THE NUCLEON SPIN POLARIZABILITY
AT ORDER $O(p^4)$ IN CHIRAL PERTURBATION THEORY

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Abstract

We calculate the forward spin-dependent photon-nucleon Compton amplitude as a function of photon energy at the next-to-leading ($O(p^4)$) order in chiral perturbation theory, from which we extract the contribution to nucleon spin polarizability. The result shows a large correction to the leading order contribution.
In recent years, the static properties of the nucleon have been under intense theoretical and experimental investigations. Examples include the various elastic form factors measurable in low-energy electroweak processes and parton distributions accessible through hard scattering. The nucleon polarizabilities comprise yet another type of observable which characterizes the response of the nucleon when exposed to external electromagnetic fields. They can be measured through low-energy Compton scattering, which has become feasible recently thanks to novel advances in experimental technology. For a summary of theoretical and experimental progress in studying the polarizabilities, the reader can consult Ref. [1].

In this letter, we study the spin polarizability of the nucleon, which we define here via a spin-dependent forward Compton amplitude. The photon-nucleon forward Compton amplitudes are related to

\[ T^{\mu\nu} = i \int d^4\xi e^{i\eta\xi} \langle PS|TJ^\mu(\xi)J^\nu(0)|PS\rangle. \]

where \( |PS\rangle \) is the covariantly-normalized ground state of a nucleon of momentum \( P^\mu \) and spin polarization \( S^\mu \). \( J^\mu = \sum_i e_i \bar{\psi}_i \gamma^\mu \psi_i \) is the electromagnetic current (with \( \psi_i \) the quark field of flavor \( i \) and \( e_i \) its charge in units of the proton charge). The four-vector \( q^\mu \) is the photon four-momentum. Using Lorentz symmetry, parity and time-reversal invariance, one can express the spin-dependent (\( \mu\nu \) antisymmetric) part of \( T^{\mu\nu} \) in terms of two scalar functions:

\[ T^{[\mu\nu]}(P, q, S) = -i\epsilon^{\mu\nu\alpha\beta} q_\alpha \left[ S_\beta S_1(\nu) + (M \nu S_\beta - S \cdot q P_\beta) S_2(\nu) \right], \]

where \( \nu \) is the energy of the real photon and \( M \) is the nucleon mass (\( \epsilon^{0123} = +1 \)). \( S_{1,2}(\nu) \) are the spin-dependent, invariant Compton amplitudes. Of course, in real photon scattering \( S_2(\nu) \) decouples and one can measure the \( S_1(\nu) \) amplitude only. The relation between \( S_1 \) and the traditional amplitude \( f_2(\nu) \) (see Ref. [2]) is

\[ f_2(\nu) = \frac{\alpha_{em}}{2} S_1(\nu). \]

Through crossing symmetry, it is easy to see \( S_1(\nu) \) is even in \( \nu \). The spin-dependent polarizability \( \gamma \) is defined as

\[ \gamma = \frac{df_2(\nu)}{d\nu^2} \bigg|_{\nu=0}, \]

which is just the slope of \( f_2(\nu) \) at \( \nu^2 = 0 \).

Since the spin polarizability is a low-energy observable, it is natural to explore its physical content in chiral perturbation theory (\( \chiPT \)), or more broadly in low-energy effective theories. In \( \chiPT \), one considers the pion mass \( m_\pi \) and the external three-momentum \( \vec{p} \) small compared to any other scales in the problem. In low-energy effective field theories one considers expansions also in terms of other small parameters, such as the mass difference \( \Delta \) between the nucleon and delta resonance. Here the expansion parameter is generically denoted as \( \epsilon \). Bernard, Kaiser, Kambor, and Meissner have studied the spin polarizability in \( \chiPT \) [3]. They found that at the leading-order (\( O(p^3) \))
\begin{equation}
\gamma_{p,n}(p^3) = \frac{\alpha_{\text{em}} g_A^2}{24\pi^2 f^2 \pi^2 m^2} = 4.4 \cdot 10^{-4} \text{ fm}^4 ,
\end{equation}

which diverges as $1/m^2$ in the chiral limit. The above result was obtained formally in the limit of the nucleon mass going to infinity (heavy-baryon chiral perturbation theory) \[4\], a trick to make power counting manifest in Feynman diagrams.

Being a spin-dependent quantity, the spin-polarizability undoubtedly receives a large contribution from the delta resonance. In pure $\chi$PT, the delta resonance contributes through counter terms and the effect appears at $\mathcal{O}(p^5)$ (see the discussion below). However, the nucleon and delta mass difference is of order $m_\pi$ and vanishes in the large-$N_c$ (number of colors) limit. Therefore it is reasonable to include the delta resonance explicitly and consider an expansion with $\Delta$ considered a small parameter. Using $\epsilon$ to denote $m_\pi$, $\Delta$, and any small momentum, one finds an $\mathcal{O}(\epsilon^3)$ contribution from the tree diagram with an intermediate delta,

