Impact of long-range interactions on the disordered vortex lattice

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The interaction between the vortex lines in a type-II superconductor is mediated by currents. In the absence of transverse screening this interaction is long-ranged, stiffening up the vortex lattice as expressed by the disperseive elastic moduli. The effect of disorder is strongly reduced, resulting in a mean-squared displacement correlator \( \langle u^2(\mathbf{R}, \mathbf{L}) \rangle \equiv \langle (\mathbf{u}(\mathbf{R}, \mathbf{L}) - \mathbf{u}(0, 0))^2 \rangle \) characterized by a mere logarithmic growth with distance. Finite screening cuts the interaction on the scale of the London penetration depth \( \lambda \) and limits the above behavior to distances \( R < \lambda \). Using a functional renormalization group (RG) approach, we derive the flow equation for the disorder correlation function and calculate the disorder-averaged mean-squared relative displacement \( \langle u^2(R) \rangle \propto \ln^{2\sigma}(R/a_0) \).

The logarithmic growth \((2\sigma = 1)\) in the perturbative regime at small distances \([A.I. Larkin and Yu.N. Ovchinnikov, J. Low Temp. Phys. 34, 409 (1979)]\) crosses over to a sub-logarithmic growth with \(2\sigma = 0.348\) at large distances.

I. INTRODUCTION

Disordered elastic systems have attracted much attention in the last decade. They have applications in various physical systems, such as charge density waves, domain walls in Ising-ferromagnets, and vortices in type-II superconductors. The most interesting physical properties of such disordered elastic systems are their structural (static) order and their dynamical response under external forces. The dynamics involves a finite critical force with creep at small- and depinning at large forces. Here, we concentrate on the structural aspects as expressed through the displacement correlator \( \langle u^2(\mathbf{r}) \rangle \equiv \langle (\mathbf{u}(\mathbf{r}) - \mathbf{u}(0))^2 \rangle \) (angular brackets denote average over disorder). In general, the mean-squared displacement grows with distance as \( \langle u^2(\mathbf{r}) \rangle \propto |\mathbf{r}|^{2\zeta} \), where \( \zeta \) is the wandering exponent characterizing the roughness of the elastic manifold; for \( \zeta > 0 \) the disorder is relevant and the manifold is rough.

The generic model describing the competition between elasticity, quenched disorder, and possibly thermal fluctuations involves an isotropic homogeneous elastic medium (manifold) characterized by \( d \) internal dimensions (\( \mathbf{r} \)) and an \( N \)-component vector field for the displacement \( \mathbf{u}(\mathbf{r}) \) (\( |d + N| \)-model). The system we consider in the following is the flux-line lattice in a type-II superconductor submitted to quenched disorder. The distortions of the lattice are described by three internal dimensions \( \mathbf{r} \equiv (\mathbf{R}, \mathbf{L}) \) and a two-component displacement field \( \mathbf{u}(\mathbf{r}) \) (\( 3 + 2 \)-model).

A closer inspection shows that the underlying physical properties of the vortex lattice infer interesting modifications of the generic structural characteristics, both quantitatively and even qualitatively (a similar type of specific properties is found in other systems, e.g., in Wigner crystals). The underlying structure of the vortex lattice introduces a number of additional length scales in the problem: i) On a microscopic level, the flux-line lattice is built from interacting lines; their arrangement in a periodic fashion with distance \( a_0 \) drastically reduces the effect of disorder on large scales. The line-nature leads to a torsional elasticity with the shear modulus different from the compression and tilt modulus; this introduces a weak dependence of the wandering exponent upon the ratio of compression and shear modulus. ii) Long-range interactions between the vortex lines stiffen up the lattice as expressed by dispersive compression and tilt modulus thereby qualitatively reducing the effect of disorder; in the presence of transverse screening, this effect is limited to scales below the London penetration length \( \lambda \). iii) The long-range interaction between the vortices and the defects implies a long-range correlation in the disorder landscape, modifying the physics of thermal depinning.

In this paper, we focus on the effect of the long-ranged interaction between the vortices, see iii above. This interaction plays an important role in the short- to intermediate distance regime before being cut off by transverse screening at \( R \sim \lambda \) in transverse, \( L \sim \lambda^2/a_0 \) in longitudinal direction. Let us put the results for the displacement correlator into context and follow its evolution from small to large scales:

Because of the line-nature of the vortex lattice we assume weak point-like disorder; the effect of the interaction changes the algebraic growth of the displacement correlator \( \langle u^2(\mathbf{r}) \rangle \) into a mere logarithmic growth, as can be seen in lowest-order perturbation theory valid at small distances \( R < R_c \) and in the weak pinning limit,

\[
\langle u^2(R) \rangle = \xi^2 \frac{\ln[R/a_0]}{\ln[R_c/a_0]}.
\] (1)

Here, \( \xi \) is the correlation length of the disorder landscape properly defined in Eq. (11) (for point-like disorder the scale \( \xi \) is of the order of the vortex core dimension) and \( \alpha \) is a number of order unity. Later, we will obtain
\( \alpha \approx c/\sqrt{\pi} \), where \( c = e^{-\gamma} \) and \( \gamma \approx 0.577 \) is the Euler constant. The result \( \langle u^2(R) \rangle \) in the longitudinal direction is obtained from the above equation via the substitution \( R \rightarrow [(c/\sqrt{\pi})a_0L]^{1/2} \). The dependence on the disorder strength has been encoded in the collective bundle pinning radius \( R_c \) defined via \( \langle u^2(R_c) \rangle = \xi^2 \); for suitably weak disorder we have \( a_0 < R_c < \lambda \) and we will discuss other cases later. The long-range nature of the interaction between vortices is relevant for

\[
\alpha_0 < R < \lambda, \quad a_0 < L < \lambda^2/a_0. \tag{2}
\]

When \( \langle u^2(r) \rangle \) increases beyond \( \xi \) at \( R_c \) the appearance of multiple minima leads to a breakdown of the perturbative analysis.\(^{17}\) For \( R_c < R < \lambda \), we use the renormalization group (RG) to calculate the mean-squared displacement and find the result

\[
\langle u^2(R) \rangle \sim \xi^2 \left( \frac{\ln[R/a_0]}{\ln[R_c/a_0]} \right)^{2\sigma}, \tag{3}
\]

exhibiting a sub-logarithmic growth with \( \sigma \approx 0.174 \) (to the precision assumed here, we drop the numerical \( \alpha \) from Eqs. 9 to 11); the substitution \( R \rightarrow \sqrt{a_0L} \) again provides the result along the longitudinal direction. Equation 9 is the central result of this paper and will be derived below.

