Study of angular momentum effects in fission

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(Dated: October 10, 2020)

Background: The role of angular momentum in fission has long been discussed but the observable effects are difficult to quantify. Purpose: We discuss a variety of effects associated with angular momentum in fission and present quantitative illustrations. Methods: We employ the fission simulation model FREYA which is well suited for this purpose because it obeys all conservation laws, including linear and angular momentum conservation at each step of the process. We first discuss the implementation of angular momentum in FREYA and then assess particular observables, including various correlated observables. We also study potential effects of neutron-induced fission of the low-lying isomeric state of $^{235}$U relative to the ground state. Results: The fluctuations inherent in the fission process ensure that the spin of the initial compound nucleus has only a small influence on the fragment spins which are therefore nearly uncorrelated. There is a marked correlation between the spin magnitude of the fission fragments and the photon multiplicity. We also consider the dynamical anisotropy caused by the rotation of an evaporating fragment and study especially the distribution of the projected neutron-neutron opening angles, showing that while it is dominated by the effect of the evaporation recoils, it is possible to extract the signal of the dynamical anisotropy by means of a Fourier decomposition. Finally, we note that the use of an isomeric target, $^{235}$U(n$_{th}$,f), may enhance the symmetric yields and can thus result in higher neutron multiplicities for low total fragment kinetic energy.

I. INTRODUCTION

The role of angular momentum in nuclear fission has long been a topic of central interest, dating back over sixty years. Experimental evidence suggests that the primary fission fragments on average carry spins of magnitude $5-7\hbar$ aligned roughly perpendicular to the fission axis $\hat{1}$. The associated fragment rotation generally causes the neutron evaporation to be anisotropic $\hat{2}$, which may affect a variety of neutron-related observables, including neutron spectra, angular distributions, and directional correlations $\hat{4}$, as well as attempts to find evidence of scission neutrons $\hat{2}$ $\hat{5}$ $\hat{6}$. The fragment angular momentum also influences the photon radiation $\hat{7}$.

For the purpose of elucidating the various ways angular momentum enters into the fission process and to quantitatively ascertain the effect on observables of particular interest, the present study utilizes the event-by-event fission simulation model FREYA $\hat{8}$ $\hat{9}$, which uses Monte Carlo techniques to generate large samples of complete fission events. An important advantage of employing FREYA is that all conservation laws are obeyed throughout each fission event, including those affecting the angular momentum directions during the evaporation cascades.

Section II describes how angular momentum is treated in FREYA. Section III addresses observables of particular interest. Then Sec. IV describes potential observable differences in $^{235}$U(n$_{th}$,f) arising if the target nucleus is in its isomeric state. Our concluding remarks are presented in Sec. V.

II. TREATMENT OF ANGULAR MOMENTUM IN FREYA

FREYA is a Monte Carlo model that is capable of quickly generating large samples of complete fission events, namely the full kinematic information for the two prompt product nuclei and all prompt neutrons and photons in each event. With these large samples, it is straightforward to extract any observable of interest. Because each fission event conserves mass, charge, energy, linear and angular momentum, and spin, any inherent correlations between various quantities are preserved, thus making FREYA particularly well suited for determining how angular momentum affects various final states.

Angular momentum enters at several stages during the fission process. In the following section, we describe how it is treated in FREYA.

A. Preparation

During the first stage of a neutron-induced fission event, a neutron of momentum $p_0$ impinges on the target nucleus, in the present case $^{235}$U. The associated impact parameter $\rho$ is chosen randomly from a disk, perpendicular to the direction of motion, with a radius equal to that of the nucleus. Thus the neutron introduces a linear momentum of $p_0$ and an angular momentum of $\rho \times p_0$.

The incoming neutron may cause the emission of a pre-equilibrium neutron. While pre-equilibrium processes grow increasingly likely at higher energies, they are unimportant at thermal energies. If pre-equilibrium emission does occur, the appropriate reduction is made of the mass.
number as well as the linear and angular momenta of the residual system which is assumed to subsequently relax to a compound nucleus which can ultimately fission.

If sufficiently excited, the compound nucleus may evaporate one or more neutrons before fission occurs, according to the energy-dependent branching ratio $\Gamma_n/\Gamma_t$. Such pre-fission evaporation is treated the same way as the post-fission evaporation from the fission fragments, see Sec. [1] For each evaporation, FREYA reduces the excitation energy of the daughter nucleus and changes its linear and angular momenta as dictated by conservation laws.

At the end of the pre-fission evaporation chain, we arrive at the fissioning nucleus with angular momentum $S_0$ which has generally been reoriented relative to $\rho \times p_0$ due to the spin recoils, by about 24° on average for 235U(n,f) for an incoming neutron energy of $E_n = 20$ MeV.

### B. Scission

The second stage of the fission process is the evolution of the pre-fission compound nucleus to two well-separated and fully-accelerated primary fragments, each of which is also in equilibrium. It is not the purpose of FREYA to model how this complicated time-dependent many-body process develops. That challenging problem was recently reviewed in Ref. [10] and a broader review of the recent experimental and theoretical progress in fission can be found in Ref. [11]. Rather, FREYA generates an ensemble of possible outcomes based primarily on the input provided. Generally, the FREYA input is based on experimental data, but a certain degree of modeling is necessary because the data sets are often incomplete. For example, while experimental data for the fragment mass distribution $Y(A)$ and the associated mean total fragment kinetic energy $\text{TKE}(A)$ is often available for thermal neutron-induced fission, the energy dependence of these functions is usually not well measured.