\begin{equation}
\gamma_{p,n}(\epsilon^3)_{\Delta-\text{tree}} = -\frac{\alpha_{\text{em}} G_1^2}{9 M^2 \Delta^2} = -2.4 \cdot 10^{-4} \text{ fm}^4,
\end{equation}

where $G_1$ is a spin-flip $N$-$\Delta$-$\gamma$ coupling and its numerical estimate of 3.85 is taken from Ref. \[5\] (in the large $N_c$ limit, it takes the value of $3/(2\sqrt{2})\kappa_V$). Not surprisingly, the above contribution is large and negative and almost entirely cancels the one-loop chiral contribution. At the same order, the delta resonance also contributes through one-loop intermediate states:

\begin{equation}
\gamma_{p,n}(\epsilon^3)_{\Delta-\text{loop}} = \frac{\alpha_{\text{em}} g_A^2}{3\pi(4\pi f_\pi)^2} \left\{ \frac{2}{m^2_\pi} - \frac{\Delta^2 + 2m^2_\pi}{(\Delta^2 - m^2_\pi)^2} + \frac{3\Delta m^2_\pi}{(\Delta^2 - m^2_\pi)^{3/2}} \ln \left[ \frac{\Delta}{m_\pi} + \sqrt{\frac{\Delta^2}{m^2_\pi} - 1} \right] \right\}.
\end{equation}

With realistic parameters, the above contribution is much smaller than the other two contributions. The total $\mathcal{O}(\epsilon^3)$ result is different from that in Ref. \[3\] because of the nonrelativistic expansion and the newly-fitted $G_1$ parameter.

In this letter, we report a calculation at next-to-leading order ($\mathcal{O}(p^4)$) in chiral perturbation theory. The result has a comparable size to that at $\mathcal{O}(p^3)$ and the sign is opposite. We have not considered the tree-level delta contribution at $\mathcal{O}(\epsilon^4)$, as it involves a number of unknown parameters on which one has no firm theoretical handle. The delta-loop contribution can be evaluated at this order, but we expect its contribution to be small.

Before we present the details and the result of our calculation, it is useful to recall some of the standard infrared power counting techniques in effective field theories as formulated in the heavy-baryon approach. The full lagrangian (including nucleon, delta, photon, and pion fields) can be expanded:

\[\mathcal{L} = \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \mathcal{L}^{(3)} + \mathcal{L}^{(4)} + \ldots\]

where $\mathcal{L}^{(n)}$ contains terms of order $\epsilon^n$, with one power of $\epsilon$ assigned to each derivative, pion mass, photon field, nucleon-delta mass difference, etc. The infrared power of a Feynman
diagram is generated from the vertices and the propagators. For polarized Compton scattering, Feynman diagrams start at $O(\epsilon^3)$. For tree diagrams at this order, one needs to consider vertices at $O(\epsilon^n)$ with $n \leq 3$. For one-loop diagrams, we need to consider vertices at $O(\epsilon)$ only. In general, at order $O(\epsilon^n)$, one needs to consider vertices at order $O(\epsilon^{n-2\ell})$ for diagrams of $\ell$-loops.

In the definition of a physical observable, there is a certain number of infrared factors involved. For instance, the spin polarizability is defined from a Compton amplitude which contains two factors of photon fields, one factor of photon momentum $q$, and two powers of photon energy $\nu$. Therefore, when Feynman diagrams are calculated at $O(\epsilon^n)$, the contribution to the spin polarizability has infrared power $\epsilon^{n-5}$. Since the spin-dependent polarizability starts at $O(\epsilon^3)$, we expect $\gamma$ to start at order $\epsilon^{-2}$. In other words, we have the following expansion in infrared parameters,

$$\gamma = c_{-2}\epsilon^{-2} + c_{-1}\epsilon^{-1} + c_0 + c_1\epsilon + \ldots . \quad (9)$$

The calculation presented in Ref. [3] corresponds to $c_{-2}$. In this paper, we are interested in $c_{-1}$. Since the delta contribution at this order involves new unknown parameters, we mainly concentrate on how to get the nucleon contribution to $c_{-1}$ in chiral perturbation theory.

We immediately rule out the tree nucleon contribution to $c_{-1}$ because the tree cannot possibly contain the infrared parameter $m_\pi$ in the denominator. For this reason, the tree nucleon diagram starts to contribute to the spin polarizability only at $O(p^5)$. Therefore we need to consider only the one-loop nucleon diagram at $O(p^4)$. As we have discussed above, this involves vertices at $O(p)$ and $O(p^2)$. The Feynman rules in a general electromagnetic gauge can be easily derived from the lagrangian, for instance, in Ref. [3].