On larger length scales, the static properties of the disordered flux-line lattice are well understood.\(^{10,11,12}\) At a distance \( R_a \) the displacements become of the order of the lattice constant \( a_0 \), \( \langle u^2(R_a) \rangle \sim a_0^2 \), and the effect of disorder is reduced since the flux-lines no longer probe independent disorder realizations. In the absence of transverse screening the correlator then has an even slower growth \( \langle u^2(R) \rangle \propto \ln \ln[R/R_a] \).\(^{13}\) In a real system, however, the vortex-vortex interaction is usually screened on scales \( \lambda \) smaller than \( R_a \), the lattice gets softer and therefore rougher. For \( \lambda < R < R_a \) (random manifold regime) one obtains\(^3\)

\[
\langle u^2(R) \rangle \sim \xi^2 \left( \frac{R/\lambda}{\ln[R_c/a_0]} \right)^{2\gamma}. \tag{4}
\]

The result in longitudinal direction can be obtained via the substitution \( R \rightarrow a_0L/\lambda \) (the anisotropy is modified as compared to the case \( R < \lambda \) above). The functional RG calculation\(^{14}\) provides a value of the roughness exponent \( \gamma \) which depends weakly on the ratio of the compression \( c_{11} \) and shear \( c_{66} \) moduli, assuming the value \( \gamma \approx 0.174 \) in the limit \( c_{11} \gg c_{66} \) relevant in the vortex lattice. Finally, periodicity becomes important at \( R_a \) (Bragg Glass regime) and one recovers a logarithmic growth\(^{10,11,12}\)

\[
\langle u^2(R) \rangle \sim a_0^2 \ln \left( \frac{R}{R_a} \right), \tag{5}
\]

where the prefactor of the logarithm does not depend on the disorder strength (the latter is encoded in \( R_a \)). The physical properties of the vortex lattice, its periodicity and the long-ranged interaction between the constituents, thus qualitatively reduce the effect of disorder.

The paper is organized as follows: In the next section we present the model and perform the perturbative calculation which provides the starting point for the renormalization group analysis. Section III is devoted to the derivation of the flow equation for the disorder correlator and the calculation of the effective mean-squared relative displacement. We summarize and conclude in section IV.

### II. THE MODEL

We consider an isotropic type-II superconductor in the mixed state and use a coordinate system with the magnetic field \( B \) applied along the \( \hat{z} \)-axis. In the absence of disorder the vortices form a triangular Abrikosov lattice with a lattice constant \( a_\perp = 2a_0^2/\sqrt{3} \), and \( a_\parallel = \Phi_0/B \) (\( \Phi_0 = hc/2e \) is the magnetic flux quantum). The disordered vortex system is described by a free energy involving both elastic and pinning parts. Within the continuum elastic theory, the elastic part takes the form

\[
F_{\text{el}}[\mathbf{u}] = \frac{1}{2} \int d^3k \left( \frac{2\pi}{k_4} \right)^2 \left[ c_{11}(k) \mathbf{K} \cdot \mathbf{u}(k) \right]^2 + c_{66}[\mathbf{K}_\perp \cdot \mathbf{u}(k)]^2 + c_{44}(k)k_4^2[\mathbf{u}(k)]^2, \tag{6}
\]

where \( \mathbf{k} = (k_x, k_y, k_z) = (k_x, k_y, k_z), \mathbf{K}_\perp = (k_y, -k_x) \), and with the integration defined over the lattice Brillouin zone. The compression and tilt moduli are strongly dispersive, i.e., they depend on the wave vector \( \mathbf{k} \) of the distortion,

\[
c_{11}(k) \approx c_{44}(k) \approx \frac{\hat{c}_{44}}{1+\lambda^2k^2}, \tag{7}
\]

where \( k = |\mathbf{k}| \). The dispersion is due to the long-range current-mediated interaction between the vortices and is relevant in the intermediate distance regime,

\[
a_0 < R < \lambda, \quad a_0 < L < \lambda^2/a_0 \tag{8}
\]

(\( r = (R, L), R = |R| \)), before it is cut on the scale of the penetration depth \( \lambda \); we will restrict ourselves to this intermediate regime in the following. As usual, we ignore the weak dispersion in the shear modulus \( c_{66} \),

\[
c_{66} \approx \frac{\Phi_0 B^2}{(8\pi \lambda)^2}. \tag{9}
\]

The static Green function corresponding to the elastic energy\(^3\) reads \( (\mu, \nu = x, y) \)

\[
G^{\mu\nu}(k) = \frac{P_{11}(k)}{c_{11}(k)k^2 + c_{44}(k)k_4^2} \tag{10}
\]

\[
+ \frac{P_{66}(k)}{c_{66}k^2 + c_{44}(k)k_4^2}.\]
with the longitudinal and transverse projection operators \( P^\mu_L(K) = K_\nu K_\mu / K^2 \) and \( P^\mu_T(K) = \delta^{\mu\nu} - P^\mu_L(K) \), respectively (\( K \equiv |K| \)). In the regime we are interested in, the compression modes involve large energies (\( c_1 \gg c_{66} \)) and the longitudinal part of the Green function can be neglected; we denote the transverse part by \( G^\mu_{TT} \).

Assuming weak point-like defects, we describe the disorder term in the free energy via the pinning energy density \( E_{\text{pin}}(r, u(r)) \),

\[
F_{\text{pin}}[u] \equiv \int d^3r \ E_{\text{pin}}(r, u(r)).
\]  

(11)

The pinning energy density \( E_{\text{pin}} \) derives from the disorder term by minimizing the free energy functional \( F[u] = F_{\text{el}} + F_{\text{pin}} \) and is characterized by the disorder correlator,

\[
\langle E_{\text{pin}}(r, u)E_{\text{pin}}(r', u') \rangle = \delta^3(r - r')K_0(u - u'),
\]  

(12)

where \( K_0 \) is proportional to the disorder strength and is a short-range function decaying over the characteristic size \( \xi \) of the vortex core. For the disordered vortex lattice the correlator has an algebraic decay \( K_0 \propto (1/u^2) \ln(u/\xi) \) for \( u < \lambda \). We consider weak disorder where the vortices are pinned in bundles of size \( R_c > a_0 \); this contrasts with single vortex pinning realized for larger disorder strength to be discussed later. The above description remains valid as long as the disorder-induced displacements remain small, \( (u^2) < a_0^2 \), such that the vortices probe a different random environment upon deformation. As the displacement becomes of the order of \( a_0 \) on large scales, one has to take the periodicity of the lattice into account.

