Effects related to spacetime foam in astrophysics

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ABSTRACT
During the quantum stage, spacetime had the foam-like structure. When the Universe cools down, the foam structure tempers and does not disappear, therefore the Friedman model represents only an idealization. We show that the large scale observational effects of the foamed structure appear as the Dark Matter and Dark Energy phenomena. We also examine the scattering of free particles on the foam-like structure, and show that this scattering explains the diffuse component of the X-ray background, and provides a simple picture for the origin of Gamma-ray bursts.

Key words: galaxies: kinematics and dynamics – galaxies: spiral – dark matter.

1 INTRODUCTION
Modern astrophysics faces a rather difficult period. The perpetual improvement of the observation technic provides more and more inconsistencies with the standard Λ Cold Dark Matter (ΛCDM) models. One of the first indications was given ten years ago by Persic, Salucci & Stel 1996, when the empirical Universal rotation curve (URC) for spirals was constructed and the existence of a smooth core in the distribution of Dark Matter (DM) in galaxies was shown. Now it is quite well established that CDM cannot explain the observed cored distribution of DM in galaxies (e.g., see Gentile et al., 2004 and references therein; Weldrake, de Blok & Walter 2003; de Blok & Bosma 2002; for spirals and Gerhard et al., 2001; Borriello, Salucci & Danese 2003 for ellipticals). At very large scales the observed Universe expansion seems to be driven by a very strange state of matter — this is the so-called Dark Energy phenomenon which still has not found an adequate description. It looks like we are trying to describe the essentially non-linear phenomena of DM and DE by simplistic linear models (a cold non-baryonic dust and the Lambda term).

In this letter we show that all the variety of the DM phenomena (including the DE effect) can be straightforwardly explained by the presence of a certain topological bias (e.g., see Kirillov 2006 and references therein). We describe the theoretical origin of such bias and study the simplest effects. In particular, we demonstrate that a number of long-standing astrophysical puzzles acquires a quite simple interpretation.

The bias results from quantum gravity effects. In the past, the Universe went through a quantum period when topology underwent fluctuations and spacetime had a foamed structure (Wheeler 1964). During the expansion, the Universe cools down, quantum gravity processes stop, and the topological structure of space freezes. One can easily imagine a picture in which space consists of a huge set of wormholes glued randomly together, while the isotropic and homogeneous Friedman space appears only as the result of an averaging of this chaotic structure. In other words, once spacetime foam is taken into account it becomes unnatural to believe that our Universe has a simple topological structure on scales which require an enormous extrapolation.

The immediate effect of the large-scale foamed structure is that free particles undergo a certain elastic scattering. Indeed, if we consider a discrete source for radiation or gravity, some portion of photons (or gravitons) will be scattered in space by the randomly placed wormholes. In other words, along with every single image in the sky we always have to observe a specific halo.

Moreover, when the actual volume of the Universe disagree with the volume of the Friedman space, we shall always either overestimate (when using the Gauss divergence theorem), or underestimate the actual intensity of a source. To clarify the first case, consider a toy example where the space is a cylinder of a radius R. The metric is the same as for the standard flat Friedman model

\[ ds^2 = dt^2 - a^2(t) (dx^2 + dy^2 + dz^2) \]  

but one of coordinates, say z, can be periodic (z + R = z). Then on scales r ≪ R we observe the standard Newton’s potential for a point mass \( \phi \sim M/r \), while for larger scales r ≫ R the compactification of one dimension will result in the crossover of the potential to \( \phi \sim M/R \ln r \). Thus, if we mistakenly think about this space that it is the ordinary

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There are no doubts though that the concept of the flat manifold works well at laboratory scales, up to the size of, say, the Solar system.
we will see that the intensity of the source increases as \( M(r) \sim M^2 \). So we will conclude that the source is surrounded with a halo of Dark Matter. Vice versa, if we think that we live on the cylinder and expect the potential \( \phi \sim \frac{M}{r} \) in \( r \), while the actual space is \( R^3 \), we will observe the decreasing dynamic mass \( M(r) \sim M^2 \), i.e., we will conclude the presence of a strange form of matter (or better to say anti-matter). In the last case the halo carries the negative density which represents the so-called Dark Energy phenomenon.

