Large Lepton Mixing and Nonsymmetric Mass Matrices with Flavor $2 \leftrightarrow 3$ Symmetry

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Abstract

We analyze the lepton sector of a recently proposed nonsymmetric mass matrix model. Our model gives a unified description of quark and lepton with the same texture form based on an extended flavor $2 \leftrightarrow 3$ symmetry with a phase. By investigating possible types of assignment for masses, we find that the model can lead to large lepton mixings which are consistent with experimental values. We also find that the model predicts a very small value, $1.3 \times 10^{-10}$, for the lepton mixing matrix element square $|U_{13}|^2$. The $CP$ violating phases in the lepton mixing matrix and a suppression of the averaged neutrino mass in the neutrinoless double beta decay are also predicted.

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I. INTRODUCTION

In order to explain a nearly bimaximal lepton mixing ($\sin^2 2\theta_{12} \sim 1$, $\sin^2 2\theta_{23} \simeq 1$) observed from neutrino oscillation experiments [1], mass matrices with various structures have been investigated in the literature. For example, mass matrices with texture zeros [2]–[4], with a flavor $2 \leftrightarrow 3$ symmetry [5]–[17], and so on have been proposed. Recently we have proposed [18] a following nonsymmetric mass matrix model for all quarks and leptons based on an extended flavor $2 \leftrightarrow 3$ symmetry with one phase:

$$
M_f = \begin{pmatrix}
0 & a_f e^{-i\phi_f} & a_f \\
& a'_f e^{-i\phi_f} & b_f e^{-2i\phi_f} \\
& a'_f & (1 - \xi_f) b_f
\end{pmatrix}, \quad (f = u, d, \nu, e, (D, \text{ and } R)) \quad (1.1)
$$

where $a_f, b_f, \xi_f$, and $a'_f$ are real parameters and $\phi_f$ is a phase parameter. Here, $M_u, M_d, M_\nu$, and $M_e$ are mass matrices for up quarks ($u, c, t$), down quarks ($d, s, b$), neutrinos ($\nu_e, \nu_\mu, \nu_\tau$) and charged leptons ($e, \mu, \tau$), respectively. The mass matrices $M_D$ and $M_R$ are, respectively, the Dirac and the right-handed Majorana neutrino mass matrices, which are included in the model if we assume the seesaw mechanism [19] for neutrino masses. In this model, we assume that all the mass matrices for quarks and leptons have this common structure, which is against the conventional picture that the mass matrix forms in the quark sector take somewhat different structures from those in the lepton sector. In our previous works [18], we have pointed out that this structure leads to reasonable values for the Cabibbo–Kobayashi–Maskawa (CKM) [20] quark mixing, if we use a specific assignment of the quark masses.

In this paper, we shall discuss the Maki-Nakagawa-Sakata-Pontecorv (MNSP) [21] lepton mixing of the model by assuming that neutrinos are the Majorana particles. Under this assumption, the neutrino mass matrix $M_\nu$ should be symmetric. Namely, we further assume

$$
a_\nu = a'_\nu. \quad (1.2)
$$

In the scenario that the neutrino mass matrix is constructed via the seesaw mechanism, i.e. $M_\nu = -M_D^T M_R^{-1} M_D$, the structure of $M_\nu$ mentioned above is alternatively realized by using the following two assumptions: (i) The mass matrices $M_D$ and $M_R$ have the same extended flavor $2 \leftrightarrow 3$ symmetry in Eq. (1.1) with identical phase parameters, i.e. $\phi_D = \phi_R \equiv \phi_\nu$. (ii) $M_D$ and $M_R$ are proportional to each other, except for their (2,1) and (3,1) elements.
On the other hand, $M_e$ is assumed to have the above nonsymmetric structure given in Eq. (1.1). Namely, in this paper, the mass matrices $M_e$ and $M_\nu$ are assumed to have the following forms:

$$
M_e = \begin{pmatrix}
0 & a_e e^{-i \phi_e} & a_e \\
a'_e e^{-i \phi_e} & b_e e^{-2i \phi_e} & (1 - \xi_e) b_e \\
a'_e & (1 - \xi_e) b_e & b_e
\end{pmatrix},
$$

(1.3)

$$
M_\nu = \begin{pmatrix}
0 & a_\nu e^{-i \phi_\nu} & a_\nu \\
a_\nu e^{-i \phi_\nu} & b_\nu e^{-2i \phi_\nu} & (1 - \xi_\nu) b_\nu \\
a_\nu & (1 - \xi_\nu) b_\nu & b_\nu
\end{pmatrix},
$$

(1.4)

where $\phi_e$ and $\phi_\nu$ are phase parameters.