The equilibrium configuration \( u(r) \) of the system is obtained by minimizing the free energy functional \( F[u] = F_{\text{el}} + F_{\text{pin}} \) and is implicitly given by

\[
\langle u^\mu(r) \rangle = -\int d^3r' \ G^{\mu\nu}(r - r') \frac{\partial}{\partial u^\nu} E_{\text{pin}}(r', u(r')) \equiv u^\nu(r),
\]  

(13)

where \( G^{\mu\nu}(r) \) is the Fourier transform of the Green function \( G_{\text{el}} \). This ‘equation of state’ allows for calculating the mean-squared displacement correlator \( \langle u^2(r) \rangle \).

### A. Perturbation theory

For small displacements \( \langle u^2(r) \rangle < \xi^2 \), the \( u \) dependence in the force \( -\partial_{u^\nu} E_{\text{pin}} \) can be neglected in \( \text{13} \), and the mean-squared relative displacement can be written in the form

\[
\langle u^2(r) \rangle \approx -2K_0''(0) \int \frac{d^3k}{(2\pi)^3} \left( 1 - \cos kr \right) 
\times G^{\mu\nu}(k)G^{\mu\nu}(-k),
\]  

(14)

with the integration restricted to the lattice Brillouin zone in the plane.

We denote derivatives of the correlator with respect to \( u^\nu \) by a superscript,

\[
K_0' \equiv \frac{\partial}{\partial u^\nu} K_0,
\]  

and sum over indices that appear twice. Equation \( \text{14} \) simplifies since \( K_0 \) is isotropic: \( K_0''(0) = K_0''(0) = K_0''(0) = 0 \) (the primes denote derivatives of \( K_0 \) with respect to \( u \equiv |u| \)). Inserting the transverse part of the Green function \( G_{\text{el}} \), we obtain

\[
\langle u^2(r) \rangle \approx -2K_0''(0) \int \frac{d^3k}{(2\pi)^3} \left( 1 - \cos kr \right) 
\times \frac{1}{c_{66}K^2 + c_{44}/(K^2 + c_{44}^2/\lambda^2)}. \]  

(15)

In the intermediate regime, \( a_0 < R < \lambda, a_0 < L < \lambda^2/a_0 \), the relevant values of \( k_z \) are much smaller than \( K \). Hence, we can approximate \( c_{44}(k) = c_{44}/(1 + \lambda^2k^2) \approx c_{44}/\lambda^2K^2 \) and obtain

\[
\langle u^2(r) \rangle \approx -2K_0''(0) \int \frac{d^3k}{(2\pi)^3} \left( 1 - \cos(kr) \right) \lambda^4K^4 
\times \frac{1}{c_{66}K^2 + c_{44}/\lambda^2}. \]  

(16)

The divergence of the Green function as \( k \to 0 \) is cut off by the numerator on the scale \( k \sim 1/r \). For large wave vectors, the integral is limited by \( k \sim 1/a_0 \) (bundle pinning regime). In a first analysis, we therefore replace \( \text{16} \) by

\[
\langle u^2(r) \rangle \approx -1/2K_0''(0) \int \frac{d^2K dq}{\pi^2} \frac{K^4}{[K^4 + q^2]^2}, \]  

(17)

where \( q \equiv k_z \sqrt{c_{44}/c_{66}/\lambda} \approx k_z4/\sqrt{\lambda/a_0} \) and

\[
I \equiv \frac{\lambda}{4\pi c_{66} \sqrt{c_{66}c_{44}}}. \]  

(18)

The integral in \( \text{17} \) is essentially four-dimensional,

\[
2 \frac{K^4d^2K dq}{\pi^2 [K^4 + q^2]^2} \approx \frac{1}{\pi} \frac{d^2(K^2) dq}{2[K^2]^2 + q^2} \approx \frac{1}{\pi} \frac{d^2y}{y^3} \]  

(19)

(18)

The symbol \( \hat{=} \) reminds us that we are using \( d^2x = x^2S_d dx \), with \( S_d \) the surface of the \( d \)-dimensional sphere, and has a logarithmic divergence at small \( y \equiv (K^4 + q^2)^{1/2} \). The integral \( \text{17} \) is dominated by values \( q \sim K^2 \); for \( R^2 < a_0 \) the small scale cut-off is given by \( y \sim 1/R^2 \), while a cut-off \( y \sim 1/a_0L \) applies in the opposite limit \( L \gg R^2/a_0 \). We therefore obtain

\[
\langle u^2(r) \rangle \approx \frac{1}{2K_0''(0)} \ln \left( \frac{\rho(r)}{a_0} \right), \]  

(19)

where the function \( \rho(r) \) assumes the limits

\[
\rho(R^2 > a_0L) = R/\alpha, \]  

\[
\rho(R^2 \ll a_0L) = (a_0L)^{1/2}/\alpha', \]  

(20)

with \( \alpha \) and \( \alpha' \) numbers of order unity; the proper interpolation for arbitrary values of \( R \) and \( L \) is beyond the accuracy of the present analysis. Note that as a consequence of the dispersion in the tilt modulus, the lengths along the longitudinal (\( L \)) and transverse (\( R \)) directions scale differently, \( L \sim R \sqrt{c_{44}/c_{66}} \sim R^2/a_0 \) for \( R < \lambda \), \( L < x^2/a_0 \). The logarithmic growth of \( \text{19} \) is a consequence of the dispersion in the tilt modulus (and hence of
the long-range interaction), effectively lifting the problem to four dimensions where the disorder is only marginally relevant\cite{footnote1}, the prefactor of the logarithm is proportional to the disorder strength.