After the neutron and proton numbers of the primary fragments have been selected, their angular momenta are sampled from the statistical distribution of the dinuclear rotational modes at scission [12]. The total angular momentum of the system after scission is given by $S_i = S_L + S_H + L$, namely the sum of the two individual light and heavy fragment spins and the orbital angular momentum of the two-fragment system, $L = R \times P$, where $R = R_L - R_H$ is the fragment separation and $P = \mu(V_L - V_H)$ is the associated momentum, equal to the reduced mass $\mu = M_L M_H/(M_L + M_H)$ times their relative velocity. The overall rotation of the dinuclear complex determines the average values of the fragment spins, $\bar{S}_i = (I_i/I) S_0$, as well as the average of their relative angular momentum, $\bar{L} = (I_R/I) S_0$. Here $I_i$ is the moment of inertia of fragment $i = L, H$, $I_R = \mu R^2$ is the moment of inertia for the relative motion, and $L = I_L + I_H + I_R$ is the total moment of inertia. The nuclear moments of inertia are taken to be 50% of the rigid-body values, as is commonly done.

Because the system is excited at the time of scission, the intrinsic rotational modes are expected to be agitated and the actual angular momenta will therefore fluctuate, $S_i = S_0 + \delta S_i$. In order to sample the spin fluctuations, $\delta S_i$, FREYA brings the rotational energy, 

$$E_{\text{rot}} = S_i^2/2I_L + S_H^2/2I_H + L^2/2I_R,$$  \hspace{1cm} (1)

onto normal form [3,12]. A binary system generally has six normal modes of rotation [13]. These are tilting and twisting, in which the fragments rotate in the same or in the opposite sense around the dinuclear axis $\hat{z} = R/R$, and wriggling and bending, in which they rotate in the same or the opposite sense around an axis perpendicular to the dinuclear axis [14,15]. The two latter modes are each doubly degenerate (corresponding to rotations around $\hat{x}$ and $\hat{y}$, for example.) Only the perpendicular modes (wriggling and bending) are considered by FREYA [3,12], because the agitation of the first two tends to be suppressed due to the constricted neck [13].

The rotational energy associated with the four perpendicular modes can be written on normal form as

$$E_{\text{rot}}^\pm = s_+^2/2I_L + s_-^2/2I_L,$$  \hspace{1cm} (2)

where $s_+$ represents wriggling and $s_-$ represents bending. The corresponding moments of inertia are [13]

$$I_+ = (I_L + I_H)I/I_R, \quad I_- = I_L I_H/(I_L + I_H).$$  \hspace{1cm} (3)

FREYA then samples $s_\pm$ from a distribution of statistical form, $P(s_\pm) \sim \exp(-s_\pm^2/2I_\pm T_S)$, where $T_S$ is the effective spin temperature (explained below).

The resulting angular momenta of the individual fragments are subsequently obtained as

$$S_L = \bar{S}_L + \delta S_L = (I_L/I) S_0 + (I_L/I_R)s_+ + s_-,$$  \hspace{1cm} (4)

$$S_H = \bar{S}_H + \delta S_H = (I_H/I) S_0 + (I_H/I_+)s_+ - s_-.$$  \hspace{1cm} (5)

The fluctuations $s_+$ and $s_-$ are oriented randomly in the plane perpendicular to the dinuclear axis. The wriggling mode adds parallel fluctuations to the fragment spins, while the contributions from the bending mode are antiparallel. There is thus no simple relationship between the direction of the two resulting fragment spins, $S_L$ and $S_H$, as we now discuss in more detail.

As brought out in Eqs. (4)-(5), there are two distinct contributions to the angular momentum of a primary fragment, namely the fragment’s share of the overall rotation of the dinuclear complex, $\bar{S}_i$, and the fluctuations received at scission, $\delta S_i$. It is important to recognize that the latter generally dominates.

To understand this important feature, we first note that the angular momentum brought in by a thermal neutron is negligible, $S_0 \approx 0.34h$ on average. At an incoming energy of $E_n = 20$ MeV the angular momentum of the fissioning compound nucleus is $S_0 \approx 5h$. However, at scission where the total angular momentum is divided up between the two fledgling fragments and their relative
motion, the former acquire only small fractions because their moments of inertia are relatively small, $I_i \ll I_R$. Consequently, even for $E_n = 20$ MeV, this contribution amounts to only $\approx 0.26\hbar$ on average for $^{235}\text{U}(n,f)$ (this would increase to $\approx 0.48\hbar$ if the full rigid body values were used for the fragment moments of inertia).