This article is organized as follows. In Sec. II, we discuss the diagonalization of mass matrix of our model. The analytical expressions of the lepton mixings and phases of the model are given in Sec. III. Sec. IV is devoted to a summary.

II. DIAGONALIZATION OF MASS MATRIX

A. mass matrix for charged leptons

The diagonalization of the mass matrix for the charged leptons $M_e$ can be done similarly to the case of the mass matrices for up and down quarks. These mass matrices have common structure given by $M_f$ in Eq. (1.1). Thus let us present a method for diagonalization of $M_f$, which is treated in Ref [18] in details.

First, let us mention free parameters in the mass matrix. There are five parameters, $a_f$, $a'_f$, $b_f$, $\xi_f$, and $\phi_f$ in $M_f$. Even if we fix three eigenvalues, $m_{i_f}$, of $M_f$ by the observed fermion masses, there still exist two free parameters. As the free parameters, let us choose a parameter $\alpha_f$ defined by

$$
\alpha_f \equiv \frac{a'_f}{a_f},
$$

(2.1)

and a phase parameter $\eta_f$ defined in Fig. 1 independently of mass eigenvalues $m_{i_f}$. Note that for the neutrino mass matrix $M_\nu$ we assume $\alpha_\nu = 1$ as mentioned above.

Second, let us discuss an unitary matrix $U_{L_f}$ which diagonalizes $M_f M_f^\dagger$. The explicit expression of $U_{L_f}$ depends on the following three types of assignment for $m_{i_f}$:
(i) Type A:

In this type, the mass eigenvalues $|m_{1f}|$, $m_{2f}$, and $m_{3f}$ of $M_f$ are allocated to the masses of the first, second, and third generations, respectively. (i.e. $|m_{1f}| \ll m_{2f} \ll m_{3f}$.) In this type, the $M_f M_f^\dagger$ is diagonalized as

$$U_{L_f}^\dagger M_f M_f^\dagger U_{L_f} = \text{diag} \left( m_{1f}^2, m_{2f}^2, m_{3f}^2 \right),$$

(2.2)

by an unitary matrix $U_{L_f}$ given by

$$U_{L_f} = P_f^\dagger O_f.$$  

(2.3)

Here $P_f$ is the diagonal phase matrix expressed as

$$P_f = \text{diag} \left( 1, e^{i(\phi_f - \varphi_f)}, e^{-i\varphi_f} \right),$$

(2.4)

where $\varphi_f$ and $\phi_f$ are given by

$$\cos \varphi_f = \frac{|X_f| - m_{3f} \cos \eta_f}{\sqrt{|X_f|^2 + m_{2f}^2 - 2m_{3f}|X_f| \cos \eta_f}},$$

(2.5)

$$\cos \phi_f = \frac{|X_f|^2 - m_{3f}^2}{\sqrt{(|X_f|^2 + m_{2f}^2)^2 - 4m_{3f}^2|X_f|^2 \cos^2 \eta_f}}.$$  

(2.6)

Here $X_f$ is defined by $X_f \equiv b_f + (1 - \xi_f) b_f e^{i\phi_f} \equiv |X_f| e^{i\varphi_f}$, and $|X_f|$ is expressed in term of $\alpha_f$ and $m_{if}$ as

$$|X_f|^2 = m_{1f}^2 + m_{2f}^2 - |m_{1f}| m_{2f} \left( \frac{1 + \alpha_f^2}{\alpha_f} \right).$$

(2.7)

In Eq. (2.3), $O_f$ is the orthogonal matrix given by

$$O_f \equiv \begin{pmatrix} c_f & s_f & 0 \\ \frac{s_f}{\sqrt{2}} & \frac{c_f}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{s_f}{\sqrt{2}} & \frac{c_f}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix},$$

(2.8)

where

$$c_f = \sqrt{\frac{m_{2f}^2 - |m_{1f}| m_{2f}}{m_{2f}^2 - m_{1f}^2}}, \quad s_f = \sqrt{\frac{|m_{1f}| m_{2f}}{m_{2f}^2 - m_{1f}^2}}.$$  