The simplifications made in \cite{footnote1} do not allow for a determination of the numericals $\alpha$, $\alpha'$, and the shape $\rho(r)$; in the following we perform a more accurate calculation of the original expression \cite{footnote1} in order to obtain a precise result for the argument under the logarithm in \cite{footnote1}. We approximate the hexagonal lattice Brillouin zone by the circular version $K < K_{\text{BZ}} = \sqrt{\pi}/a_0$; using cylindrical coordinates $k = (K, \phi, k_z)$, the angular integration in \cite{footnote1} can be carried out directly and the integration over the $k_z$-axis can be performed using residues, after which
\[
\frac{\langle u^2(R) \rangle}{I K_0^2(0)} \approx - \frac{K_{\text{BZ}}}{\int_0^{K_{\text{BZ}}} dK 1 - J_0(KR) e^{-a_0 L K^2/4\sqrt{\pi}} K e^{-cK^2/K_{\text{BZ}}^2}}. \quad (21)
\]

For $R$ or $L$ are much larger than $a_0$, the integral in \cite{footnote1} can be approximated by
\[
A(R, L) = \int_0^{\infty} dK \frac{1 - J_0(KR) e^{-a_0 L K^2/4\sqrt{\pi}} e^{-cK^2/K_{\text{BZ}}^2}}{K},
\]
where the constant $c$ is fixed by demanding
\[
\frac{K_{\text{BZ}}}{\int_{\delta} dK K} = \frac{1}{\sqrt{\pi}} e^{-cK^2/K_{\text{BZ}}^2}
\]
for $\delta \to 0$; one obtains $c = e^{-\gamma} \approx 0.56$, where $\gamma \approx 0.577$ is the Euler constant.

The integral $A(R, L)$ exhibits a logarithmic behavior for small $K$ which then is cut off either by $R$ or by $\sqrt{La_0}$. The approximation of a circular Brillouin zone (see above) is correct up to a factor of order unity under the logarithm.

We directly evaluate the integral $A(R, L)$ and obtain
\[
A(R, L) = \ln \left( \frac{R}{(c/\sqrt{\pi}) a_0} \right) + \frac{1}{2} \Gamma \left( 0, \frac{\pi R^2}{ca_0^2 + \sqrt{\pi} a_0 L} \right) \quad (\Gamma(0, x) = \int_x^{\infty} dt e^{-t}/t \text{ is the incomplete gamma function}),
\]
(assuming a cut-off at $R$ and $L$. In the limit of large $R$, $R^2 \gg 2ca_0^2/\pi + 2a_0 L/\sqrt{\pi}$, the above result simplifies to
\[
A(R, L) \approx \ln \left( \frac{R}{(c/\sqrt{\pi}) a_0} \right). \quad (22)
\]
while in the opposite limit of large $L$, $R^2 \ll 2ca_0^2/\pi + 2a_0 L/\sqrt{\pi}$ (and $L \gg ca_0/\sqrt{\pi}$) one obtains
\[
A(R, L) \approx \ln \left( \frac{L}{(c/\sqrt{\pi}) a_0} \right)^{1/2}. \quad (23)
\]
The present, more accurate, analysis then allows for the definition of the cut-off function $\rho(R, L)$ for arbitrary values of $R$ and $L$ within the perturbative regime,
\[
\rho(R, L) = a_0 e^{A(R, L)}. \quad (24)
\]
Comparing the above results with \cite{footnote1} and \cite{footnote2}, we find the numericals $\alpha \approx c/\sqrt{\pi} \approx 0.32$ and $\alpha' = a_1/\sqrt{\pi} \approx 0.56$.

Defining the collective pinning radius $R_c$ via
\[
\langle u^2(R_c) \rangle = \xi^2, \quad (25)
\]
we obtain \cite{footnote1} with the given value of the numerical $\alpha$.

The above perturbative results are valid within the collective pinning regime $R < R_c$, which is equivalent to $\rho(R, L) < R_c/a_0 \approx 3.16 R_c$. Beyond this regime, the perturbative result \cite{footnote1} breaks down because of the appearance of multiple minima as the displacement $\langle u^2(r) \rangle^{1/2}$ increases beyond the characteristic scale $\xi$ of the disorder potential \cite{footnote1}. Along the longitudinal direction, this breakdown occurs at the larger scale $L^c_c \approx 3.16 R^2_c/a_0 > R_c$.

Within the present perturbative approach the length scale $\xi$ appearing in \cite{footnote2} has been introduced ad hoc — this will be different in the functional RG treatment extending the discussion to larger length scales beyond $R_c$.

### III. Functional RG Equation

The logarithmic behavior of the displacement correlator \cite{footnote1} provides the motivation for applying the renormalization group (RG); we briefly summarize the main steps in the derivation of the functional RG equation for the disorder correlator $K_0(u)$ using the real-space renormalization procedure introduced in Ref.\cite{footnote1}. We aim at finding the effective disorder correlator $\xi$ on large scales ($\xi < R < \lambda$, $L^b_c < L < \lambda^2/a_0$). The elastic coefficients are not renormalized as the pinning part of the system is invariant under the ‘tilt’ $u(k) \to u(k) + v(k)$ for any vector-function $v$\cite{footnote1,footnote2,footnote3}.

Assuming renormalization up to a distance $(R_1, L_1)$, we define the displacement $u_1$, the energy $E^{(2)}_{\text{pin}}$, and the correlator $K_1$ at the scale $(R_1, L_1)$. We perform the RG step to $R_2 > R_1$, $L_2 > L_1$, and split $u_1 = u_2 + w$ (using the self-consistency equation \cite{footnote1} into a far-field
\[
u^2_2(r) = - \int_{\Omega^+} d^3 r' G^{uu}(r - r') \partial_{u'} E^{(1)}_{\text{pin}}(r', u_1(r'))
\]
with $\Omega^+ = \{ |R - R'| > R_2; L - L' > L_2 \}$, and a near-field contribution,
\[
u^w(r) = - \int_{\Omega} d^3 r' G^{uu}(r - r') \partial_{u'} E^{(1)}_{\text{pin}}(r', u_1(r'))
\]
where $\Omega = \{ R_1 < |R - R'| < R_2; L_1 < L - L' < L_2 \}$. We express the effective free energy $E^{(2)}_{\text{pin}}(u_2, r, r)$ for the far-field contribution as a Taylor expansion in the near-field $w(r)$ and, assuming Gaussian disorder, determine the effective pinning energy correlator
\[
K_2(u_2 - u_2') \delta(r - r') \equiv \langle E^{(2)}_{\text{pin}}(r, u_2) E^{(2)}_{\text{pin}}(r', u_2') \rangle
\]
on the scale $(R_2, L_2)$. Evaluating the terms to second
order in $K_1$ (first loop), one obtains

$$K_2(u_2) = K_1(u_2) + \mathcal{I}_{\mu\nu\rho\kappa}^{\mu\nu\rho\kappa} \left[ \frac{1}{2} K_1^{\mu\rho} K_1^{\nu\kappa} - K_1^{\mu\nu}(0) K_1^{\rho\kappa} \right],$$