The magnitude of the spin fluctuations is governed by the degree of internal excitation at scission, $E_{\text{sc}}^i$. Because this quantity depends on how much of the total excitation energy, TXE, is tied up in distortion energy, it is not readily available and FREYA therefore employs an effective value given by $c_S^2$TXE = $a_0 I_i^2$, where the reduction factor $c_S$ is a parameter in FREYA and the level-density parameter $a_0$ is calculated assuming a back-shifted Fermi gas [10]. [For $^{235}\text{U}(n,f)$ FREYA uses $c_S = 0.87$; one can elucidate the effects of the spin fluctuations by varying $c_S$, see Ref. [7].] Starting from $\approx 22$ MeV for thermal fission, TXE increases steadily to $\approx 40$ MeV at $E_n = 20$ MeV, so the effective spin temperatures $T_S$ is in the range $0.85 - 1.15$ MeV. The mean spin fluctuations $\langle (\delta S_i^2) \rangle^{1/2}$ are then $4.8\hbar/6.4\hbar$, for the light/heavy fragment in thermal fission and $5.8\hbar/7.1\hbar$ for $E_n = 20$ MeV (or about two units more if the rigid body moments of inertia were used). Thus, generally, the spin fluctuation is over an order of magnitude larger than the aligned component. Consequently, the fragment spins are primarily determined by the fluctuations acquired at scission.

As discussed above, the fragments themselves inherit only a small fraction of the total angular momentum in the system, with the main part going to the relative motion. Because the spin fluctuations dominate over the averages, there is very little correlation remaining between the directions of the resulting total fragment spins $S_i$ and the direction of the overall angular momentum, $S_0$, aside from them all being perpendicular to the dinuclear axis. (Note that the relative orbital angular momentum $L$ is adjusted to counteract the bending-mode fluctuations, ensuring conservation of the total angular momentum.)

Furthermore, the two individual fragment spins are also rather uncorrelated. Indeed, ignoring the small aligned component $S_0$ (see Eqs. (4) and (5)), i.e. assuming $S_i \approx \delta S_i$, we find

$$\langle S_i^2 \rangle \approx \langle I_i^2 / I_i^2 + I_+ \rangle = 2 \left( \langle I_i^2 / I_i^2 + I_+ \rangle + I_- \right) T_S = 2I_i (1-I_i/I) T_S \approx 2I_i T_S. \tag{6}$$

Thus the resulting fragment spins are approximately equivalent to statistical sampling without preserving any conservation laws. The ratio between the wiggling and bending terms in Eq. (6) is $\approx I_H/I_L$ for $\langle S_i^2 \rangle$, while it is $\approx I_H/I_L$ for $\langle S_i^2 \rangle$. Thus the two types of modes contribute about equally to the fragment spins. It then follows that their directional correlation is rather weak.

This overall weak directional correlation can be quantitatively observed in Fig. 1 which shows $P(\Phi_{LH})$, the distribution of the opening angle between the two spins. For a given mass partition, this distribution does not depend on the incident energy because the spin fluctuations scale with $T_S$. Furthermore, because the moments of inertia of the fragments are so small compared to the relative moment of inertia there is also very little dependence on the mass partition. Thus $P(\Phi_{LH})$ is a fairly universal function. We note that it can be represented to a very good approximation as $P(\Phi_{LH}) \approx 1 + f_2 \cos \Phi_{LH}$, with Fourier amplitude $f_2 \approx -0.082$. Thus the two fragment spins have a slight preference for being oppositely directed, with $P(180^\circ)/P(0^\circ) \approx 1.18$.

It should be noted that because the wriggling mode contributes parallel spin fluctuations, conservation of angular momentum causes the orbital angular momentum $L$ to be affected oppositely, $\delta L = -(I_R/I)s_+$. This changes not only the magnitude of $L$ but also its orientation. Therefore the plane of the relative fragment motion (the exit plane) generally differs from the impact plane. FREYA takes this into account when calculating the orbital Coulomb trajectory of the receding fragments. Furthermore, because the Coulomb trajectory is hyperbolic, the asymptotic direction of the relative fragment motion differs from the orientation of the system at scission. However, as a result of the relative slowness of the orbital fragment motion at scission and the strong radial acceleration from the mutual Coulomb repulsion, the associated reorientation angle is rather small, amounting typically to about $2^\circ$.

### C. Fragment de-excitation

After their formation and acceleration, the excited primary fragments undergo a sequence of decay processes. FREYA first considers neutron evaporation, starting by calculating the available statistical excitation energy for each fragment, $Q_i = E_i^* - E_i^\text{rot}$, where $E_i^*$ is its total excitation energy and its rotational energy is given by $E_i^\text{rot} = S_i^2/2I_i$. If the statistical energy exceeds the neutron separation energy, $S_n$, then evaporation can occur.
The neutron is evaporated with a black-body spectrum from a randomly selected point on the surface of the rotating fragment. The local rotational velocity of the surface element adds a centrifugal boost to the neutron. The daughter fragment absorbs the resulting linear and angular momentum recoils. This procedure is iterated as long as evaporation is energetically allowed.

The centrifugal boost from the fragment rotation causes the angular distribution of the evaporated neutrons to be anisotropic, with an enhancement in the equatorial plane, as illustrated in Fig. 2. The degree of bulging may be expressed in terms of the so-called dynamical anisotropy 6.

\[ A \equiv \left[ \frac{dN_n}{d\Omega_{nS}} \right]_{\theta_{nS}=90^\circ} / \left[ \frac{dN_n}{d\Omega_{nS}} \right]_{\theta_{nS}=0^\circ} - 1, \quad (7) \]

which is \( \approx 0.093 \) for \( ^{235}\text{U}(n_{th},f) \). The fragment evaporation chains lead to a reorientation of the fragment spins by \( \approx 13^\circ \) on average, while the spin magnitudes are reduced only very slightly, by \( \approx 0.06 \hbar \) on average.