(2.9)

It should be noted that the mixing angles are functions of only $\alpha_f$, since the $m_{if}$ is fixed by the experimental fermion mass values. We find from Eq. (2.6) that $\phi_f \approx \pm \pi$ for $m_{1f}^2 \ll m_{2f}^2 \ll m_{3f}^2$ in this type A assignment.
(ii) Type B:
In this type, the mass eigenvalues $|m_{1f}|$, $m_{3f}$, and $m_{2f}$ are allocated to the masses of the first, second, and third generations, respectively. (i.e. $|m_{1f}| \ll m_{3f} \ll m_{2f}$.) The $M_f M_f^\dagger$ is diagonalized as

$$U_{Lf}^\dagger M_f M_f^\dagger U_{Lf} = \text{diag} \left( m_{1f}^2, m_{3f}^2, m_{2f}^2 \right).$$

(2.10)

by an unitary matrix $U_{Lf}$ given by

$$U_{Lf} = P_f^\dagger O'_f.$$  

(2.11)

Here $O'_f$ is obtained from $O_f$ by exchanging the second row for the third one as

$$O'_f \equiv \begin{pmatrix} c_f & 0 & s_f \\ -\frac{s_f}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{c_f}{\sqrt{2}} \\ -\frac{s_f}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{c_f}{\sqrt{2}} \end{pmatrix}.$$  

(2.12)

(iii) Type C:
In this type, the mass eigenvalues $m_{3f}$, $|m_{1f}|$, and $m_{2f}$ are allocated to the masses of the first, second, and third generations, respectively. (i.e. $m_{3f} \ll |m_{1f}| \ll m_{2f}$.) In this type, we have

$$U_{Lf}^\dagger M_f M_f^\dagger U_{Lf} = \text{diag} \left( m_{3f}^2, m_{1f}^2, m_{2f}^2 \right),$$

(2.13)

where

$$U_{Lf} = P_f^\dagger O''_f.$$  

(2.14)

Here, the orthogonal matrix $O''_f$ is given by

$$O''_f \equiv \begin{pmatrix} 0 & c_f & s_f \\ \frac{1}{\sqrt{2}} & -\frac{s_f}{\sqrt{2}} & \frac{c_f}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{s_f}{\sqrt{2}} & \frac{c_f}{\sqrt{2}} \end{pmatrix}.$$  

(2.15)

This type is not so useful to get the reasonable lepton mixing values.

B. mass matrix for neutrinos

Since the mass matrix for the Majorana neutrinos $M_\nu$ is symmetric, $M_\nu$ is diagonalized as follows depending on the following three types of assignments for the neutrino mass $m_\nu$:
(i) Type A:

In this type, the mass eigenvalues \( m_1, m_2, \) and \( m_3 \) of \( M_\nu \) are allocated to the masses of the first, second, and third generations, respectively. In this type, \( M_\nu \) is diagonalized as

\[
U_\nu^\dagger M_\nu U_\nu^* = \text{diag}(m_1, m_2, m_3), \tag{2.16}
\]

where \( m_i(i = 1, 2, \text{and } 3) \) are real positive neutrino masses. The unitary matrix \( U_\nu \) is described as

\[
U_\nu = P_\nu^\dagger O_\nu Q_\nu. \tag{2.17}
\]

Here, in order to make the neutrino masses \( m_i \) to be real positive, we introduce a diagonal phase matrix \( Q_\nu \) defined by

\[
Q_\nu \equiv \text{diag} \left( e^{-i(\phi_\nu - \pi)/2}, e^{-i(\phi_\nu)/2}, e^{-i(\phi_\nu - \eta_\nu + \pi)/2} \right). \tag{2.18}
\]

The diagonal phase matrix \( P_\nu \) and the orthogonal matrix \( O_\nu \) are obtained from Eqs. (2.4) – (2.7) and (2.8) – (2.9) with \( f = \nu \) by replacing \( |m_{1f}|, m_{2f}, \text{and } m_{3f} \) with \( m_1, m_2, \text{and } m_3 \), respectively and by setting \( \alpha_\nu = 1 \).