(26)

where

$$\mathcal{I}_{\mu\nu\rho\kappa}^{\mu\nu\rho\kappa} = \int d^3 r' \, G^\mu\nu(r - r') G^\rho\kappa(r - r').$$

(27)

The Green function is most conveniently expressed in Fourier space,

$$\mathcal{I}_{\mu\nu\rho\kappa}^{\mu\nu\rho\kappa} = \int \Omega' (2\pi)^3 \, G^\mu\nu(k) G^\rho\kappa(k),$$

(28)

where $\Omega'$ is the appropriate $k$-space integration domain corresponding to $\Omega$ (see below). Since $c_{11} \gg \epsilon_{06}$ we can neglect the longitudinal part of the Green function and inserting the transverse part $G_T$ into the tensorial structure of the integral can directly be calculated using cylindrical coordinates $(K, \phi, z)$,

$$\int_0^{2\pi} d\phi \, \mathcal{P}_T^\mu\nu (K) \mathcal{P}_T^\rho\kappa (K) = \frac{2\pi}{8} \Delta_{\mu\nu\rho\kappa},$$

(29)

where

$$\Delta_{\mu\nu\rho\kappa} = \delta_{\mu\nu} \delta_{\rho\kappa} + \delta_{\mu\rho} \delta_{\nu\kappa} + \delta_{\mu\kappa} \delta_{\nu\rho}$$

(30)

(for the isotropic situation with $G^\mu\nu \propto \delta_{\mu\nu}$ one obtains $\mathcal{I}_{\mu\nu\rho\kappa}^{\mu\nu\rho\kappa} \propto \delta_{\mu\nu} \delta_{\rho\kappa}$).

Equation (28) resembles the equation for the displacement correlator (14); following Ref. 24 we approximate the term $1 + \lambda^2 k^2$ in $c_{44}(k)$ by $\lambda^2 K^2$ and use the fact, that the resulting integral is effectively four-dimensional, cf. (17). Performing the RG-step only in transverse direction ($L_2 = L_1$), the $k_z$-integration extends over the whole axis, while the $K$-integration is cut-off at $K_2 \sim 1/R_2$ and $K_1 \sim 1/R_1$; in the opposite case, the integral is cut by $k_{z2} \sim 1/L_2$ and $k_{z1} \sim 1/L_1$. The final result is conveniently expressed through the function $\rho(r)$ as defined in (20)

$$I_{\mu\nu\rho\kappa}^{\mu\nu\rho\kappa} = I \Delta_{\mu\nu\rho\kappa} \ln \left( \frac{\rho(R_2, L_2)}{\rho(R_1, L_1)} \right);$$

(31)

note that, given the accuracy of the present analysis, we do not make use of the result (24) which has been derived within the perturbative regime only. We define the flow parameter

$$l \equiv \ln(\rho(R, L)/a_0),$$

(32)

where the starting point of the flow is chosen by matching our RG analysis with the result from perturbation theory, cf. (19).

Inserting (31) into (20) yields the RG equation for the effective pinning energy correlator,

$$\partial_l K_l(u) = \frac{I}{16} \Delta_{\mu\nu\rho\kappa} \left( \frac{1}{2} K_l^{\mu\rho} K_l^{\nu\kappa} - K_l^{\mu\nu}(0) K_l^{\rho\kappa} \right).$$

(33)

Summing over pairs of indices, this simplifies to

$$\partial_l K_l = I \left( \frac{1}{16} K_l^{\mu\nu} K_l^{\mu\nu} - 2 K_l^{\mu\nu} K_l^{\mu\nu}(0) + \frac{1}{2} K_l^{\mu\rho} K_l^{\nu\kappa} - K_l^{\mu\nu}(0) K_l^{\rho\kappa} \right).$$

(34)

The first line of this RG equation contains the terms that are obtained for an isotropic situation$^{23,25}$ with $G^\mu\nu \propto \delta_{\mu\nu}$; the terms in the second line arise due to the transverse structure of the Green function entering (24). Up to scaling terms, an equation equal to (33) is obtained in the non-dispersive $[3+2]$-case for $c_{11} \gg \epsilon_{06}$, while a different tensorial structure is found assuming arbitrary values of $c_{11}$ and $\epsilon_{06}$, cf. Ref. 13.

In the regime we are interested in, the correlator is isotropic, $K_l(u) = K_l(u)$, and the derivatives can be rewritten as

$$K_l^{\mu\nu}(u) = \left( K_l^{\mu\nu}(u) - \frac{K_l^\prime(u)}{u} \right) \frac{u^{\mu\nu}}{u^2} + \frac{K_l^\prime(u)}{u} \delta_{\mu\nu},$$

$$K_l^{\mu\nu}(0) = K_l^\prime(0) \delta_{\mu\nu},$$

(35)

the RG equation (33) then takes the form

$$\partial_l K_l(u) = I \left[ \frac{3}{2} \left( K_l^\prime \right)^2 + K_l^\prime \frac{K_l}{u} + \frac{3}{2} \left( \frac{K_l^\prime}{u} \right)^2 \right] - 4 K_l^\prime (0) \left( K_l^\prime + \frac{K_l}{u} \right),$$

(36)

where the primes denote derivatives with respect to $u = |u|$. Taking four derivatives with respect to $u$ and assuming an analytic correlator, one obtains

$$\partial_l K_l^{(4)}(u) = I \left[ K_l^{(4)}(0) \right]^2.$$

At the scale $l_c$, the fourth derivative of the correlator becomes infinite at the origin, $K_l^{(4)}(0) \rightarrow \infty$, and the second derivative develops a cusp singularity. This non-analyticity provides a definition for the collective pinning radius $R_c$ which depends on the fourth derivative $K_l^{(4)}(0)$ of the initial correlator,

$$l_c = \frac{1}{IK_l^{(4)}(0)} \equiv \ln \left[ R_c/a_0 \right].$$

(37)

Next, we determine the static structure of the vortex lattice on large scales. As for the disorder correlator $K_l$, we use perturbation theory to find the renormalized displacement field $\langle u^2(r) \rangle$ on scale $r$. We split the integral (17) into shells and replace $K_l^\prime(0)$ in each shell by the scale-dependent disorder correlator $K_l^{\mu\nu}(0)$. Integrating over all shells we obtain

$$\langle u^2(r) \rangle \approx - I \int_{ln(a_0)}^{ln(R)} dln' K_l^{\mu\nu}(0).$$

(38)

As in (17), the integral is cut off by the function $\rho(r)$, which is defined in (20) via its limits for $R^2$ much larger or much smaller than $a_0 L$. 
IV. ANALYSIS OF THE RG FLOW

Analyzing (35), the disorder correlator \( K_l \) flows towards zero on large scales, indicating that the disorder is only marginally relevant. Here, we are interested in learning how \( K_l''(0) \) flows to zero with increasing \( l \) in order to integrate the equation (38) for the displacement correlator. We will see that a proper rescaling of the displacement field \( \mathbf{u}(\mathbf{r}) \) and of the correlator \( K_l \) maps the flow equation (38) describing the marginal situation to the flow equation in one dimension less (39); this will allow us to make use of results derived from an \( \epsilon = 4 - d \)-expansion at the value \( \epsilon = 1 \).