If we assume the thermal equilibrium state during the quantum period of the evolution of the Universe, we should expect that the actual space is homogeneously filled with matter. Then the direct count of the number of baryons on large scales reflects the behavior of the actual physical volume. Observations suggest that at least up to \( 200 \text{Mpc} \) the number of baryons within the radius \( r \) behaves as \( N(r) \sim r^D \) with \( D \approx 2 \) (Pietronero 1987; Ruffini, Song & Taraglio 1988, Labini, Montuori & Pietronero 1998). This indicates that the actual volume also behaves as \( V_{phys}(r) \sim r^D \) and therefore the apparent luminosity for a discrete source should decay with distances as \( \ell(r) \sim L/r^{D-1} \). In the Friedman model we expect \( r^D\ell(r) = L = \text{const} \), while the actual value increases as \( r^D\ell(r) = Lr^D - Lr \). Moreover, most of the luminosity has to come as a diffuse radiation (scattered on the nontrivial topology, i.e., \( \ell(r) = \ell_{vis} + \ell_{diff} \)), where \( \ell_{vis} = L/r^2 \) comes from discrete sources. And this is exactly what we actually observe: we see spherical halos of Dark Matter around galaxies and, along with discrete X-ray sources, we see diffuse X-ray background (e.g., see the book of Sarazin 1988). In this picture the ratio of the two components (the diffuse background and discrete sources) should repeat the ratio of DM and baryons, which is also in agreement with observations.

Another important evidence for the random foamed structure of space can be the existence of Gamma-ray bursts (GRB) - brief flashes of \( \gamma \)-ray light which frequently are accompanied with an afterglow (e.g., see, van Paradijs, et al., 2000 and references therein). Such flashes come from supernovae explosions and represent an essentially non-stationary process. Mechanisms suggested are controversial. In our picture of the foamed space, the afterglow represents the basic signal, while flashes appear due to the stochastic (spontaneous) interference on the foamed structure. We note that if such interpretation is correct then analogous flashes (from non-stationary sources) should exist for all frequencies, however our Galaxy seems to hide such effects.

### 2 ORIGIN OF THE TOPOLOGICAL BIAS

Consider any physical Green function for particles \( G(X_1, X_2) \), where \( X = (t, x) \). Inhomogeneity of the topological structure of space means that this function cannot be reduced to the form \( G(X_1 - X_2) \). Moreover, unlike the Friedman space, the complicated foam-like manifold cannot be covered by a single coordinate map, and therefore coordinates \( X \) require some specification.

First, we note that the Universe looks to be homogeneous and isotropic. Therefore, we can use coordinates of the Friedman space which represent an extrapolation of our Solar coordinate system. In terms of such coordinates we can speak about the probability of different realizations of the actual topological structure of space. Indeed, starting many times form the same quantum thermal equilibrium state we, at the end of quantum era (when the topology has been tempered), will obtain different realizations of the space structure. The probability of different realizations is defined by the density matrix which was evaluated in (Kirillov & Turaev, 2002, Kirillov 2003). The homogeneity and isotropy mean that upon an averaging over different realizations of topology, the Green functions acquire the homogeneous structure

\[
G(t_1, t_2, |x_1 - x_2|) = \langle G(X_1, X_2) \rangle.
\]

The Green functions for the Friedman Universe can be easily evaluated \( G_0(X_1, X_2) = G_0(t_1, t_2, |x_1 - x_2|) \) while the true Green functions can be directly measured from observations (e.g., the fluctuation-dissipation theorem relates Green functions with field fluctuations in a medium). Thus, if the topological structure of the actual space differs from that of the Friedman space, there appears an observable discrepancy which can be interpreted as a bias of sources \( \delta(X_1 - X_2) \rightarrow K(X_1, X_2) \). Indeed, let us relate the true Green function \( G \) with the function \( G_0 \) by

\[
G(X_1, X_2) = \int G_0(X_1, Y) K(Y, X_2) dY,
\]

where \( dX = \sqrt{-\delta} d^3x \). Then, if we assume that the Friedman model is correct and the true Green function is \( G_0(X_1, X_2) \), then we shall see that every point source becomes distributed in space, i.e. \( \delta(X_1 - X_2) \rightarrow K(X_1, X_2) \).