(ii) Type B:

In this type, the mass eigenvalues \( m_1, m_3, \) and \( m_2 \) are allocated to the masses of the first, second, and third generations, respectively. In this type, \( M_\nu \) is diagonalized as

\[
U_\nu^\dagger M_\nu U_\nu^* = \text{diag}(m_1, m_3, m_2). \tag{2.19}
\]

The unitary matrix \( U_\nu \) is described as

\[
U_\nu = P_\nu^\dagger O'_\nu Q'_\nu. \tag{2.20}
\]

Here the diagonal phase matrix \( Q'_\nu \) is defined by

\[
Q'_\nu \equiv \text{diag} \left( e^{-i(\phi_\nu - \pi)/2}, e^{-i(\phi_\nu - \eta_\nu + \pi)/2}, e^{-i(\phi_\nu)/2} \right). \tag{2.21}
\]

The orthogonal matrix \( O'_\nu \) is obtained from Eqs. (2.8) and (2.9) with \( f = \nu \) by replacing \( |m_{1f}|, m_{2f}, \text{and } m_{3f} \) with \( m_1, m_3, \text{and } m_2 \), respectively and by setting \( \alpha_\nu = 1 \).

(iii) Type C:
In this type, the mass eigenvalues $m_3$, $m_1$, and $m_2$ are allocated to the masses of the first, second, and third generations, respectively. In this type, $M_\nu$ is diagonalized as

$$U_\nu^\dagger M_\nu U_\nu^* = \text{diag}(m_3, m_1, m_2).$$

(2.22)

The unitary matrix $U_\nu$ is described as

$$U_\nu = P_\nu^\dagger O_\nu'' Q_\nu''.$$  

(2.23)

Here the diagonal phase matrix $Q_\nu''$ is defined by

$$Q_\nu'' \equiv \text{diag} \left( e^{-i(\phi_\nu - \eta_\nu + \pi)/2}, e^{-i(\phi_\nu - \pi)/2}, e^{-i(\phi_\nu)/2} \right).$$

(2.24)

The orthogonal matrix $O_\nu''$ is obtained from Eqs. (2.8) and (2.9) with $f = \nu$ by replacing $|m_{1f}|$, $m_{2f}$, and $m_{3f}$ with $m_3$, $m_1$, and $m_2$, respectively and by setting $\alpha_\nu = 1$.

These types B and C are not so useful to get the reasonable lepton mixing values.

### III. MNSP LEPTON MIXING MATRIX

Now let us discuss the MNSP lepton mixing matrix of the model by taking the type A, the type B, and the type C assignments for charged leptons and neutrinos. We find that the assignment that is consistent with the present experimental data is only one case. Namely, the case in which type B assignment for charged leptons and type A for neutrinos are taken. The other possible cases fail to reproduce consistent lepton mixing. In this case, we obtain the MNSP lepton mixing matrix $U$ as follows.

$$U = U_{Le}^\dagger U_\nu = O_e^T P_\nu P_\nu^\dagger O_\nu Q_\nu = O_e^T P O_\nu Q_\nu$$

$$= \begin{pmatrix}
    c'_e c_\nu + \rho_\nu s'_e s_\nu & c'_e s_\nu - \rho_\nu s'_e c_\nu - \sigma_\nu s'_e \\
    \sigma_\nu s_\nu & -\sigma_\nu c_\nu - \rho_\nu \\
    s'_e s_\nu - \rho_\nu c'_e s_\nu & s'_e c_\nu + \rho_\nu c'_e c_\nu & \sigma_\nu c'_e
\end{pmatrix} Q_\nu,$$

(3.1)

where

$$s'_e = \sqrt{\frac{|m_e|m_\tau - m_e^2}{m_\tau^2 - m_e^2}}, \quad c'_e = \sqrt{\frac{m_\tau^2 - |m_e|m_\tau}{m_\tau^2 - m_e^2}},$$

$$s_\nu = \sqrt{\frac{|m_1|}{m_2 + |m_1|}}, \quad c_\nu = \sqrt{\frac{m_2}{m_2 + |m_1|}}.$$  

(3.2)
Here the phase matrix $Q_\nu$ is shown in Eq. (2.18), and we have put

$$P \equiv P e^P^\dagger \equiv \text{diag}(1, e^{i\delta_2}, e^{i\delta_3}).$$  \hspace{1cm} (3.3)

The orthogonal matrices $O_\nu$ and $O_\sigma$ are obtained from Eq. (2.8) and Eq. (2.12), respectively. Here we denote the lepton masses $(m_1, m_3, f)$ as $(m_e, m_\tau, m_\mu)$ for $f = e$, and as $(m_1, m_2, m_3)$ for $f = \nu$. Note also that $\alpha = 1$.