Following (35) we need to integrate the flow equation for \( K_l''(0) \). Differentiating (34) or (36) twice with respect to \( u \) and evaluating the result at \( u = 0 \) one obtains

\[
\partial_l K_l''(0) = \frac{3 l}{16} K_l''''(0)^2. \tag{39}
\]

In the Larkin regime, \( R < R_c, L < L^c_l \), the disorder correlator is an analytic function \( K_l''''(0^+) = 0 \) and hence \( K_l''''(0) = K_l''''(0)^{+} \) remains constant; integrating (39) we confirm the perturbative result (19),

\[
\langle u^2(R_c) \rangle = \frac{K_l''''(0)^2}{K_l''''(0)} \equiv \xi^2; \tag{41}
\]

this precise definition of \( \xi \) naturally appears within the renormalization group framework.

Going beyond \( l_c \), it is more convenient to investigate the asymptotic flow of \( K_l(u) \), rather than solving Eq. (39). Since the magnitude \( K_l(0) \) decreases and the function \( K_l(u) \) broadens, we perform the rescaling

\[
\frac{l}{16} K_l(u) = \frac{1}{l} f_l \left( \frac{u}{\tilde{u}} \right) \tag{42}
\]

in order to extract the characteristic geometry of the correlator (its height and width). The simplified flow equation for \( f_l \) then will converge to a regular and finite function \( f^* \) for large \( l \) such that we arrive at a proper asymptotic scaling form for \( K_l \sim 1 - l^{-\sigma} f^*(u/\tilde{u}) \). Inserting (42) into (39), the resulting flow equation involves terms proportional to \( l^{-\tau - 1} \) and \( l^{-2\sigma - 4\alpha} \); we require these terms to scale the same way in order to achieve a flow \( f_l \rightarrow f^* \) and hence we fix \( \tau = 1 - 4\sigma \). The flow equation for \( f_l \) then reads

\[
\frac{\partial}{\partial \ln l} f_l = (1 - 4\sigma) f_l + \frac{3}{2} \left( \frac{f_l'}{u} \right)^2 + 3 \left( \frac{f_l''}{u} \right)^2 + \frac{3}{2} \left( \frac{f_l'''}{u} \right)^2 + \frac{3}{2} \left( \frac{f_l''''}{u} \right)^2 \tag{43}
\]

where the first two terms on the rhs arise due to the explicit \( l \)-scaling in (42) and the primes denote derivatives with respect to \( \tilde{u} \equiv u/l^\sigma \). As we will see, this equation is the same (up to \( \ln l \rightarrow l \)) as the (first loop) flow equation for the rescaled disorder correlator \( R_l \) in the non-dispersive \([3+2]-\)case in an \( \epsilon = 4 - d \)-expansion, cf. Eq. (47) below.

In the dispersive \([3+2]-\)case, the system is at the upper critical dimension and the relevant integral for \( f_{\mu\nu\rho\kappa} \), cf. (24), exhibits a logarithmic behavior, cf. (31). This leads to the flow equation in the form \( \partial_l K_l = \mathcal{O}_{1-loop}(K_l^2) \). In the non-dispersive case the upper critical dimension is \( d = 4 - \epsilon \) the integral (27, 31) has an algebraic behavior

\[
f_{\mu\nu\rho\kappa} = \frac{\tilde{l}}{16} \Delta_{\mu\nu\rho\kappa}^{\epsilon \epsilon}(\rho(\mathbf{r}_2) - \rho(\mathbf{r}_1)^\gamma) \tag{44}
\]

(with \( \tilde{l} = 2 l/\pi \lambda \) for \( \epsilon = 1 \)), leading to the flow equation

\[
\partial_l R_l = \mathcal{O}_{1-loop}(K_l^2), \tag{45}
\]

(here, \( c_{\kappa \kappa} \approx c_{44} \) and therefore \( \rho(R, L) \) is different from (20): \( \rho(R, 0) \sim R, \rho(0, L) \sim a_0 L/\lambda \)). Next, we rescale the correlator \( K_l = \exp(-\zeta l) K_{l0} \) and obtain the flow equation in the form \( \partial_l K_l(u) = \epsilon K_l + \mathcal{O}_{1-loop}(K_l^2) \). Finally, we perform the rescaling

\[
\frac{l}{16} K_l(u) = e^{-\zeta l} R_l \left( u e^{-\zeta l} \right) \tag{46}
\]

corresponding to (42) above if we replace \( l \rightarrow \exp(\tilde{l}) \); the latter accounts for the change from a logarithmic to an algebraic behavior when going away from the marginal dimension. Carrying out the above sequence of steps, we arrive at the standard flow equation (14) (we define \( \tilde{u} = u \exp(-\zeta l) \) and assume \( c_{11} \gg c_{66} \) (the primes denote derivatives with respect to \( \tilde{u} \))

\[
\frac{\partial}{\partial l} R_l = (\epsilon - 4\zeta) R_l + \zeta \tilde{u} R_l' + \frac{3}{2} R_l''^2 + \frac{3}{2} R_l'' + \frac{3}{2} \left( \frac{R_l'''}{u} \right)^2 \tag{47}
\]