$S_1(\nu)$ is calculated as a function of the photon energy $\nu$. At $O(p^4)$, there are 20 nonzero Feynman diagrams and their close relatives. These diagrams are shown in Fig. 1, where the cross in each diagram represents an insertion from $L^{(2)}$. We find

$$S_1^{O(p^4)}(\nu) = \frac{g_\gamma^2 m_\pi^4}{192\pi^2 f_\pi^2 M \nu^2 (m_\pi^2 - \nu^2)} \left[ \pi \left( 1 + \frac{9}{2} \kappa_S + \frac{25}{2} \kappa_V - \frac{1}{2} \kappa_S - \frac{9}{2} \kappa_V \right) \tau^3 \right]$$

$$\times \left[ 1 - \left( \frac{\nu}{m_\pi} \right)^2 \right] - 8 \left( -2 + 2\kappa_V - 2(2 + \kappa_S)\tau^3 + \left( \frac{\nu}{m_\pi} \right)^2 (1 - 4\kappa_V + (2 + \kappa_S)\tau^3) \right)$$

$$+ \left( \frac{\nu}{m_\pi} \right)^4 \left( (4 + 2\kappa_V) + (2 + \kappa_S)\tau^3 \right) \arccos \left( -\frac{\nu}{m_\pi} \right)$$

$$+ 12\tau^3 \kappa_S \nu m_\pi \left[ 1 - \left( \frac{\nu}{m_\pi} \right)^2 \right] \arccos^2 \left( -\frac{\nu}{m_\pi} \right] + (\nu \rightarrow -\nu) , \quad (10)$$

where $\kappa_S = -0.120$ and $\kappa_V = 3.706$ are the experimental values of the isoscalar and isovector anomalous magnetic momentum of the nucleon, respectively. $S_1(\nu)$ has a pole at $\nu = 0$ which comes from the nucleon elastic contribution. According to the discussion in Ref. [7], this pole can be subtracted away, with the remainder representing the inelastic contribution (denoted with an overline). A plot of $\overline{S}_1^{O(p^4)}(\nu)$ is shown in Fig. 2.

As a check of our result, we set $\nu = 0$ :

$$\overline{S}_1^{O(p^4)}(0) = \frac{g_\gamma^2 m_\pi}{8\pi f_\pi^2 M} \left( \kappa_S \tau^3 + \kappa_V \right) . \quad (11)$$
FIG. 1. The diagrams that contribute to $S_{1,2}(\nu, Q^2)$ at NLO in heavy baryon $\chi$PT. Obviously, the diagrams from hermiticity and crossing must also be included. The cross indicates an insertion from $\mathcal{L}^{(2)}$.

FIG. 2. $S_1^{O(p^4)}(\nu)$ as a function of $\nu$ below the pion threshold. This contribution dominates the leading order for both proton (solid) and neutron (dashed) targets. We see the beginning of the characteristic cusp at threshold.
This is exactly the result required by the low-energy theorem, which states that to all orders in perturbation theory

$$S_1(0) = -\frac{\kappa^2}{M^2}.$$  \hspace{1cm} (12)

Using the known result that $$\kappa_V = \kappa_V^0 - \frac{g_A^2 m_\pi M}{4\pi f_\pi^2} + \mathcal{O}(p^2),$$ where $$\kappa_V^0$$ is the isovector anomalous magnetic moment in the chiral limit, we see that $$S_1$$ must be Eq. (11) at order $$\mathcal{O}(p^4)$$. Expanding Eq. (10) as a series in $$\nu^2$$, we get the contribution to $$\gamma$$

$$\gamma^{\mathcal{O}(p^4)} = -\frac{\alpha_{em} g_A^2 [(15 + 3\kappa_V) + (6 + \kappa_S)\tau^3]}{192\pi f_\pi^2 m_\pi M}. \hspace{1cm} (13)$$

Numerical results can be obtained by substituting in the physical values of the parameters :

$$\gamma_p^{\mathcal{O}(p^4)} = -8.2 \cdot 10^{-4} \text{ fm}^4$$

$$\gamma_n^{\mathcal{O}(p^4)} = -5.2 \cdot 10^{-4} \text{ fm}^4 \hspace{1cm} (14)$$

One notices immediately that these numbers are somewhat larger than those from order $$\mathcal{O}(p^3)$$. This certainly calls the convergence of the chiral expansion into question. On the other hand, one may argue that the chiral terms at even and odd orders are fundamentally different as exemplified by the difference in the overall $$\pi$$ factors; it is possible that the next terms are much smaller than the terms considered here.

Finally, we come to the $$\Delta$$ contribution at $$\mathcal{O}(\epsilon^4)$$. In contrast to the nucleon intermediate states, there is a contribution at tree order because we treat the mass difference $$\Delta$$ as small. Indeed, the mass difference $$\Delta$$ appears in the denominator of the delta propagator. The tree contributions come with one insertion of the vertices from $$\mathcal{L}^{(2)}$$ and $$\mathcal{L}^{(3)}$$. Since $$\mathcal{L}^{(3)}$$ contains a number of unknown parameters, we will not consider its contribution here.

One can also consider the one-loop contribution with intermediate delta states. In this case, only one insertion from $$\mathcal{L}^{(2)}$$ is needed. However, considering the fact that the delta-loop contribution is small at $$\mathcal{O}(p^3)$$, we suspect that the contribution at this level is also small. Therefore, it has been neglected.

To summarize, we have calculated the spin polarizability of the nucleon at $$\mathcal{O}(p^4)$$. The result indicates a large correction to the $$\mathcal{O}(p^3)$$ value.

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