The parameters $\rho_\nu$ and $\sigma_\nu$ in Eq. (3.1) are defined by

$$\rho_\nu = \frac{1}{2}(e^{i\delta_3} + e^{i\delta_2}) = \cos \left(\frac{\delta_\nu - \delta_\nu}{2}\right) \exp i \left(\frac{\delta_\nu + \delta_\nu}{2}\right),$$  \hspace{1cm} (3.4)

$$\sigma_\nu = \frac{1}{2}(e^{i\delta_3} - e^{i\delta_2}) = \sin \left(\frac{\delta_\nu - \delta_\nu}{2}\right) \exp i \left(\frac{\delta_\nu + \delta_\nu}{2} + \frac{\pi}{2}\right).$$  \hspace{1cm} (3.5)

Note that the phases of $\rho_\nu$ and $\sigma_\nu$ are

$$\arg \rho_\nu = \begin{cases} \frac{\delta_\nu + \delta_\nu}{2} & \text{for } \cos \left(\frac{\delta_\nu - \delta_\nu}{2}\right) > 0, \\ \frac{\delta_\nu + \delta_\nu}{2} + \pi & \text{for } \cos \left(\frac{\delta_\nu - \delta_\nu}{2}\right) < 0, \end{cases}$$  \hspace{1cm} (3.6)

$$\arg \sigma_\nu = \begin{cases} \frac{\delta_\nu + \delta_\nu}{2} + \frac{\pi}{2} & \text{for } \sin \left(\frac{\delta_\nu - \delta_\nu}{2}\right) > 0, \\ \frac{\delta_\nu + \delta_\nu}{2} - \frac{\pi}{2} & \text{for } \sin \left(\frac{\delta_\nu - \delta_\nu}{2}\right) < 0. \end{cases}$$  \hspace{1cm} (3.7)

By using Eqs. (3.3) and (2.4), the phases $\delta_\nu$ and $\delta_\nu$ in our model are given by

$$\delta_\nu = \phi_\nu - \phi_e = (\phi_\nu - \phi_e),$$  \hspace{1cm} (3.8)

$$\delta_\nu = \phi_\nu - \phi_e.$$  \hspace{1cm} (3.9)

Here the phases $\phi_e$, $\phi_e$, $\phi_\nu$, and $\phi_\nu$ are expressed as

$$\cos \phi_e = \frac{|X_e|^2 - m_\mu^2}{\sqrt{(|X_e|^2 + m_\mu^2)^2 - 4m_\mu^2|X_e|^2 \cos^2 \eta_e}},$$  \hspace{1cm} (3.10)

$$\cos \phi_\nu = \frac{|X_\nu|^2 - m_3^2}{\sqrt{(|X_\nu|^2 + m_3^2)^2 - 4m_3^2|X_\nu|^2 \cos^2 \eta_\nu}},$$  \hspace{1cm} (3.11)

$$\cos \phi_e = \frac{|X_e| - m_\mu \cos \eta_e}{\sqrt{|X_e|^2 + m_\mu^2 - 2m_\mu |X_e| \cos \eta_e}},$$  \hspace{1cm} (3.12)

$$\cos \phi_\nu = \frac{|X_\nu| - m_3 \cos \eta_\nu}{\sqrt{|X_\nu|^2 + m_3^2 - 2m_3 |X_\nu| \cos \eta_\nu}},$$  \hspace{1cm} (3.13)

where

$$|X_e|^2 = m_e^2 + m_\tau^2 - |m_e|m_\tau \left(\frac{1 + \alpha_\tau^2}{\alpha_e}\right),$$  \hspace{1cm} (3.14)

$$|X_\nu|^2 = m_1^2 + m_2^2 - 2|m_1|m_2.$$  \hspace{1cm} (3.15)
In the following discussions we consider the normal mass hierarchy $|m_1| < m_2 \ll m_3$ for the neutrino masses. Then the evolution effects can be ignored. Scenarios in which the neutrino masses have the quasi degenerate or the inverse hierarchy are denied from Eqs. (3.21) and (3.26).