where we have used \( \alpha = -4\zeta \). For \( \epsilon = 1 \), this equation is identical to (14) if we replace \( \partial \ln l \rightarrow \partial l \) and \( \sigma \rightarrow \zeta \). Comparing to standard derivations of flow equations of the type (17), cf. Refs. 22 and 13, here we have delayed all the rescaling of the field \( u \) and of the coupling function \( K_l \) to the very end of the calculation. The rescaling (16) then again is motivated by the desire for a flow equation for \( R_l(\tilde{u}) \) admitting a fix-point function \( R^*(\tilde{u}) \) corresponding to \( \zeta_{1/2} \approx 0.174 \) as found in Ref. 13 for \( c_{11} \gg c_{66} \) (we denote by \( c_{d,n} \) the one-loop value of the wandering exponent for the \([d + N]-\)model).
We briefly remind the arguments of Ref. 23 providing the proper fixed-point function $R^*(\tilde{u})$ and the wandering exponent $\zeta$. Depending on the starting conditions, the numerical integration of the fixed-point equation for $R^*$ provides three classes of decaying solutions: for $\zeta < \zeta_{3,2}^l$, the function $R^*(\tilde{u})$ crosses the $\tilde{u}$-axis at least once. Choosing $\zeta > \zeta_{3,2}^l$, the solution is always positive and decays in a power law fashion for large $\tilde{u}$. Finally, for $\zeta = \zeta_{3,2}^l$, $R^*(\tilde{u})$ is positive as well and decays exponentially. The first class of solutions is forbidden by the flow as a positive initial correlator remains positive under the RG flow, hence $\zeta \geq \zeta_{3,2}^l$. On the other hand, any algebraic tail $R_l(\tilde{u}) \sim A_l/\tilde{u}^\gamma$ vanishes under the flow,
\[
\frac{\partial}{\partial l} A_l = (1 - 4\zeta - \gamma\zeta) A_l \leq (1 - 4\zeta_{3,2}^l - \gamma\zeta_{3,2}^l) A_l,
\]
provided that $\gamma > 1/\zeta_{3,2}^l - 4$ (this is the usual power-law separating short- from long-range correlators); for our correlator with $K_0 \propto 1/\tilde{u}^\delta$, we indeed obtain that the tail disappears under the flow and the only acceptable solution is the exponentially decaying short-range fix-point function realized for $\zeta = \zeta_{3,2}^l \approx 0.174$.

Next, we can make use of the analogy between the dispersive four-dimensional and the non-dispersive three-dimensional case discussed above: the fix-point equations for $R^*$ and for $f^*$ are identical and we obtain an exponent $\sigma = \zeta_{3,2}^l \approx 0.174$ describing the power-law type evolution of the correlator’s width, cf. 172. An important difference appears when considering higher-loop corrections: such corrections to $R^*$ and the roughness exponent $\zeta$ are relevant for the $\epsilon$-expansion since under the transformation each higher-loop term appears with an additional factor $\exp[-(\alpha + 4\zeta) l] = 1$. On the other hand, in the marginal situation the higher-loop corrections to $f^*$ are irrelevant; higher-loop terms come with an additional factor $l^{-(\zeta + 4\alpha)} = l^{-1}$ and thus can be neglected at large $l$. This happens because at the upper critical dimension the correlator $K_l$ flows to zero, and higher order corrections in $K_l$ flow to zero faster and can be neglected, whereas $\tilde{K}_l$ (for $\epsilon > 0$) flows towards infinity, such that higher order terms may be important (the growth in amplitude and width turn out to compensate each other). The above one-loop value for $\sigma$ thus is exact.

This is similar to what happens in critical phenomena, where the leading singularity can be calculated exactly in one loop at the critical dimension.

We now are in a position to determine the displacement correlator $\langle u^2(\mathbf{r}) \rangle$ on large scales following 35. Taking the second derivative of (42) and using $f_l \to f^*$, we obtain $\langle u^2(\mathbf{r}) \rangle \sim (16/\epsilon)^{\alpha/2} (l^{-1/2})^{1-2\sigma}$. We assume that this asymptotic behavior crosses over to the solution at small distances at $l \sim l_c$. Using $K^*_l(0) = K^*_0(0)$, we obtain the large $l$ behavior
\[
\langle u^2(\mathbf{r}) \rangle \sim l_c^{-1-2\sigma}.
\]
Integrating 35, we obtain the displacement correlator at large distances
\[
\langle u^2(\mathbf{r}) \rangle \sim I \int_{l_c}^l dl' \left[-K''_l(0)\right] \\
\sim -IK^*_0(0) \int_{l_c}^l dl' \left(\frac{l_c}{l}ight)^{1-2\sigma}.
\]
Using (41) and $l_c = \ln(R_0/\alpha a_0)$, we finally arrive at
\[
\langle u^2(\mathbf{r}) \rangle \sim \xi^2 \left(1 + (\zeta_{4,1}^l(0))/l_c\right)^{2\sigma}_2
\]
with $\rho$ given by 20. Such a sub-logarithmic behavior has already been conjectured in Ref. 22 here, we have put the sub-logarithmic growth on a firm basis and have accurately calculated the value of the exponent.

The sub-logarithmic growth found in (49) above has to be cut off as the interaction between the vortices is screened on distances beyond the penetration depth $\lambda$ or when the displacement field increases beyond the lattice constant $a_0$. If the latter condition $(l_c^2(R))/l < a_0$ is realized within the dispersive regime the growth in the displacement field turns even slower: beyond $u > a_0$ the effective disorder correlator becomes a periodic function in $u$, implying that $\sigma = 0$ in (12) and hence $K^*_l(0) \propto 1/\epsilon$; the integration in (18) then leads to a growth $\langle u^2(\mathbf{r}) \rangle \sim \ln(\rho(\mathbf{r})/\rho(\mathbf{R}_0))$. However, typically one would expect that screening becomes relevant before periodicity in $K_0$ and the displacement correlator follows the evolution dictated by the usual non-dispersive random manifold- and Bragg Glass regimes, cf. 14 and 18.

The above results have been derived within a weak collective pinning approach with $R_c > a_0$ and are valid on scales $\rho(\mathbf{r}) > a_0$. On the other hand, increasing the disorder strength, the collective pinning radius $R_c$ drops below $a_0$ and vortices become individually pinned. Within this single vortex pinning regime, finite segments of length $L_c \sim \langle u^2(\mathbf{r}) \rangle$ are pinned by the collective action of defects; here, $k_0(u)$ denotes the pinning energy correlator for an individual vortex line and $\xi_0 = (\Phi_0/4\pi\lambda)^2$ is the short wave-length line tension. Given the same underlying disorder potential, the single vortex- and vortex lattice correlators are related via $k_0(u) = a_0^2 K_0(\epsilon_0)$. The collective pinning length $L_c$ and the collective pinning radius $R_c$ then are formally related via
\[
R_c \sim a_0 \exp \left[ \text{const.} \left(\frac{L_c}{a_0}\right)^3 \right].
\]
cf. 317; depending on the disorder strength, either $L_c < a_0$ or $R_c > a_0$ is the physically relevant quantity. On the other hand, the length $L_c$ as defined via $k_0(L_c)$ is a convenient parameter quantifying the strength of the disorder.