After evaporation has ceased, the resulting product nucleus disposes of its remaining excitation energy and angular momentum by photon emission. First the statistical excitation energy is radiated away by emission of E1 and M1 photons, each one changing the nuclear spin by one unit. The statistical radiation brings the system to the yrast line, where \( E_t^\gamma = E_{\text{tot}} \). The nucleus then starts emitting stretched quadrupole photons. At some point, the excitation reaches the regime tabulated in the RIPL database 17 and FREYA then simulates those transitions until the ground state, or a sufficiently long-lived isomeric state, is reached. (The further fate of a prompt product nucleus due to \( \beta \) processes is not yet considered in FREYA.)

For the present study, it is interesting to note that there is a relatively tight correlation between the initial fragment spin magnitude and the number of photons emitted, as brought out in Fig. 3. This relationship is fairly universal: it is approximately independent of the incident neutron energy and it changes by only a fraction of a unit between \( ^{235}\text{U}(n,f) \) and Cf(sf). When the combined spin magnitudes, \( S = S_t + S_H \), exceed \( \approx 6 \hbar \), there is a clear increase in \( N_\gamma \) with \( S \), with roughly one additional photon emitted for each additional unit of total fragment angular momentum. As may be expected, the relationship is sensitive to the degree of reduction of the fragment moment of inertia, \( c_I = I/I_{\text{rigid}} \). For \( c_I = 0.5 \), the value used throughout the present study, the slope for large \( S \) is \( dN_\gamma/dS \approx 0.84 \), while it is \( \approx 1.06 \) for \( c_I = 0.3 \). Because of this feature, the measurement of the photon multiplicity may, to some degree, substitute for the measurement of the total fragment angular momenta (see Fig. 6).

**III. RESULTS**

In this section, results are presented for observables that may elucidate behavior caused by the fragment angular momenta. We begin with observables related to the neutron distribution relative to the direction of the primary fragment. We then discuss neutron-neutron correlations gated on the fragment angular momentum, using photon multiplicity as a proxy. Finally, we examine a recently proposed observable 6 based on the transverse neutron motion.

As mentioned in the previous section, the neutrons evaporated from rotating fission fragments have a slight preference for emission perpendicular to the angular momentum due to the centrifugal boost. Simulations with
\[ \frac{dN_n}{d\Omega_{nS}} \sim 1 + \alpha_2 P_2(\cos \theta_{nS}) + \alpha_4 P_4(\cos \theta_{nS}) + \ldots, \quad (8) \]

where the polar angle \( \theta_{nS} \), defined with respect to the direction of the mother fragment spin, is well described by the second-order Legendre approximation (see Fig. 2). When averaging over all events (i.e. over the impact parameter, mass, charge, and TKE, as well as the associated evaporation cascades), FREYA gives \( \alpha_2 = -0.061 \) and \( \alpha_4 = 0.0056 \) for \( ^{235}\text{U}(n_{th},f) \).

Because the orientation \( \chi \) of the fragment spin in the plane perpendicular to the fragment motion is unknown, averaging over \( \chi \) turns the inherently oblate emission pattern in Eq. (8) into a prolate shape with its symmetry axis along the fragment direction. In the frame of the moving fragment, the distribution of \( \theta_{nF} \), the angle between the neutron velocity and that of the fragment, is

\[ \langle \frac{dN_n}{d\Omega_{nF}} \rangle \chi \sim 1 - \alpha'_2 P_2(\cos \theta_{nF}) + \alpha'_4 P_4(\cos \theta_{nF}) + \ldots. \quad (9) \]

It can be generally shown that the coefficients in Eq. (9) are related to those in Eq. (8) by

\[ \alpha'_2 = -\frac{3}{2} \alpha_2, \quad \alpha'_4 = \frac{1}{8} \alpha_4, \quad \alpha'_6 = -\frac{5}{16} \alpha_6, \quad \text{and so on.} \]

### A. Angular distribution

The distributions in Eqs. (8) and (9) are not directly observable and must be transformed to the laboratory frame, with the associated boost velocity depending on the mass and kinetic energy of each fragment. Figure 3 shows a contour plot of the velocity distribution of the neutrons from both fragments in each event, \( dN_n/\delta^3 \mathbf{u} \), for \( ^{235}\text{U}(n_{th},f) \). Even though the distribution retains the axial symmetry of the contributions from each fragment, it is forward-backward asymmetric because the light fragment moves faster and tends to evaporate more neutrons. The circles centered at the origin represent constant neutron kinetic energies of 1 and 2 MeV. They make it apparent that introducing an energy threshold will enhance the forward-backward character of the emission pattern.