In order to reproduce the large lepton mixing between the second and third generation, we now choose specific values of the parameters $\alpha_e$ and $\eta_e$ such that

$$
\alpha_e = \frac{m^2_e - m^2_\mu + m^2_\tau + \sqrt{(m^2_e - m^2_\mu + m^2_\tau)^2 - 4m^2_em^2_\tau}}{2m_em_\tau} \simeq \frac{m_\tau}{m_e} \left[ 1 - \left( \frac{m_\mu}{m_\tau} \right)^2 \right], \tag{3.16}
$$

$$
\cos^2 \eta_e \neq 1. \tag{3.17}
$$

Then, we obtain

$$
\varphi_e \simeq \frac{\pi - \eta_e}{2}, \quad \varphi_\nu \simeq \pi - \eta_\nu, \quad \phi_e \simeq \begin{cases} 
\frac{\pi}{2} & \text{for } 0 < \eta_e < \pi \\
\frac{3\pi}{2} & \text{for } \pi < \eta_e < 2\pi
\end{cases}, \quad \phi_\nu \simeq \pi, \tag{3.18}
$$

and

$$
|\rho_\nu| \simeq \frac{1}{\sqrt{2}}, \quad |\sigma_\nu| \simeq \frac{1}{\sqrt{2}}, \quad s'_e \simeq \frac{|m_e|m_\mu}{m^2_\tau} = 1.63 \times 10^{-5}, \quad c'_e \simeq 1. \tag{3.19}
$$

Thus, the explicit magnitudes of the components of $|U_{ij}|$ are obtained as

$$
|U_{11}| \simeq \sqrt{\frac{m^2_1}{m^2_2 + m^2_1}}, \quad |U_{12}| \simeq \sqrt{\frac{m^2_1}{m^2_2 + m^2_1}}, \quad |U_{13}| \simeq \frac{1}{\sqrt{2}} \frac{|m_e|m_\mu}{m^2_\tau},
$$

$$
|U_{21}| \simeq \frac{1}{\sqrt{2}} \sqrt{\frac{m^2_1}{m^2_2 + m^2_1}}, \quad |U_{22}| \simeq \frac{1}{\sqrt{2}} \sqrt{\frac{m^2_2}{m^2_2 + m^2_1}}, \quad |U_{23}| \simeq \frac{1}{\sqrt{2}},
$$

$$
|U_{31}| \simeq \frac{1}{\sqrt{2}} \sqrt{\frac{m^2_1}{m^2_2 + m^2_1}}, \quad |U_{32}| \simeq \frac{1}{\sqrt{2}} \sqrt{\frac{m^2_2}{m^2_2 + m^2_1}}, \quad |U_{33}| \simeq \frac{1}{\sqrt{2}}. \tag{3.20}
$$

In Fig. 2, we present more detailed predicted values for $|U_{23}|$ in the $\eta_e - \eta_\nu$ parameter space, by taking the value for $\alpha_e$ given in Eq. (3.16). It can be seen from Fig. 2 that the large mixing angle between the second and third generation is well realized in the model if we use the specific values of $\alpha_e$ given in Eq. (3.16).

As seen from Eq. (3.20), the neutrino oscillation angles of the model are related to the lepton masses as follows:

$$
\tan^2 \theta_{\text{solar}} = \frac{|U_{12}|^2}{|U_{11}|^2} \simeq \frac{m_1}{m^2_2}, \tag{3.21}
$$

$$
\sin^2 2\theta_{\text{atm}} = 4|U_{23}|^2|U_{33}|^2 \simeq 1, \tag{3.22}
$$

$$
|U_{13}|^2 \simeq \frac{1}{2} \left( \frac{m_em_\mu}{m^2_\tau} \right)^2. \tag{3.23}
$$
It should be noted that the present model leads to the same results for $\theta_{\text{solar}}$ and $\theta_{\text{atm}}$ as the model in Ref.\[13\], while a different feature for $|U_{13}|^2$ is derived.