In general the bias is a random function which includes effects of the scattering on topology and depends on the specific realization of the topological structure. The averaging over different realizations gives the isotropic and homogeneous form of the bias \( \langle K(X_1, X_2) \rangle \rightarrow \langle K(X_1, X_2) \rangle \). Thus, any physical source \( J(X) \) acquires a specific bias (an additional distribution in space)

\[
J(X) \rightarrow \tilde{J}(X) = \int K(X, Y) J(Y) dY.
\]

For a constant source the bias can be taken in the form

\[
\delta(r - r') \rightarrow \delta(r - r') + b(\vec{r}, \vec{r}'),
\]

where \( b(r_1, r_2) \) describes effects of the scattering on the nontrivial topological structure and represents an effective random halo around any source (e.g., Dark Matter or the effective distributed source for the diffuse radiation). For galaxies the bias, which fits observations quite well, takes the form (Kirillov & Turaev 2006)

\[
\tilde{E}(\vec{r}, \vec{r}') = \frac{\mu}{2\pi^2 |\vec{r} - \vec{r}'|} (1 - \cos(\mu |\vec{r} - \vec{r}'|)),
\]

where \( \mu = \pi/(2R_0) \) defines the scale at which DM starts to show up.

\footnote{In a nonstationary Universe the dependence on time cannot be reduced to a single argument \( t_1 - t_2 \).}

\footnote{For the sake of simplicity we neglect here the fact that the Universe expands and therefore the bias includes a dependence on time \( b(t, r, r') \).}
From (6) one can find that the total mass within the radius \( r \) increases as \( M (r) \sim M (1 + r/R_0) \). This follows immediately from the Gauss divergence theorem for the standard Newton potential with a source of the form (5). Analogous conclusion can be made for the luminosity of a point source (i.e., for a galaxy or an X-ray source). Basically, this represents the main reason of why we overestimate the number of sources for radiation (the number density of baryons) and accept the picture of the homogeneous distribution of sources (i.e., of the Friedman Universe). If we accept the foamed picture of space, then at the first sight this will look as a violation of the conservation law for the mass or the energy. However, as it was already pointed out in the previous section, in general the Friedman coordinates cannot match properly the actual space. This fact can be expressed by a discrepancy between the value of the physical volume \( V_{\text{phys}} (x, r) \) within the radius \( r \) around a point \( x \) and that of the Friedman (coordinate) space \( V_F (r) \sim r^3 \). The actual volume represents a random function of \( x \) and \( r \), which depends on the foamed structure of the actual space. Upon averaging over different realization it becomes function of \( r \) only, i.e., \( V_{\text{phys}} (x, r) \equiv V_{\text{phys}} (r) \). In general there can be both situations: when in some range of distances \( V_{\text{phys}} (r) \sim r^D \) with \( D < 3 \), (where \( D \) is the effective dimension, e.g., see Kirillov 2003 ) and that with \( D > 3 \). In the first case we should always observe an excess of sources. And this is exactly what we do starting from the galaxy scales - the so-called Dark Matter phenomenon. Hence, we should accept the fractal distribution of baryons with \( D \sim 2 \) from scales, say, a few \( 100 \) pc to about \( 200 \) Mpc. In particular, the dimension \( D \approx 2 \) explains geometrically the origin of observed flat rotation curves in galaxies which follows from the bias (6). Moreover, the empirical Tully-Fisher law (Tully & Fisher 1977) \( L \sim V^3 \) with \( \beta \approx 4 \) (where \( L \) and \( V \) are the luminosity and rotation velocity of spirals) gives indirect evidence for the fractal picture with \( D \approx 2 \), for as it was shown recently by (Kirillov & Turaev 2006) \( \beta = 2D/(D - 1) \). In such a geometry a point source gives an excess of the energy flux \( \ell (r) \sim L/4\pi G r^2 \) \( \sim r^{D-1} \) (i.e., \( L = L (r) \sim r^{3-D} \)) and, therefore, the fractal distribution of sources produces a homogeneous and isotropic background (as if sources were homogeneously distributed in the Friedman space) exactly like the fractal distribution of baryons forms the homogeneous background of the Dark Matter (Kirillov 2006).