The resulting angular distribution with respect to the direction of the light fragment, \( P(\cos \theta_{nL}) \), follows from the dumbbell-shaped distribution shown in Fig. 4. The boost enhances the yield in the forward direction, \( \cos \theta_{nL} > 0 \), and depletes it in the backward direction, \( \cos \theta_{nL} < 0 \), as shown in Fig. 5 for \( ^{235}\text{U}(n_{th},f) \). The black curve shows the standard result, including the rotational boost, while the black circles show the effect of omitting that boost. No energy cut is applied to the emitted neutrons in either scenario. Also shown are the results for two different scenarios when either only neutrons above 2 MeV or below 1 MeV are considered. [As expected from Fig. 4, the 2 MeV threshold enhances the relative yield in the forward direction whereas the 1 MeV upper bound reduces the anisotropy. In all three scenarios, there is little visible effect of the rotation. The largest deviations (still barely noticeable) occur near 0° and 180° and for the lowest neutron energies, which are the hardest to measure. Thus, even though there is a clear effect of the fragment rotation on the inherent neutron emission pattern, as expressed in Eqs. (8) and (9), it has a minimal influence on the observable angular distribution \( dN_n/d\cos \theta_{nL} \). It thus seems unlikely that the fragment rotation can be determined experimentally on the basis of the one-neutron distribution alone.]
B. Gated angular correlations

We now investigate the sensitivity of the neutron-neutron angular correlations to the angular momenta of the fragments. Because there is a close correlation between the fragment angular momentum and the total photon multiplicity, as shown in Fig. 3, this type of measurement could provide additional information on event-averaged neutron-photon correlations beyond those measured in Refs. [18–20] which do not provide a very clear picture. All three previous measurements are based on mass-averaged neutron and photon multiplicities in different TKE bins. Nifenecker et al. [18] suggested a strong positive correlation, Glassel et al. [19] saw a much weaker correlation, and Wang et al. [20] determined a more complex correlation by studying the correlation in different mass regions. Here we propose measuring the two-neutron angular correlations gating on the total photon multiplicity.

![Graph](image)

**FIG. 6**: (Color online) Two-neutron angular correlations are shown for $^{235}U(n_{th},f)$, gating on either the total spin magnitude $S = S_L + S_H$ (a) or the total photon multiplicity $N_\gamma$ (b). The angular correlation functions $P(\phi_{nn})$ for either “soft” neutrons ($E_n < 2$ MeV) or “hard” neutrons ($E_n > 2$ MeV) are presented in (a) for events with either low or high spin magnitude ($S \leq 7$ or $S \geq 8\hbar$, respectively) and in (b) for events with either low or high total photon multiplicity ($N_\gamma \leq 7$ or $N_\gamma \geq 8$, respectively).

The general form of the correlation function has been discussed previously [21]. It can be readily understood from the dumbbell shape of the neutron velocity distribution shown in Fig. 4 that the distribution of the opening angle $\phi_{nn}$ is enhanced near $0^\circ$ and $180^\circ$. For a more detailed understanding, we note that in the case considered, $^{235}U(n_{th},f)$, the most probable outcome (22%) is that each fragment emits one neutron, which contributes near $180^\circ$. It is nearly as likely that the light fragment emits two neutrons while the heavy fragment emits one, again yielding two contributions near $180^\circ$ and one near $0^\circ$ (17% for each). The greater number of contributions to the large-angle contribution, near $180^\circ$, results in a somewhat higher peak than at small angles, near $0^\circ$.

Figure 6(a) shows how the distribution of the opening angle between two detected neutrons, $P(\phi_{nn})$, depends on the energy of the neutrons and the combined spin magnitudes of the two primary fission fragments, averaged over all mass and charge partitions as well as the TKE distribution. Based on the combined spin magnitude, $S = S_L + S_H$, the fission events generated by FREYA are divided into either “low-spin” events ($S \leq 7\hbar$) or “high-spin” events ($S \geq 8\hbar$). A neutron angular correlation function is extracted separately for either “soft” ($E_n < 2$ MeV) or “hard” ($E_n > 2$ MeV) neutrons. It is evident that there is very little sensitivity to $S$, whereas the small-angle behavior of the correlation function depends significantly on the neutron energy. As expected from the velocity distribution in Fig. 4 the hard, energetic neutrons exhibit the expected enhancements near $0^\circ$ and $180^\circ$, whereas the soft neutrons do not display a small-angle peak.

As already discussed, although the spin magnitude $S$ is not directly observable, the photon multiplicity may, to some degree, provide a proxy. To illustrate this possibility, Fig. 6(b) shows how the results in Fig. 6(a) are modified when the combined spin magnitude $S$ is replaced by the total photon multiplicity $N_\gamma$. Here “photon-poor” events having $N_\gamma \leq 7$ replace low-spin events and “photon-rich” events with $N_\gamma \geq 8$ replace high-spin events. There is a larger sensitivity to $N_\gamma$ than to $S$. There is a somewhat stronger small-angle enhancement for the photon-poor correlations with $E_n > 2$ MeV while the large-angle peak is enhanced for low energy neutrons in photon-poor events.

C. Projected angular correlations

A recent experimental investigation [6] introduced a new analysis method for extracting a dynamical anisotropy, the bulging of the neutron emission pattern caused by the rotation of the evaporating fragment. We employ FREYA to examine this idea in this section, enabling us to assess the importance of the various effects that complicate the analysis. Because Ref. [6] studied...
increased the signal for our present studies, we have extract
the angular undulation of the projected opening angle would
amount to \( c_2 \approx 0.18\% \). However, there are two dis-
tinct anisotropic distributions in each fission event, one for each of the two fragments. Because the fragment
spins are not mutually aligned, see Fig. \( \text{I} \) the resulting
signal is correspondingly reduced. If the angle between
the two spins were totally random, which is very nearly
the case in FREYA as discussed in Sec. \( \text{III} \) then \( c_2 \) would
be reduced by a factor of two.