On the other hand, we have\[22\] a experimental bound for $|U_{13}|^2_{\text{ex}}$ from the CHOOZ\[23\], solar\[24\], and atmospheric neutrino experiments\[1\]. From the global analysis of the SNO solar neutrino experiment\[22, 24\], we have $\Delta m^2_{12}$ and $\tan^2 \theta_{12}$ for the large mixing angle (LMA) Mikheyev-Smirnov-Wolfenstein (MSW) solution. From the atmospheric neutrino experiment\[1, 22\], we also have $\Delta m^2_{23}$ and $\tan^2 \theta_{23}$. These experimental data with 3$\sigma$ range are given by

\[
|U_{13}|^2_{\text{ex}} < 0.054 ,
\]

\[
\Delta m^2_{12} = m^2_2 - m^2_1 = \Delta m^2_{\text{soli}} = (5.2 - 9.8) \times 10^{-5} \text{eV}^2 ,
\]

\[
\tan^2 \theta_{12} = \tan^2 \theta_{\text{soli}} = 0.29 - 0.64 ,
\]

\[
\Delta m^2_{23} = m^2_3 - m^2_2 \simeq \Delta m^2_{\text{atmi}} = (1.4 - 3.4) \times 10^{-3} \text{eV}^2 ,
\]

\[
\tan^2 \theta_{23} \simeq \tan^2 \theta_{\text{atmi}} = 0.49 - 2.2 .
\]

Hereafter, for simplicity, we take $\tan^2 \theta_{\text{atmi}} \simeq 1$. Thus, by combining the present model with the mixing angle $\theta_{\text{soli}}$, we have

\[
m_1 \simeq \tan^2 \theta_{\text{soli}} = 0.29 - 0.64 .
\]

Therefore we predict the neutrino masses as follows.

\[
m^2_1 = (0.48 - 6.8) \times 10^{-5} \text{eV}^2 ,
\]

\[
m^2_2 = (5.7 - 16.6) \times 10^{-5} \text{eV}^2 ,
\]

\[
m^2_3 = (1.4 - 3.4) \times 10^{-3} \text{eV}^2 .
\]

Let us mention a specific feature of the model. Our model imposes a stringent restriction on $|U_{13}|$ as

\[
|U_{13}|^2 \simeq \frac{1}{2} \left( \frac{m_e m_\mu}{m^2_\tau} \right)^2 = 1.3 \times 10^{-10} .
\]

Here we have used the running charged lepton masses at the unification scale $\mu = \Lambda_X$\[25\]: $m_e(\Lambda_X) = 0.325 \text{ MeV}$, $m_\mu(\Lambda_X) = 68.6 \text{ MeV}$, and $m_\tau(\Lambda_X) = 1171.4 \pm 0.2 \text{ MeV}$. The value in Eq.\[3.31\] is consistent with the present experimental constraints Eq.\[3.24\], however it is too small to be checked in neutrino factories in future. The very small predicted value for $|U_{13}|$ is in contrast to previously proposed model\[11, 13\].

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Next let us discuss the CP-violation phases in the lepton mixing matrix. The Majorana neutrino fields do not have the freedom of rephasing invariance, so that we can use only the rephasing freedom of $M_e$ to transform Eq. (3.31) to the standard form

$$U_{\text{std}} = \text{diag}(e^{i\alpha_1^e}, e^{i\alpha_2^e}, e^{i\alpha_3^e}) U$$

$$= \begin{pmatrix}
  c_{\nu 13} c_{\nu 12} & c_{\nu 13} s_{\nu 12} e^{i\beta} & s_{\nu 13} e^{i(\gamma - \delta_\nu)} \\
  -c_{\nu 23} s_{\nu 12} - s_{\nu 23} c_{\nu 12} s_{\nu 13} e^{i\delta_\nu} e^{-i\beta} & c_{\nu 23} c_{\nu 12} - s_{\nu 23} s_{\nu 12} s_{\nu 13} e^{i\delta_\nu} & s_{\nu 23} c_{\nu 13} e^{i(\gamma - \beta)} \\
  (s_{\nu 23} s_{\nu 12} - c_{\nu 23} c_{\nu 12} s_{\nu 13} e^{i\delta_\nu}) e^{-i\gamma} & (-s_{\nu 23} c_{\nu 12} - c_{\nu 23} s_{\nu 12} s_{\nu 13} e^{i\delta_\nu}) e^{-i(\gamma - \beta)} & c_{\nu 23} c_{\nu 13}
\end{pmatrix}.$$ (3.32)