Observations, however, suggest that the Universe accelerates (e.g., see, Riess, et al., 1998, Netterfield, et al., 2002). This should mean that starting from some scale \( R_\ast \) (larger than \( 200 \) Mpc\(^4\)) the dimension of the Universe becomes \( D > 3 \). There exist plausible theoretical reasoning of such a behavior. Indeed, on the quantum stage of the evolution of the Universe the horizon size represents an operator value \( \ell_{\text{hor}} \) which, for the thermal equilibrium state, can be characterized by a mean value \( \langle \ell_{\text{hor}} \rangle \) and a dispersion \( \sigma_{\text{hor}} \). When the topology tempers, the horizon also remains a random Gaussian function. Physically the presence of the dispersion means that there exist rather short wormholes, say, of a size \( \lambda \), which glue together sufficiently remote regions of space (e.g., at distances \( R \gg \lambda \)). Then, when we consider a ball of a radius \( r > \lambda \) (but \( r < R_\ast \)) and such a wormhole gets into the ball, the ball will simultaneously cover two regions of space separated by the distance \( R \) and, therefore, the physical volume \( V_{\text{phys}} (r) \) increases with \( r \) two times faster.

Clearly, in a foamed space such wormholes have random characteristics and they can teleport signals from extremely remote regions of space. This represents the main mechanism of why the physical volume can increase faster than the volume of the Friedman space (i.e., the dimension becomes \( D > 3 \)).

When the dimension of the actual space becomes \( D > 3 \), we naturally start to observe a lack of matter - the Dark Energy phenomenon. Every point source is surrounded with a halo which carries a negative density \( (L(r) \approx r^{3-D} \to 0 \) and \( \ell_{\text{hor}} (r) \approx -\ell_{\text{vis}} (r) < 0 \), i.e., for \( r > R_\ast \) the bias (6) should become negative \( \ell (r) < 0 \). If in some region of space the negative density \( \rho_{\text{DE}} < 0 \) dominates and we put it into the Einstein equation we immediately find that the region of the effective Friedman space accelerates

\[
\frac{d^2a}{dt^2} = -\frac{4\pi G}{3} \langle \rho_{\text{DE}} \rangle a > 0. \tag{7}
\]

Exactly like the Dark Matter phenomenon such an acceleration, however, has a fictitious character. The physics here is very simple - at large distances \( \gg R_\ast \), the larger number of teleporting wormholes appear and we observe (through such wormholes or windows) the larger number of sources which move from us much faster, than it seems to be predicted by the standard Hubble law.

Thus, we see that both phenomena - Dark Matter and Dark Energy - have common origin and carry a fictitious character, for they indicate only the discrepancy between the topological structure of the actual foamed Universe and that of the fictitious (or effective) Friedman space. This explains the fact of why so far we were unable to detect exotic particles in laboratory physics (on accelerators).

### 3 ORIGIN OF A VARIABLE NUMBER OF FIELDS

From a phenomenological background it was demonstrated previously (e.g., Kirillov 1999, 2003, Kirillov & Turaev 2005) that the non-trivial foam-like topological structure of space can be effectively described in terms of a variable number of fields or fields obeying the generalizes statistics (Kirillov 2005). In the present section we infer this fact rigorously and thereby set a new theoretical basis to our previous results.

