Cherenkov effect in the weak interactions generated by the neutrinos and new approach for estimation of neutrino mass

Beshtoev Kh. M. (beshtoev@cv.jinr.ru)
Joint Institute for Nuclear Research, Joliot Curie 6, 141980 Dubna, Moscow region, Russia.

Abstract

It is shown that if weak interactions can generate masses and polarize matter, then the Cherenkov effect induced by these interactions appears. The resonance ($v_{\nu} < c/n$) and the Cherenkov ($v_{\nu} > c/n$) effects are competitive processes and at definite neutrino energies the resonance effect will change to the Cherenkov effect and we obtain an excellent possibility of estimating neutrino masses.

1 Introduction

At present the existence of three types of neutrinos—electron ($\nu_e$), muon ($\nu_\mu$) and tau ($\nu_\tau$) neutrinos—is established [1]. Determination of masses of these neutrinos is of great interest. Experiments were carried out to estimate the electron neutrino mass with using the beta decay [2]. Also, experiments on neutrinoless double beta decays were conducted to estimate neutrino masses on the assumption that neutrinos are Majorana particles [3]. In addition, mass differences between $\nu_1$, $\nu_2$, $\nu_3$ were measured in the neutrino oscillation experiments [4], but in these experiments it is impossible to establish neutrino masses.

The suggestion that, by analogy with $K^o, \bar{K}^o$ oscillations, there could be neutrino-antineutrino oscillations ($\nu \to \bar{\nu}$), was considered by Pontecorvo [5] in 1957. It was subsequently considered by Maki et al. [6] and Pontecorvo [7] that there could be mixings (and oscillations) of neutrinos of different flavors (i.e., $\nu_e \to \nu_\mu$ transitions). Then the resonance mechanism of neutrino oscillations in matter [8] was assumed which implied that as neutrinos passed through matter,
enhancement of neutrino oscillations took place since effective masses of neutrinos change and at definite matter density they can be equal.

This work is devoted to consideration of Cherenkov radiation of neutrinos in matter and a new approach for estimation of the neutrino mass.

Cherenkov radiation can appear only when neutrinos move in matter with a velocity \( v_i > c/n_i \), \( i = \nu_e, \nu_\mu, \nu_\tau \). But at the neutrino velocity \( v_i < c/n_i \) there may take place resonance enhancement of neutrino oscillations in matter. Therefore, before considering the Cherenkov effect, we give elements of the resonance effect.

## 2 Elements of the resonance mechanism enhancement of neutrino oscillations in matter

Before consideration of the resonance mechanism it is necessary to gain an understanding of the physical nature of origin of this mechanism. As neutrinos pass through matter, there can be two processes: neutrino scattering and polarization of the matter by neutrinos. Obviously, resonance enhancement of neutrino oscillations in matter will arise due to polarization of the matter by neutrinos. If the weak interaction can generate not only neutrino scattering but also polarization of matter, then the resonance effect will exist otherwise this effect cannot exist.

In the ultrarelativistic limit, the evolution equation for the neutrino wave function \( \nu_p \) in matter has the following form [8]:

\[
i \frac{d\nu_{Ph}}{dt} = (p \hat{I} + \frac{\hat{M}^2}{2p} + \hat{W})\nu_{Ph},
\]

where \( p, \hat{M}^2, \hat{W}_i \) are, respectively, the momentum, the (nondiagonal) square mass matrix in vacuum, and the matrix, taking into account neutrino interactions in matter,

\[
\nu_{Ph} = \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}, \quad \hat{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.
\]
\[ M^2 = \begin{pmatrix} m_{\nu_e \nu_e}^2 & m_{\nu_e \nu_{\mu}}^2 \\ m_{\nu_{\mu} \nu_e}^2 & m_{\nu_{\mu} \nu_{\mu}}^2 \end{pmatrix}. \]

If we suppose that neutrinos in matter like photons in matter (i.e., the polarization at neutrino passing through matter arises) and the neutrino refraction indices are defined by the expression

\[ n_i = 1 + \frac{2\pi N}{p^2} f_i(0) = 1 + 2\frac{\pi W_i}{p}, \quad (2) \]

where \( i \) is a type of neutrinos (\( \nu_e, \nu_{\mu}, \nu_{\tau} \)), \( N \) is density of matter, \( f_i(0) \) is a real part of the forward scattering amplitude which appears owing to polarization of matter by neutrino, then \( W_i \) characterizes polarization of matter by neutrinos (i.e., it is the energy of matter polarization).

The electron neutrino (\( \nu_e \)) in matter interacts via \( W^\pm, Z^0 \) bosons and \( \nu_{\mu}, \nu_{\tau} \) interact only via the \( Z^0 \) boson. These differences in interactions lead to the following differences in the refraction coefficients of \( \nu_e \) and \( \nu_{\mu}, \nu_{\tau} \)

\[ \Delta n = \frac{2\pi N}{p^2} \Delta f(0), \quad (3) \]

\[ \Delta f(0) = \sqrt{2G_F} p, \]

\[ E_{\text{eff}} = \sqrt{p^2 + m^2} + <e\nu|H_{\text{eff}}|e\nu> \approx p + \frac{m^2}{2p} + \sqrt{2G_F N_e} \]

where \( G_F \) is the Fermi constant.