To understand the effect of the various complications
above, we start from a simplified scenario in which
the dinuclear motion remains purely radial, as would be the case if the dinuclear complex had no or-ental motion and the linear and angular momentum re-
coils were absent. In that ideal scenario, the undulation
amplitude is \( c_2 \approx 0.042\% \) if all the neutrons are included
in the analysis. In an actual experiment, there is an
energy threshold below which neutrons cannot be mea-
sured. Therefore, to conform with the experimental anal-
ysis \( \text{C} \), we use \( E_{\min} = 0.9 \text{ MeV} \) in the following. This
exclusion of the softest neutrons reduces the statistics by
about one third, while it enhances the signal somewhat,
to \( c_2 \approx 0.058\% \). Generally, the FREYA simulations sug-

The angular momenta of the primary fission fragments
are determined at scission, at which point they are as-
sumed to be perpendicular to the fission axis, the line
between the centers of the two fledgling fragments, as
described in Sec. \( \text{III} \). Subsequently, as the two fragments
are being pushed apart by their mutual Coulomb repul-
sion, the line connecting their centers rotates somewhat
due to the orbital motion of the dinuclear system. Fur-
thermore, each evaporation process changes the magni-
tude and direction of both the linear and the angular
momentum of the emitting nucleus. These effects com-
plicate the extraction of the proposed correlation signal,
as we now discuss.

We first consider a simplified scenario in which the
relative fragment motion is purely radial, so there is
no directional change of the dinuclear axis during the
Coulomb acceleration, and no recoils are imparted to
the fragments by evaporation. Then, to leading or-
der, the undulating \( \phi_{nnL} \) distribution is of the form
\[ P(\phi_{nnL}) \sim 1 + c_2 \cos 2\phi_{nnL} \] with \( c_2 > 0 \). Because the amplitude \( c_2 \) grows approximately as the three-halves power of the anisotropy \( A \), it is rather challenging to extract \( c_2 \) for small anisotropies. Therefore, to artifi-
cially enhance the signal for our present studies, we have
increased the FREYA “spin temperature” parameter \( c_S \)
from its standard value of 0.87 to 1.4. The mean light
and heavy fragment spins are then 7.3\( \hbar \) and 9.0\( \hbar \) and the angular distribution of the evaporated neutrons relative
to the spin direction of their respective mother fragments
is then characterized by an overall dynamical anisotropy of
\( A \approx 0.12 \), a value rather similar to that obtained by
Gavron \( \text{F} \).

If the neutrons are sampled from a common distri-
bution with that anisotropy, the amplitude of the an-
gular undulation of the projected opening angle would
amount to \( c_2 \approx 0.18\% \). However, there are two dis-

Thus the effect of the evaporation recoils and the effect of the dynamical anisotropy are largely independent. They can thus be extracted by performing a Fourier analysis of the distribution function,

\[ P(\phi_{nnL}) \sim 1 + c_1 \cos \phi_{nnL} + c_2 \cos 2\phi_{nnL}. \]  

This approach has the additional advantage that the Fourier coefficients can be extracted with a reasonable degree of confidence even in the presence of large statistical errors on the individual values of \( P(\phi_{nnL}) \). This is an important advantage because quite large event samples are required for the extraction of this effect (tens of millions of events are needed in the FREYA simulations).

We finally note that the effects of the evaporation recoils and the rotational boosts are modified if separate analyses are made of neutron pairs that are emitted into opposite hemispheres (one is moving forward and the other backward, as seen in the laboratory) and neutron pairs emitted into the same hemisphere (both are either moving forward or backward). The FREYA simulations provide a quantitative impression of these effects. In the present scenario, with \( c_S = 1.4 \) rather than 0.87, FREYA yields \( c_2 \approx 0.040\% \). If only pairs originating from the same hemisphere are included, then \( c_2 \) is reduced to about 0.020\%, while it is increased to about 0.164\% when only pairs from opposite hemispheres are included. For the same cases, \( c_1 \) is 1.3\%, 1.2\%, and 1.6\% respectively.

The results in the discussion so far are specific to \(^{252}\text{Cf}\) obtained with FREYA for that case, but our discussion applies to other cases as well. Figure 7 shows the results for the case of primary interest in the present study, \(^{235}\text{U}(n_{th},f)\). We note that even though 40 million events were generated, the extracted distribution exhibits a considerable degree of statistical fluctuation. Nevertheless, it is possible to extract the Fourier coefficients \( c_1 \) and \( c_2 \) with reasonable confidence and the corresponding functions \( 1 + c_1 \cos \phi_{nnL} \) and \( 1 + c_2 \cos \phi_{nnL} \) are also shown. The first one, resulting from the evaporation recoils, dominates, while the undulations of the second one, reflecting the dynamical anisotropy, is more than an order of magnitude smaller and hardly visible.

**IV. FISSION FROM ISOMERIC STATES**

In neutron-induced fission, the target nucleus is usually in its ground state prior to the arrival of the neutron. However, in certain environments, both in nature and in the laboratory, there is a finite probability that the neutron absorption happens on an excited state of the target nucleus. This possibility is particularly likely when the target nucleus has a low-lying isomeric state. A prime example is \(^{235}\text{U}\), which we shall focus on here, whose first excited state lies at \( E^* = 77 \text{ eV} \) and has a half-life of around 25 minutes \cite{22, 23}. This isomeric state, \(^{235}\text{mU}\), may readily be populated in the astrophysical environments occurring during the r process \cite{24} or in terrestrial laser-generated plasmas \cite{25}. The \(^{235}\text{mU}(n_{th},f)\) cross section was measured to be larger than the fission cross section of the ground state \cite{26}.