Here, $\alpha_i^e$ comes from the rephasing in the charged lepton fields to make the choice of phase convention. The CP-violating phase $\delta_\nu$, the additional Majorana phase $\beta$ and $\gamma$ in the representation Eq. (3.32) are calculable and obtained as

$$\delta_\nu = \arg \left[ U_{12} U_{22}^* + \frac{|U_{12}|^2}{1 - |U_{13}|^2} \right] \simeq \arg \left( \frac{U_{12} U_{22}^*}{U_{13} U_{23}^*} \right)$$

$$\simeq \varphi_e - \varphi_\nu - \frac{1}{2}(\phi_e - \phi_\nu), \simeq \begin{cases} 
-\frac{\eta_e}{2} + \eta_\nu - \frac{\pi}{4} & \text{for } 0 < \eta_e < \pi \\
-\frac{\eta_e}{2} + \eta_\nu - \frac{3\pi}{4} & \text{for } 0 < \eta_e < \pi 
\end{cases},$$ (3.33)

$$\beta = \arg \left( \frac{U_{12}}{U_{11}} \right) \simeq \frac{3\pi}{2},$$ (3.34)

$$\gamma = \arg \left( \frac{U_{13}}{U_{11}} e^{i\delta_\nu} \right) \simeq \begin{cases} 
\frac{\eta_e}{2} + \frac{\pi}{4} & \text{for } 0 < \eta_e < \pi \\
\frac{\eta_e}{2} - \frac{\pi}{4} & \text{for } 0 < \eta_e < \pi 
\end{cases},$$ (3.35)

by using the relation $m_e \ll m_\tau$. Hence, we also predict the averaged neutrino mass $\langle m_\nu \rangle$ which appears in the neutrinoless double beta decay as follows:

$$\langle m_\nu \rangle \equiv |m_1 U_{11}^2 + m_2 U_{12}^2 + m_3 U_{13}^2|$$

$$\simeq \left| m_1 \frac{m_2}{m_2 + m_1} + m_2 \frac{m_1}{m_2 + m_1} e^{3\pi i} \right| \ll m_1.$$ (3.36)

This value of $\langle m_\nu \rangle$ is too small to be observed in near future experiments.

In Fig. 2, we present more detailed predicted values for the lepton mixing matrix elements ($|U_{12}|$, $|U_{23}|$, and $|U_{13}|$), and phases ($\sin \delta_\nu$, $\sin \beta$, and $\sin \gamma$) in the $\eta_e - \eta_\nu$ parameter plane. Here we take a value given in Eq. (3.16) for the parameter $\alpha_e$ and center values given in Eq. (3.30) for neutrino masses $m_i$. 

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IV. CONCLUSION

We have analyzed the lepton mixing matrix of a recently proposed nonsymmetric mass matrix model. The model gives a universal description of quark and lepton with the same texture form \([1,1]\) based on an extended flavor \(2 \leftrightarrow 3\) symmetry including a phase \(\phi\). By using the charged lepton masses as inputs, the present model has six adjustable parameters, \(\alpha_e, \eta_e, \eta_\nu, m_1, m_2, \text{ and } m_3\) to reproduce the observed MNSP lepton mixing matrix parameters and neutrino-mass-squared differences. We have shown that only the case where the type B assignment for charged leptons and the type A for neutrinos are taken can lead to consistent values with neutrino oscillation experiments. In this case, we find that the observed large lepton mixing between the second and third generation is realized by a fine tuning of the parameter \(\alpha_e\) as given in Eq. (3.16). It is also shown that the model predicts very small value for \(|U_{13}|\), which is in contrast to previously proposed model \([1,1,1,13]\). The \(CP\) violating phases \(\delta_\nu, \beta, \text{ and } \gamma\) in the lepton mixing matrix are obtained. The decay rate of the neutrinoless double beta decay is also predicted to be almost suppressed.

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FIG. 1: The relations among the components in the mass matrix, and the definition of a free \( CP \) violating phase parameter \( \eta_f \) which is independent of the mass eigenvalues.
FIG. 2: The predicted values for the lepton mixing matrix elements ($|U_{12}|$, $|U_{23}|$, and $|U_{13}|$), and phases ($\sin \delta_\nu$, $\sin \beta$, and $\sin \gamma$) in the $\eta_e - \eta_\nu$ parameter plane for the case in which the type B assignment for charged leptons and type A for neutrinos are taken. Here we take a value given in Eq. (3.16) for the parameter $\alpha_e$ and center values given in Eq. (3.30) for neutrino masses $m_i$. 