Therefore the velocities (or effective masses) of \( \nu_e \) and \( \nu_{\mu}, \nu_{\tau} \) in matter are different. And at the suitable density of matter this difference can lead to resonance enhancement of neutrino oscillations in matter [8, 9]

\[ \sin^2 2\theta_m = \sin^2 2\theta \cdot [(\cos 2\theta - \frac{L_0}{L^0})^2 + \sin^2 2\theta]^{-1}, \quad (4) \]

where \( \sin^2 2\theta_m \) and \( \sin^2 2\theta \) characterize neutrino mixing in matter and vacuum, \( L_0 \) and \( L^0 \) are lengths of oscillations in vacuum and matter

\[ L_0 = \frac{4\pi E_{\nu} \hbar}{\Delta m^2 c^3}, \quad L^0 = \frac{\sqrt{2\pi} \hbar c}{G_F n_e}, \quad (5) \]
where $E_\nu$ is the neutrino energy, $\Delta m^2$ is the difference between squared neutrino masses, $c$ is the velocity of light, $\hbar$ is the Plank constant, $G_F$ is the Fermi constant and $n_e$ is the electron density of matter.

At resonance

$$\cos 2\theta \simeq \frac{L_0}{L^0} \quad \sin^2 2\theta_m \simeq 1 \quad \theta_m \simeq \frac{\pi}{4}.$$  \hspace{1cm} (6)

It is necessary to stress that this resonance enhancement of neutrino oscillations in matter is realized when, the neutrino velocity is smaller than the velocity of light in matter (i.e. $v_i < \frac{c}{n_i}$).

What will happen when the neutrino velocity is larger than the velocity of light in matter? Now let’s us turn to consideration of this problem.

### 3 Cherenkov effect in weak interactions generated by neutrinos in matter

The specific electromagnetic radiation produced by fast electrons moving in a medium was observed by Cherenkov in 1934 [10]. Tamm and Frank [11] showed that the charged particle must radiate when its velocity exceeds the velocity of light in the medium (see also [12] where motion of a charged particle in a medium with a constant electric permittivity (or refraction index) was considered). It is obvious that analogous radiation must take place when neutrinos move in a medium with a velocity exceeding the velocity of light in the medium if $n_i - 1 > 0$.

For realization of the mechanism of resonance enhancement of neutrino oscillations in matter the following condition must be fulfilled:

$$n_i - 1 > 0, \quad n_{\nu_e}^{SUN} - 1 = \Delta n_{\nu_e} \approx 10^{-17} \div 10^{-19}$$  \hspace{1cm} (7)

which is equivalent to the demand that the matter polarization exist at

$$v_i < \frac{c}{n_i}.$$  \hspace{1cm} (8)
For existence of the Cherenkov effect in weak interactions it is necessary to fulfil the following two conditions:

\[ n_i - 1 > 0, \]  
(9)

and

\[ v_i > \frac{c}{n_i}, \]  
(10)

where \( v_i, c \) are the velocities of the neutrino and light respectively. The first condition coincides with the resonance existence condition. The second condition means that if the neutrino moves with the velocity which is larger than the velocity of light in matter, then it will polarize matter and go away keeping in reserve this polarization which must be radiated afterwards. What will happen after that? This polarization can be taken off in two ways:

1. Via radiation of weakly interacting particles (neutrinos) or
2. Via electromagnetic radiation (polarized electrons have electrical charges and they can give electromagnetic radiation).

Since the probability of radiation of weakly interacting particles is very small, the Cherenkov radiation will be realized mainly in the form of electromagnetic radiation but not weak interaction radiation (it is clear that if energy of matter polarization is very small then neutrinos cannot be produced). It is related with the fact that weak interactions are slow processes while electromagnetic processes are fast. Energy of this radiation is

\[ E \approx W = \sqrt{2} G_F N_e, \quad (E_{SUN}^{SUN} \approx 10^{-10} \div 10^{-11} \text{eV}) \]  
(11)

where \( G_F \) is the Fermi constant, \( N_e \) is the electron density in the matter. The radiation angle \( \beta_i \) is

\[ \cos \beta_i = \frac{c}{v_i n_i} \]

If

\[ v_i < \frac{c}{n_i}, \]  
(12)
then the neutrino will polarize matter and since the velocity is smaller than the velocity of light in matter, this matter polarization will move together with this neutrino and the resonance effect can be realized.

Now we see that the resonance and Cherenkov effects are competing processes. If \( v_i < \frac{c}{n_i} \), the resonance effect will be realized and if \( v_i > \frac{c}{n_i} \), the Cherenkov effect will be realized. It is very important to remark that if in reality in weak interactions the matter polarization is present, then we obtain an excellent possibility of estimating neutrino masses using the point transition between the above two mechanisms and then the expression for the neutrino mass is

\[
m_\nu = E_{\text{trans}} \sqrt{1 - \left(\frac{1}{n_i}\right)^2}
\]

if \((n-1) \ll 1\), then

\[
m_\nu \simeq E_{\text{trans}} \sqrt{2(n_i - 1)},
\]

where \(E_{\text{trans}}\) is the neutrino energy at the point where the transition between the indicated mechanisms is realized.

### 4 Conclusion

It has been shown that if weak interactions can generate masses and polarize matter, the Cherenkov effect appears which is induced by these interactions. The resonance \( (v_\nu < c/n_i) \) and the Cherenkov \( (v_\nu \geq c/n_i) \) effects are competitive processes and at definite neutrino energies the resonance effect will change to the Cherenkov effect and we obtain an excellent possibility of estimating the neutrino masses.

It is necessary to remark that if the mechanism of MaVaN oscillations [13] is realized then the Cherenkov effect generated by neutrinos will also be realized.
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