Here we seek to identify possible observable consequences of the target nucleus being in its isomeric state rather than its ground state when the incoming neutron arrives. To do this, we carry out FREYA simulations for two different scenarios. The standard scenario corresponds to the case when the target nucleus \(^{235}\text{U}\) is in its ground state. Because it has spin \( \frac{7}{2} \), the resulting compound nucleus can have angular momentum \( S_0 = 3, 4 \text{ h} \). We consider \( S_0 = 4 \text{ h} \). In the alternative scenario, the target nucleus \(^{235}\text{U}\) is in its isomeric state at 77 eV. Because it has spin \( \frac{1}{2} \), the resulting compound nucleus can have \( S_0 = 0, 1 \text{ h} \). We consider \( S_0 = 0 \).

The potential-energy landscape for \(^{236}\text{U}\) shows a well-developed mass-asymmetric valley beyond the second saddle (which is asymmetric). In addition there is a pronounced mass-symmetric valley separated from the asymmetric valley by a down-sloping ridge. (The topography of the fission barrier landscape is brought out very well in Fig. 8 of Ref. \cite{27}. At low energy, such as occurring in \(^{235}\text{U}(n_{th},f)\), the nuclear shape evolution takes the system over the lowest barrier and, consequently, down the asymmetric valley. The resulting fragment mass distribution is therefore asymmetric and the yield at symmetry is negligible. But as the energy is increased, it becomes ever easier for the shape to surmount the ridge and enter the symmetric valley. As a consequence, the mass distribution exhibits an ever more prominent symmetric component.

A recent study using microscopic many-body level densities to guide the Brownian shape evolution \cite{28} found that the “leakage” into the symmetric valley is sensitive to the structure of the involved highly deformed nuclear shapes, in particular to their pairing correlations, which may generally be larger than the shell effects in the barrier region. As a consequence, the symmetric yield has
a delicate energy dependence (which may even be non-monotonic). Furthermore, because the employed combinatorial method [29] provides the level density for different values of the total angular momentum $S_0$, it was possible to also study the dependence of the symmetric yield on $S_0$ [28]. It was found that the fragment mass distribution is generally rather insensitive to $S_0$ for moderate values up to $10\hbar$. However, the symmetric yield is significantly enhanced for $S_0 = 0$ due to pairing effects.

This finding is of particular interest to our present study, because the isomeric state in $^{235}\text{U}$ leads to compound spins of $S_0 = 0, 1\hbar$ with about equal probability, whereas the ground state leads to $S_0 = 3, 4\hbar$. In the latter case, for which extensive experimental data exist, the symmetric yield is very small, while the results reported in Ref. [28] suggest that for $S_0 = 0$ the symmetric mass yield is about 5% of the peak yield. We wish to explore the consequences of such a possible enhancement.

We have therefore constructed a mass distribution with a suitably enhanced symmetric yield to use as FREYA input. This is relatively easily done, because the usual input mass distribution is represented as a sum of a dominant asymmetric contribution and a small symmetric component [51]. We have increased the relative weight of the symmetric term to ensure $Y(\text{symm})/Y(\text{peak}) = 0.05$ and we use this modified mass distribution when simulating the alternative scenario where the target nucleus is in its isomeric state. We have left all other FREYA inputs unchanged, including the input TKE distribution $\text{TKE}(A_H)$.

| Case | $M_1$ | $M_2$ | $M_3$ | $M_4$ |
|------|------|------|------|------|
| $^{235}\text{gs}\text{U}(n_{th}, f)$, $S_0 = 4\hbar$ | 2.39526 | 4.53167 | 6.46183 | 6.59926 |
| $^{235}\text{m}\text{U}(n_{th}, f)$, $S_0 = 0$ | 2.43391 | 4.75387 | 7.23551 | 8.62523 |

TABLE I: Factorial moments of the neutron multiplicity distribution, $M_n = (\nu(\nu - 1)\ldots(\nu - n + 1))$, for thermal fission using the ground state or the isomeric state of $^{235}\text{U}$.

It is important to note that the two fission modes, asymmetric and symmetric, have other distinct characteristic features apart from the difference in their mass splits. Of particular relevance is the fact that the scission shapes of the symmetric mode tend to be significantly more elongated than those of the asymmetric mode [28, 32]. Thus, in the symmetric mode the centers of the proto-fragments are further apart than in the asymmetric mode and the potential energy at scission is correspondingly lower. This results in larger excitation energies at scission and smaller fragment kinetic energies. Furthermore, the additional fragment excitation gained from the shape relaxation of the distorted proto-fragments [32] is also larger than in the asymmetric mode. The resulting higher excitation of the symmetric-mode primary fragments will in turn cause more neutrons to be evaporated. Consequently, one should expect the two scenarios to exhibit different relations between the number of neutrons evaporated and the total fragment kinetic energy in the region of low TKE.

