappa_3 = 0.02, \kappa_3 = 0.03$

7.4.3. Effect of amplitude ratio ($\varnothing$):
To study the effect of $\varnothing$ we keep ($De, \kappa_3, \epsilon$) fixed as $(0.020, 0.01, 9)$ respectively and the value of $\varnothing$ $(0.3, 0.35, 0.4)$ respectively the increasing of $\varnothing$ made the pressure increasing.

Figure (15) pressure gradient for different values of $\varnothing$, when $De = 0.020, \kappa_3 = 0.01, \epsilon = 9, \varnothing = 0.3, \varnothing = 0.35, \varnothing = 0.4$
7.4.4. Effect of pressure gradient at the tube entrance ($\epsilon$):

To study the effect of $\epsilon$ we keep the ($De, \kappa$, $\theta$) fixed as (0.020, 0.01, 0.3) respectively and the value of $\epsilon$ (8, 8.5, 9) respectively the increasing of $\epsilon$ made the pressure increasing.

Figure (16) pressure gradient for different values $\epsilon$, when $De = 0.020$, $\kappa = 0.01$, $\theta = 0.3$, $\epsilon = 8$, $\epsilon = 8.5$, $\epsilon = 9$

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