For the sake of simplicity we consider scalar particles and a stationary (non-relativistic) manifold. In quantum mechanics (e.g., see Feynman & Hibbs 1965) the Green function of a particle \( g(x, x') \) can be found by the integral over histories

\[
g(x, x') = C \int_{x'}^{x} \exp \left( \frac{1}{\hbar} S[x(t)] \right) dx(t), \tag{8}
\]

where \( S[x(t)] \) is the classical action for the particle, \( C \) is the normalization constant, \( x(t) \) is a trajectory in space, and \( x, x' \) are the beginning and end points of the trajectory respectively. The standard way to evaluate such an integral is to expand the action around the classical trajectory

\[
\frac{d^2a}{dt^2} = -\frac{4\pi G}{3} \langle \rho_{\text{DE}} \rangle a > 0. \tag{7}
\]
\[ x(t) = x_0 + \delta x(t), \quad S = S_{cl} + \delta^2 S + \ldots \] (it is assumed that on the classical trajectory the action reaches the minimal value) and retain the quadratic term only which gives

\[ G(x, x') \simeq C \exp \left( \frac{i}{\hbar} S_{cl} [x, x'] \right) \int \exp \left( \frac{i}{\hbar} \delta^2 S \right) D\delta x(t), \]  

where

\[ \delta^2 S = \int \int \frac{\delta^2 S}{\delta x(t) \delta x(t')} \delta x(t) \delta x(t') \, dt \, dt' \]  

and the integral acquires the Gauss form and can be explicitly evaluated.

When the topology of space is simple (e.g., in the Friedmann space) there exists a unique classical trajectory for an arbitrary choice of the end point \( x \) and \( x' \) and all other trajectories can be continuously transformed into the classical one. In other words, in Friedmann models any field (or a wave function for a free particle) has a unique quasi-classical limit. In a foamed manifold this is not correct anymore. Now any two points can be connected with a set of extremals \( x^A(t) (A = 1, 2, \ldots) \) and none of such trajectories can be continuously transformed into another\(^5\). In general, one trajectory gives the absolute minimum for the action while the rest correspond to local minima. The space of trajectories is said to split into a set of non-equivalent homotopic classes which represent one of basic topological characteristics of the foamed manifold. Thus we see that in a general foamed space any field possesses a set of trajectories as the quasi-classical limit.

However, when we describe particles (or a field) in terms of a simple topology (coordinate) space, we use some specific continuation of the field (e.g., in \( x \) \( x \) and \( x' \) and all other trajectories can be continuously transformed into the classical one. In other words, in Friedmann models any field (or a wave function for a free particle) has a unique quasi-classical limit. In a foamed manifold this is not correct anymore. Now any two points can be connected with a set of extremals \( x^A(t) (A = 1, 2, \ldots) \) and none of such trajectories can be continuously transformed into another\(^5\). In general, one trajectory gives the absolute minimum for the action while the rest correspond to local minima. The space of trajectories is said to split into a set of non-equivalent homotopic classes which represent one of basic topological characteristics of the foamed manifold. Thus we see that in a general foamed space any field possesses a set of trajectories as the quasi-classical limit.

The total Green function requires an additional summation over the homotopic classes (i.e., over fields)

\[ G(x, x') = \sum_{A} G^A(x, x'). \]  

In the limit \( x \to x' \) this sum includes effectively only one term (the shortest geodesic line), since the action for the other trajectories is much bigger, the path integral rapidly oscillates and the contribution from the rest fields (i.e., from trajectories of other homotopic classes) is negligible. In a general situation, at large distances between \( x \) and \( x' \), a finite number of homotopic classes can contribute to the sum\(^6\), i.e., those which have comparable values of the action \( S_{cl}^{A} \). Thus, we can introduce a new topological characteristic - the number of fields - \( N(x, x') \) which gives us the effective number of terms in the sum\(^5\).