This is indeed the case, as illustrated in Fig. 8 which shows the multiplicity of evaporated neutrons as a function of TKE. The results for the two cases are effectively identical for TKE $\geq 150$ MeV but begin to separate for lower values where the higher probability for fission from the symmetric mode in the isomeric state tends to yield higher neutron multiplicities at low TKE. The data from Ref. [30], also shown on Fig. 8, are consistent with both calculations above 150 MeV but tend to agree more with
the ground state calculation, within the increasing statistical uncertainties, until TKE = 140 MeV where the data set ends. If data could be taken to still lower TKE, it might be possible to better distinguish between the two scenarios. Also, if isomeric-state targets could be fashioned and used to obtain a sufficiently significant data set, it might be feasible to measure a difference between the two scenarios. However, this would require a large number of isomeric targets to obtain enough low TKE data to observe statistical differences.

On the theoretical side, these results were obtained only by an ad hoc modification of the mass yields, \(Y(A)\), to enhance the yield at symmetry, consistent with the results of Ref. [28]. An improved calculation of the shape evolution for different values of \(S_0\) would be required to obtain a more precise \(Y(A)\) distribution to use in FREYA.

The enhanced neutron multiplicity at low TKE from an isomeric target should also manifest itself in an overall larger average neutron multiplicity and this is indeed the case. The neutron multiplicity distribution can be conveniently characterized by its factorial moments, \(M_n = \langle \nu \nu - 1 \ldots \nu - n + 1 \rangle\). As shown in Table I, the average neutron multiplicity, the first factorial moment \(M_1\), is increased by 1.7%. As evident in Fig. 8, the increase in the total multiplicity comes from the low TKE events which yield the highest values of \(\nu\). The next three moments, \(M_2 - M_4\), are also shown in Table I. It is clear that fission from the isomeric state enhances the higher multiplicity moments most. This can also be observed graphically in Fig. 9 presented on a logarithmic scale to more easily distinguish the high multiplicity behavior.

V. CONCLUDING REMARKS

We have studied the role of angular momentum in the fission process to search for evidence of any quantitative effect it might have on fission observables. We employed FREYA in this study because it obeys all conservation laws throughout each step of a fission event, from scission through prompt neutron and photon evaporation. All of these analyses were carried out without changing FREYA inputs, unless otherwise noted. We have previously studied how changes in the spin temperature parameter, \(c_s\), modifies the fragment rotational energy and thus affects photon observables, see Ref. [2] for details.

We have shown that, even if the initial compound nucleus is prepared with a definite angular momentum, which endows the fragments with correspondingly aligned average spins, the spin fluctuations acquired at scission ensure that there is little correlation between the resulting fragment spins and that of the compound nucleus. Furthermore, the spins of the two fragments are also essentially uncorrelated.

We showed that the total photon multiplicity is related to the combined magnitude of the two fragment spins, especially for \(S_L + S_H > 5\hbar\), see Fig. 5. We found that this effect is almost independent of the incident neutron energy, perhaps because, in FREYA, neutrons are emitted as long as energetically possible.

We have also studied neutron observables and found that neutron emission from a rotating fragment results in an oblate emission pattern, as first discussed in Ref. [12]. The resulting dynamical anisotropy increases with the fragment spin but it is hardly sensitive to the spin of the fissioning nucleus. Because the dynamical anisotropy is relatively small (~10%), it has hardly any observable influence on the neutron angular distribution with respect to the direction of the light fragment in the laboratory, see Fig. 5.

We have particularly focused on correlation observables. We discussed using neutron-neutron correlations gated on photon multiplicity as a proxy for gating on fragment angular momentum, see Fig. 5. While we found a weak dependence of the angular correlation on photon multiplicity, we also showed that making distinction between low and high energy neutrons has a stronger impact on the correlation function. Finally, we discussed the projected neutron-neutron angular correlations proposed in Ref. [6] and found that even though the signal of the dynamical anisotropy is weak and is overwhelmed by the effect of evaporation recoils on the linear and angular momenta of the emitting fragments, it may be extracted by Fourier analysis, see Fig. 7.

In a separate analysis, we discussed possible observable effects of neutron-induced fission on the isomeric state of \(^{235}\text{U}\) instead of a \(^{235}\text{U}\) target in its ground state. For this, we employed a modified yield function \(Y(A)\) to model the enhanced symmetric yield from a spin-zero \(^{236}\text{U}\) compound nucleus obtained in Ref. [28] and found that this results in higher neutron multiplicities at low TKE, a potentially observable effect that could distinguish fission from the isomer relative to the ground state, see Fig. 5. [It would obviously be very interesting to experimentally test the enhancement of the symmetric yield for spin zero predicted in Ref. [6].]

In general, we found that angular momentum effects are subtle and are generally insensitive to the spin of the initial state. While we have primarily focused on \(^{235}\text{U}(n,f)\), we have found similar effects for other isotopes. The exception is fission from the low-lying uranium isomeric state where the predicted increase in the symmetric yield could have observable consequences if enough target material were available to accumulate sufficient statistics at low TKE.

Acknowledgments

This work was supported by the Office of Nuclear Physics in the U.S. Department of Energy under Contracts DE-AC02-05CH11231 (JR) and DE-AC52-07NA27344 (RV) and was supported by the LLNL-LDRD Program under Project No. 20-ERD-031 (RV).
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