Assuming $L_c < a_0$ and short distances $L < L_c$ the mean-squared displacement behaves as
\[
\langle u^2(L) \rangle \sim \xi^2(L/L_c)^3.
\]
the condition $\langle u^2(L_c) \rangle \sim \xi^2$ replaces the previous condition $\langle u^2(R_c) \rangle \sim \xi^2$ valid in the weak pinning regime. Going beyond the collective pinning (or Larkin-) length $L_c$ the displacement grows as

$$\langle u^2(L) \rangle \sim \xi^2 (L/L_c)^{2\zeta_{1,2}^w},$$  \hspace{1cm} (52)

where $\zeta_{1,2}^w$ is the roughness exponent for the single vortex.

Let us now connect the single vortex- with the lattice pinning regimes at the scales $L \sim a_0$ and $R \sim a_0$. In the weak pinning case ($L_c > a_0$), we match \ref{rho} with the logarithmic growth \ref{log} valid for $a_0 < \rho(r) < R_c/\alpha$, 

$$\langle u^2(r) \rangle \sim \xi^2 (a_0/L_c)^3 \ln [\rho(r)/a_0],$$

and we recover the relation $\ln[R_c/\alpha a_0] \sim (L_c/a_0)^3$, in agreement with \ref{log} above. The equation for the sub-logarithmic growth \ref{sublog} valid for $\rho(r) > R_c/\alpha$ can then be casted in the form

$$\langle u^2(r) \rangle \sim \xi^2 ((a_0/L_c)^3 \ln [\rho(r)/a_0])^{2\sigma}.$$ 

For intermediate pinning ($\xi \ll L_c < a_0$) we match \ref{rho} (valid up to $L \sim a_0$) to the sub-logarithmic growth \ref{sublog} and obtain the relation

$$\langle u^2(r) \rangle \sim \xi^2 (a_0/L_c)^{2\zeta_{1,2}^w} \ln [\rho(r)/a_0]^{2\sigma},$$

describing the growth of the displacement correlator for larger distances $\rho(r) > a_0$.

Comparing the two cases of weak and intermediate pinning in the sub-logarithmic regime we note a slightly different dependence on the disorder parameter $L_c$: in the weak pinning situation the disorder dependence is given by the factor $L_c^{-6\sigma}$, whereas for intermediate pinning the corresponding factor $L_c^{-2\zeta_{1,2}^w}$ involves a different exponent. The numerical values of the two exponents can be calculated within RG; the individual vortex is characterized by an isotropic elasticity and $\zeta_{1,2}^w \approx 0.177\epsilon$ to first loop order. This value is close to the one obtained for the anisotropic elasticity of the vortex lattice, $\zeta_{d,2} \approx 0.174\epsilon$. Therefore, the exponent $2\zeta_{1,2}^w \approx 1.062$ is similar to the value $6\sigma \approx 1.044$. However, note that the roughness exponent $\zeta_{1,2}^w$ is subject to corrections by higher-loop terms, whereas the value for $\sigma$ remains unchanged.

V. CONCLUSION

Summarizing, the long-ranged interaction between vortices as expressed by the dispersive elastic moduli of the flux-line lattice strongly reduces the effect of disorder: rather than the usual algebraic growth $\langle u^2(r) \rangle \propto r^{2\xi}$ of the mean-squared displacement correlator, one finds a mere logarithmic dependence in the perturbative regime $a_0 < \rho(r) < R_c/\alpha$, \ref{rho} with $\rho(r)$ given by \ref{rho}. On larger scales $R_c/\lambda < \rho(r) < \lambda$ we have found a sub-logarithmic law,

$$\langle u^2(r) \rangle \sim \xi^2 \left( \frac{\ln[\rho(r)/a_0]}{\ln[R_c/\alpha a_0]} \right)^{2\sigma};$$

here, the scale function $\rho(r)$ is determined to the precision given in \ref{rho} with $\alpha = \sigma^2 \approx 0.32$. The exponent $2\sigma \approx 0.348$ obtained within the one-loop RG calculation remains unchanged by higher loop corrections. The definition

$$\xi^2 = -\frac{K''_0(0)}{K''_0(0)}$$ \hspace{1cm} (53)

for the relevant length scale of the disorder landscape emerges naturally within the RG framework.

The mean-squared displacement correlator for the disordered vortex lattice then exhibits two separate regimes with a well developed translational order involving either a logarithmic or a sub-logarithmic growth of $\langle u^2(r) \rangle$, a first regime at small length scales originating from the long-ranged interaction between vortices, and a second regime at large scales which is due to the periodicity of the lattice (Bragg Glass regime, cf. Eq. \ref{sublog}). A number of experiments have been devised in order to observe this type of logarithmic order, e.g., via Bitter decoration or using small angle neutron scattering (SANS) (we note that the Bitter decoration analysis carried out at low fields does not probe the dispersive regime discussed here). The observation of a well (i.e., quasi-long range) ordered lattice then is often interpreted as evidence for Bragg-glass order. The above discussion demonstrates, that this need not to be the case, however, as the vortex lattice exhibits this type of logarithmic order on intermediate scales $a_0 < R < \lambda$, $a_0 < L < \lambda^2/a_0$ as well (note the 'cigar' shaped geometry of the dispersive regime which can reach a large extension along the longitudinal direction, i.e., parallel to the magnetic field).

A further complication of this type of analysis is found in the slow relaxation of the vortex system towards the proper glassy order: frozen-in at some higher temperature $T_{melt}$ where pinning is irrelevant, the glassy order of the vortex lattice first has to establish itself via proper relaxation through creep over the relevant barriers. During the experimental time $t$, the vortex system then can overcome barriers of size $T \ln(t/t_0)$ typically, with $t_0^{-1}$ a proper attempt frequency. With typical temperatures of order $50$ K and $\ln(t/t_0) \approx 20$, barriers $U(R) \approx 1000$ K may be overcome, producing relaxed domains of size $R$ of a few lattice constants typically; hence reaching glassy order over large scales is quite a nontrivial task.

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