For a particular realization of the topological structure the number of fields \( N(x, x') \) represents a random function of \( x \) and \( x' \). There always exists a particular basis of solutions to the wave equation for free particles \( \{f_n(x)\} \) in terms of which this function acquires the diagonal form \( N(x, x') = \sum_n N_n f_n(x) f_n(x') \), where \( N_n \) has the meaning of the number of fields at the mode \( f_n(x) \). Thus, we arrive at the situation where the number of fields becomes a variable \( N_n \) (a function of the mode \( f_n(x) \)). We note that in the homogeneous and isotropic Universe (i.e., upon averaging over possible realizations of the topological structure) the number of fields depends on coordinates in the form \( N(x, x') = \langle \delta \rangle \) and therefore it diagonalizes in the Fourier representation \( \tilde{N}(k, k') = N(k) \delta(k - k') \), where \( k \) has the meaning of the standard wave number. This allows us to write the thermal statistical distribution for \( N(k) \) derived previously in Kirillov & Turaev 2002, Kirillov 2003 and to define the form of the topological bias\(^6\), for a review and a relation to the generalized statistics see also Kirillov 2005.

### 4 CONCLUSION

In conclusion, we briefly repeat basic results. First of all, the concept of spacetime foam introduced by Wheeler turns out to be crucial in explaining the whole list of long-standing astrophysical puzzles. It gives a natural explanation to the Dark Matter and Dark Energy phenomena. In particular, it is remarkable that the Universal rotation curve (URC) constructed by (Kirillov & Turaev 2006) on the basis of the topological bias shows very good fit to the empirical URC (Persic, Salucci & Stel 1996). We stress that in the actual Universe the bias \( b(r, r') \) is a random function of both arguments which reflects the discrepancy between the topological structure of the Friedman space and that of our Universe. This function requires, in the first place, an empirical definition (predictions of \( b(r, r') \) or URC are probabilistic in nature).

As it was demonstrated in this Letter, spacetime foam explains origin and the nature of the diffuse background of radiation. The mechanism has the same nature as the origin of DM halos described in Kirillov & Turaev 2002, for the topological bias actually describes the effects of scattering of signals on the foamed structure. Moreover, it predicts that the ratio of the two components (the diffuse background and discrete sources) is the same as the ratio of DM and baryons.

The foamed picture of our Universe explains naturally the problem of missing baryons. Recall that the direct count of the number of baryons gives a very small value \( \Omega_b \sim 0.003 \) for the whole nearby Universe out to the radius \( \sim 300 h^{-1}_{50} \) Mpc (e.g., see Persic & Salucci 1992). This only means that at the radius \( \sim 300 h^{-1}_{50} \) Mpc the actual volume is ten times smaller than in the Friedman space \( (V_{300 h^{-1}_{50}} \simeq 0.1 V_F) \) which reconciles the predictions of the nucleosynthesis with such a small number of baryons.

Finally, the foam-like structure of the Universe allows to explain naturally the origin of Gamma Ray Bursts, for in a foamed space any non-stationary signal is accompanied with random short flashes at random positions (the result of the stochastic interference).
We also point out that the spacetime foam gives the possibility to explain the nature of Higg's fields and origin of the rest mass spectrum (e.g., see Kirillov 2005). Thus, there is every reason to believe that the actual Universe has the foam-like structure. Once we accept the foamed picture of the Universe, the Friedman model appears only as an approximation, as a mean statistical picture which comes from the extrapolation of the observer’s reference system. The formation of properties of the modern Universe took place during the quantum stage when quantum gravity effects ruled the World. We note also that this requires an essential revision and probably reinterpretation of most of observational data and opens a new perspective for further investigations. In particular, as it was demonstrated above, the foam-like Universe may have not the event horizon - any points in space can, in principle, be connected with a relatively short wormhole, though there always exists a characteristic distance upon which all radiation from a point source should scatter and acquire the diffuse character.

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